

## Research Article

# Energy Hole Mitigation through Cooperative Transmission in Wireless Sensor Networks

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The energy balancing capability of cooperative communication is utilized to solve the energy hole problem in wireless sensor networks. We first propose a cooperative transmission strategy, where intermediate nodes participate in two cooperative multi-input single-output (MISO) transmissions with the node at the previous hop and a selected node at the next hop, respectively. Then, we study the optimization problems for power allocation of the cooperative transmission strategy by examining two different approaches: network lifetime maximization (NLM) and energy consumption minimization (ECM). For NLM, the numerical optimal solution is derived and a searching algorithm for suboptimal solution is provided when the optimal solution does not exist. For ECM, a closed-form solution is obtained. Numerical and simulation results show that both the approaches have much longer network lifetime than SISO transmission strategies and other cooperative communication schemes. Moreover, NLM which features energy balancing outperforms ECM which focuses on energy efficiency, in the network lifetime sense.

## 1. Introduction

Extending the lifetime of wireless sensor networks (WSNs) is critical due to the limited energy supply of sensor nodes. Much research effort has been devoted to addressing this challenge through improving energy efficiency. Recently, the *energy hole* problem has been regarded as another key factor that seriously confines the network lifetime [1–3]. The cause of energy hole lies in the intrinsic many-to-one traffic pattern of WSNs. Nodes nearer to the sink carry heavier traffic loads, leading to more energy consumption. This unbalanced energy consumption phenomenon results in an energy hole around the sink. When the energy hole appears, no more data can be delivered to the sink. The network lifetime ends prematurely while a large amount of energy is unused. Experimental results in [4] show that up to 90 percent of the energy of a network would be left unused when the network lifetime is over.

Several approaches have been proposed to mitigate the energy hole problem in WSNs. Works in [5–7] deploy more

nodes in heavy-loaded areas, which is known as the nonuniform deployment scheme. Another form of this scheme is to equip nodes closer to the sink with more initial energy [8]. Topology control strategies are proposed in [9–13], in which nodes collaboratively adjust their transmission power and form a proper network topology to balance energy consumption. In all above approaches, the data are transmitted through the single-input single-output (SISO) method.

Recently, cooperative communication has been applied to WSNs to enhance network performance, such as reliability, throughput, and coverage [14–16]. Energy efficiency can also be improved through this technique by exploiting the spatial diversity gain generated [17–20]. Moreover, since it allows the distributed nodes to cooperate with each other in data transmission, the energy consumptions of cooperating nodes can be properly balanced. Some existing approaches focus on balancing energy consumption among cooperative relay nodes in dual-hop relaying networks, such as the probabilistic path selection method in [21] and the optimal power allocation strategy in [22, 23]. These approaches do

not aim at solving the energy hole problem and uneven energy dissipation still exists between source nodes and relay nodes, which would cause a reduction in lifetime. As for multihop networks with cooperative communication, a nonuniform node deployment scheme to mitigate energy hole problem is proposed in [24], which is similar to the nonuniform deployment schemes for networks with SISO transmission, as in [5–7]. In [25], the transmission BER of each cluster is adjusted according to the hops between the sink and the cluster to mitigate the energy consumption in the hotspots, as well as keeping the promised reliability. In these schemes, cooperative communication is adopted to achieve high energy efficiency rather than energy balancing. Cooperative communication is usually utilized to obtain high energy efficiency for network lifetime extension in the previous works. If cooperative communication can be applied in WSNs to solve energy hole problem by its energy balance capability and meanwhile to improve energy efficiency, network lifetime can be further extended. However, to our knowledge, cooperative communication has not been applied to mitigate energy holes so far.

This paper utilizes the energy balancing capability of cooperative communication to mitigate energy holes and consequently extend network lifetime in WSNs. In our proposed cooperative transmission strategy, when one node is relaying data transmitted from the node further away from the sink, these two nodes would cooperate with each other to forward the data towards the sink. The data are transmitted through cooperative multi-input single-output (MISO) method, so that the energy burden of nodes closer to the sink can be shared by those further away. Here, a cooperative MISO system refers to a virtual antenna array contributed by multiple sensor nodes with each having a single antenna, instead of a cooperative system with nodes equipped with multiple antennas. In this way, energy consumption can be balanced over the network to solve the energy hole problem.

Our method can be applied to the networks with arbitrary node distribution, while schemes proposed in [5–7] require manual deployment with a prohibitive cost and restricted access to the network area for sensor node deployment. The topology control strategies in [9–13] are confined by the maximum transmitting power limitation of wireless sensor nodes when building the topology. This limitation can be relaxed in our method as for a certain transmission distance; lower transmitting power is required in cooperative communication compared to that in SISO transmission. Our method can also solve the problem of the uneven energy consumption between source nodes and relay nodes, which is ignored in [21–23]. Although cooperative communication is also exploited in other works such as [24] to improve energy efficiency, it is different from our approach since the energy hole problem is mitigated in [24] by using a nonuniform node deployment scheme rather than cooperative communication itself.

To enhance the network lifetime performance of the proposed cooperative transmission strategy, optimal power allocation among cooperative nodes is investigated. Two optimization approaches aimed for network lifetime maximization and energy consumption minimization are studied,

respectively, and compared with SISO transmission strategies for energy hole mitigation and other cooperative communication schemes. We also intend to show the performance difference between the two approaches under the same cooperative communication strategy for network lifetime extension. Numerical and simulation results show that the proposed cooperative transmission strategy with both optimization approaches can effectively mitigate the energy hole problem and achieve considerably longer network lifetime than the SISO transmission strategies for energy hole mitigation and other cooperative communication schemes. In addition, it is demonstrated that the optimal power allocation approach for network lifetime maximization outperforms the approach for energy consumption minimization in terms of network lifetime extension performance.

The rest of this paper is organized as follows. Section 2 states the energy hole problem and proposes the cooperative transmission strategy for energy hole mitigation. The energy model for the proposed strategy is described in Section 3. The optimal power allocation is studied in Section 4. Section 5 presents the performance analysis. Finally, we conclude our work and give an outlook for future research in Section 6.

Some of the notations that will be used in this paper are summarized here.

$E_{k,t}^{(i,j)}$ ,  $E_{k,r}^{(i,j)}$  represent energy consumed by nodes in corona  $C_k$  in transmission and reception, respectively, where nodes in coronas  $C_i$  and  $C_j$  transmit data together through the cooperative MISO method. The corona is defined in Section 2.1.

$E_{k,t}^{(i)}$ ,  $E_{k,r}^{(i)}$  represent energy consumed by nodes in  $C_k$  in transmission and reception, respectively, where nodes in  $C_i$  transmit data through the SISO method.

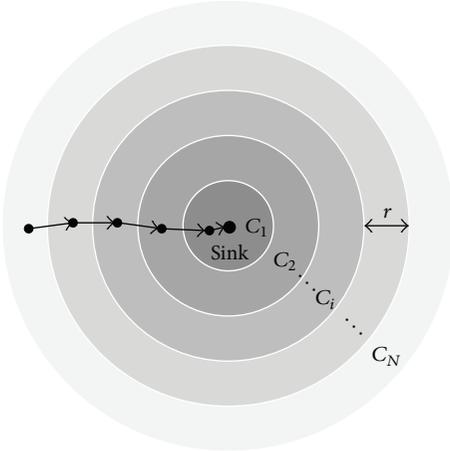
$P_k^{(i,j)}$  represents power consumed by the power amplifier (PA) of nodes in  $C_k$  in transmission, where nodes in  $C_i$  and  $C_j$  transmit data together through the cooperative MISO method.

$\gamma_k^{(i,j)}$ ,  $\bar{P}_k^{(i,j)}$  represent instantaneous signal-to-noise ratio (SNR) and the average bit error rate (BER) received at nodes in  $C_k$ , respectively, where nodes in  $C_i$  and  $C_j$  transmit data together through the cooperative MISO method.

## 2. Energy Hole Mitigation

**2.1. Energy Hole Problem.** Consider a wireless sensor network, where sensor nodes dispersed over a monitoring area report their sensed data to a sink. It is assumed that all the nodes are uniformly deployed with density  $\rho$ . The monitoring area is divided into  $N$  concentric coronas  $\{C_i, i = 1, 2, \dots, N\}$  centered at the sink with the same width  $r$  (see Figure 1), as in [5, 12]. The sink node is denoted as  $C_0$ .

Every node in the network generates and sends data to the sink hop by hop and the hop distance is  $d = r$ . To save energy, each node prefers to transmit the data to the relay node that lies in the linear route to the sink. As a result, the network can be depicted using a tree topology as used in [26], with the sink node in the middle being the root, branching out from inner coronas to outer coronas. In particular, nodes in  $C_1$  would transmit data directly to the sink, and the actual transmission

FIGURE 1: A network consisting of  $N$  coronas.

distance from nodes in  $C_1$  to the sink  $d \leq r$ . For simplicity but without losing generality, the transmission distance of nodes in every corona is assumed to be  $r$ , that is, no more than the maximal distance of successful transmission; thus nodes in  $C_1$  will exhaust energy first as discussed below. If nodes in  $C_1$  transmit data with the minimum power for the actual transmission distance to the sink while nodes in other coronas transmit data with the power for distance of  $r$ , nodes in  $C_2$  may first use up their energy. The data-gathering process is divided into time slots. In each time slot, each node generates  $l$  bits data and transmits the generated data with a data rate. Nodes are duty-cycled to save energy as applied in [27]; that is, when transmitting data, nodes are in the active state; after finishing transmitting, they turn into the sleep state until the next time slot begins.

In the conventional SISO transmission strategy, all data are transmitted through the SISO method. Nodes in  $C_i$  all together need to transmit both  $L_i$  bits data generated by themselves and  $L_{i+1,i}$  bits data transmitted from the nodes in  $C_{i+1}$ , which is generated by the nodes in outer coronas  $\{C_j \mid i+1 \leq j \leq N\}$ , to the nodes in  $C_{i-1}$ , where

$$L_i = [i^2 - (i-1)^2] \pi r^2 \rho l; \quad (1a)$$

$$L_{i+1,i} = (N^2 - i^2) \pi r^2 \rho l. \quad (1b)$$

Nodes in  $C_{i-1}$  receive all these data,  $L_i + L_{i+1,i}$  bits in total, and continue to transmit them towards the sink. Note that nodes in  $C_N$  only need to transmit their own data of  $L_N$  bits.

In corona  $C_i$ , the average traffic load per node,  $\bar{l}_{i,i-1}$ , is given by

$$\bar{l}_{i,i-1} = \frac{L_i + L_{i+1,i}}{(2i-1) \pi r^2 \rho} = \frac{N^2 - (i-1)^2}{2i-1} l. \quad (2)$$

$\bar{l}_{i,i-1}$  increases with decreasing  $i$ . Therefore, nodes in  $C_1$  carry the heaviest traffic load and consume energy the fastest. When these nodes exhaust their energy, an energy hole appears.

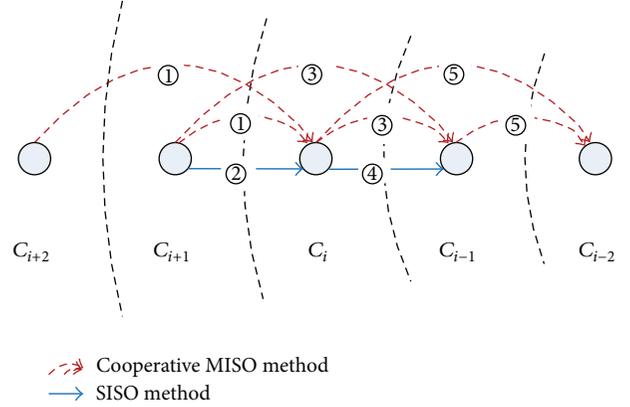


FIGURE 2: Cooperative transmission strategy.

**2.2. Cooperative Transmission Strategy.** In a multihop network, each node transmits the data generated by it and forwards the data from the previous-hop node to the next-hop node. We propose a cooperative transmission strategy, in which each node transmits its own data to the next-hop node by SISO method, but, for the data from its previous-hop node, it forwards the data to the next-hop node with the help of the previous-hop node by cooperative MISO method. The cooperative MISO method can reduce the energy consumption and balance it among the nodes in different coronas. The energy hole problem will be mitigated and the network lifetime can be improved.

The Alamouti space-time code and the maximum ratio combining technique are used for cooperative MISO transmission to achieve the spatial diversity gain. The details of the proposed strategy are illustrated in Figure 2. There are five phases for the data reception and transmission process of our proposed cooperative transmission strategy.

- (i) *Phase 1:* nodes in  $C_i$  receive  $L_{i+2,i+1}$  bits data relayed by nodes in  $C_{i+1}$  with the cooperation of nodes in  $C_{i+2}$  through the cooperative MISO method.
- (ii) *Phase 2:* nodes in  $C_i$  receive  $L_{i+1}$  bits data generated and transmitted by nodes in  $C_{i+1}$ . The transmission is through the SISO method.
- (iii) *Phase 3:* instead of forwarding  $L_{i+1,i} = L_{i+2,i+1} + L_{i+1}$  bits data to nodes in  $C_{i-1}$  alone, nodes in  $C_i$  transmit these data through the cooperative MISO method with the nodes in  $C_{i+1}$ .
- (iv) *Phase 4:* nodes in  $C_i$  transmit  $L_i$  bits data generated by themselves to nodes in  $C_{i-1}$  through the SISO method.
- (v) *Phase 5:* nodes in  $C_i$  help their relay nodes in  $C_{i-1}$  to forward the  $L_{i,i-1} = L_{i+1,i} + L_i$  bits data, which they sent to nodes in  $C_{i-1}$  previously in Phase 3 and Phase 4, to nodes in  $C_{i-2}$ . This transmission is also through the cooperative MISO method.

Note that nodes in  $C_N$  do not need to relay data from outer coronas and there are no Phases 1–3 for them. Phase 5 does not exist for nodes in  $C_1$  since they have no relay node to help.

In our cooperative transmission strategy, nodes in  $C_i$  would participate in two cooperative MISO transmissions, respectively, with nodes in  $C_{i+1}$  and  $C_{i-1}$ . In Phase 3, nodes in  $C_i$  are helped by nodes in  $C_{i+1}$  to transmit data to nodes in  $C_{i-1}$ . The energy burden of nodes in  $C_i$  can be shared by nodes in the outer corona  $C_{i+1}$ . In phase 5, nodes in  $C_i$  help nodes in  $C_{i-1}$  to transmit data to nodes in  $C_{i-2}$ ; thus, they share the energy burden of nodes in the inner corona  $C_{i-1}$ . If the transmitting power is allocated appropriately between nodes in  $C_{i+1}$  and  $C_i$  and between nodes in  $C_i$  and  $C_{i-1}$ , the energy consumption over the network can be balanced. As a result, energy hole problem can be mitigated or even avoided by our cooperative transmission strategy.

### 3. Energy Model

According to the description of the proposed cooperative transmission strategy in the previous section, the energy consumption of nodes in  $C_i$  in one time slot,  $E_i$ , is the sum of the energy consumption in Phases 1–5, that is,  $E_{i,r}^{(i+2,i+1)}$ ,  $E_{i,r}^{(i+1)}$ ,  $E_{i,t}^{(i+1,i)}$ ,  $E_{i,t}^{(i)}$ , and  $E_{i,t}^{(i,i-1)}$ , correspondingly:

$$E_i = \begin{cases} E_{i,r}^{(i+2,i+1)} + E_{i,r}^{(i+1)} + E_{i,t}^{(i+1,i)} \\ \quad + E_{i,t}^{(i)} + E_{i,t}^{(i,i-1)}, & i = 2, 3, \dots, N-1 \\ E_{i,r}^{(i+2,i+1)} + E_{i,r}^{(i+1)} + E_{i,t}^{(i+1,i)} + E_{i,t}^{(i)}, & i = 1 \\ E_{i,t}^{(i)} + E_{i,t}^{(i,i-1)}, & i = N. \end{cases} \quad (3)$$

The energy consumed during reception comes from circuitry only. Similar to [17], the energy consumption of nodes in  $C_i$  in Phases 1 and 2,  $E_{i,r}^{(i+2,i+1)}$  and  $E_{i,r}^{(i+1)}$ , can be calculated as follows:

$$E_{i,r}^{(i+2,i+1)} = L_{i+2,i+1} E_{\text{cr}}; \quad (4a)$$

$$E_{i,r}^{(i+1)} = L_{i+1} E_{\text{cr}}, \quad (4b)$$

where  $E_{\text{cr}}$  represents the energy consumption of circuitry for receiving one bit, which is the same for data reception through cooperative MISO and SISO methods.

The energy consumed during transmission is contributed by PA and other circuitry. In Phase 3, the energy consumption of nodes in  $C_i$  for MISO transmission,  $E_{i,t}^{(i+1,i)}$ , can be given by

$$E_{i,t}^{(i+1,i)} = L_{i+1,i} \left( E_{\text{ct}} + \frac{P_i^{(i+1,i)}}{R} \right), \quad (5)$$

where  $E_{\text{ct}}$  represents the energy consumption of circuitry for transmitting one bit and  $R$  is the transmission rate.  $P_i^{(i+1,i)}$  is the PA power consumed by an arbitrary node in  $C_i$  to transmit data to the corresponding node in  $C_{i-1}$  through the cooperation with their previous-hop node in  $C_{i+1}$ . The PA power is assumed to be the same for the nodes in the same corona since all the nodes in a corona will choose the nodes in the next corona with the same distance to transmit the data in our cooperative transmission strategy and the same BER requirement need be satisfied for all the transmissions.

Under BPSK modulation, the average BER received at a node in  $C_{i-1}$  is given by [28]

$$\bar{P}_{i-1}^{(i+1,i)} = \varepsilon_{\text{h}} \left\{ Q \left( \sqrt{2\gamma_{i-1}^{(i+1,i)}} \right) \right\}, \quad (6)$$

where  $\varepsilon_{\text{h}}\{x\}$  denotes the expectation of  $x$  with the channel and  $Q(x)$  is the  $Q$ -function.  $\gamma_{i-1}^{(i+1,i)}$  represents the instantaneous received SNR at a node in  $C_{i-1}$ , which can be expressed as

$$\gamma_{i-1}^{(i+1,i)} = \frac{P_i^{(i+1,i)} |h_{i,i-1}|^2 + P_{i+1}^{(i+1,i)} |h_{i+1,i-1}|^2}{N_0 W}, \quad (7)$$

where  $P_{i+1}^{(i+1,i)}$  is the PA power consumed by a node in  $C_{i+1}$  in cooperation with a node in  $C_i$ .  $N_0$  is the noise power spectral density and  $W$  is bandwidth.  $h_{i,i-1}$  is the channel coefficient from the node in  $C_i$  to that in  $C_{i-1}$  and  $h_{i+1,i-1}$  is that from  $C_{i+1}$  to  $C_{i-1}$ . The transmission channel is assumed to be independent frequency-flat Rayleigh fading channel and  $h_{i,i-1}$  and  $h_{i+1,i-1}$  are complex Gaussian random variables with zero mean and variances  $\sigma_{i,i-1}^2$  and  $\sigma_{i+1,i-1}^2$ , respectively. It is assumed that the path loss from a node in  $C_i$  to that in  $C_j$  is

$$\sigma_{i,j}^2 = K \cdot d_{i,j}^{-\alpha}, \quad (8)$$

where  $\alpha$  is the path loss exponent,  $d_{i,j}$  is the transmission distance from  $C_i$  to  $C_j$  and  $K$  denotes the factor of proportionality. Since all the nodes in corona  $C_i$  choose the nodes in corona  $C_j$  with the same distance  $d_{i,j}$  to transmit the data, the channel coefficients of transmitter-receiver pairs from  $C_i$  to  $C_j$  are identically distributed and, hence, the average BER is the same for the nodes in the same corona. The instantaneous channel coefficients of the transmitter-receiver pairs may be different; however, it does not influence the calculation of the average BER. Therefore, the same notation of instantaneous channel coefficient  $h_{i,j}$  is used for different transmitter-receiver pairs from  $C_i$  to  $C_j$  for simplicity.

By substituting (7) into (6), the average BER  $\bar{P}_{i-1}^{(i+1,i)}$  can be represented as a function of  $P_i^{(i+1,i)}$  and  $P_{i+1}^{(i+1,i)}$ . Under high SNR, it can be approximated as [28]

$$\bar{P}_{i-1}^{(i+1,i)} = \frac{N_0^2 W^2}{P_i^{(i+1,i)} P_{i+1}^{(i+1,i)} \sigma_{i,i-1}^2 \sigma_{i+1,i-1}^2}. \quad (9)$$

For successful reception, the average BER must achieve a predetermined threshold  $\bar{p}_{\text{th}}$ .

In Phase 4, energy consumption  $E_{i,t}^{(i)}$  for SISO transmission is given by

$$E_{i,t}^{(i)} = L_i \left( E_{\text{ct}} + \frac{P_i^{(i)}}{R} \right), \quad (10)$$

where  $E_{\text{ct}}$  is the energy consumption of circuitry for transmitting one bit through the SISO method, which is the same as that for MISO transmission.  $P_i^{(i)}$  is the PA power consumed by nodes in  $C_i$ . The instantaneous received SNR is

$$\gamma_{i-1}^{(i)} = \frac{P_i^{(i)} |h_{i,i-1}|^2}{N_0 W}. \quad (11)$$

The average BER can be obtained approximately as

$$\bar{P}_{i-1}^{(i)} = \varepsilon_h \left\{ Q \left( \sqrt{2\gamma_{i-1}^{(i)}} \right) \right\} = \frac{N_0 W}{P_i^{(i)} \sigma_{i,i-1}^2} \quad (12)$$

and also need to achieve the BER requirement  $\bar{P}_{th}$  for successful reception.

The minimum value of  $P_i^{(i)}$  achieving the BER requirement can be obtained as

$$P_i^{(i)} = \frac{N_0 W}{\bar{P}_{th} \sigma_{i,i-1}^2}. \quad (13)$$

In Phase 5, nodes in  $C_i$  help their relay nodes in  $C_{i-1}$  to transmit data to nodes in  $C_{i-2}$ . This transmission is also through the cooperative MISO method. Similarly, the energy consumption of nodes in  $C_i$ ,  $E_{i,t}^{(i,i-1)}$ , can be expressed as a function of  $P_i^{(i,i-1)}$  and the average BER constraint of  $P_i^{(i,i-1)}$  and  $P_{i-1}^{(i,i-1)}$  can be derived.

In summary, the energy consumption of nodes in  $C_i$ ,  $E_i$ , is determined by the following power variables:  $P_i^{(i+1,i)}$  in Phase 3,  $P_i^{(i)}$  in Phase 4, and  $P_i^{(i,i-1)}$  in Phase 5. The PA power of SISO transmission  $P_i^{(i)}$  is an independent variable and its optimal value can be calculated by (13). However, the values of PA power of cooperative MISO transmission consumed by nodes in different coronas are coupled with each other; that is,  $P_i^{(i+1,i)}$  is coupled with  $P_{i+1}^{(i+1,i)}$  and  $P_i^{(i,i-1)}$  with  $P_{i-1}^{(i,i-1)}$  in the constraints of BER. We can formulate an optimization problem to allocate  $P_i^{(i+1,i)}$ ,  $i = 1, 2, \dots, N-1$ , and  $P_i^{(i,i-1)}$ ,  $i = 2, 3, \dots, N$ , to improve lifetime extension performance of the network.

#### 4. Power Allocation

In this section, we study the optimal power allocation among cooperating nodes to enhance the performance of the cooperative transmission strategy we propose. The optimization problems for network lifetime maximization and for energy consumption minimization are considered, respectively.

*4.1. Power Allocation for Network Lifetime Maximization (NLM).* The primary motivation of mitigating energy hole in WSNs is to prolong the network lifetime. Therefore, we first formulate the power allocation of  $P_i^{(i+1,i)}$ ,  $i = 1, 2, \dots, N-1$ , and  $P_i^{(i,i-1)}$ ,  $i = 2, 3, \dots, N$  as an optimization problem aiming at maximizing the network lifetime.

The network lifetime is generally defined as the duration from the very beginning of the network operation until the first node exhausts its energy. In our discussion, nodes in the same corona have the same rate of energy consumption and would completely deplete energy simultaneously. Let  $e$  denote the initial energy of each node, whereas the sink has no energy limitation. The network lifetime measured by the number of time slots is given by

$$T = \min_i \left( \frac{\varepsilon}{E_i / [(2i-1)\pi r^2 \rho]} \right), \quad (14)$$

where energy consumption of nodes in  $C_i$ ,  $E_i$ , is a function of both  $P_i^{(i+1,i)}$  and  $P_i^{(i,i-1)}$ .

Then, the NLM problem can be formulated as follows:

$$\begin{aligned} & \max_{P_i^{(i+1,i)}, P_{i+1}^{(i+1,i)}, i=1,2,\dots,N-1} & \min_i & \left( \frac{\varepsilon}{E_i / [(2i-1)\pi r^2 \rho]} \right) \\ & \text{s.t.} & & \bar{P}_{i-1}^{(i+1,i)} \leq \bar{P}_{th}, \\ & & & i = 1, 2, \dots, N-1 \\ & & & P_i^{(i+1,i)}, P_{i+1}^{(i+1,i)} > 0, \\ & & & i = 1, 2, \dots, N-1, \end{aligned} \quad (15)$$

where the first condition represents the BER constraints of  $P_i^{(i+1,i)}$  and  $P_i^{(i,i-1)}$ .

Considering that maximizing the minimum of  $\varepsilon/(E_i/[(2i-1)\pi r^2 \rho])$  equals minimizing the maximum of  $E_i/[(2i-1)\pi r^2 \rho]$ , the problem can be internally reformulated into an equivalent problem by appending additional constraints of the form  $\tau \geq E_i/[(2i-1)\pi r^2 \rho]$ , and then minimizing  $\tau$  over the optimization problem. The equivalent problem is obtained as

$$\begin{aligned} & \min_{P_i^{(i+1,i)}, P_{i+1}^{(i+1,i)}, i=1,2,\dots,N-1} & \tau \\ & \text{s.t.} & \frac{E_i}{(2i-1)\pi r^2 \rho} \leq \tau, \\ & & i = 1, 2, \dots, N \\ & & \bar{P}_{i-1}^{(i+1,i)} \leq \bar{P}_{th}, \\ & & i = 1, 2, \dots, N-1 \\ & & P_i^{(i+1,i)}, P_{i+1}^{(i+1,i)} > 0, \\ & & i = 1, 2, \dots, N-1. \end{aligned} \quad (16)$$

This problem is not convex, but we adopt Karush-Kuhn-Tucker (KKT) conditions to solve this problem. Since the KKT conditions provide necessary conditions for the optimality [29], the solution of the nonconvex problem can be obtained with the optimal selection from KKT solutions.

**Proposition 1.** *The necessary conditions for the optimality of problem (16) are*

$$\frac{E_i}{(2i-1)\pi r^2 \rho} = \tau, \quad i = 1, 2, \dots, N; \quad (17a)$$

$$\bar{P}_{i-1}^{(i+1,i)} = \bar{P}_{th}, \quad i = 1, 2, \dots, N-1; \quad (17b)$$

$$P_i^{(i+1,i)}, P_{i+1}^{(i+1,i)} > 0, \quad i = 1, 2, \dots, N-1. \quad (17c)$$

*Proof.* See Appendix A.  $\square$

We choose  $N - 1$  out of  $N$  equations in (17a), except for the one when  $i = N$ , and combine them with (17b) to derive power variables as functions of  $i$ ,  $N$ , and  $\tau$ :

$$P_i^{(i+1,i)} = f_1(i, N, \tau), \quad i = 1, 2, \dots, N - 1; \quad (18a)$$

$$P_{i+1}^{(i+1,i)} = f_2(i, N, \tau), \quad i = 1, 2, \dots, N - 1, \quad (18b)$$

where

$$\begin{aligned} f_1(i, N, \tau) &= g_1(i, N, \tau) - q(i) \left[ \frac{g_2(i-1)g_3(i, N)}{f_1(i-1, N, \tau)} + g_4(i, N) \right]; \\ f_2(i, N, \tau) &= (g_2(i)) \times (\{g_1(i, N, \tau) - q(i) \\ &\quad \times [g_3(i, N) f_2(i-1, N, \tau) + g_4(i, N)]\})^{-1}. \end{aligned} \quad (19)$$

Here,

$$\begin{aligned} g_1(i, N, \tau) &= \frac{(2i-1)\tau R}{(N^2 - i^2)l} - E_{\text{cr}}R \\ &\quad - E_{\text{ct}}R \frac{N^2 - (i-1)^2}{N^2 - i^2} - \frac{N_0 W}{\bar{p}_{\text{th}} \sigma_{i,i-1}^2} \frac{2i-1}{N^2 - i^2}, \\ &\quad i = 1, 2, \dots, N - 1; \\ g_2(i) &= \frac{N_0^2 W^2}{\bar{p}_{\text{th}} \sigma_{i+1,i-1}^2 \sigma_{i,i-1}^2}, \quad i = 1, 2, \dots, N - 1; \\ g_3(i, N) &= \frac{N^2 - (i-1)^2}{N^2 - i^2}, \quad i = 1, 2, \dots, N - 1; \\ g_4(i, N) &= \frac{N^2 - (i-1)^2}{N^2 - i^2} E_{\text{ct}}R, \quad i = 1, 2, \dots, N - 1; \\ q(i) &= \begin{cases} 0, & i = 1 \\ 1, & i = 2, 3, \dots, N - 1. \end{cases} \end{aligned} \quad (20)$$

Substituting  $P_N^{(N,N-1)}$  calculated by the iteration equation (18b) into (17a) when  $i = N$ , we have

$$f_2(N, N, \tau) - \frac{\tau R}{l} - 2E_{\text{ct}}R \frac{N-2}{2N-1} - \frac{N_0 W}{\bar{p}_{\text{th}} \sigma_{i,i-1}^2} = 0. \quad (21)$$

Given  $N$ , the minimum  $\tau^*$  of optimization problem (16) can be obtained by (21) using the Newton-Raphson method. Thus, the set of optimal power values can be obtained by substituting  $\tau^*$  into (18a) and (18b). Finally, we check whether the optimal power values satisfy (17c) or not.

When the solution that satisfies the KKT conditions does not exist, there will be no optimal solution to the NLM problem. No optimal solution means that the perfect energy

balance among the nodes in different coronas cannot be achieved; that is, (17a) in KKT conditions cannot be held. Consider that the nodes in the outermost corona  $C_N$  do not need to forward the data from other nodes and have the minimal traffic load in the network. The nodes in  $C_N$  cannot consume energy at the same rate as those in the inner coronas. Thus, we relax the condition in (17a) and assume that this condition is satisfied only when  $i = 1, 2, \dots, N - 1$ . Considering the first constraint of the NLM problem (16) when  $i = N$ , the relaxed conditions which will lead to a suboptimal solution of the NLM problem can be expressed as follows, according to (17a), (17b), and (17c),

$$\frac{E_i}{(2i-1)\pi r^2 \rho} = \tau, \quad i = 1, 2, \dots, N - 1; \quad (22a)$$

$$\bar{p}_{i-1}^{(i+1,i)} = \bar{p}_{\text{th}}, \quad i = 1, 2, \dots, N - 1; \quad (22b)$$

$$P_i^{(i+1,i)}, P_{i+1}^{(i+1,i)} > 0, \quad i = 1, 2, \dots, N - 1; \quad (22c)$$

$$\frac{E_N}{(2N-1)\pi r^2 \rho} < \tau. \quad (22d)$$

Thus, we have a solution subspace constricted by conditions in (22a)–(22d). To obtain suboptimal solution in the subspace, a searching algorithm is proposed, in which we keep decreasing  $\tau$  through the following iteration procedure. The details of the algorithm are described as follows.

- (i) *Step 1:* initialize  $\tau(0) = \epsilon$ ,  $k = 0$ .
- (ii) *Step 2:* given  $\tau(k)$ , a set of power values is derived from (18a) and (18b) by substituting  $\tau(k)$  in place of  $\tau$ .
- (iii) *Step 3:* if conditions in (22c) or (22d) cannot be met, the minimum  $\tau^*$  is obtained by  $\tau^* = \tau(k) + \Delta\tau$ . The suboptimal solution is derived from (18a) and (18b) by substituting  $\tau^*$  in place of  $\tau$ . Otherwise, let  $\tau(k+1) = \tau(k) - \Delta\tau$ ,  $k = k + 1$ , and go to *Step 2*.

**4.2. Power Allocation for Energy Consumption Minimization (ECM).** We then present another optimization problem for power allocation aiming at minimizing the energy consumption of the network. The optimization problem for ECM is formulated as follows:

$$\begin{aligned} \min_{P_i^{(i+1,i)}, P_{i+1}^{(i+1,i)}, i=1,2,\dots,N-1} & \sum_{i=1}^N E_i \\ \text{s.t.} & \bar{p}_{i-1}^{(i+1,i)} \leq \bar{p}_{\text{th}}, \\ & i = 1, 2, \dots, N - 1 \\ & P_i^{(i+1,i)}, P_{i+1}^{(i+1,i)} > 0, \\ & i = 1, 2, \dots, N - 1. \end{aligned} \quad (23)$$

Similarly, the optimization problem (23) can be solved by KKT conditions.

**Proposition 2.** Based on KKT conditions, the optimal power allocation for ECM is given by

$$P_i^{(i+1,i)*} = P_{i+1}^{(i+1,i)*} = \frac{N_0 W}{\sqrt{\bar{P}_{th} \sigma_{i+1,i-1} \sigma_{i,i-1}}}, \quad (24)$$

$$i = 1, 2, \dots, N - 1.$$

*Proof.* See Appendix B.  $\square$

With the optimal power allocation for ECM, the network lifetime is given by

$$T = (\epsilon) \left( (N^2 - 1) I E_{cr} + N^2 I E_{ct} + \frac{(N^2 - 1) N_0 W I}{\sqrt{\bar{P}_{th} \sigma_{2,0} \sigma_{1,0} R}} + \frac{N_0 W I}{\bar{P}_{th} \sigma_{1,0}^2 R} \right)^{-1}. \quad (25)$$

For SISO transmission, the nodes in the innermost corona  $C_1$  will exhaust their energy first. The network lifetime of SISO transmission can be given by

$$T_{SISO} = \frac{\epsilon}{(N^2 - 1) I E_{cr} + N^2 I E_{ct} + (N^2 N_0 W I / \bar{P}_{th} \sigma_{1,0}^2 R)}. \quad (26)$$

Comparing this equation with (25), we can find that the network lifetime of cooperative transmission under ECM approach is longer than that of SISO transmission if

$$\bar{P}_{th} < \left( \frac{\sigma_{2,0}}{\sigma_{1,0}} \right)^2. \quad (27)$$

Under our setting,  $\sigma_{2,0}/\sigma_{1,0} = (1/2)^{\alpha/2}$ . Normally,  $\bar{P}_{th}$  takes the value of  $10^{-3}$  or lower and  $\alpha \in [2, 4]$ . Therefore, the condition (27) can be satisfied and ECM approach achieves longer network lifetime than SISO transmission. Since NLM approach outperforms ECM approach, it also achieves longer network lifetime than SISO transmission.

## 5. Numerical and Simulation Results

In this section, numerical and simulation results of the proposed cooperative transmission strategy are presented. Based on [5, 17], the system parameters are assigned and listed in Table 1.

In Figure 3, we compare the network lifetime achieved by the cooperative transmission strategy with that by the conventional SISO transmission strategy. The network lifetime is plotted against the number of coronas  $N$  in the network. It can be found that the cooperative transmission strategy achieves much longer network lifetime than the conventional SISO transmission strategy over the entire range of  $N$ , as the cooperative transmission strategy can balance energy consumption as well as improve energy efficiency

TABLE 1: System parameters.

$\epsilon = 300$ J	$R = 10$ kbit/s
$E_{cr} = E_{ct} = 50$ nJ/bit	$K = -31.54$ dB
$N_0 = -171$ dBm/Hz	$\alpha = 3$
$W = 10$ kHz	$r = 50$ m
$l = 1$ kbit/timeslot	$\bar{P}_{th} = 10^{-3}$
$\rho = 0.05$ nodes/m <sup>2</sup>	

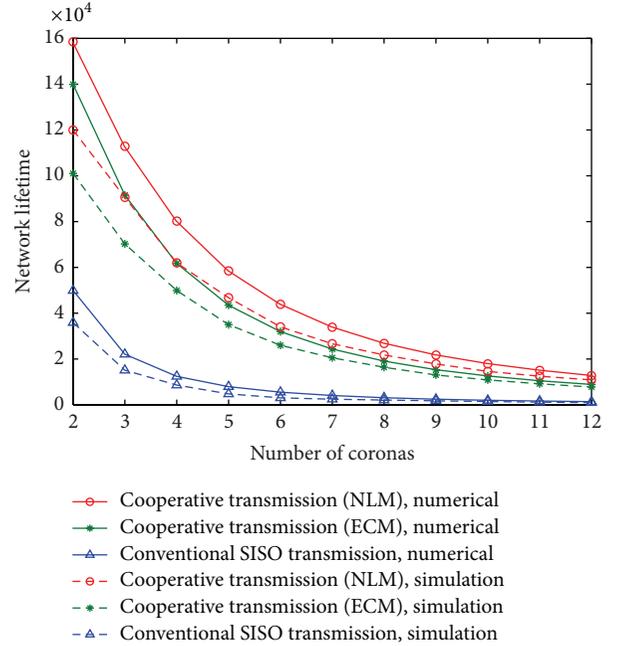


FIGURE 3: Network lifetime versus number of coronas.

of data transmission. It can also be observed that, under the cooperative transmission strategy, NLM power allocation outperforms ECM in network lifetime extension, since NLM puts the emphasis on energy balancing (see (17a)) while ECM focuses on energy efficiency improvements. Besides, the network lifetime drops as the network scale expands. In a larger network, the distance to the sink from nodes in outer coronas is longer; thus more energy is required to transmit data to the sink. On the other hand, the traffic load unbalance among coronas is greater and the energy hole problem is more serious for a larger network. From Figure 3, we can see that the network lifetime obtained by simulation is lower than that by calculation. In calculation, the sensor nodes are assumed to be perfectly evenly distributed while in simulation they are randomly deployed in the monitoring area. Therefore, the transmission distance used in simulation cannot be the same throughout the network and the traffic loads of the nodes in the same corona are not identical either, which results in a reduction in network lifetime for both SISO and cooperative transmission strategies.

To better illustrate the performance of network lifetime extension, we plot the ratio of the network lifetime achieved by the cooperative transmission strategy to that achieved by the conventional SISO transmission strategy versus

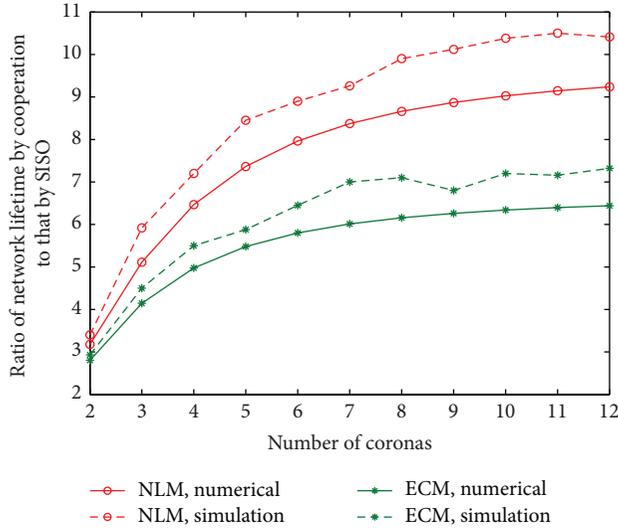


FIGURE 4: Ratio of the network lifetime achieved by cooperative transmission to that achieved by SISO transmission versus number of coronas.

the number of coronas  $N$  in Figure 4. It can be seen that the cooperative transmission strategy achieves larger extension of network lifetime as the scale of the network expands. Furthermore, the lifetime extension of NLM power allocation is higher than that of ECM, and the gap between them increases when network becomes larger. In a 10-corona network, the ratio of lifetime by NLM to that by SISO is up to 9 and that of ECM is about 6. It implies that balancing energy consumption is more effective than improving energy efficiency in terms of network lifetime extension, especially in large-scale networks. From Figure 4, we can also see that the lifetime extension of NLM and ECM obtained by simulation is higher than those by calculation, since in simulation the uneven node distribution results in the reduction in network lifetime of SISO transmission is more notable than that of cooperative transmission due to the uneven node distribution.

The residual energy ratio is plotted in Figure 5, which is the ratio of energy remained when the network lifetime ended to the sum of the initial energy of all the nodes in the network. We only present the numerical results in Figures 5–7 and the simulation results are similar and hence omitted. From Figure 5, we can see that the residual energy ratio of the conventional SISO transmission strategy exceeds 90% in the networks with more than 7 coronas. That is consistent with the experimental results in [4]. It can be observed that the residual energy ratio under the cooperative transmission strategy with ECM is lower than that under the conventional SISO transmission strategy. This is due to the intrinsic energy balance capability of the cooperative communication. Moreover, the residual energy ratio of the cooperative strategy with NLM is the minimum among the three methods, since NLM further balances the energy consumption by proper power allocation to mitigate the energy hole problem.

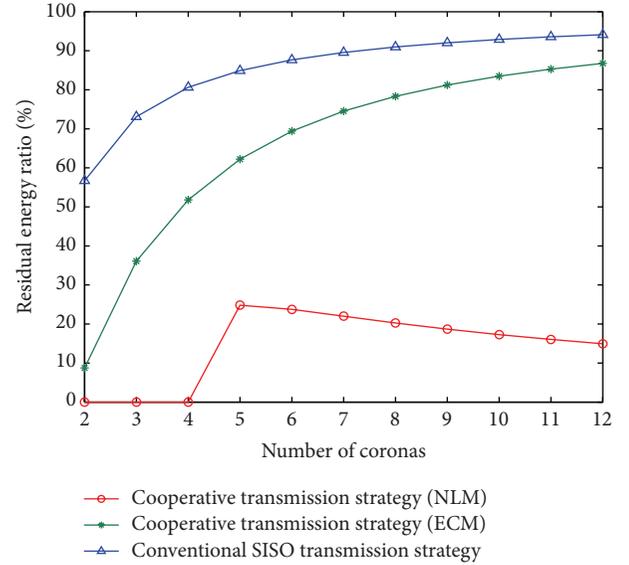


FIGURE 5: Residual energy ratio versus number of coronas.

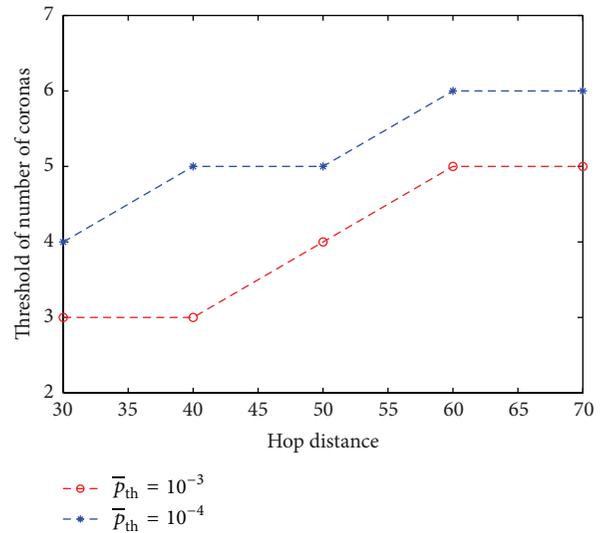


FIGURE 6: Threshold of number of coronas versus hop distance for different BER requirements.

It can be found that the residual energy ratio of NLM is 0 when the number of coronas  $N$  is no more than 4 in our scenario. In this case, energy hole problem can be avoided and optimal power allocation can be obtained. Otherwise, the residual energy ratio is not zero but very small. The energy hole problem can only be mitigated by applying a suboptimal power allocation. The residual energy only exists in the outermost corona when using NLM, according to the derivation of suboptimal power allocation. As the number of coronas increases, the total initial energy of the network increases while the residual energy in the outermost corona keeps relatively constant. Therefore, the residual energy ratio drops with the number of coronas for NLM. For ECM and SISO, the number of coronas is determined by the lifetime

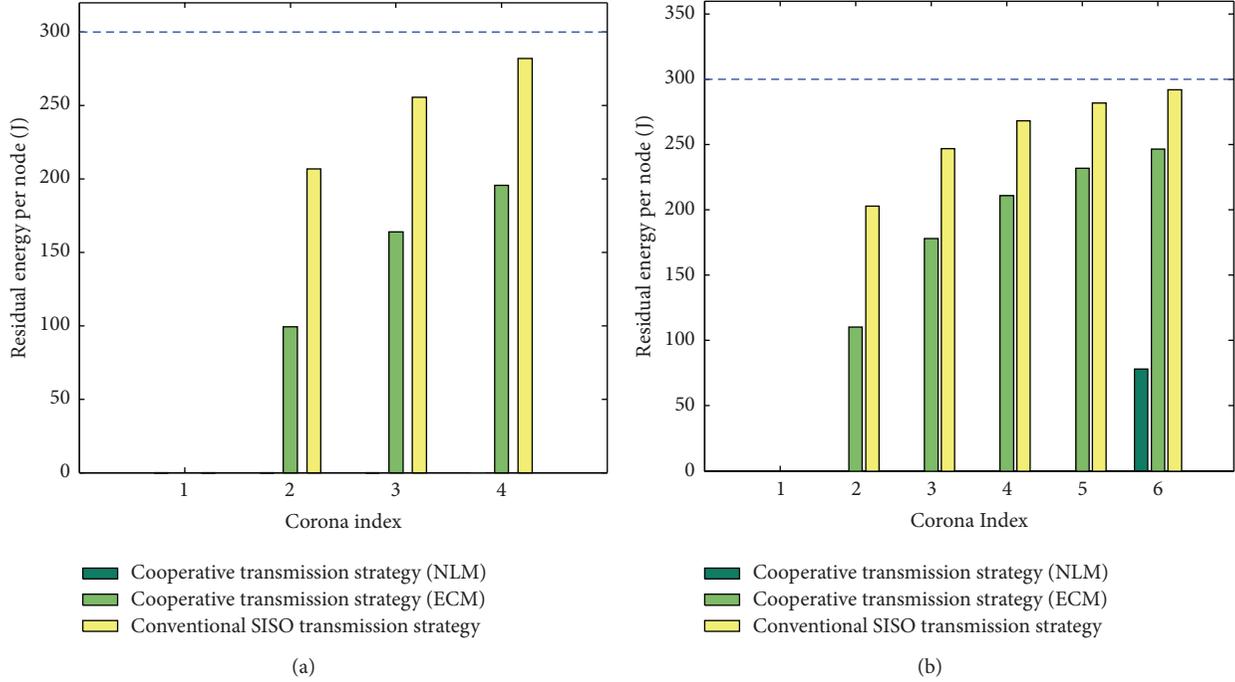


FIGURE 7: Residual energy per node of each corona. (a) 4-corona network and (b) 6-corona network.

of nodes in the innermost corona  $C_1$ . As the number of coronas increases, the traffic load carried by each node in  $C_1$  increases and the network lifetime decreases due to the shortened lifetime of nodes in  $C_1$ . The shortened lifetime will make the network have more residual energy when it ends its operation. This is why the residual energy ratio increases with the number of coronas in the case of ECM and SISO.

There is a threshold of the number of coronas, denoted by  $N_{th}$ , in NLM problem. When the number of coronas  $N \leq N_{th}$ , the solution using the KKT conditions exists and there is an optimal solution to the NLM problem; otherwise, no solution satisfying the KKT conditions exists and, hence, the solution using the relaxed KKT conditions is used as a suboptimal solution to the NLM problem. The threshold  $N_{th}$  is determined by the hop distance,  $r$ , and the required BER,  $\bar{p}_{th}$ , as shown in Figure 6. It can be seen that  $N_{th}$  increases with the increase of  $r$  and decrease of  $\bar{p}_{th}$ . In other words, with longer hop distance and higher BER requirement, the transmitting power of corresponding nodes will increase and this can help in balancing energy consumption over the network. In addition, the large  $N_{th}$  will allow the optimal solution proposed to mitigate the energy hole problem.

We further study the residual energy distribution in the network when the network lifetime ends. Considering that the energy hole problem can be avoided when the number of coronas  $N \leq 4$  and can only be mitigated when  $N > 4$ , we plot the residual energy per node of each corona under cooperative transmission strategy with NLM, ECM, and conventional SISO transmission strategy in a 4-corona network (Figure 7(a)) and a 6-corona one (Figure 7(b)). The dashed line represents the initial energy assigned to each node. In Figure 7(a), it can be seen that the cooperative transmission

strategy with NLM exhausts energy in all coronas. This means that the optimal power allocation of NLM effectively balances the energy consumption among coronas, and the energy hole problem is completely avoided. For both the cooperative transmission strategy with ECM power allocation and the conventional SISO transmission strategy, the energy in  $C_1$  is used up and an energy hole appears there. We can find that residual energy of the cooperative transmission strategy with ECM being lower than that of the conventional SISO transmission strategy in all other coronas,  $C_2$  to  $C_4$ . It demonstrates that the cooperative communication can balance energy consumption intrinsically, although ECM power allocation focuses on energy efficiency improvement. As expected, the further away the nodes from the sink, the greater the residual energy. In Figure 7(b), we observe that, for the cooperative transmission strategy with NLM, all nodes exhaust their energy but those in the outermost corona  $C_6$ . In this case, the suboptimal power allocation for NLM significantly mitigates the energy hole problem.

In the cooperative transmission strategy, each node participates in two cooperative MISO transmissions, that is, the transmissions in Phases 3 and 5, as described in Section 2.2. The power allocation of one node between these two cooperative MISO transmissions,  $P_i^{(i+1,i)}$  and  $P_i^{(i,i-1)}$  correspondingly, in different coronas in a wireless sensor network is shown in Figure 8. It can be seen that there is no  $P_1^{(1,0)}$  in the innermost corona  $C_1$ , and no  $P_N^{(N+1,N)}$  in the outermost corona  $C_N$ , since there is no Phase 5 and Phase 3 for them, respectively. It can also be seen that, for ECM, the transmitting power of a node is equally allocated in the two cooperative MISO transmissions that the node participates in. For NLM, lower power is allocated when a node is helped by its previous-hop

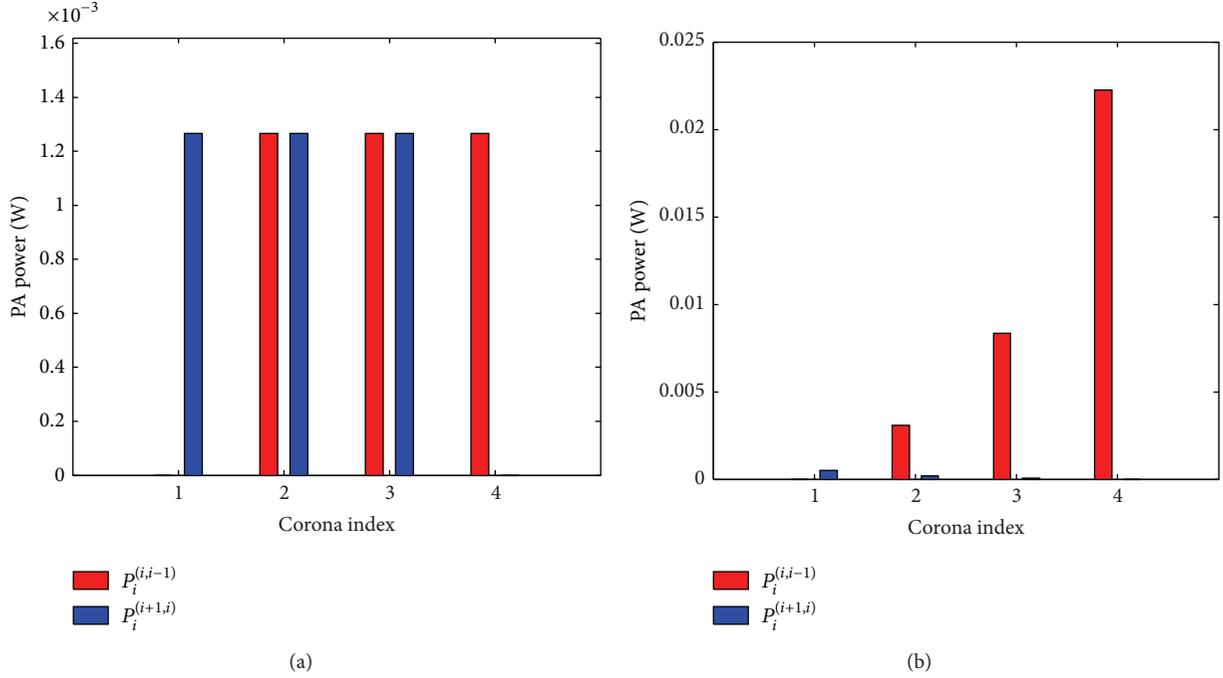


FIGURE 8: Power allocations of nodes in each corona which participate in two cooperative MISO transmissions in a wireless sensor network. (a) ECM and (b) NLM.

node in cooperative MISO transmission, while higher power is allocated when it helps its next-hop node. This is due to the fact that a node in the network has heavier traffic load than its previous-node and lighter load than its next-hop node. Moreover, for NLM, the total power per node  $P_i^{(i+1,i)} + P_i^{(i,i-1)}$  is unevenly allocated among different coronas; that is, higher power is allocated to nodes in the outer coronas compared to those in the inner ones. In other words, the power is unevenly allocated among nodes with different distance to the sink and it is equally allocated for ECM. Therefore, NLM can effectively balance the energy consumption over the network, while ECM cannot.

In the above discussions, the hop distance is fixed. It is valuable to study the influence of hop distance on network lifetime, as shown in [18]. In our study, the hop distance is chosen as the width of each corona, and it would decrease with increasing the number of coronas  $N$  in a fix-radius network. In Figure 9, we plot network lifetime versus the number of coronas in a network with radius of 500 m. Network lifetime increases firstly and reduces afterwards as  $N$  increases for both the cooperative and the conventional SISO transmission strategies. To show the performance of the cooperative transmission strategy clearly, the decrease side of network lifetime in the conventional SISO transmission strategy is omitted in Figure 9. We can explain this as follows. In a fix-radius network, the increase of the number of coronas leads to the decrease of the hop distance. Short hop distance decreases the energy consumption of a single hop. Thus, the network lifetime increases with the number of coronas increasing. On the other hand, as the number of coronas increases the traffic load in the inner coronas will increase.

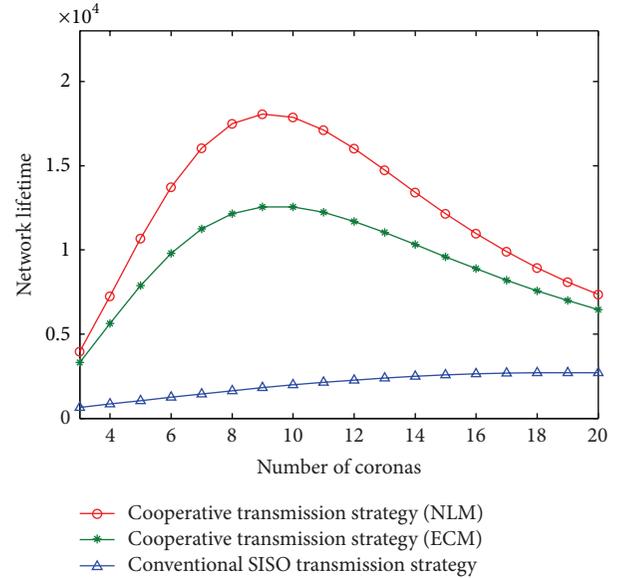


FIGURE 9: Network lifetime versus number of coronas in a network with radius of 500 m.

Thus, the network lifetime will decrease after the number of coronas exceeds a certain value. The impact of hop distance on the performance of cooperative transmission strategy implies that hop distance can be optimized to further extend the network lifetime.

We compare network lifetime performance of the proposed cooperative transmission strategy with SISO transmission strategies used for energy hole mitigation. Two typical

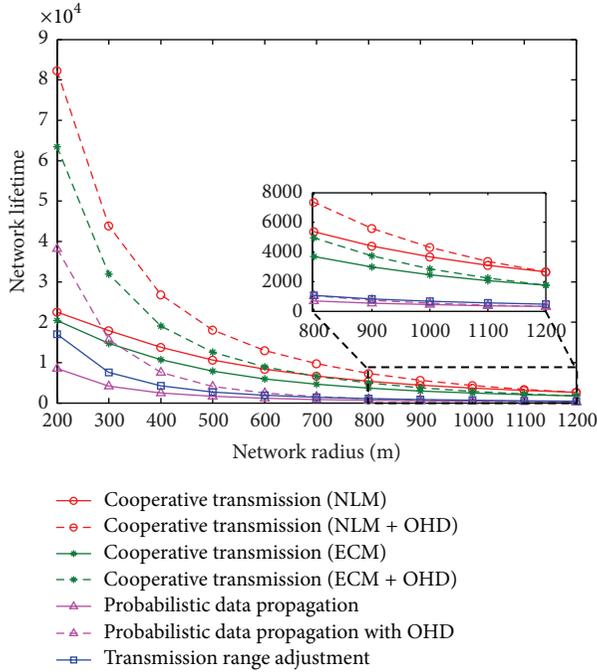


FIGURE 10: Network lifetime versus network radius for proposed cooperative transmission strategy and different SISO transmission strategies.

algorithms under the topology control strategy are considered, for example. One of them is the probabilistic data propagation algorithm, which allocates traffic load between hop-by-hop transmission mode and direct transmission mode, as in [9, 10]. Another one is the transmission range adjustment algorithm which adjusts the transmission distance of each hop to balance the energy consumption, as in [11, 12]. Figure 10 demonstrates the network lifetime achieved by the cooperative transmission strategy and the topology control strategy with noncooperative SISO transmission. The results shown by the solid line are obtained by assuming a fixed hop distance  $r = 100$  m. NLM, ECM, and the probabilistic data propagation algorithm are also combined with the optimized hop distance (OHD) method and have their corresponding results shown by the dashed lines. In the transmission range adjustment algorithm, however, the hop distance is varied and decided by itself. It can be seen from Figure 10 that the cooperative transmission strategy significantly outperforms the topology control strategy, regardless of whether or not the hop distance is optimized. In a network with radius of 1000 m, the lifetime of NLM without OHD is 800% and 430% higher than that of probabilistic data propagation without OHD and transmission range adjustment, respectively, and the lifetime of ECM without OHD is 520% and 260% higher than that of probabilistic data propagation without OHD and transmission range adjustment, respectively. Better performance can be achieved by any of the approaches or algorithms that apply the optimized hop distance method.

Finally, the proposed cooperative transmission strategy for energy hole mitigation is compared with two other cooperative communication schemes. One of them is cooperative

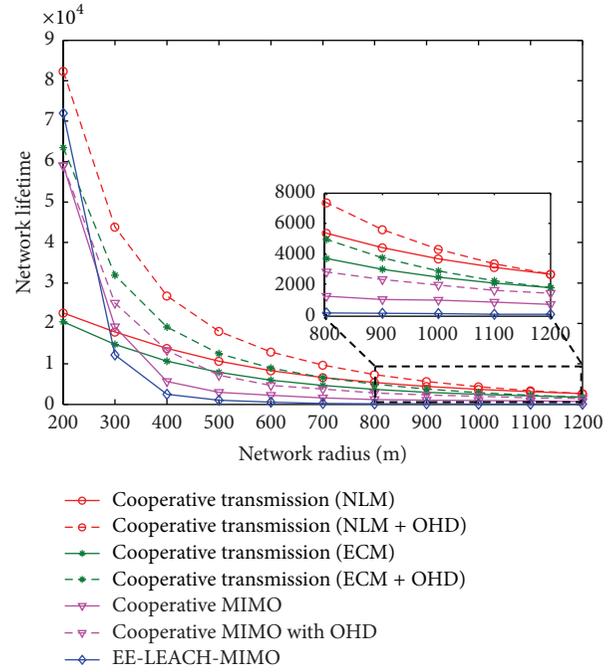


FIGURE 11: Network lifetime versus network radius for proposed cooperative transmission strategy and other cooperative communication schemes.

MIMO proposed in [17], where individual single-antenna nodes cooperate to form multiple-antenna transmitters and cooperatively transmit the information to a relay node; then multihop-based routing is used to forward the data to its final destination. The other is the EE-LEACH-MIMO scheme, which integrates MIMO technology into the LEACH algorithm, and considers the location and the residual energy of each node when the cluster heads for clustering and cooperative nodes for the MIMO system are selected [19]. In Figure 11, the network lifetime of NLM, ECM, and cooperative MIMO is shown by the solid line, and its alternative one with OHD is shown by the dashed lines. It can be seen from the figure that when the network radius is larger than 400 m, the proposed cooperative transmission strategy outperforms other cooperative communication schemes, regardless of whether or not the hop distance is optimized. In a network with radius of 1000 m, the lifetime of NLM without OHD is 300% and 1600% higher than that of cooperative MIMO without OHD and EE-LEACH-MIMO, respectively, and the lifetime of ECM without OHD is 170% and 1100% higher than that of cooperative MIMO without OHD and EE-LEACH-MIMO, respectively. In the EE-LEACH-MIMO scheme, the cluster head and its cooperative nodes transmit the data directly to the sink no matter how far they are from the sink. As a result, its lifetime in a large network is extremely low and even lower than that of SISO transmission. When the network radius is small, the lifetime of the proposed cooperative transmission strategy is lower than that of cooperative MIMO and EE-LEACH-MIMO. In a small network with very few hops, the energy efficiency of the proposed cooperative transmission strategy is low since the data from the first

coronas are transmitted to the sink directly with the SISO method. However, in a larger network with more hops, where the traffic unbalance is more severe, the proposed cooperative transmission strategy can perform better than cooperative MIMO and EE-LEACH-MIMO in terms of lifetime extension since it can balance energy consumption effectively throughout the network.

## 6. Conclusion and Future Works

In WSNs, both energy efficiency and the energy hole problem are key factors affecting network lifetime. In this paper, a cooperative transmission strategy is proposed to mitigate energy hole problem by exploring the network energy balancing capability and improving energy efficiency simultaneously. Optimal power allocations for NLM and ECM are studied. Numerical and simulation results show that network lifetime achieved by the cooperative transmission strategy is much longer than that by applying SISO transmission for energy hole mitigation and other cooperative communication schemes. In the proposed cooperative transmission strategy, NLM power allocation outperforms ECM in terms of network lifetime extension. In a 10-corona network, the ratio of lifetime by NLM to that by SISO is up to 9, compared to about 6 for ECM. The unique feature that makes the NLM approach superior to other schemes or algorithms in mitigating the energy hole problem is the two-tier power allocation strategy, that is, uneven power allocation between two cooperative MISO transmissions for nodes in a specific corona and uneven power allocation for intercorona nodes as well. NLM also utilizes cooperative communication which has higher energy consumption efficiency inherently than noncooperative transmission. Our method can be applied to the networks with arbitrary node distribution and the maximum transmitting power limitation can be relaxed since cooperative communication is introduced here.

The implementation of proposed cooperative transmission requires a central controller to collect channel information and conduct power allocation. The additional overhead for the central control can be reduced by introducing a distributed control method at the cost of a slight performance degradation. As we have seen, combining cooperative transmission with optimized hop distance can achieve better performance in network lifetime extension. When allowing the distance of each hop to be adjustable, we will be able to further improve the performance in conjunction with the cooperative transmission strategy.

## Appendices

### A. Proof of Proposition 1

The lagrangian function associated with (16) is defined by

$$\Gamma(\tau, \mathbf{P}, \boldsymbol{\nu}, \boldsymbol{\mu}) = \tau + \sum_{i=1}^N \nu_i \left( \frac{E_i}{(2i-1)\pi r^2 \rho} - \tau \right) + \sum_{i=1}^{N-1} \mu_{i-1} (\bar{P}_{i-1}^{(i+1,i)} - \bar{P}_{\text{th}}), \quad (\text{A.1})$$

where  $\mathbf{P} = [P_1^{(2,1)}, P_2^{(2,1)}, P_2^{(3,2)}, P_3^{(3,2)}, \dots, P_{N-1}^{(N,N-1)}, P_N^{(N,N-1)}]$  and the multipliers  $\boldsymbol{\nu} = [\nu_1, \dots, \nu_N]$ ,  $\boldsymbol{\mu} = [\mu_0, \dots, \mu_{N-2}]$ .

The KKT optimality conditions is given by

$$\frac{E_i}{(2i-1)\pi r^2 \rho} - \tau \leq 0, \quad i = 1, 2, \dots, N; \quad (\text{A.2a})$$

$$\bar{P}_{i-1}^{(i+1,i)} - \bar{P}_{\text{th}} \leq 0, \quad i = 1, 2, \dots, N-1; \quad (\text{A.2b})$$

$$P_i^{(i+1,i)}, P_{i+1}^{(i+1,i)} > 0, \quad i = 1, 2, \dots, N-1; \quad (\text{A.2c})$$

$$\nu_i \geq 0, \quad i = 1, 2, \dots, N; \quad (\text{A.2d})$$

$$\mu_{i-1} \geq 0, \quad i = 1, 2, \dots, N-1; \quad (\text{A.2e})$$

$$\nu_i \left( \frac{E_i}{(2i-1)\pi r^2 \rho} - \tau \right) = 0, \quad i = 1, 2, \dots, N; \quad (\text{A.2f})$$

$$\mu_{i-1} (\bar{P}_{i-1}^{(i+1,i)} - \bar{P}_{\text{th}}) = 0, \quad i = 1, 2, \dots, N-1; \quad (\text{A.2g})$$

$$\frac{\partial \Gamma}{\partial P_i^{(i+1,i)}} = \nu_i \frac{N^2 - i^2}{2i-1} \frac{l}{R} - \mu_{i-1} \frac{N_0^2 W^2}{\sigma_{i+1,i-1}^2 \sigma_{i,i-1}^2 P_{i+1}^{(i+1,i)} P_i^{(i+1,i)^2}} = 0, \quad i = 1, 2, \dots, N-1; \quad (\text{A.2h})$$

$$\frac{\partial \Gamma}{\partial P_{i+1}^{(i+1,i)}} = \nu_{i+1} \frac{N^2 - i^2}{2i+1} \frac{l}{R} - \mu_{i-1} \frac{N_0^2 W^2}{\sigma_{i+1,i-1}^2 \sigma_{i,i-1}^2 P_i^{(i+1,i)} P_{i+1}^{(i+1,i)^2}} = 0, \quad i = 1, 2, \dots, N-1; \quad (\text{A.2i})$$

$$\frac{\partial \Gamma}{\partial \tau} = 1 - \sum_{i=1}^N \nu_i = 0. \quad (\text{A.2j})$$

If an arbitrary  $\nu_i = 0$ , we can derive that  $\mu_{i-1} = 0$  by (A.2h). Then,  $\nu_{i+1} = 0$  can be derived by (A.2i). By substituting  $i+1$  in place of  $i$  in (A.2h), we can further derive that  $\mu_i = 0$ . Continuing the analogy, we can derive that  $\nu_j = 0$ ,  $j = i+1, i+2, \dots, N$ , and  $\mu_{j-1} = 0$ ,  $j = i, i+1, \dots, N-1$ . On the other hand, when  $\nu_i = 0$ , we can derive that  $\mu_{i-2} = 0$  by substituting  $i$  in place of  $i+1$  in (A.2i). Similarly, it can be derived that  $\nu_j = 0$ ,  $j = 1, 2, \dots, i-1$ , and  $\mu_{j-1} = 0$ ,  $j = 1, 2, \dots, i-1$ . In sum, if an arbitrary  $\nu_i = 0$ , it can be derived that  $\nu_i = 0$ ,  $i = 1, 2, \dots, N$ , and  $\mu_{i-1} = 0$ ,  $i = 1, 2, \dots, N-1$ . Similarly, if an arbitrary  $\mu_{i-1} = 0$ , the same result can be derived. However, this result contradicts the condition in (A.2j). Therefore, we find that, to satisfy the KKT conditions, all multipliers  $\nu$  and  $\mu$  should be positive. Combining that with (A.2g) and (A.2f), it can be derived that the optimal solution  $P$  should satisfy the conditions

$$\frac{E_i}{(2i-1)\pi r^2 \rho} = \tau, \quad i = 1, 2, \dots, N; \quad (\text{A.3})$$

$$\bar{P}_{i-1}^{(i+1,i)} = \bar{P}_{\text{th}}, \quad i = 1, 2, \dots, N-1.$$

Moreover, we combine these conditions with other KKT conditions which we have not used before, that is, (A.2a), (A.2b), and (A.2c). It can be found that the solution which satisfies conditions in (A.3) would always satisfy the conditions in (A.2a) and (A.2b). Thus, the necessary conditions can be derived by

$$\frac{E_i}{(2i-1)\pi r^2 \rho} = \tau, \quad i = 1, 2, \dots, N; \quad (\text{A.4a})$$

$$\bar{p}_{i-1}^{(i+1,i)} = \bar{p}_{\text{th}}, \quad i = 1, 2, \dots, N-1; \quad (\text{A.4b})$$

$$P_i^{(i+1,i)}, P_{i+1}^{(i+1,i)} > 0, \quad i = 1, 2, \dots, N-1. \quad (\text{A.4c})$$

## B. Proof of Proposition 2

The lagrangian function associated with (23) is defined by

$$\Gamma(\mathbf{P}, \boldsymbol{\mu}) = \sum_{i=1}^N E_i + \sum_{i=1}^{N-1} \mu_{i-1} (\bar{p}_{i-1}^{(i+1,i)} - \bar{p}_{\text{th}}), \quad (\text{B.1})$$

with the multipliers  $\boldsymbol{\mu} = [\mu_0, \dots, \mu_{N-2}]$ . And  $\mathbf{P} = [P_1^{(2,1)}, P_2^{(2,1)}, \dots, P_{N-1}^{(N,N-1)}, P_N^{(N,N-1)}]$ .

The KKT optimality conditions is given by

$$\bar{p}_{i-1}^{(i+1,i)} - \bar{p}_{\text{th}} \leq 0, \quad i = 1, 2, \dots, N-1; \quad (\text{B.2a})$$

$$P_i^{(i+1,i)}, P_{i+1}^{(i+1,i)} > 0, \quad i = 1, 2, \dots, N-1; \quad (\text{B.2b})$$

$$\mu_{i-1} \geq 0, \quad i = 1, 2, \dots, N-1; \quad (\text{B.2c})$$

$$\mu_{i-1} (\bar{p}_{i-1}^{(i+1,i)} - \bar{p}_{\text{th}}) = 0, \quad i = 1, 2, \dots, N-1; \quad (\text{B.2d})$$

$$\begin{aligned} \frac{\partial \Gamma}{\partial P_i^{(i+1,i)}} &= (N^2 - i^2) \frac{\pi r^2 \rho l}{R} \\ &\quad - \mu_{i-1} \frac{N_0^2 W^2}{\sigma_{i+1,i-1}^2 \sigma_{i,i-1}^2 P_{i+1}^{(i+1,i)} P_i^{(i+1,i)^2}} = 0, \quad (\text{B.2e}) \\ &\quad i = 1, 2, \dots, N-1; \end{aligned}$$

$$\begin{aligned} \frac{\partial \Gamma}{\partial P_{i+1}^{(i+1,i)}} &= (N^2 - i^2) \frac{\pi r^2 \rho l}{R} \\ &\quad - \mu_{i-1} \frac{N_0^2 W^2}{\sigma_{i+1,i-1}^2 \sigma_{i,i-1}^2 P_i^{(i+1,i)} P_{i+1}^{(i+1,i)^2}} = 0, \quad (\text{B.2f}) \\ &\quad i = 1, 2, \dots, N-1. \end{aligned}$$

Since  $P_i^{(i+1,i)}$  and  $P_{i+1}^{(i+1,i)}$  are interchangeable in the KKT conditions, the optimized value of  $P_i^{(i+1,i)}$  and  $P_{i+1}^{(i+1,i)}$  should be equal,

$$P_i^{(i+1,i)*} = P_{i+1}^{(i+1,i)*}, \quad (\text{B.3})$$

at the extreme point. From (B.2e) and (B.2f), it can be derived that  $\mu_{i-1} \neq 0, i = 1, 2, \dots, N-1$ . Combining (B.2d), we can further derive that

$$\bar{p}_{i-1}^{(i+1,i)} - \bar{p}_{\text{th}} = 0, \quad i = 1, 2, \dots, N-1. \quad (\text{B.4})$$

Thus, the optimal solution can be obtained as

$$P_i^{(i+1,i)*} = P_{i+1}^{(i+1,i)*} = \frac{N_0 W}{\sqrt{\bar{p}_{\text{th}} \sigma_{i+1,i-1} \sigma_{i,i-1}}}, \quad i = 1, 2, \dots, N-1. \quad (\text{B.5})$$

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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