Modelling Cracking Damage of Asphalt Mixtures under Compressive Monotonic and Repeated Loads using Pseudo J-integral Paris' Law [1]

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Modelling Cracking Damage of Asphalt Mixtures under Compressive Monotonic and Repeated Loads using Pseudo J-integral Paris' Law

Abstract Field observations and mechanical analyses have shown that cracks accompany rutting in asphalt mixtures under external compressive loads. This study aims to model crack growth in asphalt mixtures under compressive monotonic and repeated loads. With using energy equilibrium and viscoelastic Griffith fracture criterion, a damage density characterizing the cracks in mixtures is derived as a function of stress, nonlinear viscofracture strain, asphalt film thickness and bond energy. Crack evolution is modelled by pseudo J-integral Paris' law. Six types of asphalt mixture were tested by monotonic compressive strength tests at 40°C. Two were further tested at four more temperatures and loading rates, respectively. Repeated load test results for the same mixtures were obtained from previous studies. The different shape of the damage density curve (S-shape for monotonic load and increasing exponential shape for repeated load) demonstrates the dependence of damage growth on loading mode, due to the different energy release rate. Pseudo J-integral Paris' law can model the crack growth in mixtures and capture the post-peak softening behaviour under a monotonic load. The Paris' law coefficients (A and n) are independent of loading mode (monotonic or repeated), rate or temperature. They are fundamental material properties and can be used to predict crack growth under varying loading and temperature conditions.

Keywords: asphalt mixture, crack growth, damage density, Paris' law, pseudo J-integral

Introduction

Field and laboratory observations have shown that cracks accompany rutting in asphalt layers when subjected to external compressive loads (Underwood, Yun, & Kim, 2011; Wang et al., 2003). Mechanical analyses demonstrate that the post-peak softening behaviour under a monotonic compressive load and the tertiary flow of rutting under a repeated compressive load result from cracking alongside plastic deformation (Ramsamooj & Ramadan, 1999; Zhang, Luo, & Lytton, 2012; Zhang, Luo, & Lytton, 2013). Crack initiation criteria have been developed using viscoelastic Griffith theories for asphalt mixtures in compression and in tension, respectively (Luo, Luo, & Lytton, 2015; Zhang, Luo, & Lytton, 2014). It was found that cracks start to grow from the peak stress under a monotonic load or from the flow number under a repeated load. Bond energy and tensile or compressive strength of asphalt mixtures can be predicted using the crack initiation criteria.

Crack evolution in the tertiary stage of rutting under the repeated load has been studied in previous studies (Zhang et al., 2013; Zhang, Luo, & Lytton, 2014). In these studies, a damage density (ξ in Eq.1) was employed to quantify the damage caused by randomly distributed cracks and air voids in the mixture based on continuum damage mechanics (CDM) (Kachanov, 1986; Lemaitre & Desmorat, 2005). A pseudo J-integral based Paris' law (Eq.2) was used to predict the crack growth in the asphalt mixtures based on Schapery's viscoelastic damage theory (Schapery, 1984). The Paris' law coefficients (*A* and *n* in Eq.2) were obtained from repeated tests for different mixtures. However, the repeated load tests are very time-consuming (e.g., taking hours to days when the load level is relatively low) and the analyses are complicated to obtain the coefficients. In the meantime, the monotonic load test (e.g. constant strain rate strength test) is time-saving (e.g., usually taking a few minutes) and can provide sufficient information for obtaining material fundamental properties.

This study hypothesizes that the pseudo J-integral based Paris' law can characterize the post-peak softening behaviour under a monotonic compressive load, and also hypothesizes that the Paris' law coefficients (*A* and *n*) are fundamental material properties which are independent of loading mode, rate, and temperature. If these hypotheses are verified, the Paris' law will enable the prediction of the crack growth in different loading and environmental conditions, e.g., using the *A* and *n* of an asphalt mixture obtained from monotonic tests at one temperature in the Paris' law to predict the crack growth of the same mixture at different loading and temperature conditions.

Crack Evolution Models for Asphalt Mixtures

The pseudo J-integral Paris' law (Eq.2) based on damage density (Eq.1) is used to characterize the crack evolution of asphalt mixtures.

$$\xi = \frac{A_L}{A_0} = \frac{m\pi c^2}{A_0} = 1 - \frac{\sigma_A}{\sigma_T} \quad (1)$$

$$\frac{d\xi}{dN} \ or \ \frac{d\xi}{dT} = A\left(\Delta J_R\right)^n \quad (2)$$

where ξ = damage density; $A_L = m\pi c^2$ is lost area due to cracks in a cross section, and *m* = number of cracks, *c* = mean crack radius which is the average radius of all cracks in an asphalt mixture; A_0 = total area of the cross section. *A* and *n* = Paris' law coefficients; *N* = number of load cycles; *T* = loading time; $d\xi/dT$ and $d\xi/dN$ are used for modelling crack damage growth under monotonic and repeated load, respectively. ΔJ_R = incremental pseudo J-integral; σ_A = apparent (measured) stress acting on A_0 and σ_T = effective (true) stress acting on intact material area ($A_0 - A_L$). Note that damage density has enabled the connection between crack size and stress responses, as shown in Eq. 1. The last term in Eq. 1 is obtained by force balance between the apparent configuration and the effective configuration, i.e., $F = \sigma_A A_0 = \sigma_T\left(A_0 - A_L\right)$. It also worth noting that the Paris' law in Eq. 2 utilizes the pseudo J-integral which has addressed the effects of crack size, stress and the modulus, while eliminated the viscous effect on the crack growth. Physically, pseudo J-integral equals to the energy release rate resulting from crack growth while does not include the energy dissipation for viscoelastic relaxation.

Principles of strain decomposition, redistribution and equilibrium of stored energy are employed to determine ξ and ΔJ_R so that *A* and *n* can be computed from a monotonic compressive test. The strain decomposition was developed by the authors (Zhang et al., 2012) based on extended elastic-viscoelastic correspondence principle (Schapery, 1984). It has been implemented for repeated load tests and monotonic load tests (Zhang, Luo, & Lytton, 2014; Zhang, Luo, Luo, et al., 2014). Pseudostrain is determined by:

$$\varepsilon^R\left(t\right) = \frac{1}{E_R}\int_{0^-}^{t} E\left(t-s\right)\frac{d\varepsilon\left(s\right)}{ds}ds \quad (3)$$

where ε^R = pseudostrain. $\varepsilon(s)$ = measured total strain. *s* = the time before current time *t*. $E(t)$ is the relaxation modulus determined from creep compliance to be obtained by compressive creep tests. E_R is a reference modulus which is derived to be the Young's modulus when used for strain decomposition (Zhang et al., 2012). As shown in Fig.1, using the pseudo-strain (ε^R), the total strain (ε) is decomposed into viscous strain (ε^{vi}), viscoplastic strain (ε^{vp}), viscofracture strain (ε^{vf}) and nonlinear elastic strain (ε^{ne}). It is demonstrated that $\varepsilon^R = \varepsilon - \varepsilon^{vi} = \varepsilon^{vp} + \varepsilon^{vf} + \varepsilon^{ne}$ and the $\sigma \sim \varepsilon^R$ relation is a linear line with a slope of Young's modulus (E_0) before the viscoplastic yielding, as demonstrated in the literature (Zhang, Luo, Luo, et al., 2014).

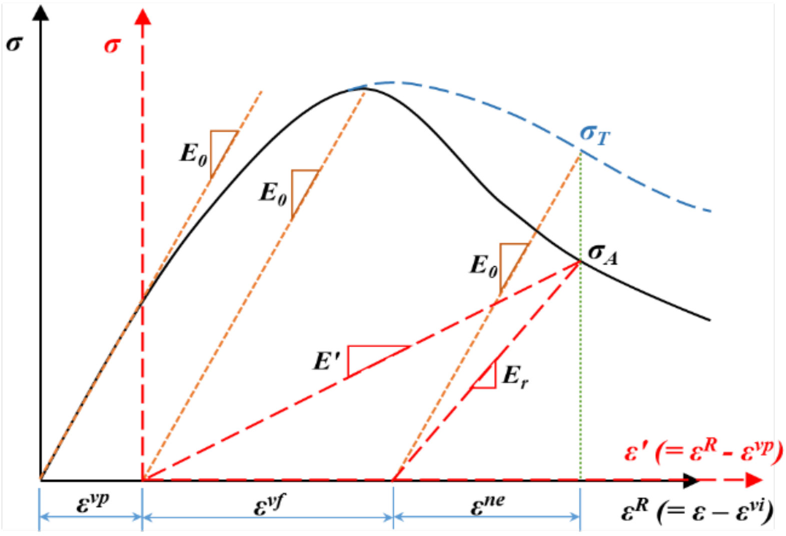

Figure 1. Stress vs. pseudo-strain relation.

A viscoplastic yielding analysis indicates that, under a monotonic load, the dissipated pseudo strain energy (DPSE) is used for permanent deformation before the peak stress, whereas it is consumed for crack growth after the peak (Zhang, Luo, Luo, et al., 2014). Thus if a nonlinear strain is defined as $\varepsilon' = \varepsilon^R - \varepsilon^{vp} = \varepsilon^{vf} + \varepsilon^{ne}$ where ε^{vp} is the viscoplastic strain at the peak, the post-peak softening is represented as a nonlinear elastic cracking damage process. The nonlinearity is due to that the crack growth rate is nonlinearly related to the energy release rate following the Paris' low in Eq.2. In the $\sigma \sim \varepsilon'$ coordinate, E' and E_r are defined as secant modulus and recovery modulus, as shown in Fig.1. Correspondingly, a true stress exists in the effective configuration which contains the undamaged material with a modulus of E_0. Note that $E_r < E_0$ due to the softening effect caused by cracks existing in the apparent configuration.

The redistribution principle of the stored energy is formulated in $\sigma \sim \varepsilon'$ based on Griffith fracture theory and shown in Eq.4. Recall that the post-peak softening has become a nonlinear elastic cracking damage process in the $\sigma \sim \varepsilon'$ coordinate after the strain decomposition, thus the Griffith fracture theory can be applied for modelling this post-peak softening behaviour. Based on this theory, the total stored energy for a cracked material = the total stored energy for the un-cracked material – released energy due to crack growth + stored energy on the crack surfaces, as that represented in Eq.4. Note that this equilibrium becomes valid only when energy is primarily dissipated for creating new crack surfaces, while no significant energy has been dissipated for other physics, e.g., viscoelastic relaxation and/or viscoplastic deformation, which is the case when the $\sigma \sim \varepsilon'$ constitutive relation is used in this study.

$$\Pi = \frac{\sigma_A^2}{2E'}V_0 = \frac{\sigma_A^2}{2E_0}V_0 - \frac{\sigma_A^2}{2E_0}\cdot V_R \cdot 2m + \Delta G \cdot 2m \cdot S_w \tag{4}$$

where Π = total stored energy of a representative volume element (RVE). $V_0 = A_0t$ = volume of RVE; t = thickness of RVE that can be interpreted as asphalt film thickness. V_R = volume of the material that releases energy for creating new cracks; ΔG = bond energy of an asphalt mixture that is the work of adhesion or cohesion per unit of crack surface area. S_w = crack surface area. Under a compressive load, new wing cracks are generated along the direction of external load and it was obtained that $V_R = \pi c^3/12$ and $S_w = 7\pi^2c^2/48$ (Zhang, Luo, Luo, et al., 2014). It is understood that the growth of cracks in an asphalt mixture (with air voids as pre-existing cracks) is not continuous but a step-wise process. Thus the Griffith crack initiation criterion applies to each crack growth step and determines when a next-step crack growth will occur. Applying the Griffith criterion, i.e., $\partial\Pi/\partial c = 0$, to Eq.4 obtains:

$$c = \frac{7\pi E_0 \cdot \Delta G}{3\sigma_A^2} \tag{5}$$

Substituting $\sigma_A = E'\varepsilon'$ and Eq.5 in Eq.2 yields the damage density as:

$$\xi = \frac{72t}{7\pi \cdot \Delta G}\left(\frac{1}{2}\sigma_A\varepsilon' - \frac{\sigma_A^2}{2E_0}\right) \tag{6}$$

The energy equilibrium principle is originated from CDM theory and hypothesizes that the stored energy in a damaged (apparent) material equals to an idealized undamaged (true) configuration, i.e., $\sigma_A^2/2E' = \sigma_T^2/2E_0$. Substituting this equation into Eq. 1 ($\xi = 1-\sigma_A/\sigma_T$) yields the first relation in Eq.7. Since no crack exists in the true configuration, the material behaves elastically leading to an elastic unloading with a slope of E_0 and an elastic strain, as shown in Fig.1. Thus, $\varepsilon^{ne} = \sigma_T/E_0 = \sigma_A/E_r$. Substituting this equation into Eq.1 results in the second relation in Eq.7.

$$\begin{cases} E' = E_0\left(1-\xi\right)^2 \\ E_r = E_0\left(1-\xi\right) \end{cases} \tag{7}$$

The incremental pseudo J-integral is equivalent to the energy release rate that is the dissipated work per unit crack surface created. Based on this relationship the pseudo J-integral is calculated by:

$$\Delta J_R = \frac{\partial\left(DPSE^{vf}\cdot V\right)}{\partial\left(2m\pi c^2\right)} \approx \frac{H}{2}\frac{\partial}{\partial\xi}\left(\frac{\sigma_A^2}{E'} - \frac{\sigma_A^2}{2E_r}\right) \tag{8}$$

where $DPSE^{vf}$ = dissipated pseudo-strain energy density for crack growth which is computed as $\int_0^t \sigma_A d\varepsilon' - \sigma_A^2/2E_r$ based on $\sigma \sim \varepsilon'$ diagram in Fig.1; Note that $DPSE^{vf}$ is the energy (density) which is dissipated solely for crack growth, which excludes the energy dissipation for viscoelastic relaxation. $V=A_0H$ is volume of the lab sample and H = sample height. Note that H is used here rather than the asphalt film thickness, t (used in Eq.4) as the crack model (Eq.2) is now applied to a real lab sample and ξ quantifies the overall damage caused by all cracks in the sample. In Eq.8, the $2m\pi c^2$ computes the total area of the cracks in the sample, which is equal to $A_0\,\xi$.

Substituting Eqs. 7 and 8 in Eq.2 yields the crack evolution for a laboratory asphalt mixture sample under a monotonic compressive load:

$$\frac{d\xi}{dT} = A\left[\frac{H}{2}\frac{\sigma_A^2}{2E_0}\frac{(3+\xi)}{(1-\xi)^3}\right]^n \tag{9}$$

Note that Eq. 9 can be extended to multi-axle loads by replacing $\sigma_A^2/2E_0$ with $(\sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2)/(2E_0) + 2(1+\upsilon_0)(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2)/E_0$, where υ_0 is an elastic Poisson's ratio.

Materials and Laboratory Tests

Six types of asphalt mixture were fabricated using one binder (PG67-22) at two air void contents (4% and 7%) and conditioned for three aging periods (unaged, 3-month and 6-month aged at 60°C continuously). A commonly-used Texas Hanson limestone was selected and the gradation of the aggregates was determined based on a Type C (coarse surface) dense gradation specified by the Texas Department of Transportation (TxDOT, 2004). The optimum asphalt content was calculated based on the TxDOT test procedure (TxDOT, 2008) and was determined as 4.4%. The asphalt mixture specimens were compacted using the Superpave gyratory compactor to a cylindrical sample with 150 mm in diameter and 175 mm in height. Then the samples were cored to 100 mm in diameter and cut to 150 mm in height. The target air void contents had two levels including 4% and 7%. The obtained air void content for test samples were found to be within ±0.5% of the target air void content. The unaged asphalt concrete specimens were tested once they were fabricated. The other specimens were stored in an aging room and aged at a constant temperature of 60°C for 3 months and 6 months, respectively. All specimens were put in an environmental chamber at the testing temperature for at least 3 hours to reach the equilibrium temperature before testing.

A Universal Testing Machine (UTM) was used to perform the laboratory tests on the asphalt mixtures. Fig. 2 shows the uniaxial compressive test set-up configuration which remain the same for the UCC, UCS and the repeated load tests. Uniaxial compressive creep (UCC) tests were firstly conducted for all samples at 40kPa and 40°C for 120 sec to obtain the viscoelastic properties (i.e., relaxation modulus) used for strain decomposition. Note that the creep test time and load level were selected to ensure the total strain of the sample was remained below 150με which is hypothesized as the upper limit for the linear viscoelasticity of asphalt mixture in compression (Levenberg & Uzan, 2004). A creep compliance was firstly obtained from the UCC tests and then converted to the relaxation modulus using viscoelastic theories. Uniaxial compressive strength (UCS) tests were performed for all samples at a constant strain rate of 311 με/sec (a monotonic load) and at 40°C. To evaluate temperature and rate effect, UCC and UCS tests were conducted on one of the mixtures (4%, 6-month aged) at 4 more temperatures (45, 50, 55, and 60°C) and on another mix (7%, 6-month aged) at 4 more loading rates (18, 65, 622, and 1074με/sec). Repeated load tests were performed on the same types of mixtures using a sinusoidal wave load at 600 kPa and 1 Hz until the sample collapsed or the deformation transducer reached its limit. Note that the repeated load test results on the same types of mixtures were reported in a previous study (Zhang, Luo, & Lytton, 2014) and are employed here for comparison. It is also emphasized that the repeated load test applied the sinusoidal wave load without any rest periods between load cycles. Thus the healing effect was not considered in the cracking damage modelling.

Figure 2. Laboratory test configuration for uniaxial compressive creep, strength and repeated load tests.

Results and Discussions

Fig.3 shows a typical curve of the damage density calculated by Eq.6 using ΔG results from the previous study (Zhang, Luo, Luo, et al., 2014) and assuming t = 10 μm. A single film thickness was assumed for all asphalt mixtures due to that one binder content was determined for the mixtures. This film thickness is consistent with the results reported in the literature (Hmoud, 2011; Sengoz & Agar, 2007). Fig. 3 shows that the damage density is an increasing S-shape curve in the strain rate controlled monotonic load test. Its slope (i.e., damage density rate) increases and then decreases with loading time, as shown in Fig.4, which confirms the damage density curve as an increasing S-shape curve with loading time under a monotonic load. In comparison, the damage density has an exponential shape with an increasing slope in stress controlled repeated load tests, as shown in Fig. 5 which is reproduced from the previous study (Zhang, Luo, & Lytton, 2014). Thus the different shapes of damage density in monotonic and repeated load tests demonstrates that the growth of damage density is dependent of the loading mode. Fundamentally this is due to the different energy release rates (equivalent to the incremental pseudo J-integral) in different loading modes, resulting distinct growth rate of the damage density according to the damage evolution law in Eq.2.

Fig.3 also shows that the effective (true) stress σ_T (calculated by Eq.1 using ξ) is greater than the apparent (measured) stress σ_A due to crack growth during the post-peak softening process. Fig.4 shows that the mean crack radius (determined by Eq.5) increases from 0.85 mm to 2.1 mm with loading time. Note that the asphalt mixture shown in Fig.4 has an original air void content of 7% and this mixture demonstrates an initial mean crack radius of 0.85mm, which corresponds to an air void content within a range from 6% to 9% based on the direct measurements of X-ray computed tomography reported in the literature (Kassem et al., 2008; Masad et al., 2002). The match of damage density rate in Fig.4 between calculated values by Eq.6 and predicted values from Eq.9 proves that pseudo-J integral Paris' law can capture the post-peak softening behaviour and can model the crack growth in asphalt mixtures under a monotonic load. Therefore a monotonic loading test can be used to obtain the Paris' law coefficients (*A* and *n*), which are shown in Fig. 6 for the tested asphalt mixtures in this study.

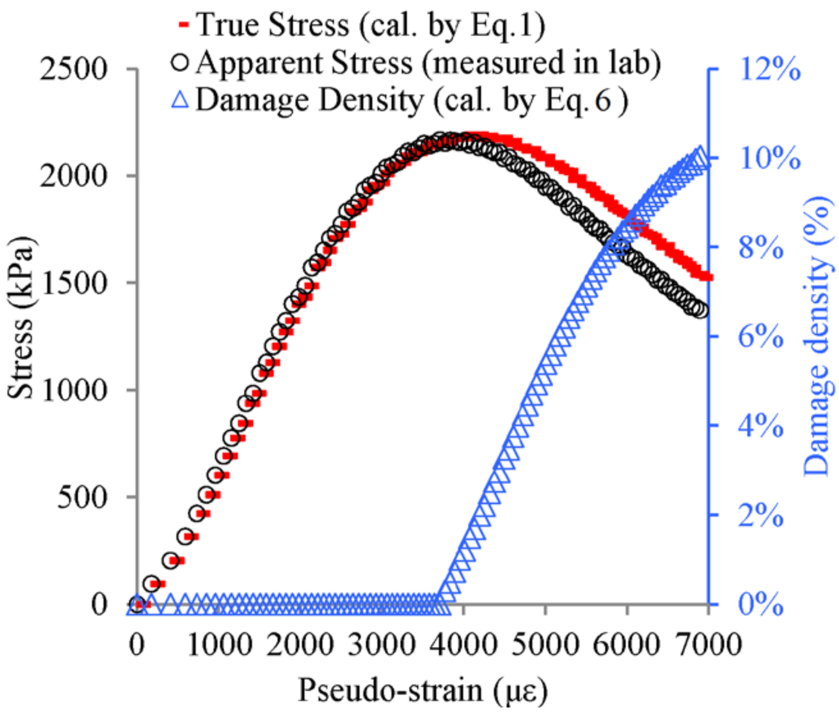

Figure 3. True stress, apparent stress and damage density vs. pseudo-strain for an asphalt mixture (4% air void content and 6-month aged).

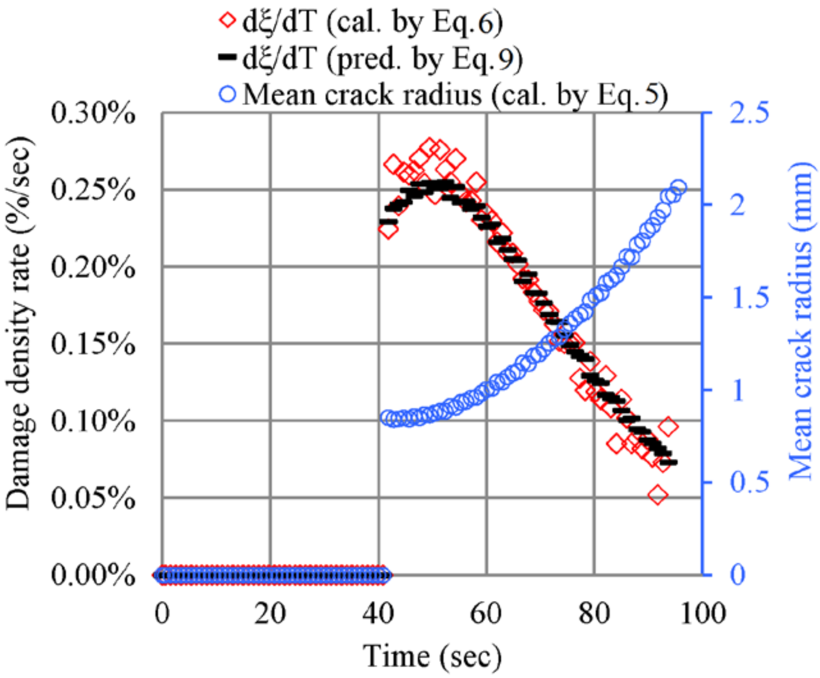

Figure 4. Calculated vs. predicted damage density rate, and mean crack radius of an asphalt mixture (7% air void content and 3-month aged)

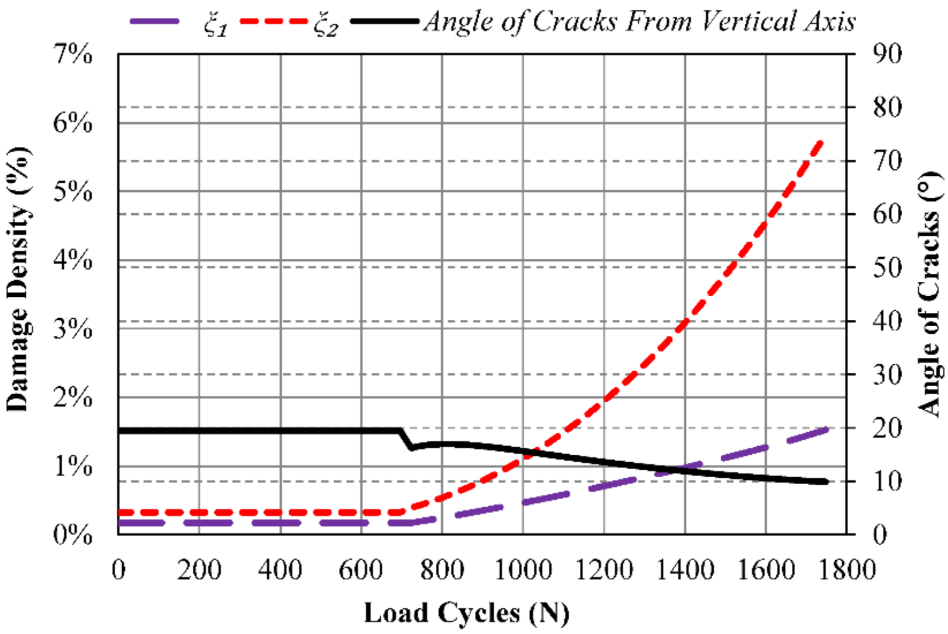

Figure 5. Damaged density (ξ_1 and ξ_2 are damage density in axial and radial directions) vs. loading cycles in a repeated load test for an asphalt mixture (reproduced from Zhang, Luo, & Lytton (2014) with permission from ASCE).

Fig.6 shows the Paris' law coefficients (*A* and *n*) for the six different asphalt mixtures determined from the monotonic load tests and the repeated load tests. Fig. 7 shows the Paris' law coefficients (*A* and *n*) of an asphalt mixture at different loading rates and temperatures. Note that *A* and *n* for repeated load tests have been reported in the previous study (Zhang, Luo, & Lytton, 2014). It is found from Fig. 6 that, when the asphalt mixture become stiffer (e.g. due to a lower air void content or a longer ageing period), the Paris' law coefficient (*A*) becomes decreased and the Paris' law exponent (*n*) will increase. More importantly, Fig. 6 demonstrates that the Paris' law coefficients (*A* and *n*) for the six types of asphalt mixture determined from monotonic load tests are statistically comparable to that from repeated load tests at the same temperatures. Fig.6 indicates that the Paris' law coefficients of a specific asphalt mixture do not vary significantly with either temperatures or loading rates.

One can conclude from Figs. 6 and 7 that Paris' law coefficients are fundamental material properties and independent of temperature, loading mode or loading rate. Note that this conclusion is valid only when pseudo J-integral (J_R) is used in Paris' law. It is because J_R ($J_R = \int_0^t D(t-s)\frac{dK^2}{ds}ds$) has addressed and taken into account the temperature effect by creep compliance (*D(t)*) and accounted for loading (mode or rate) effects by stress intensity factor (*K*). If the stress intensity factor rather than the pseudo J-integral is used in the Paris' law, the stress intensity factor based Paris' law coefficients will not be fundamental material properties and will vary with temperature, as reported in the literature (Jacobs, 1995; Zhou et al., 2007). It also must be noted that, in this study, the pseudo J-integral was determined based on its equivalence to the energy release rate that is the dissipated work per unit crack surface created (i.e., Eq.8) rather than using the stress intensity factor and creep compliance. The independence of Paris' law coefficients of temperature and loading has been verified by another study done by the authors (Luo, Zhang, & Lytton, 2015), in which it was demonstrated that the pseudo J-integral based Paris' law coefficients are function of asphalt mixture material properties such as air void content, binder content, aggregate gradation, instantaneous modulus and relaxation modulus exponent, which are independent of temperature or loading rate.

Since repeated load tests are costly and time-consuming, monotonic load tests (e.g., compressive strength tests) are recommended to determine the Paris' law coefficients for an asphalt mixture. Then the coefficients can be implemented in Paris' law using an accurate pseudo J-integral to predict the crack growth of the asphalt mixture under different loading and temperature conditions. However, it is noted that the Paris' law coefficients are different for asphalt mixtures in tension and in compression as cracks grow differently in the two cases. Nevertheless, the same methods (using the energy redistribution principle, the viscoelastic Griffith criterion and pseudo J-integral Paris' law) can be used to model the crack growth in tension, or other mixed mode of loading (Luo, Luo, et al., 2015; Luo, Zhang, & Lytton, 2016).

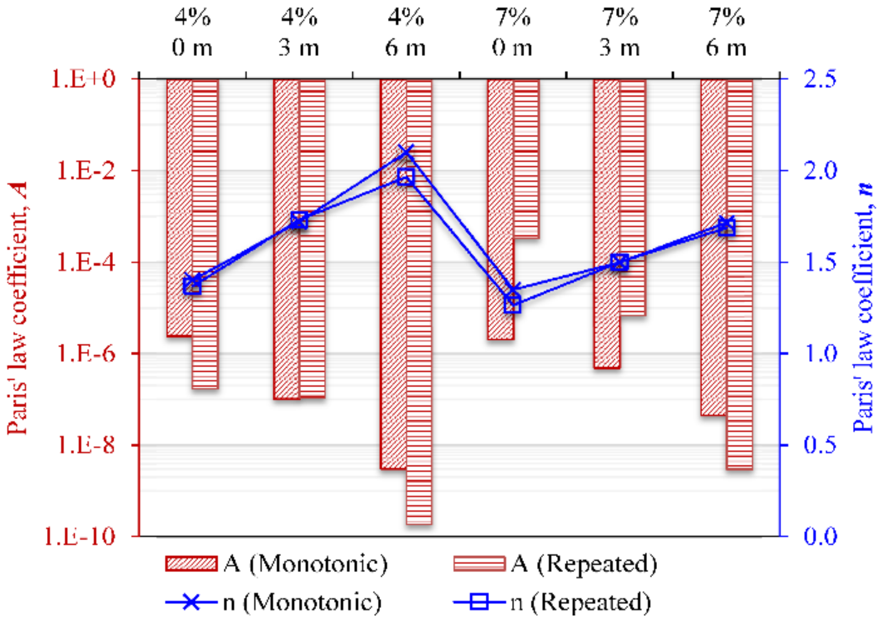

Figure 6. Paris' law coefficients obtained from monotonic and repeated load test for different mixtures.

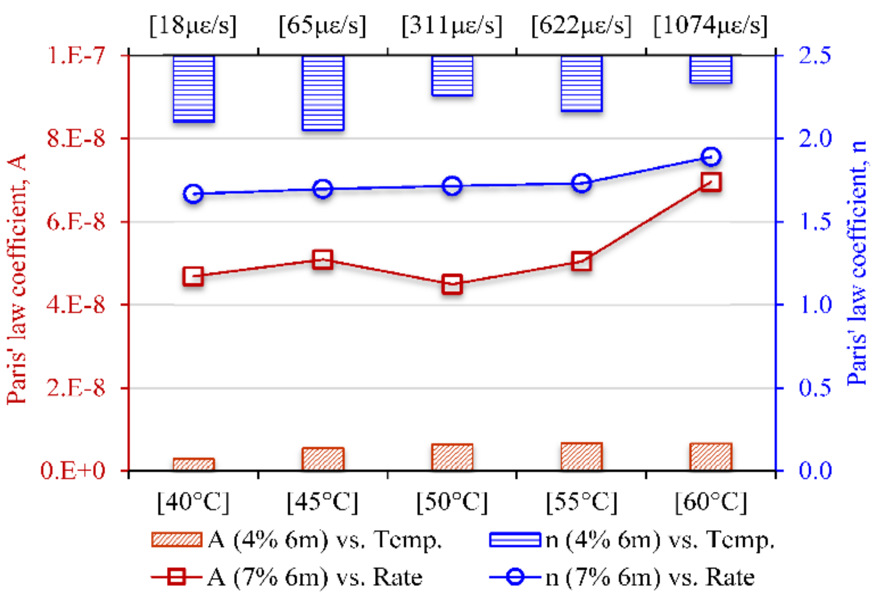

Figure 7. Paris' law coefficients at different temperatures and loading rates.

Summary and Conclusions

Cracking damage evolution in asphalt mixture is investigated under monotonic and repeated load in compression. Research questions are raised regarding if the Paris' law coefficients are fundamental material properties that are solely dependent of material itself while independent of temperature, loading mode or loading rate. With using energy equilibrium and viscoelastic Griffith fracture criterion, a damage density characterizing the cracks in a mixture is derived as a function of stress, nonlinear viscofracture strain, asphalt film thickness and bond energy. Pseudo J-integral based Paris law is used to model the evolution of the cracking damage. Laboratory monotonic and repeated test results are reported for six asphalt mixtures with two different air void contents and three ageing periods. Two of the six mixtures were further tested at four more temperatures and four more loading rates, respectively. Conclusions were drawn from the test results as below:

(1) Damage density shows an increasing S-shape curve with loading time under a strain-rate controlled monotonic load and an increasing exponential curve with loading cycles under a stress controlled repeated load. This demonstrates that the growth of damage density is dependent of the loading mode due to the different energy release rates in different loading modes.
(2) Pseudo J-integral based Paris' law can capture the post-peak softening behaviour and model the crack growth in asphalt mixtures under a monotonic load.
(3) Pseudo J-integral based Paris' law coefficients (A and n) are fundamental material properties and independent of temperature, loading mode (monotonic or repeated) or loading rate.
(4) A stiffer asphalt mixture (e.g. with a lower air void content or a longer ageing period) tends to have a lower Paris' law coefficient (A) and a higher Paris' law exponent (n).

Based on the above conclusions, the monotonic load test (e.g. constant strain rate strength test) is recommended for the determination of Paris' law coefficients of an asphalt mixture since it is a simple and quick test. The determined Paris' law coefficients can be used for predicting the cracking damage of the same asphalt mixture at different loading modes, rates and temperatures. Prediction of rutting and cracking in the field requires a coupled 3D modelling of viscoplastic deformation and cracking damage including their initiation and evolution in a mixture, which is being done and will be presented in a future study.

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