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Xue Luo, Fan Gu, Yuqing Zhang, Robert L. Lytton, Dan Zollinger

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Mechanistic-Empirical Models for Better Consideration of Subgrade and Unbound Layers Influence on Pavement Performance

Xue Luo, Ph.D.

Associate Research Scientist
Center for Infrastructure Renewal
Texas A&M University System

3135 TAMU, CEOB 508B, College Station, Texas 77843
Phone: (979) 458-8535, Email: xueluo@tamu.edu

Fan Gu, Ph.D.

Postdoctoral Researcher
National Center for Asphalt Technology
Auburn University

277 Technology Parkway, Auburn, AL 36830
Phone: (334)-844-6251, Email: fzg0014@auburn.edu

Yuqing Zhang, Ph.D.

Lecturer
School of Engineering and Applied Science
Aston University

Aston Triangle, Birmingham, B4 7ET, U.K.
Phone: +44 (0) 121-204-3391, Email: y.zhang10@aston.ac.uk

Robert L. Lytton, Ph.D., P.E.

Professor, Fred J. Benson Chair
Zachry Department of Civil Engineering
Texas A&M University

3136 TAMU, CEOB 503A, College Station, Texas 77843
Phone: (979) 845-9964, Email: r-lytton@civil.tamu.edu

Dan Zollinger, Ph.D., P.E.

Professor
Zachry Department of Civil Engineering
Texas A&M University

3136 TAMU, CEOB 501D, College Station, Texas 77843
Phone: (979) 845-9918, Email: d-zollinger@tamu.edu

Abstract

It has been reported that the pavement performance predicted by the current mechanistic-empirical pavement design shows low or no sensitivity to subgrade and unbound layers. This issue has raised wide attention. Targeting this problem, this paper summarizes the process used by the authors to find better models of the influence of subgrade and unbound base course layers on the performance of flexible and rigid pavements. A comprehensive literature review is first conducted and the findings are categorized. It is found that the resilient modulus, permanent deformation, shear strength, and erosion are key factors. In particular, the properties that provide greater sensitivity are 1) the moisture-dependency of the modulus, shear strength, and permanent deformation; 2) stress-dependency of the modulus and permanent deformation; and 3) cross-anisotropy of the modulus. A number of unbound layer/subgrade models have been located and categorized. Three criteria are developed to identify the candidate models in terms of the degree of susceptibility, degree of accuracy, and ease of development. The first two criteria are used to evaluate the collected unbound layer/subgrade models, while associated development and implementation issues are planned as subsequent work. Two models that the authors previously developed are selected as examples to illustrate the improvement of the performance prediction, including the moisture-sensitive, stress-dependent, and cross-anisotropic modulus model for unbound layers and stress-dependent mechanistic-empirical permanent deformation model for unbound base layers. These two models are verified through laboratory tests and numerical simulations. Moreover, they are compared to their counterparts in the AASHTOWare Pavement ME Design. The advantages of accuracy and sensitivity to the operational conditions (e.g. moisture, traffic stress, and load-induced/particle-induced anisotropy) are obvious. In addition to these two models, the development of the shear strength model and erosion model are sketched. The candidate models need further development and implementation, which address issues such as hierarchical inputs, calibration/validation, and implementation. These are the on-going and planned work on this topic to better incorporate the influence of subgrade and unbound layers so as to contribute to the improvement of pavement designs.

Keywords: mechanistic-empirical models; resilient modulus; permanent deformation; shear strength; erosion

1. Introduction

A pavement is a composite structure that usually consists of a surface layer, a base layer, and subgrade. The surface layer can be asphalt concrete for flexible pavements, or cement concrete for rigid pavements. The base course can be an unbound layer or stabilized by cementitious materials. It is well known that the performance of a pavement heavily depends on the behaviors of the surface layer. For example, the behaviors of asphalt concrete under repeated traffic loading or under various environmental conditions are studied to characterize and predict different pavement distresses, such as fatigue cracking and permanent deformation. It has also been recognized that the performance of flexible and rigid pavements is closely related to the characteristics of unbound layers and subgrade. However, recent research studies indicate that the performance predicted by the AASHTOWare Pavement ME Design shows low or no sensitivity to these underlying layers (Schwartz et al., 2011). The specific performance indicators used in the Pavement ME Design includes (AASHTO, 2008): 1) total rutting, load-related cracking, thermal cracking, and smoothness for flexible pavements; and 2) transverse cracking, faulting, punchouts, crack width, and smoothness for rigid pavements. Schwartz et al. (2011) investigated the sensitivity of the properties of unbound layers and subgrade to these performance indicators. The properties examined for flexible pavements include: 1) the resilient modulus, thickness, soil water characteristic curve (SWCC), and Poisson's ratio for unbound layers; and 2) resilient modulus, SWCC, Poisson's ratio, liquid limit, plasticity index, percent passing No. 200, and groundwater depth for subgrade. The properties examined for rigid pavements include: 1) resilient modulus, thickness, erodibility index, load transfer efficiency (LTE), and base slab friction for unbound layers; and 2) resilient modulus and groundwater depth for subgrade. The degree of sensitivity of each property is defined to reflect how the performance indicator is affected by this property. The final results reveal that the performance predicted by the Pavement ME Design generally shows low or no sensitivity to the inputs from subgrade and unbound layers. In particular, the following cases become the major problems:

- Total rutting in flexible pavements is marginally sensitive to resilient modulus and SWCC of unbound layers and subgrade, non-sensitive to thickness of unbound layers;
- Load-related cracking in flexible pavements is non-sensitive to SWCC of unbound layers, marginally sensitive to SWCC of subgrade;

- Faulting in Jointed Plain Concrete Pavement (JPCP) is marginally sensitive to resilient modulus and erodibility, non-sensitive to thickness of unbound layers; and
- Transverse cracking in JPCP is marginally sensitive to resilient modulus, thickness and erodibility of unbound layers.

Targeting these problems, it is the aim of this paper to find the probable reasons for this low/no sensitivity, and propose an approach to identify candidate solutions to improve the models of the influence of subgrade and unbound layers on the performance of flexible and rigid pavements. In order to achieve this goal, the following section first presents a comprehensive review of the modeling of these underlying layers in the existing literature. The factors that affect the pavement performance and the models available to address these effects are listed. Then the evaluation and screening criteria are used to identify candidate models for better incorporating these influences. As examples of our selection process, two models of unbound layers/subgrade are selected, reviewed, and compared to those in the Pavement ME Design in the next few sections. Note that the models reviewed herein are developed previously, so they are briefly discussed and more details can be found in relevant literature. After that a roadmap for future model development and implementation is presented. Finally, a conclusion and recommendation section summarizes these results.

2. Modeling of unbound layers and subgrade in flexible and rigid pavements

In this section, the literature review is conducted to identify the probable root causes for the problems pointed out in the Introduction section. More specifically, the factors of unbound layers and subgrade relevant to pavement performance as well as the models developed to account for these factors are collected and discussed in the following two subsections respectively. Finally, a discussion is presented about how to evaluate and screen candidate models for further considerations.

2.1. Influence of unbound layers and subgrade on performance of flexible and rigid pavements

The introduction section listed the inputs of unbound layers and subgrade required in Pavement ME Design for predicting performance of flexible and rigid pavements. However, besides these parameters, recent studies have identified the pavement performance to be significantly affected

by other characteristics of the underlying layers. According to a comprehensive literature review, these factors can be divided into the following categories:

- Material properties (e.g., modulus, shear strength)
- Material behaviors responding to traffic and environmental (temperature and moisture) conditions (e.g., permanent deformation and erosion)
- Structural characteristics (e.g., thickness of unbound layers)

Tables 1 to 4 give a brief summary of how each performance indicator is influenced by the factors of unbound layers and subgrade. The relevant literatures are also given in these tables. Some abbreviations in Tables 1 to 4 are defined herein: IRI is International Roughness Index and CRCP is Continuously Reinforced Concrete Pavement.

2.2. Category of unbound layer and subgrade models for performance influence

Tables 1 to 4 demonstrate a variety of characteristics of unbound layers and subgrade that affect the performance of flexible and rigid pavements. Based on these results, the search is performed to identify the empirical or mechanistic-empirical models that address such influence. More specifically, the models are elaborated and categorized as follows with relevant literatures.

- Resilient modulus models of unbound layers and subgrade
 - 1) Empirical regression models (AASHTO 1993; ARA, 2004)
 - 2) Nonlinear stress-dependent models (Seed et al. 1967; Hicks and Monismith 1971; Thompson and Robnett 1979; Drumm, 1990; Uzan 1985; Witczak and Uzan 1988; Witczak 2003; Lade and Nelson 1987)
 - 3) Moisture-sensitive models (AASHTO 2008)
 - 4) Moisture-sensitive and stress-dependent models (Oloo and Fredlund 1998; Lytton et al. 1993; Lytton 1995; Sahin et al. 2013; Yang et al. 2005; Liang et al. 2008; Cary and Zapata 2011; Gupta et al. 2007; Oh and Fernando 2011)
 - 5) Stress-dependent and cross-anisotropic models (Al-Qadi et al. 2010; Tutumluer and Thompson 1997)
 - 6) Moisture-sensitive, stress-dependent, and cross-anisotropic model (Gu et al. 2016a)
 - 7) Regression models for stress-dependent model coefficients (Yau and Von Quintus 2002)

- 8) Regression models for moisture-sensitive and stress-dependent model coefficients (Gu et al. 2015b)
- Permanent deformation models of unbound layers and subgrade
 - 1) Non-stress-dependent mechanistic-empirical models (Kenis 1977; Uzan 2004; Tseng and Lytton 1989; Ayres and Witczak 1998)
 - 2) Stress-dependent mechanistic-empirical models (ARA, 2004; Uzan 2004; Korkiala-Tanttu 2009; Chow et al. 2014; Gu et al. 2015a)
 - 3) Regression models for Pavement ME Design model coefficients (Tseng and Lytton 1989; ARA, 2004; Epps et al. 2014)
 - Shear strength models of unbound layers and subgrade
 - 1) Non-moisture-sensitive models (Lambe and Whitman 1969)
 - 2) Moisture-sensitive models (Abramanto and Carvalho 1989; Fredlund and Rahardjo 1993; Titus-Glover and Fernando 1995; Öberg and Sällfors 1997; Fredlund et al. 1996; Vanapalli et al. 1996; Rassam and Cook 2002; Epps et al. 2014)
 - Erosion models of unbound layers
 - 1) Empirical models (Rauhut et al. 1982; Markow and Brademeyer 1984; Larralde 1984; Van Wijk 1985; PCA 2008; ARA 2004)
 - 2) Mechanistic-empirical models (Jung and Zollinger 2011)
 - Foundation models of subgrade
 - 1) No-shear models (Winkler 1867; Filonenko-Borodich 1940; Hetenyi 1950)
 - 2) Shear-included models (Pasternak 1954; Kerr 1965)

2.3. Criteria to evaluate and screen unbound layer and subgrade models

Based on the literature review summarized above, alternative models are available and have the potential to serve as the enhancements to Pavement ME Design, which can improve the considerations of the influence of the underlying layers on pavement performance. In order to evaluate and select candidate models, the following three criteria must be noticed:

- Susceptibility criterion;
- Accuracy criterion; and
- Development criterion.

Table 1. Influential Factors of Unbound Layers on Performance of Flexible Pavements

Performance Indicators	Material Properties			Material Behaviors		Thickness
	Modulus		Shear Strength	Permanent Deformation		
	Magnitude	Cross-Anisotropy			Moisture Sensitivity	
Total Rutting	Total rutting decreases as modulus increases (Shahji 2006; Masad and Little 2004)	The amount of permanent deformation significantly increases when anisotropic properties are used (Masad et al. 2006)	Modulus has a high sensitivity in change of matric suction that represents moisture susceptibility; high degree of moisture change causes decrease of the modulus (Witczak et al. 2000; Butalia et al. 2003; Wolfe and Butalia 2004; Gupta et al. 2007; Cary and Zapata 2011)	Shear strength directly affects total rutting; it decreases as shear strength increases (Brown 1996; Theyse et al. 2007; Núñez et al. 2004; Zhou et al. 2010; Gabr and Cameron, 2012; Chow et al. 2014)	Total rutting increases as permanent deformation of unbound base course increases (Cerni et al. 2012)	Rutting decreases with increase of the thickness of the base layer (Masad and Little 2004)
Load-related Cracking (Alligator & Longitudinal)	Load-related cracking would easily occur with reduced modulus (Cerni et al. 2012; Masad and Little 2004)	Use of cross-anisotropy of unbound base course results in less estimated fatigue cracking life (Adu Osei et al., 2001)		A larger shear strength improves the integrality of supporting layers and also resistance to load-related cracking (AASHTO 2008)	N/A	The resistance to load-related cracking would be enlarged with thick unbound layers (Shahji 2006; Masad and Little 2004)
Thermal Cracking	Thermal cracking is accelerated by loss of modulus (Sahin et al. 2013)	N/A		N/A	N/A	The greater thickness of the base layer possibly helps alleviate the severity of thermal cracking (Carpenter and Lytton 1977)
Smoothness (IRI)	IRI decreases with the increase of base modulus (Masad and Little 2004)	Cross-anisotropy affects total rutting and cracking, which leads to the change of IRI (Masad and Little, 2004)		High shear strength results in low IRI values (AASHTO, 2008; Chow et al., 2014)	Permanent deformation of unbound base is a major distress resulting in increase of surface roughness (Zhou et al. 2007)	Change of IRI diminishes with increase of thickness of the base layer (Masad and Little 2004)

Table 2. Influential Factors of Unbound Layers on Performance of Rigid Pavements

Performance Indicators	Material Properties			Material Behaviors		Thickness	
	Modulus		Shear Strength	Erosion	Permanent Deformation		
	Magnitude	Cross-Anisotropy					Moisture Sensitivity
Transverse Cracking (JPCP)	Transverse cracking would be promoted with low modulus of unbound layers (Hansen and Jensen 2001; Shahji 2006)	Cross-anisotropy greatly affects stress/strain and cracking (Adu-Osei et al. 2001; Masad et al. 2006)	Modulus has a high sensitivity in change of matric suction that represents moisture susceptibility; high degree of moisture causes decrease of the modulus (Cary and Zapata 2011; Sahin et al. 2013; Gu et al. 2016a)	High shear strength prevents occurrence of transverse cracking (Cleveland et al. 2002; Lytton et al. 2010)	N/A	N/A	Thickness of baser layer directly affects amount of transverse cracking (Shahji 2006)
Faulting (JPCP)	Loss of modulus of unbound base course lead to development of faulting (Jung and Zollinger 2011)	N/A		Increase of shear strength inhibits the development of faulting (Jung and Zollinger 2011)	Development of erosion accelerates faulting (Jeong and Zollinger 2001; Jung et al. 2010b; Jung and Zollinger 2011)	Greater permanent deformation of unbound base leads to higher potential of faulting (Bakhsh and Zollinger 2014a)	Faulting decreases with high base thickness (Shahji 2006)
Punchouts (CRCP)	Reduction of modulus of unbound base course causes punchouts (Jung et al. 2012; Vandenbossche et al. 2012; Rao and Darater 2013)	N/A		Potential for punchouts is greater when shear strength decreases (Jeong and Zollinger 2001; Jung et al. 2012)	Erosion intensifies punchout (Jeong and Zollinger 2001; Jung et al. 2012; Ren et al. 2013)	N/A	Increase of thickness is an effective method to control punchouts (Shahji 2006)
LTE (JPCP & CRCP)	A higher modulus of unbound base layer improves LTE (Jeong and Zollinger 2001; Jung et al. 2009)	N/A		Unbound layers with high shear strength have good LTE (Jung and Zollinger 2011)	Development of erosion causes low LTE (Jeong and Zollinger 2001; Jung et al. 2010b)	N/A	Increase of thickness helps improve LTE (Jeong and Zollinger 2001)
Smoothness (IRI) (JPCP & CRCP)	IRI decreases with increase in modulus of base layer (Shahji 2006)	Cross-anisotropy affects cracking and so IRI (Adu-Osei et al. 2001; Masad et al. 2006)		Increase of shear strength of base layer diminishes roughness (Byrum and Perera, 2005; Bakhsh 2014)	Erosion aggravates IRI (Jeong and Zollinger 2001; Jung et al. 2010a,b)	Permanent deformation of unbound base increases roughness (Zhou et al. 2007)	IRI decreases with increase in base layer thickness (Shahji 2006)

Table 3. Influential Factors of Subgrade on Performance of Flexible Pavements

Performance Indicators	Material Properties			Material Behaviors	
	Modulus			Shear Strength	Permanent Deformation
	Magnitude	Cross-Anisotropy	Moisture Sensitivity		
Total Rutting	Total rutting decreases as modulus increases (Gupta et al. 2007)	Use of nonlinear anisotropic model of subgrade affects stress/strain distribution, and then influences the inputs in distress prediction models (Yu and Dakoulas 1993; Oh et al. 2006; Masad et al. 2006)	A higher soil suction generates a larger modulus of subgrade (Khoury and Zaman 2004; Yang et al. 2005; Sawangsuriya et al. 2008; Sawangsuriya et al. 2009; Khoury et al. 2010; Vanapallia and Han 2013)	Total rutting decreases as shear strength of subgrade increases (Li et al. 2011)	Total rutting increases as permanent deformation of subgrade augments (Li et al. 2011)
Load-related Cracking (Alligator & Longitudinal)	Resistance to load-related cracking would be enhanced with increase of modulus of subgrade (Gupta et al. 2007; Shahji 2006; Schwartz et al. 2013)			N/A	Lower permanent deformation of subgrade reduces the probability of load-related cracking (Oh et al. 2007)
Thermal Cracking	N/A	N/A	N/A	N/A	Thermal cracking is related to shrinkage of supporting subgrade soils; high permanent deformation would reduce the resistance to thermal cracking (Sahin et al. 2013)
Smoothness (IRI)	IRI has a negative relation with modulus of subgrade (Masad and Little 2004; Vaillancourt et al. 2014)	Use of nonlinear anisotropic model of subgrade affects stress/strain distribution, and then influences the inputs in distress prediction models (Yu and Dakoulas 1993; Oh et al. 2006; Masad et al. 2006)	Soil suction is a major factor for prediction of subgrade modulus (Yang et al. 2005)	Decrease of shear strength of subgrade results in loss of smoothness (AASHTO 2008)	High permanent deformation exacerbates the roughness of pavement (Sahin et al. 2013; Zhou et al. 2007)

Table 4. Influential Factors of Subgrade on Performance of Rigid Pavements

Performance Indicators	Material Properties			Material Behaviors	
	Modulus		Moisture Sensitivity	Shear Strength	Permanent Deformation
	Magnitude	Cross-Anisotropy			
Transverse Cracking (JPCP)	Increasing modulus of subgrade would reduce transverse cracking (Hansen and Jensen 2001; Shahji 2006)	Cross-anisotropy affects stress/strain and then influences the inputs in distress prediction models (Adu-Osei et al. 2001; Masad et al. 2006)	Soil suction is a major factor for the prediction of modulus of subgrade materials; a higher soil suction generates a larger modulus of subgrade (Yang et al. 2005)	Increase of shear strength of subgrade raises the resistance of transverse cracking (AASHTO 2008)	High permanent deformation leads to loss of supporting layers, which could cause development of transverse cracking (AASHTO 2008)
Faulting (JPCP)	Increase in modulus of subgrade causes a decrease in faulting (Velasquez et al. 2009)			Higher shear strength of subgrade layer helps improve resistance to faulting (Bakhsh and Zollinger 2014a)	High permanent deformation of subgrade increases the possibility of faulting (Huang 1993; AASHTO 2008)
Punchouts (CRCP)	Punchout increases with low k-value of subgrade (Jung et al. 2012; Vandenbossche et al. 2012)	N/A		Punchout is accelerated with lower shear strength of subgrade (Jung et al. 2012)	Increase of permanent deformation of subgrade makes poorer LTE; thus leads to development of punchouts (Huang 1993)
LTE (JPCP & CRCP)	LTE is increased by high modulus of subgrade (Jeong and Zollinger 2001)	N/A		Increase of shear strength improves LTE (Jeong and Zollinger 2001)	Loss of LTE occurs with high permanent deformation of subgrade (Jung et al. 2012)
Smoothness (IRI) (JPCP & CRCP)	IRI value diminishes with the increase in subgrade modulus (Shahji 2006)	Cross-anisotropy affects cracking/faulting and so IRI (Adu-Osei et al. 2001; Masad et al. 2006)		Improvement of shear strength of subgrade layer could increase smoothness (Bakhsh 2014)	Rutting generated from permanent deformation of subgrade is associated with increased roughness (Zhou et al. 2007)

The susceptibility criterion refers to how the model responds to the changes in the operational conditions, including moisture, heat, traffic stress, and load-induced/particle-induced anisotropy. As listed in Tables 1 to 4, the performance of flexible and rigid pavement is closely related to the operational conditions of unbound layers and subgrade. For example, as shown in Table 1, a flexible pavement is more susceptible to the load-related cracking (alligator and longitudinal cracking) when the modulus of the base course decreases. When the cross-anisotropy is taken into account, the fatigue life is normally shorter than when using an isotropic modulus for the base course. In addition, the modulus of the base course significantly reduces as the degree of moisture increases, which results in more severe load-related cracking. Based on the results in Tables 1 to 4, each unbound layer/subgrade model should be evaluated under these operational conditions.

The accuracy criterion refers to how close the predictions made by an unbound layer/subgrade model are to the actual behaviors of these underlying materials. More specifically, the model should be verified by comparing to the laboratory measurements on unbound layer and subgrade materials. In addition, the model needs to be compared with the performance prediction that is made by its counterpart in Pavement ME Design through a sensitivity analysis.

The development criterion refers to the efforts required to develop, validate, and test the unbound layer/subgrade model for the enhancements of Pavement ME Design. It is used to ensure that essential development issues can be identified and solved (e.g., whether the data elements that are needed for the model are available and/or whether the test methods and equipment that are needed to provide inputs for the model are available). Furthermore, the model can be validated by making predictions of the observed performance of pavements in the Long-Term Pavement Performance (LTPP) database and/or from state departments of transportation (DOTs). This criterion serves as the basis of the development and implementation of enhancements for Pavement ME Design.

With respect to the scope of this study, the first two criteria are used to evaluate the aforementioned unbound layer/subgrade models. In other words, this study only focuses on the susceptibility and accuracy of the models. As identified in Tables 1 to 4, the resilient modulus models should be particularly those that incorporate the effects of the level of moisture in addition to the traffic-related stresses. The anisotropy of the base course also needs to be reflected in a separate model for the vertical modulus and the horizontal modulus. The

permanent deformation models should be sensitive to the changes of properties and thickness of the underlying layers. The shear strength models should be especially those models that include the effects of moisture as well as traffic-related stresses on the shear strength of the material. The erosion models should be those that are mechanical-empirical in nature.

It is worth mentioning that the thickness of the base course is also a critical input in the pavement design. However, the current investigations indicate that the performance predicted by the Pavement ME design shows low sensitivity to the thickness of the base layer. This problem can be solved by choosing an unbound/subgrade model when the susceptibility and accuracy criteria are satisfied. The moisture-sensitive, stress-dependent, and cross-anisotropic modulus models; moisture-sensitive shear strength models; stress-dependent mechanistic-empirical permanent deformation models; and mechanistic-empirical erosion models could contribute to this category.

In the following sections, several models are selected as examples to demonstrate the factors that must be included in modeling unbound layers and subgrade and the improvements that are achieved as compared with the Pavement ME Design models.

3. Moisture-sensitive, stress-dependent, and cross-anisotropic modulus model for unbound layers

This section presents an example resilient modulus model for unbound base layers considering the effects of moisture and nonlinear stress distribution in the base course as well as the anisotropic features of this layer.

3.1. Model development and verification for resilient modulus

The resilient modulus model introduced herein is the one recently developed by the authors (Gu 2015c, Gu et al. 2016a) for unbound base courses. It considers both nonlinear cross-anisotropic behavior and moisture-sensitive characteristics, and incorporating the proposed constitutive model into the finite element model of the base layer to quantify the influence of moisture content on the pavement performance. More specifically, the saturation factor and the matric suction of the unsaturated unbound aggregates are applied to the proposed constitutive model to reflect the moisture dependence. Additionally, a new user-defined material subroutine (UMAT) is developed to characterize the moisture-sensitive and stress-dependent nonlinear cross-

anisotropic behavior of base materials in the software ABAQUS (Gu et al. 2016b). The formulation of the model is given as follows:

$$M_R^V = k_1 P_a \left(\frac{I_1 - 3\theta f h_m}{P_a} \right)^{k_2} \left(\frac{\tau_{oct}}{P_a} \right)^{k_3} \quad (1)$$

$$s = \frac{M_R^H}{M_R^V} ; r = \frac{G_{VH}}{M_R^V} \quad (2)$$

where M_R^V is the resilient modulus in the vertical direction; I_1 is the first invariant of stress tensor; τ_{oct} is the octahedral shear stress; P_a is the atmospheric pressure; θ is the volumetric water content; f is the saturation factor, $1 \leq f \leq \frac{1}{\theta}$; h_m is the matric suction; k_1 , k_2 , and k_3 are regression coefficients; M_R^H is the resilient modulus in the horizontal direction; G_{VH} is the shear modulus in the horizontal-vertical plane; and s and r are the modulus ratios.

In order to verify the accuracy of the modulus model in Equation 1, the repeated load triaxial tests are conducted on two selected materials at three different moisture contents. The matric suction value in Equation 1 is obtained from the filter paper test. Figure 1 presents the comparison between the predicted moduli using Equation 1 and the measured moduli from the triaxial tests. The model prediction provides a good agreement with the test measurements. This indicates that the constitutive model proposed in Equation 1 is able to reflect the moisture-sensitive and stress-dependent behavior of unbound aggregates. After verification, Equations 1 and 2 are coded into a UMAT to develop a moisture-sensitive and stress-dependent nonlinear program that incorporates cross-anisotropy.

Using this moisture-sensitive and stress-dependent nonlinear cross-anisotropic program, a numerical study is conducted on a typical flexible pavement structure to examine its capability to reflect the influence of unbound base on the pavement performance. The pavement structure, finite element model, and modeling parameters are given in Figure 2.

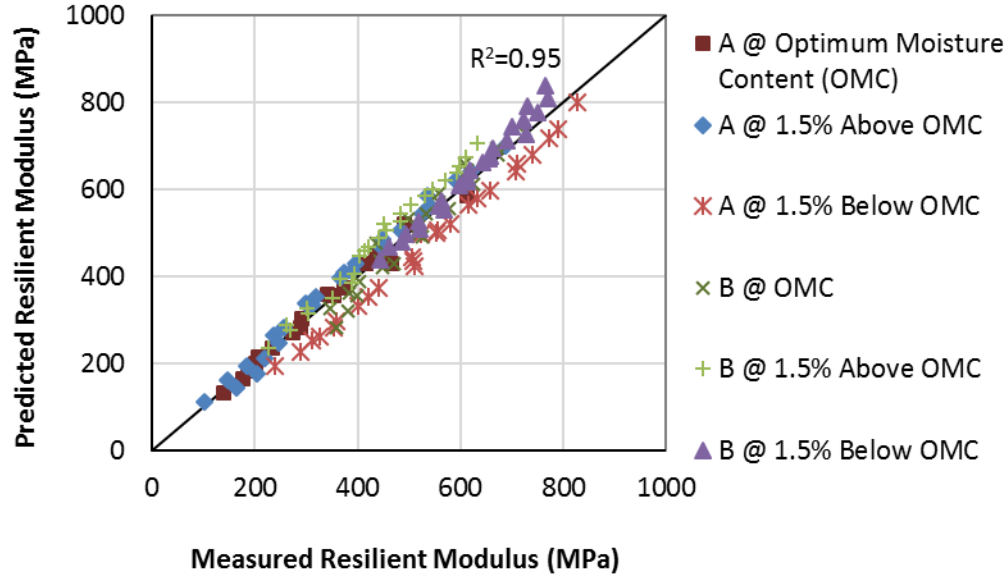
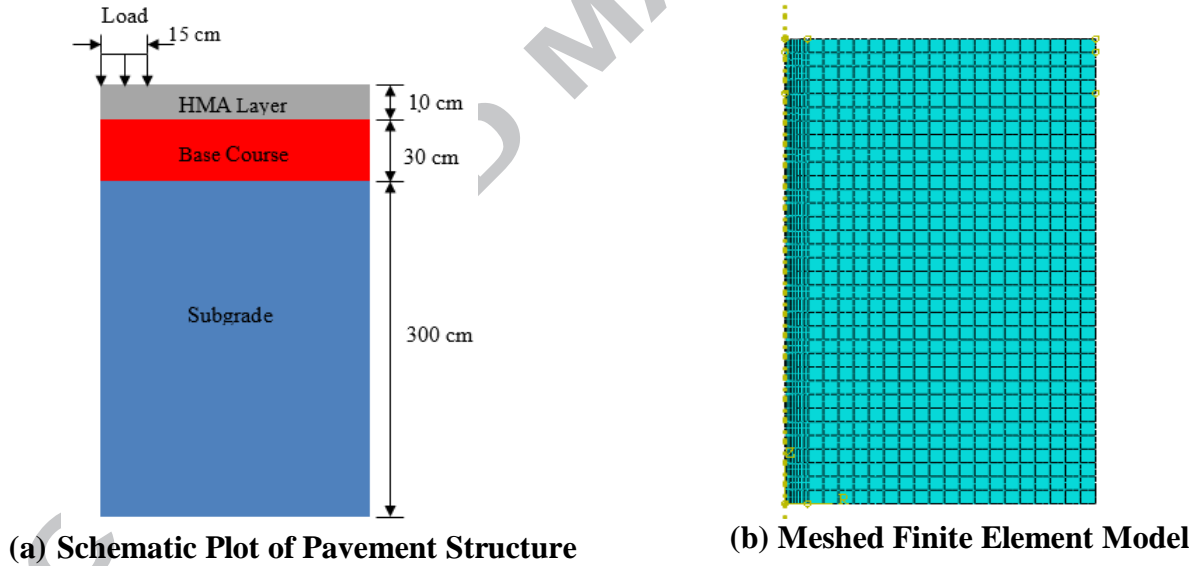


Figure 1. Comparison of Predicted and Measured Resilient Moduli for Unbound Base Materials (A, B stand for 2 types of unbound aggregates)

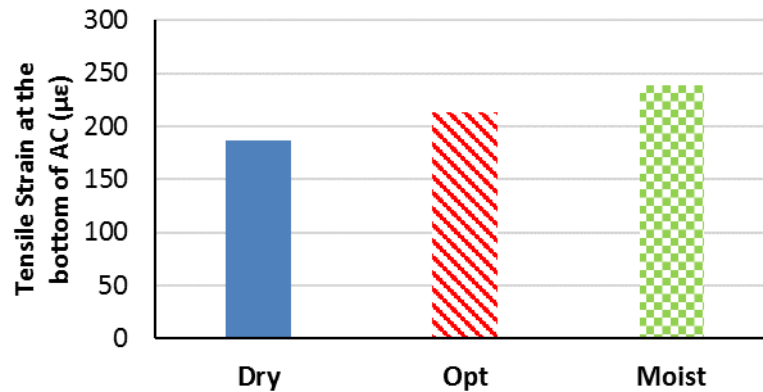


Traffic Load	565 kPa (9 kips)		
Base Moisture Conditions	Moist (1.5% above optimum)	Optimum	Dry (1.5 below optimum)
Material Properties	HMA layer	Viscoelastic	
	Unbound base course	Nonlinear cross-anisotropic & moisture-sensitive	
	Subgrade	Elastic	

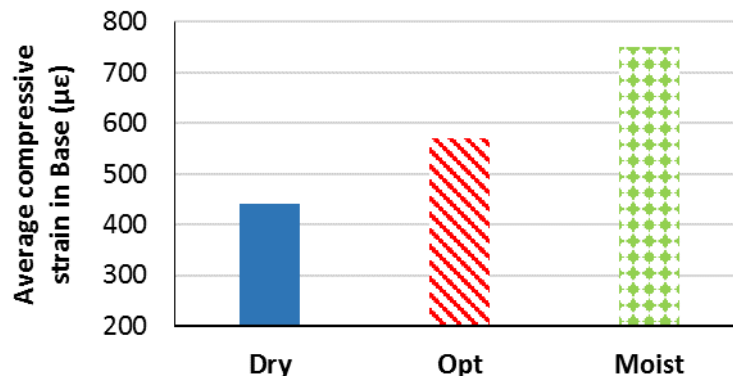
(c) Modeling Parameters

Figure 2. Finite Element Modeling Using Moisture-sensitive and Stress-dependent Nonlinear Cross-anisotropic Program

The tensile strain at the bottom of the asphalt layer and the compressive strain in the base course are obtained from the numerical modeling, as shown in Figures 3 and 4. The increase of the moisture content in the base course significantly increases the tensile strain at the bottom of the asphalt layer; it also leads to an increase of the compressive strain in the base course. The incorporation of cross-anisotropy of base materials results in an increase of both tensile strain at the bottom of the asphalt layer and compressive strain in the base course. According to the fatigue life prediction equation and rut depth equation in Pavement ME Design, the fatigue life and the rut depth of this pavement change accordingly. The results of these pavement responses indicate that the proposed model and program demonstrate the desired influence of moisture of base materials and the resulting change of the stress state in the base course on pavement performance. The model and program also reflect the fact that granular base materials exhibit cross-anisotropic behaviors that affect the performance of pavements.

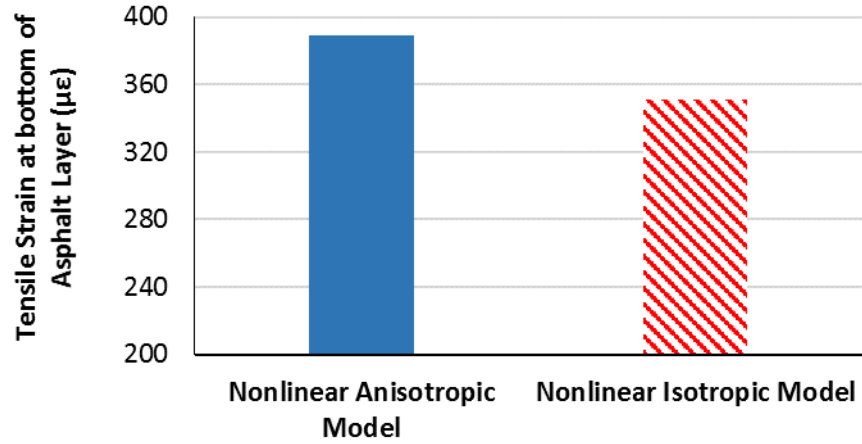


(a) Tensile Strain at Bottom of Asphalt Layer to Predict Fatigue Life

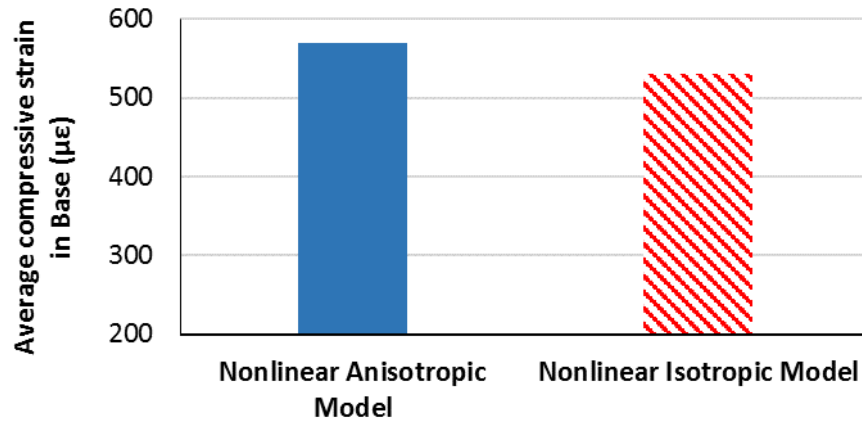


(b) Compressive Strain in Unbound Base to Predict Rutting

Figure 3. Demonstration of Effect of Moisture on Pavement Performance



(a) Tensile Strain at Bottom of Asphalt Layer to Predict Fatigue Life



(b) Compressive Strain in Unbound Base to Predict Rutting

Figure 4. Demonstration of Effect of Cross-Anisotropy on Pavement Performance

3.2. Comparison with Pavement ME Design modulus models

The modulus models currently used in Pavement ME Design are the following (AASHTO 2008):

Generalized modulus model:

$$M_R = k_1 P_a \left(\frac{\theta}{P_a} \right)^{k_2} \left(\frac{\tau_{oct}}{P_a} + 1 \right)^{k_3} \quad (3)$$

Modulus model considering moisture sensitivity:

$$\log \frac{M_R}{M_{Ropt}} = a + \frac{b - a}{1 + \exp \left[\ln \frac{-b}{a} + k_m (S - S_{opt}) \right]} \quad (4)$$

where M_R is resilient modulus; θ is bulk stress; τ_{oct} is octahedral shear stress; P_a is atmospheric pressure; k_1, k_2, k_3 are regression coefficients determined from laboratory test data; $M_{R_{opt}}$ is resilient modulus at a reference condition; a, b, k_m are regression coefficients determined from test data; and $(S - S_{opt})$ is variation in degree of saturation. The accuracy of predictions made by Equation 1 is compared with that by Equation 3 and by Equation 4, respectively. Figure 5 shows an example of comparison of the prediction between the proposed model and the generalized model in which the matric suction is ignored. It can be seen that the correlation between the predicted resilient moduli and the measured values are significantly improved when the matric suction is included. Figure 5 also shows the comparison between the proposed model and the model in Equation 4. It is obvious that the proposed model provides a more accurate prediction of the changes in resilient modulus because of changes in moisture. This is because the model in Equation 4 assumes the moisture condition and stress state are independent, whereas the proposed model considers the influence of the moisture variation on the stress state in terms of matric suction.

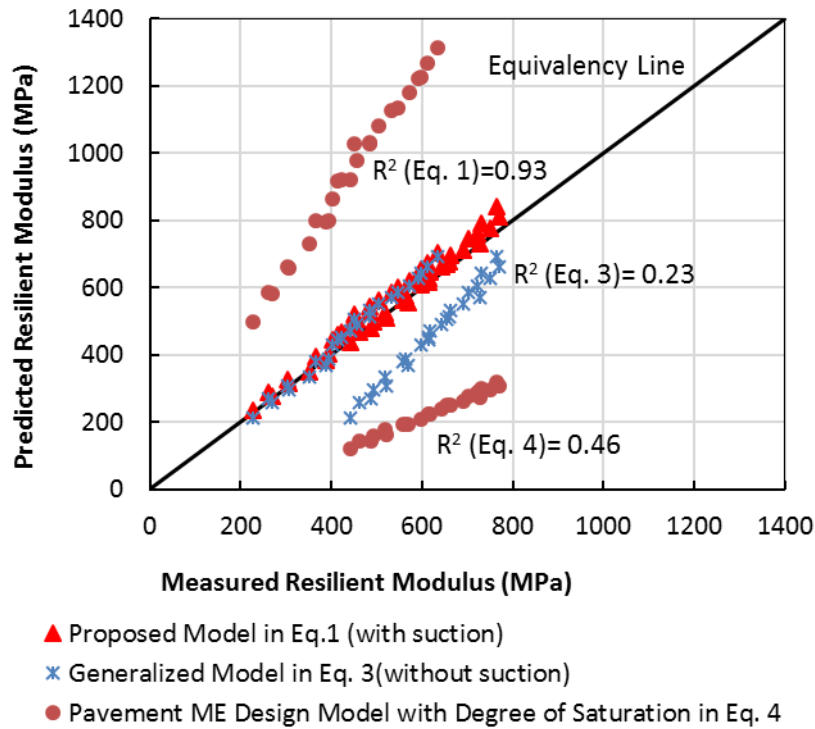


Figure 5. Comparison between Proposed Resilient Modulus Model and Pavement ME Design Models

4. Stress-dependent mechanistic-empirical permanent deformation model for unbound base layers

This section presents an example of the permanent deformation model for unbound base layers considering the nonlinear stress distribution in the base course as well as the effects of the moisture.

4.1. Model development and verification for permanent deformation

The authors have recently developed a new mechanistic-empirical rutting model (Gu et al. 2015a, Gu et al. 2016c) for unbound granular materials, which is capable of predicting the permanent deformation behavior at different stress states using the single-stage test protocol. The formulation of the model is given as follows:

$$\varepsilon_p = \varepsilon_0 e^{-\left(\frac{\rho}{N}\right)^\beta} \left(\sqrt{J_2}\right)^m (\alpha I_1 + K)^n \quad (5)$$

$$\alpha = \frac{2 \sin \phi}{\sqrt{3}(3 - \sin \phi)} \quad (6)$$

$$K = \frac{c \cdot 6 \cos \phi}{\sqrt{3}(3 - \sin \phi)} \quad (7)$$

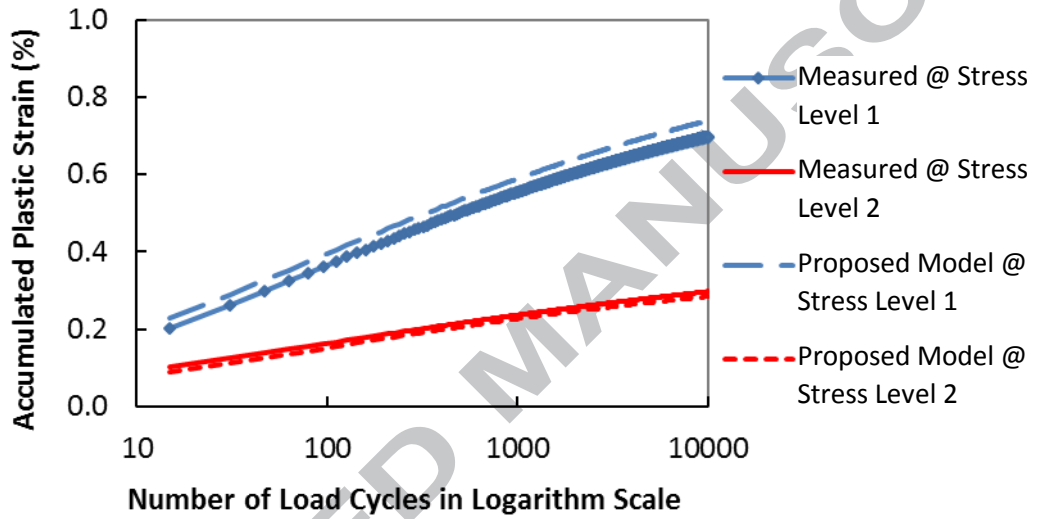
where J_2 is the second invariant of the deviatoric stress tensor; I_1 is the first invariant of the stress tensor; ε_0 , ρ , β , m , and n are model coefficients; c and ϕ are cohesion and friction angle, respectively. In this model, the two terms, $\sqrt{J_2}$ and $\alpha I_1 + K$, are incorporated into the Tseng-Lytton model (Tseng and Lytton 1989), which is used to reflect the influence of a stress state on the permanent deformation of unbound materials. Two types of tests are needed to determine the coefficients in Equation 5:

- Triaxial compressive strength tests to determine the cohesion c and friction angle ϕ ;
- Repeated load triaxial tests at multiple stress levels to determine the coefficients ε_0 , ρ , β , m , and n .

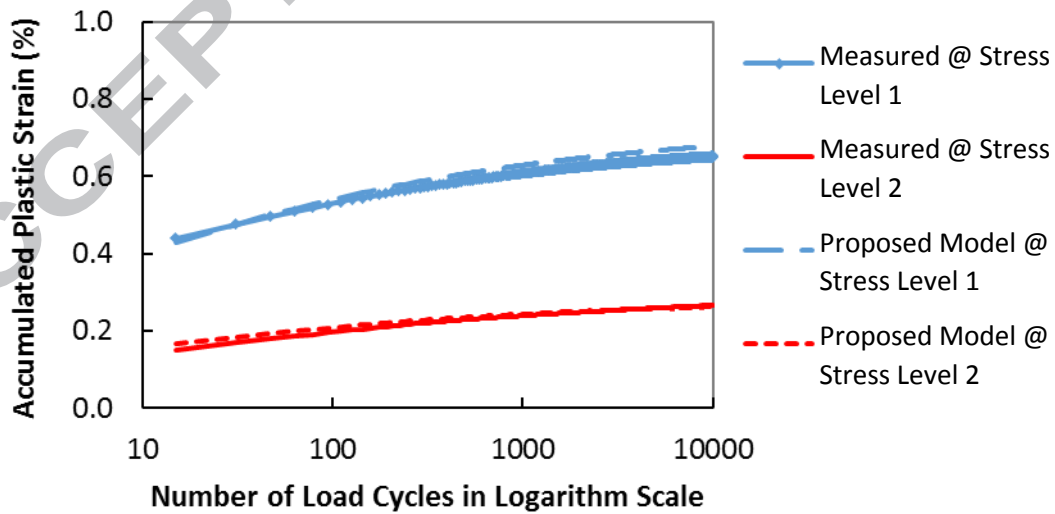
The triaxial compressive strength test is a standard test used to determine the shearing resistance of base materials, which is documented in Tex-117-E (TxDOT 2010). The repeated load triaxial

test is performed on cylindrical aggregate specimens using the universal testing machine (UTM) with a rapid triaxial test (RaTT) cell.

The accuracy of the model proposed in Equation 5 is validated by comparing the predicted permanent deformation curves to those measured from the tests. Two types of unbound materials are selected: a granite aggregate and a limestone aggregate. Both types were subjected to the test protocol above. Two stress levels were used in the repeated load triaxial tests. The results of comparisons, shown in Figure 6, indicate that the proposed model matches well with the measured permanent deformation curves.



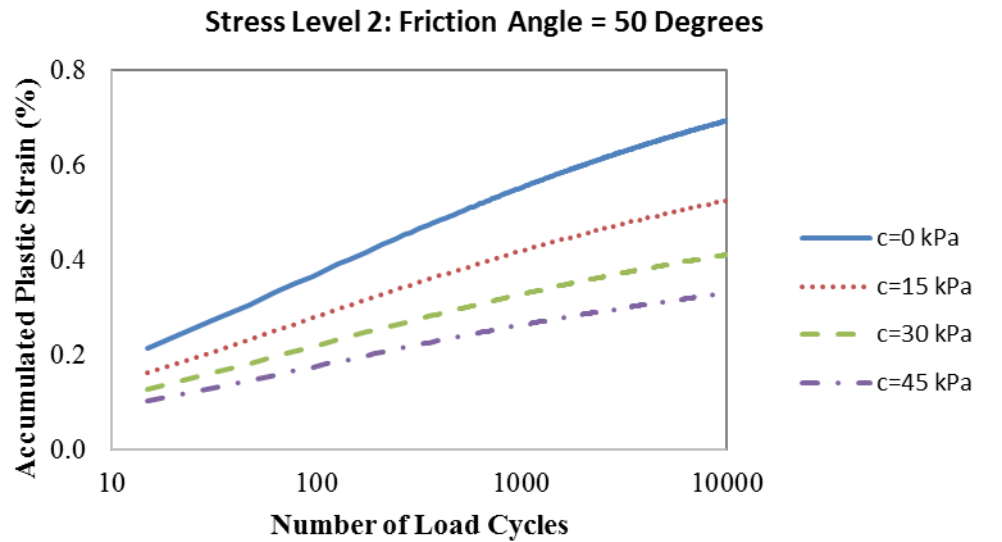
(a) Granite Aggregates



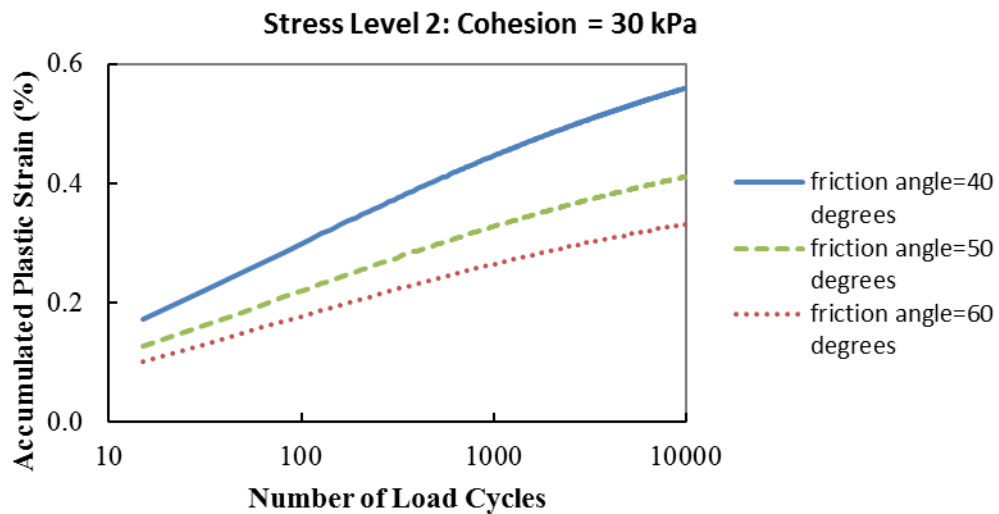
(b) Limestone Aggregates

Figure 6. Validation of Accuracy of Proposed Permanent Deformation Model

The sensitivity of the model proposed in Equation 5 is also examined to determine whether it reflects the influence of properties of unbound materials as well as moisture conditions. Figure 7 shows how the permanent deformation varies as the cohesion and friction angle of unbound aggregates changes. It is known that moisture affects the cohesion of unbound materials. Therefore, the proposed model is able to discriminate the effects of the cohesion and friction angle as well as moisture on permanent deformation behaviors of unbound aggregate materials.



(a) Change of Cohesion



(b) Change of Friction Angle

Figure 7. Sensitivity Analysis of Proposed Permanent Deformation Model

4.2. Comparison with Pavement ME Design permanent deformation models

The regression models for Pavement ME Design model coefficients refer to the models for the coefficients in the rutting model currently used in Pavement ME Design. The Pavement ME Design rutting model is:

$$\Delta_p = \beta_s \left(\frac{\varepsilon_0}{\varepsilon_r} \right) e^{-\left(\frac{\rho}{N}\right)^\beta} \varepsilon_v h \quad (8)$$

where Δ_p is permanent deformation for the layer; ε_r is resilient strain imposed in the laboratory test; ε_v is the average vertical resilient strain in the layer; h is the thickness of the layer; N is the number of traffic repetitions; ε_0 , ρ , β are model coefficients; and β_s is the global calibration coefficient, 1.673 for granular materials and 1.35 for subgrade soils. The Pavement ME Design also provides the models to predict the coefficients as shown below:

$$\frac{\varepsilon_0}{\varepsilon_r} = \frac{\left(e^{\rho^\beta} \times 0.15 \right) + \left(e^{\left(\frac{\rho}{10^9}\right)^\beta} \times 20 \right)}{2} \quad (9)$$

$$\log \beta = -0.61119 - 0.017638W_c \quad (10)$$

$$\rho = 10^9 \times \left(\frac{-4.89285}{\left[1 - \left(10^9 \right)^\beta \right]} \right)^{\frac{1}{\beta}} \quad (11)$$

where W_c is the water content. Similarly, the authors also developed models to predict ε_0 , ρ , β , which are given below (Epps et al. 2014):

$$\ln \varepsilon_0 = 10.24 - 0.03MBV + 0.10pfc + 0.88a_A - 3.95 \ln \lambda_T \quad (12)$$

$$\ln \rho = 6.74 + 0.02MBV + 0.04pfc - 0.85a_G + 0.03\lambda_G - 0.13a_T \quad (13)$$

$$\ln \beta = 10.17 - 2.75 \ln \gamma_d - 0.05pfc - 2.00a_G - 1.61 \ln \lambda_A - 0.34a_T \quad (14)$$

where MBV is the methylene blue value; pfc is the percent fines content; λ_T is the scale factor of the texture index; a_A is the shape factor of the angularity index; λ_A is the scale factor of the angularity index; γ_d is the dry density; a_G is the shape factor of gradation; a_T is the shape

factor of texture index; and λ_G is the scale factor of gradation. The parameters on the right side of Equations 12 to 14 are performance-related base course properties, which are a mixture of those that can only be measured in the laboratory and others that can also be measured in the field. The properties that can be measured in the laboratory include the dry density, the gradation, and the measures of shape, angularity, and texture. The properties that can be measured in the field contain the methylene blue value, the percent fines content, and the water content. The corresponding tests include the methylene blue test, aggregate imaging system (AIMS) test, and percent fines content test. The Grace methylene blue test method is used to determine the MBVs of base materials (W.R. Grace & Co. 2010). The AIMS device utilizes image acquisition hardware, high-resolution camera, microscope and others to characterize the morphology of coarse aggregates (Masad 2005). The percent fines content test is performed by a Horiba laser scattering particle size distribution analyzer (Sahin 2011). More details regarding these tests can be found in Gu et al. (2015b).

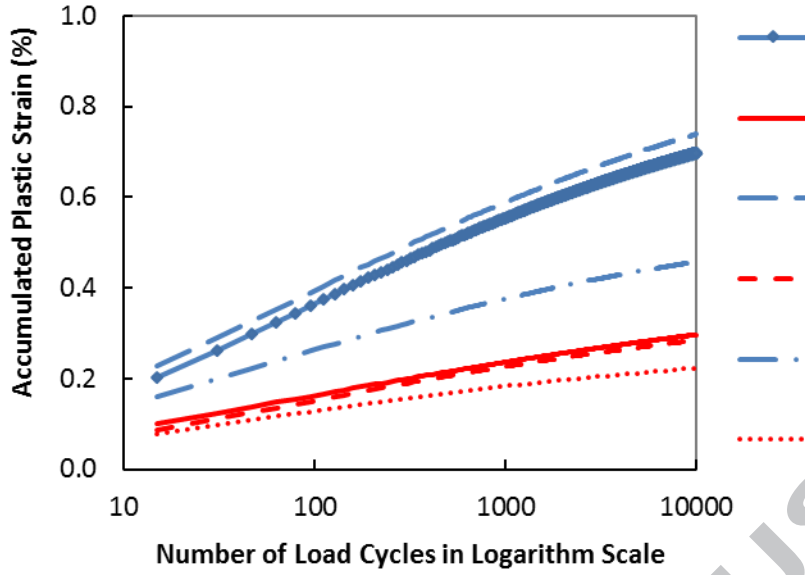
Based on the models presented above, there are two options to perform the comparison between the proposed and Pavement ME Design models:

- 1) Compare Equation 5 with the Pavement ME Design rutting model (i.e., Equation 8).
- 2) Compare Equations 12 to 14 with the coefficient models in Pavement ME Design (i.e., Equations 9 to 11).

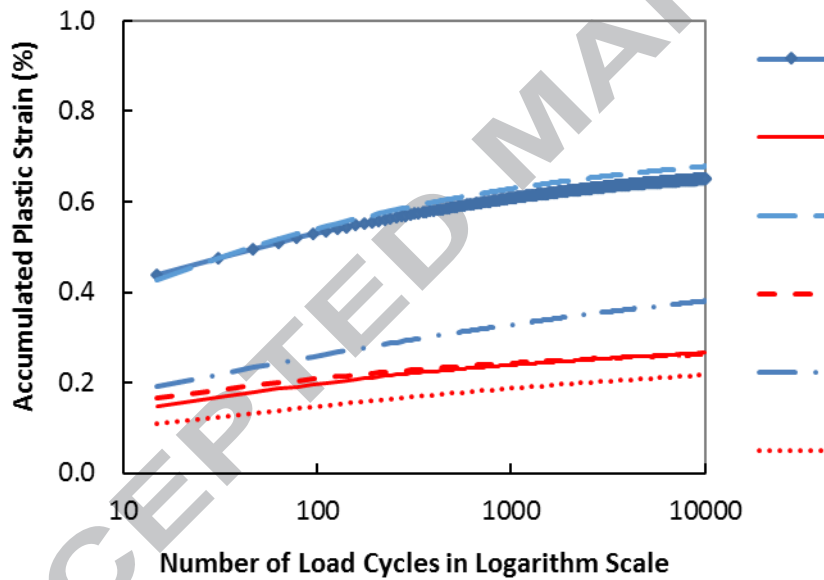
Each aspect is elaborated as follows.

Proposed permanent deformation model versus Pavement ME Design rutting model

First, the same data presented in Figure 6 are utilized to compare the predicted permanent deformations by the proposed model in Equation 5 and the Pavement ME Design model in Equation 8 to the measured values from the tests, as shown in Figure 8. It is clear that the Pavement ME Design model underestimates the permanent deformation of the tested unbound granular materials.



(a) Granite Aggregates



(b) Limestone Aggregates

Figure 8. Validation of Accuracy of Proposed Permanent Deformation Model and Pavement ME Design Rutting Model

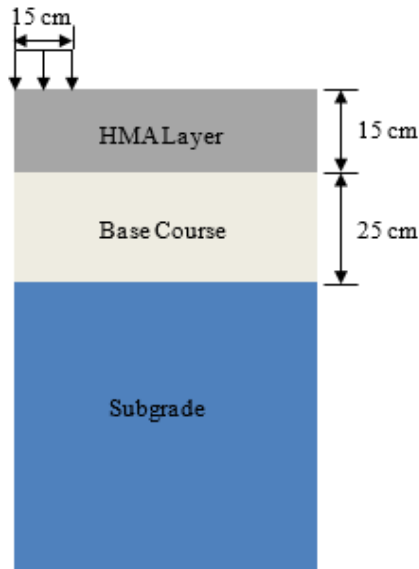
Second, a numerical study is conducted through the finite element software, ABAQUS, to demonstrate how these two models predict rutting in pavement structures. A typical flexible pavement structure was selected as shown in Figure 9 with the input material properties. The UMAT developed and mentioned above was implemented in ABAQUS to characterize the

nonlinear cross-anisotropic behaviors of the base layer. After obtaining the stress and strain distributions in the base layer from the finite element modeling, the multi-layered incremental approach is employed to compute the total rut depth, as shown in the following equations:

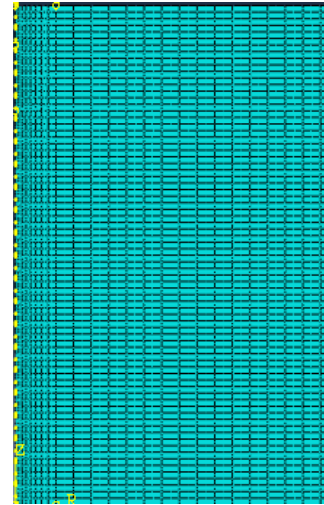
$$\delta_{newME}(N) = \int_0^h \varepsilon_0 e^{-\left(\frac{\rho}{N}\right)^\beta} \left(\sqrt{J_2(z)} \right)^m (\alpha I_1(z) + K)^n dz \quad (15)$$

$$\delta_{ME}(N) = \int_0^h \frac{\varepsilon_0}{\varepsilon_r} e^{-\left(\frac{\rho}{N}\right)^\beta} \varepsilon_v(z) dz \quad (16)$$

where δ_{newME} and δ_{ME} is the total rut depth in the base course calculated by the proposed new mechanistic-empirical model and by the Pavement ME Design model, respectively, h is the thickness of the base layer, and z is the depth within the base layer



(a) Schematic Plot of Pavement Structure



(b) Meshed Finite Element Model

Traffic Load	9, 12, and 16 kips	
Material Properties	HMA layer	$E_{HMA}=2400 \text{ MPa}; \nu_{HMA}=0.35$
	Unbound base course	$k_1=1081, 1281, \text{ and } 1481; k_2=0.81; k_3=-0.08; s=0.45; r=0.35; \nu_{yx}=0.38; \nu_{xx}=0.43$
	Subgrade	$E_{SG}=69 \text{ MPa} \quad \nu_{SG}=0.4$

(c) Modeling Parameters

Figure 9. Finite Element Modeling to Predict Accumulated Rut Depth in Unbound Base Course

Figure 10 presents the results of the computed total rut depths by the Pavement ME Design model and the proposed model when the pavement is subjected to 100,000 traffic loading cycles. That figure shows that the rut depth predicted by the proposed model is higher than that by the Pavement ME Design model. This is consistent with the results shown in Figure 8 when comparing to the laboratory measurements.

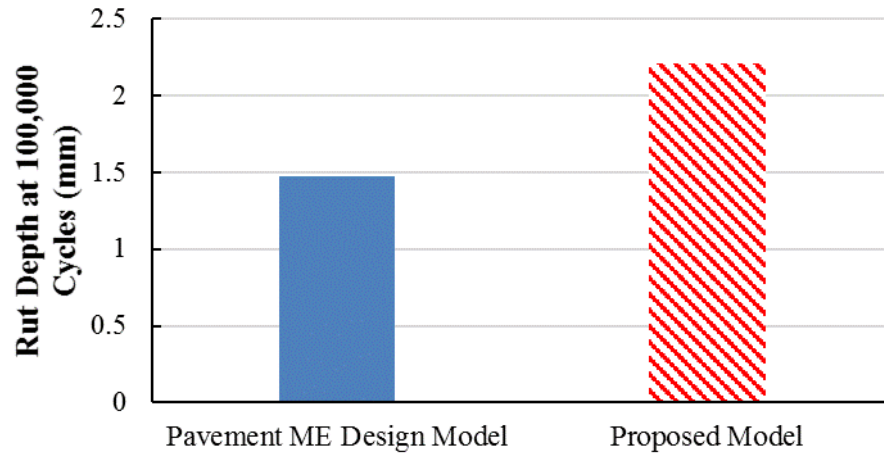


Figure 10. Computation of Rut Depth Using Proposed Model and Pavement ME Design Model by Finite Element Modeling

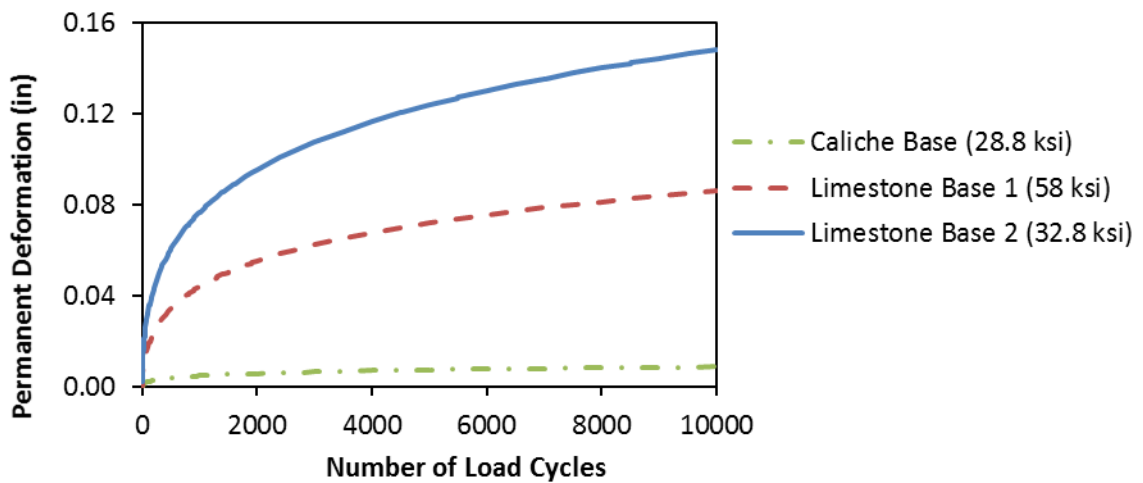
Proposed coefficient models versus Pavement ME Design coefficient models

The difference between the proposed (Equations 12 to 14) and Pavement ME Design coefficient models is that the former employs parameters that are directly related to the performance, while the latter relies on just the water content. In order to compare the accuracy of these two types of coefficient models, the authors utilize the laboratory test results of three types of unbound base materials (caliche and two limestone bases) in terms of the resilient modulus and permanent deformation. Table 5 lists the performance-related base course properties that are measured in the laboratory, including the dry density (γ_d), water content (w), MBV, pfc, and shape parameter a and scale parameter λ for aggregate gradation, angularity, shape, and texture. The subscripts “G”, “A”, “S”, and “T” stand for gradation, angularity, shape, and texture, respectively. The model coefficients were calculated by the equations above, and then the permanent deformation is predicted with the same rutting model (the Pavement ME Design model in Equation 8). The results are given in Figure 11. The permanent deformation predicted using the proposed coefficient models (Equations 12 to 14) varies significantly with the change of the base modulus. However, the permanent deformation predicted using the Pavement ME Design approach is

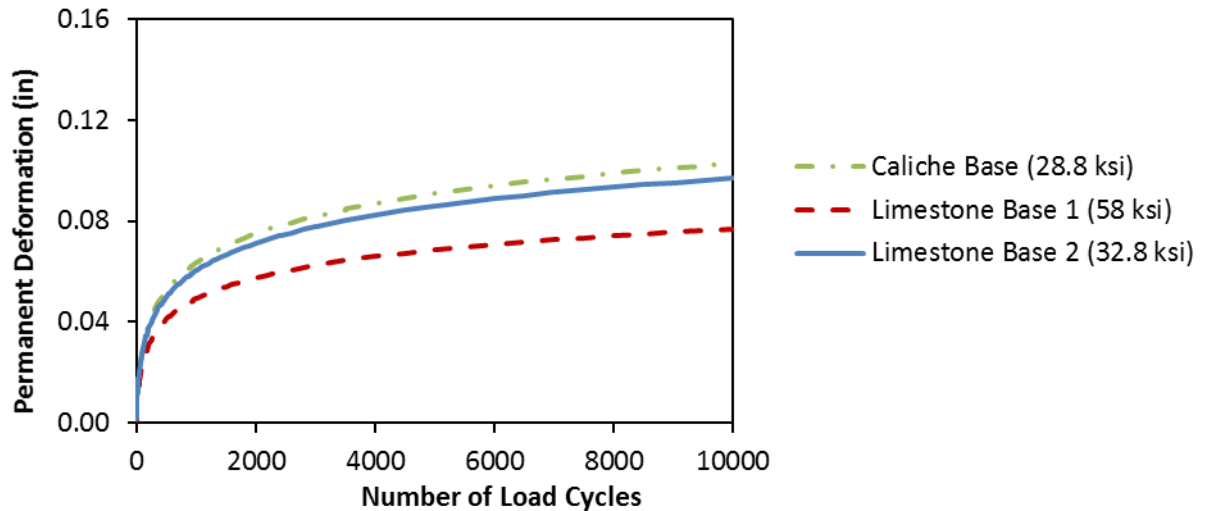
much less sensitive to the base modulus. As a result, the rutting model coefficients are not appropriately calculated in Pavement ME Design, so they do not sufficiently reflect the influence of base modulus on the rutting deformation.

Table 5. Measured Performance-Related Base Course Properties

Material Type	γ_d (kg/m ³)	w (%)	MBV (mg/g)	pfc (%)	Gradation		Angularity		Shape		Texture	
					a_G	λ_G	a_A	λ_A	a_S	λ_S	a_T	λ_T
Limestone 1	2246	6.2	16.4	12.7	0.93	10.3	5.10	3072.9	3.65	8.0	1.96	171.5
Limestone 2	2233	7.1	7.6	15.8	0.85	13.1	4.53	3210.5	4.63	8.0	1.86	138.8
Caliche	2092	7.7	18.5	22.8	0.75	9.9	3.25	3633.4	4.27	8.2	2.87	253.9



(a) Predicted by Proposed Coefficient Models



(b) Predicted by Pavement ME Design Coefficient Models

Figure 11. Comparison of Permanent Deformation Predicted by Proposed Coefficient Models and Pavement ME Design Coefficient Models

5. Roadmap for future development and implementation

The resilient modulus and permanent deformation models reviewed above serve as two examples about selecting candidate models to improve the Pavement ME Design in terms of sensitivity of the unbound layers and subgrade. This section will sketch the further development to enhance other aspects in modeling these underlying layers as well as associated implementation issues and possible solutions.

5.1. Development for moisture-sensitive shear strength model and mechanistic-empirical erosion model

The Pavement ME Design takes the elastic behavior of unbound layers and subgrade as the major concern in the design, but little attention has been paid to their shear strength. This needs to be improved because the shear strength of underlying layer materials is closely related to the pavement performance as shown in Tables 1 to 4. In light of the critical role of shear strength in performance prediction, it is highly desirable to incorporate the shear strength in mechanistic-empirical design of both flexible and rigid pavements. Furthermore, the impact of moisture variations to the shear strength must be taken into account. The detrimental effects of moisture on shear strength are shown in Oloo (1994). As the water content increases by a small amount, the shear strength decreases significantly depending on the magnitude of the normal stress. Such

a reduction accelerates shear failure and intensifies rutting in flexible pavements and erosion in rigid pavements.

The general shear strength model is defined according to the Mohr-Coulomb failure envelope, which is determined from triaxial tests on laboratory molded specimens. In the presence of water, the general shear strength model can be expressed in the following way (Fredlund and Rahardjo, 1993):

$$\tau = c + \sigma_n \tan \phi = \left[c' + (u_a - u_w) \tan \phi^b \right] + \sigma_n \tan \phi \quad (17)$$

where τ is the shear stress; c is the total cohesion; σ_n is the normal stress on the failure plane; ϕ is the angle of internal friction; c' is the effective cohesion; u_a is the pore air pressure; u_w is the pore water pressure; $(u_a - u_w)$ is the matric suction; and ϕ^b is the angle indicating the rate of increase in shear strength relative to the matric suction. To make the shear strength model more applicable in the pavement design, the authors plan to develop prediction models for the shear strength parameters c and ϕ . In this way, the shear strength of unbound layers and subgrade can be estimated using common design inputs in the absence of triaxial test data.

Another model that will be developed is the mechanistic-empirical erosion models of unbound layers and subgrade. Considering that most of the existing erosion models are empirical in nature, a mechanistic-empirical model was developed at Texas A&M University (Jeong and Zollinger 2001; Jung and Zollinger 2011) to characterize erosion in rigid pavements. The model considers the major factors responsible for erosion, including traffic load and speed, temperature variations, moisture infiltration, stiffness of unbound layers and subgrade, interfacial bonding between concrete slab and unbound layers, and permeability of the concrete slab. The formulation of the model is as follows:

$$f(\%Erosion) = f_0 e^{-\left(\frac{\rho}{D(N)-v}\right)^\beta} \quad (18)$$

where f_0 is the maximum faulting; $f(\%Erosion)$ is the level of faulting; ρ is the scale calculation factor based on laboratory erosion test; $D(N)$ is the damage after N load repetitions; v is the time delay before the appearance of visible (measurable) damage; and β is the shape factor related to the erosion rate. The values of ρ , v , and β depend on the base course characteristics beneath the concrete layer. The damage function $D(N)$ quantifies the combined

effect of curling due to thermal gradient, warping due to moisture gradient, and permanent deformation in the supporting base layer. A laboratory test is designed to measure the erodibility of a subbase or a subgrade material using the Hamburg wheel-tracking device (HWTDD) in the laboratory. Detailed procedures were documented in Jung et al. (2010a).

Based on the HWTDD test results above, the authors will develop a model to predict the critical erosion depth in rigid pavements. The critical erosion depth is defined as the critical value of the erosion depth at which erosion begins to accelerate, as illustrated in Figure 12. The critical erosion occurs at the point of inflection, which is the critical point where the curvature of the erosion depth curve changes from negative to positive. The erosion depth curve is expressed by the following mathematical form:

$$N = N_{\infty} e^{-\left(\frac{\rho_e}{D_e}\right)^{\beta_e}} \quad (19)$$

where N is the number of load cycles in a HWTDD test; N_{∞} is the number of load cycles to failure due to erosion; D_e is the erosion depth; and ρ_e and β_e are model coefficients.

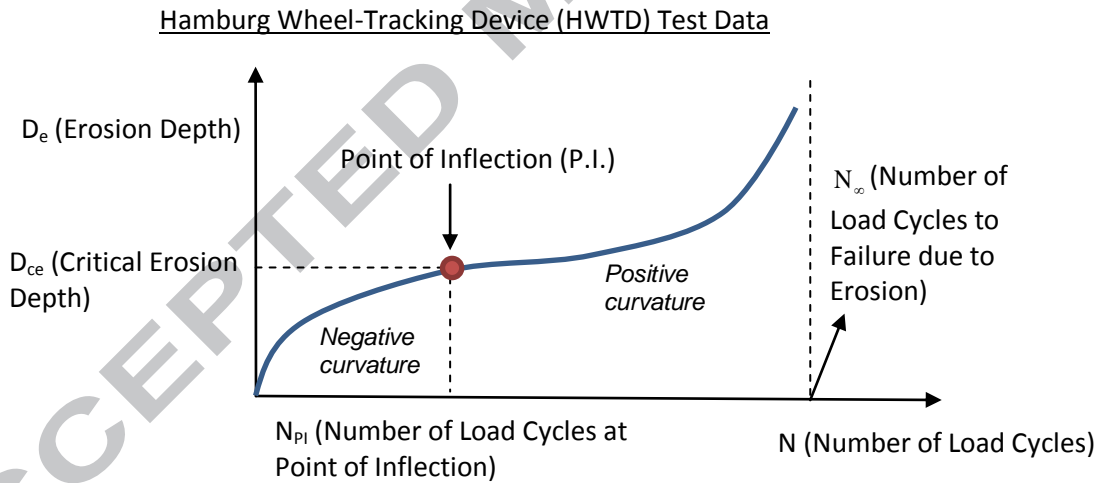


Figure 12. Illustration of the Concept of Critical Erosion Depth

Pavement ME Design currently takes an empirical approach to address the effect of erosion by classifying the base or subbase materials into five groups. By making use of the erosion data on erodible base course materials, and the equations presented above, it will be possible to identify the critical erosion depth and the number of load cycles to reach that critical depth. Therefore, the mechanistic-empirical erosion model proposed above is superior to the

empirical approach in Pavement ME Design. The mechanistic-empirical approach takes into account the major factors that affect erosion and quantifies erodibility of subbase/subgrade materials.

5.2. Possible implementation issues and actions

In order to implement the enhanced models mentioned above, there are some issues that all of these enhanced models will have in common:

- 1) New data elements that are needed for the enhanced model;
- 2) New test methods and equipment that are needed to provide inputs for the enhanced model;
- 3) The efforts in testing and data analysis that will be required by the enhanced model;
- 4) The potential that the properties required for the enhanced model can be catalogued;
- 5) The effort to implement the enhanced model into the Pavement ME Design software;
- 6) The time and costs associated with the implementation of the enhanced model;
- 7) The effort of calibrate and validate the enhanced model;
- 8) The expected realism of the predictions to be made with the enhanced model;
- 9) The relative priorities of implementing the completed enhanced models; and
- 10) Possible future desirable enhancements to the enhanced models.

As a continuation of this study, the implementation issues associated with each model that needs further development will be addressed along with possible actions that can be taken in response to these issues. In this study, the authors take the modulus model presented in Section 3 as an example to discuss some major implementation issues and possible actions.

To be compatible with the Pavement ME Design, the authors envision three levels of inputs for the moisture-sensitive, stress-dependent, and cross-anisotropic modulus model:

- Level 1: k_1 , k_2 , k_3 , s and r as measured from the repeated load resilient modulus tests;
- Level 2: k_1 , k_2 , k_3 , s and r as predicted by water content, dry density, plasticity index, and other simple material properties, or by methylene blue value, percent fines content, angularity index, shape index, and other performance-related properties;
- Level 3: default values for AASHTO classes of base course.

The test protocol with the analysis methods will be provided to obtain Level 1 inputs. For Level 2 inputs, depending on the available data collected from the existing database or literature,

Artificial Neural Network (ANN) models will be developed for k_1 , k_2 , k_3 , s and r , respectively using simple material properties or performance-based properties. These ANN prediction equations will be used to provide Level 2 inputs considering the high accuracy of the ANN algorithm. Based on our test results and collected data, default values of the modulus will be recommended for Level 3, the accuracy of which can be enhanced by the tabulation of typical values.

Furthermore, the authors will compose an external subroutine that is compatible with the Pavement ME Design software. The unbound base course subroutine will require inputs from the current Pavement ME Design of the traffic, pavement structure, variation of degree of saturation, and material properties. The new inputs that will be required by the base course subroutine include the suction versus water content coefficients and modulus model coefficients at different levels (Level 1, Level 2, and Level 3). The base course subroutine contains the stress-dependent, moisture-sensitive, cross-anisotropic constitutive equations for the resilient modulus. It will make use of the stress state produced by the Pavement ME Design software to calculate the stress-dependent and moisture-sensitive resilient modulus in the vertical direction and that in the horizontal direction. The golden mean of the vertical and horizontal moduli is computed and compared to the input modulus value of the Pavement ME design. An iteration process will be employed to make the input of the Pavement ME Design match the output modulus of the base course subroutine.

6. Conclusions and recommendations

This paper targets the problem that the current Pavement ME Design does not sufficiently reflect the influence of unbound layers and subgrade on the performance of flexible and rigid pavements. Through a wide literature review and on the basis of our previous investigations on this subject, the following findings are derived.

The prerequisite to understand why the pavement performance shows low/no sensitivity to these underlying layers is to find out the material properties/behaviors that play an important role. It is found that for flexible pavements the resilient modulus, shear strength, and permanent deformation are the key factors. For rigid pavements, the resilient modulus, shear strength, erosion, and permanent deformation of unbound layers or subgrade are critical factors. In particular, it lacks considerations including: a. moisture-dependency of the modulus, shear

strength, and permanent deformation; b. stress-dependency of the modulus and permanent deformation; and c. cross-anisotropy of the modulus.

Among numerous models that have been developed for unbound layers and subgrade, the selection of an appropriate one relies on three criteria: a. the degree of susceptibility, which indicates how the model responds to the changes in the operational conditions, including moisture, heat, traffic stress, and load-induced/particle-induced anisotropy; b. the degree of accuracy, which refers to how close the predictions made by an unbound layer/subgrade model are to the actual behaviors of these underlying materials; and c. the ease of development, which means the efforts required to develop, validate, and test the unbound layer/subgrade models.

The resilient modulus model discussed in this study reflects the intent of the authors to characterize the moisture-dependency, stress-dependency, and cross-anisotropy of the modulus of unbound base layers. The model is verified by laboratory tests and numerical simulations. By comparing with the Pavement ME Design models, the advantage of accuracy and moisture-sensitivity is obvious.

The permanent deformation model introduced in this study includes our consideration of the stress-dependency and moisture-sensitivity of permanent deformation of unbound base courses. Compared to the rutting models in the Pavement ME Design, the advantages of the proposed model are verified using laboratory tests and numerical simulations. Furthermore, the authors proposed to improve the models for the Pavement ME Design rutting model coefficients, and developed new prediction models for these coefficients. Increasing the accuracy of these coefficients also leads to an enhanced sensitivity of permanent deformation.

Due to the limit of the paper length, the authors selected these models as mentioned above, which act as examples to illustrate how to enhance the Pavement ME Design by including the key properties of unbound layers and subgrade. Planned as future work, the candidate models will be further developed and implemented, and more new models of subgrade and unbound layers will be presented. For instance, the hierarchical inputs at Level 1, Level 2, and Level 3 for each model should be provided. Another example of development is how these models associated with the properties will be calibrated and validated with the observed performance data on in-service pavements. Moreover, the implementation issues associated with each proposed model should be addressed along with possible actions that can be taken in response to these issues. These are the on-going and planned work, which will further contribute to the

improvement of pavement designs so as to better incorporate the influence of subgrade and unbound layers.

Acknowledgements

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References:

- Abramento, M., and S. Carvalho. 1989. "Geotechnical Parameters for Study of Slope Instabilization at Serra do Mar-Brazillan Southeast." In *Proceedings of the 12th International Conference on Soil Mechanics and Foundation Engineering*, Rio de Janeiro, Vol. 3, pp. 1599–1602.
- Adu-Osei, A., D.N. Little, and R.L. Lytton. 2001. "Cross-Anisotropic Characterization of Unbound Granular Materials." In *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 1757, No. 1, pp. 82–91.
- Al-Qadi, I.L., H. Wang, and E. Tutumluer. 2010. "Dynamic Analysis of Thin Asphalt Pavements by Using Cross-Anisotropic Stress-Dependent Properties for Granular Layer." In *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 2154, No. 1, pp. 156–163.
- American Association of State Highway and Transportation Officials (AASHTO). 1993. *Guide for Design of Pavement Structure, Volume I & II*. Highway Sub-committee on Design, Washington, DC.
- American Association of State Highway and Transportation Officials (AASHTO). 2008. *Mechanistic-Empirical Pavement Design Guide—A Manual of Practice*. Interim Edition, Washington, DC.
- ARA, Inc. 2004. "Guide for Mechanistic-Empirical Design of New and Rehabilitated Pavement Structures." *Final Report, NCHRP Project 1-37A*. Transportation Research Board, National Research Council, Washington, DC.
- Bakhsh, K.N. 2014. "Design Methodology for Subgrades and Bases under Concrete Roads and Parking Lots." PhD diss., Texas A&M University, College Station, TX.
- Bakhsh, K.N., and D. Zollinger. 2014a. "Faulting Prediction Model for Design of Concrete Pavement Structures." *Pavement Materials, Structures, and Performance*, pp. 327–342. doi: 10.1061/9780784413418.033
- Bakhsh, K. N., and D. Zollinger 2014b. "Test Methods & Results of Erosion Potential of Commonly Used Subgrade and Base Materials." Technical report to Ready Mixed Concrete Research and Education Foundation, Texas A&M Transportation Institute, College Station, TX.
- Butalia, T.S., J. Huang, D.G. Kim, and F. Croft. 2003. "Effect of Moisture Content and Pore Water Pressure Buildup on Resilient Modulus of Cohesive Soils in Ohio." *ASTM Special Technical Publication, No. 1437*, pp. 70–84.
- Byrum, C.R., and R.W. Perera. 2005. "The Effect of Faulting on IRI Values for Jointed Concrete Pavements." Paper presented at the Proc. 8th International Conference on Concrete Pavements, International Society for Concrete Pavements, Bridgeville, PA, and Purdue University, West Lafayette, IN.
- Cary, C.E., and C.E. Zapata. 2011. "Resilient Modulus for Unsaturated Unbound Materials." *Road Materials and Pavement Design*, Vol. 12, No. 3, pp. 615–638.

- Cerni, G., F. Cardone, A. Virgili, and S. Camilli. 2012. "Characterisation of Permanent Deformation Behaviour of Unbound Granular Materials under Repeated Triaxial Loading." *Construction and Building Materials*, Vol. 28, No. 1, pp. 79–87. doi: <http://dx.doi.org/10.1016/j.conbuildmat.2011.07.066>
- Carpenter, S.H., and R.L. Lytton. 1977. "Thermal Pavement Cracking in West Texas: Final Report." *Research Report 18-4F*. Texas Transportation Institute, Texas A&M University, College Station, TX.
- Chow, L.C., D. Mishra, and E. Tutumluer. 2014. "Framework for Development of an Improved Unbound Aggregate Base Rutting Model for Mechanistic–Empirical Pavement Design." In *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 2401, No. 1, pp. 11–21.
- Cleveland, G.S., J.W. Button, and R.L. Lytton. 2002. "Geosynthetics in Flexible and Rigid Pavement Overlay Systems to Reduce Reflection Cracking." *Research Report 1777-1*. Texas Transportation Institute, The Texas A&M University System, College Station, TX.
- Drumm, E.C. 1990. "Estimation of Subgrade Resilient Modulus from Standard Tests." *Journal of Geotechnical Engineering*, Vol. 116, No 5, pp. 774–789.
- Epps, J., S. Sebesta, B. Hewes, H. Sahin, R. Luo, J. Button, R.L. Lytton, C.A. Herrera, R. Hatcher, and F. Gu. 2014. "Development of a Specification for Flexible Base Construction." *Research Report No. FHWA/TX-13/0-6621*. Texas A&M Transportation Institute, College Station, TX.
- Filonenko-Borodich, M.M. 1940. "Some Approximate Theories of the Elastic Foundation." *Uchenye Zapiski Moskovskogo Gosudarstvennogo Universiteta. Mekhanika*, No. 46, pp. 3–18 (in Russian).
- Fredlund, D.G., and H. Rahardjo. 1993. *Soil Mechanics for Unsaturated Soils*. John Wiley & Sons, Inc., New York, NY.
- Fredlund, D.G., A. Xing, M.D. Fredlund, and S.L. Barbour. 1996. "The Relationship of the Unsaturated Soil Shear to the Soil-Water Characteristic Curve." *Canadian Geotechnical Journal*, Vol. 33, No. 3, pp. 440–448.
- W.R. Grace & Co. 2010 "Grace Rapid Clay Test Kit: Step-by-Step Procedure." Grace Custom Aggregate Solutions. Cambridge: W.R. Grace & Co.
- Gu, F., Y.Q. Zhang, C.V. Drodody, R. Luo, and R.L. Lytton. 2015a. "Development of a New Mechanistic-Empirical Rutting Model for Unbound Granular Material." *Journal of Materials in Civil Engineering*, Vol. 28, No. 8, 04016051-1 - 04016051-10.
- Gu, F., H. Sahin, X. Luo, R. Luo, and R. Lytton. 2015b. "Estimation of Resilient Modulus of Unbound Aggregates Using Performance-Related Base Course Properties." *Journal of Materials in Civil Engineering*, Vol. 27, No. (6), 04014188.
- Gu, F. 2015c. "Characterization and Performance Prediction of Unbound Granular Bases with and without Geogrids in Flexible Pavements." *Ph.D. Dissertation*, Texas A&M University, College Station, Texas.
- Gu, F., Luo, X., Zhang, Y., and Lytton, R. L. 2016a. "Modelling of Unsaturated Granular Materials in Flexible Pavements." *E3S Web of Conferences*, Vol. 9, 20002.

- Gu, F., Luo, X., Luo, R., Lytton, R. L., Hajj, E. Y., and Siddharthan, R. V. 2016b. "Numerical Modeling of Geogrid-Reinforced Flexible Pavement and Corresponding Validation Using Large-Scale Tank Test." *Construction and Building Materials*, Vol. 122, 214-230.
- Gu, F., Zhang, Y., Luo, X., Luo, R., and Lytton, R. L. 2016c. "Impact of Geogrid on Cross-Anisotropy and Permanent Deformation of Unbound Granular Materials." In *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 2580, No. 1, pp. 34-46.
- Gupta, S., A. Ranaivoson, T. Edil, C. Benson, and A. Sawangsuriya. 2007. "Pavement Design Using Unsaturated Soil Technology." Report No. MN/RC-2007-11. Final Research Report submitted to Minnesota Department of Transportation, University of Minnesota, Minneapolis, MN.
- Hansen, W., and E.A. Jensen. 2001. "Transverse Crack Propagation of JPCP as Related to PCC Toughness." *Research Report: RC-1404*. Michigan Department of Transportation, Lansing, MI.
- Hetenyi, M. 1950. "A General Solution for the Bending of Beams on an Elastic Foundation of Arbitrary Continuity." *Journal of Applied Physics*, Vol. 21, pp. 55-58.
- Hicks, R.G., and C.L. Monismith. 1971. "Factors Influencing the Resilient Properties of Granular Materials." In *Transportation Research Record*, Vol. 345, Transportation Research Board, National Research Council. Washington, DC, pp. 15-31.
- Huang, Y.H. 1993. *Pavement Analysis and Design*. Prentice Hall, Englewood Cliffs, NJ.
- Jeong, J.H., and D.G. Zollinger. 2001. "Characterization of Stiffness Parameters in Design of Continuously Reinforced and Jointed Pavements." In *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 1778, pp. 54-63.
- Jung, Y.S., D.G. Zollinger, M. Won, and A.J. Wimsatt. 2009. "Subbase and Subgrade Performance Investigation for Concrete Pavement." *Research Report 6037-1*. Texas Transportation Institute, Texas A&M University, College Station, TX.
- Jung, Y.S., D.G. Zollinger, B.H. Cho, M. Won, and A.J. Wimsatt. 2010a. "Subbase and Subgrade Performance Investigation and Design Guidelines for Concrete Pavement." *Research Report No. FHWA/TX-12/0-6037-2*. Texas Transportation Institute, The Texas A&M University System, College Station, TX.
- Jung, Y.S., D.G. Zollinger, and A.J. Wimsatt. 2010b. "Test Method and Model Development of Subbase Erosion for Concrete Pavement Design." In *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 2154, pp. 22-31.
- Jung, Y.S., and D.G. Zollinger. 2011. "New Laboratory-Based Mechanistic-Empirical Model for Faulting in Jointed Concrete Pavement." In *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 2226, No. 1, pp. 60-70.
- Jung, Y.S., D.G. Zollinger, and B.M. Ehsanul. 2012. "Improved Mechanistic-Empirical Continuously Reinforced Concrete Pavement Design Approach with Modified Punchout Model." In *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 2305, No. 1, pp. 32-42.

- Jung, Y.S., D.G. Zollinger, and B.M. Ehsanul. 2012. "Improved Mechanistic-Empirical Continuously Reinforced Concrete Pavement Design Approach with Modified Punchout Model." In *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 2305, No. 1, pp. 32–42.
- Kenis, W.J. 1977. "Predictive Design Procedures, VESYS User's Manual." *Final Report, No. FHWA-RD-77-154*. Federal Highway Administration, McLean, VA.
- Kerr, A. 1965. "A Study of a New Foundation Model." *Acta Mechanica*, Vol. 1, No. 2, pp. 135–147. doi: 10.1007/BF01174308
- Khoury, N.N., and M.M. Zaman. 2004. "Correlation between Resilient Modulus, Moisture Variation, and Soil Suction for Subgrade Soils." In *Transportation Research Record*, Vol. 1874, pp. 99–107. Transportation Research Board, Washington, DC.
- Khoury, C., G.A. Miller, and Y.N. Abousleiman. 2010. "Effect of Suction Hysteresis on Resilient Modulus of Fine-grained Cohesionless Soil." *8th Int. Conf. Bearing Capacity Roads, Railways, and Airfields*, Univ. of Illinois–UrbanaChampaign, Champaign, IL, pp. 71–78.
- Korkiala-Tanttu, L. 2009. "Verification of Rutting Calculation for Unbound Road Materials." In *Proceedings of the Institution of Civil Engineers-Transport*, Vol. 162, No. 2, pp. 107–114. doi: 10.1680/tran.2009.162.2.107
- Lade, P.V., and R.D. Nelson. 1987. "Modeling the Elastic Behavior of Granular Materials." *International Journal for Numerical and Analytical Methods in Geomechanics, Volume II*, pp. 521–542.
- Lambe, T.W., and R.V. Whitman. 1969. *Soil Mechanics*. Wiley, New York.
- Li, Q., H.J. Lee, and S.Y. Lee. 2011. "Permanent Deformation Model Based on Shear Properties of Asphalt Mixtures Development and Calibration." In *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 2210, pp. 81–89.
- Larralde, J. 1984. "Structural Analysis of Ridge Pavements with Pumping." PhD diss., Purdue University, West Lafayette, IN.
- Liang, R., S. Rabab'ah, and M. Khasawneh. 2008. "Predicting Moisture-Dependent Resilient Modulus of Cohesive Soils Using Soil Suction Concept." *Journal of Transportation Engineering*, Vol. 134, No. 1, pp. 34–40.
- Lytton, R.L., J. Uzan, E.G. Fernando, R. Roque, D. Hiltunen, and S.M. Stoffels. 1993. "Development and Validation of Performance Prediction Models and Specifications for Asphalt Binders and Paving Mixes." *Strategic Highway Research Program, SHRP A-357*. National Research Council, Washington, DC.
- Lytton, R.L. 1995. "Foundations and Pavements on Unsaturated Soils." Paper presented at the *Proceedings of the First International Conference on Unsaturated Soil*, Paris.
- Lytton, R.L., F.L. Tsai, S.I. Lee, R. Luo, S. Hu, and F. Zhou. 2010. "Models for Predicting Reflection Cracking of Hot Mix Asphalt Overlays." *NCHRP Report 669*. Transportation Research Board, Washington, DC.

- Markow, M.J., and B.D. Brademeyer. 1984. "EAROMAR Version 2." *Final Technical Report FHWA/RD-82/086*. Federal Highway Administration, U.S. Department of Transportation.
- Masad, S.A., and D.N. Little. 2004. "Sensitivity Analysis of Flexible Pavement Response and AASHTO 2002 Design Guide to Properties of Unbound Layers." Research Report ICAR 504-1. International Center for Aggregates Research, Austin, TX.
- Masad, E. A. 2005. "Aggregate Imaging System (AIMS): Basics and Applications." Final Rep. No. FHWA/TX-05/5-1707-01-1, Texas A&M Transportation Institute, College Station, TX.
- Masad, S., D. Little, and E. Masad. 2006. "Analysis of Flexible Pavement Response and Performance Using Isotropic and Anisotropic Material Properties." *Journal of Transportation Engineering-ASCE*, Vol. 132, No. 4, pp. 342–349.
- Öberg, A.L., and G. Sällfors. 1997. "Determination of Shear Strength Parameters of Unsaturated Silts and Sands based on the Water Retention Curve." *Geotechnical Testing Journal*, Vol. 20, No. 1, pp. 40–48.
- Oh, J.H., R.L. Lytton, and E.G. Fernando. 2006. "Modeling of Pavement Response Using Nonlinear Cross-Anisotropy Approach." *Journal of Transportation Engineering-ASCE*, Vol. 132, No. 6, pp. 458–468.
- Oh, J., E.G. Fernando, and R.L. Lytton. 2007. "Evaluation of Damage Potential for Pavements Due to Overweight Truck Traffic." *Journal of Transportation Engineering-ASCE*, Vol. 133, No. 5, pp. 308–317.
- Oh, J., and E.G. Fernando. 2011. "Comparison of Resilient Modulus Values Used in Pavement Design." *Report BDL76-1*. Texas Transportation Institute, The Texas A&M University System, College Station, TX.
- Oloo, S.Y. 1994. "A bearing capacity approach to the design of low-volume traffic roads." Ph.D. dissertation, University of Saskatchewan, Saskatoon, Canada.
- Oloo, S.Y., and D.G. Fredlund. 1998. "The Application of Unsaturated Soil Mechanics Theory to the Design of Pavements." In *Proc. 5th Intl. Conf. on the Bearing Capacity of Roads and Airfields*, Trondheim, Norway, pp. 1419–1428.
- Pasternak, P.L. 1954. "On a New Method of Analysis of an Elastic Foundation by Means of Two Foundation Constants." *Gosudarstvennoe Izdatel'stro Literaturi po Stroitel'stru i Arkhitekture*, Moscow, USSR (in Russian).
- PCA. 2008. "Pavement—Soil Cement." Portland Cement Association, Skokie, IL. http://www.cement.org/pavements/pv_sc.asp. Accessed: October 1, 2008.
- Rao, C., and M.I. Darter. 2013. "Enhancements to Punchout Prediction Model in Mechanistic-Empirical Pavement Design Guide Procedure." In *Transportation Research Record*, Vol. 2367, pp. 132–141.
- Rassam, D.W., and F.J. Cook. 2002. "Predicting the Shear Strength Envelope of Unsaturated Soils." *Geotechnical Testing Journal*, Vol. 28, pp. 215–220.

- Rauhut, J.B., R.L. Lytton, and M.I. Darter. 1982. "Pavement Damage Functions for Cost Allocation, Vol. 1, Damage Functions and Load Equivalency Factors." *Report No. FHWA/RD-82/126*. Federal Highway Administration, Washington, DC.
- Ren, D.Y., L. Houben, and L. Rens. 2013. "Cracking Behavior of Continuously Reinforced Concrete Pavements in Belgium Characterization of Current Design Concept." In *Transportation Research Record*, Vol. 2367, pp. 97–106.
- Sahin, H. 2011. "Characterization of Expansive Soil for Retaining Wall Design." M.S. thesis, Texas A&M Univ., College Station, TX.
- Sahin, H., F. Gu, Y. Tong, R. Luo, and R.L. Lytton. 2013. "Unsaturated Soil Mechanics in the Design and Performance of Pavements." *Advances in Unsaturated Soils*, pp. 87–99.
- Sawangsurinya, A., T.B. Edil, and C.H. Benson. 2009. "Effect of Suction on Resilient Modulus of Compacted Fine-Grained Subgrade Soils." In *Transportation Research Record*, Vol. 2101, pp. 82–87. doi: Doi 10.3141/2101-10
- Schwartz, C.W., Li, R., Kim, S., Ceylan, H., and Gopalakrishnan, K. (2011). "Sensitivity Evaluation of MEPDG Performance Prediction." *Final Report of NCHRP 1-47 project*, University of Maryland and Iowa State University.
- Schwartz, C.W., R. Li, H. Ceylan, S. Kim, and K. Gopalakrishnan. 2013. "Global Sensitivity Analysis of Mechanistic–Empirical Performance Predictions for Flexible Pavements." In *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 2368, No. 1, pp. 12–23.
- Seed, H.B., F.G. Mitry, C.L. Monismith, and C.K. Chan. 1967. "Prediction of Flexible Pavement Deflections from Laboratory Repeated-Load Tests." *NCHRP Report No. 35*. National Cooperative Highway Research Program.
- Shahji, S. 2006. "Sensitivity Analysis of AASHTO's 2002 Flexible and Rigid Pavement Design Methods." Master of Sc. in the Dept. of Civil & Environmental Engineering—College of Engineering and Computer Science—University of Central Florida. Orlando, FL.
- Thompson, M.R., and R.L. Robnett. 1979. "Resilient Properties of Subgrade Soils." *Transportation Engineering Journal, ASCE*, Vol. 105, No. TE1, pp. 71–89.
- Titus-Glover, L., and E.G. Fernando. 1995. "Evaluation of Pavement Base and Subgrade Material Properties and Test Procedures." *Research Report 1335-2*. Texas Transportation Institute, The Texas A&M University System, College Station, TX.
- Tseng, K.H., and R.L. Lytton. 1989. "Prediction of Permanent Deformation in Flexible Pavement Materials." In *Implication of Aggregates in the Design, Construction, and Performance of Flexible Pavement, ASTHMA STP 1016, ASTM, Philadelphia, PA.*, pp. 154–172.
- Tutumluer, E., and M.R. Thompson. 1997. "Anisotropic Modelling of Granular Bases in Flexible Pavements." *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 1577, Transportation Research Board. National Research Council. Washington, DC.
- TxDOT. 2010. "Test Procedure for Triaxial Compression for Disturbed Soils and Base Materials." *TEX-117-E*, Texas Department of Transportation, Austin, TX.

- Uzan, J. 1985. "Characterization of Granular Material." In *Transportation Research Record*, Vol. 1022, pp. 52–59. TRB. National Research Council, Washington, DC.
- Uzan, J. 2004. "Permanent Deformation in Flexible Pavements." *Journal of Transportation Engineering*, Vol. 130, No. 1, pp. 6–13.
- Vandenbossche, J.M., S. Nassiri, L.C. Ramirez, and J.A. Sherwood. 2012. "Evaluating the Continuously Reinforced Concrete Pavement Performance Models of the Mechanistic-Empirical Pavement Design Guide." *Road Materials and Pavement Design*, Vol. 13, No. 2, pp. 235–248.
- Vanapalli, S.K., and Zhong Han. 2013. "Prediction of the Resilient Modulus of Unsaturated Fine Grained Soils." Paper presented at the *Proceedings of International Conference on Advances in Civil Engineering*, AETACE.
- Vaillancourt, M., L. Houy, D. Perraton, and D. Breysse. 2014. "Variability of Subgrade Soil Rigidity and its Effects on the Roughness of Flexible Pavements: A Probabilistic Approach." *Materials and Structures*, pp. 1–10.
- Van Wijk, A.J. 1985. "Rigid Pavement Pumping: (1) Subbase Erosion and (2) Economic Modeling." *Joint Highway Research Project File 5-10*. School of Civil Engineering, Purdue University, West Lafayette, IN.
- Velasquez, R., K. Hoegh, I. Yut, N. Funk, G. Cochran, M. Marasteanu, and L. Khazanovich. 2009. "Implementation of the MEPDG for New and Rehabilitated Pavement Structures for Design of Concrete and Asphalt Pavements in Minnesota." *MnDOT Report No. MN/RC 2009-06*. University of Minnesota—Twin Cities, St. Paul.
- Winkler, E. 1867. *Die Lehre von der Elasticitaet und Festigkeit*. Prag, Dominicus.
- Witczak, M.W. and J. Uzan. 1988. "The Universtiy Airport Pavement Design System Report I of V: Granular Material Characterization." Department of Civil Engineering, University of Maryland, Silver Spring, MD.
- Witczak, M.W., D. Andrei, and W.N. Houston. 2000. "Resilient Modulus as Function of Soil Moisture—Summary of Predictive Models." Development of the 2002 Guide for the Development of New and Rehabilitated Pavement Structures. NCHRP 1-37A, Inter Team Technical Report (Seasonal 1), Arizona State University, Tempe, AZ.
- Witczak, M.W. 2003. "Harmonized Test Methods for Laboratory Determination of Resilient Modulus for Flexible Pavement Design." *NCHRP 1-28A*, Final Report, Arizona State University, Tempe, AZ.
- Wolfe, W., and T. Butalia. 2004. "Continued Monitoring of SHRP Pavement Instrumentation Including Soil Suction and Relationship with Resilient Modulus." *Report No. FHWA/OH-2004/007*. U.S. Department of Transportation, Federal Highway Administration, Washington, DC.
- Yang, S.-R., W.-H. Huang, and Y.-T. Tai. 2005. "Variation of Resilient Modulus with Soil Suction for Compacted Subgrade Soils." In *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 1913, No. 1, pp. 99–106.

- Yau, A., and H.L. Von Quintus. 2002. "Study of LTPP Laboratory Resilient Modulus Test Data and Response Characteristics." *Report No. FHWA-RD-02-051*. Federal Highway Administration Research, U.S. Department of Transportation.
- Yu, S.Y., and P. Dakoulas. 1993. "General Stress-Dependent Elastic-Moduli for Cross-Anisotropic Soils." *Journal of Geotechnical Engineering-ASCE*, Vol. 119, No. 10, pp. 1568–1586. doi: 10.1061/(ASCE)0733-9410(1993)119:10(1568)
- Zhou, F., E.G. Fernando, and T. Scullion. 2007. "A Review of Performance Models and Test Procedures with Recommendations for Use in the Texas M-E Design Program." *Research Report No. FHWA/TX-08/0-5798-1*. Texas Transportation Institute, The Texas A&M University System.