PREDICTION EQUATIONS FOR CUTTING FORCES WHILE OBLIQUE MACHINING EN - 8 STEELS EMPLOYING "RESPONSE SURFACE METHODOLOGY".

BY

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LIST OF SYMBOLS

SECTION 2.2.

S_s - Shear Strength of Work-piece material.
f - Feed/Revolution.
d - Depth of cut.
λ - Chip Compression Factor (Ratio of chip thickness 'after' to 'before' cutting).
τ - Friction Angle.
α - True Rake Angle.
ϕ - Shear Plane Angle.
C_p - Unit cutting force for an area of cut 0.001 in^2.
Z_p - Slope of double logarithmic plot of C_p against area of cut.
A - Area of cut in units of 0.001 in^2.
G - Slenderness Ratio9(Ratio of depth of cut to feed/rev.)
g_p - Exponent to Slenderness Ratio.

SECTION 2.3.

v - Cutting velocity.
λ - Angle of inclination of Main Cutting Edge.
γ - Rake Angle in Normal plane.
η - Angle of deflection of chip from main normal plane.
ϕ - Main Plan Angle.
δ - Clearance Surface Wear-land width.
$A_{2.5}$- Estimate of Shear Strength of Work-piece material.
ϵ - Cutting Ratio(Ratio of chip thickness 'before' to 'after' cutting).
$\bar{\epsilon}$ - Percentage reduction in cross-sectional area of the chip after cutting.
σ - Tensile Strngth of the work-piece material.
 - Mean coefficient of Friction.
C - Constant sum of Shear Plane Angle and Angle of Action.

SECTIONS 4.1 TO 8.2.

x_1 -- Cutting Speed Variable.
x_2 - Feed Rate Variable.
x_3 - Depth of Cut Variable.

SUMMARY.

Investigations have confirmed the existence of a linear relationship between the resultant of the two power components of cutting force and each of the three variables of (1) Depth of cut (2) Feed-rate and (3) Cutting speed in the double logarithmic scale during oblique machining of a plain carbon steel on a centre lathe.

A linear multiple regression model was therefore postulated between these three variables and the cutting force response, in their logarithmic transformed state, for prediction purposes of the latter.

An experimental design involving two replicates of fifteen 'different' cutting conditions and nine 'repeats' was employed to formulate a first order three-variable prediction equation using the technique of "Response Surface Methodology". (1*) This was then extended to include the quadratic effects of the three variables and also their first order interactions. The three variable first order equation was chosen for prediction purposes on statistical significance basis. A control test series of 30 cuts was then carried out to check the adequacy of predicted forces. The largest variation was only 7%.

Some results on the pattern of variation of forces with tool wear and that of surface texture with tool wear were also presented.

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CHAPTER 1.

1. INTRODUCTION AND SCOPE OF THE PRESENT INVESTIGATION.

1.1 INTRODUCTION.

Cutting forces were, originally, attempted to be studied systematically by F.W. Taylor (2)* in the U.S.A. Apart from the various easily identified factors, there were other influencing variables like the condition of the cutting edge, tool wear, non-uniformity of the tool and workpiece material properties, inhomogeneties in the workpiece structure, and machine conditions which affected the cutting forces significantly. Even if these factors were identified, they were difficult to be measured quantitatively. This aspect made the repeatability of results during investigations so difficult that early investigators like Taylor thought it not worthwhile to spend any efforts in this direction. (2)*.

The chief factors affecting cutting forces, in the case of cutting by single point tools, are:

- (i) Depth of cut.
- (ii) Feed rate.
- (iii) Cutting speed.
- (iv) Radius of tool tip.
- (v) Hardness, Strength, Ductility and work-hardening properties of the workpiece material.
- (vi) Conditions at the tool tip - workpiece interface.
  - (a) Sharpness of cutting edge.
  - (b) Tool wear at flank, clearance and rake faces.
  - (c) Extent and nature of built-up edge, if any present.
  - (d) Friction between tool faces and workpiece.
  - (e) Temperature at the cutting zone.
- (vii) Tool Geometry.
- (viii) Properties of coolants used, if any.
- (ix) Hardness and Strength of Tool material.

Under production environments, a knowledge about the cutting forces is important for the following reasons:

- (i) Influences the capacity and conditions of the lathe, its tools and attachments.
- (ii) Affects the power consumption.

(iii) Helps to decide the machinability of any particular workpiece material.

(iv) Affects the quality of surface finish and precision of the machined parts in an indirect manner.

However, the most important use of a knowledge about the extent and direction of cutting forces and the variables affecting them is while designing the machine tool itself. A reliable estimate of the cutting forces that a machine tool would be subjected to, under the worst combination of circumstances that the machine is intended for, is necessary for the effective and efficient design of the structures that constitute it so as to possess sufficient strength and rigidity. Only then the machine could be considered as of good technical design and would be economical in its performance. While writing up the operation manuals for the various machines, this knowledge is vital for safety and effectiveness during use. Thus, lighter, faster and safer machine tools have resulted from a knowledge of cutting forces. This knowledge would also enable parts intended for machining to be designed to withstand cutting forces thereby reducing deflections and consequently leading to higher accuracies and productivity.

A great majority of the workers, in the past, have confined themselves to treating the forces during orthogonal cutting only. Even in these cases, the equations are complex, difficult to comprehend and calculations very much involved to be of easy, day-to-day use. Thus there is a need for a simple, easily comprehensible equation for calculating cutting forces reliably. Also, in these days of advanced technology, the number of materials used is great many. It is, therefore, equally important to be able to formulate a reliable equation with a minimum expenditure of resources for different materials. With regard to both these requirements, the investigations presented filled in the needs adequately.

1.2 SCOPE OF THE PRESENT INVESTIGATIONS.

In the present investigation, it was decided to include the following important factors affecting cutting forces:-

(a) Depth of cut.

(b) Feed rate.

(c) Cutting speed.

POWER COMPONENTS OF CUTTING FORCE

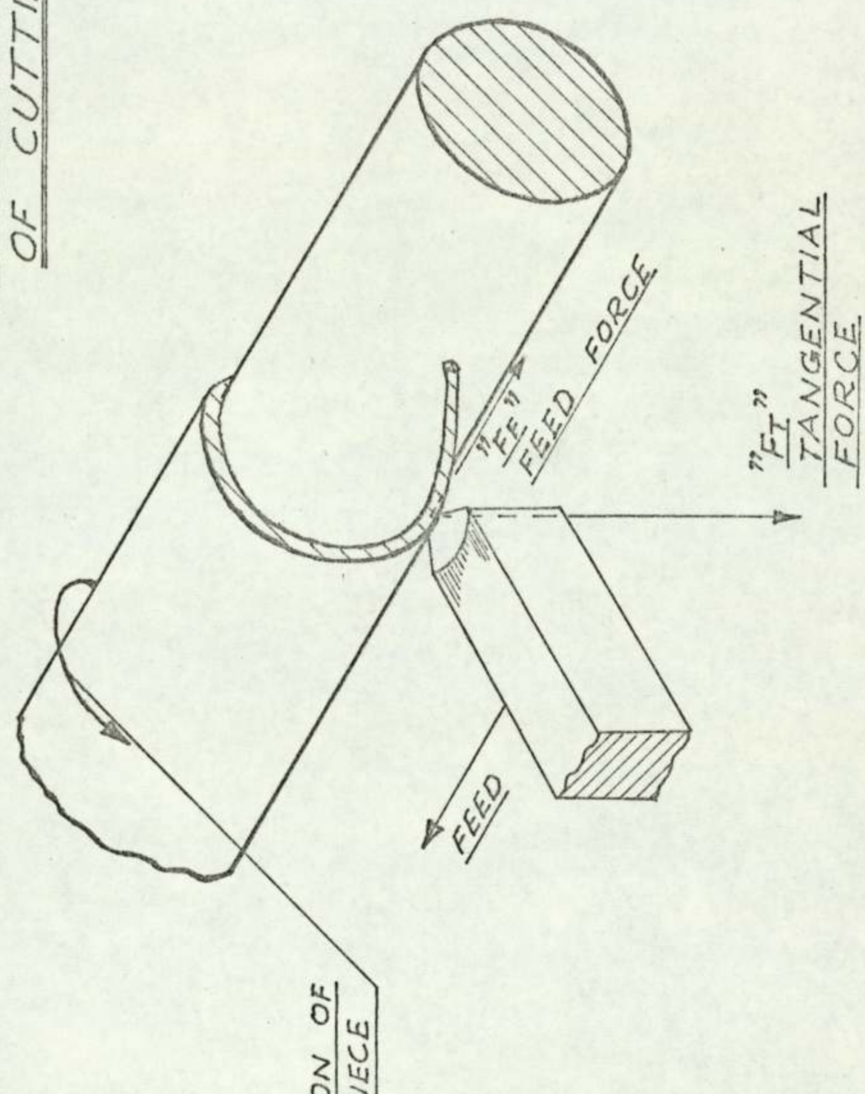

FIG. 1

The resultant of the two power components of cutting forces (i.e. those of the 'Tangential' force and the 'Feed' force (Fig.1)) was observed to be plotting linearly in double logarithmic scale to each of the above three factors, individually, in certain practical ranges of values and therefore offered scope for fitting a plane in a four dimensional space using multiple regression analysis.

The interactions between pairs of the above three variables and their quadratic effects were then assumed to exist and the regression analysis was developed to include them as well.

In each of the above cases the equation to the mean square plane was formulated employing "Response Surface Methodology". (Sect.5.1) The significance of the contribution made by each variable and their combinations was tested statistically and the final form of the prediction equation was chosen.

A further series ot cutting tests was run measuring the actual cutting forces. The forces were also calculated using the accepted prediction equation and these were compared with the observed ones.

All tests were intended to exclude any large effects of built-up edge formation and therefore the lowest value of cutting speed chosen was 150 f.p.m.

The range of cutting speeds varied from 150 f.p.m. to 1050 f.p.m., that of feeds from 0.0022 i.p.r. to 0.0200 i.p.r. and that of depth of cuts from 0.015 inch to 0.105 inch.

All cuts were carried out 'dry' without the use of coolants.

Before the commencement of the test series, the pattern of variation of cutting forces with tool wear was studied and the appropriate portion of tool life was chosen to minimize this effect.

The workpiece material in all tests was EN 8 steel of 220 - 235 Brinell Hardness in the forms of hot rolled bars of diameter 2½ to 3½ inches.

The tool material employed was of the sintered carbide variety manufactured by Messrs. Wickman Wimet Ltd., with the following specifications:-

Manufacturer's grade:-	XL3
Chemical Composition:	9% Cobalt.
	9% Titanium Carbide.
	12% Tantalum Carbide.
	70% Tungsten Carbide.

Hardness: 1450 V.P.N.

Rupture Strength: 280,000 lbf / $in.^2$.

Modulus of Elasticity: 74 x 10^6 lbf / in^2.

The measured surface finish on the rake face of the tips varied between 10 - 15 micro-inches C.L.A. value.

The dimensions of the square tips were:

Sides: 0.5 in.

Thickness: 3/16 in.

The geometry of the tool when mounted on the tool-post mass of the dynamometer was constant at values of:

Back Rake Angle - 5^o (Negative).

Side Rake Angle - 5^o (Negative).

Front Clearance Angle - 5^o.

Side Clearance Angle - 5^o

Side Cutting Edge Angle - 16^o

Tool-nose radius - 0.040 inch.

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CHAPTER 2.

2. SUMMARY SURVEY OF SOME IMPORTANT PREVIOUS WORKS.

2.1 GENERAL SURVEY (*3)

The earliest recorded attempts to measure cutting forces were those of R.H. Smith using a dead weight balance in 1882. In 1892 A.Haussner built a planing dynamometer using a stiff spring and in 1893 K.A. Zvorykin published his results on extensive theoretical and experimental work using a hydraulic dynamometer in planing operations. Other early workers were Nicolson (1903), F.W. Taylor (1907), G. Linder (1904) etc..

Among the more recent workers are the names of G.Schlesinger, V. Piispanen, H.Ernst, M.E.Merchant, M. Kronenberg, N.N. Zorev, and M.C. Shaw. All of them and many more have contributed much to the theory and development of metal cutting mechanics in general and specially to the development and/or applications of dynamometry in metal cutting operations.

One of the earliest people to formulate prediction equation for cutting forces were O.W. Boston and C.E. Kraus (*4). In addition to developing a number of empirical equations for varying rake angles in the case of orthogonal cutting, they have given the following two equations for oblique cutting.

Tool Specifications.	Tangential Force.	Feed Force.	Radial Force.
8-14-6-6 - 45-3/64	$88{,}000\ f^{0.74}\ d^{0.93}$	$31{,}400\ f^{0.66}\ d^{1.12}$	$40{,}700\ f^{0.77} d^{1.0}$
8-14-6-6 - 30-3/64	$120{,}000\ f^{0.8}\ d^{1.0}$	$36{,}500\ f^{0.6}\ d^{1.28}$	$14{,}500\ f^{0.68} d^{0.84}$

f - feed rate.

d - depth of cut.

TABLE 1.

The limitations of these two equations were that they involved only two variables viz: feed and depth of cut and the experimental design was such that the effect of each of these was determined separately and combined to give the equations. The tools employed were of the high speed steel material 3/8 inch square section, and during the determination a constant cutting speed of 80 f.p.m. was employed. The work material was a 0.21 % carbon plain carbon steel in the form of annealed bars of 4 - 6 inch diameter. Boston and Kraus had found that the speed effect in the range of 26 - 320 f.p.m. on forces was negligible.

Some German workers have also formulated predictive equations for cutting forces.

However, the most outstanding among the well known works in the field of metal cutting force investigations are those due to

(a) M. Kronenberg (5*)
(b) N.N. Zorev. (6*)

It would be, therefore, appropriate to briefly review the ideas of these two authorities.

2.2. SUMMARY OF WORK BY M. KRONENBERG IN RELATION TO CUTTING FORCES.

As a first approximation for the 'Tangential' and 'Feed' cutting forces, Kronenberg developed the following formulae:

$$\text{Tangential Force, } P_1 = Ss \times f \times d \times \frac{\lambda \cos(\tau - \alpha)}{\cos(\tau - \alpha + \phi)}$$

$$\text{Feed Force, } P_2 = Ss \times f \times d \times \frac{\lambda \sin(\tau - \alpha)}{\cos(\tau - \alpha + \phi)}$$

Where:

Ss = Shear strength of the workpiece material.
f = feed/rev.
d = depth of cut.
λ = Chip compression factor.
τ = Friction Angle.
α = True Rake Angle.
ϕ = Shear plane Angle.

The chip compression factor, in addition to being the ratio of the chip thicknesses before and after cutting, is also equal to the ratio of the cutting velocity to the velocity of chip flow. Thus, the expression for forces has taken into account:

(a) The dimensions of the cut.
(b) The kinetics of the cutting action.
(c) Friction effects.
(d) Some aspects of tool geometry.

By analyzing the data of G. Altmeyer and H. Krapf of the Soviet Union (5*) Kronenberg has shown that this formula yielded values which were in closer agreement to observed ones than those obtained by using others developed by Piipsanen, Hucks and Krystoff.

Kronenberg has emphasized that (a) Unit cutting force and (b) Metal removal rate per minute per h.p., both referred to the same fundamental quantity but bearing an inverse proportionality relationship between them.

As further refinements Kronenberg proceeded to develop the following two laws:

1. ELEMENTARY CUTTING FORCE LAW.

According to this law, the Tangential Force P_1 is given by -

$$P_1 = \frac{C_p \times (1000A)^{1-Z_p}}{1000} \text{ lbf} \quad - - - - \text{(i)}$$

where:

A = Area of cut (i.e. f x d) expressed in units of 0.001 in.^{2}

C_p = Cutting force registered while removing a chip of cross-sectional area 0.001 in.^{2} for a particular work-piece material.

Z_p = Slope of the double logarithmic plot of unit cutting force against chip cross-sectional area (0.001 in.^{2}) for a particular work-piece material.

Kronenberg has high-lighted the fact that the actual cross-sectional area of the chip is smaller than the nominal area (i.e. f x d) and has included some typical correction graphs (5*) to compensate the effects of plastic deformation during cutting.

He has also furnished a table (Table 52 of (5*)) giving the comparative values of 'C_p' and 'Z_p' calculated from the data of several investigators and commented on disagreement in relevant instances.

2. EXTENDED CUTTING FORCE LAW.

(i) The basic formula for the Tangential force P_1 according to this law is:

$$P_1 = \frac{C_p \, (G/5)^{g_p} \, (1000A)^{1 - Z_p}}{1000} \text{ lbf} \quad - - - \text{(ii)}$$

Where:

A is the chip cross-sectional area in 0.001 in.^{2} units.

C_p is the cutting force in the case of a cut of 0.001 in.^{2} cross-sectional area for a particular material.

Z_p is half the sum of the exponents 'u' and 'v' of feed and depth of cut respectively in the equation for Unit Cutting force,

$K_s = \frac{C_1}{f^u d^v}$ where C_1 is a constant for a feed of 1 i.p.r. and depth of cut of 1 inch obtained by extrapolation.

g_p is half the difference of the exponents of 'u' and 'v' referred in the case of 'Z'_p'

G is the Slenderness Ratio (Ratio of depth of cut to feed/rev).

In this formula the effect of the "Slenderness Ratio" (i.e. Ratio of 'depth of cut' to 'feed-rate') of the cut is, additionally, taken into consideration. However, it was pointed out that the effect due to variations in the Slenderness Ratio is, by far, smaller than that due to the area of cut.

By analyzing the cutting force data of Taylor, Boston and Krauss, Holmes, R. Cave, A.S.M.E., A.W.F - 158 and Dawhil - Dinglinger, Kronenberg has shown fair agreement between the values of 'C'_p', 'g_p' and 'Z'_p' in the case of a few materials. (5*)

He has also indicated that it should be possible to prepare systematic tables for practical cutting force data by the application of the above formula after providing for the inclusion of "Hardness of Workpiece" and "True Rake Angle of Tool" factors.

In the case of Steels and Cast Irons, Kronenberg has, further, given a Table of values (Tables 8a, b, and c, and 9a, b and c, of the Appendix in (5*) for calculation of cutting forces using the "Extended Cutting Force Law".

(ii) <u>The inclusion of the effect of hardness</u> of different grades of the same basic material (e.g. Plain carbon steels, Straight Cast Irons etc.) has been next done by further splitting of the 'C_p' values applied in the basic formula above.

The 'C'_p' values for different grades of the same material were plotted against their respective Brinell Hardness numbers (B.H.N) in a double logarithmic graph. These points were found to lie on a straight line. The slope of this line was determined (Say 'n') and the line extrapolated to form an intercept (Say 'C_{p1}') on the 'C_p' axis. (Fig. 169 of ($\overset{*}{5}$)).

Then:

$$C_p = C_{p\,1}\,(B.H.N.)^n \quad - - - - \quad (iii)$$

The value of 'C_p' thus calculated could be substituted in Equation (ii) to include this effect.

(iii) The inclusion of the effect of True Rake Angle

For identical tools with varying Rake angles, it has been shown by Kronenberg from Stanton and Heyde data (Fig. 182 of ($\overset{*}{5}$)), that the Tangential force plots linearly with True Rake Angle in a double logarithmic graph for Ni-Steel. Similar to the case of 'Hardness' inclusion, by extrapolating the line, in the case of a cut with a cross sectional area of 0.001 in.^{2}, it would be possible to express the constant $C_{p\,1}$ as

$$C_{p\,1} = C_b\,(\beta/50)^m \quad - - - - - - \quad (iv)$$

Where:

C_b is the intercept of the plot.
m is the slope of the plot
β is the lip angle (90^o - (Rake Angle + Clearance Angle))
50^o is the smallest practical lip angle.

(iv) Procedure showing the inclusion of all the effects for calculation of Tangential Force.

In the case of a tool with a specific clearance angle, say 10^o (i.e. Lip Angle + Rake Angle = 80^o), Kronenberg has indicated the following method of arriving at the final value of C_p (to be applied in Equation (i)) taking into consideration the Lip Angle and Hardness factors:

(i) To find out the proportion (say x) of the 'C_p' cutting force constant for tools with a Lip Angle (say β_1°), which is due to the Lip Angle, write

$$C_p = x\,(\beta_1/50)^{m_1} \text{ for any particular material.}$$

(ii) From Table 52 of ($\overset{*}{5}$), find out the quoted value of C_p for the same material (Say C_{p_x}). Then:

$$x = \frac{C_{p_x}}{(\beta_1 / 50)^{m_1}}$$

Now m_1 can be determined from Fig. 194 of (5*) for the same material. Therefore x can be calculated and C_p can be determined.

(iii) As C_p value is also influenced by the Hardness factor, the 'x' value calculated should contain this effect also. From Fig. 169 of (5*), for the same material one can express $C_p = C_{p_1} (B.H.N.)^{n_1}$. (Equation (iv)). Since B.H.N., C_{p_1} and n_1, are known, in order to resolve the 'x' we could write

$$y = \frac{x}{C_{p_1} (B.H.N.)^{n_1}}$$ and 'y' can be calculated.

The factor 'x' could, thus, be expressed as

$$x = y \times C_{p_1} (B.H.N.)^{n_1}$$

(iv) Replace x in Step (i) to yield:

$$C_p = y \times C_{p_1} (B.H.N.)^{n_1} \times (\beta_1 / 50)^{m_1}$$

(v) Obtain Z_p from Table 52 of (5*) for the same material.

By substituting the values of C_p, Z_p and A in Equation (i) the Tangential force P_1 can be calculated.

CRITICISM AND COMMENTS.

The chief limitation of Kronenberg's analysis is that the laws apply to only orthogonal cutting. The various investigators whose data have been analyzed by Kronenberg observed independence between cutting forces and cutting velocity. In the case of oblique cutting with negative rake carbide tools, it has been observed during the current work that there is significant double logarithmic inverse linear relationship between the resultant of the two power components of the cutting forces ('Tangential' and 'Feed') and cutting speed. Therefore, the validity of neglecting this effect in the above laws could be open to criticism.

Another difficulty is that there is no unequivocal agreement between the 'C_p' and 'Z_p' values obtained by using the data from various investigators. Thus, the value of the calculated forces could vary significantly depending upon whose data has been used. Unless a separate series of cutting tests was specifically carried out to evaluate these parameters reliably, the calculated values could be in error. Even in this case, generality of results could not be hoped for because of the lack of complete agreement between materials specifications under various national standards.

However, Kronenberg's formulae have opened up the vista in the right direction and it could be anticipated that future investigators would bring in further refinements and extensions to enhance its usefulness.

2.3 SUMMARY OF N.N.ZOREV'S WORK ON "CALCULATION OF CUTTING FORCE COMPONENTS". (6*)

Zorev has suggested calculation of cutting forces components under two separate methods:-

(a) Cutting force components for a constant tool life.

(b) Cutting force components from the 'Cutting Ratio.'

(a) Force components at constant tool life could be calculated as below:

(i) From a knowledge of the following:

1). Tensile Strength of the material cut.

2). Depth of cut.

3). Rake angle on tool.

and 4). Type of tool material used, refer to the Nomogram (Fig.328 of (6)), to determine the 'Specific work of Chip formation' (Q_c).

(ii) From a knowledge of (2), (3) and (4) above, refer to the Nomogram (Fig. 329 of (6*)) to determine the mean coefficient of friction (μ) between the tool and workpiece material.

(iii) The angle of flow of chip ϑ could then be determined from the formula:

$$\vartheta = \eta - v^{-0.8} \text{ arc tan } (\tan \lambda . \text{Cos} \eta + \tan \gamma \text{ Sin } \eta)$$

where

v is the cutting velocity (m/min).

λ is the angle of inclination of the main cutting edge.

γ is the rake angle in the main normal plane.

η is the angle of deflection of the chip from the main normal plane which is found from the Nomogram (Fig.330 of (6*))from a knowledge of the depth of cut, feed, length of transition cutting edge, radius of curvature of transition cutting edge and main plan angle ϕ.

(iv) Calculate

1) $$K_x = \frac{\left[(\mu \cos\gamma \cos\vartheta - \sin\gamma)\sin\phi - \left[(\cos\gamma + \mu\cos\vartheta \sin\gamma) + \mu\sin\vartheta\cos\lambda\right]\cos\phi\right]}{\left[\cos\gamma + \mu\cos\vartheta\sin\gamma)\cos\lambda - \mu\sin\vartheta\sin\lambda\right]}$$

and

2) $$K_y = \frac{\left[(\mu\cos\vartheta\cos\gamma - \sin\gamma)\cos\phi + \left[(\cos\gamma + \mu\cos\vartheta\sin\gamma)\sin\lambda + \mu\sin\vartheta\cos\lambda\right]\sin\phi\right]}{\left[\cos\gamma + (\mu\cos\vartheta\sin\gamma)\cos\lambda - \mu\sin\vartheta\sin\lambda\right]}$$

(v) The projections of the Chip formation forces P''_z, P''_y, & P''_x (Tangential force, feed force, and radial force) are then determined from the application of following formulae

(a) $P''_z = Q_c \cdot S.t.$

(b) $P''_y = Q_c . k_y . S \times t.$

(c) $P''_x = Q_c \times k_x . S \times t.$

Where S = Feed/rev. in mm.

t = depth of cut - mm.

and P''_z, P''_x, P''_y are in Kgmf.

(vi) The projections of the forces on the Clearance Face P'_z, P'_y and P'_x (clearance face forces in the directions of cutting, feed and radial forces respectively) are then determined as follows:-

$$P'_z = q^1_N \, \mu \, \delta \left(\frac{t}{\sin\phi} + \gamma \tan \phi/_2 + S\right)$$

$$P'_y = q^1_N \, \delta \, (t.\cot\phi + \tan\phi/_2 + S)$$

$$P'_x = q^1_N . \delta . t$$

Where γ, t, s, ϕ are known values from above, δ is the clearance surface wearland width in m.m.,

q^1_N is the specific normal pressure on the clearance face of the tool obtained from the Nomogram (Fig. 322 of (6*) with a knowledge of the Hardness of work material and depth of cut. (Kgmf/mm.^{2})

(vii) The total components of the cutting forces (Chip formation forces and Clearance face forces) are then obtained from

$$P_z = P''_z + P'_z$$

$$P_y = P''_y + P'_y$$

and $P_x = P_x'' + P_x'$

(b) FORCE COMPONENTS FROM CUTTING RATIO.

Calculation proceeds by the following steps:-

(i) The specific work of chip formation (Q_c) is given by the formula

$$Q_c = A_{2.5} \left[\frac{(\varepsilon - \sin\gamma)}{\cos\gamma} + \tan C \right]$$

Where $A_{2.5}$ is an estimate of shear strength of the workpiece material given by

$$\frac{0.6\,\sigma}{1 - 1.7\,\bar{\varepsilon}}$$ (σ is the tensile strength & $\bar{\varepsilon}$ is the reduction in cross sectional area)

ε is the Cutting Ratio.

C is the constant sum of the shear plane angle and angle of action (Angle between the resultant force and the cutting speed vector) for any particular material (values of 'C' are quoted in a table by Zorev).

(ii) Compute the mean coefficient of friction (μ) from the equation

$$\mu = \tan\left[C - \gamma - \arctan\left(\frac{\cos\gamma}{\varepsilon - \sin\gamma}\right)\right]$$

Steps (iii) to (vii) are identical as in the case of calculations for constant tool life.

The advantages quoted by Zorev in the case of calculations of force from the Cutting Ratio are as follows:-

(a) No special apparatus is necessary to determine the Cutting ratio.
(b) The Cutting Ratio can be measured on any machine tool without any preliminary set up.
(c) The Cutting Ratio can be measured when using any type of tool.
(d) Measurement of Cutting Ratio is a simple operation which does not require any high skill.
(e) Reproducibility of results can be easily checked on the spot during measurement of Cutting Ratio.
(f) After having determined the Cutting Ratio under one set of cutting conditions corresponding to a given tool life, it could be used to calculate cutting forces for other sets of cutting conditions corresponding to the same tool life.

Zorev has further given a comparative table of actual experimental values of cutting forces and the calculated values for a series of 44 tests. Taking into consideration the possible experimental errors, the agreement between the theoretical and experimental values appeared to be fairly good.

CRITICISM AND COMMENTS.

The chief merit of Zorev's formulae are that they are the most theoretical ones which have been developed whereas most of the others are purely empirical ones. However, the calculations could sometimes be involved and time consuming especially in the case of oblique cutting. But this has been partially overcome by the author by giving ready reference Nomograms for computing the components of forces. The limitation of these Nomograms is that they pertain to materials and tools according to Soviet Standards and unless one is quite sure of the equivalence of materials the force values obtained would be erroneous. Another deficiency might be that the feed rate which affects the size of the cut has not been catered for in the formulae in any direct manner except that it entered in the calculation of the direction of chip flow in some indirect manner.

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CHAPTER 3.

3. EXPERIMENTAL APPARATUS AND THEIR CHARACTERISTICS.

3.1. DYNAMOMETER - GENERAL REQUIREMENTS: (7)*

The principle on which all dynamometers are based is one of measuring the strains produced in relevant structures constituting it consequent to the action of applied cutting forces.

The chief requirements in a dynamometer are:-

(1) Sensitivity.

It is desirable for the dynamometer to have a high sensitivity to applied forces.

(2) Rigidity.

The dynamometer should be rigid enough such that

(a) The applied forces do not deform its elements plastically.

(b) The elastic deflections in elements should be such that they do not substantially alter their size, shape or geometry.

(3) Natural Frequency.

In order that the force indications are not influenced by the cyclic motions involved in cutting operations, the natural frequency of the tool-post mass of the dynamometer must be considerably higher than the frequency of exciting vibrations.

(4) The 'flat frequency response' of the sensing elements, if any used, should be adequate for the dynamic ranges required.

(5) Cross Sensitivity.

In general, as a dynamometer should measure more than one component of the cutting forces, the cross sensitivity between these components should be as small as possible.

(6) Linear Calibration.

It is desirable for the dynamometer to have a linear calibration for all the components of forces in the required ranges.

(7) Repeatability of Readings.

A dynamometer should be stable with respect to time, temperature and humidity.

The dynamometer employed during the investigations was found to satisfy the above requirements quite well.

3.2. P.E.R.A. DYNAMOMETER.

The dynamometer used in the present investigations was designed and built by the Production Engineering Research Association of Great Britain. It was designed to measure up to a maximum of 2000 lbf, with the lowest gain setting, of each of the three components of 'Tangential', 'Feed', and 'Radial' cutting forces. The output sensitivity could be increased by using increased gain settings of the output amplifier, with a consequent reduction in range of loading. With the maximum gain setting, the measuring range was 200 lbf and this setting was employed for most of the readings during the investigations (with the exception of larger forces in which case the next lower gain setting was employed).

The mechanical details of the dynamometer tool-post mass were shown in Fig. (i) of the Appendix. The force sensing elements were a set of 24 strain gauges wired up in three Wheatstone bridge configurations, each bridge measuring one component of the forces (Fig.(ii) of Appendix). Temperature compensation was also achieved by the use of compensating gauges in the bridge configuration.

The chief difference of this dynamometer from others using similar principles is that all the three components of forces were measured by the strains of a single set of ligaments (4 ligaments forming sectors of a circular annalus). In other dynamometers, usually, rings located at different orientations are used for individual components of forces. While all the strain gauges for measuring the 'Tangential' and 'Feed' forces were bonded radially (with longer axis of the gauge parallel to the circumference of annalus), those for measuring the 'Radial' force were bonded parallel to the longitudinal axis, along the ligaments. Also, the temperature compensating gauges for the 'Radial' bridge were located on 4 dummy ligaments.

The output from each of these three bridges was fed on to a stable variable gain amplifier (Manufactured by Fylde Electronics, Ltd.) After amplification, the output was indicated on a voltmeter with a full scale deflection of $\pm$ 5 volts whose dial was marked into a 100 divisions equivalent to the full scale deflection. To the front of the ligament arm was attached a recessed plate for taking in the tool holder. The strain gauge and ligament assembly were housed in a water proof casing with provision for terminals at its rear end for the three output signals from the individual bridges. The tool holder could be clamped into the recess at the front of the mass by means of three cap-head screws. The tool holder, in turn, incorporated clamping arrangements for 'throw away' type of square carbide insert tool tips. (Fig. (iii) of Appendix). The square tip was seated inside a square slot and clamped by means of a single cap-head screw. The tool-post mass, tool holder, three calibration tools and the console containing amplifiers and dials were displayed in Plate I.

3.3. CALIBRATION OF P.E.R.A. DYNAMOMETER.

In addition to the tool holder, three specially designed calibration tools were also provided. These were, principally, of the same external dimensions as the tool holder and fitted exactly in the recess provided in the front plate of the tool-post mass. Individually, these three could also be fixed into the slot, by means of the three cap-head screws provided, during calibration. The 'Tangential' calibration tool was fixed for 'Tangential' force calibration and one of the other two tools respectively for the calibration of the other two forces.

The calibration of the dynamometer was achieved in a "Denison" Tensile Testing machine by the application of known compressile loadings vertically, but with the dynamometer in three different orientations. Plate 2 showed the set up on the Denison machine during calibration. The critical aspect of the three calibration tools was that they contained three small conical cavities at relevant locations such that when forces were applied at these locations, through a small diameter hardened steel ball placed on these cavities, the points of application of the three forces were exactly identical to the points of application of the three forces on the tool tip during cutting operations. This aspect ensured complete static similarity of conditions during calibration and cutting operations. This was an essential condition to be satisfied for accurate calibration.

With the calibration tool in position, the tool post mass was straddled down rigidly to the base of the Denison machine by means of a liberally proportioned straddling plate and two bolts. As the range in which calibration required in the present investigations did not exceed 500 lbf., this method of clamping down was found to be adequate. To the bottom of the movable platform (sliding along vertical columns of the machine to apply known loads) was fixed a

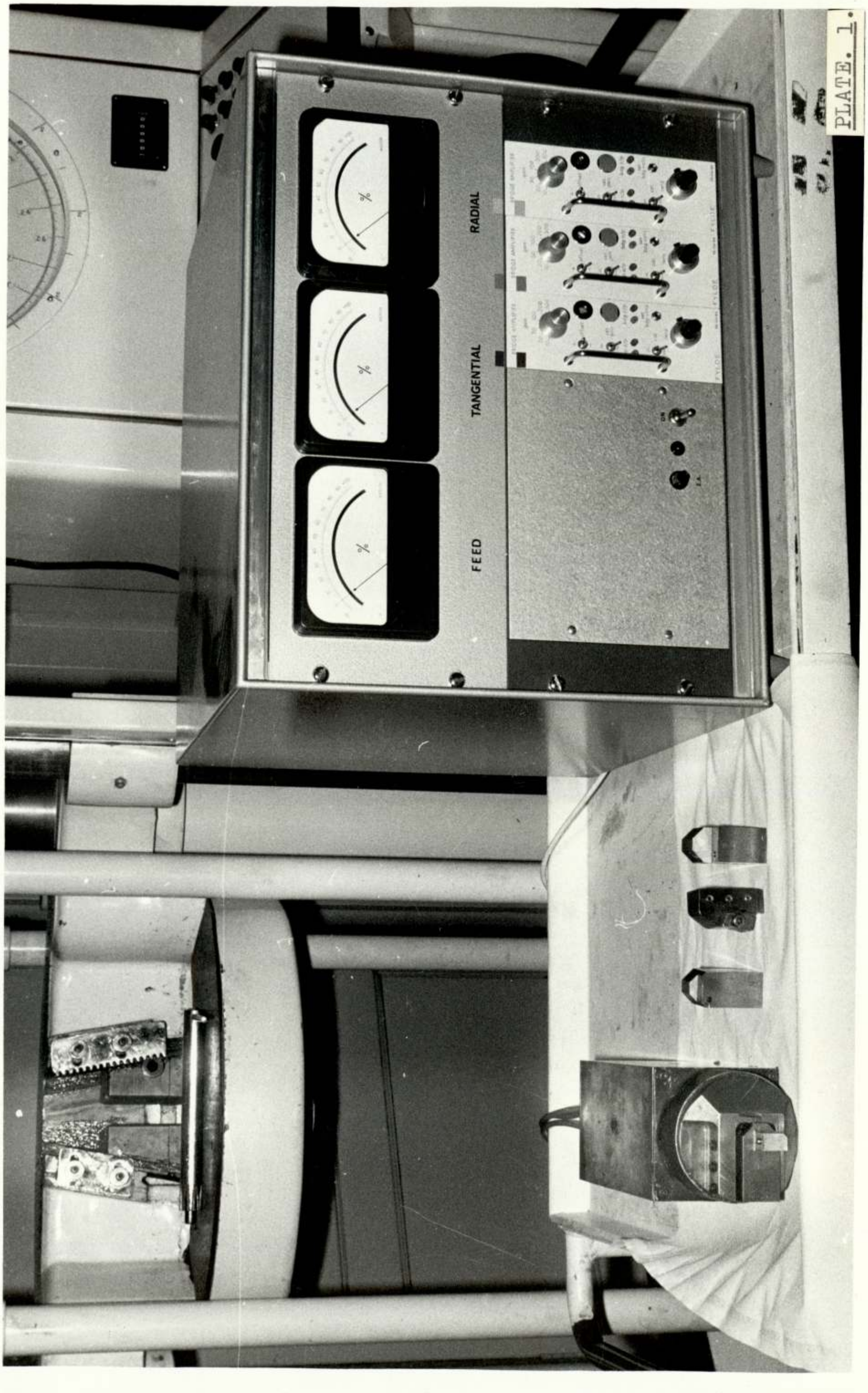


PLATE. 1.

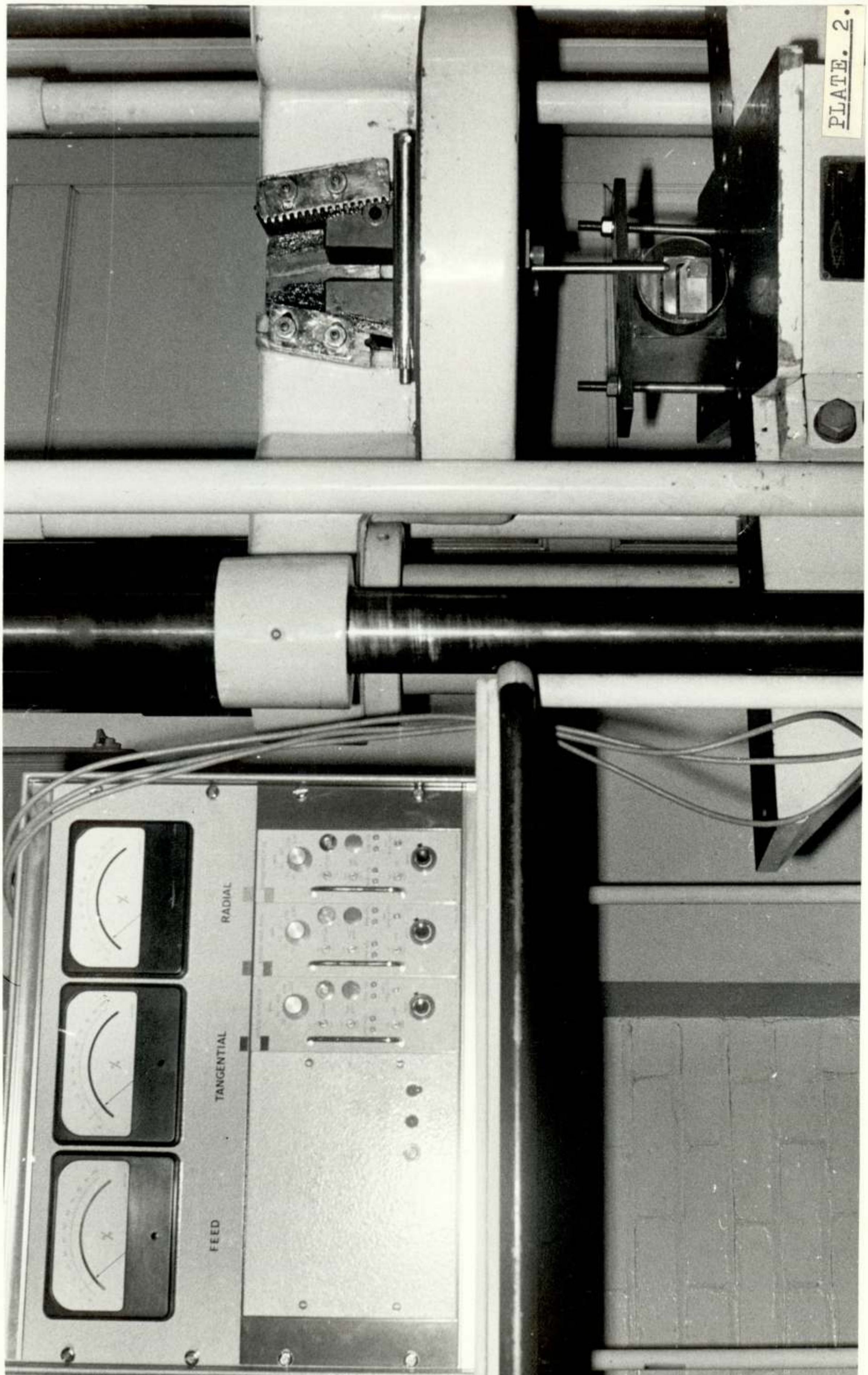


PLATE. 2.

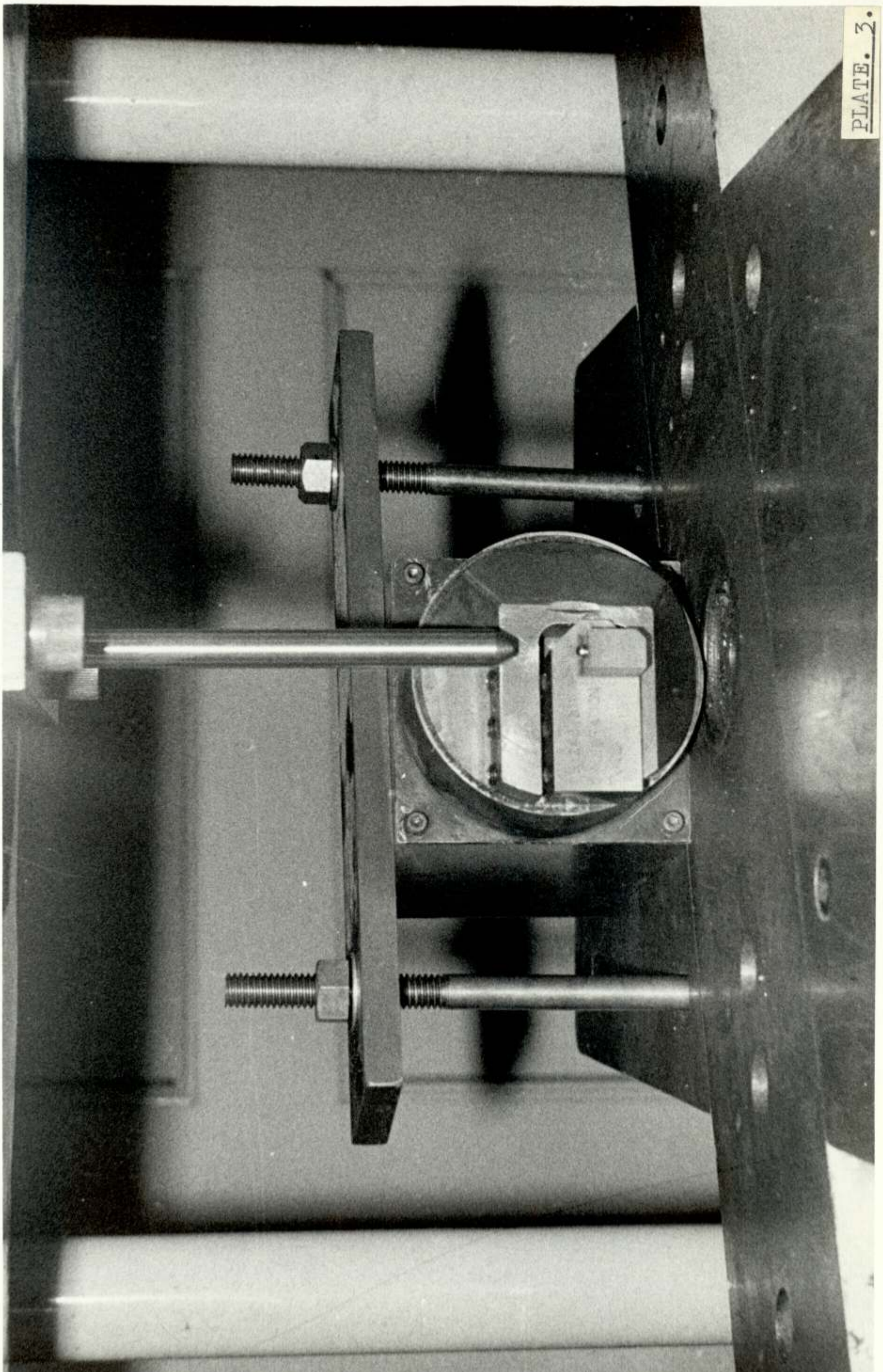
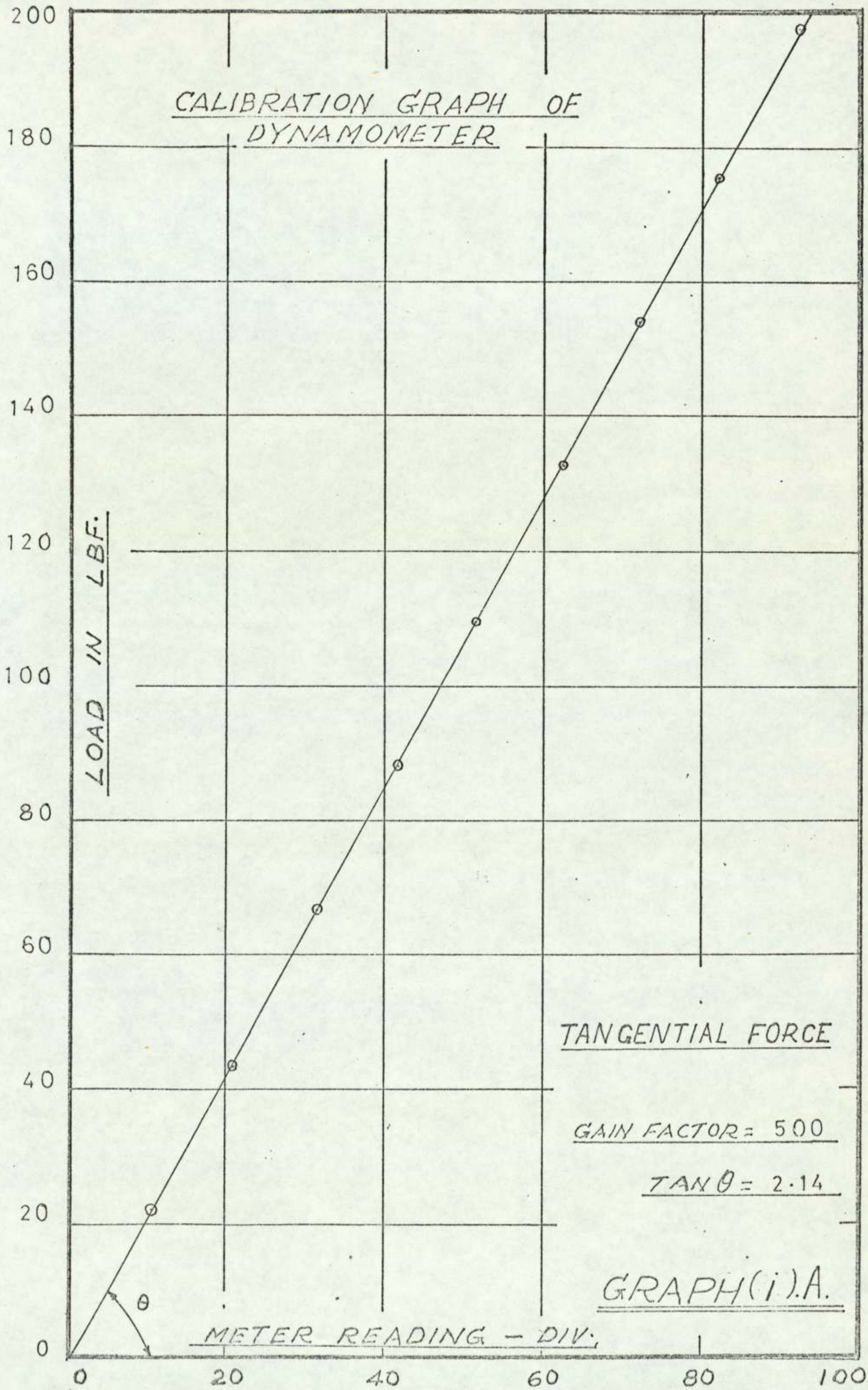
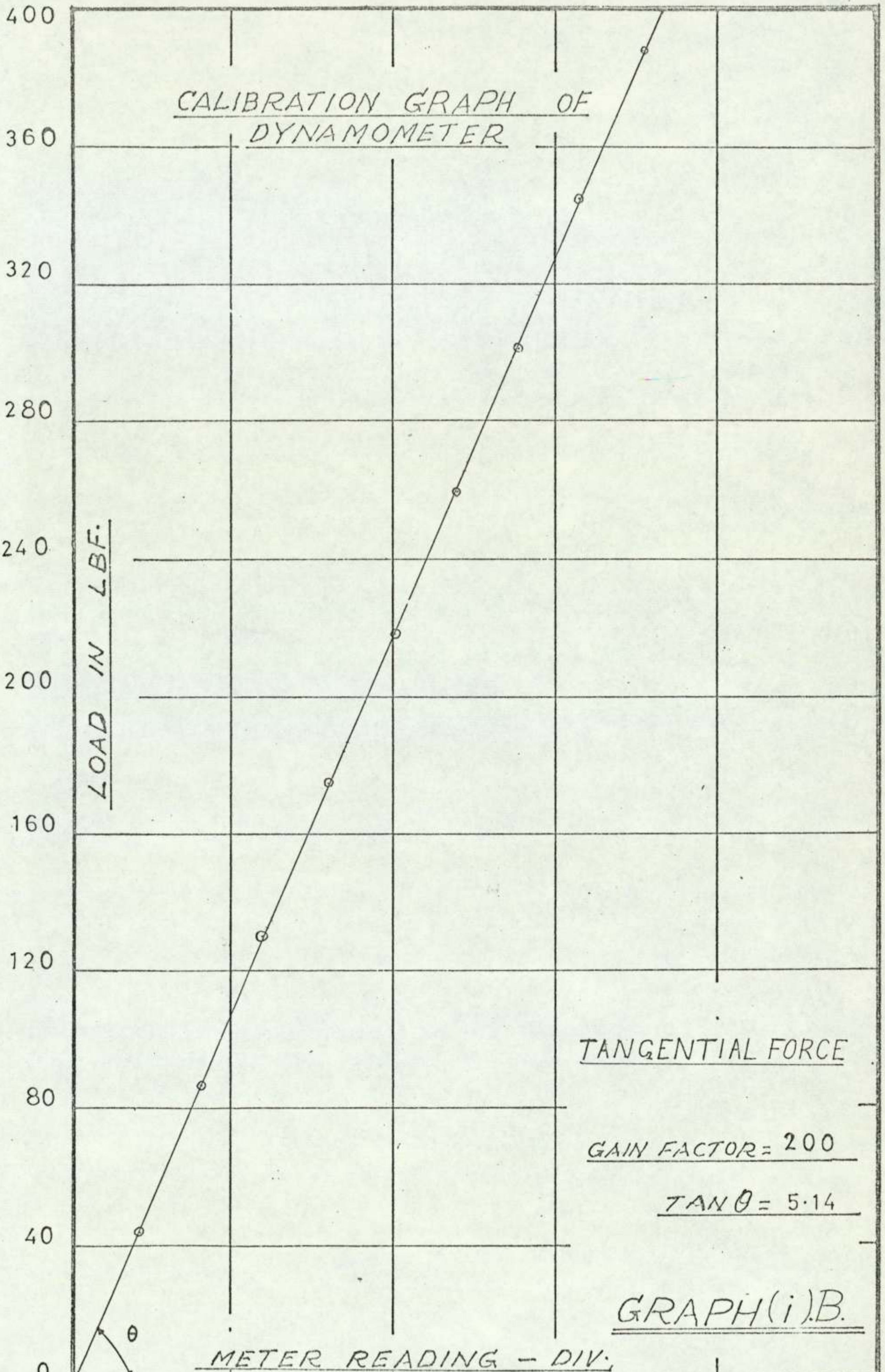


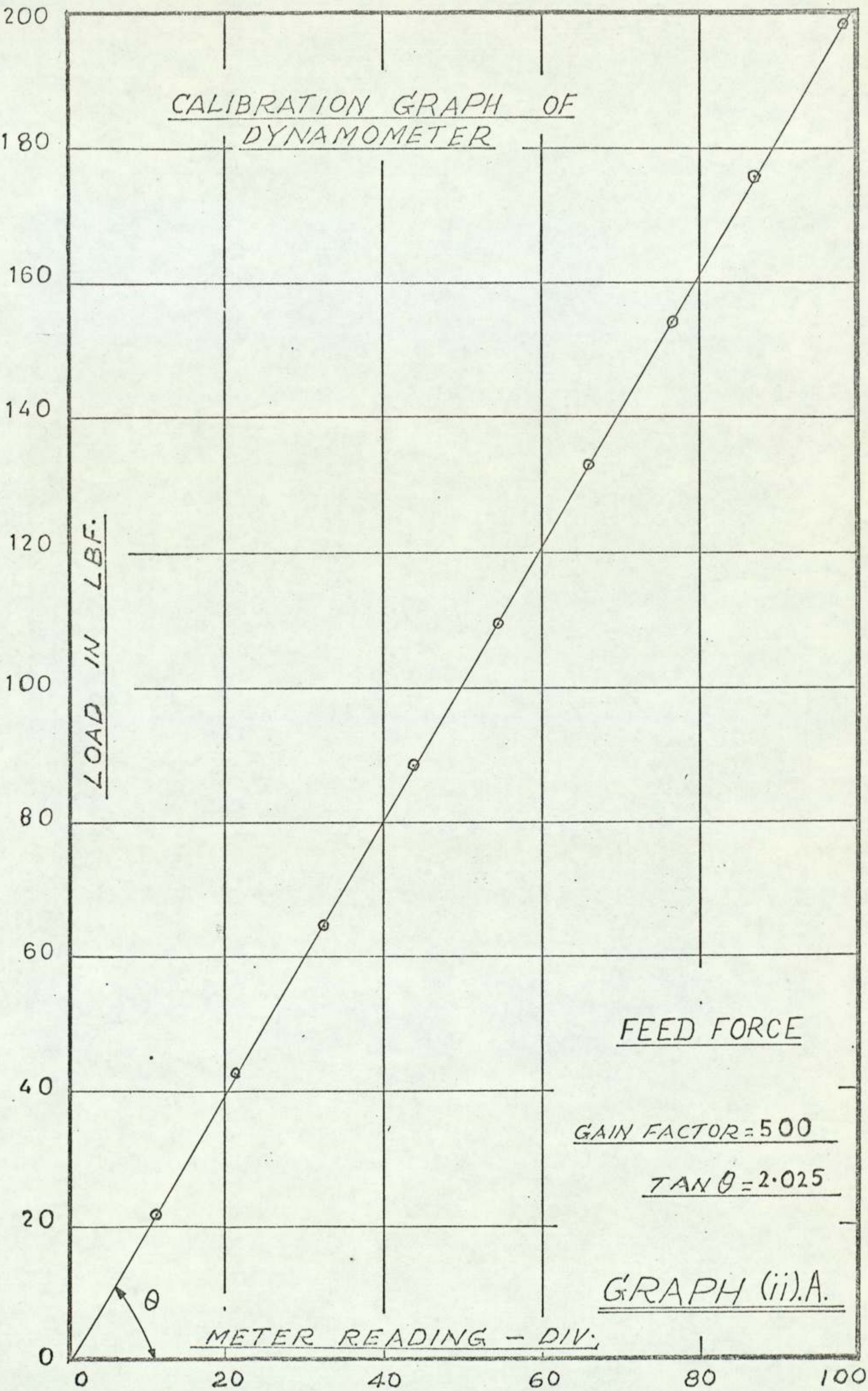
PLATE. 3.


CALIBRATION GRAPH OF DYNAMOMETER
LOAD IN LBF.
METER READING - DIV.
TANGENTIAL FORCE
GAIN FACTOR = 500
TAN θ = 2·14
θ
0
20
40
60
80
100
120
140
160
180
200
0
20
40
60
80
100

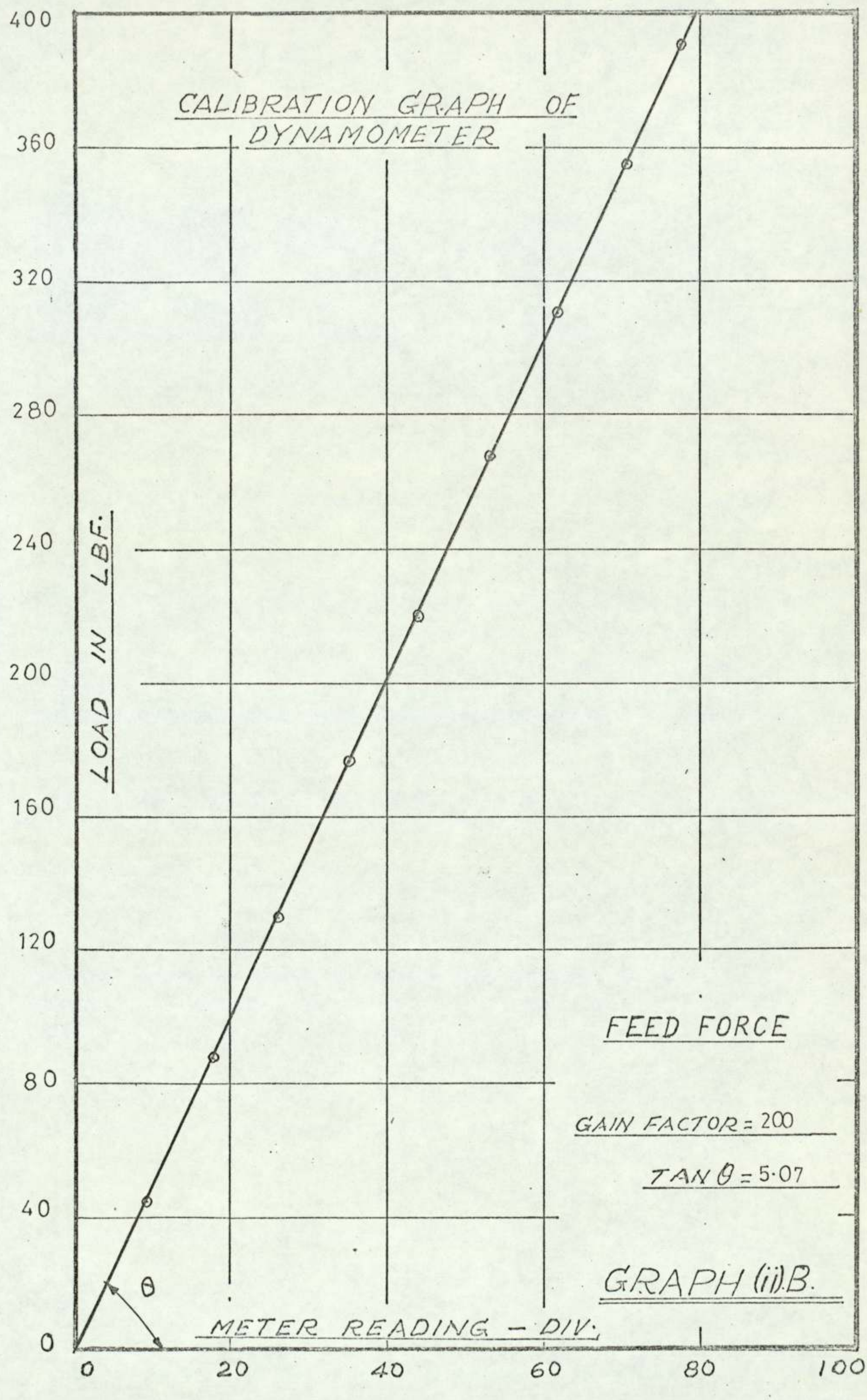
GRAPH (i).A.

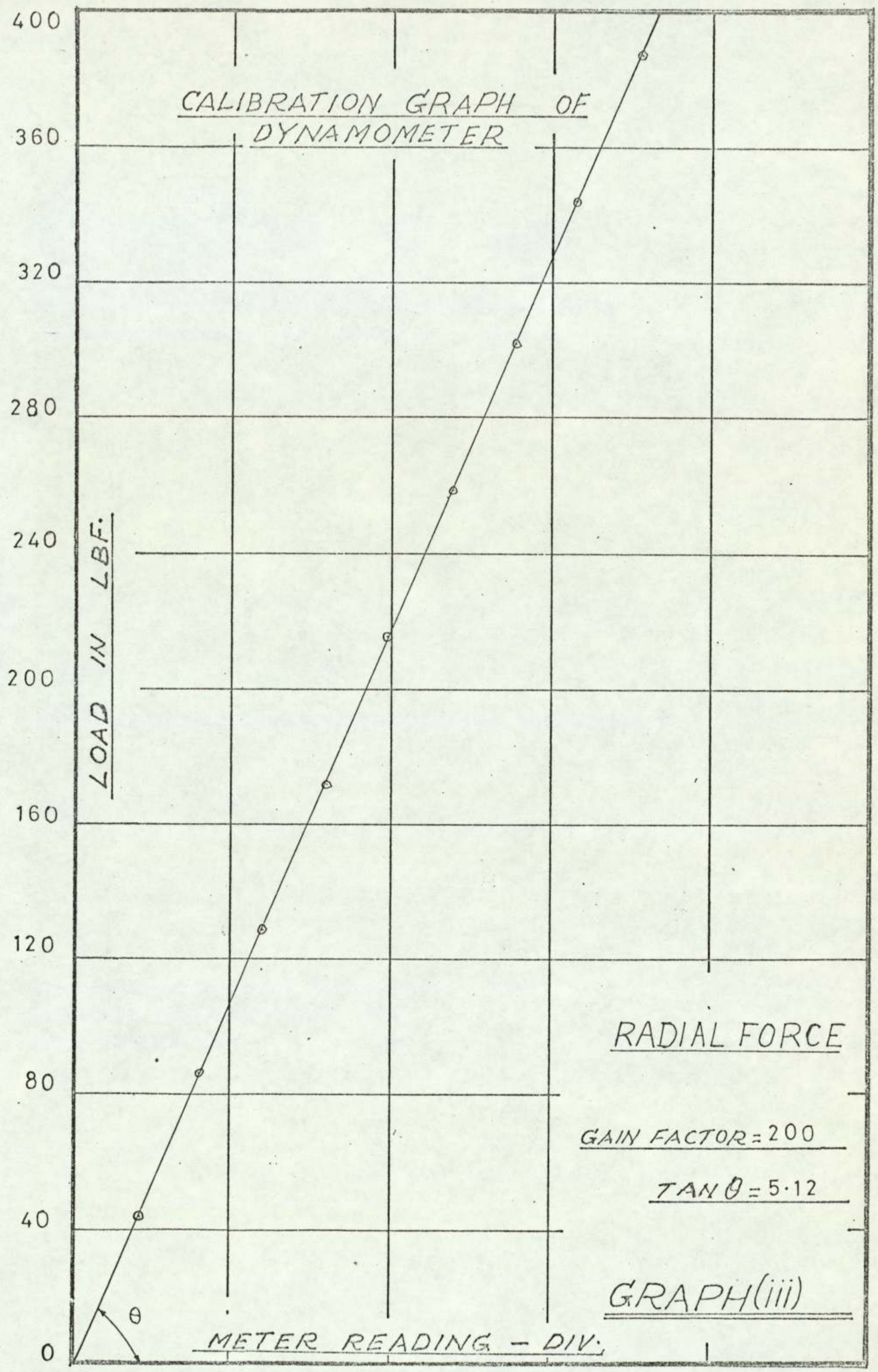

CALIBRATION GRAPH OF DYNAMOMETER
LOAD IN LBF.
METER READING - DIV.
TANGENTIAL FORCE
GAIN FACTOR = 200
TAN θ = 5·14
θ
0
40
80
120
160
200
240
280
320
360
400
0
20
40
60
80
100

GRAPH (i).B.


CALIBRATION GRAPH OF DYNAMOMETER
LOAD IN LBF.
METER READING - DIV.
FEED FORCE
GAIN FACTOR = 500
TAN θ = 2·025
θ
0
20
40
60
80
100
120
140
160
180
200
0
20
40
60
80
100

GRAPH (ii).A.


CALIBRATION GRAPH OF DYNAMOMETER
LOAD IN LBF.
400
360
320
280
240
200
160
120
80
40
0
FEED FORCE
GAIN FACTOR = 200
TAN θ = 5·07
GRAPH (ii).B.
θ
METER READING - DIV.
0
20
40
60
80
100


CALIBRATION GRAPH OF
DYNAMOMETER
LOAD IN LBF.
METER READING - DIV.
RADIAL FORCE
GAIN FACTOR = 200
TAN θ = 5·12
GRAPH(iii)
θ
0
40
80
120
160
200
240
280
320
360
400
0
20
40
60
80
100

short vertical ¾ inch diameter high Tensile steel rod. One end of this rod was shaped to a 60 degree truncated cone. (Plate 3). A 0.125 mm diameter hardened steel ball was placed on the conical cavity provided on the calibration tool. Plate 3 showed a 'close-up' view of the clamping arrangement and the method of application of known loads to the dynamometer during calibration of 'Tangential' forces. The tool-post mass was laid on its one side and clamped for 'Feed' force calibration. It was stood vertically on two thick parallels of sufficient height (because the terminals for the bridge output were protruding from the plate at the rear of the tool-post mass) for the calibration of 'Radial' forces.

Calibration of the dynamometer for the three forces with the highest gain setting of X 500 and the next lower X 200 was carried out and the results were shown in Graphs (i) - (iii). For all the three forces and with both gain settings, the graphs were found to be linear. The points showed no 'hysterisis' effects while 'loading'and 'unloading' and the calibration was checked once during the investigations after a lapse of about 90 days and did not show any significant variations.

During the calibration tests, it was also established that the cross sensitivity between the three forces was less than one per cent.

3.4 DETERMINATION OF NATURAL FREQUENCY OF THE TOOL-POST MASS OF THE P.E.R.A. DYNAMOMETER.

It was primarily important to ensure that the natural frequency of the tool-post mass was well above the exciting frequency likely to be encountered during cutting. Therefore, the determination of this aspect was carried out. The Block Diagram (Fig.2) showed the instrumentation and set up for this test.

The scheme of testing involved application of vibrations of known frequencies, at constant input (the input was controlled by means of a feed back circuit), on to the tool-post mass which was mounted on a vibrating pad actuated by a Power Amplified Oscillator and monitoring the response of these vibrations from an Accelerometer bonded on top of the mass. The output from the accelerometer could be amplified and impedence matched by means of a Charge Amplifier and fed to an R.M.S. Voltmeter. The frequency of the applied vibrations could be varied continuously from 1 c.p.s. upwards by changing the oscillator frequency. While scanning the applied range

BLOCK DIAGRAM FOR DETERMINING THE NATURAL FREQUENCY OF THE TOOL-POST MASS OF DYNAMOMETER.

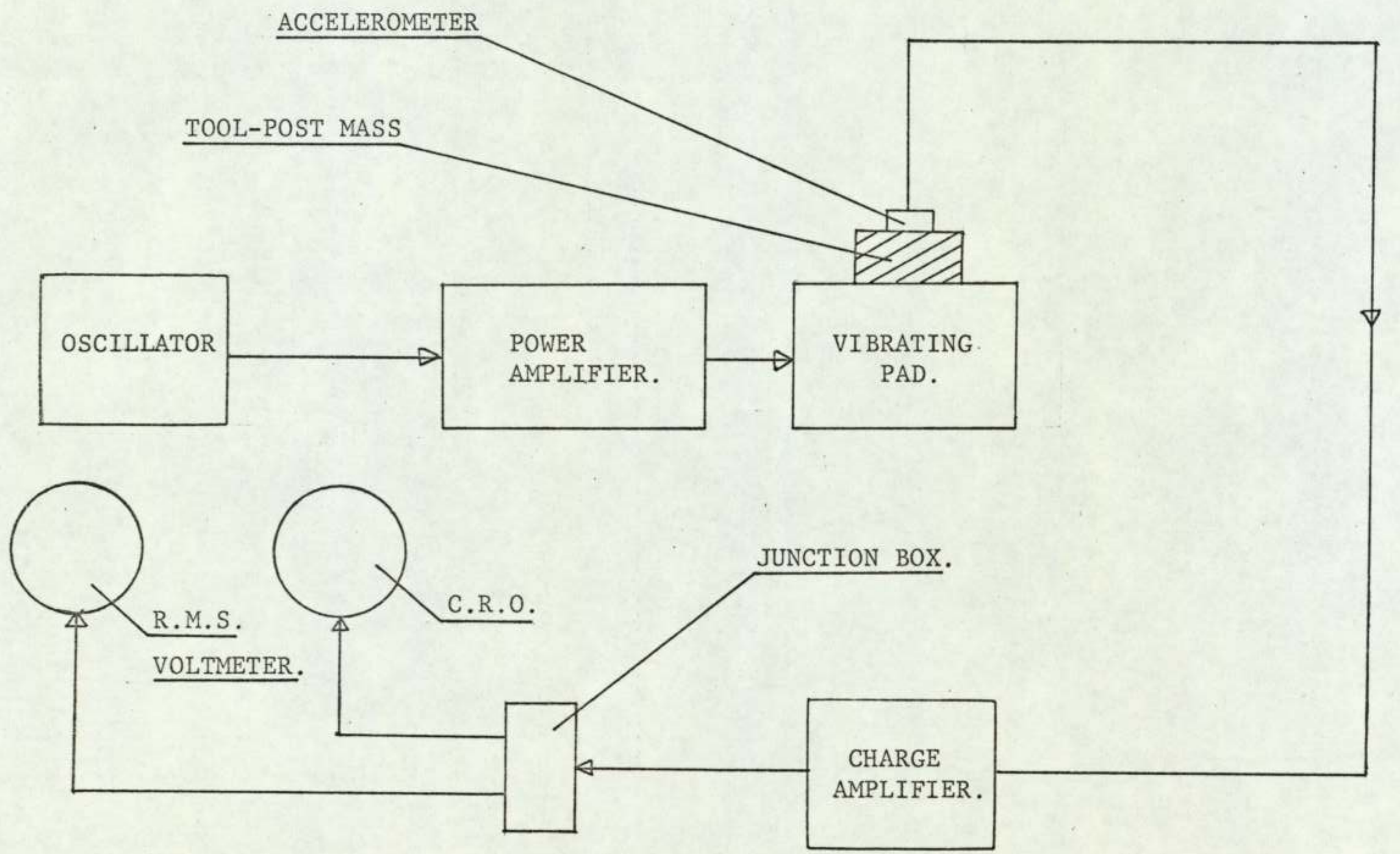


FIG.2 .

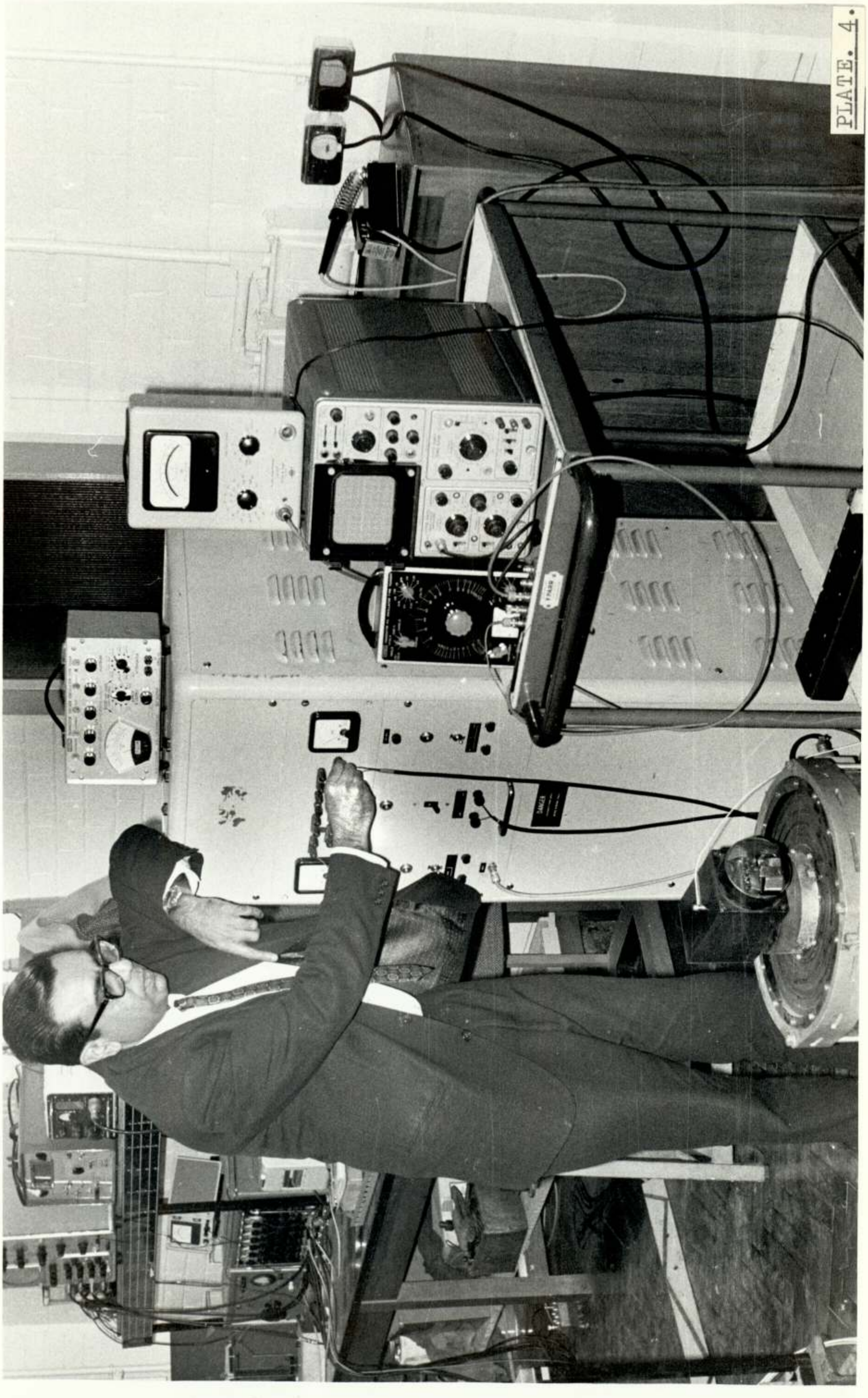


PLATE. 4.

of frequency in this manner, the R.M.S. Voltmeter output could be monitored until the peak reading occurred. This effect could also be counter-checked by means of a Cathode Ray Oscilloscope display.

According to this scheme, it was essential to initially determine the frequency of resonance condition of the vibrating pad itself before testing the dynamometer mass. Therefore, this was first investigated by placing the accelerometer straight on to the pad and scanning the frequency spectrum continuously monitoring the Voltmeter and Oscilloscope outputs. In this manner the resonance frequency of the plain vibrating pad was determined to be in the region of 3200 c.p.s.

In addition to this information, it was also known that the 'flat frequency responses' of the individual instruments used were as follows:

Oscillator - 0 to 150 K Hz.
Power Amplifier - 50 Hz. to 5 K Hz.
Accelerometer - 0 to 20 K Hz.
Charge Amplifier - 2 Hz. to 20 K Hz.
Oscilloscope - 1 Hz. upwards.

It was thus ensured that all spurious effects due to resonance conditions developed in the test equipment and its instrumentation system were eliminated.

Having established these facts, varying known frequencies were then applied to the tool-post mass through the vibrating pad. The experimental set up employed could be viewed in Plate 4. The fundamental resonance peak of the dynamometer mass was determined to be in the range of 335 to 355 c.p.s. It was observed that resonance peak value of the vibrations varied slightly (within the range of 335 - 355 c.p.s.) depending upon the location of the accelerometer along the top face of the mass. After the test series, a mean value of about 345 c.p.s. was accepted as the natural frequency of the tool post mass.

During cutting operations, the largest possible rotational speeds that could be attained on the lathe was only of the order of 30 revolutions per second. It was therefore concluded that dynamic effects due to exciting vibrations during cutting could be ignored.

3.5 The Lathe on which the investigations were carried out was manufactured by George Swift and Sons, Ltd., Halifax. It was a model 12 V 6 centre lathe with a maximum length between centres of five feet nine inches, and swing over bed of 22½ inches (8*). Even though the feed rates available in the required range was limited to five, the main advantage was the continuously variable speed drive. The variable speed drive motor was made by Metropolitan Vickers of Manchester and had a range of speeds from 20 -1100 r.p.m. with standard pulleys. With the fitting of special pulleys the rotational speeds of the spindle could be increased up to 2000 r.p.m. The horsepower developed by the main motor at 1440 r.p.m. was 30 and at 483 r.p.m. 10. The standard tool-post of this lathe along with its base was removed and an auxiliary base with a flat top was fitted in its place. Along the flat top of this base was drilled and tapped four holes of ½" diameter B.S.W. at appropriate positions to take in 4 dynamometer clamping bolts. Two strips of ½" x 1" sections were also bolted across it at the correct distance apart for proper location of the dynamometer tool-post mass. After adjusting the height of the tool-post mass with 'ground' strips such that the tool tip was at the centre line of the work piece, and properly aligning it such that the tool motion, while transversely fed, was truly perpendicular to the axis of the workpiece, the tool-post mass was clamped down to the base by means of two thick mild steel plates. The whole clamping arrangements in relation to the workpiece, a portion of the lathe and the console cabinet could be viewed in Plate 5.

Workpieces, in all cases, were hot rolled bars of EN - 8 steel within a hardness range 220 - 235 B.H.N., of 2½" - 3½" diameter and mounted between a three jaw chuck and a 'live' centre. As the workpiece got warmed up after some cutting, the adjustable live centre clamp was frequently released and reclamped to relieve excessive clamping pressures. The Chemical Analysis of typical EN - 8 steel used in the investigations was as follows:-

Carbon - 0.44 %
Sulphur - 0.053%
Silicon - 0.25 %
Phosphorous - 0.031 %
Manganese - 0.81 %

--oo000oo--

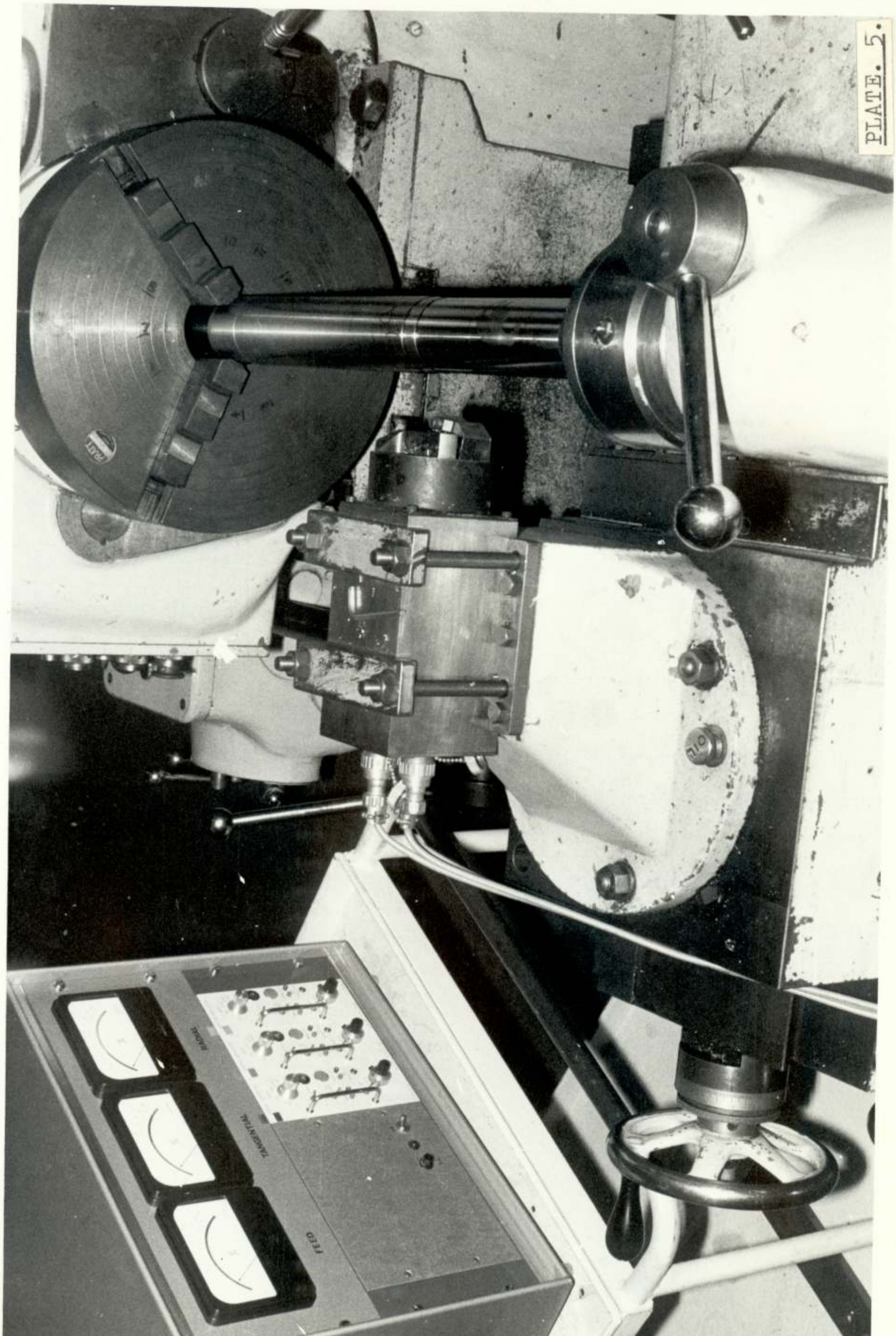


PLATE. 5.

CHAPTER 4.

4. PRELIMINARY INVESTIGATIONS AND PROVING OF FUNCTIONAL RELATIONSHIP.

4.1 ORIGINS OF THE WORK.

During machining of plain carbon (medium carbon) steels, it was observed that the logarithms of the resultant of the two power components ("Tangential" and "Feed") of cutting forces were plotting linearly with the logarithms of each of the following cutting variables

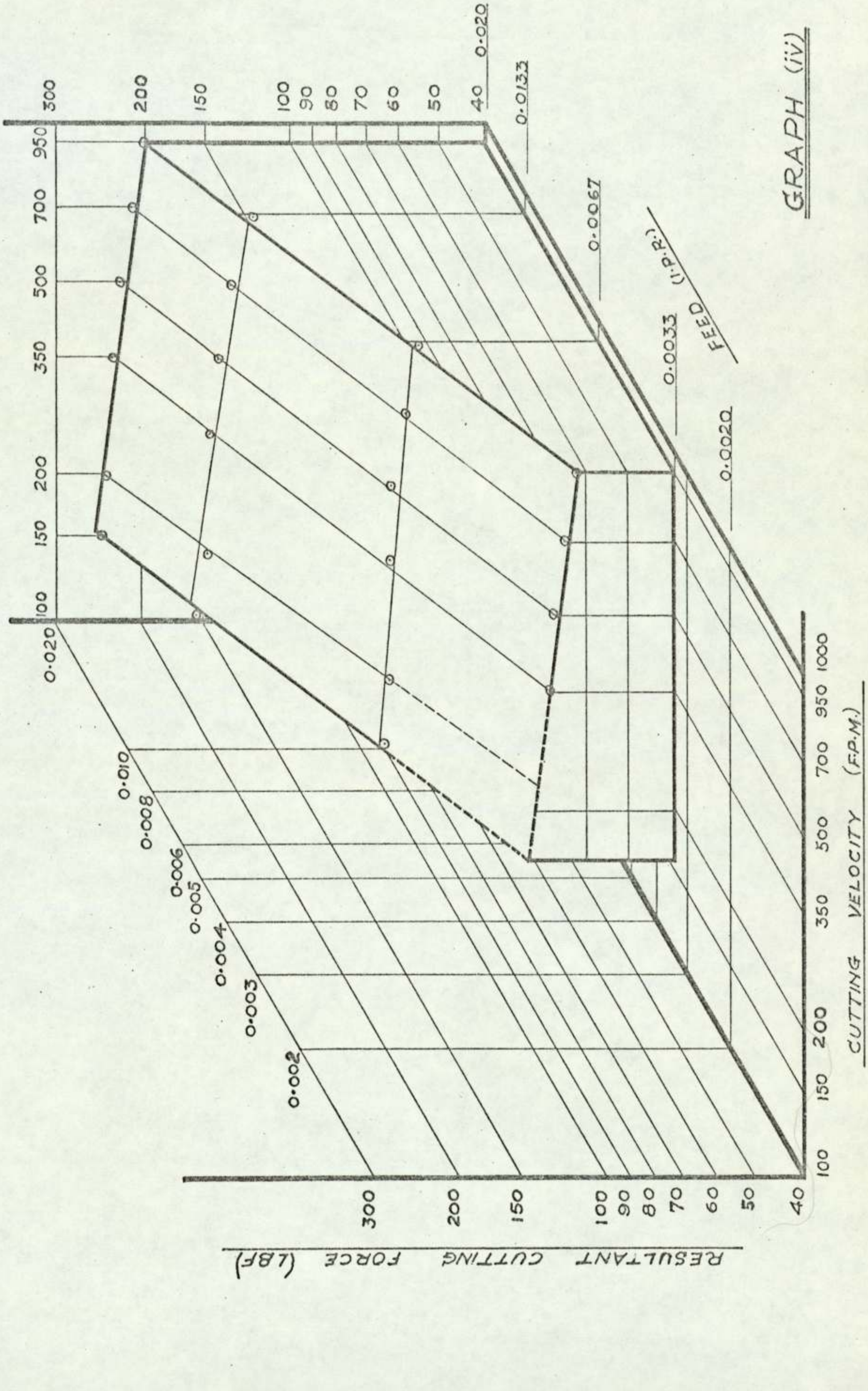
(i) Cutting speed.
(ii) Feed rate.
(iii) Depth of cut.

It was also observed that the radial force variation showed a tendency to increase with higher 'feeds' and 'depths of cuts' and decrease with increases of 'cutting speed'. But the pattern of variation was not systematic enough to be included for the calculation of resultant force (probably because the radial forces were very much sensitive to minor variations in the tool tip conditions like the sharpness of the cutting edge). Also, the radial force would not affect the power consumption as there is no velocity associated with it. It was therefore decided to omit the radial force from the calculation of resultant force. However, the values of this force was recorded in all cases for the sake of completeness.

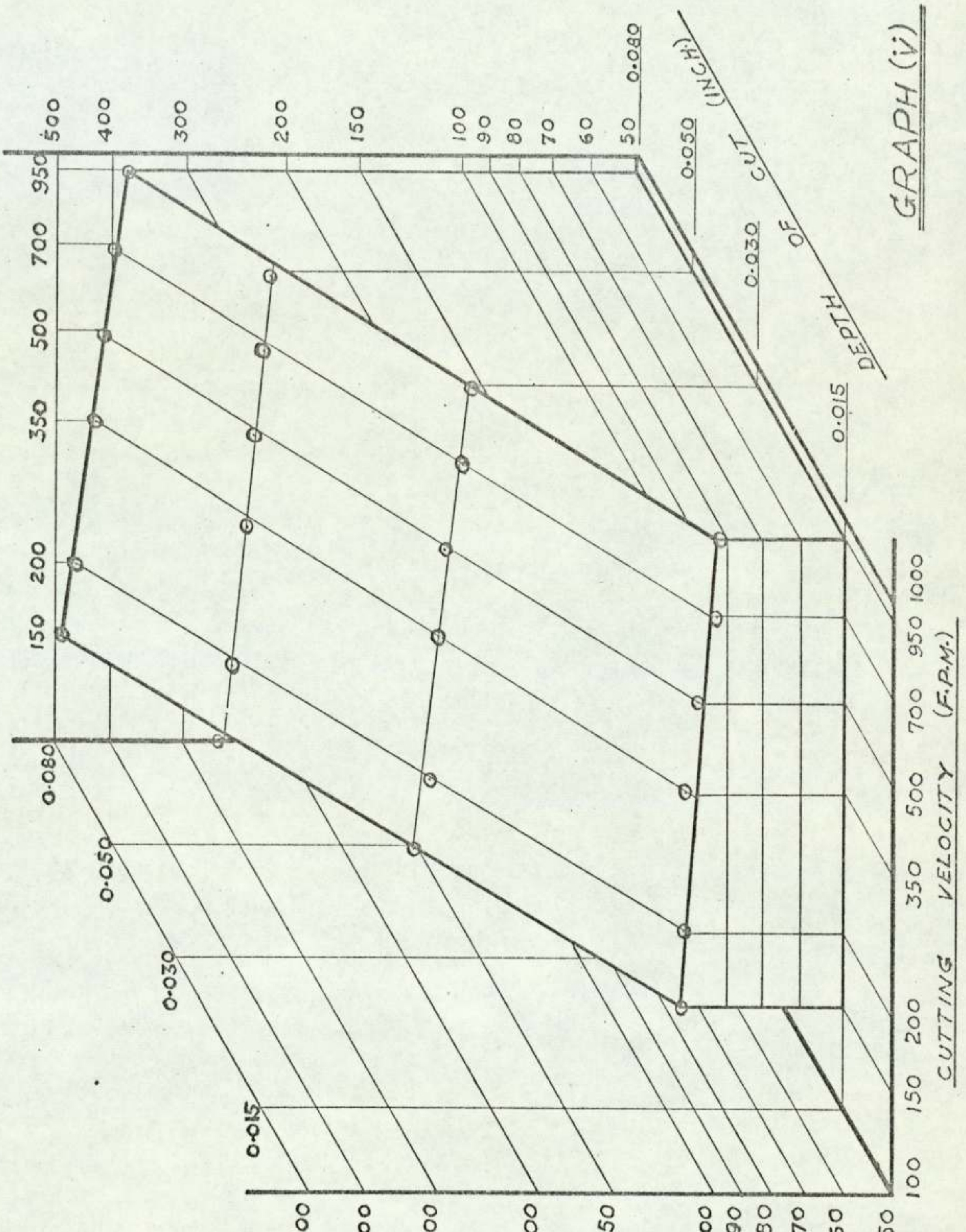
Having a reasonable idea about the relationship of the response (resultant cutting force) to the individual variables, two sets of experiments were planned. The first was to vary 'Speed' and 'Feed' and record the resultant cutting force. The second was to vary the 'Speed' and 'Depth of cut' and to record the response. The results of these two experiments were plotted in two "three dimensional" logarithmic graphs (Graphs (iv) and (v)).

From Graph (iv) it could be observed that the responses lie reasonably well in a plane in a three dimensional logarithmic co-ordinate system with the exception of very low values of 'Feed' and 'Speed'. From Graph (v) it could be seen that all the responses lie reasonably well on a plane (in the entire range investigated) in the case of 'speed' and 'depth of cut' variables.

The discrepancy in the low range values of 'Feed' and 'Speed' (Graph (iv)) could be attributed to the formation of built-up edge


RESULTANT CUTTING FORCE (LBF)
40
50
60
70
80
90
100
150
200
300
CUTTING VELOCITY (F.P.M.)
100
150
200
350
500
700
950
1000
FEED (I.P.R.)
0.0020
0.0033
0.0067
0.0133
0.020
0.002
0.003
0.004
0.005
0.006
0.008
0.010

GRAPH (iv)



RESULTANT CUTTING FORCE (LBF)
500
400
300
200
150
100
90
80
70
60
50
CUTTING VELOCITY (F.P.M.)
100
150
200
350
500
700
950
1000
DEPTH OF CUT (INCH)
0·015
0·030
0·050
0·080
GRAPH (V)

at the cutting edge of the tool. On a new tip, the forces were observed to be substantially higher, but this was only a short lived transient condition and as the built-up edge stabilized (which took only a few seconds), the cutting forces dropped to a much lower value. However, there was no such noticeable effect with low values of 'Speed' and 'Depth of cut'. Therefore, it could be concluded that built-up edge formation was more favoured by the combination of low 'Speeds' and low 'Feeds'. The ranges chosen for the three variables of 'Speed', 'Feed' and 'Depth of cut' included the commonly used ones.

In order to extend the range in certain cases, and also to confirm the observed relationship between the three variables and the response a "Latin Square" type of experiment (9*) was carried out next. It was also decided, at this stage, to choose EN - 8 as the work material.

The choice was purely arbitrary except that it is a typical medium carbon steel which is widely used in industry for general production purposes.

4.2 "LATIN SQUARE" SEARCH FOR FUNCTIONAL RELATIONSHIP. (9*)

From the preliminary set of graphs, it was seen that the relationship between the Resultant of the two cutting force components and the Speed, Feed and Depth of cut parameters were very nearly linear in the double logarithmic scale within the ranges investigated. i.e. The relationship could be expressed as:

$$F = k \times f^a \times v^b \times d^c \text{ ----------------------------------(i)}$$

where

F is the Resultant cutting force in lbf.

v is the velocity of cutting in fpm.

f is the feed in inches per rev.

d is the depth of cut in inches.

a, b and c are the logarithmic slopes.

k is the antilog of the intercept of the Response Surface with the force axis in a 4 dimensional logarithmic co-ordinate system.

Through a balanced experiment such as the "Latin Square" and with the assumption of the functional relationship as in equation (i), it should be possible to evaluate the constant 'k'. Consistency or otherwise of the 'k' values obtained from different combinations of cutting conditions employed in such an experiment would also indicate within reasonable limits whether the functional relationship assumed was valid or not.

Consider a Latin Square experiment involving variations of speed, feed and depth of cut at 3 levels and the resultant cutting force as the response, below:

Feed \ Speed	V_1	V_2	V_3
f_1	F_1 (d_1)	F_4 (d_2)	$F_7(d_3)$
f_2	F_2 (d_2)	F_5 (d_3)	$F_8(d_1)$
f_3	F_3 (d_3)	F_6 (d_1)	$F_9(d_2)$

* 'd's indicate the 3 levels of depth of cut.

The general form of equation (i) can be written as:-

$$F = \emptyset_1 (f) \cdot \emptyset_2 (v) \cdot \emptyset_3 (d) \text{ ------------------------------(ii)}$$

where

$\emptyset_1, \emptyset_2, \emptyset_3$ indicates functions.

Taking logs of both sides of equation (ii) :

$$\log F = \log \emptyset_1 (f) + \log \emptyset_2 (v) + \log \emptyset_3 (d)$$

Applying this relationship to the top row of the Latin Square matrix:

$$\log F_1 = \log \emptyset_1(f_1) + \log \emptyset_2 (v_1) + \log \emptyset_3(d_1)$$
$$\log F_4 = \log \emptyset_1(f_1) + \log \emptyset_2 (v_2) + \log \emptyset_3(d_2)$$
$$\log F_7 = \log \emptyset_1(f_1) + \log \emptyset_2 (v_3) + \log \emptyset_3(d_3)$$

Adding :-

$$\sum \log F_{(f_1)} = 3 \log \emptyset_1 (f_1) + \log \left[\emptyset_2 (v1) + \emptyset_2(v_2) + \emptyset_2(v_3)\right] + \log \left[\emptyset_3 (d_1) + \emptyset_3 (d_2) + \emptyset_3 (d_3)\right]$$

i.e. $$\log \emptyset_1 (f_1) = \frac{\sum \log F_{(f_1)}}{3} - Q \text{ ---------------------------(iii)}$$

where:

$$Q = \frac{\log \left[\emptyset_2 (v_1) + \emptyset_2 (v_2) + \emptyset_2 (v_3)\right] + \log\left[\emptyset_3(d_1) + \emptyset_3(d_2) + \emptyset_3 (d_3)\right]}{3}$$

Similar treatment for the other two rows would yield:

$$\log \emptyset_1 (f_2) = \frac{\sum \log F_{(f_2)}}{3} - Q \text{ -----------------------------(iv)}$$

and $$\log \emptyset_1 (f_3) = \frac{\sum \log F_{(f_3)}}{3} - Q \text{-----------------------------(v)}$$

From the above, it is evident that when the logarithms of the response are numberically averaged over a single 'feed' (f), 'speed' (v) or 'depth of cut' (d) level, the effects of those factors that are changing ('speed' and 'depth of cut' in the case examined) would remain the same from one feed level to the other. Thus, all changes in the log-average of the responses are wholly due to the effect of 'feed' alone.

The above proof could be extended to show that when a similar averaging occurs over the three 'speed' levels and also over the three 'depth of cut' levels, the responses are wholly due to the respective effects alone. It is evident that the proof could be extended to the case of more than three levels as long as the experiment is balanced.

From equations (iii), (iv), and (v), it follows that:

$$F_{(f)} = k_1 \emptyset_1 (f)$$

i.e. $$\emptyset_1 (f) = \frac{F_{(f)}}{k_1} \text{ ---------------------------------(vi)}$$

where:

$F_{(f)}$ is the antilog of $\frac{\sum \log F_{(f)}}{n}$

(n being the number of levels considered).

k_1 is the antilog of Q in equations (iii), (iv), and (v).

Similar considerations of 'speed' and 'depth of cut' effects yield the equations:

$$F(v) = k_2 \emptyset_2 (v)$$

i.e. $$\emptyset_2 (v) = \frac{F (v)}{k_2} \text{ -------------------------------------(vii)}$$

and $$F_{(d)} = k_3 \emptyset_3 (d)$$

i.e. $$\emptyset_3 (d) = \frac{F(d)}{k_3} \text{ ---------------------------------(viii)}$$

From equation (ii) :

$$F = \emptyset_1 (f) \times \emptyset_2 (v) \times \emptyset_3 (d)$$

Therefore, combining equation (ii) with equations (vi), (vii) and (viii):

$$F = \frac{F(f) \times F (v) \times F (d)}{k_1 \times k_2 \times k_3} \text{ --------------------------------(ix)}$$

$= K \times F(f) \times F(v) \times F(d)$

where

$$K = (k_1 \times k_2 \times k_3)^{-1}$$

If the response is measured under known conditions F(f), F(v) and F(d) could be calculated by the log averaging process and 'K' can be evaluated as below:-

LATIN SQUARE ANALYSIS ON LOG-TRANSFORMED DATA.

Serial No. of Test	$F_{F(lbf)}$	$F_{T(lbf)}$	$F_{R(lbf)}$	$F_{\sqrt{T^2 + F^2}(lbf)}$
1	108.4.	138.8	122.9	176.1
2	95.2	159.5	135.7	185.8
3	58.7	138.6	133.1	150.5
4	28.4	107	122.9	110.7
5	228.2	552	373.8	597.3
6	71.9	97.4	87.0	121.1
7	48.6	87.7	92.2	100.3
8	25.3	79.2	87.0	83.1
9	266.2	406.1	291.8	485.6
10	177.5	401	307.2	438.5
11	38.5	56.7	58.9	68.5
12	22.8	50.3	56.3	55.2
13	158	251.9	189.5	297.4
14	168.1	303.3	307.2	346.8
15	89.1	226.2	220.2	243.1
16	10.1	23.5	15.4	25.6.

LATIN SQUARE OF 'RESPONSE' AND 'CUTTING CONDITIONS'

Feed \ Speed	150	300	600	900
0.0033	(16) 25.6-d_1*	(11)68.5-d_2	(6)121.1-d_3	(1)176.1-d_4
0.0067	(13)297.4-d_4	(12)55.2-d_1	(7)100.3-d_2	(2)185.8-d_3
0.0133	(14)346.8-d_3	(9)485.6-d_4	(8)83.1-d_1	(3)150.5-d_2
0.0200	(15)243.1-d_2	(10)438.5-d_3	(5)597.3-d_4	(4)110.7-d_1

* d_1 = 0.015 in. depth of cut.
d_2 = 0.030 in. depth of cut.
d_3 = 0.060 in. depth of cut.
d_4 = 0.090 in. depth of cut.

Taking logs of Responses:-

					Total	Average.	Antilog.
	1.408	1.836	2.083	2.246	7.573	1.893	78.2.
	2.473	1.742	2.001	2.271	8.487	2.122	132.4
	2.540	2.686	1.920	2.178	9.324	2.331	214.3
	2.386	2.642	2.776	2.044	9.848	2.462	289.7
Totals	8.807	8.906	8.780	8.739	7.114	1.779	60.1
Average	2.202	2.227	2.195	2.185	8.401	2.100	125.9
Antilog.	159.2	169.0	156.7	153.1	9.536	2.384	242.1
					10.181	2.545	350.8

$$K_1 = \frac{176.1}{153.1 \times 78.2 \times 350.8} = \frac{176.1}{4199924.9} = 4.1929 \times 10^{-5}$$

$$K_2 = \frac{185.8}{153.1 \times 132.4 \times 242.1} = \frac{185.8}{4907473.5} = 3.7861 \times 10^{-5}$$

- -

$$K_{16} = \frac{25.6}{78.2 \times 159.2 \times 60.10} = \frac{25.6}{748211.3} = 3.4215 \times 10^{-5}$$

The various K values are:-

3.4215	4.1169	4.1510	4.1929
4.0221	4.1048	3.8399	3.7861
4.1987	3.8222	4.0529	3.6435
4.1867	3.6995	3.7507	4.1529

Range of K values = $4.1987 - 3.4215 = 0.7772 \times 10^{-5}$

$$\bar{K} = \frac{63.1423}{16} = 3.9464 \times 10^{-5}$$

Max. Variation from $\bar{K} = 10^{-5}(3.9464 - 3.4215) = 0.5249 \times 10^{-5}$

$$= \frac{0.5249 \times 100}{3.9464} = \underline{\underline{13.3\%}} \text{ of the mean value.}$$

As judged from the variations in 'K' values, the assumption regarding the functional relationship did not seem to be an unreasonable one.

A part of the deviations could be attributed to the uncertainty in taking the Dynamometer readings as a certain amount of flicker in the needle, especially at low speeds and light cuts, could not be eliminated.

A second reason could be due to the difficulty of keeping the cutting velocity at the exact values necessary as there occurred a slight but varying (depending upon the area of cut) fall in the r.p.m. of the spindle when a cut is actually applied even though compensation has been allowed.

A third reason could be due to the differences in the clamping pressure of the bar (between the chuck and a live-centre) due to expansion of the workpiece by heating. This effect was observed to exist and was watched for during the investigation.

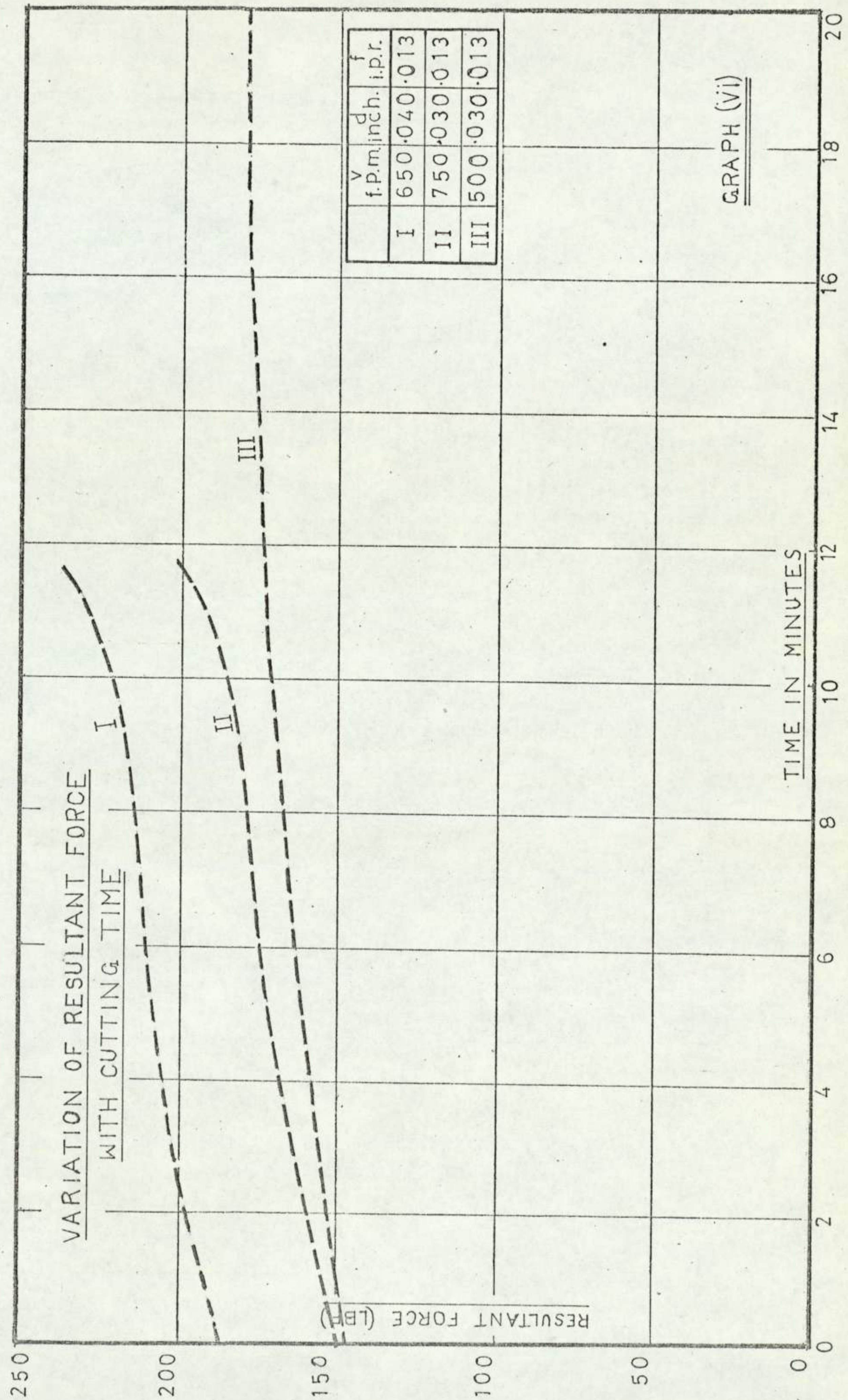
Finally, it could be noted that the lowest value of 'K' (3.4215) was obtained at a cutting speed of 150 f.p.m., a feed of 0.0033 i.p.r. and a depth of cut of 0.015 inch. This combination of conditions, among all the ones used, was obviously the worst one for promoting a strong tendency for the formation of built-up edge and consequent decrease of forces. Ignoring this one value, it could be seen the 'K' values were in much closer agreement and the maximum variation was only 5.5%.

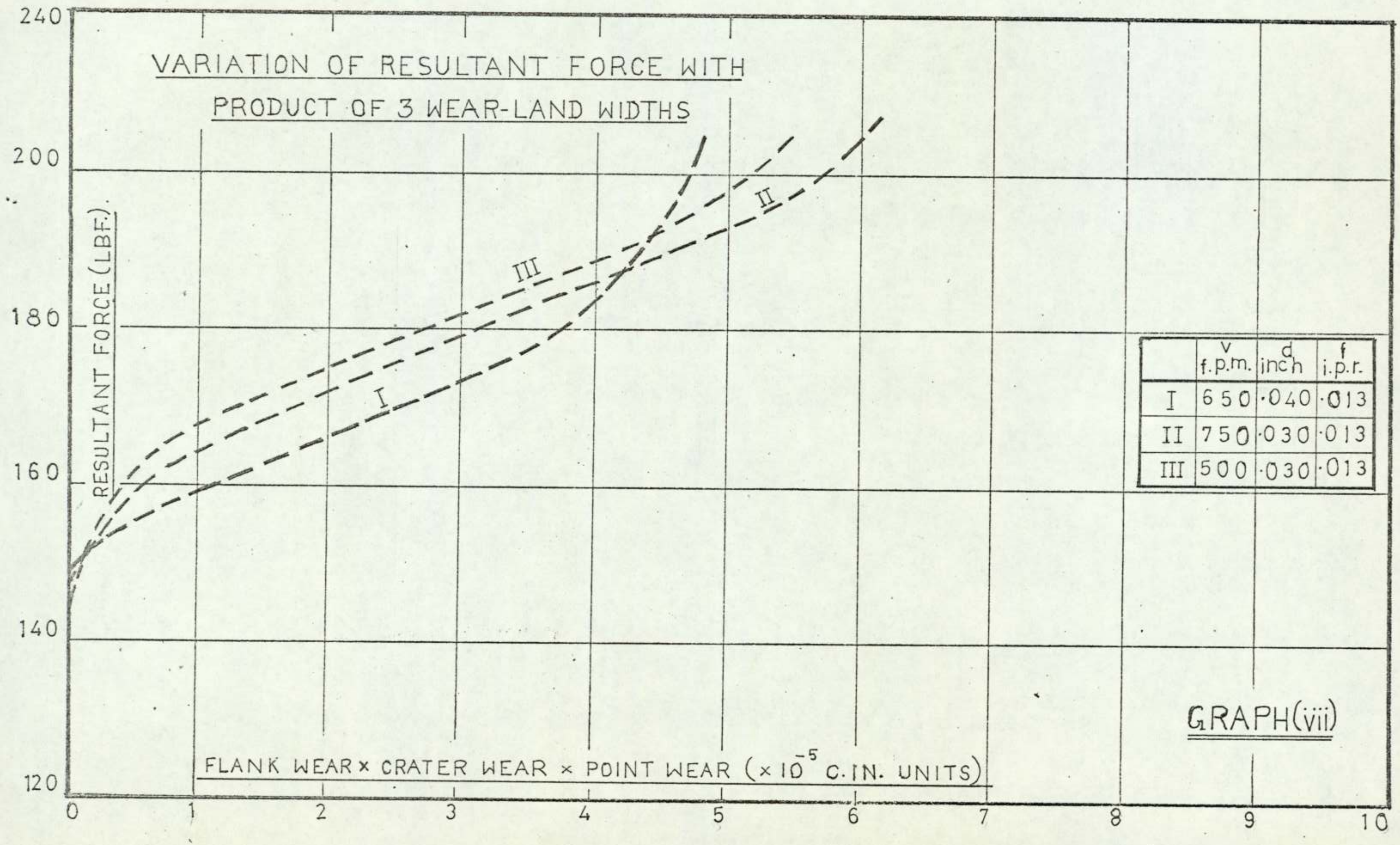
From the foregoing, it was concluded that the linear relationship of the variables and the response (in their logarithmic transformed state) existed. A linear multiple regression model was therefore postulated.

4.3 CHOOSING THE RANGE OF TOOL LIFE FOR THE CUTTING TESTS.

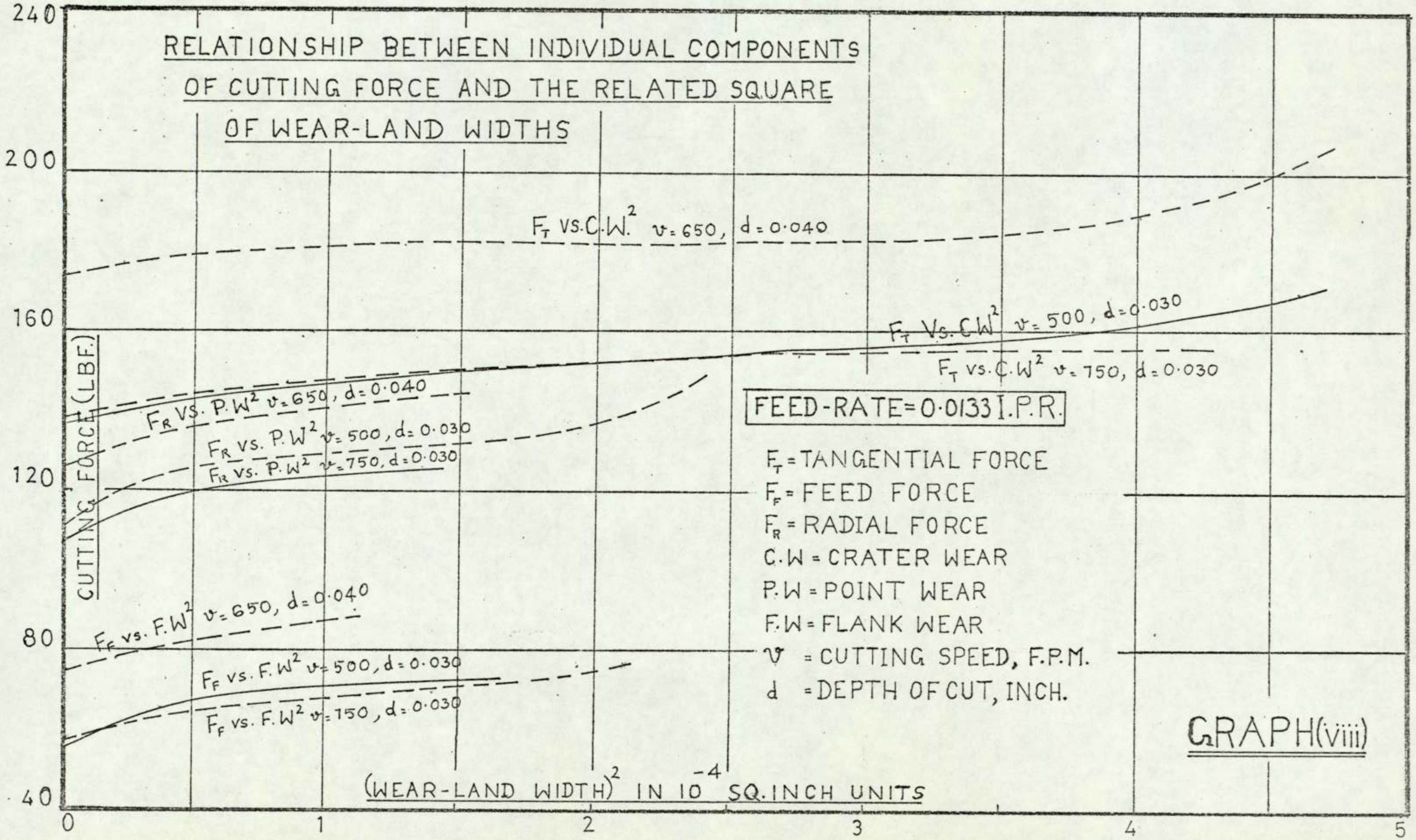
It was evident from the investigations so far that the cutting forces were time dependent on tool wear. In order to minimize this effect, it was necessary to find out the range under which this effect was less. The variation of cutting forces with cutting time (tool wear) under typical cutting conditions were presented in graphical form in Graph (vi).

It was observed that the forces varied with tool wear considerably more with higher values of area of cut and also with higher cutting speeds.


VARIATION OF RESULTANT FORCE
WITH CUTTING TIME
RESULTANT FORCE (LBF)
TIME IN MINUTES
GRAPH (Vi)
I
II
III
v f.p.m. | d inch. | f i.p.r.
I 650 ·040 ·013
II 750 ·030 ·013
III 500 ·030 ·013
0
50
100
150
200
250
2
4
6
8
10
12
14
16
18
20


VARIATION OF RESULTANT FORCE WITH
PRODUCT OF 3 WEAR-LAND WIDTHS
RESULTANT FORCE (LBF.)
240
200
180
160
140
120
0
1
2
3
4
5
6
7
8
9
10
I
II
III
v f.p.m.
d inch
f i.p.r.
I 650 ·040 ·013
II 750 ·030 ·013
III 500 ·030 ·013
FLANK WEAR × CRATER WEAR × POINT WEAR (× 10^-5 C.IN. UNITS)

GRAPH(vii)


RELATIONSHIP BETWEEN INDIVIDUAL COMPONENTS OF CUTTING FORCE AND THE RELATED SQUARE OF WEAR-LAND WIDTHS
F_T VS. C.W.^2 v = 650, d = 0·040
F_T Vs. C.W^2 v = 500, d = 0·030
F_T vs. C.W^2 v = 750, d = 0·030
F_R VS. P.W^2 v = 650, d = 0·040
F_R vs. P.W^2 v = 500, d = 0·030
F_R vs. P.W^2 v = 750, d = 0·030
F_F vs. F.W^2 v = 650, d = 0·040
F_F vs. F.W^2 v = 500, d = 0·030
F_F vs. F.W^2 v = 150, d = 0·030
FEED-RATE = 0·0133 I.P.R.
F_T = TANGENTIAL FORCE
F_F = FEED FORCE
F_R = RADIAL FORCE
C.W = CRATER WEAR
P.W = POINT WEAR
F.W = FLANK WEAR
v = CUTTING SPEED, F.P.M.
d = DEPTH OF CUT, INCH.
CUTTING FORCE (LBF.)
240
200
160
120
80
40
0
1
2
3
4
5
(WEAR-LAND WIDTH)^2 IN 10^-4 SQ.INCH UNITS
GRAPH(viii)

It was decided to choose the portion of tool life period between five to ten minutes for speeds above 600 f.p.m. and five to fifteen minutes for lower cutting speeds, both for the experimental tests and also the proving tests. However, at values of 'feeds' 0.0133 i.p.r. and 'depths of cut' 0.050 in. and greater the tool life portion was again chosen to lie between five to ten minutes. These choices were made in order to reduce the wear dependent variations in forces to a minimum.

Employing the cutting force-tool wear data, it was further attempted to discover the pattern of rupture conditions of the workpiece material at the tool tip.

Graph (vii) showed a plot of the resultant cutting force against the product of three wear land widths (i.e. product of Crater Wear, Point Wear and Flank Wear land widths taken as a parameter representing volumetric wear of tool.) The wear-land widths were measured using an O.M.T. '385' tool-makers' microscope with a discrimination of 0.01 m.m. The approximate parallellism between sectors of the three curves, for three different cutting conditions was an interesting phenomena observed.

In Graph (viii), the individual components of the cutting forces were plotted against the respective squares of the wear land (e.g. Square of the Crater wear land against 'Tangential' force, etc..) Most of these curves displayed an approximately parallel sector. This was indicative of the fact that the rupture of the workpiece occurred at approximately equal values of stresses at Flank, Point and Top Rake faces of the tool during cutting.

In all the three tool wear force tests, the cutting was carried on till the failure of the tool tip occurred. In all the three instances, apart from the width of the wear-land, the actual breakdown of the tool tip occurred at the blending of the radii with the straight portion of the front and side cutting edges. Plates 6 - 8 showed the crater view of the tool tips at failure.

However, the information contained in Graphs (vii) and (viii) are in no way conclusive or adequate for any decisive inference to be drawn, but were presented only by way of highlighting a possible area of search.

Graph (ix) showed the association observed between the 'point' wear and the C.L.A. value of the surface texture obtained on a 'turned' workpiece for one set of cutting conditions. Here again, it was merely

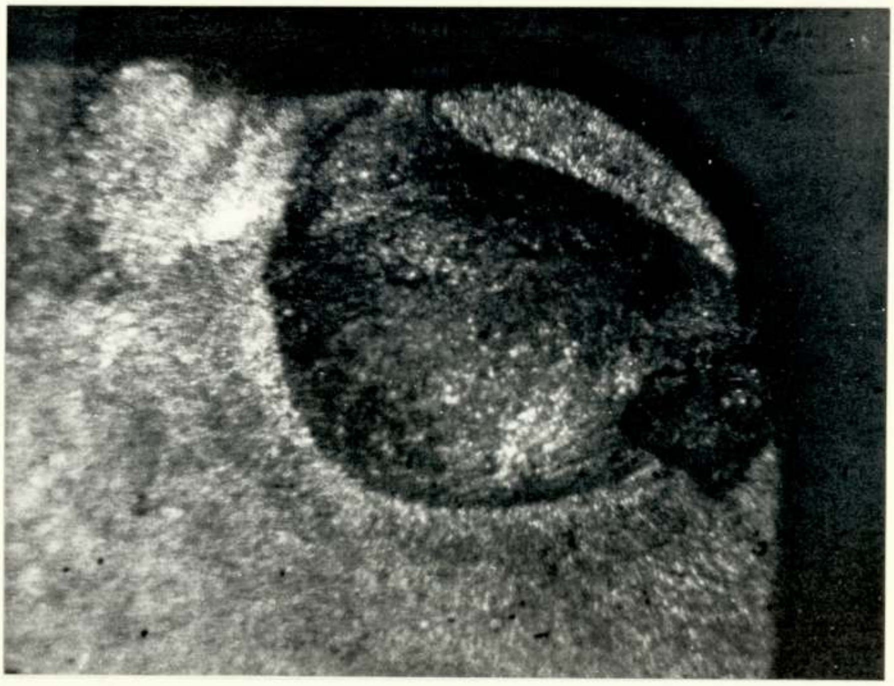
MICROPHOTOGRAPH OF TOOL-TIP AT FAILURE.
(×50)

PLATE. 6.

v = 650 f.p.m.
f = 0.0133 i.p.r.
d = 0.040 inch.

PLATE. 7.

v = 500 f.p.m.
f = 0.0133 i.p.r.
d = 0.030 inch.



MICROPHOTOGRAPH OF TOOL-TIP AT FAILURE.

(×50)

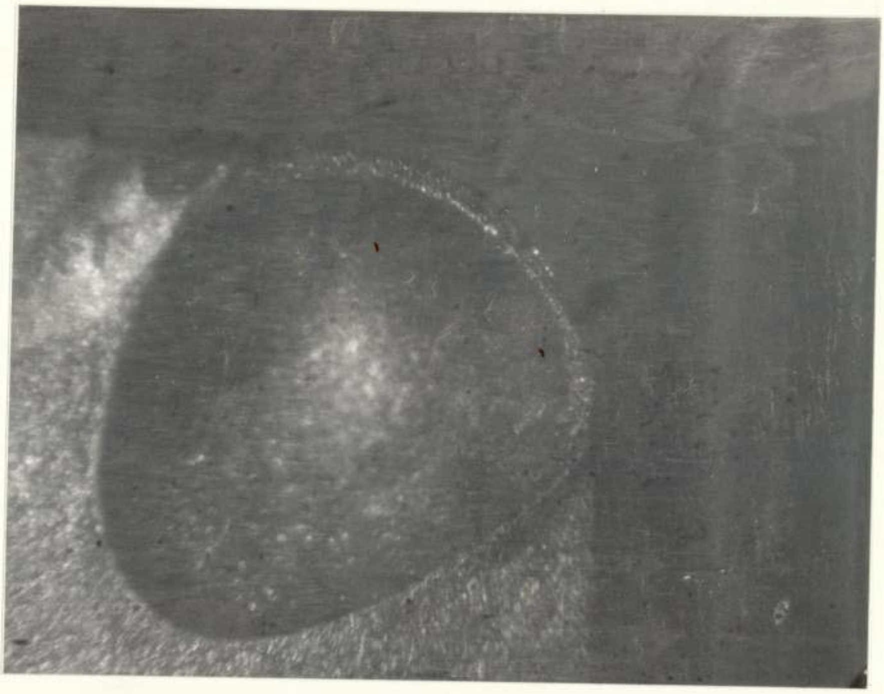
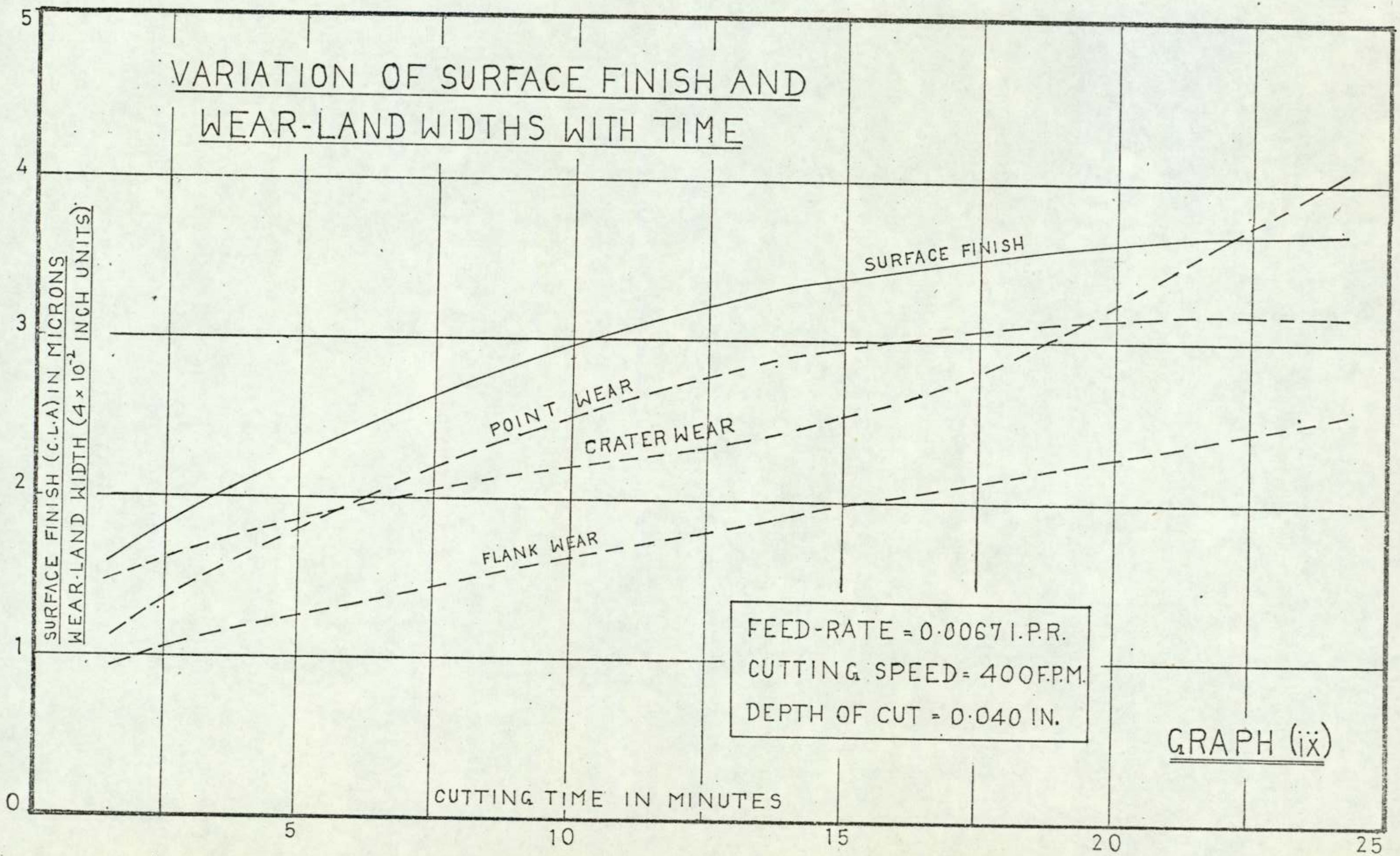


PLATE. 8.

v = 750 f.p.m.
f = 0.0133 i.p.r.
d = 0.030 inch.


VARIATION OF SURFACE FINISH AND
WEAR-LAND WIDTHS WITH TIME
SURFACE FINISH (C.L.A) IN MICRONS
WEAR-LAND WIDTH (4 × 10⁻² INCH UNITS)
0
1
2
3
4
5
SURFACE FINISH
POINT WEAR
CRATER WEAR
FLANK WEAR
FEED-RATE = 0·0067 I.P.R.
CUTTING SPEED = 400 F.P.M.
DEPTH OF CUT = 0·040 IN.
GRAPH (ix)
CUTTING TIME IN MINUTES
5
10
15
20
25

indicative that the surface texture might be correlated to the 'point' wear and not to either of 'crater' or 'flank' wear in a direct manner. The C.L.A. values of the surface texture were measured by a Model 3 Talysurf.

--oo000oo--

CHAPTER 5.

EXPERIMENTAL DESIGN AND RESULTS.

5.1 DETAILS OF EXPERIMENTAL DESIGN.

According to Bartee (10*), the general methodology to be adopted in an experimental search should follow the outline below:

I. Analysis Phase.

1. Formulating the experimental problem.
2. Analysis of the experiment.

II. Synthesis Phase.

3. Design of the experimental model.
4. Design of analytical model.

III. Evaluation Phase.

5. Conducting the experiment.
6. Deriving a solution from the model.

Having gone through Stages 1, 2 and 4, it was then necessary to revert to Stage 3, namely the "Design of the experimental model".

The experimental design should aim at optimizing the yield from it. In a poorly planned experiment the addition of a vast amount of data rarely lead to improve the effectiveness of the investigation. The classical approach of studying the effect of one variable at a time has two main disadvantages -

(a) Excessively 'time and resources - consuming'.
(b) Assumption of absence of 'interaction' between independent variables.

With a view of overcoming these two limitations, the technique of "Response Surface Methodology" was adopted for devising the experimental plan. This could simply be defined as fitting a 'least-square' surface along which the 'responses' lay in a multi-dimensional space constituted by the number of concomitant variables, by means of an optimum experimental design. In this case, each combination of levels of the concomitant variables (factors) corresponded to a point in the factor space and the pattern of such points employed to elucidate the response surface is called the experimental design (11*). Experimental designs are, in other words, chosen patterns of experimental points in a multi-factor space. If the experimental design is inadequate, it would result not only in loss of accuracy while estimating the constants, but might not even allow certain constants to be estimated at all.

By detailed considerations G.E.P. Box (11*) has indicated that a 'central' composite design (Fig. 3.) has been used in many practical situations and found to be reasonably efficient. S.M. Wu and R.N. Meyer also have utilized the same experimental design in their investigations on "Cutting-tool Temperature Prediction Equation" (12*).

According to Box (13*), composite designs to determine effects up to second order could be built up from a complete two-level factorial one. Thus, in the case of 3 factors, the first order and second order inter-action effects could be estimated by a basic two level design consisting of 2^3 experimental points. One effective way of estimating, additionally, the quadratic effects was by adding seven further experimental points one at the centre and remaining six in pairs along the co-ordinate axes at chosen intervals. The fundamental experimental design, therefore, consisted of the first eight points corresponding to the 2^3 factorial, the next six corresponding to the axial points and finally, the point at the centre, altogether thus comprising of 15 distinct combinations of cutting conditions.

The range of each of the cutting conditions was governed by practical considerations. In the first instance the range had to be commensurate with the general requirements in practice. The second factor was the availability of these ranges in the centre-lathe used during the experiment. It was observed that the variations of the cutting conditions possible on the lathe embraced substantially the ranges used in practice. The different levels of each independant variable (namely, each of the cutting conditions of 'speed', 'feed-rate' and 'depth of cut) was spaced at approximately -4/3, - 1, 0, 1, 4/3 ratios in a coded scale. (e.g. For cutting speed, 400 f.p.m. level was taken as zero and the five levels were 150, 200, 400, 800 and 1050 f.p.m. respectively. The arrangement of levels in this manner enabled orthogonality between the variables in the 'cutting conditions' matrix (Appendix).

As stated earlier fifteen experimental points were sufficient to predict both (a) First order and (b) second order 'prediction' equations. But in order to test the 'goodness of fit' of the least-

EXPERIMENTAL DESIGN AND TRIAL NUMBERS

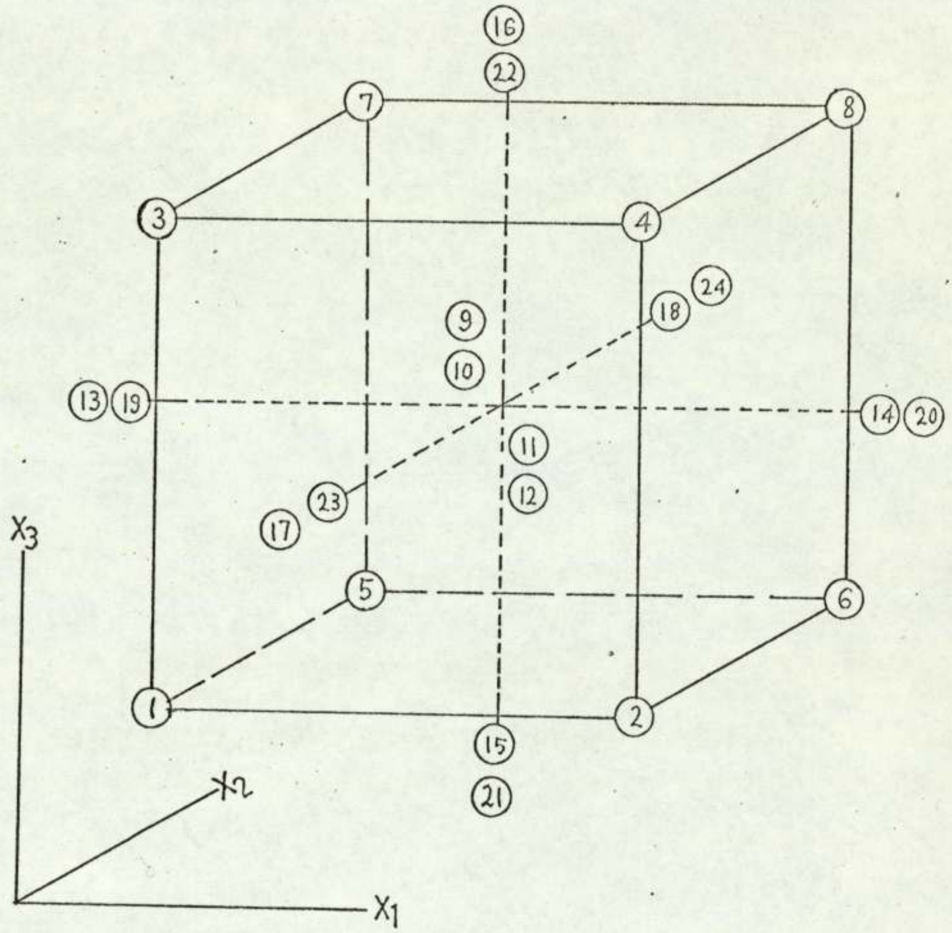


FIG. 3.

square response plane (Appendix), it was necessary to obtain an unbiased estimate of the experimental error. To obtain this, 9 repetitions,3 at the centre point and 6 at the augment points were also included into the basic experimental design. The whole experimental design, thus was composed of 24 cutting tests, and the designated trial numbers of the design are as in Fig. 3. In order to increase the accuracy of estimation of constants, the 24 tests were replicated once. Hence the initial tests consisted of an 'Original' set of 24 cuts and a 'Replicate' set of the same number. The tests were carried out in a random order as indicated by test sequence numbers in Table. 2.

The randomization of the test sequence numbers was done by reference to (14*) which yielded random sequencing of 1 - - - - 24 numbers.

5.2 EXPERIMENTAL RESULTS.

The individual cutting conditions employed together with the test results were presented in Table. 2. overleaf.

TABLE 2.

TEST SEQUENCE NO.	EXPERIMENTAL DESIGN POINT NO.	CUTTING CONDITIONS. SPEED F.P.M.	FEED-RATE I.P.R.	DEPTH OF CUT INCH.	FORCES IN LBF 'ORIGINAL' RESULTS. F_{FEED}	F_{TANG}	$\sqrt{F^2 + T^2}$	'REPLICATE' RESULTS F_{FEED}	$F_{TANG.}$	$\sqrt{F^2 + T^2}$
14	1	200	0.0033	0.020	20.8	37.5	42.8	18.2	38.5	42.5
21	2	800	0.0033	0.020	24.3	38.6	45.6	24.3	38.5	45.5
11	3	200	0.0033	0.080	101.3	139.1	172.1	101.3	137.0	170.4
15	4	800	0.0033	0.080	95.2	120.8	153.8	97.2	120.8	155.0
18	5	200	0.0133	0.020	42.5	109.1	117.1	37.5	104.9	111.4
5	6	800	0.0133	0.020	36.5	102.8	109.1	38.5	100.3	107.4
3	7	200	0.0133	0.080	218.0	385.4	442.7	221.8	380.3	440.2
17	8	800	0.0133	0.080	160.0	334.1	370.4	164.0	349.5	386.1
4	9	400	0.0067	0.040	68.9	113.1	132.4	70.4	115.6	135.3
22	10	400	0.0067	0.040	67.8	118.2	136.3	65.9	113.4	131.5
6	11	400	0.0067	0.040	67.8	115.6	134.0	65.9	115.6	133.1
24	12	400	0.0067	0.040	65.9	115.6	133.1	67.8	115.6	134.0
8	13	150	0.0067	0.040	70.9	119.8	139.2	72.9	118.8	139.3
19	14	1050	0.0067	0.040	68.9	107.9	128.0	65.8	109.1	127.4
2	15	400	0.0067	0.015	23.3	48.9	54.2	24.3	51.4	56.9
23	16	400	0.0067	0.105	167.1	259.6	308.8	169.5	272.4	320.8

TABLE 2 (cont..)

TEST SEQUENCE NO.	EXPERIMENTAL DESIGN POINT NO.	CUTTING CONDITIONS. SPEED F.P.M.	FEED-RATE I.P.R.	DEPTH OF CUT INCH.	FORCES IN LBF 'ORIGINAL' RESULTS F_{FEED}	F_{TANG}	$F \sqrt{F^2 + T^2}$	'REPLICATE' RESULTS. F_{FEED}	$F_{TANG.}$	$F \sqrt{F^2 + T^2}$
12	17	400	0.0022	0.040	42.0	51.5	66.5	39.0	52.4	65.3
7	18	400	0.0200	0.040	102.3	262.1	281.4	101.3	267.3	285.8
13	19	150	0.0067	0.040	73.9	123.1	143.5	72.9	123.1	143.0
1	20	1050	0.0067	0.040	70.4	111.3	131.8	70.4	112.4	132.6
10	21	400	0.0067	0.015	21.3	52.4	56.5	23.3	50.3	55.4
16	22	400	0.0067	0.105	168.6	272.4	320.4	165.5	264.8	312.3
9	23	400	0.0022	0.040	39.5	59.4	71.3	38.5	58.9	70.3
20	24	400	0.0200	0.040	103.3	251.9	272.3	102.2	257.0	276.6

CHAPTER 6.

ANALYSIS OF RESULTS AND SELECTION OF FINAL PREDICTION EQUATION.

6.1 FIRST AND SECOND ORDER PREDICTION EQUATIONS.

The observed results of the replicated experiment were analyzed using an I.C.L. 1900 Electronic computer. The theory of multiple regression analysis was given in the Appendix. The experimental results and the cutting conditions (five levels of each of the Variables of Speed, Feed rate and Depth of cut) were initially transformed by the computer to their natural logarithms. Using the available standard procedure for multiple regression analysis (I.C.L. 1900 Stats. Package), each of the two separate sets of data was processed.

The experimental design employed being an orthogonal one, it was perfectly valid to average out the results obtained from the two sets to yield a mean prediction equation. This was done to increase the precision and generality of the prediction within the experimental premises laid down earlier.

Analysis of the 'Original' set of data yielded the following first order prediction equation:

$$y = 87{,}622\ x_1^{-0.0486}\ x_2^{0.648}\ x_3^{0.912} \qquad \text{- - - - - - (i)}$$

where:

- y is the resultant power component of the cutting forces in lbf.
- x_1 is the cutting speed in f.p.m.
- x_2 is the feed rate in i.p.r.
- x_3 is the depth of cut in inches.

Analysis of the 'Replicate' set of data yielded:

$$y = 86{,}000\ x_1^{-0.039}\ x_2^{0.652}\ x_3^{0.920} \qquad \text{- - - - (ii)}$$

These two expressions defined the equations to the least square "Response Surface" in a four dimensional space.

The constant and the exponents were then averaged to yield the mean equation of the response surface as:

$$y = 86{,}811\ x_1^{-0.044}\ x_2^{0.650}\ x_3^{0.916} \qquad \text{- - - - - (iii)}$$

The analysis was then further extended to include the quadratic effects of the three variables and their first order interaction in pairs. The respective equations obtained from the two sets of data were:

Equation from 'Original' Data:

$$y = 3,063,900\ x_1^{-0.655}\ x_2^{1.153}\ x_3^{1.213}\ (x_1^2)^{-0.009}\ (x_2^2)^{0.024}\ (x_3^2)^{-0.011}\ (x_1.x_2)^{-0.052}\ (x_1.x_3)^{-0.074}\ (x_2.x_3)^{0.014} \quad \text{- -(iv)}$$

Equation from 'Replicate' Data:

$$y = 1,687,400\ x_1^{-0.431}\ x_2^{1.102}\ x_3^{1.333}\ (x_1^2)^{-0.001}\ (x_2^2)^{0.017}\ x_3^{2^{-0.006}}\ (x_1.x_2)^{-0.040}\ (x_1.x_3)^{-0.064}\ (x_2.x_3)^{0.013} \quad \text{-- (v)}$$

These two expressions referred to the least square response surface in a ten-dimensional space.

The constant and the exponents were then averaged to yield the mean equation of the response surface as:-

$$y = 2,375,600\ x_1^{-0.543}\ x_2^{1.128}\ x_3^{1.273}\ (x_1^2)^{-0.005}\ (x_2^2)^{0.021}\ (x_3^2)^{-0.008}\ (x_1.x_2)^{-0.046}\ (x_1.x_3)^{-0.069}\ (x_2.x_3)^{0.014} \quad \text{--(vi)}$$

6.2 TESTS FOR ADEQUACY AND 'FIT' OF THE FIRST ORDER EQUATION.

The relevant information contained in computer print-outs, in the case of both sets of log-transformed data, were as below:

ORIGINAL DATA.

Number of degrees of freedom - 20

Variable Name	Regression Coefficient.	Standard Error	T-Statistics.
x_1	0.0486387	0.0141735	3.43
x_2	0.6484681	0.0132948	48.78
x_3	0.9119176	0.0141735	64.34

Error Sum of Squares - 0.0306569

Residual Error - 0.0391516

Multiple Correlation Coefficient - 0.998

Intercept Term - 11.3808098.

TABLE . 3.

REPLICATE DATA

Number of degrees of Freedom - 20

Variable Name	Regression Coefficient.	Standard Error.	T-Statistics
x_1	0.0386603	0.0137236	2.82
x_2	0.6517568	0.0128095	50.88
x_3	0.9196497	0.0137236	67.01

Error Sum of Squares - 0.0287418

Residual Error - 0.0379090

Multiple Correlation Coefficient - 0.999

Intercept Term - 11.3620125

TABLE . 4

The T - Statistics in the first case indicated that the cutting speed was a significant factor at 1% level - and in the second case at 2% level.

Thus, in all cases the contribution made by each of the variables to the regression equation was determined to be significant. The coefficient of multiple correlation in both cases were assertive that the three parameters considered explained the variations in cutting forces adequately. It was, therefore, decided to subsequently use the information from the 9 repeat points during the test to check whether any 'lack of fit' existed between the first order regression equation and the data points (Ref. Appendix for theory).

The scheme of checking this was best laid down by Draper and Smith (15*) in diagramatic form in Fig. 4.

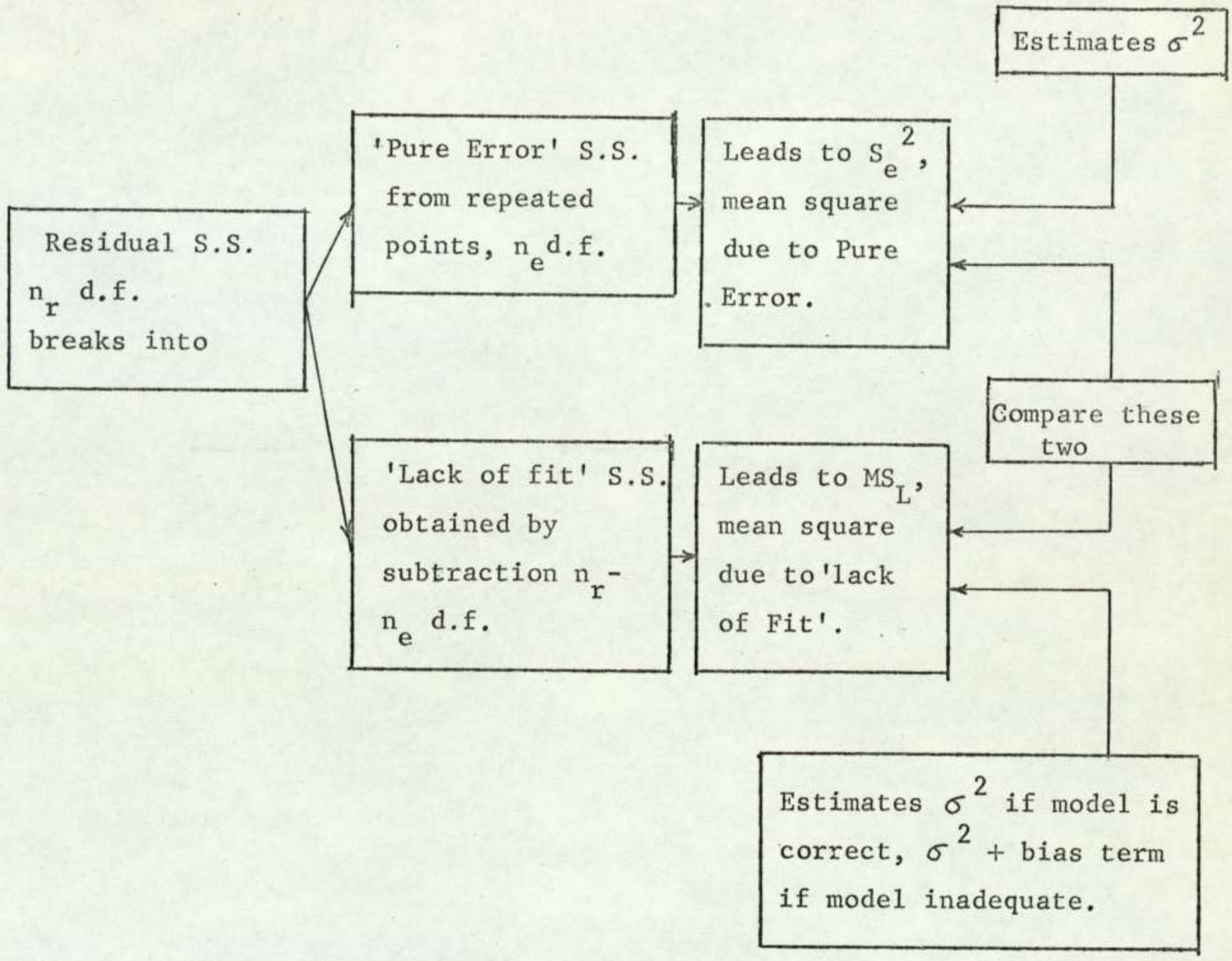


FIG.4.

The specimen calculation in $\log_e$- transformed responses from the 'Original' data was as follows:

Pure Error Sum of Squares
from 4 repeats 9, 10, 11 & 12:

$$(0.488583^2 + 0.491486^2 + 0.489784^2 + 0.489110^2) - 4 \times \left[\frac{0.488583 + 0.491486 + 0.489784 + 0.489110}{4}\right]^2$$

$= 0.000,380,644,1$

Pure error Sum of Squares from 2 repeats 13 & 19

$$= (0.493591^2 + 0.496634^2) - 2\left[\frac{0.493591 + 0.496634}{2}\right]^2$$

$$= \tfrac{1}{2}(0.496634 - 0.493591)^2$$

$= 0.000,462,992,5$

Similarly, the respective quantities were:

From repeats 14 & 20 = 0.000,428,073,8

From repeats 15 & 21 = 0.000,429,045,0

From repeats 16 & 22 = 0.000,540,547,2

From repeats 17 & 23 = 0.000,863,616,8

From repeats 18 & 24 = 0.000,680,067,2

Therefore, total Pure error Sum of Squares was 0.005,784,986,6 with 9 degrees of freedom.

Thus, the Analysis of Variance Table showing 'lack of fit' was constructed:

Source	d.f.	Sum of Squares.	Mean Square.	F. ratio.
Residual	20	0.030,656,9		
Lack of Fit.	11	0.024,871,9	0.002,261,1)	
Pure Error.	9	0.005,785,0	0.000,642,8)	3.52

TABLE . 5.

Table 2 gives the Analysis of Variance showing 'lack of fit' for the 'Replicate' data.

Source	d.f.	Sum of Squares.	Mean Square.	F. ratio
Residual	20	0.028,741,8		
Lack of fit.	11	0.023,225,0	0.002,111,4)	
Pure error.	9	0.005,516,8	0.000,613,0)	3.44

TABLE . 6.

In both the above cases reference to F - tables indicated that for the respective degrees of freedom the 'lack of fit' observed was not significant at 2.5% level (F (0.025,11,9) = 3.92). Thus it could be concluded that the first order equation involving three variables only was a sufficiently reasonable predictor.

This fact was again confirmed during the proving test series when only one out of thirty points was found to be outside the 95% confidence limits of the response predicted by this equation.

The values obtained from the computer print-outs for both sets of first order results revealed that the Residual Error expressed as a percentage of the Mean Response was also quite small (less than 0.8 % in both cases), and furnished further support for the adequacy of the three-variable prediction equation.

6.4 LIMITATIONS OF THE SECOND ORDER EQUATION REVEALED DURING THE ANALYSIS.

Considering the exponents of the second order equation, it could be observed that the relative size of the second order effects (the largest of these from the mean equation was 0.069) was quite small compared to the first order effects (the least of these from the mean-equation was 0.543). Also from the computer print-outs for regression equations involving 9 variables, it was observed that the T- statistics did not show any marked significance of the second order effects in most cases. It was further observed that the improvement in the coefficient of multiple correlation (this was 0.999 for both sets of data) with the addition of 6 more effects was insignificant (to the extent of 0.001 in the case of 'Original' data only). It could not, therefore, be considered worthwhile to include the second order effects for the formulation of prediction equation.

6.3 ANALYSIS OF RESIDUALS FROM THE FIRST ORDER PREDICTION EQUATION.

The computer print-outs also contained an analysis of residuals (i.e. $y_{obs.} - y_{est.}$). While postulating a multiple linear regression model, the assumption made was that the errors were independent, have zero mean, a constant variance and a normal distribution. The assumption of normality of the distribution validates the use of 'T' and 'F' - ratios for significance testing. Thus, if the postulated model was correct, the residuals should not exhibit any contradiction to the assumptions. This aspect was ascertained by the following checks.

1. For the 'Original' set of results, there were 12 positive ($y_{obs.} - y_{est.}$) and 12 negative residuals. For the 'Replicate' set there were 10 positive and 14 negative residuals. The positive and negative residuals in both sets appeared to have occurred randomly (qualitative assessment) and did not reveal any time-dependent trends.

2. The $\frac{e_i}{S}$ (where e_i denotes the residual and 'S' is the residual error) i = 1, 2 -------24 ratio was in all cases, except one, less than 2. Since 95% of an N (0,1) distribution lay within the limits (-2, 2), the assumption of normality of errors was not violated as revealed by this test.

3. The ratio $\frac{y_{est.}}{y_{obs.}}$ was in all cases found to lie between 1.027 and 0.982 and was less than or greater than unity in a random manner revealing no systematic trends.

4. The computed value of the Durban-Watson 'D' Statistic (16*) defined by $D = \frac{\sum_{i=1}^{n} (e_i - e_{i-1})^2}{\sum_{i=1}^{n} e_i^2}$ was in both cases less than the tabulated value for the respective number of variables involved. This 'D' statistic was printed out as part of the 'residual analysis' from the computer and only comparison of values tabulated in (17*) was made.

All the above checks were satisfactory and validated the postulation of a multiple linear regression model about the logarithmic transformed data.

It was, therefore, decided to accept the mean first order equation for the 'Response Surface', $y = 86{,}811\, x_1^{-0.044}\, x_2^{0.650}\, x_3^{0.916}$, as the prediction equation for the resultant power component of the cutting forces under the cutting conditions stipulated.

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CHAPTER 7

EVALUATION OF PREDICTION EQUATION BY PROVING TESTS.

7.1 ASPECTS FOR EVALUATION.

After having accepted the final form of 'Prediction' Equation for forces, it was then necessary to verify the same. It was also important to check whether the time-dependent trends in cutting forces were predominant. With these two objectives in view, a series of 30 control tests were planned. The chief limitation in devising the test series was the tool-wear. At higher speeds and at larger areas of cut, the tool-wear was significant. It was, therefore, decided to employ 4 corners of the same square tip, in rotation, serially for each test. This procedure would distribute the wear to 4 different cutting edges and simultaneously help to discover presence of time-trends, if any prevailed, as the tests progressed successively. If the cutting forces increased significantly with wear, it should be possible to observe the increases systematically while comparing the theoretical and experimental values of cutting forces. However, the most unfavourable combinations of high 'speeds' and large 'areas of cut' (either due to large 'feed-rates' or large 'depth of cuts') were avoided to eliminate early failure of tool-tips.

7.2 VERIFICATION OF PREDICTION EQUATION.

The cutting conditions employed and the observed cutting forces were given in Table.7.

Further, the resultant cutting force (power components) were calculated using the accepted form of prediction equation. These values together with the percentage differences between the 'observed' and 'predicted' values were presented in Table.8.

It was observed that the largest deviation occurred in Test No. 14 and was 7.2 %.

In order to ascertain whether a deviation of this order was acceptable or not, confidence limit regions of the regression plane for individual observations were then calculated (Appendix - Theory of Regression Analysis).

The 95% Confidence Limits for an individual observation for any point $x_1 = p$, $x_2 = q$ and $x_3 = r$ were:

$$y \pm t_{(26,0.975)}.s.\sqrt{\underline{X}_o^{'}\ (\underline{X}^{'}\underline{X})^{-1}\underline{X}_o}$$

TABLE . 7

Test No.	Cutting Conditions. Speed f.p.m.	Feed-rate i.p.r.	Depth of cut inch.	Observed Resultant cutting force lbf.
1	1000	0.0033	0.100	184.0
2	1000	0.0067	0.080	236.8
3	1000	0.0133	0.060	289.7
4	1000	0.0200	0.040	276.2
5	800	0.0033	0.100	185.7
6	800	0.0067	0.080	237.8
7	800	0.0133	0.060	282.6
8	800	0.0200	0.040	274.8
9	800	0.0022	0.020	34.8
10	600	0.0033	0.100	185.5.
11	600	0.0067	0.080	242.0
12	600	0.0133	0.060	287.6
13	600	0.0200	0.040	275.1
14	600	0.0022	0.020	36.8
15	500	0.0033	0.100	188.4
16	500	0.0067	0.080	243.2
17	500	0.0133	0.060	295.1
18	500	0.0200	0.040	274.7
19	400	0.0033	0.100	193.2
20	400	0.0067	0.080	247.7
21	400	0.0133	0.060	307.7
22	400	0.0200	0.040	275.1
23	200	0.0033	0.100	200.0
24	200	0.0067	0.080	268.2
25	200	0.0133	0.060	316.6
26	200	0.0200	0.040	292.0
27	1000	0.0033	0.080	155.8
28	1000	0.0022	0.060	94.8
29	1000	0.0067	0.040	124.2
30	1000	0.0133	0.020	107.7

TABLE . 8

Test No.	Observed Force lbf	Predicted Force. lbf.	Difference	% Difference.
1	184.0	190.7	+ 6.7	+ 3.5
2	236.8	264.0	+ 7.2	+ 3.0
3	289.7	293.4	+ 3.7	+ 1.3
4	276.2	264.1	-12.1	- 4.6
5	185.7	192.6	+ 6.9	+ 3.6
6	237.8	246.4	+ 8.6	+ 3.5
7	282.6	297.1	+14.5	+ 4.9
8	274.8	266.7	- 8.1	- 3.0
9	34.8	33.9	- 0.9	- 2.7
10	185.5	195.1	+ 9.7	+ 5.0
11	242.0	249.5	+ 7.5	+ 3.0
12	287.6	300.8	+13.2	+ 4.4
13	275.1	270.1	- 5.0	- 1.9
14	36.8	34.3	- 2.5	- 7.2
15	188.4	196.7	+ 8.3	+ 4.2
16	243.2	251.2	+ 8.0	+ 3.2
17	295.1	303.3	+ 8.2	+ 2.7
18	274.7	272.3	- 2.4	- 0.9
19	193.2	198.6	+ 4.4	+ 2.2
20	247.7	254.0	+ 6.3	+ 2.5
21	307.7	306.0	- 1.7	- 0.5
22	275.1	274.9	- 0.2	
23	200.0	204.7	+ 4.7	+ 2.3
24	268.2	261.9	- 6.3	- 2.4
25	316.6	315.7	- 0.9	- 0.3
26	292.0	283.5	- 8.5	- 3.0
27	155.8	155.5	- 0.3	
28	94.8	91.8	- 3.0	- 3.3
29	124.2	129.3	+ 5.1	+ 3.9
30	107.7	107.5	- 0.2	

TABLE. 9.

Test No.	Predicted Force. (lbf.)	Observed Force. (lbf).	Upper Confidence. Limit.	Lower Confidence Limit.
1	190.7	184.0	205.5	177.0
2	244.0	236.8	262.7	227.0
3	293.4	289.7	316.2	273.4
4	264.1	276.2	284.5	255.1
5	192.6	185.7	207.3	179.0
6	246.4	237.8	264.8	229.1
7	297.1	282.6	316.5	276.3
8	266.7	274.8	287.1	247.7
9	33.9	34.8	36.8	31.1
10	195.1	185.5	209.8	181.3
11	249.5	242.0	268.0	232.3
12	300.8	287.6	333.1	279.9
13	270.1	275.1	290.6	251.0
14	34.3	36.8*	36.7	31.9
15	196.7	188.4	211.6	182.8
16	251.2	243.2	270.1	233.1
17	303.3	295.1	325.8	282.2
18	272.3	274.7	292.9	253.0
19	198.6	193.2	213.8	184.5
20	254.0	247.7	272.9	236.4
21	306.0	307.7	329.2	284.9
22	274.9	275.1	295.9	255.5
23	204.7	200.0	221.3	189.4
24	261.9	268.2	282.8	243.0
25	315.7	316.6	340.6	292.7
26	283.5	292.0	306.1	262.5
27	155.5	155.8	167.3	144.4
28	91.8	94.8	101.1	85.2
29	129.3	124.2	139.1	120.2
30	107.5	107.7	116.1	99.5

where:

y is the predicted value.

t is the relevant 'T' statistic from Tables.

s if the best available estimate of the standard error. (This was calculated using the Residual Sum of Squares between the theoretical and observed responses).

$\underline{X}'_o$ is (1, ln p, ln q, ln r) matrix.

$\underline{X}'$ is the (4 x 30) $\log_e$ transformed 'cutting conditions' matrix.

The calculation of $\underline{X}'_o (\underline{X}'\underline{X})^{-1} \underline{X}_o$ involved multiplication inversion of large matrices to yield a row-vector of 30 numbers. To achieve this step, a computer programme was developed using available matrix 'multiplication' and 'inversion' procedures (Appendix). The upper and lower confidence limits for the 30 individual observations were as in Table. 9 .

From Table. 9 , it was observed that for the entire series of tests, the actual cutting forces were within the confidence limits with the exception of Test No. 14 (*). Thus, the validity of the prediction equation was verified by the test series.

7.3 ESTABLISHING THE ABSENCE OF 'TIME-TRENDS'.

Draper and Smith (15*) have quoted Swed and Eisenhart (18*) and indicated a method by which the significance of any time-trends in a set of data could be tested.

This involved counting the number of positive and negative residuals and also counting the number of groups of 'like' residuals in the time sequence of tests. If $'n'_1$, were the number of positive residuals, $'n'_2$ the number of negative residuals and 'u' the number of groups of 'like' residuals they have quoted that the 'mean' (μ) and 'variance' (σ^2) of the discrete distribution of 'u' were given by:

$$\mu = \frac{2\, n_1 \cdot n_2}{n_1 + n_2} + 1$$

$$\text{and } \sigma^2 = \frac{2\, n_1 . n_2\, (2\, n_1 . n_2 - n_1 - n_2)}{(n_1 + n_2)^2\, (n_1 + n_2 - 1)}$$

Then, the test unit normal deviate

$$z = \frac{u - \mu + \frac{1}{2}}{\sigma}$$

could be formed and compared against the tabulated 'Area under normal curve' values.

The time sequence signs of residuals in the Proving Test numbers (1) to (30) were as follows:

(+++) (-) (+++) (--) (+++) (--) (+++) (-) (++) (--) (+) (-----) (+) (-)

Here, $n_1 = 16$

$n_2 = 14$

$u = 14$

$\therefore$ Mean of the distribution of 'u' = $\mu = \frac{239}{15}$

and Variance of the distribution of 'u' = $\sigma^2 = \frac{187,264}{261,00}$

$= 7.17487$

$\therefore \quad \sigma = \sqrt{7.17487} = 2.6786$

$\therefore$ The test normal variate $z = -\frac{43}{80.36} = -0.5351$.

This small value of the unit normal deviate indicated that the idea of randomness in the arrangement of signs was not to be rejected. This test furnished negative evidence against the presence of any time-trends.

It was therefore concluded that the results from 'Proving tests' satisfied both aspects for evaluation of the accepted prediction equation.

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CHAPTER 8.

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK.

8.1 CONCLUSIONS.

1. It was found that the resultant of the power components of the cutting forces, when plotted against 'speed' and 'feed-rate' on a three-dimensional logarithmic scale,lay in approximate plane within the ranges considered (Section 1.1). Exceptions to the above did, however occur at combinations of very low values of 'speed' and 'feed-rate'. The departure from planar response for low values of 'speed' and 'feed' was due to the effect of built-up edge formation at the tool tip (Graph iv).

2. Similarly, when plotting the resultant force against 'speed' and 'depth of cut', the values lay in an approximate plane for the entire range considered. (Graph v).

3. Within the range of this investigation the resultant of cutting forces considered (y) was a power function of (i) cutting speed (x_1), (ii) feed-rate (x_2) and (iii) depth of cut (x_3) in accord with the general equation $y = k.x_1^a.x_2^b.x_3^c$ (where k, a, b, c are constants for a particular material), the influence of the depth of cut being the most and that of cutting speed the least.

4. Previous workers (5) have contended that the effect of 'cutting speed' on forces was negligible. But, this programme of research has revealed that it was not so for the conditions laid down (Section 1.2). The investigation has proved that the 'cutting speed' influenced the resultant cutting force in a significant manner with an inverse linear double logarithmic relationship.

5. The mean first order prediction equation for cutting forces, in the case of EN-8 steel was formulated as:

$$y = 86,811\, x_1^{-0.044} \cdot x_2^{0.650} \cdot x_3^{0.916}$$

where:

y = Resultant power component of cutting forces in lbf.

x_1 = Cutting speed in f.p.m.

x_2 = Feed-rate in i.p.r.

x_3 = Depth of cut in inches.

This equation was determined to be a sufficient predictor for forces by suitable statistical tests and compared favourably with the two variable equations given by Boston and Kraus (4)*. The equation was also verified by means of a proving test series.

6. When a second order regression model was postulated to include the quadratic and interaction terms of the three variables, with the same experimental plan, the mean equation was determined as:

$$y = 2,375,600 \quad X_1^{-0.543} \cdot X_2^{1.128} \cdot X_3^{1.273} \cdot (X_1^2)^{-0.005} \cdot (X_2^2)^{0.021}$$

$$(X_3^2)^{-0.008} \cdot (X_1.X_2)^{-0.046} \cdot (X_1.X_3)^{-0.069} \cdot (X_2.X_3)^{0.014}$$

The exponents of the quadratic and interaction terms revealed that these effects were small compared to the first order effects. However, it was observed from the computer analysis that there was a very high degree of correlation between the cross-product term $(X_2.X_3)$ taken singly, and the response.

7. The power of "Response Surface Methodology" approach was highlighted by the results of current investigations. With a small number of 24 cutting tests, it was proved to be possible to fully formulate both the first and second order prediction equations and also assess their relative importance and precision for any material, reliably. The method could be used, with advantage, in situations such as prediction of "Surface Texture" or "Machinability" with considerable economy in comparison to the classical approach of studying the effect of one variable at a time. In addition, the method enabled one to do without the assumption of absence of interaction between different parameters.

8. The effectiveness and reliability of the statistical checks used for testing the adequacy of the first order model were established by the results of the proving test series during which only one (out of 30) lay outside the 95% Confidence Limits for individual observations.

9. The effect of built-up edge phenomenon was more predominant with combinations of low 'speeds' and low 'feed-rates' than with combinations of low 'speeds' and low 'depths of cut'.

10. The radial component of the cutting forces was found to have a tendency to increase with larger areas of cut, but was very much sensitive to conditions at the cutting edge and did not show high

possibilities of repeatability.

11. The P.E.R.A. design of the three dimensional Strain-Gauge type of dynamometer adequately satisfied the general requirements for a 'turning' dynamometer in the range of 0 - 500 lbf. for all three components of cutting forces (Section 3.1).

8.2 SUGGESTIONS FOR FURTHER WORK.

1. During the investigations, evidence was found to suggest that the hardness of the work-material (in the case of plain carbon steels) also had a logarithmic linear relationship with the resultant cutting force. Other variables like Tool Geometry (specially the Rake Angle and Tool-nose Radius) also could be attempted for inclusion into the regression model after suitable transformations. Further investigations with the addition of these new variables could lead to formulation of a more generalized prediction equation for cutting forces.

2. The variations of cutting forces tended to be a minimum within a tool-life range of 5 - 15 minutes for low speeds and small area of cuts and 5 - 10 minutes for higher speeds and larger areas of cuts (Graph (vi)). However, the definitive pattern and the significance of these variations has yet to be determined. Further, the rupture stresses of the work-material at three faces of the tool (Top rake, Front cutting edge and Side cutting edge) showed a tendency to be approximately equal (Graph (viii)). Additional investigations are needed in these areas.

3. The high correlation between the cross-product term $x_2.x_3$ ('feed-rate' x 'depth of cut') and the cutting forces could be further investigated to ascertain whether this term alone would yield a sufficient predictor equation and compare the results thus obtained with those from the currently accepted equation.

4. There was some evidence to suggest that the surface texture (C.L.A. value) on the work-piece tended to be correlated to 'point' wear more than 'flank' or 'crater' wear. Investigations up until now did not seem to have taken this aspect quantitatively into consideration for evaluating surface finish. Future work could be done in this direction.

5. "Response Surface Methodology" approach could be effectively

used for prediction of 'Surface Texture', 'Machinability' and such other excessively 'time and other resources' consuming studies with remarkable success and economy. It is felt that a particular area in which this technique could be applied quite successfully is in investigations concerning surface wear patterns during the operative or functional stage of components once the parameters affecting it have been sufficiently identified to be measured and their individual relationships to the 'wear pattern' established. The smallness of the number of tests required is of tremendous advantage in such a time-consuming study.

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APPENDIX I

THEORY OF REGRESSION ANALYSIS.

APPENDIX.1

1. THEORY OF MULTIPLE REGRESSION ANALYSIS. (15*, 19*, 20*, 21*)

One of the most frequently used types of statistical analysis is the one which is provided by the theory of regression and correlation. In situations where data is available on one or more variables affecting a dependent variable, whether the existing relationships are linear or otherwise, an analysis of regression would help to reveal and formulate the relationships. The availability of suitably programmed electronic computers have accelerated the use of this technique in a variety of applications. In cases where the relationship between the dependent variable and the independent variables are linear, the analysis is straightforward. In a vast majority of cases where the relationship is not linear, it is possible to linearize by means of suitable transformations. One of the most common transformations employed is the logarithmic one.

In the present investigations, it has been established by preliminary trials that the relationships between the resultant power component of the cutting forces and each of the three independent variables could be linearized by double logarithmic transformations. The analysis, in this case, is also straightforward except that instead of the values of the variables, their logarithms are considered and the final results are expressed in their original form. As the independent variables were more than one, this was a case of multiple regression analysis. The matrix algebra method of regression analysis was easily amenable to computer calculations and was employed for the analysis. The theory underlying this method could be summarized as below.

The object, in the case of a multiple regression model, is to find the equation to the least square plane between the response and the concomitant variables.

A matrix is a rectangular array of symbols or numbers and is denoted by underlined capital letters in this Thesis. The size of a matrix is denoted by 'm x n' matrix indicating that there are 'm' rows and 'n' columns. A row vector is '1 x n' matrix and a column vector is 'm x 1' matrix. The transpose of a matrix $\underline{A}$ is denoted by $\underline{A}'$ and its inverse $\underline{A}^{-1}$.

The theory would be explained, for the sake of simplicity, for the case of two independent variables and a response though the method could be extended to any number of independent variables.

Let the vector of the responses observed in a series of 'm' tests be denoted by $\underline{Y}$ ('m x 1' matrix) and the corresponding matrix of independent variables x_1 and x_2 be denoted by $\underline{X}$ ('m x 2' matrix).

In the case of linear relationship between one response and one independent variable, the relationship could be expressed as $y = c + px$ where 'p' is the slope of the line and 'c' its intercept. Thus, in the case of two parameters, there would be two constants or coefficients to be evaluated. In the present case, there would be three constants to be evaluated. These could be denoted by a '1 x 3' matrix ($\underline{B}$). The equation to be formulated would be that of a plane from which the sum of the squares of deviations to the individual observation points in the simple space is the least. Thus, the relationship formulated would not be exact for all the individual cases and would contain an error term denoted by $\underline{\varepsilon}$ ('m x 1' matrix). The regression model could thus be expressed by the matrix equation:

$$\underline{Y} = \underline{B}\,\underline{X} + \underline{\varepsilon}$$

METHOD OF CALCULATION OF COEFFICIENTS.

In the case of a two variable problem ($y = c + px$), we have referred to the calculation of two constants 'c' and 'p'. The constant 'c' could be considered as the coefficient of unity. Thus, the introduction of a column of ones as x_o in the observation matrix of independent variables would make things simpler by treating them all as coefficients of variables. The regression model would, therefore, be:

$$\underset{\underline{Y}}{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ - \\ - \\ - \\ y_m \end{bmatrix}} = \underset{\underline{B}}{\begin{bmatrix} \beta_o \\ \beta_1 \\ \beta_2 \end{bmatrix}} \underset{\underline{X}}{\begin{bmatrix} x_{10}\ (1) & x_{11} & x_{12} \\ x_{20}\ (1) & x_{21} & x_{22} \\ x_{30}\ (1) & x_{31} & x_{32} \\ - & - & - \\ - & - & - \\ - & - & - \\ x_{mo}\ (1) & x_{m1} & x_{m2} \end{bmatrix}} + \underset{\underline{\varepsilon}}{\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ - \\ - \\ - \\ \varepsilon_m \end{bmatrix}}$$

If the transpose of the matrix $\underline{X}$ (i.e. $\underline{X}'$) - a '3 x m' matrix - is post multiplied by the matrix $\underline{X}$, it would yield a 3 x 3 matrix with the following terms:

$$\text{i.e. } \underline{X}'\underline{X} = \begin{bmatrix} m & \sum_{i=1}^{m} x_{i1} & \sum_{i=1}^{m} x_{i2} \\ \sum x_{i1} & \sum x_{i1}^2 & \sum (x_{i1}.x_{i2}) \\ \sum x_{i2} & \sum (x_{i1}.x_{i2}) & \sum x_{i2}^2 \end{bmatrix}$$

Similarly:

$$\underline{X}'\underline{Y} = \begin{bmatrix} \sum Y_i \\ \sum x_{i1} Y_i \\ \sum x_{i2} Y_i \end{bmatrix}$$

Applying matrix algebra the matrix $\underline{B}$ could be evaluated as:

$$\underline{B} = \left[\underline{X}'\underline{X}\right]^{-1} \underline{X}'\underline{Y}$$

where '$\underline{B}$' is the vector of coefficients
and $\left[\underline{X}'\underline{X}\right]^{-1}$ is the inverse of $\underline{X}'\underline{X}$

The coefficients thus evaluated are the estimates of the population coefficients β_0, β_1, & β_2, and could be, therefore, denoted by b_o, b_1 and b_2.

The solution for $\underline{B}$ can be obtained only if $\underline{X}'\underline{X}$ is nonsingular so that its inverse exists.

PRECISION OF ESTIMATED REGRESSION.

In order to assess the extent of utility of the relationship expressed by the regression equation, it would be necessary to find out its associated precision. If the estimated value of the response (under a certain set of conditions) given by the regression equation is denoted by Y_i and its true value by Y_i, then:

$$Y_i - \hat{Y}_i = (Y_i - \bar{Y}) - (\hat{Y}_i - \bar{Y})$$

By squaring both sides and summing from i = 1 to m and simplifying

$$\sum (Y_i - \bar{Y})^2 = \sum (Y_i - \hat{Y}_i)^2 + \sum (\hat{Y}_i - \bar{Y})^2$$

i.e. (Sum of the Squares (denoted by) (S.S.) of deviations of observations from its overall mean) = (S.S. of deviations of observations from its predicted values) + (S.S. of deviations of predicted values from the overall mean.)

The above statement could be given the following notation:
S.S. about mean = S.S. due to regression + S.S. about regression
(Total Corrected S.S.) (Residual) (Regression).

Any sum of squares is associated with its 'degrees of freedom' which in the above case would be (m-1), k and (m-k-1) respectively for the three quantities.

The square root of the ratio

$\frac{(\text{S.S. due to regression})}{(\text{S.S. about mean})}$ is called the "coefficient of Multiple Regression" (R) and the nearer it is to unity, the better would be the agreement between the predicted and experimental values.

The ratio $\frac{(\text{S.S. due to regression}/k)}{(\text{S.S. about regression}/(m-k-1))}$ indicates an estimate of the precision of the prediction and is to be compared against the relevant 'F' - Statistics for determining the statistical significance of the regression equation.

The term $\sum(Y_i - \bar{Y})^2$ is called the Corrected S.S. and is equal to $\sum Y_i^2 - \frac{(\sum Y_i)^2}{m}$. Therefore $\sum Y_i^2$ could be logically called the Uncorrected S.S. and $\frac{(\sum Y_i)^2}{m}$ the correction factor.

Now, if the centre of the co-ordinate system in which the values of 'x's and 'Y's are plotted is made to coincide with $\bar{Y}$, the constant term (intercept) b_o would be zero and therefore $\frac{(\sum Y_i)^2}{m}$ would be zero. Thus, the correction term $\frac{(\sum Y_i)^2}{m}$ is 'due to b_o' and could therefore be denoted as S.S. (b_o) and it has only a single degree of freedom.

Similarly, the product of a particular coefficient (e.g. b_1) and the co-variance of the variable associated with the coefficient (x_1 in this case) to the response variable (Y) gives the sum of squares for b_1 after allowance has been made for b_o. This is indicated by the notation S.S. (b_1/b_o) and has again got a single degree of freedom. Each of the above quantities S.S. (b_o) and S.S. (b_1/b_o) could be tested for statistical significance by comparison against Residual S.S. and reference to 'F' tables (or the square root of the ratio as has been done in the computer print-out, to 'T' -tables).

This would enable one to decide about the usefulness or otherwise of adding each variable into the regression equation.

Thus, for a Regression Analysis, the Analysis of Variance Table would be:

SOURCE	S.S.	D.F.	M.S.
Regression (b_o)	$S.S.(b_o) = \frac{\sum Y_i^2}{m}$	1	
Regression (b_1/b_o)	$S.S.(b_1/b_o) = b_1\left[\sum x_i Y_i - \frac{\sum x_i . \sum Y_i}{m}\right]$	1	
Residual	S.S.(R) by Subtraction.	m-2	$s^2 = \frac{S.S.(R)}{m-2}$
Total Uncorrected for mean	$\sum Y_i^2$	m	

TABLE NO. A1(i)

Also, we have:

(i) $$S.S.(b_o) = \frac{\sum Y_i^2}{m} = m\,\bar{Y}^2$$

(ii) $$S.S.(b_1/b_o) = b_1 \left[\sum x_i Y_i - \frac{\sum x_i . \sum Y_i}{m}\right]$$

$$= b_1 \left[\sum x_i Y_i - m.\bar{X}.\bar{Y}\right]$$

$$\therefore \quad S.S.(b_o) + S.S.(b_1/b_o) = b_1 \sum x_i Y_i - b_1 .m.\bar{X}.\bar{Y} + m\,\bar{Y}^2$$

$$= b_1 \sum x_i Y_i + m\,\bar{Y}(\bar{Y} - b_1\bar{X})$$

$$= b_1 \sum x_i Y_i + b_o \sum Y_i$$

$$= (b_o\ b_1)\begin{bmatrix}\sum Y_i \\ \sum x_i Y_i\end{bmatrix}$$

$$= \underline{B}'\ \underline{X}'\ \underline{Y}$$

Therefore, using matrix notation, the Analysis of Variance Table could be written as: (Table No. A1 (ii))

Also, $$R^2 = \frac{\underline{B}'\ \underline{X}'\ \underline{Y} - m\bar{Y}^2}{\underline{Y}'\underline{Y} - m\,\bar{Y}^2}$$

SOURCE	S.S.	D.F.	M.S.
Regression (b_o)	$m\bar{Y}^2$	1	
Regression (b_1/b_o)	$\underline{B}'\underline{X}'\underline{Y} - m\bar{Y}^2$	1	
Residual.	$\underline{Y}'\underline{Y} - \underline{B}'\underline{X}'\underline{Y}$	m - 2	$s^2 = \frac{\underline{Y}'\underline{Y} - \underline{B}'\underline{X}'\underline{Y}}{m - 2}$
Total	$\underline{Y}'\underline{Y}$	m	

TABLE NO. A1(ii)

ANALYSIS OF 'RESIDUALS' TO TEST WHETHER 'LACK OF FIT' IS SIGNIFICANT.

The regression plane is 'fitted' on the assumption that the postulated model is correct. This can be tested by further analysis. If $e_i = Y_i - \hat{Y}_i$ (where Y_i and $\hat{Y}_i$ are the observed and predicted values respectively) is the residual at x_i, this residual should contain the information on the ways in which the fitted model fails to explain properly the variations in the response.

Let $\eta_i = E\,(Y_i)$ denote the value of Y_i given by the true model at $x = x_i$.

$$
\begin{aligned}
\text{Then, } Y_i - \hat{Y}_i &= (Y_i - \hat{Y}_i) - E\,(Y_i - \hat{Y}_i) + E\,(Y_i - \hat{Y}_i) \\
&= \left[(Y_i - \hat{Y}_i) - [\eta_i - E(\hat{Y}_i)]\right] + [\eta_i - E(\hat{Y}_i)] \\
&= q_i + p_i
\end{aligned}
$$

$$\text{where } q_i = \left[(Y_i - \hat{Y}_i) - [\eta_i - E\,(\hat{Y}_i)]\right]$$

$$\text{and } p_i = [\eta_i - E(\hat{Y}_i)]$$

The quantity 'p_i' is the bias error at $x = x_i$. If the model is correct, $E\,(\hat{Y}_i) = \eta_i$ and p_i would be zero or conversely if incorrect 'p_i' would have a value. The quantity 'q_i' is a random variable irrespective of the validity of the postulate and can be shown to be correlated with an expected mean value of $(n - 2)\sigma^2$ where $V(Y_i) = V\,(\varepsilon_i) = \sigma^2$. It can also be further shown that the residual mean square value of $\frac{1.}{n - 2}\left[\sum_{i=1}^{m} (Y_i - \hat{Y}_i)^2\right]$ has an expected value of σ^2 (Error variance) if the model is correct and $\sigma^2 + \sum p_i^2/n - 2$ otherwise. If the model is correct, i.e. $p_i = 0$, then the residuals are correlated random deviations q_i and the residual mean square can be used as an estimate of σ^2.

However, if the model is not correct, then $p_i \neq 0$ and the residuals would contain both random (q_i) and systematic (p_i) components and would tend to be comparatively larger. If a prior estimate of σ^2 is available from previous observations, this can be compared against the residuals obtained and the ratio checked by means of an F - test for significance. If by the F - test it is found that the residuals obtained is great, it could be concluded that there is significant 'lack of fit' and the postulated model has to be reconsidered. However, if no prior estimate of σ^2 is available, repeat measurements of Y can be made at specified levels of 'x's and from the information obtained from these repeats σ^2 can be estimated. Such an estimate is denoted S.S. "pure error" because it contains only the random variations affecting the response from identical conditions. For this reason nine repeat points , at various levels, have been included in the experimental design devised.

If, Y_{11}, Y_{12}, - - - $Y_{1\,n_1}$ are n_1 repeats at x_1,

Y_{21}, Y_{22}, - - - $Y_{2\,n_2}$ are n_2 repeats at x_2,

- -

Y_{k1}, Y_{k2}, - - - $Y_{k\,n_k}$ are n_k repeats at x_k

then the Mean Square for pure error is:

$$s_e^2 = \frac{\sum_{i=1}^{k} \sum_{u=1}^{n_i} (Y_{iu} - \bar{Y}_i)^2}{\sum_{i=1}^{k} (n_i - k)}$$

Once the S.S. pure error has been evaluated, the 'lack of fit' S.S. can be found by subtraction from 'Residual'S.S. and the 'F' ratio can be formulated for significance testing after dividing both by their respective 'degrees of freedom.'

Therefore, the residual error S.S. evaluated in Table can further be split into Pure 'error' and 'lack of fit' S.S. and the ratio $\left(\frac{\text{Mean Square 'Lack of fit'}}{\text{Mean Square 'Pure Error'}}\right)$ formulated to decide whether there is any evidence to indicate any inadequacy of the postulated model.

VARIANCES AND CO-VARIANCE OF b_o and b_1 BY MATRIX CALCULATION.

For a function, $F = a_1 Y_1 + a_2 Y_2 + - - - a_m Y_m$, the variance is given by:

$$V(F) = a_1^2 V(Y_1) + a_2^2 V(Y_2) + \cdots\cdots a_m^2 V(Y_m)$$

If all the 'Y's are uncorrelated, the 'a's are constants and $V(Y_i) = \sigma^2$, then,

$$V(F) = (a_1^2 + a_2^2 + \cdots a_m^2) \cdot \sigma^2$$
$$= (\Sigma a_i^2)\,\sigma^2 \quad \text{------------(i)}$$

In the case of 'centered' data, i.e. the origin of the co-ordinate system is transferred to $(\bar{x}, \bar{Y})$, it is easy to see that b_o is zero and

$$b_1 = \frac{\Sigma (x_i - \bar{x})(Y_i - \bar{Y})}{\Sigma (x_i - \bar{x})^2}$$
$$= \frac{\Sigma (x_i - \bar{x})\, Y_i}{\Sigma (x_i - \bar{x})^2} \quad \text{------------(ii)}$$

(Since $\Sigma (x_i - \bar{x}) = 0$)

Applying Equation (i) into (ii) and after reduction:

$$V(b_1) = \frac{\sigma^2}{\Sigma (x_i - \bar{x})^2} \quad \text{------------(iii)}$$

Similarly, $V(b_o) = V(\bar{Y} - b_1 \bar{x})$

$$= \frac{\sigma^2}{m} + \frac{\sigma^2 \bar{x}^2}{\Sigma (x_i - \bar{x})^2}$$
$$= \frac{\sigma^2 \Sigma x_i^2}{m \Sigma (x_i - \bar{x})^2} \quad \text{------------(iv)}$$

Covariance $(b_o, b_1) = \text{Cov.} \left[(\bar{Y} - b_1 \bar{x}), b_1 \right]$

$$= -\bar{x}\; V(b_1)$$
$$= \frac{-\bar{x}\,\sigma^2}{\Sigma (x_i - \bar{x})^2} \quad \text{------------(v)}$$

Thus, the variance - covariance of the vector $\underline{B}$ could be written as:

$$V(\underline{B}) = V \begin{pmatrix} b_o \\ b_1 \end{pmatrix} = \begin{bmatrix} V(b_o) & \text{Cov}(b_o b_1) \\ \text{Cov}(b_o b_1) & V(b_1) \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\sigma^2 \sum x_i^2}{m \sum (x_i - \bar{x})^2} & -\dfrac{\bar{x} \cdot \sigma^2}{\sum (x_i - \bar{x})^2} \\ -\dfrac{x \quad \sigma^2}{\sum (x_i - \bar{x})^2} & \dfrac{\sigma^2}{\sum (x_i - \bar{x})^2} \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} \dfrac{\sum x_i^2}{m\sum (x_i - \bar{x})^2} & -\dfrac{\bar{x}}{\sum (x_i - \bar{x})^2} \\ -\dfrac{x}{\sum (x_i - \bar{x})^2} & \dfrac{1}{\sum (x_i - \bar{x})^2} \end{bmatrix}$$

The matrix in the above case is the same as $[\underline{X}' \ \underline{X}]^{-1}$ matrix in the case of a two variable (one dependent and one independent) set.

i.e. $V(\underline{B}) = (\underline{X}'\underline{X})^{-1} \cdot \sigma^2$ - - - - - - - - - - - - - - - - - (vi)

Therefore, the variance of each individual coefficient is obtained by multiplying the variance of the response with the corresponding diagonal term of the $(\underline{X}'\underline{X})^{-1}$ matrix.

VARIANCE OF $\hat{Y}$ USING MATRIX METHOD.

$$Y = b_o + b_1 x_1$$

In a particular case, let the vector $\underline{X}_1$ be $\begin{bmatrix} 1, \\ x_1 \end{bmatrix}$

i.e. $\underline{X}_1' = (1, x_1)$

Using matrix notation, we can denote

$Y = b_o + b_1 x_1$ as $\underline{Y} = (1, x_1) \begin{bmatrix} b_o \\ b_1 \end{bmatrix}$

$= \underline{X}_1' \ \underline{B} = \underline{B}' \ \underline{X}_1$

Since $\hat{Y}$ is a linear combination of random variables b_o, b_1, it could be stated that

$$V(Y) = V(b_o) + 2 x_1 \text{Cov.} (b_o, b_1) + x_1^2 \ V(b_1)$$

It could be verified by working out the indicated matrix and the vector products, the above expression would be:

$$V(Y) = \begin{bmatrix} 1 & , x_1 \end{bmatrix} \begin{bmatrix} V(b_o) & \text{Cov}(b_o, b_1) \\ \text{Cov}(b_o, b_1) & V(b_1) \end{bmatrix} \begin{bmatrix} 1 \\ x_1 \end{bmatrix}$$

$= \underline{X}_1' \left[\underline{X}'\underline{X}\right]^{-1} \underline{X}_1 \cdot \sigma^2 .$

The above expression could be generalized for any variable x_k as :

$$V(Y_k) = \underline{X}'_k \left[(\underline{X}'\underline{X}\right]^{-1} \underline{X}\, k. \sigma^2 \text{ - - - - - - - - - - - - - - - - (vii)}$$

It is also known that an estimate of σ^2 is, in all cases, provided by the Residual S.S. which could, therefore be given the notation S^2 (Tables A1(i) and A1(ii)).

It could be verified that the variance of Y_k is a minimum at $x_k = \overline{x}$ and increases as x_k values move away from $\overline{x}$ in both directions. This indicates that one might make the best prediction in the neighbourhood of the 'middle' range of the sample space.

CONFIDENCE LIMITS FOR THE VALUE OF Y GIVEN A SPECIFIC SET OF 'x's.

(i) For the mean Value of Y

The (1 - α) Confidence limits of the true mean value of Y at x_k are given by:

$$Y \pm t_{(m - k - 1,\ 1 - \alpha/2)} \quad S.\sqrt{\underline{X}_k' \ (\underline{X}'\underline{X})^{-1} \ \underline{X}_k}$$

Where 't' is the tabulated values obtained from the T - tables.

It is obvious that the confidence region of Y tends to get larger when one moves away from $\overline{Y}$. The confidence limits form part of hyperboloid surfaces on either side of the least square plane in a multiple regression problem.

(ii) For an Individual Observation Given a Specific Set of 'x's.

The (1 - α) confidence limits of a mean of 'n' observations of Y is given by:

$$Y \pm t_{(m - k - 1,\ 1 - \alpha/2)} \ . \ S\sqrt{1/n + \underline{X}_k' \ (\underline{X}'\underline{X})^{-1} \ X_k}$$

Therefore, for an individual observation of Y, the confidence limits are:-

$$Y \pm t \ (m - k - 1,\ 1 - \alpha/2).\ S.\sqrt{1 + \underline{X}_k' \ (\underline{X}'\underline{X})^{-1} \ \underline{X}_k}$$

EFFECTS OF HAVING ORTHOGONAL COLUMNS IN THE X MATRIX.

In a regression problem having parameters β_o, β_1, and β_2, it is possible to calculate the sum of squares due to any one variable such as S.S.(b_1) from model $Y = \beta_1 X_1 + \varepsilon$

S.S.(b_1/b_o) from model $Y = \beta_o + \beta_1 X_1 + \varepsilon$

S.S.$(b_1/b_o, b_2)$ from model $Y = \beta_o + \beta_1 X_1 + \beta_2 X_2 +$

These quantities would all have, generally, different values except in the case when 'β_1' column of the $\underline{X}$ matrix is orthogonal to both 'β_o' and 'β_2' columns. In the latter case, all three values of the sums of squares would be identical.

i.e. S.S. (b_i) = S.S. $(b_i$/any set of 'b_j's $j \neq i)$

It can also be proved that when replicate sets of responses due to an orthogonally spaced levels of influencing variables are available the regression coefficients can be averaged to obtain better estimates.

It is therefore, of definite advantage to orthogonalize the data matrix by choosing equal logarithmic increments symmetrically on either side of a coded zero mean value with each independent variable. This has been done in the present investigations with specific advantages.

FIRST AND SECOND ORDER REGRESSION MODELS.

When the terms of a regression model are all linear, it is caled a 'first order' model.

When the regression equation contains quadratic and interaction effects also, it is called a second order model. A complete second order model involving 'k' influencing variables would involve evaluating $\frac{1}{2}(k^2 + 3k) + 1$ coefficients.

With the aid of modern high speed electronic computers, the analysis of a second order model is identical to a first order one by simply treating each second order effect as a separate variable.

ADAPTATION OF MATRIX METHOD FOR HIGHER ACCURACY DURING COMPUTER ANALYSIS.

When data is treated in the normal manner, the $\underline{X}'\underline{X}$ matrix may contain numbers of widely varying orders. While inverting such a matrix, large 'rounding-off' errors would occur. (25*).

An effective way of overcoming this difficulty is by 'centering' of the data and scaling down. In this case, all terms of the $\underline{X}'\underline{X}$ matrix would lie between -1 and +1. The procedure involved is as follows.

If Z_1, Z_2 - - - - Z_n are some functions of the independent variables $x_1, x_2,$ - - - - x_n influencing the response Y, the regression model can be written as

$$Y = \beta_o + \beta_1 Z_1 + \beta_2 Z_2 + --- \beta_n Z_n + \varepsilon$$

Let $\overline{Z}_1, \overline{Z}_2$ - - $\overline{Z}_n$ be the means of the various 'Z' columns in the data matrix.

Then the model can be rewritten as:

$$Y = \beta_o + \beta_1 \overline{Z}_1 + \beta_2 \overline{Z}_2 + - \beta_n \overline{Z}_n + \beta_1(Z_1 - \overline{Z}_1) + \beta_2(Z_2 - \overline{Z}_2) + - - - \beta_n(Z_n - \overline{Z}_n) + \varepsilon$$

If we write $z_i = Z_i - \overline{Z}_i$, i = 1, 2 - - - n and $\beta'_o = \beta_o + \beta_1 \overline{Z}_1 + \beta_2 \overline{Z}_2 + - - - - \beta_n \overline{Z}_n$, the model can be expressed as

$$Y = \beta'_o + \beta_1 z_1 + \beta_2 z_2 + - - - \beta_n z_n + \qquad - - - (viii)$$

Now, if we make the same transformation on the data as made on the variables above, i.e. $z_{ji} = Z_{ji} - \overline{Z}_j$ j = 1, 2 - - n and i = 1, =, ---- m (assuming there are m rows in the data matrix), it follows that $\overline{z}_j = 0$ for j = 1, = ---- m. By differentiating the residual sum of squares with respect to β'_o, the first 'normal equation' could be obtained which would yield:

$$\beta'_o + \beta_1 \overline{z}_1 + \beta_2 \overline{z}_2 + - - - \beta_n \overline{z}_n = \overline{Y}$$

But $\overline{z}_1, \overline{z}_2$ - - - $\overline{z}_n$ are all zero.

Therefore, $\beta'_o = \overline{Y}$.

Thus the model indicated by Equation (viii) can be written as:

$$Y - \overline{Y} = \beta_1(Z_1 - \overline{Z}_1) + \beta_2 (Z_2 - \overline{Z}_2) + - - - + \varepsilon$$

Taking the simple example of a two variable and response case, with 'n' rows of data matrix, the model, after 'centering' can be presented as:

$$Y - \overline{Y} = \beta_1(Z_1 - \overline{Z}_1) + \beta_2(Z_2 - \overline{Z}_2) + \varepsilon$$

If we denote $\sum_{i=1}^{n} (Z_{ji} - \overline{Z}_j)(Z_{li} - \overline{Z}_l)$, j, l = 1, 2, as S_{jl}, the $\underline{X}'\underline{X}$ matrix could be seen to consist of $\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$.

These numbers could be of widely varying orders.

If we transform the 'centered' data by:

$$x_{ji} = \frac{Z_{ji} - \overline{Z}_j}{S_{jj}^{\frac{1}{2}}}, \; j = 1, 2 \text{ and}$$

$$Y_i = \frac{Y_i - \overline{Y}}{S_{yy}^{\frac{1}{2}}} \quad \text{where } S_{yy} = \sum_{i=1}^{n} (Y_i - \overline{Y})^2$$

and make similar transformations on the variables Z_1, Z_2 and Y by dropping the 'i' suffix throughout, this would yield a new form of the centered model as below:

$$Y.S_{yy}^{\frac{1}{2}} = \beta_1.S_{11}^{\frac{1}{2}} x_1 + \beta_2.S_{22}^{\frac{1}{2}} x_2 + \varepsilon$$

i.e. $$Y = \beta 1.\frac{S_{11}^{\frac{1}{2}}}{S_{yy}^{\frac{1}{2}}}.x_1 + \beta 2.\frac{S_{22}^{\frac{1}{2}}}{S_{yy}^{\frac{1}{2}}}.x_2 + \varepsilon$$

$$= \alpha_1.x_. + \alpha_2.x_2 + \varepsilon \text{ - (ix)}$$

where:

$$\alpha_1 = \beta_1.\frac{S_{11}^{\frac{1}{2}}}{S_{yy}^{\frac{1}{2}}} \text{ and}$$

$$\alpha_2 = \beta 2.\frac{S_{22}^{\frac{1}{2}}}{S_{yy}^{\frac{1}{2}}}$$

These new coefficients α_1 and α_2 are scaled forms of the original coefficients β_1 and β_2 respectively, the ratios $\left[\frac{S_{11}}{S_{yy}}\right]^{\frac{1}{2}}$ and $\left[\frac{S_{22}}{S_{yy}}\right]^{\frac{1}{2}}$ being the scaling factors.

As a consequence of the transformations effected, the $\underline{X}'\underline{X}$ matrix for the problem would now become:

$$\begin{bmatrix} 1 & \dfrac{S_{12}}{(S_{11}.S_{22})^{\frac{1}{2}}} \\ \dfrac{S_{21}}{(S_{11}.S_{22})^{\frac{1}{2}}} & 1 \end{bmatrix} \quad \text{(Also, } (S_{12} = S_{21})$$

Therefore, in the general case of more than two variables, the $\underline{X}'\underline{X}$ matrix would consist of a symmetric matrix with diagonal terms unity and the other terms as below:

$$\begin{bmatrix} 1 & \frac{S_{12}}{(S_{11}.S_{22})^{\frac{1}{2}}} & \frac{S_{13}}{(S_{11}.S_{33})^{\frac{1}{2}}} & \cdots & \frac{S_{1n}}{(S_{11}.S_{nn})^{\frac{1}{2}}} \\ \frac{S_{21}}{(S_{22}.S_{11})^{\frac{1}{2}}} & 1 & \frac{S_{23}}{(S_{22}.S_{33})^{\frac{1}{2}}} & \cdots & \frac{S_{2n}}{(S_{22}.S_{nn})^{\frac{1}{2}}} \\ \frac{S_{31}}{(S_{33}.S_{11})^{\frac{1}{2}}} & \frac{S_{32}}{(S_{33}.S_{22})^{\frac{1}{2}}} & 1 & \cdots & \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{S_{n1}}{(S_{nn}.S_{11})^{\frac{1}{2}}} & \cdots & \cdots & \cdots & 1 \end{bmatrix}$$

Where:

$$S_{jy} = S_{yj} = \sum_{i=1}^{n} (Z_{ji} - \bar{Z}_j)(Y_i - \bar{Y})$$

$$j = 1, 2 \cdots m$$

By definition of the coefficient of correlation between any two variables x and Y

$$\Upsilon_{xY} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(Y_i - \bar{Y})}{\left[\sum_{i=1}^{n} (x_i - \bar{x})^2 . \sum_{i=1}^{n} (Y_i - \bar{Y})^2\right]^{\frac{1}{2}}}$$

From this definition, it follows that each of the terms of the above $\underline{X}'\underline{X}$ matrix denotes a coefficient of correlation between a pair of variables and therefore the matrix can be re-written as:

$$\begin{bmatrix} 1 & r_{12} & r_{13} & \cdots & r_{1n} \\ r_{21} & 1 & r_{23} & \cdots & r_{2n} \\ r_{31} & r_{32} & 1 & \cdots & \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{n1} & \cdots & \cdots & \cdots & 1 \end{bmatrix}$$

Since $r_{jy} = r_{yj}$, this is a symmetric matrix and is termed the "correlation matrix".

The correlation matrix in the case of a two variable problem would be:

$$\begin{bmatrix} 1 & r_{12} \\ r_{21} & 1 \end{bmatrix}$$

The normal equations for the scaled model could now be expressed in matrix form as:

$$\begin{bmatrix} 1 & r_{12} \\ r_{21} & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} r_{1y} \\ r_{2y} \end{bmatrix}$$

Where a_1 and a_2 are the least square estimates of α_1 and α_2 in Equation (ix).

The least square solution of the transformed coefficients a_1 and a_2 are thus given by:

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} r_{1y} \\ r_{2y} \end{bmatrix} \begin{bmatrix} 1 & r_{12} \\ r_{21} & 1 \end{bmatrix}^{-1}$$

By inspection of the above matrices, it is evident that each of the terms of the right hand side product-matrix would be between - 1 and + 1. Thus, the order of all numbers being the same, rounding-off errors would be a minimum.

The solution of the coefficients b_o, b_1 and b_2, in the original form is obtained from a_1 and a_2 by applying the inverse relationships:

$$b_1 = a_1 \left[\frac{S_{yy}}{S_{11}}\right]^{\frac{1}{2}}$$

$$b_2 = a_2 \left[\frac{S_{yy}}{S_{22}}\right]^{\frac{1}{2}}$$

and $b_o = \overline{Y} - b_1 \overline{Z}_1 - b_2 \overline{Z}_2$

where:

Z_1, Z_2 and Y are some functions of the investigated variables.

In the presented analysis, the digital computer not only reconverted the 'a's to 'b's, but also reversed the scaling process and printed out the coefficients in terms of logarithms of the original variables.

The computer calculated the correlation matrix and initially chose the independent variable (say, x_k) which was most correlated with the response (Y) and calculated the coefficients b_o, b_k and also the coefficient of correlation between x_k and Y. Further, it regressed the residuals against each of the remaining concomitant variables and next selected the one with the greatest coefficient of partial correlation (say x_p) and included it in the regression model giving b_o, b_k, b_p and the coefficient of multiple correlation and certain other statistics. The process was repeated until all the variables had been similarly considered in terms of decreasing order of contribution to explain the variation between the observed and calculated values of the response. With the addition of each variable, the computer print-outs also included the variances and the T - Statistics for determining the significance of each variable and also the partial correlation coefficients of the variables yet to be included.

The programme also yielded a full-scale analysis of the residuals at each stage giving first and second differences between observed and calculated values of the response and certain other ratios of importance in discovering concealed trends, if any, in the residuals.

ooOOOoo

A P P E N D I X   II

DETAILS OF P.E.R.A. DYNAMOMETER.

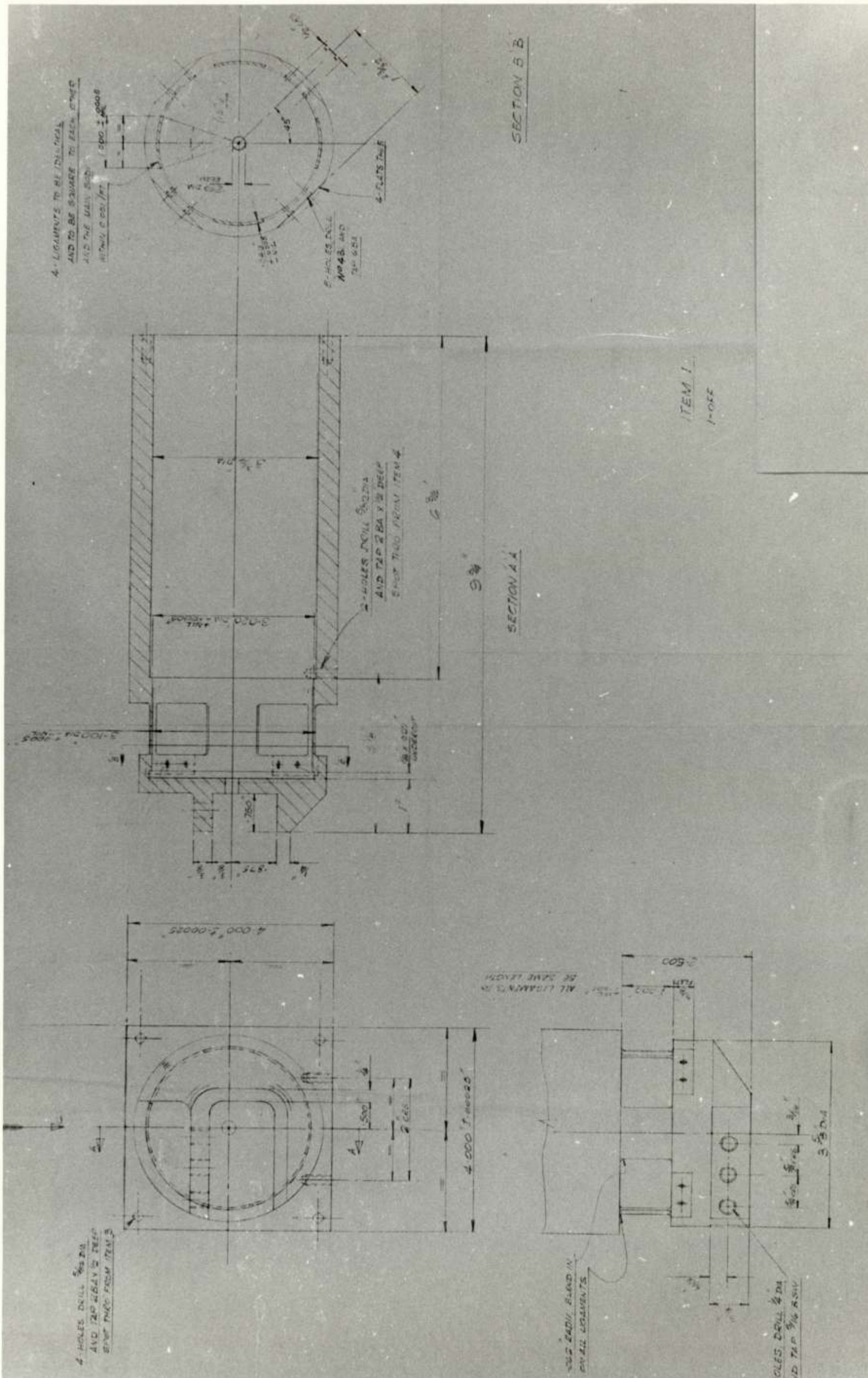


FIG. (i).

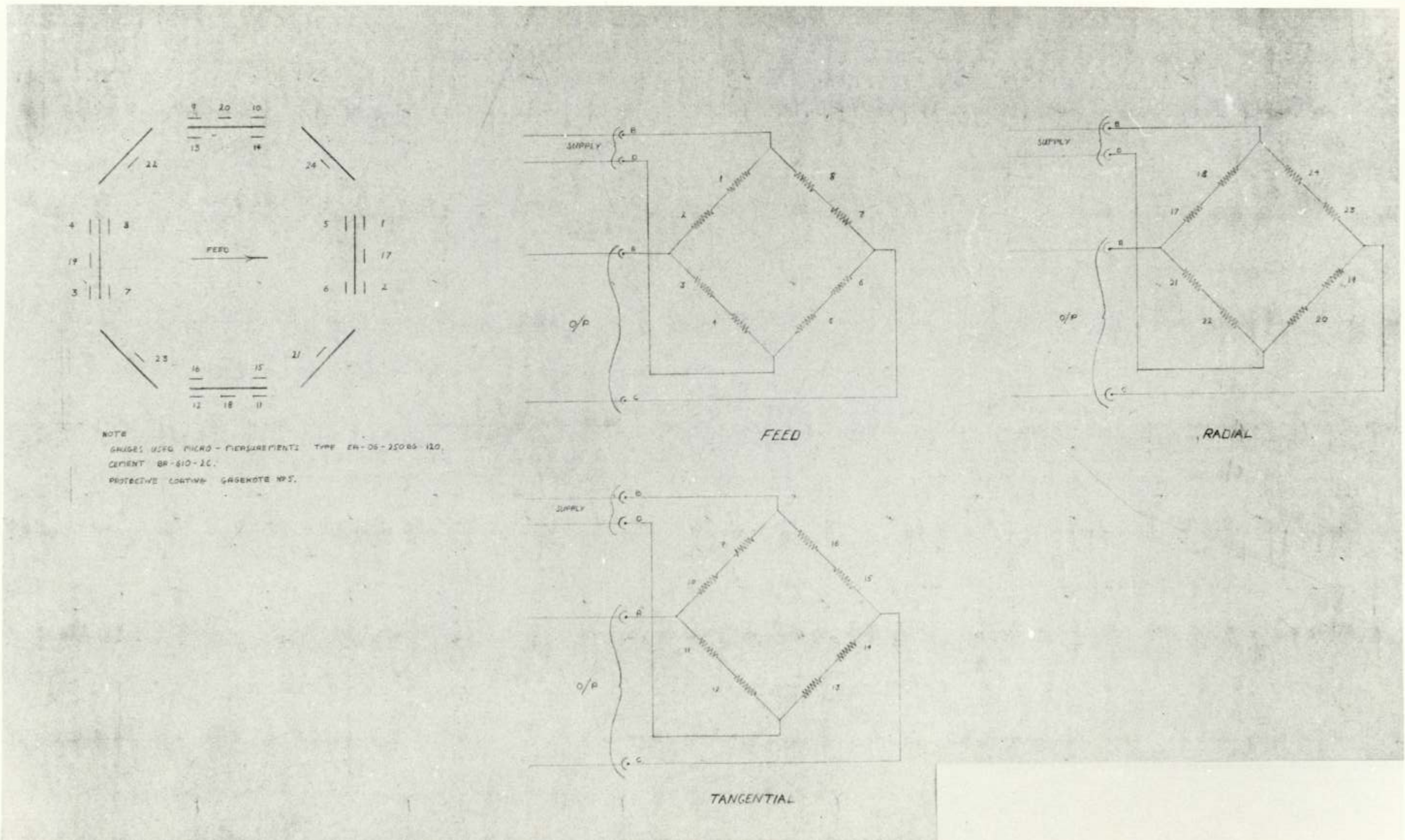
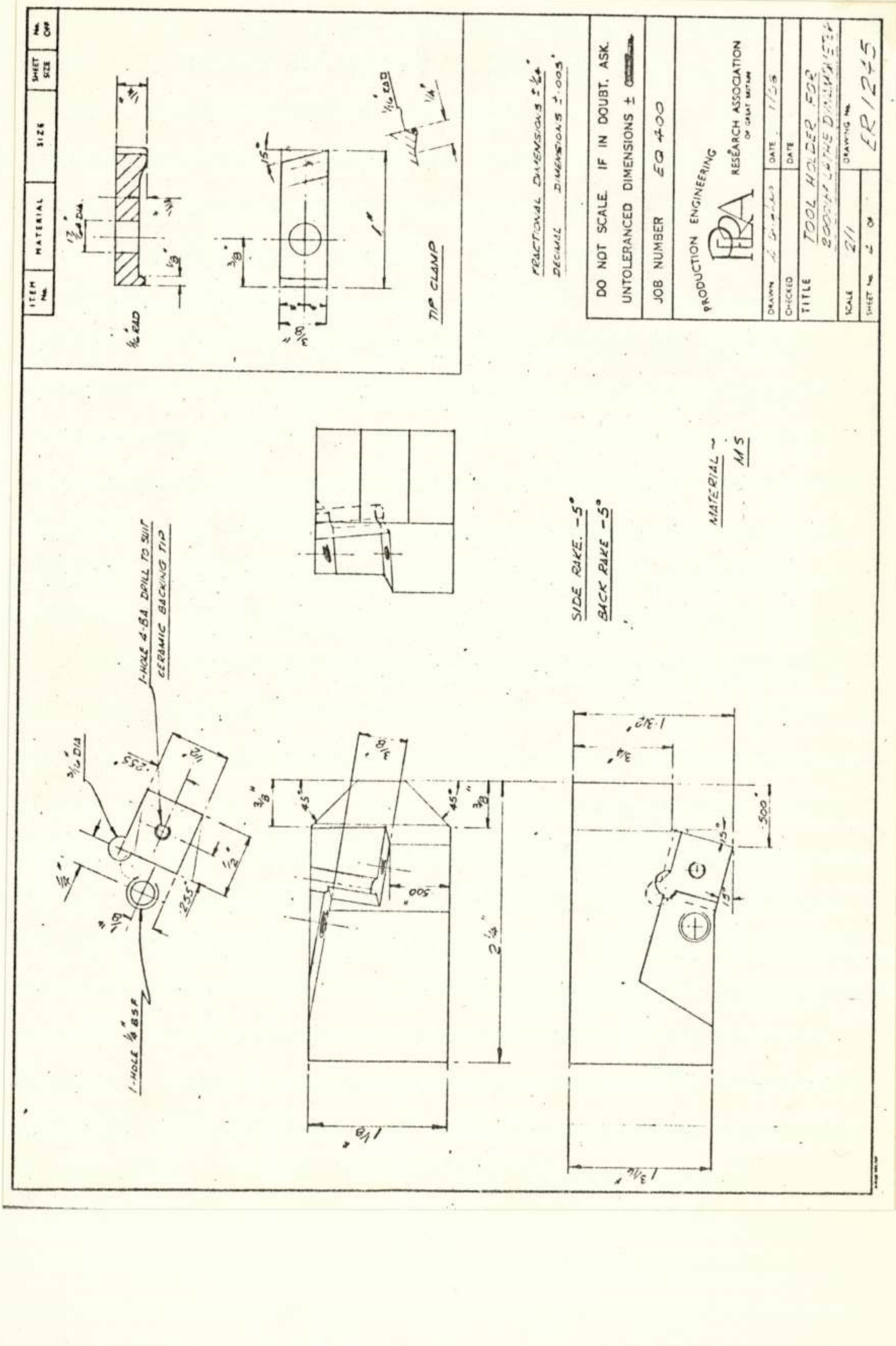

FEED
SUPPLY
O/P
FEED
RADIAL
TANGENTIAL
NOTE
GAUGES USED MICRO-MEASUREMENTS TYPE EA-06-250BG-120.
CEMENT BR-610-2C.
PROTECTIVE COATING GAGEKOTE No5.

FIG. (ii).

FIG. (iii).


1-HOLE 4-BA DRILL TO SUIT CERAMIC BACKING TIP
1-HOLE 1/4" BSF
TIP CLAMP
SIDE RAKE -5°
BACK RAKE -5°
MATERIAL ~ M S
FRACTIONAL DIMENSIONS ±1/64"
DECIMAL DIMENSIONS ±.005"
DO NOT SCALE. IF IN DOUBT, ASK.
UNTOLERANCED DIMENSIONS ±
JOB NUMBER EQ 400
PRODUCTION ENGINEERING RESEARCH ASSOCIATION OF GREAT BRITAIN
TITLE TOOL HOLDER FOR

A P P E N D I X III

COMPUTER PROGRAMME FOR CALCULATING CONFIDENCE LIMITS FOR PROVING TEST SERIES.

```
STATEMENT
  0         'LIBRARY' (ED. SUB GROUP MATX)
  0         'TRACE' 2
  0         'BEGIN'
  1        'INTEGER' 1.M1.N1;
  1        'INTEGER'   NUMBER ;
  2         START:
  3         SELECT OUTPUT (0) :
  4         SELECT INPUT (0) :
  5         NUMBER: = READ:
  6              N1 = READ:
  7              M1: = READ:
  8              'BEGIN'
  8              'ARRAY' XO(1:N1.1:M1).XT(1:M1.1:N1).XP(1:M1.1:M1):
  8              'PROCEDURE' MX READ(A.ML.MU.NL.NU):
 10              'VALUE' ML.MU.NL.NU:
 11              'INTEGER' ML.MU.NL.NU: 'ARRAY' A: 'ALGOL':
 13              'PROCEDURE' MXPRINT (A.ML.MU.NL.NU.P.Q):
 15              'VALUE' ML.MU.NL.NU.P.Q:
 16              'INTEGER' ML.MU.NL.NU.P.Q: 'ARRAY' A: 'ALGOL'
 18              'PROCEDURE' MXINV1 (A.NL.NU):
 20              'VALUE' NL.NU:
 21              'INTEGER' NL.NU:  'ARRAY' A: 'ALGOL':
 23              'PROCEDURE' MXTRAN(A.B.ML.MU.NL.NU):
 25              'VALUE' ML.MU.NL.NU:
 26              'INTEGER' ML.MU.NL.NU: 'ARRAY' A.B: 'ALGOL'.
 28              'PROCEDURE' MXPROD (A.B.C.ML.MU.NL.NU.LL.LU):
 30              'VALUE' ML.MU.NL.NU.LL.LU:
 31              'INTEGER' ML.MU.NL.NU.LL.LU: 'ARRAY' A.B.C.: 'ALGOL'.
 33             MXREAD(XO.1.N1.1.M1):
 35             MXTRAN(XT.XO.1.N1.1.M1):
 36             MXPROD(XP.XT.XO.1.M1.1.N1.1.M1):
 37             MXINV1(XP.1.M1):
 38        WRITETEXT('(''('C')'(X'X)-1')'):
 39        NEW LINE (1):
 40             MXPRINT(XP.1.ML.1.M1.5.5):
 41             'BEGIN'
 41                  'INTEGER' N.M:
 41             N: = READ:
```

```
                M: = READ:
            'FOR' 1: = 1 ' STEP' 1 ' UNTIL' NUMBER 'DO'
                'BEGIN'
                    'ARRAY' X(1:N.1:M).XTT(1:M.1:N).XW(1:M.1:N)
                       XA(1:M.1:M)
                    MXREAD(X.L.N.L.M):
                WRITETEXT('(''('C')'MATRIX X'('2C)'')'):
                 MXPRINT(X.1.N.1.M.5.5):
            'IF' N'NE'M1 'THEN' 'GOTO' ERROR:
                MXTRAN (XTT.X.1.N.1.M):
                MXPROD(XW.XTT.XP.1.M.1.N.1.N):
                MXPROD(XA.XW.X.1.M.1.N.1.M):
                MXPRINT(XA.1.M.1.M.5.5):
            'GOTO' FIN:
      ERROR:WRITETEXT('(''('C')'MATRICES %WRONGLY%DIMENSIONED')'):
            FIN:NEWLINE(5):
         'END' OF 1 LOOP:
            'END':
        'END' OF INNER BLOCK:
```