The INFLUENCE of COMPOSITION, TEMPERATURE and STRAIN RATE on the DEFORMATION of F.C.C. METALS and ALLOYS

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by

JOHN FRANCIS HILL, M.Sc., A.C.T.(Birm.), A.I.M.

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SYNOPSIS

Various methods of assessing deformation behaviour are considered and compared, and torsion is selected as a convenient means of obtaining data to high strains, over a range of temperatures and strain rates. Parameters which describe the most relevant features of torsion test results are suggested.

The way in which the descriptive parameters are affected by changes in composition, grain size and conditions of testing are investigated by multiple regression analysis for a number of pure metals and single phase f.c.c. alloys. A series of equations are produced which are applicable over the whole range of compositions considered. Based on the equations derived it is suggested the most important factors relating to composition are shear modulus, Burger's vector and stacking fault energy. Structure may be described by the reciprocal square root of grain size, and of the process variables temperature is shown to be much more influential than strain rate within the range of values used.

Comparison of the regression equations shows that the process termed restoration, by which the strains induced by work-hardening are relieved, is predominantly recovery by dislocation climb rather than recrystallisation involving grain boundary migration.

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1. INTRODUCTION

By far the greater proportion of all the metal which is processed and marketed by the world's industries is subjected to some form of mechanical deformation, either during its preparation for, or when put into service. With the inevitable evolution of modern industry, and the increasing competition for market space amongst the rapidly extending range of new materials, metallic and non-metallic, attention is being focussed more and more sharply on the means of carrying out mechanical deformation. In particular the use of computer control, already becoming established in the production of steel semi-finished products[1,2], will demand a newer and more quantitative appreciation of the behaviour of metals during deformation. The automatic control of industrial scale mechanical working processes is based upon predicting the response of the metal being worked under a given set of conditions, or alternatively, the desired response might be predetermined and the optimum conditions for achieving the response are then sought. In those cases where the range of materials being processed is small or the choice between different methods of processing is limited the information necessary for effective control may be determined empirically. Circumstances frequently arise, however, when the empirical determination of performance, by laboratory tests for example, is restricted by time or because the material being considered has not been produced in sufficient quantities or in suitable form, or, indeed, for many other reasons. Then the prediction of the response of a given metal or alloy to a process or range of processes, under different conditions of temperature and strain rate is a most difficult task, and might well be possible only in a qualitative sense.

The problem of prediction can be resolved into three components[3,4], each of which is basically independent. It is necessary to consider the influence of:

1. the material to be processed, in terms of its composition and structure,
2. the process variables, which can usually be reduced to two, namely temperature and strain rate,

and 3. the process itself, which is a means of applying the stresses, appropriate in magnitude and direction, to bring about the desired change of shape in the work material.

Analyses have been made of a very wide range of mechanical working processes[5] and have been shown to provide an effective basis for process control[6,7]. It is not intended to pursue this aspect further, therefore, although newer, improved mathematical models of mechanical working processes are not only possible but are, in fact, being developed[3,8].

In this project it is intended to provide a study of 1. - the material, and 2. - the process variables, and their effect on the overall system. Since virtually all descriptions of mechanical working processes rely upon some form of stress-strain curve, the project has been based upon an investigation of this relationship. A means of describing the relationship has been sought, and the effects of different chemical compositions and structures have been examined over a range of temperatures and strain rates. By the use of regression analysis an attempt has been made to provide a model to facilitate the prediction of the parameters of the stress-strain relationship. It is suggested that such a model, used in conjunction with an appropriate model of a mechanical working process could make possible preliminary investigations (for example, by computer simulation) of the effects of modifications in process practice or the introduction of new material compositions, and would identify those parameters which are of greatest importance in establishing automatic control procedures.

It seems reasonable to assume that the parameters of the model and the factors influencing the form of the stress-strain curve will depend upon the mechanisms of deformation involved, and this in turn depends, amongst other factors, upon the crystal structure. In order to avoid the complication of partitioning deformation between different crystal structures within one aggregate, or of attempting to devise a model which would cope with a number of different mechanisms of deformation, the investigation has been confined to single phase materials of one crystal group, viz. face centred cubic. This group was selected because of its commercial importance, including as it does, not only copper, nickel and aluminium and many of their alloys, but also many types of steel, particularly at elevated temperatures. Additionally, the considerable amount of research which has been carried out on this group[9] provides a comprehensive summary of the variables which are most likely to provide an effective basis for the model.

1.2. The Strategy of the Investigation

In the following sections the objectives outlined above are approached systematically.

1.2.1. As a first step alternative methods of predicting performace are considered in section 2.1. and reasons presented for selecting techniques based on the stress/strain relationship. In subsequent sections (2.2. and 2.3.) the mechanisms giving rise to the particular shape of stress/strain curve are described, and methods available for the experimental measurement of the relationship, and the determination of the appropriate parameters, are considered.

Consideration is then given to the factors likely to exert an influence on the selected parameters in section 2.4. Of those relating to the material the largest group comprises those factors which are associated with chemical composition. The effects of structure, having been minimised by restricting the investigation to single phase materials of one crystal type are reduced to grain-size and subgrain-size dependence.

Changes in process conditions are confined to geometry, temperature and strain rate. The first of these has an influence only on the magnitude and direction of stresses and so is related to the specific process being used and falls outside the terms of this investigation. Temperature and strain rate are usually variable within any process and the influence of each of them is considered in sections 2.4.3. and 2.4.4.

In the final section of the review the reasons for using regression analysis are given and the techniques available with their relative advantages and disadvantages are considered.

The techniques adopted in the selection and preparation of the materials being investigated, together with the experimental programme and methods of testing and analysis are presented in section 3.

Following the results, section 4, the discussion is contained in section 5. This takes the form of

(i) an assessment of the model produced by the investigation in terms of its reliability and its usefulness as a basis for the prediction of the performance of single phase f.c.c. metals in mechanical working processes,

(ii) the insight gained into the processes of work hardening and restoration from consideration of the variables found to make a significant contribution to the proposed model, is then discussed, with particular regard to the role of stacking fault energy.

The conclusions are summarised in section 6.

2. LITERATURE SURVEY AND REVIEW

2.1. Conventional Methods for Prediction

The measures currently available for predicting the performance of metals during mechanical working are conveniently divided into
(i) mechanical testing and
(ii) other techniques.
Of these division (i) may be sub-divided[10] into
(a) Simulative or scaled down working tests and
(b) Laboratory tests.

2.1.1. Simulative Tests

Scaled down working tests have the advantage of more or less exactly reproducing the stress system of the appropriate full scale process. Apart from minor disadvantages such as the uncertainty of thermal conditions, particularly with regard to the influence of strain rate, the use of scaled down tests is limited mainly by cost and convenience. The test equipment involved in scaled down simulation of rolling or tube making, for example, is difficult to obtain and expensive both to install and operate. In addition test-pieces are likely to be relatively large and expensive to prepare. The usefulness of the information derived is limited to a specific process, and in order to assess performance under a number of different working conditions it is necessary to carry out a series of tests. In order to assess likely performance in a number of different processes it is necessary to carry out a series of tests for each one.

2.1.2. Laboratory Tests

In contrast the use of laboratory tests is not specific to any one working process and is usually much more convenient and inexpensive. The information obtained, however, is less likely to be of direct use in the way that the results of simulative tests are. Reviews of laboratory testing techniques[10,11] indicate that useful correlations may sometimes be obtainable between performance in specific tests and processes, but that the discrimination offered by this approach is rarely as critical as is desired. It is considered that the main function of tests carried out for this purpose is to check the consistency of successive batches of material. Success or failure in the laboratory test by no means guarantees a similar performance in the working process.

The main use of laboratory tests, and the purpose to which they are best suited, is to determine the relationship between stress and strain for any selected material. This may then form the basis of mathematical predictions of performance, since all mechanical working operations are concerned with inducing strain in the work-material. A further advantage of this approach is that the information may be more widely applied, e.g. to other forms of mechanical deformation such as high temperature creep[12], or to the calculation of residual stresses[13].

2.1.3. Other Techniques

The use of techniques other than mechanical testing to provide information is principally to supplement results already available. The information provided is frequently qualitative or semi-quantitative. Micro-examination in order to determine the amount of non-metallic or inter-metallic inclusions, for example, may divide batches of metals into 'clean' or 'dirty' but will not provide any information on which quantitative predictions of performance may be based.

The influence of grain size on strength has also been investigated(refer to section 2.4.2.). It has been found that the relationship between flow stress and grain size may be expressed by an equation of the form:

$$\sigma = \sigma_o + K D^{-\frac{1}{2}} \qquad \text{(2.1)}$$

No attempt has been made to express this relationship in a form which is both quantitative and general to a wide range of metals and alloys. Similarly the effect of increasing the proportion of solute in a particular system has been investigated on many occasions, but no generalised form of relationship has been found between the amount of alloying additions and flow stress.

It seems, therefore, that although such factors as chemical composition and microscopic structure are readily determinable and are known to influence behaviour during mechanical working there is no simple or convenient method of anticipating their influence upon a particular process. Since there are no generalised quantitative data on the influence of these variables it is not easily possible to determine the relative importance of each or any of them with regard to mechanical working.

In summary it emerges that the behaviour of a metal or alloy when subjected to mechanical working is a function of (a) the material and (b) the process.

The most important deficiences in existing knowledge would appear to be:

(i) what characteristics of the material are most important in determining its behaviour.

(ii) in what manner these characteristics affect the behaviour.

(iii) how the material and process interact.

2.2. The Stress/Strain Relationship

2.2.1. Single crystals

During the plastic deformation of single crystals of f.c.c. metals three discreet stages occur, distinguishable on the stress-strain diagram as shown in figure 2.1.

Stage I, the region of easy glide[14] is characterised by a low rate of work hardening and it is generally considered that dislocations produced by the deformation leave the crystal at the surface.

Stage II, the region of linear or rapid hardening shows a higher work hardening rate, the slope of which is observed[15] to be approximately independent of applied stress, temperature, crystallographic orientation or impurity content.
The ratio:

$$\frac{d\sigma/d\epsilon}{\mu}$$

where μ is the shear modulus, is of the same order of magnitude for all f.c.c. metals, viz. 5×10^{-3}. Smallman suggests that the characteristic feature of stage II deformation is that slip occurs on both the primary and secondary slip systems, giving rise to lattice imperfections such as forest dislocations, Lomer-Cottrell barriers and jogs at the points where dislocations intersect.

During stage III, the region of dynamic recovery, the stress-strain relationship is approximately parabolic with a decreasing rate of work hardening, until some limiting stress is reached at which some mechanism such as fracture or 'restoration' interferes. Within this stage it appears that dislocations held up in stage II are able to move by some process which had previously been suppressed. The mechanism involved appears to be that of

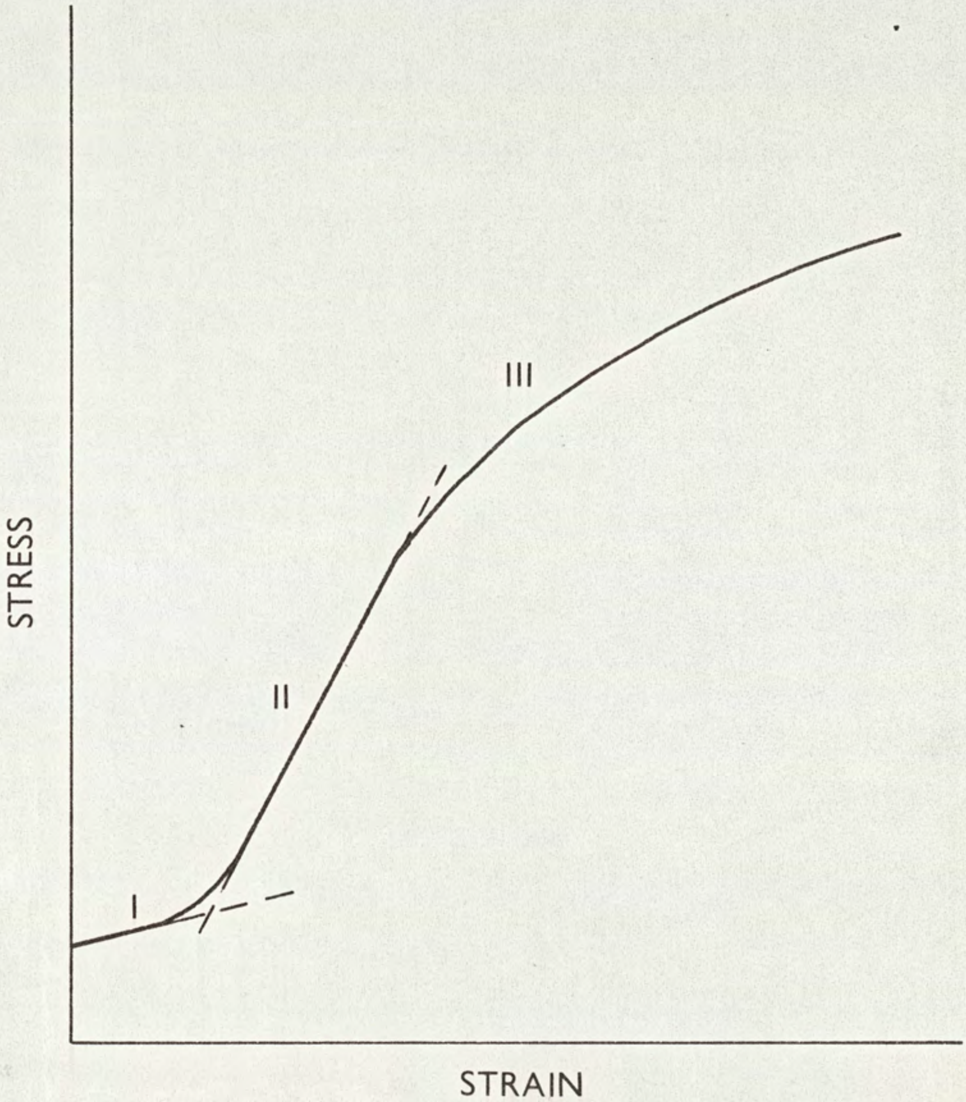


Figure 2.1. Showing three stages of single crystal stress/strain curve.

cross-slip[15] by means of which a screw dislocation glides into another slip plane having a slip direction in common with the original slip plane.

2.2.2. Deformation Twinning

Although deformation in f.c.c. metals and alloys is principally by slip, involving atomic movements which are (approximately) increments of whole lattice vectors, it is possible for deformation to occur by twinning[15]. In this case the atomic movements are much less than those involved in slip, although the atoms in each plane are moved by an amount equal to that of the atoms in adjacent planes.

The contribution to the overall deformation, which is made by twinning is usually very small and in the case of aluminium it appears[16,17] that twinning does not occur even under the most favourable conditions.

In the metals which are the subject of this investigation twinning only occurs at high stresses[18] and consequently it is not normally a deformation mode at room temperature or above. The onset of twinning is readily determined in most metals because the test load drops suddenly when the twin stress is reached[18 19], as shown in figure 2.2. Load drops are not observed in all cases, however, and in copper alloys containing more than 20% zinc or 8 atomic % aluminium none has been observed under any conditions of testing, although Venables[18] has demonstrated the presence of deformation twins by selected area diffraction on the electron microscope. It is suggested[18] that the lack of load drops indicate that twins, once nucleated are unable to propagate through the the crystal.

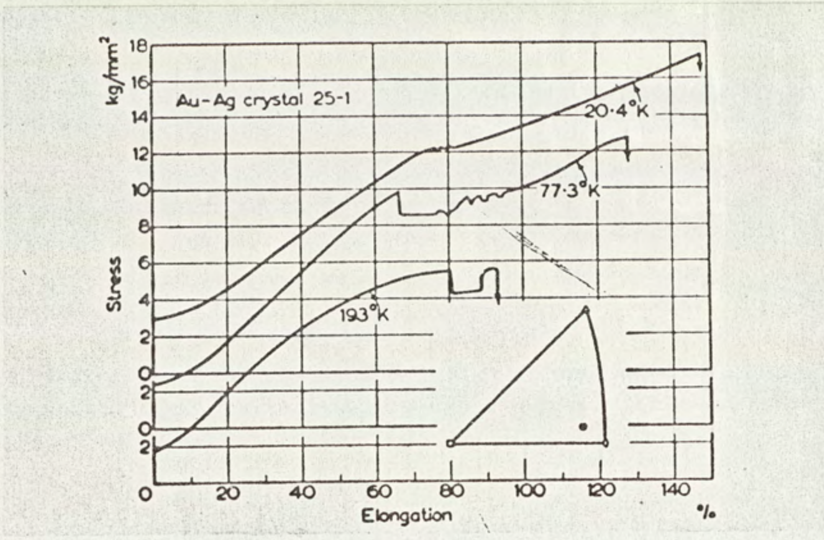


Figure 2.2. Load drops due to deformation twinning - ref. 18.

One further consequence of twinning which is possible is the brittle, cleavage type of fracture[15]. It appears that a twin, like a grain boundary, may present a strong barrier to slip and a crack can be initiated by the pile-up of slip dislocations at the twin interface.

2.2.3. Polycrystalline Aggregates

In that the mechanisms by which plastic deformation can occur within a crystal remain the same, the deformation of individual crystals within a polycrystalline aggregate is subject to the same laws as the deformation of an isolated single crystal. However, the requirement that physical continuity must be maintained between adjacent crystals or grains imposes further restrictions which increase the resistance to deformation of the aggregate as a whole. Since the presence of a grain boundary automatically implies a difference in crystallographic orientation it follows that any grain which is ideally orientated for deformation to occur, relative to the direction of applied stress, must be bounded by other grains which are less favourably orientated, and therefore inhibit the deformation process.

Taylor[20] related the stress-strain curves for f.c.c. single crystals and polycrystalline specimens by mathematical synthesis. The synthesis involved the division of a crystallographic unit triangle into forty-four equal areas and from these the slip systems involving the lowest values of work done were selected. Although certain assumptions made by Taylor were not justified, for example it was assumed that strain throughout the aggregate was homogeneous and this is almost never attained, experimental evidence[21,22] has confirmed the accuracy of the work. It would seem desirable that any mathematical expression relating stress and strain for a given metal should recognise the mechanisms by which deformation is occurring. This is likely to improve the possibility of discovering the influence of metallurgical variables on the form of the expression.

2.2.4. Mathematical Models

The best known and most widely used equation relating stress and strain is that due to Ludwik[23]:

$$\sigma = \sigma_c - K\epsilon^n \qquad \text{\_\_\_\_\_\_\_\_\_\_} \quad (2.2.)$$

although the first term on the right hand side is usually dropped and the equation expressed as:

$$\sigma = K\epsilon^n \qquad \text{\_\_\_\_\_\_\_\_\_\_} \quad (2.3.)$$

where K and n are constants.

Support for the use of this equation has come from a number of workers in this field including Holloman[24], Feltham[25], Nadai[26] and Hodierne[27]. This support is based upon the fact that when the logarithm of the true stress is plotted against the logarithm of true strain the values lie close to a straight line, i.e.

$$\log\sigma = n\log\epsilon + \log K \qquad \text{\_\_\_\_\_\_\_\_\_\_} \quad (2.4.)$$

where K is the stress at unit strain,

and this relationship is equivalent to (2.3.) above. Although the equation is adequate for many purposes it has certain distinct disadvantages. Part of the general applicability of the expression is derived from the wide range of shapes which curves satisfying the equation can take. Where n = 1 the σ/ϵ relationship is linear, while for very small values of n the curve virtually conforms to a right angle bend. In order to fit the expression to experimentally derived data, therefore, both of the constants in equation (2.3.) may vary and it is most difficult to relate the variations in each to variations in specific material properties or test conditions.

The use of a single term to describe the interdependance of stress and strain implies a single mechanism throughout and this is clearly inappropriate. The result might sometimes be a disproportionate contribution by the elastic portion of the relationship to the plastic curve.

One further, minor disadvantage lies in the fact that in the 'popularised' version of Ludwik's equation it is implied mathematically, that the material may be strained to an indefinite extent, giving rise to an infinite value of stress[28].

The empiricial nature of the relationship led Voce[28] to propose a new relationship based on the log/log plot of stress versus strain. It was noted that distributions of experimental points could be more accurately described by a series of three straight lines rather than one. (Holloman[29] had previously published similar results but had not paid any great attention to this facet of the work.) The smoothed version of this series, described by Voce[30] as 'an italicised integration sign' was expressed by an equation of the form:

$$\sigma = \sigma_\infty - (\sigma_\infty - \sigma_o) \exp - (\epsilon / \epsilon_c) \quad \text{\_\_\_\_\_\_\_\_} \quad (2.5.)$$

where σ_o is the threshold stress for plastic deformation

σ_∞ is the maximum stress

and ϵ_c is the 'characteristic' strain.

The improvement offered by this equation over that of (2.3.) clearly goes some way to satisfying the objections to Ludwik's expression. Equation (2.5.) suffers from the disadvantage that may be difficult to determine, particularly if the material under test has restricted ductility. Furthermore the division into three straight lines is frequently as arbitrary as Ludwik's method of using only one, since the relationship, even on a double logarithmic plot may include at least one curved region. An alternative approach is the attempt to improve upon Ludwik's work was adopted by Bell[22]. Analysing more than three hundred stress-strain curves of f.c.c. metals produced by other workers he chose to fix the value of n in the power equation at 0.5, giving the expression:

$$\sigma = K\epsilon^{\frac{1}{2}} \qquad (2.6.)$$

While this device still fails to remove many of the objections to equation (2.3.) it does reduce the constants which may reflect variations in materials or conditions to one, and this is a most useful improvement.

An examination of Bell's results reveals that the calculated values based on equation (2.6.) most adequately fit the experimental values at higher values of strain. In view of the remarks previously made regarding the contribution due to the elastic portion of the curve this is rather to be expected.

2.2.5. Stress/Strain Maxima

In all of the mathematical models considered it was implicitly assumed that the maximum stress would be attained at maximum strain, i.e. at fracture. This is by no means always the case[9,31], however. At elevated temperatures, i.e. those in excess of approximately 0.5 Tm, the stress/strain curve usually shows a maximum stress at some strain before the maximum. It is generally considered that the stage at which maximum stress occurs is followed by a process which Hardwick[32] described as 'restoration'. The term 'restoration' was used to avoid describing the process as either recovery or recrystallisation, since it was not certain to what extent each of these mechanisms of softening was involved. Hardwick presented metallographic evidence which, he suggested, indicated that materials of high stacking fault energy, e.g. Al, tend to soften by a process of recovery, i.e. dislocation cross-slip and climb, while the low stacking fault energy metals, e.g. Cu, tend to soften by recrystallisation, a diffusion-controlled process. Metals of intermediate stacking fault energy e.g. Ni, soften by a combination of these two mechanisms, he suggested. However, more recent estimates of the stacking fault energy of Ni[33] indicate that it is in fact higher than that for Al, and this must invalidate some of Hardwick's suggestions.

From measurements made at Sheffield[34] on the time required for a highly strained metal specimen to begin recrystallisation it appears that metallographic evidence for recovery or recrystallisation must be very carefully interpreted, since important structural changes can occur between the cessation of deformation and the lowering of the specimen temperature to ambient or thereabouts.

Stuwe[35] has suggested that except at low strain-rates recovery mechanisms are adequate to describe the softening observed. He postulated[36] that the vast increase in the number of point defects produced during deformation assists the movement of edge dislocations away from their slip planes and enables a steady-state condition of constant dislocation density to be set up.

Sellars and Tegart[38], supporting the view that recrystallisation can occur during deformation point out the cyclical form of stress-strain curve sometimes observed at low strain rates (Rossard and Blain observed a similar form of curve) and suggest that the cycle is similar to that occurring in creep[37]. The form would be explained by a repeated cycle of work-hardening and recrystallisation.

Whatever the form of restoration occurring it seems clear that the stress maximum must be regarded as a limiting value for the equations previously considered. Where the strain to failure is less than the projected strain to restoration the stress at failure will then limit the relationship.

2.2.6. Deformation at Constant Stress

A feature of deformation carried out at high temperature is the protracted region of strain at virtually constant stress[9,31]. There is general agreement[9] that the processes of work hardening and restoration achieve an equilibrium condition during this stage. In many cases the steady state stress is approximately equal to the maximum stress, and this is logically to be expected from the explanations in the foregoing section.

Generally, however, the steady state stress is less than the maximum stress and the difference appears to be due to some form of activation energy which is required to initiate the process of restoration.

Where the stress/strain equation is to be applied to processes involving high values of strain, therefore, the value of steady state stress is clearly an important part of the expression.

2.3. Measurement of Stress/Strain Relationships

There are a number of methods available for determining stress/strain relationships[11], or those parameters of the relationships required, in metals. The principal advantages and disadvantages of the best established methods are presented below:

2.3.1. The Tension Test

This is almost certainly the most widely used and best known mechanical test for metals and it is not considered necessary to describe the test here.

The chief advantages of the test are its relative simplicity and the ready availability of suitable test equipment.

The major disadvantages stem from the relatively low strain rates which are commonly used, and from development of plastic instability at quite an early stage in the test. Although corrections may be made to the measured stress[39,40] to allow for the triaxial stresses arising in the necked region of the specimen, these require measurements to be made of the changing geometry of the specimen and this is clearly not compatible with high rates of strain. Additionally, the volume of metal in the neck is likely to become very small with the consequent likelihood of increased experimental errors.

2.3.2. The Compression Test

A range of tests is available under this heading, from simple upsetting, to those due to Polakowski[41] or Alder and Phillips[42] were intended to overcome the complications in the stress system, and the increased tendency to failure caused by barrelling of the specimen. The use of indented specimens as suggested by the former still leads to a more complex stress system, however, while his technique of remachining the specimen to remove the barrel, at intervals, does not lend itself particularly well to either high temperatures or high strain rates.

The cam plastometer used by Alder and Phillips overcame some of the difficulties due to the changing specimen geometry during the test, but did not prevent barrelling.

Plane strain compression[43] is not susceptible to barrelling in the manner that compression of a plane cylindrical specimen is, but the large surface to volume ratio of the specimen causes the frictional restrains to exercise a disproportionate influence on the test. It is of interest to note that Bailey and Singer[31] used a cam plastometer in conjunction with the plane strain compression test, and achieved reductions in thickness up to 90%. No reference is made in their work to the effect of the widely differing width/thickness ratios achieved by this method, although Watts and Ford[44] suggest that this has an important influence on the results.

2.3.3. <u>The Bend Test</u>

This form of test is extremely limite in its usefulness due to the geometric limitation of strain imposed by being unable to bend the specimen continuously more than 180°. Edge effects may be reduced by the use of a suitably large width/thickness ratio, but the stress system is a complex and variable combination of compression and tension.

The use of this technique is limited in practice to brittle materials[11].

2.3.4. <u>The Torsion Test</u>

In the torsion test a specimen, similar in form to that used in the tension test, is strained by being twisted about its longitudinal axis. The stresses developed are virtually pure shear.

Much interest has recently been shown in the torsion test[45] mainly because the test-piece, provided that its length is constrained, retains its original shape throughout the test. This characteristic obviates the need for additional measurements during the test and so allows high strain rates to be used, the constant geometry of the test-piece also gives a constant strain rate during each test, without the need for special techniques such as are used in the cam plastometer, for example.

In the absence of such phenomena as barrelling or necking the entire gauge length is deformed more or less uniformly. Where local variations in deformation occur it is on a micro-scale and is unlikely to exert any influence on the stress-strain relationship[45]. The volume of metal deformed is, therefore, not restricted as in other tests and this improves the experimental accuracy by minimising the influence of inclusions or other defects.

One disadvantage encountered in torsion testing is due to the influence of a stress developed in the axial direction if the specimen is constrained (if the constraint is removed the specimen length and shape tend to change).

The causes of this axial effect are not understood at all clearly. Earlier explanations based on analogy with the shortening of a towel when twisted, or on thermal expansion or contraction have been thoroughly investigated[46] and discarded. Dragan[46] suggested that competing mechanisms of intra-crystalline slip and grain-boundary slip might be involved, but offered no experimental evidence in support of his suggestion. More recent investigations[48] appear to indicate an association between the axial stress and crystallographic preferred orientation, but this work is incomplete.

Although there is evidence that axial stresses developed during torsion testing influence the strain which the material is capable of sustaining without failure, it seems unlikely they have any significant effect upon the shear stress/shear strain curve[47].

Since the torsion test was selected for the work presented here a more detailed analysis is presented in the section on Experimental Method.

2.3.5. Equivalence of Test Results

The stress-strain curves of various metals determined by different testing methods have been compared by several workers. Holloman[24], Hodierne[47], Bailey and Singer[31], Jonas et al[9] and Watts and Ford[44] have all shown that data obtained variously from plane strain compression, axisymmetric compression, tension and torsion produce stress-strain curves that are equivalent within reasonable limits of confidence, when established methods of conversion are employed.

The most widely accepted method[47] of conversion between tensile and shear systems are those based upon the von Mises yield criterion and the concept of ideal work, due to Hill[49]:

$$\sigma = \sqrt{3}\,\tau \qquad \text{\_\_\_\_} \quad (2.7.)$$

$$\epsilon = \gamma / \sqrt{3} \qquad \text{\_\_\_\_} \quad (2.8.)$$

$$\dot{\epsilon} = \dot{\gamma} / \sqrt{3} \qquad \text{\_\_\_\_} \quad (2.9.)$$

2.4. Variables Influencing the Stress/Strain Relationship

The selection of variables to be considered for inclusion in the model of the stress/strain relationship can most conveniently be based on the various theoretical equations and proposed mechanisms of deformation developed from previous research programmes.

2.4.1. The material

Even between single-phase materials belonging to the same crystal-structure group differences in the stress/strain relationship arise due to different chemical compositions and microstructures. It is desirable that the inclusion of variables relating to and defining these properties should be based upon functional relationships between themselves and the dependent variable.

(a) Atomic proportion

The simplest description of chemical composition is one based on the atomic percentage of solute present. This is extremely easy to determine, but it is a naive method, the chief weakness of which is that it fails to distinguish between pure metals, representing aluminium, copper and nickel, for example, in exactly the same way.

(b) Shear strength (Gb)

Early studies of deformation in metals, due to Taylor[20], suggested both the parabolic relationship between stress and strain, and the influence of compositional variables on the form of the curve. The equation was:

$$\tau = a\, G(b\gamma/l\,)^{0.5} \qquad \text{———} \quad (2.10.)$$

in which G = modulus of shear

b = Burger's vector

l = mean free path between obstacles

and a = a constant

The importance of the modulus of shear and Burger's vector has been confirmed on the basis the stress required for the intersection of dislocations[52], and a dimensionality argument due to Nabarro et. al.[53], while Kovac [51] developed an argument based on the production of lattice defects which confirmed Nabarro's findings for stage II deformation, viz.:

$$\tau = a'\ G\ b\ N^{\frac{1}{2}} \qquad \text{———} \quad (2.11)$$

where N is dislocation density

and $^1/_7 < a' < \frac{1}{2}$,

but suggested a rather different relationship in stage III of the form:

$$\tau = \beta\ G\ b^{2/3}\ N^{\frac{1}{3}} \qquad \text{———} \quad (2.12.)$$

although contemporary studies by Brydges[54] were in agreement with equation (2.11.).

(c) Stacking fault energy

The critical stress at which stage III deformation occurs is difficult to identify in polycrystalline materials and most of the work relating to this subject has been carried out on single crystals. Electron microscope studies on f.c.c. metals led Diehl et. al.[55] to conclude that the transition to stage III-type deformation was the result of thermally activated cross-slip, a view which is now generally accepted[15]. Several models of the cross-slip process have been proposed[56,57,58] but in all cases it is a pre-requisite that partial or extended dislocations of the type described by Heidenreich and Shockley[59] should first combine to form undissociated dislocations. This necessitates overcoming the repulsive force-F of the two partial dislocations, which Cottrell[60] calculated as:

$$F \sim G\,a^2/_{24\pi r} - \gamma_{SF} \qquad \text{——— (2.13.)}$$

where γ_{SF} is the stacking fault energy of the faulted region, and 'a' is the lattic ~~param~~eter.

More recently Copley and Kear[61] claimed that the ~~interaction~~ of Shockley partials with the lattice could be regarded as a frictional drag which is also associated with the force due to the creation or annihilation of the stacking fault region.

(d) <u>Other variables</u>

In a review of factors affecting the high temperature strength of polycrystalline solids Sherby[62] suggested that partial dislocations were not a significant factor, and that mechanisms for dislocation climb based on thermally activated diffusion controlled the deformation. He suggested that the flow stress -σ at a constant strain rate and grain size could be written as:

$$\sigma = \text{constant}.\ G.D_d^{-1/5} \qquad \text{——— (2.14.)}$$

where D_d is the self diffusion rate according to the equation[64]:

$$D_d = D_o \exp - \frac{(Ko + V)\,Tm}{T} \qquad \text{———} \quad (2.15.)$$

where $D_o = 1$, Tm = absolute melting temperature, T = temperature of test, V = valence and K_o is a constant = 17 for f.c.c. metals.

On the other hand, Sherby[65] also lists a fine stable grain size as contributing to strength at high temperatures, a property which was subsequently shown[66] to depend upon stacking fault energy in work with which he as associated.

Other variables which Sherby considered to be of importancy apart from elastic modulus, were valence and a high melting temperature.

2.4.2. Structure

(a) Grain size

Probably the most widely researched relationship in metallurgy is that between flow stress and grain size. Since the work of Hall[67] and Petch[68] giving rise to the empirically determined equation:

$$\sigma_f = \sigma_o + KD^{-\frac{1}{2}} \qquad \text{———} \quad (2.16.)$$

a number of other workers in this subject have proposed[69,70] mechanisms explaining the relationship.

Work in confirmation of the equation has been based on a wide range of materials and Floreen and Westbrook[71], for example, list eighteen references to such studies.

(b) Other structure factors

Other factors to be considered include sub-grain size, misorientation between grains or sub-grains and dislocation density. Values of dislocation density were not established in the work reported here and there are obvious and serious difficulties in remedying this. The same is true of intergranular misorientation, and it seems unlikely that the contribution of this factor is an important one[9]. Sub-grain size on the other hand, has been shown[72] to have a considerable influence although it seems[9] that a stable sub-grain size is not developed until strains of up to 0.3 at strain rates of 0.05 to 1.0 sec.$^{-1}$. Since the strain at which maximum stress occurs is about 0.3 in the metals tested and the sub-grain size is unstable up to this point, the determination of an appropriate size is, therefore, rather difficult. Furthermore, the sub-grain size is clearly influenced by stacking fault energy[73,66], a parameter already considered.

2.4.3. Temperature

Studies on the influence of temperature on deformation behaviour have followed several different lines of approach which have, in most cases, also involved the effect of strain rate[9].

Attempts to produce an empirical relationship between temperature and stress/strain parameters have met with mixed success. In some cases[31, 74] no meaningful relationship could be established at all, while other workers[42] simply commented that 'The stress ...... varied with temperature in a complex manner .....'.

Some studies, however, particularly those directed towards creep mechanisms, have established clear relationships between metal behaviour and temperature. Sellars and Tegart[75] developed an equation, applicable over a wide range of stresses and strain rates, of the form:

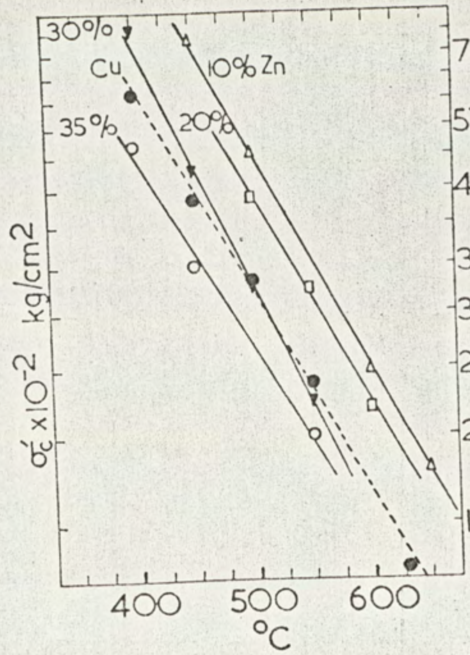
$$\dot{\epsilon} = A(\sinh \alpha \sigma)^n \exp(-Q/RT) \qquad \text{\_\_\_\_\_} \quad (2.17.)$$

where A, α and n are temperature-independent constants, and Q is an activation energy.

Feltham and Copley[25] identified a critical stress - σ_c above which Cottrell-Lomer locking becomes less effective in inhibiting slip. They demonstrated that this stress was linearly dependent upon temperature (see figure 2.3.). The absolute values of σ_c were clearly affected by composition although the slope of the line σ_c /°C appeared, from the published results, to be composition insensitive. From the report[25] that slip bands were observed in creep at stresses above the critical value, but not below, it seems likely that the high stress deformation is analogous to stage III deformation in the stress/strain curve.

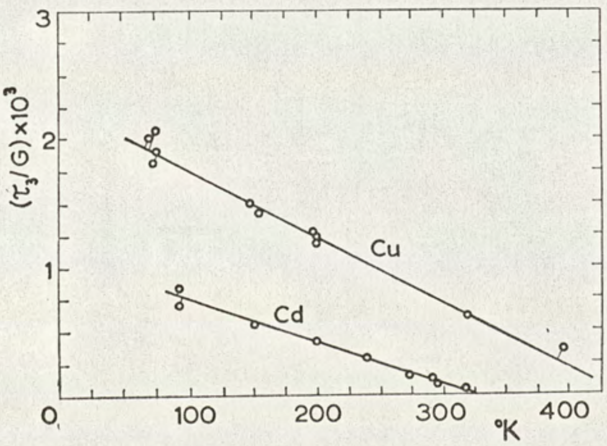
In subsequent investigations[76,77] of the cross-slip process Feltham suggested a mechanism whereby the mutual annihilation of jogged screw dislocations leads to the observed decrease in work hardening rate during stage III deformation. The transition stress at which stage III deformation is initiated was shown in the earlier paper[76] to depend upon the logarithm of temperature. Later, however, it was shown[77] that the relationship between the transition stress and temperature could be expressed as:

$$\tau_{III}/G = \text{const. } T \qquad \text{\_\_\_\_\_} \quad (2.18.)$$



Temperature dependence of the critical tensile stress σ_c'.

(a) Cu and Cu/Zn alloys - ref 76.



The temperature dependence of the ratio τ_3/G in copper and cadmium crystals,

(b) Cu and Cd - ref. 77.

Figure 2.3. Influence of temperature on critical stress for 'cross-slip'

This relationship being the one which the proposed dynamic recovery model requires. Bearing in mind the exponential form of the dependence of G the shear modulus on temperature (see Appendix A) the logarithmic relationship determined in the earlier paper would also be expected from this model.

In their review of strength under hot working conditions Jonas et. al.[9] point out the similarity of the activation energies for hot working, creep and self-diffusion. This suggests that rate-controlling mechanism involves the formation and migration of vacancies in the manner proposed by Feltham[77]. It is pointed out, however, that at strain rates in excess of those normally associated with creep behaviour, experiments to confirm alternative models of deformation mechanisms, such as the climb theory of Weertman[78] and Dorn[79] or the network model of McLean[80], are extremely difficult.

The possibility of dynamic recrystallisation as the means of restoration during hot working has also been proposed but no formal theory of an appropriate mechanism has been suggested[9]. Clearly such a process would also depend upon diffusion and would be expected to be sensitive to strain rate.

2.4.4. Strain Rate

It was stated in section 2.4.3. that the influence of temperature and strain rate have, in most investigations, been considered concurrently. The principal reasons for this are:

(a) most thermally activated processes occurring in metals are diffusional, and therefore dependent upon time as well as temperature. The time available for diffusion to occur is limited by the speed with which other processes, usually stress-activated, take place, and these are directly dependent upon the strain rate;

(b) the thermal energy available may be increased by the heat of deformation produced, particularly at the higher rates of strain when the time available for heat to be lost to the surroundings is limited.

The association between time and temperature has led a number of workers[75,81] to produce parameters composed of both temperature and strain rate. MacGregor and Fisher[81] suggested a velocity-modified temperature, T^1, such that:

$$T^1 = T(1 - k \ln \dot{\epsilon} / \dot{\epsilon}_o) \qquad \text{(2.19.)}$$

where T = absolute testing temperature,

$\dot{\epsilon}$ = true strain rate and

$\dot{\epsilon}_o$ = unit strain rate (taken as $10^{-3}\,\text{sec.}^{-1}$)

Although qualitatively correct the results of later investigators[42,31] were not able to confirm this equation. The temperature-compensated strain-rate parameter Z, proposed by Sellars and Tegart[75] has been confirmed[9], over a wide range of strain rates, as:

$$Z = \dot{\epsilon} \exp (Q/RT) \qquad \text{(2.20.)}$$

As with many of the studies of temperature-dependence, however, this relationship has only been demonstrated for steady-state deformation and its value in conditions of transient deformation are in some doubt[9].

A much earlier study[23] suggested that the stress to a given strain could be represented by the semi-logarithmic relationship:

$$\sigma = A \ln \dot{\epsilon} + \sigma_0 \qquad \text{\_\_\_\_\_\_ (2.21)}$$

Alder and Philips[42] were able to confirm the equation, but found that a power relationship of the form:

$$\sigma = \sigma_0 \dot{\epsilon}^{n} \qquad \text{\_\_\_\_\_\_ (2.22.)}$$

was equally suitable. In addition to their own results they quoted the work of many other researchers whose evidence also seemed divided between the alternatives offered by equations 2.21 and 2.22. In this and later work[31] it was shown that both σ σ_o and n in equation 2.22 varied with strain and temperature as well as chemical composition.

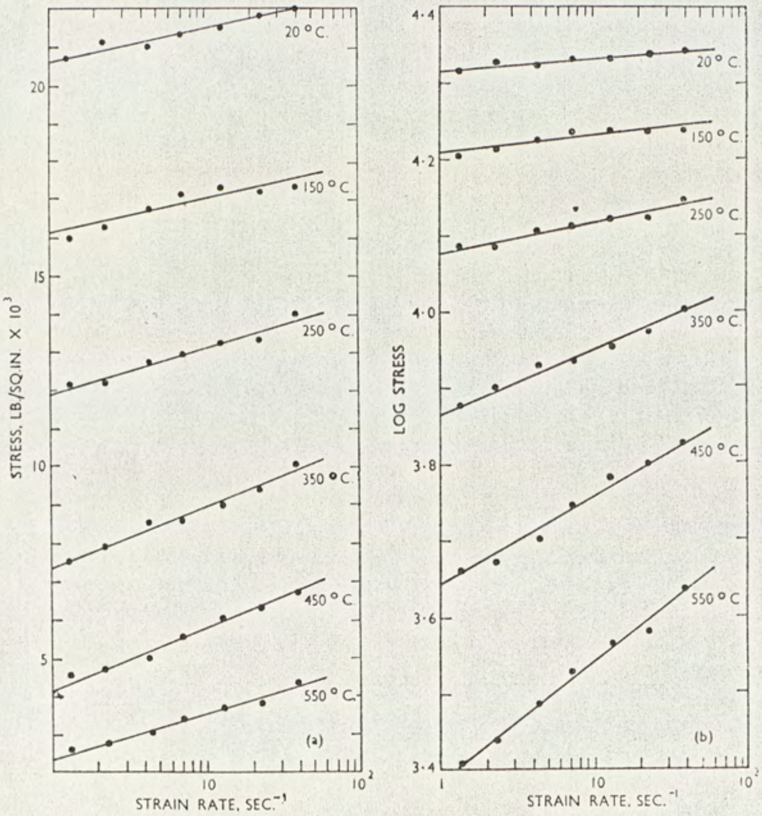
Feltham's model of cross-slip[76,77] predicts an equation of the form:

$$\tau_{III} = \ln \dot{\gamma} \text{ (const. (KT/Q))} \qquad \text{\_\_\_\_\_\_ (2.23.)}$$

(see figure 2.4.)

where τ_{III} is the stress for transition to stage III deformation, $\dot{\gamma}$ is shear strain rate, T is temperature and Q is an activation energy.

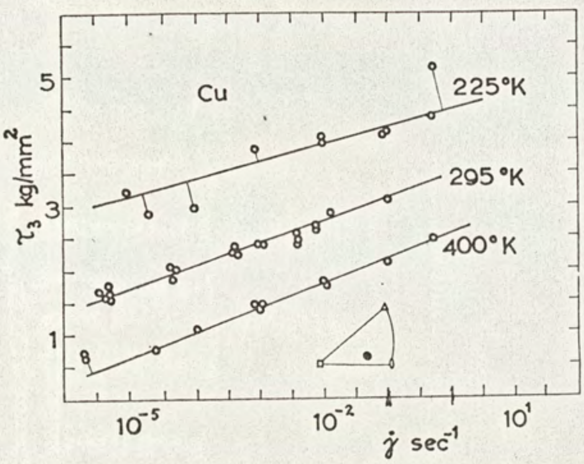
The magnitude of the strain rate effect is such that, from Bailey and Singer[31], an increase in the flow stress of aluminium at 600°C. and a strain of 2.3, from approximately 1000 p.s.i. to 1800 p.s.i. would occur over the range of strain rates being considered in the current investigation. At room temperature strain rate has no measurable influence on the flow stress. The results of Alder and Philips confirm this value for aluminium, and suggest an increase of greater than 2,000 p.s.i. for copper at the same strain and 750°C.



Effect of Strain Rate on the Stress Required to Compress Aluminium to 40% Reduction at Various Temperatures.

(*a*) σ *v.* $\log_{10}\dot{\epsilon}$; (*b*) $\log_{10}\sigma$ *v.* $\log_{10}\dot{\epsilon}$.

(a) Aluminium - ref. 42.



The dependence of the shear stress τ_3 on the shear rate in copper crystals of the same orientation deformed at three temperatures. The points represent values measured by Berner (1960).

(b) Copper - ref. 77.

Figure 2.4. Influence of strain rate on flow stress.

2.4.5. Summary and Conclusions

There is considerable theoretical and practical evidence in support of the view that the relationship between stress and strain is parabolic within stage III, the region embracing most of the deformation in polycrystalline aggregates of f.c.c. metals. If this relationship is adopted (i.e. the strain exponent is fixed at $\frac{1}{2}$) then, referring to equation (2.6.), the rate of work hardening can be measured in terms of the coefficient K. In order to define stage III of the curve it is then only necessary to determine the upper and lower limits.

The upper limit of the relationship is conveniently measured since the stress at which restoration begins marks the end of the conventional stage III behaviour and the maximum strength of the metal.

The lower limit is ideally measured by τ_{III} - but this is very difficult to define in a polycrystalline sample, and if the stacking fault energy, or the temperature, or both are high the material may cross-slip at a very early stage (e.g. aluminium). The difficulties of identifying and measuring τ_{III}, when its value is approximately equal to the stress at which plastic deformation first occurs, may be avoided by extrapolating the curve of stage III back to its intersection with the stress axis, at the point indicated by τ_o in figure 2.5. Even in the worst conditions the strain over which extrapolation is necessary is much smaller than the strain in stage III. τ_o should, therefore, be approximately equivalent to τ_{III}, although it will always tend to underestimate

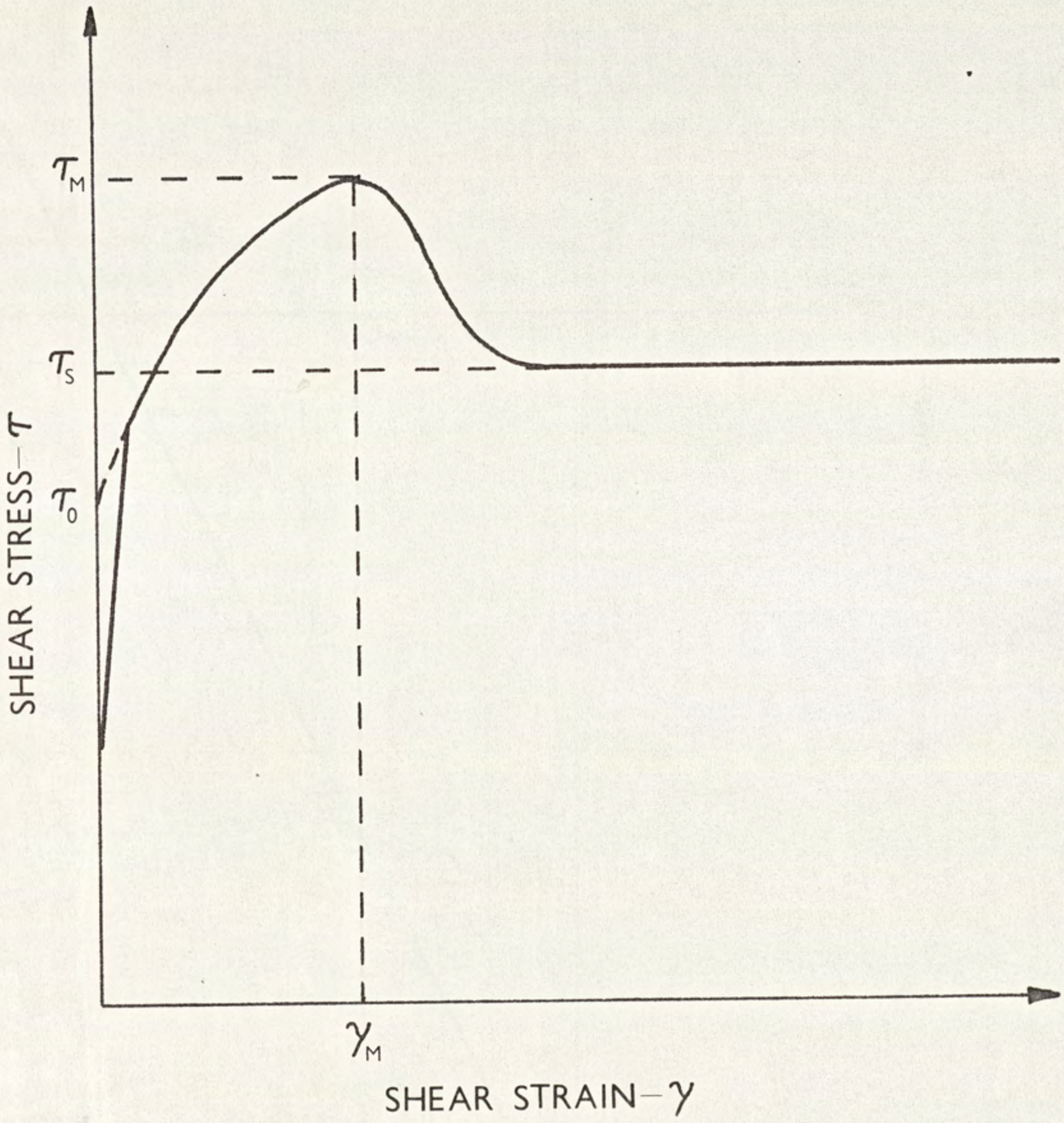


Figure 2.5.(a)Showing parameters of shear stress- shear strain relationship.

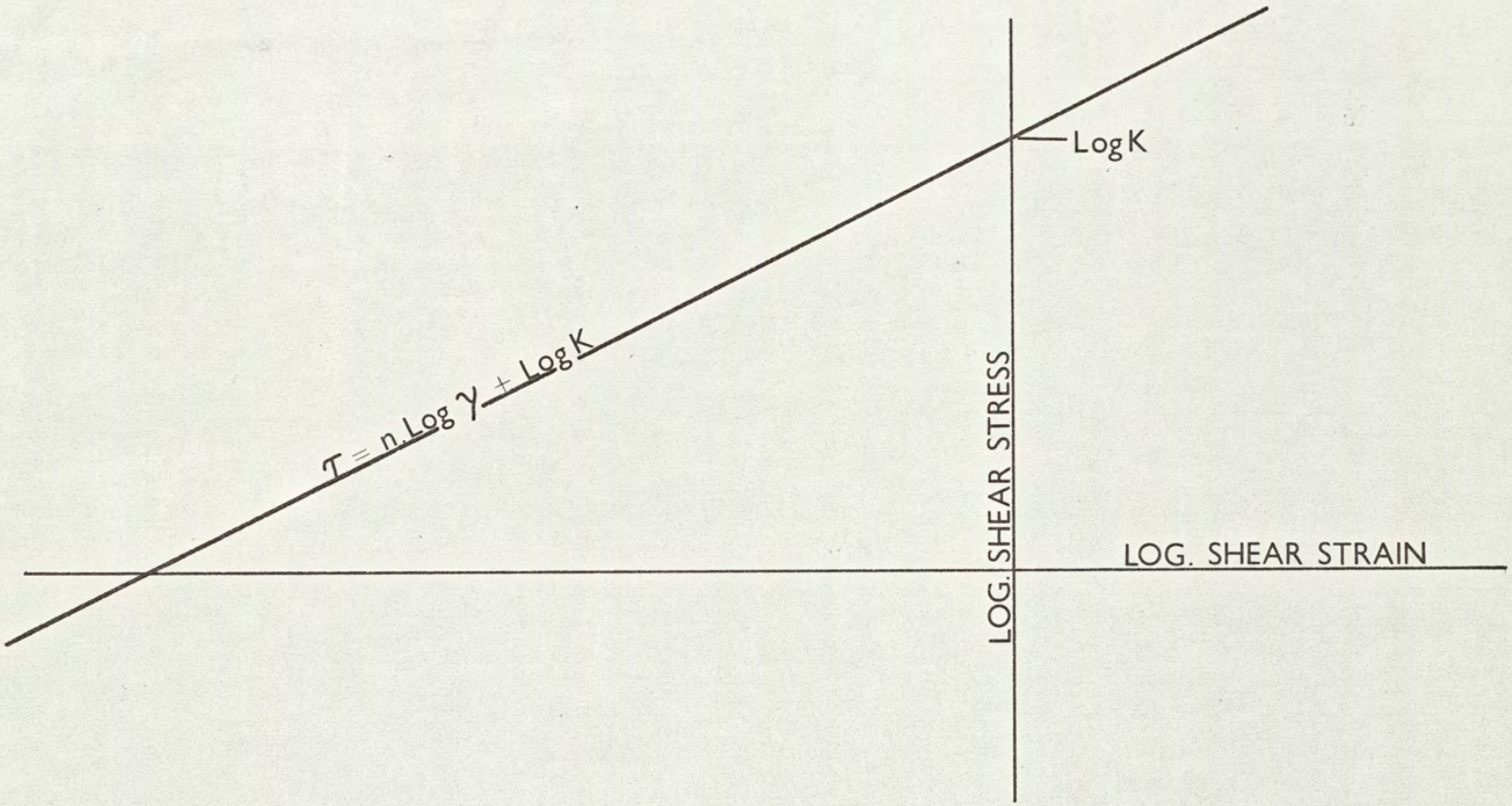


Figure 2.5.(b)Showing Log/log plot of shear stress versus shear strain.

rather than over-estimate the true value, and the error variance is inherently greater than that for τ_m or τ_s because of the extrapolation.

The one other parameter which is essential to the completion of the model at high temperatures in the steady-state stress - τ_s.

Of the material variables which are likely to influence τ_o, τ_m, τ_s and K the product term G.b is directly related to the stress required to move a dislocation and so is likely to be of considerable importance. Similarly, the stacking fult energy, which controls the degree of separation of partial dislocations, is likely to affect both the flow stress and the rate of work-hardening.

Among the variables, Atomic % of solute is worth considering because of the ease with which it may be determined. The electron/atom ratio may offer a guide to the influence of chemical interactions, and the diffusion coefficient - D_d will be important where thermally activated diffusion contributes to the relief of lattice strain by reducing the number of lattice defects.

The influence of temperature may be expected to be quite large, even after its effect on the modulus of shear is taken into account, and it seems likely, from section 2.4.3., that the parameters of the stress/strain model could depend upon T/T_m or $G.T/T_m$.

The strain rate, compared with the model parameters on the basis of a semi-logarithmic or of a power relationship seems likely to have a small but significant effect. Aluminium at 600°C, for example almost doubles in strength over the range of strain rates which are available in this investigation. There appears to be a slight preference for the semi-logarithmic function (the data of Alder and Phillips[42] supports this view on the basis of comparing the correlation coefficients produced by the two expressions, for example, despite the conclusions which those workers reached) although either appears to fit most of the available evidence.

2.5. Regression Analysis

The application of regression analysis is based upon a postulated mathematical model of the physical situation. In particular the hypothesis proposed is that the observed values of a dependent or response variable - Y may be expressed as linear functions of one or more independent or control variables $X_1, X_2, \ldots\ldots, X_p$, with residual errors which are normally and independently distributed with constant variance and mean equal to zero

The technique of regression analysis involving formulation of the model, calculating its parameters and assessing its accuracy and reliability, have been presented in a number of texts[82]. In particular, the treatment by Draper and Smith[83] provides a thorough and authoritative approach to the subject and has been adopted here as the major reference.

2.5.1. The Regression Model

The regression model may be expressed in the form:

$$Y_\alpha = \beta_1 X_1 + \beta_2 X_2 + \ldots\ldots + \beta_p X_p + \epsilon_x \quad (2.24.)$$

where $X_1, X_2, \ldots\ldots X_p$ have particular known values for each Y_α, say, $X_{1\alpha}, X_{2\alpha} \ldots\ldots\ldots, X_{p\alpha}$. Usually X_1 is equal to unity for all values of Y_α unless there is reason to believe that Y_α equals zero when all of the control variables equal zero.

Implicit in this form of model is the assumption that the regression of Y on any X_i is linear with constant slope when the other variables X_j, $(j \neq i)$ are constant, whatever the value of all the other variables. Deviations from linearity, either inherent or due to interaction between the effects of independent variables can sometimes be taken into account by the inclusion of second order terms, e.g. X_i^2, $X_i X_j$.

2.5.2. Parameters of the Model

In order to complete the model it is necessary to estimate the β - parameters, or regression coefficients, in equation (2.24.)

The techniques for carrying out the estimation are based upon a theorem due to Markoff which states that the best linear unbiased estimates of the constants $\beta_1, \beta_2, \ldots, \beta_p$ are those that minimise the sum of squares

$$\Sigma_\alpha (Y_\alpha - \beta_1 X_{1\alpha} - \beta_2 X_{2\alpha} - \ldots\ldots - \beta_p X_{p\alpha})^2 \qquad (2.25.)$$

If (2.25) is differentiated with respect to β_1 and equated to zero it becomes:

$$\beta_1 \Sigma X^2_{1\alpha} + \beta_2 \Sigma X_{1\alpha} + \ldots + \beta_p \Sigma X_{p\alpha} = \Sigma Y_\alpha X_{1\alpha} \qquad (2.26.)$$

If this is repeated for each β_i in turn a series of equations are produced, equal in number to the number of regression coefficients to be estimated, of the form:

$$\begin{aligned} \beta_1 \Sigma X^2_{1\alpha} + \beta_2 \Sigma X_{2\alpha} + \ldots + \beta_p \Sigma X_{1\alpha} X_{p\alpha} &= \Sigma Y_\alpha X_{1\alpha} \\ \beta_1 \Sigma X_{1\alpha} X_{2\alpha} + \beta_2 \Sigma X^2_{2\alpha} + \ldots + \beta_p \Sigma X_{2\alpha} X_{p\alpha} &= \Sigma Y_\alpha X_{2\alpha} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \beta_1 \Sigma X_{1\alpha} X_{p\alpha} + \beta_2 \Sigma X_{2\alpha} X_{p\alpha} + \ldots + \beta_p \Sigma X^2_{p\alpha} &= \Sigma Y_\alpha X_{p\alpha} \end{aligned} \qquad (2.27.)$$

These equations are usually referred to as the 'normal equations'.

Provided that the X - values are not such that one or more linear function of them is equal to zero, a unique solution exists to the set of simultaneous equations given in (2.27.). The solution provides a set of estimates - b_i, of the p values of β_i, specific to the sample of observations on which they are based. Maintaining the assumption that variations of the observations about the line are normal, that is, that the errors e_i are all from the same normal distribution, it can be shown[83] that 100 $(1 - \alpha)$% confidence limits can be assigned to β_i by

$$b_i \pm t_{(n - m - 1,\ 1 - \frac{1}{2}\alpha)} .\ \text{S.E.}\ (b_i) \qquad \text{—— (2.28.)}$$

where $t_{(n - m - 1,\ 1 - \frac{1}{2}\alpha)}$ is the $(1 - \frac{1}{2}\alpha)$ percentage point of a t - distribution with $(n - m - 1)$ degrees of freedom, m is the number of independent variables in the regression model and S.E. (b_i) is the standard error of b_i.

It is now almost universally accepted that the methods of matrix algebra provide the most convenient means of solving the normal equations. This is not only because of the ease with which general purpose algorithms can be developed for regression work, but also because of additional information which becomes readily available when matrix methods are employed.

Reverting to matrix notation, therefore, the mathematical model under consideration can be written as:

$$\underline{Y} = \underline{X}\ \underline{\beta} + \underline{e} \qquad \text{—— (2.29.)}$$

where $\underline{Y}$ is an $(n \times 1)$ vector of observations

$\underline{X}$ is an $(n \times p)$ matrix of known form

$\underline{\beta}$ is a $(p \times 1)$ vector of parameters

$\underline{e}$ is an $(n \times 1)$ vector of errors.

It can be shown (see, for example, Placket[84]) that the normal equations (2.27.) can be rewritten:

$$(\underline{X'X})\ \underline{b} = X'Y \qquad \text{\_\_\_\_\_ (2.30.)}$$

where $\underline{X}'$ is the transpose of $\underline{X}$ and

$\underline{b}$ is the least squares estimate of _ .

The matrix $(\underline{X'X})$ is inherently symmetrical. Provided, therefore, that it is also non-singular, i.e. that none of the normal equations are inter-dependent, it is possible to produce the inverse $(X'X)^{-1}$ of $(X'X)$. The solution of the normal equations can then be written:

$$\underline{b} = (\underline{X'X})^{-1}\ \underline{X'Y} \qquad \text{\_\_\_\_\_ (2.31.)}$$

Draper and Smith point out that the solution $\underline{b}$ has the following properties:

1. It is an estimate of β which minimises the error sum of squares $\underline{e'e}$ irrespective of any distribution properties of the errors.

2. The estimates of $\underline{b}$ are linear functions of the observations $Y_1, Y_2, \ldots, Y_n$, and provide unbiased estimates of the elements of _ which have the minimum variances (of any linear functions of the Y's which provide unbiased estimates), irrespective of distribution properties of the rrors.

A number of well-established methods are available for inverting matrices[86]. The method due to Woolf[85] offers certain advantages, however. In particular, the ease with which individual variables may be added to or removed from the reciprocal matrix is of benefit in carrying out the tests of significance described in 2.6. One shortcoming of this method (which could apply equally to any other method) is the risk of ill-conditioning. This is particularly likely in iterative processes for matrix inversion, and arises mainly due to rounding-off of values during intermediate stages. If different variables have widely different values (e.g. temperature and grain size) the likelihood of ill-conditioning is increased, even when the calculations are carried out on a computer. These problems can be countered, however, by replacing the matrix of cross-products and squares by a matrix of correlation coefficients, since these values must all lie between -1 and +1, by definition. The replacement is easily carried out since, if the j th element in the i th column of the matrix of cross-products and squares is a_{ij} its counterpart r_{ij} in the matrix of correlation coefficients is obtained by:

$$r_{ij} = \frac{a_{ij}}{(a_{ii} \cdot a_{jj})^{\frac{1}{2}}} \qquad \text{———} \; (2.32.)$$

Then, if the standard deviation of Y is S_y and that of X_i is S_i, and the element of the reciprocal matrix corresponding to r_{ij} in the matrix of correlation coefficients is r^{ij} the following values may be derived:

$$b_i = \frac{r^{yi}}{r^{yy}} \cdot \frac{S_i}{S_y} \qquad \text{———} \; (2.33)$$

The standard error of b_i, denoted by S.E.(b_i):

$$\text{S.E.}(b_i) = ((r^{yy}\, r^{ii} - r^{yi2})/(n - m - 1))^{0.5} \cdot \frac{r^{yi}}{b_i} \qquad (2.34)$$

2.6. Assessing the Model

Assessment of the model involves two requirements:

1. assessment of the model as an entity
2. assessment of the contribution of each individual variable to the model.

2.6.1. The Whole Model

It is convenient to consider first the value $\hat{Y}_i - Y_i$, since this indicates the extent of the disagreement between the observed value Y_i and the value $\hat{Y}_i$ predicted by the model. This value can be divided:

$$\hat{Y}_i - Y_i = (Y_i - \bar{Y}) - (\hat{Y}_i - \bar{Y}) \qquad \text{(2.35.)}$$

If both sides are squared and summed from i = 1 to n this becomes

$$\Sigma (\hat{Y}_i - Y_i)^2 = \Sigma (Y_i - \bar{Y})^2 - \Sigma (\hat{Y}_i - \bar{Y})^2 \qquad \text{(2.36.)}$$

which can be rewritten

$$\Sigma (Y_i - \bar{Y})^2 = \Sigma (\hat{Y}_i - Y_i)^2 + \Sigma (\hat{Y}_i - \bar{Y})^2 \qquad \text{(2.37.)}$$

This is the mathematical expression of the statement:

the total sum of squares about the mean = sum of squares about the regression + sum of squares due to the regression

The usefulness of the regression equation as a predictor may then be assessed by considering the ratio (S.S. due to regression/S.S. about the mean). The more nearly that this ratio, designated R^2, approaches unity the better is the predictive capability of the regression equation. The parameter R is termed the Multiple Correlation Coefficient. Using the Woolf method and invertine the matrix of correlation coefficients the value of R^2 is very conveniently obtained:

$$R^2 = 1 - 1 / r^{11} \qquad \text{(2.38.)}$$

It is not sufficient, however, to consider R^2 in isolation since any variable added to the equation will increase the regression S.S. until R^2 equals unity when the number of independent variables is equal to one less than the number of observations. Any sum of squares has associated with it a number of degrees of freedom indicating the number of independent observations from which it has been compiled. Using the various sums of squares and their associated degrees of freedom it is possible to construct a table of the analysis of variance in the following form:

Source	Sum of Squares	Degrees of Freedom	Mean Square
Regression	$R^2.\sum_{i=1}^{n}(Y_i - Y)^2$	m	MS_R
About regression (residual)	By subtraction	n - m - 1	S^2
About mean	$\sum Y_i^2 - \frac{(\sum Y_i)^2}{n}$	n - 1	

One further quantity required to assess the overall model is the 'pure error' mean square. This quantity, which is, again, computed from the sum of squares of deviations from the mean, divided by the number of degrees of freedom, is best estimated from repeat observations of the dependent variable at each set of independent variables. Where this estimate is not available for any reason (for example, if the number of experiments required is prohibitive) other methods of obtaining an estimate might be possible . The only difference between certain experiments might be due to some variable which has been shown to have no significant effect on the regression model, for instance.

Having been estimated the 'pure error' sum of squares may be introduced into the analysis of variance table as shown in figure 2.6. . from Draper and Smith[83]. If the residual mean square is significantly greater than the pure error mean square, as determined by an F-test, there is said to be a lack of fit and the model is considered incomplete in its existing form.

If the residual mean square is not significantly greater than the pure error mean square, however, this indicates that there are no grounds for doubting the adequacy of the model, and both pure error and lack of fit mean squares can be used as estimates of σ^2.

Finally, the residual values $\hat{Y}_i - Y_i$ should be examined. The residuals are the set of values e_i in (2.24.) and certain assumptions have been made concerning them. In particular, the assumption that the errors are normally distributed is necessary for making F-tests. Since the criteria for assessing the model are based on F-tests it is necessary to confirm that the data do not contradict the assumptions. There are a number of methods for examining the residuals including

(a) graphical methods where the residuals are plotted against the fitted values and against any other variables (including those in the adopted model) which might introduce bias;

(b) comparison with the normal distribution to discover extremes of skewness or curtosis.

2.6.2. The Individual Variables

The assessment of each of the independent variables is aimed at only including in the model those variables which make a significant contribution. The final objective of finding a compromise between a large number of independent variables,

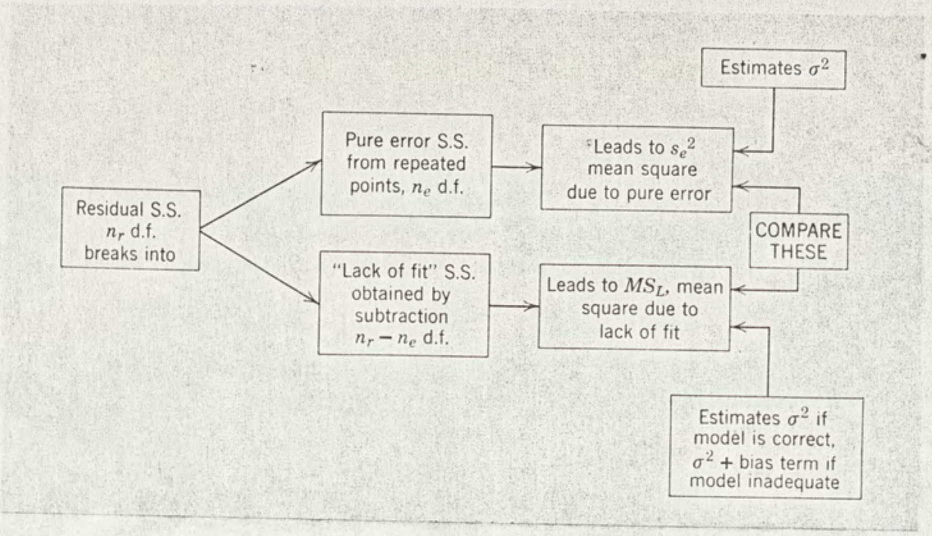


Figure 2.6. Schematic presentation of analysis of variance - ref. 83.

with the resulting high value of R^2, and the efficiency and convenience of a small number of predictor variables is usually referred to as selecting the 'best' regression equation. In their paper introducing the technique of element analysis Newton and Spurrell[87] suggest that an alternative objective of regression analysis to learn about the 'operation' of the process being studied. In this case it is desired to identify those variables which are important in controlling the process and which independently have as large effects upon the residual sum of squares as possible. They suggest that this approach is conveniently described as 'operational'.

Various methods of selecting the 'best' sub-set have been suggested with a number of different criteria of 'bestness'.

In the 'all regressions' method it is necessary to compute all $2^p - 1$ possible combinations of the p variables. If the residual mean square is plotted against the number of variables in the equation, it will usually be found that the value attains an approximately constant level at some number of variables less than p. The number of variables at which this levelling-off occurs is a guide to the size of the eventually adopted equation. It is worth mentioning here that the value of residual mean square at which the above mentioned plot levels off may provide an estimate of the error mean square - σ, if no better estimate (such as duplicate test results) is available. The highest value of R^2, i.e. the highest regression sum of squares, attained at the appropriate number of variables is one criterion for indicating the best sub-set.

One major disadvantage of this approach is that it is extremely laborious once 'p' becomes large. Even adopting the approach suggested by Garside[88] whereby all possible sub-sets may be compared 'in a more efficient way than performing the calculation 'ab initio'', a running time of greater than four minutes was required on an Atlas computer (a very powerful machine) to deal with thirteen variables. Indeed, even on Atlas the maximum number of variables which can be dealt with, irrespective of time, is 48 if the program developed by Garside is used.

Other methods of assessing the individual variables ~~may be~~ based on the concept of 'the additional sum of squares'. This quantity is the amount by which the regression sum of squares is increased as a result of adding the particular variable last to the regression. If the independent variables in the equation are truly independent of each other, i.e. the non-diagonal elements of the matrix of correlation coefficients (except those including the dependent variable) are all zero, the matrix is said to be orthogonal. In this case the additional sum of squares is an accurate and unambiguous indication of the relative importance of each of the variables. Moreover, a comparison of this value with the error variance - σ^2, by means of an F-test, provides an indication of the statistical significance of the contribution that the variable concerned makes to the regression sum of squares. Where the matrix of correlation coefficients is non-orthogonal, however, the additional sum of squares due to any variable depends upon the other variables which are included in the regression equation and is sensitive to the order of inclusion or exclusion of variables.

There are, basically, two alternative approaches to the process of selection. The first is to add into the regression equation, at each stage, the variable which makes the greatest addition to the regression sum of squares. The alternative is to begin with all possible variables included in the equation and to remove them, one by one in order of the contribution to the regression sum of squares, beginning with the smallest. In each case the equation is considered to be complete when no variable which could be added to the equation makes a statistically significant controbution and no variable in the equation can be removed without reducing the regression sum of squares by an amount which is significant when compared with σ^2. Neither of these methods can be presented as being indisputedly better than the other, but both have the disadvantage, mentioned earlier, that the significance of some variables may be greatly influenced by the other variables present in the equation. In order to counter this disadvantage techniques have been suggested the best known probably being that due to Efroymson[89], in which the variables already included (in the case of forward selection) or excluded (in the case of backward elimination) are re-considered in order to see if their status has changed, at each stage of selection.

A refinement of the approach whereby the additional sum of squares is the only criterion of selection has been proposed and developed by Newton and Spurrell[87,90,91,92]. They advocate a technique based upon 'element analysis'. The additional sum of squares is described by Newton and Spurrell as the primary element, and additionally they defined quantities termed 'secondary elements' which cannot be attributed directly to any individual variable but are a measure of the correlation between variables. The secondary elements, which may be positive or negative, are added to the regression sum of squares in the presence of either of the variables with which they are associated. The secondary elements, therefore, provide supplementary criteria in selecting variables. The optimum equation will contain those variables having the largest primary elements, all of which should be statistically significant at the chosen level of probability, and the least positive secondary elements consistent with this criterion. Where a choice appears necessary between variables having equal or nearly equal effects, that which has the highest secondary elements in association with the other variables will produce the higher value of R^2, while that with the lowest secondary elements will be the 'more orthogonal'.

2.6.3. Summary and Conclusions

Regression analysis may serve two purposes; (a) it provides a model, on the basis of which predictions concerning the behaviour of a 'response' variable may be made from a knowledge of the values of appropriate 'predictor' variables,
(b) it provides information relating to the importance of the predictor or independent variables in their influence upon some process, of which the response variable may be some form of measure.

The use of regression analysis would seem to be appropriate when the errors (which arise in any programme of experiments) can reasonably be expected to be approximately normally distributed, when the response variable depends upon a series of influences which may be independently identified and when those influences may be represented by a linear response of the dependent variable. From the survey of the mechanisms which influence the form of the stress-strain curve in metals all of the above conditions appear to apply in the present investigation. The influences which might be expected to affect the parameters outlined in section 2.4.5. include the stress required for dislocation movement, the stress required to 'close' an extended dislocation, the 'misorientation' stress due to using a polycrystalline specimen, etc. All of these influences are independent although clearly they are each governed, in turn, by a limited number of factors, such as composition, temperature, etc.

Since the main purpose of this investigation is to suggest a form of model of the behaviour of a particular group of metals during deformation, the manner in which the various control variables exert their influence is of limited importance. It is sufficient, therefore, to study the effect of each of the possible predictors and to classify their effects as 'significant' or 'not significant'. Clearly, the predictors which are shown to have a 'significant' effect provide some guide to the mechanisms taking place.

Selection of the significant predictor variables is most frequently based upon the 'additional sum of squares', i.e. the amount by which the regression sum of squares is increased when the variable to be considered is added to the regression equation. In circumstances where the matrix of correlation coefficients is not orthogonal the choice of variables may be strongly affected by the order of addition or deletion of variables in the equation, and some supplementary criterion, i.e. extra to the 'additional sum of squares', such as the element analysis methods proposed by Newton and Spurrell, can be of benefit.

The efficacy of the adopted model may be assessed either in terms of R^2 - a measure of the extent to which the total variation in the dependent variable may be explained by the regression equation, or by an F-test which would indicate the sub-set of variables having, <u>on average</u>, the greatest predictive capability.

3. EXPERIMENTAL METHOD

3.1. Introduction

In order to fulfill the main objective of the investigation, namely the construction of an appropriate predictive model of the stress-strain relationship, a study was made of a wide range of materials under a wide range of experimental conditions. The materials chosen varied from commercially pure metals to alloys containing up to 25% of solute, while the test temperatures ranged from that of liquid nitrogen up to values close to the melting points of the materials under study.

The test programme required the production of suitable metals and alloys, from their particular constituents, in a form suitable for machining into test-pieces. Each of the individual tests, which numbered well in excess of one hundred, was carried out the torsion machine described in section 3.3.2. and the output, in the form of a load vs. time trace, measured manually, approximately thirty to forty readings being taken. The equipment was calibrated at frequent intervals in order to ensure the highest possible degree of accuracy.

The computer programmes in Appendices B and C were developed specifically for use in the analysis of the results of this investigation.

3.2. Materials

3.2.1. Introduction

The materials used in this investigation were selected in order to satisfy two main criteria:

(i) They should belong to the same crystal-structure group.

(ii) They should provide as wide as possible a range of values of each of the experimental parameters.

The crystal group chosen to satisfy (i) above was the face centred cubic. This choice was due to the commercial and industrial importance of the group and the availability of reference data of relevance to studies of deformation.

With reference to (ii) the metals Ni, Cu and Al provide a satisfactory range of elastic properties, melting temperatures and electron/atom ratios. In order to include metals of low, as well as high, stacking fault energy, without resorting to precious metals, it was necessary to include some alloys. The most convenient, for their range of solubility, availability of stacking fault energy data and the ease with which they could be obtained in suitable form, are those based on Cu. To avoid having alloys from one system only, Cu/Zn and Cu/Al alloys were used.

The nominal copositions and identification codes for the materials used are as follows:-

			Code
Nickel	- commercially pure	-	N
Aluminium	- " "	-	A
Copper	- " "	-	C
Copper/10% Zinc		-	CZ1
Copper/20% Zinc		-	CZ2
Copper/25% Zinc		-	CZ3
Copper/5% Aluminium		-	CA1

3.2.2. <u>Material Preparation</u>

The Nickel was very kindly supplied by Messrs. International Nickel in the form of nominally $\frac{5}{8}$ inch diameter hot swaged bar, ready to be machined into test-pieces.

The other materials were supplied in ingot form as high purity basis metal (Aluminium and Copper) and 50/50 Cu/Zn and 80/20 Cu/Al. The metals, with appropriate alloying additions, were melted in a town's gas-fired reverberatory furnace and chill cast into 2 inch diameter steel moulds.

Hot forging to a nominal $\frac{3}{4}$ inch diameter was carried out by Messrs. High Duty Alloys Ltd., according to the schedule in Table 3.1.

The bars were then cold rolled to $\frac{1}{2}$ inch. diameter in grooved rolls and annealed.

Samples were taken from the bars for chemical analysis. The copper-contents of the copper-based alloys were determined by electrolysis and all other determinations were made by Messrs. I.M.I. Ltd., using atomic-absorption spectrophotometry. The full analyses are given in Table 3.2.

Material Code	Method of Preparation
C, CZ1, CA1	Heated 900°C. - Forged to 1" sq. - Reheat 900°C. - Forged to $\frac{3}{4}$" sq. - remove corners - finish in half-round tools to $\frac{3}{4}$" dia.
CZ2, CZ3	As above but forging temp. 850°C.
A	As above but forging temp. 430°C.

Table 3.1. Preparation of materials for machining.

MAT^{L}.	Cu	Ni	Al	Zn	Sn	Pb	Fe	Mn	Sb	Bi	As	Si	P
N	n.d.	≮ 99.7											
A	< .005	<.05	rem.	<.05	<.03	<.05	<.02	<.01	n.d.	n.d.	n.d.	<.03	n.d.
C	rem.	.0002	≪.002	<.001	.0002	.0004	<.0005	<.0002	<.0001	0.0002	<.001	<.001	≪.02
CZ1	88.85	0.02	<.007	rem.	0.02	0.04	~0.01	<.003	<.001	<.0007	0.1	<.0004	<.007
CZ2	79.20	0.02	<.007	rem.	0.02	0.04	0.08	<.003	<.001	<.0007	0.1	<.004	<.007
CZ3	73.55	0.02	<.007	rem.	0.02	0.06	0.06	<.003	<.001	.002	0.1	<.004	<.007
CA1	95.90	<.005	rem.	<.01	0.006	<.005	0.04	<.003	<.001	<.0007	<.004	0.006	<.007

Table 3.2. Chemical analysis of materials (n.d. = not determined, rem. = remainder)

3.2.3. Modulus of Shear - G

Values of the modulus of shear for the alloys used in the investigation were calculated using the equations derived in Appendix A. In all cases where it was possible confirmation was obtained from tabulated data[65].

3.2.4. Stacking Fault Energy - γ_{SF}

The determination of stacking fault energies is subject to some uncertainty since most methods of measurement are based upon assumptions, the validity of which may be difficult to establish. Attempts to circumvent the difficulties have led to a number of techniques of measurement based on observations of dislocation nodes[93], the stress/strain curves of single crystals[94], the surface energy of twins[95], the evaporation of stacking fault loops[96] and tetrahedra[97], or the assessment of preferred orientation in cold rolled sheet[98].

The values of stacking fault energy adopted for this investigation are as follows:

N	-	240 ergs. $cm.^{-2}$	(ref. 99)
A	-	135 " "	(ref. 96)
C	-	85 " "	(ref. 99)
CZ1	-	42 " "	(ref.100)
CZ2	-	26 " "	(ref.100)
CZ3	-	19 " "	(ref.100)
CA1	-	10 " "	(ref.100)

In each case it was considered that sufficient confirmatory evidence is available, both in the references quoted and the previous work to which they, in turn, refer to accept them as authoritative estimates.

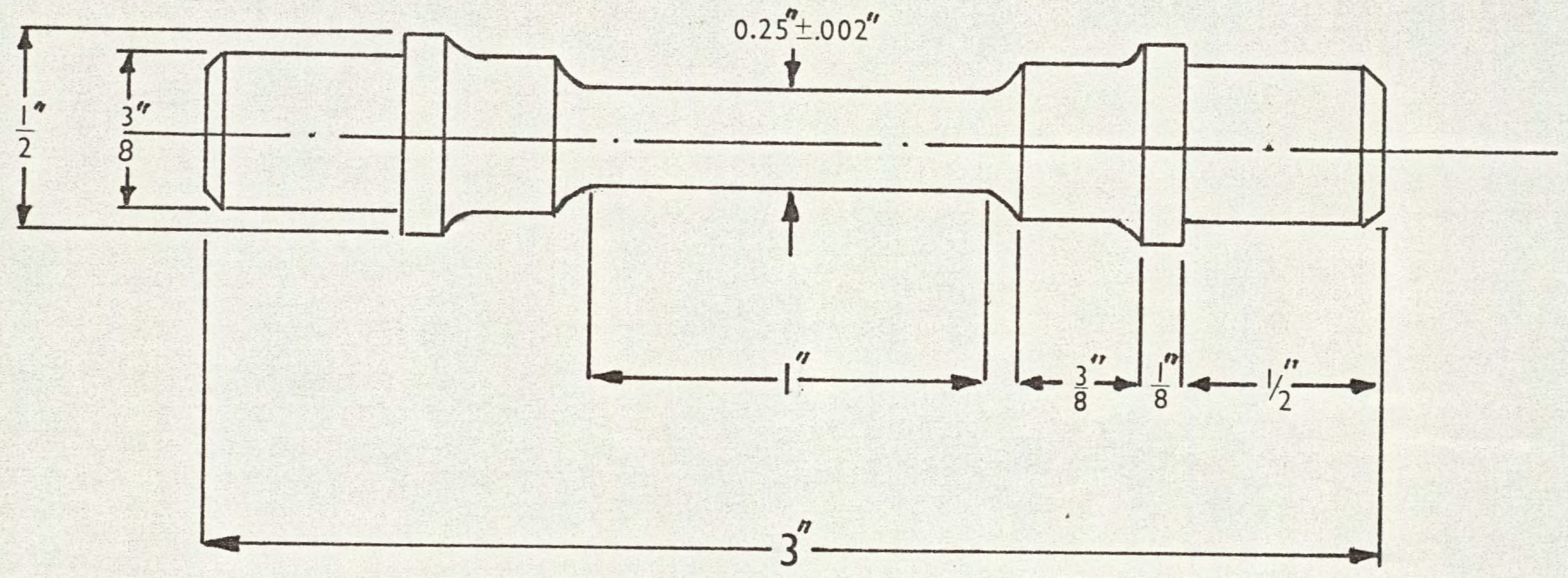


Figure 3.1. Showing dimensions of Torsion Test Piece.

3.3. The Torsion Test

3.3.1. Introduction

In selecting the test conditions two forms of specimen geometry are possible, i.e. hollow or solid specimens.

The strain applied to a test-piece deformed in torsion varies with the radius of the specimen. The use of a thin-walled tubular specimen, as suggested by Hodierne [36], simplifies the calculations involved in producing a stress-strain curve from torque-twist measurements as the strain can be assumed to be uniform across the wall thickness. This ease of calculation is obtained at the cost of some practical restrictions, however. The tubular form of specimen is susceptible to buckling, particularly during high temperature tests when the elastic modulus is lowered, and must be kept to a low length to diameter ratio. Some of the advantages of torsion testing are thus lost, since the volume of metal tested is greatly reduced and the influence of blending radii at the ends of the gauge length may be quite significant.

It was decided, therefore, to use a solid specimen in these tests. The gauge length to diameter ratio was fixed at 4:1, rather arbitrarily, to coincide with that being used elsewhere [12] in anticipation of the possibility of comparison of the results.

The gauge length was then fixed at one-inch since this appeared, after a number of trials, to be the greatest length over which uniformity of heating could be guaranteed with the available heating equipment. The specimen dimensions are given in figure 3.1.

3.3.2. The Torsion Machine

The torsion machine used is shown in plates 3.1. and 3.2. It was basically that described by Dragan, but the facilities available for the application of axial stresses and measurement

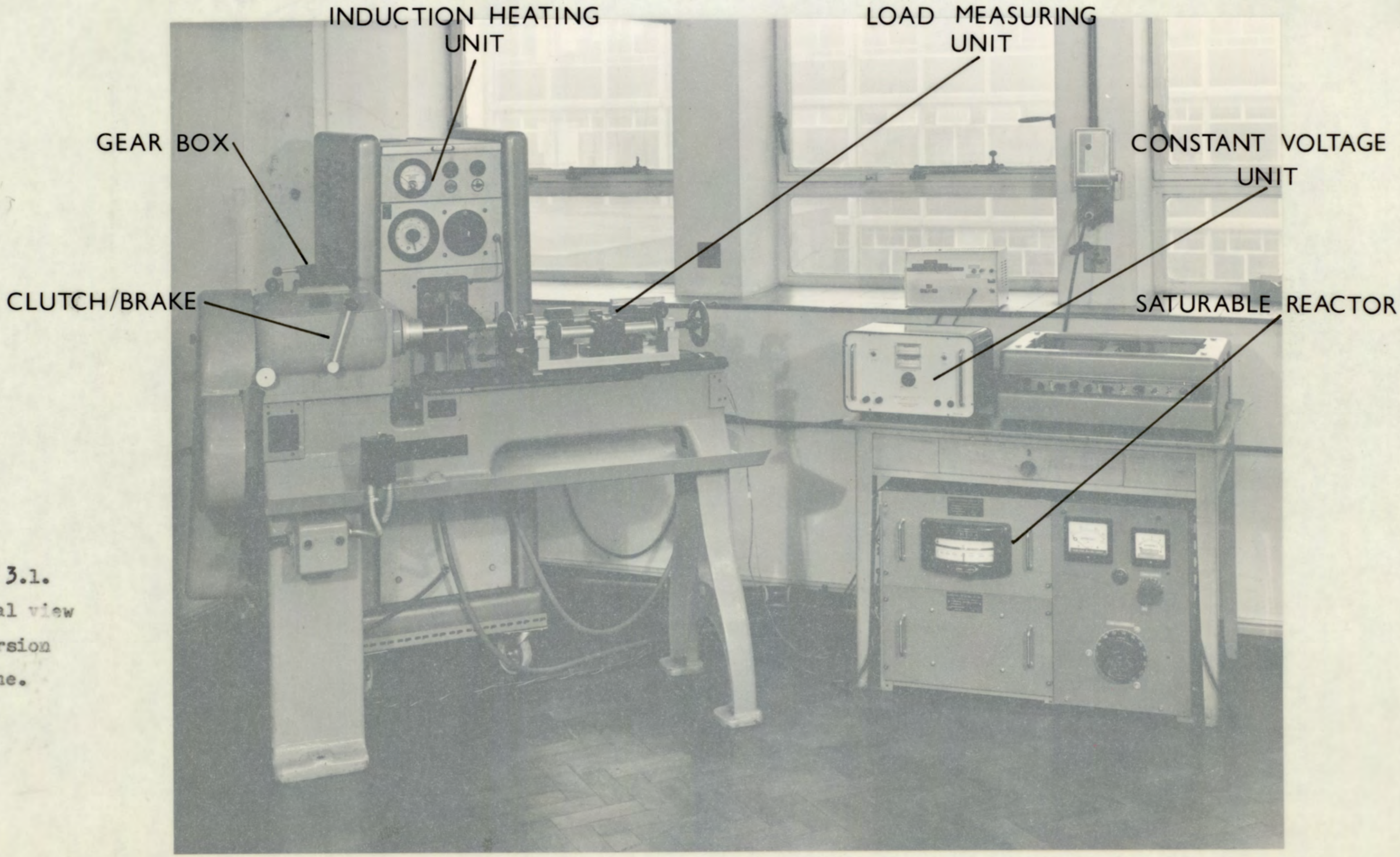
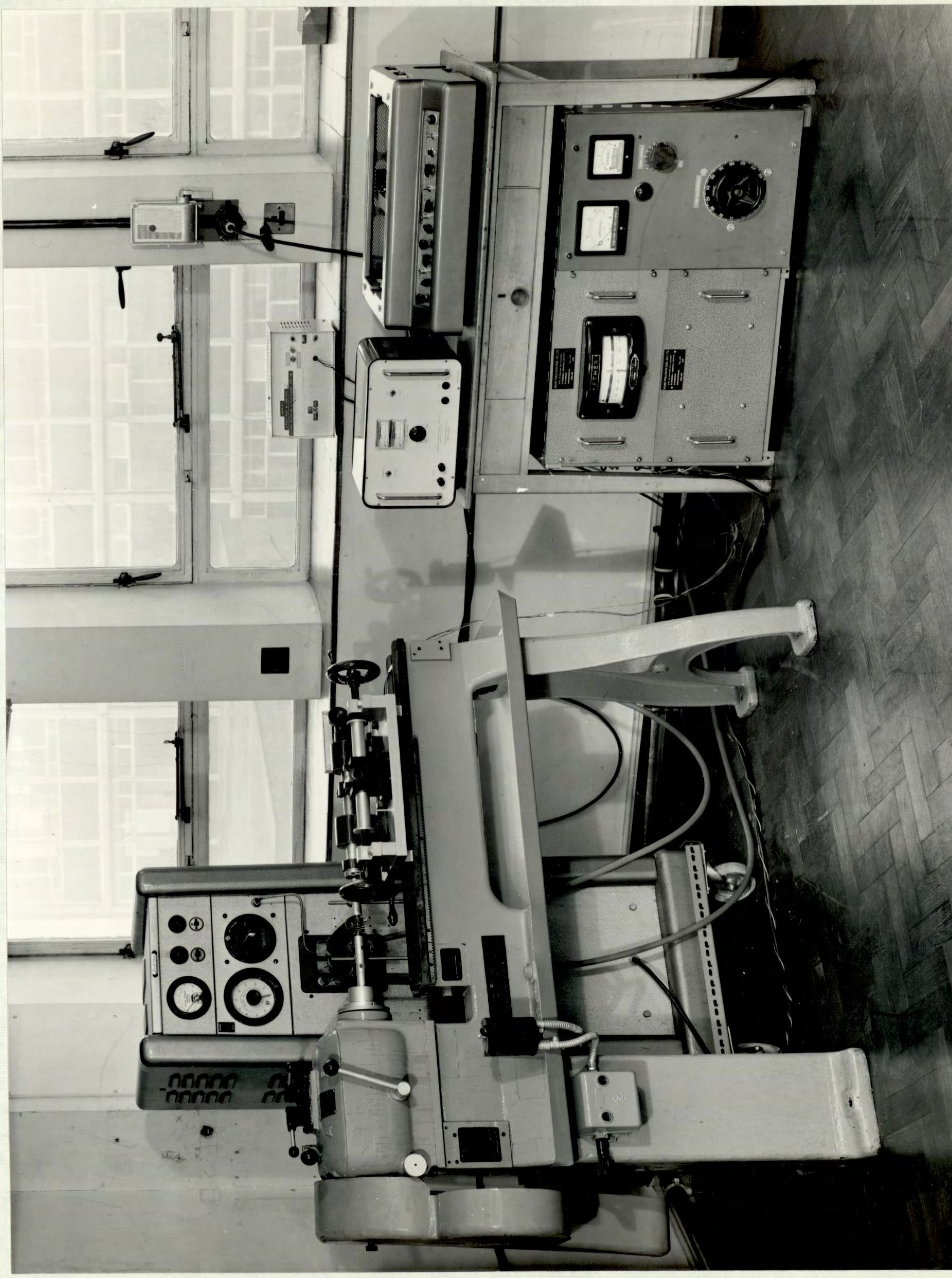


Plate 3.1.
General view
of Torsion
Machine.



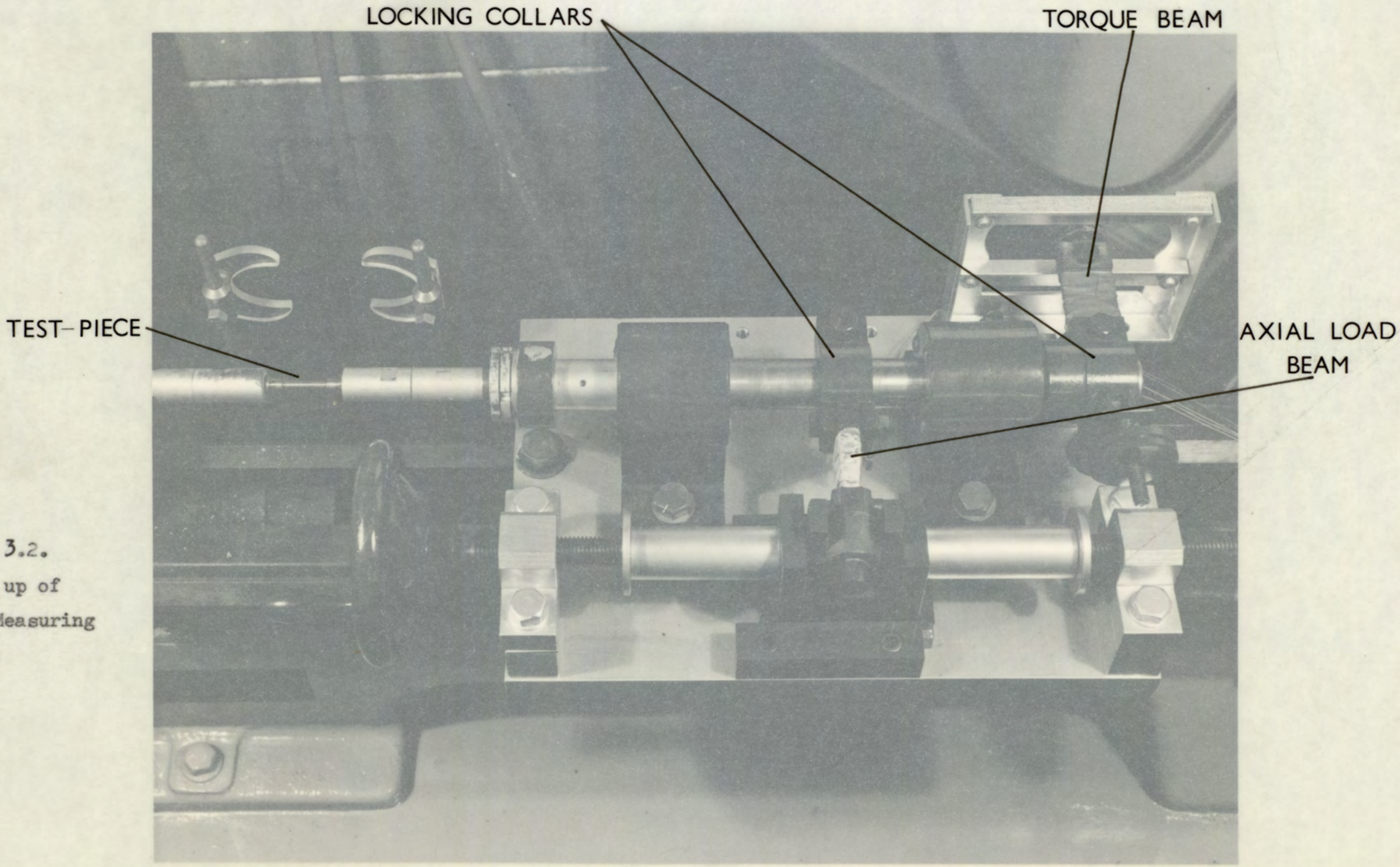
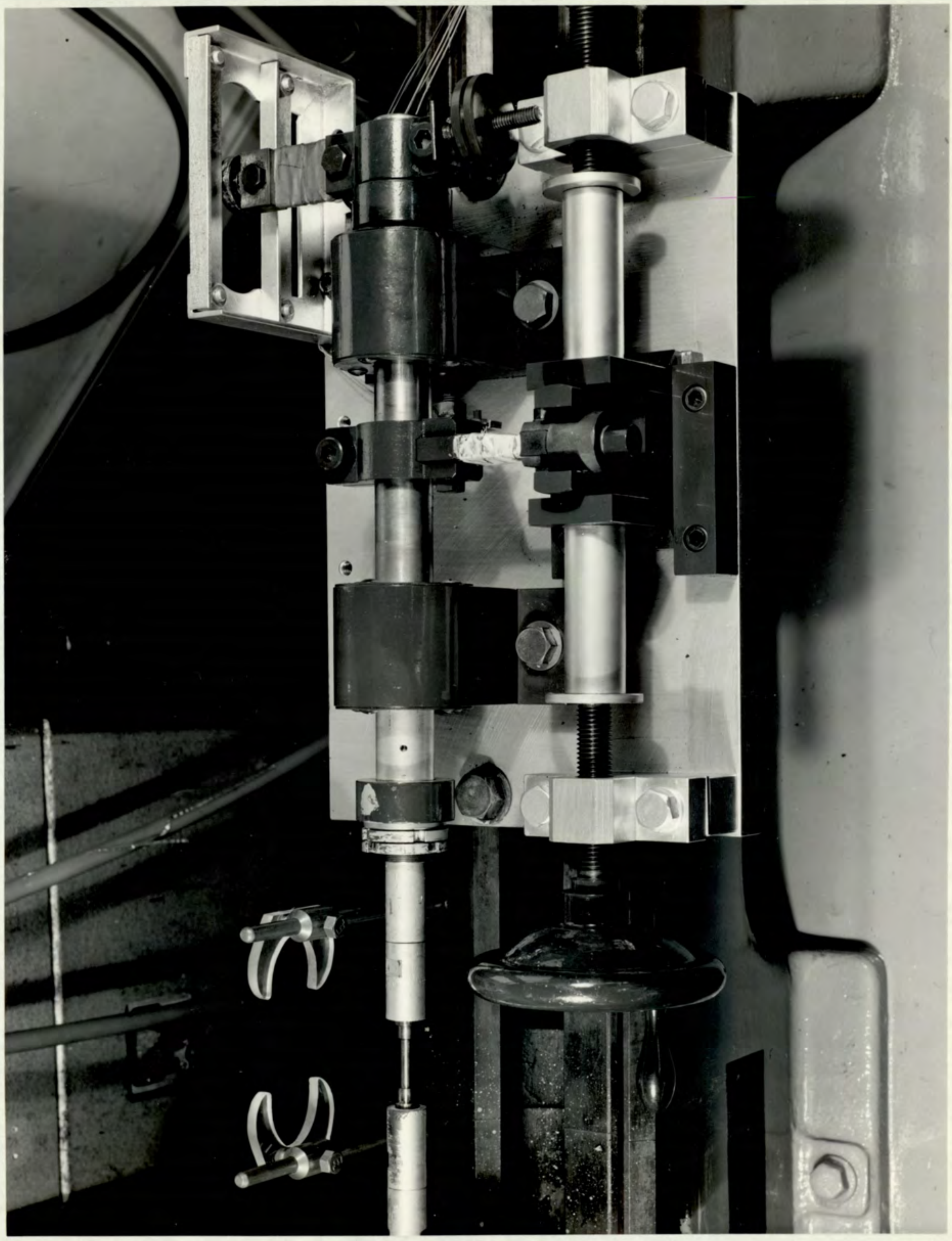


Plate 3.2.
Close up of
Load Measuring
Unit.



of axial strains involved in Dragan's investigations, were not used. Effectively, therefore, the apparatus comprised a rotating shaft driven by an electric motor through a clutch/brake and gear-box mounted at one end of a lathe bed. A movable table, attached to the lathe bed, carried a similar shaft, aligned with the first, and leading to two cantilever beams, one opposing the torsional and the other opposing the axial movement of the shaft. The beams were attached to the shaft by split collars so that the whole assembly could be locked when the specimen was in place, screwed into and connecting the two shafts, avoiding the imposition of stresses prior to testing.

The loads developed during the test were measured by means of strain gauges attached to the cantilever beams. The signals from a constant voltage source, via the strain gauges, were recorded using a Souther Electronics Ltd. S.E. 2005 Ultra-Violet recorder with rotating-mirror c-40 type galvanometers.

As the nominal speeds provided by the lathe gear-box were not accurately listed and depended to some extent upon local voltage fluctuations, the revolutions per minute of the drive shaft were recorded during each test, by means of a sliding electrical contact acting on a split copper disc which was attached to the shaft. Contact was broken at the split in the disc once in each revolution, causing a break in an otherwise continuous line produced on the output from the U.V. recorder.

The U.V. - sensitive photographic paper providing the record could be output from the recorder at any of fifteen discrete speeds between 1.25 and 2 x 10^3 mm.sec.$^{-1}$ In addition a timing device providing a line across the chart at intervals of 1.0, 0.1 or 0.01 second. A schematic representation of a typical trace is shown in Fugure 3.2.

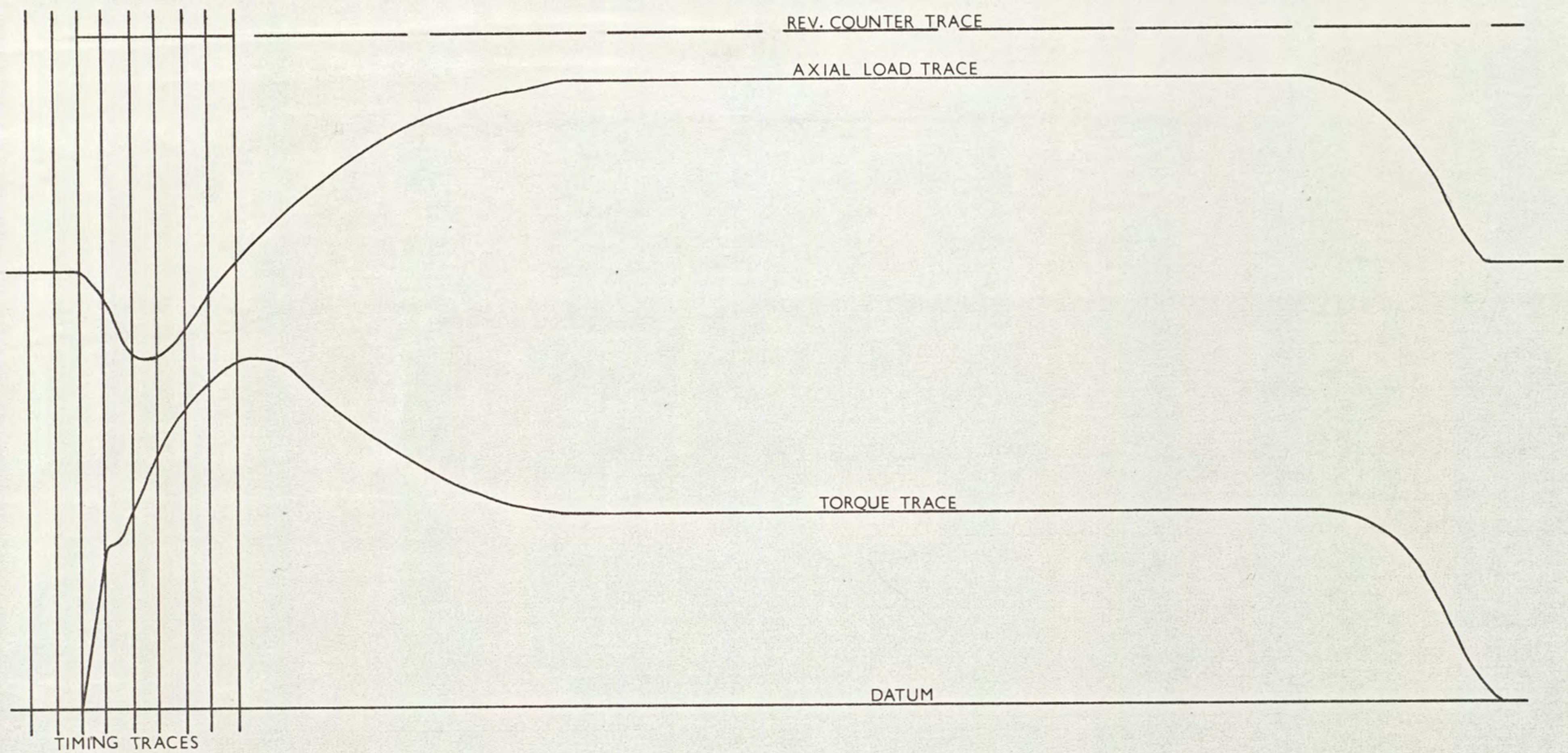


Figure 3.2. Schematic illustration of torsion-test output.

3.4. Grain Size

Grain sizes were measured using a Cambridge Instrument Co. Quantimet Image Analysing Computer. The instrument scans a metal specimen which has been prepared for metallographic examination in conventional manner, and projects an image onto a television screen. A threshold control enables an optical intensity to be selected below which the Quantimet will defect features. The on-line analogue computer registers the vertical height of intersection occurring between the television scanning lines and features of less than the selected threshold intensity. As the horizontal scans are equi-spaced the number of intersections is proportional to the total vertical length projected in the horizontal direction.

The mean linear distance between intersections - D, may then be calculated using the method of Hilliard [62]:

$$D = \frac{L}{P \times M} \qquad (3.1.)$$

where L = the length of a single scan.

P = the meter reading of intersections.

M = the magnification.

If required the mean linear intercept - D may be converted to A.S.T.M. grain size by the equation [62]:

$$\text{A.S.T.M. No.} = (6.64 \log D^{-1}) - 10 \qquad (3.2)$$

This technique makes it necessary to prepare the specimens very carefully for examination as the Quantimet is virtually unable to distinguish between grain boundaries and scratches or similar defects.

In this investigation specimens were mounted in a cold-setting moulding compound and ground by hand on successively finer grades of emery paper, prior to final polishing and etching as in table, 3.3.

Grain sizes, expressed as mean linear intercepts, were determined on specimens cut from the bar stock before machining. Further determinations were carried out on the un-deformed shoulders of testpieces which had been torsion tested at high temperatures. The latter measurements were compared with those made on the original stock using Students' 't' test, to assess whether or not grain growth was likely to have occurred during the test or heating to test temperature. In no case was there any evidence of a significant increase in grain size.

Material	Polish	Etch
Nickel	$\frac{1}{4}$ diamond	Fry's reagent
Aluminium	Electropolish 70% Methyl Alcohol 20% Perchloric Acid 10% Glycerine	Anodise 2% HF. 49% Methyl Alcohol 49% water 30v for 10 mins.
Copper and copper alloys	Electropolish 30% Phosphoric Acid	alcoholic ferric chloride

Table 3.3. Preparation of Metallographic Specimens

3.5. Elevated Temperature Tests

3.5.1. Introduction

Tests at greater than room temperature were facilitated by heating the torsion test-piece in situ, using a 3 KW, 4KHz high frequency induction heating unit. The induction coil, which surrounded the test-piece, was of $\frac{1}{8}$ - inch diameter O.F.H.C. tube, and after a series of trials to obtain the best heating rate and temperature distribution the number of turns in the coil was fixed at five, each of $\frac{5}{8}$ - inch internal diameter. This gave an inductive couple of $\frac{1}{8}$ - inch at the test-piece shoulders, compensating the heat loss into the grips. Heating times varied depending upon the temperature of testing and the material being heated, but were never in excess of ten minutes and rarely exceeded five minutes. The greatest difficulty encountered was in heating the pure Cu test-pieces, and for these the induction coil was silver plated to reduce the electrical resistance in the surface and improve the inductive couple.

High frequency induction heating was chosen mainly because the high heating rates avoid grain growth, as shown previously (section 3.4.). In addition the specimen is visible during deformation, it is exposed so that rapid quenching is possible without the need to extricate it from a furnace, and the amount of incidental heat in test-piece holders etc. is minimised.

3.5.2. Temperature Measurement

Difficulties arise in the measurement of temperature when the test-piece is not contained in a furnace, as there is a constant heat loss from the specimen surface. A conventional thermocouple can give rise to considerable variation in the indicated temperature depending upon the degree of contact with the specimen surface

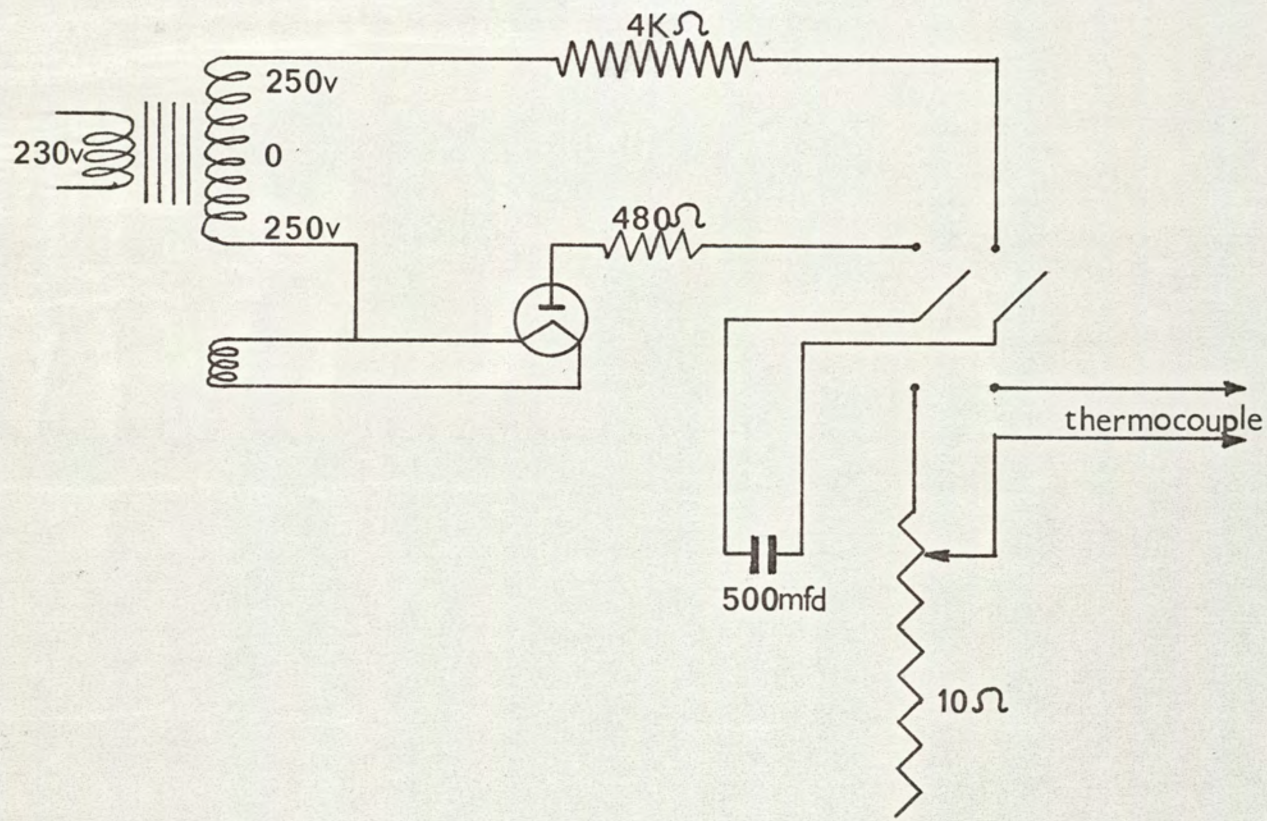


Figure 3.3. Circuit diagram for discharge welder.

the size of thermocouple bead, the temperature gradient across the bead and the position of the true hot junction. In setting-up the equipment for a test, a thermocouple with a flattened bead approximately one-tenth of an inch across and one-hundredth of an inch thick, was held in contact with the specimen surface using a low conductivity ceramic rod. The separations of the turns in the induction coil were then altered until the temperature variation along the test-piece gauge length was less than 5°C.

This type of thermocouple was not suitzble for temperature measurement and control during testing because of the difficulty of maintaining a firm and constant pressure as the test-piece was twisted, and the deflection caused by attempting to do so. The problem was overcome by using an open-ended thermocouple, the arms of a Pt/Pt 13% Rh couple being attached separately to the test-piece surface approximately 0.2 inch apart. This had the added advantage of minimising errors due to the conduction of heat away from the hot junction down the thermocouple arms. The separate arms were attached using a discharge welder, the circuit diagram of which is shown in figure 3.3, in which a condensor was charged and then rapidly discharged through the thermocouple/test-piece interface. The rapid local heating allowed the leads to fuse to the surface of the test-piece. Damage to the test-piece was very slight and failure did not occur in the immediate vicinity of the discharge marks in any of the tests recorded.

3.5.3. Temperature Control

The temperature was controlled by a saturable reactor in series with the high-frequency unit. The circuitry is shown schematically in figure 3.4.

The output current from the test-piece thermocouple was led to a moving coil galvanometer which gave temperature indications on a conventional curved scale. Attached to the galvanometer, behind the scale, was a small metal flag. A movable pointer enabled the desired temperature to be selected and a photo-electric cadmium cell was attached to the pointer, together with a light source. The output from the photo-electric cell was amplified and fed to the reactor core. By saturating the core this amplified d.c. current allowed the full mains output to be led into the high-frequency unit.

As the thermocouple approached the selected temperature the flag on the galvanometer was interposed between the cadmium cell and the light source reducing the output to the magnetic applifier and thence to the core. The drop in inductive saturation of the core then reduced the flow of current to the high-frequency unit.

As the cadmium cell output was progressively reduced, rather than simply being switched on and off, the high-frequency output was under fine control and temperature fluctuations were generally not greater than $\pm$ 1°C.

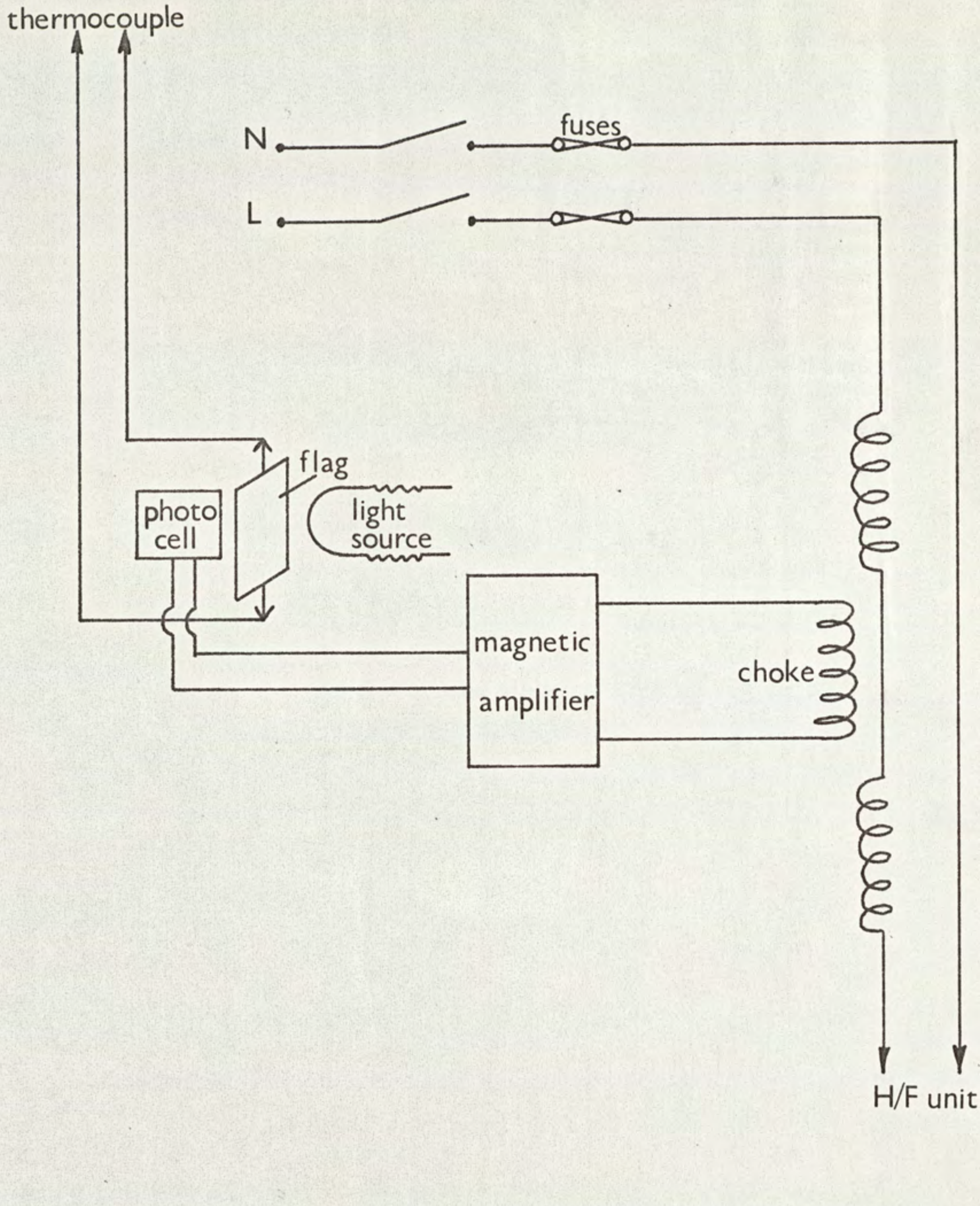


Figure 3.4. Scehmatic diagram of saturable reactor circuit.

3.6. Low Temperature Tests

In order to cover as wide a range of temperature as possible some tests were carried out at less than room temperature. This was facilitated by immersing the test-piece, and a length of the grips, in an appropriate coolant. A diagram of the apparatus is shown in figure 3.5.

The low temperature bath was filled with coolant and the assembly allowed to stand until the fluid was completely quiescent with no indication of boiling in the vicinity of the test-piece. The test was then carried out with the bath still full of coolant. Two coolants only were used giving the following test temperatures:

Liquid Nitrogen	$77^{o}K$
Solid CO_2 in Methyl Alcohol	$198^{o}K$

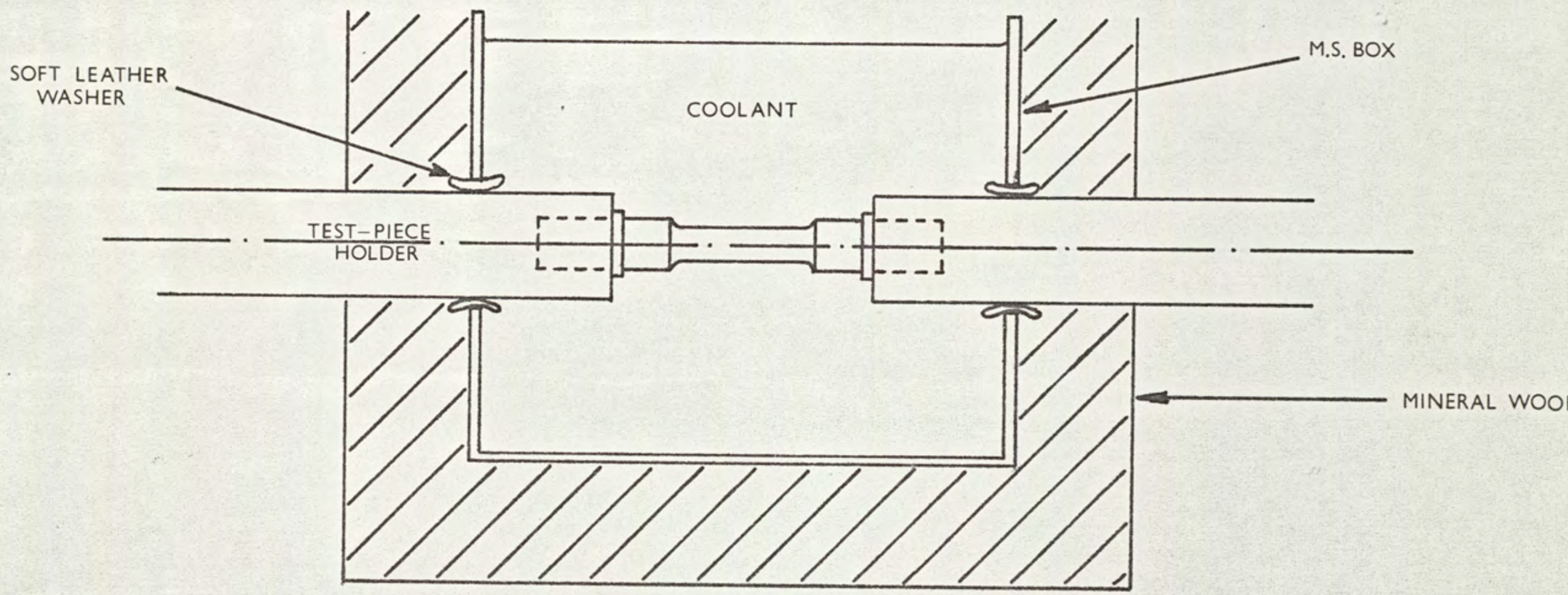


Figure 3.5. Scehmatic drawing of low temperature test bath.

3.7. Strain Rate

The range of strain rates available on the torsion machine used in this investigation was from 17 $min.^{-1}$ to 390 $min.^{-1}$ (i.e. 0.28 $sec.^{-1}$ to 6.5 $sec.^{-1}$). While this range is narrow compared with those of various other investigations[9,31,42] it embraces the lower end of the strain rates encountered in mechanical working processes and, as explained in section 2.4.4., a similar range of strain rates has been associated with significant variations of flow stress reported in previous work.

3.8. Experimental Programme

3.8.1. Introduction

The programme of the investigation was basically of partial factorial design, but whereas the ideal configuration calls for all of the independent variables to be orthogonal to one another, practical limitations made some compromise necessary. The most important are detailed in section 3.8.2. below.

In order to study the influence of the material, structure and process variables on the steady state stress - τ_s it was necessary to select those tests in which a clearly discernible steady state region occurred in the stress-strain curve. In tables 3.3. through 3.10. in which the full programme of tests is listed, those which resulted in a clear steady state region are indicated by a letter C, and henceforth are referred to as C-type tests.

Similarly, only those test results which displayed a clear maximum in the stress-strain curve, i.e. those in which the strain to failure - γ_F was greater than the strain to maximum stress - γ_m were suitable for studies involving maximum stress - τ_m. These tests are indicated in the tables 3.3. through 3.10. by the letter B. The B-type tests include all of the C-type tests.

In some tests failure occurred before any clear sign of restoration could be distinguished. A number of these failures occurred in circumstances under which it seemed likely that deformation twinning had taken place viz. low temperatures and high strain rates, with brittle, cleavage-type fracture surfaces. These tests, together with those in which load-drops were observed, apparently similar to those referred to in section 2.2.2., were designated A.

The A-type tests are shown in the following tables, although they were not analysed further since the exact mechanisms involved in the deformation had not been determined at all rigorously.

3.8.2. <u>Limitations Imposed on Programme</u>

The selection of alloys with stacking fault energy greater than approximately 40 ergs. cm^{-2} is difficult as small changes in chemical composition can lead to considerable changes in stacking fault energy when the solute content is low. Clearly the number of suitable pure metals is limited and consequently the intervals between values greater than about 40 $ergs/cm^2$ are large and irregular.

Both the intervals and the minimum and maximum limits of the homologous temperature - θ varied from material to material. The limits were imposed by the capacity of the available heating equipment and types of refrigerant used. Intervals between test temperatures were chosen to give an approximately uniform distribution between the limits, but greater accuracy of temperature measurement was possible when the controller on the saturable reactor was set to the nearest unit of ten degrees Centigrade.

In some cases there was a slight tendency for the tests at high temperature to be associated with high strain rate - $\dot{\gamma}$. This arose because the high number of revolutions to failure under these conditions frequently caused the thermocouple to break, and at low strain rates the temperature was uncontrolled for an unacceptably long period. There is, similarly, a slight imbalance due to the tendency of some materials to low ductility at low temperatures and strain rates. Examination of tables 3.14 and 3.15 reveals that the coefficients of correlation between temperature and strain rate were still very small, however.

Orthogonality was sometimes unobtainable because variables were not truly independent. For example, having chosen a material for the required value of stacking fault energy and selected the test temperature the value of elastic modulus is fixed. Similarly the electron/atom ratio could not be selected independently of stacking fault energy and elastic modulus, although the very low correlation with the former is perhaps surprising.

Material	-	Ni.
Melting temperature	-	1728 $^{\circ}K$
Stacking fault energy	-	240 $ergs.cm.^{-2}$
Electron/atom ratio	-	0.60.
Burgers vector	-	2.49A
Grain size	-	0.078 cm.

		Shear Strain Rate (Min^{-1})			
θ	$G \times 10^{-2}$ (t.s.i.)	17	110	150	245
0.172	53.64	A A	A		A
0.274	50.53		B		
0.395	46.36	C	C		B
0.621	37.12	B			C
0.708	32.98				C

Table 3.4. Experimental design for nickel tests.

Material	-	Al
Melting temperature	-	933 °K
Stacking fault energy	-	135 ergs. cm^{-2}
Electron/atom ratio	-	3.00
Burgers vector	-	2.87A
Grain size	-	0.073cm

θ	$G \times 10^{-2}$ (t.s.i.)	Shear strain rate (min^{-1}) 17	110	150	245
0.083	18.08	A	A		A
0.212	17.29	A	B		B
0.314	16.59				B
0.319	16.55	B		B	B
0.561	14.57		C	C	
0.721	12.97		C	C	B
0.829	11.74		C		
0.936	10.38			C	

Table 3.5. Experimental design for aluminium tests.

Material	-	Cu.
Melting temperature	-	1356 °k
Stacking fault energy	-	85 ergs. cm.$^{-2}$
Electron/Atom ratio	-	1.00
Burgers vector	-	2.55A
Grain size	-	0.031 cm.

		Shear Strain Rate (min.$^{-1}$)			
θ	G x 10^{-2} (t.s.i.)	17	110	150	245
0.057	34.96	A		A	A
0.219	32.42	B		B	B
0.386	29.34	C		C	C
0.496	27.00	C		C	C
0.533	26.16	C			
0.570	25.28	C		C	C
0.644	23.44				C
0.718	21.45				C
0.791	19.32				C
0.865	17.02				C
0.902	15.80				C

Table 3.6. Experimental design for copper tests - (a)

Material	-	Cu.
Melting temperature	-	1356°K
Stacking fault energy	-	85 ergs. cm.$^{-2}$
Electron/atom ratio	-	1.00
Burgers vector	-	2.55A
Grain size	-	D (cm)

θ	G x 10^{-2} (t.s.i.)	Shear Strain Rate (min^{-1}) 24.5	245
D = 0.046			
0.218	32.42		B
0.386	29.34	C	
0.496	27.00	C	
D = 0.052			
0.386	29.34	C	
0.496	27.00	C	
D = 0.059			
0.218	32.42		B
0.386	29.34	C	
0.496	27.00	C	

Table 3.7. Experimental design for copper tests - (b)

Material	-	Cu/11% Zn.
Melting temperature	-	1293
Stacking fault energy	-	42 ergs. cm.$^{-2}$
Electron/atom ratio	-	1.11
Burgers vector	-	2.48A
Grain size	-	0.025 cm.

θ	G x 10^{-2} (t.s.i.)	17	60	110	150	245	390
0.060	31.96	A			A		
0.227	29.62	A			B	A	
0.230	29.57	B					
0.366	27.34			B		B	
0.520	24.41		AA	A		B	
0.598	22.76	B		B			
0.737	19.47					C	
0.753	19.07	B		C			
0.799	17.85	B					C

Table 3.8. Experimental design for copper/10% zinc tests.

Material	-	Cu/20% Zn
Melting temperature	-	$1240^{\circ}K$
Stacking fault energy	-	26 ergs. $cm.^{-2}$
Electron/atom ratio	-	1.20
Burgers vector	-	2.42A
Grain size	-	0.022 cm.

θ	$G \times 10^{-2}$ (t.s.i.)	17	40	60	245	390
0.239	27.33		B			
0.381	25.19				A	
0.543	22.38	B			B	
0.623	20.79	C		C		
0.704	19.07	C				
0.785	17.21					C
0.849	15.61	C				

Table 3.9. Experimental design for copper/20% zinc tests.

Material - Cu/25% Zn
Melting temperature - 1208 °K
Stacking fault energy - 19 ergs. $cm.^{-2}$
Electron/atom ratio - 1.26
Burgers vector - 2.37A
Grain size - 0.045 cm.

θ	G x 10^{-2} (t.s.i.)	Shear Strain Rate ($min.^{-1}$) 17	110	245	390
0.247	26.01			B	
0.392	23.95	B		A	
0.557	21.12	B	B		
0.640	19.66		C	C	C
0.723	17.90				C
0.805	16.15				C
0.888	14.16				C

Table 3.10. Experimental design for copper/25% zinc tests

Material	-	Cu/4% Al.
Melting temperature	-	1330 °K
Stacking fault energy	-	10 ergs. cm.$^{-2}$
Electron/atom ratio	-	1.18
Burgers vector	-	2.56A
Grain size	-	0.017 cm.

θ	$G \times 10^{-2}$ (t.s.i.)	Shear Strain Rate (min^{-1}) 17	110	150	245
0.223	31.23	A	A	A	
0.356	28.92	B	B	B	
0.506	25.91	C	C		B
0.656	22.41	C	C		C
0.957	13.61			C	

Table 3.11. Experimental design for copper/4% aluminium tests.

3.9. Experimental Method

The procedure for torsion testing a specimen and analysing the results as detailed in the following sections is for tests carried out at greater than room temperature. Apart from the steps directly concerned with heating the test-piece, however, all of the tests were carried out by the same technique.

3.9.1. Setting-up

One of the threaded ends of the test-piece was screwed into the holder at the end of the load-measuring assembly, by gripping the half-inch diameter collar adjacent to the thread. The test-piece was tightened until the screw-in load was sufficient to cause a full-scale deflection of the galvanometer trace. The split collars were then released and the table carrying the load measuring assembly slid along, until the test-piece passed through the induction coil before being screwed into the holder attached to the lathe spindle. The force necessary to screw the test-piece in at this end, sufficiently to avoid further movement during the test, could only be estimated from that needed at the end already attached.

The Pt/Pt-Rh thermocouple was threaded into the induction coil, parallel with its longitudinal axis, and welded in place by charging the welder and then pressing the ends of the leads against the test-piece surface using a porcelain rod between the turns of the coil. The leads of the thermocouple were then attached to the saturable reactor, which was set to the appropriate test temperature.

Finally the appropriate test speed was selected on the lathe gear box, and the paper output speed and timing interval on the ultra-violet recorder were set to give timing marks separated by between two and five millimetres.

3.9.2. Testing

The induction heating unit was switched on and the test-piece raised to test temperature. Between one and two minutes were allowed for the temperature to stabilise and the locking collars on the load measuring assembly were tightened. The paper output on the ultra-violet recorder was switched on and the motor was engaged by the clutch, driving the lathe spindle and twisting the test-piece.

Failure of the test-piece was indicated by the galvanometer trace on the recorder dropping rapidly to its datum line. When this occurred the induction heating unit was immediately switched off and the test-piece quenched with water between the turns of the induction coil.

3.9.3. Translation

From the ultra-violet recorder chart the height of the torque trace shown in figure 3.2. was measured along selected timing trace lines. The lines were arbitrarily selected, close together where the curve sloped steeply and further apart elsewhere.

The deviations of the axial load trace corresponding with each torque trace measurement were also noted.

The point at which the torque trace first deviated from its datum was taken as the datum point for the strain measurements. The chart length was measured between this datum and each timing trace at which a torque reading was taken. The length along the chart to the point of failure was also recorded.

When the measurements from a test record had been assembled a heading was added giving details of the material, chart speed, torque calibration constant, axial load calibration constant and test conditions. The total number of observations and the number of the first reading clearly identifiable as being within the parabolic region of the curve were also added.

The complete set of data was then punched on to five-hole paper tape in preparation for processing by computer. A typical example of one set of data is given in table 3.16.

3.9.4. Computation

The purposes of computation were to convert the test data to shear stress - shear strain data, and to derive the parameters which define the stress-strain curve.

Values of shear stress - were calculated using the method of Sellars and Tegart[12], i.e.

$$= \frac{3T}{2\pi r^3} \qquad \text{———— (3.3.)}$$

where T = torque

and r = specimen radius.

The value of T was determined from the height of the torque trace - h and the calibration factor - K.

$$T = h.K \qquad \text{———— (3.4.)}$$

Equation (3.3.) then becomes:

$$= \frac{3hK}{2\pi r^3} \qquad \text{———— (3.5.)}$$

Since all of the values except h are constants this may be re-written in two stages, thus:

$$\frac{3K}{2\pi r^3} = \text{const.} \qquad (3.6.)$$

$$h \times \text{const.} = \tau \qquad (3.7.)$$

or, in ALGOL computer language

```
CONST: = (3 * K) / (2* 3.14159 * R * * 3)     (3.8.)
TORR:  = H * CONST                            (3.9.)
```

Shear strain - was calculated[27] using the equation

$$\gamma = \frac{r\theta}{l} \qquad (3.10.)$$

where θ = specimen rotation in radians.

l = gauge length.

was determined from readings of chart length - d, thus:

since 2π radians = 360°

$$\theta = 2\pi \times \frac{(\text{machine speed in r.p.m.})}{(\text{time in minutes})} \qquad (3.11.)$$

$$\theta = 2\pi\,(\text{r.p.m})\,/\,\text{d.}\ \left(\frac{1}{\text{chart speed}}\right) \qquad (3.12.)$$

$$\theta = 2\pi \times (\text{r.p.m.}) \times \text{chart speed/d} \qquad (3.13.)$$

As in the calculations of shear stress, all of the values except one are constant and equation (3.13.) may be re-written:

$$\frac{2\pi \times (\text{r.p.m.}) \times \text{chart speed} \times r}{l} = \text{const.} \qquad (3.14.)$$

$$\frac{\text{const.}}{d} = \gamma \qquad (3.15.)$$

or, in ALGOL:

```
CONST 2 : = 2 * 3.14159 * RPM * CS * R/L     (3.16.)
GAMMA   : = CONST 2/D                        (3.17.)
```

From the values of stress and strain computed as shown above, using the program in Appendix B, the parameters chosen to define the stress-strain relationship were extracted. The values of maximum stress - τ_M strain at maximum stress - γ_M and the steady state stress - τ_S were read directly from the computer output.

Using the method of least squares the coefficients of the equation:

$$\tau = b_o + b_1\sqrt{\gamma} \qquad \text{———— (3.18.)}$$

were found. The value of b_o was recorded as τ_o - the intercept of the extrapolated parabolic curve with the stress axis, and the value of b_1 was recorded as K - the slope of the curve.

When all of the parameters were available for all of the tests these were punched onto five-hole paper tape, together with the appropriate identification and data from the independent variables, for analysis.

N	T	R	r	l	K	S	KAX	CODE
32	+196	22.2	0.25	1.0	31.8	200.0	0.0318 £	AL

h	d	a	
2.0	0.53	0.0	
4.0	0.91	0.04	
6.0	0.98	0.03	
8.0	0.99	0.02	
10.0	1.02	0.0	
20.0	1.07	-0.01	
30.0	1.12	-0.02	
40.0	1.16	-0.03	
50.0	1.21	-0.03	
100.0	1.38	-0.12	
150.0	1.54	-0.12	
200.0	1.66	-0.31	
250.0	1.77	-0.38	
300.0	1.86	-0.45	
350.0	1.95	-0.49	
400.0	2.03	-0.55	
500.0	2.16	-0.73	
600.0	2.25	-0.95	
700.0	2.32	-1.15	
800.0	2.39	-1.32	
900.0	2.44	-1.47	
980.0	2.49	-1.56	
1500.0	2.69	-1.89	
2000.0	2.88	-1.72	
2500.0	3.07	-1.26	
3000.0	3.25	-0.90	
3500.0	3.41	-0.62	
4000.0	3.54	-0.56	
4500.0	3.68	-0.56	
5000.0	3.79	-0.74	
5500.0	3.89	-0.90	
5575.0	3.90	-0.95	5575.0

N = no. of observations
T = temperature
R = rev./min.
r = radius
l = length
K = calibration factor
S = chart speed
KAX = axial load calibration factor

Table 3.16. Typical data from torsion test

3.10. Method of Analysis

In order to carry out the regression analysis a system of computer subroutines were developed. They employ the I.C.L. Egdon version of FORTRAN (known as EGTRAN) and make use of standard routines in the Egdon library for such facilities as input, output, function generation etc. The package of subroutines, named STATPAC, are presented in Appendix C, the purpose of each subroutine is outlined in 3.10.1.

3.10.1. The STATPAC Subroutines

The following subroutines, included in the STATPAC package, were developed using the equations presented in section 2.5.

INPUT 1 — reads in the number of observations and the number of variables, followed by the (square) matrix of correlation coefficients.

INREG — reads in the subscript of the dependent variable, the number of independent variables and the subscripts of each of them.

START — begins the process of matrix inversion by making r^{11} equal to 1.0 (i.e. the reciprocal of r_{yy}.)

ADDVAR — increases the order of the reciprocal, matrix by one, using the 'add-a-variate' method of Woolf[85].

COEFFS — calculates the regression coefficients as in equation 2.31, the standard errors of the coefficients, using equation 2.34 and the constant term - b_o.

ANOVA — carries out the analysis of variance, partitioning the total sum of squares into those due to the regression and the residual. The variance ratio - F is computed using the residual mean square as an estimate of the 'pure error' variance.

ELANAL performs an element analysis using the Newton and Spurrell method[91,92], and calls upon the following four subroutines in order to do so.

PRIMEL calculates the primary element, i.e. the additional sum of squares, from: b_i^2 / r^{ii} after the values in the reciprocal matrix have been corrected for the presence of the dependent variable.

REMVAR removes any nominated variable from the reciprocal matrix.

PRIMT prints out the primary elements and the ratio of the additional mean square (d.f. = 1) to the residual mean square.

PREST calculates the secondary elements as in references 91 and 92 and prints them out.

COVAR computes the matrix of covariances from the reciprocal matrix.

3.10.2. The Selection Process

The first stage adopted in the selection process was to specify the dependent variables and all of those independent variables which might be used as predictors.

The dependent variables were designated:

1. The intercept stress - τ_o
2. The maximum stress - τ_m
3. The steady-state stress - τ_s
4. The work-hardening coefficient - K
5. The strain at maximum stress - γ_m.

The independent variables were all based upon nine initial parameters and were functions of from one up to five of these. The initial parameters were:

(a) atomic proportion of solute - At

(b) melting temperature - Tm

(c) Burger's vector - b

(d) stacking fault energy - γ_{SF}

(e) modulus of shear - G

(f) electron/atom ratio - ea

(g) grain size - D

(h) test temperature (absolute) - T

(j) strain rate - $\dot{\gamma}$

A number of the functions were based on the parameters discussed in section 2, while others were simply product terms which might be expected to make a contribution by 'correcting' non-linear relationships or where interactions between independent variables occur.

The complete list of variables is given in table 3.17, the numbers by which they are designated have no significance and arise entirely from a convenient series of computations.

No attempts were made to classify the variables in terms of the likelihood of being effective predictors. It was considered, on the basis of the evidence presented in the literature review, that the modulus of shear - G and the Burger's vector - b were more likely to provide a suitable description of the chemical composition. Stacking fault energy has a special role in the deformation of f.c.c. metals, and it was considered that some function of this property was extremely likely to make a significant contribution to the model.

It was not considered that sufficient evidence exists for clear distinctions to be made between many of the other variables listed in table 3.17.

The matrix of correlation coefficients required by the STATPAC routines was computed from the raw data using library programs on the I.C.L. KDF9 computer at the University of Birmingham, and output on punched cards. The punched cards were then used as input media for the selection stage.

The correlation matrices for the B-type and C-type tests, respectively, are presented, together with the means and standard deviations of the variables, in tables 3.12 and 3.15.

The first variable entered into the regression equation was that having the highest correlation coefficient in association with the dependent variable. The equation relating the dependent to the independent variable was computed followed by the analysis of variance. Each of the variables not included in the equation was then considered in turn and the additional sum of squares which would result from adding that variable to the equation was calculated, together with the secondary element. The variable making the greatest contribution was added to the equation and the full equation, analysis of variance and element analysis were computed and printed. If any independent variable had ceased to make a significant contribution, following the addition of the newest variable to the equation, it was deleted. Otherwise the cycle was repeated until no variable was omitted which could make a significant contribution to the regression sum of squares.

Up to ten variables might be expected to make a contribution and it was decided to include only those whose primary element was significant at the 0.05 level of probability. This would require a ratio between the primary element (or additional sum of squares) and the residual mean square of at least 4.0. That is to say that the 'Partial F' value would need to equal or exceed 4.0.

1. τ_o
2. τ_m
3. τ_s
4. K
5. γ_m
6. At
7. T
8. b
9. $1/\gamma_{sf}$
10. G
11. e
12. $D^{-\frac{1}{2}}$
13. θ
14. $\ln\dot{\gamma}$
15. G/γ_{sf}
16. G^2/γ_{sf}
17. $At.D^{-\frac{1}{2}}$
18. $T.D.^{-\frac{1}{2}}$
19. $b.D^{-\frac{1}{2}}$
20. $D^{-\frac{1}{2}}/_{sf}$
21. $G.D^{-\frac{1}{2}}$
22. $e\ .D^{-\frac{1}{2}}$
23. $\theta.D^{-\frac{1}{2}}$
24. $\ln\dot{\gamma}.D^{-\frac{1}{2}}$
25. $At.\theta$
26. $T.\theta$
27. $b.\theta$
28. θ/γ_{sf}
29. $G.\theta$
30. $e\ .\theta$
31. $\ln\dot{\gamma}.\theta$
32. $At.\ \ln\dot{\gamma}$
33. $T.\ \ln\dot{\gamma}$
34. $b.\ \ln\dot{\gamma}$
35. $\ln\dot{\gamma}/\gamma_{sf}$
36. $G.\ \ln\dot{\gamma}$
37. $e\ .\ln\dot{\gamma}$
38. $G.D_d^{-1/5}$
39. $G.b/\gamma_{sf}.$
40. G.b
41. $G.b.D^{-\frac{1}{2}}/\gamma_{sf}$
42. $G.b.D^{-\frac{1}{2}}$
43. $G.b^2.D^{-\frac{1}{2}}$
44. $G.b^2/\gamma_{sf}$

Table 3.17 Variables to be considered in regression analysis.

Note: The values indicated by G are in fact $G \times 10^{-2}$.

MEANS		STANDARD DEVIATIONS	
1	4.1153	1	2.8120
2	7.9439	2	4.6153
3	2.7620	3	3.1015
4	3.6321	4	2.5225
5	2.8874	5	2.8952
6	7.5746	6	9.3293
7	681.4118	7	263.8678
8	2.5484	8	0.1443
9	0.0300	9	0.0298
10	24.2685	10	7.8798
11	1.3353	11	0.6944
12	5.4175	12	1.3227
13	0.5309	13	0.1985
14	4.4976	14	1.1388
15	0.6945	15	0.7375
16	16.9296	16	19.4994
17	45.2452	17	52.1914
18	3762.9841	18	1839.7829
19	13.7308	19	3.1803
20	0.1895	20	0.2278
21	130.4300	21	44.9250
22	6.8755	22	2.4611
23	2.9191	23	1.3882
24	24.1726	24	8.1901
25	4.4867	25	6.0643
26	410.1920	26	276.8439
27	1.3485	27	0.5045
28	0.0167	28	0.0182
29	12.0165	29	3.8848
30	0.7046	30	0.4668
31	2.4206	31	1.2204
32	34.0634	32	45.7191
33	3094.0449	33	1586.4706
34	11.4784	34	3.0174
35	0.1337	35	0.1391
36	107.6511	36	42.9327
37	6.1262	37	3.8638
38	7572.9052	38	27994.7384
39	1.7414	39	1.8765
40	61.4829	40	19.2767
41	11.1356	41	14.7025
42	329.4981	42	111.3896
43	74.3082	43	115.1737
44	0.7440	44	1.5888

Table 3.12 Means and Standard Deviations - B-type tests.

MEANS		STANDARD DEVIATIONS	
1	2.8975	1	1.8099
2	5.6842	2	3.1565
3	4.5202	3	2.7862
4	2.9725	4	2.0333
5	136.7837	5	961.1163
6	6.6275	6	9.4720
7	803.5769	7	199.6864
8	2.5525	8	0.1404
9	0.0286	9	0.0294
10	22.6879	10	7.6463
11	1.3144	11	0.6840
12	5.3604	12	1.2881
13	0.6302	13	0.1588
14	4.5703	14	1.1305
15	0.6047	15	0.6483
16	13.4743	16	15.2555
17	39.3394	17	53.1123
18	4378.5515	18	1706.5639
19	13.6141	19	3.1081
20	0.1797	20	0.2243
21	120.3534	21	40.6473
22	6.7150	22	2.4537
23	3.4079	23	1.2757
24	24.3386	24	8.1184
25	4.7338	25	6.9168
26	532.4564	26	250.4250
27	1.6095	27	0.4229
28	0.0190	28	0.0205
29	13.3812	29	2.8592
30	0.8618	30	0.5728
31	2.9659	31	1.2260
32	32.1742	32	49.7986
33	3766.8556	33	1566.1437
34	11.6661	34	2.9234
35	0.1313	35	0.1392
36	100.7534	36	36.7283
37	6.1311	37	3.7817
38	6.1236	38	11.7223
39	1.5172	39	1.6458
40	57.5744	40	18.8445
41	9.6063	41	12.9163
42	304.7089	42	101.5063
43	63.6521	43	101.2923
44	0.6308	44	1.3667

Table 3.13 Means and Standard Deviations - C-type tests.

4. RESULTS

Since very few 'true' replicate tests were carried out estimates of the 'true' error variance were made using near replicates. The effect of strain rate has been shown to be very small and the contribution made by the untransformed values of this parameter proved to be insignificant. Estimates of the error variance, therefore, based on tests which vary only in terms of strain rate are presented in table 4.1.

The results of the regression analyses are contained in the sets of tables numbered 4.2. through 4.8. The values are presented as a decimal quantity multiplied by a stated power of 10 (E - format in FORTRAN computer language).

In table 4.2 the regression equation for the maximum stress - τ_m is shown for the B-type tests, at each stage of development. Accompanying the equation, at each stage, is the overall analysis of variance and, on the facing page, the results of element analysis. In the interests of clarity the alternative variables with their associated primary and secondary elements have not been shown. The variable added to the equation at each stage was usually that with the highest primary element. In a very small number of cases each of two alternative variables was added separately, but invariably one of the two was clearly established as the more significant at a later stage of the analysis.

In table 4.3 the results are presented in similar manner for the analysis concerning the intercept stress - τ_o, and in table 4.4 those for the steady-state stress - τ_s for the C-type tests.

Tables 4.5 and 4.6 contain the analyses for the maximum and intercept stresses, respectively, for the C-type tests. The results of element analysis are not presented in total for these tests because of their similarity to their counterparts for the B-type tests.

The work-hardening coefficient - K is considered for the C-type tests in table 4.7 and for the B-type tests in 4.8. In each of these tables alternative solutions are presented, those in table 4.7 arising as a normal consequence of the analytic technique, but those in table 4.8 being a check for confirmation over the wider temperature range covered by the B-type tests.

The numbers of the variables are as in table 3.11, but for clarity the variables appearing in the following tables are listed here with their identifying numbers:

1.	τ_o	27.	$b.\theta$
2.	τ_m	29.	$G.\theta$
3.	τ_s	36.	$G.\ln\dot{\gamma}$
4.	K	38.	$G.D_d^{-1/5}$
8.	b	39.	$G.b/\gamma_{sf}$
13.	θ	41.	$G.b.D^{-\frac{1}{2}}/\gamma_{sf}$
16.	G^2/γ_{sf}	42.	$G.b.D^{-\frac{1}{2}}$

Source	Sum of Squares	d.f.	Mean Square
τ_o (total)	664.22	84	7.91
τ_o (error)	180.63	35	2.15
τ_m (total)	1789.28	84	21.30
τ_m (error)	57.31	35	0.68
K (total)	534.49	84	6.36
K (error)	97.24	35	2.78
τ_o (total)	167.06	51	3.28
τ_o (error)	81.67	23	1.60
τ_m (total)	508.14	51	9.96
τ_m (error)	29.84	23	0.59
τ_s (total)	359.91	51	7.76
τ_s (error)	38.99	23	0.76
K (total)	210.85	51	4.13
K (error)	29.15	14	2.08

Table 4.1 Estimates of total and 'error' mean square values.

TABLE 4.2.

Regression equations for maximum stress - B-type tests

On alternate pages are presented (a) the primary and secondary elements and (b) the regression equation and analysis of variance.

The regression coefficients are presented in the column headed 'B' and the standard error in the column headed 'S.E.(B)'.

All of the non-integer values are presented in the FORTRAN E-format that is to say as a decimal value raised to the indicated power of ten.

The variables are identified by their index numbers as in table 3.17, the relevant values being repeated on page 85.

ELEMENT ANALYSIS

PRIMARY ELEMENT 27 0.10833+04 PARTIAL F 127.36
SECONDARY ELEMENTS
27
0.

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
2	DEPENDENT	
27	-0.71183D+01	0.63075D+00
CONSTANT	0.17543D+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.108330D+04	1	0.108330D+04
RESIDUAL	0.705980D+03	83	0.850579D+01
TOTAL	0.178928D+04	84	0.213010D+02
F VALUE	127.36		
MULTIPLE R	0.7781		
R SQUARED	0.6054		

ELEMENT ANALYSIS

```
PRIMARY ELEMENT     27        0.10899D+04                              PARTIAL F      215.60
SECONDARY ELEMENTS
  27          8
  0.          -0.66028D+01

PRIMARY ELEMENT      8        0.29146D+03                              PARTIAL F       57.66
SECONDARY ELEMENTS
  27          8
 -0.66028D+01  0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
2	DEPENDENT	
27	-0.71400D+01	0.48627D+00
8	-0.12909D+02	0.17001D+01
CONSTANT	0.50469D+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.137476D+04	2	0.687381D+03
RESIDUAL	0.414522D+03	82	0.505514D+01
TOTAL	0.178928D+04	84	0.213010D+02
F VALUE	135.98		
MULTIPLE R	0.8765		
R SQUARED	0.7683		

ELEMENT ANALYSIS

```
PRIMARY ELEMENT    27     0.88243+03                         PARTIAL F   297.86
SECONDARY ELEMENTS
  27          8           16
  0.          -0.38718+02  0.20748+03

PRIMARY ELEMENT    8      0.18521+03                         PARTIAL F    62.52
SECONDARY ELEMENTS
  27          8           16
 -0.38718+02  0.          0.10625+03

PRIMARY ELEMENT    16     0.17456+03                         PARTIAL F    58.92
SECONDARY ELEMENTS
  27          8           16
  0.20748+03  0.10625+03  0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
2	DEPENDENT	
27	-0.65570D+01	0.37992D+00
8	-0.10569D+02	0.13367D+01
16	0.77393D-01	0.10082D-01
CONSTANT	0.42410D+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.154932D+04	3	0.516440D+03
RESIDUAL	0.239965D+03	81	0.296253D+01
TOTAL	0.178928D+04	84	0.213010D+02
F VALUE	174.32		

MULTIPLE R	0.9305
R SQUARED	0.8659

ELEMENT ANALYSIS

```
PRIMARY ELEMENT      27        0.81467+02                                  PARTIAL F      54.32
SECONDARY ELEMENTS
  27           8             16            13
  0.           0.85650+02    0.26017+02    0.80096+03

PRIMARY ELEMENT       8        0.23602+03                                  PARTIAL F     157.37
SECONDARY ELEMENTS
  27           8             16            13
  0.85650+02   0.            0.90856+02   -0.50814+02

PRIMARY ELEMENT      16        0.13880+03                                  PARTIAL F      92.55
SECONDARY ELEMENTS
  27           8             16            13
  0.26017+02   0.90856+02    0.            0.35752+02

PRIMARY ELEMENT      13        0.11998+03                                  PARTIAL F      80.00
SECONDARY ELEMENTS
  27           8             16            13
  0.80096+03  -0.50814+02    0.35752+02    0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
2	DEPENDENT	
27	0.31065D+02	0.42150D+01
8	-0.30863D+02	0.24603D+01
16	0.69530D-01	0.72274D-02
13	-0.97147D+02	0.10861D+02
CONSTANT	0.95103D+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.166930D+04	4	0.417325D+03
RESIDUAL	0.119983D+03	80	0.149979D+01
TOTAL	0.178928D+04	84	0.213010D+02
F VALUE	278.26		

MULTIPLE R 0.9659
R SQUARED 0.9329

ELEMENT ANALYSIS

```
PRIMARY ELEMENT      27      0.86115E+02                                          PARTIAL F     62.64
SECONDARY ELEMENTS
  27             8              16              13              29
  0.          0.47194E+02   0.27293E+02   0.47045E+03  -0.46480E+01

PRIMARY ELEMENT       8      0.24656E+03                                          PARTIAL F    179.35
SECONDARY ELEMENTS
  27             8              16              13              29
  0.47194E+02  0.            0.94684E+02  -0.75428E+02  -0.10538E+02

PRIMARY ELEMENT      16      0.13473E+03                                          PARTIAL F     98.00
SECONDARY ELEMENTS
  27             8              16              13              29
  0.27293E+02  0.94684E+02  0.             0.36296E+02   0.40745E+01

PRIMARY ELEMENT      13      0.12307E+03                                          PARTIAL F     89.52
SECONDARY ELEMENTS
  27             8              16              13              29
  0.47045E+03 -0.75428E+02  0.36296E+02   0.            -0.30907E+01

PRIMARY ELEMENT      29      0.11376E+02                                          PARTIAL F      8.28
SECONDARY ELEMENTS
  27             8              16              13              29
 -0.46480E+01 -0.10538E+02  0.40745E+01  -0.30907E+01   0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
2	DEPENDENT	
27	0.32055+02	0.40501+01
8	-0.32770+02	0.24470+01
16	0.68580-01	0.69275-02
13	-0.98489+02	0.10409+02
29	-0.12812+00	0.44539-01
CONSTANT	0.10089+03	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.168068+04	5	0.336135+03
RESIDUAL	0.108606+03	79	0.137477+01
TOTAL	0.178928+04	84	0.213010+02
F VALUE	244.50		

MULTIPLE R 0.9692
R SQUARED 0.9393

ELEMENT ANALYSIS

```
PRIMARY   ELEMENT    27        0.77782E+02                                               PARTIAL  F        59.84
SECONDARY  ELEMENTS
  27           8            16            13            29            36
  0.           0.55582E+02  0.29387E+02  0.15740E+03  0.47697E-01  0.83334E+01

PRIMARY   ELEMENT     8        0.22831E+03                                               PARTIAL  F       175.66
SECONDARY  ELEMENTS
  27           8            16            13            29            36
  0.55582E+02  0.           0.98514E+02 -0.49919E+02 -0.14084E+02  0.18245E+02

PRIMARY   ELEMENT    16        0.13932E+03                                               PARTIAL  F       107.19
SECONDARY  ELEMENTS
  27           8            16            13            29            36
  0.29387E+02  0.98514E+02  0.           0.39298E+02 -0.70042E+00 -0.45874E+01

PRIMARY   ELEMENT    13        0.10693E+03                                               PARTIAL  F        82.28
SECONDARY  ELEMENTS
  27           8            16            13            29            36
  0.15740E+03 -0.49919E+02  0.39298E+02  0.           0.64499E+01  0.16137E+02

PRIMARY   ELEMENT    29        0.18329E+02                                               PARTIAL  F        14.10
SECONDARY  ELEMENTS
  27           8            16            13            29            36
  0.47697E-01 -0.14084E+02 -0.70042E+00  0.64499E+01  0.          -0.69523E+01

PRIMARY   ELEMENT    36        0.72281E+01                                               PARTIAL  F         5.56
SECONDARY  ELEMENTS
  27           8            16            13            29            36
  0.83334E+01  0.18245E+02 -0.45874E+01  0.16137E+02 -0.69523E+01  0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
2	DEPENDENT	
27	0.30759D+02	0.39761D+01
8	-0.31906D+02	0.24073D+01
16	0.70026D-01	0.67637D-02
13	-0.93663D+02	0.10326D+02
29	-0.19087D+00	0.50827D-01
36	0.93010D-02	0.39441D-02
CONSTANT	0.97608D+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.168790D+04	6	0.281317D+03
RESIDUAL	0.101378D+03	78	0.129973D+01
TOTAL	0.178928D+04	84	0.213010D+02
F VALUE	216.44		
MULTIPLE R	0.9713		
R SQUARED	0.9433		

TABLE 4.3.

Regression equations for intercept stress - B-type tests

ELEMENT ANALYSIS

PRIMARY ELEMENT 27 0.28531*+03 PARTIAL F 62.50

SECONDARY ELEMENTS

27

0.

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
1	DEPENDENT	
27	-0.36531D+01	0.46209D+00
CONSTANT	0.90415D+01	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.285314D+03	1	0.285314D+03
RESIDUAL	0.378903D+03	83	0.456510D+01
TOTAL	0.664217D+03	84	0.790734D+01
F VALUE	62.50		

MULTIPLE R	0.6554
R SQUARED	0.4295

ELEMENT ANALYSIS

```
PRIMARY ELEMENT      27        0.28727D+03                      PARTIAL F      83.66
SECONDARY ELEMENTS
  27          8
  0.        -0.19597D+01

PRIMARY ELEMENT       8        0.97318D+02                      PARTIAL F      28.34
SECONDARY ELEMENTS
  27          8
 -0.19597D+01  0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
1	DEPENDENT	
27	-0.36657D+01	0.40078D+00
8	-0.74593D+01	0.14012D+01
CONSTANT	0.28068D+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.382632D+03	2	0.191316D+03
RESIDUAL	0.281585D+03	82	0.343396D+01
TOTAL	0.664217D+03	84	0.790734D+01
F VALUE	55.71		
MULTIPLE R	0.7590		
R SQUARED	0.5761		

ELEMENT ANALYSIS

```
PRIMARY ELEMENT    27      0.23907⏨+03                    PARTIAL F     77.84
SECONDARY ELEMENTS
  27           8          16
  0.          -0.12371⏨+02  0.48204⏨+02

PRIMARY ELEMENT     8      0.68871⏨+02                    PARTIAL F     22.42
SECONDARY ELEMENTS
  27           8          16
 -0.12371⏨+02  0.          0.28447⏨+02

PRIMARY ELEMENT    16      0.32800⏨+02                    PARTIAL F     10.68
SECONDARY ELEMENTS
  27           8          16
  0.48204⏨+02  0.28447⏨+02  0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
1	DEPENDENT	
27	-0.34129D+01	0.38684D+00
8	-0.64450D+01	0.13610D+01
16	0.33548D-01	0.10266D-01
CONSTANT	0.24574D+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.415432D+03	3	0.138477D+03
RESIDUAL	0.248785D+03	81	0.307142D+01
TOTAL	0.664217D+03	84	0.790734D+01
F VALUE	45.09		
MULTIPLE R	0.7909		
R SQUARED	0.6254		

ELEMENT ANALYSIS

```
PRIMARY  ELEMENT     27        0.30401■+02                                      PARTIAL  F       11.79
SECONDARY  ELEMENTS
  27              8          16            13
  0.             0.26040■+02  0.65805■+01  0.20867■+03

PRIMARY  ELEMENT      8        0.85011■+02                                      PARTIAL  F       32.97
SECONDARY  ELEMENTS
  27              8          16            13
  0.26040■+02  0.            0.23285■+02  -0.16140■+02

PRIMARY  ELEMENT     16        0.23929■+02                                      PARTIAL  F        9.28
SECONDARY  ELEMENTS
  27              8          16            13
  0.65805■+01  0.23285■+02  0.             0.88715■+01

PRIMARY  ELEMENT     13        0.42494■+02                                      PARTIAL  F       16.48
SECONDARY  ELEMENTS
  27              8          16            13
  0.20867■+03  -0.16140■+02  0.88715■+01  0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
1	DEPENDENT	
27	0.18977E+02	0.55268E+01
8	-0.18523E+02	0.32260E+01
16	0.28869E-01	0.94769E-02
13	-0.57815E+02	0.14242E+02
CONSTANT	0.55933E+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.457927E+03	4	0.114481E+03
RESIDUAL	0.206290E+03	80	0.257863E+01
TOTAL	0.664217E+03	84	0.790734E+01
F VALUE	44.40		

MULTIPLE R 0.8303

R SQUARED 0.6894

ELEMENT ANALYSIS

PRIMARY ELEMENT 27 0.32888+02 PARTIAL F 13.11

SECONDARY ELEMENTS

27	8	16	13	29
0.	0.16230+02	0.68984+01	0.10453+03	-0.24871+01

PRIMARY ELEMENT 8 0.93007+02 PARTIAL F 37.06

SECONDARY ELEMENTS

27	8	16	13	29
0.16230+02	0.	0.24667+02	-0.22579+02	-0.79958+01

PRIMARY ELEMENT 16 0.22571+02 PARTIAL F 8.99

SECONDARY ELEMENTS

27	8	16	13	29
0.68984+01	0.24667+02	0.	0.89452+01	0.13580+01

PRIMARY ELEMENT 13 0.44082+02 PARTIAL F 17.57

SECONDARY ELEMENTS

27	8	16	13	29
0.10453+03	-0.22579+02	0.89452+01	0.	-0.15873+01

PRIMARY ELEMENT 29 0.80501+01 PARTIAL F 3.21

SECONDARY ELEMENTS

27	8	16	13	29
-0.24871+01	-0.79958+01	0.13580+01	-0.15873+01	0.

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
1	DEPENDENT	
27	0.19809D+02	0.54718D+01
8	-0.20127D+02	0.33060D+01
16	0.28070D-01	0.93594D-02
13	-0.58944D+02	0.14063D+02
29	-0.10778D+00	0.60174D-01
CONSTANT	0.60807D+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.465977D+03	5	0.931953D+02
RESIDUAL	0.198240D+03	79	0.250937D+01
TOTAL	0.664217D+03	84	0.790734D+01
F VALUE	37.14		
MULTIPLE R	0.8376		
R SQUARED	0.7015		

```
                                                ELEMENT   ANALYSIS
                                                ------------------

PRIMARY  ELEMENT    27        0.24030D+02                                              PARTIAL  F      11.25
SECONDARY  ELEMENTS
  27          8          16          13          29          36
  0.          0.25134D+02 0.72218D+01 -0.61189D+00 0.33213D-01 0.88580D+01

PRIMARY  ELEMENT     8        0.75252D+02                                              PARTIAL  F      35.24
SECONDARY  ELEMENTS
  27          8          16          13          29          36
  0.25134D+02 0.          0.25555D+02 0.21875D+01 -0.11786D+02 0.17754D+02

PRIMARY  ELEMENT    16        0.27476D+02                                              PARTIAL  F      12.87
SECONDARY  ELEMENTS
  27          8          16          13          29          36
  0.72218D+01 0.25555D+02 0.          0.90907D+01 -0.39129D+00 -0.49049D+01

PRIMARY  ELEMENT    13        0.29074D+02                                              PARTIAL  F      13.62
SECONDARY  ELEMENTS
  27          8          16          13          29          36
 -0.61189D+00 0.21875D+01 0.90907D+01 0.          0.41081D+01 0.15007D+02

PRIMARY  ELEMENT    29        0.28776D+02                                              PARTIAL  F      13.48
SECONDARY  ELEMENTS
  27          8          16          13          29          36
  0.33213D-01 -0.11786D+02 -0.39129D+00 0.41081D+01 0.         -0.20726D+02

PRIMARY  ELEMENT    36        0.31692D+02                                              PARTIAL  F      14.84
SECONDARY  ELEMENTS
  27          8          16          13          29          36
  0.88580D+01 0.17754D+02 -0.49049D+01 0.15007D+02 -0.20726D+02 0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
1	DEPENDENT	
27	0.17097⏨+02	0.50964⏨+01
8	-0.18318⏨+02	0.30856⏨+01
16	0.31098⏨-01	0.86692⏨-02
13	-0.48839⏨+02	0.13235⏨+02
29	-0.23916⏨+00	0.65146⏨-01
36	0.19476⏨-01	0.50552⏨-02
CONSTANT	0.53920⏨+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.497668⏨+03	6	0.829447⏨+02
RESIDUAL	0.166549⏨+03	78	0.213524⏨+01
TOTAL	0.664217⏨+03	84	0.790734⏨+01
F VALUE	38.85		
MULTIPLE R	0.8656		
R SQUARED	0.7493		

TABLE 4.4.

Regression equations for steady-state stress - C-type tests

ELEMENT ANALYSIS

PRIMARY ELEMENT 27 0.26478₁₀+03 PARTIAL F 100.96
SECONDARY ELEMENTS
27
0.

CORRELATIONS

1	1.0000	0.8391	-0.1459	0.2087	0.1070	0.2496	-0.5053	-0.3789	0.2470	0.5318	-0.3433	0.1564	-0.6066	0.0344	0.3557	0.4248	0.2433	-0.3478	0.0982	0.2175	0.5557	-0.2508	-0.4001	0.1635	0.0235	-0.5632	-0.6554	0.0130	-0.2205	-0.5658	-0.4670	0.2414	-0.3949	-0.0548	0.2606	0.4967	-0.2933	0.0548	0.3459	0.4989	0.2932	0.5315	0.2582	0.2293
2	0.8391	1.0000	-0.0737	0.6456	0.2102	0.2136	-0.5953	-0.3990	0.3009	0.6181	-0.4087	0.2660	-0.7256	-0.1472	0.4471	0.5393	0.2290	-0.3884	0.2124	0.2886	0.7303	-0.2546	-0.4573	0.0921	-0.0334	-0.6694	-0.7781	0.0062	-0.2506	-0.6828	-0.6263	0.1617	-0.5311	-0.2312	0.2472	0.4474	-0.4117	0.1345	0.4394	0.5891	0.3958	0.7119	0.3640	0.3250
3	-0.1459	-0.0737	1.0000	0.0946	-0.0360	-0.0711	0.1842	-0.1691	0.1219	0.1437	-0.2929	0.1047	0.1077	-0.2113	0.1151	0.0917	-0.0672	0.1851	0.0887	0.1293	0.1699	-0.2471	0.1413	-0.0717	0.0099	0.0624	0.0824	0.1742	0.3368	-0.1434	0.0016	-0.0413	0.0593	-0.2477	0.0791	-0.0476	-0.3173	-0.2406	0.1170	0.1364	0.1184	0.1709	0.1196	0.1265
4	0.2087	0.6456	0.0946	1.0000	-0.0443	0.2846	-0.2392	-0.3585	0.3811	0.2880	-0.3159	0.3970	-0.3282	-0.3326	0.4735	0.5103	0.3291	-0.0540	0.3538	0.3766	0.5556	-0.0740	-0.0982	0.0184	0.1007	-0.3227	-0.3763	0.1661	-0.0266	-0.4142	-0.4098	0.1570	-0.3343	-0.3940	0.2514	0.0537	-0.3972	0.0186	0.4658	0.2544	0.4414	0.5365	0.4188	0.3740
5	0.1070	0.2102	-0.0360	-0.0443	1.0000	-0.3650	-0.4311	0.1923	-0.3074	0.3047	0.0303	-0.2752	-0.4860	-0.2106	-0.2373	-0.1662	-0.3918	-0.4649	-0.2484	-0.2836	0.0856	-0.1549	-0.5011	-0.3303	-0.3549	-0.4246	-0.4675	-0.3642	-0.2587	-0.2795	-0.4649	-0.3172	-0.4444	-0.1636	-0.3023	0.0941	-0.0402	0.3337	-0.2289	0.3441	-0.2192	0.1113	-0.2089	-0.1898
6	0.2496	0.2136	-0.0711	0.2846	-0.3650	1.0000	0.1795	-0.6536	0.5483	-0.2924	-0.1552	0.3412	0.2513	-0.0003	0.4340	0.3184	0.9715	0.2910	0.2200	0.4138	-0.0172	0.0881	0.3526	0.2255	0.9328	0.2068	0.1457	0.5647	-0.0083	0.0153	0.1938	0.9433	0.1465	-0.1495	0.5448	-0.2359	-0.1477	-0.1778	0.4018	-0.3894	0.3062	-0.0908	0.2412	0.1904
7	-0.5053	-0.5953	0.1842	-0.2392	-0.4311	0.1795	1.0000	-0.3494	0.1304	-0.2540	-0.3429	0.2047	0.9245	0.0977	-0.0127	-0.1192	0.1937	0.8574	0.1521	0.1246	-0.1823	-0.2280	0.8232	0.2279	0.3649	0.9630	0.8784	0.3954	0.7907	0.2791	0.7689	0.1984	0.8458	0.0110	0.1643	-0.1613	-0.2668	-0.4481	-0.0176	-0.3071	-0.0030	-0.2162	0.0041	0.0017
8	-0.3789	-0.3990	-0.1691	-0.3585	0.1923	-0.6536	-0.3494	1.0000	-0.3061	-0.3180	0.8366	-0.3923	-0.1559	0.1038	-0.2670	-0.2226	-0.6329	-0.3971	-0.2253	-0.2222	-0.4447	0.6217	-0.3093	-0.2191	-0.6095	-0.2375	-0.0059	-0.3222	-0.4671	0.5336	-0.0754	-0.6167	-0.2376	0.3316	-0.3001	-0.2173	0.7747	0.3046	-0.2403	-0.2046	-0.1666	-0.3625	-0.1146	-0.0525
9	0.2470	0.3009	0.1219	0.3811	-0.3074	0.5483	0.1304	-0.3061	1.0000	-0.1408	-0.1578	0.6886	0.1411	-0.0333	0.9652	0.8941	0.6172	0.4350	0.6763	0.9816	0.3889	0.2979	0.4457	0.4610	0.4891	0.1233	0.0940	0.9099	0.0321	-0.0442	0.0954	0.5137	0.0929	-0.1049	0.9390	-0.1335	-0.1610	-0.1707	0.9605	-0.1774	0.9378	0.3797	0.9153	0.9037
10	0.5318	0.6181	0.1437	0.2880	0.3047	-0.2924	-0.2540	-0.3180	-0.1408	1.0000	-0.6289	-0.1002	-0.5551	-0.1669	0.0130	0.1444	-0.2852	-0.3178	-0.1488	-0.0942	0.6793	-0.7255	-0.4836	-0.1738	-0.3802	-0.4382	-0.5951	-0.2966	0.3246	-0.7486	-0.5109	-0.2935	-0.2848	-0.2346	-0.1714	0.7453	-0.5994	-0.0116	0.0173	0.9927	0.0211	0.6725	0.0259	0.0283
11	-0.3433	-0.4087	-0.2929	-0.3159	0.0303	-0.1552	-0.3429	0.8366	-0.1578	-0.6289	1.0000	-0.3903	-0.0318	0.1526	-0.1935	-0.2163	-0.1742	-0.3818	-0.2684	-0.1588	-0.6852	0.8140	-0.2189	-0.1996	-0.1395	-0.1730	0.0878	-0.1470	-0.6305	0.7169	0.0441	-0.1388	-0.2078	0.3421	-0.1385	-0.4469	0.9295	0.2943	-0.1843	-0.5520	-0.1646	-0.6354	-0.1468	-0.1005
12	0.1564	0.2660	0.1047	0.3970	-0.2752	0.3412	0.2047	-0.3923	0.6886	-0.1002	-0.3903	1.0000	0.1631	-0.1281	0.6842	0.6425	0.4983	0.6442	0.9840	0.7449	0.6342	0.2140	0.6270	0.6254	0.2955	0.1868	0.1069	0.6305	0.0881	-0.2107	0.0495	0.2611	0.0963	-0.2112	0.6121	-0.1371	-0.4004	-0.1935	0.6856	-0.1406	0.7190	0.6226	0.7219	0.6344
13	-0.6066	-0.7256	0.1077	-0.3282	-0.4860	0.2513	0.9245	-0.1559	0.1411	-0.5551	-0.0318	0.1631	1.0000	0.1450	-0.0327	-0.1647	0.2563	0.8018	0.1376	0.1173	-0.4145	0.0747	0.8494	0.2213	0.4456	0.9602	0.9867	0.4227	0.5469	0.5933	0.8508	0.2708	0.8077	0.1021	0.1826	-0.3899	0.0223	-0.4276	-0.0385	-0.5965	-0.0283	-0.4410	-0.0238	-0.0227
14	0.0344	-0.1472	-0.2113	-0.3326	-0.2106	-0.0003	0.0977	0.1038	-0.0333	-0.1669	0.1526	-0.1281	0.1450	1.0000	-0.0796	-0.1051	-0.0532	0.0190	-0.1142	-0.0590	-0.2189	0.0784	0.0410	0.6752	0.0525	0.1654	0.1572	0.0336	-0.0482	0.1897	0.6155	0.2237	0.5842	0.9719	0.2226	0.5080	0.4806	0.1021	-0.0800	-0.1611	-0.0868	-0.2140	-0.0882	-0.0675
15	0.3557	0.4471	0.1151	0.4735	-0.2373	0.4340	-0.0127	-0.2670	0.9652	0.0130	-0.1935	0.6842	-0.0327	-0.0796	1.0000	0.9792	0.5135	0.2986	0.6821	0.9670	0.5344	0.2536	0.2898	0.4242	0.3252	-0.0490	-0.0707	0.7773	0.0019	-0.1590	-0.0623	0.3904	-0.0434	-0.1396	0.8821	-0.0302	-0.2072	-0.1481	0.9993	-0.0129	0.9847	0.5339	0.9670	0.9575
16	0.4248	0.5393	0.0917	0.5103	-0.1662	0.3184	-0.1192	-0.2226	0.8941	0.1444	-0.2163	0.6425	-0.1647	-0.1051	0.9792	1.0000	0.4026	0.1790	0.6488	0.9126	0.6244	0.1996	0.1541	0.3763	0.1819	-0.1730	-0.1936	0.6432	-0.0202	-0.2401	-0.1759	0.2736	-0.1395	-0.1536	0.8005	0.0648	-0.2344	-0.1232	0.9819	0.1286	0.9754	0.6311	0.9630	0.9572
17	0.2433	0.2290	-0.0672	0.3291	-0.3918	0.9715	0.1937	-0.6329	0.6172	-0.2852	-0.1742	0.4983	0.2563	-0.0532	0.5135	0.4026	1.0000	0.3835	0.3887	0.5182	0.0954	0.1677	0.4415	0.2972	0.9023	0.2169	0.1549	0.6219	0.0070	0.0027	0.1631	0.8888	0.1250	-0.1935	0.5893	-0.2556	-0.1838	-0.1887	0.4856	-0.3780	0.4146	0.0266	0.3616	0.2927
18	-0.3478	-0.3884	0.1851	-0.0540	-0.4649	0.2910	0.8574	-0.3971	0.4350	-0.3178	-0.3818	0.6442	0.8018	0.0190	0.2986	0.1790	0.3835	1.0000	0.6066	0.4611	0.0979	0.0041	0.7774	0.4854	0.4311	0.8430	0.7440	0.6462	0.5861	0.1484	0.6196	0.2635	0.6845	-0.0735	0.4261	-0.2459	-0.3363	-0.3867	0.2965	-0.3719	0.3274	0.0705	0.3368	0.2936
19	0.0982	0.2124	0.0887	0.3538	-0.2484	0.2200	0.1521	-0.2253	0.6763	-0.1488	-0.2684	0.9840	0.1376	-0.1142	0.6821	0.6488	0.3887	0.6066	1.0000	0.7534	0.6010	0.3290	0.6004	0.6228	0.1807	0.1507	0.1069	0.6092	0.0156	-0.1347	0.0356	0.1455	0.0598	-0.1594	0.5970	-0.1710	-0.2883	-0.1493	0.6893	-0.1690	0.7391	0.6057	0.7526	0.6742
20	0.2175	0.2886	0.1293	0.3766	-0.2836	0.4138	0.1246	-0.2222	0.9816	-0.0942	-0.1588	0.7449	0.1173	-0.0590	0.9670	0.9126	0.5182	0.4611	0.7534	1.0000	0.4657	0.3282	0.4599	0.4836	0.3608	0.1099	0.0844	0.8823	0.0530	-0.0582	0.0597	0.3720	0.0716	-0.1094	0.9074	-0.1074	-0.1701	-0.1599	0.9687	-0.1174	0.9716	0.4678	0.9638	0.9482
21	0.5557	0.7303	0.1699	0.5556	0.0856	-0.0172	-0.1823	-0.4447	0.3889	0.6793	-0.6852	0.6342	-0.4145	-0.2189	0.5344	0.6244	0.0954	0.0979	0.6010	0.4657	1.0000	-0.3295	-0.0287	0.3086	-0.1472	-0.3292	-0.4720	0.1705	0.1903	-0.7168	-0.4330	-0.0705	-0.2511	-0.3124	0.3058	0.4586	-0.6790	-0.1071	0.5405	0.6553	0.5669	0.9955	0.5721	0.5141
22	-0.2508	-0.2546	-0.2471	-0.0740	-0.1549	0.0881	-0.2280	0.6217	0.2979	-0.7255	0.8140	0.2140	0.0747	0.0784	0.2536	0.1996	0.1677	0.0041	0.3290	0.3282	-0.3295	1.0000	0.1667	0.1775	0.0734	-0.0611	0.1631	0.2717	-0.6021	0.6304	0.0806	0.0525	-0.1564	0.2229	0.2686	-0.5582	0.7319	0.1832	0.2633	-0.6724	0.3018	-0.2866	0.3207	0.3155
23	-0.4001	-0.4573	0.1413	-0.0982	-0.5011	0.3526	0.8232	-0.3093	0.4457	-0.4836	-0.2189	0.6270	0.8494	0.0410	0.2898	0.1541	0.4415	0.7774	0.6004	0.4599	-0.0287	0.1667	1.0000	0.4823	0.4986	0.8479	0.8063	0.6681	0.4589	0.3172	0.6663	0.3227	0.6666	-0.0313	0.4402	-0.3735	-0.1860	-0.3786	0.2866	-0.5338	0.3137	-0.0543	0.3207	0.2774
24	0.1635	0.0921	-0.0717	0.0184	-0.3303	0.2255	0.2279	-0.2191	0.4610	-0.1738	-0.1996	0.6254	0.2213	0.6752	0.4242	0.3763	0.2972	0.4854	0.6228	0.4836	0.3086	0.1775	0.4823	1.0000	0.2306	0.2577	0.1883	0.4726	0.0475	-0.0382	0.5096	0.3533	0.5275	0.5883	0.6271	0.3047	0.0460	-0.0826	0.4253	-0.1990	0.4439	0.3040	0.4450	0.3959
25	0.0235	-0.0334	0.0099	0.1007	-0.3549	0.9328	0.3649	-0.6095	0.4891	-0.3802	-0.1395	0.2955	0.4456	0.0525	0.3252	0.1819	0.9023	0.4311	0.1807	0.3608	-0.1472	0.0734	0.4986	0.2306	1.0000	0.4033	0.3382	0.6182	0.1050	0.1257	0.3680	0.9116	0.3147	-0.0917	0.5125	-0.2821	-0.1135	-0.1850	0.2964	-0.4708	0.2164	-0.2156	0.1622	0.1232
26	-0.5632	-0.6694	0.0624	-0.3227	-0.4246	0.2068	0.9630	-0.2375	0.1233	-0.4382	-0.1730	0.1868	0.9602	0.1654	-0.0490	-0.1730	0.2169	0.8430	0.1507	0.1099	-0.3292	-0.0611	0.8479	0.2577	0.4033	1.0000	0.9296	0.4146	0.6069	0.4251	0.8330	0.2321	0.8533	0.1019	0.1715	-0.2765	-0.0964	-0.3486	-0.0539	-0.4838	-0.0393	-0.3583	-0.0321	-0.0337
27	-0.6554	-0.7781	0.0824	-0.3763	-0.4675	0.1457	0.8784	-0.0059	0.0940	-0.5951	0.0878	0.1069	0.9867	0.1572	-0.0707	-0.1936	0.1549	0.7440	0.1069	0.0844	-0.4720	0.1631	0.8063	0.1883	0.3382	0.9296	1.0000	0.3695	0.8455	0.6932	0.8467	0.1686	0.7761	0.1491	0.1346	-0.4185	0.1329	-0.4078	-0.0725	-0.6203	-0.0508	-0.4865	-0.0383	-0.0284
28	0.0130	0.0062	0.1742	0.1661	-0.3642	0.5647	0.3954	-0.3222	0.9099	-0.2966	-0.1470	0.6305	0.4227	0.0336	0.7773	0.6432	0.6219	0.6462	0.6092	0.8823	0.1705	0.2717	0.6681	0.4726	0.6182	0.4146	0.3695	1.0000	0.1689	0.1149	0.3453	0.5550	0.3334	-0.0457	0.8896	-0.2279	-0.1269	-0.2121	0.7707	-0.3408	0.7525	0.1534	0.7343	0.7221
29	-0.2205	-0.2506	0.3368	-0.0266	-0.2587	-0.0083	0.7907	-0.4671	0.0321	0.3246	-0.6305	0.0881	0.5469	-0.0482	0.0019	-0.0202	0.0070	0.5861	0.0156	0.0530	0.1903	-0.6021	0.4589	0.0475	0.1050	0.6069	0.8455	0.1689	1.0000	-0.1436	0.3981	0.0013	0.5990	-0.1559	0.0341	0.2403	-0.5623	-0.4725	-0.0000	0.2751	0.0157	0.1571	0.0255	0.0303
30	-0.5658	-0.6828	-0.1434	-0.4142	-0.2795	0.0153	0.2791	0.5336	-0.0442	-0.7486	0.7169	-0.2107	0.5933	0.1897	-0.1590	-0.2401	0.0027	0.1484	-0.1347	-0.0582	-0.7168	0.6304	0.3172	-0.0382	0.1257	0.4251	0.6932	0.1149	-0.1436	1.0000	0.5563	0.0388	0.3104	0.3075	-0.0079	-0.5324	0.6993	-0.1478	-0.1556	-0.7184	-0.1361	-0.6969	-0.1208	-0.0860
31	-0.4670	-0.6263	0.0016	-0.4098	-0.4649	0.1938	0.7689	-0.0754	0.0954	-0.5109	0.0441	0.0495	0.8508	0.6155	-0.0623	-0.1759	0.1631	0.6196	0.0356	0.0597	-0.4330	0.0806	0.6663	0.5096	0.3680	0.8330	0.8467	0.3453	0.3981	0.5563	1.0000	0.3381	0.9506	0.5638	0.2627	-0.0611	0.2485	-0.2997	-0.0674	-0.5408	-0.0653	-0.4513	-0.0638	-0.0491
32	0.2414	0.1617	-0.0413	0.1570	-0.3172	0.9433	0.1984	-0.6167	0.5137	-0.2935	-0.1388	0.2611	0.2708	0.2237	0.3904	0.2736	0.8888	0.2635	0.1455	0.3720	-0.0705	0.0525	0.3227	0.3533	0.9116	0.2321	0.1686	0.5550	0.0013	0.0388	0.3381	1.0000	0.2847	0.0641	0.5986	-0.1147	-0.0531	-0.1653	0.3596	-0.3847	0.2624	-0.1391	0.1982	0.1615
33	-0.3949	-0.5311	0.0593	-0.3343	-0.4444	0.1465	0.8458	-0.2376	0.0929	-0.2848	-0.2078	0.0963	0.8077	0.5842	-0.0434	-0.1395	0.1250	0.6845	0.0598	0.0716	-0.2511	-0.1564	0.6666	0.5275	0.3147	0.8533	0.7761	0.3334	0.5990	0.3104	0.9506	0.2847	1.0000	0.4952	0.2526	0.1212	0.0116	-0.3240	-0.0480	-0.3254	-0.0422	-0.2761	-0.0387	-0.0284
34	-0.0548	-0.2312	-0.2477	-0.3940	-0.1636	-0.1495	0.0110	0.3316	-0.1049	-0.2346	0.3421	-0.2112	0.1021	0.9719	-0.1396	-0.1536	-0.1935	-0.0735	-0.1594	-0.1094	-0.3124	0.2229	-0.0313	0.5883	-0.0917	0.1019	0.1491	-0.0457	-0.1559	0.3075	0.5638	0.0641	0.4952	1.0000	0.1367	0.4291	0.6417	0.1715	-0.1340	-0.2030	-0.1231	-0.2889	-0.1123	-0.0792
35	0.2606	0.2472	0.0791	0.2514	-0.3023	0.5448	0.1643	-0.3001	0.9390	-0.1714	-0.1385	0.6121	0.1826	0.2226	0.8821	0.8005	0.5893	0.4261	0.5970	0.9074	0.3058	0.2686	0.4402	0.6271	0.5125	0.1715	0.1346	0.8896	0.0341	-0.0079	0.2627	0.5986	0.2526	0.1367	1.0000	-0.0042	-0.0539	-0.1555	0.8765	-0.2085	0.8482	0.2953	0.8238	0.8139
36	0.4967	0.4474	-0.0476	0.0537	0.0941	-0.2359	-0.1613	-0.2173	-0.1335	0.7453	-0.4469	-0.1371	-0.3899	0.5080	-0.0302	0.0648	-0.2556	-0.2459	-0.1710	-0.1074	0.4586	-0.5582	-0.3735	0.3047	-0.2821	-0.2765	-0.4185	-0.2279	0.2403	-0.5324	-0.0611	-0.1147	0.1212	0.4291	-0.0042	1.0000	-0.2208	0.0572	-0.0270	0.7412	-0.0271	0.4548	-0.0239	-0.0143
37	-0.2933	-0.4117	-0.3173	-0.3972	-0.0402	-0.1477	-0.2668	0.7747	-0.1610	-0.5994	0.9295	-0.4004	0.0223	0.4806	-0.2072	-0.2344	-0.1838	-0.3363	-0.2883	-0.1701	-0.6790	0.7319	-0.1860	0.0460	-0.1135	-0.0964	0.1329	-0.1269	-0.5623	0.6993	0.2485	-0.0531	0.0116	0.6417	-0.0539	-0.2208	1.0000	0.3047	-0.1990	-0.5293	-0.1837	-0.6334	-0.1680	-0.1190
38	0.0548	0.1345	-0.2406	0.0186	0.3337	-0.1778	-0.4481	0.3046	-0.1707	-0.0116	0.2943	-0.1935	-0.4276	0.1021	-0.1481	-0.1232	-0.1887	-0.3867	-0.1493	-0.1599	-0.1071	0.1832	-0.3786	-0.0826	-0.1850	-0.3486	-0.4078	-0.2121	-0.4725	-0.1478	-0.2997	-0.1653	-0.3240	0.1715	-0.1555	0.0572	0.3047	1.0000	-0.1422	0.0311	-0.1348	-0.0801	-0.1273	-0.1151
39	0.3459	0.4394	0.1170	0.4658	-0.2289	0.4018	-0.0176	-0.2403	0.9605	0.0173	-0.1843	0.6856	-0.0385	-0.0800	0.9993	0.9819	0.4856	0.2965	0.6893	0.9687	0.5405	0.2633	0.2866	0.4253	0.2964	-0.0539	-0.0725	0.7707	-0.0000	-0.1556	-0.0674	0.3596	-0.0480	-0.1340	0.8765	-0.0270	-0.1990	-0.1422	1.0000	-0.0047	0.9901	0.5433	0.9752	0.9671
40	0.4989	0.5891	0.1364	0.2544	0.3441	-0.3894	-0.3071	-0.2046	-0.1774	0.9927	-0.5520	-0.1406	-0.5965	-0.1611	-0.0129	0.1286	-0.3780	-0.3719	-0.1690	-0.1174	0.6553	-0.6724	-0.5338	-0.1990	-0.4708	-0.4838	-0.6203	-0.3408	0.2751	-0.7184	-0.5408	-0.3847	-0.3254	-0.2030	-0.2085	0.7412	-0.5293	0.0311	-0.0047	1.0000	0.0095	0.6595	0.0214	0.0315
41	0.2932	0.3958	0.1184	0.4414	-0.2192	0.3062	-0.0030	-0.1666	0.9378	0.0211	-0.1646	0.7190	-0.0283	-0.0868	0.9847	0.9754	0.4146	0.3274	0.7391	0.9716	0.5669	0.3018	0.3137	0.4439	0.2164	-0.0393	-0.0508	0.7525	0.0157	-0.1361	-0.0653	0.2624	-0.0422	-0.1231	0.8482	-0.0271	-0.1837	-0.1348	0.9901	0.0095	1.0000	0.5783	0.9965	0.9861
42	0.5315	0.7119	0.1709	0.5365	0.1113	-0.0908	-0.2162	-0.3625	0.3797	0.6725	-0.6354	0.6226	-0.4410	-0.2140	0.5339	0.6311	0.0266	0.0705	0.6057	0.4678	0.9955	-0.2866	-0.0543	0.3040	-0.2156	-0.3583	-0.4865	0.1534	0.1571	-0.6969	-0.4513	-0.1391	-0.2761	-0.2889	0.2953	0.4548	-0.6334	-0.0801	0.5433	0.6595	0.5783	1.0000	0.5890	0.5369
43	0.2582	0.3640	0.1196	0.4188	-0.2089	0.2412	0.0041	-0.1146	0.9153	0.0259	-0.1468	0.7219	-0.0238	-0.0882	0.9670	0.9630	0.3616	0.3368	0.7526	0.9638	0.5721	0.3207	0.3207	0.4450	0.1622	-0.0321	-0.0383	0.7343	0.0255	-0.1208	-0.0638	0.1982	-0.0387	-0.1123	0.8238	-0.0239	-0.1680	-0.1273	0.9752	0.0214	0.9965	0.5890	1.0000	0.9902
44	0.2293	0.3250	0.1265	0.3740	-0.1898	0.1904	0.0017	-0.0525	0.9037	0.0283	-0.1005	0.6344	-0.0227	-0.0675	0.9575	0.9572	0.2927	0.2936	0.6742	0.9482	0.5141	0.3155	0.2774	0.3959	0.1232	-0.0337	-0.0284	0.7221	0.0303	-0.0860	-0.0491	0.1615	-0.0284	-0.0792	0.8139	-0.0143	-0.1190	-0.1151	0.9671	0.0315	0.9861	0.5369	0.9902	1.0000

Table 3.14 Matrix of Correlation Coefficients - B-type tests.

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
3	DEPENDENT	
27	-0.53879₁₀+01	0.53621₁₀+00
CONSTANT	0.13192₁₀+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.264782₁₀+03	1	0.264782₁₀+03
RESIDUAL	0.131126₁₀+03	50	0.262252₁₀+01
TOTAL	0.395908₁₀+03	51	0.776291₁₀+01
F VALUE	100.96		
MULTIPLE R	0.8178		
R SQUARED	0.6688		

CORRELATIONS

1	1.0000	0.7710	0.6751	0.2216	-0.2114	0.2492	-0.3686	-0.4822	0.2595	0.4987	-0.4619
2	0.7710	1.0000	0.9607	0.7209	-0.2324	0.0566	-0.5037	-0.4331	0.2899	0.6942	-0.5618
3	0.6751	0.9607	1.0000	0.7368	-0.2121	0.0333	-0.5486	-0.3598	0.2969	0.6016	-0.4869
4	0.2216	0.7209	0.7368	1.0000	-0.1701	-0.0140	-0.2927	-0.2322	0.3447	0.4740	-0.3774
5	-0.2114	-0.2324	-0.2121	-0.1701	1.0000	-0.0986	-0.0224	0.3166	-0.1015	-0.1998	0.3447
6	0.2492	0.0566	0.0333	-0.0140	-0.0986	1.0000	0.2974	-0.6338	0.5852	-0.3490	-0.1027
7	-0.3686	-0.5037	-0.5486	-0.2927	-0.0224	0.2974	1.0000	-0.3315	0.2226	-0.3080	-0.2369
8	-0.4822	-0.4331	-0.3598	-0.2322	0.3166	-0.6338	-0.3315	1.0000	-0.3245	-0.3133	0.8229
9	0.2595	0.2899	0.2969	0.3447	-0.1015	0.5852	0.2226	-0.3245	1.0000	-0.1998	-0.1304
10	0.4987	0.6942	0.6016	0.4740	-0.1998	-0.3490	-0.3080	-0.3133	-0.1998	1.0000	-0.6596
11	-0.4619	-0.5618	-0.4869	-0.3774	0.3447	-0.1027	-0.2369	0.8229	-0.1304	-0.6596	1.0000
12	0.1366	0.2412	0.2650	0.3482	-0.1807	0.3126	0.2763	-0.3775	0.6926	-0.1282	-0.3755
13	-0.5581	-0.7772	-0.7659	-0.5254	0.1738	0.3703	0.8205	0.0385	0.2068	-0.7554	0.3074
14	-0.1883	-0.3459	-0.4913	-0.3266	0.0152	0.1760	0.4177	0.0028	0.0125	-0.3397	0.1602
15	0.3374	0.4523	0.4611	0.5154	-0.1122	0.4609	0.0806	-0.2882	0.9659	-0.0494	-0.1789
16	0.3791	0.5685	0.5788	0.6332	-0.1146	0.3262	-0.0382	-0.2412	0.8913	0.0950	-0.2155
17	0.2342	0.0696	0.0586	0.0389	-0.1044	0.9729	0.3198	-0.6089	0.6532	-0.3465	-0.1163
18	-0.1544	-0.2152	-0.2232	-0.0435	-0.1254	0.3703	0.7883	-0.3989	0.5687	-0.3159	-0.3355
19	0.0594	0.1867	0.2242	0.3355	-0.1351	0.1917	0.2266	-0.2097	0.6719	-0.1753	-0.2560
20	0.2199	0.2945	0.3080	0.3868	-0.0956	0.4499	0.2119	-0.2385	0.9812	-0.1507	-0.1339
21	0.5005	0.7832	0.7398	0.7014	-0.2645	-0.1190	-0.1842	-0.4368	0.3121	0.7021	-0.7302
22	-0.3834	-0.4322	-0.3415	-0.1767	0.2498	0.1282	-0.0661	0.6105	0.3245	-0.7704	0.8194
23	-0.2479	-0.3582	-0.3417	-0.1625	-0.0384	0.4463	0.7460	-0.2591	0.5809	-0.5415	-0.0944
24	0.0040	-0.0641	-0.1664	-0.0225	-0.1206	0.3115	0.4948	-0.2733	0.4709	-0.3094	-0.1847
25	0.1730	-0.0225	-0.0434	-0.0917	-0.0964	0.9863	0.3548	-0.6272	0.5510	-0.3762	-0.1004
26	-0.4821	-0.6551	-0.6648	-0.4403	0.0594	0.3226	0.9557	-0.1659	0.2157	-0.5368	-0.0001
27	-0.6451	-0.8449	-0.8178	-0.5599	0.2534	0.2014	0.7092	0.2755	0.1184	-0.7933	0.4896
28	0.1513	0.1011	0.1014	0.1454	-0.0887	0.6275	0.3811	-0.3368	0.9617	-0.3044	-0.1064
29	0.1214	0.1861	0.0566	0.2482	-0.1793	-0.0435	0.5248	-0.5088	0.0064	0.5980	-0.7063
30	-0.5664	-0.7150	-0.6519	-0.5023	0.3964	0.0066	0.0643	0.7150	-0.0698	-0.7636	0.9363
31	-0.4421	-0.6508	-0.7164	-0.5074	0.1050	0.3074	0.7306	0.0158	0.1171	-0.6294	0.2569
32	0.2408	0.0355	-0.0087	-0.0837	-0.0910	0.9602	0.2977	-0.6141	0.5363	-0.3441	-0.0906
33	-0.3333	-0.4839	-0.5869	-0.3541	-0.0129	0.2595	0.8547	-0.2174	0.1243	-0.3531	-0.0873
34	-0.2994	-0.4437	-0.5681	-0.3746	0.0865	0.0185	0.3298	0.2438	-0.0687	-0.4065	0.3539
35	0.2687	0.2079	0.1707	0.1961	-0.0976	0.6350	0.2911	-0.3562	0.9407	-0.2566	-0.1118
36	0.3091	0.4025	0.2006	0.2600	-0.1738	-0.1978	0.0368	-0.3064	-0.1661	0.6924	-0.5242
37	-0.4590	-0.6071	-0.5879	-0.4567	0.2946	-0.0399	-0.0916	0.7380	-0.1226	-0.6861	0.9459
38	0.4334	0.5677	0.5507	0.1535	-0.0705	-0.3330	-0.5625	-0.0403	-0.2794	0.6535	-0.2769
39	0.3264	0.4510	0.4618	0.5214	-0.1082	0.4290	0.0730	-0.2620	0.9607	-0.0434	-0.1712
40	0.4612	0.6726	0.5876	0.4684	-0.1767	-0.4347	-0.3575	-0.2138	-0.2368	0.9944	-0.5954
41	0.2668	0.4186	0.4358	0.5277	-0.0945	0.3334	0.0817	-0.1836	0.9361	-0.0376	-0.1491
42	0.4704	0.7705	0.7336	0.7046	-0.2478	-0.1905	-0.2199	-0.3578	0.2951	0.6964	-0.6841
43	0.2297	0.3963	0.4167	0.5260	-0.0837	0.2693	0.0831	-0.1297	0.9124	-0.0305	-0.1305
44	0.2161	0.3834	0.4009	0.5144	-0.0643	0.2313	0.0685	-0.0753	0.9038	-0.0248	-0.0872

1	0.1366	-0.5581	-0.1883	0.3374	0.3791	0.2342	-0.1544	0.0594	0.2199	0.5005	-0.3834
2	0.2412	-0.7772	-0.3459	0.4523	0.5685	0.0696	-0.2152	0.1867	0.2945	0.7832	-0.4322
3	0.2650	-0.7659	-0.4913	0.4611	0.5788	0.0586	-0.2232	0.2242	0.3080	0.7398	-0.3415
4	0.3482	-0.5254	-0.3266	0.5154	0.6332	0.0389	-0.0435	0.3355	0.3868	0.7014	-0.1767
5	-0.1807	0.1738	0.0152	-0.1122	-0.1146	-0.1044	-0.1254	-0.1351	-0.0956	-0.2645	0.2498
6	0.3126	0.3703	0.1760	0.4609	0.3262	0.9729	0.3703	0.1917	0.4499	-0.1190	0.1282
7	0.2763	0.8205	0.4177	0.0806	-0.0382	0.3198	0.7883	0.2266	0.2119	-0.1842	-0.0661
8	-0.3775	0.0385	0.0028	-0.2882	-0.2412	-0.6089	-0.3989	-0.2097	-0.2385	-0.4368	0.6105
9	0.6926	0.2068	0.0125	0.9659	0.8913	0.6532	0.5687	0.6719	0.9812	0.3121	0.3245
10	-0.1282	-0.7554	-0.3397	-0.0494	0.0950	-0.3465	-0.3159	-0.1753	-0.1507	0.7021	-0.7704
11	-0.3755	0.3074	0.1602	-0.1789	-0.2155	-0.1163	-0.3355	-0.2560	-0.1339	-0.7302	0.8194
12	1.0000	0.1450	-0.1098	0.6923	0.6500	0.4624	0.7919	0.9840	0.7533	0.5880	0.2199
13	0.1450	1.0000	0.4769	0.0252	-0.1299	0.3777	0.6238	0.1465	0.1715	-0.5908	0.4236
14	-0.1098	0.4769	1.0000	-0.0794	-0.1560	0.0910	0.1808	-0.1160	-0.0476	-0.3713	0.0997
15	0.6923	0.0252	-0.0794	1.0000	0.9774	0.5397	0.4605	0.6819	0.9686	0.4639	0.2690
16	0.6500	-0.1299	-0.1560	0.9774	1.0000	0.4103	0.3469	0.6493	0.9134	0.5710	0.2002
17	0.4624	0.3777	0.0910	0.5397	0.4103	1.0000	0.4825	0.3529	0.5530	-0.0192	0.2075
18	0.7919	0.6238	0.1808	0.4605	0.3469	0.4825	1.0000	0.7581	0.6025	0.2014	0.1421
19	0.9840	0.1465	-0.1160	0.6819	0.6493	0.3529	0.7581	1.0000	0.7538	0.5546	0.3306
20	0.7533	0.1715	-0.0476	0.9686	0.9134	0.5530	0.6025	0.7538	1.0000	0.3962	0.3537
21	0.5880	-0.5908	-0.3713	0.4639	0.5710	-0.0192	0.2014	0.5546	0.3962	1.0000	-0.4110
22	0.2199	0.4236	0.0997	0.2690	0.2002	0.2075	0.1421	0.3306	0.3537	-0.4110	1.0000
23	0.7513	0.7440	0.2194	0.4460	0.3079	0.5547	0.9616	0.7352	0.6003	0.0049	0.3720
24	0.6219	0.4287	0.6929	0.4016	0.3167	0.3534	0.6943	0.6056	0.4729	0.1582	0.1848
25	0.2922	0.4282	0.2035	0.4036	0.2564	0.9561	0.4003	0.1714	0.4155	-0.1670	0.1170
26	0.2306	0.9362	0.4548	0.0399	-0.1041	0.3388	0.7538	0.2051	0.1950	-0.3890	0.1527
27	0.0453	0.9704	0.4546	-0.0457	-0.1821	0.2144	0.5004	0.0867	0.1053	-0.6686	0.5482
28	0.6627	0.3780	0.0977	0.8613	0.7415	0.6876	0.6701	0.6358	0.9330	0.1699	0.3322
29	0.0836	0.0027	0.0725	0.0462	0.0837	-0.0239	0.3141	0.0050	0.0381	0.4446	-0.6768
30	-0.3014	0.5852	0.2708	-0.1655	-0.2397	-0.0042	-0.1065	-0.1997	-0.0820	-0.7994	0.8020
31	-0.0044	0.8632	0.8411	-0.0445	-0.1789	0.2551	0.4607	-0.0091	0.0583	-0.5741	0.2732
32	0.1994	0.3729	0.3386	0.4035	0.2663	0.8910	0.3000	0.0799	0.3826	-0.1887	0.0680
33	0.0834	0.7568	0.8170	-0.0120	-0.1238	0.2186	0.5723	0.0466	0.0824	-0.3184	-0.0339
34	-0.1917	0.4760	0.9701	-0.1496	-0.2128	-0.0550	0.0859	-0.1573	-0.1047	-0.4639	0.2475
35	0.5895	0.2902	0.2619	0.8690	0.7685	0.6620	0.5523	0.5569	0.8924	0.1822	0.2799
36	-0.1499	-0.3613	0.4157	-0.0855	-0.0043	-0.2426	-0.1288	-0.1992	-0.1552	0.4283	-0.6426
37	-0.3899	0.4161	0.4643	-0.1960	-0.2522	-0.0838	-0.2605	-0.2856	-0.1472	-0.7758	0.7534
38	-0.2305	-0.7113	-0.3236	-0.1887	-0.0824	-0.3494	-0.4823	-0.2395	-0.2530	0.3775	-0.4396
39	0.6950	0.0173	-0.0877	0.9993	0.9809	0.5121	0.4582	0.6903	0.9700	0.4727	0.2777
40	-0.1612	-0.7798	-0.3538	-0.0768	0.0776	-0.4283	-0.3635	-0.1904	-0.1755	0.6847	-0.7264
41	0.7308	0.0220	-0.1133	0.9846	0.9762	0.4407	0.4885	0.7434	0.9716	0.5036	0.3192
42	0.5753	-0.6091	-0.3806	0.4555	0.5707	-0.0867	0.1739	0.5574	0.3901	0.9959	-0.3734
43	0.7355	0.0220	-0.1271	0.9668	0.9652	0.3886	0.4942	0.7590	0.9628	0.5131	0.3395
44	0.6592	0.0189	-0.0988	0.9613	0.9640	0.3343	0.4375	0.6909	0.9499	0.4690	0.3387

1	-0.2479	0.0040	0.1730	-0.4821	-0.6451	0.1513	0.1214	-0.5664	-0.4421	0.2408	-0.3333
2	-0.3582	-0.0641	-0.0225	-0.6551	-0.8449	0.1011	0.1861	-0.7150	-0.6508	0.0355	-0.4839
3	-0.3417	-0.1664	-0.0434	-0.6648	-0.8178	0.1014	0.0566	-0.6519	-0.7164	-0.0087	-0.5869
4	-0.1625	-0.0225	-0.0917	-0.4403	-0.5599	0.1454	0.2482	-0.5023	-0.5074	-0.0837	-0.3541
5	-0.0384	-0.1206	-0.0964	0.0594	0.2534	-0.0887	-0.1793	0.3964	0.1050	-0.0910	-0.0129
6	0.4463	0.3115	0.9863	0.3226	0.2014	0.6275	-0.0435	0.0066	0.3074	0.9602	0.2595
7	0.7460	0.4948	0.3548	0.9557	0.7092	0.3811	0.5248	0.0643	0.7306	0.2977	0.8547
8	-0.2591	-0.2733	-0.6272	-0.1659	0.2755	-0.3368	-0.5088	0.7150	0.0158	-0.6141	-0.2174
9	0.5809	0.4709	0.5510	0.2157	0.1184	0.9617	0.0064	-0.0698	0.1171	0.5363	0.1243
10	-0.5415	-0.3094	-0.3762	-0.5368	-0.7933	-0.3044	0.5980	-0.7636	-0.6294	-0.3441	-0.3531
11	-0.0944	-0.1847	-0.1004	-0.0001	0.4896	-0.1064	-0.7063	0.9363	0.2569	-0.0906	-0.0873
12	0.7513	0.6219	0.2922	0.2306	0.0453	0.6627	0.0836	-0.3014	-0.0044	0.1994	0.0834
13	0.7440	0.4287	0.4282	0.9362	0.9704	0.3780	0.0027	0.5852	0.8632	0.3729	0.7568
14	0.2194	0.6929	0.2035	0.4548	0.4546	0.0977	0.0725	0.2708	0.8411	0.3386	0.8170
15	0.4460	0.4016	0.4036	0.0399	-0.0457	0.8613	0.0462	-0.1655	-0.0445	0.4035	-0.0120
16	0.3079	0.3167	0.2564	-0.1041	-0.1821	0.7415	0.0837	-0.2397	-0.1789	0.2663	-0.1238
17	0.5547	0.3534	0.9561	0.3388	0.2144	0.6876	-0.0239	-0.0042	0.2551	0.8910	0.2186
18	0.9616	0.6943	0.4003	0.7538	0.5004	0.6701	0.3141	-0.1065	0.4607	0.3000	0.5723
19	0.7352	0.6056	0.1714	0.2051	0.0867	0.6358	0.0050	-0.1997	-0.0091	0.0799	0.0466
20	0.6003	0.4729	0.4155	0.1950	0.1053	0.9330	0.0381	-0.0820	0.0583	0.3826	0.0824
21	0.0049	0.1582	-0.1670	-0.3890	-0.6686	0.1699	0.4446	-0.7994	-0.5741	-0.1887	-0.3184
22	0.3720	0.1848	0.1170	0.1527	0.5482	0.3322	-0.6768	0.8020	0.2732	0.0680	-0.0339
23	1.0000	0.6844	0.4787	0.7847	0.6493	0.6953	0.0897	0.1368	0.5494	0.3722	0.5572
24	0.6844	1.0000	0.3219	0.4875	0.3409	0.5199	0.1150	-0.0508	0.6284	0.3738	0.6817
25	0.4787	0.3219	1.0000	0.3912	0.2562	0.6254	-0.0416	0.0287	0.3618	0.9588	0.3127
26	0.7847	0.4875	0.3912	1.0000	0.8593	0.3989	0.2624	0.3030	0.8225	0.3278	0.8437
27	0.6493	0.3409	0.2562	0.8593	1.0000	0.2786	-0.1132	0.7367	0.8311	0.2078	0.6729
28	0.6953	0.5199	0.6254	0.3989	0.2786	1.0000	0.0122	0.0069	0.2710	0.5915	0.2698
29	0.0897	0.1150	-0.0416	0.2624	-0.1132	0.0122	1.0000	-0.5758	0.0445	-0.0468	0.3824
30	0.1368	-0.0508	0.0287	0.3030	0.7367	0.0069	-0.5758	1.0000	0.4900	0.0201	0.1611
31	0.5494	0.6284	0.3618	0.8225	0.8311	0.2710	0.0445	0.4900	1.0000	0.4194	0.9250
32	0.3722	0.3738	0.9588	0.3278	0.2078	0.5915	-0.0468	0.0201	0.4194	1.0000	0.3669
33	0.5572	0.6817	0.3127	0.8437	0.6729	0.2698	0.3824	0.1611	0.9250	0.3669	1.0000
34	0.1573	0.6093	0.0463	0.4054	0.5115	0.0116	-0.0496	0.4364	0.8201	0.1759	0.7410
35	0.5731	0.6150	0.6164	0.2974	0.1902	0.9421	0.0036	-0.0274	0.3169	0.6556	0.3148
36	-0.3204	0.2398	-0.2090	-0.1663	-0.4167	-0.2093	0.6572	-0.5428	0.0055	-0.1020	0.2687
37	-0.0324	0.0475	-0.0276	0.1332	0.5729	-0.0725	-0.6136	0.9228	0.4967	0.0304	0.1772
38	-0.5771	-0.3545	-0.3301	-0.5926	-0.6878	-0.3378	-0.0426	-0.4188	-0.5758	-0.3103	-0.4931
39	0.4423	0.3983	0.3722	0.0325	-0.0472	0.8539	0.0446	-0.1612	-0.0539	0.3710	-0.0212
40	-0.5820	-0.3422	-0.4611	-0.5749	-0.7933	-0.3470	0.5547	-0.7171	-0.6523	-0.4272	-0.3919
41	0.4692	0.4058	0.2803	0.0414	-0.0241	0.8283	0.0551	-0.1398	-0.0684	0.2634	-0.0319
42	-0.0184	0.1433	-0.2379	-0.4163	-0.6676	0.1473	0.4113	-0.7665	-0.5905	-0.2572	-0.3458
43	0.4730	0.4000	0.2189	0.0433	-0.0113	0.8046	0.0613	-0.1229	-0.0771	0.1946	-0.0392
44	0.4181	0.3667	0.1817	0.0332	-0.0011	0.7935	0.0570	-0.0852	-0.0590	0.1753	-0.0294

1	-0.2994	0.2687	0.3091	-0.4590	0.4334	0.3264	0.4612	0.2668	0.4704	0.2297	0.2161
2	-0.4437	0.2079	0.4025	-0.6071	0.5677	0.4510	0.6726	0.4186	0.7705	0.3963	0.3834
3	-0.5681	0.1707	0.2006	-0.5879	0.5507	0.4618	0.5876	0.4358	0.7336	0.4167	0.4009
4	-0.3746	0.1961	0.2600	-0.4567	0.1535	0.5214	0.4684	0.5277	0.7046	0.5260	0.5144
5	0.0865	-0.0976	-0.1738	0.2946	-0.0705	-0.1082	-0.1767	-0.0945	-0.2478	-0.0837	-0.0643
6	0.0185	0.6350	-0.1978	-0.0399	-0.3330	0.4290	-0.4347	0.3334	-0.1905	0.2693	0.2313
7	0.3298	0.2911	0.0368	-0.0916	-0.5625	0.0730	-0.3575	0.0817	-0.2199	0.0831	0.0685
8	0.2438	-0.3562	-0.3064	0.7380	-0.0403	-0.2620	-0.2138	-0.1836	-0.3578	-0.1297	-0.0753
9	-0.0687	0.9407	-0.1661	-0.1226	-0.2794	0.9607	-0.2368	0.9361	0.2951	0.9124	0.9038
10	-0.4065	-0.2566	0.6924	-0.6861	0.6535	-0.0434	0.9944	-0.0376	0.6964	-0.0305	-0.0248
11	0.3539	-0.1118	-0.5242	0.9459	-0.2769	-0.1712	-0.5954	-0.1491	-0.6841	-0.1305	-0.0872
12	-0.1917	0.5895	-0.1499	-0.3899	-0.2305	0.6950	-0.1612	0.7308	0.5753	0.7355	0.6592
13	0.4760	0.2902	-0.3613	0.4161	-0.7113	0.0173	-0.7798	0.0220	-0.6091	0.0220	0.0189
14	0.9701	0.2619	0.4157	0.4643	-0.3236	-0.0877	-0.3538	-0.1133	-0.3806	-0.1271	-0.0988
15	-0.1496	0.8690	-0.0855	-0.1960	-0.1887	0.9993	-0.0768	0.9846	0.4555	0.9668	0.9613
16	-0.2128	0.7685	-0.0043	-0.2522	-0.0824	0.9809	0.0776	0.9762	0.5707	0.9652	0.9640
17	-0.0550	0.6620	-0.2426	-0.0838	-0.3494	0.5121	-0.4283	0.4407	-0.0867	0.3886	0.3343
18	0.0859	0.5523	-0.1288	-0.2605	-0.4823	0.4582	-0.3635	0.4885	0.1739	0.4942	0.4375
19	-0.1573	0.5569	-0.1992	-0.2856	-0.2395	0.6903	-0.1904	0.7434	0.5574	0.7590	0.6909
20	-0.1047	0.8924	-0.1552	-0.1472	-0.2530	0.9700	-0.1755	0.9716	0.3901	0.9628	0.9499
21	-0.4639	0.1822	0.4283	-0.7758	0.3775	0.4727	0.6847	0.5036	0.9959	0.5131	0.4690
22	0.2475	0.2799	-0.6426	0.7534	-0.4396	0.2777	-0.7264	0.3192	-0.3734	0.3395	0.3387
23	0.1573	0.5731	-0.3204	-0.0324	-0.5771	0.4423	-0.5820	0.4692	-0.0184	0.4730	0.4181
24	0.6093	0.6150	0.2398	0.0475	-0.3545	0.3983	-0.3422	0.4058	0.1433	0.4000	0.3667
25	0.0463	0.6164	-0.2090	-0.0276	-0.3301	0.3722	-0.4611	0.2803	-0.2379	0.2189	0.1817
26	0.4054	0.2974	-0.1663	0.1332	-0.5926	0.0325	-0.5749	0.0414	-0.4163	0.0433	0.0332
27	0.5115	0.1902	-0.4167	0.5729	-0.6878	-0.0472	-0.7933	-0.0241	-0.6676	-0.0113	-0.0011
28	0.0116	0.9421	-0.2093	-0.0725	-0.3378	0.8539	-0.3470	0.8283	0.1473	0.8046	0.7935
29	-0.0496	0.0036	0.6572	-0.6136	-0.0426	0.0446	0.5547	0.0551	0.4113	0.0613	0.0570
30	0.4364	-0.0274	-0.5428	0.9228	-0.4188	-0.1612	-0.7171	-0.1398	-0.7665	-0.1229	-0.0852
31	0.8201	0.3169	0.0055	0.4967	-0.5758	-0.0539	-0.6523	-0.0684	-0.5905	-0.0771	-0.0590
32	0.1759	0.6556	-0.1020	0.0304	-0.3103	0.3710	-0.4272	0.2634	-0.2572	0.1946	0.1753
33	0.7410	0.3148	0.2687	0.1772	-0.4931	-0.0212	-0.3919	-0.0319	-0.3458	-0.0392	-0.0294
34	1.0000	0.1642	0.3295	0.6283	-0.3262	-0.1513	-0.3963	-0.1559	-0.4542	-0.1557	-0.1166
35	0.1642	1.0000	-0.0554	-0.0194	-0.3033	0.8595	-0.3003	0.8174	0.1590	0.7850	0.7851
36	0.3295	-0.0554	1.0000	-0.3334	0.3320	-0.0855	0.6755	-0.0931	0.4155	-0.0942	-0.0754
37	0.6283	-0.0194	-0.3334	1.0000	-0.3322	-0.1918	-0.6328	-0.1820	-0.7376	-0.1705	-0.1208
38	-0.3262	-0.3033	0.3320	-0.3322	1.0000	-0.1826	0.6701	-0.1834	0.3876	-0.1791	-0.1723
39	-0.1513	0.8595	-0.0855	-0.1918	-0.1826	1.0000	-0.0674	0.9899	0.4674	0.9750	0.9706
40	-0.3963	-0.3003	0.6755	-0.6328	0.6701	-0.0674	1.0000	-0.0521	0.6882	-0.0385	-0.0272
41	-0.1559	0.8174	-0.0931	-0.1820	-0.1834	0.9899	-0.0521	1.0000	0.5066	0.9966	0.9892
42	-0.4542	0.1590	0.4155	-0.7376	0.3876	0.4674	0.6882	0.5066	1.0000	0.5216	0.4822
43	-0.1557	0.7850	-0.0942	-0.1705	-0.1791	0.9750	-0.0385	0.9966	0.5216	1.0000	0.9926
44	-0.1166	0.7851	-0.0754	-0.1208	-0.1723	0.9706	-0.0272	0.9892	0.4822	0.9926	1.0000

Table 3.15 Matrix of Correlation Coefficients - C-type tests.

ELEMENT ANALYSIS

```
PRIMARY  ELEMENT    27        0.22128D+03                                     PARTIAL  F       87.88
SECONDARY  ELEMENTS
  27          8
  0.          0.43503D+02

PRIMARY  ELEMENT     8        0.77499D+01                                     PARTIAL  F        3.08
SECONDARY  ELEMENTS
  27          8
  0.43503D+02  0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
3	DEPENDENT	
27	-0.51238₁₀+01	0.54656₁₀+00
8	-0.28883₁₀+01	0.16463₁₀+01
CONSTANT	0.20139₁₀+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.272532₁₀+03	2	0.136266₁₀+03
RESIDUAL	0.123376₁₀+03	49	0.251788₁₀+01
TOTAL	0.395908₁₀+03	51	0.776291₁₀+01
F VALUE	54.12		
MULTIPLE R	0.8297		
R SQUARED	0.6884		

ELEMENT ANALYSIS

```
PRIMARY  ELEMENT     27        0.18854D+03                              PARTIAL  F     166.49
SECONDARY  ELEMENTS
  27            8             16
  0.            0.19284D+02   0.32743D+02

PRIMARY  ELEMENT      8        0.10971D+01                              PARTIAL  F       0.97
SECONDARY  ELEMENTS
  27            8             16
  0.19284D+02   0.            0.66528D+01

PRIMARY  ELEMENT     16        0.69019D+02                              PARTIAL  F      60.95
SECONDARY  ELEMENTS
  27            8             16
  0.32743D+02   0.66528D+01   0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
3	DEPENDENT	
27	-0.47663D+01	0.36939D+00
8	-0.11096D+01	0.11273D+01
16	0.79186D-01	0.10143D-01
CONSTANT	0.13957D+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.341551D+03	3	0.113850D+03
RESIDUAL	0.543576D+02	48	0.113245D+01
TOTAL	0.395908D+03	51	0.776291D+01
F VALUE	100.53		
MULTIPLE R	0.9288		
R SQUARED	0.8627		

ELEMENT ANALYSIS

```
PRIMARY  ELEMENT    27        0.43156⏨+01                          PARTIAL  F      4.28
SECONDARY  ELEMENTS
  27           8             16            13
  0.          0.11715⏨+02  0.26601⏨+00  0.18422⏨+03

PRIMARY  ELEMENT     8        0.77456⏨+01                          PARTIAL  F      7.68
SECONDARY  ELEMENTS
  27           8             16            13
  0.11715⏨+02  0.            0.23492⏨+01 -0.66485⏨+01

PRIMARY  ELEMENT    16        0.68451⏨+02                          PARTIAL  F     67.91
SECONDARY  ELEMENTS
  27           8             16            13
  0.26601⏨+00  0.23492⏨+01  0.            0.56791⏨+00

PRIMARY  ELEMENT    13        0.69828⏨+01                          PARTIAL  F      6.93
SECONDARY  ELEMENTS
  27           8             16            13
  0.18422⏨+03 -0.66485⏨+01  0.56791⏨+00  0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
3	DEPENDENT	
27	0.17591⏨+02	0.85013⏨+01
8	-0.17174⏨+02	0.61954⏨+01
16	0.78866⏨-01	0.95703⏨-02
13	-0.57328⏨+02	0.21781⏨+02
CONSTANT	0.55110⏨+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.348534⏨+03	4	0.871334⏨+02
RESIDUAL	0.473748⏨+02	47	0.100797⏨+01
TOTAL	0.395908⏨+03	51	0.776291⏨+01
F VALUE	86.44		
MULTIPLE R	0.9383		
R SQUARED	0.8803		

ELEMENT ANALYSIS

```
PRIMARY  ELEMENT    27        0.61065⏨+01                                         PARTIAL  F        6.88
SECONDARY  ELEMENTS
  27             8          16           13           29
  0.          0.12888⏨+02  0.62330⏨+00  0.18075⏨+03 -0.17909⏨+01

PRIMARY  ELEMENT     8        0.11176⏨+02                                         PARTIAL  F       12.58
SECONDARY  ELEMENTS
  27             8          16           13           29
  0.12888⏨+02  0.           0.33862⏨+01 -0.73874⏨+01 -0.34309⏨+01

PRIMARY  ELEMENT    16        0.66391⏨+02                                         PARTIAL  F       74.75
SECONDARY  ELEMENTS
  27             8          16           13           29
  0.62330⏨+00  0.33862⏨+01  0.           0.10192⏨+01  0.20592⏨+01

PRIMARY  ELEMENT    13        0.91595⏨+01                                         PARTIAL  F       10.31
SECONDARY  ELEMENTS
  27             8          16           13           29
  0.18075⏨+03 -0.73874⏨+01  0.10192⏨+01  0.          -0.21767⏨+01

PRIMARY  ELEMENT    29        0.65195⏨+01                                         PARTIAL  F        7.34
SECONDARY  ELEMENTS
  27             8          16           13           29
 -0.17909⏨+01 -0.34309⏨+01  0.20592⏨+01 -0.21767⏨+01  0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
3	DEPENDENT	
27	0.21217$_{10}$+02	0.80915$_{10}$+01
8	-0.21338$_{10}$+02	0.60153$_{10}$+01
16	0.77752$_{10}$-01	0.89929$_{10}$-02
13	-0.66564$_{10}$+02	0.20728$_{10}$+02
29	-0.14748$_{10}$+00	0.54433$_{10}$-01
CONSTANT	0.67712$_{10}$+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.355053$_{10}$+03	5	0.710106$_{10}$+02
RESIDUAL	0.408553$_{10}$+02	46	0.888159$_{10}$+00
TOTAL	0.395908$_{10}$+03	51	0.776291$_{10}$+01
F VALUE	79.95		
MULTIPLE R	0.9470		
R SQUARED	0.8968		

ELEMENT ANALYSIS

```
PRIMARY  ELEMENT    27       0.60743⏨+01                                           PARTIAL  F       7.21
SECONDARY  ELEMENTS
  27            8            16           13           29           36
  0.           0.12320⏨+02  0.56427⏨+00  0.15472⏨+03 -0.37488⏨+00  0.32265⏨-01

PRIMARY  ELEMENT     8       0.11049⏨+02                                           PARTIAL  F      13.12
SECONDARY  ELEMENTS
  27            8            16           13           29           36
  0.12320⏨+02  0.           0.30748⏨+01 -0.82786⏨+01 -0.57188⏨+00  0.12792⏨+00

PRIMARY  ELEMENT    16       0.60776⏨+02                                           PARTIAL  F      72.18
SECONDARY  ELEMENTS
  27            8            16           13           29           36
  0.56427⏨+00  0.30748⏨+01  0.           0.11216⏨+01 -0.52197⏨+00  0.56152⏨+01

PRIMARY  ELEMENT    13       0.93601⏨+01                                           PARTIAL  F      11.12
SECONDARY  ELEMENTS
  27            8            16           13           29           36
  0.15472⏨+03 -0.82786⏨+01  0.11216⏨+01  0.          -0.39842⏨+00 -0.20058⏨+00

PRIMARY  ELEMENT    29       0.58330⏨+00                                           PARTIAL  F       0.69
SECONDARY  ELEMENTS
  27            8            16           13           29           36
 -0.37488⏨+00 -0.57188⏨+00 -0.52197⏨+00 -0.39842⏨+00  0.           0.59362⏨+01

PRIMARY  ELEMENT    36       0.29658⏨+01                                           PARTIAL  F       3.52
SECONDARY  ELEMENTS
  27            8            16           13           29           36
  0.32265⏨-01  0.12792⏨+00  0.56152⏨+01 -0.20058⏨+00  0.59362⏨+01  0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
3	DEPENDENT	
27	0.21161$_{10}$+02	0.78784$_{10}$+01
8	-0.21217$_{10}$+02	0.58572$_{10}$+01
16	0.75249$_{10}$-01	0.88570$_{10}$-02
13	-0.67302$_{10}$+02	0.20186$_{10}$+02
29	-0.59029$_{10}$-01	0.70921$_{10}$-01
36	-0.10084$_{10}$-01	0.53727$_{10}$-02
CONSTANT	0.67824$_{10}$+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.358019$_{10}$+03	6	0.596698$_{10}$+02
RESIDUAL	0.378895$_{10}$+02	45	0.841989$_{10}$+00
TOTAL	0.395908$_{10}$+03	51	0.776291$_{10}$+01
F VALUE	70.87		
MULTIPLE R	0.9509		
R SQUARED	0.9043		

ELEMENT ANALYSIS

```
PRIMARY   ELEMENT     27        0.37319⏨+01                                               PARTIAL   F      4.83
SECONDARY   ELEMENTS
  27             8            16            13            29            36            38
  0.            0.12646⏨+02  0.26362⏨+01  0.60311⏨+02 -0.80217⏨-01  0.42882⏨+00  0.23423⏨+01

PRIMARY   ELEMENT      8        0.73671⏨+01                                               PARTIAL   F      9.53
SECONDARY   ELEMENTS
  27             8            16            13            29            36            38
  0.12646⏨+02  0.                0.60813⏨+01 -0.39147⏨+01  0.69716⏨-01  0.73829⏨+00  0.36814⏨+01

PRIMARY   ELEMENT     16        0.64659⏨+02                                               PARTIAL   F     83.67
SECONDARY   ELEMENTS
  27             8            16            13            29            36            38
  0.26362⏨+01  0.60813⏨+01  0.              0.40747⏨+01 -0.17997⏨-01  0.37130⏨+01 -0.38826⏨+01

PRIMARY   ELEMENT     13        0.57983⏨+01                                               PARTIAL   F      7.50
SECONDARY   ELEMENTS
  27             8            16            13            29            36            38
  0.60311⏨+02 -0.39147⏨+01  0.40747⏨+01  0.             -0.69732⏨-01  0.37453⏨+00  0.35617⏨+01

PRIMARY   ELEMENT     29        0.82597⏨-01                                               PARTIAL   F      0.11
SECONDARY   ELEMENTS
  27             8            16            13            29            36            38
 -0.80217⏨-01  0.69716⏨-01 -0.17997⏨-01 -0.69732⏨-01  0.              0.53013⏨+01  0.50070⏨+00

PRIMARY   ELEMENT     36        0.43324⏨+01                                               PARTIAL   F      5.61
SECONDARY   ELEMENTS
  27             8            16            13            29            36            38
  0.42882⏨+00  0.73829⏨+00  0.37130⏨+01  0.37453⏨+00  0.53013⏨+01  0.             -0.13665⏨+01

PRIMARY   ELEMENT     38        0.38859⏨+01                                               PARTIAL   F      5.03
SECONDARY   ELEMENTS
  27             8            16            13            29            36            38
  0.23423⏨+01  0.36814⏨+01 -0.38826⏨+01  0.35617⏨+01  0.50070⏨+00 -0.13665⏨+01  0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
3	DEPENDENT	
27	0.17065₁₀+02	0.77657₁₀+01
8	-0.17913₁₀+02	0.58017₁₀+01
16	0.80205₁₀-01	0.87685₁₀-02
13	-0.55046₁₀+02	0.20096₁₀+02
29	-0.22831₁₀-01	0.69836₁₀-01
36	-0.12438₁₀-01	0.52533₁₀-02
38	0.36578₁₀-01	0.16312₁₀-01
CONSTANT	0.57720₁₀+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.361905₁₀+03	7	0.517007₁₀+02
RESIDUAL	0.340036₁₀+02	44	0.772809₁₀+00
TOTAL	0.395908₁₀+03	51	0.776291₁₀+01
F VALUE	66.90		

MULTIPLE R	0.9561
R SQUARED	0.9141

TABLE 4.5.

Regression equations for maximum stress - C-type tests

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
2	DEPENDENT	
27	-0.63063₁₀+01	0.56464₁₀+00
CONSTANT	0.15834₁₀+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.362737₁₀+03	1	0.362737₁₀+03
RESIDUAL	0.145401₁₀+03	50	0.290801₁₀+01
TOTAL	0.508138₁₀+03	51	0.996349₁₀+01
F VALUE	124.74		
MULTIPLE R	0.8449		
R SQUARED	0.7139		

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
2	DEPENDENT	
27	-0.58605₁₀+01	0.54646₁₀+00
8	-0.48738₁₀+01	0.16460₁₀+01
CONSTANT	0.27557₁₀+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.384805₁₀+03	2	0.192403₁₀+03
RESIDUAL	0.123333₁₀+03	49	0.251700₁₀+01
TOTAL	0.508138₁₀+03	51	0.996349₁₀+01
F VALUE	76.44		

MULTIPLE R	0.8702
R SQUARED	0.7573

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
2	DEPENDENT	
27	-0.54846$_{10}$+01	0.34350$_{10}$+00
8	-0.30033$_{10}$+01	0.10483$_{10}$+01
16	0.83275$_{10}$-01	0.94321$_{10}$-02
CONSTANT	0.21055$_{10}$+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.461135$_{10}$+03	3	0.153712$_{10}$+03
RESIDUAL	0.470033$_{10}$+02	48	0.979234$_{10}$+00
TOTAL	0.508138$_{10}$+03	51	0.996349$_{10}$+01
F VALUE	156.97		
MULTIPLE R	0.9526		
R SQUARED	0.9075		

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
2	DEPENDENT	
27	0.26041₁₀+02	0.71080₁₀+01
8	-0.25656₁₀+02	0.51800₁₀+01
16	0.82823₁₀-01	0.80018₁₀-02
13	-0.80839₁₀+02	0.18211₁₀+02
CONSTANT	0.79087₁₀+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.475020₁₀+03	4	0.118754₁₀+03
RESIDUAL	0.331184₁₀+02	47	0.704647₁₀+00
TOTAL	0.508138₁₀+03	51	0.996349₁₀+01
F VALUE	168.53		
MULTIPLE R	0.9669		
R SQUARED	0.9348		

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
2	DEPENDENT	
27	$0.26876_{10}+02$	$0.72471_{10}+01$
8	$-0.26615_{10}+02$	$0.53875_{10}+01$
16	$0.82567_{10}-01$	$0.80544_{10}-02$
13	$-0.82965_{10}+02$	$0.18565_{10}+02$
29	$-0.33943_{10}-01$	$0.48752_{10}-01$
CONSTANT	$0.81987_{10}+02$	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	$0.475365_{10}+03$	5	$0.950730_{10}+02$
RESIDUAL	$0.327731_{10}+02$	46	$0.712458_{10}+00$
TOTAL	$0.508138_{10}+03$	51	$0.996349_{10}+01$
F VALUE	133.44		

MULTIPLE R	0.9672
R SQUARED	0.9355

ELEMENT ANALYSIS

```
PRIMARY  ELEMENT    27        0.98427⏨+01                                          PARTIAL  F       15.14
SECONDARY  ELEMENTS
  27          8           16          13          29          36
  0.          0.17652⏨+02  0.81237⏨+00  0.15210⏨+03 -0.11494⏨+01 -0.44311⏨-01

PRIMARY  ELEMENT     8        0.17557⏨+02                                          PARTIAL  F       27.00
SECONDARY  ELEMENTS
  27          8           16          13          29          36
  0.17652⏨+02  0.          0.43721⏨+01 -0.10189⏨+02 -0.20782⏨+01 -0.17054⏨+00

PRIMARY  ELEMENT    16        0.78077⏨+02                                          PARTIAL  F      120.07
SECONDARY  ELEMENTS
  27          8           16          13          29          36
  0.81237⏨+00  0.43721⏨+01  0.          0.15494⏨+01 -0.16241⏨+01 -0.32082⏨+01

PRIMARY  ELEMENT    13        0.13950⏨+02                                          PARTIAL  F       21.45
SECONDARY  ELEMENTS
  27          8           16          13          29          36
  0.15210⏨+03 -0.10189⏨+02  0.15494⏨+01  0.          -0.11947⏨+01  0.27921⏨+00

PRIMARY  ELEMENT    29        0.28362⏨+01                                          PARTIAL  F        4.36
SECONDARY  ELEMENTS
  27          8           16          13          29          36
 -0.11494⏨+01 -0.20782⏨+01 -0.16241⏨+01 -0.11947⏨+01  0.          -0.24908⏨+01

PRIMARY  ELEMENT    36        0.35100⏨+01                                          PARTIAL  F        5.40
SECONDARY  ELEMENTS
  27          8           16          13          29          36
 -0.44311⏨-01 -0.17054⏨+00 -0.32082⏨+01  0.27921⏨+00 -0.24908⏨+01  0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
2	DEPENDENT	
27	$0.26937_{10}+02$	$0.69237_{10}+01$
8	$-0.26746_{10}+02$	$0.51474_{10}+01$
16	$0.85290_{10}-01$	$0.77838_{10}-02$
13	$-0.82162_{10}+02$	$0.17739_{10}+02$
29	$-0.13016_{10}+00$	$0.62326_{10}-01$
36	$0.10970_{10}-01$	$0.47217_{10}-02$
CONSTANT	$0.81866_{10}+02$	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	$0.478875_{10}+03$	6	$0.798125_{10}+02$
RESIDUAL	$0.292630_{10}+02$	45	$0.650290_{10}+00$
TOTAL	$0.508138_{10}+03$	51	$0.996349_{10}+01$
F VALUE	122.73		

MULTIPLE R	0.9708
R SQUARED	0.9424

TABLE 4.6.

Regression equations for intercept stress - C-type tests

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
1	DEPENDENT	
27	-0.27609₁₀+01	0.46247₁₀+00
CONSTANT	0.73411₁₀+01	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.695238₁₀+02	1	0.695238₁₀+02
RESIDUAL	0.975389₁₀+02	50	0.195078₁₀+01
TOTAL	0.167063₁₀+03	51	0.327574₁₀+01
F VALUE	35.64		
MULTIPLE R	0.6451		
R SQUARED	0.4162		

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
1	DEPENDENT	
27	-0.23724₁₀+01	0.44225₁₀+00
8	-0.42474₁₀+01	0.13321₁₀+01
CONSTANT	0.17557₁₀+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.862834₁₀+02	2	0.431417₁₀+02
RESIDUAL	0.807793₁₀+02	49	0.164856₁₀+01
TOTAL	0.167063₁₀+03	51	0.327574₁₀+01
F VALUE	26.17		
MULTIPLE R	0.7187		
R SQUARED	0.5165		

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
1	DEPENDENT	
27	$-0.22576_{10}+01$	$0.43003_{10}+00$
8	$-0.36764_{10}+01$	$0.13124_{10}+01$
16	$0.25419_{10}-01$	$0.11808_{10}-01$
CONSTANT	$0.15573_{10}+02$	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	$0.933950_{10}+02$	3	$0.311317_{10}+02$
RESIDUAL	$0.736676_{10}+02$	48	$0.153474_{10}+01$
TOTAL	$0.167063_{10}+03$	51	$0.327574_{10}+01$
F VALUE	20.28		

MULTIPLE R 0.7477

R SQUARED 0.5590

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
1	DEPENDENT	
27	0.17622$_{10}$+02	0.10196$_{10}$+02
8	-0.17961$_{10}$+02	0.74305$_{10}$+01
16	0.25134$_{10}$-01	0.11478$_{10}$-01
13	-0.50975$_{10}$+02	0.26123$_{10}$+02
CONSTANT	0.52166$_{10}$+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.989160$_{10}$+02	4	0.247290$_{10}$+02
RESIDUAL	0.681466$_{10}$+02	47	0.144993$_{10}$+01
TOTAL	0.167063$_{10}$+03	51	0.327574$_{10}$+01
F VALUE	17.06		
MULTIPLE R	0.7695		
R SQUARED	0.5921		

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
1	DEPENDENT	
27	0.20372$_{10}$+02	0.10159$_{10}$+02
8	-0.21119$_{10}$+02	0.75519$_{10}$+01
16	0.24289$_{10}$-01	0.11290$_{10}$-01
13	-0.57981$_{10}$+02	0.26023$_{10}$+02
29	-0.11186$_{10}$+00	0.68338$_{10}$-01
CONSTANT	0.61725$_{10}$+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.102667$_{10}$+03	5	0.205334$_{10}$+02
RESIDUAL	0.643956$_{10}$+02	46	0.139990$_{10}$+01
TOTAL	0.167063$_{10}$+03	51	0.327574$_{10}$+01
F VALUE	14.67		

MULTIPLE R	0.7839
R SQUARED	0.6145

ELEMENT ANALYSIS

```
PRIMARY  ELEMENT    27       0.56549⏨+01                                                PARTIAL  F      4.07
SECONDARY  ELEMENTS
  27           8             16             13             29             36
  0.           0.16929⏨+02   0.18834⏨+00    0.18225⏨+02   -0.12606⏨+01   -0.24799⏨-01

PRIMARY  ELEMENT     8       0.11048⏨+02                                                PARTIAL  F      7.96
SECONDARY  ELEMENTS
  27           8             16             13             29             36
  0.16929⏨+02  0.            0.10545⏨+01    0.45185⏨+01   -0.25656⏨+01   -0.99816⏨-01

PRIMARY  ELEMENT    16       0.74237⏨+01                                                PARTIAL  F      5.35
SECONDARY  ELEMENTS
  27           8             16             13             29             36
  0.18834⏨+00  0.10545⏨+01   0.             0.33081⏨+00   -0.80095⏨+00   -0.94480⏨+00

PRIMARY  ELEMENT    13       0.68057⏨+01                                                PARTIAL  F      4.90
SECONDARY  ELEMENTS
  27           8             16             13             29             36
  0.18225⏨+02  0.45185⏨+01   0.33081⏨+00    0.            -0.12224⏨+01    0.14390⏨+00

PRIMARY  ELEMENT    29       0.56008⏨+01                                                PARTIAL  F      4.03
SECONDARY  ELEMENTS
  27           8             16             13             29             36
 -0.12606⏨+01 -0.25656⏨+01  -0.80095⏨+00   -0.12224⏨+01    0.            -0.18498⏨+01

PRIMARY  ELEMENT    36       0.19139⏨+01                                                PARTIAL  F      1.38
SECONDARY  ELEMENTS
  27           8             16             13             29             36
 -0.24799⏨-01 -0.99816⏨-01  -0.94480⏨+00    0.14390⏨+00   -0.18498⏨+01    0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
1	DEPENDENT	
27	0.20417$_{10}$+02	0.10117$_{10}$+02
8	-0.21217$_{10}$+02	0.75215$_{10}$+01
16	0.26299$_{10}$-01	0.11374$_{10}$-01
13	-0.57389$_{10}$+02	0.25921$_{10}$+02
29	-0.18291$_{10}$+00	0.91073$_{10}$-01
36	0.81002$_{10}$-02	0.68994$_{10}$-02
CONSTANT	0.61635$_{10}$+02	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.104580$_{10}$+03	6	0.174301$_{10}$+02
RESIDUAL	0.624818$_{10}$+02	45	0.138848$_{10}$+01
TOTAL	0.167063$_{10}$+03	51	0.327574$_{10}$+01
F VALUE	12.55		
MULTIPLE R	0.7912		
R SQUARED	0.6260		

TABLE 4.7.

Regression equations for work-hardening coefficient - C-type tests

ELEMENT ANALYSIS

```
PRIMARY  ELEMENT    16        0.84539ø+02                                  PARTIAL  F        33.46
SECONDARY  ELEMENTS
  16
   0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
4	DEPENDENT	
16	0.84395⏨-01	0.14589⏨-01
CONSTANT	0.18353⏨+01	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.845386⏨+02	1	0.845386⏨+02
RESIDUAL	0.126311⏨+03	50	0.252622⏨+01
TOTAL	0.210850⏨+03	51	0.413431⏨+01
F VALUE	33.46		

MULTIPLE R	0.6332
R SQUARED	0.4009

ELEMENT ANALYSIS

```
PRIMARY  ELEMENT     16        0.16698⏨+02                                        PARTIAL  F        9.14
SECONDARY  ELEMENTS
  16           42
  0.           0.67841⏨+02

PRIMARY  ELEMENT     42        0.36838⏨+02                                        PARTIAL  F       20.17
SECONDARY  ELEMENTS
  16           42
  0.67841⏨+02  0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
4	DEPENDENT	
16	0.45676⏨-01	0.15105⏨-01
42	0.10196⏨-01	0.22701⏨-02
CONSTANT	-0.74986⏨+00	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.121376⏨+03	2	0.606883⏨+02
RESIDUAL	0.894731⏨+02	49	0.182598⏨+01
TOTAL	0.210850⏨+03	51	0.413431⏨+01
F VALUE	33.24		

MULTIPLE R	0.7587
R SQUARED	0.5757

ELEMENT ANALYSIS

```
PRIMARY  ELEMENT     16         0.29983D+02                                       PARTIAL  F       21.73
SECONDARY  ELEMENTS
  16           42             39
  0.           0.52635D+02 -0.13285D+02

PRIMARY  ELEMENT     42         0.46748D+01                                       PARTIAL  F        3.39
SECONDARY  ELEMENTS
  16           42             39
  0.52635D+02  0.             0.32163D+02

PRIMARY  ELEMENT     39         0.23238D+02                                       PARTIAL  F       16.84
SECONDARY  ELEMENTS
  16           42             39
 -0.13285D+02  0.32163D+02  0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
4	DEPENDENT	
16	0.34104⏨+00	0.73162⏨-01
42	0.44530⏨-02	0.24193⏨-02
39	-0.25850⏨+01	0.62992⏨+00
CONSTANT	0.94239⏨+00	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.144615⏨+03	3	0.482048⏨+02
RESIDUAL	0.662352⏨+02	48	0.137990⏨+01
TOTAL	0.210850⏨+03	51	0.413431⏨+01
F VALUE	34.93		

MULTIPLE R	0.8282
R SQUARED	0.6859

```
                                                   ELEMENT   ANALYSIS
                                                   ------------------

PRIMARY  ELEMENT     16        0.13220⏨+02                          PARTIAL  F       7.10
SECONDARY  ELEMENTS
  16           42             38
  0.           0.75331⏨+02   0.34779⏨+01

PRIMARY  ELEMENT     42        0.27929⏨+02                          PARTIAL  F      15.00
SECONDARY  ELEMENTS
  16           42             38
  0.75331⏨+02  0.             0.89089⏨+01

PRIMARY  ELEMENT     38        0.71567⏨-01                          PARTIAL  F       0.04
SECONDARY  ELEMENTS
  16           42             38
  0.34779⏨+01  0.89089⏨+01   0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
4	DEPENDENT	
16	0.44367⏨-01	0.16653⏨-01
42	0.10478⏨-01	0.27058⏨-02
38	-0.37844⏨-02	0.19306⏨-01
CONSTANT	-0.79488⏨+00	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.121448⏨+03	3	0.404827⏨+02
RESIDUAL	0.894016⏨+02	48	0.186253⏨+01
TOTAL	0.210850⏨+03	51	0.413431⏨+01
F VALUE	21.74		
MULTIPLE R	0.7589		
R SQUARED	0.5760		

ELEMENT ANALYSIS

```
PRIMARY  ELEMENT    16        0.33659▪+02                                       PARTIAL  F      24.75
SECONDARY  ELEMENTS
  16           42           38             41
  0.           0.15174▪+02 -0.48127▪+01 -0.20439▪+02

PRIMARY  ELEMENT    42        0.25730▪+02                                       PARTIAL  F      18.92
SECONDARY  ELEMENTS
  16           42           38             41
  0.15174▪+02  0.          -0.51334▪+01   0.21994▪+01

PRIMARY  ELEMENT    38        0.51497▪+01                                       PARTIAL  F       3.79
SECONDARY  ELEMENTS
  16           42           38             41
 -0.48127▪+01 -0.51334▪+01  0.            -0.50781▪+01

PRIMARY  ELEMENT    41        0.25496▪+02                                       PARTIAL  F      18.75
SECONDARY  ELEMENTS
  16           42           38             41
 -0.20439▪+02  0.21994▪+01 -0.50781▪+01   0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
4	DEPENDENT	
16	0.28143$_{10}$+00	0.56564$_{10}$-01
42	0.10066$_{10}$-01	0.23139$_{10}$-02
38	-0.35041$_{10}$-01	0.18005$_{10}$-01
41	-0.28732$_{10}$+00	0.66352$_{10}$-01
CONSTANT	-0.91195$_{10}$+00	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.146944$_{10}$+03	4	0.367361$_{10}$+02
RESIDUAL	0.639054$_{10}$+02	47	0.135969$_{10}$+01
TOTAL	0.210850$_{10}$+03	51	0.413431$_{10}$+01
F VALUE	27.02		

MULTIPLE R 0.8348

R SQUARED 0.6969

TABLE 4.8.

Regression equations for work-hardening coefficient - B-type tests

ELEMENT ANALYSIS

PRIMARY ELEMENT 16 0.13919E+03 PARTIAL F 29.22
SECONDARY ELEMENTS
16
0.

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
4	DEPENDENT	
16	0.660144D-01	0.12211D-01
CONSTANT	0.25145D+01	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.139185D+03	1	0.139185D+03
RESIDUAL	0.395307D+03	83	0.476274D+01
TOTAL	0.534493D+03	84	0.636301D+01
F VALUE	29.22		

MULTIPLE R	0.5103
R SQUARED	0.2604

ELEMENT ANALYSIS

```
PRIMARY ELEMENT      16         0.26192 +02                              PARTIAL F        6.06
SECONDARY ELEMENTS
  16          42
  0.          0.11299 +03

PRIMARY ELEMENT      42         0.40851 +02                              PARTIAL F        9.45
SECONDARY ELEMENTS
  16          42
  0.11299 +03 0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
4	DEPENDENT	
16	0.36917⏨-01	0.14998⏨-01
42	0.80709⏨-02	0.26254⏨-02
CONSTANT	0.34776⏨+00	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.180036⏨+03	2	0.900181⏨+02
RESIDUAL	0.354456⏨+03	82	0.432264⏨+01
TOTAL	0.534493⏨+03	84	0.636301⏨+01
F VALUE	20.82		
MULTIPLE R	0.5804		
R SQUARED	0.3368		

ELEMENT ANALYSIS

```
PRIMARY ELEMENT    16        0.44690E+01                         PARTIAL F      1.03
SECONDARY ELEMENTS
  16          42          39
  0.          0.37276E+02 0.21723E+02

PRIMARY ELEMENT    42        0.23644E+02                         PARTIAL F      5.42
SECONDARY ELEMENTS
  16          42          39
  0.37276E+02 0.          0.17207E+02

PRIMARY ELEMENT    39        0.13208E+01                         PARTIAL F      0.30
SECONDARY ELEMENTS
  16          42          39
  0.21723E+02 0.17207E+02 0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
4	DEPENDENT	
16	0.79127⏨-01	0.78153⏨-01
42	0.71877⏨-02	0.30865⏨-02
39	-0.41301⏨+00	0.75037⏨+00
CONSTANT	0.64337⏨+00	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.181357⏨+03	3	0.604523⏨+02
RESIDUAL	0.353136⏨+03	81	0.435970⏨+01
TOTAL	0.534493⏨+03	84	0.636301⏨+01
F VALUE	13.87		
MULTIPLE R	0.5825		
R SQUARED	0.3393		

ELEMENT ANALYSIS

```
PRIMARY ELEMENT    16        0.27829*+02                         PARTIAL F      6.43
SECONDARY ELEMENTS
  16          42          38
  0.          0.11477*+03 -0.16371*+01

PRIMARY ELEMENT    42        0.40925*+02                         PARTIAL F      9.45
SECONDARY ELEMENTS
  16          42          38
  0.11477*+03 0.          -0.74431*-01

PRIMARY ELEMENT    38        0.36766*+01                         PARTIAL F      0.85
SECONDARY ELEMENTS
  16          42          38
 -0.16371*+01 -0.74431*-01 0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
4	DEPENDENT	
16	0.38223D-01	0.15078D-01
42	0.80783D-02	0.26278D-02
38	0.75306D-05	0.81730D-05
CONSTANT	0.26619D+00	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.183713D+03	3	0.612376D+02
RESIDUAL	0.350780D+03	81	0.433061D+01
TOTAL	0.534493D+03	84	0.636301D+01
F VALUE	14.14		
MULTIPLE R	0.5863		
R SQUARED	0.3437		

ELEMENT ANALYSIS

```
PRIMARY   ELEMENT     16          0.32959+02                                        PARTIAL  F          7.98
SECONDARY   ELEMENTS
  16            42            38            41
  0.            0.35958+02    0.83483+00   -0.51300+01

PRIMARY   ELEMENT     42          0.27638+02                                        PARTIAL  F          6.69
SECONDARY   ELEMENTS
  16            42            38            41
  0.35958+02    0.           -0.29497+00    0.13287+02

PRIMARY   ELEMENT     38          0.25633+01                                        PARTIAL  F          0.62
SECONDARY   ELEMENTS
  16            42            38            41
  0.83483+00   -0.29497+00    0.            0.11133+01

PRIMARY   ELEMENT     41          0.20300+02                                        PARTIAL  F          4.91
SECONDARY   ELEMENTS
  16            42            38            41
 -0.51300+01    0.13287+02    0.11133+01    0.
```

REGRESSION EQUATION

VARIABLE	B	S.E.(B)
4	DEPENDENT	
16	0.15718»+00	0.55645»-01
42	0.68033»-02	0.26302»-02
38	0.63030»-05	0.80016»-05
41	-0.15579»+00	0.70277»-01
CONSTANT	0.41656»+00	

ANALYSIS OF VARIANCE

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE
REGRESSION	0.204013»+03	4	0.510032»+02
RESIDUAL	0.330480»+03	80	0.413100»+01
TOTAL	0.534493»+03	84	0.636301»+01
F VALUE	12.35		

MULTIPLE R	0.6178
R SQUARED	0.3817

SUMMARY

The final least-squares regression equations for the parameters used to describe the stress-strain curve are presented below. In general the independent variables included are significant at a level of probability of error less than 0.05. In the small number of cases where probabilities greater than this have been adopted the reasons for doing so will be explained in Section 5: Discussion.

For the B-type tests the equations are:

$$\tau_o = 53.92 + 17.10\, b.\theta - 18.32b + 0.031\, G^2/\gamma_{SF} - 48.84\theta - 0.239G.\theta + 0.019\, G.\ln\dot{\gamma} \qquad (4.1)$$

$$\tau_M = 97.61 + 30.76\, b.\theta - 31.91b + 0.070\, G^2/\gamma_{SF} - 93.66\theta - 0.191\, G.\theta + 0.009\, G.\ln\dot{\gamma} \qquad (4.2)$$

$$K = 0.42 + 0.16\, G^2/\gamma_{SF} + 0.007\, G.b.D^{-\frac{1}{2}} + 0.63 \times 10^{-5}\, GD_d^{-1/5} - 0.16\, G.b.D^{-\frac{1}{2}}/\gamma_{SF} \qquad (4.3)$$

and for the C-type tests:

$$\tau_o = 61.64 + 20.42\, b.\theta - 21.22b + 0.026\, G^2/\gamma_{SF} - 57.39\theta - 0.183\, G.\theta + 0.008\, G.\ln\dot{\gamma} \qquad (4.4)$$

$$\tau_M = 81.87 + 26.94\, b.\theta - 26.75\, b + 0.085\, G^2/\gamma_{SF} - 82.16\theta - 0.130\, G.\theta + 0.011\, G.\ln\dot{\gamma} \qquad (4.5)$$

$$\tau_S = 57.72 + 17.07\, b.\theta - 17.91\, b + 0.080\, G^2/\gamma_{SF} - 55.05\theta - 0.023\, G.\theta - 0.012\, G.\ln\dot{\gamma} + 0.037\, G.D_d^{-1/5} \qquad (4.6)$$

$$K = -0.91 + 0.28\, G^2/\gamma_{SF} + 0.010\, G.b.D^{-\frac{1}{2}} - 0.035\, G.D_d^{-1/5} - 0.287\, G.b.D^{-\frac{1}{2}}/\gamma_{SF} \qquad (4.7)$$

where: τ_o = intercept stress - t.s.i.

τ_M = maximum stress - t.s.i.

τ_S = steady state stress - t.s.i.

K = work hardening rate.

b = Burger's vector - $\mathring{A}$

θ = ratio of testing temperature to melting temperature.

G = shear modulus - t.s.i. x 10^{-2}

γ_{SF} = stacking fault energy - dynes. cm^{-2}

$\dot{\gamma}$ = strain rate - $min.^{-1}$

D = mean structure size - mm.

D_d = diffusion coefficient - k.cal. $mole^{-1}$

5. DISCUSSION

5.1. Experimental Technique

The technique of of conducting the torsion tests proved to be generally satisfactory, bearing out the anticipated advantages of test-piece stability, ease of heating to test temperature, etc. The values of maximum stress and steady state stress are parameters which are clearly identifiable and the experimental error, indicated in table 4.1 by their respective variances (or mean squares) ranged from approximately 3 per.cent. up to a maximum of 10 per cent. of the total variance.

As was expected the error variance for the intercept stress - τ_0 was much greater than for the other two stress parameters. In fact the variance proved to be approximately three times as great as for the maximum stress, a difference which is statistically significant at the 0.01 level of probability. Even this level of variability would probably be considered acceptable under many circumstances where stress determinations were required, however, the observation value being reproduced to within $\pm$ 3 t.s.i. for 95 per cent. of single determinations carried out (compared with $\pm$ 1.6 t.s.i. for the maximum stress).

The variability in K - the work hardening coefficient must be regarded as disappointingly high, the error variance representing approximately half of the total variance in this parameter, for the B-type tests. In view of the levels of reproducibility which may be inferred from the results of other workers in this field (values are rarely stated), it must be considered that the experimental technique adopted was responsible for much of the variability. Since the axial loads which develop during torsion testing can be either tensile or compressive it is necessary to provide constraint in both directions. In the current tests this was achieved by using test-pieces with threaded ends, which made machining simple and convenient. This method carries the inevitable risk of the specimen being twisted further into the threaded grips, however, giving rise to an indicated value of strain which is higher than the true value. Although care was taken to tighten the specimen into the grips to the point where the errors described were considered to be very unlikely, the high values of error variance cast some doubt on the effectiveness of this practice. If this kind of fault did arise, in fact, it must affect the validity of the final regression equation for K since the true distribution of experimental errors is likely to be strongly skewed, the tendency being always to underestimate the value of K.

5.2. Regression Analysis

The primary objective of the investigation reported here has been stated as that of producing a mathematical model of the stress-strain relationship, in a selected class of metals, such as would allow useful predictions of behaviour to be made. In order to be useful the predictions should be subject to the lowest possible error variance, that is the probability of encountering any particular value of the response variable should decrease as that value deviates to a greater extent from the 'true' value. Since the conventional methods of producing a regression equation are based upon minimising the sum of the squared values of the deviations about the regression (i.e. the residual variance) these techniques would seem to offer an attractive and convenient means of achieving the criterion of 'usefulness' stated above.

The use of regression techniques is based upon certain assumptions, the validity of which must be assessed in order to justify the techniques adopted. In making this assessment it is convenient to consider first the three parameters of stress - τ_m, τ_o and τ_s.

5.2.1. The Stress Parameters

(a) The maximum stress

The first parameter to be considered is the maximum stress for the B-type tests. The order of inclusion of independent variables was:

$b \cdot T/T_m$

b

G^2/γ_{sf}

T/T_m

At this stage the proportion of the total sum of squares which could be attributed to the regression was 0.933 (that is R^2 equalled 0.933), and the ratio of regression mean square to residual mean square (the 'total F' ratio) was 278.26, the highest F-value obtained. On the basis of achieving the highest average response this equation might be considered as the most 'efficient'. The ratio of the residual mean square to the independently estimated error mean square was 2.2 at this stage, however, which indicates a 'lack of fit' contribution to the residual, which is significant at the 0.01 level of probability.

Continuing the selection process, therefore, two more variables were added to the equation:

$G.\ T/T_m$

and $G.\ln\dot{\gamma}$ This increased R^2 to 0.943, an increase of only one per cent., but the residual mean square was reduced to 1.29 which just fails to exceed the .05 level of significance and may, therefore, be regarded as an estimate of the true error variance. No other variable of those listed had a primary element which was significant at the 0.05 level of probability at this stage.

One of the basic assumptions of the regression model is that the residual errors are normally distributed with mean equal to zero. In figure 5.1.(a) the residual errors as shown, in histogram form, compared with the curve of a true Normal

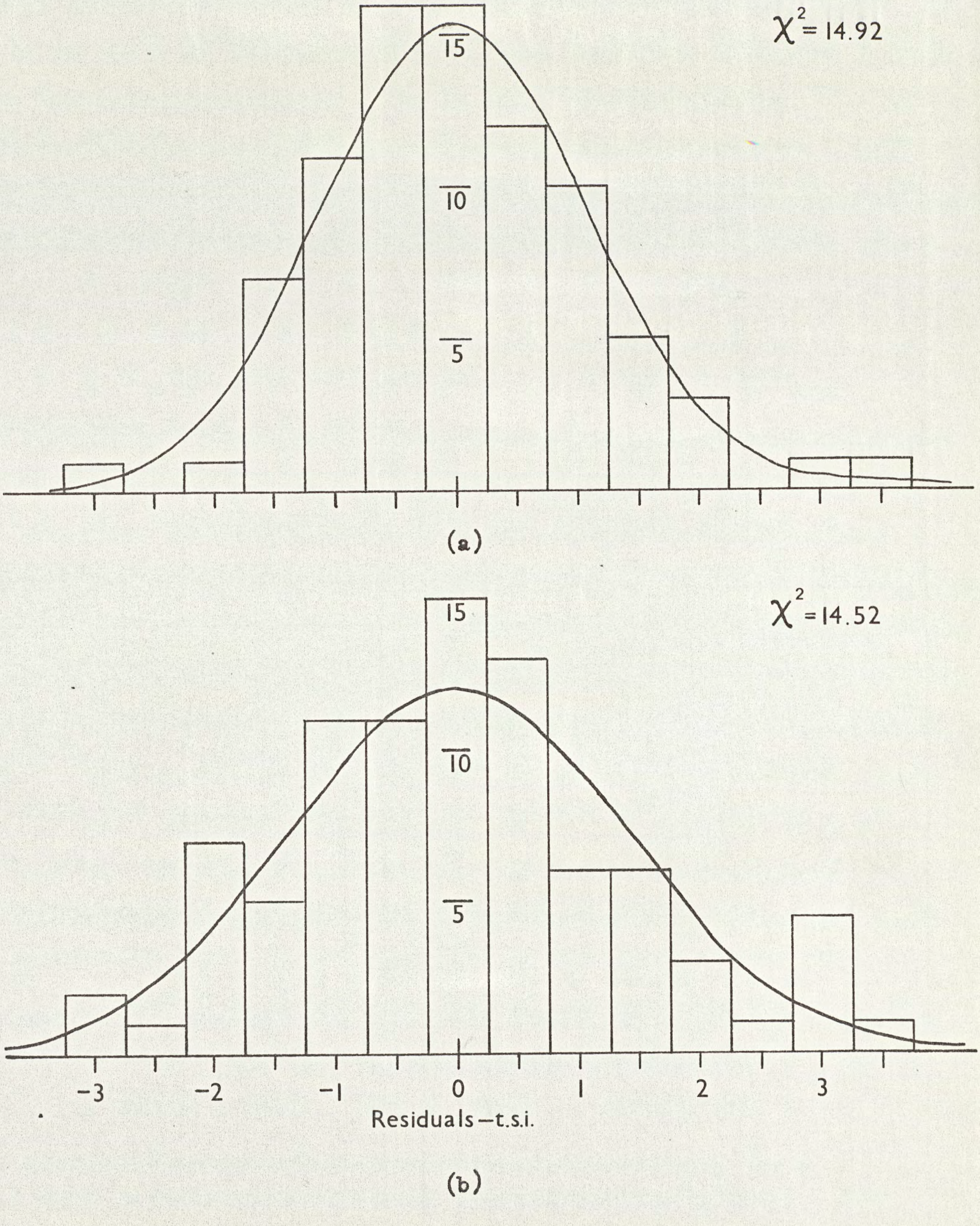


Figure 5.1. Comparison of distribution of residual errors with curve for true Normal distribution:

(a) maximum stress

(b) intercept stress

distribution with mean equal to zero and variance equal to 1.19. (The individual differences between the predicted and observed values were calculated independently and their mean and variance computed at 0.008 and 1.19 respectively. Although these differ from the predicted values of zero and 1.29 the differences are sufficiently small to be regarded as arithmetic errors, arising out of 'rounding' for example, and may be ignored).

It will be seen that the fit is quite good by visual examination. The CHI - SQUARED value for this diagram computed from the sum of all the $\frac{(O - E)^2}{E}$ terms* for each of the blocks in the histogram is about 14 and this value has a probability of between 0.5 and 0.4 from the Chi-squared tables. That is to say, that if observations had been selected at random from a population with a true Normal distribution, in order to construct the histogram, the likelihood of obtaining a distribution <u>more</u> closely resembling the true curve would have almost the same as the likelihood of obtaining one <u>less</u> closely resembling the true curve. It can quite reasonably be assumed, therefore, that the residual errors are normally distributed.

A further requirement of the regression model is that the errors are not correlated in any way with any of the variables in the regression. Where the variables may be classified into separate groups the absence or otherwise of correlations may be examined by calculating the mean and variance for each group.

* O - observed value.

E - expected (from Normal distribution) value.

These statistics should not then differ significantly from those for all of the groups added together, i.e. from those for the overall residual errors. In table 5.1 the means and variances of the residual errors in maximum stress are shown for the B-type tests together with their 't' values and 'F' values.

In no case was the t-value significant at 0.05 level of probability, confirming that the errors for different compositions are uniformly distributed about the mean, within the range of variation to be expected due to random sampling variations. Among the F-values only that for the nickel specimens was greater than the value for significance at the 0.05 level, and this was much less than the value necessary for significance to be established at the 0.01 level of probability.

It seems reasonable to accept, therefore, that the errors are independent of composition.

Where the variable which is being considered cannot be divided into groups, but is continuously variable the technique for assessing the relationship with the residual errors is different. The most convenient method is to plot the residual value (the difference between the predicted and observed values) against the predictor variable. An example of the residual errors plotted against the homologans temperature - T/T_m is shown in figure 5.2, for the B-type tests. It can be seen that there is no prominent trend to the distribution

SOURCE	MEAN	VARIANCE	d.f.	t-Value	F-Value
Total	0.008	1.195	84	-	-
N	0.524	3.118	6	1.118	2.61
A	0.094	0.567	11	0.259	2.11
C	-0.409	1.031	25	-1.715	1.16
CZ1	0.629	0.555	11	1.88	2.15
CZ2	-0.064	1.419	7	-0.175	1.19
CZ3	-0.182	1.488	9	-0.508	1.25
CA1	0.148	0.913	9	0.363	1.31

Table 5.1. Comparison of residual errors for chemical composition. Maximum stress, B-type tests.

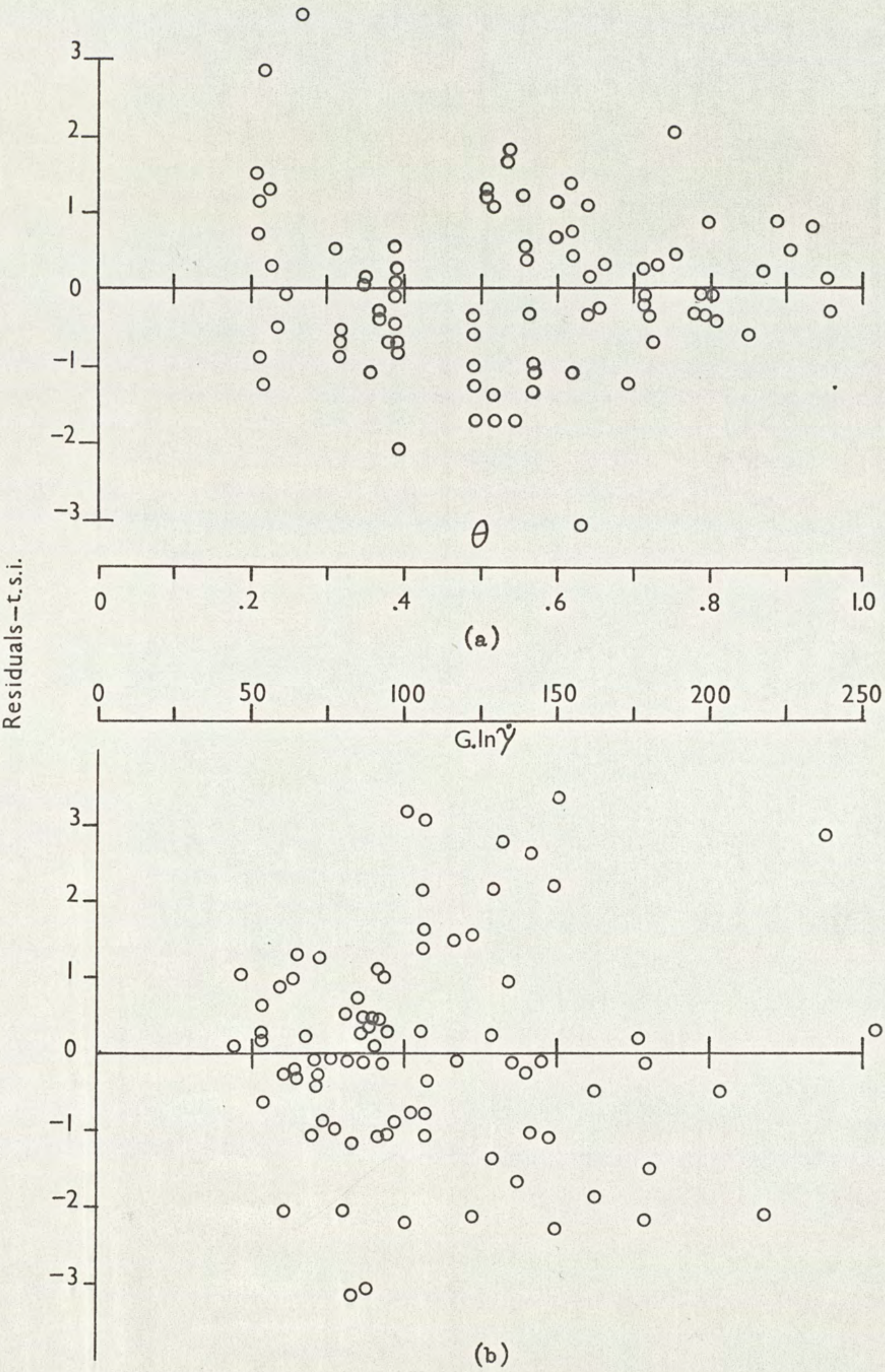


Figure 5.2. Distribution of residual errors for B-type tests:

(a) maximum stress versus T/T_m

(b) intercept stress versus $G.\ln\dot{\gamma}$

and that the residual values are distributed in an apparently random manner along the range of temperatures used. Comparison of the residuals with other predictor variables produced diagrams which were broadly similar to the one shown.

(b) <u>The intercept stress.</u>

The correlation coefficient between maximum stress and intercept stress is 0.839 (which may be compared with a value of 0.357 necessary to establish significance at the 0.001 level of probability). It is evident, therefore, that the two parameters are closely associated, and that the relationship might well be more clear but for the inherently high variability in intercept stress. It is hardly surprising, therefore, that the same variables exert an influence on both. In this case the residual mean square from the regression equation proved to be equal to the independently determined error mean square.

Figure 5.1 (b) shows the distribution of the residual errors in intercept stress compared with the theoretical curve for the Normal distribution. The value of chi-squared is, again, about 14, establishing that the residual values conform closely with a Normal distribution.

Table 5.2 shows the means, variances, t - and F - values in similar manner to those presented in table 5.1 for the maximum stress. In this case the only t - value of significance was that for the Cu tests where the mean value differed from the true mean by an amount which was significant at the 0.05 level of significance. The variance for the residual values from the Al tests was shown, by its F-value, to be significantly less than the overall variance.

SOURCE	MEAN	VARIANCE	d.f.	t-Value	F-Value
Total	-0.004	1.982	84	-	-
N	0.455	3.944	6	0.791	1.99
A	0.252	0.362	11	1.095	5.47
C	-0.641	1.117	25	-2.112	1.77
CZ1	0.890	1.716	11	2.005	1.16
CZ2	-0.385	1.574	7	-0.730	1.26
CZ3	0.276	3.915	9	0.561	1.98
CA1	-0.007	2.379	9	-0.004	1.20

Table 5.2. Comparison of residual effors for chemical composition. - Intercept stress, B-type tests.

This is not wholly unexpected in view of the fact that aluminium cross-slips almost immediately during deformation at room temperature and above. The distance over which the stage 111 curve needs to extrapolated in order to intercept the stress axis is, therefore, much less and the inaccuracy introduced by this manoevre is virtually eliminated.

The plot of residual errors against the G. ln$\dot{\gamma}$ variable is, similarly, one displaying an apparently random distribution, as were those for all of the other variables in the equation.

(c) <u>The C-type tests</u>

A comparison of the equations for maximum stress and for intercept stress in the B-type and C-type tests respectively shows that there is little discrepancy between the two ranges of temperature. For the maximum stress equation all of the regression coefficients for the C-type tests differed by less than one standard deviation from the results for the B-type tests.

In the case of the intercept stress equation the regression coefficients for the two variables which were added last differed by greater than one but less than two standard deviations between the B-type and C-type tests. For the other four variables the discrepancy between the two groups of tests was again less than one standard deviation in each case.

Since the fifty-two C-type tests were drawn from the eighty-five B-type tests, and the residual plot against temperature showed no consistent trends of any kind, this general agreement between the two types of test is to be expected. It does tend to confirm the evidence of the residual plot, however.

(d) <u>The Steady-state stress</u>

The C-type tests were selected from the B-type tests on the basis of having a clearly defined steady state region of deformation. It is only within this group of tests that the steady-state stress may be studied, therefore.

The correlation coefficient between this parameter and the maximum stress is, again, very high at 0.961. The same variables as for the other two stress parameters have been added into the equation, but it was found that when variable 36 was added the contribution due to variable 29 became insignificantly small. The normal practice would have been to eliminate 29 from the equation at this stage but it was decided to retain it in order to maintain the similarity with the equations for the other stress parameters.

One other difference that was noted between this and the other stress parameters was that the variable $G.D_d^{-1/5}$ (where D_d is the diffusion coefficient) was found to have a significant effect.

The residual mean square for the steady state stress equation was shown to be almost exactly equal to the independently estimated error variance, and the other assessments of the residual values confirmed that they were in accordance with the previously mentioned assumptions of distribution and independance.

5.2.2. Accuracy and Reliability

(a) Accuracy The accuracy of prediction of a value of the dependent variable is subject to the error variance as estimated by S^2 - the residual mean square. While S^2 is based on deviations from the mean, which is independent of the estimated regression coefficients, the regression coefficients are not independent of one another (except in the case of an orthogonal matrix of correlation coefficients). The value of S^2 must, therefore, be increased to allow for this additional uncertainty.

Davies [120] gives the equation for the adjusted variance as:

$$V(Y) = V(\bar{y}) + (X_1 - \bar{x}_1)^2 \; V(b_1) + \ldots\ldots + (X_p - \bar{x}_p)^2 \, V(b_p) + 2\,(X_1 - \bar{x}_1)\,(X_2 - \bar{x}_2) \operatorname{cov}\,(b_1\, b_2) + \ldots\ldots \qquad - (5.1.)$$

where $V(Y)$ is the variance of the value Y, predicted from X_1, X_2, ..., X_p, and cov $(b_1 b_2)$ is the covariance of b_1 and b_2.

Since the covariance is calculated from:

$$\operatorname{cov}\,(b_i b_j) = S^2 \, a^{ij} \qquad - (5.2)$$

where a^{ij} is the appropriate element from the reciprocal matrix, and since

$$a^{ij} = r^{ij} \,/\, \left(\sum (x_i - \bar{x}_i)^2 \sum (x_j - \bar{x}_j)^2\right)^{\frac{1}{2}} \qquad - (5.3)$$

where r^{ij} is the equivalent element in the reciprocal correlation matrix, we may write:

$$n\ x.\ (X_i - \bar{x}_i)\,(X_j - \bar{x}_j) \operatorname{cov}\,(b_i b_j) = r^{ij} \; \frac{\sum (x_i - \bar{x}_i)\,(x_j - \bar{x}_j)}{\left(\sum (x_i - \bar{x}_i)^2 \sum (x_j - \bar{x}_j)^2\right)^{\frac{1}{2}}} \qquad - (5.4)$$

The term associated with r^{ij} is equivalent to r_{ij} and so substituting in equation 5.1. we have:

$$V(Y) = V(\bar{y}) + {}^{1}/_{n}\,(r^{11}\,r_{11} + \ldots\ldots + r^{pp}r_{pp} + 2\,r^{12}\,r_{12} + \ldots\ldots) \qquad - (5.5.)$$

In fact, the additional variance from this source is relatively small in the present case. For the intercept stress the value is 0.119 giving:

$$V\,(\tau_o) \;=\; 2.254.$$

while for the maximum and steady state stresses the values are 0.288 and 0.339, respectively, giving:

$$V\,(\tau_m) \;=\; 1.588$$
$$V\,(\tau_s) \;=\; 1.112$$

In repeated tests, therefore, experimental values of the intercept stress would be expected to lie within $\pm$ 2.9 t.s.i. of the predicted value in 95 per cent. of cases. For the maximum and steady-state stresses the corresponding values are $\pm$ 2.5 t.s.i. and $\pm$ 2.1 t.s.i. respectively.

(b) <u>Reliability</u>. The criteria of 'bestness' in selecting a sub-set of predictor variables for regression equations are sometimes imprecise and may not be wholly compatible. Some of the more important ones may be worth mentioning here.

(i) If the regression equation is to be used for the purposes of prediction for further values, i.e. additional to those on which the analysis is based, it is desirable that the value of R^2 should be high. Once the residual mean square has been reduced until it equals the error variance, however, there is little or no advantage in further increasing R^2, since the additional variance due to covariance between the predictor variables is unlikely to be significantly reduced by the addition of non-significant terms into the regression.

(ii) It is desirable that all of the terms in the regression should be statistically significant when assessed on the basis of their primary elements.

(iii) It is usually desirable that the number of predictor variables should be as small as is compatible with (i) above.

Even within the constraints imposed by these criteria there may be more than one solution in the search for the 'best' equation and in the absence of 'a priori' evidence no means may be available for distinguishing between the alternatives.

In the present study the available evidence indicates a very close similarity in the equations relating to the three stress parameters. Differences between the results for B-type and C-type tests, where these are appropriate, appear to be no greater than would be expected from random sampling variations. Furthermore the examination of the residual values appears to confirm that there is no undue bias in their distribution and that the error variance is constant over the range of experimental conditions.

One further source of difficulty may arise from singularity, or near-singularity, of the matrix which is inverted to produce the final equation. In extreme cases this is revealed by giving rise to values of the standard errors of the regression coefficients which are indeterminate, (usually because they require a division by zero). Even circumstances less extreme than this can be problematic, however, but it is considered that the information derived from the table of primary and secondary elements give ample warning of this danger.

The technique by which selection was carried out ensures that when highly correlated variables are included in the regression each of them makes a contribution, significantly greater than that which is likely to arise by chance, and which would not be made in the absence of that variable. Consider, for example, the table of elements in table 4.2 (between pages 87 and 88). Two of the variables included in this table are 27 ($b.^T/_{T_m}$) and 13 (T/T_m) and the simple correlation coefficient between these two is equal to 0.987. This indicates that the contribution to the regression sum of squares which is due to the presence of variables 27 and 13 is the sum of these quantities, viz. 342.112 and of this total 157.40, approximately half, cannot be unambiguously ascribed to either of the individual variables. If variable 27 is eliminated from the regression, therefore, the apparent contribution due to variable 13 will increase by 157.40, but the total regression sum of squares will be reduced by 77.782, a quantity which is almost sixty times as great as the estimated error variance. On this basis it is clearly justifiable to include both variable 27 and variable 13, despite their high correlation. Since this practice has been adopted throughout the analysis it is considered that the variables included in the various equations are justified and the equations presented are valid interpretations of the experimental data.

5.2.3. The Work-Hardening Coefficient - K

The same technique was applied to this parameter as to the stress parameters and the results obtained are presented in tables 4.7 and 4.8.

For the C-type tests variables Nos. 16 and 42 were added to the regression, in that order, and accounted for 0.076 of the variance in K. The further addition of variable 39 increased R^2 to 0.686, with a reduction of the residual mean square to a value less than that of the independently estimated error variance.

If the variable containing the diffusion coefficient - 38, is forced into the equation, that is to say that it is included despite not being justified on the basis of making a significant contribution to the regression sum of squares, variable 39 becomes insignificant. Variable 41 is then found to be significant at the 0.05 level of probability. This solution had a value of R^2 equal to 0.697, with a residual mean square of 1.361 (compared with the independently estimated error variance of 2.08). The retention of variable 38 may be justified because of the strong negative secondary elements between this and all of the other variables in the equation. Newton and Spurrell[91] suggest in their 'Rule 4(a)' '....both variables must be kept in the regression because they are complementary in their effect on the dependent variable' They also make it clear that the inter-relationship may be algebraic rather than operational.

The distribution of residual values of work-hardening coefficient is shown in figure 5.3, compared with Normal curve. Once again the Chi-squared value (equal to 7.2) lies between the tabulated values for probabilities of 0.75 and 0.50 and can be regarded as confirmation that the errors are normally distributed. One outlying value of 4.4 has been discarded for the purpose of this comparison, and a repeat observation under the same experimental conditions had a value of 0.73.

Figure 5.4 shows a plot of the residual values against T/T_m and in table 5.3 the influence of composition on the residual values is, similarly, shown to insignificant, when the outlying value referred to above is excluded. (If this value is not excluded the variance within the CZ3 specimens is significantly greater than the overall error mean square. The rejection of this single value from the fifty-two experiments is considered to be justified, however, particularly since the repeat observation lay very much closer to the mean).

The application of the same technique of analysis to the B-type test data was a great deal less successful, however, and the best value of R^2 was only 0.382. If the equation produced for the C-type tests is applied to the B-type data the residual errors of the predictions not included in the C-type tests have a mean which still equals zero, but the mean square rises to approximately 12. This nearly ten-fold increase can be partly explained in terms of the shortcomings of the experimental technique mentioned in section 5.1, and is clearly more likely to occur at lower temperatures

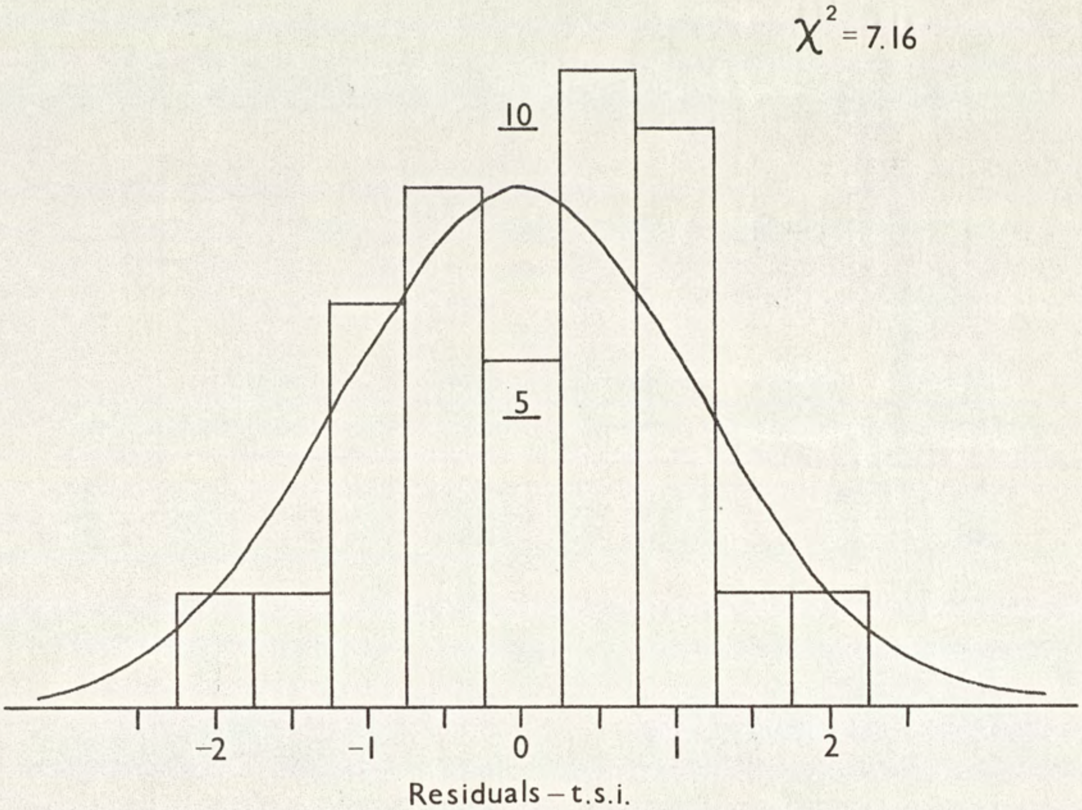


Figure 5.3. Comparison of distribution of residual errors for the work-hardening coefficient with true Normal curve.

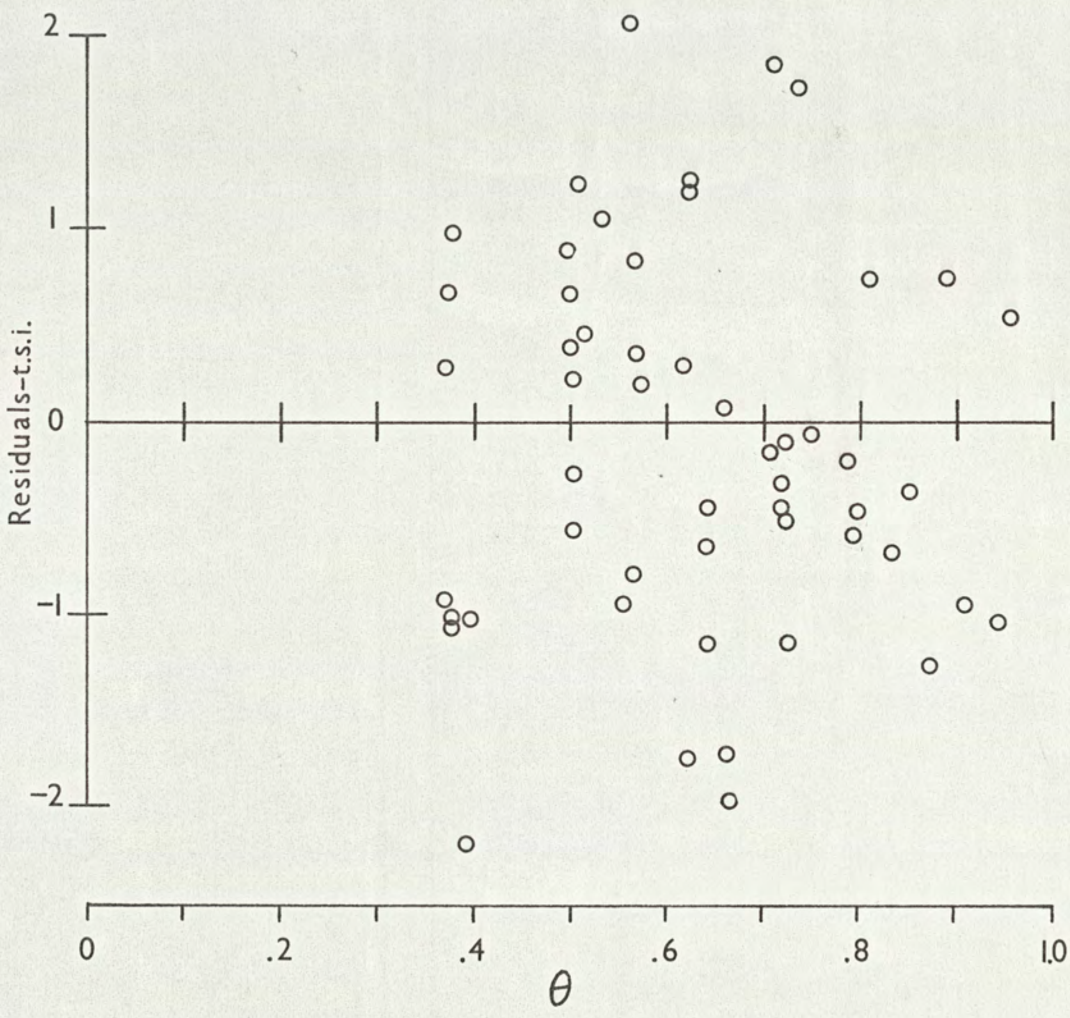


Figure 5.4. Distribution of residual errors for work-hardening coefficient versus $^{T}/T_{m}$ - C-type tests.

SOURCE	MEAN	VARIANCE	d.f.	t-Value	F-Value
Total	0.001	1.360	51	-	-
N	0.009	3.744	3	-0.01	2.82
A	-0.219	1.133	6	0.47	1.17
C	-0.093	0.587	20	0.34	2.26
CZ1	0.409	1.368	2	-0.58	1.03
CZ2	0.165	0.421	4	-0.31	3.15
CZ3	0.665	3.894	5	-1.20	2.93
CZ3 *	-0.296	0.974	4	0.55	1.36
CA1	-0.235	1.718	5	0.47	1.29

* After rejection of one outlying value.

Table 5.3. Comparison of residual errors for chemical composition. - Work-hardening Coefficient, C-type tests.

when the stresses developed are generally higher. The cases where K is overestimated can clearly not be explained in this way. One possible explanation is that the low temperatures and resultant high stresses encourage deformation twinning, to an extent which does not disqualify the result by the standards of acceptance adopted, but because twin propagation is easier than twin nucleation it may reduce the overall work-hardening rate. This is further supported by the fact that in the cases of the high stacking fault energy metals the tendency is to underestimate the rate of work-hardening, while the lower stacking fault energy metals, where twinning is easier, have usually lower work-hardening rates than the predicted values.

Using the same method for calculating variance as in the previous section the additional variance is 0.30 so that the total variance V(K) is equal to 1.66. The 95 per cent. confidence limits for a prediction of K are $\pm$ 2.5 t.s.i. for the C-type tests. No estimates were made for the B-type tests.

5.3. Work Hardening and Restoration.

The variables in the regression equations are selected on the basis of statistical criteria rather than mechanistic arguments. It would be unwise, therefore, to attempt a full explanation of the variables in the final equations on the basis of likely mechanisms. None-the-less, since the variables considered for the regression equations were selected for consideration from suggested mechanisms, some insight is gained by examining the predictor variables in the equation.

5.3.1. The Stress Parameters

The parameter of intercept stress has been proposed here as an approximation of the critical stress for the onset of stage III deformation. It has the advantage of being calculated from the stage of deformation which predominates in most f.c.c. metals, especially at elevated temperatures. Work by Dillamore, Smallman and Roberts[98] confirmed the dependence of deformation textures in f.c.c. metals on the extent of dislocation cross-slip which is associated with stage III deformation. This information was used to confirm the relationship between intercept stress and cross-slip in a final year undergraduate project[119] supervised by the author of this thesis. In this work it was demonstrated that the intensity of certain features of the pole figure of crystallographic preferred orientation were strongly correlated with the intercept stress determined as in the investigations reported here. The work was carried out on a range of austenitic steels at elevated temperatures.

The high correlations between intercept stress and the other two stress parameters, together with the close similarities between the regression equations which were derived for all three parameters suggest that the factors controlling the onset of stage III deformation also control the restoration processes occurring later in the stress-strain curve.

(a) <u>Stacking fault energy.</u>

Variable 16. - G^2/γ_{sf} is a measure of the stress required to bring together the two 'halves' of a dissociated dislocation, which is a necessary pre-requisite to cross-slip. It should be noted that dislocation glide not involving cross-slip can occur without dissociated dislocations re-combining.

Evidently, therefore, whatever additional mechanisms may contribute to the restoration process, that of dislocation cross-slip is a major controlling mechanism, contributing a primary element which is significant at the 0.001 level of probability.

(b) <u>Shear modulus</u>

The importance of shear modulus is also evident from the variables in the regression equations since this property occurs in three out of the six terms for the intercept and maximum stresses and four out of the seven terms in steady state stress equation.

The values of G used in this study have already been compensated for temperature, and it seems reasonable to assume that it simply provides a measure of the stress required to cause atomic movement in the lattice and to enable the unit movement of dislocations.

(c) <u>Burger's vector</u>

It is, perhaps, a little surprising that none of the terms involving the product term G.b proved to be significant since the 'unit' distance over which atoms must move in order to establish plastic deformation is the Burger's vector of the metal. The Burger's vector does make a significant contribution, however, but in a manner which is rather difficult to explain. In each of the equations for the stress parameters the regression coefficients for b and for b. $^{T}/T_m$ are effectively equal. Certainly the differences are invariably much less than one standard deviation. If the regression coefficient is B the effect of Burger's vector can be written as:

$$\tau = B(b - b.\ ^{T}/_{T_m}) \qquad -(5.1.)$$

Since T/T_m is invariably less than unity and B is invariably negative, it follows that the stress parameter increases with decreasing values of Burger's vector. The contribution of the terms b and b. T/T_m to the regression sum of squares, calculated by adding the two primary elements and the secondary element associated with both, is approximately one third of the total. This is clearly a most significant contribution.

One possible explanation of the anomaly whereby the stress decreases as the measure of unit strain increases, is that the Burger's vector provides a guide to some other composition-dependent property. It is not apparent what this property might be, however, and this suggestion does not provide any real insight into the mechanisms occurring.

(d) Temperature

The influence of temperature is strongly negative, as was expected from all previous deformation studies. It is not clear why both T/T_m and G. $T/_{T_m}$ exert a significant and separate influence, although it may be that they are complementary in correcting for some deviation from linearity in the temperature dependence of stress. Alternatively the two terms might reflect two different mechanisms.

If the removal of the effects of work hardening depends upon the migration of vacancies as has been suggested in the literature survey, it is to be expected that the term $G. T/T_m$ would be of importance[77]. The likelihood of this mechanism being involved is supported by the fact that when $G.Dd^{-1/5}$ becomes important to the regression, as in the case of the steady-state stress, the contribution from $G.T/T_m$ declines to almost nothing.

The T/T_m term remains significant whether or not the diffusion term is included and probably reflects the easier movement of dislocations, independantly of elastic stiffness, in the reduced stability of the lattice at higher temperature.

(e) <u>Strain rate</u>

The effect of strain rate, as was suggested earlier, proved to be small, but significant. The simple semi-logarithmic relationship was not sufficient to describe the stress dependence, but the product of (log strain rate) and shear modulus was highly significant in all cases.

In the case of steady-state stress the relationship with strain rate was negative, no doubt reflecting the increased temperature resulting from adiabatic heating at the higher values of strain.

It would seem that the similarity between steady-state deformation and deformation under creep conditions postulated by Sellars and Tegart[75] was justified. Moreover, the influence of the diffusion coefficient suggested by Sherby[63] mainly on the basis of creep studies, was also shown to apply to steady-state deformation.

Sherby's hypothesis that cross-slip is of no importance in studies of high temperature strength was clearly contradicted by the results of the present study, however.

(f) Grain size

It is somewhat surprising that none of the functions of grain size which were considered proved to be significant. In this study the annealing treatments given to the copper alloys were all similar, and it seems likely that the correlations between $D^{-\frac{1}{2}}$ and the various functions of stacking fault energy might have arisen because of the dependence of the stable sub-grain size on the latter property[66]. Since all of the materials were given similar times at similar temperatures the final grain size would depend upon the size of the nucleus, i.e. the stable sub-grain size. It might be, therefore, that in the present study the effect of grain size has been masked by its association with stacking fault energy. It must be admitted, however, that this was not revealed by the size of the secondary elements which arose.

5.3.2. The Work-Hardening Coefficient

At temperatures greater than about 0.4 Tm, the range covered by the C-type tests in this study, the work-hardening coefficient appears to confirm the theoretical calculations for the stress to move a dislocation. The non-significant contribution of $G.D_d^{-1/5}$ and the negative value of the regression coefficient associated with this variable confirms that the process of diffusion has very little influence on the rate of work-hardening.

The term $GbD^{-\frac{1}{2}}$ is associated with the concentration of stresses by the presence of grain boundaries. The influence of stacking fault energy is also in agreement with that suggested by theoretical models.

The absence of any temperature dependence is an interesting feature of the relationship, and it suggests that the influence which is exerted by temperature is assoc iated with the lowering of the shear modulus (which has been compensated for temperature in this study).

At temperatures below 0.4 Tm with metals of low stacking fault energy it appears that deformation twinning can cause a considerable drop in the rate of work-hardening, even in the absence of such overt signs of twinning as sudden load drops or cleavage fracture. Shortcomings in the experimental technique in this range of temperature prevent any quantitative estimate of the extent of the effect of twinning.

5.4. Summary

It has been proposed in this study that the stress-strain relationship in single-phase f.c.c. metals and alloys may be described by an equation of the form:

$$\tau = K\sqrt{\gamma} + \tau_o \qquad - \quad (5.7)$$

up to some limiting stress $\longrightarrow \tau_m$

At temperatures greater than about 0.4 T_m a further stage of deformation may occur during which the flow stress remains constant at a value τ_s while the values of strain encountered are very high.

The value of τ_o is suggested as an approximation to the critical stress for the onset of stage III deformation, with the reservation that inaccuracies can occur if the value of stacking fault energy is low.

Regression analysis techniques have been used to provide a means of predicting the values of the parameters of equation 5.7 on the basis of the chemical composition and grain size of the metal, and the experimental conditions of temperature and strain rate. The standard deviations of the predicted values of the stress parameters have been estimated at 1.50, 1.26 1.06 t.s.i. for τ_o, τ_m and τ_s respectively, which values should normally be within the level of accuracy of experimental measurement.

The value of the work-hardening coefficient -K may also be predicted with a standard deviation of the observed values about the predicted value of 1.29 t.s.i. for temperatures greater than 0.4 T_m. At lower temperatures the influence of deformation twinning may significantly lower the work-hardening rate of low stacking fault energy metals, but limitations in the experimental technique prevented any assessment of this effect.

From an examination of the influence of stacking fault energy, temperature and diffusion coefficient it appears that restoration depends upon similar mechanisms to those causing stage III deformation, i.e. principally thermally activated dislocation cross-slip and climb. It also appears that this mechanism plays an important role in steady-state deformation, which is analogous to high-stress creep deformation.

6. CONCLUSIONS

1. Parameters have been identified which enable the main features of the shear stress -shear strain relationship to be defined.

2. The dependence of each of the parameters on factors related to composition, structure and conditions of deformation have been investigated by multiple regression analysis.

3. It is suggested that accurate predictions may be made of the likely behaviour of single phase f.c.c. metals from equations derived by the analysis.

4. Evidence suggests that deformation twinning may reduce work hardening rates in lower stacking fault energy materials at temperatures below 0.4 T_m, even in the absence of such indications as load drops or cleavage fracture.

5. It appears that the predominant means of reducing the effects of work-hardening, within the range of temperatures and strain rates studied, is recovery by thermally assisted dislocation cross-slip and climb.

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APPENDIX A

Modulus of Shear

Any discussion of the relative behaviour of different metals when subjected to various stresses or strains must take into account the elastic moduli of the metals concerned. The elastic moduli give an indication of the basic resistance offered by a material to any attempt to displace atoms of that material relative to one another. In fact the modulus of elasticity is so closely related to the interatomic binding forces that Born[101] was able to predict moduli for simple ionic materials from a consideration of their attractive charges. Since bulk deformation is the result of local shearing of atoms relative to one another[102], the modulus of shear is of prime interest in deformation studies, although this value is simply related to the tensile and bulk moduli via Poisson's ratio, ~~thus~~:

$$G = \frac{E}{2(1 + \nu)} \qquad \text{(A.1.)}$$

$$K = \frac{E}{3(1 - 2\nu)} \qquad \text{(A.2.)}$$

For most metals is approximately 0.33[103] so that

$$K \sim E \qquad \text{(A.3.)}$$

$$G \sim \tfrac{3}{8} E \qquad \text{(A.4.)}$$

The linear interdependance of stress and strain according to Hooke's Law for elastically isotropic solids can be resolved into separate components to describe the behaviour of essentially anisotropic solids, such as the single crystals of metals[104].

Using Cartesian co-ordinate rotation the six possible principal strain components can be expressed in terms of the six possible principal stress components (three acting normally and three acting is shear):

$$
\begin{aligned}
e_{xx} &= s_{11}\sigma_x + s_{12}\sigma_y + s_{13}\ \sigma_z + s_{14}\ \tau_{zx} + s_{15}\ \tau_{xy} + s_{16}\tau_{yz} \\
e_{yy} &= s_{21}\sigma_x + s_{22}\sigma_y + s_{23}\ \sigma_z + s_{24}\ \tau_{zx} + s_{25}\ \tau_{xy} + s_{26}\tau_{yz} \\
e_{zz} &= s_{31}\sigma_x + s_{32}\sigma_y + s_{33}\ \sigma_z + s_{34}\ \tau_{zx} + s_{35}\ \tau_{xy} + s_{36}\tau_{yz} \\
e_{xy} &= s_{41}\sigma_x + s_{42}\sigma_y + s_{43}\ \sigma_x + s_{44}\ \tau_{zx} + s_{45}\ \tau_{xy} + s_{46}\tau_{yz} \qquad (A.5.) \\
e_{yz} &= s_{51}\sigma_x + s_{52}\sigma_y + s_{53}\ \sigma_x + s_{54}\ \tau_{zx} + s_{55}\ \tau_{xy} + s_{56}\tau_{yz} \\
e_{zx} &= s_{61}\sigma_x + s_{62}\sigma_y + s_{63}\ \sigma_x + s_{64}\ \tau_{zx} + s_{65}\ \tau_{xy} + s_{66}\tau_{yz}
\end{aligned}
$$

where the constants s_{11}, s_{12}, ... s_{ij} are known as the elastic compliances.

It follows, then that the stress components can be expressed as linear functions of the strain components:

$$
\begin{aligned}
\sigma_x &= c_{11}e_{xx} + c_{12}\ e_{yy} + c_{13}e_{zz} + c_{14}\gamma_{yz} + c_{15}\ \gamma_{zx} + c_{16}\gamma_{xy} \\
\sigma_y &= c_{21}e_{xx} + c_{22}\ e_{yy} + c_{23}e_{zz} + c_{24}\gamma_{yz} + c_{25}\ \gamma_{zx} + c_{26}\gamma_{xy} \\
\sigma_z &= c_{31}e_{xx} + c_{32}\ e_{yy} + c_{33}e_{zz} + c_{34}\gamma_{yz} + c_{35}\ \gamma_{zx} + c_{36}\gamma_{xy} \qquad (A.6.) \\
\tau_{yz} &= c_{41}e_{xx} + c_{42}\ e_{yy} + c_{43}e_{zz} + c_{44}\gamma_{yz} + c_{45}\ \gamma_{zx} + c_{46}\gamma_{xy} \\
\tau_{zx} &= c_{51}e_{xx} + c_{52}\ e_{yy} + c_{53}e_{zz} + c_{54}\gamma_{yz} + c_{55}\ \gamma_{zx} + c_{56}\gamma_{xy} \\
\tau_{xy} &= c_{61}e_{xx} + c_{62}\ e_{yy} + c_{63}e_{zz} + c_{64}\gamma_{yz} + c_{65}\ \gamma_{zx} + c_{66}\gamma_{xy}
\end{aligned}
$$

where the constants c_{11}, c_{12}, ... c_{ij} are known as the elastic stiffness constants.

It will be apparent that for any metal any anisotropy of the single crystal will be transferred to the polycrystalline aggregate to an extent which is mainly dependent upon the degree of crystallographic preferred orientation of the aggregate. It is partly for this reason that a number of workers have conducted measurements only on single crystals[105]. The prediction of values of moduli of polycrystalline metals from the single crystal data is greatly simplified by the symmetry within metal crystals. The thirty-six values of elastic compliances or of elastic stiffness constants, therefore, can be reduced first to twenty-one by the symmetry of the matrix on the interchange of the double indices, i.e. $s_{ij} = s_{ji}$ and $c_{ij} = c_{ji}$. A further reduction in the number of independant constants may be possible due to the physical symmetry of particular crystal classes, so that there are nine independant constants for orthorhombic crystals, five for hexagonal and three for cubic systems.

The predictions from single crystal data, however, are complicated by the need to compromise between the assumptions of uniform local strain and uniform local stress[106]. The former assumption by Voigt[107] and the latter by Reuss[108] giving rise to the following expressions, respectively:

Voigt's Averages:

$$K_v = \tfrac{1}{3} (F + 2G)$$

$$G_v = {}^1/_5 (F - G + 3H) \qquad (A.7.)$$

$$Ev = \frac{(F - G + 3H)(F + 2G)}{(2F + 3G + H)}$$

where

$$F = \tfrac{1}{3} (C_{11} + C_{22} + C_{33})$$

$$G = \tfrac{1}{3} (C_{12} + C_{33} + C_{13})$$

$$H = \tfrac{1}{3} (C_{44} + C_{55} + C_{66})$$

Reuss's Averages:

$$(K_R)^{-1} = 3 (F' + 2G')$$

$$(G_R)^{-1} = {}^1/_5 (4F' - 4G' + 3H') \qquad (A.8.)$$

$$(E_R) = {}^1/_5 (3F' + 2G' + H')$$

where $F' = \frac{1}{3}(S_{11} + S_{22} + S_{32})$

$G' = \frac{1}{3}(S_{12} + S_{23} + S_{13})$

$H' = \frac{1}{3}(S_{44} + S_{55} + S_{66})$

In table A.1. from Tegart[106] the Voigt and Reuss averages for four cubic metals are compared with experimental values from Fine[109]. It will be seen that the experimental values can usually be most closely estimated from the mean of the Voigt and Reuss values, and that in the case of aluminium, which is nearly elastically isotropic, the two theoretical values agree very closely. For most materials, however, the discrepancy between the Voigt and Reuss averages, and the relationship of these to experimental values suggests that values of modulus determined from polycrystalline specimens are no less reliable than those from single crystal data.

Metal	G_{Reuss}	G_{Voigt}	G_{Expt}
Al	2.65	2.65	2.72
Cu	4.08	5.51	4.64
Au	2.45	3.16	2.82

Table A.1. Comparison of Voigt and Reuss averages with experimental results.

Effects of Temperature and Composition

Since elastic moduli are closely related to atomic binding forces it follows that any factor having an influence on the binding forces will also affect the elastic properties. In practice the most important factors are temperature and composition, although any change in crystal structure, e.g. allotropic changes, phase changes, order - disorder reactions, leads to a discontinuity in the rate of change due to either of these[106].

In a review of the literature referring to this subject Mack[110] found that the most successful empirical formulae relating moduli to temperature were those of Portevin[111]:

$$E = KT_m a / V^b \qquad (A.9.)$$

where K, a and b are constants and V is the specific volume and Andrews[112]:

$$E = V^{-\alpha} \quad A \exp \quad (-\beta \; T/Tm) \qquad (A.10.)$$

Andrews suggested that in his formula the constants and had two sets of values to take account of the increasing influence of grain boundary slip at values of T/Tm greater than 0.5. In practice it is found that, for f.c.c. metals at least, this is a refinement of doubtful value since the extent to which the formula describes the experimental results is generally within the limits of experimental accuracy even when using only one set of constants.

Koster and Rauscher[113] found that within the limits of solid solubility for binary systems Young's modulus at room temperature varied linearly with the atomic solute content. Following on this work Smith[114] showed that a close relationship exists between the rate of decrease of Young's modulus at room temperature and the solidus temperature for a number of binary alloy systems based on Cu or Ag.

An examination of the curves of modulus versus temperature, produced by Koster[115] and Koster and Rauscher[113] shows that the moduli for f.c.c. metals at $T/T_m = 1$ usually occur in the order of their melting temperatures (or solidus temperatures for alloys). This may be appreciated, at least qualitatively, by assuming that when T/T_m is slightly greater than 1 the compressibility of the liquid metal is almost entirely dependent upon the temperature in °K, i.e. it is independent of the melting temperature. For any given metal, therefore, at $T/T_m = 1$ the thermal energy is still of considerable significance in comparison with the dminished influence of the metallic bond. It follows, then, that the modulus at $T/T_m = 1$ is closely related to the absolute value of T_m.

In view of the demonstration by Koster[115] and Smith[114] that the modulus at room temperature varies approximately as the melting temperature it seems that the slope of the modulus/ temperature curve for any given metal must also vary with the melting temperature.

Following on from Andrews' formula therefore, Young's modulus for any f.c.c. metal at any temperature up to the melting temperature might be expected to follow a general formula of the type:

$$E = f\,(T_m).\ \exp\ (T/T_m) + f^1\,(T_m) \qquad (A.11.)$$

where $f\,(T_m)$ and $f^1\,(T_m)$ are functions of the melting temperature.

On carrying out a line-fitting exercise, by the method of least squares, on the results of Koster and Rauscher[113], Nishiyama[116] and Smith[114] it appears that the elastic moduli of a number of f.c.c. metals and alloys, over a wide range of temperatures, may be described by the formula:

$$\frac{E}{1000} = (A.\exp\,(T_m/10^3) + c)\,\exp\,(T/T_m) + (B.\exp\,(T_m/10^3)) + d \quad (A.12.)$$

From the constancy of the relationships expressed in equations (A.1.) and (A.2.) it is evident that the same general formula applies for the bulk and shear moduli as for the modulus of tension.

In Figure A.1. measured values of shear modulus are compared with those predicted by (A.12.) with the constants evaluated, thus:

$$A = -0.625$$
$$c = 1.0625$$
$$B = 1.912$$
$$d = -2.481$$

Clearly the formula provides an accurate method of predicting the moduli of pure metals and alloys over a range of temperatures.

The constants listed above do not enable accurate predictions to be made for the noble metals, and this is in accord with the results of both Smith and Koster who found that the moduli of gold and silver were lower than expected. The bulk moduli for these metals are high in relation to their moduli in shear and in tension. An alternative way of expressing this is to say that Poisson's ratio for the noble metals is usually higher than for the more common metals. Hume-Rothery[117] suggested that this was due to the ease with which the noble metals could be polarised, reducing

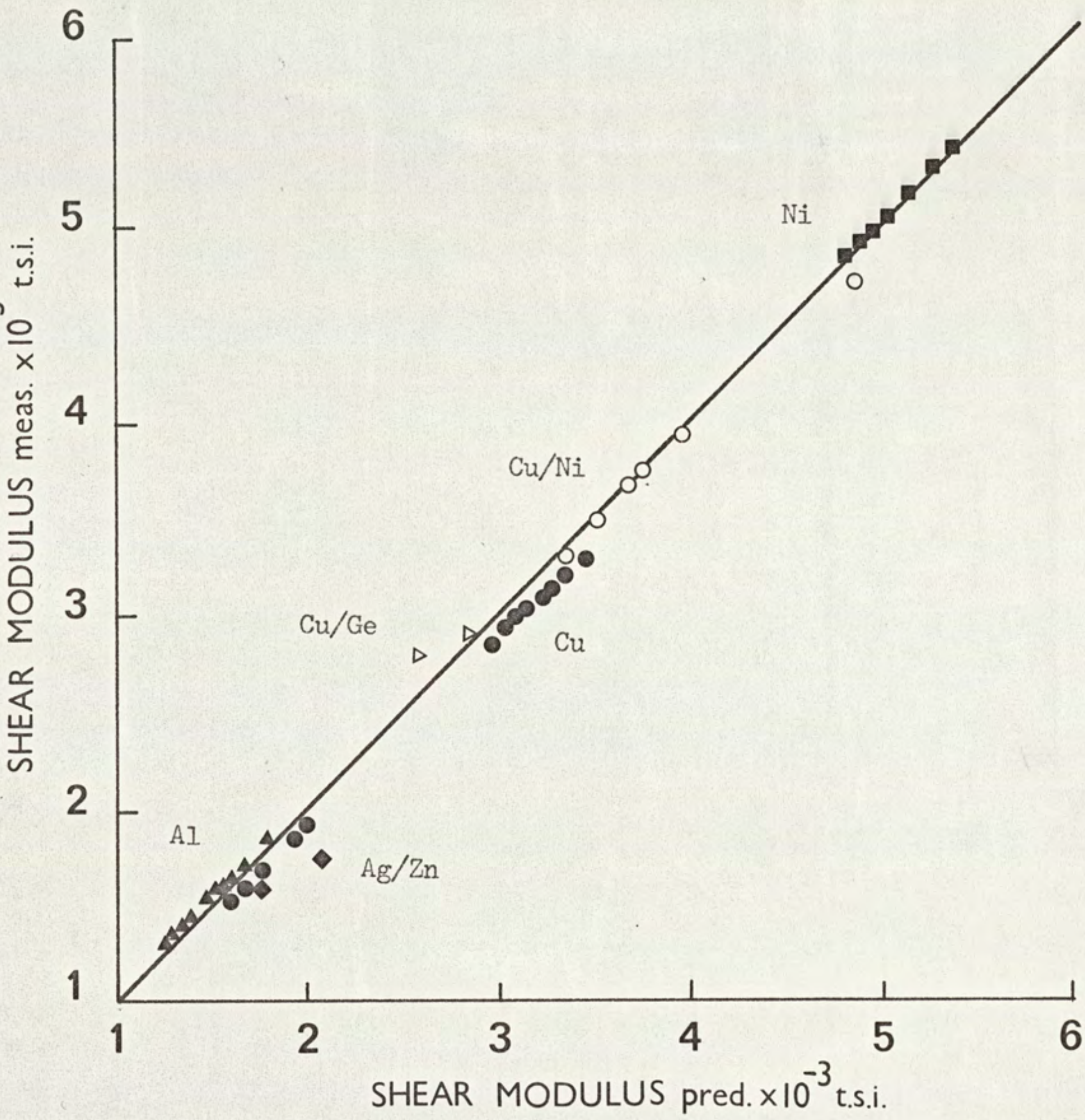

Figure A.1. Comparison of measured shear modulus and shear modulus predicted from equation A.12.

the uniaxial strength. In any event, the accuracy of the formula as a means of predicting moduli for metals with Poisson's ratio outside the range 0.31 - 0.35, may be improved by multiplying the result by a correction factor equal to:

$$\frac{2(1+\nu)}{3(1-2\nu)} \quad 0.38 \qquad (A.13)$$

Typically, these correction factors are:

Au (ν = 0.42)	2.28
Pt (ν = 0.39)	1.68
Ag (ν = 0.37)	1.30

Values of shear modulus for gold and for two platimum/copper alloys have been calculated for a range of temperatures and are shown plotted against the measured values in Figure A.2. Clearly, although the agreement is generally good, there is a systematic deviation with temperature, suggesting that Poisson's ratio is not independent of temperature for these materials. There is little published information on the temperature dependance of Poisson's ratio, but Harris and Watkins[87] showed that for steels the only effect of temperature was to increase the solubility of alloying elements present with a consequent increase in .

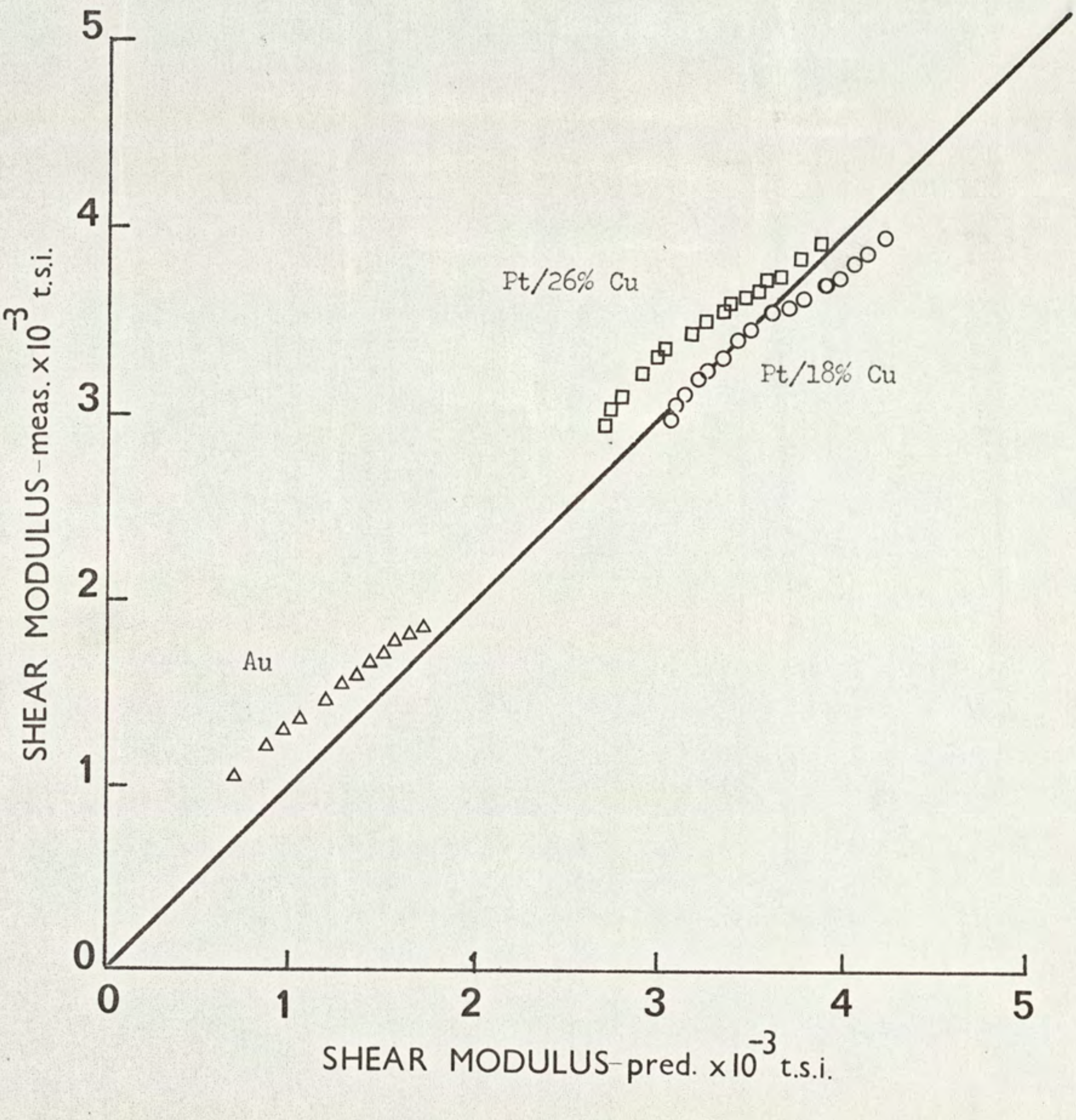

Figure A.2. Comparison between measured and predicted values of shear modulus for noble metals.

APPENDIX B

```
DATANAL
BEGIN INTEGER TEMP, MIN, MAX, Q,A,NU,N'
REAL REV,D,L,K,KAX,RATE,RADF,TORRF2,XSUM,YSUM,PROD,
     XSQ,YSQ,XMEAN,YMEAN,RT,M,CS,C,COR,DUMMY1,DIST,H'
ARRAY STR(1:50),G,T(1:200)'
SWITCH SW:=ONE'
SAMELINE'
ONE:PUNCH(1)'
READ N,TEMP,REV,D,L,K,CS,KAX,READER(2),MIN,MAX'
Q:=1'
INSTRING(STR,Q)'
RATE:=3.14159*REV*D/L'
Q:=1'
PRINT ££R2015??'
OUTSTRING(STR,Q)'
PRINT PREFIX(££S5??),TEMP,RATE'
RADF:=2*3.14159*(REV/60)/CS'
TORRF2:=1/(2*3.14159*((D/2)**3)*2240)'
FOR A:=1STEP 1 UNTIL N DO
BEGIN READ DIST,H,DUMMY1'
     IF A+1 GREQ MIN AND A+1 LESSEQ (MAX+2) THEN
     BEGIN G(A):=DIST*RADF'
           T(A):=K*H
     END
END'
XSUM:=YSUM:=PROD:=XSQ:=YSQ:=0.0'
FOR Q:=MIN STEP 1 UNTIL MAX DO
BEGIN RT:=TORRF2*(3*T(Q)+(G(Q)*((T(Q+1)-T(Q-1))/
         (G(Q+1)-G(Q-1)))))'
     XSUM:=XSUM+SQRT(0.125*G(Q))'
     YSUM:=YSUM+RT'
     PROD:=PROD+SQRT(0.125*G(Q))*RT'
     XSQ:=XSQ+0.125*G(Q)'
     YSQ:=YSQ+RT*RT
END'
NU:=MAX-(MIN-1)'
XMEAN:=XSUM/NU'
YMEAN:=YSUM/NU'
M:=(PROD-((XSUM"YSUM)/NU))/(XSQ-((XSYM**2)/NU))'
C:=((XSUM*PROD)-(YSUM*XSQ))/((XSUM**2)-(NU*XSQ))'
COR:=((PROD-(XMEAN*YMEAN))/NU)/((SQRT(((XSQ-(XMEAN**2))/NU)))*
     (SQRT(((YSQ-(YMEAN**2))/NU))))'
PRINT ££L2?SLOPE££S9?=££S3??,ALIGNED(3,4),M,££L1?TORR(GAMMA 0)=,
     ££S3??,C,££L1?GAMMA(TORR 0) = ££S3??,(0-C/M),££L1??,
     £CORRELATION££S3?=££S3??,COR,££S5?DEG.FREE. =?,DIGITS(3).

NU-1'
READ DIST'
PUNCH(3)'
PRINT ££L1?FINISHED ?'
Q:=1'
OUTSTRING(STR,Q)'
PRINT FREEPOINT(6), PREFIX(££S5??),TEMP,RATE'
GOTO ONE
END OF PROGRAM'
```

Appendix C

In the following pages are presented the subroutines used in the regression analysis.

The first section is the 'main' program used to produce the output in the Results section.

The subroutines are in the EGTRAN dialect of FORTRAN and each subroutine is followed by the number of computer words of instructions, and the names of the subroutines which are required from the EGTRAN library.

```
JOBHILL/ABIM/STATPAC4C/IM00/10                          22/10/70   04.24.02

                 *XEQ
                 *CARDLIST
                 *STORAGE//14000

JOB ORGANISER ENTERED  04.24.40             22/10/70
                 *CHAIN1
                 *FORTRAN

EGTRAN COMPILER          MARK NO. 302           DATE      22/10/70              TIME
      PUBLIC C,NXS,NX,K
      DIMENSION C(50,50),NX(50),PRI(50),SEC(50)
      READ 5,ILIM,JLIM
    5 FORMAT(2I4)
      DO 100 II=1,ILIM
      CALL INPUT1
      DO 90 JJ=1,JLIM
      CALL INREG
      CALL START
      NNXS=NXS
      NXS=0
      DO 10 M=1,NNXS
      NXS=NXS+1
      CALL ADDVAR(M)
      CALL COEFFS
      CALL ANOVA
   10 CALL ELANAL
      CALL COVAR
   90 CONTINUE
  100 CONTINUE
      CALL EXIT
      END

       ROUTINE COMPILED
       TIME  LESS THAN 2 SECS
       NUMBER OF INSTRUCTION WORDS 26
       * * * * * * * * * * * * * * * * * *

NUMBER OF INSTRUCTION WORDS 0025

EXTERNAL ROUTINES REQU.
                          WONT30  WONT31  WONT32  INPUT1   INREG    START  ADD
                 ELANAL   COVAR    EXIT
                 *FORTRAN

EGTRAN COMPILER          MARK NO. 302           DATE      22/10/70              TIME
      SUBROUTINE INPUT1
C TO READ IN NUMBER OF VARIABLES, NUMBER OF OBSERVATIONS, MEANS, STANDARD
C DEVIATIONS AND CORRELATION MATRIX
      PUBLIC NV,NO,XBAR,SX,R
      DIMENSION XBAR(50),SX(50),R(50,50)
      READ 101,NV,NO
      READ 102,(XBAR(I),I=1,NV)
      READ 102,(SX(I),I=1,NV)
      DO 99 I=1,NV
      READ 103,(R(I,J),J=1,NV)
   99 CONTINUE
  101 FORMAT(2I4)
```

```
  102 FORMAT(7F10.4)
  103 FORMAT(10F7.4)
      RETURN
      END

       ROUTINE COMPILED
       TIME  LESS THAN 2 SECS
       NUMBER OF INSTRUCTION WORDS 36
       * * * * * * * * * * * * * * * *


NUMBER OF INSTRUCTION WORDS 0036


EXTERNAL ROUTINES REQU.
                INPUT1  WONT30  WONT31  WONT32
                *FORTRAN

EGTRAN COMPILER          MARK NO. 302          DATE     22/10/70          TIME
      SUBROUTINE INREG
C TO READ IN PARAMETERS OF REGRESSION EQUATION
      PUBLIC NX, NY,NXS
      DIMENSION NX(50)
      READ 401,NY,NXS
      READ 402,(NX(I),I=1,NXS)
  401 FORMAT(2I4)
  402 FORMAT(18I4)
      RETURN
      END

       ROUTINE COMPILED
       TIME  LESS THAN 1 SEC
       NUMBER OF INSTRUCTION WORDS 15
       * * * * * * * * * * * * * * * *


NUMBER OF INSTRUCTION WORDS 0015


EXTERNAL ROUTINES REQU.
                INREG  WONT30  WONT31  WONT32
                *IDENTIFIERSTART
                *PUNCH
                *FORTRAN

EGTRAN COMPILER          MARK NO. 302          DATE     22/10/70          TIME
      SUBROUTINE START
      PUBLIC C
      DIMENSION C(50,50)
      C(1,1)=1.0
      RETURN
      END

       ROUTINE COMPILED
       TIME  LESS THAN 1 SEC
       NUMBER OF INSTRUCTION WORDS 4
       * * * * * * * * * * * * * * * *


NUMBER OF INSTRUCTION WORDS 0004
```

```
EXTERNAL ROUTINES REQU.
                  START
                  *IDENTIFIERADDVAR
                  *PUNCH
                  *FORTRAN

EGTRAN COMPILER          MARK NO. 302          DATE     22/10/70          TIME
      SUBROUTINE ADDVAR(M)
C TO ADD VARIABLE L TO RECIPROCAL MATRIX OF ORDER K-1
      PUBLIC K,NX,P,C,R,NY
      DIMENSION C(50,50),R(50,50),NX(50),P(50),A(50)
     PK=M+1
     PL=NX(M)
     PNY=NY
  207PA(1)=R(NY,L)
  209PIF(K.LE.2) GO TO 206
      DO 205 I=2,M
  208PJ=NX(I-1)
  205PA(I)=R(J,L)
  206 CONTINUE
      DO 200 N=1,M
      P(N)=0.0
      DO 201 I=1,M
  201 P(N)=P(N)+A(I)*C(N,I)
  200PP(N)=P(N)*(-1.0)
      G=R(L,L)
      DO 202 I=1,M
  202 G=G+A(I)*P(I)
     PC(K,K)=1.0/G
      DO 204 I=1,M
     PC(I,K)=P(I)*C(K,K)
  204 C(K,I)=C(I,K)
     PDO 203 I=1,M
     PDO 203 J=1,M
  203PC(I,J)=C(I,J)+C(I,K)*P(J)
      RETURN
      END

       ROUTINE COMPILED
       TIME  LESS THAN 3 SECS
       NUMBER OF INSTRUCTION WORDS 99
       * * * * * * * * * * * * * * * *


NUMBER OF INSTRUCTION WORDS 0098


EXTERNAL ROUTINES REQU.
                  ADDVAR  DONTQ8
                  *IDENTIFIERCOEFFS
                  *PUNCH
                  *FORTRAN

EGTRAN COMPILER          MARK NO. 302          DATE     22/10/70          TIME
      SUBROUTINE COEFFS
      PUBLIC SX,C,XBAR,NO,NXS,NY,NX
      DIMENSION C(50,50),SX(50),XBAR(50),NX(50)
      PRINT 601
      PRINT 602
      PRINT 603
```

```
      PRINT 604
      BO=XBAR(NY)
      PRINT 607,NY
      DO 690 L=1,NXS
      J=NX(L)
     PSCALE=SX(NY)/SX(J)
     PI=L+1
     PBI=((-1.0)*C(1,I)/C(1,1))*SCALE
      SEB=SQRT((C(1,1)*C(I,I)-C(1,I)**2)/(NO-NXS-1))*SCALE/C(1,1)
      BO=BO-BI*XBAR(J)
  690 PRINT 605,J,BI,SEB
      PRINT 606,BO
  601 FORMAT(1H1,50X,20HREGRESSION  EQUATION)
  602 FORMAT(1H ,50X,20H----------  --------)
  603 FORMAT(1H0,35X,8HVARIABLE,16X,1HB,15X,7HS.E.(B))
  604 FORMAT(1H ,35X,8H--------,16X,1H-,15X,7H-------)
  605 FORMAT(1H0,38X,I2,13X,E12.5,7X,E12.5)
  606 FORMAT(1H0,35X,8HCONSTANT,10X,E12.5)
  607 FORMAT(1H0,38X,I2,14X,9HDEPENDENT)
      RETURN
      END

       ROUTINE COMPILED
       TIME  LESS THAN 3 SECS
       NUMBER OF INSTRUCTION WORDS 54
       * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *


NUMBER OF INSTRUCTION WORDS 0053


EXTERNAL ROUTINES REQU.
                COEFFS  WONT40  WONT42  WONT41   SQRT  DONT83  DONT53
                *IDENTIFIERANOVA
                *PUNCH
                *FORTRAN

EGTRAN COMPILER          MARK NO. 302          DATE    22/10/70          TIME
      SUBROUTINE ANOVA
      PUBLIC C,NXS,NO,SX,NY
      DIMENSION C(50,50),SX(50)
      REAL MULTR
      NTOT=NO-1
      NRES=NO-NXS-1
      RSQ=1.0-1.0/C(1,1)
     PMULTR=(SQRT(RSQ))
      SSTOT=SX(NY)**2*NTOT
      SSREG=RSQ*SSTOT
      SSRES=SSTOT-SSREG
      REGMS=SSREG/NXS
      RESMS=SSRES/NRES
      TOTMS=SSTOT/NTOT
      F=REGMS/RESMS
      PRINT 599
  599 FORMAT(1H0)
      PRINT 501
      PRINT 502
      PRINT 503
      PRINT 504
      PRINT 505
      PRINT 506,SSREG,NXS,REGMS
```

```
      PRINT 507
      PRINT 506,SSRES,NRES,RESMS
      PRINT 508
      PRINT 506,SSTOT,NTOT,TOTMS
  501 FORMAT(1H0,49X,22HANALYSIS  OF  VARIANCE)
  502 FORMAT(1H ,49X,22H----------------------)
  503 FORMAT(1H0,34X,6HSOURCE,9X,21HSUM OF SQUARES   D.F.,5X,11HMEAN SQU
     1ARE)
  504 FORMAT(1H ,34X,6H------,9X,21H--------------   ----,5X,11H--------
     1---)
  505 FORMAT(1H0,34X,16HREGRESSION      )
  506 FORMAT(1H+,50X,E12.6,5X,I2,5X,E12.6)
  507 FORMAT(1H ,34X,16HRESIDUAL        )
  508 FORMAT(1H ,34X,16HTOTAL           )
      PRINT 511,F
      PRINT 509,MULTR
      PRINT 510,RSQ
  509 FORMAT(1H0,34X,10HMULTIPLE R,10X,F6.4)
  510 FORMAT(1H ,34X,9HR SQUARED,11X,F6.4)
  511 FORMAT(1H ,34X,7HF VALUE,13X,F6.2)
      RETURN
      END

       ROUTINE COMPILED
       TIME  LESS THAN 4 SECS
       NUMBER OF INSTRUCTION WORDS 66
       * * * * * * * * * * * * * * * *


NUMBER OF INSTRUCTION WORDS 0065


EXTERNAL ROUTINES REQU.
                ANOVA    SQRT  DONT83  DONT53  WONT40  WONT42  WONT41
               *IDENTIFIERELANAL
               *PUNCH
               *FORTRAN

EGTRAN COMPILER          MARK NO. 302          DATE    22/10/70          TIME
      SUBROUTINE ELANAL
      PUBLIC K,C
      DIMENSION CC(50,50),C(50,50),PRI(50),SEC(50)
      PRINT 801
      CALL PRIMEL(C,PRI)
      DO 800 L=2,K
      CALL PRIMT(L,PRI)
      CALL REMVAR(L,CC)
      CALL PRIMEL(CC,SEC)
      CALL PREST(SEC,PRI,L)
  800 CONTINUE
  801 FORMAT(1H1,54X,18HELEMENT   ANALYSIS/1H ,54X,18H------------------
     1)
      RETURN
      END

       ROUTINE COMPILED
       TIME  LESS THAN 2 SECS
       NUMBER OF INSTRUCTION WORDS 16
       * * * * * * * * * * * * * * * *
```

```
NUMBER OF INSTRUCTION WORDS 0016


EXTERNAL ROUTINES REQU.
                ELANAL  WONT40  WONT42  PRIMEL   PRIMT  REMVAR   PREST
                *IDENTIFIERPRIMEL
                *PUNCH
                *FORTRAN

EGTRAN COMPILER         MARK NO. 302          DATE     22/10/70         TIME
      SUBROUTINE PRIMEL(RMAT,ELT)
      PUBLIC SX,NY,NO,K
      DIMENSION SX(50),ELT(50),RMAT(50,50)
      SSY=SX(NY)**2*(NO-1)
     PM=K-1
      DO 701 J=1,M
     PI=J+1
     PRMATT=RMAT(I,I)
      IF(RMATT.GT.0.00001) GO TO 700
      ELT(J)=0.0
      GO TO 701
  700PELT(J)=SSY/(RMAT(I,I)/(RMAT(1,I)/RMAT(1,1))**2-RMAT(1,1))
  701 CONTINUE
      RETURN
      END

       ROUTINE COMPILED
       TIME  LESS THAN 2 SECS
       NUMBER OF INSTRUCTION WORDS 48
       * * * * * * * * * * * * * * * *


NUMBER OF INSTRUCTION WORDS 0048


EXTERNAL ROUTINES REQU.
                PRIMEL  DONT83  DONT53  DONTQ2
                *IDENTIFIERPRIMT
                *PUNCH
                *FORTRAN

EGTRAN COMPILER         MARK NO. 302          DATE     22/10/70         TIME
      SUBROUTINE PRIMT(L,PRI)
      PUBLIC C,SX,NY,NO,NX,NXS
      DIMENSION PRI(50),SX(50),NX(50),C(50,50)
      I=L-1
     PPARTF=PRI(I)/((1.0/C(1,1)*SX(NY)**2*(NO-1))/(NO-NXS-1))
      PRINT 900, NX(I),PRI(I),PARTF
  900 FORMAT(1H0,16HPRIMARY  ELEMENT,3X,I3,3X,E15.5,37X,10HPARTIAL  F,F1
     11.2)
      RETURN
      END

       ROUTINE COMPILED
       TIME  LESS THAN 2 SECS
       NUMBER OF INSTRUCTION WORDS 32
       * * * * * * * * * * * * * * * *


NUMBER OF INSTRUCTION WORDS 0031
```

```
EXTERNAL ROUTINES REQU.
                  PRIMT  DONT83  DONT53  WONT40  WONT41  WONT42
                  *IDENTIFIERREMVAR
                  *PUNCH
                  *FORTRAN

EGTRAN COMPILER        MARK NO. 3O2            DATE    22/10/70          TIME
      SUBROUTINE REMVAR(L,CC)
C TO REMOVE VARIABLE L FROM RECIPROCAL MATRIX OF ORDER K
      PUBLIC R,C,K
      DIMENSION R(50,50),C(50,50),CC(50,50),P(50)
      DO 300 N=1,K
  300PP(N)=C(N,L)/C(L,L)
      DO 301 I=1,K
      DO 301 J=1,K
  301PCC(I,J)=C(I,J)-P(J)*C(I,L)
      RETURN
      END

       ROUTINE COMPILED
       TIME  LESS THAN 2 SECS
       NUMBER OF INSTRUCTION WORDS 43
       * * * * * * * * * * * * * * * *


NUMBER OF INSTRUCTION WORDS 0042


EXTERNAL ROUTINES REQU.
                  REMVAR
                  *IDENTIFIERPREST
                  *PUNCH
                  *FORTRAN

EGTRAN COMPILER        MARK NO. 3O2            DATE    22/10/70          TIME
      SUBROUTINE PREST(SEC,PRI,L)
      PUBLIC K,NX
      DIMENSION SEC(50),PRI(50),NX(50)
      PRINT 1000
      M=K-1
      I=1
      LIM=10
 1005 IF(LIM.LT.M) GO TO 1004
      LIM=M
 1004 PRINT 1001,(NX(N),N=I,LIM)
      J=L-1
      DO 1002 N=I,LIM
     PSEC(N)=SEC(N)-PRI(N)
      IF(N.NE.J) GO TO 1002
      SEC(N)=0.0
 1002 CONTINUE
      PRINT 1003,(SEC(N),N=I,LIM)
      IF(LIM.EQ.M) GO TO 1006
      I=I+10
      LIM=LIM+10
      GO TO 1005
 1006 CONTINUE
 1000 FORMAT(1H ,19HSECONDARY  ELEMENTS)
 1001 FORMAT(1H ,10(I4,9X))
 1003 FORMAT(1H ,10E13.5)
```

```
      RETURN
      END

       ROUTINE COMPILED
       TIME  LESS THAN 3 SECS
       NUMBER OF INSTRUCTION WORDS 62
       * * * * * * * * * * * * * * * *


NUMBER OF INSTRUCTION WORDS 0062


EXTERNAL ROUTINES REQU.
                   PREST  WONT40  WONT42  DONTQ6  WONT41  DONTQ5  DONT57
                 *FORTRAN

EGTRAN COMPILER          MARK NO. 302          DATE      22/10/70          TIME
      SUBROUTINE COVAR
      PUBLIC C,NXS,NO,SX,NY,NX,K
      DIMENSION C(50,50),COV(50,50),NX(50),SX(50)
      PRINT 902
  902 FORMAT(1H1)
      DO 900 L=2,K
      LI=NX(L-1)
      SSQXI=SX(LI)*SX(LI)*(NO-1)
      DO 900 LL=2,L
      LJ=NX(LL-1)
      SSQXJ=SX(LJ)*SX(LJ)*(NO-1)
      I=L
      J=LL
      COV(I,J)=(C(I,J)/SQRT(SSQXI*SSQXJ))
      PRINT 901,(COV(I,J),J=1,L)
  901 FORMAT(13F10.5)
  900 CONTINUE
      RETURN
      END

       ROUTINE COMPILED
       TIME  LESS THAN 2 SECS
       NUMBER OF INSTRUCTION WORDS 42
       * * * * * * * * * * * * * * * *


NUMBER OF INSTRUCTION WORDS 0042


EXTERNAL ROUTINES REQU.
                   COVAR  WONT40  WONT42  DONT83   SQRT  WONT41
                 *PREDATA

JOB ORGANISED  04.25.24          22/10/70

COMPOSER VERSION  POSEUPDATER4    DATED 17/09/70
*PREDATA



      PUBLIC NV,NO,XBAR,SX,R,NX,NY,NXS,C,K,P

      DIMENSION XBAR(50),SX(50),R(50,50),NX(50),C(50,50),P(50)
```

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J.F.H.

GLOSSARY OF TERMS AND SYMBOLS

N	Nickel
A	Aluminium
C	Copper
CZI	Copper/10% Zinc
CZ2	Copper/20% Zinc
CZ3	Copper/25% Zinc
CA1	Copper/4% Aluminium

σ	Tensile stress
ϵ	tensile strain
τ	shear stress
γ	shear strain
τ_m	maximum shear stress
γ_m	shear strain at maximum shear stress
τ_o	intercept shear stress, i.e. shear stress at shear strain = 0
τ_s	steady state shear stress
γ	shear strain rate
ϵ	tensile strain rate
K	rate of work hardening
E	Young's modulus
G	modulus of shear
T	temperature of test
T_m	melting temperature
θ	homologous temperature, i.e. T/T_m
D	grain size (mean linear intercept)
γ_{SF}	stacking fault energy
e	electron/atom ratio
ϕ	degress of freedom
ν	Poisson's ratio