Comparison of Bit Error Rate Estimation Methods for QPSK CO-OFDM Transmission

Son Thai Le, Mary E. McCarthy, Naoise Mac Suibhne, Elias Giacoumidis, Nick J. Doran, Andrew D. Ellis and Keith J. Blow

***Abstract*—In this letter, we experimentally study the statistical properties of a received QPSK modulated signal and compare various bit error rate (BER) estimation methods for coherent optical orthogonal frequency division multiplexing (CO-OFDM) transmission. We show that the statistical BER estimation method based on the probability density function of the received QPSK symbols offers the most accurate estimate of the system performance.**

***Index Terms*— Bit error rate, coherent detection, coherent optical transmission, orthogonal frequency division multiplexing.**

I. INTRODUCTION

COHERENT optical orthogonal frequency division multiplexing (CO-OFDM) is considered as a promising candidate for future long-haul high capacity transmission systems [1, 2]. CO-OFDM provides an efficient way to compensate for inter-symbol interference caused by both chromatic dispersion (CD) and polarization-mode dispersion (PMD) while using simplified equalization scheme [1, 3]. The design, development, and operation of CO-OFDM systems all require simple, efficient and reliable methods of performance evaluation.

The bit error rate (BER) in CO-OFDM systems can be estimated in numerical investigations using Monte Carlo simulation and in experiments by directly counting the number of errors at the receiver. The corresponding Q-factor is calculated using the inverse complementary error function. However, this method relies on a large number of statistical samples and is time-consuming, especially if the signal quality is high or extensive optimization is required. Therefore, it is highly desirable and practically important to develop efficient indirect numerical and statistical methods for evaluating CO-OFDM system performance.

For coherent communication systems utilizing multi-level amplitude and phase signals, the error vector magnitude (EVM) is commonly used as a fast measure of the received digital signal's quality [4, 5]. The EVM describes the effective distance of the received complex symbol from its ideal position in the constellation diagram. In an additive white Gaussian noise channel the association between EVM and BER has been determined theoretically [5]. The EVM can also be estimated without knowing the transmitted data by performing hard decision on the received symbols. Based on the assumption that in-phase and quadrature components of the received QPSK signal have Gaussian distribution [6], a few other relevant methods of evaluating the signal quality in QPSK system have been proposed and investigated in [7]. Recently, a novel statistical BER estimation method for QPSK CO-OFDM transmission based on the probability density function of the received QPSK symbols' phase has also been proposed in [8].

In this paper, we experimentally study the statistical properties of a QPSK modulated OFDM signal and compare for the first time different BER estimation methods for wavelength division multiplexing (WDM) CO-OFDM transmission. We show that the distribution of the received QPSK symbols' phase in each quadrant of the constellation diagram is essentially Gaussian. Furthermore, the statistical approach [8] offers the most accurate estimate of the system performance in comparison with other well-known approaches.

II. EXPERIMENTAL SETUP AND RESULTS

For studying the statistical properties of a QPSK modulated OFDM signal and comparison of different BER estimation methods, we set up a WDM CO-OFDM transmission system as shown in Fig. 1. This comprised a laser grid of five standard DFB lasers on a 100 GHz grid which were substituted in turn by a 100 kHz linewidth external cavity laser. The DFBs were located between 193.5 to 193.9 THz. Twenty additional loading channels (10 GHz bandwidth) were generated using an ASE source which was spectrally shaped using a WaveShaper wavelength selective switch (WSS). These loading channels were spread symmetrically around the test wavelengths so that the total bandwidth of the transmission signal was 2.5 THz. A wideband filter was used to filter out of band ASE noise at the transmitter. The transmission path is an acousto-optic modulator based re-circulating loop consisting of 4 x 100 km spans of Sterlite OH-LITE (E) fibre, having 18.9 to 19.5 dB insertion loss (per 100 km span) and dual stage amplifiers (EDFA, 5dB-6 dB of noise figure). The loop switch was located in the mid-stage of the first EDFA and a gain flattening

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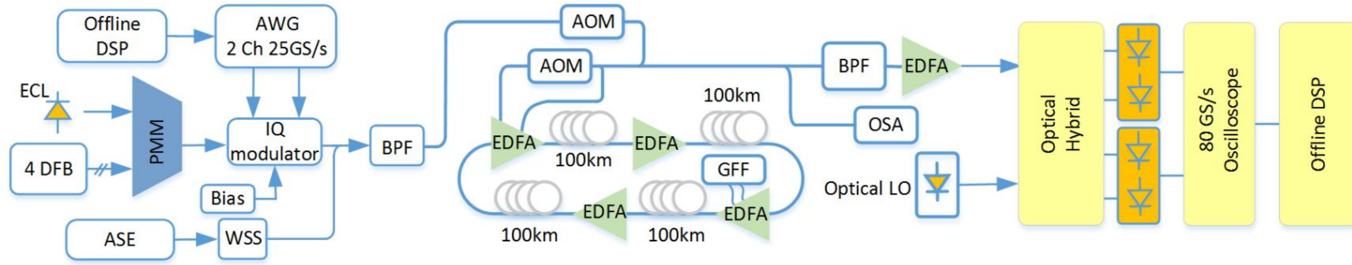

Fig. 1: Schematic of experimental setup of WDM CO-OFDM transmission. ECL: external cavity laser, PMM: polarization maintaining multiplexer, WSS: wavelength selective switch, DFB: distributed feedback laser, BPF: band-pass filter (optical), AOM: acousto-optic modulator, GFF: gain flattening filter, OSA: optical spectrum analyser, LO: local oscillator (optical), EDFA: Erbium-doped fibre amplifier.

filter was placed in the mid stage of the third EDFA. After fibre propagation the signal was filtered using a 4.2 nm flat topped filter and coherently detected. The received electrical signals were then sampled by a real-time oscilloscope at 80 GS/s and processed offline in MATLAB.

The OFDM signals (400 symbols each of 20.48 ns length, 2% cyclic prefix) encoded with QPSK modulation format were generated offline in MATLAB using an IFFT size of 512, where 210 subcarriers were filled with data and the remainder zeros giving a potential line rate of 20 Gb/s per channel. The DSP at the receiver included combining x- and y-polarizations using the maxima-ratio combining method [9], frequency offset compensation, chromatic dispersion compensation using a frequency domain equalizer (overlap-and-save method), channel estimation and equalization with the assistance of initial training sequence (2 training symbols every 100 symbols), common phase error (CPE) compensation by distributing 8 pilots uniformly across the OFDM band [10], giving a net data rate of 17.4 Gbit/s.

Figure 2 shows the histograms of in-phase and quadrature components of the received QPSK signal for the center channel. The Gaussian fitting is obtained by calculating the mean and standard deviation (STD) of the received statistical samples ($\sim 8\times10^5$ in total). Herein, the well-known Kolmogorov-Smirnov test (K-S test) was applied to define if a statistical signal has a Gaussian-like distribution. The Kolmogorov-Smirnov statistic (KSSTAT) for a given cumulative distribution function *F(x)* is defined as:

$$D = \sup\left|F_n(x) - F(x)\right| \tag{1}$$

where sup is the supremum, $F_n(x)$ is the empirical distribution function for *n* observations of the statistical signal. A statistical signal can be assumed to have a Gaussian distribution if $D \le 0.05$. The Gaussian fitting and KSSTAT values shown in Fig. 2 indicate that at this power level the nonlinear interference noise (NLIN) in CO-OFDM transmission deviates from Gaussian distribution. The obtained result herein agrees well with a recent study reported in [11], indicating that the Gaussian assumption of NLIN, which is the key in the derivation of closed-form expression for the nonlinear performance of CO-OFDM [12] is, in general, not satisfied exactly. On the other hand, as shown in Fig. 3, the distribution of the received QPSK symbols' phase in each quadrant of the constellation diagram is essentially Gaussian. This result agrees well with numerical results presented in [8], indicating that the nonlinear interaction of the ASE noise and signal induces the distribution of QPSK phases in OFDM systems (rather than the in-phase/quadrature components) to be Gaussian.

Next, we investigate the performance of various BER estimation methods. Herein, we take into account the data-aided EVM (Q(EVM1), nondata-aided EVM (Q(EVM2)), two relevant methods proposed in [7] (Q-factor 1, Q-factor 2 or Q1, Q2) and the statistical method proposed in [8, 13] (Q-factor 3 or Q3).

The Q1, Q2 methods are based on the assumption that the four components of a QPSK signal are Gaussian distributed. Following the same well known approach for calculating the conventional Q-factor for on-off-keying signals, the Q1 method defines the Q-factors of the in-phase and quadrature components of the received QPSK signals by:

$$Q_{\mathrm{Re}} = \frac{\left\langle c_{k,\mathrm{Re}}(c_{k,\mathrm{Re}} > 0)\right\rangle - \left\langle c_{k,\mathrm{Re}}(c_{k,\mathrm{Re}} < 0)\right\rangle}{\sigma(c_{k,\mathrm{Re}} > 0) + \sigma(c_{k,\mathrm{Re}} < 0)} \tag{2}$$

$$Q_{\mathrm{Im}} = \frac{\left\langle c_{k,\mathrm{Im}}(c_{k,\mathrm{Im}} > 0)\right\rangle - \left\langle c_{k,\mathrm{Im}}(c_{k,\mathrm{Im}} < 0)\right\rangle}{\sigma(c_{k,\mathrm{Im}} > 0) + \sigma(c_{k,\mathrm{Im}} < 0)} \tag{3}$$

where $\sigma(\cdot)$ denotes the STD of the statistical samples, $<\cdot>$ denotes the expectation operator, $C_{\mathrm{k,Re}}$, $C_{\mathrm{k,Im}}$ are the real and imaginary parts of the kth received QPSK symbol (C_{k}). The BER then can be obtained by using the estimations from both in-phase and quadrature components:

$$BER = \left\langle \frac{1}{2} erfc\left(\frac{Q_{\mathrm{Re}}}{\sqrt{2}}\right), \frac{1}{2} erfc\left(\frac{Q_{\mathrm{Im}}}{\sqrt{2}}\right)\right\rangle \tag{4}$$

The Q2 method is based on the estimation of the ratio between the mean and the STD value of each constellation point. For the symbol in the first quadrant, the Q-factors are:

$$Q_{1,\mathrm{Re}} = \frac{\left|\left\langle c_{k,\mathrm{Re}}(c_{k,\mathrm{Re}} > 0, c_{k,\mathrm{Im}} > 0)\right\rangle\right|}{\sigma(c_{k,\mathrm{Re}} > 0, c_{k,\mathrm{Im}} > 0)} \tag{5}$$

$$Q_{1,\mathrm{Im}} = \frac{\left|\left\langle c_{k,\mathrm{Im}}(c_{k,\mathrm{Re}} > 0, c_{k,\mathrm{Im}} > 0)\right\rangle\right|}{\sigma(c_{k,\mathrm{Re}} > 0, c_{k,\mathrm{Im}} > 0)} \tag{6}$$

The overall BER can be obtained by using $Q_{i,\mathrm{Re}}$ and $Q_{i,\mathrm{Im}}$, i = 1,2,3,4 of all the constellation symbols:

$$BER = \left\langle \frac{1}{2} erfc\left(\frac{Q_{i,\mathrm{Re}}}{\sqrt{2}}\right), \frac{1}{2} erfc\left(\frac{Q_{i,\mathrm{Im}}}{\sqrt{2}}\right) \right\rangle \qquad (7)$$

In contrast to Q1 and Q2 the statistical method Q3 [8] is based on the assumption that the received QPSK symbols' phases in CO-OFDM system are Gaussian distributed. As a result, Q3 estimates the BER as:

$$BER = \frac{1}{8}\sum_{k=1}^{4}\left[erfc\left(\frac{\phi_k - \theta_k + \pi/4}{\sigma_k\sqrt{2}}\right) + erfc\left(\frac{\theta_k + \pi/4 - \phi_k}{\sigma_k\sqrt{2}}\right)\right] \qquad (8)$$

where ϕ_k and σ_k denote the means and standard deviations of the received phases in the kth quadrant ($k=1,2,3,4$), θ_k is the phase angle of the kth ideal QPSK symbol, and *erfc* is the scaled complementary error function.

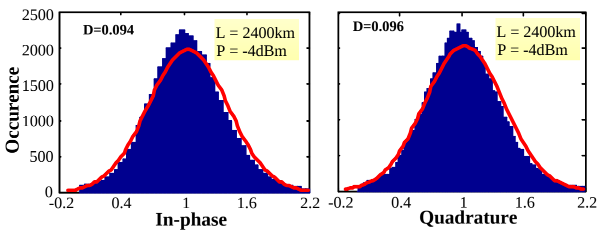

Fig. 2. Histograms of in-phase and quadrature components of the received QPSK symbols in the first quadrant. Gaussian fitting is superimposed to each histogram; KSSTAT values are also included in each histogram.

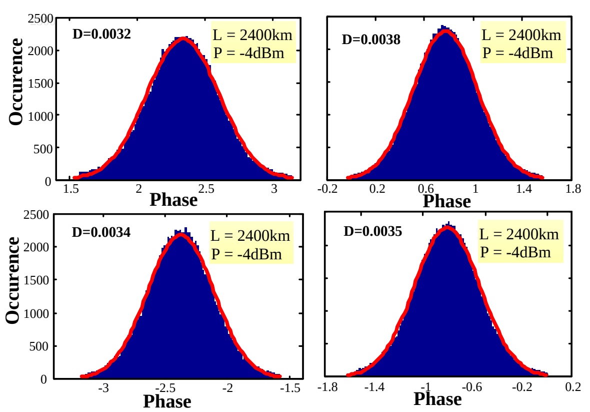

Fig. 3. Histogram of the received QPSK symbols' phase of the center channel in four quadrants of the constellation diagram.

The aforementioned BER estimation methods for WDM CO-OFDM transmission are compared in Fig. 4 (a) for the center channel and in Fig. 5 for the #2 channel. Similar results, which were obtained for other modulated channels, are not shown here. The blue line with circle markers (Q(BER)) is the reference result derived directly from the BER from error counting following OFDM processing of 10 recorded traces (~10^6 bits in total) for each data point. The red line (Q3) shows the result obtained using the expression (8) [8]. In Fig. 4 only a small mismatch (< 0.2 dB) between Q(BER) and Q3 is observed, indicating that this BER estimation method is highly accurate. In addition, as Q3 is based on the assumption that the received symbols' phases are Gaussian distributed, this method is tolerant to residual CPE as the residual CPE, which is common to all subcarriers, affects only the mean but not the variance of the symbols' phases. This phenomenon is confirmed by the simulated results for the back-to-back case (AWGN channel) shown in the Fig. 6. Without the laser phase noise, Q3 offers slightly worse performance in comparison to other methods because in the AWGN channel the symbols' phases do not follow a Gaussian distribution [14]. However, in the presence of the laser phase noise Q3 offers the best performance (Fig. 6(b)), because the random phase noise makes the distribution of the QPSK phases conforms more closely to a Gaussian distribution as a result of the central limit theorem. As a result, Q3 still offers an excellent performance even in the ASE limited regime.

On the other hand, all the other BER estimation methods, namely EVM (data-aided, nondata-aided), Q-factor 1 and Q-factor 2, overestimate the system performance by approximatelly 0.7 to 1dB. Morever, unlike Q3 method, Q(EVM), Q1, Q2 methods are sensitive to residual CPE because the residual CPE strongly affects the ditributions of the in-phase and quadrature components of the QPSK signal.

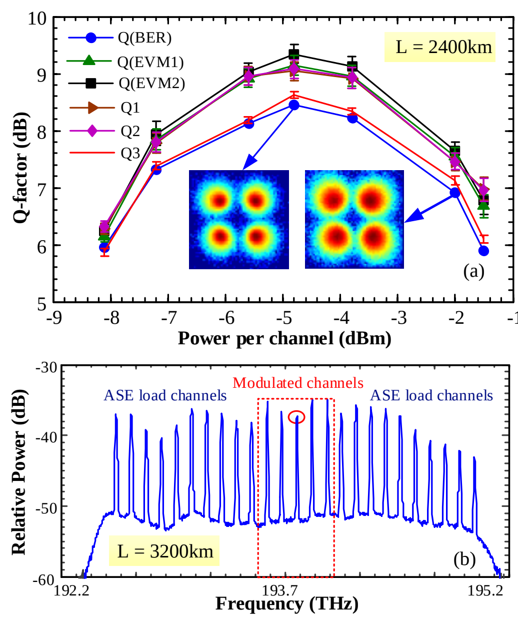

Fig. 4. (a) - Q-factor values for the center channel as a function of the launch power at 2400km, (b) – received optical spectrum at 3200km.

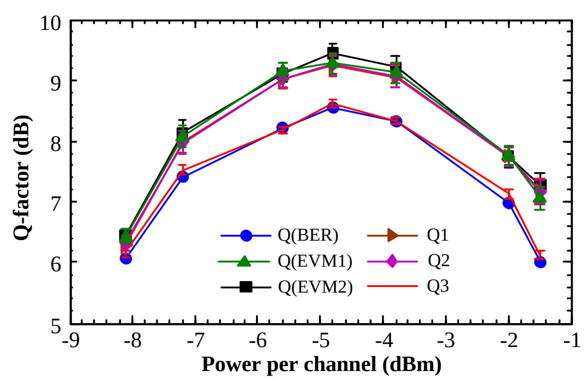

Fig. 5. Q-factor values for the #2 channel as a function of the launch power at 2400km.

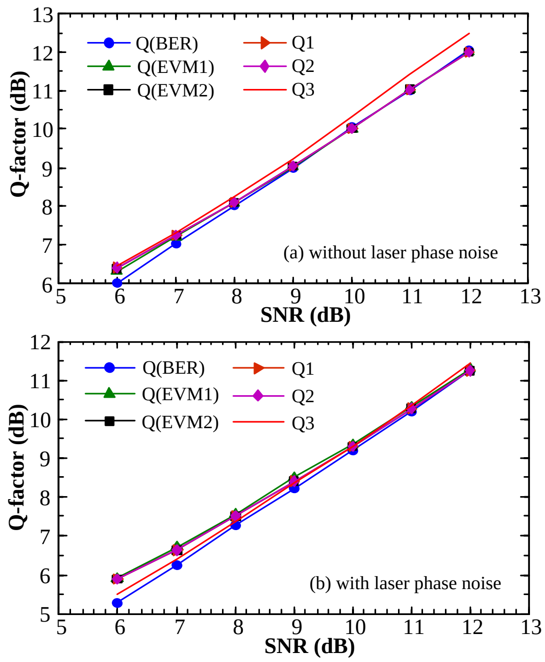

Fig. 6. Performance comparison of BER estimation methods in the back-to-back case (simulation results), without (a) and with (b) the laser phase noise, the combined laser linewidth is 200 kHz.

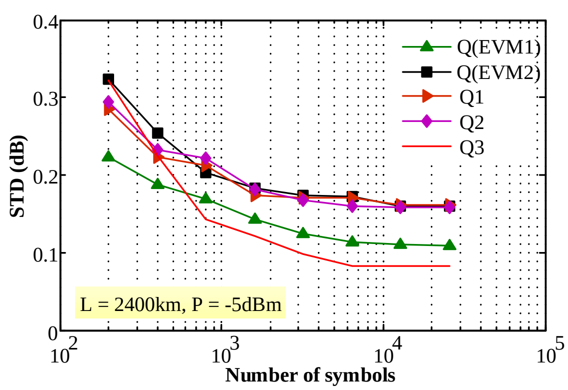

Fig. 7. STD of the Q-factor obtained using different BER estimation methods as a function of the number of processed symbols.

Figure 7 shows the STDs obtained using considered here BER estimation methods as a function of the number of processed statistical symbols in each trace (N) when processing 20 recorded traces. When N > 10^3 the STD of Q3, which is only around 0.1 dB, is the smallest among those obtained with BER estimation methods studied here. This result indicates that the statistical BER estimation method proposed in [8] can be applied effectively with a relatively small number of received symbols (~10^3) in comparison with other methods, offering a fast and accurate BER estimation method for QPSK CO-OFDM transmission.

III. Conclusion

We have experimentally investigated the statistical properties of QPSK signal and compare various BER estimation methods for WDM CO-OFDM transmission. Experimental results reveal that the most accurate estimate of the system performance was achieved with the statistical method based on a Gaussian approximation of the received phase noise statistics.

Acknowledgment

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