Guided modes in non-Hermitian optical waveguides

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We study guided modes in non-Hermitian optical waveguides with dielectric layers having either gain or loss. For the case of a three-layer waveguide, we describe stationary regimes for guided modes when gain and loss compensate each other in the entire structure rather than in each layer. We demonstrate that, by adding a lossless dielectric layer to a double-layer waveguide with the property of parity-time ($\mathcal{PT}$) symmetry, we can control a ratio of gain and loss required to support propagating and nondecaying optical guided modes. This novel feature becomes possible due to the modification of the mode structure, and it can allow using materials with a lower gain to balance losses in various optical waveguiding structures. In addition, we find a non-$\mathcal{PT}$-symmetric regime when all guided modes of the system have their losses perfectly compensated.

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I. INTRODUCTION

Quantum mechanics is based on the widely accepted postulate that all physical observables should correspond to real eigenvalues, and the use of Hermitian operators ensures that the system possesses an entirely real eigenvalue spectrum [1,2]. However, the Hermitian operators are not the only operators to possess real spectra. Some years ago, Bender and Boettcher [3,4] suggested that there exist other classes of non-Hermitian Hamiltonians that can possess real eigenvalue spectra, provided they possess the so-called *parity-time* ($\mathcal{PT}$) *symmetry*. Moreover, there are a number of complex potentials that possess real spectrum, which are not $\mathcal{PT}$ symmetric [5].

Due to a close analogy between the linear equations of quantum mechanics and the equations for slowly varying amplitudes in optics, similar $\mathcal{PT}$-induced phenomena can be observed in optical systems with gain and loss, as was suggested theoretically and also verified in experiment with optical couplers [6–9]. To achieve a balance between gain and loss in optics, active and passive regions of an optical system should be placed symmetrically with respect to each other, and the refractive index of the system should satisfy the relation $n(x) = n^*(-x)$.

In a majority of the subsequent studies of $\mathcal{PT}$-symmetric optical systems [10], researchers paid attention to two main features of such systems: real spectra of dissipative systems and the symmetry-breaking transition between the $\mathcal{PT}$-symmetric regimes, when all eigenvalues are real, and $\mathcal{PT}$-symmetry-broken regimes, when some of the eigenvalues become complex [11,12].

Importantly, it was also shown that the $\mathcal{PT}$-symmetry for non-Hermitian systems is neither a sufficient condition nor a necessary condition to realize a real spectrum [13]. Thus, the concept of *pseudo-Hermiticity*, a condition for real spectra of non-Hermitian systems, was introduced [13]. Recently, it was also shown that nonsymmetric waveguides with gain and loss can couple and provide loss compensation for at least one mode [14].

In optics, the topic of $\mathcal{PT}$ symmetry is closely related to the studies of various structures with gain. For example, from the conventional point of view, it is reasonable to expect that by adding gain to the waveguiding structure one can control the characteristics of the propagating modes, as was shown in Ref. [15]. In plasmonic structures, waveguiding is suppressed by losses particularly strongly. There is a search in either optimizing the geometry for these structures [16] or using novel materials [17]. Clearly, such approaches try to minimize losses, and one needs gain materials to compensate losses in plasmonic structures (see, e.g., Refs. [18–21]).

Recently, Suchkov *et al.* [22] investigated pseudo-Hermitian (PH) optical couplers and compared their properties with those of $\mathcal{PT}$-symmetric couplers. They revealed that the mode spectrum can be entirely real even without $\mathcal{PT}$ symmetry, provided the waveguides in a coupler are placed in a special order. Being inspired by those findings, here we study three-layer non-Hermitian dielectric waveguides with gain and/or loss (e.g., those shown in Fig. 1). We choose the three-layer structure since the additional parameters allow one to achieve a wider range of regimes as compared to two-layer structures, which were mostly studied up to now. For the case of three-layer waveguides, we describe the stationary regimes when gain and loss compensate each other globally but not locally. We reveal that this system, even being non-$\mathcal{PT}$ symmetric, supports different types of asymmetric modes and allows additional functionalities and control of the guided modes. We believe that our approach can be useful for reducing the value of gain for balancing losses in optical waveguides.

II. THREE-LAYER WAVEGUIDES

We consider a three-layer waveguide placed in a free space, as shown schematically in Fig. 1. Each layer i has a thickness d_i and can have an arbitrary complex index of refraction. In the examples given below we assume that layers are of the same thickness, $d_i = d$. We use the $\exp(-i\omega t)$ time convention, and in this convention the positive imaginary part of the refractive index describes lossy media, while negative values of this quantity correspond to gain media. We look for TE-guided modes, which have one nontrivial electric field component (E_y) and two magnetic field components (H_x, H_z). Modes of the structure have the form $E_y = E(x)\exp(i\beta z)$, where β is the mode wave number, and the mode profile E is described

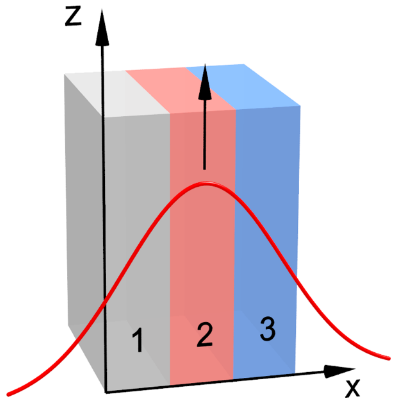

FIG. 1. Schematics of a three-layer non-Hermitian waveguide. Each layer can be either passive or exhibit gain or loss. For visual identification, we use red tint to denote gain layers, blue to denote loss layers, and grey to denote passive layers.

by the equation

$$\frac{d^2E}{dx^2} + \frac{\omega^2}{c^2}[\varepsilon(x) - \beta^2]E = 0. \tag{1}$$

Following the standard procedure for the mode finding, we write solutions in each layer and in the surrounding vacuum, and in order to find the unknown constants we apply the boundary conditions of the continuity of the tangential components of the electric and magnetic fields. There are eight unknown constants of integration and a set of eight linear equations for these unknowns. The set of linear equations has nontrivial solutions when the determinant of the matrix of the coefficients of this set vanishes. We explicitly write this matrix as

$$\hat{M} = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ \kappa_0 & -ik_1 & ik_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{ik_1d_1} & e^{-ik_1d_1} & -1 & -1 & 0 & 0 & 0 \\ 0 & k_1e^{ik_1d_1} & -k_1e^{-ik_1d_1} & -k_2 & k_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{ik_2d_2} & e^{-ik_2d_2} & -1 & -1 & 0 \\ 0 & 0 & 0 & k_2e^{ik_2d_2} & -k_2e^{-ik_2d_2} & -k_3 & k_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{ik_3d_3} & e^{-ik_3d_3} & -1 \\ 0 & 0 & 0 & 0 & 0 & ik_3e^{ik_3d_3} & -ik_3e^{-ik_3d_3} & \kappa_0 \end{bmatrix}, \tag{2}$$

where $\kappa_0^2 = (\beta^2 - \omega^2/c^2)$ and $k_i^2 = \varepsilon_i\omega^2/c^2 - \beta^2$ are the transverse wave numbers in each medium.

As we mentioned above, the wave numbers of the localized modes are found from the equation

$$\det(\hat{M}) = 0. \tag{3}$$

In general, this equation cannot be solved analytically; therefore, in what follows we solve it numerically in order to find the mode wave numbers β. To find regimes when conservative modes exist in this structure, we fix parameters of the first layer, $n_1 = 2 + 0.1i$, and also fix the real parts of the refractive indices of the two remaining layers at 2. Then, we scan the plane of parameters of imaginary parts of the layers 2 and 3 [$\mathrm{Im}(n_2)$,$\mathrm{Im}(n_3)$] in order to find points at which there is a solution to Eq. (3) with real β. The examples of this search are shown in Fig. 2, where we demonstrate the cases for three values of layer thickness d. For thin layers, $d = 100$ nm, there is just one mode, and its losses can be compensated for parameters shown by the line in Fig. 2(a). As we make the layers thicker, more modes appear, and corresponding parameters required to compensate their attenuation due to losses are shown by two (for $d = 200$ nm) and three (for $d = 300$ nm) curves in the Figs. 2(b) and 2(c), respectively. In these two cases, all the curves intersect in a point on the vertical axis. This point corresponds to the case when the middle layer is passive, and $n_1 = n_3^*$, where the star denotes complex conjugation. This coincides with the condition of classic optical $\mathcal{PT}$ symmetry, when the index of refraction satisfies the condition $n(x) = n(-x)^*$ (with $x = 0$ corresponding to the center of our structure). The mode structure for the case A shown in Fig. 2(b) is shown in Fig. 3(a). It has a symmetric amplitude distribution, while the phase shows some gradient indicating the energy flow from an active layer to a lossy layer.

Cases B and C are quite remarkable, and they are offering *a new mechanism* for controlling the required balance between gain and loss in two nonconservative layers. Indeed, if we have two layers of the same thickness, then the condition of

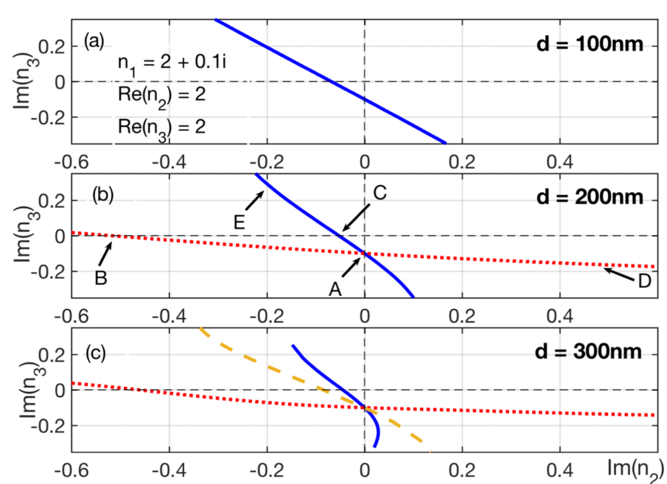

FIG. 2. Location of the energy-conserving modes on the plane of parameters of [$\mathrm{Im}(n_2)$,$\mathrm{Im}(n_3)$] for three different values of layer thickness d: (a) $d = 100$ nm, (b) $d = 200$ nm, and (c) $d = 300$ nm. In panel (b) points A, B, and C show the special cases, and points E and D correspond to general cases, which are discussed in the text.

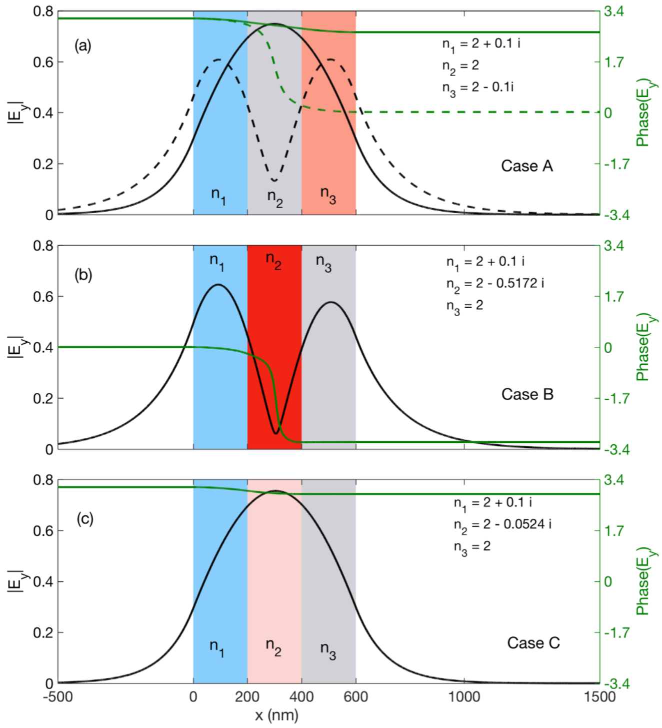

FIG. 3. Mode structure for three special cases; shown are the electric field amplitudes and phases for (a) a degenerate $\mathcal{PT}$-symmetric case with parameters corresponding to point A in Fig. 1 and (b,c) two cases corresponding to points B and C in Fig. 1, when one of the layers is passive. Parameters of the structures are shown in the corresponding figure panels.

usual $\mathcal{PT}$ symmetry requires that the amount of gain in one of the layers is equal to the loss in another layer. Now, we can attach the third layer to the structure, and due to a change in the mode profile the amount of the required gain can be either larger [case B, Fig. 3(b)] or smaller [case C, Fig. 3(c)]. In the former case, the amount of gain is characterized be the imaginary part of the index of refraction, $\mathrm{Im}(n_3) \approx -0.517$, while in the latter case, it is -0.0524, whose magnitude is almost twice smaller than the loss coefficient $\mathrm{Im}(n_1) = 0.1$. This is achieved by having larger field intensities in the gain layer as compared to the field in the lossy layer.

Finally, in a more general case, the modes have a complicated structure shown in Fig. 4, where we show two typical modes corresponding to the two dispersion curves. One of the modes resembles the fundamental mode of dielectric waveguides with just one maximum, while another one is double humped.

Equation (3) has more than one solution. We study one case of fixed parameters, case E: $n_1 = 2 + 0.1i$, $n_2 = 2 - 0.2075i$, and $n_3 = 2 + 0.3098i$. In this case, we plot $\det(\hat{M})$ on the complex plane of wave numbers in Fig. 5. We observe that there are several zeros that correspond to the solutions of Eq. (3). There is one solution that corresponds to the mode that propagates without loss (marked by a red cross), and there are multiple solutions with complex wave numbers corresponding to the modes that decay away from the source. Thus, we can conclude that our system provides energy conservation just for one mode, whereas other modes experience attenuation.

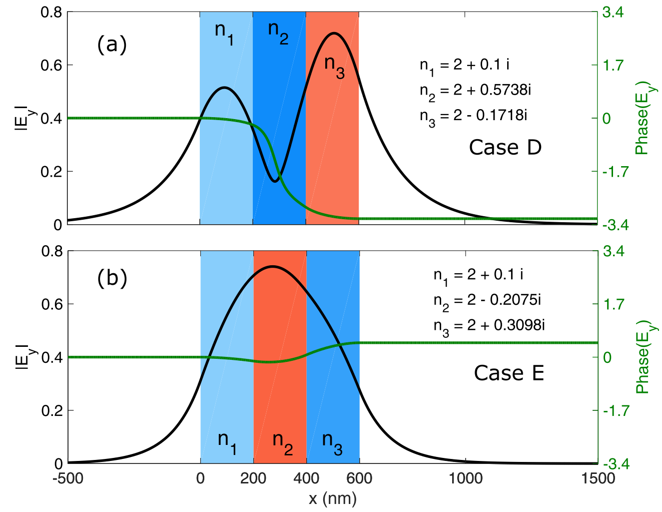

FIG. 4. Mode structure for two general cases. Shown are the electric field amplitudes and phases; (a) and (b) correspond to points E and F in Fig. 1, respectively.

Figure 6 shows the parameter plane of the imaginary parts $[\mathrm{Im}(n_2),\mathrm{Im}(n_3)]$ for the asymmetric case, when $\mathrm{Re}(n_3) = 2.2$, while n_1 and n_2 are the same as above. Two curves corresponding to the two modes of the system still intersect at one point, but this point is now not on the $\mathrm{Im}(n_2) = 0$ axis, as it was in the previously considered symmetric case. Remarkably, this regime now possesses the same properties as the $\mathcal{PT}$-symmetric case, i.e., both modes of the system have real eigen wave numbers, but the system is not $\mathcal{PT}$ symmetric. Thus, we have revealed novel regimes in nonsymmetric structures when all modes have their losses perfectly compensated by gain.

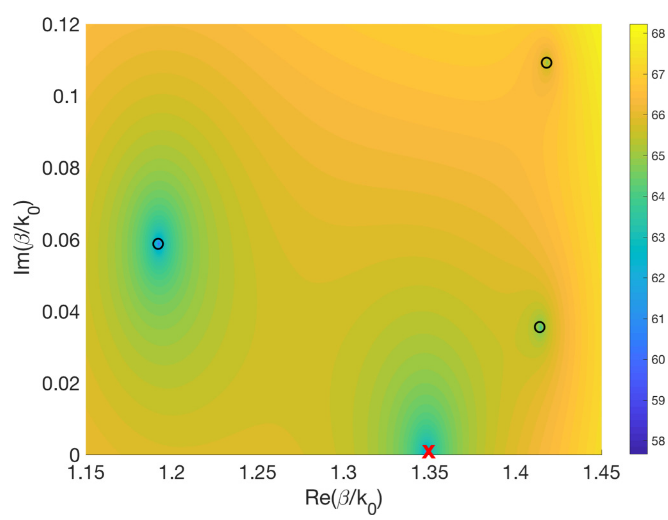

FIG. 5. Determinant of the matrix M in the logarithmic scale on the plane of complex wave numbers. Shown are the points of the stationary propagating mode (marked by a red "x") and the nonpropagating modes (marked by black circles).

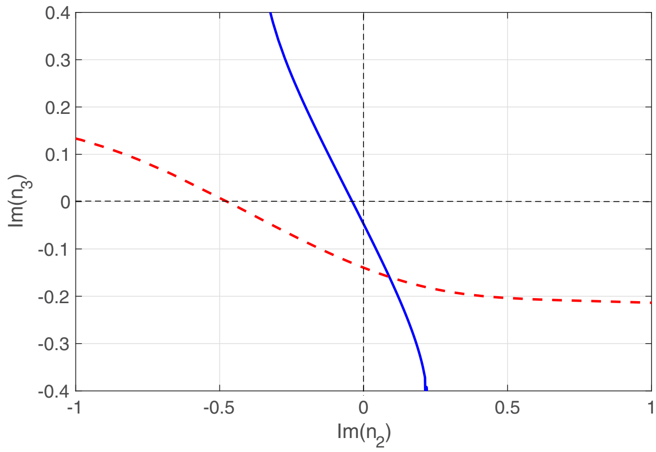

FIG. 6. Location of the energy conserving modes on the plane of parameters of [Im(n_2),Im(n_3)] for the asymmetric case. Parameters are $d = 200$ nm, $n_1 = 2 + 0.1i$, Re(n_2) = 2, and Re(n_3) = 2.2.

III. CONCLUSION

We have studied the guiding properties of three-layer non-Hermitian dielectric waveguides with gain and loss. We have revealed that the functionalities of conventional $\mathcal{PT}$-symmetric optical waveguides can be expanded substantially by adding an additional dielectric layer and extending the structure into a broader class of non-Hermitian systems to control a ratio of gain and loss required to support propagating and nondecaying guided modes. Our approach can be useful for a design of novel types of waveguiding systems with low-gain materials for the loss compensation.

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