HYDROSTATIC EXTRUSION OF STEEL

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SUMMARY

A theoretical method for analysing the plastic flow of metals through a simple cone die based on the total work of deformation is discussed.

An apparent strain is defined by equating an area under the equivalent stress strain diagram to the work done per unit volume of material. The strain range covered by this area is called the 'apparent strain' ~~since it~~ which is the equivalent strain ~~which~~ corresponding to the total external work done and not the actual mean equivalent strain undergone by the material during deformation. The apparent strain theory is based on Siebel's and Pugh's methods and includes assumptions which permit its application to processes which involve friction and work hardening materials.

It is assumed in this theory that the mean equivalent strain induced by the plastic flow of metals through a die depends only on the geometry of the process and not the way in which the forming loads are applied. This permits its application to all forms of extrusion and drawing. The theory has given good agreement with experimental results obtained by hydrostatic extrusion in which the forming loads consist of combinations of oil pressure and direct loads applied to either the billet or product.

The experimental work has shown the feasibility of extruding steel bar and tube by simple hydrostatic extrusion [in which the billet is extruded by the sole actions of the oil pressure] proportionally augmented hydrostatic extrusion [in which the extrusion effect of the oil pressure is increased] and augmented hydrostatic extrusion [in which the oil pressure is aided by an externally applied force]. The technological difficulties associated with these methods are discussed and their possible fields of application are outlined.

It has been shown that product augmented hydrostatic extrusion, in which the oil pressure is assisted by a load applied to the product, is a useful and versatile metal forming process which has application to the manufacture of steel tube. This method permits large reductions at product speeds comparable with those obtained by conventional drawing. The limits to the extrusion ratio which can be achieved are fully discussed.

CONTENTS

1. INTRODUCTION

Large sections of industry are concerned with the forming of bars and tube by causing them to flow plastically through a die. This is achieved by a variety of processes which may be used to produce elongation and change in shape. It is interesting to consider, without regard to any particular process, the problems presented by this type of forming operation. The essential requirement is to apply loads to the workpiece in such a way that they produce the work necessary for plastic deformation. These may be applied to the feed material acting towards the die, to the product acting away from the die and to the feed material and the product simultaneously. The technological difficulties in applying these loads in all but the case of fluid pressure are such that they cannot be applied close to the die where plastic deformation takes place. It is, therefore, necessary to apply the loads to the workpiece at some distance from the die and for the workpiece to sustain the applied loads in order to transmit them to the deformation zone. If the workpiece cannot be supported its strength limits the forming load and the work input. This occurs during drawing and open die extrusion, the process in which a direct load is applied to the end of a short unsupported billet to cause it to extrude through the die. In these cases the loaded portion of the workpiece must remain elastic so that uncontrolled plastic flow does not take place. With these operations the area reduction which can be achieved is limited to approximately 50% and is independent of the material formed. With soft materials this reduction can be achieved more readily by other methods but with hard materials, such as steel, the reduction obtained is commercially attractive. It is for this reason that drawing of steel tube, for example, is still undertaken on a large scale. Also, drawing permits the reduction of long lengths of feed material which is unlike extrusion.

To obtain large reductions it is necessary to ~~completely~~ support the work piece completely so that the strength of the material being formed does not pose a limit to the forming loads. This is achieved in conventional extrusion by surrounding the billet with a thick cylinder which prevents uncontrolled radial flow of the billet. However, the presence of the cylinder introduces a second problem. As the extrusion load is applied some radial plastic flow will take place until the billet and the container are in contact. During this ~~upsetting~~ operation a high contact pressure is produced which introduces a frictional load to the movement of the billet. This frictional load can represent a significant proportion of the total extrusion load and will depend on the length of the billet. To keep it within reasonable proportions it is usual to limit the length-diameter ratio of the billet to approximately 6:1. This billet geometry is also necessary to prevent buckling of the extrusion ram, through which the extrusion load is transmitted to the billet. With soft materials and hard materials which are heated, large extrusion ratios can be achieved by conventional extrusion which enable long products to be produced from the relatively short billets, but with hard materials which have to be cold worked high extrusion ratios are not possible and only short product lengths can be produced. It is for this reason that conventional cold extrusion of steel is limited to the manufacture of specialised products which are relatively short and why it is not applied to the extrusion of steel tubes where long product lengths are required.

In order to overcome this billet length limitation it is necessary to prevent the billet ~~upsetting~~ expanding until it fills the extrusion container. However, if high forming loads are to be applied it is still essential to support the billet. These two requirements are achieved by surrounding the billet by a liquid. In this form of extrusion, known as hydrostatic extrusion, the ram pressurizes the liquid which in turn applies the forming load and supports the billet. The fact that the extrusion pressure is transmitted through a liquid gives this form of extrusion many characteristics which are not possessed by conventional extrusion. One of the beneficial effects is that the liquid, usually an oil, is carried between the die and the deformation zone. With round bar to round bar extrusion through cone dies this helps with lubrication to such an extent that extrusion is almost frictionless.

The above indicates that the use of fluid pressure is a~~n~~ ~~interesting~~ useful way of overcoming some of the limitations of the more conventional methods of extrusion but it is a complex method which has some limiting features. In the form of extrusion described, known as simple hydrostatic extrusion, detrimental effects are introduced due to the compressibility of the liquid. Extrusion is achieved by transmitting the extrusion pressure through the liquid which behaves as a 'liquid spring' and in addition the cylinder expands elastically. The extrusion rate is not, therefore, directly controlled and relative movement between the billet and the extrusion ram is possible giving rise to an unsteady mode of extrusion known as 'stick-slip'. This is characterised by an intermittent forward motion of the billet and oscillations in the oil pressure, this is discussed in the Appendix, section 8.1. Another limiting feature is presented by the large quantity of energy stored in the compressed oil. When the billet is completely extruded this energy is released in a blast of oil which causes the product to accelerate and inflicts damage to the rear end of the product. The product may be successfully decelerated by a water filled catching device but this is another complication which does not always guarantee an undamaged product.

In both conventional and hydrostatic extrusion the forming load is not limited by the strength of the billet material but it is limited by the capacity of the extrusion press and the strength of the extrusion container. In hydrostatic extrusion the container is more heavily stressed than the container of a conventional extrusion press. This occurs since, in the latter process the billet container contact pressure is less than the extrusion pressure whilst in hydrostatic extrusion they are the same. As these vessels are subjected to one pressure application per extrusion their fatigue lives become a significant factor in the economics of extrusion. The pressure transmitting fluid used in hydrostatic extrusion also reduces fatigue life since it enters the fatigue crack and aids its growth.

To overcome some of the difficulties of simple hydrostatic extrusion, augmented hydrostatic extrusion has been developed at the Reactor Fuel Element Laboratory, U.K.A.E.A. Augmented hydrostatic extrusion differs from simple hydrostatic extrusion in that direct loads are applied to the workpiece in addition to the oil pressure. In this form of extrusion the work is shared between the oil pressure and the direct load. It is possible to consider augmented hydrostatic extrusion to be the combination of simple hydrostatic extrusion and conventional extrusion or drawing. When an augmented load is applied to the billet, giving the combination of simple hydrostatic and open die extrusion, the process is known as billet augmented hydrostatic extrusion. When an augmenting load is applied to the product, giving the combination of simple hydrostatic extrusion and drawing, this process is known as product augmented hydrostatic extrusion or hydrostatic drawing. Augmenting loads may be applied to the billet and product simultaneously giving the process known as fully augmented hydrostatic extrusion. For these definitions see Figure 1.

In all augmented hydrostatic extrusion processes it is necessary to make contact with the workpiece so that the augmenting loads may be applied. This gives direct control over the extrusion rate and eliminates the stick-slip mode of extrusion. As the extrusion rate is controlled it is a simple matter to terminate extrusion with a discard so that the energy in the oil is not released at the end of extrusion. In product augmented hydrostatic extrusion it has been possible to phase-out the oil pressure at the end of extrusion in a controlled manner so that complete extrusion is achieved without the uncontrolled release of the energy in the oil.

In augmented hydrostatic extrusion the assistance given by the augmenting load to the work of deformation is limited by the same factors as govern~~ing~~ the limits of drawing and open die extrusion, described earlier. The assistance, therefore, depends on the strength of the material being formed, greater assistance being available when the material is hard.

There is one important difference between billet and product augmented hydrostatic extrusion. In the form of billet augmented hydrostatic extrusion, achieved by the Fielding and Platt 1600/80 'HYDROSTAT', reference 1, the load is applied to the billet by a ram acting in the extrusion container. For the billet to remain stable its length-diameter ratio must be small. With product augmented hydrostatic extrusion the augmenting load does not induce an instability which is dependent on length and, therefore, long billets may be extruded by this method. This is discussed further in section 6.2.d.

In addition to these principal arrangements of hydrostatic extrusion devices it is possible to use mandrels to achieve an augmenting effect in what is otherwise a press for simple hydrostatic extrusion. The oil pressure acts on these mandrels in such a way that its effect is intensified to increase the extrusion effect which permits a higher extrusion ratio to be achieved for a given oil pressure. This form of augmented hydrostatic extrusion, in which the augmenting stress is proportional to the oil pressure, is known as proportional augmented hydrostatic extrusion and is discussed later.

The main object of this work has been to investigate the hydrostatic extrusion of some of the steels of interest to the U.K.A.E.A., with particular emphasis on the production of round tube. Figure 2 shows some of the methods by which rod and tube can be formed by hydrostatic extrusion. Only those methods which have potential as a production process have been fully investigated but feasibility trials have been carried out on most of the methods shown.

To fulfil the main aim of this work it was necessary to develop equipment for the modification of the Fielding and Platt 200/50 'HYDROSTAT' to enable product augmentation to be achieved. In addition, it was necessary to develop several tool sets to achieve tool induced augmentation.

As well as fulfilling these practical aims this work has led to the development of a ~~new~~ method of analysis. This method is a modification of the methods adopted by both Pugh and Siebel, discussed later. The main objectives in developing this analytical method have been to make an adaptable approach to the many methods of employing hydrostatic extrusion and for the method to be realistic when applied to axi-symmetric processes. At the present stage of development of analytical methods it has not been possible to obtain exact solutions, hence the method devised, which is called the "apparent strain method", includes some simplifying assumptions. These assumptions are justified by the ease with which the apparent strain method has predicted some of the observed hydrostatic extrusion characteristics ~~phenomena~~. The apparent strain method enables work hardening, redundant work and friction effects to be taken into account.

The apparent strain method predicts both the oil pressure and the augmenting stresses required for a given hydrostatic extrusion process. It equates the work done per unit volume of material to an area under the equivalent stress-strain diagram. The strain range covered by this area is called the 'apparent strain'. This name is given since it is the strain obtained from the total work done which differs from the mean equivalent strain experienced by the deformed material when friction is present, see Figure 3.

2. NOTATION

VARIABLES

A	=	area
a	=	radius of the mandrel or a constant
B	=	Sachs friction factor = $\mu/\tan(\alpha)$
b,c,d	=	constants
K	=	billet diameter ratio
K_1	=	product diameter ratio
D	=	diameter
e	=	natural logarithm base
F,H,I,C	=	function
f	=	slope of the equivalent stress-strain diagram
h	=	thickness
j	=	scalar quantity
k	=	plane strain, shear stress at the yield point
L	=	die bursting force
l	=	length
M	=	ratio of the coefficients of friction at the inner and outer surfaces of a tube
m	=	mass
N	=	dimensionless number
n	=	index or integer value
P	=	force
p	=	oil pressure
R	=	extrusion ratio
r	=	radius
s	=	distance
t	=	time

U	=	billet velocity
V	=	velocity
v	=	change in volume from original value i.e. $VOL = VOL_o - v$
VOL	=	volume (V is used in Appendix I)
W	=	total work done
w	=	work done per unit volume
Y	=	yield as flow stress
y	=	displacement

α	=	semi die angle
β	=	work hardening factor and semi angle of billet taper
ϵ	=	strain
$\bar{\epsilon}$	=	equivalent strain
ζ	=	dimensionless stress or pressure i.e. $\frac{\sigma}{Y_m}$ or $\frac{p}{Y_m}$
η	=	efficiency or viscosity
θ	=	angle
λ	=	damping coefficient
μ	=	coefficient of friction
ρ	=	radius of curvature and density
σ	=	stress
$\bar{\sigma}$	=	equivalent stress
σ'	=	deviatoric stress
τ	=	shear stress
ϕ	=	angle
χ	=	extrinsic factor
ψ	=	friction factor
ϖ	=	augmenting ratio i.e. $\frac{\sigma_a}{p}$ or $\frac{\sigma_p}{p}$

SUFFICES

1,2,3	=	principal values and function numbers
A	=	apparent value
B, b	=	acting on billet
D	=	die surface
e	=	equivalent value
F	=	frictional
f	=	final value
H	=	homogeneous
M	=	mandrel surface
m	=	mean value
o	=	original value
P, a	=	acting on product
R	=	redundant
S	=	steady value
ULT	=	ultimate value
x	=	along axis of the die

PRIMES

*	=	without friction
p	=	plastic component

3. REVIEW OF SOME EXISTING METHODS OF ANALYSIS

3.a. SACHS ANALYSIS

One of the earliest methods for analysing the extrusion process was due to Sachs (2). This is based on the solution of the equation of equilibrium and the yield criterion ~~plasticity condition~~ and ~~as such it represents a statically determinate solution since it~~ does not rely on the validity of a theory of plastic flow.

Sachs assumed,

(1) All deformation is homogeneous,

(2) The material is perfectly plastic,

(3) The material obeys either the Tresca or Maxwell yield condition,

(4) Friction obeys Coulomb's laws, and

(5) The coefficient of friction remains constant over the interface between the die and the deformation zone.

The first assumption is the main limitation to the application of this analysis. Experiments have shown that redundant work accounts for a considerable part of the total plastic work of deformation, but it is difficult to include redundant work with this analytical approach. Green (3) has suggested a method for correcting Sachs' solution to take into account redundant work for plane strain processes but this method cannot be easily extended. ~~to axi-asymmetrical processes~~.

The second assumption also poses limitations for all but a few commercially unimportant materials. However, Sachs suggests that his solutions can be applied to work hardening materials by using the average of the ultimate tensile stresses measured before and after the process.

The following is Sachs' solution for wire and bar extrusion.

Figure 4a shows the forces considered to act in an element of material in the deformation zone, from which the equation of equilibrium becomes

$$D\,d\sigma_x + 2\sigma_x\,dD + 2\sigma_D\,dD\left[1 + \frac{\mu}{\tan(\alpha)}\right] = 0$$

Both the Tresca and Maxwell yield criteria give the same relationship between the principal stresses, namely

$$\sigma_x + \sigma_D = \sigma_Y$$

By substitution and integration for the case with zero front pressure the Sachs solution for wire and bar extrusion becomes

$$\frac{\sigma_{xB}}{\sigma_Y} = \frac{1+B}{B}\left(1 - \left[\frac{D_B}{D_a}\right]^{2B}\right) \qquad \ldots(1)$$

where $$B = \frac{\mu}{\tan(\alpha)}$$

Another limitation to the application of Sachs analysis is in deciding on a suitable value of the coefficient of friction. Early experimenters used Sachs analysis to estimate the coefficient of friction but it was quickly realised that larger errors were incurred due to the omission of redundant work. Lancaster and Rowe (4) using Green's correction for redundant work were able to make more realistic estimates of the coefficient of friction involved in plane strain drawing and compared them with the results of split die experiments.

Davis and Dokas (5) modified Sachs analysis to take into account work hardening.

3.b. THE HILL & TUPPER (6) METHOD OF ACCOUNTING FOR FRICTION

This is based on the equilibrium of the whole of the deformation zone. Figure 4b shows the forces considered.

The coefficient of friction is taken to be the ratio of the mean surface shearing stress to the mean die pressure. This is different from Sachs definition which defines the coefficient of friction as the ratio of the shear stress to the contact pressure at a point on the surface of the deformation zone. When comparing these methods it should be realised that they will only give the same result when the coefficient of friction remains constant.

The equation of equilibrium becomes

$$P = (A_o - A)(1 + \mu.\cot\alpha)(\sigma_D)_m \qquad \text{...(2)}$$

It follows that if the mean die pressure is assumed to be independent of frictional effects the relationship between the force required to carry out a metal working process with and without friction is

$$P = (1 + \mu.\cot\alpha)\, P^*$$

This simple analysis has led to a very useful experimental method for measuring directly the coefficient of friction, as defined above. This method requires the forming load and the die bursting force to be measured simultaneously. The die bursting force is measured by a load cell and a split die. The coefficient of friction is given by

$$\mu = \frac{1 - \left(\frac{\pi.L}{P}\right)\tan\alpha}{\tan\alpha - \left(\frac{\pi.L}{P}\right)}$$

This has been used successfully by Christopher and Naylor (7).

3.c. SIEBEL'S TREATMENT OF REDUNDANT WORK (8)

Siebel in his analysis of wire drawing considered redundant work to be due to the material passing through two shear surfaces at the entry and exit sections of the die. He considered these to be two spherical surfaces having their centres on the virtual apex of the die. These shear surfaces have the effect of changing the direction of flow as the material enters and leaves the conical die passage.

The following is assumed,

(1) The material is perfectly plastic,

(2) The material obeys the Tresca yield condition,

(3) Friction obeys Coulomb's laws,

(4) The coefficient of friction is constant over the surface of the deformation zone,

(5) That homogeneous, redundant and frictional work are additive, and

(6) The mean die pressure is equal to the yield stress.

From assumptions (5) and (6),

$$W = W_H + W_R + W_F \qquad \ldots(3)$$

and $$(\sigma_D)_m = Y$$

By considering the equilibrium of the whole of the deformation zone Siebel arrived at an equation similar to equation (2) and modified it in accordance with the assumptions made, giving.

$$\frac{P}{A} = W_H + W_F = \left(1 + \frac{\mu}{\alpha}\right) . Y . \ln\left(\frac{A_o}{A}\right)$$

This is a first order approximation, since $\cot(\alpha) \doteqdot \frac{1}{\alpha}$

for small semi die angles

and $$A_o - A \doteqdot A . \ln\left(\frac{A_o}{A}\right)$$

Figure 7 shows the mode of deformation assumed.

By consideration of the rate at which redundant work is done at the spherical surfaces

$$U.\pi r_o^2 . W_R = 2\left[\frac{2\pi.k.U\alpha}{r_o}\int_0^{r_o} r^2\, dr\right]$$

since $\theta = \alpha . \frac{r}{r_o}$ for small semi die angles,

giving $$W_R = \frac{2\alpha}{3} . Y$$

By addition the total work of deformation per unit volume of material is

$$w = Y\left[\left(1 + \frac{\mu}{\alpha}\right).\ln\left(\frac{A_o}{A}\right) + \frac{2\alpha}{3}\right] \qquad ...(4)$$

3.d. PUGH'S MEAN STRAIN ANALYSIS (9) VOL IIIB

This was used to analyse simple hydrostatic extrusion of round bar.

He assumed,

(1) The material obeys Tresca's yield condition,

(2) The coefficient of friction is constant over the surface of the deformation zone,

(3) That homogeneous, redundant and frictional work are additive, and

(4) The mean die pressure is independent of the coefficient of friction.

From assumption (3) the total extrusion pressure may be written

$$p = p_H + p_R + p_F \qquad ...(5)$$

As this is an analysis of simple hydrostatic extrusion

$$w = p, \quad w_H = p_H, \quad w_R = p_R \text{ and } w_F = p_F$$

and, therefore, equations (3) and (5) are identical.

Pugh, as Siebel, assumed for simplicity of analysis that redundant work was done by the material shearing over two surfaces within the billet at the entry and exit sections of the die. According to Pugh's model, as the material passes through the deformation zone it is sheared at the inlet section, reduced in the conical die passage homogeneously and then sheared at the exit section. During the passage of the material through the die, work is dissipated by friction at the die face.

Using this model Pugh described the components of equation (5) in terms of areas under the equivalent stress-strain diagram thus,

$$p = \int_0^{\epsilon_1} Y\,d\epsilon + \int_{\epsilon_1}^{\epsilon_2} Y\,d\epsilon + \frac{\mu . R . \ln(R)}{(R-1) . \sin\alpha}\int_{\epsilon_1}^{\epsilon_2} Y\,d\epsilon + \int_{\epsilon_2}^{\epsilon_3} Y\,d\epsilon \qquad \ldots(6)$$

$$= \int_0^{\epsilon_3} Y\,d\epsilon + \frac{\mu . R . \ln(R)}{(R-1) . \sin\alpha}\int_{\epsilon_1}^{\epsilon_2} Y\,d\epsilon$$

where

ϵ_1 = mean strain induced by shearing at the inlet section

ϵ_2 = mean strain induced after the material has passed through the conical die passage

= $\epsilon_1 + \ln(R)$

and

ϵ_3 = mean final strain after the material has left the deformation zone by passing through the outlet shear plane

= $2\epsilon_1 + \ln(R)$

It is important to this theory to distinguish between the work done at the inlet and outlet shear surfaces since these are not equal, due to work hardening of the material. This did not arise in Siebel's analysis since he considered the material to be perfectly plastic.

The third component of equation (6) represents the work done in overcoming friction. Figure 4c shows the forces considered in arriving at this expression.

The rate at which work is dissipated by friction may be written thus,

$$p_F \cdot \pi \cdot r_o^2 \cdot U = \int_{r_F}^{r_o} 2\pi \cdot r \cdot \mu \, (\sigma_D)_m \cdot V_D \cdot dS_D \qquad \ldots(7)$$

where V_D = velocity of sliding.

Pugh considered the velocity of sliding to be equal to the mean billet velocity at any section, giving

$$V_D = U \left(\frac{r_o^2}{r^2} \right)$$

By substituting and integration equation (7) reduces to

$$p_F = \frac{\mu (\sigma_D)_m \ln(R)}{\sin(\alpha)} \qquad \ldots(8)$$

Hill and Tupper (6) have shown that provided the coefficient of friction is small then the pressure on the die is little affected by friction, this is discussed later. The mean die pressure for a frictionless process is

$$(\sigma_D)_m^* = \frac{R}{R-1} \cdot p_H$$

so that after substitution equation (8) becomes:

$$p_F = \frac{\mu \cdot R \cdot \ln(R)}{(R-1) \cdot \sin(\alpha)} \int_{\epsilon_1}^{\epsilon_2} Y \, d\epsilon$$

since it is assumed that

$$(\sigma_D)_m = (\sigma_D)_m^*$$

and

$$p_H = \int_{\epsilon_1}^{\epsilon_2} Y \, d\epsilon$$

To estimate redundant work, Pugh used three upper bound solutions obtained by considering the shear surfaces to be,

1. Simple cross sections
2. Conical surfaces, optimised to give minimum work
3. Spherical surfaces with their centres on the virtual apex of the die.

By comparison with experimental results, obtained by simple hydrostatic extrusion of round bar, Pugh has shown that the spherical shear surfaces give the best estimate of redundant strain. Pugh's analysis of these surfaces differs from Siebel's only in that large die angles were considered, giving,

$$\epsilon_1 = \frac{1}{2}\left(\frac{\alpha}{\sin^2(\alpha)} - \cot(\alpha)\right) \qquad \ldots(9)$$

In order to improve agreement with experimental results, Pugh recommends that this be modified to,

$$\epsilon_1 = 0.462\left(\frac{\alpha}{\sin^2(\alpha)} - \cot(\alpha)\right) \qquad \ldots(10)$$

4. THEORY

4.1. GENERAL

4.1.a. WORK DONE BY EXTERNAL FORCES

There are three ways in which external forces can be applied to material which is being plastically deformed in a die passage so that the forces contribute to the work done.

A force may be applied to the back face of the billet acting towards the die. Such forces are induced in conventional extrusion and billet augmented hydrostatic extrusion. In the latter process this force is limited by the buckling or crushing strength of the billet. In conventional extrusion, the billet is prevented from buckling or swelling by a close fitting container. However, as previously stated ~~high~~ frictional forces exist at the billet container interface which increases the work required to obtain a given extrusion ratio.

A force may be applied to the product acting away from the die. Such forces are induced by drawing and product augmented hydrostatic extrusion. These forces are limited by the plastic instability of the product in tension.

Hydrostatic pressure may be applied to the billet as in hydrostatic extrusion. The work effect of this pressure is identical to an equivalent stress applied by conventional extrusion to the back face of the billet. As buckling or swelling is not induced by hydrostatic pressure the configuration of the material being formed does not pose a limit to the pressure which may be used.

Figure 2a shows fully augmented hydrostatic extrusion with augmenting forces acting on both the billet and the product. This process gives a greater work input than any other and includes all the possible forming loads.

The combined work done per unit time is

$$\frac{dW}{dt} = U.F_B + U.A_o.p + V_p.F_p$$

and the work done per unit volume of material is

$$w = \sigma_B + p + \sigma_p \qquad \ldots(11)$$

since, by the concept of volume ~~consistency~~ constancy

$$U.A_o = V_p.A_p \quad \text{and} \quad w.U.A_B = \frac{dW}{dt}$$

4.1.b. YIELDING AND PLASTIC WORK

Many attempts have been made to define the relationship between the principal stresses at the yield point, such relationships are referred to as ~~plasticity~~ yield ~~conditions~~ criterion. Taylor and Quinney (10) along with many others have shown that the Maxwell yield condition is the simplest relationship which fits experimental results for ductile materials. This may be written in the form,

$$Y = \sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]}$$

$$\bar{\sigma} = \sqrt{\frac{3}{2}\left[\sigma_1'^2 + \sigma_2'^2 + \sigma_3'^2\right]} \qquad \ldots(12)$$

when $\bar{\sigma}$ is numerically equal to the yield stress measured in simple tension or compression and depends on the amount of pre-strain. This criterion implies that the yield stress is independent of the strain path and the hydrostatic component of the stress state.

When a material undergoes plastic deformation the resistance to deformation increases. This is known as work hardening. During work hardening the yield locus defined by equation (12) enlarges. One hypothesis which governs this is that the degree of work hardening depends on the total plastic work done, giving

$$\bar{\sigma} = I(w^p) \qquad \ldots(13)$$

To use the stress and strain increments as a means of obtaining an expression for plastic work it is necessary to have a theory of plastic flow. One of the most significant of these theories is due to Saint Venant (1870) which was later extended by Levy and von Mises. Three ideas are implicit in this theory.

(1) The deviator stress tensor and the deviator strain increment tensor are co-axial,

(2) The corresponding terms of the two tensors are proportional, and

(3) The volume of the material remains constant.

A material which simultaneously satisfies these requirements is often referred to as a Levy-Mises solid.

Assuming the validity of this theory the increment of plastic work may be expressed as,

$$dw^p = \bar{\sigma} . d\epsilon^p \qquad \ldots(14)$$

where

$$d\bar{\epsilon}^p = \sqrt{\frac{2}{9}\left[\left(d\epsilon_1^p - d\epsilon_2^p\right)^2 + \left(d\epsilon_2^p - d\epsilon_3^p\right)^2 + \left(d\epsilon_3^p - d\epsilon_1^p\right)^2\right]} \qquad \ldots(15)$$

Equations (13) and (14) may be combined to give

$$\bar{\sigma} = H\int d\bar{\epsilon}^p \qquad \ldots(16)$$

When analysing metal forming processes it is usually permissible to ignore the elastic components of the total strain, so that in equations 14-16

$$d\bar{\epsilon}^p = d\bar{\epsilon}$$

$$\ldots(17)$$

and

$$\bar{\epsilon}^p = \bar{\epsilon}$$

These equations are sufficient to enable the total plastic work done during any forming operation to be determined. However, at the present stage of development of analytical methods some experimental method must be employed to obtain detailed information of the strain state throughout the deformation zone. This can be done with some success by using the deforming grid method or more accurately by using the Moire method of strain analysis. When this information is available equation (14) must be integrated along each flow path and the total work done found by summation. This is an involved and lengthy process and, therefore, without the aid of a digital computer is of limited usefulness.

4.1.c. MATERIAL CHARACTERISTICS

Before any analysis can begin the characteristics of the material to be formed must be determined. To do this it is necessary to assume that the material has the following properties.

(1) Its plastic flow characteristics are those required by a Levy-Mises solid.

(2) The material is isotropic.

(3) The Bauschinger effect is negligible.

It would appear that this limits the application of these theories to an extent which makes them of little use. In practice this is not so for many of the common metals will fulfil the requirements of (1) and (2) when they are in the annealed state and if the process does not induce a reversal of plastic strain the Bauschinger effect can be neglected.

Any testing procedure must enable equations (15) and (16) to be solved. This is readily achieved if the principal strains induced are proportional to each other. For this special case equations (15) and (17) may be written in the form

$$\frac{d\bar{\epsilon}}{d\epsilon_1} = \sqrt{\frac{2}{9}\left[\left(1-\frac{d\epsilon_2}{d\epsilon_1}\right)^2+\left(\frac{d\epsilon_2}{d\epsilon_1}-\frac{d\epsilon_3}{d\epsilon_1}\right)^2+\left(\frac{d\epsilon_3}{d\epsilon_1}-1\right)^2\right]}$$

If it is assumed that $\bar{\epsilon}^p = \bar{\epsilon}$ and $\epsilon_1^p = \epsilon_1$ etc.

This shows that when proportional straining exists i.e.

$$\frac{d\epsilon_2}{d\epsilon_1} = \text{constant} \quad \text{and} \quad \frac{d\epsilon_3}{d\epsilon_1} = \text{constant}$$

then

$$d\bar{\epsilon} = j \cdot d\epsilon_1 \qquad \ldots(18)$$

and equation (16) may be written,

$$\bar{\sigma} = H(\bar{\epsilon}) = H(j \cdot \epsilon_1) \qquad \ldots(19)$$

Proportional straining over large plastic strains can be achieved by three principal testing methods.

(1) Simple compression - Pugh (9) Vol IIIA

(2) Plane strain compression - Watts and Ford (11)

(3) Balance Bi-axial tension - Mellor (12)

4.2. APPARENT STRAIN

4.2.a. INTRODUCTION

This is a theoretical method which attempts to deal with all metal working processes which involve the plastic deformation of metal in a conical die passage and in particular it is an attempt to analyse augmented hydrostatic extrusion. It is applicable to processes involving materials which work harden and a valid solution can be obtained from a knowledge of the equivalent stress-strain diagram, the geometry of the process and the mean coefficient of friction. It is similar to the methods adapted by both Pugh and Siebel.

When friction is present the apparent strain differs from the mean strain experienced by the deformed material. This arises because work, which may be considerable, is done in overcoming the frictional resistance to plastic flow and this is accounted for in this method by an area under the equivalent stress-strain diagram, see Figure 3. The differences between the apparent and mean equivalent strains is called the frictional strain and is the imaginary component of the apparent strain.

The use of the apparent strain has been justified experimentally for hydrostatic extrusion in which the frictional effects are small. For this process the apparent strain has been shown to be independent of the equivalent stress-strain characteristics of the material and to depend only on the geometry of the process.

In this apparent strain theory it is postulated that:

(a) The presence of small frictional effects between the tools and the work piece has negligible effect on the plastic work done.

(b) The plastic work required, depends only on the geometry of the forming operation and the flow characteristics of the material, but not on the process employed to do the plastic work.

(c) The plastic work done by a non-homogeneous mode of deformation on a work hardening material is proportional to the area under the equivalent stress strain diagram.

(d) The work done against friction can be estimated from the work of sliding based on the mean die pressure, mean coefficient of friction and the distribution of the velocity of sliding.

(e) In tube forming processes the mean pressure between the mandrel and the deformation zone is equal to the mean die pressure.

4.2.b. GENERALISED THEORY

The apparent strain is defined by generalising the validity of equations (14) and (17) thus,

$$w = \int_0^{\bar{\epsilon}_A} \bar{\sigma}\, d\bar{\epsilon} = Y_m \cdot \bar{\epsilon}_A \qquad \ldots(20)$$

Assumption (a) enables the apparent strain to be considered to be made up of two components, the mean induced equivalent strain and the frictional strain. When the effect of friction on the plastic work done can be neglected the mean induced equivalent strain becomes independent of frictional effects. It is, therefore, possible to write

$$\bar{\epsilon}_A = \bar{\epsilon}_m + \bar{\epsilon}_F \qquad \ldots(21)$$

The work done per unit volume of material by the external forces acting during fully augmented hydrostatic extrusion is given by equation (11). Assumption (b) allows this to be combined with equation (20), so

$$Y_m \cdot \bar{\epsilon}_A = \sigma_B + p + \sigma_P$$

$$\ldots(22)$$

or $$\bar{\epsilon}_A = \zeta_B + \zeta + \zeta_P$$

This general discussion will be limited to bar extrusion and to tube extrusion in which the bore change is small. When large bore changes occur the longitudinal force acting on the work piece includes a component which is produced by the oil pressure acting in the bore. As this adds an extra complication it is omitted in the following for simplicity.

The longitudinal force exerted by the die pressure, mandrel pressure and their frictional components is given by

$$P = \pi (r_B^2 - r_P^2)(1 + \mu . \cot\alpha)(\sigma_D)_m + \mu . (\sigma_D)_m A_M \quad ...(23)$$

The last term in equation (23), which is due to the frictional force between the deformation zone and the mandrel, does not apply to bar extrusion. For the processes considered, this longitudinal force can be written in terms of the externally applied forces, thus

$$P = \sigma_B . A_B + p . A_B + \sigma_P . A_P \quad ...(24)$$

or $$\frac{P}{Y_m . A_B} = \zeta_B + \zeta + \frac{1}{R} . \zeta_P$$

Equations (22), (23) and (24) may be combined to give

$$I_1 \frac{(\sigma_D)_m}{Y_m} = \bar{\epsilon}_A - \zeta_P \left(\frac{R - 1}{R} \right) \quad ...(25)$$

The work done against friction can be obtained from the definition of the friction strain thus

$$W_F = (Y_m)_F . \bar{\epsilon}_F \quad ...(26)$$

where $(Y_m)_F$ is the mean flow stress over the strain range $\bar{\epsilon}_m \sim \bar{\epsilon}_A$ see Figure 3.

The frictional work can also be obtained from the work of sliding produced at the interface between the die and the billet, thus

$$W_F = \frac{\mu (\sigma_D)_m U \left[\sum_0^{A_D} \frac{(\sigma_D)}{(\sigma_D)_m} . \frac{V_D}{U} . dA_D + \sum_0^{A_M} \frac{(\sigma_D)}{(\sigma_D)_M} . \frac{V_M}{U} . dA_M \right]}{(\dot{VOL})} \quad ...(27)$$

Assumption (d) enables this to be simplified since it gives

$$\frac{\sigma_D}{(\sigma_D)_m} = 1.0$$

Combining these equations gives

$$\bar{\epsilon}_F = I_2 \frac{(\sigma_D)_m}{(Y_m)_F} \qquad \ldots(28)$$

where

I_2 = function of the mean coefficient of friction and the process variables. Substituting equation (28) in (25) gives

$$\bar{\epsilon}_A - \zeta_P\left(\frac{R-1}{R}\right) = \frac{1}{\psi}\cdot\bar{\epsilon}_F \qquad \ldots(29)$$

where $$\psi = \frac{I_2}{I_1}.\beta$$

Equation (29) may be written in the form

$$\bar{\epsilon}_A = \frac{\bar{\epsilon}_m}{(1-\psi)} - \zeta_P\left(\frac{R-1}{R}\right)\cdot\frac{\psi}{(1-\psi)} \qquad \ldots(30)$$

and from equation (25) and (30)

$$(\sigma_D)_m = \frac{Y_m\left[\bar{\epsilon}_m - \zeta_P\left(\frac{R-1}{R}\right)\right]}{I_1(1-\psi)} \qquad \ldots(31)$$

The apparent strain theory includes a work hardening factor, β. This is the ratio of the mean flow stress over the strain range equal to the apparent strain to the mean flow stress over the strain range $\bar{\epsilon}_m \sim \bar{\epsilon}_A$. The latter depends not only on the material characteristics but also on the process and the amount of friction present, see Figure 3. For many processes where the coefficient of friction is small the strain range $\bar{\epsilon}_m \sim \bar{\epsilon}_A$ is also small and the work hardening factor approaches the ratio of the mean flow stress to the instantaneous flow stress at any strain. This latter definition is a close approximation to the one defined in the theory and is a true material characteristic. The approximate work hardening factor is defined,

$$\beta = \frac{\int_0^{\bar{\epsilon}_A} \bar{\sigma}\, d\bar{\epsilon}}{(\bar{\sigma})_{\bar{\epsilon}=\bar{\epsilon}_A}}$$

since $\bar{\sigma} = Y$

For many of the steels used the equivalent stress-strain curve fits the empirical law,

$$\bar{\sigma} = (Y)_{\bar{\epsilon}=1} \cdot \bar{\epsilon}^{n}$$

which gives

$$\beta = \frac{1}{1+n} \qquad \ldots(32)$$

The above is a general approach which can be applied to all hydrostatic extrusion processes. When large bore changes occur the expression for the longitudinal force in terms of the process stresses equation (24) needs modification, but the remainder of the method remains unaltered.

4.3. APPLICATION OF THE APPARENT STRAIN THEORY TO HYDROSTATIC EXTRUSION PROCESSES

4.3.a. ROUND BILLETS TO ROUND BAR, FIGURE 2a

To obtain a general solution, fully augmented extrusion will be considered, equation (25) is applicable where

$$(I_1)_a = \frac{R-1}{R}\left[1 + \mu.\cot(\alpha)\right]$$

Note, the last term in equation (23) does not apply as a solid billet is considered.

To solve equation (28) it is necessary to know the variation of the velocity of sliding across the deformation zone. This has been measured experimentally and will be discussed later. An alternative theoretical approach is to determine the variation from a model of deformation. One of the most important models of deformation is that discussed by Siebel in his work on wire drawing, see Figure 5. The distribution of the velocity of sliding for Siebel's model is

$$V_D = U \cdot \left(\frac{r_B}{r}\right)^2 \cdot \cos(\alpha)$$

When this distribution is substituted into equation (28),

$$(I_2)_a = \mu \cdot \cot\alpha \cdot \ln(R)$$

The general solution for bar extrusion becomes

$$\bar{\epsilon}_A = \frac{\bar{\epsilon}_m}{(1-\psi)} - \zeta_P \cdot \left(\frac{R-1}{R}\right) \cdot \frac{\psi}{1-\psi} \qquad \ldots(30)$$

where

$$\psi = \left(\frac{I_2}{I_1}\right)_a \cdot \beta = -\frac{\mu \cdot \cot\alpha \cdot \beta \cdot \left(\frac{R}{R-1}\right) \cdot \ln(R)}{(1 + \mu \cdot \cot\alpha)}$$

and from equation (31) 1 and not I

$$(\sigma_D)_m = \frac{Y_m \left[\left(\frac{R}{R-I}\right) \cdot \bar{\epsilon}_m - \zeta_P \right]}{(1-\psi)(1 + \mu \cdot \cot\alpha)} \qquad \ldots(31a)$$

The mean equivalent strain, $\bar{\epsilon}_m$, can either be estimated from experimental results or from a suitable upper bound solution. Both these approaches are discussed later. The friction factor is shown in Figure 6.

4.3.b. THE EXTRUSION OF ROUND BILLETS TO ROUND BAR BY DIFFERENTIAL PRESSURE HYDROSTATIC EXTRUSION, SEE FIGURE 1.

Differential pressure hydrostatic extrusion is a process in which the billet is hydrostatically extruded into a vessel which contains a liquid at a lower pressure than that of the oil in the extrusion container. The billet is extruded by the action of the differential pressure which exists across the deformation zone. This process is used to increase the ductility of materials which exhibit a brittle-ductile transition when subjected to an all-round compressive stress state. The back-pressure required to bring about this change in the ductility differs from one material to another.

It is of value to analyse this process using the apparent strain method, even though the materials for which this process has advantages will only have characteristics which approximate to those required by the analysis when acted on by an 'all-round' compressive stress state and are, therefore, difficult to measure.

For this particular process the work done by the external forces per unit volume of material is

$$W = \sigma_B + p - p_p \qquad \text{since} \qquad \sigma_p = -p_p$$

Proceeding in the manner outlined above the general solution becomes

$$\bar{\epsilon}_A = \frac{\bar{\epsilon}_m}{(1-\psi)} + \frac{p_p}{Y_m}\cdot\left(\frac{R-1}{R}\right)\cdot\frac{\psi}{1-\psi}$$

and

$$(\sigma_D)_m = \frac{Y_m\left[\left(\frac{R}{R-1}\right)\cdot\bar{\epsilon}_m + \frac{p_p}{Y_m}\right]}{(1-\psi)(1+\mu.\cot\alpha)}$$

4.3.c. ANALYTICAL DIFFICULTIES PRESENTED BY THE EXTRUSION OF ROUND TUBE WITH SMALL BORE CHANGES

There are several different methods of extruding a tube. Three methods are achieved when different tool sets are used in a simple hydrostatic extrusion press. The tube may be extruded by the sole action of oil pressure over a stationary mandrel, Figure 2b, or it may be extruded over a travelling mandrel which is attached to a head at the back of the billet, Figure 2c. This type of tool set produces proportional billet augmentation. Or finally, it may be extruded over a travelling mandrel fixed to the product, Figure 2d, this tool set produces proportional product augmentation.

In addition to these methods which arise out of the use of different tool sets, hydrostatic tube extrusion may be augmented by loads applied by specially designed extrusion presses. This latter type of augmentation may be applied when the mandrel is stationary or travelling. Also, combinations of machine applied and tool induced augmentation may be used.

From all these methods three different analytical problems arise. These are due to the different relative motions produced by stationary, billet-fixed and product-fixed mandrels, between the mandrel and the deformation zone. These methods will now be considered separately.

4.3.d. ROUND TUBE OVER A STATIONARY MANDREL WITH SMALL BORE CHANGES, FIGURE 2b.

To obtain a general solution the fully augmented method will be considered.

Using assumption (e), section 4.2.a., equation (23) may be written

$$\frac{P}{A_o} = (\sigma_D)_m \left[\frac{K^2 - K_1^2}{K^2 - 1}(1 + \mu.\cot\alpha) + \mu.\cot\alpha.2\left(\frac{K - K_1}{K^2 - 1}\right)\right] \qquad \ldots(23d)$$

Combining equations (22), (23d) and (24) gives (25), where

$$(I_1)_d = \frac{K^2 - K_1^2}{K^2 - 1}(1 + \mu.\cot\alpha) + \mu.\cot\alpha.2\left(\frac{K - K_1}{K^2 - 1}\right)$$

To solve equation (28) it is necessary to know the variation of the velocity of sliding across the deformation zone both at the interface between the die and the deformation zone and its interface with the mandrel. These distributions may be determined experimentally or they may be obtained from a model of deformation. One of the simplest models of deformation is that obtained by assuming the material to enter and leave the deformation zone by passing through circular shear planes having centres on the point of intersection between the outer surface of the mandrel and the projected die surface, see Figure 7. This produces a model compatible with Siebel's for wire drawing. Assuming the validity of this model, the distributions of the relative motions between the deformation zone and both the die and mandrel become,

$$V_D = U \cdot \frac{A_o}{A} \cdot \cos\alpha$$

$$V_M = U \cdot \frac{A_o}{A}$$

Substituting these distributions into equation (27) gives

$$w_F = \mu\,(\sigma_D)_m \cdot \cot\alpha \left[\ln\left(\frac{K^2-1}{K_1^2-1}\right) + \ln\left(\frac{K-1}{K_1-1}\right) + \ln\left(\frac{K+1}{K_1+1}\right) \right] \quad \ldots(27d)$$

Equation (28) is, therefore, applicable where

$$(I_2)_d = \mu \cdot \cot\alpha \cdot 2 \cdot \ln\left(\frac{K-1}{K_1-1}\right)$$

The general solution becomes

$$\bar{\epsilon}_A = \frac{\bar{\epsilon}_m}{(1-\psi)} - \zeta_P \cdot \left(\frac{K^2-K_1^2}{K^2-1}\right) \cdot \frac{\psi}{1-\psi} \quad \ldots(30)$$

where

$$\psi = \left(\frac{I_2}{I_1}\right)_d \cdot \beta = \frac{\mu \cdot \beta \cdot \cot\alpha \cdot 2 \cdot \ln\left(\frac{K-1}{K_1-1}\right)}{\left[\frac{K^2-1}{K_1^2-1} \cdot (1+\mu\cot\alpha) + \mu \cdot \cot\alpha \cdot 2\left(\frac{K-K_1}{K^2-1}\right)\right]}$$

This friction factor is shown in Figure 6 and the mean die pressure becomes

$$(\sigma_D)_m = \frac{Y_m \left[\bar{\epsilon}_m - \zeta_p \cdot \left(\frac{K^2 - K_1^2}{K^2 - 1} \right) \right]}{(1-\psi)\left[\frac{K^2 - K_1^2}{K^2 - 1} (1 + \mu . \cot\alpha) + \mu . \cot\alpha . 2 \left(\frac{K - K_1}{K^2 - 1} \right) \right]}$$

...(31d)

4.3.e. ROUND TUBE OVER A BILLET-FIXED, TRAVELLING MANDREL WITH SMALL BORE CHANGES, FIGURE 2c

To obtain a general solution the fully augmented method will be considered. Often the only billet augmentation applied will be that produced by the oil pressure acting on the mandrel, for which

$$\zeta_B = \varpi_B . \zeta \quad \text{since} \quad \varpi_B = \frac{1}{K^2 - 1}$$

Here friction between the mandrel and the deformation zone does not affect the longitudinal equilibrium of the work piece, hence equation (23) may be simplified to give,

$$\frac{P}{A_0} = \frac{R-1}{R} \left(1 + \mu . \cot\alpha \right) (\sigma_D)_m = (I_1)_e . (\sigma_D)_m$$

...(23e)

The same model of deformation will be considered as that adopted above. The relative motion between the mandrel and the deformation zone will be different from that produced by a stationary mandrel. This arises because of the forward motion of the mandrel. The distributions of the velocity of sliding between the deformation zone and both the die and mandrel become,

$$V_D = U.\frac{A_o}{A}.\cos\alpha$$

$$V_m = U\left(\frac{A_o}{A} - 1\right)$$

Using these distributions equation (28) is obtained where

$$\left(I_2\right)_e = \mu.\cot\alpha\left[\ln\left(\frac{K-1}{K_1-1}\right) - \left(\frac{K-K_1}{K^2-1}\right)\right]2$$

The general solution becomes

$$\bar{\epsilon}_A = \frac{\bar{\epsilon}_m}{(1-\psi)} - \zeta_p.\left(\frac{K^2-K_1^2}{K^2-1}\right).\frac{\psi}{1-\psi} \qquad \ldots(30)$$

where

$$\psi = \left(\frac{I_2}{I_1}\right)_e.\beta = \frac{\mu\cot\alpha.\beta.2\left[\ln\left(\frac{K-1}{K_1-1}\right)-\left(\frac{K-K_1}{K^2-1}\right)\right]}{\frac{K^2-K_1^2}{K^2-1}\left(1+\mu.\cot\alpha\right)}$$

and the mean die pressure is

$$(\sigma_D)_m = \frac{Y_m\left[\left(\frac{K^2-1}{K^2-K_1^2}\right)\bar{\epsilon}_m - \zeta_p\right]}{(1-\psi)(1+\mu\cot\alpha)} \qquad \ldots(31e)$$

The friction factor is shown in Figure 8.

There are two possible designs of billet-fixed, travelling mandrel. One incorporates a solid head and the other a port which passes through the head to allow the oil to enter the bore of the billet. The former design is attractive since it leads to easy sealing, but it has the disadvantage that it induces a higher frictional effect at the bore, since the high pressure oil is not available to assist with bore lubrication.

The above analysis applies to the mandrel which allows the oil to enter the bore of the billet, since this arrangement leads to approximately equal coefficients of friction at both the outer and inner surfaces of the deformation zone. When the travelling mandrel with a solid head is used and the coefficients of friction at the outer and inner surfaces differ and the general solution becomes

$$(I_2)_e = \mu_1 \cdot \cot\alpha \left[(M+1).\ln\left(\frac{K-1}{K_1-1}\right) - (M-1).\ln\left(\frac{K+1}{K_1+1}\right) - 2M\left(\frac{K-K_1}{K^2-1}\right)\right]$$

where

$$M = \frac{\mu_2}{\mu_1}$$

μ_2 = coefficient of friction at the bore

μ_1 = coefficient of friction at the outer surfaces

and

$$\bar{\epsilon}_A = \frac{\bar{\epsilon}_m}{(1-\psi)} - \zeta\left(\frac{K^2-K_1^2}{K^2-1}\right)\frac{\psi}{1-\psi}$$

where

$$\psi = \left(\frac{I_2}{I_1}\right)_e.\beta = \frac{\mu_1.\beta.\cot\alpha\left[(M+1).\ln\left(\frac{K-1}{K_1-1}\right) - (M-1).\ln\left(\frac{K+1}{K_1+1}\right) - 2M.\left(\frac{K-K_1}{K^2-1}\right)\right]}{\frac{K^2-K_1^2}{K^2-1}.(1+\mu_1.\cot\alpha)}$$

and the mean die pressure is given by

$$(\sigma_D)_m = \frac{Y_m\left[\left(\frac{K^2-1}{K^2-K_1^2}\right)\bar{\epsilon}_m - \zeta_D\right]}{(1-\psi)(1+\mu_1.\cot\alpha)}$$

4.3.f. ROUND TUBE OVER A PRODUCT-FIXED, TRAVELLING MANDREL WITH SMALL BORE CHANGES, FIGURE 2d

To produce a general solution the fully augmented method will be considered. Often the only product augmentation applied will be that produced by the oil pressure acting on the mandrel, for which

$$\zeta_p = \varpi_p \cdot \zeta$$

where

$$\varpi_p = \frac{1}{K_1^2 - 1}$$

As before, friction between the mandrel and the deformation zone will not affect the longitudinal equilibrium, hence equation (23) may be simplified to give,

$$\frac{F}{A_o} = \frac{R-1}{R} \cdot \left(1 + \mu \cot \alpha \right) (\sigma_D)_m = (I_1)_f \cdot (\sigma_D)_m$$

The same model of deformation will be adopted as in the previous two analyses, but the relative motion between the mandrel and the deformation will be different since in this case the mandrel is attached to the product.

The distributions of the velocity of sliding between the deformation zone both the die and the mandrel become

$$V_D = U \cdot \frac{A_o}{A} \cdot \cos \alpha$$

$$V_M = U \left[R - \frac{A_o}{A} \right]$$

Using these distributions equation (28) is obtained in which,

$$(I_2)_f = \mu \cdot \cot \alpha \cdot 2 \left[\ln \left(\frac{K+1}{K_1+1} \right) + \left(\frac{K - K_1}{K_1^2 - 1} \right) \right]$$

The general solution becomes

$$\bar{\epsilon}_A = \frac{\bar{\epsilon}_m}{(1 - \psi)} - \zeta_p \cdot \left(\frac{K^2 - K_1^2}{K^2 - 1} \right) \cdot \frac{\psi}{1 - \psi} \qquad \ldots (30)$$

where

$$\psi = \left(\frac{I_2}{I_1}\right)_f \cdot \beta = \frac{\mu \cdot \cot\alpha \cdot \beta \cdot 2\left[\ln\left(\frac{K+1}{K_1+1}\right) + \left(\frac{K-K_1}{K_1^2-1}\right)\right]}{\frac{K^2-K_1^2}{K^2-1} \cdot (1 + \mu \cdot \cot\alpha)}$$

and the mean die pressure is

$$(\sigma_D)_m = \frac{Y_m\left[\left(\frac{K^2-1}{K^2-K_1^2}\right)\bar{\epsilon}_m - S_o\right]}{(1-\psi)(1 + \mu\cot\alpha)} \qquad \ldots(31f)$$

The friction factor is shown in Figure 8.

4.4. THE MEAN INDUCED STRAIN

This is the mean plastic strain undergone by the material during extrusion. It can be estimated either from experimental results, see section 5.7.a., or it can be said to exist between upper and lower bounds which are determined from a model of deformation. The experimental method will be discussed later but the upper and lower bounds will now be considered for some simple models of deformation.

4.4.a. THE LOWER BOUND, GENERAL

The work required to deform a billet plastically is minimised if plane deformation occurs, i.e. a mode of deformation in which plane sections remain plane and at right angles to the work piece axis. Such a mode of deformation gives the lower bound for the work required to change the shape of the billet to that of the product.

With this mode of deformation the mean strain induced is given by

$$(\bar{\epsilon}_m)_{MIN.} = \ln\left(\frac{A_B}{A_P}\right)$$

for bar extrusion this may be written

$$(\bar{\epsilon}_m)_{MIN.} = 2.\ln\left(\frac{r_B}{r_P}\right)$$

and for tube extrusion

$$(\bar{\epsilon}_m)_{MIN.} = \ln\left(\frac{K^2-1}{K_1^2-1}\right)$$

4.4.b. THE UPPER BOUND, GENERAL

The upper bound is determined from an idealised model of deformation which includes redundant work. The model must be consistent with the known velocity field and must give a continuous mode of deformation.

During the extrusion of bar and tubes through a cone die plane sections do not remain plane. Shearing takes place within the deformation zone which produces distortion of a rectilinear grid. When this occurs redundant work is said to take place which increases the total plastic work done. Experience has shown that the redundant work depends on the geometry of the extrusion process. An upper bound may be determined for the plastic work done by adding to the work of homogeneous deformation the redundant shear work. The work done in shearing the material is calculated from the model of deformation which includes a number of shear planes which are consistent with the flow of metal into and out of the deformation zone.

Two simple models for bar and tube extrusion, with small bore changes, will now be considered.

4.4.c. THE UPPER BOUND FOR BAR EXTRUSION

Figure 5 shows the model of deformation. The material enters the deformation zone by passing through an inlet shear plane, which is assumed to be a spherical surface with its centre on the virtual apex of the die. As the normal velocities on either side of the inlet shear plane are equal the material flows through the deformation zone towards the virtual apex of the die with a velocity which differs from one conical element to another. To achieve this the material must shear throughout the conical zone. The material leaves the deformation zone by passing through an outlet shear plane which is again assumed to be a spherical surface with its centre on the virtual apex of the die.

As the inlet and outlet shear planes are identical to those assumed by Pugh (9) the shear work done has already been determined in section 3.d. and is given by

$$W_1 = 2k\left[\frac{\alpha}{\sin^2\alpha} - \cot\alpha\right] \qquad \ldots(33)$$

The normal velocity of flow to the inlet shear planes is given by

$$V = U.\cos\theta$$

and at the outlet shear plane by

$$V = U.\left(\frac{r_B}{r_D}\right)^2.\cos\theta$$

It is consistent with these boundary conditions to assume that the distribution of the velocity of flow within the conical portion of the deformation zone is given by

$$V = U.\left(\frac{r_B}{r}\right)^2.\cos\theta \qquad \ldots(34)$$

It will be noticed that this is consistent with the assumed velocity of sliding at the surface used in the apparent strain method, section 4.3.d., a distribution which is discussed later. The shear work associated with this distribution of velocity will now be determined. The rate of doing shear work is given by

$$\frac{\partial w}{\partial t} = k . \sum_{0}^{n} V_n . \delta S_n$$

Considering a small element within the conical portion of the deformation zone gives

$$\delta\left(\frac{\partial w}{\partial t}\right) = k \sum \left(V_{(\theta+\delta\theta)} - V_\theta \right) r \, d\phi \, d\rho \qquad \ldots(35)$$

where

$$V_{(\theta+\delta\theta)} - V_\theta = - \frac{\partial V}{\partial \theta} . d\theta \qquad \ldots(36)$$

combining equations (32), (35) and (36) gives

$$\frac{dw}{dt} = k U \rho_0^2 \int_0^{\alpha} \int_{\rho_f}^{\rho_0} \int_0^{2\pi} \left(\frac{\sin^2\theta}{\rho} \right) d\phi \, d\rho \, d\theta$$

and

$$w_2 = 2k . \frac{1}{4} \left[\frac{\alpha}{\sin^2\alpha} - \cot\alpha \right] . \ln(R) \qquad \ldots(37)$$

The additional work of homogeneous deformation is given by

$$w_3 = 2k . \ln(R)$$

The total work done per unit volume of material becomes

$$w = w_1 + w_2 + w_3 = 2k\left[\left(\frac{\alpha}{\sin^2\alpha} - \cot\alpha \right) + \left(1 + \frac{1}{4}\left\{ \frac{\alpha}{\sin^2\alpha} - \cot\alpha \right\} \right) \ln(R) \right]$$

which gives

$$\bar{\epsilon}_m = \frac{\alpha}{\sin^2\alpha} - \cot\alpha + \left[1 + \frac{1}{4}\left(\frac{\alpha}{\sin^2\alpha} - \cot\alpha\right)\right]\ln(R) \quad \ldots(38)$$

for small angles where $\alpha \doteqdot \sin\alpha$ this may be simplified to give

$$\bar{\epsilon}_m = \frac{1 - \cos\alpha}{\sin\alpha} + \left[1 + \frac{1}{4}\left(\frac{1-\cos\alpha}{\sin\alpha}\right)\right]\ln(R) \quad \ldots(39)$$

4.4.d. THE UPPER BOUND FOR TUBE EXTRUSION

Figure 7 shows the model of deformation assumed. The material enters the deformation zone by passing through a shear plane formed by the revolution of an arc of a circle, with its centre on the point of intersection between the outer mandrel surface and the projected die surface. As with the model for bar extrusion the material flows through the conical deformation zone towards the centres of the inlet and outlet shear planes with a velocity which differs from one conical element to another. This produces shearing within the conical zone. The material leaves the deformation zone by passing through an outlet shear plane which is similar to that at inlet. The inlet and outlet shear planes have common centres.

The shear work done at the outlet and inlet shear planes will now be considered. The velocity along the inlet shear planes is given by

$$V = U\sin\theta$$

which produces the following shear work

$$W_1 = 2k\left[\frac{1}{2}\left(\frac{K-1}{K+1}\right)\left\{\frac{\alpha}{\sin^2\alpha} - \cot\alpha\right\} + \frac{1}{K+1}\left\{\frac{1-\cos\alpha}{\sin\alpha}\right\}\right]$$

The distribution of velocity along the outlet shear plane is

$$V = U.\left(\frac{r_0^2 - a^2}{r_1^2 - a^2}\right).\sin\theta$$

and the shear work associated with this distribution,

$$w_2 = 2k\left[\frac{1}{2}\left(\frac{K_1-1}{K_1+1}\right)\left\{\frac{\alpha}{\sin^2\alpha} - \cot\alpha\right\} + \frac{1}{K_1+1}\left\{\frac{1-\cos\alpha}{\sin\alpha}\right\}\right]$$

By addition the combined shear work at the inlet and outlet shear planes becomes,

$$w = 2k\left[\frac{(KK_1-1)\left(\frac{\alpha}{\sin^2\alpha} - \cot\alpha\right) + (K+K_1+2)\left(\frac{1-\cos\alpha}{\sin\alpha}\right)}{(K+1)(K_1+1)}\right]$$

When the die angle is small $\alpha \doteqdot \sin\alpha$ gives,

$$w = 2k\left(\frac{1-\cos\alpha}{\sin\alpha}\right)$$

and when the mandrel diameter is small

$$w = 2k\left(\frac{\alpha}{\sin^2\alpha} - \cot\alpha\right)$$

The velocity of flow normal to the inlet shear plane is

$$V = U\cos\theta$$

and normal to the outlet shear plane,

$$V = \left(\frac{r_0^2 - a^2}{r_D^2 - a^2}\right).U.\cos\theta$$

It is consistent with these boundary conditions to assume that the distribution of the velocity of flow within the conical portion of the deformation zone is

$$V = \left(\frac{r_0^2 - a^2}{r^2 - a^2}\right).U.\cos\theta$$

This is the same form of distribution assumed for the surface velocity of sliding in the apparent strain method, sections 4.3.d, e and f.

The shear work associated with this distribution will now be determined. The rate of doing shear work may be found using the method outlined above, this gives

$$\frac{dW}{dt} = kU\rho_o(\rho_o \sin\alpha + 2a)\int_0^{\alpha}\int_{\rho_f}^{\rho_o}\int_0^{2\pi}\left[\frac{\rho\sin^2\theta + a\sin\theta}{\rho(\rho\sin\alpha + 2a}\right]d\phi\, d\rho\, d\theta$$

The shear work per unit volume of material becomes

$$w_3 = 2k.\frac{1}{2}\left[\left(\frac{\alpha}{\sin^2\alpha} - \cot\alpha\right)\ln\left(\frac{K+1}{K_1+1}\right) + \left(\frac{1-\cos\alpha}{\sin\alpha}\right)\left(\ln\left\{\frac{K-1}{K_1-1}\right\} - \ln\left\{\frac{K+1}{K_1+1}\right\}\right)\right]$$

...(40)

When the die angle is small, so that $\alpha \doteq \sin\alpha$, this may be simplified to give

$$w_3 = 2k.\frac{1}{2}\left(\frac{1-\cos\alpha}{\sin\alpha}\right)\ln\left(\frac{K-1}{K_1-1}\right) \qquad ...(41)$$

The plastic work of homogeneous deformation is given by

$$w_4 = 2k.\ln\left(\frac{K^2-1}{K_1^2-1}\right) \qquad ...(42)$$

The total work done is given by summation, which gives, when the die angle is small,

$$w = 2k\left[\left(\frac{1-\cos\alpha}{\sin\alpha}\right)\left(1 + \frac{1}{2}\ln\left\{\frac{K-1}{K_1-1}\right\}\right) + \ln\left(\frac{K^2-1}{K_1^2-1}\right)\right]$$

and the mean equivalent strain becomes

$$\bar{\epsilon}_m = \left(\frac{1-\cos\alpha}{\sin\alpha}\right)\left[1 + \frac{1}{2}\ln\left\{\frac{K-1}{K_1-1}\right\}\right] + \ln\left(\frac{K^2-1}{K_1^2-1}\right) \qquad \ldots(43)$$

The mean induced strains for both bar and tube extrusion are shown in Figure 9a and b, respectively.

4.5. EFFECTS DUE TO THE MAGNITUDES OF THE AUGMENTING STRESSES

The analyses on section 4.3., only apply to processes in which the augmenting stresses are of elastic order of magnitude. It will be shown that the augmenting stresses can be of plastic order of magnitude, this particularly applies to billet augmentation. With augmenting stresses of plastic order of magnitude dimensional changes occur in the work piece which are not due to deformation in the die. With billet augmentation of this order of magnitude the extrusion ratio and the mean flow stress increase. These effects will now be considered.

4.5.a. BILLET AUGMENTATION, GENERAL

If the augmenting stress exceeds the yield stress of the material the billet is compressed before it enters the die. This causes the billet to expand, the extrusion ratio to increase and the billet to work harden. All these factors increase the work which must be done during extrusion. It follows that a type of instability can arise when the increase in the work required is greater than that obtained by increasing the augmenting stress. When this occurs extrusion cannot take place by increasing the augmentation alone. If this is attempted the billet will expand until it fills the extrusion container.

There are two possible ways in which billet augmentation may be used. The oil pressure may remain constant while augmentation is applied or the oil pressure and the augmentation may be applied simultaneously. The latter case arises with devices which apply proportional augmentation i.e. augmentation which is a fixed proportion of the oil pressure. These two methods of applying the augmentation lead to different results and must be treated separately. Also, differences arise between billet augmentation applied to solid and hollow billets. This is due to the influences of mandrel friction on the expansion of the billet.

4.5.b. BILLET AUGMENTED HYDROSTATIC EXTRUSION OF SOLID BILLETS

The upper limit to the augmenting stress which may be applied before instability occurs may be estimated readily of the increase in the mean flow stress due to the application of the augmenting stress is ignored. This simplified approach leads to a graphical means of determining the upper limit.

Equation (22) may be written

$$p + \sigma_B = Y_m \, . \, \bar{\varepsilon}_A \quad , \text{ since } \quad \sigma_p = 0$$

It will be shown later that an empirical law exists between the apparent strain and the extrusion ratio, which is of the form

$$\bar{\varepsilon}_A = c + d \, . \ln(R) \qquad \ldots(44)$$

Combining these equations gives

$$p + \sigma_B = a + b \, . \ln(R)$$

since $\qquad Y_m \doteqdot \text{constant}$

giving

$$\frac{d\sigma_B}{d(\ln R)} = b \qquad \ldots(45)$$

As the augmenting stress increases let the cross sectional area of the billet increase from A_o' to A_o. Assuming the resultant deformation is homogeneous the equivalent compressive strain induced is

$$\bar{\epsilon} = \ln\left(\frac{A_o}{A_o'}\right) = \ln(R) - \ln(R_o) \qquad \ldots(46)$$

where

R_o = extrusion ratio before augmentation is applied

R = extrusion ratio to be achieved after augmentation is applied.

The resultant increase in the augmenting stress may be determined from the equivalent stress-strain characteristic of the material, thus

$$\frac{d\bar{\sigma}}{d\bar{\epsilon}} = \frac{d\sigma_B}{d(\ln R)} = f$$

Instability will occur when these two rates of change are equal, giving the instability condition

$$b = f \quad \text{or} \quad \frac{d\sigma_B}{d(\ln R)} = \frac{d\bar{\sigma}}{d\bar{\epsilon}} \qquad \ldots(47)$$

The actual magnitude of the maximum augmenting stress can be determined by the simple graphical construction shown in Figure 10.
The load which must be applied to achieve this maximum augmentation is given by

$$\text{Load} = A_o' \left(\sigma_B\right)_{MAX} \cdot e^{\left[\frac{(\sigma_B)_{MAX}}{(Y)_{\bar{\epsilon}=1}}\right]^{\frac{1}{n}}} \qquad \ldots(48)$$

when the equivalent stress-strain diagram fits the empirical law

$$\bar{\sigma} = (Y)_{\bar{\epsilon}=1} \cdot \bar{\epsilon}^{\,n} \qquad \ldots(49)$$

If the increase in the mean flow stress due to the application of the augmenting stress cannot be ignored the above approach is inaccurate. An exact analytical solution is difficult to obtain but the augmenting stress required for a given oil pressure and extrusion ratio can be determined by an iterative process. An outline of the iterative process is given below for a material which has the characteristics of equation (49).

1 $$\bar{\epsilon}_m = F(\alpha, R) \qquad \text{initially take } R = R_o$$

2 $$Y_m = \frac{(Y)_{\bar{\epsilon}=1}}{1+n}\left[\frac{(\bar{\epsilon}_m + \bar{\epsilon})^{n+1} - \bar{\epsilon}^{n+1}}{\bar{\epsilon}_m}\right] \qquad \text{initially take } \bar{\epsilon}=0$$

3 $$\psi = F_1(\mu, \beta, \alpha, R) \qquad \text{where} \qquad \beta = \frac{Y_m}{(Y)_{\bar{\epsilon}=1}(\bar{\epsilon}_m + \bar{\epsilon})^n}$$

4 $$\sigma_B = \frac{Y_m \bar{\epsilon}_m}{1-\psi} - p$$

5 $$\bar{\epsilon} = \left(\frac{\sigma_B}{[Y]_{\bar{\epsilon}=1}}\right)^{\frac{1}{n}}$$

6 $$R = R_o . e^{\bar{\epsilon}}$$

This is repeated until the augmenting stress is obtained to the required degree of accuracy. A computer program for this calculation is discussed in the Appendix, section 10.3.a. and a typical result is shown in Figure 11.

With proportional augmentation the billet augmenting stress depends on the oil pressure, the constant of proportionality and the expanded cross sectional area of the billet. When the augmenting stress is of elastic order of magnitude it is given by

$$\sigma_B = \varpi_B . p$$

When billet expansion occurs this must be modified to the form

$$\sigma_B = \frac{R_o}{R} . \varpi_B . p \qquad \ldots(50)$$

Substituting this into equation (22) gives

$$p\left(1 + \frac{R_o}{R}.\varpi_B\right) = Y_m.\bar{\epsilon}_A$$

Again an exact analytical solution to this equation is difficult to obtain but a solution can be obtained by the iterative process outlined below,

1 $\bar{\epsilon}_m = F(\alpha, R)$ initially take $R = R_o$

2 $Y_m = \dfrac{(Y)_{\bar{\epsilon}=1}}{1+n}\left[\dfrac{(\bar{\epsilon}_m + \bar{\epsilon})^{n+1} - \bar{\epsilon}^{n+1}}{\bar{\epsilon}_m}\right]$ initially take $\bar{\epsilon} = 0$

3 $\psi = F_1(\mu, \beta, \alpha, R)$ where $\beta = \dfrac{Y_m}{(Y)_{\bar{\epsilon}=1}(\bar{\epsilon}_m + \bar{\epsilon})^n}$

4 $p = \dfrac{Y_m\,\bar{\epsilon}_m}{(1-\psi)(1 + \frac{R_o}{R}.\varpi_B)}$

5 $\sigma_B = \dfrac{R_o}{R}.\varpi_B.p$

6 $\bar{\epsilon} = \left(\dfrac{\sigma_B}{[Y]_{\bar{\epsilon}=1}}\right)^{\frac{1}{n}}$

7 $R = R_o.e^{\bar{\epsilon}}$

By repeated calculation the oil pressure can be obtained to any degree of accuracy, see Figure 12. A computer program for this calculation is discussed in the Appendix, see section 10.3.b.

4.5.c. BILLET AUGMENTED HYDROSTATIC EXTRUSION OF HOLLOW BILLETS

In the analysis of billet augmented hydrostatic extrusion of hollow billets mandrel friction can increase the amount of billet swelling induced. This will now be discussed for the three main types of mandrel.

Stationary mandrels, Figure 2b

The methods outlined for the analysis of proportional billet augmented hydrostatic extrusion of solid billets apply to tube extrusion when a stationary mandrel is employed, since mandrel friction does not influence billet swelling. An empirical law exists between the apparent strain and the extrusion ratio and is of the form

$$\bar{\epsilon}_A = c + d.\ln(R)$$

Billet-fixed, travelling mandrel, Figure 2c

With this design of mandrel the longitudinal stress acting in the billet is made up of two components. One component is due to the intensification of the oil pressure and the other is due to the frictional force acting between the mandrel shank and the deformation zone. This frictional force puts the mandrel shank in tension and the walls of the hollow billet in compression. It does not add to the work done on the billet but it increases billet swelling when the total billet stress is of plastic order of magnitude.

When the mean mandrel pressure is assumed equal to the mean die pressure, the total billet stress is given by

$$\sigma_B = \left(\frac{1}{K^2-1}\right).p + \mu.\cot\alpha.2\left(\frac{K-K_1}{K^2-1}\right)(\sigma_D)_m \qquad ...(51)$$

Note:- When the mandrel incorporates a solid head $\mu = \mu_2$

Again an analytical solution is difficult to obtain but the extrusion pressure can be determined by the following iterative procedure.

1 $$\bar{\epsilon}_m = F(\alpha, K, K_1)$$

2 $$Y_m = \frac{(Y)_{\bar{\epsilon}=1}}{1+n}\left[\frac{(\bar{\epsilon}_m + \bar{\epsilon})^{n+1} - \bar{\epsilon}^{n+1}}{\bar{\epsilon}_m}\right]$$ initially take $\bar{\epsilon}=0$

3 $$\psi = F_1(\mu, \beta, \alpha, K, K_1)$$ where $\beta = \dfrac{Y_m}{(Y)_{\bar{\epsilon}=1}(\bar{\epsilon}_m+\bar{\epsilon})^n}$

4 $$p = \frac{Y_m\ \bar{\epsilon}_m}{(1-\psi)(1+\{\frac{1}{K^2-1}\})}$$

5 $$R = \frac{K^2-1}{K_1^2-1}$$

6 $$(\sigma_D)_m = \frac{Y_m\left(\frac{K^2-1}{K^2-K_1^2}\right)\bar{\epsilon}_m}{(1-\psi)(1+\mu.\cot\alpha)}$$

7 $$\sigma_B = \left(\frac{1}{K^2-1}\right)p + \mu.\cot\alpha.2\left(\frac{K-K_1}{K^2-1}\right)(\sigma_D)_m$$

8 $$\bar{\epsilon} = \left(\frac{\sigma_B}{[Y]_{\bar{\epsilon}=1}}\right)^{\frac{1}{n}}$$

9 As small bore changes are considered the billet will swell on the outside diameter only. Let the billet radii ratio increase from K to K' as the augmenting stress is applied.

giving

$$K = \sqrt{e^{\bar{\epsilon}}(K'^2-1) + 1}$$

This procedure is repeated until the oil pressure is obtained to the required degree of accuracy, Figure 13 shows a typical result. A computer program for this calculation is given in the Appendix, see section 10.3.c.

Product-fixed, travelling mandrel, Figure 2d

As for the stationary mandrel this design of mandrel may be treated in the same manner as the extrusion of a solid billet.

When describing augmented estrucion it is useful to use the term 'equivalent extrusion pressure'. That is an oil pressure which if acting alone could perform the same process. If the billet augmenting stress is of elastic order of magnitude this is the sum of the oil pressure and the billet augmenting stress. If the augmenting stress is of plastic order of magnitude the equivalent extrusion pressure is less than the sum of the process stresses. The difference is due to the swelling of the billet. In either case if the increase in the mean flow stress is ignored the equivalent extrusion pressure is given by

$$p_e = p + \frac{R_o}{R} . \sigma_B \qquad \ldots(52)$$

4.5.d. PRODUCT AUGMENTATION

Product augmentation is achieved by applying a tensile load to the product. The limit to the augmenting stress which can be applied is governed by the tensile strength of the product, which is given by the well known instability expression

$$\left[\frac{d\bar{\sigma}}{d\bar{\varepsilon}} \right]_p = \sigma_p$$

As the product is work hardened this instability condition must be determined from the characteristics of the material extruded. However, as the reduction normally obtained will work harden the product above the tensile instability point corresponding to the condition of the material before it is extruded, the ultimate tensile strength of the product will be approximately equal to the equivalent flow stress of the material as it issues from the die. This is given by

$$(\sigma_p)_{MAX} = (\sigma_{ULT})_p \doteqdot (Y)_{\bar{\epsilon}=1} \cdot \bar{\epsilon}_m^n$$

It has been shown that an empirical law exists between the mean induced strain and the extrusion ratio which is of the form

$$\bar{\epsilon}_m = c + d \cdot \ln(R)$$

The maximum product augmenting stress which may be applied is approximately given by

$$(\sigma_p)_{MAX} = (Y)_{\bar{\epsilon}=1} \cdot \left(c + d \cdot \ln[R]\right)^n$$

and the corresponding maximum tensile load by

$$\text{Load} = \frac{A_o}{R} \cdot (\sigma_p)_{MAX}$$

With proportional product augmentation the ultimate tensile strength of the product will be the limiting factor to the maximum reduction which can be obtained. This limit will now be considered for tube extrusion over a product-fixed, travelling mandrel when small bore changes are produced, Figure 2d.

The maximum tensile stress induced in the product is proportional to the thrust exerted by the mandrel. This thrust depends on two factors. It depends on the pressure thrust exerted by the oil pressure acting on the mandrel and also on the frictional force between the mandrel and the bore of the deformation zone. The relative motion between the mandrel and the deformation zone is such that the frictional force opposes the thrust exerted by the oil pressure. The tensile stress induced in the product becomes

$$\sigma_p = \left(\frac{1}{K_1^2-1}\right) \cdot p - \mu \cdot \cot\alpha \cdot 2\left(\frac{K-K_1}{K_1^2-1}\right)(\sigma_D)_m \qquad \ldots(53)$$

The maximum reduction is given when this stress is equal to the ultimate strength of the product. As in previous cases this upper limit is difficult to determine in analytical form but it can be obtained to any degree of accuracy by the iterative procedure outlined below,

1 A suitable reduction is selected. Initially the reduction must be below the maximum obtainable.

2 $$K_1 = \sqrt{\left(\frac{K^2-1}{R}\right) + 1}$$

3 $$\bar{\epsilon}_m = F(\alpha, K, K_1)$$

4 $$Y_m = \frac{(Y)_{\bar{\epsilon}=1}}{1+n}\left(\bar{\epsilon}_m\right)^{1+n}$$

5 $$p = \frac{Y_m \cdot \dfrac{\epsilon_m}{(1-\psi)}}{\left\{1 + \frac{1}{K_1^2-1}\left[1 + \left(\frac{K^2-K_1^2}{K^2-1}\right)\frac{\psi}{1-\psi}\right]\right\}}$$

6 $$\varsigma_p = \frac{\omega_p \cdot p}{Y_m} = \left(\frac{1}{K_1^2-1}\right)\frac{p}{Y_m}$$

7 $$(\sigma_D)_m = \frac{Y_m\left[\left(\frac{K^2-1}{K^2-K_1^2}\right)\bar{\epsilon}_m - \varsigma_p\right]}{(1-\psi)(1+\mu \cdot \cot\alpha)}$$

8 $$\sigma_p = \left(\frac{1}{K_1^2-1}\right) \cdot p - \mu \cdot \cot\alpha \cdot 2\left(\frac{K-K_1}{K_1^2-1}\right)(\sigma_D)_m$$

9 $$(\sigma_p)_{MAX} = (Y)_{\bar{\epsilon}=1} \cdot \bar{\epsilon}_m^{\,n}$$

10 When $\sigma_p = (\sigma_p)$ max the reduction chosen is the maximum obtainable.

The computer program for this procedure is given in section 10.3.d., and a typical result is shown in Figure 14.

As it is necessary to preserve the dimensional accuracy of the product it is unlikely that product augmenting stresses will be chosen which produces permanent changes in the dimensions of the product. In addition, as the product is work hardened the strain range between the yield point and the tensile instability point will be so small that it will not be practical to operate over this range. It follows that product augmenting stresses are always going to be of elastic order of magnitude. For this type of augmentation the equivalent extrusion pressure is

$$p_e = p + \sigma_p$$

4.6. THE USE OF THE APPARENT STRAIN METHOD IN DETERMINING PROCESS EFFICIENCY

Many attempts have been made to specify a suitable parameter which may be used to indicate the mechanical efficiency of metal working operations. It is essential that this parameter be dimensionless and largely independent of the mechanical properties of the material being formed in order that the parameter may be used to compare one process with another or to compare the mechanical performance of one-process with the performance of the same process operating under different conditions.

To determine efficiency it is necessary to know the performance of the ideal metal working operation. One of the most suitable parameters which indicates the performance of the operation is the work done per unit volume of material because the existence of any process inefficiency, such as friction or ~~undue~~ redundant shearing, will increase the work done. Using this as a measure of the performance, the ideal metal working operation will be the operation which requires the minimum work to carry out the reduction. Such an operation is achieved when deformation is frictionless and homogeneous and the work done per unit volume of material is

$$W_H = Y'_m \cdot \ln(R)$$

where

Y'_m = mean yield stress over the strain range $0 \sim \ln(R)$

It must be emphasised that this measure of ideality is mechanical since metallurgical defects may be produced by a frictionless, homogeneous process if the material deformed lacks ductility, etc. In addition, the work required to achieve any degree of plastic deformation can be reduced by raising the temperature of the material. Any meaningful definition of efficiency must, therefore, refer to isothermal conditions, or more accurately to processes carried out on similar materials at the same initial temperature.

It has been shown Sect., 4.2b. that the apparent strain gives a convenient method for describing the work done during a process which induces a non-homogeneous mode of deformation when friction is present. The work done per unit volume of material during such a process may be written,

$$W = Y_m \bar{\epsilon}_A$$

where

Y_m = mean yield stress over the strain range $0 \sim \bar{\epsilon}_A$

This work quantity may be considered to be made up of two components, the useful work and the extrinsic work. The useful component is the minimum work required to achieve a given amount of plastic deformation, it is equivalent to the work done during frictionless, homogeneous deformation. The extrinsic work is that which is done in excess of the minimum work and is the work done which although necessary for a given condition of plastic flow is inessential as it can usually be reduced by better lubrication and by changing tool design. The extrinsic work is, therefore, a measure of the inefficiency of the metal working operation.

An obvious dimensionless parameter which is a measure of the inefficiency of a metal working operation is given by expressing the extrinsic work as a percentage of the useful work, thus

$$\eta = \frac{Y_m \bar{\epsilon}_A - Y_m' . \ln(R)}{Y_m' \ln(R)}$$

The inadequacy of this parameter is best illustrated by considering its application to the deformation of a material which is rigid plastic and non-work hardening, so that $Y_m = Y_m'$, giving

$$\eta = \frac{\bar{\epsilon}_A - \ln(R)}{\ln(R)}$$

or

$$\eta = \frac{\left(\frac{1}{1-\psi}\right)\bar{\epsilon}_m - \zeta_P\left(\frac{R-1}{R}\right)\frac{\psi}{1-\psi} - \ln(R)}{\ln(R)}$$

where $\bar{\epsilon}_m = a + b.\ln(R)$ and $b \doteqdot 1$

When friction is small, as is the case with hydrostatic extrusion of bar

$\psi \rightarrow 0$ and $\eta \rightarrow \dfrac{a}{\ln(R)}$

This shows that the efficiency depends on the extrusion ratio even though the basic cause of the inefficiency, the distribution of redundant shearing, remains constant. The dependency of the efficiency on the extrusion ratio makes it inconvenient.

A better measure of identity is found in what will be called the extrinsic factor. This is found by dividing the extrinsic work by the mean yield stress thus,

$$\chi = \frac{Y_m \bar{\epsilon}_A - Y_m' \ln(R)}{Y_m}$$

when applied to the deformation of a rigid plastic and non-work hardening material the extrinsic work factor simplifies to become

$$\chi = \left(\frac{1}{1-\psi}\right)\bar{\epsilon}_m - \varsigma_p\left(\frac{R-1}{R}\right)\frac{\psi}{1-\psi} - \ln(R) \qquad \ldots(54)$$

where $\bar{\epsilon}_m = a + b.\ln(R)$ and $b \doteq 1$

For hydrostatic extrusion of bar where $\psi \to 0$ ~~and~~

$\chi \to a$ (constant)

The approximate extrinsic factor, equation (54), is a useful comparator even when applied to processes carried out on work hardening materials. It is, however, difficult to convert to total work.

The effect of friction on the extrinsic factor is best illustrated by applying it to the process of simple hydrostatic extrusion in which friction is small, for this particular process equation (54) gives

$$\chi = a(1+\psi) + \psi.\ln(R) \qquad \ldots(55)$$

5. RESULTS

5.1. EQUIPMENT

5.1.a. THE FIELDING AND PLATT 200/50 'HYDROSTAT'

All experimental work was carried out on the Fielding and Platt 200/50 'HYDROSTAT'. This press was developed and built under an agreement between the U.K.A.E.A. and Fielding and Platt in 1964. The following is an extract from Green's (3) paper and because it describes this press so completely it is included here unabridged.

"A simplified cross-section of the machine is shown in Fig. 15. It may be conveniently described in two parts: the low-pressure (L.P.) assembly and the high-pressure (H.P.) assembly. The plunger (A) transmits axial load between the two. The L.P. assembly comprises a vertical double-acting piston (B) of 200 tonf thrust, which is driven downwards, on the working stroke, by stored nitrogen-gas pressure, or upwards, on the return stroke, by pumped hydraulic oil. Over a full-length working stroke the gas pressure drops by 4%, but is recompressed to its original value of 2580 lbf/in^2 on return strokes.

To effect a working stroke, oil is exhausted from the underside of the main piston through the metering valve (C), which controls the rate of oil flow to give a wide range of piston speeds up to 20 in/sec.

The nitrogen bottle is connected through an isolating valve to the main piston. The system is thus similar to a conventional gas/liquid accumulator installation, except that gas/liquid separation take place at the head of the main piston instead of in a subsidiary separating bottle. This simplifies the overall installation and contributes to the easy achievement of high piston speeds.

The oil and nitrogen have independent seals on the piston head, with an atmospheric vent between the two. This gas seal had previously been developed by Fielding and Platt for leak-proof usage in other high-speed applications. To ensure long seal life and to minimize wear, the bore of the main cylinder is plated with chromium on nickel.

The metering valve controlling the exhaust oil flow is actuated by air cylinders which open or close the valve according to electrical impulse from the control circuit. The opening of the valve is limited by a hand-operated adjustable stop which allows a maximum exhaust area through the valve of 4.9 in^2, giving an oil speed of 424 in^3/sec at a piston speed of 20 in/sec.

The valve is opened by push-button control and is closed at the end of a predetermined piston stroke by a variable position electrical limit switch. The valve may be closed before the set stroke is completed by selecting the piston return position on the return and by-pass valve. The metering valve also operates on a fail-safe system in that it will close should the guards be inadvertently opened, if any electrical failure occurs, or if the emergency stop-button is pressed. The return and by-pass valve also serves to by-pass hydraulic oil back to the suction tank at times other than when the piston is being returned.

The lower end of the intensifying plunger (A) (Fig. 15) is fitted with a high-pressure seal assembly (D), and is positioned on the sealed spigot of the die block (F) into which fit any of a variety of extrusion dies (G) and the horizontal extrusion container (H). This container and the die-support plug (L) screw into opposite sides of the die block (F). The components F, G and H are all fitted with pressure seals of the O-ring and mitre-ring type. All the high-pressure components were designed to take a working pressure of 110,000 lbf/in^2.

The end of the extrusion container is sealed with an O-ring and mitre backing ring which remain permanently in position in the container. The removable plug (J) completes the end closure and is held in position by the breech-type assembly (K) for quick and easy recharging of billets.

The high-pressure medium used is caster oil, (a mixture of castor oil and 10% methylated spirits is now preferred), which is viscous enough to prevent any loss whatsoever from the container (E) through the relatively small vertical hole in the die block (F) during billet recharging. Also, when entering a new billet through the breech it is necessary to apply a little force to displace the sluggish liquid in vessel (H). Die changing may also be carried out without any appreciable loss from the container (E).

The usual method of refilling a hydrostatic-extrusion vessel with oil is by removing the plunger and seals. To improve this and at the same time meet the specified requirement of automatic filling, the seal assembly shown in Fig. 16 was developed. Here, the withdrawal forces are transmitted directly to the main mitre ring by the thrust ring (A). The inclusion of this ring has produced a secondary leak path which is sealed off by the stationary O- and mitre-ring seals (B). The assembly is permitted radial movement relative to the plunger, which is somewhat smaller in diameter than the vessel bore. The assembly is loosely attached to the plunger by the withdrawal bolt (C) and is permitted axial movement relative to the plunger. The central hole in the assembly, through which passes the withdrawal bolt, provides a third possible leak path which is sealed with the mitre ring (D).

In operation, all leak paths are closed when the assembly is moved against the oil pressure, as in Fig. 16(a). On withdrawal, the assembly lags behind the plunger, as in Fig. 16(b), and oil is drawn into the bore, through the ample passages shown, from a reservoir (E) formed at the immediate top of the pressure vessel. This valve action means that extrusion billets larger in volume than the swept plunger volume may be "pumped" through, if necessary, by repeated strokes of the machine.

Another advantage is that the seal has an air-venting action, since, if the plunger is withdrawn from a pressure, the amount of friction between the assembly and the vessel wall allows the seal to open whilst some pressure still exists, as in Fig. 16(c). This immediately blows out, through the reservoir, any air that may have collected in the upper part of the vessel.

The main mitre ring may be made of copper but nylon is surprisingly effective in this role. This is important in preventing wear in the main high-pressure cylinder."

5.1.b. MODIFICATIONS TO THE 200/50 'HYDROSTAT'

The 200/50 'HYDROSTAT' was modified to allow product augmentation and to allow complete extrusion without the release of the oil. These modifications were carried out in two parts, the first was to construct an attachment to allow a tensile force to be exerted on the product and the second was to develop equipment which allowed the high pressure oil in the extrusion container to expand during the final stages of extrusion, thus allowing complete extrusion without the usual release of high pressure oil.

Figure 17 shows a simplified horizontal cross-section through the extrusion container with the drawing attachment in position.

The drawing attachment was made readily detachable from the 'HYDROSTAT' to enable easy conversion of the press to its original form. The drawing load reaction was transmitted directly to the die block through the main frame (N) via the Saddle member (M), the latter component being in direct contact with the die block. The drawing load acted on the neutral axis of the main frame, thus eliminating bending. This arrangement minimised the loads carried by the press foundations and the drawing attachment supports.

The drawing attachment was supported on two screw jacks, which ensured alignment with the axis of the extrusion container in the horizontal plane and allowed vertical movement for levelling. It was held in position by a clamp acting between the base plate of the press and the main frame of the drawing attachment.

The drawing attachment, see Figure 18, was capable of exerting a maximum tensile load of 10 tonf over a stroke of 7 ft. This load was produced by the low pressure hydraulic cylinder (Q) and measured by a load cell situated between the main carriage (P) and the cylinder ram (R). The hydraulic power was supplied to the cylinder from a standard power pack, see Figure 19, which incorporated two flow control valves, permitting drawing speeds in the ranges 0-10 and 0-50 ft/minute.

The tag of the billet was gripped by the hardened jaws (O), mounted in the main carriage of the unit and power operated by a pneumatic cylinder to permit remote control. The carriage (O) had provision for a mandrel which passed down the centre of the grips to support the bore of the product during tube extrusion.

The high pressure components of the 'HYDROSTAT' were not modified and were as described above.

The second part of the modifications, to permit 'high pressure phase out', involved modifications to the low pressure hydraulic circuit of the 'HYDROSTAT'. Before these modifications are described the principles of the 'phase out' process will be described.

The aim of the phase-out process was to eliminate the oil pressure in the extrusion container during the final stages of extrusion, so that the billet could be completely extruded without the usual release of high pressure oil. To achieve this, the end of the billet was tapered to a smaller cross-section, see Figure 20. The size of the end section was such that it could be drawn through the die. During steady state extrusion the full section of the billet was extruded by the combined action of the oil pressure and the drawing load. The response of the 'HYDROSTAT' was sufficient to keep the oil pressure constant over the whole range of drawing speeds.

When the tapered section of the billet entered the die the total extrusion pressure required reduced. This resulted in a reduction of the drawing load, since the oil pressure remained constant. Without further action this would have continued until the billet was extruded by the sole action of the oil pressure, resulting in the unwanted release of the high pressure oil. To prevent this the drawing load was only allowed to fall to a predetermined low value, at which the main ram of the 'HYDROSTAT' was retracted ~~returned~~, allowing the oil in the extrusion container to expand and the pressure to fall. To compensate for this loss the drawing load increased. This continued until a predetermined maximum drawing load was obtained, at which the upward movement of the main press ram was stopped. This resulted in a second phase during which the drawing load fell as a result of the tapered section of the billet passing through the die. On reaching, for the second time, the lower limit of the drawing load the pressure phase-out cycle was repeated. This continued until all the oil pressure had been removed from the extrusion container. Extrusion ended by drawing the end section of the billet through the die.

Figure 21 shows the flow diagram for the 200/50 'HYDROSTAT', the drawing attachment, the pressure 'phase out' equipment and the main control circuits.

As described, the movement of the main ram of the 'HYDROSTAT' was controlled by the low pressure oil in the annular space under the main piston. During the pressure 'phase out' cycle it was necessary to recharge this annular space to cause the ram to move upwards to allow the high pressure oil to expand. This was achieved by injecting oil from an accumulator through a metering valve, see Figure 22.

Before extrusion began the whole of the low pressure system was charged. The main pump delivered oil to the annular space, at a pressure of 4400 lbf/in^2. When the ram was fully returned the pressure rose to 4800 lbf/in^2. At this pressure the oil passed through the non-return valve into the accumulator. Once this had been filled the pressure rose further to 5000 lbf/in^2, which actuated the pressure switch and returned the solenoid controlled two-way valve to its by-pass position. In this state the press was ready for extrusion.

When the main section of the billet passed into the die the full oil pressure was applied in the extrusion container and the drawing attachment exerted its normal working load. To achieve this, the main exhaust valve opened, which relieved the oil pressure in the annular space, and the pump on the drawing attachment power pack delivered oil to the hydraulic cylinder at a rate which was regulated by number 1, flow control valve. The oil from the full bore of the cylinder returned to the reservoir via the by-pass valve. Towards the end of extrusion the by-pass valve closed and the flow of oil from the hydraulic cylinder was regulated by number 2 flow control valve. It was arranged that this reduced the speed of extrusion.

When the tapered section of the billet entered the die the drawing load fell and activated the 'minimum' relay via the electrical control unit, see Figure 23. This closed the main exhaust valve and opened the metering valve between the accumulator and the annular space of the 'HYDROSTAT'. A proximity switch was fitted to the exhaust valve to ensure that it closed before the metering valve opened. With oil passing from the accumulator into the annulus the main ram moved upwards, causing the oil pressure in the extrusion container to fall and the drawing load to increase. At the predetermined drawing load the 'maximum' relay operated which closed the metering valve and preventing further movement of the main ram. This cycle was repeated until all the pressure had been removed from the extrusion container.

5.2. TOOL SYSTEMS

5.2.a. DIE ASSEMBLIES

All dies were of the simple cone type and formed part of a die assembly, see Figure 20. Included die angles of 10, 15, 20, 25 and 45 degrees were used and the dies were made from a 12% Cr, 1.5-2.0% C tool steel.

The die assemblies consisted of five parts. The die was supported by a hardened EN 25 sealing plug which also carried the copper mitre-ring and rubber 'O' ring seal. The oil seal between the die and the sealing plug was achieved by an outer copper or mild steel tube, as shown. The end thrust exerted on the die assembly was carried by a screwed plug.

With this arrangement the outer surface of the die was subjected to radial compression, exerted by the oil pressure, acting on the outer tube, this gave support to the die.

5.2.b. MANDRELS

Three different mandrel designs were used to achieve both simple and proportionally augmented hydrostatic extrusion. Figure 24 shows the stationary and the billet-fixed travelling mandrels used to induce simple and proportional billet augmentation. The third design used to produce proportional product augmentation consisted of simple bars of hardened and ground true steel made in lengths longer than the product to be produced.

The stationary mandrels each consisted of a shank, which was long enough to pass from the rear end of the extrusion container to the throat of the die, and a head. This head contained the copper mitre-ring and rubber 'O' ring high pressure seal and during extrusion replaced the movable plug (J) shown in Figure 15. The mandrels were made with various shank diameters from both 12% Cr, 2.0% C and 18% W, 4% Cr, 1% V electroslay-melted, forged tool steel. The mandrels were fully hardened, and tempered in a way which optimised hardness at the tip and improved ductility near the mandrel head.

The stationary mandrel presented a sealing problem at the billet-mandrel interface. Three ways of achieving the necessary oil seal were investigated.

It was found sufficient to preform the nose of the billet, to give an interference fit with the mandrel. This method had the advantage that the oil acted continuously on both the inner and outer billet surfaces, thus assisting with lubrication throughout extrusion. Its disadvantage was that it necessitated an extra forming operation.

A permanent rear end seal was used which moved with the billet and passed over the mandrel shank during extrusion. This seal, although effective, often led to the collapse of the billet on to the mandrel and excluded the oil from the bore which increased the problem of bore lubrication.

The most successful seal was a rear end attachment which only sealed up to a predetermined intermediate oil pressure, see Figure 24a It consisted of a cylindrical part which was free to slide over the shank of the mandrel. A copper mitre and rubber 'O' ring seal was contained at the rear of the attachment, this prevented oil from entering the bore. The section between the seal housing and the billet consisted of a cylinder which had a small wall thickness and was an integral part with the seal housing. Initially the attachment was held against the billet by a coil spring acting between the head of the mandrel and the attachment. During pressurisation the attachment followed the billet forward until the surrounding pressure was sufficient to elastically collapse the thin section on to the mandrel, the attachment was designed so that this occurred at the desired intermediate oil pressure. This collapse locked the attachment in position and allowed the billet to move away. Once the seal was broken the oil entered the bore of the billet but could not escape as the front end of the billet had formed on to the mandrel. This attachment eliminated the need to preform the billet.

The billet-fixed travelling mandrels, see Figure 24b, were made with various shank diameters and each with a shank length of 12 in. The shanks were given a 0.010 in. taper to relieve mandrel drag and to facilitate the withdrawal of the mandrel.

The heads of the mandrels were made in two parts. The main part was an integral piece with the shank and housed a non-return hydraulic valve. When this valve was assembled it allowed the oil trapped in the bore of the billet to pass into the extrusion container at the end of extrusion. When the valve parts were removed, the valve passages allowed the oil from the extrusion container to pass into the bore of the billet to improve lubrication. With this latter arrangement it was necessary to have an initial seal between the nose of the billet and the mandrel shank. The other part of the head was a loose annular piece which passed over the mandrel shank. It produced a square shoulder to give a good contact with the billet and contained a softened ring which acted as a buffer when the head made contact with the die. These mandrels were made from 12% Cr, 2.0% C tool steel, heat treated to optimise hardness.

5.3. BILLETS

The billets were made from the materials shown in Table I and also mild steel.

Material	C	Cr	Ni	Mo	Co	Si	Mn	Ti	Al	No.	Fe	N
AISI 304	0.08	18-20	8-12	-	-	1.0	2.0	-	-	-	Balance	-
AISI 316	0.08	16-18	10-14	2-3	-	1.0	2.0	-	-	-	Balance	-
20/25/ND	0.01-0.05	19-21	24-26	-	-	0.75	0.85	-	-	C x 10	Balance	-
PE 16	0.10	15-18	42-45	2.5-4.0	2.0 Max	-	-	0.9-1.5	0.9-1.5	-	Balance	-
PE 13 Hastelloy X	0.05-1.5	20.5-23	Balance	8-10	0.5-2.5	1.0	1.0	-	-	-	17 20	0.2-1.0

Table No. 1. Materials extruded

The design of the billets varied considerably and depended on the extrusion method to be employed.

A common feature of most of the billets was that they were all machined with a front end nose cone mainly with included angle of 14 degrees. The only exceptions were the billets for extrusion through cone dies with an included angle of 10 degrees and these billets had a nose cone with an 8 degree included angle. All the billets were made from bar material to a fine finish, free from machining marks, and annealed after machining according to the recommendations of the suppliers. This heat treatment reduced the material to its softest condition and removed the surface residual stresses introduced by machining.

Most of the solid billets used for simple hydrostatic extrusion were 1 in. dia. and 12 in. long. Billets in Nimonic PE 13 and 16 were $\frac{1}{2}$ in. dia., as material of this size was readily available.

Billets for product augmented hydrostatic extrusion of bar had two parallel sections joined by a truncated conical section, with an included angle of 14 degrees. The main rear end parallel section was 1 in. dia. and 17 to 19 in. long. The diameter of the front parallel section varied from one material to another. This diameter was chosen so that the section could be extruded by simple hydrostatic extrusion. As the maximum oil pressure was limited to 50 tonf/in^2, the extrusion ratio achieved during the extrusion of this section, was limited to 1.5-1.8. The overall length of the billet did not exceed 28 in.

The dimensions of the hollow billets for extrusion by both simple and proportional augmented hydrostatic extrusion, were standardized. The bore diameter was always 0.015 in. larger than the mandrel diameter. As most of the extrusion trials were carried out using a 0.500 in. dia. mandrel the most common bore size was 0.515 in. The outside diameters of the hollow billets were fixed by considering the maximum billet augmenting stresses induced by the billet-fixed, travelling mandrels. The maximum billet augmenting stress for mild steel was limited to 40,000 lbf/in^2, which gave a billet outside diameter of 1.00 in. and 55,000 lbf/in^2 for the AISI 316, AISI 304 and 20/25/Nb stainless steels, which gave an outside diameter of 29/32 in. (0.908 in.). A billet is shown in Figure 24b.

The hollow billets used for product augmented hydrostatic extrusion had a uniform bore and outer surfaces which were similar to the solid billets described above, see Figure 24a. The outside diameters of the main rear end parallel sections were the same as those used for the billets for proportional billet augmented hydrostatic extrusion. The outside diameters of the front parallel sections were fixed by limiting the extrusion ratio achieved to 1.4-1.6. The overall length of these billets did not exceed 28 in. During complete product augmented hydrostatic extrusion the billets were as described with various tapers machined at the rear end of the billets.

Some feasibility trials were carried out, on methods for reducing both the bore diameter and the overall thickness, using billets with a 0.838 in. outside diameter and 0.714 in. bore in annealed 18/8 (AISI 304) austenitic stainless steel. Billets of these dimensions were also used for the trials on hydrostatic extrusion without a mandrel, in which the front end was sealed and the high pressure oil was allowed to enter the bore of the product. These billets were made by swaging a front nose cone, with an included angle of approximately 14 degrees, on to tube lengths 15 in. long, until the front end was virtually closed. The end was sealed by welding.

5.4. LUBRICANTS

All lubrication was achieved by applying surface coats of suitable lubricants to the billets prior to extrusion. The technique of using suspensions of a lubricant in the pressure transmitting medium was not attempted.

For billets made from stainless steel an oxylate and soap lubricant was used, this followed established commercial practice. The Pyrene Bonderite SS4 and Bonderlube 350 were used to apply these coats.

A special, laboratory size tank was made to contain the Bonderite SS4 solution. This consisted of a stainless steel container 9 in. dia. and 20 in. high, heated by a Nimonic clad electrical resistance heating coil. The temperature could be maintained constant to $\pm$ 5°F up to a maximum of 180°F, by a thermostatic control. This tank was provided with lip extraction and contained approximately 4 gallons of solution.

The Bonderite SS4 solution was made by the addition of the following chemicals to 4 gallons of water.

6 lb of Bonderite SS4 IMU powder

0.24 lb of Bonderite SS4 accelerater powder

The solution was prepared by adding the Bonderite SS4 IMU powder to the 4 gallons of water, pre-heated to 130°F. Once this had dissolved the solution was heated to its working temperature of 180°F. The Bonderite SS4 accelerater powder was then added. This produced a 40 point solution.

The Bonderlube 350 solution was contained in a commercially available, bench mounted laboratory tank, made by Laboratory Thermal Equipment Ltd. This was 16 in. long, 12 in. wide and 16 in. deep. It was fitted with a heating device, capable of maintaining the temperature constant to $\pm$ 5°C up to a maximum of 150°C and a mechanical adgitator. The working capacity of this tank was approximately 10 gallons. A 20-25% solution was made by adding 25 lbs of Bonderlube 350 powder to 10 gallons of water, which had been pre-heated to its normal working temperature of 80°C.

The stainless steel billets were lubricated by the following sequence of operations.

1. Degrease with Ethyl Methyl Ketone.
2. Bonderite SS4, applied by immersion for 10 minutes.
3. Cold water rinse
4. Bonderlube 350, applied by immersion for 10 minutes.
5. Air dry.

The mild steel billets were lubricated by a coat of Evo-stik and Teflon. This lubricant had been previously used for the hydrostatic extrusion of non-ferrous metals and was found to be adequate for mild steel.

Evo-stik was developed as an 'impact' adhesion and applied to the surface of the billet in a 50% dilute solution with Ethyl Methyl Ketone. This was contained in a vertical tube 2 in. dia. and 30 in. high. The billets were withdrawn from the solution by a motorised hoist at a speed of approximately 8 in. per minute, through an electrical resistance coil heater. This produced a uniform coat which had a rubber-like consistency and was slightly tacky. The lubricant was completed by spraying a coat of Flucalube-H on to the layer of Evo-stik from an aerosol container. Flucalube-H was a proprietry brand of lubricant and contained a Teflon powder in a Freon suspension. This assisted with lubrication and eliminated the tackiness of the Evo-stik coat.

5.5. COMPRESSION TESTS AND MECHANICAL PROPERTIES

The mechanical properties of the materials to be extruded were determined by simple compression tests carried out on a grade A, Universal, 50 tonf capacity, Denison testing machine. The tests were carried out between hardened and tempered EN 25 platens which had tungsten carbide inserts on the forming surfaces.

The compression test specimens were 0.2 in. dia. by 0.3 in. long, machined from bar stock. After machining they were given the same annealing heat treatment as that given to the billets.

The compression tests were carried out using the load increment technique and P.T.F.E. tape as a lubricant. After the load had been applied the specimen was removed from the machine and the diameter and length measured with a hand micrometer. The lubricating tape was replaced before the next load, an increment larger than the previous load, was applied. This was repeated up to the maximum capacity of the testing machine. The use of P.T.F.E. as a lubricant minimised barrelling to an extent which eliminated the need to re-machine the specimen. Figure 25 shows the results of these tests.

The equivalent stress and strain were calculated from the instantaneous load the specimen dimensions as follows,

$$\bar{\sigma} = \frac{4L}{\pi d^2}$$

and

$$\bar{\epsilon} = \ln\left(\frac{h_o}{h}\right)$$

The experimentally determined stress-strain curves agreed with the following empirical law over a wide range of strain.

$$\bar{\sigma} = (Y)_{\bar{\epsilon}=1} \cdot \bar{\epsilon}^n$$

Material	$(Y)_{\bar{\epsilon}=1}$ lbf/in^2	n
Mild steel	112000	0.247
20/25/NB S.S.	167000	0.320
AISI 316	203000	0.350
AISI 304	231000	0.570

Table No. 2. Material characteristics over the strain range 0-1.5

It had beed noticed by Pugh and colleagues at the National Engineering Laboratory that the ratio, given by extrusion pressure divided by the natural logarithm of the extrusion ratio, gave a linear relationship when plotted against the hardness number. This was interpreted to imply that some single and unique material parameter influenced the extrusion pressure. The empirical law, quoted above, gave two parameters which conflicted with this idea. As a result it was decided to look for a relationship between these two parameters, Figure 26 shows the result. Extra points were obtained from empirical laws, obtained from previously tested non ferrous materials, and from the literature. A definite trend was observed for annealed materials.

5.6. METHODS OF MEASUREMENT

Measurements of the extrusion parameters on the 200/50 'HYDROSTAT' were made with suitable electrical transducers and displayed on a four channel oscilloscope against a time base. This display was photographed by a polaroid camera for permanent records.

The following parameters were measured:

1. Oil pressure in the extrusion container
2. Drawing load
3. Annulus oil pressure
4. Main ram velocity
5. Product displacement

The oil pressure in the extrusion container was measured by a Bourden pressure gauge, which had the range 0-150,000 lbf/in^2. This was converted to an electrical signal by measuring the displacement of the Bourden tube with a linear transducer. The electronic recording device had maximum and minimum calibrating circuits for fixing the range on the oscilloscope.

The drawing load was measured by electrical resistance strain gauges mounted on a 10 tonf capacity load cell, this was fully temperature compensated. Maximum and minimum calibrating circuits were also provided for fixing the working range.

The annulus oil pressure was recorded by a standard, commercial low pressure transducer.

The ram velocity was converted to a D.C. electrical signal, by means of a small dynamo transducer and displayed directly on the oscilloscope without amplification.

The product displacement was obtained by equally spaced photo-electrical cells mounted on a run-out table. These cells operated a trigger circuit which gave a 'blip' type display on the oscilloscope. They could not be used when the drawing unit was in use.

5.7. EXTRUSION RESULTS

5.7.a. SIMPLE HYDROSTATIC EXTRUSION OF BAR AND TUBE - THE EFFECTS OF REDUCTION

The experimental method employed to achieve simple hydrostatic extrusion of bar was as follows. After applying a suitable lubricant the billet was loaded into the extrusion container via the sealing device and rammed forward to push the nose of the billet into the die. The space behind the billet was filled by mild steel filler rods, 1 in. dia., and a short coil spring to ensure billet-die contact. The movable plug was inserted into the end of the extrusion container and the breech device closed.

The setting of the exhaust valve was adjusted to give a suitable extrusion speed. The exhaust valve setting controlled the extent the valve opened during extrusion and, therefore, the flow of oil from the annular space and in turn the extrusion speed. Initially the speed setting was chosen arbitrary and an extrusion undertaken. If the stick-slip mode of extrusion was produced the speed setting was increased to give a steady mode of extrusion.

Before extrusion commenced a mild steel catching tube was screwed into the die support plug in front of the die and sealed at the other end by a steel cap. The catching device was filled with water. On complete extrusion the billet passed down the bore of the catching tube and was decelerated by the hydrodynamic effect. The oil which entered the tube after the billet was extruded, was allowed to escape through a drilled hole in the wall of the tube, close to the die support plug. The efficiency of the catching device improved by covering the outlet hole by a short length of tube. The catching tube had a bore diameter of 1 in. and a wall thickness of 0.250 in.

With the press assembled and the catching device in position, the main ram was allowed to descend by exhausting the low pressure oil from the annular space, through a hand operated tool setting valve. As the ram descended air was driven from the pressure intensifier through the self venting seal, as described. Once the ram made contact with the high pressure oil the self venting seal closed and compression began. This operation was terminated as soon as oil pressure was indicated on the Bourden pressure gauge. If the position of the ram showed that insufficient oil was contained in the high pressure system the main ram was returned to the top of its stroke to allow more oil to be drawn in through the self venting seal. Oil pressure was once again obtained by the tool setting valve.

After taking the necessary safety precautions, which are discussed later, the recording equipment was activated and the extrusion carried out. This was repeated for a variety of steels and extrusion ratios. Figures 27 and 28 show the results. Some of the extrusion pressures were converted to apparent strain and can be seen in Figure 29 against the natural logarithm of the extrusion ratio.

The extrusion pressure was converted to apparent strain by using the experimentally determined oil pressure and the equivalent stress-strain characteristics of the material, obtained by mechanical tests, see section 5.5. The equivalent stress-strain diagram was converted to a plastic work-strain diagram by graphical integration. The plastic work-strain diagram showed the area under the equivalent stress-strain diagram, from zero strain to the strain considered. The experimentally determined oil pressure was a measure of the total work done per unit volume, see equation 11, and was located on the plastic work axis and converted to its corresponding strain. The apparent strain- $\ln(R)$ diagram was then constructed by plotting these strains against the natural logarithm of the extrusion ratios. This procedure is shown in diagram form in Figure 30.

When a tube was extruded by simple hydrostatic extrusion several differences existed in the experimental procedure from that described above. Simple hydrostatic extrusion of a tube was only obtained when using a stationary mandrel. With this mandrel it was necessary to preform the nose of the billet to produce an oil seal at the bore. The effective rear end seal, described in the section 3, had not been developed at the time these trials were undertaken, hence, the interference sealing method was used throughout. After pre-forming the nose of the billet a suitable lubricant was applied, the billet was pushed on to the mandrel shank and the assembly loaded into the press. The mandrel head replaced the movable plug and was held in position by the breech device. The interference fit between the billet and the mandrel was sufficient to hold the billet against the die during the initial stages of compression. The remainder of the procedure was as described for the extrusion of bar. Figures 31 and 32 show the results of these extrusions. Some of the extrusion pressures were converted to apparent strain and are shown in Figure 33 against the natural logarithm of the extrusion ratio. Figure 34 shows a comparison between the oil pressure required to extrude mild steel bar and tube by simple hydrostatic extrusion.

5.7.b. SIMPLE HYDROSTATIC EXTRUSION OF BAR AND TUBE - THE EFFECTS OF DIE ANGLE

Trials were carried out to investigate the effects of die geometry on both bar and tube extrusion, and consisted of the extrusion of billets through simple cone dies of differing included die angles at a common extrusion ratio. This was repeated at different extrusion ratios and in every case covered the included die angle range 10 to 45 degrees. Pugh and colleagues at the National Engineering Laboratory had previously carried out this type of investigation for bars, one of their results is shown in Figure 35. In view of this available information only one set of bar extrusions was carried out in order that the geometric and mechanical properties of the products could be measured. In addition to oil pressure measurements the product movement, hardness, ultimate tensile strength and percentage elongation was measured for each product. The results of these tests are shown in Figure 36. The product movement was shown in a dimensionless form by dividing the difference between the product and the die throat diameters by the nominal diameter.

A number of similar trials were carried out on tube which covered a range of extrusion ratios. For one set the geometric and mechanical properties, listed above, were measured. The product movement at the bore was also measured and shown in a dimensionless form. This was obtained by dividing the difference between the product bore and mandrel diameters by the nominal mandrel diameter. Figures 37 to 40 show these results.

5.7.c. THE EXTRUSION OF TUBE WITHOUT A MANDREL - WITH A FRONT END SEAL

A number of extrusions were undertaken to investigate the feasibility and some of the basic characteristics of the hydrostatic extrusion of tube without a mandrel. The method in which the front end of the billet was sealed was investigated. The manufacture of the 18/8 austenitic stainless steel billets, used for these tests, has already been described. Each billet was lubricated and loaded into the 'HYDROSTAT' in an identical manner to the method used for a solid billet. Care was taken to ensure that the filler rod did not inadvertently make an oil seal with the open rear end of the billet. Figure 41 shows the results of these tests. As this form of hydrostatic extrusion was heavily proportionally product augmented the effective extrusion pressure was determined using the following expression

$$p_e = p\left[1 + \left(\frac{1}{K_1^2 - 1}\right)\right]$$

Several reductions in diameter were attempted until the product failed by bursting. It was noticed that the product failed at the swaged nose and that failure was probably initiated by damage caused by swaging. With greater care during the forming of the nose a larger diameter reduction than the one indicated may be possible.

5.7.d. THE EXTRUSION OF TUBE WITHOUT A MANDREL - WITH REAR END SEAL

Extrusions were carried out on some of the billets designed for simple hydrostatic extrusion by sealing the rear end with a special cap. The experimental procedure was similar to that used to extrude a solid bar. After the billet had been lubricated the end cap was fitted and the billet loaded into the extrusion container. The procedure followed that previously described.

Two sets of extrusions were carried out. In one set the diameter reduction was increased, with the die angle constant, until the billet collapsed by the action of external pressure. In the second set the included die angle was varied over the range 10 to 45 degrees at a common diameter reduction. Figures 42 and 43 show the results of these tests. This form of hydrostatic extrusion was proportionally billet augmented hence the effective extrusion pressure was determined using the following expression:

$$p_e = p\left[1 + \left(\frac{1}{K^2 - 1}\right)\right]$$

5.7.e. PROPORTIONAL AUGMENTED HYDROSTATIC EXTRUSION - BILLET-FIXED, TRAVELLING MANDREL

Extrusion was carried out to investigate the properties of proportional augmented hydrostatic extrusion of tubes with small bore changes. This was achieved by the use of travelling mandrels.

Emphasis was given to proportional billet augmented hydrostatic extrusion since the forces applied to the billet were similar to those applied by the 1600/80 'HYDROSTAT'.

The experimental procedure for the extrusion of tube using a billet-fixed, travelling mandrel was similar to the procedure used to extrude a solid bar. The billet was lubricated and threaded over the mandrel, which in every case included a high pressure non-return valve. This type of mandrel was self sealing at the bore. The billet-mandrel assembly was loaded into the 'HYDROSTAT' as a solid billet and extruded in the manner previously described.

As this mandrel had a head larger in diameter than the product, complete extrusion was impossible. At the end of extrusion the head made contact with the die which sealed the extrusion container. As the 200/50 'HYDROSTAT' worked on a gas intensified system, extrusion was followed by further compression of the oil until maximum pressure was obtained. This can be seen in Figure 44 which shows a typical set of oil pressure-time traces for the proportional billet augmented hydrostatic extrusion of 20/25/Nb stainless steel tube.

Figure 45 compares the oil pressure required to extrude 20/25/Nb stainless steel tube by simple hydrostatic extrusion with both the oil pressure and equivalent extrusion pressure required when a billet-fixed, travelling mandrel was used. The effect of billet swelling and work hardening, induced by the application of billet augmentation, on the equivalent extrusion pressure can be seen in this figure.

To investigate the effects of increased billet augmentation further extrusions were carried out using billets which had a reduced wall thickness. These were made from 20/25/Nb stainless steel and had a diameter ratio of 1:455. It was found that at the higher extrusion ratios attempted the instantaneous area reduction varied significantly due to the taper in the mandrel shank. Figure 46 shows the mean oil pressure recorded against the natural logarithm of the mean extrusion ratio. As the head of the mandrel excluded the pressure transmitting oil from the bore, friction at the inner surface was larger than at the outer surface. Figure 46 shows a comparison between the experimental results and the apparent strain predictions, made by assuming the coefficient of friction at the bore to be higher than that at the outer surface by fixed ratios and the coefficient of friction at the outer surface to be 0.005.

Figure 47 shows a comparison between the oil pressure required to extrude 20/25/Nb stainless steel over a billet-fixed, travelling mandrel for two billet sizes.

5.7.f. PROPORTIONAL AUGMENTED HYDROSTATIC EXTRUSION - PRODUCT-FIXED, TRAVELLING MANDREL

One series of extrusions were carried out using a product-fixed, travelling mandrel. Figure 48 shows the oil pressures obtained along with some apparent strain predictions based on various coefficients of friction. The billets were made by swaging the front end of machined hollows to a reduced bore size and annealed before extrusion. Figure 44 shows a comparison between the oil pressures required to extrude AISI 316 stainless steel tubes by the three principal mandrel designs.

It was found difficult to realise the greatest reduction which was supposed possible, from considerations of augmenting stresses and the work hardened strength of the product. This was because the initial geometry at high reductions did not induce the same degree of augmentation that could be produced when extrusion began, since the pre-formed billet nose did not allow the mandrel to pass beyond the throat of the die, this is discussed later. At small reductions this difficulty did not arise.

Extrusion by a product-fixed, travelling mandrel was curtailed due to the danger of mandrel ejection under pressure. This occurred several times and resulted in the mandrel penetrating the guard of the machine. On one occasion the mandrel penetrated several mild steel components, resulting in a total penetration of 0.75 in. In view of these difficulties it was decided that the danger of using this mandrel outweighed the other advantages it offered.

5.7.g. PRODUCT AUGMENTED HYDROSTATIC EXTRUSION - EFFECTS OF REDUCTION

With product augmented hydrostatic extrusion the maximum reduction was limited by the tensile strength of the tag. To increase the tag strength a system was adopted in which it was extruded by the action of the oil pressure alone. This not only work hardened the tag but it also minimised billet pre-forming.

With bar extrusion, pre-extrusion procedure was the same as that previously described for simple hydrostatic extrusion of bar. A high speed setting was used to eliminate 'stick-slip' during the extrusion of the tag.

It was found necessary to use a product straightening device in order to keep the tag on the centre line of the drawing unit. This consisted of a guide bush which screwed into the back of the die retaining plug approximately 6 in. from the die.

When the tag had been extruded, the oil pressure in the extrusion container rose to its maximum value of 112,000 lbf/in^2. From inspection of the billets taken from the extrusion container at this point in the procedure, the die only made contact with the change in section of the billet over a short length. The remainder of the truncated section was clear of the die face, since its included angle was smaller than that of the die. This geometry allowed the drawing load to be applied without a peak initial load at the beginning of the second phase of extrusion.

The carriage of the drawbench was returned to the beginning of its working stroke and the tag gripped by the pneumatically operated grips. The drawing load was applied and extrusion re-commenced. In the early stages of these trials this was done at a high drawbench speed but this frequently led to a snatching action which often fractured the tag. Later this was overcome by applying the drawload at a low speed setting and changing the speed to its high value by manual control to reduce cycle times. This procedure was completely successful and overcame the problem of premature tag fracture.

To test the controllability gained over the extrusion rate, the speed was reduced for a second time, by means of a limit switch operated by the position of the carriage. It was found possible to vary the speed during extrusion over a wide range. A typical speed change was from 25 to 1.25 ft/minute without producing any form of instability.

Two observations were made during the change in speed. The drawing load always reduced with the speed and a peak draw load was obtained when changing from the low to high speeds. This was attributed to the characteristics of the hydraulic control on the drawbench power peak, but it did prevent speed change when the drawing stress was large. Extrusion was terminated by a limit switch which was positioned to prevent complete extrusion of the billet.

Figure 50 shows a comparison between the oil pressure required to extrude mild steel bar with the sum of oil pressure and drawing stress required during product augmented hydrostatic extrusion. These results were obtained when the extrusion ratio achieved over the tag section was 1.5 : 1. In every case the lowest drawing stress, obtained at the low drawing speed, was taken. It was noticed that the oil pressure in the extrusion container fell to approximately 100,000 lbf/in^2 during augmented extrusion, this was attributed to seal friction.

The procedure for extruding a tube by product augmented hydrostatic extrusion was similar to that described above. The rear end seal, described in section 3, was used throughout to seal the bore of the billet with the mandrel.

Mandrel fracture occurred when extruding tube in some of the hard materials. This was ~~partly~~ due partly to the difficulty of lubricating the bore effectively and to the combined effect of excessive stress induced in the mandrel and high contact pressure over the deformation zone. It appeared that the combined action of these loads reached a maximum value just inside the deformation zone, since mandrel failures always occurred at this point. The fractured surfaces showed evidence of low cycle fatigue.

5.7.h. PRODUCT AUGMENTED HYDROSTATIC EXTRUSION - COMPLETE EXTRUSION BY THE PRESSURE PHASE-OUT METHOD

The response of the pressure phase-out equipment was investigated by carrying out a series of decompression tests. These involved the removal of the oil pressure from the extrusion container by injecting low pressure oil into the annular chamber of the 'HYDROSTAT' from the separate accumulator with the extrusion container sealed by a solid die.

This was repeated for several flow value settings. Figure 51a shows the results of these tests for three arbitrary value settings.

During extrusion no difficulty was experienced in phasing-out the oil pressure in the extrusion container when the rear end of the billet was tapered. Most of these tests were carried out with included rear end tapers of 2 degrees and metering valve settings of 6.5. Figure 51b shows a typical oil pressure and drawing load - time records from one of these tests.

Some trials were carried out to investigate the feasibility of using the pressure-phase out method when the billets had square ends and simple cone dies were used. The difficulties associated with this are discussed in the Appendix, section 8.2. Figure 52 shows the results of these trials for different metering valve settings, all were unsuccessful.

As discussed in the Appendix, phase-out under these conditions requires an ever increasing rate of decompression. The characteristics of the phase-out equipment gave a pressure-time characteristic in which the rate of decompression fell, the effect was to increase the rate of reduction of the draw load in the latter stages of extrusion, which completely defeated the response of the equipment.

5.8. SAFETY PRECAUTIONS

No account of work on hydrostatic extrusion would be complete without mention of the safety precautions adopted.

Hydrostatic extrusion is potentially dangerous because of the high oil pressure involved, and the large amount of energy stored in the oil due to compressibility effects.

On the 200/50 'HYDROSTAT' a two guard system was used. This consisted of a mild steel guard around the high pressure section of the press and a personnel protection barrier between the operator and the press.

The press guard completely surrounded the high pressure section and was firmly bolted to a rigid frame. Several high pressure stop valves were incorporated in the press. These were positioned inside the press guard and never manually operated when subjected to pressure. When operation under pressure was necessary a remote controlled servo-mechanism was used.

The control consul was positioned to the rear of the press, to give an adequate view of the drawing unit and the pressure gauges mounted close to the press guard. The personnel protection barrier was rigidly fastened to the walls and floor immediately in front of the control consul. It was made from $\frac{3}{8}$ in. thick mild steel plate bolted to an angle iron frame and included three 1 in. thick armour plate glass windows. The size of the barrier was determined by considering all the possible trajectories the products could take from the extrusion area.

These physical precautions were backed by regulations which limited the movement of experimental staff both within, and to and from the laboratory when high pressures were developed. The experimental staff were forbidden to enter the press area whilst pressure greater than 5000 lbf/in^2 were developed. This limit was fixed to allow the use of the tool setting valve whilst operators observed the press with the guard doors open. Doors leading to and from the laboratory were locked when high pressures were developed. External warning lights were fitted to indicate the danger.

An alarm system was installed which could be switched on from a number of points close to control units of the various items of equipment. The alarm set off horns in offices and laboratories close to the extrusion laboratory and several key personnel were advised as to the action necessary should the alarm be switched on.

6. DISCUSSION

6.1. THEORETICAL ASPECTS

6.1.a. ACCURACY OF THE ASSUMPTIONS MADE IN THE APPARENT STRAIN METHOD

The apparent strain method was developed in an attempt to deal with axi-symmetric problems and work hardening materials. The best approach existing when this work was initiated, flexible enough to permit application to augmented hydrostatic extrusion, was that developed by Pugh (9). It was decided to base the apparent strain method in that of Pugh's and at the same time review some of the assumptions made. This led to the assumptions listed in section 4.2., and discussed now in the light of existing evidence.

Assumption (a) states that, the presence of small frictional effects between the tools and the work piece has negligible effect on the plastic work done. This implies that the surface shear stress induced by the presence of friction does not effect the mode of plastic flow within the work piece. This is obviously only true when the surface shear stress is small. As this is the situation with hydrostatic extrusion the assumption may be considered valid. With large frictional effects the surface of the deformation zone will be sheared to an extent which affects the mode of deformation deep within the work piece and will, therefore, affect the plastic work done. This limitation may prevent the more general application of the apparent strain method.

Assumption (b) states that, the plastic work required depends only on the geometry of the forming operation and the flow characteristics of the material, but not on the process employed to do the plastic work. This implies that the plastic work done during hydrostatic extrusion is not affected by the addition of augmentation. This is only strictly true when the magnitudes of the augmenting stresses are small and elastic in order of magnitude.

The important exception is in the case of billet augmentation when the stress is plastic order of magnitude and billet swelling and work hardening takes place, the application of augmentation directly increases the plastic work done on extrusion through the die. It is for this reason that billet augmentation is given special treatment. This treatment divides the analysis into two parts, the swelling of the billet on application of augmentation and the reduction of the expanded billet in the die. The first is treated by considering compression of the billet to be homogeneous and the second by considering the flow of the metal through the die to obey the assmuptions made in the apparent strain method. This approach allows the apparent strain method to be applied to this special case.

When billet augmentation is dealt with in the way described, assumption (b) is then accurate in all cases to the first order of magnitude.

However, some second order effects have been observed which limit its general validity. When a billet is extruded by simple hydrostatic extrusion it was noticed that the inlet edge of the deformation zone was well rounded. This was less pronounced when the billet was extruded using billet augmentation and indicated some small changes in the mode of deformation.

During product augmented hydrostatic extrusion of some non-circular sections it was observed that the corners of the cross sections were rounded. This rounding increased with the magnitude of the product augmenting stress and altered the extrusion ratio.

Some earlier work had been undertaken to obtain Moire interference patterns on the diametral plane of some copper billets extruded by simple hydrostatic extrusion. These indicated the existence of deformation well outside the zone enclosed within the die and also they indicated a broad band of deformation around the outer edge of the deformation zone. This latter band approximated to the single shear plane assumed in the Siebel model of deformation. The deformation which takes place in these areas and the plastic work done must be effected to some extent by the shape of the equivalent stress-strain curve.

As agreement between the apparent strains obtained by the use of different materials was good, it is concluded that the effects of the phenomena described above are small and can be neglected when determining plastic work. Also, good agreement was obtained when comparing the oil pressure obtained by simple hydrostatic extrusion with the sum of oil pressure plus drawing stress induced by product augmented hydrostatic extrusion of round bar. This shows the equivalence of the plastic work done.

Assumption (c) states that, the plastic work done by a non-homogeneous mode of deformation on a work hardening material is proportional to the area under the equivalent stress-strain diagram over the strain range equal to the mean equivalent strain induced. Hydrostatic extrusion is a forming method by which the accuracy of this statement can be assessed. This arises since the frictional effect produced by the well lubricated simple hydrostatic extrusion of round bar is so small that it can be neglected. Under these circumstances the oil pressure is a direct measure of the plastic work done, see Pugh's mean equivalent strain method (9). The results obtained by simple hydrostatic extrusion are in good agreement with each other and adequately justify this assumption.

Assumption (d) states that, the work done against friction can be estimated from this work of sliding based on the mean die pressure, mean coefficient of friction and the distribution of the velocity of sliding.

The method of determining the frictional work from a knowledge of the contact pressure, the actual coefficient of friction and the velocity of sliding is a precise method. However, the exact magnitude and distribution of the factors involved are not known and it will not, of course, give any additional plastic work done as a consequence of the presence of the surface shear stress.

Only the mean contact pressure can be found by the apparent strain method. The use of this value leads to inaccuracies.

It has been assumed throughout that the coefficient of friction is constant across the deformation zone and that it only depends on the materials that make up the sliding surfaces and the lubricant. This obviously over simplifies the real situation. During extrusion the surface of the work piece is elongated, which reduces the thickness of the lubricant layer. This is bound to increase the coefficient of friction for sections close to the die throat and for large reductions. Figures 35 and 37 show that when the die angle is small the extrusion pressure increases but the ultimate tensile strengths of the products do not show the same increase, see Figures 36 and 40. The increase in extrusion pressure must, therefore, be produced by an increased frictional effect. This is probably due to the much increased area of contact between the billet and the die, at small die angles.

Little is known about the velocity of sliding. In Pugh's mean strain method the velocity of sliding is taken to be the mean velocity at the section considered, whereas, in the apparent strain method it is taken to be the cosine component of the mean velocity, to be compatible with the models of deformation. Some simple measurements were undertaken to determine the accuracy of these assumptions by measuring the spacings of circumferential lines over the surface of the deformation zone, Figure 53. These lines were machined into the surface of the billet and were initially equally spaced. The mean velocity mid way between two of the lines was found, as a ratio of the billet velocity, by dividing the spacing by the initial pitch of the lines. This type of test indicated that near to the inlet section of the die the distribution agreed with Pugh's mean velocity distribution but near the die throat the cosine component was a better estimate. This can be explained by the fact that deformation at inlet to the deformation zone does not consist of a single shear plane and, therefore, there is no discontinuity in the surface velocity. Such a shear plane does exist at the outlet section of the deformation zone, therefore, a discontinuity in surface velocity also exists and the cosine component of the mean velocity is the better estimate in the outlet region.

To summarise these effects, all the three relevant parameters are over-estimated near the inlet to the deformation zone and underestimated near the outlet, this leads to inaccuracies.

Assumption (e) states that, in tube forming processes the mean pressure between the mandrel and the deformation zone is equal to the mean die pressure. This is a commonly held assumption but no attempt was made to check its validity with respect to the applications considered. It is probably only true when the wall thickness and die semi-angle are small.

To make a completely theoretical prediction by the apparent strain method it is necessary to determine the mean induced equivalent strain analytically. This has been done by considering the upper and lower bounds for some simple models of deformation. The results showed the upper bound estimate to be the most accurate. For bar extrusion the Siebel model has been shown to give a result which compares very favourably with the experimental results. From the investigations carried out into the distribution of the velocity of sliding and the mode of deformation it was evident that the real mode of deformation was very much more complex than that predicted by the Siebel model. Its accuracy when compared with experimentally induced strains must therefore be considered somewhat fortuitous. The model devised for tube extrusion appeared to lead to results which were reasonable by comparison with the experimental results but the existence of an increased frictional effect made precise comparison much more difficult. In the upper bound solutions for both bar and tube the effect of shear between the elements of material flowing between the inlet and outlet shear planes were estimated. The results show that over the die angle range considered the effect of the shear distribution is very small and can be neglected.

The most interesting features of the apparent strain method is its adaptability to be modified in the light of more accurate information. For example, if a more realistic model of deformation was devised, its plastic strain predictions could be included without affecting the rest of the procedure. It is likely that further, more detailed experimental work will reveal better estimates of the distributions of the velocity of sliding and contact die pressure. This information could be inserted into equation (27) and a better estimate of the frictional strain made.

6.1.b. COMPARISON OF ANALYTICAL METHODS

Although Sachs'(2) solution for extrusion is only of historical interest it is interesting to compare it with the apparent strain approach ~~solution~~, when the latter is simplified according to the assumptions made by Sachs. These simplifying assumptions are that deformation is homogeneous and that the metal being formed is non-work hardening, giving

$$\bar{\epsilon}_m = \ln(R)$$

$$\beta = 1.0$$

Figure 54, shows this comparison, the predictions of extrusion pressure are in very good agreement. The maximum difference over the range considered is only 10% and for small values of reduction and coefficients of friction agreement is very much better. It is not possible to compare the mean die pressures, since Sachs' solution determined the die pressure at every section in accordance with the plasticity condition whilst the apparent strain method only gives the mean value.

The Hill and Tupper's (6) method of accounting for friction is very important since it forms the basis of the theories which follow. However, their statement that the mean die pressure is independent of the coefficient of friction needs some further comment. It was originally made in an analysis of sheet drawing, their conclusion was that for sheet drawing, (the maximum reduction for which would not exceed R = 2, $\ln(2) = 0.6931$) and when the coefficients of friction were less than 0.1 the die pressure was independent of the coefficient of friction. In the apparent strain theory for hydrostatic extrusion this assumption is not held to be generally valid, yet for extrusion it agrees with Hill and Tupper's observations over the range they specified in their work on drawing. It is unrealistic to hold this assumption valid at all values of reduction and all values of the coefficient of friction.

Siebel's (8) analysis of wire and bar drawing was the first theory to include a realistic redundant strain factor and subsequent work has shown his simple model of deformation accurately predicts the work contribution made by redundant shearing. The main shortcoming of this theory lies in the assumption that the mean die pressure is equal to the yield stress and independent of both the reduction and the coefficient of friction. Although this assumption led to a simple solution it is nevertheless unrealistic. The effect of this assumption is to reduce the dependence of the forming load on the coefficient of friction. This is because the yield stress is an under-estimate of the mean die pressure and, hence, the predicted work done against friction is too small.

Pugh's work on the analysis of hydrostatic extrusion represents an important step forward in analysing the deformation of metal in a conical die passage. It is felt that in the light of this work on apparent strain, his literal interpretation of Hill and Tupper's statement on the complete independence of the mean die pressure on the coefficient of friction leads to some errors in accounting for the effects of friction. This criticism, however, does not invalidate his subsequent analysis of the hydrostatic extrusion of round bar since this metal working process is almost frictionless and assumptions on the dependence or otherwise of the mean die pressure on the coefficient of friction are irrelevant. In addition, the form of his solution is a little inconvenient since it requires graphical integration of the equivalent stress-strain diagram.

It is interesting to observe that the apparent strain theory predicts an increase in the mean die pressure with increase in the coefficient of friction. There are some exceptions to this, if the die angle and the work hardening factor are small, the apparent strain theory predicts a small decrease in mean die pressure with increase in the coefficient of friction when the reduction is small. This was predicted by Hill and Tupper (6). Lancaster and Rowe (4) during their work on sheet drawing measured the mean die pressure and the coefficient of friction at moderately large reductions by using the split die technique. These results show that the mean die pressure does increase with an increase in the coefficient of friction. Important observations in favour of the method of treating frictional effects adopted in the apparent strain theory.

6.1.c. THEORETICAL PREDICTIONS

The frictional factors for several important cases were determined and are shown in Figures 6 and 8. These enable a completely graphical solution to be made and allow the die angle and the work hardening factor to be taken into account. These figures show that for all values of the coefficient of friction the frictional effect increases with the extrusion ratio. Also, it is shown that the frictional effect for a given coefficient of friction is greater for tube than bar extrusion, obviously due to the increased area of contact between a tube and its tools over that which occurs during bar extrusion.

During proportional augmented hydrostatic extrusion of tube the different mandrel designs have an influence on the magnitude of the frictional factors. This is due to the different relative velocities produced between the bore and the mandrels. In practice the differences are likely to be larger than those indicated since the different relative velocities were likely to produce additional changes in the coefficient of friction. This effect is ignored in the predictions made and the coefficients of friction is considered common to all mandrel designs.

The mean equivalent strains induced by both bar and tube hydrostatic extrusion were determined using the models of deformation and the upper bound method. These are shown in Figures 9a and b. It is seen that the mean equivalent strain increases with the extrusion ratio and the included die angle. The latter parameter primarily increases the redundant component of the strain hence a family of parallel mean equivalent strain lines are obtained, one for each die angle considered.

By comparison of the predictions made for bar and tube, at a common die angle, it can be seen that the mean equivalent strains induced for these product shapes are very similar. This is due to the similarity between the models of deformation considered, but the trend supports the commonly held view that the shape of the product has little effect on the mean equivalent strain. In the case of tube extrusion the accuracy of this will probably depend on the billet diameter ratio. With thick walled tubes, such as those used in this work, the mode of deformation will be similar to that induced by bar extrusion. For thin wall tube this is unlikely to be true and the mode of deformation will approach that induced by plane strain extrusion. Hence, the equivalent mean strain predictions for tube can only be expected to apply to the extrusion of thick walled billets.

One of the most interesting aspects of the theoretical work is its application to the optimisation of augmented hydrostatic extrusion. In this work attempts were made to predict the maximum extrusion ratio obtainable for a given oil pressure and material, taking into account the instabilities which occur when augmentation is taken to its limit.

Figure 12 shows the estimates of the oil pressure required to extrude 20/25/Nb stainless steel bar, by proportional billet augmentation for various augmenting ratios. If billet swelling and work hardening could be ignored the oil pressure required would be reduced as the augmenting ratio increased and would be a linear function with the natural logarithm of the extrusion ratio. The figure shows that the effects of swelling and work hardening are very significant and that at high reductions, in excess of an extrusion ratio of 2.5 : 1, these effects limit the augmenting ratio which can be used with benefit. Figure 13 shows theoretical curves describing the same effect when extruding 20/25/Nb stainless steel tube by proportional billet augmentation induced by a billet-fixed, travelling mandrel with a solid head. In this form of extrusion the augmenting ratio is fixed by the billet radii ratio. With thin billets a higher augmenting ratio is produced than with thick billets. The combined effect of billet swelling, work hardening and increased augmenting ratio with thin billets, leads to instability at low extrusion ratios. This is shown as a region of unstable extrusion in which it is impossible to extrude by this method. This indicates that a billet-fixed, travelling mandrel can only be used with benefit on billets which have a large radii ratio. This limits its application to the initial stages of tube manufacture.

The effect of friction at the bore of the deformation zone on swelling and work hardening during proportional billet augmentation has been discussed already. Figure 46 shows predictions of the oil pressure needed to extrude hollow billets of 20/25/Nb stainless steel with a diameter ratio of 1.455 for different values of bore friction. It shows that in this particular case, in which the wall thickness is small, the frictional effects are very significant. The practical conclusion to draw from this result is that bore lubrication is very important with this design of mandrel.

Figure 55 shows the estimate of the maximum extrusion ratio obtainable by billet augmented hydrostatic extrusion obtained by using the approximate method of determining the point of instability , which neglects the work hardening effect on the mean flow stress. This is largely an experimental result, but is is interesting that the maximum extrusion ratios obtainable with steels, which differ considerably in strength, lie in a narrow range. This supports the conclusion already stated that high strength materials can be augmented to high stresses with considerable benefit from the work input point of view.

Figure 14 shows the predictions of the oil pressure required to extend 20/25/Nb stainless steel tube by proportional product augmented hydrostatic extrusion. As expected the oil pressure needed to obtain a given area reduction decreases with the billet diameter ratio, brought about by the increase in the augmenting ratio. Predictions based on the steady state conditions show that a large reduction is possible at a relatively low oil pressure and the oil pressure is lower than that needed by any other mandrel design for a given reduction. The former prediction needs some clarification. During the extrusion trials with this mandrel, difficulty was experienced in successfully starting extrusion when large reductions were attempted. In many cases the oil pressure would rise well above that predicted and result in the release of the mandrel, by causing the front end of the billet to fail in tension. It is likely that this is due to the absence of product augmentation at the early stages of extrusion. During the early stages is is most likely that the mandrel is temporarily fixed to the rear end of the deformation zone and not to the product. If this is so extrusion begins by billet augmentation. This is further supported because a higher oil pressure is required when billet augmentation is induced than that needed during product augmentation. Assuming extrusion begins in its manner suggested, work piece failure will occur as soon as the front end of the mandrel passes through the die throat and product augmentation is established. This failure would be caused by the excess oil pressure causing failure of the product in tension and would result in the release of the mandrel. The effect of these starting conditions is to reduce the maximum extrusion ratio obtainable by the method. The prediction of this maximum extrusion ratio is shown in Figure 14 and was obtained by considering the oil pressure needed to cause product failure and subsequently the maximum reduction this oil pressure could achieve by proportional billet augmentation. This involved a step by step calculation until the result was obtained to the required degree of accuracy.

Figure 11 shows the effect of billet augmentation, in which the augmenting stress is independent of the oil pressure, on the extrusion of 20/25/Nb stainless steel bar. In this form of extrusion billet swelling and work hardening will only occur on application of augmentation after the maximum oil pressure has been reached. It shows that this form of extrusion can increase the maximum extrusion ratio obtainable for a given oil pressure, but that diminishing returns are obtained as the magnitudes of the augmenting stress is increased. It shows further the obvious conclusion that a higher maximum reduction is obtained when the oil pressure is increased.

Figure 56 shows the estimate of the maximum extrusion ratio which is obtainable by product augmented hydrostatic extrusion of steel tube. It is assumed that the tag is extruded by the sole action of the oil pressure before augmentation is applied and that an extrusion ratio of 1.6 : 1 is achieved over the tag section. The maximum augmenting stress is taken to be the flow stress of the material as it leaves the die during the extrusion of the tag. It is seen that the maximum extrusion ratios obtainable lie within a narrow range, in spite of the differing strengths of the materials considered. This result is similar to that obtained by considering the limits for billet augmented hydrostatic extrusion but shows that a slightly larger reduction is possible by product augmentation, other factors being equal.

One of the interesting features of the apparent strain theory is that it clearly indicates that when friction is present the extrusion pressure and mean die stress are reduced when some of the work is done by product augmentation, see equation 30. There are however, other practical considerations which tend to eliminate this advantage, these will be discussed in the next section.

In section 4.6 the use of the apparent strain method to determine process efficiency in terms of an extrinsic factor is discussed. This has been used to predict the extrinsic factor for the simple hydrostatic extrusion of both bar and tube and also the extrinsic factors obtained at a common reduction but differing die angles for tube extrusion, see Figures 57 and 58. The higher the extrinsic factor, the lower the efficiency and the more 'unnecessary' work is done. Figure 58 shows clearly that the extrinsic factor is made up of redundant shear work and work against friction. It shows also that at small die angles the extrinsic factor increases due to an increased frictional effect. This increased effect occurs at all values of the coefficient of friction.

6.1.d. COMPARISON BETWEEN EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS

Figure 29 shows the comparison between the apparent strains obtained by the simple hydrostatic extrusion of round bar and from the theory. It is seen that the experimental points are consistent with each other which indicates the uniqueness of the apparent strain. The results obtained from mild steel are consistently higher than those obtained with other materials, this is due to the use of Evo-stik as a lubricant which from practical experience is believed to give a higher coefficient of friction. The figure also shows that the mean coefficient of friction is very small and approximately 0.01 for mild steel coated with Evo-stik and is negligible for stainless steel coated with an oxylate coat and soap. These coefficients are much lower than those obtained from other methods of forming, such as drawing, which emphasises the assistance given by the pressure transmitting oil to lubrication.

Figure 33 shows a similar comparison for the simple hydrostatic extrusion of tube with the same materials and lubricants. In every case a higher mean coefficient of friction is indicated than that obtained by bar extrusion. It should be made clear that this could arise because of inaccuracies in the estimates of the mean induced strain or real increases in the coefficient of friction. Some evidence exists which suggest that the latter factor is the most significant in accounting for the apparent increase. There was a tendancy for the bore to 'pick up' with the mandrel, this indicated that the metal surfaces at the bore were coming into contact. This leads to higher coefficients of friction. The figure shows that the mean coefficients of friction were 0.010-0.015 for mild steel lubricated with Evo-stik and 0.005-0.010 for stainless steel lubricated with an oxylate coat and soap.

Experimental evidence to support the theoretical prediction that the equivalent extrusion pressure is increased when proportional billet augmentation is used is shown in Figure 45. It is seen that a significant increase in the extrusion pressure is obtained even though the augmenting ratio of 0.477 is small.

Figure 46 shows a similar result for the higher augmenting ratio of 0.87. It clearly shows that the effect of friction is very significant in this form of extrusion. The theoretical predictions were based on a coefficient of friction of 0.005 in the outer surface and various coefficients at the bore, expressed as a ratio of the coefficient acting on the outer surface. Comparison shows that the coefficient of friction at the bore was within the approximate range 0.025-0.035. As the pressure transmitting oil was excluded from the bore the lubricating conditions were the same as that which exists during conventional drawing. This result is in good agreement with the coefficients obtained, by other workers, by drawing.

Figure 48 shows a comparison between some of the results obtained by proportional product augmented hydrostatic extrusion of tube and theoretical predictions. From the results which are available it is seen that good agreement is obtained and that a coefficient of friction between 0-0.010 is obtained with stainless steel and an oxylate-soap lubricant. This result agrees well with that obtained by simple hydrostatic extrusion.

Figure 49 shows a comparison between the oil pressure required to extrude AISI 316 stainless steel tube by means of three principal mandrel designs; it also shows the theoretical predictions based on the apparent strain method, taking a common mean coefficient of friction of 0.005. It is seen that agreement is excellent. This result supports the work approach adopted in the apparent strain method.

Figure 50 shows a comparison between results obtained by both simple and product augmented hydrostatic extrusion for mild steel bar. As discussed in the previous section the apparent strain method indicates that when friction is present the extrusion pressure is reduced when some of the work is done by product augmentation. However, this figure does not show this decrease. There are two factors which are relevant to this, namely, the low value of the coefficient of friction, approximately 0.010-0.015, and the difference in extrusion speeds. It is considered that the latter point is very important since the extrusion speeds used during simple hydrostatic extrusion were high, 3 ft/sec max., and as a consequence it is likely that hydrodynamic lubrication, see section 8.1., was achieved, whilst the extrusion speeds used during product augmented hydrostatic extrusion were much lower, 2 in/sec max. At this low speed it is unlikely that hydrodynamic lubrication would occur. This difference in extrusion speed is fundamental to the processes considered, since high speeds are used with simple hydrostatic extrusion to give a smooth extrusion whilst the speed used during product augmented hydrostatic extrusion is limited by the speed of the drawing unit. The practical outcome of this difference is that the coefficient of friction is likely to be higher with product augmented hydrostatic extrusion than with simple hydrostatic extrusion. It is this increase in the coefficient of friction which is tending to eliminate the predicted fall in the extrusion pressure with product augmented hydrostatic extrusion.

Figure 57 shows the extrinsic factors for the simple hydrostatic extrusion of both bar and tube. It is seen that for bar extrusion the extrinsic factor is constant as the reduction is increased whilst for tube extrusion the extrinsic factor increases with reduction. As discussed in section 4.6. the extrinsic factor increases with reduction when friction is present. This again supports the findings that the mean coefficient of friction for tube extrusion is higher than that which occurs during bar extrusion and that bar extrusion of stainless steel which is lubricated with an oxylate and soap lubricant is almost frictionless. The constant extrinsic factor for well lubricated bar extrusion was found to be between 0.06-0.08, this compares well with that obtained from the Siebel method.

The extrinsic factor is a useful way of illustrating the effect of die angle on the unnecessary work done, see Figure 58. It is seen that the theoretical predictions show that the minimum which occurs in both the extrinsic factor and the extrusion pressure is produced by frictional effects. This has been substantiated by tensile tests carried out on the products. It is also an explanation of the fact that the minimum occurs over an included die angle range of 10-20° which is much lower than is obtained from other methods of forming and obviously due to the low coefficient of friction produced by hydrostatic extrusion. By comparing the theoretical results with those obtained by experiment it is seen that they both follow a similar trend but the points of minimum extrinsic factor differ. Correlation is however, sufficiently good to show it is probable that the actual coefficient of friction increases as the die angle decreases.

6.2. TECHNOLOGICAL ASPECTS

6.2.a. SIMPLE HYDROSTATIC EXTRUSION

Simple hydrostatic extrusion is the most straight forward method attempted. The 200/50 'HYDROSTAT' was the first press to be built specifically to carry out hydrostatic extrusion. It represented a considerable improvement over the methods which had previously been used, which consisted of special tool sets in conventional hydraulic presses.

It is considered that the cross-bore in the die block of the 200/50 'HYDROSTAT' represents a weakness in the high pressure system, particularly from the fatigue point of view. However, the cross-bore does allow an 'out-of-line' arrangement, with the press ram acting at right angles to the extrusion container. This is very convenient and may prove an essential feature of a press specifically designed to extrude steel tubes; this point is discussed further in section 6.2.e. It may be possible to minimise some of the disadvantages of the cross-bore by improved design or by pressure supporting the highly stressed regions of the die block.

The list below shows some of the important features of simple hydrostatic extrusion.

<u>Advantages</u>

I Permits the extrusion of long billets.

II Allows complete extrusion of the billet.

III Reduces frictioned resistance.

IV The equipment is simpler than that required to achieve augmented hydrostatic extrusion.

Disadvantages

I Small extrusion ratios are obtained for a given oil pressure.

II Extrusion is prone to the 'stick-slip' mode of extrusion.

III A large amount of energy is stored within the pressure transmitting oil which is released on complete extrusion.

It is seen that the advantages offered by simple hydrostatic extrusion are very attractive. The ability of the press to extrude long billets gives it considerable advantage over conventional extrusion, in which the billet length - diameter ratio is limited to approximately 6:1. However, the disadvantages offered by simple hydrostatic extrusion are important limits to its field of application. In order to obtain extrusion ratios which are commercially attractive when extruding steel, it is necessary to employ very high oil pressures. Pressures as high as 200 tonf/in^2 are proposed by those who advocate the further development of simple hydrostatic extrusion. From work which is currently being carried out on the fatigue life of pressure vessels it is clear that operation at this high oil pressure will be extremely difficult. In addition to the limits imposed on the extrusion ratio the susceptibility of the process to the stick-slip mode of extrusion is also limiting. This has been overcome with the 200/50 'HYDROSTAT' by the use of high extrusion speeds, but this has led to speeds which are undesirably high when steel is extruded. Proposals have been made by other centres of development to eliminate stick-slip by the use of damping. This is believed to be effective but inevitably leads to extra complication. The presence of large amounts of stored energy in the pressure transmitting oil makes complete extrusion unattractive. It is considered that it is better to leave a discard at the end of extrusion and decompress the oil by withdrawal of the press ram. This leads to a controlled cycle but it does eliminate one of the obvious advantages of simple hydrostatic extrusion.

It was because of these difficulties with the simple process that development of augmented hydrostatic extrusion was undertaken. The aim of this development was to attempt to retain the advantages of the simple process whilst eliminating the disadvantages discussed above, see section 6.2.d.

The techniques outlines for the simple hydrostatic extrusion of both bar and tubes were effective. In view of the high speeds which were necessary to achieve a steady mode of extrusion it was necessary to ~~completely~~ extrude the billets and use a catching device to retard them after extrusion, as previously discribed. The high extrusion speeds made it impossible to terminate extrusion satisfactorily in a controlled way and therefore extrusion to a discard was not possible.

The main process parameter for simple hydrostatic extrusion are extrusion ratio and die angle. It was found that for the materials investigated the extrusion pressure increased with the logarithm of the extrusion ratio. This shows that from the area reduction point of view an ever increasing benefit is obtained by using higher oil pressures.

The effect of die angle was investigated in detail and the results are of great interest. It was found that the extrusion pressure was dependent on the die angle and that a minimum pressure was obtained at some unique die angle, at all reductions. The die angle which gave the minimum oil pressure differed from one reduction to another. From the results available the optimum die angle can be related to the extrusion ratio thus

$$(2\alpha)_{\text{OPTIMUM}} = 9.5 + 14.2(R-1)$$

when $1 < R < 4$

This expression is approximately valid for both bar extrusion and the extrusion of thick walled tube. For die angles smaller than the optimum it has been shown that the oil pressure increases as a result of an increased frictional effect.

From mechanical tests carried out on products produced by extrusion through dies of different angles it was shown that the ultimate tensile strength increased whilst the percentage elongation decreased over the range of die angle investigated. This is brought about by an increase in the redundant work as the die angle is increased.

The comparison between the tool and product sizes showed that the differences which existed also depended on the die angle. The differences between the outside diameter of the tube and the die size increased with the die angle up to the optimum angle and then decreased, becoming negative when the included die angle was in the range 35-45 degrees at an extrusion ratio of 1.43 : 1. The difference between the bore size and the mandrel diameter showed the opposite trend and became progressively more negative as the optimum die angle was approached.

From hardness tests carried out on the surfaces of tubes produced by extrusion through dies of different die angle it was shown that large differences were produced between the outer and inner surfaces, when the die angle was large. This indicates that the redundant work is greater at the outer than at the inner surface, at these die angles.

6.2.b. HYDROSTATIC EXTRUSION OF TUBE WITHOUT A MANDREL

This was successfully achieved by sealing either the front or rear end of the billet. Both methods were very simple and they achieved considerable reduction in the bore diameter. In every case this reduction was associated with wall thickening.

The method in which the front end was sealed was shown to be applicable to relatively thin walled tubing, see section 5.7.c. If a reduction in diameter was attempted in which the work piece was over stressed, the product failed by bursting. This was associated by the release of the energy stored in the pressure transmitting oil. The energy release was never very great since the oil pressures induced were less than 25,000 lbf/in^2, but if this method is used in a production environment this potential danger would have to be guarded. It was also possible to completely extrude the tubes by this method and retard it by a catching tube. However, it was found that the extrusion speed could be very small. This means that extrusion could be easily terminated to leave a discard if this is considered desirable. The deformation induced by this type of extrusion was similar to that produced by conventional sinking and the usual defect of bore wrinkling was produced.

The method in which the rear end of the billet was sealed, was shown to be applicable to only thick walled tubes, since the pressure transmitting oil was tending to collapse the billet, see section 5.7.d.

With the trials in which the diameter reduction was increased at a common die angle it was shown that the logarithm of the extrusion ratio was only 1.5 : 1 at the greatest diameter reduction ratio obtainable. The practical significance of this is that the elongation produced during extrusion is quite small and that long extrusion containers are needed if long products are to be produced. Some of the difficulties associated with very long containers are discussed in section 6.2.e.

The trials in which the diameter reduction ratio was constant whilst the die angle varied showed that for the particular size of billet used the wall thickening was approximately constant. It is, however, realised that this result depends on the billet diameter ratio and that it is not necessarily true for all billet sizes.

6.2.c. HYDROSTATIC EXTRUSION WITH TRAVELLING MANDRELS

This type of hydrostatic extrusion was shown to produce proportional augmented hydrostatic extrusion and a larger extrusion ratio than simple hydrostatic extrusion for a given oil pressure. It would seem that this form of extrusion ought to give a greatly increased extrusion ratio for a given press capacity. Although larger reductions were achieved than had been obtained by simple hydrostatic extrusion it did not give the magnitude of increase expected. This was due to the instabilities associated with the application of the augmenting stresses, see sections 4.5.a., 5.7.e. and 6.1.c.

The principal advantage of proportional augmented hydrostatic extrusion is that it can be achieved in what is otherwise a press for simple hydrostatic extrusion. This arises since it is produced by special mandrels which do not require modification to the press.

The billet-fixed, travelling mandrel which incorporated a solid head was very easy to use since it reduced the technique of tube extrusion to the simpler technique used for bar extrusion. This arose because the mandrel was self seating. One of the limitations of this mandrel is that the application of a billet augmenting stress necessitates short thick walled billets. This means its application is limited to the initial stages of tube manufacture. When the pressure transmitting oil is excluded from the bore by the solid mandrel head lubrication problems can arise. It follows that particular care is needed to ensure that the lubricant in the bore of the billet is adequate. Another limitation is associated with this lubrication problem and is due to the excessive frictional load induced in the shank of the mandrel when the latter is parallel. This obviously increases during extrusion and can produce mandrel failure.

Even if failure does not occur it will make the withdrawal of the mandrel difficult after extrusion. This can to some extent be overcome by having a tapered mandrel shank, but this produces a tube with a tapered bore, which in some applications cannot be tolerated.

The limitations which exist with the product-fixed, travelling mandrel have already been discussed in sections 4.5.b., 5.7.f. and 6.1.c. The limitations are associated with the difficulty of obtaining the correct form of augmentation in the early stages of extrusion. This leads to a limited maximum extrusion ratio and the tendency for the mandrel to be released under pressure. In view of these limitations it is unlikely that this mandrel design will find general application.

6.2.d. AUGMENTED HYDROSTATIC EXTRUSION

As previously stated, augmented hydrostatic extrusion was developed in an attempt to overcome some of the undesirable features of simple hydrostatic extrusion. Augmented hydrostatic extrusion achieves this by assisting the oil pressure with a direct load applied either to the product or the billet. The main beneficial effect of this is that the stick-slip mode of extrusion is prevented and the extrusion rate is directly controlled. Also, with the load applied to the product it is possible to completely extrude the billet in a controlled way so that the energy stored in the pressure transmitting oil is not released but is returned to the gas accumulator, see sections 5.1.a. and 5.7.b. These features go a long way to overcome the main defects of simple hydrostatic extrusion. However, the augmented hydrostatic extrusion processes are more complex.

There is sufficient difference between product and billet augmented hydrostatic extrusion to make comparison of interest. The following table shows this comparison.

Feature	Billet Augmentation	Product Augmentation
Point of application of the augmenting load	Billet	Product
Length-diameter ratio of the billet	Less than 6:1 *	Unlimited
Maximum extrusion ratio at an oil pressure of 50 T.S.I.	3.5 - 4.5 (For steel tube)	4.0 - 5.0 (For steel tube)
Method of terminating extrusion	With discard	With discard or by complete extrusion
Billet preparation	Nosing	Nosing and tagging
Complexity of high pressure system	Complex	Simple
Complexity of other items of equipment	Simple	Complex

Table No. 3. Comparison between the features of billet and product augmented hydrostatic extrusion

* Semi continuous hydrostatic extrusion, to be discussed later, overcomes this limitation.

The different features of the two systems of augmentation indicate their fields of application. The most distinctive feature is the length-diameter ratio of the billet. Product augmentation can be applied to long billets whilst billet augmentation cannot. Long billets will occur when the material extruded is hard, hence the extrusion ratios obtainable are small, and long product lengths are required, these circumstances arise in the manufacture of steel components such as tube. Short billets will occur when the material is soft and high extrusion ratios can be achieved by a small augmenting stress, these circumstances occur in the extrusion of non-ferrous materials. This comparison shows the suitability of product augmented hydrostatic extrusion to the manufacture of steel components. It was for this reason that it was given pre-eminence in the experimental program.

The assistance given to the extrusion pressure by the augmenting load in product augmented hydrostatic extrusion depends on the maximum oil pressure available, when the tag is extruded by the sole action of the oil pressure. This is because high oil pressures permit greater reductions during the extrusion of the tag, which increase the tag strength and the maximum augmenting stress which can be satisfactorily applied. The extrusion ratio which can be achieved during tube extrusion at an oil pressure of 50 tonf/in^2 is given in the table, this can be increased to 7.0 - 9.0 if the oil pressure is increased to 80 tonf/in^2. These extrusion ratios are well above that which is necessary to make the process more attractive than existing methods of tube making.

Another advantage product augmented hydrostatic extrusion offers is that it enables the product to be completely extruded. This is discussed in Appendix II but it requires special phase-out equipment which increases the complexity of the apparatus. The experimental work has shown that complete extrusion can be readily achieved when a rear end taper is machined on to the billet, see Figure 20. If successful this will greatly enhance the attractiveness of product augmented hydrostatic extrusion.

Several non-circular solid shapes have been produced by product augmented hydrostatic extrusion such as round to square, hexagon and a gear section, at extrusion ratios up to 5:1. An extrusion ratio of 6.5 : 1 has been obtained by round to round extrusion. The extrusion of the shapes showed that the die shape is reproduced more accurately when the augmenting stress is small. At high values of augmenting stress the corners of the shapes became rounded. This effect is similar to that produced by conventional drawing and may prove to be a limit to the field of application of product augmented hydrostatic extrusion as far as complex shapes are concerned.

Another advantage possessed by product augmented hydrostatic extrusion is that the equipment which applies augmentation is outside the high pressure system and consists of conventional machinery. This leads to a reliable machine in which the high pressure system, which is the potential source of machine failure, is as simple as it can be made.

Product augmented hydrostatic extrusion does not limit the length of the billet. It thus retains one of the most important features of simple hydrostatic extrusion. With billet augmenting systems such as the 1600/80 HYDROSTAT this is not so and short billets are essential. However, this is overcome by a billet augmented semi-continuous machine, see Green and Slater. This is discussed in the next section.

One of the disadvantages of product augmented hydrostatic extrusion is that it is necessary to pre-form the tag on the billet. The method of extruding the tag by the oil pressure helps to reduce the extent of the pre-forming operation but billet preparation cannot be avoided. In industries that use conventional drawing the preparation of tags is well established as an essential operation in the manufacturing process. To these industries the need to pre-form the billet with a tag will not by itself be a feature which eliminates the other technical advantages of product augmented hydrostatic extrusion. In these industries, to which this form of hydrostatic extrusion is best suited, the economic success of the process will depend solely on the maximum extrusion ratio obtainable and the speed of operation. Many new processes are available which give tube makers extrusion ratios up to 5:1, but some of them are slow acting, cold reducing is an example of this type of process. Hydrostatic extrusion is potentially a fast operation since the development of augmented hydrostatic extrusion has been undertaken specifically to control the extrusion rate so that speeds can be reduced. The 1600/80 HYDROSTAT is the most important augmented press in existence and is designed to produce 40 products per hour. It is felt that this order of through-put should be possible with a product augmented hydrostatic extrusion machine which is specifically designed to produce a given component such as steel tube. If this is achieved then the through-put will be several orders of magnitude greater than comparable conventional machines, such as the cold reducer.

6.2.e. FUTURE DEVELOPMENTS

There are many centres at which hydrostatic extrusion is being developed. Simple hydrostatic extrusion is being developed at the National Engineering Laboratory, Great Britain and at A.S.E.A.E., Sweden. Augmented hydrostatic extrusion is being developed at R.F.L., U.K.A.E.A. and semi continuous hydrostatic extrusion at R.F.L., U.K.A.E.A. and Imperial College, London. A great deal of equipment is actively being developed and some of it is commercially available.

In view of the results of this work it is felt that the augmented hydrostatic extrusion processes are particularly applicable to the extrusion of steel. It is the future development in this field which will now be discussed in the light of existing information.

Emphasis has been given in this work to the production of steel tube. The billet augmented hydrostatic extrusion process has been shown to have limited application to this product but development of this process to the manufacture of complex shapes and short tube lengths in specialised steels is important. Such development will probably require very high oil pressures. The difficulties associated with the use of high pressures have already been discussed and future development with this process will be centred on the design of suitable high pressure containers and punches etc.

In the previous section the billet augmented semi-continuous hydrostatic extrusion processes were mentioned. The essential feature of this machine is a pressure activated, movable seal which allows a billet of unlimited length to enter the high pressure container. The process operation sequence consists of the extrusion of a short length of the long billet, the removal of the oil pressure, the repositioning of the special rear seal to take into the high pressure vessel a new length of the billet and the re-application of pressure which causes extrusion to re-commence. The long billet is extruded in a discontinuous manner with only a short length of billet being subjected to the oil pressure and augmenting load. This is a very important development since it removes the billet length limitation which exists with the previously described billet augmented process. The feasibility of this process has been demonstrated but its use as a means of working steel has not been investigated. It is felt that the extrusion of long steel bar is well within the capacity of this process but the extrusion of steel tube presents difficulties. The principal difficulty arises because the oil pressure can only be applied to the outer surface and not to the bore. This means there will be the tendency to collapse the billet on application of the oil pressure. In addition, if the bore is to be controlled some type of floating plug will have to be used. Only future development work will show whether this is possible.

The most important process to be investigated for the manufacture of steel tube is product augmented hydrostatic extrusion. This process is worthy of considerable development. It has been shown that commercially attractive extrusion ratios can be obtained at moderately low oil pressures, less than 80 $tonf/in^2$. This is an important feature since further equipment can be developed within the limits of present technology.

In relation to future development it is interesting to consider the requirements of a production type machine to operate on the product augmented system for the production of steel tube. There is tendency in the tube industry to require long lengths of tubing, this tendency is brought about by considerations arising from the economics of the inter-pass operations. A minimum product length of 30 feet should, therefore, be considered as a requirement for any further machine. If an extrusion ratio of 5:1 is required the machine will need a container 6 feet long and an operating pressure of approximately 80 $tonf/in^2$. This pressure would allow this reduction to be achieved without resorting to optimum augmenting conditions. The diameter of the containers would be at least $3\frac{1}{2}$-4 in., to enable standard hollow sizes to be accommodated. Extrusion containers of these proportions present problems, which arise out of the difficulties of delivering large quantities of high pressure oil to containers which are long compared with their diameter.

There are three methods of delivering high pressure oil to the extrusion container. The simplest method is to have the ram acting directly in the container but this can only be used when the length diameter ratio of the container is no more than 5:1. This method is, therefore, unsuitable to the machine discussed above since the length diameter ratio of the container proposed is within the range 18 to 21.

The second method is to connect the extrusion container to a secondary pressure vessel which has a length-diameter ratio of 5:1 and a volume in excess of that of the extrusion container, in order to allow for compressibility effects. For the machine discussed, this would require a secondary pressure vessel $6\frac{5}{8}$ in. diameter by 33 in. length. At an oil pressure of 80 tonf/in^2 this requires a press with a capacity of 2750 tonf which is a large installation.

The third method which is more complex than the others, is to have a unit which delivers the quantity of oil required by more than one cycle of operations, such as a pump. Pumps exist which deliver oil at pressures up to 100,000 lbf/in^2 but the volume delivery rates are small and the working lives are short. When considering the requirements of a suitable pump, the principal parameters are the rate of oil delivery and the working pressure. For the machine discussed above it would be necessary to limit the extrusion time to 1 minute per product, a suitable pumping unit would, therefore, have to deliver oil at approximately 1150 in^3/minute, this allows for compressibility effects, at a pressure of 80 tonf/in^2. These requirements cannot be met with existing pump designs. The author and A. E. Pigott have invented a mechanical device which is activated by a reciprocating press and can perform an oil pumping function, see Figure 59. One of the important considerations in the design of a high pressure pump is its fatigue life. This is a limiting consideration since the pump must undergo many more pressure cycles than any other part of the extrusion press. It is believed that the fact that the inner pressure vessel is pressure supported by the oil delivered, will lead to an extended fatigue life since the strains produced in the vessel are always compressive. The oil enters the bore of the inner pressure vessel via a low pressure pump and a non-return valve. During the pumping stroke the oil is compressed and made to flow under the bottom face of the inner vessel, thus entering the space which surrounds the inner vessel. This space forms part of the delivery passage. The inner vessel, is, therefore, fulfilling the dual function of a pressure container and a high pressure non-return valve. In order to maintain a constant supply of oil it is necessary to include a high pressure accumulator, from which oil can be drawn during the suction stroke of the pump. This is not shown in the diagram but it can either be an integral part of the main pump assembly, or a separate component. It is obvious that the reciprocating speed of such a device is limited, it is considered that 15 cycles per minute is a typical operating speed. At this cycle rate the unit for the application considered would have a cylinder diameter of 2.5 in. and a stroke of 12.5 in. At an oil pressure of 80 tonf/in^2 the press would have to have a capacity of 400 tonf and a ram speed of 6.25 in/sec. This ram speed indicates that an accumulated gas, activated press would be required.

The relative merits of the secondary pressure vessel and the pump methods are not known since much work is needed on pumping systems before the performance and reliability are assessed. However, in general the secondary pressure vessel method will be applicable when the length diameter ratio of the extrusion container is greater than 5:1 but not too large and the pumping method will be applicable when the ratio is very large.

The method of loading the billets will also be important since it will affect the design of the extrusion container. The most convenient method of loading would be via a transfer machine at the rear of the press which carries the mandrel. Such a device would withdraw the mandrel at the end of extrusion to a position at which the subsequent hollow could be threaded over the mandrel. The device would then return the mandrel and billet to the extrusion container. The transfer machine could also carry the necessary high pressure seal for the end of the container. Although this is a convenient method it does necessitate the use of a cross-bore in the extrusion container. This would be a weakness in the design and much development work is needed to improve the design of cross-bore. An all in line arrangement is obviously possible but it has disadvantages from the point of view of loading billets.

This brief review of future developments indicates some of the area in which work must be carried out and some of the future difficulties. However, there is sufficient advantage in augmented hydrostatic extrusion processes to make future development economically worth while.

6.3. SUMMARY OF THE CONCLUSIONS

The apparent strain theory

1. This is an engineering approach to the prediction of forming loads for axi-symmetric processes and materials which work harden, which takes into account all forms of redundant work.
2. The apparent strain theory only requires a knowledge of the geometry of the process, the equivalent stress-strain diagram of the material to be formed and the mechanical properties of the lubricant.
3. The apparent strain is defined as the strain range over which the area under the equivalent stress-strain diagram is equal to the total work done per unit volume of material.
4. The apparent strain differs from the mean equivalent strain experienced by the material on deformation when friction is present.

5. The apparent strain can be determined experimentally and can be estimated theoretically.

6. It has been demonstrated that the apparent strain theory gives good agreement with results obtained by both simple and augmented hydrostatic extrusion, in which the frictional effects are small.

Simple hydrostatic extrusion

1. Is a form of extrusion in which the billet is extruded by the sole action of liquid pressure.
2. It eliminates the large frictional force between the billet and the extrusion container which exists with conventional extrusion.
3. It is prone to an unsteady mode of extrusion in which the billet is extruded in a series of forward movements which are associated with oscillations in the liquid pressure, known as 'stick-slip'.
4. Complete extrusion of the billet is possible but is associated with the release of the considerable energy stored in the liquid. This accelerates the product to a high speed and inflicts considerable damage to the rear end of the product.

Augmented hydrostatic extrusion

1. Augmentation is the name given to the method of assisting the liquid pressure by an additional direct load during hydrostatic extrusion.
2. Augmentation eliminates the 'stick-slip' mode of extrusion and results in a lower liquid pressure for a given area reduction.
3. It has been shown that augmenting loads may be applied to the product, billet and the product and billet simultaneously.

4. It has been shown that a form of augmentation can be induced during tube extrusion by the use of special mandrels. This leads to an augmenting load which is proportional to the oil pressure.
5. The application of proportional augmentation is limited to thick-walled billets.

Equipment

1. The augmentation of hydrostatic extrusion by drawing, product augmented hydrostatic extrusion, has been shown to be a versatile method for forming both hollow and solid billets in steel.
2. The equipment needed to achieve product augmented hydrostatic extrusion consists of a high pressure generator, a suitable extrusion container and a drawing unit.
3. The maximum extrusion ratio obtainable by product augmented hydrostatic extrusion differs very little with the strength of the steel extruded at a liquid pressure of 50 tonf/in^2.
4. Reductions of 3.5 : 1 for steel tube and 6.0 : 1 for steel bar have been obtained by product augmented hydrostatic extrusion.
5. The extrusion rate possible by product augmented hydrostatic extrusion is comparable to that obtainable by conventional drawing.
6. The feasibility of reducing the liquid pressure in a controlled way to enable complete extrusion to be achieved without the release of a blast of liquid, the pressure 'phase-out' method, has been demonstrated for product augmented hydrostatic extrusion.
7. The pressure 'phase-out' method has been achieved when the billet is provided with a rear end taper, it is, however, considered possible to 'phase-out' a square ended billet if special die profiles are used.

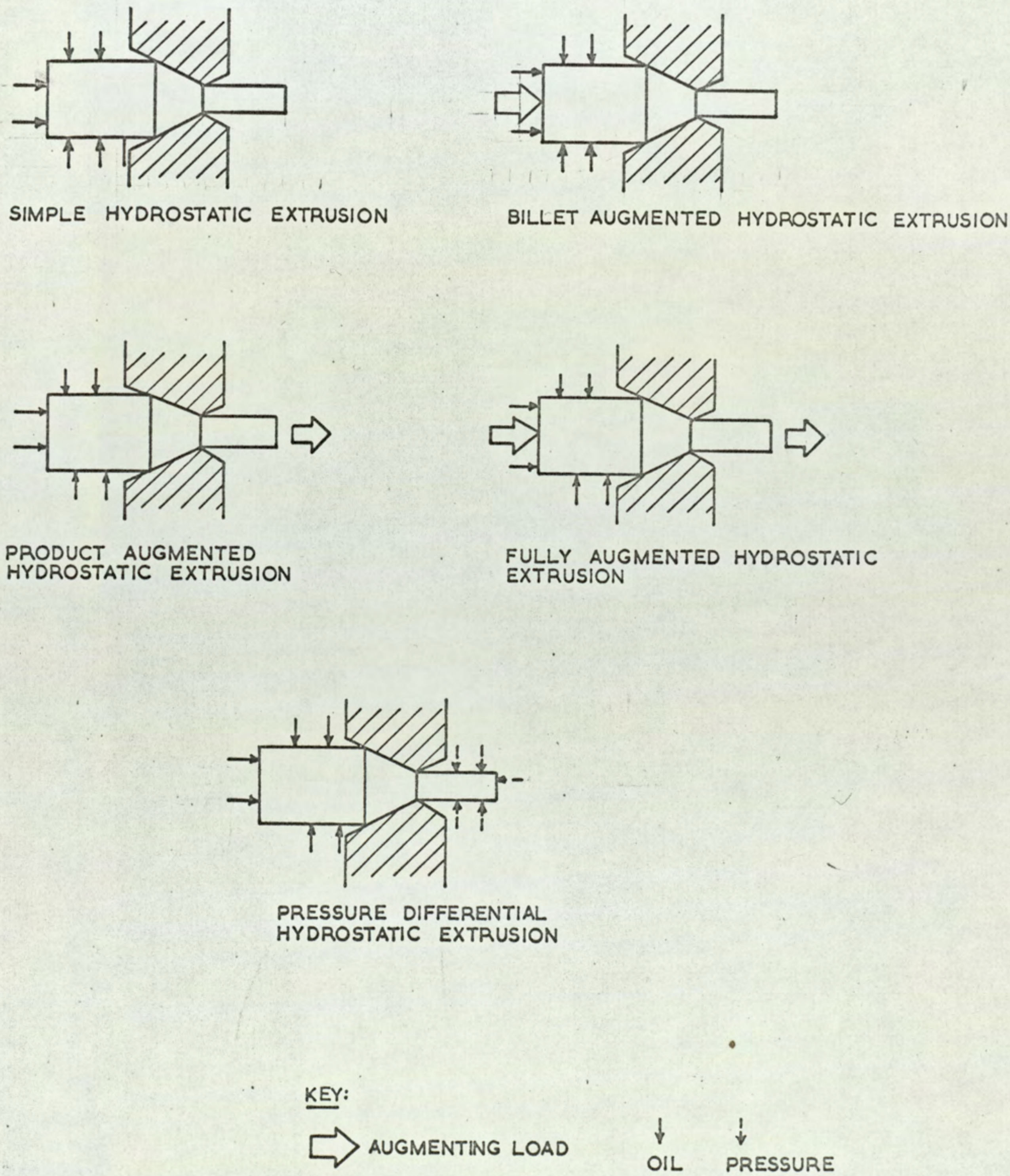


FIG.1. DEFINITIONS OF HYDROSTATIC EXTRUSION PROCESSES

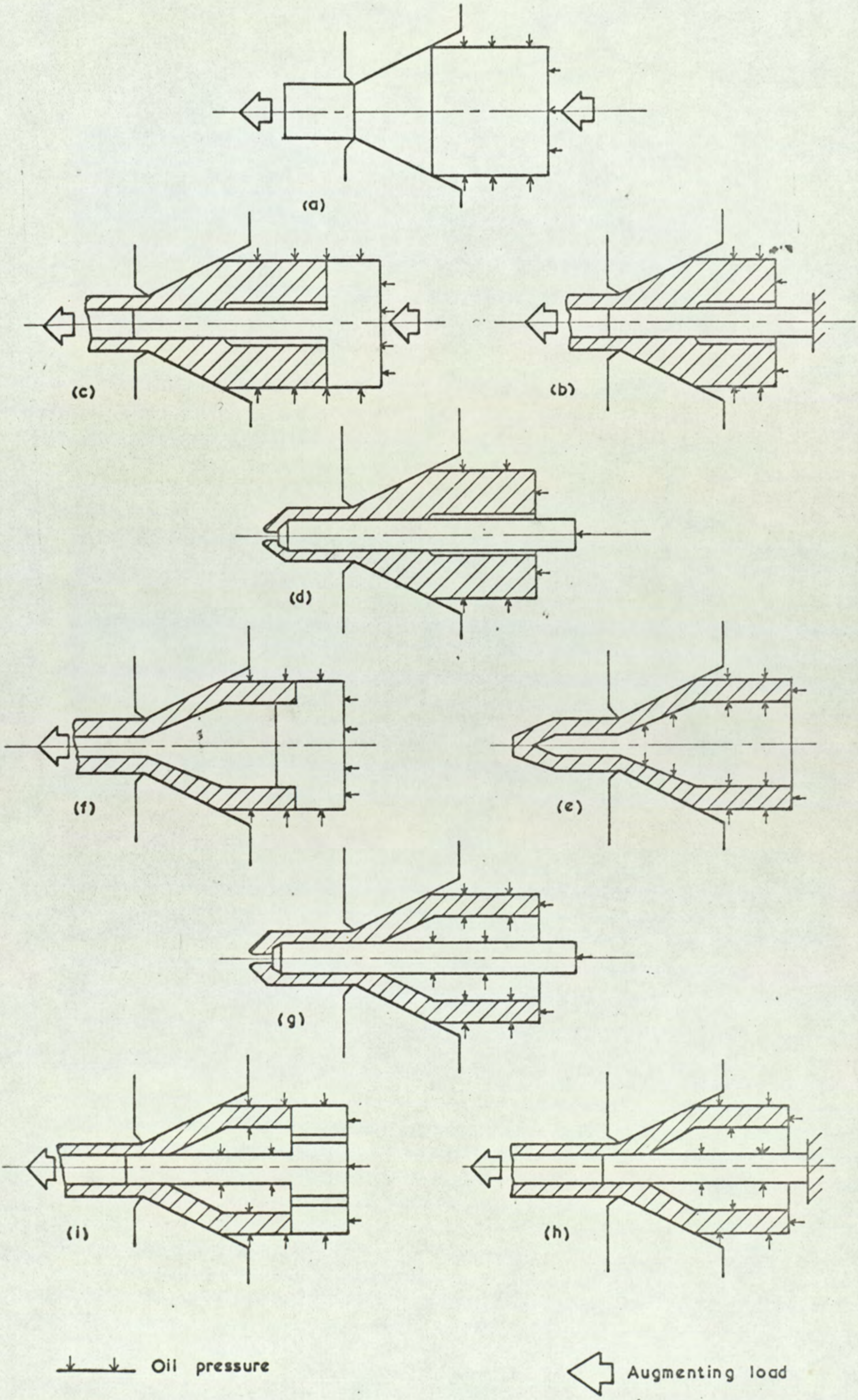


FIG.2. METHODS OF FORMING BAR AND TUBE BY HYDROSTATIC EXTRUSION

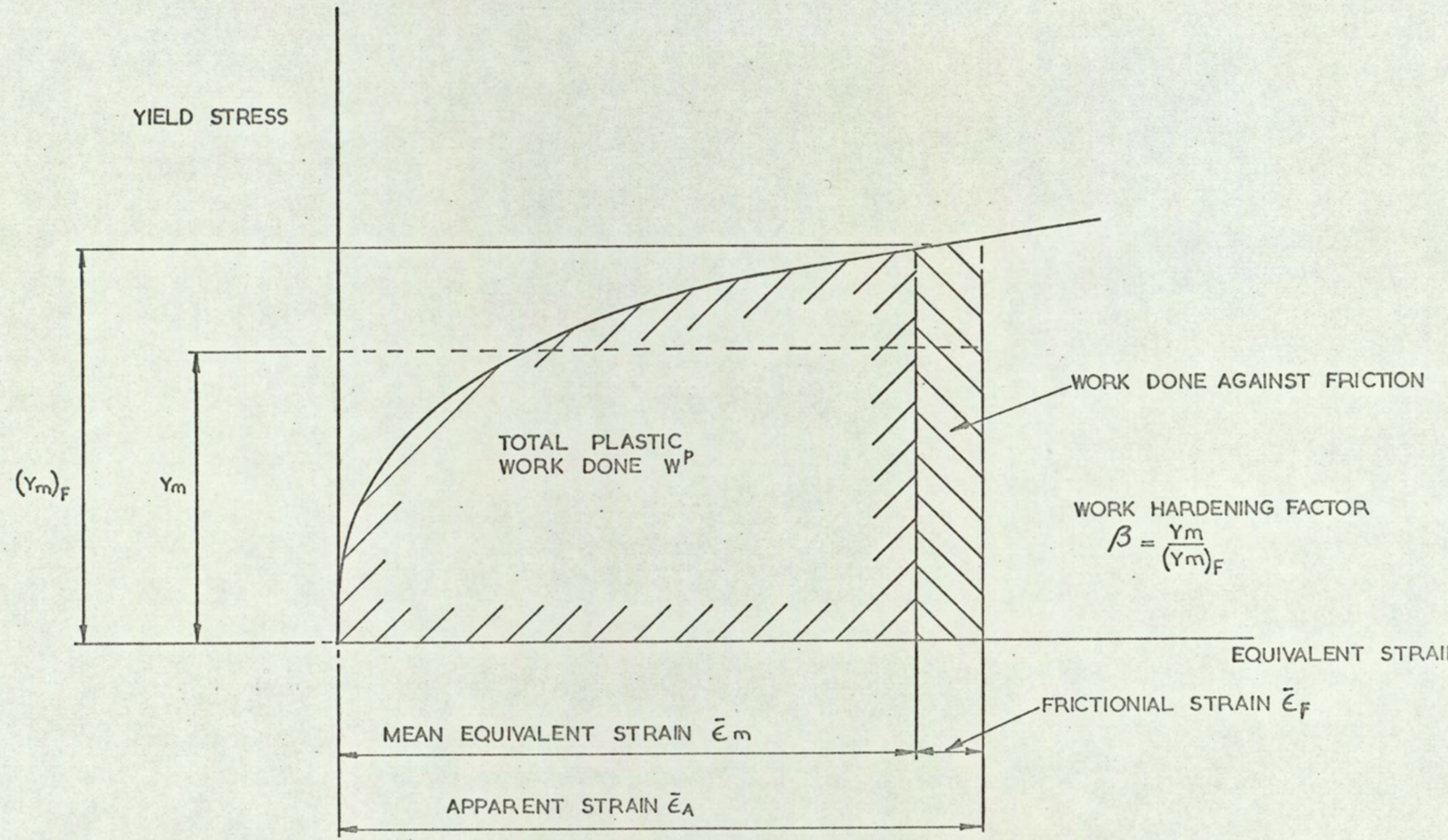


FIG. 3. DEFINITION OF TERMS USED IN THE APPARENT STRAIN THEORY

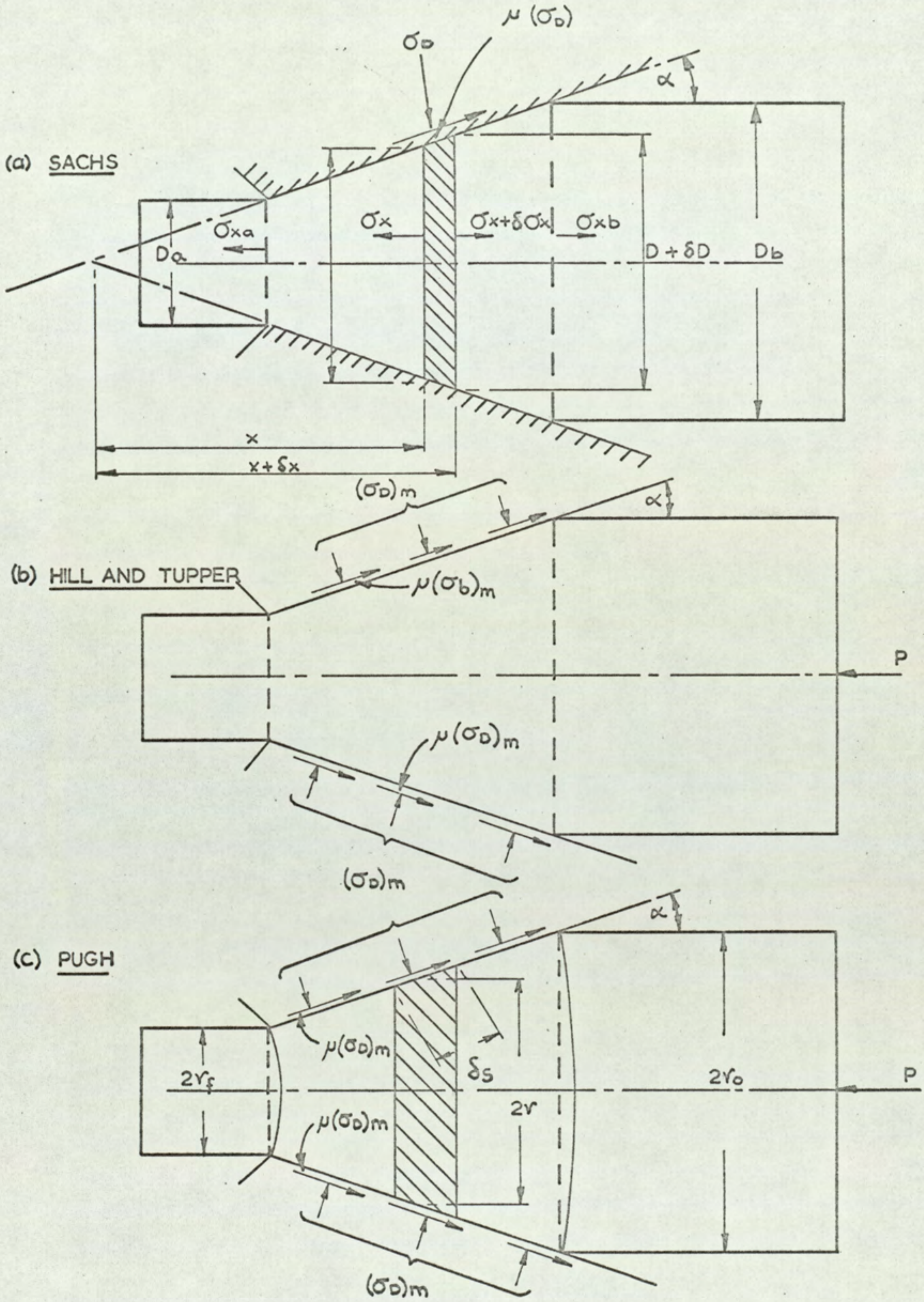


FIG.4. MODELS OF DEFORMATION DISCUSSED IN THE REVIEW

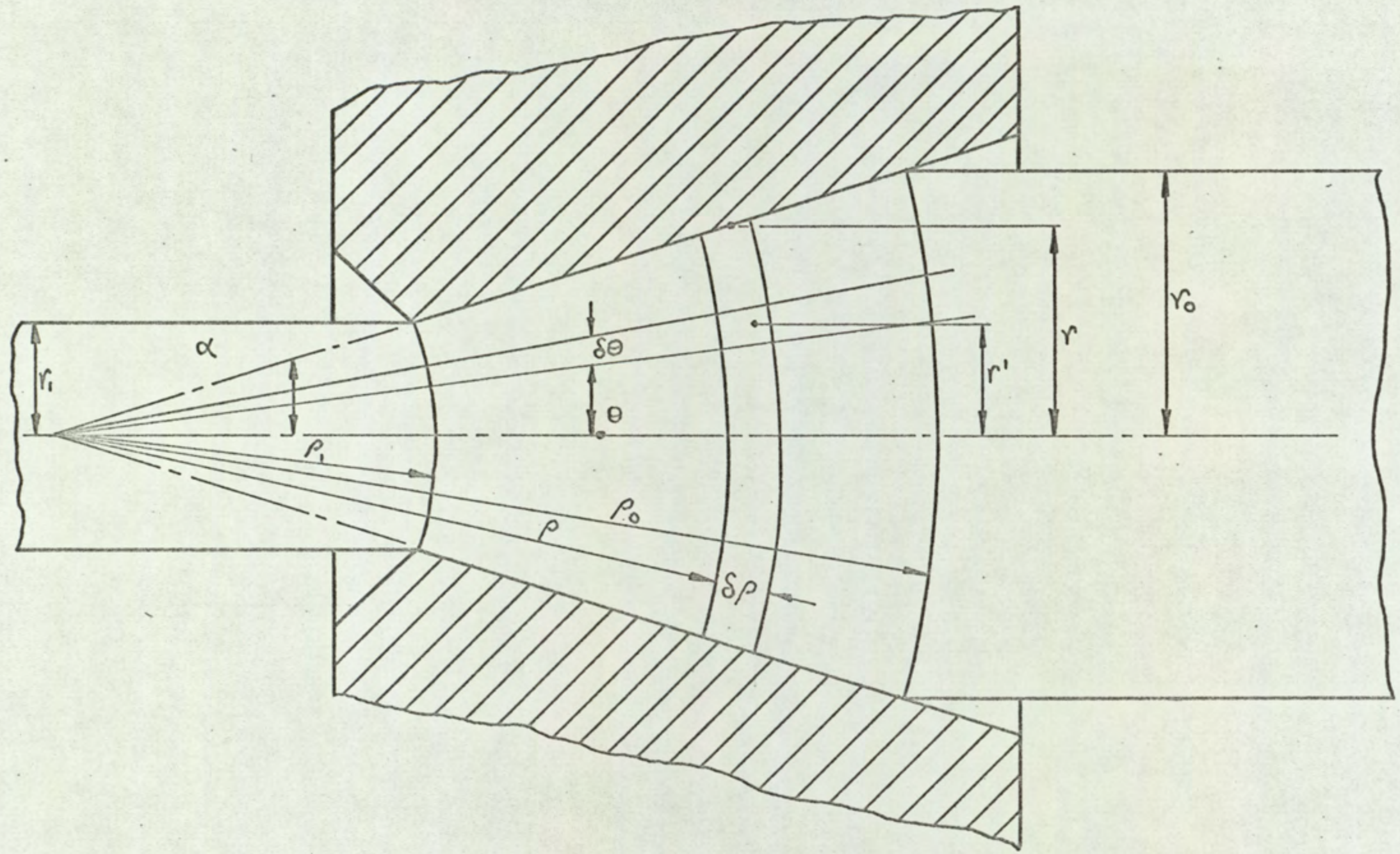
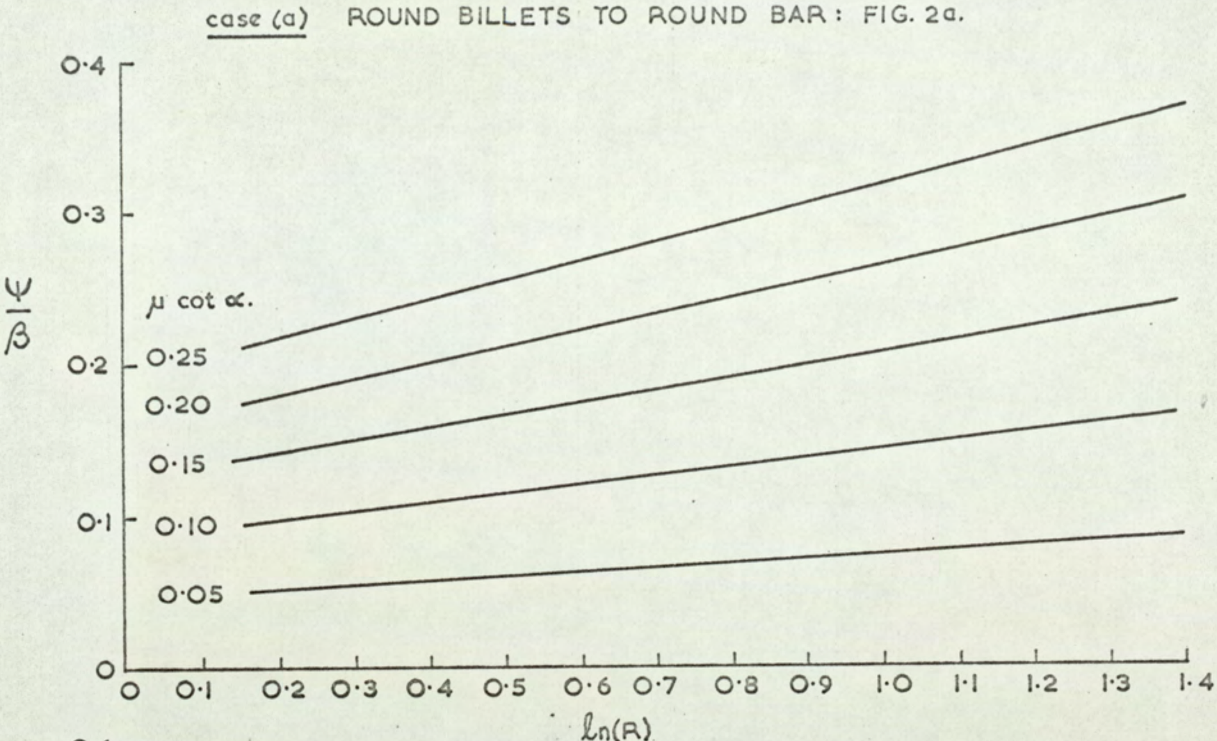


FIG. 5. MODEL OF DEFORMATION FOR BAR EXTRUSION



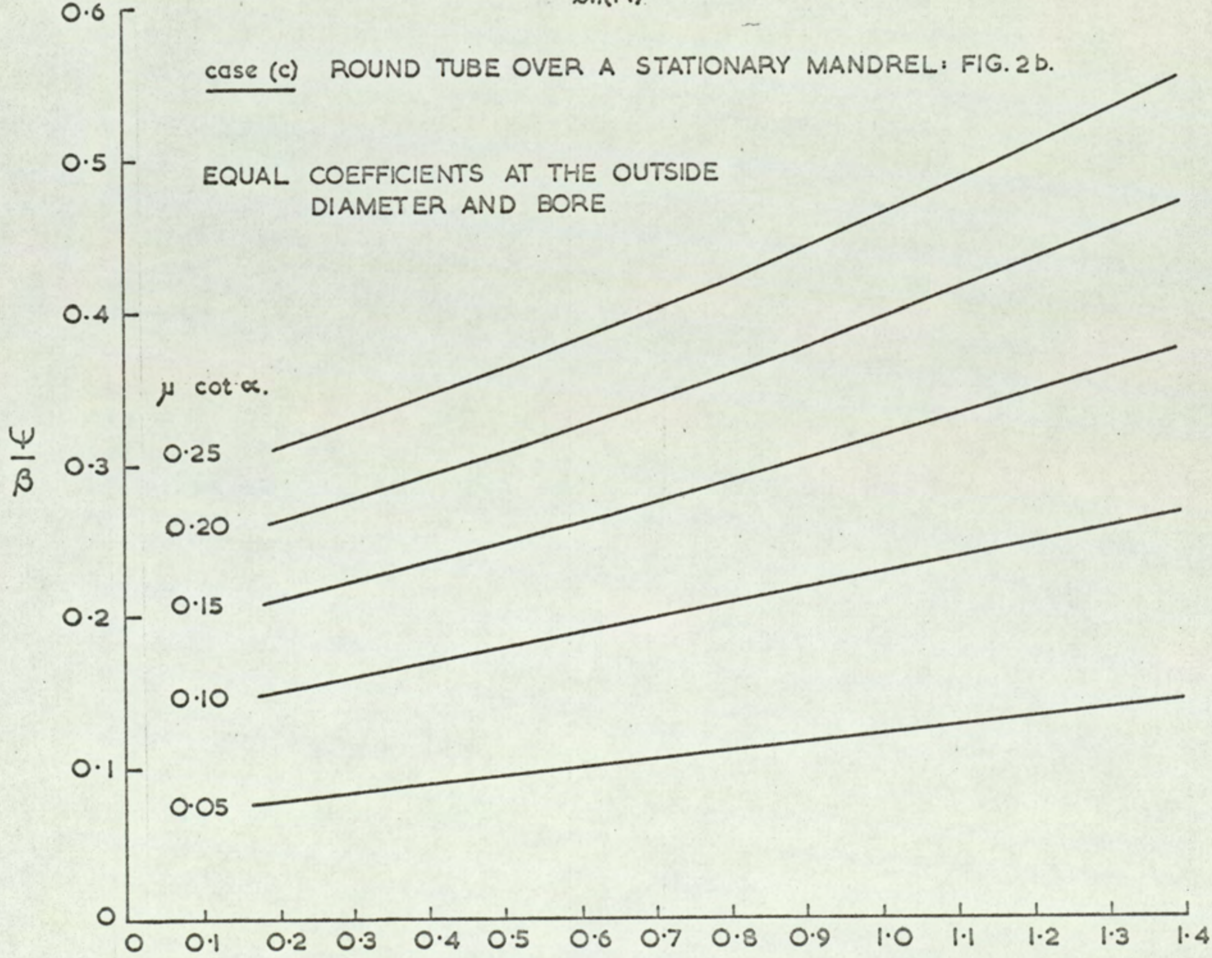


FIG.6. FRICTION FACTORS.

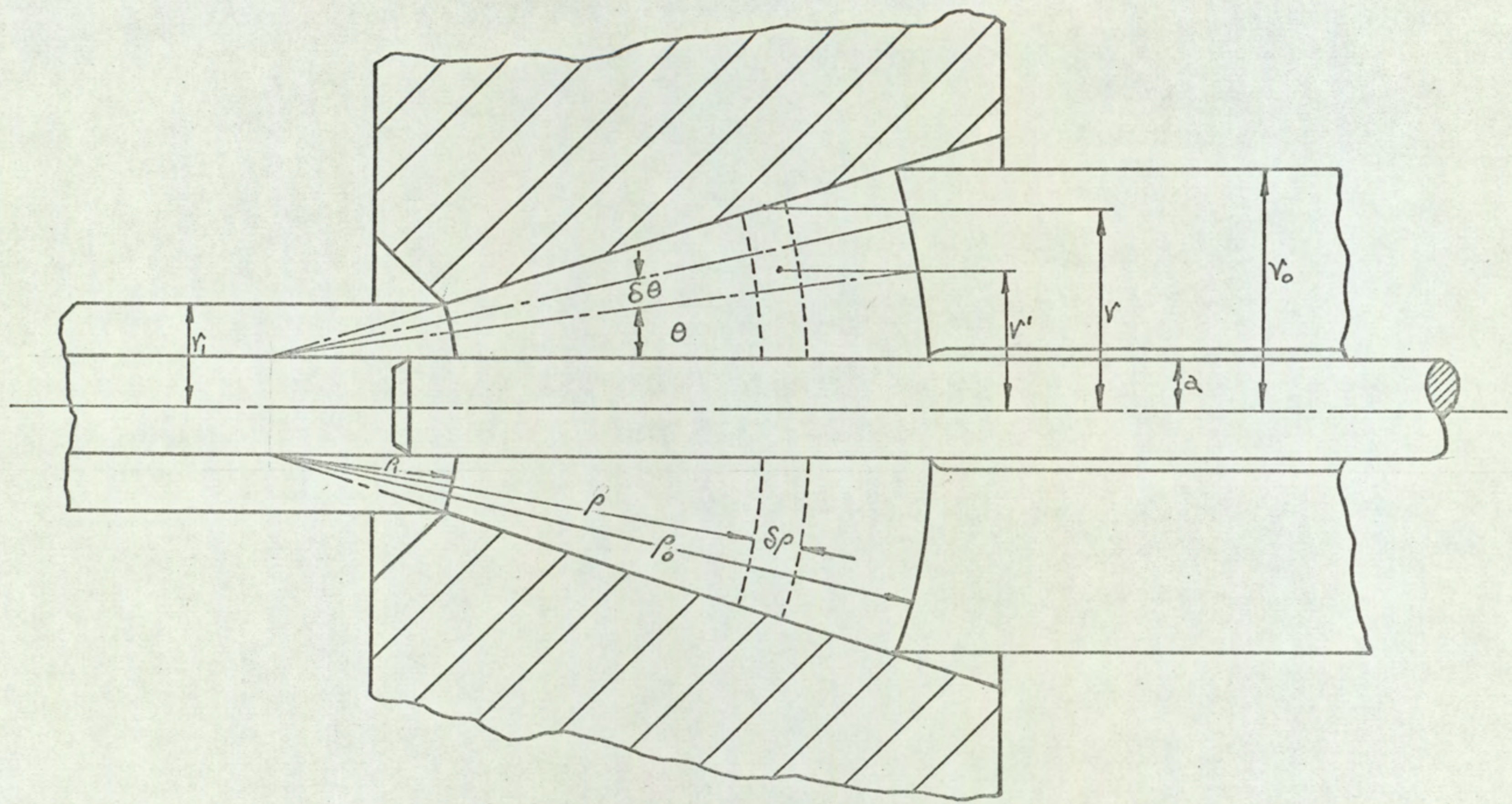
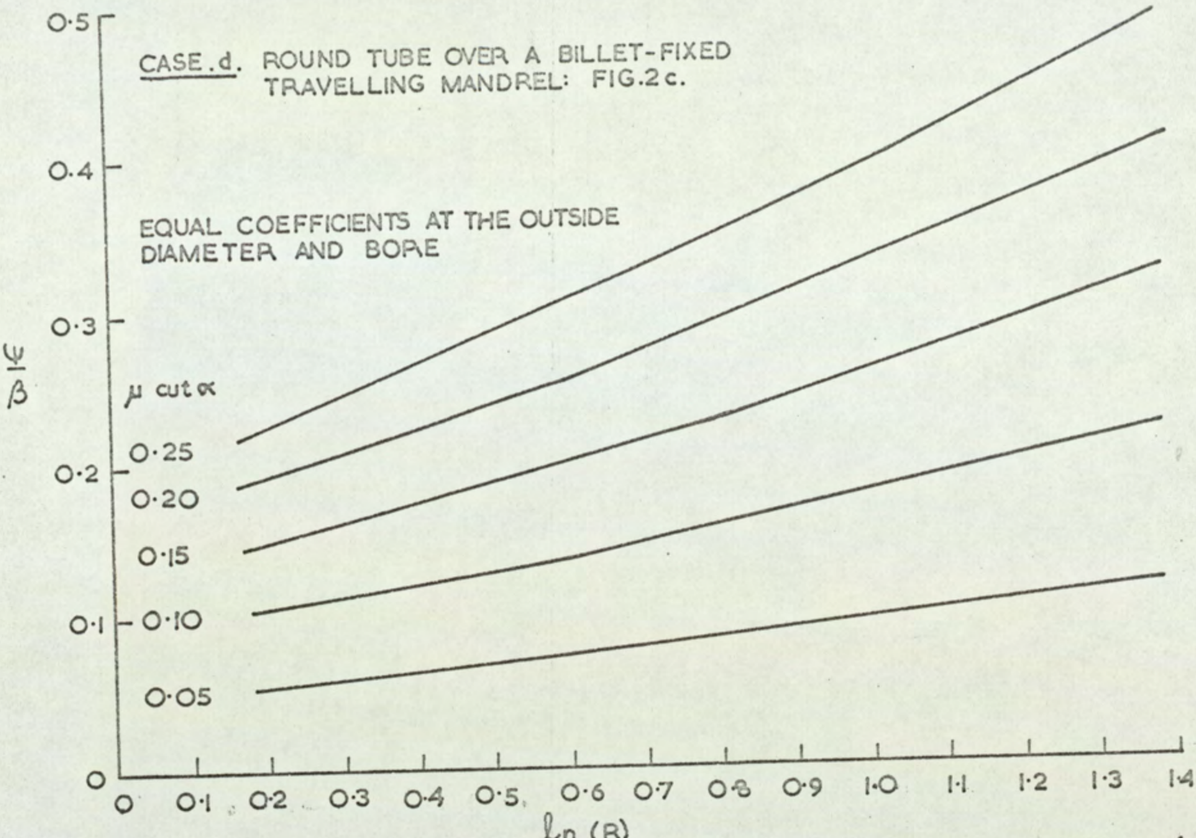


FIG. 7. MODEL OF DEFORMATION FOR TUBE EXTRUSION



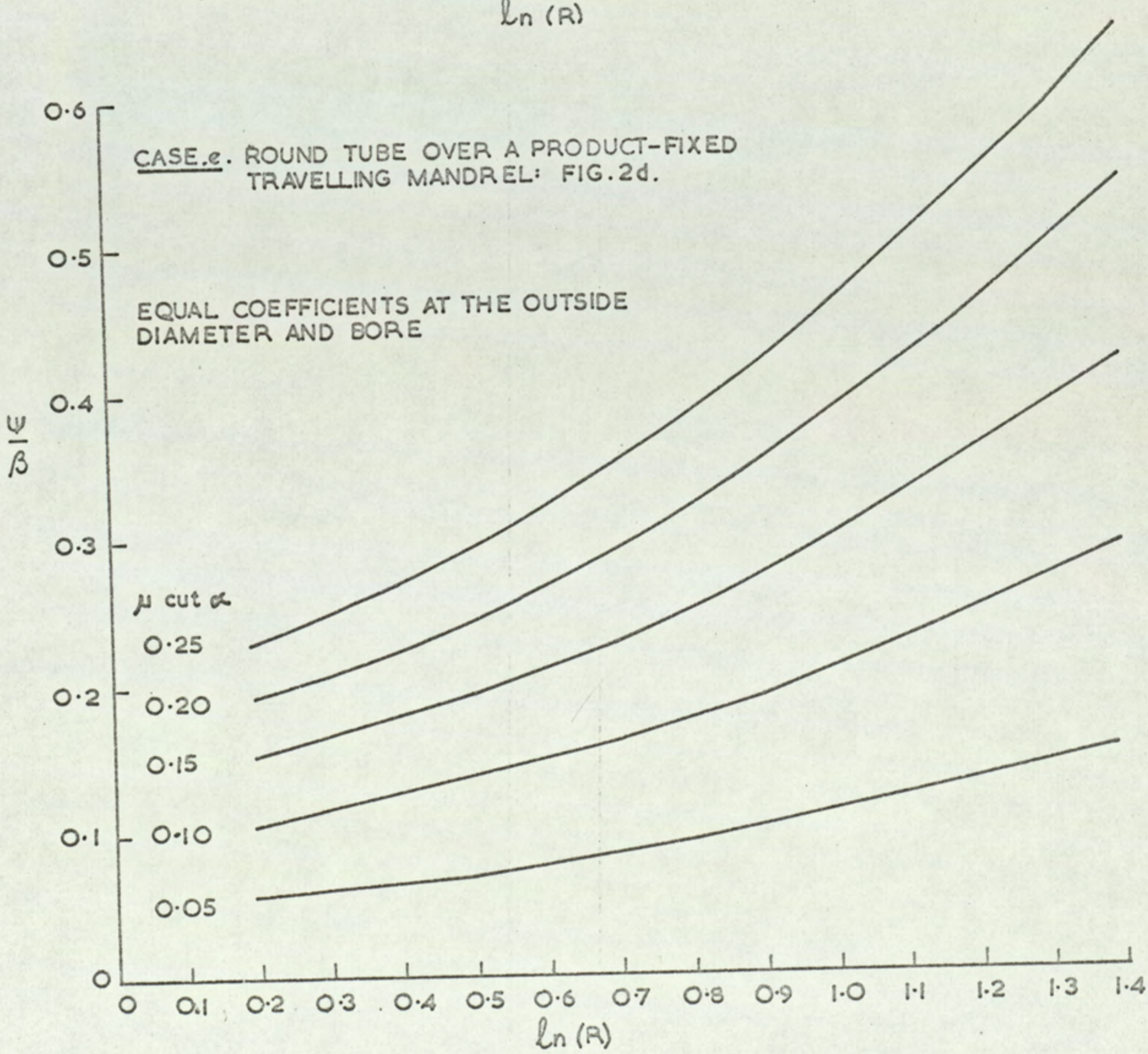


FIG.8. FRICTION FACTORS

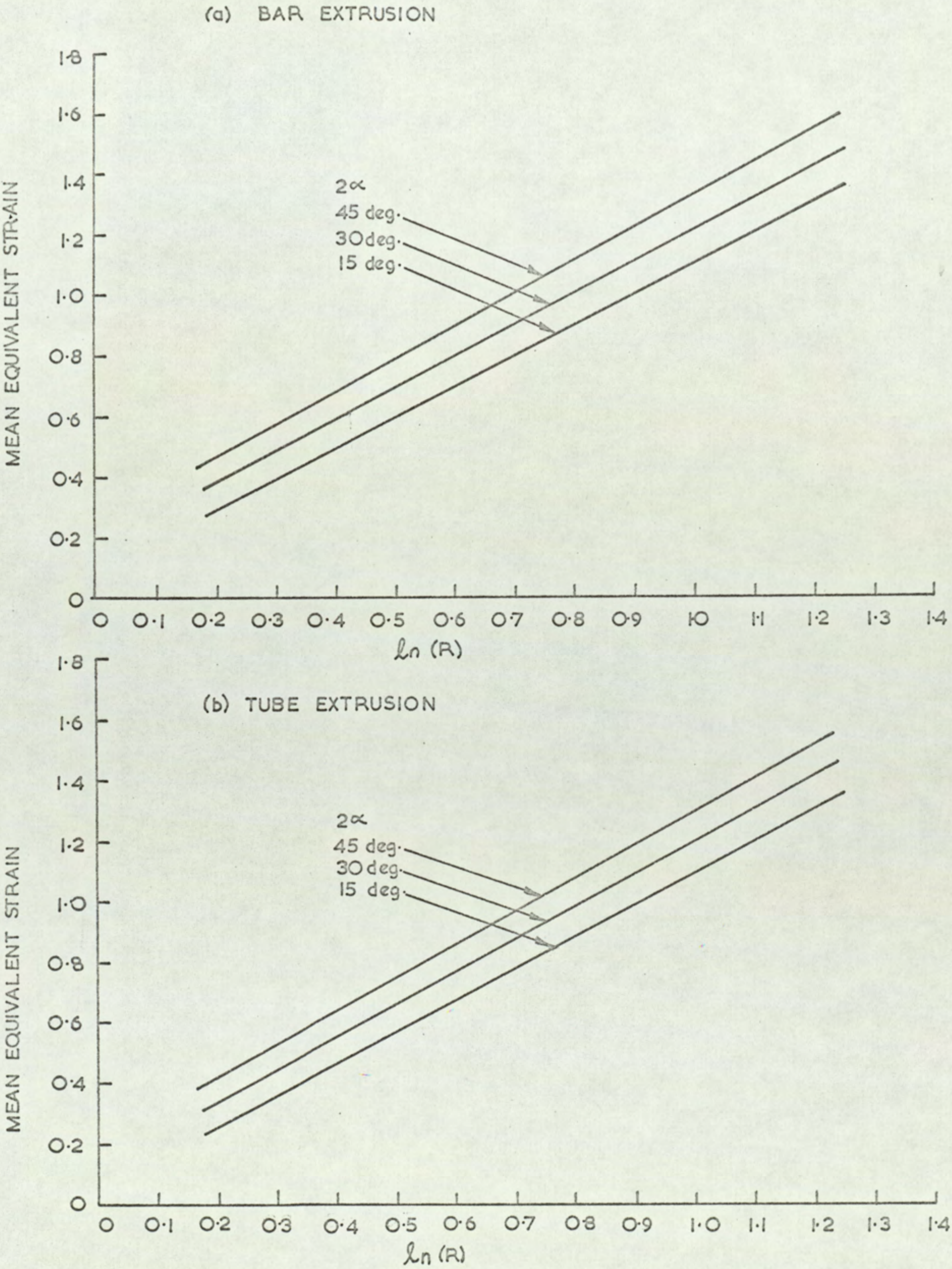


FIG. 9. THE EFFECT OF DIE ANGLE AND EXTRUSION RATIO ON THE MEAN EQUIVALENT STRAIN FOR BOTH BAR AND TUBE EXTRUSION

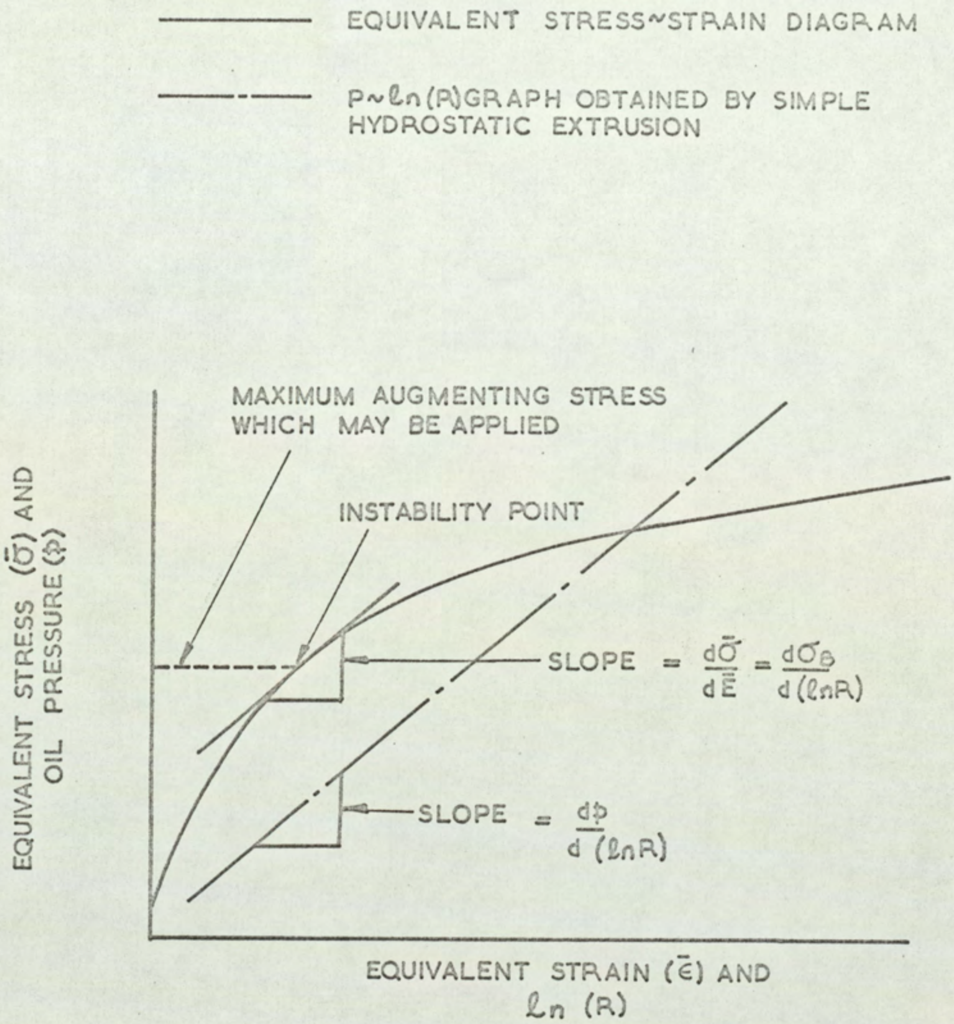


FIG. 10. GRAPHICAL METHOD FOR DETERMINING THE MAXIMUM BILLET AUGMENTING STRESS

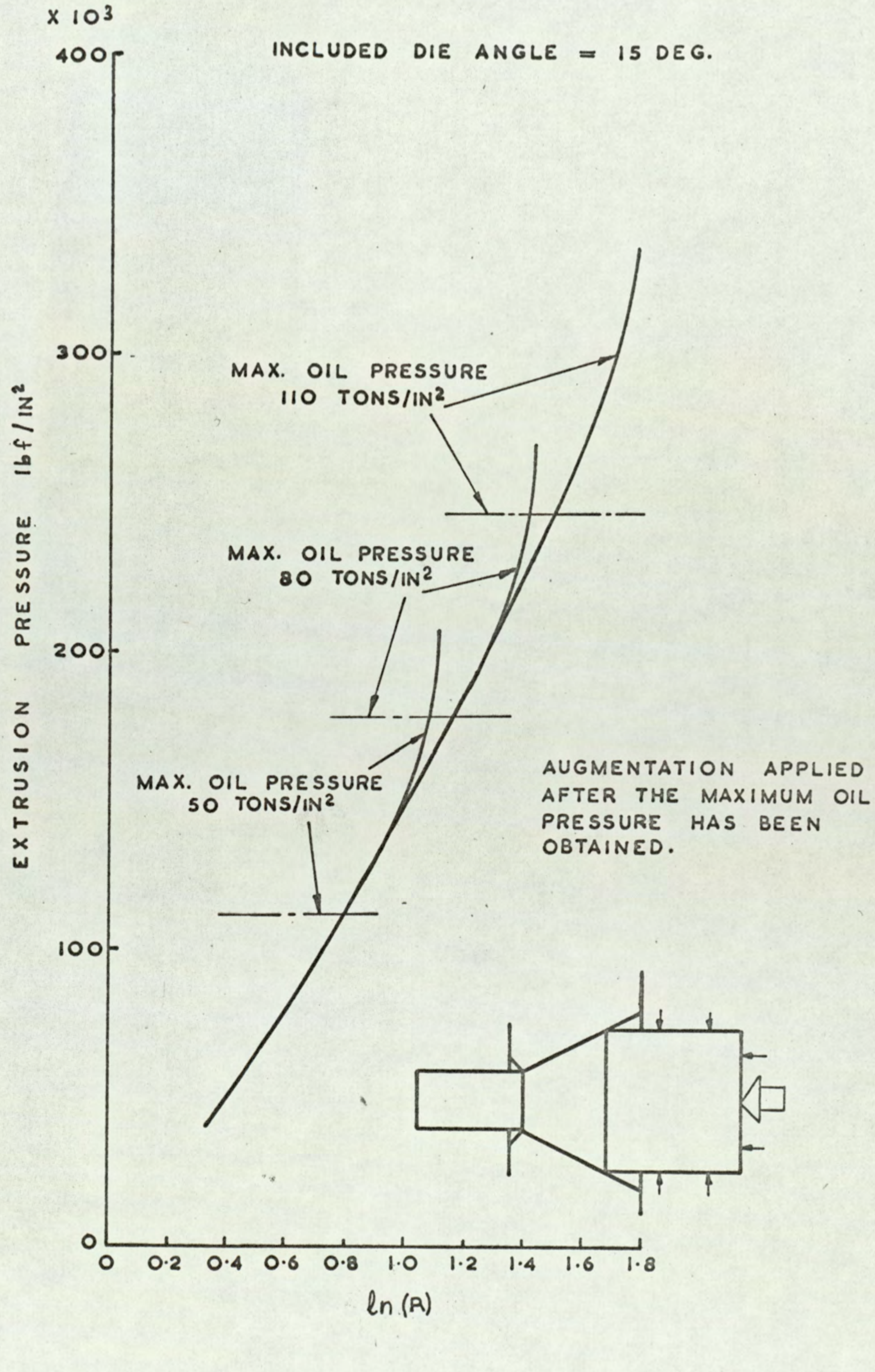
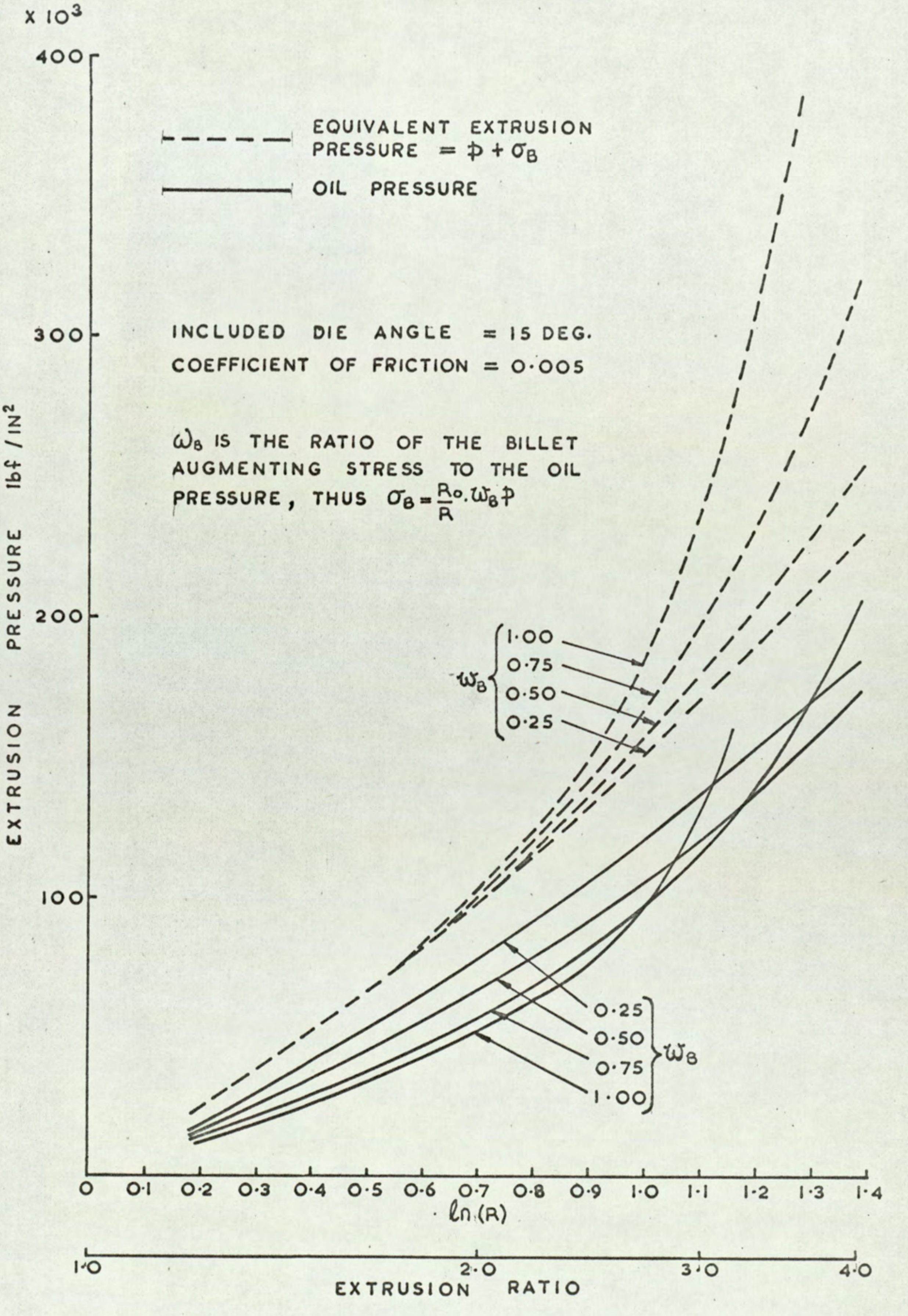


FIG.11. THEORETICAL ESTIMATE OF THE OIL PRESSURES REQUIRED FOR THE BILLET AUGMENTED HYDROSTATIC EXTRUSION OF 20/25/Nb S.S. BAR.



G.12. THEORETICAL ESTIMATE OF THE OIL PRESSURE REQUIRED FOR THE PROPORTIONAL BILLET AUGMENTATION OF 20/25/Nb STAINLESS STEEL BAR.

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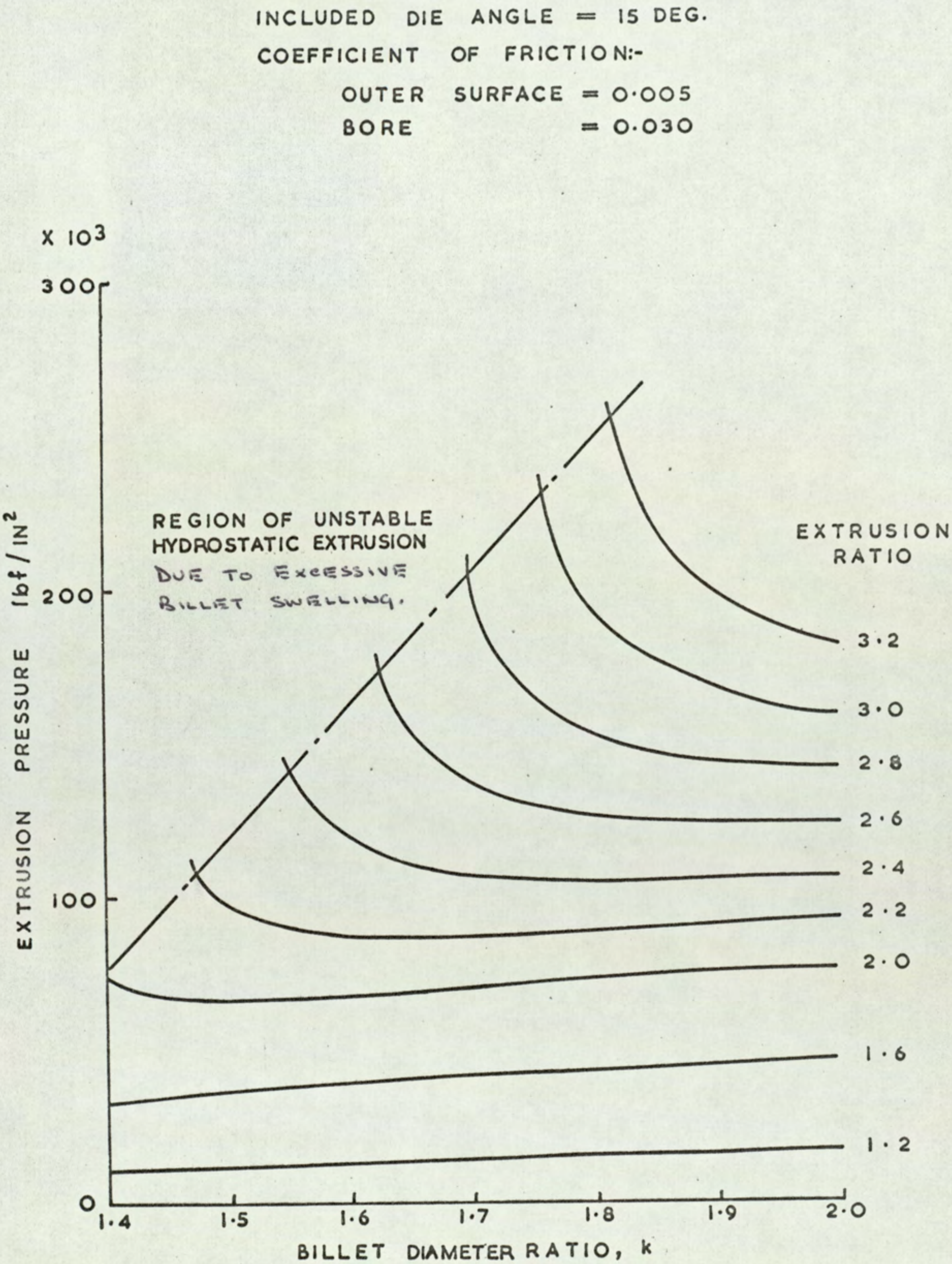


FIG.13. THEORETICAL ESTIMATE OF THE OIL PRESSURE REQUIRED FOR THE HYDROSTATIC EXTRUSION OF 20/25/Nb STAINLESS STEEL TUBE OVER A BILLET-FIXED TRAVELLING MANDREL WITH A SOLID HEAD.

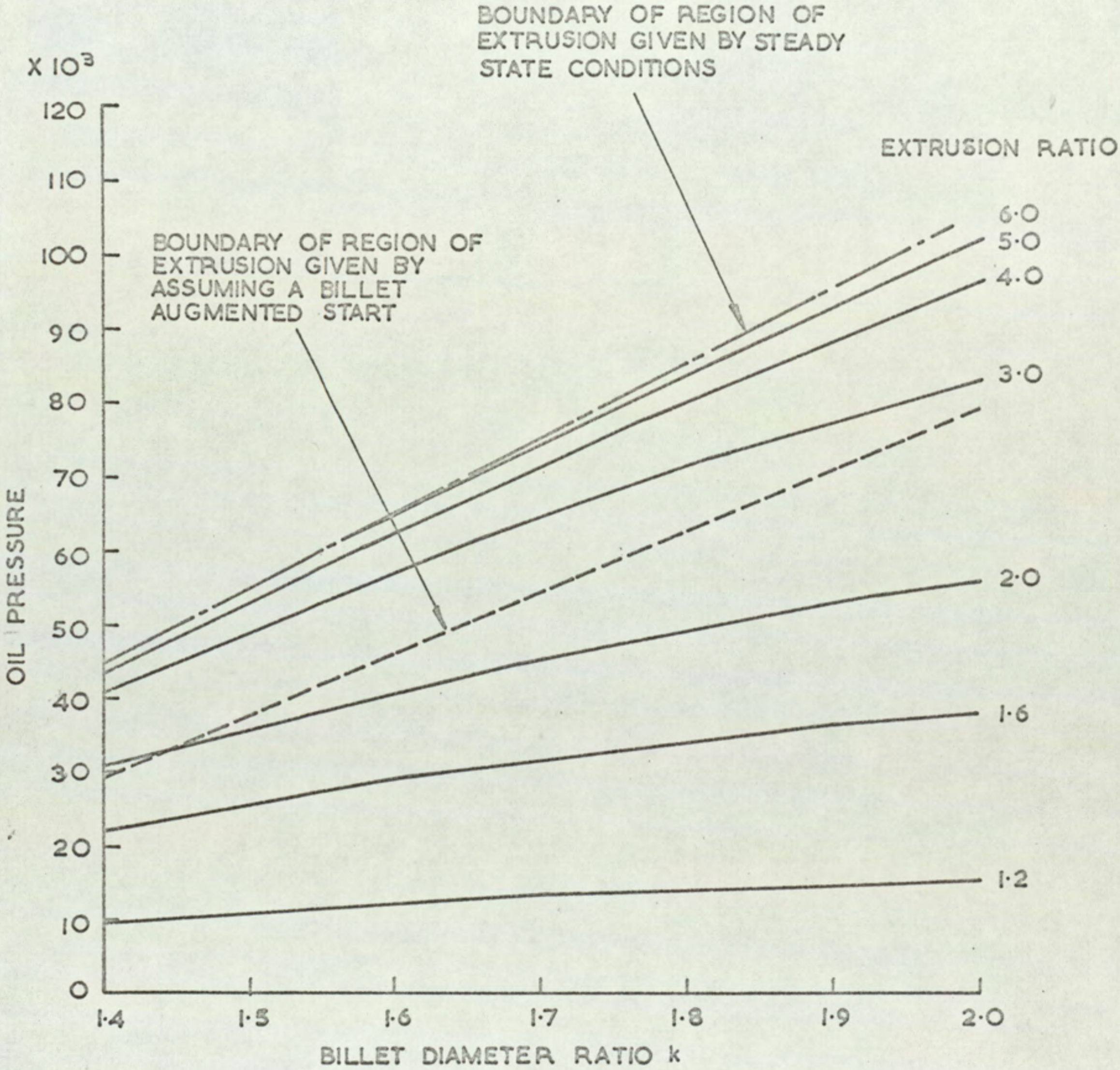


FIG. 14. PROPORTIONAL PRODUCT AUGMENTED HYDROSTATIC EXTRUSION OF 20/25/Nb STAINLESS STEEL TUBE

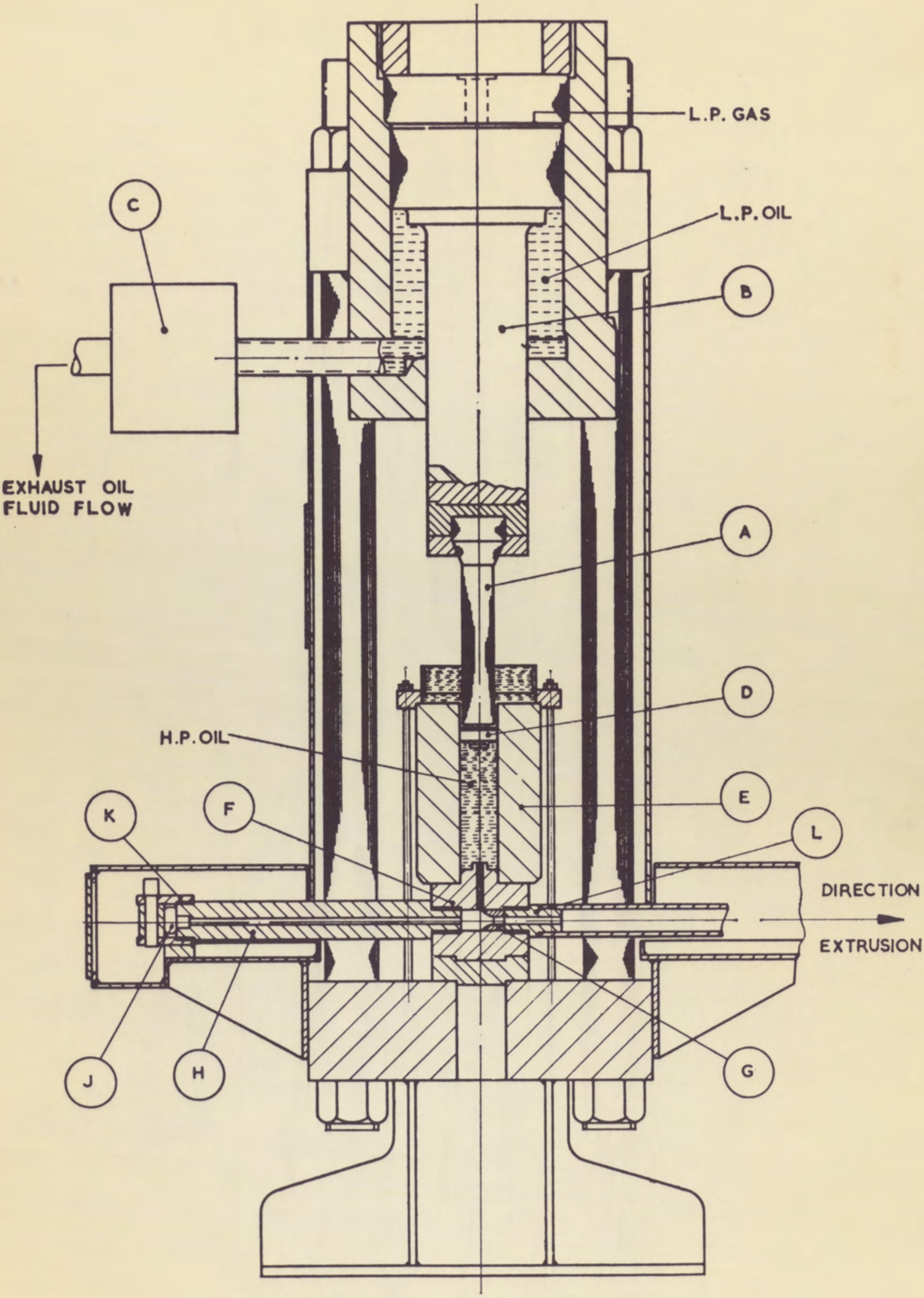


FIG. 15. DIAGRAMMATIC CROSS-SECTION OF HIGH-SPEED MACHINE

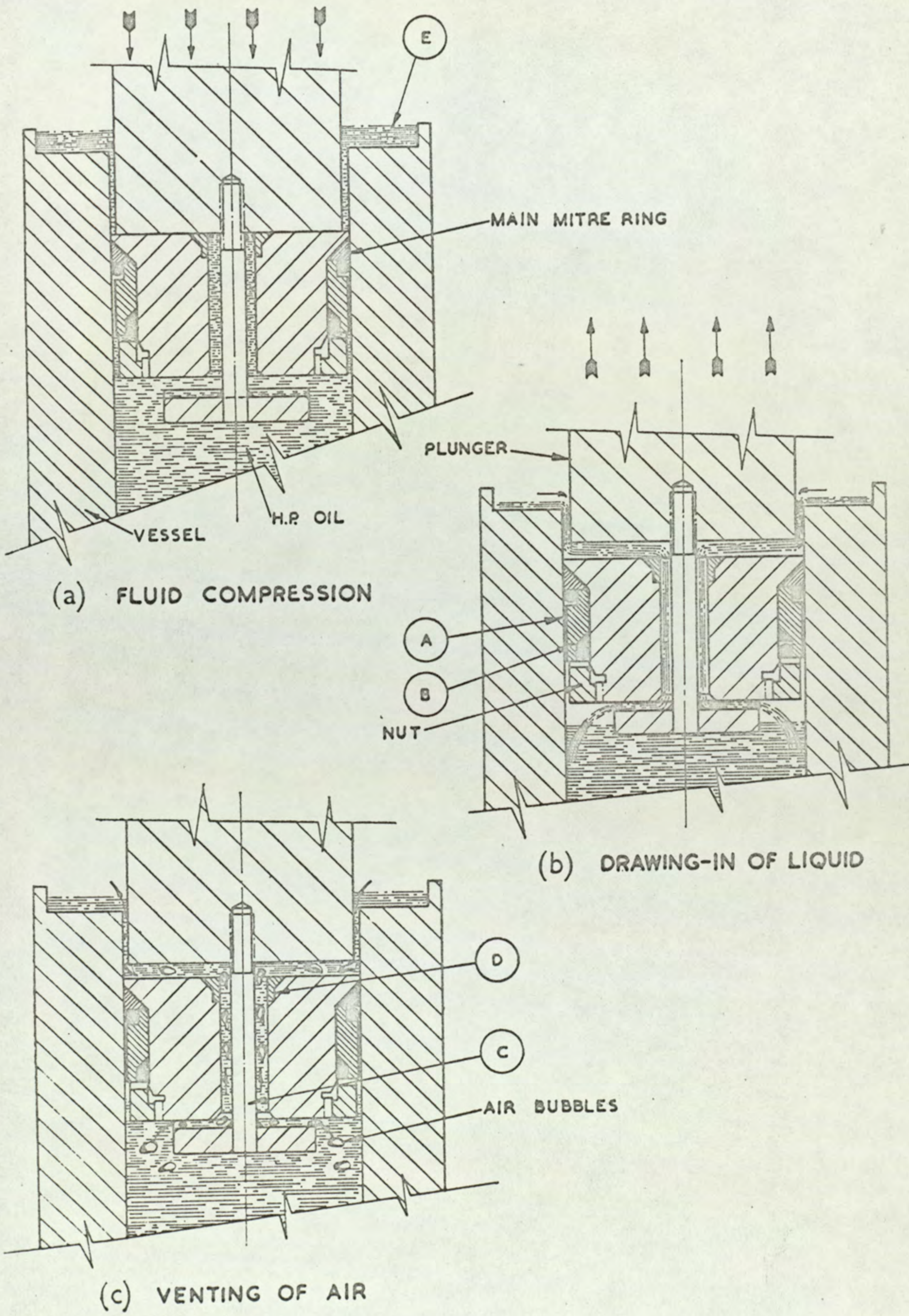


FIG. 16 (a)-(c). SELF VENTING HIGH PRESSURE SEAL

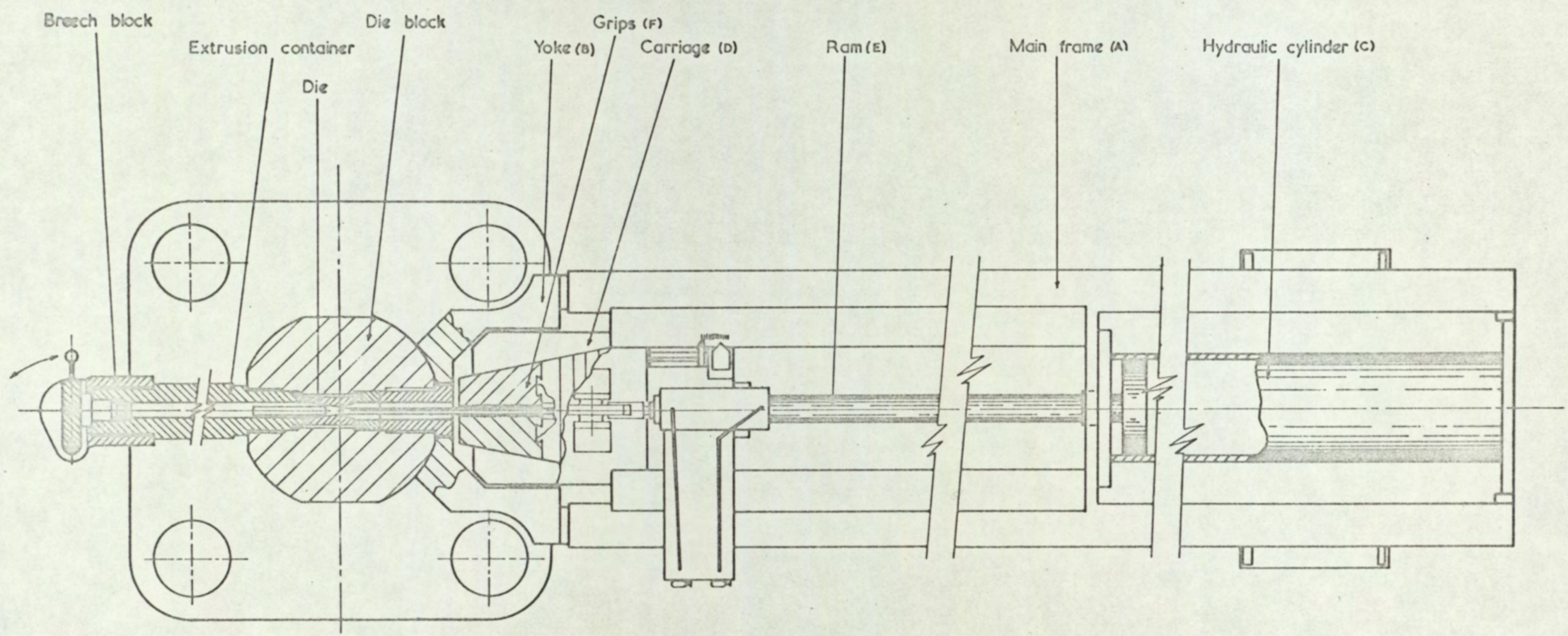


FIG.17. A HORIZONTAL SECTION PASSING THROUGH THE 200/50 HYDROSTAT AND THE DRAWING UNIT

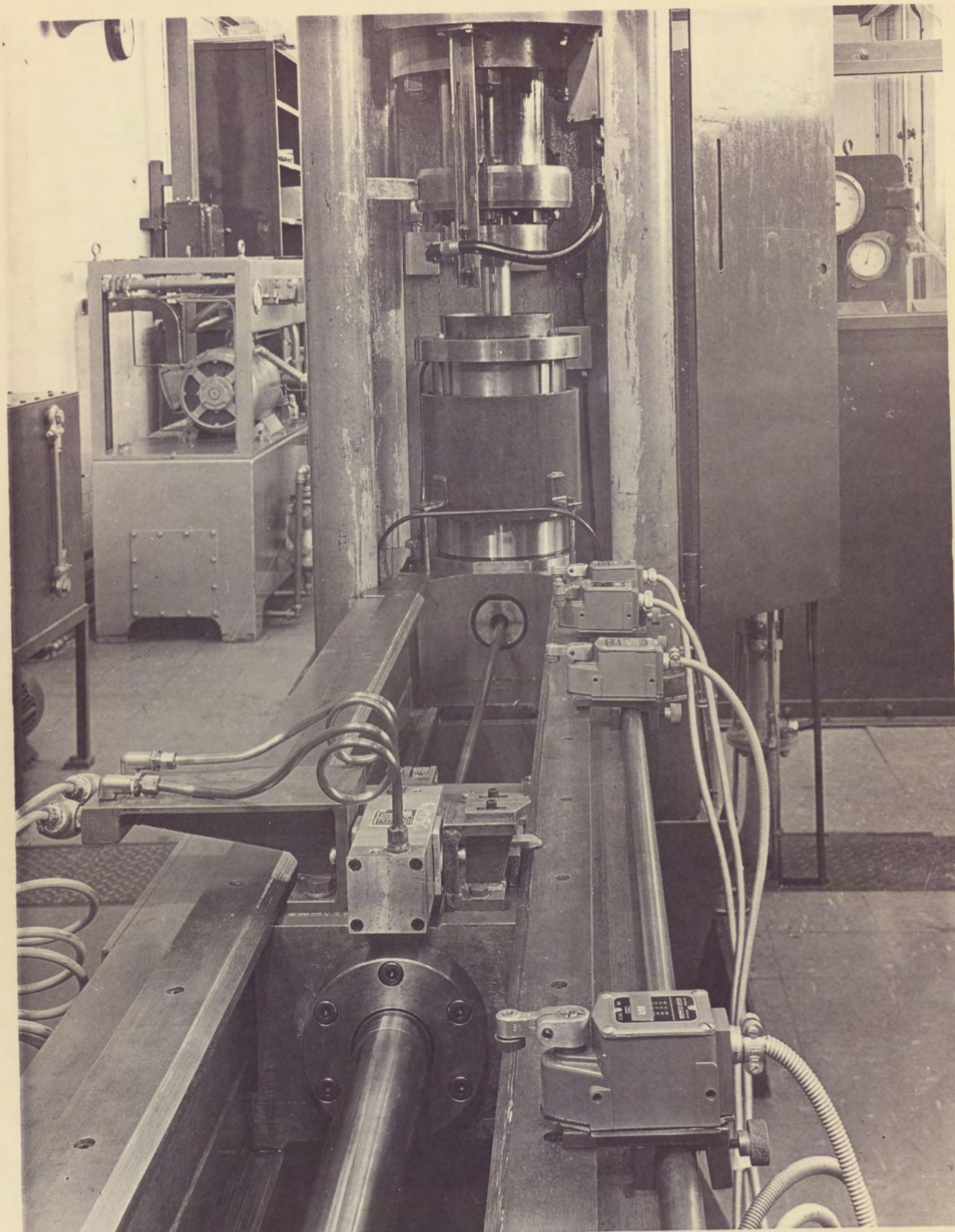


FIG. 18. MACHINE FOR PRODUCT AUGMENTED HYDROSTATIC EXTRUSION

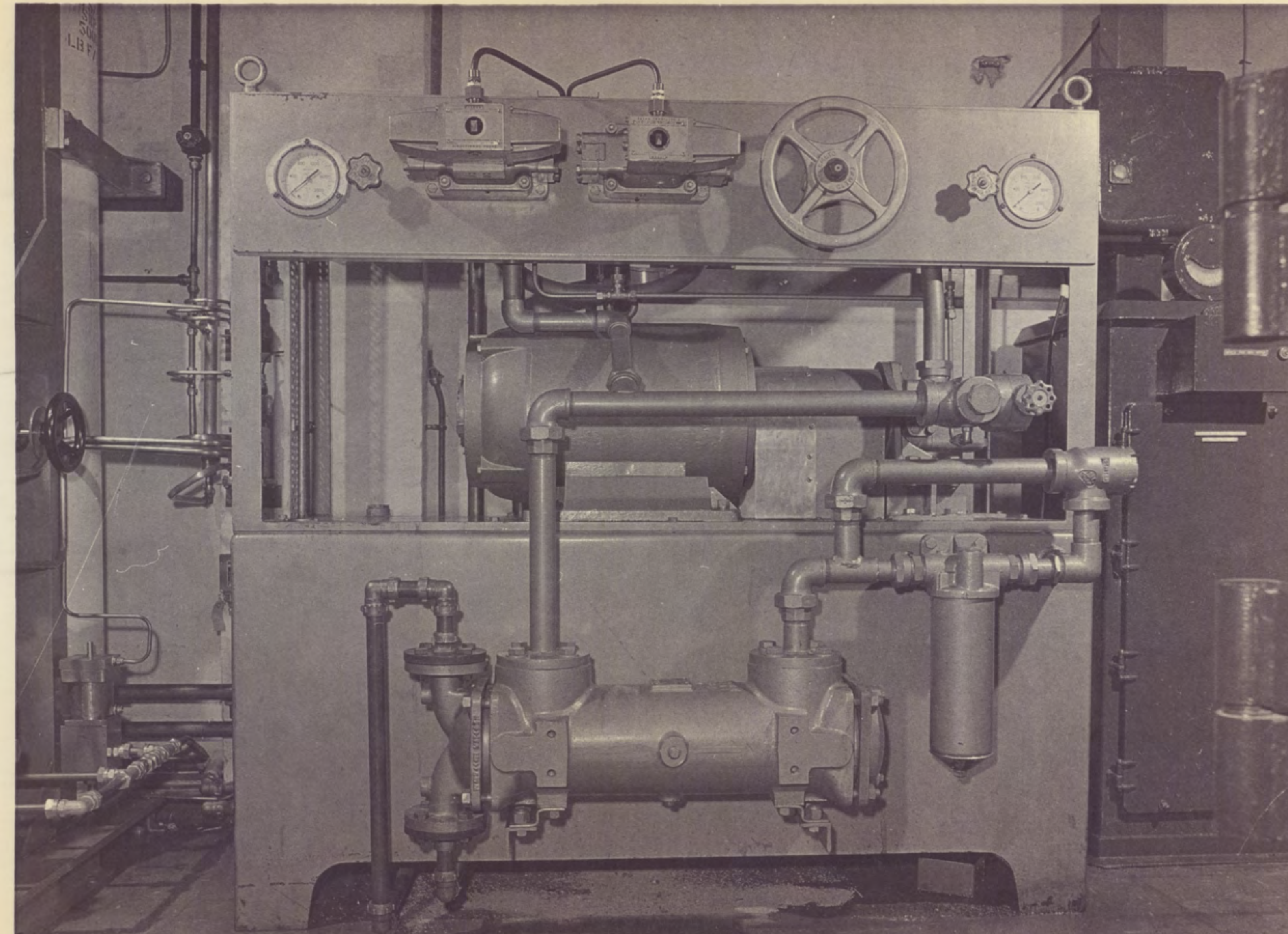


FIG. 19. POWER PACK FOR THE DRAWING UNIT

NOTE:
THE EXTRUSION CONTAINER IS ONLY SHOWN DIAGRAMMATICALLY

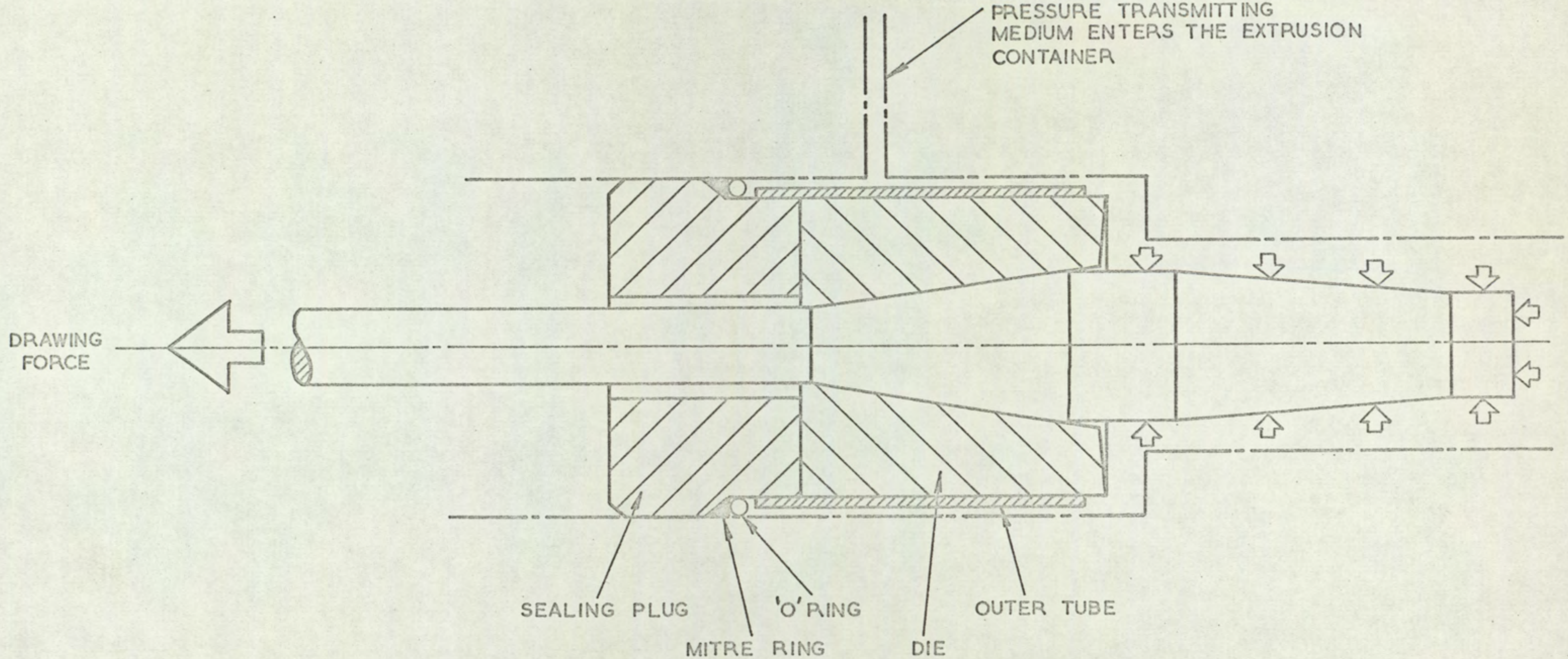


FIG. 20. BILLET AND DIE ARRANGEMENT FOR PRODUCT AUGMENTED EXTRUSION

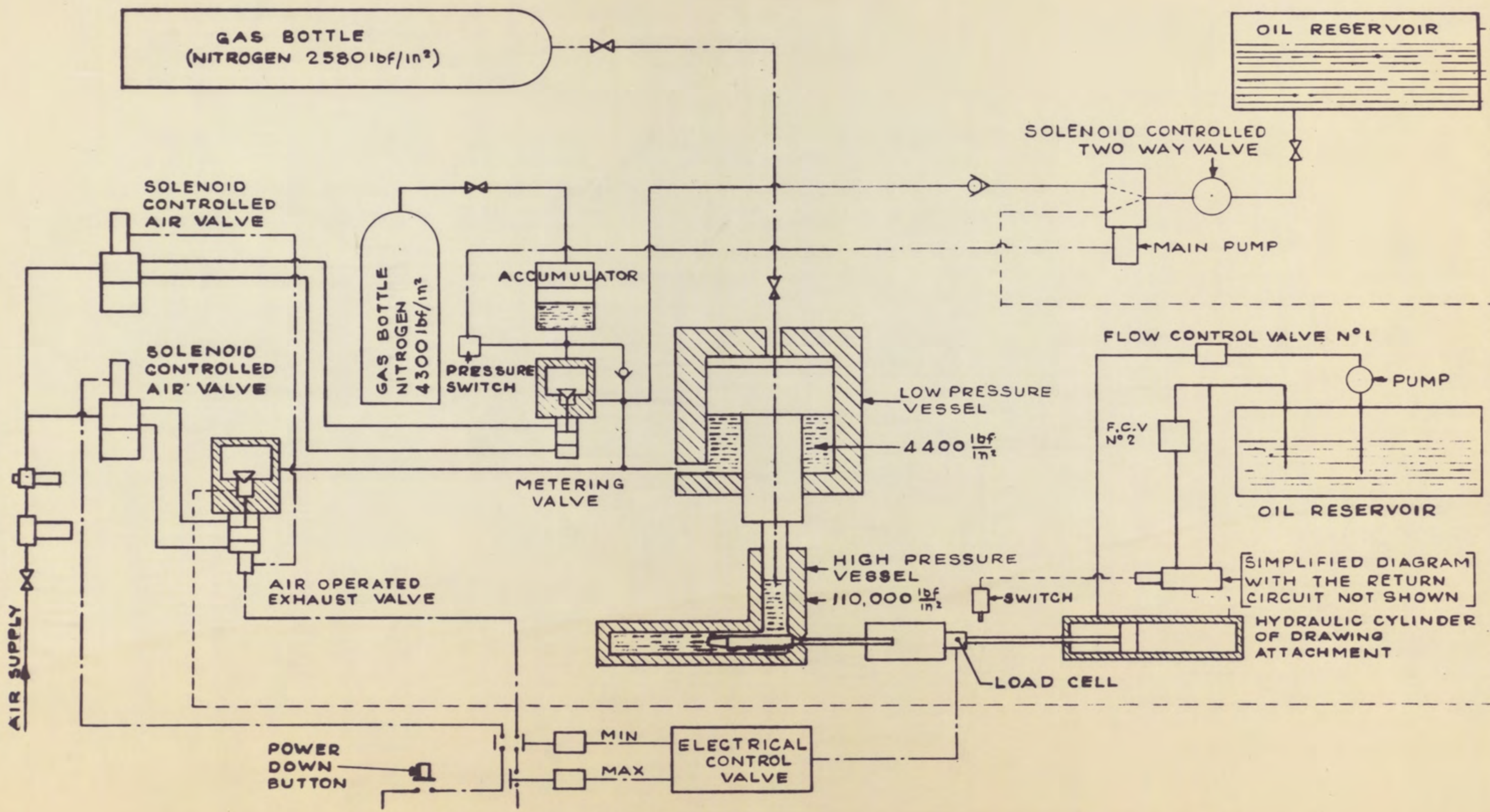


FIG. 21. HYDRAULIC FLOW DIAGRAM FOR THE 200/50 'HYDROSTAT'

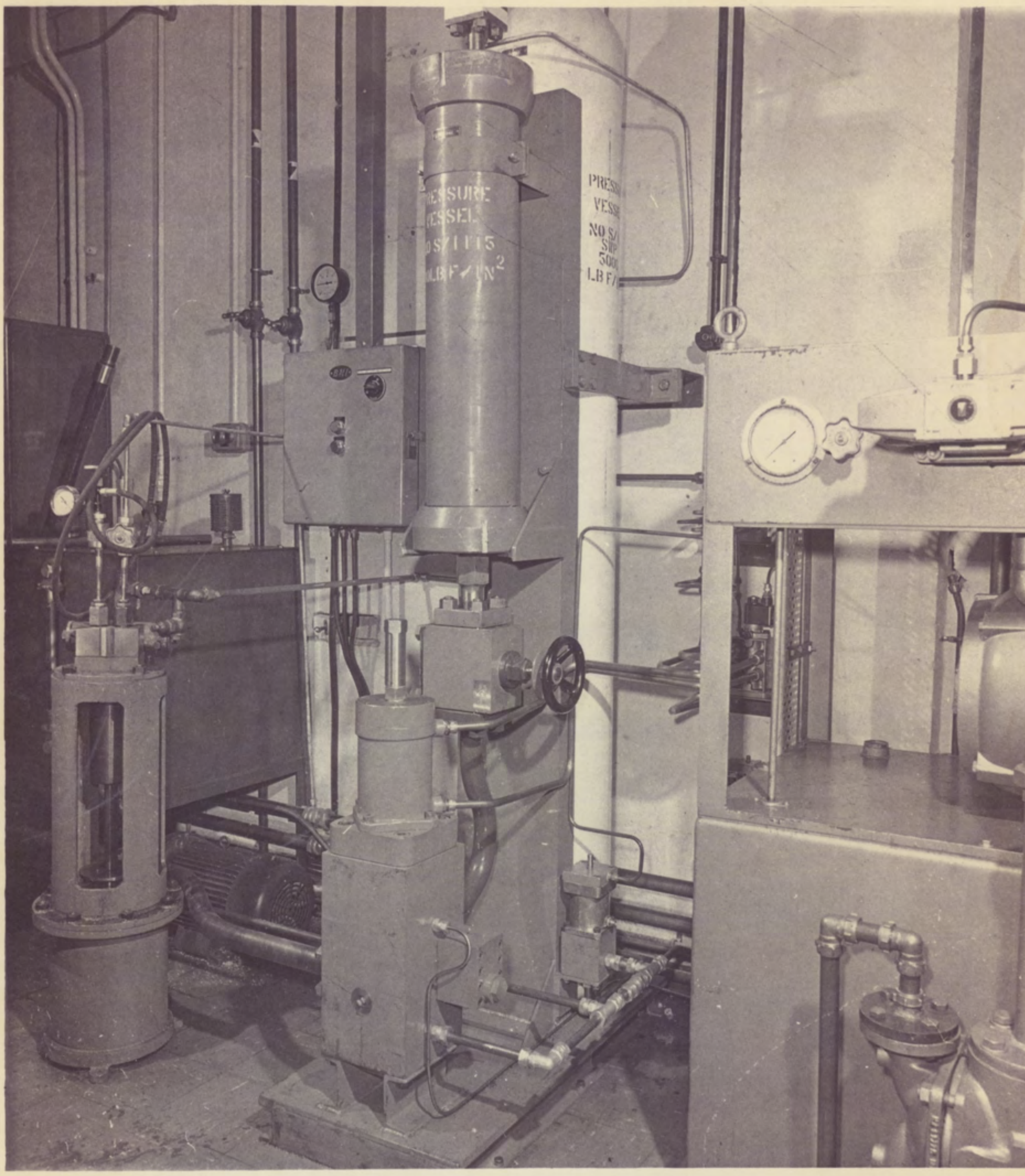


FIG. 22. PRESSURE PHASE-OUT UNIT FOR THE HYDROSTATIC DRAWBENCH

FIG. 23. CONTROL CONSUL OF THE HYDROSTATIC DRAWBENCH

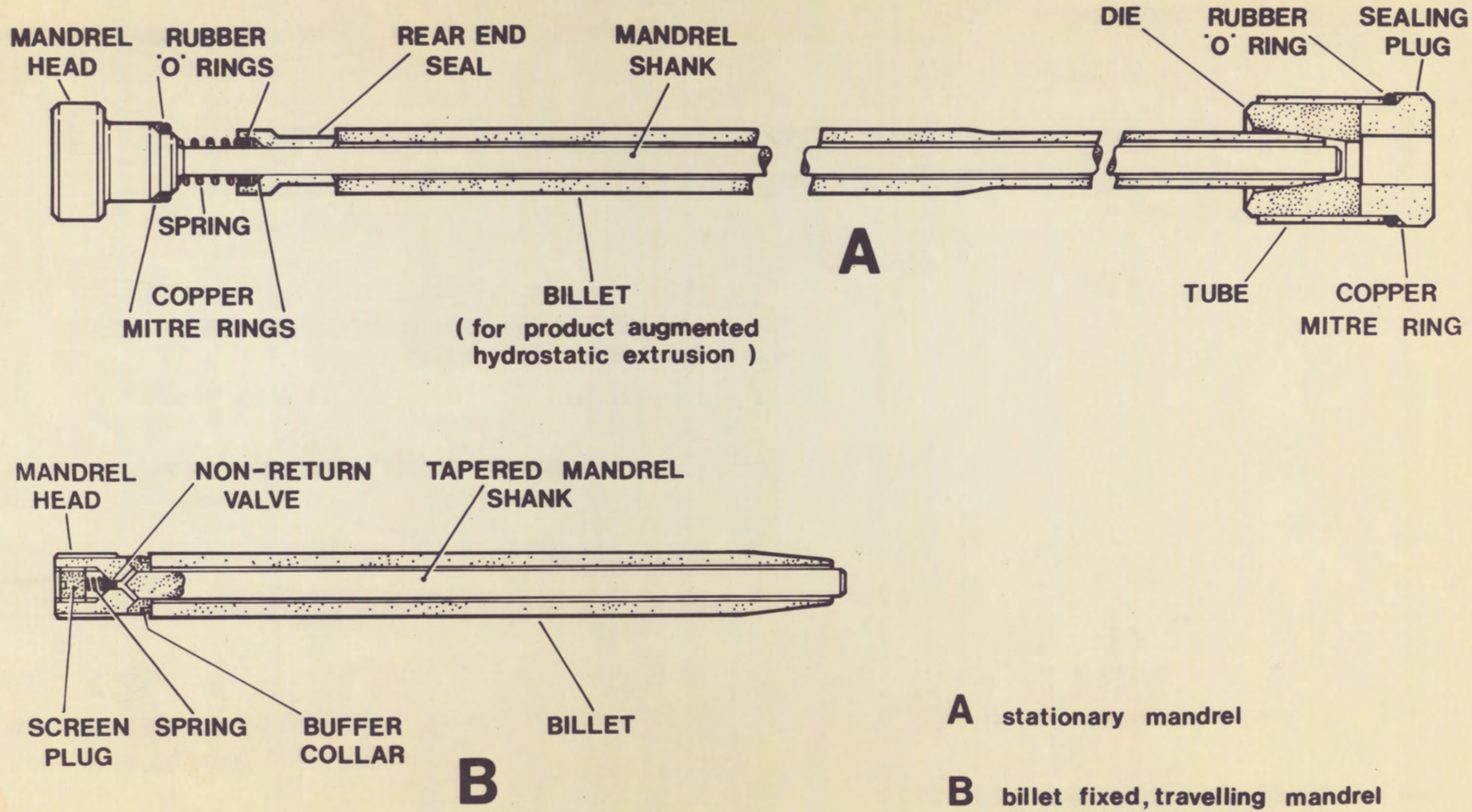


FIG. 24. MANDRELS

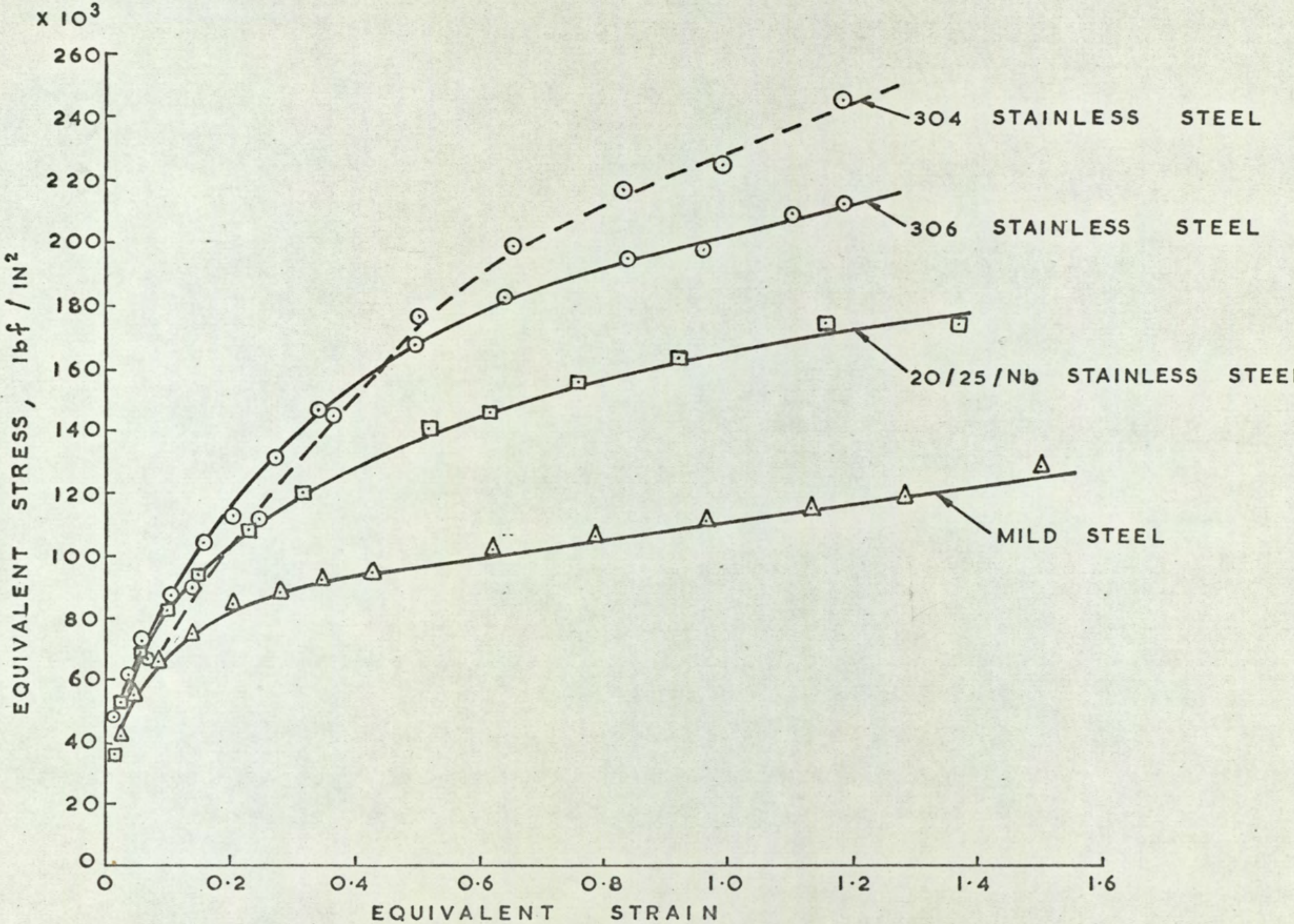


FIG.25. EQUIVALENT STRESS–STRAIN DIAGRAM FOR SOME STEELS.

EMPIRICAL EQUIVALENT STRESS ~ STRAIN LAW

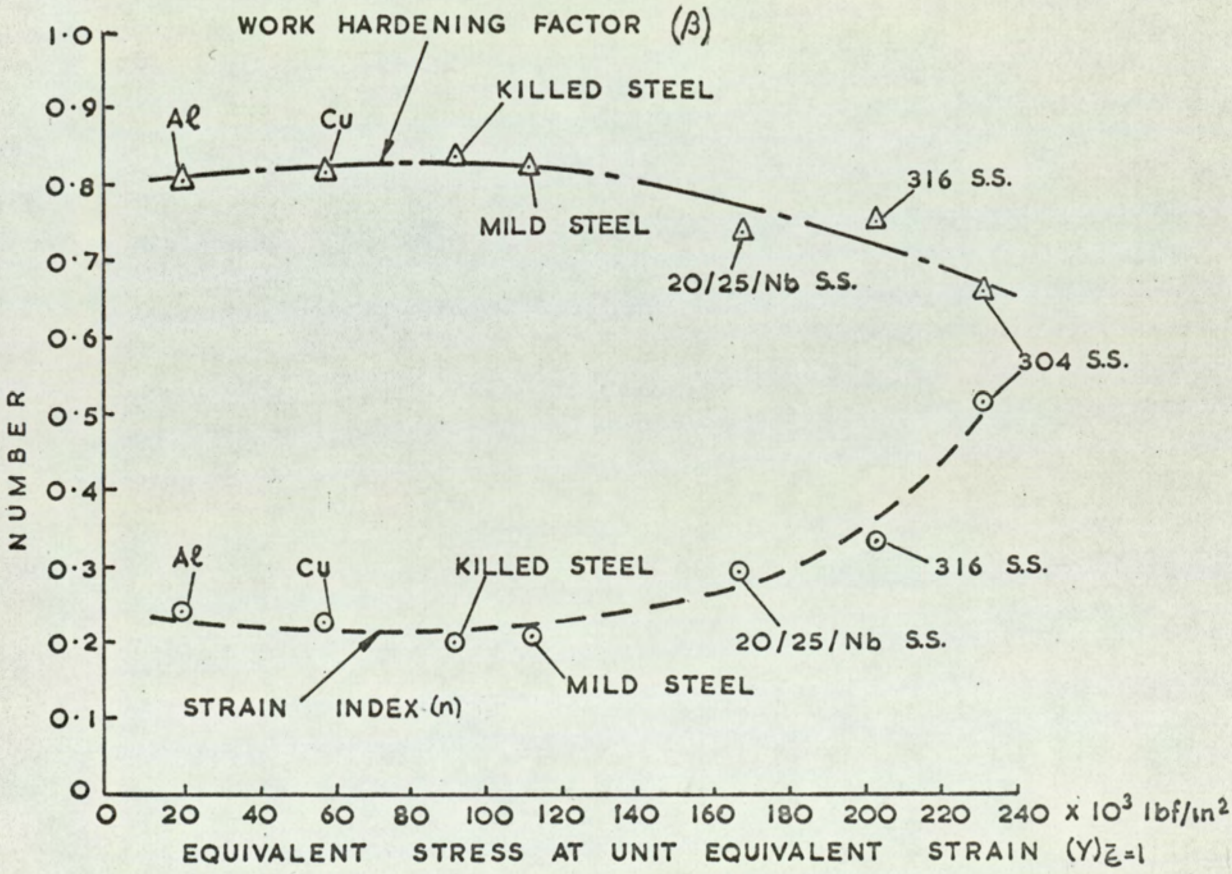
$$\sigma = (Y)_{\bar{\varepsilon}=1}\bar{\varepsilon}^n \qquad 0 < \bar{\varepsilon} < 1\cdot5$$

WORK HARDENING FACTOR β

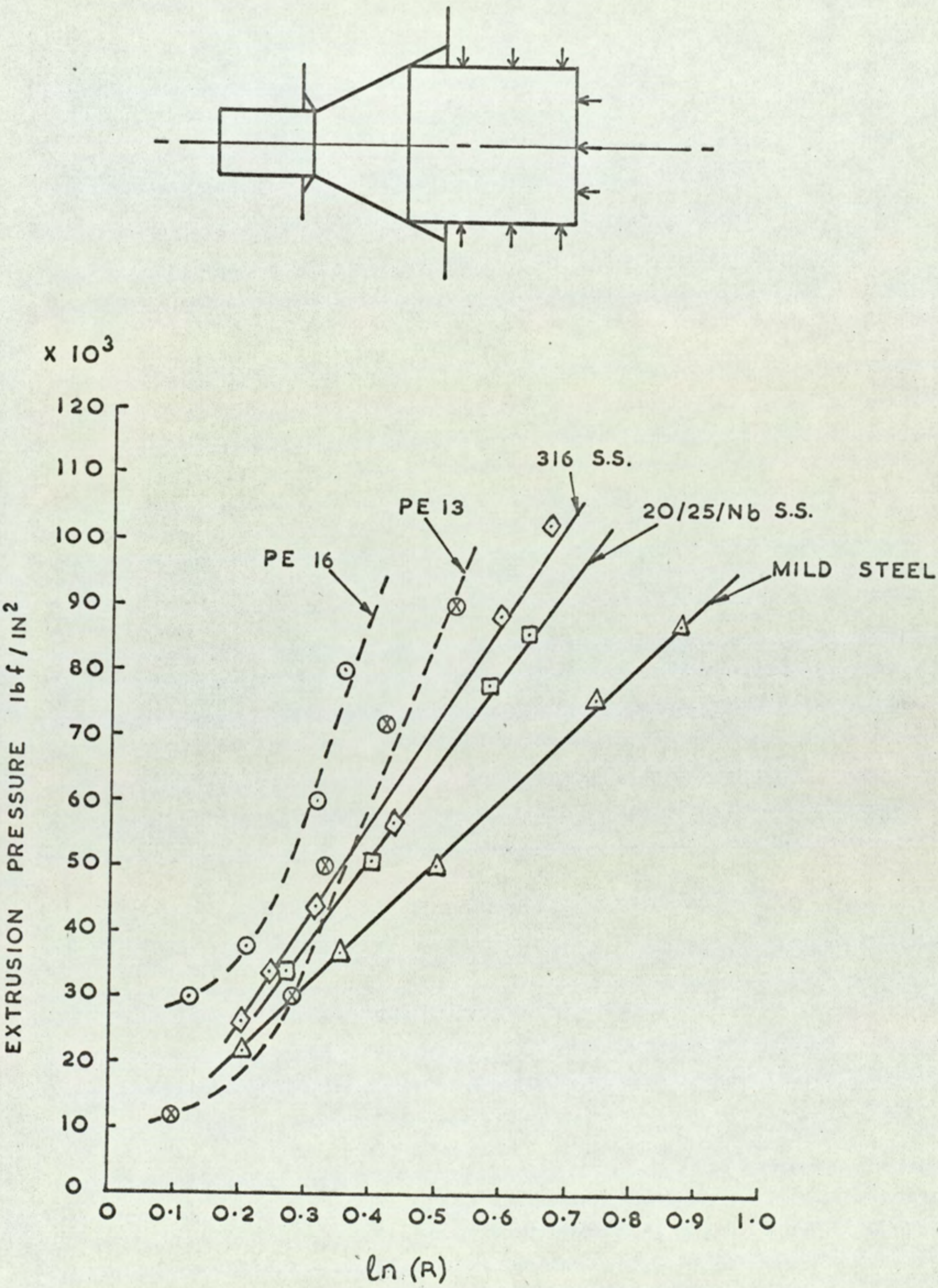
$$\beta = \frac{1}{1+n}$$

MEAN YIELD STRESS Ym

$$Y_m = \beta.(Y)_{\bar{\varepsilon}=1}.\bar{\varepsilon}.$$



G.26. MATERIAL CHARACTERISTICS FOR LARGE STRAINS. AND ANNEALED METALS



27. OIL PRESSURE REQUIRED FOR THE SIMPLE HYDROSTATIC EXTRUSION OF BAR.
(INCLUDED DIE ANGLE 15 DEG.)

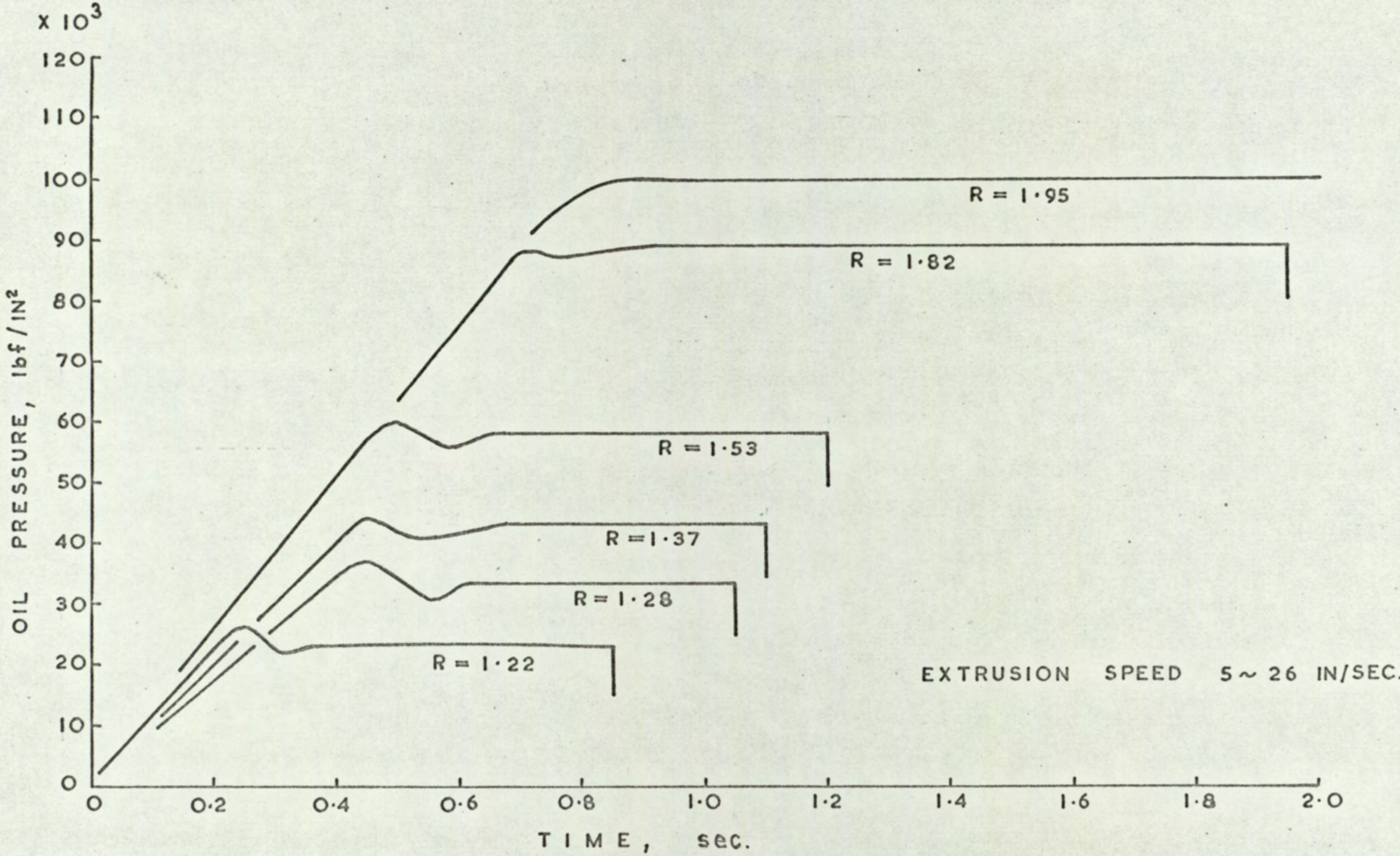


FIG. 28. OIL PRESSURE v TIME CHARACTERISTICS FOR THE SIMPLE HYDROSTATIC EXTRUSION OF A.I.S.I. 316 BAR.

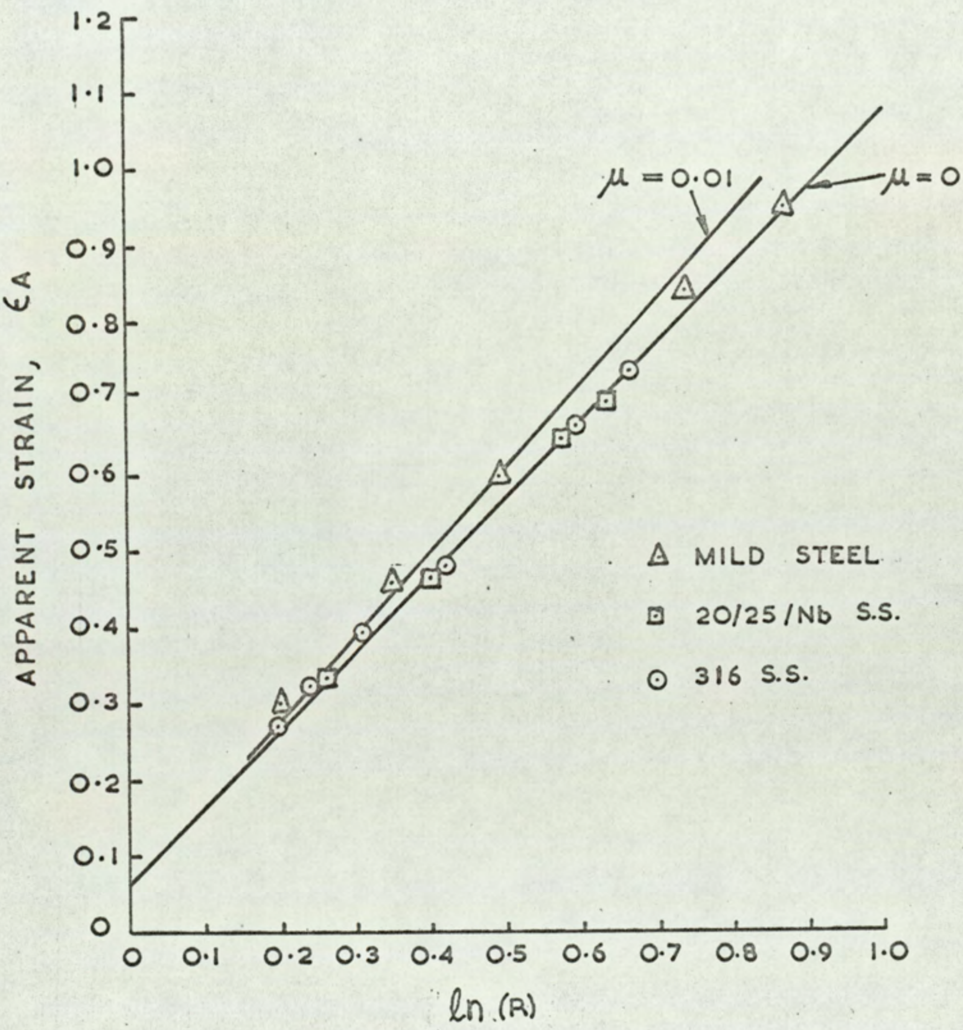


FIG. 29. APPARENT STRAIN INDUCED BY THE SIMPLE HYDROSTATIC EXTRUSION OF STEEL BAR. (INCLUDED DIE ANGLE 15 DEG.)

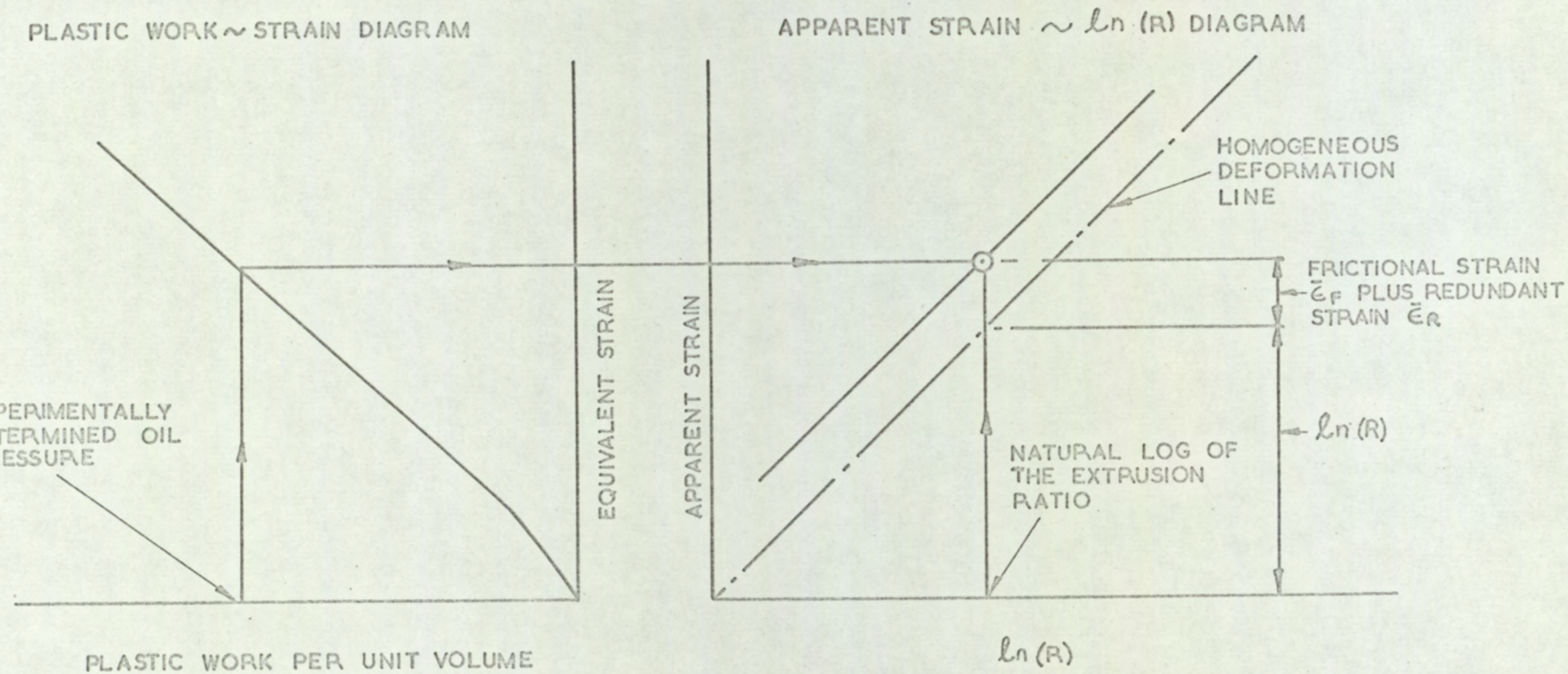
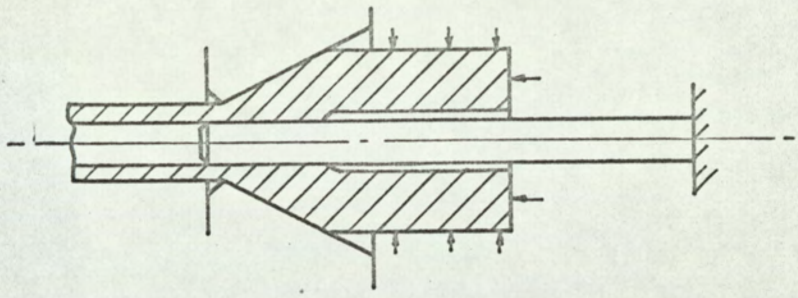


FIG.30. METHOD FOR CONSTRUCTING THE APPARENT STRAIN ~ ln(R) DIAGRAM



INCLUDED DIE ANGLE 15 DEG.
BILLET DIAMETER K = 1·76 ~ 2·0

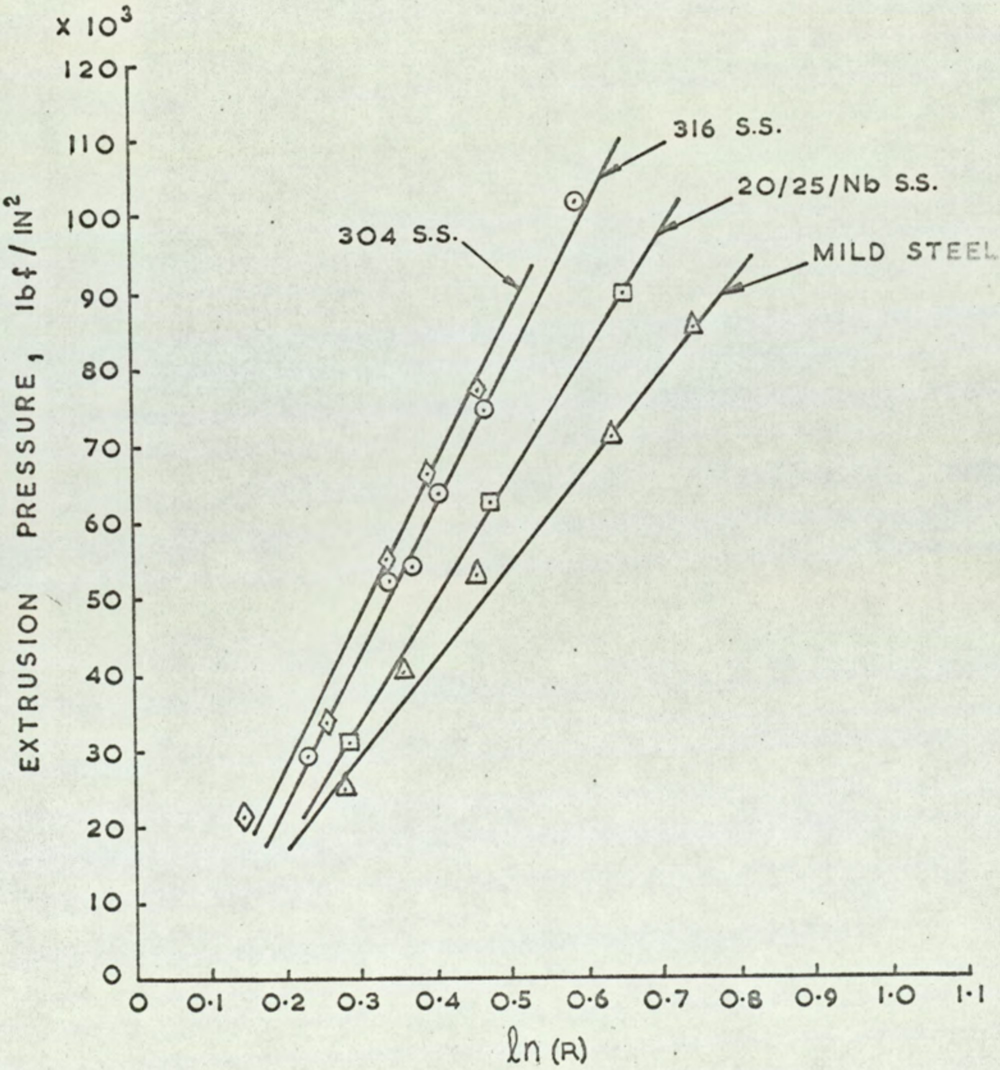
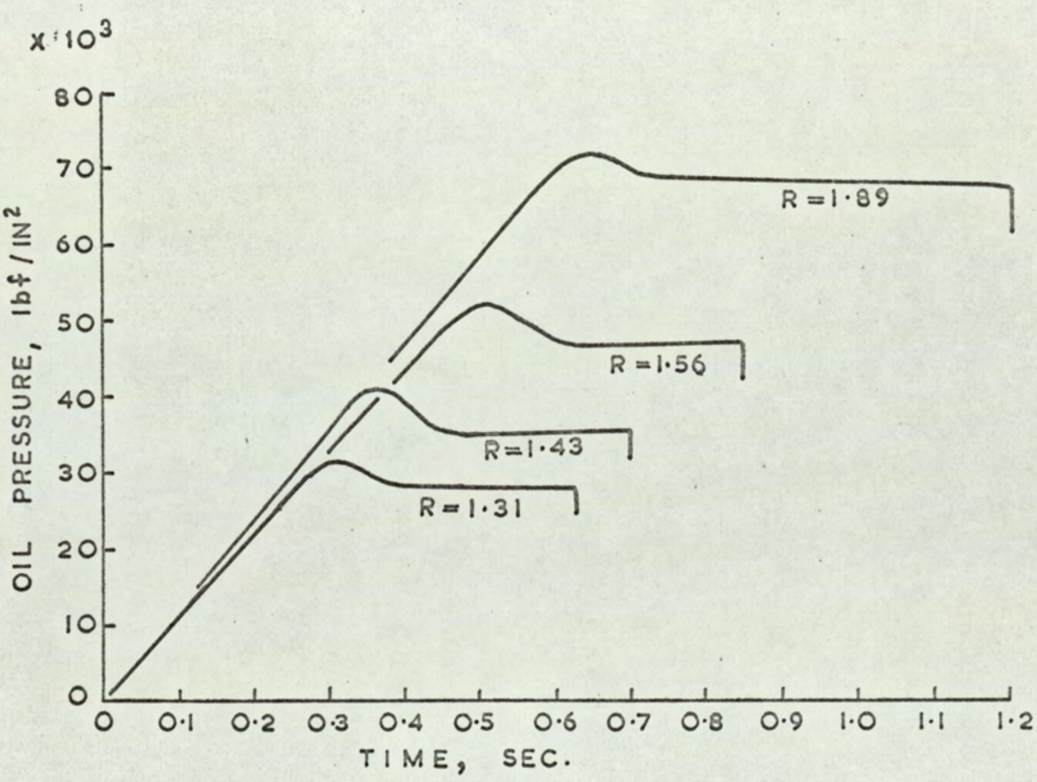


FIG. 31. OIL PRESSURES REQUIRED FOR THE SIMPLE HYDROSTATIC EXTRUSION OF TUBE.

EXTRUSION SPEED 21 ~ 36 IN/SEC.
INCLUDED DIE ANGLE 15 DEG.
BILLET DIAMETER RATIO 1·94



G.32. OIL PRESSURE ~ TIME CHARACTERISTICS FOR THE SIMPLE HYDROSTATIC EXTRUSION OF MILD STEEL TUBE.

BILLET DIA. RATIO K = 1·76 ~ 2·9
INCLUDED DIE ANGLE 15 DEG.

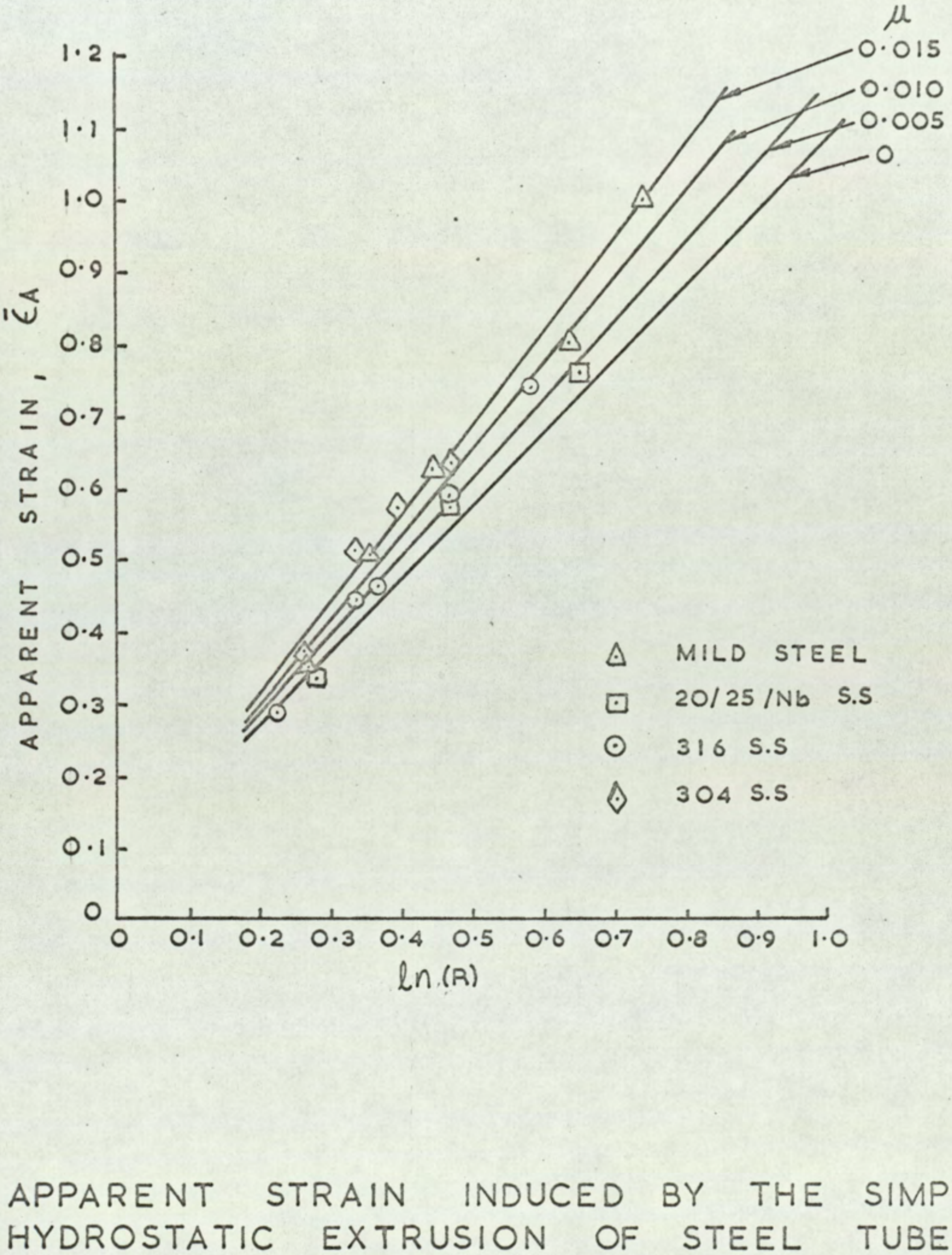


FIG.33. APPARENT STRAIN INDUCED BY THE SIMPLE HYDROSTATIC EXTRUSION OF STEEL TUBE.

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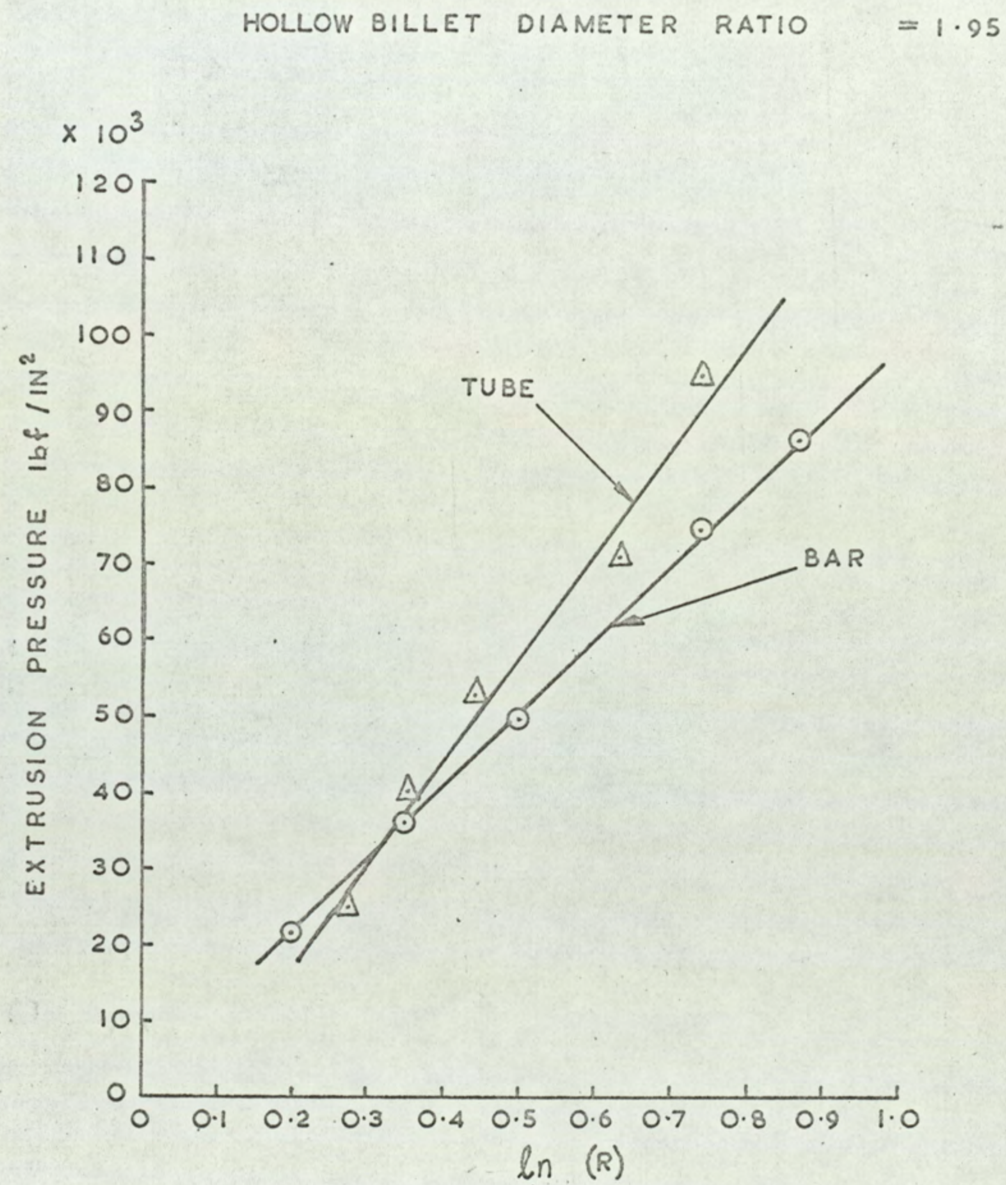


FIG.34. COMPARISON BETWEEN THE EXTRUSION PRESSURES FOR THE SIMPLE HYDROSTATIC EXTRUSION OF MILD STEEL TUBE & BAR.

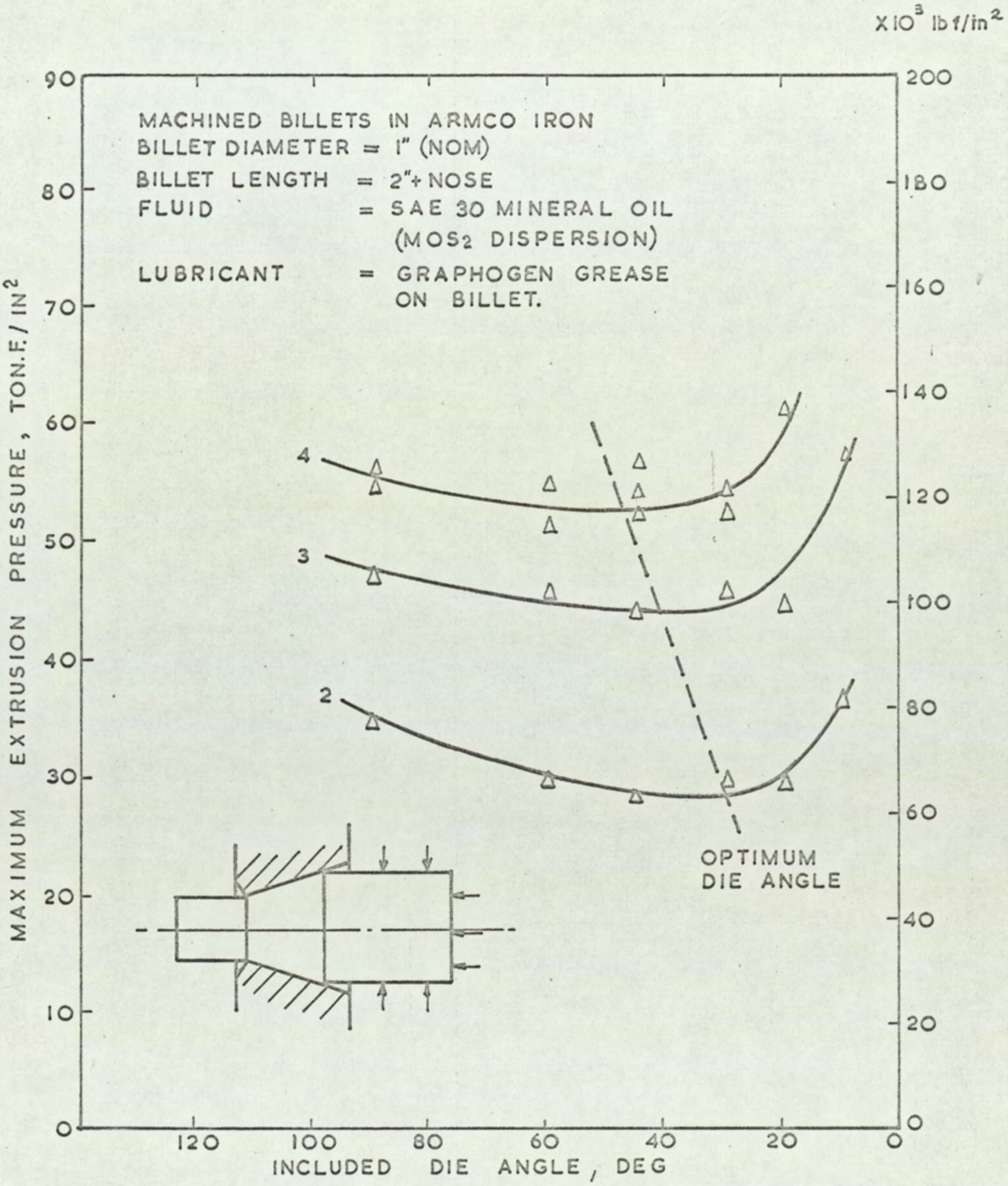


FIG. 35. THE EFFECTS OF DIE ANGLE AND EXTRUSION RATIO ON THE OIL PRESSURE REQUIRED FOR THE SIMPLE HYDROSTATIC EXTRUSION OF BAR

THIS DIAGRAM WAS TAKEN FROM 'RECENT DEVELOPMENTS IN METAL FORMING' BY PUGH, BULLEID MEMORIAL LECTURES, VOL. III B, NOTTINGHAM UNIVERSITY

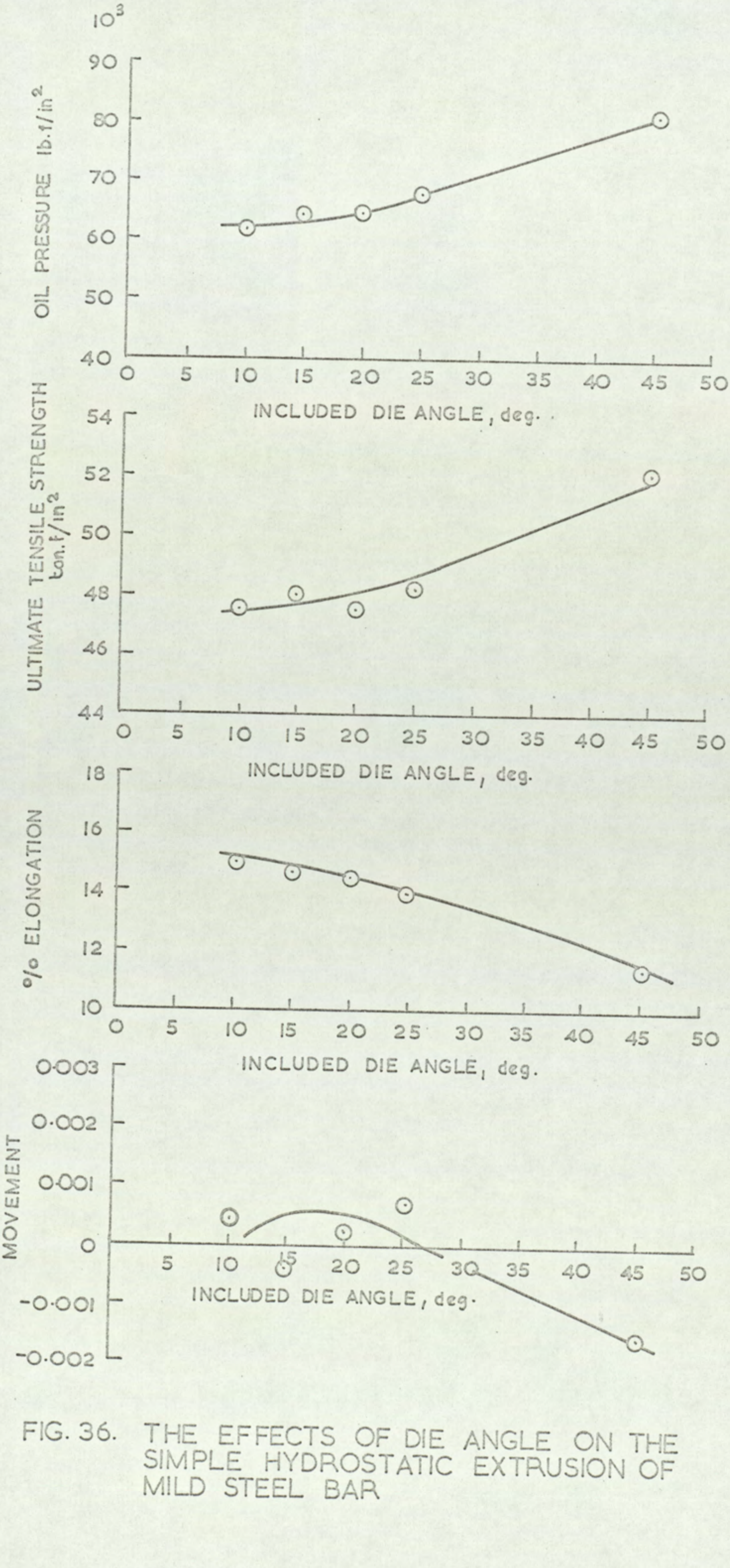
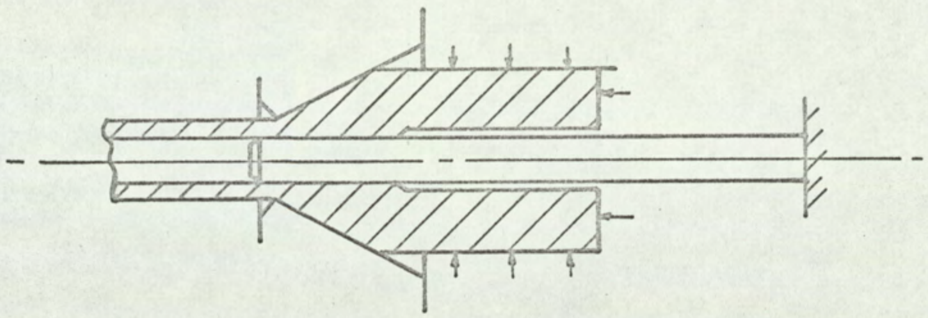


FIG. 36. THE EFFECTS OF DIE ANGLE ON THE SIMPLE HYDROSTATIC EXTRUSION OF MILD STEEL BAR



BILLET DIAMETER RATIO K = 1·76

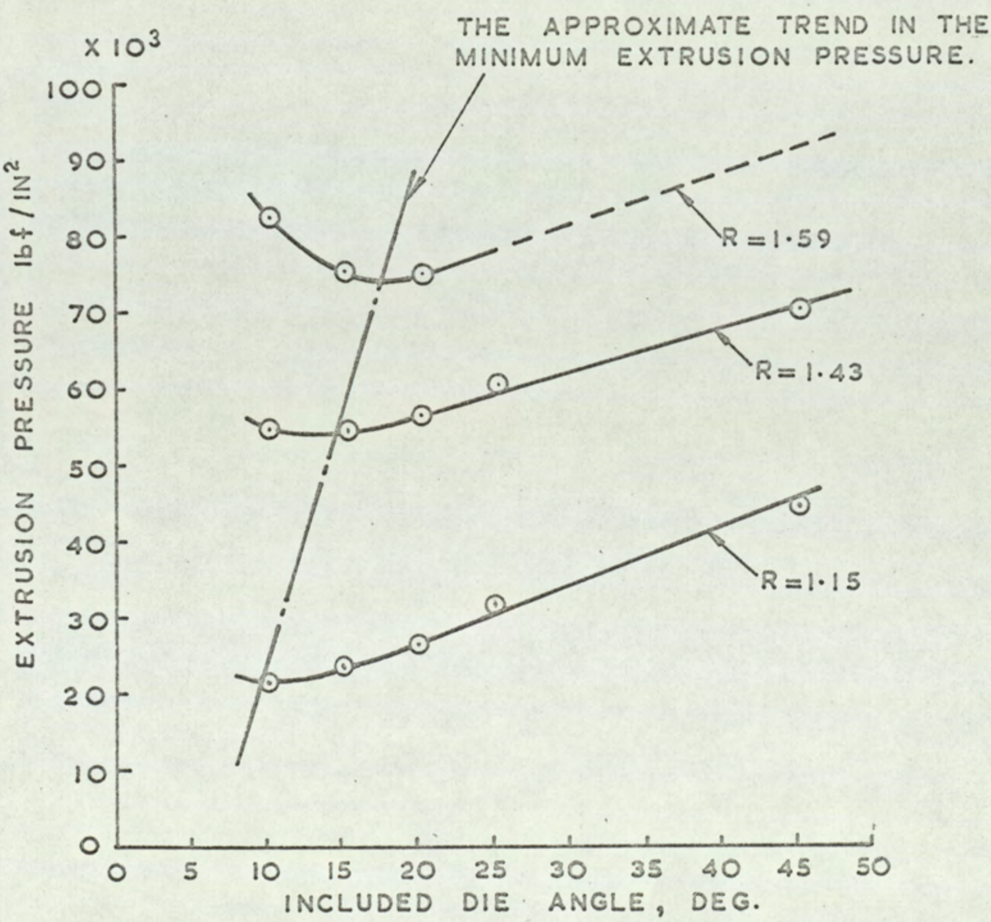
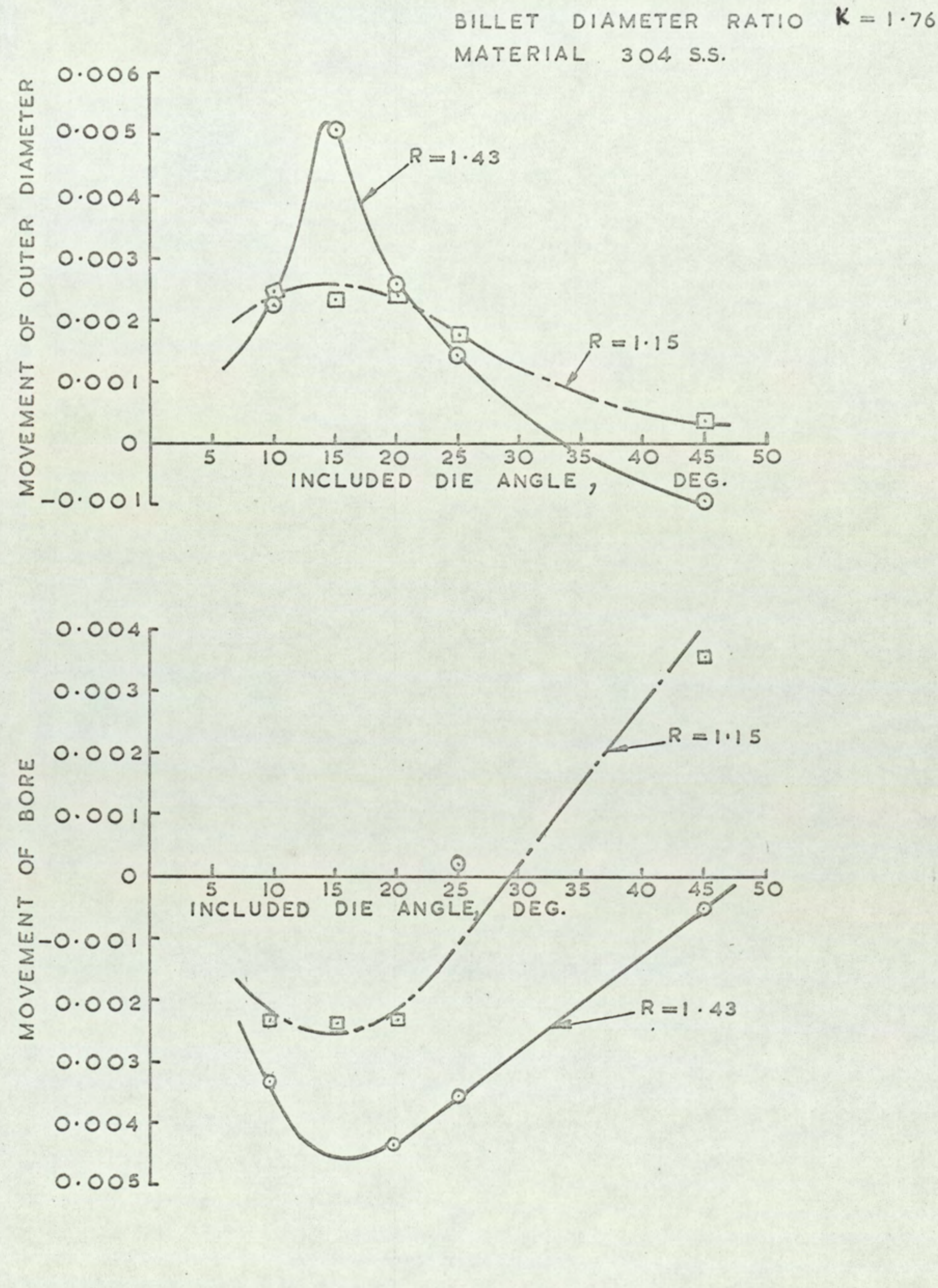


FIG.37. THE EFFECT OF DIE ANGLE AND EXTRUSION RATIO ON THE EXTRUSION PRESSURE FOR THE SIMPLE HYDROSTATIC EXTRUSION OF 304 S.S. TUBE



$$\text{MOVEMENT} = \frac{\text{PRODUCT SIZE} - \text{TOOL SIZE}}{\text{TOOL SIZE}}$$

FIG.38. COMPARISON BETWEEN PRODUCT AND TOOL DIMENSIONS FOR THE SIMPLE HYDROSTATIC EXTRUSION OF TUBE.

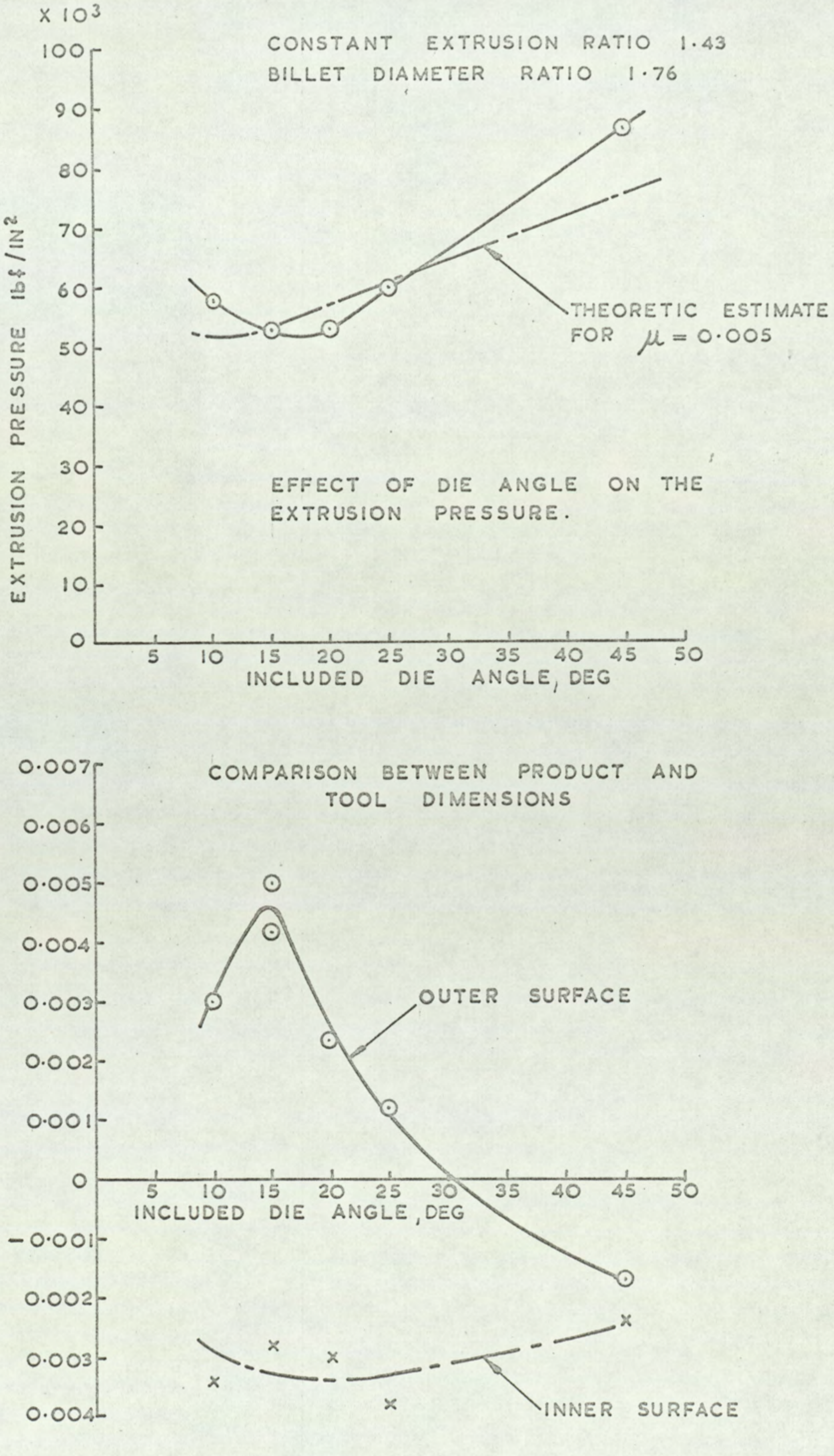


FIG.39. EFFECT OF DIE ANGLE ON THE SIMPLE HYDROSTATIC EXTRUSION OF 316 STAINLESS STEEL.

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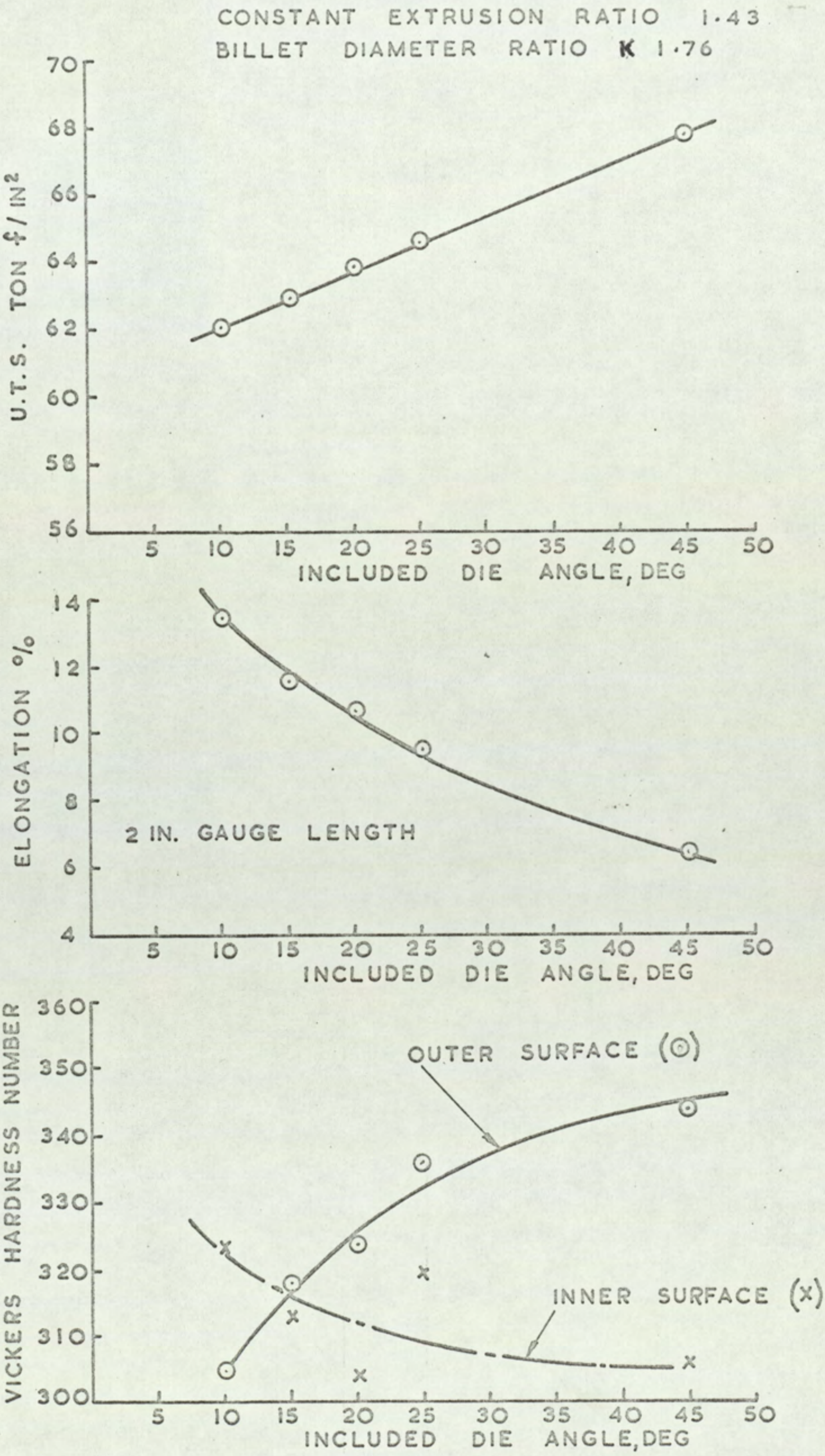
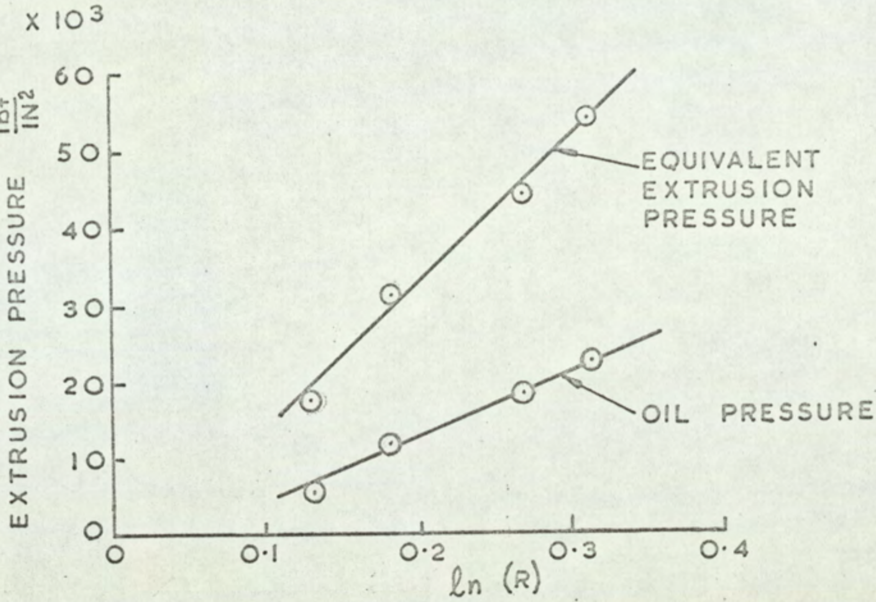
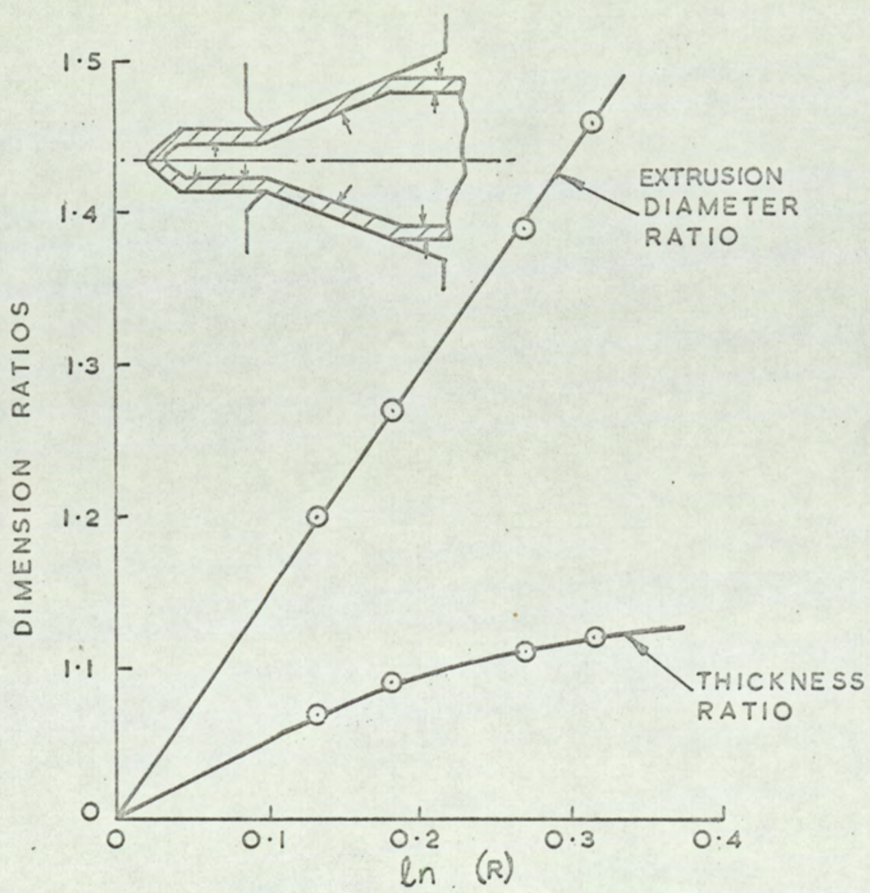


FIG.40. EFFECT OF DIE ANGLE ON THE MECHANICAL PROPERTIES OF 316 S.S. TUBE PRODUCED BY SIMPLE HYDROSTATIC EXTRUSION.





BILLET O/D = 0·838 in.
DIE ANGLE 15°
INITIAL THICKNESS = 0·0615 in.

FIG 41. THE HYDROSTATIC EXTRUSION OF ANNEALED 18/8 S.S. TUBE WITHOUT A MANDREL AND WITH THE FRONT END SEALED.

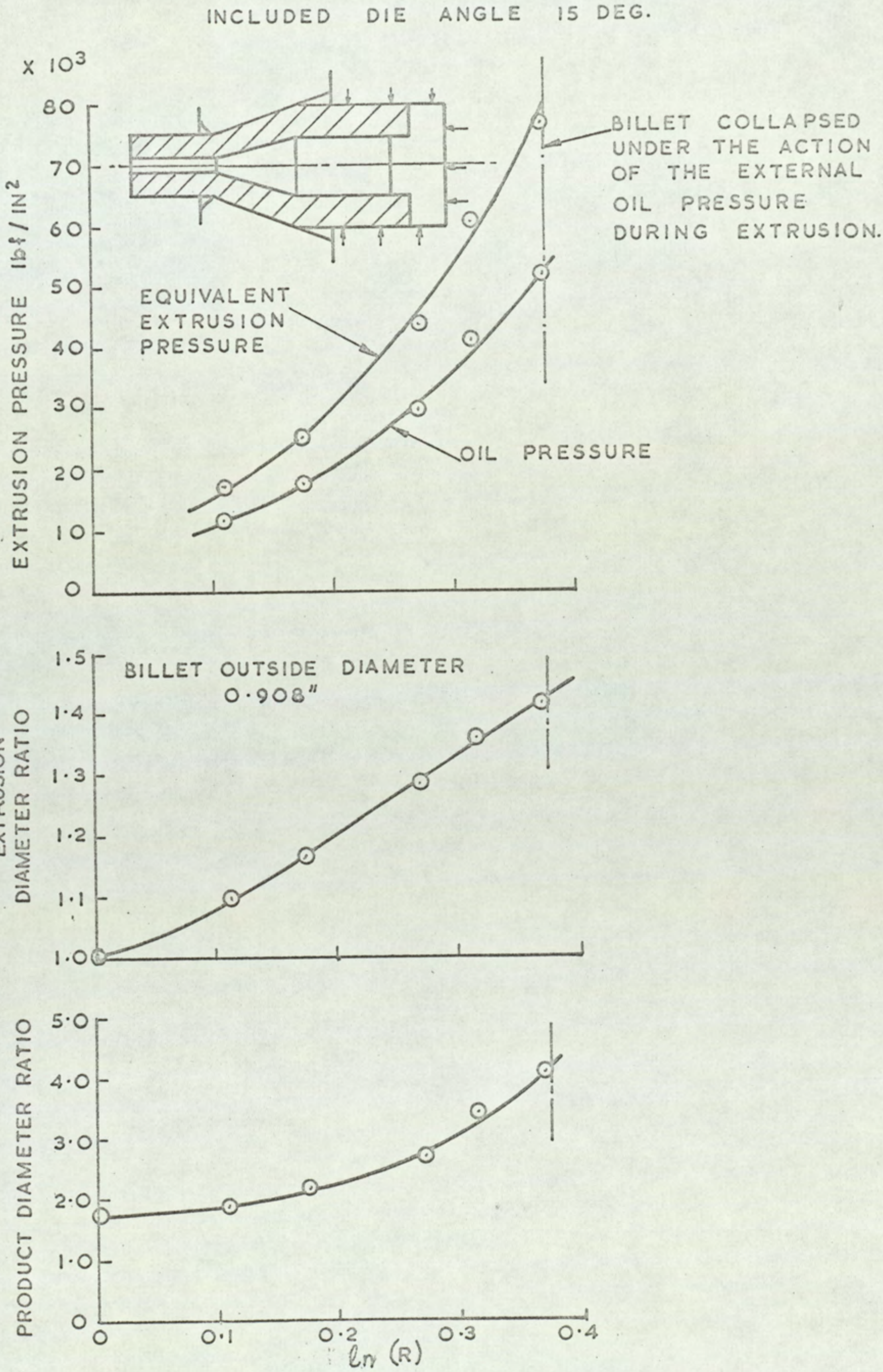
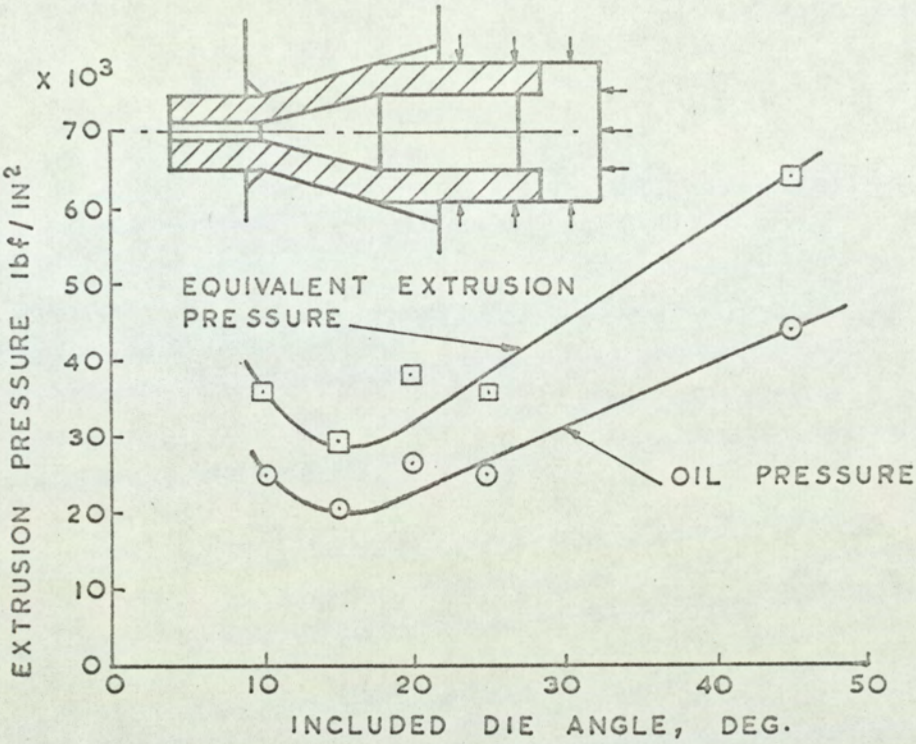
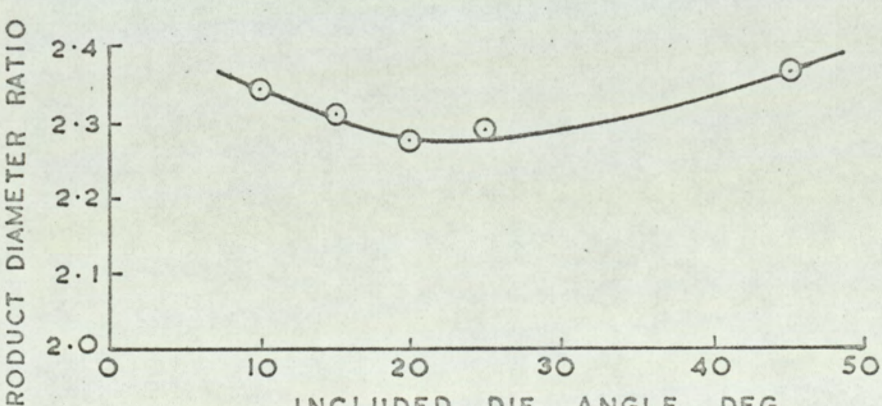


FIG.42. THE HYDROSTATIC EXTRUSION OF 20/25/Nb STAINLESS STEEL WITHOUT A MANDREL AND THE REAR END SEALED.





EXTRUSION DIAMETER REDUCTION = 1·21

BILLET DIAMETER RATIO = 1·76

FIG.43. THE EFFECT OF DIE ANGLE ON THE HYDROSTATIC EXTRUSION OF 20/25/Nb STAINLESS STEEL TUBE WITHOUT A MANDREL, AND THE REAR END SEALED.

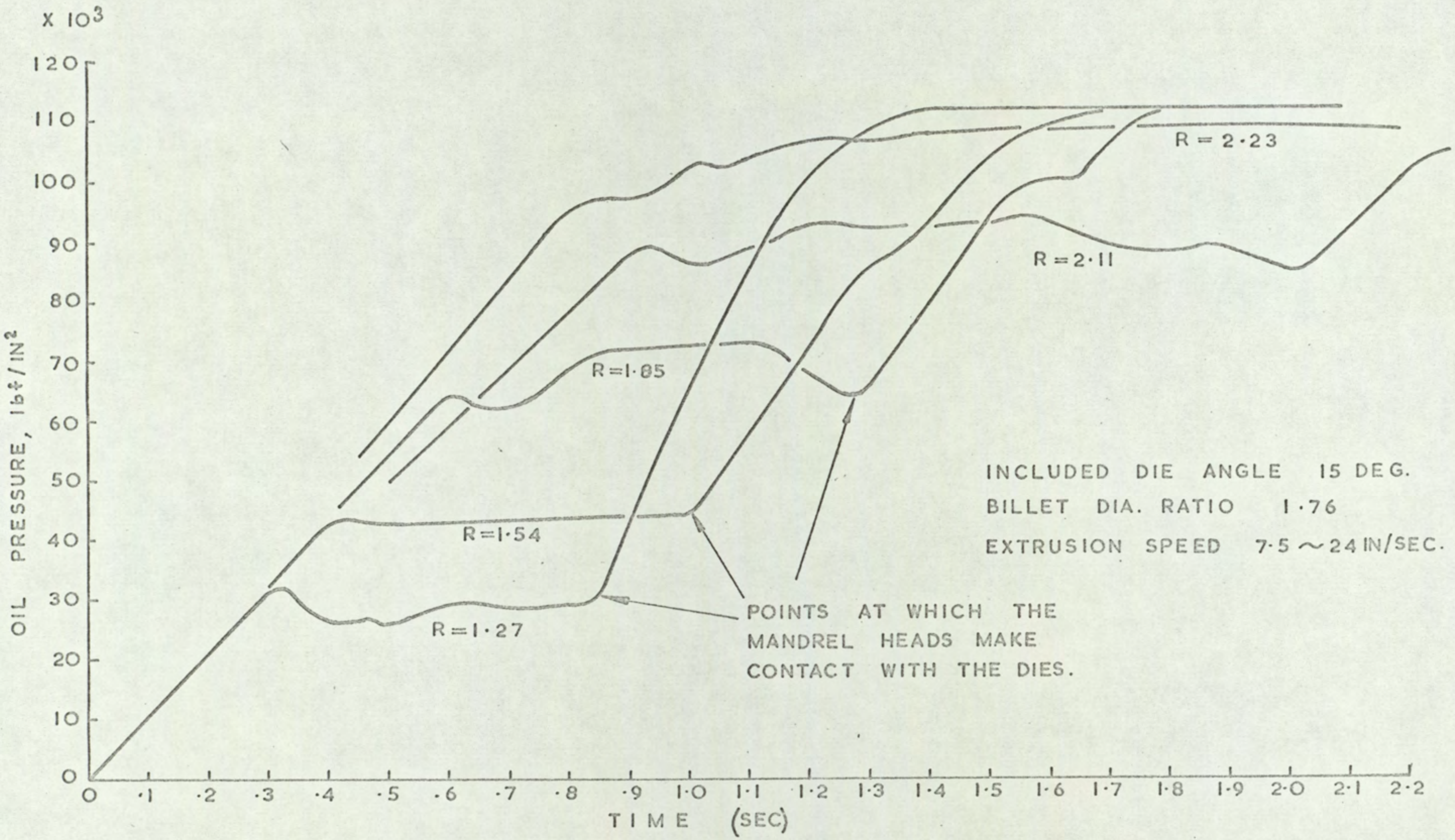


FIG.44. OIL PRESSURE ~ TIME CHARACTERISTICS FOR THE PROPORTIONAL AUGMENTED HYDROSTATIC EXTRUSION OF 20/25/Nb STAINLESS STEEL TUBE.

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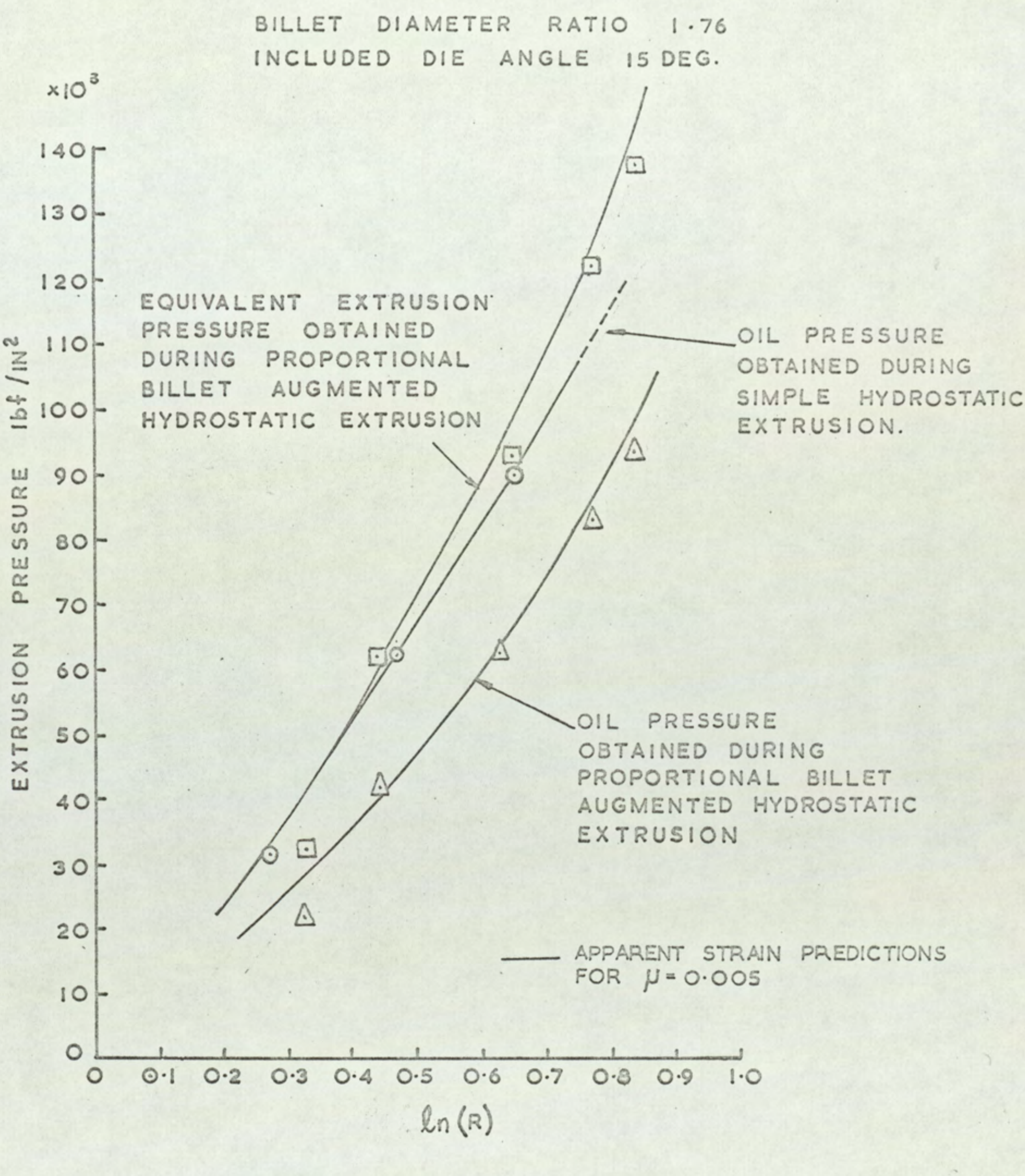


FIG 45. COMPARISON BETWEEN THE EXTRUSION PRESSURES OBTAINED DURING THE SIMPLE HYDROSTATIC EXTRUSION AND PROPORTIONAL BILLET AUGMENTED HYDROSTATIC EXTRUSION, WHEN THE AUGMENTING STRESSES ARE SMALL, FOR 20/25/Nb STAINLESS STEEL TUBE.

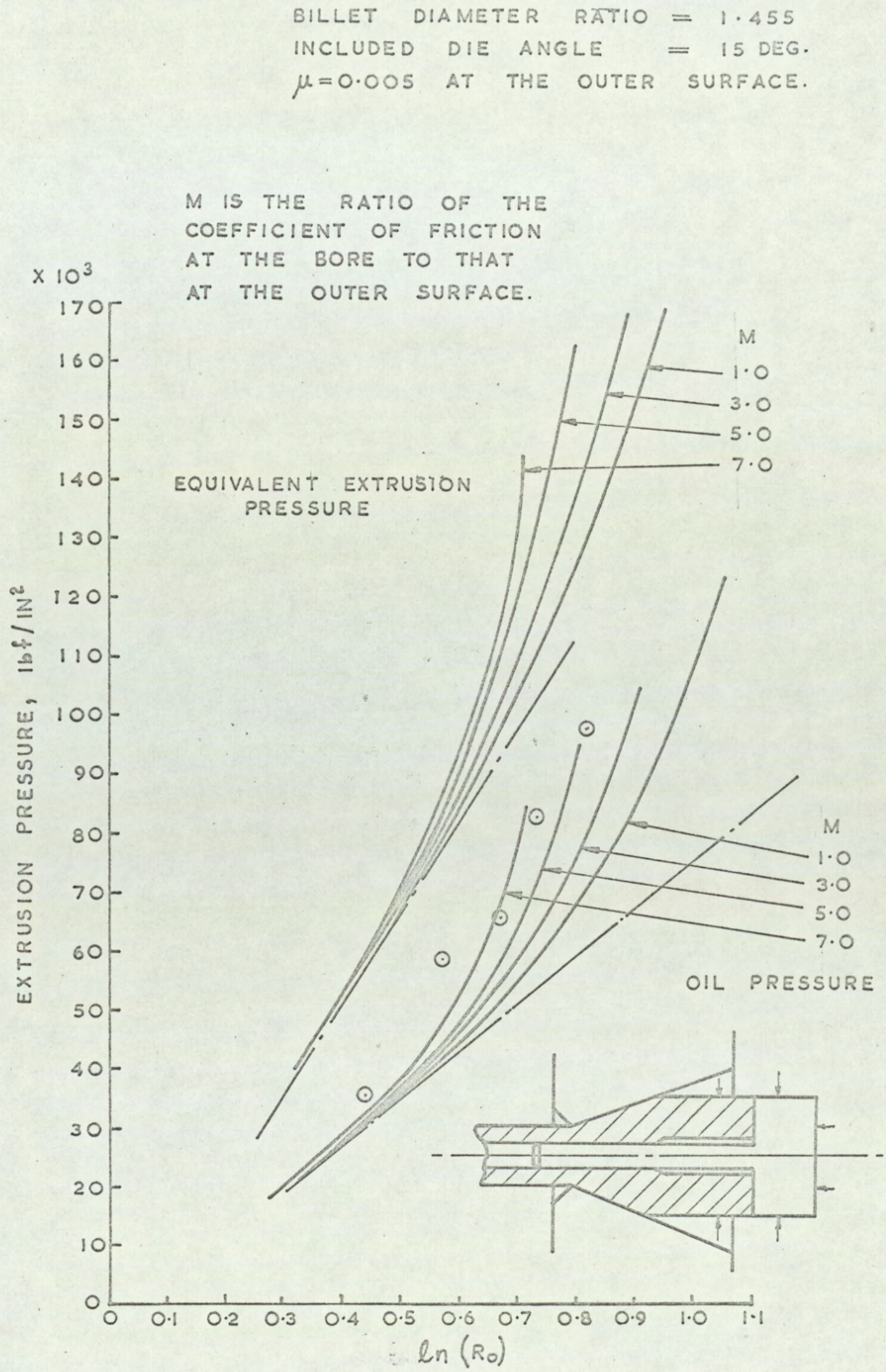


FIG. 46. EXTRUSION PRESSURE FOR THE HYDROSTATIC EXTRUSION OF 20/25/Nb TUBE OVER A BILLET FIXED TRAVELLING MANDREL.

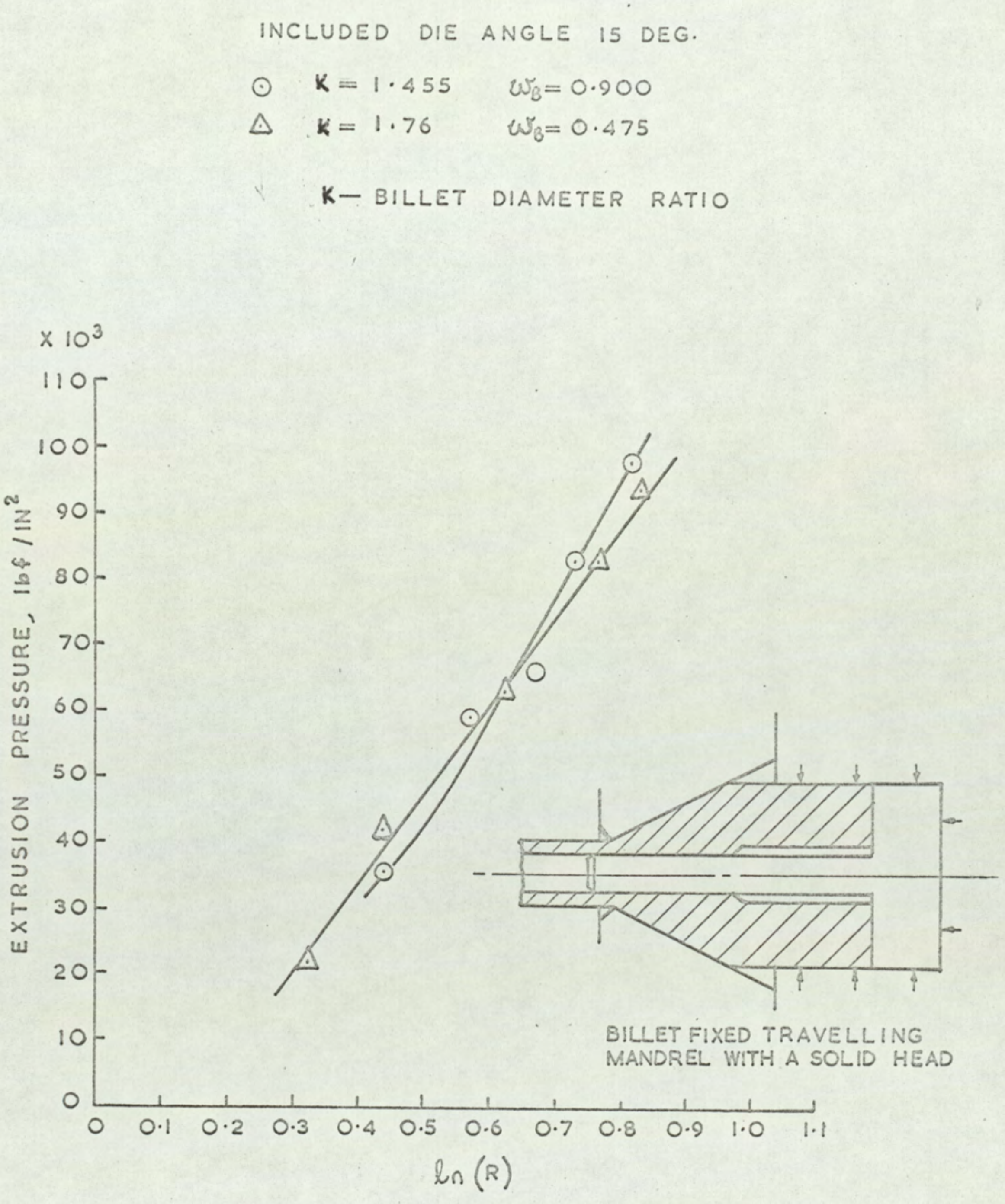


FIG.47. OIL PRESSURES FOR THE PROPORTIONAL BILLET AUGMENTED HYDROSTATIC EXTRUSION OF 20/25/Nb STAINLESS STEEL TUBE.

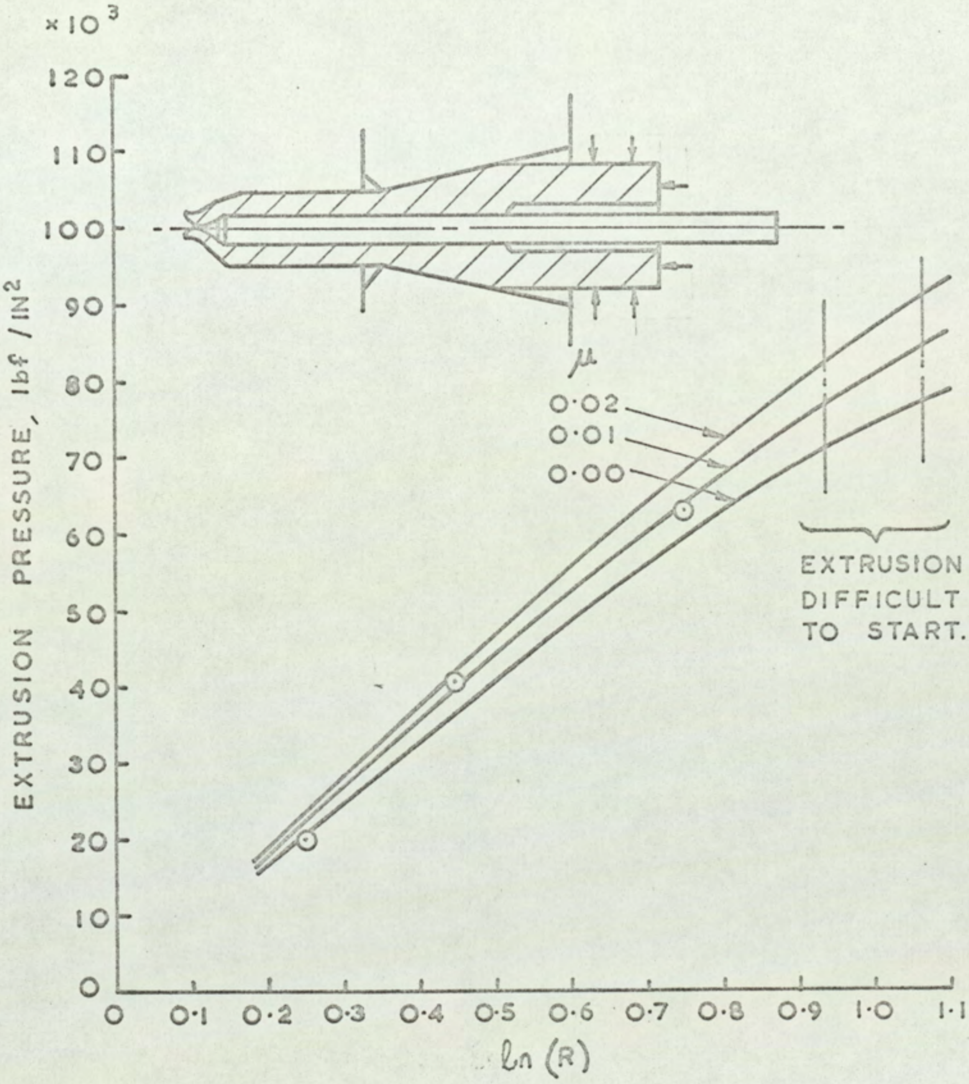


FIG48. OIL PRESSURE FOR THE HYDROSTATIC EXTRUSION OF 316 STAINLESS STEEL TUBE OVER A PRODUCT-FIXED TRAVELLING MANDREL.

BILLET DIAMETER RATIO = 1·76
INCLUDED DIE ANGLE 15 DEG.

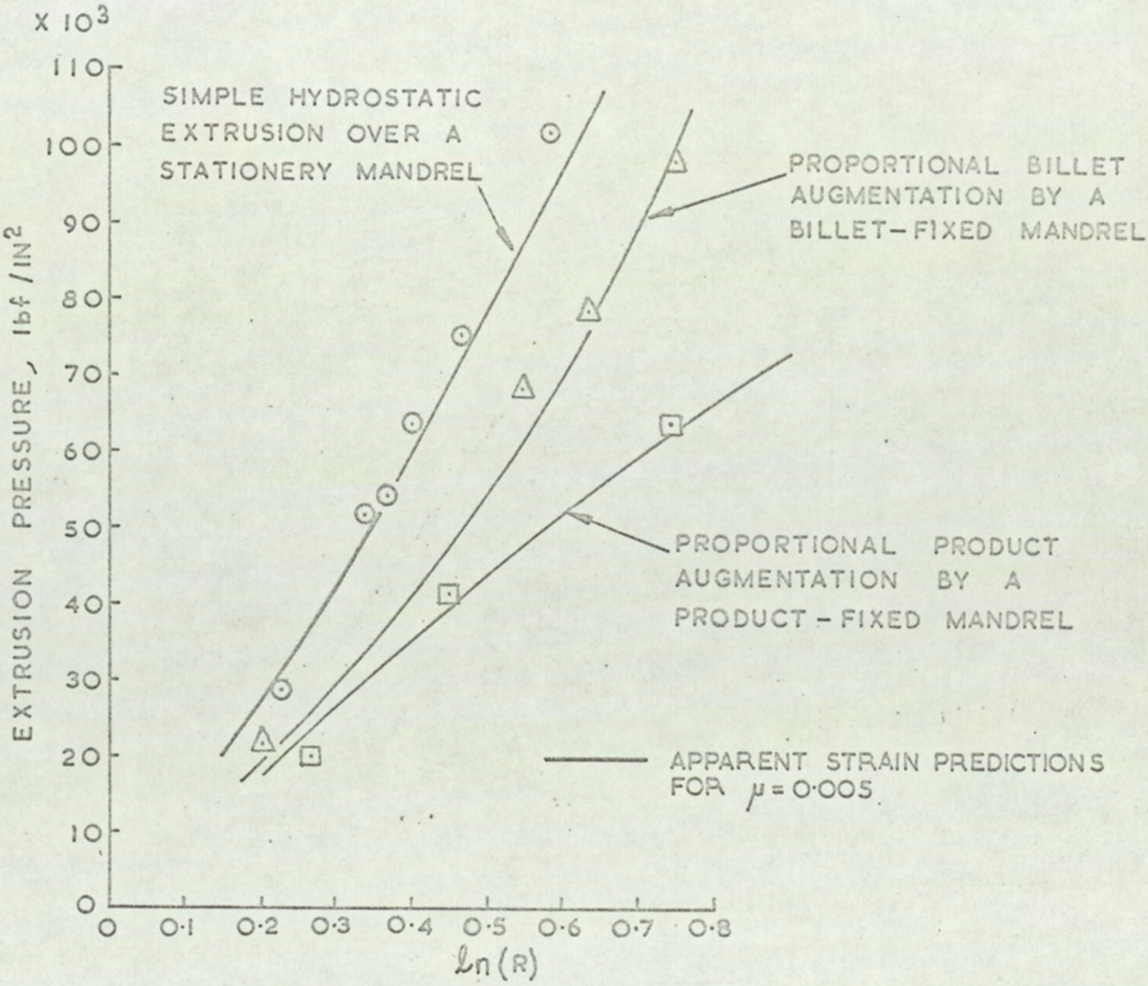


FIG 49. THE COMPARISON OF EXTRUSION PRESSURES FOR THE HYDROSTATIC EXTRUSION OF 316 STAINLESS STEEL TUBE BY THE THREE PRINCIPAL MANDREL DESIGNS.

TRG REPORT 1664 (S)

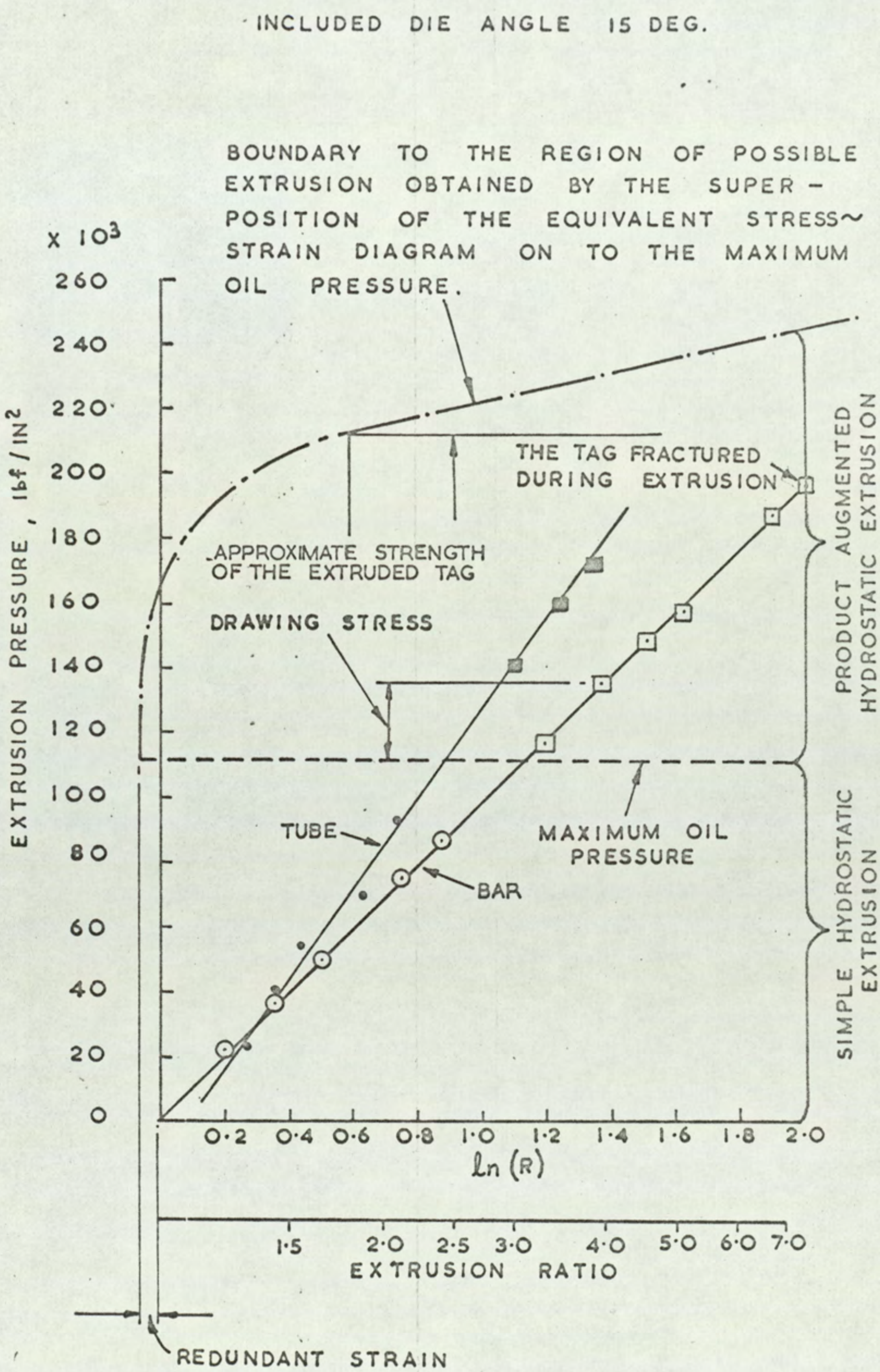


FIG.50. COMPARISON OF EXTRUSION PRESSURES OBTAINED BY BOTH SIMPLE AND PRODUCT AUGMENTED HYDROSTATIC EXTRUSION OF MILD STEEL BAR AND TUBE

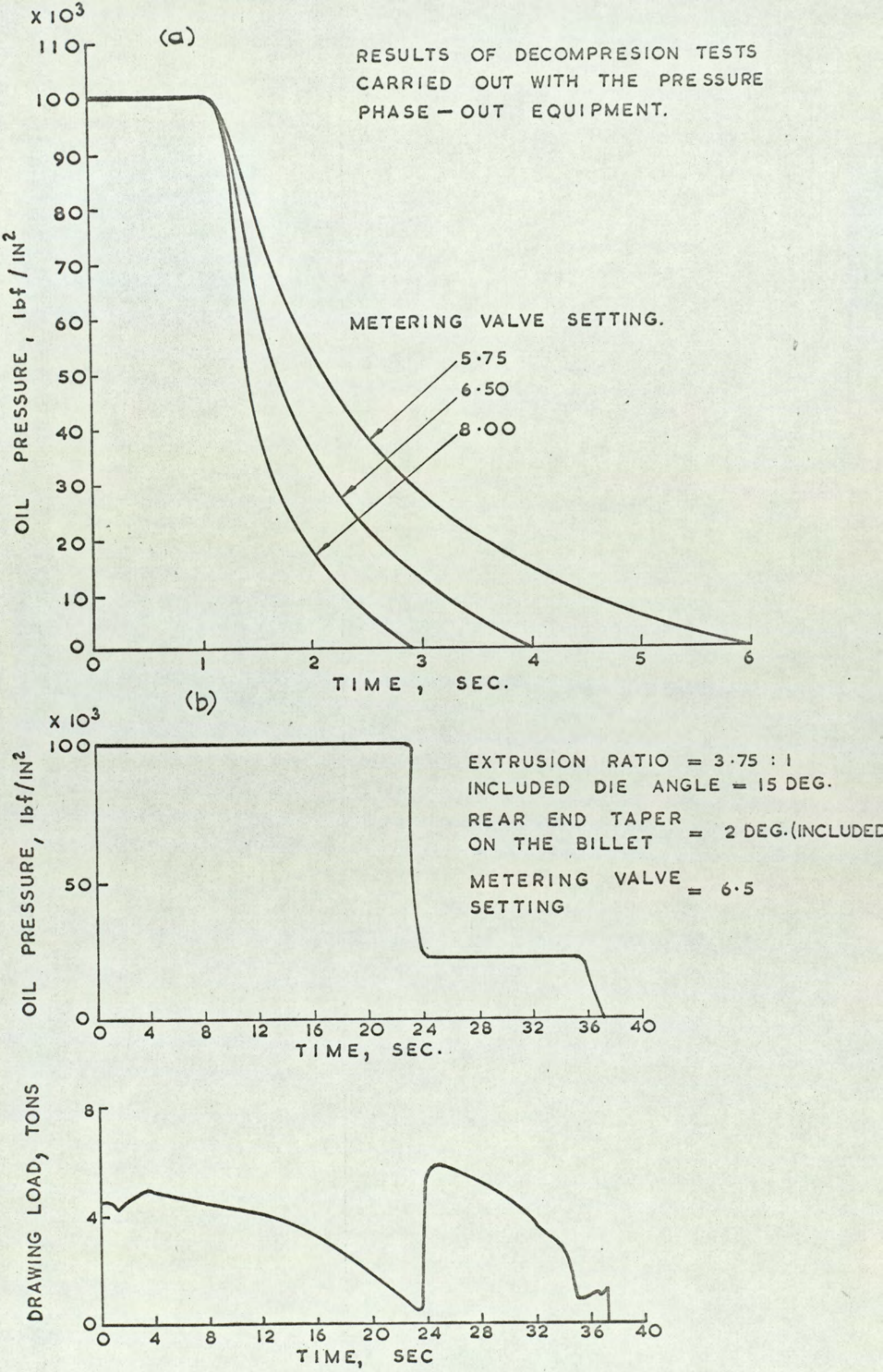


FIG.51. COMPLETE EXTRUSION OF MILD STEEL BAR BY PRODUCT AUGMENTED HYDROSTATIC EXTRUSION USING THE PRESSURE PHASE-OUT METHOD.

TRG REPORT 1664 (S)

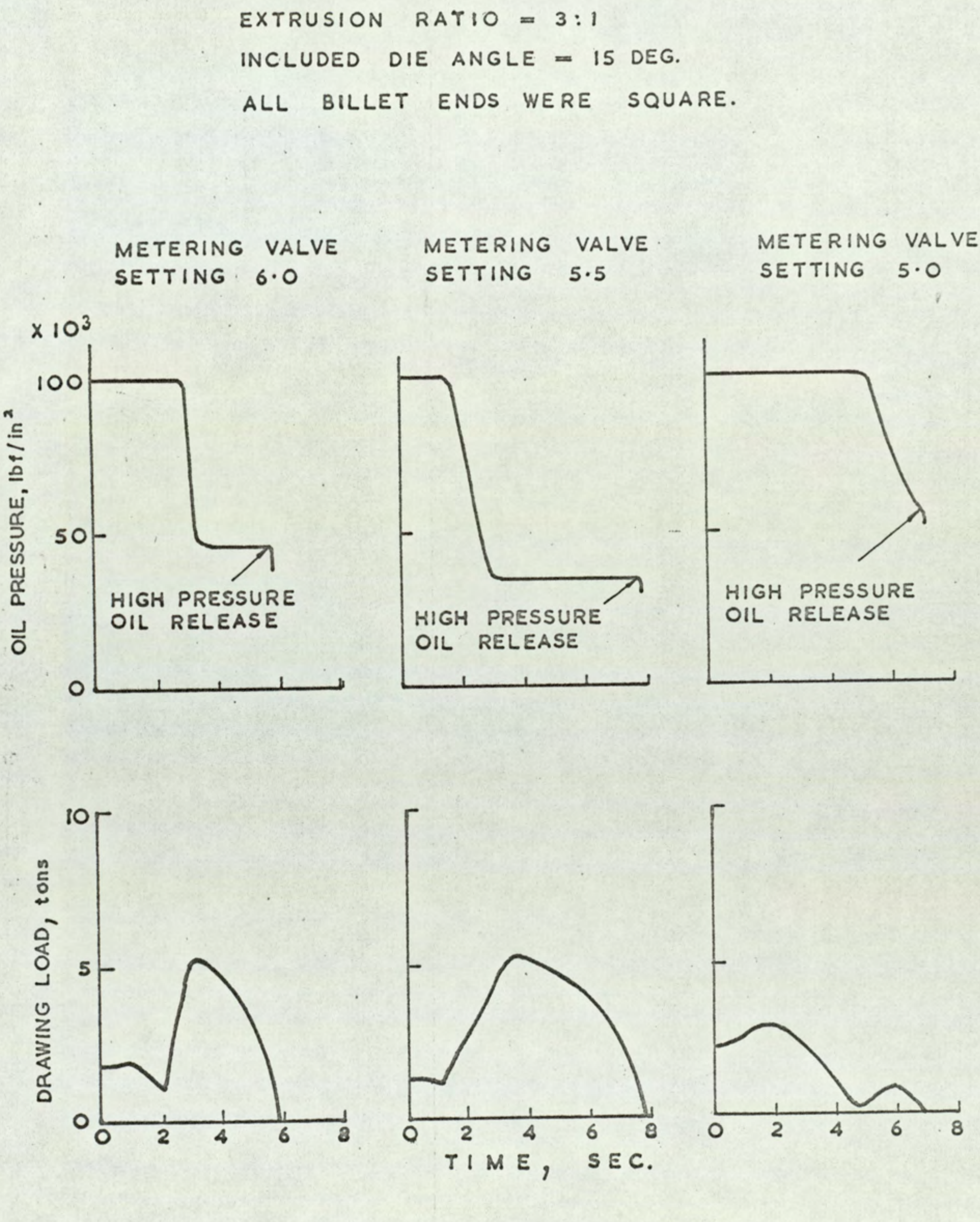


FIG.52. ATTEMPTED COMPLETE EXTRUSION OF MILD STEEL TUBE THROUGH A SIMPLE CONE DIE BY PRODUCT AUGMENTED HYDROSTATIC EXTRUSION USING THE PRESSURE PHASE – OUT METHOD.

TRG REPORT 1664 (S)

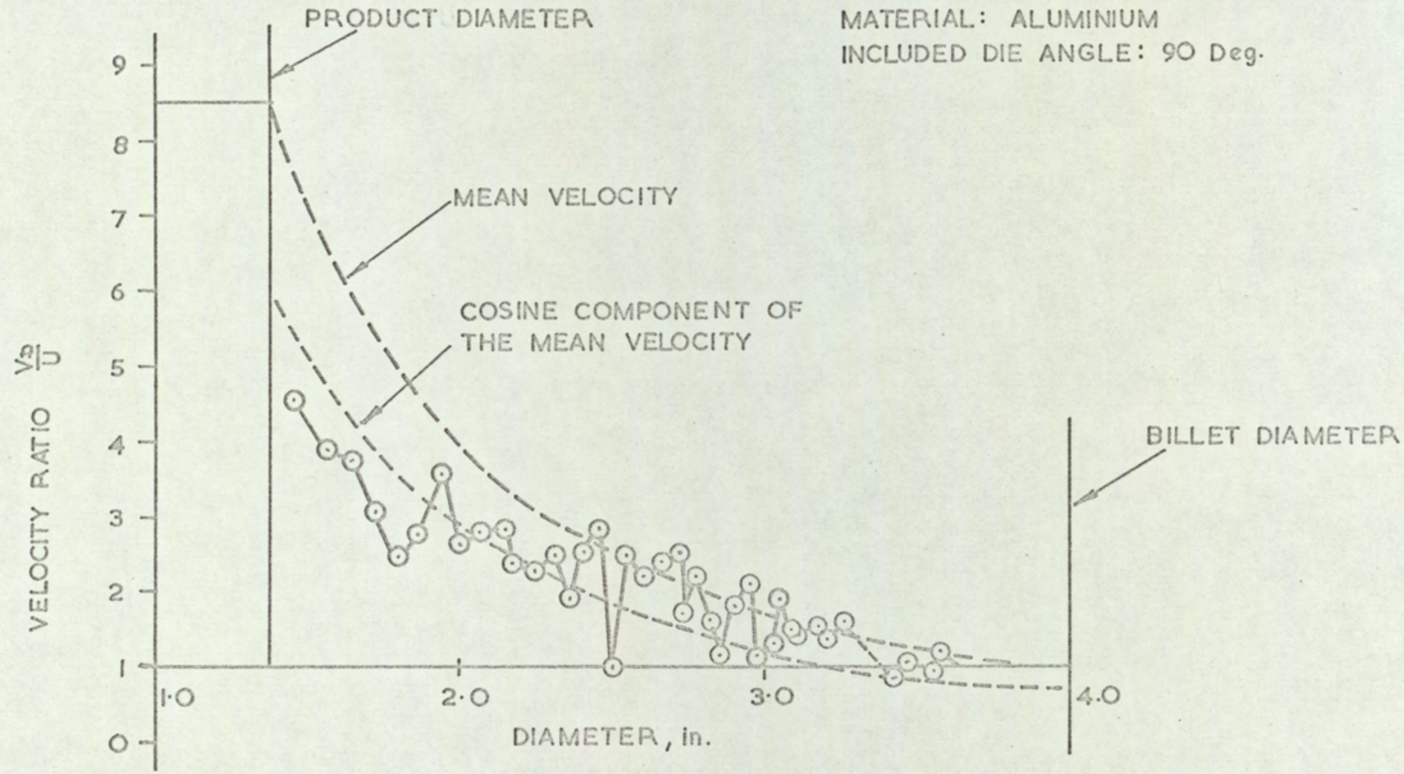


FIG.53. DISTRIBUTION OF THE VELOCITY OF SLIDING BETWEEN THE BILLET AND THE DIE WALL

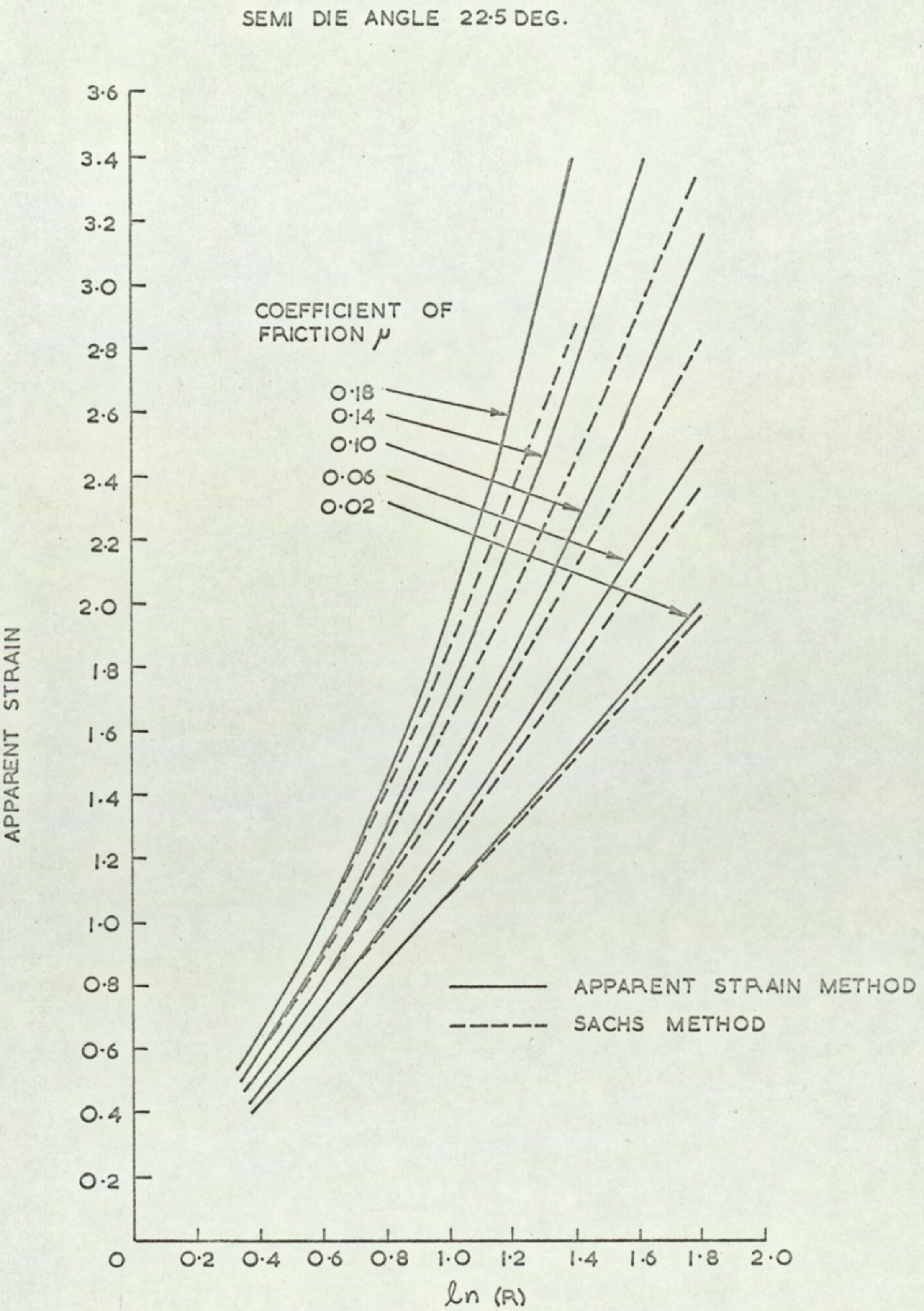


FIG. 54. COMPARISON BETWEEN THE INDUCED APPARENT STRAIN PREDICTED BY THE APPARENT METHOD AND SACHS METHOD FOR BAR EXTRUSION

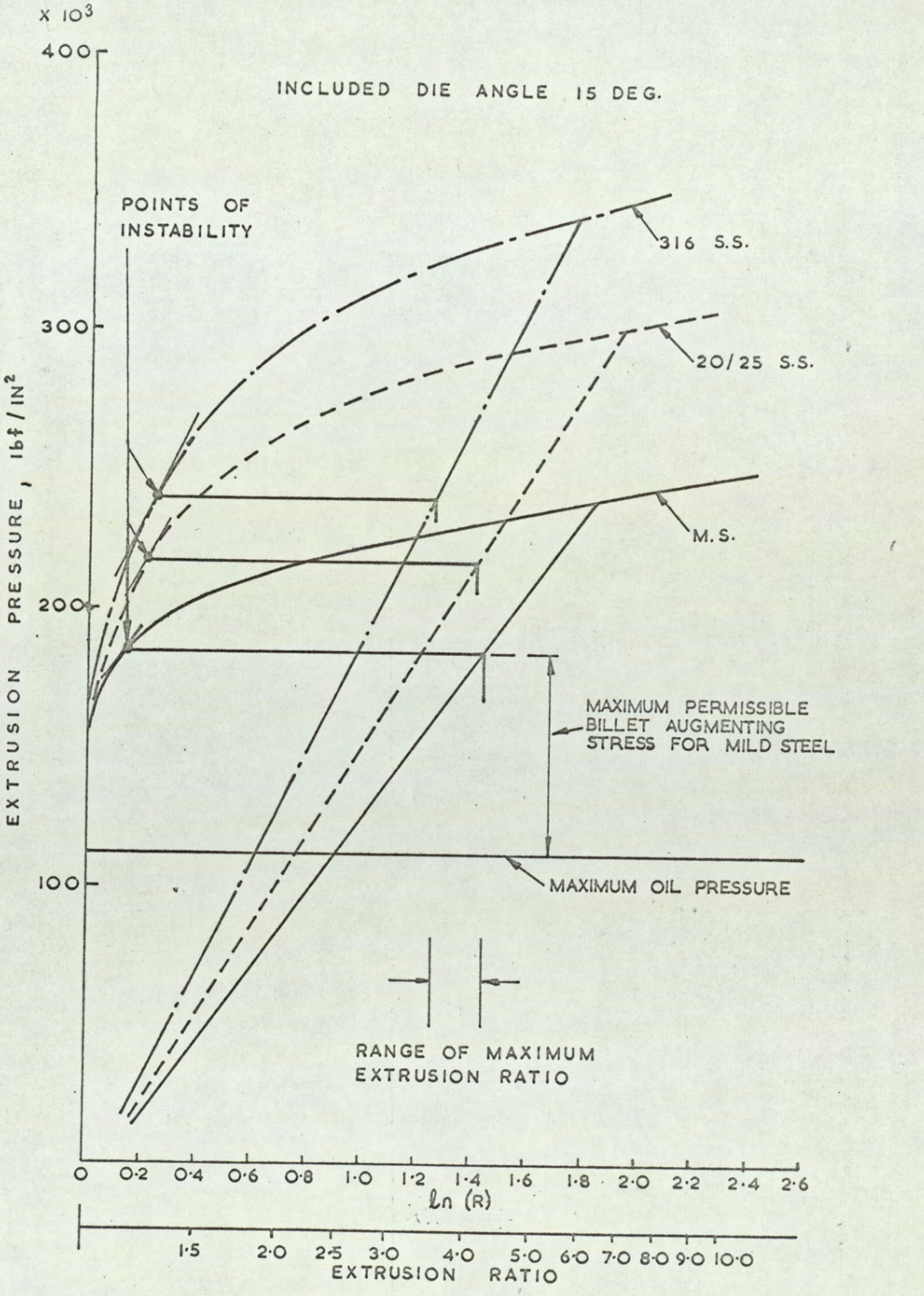


FIG. 55. EFFECT OF THE MATERIAL EQUIVALENT STRESS~STRAIN CHARACTERISTICS ON THE APPROXIMATE MAXIMUM EXTRUSION RATIO OBTAINABLE BY BILLET AUGMENTED HYDROSTATIC EXTRUSION OF TUBE.

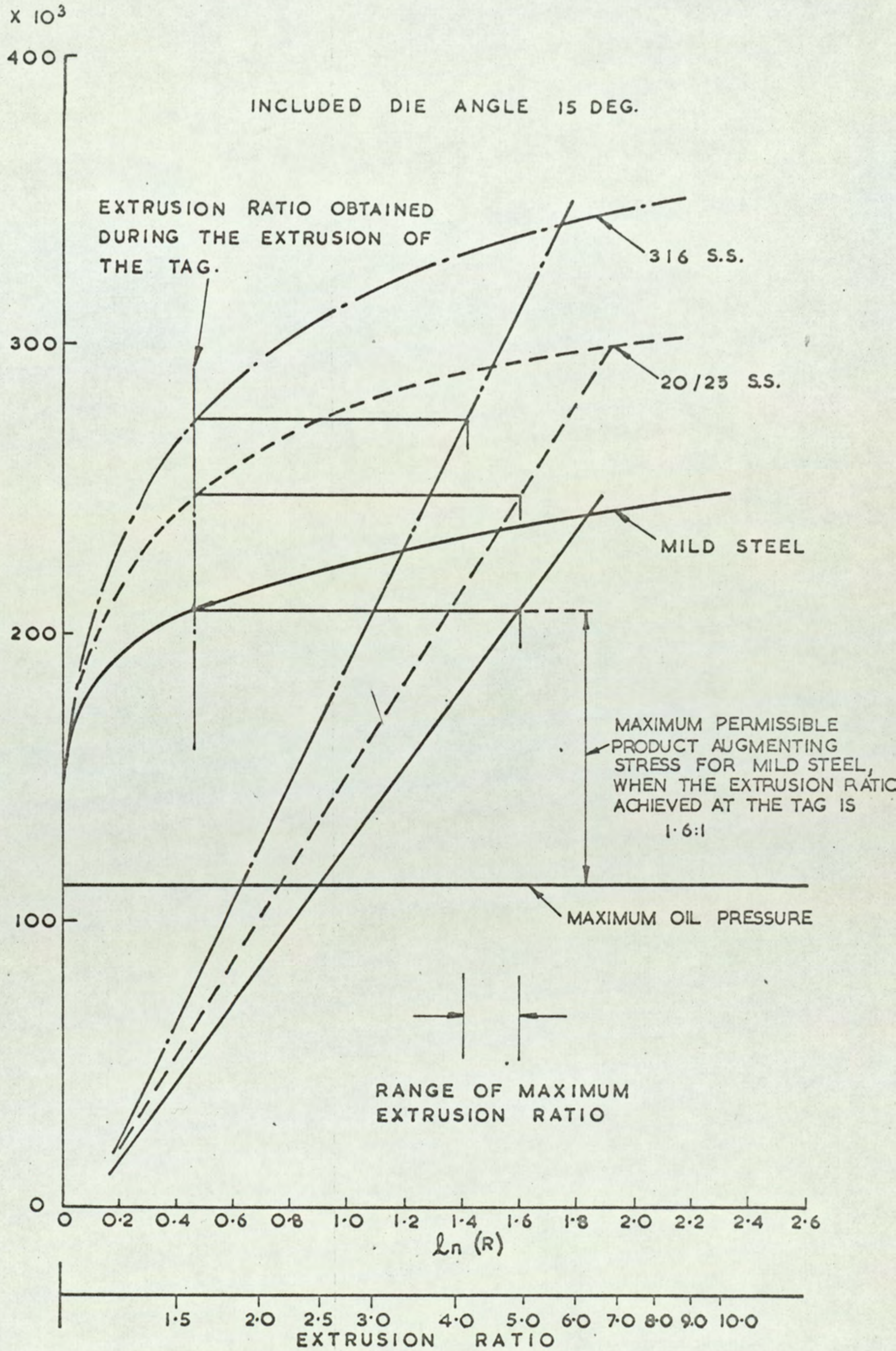


FIG.56. EFFECT OF THE MATERIAL EQUIVALENT STRESS~ STRAIN CHARACTERISTICS ON THE MAXIMUM EXTRUSION RATIO OBTAINABLE BY PRODUCT AUGMENTED HYDROSTATIC EXTRUSION OF TUBE.

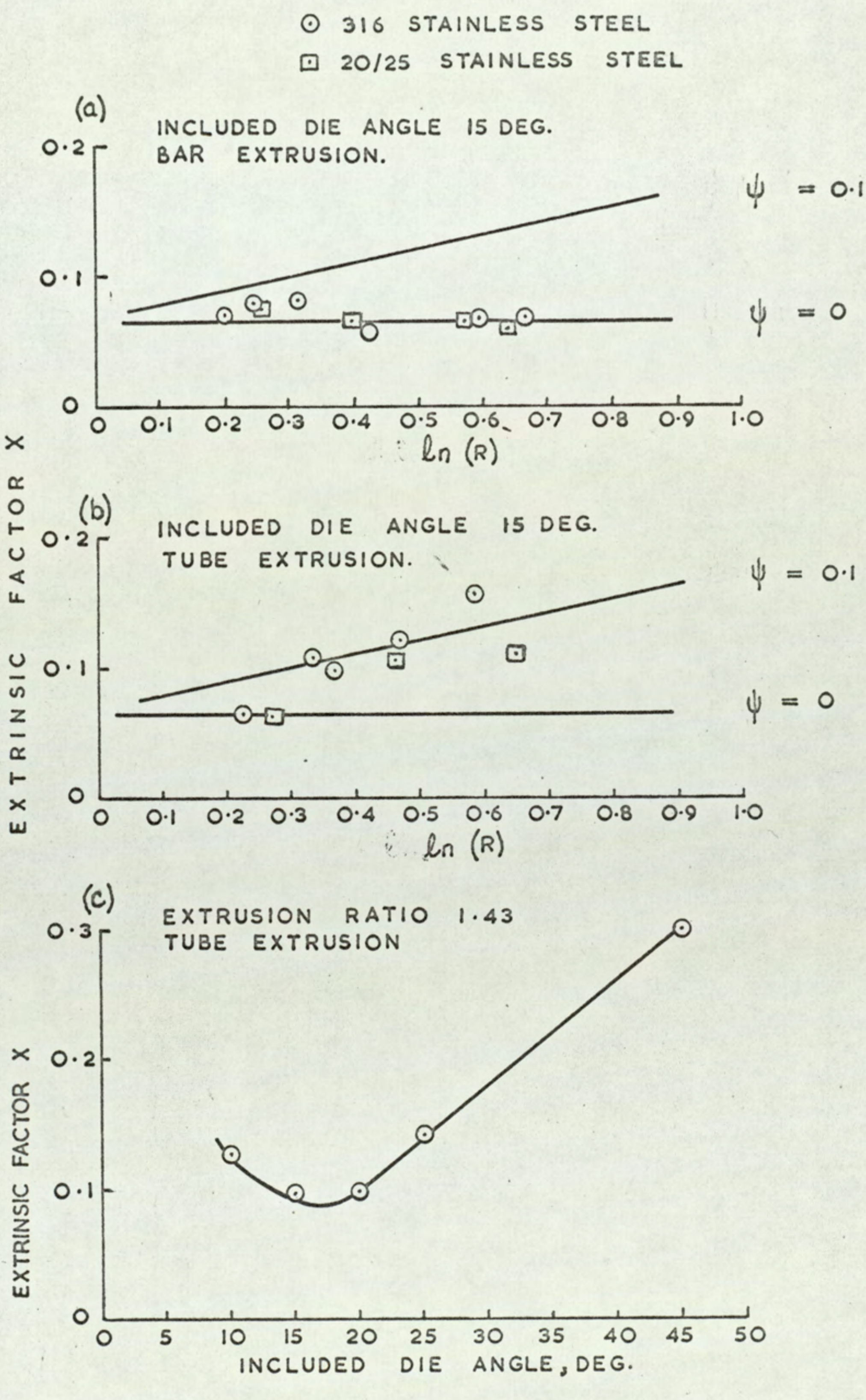


FIG.57. EFFECT OF THE PROCESS VARIABLES ON THE EXTRINSIC FACTOR INDUCED BY SIMPLE HYDROSTATIC EXTRUSION.

BILLET DIAMETER RATIO k = 1·76

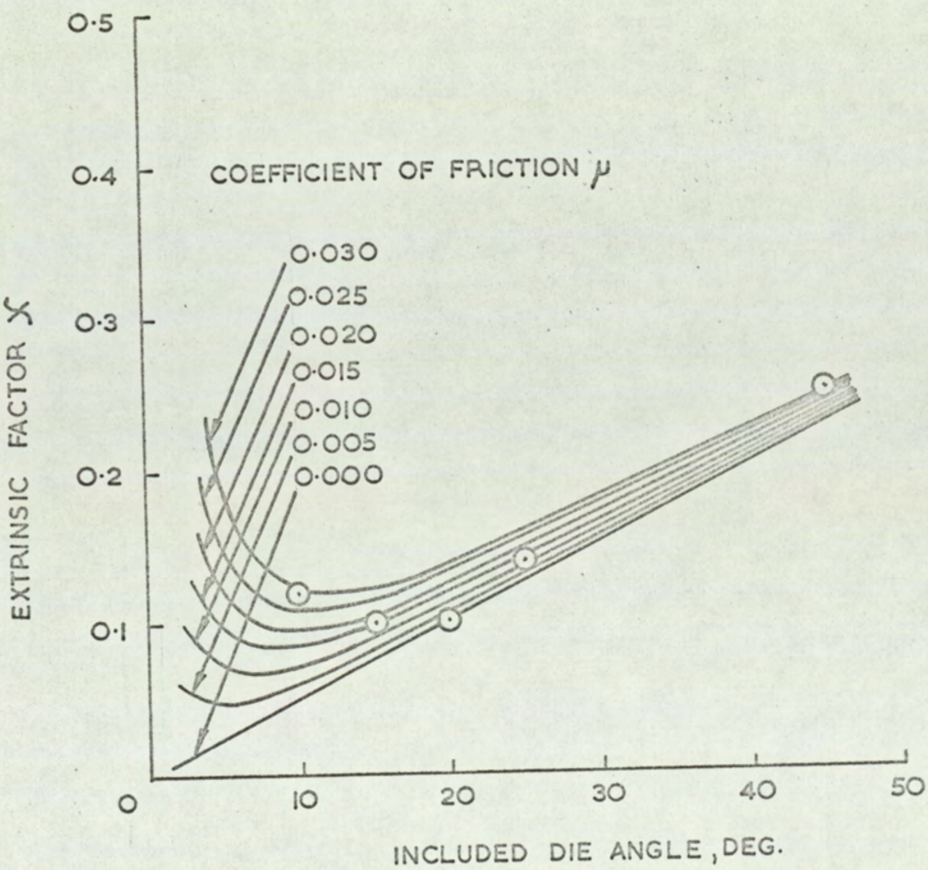


FIG. 58. THE EFFECT OF DIE ANGLE ON THE EXTRINSIC FACTOR FACTOR FOR AISI 316 TUBE HYDROSTATIC EXTRUSION

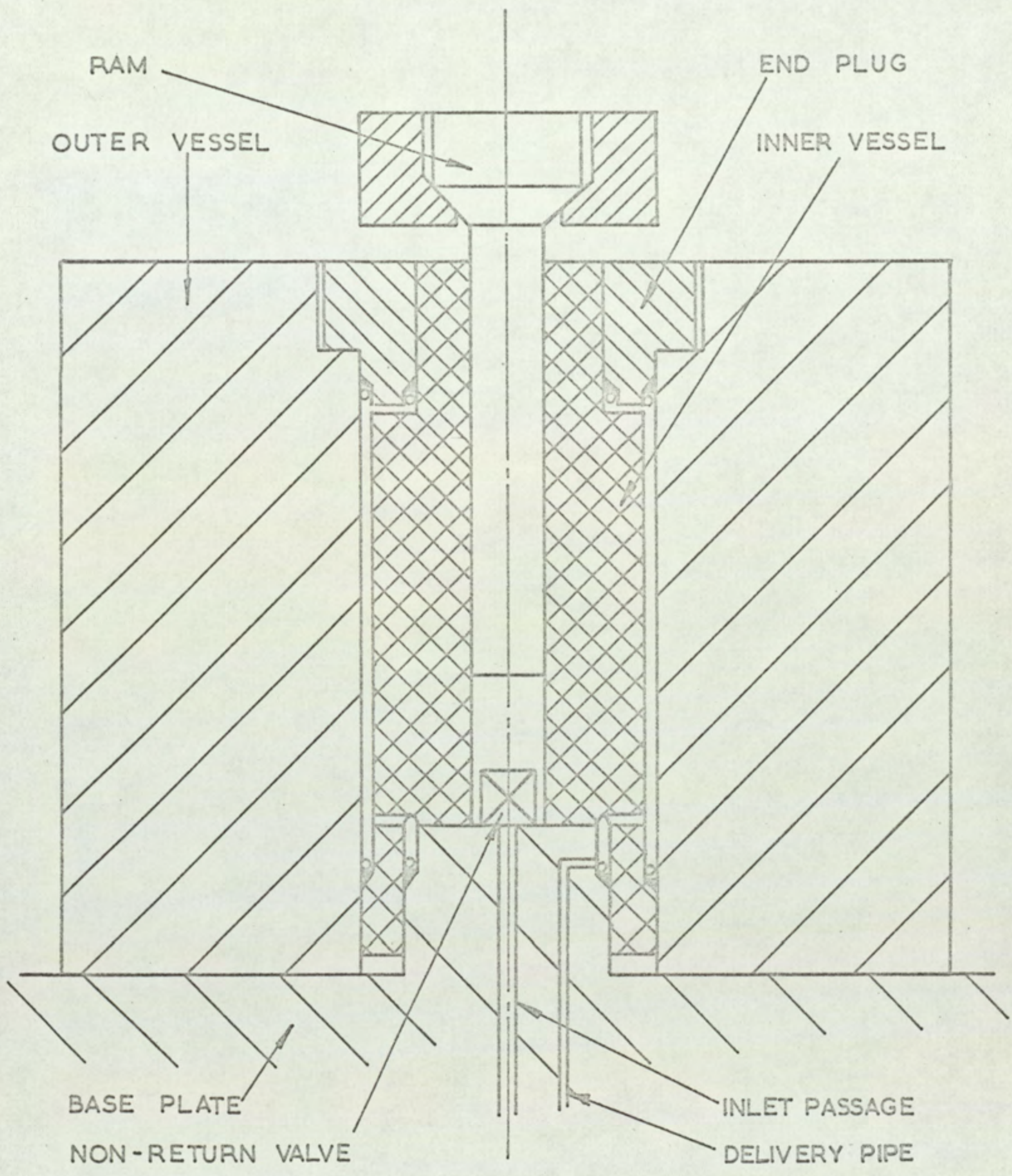
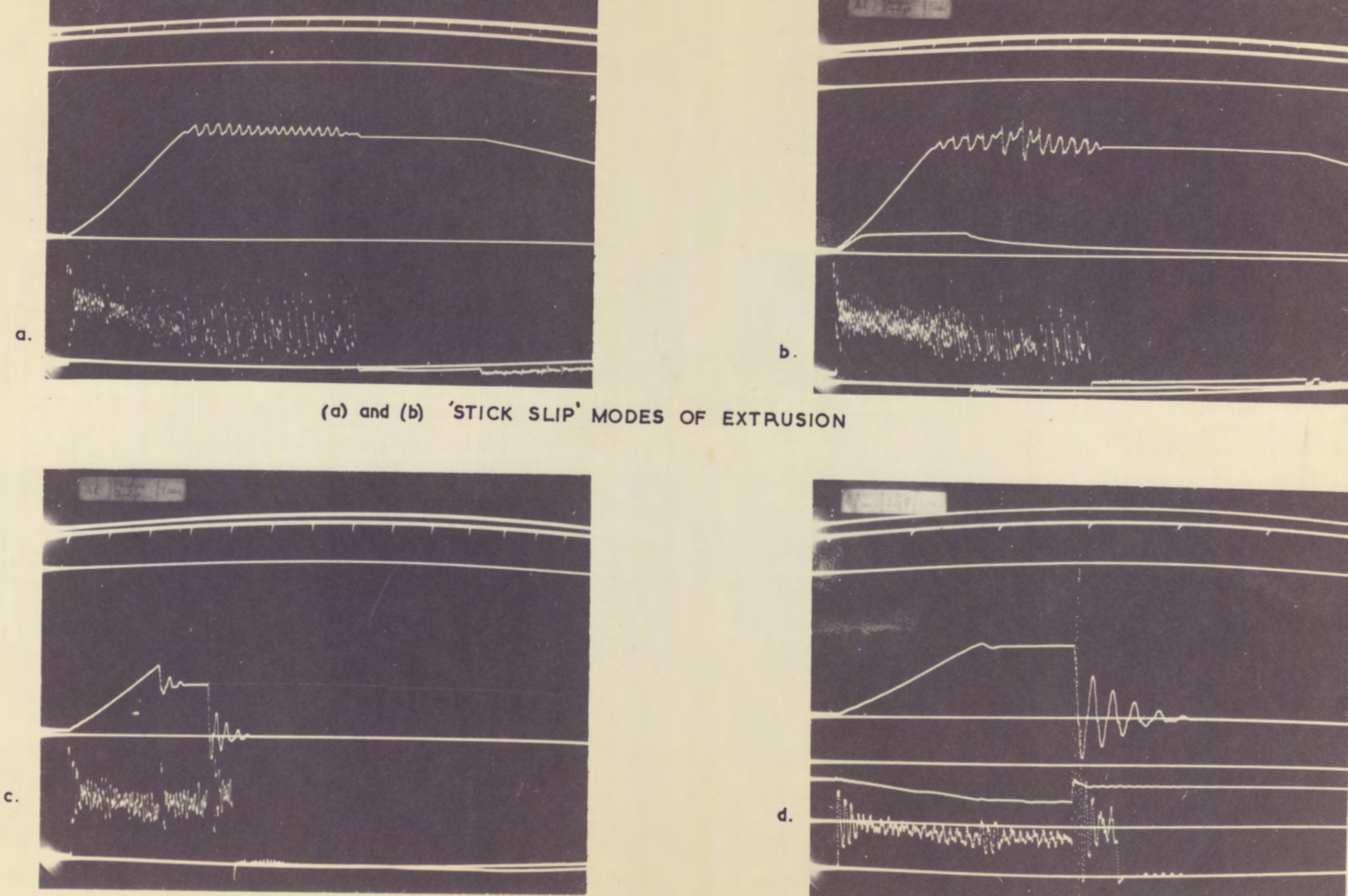


FIG. 59 ESSENTIAL FEATURES OF THE HIGH PRESSURE PUMP



(a) and (b) 'STICK SLIP' MODES OF EXTRUSION

(c) and (d) MODES OF EXTRUSION WHICH INDICATE DAMPING

FIG. 60. EXAMPLES OF OIL PRESSURE–TIME TRACES SHOWING THE DYNAMIC RESPONSE OBTAINED DURING THE SIMPLE HYDROSTATIC EXTRUSION OF SOME ALUMINIUM ALLOYS

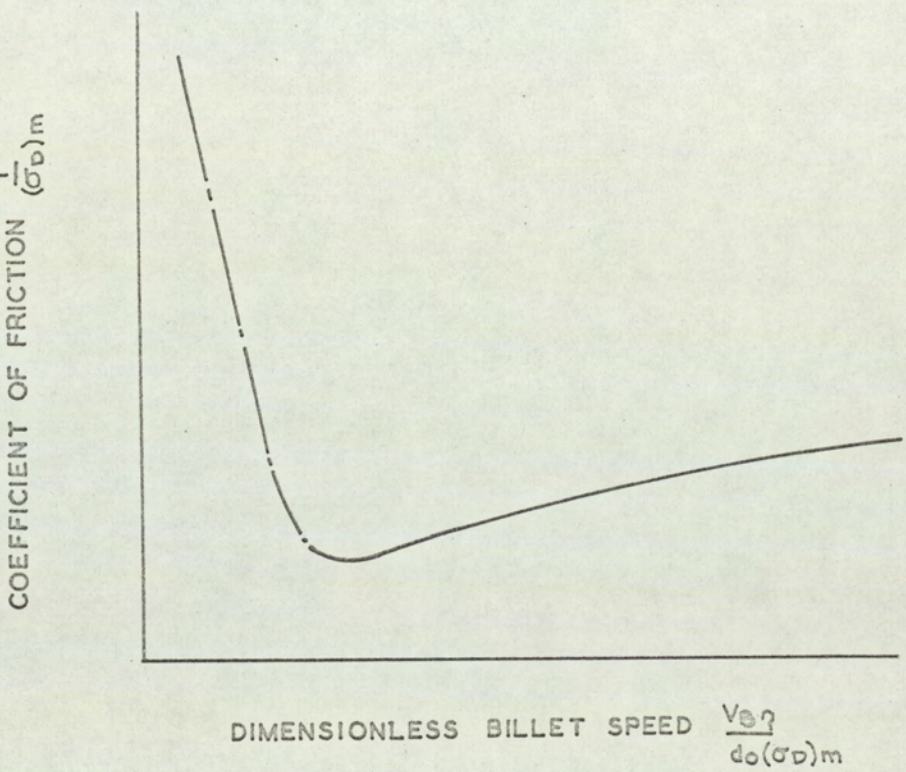


FIG. 61. PROBABLE VARIATION IN THE COEFFICIENT OF FRICTION AGAINST BILLET SPEED

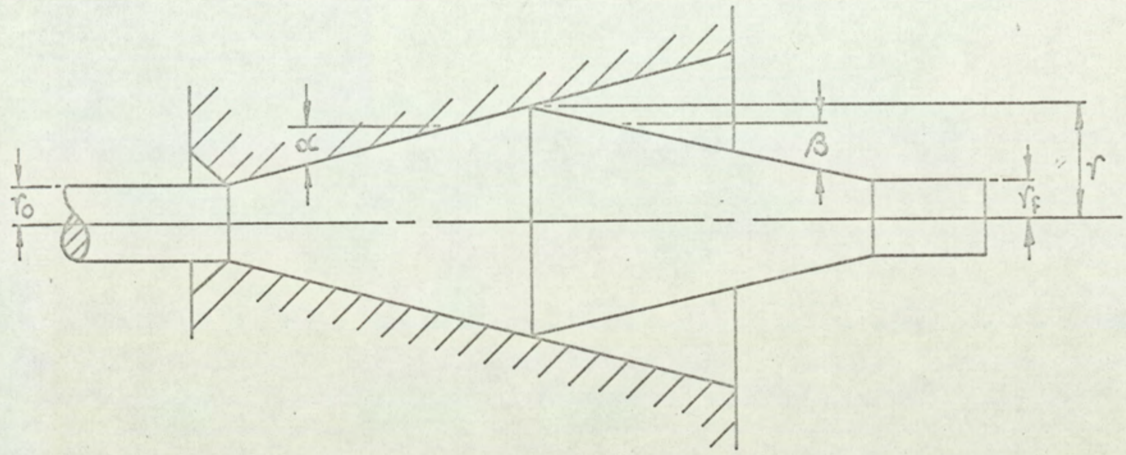


FIG. 62. SHOWS THE GEOMETRY DURING THE EXTRUSION OF THE REAR TAPER

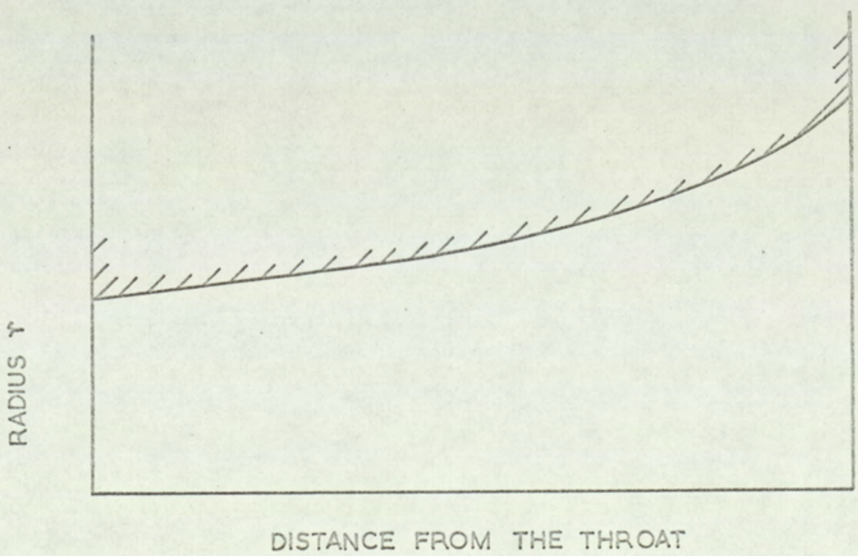


FIG. 63. SHOWS THE DIE PROFILE REQUIRED FOR A CONSTANT RATE OF PRESSURE PHASE OUT

8. APPENDICES

8.1. THE DYNAMIC BEHAVIOUR OF HYDROSTATIC EXTRUSION

During simple hydrostatic extrusion the billet moves under the influence of the forces exerted by the oil pressure, the die reaction, friction and damping. The billet is not in contact with any moving mass, which is unlike the situation which exists during augmented extrusion, and is, therefore, a body of low inertia. The movement of the billet is not directly controlled since the compressibility of the oil allows billet movement. These factors, along with the variation in the frictional resistance produced by changes in the billet speed can lead to an unsteady mode of extrusion in which the billet is extruded in a series of forward movements which are associated with oscillations in the oil pressure, see Figure 60. This mode of extrusion is known as 'stick-slip'.

Stick-slip occurs because a higher oil pressure is needed to start extrusion than the oil pressure required once extrusion has begun. The high initial oil pressure is required to overcome the static frictional resistance existing at the die-billet interference before plastic flow commences. During the transient period when the oil pressure is decreasing from its initial high value and the billet is accelerating. This acceleration is brought about by the excess oil pressure which exists as a result of the rapid fall in frictional resistance as extrusion begins. The mode of extrusion which results depends on the rate of oil compression and the degree of damping. If both these factors are small the momentum of the billet is sufficient to cause over-expansion of the oil and the oil pressure to fall below the required to maintain extrusion. This stops extrusion and results in the recompression of the oil. This sequence of events is then repeated to produce the stick-slip mode of extrusion. With small rates of oil compression the stick-slip action can be minimised by damping, see Low and Donaldson (16). The most satisfactory way of achieving steady extrusion is to employ high rates of oil compression for this does not allow the oil pressure to fall sufficiently to cause the stick-slip action. This method of hydrostatic extrusion also induces hydrodynamic lubrication which is associated with a reduction in frictional resistance.

The two methods of overcoming the stick-slip mode of extrusion will now be considered in more detail

<u>The effect of damping in the dynamic behaviour of simple hydrostatic extrusion</u>

The oil pressure required to initiate extrusion is given by

$$p_o = \frac{\bar{\epsilon}_m}{1 - \psi_{STATIC}} \quad \text{since} \quad \zeta_p = \zeta_B = 0$$

and the oil pressure needed to maintain extrusion with small billet velocities is given by

$$p_s = \frac{\bar{\epsilon}_m}{1 - \psi_{DYNAMIC}}$$

During the transient stage the oil pressure is in excess of that needed to maintain extrusion, this causes the billet to accelerate. If damping is present the equation of motion becomes

$$A_o(p - p_s) - \lambda . \frac{dy}{dt} = m . \frac{d^2y}{dt^2} \qquad \ldots(56)$$

To solve this equation it is necessary to relate the oil pressure, billet velocity and acceleration to the volume occupied by the oil. This volume may be written

$$V = V_o - \dot{v}.t + y.A_o \qquad \ldots(57)$$

where the rate of volume displacement of the ram is considered constant and V_0 is the volume of the oil when plastic flow commences at $t=0$. As we are concerned only with changes in the volume of the oil it is convenient to express the volumes thus

$$V = V_0 - v$$

where

$$v = \dot{v}t - yA_0$$

giving

$$\left.\begin{aligned} \frac{dv}{dt} &= \dot{v} - \frac{dy}{dt}.A_0 \\ \text{and} \qquad \frac{d^2v}{dt^2} &= -\frac{d^2y}{dt^2}.A_0 \end{aligned}\right\} \qquad \ldots(58)$$

To express the pressure in terms of the volume of the oil it is necessary to know the pressure-volume characteristics of the oil. It is believed sufficiently accurate to consider a linear relationship between the oil pressure and its density over the pressure range covered during the transient period, giving

$$\frac{dp}{d\rho} = c = \frac{p - p_0}{\rho - \rho_0}$$

and

$$p = c(\rho - \rho_0) + p_0$$

which may be written, if v^2 can be neglected

$$p = \frac{c\rho_0 v}{V_0} + p_0 \qquad \ldots(59)$$

Combining equations 56, 58 and 59 gives

$$A_0\left(\frac{c\rho_0 v}{V_0} + p_0 - p_s\right) - \lambda\left(\dot{v} - \frac{dv}{dt}\right)\frac{1}{A_0} = -\frac{m}{A_0}\cdot\frac{d^2v}{dt^2}$$

which is of the form

$$\frac{d^2v}{dt^2} + \frac{\lambda}{m}\cdot\frac{dv}{dt} + F_1.v = F_2 \qquad \ldots(60)$$

where

$$F_1 = \frac{\rho_o . A_o^2 . c}{m . V_o} \quad \text{and} \quad F_2 = \frac{\lambda}{m}\dot{v} - \frac{A_o^2}{m}(p_o - p_s)$$

This is a general equation of motion which governs the dynamic behaviour of simple hydrostatic extrusion and permits several modes of extrusion depending on the magnitude of the damping coefficient. The most important modes will now be considered.

<u>Simple hydrostatic extrusion with negligible damping $\lambda = 0$</u>

The general equation of motion simplifies to give

$$\frac{d^2 v}{dt^2} + F_1 v = F_3 \quad \text{where} \quad F_3 = -\frac{A_o^2}{m}(p_o - p_s)$$

and the boundary conditions are

when $t = 0$; $v = 0$

and $t = 0$; $\frac{dv}{dt} = \dot{v}$

This leads to the solution,

$$v = \frac{\dot{v}}{\sqrt{F_1}} . \sin\left(\frac{t}{\sqrt{F_1}}\right) + v_s . \cos\left(\frac{t}{\sqrt{F_1}}\right) - v_s$$

and

$$\frac{dv}{dt} = \dot{v} \cos\left(\frac{t}{\sqrt{F_1}}\right) - v_s . \sqrt{F_1} . \sin\left(\frac{t}{\sqrt{F_1}}\right)$$

By substitution into equation (58) the billet velocity becomes

$$\frac{dy}{dt} = \frac{1}{A_o}\left\{\left[1 - \cos\left(\frac{t}{\sqrt{F_1}}\right)\right]\dot{v} + v_s . \sqrt{F_1} . \sin\left(\frac{t}{\sqrt{F_1}}\right)\right\}$$

This is the general solution for the 'stick-slip' mode of extrusion. From this result the value of the parameters at the end of the principal time intervals are,

Time t	Change in volume of the oil V	Oil pressure p	$\frac{dv}{dt}$	Billet velocity $\frac{dy}{dt}$
0	0	p_0	$\dot{v}$	0
$\frac{\pi}{\sqrt{F_1}}$	$-V_s$	$p_0 - 2(p_0 - p_s)$	$-\dot{v}$	$\frac{2\dot{v}}{A_0}$
$\frac{2\pi}{\sqrt{F_1}}$	0	p_0	$\dot{v}$	0

Simple hydrostatic extrusion with damping

Two distinct modes of extrusion are possible when damping is present which depends on the magnitude of the damping coefficient.

Small coefficients of damping $\quad \lambda < 2A_0\sqrt{\frac{\rho_0 m c}{V_0}}$

As the boundary conditions are the same as in the previous case, the general solution becomes

$$V = e^{-\frac{\lambda t}{2m}}\left[\frac{\dot{v}\,2m}{F_4 . \lambda}\cdot\sin\left(t.F_4\right) + \left(\frac{\lambda\dot{v}}{\rho_0 A_0^2 c} - V_s\right)\cos\left(t.F_4\right)\right] + \frac{\lambda\dot{v}}{\rho_0 A_0^2 c} - V_s$$

where

$$F_4 = \sqrt{\left(\frac{\lambda}{2m}\right)^2 - \frac{\rho_0 A_0^2 c}{m}}$$

Large coefficient of damping $\quad \lambda > 2A_0\sqrt{\frac{\rho_0 m c}{V_0}}$

For this case the general solution is,

$$V = \left(\frac{\lambda\dot{v}}{\rho_0 A_0^2 c} - V_s\right)\left[\left(\frac{1}{\frac{F_5}{F_6} - 1}\right)e^{-F_5 t} - \left(\frac{1}{1 - \frac{F_6}{F_5}}\right)e^{-F_6 t} + 1\right]$$

where
$$F_5 = \frac{\lambda}{2m} + F_4$$

$$F_6 = \frac{\lambda}{2m} - F_4$$

These solutions show that when damping is present the steady oil pressure obtained is

$$\left(p_s\right)_{DYNAMIC} = p_s + \frac{\lambda \dot{v}}{A_o^2} \qquad \ldots(61)$$

where

$\left(p_s\right)_{DYNAMIC}$ = steady oil pressure obtained under dynamic conditions

p_s = steady oil pressure obtained when the damping resistance is small.

The above analysis has shown that the critical damping coefficient is

$$\lambda_{CRITICAL} = 2A_o\sqrt{\frac{\rho_o m c}{V_o}}$$

<u>Example</u>

Consider the simple hydrostatic extrusion of 20/25/Nb stainless steel bar. Let the pressure transmitting medium be castor oil and the mean billet velocity 4 ft/second. The billet diameter considered will be 1 in.

$A_o = \frac{\pi}{57}$ ft^2

ρ_o = 70 lb/ft^3 (castor oil at 40 tonf/in^2)

m = 2.71 x l lb

l = length of billet in feet

c = 1.88 x 10^6 ft lbf/lb (castor oil at 40 tonf/in^2)

V_o = 30 in^3 $\equiv$ 0.0174 ft^3

v = 4 ft/s

$$\lambda_{\text{CRITICAL}} = 276\sqrt{l} \cdot \frac{\text{lbf}}{\text{in}^2}$$

and the dynamic press increase = $1400\sqrt{l} \cdot \frac{\text{lbf}}{\text{in}^2} = \frac{\lambda_{\text{CRITICAL}} \cdot V}{A_o}$

The effect of billet speed on the dynamic behaviour of simple hydrostatic extrusion

One of the principal effects of high billet speed is that the oil is drawn between the surfaces of the die and billet, which produces a reduction in the frictional resistance to plastic flow. This effect is known as hydrodynamic lubrication.

The reduction in frictional resistance can be interpreted in terms of variation in the coefficient of friction. Figure 61 shows the probable variation against the billet speed, shown as a dimensionless number. When the billet speed is small little oil is drawn between the die and the billet, the frictional resistance will depend on the surface lubricant applied to the billet before extrusion. As the speed is increased the thickness of the layer of oil increases, this produces a reduction in the coefficient of friction. At higher speeds the die and billet surfaces are completely separated by the oil and hydrodynamic lubrication is fully established. When this occurs the only resistance to plastic flow is the viscious drag of the oil layer. At even higher speeds the drag on the billet will tend to increase. There will, therefore, be a minimum resistance to plastic flow at some critical billet speed.

There are three important features of hydrodynamic lubrication which are worthy of analytical assessment, they are the oil layer thickness, the shear stress applied to the surface of the billet and the coefficient of damping due to the oil layer. Some information about the form of the expressions for these quantities may be obtained by dimensional analysis, thus,

$$h, \tau, \lambda = F\left(\eta, v_B, d_o, (\sigma_D)_m\right)$$

giving

$$h = d_o F_1(N)$$
$$\tau = (\sigma_D)_m F_2(N) \qquad \text{where} \quad N = \frac{\eta v_B}{d_o (\sigma_D)_m}$$
$$\lambda = d_o \eta F_3(N)$$

The existence of this dimensionless hydrodynamic lubrication number has been independently verified by Rozner and Faupel (17) and B. Aritzur (18). The analytical work of Rozner and Faupel on the thickness of the oil layer gave the expression

$$h = \sqrt{\frac{\eta \cdot v_B l}{p_{av.}}}$$

which may be written,

$$h = d_o . C_1 . N^{\frac{1}{2}} \qquad \text{...(6}$$

where

$$C_1 = \sqrt{\frac{(\sqrt{R} - 1)}{\sqrt{R} . \sin(\alpha)}}$$

This solution may be used to obtain expressions for the shear stress and the coefficient of damping. From Newton's law of viscosity.

$$\tau = \eta \frac{dv}{dy} \doteqdot \eta \cdot \frac{v_B}{h}$$

Combining this with the above result gives,

$$\tau = \frac{(\sigma_D)_m}{C_1} . N^{\frac{1}{2}}$$

which may be written as an equivalent coefficient of friction thus

$$\mu = \frac{1}{C_1} . N^{\frac{1}{2}} \qquad \text{...(6}$$

Proceeding in a similar manner the coefficient of viscous damping becomes

$$\lambda = \frac{C_2}{C_1} \cdot d_o \cdot \eta . N^{1/2} \qquad \text{...(6}$$

where

$$C_2 = \frac{\pi \cot\alpha}{4}\left(\frac{R-1}{R}\right)$$

Example

For the simple hydrostatic extrusion of 20/25/Nb stainless steel bar through an extrusion ratio of 2:1, determine

a. the thickness of the oil layer in the die,

b. the effective coefficient of friction,

and c. the coefficient of viscous damping,

where

$(\sigma_D)_m$ = 28.8 x 10^6 lbf/ft^2

η = 2.0 lb/ft s

V_B = 4.0 ft/s

2α = 15 DEG

d_o = 1 in. = 0.083 ft

l = 1 ft

a)
$$C_3 = \sqrt{\frac{(\sqrt{R}-1)}{\sqrt{R}.\sin\alpha}} = 1{\cdot}50$$

from equation 62,

$$h = C_3\sqrt{\frac{\eta . V_B\, d_o}{(\sigma_D)_m}} = 12{\cdot}9\,\mu$$

b) from equation 63,

$$\mu = \frac{1}{C_3} \cdot N^{1/2}$$

$$N^{1/2} = \sqrt{\frac{\eta\, V_B}{d_o\,(\sigma_D)_m}} = \frac{1}{3090}$$

giving $$\mu = 0{\cdot}0002$$

c) from equation 64,

$$C_4 = \frac{\pi}{4} \cdot \cot\alpha \left(\frac{R-1}{R}\right) = 3.0$$

$$\lambda = d_o \cdot \eta \cdot \frac{C_4}{C_3} \cdot N^{1/2}$$

giving $$\lambda = 31.9 \ \frac{\text{lbf.s}}{\text{ft}}$$

8.2. SOME THEORETICAL ASPECTS OF PRESSURE PHASE-OUT DURING PRODUCT AUGMENTED HYDROSTATIC EXTRUSION

As described in the main text pressure phase-out is the process by which a billet is completely extruded by product augmented hydrostatic extrusion in a manner which does not cause a release of high pressure oil. The method relies on the fact that the oil pressure and the stress induced in the product by the drawing device are equivalent from the point of view of their capability for doing work. It is, therefore, possible to reduce the oil pressure if an equivalent increase in drawing stress can be carried by the product. This method, as previously described, can be used to completely phase-out the oil pressure at the end of extrusion if either the billet is tapered or special die profiles are employed. It should be mentioned here that if the extrusion press was capable of rapid ram movements a square-ended billet could be completely extruded through a simple cone die by the phase-out method, but the rate of pressure release would be very great.

The following is a theoretical study of some of the parameters which affect the rate of pressure release during the complete product augmented hydrostatic extrusion of a solid round billet using the pressure phase-out method. Figure 62 shows the partly extruded end of a billet which has a back taper. The diameter of the parallel end section and the die throat diameter are equal. This latter point is not essential but it makes analysis simple.

The volume of the un-extruded part of the billet between the die throat and the beginning of the parallel end section is given by

$$Vol = \frac{\pi}{3\tan\alpha}\left(r^3 - r_f^3\right) + \frac{\pi}{3\tan\beta}\left(r^3 - r_f^3\right)$$

The rate at which this changes with respect to time may be written

$$\frac{d}{dt}(Vol) = \pi r^2 \left(\frac{\tan\alpha + \tan\beta}{\tan\alpha . \tan\beta}\right)\frac{dr}{dt} \qquad ...(65)$$

The work equation may be written, using the empirical law for the total extrusion pressure required, thus

$$p + \sigma_p = a + b.\ln(R) \qquad ...(66)$$

when 'R' is the instantaneous value of the extrusion ratio and given by

$$R = \left(\frac{r}{r_f}\right)^2$$

Equation (66) may be differentiated to give

$$\frac{dp}{dt} + \frac{d\sigma_p}{dt} = \frac{2b}{r}\cdot\frac{dr}{dt} \qquad ...(67)$$

Combining equation (65) and (67) together with the law of volume consnstency, which is

$$\frac{d}{dt}(Vol) = -\pi r_f^2 u \qquad \text{since} \qquad r_o = r_f$$

gives

$$\frac{dp}{dt} + \frac{d\sigma_p}{dt} = -\frac{2bu r_f^2}{r^3}\left(\frac{\tan\alpha . \tan\beta}{\tan\alpha + \tan\beta}\right) \qquad ...(68)$$

This is the basic equation which governs the changes in oil pressure and drawing stress during the pressure phase-out period.

It is of some interest to consider the rate of change of oil pressure required to hold the drawing stress constant. This is given by

$$\frac{dp}{dt} = - \frac{2bUr_f^2}{r^3}\left(\frac{\tan\alpha \cdot \tan\beta}{\tan\alpha + \tan\beta}\right) \qquad \ldots(69)$$

If a billet with a square end is used this simplified to give

$$\frac{dp}{dt} = - \frac{2b.U.r_f^2.\tan\alpha}{r^3} \quad \text{since} \quad \beta = 90^\circ \qquad \ldots(70)$$

This shows that if a simple cone die is used, α = constant, the rate of pressure change required varies depending on the instantaneous value of the maximum radius of the billet in contact with the die. At the beginning of the phase-out period.

$$\frac{dp}{dt} = - \frac{2bU\tan\alpha}{r_o . R} \quad \text{since} \quad r = r_o$$

When R is the extrusion ratio achieved during the steady state period of extrusion. At the end of the phase-out period when the billet is about to leave the die, the rate of pressure change becomes

$$\frac{dp}{dt} = - \frac{2b.u.\tan\alpha}{r_f} \quad \text{since} \quad r = r_f$$

It follows that when a simple cone die is used to completely extrude a billet with a square end, the rate of pressure release increases as the end passes along the die. This can be overcome by using a modified die profile. It can be seen from equation (70) that if

$$\frac{\tan\alpha}{r^3} = \text{constant}$$

the rate of pressure release required as the end of the billet passes along the die is constant and depends on the value of the constant chosen. A completely independent choice of the constant is not possible for it will be governed by the maximum inlet angle which can be tolerated and the limit to the overall length of the die. Figure 63 shows a typical die profile which fulfils this requirement. It can be seen that it produces a trumpet shaped die orifice.

Example

Consider a 1 in. diameter annealed mild steel billet which is reduced by an extrusion ratio of 4:1 at a product speed of 1 in/sec. Determine the rate of pressure release required to maintain the drawing stress constant during the pressure phase-out period of product augmented hydrostatic extrusion if

(a) the die is a simple cone die with a semi-die angle of 7.5 deg. and the semi-angle of the rear end taper is 2 deg.,

(b) the die is a simple cone die with a semi-die angle of 7.5 deg. and the billet has a square end,

(c) the die has a profile given by the law $\frac{\tan\alpha}{r^3} = 0.70$ and the billet has a square end.

The empirical law for the total extrusion pressure for mild steel round bar is

$$\text{Extrusion pressure} = 2000 + 97000 \log_e R \text{ p.s.i.}$$

Solution

(a) (i) As the rear end taper begins to enter the die

$$\frac{dp}{dt} = -\frac{2bU}{r_0 R}\left(\frac{\tan\alpha . \tan\beta}{\tan\alpha + \tan\beta}\right) \qquad = -3,650 \text{ p.s.i./sec}$$

(ii) As the rear end taper begins to leave the die

$$\frac{dp}{dt} = -\frac{2bU}{r_f}\left(\frac{\tan\alpha . \tan\beta}{\tan\alpha + \tan\beta}\right) \qquad = -29,200 \text{ p.s.i./sec}$$

(b) (i) As the rear end enters the die

$$\frac{dp}{dt} = -\frac{2bU\tan\alpha}{r_0 R} \qquad = -12,800 \text{ p.s.i./sec}$$

(ii) As the rear end leaves the die

$$\frac{dp}{dt} = -\frac{2bU\tan\alpha}{r_f} \qquad = -102,000 \text{ p.s.i./sec}$$

(c) $\frac{dp}{dt} = 2bU r_f^2$ (constant) $= -8,500$ p.s.i./sec

8.3. COMPUTER PROGRAMS

This section deals with the computer programs required for the iterative procedure given in sections 4.5.a. to d. The programs are written in FORTRAN IV suitable for the ELLIOTT 4100 computer.

It was necessary to adopt a different rotation in these programs from that given in the text. Hence, to make the principal parameters in the programs clear they have been labelled.

8.3.a. PROGRAM (a) FOR THE BILLET AUGMENTATION OF BAR

The iterative procedure for this program is given in section 4.5.a. The program deals with the billet augmentation of 20/25/Nb stainless steel bar, in which the oil pressure remains constant whilst augmentation is applied. It covers the oil pressure range 50-110 tonf/in^2 and an extension ratio range 1.4 - 7.0, it determines the total extension pressure to an accuracy of $\pm$ 500 lbf/in^2. The predictions made by this program are shown in Figure 11.

8.3.b. PROGRAM (b) FOR THE PROPORTIONAL BILLET AUGMENTATION OF BAR

The iterative procedure for this program is given in section 4.5.b. The program deals with the proportional billet augmentation of 20/25/Nb stainless steel bar. It covers the extension ratio range 1.2 - 4.0 and deals with augmenting ratios in the range 0.25 - 1.0. The predictions of this program are shown in Figure 12.

8.3.c. PROGRAM (c) FOR THE PROPORTIONAL BILLET AUGMENTATION OF TUBE

The iterative procedure for this program is given in section 4.5.c. The program deals with the proportional billet augmentation of 20/25/Nb stainless steel tube over a billet-fixed, travelling mandrel, which has a solid head and gives rise to differing coefficients of friction at the inner and outer surfaces. The coefficients of friction considered to act at the inner and outer surfaces are 0.030 and 0.005, respectively. The method used to calculate the friction factor is given in section 4.3.e. The program covers the billet radii ratio range 1.2 - 2.0 and the extrusion ratio range 1.2 - 4.0. The predictions given by this program are shown in Figure 13.

8.3.d. PROGRAM d FOR THE PROPORTIONAL PRODUCT AUGMENTATION OF TUBE

The iterative procedure for this program is given in section 4.5.d. The program deals with the proportional product augmentation of 20/25/Nb stainless steel tube. It covers the billet radii ratio range 1.4 - 2.0 and the extension ratio range 1.2 - 6.0. The predictions given by this program are shown in Figure 14.

Program (a)

```
&JOB; BILLET AUGMENTATION OF 20/25/Nb BAR
&LIST; PRINTED COPY OF PROGRAM ON LINE PRINTER
      DIMENSION C(2),D(50)
      REAL PR, A, RO, R, C, E, YIELDS,BETA,STRAIN,PSI,P,BILLS,LOGRO
      INTEGER I,M,N,K,J
      DO 15 1=0,2,1
      M=50+30*1 - (Maximum oil pressure T.S.I.)
      PR=2240.0*M
      WRITE(5,1)M
    1 FORMAT(I4)
      DO 15 N=0,14,1
      RO=1.4+N*0.4 - (Initial extrusion ratio)
      A=7.5*3.1416/180.0 - (Semi die angle)
      K=0
      LOGRO=ALOG(RO)
      R=RO
      C(1)=(1.0-COS(A))/SIN(A)
      C(2)=COS(A)/SIN(A)
   10 E=C(1)+(1.0+0.25*C(1))*ALOG(R) - (Mean induced equivalent strain)
      IF(1-K)3,3,2
    2 YIELDS=167000*E**1.294/(E*1.294) - (Mean flow stress)
      BETA=1.0/1.294
      GOTO4
    3 YIELDS=16700*((E+STRAIN)**1.294-STRAIN**1.294)
      1/(E*1.294)
      BETA=YIELDS/(167000*(E+STRAIN)**0.294) - (Work hardening factor)
    4 PSI=(0.005*C(2)*BETA*(R/(R-1.0))*ALOG(R))
      1/(1.0+0.005*C(2)) - (Friction factor)
```

```
      D(K)=YIELDS*E/1.0-PSI) - (Sum of oil pressure plus augmenting stress
      IF(PR-D(K))5,6,6
    5 IF(1-K)9,9,8
    9 IF(2*RO-R)11,11,12
   12 IF(50-K)11,11,13
   13 IF(ABS(D(K)-D(K-1)).LE.500.0)GOTO6
    8 BILLS=D(K)-PR - (Billet stress)
      STRAIN=BILLS/167000.0
      STRAIN=STRAIN**3.4 - (Billet strain due to augmentation)
      R=RO*EXP(STRAIN) - (Final extrusion ratio)
      K=K+1
      GOTO 10
    6 WRITE(5,7)RO,R,LOGRO,D(K)
    7 FORMAT(F6.1,F7.2,F8.3,F11.1)
      GOTO 15
   11 WRITE(5,14)RO,R,LOGRO
   14 FORMAT(F6.1,F7.2,F8.3,10H UNSTABLE)
   15 J=K
      STOP
      END

&RUN
```

Program (b)

```
 &JOB; PROPORTIONAL  BILLET AUGMENTATION OF 20/25 BAR.
&LIST; PRINTED COPY OF PROGRAM ON LINE PRINTER.
      DIMENSION P(50), C(2)
      REAL OMEGA,RO,A,E,YIELDS,STRAIN,BETA,PSI,EXPT,LOGRO,LAM,MK,Q,SIGMA
      INTEGER I,M,K,N
      LAM=2.0
      DO 14 I=1,4,1
      OMEGA=0.25*I - (Augmenting ratio)
      WRITE(5,15) OMEGA
   15 FORMAT(F7.2)
      DO 14 M=0,14,1
      RO=1.2+M*0.2 - (Initial extrusion ratio)
      LOGRO=ALOG(RO)
      R=RO
      A=7.5*3.1416/180.0 - (Semi die angle)
      C(1)=(1.0-COS(A))/SIN(A)
      C(2)=COS(A)/SIN(A)
      N=0
      STRAIN=0.0
    1 N=N+1
      E=C(1)+(1.0+0.25*C(1))*ALOG(R) - (Mean induced equivalent strain)
      IF(N-1)2,2,3
    2 YIELDS=167000*E**1.294/(E*1.294) - (Mean flow stress)
      GO TO 4
    3 YIELDS=167000*((E+STRAIN)**1.294-STRAIN**1.294)/(E*1.294)
    4 BETA=YIELDS/(167000*(E+STRAIN)**0.294) - (Work hardening factor)
      PSI=(0.005*C(2)*BETA*(R/(R-1.0))*ALOG(R))/(1.0+0.005*C(2))
                                                        - (Friction factor)
      P(N)=(YIELDS*E)/((1.0-PSI)*(1.0+RO*OMEGA/R)) - (Oil pressure)
```

```
      BILLS=RO*OMEGA*P(N)/R - (Billet augmenting stress)
      EXTP=P(N)+BILLS - (Equivalent extrusion pressure)
      IF (N-1) 12,12,13
   13 IF (50-N) 5,5,6
    6 IF (ABS(P(N)-P(N-1)).LE.500.0) GO TO 9
      GO TO 12
    5 WRITE (5,7) RO,N,R,LOGRO
    7 FORMAT(F6.1,14,F7.2,F8.3)
      GO TO 14
    9 MK=2.0*LAM*(SQRT(R)/(1.0+SQRT(R)))
      Q=E*R/(R-1.0)
      Q=YIELDS*Q/((1.0-PSI)*(1.0+0.005*C(2))) - (Mean die pressure)
      SIGMA=Q*(MK**2.0+1.0)/MK**2.0-1.0) - (Die hoop stress)
      1-P(N)*2.0*(MK**2.0)/(MK**2.0-1.0)
      WRITE(5,10)RO,R,LOGRO,P(N),Q,SIGMA
   10 FORMAT(F6.1,F7.2,F8.3,3F11.1)
      GO TO 14
   12 STRAIN=BILLS/167000.0
      STRAIN=STRAIN**3.4 - (Billet strain due to augmentation)
      R=RO*EXP(STRAIN) - (Final extrusion ratio)
      GO TO 1
   14 K=N
      STOP
      END

&RUN
```

Program (c)

```
 &JOB; PROPORTIONAL BILLET AUGMENTATION OF 20/25 TUBE
&LIST; PRINTED COPY OF PROGRAM ON LINE PRINTER
       DIMENSION P(25),G(3),C(2),F(5),MU(2)
       REAL RO,R,A,E,YIELDS,BETA,STRAIN,PSI,Q,BILLS,EXTP,LOGRO,MU
       INTEGER N,M,J,K,S
       A=7.5*3.1416/180.0 - (Semi die angle)
       MU(1)=0.005 - (Coefficient of friction)
       M=6 - (Friction ratio)
       DO 19 N=0,8,1
       G(1)=2.00-N*0.1 - (Billet radii ratio)
       WRITE(5,5)G(1)
     5 FORMAT(F7.3)
       DO 19 J=0,14,1
       RO=1.2+J*0.2 - (Initial extrusion ratio)
       K=0
       MU(2)=0.005*M
       R=RO
       LOGRO=ALOG(RO)
       G(2)=G(1)
       G(3)=SQRT((G(1)**2.0-1.0)/RO+1)
    18 K=K+1
       F(1)=(G(2)-1.0)/(G(3)-1.0)
       C(1)=(1.0-COS(A))/SIN(A)
       E=C(1)*(1.0+0.5*ALOG(F(1)))+ALOG(R) - (Mean strain)
       IF(K-1)8,8,9
     8 YIELDS=167000*E**1.294/(E*1.294)
       BETA=1.0/1.294
       GO TO 10
     9 YIELDS=167000*((E+STRAIN)**1.294-STRAIN**1.294)/(E*1.294)
                                                   - (Mean yield stress)
```

```
   BETA=YIELDS/(167000*(E+STRAIN)**0.294) - (Work hardening factor)
10 C(2)=COS(A)/SIN(A)
   F(2)=(G(2)+1.0)/(G(3)+1.0)
   F(3)=(G(2)-G(3))/(G(2)**2.0-1.0)
   F(4)=(G(2)**2.0-G(3)**2.0)/(G(2)**2.0-1.0
   PSI=MU(1)*BETA*C(2)*((M+1)*ALOG(F(1))-(M-1)*ALOG(F(2))
                                              - (Friction factor)
   1-2.0*M*F(3))/(F(4)*(1.0+MU(1)*C(2)))
   F(5)=1.0/(G(2)**2.0-1.0)
   P(K)=YIELDS*E/((1.0-PSI)*(1.0+F(5))) - (Oil pressure)
   Q=YIELDS*E*/((1.0-PSI)*(1.0+MU(1)*C(2))*F(4)) - (Mean die stress)
   BILLS=P(K)*F(5)+MU(2)*C(2)*2.0*F(3)*Q - (Billet augmenting stress)
   EXTP=P(K)*(1.0+F(5))
   IF (K-1) 11,11,20
20 IF (25-K) 13,13,12
12 IF (2RO-R) 13,13,14
14 IF (ABS(P(K)-P(K-1)).LE.500.0)GO TO 15
   GO TO 11
13 WRITE (5,16) RO,K,R,LOGRO
16 FORMAT (F6.1,14,F7.2,F8.3)
   GO TO 19
15 WRITE (5,17) RO,K,R,LOGRO,EXTP,P(K)
17 FORMAT(F6.1,14,F7.2,F8.3,2F11.1)
   GO TO 19
11 STRAIN=BILLS/167000.0
   STRAIN=STRAIN**3.4
   R=RO*EXP(STRAIN) - (Extrusion ratio)
   G(2)=SQRT(EXP(STRAIN)*(G(1)**2.0-1.0)+1.0)
   GO TO 18
19 S=K
   STOP
   END
&RUN
```

Program (d)

```
 &JOB; PROPORTIONAL PRODUCT AUGMENTATION OF 20/25/Nb TUBE
&LIST; COPY OF PROGRAM ON LINE PRINTER
       DIMENSION G(2),F(8),C(2),S(2)
       REAL G,F,C,S,A,MU,R,LR,E,YIELD,BETA,PSI,P,H,Q
       INTEGER J,K,M
       MU=0.005 - (Mean coefficient of friction)
       A=7.5*3.1416/180.0 - (Semi die angle)
       BETA=1.0/1.320 - (Work hardening factor)
       DO 7 J=0,6,1
       G(1)=1.4+0.1*J - (Billet radii ratio)
       WRITE(5,1)G(1)
     1 FORMAT(F5.2)
       DO 7 K=0,24,1
       R=1.2+0.2*K - (Extrusion ratio)
       LR=ALOG(R)
       G(2)=SQRT((G(1)**2.0-1.0)/R+1.0) - (Product radii ratio)
       C(1)=(1.0-COS(A))/SIN(A)
       F(1)=(G(1)-1.0)/(G(2)-1.0)
       F(2)=(G(1)**2.00-1.0)/(G(2)**2.0-1.0
       E=C(1)*(1.0+0.5*ALOG(F(1)))+ALOG(F(2)) - (Mean induced equivalent
                                                                  strain)
       YIELD=E**0.320
       YIELD=167000.0*YIELD/1.320 - (Mean flow stress)
       F(3)=(G(1)+1.0)/(G(2)+1.0)
       F(4)=(G(1)-G(2))/(G(2)**2.0-1.0)
       F(5)=(G(1)**2.0-G(2)**2.0)/(G(1)**2.0-1.0
       C(2)=COS(A)/SIN(A)
       PSI=MU*C(2)*BETA*2.0*(ALOG(F(3))+F(4))
       1/(F(5)*(1.0+MU*C(2))) - (Friction factor)
```

```
      F(6)=1.0/(G(2)**2.0-1.0)
      F(7)=PSI*(G(1)**2.0-G(2)**2.00)/((G(1)**2.0-1.0)*(1.0-PSI))
      P=YIELD*E/(1.0-PSI)
      P=P/(1.0+F(6)*(1.0+F(7))) - (Oil pressure)
      H=F(6)*P/YIELD
      F(8)=1.0/F(5)
      Q=YIELD*(F(8)*E-H)/((1.0-PSI)*(1.0+MU*C(2))) - (Mean die stress)
      S(1)=P*F(6)-MU*C(2)*2.0*F(4)*Q - (Stress induced in the product)
      S(2)=E**0.320
      S(2)=S(2)*167000.0 - (Approximate ultimate tensile strength of
                            the product)
      IF (S(1)-S(2))2,3,3
    2 WRITE(5,4)R,LR,P,Q
    4 FORMAT(F6.1,F8.3,2F12.1)
      GO TO 7
    3 WRITE(5,6)R,LR
    6 FORMAT(F6.1,F8.3,9H FAILURE)
    7 M=J
      STOP
      END

&RUN
```

9. REFERENCES

1. 1600/80 ton hydrostatic extrusion press. The Engineer. Sept. 8th, 1967.
2. Hoffman, O. and Sachs, G. 1953. Introduction to the Theory of Plasticity for Engineers. McGraw-Hill Book Co. Inc.
3. Green, A.P. 1960. Plane Strain Theories of Drawing. Proc. Instn. Mech. Engrs. Vol. 174 No. 31.
4. Lancaster, P.R. and Rowe, G.N. 1963. Experimental Study of the Influence of Lubrication upon Cold Drawing under Approximate Plane-strain Conditions at Low Speeds. Proc. Instn. Mech. Engrs. Vol 178, Part 1, No. 3.
5. Davis, F.A. and Dokas, S.J. 1944. Theory of Wire Drawing. J. Appl. Mech. Vol. II. p.p. 193-198.
6. Hill, R. and Tupper, S.J. 1948. A New Theory of the Plastic Deformation in Wire Drawing. J. Iron and Steel Inst. 159. 353
7. Christopher, D.G. and Naylor, N. 1955. Promotion of Fluid Lubrication in Wire Drawing. Proc. Instn. Mech. Engrs. Vol. 169. No. 35.
8. Siebel, E. 1947. The Present State of Knowledge of the Mechanics of Wire Drawing. Stohl und Eisen 66, 171.
9. Pugh, H.Ll.D. 1965. Recent Development in Cold Forming. Bullied Memorial Lectures. University of Nottingham. Vol. III A & B
10. Taylor, G.I. and Quinney, H. 1931. The Plastic Distortion of Metals. Trans. Roy. Soc. (London) Series A, Vol. 230, p.p. 323-362.
11. Watt, A.B. and Ford, H. 1952. An Experimental Investigation of the Yielding of Strip Between Smooth Dies. Proc. Instn. Mech. Engrs. (B) IB 448.
12. Mellor, P.B. 1956. Stretch Forming Under Fluid Pressure. J. Mech. Phys. Solid. 5, 41.

13. Green, D. 1964. An Experimental High Speed Machine for the Practical Exploitation of Hydrostatic Extrusion. J. Inst. of Metals. Vol. 93, p.p. 65-70.

14. Slater, H.K. and Green, D. Augmented Hydrostatic Extrusion of Continuous Bar. (High Pressure Conference) 1967-68. Proc. Inst. Mech. Engrs. Vol. 182, Part 3C.

15. Alexander and Lengyel, B. Semi-continuous Hydrostatic Extrusion of Wire. Paper 7. Appl. Mech. Conf., Proc. Instn. Mech. Engrs. 1965-66. 180 (Part 3I), 317.

16. Low, A.H. and Donaldson, C.J.H. 1967. An Investigation of Speed Instability in Hydrostatic Extrusion. N.E.L. Report No. 289.

17. Rozner, A. and Faupel, J.H. 1964. Some Considerations of the Mechanics of Hydrostatic Extrusion. J. of the Franklin Instit., Vol. 277, No. 3.

18. Aritzur, B. Private Communication 1967. Lehigh University, U.S.A.

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