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A COST MALMQUIST PRODUCTIVITY INDEX CAPTURING GROUP PERFORMANCE

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Abstract

This paper develops an index for comparing the productivity of Groups of operating units in cost terms when input prices are available. In that sense it represents an extension of a similar index available in the literature for comparing groups of units in terms of technical productivity in the absence of input prices. The index is decomposed to reveal the origins of differences in performance of the groups of units both in terms of technical and cost productivity. The index and its decomposition are of value in contexts where the need arises to compare units which perform the same function but they can be grouped by virtue of the fact that they operate in different contexts as might for example arise in comparisons of water or gas transmission companies operating in different countries.

Key words: Data Envelopment Analysis, Malmquist index, productivity, cost efficiency.

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1. INTRODUCTION

Efficiency and productivity are major sources of economic development and a thorough understanding of the factors affecting productivity is important for managers, economists and policy makers, especially in difficult times of economic crisis where better performance is paramount for sustainability and progress. It is not surprising, therefore, that in recent decades the measurement and analysis of performance has enjoyed a great deal of interest and has seen major developments from a theoretical, methodological and empirical point of view. The measurement and analysis of efficiency and productivity evolved for a long time as independent scientific fields but in recent years the two have merged in a common framework and in this context often efficiency is incorporated in productivity analysis, which is deemed a better approach by many.

We address here the case where operating units are using multiple inputs to secure multiple outputs and input prices are exogenous and available. Further, we address the case where the units in question perform the same function, using the same types of inputs to secure the same kinds of outputs, but are operating in different contexts. One case in point is the increasing need to conduct comparisons across countries. For example see Haney and Pollitt (2012) on the international comparison of electricity transmission companies. Clearly companies performing the same function but in different countries can be grouped as operating in different contexts (e.g. on prices and regulatory regimes).Even within a given country, however, often operating units performing the same function can differ by context. For example the branches of a bank may differ in terms of scope of activities and types of clientele depending on whether they operate in a rural or urban environment. In such cases input costs, e.g. for labour and capital assets, may differ between groups of units as well as within units of a given group. Comparisons therefore of units need to isolate and measure the impact of group membership on productivity.

This issue has already been addressed by a number of authors. The concept of 'metafrontiers' has been developed to isolate group membership from 'managerial' effects on efficiency and productivity (e.g. see Battese et al., 2004, and O'Donnell et al., 2008). These approaches assume each group has its own best practice frontier but that there is a *metafrontier* which envelops all individual group frontiers. This allows one to decompose the evaluated unit's attainment into a

part attributable to the unit itself (i.e. its own management) and a part attributable to group membership. More recently Brennan et al. (2014) have considered groupings of operating units by environmental context and have developed models for estimating an index to capture the impact on productivity change attributable to the context of each grouping. They have prior notions as to more and less favourable operating contexts.

An alternative to the metafrontier approach for comparing groups of units on performance is put forth by Camanho and Dyson (2006). This approach relies on assessing units within existing groupings without recourse to a metafrontier. Recourse to a metafrontier implies that there is an expansion of technology by convexification of existing group technologies. The Camanho and Dyson (2006) approach does not make this assumption. Further, it does not distinguish between more or less favourable operating contexts. Thus the Camanho and Dyson (2006), and therefore the approach in this paper too, is less demanding of prior assumptions.

The Camanho and Dyson (2006) approach compares groups of operating units where the focus is on technical efficiency and prices of inputs or outputs either do not exist or are ignored. Our paper builds on the Camanho and Dyson (2006) approach to address the case where input prices are available and they may differ for units both within and across groups. As in their case we make no recourse to the notion of a metafrontier and make no prior assumptions as to whether operating in one group as opposed to another is necessarily advantageous and whether that holds for all input-output mixes and or scale sizes. The index developed here, as we will see later, offers a number of advantages over more traditional metafrontier based approaches for comparing groups of DMUs on performance. The advantages stem from the fact that the index developed here takes into account both technical and cost efficiency. Further, more than the metafrontier approach, it is decomposable both in cost and technical terms at several levels, enriching the insights that can be gained into group performance. We return to the advantages and drawbacks of the index at the concluding section.

Turning to measures of productivity, there are alternative approaches to quantifying productivity and a very popular one is the **Malmquist** productivity index. Malmquist's(1953) seminal work stayed unnoticed and without any applications for some time. Caves *et al.* (1982) reintroduced it to productivity measurement and subsequently, Färe and Grosskopf (1992, 1996), Grosskopf

(1993), Färe *et al.* (1989, 1994, 1996, 1997, 1998), Portela and Thanassoulis (2006, 2010) further elaborated the approach. A major extension of the index was its decomposition into a measure capturing efficiency change and one capturing technical changeover time by Färe *et al.* (1994). We refer to this here as the 'classical' Malmquist index. The part measuring efficiency change measures the shift of the individual unit relative to its frontier overtime while technical change captures the shift of the production boundary itself over time. The index can be computed in the empirical context using DEA models. Under certain conditions the Malmquist index approximates other popular indices such as the Törnqvist (1936) and the Fisher (1922) index. These two indices are easy to compute and they have been shown to be exact for general forms of technology, but in the presence of inefficiency they may provide biased (see Coelli*et al.* (1998)) estimates of productivity and thus the Malmquist index is preferable.

The classical Malmquist index Färe et al. (1994) was generally developed for cases where technical efficiency in terms of input-output levels was the focus and input prices either did not exist or were ignored. Later a parallel strand of the literature evolved which takes input prices into account where they are available. In this case an important form of efficiency, namely allocative, is contributory to productivity change in cost terms. Allocative efficiency captures the degree to which an already technically efficient production unit can further reduce its aggregate cost of securing its outputs by selecting an optimal mix of inputs given the exogenously fixed prices at which it can secure its inputs. Allocative efficiency and its change may affect performance significantly and this is important in light of empirical studies which have identified frequent instances of allocative inefficiency at production units. In such cases production units may improve over time their performance by changing the input mix they employ to produce their output. Hence the impact of allocative efficiency change on productivity change should be accounted for (Coelli*et al.* (1998)) when input prices are available. In this context, Bauer (1990) and Balk (1998) decomposed, in the econometric and index number framework respectively, productivity change so that allocative efficiency change is captured. Maniadakis and Thanassoulis (2000, 2004) developed a cost Malmquist productivity index, computed through DEA models, which is decomposed into technical change and overall efficiency change which captures costs. The index is defined in terms of cost rather than input distance functions and is applicable when producers can be assumed to be cost minimisers and input-output quantity and

input price data are available. This index has seen many applications in various settings including health care, banks, electricity units, real estate, forest product industries, and educational programmes and it has also seen further extensions (Hosseinzadeh et al. 2007).

Camanho and Dyson (2006) address the case where units can be grouped by operating context. They developed measures, based on the Malmquist index, that enable the decision making unit's internal inefficiencies to be distinguished from those associated with the group (or program) to which the unit belongs. The present paper extends this idea to show how the Cost Malmquist index of Maniadakis and Thanassoulis (2004) can be used to build on the Camanho and Dyson (2006) ideas so as to compare groups of operating units in cost terms. The paper develops an overall index that captures the relative productivity in terms of cost between units belonging to different groups. The index is then decomposed to reveal the impact of technical and allocative efficiency at group level. Information of this type would be useful for managing the performance of groups of units. It would enable managers to identify best practice across groups and this would be both in terms of technical and cost efficiency.

The remainder of this paper is organized as follows. Section 2 provides a review of literature on the classical Malmquist index; the cost Malmquist index and group (technical) Malmquist index. Section 3 develops the cost Malmquist index for comparing groups of units on productivity. Section 4 develops the decomposition of the index defined in Section 3. Section 5illustrates the index developed by means of a numerical example. Section 6concludes.

2. TECHNICAL BACKGROUNG ON MALMQUIST INDICES

2.1 Definitions

Let us assume that in time period t, producers are using inputs, $x^t \in R_+^m$ to produce outputs $y^t \in R_+^s$ and the technology of production can be captured in terms of the *input distance function* (Shephard 1953) as:

$$D_i^t(y^t, x^t) = Sup_\theta \left\{\theta : \frac{x^t}{\theta} \in L^t(y^t), \theta > 0\right\}, \qquad (1).$$

where the subscript *i* denotes input orientation and $L^t(y^t)$ is the set of input vectors x^t which can secure the output vector y^t. When input prices, $w^t \in R_+^m$, are available one may define technology in terms of the *cost function*, which is:

$$C^t(y^t, w^t) = min_{x^t}\{w^t x^t : x^t \in L^t(y^t)\} \qquad (2),$$

where $w^t\ x^t = \sum_{n=1}^{m} w_n^t\ x_n^t$ the subscript n denoting the nth input. $C^t(y^t, w^t)$ defines the minimum cost of producing a given output vector y^t given the input prices w^t and the technology of period t. The set of input vectors x^t which correspond to the scalar $C^t(y^t, w^t)$ lie on an *isocost line* which defines a *cost boundary* which is the locus of the input vectors that, given the technology and input prices, are capable of securing output y^t at the cost of $C^t(y^t, w^t)$.

2.2 The Classical Malmquist Index

Caves *et al.* (1982) adapted to productivity measurement an index that was first proposed by Sten Malmquist (1953) in the context of consumer theory. Assuming technical efficiency, the authors treated the index as a theoretical one and showed its relation to the Törnqvist (1936) quantity index, which under certain conditions is consistent with flexible representations of the technology. Färe *et al.* (1989) relaxed the assumption of technical efficiency and used the index for the first time in an empirical context. Assume two time periods t and t+1 respectively and define in each one of them technology and production as shown in the previous section. The input oriented Malmquist (IM) productivity index is as in (3):

$$IM = \left[\frac{D^t(y^{t+1}, x^{t+1})}{D^t(y^t, x^t)}\ \frac{D^{t+1}(y^{t+1}, x^{t+1})}{D^{t+1}(y^t, x^t)}\right]^{1/2} \qquad (3).$$

The distance functions in the index in (3) are defined as in (1) and with reference to the constant returns to scale (CRS) production boundary. For unit j_0, the term $1/D_o^t(x^t, y^t)$ can be computed using models such as that in (4):

$$\frac{1}{D_o^t(x^t,y^t)} = \min_{\varphi,\lambda_i} \ \varphi$$
$$subject\ to\ \sum_{j=1}^{n} \lambda_j x_{ij}^t \leq \varphi x_{ij_o}^t, i = 1, \dots, m$$
$$\sum_{j=1}^{n} \lambda_j y_{rj}^t \geq y_{rj_o}^t, r = 1, \dots, s,$$
$$\varphi, free; \lambda_j \geq 0, j = 1, \dots, n. \qquad (4).$$

Färe and Grosskopf (1994), Färe et *al.* (1997) showed that the constant returns to scale (CRS) - based index measures productivity accurately irrespective of whether the true form of the technology is CRS or variable returns to scale (VRS). The index is in effect the geometric mean of two indexes. The first uses as a reference the production boundary of period t and the second that of period t+1. When the productivity index computed by (3) is less than one it indicates progress, in the sense that lower input levels are needed to secure given output levels. An index score greater than 1 implies productivity regress and constant productivity is signalled by an index score of 1. Färe *et al.* (1989) showed how to decompose the index into technical efficiency change and technical change and Färe et *al.* (1994) showed how to decompose technical efficiency change further into a scale and a pure technical efficiency change when the technology is VRS. However, the Färe et *al.* (1994) decomposition for VRS technologies only captures technology change of the CRS frontier rather than the true (VRS) frontier. Ray and Desli (1997) develop for VRS technologies a decomposition which captures technology change with reference to the true (VRS) frontier on which the unit sits or can be projected. However, some of the components defined in the Ray and Desli (1997) decomposition may not be computable for some of the units. This happens for those units whose data may not be enveloped by data in a different period (or Group in our case). For more details on how to compute the classical Malmquist Index and its components see Thanassoulis (2001, chapter 7).

2.3 A Cost Malmquist Productivity index

Maniadakis and Thanassoulis (2004) proposed a cost Malmquist index, which is applicable when producers are cost minimizers and input prices are known. Specifically, the Cost Malmquist (CM) productivity index is as in (5):

$$CM = \left[\frac{w^t x^{t+1} / C^t(y^{t+1}, w^t)}{w^t x^t / C^t(y^t, w^t)} \frac{w^{t+1} x^{t+1} / C^{t+1}(y^{t+1}, w^{t+1})}{w^{t+1} x^t / C^{t+1}(y^t, w^{t+1})} \right]^{1/2} \qquad (5),$$

Where $w^t\ x^t$=$\sum_{n=1}^{m} w_n^t\ x_n^t$ and the cost functions $C^t(y^t, w^t)$ are with reference to the CRS technology. The cost ratios in (5) represent distances or inflation (deflation) factors in the terminology of Caves *et al.* (1982). These factors are defined in terms of input quantities in the IM index in (3) but they are defined in terms of input costs in the CM index in (5). Just as with the IM index, a CM index value less than 1 implies productivity progress, a value greater than 1 implies regress and a value of 1 indicates constant productivity in terms of aggregate cost of inputs controlling for output. The CM index can be decomposed in a similar manner tothe IM indexinto *overall efficiency change* and *cost- technical change*. Moreover, both of these components can be further decomposed into input quantity and input price components as detailed in Maniadakis and Thanassoulis (2004).

DEA can be used to compute the CM index as follows. Let us have in each time period production units *j=1,2* ,…,*n*. In period *t*, the j_0th unit employs amount $x_{ij_o}^t$ (*i=1, 2,…, m*) available at prices $w_{ij_o}^t$ *(i=1, 2,…, m*). For unit j_0 the cost of securing its output is $w^t x^t = \sum_{i=1}^{m} w_{ij_o}^t\, x_{ij_o}^t$. Similarly the costs denoted $w^{t+1}x^{t+1}$, $w^t x^{t+1}$ are respectively $\sum_{i=1}^{m} w_{ij_o}^{t+1}\, x_{ij_o}^{t+1}$ and $\sum_{i=1}^{m} w_{ij_o}^{t}\, x_{ij_o}^{t+1}$. For unit j_0, the term $C^t(y^t, w^t)$ can be computed using models such as that in (6):

$$
\begin{aligned}
C^t(y^t, w^t) &= \min_{x_i,\lambda_i} \sum_{i=1}^{m} w_{ij_o}^t x_i \\
\sum_{j=1}^{n} \lambda_j x_{ij}^t &\le x_i \ \ , i = 1, \dots, m \\
\sum_{j=1}^{n} \lambda_j y_{rj}^t &\ge y_{rj_o}^t, r = 1, \dots, s, \\
\lambda_j \ge 0, x_i \ge 0, j &= 1, \dots, n, i = 1, \dots, m.
\end{aligned}
\qquad (6).
$$

In the model above $w_{ij_o}^t$ is the price of input *i* for DMU j_0 at time period t. x_i, *i=1,2,...,m* as well as λ_j, *j=1,2,...,n* are the variables of the model. The cross period cost $C^t(y^{t+1},w^t)$ is computed using model (6) after changing *t* to *t*+1 in $y_{rj_o}^t$ (ie using period t+1 output levels for unit j_0) while the constraints and prices remain as they are, using period t data. The model in (6) relates to CRS technologies. For VRS technologies the convexity constraint $\sum_{j=1}^{n} \lambda_j$ =1 is added to the constraints to model (6).

2.4 Group performance Malmquist indices

As noted earlier Camanho and Dyson (2006) developed measures which make it possible to compare groups of decision making units (DMUs) on performance in terms of technical rather than cost efficiency. In outline, the Malmquist index for measuring Group Performance developed by Camanho and Dyson (2006) is as follows. Consider δ_A DMUs in Group *A*, using inputs $X^A \in R_+^m$ to produce outputs, $Y^A \in R_+^s$ and δ_B DMUs in Group *B*, using inputs $X^B \in R_+^m$ to produce outputs, $Y^B \in R_+^s$. The DMUs operating in Group *A* are represented by their input-output vectors as $\left(X_j^A, Y_j^A\right)$ for $j = 1, 2, ..., \delta_A$. A similar notation is used for Group *B*. $D^A\left(X_j^B, Y_j^B\right)$ represents the input distance function for DMU j of Group *B* with respect to the frontier of units in Group *A*. The Malmquist index for measuring the productivity of DMUs in Group A relative to that of DMUs in Group B, I^{AB} is defined in (7).

$$I^{AB} = \left[\frac{\left(\prod_{j=1}^{\delta_A} D^A\left(X_j^A, Y_j^A\right)\right)^{1/\delta_A}}{\left(\prod_{j=1}^{\delta_B} D^A\left(X_j^B, Y_j^B\right)\right)^{1/\delta_B}} \frac{\left(\prod_{j=1}^{\delta_A} D^B\left(X_j^A, Y_j^A\right)\right)^{1/\delta_A}}{\left(\prod_{j=1}^{\delta_B} D^B\left(X_j^B, Y_j^B\right)\right)^{1/\delta_B}}\right]^{1/2} \quad (7).$$

The superscript AB in I^{AB} is used to indicate that the distance functions of the DMUs in Group A are in the numerator of the definition of I^{AB}. The numerator of the first fraction within the square root (outer) bracket computes the geometric mean of the distance of the DMUs in Group *A* from the efficient frontier of that group. The denominator of that fraction computes the geometric mean of the DMUs in Group *B* again from the Group *A* frontier. As the frontier is constant the ratio of the geometric means concerned reflects the productivity of the DMUs in Group *A* compared to that of the DMUs in *B*. The larger the fraction value the larger the distance of the DMUs in Group *A* compared to those in Group *B* from the referent frontier used and hence the worse the productivity of the DMUs in Group *A* compared to that of the DMUs in Group *B*. The converse is the case if the fraction value is below 1 and if it is 1 then on average the DMUs in the two groups have similar productivity.

The second fraction in the square root brackets is interpreted in a similar manner, the only difference being the referent frontier used is that of the DMUs in Group *B*. Thus the overall square root value is interpreted in the same way as each one of its individual component fractions and so the larger the value of the index I^{AB} the worse the productivity of the DMUs in A compared to those in B so that those in *A* consume more input than the DMUs in *B* for the same output.

The overall productivity measure in (7) can be decomposed into the following components:

$$I^{AB} = \frac{\left(\prod_{j=1}^{\delta_A} D^A\left(X_j^A, Y_j^A\right)\right)^{1/\delta_A}}{\left(\prod_{j=1}^{\delta_B} D^B\left(X_j^B, Y_j^B\right)\right)^{1/\delta_B}} \times \left[\frac{\left(\prod_{j=1}^{\delta_A} D^B\left(X_j^A, Y_j^A\right)\right)^{1/\delta_A}}{\left(\prod_{j=1}^{\delta_A} D^A\left(X_j^A, Y_j^A\right)\right)^{1/\delta_A}} \times \frac{\left(\prod_{j=1}^{\delta_B} D^B\left(X_j^B, Y_j^B\right)\right)^{1/\delta_B}}{\left(\prod_{j=1}^{\delta_B} D^A\left(X_j^B, Y_j^B\right)\right)^{1/\delta_B}}\right]^{1/2} \quad (8).$$

The first term in (8) measures the mean distance of the DMUs in *A* from their own frontier to that of the DMUs in Group *B* from their own frontier. The ratio of these distances reflects the relative spread of the DMUs in each group. The larger the value of the first term in the RHS of (8) the further on average are the DMUs in A from their own frontier than are those of Group B from their own frontier. The value of this ratio cannot convey a measure of relative productivity of units in each group as the referent boundaries differ between the numerator and the denominator. The first fraction in the square root bracket in (8) uses the DMUs in Group *A* as referents both in the numerator and the denominator and so it captures the distance between the boundaries of groups *A* and *B*. The second fraction in the square root captures again the distance between the boundaries of the two groups of units, using this time the DMUs in Group *B* as referents. Thus the square root value is a measure of the distance of the boundaries of the two groups akin to the boundary shift in measuring productivity change over time. The larger the

value of the square root the less productive the frontier units of Group *A* compared to those of the frontier of Group *B*.

Thus in effect the Malmquist index is adapted here in order to be used in a single time period and to compare the productivity of groups of units. In this context, the Malmquist index is multiplicatively decomposed into an index reflecting the efficiency spread among DMUs operating in each group, and an index reflecting the productivity gap between the best-practice frontiers of the two groups.

3. A COST MALMQUIST INDEX FOR COMPARING GROUPS OF UNITS

We propose in this paper a Malmquist Index for comparing Groups of DMUs on productivity in terms of costs for the case where input prices are available and exogenous. That is for the case where DMUs are *price takers* in the sense that the input prices actually paid by the DMU are determined by the market which it cannot influence in any substantial way. To illustrate the derivation of this index consider δ_A DMUs in Group A, using inputs $X^A \in R_+^m$ to produce outputs $Y^A \in R_+^s$, and δ_B DMUs in Group B, using inputs $X^B \in R_+^m$ to produce outputs $Y^B \in R_+^s$. DMU j of Group A has input price vector $\boldsymbol{W_j^A}$ and $\boldsymbol{W_j^B}$ is defined in an analogous manner for DMU j of Group B. DMU j operating in Group A is represented by its input-output vector (X_j^A, Y_j^A) and input prices $\boldsymbol{W_j^A}$. A similar notation is used for DMUs in Group B. $C^A(Y_j^A, W_j^A)$ represents the minimum cost at which DMU j of Group A can secure its outputs as computed using the model in (6) with reference its input-output levels and input prices in the technology defined by DMUs in Group A. For DMUs $j = 1,2,\dots,\delta_A$ define now the cost efficiency of DMU j as:

$$CE^A(X_j^A, Y_j^A, W_j^A) = \frac{C^A(Y_j^A, W_j^A)}{W_j^A X_j^A} \qquad (9).$$

Following the Malmquist-type index developed by Maniadakis and Thanassoulis (2004) and the group comparison index of Camanho and Dyson (2006), we define a **cost Malmquist index** for comparing Groups A and B of DMUs on costs of output production, as follows.

$$CI^A = \frac{\left(\prod_{j=1}^{\delta_A} W_j^A X_j^A \Big/ C^A\left(Y_j^A, W_j^A\right)\right)^{1/\delta_A}}{\left(\prod_{j=1}^{\delta_B} W_j^B X_j^B \Big/ C^A\left(Y_j^B, W_j^B\right)\right)^{1/\delta_B}} = \frac{\left(\prod_{j=1}^{\delta_B} CE_j^{A,B}\right)^{1/\delta_B}}{\left(\prod_{j=1}^{\delta_A} CE_j^A\right)^{1/\delta_A}} \qquad (10).$$

The notation $\boldsymbol{C^A}$ denotes the referent technical frontier for computing the minimum cost for an output bundle is that defined by the DMUs in Group A. $\boldsymbol{CE^{A,B}}$ similarly denotes that the referent technical frontier for computing the cross-group cost efficiency of a DMU in Group B is that defined by the DMUs in Group A.

The within-group cost efficiency $\boldsymbol{CE^A}$ of DMU j of Group A is computed using the model in (6) as noted above. The cross-group inverse of cost efficiency $\boldsymbol{CE_j^{A,B}}$ is $\boldsymbol{W_j^B X_j^B} \Big/ \boldsymbol{C^A(Y_j^B, W_j^B)}$.

The denominator $C^A\left(Y_j^B, W_j^B\right)$ is computed using the model in (6) modified as in (11):

$$\begin{aligned} C^A\left(y_j^B \quad, w_j^B\right) &= \min_{x_i,\lambda_j} \sum_{i=1}^m w_{j_o}^B x_i \\ subject\ to\ \sum_{j=1}^n \lambda_j x_{ij}^A &\le x_i \quad, i = 1, \dots, m \\ \sum_{j=1}^n \lambda_j y_{rj}^A &\ge y_{rj_o}^B, r = 1, \dots, s, \\ \lambda_j \ge 0, x_i \ge 0, j &= 1, \dots, n, i = 1, \dots, m. \end{aligned} \qquad (11).$$

The numerator of the rightmost ratio in (10) captures the geometric mean of the cost efficiencies of the DMUs in Group *B* relative to a cost frontier based for each DMU of Group B on its own input prices applied to the technical frontier of the DMUs in Group *A*. The denominator of the rightmost ratio in (10) captures the geometric mean of the cost efficiencies of the DMUs in Group *A* relative to their own cost frontier based on their own technical frontier. Thus, since the same technical frontier (that of the DMUs in Group A) is used for the numerator and the denominator the **index** labelled CI^A in (10) captures the productivity in **cost terms** of the DMUs in Group *B* relative to that of the DMUs in Group *A* given each the input prices it faces. The larger the value of CI^A the higher on average the cost efficiency of the DMUs in Group B

compared to that of the DMUs in Group A. When $CI^A > 1$ it means in percentage terms DMUs in Group B have lower scope for cost savings than DMUs in Group A. As we have used the same referent technology boundary in computing CI^A for both groups of DMUs, had the DMUs in the two Groups had identical input prices (all DMUs and both Groups) $CI^A > 1$ would signal that for given output level the DMUs in Group B incur a lower cost and that would be due to their better productivity in technical terms. However, as the input prices may differ both in absolute terms and in the ratio they are to each other, we can only at this stage conclude that in <u>percentage</u> terms DMUs in Group B have lower scope for cost savings than DMUs in Group A when $CI^A > 1$. We turn later to a decomposition of indices of this type and to the issue of absolute input price differences between the Groups in order to gain a better insight into their relative performance in cost terms.

An index similar to that of CI^A in (10) can be defined with respect to the cost frontier of the DMUs in *B*. The index is labelled CI^B and is defined in (12).

$$CI^B = \frac{\left(\prod_{j=1}^{\delta_A} W_j^A X_j^A \Big/ C^B\left(Y_j^A, W_j^A\right)\right)^{\frac{1}{\delta_A}}}{\left(\prod_{j=1}^{\delta_B} W_j^B X_j^B \Big/ C^B\left(Y_j^B, W_j^B\right)\right)^{\frac{1}{\delta_B}}} = \frac{\left(\prod_{j=1}^{\delta_B} CE_j^B\right)^{\frac{1}{\delta_B}}}{\left(\prod_{j=1}^{\delta_A} CE_j^{B,A}\right)^{\frac{1}{\delta_A}}} \qquad (12).$$

The interpretation of the index CI^B is similar to that of CI^A. That is a value greater than 1 would mean that given the input prices of each DMU, the DMUs in Group B are more productive in cost terms than those in Group A, in the sense that in percentage terms they are closer to their minimum achievable costs than are the DMUs in Group A. A value below 1 for CI^A or CI^B means the converse in that the DMUs in Group A are more productive in cost terms than those in Group B, in the sense outlined. Finally either index having a value of 1 would suggest the two groups of units have approximately the same productivity in cost terms.

As the choice of referent technical frontier is arbitrary, in the tradition of the Malmquist index we use the geometric mean of CI^A and CI^B as in (13) to capture the productivity in cost terms of

the DMUs in Group *A* relative to that of the DMUs in Group *B*. Thus the cost Malmquist index for two groups *A* and *B* is as follows:

$$\boldsymbol{CI^{BA}} = (\boldsymbol{CI^{A}} \times \boldsymbol{CI^{B}})^{0.5} = \left[\frac{\left(\prod_{j=1}^{\delta_B} CE_j^B\right)^{\frac{1}{\delta_B}}}{\left(\prod_{j=1}^{\delta_A} CE_j^{B,A}\right)^{\frac{1}{\delta_A}}} \times \frac{\left(\prod_{j=1}^{\delta_B} CE_j^{A,B}\right)^{\frac{1}{\delta_B}}}{\left(\prod_{j=1}^{\delta_A} CE_j^{A}\right)^{\frac{1}{\delta_A}}}\right]^{0.5} \qquad (13).$$

We have used in CI^{BA} in (13) the superscript BA to indicate that the cost efficiencies of DMUs in Group B are in the numerator and those of Group A are in the denominator. With this definition of CI^{BA} a value greater than 1 would indicate that the DMUs in Group *B* are more productive in cost terms than those in Group *A* in terms of percentage of potential savings needed to reach minimum cost. *A* value below 1would indicate the converse and a value equal to 1 would suggest equal cost productivity of the DMUs in the two groups.

Clearly the CI^{BA} index does not reflect in absolute terms the cost differences between the DMUs in each Group. For example an index value of say 1.1 would indicate that controlling for output quantity and input prices, on average DMUs in Group A have 10 percentage points more scope for savings than do DMUs in Group B. Yet, in absolute terms the DMUs in Group A may be delivering a given output quantity at lower cost than those in Group B if the levels of input prices at Group A are sufficiently lower than those at Group B. In the context where units are price takers we can still deem the DMUs in Group B more 'cost effective' than those in Group A because given the input prices they face they perform better than do the DMUs in Group A. (For the case where units are not strictly price takers notions of price efficiency arise reflecting an additional component for a unit to save on aggregate costs by achieving a more favourable set of input prices. E.g. see Tone (2002), Tone and Tsutsui (2007), Camanho and Dyson (2008), and Portela and Thanassoulis (2014).)

We can, however, in the context of the units being price takers readily, adjust the CI^{BA} indexto account for absolute input price differences between Groups. For example let the mean price of input i in Group A be $\boldsymbol{W_i^A}$ and $\boldsymbol{W_i^B}$ be analogously defined for Group B. We can compute an index

$$P^{AB} = \left(\prod_{i=1}^{m} \frac{W_i^A}{W_i^B}\right)^{1/m}$$

whose value reflects the absolute magnitudes of input prices in Group A relative to those at B. For example a value of 1.1 for $\boldsymbol{P^{AB}}$ would indicate that on average input prices in Group A are about 10% higher than in Group B.

We can now adjust the CI^{BA} index to take account of the relative magnitudes of the input prices in the two Groups so as to gain a view of the potential for cost savings of DMUs in each Group in <u>absolute</u> terms. Thus define

Adj $\boldsymbol{CI^{BA} = CI^{BA}\, P^{AB}}$.

Adj CI^{BA} reflects the comparative potential for savings between Groups A and B in absolute terms. To see this note that if two DMUs one in Group A and the other in Group B have identical input-output levels and each input price of the DMU in Group A is a multiple P^{AB} of the corresponding one at the DMU in Group B then in the technology of either Group as used to compute CI^{BA}, the two DMUs will have the same technically efficiency point in terms of input levels. If we denote these inputs levels *eff* and use P^A and P^B for the vector of input prices at the DMU in Group A and B respectively, then at DMU level we can re-write either one of the components of CI^{BA} in (13) as $\frac{\frac{eff\ P^B}{Obs\ Cost\ B}}{\frac{eff\ P^A}{Obs\ Cost\ A}}$. Thus we have Adj $CI^{BA} = CI^{BA}\, \boldsymbol{P^{AB}} = \frac{\frac{eff\ P^B}{Obs\ Cost\ B}}{\frac{eff\ P^A}{Obs\ Cost\ A}} \frac{1}{\frac{1}{P^{AB}}}$ which reduces to $\frac{\frac{eff\ P^B}{Obs\ Cost\ B}}{\frac{eff\ P^B}{Obs\ Cost\ A}} = \frac{Obs\ Cost\ A}{Obs\ Cost\ B}$. This shows that AdjCI^{BA} reflects as we would expect the potential savings at DMU A relative to those at DMU B when all that differs between the two are the input prices.

A value above 1 for AdjCI^{BA} would indicate that when we take into account the relative efficiencies and magnitudes of the input prices, the DMUs in Group A have higher scope for efficiency savings than do the DMUs in Group B. The converse is the case when AdjCI^{BA} has a value below 1. The DMUs in the two groups have similar scope for efficiency savings in absolute terms when Adj CI^{BA} has a value of 1 or close**.**

However, we should use with caution the adjusted index AdjCI^{BA}. This is for a number of reasons. One is that the inputs may not be totally homogeneous across the two Groups of DMUs and so input price differences may reflect differences in quality or functionality of inputs.

Another is that mean input prices as used to compute the index $\boldsymbol{P^{AB}}$ can be significantly affected by some unusually high or low prices at certain DMUs. Above all, however, in computing both the geometric and the arithmetic means all inputs are given equal weight in $\boldsymbol{P^{AB}}$ irrespective of how much the corresponding input contributes to aggregate input costs. Notwithstanding these reservations, however, the adjusted CI^{BA} index does provide an indication of the relative aggregate cost levels of DMUs of equal cost efficiency in each Group.

4. DECOMPOSITION OF THE COST MALMQUIST INDEX FOR COMPARING GROUPS OF DMUs

The CI^{BA} index can be decomposed into *overall efficiency change- group* ($OECG^{BA}$) and *cost technical Change- group* ($CTCG^{AB}$) as follows:

$$OECG^{BA} = \frac{\left(\prod_{j=1}^{\delta_B} CE_j^B\right)^{\frac{1}{\delta_B}}}{\left(\prod_{j=1}^{\delta_A} CE_j^A\right)^{\frac{1}{\delta_A}}} \text{ and } CTCG^{AB} = \left[\frac{\left(\prod_{j=1}^{\delta_A} CE_j^A\right)^{\frac{1}{\delta_A}}}{\left(\prod_{j=1}^{\delta_A} CE_j^{B,A}\right)^{\frac{1}{\delta_A}}} \times \frac{\left(\prod_{j=1}^{\delta_B} CE_j^{A,B}\right)^{\frac{1}{\delta_B}}}{\left(\prod_{j=1}^{\delta_B} CE_j^B\right)^{\frac{1}{\delta_B}}}\right]^{0.5}$$

so that we have

$$CI^{BA} = OECG^{BA} \times CTCG^{AB} \qquad (14).$$

The ratio $\mathbf{OECG^{BA}}$ compares within-group cost efficiency spreads, the superscript BA indicating that the cost efficiencies of the DMUs in B are in the numerator and those of A in the denominator . The larger the value of this ratio the closer the DMUs in Group *B* to their own cost frontier compared to the DMUs in Group *A* and their own cost frontier. A value below 1 suggests the opposite and a value of 1 suggests on average we have a similar spread of units around their cost frontiers in both groups. The value of the ratio does not tell us which ones are more productive in cost terms as the cost frontiers are different for the two groups.

The superscript AB in $CTCG^{AB}$ indicates that the technology boundary of the DMUs of Group A is in the numerator of the ratios within $CTCG^{AB}$. The numerator of the first ratio in the RHS of the equation defining $CTCG^{AB}$ in (14) captures the geometric mean of the cost efficiency of the DMUs in Group *A* relative to their own cost frontier. The denominator captures the geometric mean of the cost efficiency again of the DMUs in Group A but relative to a cost frontier of the DMUs in Group B, using again each its own input prices. Thus as the group of DMUs is the same (Group A) the ratio of the numerator to the denominator captures the distance between the

cost frontier of the DMUs in A from that of the DMUs in B. As the same input prices are used both in the numerator and denominator for each DMU, the distance of the cost frontiers will reflect a combination of technical boundary shift and allocative efficiency change between the two Groups as illustrated in Figure 1.

Figure 1 shows a two input, single normalised output scenario in which the technical frontier of DMUs in Group A is denoted G_A and that of the DMUs in Group B G_B. The unit being assessed is A and the isocost frontiers are "isocost B" and "isocost A" relative to technical frontier G_B and G_A respectively.

The component of the first ratio in the RHS of the equation defining $CTCG^{AB}$ in (14) in relation to DMU A in Figure 1 would be $\frac{OB/OA}{OD/OA}$ = OB/OD. This ratio reflects the distance between the cost frontiers drawn on the two technical frontiers, using the same input prices. The distance between the cost frontiers reflects the shift, if any, between the technical frontiers G_A and G_B and the difference in allocative efficiency of DMU A in relation to the two Groups of DMUs, depicted by the difference between BC and DE in Figure 1.

The larger the value of first ratio in the RHS of the equation defining $CTCG^{AB}$ in (14) the closer are the isocost lines of the Group A compared to those of Group B, to the 'referent' DMUs of Group A. As the input prices used are the same irrespective of technical boundary, this would imply that for given input prices the isocost hyperplanes on Group B define a more demanding target in cost terms for a DMU than do the corresponding hyperplanes on Group A.

The second ratio in the RHS of the equation defining $CTCG^{AB}$ in (14) is interpreted in a similar manner but is using the DMUs in Group B as 'referent' to capture the distance of the isocost hyperplanes drawn on the two Group technical boundaries. Thus the geometric mean of the two ratios forming $CTCG^{AB}$ reflects the mean distance between the cost frontiers of groups *A* and *B*, akin to the boundary shift in the traditional Malmquist index where the frontiers are of the same group of units at two different points in time. Where in the traditional Malmquist index the same DMUs are differentiated by time period here the DMUs are differentiated by grouping on

context. Further, the shift as pointed out above, now reflects a combination of technical and allocative differences between the two Groups of DMUs.

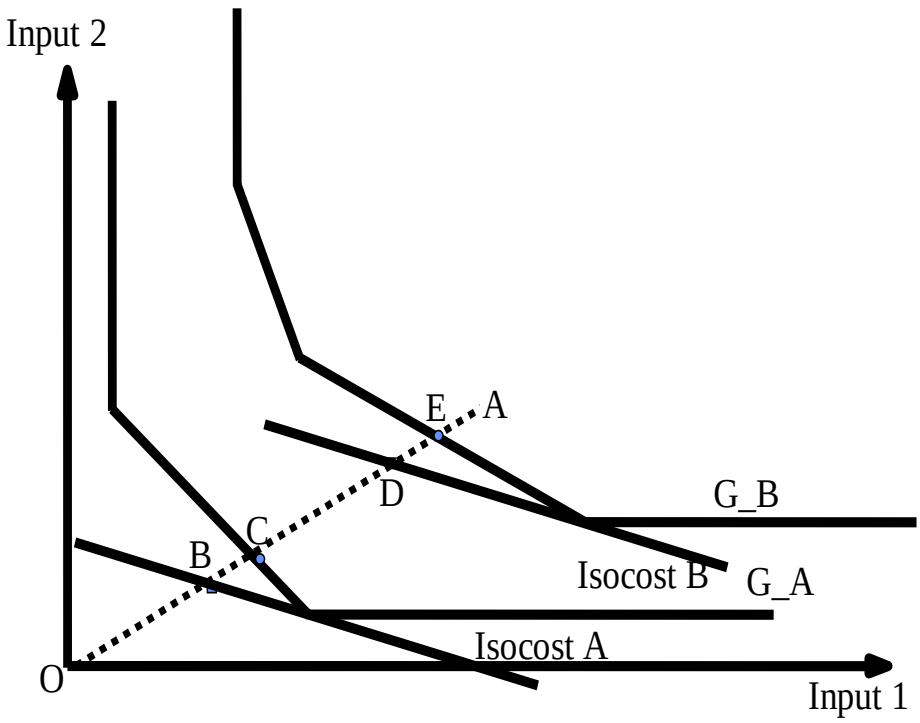

Figure 1: Illustration of the CTCG^{AB} Component of the Cost Malmquist Index

The $OECG^{BA}$ and $CTCG^{AB}$ components of the CI^{BA} index can themselves be decomposed.

The $OECG^{BA}$ component in (14) can be decomposed into *technical efficiency change – group* ($TECG^{AB}$) and *allocative efficiency change-group* ($AECG^{BA}$) as follows:

$$OECG^{AB} = \frac{\left(\prod_{j=1}^{\delta_A} D^A\left(Y_j^A, X_j^A\right)\right)^{1/\delta_A}}{\left(\prod_{j=1}^{\delta_B} D^B\left(Y_j^B, X_j^B\right)\right)^{1/\delta_B}} \times \frac{\left(\prod_{j=1}^{\delta_B} CE_j^B \times D^B\left(Y_j^B, X_j^B\right)\right)^{1/\delta_B}}{\left(\prod_{j=1}^{\delta_A} CE_j^A \times D^A\left(Y_j^A, X_j^A\right)\right)^{1/\delta_A}} = TECG^{AB} \times AECG^{BA}$$

(15).

The first component on the (first) right-hand side of (15) captures the spread of DMUs in Group *A* relative to those in Group B each one relative to their own technical as opposed to cost frontier. The spread is as found as a factor in the decomposition of the Camanho and Dyson

(2006),- see its interpretation earlier within index I^{AB} in (8). In a similar fashion, the second component in the first RHS in (15) captures **allocative efficiency change - group**, denoted $AECG^{BA}$. This can be readily seen from the fact that each component of the product in the numerator and the denominator is an allocative efficiency measure. (It is recalled the allocative efficiency of a DMU is the ratio of its overall cost efficiency to its technical efficiency, and the distance function $D^B\left(X_j^B, Y_j^B\right)$ is the inverse of the technical (Farrell) efficiency of DMU j.) Thus, $CE_j^B D^B\left(X_j^B, Y_j^B\right)$ is the allocative efficiency of DMU j in Group *B*. In view of the definition of $AECG^{BA}$ in (15) when its value is above 1 the DMUs in Group B are on average more allocatively efficient than those in Group A in the sense that the input prices are better aligned with the mix of inputs used by DMUs in Group B rather than in Group A. The converse is the case when the value of $AECG^{BA}$ is below 1.

The $\mathbf{CTCG^{AB}}$component of CI^{BA} can be decomposed into a *technical change- Group* (TCG^{BA}) and a *price-technical effect- Group* (PEG^{AB}) component**.** These are defined as follows

$$TCG^{BA} = \left[\frac{\left(\prod_{j=1}^{\delta_A} D^B\left(X_j^A, Y_j^A\right)\right)^{1/\delta_A}}{\left(\prod_{j=1}^{\delta_A} D^A\left(X_j^A, Y_j^A\right)\right)^{1/\delta_A}} \times \frac{\left(\prod_{j=1}^{\delta_B} D^B\left(X_j^B, Y_j^B\right)\right)^{1/\delta_B}}{\left(\prod_{j=1}^{\delta_B} D^A\left(X_j^B, Y_j^B\right)\right)^{1/\delta_B}}\right]^{1/2} \text{ and,}$$

$$PEG^{AB} = \left[\frac{\left(\prod_{j=1}^{\delta_A} CE_j^A D^A\left(X_j^A, Y_j^A\right)\right)^{1/\delta_A}}{\left(\prod_{j=1}^{\delta_A} CE_j^{B,A} D^B\left(X_j^A, Y_j^A\right)\right)^{1/\delta_A}} \times \frac{\left(\prod_{j=1}^{\delta_B} CE_j^{A,B} D^A\left(X_j^B, Y_j^B\right)\right)^{1/\delta_B}}{\left(\prod_{j=1}^{\delta_B} CE_j^B D^B\left(X_j^B, Y_j^B\right)\right)^{1/\delta_B}}\right]^{1/2}.$$

Thus we have $CTCG^{AB} = TCG^{BA} \times PEG^{AB}$ (16).

The superscript BA in TCG^{BA} indicates that the technology of Group B is in the numerator and that of A in the denominator. The reverse is the order of technologies in PEG^{AB}.
The TCG^{BA} in (16) is as found in the decomposition of the Camanho and Dyson (2006), index I^{AB} in (8). That is it is a measure of the distance of the technical (non cost) boundaries of the two groups. The larger the value of the component the less productive in technical rather than cost terms the frontier units of Group A compared to those of the frontier of Group B.

The PEG^{AB} component in (16) captures a form of 'allocative shift' which parallels the measure of boundary shift in the classical Malmquist index. This can be seen by noting that for example the geometric mean of the terms $CE_j^B D^B\left(X_j^B, Y_j^B\right)$in the denominator in the second ratio of the expression for PEG^{AB} in (16), is the mean allocative efficiency of the DMUs in Group B. The numerator into which this mean allocative efficiency divides is a similar mean 'allocative' efficiency measure again of the DMUs in B but relative to the technical frontier of the DMUs in

Group A, using an isocost line based on the respective input prices of the DMUs in Group B. As the same DMUs (Group B) are used both in the numerator and the denominator the ratio of the mean allocative efficiencies reveals the change or difference is the distance between the cost and technical frontiers of the DMUs in Group A in the numerator compared to the corresponding distance of the DMUs in Group B. In Figure 1 the distances compared in PEG^{AB} are illustrated by the segments BC and DE between the technical and isocost frontiers.

Looking now at the first ratio in the RHS of the definition of PEG^{AB} when the value of this ratio is above 1 the allocative efficiency of the DMUs in *A* relative to their own boundary and input prices is larger than relative to the boundary of the DMUs in B. This would suggest the input prices of the DMUs in A are more in line with the technical boundary of the DMUs in Group A rather than with that of the DMUs in Group B.

Thus overall the value of the expression for PEG reflects the change in the distance between technical and cost frontiers between the two groups of DMUs. When PEG is larger than 1 the DMUs have larger allocative efficiency relative to the Group A rather the than to the Group B technical boundary, using each one its own input prices. This in practical terms means that on average the DMUs have lower scope for cost savings by adjusting their input mix once they attain technical efficiency within Group A than they would had they been operating in Group B, each with its own input prices. The reverse would be the case when the value of PEG is below 1 and when it is 1 it means the distances between cost and technical boundaries are similar in the two groups.

Thus, in summary the overall CI^{BA} index can be decomposed as follows:

$$\begin{aligned} CI^{BA} = {} & overall\ efficiecny\ change\ group\ (OECG^{BA}) \\ & \times cost\ technical\ change\ group\ (CTCG^{AB}) \\ & = technical\ efficiecny\ change\ group\ (TECG^{AB}) \\ & \times allocative\ efficiecny\ change\ group\ (AECG^{BA}) \\ & \times technical\ change\ group\ (TCG^{BA}) \times price\ technical \\ & - effect\ group\ (PEG^{AB}) \\ & = I^{AB} \times allocative\ efficiecny\ change\ group\ (AECG^{BA}) \\ & \times price\ technical - effect\ group\ (PEG^{AB}) \end{aligned}$$

(17) or

$$\begin{aligned} CI^{BA} = OECG^{BA} \times CTCG^{AB} &= [TECG^{AB} \times AECG^{BA}] \times [TCG^{BA} \times PEG^{AB}] \\ &= I^{AB} \times AECG^{BA} \times PEG^{AB} \end{aligned}$$

(18)

The decomposition in (18) has been derived for the case of CRS technologies. In VRS technologies it is possible to derive components reflecting the shift in the VRS boundaries between Groups and also the impact of scale efficiency differences on the relative cost productivities of DMUs in the two Groups. This would be achieved by computing CI^{BA} using the VRS versions of models (6) and (11) and then modifying the first stage decomposition $CI^{BA} = OECG^{BA} \times CTCG^{AB}$ in (14) using the Ray and Desli (1997) decomposition of the Malmquist index for VRS technologies. In the interests of simplicity we do not address in this paper VRS technologies.Similarly, in the interests of simplicity we have not carried through the adjustment by P^{AB} of any one of the components of CI^{BA}.

5. NUMERICAL EXAMPLE

In this section, we use a numerical example to illustrate the information that can be gleaned through the Cost Malmquist Group index and its decomposition. Consider two groups of DMUs*A* and *B* where Group *A* has 6 DMUs and Group *B* has 5DMUs. Each DMU uses two inputs to deliver two outputs. The data are in Table 1 and Table 2 for Group A and B respectively.

Insert Table 1 and 2 here please

The Group Cost Malmquist Index CI^{BA}, defined in (13) is found to have a value of 1.05. This suggests that on average, if we control for output and input prices the DMUs in Group B are more productive in cost terms than those in Group A. Specifically, given observed input prices, and recalling that cost efficiency is defined as minimum achievable to observed aggregate cost of output, then in round figures the DMUs in Group A would need to reduce their observed aggregate costs by just under 5% (i.e. 1/1.05) in order to attain the same level of cost efficiency as the DMUs in Group B. It is recalled that this is contrasting for each group minimum achievable relative to observed aggregate costs rather than absolute costs of output in each group.

The relative price index P^{AB} is 0.548. This means that on average input prices at DMUs in Group A are about 55% of those in Group B. So the adjusted index AdjCI^{BA} is 0.548 x 1.05 = 0.575. Thus though DMUs in Group A are on average less cost efficient than those in B, because in absolute terms input prices at Group A are so much lower than those in Group B a

DMU in Group B would need to lower to 57.5% its observed aggregate costs to match an equally efficient DMU in Group A in terms of aggregate cost, controlling for output.

We need to look into the decomposition of the index in CI^{BA} in order to understand what lies behind the relative cost efficiency of the DMUs in each Group.

The first stage decomposition of the CI^{BA} index (see components in Table 3) gives the following picture:

$$CI^{BA} = OECG^{BA} \times CTCG^{AB} = 0.69 \times 1.53 = 1.05. \quad (19)$$

The component $OECG^{BA}$ in the decomposition in (19) being 0.69 suggests the DMUs in A in cost efficiency terms are clustered closer than the DMUs in B to their own cost frontier and so perform closer to their own boundary than do the DMUs in B. That is on average the DMUs in Group A are some 45% (i.e. 1/0.69) closer to their respective minimum attainable costs for their output than are the DMUs in Group B. The value of 1.53for the component $CTCG^{AB}$ suggests that on average the **cost boundaries** drawn on the technical boundary of DMUs in Group B are more demanding in terms of percent of observed costs to be cut for a DMU to attain cost efficiency than is the case for cost boundaries drawn on the technical boundary of the Group A DMUs. On average the efficient cost level set by Group B would be about 65% (i.e. 1/1.53) of what would be the case if the DMUs in Group A were to be the benchmark. This could be because the technical efficient boundary of Group B is more productive (i.e. lower input levels being needed for given output levels) and/or DMUs having generally input prices that necessitate bigger adjustments to input mix to reach allocative efficiency if the benchmarks are taken from Group B.

To better understand the performance of the DMUs in the two Groups we look further into the decompositions of the two components in (19).Table 3 shows the individual components of the overall index CI^{BA} =1.05while Table 4 shows the alternative decompositions of the index.

They are as follows:

$OECG^{BA} = TECG^{AB} \times AECG^{BA} = 0.71 \times 0.97.$

The component of 0.71 for $TECG^{AB}$ suggests that in technical efficiency terms the DMUs in A are substantially closer to their own frontier than are the DMUs in B (see expression (15) for the definition of $TECG^{AB}$). The value of 0.97 of the $AECG^{BA}$ component suggests that the allocative efficiency of the DMUs in A is on average slightly higher than that of the DMUs in B. Thus we have the DMUs in A closer to their own technical frontier with allocative efficiency close if better than that of the DMUs in B. So the dominant effect is the fact that the DMUs in A are closer to their own technical frontier than are those of Group B. So of critical significance in the comparative cost performance of the two groups will be the relative productivity of the Group A versus the Group B technical frontier. We found above that the $CTCG^{AB}$ component of the CI^{BA} index is 1.53. This as noted above strongly suggests technical boundary units in Group B perform much better than those in Group A which along with the allocative efficiency adjustments noted above set lower cost targets for efficiency, controlling for input prices and outputs.

To see this we look at the decomposition of the $CTCG^{AB}$ component (see Table 3). It is found that

$$CTCG^{AB} = TCG^{BA} \times PEG^{AB} = 1.33 \times 1.15 = 1.53.$$

The value of 1.33 for TCG suggests the boundary units in B are considerably more productive in technical (non cost) terms than those in A (i.e. 33% more output from boundary DMUs in Group B compared to boundary DMUs in Group A for given input levels). This combined with the finding earlier that the units in Group A are clustered closer to their technical efficient frontier reinforces the expectation that the units in Group B would perform better than those in A (i.e. we have efficient and non efficient Group A units clustered together in a less productive locus than the boundary units in Group B). The component of 1.15 on the other hand for PEG^{AB} suggests DMUs have lower scope to save cost once technically efficient relative to the Group Arather than the Group B boundary (with their own input prices) reinforcing again the fact that the Group B cost boundary is more demanding of performance in cost terms than that of Group A.

Thus in overall terms we conclude that the DMUs in Group B perform better in technical efficiency terms and have a more demanding cost boundary and so the overall effect is that the Group B units is more cost efficient if we treat prices as exogenous. Of course as we saw earlier Group B does suffer substantially more than Group A from high exogenous prices in absolute terms, and would need to lower input prices (if feasible) to match Group A in overall costs for given output levels.

Insert Table 3 and Table 4 please

6. CONCLUSION

This paper has put forward a new index for comparing groups of operating units on their productivity in aggregate cost terms when they use multiple inputs to secure multiple outputs and input prices are exogenous and available. The units perform the same function using the same inputs and outputs but they may differ in contextual terms, e.g. operating in different cultural or geographical areas and under different prevailing input prices. For example in the case of police forces they may be grouped respectively into those serving rural, small town, large town or cities of very large size. Though the forces may have the same objectives, the type of crime and public they face may be different by type of area as may be the input prices, including salaries for similar skilled staff. In such cases part of the performance of a unit may be attributable to its own management and operating practices while another part may be inherent to the context in which the unit operates. In particular groups of units may face different input prices, for example by geographic location within a country or indeed price differences across countries, and performance of units relative to the input prices they face is an important component of productivity to be isolated.

The index developed in the paper can be seen as an amalgamation of the index developed in Camanho and Dyson (2006) for comparing groups of units on technical efficiency terms and the cost Malmquist-type index developed by Maniadakis and Thanassoulis (2004) for capturing productivity change in cost terms when input prices are available. In this sense the index developed in this paper reflects in summary form the scope for savings at one Group of DMUs relative to another, given the input prices they each face. The index can also be adjusted to

reflect the scope for savings at one Group relative to another when we also take into account the absolute levels of the input prices the DMUs in each group face.

The unadjusted index is decomposed multiplicatively at two levels. The first level decomposition consists of two components. The first one of these reflects how far or close to their own efficient (in cost terms) boundary are the units of each set or group. The second component reflects a combination of technical boundary shift and allocative efficiency change between the two Groups of DMUs. The second level decomposition consists of four components. The first one of these reflects the spread of the units of one group around their own technical efficient frontier compared to the similar spread of the second group of DMUs. This component was also part of the decomposition of the index developed by Camanho and Dyson (2006) for comparing groups of DMUs on technical efficiency. The second component captures the mean allocative efficiency of the DMUs of one group against that of another. The third component reflects the relative productivity in technical (non cost) terms of the boundary units of one group relative to that of a second group. This component is also found in the decomposition of the index developed by Camanho and Dyson (2006), where input prices are not available. It is akin to the boundary shift in the classical Malmquist index of productivity change albeit in terms of boundaries of groups of DMUs rather than shift over time of the frontier of the same group of DMUs. A final component of the second stage reflects the comparative distance between the cost and technical efficient frontiers in each Group of DMUs.

Hitherto the method of choice for assessing group membership impact on performance has been the 'metafrontier' one. In this approach units are assessed within group, projected to the group efficient boundary, and then the distance between the group boundary and the boundary of the projected units of all groups (the metafrontier) identifies the impact of group membership on performance. Summary measures of this impact are normally computed by averaging in some manner the distances between the group and metafrontier, across the units. (For some applications using this approach see Portela and Thanassoulis (2001), Thanassoulis and Portela (2002) and Jones (2006).) The index developed in this paper offers the following advantages over the metafrontier approach:

- it compares groups both from the technical and cost perspective while the metafrontier comparisons have so far focused only on technical efficiency;

- unlike the metafrontier approach our indexrequires no assumption that it is possible to create feasible in principle DMUs through convex combinations of units (or of their efficient projections) across groups of DMUs;

- our index can be decompsoed so that Groups can be compared on overall performance in cost terms as well as on components of performance such as technical, allocative and price effects. These components have not been explored, to our knowledge, in the tarditional metafrontier approach.

The index developed on the other hand does have some disadvantages:

- The comparison is at Group level only while the metafrontier approach gives results both at group as well as at unit (DMU) level;
- Our comparisons are based on averages (geometric) of DMU efficiencies and such efficiencies do not reflect the relative sizes of the DMUs. That iscertain DMUs may be much bigger than others within a given Group and yet their efficiencies have the same weight as those of smaller DMUs. (This drawback would generally affect the metafrontier approach too,if averages are used for comparing Groups rather than units within Groups);

- The index as developed here uses radial (Farrell) efficiency measures which do not reflect any slack values of inputs or outputs. However, this drawback can be overcome by using measures of efficiency which do capture slack effects. This would require a fuller development of a related index which is not within the scope of this paper.

In conclusion the index developed enables the user to compare groups of DMUs on technical and cost productivity deriving both an overall index and components which identify at group level the origins of any differences in cost productivity. Such information makes it possible to target interventions at group level to improve performance. There remain, however, further

possibilities for enhancing the approach developed in this paper. One area concerns the decomposition of the main index (expression 13) for the case where the technology involved is VRS. This would make it possible to compare the Groups of DMUs not only in the areas covered in the decomposition developed in this paper, but also to identify the impact, if any, of scale size on the relative cost and indeed technical productivity of the units of each Group. Another area for further research is the exploration of whether bootstrapping or other approaches can be used to estimate confidence intervals on the main index in (13) and its components.

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Table 1. Input and Output data for Group *A*

DMU	Input 1	Input 2	Input price Input 1	Input price Input 2	Output 1	Output 2
1	6	5	3	2	8	11
2	4	7	2	2.5	9	12
3	3	6	4	2	8	15
4	5	2	3	2.5	7	10
5	4	7	4	2	6	13
6	8	5	2	1	5	14

Table 2.Input and Output data for Group *B*

DMU	Input 1	Input 2	Input price Input 1	Input price Input 2	Output 1	Output 2
1	4	3	4	4.5	4	7
2	5	6	3	5	2	10
3	7	4	5	4.5	5	6
4	3	3	4	5	10	14
5	2	2	4	6	8	6

Table 3. I^{AB} and CI^{AB} index and component values for two groups

I^{AB}	TCG^{AB}	$TECG^{AB}$	CI^{BA}	$OECG^{AB}$	$CTCG^{AB}$	$AECG^{AB}$	PEG^{AB}
0.95	1.33	0.71	1.05	0.69	1.53	0.97	1.15

Table 4.I^{AB} and CI^{AB} index and component values for two groups

$OECG^{AB}\times CTCG^{AB}$	$[TECG^{AB}\times AECG^{AB}]\times[TCG^{AB}\times PEG^{AB}]$	$I^{AB}\times AECG^{AB}\times PEG^{AB}$
$0.69\times1.53=1.05$	$[0.71\times0.97]\times[1.33\times1.15]=1.05$	$0.95\times0.97\times1.15=1.05$