Message-Passing Algorithms for Optimal Utilization of Cognitive Radio Networks

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Abstract—**Cognitive Radio has been proposed as a key technology to significantly improve spectrum usage in wireless networks by enabling unlicensed users to access unused resource. We present new algorithms that are needed for the implementation of opportunistic scheduling policies that maximize the throughput utilization of resources by secondary users, under maximum interference constraints imposed by existing primary users. Our approach is based on the Belief Propagation (BP) algorithm, which is advantageous due to its simplicity and potential for distributed implementation. We examine convergence properties and evaluate the performance of the proposed BP algorithms via simulations and demonstrate that the results compare favorably with a benchmark greedy strategy.**

I. INTRODUCTION

Cognitive Radio Networks (CRNs) have recently been proposed as a promising approach to improve spectrum usage, by enabling unlicensed users to access unused wireless resources [1]. The key technology is cognitive radio that can dynamically adjust its channel allocation depending on spectrum availability. Therefore, in CRNs, we distinguish between: Primary Users (PU), who have licensed rights to network resources, and (unlicensed) Secondary Users (SU) equipped with cognitive radios, who access underutilized spectrum channels. A basic requirement is that secondary users must not adversely impact the performance of primary users. This can be ensured by a coordination mechanism, under the terms of an agreement between users, but in many practical cases such extensive coordination is unfeasible. In contrast, an approach that leverages the cognitive radio capabilities consists in secondary users monitoring the spectrum and exploiting opportunistically channels unemployed by primary users. Combined with the dynamic wireless environment this poses important challenges as the channel access algorithms must be very efficient for the access to be updated continuously, and distributed to allow for uncoordinated opportunistic operation.

In this paper, we present new algorithms needed for opportunistic scheduling, which maximize the utility of the secondary users, under maximum interference constraints imposed by existing primary users. The utility can be a function of actual or stochastic traffic and communication rates, or corresponding queue lengths; such choices can be formulated with the goal to maximize the achievable network throughput using Lyapunov Optimization techniques (see the seminal paper [9], and [10] for an application in CRNs). Our aim is to select sets of links between secondary users and free primary channels that can be activated simultaneously, such as to maximize the total utility of secondary users, without causing disruptive interference to existing primary users. Hence, the optimization problem in our framework, which we describe in Section II, is an NP-hard generalization of Maximum Weight Matching (MWM), with additional interference constraints.

Our approach is based on the Belief Propagation (BP) algorithm, which is advantageous due to its simplicity and potential for distributed implementation. BP is an iterative message-passing algorithm, that was discovered independently in the information theory [3], machine learning [7] and statistical physics [6] communities. It has been used, with spectacular experimental success, in many application areas, for instance iterative decoding and combinatorial optimization, which involve graphs with cycles. The physical interpretation of message-passing algorithms and in particular belief propagation was later explained by statistical mechanics studies of disordered systems [4], [11], [5], consolidating BP and its generalizations to a very powerful tool that provides practical solutions to hard problems. Additionally, BP has been shown to solve the classical Maximum Weight Matching exactly in bipartite graphs with cycles [2].

Cognitive radio networks have attracted significant research interest (see [1] for a survey), including work on optimal spectrum scheduling. However, the problem of efficient and fully distributed throughput optimization under interference remains challenging. A recent paper [8] proposes affinity propagation based algorithms for spectrum access in CRNs, in the specific case where primary users permit secondary users access as long as they consent to act as relays. The study that is closest to our approach is [10], which introduces a general framework using the Lyapunov Optimization technique to design a scheduling policy for flow control and resource allocation in CRNs. It motivates the formulation of the optimization problem we aim to address; specifically, the policy proposed in [10] is required to solve a certain deterministic optimization problem at each time slot, which imposes a computational bottleneck when interference is taken into account. In this paper, our contribution is complementary as we focus on a more general algorithmic solution, and the problem we address can be used to solve the optimization problem discussed above, in contrast to [10] that merely presents a greedy algorithm for the simplified case of no interference.

The paper is organized as follows: we present a general

modeling framework for opportunistic cognitive radio networks and formulate the problem of optimal channel allocation with interference control in Section II; in Section III, we develop two new Belief Propagation algorithms, which can be fully decentralized, to solve the optimization problem for hard and soft interference constraints, respectively; in Section IV, we show the convergence and evaluate the performance of the proposed BP algorithms via simulation scenarios in a realistic Signal to Interference plus Noise (SINR) based model of cognitive radio networks and demonstrate that the results compare favorably with a benchmark greedy scheduling strategy.

II. MODEL

The model adopts a hierarchical access structure with primary and secondary users. The basic idea is to open channel frequencies to secondary users while considering the interference experienced by primary users (licensees). Fig 1 illustrates a CRN. Primary users (squares) are first to be served by channels. Secondary users, shown by circles, in non-cooperative scenario can not access channels if they are occupied by a primary user. We assume that information
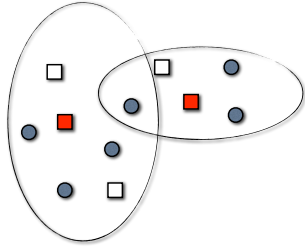

Fig. 1. A simple cognitive radio network, with 6 secondary users and 5 primary users. Red squares correspond to active primary users, white squares to idle primary users and cognitive users are shown by circles.

about primary user communications are provided reliably. Each primary user has its own channel frequency which is not shared with others. Thus, primary users can send data over their own licensed channels to their respective access points simultaneously. Secondary users do not have such channels and opportunistically try to send their data to receivers by utilizing idle primary channels. Formally we define a CRN by N secondary users and M primary users. Each secondary user may have access to a subset of primary channels (see Fig. 1) according to their Euclidean distance. Throughout this paper primary users are denoted by $\{a, b, ...\}$ and secondary users by $\{i, j, ..\}$. We define a binary metric $\mathcal{I}(i, a) = \{0, 1\}$ for each secondary user $i \in \{1, ..., N\}$ and primary channel $a \in \{1, ..., M\}$ where $\mathcal{I}(i, a) = 1$ indicates accessibility of primary channel a to secondary user i.

We consider a CRN where a subset of primary users are inactive at each time slot. Each cognitive user can access a primary channel if the channel is accessible and if its corresponding primary user is inactive. However, at the same time it should not interfere with transmissions of other active primary users. We define an interference matrix $\mathcal{G}(i, a)$ which
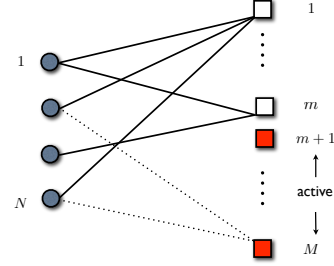

Fig. 2. A factor graph representation of CR network. Solid links represent the accessibility of the idle primary channels to secondary users ($\mathcal{I}(i, a)$). Dashed lines represent possible interference experienced at active primary users due to transmissions of secondary users ($\mathcal{G}(i, a)$).

measures the interfere experienced at primary user a when secondary user i is connected to a free channel. Note that $\mathcal{G}(i, a)$ is different from the accessibility matrix $\mathcal{I}(i, a)$ because a secondary user might not have access to a primary channel ($\mathcal{I}(i, a) = 0$) but have a non-zero interference with (active) primary user a ($\mathcal{G}(i, a) \neq 0$) when it is connected to another free channel. To each secondary user i, we assign a binary variable s_i where $s_i = 1$ indicates secondary user i connected to a free channel and $s_i = 0$ stands for the case where i is not connected to any primary channel. We now introduce a factor graph representation of the CRN (Fig. 2). It is a bipartite graph which includes secondary and primary users as different factor nodes, shown by circles and squares, respectively. Secondary and primary users are connected to each other according to the accessibility and interference matrices. We assume that the first m primary users are idle at the time and only $M - m$ primary users are active (indexed by $(m + 1, ..., M)$).

In order to distinguish which primary channel is assigned to a secondary user, we introduce variable $\sigma_{i,a} = \{0, 1\}$ for user $i \in \{1, ..., N\}$ and its accessible free primary channel $a \in \{1, ..., m\}$ such that $\sigma_{i,a} = 1$ when i is connected to a. By definition we have

$$s_i = \Theta\left(\sum_{a=1}^{m} \sigma_{i,a} \, \mathcal{I}(i, a)\right) \qquad (1)$$

where Θ is the step function. The ultimate goal is to efficiently connect secondary users to free primary channels. Secondary users might have different priority to be connected according to their different tasks. The problem then can be expressed as an optimization problem to find the best assignment $\vec{s} = \{s_1, ..., s_N\}$ such that the following cost function is minimized

$$\mathcal{H}(\vec{s}) = -\sum_{i=1}^{N} s_i c_i \qquad (2)$$

Here c_i is the task (priority) of secondary user i. Clearly secondary users with higher priority have preference to be connected. However the interference and accessibility constraints must both be satisfied which makes the problem hard. Note that it is possible to extend the current prior task vector to more

general cases where secondary users have different priorities to access different unused channels.

In what follows we describe two different optimization objectives in solving the CRN optimization problem. The first approach is based on hard constraints induced by active primary users while the second is based on soft constraints where interferences between secondary users and primary users are tolerated but the cost function is modified accordingly.

A. Model A

To formulate the hard interference constraints imposed by active primary users, we introduce a quenched variable for every primary user $\vec{\theta} = \{\theta_1, ..., \theta_M\}$ to indicate the maximum interference tolerated. We assume that the sum over interferences from all secondary users connected to free channels must be smaller than the active primary user threshold. The optimization problem then is to minimize the cost function Eq. 2 such that following conditions are satisfied, which must be solved in conjunction with Eq. 1:

- hard interference constraints imposed by active primary users

$$\sum_{i=1}^{N} \mathcal{G}(i,a) s_i \leq \theta_a \qquad \text{for } a \in \{m+1, ..., M\} \quad (3)$$

- each secondary user is connected to at most one free channel

$$\sum_{a=1}^{m} \mathcal{I}(i,a) \, \sigma_{i,a} \leq 1 \qquad \text{for } i \in \{1, ..., N\} \quad (4)$$

- and each free channel to at most one secondary user

$$\sum_{i=1}^{N} \mathcal{I}(i,a) \, \sigma_{i,a} \leq 1 \qquad \text{for } a \in \{1, ..., m\} \quad (5)$$

B. Model B

An alternative approach is to relax the hard interference constraints between secondary users and active primary channels. We modify the cost function such that the solution minimizes the interference between secondary users and active primary channels. We consider a quadratic cost of the interferences between SUs and active PUs.

$$\mathcal{H}(\vec{s}) = -\sum_{i=1}^{N} s_i c_i + \sum_{a=m+1}^{M} \frac{1}{\theta_a} \left(\sum_{i=1}^{N} \mathcal{G}(i,a) s_i \right)^2$$

$$\mathcal{H}(\vec{s}) = -\sum_{i=1}^{N} s_i c_i + \sum_{a=m+1}^{M} \frac{1}{\theta_a} \sum_{i,j=1}^{N} \mathcal{G}(i,a) \mathcal{G}(j,a) s_i s_j \quad (6)$$

where the goal is to find $\vec{s}$ that minimizes the cost function (Eq. 6), conditioned on constraints introduced in Eqs. 4 and 5. This objective represents a trade-off between maximizing the number of connected secondary users and minimizing the induced interference.

In both models, we assume a stationary state described by the Boltzmann distribution $p(\vec{s}) = \frac{1}{Z} e^{-\beta \mathcal{H}(\vec{s})}$, where the parameter β is analogous to inverse temperature and is a measure of how strictly the minimization in enforced. In the limit $\beta \rightarrow \infty$, it will be concentrated to the minimum of the cost function.

III. BELIEF PROPAGATION

In a system with a given cost function, the problem of finding the global minimum or calculating marginal probabilities can be solved approximately by the BP algorithm.
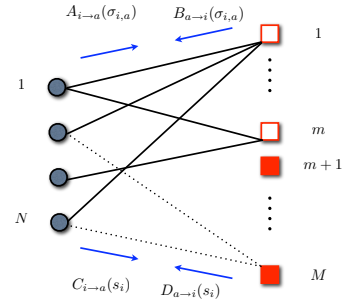

Fig. 3. Messages are exchanged among secondary users and available primary users and between secondary users and active primary users. They fulfill a set of consistent equations and correspond to the BP approximation of the corresponding CRN.

To implement BP we exchange a set of messages between the different nodes in a factor graph. The type of messages to be exchanged between secondary users and active primary users are essentially different from those exchanged between secondary users and inactive primary users. To an active primary user the important information is whether a secondary user with non-zero interference is connected to any free channel, irrespective of the channel. On the other hand, for free channels it is important to take into account the information about the specific connections (see constraints 4 and 5). We introduce four different messages as follows

- $A_{i \rightarrow a}(\sigma_{i,a})$ is a messages sent from secondary user i to free channel a with non-zero accessibility metric $\mathcal{I}(i,a) \neq 0$. It gives the probability of observing $\sigma_{i,a}$.
- $B_{a \rightarrow i}(\sigma_{i,a})$ is the reverse message sent from available primary user a to secondary user i.
- $C_{i \rightarrow a}(s_i)$ is a message sent from secondary user i to active primary user a with non-zero interference metric ($\mathcal{G}(i,a) \neq 0$). It gives the probability of secondary user to be connected (not-connected) to one of its available free channel $s_i = 1$ ($s_i = 0$).
- $D_{a \rightarrow i}(s_i)$ is the corresponding reverse message from active primary user a to secondary user i.

These BP messages fulfill a set of closed equations (Eq. 7-11).

Belief propagation equations are to be solved iteratively. We assign an initial (unbiased) condition to each message and iterate all messages until a fixed point is reached. This is a fully decentralized procedure and can be applied efficiently. Once the fixed point is reached, the marginal probabilities for each secondary user can be computed. Each marginal probability represents the probability of connecting corresponding secondary users to a free channel. Belief propagation algorithm is exact for trees and is expected to work for random graphs with locally tree-like structures. Cognitive radio networks are two dimensional networks and hence can not be considered

$$A_{i \to a}(\sigma_{i,a}) \;\propto\; \sum_{\{\sigma_{i,b}\} \backslash \sigma_{i,a}} e^{\beta s_i c_i} \prod_{b=1 \neq a}^{m} B_{b \to i}(\sigma_{i,b}) \prod_{b=m+1}^{M} D_{b \to i}(s_i)\; \mathbb{1}\left(\sigma_{i,a} + \sum_{b=1 \neq a}^{m} \sigma_{i,b}\, \mathcal{I}(i,b) \leq 1\right) \tag{7}$$

$$B_{a \to i}(\sigma_{i,a}) \;\propto\; \sum_{\{\sigma_{j,a}\} \backslash \sigma_{i,a}} \prod_{j \neq i} A_{j \to a}(\sigma_{j,a})\; \mathbb{1}\left(\sigma_{i,a} + \sum_{j \neq i} \sigma_{j,a}\, \mathcal{I}(j,a) = 1\right) \tag{8}$$

$$C_{i \to a}(s_i) \;\propto\; \sum_{\{\sigma_{i,b}\}} e^{\beta s_i c_i} \prod_{b=1}^{m} B_{b \to i}(\sigma_{i,b}) \prod_{b=m+1 \neq a}^{M} D_{b \to i}(s_i)\; \delta_{s_i, \Theta\left(\sum_{c=1}^{m} \sigma_{i,c}\, \mathcal{I}(i,c)\right)} \tag{9}$$

$$D_{a \to i}(s_i) \;\propto\; \sum_{\vec{s} \backslash s_i} \prod_{j=1 \neq i}^{N} C_{j \to a}(s_j)\, \mathbb{1}\left(\sum_{j} \mathcal{G}(j,a)\, s_j \leq \theta_a\right) \quad \text{Model A} \tag{10}$$

$$D_{a \to i}(s_i) \;\propto\; \sum_{\vec{s} \backslash s_i} \prod_{j=1 \neq i}^{N} C_{j \to a}(s_j)\; e^{-\frac{\beta}{\theta_a} \mathcal{G}(i,a) s_i \left(\sum_{j=1}^{N} \mathcal{G}(j,a) s_j\right)} \quad \text{Model B} \tag{11}$$

as random graphs. However, accessibility matrix in CRNs is not highly structured and one might expect to get promising results for Belief propagation.

Note that in order to update message $D_{a \to i}$ we have to take a summation over $2^{\mathcal{O}(K)}$ where K is the average connectivity for active primary users. For sparse interference/connectivity matrices the algorithm provides a fast solution while for dense matrices the corresponding computational complexity grows exponentially and becomes infeasible.

For Model B, we will show that computational complexity remains manageable even in the regime where active primary users experience interference from many secondary users. To simplify the BP equations we introduce the BP field messages $h_{i \to a} = \frac{1}{\beta} \ln\left(\frac{A_{i \to a}(1)}{A_{i \to a}(0)}\right), h_{a \to i} = \frac{1}{\beta} \ln\left(\frac{B_{i \to a}(1)}{B_{i \to a}(0)}\right), q_{i \to a} = \frac{1}{\beta} \ln\left(\frac{C_{i \to a}(1)}{C_{i \to a}(0)}\right), q_{a \to i} = \frac{1}{\beta} \ln\left(\frac{D_{i \to a}(1)}{D_{i \to a}(0)}\right)$. The corresponding BP equations for model B become

$$h_{i \to a} = -\frac{1}{\beta} \ln\left[e^{-\beta\left(c_i + \sum_{b=m+1}^{M} q_{b \to i}\right)} + \sum_{b=1 \neq a}^{m} e^{\beta h_{b \to i}}\right]$$

$$h_{a \to i} = -\frac{1}{\beta} \ln\left[1 + \sum_{j \neq i} e^{\beta h_{j \to a}}\right] \tag{12}$$

$$q_{i \to a} = -c_i - \sum_{b=m+1 \neq a}^{M} q_{b \to i} - \frac{1}{\beta} \ln\left[\prod_{b=1}^{m}\left(1 + e^{\beta h_{b \to i}}\right) - 1\right]$$

$$q_{a \to i} = -\frac{1}{\beta} \sum_{j \neq i} \ln\left[\frac{\exp\left(\beta\left(q_{j \to a} - \mathcal{G}(i,a)\mathcal{G}(j,a)/\theta_a\right)\right) + 1}{\exp\left(\beta q_{j \to a}\right) + 1}\right]$$

The advantage of this new version of BP is that all equations involve summations over a finite number of terms.

IV. SIMULATION RESULTS

In this section, we investigate the performance of BP and compare it to a greedy benchmark algorithm. We show how BP outperforms the latter in finding the better assignment of secondary users to the available free channels.

We consider a cell-based simulation scenario, where base stations are placed uniformly at random in a unit square (accounting for irregular urban terrain) and each base station has one associated channel allocated to a primary user. The secondary users attempt to opportunistically send data to the base-stations using idle primary channels; we thus concentrate on uplink communication.

Power attenuation at distance r is $g \cdot r^{-\alpha}$, where $\alpha \geq 2$ is the path-loss exponent (we take $\alpha = 3.5$) and g is a non-negative random variable accounting for fading and shadowing, with unit mean. We assume a Rayleigh fading inspired model, where g is distributed exponentially and is i.i.d. for each secondary user/base-station pair. We also consider a cut-off value below which the received power is assumed to be 0 (this is realistic in practice, and the sum of interference contributions below the threshold can be assumed to be taken into account as background noise). Secondary users can connect to a free channel if the SNR (signal to noise ratio) from the corresponding base-station is above a predefined threshold. Similarly the interference constraint is a different predetermined threshold.

We also propose a greedy algorithm strategy as a simple benchmark to compare with our message passing approach. For Model A, the algorithm is the following: we iteratively choose among the remaining links the one of largest weight (i.e., largest c_i) and mark it as active as long as the active links constitute a matching and the interference constraints
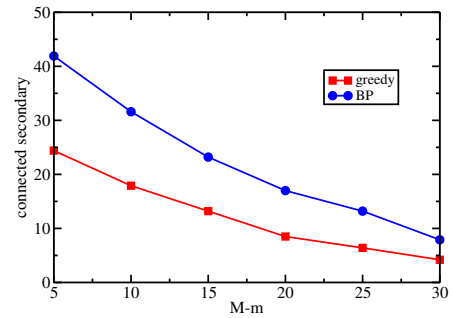

Fig. 4. The number of connected secondary users versus the number of active primary users in a CRN with $N = 100$ and $M = 50$. The blue line represents the result obtained by BP while red line represents the performance of the greedy algorithm. Results are averaged over 10 different network realizations with the same set of parameters.
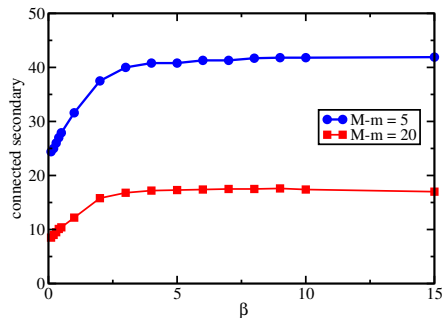
Fig. 5. Performance of BP (Model A) for different values of β. The results improve by increasing β as the optimal solution is expected to be found at $\beta \to \infty$. Simulation results show convergence as β increases.

Fig. 7. The interference between connected secondary and active primary users in a cognitive radio networks with $N = 100$, $M = 50$ and $M - m$=5. The results are obtained by running BP on 10 different networks realizations. The inset shows the number of connected secondary users.

are satisfied for all active primary users; otherwise, the link is removed from the remaining link set. To solve model B, we apply the same algorithm with a simple modification: for each link we subtract from its weight the sum of squared interference it generates and omit the interference constraints.

Through our simulations we set the priority of secondary users to 1 for all users ($c_i = 1$) therefore the absolute value of cost function in model A is equivalent to the number of connected secondary users (see Eq. 2). The higher this number is the better the algorithm performs. All interference thresholds are also set to 1 ($\theta_a = 1$). Fig. 4 compares results obtained by BP and the greedy algorithm for a cognitive radio networks with $N = 100$ secondary and $M = 50$ primary users. As we increase the number of active primary users ($M - m$), we end up with less free channel frequency to be shared with secondary users and hence the number of connected secondary users decreases. For all values of active primary users BP outperforms the greedy algorithm as is shown in the figure.

To investigate the optimal solution found by BP we plot the results obtained for various values of β (Fig. 5). As mentioned earlier, we expect to find the optimal solution by taking the limit $\beta \to \infty$. However, the number of connected secondary users converges already for $\beta \geq 5$.
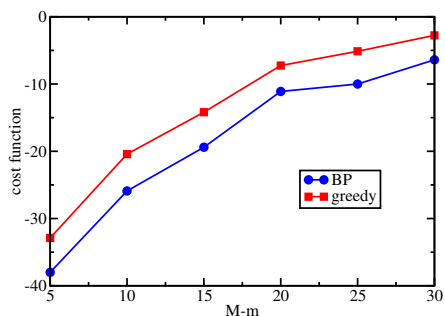

Fig. 6. The minimal cost achieved for model B (Eq. 6) in a cognitive radio networks with $N = 100$ secondary and $M = 50$ primary users. Results obtained by BP (blue red) show better performance compared to the greedy algorithm (red line). Results are averaged over 10 different network realizations with the same set of parameters.

Similar behavior is observed for Model B with soft interference constraints. Fig. 6 compares results obtained by BP and the greedy algorithm. It shows the optimal cost value obtained
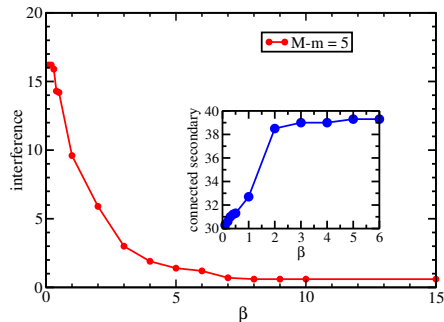
by the different methods for various numbers of active primary users. For all parameters BP finds a better solution (lower minimal cost) compared to the greedy algorithm.

The performance of BP in model B is examined by monitoring the two parts of cost function: interference and the number of connected secondary users for different values of β. As expected, the interference part of cost function ($\sum_{i,j} \sum_a G(i,a)G(j,a)s_i s_j$) vanishes as β increases. Figure 7 shows the average results for 10 realizations of cognitive networks with $N = 100$, $M = 50$ and $M - m = 5$. As we increase β the interference decreases and the number of connected secondary users increases.

V. CONCLUSION

We have applied BP to find the optimal resource utilization in cognitive radio networks. We have cast the task into a optimization problem where the number of connected secondary users is to be maximized under hard and soft interference constraints. A greedy algorithm is used as a benchmark for the BP results; the latter outperforms the former in all the experiments carried out and is computationally efficient.

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