Nonlinear Semi-Analytical Model for Simulation of Few-Mode Fiber Transmission

Filipe Ferreira, *Student Member, IEEE*, Sander Jansen, *Senior Member, IEEE*,
Paulo Monteiro, *Member*, *IEEE*, and Henrique Silva, *Member*, *IEEE*

Abstract—In this paper, a nonlinear semi-analytical model (NSAM) for simulation of few-mode fiber transmission is proposed. The NSAM considers the mode mixing arising from the Kerr effect and waveguide imperfections. An analytical explanation of the model is presented, as well as simulation results for the transmission over a two mode fiber (TMF) of 112 Gb/s using coherently detected polarization multiplexed quadrature phase-shift-keying modulation. The simulations show that by transmitting over only one of the two modes on TMFs, long-haul transmission can be realized without increase of receiver complexity. For a 6000 km transmission link, a small modal dispersion penalty is observed in the linear domain, while a significant increase of the nonlinear threshold is observed due to the large core of TMF.

Index Terms— Nonlinear Fiber Optics, Optical Fibers, Optical Fiber Communication, Multimode Fibers, Few-Mode Fibers.

I. INTRODUCTION

THE exponential increase in communications traffic is rapidly exhausting the optical fiber bandwidth available in the low loss window. In order to overcome this limit, considerable effort has been directed towards the development of advanced modulation formats, in order to enhance the spectral efficiency using denser Quadrature Amplitude Modulation (QAM) constellations [1,2]. However, modulation formats with denser constellations require a higher optical signal to noise ratio (OSNR) and have a reduced nonlinear tolerance. As a result, the feasible transmission distance of these modulation formats is significantly reduced. One radical solution is to increase the fiber core radius. However, as the fiber core radius increases the fiber becomes multimode.

Few-mode fibers (FMFs) have been proposed as a good

compromise between the decrease of the nonlinear coefficient and the introduced modal dispersion [3]. Due to the high mismatch of phase velocity between modes, these fibers present very weak mode coupling, such that long-haul single mode propagation has been already experimentally demonstrated [3]. Furthermore, FMFs have a theoretical larger information transport capacity, since each mode has a capacity which basically equals or doubles the capacity of a single mode fiber (SMF), depending of the mode degeneracy [4]. To take advantage of this additional capacity, Mode Division Multiplexing (MDM), has been proposed [5]. However, the complexity of the equalizer at the receiver is significantly higher than that for single mode transmission.

In this paper, we present a nonlinear semi-analytical model (NSAM) for simulation of FMF transmission, taking into account the mode coupling resulting from waveguide imperfections (Section II), and the Kerr nonlinear effects (Section III). Section IV provides the simulation setup and discussion of results. Conclusions are drawn in Section V.

II. WAVEGUIDE IMPERFECTIONS

The waveguide imperfections [6] caused by perturbations introduced during the fabrication process or by mechanical stresses imposed on the fiber in the field were modeled by random fluctuations of the core center position, given by: $\varepsilon_r(x,y,z) = \varepsilon_{r0}(x+\delta x(z),y+\delta y(z),z)$, where ε_r is the perturbed relative permittivity, ε_{r0} is the ideal relative permittivity, δx and δy are the random displacement of the abscissa and ordinate coordinates, respectively. The proposed model divides the fiber in multiple sections, each with a constant random displacement of the core center position. Therefore, each section has constant coupling coefficients. In order to analytically describe the crosstalk arising from the waveguide imperfections, we used the coupled wave theory [6], particularly the following coupled-mode equations:

$$\frac{\partial \tilde{A}_{\mu}}{\partial z} = -j\left[\beta_{\mu} - \beta_{0\mu}\right]\tilde{A}_{\mu} - j\frac{\beta_{\mu}}{|\beta_{\mu}|}\sum_{v} C_{\mu v}\tilde{A}_{v}e^{j(\beta_{0\mu}-\beta_{0v})z} \quad (1)$$

$$C_{\mu v} = \frac{\omega\varepsilon_0}{4}\iint_{-\infty}^{+\infty}\left[\varepsilon_r\left(x,y\right)-\varepsilon_{r0}\left(x,y\right)\right]\mathbf{E}_{\mu}\cdot\mathbf{E}_{v}^{*}dxdy \quad (2)$$

where $\tilde{A}_{\mu}(z,\omega)$ is the Fourier transform of the mode μ slowly varying field envelope A_{μ}, $\beta_{\mu}(\omega)$ is the mode μ propagation constant, and $\beta_{l\mu}$ is the lth order coefficient of a Taylor series

expansion of $\beta_\mu(\omega)$ centered at the carrier frequency ω_0. $C_{\mu\nu}$ are the coupling coefficients given by the area integral of the dot product of the electrical fields of mode μ and mode ν, over the area where the permittivity difference between the ideal fiber (ε_{r0}) and the perturbed one (ε_r) is nonzero.

Considering, the simple case of a TMF, where only the coupling between the LP_{01} mode ($\mu = 1$) and the LP_{11} mode ($\mu = 2$) is present, (1) can be written in a simpler form that can be solved analytically in each section [6]:

$$A_1(z) = e^{j(\Delta\beta/2)z}\left[\cos sz - j\frac{\Delta\beta}{2}\frac{\sin sz}{s} \quad -j\kappa\frac{\sin sz}{s}\right]\begin{bmatrix}A_1(0)\\A_2(0)\end{bmatrix}$$
(3)

$$A_2(z) = e^{-j(\Delta\beta/2)z}\left[-j\kappa^*\frac{\sin sz}{s} \quad \cos sz + j\frac{\Delta\beta}{2}\frac{\sin sz}{s}\right]\begin{bmatrix}A_1(0)\\A_2(0)\end{bmatrix}$$
(4)

where $\Delta\beta = \beta_1 - \beta_2$, $s^2 = \kappa\kappa^* + (\Delta\beta/2)^2$, and $\kappa = C_{12} = C_{21}^*$. From (3) and (4), it can be concluded that the coupling strength depends on the relation between $|\kappa|^2$ and $\Delta\beta^2$.

A uniform distribution was assumed for the random position (r_d, θ_d) of the core center. In order to get a typical value for the maximum random displacement ($r_{d,max}$), the proposed model was applied to high birefringence (HiBi) fibers and the results were compared with published experimental measurements. The characteristics of the FlatCladd PMF fibers shown in table I of [7] were considered. Their birefringence is $B = \Delta n = 2.5\times10^{-4}$, and the mode coupling parameter is $h = 5.9 \times10^{-6}$. The mode extinction ratio (MER) along the fiber length l is given as a function of h by $MER = 10\log_{10}(\tanh(hl))$ [7], which is used to fit experimental results.

Finally, the simulation results and the experimental fitted function were compared. For a $r_{d,max}$ of 1% of the core radius (r_{core}) and a 50 meter simulation step size, considering perfect launch coupling to one of the modes, a very good agreement of the evolution of the power in each of the modes along the fiber length was obtained. Therefore, this model was adopted for the transmission simulations described in section IV.

III. KERR NONLINEAR EFFECTS

The Kerr effects were modeled by deriving the nonlinear pulse propagation equation for a multimode fiber, starting by writing the induced nonlinear polarization as a function of the electric field expansion into the two orthogonal polarization components of the N orthogonal modes supported by the fiber. Following a derivation similar to the one present in [8] for single mode fibers, the generalized coupled nonlinear Schrödinger equations can then be written as:

$$\frac{\partial A_{\mu i}}{\partial z} + \beta_{1\mu i}\frac{\partial A_{\mu i}}{\partial t} - \frac{j\beta_{2\mu i}}{2}\frac{\partial A_{\mu i}}{\partial t} + \frac{\alpha_{\mu i}}{2}A_{\mu i} =$$
$$-j\left[\gamma_{\mu\mu ii}|A_{\mu i}|^2 + 2\gamma_{\mu\nu ii}\sum_{\nu\neq\mu}|A_{\nu i}|^2 + \frac{2}{3}\gamma_{\mu\nu ij}\sum_{\nu}|A_{\nu j}|^2\right]A_{\mu i}$$
(5)

where i and j are the orthogonal states of polarization of each mode μ. $A_{\mu i}(z,t)$, $\beta_{1\mu i}$, $\beta_{2\mu i}$ and $\alpha_{\mu i}$ are the slowly varying field envelope, group velocity parameter, group velocity dispersion parameter and attenuation parameter for the i polarization of the μ mode, respectively. $\gamma_{\mu\nu ij}$ is the nonlinear coupling parameter between the i polarization of mode μ and the j polarization of mode ν, which depends on the nonlinear refractive index n_2 of the silica, approximately 2.6×10^{-26} m^2/W [8], and on the intermodal effective area, and is given by:

$$\gamma_{\mu\nu ij} = n_2\frac{\omega_0}{c}\frac{\iint_{-\infty}^{+\infty}|E_{\mu i}|^2|E_{\nu j}|^2\,dxdy}{\left(\iint_{-\infty}^{+\infty}|E_{\mu i}|^2\,dxdy\right)\left(\iint_{-\infty}^{+\infty}|E_{\nu j}|^2\,dxdy\right)}$$
(6)

where $E_{\mu i}(x,y)$ is the mode field transverse distribution for the i polarization of mode μ.

In (5), the first term of the right side is responsible for modal self-phase modulation (mSPM) of the polarization i of the mode μ. The second term results in modal cross-phase modulation (mXPM) from the same polarization (i) of different waveguide modes ($\nu\neq\mu$). The third term results also in mXPM, but coming from the orthogonal polarization (j) of the same ($\nu=\mu$) or different waveguide modes ($\nu\neq\mu$).

Finally, considering simultaneously the nonlinear contribution to the permittivity and the linear perturbation of the permittivity due to fiber structure imperfections, a novel integrated model was derived. Following the reasoning used in (1) and (5), the result obtained after some algebra was:

$$\frac{\partial A_{\mu i}}{\partial z} + \beta_{1\mu i}\frac{\partial A_{\mu i}}{\partial t} - \frac{j\beta_{2\mu i}}{2}\frac{\partial A_{\mu i}}{\partial t} + \frac{\alpha_{\mu i}}{2}A_{\mu i} =$$
$$-j\left[\gamma_{\mu\mu ii}|A_{\mu i}|^2 + 2\gamma_{\mu\nu ii}\sum_{\nu\neq\mu}|A_{\nu i}|^2 + \frac{2}{3}\gamma_{\mu\nu ij}\sum_{\nu}|A_{\nu j}|^2\right]A_{\mu i}$$
$$-j\frac{\beta_{\mu i}}{|\beta_{\mu i}|}\sum_{\nu a}C_{\mu\nu ia}A_{\nu a}e^{j(\beta_{0\mu i} - \beta_{0\nu a})z}$$
(7)

In order to solve (5) and/or (7), in a nonlinear dispersive media, the split-step Fourier method was used.

IV. SIMULATION SETUP, RESULTS AND DISCUSSION

In this section, the weak nonlinearity of a TMF was evaluated through the transmission of traffic over one of the supported modes, LP_{01} or LP_{11}. The fiber considered has a step-index profile with a cladding refractive index of 1.46, a relative index gradient at the core–cladding interface $\Delta = (n_{co} - n_{cl})/n_{co} = 2.5\times10^{-3}$, and a core radius of 9 µm. By solving the Maxwell equations for these parameters, and using the weak guiding approximation, was obtained an effective mode index difference of $\Delta n \approx 1.32\times10^{-3}$, consequently a modal delay of ~4 ps/m. For this fiber the average MER obtained at the end of 75 km. using (3) and (4), is ~-20 dB, for $r_{d,max} = 0.01 \times r_{core}$. Table I gathers some of the TMF properties at 1550 nm, together with those of the SMF used as reference for performance comparison.

The simulations were performed considering the LP

approximate modes. The phase relation variations between the exact modes of the decomposition of LP_{11} (TM_{01}, TE_{01} and HE_{21}) [4], was accounted for during the simulations as polarization mode dispersion (PMD), as shown in Table I.

TABLE I
FIBER PROPERTIES AT 1550 NM.

Property	Unit	TMF LP_{01}	TMF LP_{11}	SMF LP_{01}
Dispersion	ps/(nm km)	22.64	20.86	15.35
Dispersion slope	ps/(nm^2 km)	0.063	0.053	0.057
A_{eff}	μm^2	206.7	322.9	91.4
γ - intramodal	W^{-1}/km	0.510	0.326	1.153
γ - intermodal	W^{-1}/km	0.314	0.314	-
PMD	ps/√km	0.069	0.687	0.069
Attenuation	dB/km	0.22	0.22	0.22

In the transmitter/receiver in order to launch/detect selectively one of the TMF modes, an adiabatic taper can be used to launch/detect the LP_{01}, and for the LP_{11} long-period fiber gratings (LPFG) can be used [5]. This setup introduces slight additional complexity, and during simulations their performance was considered ideal. In all transmission simulations performed, periodic amplification with 75 km spans was assumed to be realized with single stage erbium-doped fiber amplifiers (EDFA), where only the wanted mode was amplified and the unused mode was removed. The modulation format used was 112 Gb/s coherently detected polarization multiplexed (CP) quadrature phase-shift-keying (QPSK). A single channel, centered over the 1550 nm wavelength was transmitted. The receiver considered was the blind coherent digital receiver presented in [9], using twofold oversampling and frequency-domain equalization to compensate for the bulk of accumulated chromatic dispersion. After this, the signal is retimed, demultiplexed, and equalized in a butterfly 13 taps FIR filter. Finally, the phase is recovered from the constellation diagram. A 45 GHz second-order optical Gaussian filter and a 28 GHz fifth-order electrical Bessel filter were considered. The bit-error-ratio (BER) after transmission was measured using Monte Carlo simulation, considering EDFAs with a spontaneous emission factor of 2 and a gain adjusted to compensate for the total span losses.

Fig. 1 shows $\log_{10}$(BER) after 6000 km as a function of the average input power, considering a 50 meter step size and $r_{d,max} = 0.01$ x r_{core}. In the linear region, the TMF results show a power penalty of less than 1 dB, compared with the SMF, due to the linear mode coupling between LP_{01} and LP_{11} of the TMF. This low penalty (in the absence of nonlinear effects) confirms the weak coupling strength given by (3) and (4), since |κ| ranges from 0 to ~60, and $\Delta\beta \approx 5000$, and already confirmed experimentally in [3]. However, for high launch powers where the optical performance is dictated by nonlinear effects TMF clearly outperforms SMF. Compared to the nonlinear threshold of SMF an increase with ~4.5 dB and ~6.5 dB of the maximum optical launched power for a BER = 3.8x10^{-3} is observed for LP_{01} mode and LP_{11} mode, respectively. These simulations improvements are related with the higher effective area of the TMF modes, compared to the

SMF, as shown in Table I. This results demonstrate that by transmitting over only one of the two modes on TMF, long-haul transmission can be realized providing a significant increase in nonlinear threshold compared to transmission on SMF without increase of receiver complexity (except for the necessary mode converters). These enhancements are in line with the experimental results presented in [3] for transmission over the LP_{01} of a similar TMF.
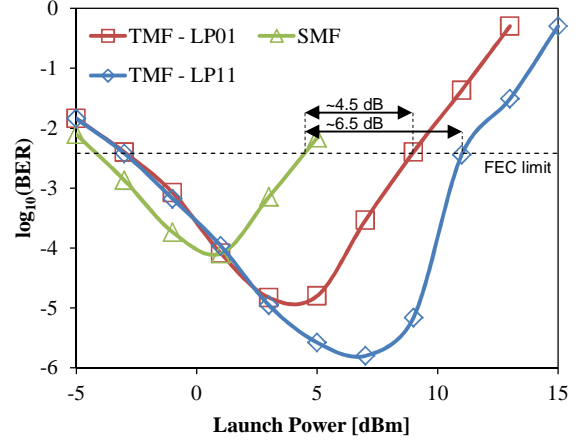

Fig. 1. $\log_{10}$(BER) as function of the mode launch power.

V. CONCLUSION

This paper proposes a NASM for simulation of FMF transmission, including the modal coupling arising from waveguide imperfections and from the Kerr nonlinear effects. Therefore, this model is a valuable tool for the development of future high-capacity multimode fiber systems.

The simulation results obtained provide a validation of the weaker nonlinearity offered by TMFs. These fibers outperformed SMFs due to a much higher effective area, and a low modal dispersion penalty, showing that TMFs can be used for long-haul transmission without increase of receiver complexity (except for the necessary mode converters).

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