Commodity Prices Rise Sharply at Turning Points

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Commodity prices depend on supply and demand. With an uneven distribution of resources, prices are high at locations starved of commodity and low where it is abundant. We introduce an agent-based model in which agents set their prices to maximize profit. At steady state, the market self-organizes into three groups: excess producers, consumers, and balanced agents. When resources are scarce, prices rise sharply at a turning point due to the disappearance of excess producers. Market data of commodities provide evidence of turning points for essential commodities, as well as a yield point for non-essential ones.

Trading | Optimization | Nash equilibrium | Turning points

Introduction

The oil price crisis of 1973 rattled the world and left persistent effects on the world economy and politics [1]. Peak periods in food price index during 2008 and 2011 coincided with incidents of food riots and instabilities across the world [2]. Clearly, prices of commodities affect our lives in many ways; they determine the economic well-being of individuals, companies, societies and the stability of governments. Besides the immediate effects on the livelihood of the average citizen, farmers need to know the prices of crops for planning their land use, manufacturers need to know when to import their raw materials, policy-makers need to decide on their agricultural stabilization schemes, and speculators would like to make a fortune in the futures market. The basic factors affecting commodity prices include supply, demand, stocks, prediction of future prices, bargaining power of the market participants, and government policies [3].

This complexity poses challenges to economists, econometricians, forecasters and researchers from other disciplines. For example, with the recent application of social network theory to economics [4], the bargaining power of the agents was found to depend on the topology of the corresponding trading networks, which determines the competition relation between suppliers and consumers [5], giving rise to price variations at equilibrium [6]. An important message conveyed from these studies is the importance of interactions between agents in the pricing process. The interactions may be achieved through auctions, bargains, assessment of marketing information, or price adjustment after repeated transactions. Furthermore, such interactions can lead to Nash equilibrium states that maximize the utility of agents through the allocation of goods [7]. This process of attaining a global stationary state through local responses to neighboring interactions can be considered a graphical game [8]. It belongs to a class of problems that reaches stationary states by passing messages between neighbors, widely applicable in areas such as statistical inference and network optimization [9].

To understand pricing behavior in the market, one should further consider the effects of uneven distribution of resources. Microscopically, the bargaining power of the agents depends not only on the connectivities with their trading partners, but also on the supply and demand. In a sufficiently well-connected market, sellers with more abundant supply may set a lower price so as to capture a larger market share, and buyers with a strong demand may accommodate a higher price so as to secure commodity provision. Macroscopically, the marketwide supply and demand determines the overall price level. The balance between supply and demand is reflected by the stock level. Indeed, the correlation between stock level and prices was recognized long time ago [10], and illustrated by the 2008 hike in grain price due to the diversion of corn to biofuel production [11]. When stocks are high, prices are insensitive to fluctuations in supply and demand, but when stocks decline to dangerous levels, prices become highly sensitive to small perturbations. A natural question is whether this change in sensitivity is gradual or abrupt. Empirically, this change in price volatility is often assumed to be gradual [12]. Alternatively, a sharp change may be envisaged similar to the one found in the famous Lewis Model describing the labor market in developing economies, in which wages rise when low-cost labor runs into shortage [13], as has been experienced in China recently [14]. If the change is sharp, it will have substantial impact on the dynamics of the economy and our preparedness for the changes to come. Furthermore, it is interesting to see whether it resembles phase transitions in many-body systems with interacting components [15] and whether existing analytical tools can elucidate this behavior.

In this paper we introduce a model of trading networks with a heterogeneous distribution of supply and demand, and study its effects on the bargaining power and pricing strategies of agents on the network. The model is analytically solvable in a fully connected network. We will consider how prices change when the availability of commodity varies, and discuss how a turning point in price emerges from the model, with its sharpness rounded by the presence of inventory. We will also

Significance

We investigate the relation between supply, demand, inventory, and the price of commodities within a systematic framework based on selfish profit-optimization. The analysis identifies a sharp turning point in commodity prices when resource availability changes. Derived for the first time from a rigorous agent-based model, the turning point is manifested in a sharply increasing anti-correlation between price and resource availability, reminiscent of phase transitions in statistical mechanics. We show that real commodity prices exhibit a similar behavior to our prediction. We identify both turning points where prices rapidly increase, and yield points indicating that commodities are no longer essential. This work brings new understanding of observed tipping points in commodity prices, and provides useful insight and tools for their prediction.

Reserved for Publication Footnotes

discuss the correspondence between results predicted by the model and trading data from various sources.

Here *inventory* carries a different meaning from stocks. Stocks refer to the excess amount of commodities left behind in the hands of the agents when their production plus inflow exceeds outflow. On the other hand, low levels of inventory are necessary for all agents to maintain a smooth operation of the system [16]. For example, industrialists need to keep an inventory of raw materials so as to streamline their manufacturing process. Dealers need to keep an inventory to facilitate sales and deliveries to anticipate sporadic transactions, and occasionally they are forced to carry inventories when faced with low seasons of sale. While inventory levels are low, they act as buffers to smoothen sharp changes in supply and demand, and represent the level of commodity agents keep to avoid running out of stock when purchasing orders arrive.

Model

We consider a network of N nodes. Each node i is connected to a set of trading partners denoted as ∂i. Unless stated otherwise, we will consider fully connected networks in this work where ∂i consists of all nodes except i. Each node is either a producer or a consumer of a commodity, with an initial capacity Λ_i randomly drawn from a distribution $\rho(\Lambda_i)$ for node $i=1,\ldots,N$. Positive Λ_i represents the amount of commodity produced per unit time by node i, whereas negative Λ_i represents the amount of commodity consumed per unit time by node i. The commodity is essential to all consumers, so that each consumer has to purchase a sufficient amount of commodity to satisfy their needs, and each producer cannot sell more commodity than its capacity. This is possible globally if the average $\langle\Lambda\rangle$ of the distribution $\rho(\Lambda)$ is positive. Let y_{ij} be the flow of commodity from node j to i. We adopt the convention that negative y_{ij} means a flow of magnitude $|y_{ij}|$ in the opposite direction. Hence the inequality $\sum_{j\in\partial i} y_{ij}+\Lambda_i \geq 0$ applies to each node i. The flows y_{ij} associated with a producer (consumer) i with a largely positive (negative) capacity are all outgoing (incoming), while the flows associated with a node with intermediate capacity may be partly outgoing and partly incoming, corresponding to their role as middle-men besides providing or consuming their own resources.

The net demand of node i is the outflow minus the capacity if the difference is positive and 0 otherwise, given by $\max\left(\sum_{j\in\partial i} y_{ji}-\Lambda_i, 0\right)$. When the argument $\sum_{j\in\partial i} y_{ji}-\Lambda_i$ changes sign, the demand has a discontinuous slope. In practice, trading nodes need to keep a provisional level of commodity so that they do not run out of stock when purchasing order arrives. Hence we propose a smoother demand ξ_i

$$\xi_i = f\left(\sum_{j\in\partial i} y_{ji} - \Lambda_i\right), \qquad [1]$$

where $f(x)$ is a function with a continuous slope, and asymptotically approaches 0 and x, respectively, in the limits $x\to\mp\infty$. For convenience, we use $f(x)=v\ln[1+\exp(x/v)]$, where v is referred to as the inventory level, but other functions may also be considered. The original demand function with a discontinuous slope at zero demand is recovered in the limit $v\to 0$. On the other hand, for finite values of v, $f(x)$ starts to deviate smoothly from 0 when x is of the same order as v. The inventory has the same effect as a fluctuating capacity Λ_i+z_i, where z_i is drawn from the distribution $P(z_i)=\frac{1}{4v}\text{sech}^2\left(\frac{z_i}{4v}\right)$.

To satisfy the demand ξ_i, node i purchases commodity from other nodes. Let r_{ij} be the fraction purchased from node j by node i, so that the amount of commodity shipped from j to i is $y_{ij}=\xi_i r_{ij}$. The fractions are determined by the prices set by neighboring nodes $k\in\partial i$ on a competitive basis. We consider fractions of the form

$$r_{ij} = \frac{F(\phi_j)}{\sum_{k\in\partial i} F(\phi_k)}, \qquad [2]$$

where ϕ_j is the price set by node j, and $F(\phi)$ is a non-negative decreasing function of ϕ. For convenience, we use the exponential form $F(\phi)=\exp(-\beta\phi)$, where β is a parameter playing the role of inverse temperature in the statistical physics literature, but other forms are also possible. When $\beta\to\infty$, r_{ij} becomes a winner-take-all function, such that the node with the lowest price becomes the sole provider of node i. In reality, agents diversify their purchases due to many factors. For example, they may have considerations other than prices such as quality and service, they may not like to be monopolized, or the cheapest choice may not be available at their moment of need. We note that β^{-1} is the scale of the price. This means that when the prices set by two suppliers differ by less than β^{-1}, the buyer would purchase from both suppliers with roughly equal weight. However, when the price difference becomes much greater the purchasing amount will differ significantly. Hence β^{-1} can be considered as the intrinsic value of a unit of commodity. For convenience, we will take $\beta=1$, so that prices are scaled in units of the intrinsic value.

Each node i calculates its price ϕ_i by minimizing its net cost E_i, which is the purchasing cost minus the sales revenue, assuming that the price of other nodes are not changed. Hence

$$E_i = \sum_{j\in\partial i} y_{ij}\phi_j - \sum_{j\in\partial i} y_{ji}\phi_i = \sum_{j\in\partial i}\xi_i r_{ij}\phi_j - \sum_{j\in\partial i}\xi_j r_{ji}\phi_i. \qquad [3]$$

The clearing and price adjustment process of this trading model with and without inventory can be simulated in the way described in the Supporting Information (SI).

Results

To minimize E_i, node (trader) i needs to assess the effects of changing its price by $\delta\phi_i$. Obviously, the sales revenue changes since the price of every unit of sold commodity changes. In addition, node i needs to know how its trading partners respond to the price change, specifically the flow change δy_{ji} in response to $\delta\phi_i$. It may obtain this knowledge through an active bargaining process, or through the passive observation of how the sales volume changes with price. Node i will then consider such messages from all neighbors before establishing its new price. In this respect, this trading network model belongs to the class of network problems solvable by passing messages [9]. The message sent from node j to i through the bargaining process is

$$a_{ij} = -\frac{\partial y_{ji}}{\partial\phi_i} = \xi_j r_{ji}(1-r_{ji}) - r_{ji}\frac{\partial\xi_j}{\partial\phi_i}. \qquad [4]$$

Since the export of node i changes, the demand ξ_i in the purchasing cost also changes. Using Eq. (1), we have

$$\frac{\partial\xi_i}{\partial\phi_i} = -f'\left(\sum_{j\in\partial i}\xi_j r_{ji} - \Lambda_i\right)\sum_{j\in\partial i} a_{ij}, \qquad [5]$$

For the term $\partial\xi_j/\partial\phi_i$ in Eq. (4), we need to consider how a price change $\delta\phi_i$ at node i induces changes in demands of all nodes, assuming that prices at other nodes are unchanged. However, the demand changes are inter-dependent. $\delta\xi_j$ induces changes in the neighbors of j, which induces changes back in $\delta\xi_j$, commonly referred to as Onsager reactions in many-body physics. As shown in the SI for fully connected networks using Green's function techniques, $\delta\xi_j$ is of the order N^{-1} of $\delta\xi_i$ for nodes j neighboring node i. Hence the second term in Eq. (4) can be neglected in the large N limit. After collecting messages from all neighbors, the price becomes

$$\phi_i = \frac{\sum_{j\in\partial i}\xi_j r_{ji}}{\sum_{j\in\partial i}\xi_j r_{ji}(1-r_{ji})} + f'(\sum_{j\in\partial i}\xi_j r_{ji} - \Lambda_i)\sum_{j\in\partial i} r_{ij}\phi_j. \quad [6]$$

When the network is fully connected, the price behavior depends on two parameters: $\phi_p \equiv \langle\phi e^{-\phi}\rangle/\langle e^{-\phi}\rangle$ being the average purchasing price, and $y\equiv\langle\xi\rangle/\langle e^{-\phi}\rangle$ termed the *demand coefficient*, playing a role in determining the demand itself. Averages denoted by the angled brackets are taken over all nodes. The price ϕ of a node becomes a unique function of its capacity Λ, bounded between the maximum price $1+\phi_p$ and minimum price 1. $\phi(\Lambda)$ is the inverse function of

$$\Lambda = ye^{-\phi} + v\ln\left(\frac{1+\phi_p-\phi}{\phi-1}\right). \quad [7]$$

We first consider the limit of zero inventory. When $v\to 0$ the price at node i depends on its capacity Λ_i as

$$\phi_i = \begin{cases} 1+\phi_p, & \Lambda_i \le ye^{-1-\phi_p}, \\ \ln\left(\frac{y}{\Lambda_i}\right), & ye^{-\phi_p-1}\le\Lambda_i\le ye^{-1}, \\ 1, & ye^{-1}\le\Lambda_i. \end{cases} \quad [8]$$

Hence there are three types of nodes. (1) Consumers ($\Lambda_i < ye^{-1-\phi_p}$) with positive demands. (2) Balanced nodes ($ye^{-1-\phi_p}\le\Lambda_i\le ye^{-1}$) with zero demands and no access resources. (3) Excess producers ($\Lambda_i > ye^{-1}$) with zero demands and excess resources.

With the inventory effect, price becomes a continuously changing function, as shown in Fig. 1(a). The three groups of nodes can still be identified, although the boundaries become fuzzy. Due to the presence of inventory, the out-flows of the balanced nodes differ from their capacities by an amount of order v. For balanced nodes with $ye^{-1-\phi_p}\le\Lambda_i\le ye^{-1-\phi_p/2}$, the out-flow is greater than the capacity by an amount of order v, whereas for balanced nodes with $ye^{-1-\phi_p/2}\le\Lambda_i\le ye^{-1}$, the out-flow is less than the capacity by an amount of the order v. These two groups will be referred to as quasi consumers and quasi producers respectively.

Solutions of the self-consistent equations for ϕ_p and y depend on the resource distribution $\rho(\Lambda)$. Considering the bounded resource production and consumption in real data we adopt distributions with upper and lower bounds. The expressions of ϕ_p and y in the limit of small v are derived in the SI for the rectangular distribution of mean $\langle\Lambda\rangle$ and width 1.

For the rectangular capacity distribution with $v=0$, the dependence of price and cost on capacity is verified by simulations shown in Figs. 2(a) and (b) respectively. In both figures, the theoretical results (dashed lines) are in excellent agreement with those obtained by solving the Nash equilibrium equations (8). As expected, the cost increases with decreasing capacity. It is interesting to note that through trading at an optimal price, even the consumers with Λ_i close to 0 can gain profit (negative cost).

When resources become increasingly tight, the purchasing price increases, as shown in Fig. S2 of SI. For the rectangular capacity distribution, ϕ_p approaches the finite value of 1.83 with an infinite slope when $\langle\Lambda\rangle$ approaches 0. When $\langle\Lambda\rangle$ falls below 0, the price diverges discontinuously. Note that excess producers exist in the range $(\sqrt{1/2}-\sqrt{\langle\Lambda\rangle})^2\le\Lambda\le 1/2+\langle\Lambda\rangle$, showing that the fraction of excess producers approaches 0 when $\langle\Lambda\rangle$ approaches 0. However, for finite values of $\langle\Lambda\rangle$, excess producers always exist in the case of zero inventory $v=0$.

When v has a small non-zero value, the price discontinuity for $v=0$ is replaced by a more refined picture in the range $\langle\Lambda\rangle\sim v$ as shown in Fig. 1(b). First, we find that when $\langle\Lambda\rangle$ is in the range $0.622v\le\langle\Lambda\rangle\ll 1$, the average price remains effectively at 1.83. When $\langle\Lambda\rangle$ falls below $0.622v$, excess producers disappear, and the price rises above 1.83. This shows that the excess producers stabilize the price by acting as a reservoir of resources. However, although resource production is still above consumption for $\langle\Lambda\rangle>0$, the holding up of resources in inventories causes the excess resources of the excess producers to dry up. The price thus experiences a *sharp turning point*. This turning point resembles a phase transition in many physical systems. Hence when $\langle\Lambda\rangle/v$ falls below the turning point, the purchasing price turns from flat to rapidly rising. However, the turning point is sharp only in the limit of van-

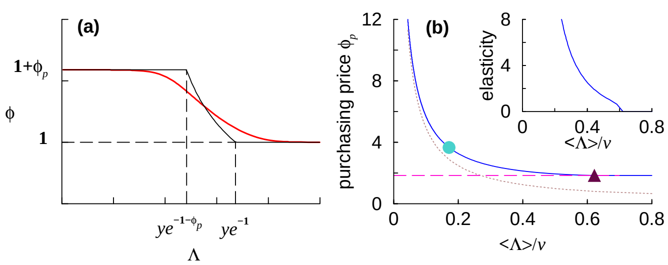

Figure 1. (a) The dependence of price on the node capacity. The red curve is given by Eq. (7) for average capacity $\langle\Lambda\rangle=0.2$ and inventory $v=0.01$. The black curve is the zero inventory limit of Eq. (8), using the same values of y and ϕ_p. (b) The dependence of purchasing price on average capacity in the regime of no excess producers (solid blue line). Pink dashed line: price in the regime with excess producers. Brown symbol: Disappearance of excess producers. Turquoise dot: Disappearance of quasi producers. Brown dotted line: asymptotic limit of vanishing capacity. Inset: The dependence of the capacity elasticity of price on the average capacity in the $v=0$ limit.

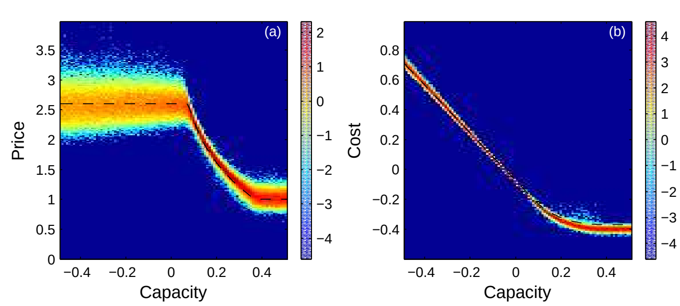

Figure 2. (a)The price distribution for different node capacities after 10,000 time steps, in markets with $v=0$, $N=100$ and 1,000 samples. Each time step consists of 200 updating cycles. The average capacity is $\langle\Lambda\rangle=0.01$. Other parameters: $\varepsilon=1$ and $\eta=0.01$. (b)The cost distribution for different node capacities with the same set of parameters as in (a). The dashed curves represent the theoretical predictions given by Eq. (8). Both distributions are in natural log scale.

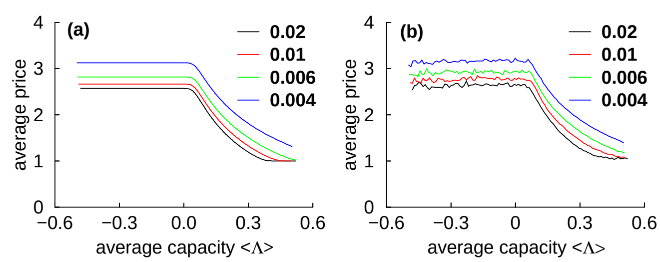

Figure 3. (a) The analytical result of the capacity dependence of the prices at different inventory levels at $v=0.01$. (b) The corresponding simulation results.

ishing v. For finite values of v, the change is smoother. The inset of Fig. 1(b) shows that the capacity elasticity of price, $-v d\phi_p/d\langle\Lambda\rangle$, has a discontinuous slope at $\langle\Lambda\rangle/v = 0.622$.

When $\langle\Lambda\rangle$ falls further below $0.171v$, and the price rises to 3.67, even the quasi producers disappear. However, since the excess resources held by the quasi producers are of the order v, the effect on the price behavior is much less pronounced. In this regime, the price rises with decreasing $\langle\Lambda\rangle$ asymptotically as $0.496v/\langle\Lambda\rangle$, and diverges when $\langle\Lambda\rangle/v$ approaches 0. Figure 3(a) shows the analytical result of the capacity dependence of the prices at different inventory levels. The prices rise rapidly when $\langle\Lambda\rangle/v$ falls below 0.622. Simulation results in Fig. 3(b) confirm the trend. The results also have an excellent agreement with those obtained by solving the Nash equilibrium equations (8).

In summary, the trading model predicts that when resources are plenty, prices are insensitive to changes in supply and demand. However, when resources become increasingly tight, prices start to become highly sensitive to these changes. This happens when excess resources are exhausted, and the market loses the buffering provided by excess producers. This mechanism is reminiscent of the Lewisian turning point, which describes the rise in wages of unskilled labor in developing economies when the labor market starts to run out of unskilled labor [13]. When the average resource is of the same order as the inventory level v, this results in a turning point in the resource dependence, where the response of the price to resource availability has a discontinuous slope. The discontinuity is smoother for finite values of v.

Comparison with Commodities Data

A common parameter to measure resource availability in commodity markets is the stocks-to-use ratio (SUR) defined as the amount of carryover stock of a commodity at the end of a period (usually a year) divided by the consumption during the same period. While conventionally SUR is expressed as a percentage, it has the dimension of time, representing the duration in which stocks will be consumed by the market (assuming that no other resources are available). SUR is an important predictive tool of commodity prices. For example, there is a strong negative correlation between cotton prices and SUR [17]. Similar trends were also observed in wheat and corn prices [12].

However, the prediction of the current trading model is more than merely the anti-correlation between the price and the SUR. It further predicts a sharp turning point from a regime of weak anti-correlation for a sufficiently large SUR to one of rapidly increasing anti-correlation with decreasing SUR. For grains, price spikes took place in recent years and were attributed to unusually low SUR [11]. A more detailed analysis is required to verify this prediction.

In general, a plot of the price of a commodity as a function of the SUR appears as a collection of scattered points, although a rough trend is often visible. One factor is that the data is gathered over many years or even decades, such that the data is interfered by many other factors, for instance changes in market needs. Here, we propose that the quality of data can be improved by defining the *SUR elasticity of price*,

$$E_p = -\frac{\text{change in price}}{\text{change in SUR}}. \qquad [9]$$

In practice, we calculate the yearly elasticity of the commodities, and sort the corresponding SUR in order. Approximately 10 data points with consecutive SUR values are clustered for regression, and the slope of the cluster is plotted as a function of the average SUR of the cluster. Remarkably, a much clearer picture often emerges from this analysis. Furthermore, it provides insights on where and why there are deviations from the prediction. In the following subsections we will illustrate this effect by considering several commodities.

Crude Oil. Using the OPEC spare production capacity and WTI crude oil prices data [18], we plot the price in the inset of Fig. 4(a). It shows the general trend of increasing price on decreasing capacity, which is obscured by fluctuations. Hence we plot E_p in Fig. 4(a) (we used the OPEC spare production capacity as the abscissa, since SUR data is unavailable). Both abscissa and ordinate are rescaled to facilitate comparison with the prediction of the pricing model. The turning point at 2.3 million barrels per day is visible. Beyond the turning point, the price is effectively independent of the spare capacity, whereas near the turning point, the price rises sharply.

Agricultural Products. SUR and price data in the U.S. market was obtained from the U.S. Department of Agriculture [19]. Figures 5(a)-(c) show the SUR dependence of the elasticity for long-grain rice, short-grain rice, cotton, and soybeans.

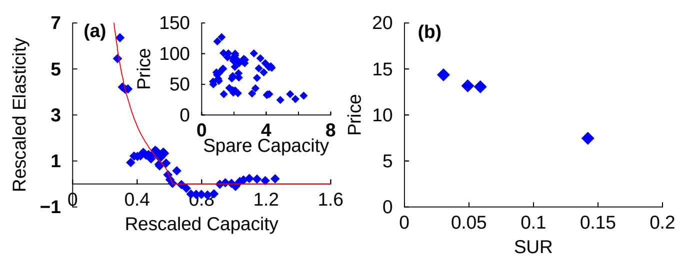

Figure 4. (a) The rescaled elasticity versus the rescaled capacity of crude oil from 1st quarter of 2001 to 4th quarter of 2014. To enable comparison with the pricing model (solid curve with a turning point at 0.622), the spare capacity is rescaled by 3.74 million barrels per day (mbpd) and elasticity by 7.79 USD/barrel/mbpd. Each plotted point represents a regression of 11 data points. Inset: WTI crude oil prices in US$@2010 per barrel versus OPEC spare production capacity in million barrels per day during the same period. (b) The price of carbon permits in Euros versus SUR from 2009 to 2012.

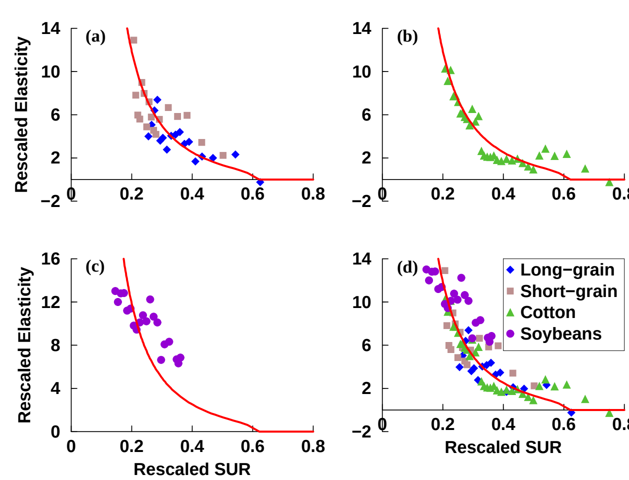

Figure 5. The rescaled elasticity versus the rescaled SUR for (a) long-grain rice and short-grain rice from 1983 to 2011, (b) cotton from 1965 to 2010, (c) soybeans from 1980 to 2012. To enable comparison with the pricing model (the solid curve with a turning point at 0.622), the elasticities and the SURs are respectively rescaled by (a) 4.72 \$@1998/cwt/y and 0.385 y for long-grain rice, and 1.86 \$@1998/cwt/y and 0.830 y for short-grain rice, (b) 14.7 cents@1998/lb/y and 0.906 y, (c) 2.18 \$@1998/bu/y and 0.474 y. Each plotted point comes from a regression of 13 data points. (d) The composite plot of the four agricultural products.

These commodities have the common feature that the elasticity shows an increasing trend with decreasing SUR. Compared with the crude oil data, these agricultural products lie in the regime of tight resources with positive elasticity.

We have also considered the data of other agricultural products. However, commodities such as honey and peanuts do not exhibit the behavior predicted by the trading model. This may be an indication that they are not essential and market demand would shrink if prices are too high.

Figure 5(d) is the composite plot of the four agricultural products illustrating their universal behavior. The plot is consistent with the prediction of our model showing that the elasticity increases with decreasing SUR. One remaining point is whether the evidence is strong enough to support the existence of a turning point, since data can probably be fitted with curves that continuously decrease with increasing SUR, such as in [12]. To provide a perspective on this point, it may be argued that the world economy has adjusted itself to the state of a low level of SUR, such that spare capacity is converted to other more efficient and profitable use of resources, rendering the turning point unobservable. In this respect, we may consider such analyses of the agricultural products are complementary to our analysis of the crude oil data.

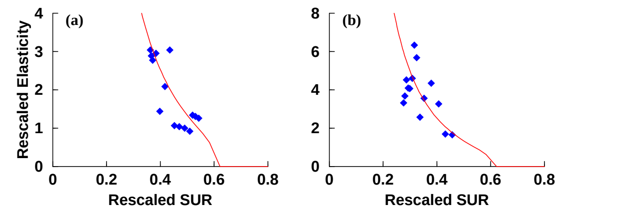

Figure 7. The rescaled elasticity versus the rescaled SUR for (a) wheat, (b) coarse grains, both from 1991 to 2012 and excluding 1995. To enable comparison with the trading model (the solid curve with a turning point at 0.622), the elasticities and SURs are rescaled by (a) 448 US$@1998/tonne/y and 0.684 y, (b) 223 US$@1998/tonne/y and 0.563 y. Each plotted point comes from a regression of 7 data points.

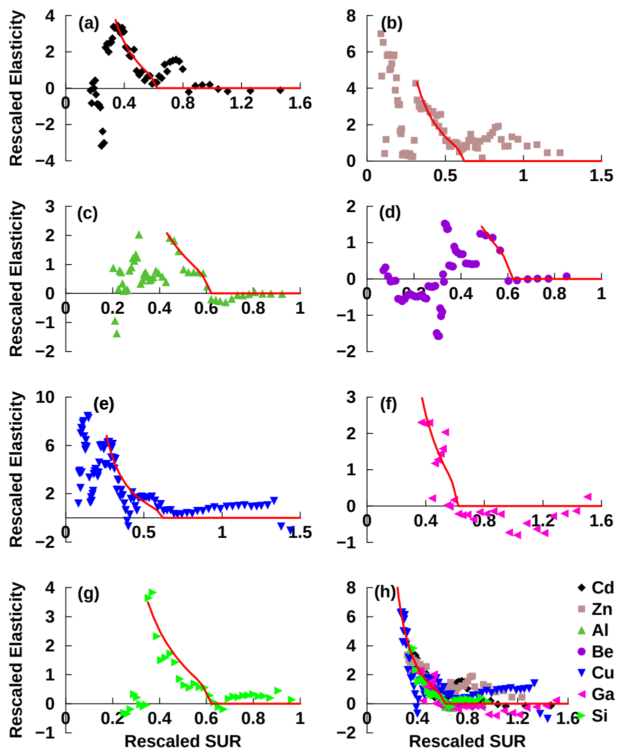

Figure 6. The rescaled elasticity versus the rescaled SUR for (a) cadmium from 1940 to 2010, (b) zinc 1915-2012, (c) aluminum 1949-2012, (d) beryllium 1941-2012, (e) copper 1900-2012, (f) gallium 1971-2012, (g) silicon 1964-2012. To enable comparison with the trading model (the solid curve with a turning point at 0.622), the elasticities and the SURs are respectively rescaled by (a) 4,440 $@1998/t/y and 0.499 y, (b) 802 $@1998/t and 0.590 y, (c) 945 $@1998/t/y and 1.38 y, (d) 143,000 $@1998/t/y and 2.06 y, (e) 962 $@1998/t/y and 0.625 y, (f) 996 $@1998/t/y and 0.144 y, (g) 2,280 $@1998/t/y and 0.203 y. Each plotted point comes from a regression of 13 data points. (h) The composite plot of the seven metals after excluding data below the yield points.

Commodity	SUR at turning point (year)	SUR at yield point (year)	Relative yield elasticity
Cadmiun	0.311	0.163	0.153
Zinc	0.367	0.184	0.438
Aluminum	0.860	0.613	0.448
Beryllium	1.28	0.995	0.289
Copper	0.390	0.168	0.266
Silicon	0.126	0.0719	0.619

Table 1. SUR at the turning and yield points and the relative yield elasticity for seven metals.

Metals. SUR and price data can be obtained from the website of U.S. Geological Survey [20]. Figures 6(a)-(g) show the elasticity-SUR plots for cadmium, zinc, aluminum, beryllium, copper, gallium, and silicon. The curves agree only partially with our trading model. Consider the example of cadmium in Fig. 6(a). The elasticity increases only up to a certain point as SUR decreases. Below that point the elasticity decreases with decreasing SUR and even negative elasticity is observed. A plausible explanation is that this commodity is only considered essential when the price is not high or the stock is sufficient. When the price becomes too high or the SUR too low, the market will no longer consider the commodity essential, and may switch to alternative commodities or at least refrain from purchasing. Note that this behavior is not present in other more essential commodities such as crude oil or wheat. Hence the maximum point of the curve reveals the maximum price that the market is willing to pay for the commodities, or the minimum SUR that the market is willing to accept. Below, we will term this point the *yield point*.

Fitting the curves of these metals with the trading model prediction, the turning points can be obtained, although in a few cases, the elasticity beyond the fitted turning point still has considerable magnitudes. All these metals also exhibit yield points. Figure 6(h) is the composite plot of the seven metals illustrating their universal behavior after excluding data points below the yield point.

We have also considered the data of other metals such as lead and nickel. They do not admit the behavior predicted by the pricing model, indicating that their prices may be affected by factors other than supply and demand.

Referring to Table , it is interesting to note that despite the wide range of commodities, the SURs of a few commodities at the turning points typically lie in the range 0.1 to 0.4 y, and their SURs at the yield points typically lie in the range 0.05 to 0.2 y. This may be an indication that real markets require a finite duration to complete transactions. To understand the typical value of the yield elasticity, we introduce the relative yield elasticity, defined as the elasticity at the yield point divided by the typical price of the commodity and multiplied by the SUR at the yield point. The typical price of the commodity is calculated to be the average price in the elastic regime (between the turning and yield points). We find that the relative yield elasticity is in the range 0.1 to 0.5. It is plausible that the market mechanism determining the price of these commodities is rather universal.

Cereal. Cereal data are available from the UN FAO yearly food outlooks [21]. These reports provided global annual average prices and major exporters' SUR for different commodities, such as wheat, coarse grains and rice. Although data from 1992 to 2012 are available, the data show a discontinuity in the SUR from 1995 to 1996. According to the report from February 2001, the discontinuity was due to significant data changes when the cereal stocks estimates in China (Mainland) were revised [21]. Hence we focus on wheat and coarse grains data from 1991 to 2010 excluding 1995 for the elasticity versus SUR plot. As shown in Figs. 7(a)-(b), both wheat and coarse grains data follow the trend predicted by our model in the elastic regime.

Carbon Trading. We further studied carbon trading in the European Union Emission Trading System [22]. A feature of this commodity is that licenses for carbon emissions have to be surrendered annually, so that surplus permits cannot be carried over to future years. In this sense, the mode of trading agrees with the assumptions of our model. EU-wide carbon permit prices can be obtained from the French stock exchange [23]. Daily prices are averaged annually. The SUR of carbon trading is defined as the EU-wide carbon emission allocation minus the actual release, divided by the actual release. In contrast with other physical commodities, negative SURs are allowed for carbon trading, but penalty was imposed on non-compliance. In practice, a negative SUR was only found for the years 2008 and 2013 when carbon trading entered phases 2 and 3 respectively. Figure 4(b) shows that the price decreases with increasing SUR. Since data points are too few, we have not attempted the elasticity plot.

Summary. Analyzing the price history of crude oil, agricultural commodities, metals, cereals, and carbon trading we found that: (1) Elasticity versus SUR plots are much more interpretable than price versus SUR. This is probably because elasticity is based on short-term price changes, whereas a good plot of price versus SUR requires the long-term independence of the environment. (2) The elasticity versus SUR plot reveals two critical points: *turning* point and *yield* point. Three regimes are identified on decreasing SUR: inelastic, elastic and yielded. (3) Different data types have different characteristics. Only non-essential commodities have yield points. Crude oil covers both the inelastic and elastic regimes due to the producers' ability to control spare capacity. Most agricultural commodities, cereals and carbon trading cover the elastic regime only. Yielded regimes are present in most metals. (4) The data support the insight gained from our trading model.

Conclusion

We propose an agent-based model in which resources available to each agent are inhomogeneous and agents set their prices to maximize profits or minimize costs. At steady state, the market self-organizes into three types of agents depending on their capacities. Excess producers have excess resources and set their prices at the intrinsic value of the commodity. They act as a buffer for price stability. Consumers have high demands of the commodity and set their prices at the highest value, since their priority is to acquire resources. Balanced agents act as mediators. Since transactions can be set up between any two agents, the market behavior, including the distributions of prices, final resources, and costs, depend on only two mean-field parameters: the purchasing price ϕ_p and the demand coefficient y. An important prediction of the model is that prices are relatively inelastic when resources are plenty, but become elastic when the available resources are below a turning point, which is triggered by the disappearance of the excess producers, analogous to the Lewisian turning point in the labor market [13]. Comparing this behavior with market data, we found supporting evidence for turning point in essential commodities, and discovered that the behavior may be modified by a yield point for non-essential commodities. However, not all commodities obey this behavior, indicating that they may be influenced by additional factors other than supply and demand. In spite of this, SURs have been identified to be useful indicators for price hikes in global cereal markets [24] and are used in trading strategies; SUR values have been suggested as rules of thumbs for forecasting price hikes commodities such as wheat, corn, and soybeans [25].

The price trends and the existence of the turning point are insensitive to the details of the model. Purchasing fractions other than the exponential function in (2), such as a power law of the prices, exhibit similar behaviors. Capacity distributions, other than the rectangular one, also exhibit similar behaviors as long as they have an upper bound. Similar predictions are also applicable to networks whose nodes have high but finite connectivities. It will be interesting to extend the agent-based approach to networks other than fully connected ones, such as those with low connectivities where Onsager reactions become significant, geographical networks where distances are important, or scale-free networks that are relevant to realistic social and technological networks [26].

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