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Granular superconductors: From the nonlinear σ model to the Bose-Hubbard description

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We modify a nonlinear σ model (NLσM) for the description of a granular disordered system in the presence of both the Coulomb repulsion and the Cooper pairing. We show that under certain controlled approximations the action of this model is reduced to the Ambegaokar-Eckern-Schön (AES) action, which is further reduced to the Bose-Hubbard (or "dirty-boson") model with renormalized coupling constants. We obtain an effective action which is more general than the AES one but still simpler than the full NLσM action. This action can be applied in the region of parameters where the reduction to the AES or the Bose-Hubbard model is not justified. This action may lead to a different picture of the superconductor-insulator transition in two-dimensional systems.

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A wide variety of experimental data on the superconductor-insulator (SI) transition in two-dimensional structures[1–4] continues to attract acute theoretical interest. The transition can be tuned by either disorder (changing with the thickness of a superconducting film) or magnetic field, thus being one of the most intensely studied examples of quantum phase transitions.[5] However, recent experiments[1,2] have challenged the very existence of the SI transition, leaving open the possibility that a dramatic drop in resistance is due to the existence of a crossover to a new metallic phase with resistance much lower than that in the normal state and to a subsequent metal-superconducting transition.[1] This situation requires a reassessment of theoretical approaches to the problem of dirty superconductors.

One of the ways to understand the problem of the SI transition is based on the so-called Bose-Hubbard (or "dirty-boson") models[6] where the superconducting phase is due to the Bose condensation of charge-2e bosons (preformed Cooper pairs) with localized vortices while the insulating phase is due to the Bose condensation of vortices with localized Cooper pairs. Another approach which captures the basic physics of granular superconductors is based on dissipative models[7] of resistively shunted charged Josephson arrays,[8–11] with the emphasis on the role of dissipation and Coulomb interaction. In both groups of models,[6–11] the transition is driven by fluctuations of the phase of the order parameter. An alternative approach is based on a microscopic description of *homogeneous* systems that incorporates both the attractive (in the Cooper channel) and repulsive electron-electron interaction in the presence of disorder into an effective field theory, the nonlinear σ model (NLσM).[12–15] The SI transition in these models is driven by fluctuations of the amplitude rather than the phase of the order parameter, and the Cooper pairing is suppressed by the repulsive interaction on the insulating side of the transition. However, experimental distinction between homogeneous and granular systems is not as strict[1] as it seemed a few years ago, and recent experimental observations[1,2] strongly suggest that the amplitude fluctuations in the vicinity of the SI transition are no less important than the phase fluctuations.

The purpose of this paper is to derive microscopically a general NLσM action that takes into account fluctuations of both amplitude and phase of the order parameter Δ, thus encompassing all the above described approaches. We further show that both the Bose-Hubbard model[6] and the dissipative models[8–11] can, in fact, be derived from this action. The two models correspond to certain simplifications made within the NLσM. The latter is more general and allows one to go beyond different limitations inevitable in the derivation of the Bose-Hubbard and dissipative models. Note that the dissipative action[8] of Ambegaokar, Eckern, and Schön (AES) has been widely used[16] in a simplified form in the context of a normal tunnel junction. This variant of the AES action has been very recently derived[17] from the NLσM describing electrons with the repulsive interaction moving in the presence of disorder. Here we will derive both the full AES action for Josephson junctions and the Bose-Hubbard model from the NLσM that includes both the attraction in the Cooper channel and the Coulomb repulsion. We shall use a new variant[18] of the NLσM which, in our opinion, considerably simplifies the calculations. Naturally, one can use (after a straightforward modification for a granular system) any version of the NLσM that includes the Cooper pairing and the Coulomb interaction, either the original Finkelstein model[12] or a more recent model[15] in Keldysh technique.

Our starting point is the standard microscopic Hamiltonian that includes a δ-correlated Gaussian random potential, the Coulomb interaction, and the BCS attraction. We consider a coarse-grained version of this Hamiltonian which corresponds to a granular superconductor. This will allow us to separate scales of fluctuations of the amplitude and the phase of the superconducting order parameter Δ. Neglecting (at some later stage) the amplitude fluctuations and making some further simplifications will lead eventually to the models[6,8] governed only by the phase fluctuations.

Let us stress again that the NLσM used here can be applied to the simultaneous description of both the amplitude and phase fluctuations of Δ that can be quite important in the relation to recent experiments.[2] Moreover, even when focusing on the phase fluctuations only, the effective functional is essentially generalized by disorder (affecting intragranular electron motion and thus leading to a different model of the phase fluctuations) and can in principle lead to a different picture of the transition.

The derivation of the NLσM from the Hamiltonian described above follows the standard steps.[19] First one averages the replicated imaginary-time fermionic action over the random potential. Then one employs the Hubbard-Stratonovich transformation to decouple three quartic (in the electron field) terms, corresponding to the disorder, the Coulomb repulsion, and the BCS attraction. Finally, by integrating out the fermionic fields, one arrives at the effective action in terms of three bosonic fields: a matrix field $\hat{\sigma}$ that decouples the disorder-induced "interaction," Φ that decouples the Coulomb repulsion, and Δ that decouples the BCS attraction,

$$\mathcal{S}[\hat{\sigma},\hat{\Delta},\Phi]=\frac{\pi\nu}{8\tau_{\rm el}}\mathrm{Tr}\,\sigma^2+\frac{1}{4\lambda_0}\mathrm{Tr}\,|\hat{\Delta}|^2+\frac{1}{2}\mathrm{Tr}\,\Phi U^{-1}\Phi -\frac{1}{2}\mathrm{Tr}\ln\left[-\hat{\xi}-\hat{t}+\frac{i}{2\tau_{\rm el}}\hat{\sigma}+i(\hat{\Delta}+\Phi+\hat{\epsilon})\right]. \tag{1}$$

Here the operator $\hat{\epsilon}$ equals $i\hat{\tau}_3\partial_\tau$ in imaginary time representation and becomes the diagonal matrix of fermionic Matsubara frequencies in frequency representation, $\hat{\xi}$ is the operator of the intragrain kinetic energy (counted from the chemical potential), and $\hat{t}$ is the tunneling amplitude matrix (i.e., the intergrain kinetic energy). All the bosonic fields are defined in the space which is convenient to think of as a direct product of the $N\times N$ replica sector, the 2×2 spin sector, the 2×2 "time-reversal" sector (introduced for a correct decoupling in the Cooper, channel for both disorder-induced and BCS interactions), and of the $m\times m$ grain sector. The symbol Tr refers both to a summation over all these matrix indices and to an integration over position $\mathbf{r}$ and the imaginary time τ. The matrix $\hat{\sigma}$ is diagonal in grain indices (called later i,j) and possesses standard symmetries in all the other sectors.[19] The pairing field $\hat{\Delta}$ is diagonal in the replica and grain indices and in $x\equiv(\mathbf{r},\tau)$, and has the following structure[18] in the time-reversal and spin sectors:

$$\hat{\Delta}(x)=|\Delta(x)|e^{(i/2)\chi(x)\hat{\tau}_3}\hat{\tau}_2^{sp}\otimes\hat{\tau}_2 e^{-(i/2)\chi(x)\hat{\tau}_3}, \tag{2}$$

where $\hat{\tau}_\alpha$ and $\hat{\tau}_\alpha^{\rm sp}$ are Pauli matrices in the time-reversal and spin sectors, respectively. The Coulomb field $\Phi(x)$ is proportional to the unit matrix in all the matrix sectors. Finally, in Eq. (1) λ_0 and $\tau_{\rm el}$ are the BCS coupling constant and the elastic mean free time, respectively, and $U\equiv U(\mathbf{r}-\mathbf{r}')$ is the Coulomb interaction.

The principal simplification for granular systems is that all the fields are spatially homogeneous inside each grain when the grains are zero dimensional, i.e., their sizes $L\lesssim\xi,L_T$ (ξ and L_T are the superconducting and thermal coherence lengths) which is equivalent to $|\Delta|,T\lesssim 1/\tau_{\rm erg}$. Then the Coulomb interaction reduces to the capacitance matrix, $U^{-1}\to C_{ij}/e^2$, and the tunneling matrix $\hat{t}=\{t_{ij}\}$ depends only on grain indices.

Now we follow the procedure of derivation of the NLσM for dirty superconductors.[18] First, we look for a saddle point of Eq. (1) with respect to $\hat{\sigma}$ separately for each grain. It can be parametrized as $\hat{\sigma}_{\rm s.p.}=S^\dagger\Lambda S$, where S is a certain matrix which also diagonalizes $\hat{\epsilon}+\Phi+\hat{\Delta}$,

$$\hat{\epsilon}_i+\Phi_i+\hat{\Delta}_i=S_i^\dagger\lambda_i S_i, \tag{3}$$

where all the matrices are diagonal in grain indices. Then the entire saddle-point manifold is parametrized as

$$\hat{\sigma}_i=S_i^\dagger Q_i S_i, \quad Q_i=U_i^\dagger\Lambda U_i, \tag{4}$$

where in the Matsubara representation $\Lambda=\mathrm{diag}\{\mathrm{sgn}\,\epsilon\}$, and matrix U defines the standard coset space.[20] Here Q is a degenerate solution to the saddle-point equation for the action (1) when $\hat{\epsilon}=0$, $\hat{\Delta}$, and Φ all vanish. The parametrization (4) "aligns" the field σ so that $\hat{\Delta}$ and Φ are taken into account in the zeroth approximation.

Now we perform a similarity transformation with matrices S and $S^\dagger$ under Tr ln in Eq. (4). As all the fields are spatially homogeneous inside each grain, S commutes with the operator $\hat{\xi}$. Then one only needs to expand the Tr ln to the first nonvanishing orders in t_{ij} and λ_i, this expansion being justified when $|t|,|\Delta|,T\ll 1/\tau_{\rm el}\ll\varepsilon_F$. Thus one arrives at the following effective action:

$$\mathcal{S}[Q,\Delta,\Phi]=\int_0^\beta d\tau\left\{\sum_i\frac{|\Delta_i|^2}{\nu\lambda_0\delta_i}+\sum_{ij}\frac{C_{ij}}{2e^2}\Phi_i\Phi_j\right\} -\sum_i\frac{\pi}{2\delta_i}\mathrm{Tr}\,\lambda_i Q_i-\frac{g_{ij}^T}{2}\sum_{ij}\mathrm{Tr}\,Q_iS_{ij}Q_jS_{ji}, \tag{5}$$

where $S_{ij}\equiv S_iS_j^\dagger$, all the fields depend on τ, Tr refers to all indices except those numerating grains, δ_i is mean level spacing in the ith grain, and the tunneling conductance is defined by $g_{ij}^T\equiv 2\pi^2|t_{ij}|^2/\delta_i\delta_j$ (which is nonzero only for neighboring grains). Both S and λ should be found from the diagonalization procedure in Eq. (3).

The next step is to represent S_i as

$$S_i=V_ie^{-(i/2)\chi_i(\tau)\hat{\tau}_3}. \tag{6}$$

This is similar to the gauge transformation suggested in Refs. 21 and used in Ref. 17 to gauge out the Coulomb field. However, one cannot gauge out two independent fields, Δ and Φ. Substituting the transformation (6) into the diagonalization condition (3), we reduce it to

$$\hat{\epsilon}+\tilde{\Phi}_i+\hat{\Delta}_i^0=V_i^+\lambda_iV_i, \tag{7}$$

where $\hat{\Delta}_i^0$ is the field (2) taken at $\chi=0$ and the field $\tilde{\Phi}$ is given in the τ representation by $\tilde{\Phi}_i\equiv\Phi_i-\frac{1}{2}\partial_\tau\chi_i$.

Both $\tilde{\Phi}$ and $|\Delta|$ are massive fields whose fluctuations are strongly suppressed. It is straightforward to show that the fluctuations of $\tilde{\Phi}$ are of order δ which is much smaller than both T and $|\Delta|$. Therefore, in the mean-field approximation in $\tilde{\Phi}$ this field can be neglected, $\tilde{\Phi}=0$. This condition is nothing more than the Josephson relation in imaginary time.[9]

This locks the fluctuations of the Coulomb field Φ with the phase fluctuations of the pairing field $\hat{\Delta}$,

$$\Phi=\tfrac{1}{2}\partial_\tau\chi, \tag{8}$$

thus reducing the action (5) to one depending only on the fields Q and Δ.

The mean-field approximation in $|\Delta_i|$ is valid for $|\Delta_i| \gg \delta$ and reduces to the standard self-consistency equation that formally follows from the variation of the action (5) with respect to $|\Delta|$. In this approximation one finds $|\Delta_i|$ to be independent of i and τ. Thus the first term in Eq. (5) becomes a trivial constant, so that the action depends on Q and χ only,

$$\mathcal{S}[Q,\chi]=\sum_{ij}\frac{C_{ij}}{8e^2}\int_0^\beta d\tau\,\partial_\tau\chi_i\partial_\tau\chi_j-\sum_i\frac{\pi}{2\delta_i}\mathrm{Tr}\,\lambda_i Q_i-\frac{g_{ij}^T}{2}\sum_{ij}\mathrm{Tr}\,Q_iS_{ij}Q_jS_{ji}\,. \tag{9}$$

Note that the field χ in this action obeys the standard boundary condition $\chi(\tau+\beta)=\chi(\tau)\,\mathrm{mod}\,2\pi$. Thus when calculating the partition function with this action, one should take into account different topological sectors corresponding to different winding numbers in χ.

The ''phase-only'' action (9) includes neither fluctuations of the amplitude of the order parameter Δ nor fluctuations of the Coulomb field beyond the Josephson relation, Eq. (8). We show below that it can be reduced to the AES action. Still, we stress that the action is more general than the AES action. Thus, in the absence of superconductivity, $\Delta\equiv 0$, it was shown[17] that the former contains a correct screening of the Coulomb interaction at low T, in contrast to the latter. This may also be important in the case when Δ is much smaller than the charging energy.

To further simplify the action (9) we note that the diagonalization conditions (7), in the absence of the $|\Delta|$ fluctuations, are the same for each grain and reduced to those solved in Ref. 18,

$$V_{\epsilon\epsilon'}=\cos\frac{\theta_\epsilon}{2}\,\delta_{\epsilon,\epsilon'}+\hat{\tau}_2^{sp}\otimes\hat{\tau}_2\sin\frac{\theta_\epsilon}{2}\mathrm{sgn}\,\epsilon\;\delta_{\epsilon,-\epsilon'}\,,$$

$$\lambda=\mathrm{diag}\sqrt{\epsilon^2+|\Delta|^2}\,\mathrm{sgn}\,\epsilon,\qquad \cos\theta_\epsilon\equiv\frac{|\epsilon|}{\sqrt{\epsilon^2+|\Delta|^2}}. \tag{10}$$

Then S_{ij} in Eq. (9) can be expressed in terms of V as

$$S_{ij}\equiv Ve^{-(i/2)\chi_{ij}\hat{\tau}_3}V^\dagger,\qquad \chi_{ij}\equiv\chi_i-\chi_j\,. \tag{11}$$

Finally note that large-$|\epsilon|$ contributions to the action (4) are strongly suppressed, while for $|\epsilon|\ll|\Delta|$ one has $\lambda=|\Delta|\Lambda$ which suppresses fluctuations of Q in each grain imposing $Q=\Lambda$. Then, all matrices in the action (9) are diagonal in the replica indices so that these indices become redundant. The diagonalization procedure [Eqs. (10) and (11)] has resolved explicitly the matrix dependence on the time-reversal and spin indices. This reduces the action to that depending only on one *scalar* bosonic field, the phase χ of the order parameter, whose arguments are the imaginary time τ, and the position index i, i.e., the grain number. Indeed, the second term in Eq. (9) reduces to a trivial constant; evaluating the tunneling term with the help of Eqs. (10) and (11), we obtain

$$\mathcal{S}[\chi]=\sum_{ij}\left\{\frac{C_{ij}}{8e^2}\int_0^\beta d\tau\,\partial_\tau\chi_i\,\partial_\tau\chi_j-2g_{ij}^T\int_0^\beta d\tau\int_0^\beta d\tau'\times g_{\mathrm{n}}^2(\tau-\tau')\cos\chi_{ij}^-+g_{\mathrm{a}}^2(\tau-\tau')\cos\chi_{ij}^+\right\}, \tag{12}$$

where $\chi_{ij}(\tau)\equiv\chi_i(\tau)-\chi_j(\tau)$,

$$\chi_{ij}^\pm\equiv\tfrac{1}{2}[\chi_{ij}(\tau)\pm\chi_{ij}(\tau')],$$

and the normal and anomalous Green's functions $g_{n,a}$ (integrated over all momenta) are given by

$$g_n(\tau)=T\sum_\epsilon\frac{\epsilon\sin\epsilon\tau}{\sqrt{\epsilon^2+|\Delta|^2}},\qquad g_a(\tau)=T\sum_\epsilon\frac{|\Delta|\cos\epsilon\tau}{\sqrt{\epsilon^2+|\Delta|^2}}. \tag{13}$$

This action coincides with that derived in Refs. 8–10. Further simplifications are possible in two limiting cases.

First, in the normal case $(\Delta=0)$, one has in Eq. (13)

$$g_a=0,\qquad g_n^2(\tau)=\frac{T^2}{\sin^2\pi T\tau}. \tag{14}$$

Then the field χ should be substituted, according to Eq. (8), by $2\int^\tau d\tau'\Phi(\tau')$. This limiting case corresponds to using the actions (12) and (14) in the context of a normal tunnel junction.[16] This is precisely the action which has been recently derived from the NLσM in Ref. 17; the functional (9) in the limit $\Delta=0$ is equivalent to the σ model of Ref. 17. Including the disorder-induced fluctuations (i.e., going beyond the $Q=\Lambda$ approximation) allows one to obtain[17] a correct low-T limit for the phase correlation function missing in the action (12).

The action (9) is more general than that considered in Ref. 17: although under the mode-locking condition (8) it depends only on the fields χ and Q, the matrix S_{ij}, Eqs. (10) and (11), reduces to a simple $U(1)$ gauge transformation as in Ref. 17 only in the limit $\Delta=0$.

The second limiting case, $T\ll|\Delta|$, is just the limit relevant in the context of the SI transition in granular superconductors. For $T=0$, the summation in Eq. (13) can be substituted by integration which yields

$$g_n(\tau)=\frac{|\Delta|}{\pi}K_1(|\Delta|\tau),\qquad g_a(\tau)=\frac{|\Delta|}{\pi}K_0(|\Delta|\tau).$$

This is also a good approximation for a low-temperature case; substituting this into Eq. (12) gives the action for the dissipative model.[9,10] Note that for $|\tau-\tau'|\ll|\Delta|^{-1}$, the main contribution in the tunneling action (12) is given by the normal term with the corresponding kernel proportional to $|\tau-\tau'|^{-2}$. The Fourier transform of this would give a term of the Caldeira-Leggett type[7] proportional to $|\omega|$.

The tunneling action (12) is nonlocal in τ. As has been noted in Ref. 9 for the case of one tunnel junction, for sufficiently large capacitance the phase χ_{ij} changes slowly in comparison with $|\Delta|^{-1}$, and in the adiabatic approximation $\chi(\tau')$ is changed by $\chi(\tau)+(\tau'-\tau)\partial_\tau\chi(\tau)$. Making such an expansion, one obtains from Eq. (12) the following local action:

$$\mathcal{S}[\chi]=\int_0^\beta d\tau\left\{\sum_{ij}\ \tfrac{1}{2}u_{ij}^{-1}\dot{\chi}_i\dot{\chi}_j-\left|\Delta\right|g_{ij}^T\cos\chi_{ij}\right\},\qquad(15)$$

where $\dot{\chi}_i\equiv\partial_\tau\chi_i$ and

$$\frac{1}{u_{ii}}\equiv\frac{C_{ii}}{4e^2}+\sum_j\frac{g_{ij}^T}{|\Delta|}\frac{3+\cos\chi_{ij}}{8},$$

$$u_{ij}^{-1}\equiv\frac{C_{ij}}{4e^2}-\frac{g_{ij}^T}{|\Delta|}\frac{3+\cos\chi_{ij}}{8}.$$

If all the self-capacitances are equal to C with $E_c\propto e^2/C$ being the charging energy, and all $g_{ij}^T=g^T$, then $u_{ii}\equiv U$ has the meaning of the renormalized charging energy. Ignoring a weak dependence of u on $\cos\chi_{ij}$ in the above relations, one obtains the renormalized charging energy:

$$U=\frac{E_c}{1+\#E_cg^T/|\Delta}.\qquad(16)$$

Here the coefficient # depends on the number of next neighbors for each grain, etc. A similar renormalization takes place for the next-neighbor off-diagonal energy u_{ij}. Now one can see that on the face of it the adiabatic approximation employed to obtain Eq. (15) is valid for $U\ll|\Delta|$. However, in the region $g^T\gg|\Delta|/E_c$, where the charging energy (16) is strongly renormalized, the instantonlike solutions[22] may be important. This may further reduce the region of applicability for the local in τ action (15).

Finally, by introducing the operator $\hat{n}$ canonically conjugate to the phase χ, one finds the Hamiltonian that corresponds to the action (15):

$$\hat{H}=\sum_{ij}\ \tfrac{1}{2}u_{ij}\hat{n}_i\hat{n}_j-|\Delta|g_{ij}^T\cos(\chi_i-\chi_j).\qquad(17)$$

This is just the Hamiltonian of the Bose-Hubbard model[6] which was first microscopically derived by Efetov[23] in the context of granular superconductors.

To conclude, we have derived the effective NLσM-type action [Eq. (5)] for a granular system with zero-dimensional grains in the presence of Coulomb interaction and superconductivity. This is the most general (in the present context) action that takes into account fluctuations of both amplitude and phase of the order parameter Δ. Neglecting fluctuations of $|\Delta|$ and fluctuations of the Coulomb field beyond the Josephson relation (8) reduces this action to the "phase-only" action (9) which still contains intragranular disorder important for the correct screening for $|\Delta|$ small compared to the charging energy. Neglecting this disorder further reduces the action (9) to that of the AES model (12). When the renormalized charging energy, Eq. (16), is much smaller than $|\Delta|$, the action (12) finally goes over to that of the Bose-Hubbard model, Eq. (17), which is widely used for the description of the superconductor-insulator transition.[6] However, the above estimations show that this reduction is parametrically justified only for the region $E_c\ll|\Delta|$ where the transition happens at $g^T\ll E_c/|\Delta|\ll 1$ which corresponds to a strongly granular system. Note finally that the most general (in the present context) action (5) describes both amplitude and phase fluctuations of the order parameter, being still considerably different from the NLσM action for homogeneous systems. We hope that using this action may eventually lead to a different phase diagram for granular superconductors.

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