

# Simple geometric interpretation of signal evolution in phase-sensitive fibre optic parametric amplifier

A.A. REDYUK,<sup>1,2,\*</sup> A.E. BEDNYAKOVA,<sup>1,2</sup> S.B. MEDVEDEV,<sup>2</sup>  
M.P. FEDORUK,<sup>1,2</sup> AND S.K. TURITSYN<sup>1,3</sup>

<sup>1</sup>*Novosibirsk State University, Novosibirsk, 2 Pirogova street, 630090, Russia*

<sup>2</sup>*Institute of Computational Technologies SB RAS, Novosibirsk, 6 Acad. Lavrentiev avenue, 630090, Russia*

<sup>3</sup>*Aston Institute of Photonic Technologies, Aston University, Aston Triangle, Birmingham B4 7ET, UK*

\*[alexey.redyuk@gmail.com](mailto:alexey.redyuk@gmail.com)

**Abstract:** Visualisation of complex nonlinear equation solutions is a useful analysis tool for various scientific and engineering applications. We have re-examined geometrical interpretation of the classical nonlinear four-wave mixing equations for the specific scheme of a phase sensitive one-pump fiber optical parametric amplification, which has recently attracted revived interest in the optical communications due to potential low noise properties of such amplifiers. Analysis of the phase portraits of the corresponding dynamical systems provide valuable additional insight into field dynamics and properties of the amplifiers. Simple geometric approach has been proposed to describe evolution of the waves, involved in phase-sensitive fiber optical parametric amplification (PS-FOPA) process, using a Hamiltonian structure of the governing equations. We have demonstrated how the proposed approach can be applied to the optimization problems arising in the design of the specific PS-FOPA scheme. The method considered here is rather general and can be used in various applications.

© 2016 Optical Society of America

**OCIS codes:** (190.4380) Nonlinear optics, four-wave mixing; (190.4410) Nonlinear optics, parametric processes; (060.2320) Fiber optics amplifiers and oscillators.

## References and links

1. M. E. Marhic, *Fiber Optical Parametric Amplifiers, Oscillators and Related Devices* (Cambridge University, 2008).
2. R. H. Stolen, "Phase-matched-stimulated four-photon mixing in silica-fiber waveguides," *IEEE J. Quantum Electron.* **11**, 100–103 (1975).
3. J. Hansryd, P. A. Andrekson, M. Westlund, J. Li, & P. O. Hedekvist, "Fiber-based optical parametric amplifiers and their applications," *IEEE J. Sel. Topics Quantum Electron.* **8**, 506–520 (2002).
4. S. Radic & C. J. McKinstrie, "Two-pump fiber parametric amplifiers," *Opt. Fiber Technol.* **9**, 7–23 (2003).
5. C. McKinstrie & S. Radic, "Phase-sensitive amplification in a fiber," *Opt. Express* **12**, 4973–4979 (2004).
6. C. J. McKinstrie, S. Radic, & A. H. Gnauck, "All-optical signal processing by fiber-based parametric devices," *Opt. Photonics News* **18**, 34–40 (2007).
7. M. E. Marhic, P. A. Andrekson, P. Petropoulos, S. Radic, C. Peucheret, & M. Jazayerifar, "Fiber optical parametric amplifiers in optical communication systems," *Laser Photonics Rev.* **9**, 50–74 (2015).
8. Z. Tong, C. Lundström, P. A. Andrekson, C. J. McKinstrie, M. Karlsson, D. J. Blessing, E. Tipsuwannakul, B. J. Puttnam, H. Toda, & L. Grüner-Nielsen, "Towards ultrasensitive optical links enabled by low-noise phase-sensitive amplifiers," *Nat. Photonics*, **5**, 430–436 (2011).
9. R. Slavik, F. Parmigiani, J. Kakande, C. Lundström, M. Sjödin, P. A. Andrekson, R. Weerasuriya, S. Sygletos, A. D. Ellis, L. Grüner-Nielsen, D. Jakobsen, S. Herstrom, R. Phelan, J. O’Gorman, A. Bogris, D. Syvridis, S. Dasgupta, P. Petropoulos, & D. J. Richardson, "All-optical phase and amplitude regenerator for next-generation telecommunications systems," *Nat. Photonics* **4**, 690–695 (2010).
10. A. A. Redyuk, M. F. C. Stephens, & N. J. Doran, "Suppression of WDM four-wave mixing crosstalk in fibre optic parametric amplifier using Raman-assisted pumping," *Opt. Express* **23**(21), 27240–27249 (2015).
11. M. F. C. Stephens, I. D. Phillips, P. Rosa, P. Harper, & N. J. Doran, "Improved WDM performance of a fibre optical parametric amplifier using Raman-assisted pumping," *Opt. Express* **23**(2), 902–911 (2015).
12. A. Bendahmane, A. Mussot, A. Kudlinski, P. Szriftgiser, M. Conforti, S. Wabnitz, & S. Trillo, "Optimal frequency conversion in the nonlinear stage of modulation instability," *Opt. Express* **23**, 30861–30871 (2015).
13. J. Kakande, R. Slavik, F. Parmigiani, A. Bogris, D. Syvridis, L. Gruner-Nielsen, R. Phelan, P. Petropoulos, & D. J. Richardson, "Multilevel quantization of optical phase in a novel coherent parametric mixer architecture," *Nat. Photonics* **5**, 748–752 (2011).

14. A. Perentos, S. Fabbri, M. Sorokina, I. D. Phillips, S. K. Turitsyn, A. D. Ellis, & S. Sygletos, "QPSK 3R regenerator using a phase sensitive amplifier," *Opt. Express* **24**, 16649–16658 (2016).
  15. S. Watanabe, F. Futami, R. Okabe, R. Ludwig, C. Schmidt-Langhorst, B. Huettl, C. Schubert, & H. Weber, "An Optical Parametric Amplified Fiber Switch for Optical Signal Processing and Regeneration," *IEEE J. Sel. Topics Quantum Electron.* **14**, 674–680 (2008).
  16. H. Steffensen, J. R. Ott, K. Rottwitt, & C. J. McKinstrie, "Full and semi-analytic analyses of two-pump parametric amplification with pump depletion," *Opt. Express* **19**(7), 6648–6656 (2011).
  17. J. R. Ott, H. Steffensen, K. Rottwitt, & C. J. McKinstrie, "Geometric interpretation of four-wave mixing," *Phys. Rev. A* **88** 043805 (2013).
  18. G. Cappellini & S. Trillo, "Third-order three-wave mixing in single-mode fibers: exact solutions and spatial instability effects," *J. Opt. Soc. Am. B* **8**, 824–838 (1991).
- 

## 1. Introduction

Optical parametric amplification (OPA) in fibers [1, 2] is a classical nonlinear phenomenon that manifests itself as a process of nonlinear energy transfer from powerful pumping wave(s) (considered as energy sources) to signal wave(s), leading to effective amplification of the latter. OPA is a particular case of a fundamental nonlinear wave mixing effect when waves of different frequencies are agitated through optical nonlinearity to generate new frequencies. In optical fiber, that is an example of media without inversion symmetry, the leading third-order nonlinearity (Kerr nonlinearity) involves interaction of four photons. Therefore, four-wave-mixing (FWM) is the general term covering various physical effects resulting from the Kerr nonlinearity.

Recently, fiber OPA has attracted a great deal of renewed interest as a promising technology for low-noise parametric amplification in high-capacity optical communications [3–12]. This recent revival of rather classical field is strongly supported by advances in the technology of highly-nonlinear fibers (HNLF) with a good figure of merit (high nonlinear coefficient with a reasonably low losses). One of the key reasons for renewed interest in fiber OPA is a potential possibility to reach a 0 dB noise figure for the in-phase components (phase sensitive amplification). Despite numerous technical challenges this makes phase-sensitive parametric amplifiers an interesting and timely research direction. Low noise in-line amplifiers would offer significant link performance improvement. Moreover, such phase-sensitive amplifiers may open up new opportunities for all-optical regeneration of coherent optical signal [9, 13–15].

In this work we will apply geometrical approach developed in [16–18] for optimisation of specific amplification schemes based on phase sensitive fiber OPA. Geometric presentation of a phase dynamics and visualisation of families of solutions is often more useful than formal exact analytical solutions. We will study energy transfer between pump and signal using intuitive phase portraits (geometric interpretation) and find optimum parameters of phase-sensitive fibre optic parametric amplifiers (PS-FOPA) leading to the maximum gain.

## 2. Theory

### 2.1. Basic equations and Hamiltonian formulation

General FWM process involves interaction of four waves at different frequencies. When applied to fiber OPA this would generally mean the use of two pump waves at different frequencies and a signal wave. The interaction of signal with two pumping waves create the fourth wave, called the idler. However, in fiber OPA a partially degenerate case (in which the frequencies of two waves/photons are equal) is also important for practical applications. In this case one pumping waves creates a pair of Stokes (lower frequency) and anti-Stokes (higher frequency) waves, with frequencies symmetrically located with respect to the frequency of the pump.

In case of one-pump optical parametric amplification degenerate three-wave mixing process between three continuous waves is described by the well-known three coupled differential equations for the powers  $P_i$  ( $i = 1, 2, 3$ ) and one for the relative phase of the interacting waves

$$\theta = \Delta\beta z + \theta_2 + \theta_3 - 2\theta_1:$$

$$\frac{dP_1}{dZ} = -2\frac{dP_2}{dZ} = -2\frac{dP_3}{dZ} = -4P_1\sqrt{P_2P_3}\sin\theta, \quad (1)$$

$$\frac{d\theta}{dZ} = \frac{\Delta\beta}{\gamma} + (2P_1 - P_2 - P_3) + P_1\sqrt{P_2P_3}\left(P_2^{-1} + P_3^{-1} - 4P_1^{-1}\right)\cos\theta, \quad (2)$$

where  $P_1$  denote pump power,  $P_2$  and  $P_3$  are signal and idler powers,  $\theta_i$  is the phase of  $i$ th wave,  $\Delta\beta = \beta_2 + \beta_3 - 2\beta_1$  is the propagation constant mismatch,  $\gamma$  is the coefficient of nonlinearity,  $Z = \gamma z$  is the normalised distance.

The system of Eqs. (1)-(2) admits the following two invariants, preserving at any propagation distance along the fiber:

$$\delta_p = P_2 - P_3, \quad (3)$$

$$P_T = P_1 + P_2 + P_3. \quad (4)$$

We consider a one-pump phase sensitive fibre optical parametric amplification, where signal and idler wavelengths are symmetrically located around the pump wavelength. Assuming signal and idler input powers to be equal in PS-FOPA,  $P_2(0) = P_3(0)$ , the initial problem (1)-(2) can be simplified and reduced to:

$$\frac{dP_1}{dZ} = -2\frac{dP_2}{dZ} = -4P_1P_2\sin\theta, \quad (5)$$

$$\begin{aligned} \frac{d\theta}{dZ} &= \frac{\Delta\beta}{\gamma} + 2(P_1 - P_2) + P_1P_2\left(2P_2^{-1} - 4P_1^{-1}\right)\cos\theta = \\ &= \frac{\Delta\beta}{\gamma} + 2(P_1 - P_2) + (2P_1 - 4P_2)\cos\theta. \end{aligned} \quad (6)$$

The system of Eqs. (5)-(6) is a Hamiltonian system that can be presented as follows:

$$\frac{dP_1}{dZ} = \frac{\partial H}{\partial\theta} = -2P_1(P_T - P_1)\sin\theta, \quad (7)$$

$$\frac{d\theta}{dZ} = -\frac{\partial H}{\partial P_1} = 2(2P_1 - P_T)\cos\theta + 3P_1 - P_T + \frac{\Delta\beta}{\gamma}. \quad (8)$$

Here the Hamiltonian  $H$  reads:

$$H = 2P_1(P_T - P_1)\cos\theta - \frac{3}{2}P_1^2 + \left(P_T - \frac{\Delta\beta}{\gamma}\right)P_1. \quad (9)$$

After introducing new variables (following the procedure described in [18])  $p = P_1/P_T$ ,  $l = Z P_T$  and  $k = \frac{\Delta\beta}{\gamma P_T}$  the Hamiltonian (9) and Eqs. (7)-(8) can be presented in the normalised form:

$$H = 2p(1-p)\cos\theta - \frac{3}{2}p^2 + (1-k)p, \quad (10)$$

$$\frac{dp}{dl} = \frac{\partial H}{\partial\theta} = -2p(1-p)\sin\theta, \quad (11)$$

$$\frac{d\theta}{dl} = -\frac{\partial H}{\partial p} = (k-1) + 3p - 2(1-2p)\cos\theta, \quad (12)$$

where  $p$  is a contribution of the pump power to the total power lying in the range between 0 and 1,  $k$  can be interpreted as the normalised phase mismatch.

## 2.2. Stationary points of the dynamical system

First, let us find the stationary points  $(p, \theta)$  of the dynamical system (11)-(12) that describe steady state co-propagation of three nonlinearly interacting waves in such systems. This is not the focus of this paper, however, potentially, such steady state describing joint propagation of high power nonlinear waves at different wavelengths can be interesting for various applications.

There are four possibilities when the right-hand side of the Eq. (11) becomes zero:

$$p_A = 0, \quad p_B = 1, \quad \theta_C = 2\pi n, \quad \theta_D = 2\pi n + \pi, \quad n \in \mathbb{Z}.$$

Substituting these values in the right-hand side of the Eq. (12), we find equations governing the existence of stationary points:

$$\cos \theta_A = \frac{k-1}{2}, \quad \cos \theta_B = -\frac{k+2}{2}, \quad p_C = \frac{3-k}{7}, \quad p_D = k+1.$$

Now we can determine mathematical conditions for the stationary points existence.

For inner points  $C$  and  $D$ :

point  $C = (\frac{1}{7}(3-k), 2\pi n)$  lies inside the band  $p \in [0, 1]$  when  $k \in [-4, 3]$ ;

point  $D = (k+1, 2\pi n + \pi)$  lies inside the band  $p \in [0, 1]$  when  $k \in [-1, 0]$ .

For boundary points  $A$  and  $B$ :

point  $A = (0, \theta_A)$  exists when  $k \in [-1, 3]$ ;

point  $B = (1, \theta_B)$  exists when  $k \in [-4, 0]$ .

It should be mentioned that although the boundary stationary points are not involved in the dynamic process, they define the structure of the phase portrait in their vicinity.

## 2.3. Phase portraits

A phase portrait, corresponding to the Hamiltonian (10), depends on a set of stationary points. It turns out that we can obtain important information concerning dynamics of the nonlinear system analysing behaviour near critical points. The rest of the phase portrait can typically be deduced from this information. As it was shown in the section 2.2, there are three different scenarios of the system dynamics: when  $k \in [0, 3]$  only two stationary points  $A$  and  $C$  exist, when  $k \in [-1, 0]$  all points  $A, B, C$  and  $D$  exist, and finally, when  $k \in [-4, -1]$  two points  $B$  and  $C$  exist. Figure 1 demonstrates different types of phase portraits for  $k = 2$ ,  $k = -0.5$  and  $k = -2$ .

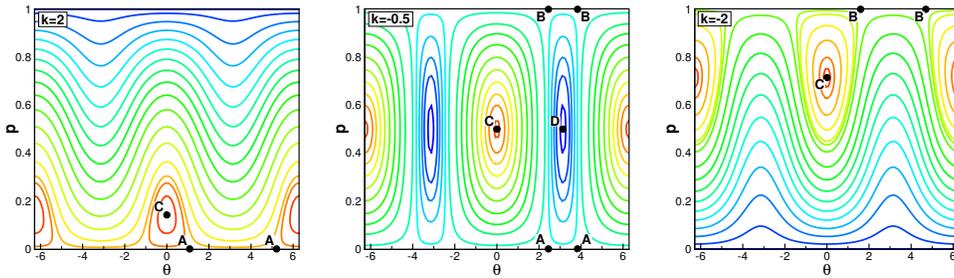


Fig. 1. Phase portraits:  $k = 2$  (left),  $k = -0.5$  (middle),  $k = -2$  (right).

The existence of the a Hamiltonian for differential Eqs. (11)-(12) can be used to reduce the problem to only one variable. Consider the Hamiltonian (10) and Eq. (11):

$$H = 2p(1-p)\cos\theta - \frac{3}{2}p^2 + (1-k)p, \quad (13)$$

$$\frac{dp}{dl} = -2p(1-p)\sin\theta. \quad (14)$$

The relative phase  $\theta$  is specifically important as it shows the direction of the power flow. Mathematical solutions governing signal amplification (and corresponding pump depletion) correspond to the regime of PS-FOPA operation that is of major interest for applications in optical communication systems. When the variable  $\theta$  takes values between 0 and  $\pi$  ( $\sin\theta$  is positive), the power is transferred from the pump to the signal and idler, whereas when  $\theta$  is between  $\pi$  and  $2\pi$  ( $\sin\theta$  is negative), the energy is spreading in the opposite direction. At some point the relative phase can change so much that the power flow is reversed and the power begins to flow back to the pump. Such behaviour is typical for nonlinear dynamical regimes described by the periodic functions. In physical terms, the system operates as a nonlinear coupler between waves propagating at different wavelengths. The phase  $\theta$  can be found from the Hamiltonian as

$$\cos\theta = \frac{H + \frac{3}{2}p^2 - (1-k)p}{2p(1-p)}. \quad (15)$$

Let us denote using (15):

$$\alpha(H, p) = \sin\theta = \pm\sqrt{1 - \cos^2\theta}. \quad (16)$$

Thus, inserting the equation for  $\theta$  in (14), we obtain the differential equation with separable variables

$$\frac{dp}{-2p(1-p)\alpha(H, p)} = dl, \quad (17)$$

where the Hamiltonian  $H$  play a role of a parameter. Due to the fact that the Hamiltonian is independent of  $l$ , its value defines the particular trajectory of the phase portrait.

The equation (17) can be rewritten in the integral form as

$$\int_{p(0)}^{p(L)} \frac{dp}{-2p(1-p)\alpha(H, p)} = \int_0^L dl \equiv L. \quad (18)$$

Explicit solution  $p(l)$  of Eq. (18) was found in [18] in terms of Jacobian elliptic functions. As we are going to examine typical optimization problems for PS-FOPA, the use of Eq. (18) is more convenient for our further analysis than its analytical solution presented in a formal rather complicated mathematical form. Equation (18) connects three key important parameters: input power  $p(0)$ , output power  $p(L)$  and length  $L$ , and implicitly, through a Hamiltonian, describes impact of the input phase  $\theta(0)$ . It is also convenient that Eq. (18) can be easily integrated numerically through e.g. trapezoidal method. In the next section, we apply this approach to several optimization problems.

### 3. Optimization of the practical PS-FOPA

Now let us consider practical implementation of idler and pump power evolution in PS-FOPA. We study waves evolution in a typical highly nonlinear fiber with the following parameters: zero dispersion wavelength  $\lambda_{zdwf} = 1564.3$  nm, dispersion slope  $S = 0.083$  ps nm<sup>-2</sup>km<sup>-1</sup> and nonlinear coefficient  $\gamma = 8.7$  (W km)<sup>-1</sup>. For the fixed input pump power  $P_1 = 3$  W and signal (idler) power  $P_2 = 0.01$  mW, signal wavelength  $\lambda_s = 1550$  nm and pump wavelength  $\lambda_p = 1565.5$  nm the value of parameter  $k$  can be calculated directly  $k = -0.7$ .

Figure 2(a) shows the phase portrait for the case  $k = -0.7$ , corresponding to the system evolution with parameters described above. It is seen that there are three types of trajectories, which we call type "1", "2" and "3". Trajectories of type "1" and "2" are closed trajectories around stationary points  $C$  and  $D$ , respectively. Characteristic evolution of the pump power, signal (idler)

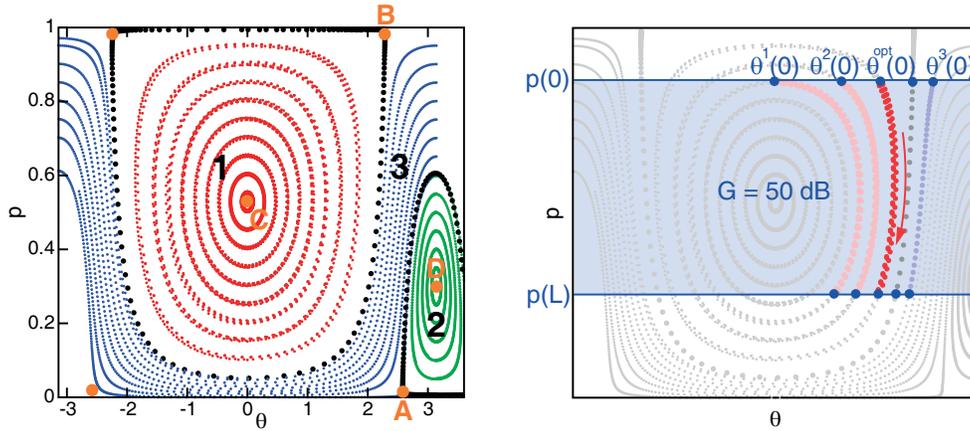


Fig. 2. (a) Phase portrait for  $k = -0.7$ . (b) Schematic depiction of the power transfer process in PS-FOPA with a fixed gain.

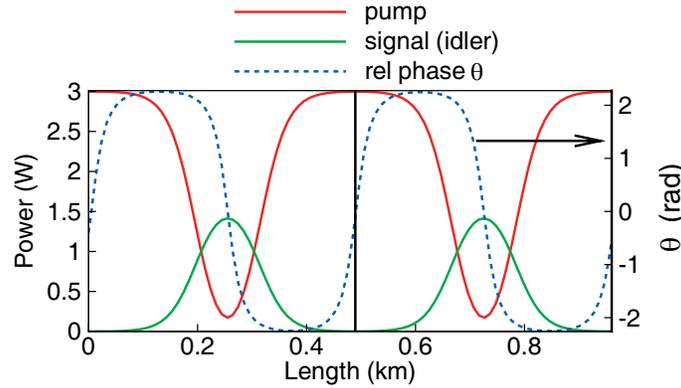


Fig. 3. Evolution of the pump power (solid red), signal (idler) power (solid green) and relative phase  $\theta$  (dashed blue) during two full periods of the 1st type trajectory.

power and relative phase  $\theta$  during two full periods of the 1st type trajectory is depicted in Fig. 3. For trajectories of type "3" phase  $\theta$  changes in a whole range of values  $[0, 2\pi]$ .

Separatrices confining trajectories of types "1", "3" and "2", "3" are depicted by black dots in Fig. 2(a). At first glance, one can think that the separatrix enclosing stationary point C, which gives the maximum signal amplification  $\Delta p = p(0) - p(L)$ , would define an optimal trajectory of signal amplification in the phase plane. In the next section we consider this regime in more details and will see that this is not necessarily truth.

### 3.1. Properties of separatrices

To analyse separatrices, first, let us make a simple transformation by adding a constant  $k + 1/2$  to the Hamiltonian (10) leading to its factorization:

$$\tilde{H}(p, \theta) = H(p, \theta) + k + 1/2 = \frac{1}{2}(1 - p)((4 \cos \theta + 3)p + 2k + 1). \quad (19)$$

Equation  $\tilde{H}(p, \theta) = 0$  defines the separatrix enclosing stationary point C. It consists of two curves: the horizontal line  $p = 1$  between the two stationary points of type B and the curve given by

equation:

$$p = -\frac{2k + 1}{4 \cos \theta + 3}. \quad (20)$$

Substitution of the expression (20) in the right-hand side of (12) yields the following ordinary differential equation:

$$\frac{d\theta}{dl} + 2 \cos \theta + 2 + k = 0, \quad (21)$$

or in the equivalent form:

$$\frac{d\theta}{-(2 \cos \theta + 2 + k)} = dl. \quad (22)$$

Equation (22) has a singularity at  $2 \cos \theta + 2 + k = 0$  (coordinate of the stationary point B), which means that infinite distance (here fiber length) is required to approach the stationary point B. It can be shown by analogy, that power flow propagation along the second separatrix enclosing stationary point D also requires an infinite fiber span length. Though, formally, a separatrix corresponds to the maximum amplification  $\Delta p = 1 - p_m$ , where  $p_m = -(2k + 1)/7$  (see eq.20), this amplification level is achieved for propagation over infinite fiber span. Therefore, in practical terms, optimisation of the amplification parameters such as gain, should be done at the finite propagation distance  $L$ . There are other trajectories on the phase plane that give required high amplification over finite fiber spans. This qualitative observation is supported by the direct numerical solution of the amplifier optimization problem presented in the next section.

### 3.2. Optimization of the PS-FOPA using geometrical approach

The first important practical design problem, that can be easily solved using Eq. (18), is a determination of HNLF length for a given input signal power, pump power and required gain. Assuming, for instance, that we are interested in 50 dB signal amplification in the shortest possible PS-FOPA. Note, that the maximum attainable gain (when all pump power is converted to the signal and idler) is limited by the power conservation law up to  $G_{max} = P_1/2P_2 + 1 \approx 51$  dB. To solve the optimization problem we should find all the trajectories on the phase plane, yielding 50 dB signal gain and then choose the trajectory, corresponding to the shortest possible length of the amplifier providing for such target gain.

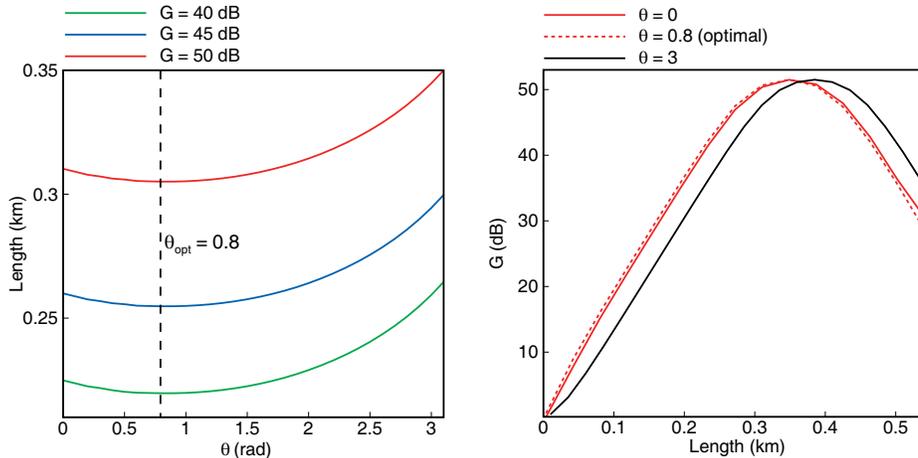


Fig. 4. (a) Dependence of PS-FOPA length on initial relative phase  $\theta(0)$  for  $G = 40$  (green), 45 (blue) and 50 (red) dB. (b) Dependence of  $G$  on PS-FOPA length for  $\theta(0) = 0$  (solid red), 0.8 (dashed red) and 3 (solid black) rad.  $P_1 = 3$  W,  $P_2 = P_3 = 0.01$  mW.

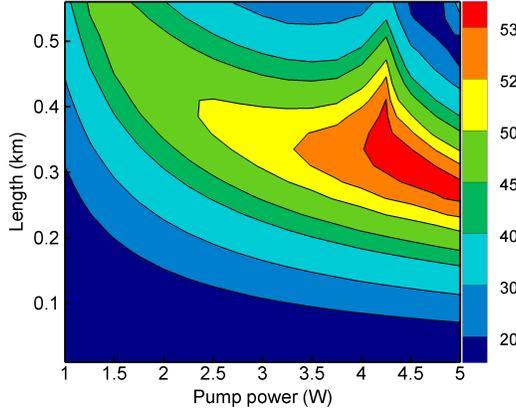


Fig. 5. Dependence of  $G$  on PS-FOPA length and pump power.  $\theta(0) = 0$ ,  $P_2 = P_3 = 0.01$  mW.

Schematic depiction of the power transfer process in PS-FOPA with the desired gain is presented in Fig. 2(b). The coordinates  $p(0) = P_1(0)/P_T$  and  $p(L) = P_1(L)/P_T$  are known and determined by the initial powers of the pump and signal (idler) and target amplification level  $G = \frac{1-p(L)}{1-p(0)}$ . The points of intersection between the trajectories and horizontal lines  $p = p(0)$  and  $p = p(L)$  determine the start and end points of the power flow. Each trajectory of interest is uniquely defined by the relative phase parameter at the fiber input  $\theta(0)$ . Only one of the trajectories corresponds to the shortest length of the amplifier (shown by red circles in Fig. 2(b)). Therefore, in order to determine the shortest possible fiber length we should find the optimum value of  $\theta(0)$ . Equation (18) provides us with the required dependence of FOPA length on  $\theta(0)$ . In the considered example  $p(0)$ ,  $p(L)$  and  $\theta(0)$  are known and the length of PS-FOPA can be straightforwardly calculated by numerical integration of (18). It can be seen from Fig. 4(a), that an optimum value of the relative phase at  $L = 0$  does exist and it is approximately equal to 0.8 for all considered gain levels  $G = 40, 45$ , and  $50$  dB. In the other words, we found the optimum trajectory, which provides us with the shortest fiber length and that does not depend on the gain variation.

Next optimization problem, that can be considered, is to determine signal gain for given input signal power, pump power and given length of HNLF. Signal gain can also be found by solving Eq. (18) for known fiber length  $L$ , initial normalised pump power  $p(0)$  and unknown power  $p(L)$  using simple bisection method. Resulting dependence of signal gain on cavity length is shown in Fig. 4(b). Figure 4(b) shows, that gain is almost the same for  $\theta(0) = 0$  and  $\theta_{opt}(0) = 0.8$ , so we can confine the optimization problem to the simplest case when  $\theta(0) = 0$ . The maximum signal gain is reached at  $L \approx 0.35$  km for the initial relative phases  $\theta(0) = 0$  and  $\theta_{opt}(0) = 0.8$ .

Finally, another design problem, that can be solved using proposed approach, is a multiparametric optimization of PS-FOPA. For example, to investigate signal gain behaviour as a function of pump power and HNLF length contour plots can be plotted solving Eq. (18) numerically using bisection method as in the previous section. Dependence of the signal gain on the initial pump power and cavity length variation is shown in Fig. 5.

#### 4. Conclusion

We have performed theoretical analysis of the classical four-wave mixing equations in the application to the specific scheme of phase sensitive one-pump fibre optical parametric amplification.

Simple geometric approach has been applied to describe the evolution of the waves, involved in the PS-FOPA process, using the Hamiltonian structure of governing equations. This method provides geometrical visualisation of the solutions that might be useful for practical qualitative and quantitative design analysis. We have demonstrated how the geometrical approach can be used for solving the optimization problems directly relevant to the design of the PS-FOPA with specific parameters.

### **Funding**

Work of AR, SBM and MPF was supported by Russian Science Foundation (14-21-00110). Work of SKT was supported by the grant of the Ministry of Education and Science of the Russian Federation (14.B25.31.0003). Work of AB was supported by the Ministry of Education and Science of Russian Federation (14.578.21.0029).