Invited Paper

Strong localization of Whispering Gallery Modes in an optical fiber via asymmetric perturbation of the translation symmetry

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ABSTRACT

Recently introduced Surface Nanoscale Axial Photonics (SNAP) is based on whispering gallery modes circulating around the optical fiber surface and undergoing slow axial propagation. In this paper we develop the theory of propagation of whispering gallery modes in a SNAP microresonator, which is formed by nanoscale asymmetric perturbation of the fiber translation symmetry and called here a nanobump microresonator. The considered modes are localized near a closed stable geodesic situated at the fiber surface. A simple condition for the stability of this geodesic corresponding to the appearance of a high Q-factor nanobump microresonator is found. The results obtained are important for engineering of SNAP devices and structures.

Keywords: whispering gallery modes, optical fiber, slow light, microresonators

1. INTRODUCTION

Different types of high Q-factor optical microresonators have been demonstrated such as microrings, microspheres, microtoroids, microbottles, microfiber loops and coils, microbubbles, etc.[1–5] Generally such devices and circuits can be created by manipulation (melting, etching, machining, looping, coiling, blowing, etc.) of uniform and tapered optical cylinders, disks, fibers, and capillaries. Recently, a new platform for fabrication of microresonators and microresonator photonic circuits and devices at the surface of an optical fiber has been introduced.[6] This platform, called Surface Nanoscale Axial Photonics (SNAP), is based on the effective nanoscale variation of the fiber radius, which combines variations of the fiber physical radius and its refractive index, and used for controlling of WGMs which circulate along the fiber surface and slowly propagate along its axis. The transverse input waveguide (microfiber) is used to launch these WGMs.

Here we develop the theory of a new type of SNAP microresonator, a nanobump microresonator, which is formed by asymmetric perturbation of the translation symmetry of an optical fiber. This resonator can be created by local directional annealing of the fiber with a CO_2 laser beam. Generally, in SNAP, the introduced local deformation of the fiber is not axially symmetric. Furthermore, this deformation can be much smaller than the original transnationally symmetric asymmetry of the fiber. Nevertheless, this nanoscale deformation can cause complete localization of WGMs and formation of the nanobump microresonator. Our theory of a nanobump microresonator is based on the theory of WGMs localized near a stable closed geodesic. For a nanobump microresonator, this geodesic coincides with a circumference at the fiber surface (Fig. 1).

The paper is organized as follows. In Section 2 we describe the theory WGMs localized in the vicinity of a stable closed geodesic at the fiber surface. In Sections 3 and 4 we derive analytical expressions for the eigenmodes and eigenvalues of a nanobump microresonator and present numerical examples. The cases of axially symmetric and axially asymmetric microresonators are considered. The results of this paper are discussed and summarized in Section 5.

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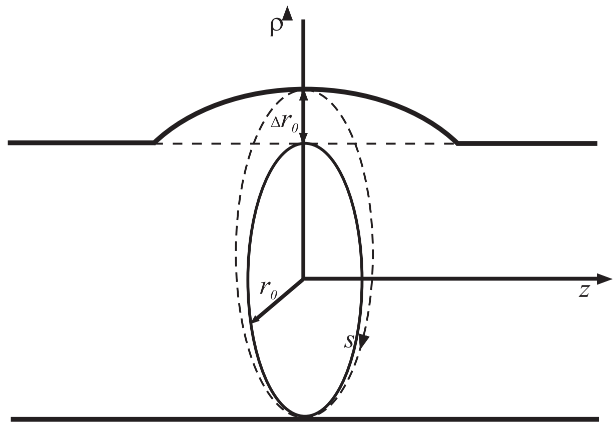

Figure 1. Illustration of a nanobump microresonator. Dashed curve is a closed geodesic (optical ray).

2. EIGENFUNCTIONS LOCALIZED IN THE VICINITY OF A CLOSED GEODESIC

WGMs localized near a closed geodesic (optical ray) at the fiber surface can be described using the frame of reference introduced in the vicinity of this geodesic, (s, ρ, z), where s is the coordinate along the geodesic, and ρ and z are the radial and axial cylindrical coordinates (Fig. 1). Here s is related to the azimuthal angle φ of the cylindrical frame of reference as $s = r_0\varphi$. Then, the asymptotic expression for the eigenfunctions localized near the geodesic has the form:[7]

$$E_{mlp}(s,\rho,z) \approx r^{-1/6}(s)\sigma^{-1/2}(s)\exp\left[\imath\int^{s}\beta(s)ds + \frac{\beta_p}{2\sigma(s)}\left(\imath\frac{d\sigma}{ds} - \frac{1}{\sigma(s)}\right)z^2\right]\times$$
$$H_m\left(\frac{\beta_p^{1/2}z}{\sigma(s)}\right)Ai\left(-\left(\frac{2\beta_p^2}{r(s)}\right)^{1/3}\rho - \zeta_l\right), \tag{1}$$

where

$$\beta(s) = 2\beta_p - \zeta_l\left(\frac{\beta_p}{2r^2(s)}\right)^{1/3} - \left(m + \frac{1}{2}\right)\frac{1}{\sigma^2(s)}. \tag{2}$$

In Eq. (1) $H_m(x)$ - Hermite polynomial; $Ai(x)$ - Airy function; ζ_l are the roots of Airy function ($\zeta_0 = 2.338, \zeta_1 = 4.088, \zeta_2 = 5.52, \ldots$); $r(s)$ - local radius of curvature of the geodesic; $\beta_p = p/r_0 \approx 2\pi n_f/\lambda$; r_0 - original fiber radius; n_f - the fiber refractive index; λ - the radiation wavelength; m, l, p - quantum numbers. Function $\sigma(z)$ in Eq. (1) is defined as

$$\sigma(s) = \sqrt{\sum_{i,j=1}^{2} a_{ij}\theta_i(s)\theta_j(s)}. \tag{3}$$

Here $\theta_{1,2}$ are the Floquet solutions of the linear differential equation:

$$\frac{d^2\theta_i}{ds^2} + K(s)\theta_i = 0, \tag{4}$$

where $K(s)$ is the Gaussian curvature of the fiber surface at the geodesic s and a_{ij} are free parameters, related by the equality:

$$\det\|a_{ij}\|\left(\theta_1\frac{d\theta_2}{ds} - \theta_2\frac{d\theta_1}{ds}\right)^2 = 1. \tag{5}$$

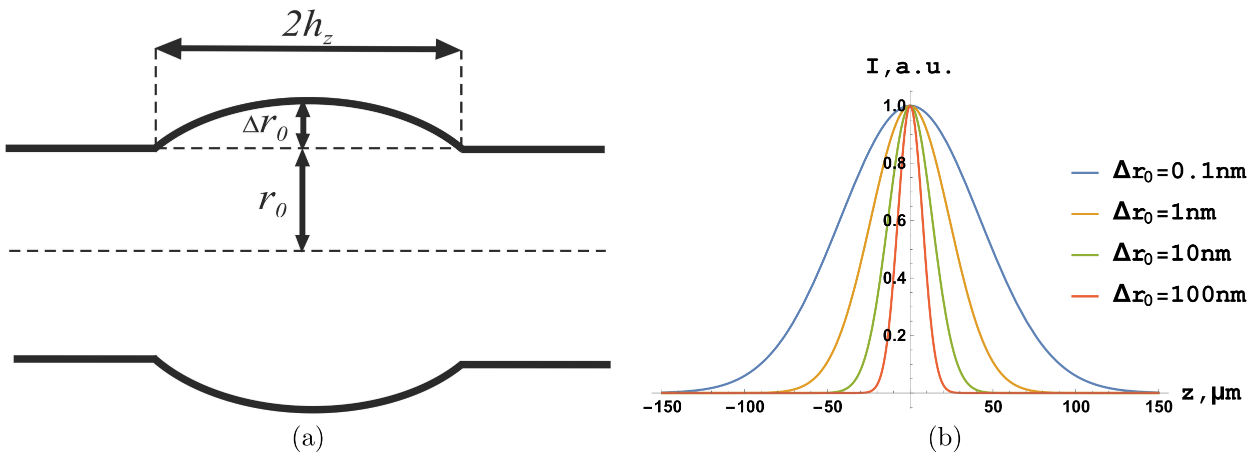

Figure 2. (a) - Illustration of axially symmetric microresonator; (b) - axial distribution of the fundamental WGM field intensity inside the axially symmetric microresonator for different values of parabolic nanobump height Δr_0. The rest parameters are: $r_0 = 40\mu m$; $h_z = 50\mu m$.

3. AXIALLY SYMMETRIC MICRORESONATOR

For the axially symmetric microresonator, illustrated in the Fig. 2(a), the Gaussian curvature $K(s) = K_0$ is independent of s and solutions of Eq. (4) are $\theta_{1,2}(s) = \exp(\pm\imath K_0^{1/2} s)$ where

$$K(s) = K_0 = \frac{1}{R_0 r_0}. \tag{6}$$

Here R_0 is the axial radius of the nanobump. The individual WGMs can be obtained by setting $a_{11} = a_{22} = 0$ and $a_{12} = a_{21} = 1/2K_0^{1/2}$. Therefore, it follows from Eq. (3) that $\sigma^2(s) = 1/K_0^{1/2}$. Substitution of these expressions into Eq. (1) yields:

$$E_{mlp}(s,\rho,z) \sim \exp\left[\imath\beta(s)s - \frac{1}{2}\left(\frac{\beta_p^2}{R_0 r_0}\right)^{1/2} z^2\right] H_m\left[\left(\frac{\beta_p^2}{R_0 r_0}\right)^{1/4} z\right] Ai\left[-\left(\frac{2\beta_p^2}{r_0}\right)^{1/3}\rho - \zeta_l\right]. \tag{7}$$

From Fig. 2 (a) it follows that if h_z is an axial length and Δr_0 is a height of the axially symmetric nanobump than R_0 can be expressed as $R_0 = h_z^2/2\Delta r_0$. Fig. 2 (b) shows the axial distribution of the fundamental WGM field intensity determined from Eq. 7. It is seen that the WGMs are strongly localized inside the resonator and the localization width weakly depends on Δr_0.

4. NANOBUMP MICRORESONATOR

Experimentally, SNAP structures are usually formed by the axially asymmetric nanoscale deformations of the optical fiber shape and refractive index[6] as illustrated in Fig. 3(a). As a model of such deformation, we consider the microresonator with the profile:

$$r(z,s) = r_0 + \frac{\Delta r_0}{\left((1+(s/h_s)^2)(1+(z/h_z)^2)\right)^2}. \tag{8}$$

Here Δr_0 is the height of the bump and parameters h_s and h_z determine the characteristic dimensions of the bump along s and z axes, respectively. It can be shown that, for small deformation, the Gaussian curvature of the fiber surface at the geodesic can be obtained from

$$K(s) = -\frac{r_{zz}(0)}{r_0}. \tag{9}$$

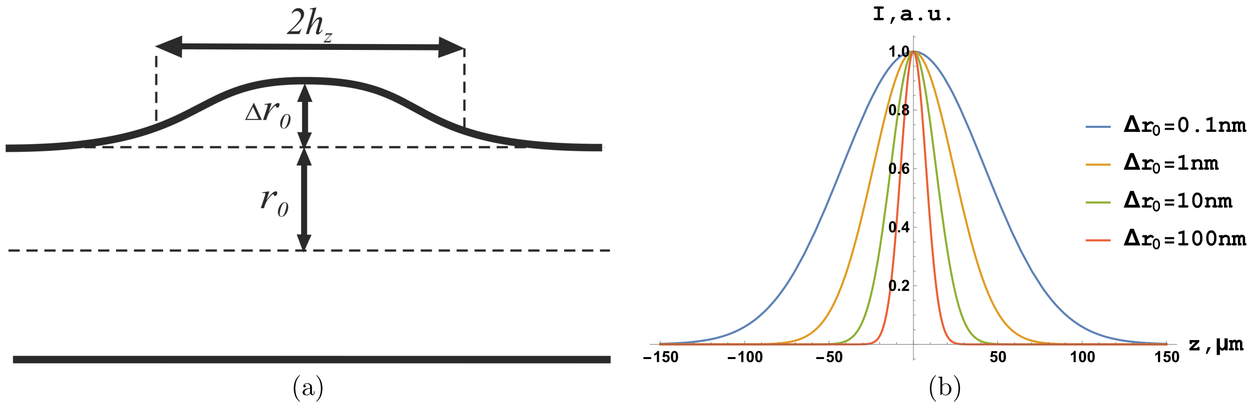

Figure 3. (a) - Illustration of nanobump microresonator; (b) - axial distribution of the fundamental WGM field intensity inside the nanobump microresonator for different values of parabolic nanobump height Δr_0. The rest parameters are: $r_0 = 40\mu$m; $h_z = 50\mu$m; $h_s = 20\mu$m.

After substitution of Eq. (8) this equation yields:

$$K(s) = \frac{4\Delta r_0 h_s^4}{r_0 h_z^2 (s^2 + h_s^2)^2}. \tag{10}$$

In this case, Eq. (4) has the form

$$(s^2 + h_s^2)^2 \theta'' + \frac{4\Delta r_0 h_s^4}{r_0 h_z^2}\theta = 0, \tag{11}$$

and can be solved analytically. As the result, we find the condition for the stability of microresonator as:

$$0 < \frac{\pi^2 \Delta r_0 h_s}{h_z^2} < 1. \tag{12}$$

Thus, the closed geodesic is stable and the WGMs are fully localized if the deformation is small. The expression for the WGMs are found in this case from Eq. (1) and

$$\sigma(s) = \frac{(h_z r_0)^{1/2}}{(\Delta r h_s)^{1/4}} - \frac{2s ArcTan\left[s/h_s\right] (\Delta r_0 h_s)^{3/4}}{r_0^{1/2} h_z^{3/2}}. \tag{13}$$

As follows from Eq. (1), function $\sigma(s)$ describes the evolution of the characteristic width of the mode along peripheral part of the fiber. From Eq. (13), this function has its minimum in the center of the bump. However, the difference between the field axial width near the bump and on the other part of the microresonator is small, so the azimuthal distribution of the WGMs is close to uniform.

Fig. 3(b) shows the distribution of fundamental WGM field intensity inside the nanobump microresonator. As in the previous case of the axially symmetric microresonator, the WGMs are strongly localized inside the resonator and the localization widths weakly depends on Δr_0.

5. CONCLUSION

We have introduced a new type of WGM resonator a nanobump microresonator and developed its theory. It is shown that infinitesimally small axially asymmetric deformation of the optical fiber is sufficient to fully localize WGMs propagating along the fiber surface. Simple conditions for the stability of a closed geodesic (optical ray), in the vicinity of which these WGMs are localized, are determined. The results obtained are important for understanding of the effect of fiber nonuniformities on the localization of WGM and for engineering of advanced nonuniform SNAP devices.

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