An allocation Malmquist index with an application in the China securities industry

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Acknowledgements

This work is financially supported by 211 Project for the Southwestern University of Finance and Economics (Phase 3), Chengdu, China. The authors would like to thank Mr. Guangdong Sun and Miss Jing He for assistant work of collecting the data. We also would like to thank to anonymous reviewers for their insightful comments and suggestions, as results the paper has been improved substantially.

An allocation Malmquist index with an application in the China securities industry

Abstract:

This paper proposes an allocation Malmquist index which is inspired by the work on the non-parametric cost Malmquist index. We first show that how to decompose the cost Malmquist index into the input-oriented Malmquist index and the allocation Malmquist index. An application in corporate management of the China securities industry with the panel data set of 40 securities companies during the period 2005-2011 shows the practicality of the propose model.

Keywords: Data Envelopment Analysis; Overall efficiency; Allocative efficiency; Allocation Malmquist index.

1. Introduction

Since Charnes *et al*. (1978) introduced Data Envelopment Analysis (DEA) as a non-parametric model for measuring the efficiency of Decision Making Units (DMUs) more than 5000 studies have been reported in this area (Emrouznejad, *et al.*, 2008 & Emrouznejad and De Witte, 2010). Non-parametric Malmquist index is also introduced by Färe *et al.* (1994) (see also Malmquist (1953) and Caves *et al*. (1982)). As results many theoretical studies as well as applications in productivity measurement are reported in the literature.

Maniadakis and Thanassoulis (2004) have developed a cost Malmquist index (CM) applicable when decision-makers can be assumed to be cost minimizers and input–output quantity and input price data are known. They have pointed out that the CM index can be decomposed as follows:

$$\begin{aligned} CM &= OEC \times CTC \\ &= TEC \times AEC \times TC \times PE \qquad (1) \\ &= IM \times AEC \times PE \end{aligned}$$

where;

OEC = overall efficiency change
CTC = cost technical change
TEC =technical efficiency change
AEC = allocative efficiency change
TC = technical change
PE = price effect
IM = input-oriented Malmquist index
AEC = allocative efficiency change

In this paper, following the study of Maniadakis and Thanassoulis (2004), a concept of an Allocation Malmquist index (AM) is proposed and three types of Malmquist indexes with their relationships in the framework of organization management are discussed. The cost Malmquist index can be decomposed into the input-oriented Malmquist index and the allocation Malmquist index. To show the applicability of the proposed model, the productivity measure in corporate management of the China securities industry, with the panel data set of 40 securities companies during the period 2005-2011, is provided and discussed. Note that the efficiencies and productivities given in this paper are calculated in the input-oriented measure; with the same manner output-oriented measures can be also defined.

There are only a few papers using DEA to study the efficiency and productivity problems of securities companies which, in fact, plays an important role in the capital market. For examples, Goldberg *et al.* (1991), Zhang *et al.* (2006), Fukuyama and Weber (2008), Zhu and Liu (2008) used DEA method, respectively, to discuss the efficiency and productivity problems of securities companies of some countries including US, Japan and China.

Zhang *et al.* (2006) studied the technical efficiency and classical Malmquist index of the US securities companies. They pointed out that the lack of research in the securities industry can be attributed mainly to industry regulators as they have not collected / published information used for analyzing the securities industry. To the best of our knowledge there is no single studying on the overall efficiency and CM of the China securities companies.

The rest of the paper is organized as follows. The methodology is discussed in Section 2. In this section we also introduce the concept of Allocation Malmquist Index and its decomposition. Section 3 presents an application in the China securities industry by using the propose model and Section 4 concludes this paper and provides direction for future research.

2. Methodology

2.1. Technical efficiency, overall efficiency and allocative efficiency

Consider a set of J decision-making units (DMUs) with n input and m output in T (t=1,…T) periods. Assume in time period t, decision-makers are using inputs $x^t \in R_+^n$, to produce outputs $y^t \in R_+^m$. Figure 1 provides an illustration of an input-distance function in period t, where two inputs, x_1^t and x_2^t, are used to produce output, y^t. The isoquant, SS', is the inner boundary of the input set reflecting the minimum input combinations that may be used to produce a given output vector in period t. In this case, the value of the distance function for a firm producing output, y^t, using the input vector defined by point A, is equal to the ratio, OA/OB.

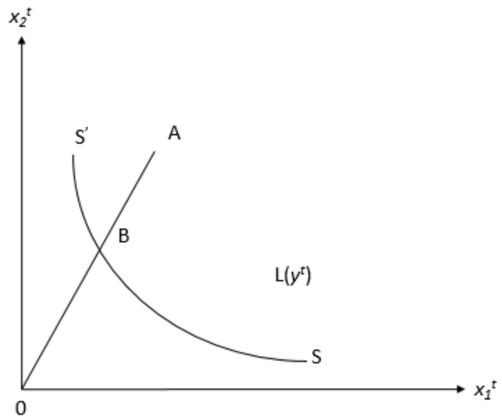

Figure 1. Input isoquant and input distance function

Consider the input distance function $D^t(y^t, x^t)$ of period t, the technical (or productive) efficiency (TE) in period t is defined as :

$$TE\left(y^t, x^t\right) = 1/D^t\left(y^t, x^t\right) \qquad (2)$$

In general, TE＜1, indicates that the DMU under assessment, comparing with other DMUs, is productively inefficient since its production is based on excessive input usage. TE=1, indicates the DMU is fully productively efficient.

As Maniadakis and Thanassoulis (2004), given input prices $w^t \in R_+^n$, lets also consider the cost function C^t (y^t, w^t) that represents the minimum cost of production of period t, the following formula represents the overall (or cost) efficiency (OE) in period t for (y^t, x^t) under input prices w^t :

$$\mathrm{OE}^t\left(y^t, x^t, w^t\right) = C^t\left(y^t, w^t\right)/w^t x^t, \quad \text{where } w^t x^t = \sum_{j=1}^{n} w_j^t x_j^t. \qquad (3)$$

In general, OE<1, indicates that the DMU under assessment, comparing with other DMUs, is cost inefficient. OE=1 indicates that the DMU is fully cost efficient.

Using above input distance and cost functions, allocative efficiency (AE) in period t for (y^t, x^t) under input prices w^t is defined as follows:

$$\mathrm{AE}^t\left(y^t, x^t, w^t\right) = D^t\left(y^t, x^t\right) \times C^t\left(y^t, w^t\right)/w^t x^t. \qquad (4)$$

The AE reflects the distance between the actual and minimum cost at which a DMU may secure its outputs once any productive inefficiency of the DMU has been eliminated. In general, AE＜1 indicates that the DMU under assessment comparing with other DMUs, is allocatively inefficient, since its production takes place at the wrong input mix in light of input prices. AE=1 indicates the DMU, comparing with other DMUs, is fully allocatively efficient.

According to Farrell (1957) the OE can be decomposed into the TE and AE (see also Cooper, Seiford and Tone, 2000):

$$\mathrm{OE}^t\left(y^t, x^t, w^t\right) = \mathrm{TE}^t\left(y^t, x^t\right) \times \mathrm{AE}^t\left(y^t, x^t, w^t\right). \qquad (5)$$

Accordingly, if a DMU is cost inefficient, it is either productively inefficient or allocatively inefficient, or both. In other words, it is either because production is based on excessive input usage or because it takes place at the wrong input mix in light of input prices, or both (Maniadakis and Thanassoulis, 2004).

2.2. Three types of Malmquist indexes

2.2.1. Input-oriented (production) Malmquist index

Let IM^t and IM^{t+1}, measures the distance of (y^{t+1}, x^{t+1}) and (y^t, x^t) from the Constant Returns to Scale (CRS) production boundary of period t, t+1, respectively. We have

$$\mathrm{IM}^t = \frac{D^t(y^{t+1}, x^{t+1})}{D^t(y^t, x^t)}, \quad \mathrm{IM}^{t+1} = \frac{D^{t+1}(y^{t+1}, x^{t+1})}{D^{t+1}(y^t, x^t)} \tag{6}$$

where IM^t and IM^{t+1} are called the input-oriented Malmquist index (IM) of period t, t+1, respectively (Maniadakis and Thanassoulis, 2004). The IM of period t, t+1 is defined by the geometric mean of IM^t and IM^{t+1}, see (7). It is well known that the IM can be decomposed into the technical (or productive) efficiency change (TEC or PEC) and (production-) technical change (TC or PTC) as follows:

$$\begin{aligned} \mathrm{IM} &= \left[\mathrm{IM}^t \times \mathrm{IM}^{t+1}\right]^{1/2} \\ &= \frac{\mathrm{TE}^t(y^t, x^t)}{\mathrm{TE}^{t+1}(y^{t+1}, x^{t+1})} \times \left[\frac{D^t(y^t, x^t)}{D^{t+1}(y^t, x^t)} \times \frac{D^t(y^{t+1}, x^{t+1})}{D^{t+1}(y^{t+1}, x^{t+1})}\right]^{1/2} \\ &= \mathrm{TEC} \times \mathrm{TC}, \end{aligned} \tag{7}$$

where TEC= $\mathrm{TE}^t(y^t, x^t) / \mathrm{TE}^{t+1}(y^{t+1}, x^{t+1})$ denotes the change of the technical efficiency between period t and t+1. The TEC indicates whether the DMU 'catches up' the production boundary when moving from period t to t + 1. In this case TEC＜1, TEC＞1 and TEC=1 imply the technical efficiency, respectively, increase, decrease and constant between period t and t+1. The TEC can be further decomposed into the scale efficiency change (SEC) and pure technical efficiency change (PTEC): TEC= SEC× PTEC (Färe *et al.* 1996, and Ray and Desli, 1997).

In (5), the TC denotes the technical change of the DMU between period t and t+1 that measures the shift of the production boundary evaluated at the inputs x^t and x^{t+1}. The progress and regress in production technology between period t and t+1 can be identified by TC$<$1 and TC$>$1, respectively. Similarly, IM$<$1 (IM$>$1) implies the input-oriented Malmquist index progress (regress) between period t and t+1.

IM is applicable when decision-makers are making production management and input–output quantity data are known. Hence, the IM index, as the classical Malmquist index of Färe *et al.* (1994), is directly concerning to *production* and can be considered as a *production Malmquist index* (PM).

2.2.2. Cost Malmquist index

In the DEA literature, the non-parametric *cost Malmquist indices*, CM^t and CM^{t+1} of period t, t+1 are defined as follows (Maniadakis and Thanassoulis, 2004):

$$\mathrm{CM}^t = \frac{w^t x^{t+1} / C^t(y^{t+1}, w^t)}{w^t x^t / C^t(y^t, w^t)}, \quad \mathrm{CM}^{t+1} = \frac{w^{t+1} x^{t+1} / C^{t+1}(y^{t+1}, w^{t+1})}{w^{t+1} x^t / C^{t+1}(y^t, w^{t+1})} \tag{8}$$

The cost Malmquist index of period t, t+1 (CM) is defined by the geometric mean of CM^t and CM^{t+1}, see (9). Using (3), we have:

$$\begin{aligned}
\mathrm{CM} &= \left[\mathrm{CM}^t \times \mathrm{CM}^{t+1}\right]^{1/2} = \left[\frac{w^t x^{t+1} / C^t(y^{t+1}, w^t)}{w^t x^t / C^t(y^t, w^t)} \times \frac{w^{t+1} x^{t+1} / C^{t+1}(y^{t+1}, w^{t+1})}{w^{t+1} x^t / C^{t+1}(y^t, w^{t+1})}\right]^{1/2} \\
&= \frac{w^{t+1} x^{t+1} / C^{t+1}(y^{t+1}, w^{t+1})}{w^t x^t / C^t(y^t, w^t)} \times \left[\frac{w^t x^t / C^t(y^t, w^t)}{w^{t+1} x^t / C^{t+1}(y^t, w^{t+1})} \times \frac{w^t x^{t+1} / C^t(y^{t+1}, w^t)}{w^{t+1} x^{t+1} / C^{t+1}(y^{t+1}, w^{t+1})}\right]^{1/2} \\
&= \frac{\mathrm{OE}^t(y^t, x^t, w^t)}{\mathrm{OE}^{t+1}(y^{t+1}, x^{t+1}, w^{t+1})} \times \left[\frac{w^t x^t / C^t(y^t, w^t)}{w^{t+1} x^t / C^{t+1}(y^t, w^{t+1})} \times \frac{w^t x^{t+1} / C^t(y^{t+1}, w^t)}{w^{t+1} x^{t+1} / C^{t+1}(y^{t+1}, w^{t+1})}\right]^{1/2} \\
&= \mathrm{OEC} \times \mathrm{CTC}.
\end{aligned} \tag{9}$$

The CM index is applicable when decision-makers are making cost management and input–output quantity and input price data are known. Similar to decomposition of the IM index of (7), the CM can be decomposed into the overall (or cost) efficiency change (OEC or CEC) and cost-technical change (CTC), see (9).

OEC = OE^t (y^t, x^t, w^t) /OE^{t+1} $(y^{t+1}, x^{t+1}, w^{t+1})$ denotes the change of the overall efficiency between period t and t+1. The OEC indicates whether the DMU ‘catches up’ the cost boundary when moving from period t to $t+1$. OEC＜1, OEC＞1 and OEC＝1 imply the overall efficiency, respectively, increase, decrease and constant between period t and t+1. Similar concept is applied to CTC that measures the shift of the cost boundary evaluated at the input prices w^t and w^{t+1}. Also the progress or regress in the cost Malmquist index between period t and t+1 can be identified by CM＜1 or CM<1, respectively.

2.2.3. Allocation Malmquist index

In the spirit of the work done by Farrell (1957), Färe *et al.* (1994) and Maniadakis and Thanassoulis (2004), the AM^t and AM^{t+1} are called the *allocation Malmquist index* of period t, t+1, respectively, can be defined as follows:

$$\begin{aligned}
\mathrm{AM}^t &= \frac{w^t x^{t+1} \big/ C^t(y^{t+1}, w^t) D^t(y^{t+1}, x^{t+1})}{w^t x^t \big/ C^t(y^t, w^t) D^t(y^t, x^t)}, \\
\mathrm{AM}^{t+1} &= \frac{w^{t+1} x^{t+1} \big/ C^{t+1}(y^{t+1}, w^{t+1}) D^{t+1}(y^{t+1}, x^{t+1})}{w^{t+1} x^t \big/ C^{t+1}(y^t, w^{t+1}) D^{t+1}(y^t, x^t)}.
\end{aligned} \tag{10}$$

The overall allocation Malmquist index of period t to period t+1 (AM), using the geometric mean of AM^t and AM^{t+1} can be decomposed as follows:

$$\begin{aligned}
\mathrm{AM} &= \left[\mathrm{AM}^t \times \mathrm{AM}^{t+1}\right]^{1/2} \\
&= \left[\frac{w^t x^{t+1} \big/ C^t(y^{t+1}, w^t) D^t(y^{t+1}, x^{t+1})}{w^t x^t \big/ C^t(y^t, w^t) D^t(y^t, x^t)} \times \frac{w^{t+1} x^{t+1} \big/ C^{t+1}(y^{t+1}, w^{t+1}) D^{t+1}(y^{t+1}, x^{t+1})}{w^{t+1} x^t \big/ C^{t+1}(y^t, w^{t+1}) D^{t+1}(y^t, x^t)}\right]^{1/2} \\
&= \frac{w^{t+1} x^{t+1} \big/ C^{t+1}(y^{t+1}, w^{t+1}) D^{t+1}(y^{t+1}, x^{t+1})}{w^t x^t \big/ C^t(y^t, w^t) D^t(y^t, x^t)} \times \\
&\quad \left[\frac{w^t x^t \big/ C^t(y^t, w^t) D^t(y^t, x^t)}{w^{t+1} x^t \big/ C^{t+1}(y^t, w^{t+1}) D^{t+1}(y^t, x^t)} \times \frac{w^t x^{t+1} \big/ C^t(y^{t+1}, w^t) D^t(y^{t+1}, x^{t+1})}{w^{t+1} x^{t+1} \big/ C^{t+1}(y^{t+1}, w^{t+1}) D^{t+1}(y^{t+1}, x^{t+1})}\right]^{1/2}
\end{aligned} \tag{11}$$

$$= \frac{\mathrm{AE}^{t}(y^{t}, x^{t}, w^{t})}{\mathrm{AE}^{t+1}(y^{t+1}, x^{t+1}, w^{t+1})} \times$$

$$\left[\frac{w^{t} x^{t} / C^{t}(y^{t}, w^{t}) D^{t}(y^{t}, x^{t})}{w^{t+1} x^{t} / C^{t+1}(y^{t}, w^{t+1}) D^{t+1}(y^{t}, x^{t})} \times \frac{w^{t} x^{t+1} / C^{t}(y^{t+1}, w^{t}) D^{t}(y^{t+1}, x^{t+1})}{w^{t+1} x^{t+1} / C^{t+1}(y^{t+1}, w^{t+1}) D^{t+1}(y^{t+1}, x^{t+1})} \right]^{1/2}$$

$$= \mathrm{AEC} \times \mathrm{ATC}.$$

Similar to the CM index, the AM index is applicable when decision-makers are making resource allocation management and the input–output quantity and input price data are known. From (11), it can be seen that similar to the IM of (7) and CM of (9), the AM can be decomposed into the allocative efficiency change (AEC) and allocation-technical change (ATC) [*].

$\mathrm{AEC} = \mathrm{AE}^{t}(y^{t}, x^{t}, w^{t}) / \mathrm{AE}^{t+1}(y^{t+1}, x^{t+1}, w^{t+1})$ that denotes the change of the allocative efficiency between period t and t+1. The AEC indicates whether the production boundary of the DMU 'catches up' the cost boundary when going from period t to t+1.

In (10), the ATC denotes the change of allocation technology of the DMU between period t and t+1 that measures the shift of the production boundary evaluated at the inputs x^t and x^{t+1} and cost boundary evaluated at the input prices w^t and w^{t+1}.

Just as with the IM and CM indexes, AM＜1, AM＞1 and AM＝1 imply the allocation Malmquist index, respectively, progress, regress and constant between period t and t+1.

2.2.4. Geometric meaning of IM, CM and AM

The IM, CM and AM and their component indexes are illustrated in Figure 2[†]. As in Maniadakis and Thanassoulis (2004), there is a case where production takes place

[*] The term of 'allocation-technical change (ATC)' in this paper was called the term of 'price effect (PE)' by Maniadakis and Thanassoulis (2004). It was also discussed, and a suggestion that 'one may view this term as AEC at industry rather than at firm level' was provided by Maniadakis and Thanassoulis (2004).

[†] The wrong printing of $w^t x^{t+1}$ and $w^{t+1} x^{t+1}$ in Fig. 2 given by Maniadakis and Thanassoulis (2004) are revised in Fig. 1 of this paper.

under CRS and two inputs, $(x_1, x_2) \in R_+^2$, are used to produce single output, $y \in R_+$. Production takes place at point G:(y^t, x^t) in period t and shifts to point B:(y^{t+1}, x^{t+1}) in period t+1. The outputs in both periods are standardized to one unit and so $y^t = y^{t+1}$. Isoq$L^t(y^t)$ and Iso$C^t(y^t, w^t)$ are the input isoquant and isocost lines in period t, respectively. Here, the cost boundary Iso$C^t(y^t, w^t)$ contains the input vectors that are capable of securing output y^t at the cost of $C^t(y^t, w^t)$. We have:

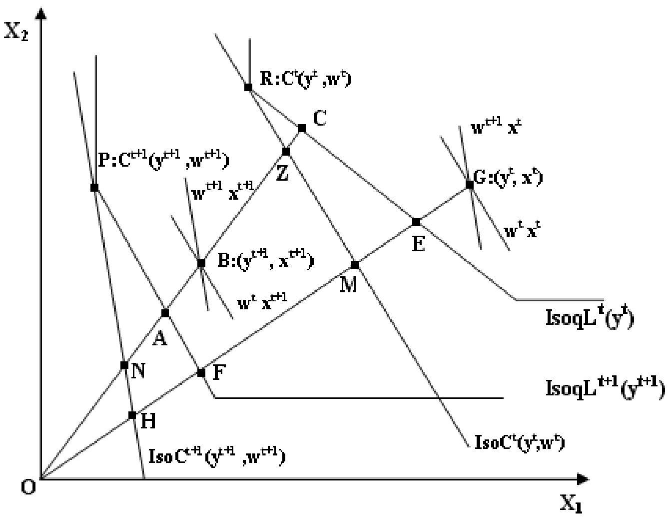

Figure 2. The input-oriented (production), cost and allocation Malmquist indexes

The decomposition of IM can be presented as follows:

$$\begin{aligned}\mathrm{IM} &= \left[\frac{\mathrm{OB/OC}}{\mathrm{OG/OE}} \times \frac{\mathrm{OB/OA}}{\mathrm{OG/OF}}\right]^{1/2} = \frac{\mathrm{OB/OA}}{\mathrm{OG/OE}} \times \left[\frac{\mathrm{OG/OE}}{\mathrm{OG/OF}} \times \frac{\mathrm{OB/OC}}{\mathrm{OB/OA}}\right]^{1/2} \\ &= \frac{\mathrm{OE/OG}}{\mathrm{OA/OB}} \times \left[\frac{\mathrm{OF}}{\mathrm{OE}} \times \frac{\mathrm{OA}}{\mathrm{OC}}\right]^{1/2} = \mathrm{TEC} \times \mathrm{TC};\end{aligned} \tag{12}$$

$$\begin{aligned}\mathrm{CM} &= \left[\frac{\mathrm{OB/OZ}}{\mathrm{OG/OM}} \times \frac{\mathrm{OB/ON}}{\mathrm{OG/OH}}\right]^{1/2} = \frac{\mathrm{OB/ON}}{\mathrm{OG/OM}} \times \left[\frac{\mathrm{OG/OM}}{\mathrm{OG/OH}} \times \frac{\mathrm{OB/OZ}}{\mathrm{OB/ON}}\right]^{1/2} \\ &= \frac{\mathrm{OM/OG}}{\mathrm{ON/OB}} \times \left[\frac{\mathrm{OH}}{\mathrm{OM}} \times \frac{\mathrm{ON}}{\mathrm{OZ}}\right]^{1/2} = \mathrm{OEC} \times \mathrm{CTC};\end{aligned} \tag{13}$$

$$
\begin{aligned}
\mathrm{AM} &= \left[\frac{\mathrm{OC}/\mathrm{OZ}}{\mathrm{OE}/\mathrm{OM}} \times \frac{\mathrm{OA}/\mathrm{ON}}{\mathrm{OF}/\mathrm{OH}}\right]^{1/2} = \frac{\mathrm{OA}/\mathrm{ON}}{\mathrm{OE}/\mathrm{OM}} \times \left[\frac{\mathrm{OE}/\mathrm{OM}}{\mathrm{OF}/\mathrm{OH}} \times \frac{\mathrm{OC}/\mathrm{OZ}}{\mathrm{OA}/\mathrm{ON}}\right]^{1/2} \\
&= \frac{\mathrm{OM}/\mathrm{OE}}{\mathrm{ON}/\mathrm{OA}} \times \left[\frac{\mathrm{OE}/\mathrm{OM}}{\mathrm{OF}/\mathrm{OH}} \times \frac{\mathrm{OC}/\mathrm{OZ}}{\mathrm{OA}/\mathrm{ON}}\right]^{1/2} = \mathrm{AEC} \times \mathrm{ATC}.
\end{aligned} \tag{14}
$$

2.2.5. Relationships among CM, IM and AM

We know that a DMU (profit or non-profit organization) needs to be managed, for best use of production, and minimum cost and resources., hence it has to accomplish goals by using available resources efficiently. In the following, we will discuss the relationships among the CM, IM and AM in the framework of organization management.

We refer readers to the Farrell decomposition (5): OE = TE×AE. Using (7) − (11), we have the relationships among CM, IM and AM, that is, CM = IM×AM as follows:

$$
\begin{aligned}
\mathrm{CM} &= \left[\frac{w^t x^{t+1}/C^t(y^{t+1},w^t)}{w^t x^t/C^t(y^t,w^t)} \times \frac{w^{t+1} x^{t+1}/C^{t+1}(y^{t+1},w^{t+1})}{w^{t+1} x^t/C^{t+1}(y^t,w^{t+1})}\right]^{1/2} \\
&= \left[\frac{D^t(y^{t+1},x^{t+1})}{D^t(y^t,x^t)} \times \frac{D^{t+1}(y^{t+1},x^{t+1})}{D^{t+1}(y^t,x^t)}\right]^{1/2} \times \\
&\quad \left[\frac{w^t x^{t+1}/C^t(y^{t+1},w^t) D^t(y^{t+1},x^{t+1})}{w^t x^t/C^t(y^t,w^t) D^t(y^t,x^t)} \times \frac{w^{t+1} x^{t+1}/C^{t+1}(y^{t+1},w^{t+1}) D^{t+1}(y^{t+1},x^{t+1})}{w^{t+1} x^t/C^{t+1}(y^t,w^{t+1}) D^{t+1}(y^t,x^t)}\right]^{1/2} \\
&= \mathrm{IM} \times \mathrm{AM}.
\end{aligned} \tag{15}
$$

Similarly, we have:

$$
\begin{aligned}
\mathrm{OEC} &= \frac{w^{t+1} x^{t+1}/C^{t+1}(y^{t+1},w^{t+1})}{w^t x^t/C^t(y^t,w^t)} \\
&= \frac{D^{t+1}(y^{t+1},x^{t+1})}{D^t(y^t,x^t)} \times \frac{w^{t+1} x^{t+1}/C^{t+1}(y^{t+1},w^{t+1}) D^{t+1}(y^{t+1},x^{t+1})}{w^t x^t/C^t(y^t,w^t) D^t(y^t,x^t)} \\
&= \mathrm{TEC} \times \mathrm{AEC}.
\end{aligned} \tag{16}
$$

Therefore we obtain:

$$
\begin{aligned}
\mathrm{CTC} &= \left[\frac{w^t x^{t+1} / C^t(y^{t+1}, w^t)}{w^{t+1} x^{t+1} / C^{t+1}(y^{t+1}, w^{t+1})} \times \frac{w^t x^t / C^t(y^t, w^t)}{w^{t+1} x^t / C^{t+1}(y^t, w^{t+1})} \right]^{1/2} \\
&= \left[\frac{D^t(y^t, x^t)}{D^{t+1}(y^t, x^t)} \times \frac{D^t(y^{t+1}, x^{t+1})}{D^{t+1}(y^{t+1}, x^{t+1})} \right]^{1/2} \times \qquad (17) \\
&\quad \left[\frac{w^t x^{t+1} / C^t(y^{t+1}, w^t) D^t(y^{t+1}, x^{t+1})}{w^t x^t / C^t(y^t, w^t) D^t(y^t, x^t)} \times \frac{w^{t+1} x^{t+1} / C^{t+1}(y^{t+1}, w^{t+1}) D^{t+1}(y^{t+1}, x^{t+1})}{w^{t+1} x^t / C^{t+1}(y^t, w^{t+1}) D^{t+1}(y^t, x^t)} \right]^{1/2} \\
&= \mathrm{TC} \times \mathrm{ATC}.
\end{aligned}
$$

Using (7), (17) and the considering that TEC=SEC×PTEC as given by Färe *et al*. (1996) (see also Ray and Desli, 1997), the CM index can be further decomposed as follows:

$$
\begin{aligned}
\mathrm{CM} &= \mathrm{OEC} \times \mathrm{CTC} \\
&= [\mathrm{TEC} \times \mathrm{AEC}] \times [\mathrm{TC} \times \mathrm{ATC}] \\
&= [\mathrm{TEC} \times \mathrm{TC}] \times [\mathrm{AEC} \times \mathrm{ATC}] \qquad (18) \\
&= \{[\mathrm{SEC} \times \mathrm{PTEC}] \times \mathrm{TC}\} \times [\mathrm{AEC} \times \mathrm{ATC}] \\
&= \mathrm{IM} \times \mathrm{AM}.
\end{aligned}
$$

where:

OEC = overall efficiency change
CTC = cost technical change
TEC =technical efficiency change
AEC = allocative efficiency change
TC = technical change
ATC = allocation-technical change
SEC = scale efficiency change
PTEC = pure technical efficiency change
TC = technical change
AEC = allocative efficiency change
ATC = allocation-technical change

Here, we need to point out that the computation of the indices and their components (the distance function *D* and cost function *C*, etc.) are based on the linear programming models provided in Section 5 of Maniadakis and Thanassoulis (2004). Software such as GAMS can be used for computing the indices. As indicated in Section 1, the decompositions in (18) provides a clearer picture of the root sources of the productivity change of the CM, IM and AM, respectively.

3. An application in the China securities industry

In this section we illustrate the proposed approach of the CM, IM and AM on the data set of 40 China's securities companies in the framework of corporate management during the period 2005-2011.

3.1 The China securities industry

As an important component of the financial industry in mainland China, the securities industry with banking industry and insurance industry are known as the "Troika". It plays a role of booster and lubricant for China's economic development and relates to the healthy development of the China's economy and securities market. The development of China's macroeconomic, investors' concern on the securities market and continual improvement of regulatory approach provide good opportunities for the development of the China securities industry. As important intermediaries in the securities market, the China securities companies were growing up in the exploration and reform, and had played the key role to the breeding and development of the whole industry.

From 2005 to 2011, under the tough external environment and internal industry comprehensive governance (China Securities Regulatory Commission, 2005), the China securities industry was developed steadily. There were 115 and 109 securities companies in the industry in 2005 and 2011, respectively. In 2011, the average assets of the securities companies totaled RMB 831.3 billion yuan (1 US$ = approx. 6.3009 yuan in 2011), RMB 513.1 billion yuan more than in 2005. In the stock market, there were 2,342 listed companies in the Shanghai and Shenzhen stock exchanges, 961 more than in 2005. See The People's Bank of China (2006−2012).

However, large increase in the Chines Securities companies is also produced a lot of problems in corporate management in the China's securities companies. Some securities companies are still inadequate in cost management, production management, resource allocation management, development scale, making-profit models, etc. Cost increase, production (service) based on excessive inputs, (human, assets, etc.) resource allocation not properly done and many other issues have affected the strength of competitiveness and capability of sustainable development of the China securities companies.

In the following, an application of the allocation Malmquist index in the mainland China securities industry is discussed; the relevant efficiencies and productivities are investigated as well.

3.2. Sample and index selections

Data used in this study is gathered from annual financial statements as published by the China securities companies from 2005 to 2011. The statements of 2006-2011 can also be found intensively in the publication of the Securities Association of China (2013).

In China, there are two kinds of the securities companies, brokerage or integrated securities companies. The former is just running securities brokerage business; the latter is running not just brokerage, but also other businesses concerning securities. Due to DEA requirement and to analyze homogenize DMUs, this paper takes only the 40 integrated securities companies as the research objects.

In China, according to Article 125 of the Securities Law (The National People's Congress, 2005), a securities firm may undertake some or all the following business operations: (a) Securities brokerage; (b) Self-operation of securities; (c) Underwriting and recommendation of securities; (d) Securities Investment consulting; (e) Financial advising relating to activities of securities trading or securities investment; (f) Securities asset management; and (g) Any other business operation concerning securities. Here, (a)−(c) consist of three traditional businesses. In addition, with gradually opening the China capital market, the securities companies have entered to (d)−(f) to expand their business space and promote the capability of business innovation. This paper classifies these three businesses as innovation business. Hence, we used the following output variables.

Outputs:

Output 1 (y_1): brokerage income (unit: 1,000,000 yuan);

Output 2 (y_2): self-operation income (unit: 1,000,000 yuan);

Output 3 (y_3): underwriting income (unit: 1,000,000 yuan);

Output 4 (y_4): innovation income (unit: 1,000,000 yuan) and

Output 5 (y_5): other income (unit: 1,000,000 yuan).

In a firm, the number of employee and the average total assets represent the human capital and total economic resource of the firm, respectively. As a securities company is a knowledge-intensive firm and referring the previous scholars' work, we used the two items as two kinds of input variables of rare resources.

Inputs:

Input 1 (x_1): the total number of employee (unit: person) and

Input 2 (x_2): the average total assets (unit: 1,000,000 yuan);

For this study we also used the following prices.

Input prices:

Input price 1 (w_1): the average employee wages (total employee wages/total number of employee) (unit: 1,000 yuan/per person) and

Input price 2 (w_2): the ratio of the revenue expenditure and average total assets (revenue expenditure/average total assets).

Table 1 below illustrates a summary statistics of the panel data of the 7 years average values of input, output variables as well as price indexes for 40 sample companies from 2005 to 2011 (see Appendix 1 for details).

Table 1: Descriptive statistics of the average values from 2005 to 2011

	y_1	y_2	y_3	y_4	y_5	x_1	x_2	w_1	w_2
Mean	1735.564	5765.492	216.888	220.483	48.791	2,258	22628.040	235.825	0.035
SD	1760.377	10261.388	324.250	402.187	57.824	1988.576	25054.957	93.701	0.008
Min	201.037	600.357	2.669	18.475	4.397	522	3646.984	126.386	0.023
Max	7438.985	41615.236	1787.119	2458.524	279.376	9.246	117986.660	556.724	0.061

Data source: Annual reports of the 40 China's securities companies from 2005 to 2011.

3.3. Results and discussion

3.3.1. Efficiencies of the China securities companies

To Show the usefulness of the proposed indices in this paper we provide and compare the efficiencies of the China securities companies during 2005 to 2011 (see Appendix 2). It can be seen that , the China securities industry in 2011 was cost inefficient (0.625) due to the industry was both of productively and allocatively inefficient, but the value of TE (=0.810) was higher than the AE's (=0.767), i.e., the TE was less influential than the AE to the OE of the industry in 2011. In 2011, 6 DMUs, including 3, 26, 27, 32, 38 and 39, were fully allocatively and fully technical efficient (AE=1 & TE=1). DMU 6, 21 and 23 were fully allocative efficient but not technically efficient.

Furthermore, at overall industry level, Table 2 summarizes the Means and Standard division of the OE, TE and AE of the 40 sample companies, in each year of 2005-2011, showing that overall the China securities industry was cost inefficient every year. The reason was that the industry was both of productively and allocatively inefficient. The TE was less influential than the AE to the OE of the China securities industry except 2005.

Table 2. The Means of the OE, TE and AE of the 40 companies from 2005 to 2011

	2005	2006	2007	2008	2009	2010	2011
OE Mean	0.705	0.321	0.590	0.645	0.571	0.615	0.625
TE Mean	0.838	0.667	0.879	0.834	0.877	0.860	0.810
AE Mean	0.839	0.465	0.665	0.772	0.641	0.717	0.767
OE SD	0.207	0.227	0.221	0.190	0.246	0.227	0.228
TE SD	0.169	0.185	0.104	0.141	0.137	0.151	0.163
AE SD	0.159	0.231	0.208	0.172	0.221	0.217	0.200

3.3.2. Malmquist indexes of the China securities companies

In the following, firstly, we use decomposition given in (11) to explain the reasons behind the productivity changes of the CM, IM and AM in the China securities industry in 2010-2011 [Note that similar results have been obtained for other periods, (2005-6, 2006-7, 2008-9 & 2009-10), in the next section we present overall results for the whole study period 2005-2011].

Table 3: The CM, IM and AM of the 40 companies between 2010 and 2011

	CM Index			IM Index			AM Index		
DMU	CM[a]	OEC	CTC	IM[b]	TEC	TC	AM[c]	AEC	ATC
1	0.853	0.671	1.272	1.141	1.042	1.095	0.747	0.643	1.161
2	1.517	1.059	1.432	1.291	1.191	1.084	1.175	0.889	1.321
3	0.383	0.360	1.061	0.405	0.529	0.766	0.945	0.682	1.386
4	0.717	0.569	1.260	0.810	0.850	0.952	0.885	0.669	1.323
5	1.305	0.871	1.497	1.183	1.164	1.016	1.103	0.748	1.474
6	1.513	1.256	1.204	1.513	1.257	1.204	1.000	1.000	1.000
7	1.496	1.100	1.360	1.290	1.209	1.067	1.159	0.910	1.274
8	2.002	1.305	1.534	1.146	1.115	1.028	1.747	1.171	1.492
9	1.991	1.256	1.585	1.499	1.105	1.357	1.328	1.137	1.168
10	1.381	0.996	1.387	1.096	1.004	1.093	1.260	0.992	1.269
11	2.232	1.490	1.499	1.352	1.185	1.141	1.651	1.257	1.313
12	1.735	1.309	1.326	1.488	1.258	1.183	1.166	1.041	1.121
13	1.017	0.866	1.174	0.900	0.824	1.093	1.130	1.052	1.074
14	1.070	0.700	1.528	1.003	1.000	1.003	1.067	0.700	1.524
15	1.393	0.994	1.401	1.289	1.155	1.116	1.081	0.861	1.256
16	1.083	0.673	1.610	0.955	0.750	1.272	1.134	0.897	1.265
17	1.332	0.928	1.435	1.162	0.944	1.231	1.146	0.983	1.166
18	1.813	1.249	1.451	1.695	1.246	1.361	1.069	1.003	1.066
19	1.194	0.740	1.613	1.002	0.949	1.056	1.191	0.780	1.528
20	1.559	1.139	1.369	1.291	1.054	1.225	1.207	1.080	1.118
21	1.018	0.760	1.340	1.040	0.787	1.321	0.979	0.965	1.015
22	1.663	1.263	1.317	1.139	1.087	1.048	1.460	1.162	1.257
23	1.455	0.898	1.621	1.452	1.095	1.326	1.002	0.820	1.223
24	1.169	0.864	1.354	1.002	0.928	1.080	1.167	0.931	1.254
25	1.032	0.632	1.633	0.950	0.979	0.971	1.086	0.645	1.682
26	1.492	0.996	1.499	1.330	1.000	1.330	1.122	0.996	1.127
27	1.054	0.613	1.719	1.122	1.000	1.122	0.940	0.613	1.533
28	1.448	0.905	1.600	1.309	1.218	1.074	1.106	0.743	1.489
29	1.498	1.006	1.489	1.381	1.177	1.173	1.084	0.854	1.269
30	2.329	2.501	0.931	2.212	2.448	0.904	1.053	1.022	1.031
31	1.989	1.211	1.642	1.665	1.170	1.423	1.194	1.035	1.154
32	0.999	1.000	0.999	0.940	1.000	0.940	1.063	1.000	1.063
33	2.048	1.803	1.136	1.599	1.428	1.120	1.281	1.263	1.014
34	1.452	0.989	1.467	1.196	1.113	1.075	1.214	0.889	1.366
35	1.426	1.000	1.425	1.341	1.164	1.153	1.063	0.860	1.236
36	1.527	1.033	1.478	1.321	0.968	1.365	1.156	1.068	1.083
37	2.263	1.768	1.280	1.247	1.062	1.174	1.815	1.664	1.090
38	0.817	1.000	0.817	0.824	1.000	0.824	0.991	1.000	0.991
39	1.700	1.000	1.700	1.700	1.000	1.700	1.000	1.000	1.000
40	1.496	0.989	1.514	1.360	1.313	1.036	1.100	0.753	1.461
G. Mean	1.362	0.986	1.382	1.200	1.067	1.125	1.135	0.924	1.229
SD	0.432	0.373	0.205	0.305	0.273	0.172	0.207	0.202	0.177

[a]CM = OEC ×CTC = IM×AM; [b]IM = TEC ×TC; [c]AM = AEC ×ATC.

According to Table 3, at overall industry level, it can be seen that between 2010 and 2011, the CM, IM and AM of the China securities industry, represented by the geometric means of the CM, IM and AM of the 40 sample companies, have relationships:

$$1.362 = 0.986 \times 1.382 = (1.067 \times 0.924) \times (1.125 \times 1.229) \quad (19)$$

$$= (1.067 \times 1.125) \times (0.924 \times 1.229) = 1.200 \times 1.135.$$

From the cost management point of view, the reason of the cost Malmquist index of the securities industry regress (CM=1.362) between 2010 and 2011 can be considered as the overall efficiency increase (OEC=0.986) and the cost technology regress (CTC=1.382) between 2010 and 2011. However, the overall efficiency increase was less influential than the cost technology regress to the CM because $0.014 = |1 - OEC| < |1 - CTC| = 0.382$. We can also see that the cost technology regress were affected by both of the production technology and allocation technology regress (TC=1.125 and ATC=1.229), and the influential effect brought by the production technology change was less than the allocation technology change.

From the productivity measurement point of view, the CM regress was due to both of the input-oriented Malmquist index and allocation Malmquist index regress (IM=1.200 and AM=1.135) of the securities industry between 2010 and 2011. The IM was more influential than the AM to the CM index.

From production management point of view, the reason of the IM of the industry regress between 2010 and 2011 was due to both of the technical efficiency decrease (TEC=1.067) and production technology regress (TC=1.125). The former was less influential than the latter to the IM index of the industry.

From resource allocation management point of view, the AM of the securities industry regress between 2010 and 2011 was mainly caused by the allocation technology regress (ATC=1.229) although the allocative efficiency increase (AEC=0.924) brought a positive effect to the AM index, see Table 4. Six DMUs, including 1, 3, 4, 21, 27 and 38, had the AM progress (AM<1). DMU 6 and 39 were the constant allocative productivity (AM=1) because these 2 DMUs were the constant allocation efficiency and constant technology (AEC=ATC=1). The other 32 DMUs were the AM regress (AM>1).

Similarly, based on Formula (11) and Table 4, we can also discuss the CM, IM and AM indexes at individual DMU level between 2010 and 2011. DMU 12, for example, had 1.735=1.488×1.166, where CM=1.735, IM=1.488 and AM=1.166. Because IM>AM, the CM of DMU 12 regress was due to both of the IM and AM regress, but the IM was more influential than the AM to the CM index between 2010 and 2011.

DMU 38, as another example, had 0.817＝0.824×0.991, where CM=0.817, IM=0.824 and AM=0.991. Since IM<AM, the CM of DMU 38 progress was due to both of its IM and AM progress, but the IM was more influential than the AM to the CM index between 2010 and 2011. From Table 2 and Appendix 2, DMU 38 was OE=TE=AE=1 in 2010 and 2011, respectively. In fact, this DMU was the largest scale securities company among the 40 sample companies: the inputs (x_1, x_2) of the number of employee and the average total assets were (15,476, 179992.56) and (13,260, 150729.03) in 2010 and 2011, respectively.

Secondly, at overall industry level, Table 4 summarizes the values of the CM, IM and AM of the China securities industry from 2005 to 2011 by the geometric means of the CM, IM and AM of the 40 sample companies.

Table 4: The G. Means of the CM, IM and AM of the 40 companies from 2005 to 2011

	CM Index			IM Index			AM Index		
Time	CM	OEC	CTC	IM	TEC	TC	AM	AEC	ATC
2005−2006	0.703	2.511	0.280	0.713	1.274	0.560	0.986	1.970	0.501
2006−2007	1.387	0.486	2.853	1.989	0.736	2.701	0.697	0.660	1.056
2007−2008	0.791	0.892	0.886	0.994	1.062	0.936	0.796	0.840	0.947
2008−2009	3.127	1.188	2.632	2.109	0.951	2.217	1.482	1.249	1.187
2009−2010	1.393	0.903	1.542	1.350	1.023	1.321	1.032	0.884	1.167
2010−2011	1.362	0.986	1.382	1.200	1.067	1.125	1.135	0.924	1.229
2005−2011	1.288	1.024	1.258	1.300	1.006	1.292	0.991	1.018	0.974

From Table 5, we can see that between 2005 and 2006, the IM progress (IM=0.713) of the China securities industry was due to its technical efficiency decrease (TEC=1.274) but production technology progress (TC=0.560). Zhu and Liu (2008) discussed the IM and its components of the China securities industry between 2005 and 2006. They concluded that the technical efficiency decrease was due to both of the scale efficiency and pure technical efficiency decrease between 2005 and 2006.

Finally, Table 5 is given the geometric means of the CM, IM and AM of the 40 securities companies between 2005 and 2011.

Table 5: The G. Means of the CM, IM and AM of the 40 companies between 2005 and 2011

	CM Index			IM Index			AM Index		
DMU	CM	OEC	CTC	IM	TEC	TC	AM	AEC	ATC
1	1.208	0.969	1.246	1.327	0.992	1.338	0.910	0.978	0.931
2	1.495	0.990	1.511	1.592	1.030	1.545	0.939	0.961	0.978
3	1.297	1.000	1.297	1.328	1.000	1.328	0.977	1.000	0.977
4	1.598	0.950	1.682	1.604	1.000	1.604	0.997	0.950	1.049
5	1.224	1.062	1.153	1.193	1.014	1.176	1.026	1.047	0.980
6	1.222	0.954	1.282	1.337	0.966	1.385	0.914	0.988	0.925
7	1.459	1.061	1.375	1.391	1.018	1.366	1.049	1.042	1.007
8	1.315	1.201	1.095	1.058	1.013	1.045	1.242	1.185	1.048
9	1.114	1.009	1.105	1.108	0.992	1.117	1.006	1.017	0.989
10	1.230	1.052	1.170	1.259	1.006	1.252	0.977	1.045	0.934
11	1.474	1.083	1.361	1.525	1.066	1.431	0.966	1.016	0.951
12	1.419	1.132	1.253	1.405	1.090	1.289	1.009	1.038	0.972
13	1.341	1.077	1.245	1.385	1.054	1.314	0.968	1.022	0.947
14	1.445	0.990	1.460	1.462	0.982	1.489	0.989	1.008	0.980
15	1.338	1.127	1.187	1.216	1.024	1.187	1.100	1.100	1.000
16	1.431	1.009	1.418	1.326	1.000	1.326	1.080	1.009	1.070
17	1.030	0.911	1.131	1.174	0.996	1.179	0.877	0.914	0.959
18	1.329	1.009	1.318	1.408	1.009	1.397	0.944	1.000	0.943
19	1.400	1.104	1.268	1.170	1.012	1.157	1.196	1.091	1.096
20	1.256	0.958	1.311	1.330	0.963	1.380	0.944	0.994	0.950
21	1.081	0.891	1.213	1.209	0.908	1.332	0.895	0.982	0.911
22	1.121	1.018	1.101	1.154	0.953	1.211	0.971	1.068	0.909
23	1.248	1.041	1.199	1.322	1.046	1.264	0.944	0.995	0.949
24	1.503	1.140	1.319	1.357	1.007	1.347	1.108	1.132	0.979
25	1.228	0.972	1.264	1.254	0.958	1.308	0.980	1.014	0.966
26	1.116	0.944	1.182	1.189	0.973	1.222	0.938	0.971	0.967
27	1.163	0.961	1.210	1.189	0.974	1.220	0.978	0.986	0.992
28	1.572	0.948	1.658	1.657	1.004	1.651	0.949	0.944	1.005
29	1.242	1.011	1.228	1.241	0.971	1.279	1.001	1.042	0.960
30	1.376	1.025	1.342	1.475	1.056	1.397	0.933	0.971	0.961
31	1.159	0.969	1.197	1.187	0.973	1.221	0.976	0.996	0.980

32	1.207	1.000	1.207	1.288	1.000	1.288	0.937	1.000	0.937
33	1.309	1.155	1.133	1.307	1.061	1.232	1.002	1.089	0.920
34	1.376	1.029	1.337	1.323	0.954	1.387	1.040	1.079	0.964
35	1.144	1.018	1.124	1.228	1.040	1.181	0.931	0.979	0.952
36	1.177	1.106	1.064	1.152	1.082	1.065	1.021	1.022	0.999
37	1.221	1.114	1.096	1.101	1.010	1.090	1.109	1.103	1.006
38	1.162	1.000	1.162	1.172	1.000	1.172	0.991	1.000	0.991
39	1.668	1.000	1.668	1.722	1.000	1.722	0.969	1.000	0.969
40	1.177	1.057	1.113	1.204	1.059	1.138	0.977	0.999	0.978
G. Mean	1.288	1.024	1.258	1.300	1.006	1.292	0.991	1.018	0.974
SD	0.150	0.069	0.154	0.154	0.038	0.151	0.075	0.053	0.040

At overall industry level, based on (11) and Table 5, it can be seen that the overall CM, IM and AM of the China securities industry between 2005 and 2011 had relationship:

$$1.288=1.024\times1.258=(1.006\times1.018)\times(1.292\times0.974) \quad (20)$$

$$= (1.006\times1.292)\times (1.018\times0.974) =1.300\times0.991,$$

where CM=1.288, OEC=1.024, CTC=1.258, TEC=1.006, TC=1.292, AEC=1.018, ATC=0.974, IM=1.300 and AM=0.991.

We also observe that the CM of the China securities industry regresses between 2005 and 2011. Since OEC<CTC, the CTC regress was the main factor to cause the CM regress. Considering CTC=TC×ATC, from (12.2), the TC was more influential than the ATC to the CTC although ATC=0.974.

The IM regress of the China securities industry between 2005 and 2011 was caused mainly by the TC rather than the TEC. However, we have the AM of the China securities industry progress between 2005 and 2011 (i.e. the ATC progress). The ATC was more influential than the AEC to the AM because $0.018=|1-AEC|<|1-ATC|=0.026$. We also have that the AM was less influential than the IM to the CM although AM=0.991.

At DMU level, from Table 5, we can see that all CM and IM of the 40 sample DMUs regress individually between 2005 and 2011. However, we also have the AM of the 26

DMUs progress, the AEC of the 15 DMUs increase and the ATC of the 32 DMUs progress between 2005 and 2011.

As another example, DMU 12 had 1.419=1.405×1.009, where CM=1.419, IM=1.405 and AM=1.009 between 2005 and 2011. Because IM>AM, the CM of DMU 12 regress was due to both of the IM and AM regress, but the IM regress was the main reason to cause the CM regress. Take DMU 38, 1.162=1.172×0.991, where CM=1.162, IM=1.172 and AM=0.991. Because $0.172=|1-IM|>|1-AM|=0.009$, the main reason of the CM of DMU 38 regress was the IM regress, i.e., IM was more influential than the AM to the CM index between 2005 and 2011.

In order to promote the development of China securities industry to be sustainable, the cost-productivity change needs to be progressed. Based on this results of this analysis, we suggest that the relevant decision-makers should make right decisions in order to raise the overall efficiency up and promote the cost technology progress, esp., pay more attention to value of allocation Malmquist index which is closely affected by both of the production-technical change and allocation-technical change.

At the same time, the decision-makers must monitor the firm to make sure that the allocation-productivity change be progressed. For this, they have to choose decisions that increase the allocation efficiency. Allocative efficiency will be achieved when scarce resources (labor, capital, etc.) with a set of given resource prices are reasonably allocated so as to maximize the required outcomes of the securities companies.

4. Conclusion

In this paper, following the study of Maniadakis and Thanassoulis (2004), a concept of an allocation Malmquist index is proposed. Three kinds of Malmquist indexes of production, cost and allocation with their relationships in the framework of organization management are discussed. It is shown that the cost Malmquist index can be decomposed into the input-oriented (production) Malmquist index and allocation Malmquist index.

An application in the China securities industry, represented by 40 companies, is provided. In terms of efficiency, we found that the China securities industry was cost inefficiently every year from 2005 to 2011. The reason was that the industry was both of productively and allocatively inefficient. Except 2005, the technical efficiency was less influential than the allocative efficiency to the overall efficiency of the China securities industry from 2006 to 2011.

In terms of productivity change, the decompositions of the three types of Malmquist indexes illustrate the reasons of the productivity change of the China securities industry from 2005 to 2011. We found that the cost Malmquist index of the China securities industry regresses between 2005 and 2011. Since the value of the overall efficiency decrease was less than the value of the cost technology regress, the cost-technical change was the main factor to cause the cost Malmquist index regress.

We also showed that the allocation Malmquist index of the China securities industry progress between 2005 and 2011 was due to the allocation technology progress. The allocation technology progress was more influential than the allocation efficiency decrease to the allocation Malmquist index.

Further work could be investigation of the economic interpretation of the allocation Malmquist index, and finding the economic root sources of the productivity change of the cost, production and allocation Malmquist indexes. The sample size discussed in this study was 40, hence, future study direction could be further investigating the performance of China securities companies by using more acquire data and large sample size, this is also a suggestion from authors that the China securities industry regulators should demand all kinds of the securities companies to release more information to the public for analyzing the securities industry and companies.

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Appendix 1: The average values of inputs, outputs and price indexes from 2005 to 2011

DMU	y_1	y_2	y_3	y_4	y_5	x_1	x_2	w_1	w_2
1	833.413	2530.219	43.929	417.154	35.789	1660	11541.702	209.577	0.051
2	1494.203	10779.663	92.089	71.349	56.398	3682	20861.757	126.580	0.035
3	201.037	1222.699	44.955	31.120	19.095	522	3927.569	146.058	0.035
4	271.665	21163.929	187.947	36.173	5.204	864	5486.974	215.936	0.042
5	1017.604	1025.349	44.306	47.948	42.180	2087	11249.782	126.386	0.040
6	1472.471	5094.234	179.923	263.822	40.502	1546	25534.718	362.614	0.029
7	920.415	2012.395	74.077	123.193	11.906	1221	10338.910	226.842	0.053
8	823.150	600.357	43.402	24.078	20.979	1316	7914.422	176.663	0.027
9	1014.326	1048.724	95.081	42.040	14.528	961	11254.658	298.551	0.036
10	2766.431	2155.358	276.461	464.217	51.048	3361	36165.250	310.170	0.029
11	701.340	1557.168	6.609	18.475	14.220	849	6950.201	192.208	0.027
12	4747.952	8384.504	410.968	331.765	82.842	6191	62848.347	304.301	0.028
13	444.495	1233.076	60.281	53.175	18.233	973	11175.943	215.217	0.026
14	1020.395	5958.265	92.061	34.670	17.455	1318	8969.044	197.507	0.061
15	588.179	893.950	140.756	68.304	46.480	1417	6625.369	178.968	0.033
16	830.196	2206.164	45.508	44.736	11.139	684	9051.694	145.068	0.035
17	444.414	749.674	13.460	22.217	4.397	560	3646.984	130.378	0.046
18	5429.641	37166.147	509.964	328.664	165.657	5862	73858.335	318.916	0.030
19	4414.831	1861.226	547.773	422.047	279.376	6833	43541.348	226.397	0.032
20	937.326	2329.450	106.764	57.841	37.873	1582	15834.828	178.240	0.029
21	4444.152	3982.711	456.708	748.430	85.351	5318	71545.517	273.298	0.029
22	1306.091	1441.602	200.676	83.744	45.271	2254	16842.504	175.026	0.032
23	5296.518	3246.132	514.339	167.074	147.746	3964	60019.720	350.579	0.037
24	443.500	1737.509	28.561	52.151	10.369	879	5302.883	207.258	0.041
25	635.541	1689.107	163.356	40.605	7.606	1206	5853.545	150.342	0.043
26	843.285	1117.804	36.606	31.986	14.343	804	8587.489	180.570	0.034
27	875.656	4292.479	904.045	154.279	37.008	2339	17141.672	261.738	0.029
28	2529.941	3432.213	89.513	104.467	68.289	3501	24084.492	159.575	0.040
29	742.377	1122.616	91.817	45.156	14.296	1102	9804.161	195.572	0.023
30	818.675	3567.947	14.576	31.248	24.753	1000	11530.623	232.617	0.034
31	4519.934	4800.326	140.083	285.456	112.182	3530	52953.421	353.960	0.025
32	251.502	1015.317	12.557	372.292	15.324	550	4286.506	217.966	0.043
33	411.403	887.137	14.421	21.398	14.638	851	4467.083	169.870	0.041
34	848.390	1813.303	45.843	64.811	12.675	1467	8711.493	157.538	0.036
35	1354.731	1414.213	111.465	361.860	21.251	1853	16048.707	327.895	0.039
36	3498.997	2026.898	574.869	668.624	57.715	3830	55020.238	369.285	0.028
37	780.893	850.901	2.669	20.302	15.014	1047	7047.485	205.739	0.028
38	7438.985	39733.866	1787.119	2458.524	203.261	9246	117986.660	453.411	0.026
39	994.093	41615.236	449.637	175.503	57.669	781	12775.468	556.724	0.033
40	1014.401	859.808	20.355	28.432	11.597	1319	8334.103	147.459	0.039
Mean	**1735.564**	**5765.492**	**216.888**	**220.483**	**48.791**	**2258**	**22628.040**	**235.825**	**0.035**
SD	**1760.377**	**10261.388**	**324.250**	**402.187**	**57.824**	**1988.576**	**25054.957**	**93.701**	**0.008**

Appendix 2: The OE, TE and AE of the 40 companies in 2005 and 2011

	2010			2011		
DMU	**OE**	**TE**	**AE**	**OE**	**TE**	**AE**
1	0.451	0.813	0.555	0.673	0.780	0.863
2	0.349	0.658	0.531	0.330	0.552	0.597
3	0.360	0.529	0.682	1.000	1.000	1.000
4	0.470	0.850	0.553	0.826	1.000	0.826
5	0.325	0.800	0.406	0.373	0.687	0.543
6	0.970	0.970	1.000	0.772	0.772	1.000
7	0.695	0.984	0.706	0.632	0.814	0.776
8	0.353	0.904	0.390	0.270	0.811	0.333
9	0.813	0.873	0.931	0.647	0.791	0.819
10	0.492	0.708	0.695	0.494	0.705	0.701
11	0.631	0.807	0.782	0.424	0.681	0.622
12	0.527	0.681	0.774	0.403	0.542	0.744
13	0.381	0.436	0.874	0.440	0.530	0.831
14	0.467	1.000 1	0.467	0.667	1.000	0.667
15	0.413	1.000	0.413	0.415	0.866	0.480
16	0.637	0.750	0.849	0.947	1.000	0.947
17	0.759	0.944	0.804	0.818	1.000	0.818
18	1.000	1.000	1.000	0.800	0.803	0.997
19	0.409	0.886	0.462	0.553	0.933	0.593
20	0.538	0.584	0.921	0.473	0.554	0.852
21	0.673	0.698	0.965	0.886	0.886	1.000
22	0.555	0.829	0.670	0.440	0.763	0.577
23	0.633	0.772	0.819	0.705	0.705	1.000
24	0.376	0.888	0.423	0.436	0.957	0.455
25	0.338	0.979	0.346	0.535	1.000	0.535
26	0.996	1.000	0.996	1.000	1.000	1.000
27	0.613	1.000	0.613	1.000	1.000	1.000
28	0.514	1.000	0.514	0.567	0.821	0.691
29	0.475	0.737	0.644	0.472	0.626	0.754
30	1.000	1.000	1.000	0.400	0.409	0.979
31	1.000	1.000	1.000	0.826	0.854	0.966
32	1.000	1.000	1.000	1.000	1.000	1.000
33	0.568	1.000	0.568	0.315	0.700	0.450
34	0.384	0.889	0.432	0.388	0.799	0.486
35	0.671	0.921	0.729	0.671	0.791	0.847
36	0.565	0.604	0.934	0.546	0.624	0.875
37	0.801	1.000	0.801	0.453	0.942	0.481
38	1.000	1.000	1.000	1.000	1.000	1.000
39	1.000	1.000	1.000	1.000	1.000	1.000
40	0.404	0.913	0.443	0.409	0.695	0.589
Mean	**0.615**	**0.860**	**0.717**	**0.625**	**0.810**	**0.767**
SD	**0.227**	**0.151**	**0.217**	**0.228**	**0.163**	**0.200**