The University of Aston in Birmingham

INVESTIGATION OF SLOW RELEASE RELAYS USING REED SWITCHES

- by -

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SUMMARY

This research concerns the design of reed relays having fast-operation and slow-release characteristics without resorting to external timing circuits.

In an electromagnetic relay of this type, additional iron is required to offset the inherently low permeance of the reed switch, if a long time constant is to be obtained. In the presence of this additional iron, means must be introduced to ensure that rapid closure of the contacts is possible, on the application of excitation. To this end, a second winding, in addition to the operating coil, is included in the relay, and the reed switches are located in the leakage field between these windings. This is the fundamental departure from normal relay practice, in that the flux across the operating gap is only a small fraction of the main flux.

A wide range of values of release m.m.f. is normally encountered in commercially available reed switches, and this gives rise to a corresponding range of release times. The relay which forms the subject of this research embodies a feature whereby, in a multi-contact version, the release of the contacts may be synchronised.

It is shown that for given operating conditions an optimum design exists for minimum relay volume. A technique for achieving this design is developed in the thesis.

Experimental relays are described and their measured performance is shown to agree well with the theory on which the design is based.

Acknowledgments

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LIST OF PRINCIPAL SYMBOLS

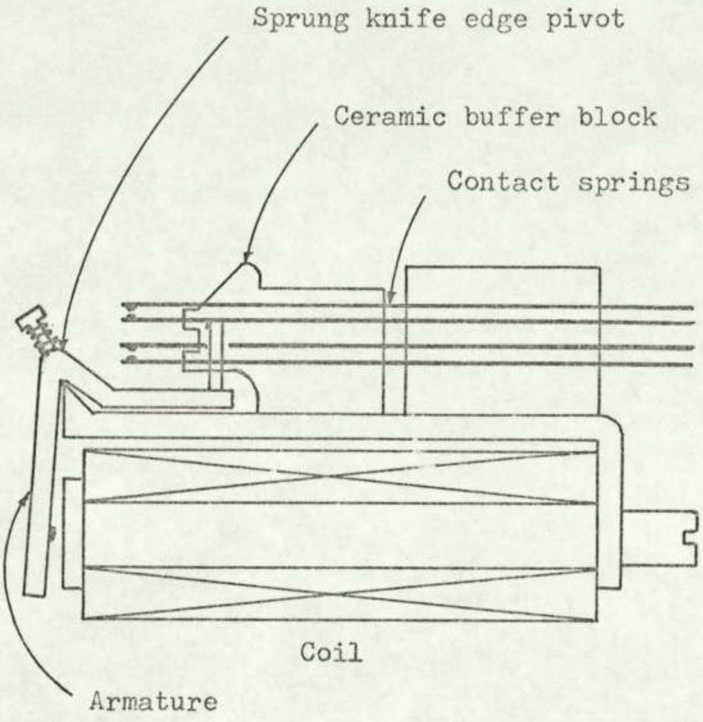
a_c	Primary coil area (cross section).
a_i	Iron core area (cross section).
g	Secondary core gap.
B	Magnetic flux density.
B_s	Magnetic flux density at knee.
H_c	Coercive m.m.f. gradient.
H_m	Maximum m.m.f. gradient.
H_s	m.m.f. gradient at knee.
I	Coil m.m.f. in general case.
i_o	Initial step of primary m.m.f.
i_1	Primary m.m.f.
i_2	Secondary m.m.f.
i	m.m.f. across reed insert.
i_e	Eddy current m.m.f.
i_r	Reed insert release m.m.f.
k	Ratio of core length to coil length in general case.
K	Unwound primary core (l_i-x).
l_c	Mean length of primary turn.
l_i	Length of primary iron path.
l_2	Length of secondary iron path.

m_1	Primary coercive m.m.f.
m_2	Secondary coercive m.m.f.
N	Number of primary coil turns.
R	Coil outer radius.
R'	External synchronising resistor.
R_1	Primary resistance referred to one turn.
R_2	Secondary resistance referred to one turn.
r	Radius of primary iron core.
S_1	Primary core reluctance.
S_2	Secondary reluctance (including gap).
s	Laplace variable.
T	Time constant in general case.
t	Time.
t_r	Release delay.
V	Supply voltage, referred to one turn.
v	Voltage across synchronising contact.
x	Primary winding length.
y	Radial thickness of coil former.
α	Composite parameter defined in equation (20).
β	Composite parameter defined in equation (21).
ρ	Effective primary coil resistivity.
μ	Core permeability for decreasing flux.
ϕ'	Core flux.
ϕ	Core flux at knee.

1. INTRODUCTION

In 1836[1] Samuel Morse found forty feet an excessive transmission distance for the correct functioning of his telegraph equipment. He conceived the idea of utilising an electromagnet to operate a switch, thus "relaying" his signal for a further forty feet.

The advent of the telephone, which required analogue, rather than digital, signals, rendered the electromagnetic relay unsuitable for augmenting the signal. However, Strowger's introduction of the automatic telephone exchange provided a new field of application for relays, associated with switching operations, and many similar functions exist in industrial and domestic control and automation. In 1932 the Post Office adopted as standard the "3000 type" relay[2] which had been recently developed by Ray and Biddlecomb,[3] (See Figure 1). A great deal of progress had been made in this development. Improved materials contributed to better performance, and numerous design innovations were introduced. These included a modified magnetic circuit associated with the elimination of pinned armature pivots, and improved reliability by the introduction of redundancy in the form of twin contacts. The "3000 type" relay is currently in



Post Office 3000 type relay.

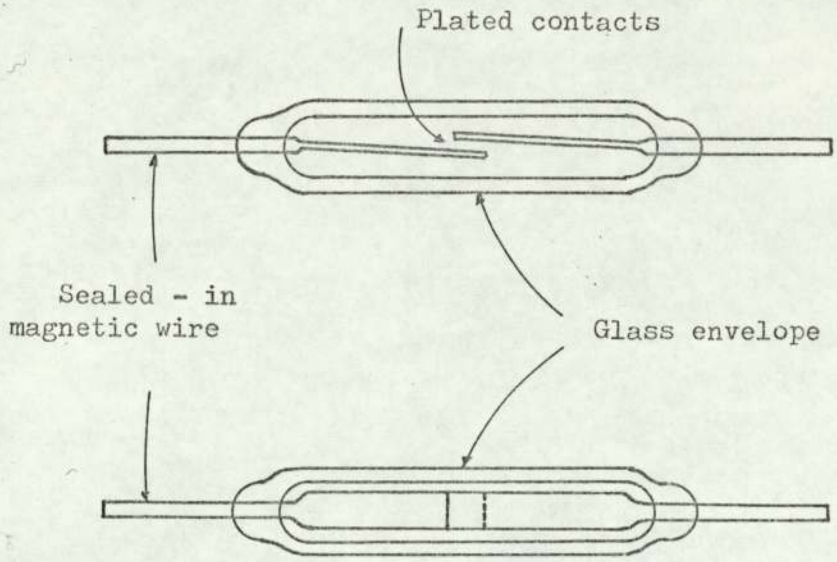
Fig. 1.

use, and it is a tribute to Strowger that the basic principle has not changed since his time.

Despite the introduction of solid state switching the electromagnetic relay has not yet been superseded entirely. It has the advantage of a low resistance in the closed state and low leakage when open. In addition, there is a high degree of isolation between the controlled circuit and the controlling circuit. Furthermore, arrangements having a large number of contacts may be operated by a single magnet system. In fact, relays are still to be found in the equipment of present day "electronic" exchanges.[4,54,55]

Notwithstanding the successful history of relays of the type described above, they have a serious disadvantage in that the contacts are exposed to the atmosphere. In unfavourable environments corrosion may be severe.[5,6,7] Enclosure of the relay in a sealed container is only a partial solution.[8,9,10,11] Extraneous matter may be successfully excluded but the relay produces its own contaminant in the form of Hydrocarbons released from the coil insulation.[12,13,14] The solution to this problem is to seal the contact assembly off from the operating system, in an enclosure filled with an inert gas. An early example is the well-known mercury switch, in which a glass capsule carries two contact pins sealed through its walls. A globule of mercury bridges the contact pins when the capsule is tilted by electromagnetic or other means. Since gravity is involved in the proper functioning of mercury switches, they are suitable only for applications in which they may be rigidly mounted on a fixed support. Improved versions of the mercury-wetted switch include at least part of the magnetic system within the envelope.[15, 16] Two magnetic reeds are attracted together by the field of a coil external to the enclosure and the contacts are kept supplied with mercury, by capillary action, from a reservoir. Relays of this type have been claimed to tolerate displacements of up to 45° from the vertical without malfunctioning.

A more versatile and compact form of enclosed switch is the dry reed switch invented by W.B. Ellwood in 1938.[17] This consists of a glass tube having a magnetic reed sealed into each end as depicted in Figure 2. The ends of these reeds overlap in the centre of the tube and are separated in the de-energised state. In the presence of a magnetic field they are attracted together to make an electrical contact.[18, 19, 20, 21, 22] The only mechanical movement which takes place is the flexure of the reeds and, in the absence of mercury, operation is possible in any position.



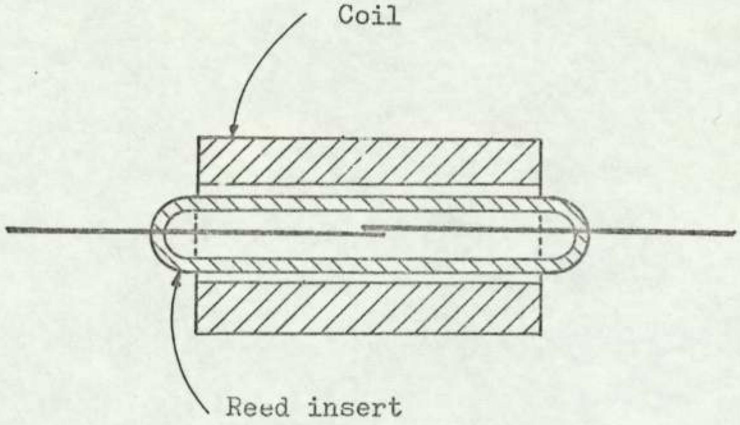
Glass-enclosed dry-reed switch.

Fig. 2.

A major factor in this development was the metallurgical research of the late 1930s[23,24] which made available magnetic alloys having a suitable coefficient of expansion for sealing into glass. The contact areas of the reeds are commonly diffused gold and an inert atmosphere is used for contact protection.[25,26,27] Operation may be by a coil wound on the outside of the glass tube as shown in Figure 3. This results in a very compact relay requiring very little power for operation.[28,29] For example, a single-contact relay measuring $\frac{7}{8}$" long and $\frac{1}{4}$" diameter requires 60mW for operation.

For multicontact operation the normal procedure is to use a single coil surrounding several reed switch "inserts". Each insert consists of a single glass capsule with its pair of magnetic reeds. If a changeover arrangement is required a third reed may be included, made from a stiff non-magnetic material against which one of the magnetic reeds bears in the de-energised state, thus forming a normally closed contact.

Apart from electromagnetic relay applications the reed switch is admirably suited for mechanical operation. The movement of a permanent magnet into the vicinity of the switch effects operation and obviates risk of mechanical wear of the moving parts.[30]



Reed relay arrangement.

Fig. 3.

The dry reed switch is inferior to the mercury wetted variety from the point of view of power handling capacity, since there is a risk of contacts becoming welded.[31] Also, contact bounce is rather worse, since the presence of mercury contributes to the damping of the reeds, and circuit maintenance is assisted by surface tension.

A very compact form of sealed relay is the diaphragm relay.[32] This consists of a coil wound on a cylindrical core and surrounded by a shell to form a "pot magnet". One contact is carried on the central core and the other on a diaphragm across the top of the "pot". This compact construction is very robust and offers good immunity to the effects of external magnetic fields.

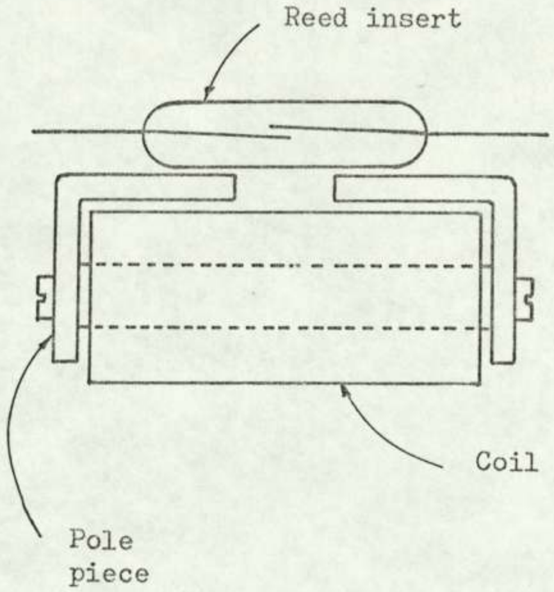
There are many applications in which control of the operate and release times may be required. For example, in telephony, a train of pulses originating from the dial mechanism represents a single digit of the number being called. A pause occurs before the next digit is dialled and the exchange equipment must be capable of distinguishing between the inter-pulse pause and the inter-digit pause. That is, a relay is required which will operate, and remain operated for the duration of a pulse train, and release before the next train is commenced. The pulse train may have a frequency as low as seven pulses per second with interruptions of up to 80% of the time, giving about 115ms. between the end of one pulse and the commencement of the next. The pause between trains of pulses is typically 200ms. plus the time taken for the caller to dial the next digit. It will be seen then that there is a requirement for a relay which will remain operated during an excitation interruption of 115ms. and release after 200ms.[33,34]

Several techniques may be used for control of the speed of conventional relays. For example, for delayed release, a slow decay of flux will be required, which implies a long time constant in the de-energised state. This is achieved by short circuiting the coil when the excitation is off. One way of doing this is to connect a diode across the coil terminals arranged to be non-conducting in the energised state. Such a diode will permit flow of coil current after removal of excitation. Since the core reluctance is lower in the released state than in the operated state, fast operation of the relay is facilitated.[35] If the additional power requirement is not an obstacle, a resistor in series with the circuit will reduce the operating time constant still further, without affecting the decay time constant. For some purposes it is desirable to obtain a time constant longer than that which can be obtained with the coil alone. If some of the available winding space is sacrificed, it can be occupied by a solid copper "slug" which may have a conductivity about twice that of a wound coil, resulting in an appreciably increased time constant.[36,37,38,39] The slug may be located at the end of the coil nearer to the moving armature, (an armature-end slug), or the end remote from the armature, (a heel-end slug). In the case of relays of similar shape to the Post Office 3000 type these alternative arrangements do not have identical properties due to the presence of leakage flux between the coil and slug. An armature end slug, since it provides an m.m.f. in the region of the operating gap, generally results in a longer delay than a heel-end slug. This difference is relatively small as far as release delay is concerned since, on de-energising, both coil and slug carry current in the same sense and the leakage flux is relatively small. When considering the initial application of excitation, however, the slug current will be of opposite sense to the coil current, and their opposing m.m.f.s will result in relatively large leakage flux. Consequently the gap flux will depend to a large extent on whether the coil or slug is nearer; the former case resulting in relatively short operating times.[40,41,42]

On considering the dynamics of reed relays, the outstanding characteristic is their high speed of response. The amount of iron in the system is very small, and with correspondingly small dimensions for the coil, short electromagnetic time constants are obtained. The small mass and high stiffness of the reeds results in very fast mechanical response. Consequently, delays of the order of one millisecond are obtained, although several bounces may occur on closing. This may occupy a further millisecond before a positive contact is established.

These inherent fast characteristics militate against the use of reed switches in relays having long time delays. The earliest publication of details of a self-contained slow-release reed relay was in 1962.[43] This described a reed relay similar in overall construction to a Post Office type 3000 having a cut-away yoke providing a gap across which the reed inserts were mounted as shown in Figure 4. Such an arrangement, having a considerable volume of additional iron, was claimed to give release times of up to 180ms.

The alternative to a self-contained slow-release



Slow-release reed relay.

Fig. 4.

relay is to use a normal reed relay operated by an electronic timing circuit. For long delays this entails the use of timers having long time constants. The reed insert is a highly reliable device, and some of the value of this reliability is lost if complex timing circuits are used. Both from this point of view, and from considerations of economy, the pursuit of a self contained design is worthy of consideration.

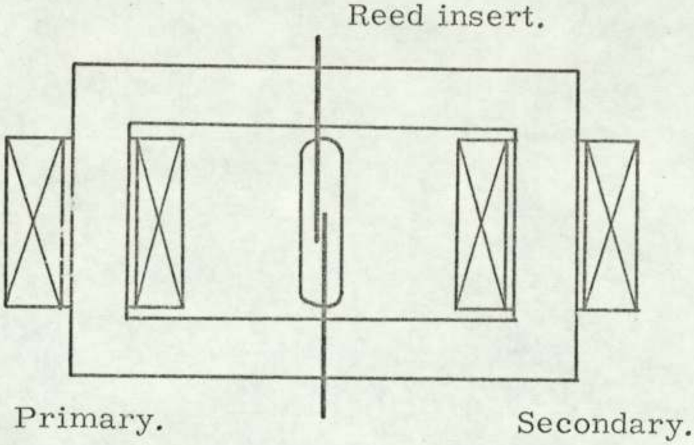
The aim of the research which forms the subject of this thesis has been to devise a reed relay having the shortest possible operate time together with release time of about 200ms.

2. PRINCIPLES OF A PROPOSED SLOW-RELEASE RELAY.

In a conventional relay the difference between the core permeances in the energised and de-energised states is an inherent advantage in constructing fast-operate/slow-release relays. These two requirements are very much in conflict in the case of a reed relay since, if sufficient iron is introduced into the system to provide the necessary time constant, the state of the reed insert will affect the permeance to a negligible extent.

The conflicting requirements of fast operation and slow release may be met by a relay shown diagrammatically in Figure 5. In this arrangement the reed insert is located across the core "window" in such a manner that the flux in the reed is only a relatively small leakage flux as far as the main core is concerned. The core carries, in addition to a primary exciting coil, a secondary winding normally in the form of a short circuited slug, so located that the operating flux for the reed insert is the leakage flux between the two windings during transient conditions. The mode of operation is described in more detail in the following paragraph.

On first applying excitation to the primary winding there will be induced in the secondary a large

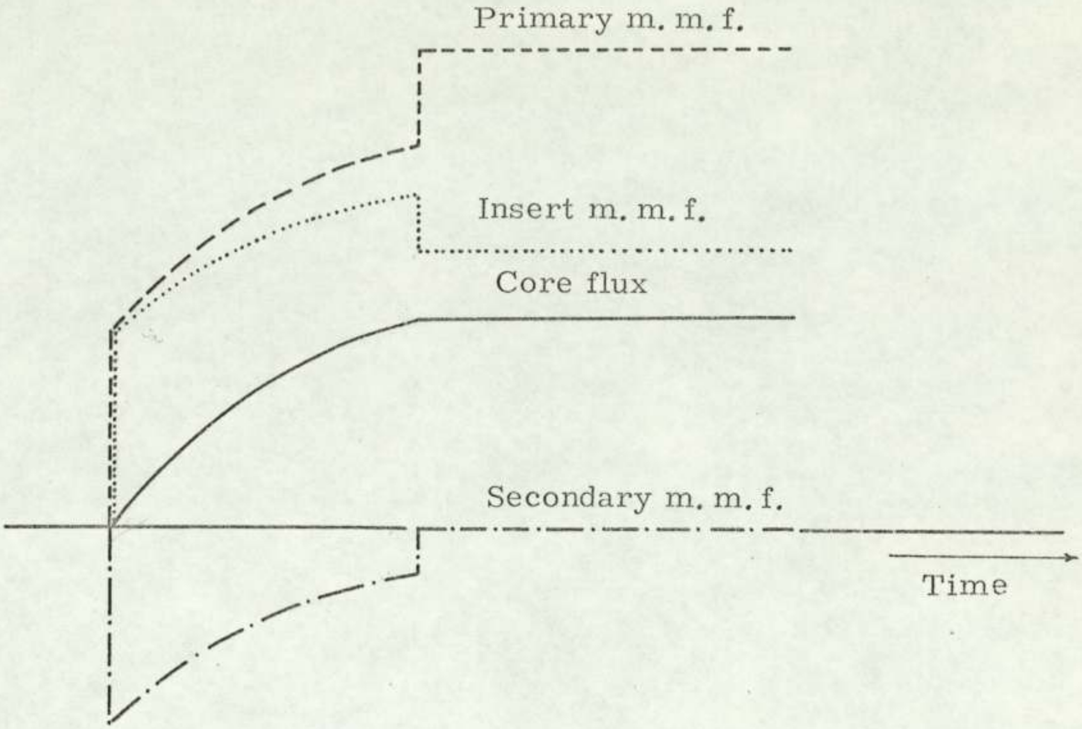


Diagrammatic Representation of Principles.

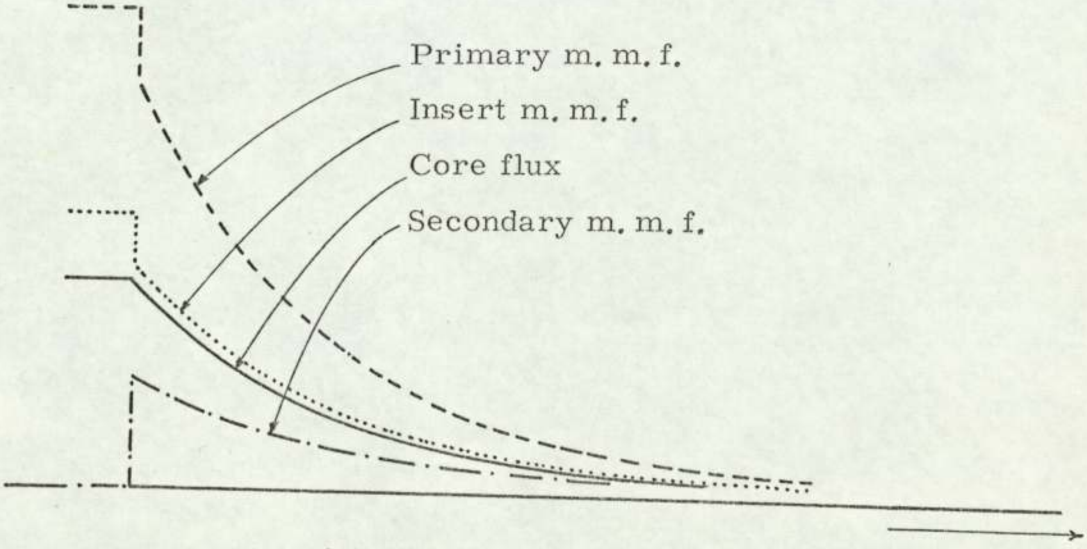
Fig. 5.

current and the primary current will rise rapidly. At this stage, the core flux will be increasing but still small, so that the primary and secondary m.m.f's. will be almost equal and in opposition. Nearly all of the primary m.m.f. will appear across the reed insert, thus effecting fast operation. This may be more clearly described by reference to Figure 6a, which is applicable to an idealised case in which the leakage flux is considered negligible compared to the core flux and the core is assumed to be linear with abrupt saturation. Since the core flux starts from zero, the m.m.f. across the reed insert will initially be equal to the primary m.m.f. which rises rapidly due to the presence of the secondary winding. Thereafter, there will be an exponential approach to a steady-state value which depends upon the ratio of primary to secondary reluctance. This exponential will be interrupted by saturation of the iron, resulting in a sudden collapse of secondary current. Associated with this collapse of secondary current there may, in some cases depending on the relay parameters, be a reduction inm.m.f. across the insert. This high transient m.m.f. together with the initial step provides a situation in which the contacts close rapidly despite the presence of a relatively long time constant in the system.

For provision of a long time constant during



(a) Energising

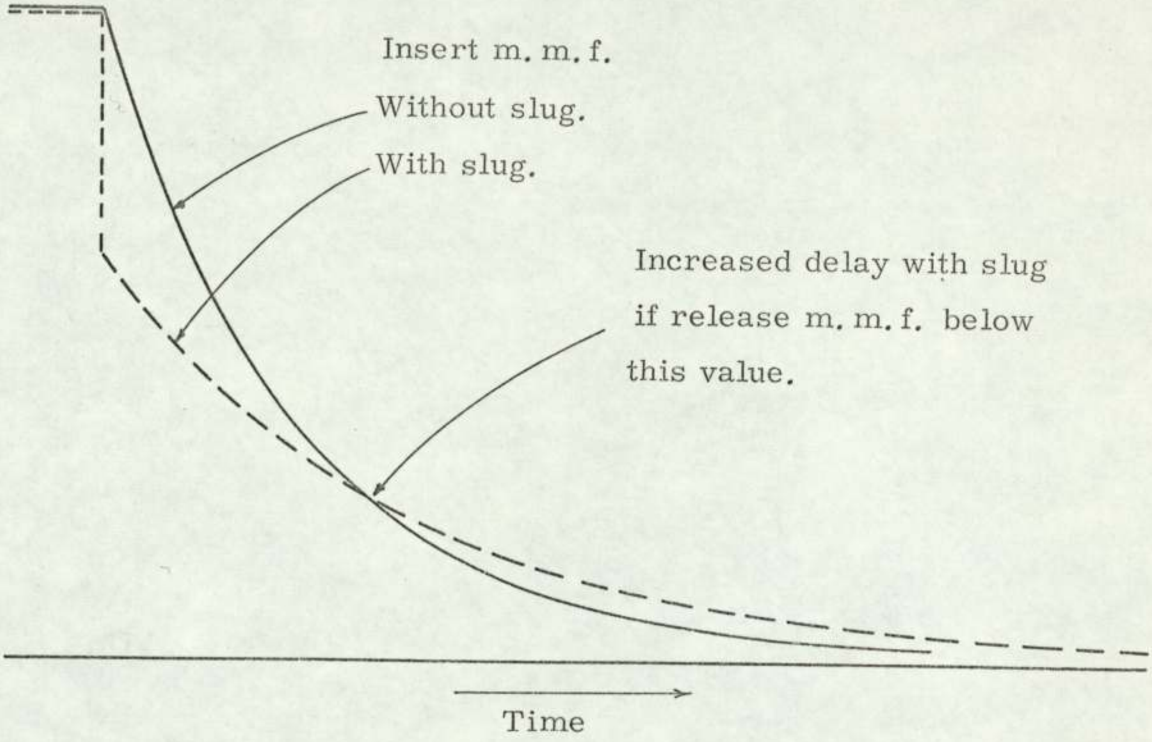


(b) De-energising

Rise and decay of flux and m. m. f.

Fig. 6.

de-energisation it is a prerequisite that a diode should be connected across the primary winding. On de-energising the primary, the m.m.f. required to sustain the core flux will be shared between the primary and secondary windings in proportion depending upon their relative conductances. There will thus be an initial sudden drop in m.m.f. across the reed insert, followed by an exponential decay as in Figure 6b. Superficially, it might appear that the sudden drop in m.m.f. arising from the presence of a secondary winding would be undesirable when attempting to obtain long release times. In fact, in some early experimental models the secondary took the form of a wound coil incorporating a diode so that it would only be effective on energising the primary. However, the increased time constant associated with a permanently short-circuited secondary can more than offset the initial sudden drop in m.m.f. This effect is illustrated in Figure 7 and more detailed discussion later in this thesis shows that for an optimum design the maximum release delay is almost independent of secondary resistance. This point is further confirmed by experimental results. The most obvious advantage arising from the use of a permanently short-circuited secondary is that a solid slug can be used which is both cheaper and more compact than a wound coil. A less

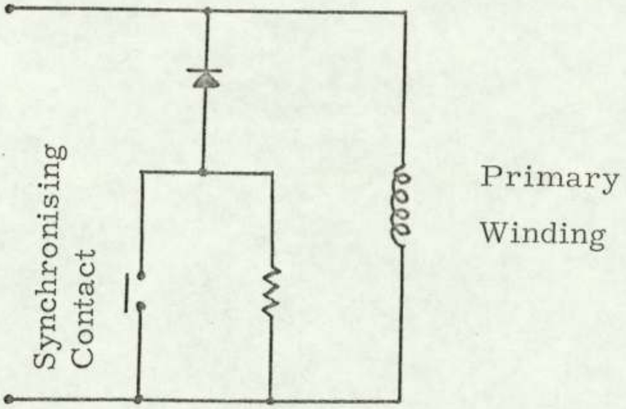


Effect of secondary slug on m. m. f. decay.

Fig. 7.

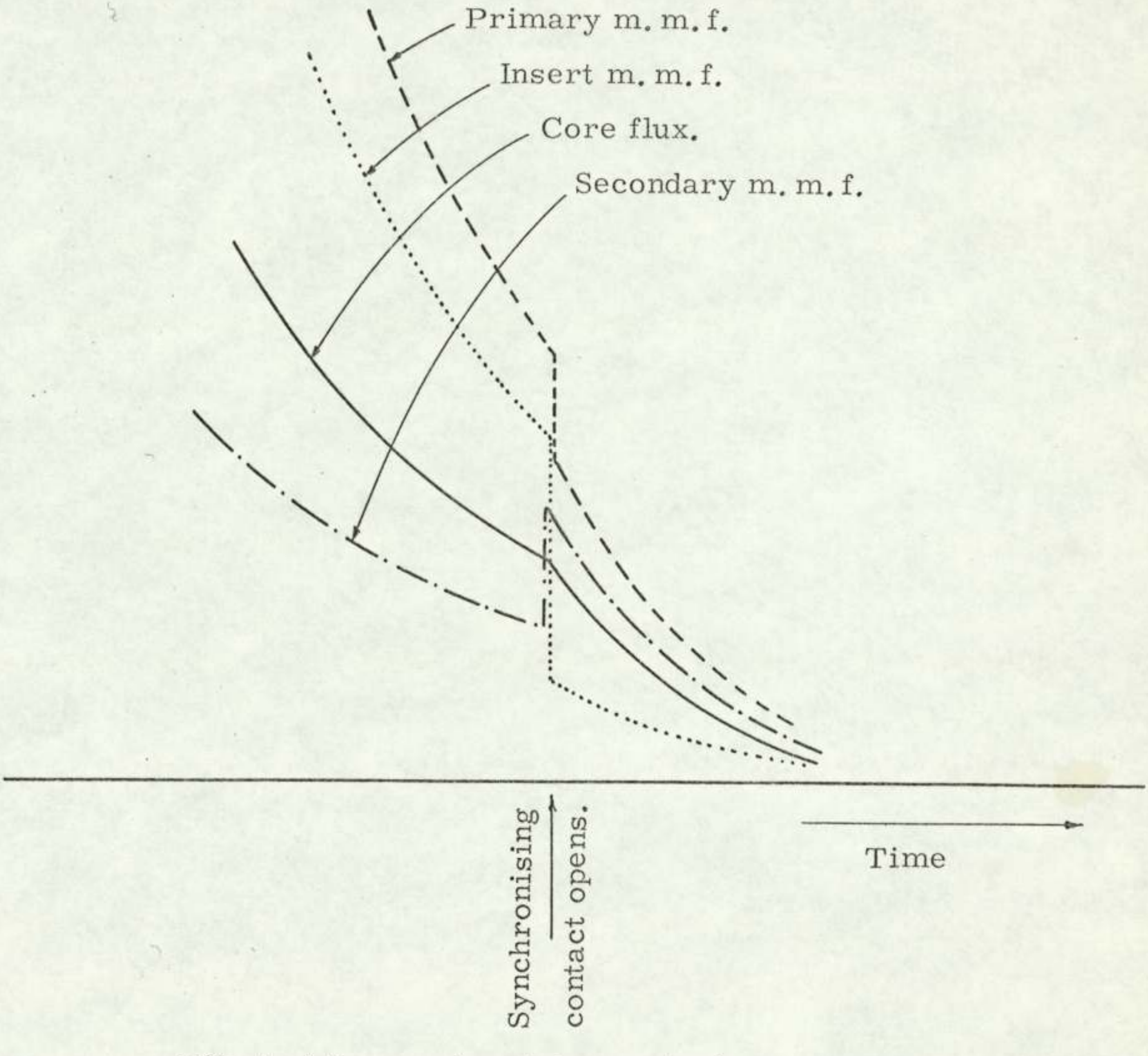
obvious advantage emerges when a multicontact relay is considered.

For multicontact operation the wide variations in release m.m.f. for reed switches may result in a considerable discrepancy between the release times of different inserts. This situation is aggravated when the inserts are magnetically in parallel, since the opening of one contact tends to increase the m.m.f. across those still closed. In such a case, the short-circuited secondary can be put to advantage by arranging the primary circuit, as shown in Figure 8, to incorporate one of the reed switches. This switch is chosen to have a higher release m.m.f. than the remainder (so that it will be the first to open) and is used to insert additional resistance into the primary circuit as shown in Figure 8. This action has a twofold effect in that, firstly, the time constant of the system is reduced and, secondly, but of greater importance, a redistribution of m.m.f. takes place as shown in Figure 9. The total m.m.f. to sustain the core flux is instantaneously constant, but the addition of resistance in the primary results in a sudden reduction in primary m.m.f. and a corresponding increase in secondary m.m.f. Consequently there is a rapid reduction in m.m.f. across the reed insert and all remaining contacts are rapidly released. Using this technique, it is in some cases possible



Synchronising circuit.

Fig. 8.



Effect of the synchronising contact.

Fig. 9.

to make the m.m.f. reach a small negative value which will counteract the residual m.m.f. of the core and can release a contact which might otherwise remain closed indefinitely. This contact to synchronise the opening of all other contacts appears to perform a rather arduous duty, but it is predicted later in this thesis (and confirmed by experiment) that the peak voltage is only about half the supply voltage, i.e. 25V on a 50V system.

Much has been done by other workers[18,23] on the dynamics of the reed switch, but since this concerns a stiff cantilever, having a small mass, moving through a short distance, the times involved are relatively short, and they have not been taken into account in this study. It is assumed that the operation of the reed switch depends solely on the applied m.m.f. and the effects of reed mass are negligible.

3. PRELIMINARY CONSIDERATIONS

Before entering into a detailed discussion of design and analysis of the performance of a particular relay it is worth giving some consideration, in general terms, to the dimensions which are likely to be required. On the assumption that an electromagnetic device which is to delay the release of a reed switch will include an amount of iron large compared to that in the reed switch itself, the switch operation may be considered to be dependent upon the m.m.f., whilst the flux in the reeds may be neglected when compared to the core flux. In general, consideration may be given to a system having a cylindrical coil of outer radius R and length x wound on an iron core of radius r, irrespective of the actual details of construction. The conductance of such a coil will be $x(R-r)/\rho\pi(R+r)$, where ρ is the effective resistivity of a wound coil, and the core permeance will be $\mu\pi r^2/kx$ where k is the ratio of core length to coil length. The time constant of such a system will be given by

$$T = \mu r^2(R-r)/\rho(R+r)k \qquad (1)$$

Inspection of equation (1) reveals that there will be an optimum value of r to give the maximum time constant for any given set of overall dimensions. Equation (1) may be rewritten in the form shown in equation (2) giving T as a function of r/R

$$T = \mu R^2 (r/R)^2 (1-r/R)/\rho\ (1+r/R)k \qquad (2)$$

For a constant value of R, differentiation of equation (2) with respect to r/R yields

$$dT/d(r/R) = \left[2R^2\mu/\rho\ k\right] \cdot \left[1-r/R-(r/R)^2\right] \ / \ \left[1-(r/R)\right]^2 \qquad (3)$$

Equating the right hand side of equation (3) to zero will give the condition for maximum T. Finite roots will be given by

$$1-r/R-(r/R)^2=0 \qquad (4)$$

and the only positive root of this equation is

$$r/R=(\sqrt{5}-1)/2=(\text{approx.})0.618 \qquad (5)$$

For any given outer radius R, the value of r to give a maximum time constant is given by equation (5) and it is interesting to note that this ratio is independent of the materials used. Substituting equation (5) in equation (2), the maximum time constant for given dimensions and material parameters will be

$$T=\left[R^2\mu/\rho k\right]\ \left[.618^2(1-.618)/(1+.618)\right]$$

$$= .09R^2\mu/\rho k \qquad (6)$$

We may now introduce

$$\mu=Bkx/I \qquad (7)$$

where B is the magnetic flux density in the core and I is the coil m.m.f. Substituting equation (7) in equation (6) and rearranging, an expression is obtained for R^2x which will be a measure of the coil volume

$$R^2x=11.1TI\rho/B \qquad (8)$$

Giving some consideration now to the release of a relay, it will be observed that neither the time constant T nor the coil m.m.f. I are directly related to the release time or release m.m.f. of the reed switch. However, if we assume linearity, we may write

$$i=I\exp(-t/T) \qquad (9)$$

where I is the initial coil m.m.f. Rearranging equation (9) gives

$$TI/ti=(T/t)\exp(t/T) \qquad (10)$$

Differentiation of equation (10) with respect to t/T shows that TI/ti has a minimum value of e when

$t/T=1$. The form of equation (10) is shown in Figure 10. If t_r and i_r are the release time and release m.m.f. respectively for a reed switch, then when $t=t_r$, i must be greater than i_r by some factor depending on the geometry of the relay, so that TI cannot be less than $et_r i_r$. When this minimum value for TI is inserted in equation (8), a minimum value for R^2x will be found

$$R^2x=30t_r i_r \rho/B_s \tag{11}$$

B_s is here substituted for B since B is the flux density corresponding to the initial coil m.m.f., I, and should clearly have the maximum possible value for full utilisation of the iron.

It now becomes possible to insert some values into equation (11), and typical figures might be

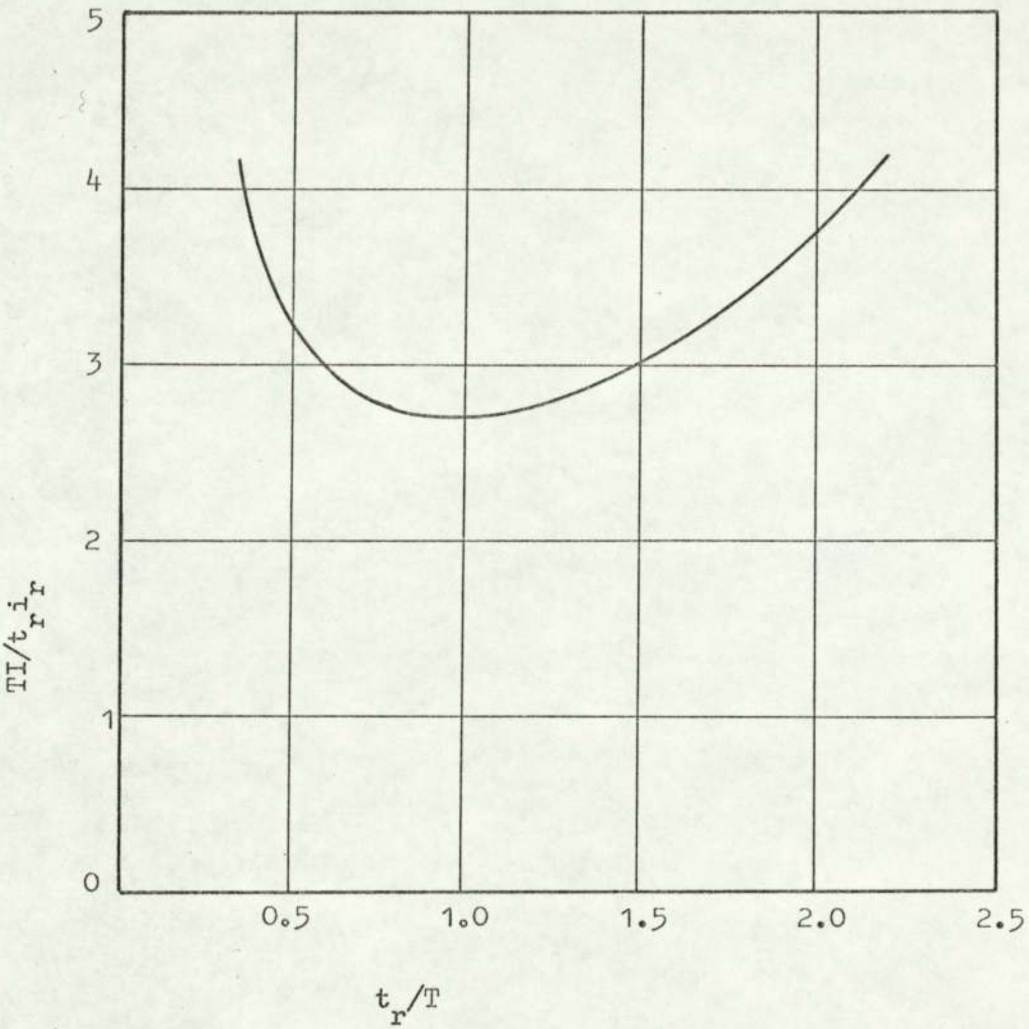
$t_r=200\text{ms}.$

$i_r=27\text{A}$

$\rho=4\times10^{-8}\Omega\text{-m}$

$B_s=1.5\text{Wb/m}^2$

hence $R^2x=4.3\times10^{-6}\text{m}^3$ approximately. (12)



Graphical representation of equation (10).

Fig. 10.

A rectangular solid to contain this coil would occupy $17.2\times10^{-6}m^3$ (approximately $1in^3$) but in assessing the space occupied by a complete relay several additional factors should be allowed for:

a) the coil m.m.f. will be greater than the m.m.f. across the reed insert

b) it may not be practicable to operate at the minimum point on Figure 10

c) space will be required for the reed inserts and any associated pole pieces

d) space will be required for a return magnetic path.

Taking the above allowances into account, we might predict that a relay to give 200ms. release time using inserts having a release m.m.f. of 27A would be likely to occupy a space of several cubic inches and this should be borne in mind in assessing the experimental models described on later pages.

These calculations are based upon overall volume as the only criterion to be considered. However, it will be observed that resistance increases continuously with increase of iron radius and, consequently, an iron radius rather less than the optimum might be considered if an increase in volume can be tolerated for the sake of a reduced power requirement.

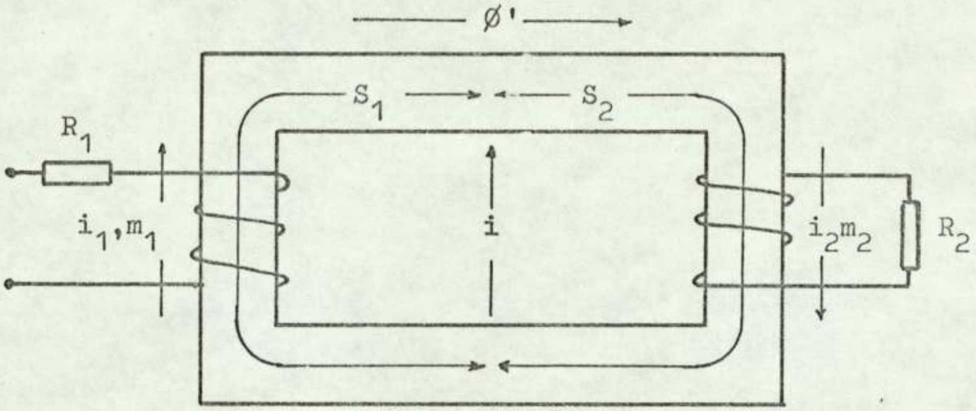
Examination of equation (11) indicates that the space occupied by the relay will not be affected by iron permeability, the only property of the iron which is of significance in this respect being the saturation flux density. However, for a given relay, the power which must be dissipated in the magnetising coil to achieve this value of flux density will depend upon the saturation m.m.f. which is required. Consequently, permeability is important from consideration of power requirement, although not from consideration of size.

4. ANALYSIS OF THE SYSTEM

Since the major objective of this work is to design a relay to have a specified release delay, the starting point taken is the analysis of the system as proposed in Section 2 and depicted in Figure 5. It will then remain to establish the requirements to give the shortest possible operate time consistent with the required release delay.

To make the problem at all tractable it is necessary to commence with certain simplifying assumptions. These are that the downward portion of the magnetising characteristic of the iron to be used may be approximated by a straight line with an abrupt change from the saturated to the unsaturated state, and that the magnetic flux in the reed inserts is small compared to that in the iron core. It is also assumed that, in the steady state, the magnetisation of the iron is carried to a point beyond the knee of the saturation curve, as the iron will not be fully utilised at any lower value of steady state flux density.

The parameters used in this analysis are depicted in Figure 11, in which i_1, i_2=Primary and secondary m.m.f.



Definition of symbols.

Figure 11.

ϕ', ϕ=Core flux and initial core flux

S_1, S_2=Primary and secondary reluctances

m_1, m_2=Primary and secondary coercive m.m.f.

R_1, R_2=Primary and secondary resistance referred to one turn.

The equations of the system may be written

$$\begin{bmatrix} -(m_1+m_2)/s \\ \phi \\ \phi \end{bmatrix} = \begin{bmatrix} 1 & 1 & -(S_1+S_2) \\ R_1 & 0 & s \\ 0 & R_2 & s \end{bmatrix} \cdot \begin{bmatrix} i_1(s) \\ i_2(s) \\ \phi'(s) \end{bmatrix} \tag{13}$$

By superposition, the m.m.f. across the reed switch is given by

$$i(s)=\left[i_1(s)+m_1/s\right] S_2/(S_1+S_2)-\left[i_2(s)+m_2/s\right] S_1/(S_1+S_2) \tag{14}$$

Solution of equation (13) for i_1 and i_2 gives

$$i_1(s)=R_2\left[\phi(S_1+S_2)-(m_1+m_2)\right] / \left[(R_1+R_2)s+R_1R_2(S_1+S_2)\right] \tag{15}$$

$$i_2(s)=R_1\left[\phi(S_1+S_2)-(m_1+m_2)\right] / \left[(R_1+R_2)s+R_1R_2(S_1+S_2)\right] \tag{16}$$

Substitution of equations (15) and (16) for i_1 and i_2 in equation (14) will yield, after some rearrangement, the expression for the reed switch m.m.f. as shown in equation (17)

$$i(s)=(R_1S_1-R_2S_2)\left[(m_1+m_2)/(S_1+S_2)-\phi\right] / \left[(R_1+R_2)s+R_1R_2(S_1+S_2)\right]$$

$$+(m_1S_2-m_2S_1)/(S_1+S_2)s \qquad (17)$$

The inverse Laplace transform of equation (17) gives an expression for i in the time domain, in which the release time t_r and release m.m.f. i_r may replace i and t respectively, yielding

$$i_r=\frac{(R_1S_1-R_2S_2)\left[(m_1+m_2)/(S_1+S_2)-\phi\right]}{(R_1+R_2)}\exp\left[\frac{-t_rR_1R_2(S_1+S_2)}{(R_1+R_2)}\right]$$

$$+(m_1S_2-m_2S_1)/(S_1+S_2) \qquad (18)$$

The intractable nature of equation (18) renders progress towards the design of a relay extremely cumbersome unless an additional simplification is introduced. However, for the analysis of a given relay, equation (18) is readily soluble for t_r. The procedure employed is to neglect coercivity, which enables the design to be pursued in an approximate form, and then return to equation (18) to obtain a calculated value for t_r. On the basis of this calculated value, a design parameter may be modified to give a closer approximation to the desired release delay. Only a very few iterations will suffice to give the final design.

Using this procedure, neglecting the coercivity terms in equation (18) the following expression is obtained

$$i_r=\emptyset(R_2S_2-R_1S_1)/(R_1+R_2).\exp\left[-t_rR_1R_2(S_1+S_2)/(R_1+R_2)\right] \qquad (19)$$

It is helpful at this stage to introduce two composite parameters α and β which are defined by

$$\alpha = i_r/\emptyset S_1 \qquad (20)$$

$$\beta = (1+S_2/S_1)/(1+R_1/R_2) \qquad (21)$$

Equation (19) may now be written in the form

$$\alpha = (\beta-1).\ \exp(-\beta R_1S_1t_r) \qquad (22)$$

For a given primary core and coil and a given reed insert, the value of α will be fixed, assuming initial saturation, whilst β will depend on the values of secondary resistance and reluctance relative to the corresponding primary quantities, as seen from the definitions of α and β in equations (20) and (21). The value of β may be controlled by variation of the secondary air gap, which will not significantly alter the overall dimensions of the relay. Differentiation of equation (22) with respect to β shows that a maximum value of release delay will occur when β has a value given by

$$\alpha = (\beta-1)\ \exp\left[-\beta/(\beta-1)\right] \qquad (23)$$

For maximum utilisation of the materials of a given relay the optimum value of β as given by equation (23), will yield, when inserted in equation (22), a release delay of

$$t_r = 1/R_1 S_1 (\beta - 1) \tag{24}$$

The release delay given by equation (24) is the maximum which can be obtained by a given primary system, assuming that β has its optimum value. As already indicated, the value of β, defined in equation (21), may be controlled by adjusting the value of S_2, irrespective of the values of S_1, R_1 and R_2. This has a twofold significance. Firstly, if S_2 includes an air gap its value has no significant effect on the amount of material used or on the space occupied by the relay. Secondly, the maximum value of t_r is constant for any value of resistance for the secondary slug, the dimensions of which may be freely established, independently of the release delay, by other considerations such as the speed of operation.

Having indicated the maximum release delay for a given core, the converse of this analysis may be applied to the design problem to achieve the minimum size of relay for a desired release delay.

Recalling the fact that, whereas β is a function of the relative values of primary and secondary parameters, α will depend upon the primary dimensions only, we should at this stage substitute for $(\beta - 1)$ in equation (24) a function of α from equation (23). However, the difficulty of performing an analytical solution of equation (23) for β in terms of α suggests an empirical reiteration of equation (23).

The graph of Figure 12, which may be readily drawn from equation (23), may be closely approximated by a straight line provided the value of α is not less than about 0.25. The validity of this assumption can readily be checked after the design is completed.

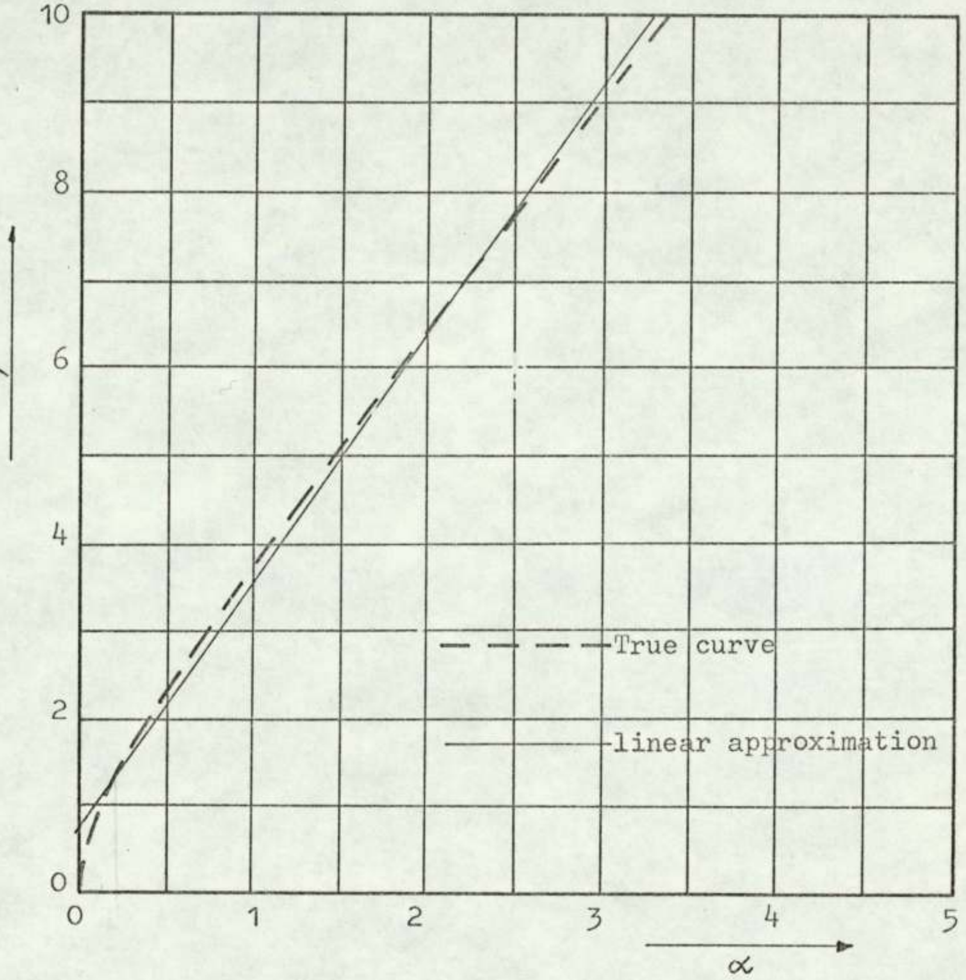
This linear approximation yields as an empirical form of equation (23)

$$(\beta-1)=0.75+2.8\alpha \tag{25}$$

so that equation (24) becomes

$$\begin{aligned} t_r &= 1/R_1S_1(0.75+2.8\alpha) \\ &= 1/R_1S_1(0.75+2.8i_r/\phi S_1) \end{aligned} \tag{26}$$

Equation (26) now expresses a relationship between the maximum release delay, the release m.m.f. and the parameters of the primary section of the relay.



Graphical representation of equation (23)

Fig. 12.

5. DESIGN TECHNIQUE

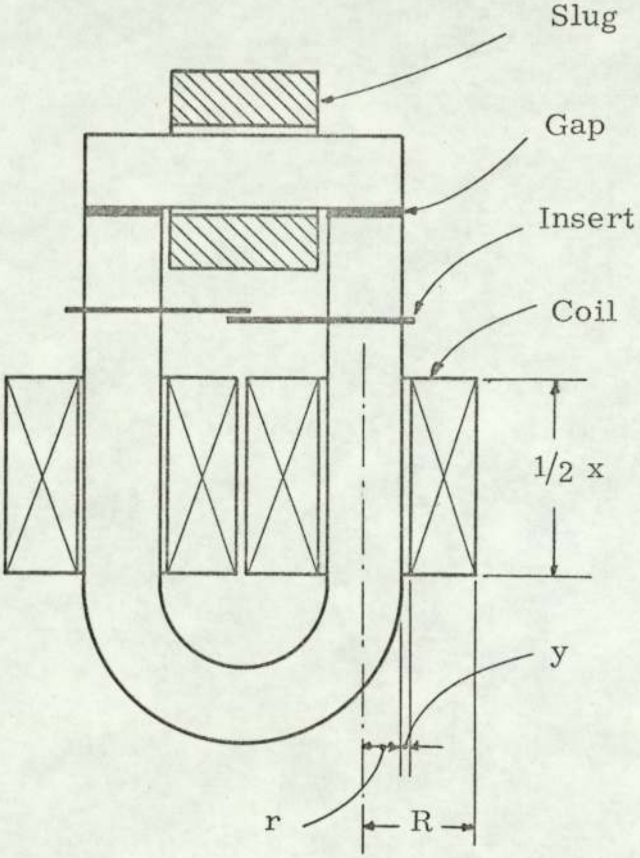
5.1 The Primary System

For design purposes, primary dimensions may be introduced into equation (26), provided always that β is adjusted to its optimum value, as indicated in equation (25), to give maximum release delay.

Let a_c and l_c represent the effective area and length of the winding and a_i and l_i represent area and length of the primary part of the iron core; then equation (26) may be rewritten, after some slight re-arrangement, as

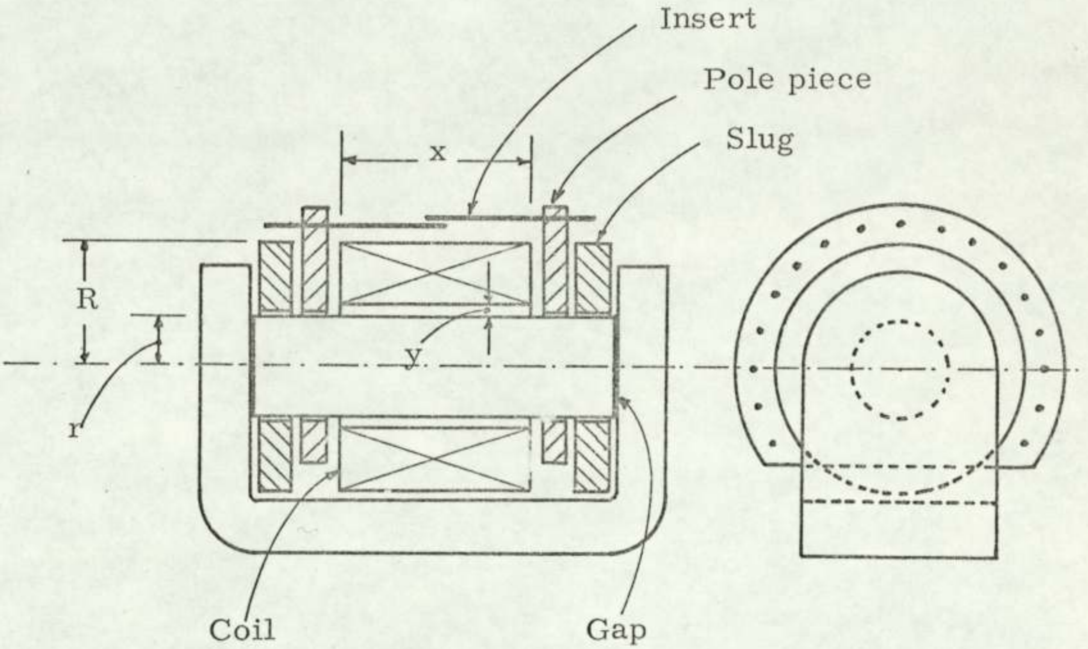
$$t_r \rho l_c (0.75 H_s l_i + 2.8 i_r) = B_x a_c a_i \tag{27}$$

Equation (27) is the basic expression which must be satisfied by the primary part of the system and from which a relay, generally of the form shown in Figure 5, may be designed. However, it cannot be further simplified until a_c, l_c, a_i and l_i are related to the actual linear dimensions of the relay and these will depend upon the practical configuration to be used. Two possible forms which have been constructed experimentally are shown diagrammatically in Figures 13 and 14, from which it can



Arrangement of Twin Coil design.

Fig. 13.



Arrangement of single coil design.

Fig. 14.

be seen that

$$a_c = x\left[R-(r+y)\right] \tag{28}$$

$$a_i = \pi r^2 \tag{29}$$

$$l_c = \pi\left[R+(r+y)\right] \tag{30}$$

$$l_i = x+K. \tag{31}$$

where K is the unwound length of the primary core.

Substitution of equations (28) to (31) in equation (27) yields, after rearrangement of the terms,

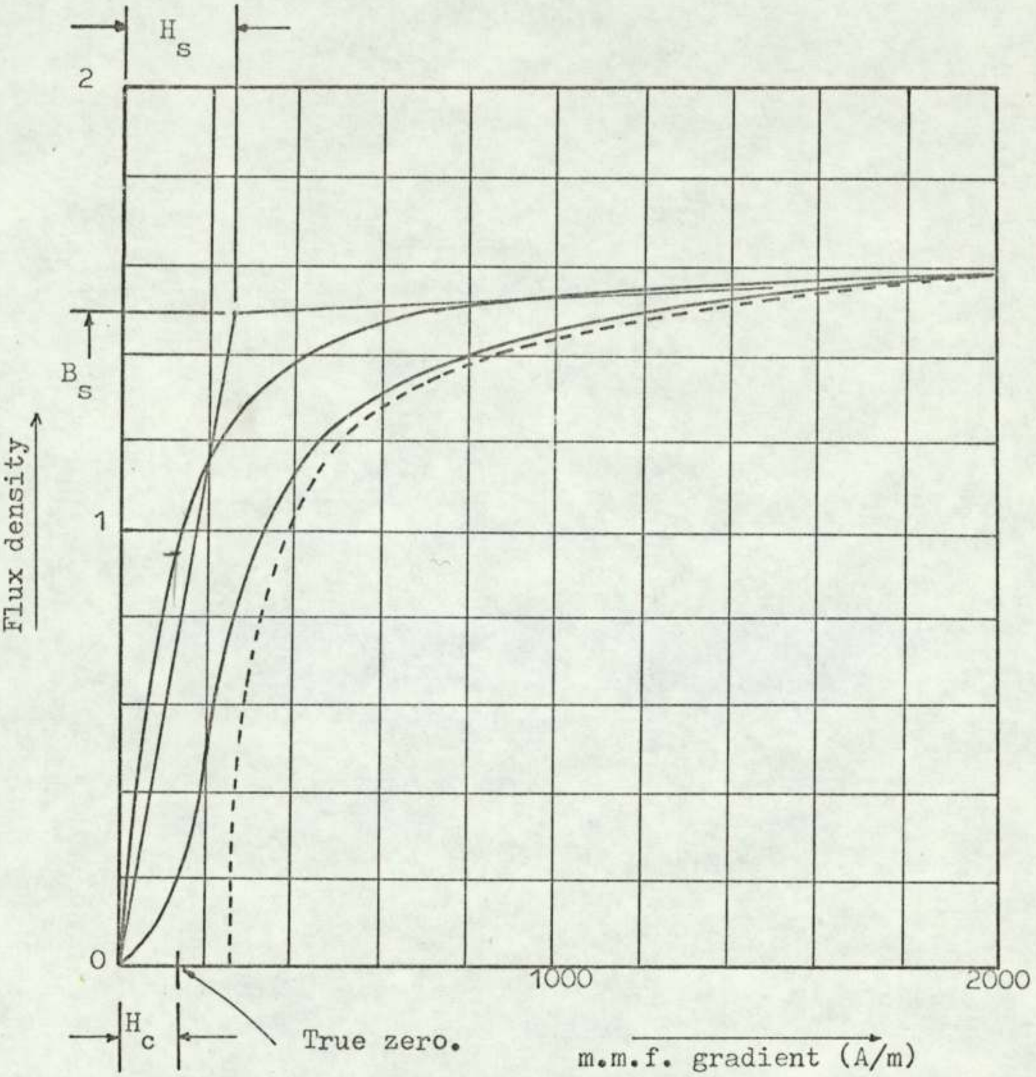
$$x\left[B_s r^2(R-r-y)/(R+r+y)t_r\rho - 0.75H_s\right] = 2.8i_r + 0.75KH_s \tag{32}$$

Before proceeding to insert numerical values into equation (32), it is worth giving some consideration to an approximate allowance for the effects of residual magnetism in the core, thus partly compensating for the approximation of equation (19). On the assumption that the secondary air gap is sufficiently large to reduce the flux density in the core to a relatively low value when de-energised, then a large proportion of the coercive m.m.f. of the primary iron will appear across the reed inserts. It merits mention in passing that, if the de-energised flux density is not small compared to the saturation value, the iron will be very inefficiently used by limitation of the total available flux swing. As an approximation then, the effect of residual may be partially taken into account by displacing the iron characteristic as shown in Figure 15 and superimposing the coercive m.m.f. which must be subtracted from the release m.m.f. of the reed insert. This is put into effect by replacing i_r by $i_r - H_c(x+K)$, thus converting equation (32) to

$$x \left[B_s r^2 (R-r-y)/(R+r+y) t_r \rho - 0.75 H_s + 2.8 H_c \right] = 2.8 i_r + 0.75 K H_s - 2.8 K H_c \qquad (33)$$

It will be observed from equation (33) that there are three major dimensions x, r, and R which, although related by this equation, are individually indeterminate. The procedure which is adopted is to make an arbitrary choice of one of the three, together with the unwound core length K, based upon purely practical considerations such as the size or number of reed inserts to be used. By examination of a graphical relationship between the other two, an optimum, or (if this is not practicable) a compromise, choice of values may be made. This is pursued in the numerical examples of section 6.

By the above means, a set of approximate dimensions



Magnetisation of iron sample.

Fig.15.

for the primary system may be obtained, based upon the simplified treatment of coercivity, and assuming that the secondary system will subsequently be designed to give optimum conditions as described by equation (23).

Returning now to equation (18), and using the approximate primary dimensions, the value of t_r may be calculated for a range of values of R_2 and S_2 giving a family of curves having the form shown in Figure 16. The peak values of t_r will be observed to be not quite independent of secondary parameters as suggested on page 36 since that suggestion was based upon the assumption of negligible coercivity. Choice of R_2 may be made by consideration of operating conditions which are discussed below and the corresponding maximum t_r may be found. If this deviates significantly from the desired value an adjustment may be made to one dimension, say coil length, and the evaluation of equation (18) repeated to give an improved value for t_r. This may be continued as an iterative process until a sufficiently close approach to the desired release time is obtained.

5.2 The Secondary System

In order to glean further information regarding the secondary system, consideration must be given to conditions during the closing of the relay. It will be recalled that, ignoring the effects of leakage flux, a step of m.m.f. will appear across the reed insert at the instant of applying excitation to the primary winding. The magnitude of this step will be given by

$$i_o=V/(R_1+R_2)$$

$$=(V/R_1)/(1+R_2/R_1) \tag{34}$$

Now, V/R_1 will be the steady-state m.m.f. of the primary winding which will have a value not greater than

$$V/R_1=H_m l_i(1+S_2/S_1) \tag{35}$$

H_m being the maximum steady-state m.m.f. gradient in the iron. Combining equations (21), (34) and (35) the ratio of primary to secondary resistance is obtained.

$$R_1/R_2=i_o/H_m l_i \beta \tag{36}$$

Having established the value of R_1, l_i and β from the primary design, an approximate value for R_2 from equation (36) enables the dimensions of the secondary slug to be calculated.

Returning to equation (21) and rearranging

$$S_2/S_1=\beta(1+R_1/R_2)-1 \tag{37}$$

which permits calculation of the secondary reluctance.

From the approximate slug dimensions, as calculated above, the length of the secondary iron path may be obtained. Then a gap is inserted to make up the total reluctance as given by equation (37). It is now possible to calculate more accurately the value of maximum m.m.f. which has previously been used in equation (35); that value having necessarily been an approximation, since S_2 included an air gap which does not saturate. This value of m.m.f. may be used to recalculate, and adjust if necessary, the value of R_2. If the consequent change in slug dimensions results in a significant change in secondary iron length, this process may be repeated, until a sufficiently close approximation is achieved. It is worth noting that the secondary resistance is not a critical value, since the only requirement is that a sufficiently large step of initial m.m.f. should appear across the reed switch to ensure rapid closure.

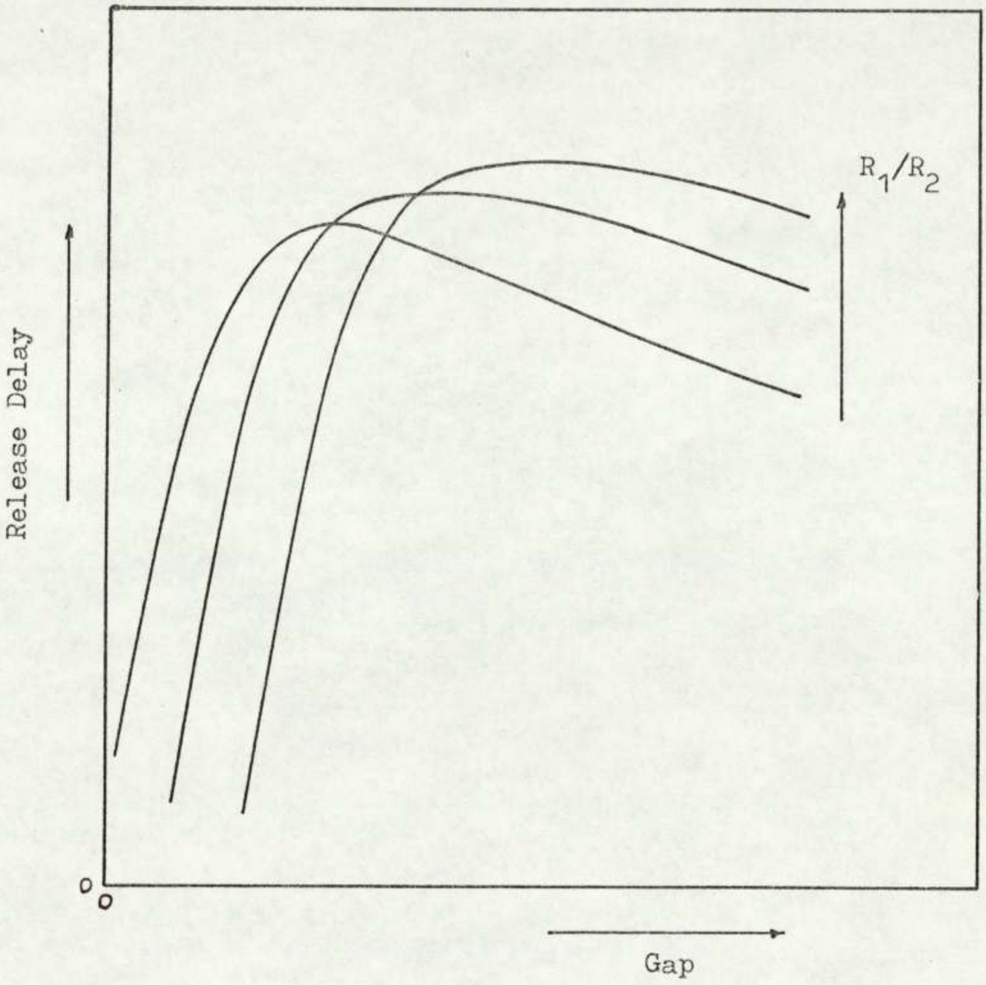
5.3 General Procedure

The methods described in Sections 5.1 and 5.2 necessarily included some approximations to avoid excessive arithmetical operations. However, a more direct technique may be used if a digital computer is used to handle the arithmetic as demonstrated in Appendix I. If, from equation (33), an approximate relationship between core radius and coil length is obtained, the value of core radius may be determined from practical considerations, leaving coil length to be confirmed. A series of curves of the form shown in Figure 16 may be calculated for a range of coil lengths and their peak values used to draw a graph of the form shown in Figure 17. In addition, equation (34) may be used to calculate the initial switch m.m.f. from a chosen value of maximum steady-state m.m.f. gradient in the iron, giving a set of curves as shown in Figure 18.

For specified values of release time and initial operating m.m.f., two curves of slug conductance against coil length may be drawn as in Figure 19. The point of intersection of these curves will be the only point at which the required operating and release conditions are both satisfied.

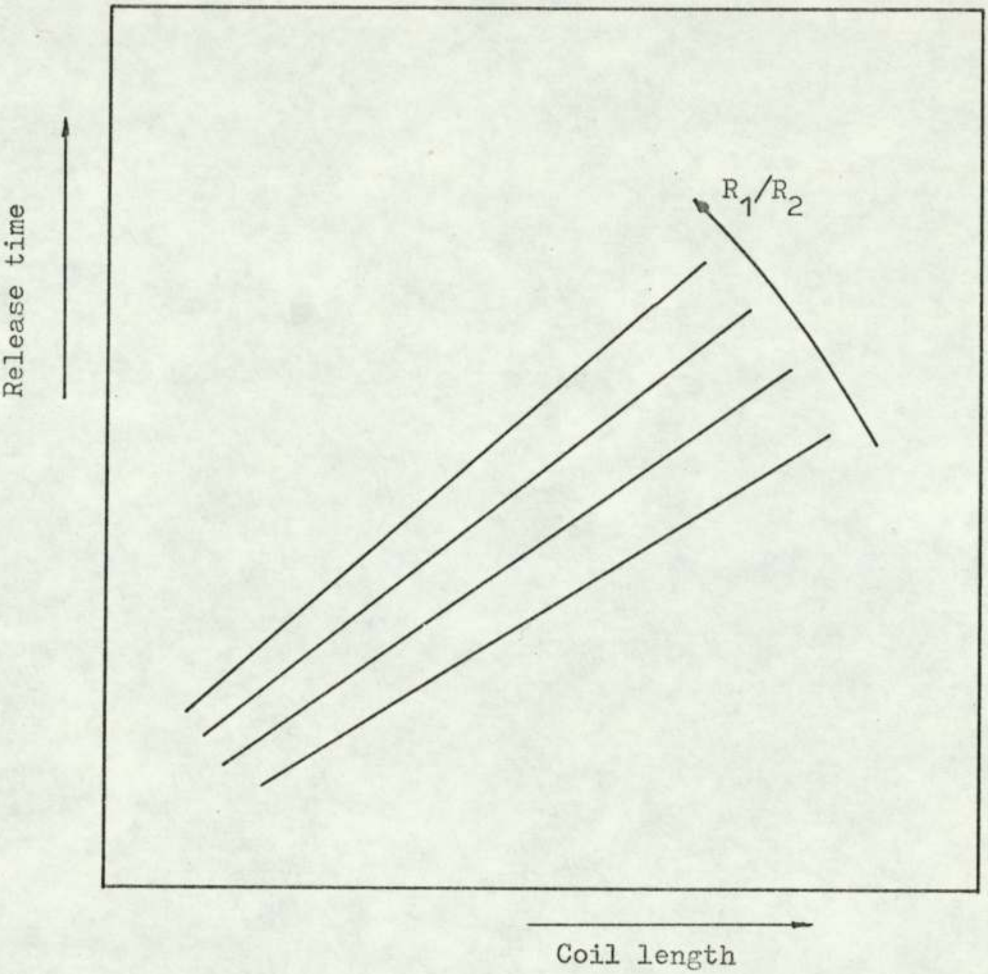
All major dimensions having been established, the remaining details, such as the number of turns of the primary winding, the total m.m.f. and the excitation power involve only simple routine calculations. The primary resistance will be given by

$$R_1=\rho\pi(R+r+y)/(R-r-y) \qquad (38)$$



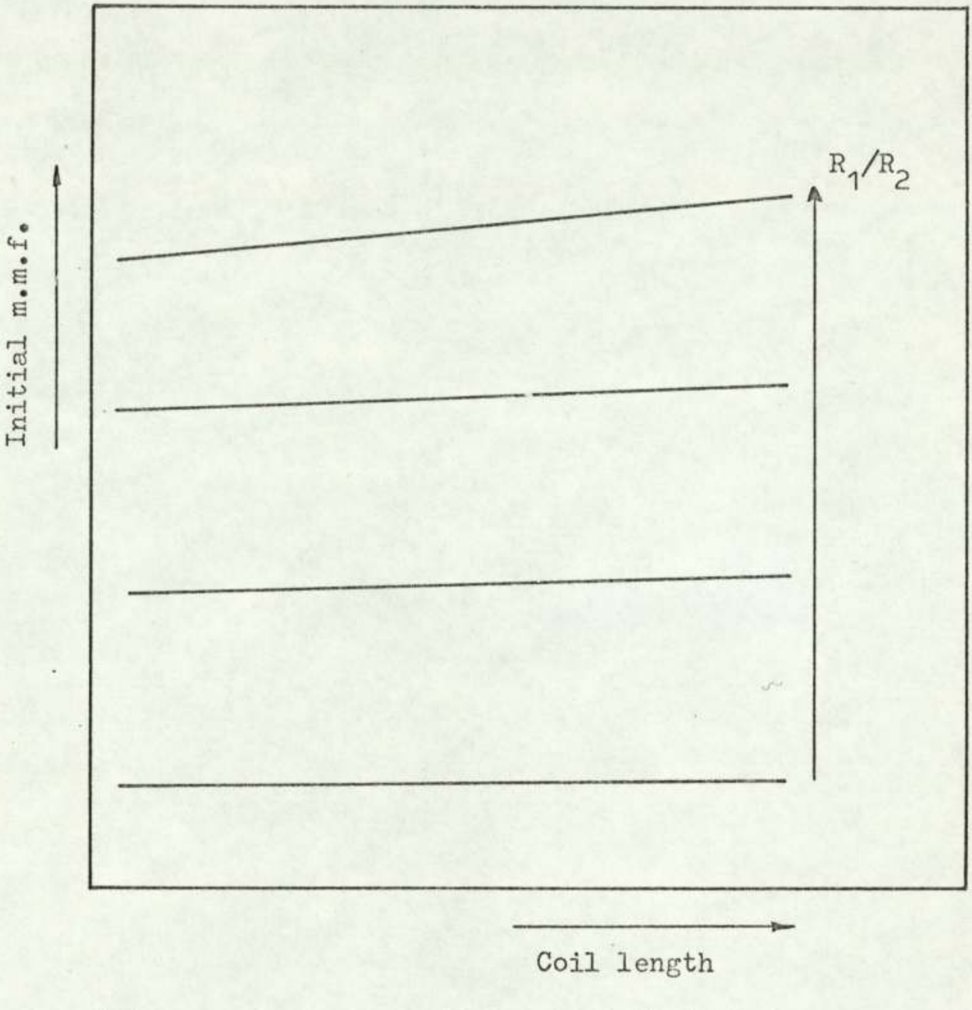
Release delay vs. gap. Typical form.

Fig. 16.



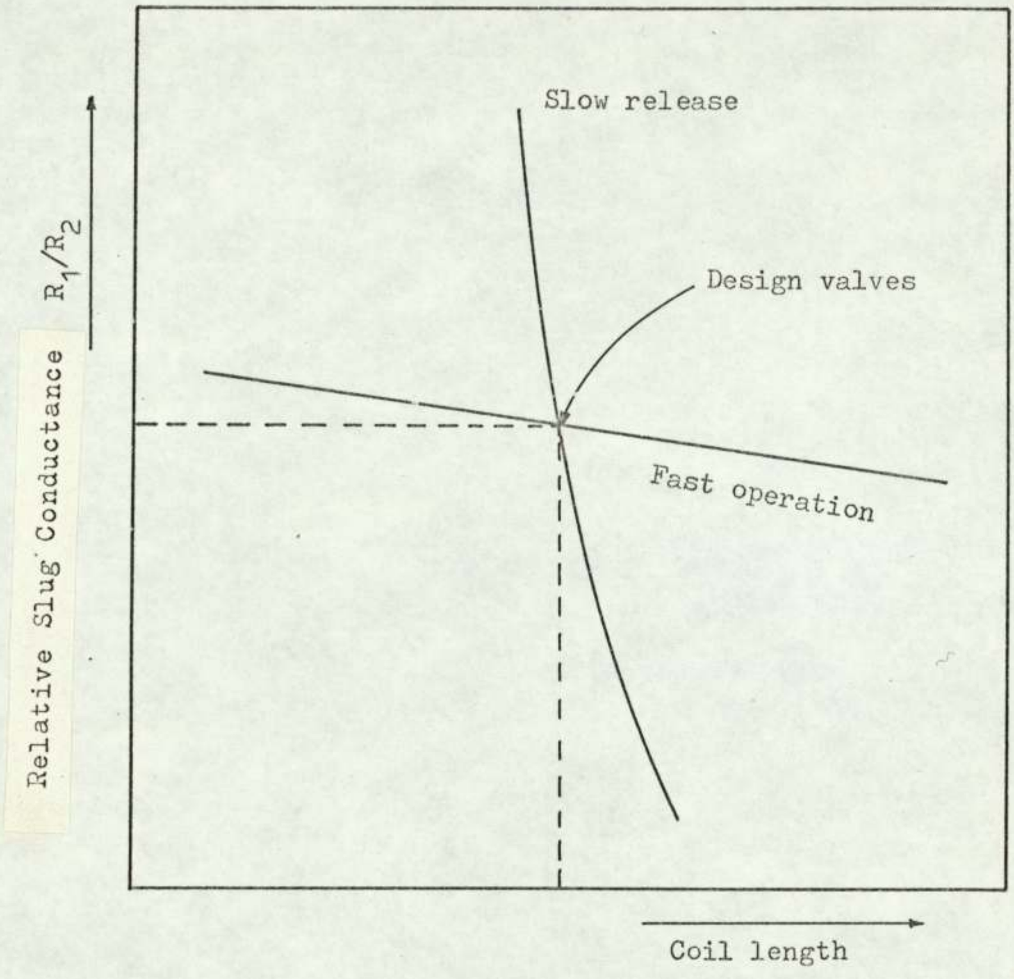
Release time vs. Coil length. Typical form.

Figure 17.



Initial m.m.f. vs. coil length. Typical form.

Fig.18.



Design Curves. Typical form.

Fig. 19.

and the resistance of the coil will be N^2R_1, and hence the coil m.m.f. will be V/NR_1. The m.m.f. will also be given by

$$(H_m \text{x} \text{total iron length})+(B_s/\mu_o \text{x} \text{gap length})$$

so that

$$V/NR_1=(H_m \text{x} \text{total iron length})+(B_s/\mu_o \text{x} \text{gap length}) \qquad (39)$$

From equation (39) the number of turns required for the primary winding may be established for any given supply voltage, and the excitation power is given by $(\text{m.m.f.})^2R_1$

5.4 SYNCHRONISING CIRCUIT

If it is assumed that the m.m.f. across the insert is to be reduced to zero by the insertion of resistance in the primary circuit as described in Section 2, the primary and secondary m.m.f. must be proportional to their respective reluctances.

$$i_1S_2=i_2S_1 \tag{40}$$

Since both windings will link the core flux, the induced e.m.f.s will be equal, giving

$$i_1(R_1+R')=i_2R_2 \tag{41}$$

where R' is the added external resistor.

Combining equations (21), (40) and (41), we get

$$\frac{R_1+R'}{R_2}=\beta(R_1/R_2+1)-1 \tag{42}$$

$$\therefore R'=\beta R_2(R_1/R_2+1)-R_2-R_1$$

$$=(R_1+R_2)(\beta-1) \tag{43}$$

Values for R_1, R_2 and β having been established, equation (43) gives a value for the resistor to be inserted in circuit by the synchronising contact.

At this stage we may predict the voltage appearing across the synchronising contact. Before opening of this contact, the total core magnetising m.m.f. will be i_1+i_2. These currents may be deduced from equation (14) together with the fact that the common flux gives rise to

$$i_1R_1=i_2R_2 \tag{44}$$

Rearranging equation (14) and neglecting terms involving coercivity gives

$$i_1S_2/S_1-i_2=i_r(S_2/S_1+1) \tag{45}$$

From solution of equations (44) and (45), the total magnetising m.m.f. is given by

$$i_1+i_2= i_r\,\frac{(S_2/S_1+1)(R_1/R_2+1)}{S_2/S_1-R_1/R_2} \tag{46}$$

and this m.m.f. will be unaltered after opening of the synchronising contact, since the core flux cannot change instantaneously. Continuing the assumption that the m.m.f. across the reed inserts is to be reduced to zero we get, by substitution of equation (40) into equation (46)

$$i_1=i_r\,\frac{R_1/R_2+1}{S_2/S_1-R_1/R_2} \tag{47}$$

and this current flowing in the resistor R' will give the voltage across the switch. By combination with equation (21), equation (47) may be rewritten

$$i_1=i_r/(\beta -1) \tag{48}$$

and hence, from the value of R' already established in equation (43), the voltage across the synchronising contact becomes

$$v=i_r(R_1+R_2) \tag{49}$$

in which i_r is the release m.m.f. of the synchronising contact.

It may be more convenient to express v in terms of the supply voltage which will be i_sR_1, where i_s is the steady-state primary m.m.f., so that equation (49) becomes

$$v/V=(i_r/i_s)(1+R_2/R_1) \tag{50}$$

The derivation of equation (50) takes no account of the effects of leakage flux linking the primary winding only, and, taking the most pessimistic view, this can be accounted for by assuming that, initially, the whole of the primary current is diverted through the synchronising resistor. From equations (44) and (45) we get, before opening

$$i_1=i_r\frac{S_2/S_1+1}{S_2/S_1-R_1/R_2} \tag{51}$$

Comparison with equation (47) indicates that i_1 before opening is β times i_1 after opening so that equation (50), which gives the voltage across the synchronising contact, would be replaced by equation (52) as follows

$$v/V=\beta(i_r/I_s)(1+R_2/R_1) \tag{52}$$

The actual value of synchronising contact voltage will fall between the values given by equations (50) and (52), depending upon the coefficient of coupling between the windings. An exact value for this voltage is not required, since it is only necessary to establish that no destructive surges can appear across the synchronising contact.

6. DESIGN OF EXPERIMENTAL MODELS

This section deals with two numerical examples arranged as shown in Figures 13 and 14. These models, although different in layout, are identical in principle, so that the foregoing theories are applicable.

The reed inserts used in this design are to a Post Office Specification as follows:-

Operate m.m.f.	30 to 58A
Release m.m.f.	15 to 27A
Envelope diameter	0.145"
Envelope length	1.125"
Lead wire diameter	0.0224"
Lead wire length	0.375"

For the core, soft iron is used, as produced by Lowmoor Alloy Steelworks Ltd. under the trade name "Super Hiperm". The analysis is .05%C; .05% mn; .05% Si. The magnetic characteristics shown in Figure 15 were obtained from measurements on a ring sample.

6.1 TWIN-COIL MODEL OF FIGURE 13.

As an example, we may consider the design of a relay arranged as in Figure 13. Basing this design on a reed insert 1.125" long, an arbitrary choice of coil radius R of 15.9 mm. (.625") is used as a starting point. This choice of R, together with a decision to construct a 10-

contact relay, dictates the value of K to be approximately 60mm, and the value of y is taken as 1.6mm. (.0625"). The coil resistivity is taken as $3.6 \times 10^{-8} \Omega$-m and, from Figure 15, the iron characteristic is assumed to be represented by

$B_s = 1.5\text{Wb/m}^2$

$H_s = 250\text{A/m}$

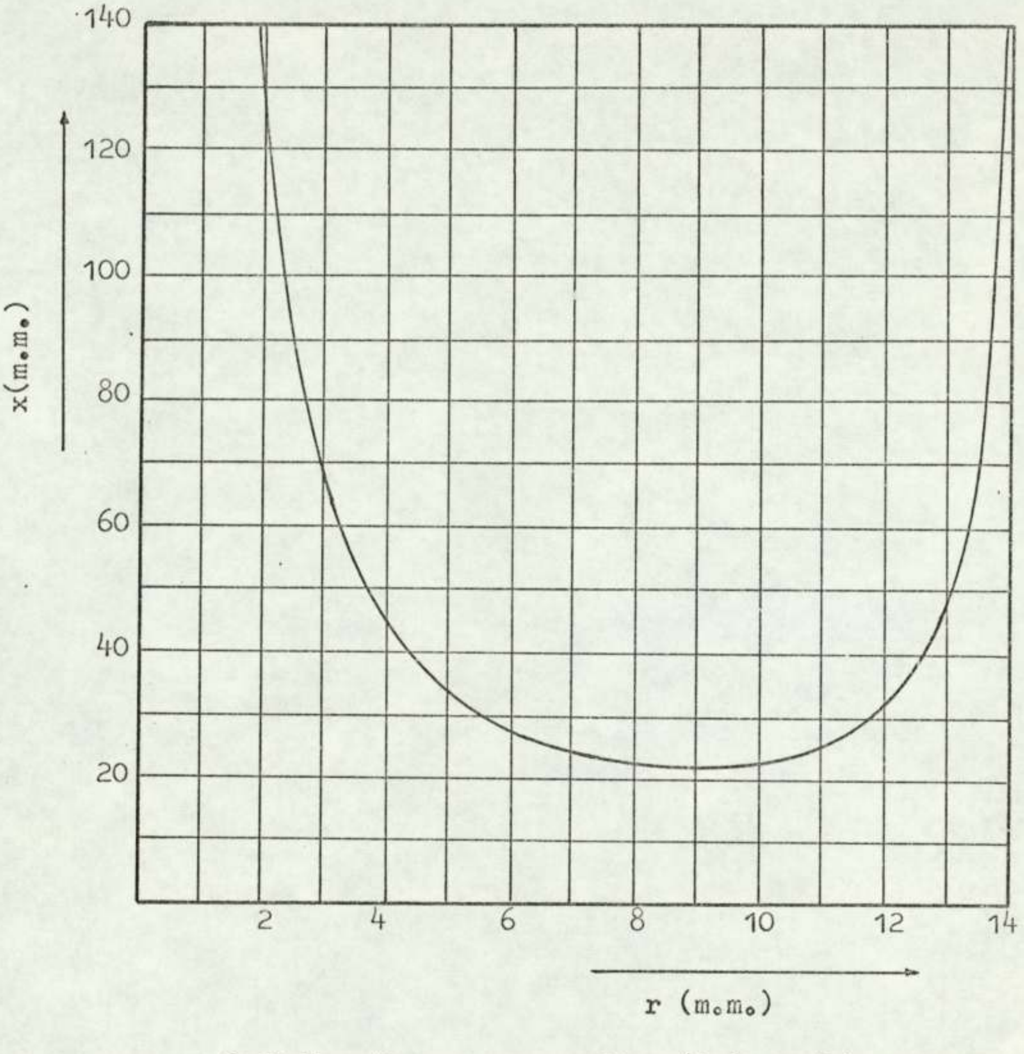
$H_c = 80\text{A/m}$

A more realistic true value of H_c would be 120 A/m but, for the purpose of making an approximate allowance for coercivity as used in equation (33), a lower value is taken, since not all of the coercive m.m.f. will appear across the reed switch. To design a relay for 210ms. release time, with a reed switch having a release m.m.f. of 26A, equation (33) may be expressed in the form

$$x\left[0.198r^2(14.3-r)/(17.5+r)+0.037\right] = 70.6 \qquad (53)$$

where x and r are expressed in millimetres.

The relationship between x and r in the above equation is shown graphically in Figure 20, from which it can be seen that a minimum axial coil length is achieved when r=9mm. approximately. This corresponds



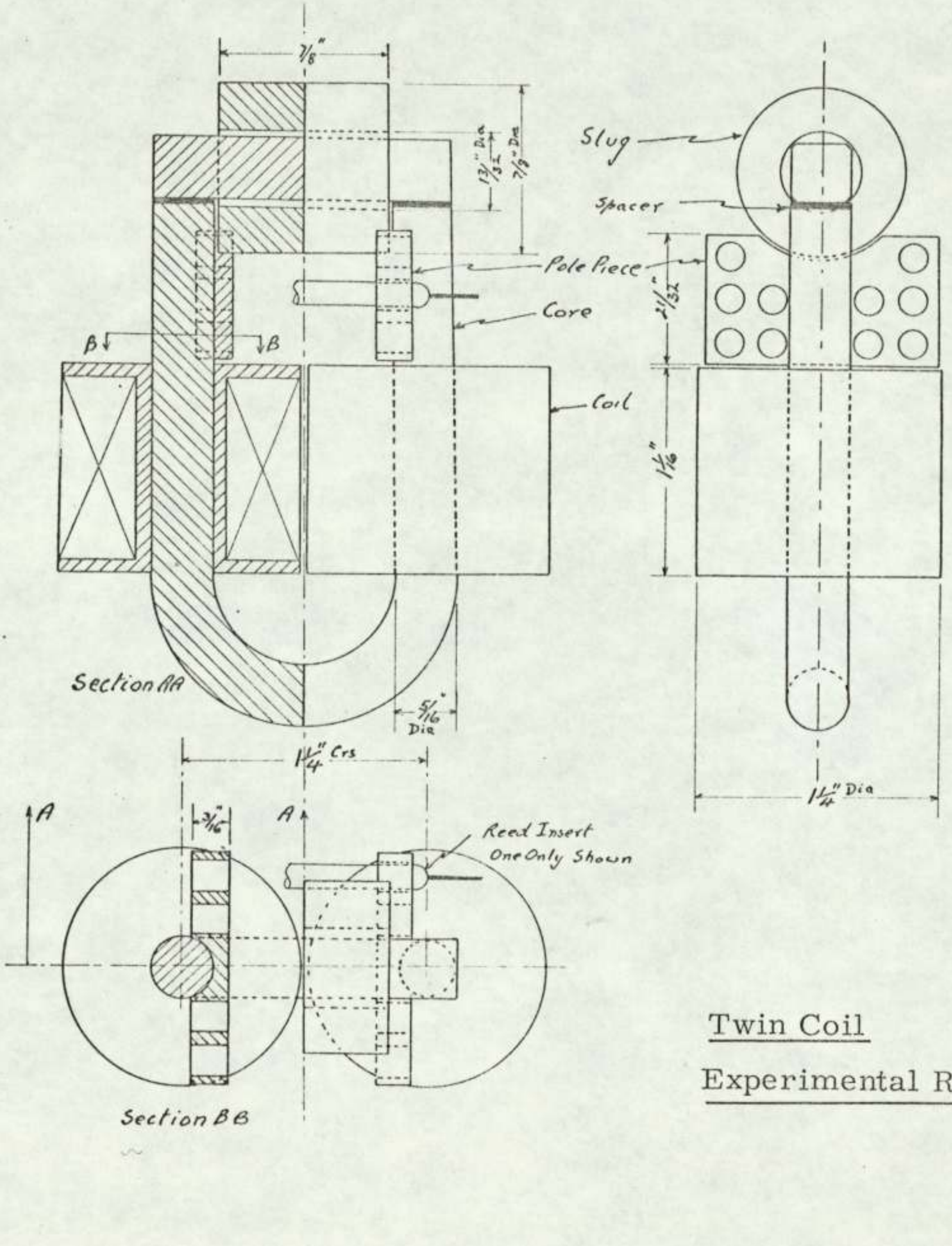
Coil length vs. core radius (twin coil).

Fig. 20.

to a core diameter of about 11/16", which gives an unreasonably large ratio of iron to copper. The disproportionately massive core would result in some difficulty in construction and in the arrangement of the pole pieces to carry the reed switches. Allowing some deviation from the minimum length to achieve reasonable proportions, a first approximation to the design is obtained by taking r=4mm. and x=45.5mm.

This gives a relay generally as shown in Figure 21, but subject to the proviso that the coil length x may be modified to make a correction for the approximations used. Pursuing the procedure outlined in Section 5.3, a return to equations (18) and (34) permits calculation of the curves of Figures 22 and 23 respectively. The details of these calculations appear in Appendix I. From Figures 22 and 23, Figure 24 may be deduced for a release time of 210ms and initial m.m.f. of 100A, their point of intersection indicating that both conditions are satisfied by a coil length of 48.3mm. and relative slug conductance of 0.55. The corresponding value of secondary gap will be seen from Table A of Appendix I to be 0.12mm.

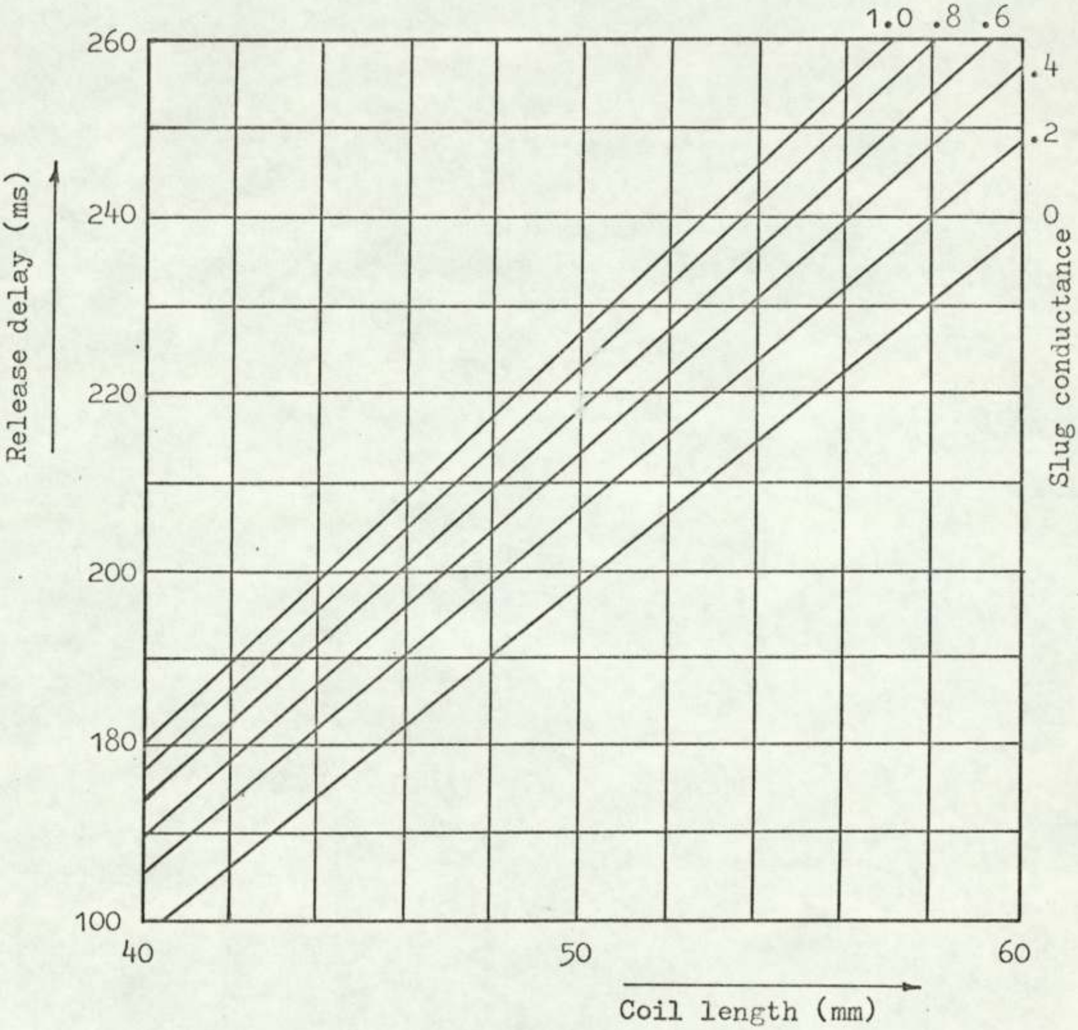
The routine calculation of m.m.f. and power having been included in Appendix I, interpolation in Table A, gives values of 273A and 388mW. respectively, from which



Twin Coil

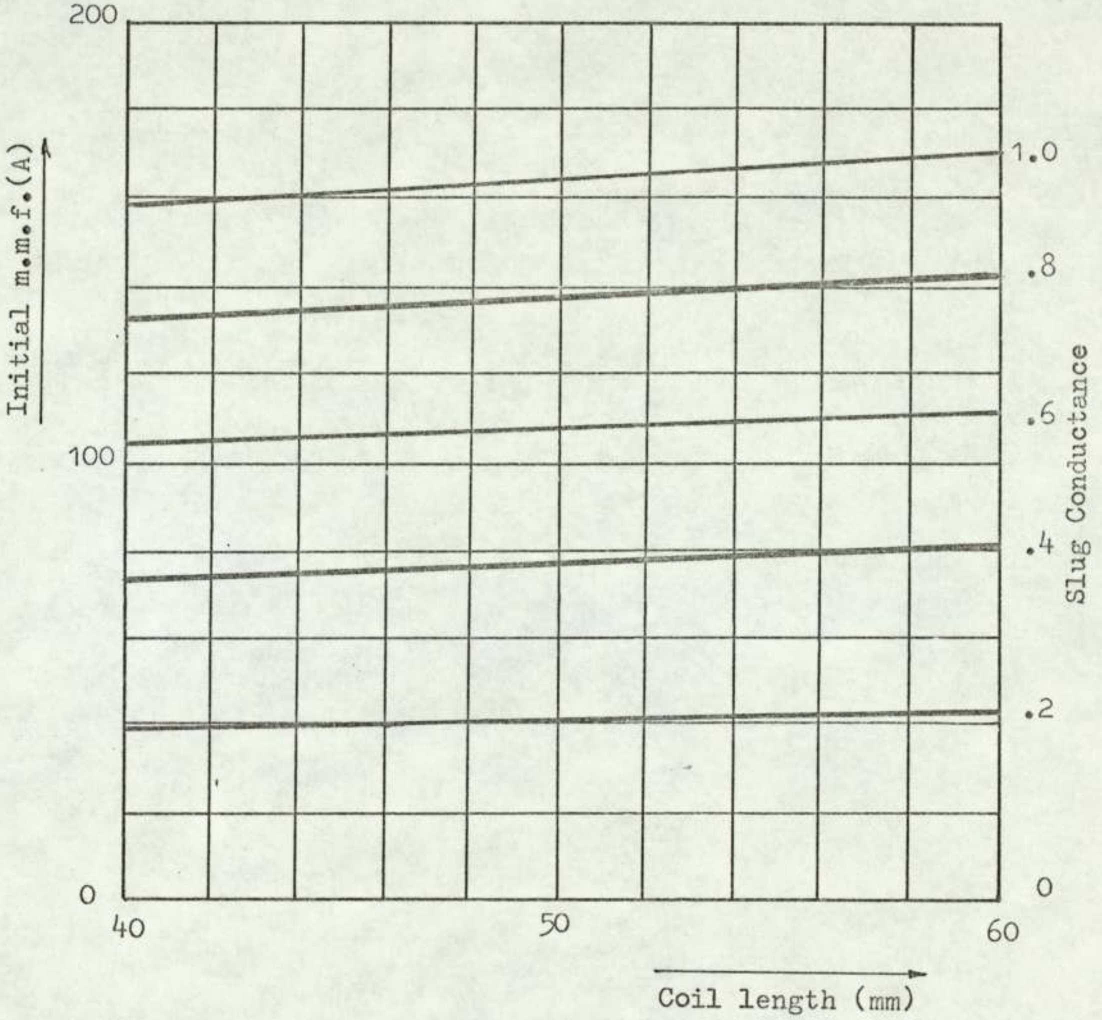
Experimental Relay.

Fig.21.



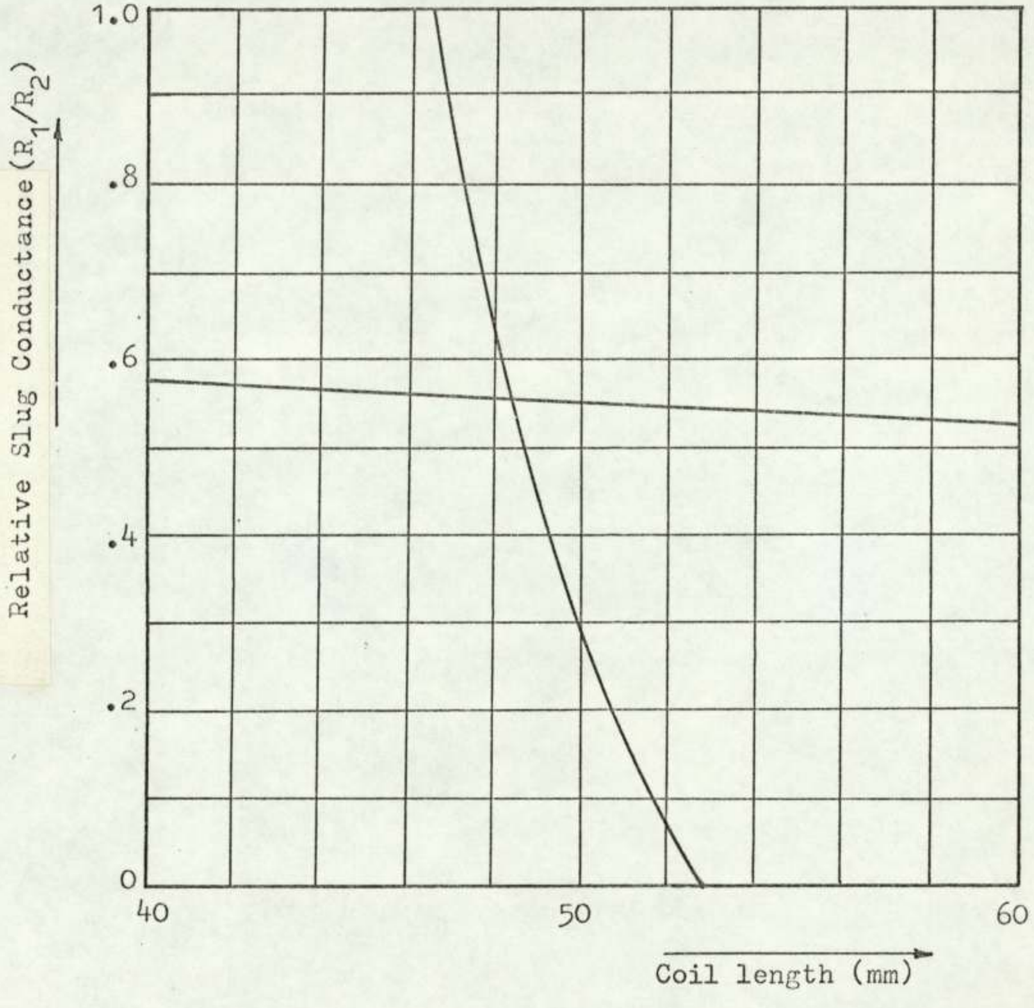
Release delay vs. coil length. Twin coil.

Fig.22



Initial m.m.f. vs. Coil length. Twin Coil.

Fig.23.



Slug conductance vs. Coil length. Twin Coil.

Fig.24.

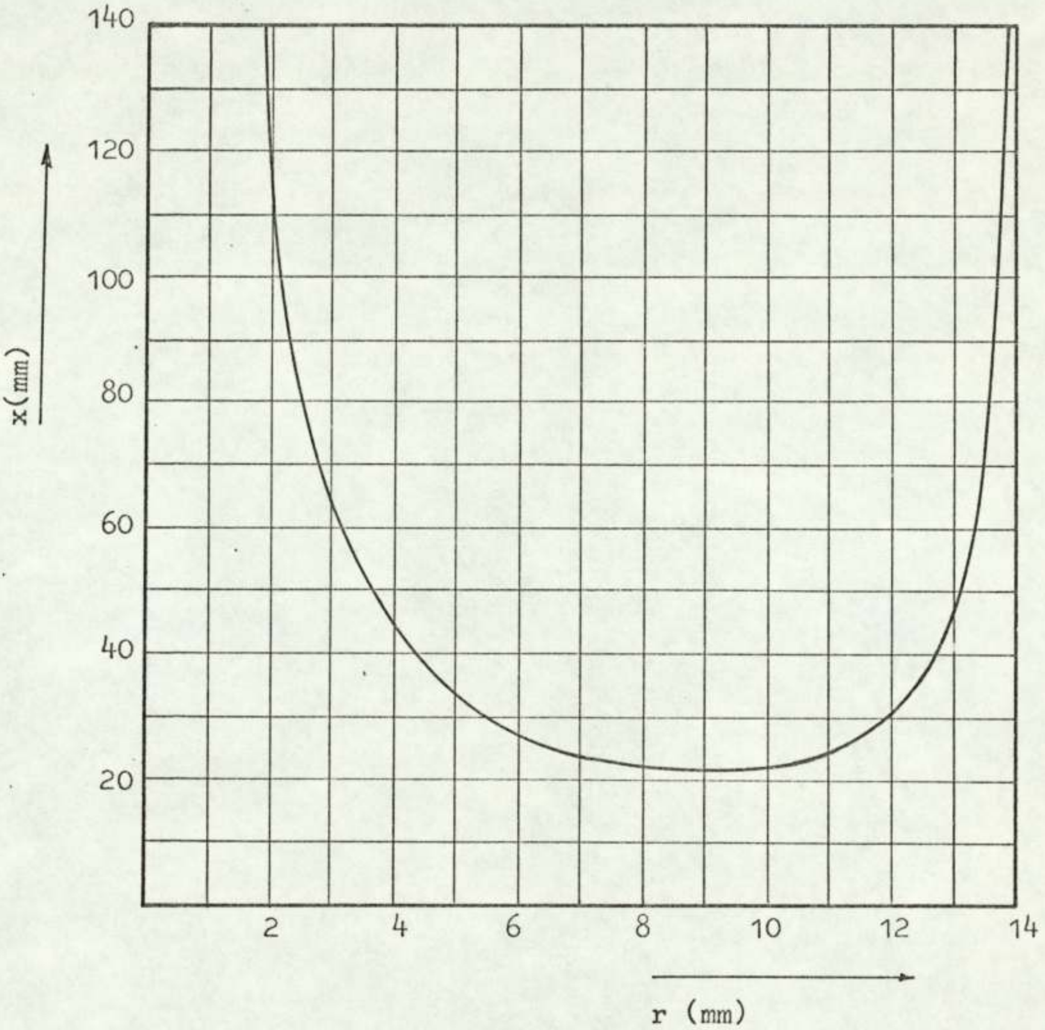
the appropriate number of turns for any given voltage may be calculated.

6.2 SINGLE-COIL MODEL OF FIGURE 14.

An alternative layout is depicted in Figure 14, for which the design may be pursued in exactly the same manner as for the twin-coil version. In this case, the constant K in equation (33) involves only the end cheeks of the coil former and an allowance for the thickness of the pole pieces. Assuming coil cheeks of 3/32" and pole pieces 1/8" thick, K becomes 8mm.(5/16"). The choice of R may be made rather arbitrarily, but it is primarily dependent upon the number of reed inserts to be used and their diameter. A value of 15.9mm. (5/8") permits the installation of at least 15, and possibly 17, inserts. For a release time of 200ms, insertion of these values into equation (33) gives

$$x\left[0.208r^{2}(14.3-r)/(17.5+r)+.037\right]=72.5 \qquad (54)$$

Following the procedure of the previous sub-section, the values of x and r in equation (54) are related by the curve of Figure 25. As before, the minimum length involves rather a bulky iron system, but a nearer approach to the optimum is possible using a core radius of 7.15mm. (9/16" diameter) and an approximate coil length of 24 mm.



Coil length vs. core radius (single coil).

Fig.25.

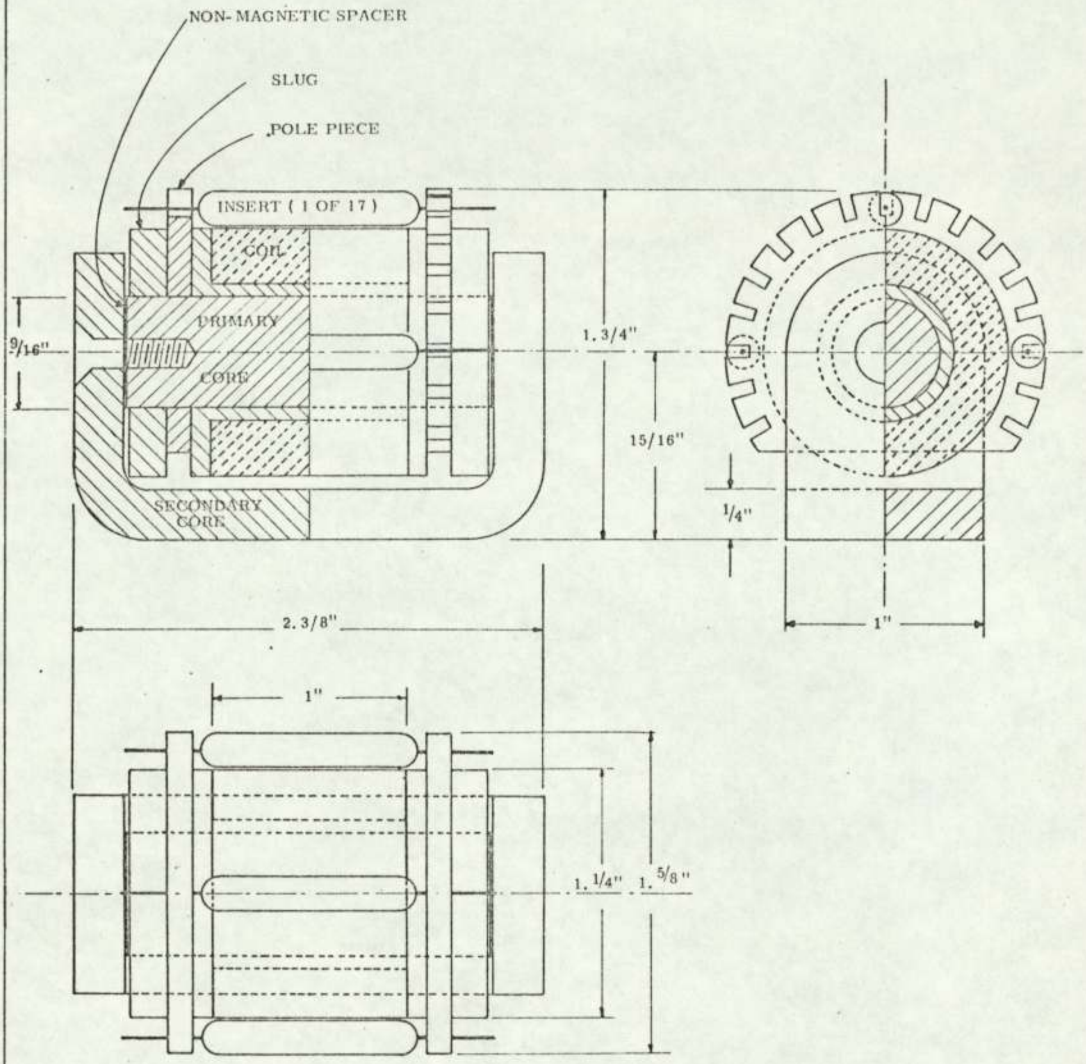
This small deviation from the minimum length also has the advantage that it simplifies the design of the pole pieces.

The design of a relay as shown in Figure 26 is continued as in the previous example, equations (18) and (34) being used to calculate the curves of Figures 27 and 28 from which Figure 29 may be derived. The design conditions in this case are satisfied by a coil length of 25mm and a relative slug conductance of 0.72 with a secondary gap of 0.11mm. See Table B of Appendix I for details. The values of m.m.f. and power are respectively 234A and 857mW.

6.3 COMPARISON OF TWIN-COIL AND SINGLE-COIL ARRANGEMENTS

Apart from the prerequisite of satisfactory performance, the criteria by which a relay may be judged fall into several categories which are not necessarily compatible. Some of these are listed below with particular reference to the twin-coil and single-coil configurations described in Sections 6.1 and 6.2.

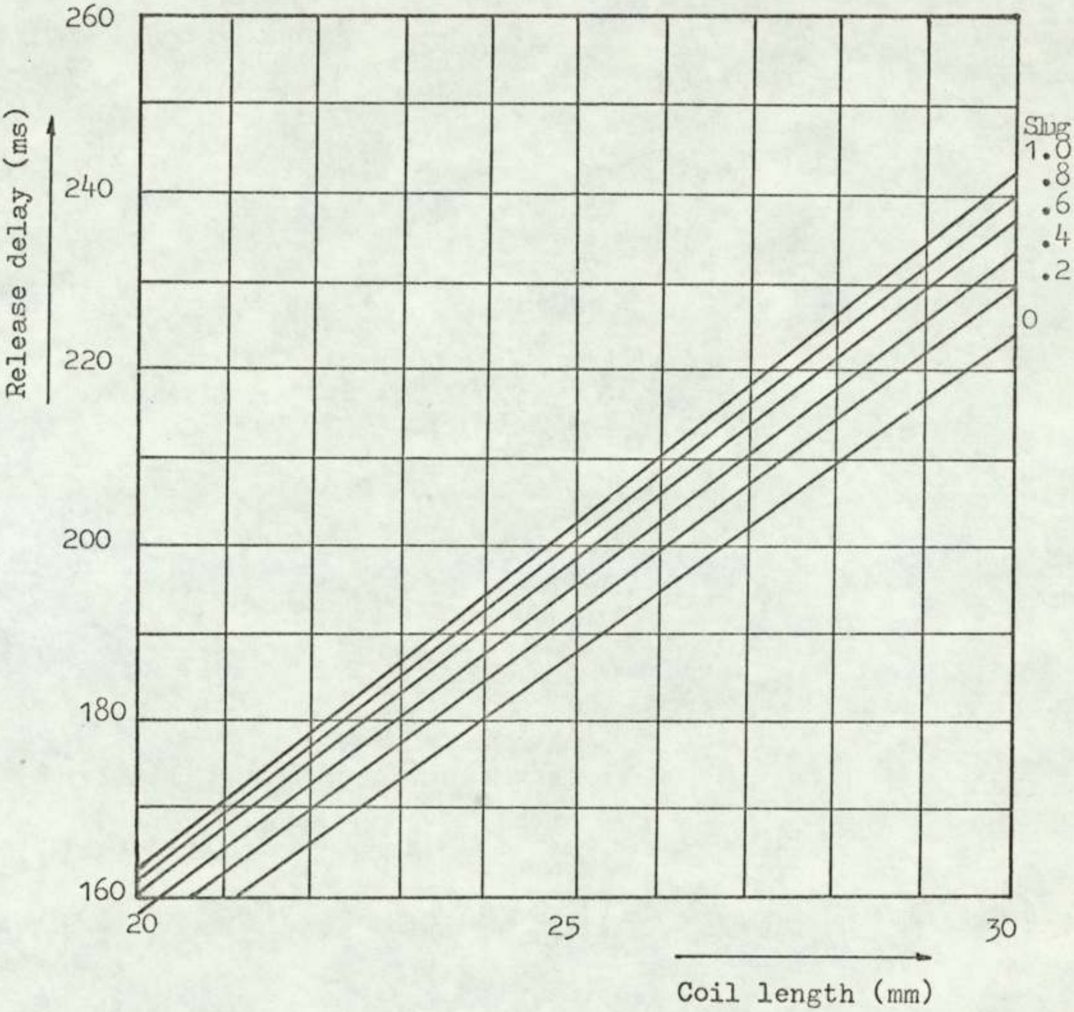
a) Volume. This has been the main criterion used in the design and an optimum value of iron radius is seen to exist, as predicted in the preliminary considerations (Figures 20 and 25). However, in the case of the twin-



Single Coil

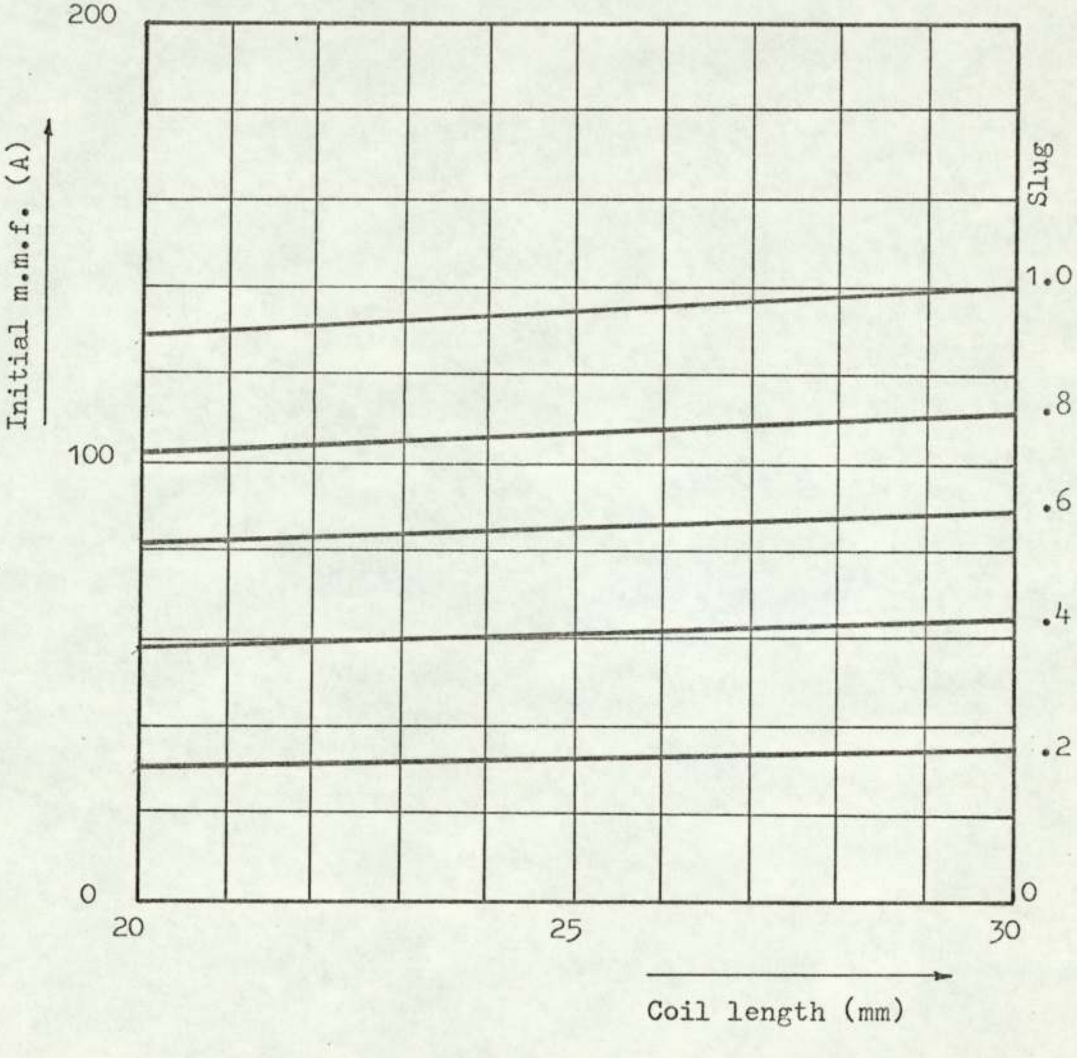
Experimental Relay.

Fig.26.



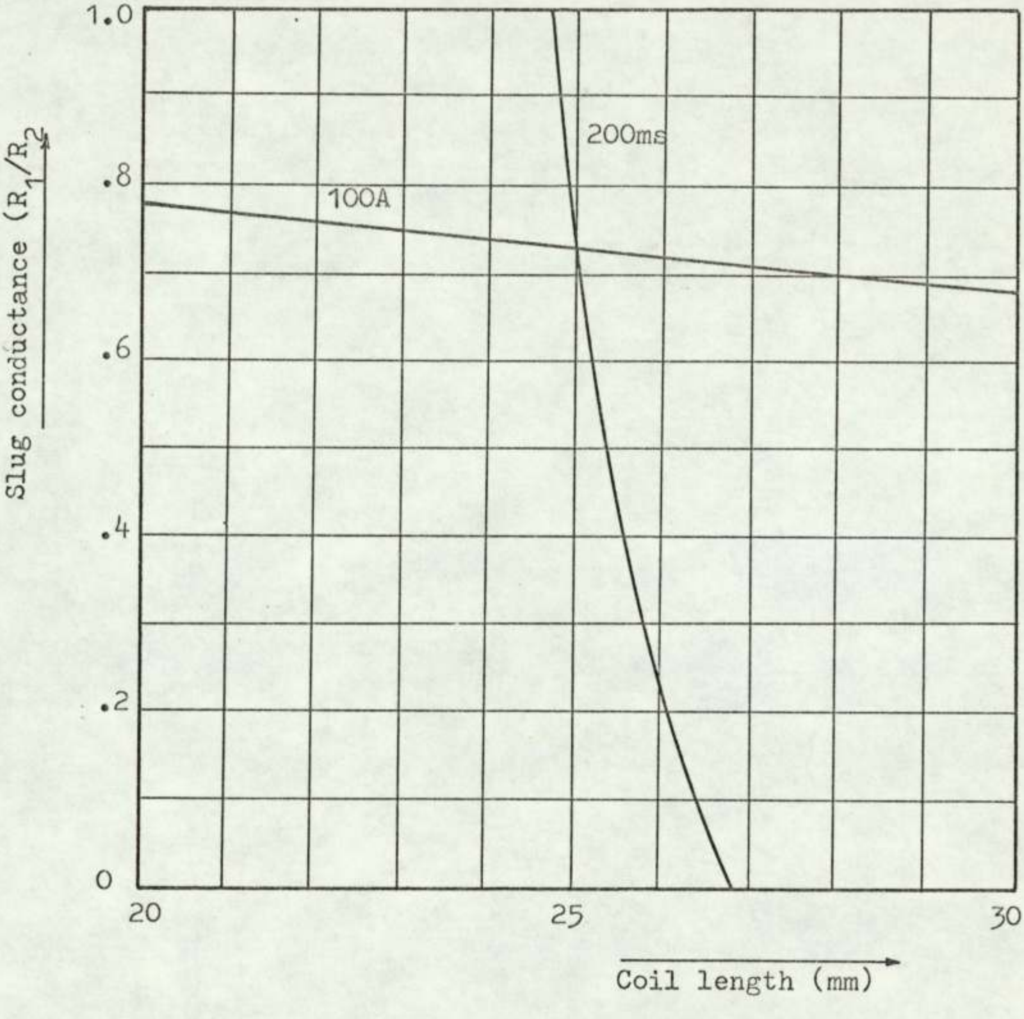
Release delay vs. coil length.
Single coil.

Fig.27.



Initial m.m.f. vs. coil length
Single coil.

Fig.28.



Slug conductance vs. coil length.
Single coil.

Fig.29.

coil model this optimum would be hard to achieve, whereas no such difficulty is presented by the single-coil construction. Furthermore, the twin-coil model is somewhat ungainly, and some space is wasted by the "U" shape of the core. An arrangement having a flat soleplate would require additional excitation power due to the introduction of joints into the primary iron.

b) Ease of construction. This will be closely related to cost and here the single coil would appear to have a definite advantage. Apart from the fact that only one coil is required, the number of components required for assembly is likely to be less than for the twin-coil construction, since the primary core is a simple cylinder onto which the pole pieces and slugs can be pressed.

c) Accessibility of reed inserts. Here again the single coil has the advantage, since a large number of inserts can be arranged around the periphery of the coil, all of which are readily accessible for replacement.

d) Power requirement. Although the single-coil model permits a nearer approach to the optimum radius to give minimum coil volume, this is done at the expense of increased power, since a considerable reduction in winding cross-sectional area is required to give a relatively small reduction in overall volume. It will be recalled that this conflict became evident in the preliminary considerations of Section 3, which were quite general and without reference to any particular relay, so that whatever form the relay may take it will be possible to reduce the exciting power by reducing the core radius, provided the consequential increase in coil length is permissible.

6.4 SENSITIVITY TO PARAMETER VARIATIONS

In order to gain some insight into the importance of accuracy in specifying the individual parameters, it is helpful to consider the change in release time arising from variations of each parameter from its designed value. This is conveniently done by considering the per-unit change of release time $\delta t_r/t_r$ which will arise from a per-unit change of a parameter $\delta p/p$, where p represents any parameter. The ratio of these quantities $\frac{p}{t_r} \cdot \frac{\partial t_r}{\partial p}$

is a normalised partial derivative which is a direct measure of the sensitivity of release time to parameter variations and gives a useful indication of the significance of individual parameters in the design calculation.

This partial derivative of t_r can be calculated from equation (18) and this is executed numerically in

Appendix II, the values applicable to the two relays of Figures 21 and 26 being listed below

PARAMETER	NORMALISED DERIVATIVE	
	Twin Coil	Single Coil
Coil length	1.024	0.967
Coil radius	0.795	1.556
Core radius	1.415	0.695
B saturation	0.974	0.989
H saturation	-0.296	-0.105
H coercive	0.299	0.024
Coil resistivity	-0.995	-0.995
Release m.m.f.	-0.981	-0.916
Slug conductance	0.070	0,038
Constant K	0.044	0.002
Constant A	-0.071	-0.055
Constant B	-0.006	-0.011
Constant M	0.000	-0.022

Some interesting observations may be made on the derivative values as listed above.

Coil length x. As might be expected the coil length is of major importance, values near to unity being obtained.

Coil Radius R. This is also of major importance. The value of the normalised derivative reaches a value of approximately 2 when the iron radius has the optimum value indicated in Figures 20 and 25, but is rather less when the iron radius is below the optimum. This is borne out by comparison between values for single-coil and twin-coil models.

Core Radius r. In these experimental models, the value of r is highly significant. However, had it been practicable to use the optimum value of r from Figures 20 and 25, the normalised derivative with respect to r would of course have become zero.

Saturation density B_s and m.m.f. gradient H_s. These are discussed together, as they are closely related. The release delay is nearly proportional to the saturation flux density, as indicated by the derivative being near to unity. However, the saturation m.m.f. gradient has a relatively small effect, particularly in the case of the single-coil model, which has a very short primary iron length, giving a normalised derivative of only -0.105. A point of considerable practical significance emerges, i.e. that high flux density is of considerably greater importance than low m.m.f. gradient, a fact which may influence the choice of core material. Furthermore, since a value for H_s is not easily estimated from a B-H curve, it is worth noting that any inaccuracy will result in only a minor error in release time.

Coercivity H_c. This is of relatively minor importance in the case of the single-coil relay with its short primary core, but, as would be expected, the increased core length of the twin-coil model greatly increases the effect of coercivity. It will be observed that the positive derivative indicates an increase in delay time with increased coercivity, or alternatively, reduced size for the same delay. Any attempt to exploit this situation by using a "harder" material is likely to result in permanent sticking of contacts.

Coil resistivity ρ. The value of -0.995 for the normalised derivative indicates that the release time is almost proportional to resistivity. Since this is an effective resistivity for a wound coil, any improvement in winding space factor will have a significant effect on the release time, or alternatively on the size of relay for a given release time.

Reed release m.m.f. i_r. This is an important parameter, giving a derivative near to unity. It is likely to be the major source of inaccuracy, since all other parameters of similar significance are predictable to a much higher degree of precision.

Slug Conductance. This has only a small effect on the release time, its primary purpose being to ensure fast operation. In fact, the simple theory in which coercivity was neglected, indicated that release time would be independent of slug conductance.

Constants K, A, B and M. These constants, as defined in Appendix I, are used in the design calculations to make allowance for the space occupied by pole pieces, insulation and secondary slug. The derivatives with respect to these constants are very small, indicating that their approximate nature still enables adequate accuracy to be achieved.

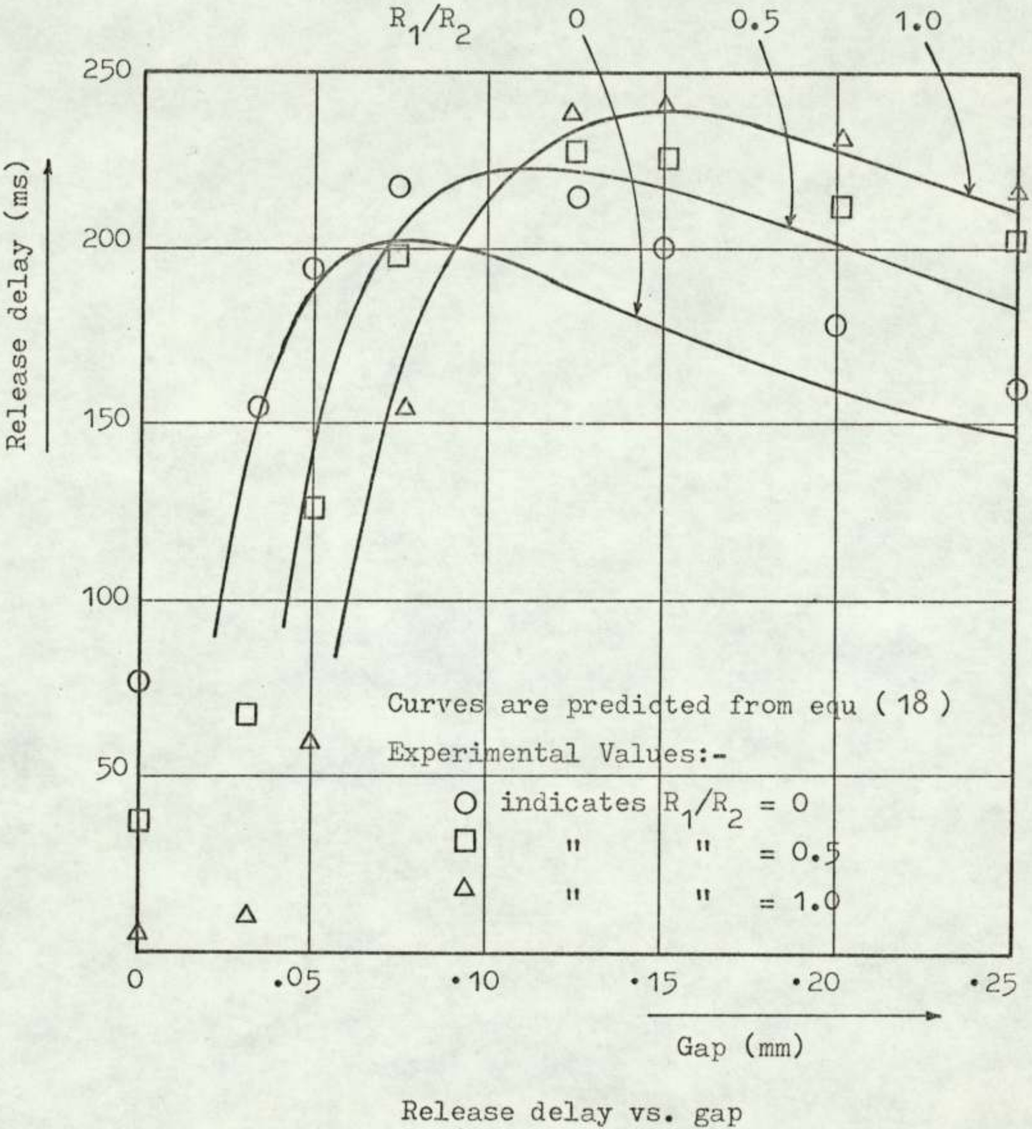
7. PERFORMANCE OF EXPERIMENTAL MODELS

The relays shown in Figures 21 and 26 (pp. 62 and 69) approximate closely to the designs of Sections 6.1 and 6.2, but space has been allowed to fit slugs larger than the design values for experimental purposes. By this means, curves of release time against secondary gap may be experimentally determined for various values of slug conductance to indicate the performance of the relay over a wide range of operating conditions. Comparison of these curves with the corresponding predicted curves, as calculated in Appendix III from the basic design equation (equation (18)), will indicate the reliability of the design technique used.

Figure 30 shows curves of release time against gap, as predicted in Appendix III, from the dimensions of the twin-coil relay of Figure 21. Superimposed upon this curve are some points, obtained experimentally, from the same relay.

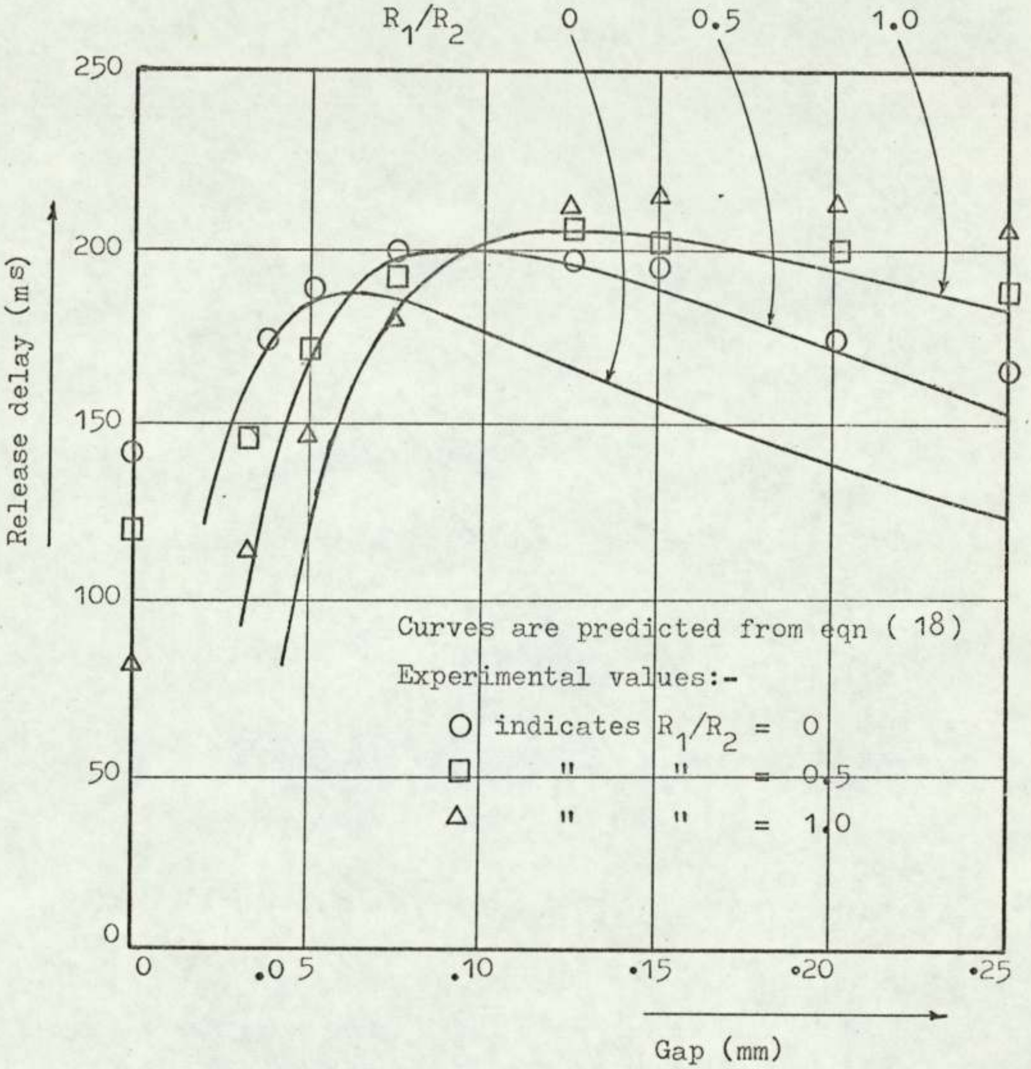
Figure 31 shows similar results for the single-coil relay of Figure 26.

The general forms of measured and calculated curves are in reasonable agreement, the maximum release delay exceeding the predicted value by between 2.5% and



Release delay vs. gap
(Twin Coil)

Fig.30.



Release delay vs. gap
Single Coil

Fig.31.

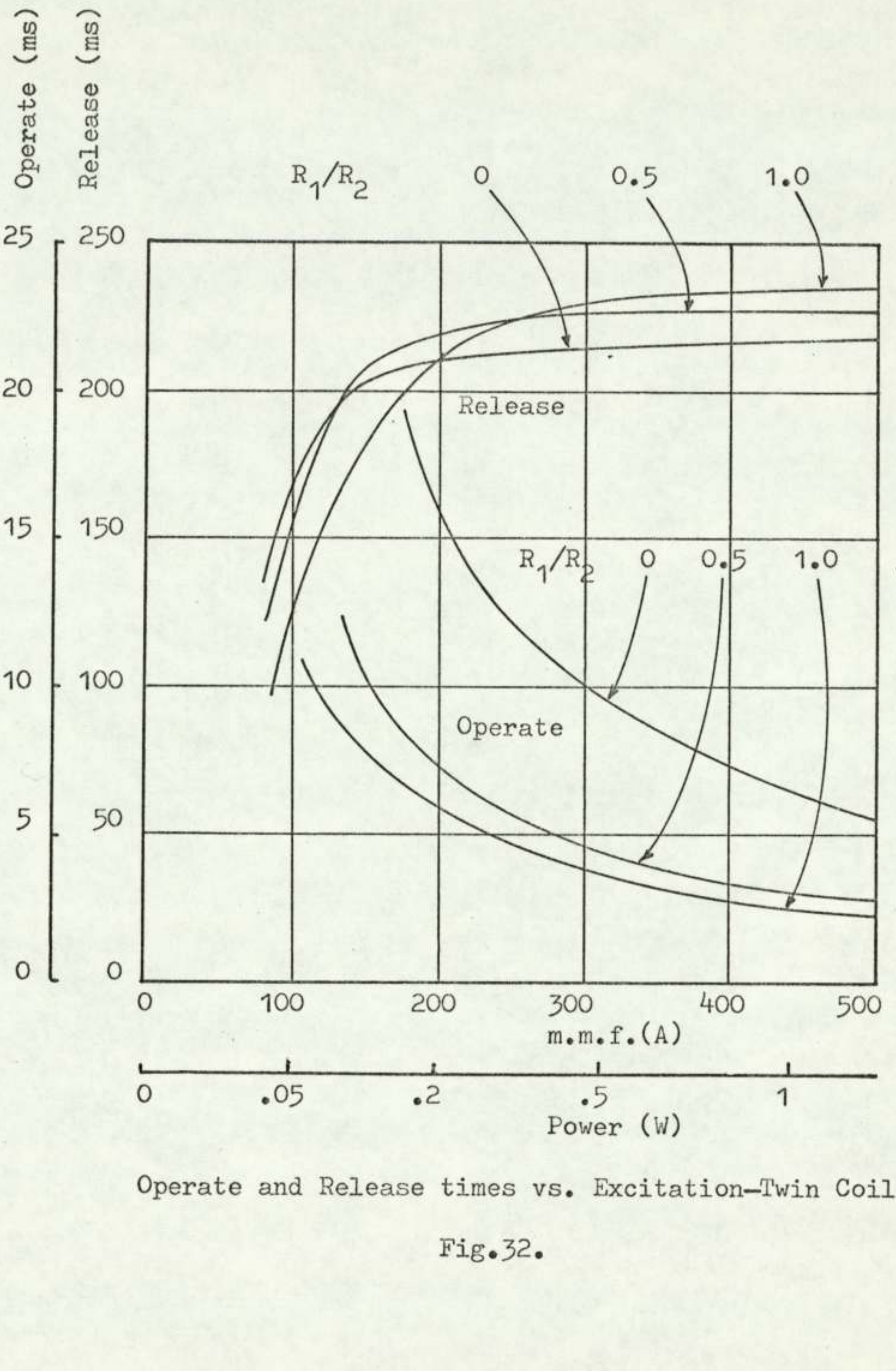
5%, although in the case of zero slug conductance the error is somewhat larger. This is only of academic interest, the practical application of this type of relay depending upon the presence of a short circuited secondary. It is probably accounted for by eddy currents in the secondary part of the iron core giving the effect of a slug of low conductance. The simplified treatment of eddy currents in Appendix IV assumes the value of H to be nearly uniform across the section of the core. This implies that the core conductance is small compared to the winding conductance, which is certainly not true if the latter is zero.

For both examples, it will be observed that the maximum value of release time is only slightly affected by the secondary slug conductance. As already mentioned on page 36, this effect is entirely absent but for the influence of residual magnetism. It is interesting to observe at this stage that, had the secondary reluctance been set to its design value by lengthening the core rather than by the introduction of a non-magnetic gap, then the magnetic field due to coercivity would reside entirely within the iron and the reed switch would be unaffected. This, of course, would not be a practicable arrangement, since the overall dimensions would be unnecessarily large. Furthermore, the iron would be inefficiently used, since the consequent high level of residual flux density would seriously limit the total flux swing between saturation and zero excitation.

Figures 32 and 33 show curves of operate and release times against excitation for three values of secondary conductance, the secondary gap being set for maximum release delay in each case. As has already been seen from Figures 30 and 31, a slightly increased release delay is obtained with higher values of secondary conductance, corresponding to larger secondary gaps.

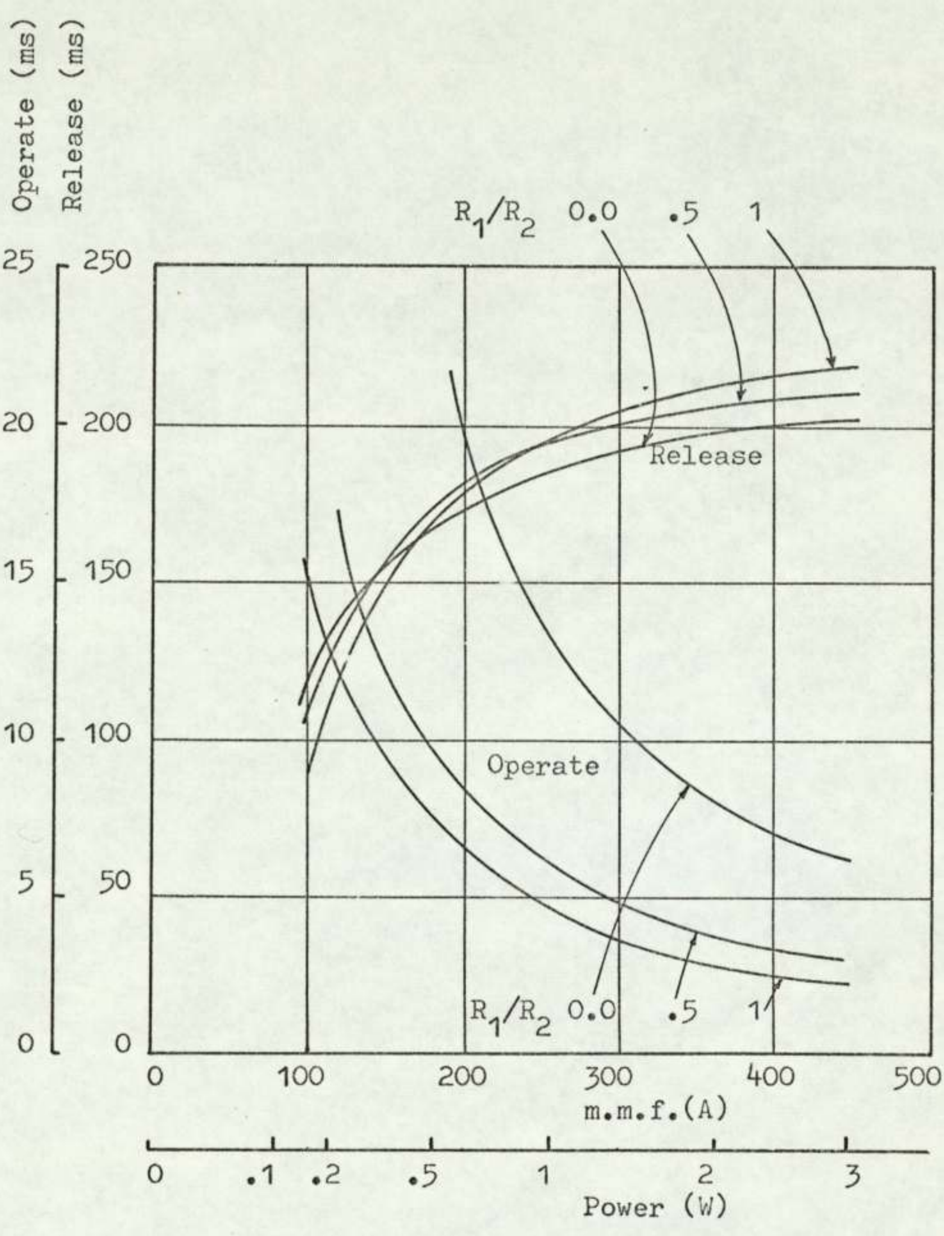
The region of constant release delay corresponds to saturation of the core and, as would be expected, a higher excitation is required to achieve this condition for the curves having a larger secondary gap.

Examination of the operate curves shows the effect of the secondary slug in reducing the operate time appreciably. It will be observed that operating times down to about 3ms can be obtained using a secondary slug having a resistance approximately equal to the designed value. Only a small additional improvement is obtained by using a slug of appreciably lower resistance. In general, the operate time in the absence of a secondary slug is about 2.5 times that obtained with a slug. Since saturation of the core has no effect on operate times, increased excitation beyond that required for saturation continues to increase the



Operate and Release times vs. Excitation–Twin Coil.

Fig.32.



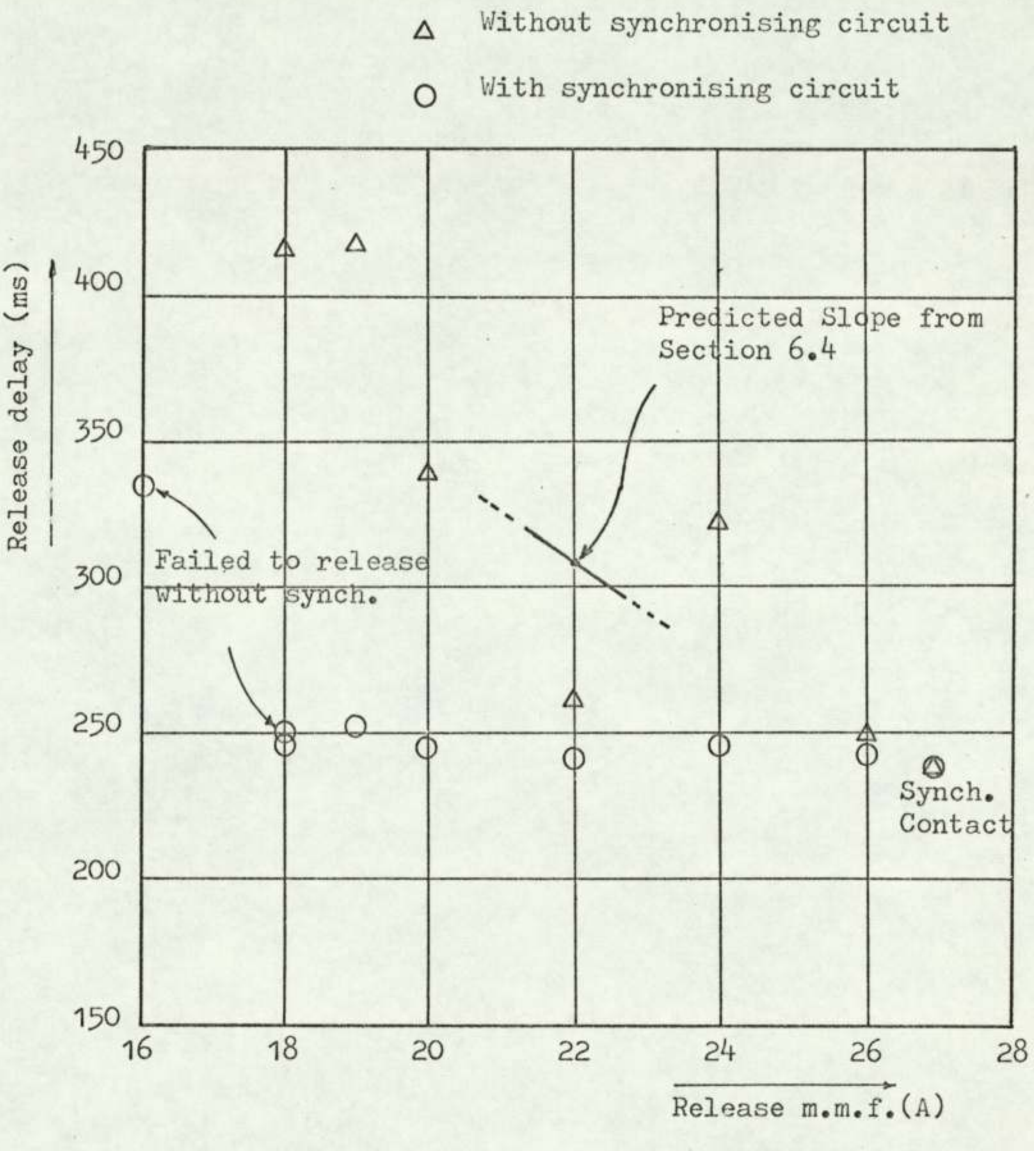
Operate and Release times vs. excitation—Single Coil.

Fig.33.

speed of operation which is consequently limited by the power available.

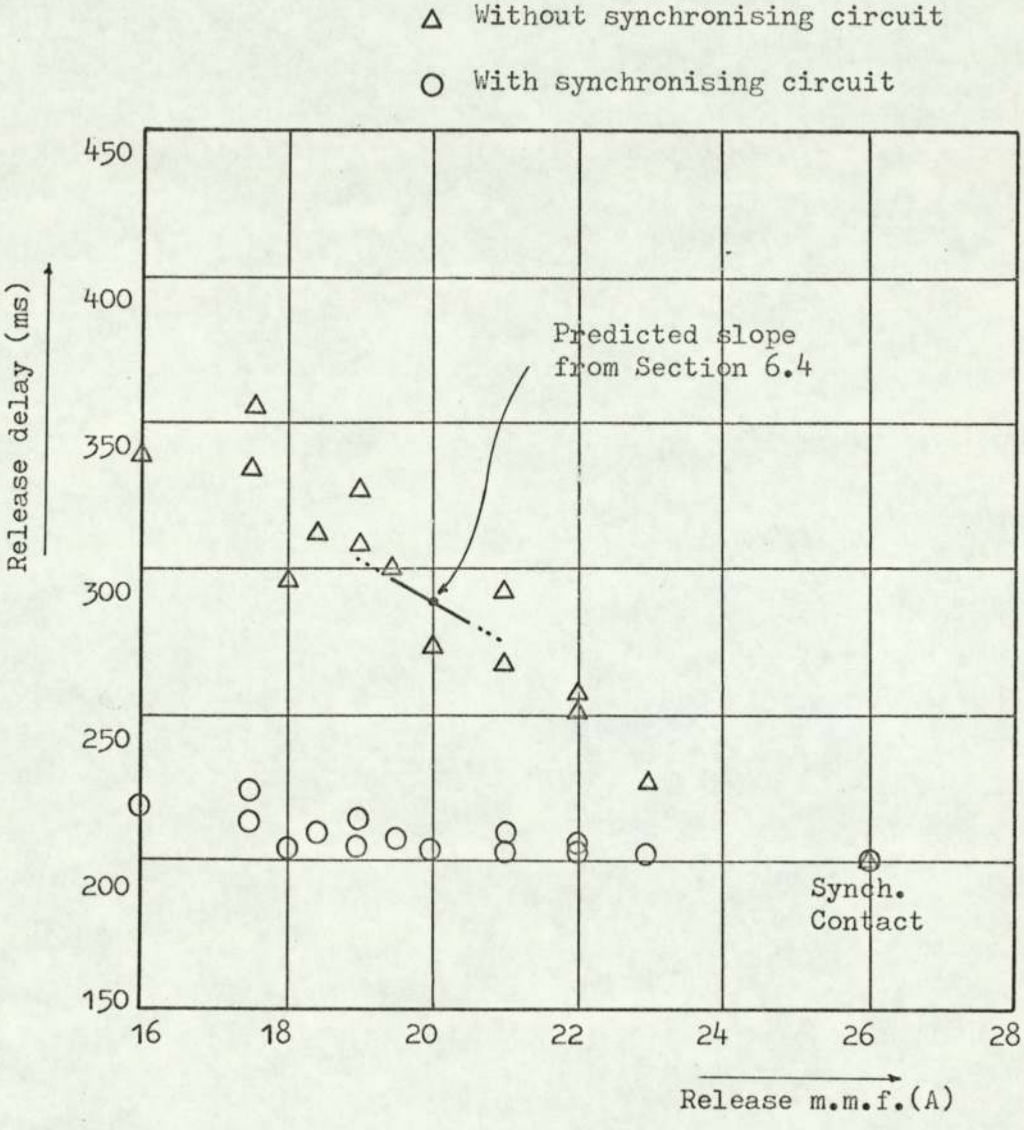
It was predicted on page 61 that the twin-coil model should require an excitation m.m.f. of 273A and power of 388mW and examination of Figure 32 shows these values to be quite adequate for saturation. In the case of the single-coil relay the predicted figures were 234A and 857mW which agree reasonably well with Figure 33, although in this case the onset of saturation is not so clearly defined. No truly satisfactory explanation of this difference can be suggested, although premature saturation of the corners of the cylindrical section of the core of the single-coil model is likely to occur at the point where it joins the secondary iron yoke, since the latter forms a relatively wide flange.

The wide variation in characteristics which is encountered with reed inserts inevitably gives rise to a considerable variation in release time in any system in which the m.m.f. decays slowly. Figures 34 and 35 indicate the extent of this variation for the two models. The variation of release time with insert m.m.f. is not sufficiently consistent to draw a curve showing a definite relationship, but the diagrams indicate the general trend. Superimposed upon these diagrams is the predicted slope,



Effect of synchronising circuit (twin coil).

Fig.34.



Effect of Synchronising circuit (single coil)

Fig.35.

as calculated from the normalised partial derivative of Section 6.4, which agrees well with the measured results. The twin-coil model, with its relatively long primary iron path gives a sufficiently high residual m.m.f. to result in the more sensitive inserts failing to release. The same diagrams include the corresponding release times when the first contact to release is used as a synchronising contact to insert a resistor into the primary circuit as described in Sections 2 and 5.4. All contacts are released within a period of about 20 ms, which is a marked improvement on the wide spread of delay times which occurs without insertion of additional primary resistance.

Equations (50) and (52) give alternative expressions for the peak voltage expected to appear across the synchronising contact. Both are approximations in that they assume the m.m.f. to be reduced to zero. Equation (50) makes the further approximation of neglecting leakage flux. Equation (52), including leakage flux, is likely to give an unrealistically high value, since the rate of rise of voltage is likely to be limited by eddy currents in the iron and the effects of inter-turn capacitance in the winding. Since the duration of the transient associated with the leakage flux is likely to be of fairly short duration, these effects might be expected to reduce the voltage to a value little greater than that given by equation (50). Applying equation (50) to the twin-coil model and taking $R_1/R_2=0.5$ (the lowest value used) and $i_s=400A$ (from Figure 32) we get $v/_V=0.195$. Using equation (52), the corresponding value is $v/_V=0.86$. The highest value obtained in experiments was 0.52, falling as expected between the two calculated figures. For a relay operating from a 50V supply this would give a contact voltage of 25.1V, which should be quite acceptable.

In the case of the single-coil relay, taking $R_1/R_2=0.5$ and $i_s=350$, we get $v/_V=0.22$ from equation (50) and 2.3 from equation (52). In this case, the measured value is 0.235 which is very near to the lower figure of 0.22 calculated from equation (50). This relatively low value for the single-coil model is not unexpected, since its construction should give a lower leakage inductance and its larger core radius provide for increased core eddy currents.

8. CONCLUDING COMMENTS

By virtue of its low permeance the reed switch is inherently suited to relay applications where fast response is required. The preliminary considerations of Section 3 indicate that a slow-release reed relay will occupy a minimum volume of several cubic inches. i.e. It will be of a size comparable to a "3000 type" relay.

The objective of the work described in this thesis has been to devise a relay, to provide a long release delay, without sacrificing the fast operation which is possible with reed switches. In general these two requirements are in conflict, but the proposed system can achieve a release delay of 200 ms with an operate delay of only 3 ms. The salient feature of the design lies in the location of the reed insert in the leakage field between two windings. In consequence, the main flux linking these windings is very much larger than the operating flux across the reed gap. Two experimental relays have been constructed, differing in layout, but identical in principle.

In this study the primary criterion on which design has been based is the space occupied by a completed relay. The twin-coil and single-coil experimental relays described in Section 7 could be contained in rectangular spaces of 10ins^3 and 6.7ins^3 respectively. These volumes may be compared with about 8ins^3 for a Post Office "3000 type" relay. The experimental models carried 10 contacts and 15 contacts respectively so that the single-coil version, in terms of "volume per contact", is by far the better arrangement. This arises from the fact that a closer approach to the optimum core radius may be achieved than is possible with twin coils.

Depending upon practical considerations regarding the application of this type of relay, criteria other than volume might well be more significant. In this connection it is interesting to note, as has been pointed out on earlier pages, that there is a clear conflict between small volume and low power input, since the iron radius which gives the minimum volume limits the available winding space. This point is borne out by comparison of the experimental models, the twin-coil version having a relatively large volume, since its construction precludes a close approach to the optimum radius, whilst its power requirement is less than half that of the single-coil version.

The analysis of a relay of this general type is straightforward, as shown in Section 4, provided some approximation is permitted regarding linearity of the iron core. The reliability of this analysis is demonstrated by comparison of the predicted curves of Figures 32 and 33.

with the corresponding experimental results. The measured release delays agree well with the form of the calculated curves. The delay for optimum gap, (which is the only value of practical significance) exceeds the calculated value by between 2.5% and 5%.

The expression for release time which emerges from the analysis of Section 4 is not readily applicable to a design calculation. However, a simple design technique emerges, if the coercivity of the iron core is neglected. In the absence of coercivity the release delay is independent of secondary gap length or slug conductance, provided they are related by equation (21). This approximation permits the elimination of secondary parameters, which greatly simplifies the design of the primary dimensions in an approximate form. After prediction of the actual release delay for the approximate design, an adjustment may be made to coil length, to give a closer approximation, if desired. That the optimum release delay depends only slightly on secondary parameters is confirmed by the experimental results of Figures 32 and 33. Having established the primary dimensions, from the slow-release requirement, the secondary part of the system is derived from the need for fast operation.

The major source of inaccuracy in a relay of this type is the wide variation in reed switch characteristics. Typically, a range of release m.m.f. of two-to-one may be encountered. Selection of reed switches to closer tolerances is likely to be rendered impracticable from the point of view of economics. However, the selection of one insert per relay could be acceptable. If the selected insert is chosen to have a release m.m.f. just above the upper limit acceptable for general use, then it may be used to synchronise the release of all other contacts. This function is described in some detail in Section 2. The experimental results demonstrate that the synchronising circuit releases all contacts within a period of about 20ms, i.e. the slowest contact has a release time only 10% longer than the fastest. In the absence of the synchronising circuit some of the more sensitive inserts may fail to release at all, due to the effects of core coercivity. No special precautions are required for protection of the synchronising contact since both theory and experiment indicate that destructive e.m.fs. do not appear across it.

For the relays described, the core material is soft iron. It has been indicated in Section 6.4 that the relay volume is affected only slightly by permeability, saturation flux density being the only significant iron parameter. However, in choosing a core material, it must be borne in mind that, for a given relay, the excitation m.m.f. will be proportional to H_s, and consequently, the power requirement will be approximately inversely proportional to the square of the permeability.

Either of the two forms described could be adapted for manufacture, although the single coil version, with its simpler pole-piece arrangement and more compact form, is likely to be less costly. Ease of mounting and accessibility of reed inserts would be further points in its favour as compared to the twin coil form. However, if low power requirement is deemed to be more important than small size a smaller core cross section is required, with a consequent increase in coil length. This would require a more complicated pole-piece for the single-coil form, and in such a case the twin-coil version could be more suitable. Replacement of the "U"-shaped core by two parallel cylinders joined by a flat sole plate would facilitate mounting. This introduction of two joints into the primary iron circuit could, however, result in an increased power requirement, thereby defeating the purpose of the twin-coil configuration.

In a production version of such a relay it is probable that an improved winding space factor could reduce the effective value of coil resistivity by up to 10% with either a corresponding reduction in coil length or a reduction in power requirement.

BIBLIOGRAPHY.

1. MORSE, S.F.B. Letters and Journals.

2. BARKER, R. Design of Relays for Automatic telephone exchange equipment with special reference to relay type 3000. P.O.E.E. Jour. v.26 1933.

3. RAY, F.I. and BIDDLECOMBE, A.N. A proposed Post Office standard telephone relay. P.O.E.E. Jour. v.24, 1931.

4. BRYAN, D.G. and BENSON, D.L. Evolution of reed relay telephone exchanges. I.E.E. Conf. on Telecommunications. April 1969.

5. YOUNG, J.S. Miniature relays. P.O.E.E. Jour. v.39, 1946.

6. EVANS, U.R. Electrical Contacts - the effect of atmospheric corrosion. Metal Industry v.73, 1948.

7. SUMMERS, N.D. Dirt in relays; its cause, effect and control. Electronic Components, 1966.

8. BANNOCHIE, J.G. and FURSEY, R.A.E. Sealed contact relays. J.Brit.I.R.E., v21, 1961.

9. COTTERILL, J. Sealed contacts. Component Technology, v.2, 1966.

10. JUPTER, W.F. Degassing of sealed relays. 8th Nat. Conf. on Electromagnetic relays. Oklahoma, 1961.

11. MARTIN, P.N. Relay contacts in sealed enclosures with no organic materials. 4th Symposium on Electromagnetic relays. Oklahoma, 1956.

12. HERMANCE, H.W. and EGAN, T.F. Organic deposits on precious metal contacts. Bell Syst. tech.J. v.37, 1958.

13. CHASTON, J.C. Some effects of surface films on contact behaviour. Proc. I.E.E. v.100, 1954.

14. BRUMFIELD, R.M. Comparison of sealed and unsealed relays. 1st Symposium on Electromagnetic relays. Oklahoma, 1953.

15. British Pat. 543935, 1941.

16. GAYFORD, M.L. Modern Relay Techniques. Newnes-Butterworth, 1969.

17. ELLWOOD, W.B. U.S. Patent 2,289,830. 1938.

18. PEEK, R.L. Magnetisation and pull characteristics of mating magnetic reeds. Bell Syst. tech. J. v.40, 1961.

19. ELLWOOD, W.B. A glass enclosed relay. Electrical Engineering, v.66, 1947.

20. HOVGAARD, O.M. Capabilities of sealed relay contacts. A.I.E.E. trans., Pt I, v.75, 1956.

21. MORGAN, R. The reed switch - a unique component. Automation. v.4, 1957.

22. RENSCH, H. Characteristics and applications of reed contacts. Electrical Communication. v.40, 1965.

23. HOVGAARD, O.M. and PERREAULT, G.E., Development of reed switches and relays. Bell Syst. tech J. v.34, 1955.

24. Research promises cheaper reed relays. Electrical Review 6 Jan., 1967.

25. LEVKOWETZ, H. The dry reed switch - a sealed contact of great reliability. Ericsson Review, v.39, 1962.

26. FENNEL, T.R., Relationship between wear debris and failure mechanism of the reed switch. Proc. of the Int. Conf. on Electromagnetic Relays, Tohoku, Oct.1963.

27. HOVGAARD, O.M. and FONTANA, W.J. A versatile miniature switching capsule, Proc. Electronic Components Conf., 1959.

28. WADGE, R.J.A. Coil operation of reed switches. Design Engineering, Dec. 1969.

29. Reed Relay Survey. Electronic Design v.11, 1963.

30. FOSTER, M.R. Permanent Magnet operation of reed switches. Design Engineering Dec. 1969.

31. SHOBERT, E.I. Welding or sticking of electrical contacts. Proc. A.S.T.M., v.47, 1947.

32. WOODHEAD, H.S. The diaphragm relay. Electronic Components. v.9, 1968.

33. ATKINSON, J., Telephony. Pitmans. 1950.

34. BROWN, G.W. Automatic telephony. Pitmans. 1930.

35. PEEK, R.L. and WAGAR, H.N. Switching relay design. Van Nostrand, 1955.

36. LOGAN, M.A. Dynamic Measurements on Electromagnetic Devices. Bell Syst. tech. J., v.32, 1953.

37. WAGAR, H.N. Slow acting relays. Bell Labs. Record. v.26, 1948.

38. AHLBERG, C. On the dynamic characteristics of the electromagnetic relay. Ericsson Rev. v.42, 1965.

39. EKELOF, S. A study of telephone relays. Ericsson Tech. v.9, 1953.

40. EKELOF, S. The development and decay of the magnetic flux in a non-delayed telephone relay. Ericsson Tech. v.13, 1957.

41. PEEK, R.L. Estimation and control of the operate time of relays. Bell Syst. tech. J. v.33, 1954.

42. PEEK, R.L. Principles of slow release relay design. Bell Syst. tech. J. v.33, 1954.

43. British Patent 1,013,789. 1962.

44. KELLER, A.C. Recent developments in Bell Systems relays - particularly sealed and miniature. Bell Syst. tech. J. v.43, 1964.

45. Reed switches and relays. Instruments and Control Systems. v.38, 1965.

46. COTTERHILL, J. Sealed contacts. Component Technology. v.2, 1966.

47. KELLER, A.C. A new general purpose relay. Bell Syst. tech.J. v.31, 1952.

48. MAHLER, P. The effects of contamination in relay performance. 8th Nat. Conf. on electromagnetic relays. Oklahoma, 1960.

49. SUMMERS, N.D. Relay design. Component Technology. v.1, 1965.

50. LOGAN, M.A. Estimation and control of the operate time of relays. Bell Syst.tech.J. v.33, 1954.

51. VITOLA, J. and BREIKNER, J.P. Dynamic noise generation in reed switch contacts. Electronic Industries, 1964.

52. WILMAN, C.W. Electrical contact resistance between magnetically attracted bodies. Nature, v.172, 1953.

53. KARO, D. and DHAR, P. The calculation of the dynamic characteristics of electromagnetic relays with the aid of an analogue computer. I.E.E. Conf. on Computer aided design. 1969.

54. ELEY, A.C. and LOWE, W.T. Reed relays for exchange systems. P.O.E.E. Jour. v.60, 1967.

55. ROGERS, B.H.E. and RIDGWAY, W.L. Dry-reed relays - Post Office relays type 14 and type 15. P.O.E.E. Jour. v.58, 1965.

APPENDIX I

COMPUTATION OF DESIGN CURVES

In general, the performance of a relay is specified by equation (18) which may be rewritten as

$$t_r=\left(\frac{R_1/R_2+1}{R_1(S_1+S_2)}\right)\log_e\left(\frac{(S_2-S_1R_1/R_2)\quad\left[\phi-(m_1+m_2)/S_1+S_2)\right]}{(1+R_1/R_2)\quad\left[i_r-(m_1S_2-m_2S_1)/(S_1+S_2)\right]}\right) \tag{55}$$

in which

$S_1=l_i/\pi r^2\mu_r$

$S_2=S_1(l_2+\mu_r g)/l_i$

$R_1=\rho\pi(R+r+y)/x(R-r-y)$

$\phi=B_s\,\pi r^2$

$m_1=H_c l_i$

$m_2=H_c l_2$

$l_i=x+K$

$l_2=Mx+A+BxR_1/R_2$

Here A, B and M are constants which can be readily established from the geometry of a particular relay once the approximate design of Sections 5.1 and 5.2 is completed. The only indeterminate quantities in equation (55) are x, R_1/R_2 and g, but, for an optimum design, g will be dependent upon R_1/R_2. The program on page 104 , which is written in "Focal" (Digital Equipment Co.'s conversational language) evaluates equation (55) for increasing values of g until a maximum release time is found, and this value is printed out. This process is repeated for a range of values of x and R_1/R_2 in the region of the approximate design values, and from the results a graph of the form shown in figure 17 is drawn.

For each solution to the above computation the value of the initial step in reed switch m.m.f. which occurs on energisation may be calculated from equation (34) which, for this purpose, may be written in the form

$$i_o = \left[H_m(l_i + l_2) + B_x g/\mu_o\right] / (1 + R_2/R_1) \qquad (56)$$

Evaluation of equation (56) enables a graph having the form of figure 18 to be drawn.

In addition, the value of total exciting m.m.f. and input power are calculated, from which the m.m.f. and power required for the final design may be predicted.

Explanatory comments on program

Lines 1.15 to 1.32	Request input data.
3.10 to 4.42	Calculate release time for gap increments of 0.01mm.
4.43 to 4.45	Compare with time for previous gap value to find maximum.
4.50 to 4.70	Calculate excitation m.m.f., initial m.m.f. step and power required.
7.20	Type result when maximum time is found.
6.24	Repeat for a range of coil lengths.
2.20	Repeat for a range of slug conductance.

RELAY DESIGN PROGRAM

```
01.05T "                RELAY DESIGN",!
01.15A "                RELEASE MMF",IR
01.16A "                COIL RADIUS",R2
01.17A "                CORE RADIUS",R1
01.18A "                B SATN     ",BS
01.19A "                H SATN     ",HS
01.20A "                COERCIVITY ",HC
01.21A "                H MAX      ",HM
01.22A "                RESISTIVITY",ROC
01.23A "                CONSTANT K ",K
01.24A "                CONSTANT A ",A
01.25A "                CONSTANT B ",B.
01.26A "                CONSTANT M ",M
01.27A "                VARY SLUG FROM ",C
01.28A "                IN STEPS OF    ",D
01.29A "                TO             ",E
01.30A "                COIL LNTH FROM ",L
01.31A "                IN STEPS OF    ",H
01.32A "                TO             ",J
02.10S MUR=BS/[HS*4E-7*3.14159]
02.20F SLG=(1E-5)+C,D,E+.01;D 6
02.30Q
03.01S L1=(X+K)/1000
03.03S L2=(M*X+A+B*X*SLG)/1000
03.04S M1=HC*L1;S M2=HC*L2
03.05S S1=L1/<3.14159*R1*R1*4*3.14159E-13*MUR>
03.06S TP=-1E10;S Y=-1
03.10F P=0,.01,.25;DO 4
04.06I (Y)4.08;R
04.08S G=P
04.10S S2=S1*(L2+MUR*G/1000)/L1
04.20S PHI=BS*3.14159*R1*R1*1E-6
04.22S RC=ROC*3.14159*<R1+R2+1.6>/<X*1E5*(R2-R1-1.6)>
04.23S RS=RC/SLG
04.31S Z=<PHI-(M1+M2)/(S1+S2)>*<RS*S2-RC*S1>/(RC+RS)
04.32S Z=Z/<IR-(M1*S2-M2*S1)/(S1+S2)>
04.41S TR=(RC+RS)/<RC*RS*(S1+S2)>
04.42S TR=TR*FLOG(Z)
04.43I (TP-TR)4.44,4.44;S Y=1;RETURN
04.44I (-TR)4.45;S TR=0;G 4.5
04.45S TP=TR
04.50S MMF=[HM*(L1+L2)+BS*G/(4E-4*3.14159)]
04.60S IO=MMF/(1+1/SLG)
04.70S POWER=MMF↑2*RC*1000
06.10T %3.02,!,"                 SLUG CONDUCTANCE=",SLG
06.12T " TIMES COIL CONDUCTANCE",!
06.13T "              "
06.20T " TIME       GAP  LENGTH   IO     MMF   POWER",!
06.24F X=L,H,J;D 7
06.26R
07.10D 3;T "          "
07.20T %5.02,TP*1E3,G-.01,X,IO,MMF,POWER,!
*
```

NUMERICAL EXAMPLES

Considering the design of Section 6.1 the length of the secondary iron path will be approximately 63mm., to allow space for the pole pieces to carry the reed switches, plus an allowance for the slug. Having decided upon a core radius of 4mm. and a coil spacing of 31.8mm. the core length corresponding to twice the slug thickness will be approximately $.215 x R_1/R_2$, assuming a copper slug having a resistivity of $1.75 x 10^{-8} \Omega$-m. Using these values, together with the dimensions already established evaluation of equations (38), (39), (55) and (56) gives the set of figures shown in Table A from which figures 22 and 23 may be drawn.

For the design of Section 6.2 the secondary length will be 63.7 mm. for ends and pole pieces, plus $0.726 x R_1/R_2$ for slugs, plus x for the return path under the coil. The procedure is identical to the previous example the results being tabulated in Table B and shown graphically in figures 27 and 28.

TABLE A

Twin Coil Model

```
RELAY DESIGN
RELEASE MMF:26
COIL RADIUS:15.9
CORE RADIUS:4
B SATN      :1.5
H SATN      :250
COERCIVITY :120
H MAX       :800
RESISTIVITY:3.6
CONSTANT K :60
CONSTANT A :63
CONSTANT B :.215
CONSTANT M :0
VARY SLUG FROM :0
IN STEPS OF     :.2
TO              :1.0
COIL LNTH FROM :40
IN STEPS OF     :2
TO              :60

SLUG CONDUCTANCE= 0.00 TIMES COIL CONDUCTANCE
 TIME      GAP  LENGTH    IO    MMF   POWER
 158.42   0.08  40.00   0.00 225.89 301.16
 166.36   0.08  42,00   0.00 227.49 290.90
 174.30   0.08  44.00   0.00 229.09 281.60
 182.24   0.08  46.00   0.00 230.69 273.13
 190.19   0.08  48.00   0.00 232.29 265.39
 198.14   0.08  50.00   0.00 233.89 258.30
 206.09   0.08  52.00   0.00 235.49 251.77
 214.05   0.08  54.00   0.00 237.09 245.75
 222.01   0.08  56.00   0.00 238.69 240.19
 229.97   0.08  58.00   0.00 240.29 235.02
 237.94   0.08  60.00   0.00 241.89 230.22

SLUG CONDUCTANCE= 0.20 TIMES COIL CONDUCTANCE
 TIME      GAP  LENGTH    IO    MMF   POWER
 164.91   0.09  40.00  39.87 239.21 337.71
 173.22   0.09  42.00  40.15 240.88 326.13
 181.54   0.09  44.00  40.43 242.54 315.63
 189.86   0.09  46.00  40.70 244.21 306.08
 198.20   0.09  48.00  40.98 245.88 297.35
 206.54   0.09  50.00  41.26 247.55 289.34
 214.89   0.09  52.00  41.54 249.22 281.98
 223.24   0.09  54.00  41.82 250.89 275.18
 231.61   0.09  56.00  42.10 252.56 268.90
 239.99   0.09  58.00  42.37 254.23 263.07
 248.37   0.09  60.00  42.65 255.89 257.65
```

(Continued.......

TABLE A (Continued....

```
SLUG CONDUCTANCE= 0.40 TIMES COIL CONDUCTANCE
 TIME       GAP   LENGTH    IO     MMF   POWER
 169.77    0.10   40.00   72.15 252.52 376.34
 178.37    0.11   42.00   76.06 266.19 398.29
 187.04    0.11   44.00   76.55 267.93 385.16
 195.73    0.11   46.00   77.05 269.67 373.21
 204.44    0.11   48.00   77.55 271.41 362.28
 213.16    0.11   50.00   78.04 273.14 352.26
 221.91    0.11   52.00   73.54 274.88 343.04
 230.67    0.11   54.00   79.04 276.62 334.52
 239.46    0.11   56.00   79.53 278.36 326.64
 248.27    0.11   58.00   80.03 280.09 319.32
 257.10    0.11   60.00   80.53 281.83 312.52
```

```
SLUG CONDUCTANCE= 0.60 TIMES COIL CONDUCTANCE
 TIME       GAP   LENGTH    IO     MMF   POWER
 173.93    0.12   40.00  104.16 277.77 455.36
 182.83    0.12   42.00  104.84 279.58 439.34
 191.75    0.12   44.00  105.52 281.38 424.80
 200.69    0.12   46.00  106.20 283.19 411.57
 209.65    0.12   48.00  106.87 284.99 399.47
 218.64    0.12   50.00  107.55 286.80 388.37
 227.65    0.12   52.00  108.23 288.61 378.15
 236.68    0.12   54.00  108.91 290.41 368.72
 245.74    0.12   56.00  109.58 292.22 359.98
 254.83    0.12   58.00  110.26 294.03 351.88
 263.94    0.12   60.00  110.94 295.83 344.34
```

(Continued......

TABLE A (Continued....

```
SLUG CONDUCTANCE= 0.80 TIMES COIL CONDUCTANCE
 TIME       GAP  LENGTH    IO     MMF  POWER
 177.24    0.13   40.00 129.37 291.08 500.06
 186.33    0.13   42.00 130.20 292.96 482.40
 195.45    0.13   44.00 131.04 294.83 466.39
 204.59    0.14   46.00 137.18 308.64 488.89
 213.82    0.14   48.00 138.01 310.52 474.23
 223.08    0.14   50.00 138.84 312.39 460.77
 232.38    0.14   52.00 139.68 314.27 448.39
 241.70    0.14   54.00 140.51 316.14 436.95
 251.07    0.14   56.00 141.34 318.02 426.35
 260.46    0.14   58.00 142.18 319.89 416.52
 269.90    0.14   60.00 143.01 321.77 407.37
```

```
SLUG CONDUCTANCE= 1.00 TIMES COIL CONDUCTANCE
 TIME       GAP  LENGTH    IO     MMF  POWER
 180.08    0.15   40.00 158.17 316.33 590.57
 189.39    0.15   42.00 159.14 318.27 569.38
 198.74    0.15   44.00 160.11 320.22 550.16
 208.11    0.15   46.00 161.08 322.16 532.65
 217.52    0.15   48.00 162.05 324.11 516.64
 226.97    0.15   50.00 163.03 326.05 501.94
 236.46    0.15   52.00 164.00 327.99 488.41
 245.98    0.15   54.00 164.97 329.94 475.91
 255.54    0.15   56.00 165.94 331.88 464.34
 265.14    0.15   58.00 166.91 333.83 453.59
 274.77    0.15   60.00 167.89 335.77 443.59
```

TABLE B

Single Coil Model

```
RELAY DESIGN
RELEASE MMF:26
COIL RADIUS:15.9
CORE RADIUS:7.15
B SATN      :1.5
H SATN      :250
COERCIVITY :120
H MAX       :800
RESISTIVITY:3.6
CONSTANT K :8
CONSTANT A :63.7
CONSTANT B :.726
CONSTANT M :1.0
VARY SLUG FROM :0
IN STEPS OF     :0.2
TO              :1.0
COIL LNTH FROM :20
IN STEPS OF     :1.0
TO              :30

SLUG CONDUCTANCE= 0.00 TIMES COIL CONDUCTANCE
 TIME       GAP  LENGTH    IO    MMF  POWER
 151.77    0.06  20.00    0.00 160.98 505.22
 159.06    0.06  21.00    0.00 162.58 490.77
 166.32    0.06  22.00    0.00 164.18 477.73
 173.55    0.06  23.00    0.00 165.78 465.91
 180.76    0.06  24.00    0.00 167.38 455.16
 187.94    0.06  25.00    0.00 168.98 445.34
 195.10    0.06  26.00    0.00 170.58 436.36
 202.23    0.06  27.00    0.00 172.18 428.12
 209.33    0.06  28.00    0.00 173.78 420.54
 216.41    0.06  29.00    0.00 175.38 413.55
 223.46    0.06  30.00    0.00 176.98 407.09

SLUG CONDUCTANCE= 0.20 TIMES COIL CONDUCTANCE
 TIME       GAP  LENGTH    IO    MMF  POWER
 155.07    0.08  20.00   31.20 187.18 683.02
 162.59    0.08  21.00   31.48 188.89 662.48
 170.08    0.08  22.00   31.77 190.61 643.91
 177.56    0.08  23.00   32.06 192.33 627.06
 185.01    0.08  24.00   32.34 194.04 611.70
 192.44    0.08  25.00   32.63 195.76 597.67
 199.85    0.08  26.00   32.91 197.47 584.80
 207.24    0.08  27.00   33.20 199.19 572.97
 214.60    0.08  28.00   33.49 200.91 562.07
 221.95    0.08  29.00   33.77 202.62 552.00
 229.27    0.08  30.00   34.06 204.34 542.68
```

(Continued.....

TABLE B (Continued.....

```
SLUG CONDUCTANCE= 0.40 TIMES COIL CONDUCTANCE
 TIME      GAP  LENGTH    IO    MMF   POWER
 157.81   0.09   20.00  57.56 201.44 791.06
 165.47   0.09   21.00  58.08 203.27 767.16
 173.12   0.09   22.00  58.60 205.10 745.55
 180.75   0.09   23.00  59.13 206.93 725.93
 188.36   0.09   24.00  59.65 208.77 708.06
 195.95   0.09   25.00  60.17 210.60 691.72
 203.52   0.09   26.00  60.70 212.43 676.74
 211.07   0.09   27.00  61.22 214.26 662.97
 218.59   0.09   28.00  61.74 216.10 650.27
 226.10   0.09   29.00  62.27 217.93 638.54
 233.59   0.09   30.00  62.79 219.76 627.68
```

```
SLUG CONDUCTANCE= 0.60 TIMES COIL CONDUCTANCE
 TIME      GAP  LENGTH    IO    MMF   POWER
 159.84   0.10   20.00  80.89 215.70 907.02
 167.62   0.10   21.00  81.62 217.65 879.51
 175.39   0.10   22.00  82.35 219.59 854.63
 183.14   0.10   23.00  83.08 221.54 832.04
 190.86   0.10   24.00  83.81 223.49 811.46
 198.57   0.10   25.00  84.54 225.44 792.65
 206.26   0.10   26.00  85.27 227.39 775.39
 213.93   0.10   27.00  86.00 229.34 759.52
 221.58   0.10   28.00  86.73 231.28 744.90
 229.22   0.10   29.00  87.46 233.23 731.38
 236.83   0.10   30.00  88.19 235.18 718.86
```

(Continued........

TABLE B (Continued....

```
SLUG CONDUCTANCE= 0.80 TIMES COIL CONDUCTANCE
 TIME       GAP  LENGTH   IO     MMF  POWER
 161.40    0.11  20.00 102.20 229.96 1030.9
 169.27    0.11  21.00 103.12 232.02 999.53
 177.13    0.11  22.00 104.04 234.09 971.15
 184.97    0.11  23.00 104.96 236.15 945.39
 192.79    0.11  24.00 105.87 238.22 921.91
 200.59    0.11  25.00 106.79 240.28 900.44
 208.40    0.12  26.00 113.02 254.28 969.65
 216.20    0.12  27.00 113.93 256.35 948.96
 223.99    0.12  28.00 114.85 258.41 929.87
 231.76    0.12  29.00 115.77 260.48 912.21
 239.51    0.12  30.00 116.69 262.54 895.84
```

```
SLUG CONDUCTANCE= 1.00 TIMES COIL CONDUCTANCE
 TIME       GAP  LENGTH   IO     MMF  POWER
 162.73    0.13  20.00 128.08 256.15 1279.2
 170.71    0.13  21.00 129.17 258.33 1239.1
 178.67    0.13  22.00 130.26 260.51 1202.8
 186.61    0.13  23.00 131.35 262.70 1169.9
 194.54    0.13  24.00 132.44 264.88 1139.8
 202.46    0.13  25.00 133.53 267.06 1112.3
 210.36    0.13  26.00 134.62 269.24 1087.1
 218.24    0.13  27.00 135.71 271.42 1063.8
 226.12    0.13  28.00 136.80 273.60 1042.4
 233.97    0.13  29.00 137.89 275.78 1022.6
 241.82    0.13  30.00 138.98 277.96 1004.2
```

*

APPENDIX II

COMPUTATION OF PARTIAL DERIVATIVE

Sensitivity to parameter errors in the design of Section 6 is obtained by inserting in equation (55) of Appendix I, all details of the final design. To execute the computation of the partial derivative by numerical means each parameter in turn is varied from its design value by 1% and the resultant change in release time is expressed as a percentage of the basic release time.

Since this procedure involves evaluation of the same expression as appears in Appendix I the main part of the programme is similar in that the gap is varied on successive runs to search for an optimum value. This means that when a parameter is varied the gap may also be changed. However, since the optimum gap has been used in each case a small variation will introduce only an infinitesimal error.

The results of this calculation are listed in Tables C and D for the two relays described.

Explanatory comments on program

Lines 1.18 to 1.31	Request input data.
3.01 to 4.42	Calculate release time for gap measurements of 0.01mm.
4.43 to 4.45	Compare with time for previous gap value to find maximum.
6.02 to 6.80	Vary each parameter in turn by 1% to find twc values of release time.
9.10	For each parameter change in group-6 lines, calculate and type out the normalised partial derivative.

PARAMETER VARIATION PROGRAM

```
C FOCAL V3A
01.05T "                    RELAY DESIGN",!
01.06T "                    PARAMETER VARIATION",!!
01.18A "                    COIL RADIUS",R2
01.20A "                    CORE RADIUS",R1
01.21A "                    COIL LENGTH",X
01.22A "                    B SATN     ",BS
01.23A "                    H SATN     ",HS
01.24A "                    COERCIVITY ",HC
01.25A "                    RESISTIVITY",ROC
01.26A "                    RELEASE MMF",IR
01.27A "                    CONSTANT K ",K
01.28A "                    CONSTANT A ",A
01.29A "                    CONSTANT B ",B
01.30A "                    CONSTANT M ",M
01.31A "                    SLUG COND. ",SLG
02.20T !,"                    PARAMETER       "
02.22T "DERIVATIVE",!;D 6
02.30Q
03.01S L1=(X+K)/1000;S MUR=BS/[HS*4*3.14159E-7]
03.03S L2=(M*X+A+B*X*SLG)/1000
03.04S M1=HC*L1;S M2=HC*L2
03.05S S1=L1/<3.14159*R1*R1*4*3.14159E-13*MUR>
03.06S TP=-1E10;S Y=-1
03.10F P=0,.01,.25;DO 4
04.06I (Y)4.08;R
04.08S G=P
04.10S S2=S1*(L2+MUR*G/1000)/L1
04.20S PHI=BS*3.14159*R1*R1*1E-6
04.22S RC=ROC*3.14159*<R1+R2+1.6>/<X*1E5*(R2-R1-1.6)>
04.23S RS=RC/SLG
04.31S Z=<PHI-(M1+M2)/(S1+S2)>*<RS*S2-RC*S1>/(RC+RS)
04.32S Z=Z/<IR-(M1*S2-M2*S1)/(S1+S2)>
04.41S TR=(RC+RS)/<RC*RS*(S1+S2)>
04.42S TR=TR*FLOG(Z)
04.43I (TP-TR)4.44,4.44;S Y=1;RETURN
04.44I (-TR)4.45;S TR=0;RETURN
04.45S TP=TR
```

(Continued......

(Continued.....

```
06.02T "                     COIL LENGTH    "
06.04S X=X*1.01;D 3;S T1=TP
06.06S X=X/1.01;D 3;S T2=TP;D 9
06.08T "                     COIL RADIUS    "
06.10S R2=R2*1.01;D 3;S T1=TP
06.12S R2=R2/1.01;D 3;S T2=TP;D 9
06.14T "                     CORE RADIUS    "
06.16S R1=R1*1.01;D 3;S T1=TP
06.18S R1=R1/1.01;D 3;S T2=TP;D 9
06.20T "                     B SATN         "
06.22S BS=BS*1.01;D 3;S T1=TP
06.24S BS=BS/1.01;D 3;S T2=TP;D 9
06.28T "                     H SATN         "
06.30S HS=HS*1.01;D 3;S T1=TP
06.32S HS=HS/1.01;D 3;S T2=TP;D 9
06.34T "                     COERCIVITY     "
06.36S HC=HC*1.01;D 3;S T1=TP
06.38S HC=HC/1.01;D 3;S T2=TP;D 9
06.40T "                     RESISTIVITY    "
06.42S ROC=ROC*1.01;D 3;S T1=TP
06.44S ROC=ROC/1.01;D 3;S T2=TP;D 9
06.46T "                     RELEASE MMF    "
06.48S IR=IR*1.01;D 3;S T1=TP
06.50S IR=IR/1.01;D 3;S T2=TP;D 9
06.52T "                     CONSTANT K     "
06.54S K=K*1.01;D 3;S T1=TP
06.56S K=K/1.01;D 3;S T2=TP;D 9
06.58T "                     CONSTANT A     "
06.60S A=A*1.01;D 3;S T1=TP
06.62S A=A/1.01;D 3;S T2=TP;D 9
06.64T "                     CONSTANT B     "
06.66S B=B*1.01;D 3; S T1=TP
06.68S B=B/1.01;D 3;S T2=TP;D 9
06.70T "                     CONSTANT M     "
06.72S M=M*1.01;D 3;S T1=TP
06.74S M=M/1.01;D 3;S T2=TP;D 9
06.76T "                     SLUG COND.     "
06.78S SLG=SLG*1.01;D 3;S T1=TP
06.80S SLG=SLG/1.01;D 3;S T2=TP;D 9
09.10S D=200*(T1-T2)/(T1+T2);T %5.04,D,!

*
```

TABLE C

```
RELAY DESIGN
PARAMETER VARIATION

COIL RADIUS:15.9
CORE RADIUS:4
COIL LENGTH:48.3
B SATN     :1.5
H SATN     :250
COERCIVITY :120
RESISTIVITY:3.6
RELEASE MMF:26
CONSTANT K :60
CONSTANT A :63
CONSTANT B :.215
CONSTANT M :0
SLUG COND. :.55

PARAMETER      DERIVATIVE
COIL LENGTH     1.0236
COIL RADIUS     0.7951
CORE RADIUS     1.4151
B SATN          0.9736
H SATN         -0.2963
COERCIVITY      0.2986
RESISTIVITY    -0.9950
RELEASE MMF    -0.9805
CONSTANT K      0.0435
CONSTANT A     -0.0708
CONSTANT B     -0.0064
CONSTANT M      0.0000
SLUG COND.      0.0698
```

*

G

TABLE D

```
RELAY DESIGN
PARAMETER VARIATION

COIL RADIUS:15.9
CORE RADIUS:7.15
COIL LENGTH:25
B SATN     :1.5
H SATN     :250
COERCIVITY :120
RESISTIVITY:3.6
RELEASE MMF:26
CONSTANT K :8
CONSTANT A :63.7
CONSTANT B :.726
CONSTANT M :1.0
SLUG COND. :0.72

PARAMETER   DERIVATIVE
COIL LENGTH  0.9672
COIL RADIUS  1.5564
CORE RADIUS  0.6954
B SATN       0.9891
H SATN      -0.1048
COERCIVITY   0.0236
RESISTIVITY -0.9950
RELEASE MMF -0.9155
CONSTANT K   0.0016
CONSTANT A  -0.0551
CONSTANT B  -0.0113
CONSTANT M  -0.0216
SLUG COND.   0.0376
```

```
*G
```

APPENDIX III

PREDICTION OF CHARACTERISTICS

Calculation of time delay for a range of secondary gap based upon equation (55) for a particular relay is comparatively straightforward and is useful as confirmation of the design equations since the gap can readily be varied experimentally.

The programme is similar to the previous one except that, in this case, the release time is typed out for each value of gap, Tables E and F referring to the twin coil and single coils respectively. These tables are depicted graphically in figures 30 and 31.

RELAY ANALYSIS PROGRAM

```
01.05T "             RELAY ANALYSIS",!!
02.08A "             COIL LENGTH      ",X
02.09A "             H SATN           ",HS
02.11A "             COIL RADIUS      ",R2
02.12A "             COIL RESISTIVITY ",ROC
02.13A "             COERCIVITY       ",HC
02.14A "             RELEASE MMF      ",IR
02.15A "             CORE RADIUS      ",R1
02.16A "             B SATN.          ",BS
02.17A "             CONSTANT K       ",K
02.18A "             CORE LENGTH TOTAL",L
02.20S MUR=BS/[HS*4E-7*3.14159]
02.40S L1=(X+K)/1000;S L2=(L/1000)-L1
02.44T !,"             SLUG",!
02.45T "             CONDUCTANCE="
02.47T "     0.0       0.5       1.0",!!
02.49T "              GAP(MM)    TIME(MS)"
02.51T "  TIME(MS)  TIME(MS)"
02.80F G=0,.01,.25;D 3
02.90Q
03.04S M1=HC*L1;S M2=HC*L2
03.05S S1=L1/<3.14159*R1*R1*4*3.14159E-13*MUR>
03.07T %3.02,!,"                ",G,"   "
03.10F SLG=1E-6,.5,1.01;D 4
04.10S S2=S1*(L2+MUR*G/1000)/L1
04.20S PHI=BS*3.14159*R1*R1*1E-6
04.22S RC=ROC*3.14159*<R1+R2+1.6>/<X*1E5*(R2-R1-1.6)>
04.23S RS=RC/SLG
04.31S Z=<PHI-(M1+M2)/(S1+S2)>*<RS*S2-RC*S1>/(RC+RS)
04.32S Z=Z/<IR-(M1*S2-M2*S1)/(S1+S2)>
04.41S TR=(RC+RS)/<RC*RS*(S1+S2)>
04.42S TR=TR*FLOG(Z)
04.50T %4.01,"    ",TR*1E3

*
```

TABLE E

```
RELAY ANALYSIS

COIL LENGTH       :50
H SATN            :250
COIL RADIUS       :15.9
COIL RESISTIVITY  :3.6
COERCIVITY        :120
RELEASE MMF       :25
CORE RADIUS       :4
B SATN.           :1.5
CONSTANT K        :60
CORE LENGTH TOTAL:184

SLUG
CONDUCTANCE=      0.0         0.5         1.0

   GAP(MM)      TIME(MS)    TIME(MS)    TIME(MS)
   0.00         -331.6      -1417       -1645
   0.01         - 46.3      -485.6      -1764
   0.02           79.3      -154.6      -636.9
   0.03          141.3         7.8      -263.5
   0.04          173.6        97.4      - 71.7
   0.05          190.4       149.8        40.3
   0.06          198.5       181.5       110.0
   0.07          201.7       200.8       155.1
   0.08          201.8       212.3       184.9
   0.09          200.2       218.9       204.7
   0.10          197.4       222.1       217.9
   0.11          194.0       223.1       226.5
   0.12          190.2       222.7       231.9
   0.13          186.2       221.1       234.9
   0.14          182.1       218.9       236.3
   0.15          178.0       216.2       236.4
   0.16          174.0       213.2       235.6
   0.17          170.1       209.9       234.2
   0.18          166.3       206.6       232.3
   0.19          162.6       203.2       230.1
   0.20          159.0       199.8       227.6
   0.21          155.6       196.4       224.9
   0.22          152.3       193.0       222.1
   0.23          149.1       189.7       219.2
   0.24          146.0       186.5       216.3
   0.25          143.0       183.3       213.4*
```

TABLE F

```
RELAY ANALYSIS

COIL LENGTH        :25
H SATN             :250
COIL RADIUS        :15.9
COIL RESISTIVITY   :3.6
COERCIVITY         :120
RELEASE MMF        :25
CORE RADIUS        :7.15
B SATN.            :1.5
CONSTANT K         :8
CORE LENGTH TOTAL:149

SLUG
CONDUCTANCE=       0.0        0.5         1.0

   GAP(MM)       TIME(MS)   TIME(MS)   TIME(MS)
   0.00          -209.6     -662.1     -1272
   0.01            25.3     -203.1     -526.1
   0.02           118.0     -  7.5     -200.1
   0.03           159.2       89.3     - 32.4
   0.04           177.9      141.2       62.4
   0.05           185.6      170.2      119.2
   0.06           187.7      186.4      154.5
   0.07           186.7      195.2      176.8
   0.08           184.1      199.4      190.8
   0.09           180.5      200.8      199.6
   0.10           176.4      200.4      204.7
   0.11           172.1      198.8      207.4
   0.12           167.8      196.5      208.5
   0.13           163.5      193.6      208.3
   0.14           159.3      190.5      207.2
   0.15           155.2      187.2      205.6
   0.16           151.3      183.8      203.6
   0.17           147.5      180.5      201.2
   0.18           143.9      177.1      198.7
   0.19           140.5      173.7      196.1
   0.20           137.2      170.5      193.3
   0.21           134.1      167.3      190.6
   0.22           131.1      164.2      187.8
   0.23           128.2      161.2      185.0
   0.24           125.5      158.2      182.3
   0.25           122.8      155.4      179.5*
```

APPENDIX IV

EFFECT OF EDDY CURRENTS IN THE CORE

In a conventional relay where the operating flux is the main core flux the effects arising from the m.m.f. associated with induced core currents cannot be ignored. However, in the reed relay under consideration, conditions are modified by the fact that the flux which operates the reed switch is, as far as the main core is concerned, a relatively minor leakage flux which is defined by the m.m.f. between the pole pieces.

Referring to figure 36, for the dotted path which embraces the coil current i_1 and part of the eddy currents i_e

$$\oint H.dl = i_1 + i_e \qquad (57)$$

and for an idealised case, neglecting end effects, the m.m.f. across the reed switch will be given by

$$i = i_1 + i_e - (B + \delta B) l/\mu \qquad (58)$$

where δB is the modification to the flux density arising from eddy currents.

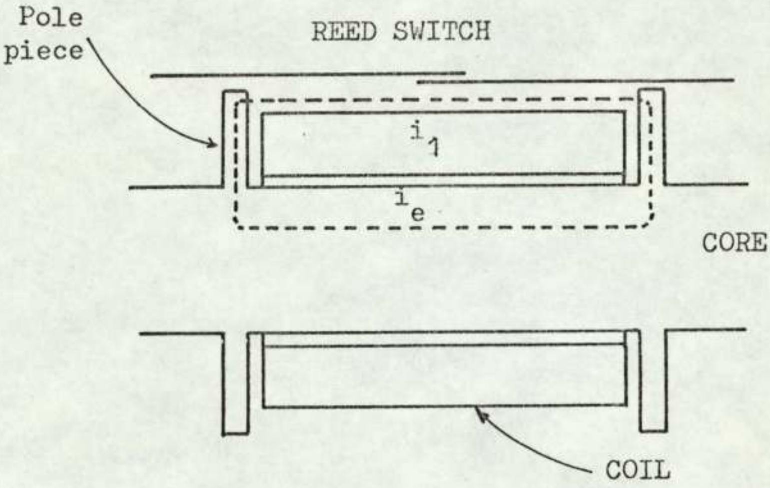


Fig.36.

Now if it is assumed that for slow rates of change the flux density across the section of the core is nearly constant, (i.e. $B \gg \delta B$), and if the core is considered as a series of concentric cylindrical shells, then the e.m.f. induced in any shell will be given by

$$e=(d\phi/dt)(x^2/r^2) \tag{59}$$

where x and r are the shell and core radii respectively. The corresponding current will be

$$\delta i=(d\phi/dt)(1/2\pi r^2\rho_i)x\,\delta x \tag{60}$$

so that

$$i_e=(d\phi/dt)(1/4\pi r^2\rho_i)\left[x^2\right]_x^r$$

$$=(d\phi/dt)(1/4\pi\rho_i)(1-x^2/r^2) \tag{61}$$

The value for i_e given in equation(61) will be the eddy current linking a shell of radius x, so that the m.m.f. gradient at this radius will be modified by an amount

$$\delta H=(d\phi/dt)/4\pi\rho_i\{1-x^2/r^2\} \tag{62}$$

so that the flux density will be modified by

$$\delta B=(d\phi/dt)\mu/4\pi\rho_i\{1-x^2/r^2\} \tag{63}$$

By substitution of equations (61) and (63) in equation (58) the m.m.f. across the reed switch becomes

$$i=i_1+(d\phi/dt)(1-x^2/r^2)l/4\pi\rho_i$$

$$-\left[B+(d\phi/dt)(1-x^2/r^2)\mu/4\pi\rho_i\right]l/\mu$$

$$=i_1-Bl/\mu \qquad (64)$$

For slow rates of change of flux equation (64) indicates that for an idealised case the eddy currents, although modifying the core flux, do not affect the m.m.f. appearing across the reed switch.