

Design and Analysis of Distributed Utility Maximization Algorithm for Multihop Wireless Network with Inaccurate Feedback

Shengbin Liao and Jianhua He

Abstract—Distributed network utility maximization (NUM) is receiving increasing interests for cross-layer optimisation problems in multihop wireless networks. Traditional distributed NUM algorithms rely heavily on feedback information between different network elements, such as traffic sources and routers. Due to the distinct features of multi hop wireless networks such as time varying channels and dynamic network topology, the feedback information is usually inaccurate, which represent as a major obstacle for distributed NUM application to wireless networks. The questions to be answered include if distributed NUM algorithm can converge with inaccurate feedback, and how to design effective distributed NUM algorithm for wireless networks. In this paper we firstly use the infinitesimal perturbation analysis technique to provide an unbiased gradient estimation on the aggregate rate of traffic sources at the routers based on locally available information. Based on that we propose a stochastic approximation algorithm to solve the distributed NUM problem with inaccurate feedback. We then prove that the proposed algorithm can converge to the optimum solution of distributed NUM with perfect feedback under certain conditions. The proposed algorithm is applied to the joint rate and media access control problem for wireless networks. Numerical results demonstrate the convergence of the proposed algorithm.

Index Terms—Network utility maximization, multi hop wireless networks, dual decomposition, gradient estimation, noise, infinitesimal perturbation analysis.

I. INTRODUCTION

A. Motivation

THE network utility maximization (NUM) model provides a new theoretical foundation for network architectural decisions and cross-layer optimization for wireline networks [1]. From the NUM model a basic distributed NUM algorithm has been developed to maximize aggregate source utility based on the dual decomposition theory [2 – 9]. The basic distribute NUM algorithm is consisted of a link algorithm which updates a shadow price signaling congestion measure at routers, and a source algorithm that adapts the source rate to congestion along the route path for the traffic. The convergence of the algorithm was presented in [1] and [2], which are similar to gradient-descent method based analysis.

In this paper we consider the application of distributed NUM framework to solve the cross-layer optimisation and resource allocation problems in multi hop wireless networks.

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With the distributed operation approach and appropriate changes, it is believed that distributed NUM model could provide viable solutions to these problems. However, as distributed NUM algorithms rely heavily on feedback information between different network elements, such as traffic sources and routers, a major problem for distributed NUM algorithms to be used in multi hop wireless networks is on the inaccurate feedback information. The inaccurate feedback can be caused by several reasons, such as time varying and capacity limited wireless channels, and dynamic network topology. For example, with multi hop wireless networks, the aggregate rate of traffic sources used by the link algorithm is hard to be estimated accurately and the feedback messages could be delayed or lost. The inaccurate feedback can lead to a large network performance degrade [10].

We take the network shown in Fig.1 as an example to illustrate the above problems. Assume that three traffic flows (path A to F for flow 1, path G to F for flow 2 and path E to F for flow 3) share the wireless link EF with source rate x_1 , x_2 and x_3 . To update link EF price, router E needs to know the aggregate rate of sources passing through it. However the aggregate source rate over link EF observed by router E is $x_1 + x_2 + x_3$, where x_1 , x_2 and x_3 denotes the estimated source rate for flows 1 to 3, respectively. There can be a deviation between the actual and the estimated aggregate source rate. However, the convergence analysis presented in [1] and [2] assumed that the aggregate source rates over all the links can be accurately obtained, for example $x_1 + x_2 + x_3 = x_1' + x_2' + x_3'$ will be implicitly assumed in Fig.1. On the other hand, shadowing price feedback from the routers to the traffic sources such as A and G in Fig.1 may also be delayed due to message queuing or message dropping, which results in an inaccurate indication of the link congestion measure and can degrade the network performances.

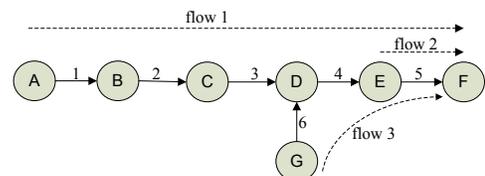


Fig. 1. An example multihop wireless networks

With the inaccurate feedback present in wireless networks as a major obstacle for the application of distributed NUM, the following two research questions need to be answered:

1) how to extend the distributed NUM algorithm for wireline networks to make it work for wireless networks with inaccurate feedback? and 2) if the distributed NUM algorithm can converge with inaccurate feedback? In this paper we firstly use the infinitesimal perturbation analysis (IPA) technique to provide an unbiased gradient estimation on the aggregate rate of traffic sources at the routers based on the locally available information, and analyse the impact of inaccurate feedback on the convergence of distributed NUM algorithms. IPA technique has been shown to be capable of providing performance sensitivities (or gradients) from observed values of a single sample path [11]. Based on the above analysis we propose a stochastic approximation algorithm to solve the distributed NUM problem with inaccurate feedback for multi hop wireless networks. We then proved that the proposed algorithm can converge to the optimum solution of distributed NUM with perfect feedback under certain conditions. The proposed algorithm is applied to investigate the joint rate and media access control problem for multi hop wireless networks. Numerical results demonstrate that the effectiveness and convergence of the proposed algorithm for multi hop wireless networks.

B. Related work

In the literature there have been many extended NUM models and resultant distributed algorithms proposed for network architectural decisions, cross-layer optimization and resource allocation in wireline and wireless networks [3 – 13]. All of these works assumed or implicitly assumed that the gradient estimation and feedback prices are perfect, i.e., the rate of every source passing through every link can be accurately estimated and every source can obtain the exact prices of links along its route path. However such assumptions are not true in practical wireless networks. There were some preliminary works which studied the stochastic stability and impacts of random errors on the distributed NUM algorithm. In Kelly’s seminal work [1], the authors studied the stochastic stability by introducing linear stochastic perturbations in the algorithm to represent random network loads. On the other hand, [14] and [15] considered NUM problems with time-varying channel state information.

[16], [17] and [18] studied the impact of inaccurate gradient estimation (or random error) on the NUM algorithms, which are most close to our work. In [16], the authors added an estimation error into the link price and then treated the estimation error as inaccuracy of the gradient in the distributed NUM algorithm. In contrast, in this paper the observed values of the link price are averaged over the real-time sample path and then the mean value is used as the estimation of gradient. As illustration next, our link price estimation is unbiased. The estimation in [16] can not be guaranteed to be unbiased and the possible measurement noise at link was not considered.

In [17], the aggregated load was estimated through online measurement with non-negligible noises, which is similar to our process. However, the authors in [17] added a random item as noise to the online observed value but did not consider the estimation error of link price.

In [18], the authors divided theoretically the gradient estimation procedure into two parts, namely biased estimation and martingale difference noise [22]; moreover, as mentioned in [18], their gradient estimation may be biased or unbiased. It is difficult for online implementation. However, our proposed method can provide easy unbiased estimation based on online observations by using IPA and it is amenable to online implementation.

The rest of the paper is organized as follows. After a brief description of IPA in Section II, Section III presents the basic NUM model and the distributed NUM algorithm. Section IV provides the gradient estimation based on the IPA and the convergence analysis of stochastic approximation of the distributed NUM algorithm in present of noisy feedbacks. In Section V, we apply the proposed method and algorithm to joint rate and media access control problem for multi hop wireless networks. Section VI concludes the paper.

II. PRELIMINARIES

Let θ be a parameter of a discrete-event dynamic system (DEDS); and ω be a random vector representing all of the randomness in the system; the underlying probability space is denoted as $(\Omega, \mathcal{F}, \mathcal{P})$. Typically, the components of ω are independently and identically distributed random variables. Let $h = h(\theta, \omega)$ be the general representation of a sample performance of interest obtained from a sample path realization. In many cases, there may exist more meaningful representation of θ and ω . For example, in this paper, θ could be the rate of a user’s transmission and ω could be the stochastic noisy.

Let $T_0 = 0, T_1, \dots, T_n, \dots$ be the sequence of the state transition instances. We consider a sample path of the system in a finite period $[0, T_N)$. The performance measured on this sample path is denoted as $h_N(\theta, \omega)$. In most cases, we are interested in the expected value of the performance $\bar{h}_N(\theta) = E[h_N(\theta, \omega)]$, and sensitivity analysis is concerned with estimating $\partial \bar{h}_N(\theta) / \partial \theta$, if we assume that $E[h_N(\theta, \omega)]$ exists, where E denotes the expectation with respect to the probability measure \mathcal{P} .

The goal of perturbation analysis is to obtain the performance derivative with respect to θ by analyzing a single sample path (θ, ω) . That is, we want to derive an quantity based on a sample path (θ, ω) and use it as an estimate of $\partial \bar{h}_N(\theta) / \partial \theta$. In infinitesimal perturbation analysis[11], the quantity is estimated as $(\partial h_N(\theta, \omega) / \partial \theta)$, where the sample derivative calculated from a single sample path is defined as

$$\frac{\partial h_N(\theta, \omega)}{\partial \theta} = \lim_{\Delta \theta \rightarrow 0} \frac{h_N(\theta + \Delta \theta, \omega) - h_N(\theta, \omega)}{\Delta \theta}. \quad (1)$$

This quantity is called the infinitesimal perturbation analysis estimate. The estimate is said to be unbiased if and only if

$$E\left[\frac{\partial h_N(\theta, \omega)}{\partial \theta}\right] = \partial \bar{h}_N(\theta) / \partial \theta = \frac{\partial E[h_N(\theta, \omega)]}{\partial \theta}. \quad (2)$$

This means that the unbiasedness of the IPA is equivalent to the interchangeability of the two operators E and $(\partial / \partial \theta)$. The primary application of IPA is gradient estimation, typically, a sample path (θ, ω) is observed, and then a perturbed path $(\theta +$

$\Delta\theta, \omega$) is constructed for estimating the gradient $\partial \bar{h}_N(\theta)/\partial\theta$ with $\Delta\theta \rightarrow 0$ and changes of $\bar{h}_N(\theta)$ due to a finite perturbation with $\Delta\theta \neq 0$. The interchangeability, i.e., eq(2), does not always hold for all systems and performance measures, but this can be satisfied by imposing some conditions on $h(\theta, \omega)$ [23]. Therefore, the two basic issues for IPA are^[24]:

(i) To develop a simple algorithm that determines the sample derivative (1) by analyzing a single sample path of a discrete event system; and

(ii) To prove that the sample derivative is unbiased, i.e., the interchangeability (2) holds.

III. PROBLEM FORMULATION

A. Optimal flow control via network utility maximization

Consider a communication network with J links, each with a fixed capacity of c_j bps for $j \in [1, J]$. Let a route r be a non-empty subset of J , and R be the set of possible routes. Set $A_{jr} = 1$ if $j \in r$, so that link j lies on route r , and set $A_{jr} = 0$ otherwise. This defines a 0-1 routing matrix $A = (A_{jr}, j \in J, r \in R)$.

Associate a route r with a user, and suppose that if a rate x_r is allocated to user r then this has utility $U_r(x_r)$ to the user. Assume that the utility $U_r(x_r)$ is increasing, strictly concave and continuously differentiable over the range $m \leq x_r \leq M$, where m and M are nonnegative constants.

Let $\mathbf{U} = (U_r(x_r), r \in R)$ and $\mathbf{C} = (c_j, j \in J)$. Under this model the network seeks a rate allocation $\mathbf{x} = (x_r, r \in R)$ which solves the following optimization problem.

$$\begin{aligned} \max \quad & \sum_{r \in R} U_r(x_r) \\ \text{subject to} \quad & \mathbf{Ax} \leq \mathbf{C} \\ & \mathbf{x} \geq 0 \end{aligned} \quad (3)$$

The above problem (3) is usually referred to as basic NUM problem, which is first presented by Kelly *et al.*[1] and is extended by Low *et al.*[2]. Over the past few years, the basic NUM model has been extended to much richer varieties and has found many applications in wired and wireless networks.

B. Solving Optimization Problem using Lagrange Duality

To solve problem (3) directly, we have to know some global information such as the utility functions and routes of all the users in the network. Typically, this information is not available. Thus, it is important to devise distributed solutions, where each user adapts its transmission rate based only on local information. In the rest of this section, we will describe distributed algorithms based on the Lagrangian dual decomposition.

To this end, we first form the Lagrangian for problem (3) as:

$$L(\mathbf{x}, \boldsymbol{\mu}) = \sum_{r \in R} U_r(x_r) - \boldsymbol{\mu}^T (\mathbf{Ax} - \mathbf{C})$$

where $\boldsymbol{\mu} = (\mu_j, j \in J)$ is a vector of Lagrange multipliers associated with flow constraints on routes of all the users.

Then, the Lagrangian dual function for problem (3) is

$$\begin{aligned} D(\boldsymbol{\mu}) &= \max_{x_r \geq 0} L(\mathbf{x}, \boldsymbol{\mu}) \\ &= \max_{x_r \geq 0} \left\{ \sum_{r \in R} U_r(x_r) - \boldsymbol{\mu}^T (\mathbf{Ax} - \mathbf{C}) \right\} \\ &= \max_{x_r \geq 0} \left\{ \sum_{r \in R} U_r(x_r) - x_r \sum_{j \in J} A_{jr} \mu_j \right\} + \boldsymbol{\mu}^T \mathbf{C} \end{aligned} \quad (4)$$

Thus, the dual problem for primal problem (3) is

$$\min_{\boldsymbol{\mu} \geq 0} D(\boldsymbol{\mu}) \quad (5)$$

In the dual formulation, Lagrange multiplier μ_j can be interpreted as congestion price on link j for violating the corresponding constraint. A key observation is that all sources can compute their optimal rate individually, based on the total congestion price $\sum_{j \in J} A_{jr} \mu_j$, using the following source rate algorithm

$$x_r = \arg \max_{x_r \geq 0} \left\{ \sum_{r \in R} U_r(x_r) - x_r \sum_{j \in J} A_{jr} \mu_j \right\} \quad (6)$$

To solve the dual problem (5), one can use the following projected gradient method

$$\mu_j(t+1) = \left[\mu_j(t) - \alpha_j (c_j - \sum_{j \in J} A_{jr} x_r) \right]^+ \quad (7)$$

where α_j is a positive scalar stepsize, and $[a]^+$ denotes the projection of a onto the set R^+ of non-negative real numbers.

According to the above problem formulation, the solution of the optimal source rates and congestion prices of links can be solved iteratively in equation (6) and (7), respectively. This suggests treating the network links and the sources as processors in a distributed computation system to solve the dual problem (5). When strong duality holds, the primal problem can be equivalently solved by solving the dual problem.

The algorithm based on (6)-(7) is called primal-dual algorithm, which can be abstracted as the following general form on primal variables \mathbf{x} and dual variables $\boldsymbol{\mu}$ with time index t :

Deterministic primal-dual algorithm (DPDA): In the primal-dual algorithm, the users rates x_r and the link prices μ_j are updated on the same time scale:

$$x_r(t+1) = \left[x_r(t) + \alpha(t) (L_{x_r}(x(t), \mu(t))) \right]^+, \quad \forall r \in R \quad (8)$$

$$\mu_j(t+1) = \left[\mu_j(t) - \beta(t) (L_{\mu_j}(x(t), \mu(t))) \right]^+, \quad \forall j \in J \quad (9)$$

where $\alpha(t)$ and $\beta(t)$ are positive scalar stepsizes, $L_{x_r}(\cdot)$ and $L_{\mu_j}(\cdot)$ are the gradients or subgradients of L with respect to x_r and μ_j , respectively.

It is clear that to implement the distributed algorithms (8)-(9), a critical issue is computing the gradients or subgradients of L , which depend on some feedback information. For example, in algorithms (6)-(7), each source needs to obtain the link congestion prices along its route path to update its data rate and each router needs to estimate the aggregated source rates and update link congestion price. But in practical

applications, due to external effects such as fading, user mobility, packet queueing and dropping, feedback messages are likely to be delayed and may not accurately reflect the actual network conditions. In the presence of noisy feedback, the gradients or subgradients $L_{x_r}(\cdot)$ and $L_{\mu_j}(\cdot)$ are stochastic. Let $\hat{L}_{x_r}(\cdot)$ and $\hat{L}_{\mu_j}(\cdot)$ denote the corresponding estimators of the gradients or subgradients $L_{x_r}(\cdot)$ and $L_{\mu_j}(\cdot)$. If the gradients or subgradients are estimators in deterministic primal-dual algorithm, then, the stochastic version of the algorithm is given as follows.

Stochastic primal-dual algorithm:

$$x_r(t+1) = \left[x_r(t) + \alpha(t)(\hat{L}_{x_r}(x(t), \mu(t))) \right]^+, \quad \forall r \in R \quad (10)$$

$$\mu_j(t+1) = \left[\mu_j(t) - \beta(t)(\hat{L}_{\mu_j}(x(t), \mu(t))) \right]^+, \quad \forall j \in J \quad (11)$$

In what follows, we shall focus on the estimation of gradients or subgradients under which the general algorithm, in the presence of noisy feedback, converges to the equilibrium point obtained by deterministic algorithms.

IV. STOCHASTIC APPROXIMATION ALGORITHM AND CONVERGENCE ANALYSIS

A. Stochastic approximation algorithm

The seminal work in stochastic approximation algorithms was conducted by Robbins and Monro on finding the root of a function when the function is unknown and only noise-corrupted observations at arbitrary values of the argument can be made. The basic stochastic approximation algorithm is analogous to the steepest-descent gradient method in deterministic optimization, except that here the gradient does not have an analytic expression and must be estimated. SA algorithms based on the steepest-descent gradient method are of the form:

$$\theta(t+1) = \left[\theta(t) + \alpha(t)g(t) \right]^+ \quad (12)$$

Where an approximation $\theta(t)$ for the optimal solution is updated to $\theta(t+1)$ using an estimator $g(t)$ of the gradient of the objective function evaluated at $\theta(t)$, and $\alpha(t)$ is a sequence of positive scalar stepsizes such that

$$\alpha(t) > 0, \alpha(t) \rightarrow 0, \sum_t \alpha(t) = \infty, \text{ and } \sum_t \alpha(t)^2 < \infty \quad (13)$$

In what follows, we will study the SA method with IPA gradient estimation, as applied to the algorithms (10)-(11) to approximate the optimal solution obtained by the algorithms (8)-(9). Let ω, ψ are the stochastic noisy presented in the algorithms (10)-(11), then, $\hat{L}_{x_r}(\cdot)$ and $\hat{L}_{\mu_j}(\cdot)$ can be expressed as $\hat{L}_{x_r}(x(t), \mu(t); \omega)$ and $\hat{L}_{\mu_j}(x(t), \mu(t); \psi)$. Assume that ω, ψ are continuous random variables that are finite with probability 1. As noted in section II, we would like to determine conditions under which IPA is valid, that is,

$$L_{x_r}(x(t), \mu(t)) = \frac{\partial E[\hat{L}(x(t), \mu(t); \omega)]}{\partial x_r} = E\left[\frac{\partial \hat{L}(x(t), \mu(t); \omega)}{\partial x_r}\right], \forall r \in R \quad (14)$$

$$L_{\mu_j}(x(t), \mu(t)) = \frac{\partial E[\hat{L}(x(t), \mu(t); \psi)]}{\partial \mu_j} = E\left[\frac{\partial \hat{L}(x(t), \mu(t); \psi)}{\partial \mu_j}\right], \forall j \in J \quad (15)$$

Typically, the condition (13) guarantees convergence of the algorithm (12). It is clear from the condition (13) that the decreasing stepsizes imply that the rate of change of $\theta(t)$ slows down as t goes to infinity. The idea is that the decreasing stepsizes would provide an implicit averaging of the observations. The linear least squares estimator of the mean value of a random variable can be used to explain how the decreasing stepsizes actually leads to an averaging of the observations. For example, let $\varphi(t)$ be a sequence of i.i.d, random variables with finite variance and unknown mean value of $\bar{\theta}$, given observation $\varphi(i), 1 \leq i \leq n$, then, the linear least squares estimator of $\bar{\theta}$ is

$$\theta(n) = \sum_{i=1}^n \frac{\varphi(i)}{n} \quad (16)$$

From equation (16), we have,

$$\begin{aligned} \theta(n+1) &= \sum_{i=1}^{n+1} \frac{\varphi(i)}{(n+1)} = \sum_{i=1}^n \frac{\varphi(i)}{(n+1)} + \frac{\varphi(n+1)}{(n+1)} \\ &= \frac{n}{n+1} \sum_{i=1}^n \frac{\varphi(i)}{n} + \frac{\varphi(n+1)}{(n+1)} \\ &= \sum_{i=1}^n \frac{\varphi(i)}{n} - \frac{1}{n+1} \sum_{i=1}^n \frac{\varphi(i)}{n} + \frac{\varphi(n+1)}{(n+1)} \\ &= \theta(n) + \alpha(n)[\varphi(n+1) - \theta(n)] \end{aligned} \quad (17)$$

Where $\theta(0) = 0$ and $\alpha(n) = 1/(n+1)$. Thus, equation (16) is equivalent to equation (17), i.e., the use of decreasing stepsizes $\alpha(n) = 1/(n+1)$ yields an estimator that is equivalent to that obtained by a direct averaging of the observations. This is driving us to update the algorithms (10)-(11) by averaging the estimators of the gradient from past observational information. Based on this, the SA algorithm to search for the optimal solution obtained by the algorithms (8)-(9) using an IPA estimator is as follows

Stochastic approximation of deterministic primal-dual algorithm (SPDA):

- Initialization: Choose the initial values of dual variables $\mu_j(0), \forall j \in J$ at each iteration $t = 0, 1, 2, \dots$
- SA algorithm for source rate updating:

$$\hat{L}_{x_r}(x(t), \mu(t); \omega(t)) = \frac{1}{t+1} \sum_{i=0}^t (\hat{L}_{x_r}(x(i), \mu(i); \omega(i))), \forall r \in R \quad (18)$$

$$x_r(t+1) = \left[x_r(t) + \alpha(t)(\hat{L}_{x_r}(x(t), \mu(t); \omega(t))) \right]^+, \forall r \in R \quad (19)$$

- SA algorithm for link shadow price updating:

$$\hat{L}_{\mu_j}(x(t), \mu(t); \psi(t)) = \frac{1}{t+1} \sum_{i=0}^t (\hat{L}_{\mu_j}(x(i), \mu(i); \psi(i))), \forall j \in J \quad (20)$$

$$\mu_j(t+1) = \left[\mu_j(t) - \alpha(t)(\hat{L}_{\mu_j}(x(t), \mu(t); \psi(t))) \right]^+, \forall j \in J \quad (21)$$

Here we use the same stepsize $\alpha(t)$ in (19) and (21) for the sake of convenience and the simplicity of the notations, this can not incur an loss of generality.

B. The validity of the IPA gradient estimator

In this section we establish the unbiasedness of the IPA gradient estimators (18) and (20) for the above stochastic approximation algorithm. To this end, we impose the following assumption:

A1. Assume that the utility function $U_r(x_r)$ is increasing, strictly concave and twice continuously differentiable over the range $m \leq x_r \leq M$, where m and M are nonnegative constants.

A2. The sequences of noise terms $\{\omega(t)\}$ and $\{\psi(t)\}$ in the estimate of $L_{x_r}(x(t), \mu(t))$ and $L_{\mu_j}(x(t), \mu(t))$ are independent across iterations, moreover, such that

$$\begin{aligned} E[\hat{L}_{x_r}(x(t), \mu(t); \omega)|F_t] &= L_{x_r}(x(t), \mu(t)) + \omega(t), \forall t \geq 0 \\ E[\hat{L}_{\mu_j}(x(t), \mu(t); \psi)|F_t] &= L_{\mu_j}(x(t), \mu(t)) + \psi(t), \forall t \geq 0 \end{aligned}$$

where F_t denotes the σ -algebra filtration generated by $\{(x_r(i), \mu_j(i)), \forall i \leq t\}$.

Theorem 1: Under condition **A1** and **A2**, then, the IPA estimators (18) and (20) in algorithms (18)-(21) are unbiased.

Proof: See Appendix A.

C. Convergence analysis

In this section, we show that the proposed stochastic approximation algorithm converges with probability one to the optimal points under some conditions. To this end, we impose the following assumptions:

A3. The stepsize $\alpha(t)$ is chosen to satisfy the equation (13).

A4. $\sup_t E\|\hat{L}_{x_r}(x(t), \mu(t); \omega)\|^2 < \infty$,
 $\sup_t E\|\hat{L}_{\mu_j}(x(t), \mu(t); \psi)\|^2 < \infty$.

A5. The noise terms $\{\omega(t)\}$ and $\{\psi(t)\}$ in the estimate of $L_{x_r}(x(t), \mu(t))$ and $L_{\mu_j}(x(t), \mu(t))$ are such that

$$\sum_{t=0}^{\infty} |\alpha(t)\omega(t)| < \infty \text{ w.p.1, } \sum_{t=0}^{\infty} |\alpha(t)\psi(t)| < \infty \text{ w.p.1}$$

Theorem 2: Under conditions **A1-A5**, the iterative sequences $\{(x_r(t), \mu_j(t)), t = 1, 2, \dots\}$, generated by **SPDA**, converge with probability one to the optimal solution of primary problem generated by **DPDA**.

Proof: The convergence analysis technique here is inspired by those provided in [19] [20] [18]. These proofs all consist of two steps. The difference between our proof and [19] is that our proof uses the saddle points of Lagrangian function to build the recurrent area of the iterative sequences generated by the algorithm, while the optimal dual solutions were directly adopted for this in [19]. The difference between our proof and [20] [18] is that we directly give an unbiased gradient estimation, while the gradient estimation was divided into two parts of the biased estimation and the martingale difference noise [22] in [20] and [18]. The complete proof please refer to appendix B.

V. APPLICATION TO JOINT RATE CONTROL AND MEDIA ACCESS CONTROL

In this section, we apply the proposed the estimation method and the approximate algorithm to the cross-layer design problem for joint rate control and media access control in wireless networks, which has been a subject of strong interests in the past decade [21][3]. Here we focus on the impact of estimation errors on the convergence of distributed NUM algorithm. Specially, we consider a multi-hop wireless network with network topology as shown in Fig.1, and assume all the links have the same capacity with $c_l^0 = 1, l \in L$, and all flows r have the same utility function $U_r(x_r) = \log(x_r)$. We further assume that there are three network flows, which are $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F$, $B \rightarrow C \rightarrow D \rightarrow E \rightarrow F$ and $G \rightarrow D \rightarrow E \rightarrow F$. Thus, we have routing matrix R as follows

$$\mathbf{R} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

We assume that each node has the same transmission and interference range, and use a conflict graph [21] to capture the contention relations among the links. In a conflict graph, each vertex represents a link and an edge between two vertices denotes that transmission along those links contend with each other and these links cannot transmit at the same time. In a conflict graph, a complete subgraph is referred to as a clique. A maximal clique is defined as a clique that is not contained in any other cliques, the vertices in a maximal clique represent a maximal set of mutually contending wireless links, along which at most one flow can transmit at any given time. Fig.2 shows the conflict graph of Fig.1.

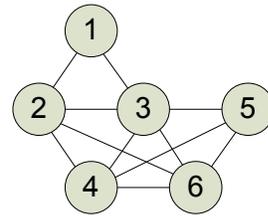


Fig. 2. The conflict graph of Fig.1

Assume the contention graph can be decomposed into N maximal cliques, each clique n contains $L_n \subset L$ links. Based on the concept of clique, we define the contention matrix $F = (F_{nl, n \in N, l \in L})$ as in [21], $F_{nl} = 1$ if $l \in L$ and $F_{nl} = 0$ otherwise. Therefore, from Fig.2, we have the contention matrix F for Fig.1 as follows

$$\mathbf{F} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Thus, the cross-layer optimization problem for joint rate and MAC control can be formulated as following:

$$\begin{aligned} & \max_{x_r \geq 0} \sum_r U_r(x_r) \\ & \text{subject to } Rx \leq c \quad Fc \leq 1 \\ & \quad c = (c_l, l \in L), c_l = 0 \text{ or } c_l^0 \end{aligned} \quad (22)$$

A. Distributed algorithm

As illustrated in Section III, we use a dual decomposition approach to solve problem (22). By relaxing the first constrain, the dual function is given by

$$\begin{aligned} D(p) &= \max_{x_r, c_l} \sum_r U_r(x_r) + p^T(c - Rx) \\ & \text{subject to } Fc \leq 1 \quad c_l = 0 \text{ or } c_l^0 \end{aligned} \quad (23)$$

Where the p^T is the vector of Lagrange multipliers with an interpretation of *congestion price*. Thus, the dual problem of the primary problem (22) can be defined as

$$\min_{p \geq 0} D(p) \quad (24)$$

To solve the dual problem (24), from the problem (23) we can know that this maximization can be decomposed into the congestion control problem

$$\max_{x_r} \sum_r U_r(x_r) - p^T Rx \quad (25)$$

and the scheduling problem

$$\begin{aligned} & \max_c p^T c \\ & \text{subject to } Fc \leq 1 \quad c_l = 0 \text{ or } c_l^0 \end{aligned} \quad (26)$$

Therefore, we can obtain a distributed NUM algorithm as shown below.

Joint rate and congestion price control algorithm (RPCA):

- Initialization: Choose the initial values of dual variables $p_l(0), \forall l \in L$ at each iteration $t = 0, 1, 2, \dots$
- Each source r updates its rate as:

$$x_r(t+1) = U_r'^{-1} \left(\sum_l p_l(t) R_{lr} \right) \quad (27)$$

- Each link l updates its congestion price as:

$$p_l(t+1) = [p_l(t) + \gamma(t) \left(\sum_r R_{lr} x_r(t+1) - c_l(t+1) \right)]^+ \quad (28)$$

Remark: The above distributed NUM (27)-(28) was first presented in [21]. It is noted that it uses an implicit assumption, i.e., each source r can exactly obtain the congestion prices $(p_l, l \in r)$ along its route path and each link l can exactly estimate the aggregate source rate $\sum_r R_{lr} x_r$ through it. However, it is difficult to obtain these information exactly in practice implementation. Thus, next we consider the impact of estimation errors on the convergence of the above distributed NUM.

The stochastic approximation of the above distributed NUM with estimation errors by using IPA as in the algorithm **SPDA** can be formulated as follows.

Stochastic Approximation of Joint rate and congestion price control algorithm (SARP):

- Initialization: Choose the initial values of dual variables $p_l(0), \forall l \in L$

At each iteration $t = 0, 1, 2, \dots$

- Each source r estimate the link prices along its route path:

$$p_l(t) = \frac{1}{t+1} \sum_{i=0}^t (p_l(x(i); \omega(i))), \quad \forall r \in R \quad (29)$$

- Each source r updates its rate as:

$$x_r(t+1) = U_r'^{-1} \left(\sum_l p_l(t) R_{lr} \right) \quad (30)$$

- Each link l estimates the source rate through it:

$$x_r(t+1) = \frac{1}{t+1} \sum_{i=0}^t (x_r(i); \psi(i)), \quad \forall j \in J \quad (31)$$

- Each link l updates its congestion price as:

$$p_l(t+1) = [p_l(t) + \gamma(t) \left(\sum_r R_{lr} x_r(t+1) - c_l(t+1) \right)]^+ \quad (32)$$

B. Numerical results

In this subsection, we show the convergence performance of the algorithm **SARP** and compare the result with algorithm **RPCA** with two network scenarios. We assume that feedback noise used in our experiments is zero mean white Gaussian noise with standard deviation σ .

The first considered network topology is depicted in Fig.1. In Fig.3 and Fig.4 the convergence of the source rates for the three flows presented in Fig.1 is plotted for RPCA algorithm with stepsize of 0.05 and 0.1, respectively. Fig.5 and Fig.6 present convergence behavior for SARP algorithm with $\sigma = 0.05$, and stepsize of 0.05 and 0.1, respectively. Fig.7 and Fig.8 present convergence behavior for SARP algorithm with $\sigma = 0.1$, and stepsize of 0.05 and 0.1, respectively.

From Fig.3 and Fig.4, we can see that all the three flows almost converge to the same values, respectively. However, with stepsize $\gamma = 0.1$ the convergence speed is faster compared to the case with stepsize $\gamma = 0.05$. This conclusion still holds even when feedback noise is presented in the algorithm **SARP**, which can be observed from Fig.5 to Fig.8.

Furthermore, Fig.3 to Fig.8 show that algorithm **SARP** almost converges to the same optimal values of algorithm **RPCA**, although there is about 5% introduced feedback noise. This validates that the distributed NUM algorithms are robust to stochastic perturbations. But from Fig.5 to Fig.8, we can also observe that feedback noise can cause oscillations around the optimal values. An increase in the standard deviation σ causes a larger oscillation. According to the above observations we believe efficient error control techniques should be applied to minimize the adverse impact of feedback noise.

Next we consider another network topology as depicted in Fig.9. It is assumed that there are three flows $A \rightarrow B \rightarrow D \rightarrow E$, $B \rightarrow C \rightarrow E$ and $C \rightarrow E$. In Fig.10 and Fig.11 the convergence of the source rates for the three flows is shown for RPCA algorithm with stepsize of 0.05 and 0.1, respectively. From Fig.10 and Fig.11, we can see that all the three flows almost converge to the same values again. However, with stepsize $\gamma = 0.1$ the convergence speed is faster compared to the case with stepsize $\gamma = 0.05$.

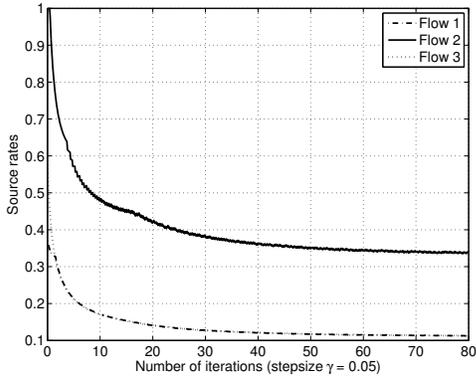


Fig. 3. The convergence of algorithm **RPCA** with $\gamma = 0.05$

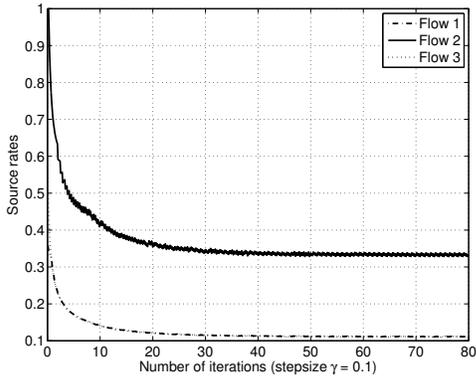


Fig. 4. The convergence of algorithm **RPCA** with $\gamma = 0.1$

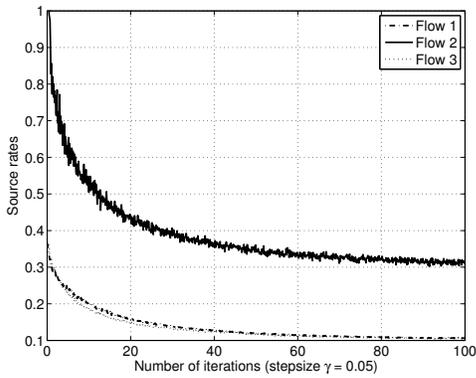


Fig. 5. The convergence of algorithm **SARP** with $\sigma = 0.05$ and $\gamma = 0.05$

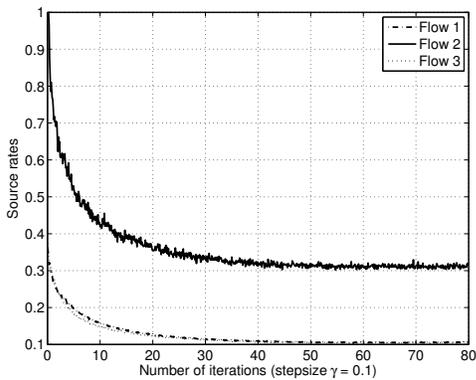


Fig. 6. The convergence of algorithm **SARP** with $\sigma = 0.05$ and $\gamma = 0.1$

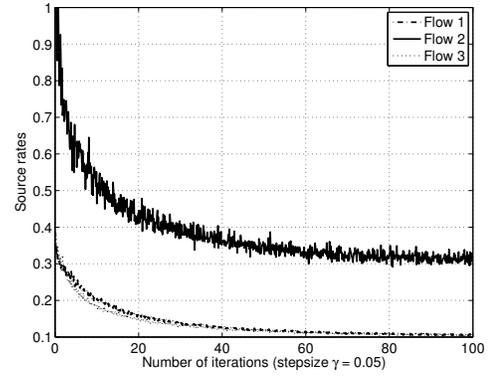


Fig. 7. The convergence of algorithm **SARP** with $\sigma = 0.1$ and $\gamma = 0.05$

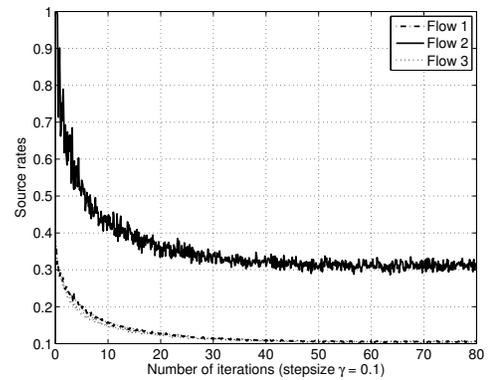


Fig. 8. The convergence of algorithm **SARP** with $\sigma = 0.1$ and $\gamma = 0.1$

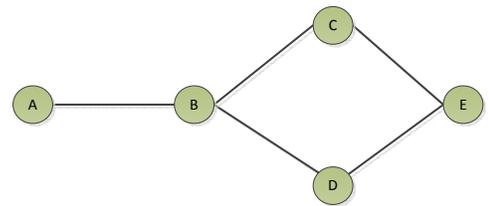


Fig. 9. The second network topology

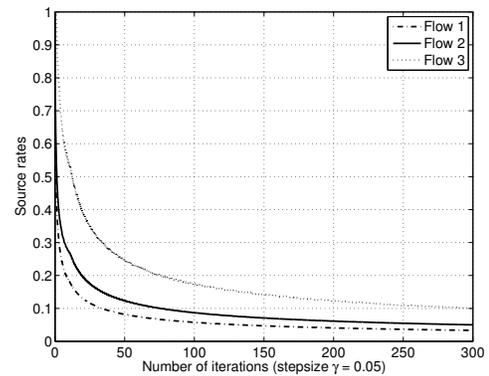


Fig. 10. The convergence of algorithm **RPCA** with $\gamma = 0.05$

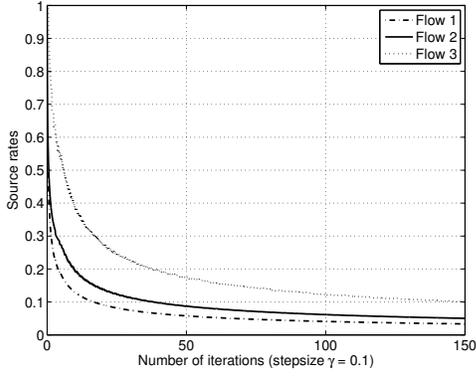


Fig. 11. The convergence of algorithm **RPCA** with $\gamma = 0.1$

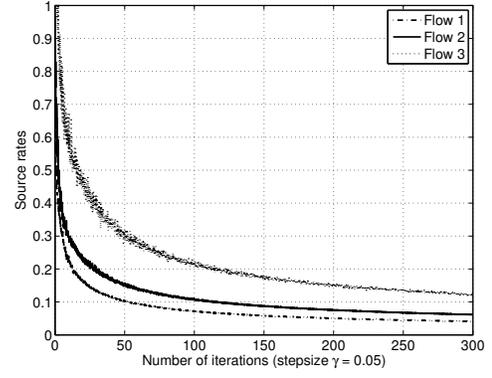


Fig. 14. The convergence of algorithm **SARP** with $\sigma = 0.1$ and $\gamma = 0.05$

Fig.12 and Fig.13 show convergence performance for SARP algorithm with $\sigma = 0.05$, and stepsize of 0.05 and 0.1, respectively. Fig.14 and Fig.15 show convergence performance for SARP algorithm with $\sigma = 0.1$, and stepsize of 0.05 and 0.1, respectively. Fig.10 to Fig.15 also show that algorithm **SARP** almost converges to the same optimal values of algorithm **RPCA** even with some introduce feedback noise. These performances are similar to those presented in Fig.3 to Fig.8. These validate the conclusion that our proposed distributed NUM algorithms are robust to stochastic perturbations.

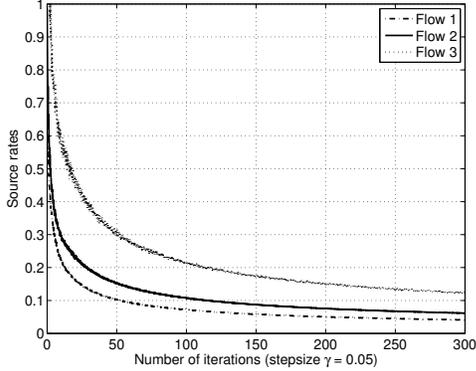


Fig. 12. The convergence of algorithm **SARP** with $\sigma = 0.05$ and $\gamma = 0.05$

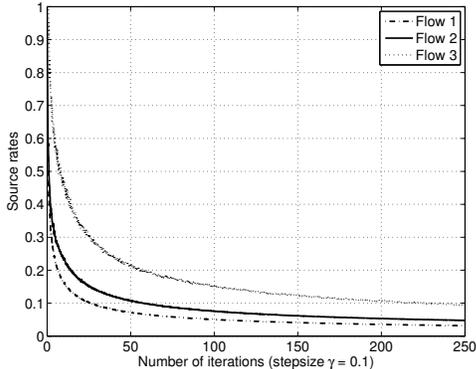


Fig. 13. The convergence of algorithm **SARP** with $\sigma = 0.05$ and $\gamma = 0.1$

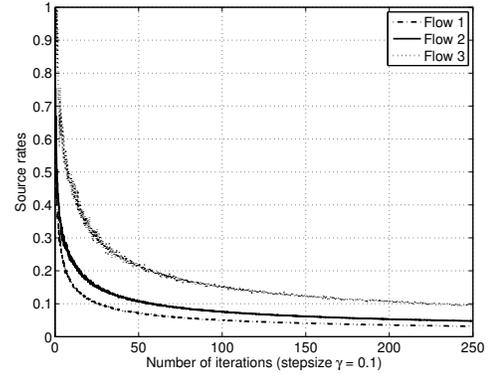


Fig. 15. The convergence of algorithm **SARP** with $\sigma = 0.1$ and $\gamma = 0.1$

VI. CONCLUSION

Distributed NUM has found many applications to the resource allocation and cross layer design optimization problems for wireline networks. In this paper we consider the application of distributed NUM for multiple wireless networks. Especially we are interested in the impacts of inaccurate feedback on distributed NUM algorithms and design of effective distributed NUM algorithms for multi hop wireless networks with inaccurate feedback. To address these problems we firstly proposed a gradient estimation method based on the infinitesimal perturbation analysis technique, which can provide an unbiased estimation based on the locally observed data. Then we proposed a stochastic approximation algorithm to solve the distributed NUM problem when feedback noise (due to the inaccurate aggregate rate estimation and feedback prices) is present. The convergence of the proposed algorithm to the optimal solution of the distributed NUM problem was proved. We applied the proposed algorithm to a joint rate and media access control problem in multihop wireless networks. Numerical results validate our theoretical analysis and showed that the proposed distributed algorithm is robust to feedback noises.

APPENDIX A
PROOF OF THEOREM 1

Proof: In the case $\hat{L}_{x_r}(x(t), \mu(t); \omega(t))$, we have, for given $\mu(t)$ and $\Delta x_r(t)$,

$$\begin{aligned} \Delta \hat{L}_{x_r} &= \hat{L}_{x_r}(x(t) + \Delta x_r(t), \mu(t); \omega(t)) - \hat{L}_{x_r}(x(t), \mu(t); \omega(t)) \\ &= \frac{1}{t} \sum_{i=0}^{t-1} (\hat{L}_{x_r}(x(i) + \Delta x_r(i), \mu(i); \omega(i)) - \hat{L}_{x_r}(x(i), \mu(i); \omega(i))) \\ &= E[\hat{L}_{x_r}(x(t) + \Delta x_r(t), \mu(t); \omega) | F_t] - E[\hat{L}_{x_r}(x(t), \mu(t); \omega) | F_t] \\ &= L_{x_r}(x(t) + \Delta x_r(t), \mu(t)) + \omega(t) - L_{x_r}(x(t), \mu(t)) - \omega(t) \\ &\leq K |\Delta x_r(t)| \end{aligned}$$

where K is a constant, the last two steps follow from conditions **A2** and **A1**, respectively. Thus, $\Delta \hat{L}_{x_r}$ is Lipschitz continuous and the unbiasedness result follows directly from the known fact (see [25], Lemma A2, p.70) that an IPA derivative is unbiased if (i) The sample derivative exists w.p.1, and (ii) The random function is Lipschitz continuous and the Lipschitz constant has a finite first moment.

Similarly, we can prove $\hat{L}_{\mu_j}(x(t), \mu(t); \psi(t))$ is unbiased. Moreover, the linear constraints in primary problem (3) can be extended to more general nonlinear cases which are convex, continuously differentiable functions, the unbiasedness result still holds. ■

APPENDIX B
PROOF OF THEOREM 2

Proof: Let (\mathbf{x}^*, μ^*) is the optimal solution to **DPDA**, then (\mathbf{x}^*, μ^*) is a saddle point for Lagrange function $L(\mathbf{x}, \mu)$ of primary problem, it follows

$$L(\mathbf{x}, \mu^*) \leq L(\mathbf{x}^*, \mu^*) \leq L(\mathbf{x}^*, \mu) \quad (33)$$

Define the function $V(\cdot, \cdot)$ as follows:

$$V(\mathbf{x}, \mu) \doteq \|\mathbf{x} - \mathbf{x}^*\|^2 + \|\mu - \mu^*\|^2 \quad (34)$$

where $\|\cdot\|$ denotes the *Euclidean* norm.

For any given γ , define the set H_γ as follows:

$$H_\gamma \doteq \{(\mathbf{x}, \mu) : L(\mathbf{x}^*, \mu) - L(\mathbf{x}, \mu^*) \leq \gamma\} \quad (35)$$

In the following, the proof consists of two steps.

Step 1 we will show that $\forall \gamma, H_\gamma$ is recurrent for $\{(\mathbf{x}(t), \mu(t))\}$.

Step 2 we will show that $(\mathbf{x}(t), \mu(t))$ eventually reside in H_γ almost surely.

Step 1: From equation (18) and (19), we have

$$\begin{aligned} \|\mathbf{x}(t+1) - \mathbf{x}^*\|^2 &= \left\| \left[\mathbf{x}(t) + \alpha(t)(\hat{\mathbf{L}}_{x(t)}(x(t), \mu(t); \omega(t))) \right]^+ - \mathbf{x}^* \right\|^2 \\ &\leq \|\mathbf{x}(t) + \alpha(t)(\hat{\mathbf{L}}_{x(t)}(x(t), \mu(t); \omega(t))) - \mathbf{x}^*\|^2 \\ &= \|\mathbf{x} - \mathbf{x}^*\|^2 + 2\alpha(t)(\mathbf{x} - \mathbf{x}^*)^T \hat{\mathbf{L}}_{x(t)}(x(t), \mu(t); \omega(t)) \\ &\quad + \alpha^2(t) \|\hat{\mathbf{L}}_{x(t)}(x(t), \mu(t))\|^2 \\ &= \|\mathbf{x} - \mathbf{x}^*\|^2 + 2\alpha(t)(\mathbf{x} - \mathbf{x}^*)^T [L_{x(t)}(t) + \omega(t)] \\ &\quad + \alpha^2(t) \|\hat{\mathbf{L}}_{x(t)}(x(t), \mu(t); \omega(t))\|^2 \end{aligned}$$

where we use the fact the projection $[\cdot]^+$ is non-expansive [26] in the above inequality and condition **A2** in the last step. $L_{x(t)}(t)$ and $\omega(t)$ are the vectors of $L_{x_r}(t)$ and $\omega_r(t)$, respectively. Similarly, with equation (20) and (21), we have

$$\begin{aligned} \|\mu(t+1) - \mu^*\|^2 &\leq \|\mu(t) - \alpha(t)(\hat{\mathbf{L}}_{\mu(t)}(x(t), \mu(t); \psi(t))) - \mu^*\|^2 \\ &= \|\mu - \mu^*\|^2 - 2\alpha(t)(\mu - \mu^*)^T \hat{\mathbf{L}}_{\mu(t)}(x(t), \mu(t); \psi(t)) \\ &\quad + \alpha^2(t) \|\hat{\mathbf{L}}_{\mu(t)}(x(t), \mu(t); \psi(t))\|^2 \\ &= \|\mu - \mu^*\|^2 - 2\alpha(t)(\mu - \mu^*)^T [L_{\mu(t)}(t) + \psi(t)] \\ &\quad + \alpha^2(t) \|\hat{\mathbf{L}}_{\mu(t)}(x(t), \mu(t); \psi(t))\|^2 \end{aligned}$$

By the assumption **A4**, we know that both $\|\hat{\mathbf{L}}_{x(t)}(x(t), \mu(t); \omega(t))\|^2$ and $\|\hat{\mathbf{L}}_{\mu(t)}(x(t), \mu(t); \psi(t))\|^2$ are bounded. Without loss of generality, we can assume that $\|\hat{\mathbf{L}}_{x(t)}(x(t), \mu(t); \omega(t))\|^2 \leq C_1$ and $\|\hat{\mathbf{L}}_{\mu(t)}(x(t), \mu(t); \psi(t))\|^2 \leq C_2$, where C_1 and C_2 are positive constants. According to this and the above inequalities, we have

$$\begin{aligned} V(\mathbf{x}(t+1), \mu(t+1)) &= \|\mathbf{x}(t+1) - \mathbf{x}^*\|^2 + \|\mu(t+1) - \mu^*\|^2 \\ &\leq V(\mathbf{x}(t), \mu(t)) + 2\alpha(t)[(\mathbf{x} - \mathbf{x}^*)^T L_{x(t)}(x(t), \mu(t)) \\ &\quad - (\mu - \mu^*)^T L_{\mu(t)}(x(t), \mu(t))] + 2\alpha(t)(\mathbf{x} - \mathbf{x}^*)^T \omega(t) \\ &\quad - 2\alpha(t)(\mu - \mu^*)^T \psi(t) + 2\alpha^2(t)(C_1 + C_2) \end{aligned} \quad (36)$$

In the following we assume that $(x(t), \mu(t)) \notin H_\gamma$ to build a contradiction and to verify the conclusion of **Step 1** with an indirect method. Recall the definition of H_γ , we have

$$L(\mathbf{x}, \mu^*) - L(\mathbf{x}^*, \mu) \leq -\gamma \quad (37)$$

Since $L(\mathbf{x}, \mu)$ is concave in \mathbf{x} and convex in μ , $L_{x(t)}(x(t), \mu(t))$ and $L_{\mu(t)}(x(t), \mu(t))$ are the gradient or subgradient vectors of $L(\mathbf{x}, \mu)$ with respect to \mathbf{x} and μ respectively, thus we have

$$(\mathbf{x} - \mathbf{x}^*)^T L_{x(t)}(x(t), \mu(t)) \leq L(\mathbf{x}, \mu) - L(\mathbf{x}^*, \mu) \quad (38)$$

$$-(\mu - \mu^*)^T L_{\mu(t)}(x(t), \mu(t)) \leq L(\mathbf{x}, \mu^*) - L(\mathbf{x}, \mu) \quad (39)$$

By the summation of the above inequalities (38) and (39), and with inequality (37), we have

$$(\mathbf{x} - \mathbf{x}^*)^T L_{x(t)}(x(t), \mu(t)) - (\mu - \mu^*)^T L_{\mu(t)}(x(t), \mu(t)) \leq -\gamma \quad (40)$$

Therefore, combining with (36), it yields that

$$\begin{aligned} V(\mathbf{x}(t+1), \mu(t+1)) &\leq V(\mathbf{x}(t), \mu(t)) - 2\alpha(t)\gamma + 2\alpha(t)(\mathbf{x} - \mathbf{x}^*)^T \omega(t) \\ &\quad - 2\alpha(t)(\mu - \mu^*)^T \psi(t) + 2\alpha^2(t)(C_1 + C_2) \end{aligned} \quad (41)$$

And then,

$$\begin{aligned} E[V(\mathbf{x}(t+1), \mu(t+1)) | F_t] &\leq V(\mathbf{x}(t), \mu(t)) - 2\alpha(t)\gamma + 2\alpha(t)(\mathbf{x} - \mathbf{x}^*)^T \omega(t) \\ &\quad - 2\alpha(t)(\mu - \mu^*)^T \psi(t) + 2\alpha^2(t)(C_1 + C_2) \\ &\leq V(\mathbf{x}(t), \mu(t)) - 2\alpha(t)\gamma + 2\alpha(t)\|(\mathbf{x} - \mathbf{x}^*)\| \|\omega(t)\| \\ &\quad + 2\alpha(t)\|(\mu^* - \mu)\| \|\psi(t)\| + 2\alpha^2(t)(C_1 + C_2) \\ &\leq V(\mathbf{x}(t), \mu(t)) - 2\alpha(t)\gamma + O(\alpha(t)(\|\omega(t)\| + \|\psi(t)\|)) + O(\alpha^2(t)) \end{aligned} \quad (42)$$

Next, we need the following proposition from [27, Prop. 4.2, p. 148] to verify the recurrence of H_γ for $\{(\mathbf{x}(t), \mu(t))\}$.

Proposition: (Supermartingale convergence theorem) Let Y_t, X_t , and $Z_t, t = 0, 1, 2, \dots$, be three sequences of random variables and let $F_t, t = 0, 1, 2, \dots$, be sets of random variables such that $F_t \subset F_{t+1}$ for all t . Suppose that:

(a) The random variables Y_t, X_t , and Z_t are nonnegative, and are functions of the random variables in F_t .

(b) For each t , we have $E[Y_{t+1}|F_t] \leq Y_t - X_t + Z_t$.

(c) There holds $\sum_{t=0}^{\infty} Z_t < \infty$.

Then, we have $\sum_{t=0}^{\infty} X_t < \infty$, and the sequence Y_t converges to a nonnegative random variable Y , with probability 1.

Then by applying condition **A5** and the above proposition to inequality (42), we have $\sum_t \alpha(t)\gamma < \infty$, which contradicts the conditions **A3**. Thus, $(\mathbf{x}(t), \mu(t)) \in H_\gamma$ for infinitely many t with probability one, i.e., H_γ is recurrent for $\{(\mathbf{x}(t), \mu(t))\}$.

Step 2: With (36), for any $n \geq t + 1$, it follows

$$\begin{aligned} & V(\mathbf{x}(n), \mu(n)) \\ & \leq V(\mathbf{x}(t), \mu(t)) + 2 \sum_{i=t}^{n-1} \alpha(i) [(\mathbf{x}(i) - \mathbf{x}^*)^T L_{x(i)}(x(i), \mu(i)) \\ & \quad - (\mu(i) - \mu^*)^T L_{\mu(i)}(x(i), \mu(i))] + 2 \sum_{i=t}^{n-1} \alpha(i) (\mathbf{x}(i) - \mathbf{x}^*)^T \omega(i) \\ & \quad - 2 \sum_{i=t}^{n-1} \alpha(i) (\mu(i) - \mu^*)^T \psi(i) \\ & \quad + 2(C_1 + C_2) \sum_{i=t}^{n-1} \alpha^2(i) \end{aligned} \quad (43)$$

From (33) and (40), for any $i \geq t + 1$, we have that

$$\begin{aligned} & (\mathbf{x}(i) - \mathbf{x}^*)^T L_{x(i)}(x(i), \mu(i)) - (\mu(i) - \mu^*)^T L_{\mu(i)}(x(i), \mu(i)) \\ & \leq 0 \end{aligned} \quad (44)$$

Thus, we have

$$\begin{aligned} & V(\mathbf{x}(n), \mu(n)) \\ & \leq V(\mathbf{x}(t), \mu(t)) + 2 \sum_{i=t}^{n-1} \alpha(i) (\mathbf{x}(i) - \mathbf{x}^*)^T \omega(i) \\ & \quad - 2 \sum_{i=t}^{n-1} \alpha(i) (\mu(i) - \mu^*)^T \psi(i) \\ & \quad + (C_1 + C_2) \sum_{i=t}^{n-1} \alpha^2(i) \end{aligned} \quad (45)$$

From conditions **A3** and **A5**, we then have that

$$\lim_{t \rightarrow \infty} (C_1 + C_2) \sum_{i=t}^{n-1} \alpha^2(i) = 0 \quad (46)$$

$$\lim_{t \rightarrow \infty} \sum_{i=t}^{n-1} \alpha(i) (\mathbf{x}(i) - \mathbf{x}^*)^T \theta_{\mathbf{x}}(i) = 0 \quad (47)$$

$$\lim_{t \rightarrow \infty} \sum_{i=t}^{n-1} \alpha(i) (\mu(i) - \mu^*)^T \varphi_{\mu}(i) = 0 \quad (48)$$

Combining (46), (47) and (48), it follows that w.p.1, for any $\epsilon > 0$, after $\{(\mathbf{x}(t), \mu(t))\}$ returns to H_γ for some sufficiently large t ,

$$\begin{aligned} & 2 \sum_{i=t}^{n-1} \alpha(i) (\mathbf{x}(i) - \mathbf{x}^*)^T \omega(i) \\ & \quad - 2 \sum_{i=t}^{n-1} \alpha(i) (\mu(i) - \mu^*)^T \psi(i) \\ & \quad + (C_1 + C_2) \sum_{i=t}^{n-1} \alpha^2(i) \leq \epsilon \end{aligned} \quad (49)$$

Therefore, applying the above inequality to (45), we have

$$V(\mathbf{x}(n), \mu(n)) \leq V(\mathbf{x}(t), \mu(t)) + \epsilon, \forall n \geq t + 1$$

Since this holds for arbitrarily small $\epsilon > 0$, thus $\{(\mathbf{x}(t), \mu(t))\}$ can not move far away from H_γ with arbitrarily small $\gamma > 0$, it follows that $(\mathbf{x}(t), \mu(t))$ converges to the optimal solution (\mathbf{x}^*, μ^*) w.p.1 ■

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