Assessing the Queuing Process Using Data Envelopment Analysis: An Application in Health Centres

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Abstract

Queuing is one of the very important criteria for assessing the performance and efficiency of any service industry, including healthcare. Data Envelopment Analysis (DEA) is one of the most widely-used techniques for performance measurement in healthcare. However, no queue management application has been reported in the health-related DEA literature. Most of the studies regarding patient flow systems had the objective of improving an already existing Appointment System. The current study presents a novel application of DEA for assessing the queuing process at an Outpatients' department of a large public hospital in a developing country where appointment systems do not exist. The main aim of the current study is to demonstrate the usefulness of DEA modelling in the evaluation of a queue system. The patient flow pathway considered for this study consists of two stages; consultation with a doctor and pharmacy. The DEA results indicated that waiting times and other related queuing variables included need considerable minimisation at both stages.

Keywords: Data Envelopment Analysis, Healthcare, Queuing, Patient Flow, Appointment Scheduling System.

1. Introduction

The existence of any nation is a function of the survival of its citizens, which in turn depends on the provision of the health care facilities (Ramanathan 2005; Adeleke *et al*., 2009). Factors such as variability in patients' medical complaints or reason for visit as well as corresponding diagnosis proposed by doctors after close examination daily and weekly workload fluctuations and multiple objectives within and between departments, make it extremely challenging to understand the operational performance of a large hospital subsystem (Matta and Patterson, 2007). Given the increasing cost pressures, complexity of diseases and increased demand of quality and efficacy, efficient and smooth provision of health services is becoming extremely important (Ramanathan 2005; Mehandiritta 2011). Despite these challenges, there is a need of continuous assessment of the operational efficiency of hospitals which allows the decision-makers to develop a better understanding of the management effectiveness, and to provide valuable insights into improving resource allocation (Chuang *et al*., 2011).

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There are a number of aspects which indicate fast and efficient services provided by the health organizations around the globe. Some of these include patient throughput, short length of stay at the clinic and low clinic overtime, while maintaining adequate staff utilization rates and low physician idle times. In addition to these factors, low patient waiting time is a significant aspect which demonstrates efficient and effective health services. Excessive waiting and service times cause dissatisfaction among the patients, leading to reduced customer demand, causing them to leave the system without receiving the service and can constitute a barrier to effective treatment (Jun *et al*., 1999; Biju and Naeema 2011; Yeboah & Thomas 2010; Silva and Serra 2008).

1.1 *Excessive Queuing in Developing Countries*

The developing countries of the world face multiple challenges mainly including   high population, low literacy rates, high rate of incidence of diseases and inadequate funding. The healthcare delivery systems in these countries face various issues such as huge number of patients, shortage of physicians and other health personnel, lack of training and post-training supervision of health personnel, late arrival of doctors, lack of medical equipment and aftercare facilities, and improper allocation of resources; resulting in overloading in health facilities and long wait times, , making the life of patients quite miserable (Babes and Sarma 1991; Manzi et al 2014; Mensah *et al* 2015). Almost all large public hospitals in developing countries provide primary and tertiary care facilities to all patients. They do not have General Practitioner (GP) referral systems, unlike developed nations and even for a minor ailment, the patients have to visit these large unwieldy hospitals. Also, the patient flow system in these hospitals consists of different stages from entry till exit. Behind simple broad pathways, immense paper work, and cumbersome admission and lay procedures are involved. Due to these factors, patients experience excessive queuing and the management faces the challenge of dealing with excess demand. There is a need to assess the efficiency of the queuing system by identifying factors which cause long waiting at each stage.

Most of the healthcare institutions in the developed world have a dedicated Appointment System (AS), which not only assists the hospital management to deal with patient flow and organize resources, but also facilitates patients. In numerous developing nations, such as Pakistan, almost all health organizations lack a set Appointment System, resulting in long waiting. The former works have emphasized on evaluating and improving patient flow and scheduling (Bhattacharjee and Ray, 2014) in different departments of the hospital including inpatients (Proudlove *et al*., 2007), outpatients (Cayirli and Veral, 2003), emergency (Gul and Guneri, 2015) and surgical/operations (May *et al*., 2011). However, there are only a limited number of studies which particularly assessed the flow of walk-in patients in health centres (Fetter and Thompson 1966; Rising *et al.,* 1973; Ashton *et al.,* 2005; Cayirli and Gunes 2014). Additionally, these few studies have been conducted in developed countries only and there are almost negligible studies in the developing world. Therefore, there is a dire need to

evaluate a queue system such that it caters for the requirements whilst keeping in mind the present idiosyncrasies; that is all patients are walk-in and Appointment Systems are non-existent.

1.2 *Main Aim and Objectives of the Current Study*

Various studies have been conducted which emphasized on performance assessment of health institutions with respect to patient flow (Bhattacharjee and Ray 2014). There are a number of different Operational Research (OR) techniques which have been used to assess the queuing problems or patient flow systems including, Queuing theory (Fomundam and Hermann, 2007; Mayhew and Smith, 2008; Lakshmi and Sivakumar, 2013), Discrete-event Simulation (Jun *et* al., 1999; Cayirli and Veral, 2003; Gunal and Pidd, 2010; Konrad *et al.,* 2013), System Dynamics (Lane *et al.,* 2000; Brailsford *et al.,* 2004; Gunal 2012) in addition to others. However, none of the queue management studies have used the performance measurement technique of Data Envelopment Analysis (DEA).

DEA is a non-parametric approach, used to assess the efficiency of a decision-making unit (DMU) by modelling the relationship between multiple inputs and outputs (Chuang *et al.,* 2011; Pelone *et al*., 2015). DEA has proven to be an effective and versatile tool for health care efficiency measurement (O'Neill *et al.,* 2008), in addition to applications in other fields such as education, banking, computer engineering, tourism, sports and some others. Liu *et al* (2013) provide a detailed survey of DEA applications in various fields. Among various healthcare applications, DEA has been mostly applied to the efficiency comparison of hospitals (Nunamaker 1983; Parkin and Hollingsworth, 1997; Ouellette and Vierstraete, 2004; Akazili *et al*., 2008; Flokou *et al.,* 2011; Kawaguchi *et al*., 2014). These studies have included a varied set of inputs and outputs in the DEA model which best represent the efficiency criteria. However, none of the previous studies included the queuing problem as part of the performance assessment. Also, very few studies have been carried out in the developing world. The present study represents a novel application of DEA for assessment of the queuing system of a large public hospital in a developing country.

The current study specifically aims at the formulation of a DEA model, using queuing variables, in order to demonstrate its usefulness in the evaluation of the current patient flow system. The objectives of the current study are: (i) to identify excessive varying wait times of patients, (ii) to identify relevant queuing variables affecting the waiting times which need to be included in the DEA model, and (iii) to assess the usefulness of DEA Modeling in analyzing a queuing system.

The remainder of this paper is structured into further four sections. The second section highlights some of the DEA applications in health. The third section describes the development of the DEA model. We have also demonstrated applicability of the proposed model in application of waiting time of patient follows. The fourth section explains the significance of these results for improving the current system including some propositions for the betterment of the patient flow system. The final section explores the opportunities for future research.

2. Data Envelopment Analysis

2.1 DEA Applications in Healthcare

Since the early 1980s, efficiency analysis has been used to measure and analyse the productive performance of health care services (Hollingsworth 2008). Efficiency measurement represents the first step towards the evaluation of a coordinated health care system, and constitutes one of the basic means of audit for the rational distribution of human and economic resources (Ramanathan 2005). One of the most-widely used techniques in this respect is DEA. Nunamaker (1983) and Sherman (1984) were the first published papers which applied DEA in the field of healthcare. Since then, DEA has enjoyed numerous healthcare applications. Some studies have provided a comprehensive review of DEA applications in health (Liu *et al.,* 2013b; Hollingsworth 2012; Hollingsworth 2008; Worthington 2004).

The DEA literature consists of efficiency measurement of different units of analysis within the healthcare sector. These include private versus public hospitals (Chang *et al*., 2004; Jehu-Appiah *et al*., 2015), primary healthcare centres in urban versus rural municipalities (Ramirez-Valdivia *et al*., 2012), acute care hospitals (Hollingsworth and Parkin 1995; Ersoy *et al*., 1997; Puig-Junoy 2000; Tsai and Molinero 2002), family health services authorities (FHSAs) (Salinez-Jaminez and Smith 1996) and district health authorities (DHAs) (Thanassoulis *et al*., 1996). A few other studies were concerned with the efficiency comparison of physicians (Chilingerian and Sherman 1997; Wagner *et al*., 2003), nurses (Osman *et al*., 2011), ambulatory surgery centers (ASCs) (Lewis *et al.,* 2011), immunization activities in primary care practices (Rouse *et al*., 2011) and dialysis facilities (Flokou *et al*., 2011). None of these studies comprise of efficiency analysis of the patient flow system in a hospital. The objective of the current study is to assess the queuing system in a busy public hospital of a developing country using DEA, demonstrating its effectiveness in the assessment of a patient flow system.

2.2 Theoretical Background and DEA Framework

DEA is a non-parametric efficiency measurement technique which does not require any a priori assumptions of the functional form of a production function. It is used to assess the efficiency levels of different units of analysis, known as decision-making units (DMUs), using multiple inputs and outputs. The DEA framework was first developed by Charnes *et al.,* (1978), also known as CCR Model. Later on, the traditional DEA model was extended by Banker *et al.,* (1984), most popularly known as the BCC Model.

The basic concept of DEA comes from the production frontier approach as discussed by Farrell (1957). DEA allows for the construction of an 'efficient frontier' formed by a set of DMUs that demonstrate best practices. It then allocates the efficiency level to all units based on their distances from the efficient frontier. Units which lie on the frontier are said to be 100% efficient, while others which are away from the frontier are inefficient, with efficiency scores below 100%. DEA identifies a

reference set for each inefficient DMU, which comprises of corresponding efficient DMUs that can be used as benchmarks for improvement. DEA also allows for calculation of target values of inputs and outputs for a particular inefficient DMU, which shows the amount required for each of these variables to increase the efficiency level to 100% (Liu *et al.,* 2013a; Lee and Kim 2014).

A DEA model could either be input-oriented or output-oriented. Basically, an input-oriented model is concerned with minimizing the quantity of inputs given that the same amount of outputs is produced. An output-oriented model focuses on maximizing the quantity of output produced subject to the condition that the amount of inputs remains unchanged (Charnes *et al.,* 1978; Banker *et al.,* 1984).

A case of two-input and one-output model can be represented graphically as shown in Figure 1. There are four DMUs, A_1, A_2, A_3 and A_4, each producing the same amount of output, O_1 while using different combinations of inputs, I_1 and I_2. This is an 'input-oriented' model as the objective is to reduce the number of inputs given the same level of output. The line SS' is the estimated efficient frontier for the four DMUs.

Three of the DMUs, A_1, A_3 and A_4 are 100% efficient as they are lying on the frontier. However, A_2 is inefficient as it consumes more resources as compared to other DMUs. A_2 can become 100% efficient if it operates as point M (at the efficiency frontier). Therefore, the efficiency of A_2 at this point is OM/OA_2. This type of efficiency is known as the technical efficiency of a unit. A_1 and A_3 are the 'efficient peers' of A_2, that is, the units which act as a source of guidance for the inefficient unit A_2 to improve its operational efficiency. The DMUs representing the efficient peers are the ones which have the most closely related input/output combination to that of the particular inefficient unit.
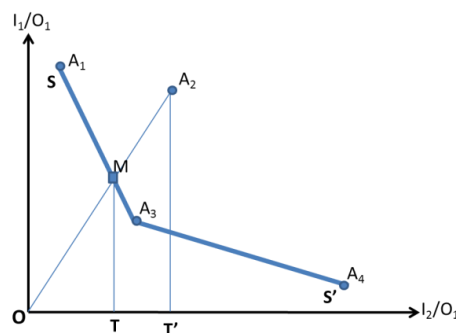
Figure 1: Measuring efficiency graphically for the input-oriented DEA model

However, when the number of inputs and outputs increase, the model becomes more complicated and is solved by constructing a linear programming model.

2.3 Mathematical Representation of DEA

In general terms, the efficiency is defined as the ratio of output to input. Mathematically, the technical efficiency in DEA is defined as the ratio of the weighted sum of outputs to the weighted sum of

inputs. Assuming there are 'n' DMUs, then for any DMU_0 the efficiency score which lies between 0 and 1 (0 to 100%) is defined as $0 \leq \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}} \leq 1$ where;

  s: total number of outputs

  m: total number of inputs

  $\mathbf{y_{rj}}$: amount of 'r^{th}' output used by DMU_j where $r = 1,...,s$ and $j = 1,...,n$

  $\mathbf{x_{ij}}$: amount of 'i^{th}' input used by DMU_j where $i = 1,...m$ and $j = 1,...,n$

  $\mathbf{y_{r0}}$: amount of output r used by the DMU under assessment

  $\mathbf{x_{i0}}$: amount of input i used by the DMU under assessment

  $\mathbf{u_r}$: weight assigned by DEA to output r

  $\mathbf{v_i}$: weight assigned by DEA to input i

Using the above concept Charnes *et al.,* (1978) transferred this to the following linear programming for measuring efficency of DMU_0.

In this mathematical programming, the objective is to obtain the weights u_r and v_i which maximize this ratio of weighted outputs to weighted inputs for DMU_0. The basic idea behind weight allocation is that each unit might value inputs and outputs differently and therefore adopt different weights. Hence, each unit should be allowed to adopt the most favourable weights in order to maximize its efficiency ratio. A dual to model 1 is presented in Model 2.

Model 1: CCR Multiplier Model

Maximize $h_0 = \sum_{r=1}^{s} u_r y_{r0}$
subject to:
$\sum_{i=1}^{m} v_i x_{i0} = 1$
$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \; ; \forall j$
$u_r, v_i \geq 0 \qquad ; \; \forall i,r$

Model 2: CCR Envelopment Model

Minimize $Z - (\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^-)$
subject to:
$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{r0} \; ; \forall r$
$\sum_{j=1}^{n} \lambda_j x_{ij} - Z x_{i0} + s_i^- = 0; \forall i$
$\lambda_j, s_r^+, s_i^- \geq 0 \; ; \forall j,r,i$

In Model 2, if the value of h_0 is 1, and all slacks are zero, then this DMU_0 is efficient relative to other DMUs in the set. In case h_0 is less than 1, this unit is inefficient when compared with others. The original CCR Model operated under the assumption of constant return to scale (CRS). This traditional model was later extended by Banker *et al.,* (1984) to incorporate variable return to scale (VRS) by adding the following additional constraint in Model (2):

$\sum_{j=1}^{n} \lambda_j = 1$

This new constraint was introduced in order to determine whether a firm is operating in the region of increasing, constant or decreasing returns to scale (in multiple input and multiple output situation) utilizing the concept of Sheppard's distance function as shown by Banker *et al.,* (1984) (Emrouznejad and Cabanda 2014).

3. A DEA Model for the Assessment of a Queuing System

3.1 Patient pathway considered for the current study

The Outpatients' department (OPD)of one of the typical public hospitals in Pakistan was observed to identify the major waiting times, where all patients are walk-in. There are different pathways that a patient can follow from the entry till exit. One of them (as shown in Figure 2) is that a patient arrives at the OPD in order to consult with the General Duty Medical Officer (GDMO) and after consultation proceeds to the Pharmacy. In this case, the patients have to wait at two stages, the OPD and Pharmacy. The observations at these two stages showed that the patients have to suffer extensive waiting times. Therefore, there is a dire need to assess the efficiency of this patient pathway mainly in terms of waiting times of patients, and also identify other factors that affect these waits The DEA modelling was utilized in order to evaluate the current queuing system and to provide guidelines for improvement.
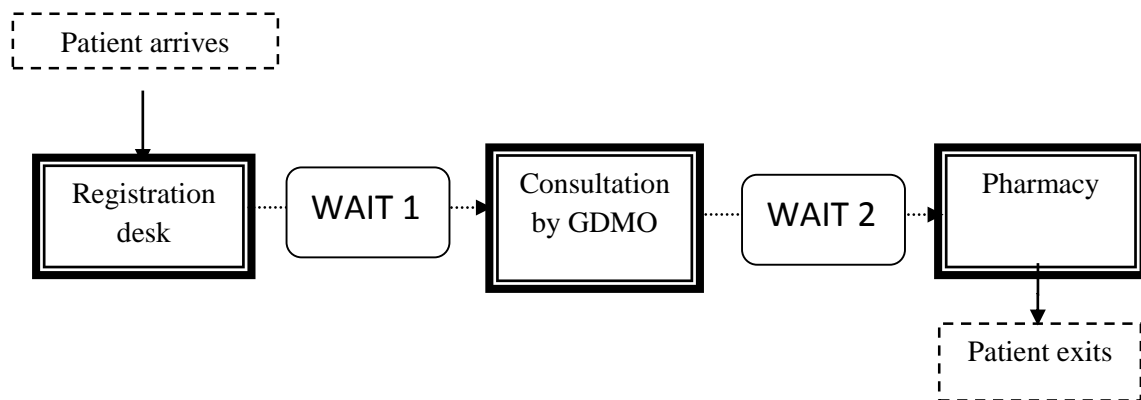
Figure 2: Patient flow system representing two types of waiting times

3.2 Selection of input and output variables in the DEA model

Selection of relevant inputs and outputs is a common issue in DEA (Harrison *et al.,* 2004). Inclusion of inputs and outputs does not only affect the results but also the ability of a particular methodology to provide useful information (Hollingsworth and Parkin 1995). However, the input and output variables included in the DEA model and their interpretation depend on the nature of the problem under consideration. Most of the DEA studies have been conducted in order to compare the efficiency of hospitals. For this purpose, these studies have considered various inputs and outputs. Table 1 shows the most commonly used inputs and outputs in previous DEA studies when comparing hospitals.

Table 1: Most Common Health-related inputs and outputs in DEA

Output variables	Input variables
Number of Outpatient visits *(Bwana 2015 ; Flokou et al., 2011; Masiye et al., 2006 ; Ramanathan 2005*; *Zuckerman et al., 1994*)	Number of beds *(Bwana 2015; Kose et al., 2014, Al-Shammari 1999; Akazili et al., 2008; Harrison et al., 2004; Puig-Junoy 2000; Weng et al., 2009; Magnussen 1996)*

Number of Inpatient visits *(Chuang et al., 2011; Butler and Li 2005; Tsai and Molinero 2002; Zere et al., 2001; Banker et al., 1986 ; Kawaguchi et al., 2014)*	Number of doctors *(Grosskopf and Valdmanis 1987; Kirigia et al., 2008; Chang et al., 2004; Huang et al., 1989; Prior 2006; Kose et al., 2014; Dotoli et al., 2015)*
Number of major and minor surgeries *(Al-Shammari 1999; Chang et al., 2004 ; Puig-Junoy 2000 ; Gerdtham et al., 1999 ; Dotoli et al., 2015)*	Number of nurses *(Chuang et al., 2011; Masiye et al., 2006; Hollingsworth and Parkin 1997; Byrnes and Valdmanis 1993; Kawaguchi et al., 2014)*
Number of emergency visits *(Butler and Li 2005 Grosskopf and Valdmanis 1987; Linna 1998; Kang et al., 2014)*	Other medical and non-medical staff members *(Dotoli et al., 2015; Kirigia et al., 2004; Huang et al., 1989; Hollingsworth and Parkin 1995; Ramanathan 2005; Valdmanis 1992)*
Number of discharges *(Blank and Van Hulst 2011; Puig-Junoy 2000; Hollingsworth and Parkin 1997; Kose et al., 2014; Dotoli et al., 2015)*	Costs: - Expenditures on supplies *(Akazili et al., 2008; Kirigia et al., 2008 ; Kawaguchi et al., 2014)* - Total inpatient costs *(Nunamaker 1983)* - Operating expenses *(Harrison et al., 2004; Tsai and Molinero 2002; Zere et al., 2001; Byrnes and Valdmanis 1993)*

Considering the health-related queue management studies, different researchers have used a number of variables and parameters to assess and improve a particular queuing system. Table 2 shows the most commonly used variables for queuing analysis in healthcare.

Table 2: Most Commonly Used Queuing Variables Used in Healthcare Studies

Queuing variables	Definition
Waiting time	- Difference between arrival time and the time consultation/service begins *(Adeleke et al., 2008;Gul and Guneri 2015; Sorup et al., 2015)* - Difference between arrival time and appointment time *(Harper and Gamlin 2003)* - Waiting time for early or late arrival for appointment *(O'Keefe (1998); Zhu et al., 2012)* - Response time *(Aboueljinane et al., 2013)*
Service Time	- Difference between the time when consultation started and when it ended *(Welch 1964; Hill-Smith 1989; Brahimi and Worthington 1991; Adeleke et al 2009; Harper and Gamlin 2003; Khori et al., 2012; Mankowska et al., 2014)*
Number of Resources	- The number of resources available upon arrival of patients mainly including personnel (doctors/nurses/other staff) and beds *(Lane et al., 2000;Griffiths et al., 2005; Gunal 2012;Mankowska et al., 2014; Gul and Guneri 2015)*
No-shows	- When patients do not arrive for their appointments *(Hassin and Mendel 2008; Klassen and Yoogalingham 2009; Feldman et al., 2014)*
Average Queue length	- The length of the queue at a particular time *(Huarng et al 1996; Cote 1999; Silvester et al., 2004; Gul and Guneri 2015)*
Idle time of doctors	- The amount of time that the doctors are not busy *(Fetter and Thompson 1966; Klassen and Rohlder 1996; Cayirli et al., 2008)*

Taking into account the variables considered by the DEA and queue management studies, and the objective of the current study which is to assess the patient flow system using DEA, the inputs and outputs for the proposed DEA-queuing Model are shown in Table 4. Since the objective is to improve the efficiency of the queue system, an input-oriented DEA model will be run in order to reduce the

waiting time of patients as well as other inputs, while keeping the level of availability of the doctors at the same level. The proposed Model is described in the following sections.

Table 3: Inputs and outputs for the proposed DEA-queuing Model

Inputs	Outputs
Waiting to be seen by GDMO (WAIT1)	Number of GDMOs (NuGDMO)
Waiting at the Pharmacy (WAIT2)	Number of Pharmacists (NuPHAR)
Length of Queue for WAIT1 (LEQ1)	
Length of Queue for WAIT2 (LEQ2)	
Consultation Time (CONS)	
Service Time at the Pharmacy (SERT)	

3.2.1) Assessment of the Queuing System Using Patient level Data in the DEA Model

In order to assess the queuing process, it is extremely important to look at the input and output of each individual patient. The DEA modelling provides detailed analysis for each and every unit of analysis in the dataset. This sort of in-depth analysis is not provided in any of the other OR techniques. Also, it is extremely important to include patients as the 'units of analysis' in the DEA model to analyse the queuing process. The utilization of patient level data is significant as it allows to pinpoint the extreme wait times experienced by patients and the reasons behind them. Usually with other DEA studies, the whole organization is considered as a DMU (for example, hospitals, banks, schools and so on), where one is compared with another. However, the objective of the current study is to evaluate the queuing process in 'one' large public hospital. Therefore, data regarding each individual patient will be gathered and a comparison among them will assist in evaluating the overall queue system.

This queue assessment represents a distinctive application of DEA modelling, moving away from its traditional usage in analysing a pure 'production' process, where inputs are turned into outputs. The current study aims to demonstrate that DEA has other applications in addition to its 'traditional' use.

3.2.2) Outputs of the DEA Model: Number of GDMOs and Pharmacists

It can be observed from Table 1 that none of the health-related DEA studies have used any outputs which are related to the queuing of patients. Most of the variables included in Table 1 cannot be used in the current study since the main objective in these studies is to assess the efficiency level of each 'hospital'; therefore, the inputs and outputs represent aggregate values recorded for every individual DMU (hospital). For instance, 'number of doctors' represents the total number of doctors working at the hospital. However, in the current study, the goal is to identify the efficiency of the current patient flow system in the Outpatients' department within one hospital. Therefore, the value of the number of GDMOs and Pharmacists will be recorded at the time each individual patient arrived at the outpatients' department and pharmacy respectively.

Both these variables will be included in the model as outputs. It can be observed from Table 1 that in DEA studies, staff members, either medical or non-medical, are one of the most widely used inputs. In these studies, the number of personnel (doctors, nurses etc) is included as an input with the objective to minimize the number of staff in order assess and improve the efficiency of a hospital. However, the main goal of the current study is to evaluate and reduce the excessive waiting time of patients. Therefore, the objective is to identify that to what extent does the wait time needs to be reduced given the fixed number of doctors working at that particular time when a patient arrived. Therefore, from this perspective, the number of GDMOs and pharmacists are included as outputs in order to analyse the required reduction in excessive wait times..

Considering these variables as outputs will assist the management in appropriate staff allocation. This information can be used by managers to ensure that an adequate number of staff is working when there are more patients in order to minimize the waiting time.

3.2.3 Inputs of the DEA Model

Waiting to be seen by the GDMO and at the Pharmacy

Almost all prior queuing studies are associated with improving an existing appointment system. Therefore, waiting times are mostly defined with respect to the appointment times as shown in Table 2. As indicated before, the goal of the present study is to formulate a DEA model in a public hospital of a third-world country where there are no appointments. Therefore, the waiting times of 'walk-in' are described differently as compared to previous works. The waiting time to be examined by the GDMO (WAIT 1) is the difference between the time when a patient enters the consultation room and the time of entry at the reception of the Outpatients' Department. The waiting time at the Pharmacy (WAIT 2) is considered as the difference between the time when a patient arrives at one of the pharmacy counters to obtain service and the time of entry at the pharmacy reception.

The wait time of each patient will be recorded to determine the efficiency level of the pathway followed by each individual patient. Both waiting times are included as inputs in the model. Due to lack of resources and overloading of patients in developed nations, there are excessive wait times of the patients. Hence, the current DEA model aims to identify the excessive wait times, which might vary from one patient to another, with the objective of minimizing them. This analysis will assist in identifying high wait times experienced by patients and the factors affecting them, so that appropriate measures can be taken to improve the existing queuing situation.

Also, the waiting time of patients is strongly related to the number of doctors available. Table 3 indicates that waiting times and number of personnel have been considered jointly in most studies to assess the queuing situation. In most of the previous works, the objective is to 'minimize' the number of doctors and observe its effect on the wait times of patients. However, in the current study, the

objective is different in that the relationship of wait time and the number of doctors/pharmacists available is expressed such that wait time is an input and the number of personnel is an output. The idea is to allow for the minimization of wait time given the current availability of doctors and pharmacists, so that proper measures for staff allocation can be undertaken to control the queue problem

Length of Queue for WAIT 1 and WAIT 2

As indicated by Table 2, average queue length has been considered as a significant variable for analysing a particular queuing system. In the current study, queue length is defined as the number of people already waiting when a patient joins the queue. The length of queue is an indicator of an overload in a particular queue system. If more patients are waiting at the time of arrival of a particular patient, the longer will be the queue, hence leading to excessive wait time of the current patient. Therefore, this variable has been included in the DEA model, to observe the extent of overload in the queue system when a patient arrived and its effect on the wait time.

Moreover, the length of queue indicates the level of efficiency of the current system, by highlighting the lack of availability of adequate staff members. Therefore, the queue lengths at both stages, consultation with the GDMO (LEQ1) and pharmacy (LEQ2), will be included as inputs with number of personnel as outputs.

Consultation time and service time at the Pharmacy

One of the most widely used criteria for queuing analysis is the service time as specified in Table 2. In the current study, consultation time has been included in the DEA model as a proxy to service time for WAIT 1 and WAIT 2. It is assumed that the patients arriving at the Outpatients' department have similar medical issues. Hence the actual consultation and diagnosis are similar for all patients.

The consultation in the public hospitals comprises of a number of miscellaneous tasks in addition to the 'actual' consultation. These tasks might include data entry (such as demographic details of patients), carrying out routine activities (such as measuring blood pressure, pulse rate, temperature etc), preparing prescriptions for medicines and some others. These additional tasks take a considerable amount of time during the actual consultation and as a result, increases the waiting time of the subsequent patients. Therefore, the consultation time has been included as an input in the DEA model, in order to pinpoint high consultation times and its effect on the wait time and queue size. The idea is to bring to the attention to the management as to the causes of high consultation so that appropriate measures can be undertaken.

At the Pharmacy, the service time should be minimal since the patients just need to collect their medicines. Excessive time for the distribution of medicines is a sign of inefficiency and requires some intervention. There might be a number of reasons for delayed response at the Pharmacy such as

understanding the given prescriptions, bringing medicines from different sections of the store and some others. Therefore, service time at Pharmacy is included as an input since the objective is to identify the causes of delayed response at the pharmacy and minimize the wait by taking appropriate actions.

Consultation time and service time at the pharmacy are strongly related to the number of personnel available. This evaluation allows for highlighting the issue that increased service times coupled with inadequate staff members at both stages increase the wait times of subsequent patients, hence reducing the efficiency of the system. Therefore, these are included as inputs in this model with number of servers as outputs.

Consultation time included as an input with a pre-specified priority

A number of extensions of the traditional DEA model has been proposed in the DEA literature, and one of them is the inclusion of preference information of the decision-makers. The DEA literature shows that several attempts have been made to incorporate preference information, mainly with the objective of target setting (Halme *et al*., 1999). Golany (1988) proposed the idea that the preference information should be considered in order to allow for selecting the most appropriate efficient output target among different alternatives. Thanassoulis and Dyson (1992) have developed models for target setting which include varying preferences over improvement to the levels of individual inputs and outputs. Zhu (1996) constructed weighted non-radial DEA models that incorporate preference structures, in order to acquire efficient targets and corresponding efficiency scores for each DMU. Athanassopoulos *et al*., (1999) developed a scenario-based process for estimating targets using DEA which is in line with the decision makers' set preferences for the strategies to be adopted.

The traditional DEA assumes that all inputs and outputs are equally important. However, in few cases, some variables might be preferred over others (Halme *et al*., 1999). Thanassoulis and Dunstan (1994) applied the target setting models with different preferences in the education sector where the mean GCSE score output had a higher priority than the output of the percentage of pupils who got a placement on leaving school. In a study on the comparison of suppliers' performance, Liu *et al.,* (2000) considered distance and supply variety as exogenously fixed input variables. Martic and Savic (2001) provided a comparison of different regions in Serbia with respect to social-economic development, considering the input variables of arable area and population as exogenously fixed. Thanassoulis *et al*., (1995) conducted a study on the provision of prenatal care in England, and considered five output quality measures as exogenously fixed. A modified DEA model with weight restrictions was adopted to reflect preferences over these output quality measures. In the present case scenario, consultation time is added in the model with a pre-specified priority.

There are negligible studies in health which incorporate preference information. In the field of healthcare, the inclusion of this preference information is crucial, since some factors indicate the 'quality' of health services provided and affects patient satisfaction. Therefore, these health variables should only be minimized up to a certain level. Hence, by incorporating priorities for some health variables, DEA analysis provides targets which are achievable and realistic, and reflect the preferences of the policy makers. In the current study, the consultation time is considered with a priority. Consultation with a doctor is the most important service provided by a hospital which reflects the level of patient care and is an indicator of patient satisfaction. Therefore, it should not be allowed to decrease below a 'certain' level as it might reflect badly on the quality of service provided. However, considering other pre and post diagnosis activities as part of the consultation, the time can be reduced such that it only consists of the 'actual' consultation. The other activities can take place either before or after the consultation. This will minimize the overall consultation time and reduce the wait time of subsequent patients.

After discussion with the hospital administrators in the index hospital and considering their expert opinion, it was decided that the level of Consultation time should not be allowed to decrease below 50%. The administrators were of the opinion that although there are other tasks carried out during the 'actual' consultation, but still the total Consultation time should not be allowed to reduce for more than 50% due to its significant nature in the provision of health services.

The inclusion of preference information highlights one of the key advantages of DEA modelling. DEA has the ability to incorporate variables with a certain priority, leading to a better representation of a real-life scenario. Furthermore, the priority level for a certain variable can be changed in order to reflect the preference of administrators.

Therefore, a modified DEA model is shown below:

Model 3: Priority setting in the CCR model

Max q

subject to:

$$\sum_{j=1}^{n} x_{ij} \lambda_j + s_i^- = x_{ij_0} (1 - w_i q) \qquad i = 1,\ldots,m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{rj_0} (1 + w_r q) \qquad r = 1,\ldots,s$$

Where:

x_{ij} and y_{rj} are the ith input and the rth output level at DMU_j,

w_i and w_r are the specified priorites,

j_0 is the DMU being assessed.

In order to estimate the radial efficiency of DMU_0 under input minimisation we set $w_i = 100\%$; $\forall\, i$ & $w_r = 0$; $\forall\, r$. (*This conditions are reversed in case of an output maximisation problem;. i.e.* $w_i = 0$; $\forall\, i$ & $w_r = 100\%$; $\forall\, r$)

Now, for the current study, an input-oriented DEA model will be utilized (see Section 4 for details), and only consultation time is added as a priority input variable (with a priority of 50%). Therefore, the above priorities will be re-defined as follows:

$w_i = 100\%$ for i = WAIT 1, WAIT 2, LEQ 1, LEQ 2 and SERT

$w_i = 50\%$ for i = CONS

$w_r = 0$ $\forall\, r$

Model (3) is solved in two stage, where in the second stage we maximise the sum of slacks, as follows, over the same constraints.

$$\text{Max} \sum_{r=1}^{s} s_r^+ F_r^+ + \sum_{i=1}^{m} s_i^- F_i^-$$

Where F_i^- and F_r^+ are the specified priorities **.**

4. Results and discussion

The results obtained from the DEA analysis are expected to highlight the excessive varying waiting times of patients and the factors affecting them in the present queuing system.

For this purpose, some preliminary observations regarding the arrival pattern of patients and the overall queue system were conducted in the busiest outpatients' department of a large public hospital in Pakistan. These observations were conducted for one full week during different times of the day. These initial observations were then used to simulate a dataset for 200 patients, which represents the patient flow in a busy outpatients' department of a public hospital in a developing country in one day. This simulated dataset was then utilized for DEA analysis. In order to conduct DEA analysis, PIM-DEA software[2] (Version 3.2) was employed

Figure 3 demonstrates that 35 units (out of 200) are 100% efficient whereas the remaining 165 units are inefficient. However, units with different 'inefficiency' levels can have varying input/output characteristics. Therefore, to develop a better understanding of the DEA results, the inefficiency levels are further divided into different ranges.

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ww.DEAsoftware.co.uk
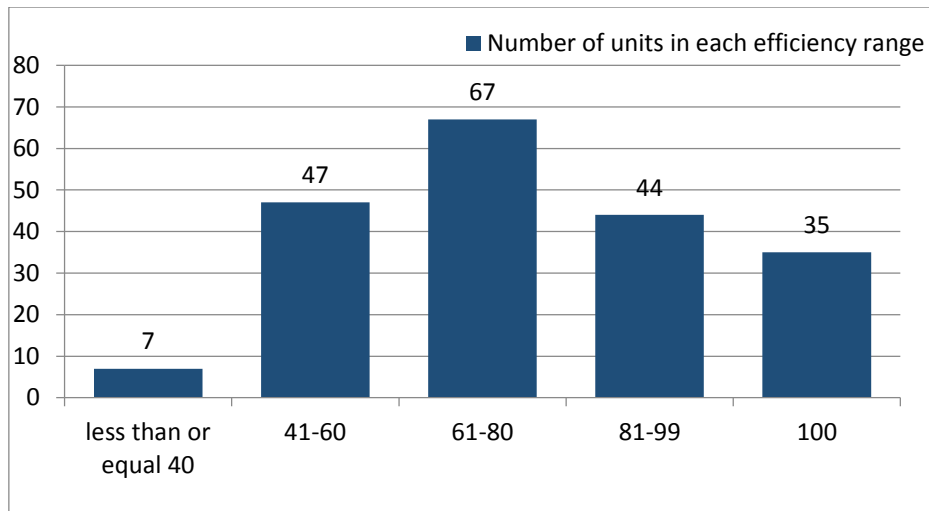
Figure 3: Number of units lying in different efficiency ranges

To show the usefulness of the proposed approach, summary results of some patientsare shown in Table 4, representing the actual and target values of the inputs. The target values indicate the amount by which these inputs can be decreased in order to increase the efficiency level to 100%. This information is very useful since it provides a guideline to the administrators to take appropriate measures for reducing the waiting time in this busy hospital.

Table 4: DEA results for selected patients in an input-oriented model

Patient No	Eff (%)	CONS		LEQ1		WAIT1		LEQ2		WAIT 2		SERT	
		Actual	Target	Actual	Target	Actual	Target	Actual	Target	Actual	Target	Actual	Target
P1	71	20	17	11	8	177	103	29	21	84	60	5	4
P5	100	43	43	9	9	82	82	22	22	38	38	5	5
P17	86	40	26	11	9	96	66	16	14	66	56	7	4
P32	44	24	13	8	4	71	31	18	8	103	45	5	2
P38	58	23	18	13	6	117	44	17	10	134	43	4	2
P56	23	21	11	13	3	144	33	21	5	224	51	7	2
P64	53	16	12	8	4	114	51	24	13	170	87	5	3
P75	100	32	32	10	10	64	64	14	14	66	66	2	2
P81	40	39	14	10	4	59	23	27	9	138	55	5	2
P87	89	32	30	11	9	63	56	20	14	112	73	4	4
P95	100	29	29	8	8	42	42	11	11	84	84	7	7
P111	39	37	18	13	5	132	44	14	5	170	66	5	2
P120	91	13	12	10	9	135	63	23	19	102	93	4	4
P183	70	15	13	5	4	105	74	29	17	136	69	5	3
P194	67	39	21	8	5	174	60	16	11	167	112	4	3

For instance, consider P5, although Wait 1 and Queue length 2 are higher, but Consultation time, Queue length 1 and Wait 2 are lower, which leads to P5 being 100% efficient. For P95, Wait 2 has slightly higher values with lower values for all other inputs. P75 has the lowest values for all inputs among the three efficient units. Based on these results, it can be said that Consultation time of around 30 minutes, Queue length 1 and 2 of around 10 patients and Wait 1 and 2 of around an hour represents an efficient patient flow system. However, slightly higher values of some the inputs while reduced values of others can compensate each and result in an overall efficient unit.

Considering the inefficient units, P56 is the most inefficient among all with an efficiency level of just 23%. One of the reasons is the extremely high value of Wait 2 which is nearly 4 hours. Also, Wait 1 is around 2 hours. Hence, although consultation time and queue length 1 are not that high but the target values show that they should be reduced to 11 and 3; respectively. In order to balance high waiting times, other inputs should be reduced considerably with the purpose of making the process efficient. For P81, Wait 2 is over 2 hours; hence although Wait 1 is almost an hour, it needs to be reduced further in order to increase the efficiency level of the unit. Also, the consultation time and queue length 1 and 2 need to be minimized. Similar observations can be made about other patients which have efficiency levels less than or around 50% such as P32, P38, P64, and P111. The service time for units with efficiency level around 50% is around 5 minutes. However, due to extremely high waiting times, even this service time has a target value of 2 minutes. This is possible as the medicines are already prepared and just need to be handed over to the patients. Therefore, this process could be further speeded up. However, it can be observed that patients have to wait for nearly 4 hours (in this case especially) just to collect medicines which hardly takes 3 minutes on average.

For units of analysis which have efficiency levels of nearly 90%, such as P22, P87 and P120, it can be seen that the target values for queue length 1 is between 6 and 10 while that of queue length 2 is between 14 and 19. These results are different from those units which have efficiency levels of less than 50% as the target values are much lower than the actual values. Wait 2 has high values of nearly two hours for these units. However, for patients with around 90% efficiency, this waiting time is at most around an hour and a half..

The results show the effectiveness of the DEA model for identifying queuing problems in a busy public hospital. The values indicated by 100% efficient units (patients) displays the input values which can act as a benchmark for the inefficient units in order to render the queuing system efficient. The comparison among the actual and target values of wait times and other related queuing variables for different patients were useful in identifying the high and low values for all variables which rendered the 'whole' queue system inefficient. This information provided by DEA analysis proved to be extremely useful for the administrators as it provides evidence that certain patients have waiting time and other values as compared to others, which leads to a varying efficiency level of the queue system as a whole. DEA modelling in this regard, provided a comprehensive analysis of the queuing process, by evaluating the pathway followed by each individual patient. Therefore, DEA modelling allowed for identifying the wait times and other related factors for each patient separately. Most of the other OR techniques rely on average values which lack this level of detail as provided by DEA, by assessing the wait times of each patient. The patient flow system in public hospitals of developing countries is complex since all patients are walk-in and the number of patients is not fixed, leading to extreme variability in results. This in-depth analysis is crucial since the management can identify the

varying wait times and other related factors so that they can take appropriate measures considering the whole queuing process, with information about the pathway followed by each patient.

5. Conclusion and Recommendations

The current study represents a distinctive application of DEA for queue management. The main objective is to develop a DEA model for assessing the queuing system of an Outpatients' department of a large public hospital in a developing country, where patients do not have pre-booked appointments. The analysis of this model plays a vital role in providing information to the hospital management about the extensive waiting times and associated factors that need to be addressed in order to improve the queuing system. Mainly, the results of the proposed model showed that the units which have less than 50% efficiency level have high waiting times. The target values demonstrate that immense reduction is required in waiting time values and other associated factors in order to increase the efficiency level to 100%.

Furthermore, an extended DEA model was utilized, which allowed for the inclusion of consultation time with a preference. The ability of DEA modelling to incorporate this preference information provides a more realistic representation of a service, such as healthcare. Hence, the model developed in the current study is dynamic in the sense that it can be adopted by any public hospital, with either the same or different level of priority of any variable depending on the preference by the administrators.

Considering the DEA results, the management can approach the problem of extensive waiting times by adopting appropriate staff scheduling strategies. For instance, more staff members (GDMOs, pharmacists and other medical/non-medical personnel) are required to be present during the busiest time periods to cater for the high patient flow. Therefore, some additional information regarding the patient flow during different times of a day or different days of the week is required to identify the busiest times. The consultation might be reduced by identifying different activities that take place during this time. It might be possible that these routine tasks (such as temperature, blood pressure, other pre-diagnosis information from the patient etc) are conducted prior to when the 'actual' consultation starts. This will allow for proper effective treatment of the patients, leading to increased satisfaction. The analysis has shown that long waiting times are involved at the pharmacy. Therefore, it might be possible to set up a small pharmacy in the Outpatients' department consisting of the most frequently prescribed medicines. This will greatly reduce the load in the main Pharmacy. The main Pharmacy can be utilized to provide more uncommon or rare medicines. In this way, the flow of patients will be distributed, hence reducing the waiting times. The current study provides evidence of the usefulness of DEA modelling for queuing analysis. However, this is the first step to assess a bottlenecked patient flow system in a developing country, using DEA analysis. A detailed study

specifically aiming at evaluating some other factors affecting the queuing system in a developing country hospital with additional supporting data is required, in order to provide specific recommendations for improvement.

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