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PORTFOLIO ANALYSIS

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In this dissertation, the techniques of portfolio analysis have been considered and the merits of the various models found in the literature are discussed. Portfolio analysis involves the determination of the most suitable combination of specific shares that should be included in the portfolio of an investor at any point in time.

The main purpose of this study has been to compare the suitability of the following five methods of portfolio selection:

- 1) The Filter Method
- 2) The Valuation Method
- 3) The Markowitz Model
- 4) The Sharpe Model
- 5) A Monte Carlo Technique.

Of these methods the Markowitz and the Sharpe approaches might be termed as classical models - and the Valuation and Monte Carlo methods as developments proposed and tested by the author. The ad-hoc methods were used as a crude strategy against which the efficiency of the more complex method could be measured empirically.

Incidental to the main study, an examination of certain of the constructs of Investment Analysis was necessary (e.g. a test of the "Random walk hypothesis").

The Markowitz and the Sharpe models have each been developed into a composite routine. The algorithms were compared with programmes utilising more general quadratic programming routines.

The Markowitz and Sharpe models, as developed, appeared to be consistently better than other methods. Since

the Sharpe model is considerably cheaper to operate than Markowitz's model, it may be concluded that Sharpe's model is the most efficient of those methods tested.

█ IN LOVING MEMORY OF

MY MOTHER █

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CHAPTER I.

INTRODUCTION

1) "October. This is one of the peculiarly dangerous months to speculate in stocks. The others are July, January, September, April, November, May, March, June, December, August and February" - Mark Twain.

All the models described in the literature for portfolio investment analysis rely on historical data relation to compare performance e.g. Share prices.

The term 'portfolio' means a list of "good" stocks and shares. It is a balanced whole, and at the same time it protects the investor in volatile situations. A portfolio analyst generally starts with information concerning individual shares, and he ends up with conclusions concerning portfolios. The job of the portfolio analyst is to find that portfolio which best meets the objectives of the investor.

The information regarding various types of shares constitutes one of the most important raw materials of a portfolio analyst. One source of such information is the Stock Exchange. The Stock Exchange is the market in which shares are bought and sold. It enables capital accumulation by governments, municipalities and public companies. It is one of the major institutions on which the national and the international economies are based. The Stock Exchange is notoriously short sighted, and it is inclined to deal with the future regardless of past experiences.

The price of a share, like the price of anything else, depends on supply and demand on a large number of factors, for example, past performance of the company, rate of dividend paid, long term prospects of the company and the market in which it operates.

Many investors, both private and institutional, buy and sell shares in the Stock Exchange through their brokers.

The collection of shares that an investor owns at any time is referred to as his portfolio. Owing to the wide diversity in personal and financial circumstances, the owners of portfolios have different requirements. Some need high income and no capital growth, while for others, it is the reverse. The optimum, however, would require a mixture of these two extremes.

Whatever the investor's requirements may be regarding the performance of his portfolio, it is certain that he will wish to base his selection of shares on the fullest information available. This information may take one of the following two forms:

- (i) Historical data
- (ii) Personal assessment.

The theory of portfolio selection involves the precise determination of the combination of specific shares that should be in the portfolio of an investor at any point in time.

In recent years, a great deal of theoretical and empirical work has been done to formulate mathematical models of portfolio behaviour. The objective of these models has been to suggest specific ways of selecting the components of a share portfolio for a given level of risk.

The aim of this dissertation is to find a suitable forecasting technique and to develop computer application programmes for the classical methods of portfolio selection. This dissertation also deals with 'Investment Analysis' in which, amongst other things, past annual rates of return are used for forecasting the next year's performance of individual shares.

Chapters II and III describe the essential elements of investment analysis. In the first part of Chapter II, the use of multiple linear regression analysis for price forecasting is discussed. Later in the chapter, the validity of

random walk hypothesis is examined.

Chapter III presents methods of portfolio selection based on the consideration of a single share. For purposes of comparison, several ad-hoc methods are proposed in the first section. Alexander's⁽¹⁾ filter technique is reviewed and applied in the second section. In the last part of the chapter, a new model of portfolio selection, known as the valuation method, is proposed.

Portfolio theory is concerned with decisions involving outcomes that cannot be predicted with complete certainty. The essential elements of portfolio theory were developed by Markowitz⁽¹⁶⁾ in 1952. In 1958, Tobin⁽²⁴⁾ made the first practical attempt to use the theory for a positive capital market model. The actual method put forward by Markowitz was not very practical. Sharpe's⁽²²⁾ diagonal model was able to overcome the computational problem that arose in the Markowitz model.

Since then, many others have contributed to this study. It may be noted that Markowitz's contribution in this field is fundamental in nature. Others have extended, modified and tested his original theory, but the essence still remains unchanged.

In Chapter IV, the classical models of the Markowitz and the Sharpe are discussed, and then, based on the above models, algorithms and computer programmes much more efficient than those in existence, have been developed.

In Chapter V, a new simulation model (based on Sharpe's model) has been developed. This hopefully will prove to be an improvement on the classical methods.

Data collection techniques, empirical results and a discussion of these are given in Chapter VI.

The algorithms of the Markowitz and the Sharpe models

are developed in appendices A1 and (A2,A3) respectively, while appendices A4, A5 and A6 contain the flow charts of the programmes used in the models of the Markowitz, Sharpe and filter methods.

Appendix A7 contains the programme codes used in this dissertation and the specimen outputs of the programmes. All the programmes are written in Fortran IV.

1.1) Achievements and Conclusions.

a) Multiple Regression Analysis.

The use of general economic indicators such as Gross National Product is not satisfactory. The share index indicators are more useful.

b) The Random Walk Model.

The random walk hypothesis appears to be valid in the short term. An attempt was made to fit a normal distribution to share price fluctuations, but the fit was not good. The fit was improved by using logarithms of share price fluctuations, but it still remains, in general, unsatisfactory.

c) The Ad-hoc Methods.

The ad-hoc methods are used as crude measuring device, against which the sophisticated methods could be measured empirically. Results obtained are better than expected.

d) The Filter Method.

The effectiveness of Filter Method depends upon the transaction cost.

e) The Valuation Method.

The Valuation Method is not yet fully developed as regards the most appropriate combination of risk and

1.1) contd.

e) contd.

growth factors. However, the present investigation indicated the usefulness of this approach to further research.

f) The Markowitz and the Sharpe Models.

The algorithms and computer programmes based on the above models have been satisfactorily developed. These programmes can also be employed as predictors of share market.

g) A Monte Carlo Technique.

The Monte Carlo model is found to be a good method of share selection. Its disadvantage lies in its high cost.

h) Comparisons.

The empirical results show that the Sharpe and the Markowitz models are the most efficient amongst all the methods considered. The Sharpe model is preferred to the Markowitz model because of its smaller computing cost.

CHAPTER II.

INVESTMENT ANALYSIS

2.1) Investment Analysis.

2.1.1) Introduction.

As mentioned in the introduction, investment analysis is concerned with studying the behaviour of a single share. The history of investment analysis is somewhat simpler than that of portfolio analysis - where emphasis is on the interaction between the component shares in the portfolio.

The classic paper by Bachiler⁽²⁾ is essentially taken to be one of the pillars of the study of investment analysis. It is usual to consider investment analysis prior to portfolio analysis. In the crude sense investment analysis is used to censor individually each share that goes on a broker's 'acceptable' list (e.g. only those shares with a certain capital, dividend growth plus earnings may be acceptable). In the more sophisticated models, investment analysis may be used to calculate lagged regression forecasts for input into Markowitz or Simulation analyses.

There are two main schools of thought amongst investment analysts. Some believe that the share prices may be predicted from a technical or mechanical study of past prices. Others believe that share prices vary in a random fashion.

In this chapter we shall attempt to present evidence of both types of study and draw conclusions as to the most acceptable on the basis of empirical verification.

The first part of the chapter will consider the regression model of share prices and the second part will deal with the random walk model.

2.1.2) Economic and Sector Analysis - including some evaluation of the equations as predictors.

In this section a statistical method of forecasting will be described. The analysis of the stock market requires the study of an ordered sequence of share data. The sequence of closing price daily, weekly, monthly and quarterly intervals may be studied.

There are two types of forecasting indicator:-

- a) General or economic relating to the whole of the economy, for example forecasting the Gross National Product (G.N.P) for the next year as a measure of the economic climate in general.
- b) Specific indicators relating to the particular subdivision of the economy in which the share of interest is to be found.

In order to predict the share prices, multiple linear regression analysis is used. Cohen, Gomme and Kendall⁽³⁾ used lagged economic relationships for forecasting future share prices. They used regression analysis for varying time lags and calculated multiple regression. They reported that the lagged relationships produced a good forecast and concluded it deserved serious consideration for short term (one or two years) economic forecasting.

Regression fit.

To fit the data in the equation

$$P = a_0 + a_1x_1 + \dots + a_6x_6$$

where

P = Share price

a_i = Regression Constants

x_i = Different variables as described below

2.1.2) contd.

The following data are used for x_i

- 1) Gross National Product
- 2) The Unemployment Rate
- 3) United Kingdom Car Production
- 4) The Bank Rate
- 5) Financial Times Index (F.T.I)
- 6) Special Share Index (Industrial Share Price Index)
 - a) Chemicals
 - b) Engineering
 - c) Insurance
 - d) Motor
 - e) Newspapers
 - f) Household Goods
 - g) Textiles
 - h) Tobacco
 - i) Investment Trust
 - j) Electrical
 - k) Food Manufacturing
 - l) Banks
 - m) Wines and Spirits
 - n) Stores
 - o) Oils
 - p) Electronics, Radio and T.V.
 - q) Machine Tools

The quarterly data of fifty equity shares for the years 1968 to 1970 were computed to get multiple correlation coefficients and 't' statistics by using different parameters. This is shown in Table 1.

The results will be discussed in Chapter VI.

2.2) The Random Walk Theory of Stock Market Price.

2.2.1) Introduction.

For many years economic statisticians have been interested in the subject of price fluctuations in the stock market. One important model that has recently evolved from this research in the theory of random walks or the random walk-hypothesis. This hypothesis supposes that stock prices respond quickly to new information as it becomes available and that new items of information enter the market place in a random fashion. Specifically, periodic changes in price - hourly, daily, weekly or monthly are independent of price changes during equivalent preceding periods. In other words day to day fluctuations in stock market prices are indistinguishable from numbers produced by some chance process.

The random walk hypothesis does not deny the possibility of correctly forecasting stock prices. It accepts the principle that investment analysts who can forecast company earning and dividends accurately should do a fairly good job of forecasting stock prices. What it denies is that the analysis of past data, particularly past market price data, can produce better than chance price forecasts. Most of the statistical investigations of the random walk hypothesis confirm the belief that successive price changes are statistically independent.

According to Levey⁽¹³⁾ there are several methods of testing the random walk hypothesis to find whether technical analysis is a desirable method of earning income in the stock market. Two of these methods

2.2.1) contd.

are statistical in nature. First, serial correlation studies attempt to measure the relationship between successive first difference in share prices, and second, runs analyses attempt to compare actual lengths of runs with mathematically determinable expected length. (Runs analysis measure the number of steps (or changes) taken in the same direction and compare this observed number with the number that would have occurred if the process is purely random). Academics who have performed these two types of statistical investigation have been virtually unanimous in their conclusions. They have contended that stock market price variations do follow a random walk and thus technical analysis cannot be used to produce a profit greater than those which would be produced by random selection.

There are many serious weaknesses in statistical test. There are as follows:-

- a) They are not able to detect the nonlinear patterns which the chartists claim exist.
- b) In runs analyses, they are too inflexible.
- c) Statistical tests are very difficult to interpret. Just how large must a serial correlation coefficient be to determine that technical analysis can produce satisfactory profits? Just how much just the actual length of run exceed the expected length of run to determine that the market is sufficiently non-random to allow a profitable forecasting of future prices based upon past prices? (Indirectly connected with this problem of interpretation is the inability of the statistical tests to account for brokerage

2.2.1) contd.

c) contd.

commissions. It would be far better to produce results in terms of pounds and pence than in terms of serial correlation coefficients and related statistical measures).

- d) The statistical tests do not consider the element of risk.
- e) They fail to correct for the co-movement of stock prices.
- f) Statistical tests are often not understandable to market practitioners who do not have any extensive background in statistics and mathematics.

Levy suggested simulation method of testing the random walk hypothesis that avoids these weaknesses of statistical test. Simulation has these advantages: it can detect nonlinear patterns, it can test for relative strength (thus filtering out the co-movement of stock prices), it can take into account brokerage commissions and express results in terms of pounds and pence, and it can measure the risk of given investment strategies. Very little work has been done in the area of simulation. Cootner⁽⁴⁾ has tested average trend indicators and Alexander⁽⁴⁾ has tested a device known as the filter technique. Their conclusions have been similar - that these technical strategies can produce profits which are greater than the profits which would be produced by naive buy-and-hold policy but that the superior profits are not sufficient to cover transaction costs. Cootner's and Alexander's tests were certainly far from exhaustive. In fact, whether one uses statistical testing or simulation,

2.2.1) contd.

the random walk hypothesis cannot be proved. All that can be indicated is that the specific strategies investigated do not yield greater profits than those which would be yielded by a random method of selection.

2.2.2) Serial Correlation of Price Difference.

The first and most obvious test of the random walk model is to look at estimates of serial correlation for various share prices and various values of time.

Let given price data be P_t ($t = 1, 2 \dots, n$), then estimates of serial correlations are found by forming the expressions

$$X_t = P_t - P_{t-1}$$

$$\bar{X} = \frac{1}{n-1} \sum_{t=1}^{n-1} X_t$$

$$X' = X_t - \bar{X} \text{ and then}$$

$$r_s = \frac{\sum X'_t X'_{t-s}}{\sum (X'_t)^2}$$

where

$$X_t = \text{price difference between the time}$$

$$\bar{X} = \text{mean price difference}$$

$$\text{and } r_s = \text{serial correlation.}$$

A number of writers have found estimates of serial correlation coefficients for various values of time.

The estimates are consistently near zero and so are generally in accordance with the random walk

2.2.2) contd.

model. It is rather difficult however to decide whether the value obtained gives any indication of being significantly non-zero. For every large sample, if the true value is zero, every estimate should be approximately normally distributed with mean zero and variance n^{-1} .

There is no consensus among the studies about whether or not there is a predominance of estimates of one sign. Fama⁽⁷⁾ (T = one day) and Kendall⁽¹¹⁾ (T = one week) found the majority of their estimates to be positive, whereas Cootner⁽⁴⁾ and Moor⁽¹⁹⁾ (T = one week) and King⁽¹²⁾ (T = one month) as well as Fama (T = four and nine days) found negative estimates to be most common.

In our estimates we got four positives in 1959 and one positive in 1964 and one positive in 1968 out of nine shares. The average values of correlations are shown in Table 2.

2.2.3) The Random Walk Model.

This model originates from considerations of short term change in share prices. To illustrate, suppose we wish to know tomorrow's price of common stocks. To know tomorrow's price, it is sufficient to know what particular increment (Δ) to add (or subtract from) today's price, i.e.

$$P_{t+1} = P_t + \Delta$$

where

P_{t+1} = price as of time t+1

P_t = price of time t

Δ = increment of change in price.

On the listed stock exchanges, prices are

2.2.3) contd.

set by buyers and sellers. There are no secret deals. Under these conditions, with the price (P_t) being set in a free market, and with all interested parties knowing P_t , then the price (P_t) is used as a starting point for negotiation of price of the next trade, P_{t+1} . It is the change and not the absolute value, which constitutes the fundamental element in price determination in the short run. If the change (Δ) is known and is constant, then since P_t is always known, tomorrow's price (P_{t+1}) is known.

Price change (Δ) can be measured in different ways; for example:

- a) Simple arithmetic change: $\Delta = P_{t+1} - P_t$
- b) Percentage change: $\Delta = \frac{(P_{t+1} - P_t)}{P_t} = \left(\frac{P_{t+1}}{P_t}\right) - 1$
- c) Relative change: $\Delta = (P_{t+1}/P_t)$
- d) Logarithm of relative change:

$$\Delta = \log(P_{t+1}/P_t) = \log(P_{t+1}) - \log(P_t)$$

It is also possible to vary the length of the difference interval:-

- (i) Daily changes
- (ii) Weekly changes
- (iii) Monthly changes.

The general random walk model involves three distinct hypotheses:

- 1) Any given set of change in price conforms to some probability distribution. The particular shape of the distribution is not specified by the theory but is supposed by many authorities to be normal.

2.2.3) contd.

- 2) The probability distribution is stationary.
- 3) Successive price changes are independent of each other
Stationarity.

This implies that there is no trend in either the mean or the variance of the distribution of change over time. The total difference in price over some time span ($t - \epsilon$) must equal the sum of the incremental difference of the span, i.e.

$$P_t - P_{t-\epsilon} = \sum_{j=t-\epsilon+1}^t \Delta_j$$

By stationarity we mean that if the process generating the observed changes in price is stationary, then neither the particular starting time (t) nor the span of time ($t-\epsilon$) is of particular significance in determining either the mean or the variance distribution of changes. If stock market prices do change in a purely random way, then the average change over some time interval (ϵ) will be zero. The general consensus among authorities is that the empirical distributions of short-term changes in price of risky shares are in fact approximately stationary.

Assumption of Independence.

In statistical terms independence means that the probability distribution for the price changes during time period t is independent of the sequence of price changes during the previous time periods. In other words, independence means no serial correlation between successive changes in price.

2.2.4) Empirical Evidence.

Empirical evidence to support the assumption of independence has been presented by many scholars. Fama⁽⁸⁾ for example examined independence between successive price changes for each of the 30 stocks contained in the Dow Jones Industrial average. For each individual stock and for the group as a whole, Fama studied differencing interval of 1,4,9 and 16 days using methods of

- (1) Serial correlation measures
- (2) Runs analysis
- (3) Alexander's filter technique.

M.G.Kendall⁽¹¹⁾ studied time series of various Δ_t by applying standard statistical procedures. He used 22 different industry aggregate price series, taken at weekly intervals from 486 to 2387 terms, to study the behaviour of short term changes in price. He concluded that there is no reason to hope to be able to predict price movements on the exchange for a week ahead, using historical price alone.

Further work by Osborne⁽²⁰⁾ strengthens the random walk hypothesis from a different point of view. While Kendall worked with serial correlations for each series separately, Osborne worked with general effect of price changes. Osborne, however, was concerned with the logarithm at price differences. It must be noted that the Osborne measurements do not concern trends in prices of stocks but merely the statistical distribution of the changes in the logarithms, which as Osborne pointed out, correspond quite closely to percentage changes.

Bachelier⁽²⁾, the pioneer in speculative

2.2.4) contd.

price behaviour as a random walk, derived the theoretical properties of the distribution of changes in the prices. Unfortunately Bachelier's work did not receive much attention from economists, and in fact his model was independently derived by Osborne⁽²¹⁾ over fifty years later. The Bachelier-Osborne model assumes that price changes from transaction to transaction in an individual share are independent, identically distributed random variables. It assumes that transactions are fairly uniformly spread across time, and that the distribution of price changes from transaction to transaction has finite variance. If the number of transactions per day, week or month is very large, then price changes across these differencing intervals will be sums of many independent variables. Under these conditions the central-limit theorem leads us to expect that the daily, weekly and monthly price changes will each have normal distribution.

2.2.5) The Stable Paretian Model.

A particularly interesting and controversial model of stock price changes is the Stable Paretian Model in which log price is assumed to be a random walk but the changes come from a distribution with infinite variance. Mandelbort⁽¹⁵⁾ is the originator and chief protagonist of this model and Fama^(5,6) has provided the best expositions and empirical evidence in favour of it. The Stable Paretian distribution was first studied by Levy⁽¹³⁾ and arises in studies of some general forms of the central limit theorem.

It may be shown that if x_1, x_2, \dots, x_n are independently and identically distributed random variables

2.2.5) contd.

then as n tends to infinity the distribution of their sum, if it exists at all, will be a member of the stable Paretian family.

The logarithm of the characteristic function for the stable Paretian family of distributions is

$$\begin{aligned} \log \phi(t) &= \log \int_{-\infty}^{\infty} \exp(iut) dP(\bar{u} < u) \\ &= i\delta t - \gamma |t|^\alpha [1 + i\beta(t+|t|) \tan(\alpha\pi/2)]. \end{aligned}$$

The above distributions have four parameters; α, β, δ and γ .

- 1) A measure of the height of the extreme tail areas of the distribution which we shall call the characteristic exponent α .
- 2) An index of skewness, β .
- 3) A location parameter which we shall call δ .
- 4) A scale parameter henceforth called γ .

α is the most important among the four when the characteristic exponent α is greater than 1, the location parameter δ is the expectation or mean of the distribution. The scale parameter γ can be any positive real number, but β , the index of skewness, can take any value in the range $-1 \leq \beta \leq 1$. When $\beta = 0$ the distribution is symmetric. When $\beta < 0$ the distribution is skewed to the left and when $\beta > 0$ it is skewed to the right. The absolute value of β is a measure of skewness.

The characteristic exponent α of a Stable Paretian distribution determines the height of, or total probability contained in the extreme tails of the distribution. The values of α lies $0 < \alpha \leq 2$. In the special case $\alpha = 2$,

2.2.5) contd.

the Stable Paretian distribution is normal. When α is in the interval $0 < \alpha < 2$, the extreme tails of the Stable Paretian distributions are higher than those of the normal distribution, and the total probability in the extreme tails is larger than the smaller value of α . The most important consequences of this is that the variance exists (i.e. is finite) only the extreme case $\alpha = 2$. The mean, however, exists as long as $\alpha > 1$.

Mandelbort's hypothesis states that for distributions of price changes in speculative series, α is in the interval $1 < \alpha < 2$, so that the distributions have means but their variances are infinite. The Gaussian hypotheses, on the other hand, state that α is exactly equal to 2. Thus both hypotheses assume that the distribution is Stable Paretian. The disagreement between them concerns the value of α .

Then the general notation of the theory of random walks in share prices is that the changes in price conform to some probability distribution.

In conclusion we can say if a price series obeys a random walk, it is statistically completely characterized by the distribution of price differences. It is, therefore, important to consider the properties of this distribution.

The results will be discussed in Chapter VI. Computer programmes numbers 2 and 3 are used in these models and the specimen output of programme number 2 is given in Appendix A7.

C H A P T E R I I I .

METHODS OF PORTFOLIO CHOICE BASED ON SINGLE
SHARE CONSIDERATIONS .

3) Introduction.

This chapter deals with the choice of portfolio based on single share consideration (i.e. no correlations taken into account). In Section 3.1 ad-hoc rules for comparison purposes are noted. In Section 3.2 the optimum filter rule has been discussed.

3.1) AD-HOC METHODS OF ASSESSMENT.

3.1.1) Introduction.

In order to measure portfolio performances as well as to compare methods of analysis to see which is best, some absolute standard is necessary. Therefore, ad-hoc methods are suggested as standards against which other methods have been compared. In the ad-hoc method it is assumed that mean and variance imply measures of growth and risk respectively.

3.1.1.1) Proposed rules for investing in individual share.

- a) Paying no attention to the risk the whole capital is invested in the share with highest expected return.
- b) Without any consideration to the risk the whole capital is invested in a share with medium expected return.
- c) The whole capital invested in share with lowest variance (no consideration for the expected returns).
- d) The whole capital is invested in share with medium variance.

3.1.1.2) Proposed rules for investing in more than one share.

- a) Invest equal proportions in each share.
- b) Invest equally in three shares with highest rates of return.

3.1.1.2) contd.

- c) Rank the shares according to highest mean rates of return and invest in the shares according to their respective ranks.
- d) Rank the shares according to lowest variance and invest in share according to their respective ranks.
- e) Rank the shares according to mean rate of return/standard deviation and invest in shares according to their respective ranks.

The mean rate of return, variance and the standard deviation is calculated for each of 50 and 9 shares on the basis of quarterly data for the years 1968 to 1970 and 1971.

The results and the comparison will be discussed in Chapter VI.

3.2) Optimum Filter Rule.

Some people e.g. Alexander, Cootner believe that share prices fluctuate randomly within certain limits. Outside these limits abnormal and optimum variations may be observed. Alexander investigated certain mechanical methods of stock trading known as Filter rules, and suggested that the use of such rules provided worthwhile profits greater than those obtainable, on the average, by a simple buy-and-hold policy.

The principal aim of investors dealing in stocks and shares is to buy and sell the right shares at the right time.

Alexander proposed and tested various filter rules which can be described as follows:

3.2) contd.

If the daily closing price of a share moves up at least X percent, buy and hold the share until its price moves down at least X percent from a subsequent high, at which time simultaneously sell and go short. The short position is maintained until the daily closing price rises at least X percent above a subsequent low at which time one covers and buys. Ignore moves of less than X percent. The more stringent the filter, the fewer losses are made, but the filter can be set with $X = 5\%$, $X = 10\%$ and $X = 2\%$, etc. Alexander tested filters ranging from 5% to 50% over time periods from 1897 to 1959. Alexander's conclusion was consistent with beliefs held by technical analysts of the market and some specialists on the trading floor of the exchange: trends do exist and, once started, tend to both challenge the ability of traders to make profits by applying Alexander's rules.

Now from the above set rule it is possible to wait for a) rise for sell, wait for b) fall in price before buying. If we have n numbers of values for financial index, we can find the most profitable combinations of (a) and (b), based on past experiences and see if these continue into the future.

Here we will investigate the combination for $\pounds \frac{1}{K}$ (where K = number of shares) available for investment and must be realised on the last of the n days at the latest.

Figure (1) helps to explain our aim. The following flow diagram [Appendix A5] explains the construction and operation of the computer programming. The results obtained are given in Tables [5-14].

The optimum filter rule method is carried out using daily shares index of nine shares from 1959 to 1968. The results are discussed in Chapter VI.

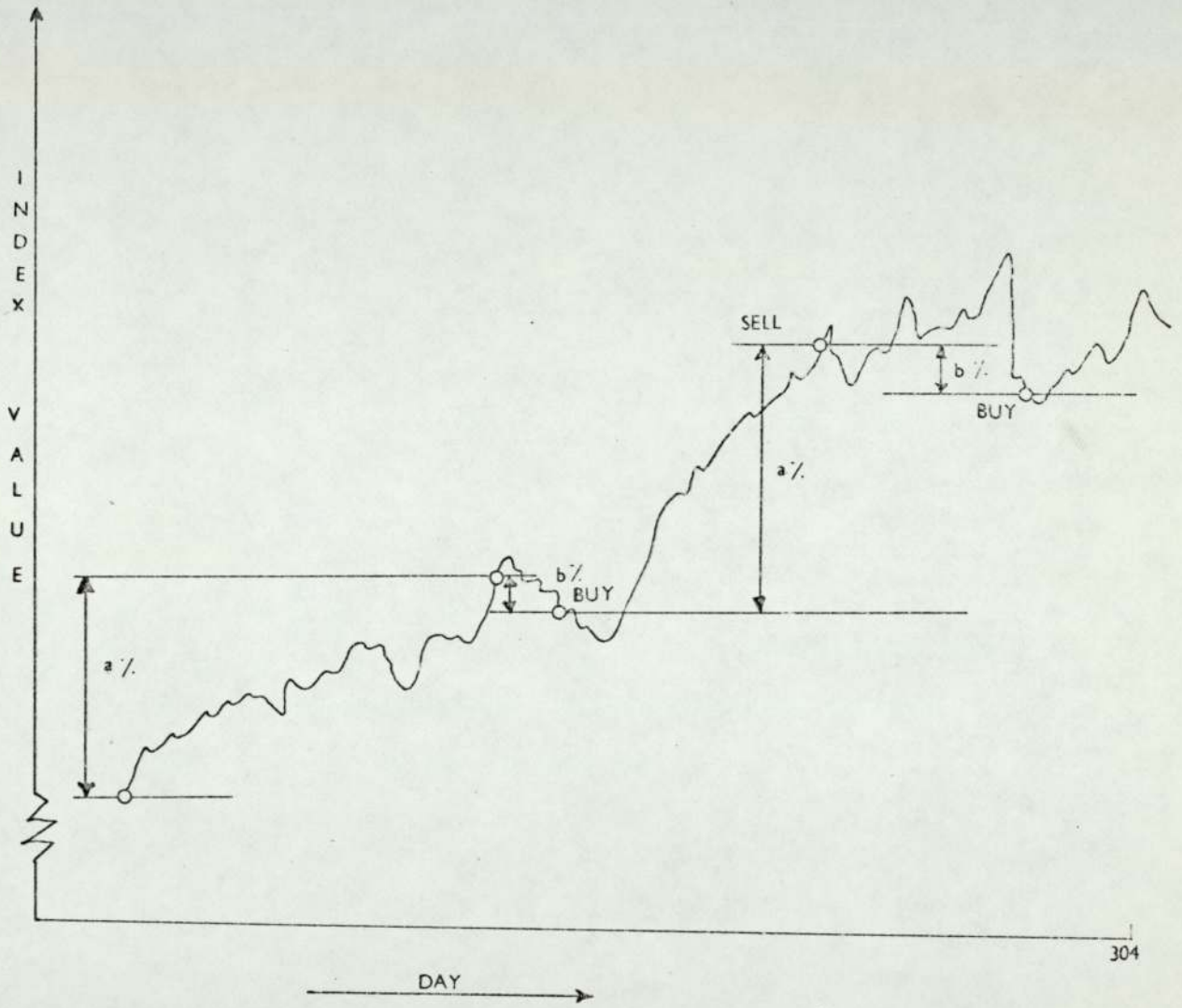


Figure - 1.

3.3) A New Model for Portfolio Selection.

3.3.1) Introduction.

Company statistics give us some personal measure of performance. Whereas share prices reflect to some extent market fluctuation - balance sheets do not.

Traditional methods of portfolio analysis use functions of share price data to measure expected earnings and risk. This "new" method of portfolio analysis extracts measures of risk and growth from balance sheet data. A criteria is then suggested for combining these measures into a single measure of 'attractiveness'.

The Price/Earnings (P/E) ratio is sometimes used by economists as a measure of a company's figure expectations. If the P/E ratio is high one may argue that a firm is relatively 'Safe' and hence the investor will have to wait sometime before any profit is realised. Alternatively if the P/E ratio is low the potential buyer expects to receive his return quickly and this can be interpreted as a long term lack of confidence.

If the above reasoning is accepted then it follows that the inverse of P/E ratio may be used as a measure of risk. This in itself is not meaningful but we shall show in the following sections that the inverse of P/E may be used to compute a discount rate which will effectively quantify the risk associated with a share.

3.3.2) Digression in Present Value.

Most people would choose without difficulty the options between £5 and £10 now - but they might have some difficulties in choosing between receiving £5 now and £10 in five years time.

3.3.2) contd.

If they were rational they would argue that since the going rate at which they could invest was $100i\%$ then $\pounds X$ invested now would then yield a sum of $\pounds X(1+i)^5$ in five years time. If this $X(1+i)^5 = \pounds 10$ then we might say that $X = \frac{\pounds 10}{(1+i)^5}$ was the discounted present value of $\pounds 10$ in five years time.

The rational person would then compare $\pounds 5$ and $\frac{\pounds 10}{(1+i)^5}$ in order to decide between the alternatives.

Suppose two shares A and B have cash flow pattern (a_1, a_2, \dots, a_n) and (b_1, b_2, \dots, b_n) for the time intervals $t = 1, 2, \dots, n$ respectively. Then the discounted present value (D.P.V) of the above shares cash flow will

be

$$(D.P.V)_A = \sum_{t=1}^n \frac{a_t}{(1+i)^t}$$

and

$$(D.P.V)_B = \sum_{t=1}^n \frac{b_t}{(1+i)^t}$$

The actual value of i is important. The following table notes the present values of $\pounds 100$ receivable in 10 years time for various values of i .

<u>Rate of i</u>	<u>$\pounds 100$ receivable in 10 years time.</u>
0.05	61.39
0.10	38.55
0.15	24.72
0.20	16.15
0.25	10.74
0.50	1.73

3.3.3) The Valuation Method.

The standard economic theory of share valuation suggests that the present value of a share is equivalent to the discounted present value of all its future earnings.

If we make the assumption that the value of a share is simply its market price and that earnings are going to stay constant, then the following holds:

$$P = E + \frac{E}{(1+i)} + \frac{E}{(1+i)^2} + \dots$$

i.e.
$$P = E \sum_{t=0}^{\infty} V^t$$

where $V = \frac{1}{1+i}$ is the discount rate and i is the 'Interest rate'.

It is reasonable to assume in general that $0 < V < 1$ so that

$$P = \frac{E}{1-V} \quad \text{and} \quad P/E = \frac{1}{1-V}$$

The discount rate, or measure of risk is then simply the inverse of P/E ratio, subtracted from unity. As a measure of growth it was decided to use the average price growth rate over the last three years combined.

3.3.4) Combining the Growth Rate with Risk factor.

If i_g is the growth rate of g th share, ($g = 1, 2, \dots, n$) and V_g is the risk factor for the g th share ($g = 1, 2, \dots, n$) an acceptable measure is given by their product $(1 + i_g)V_g$. The n shares should be ranked according to this criteria (the g th share receiving rank K_g). The proportions of capital X_g ($g = 1, 2, \dots, n$), then

3.3.4) contd.

the proportion is

$$X_g = \frac{K_g}{\sum_{i=1}^n K_g}$$

For the above analysis quarterly data of nine shares from 1968 to 1971 was used. The results and the comparison will be discussed in Chapter VI.

CHAPTER IV

CLASSICAL APPROACH.

4) Introduction.

This Chapter reviews the practical concepts of portfolio analysis. Sections 4.1 and 4.2 summarise the work of Markowitz and Sharpe in portfolio analysis. Appendices A1, A2, and A3 give detailed proof of algorithms of the Markowitz and the Sharpe models developed. Appendices A4, A5 and A7 give the flow charts and the computing codes with the specimen outputs respectively for making the computations involved in Markowitz's quadratic programming and Sharpe's simplified diagonal models in this investigation.

4.1.1) Economic Model of Portfolio Selection.

The share prices fluctuate in such a random fashion that it is almost impossible to formulate any kind of rigid mathematical model. The aim of the mathematical formulations so far attempted was to get a procedure to predict maximum expected gain and minimum risk in a certain portfolio.

The portfolio analyst has to deal with clients looking for varied financial goals. Some clients want large capital growth, some high dividend return whilst the majority settle for a combination. The analyst, therefore, has to find a suitable utility function capable of combining capital, time, risk and return to suit the investors' particular circumstances.

Let R be a random variable with mean μ and variance σ^2 .

Let the utility function be $u(R)$, Then by Taylor's theorem:

$$u(R) = u(\mu) + (R-\mu) u'(\mu) + \frac{(R-\mu)^2}{2!} u''(\mu) + \dots$$

4.1.1) contd.

Taking expectations on sides and ignoring term greater than the second order we obtain

$$E[u(R)] = E[u(\mu)] + \frac{\sigma^2}{2} u''(\mu) = u(\mu) + \frac{\sigma^2}{2} u''(\mu)$$

[Since utility is unique up to a linear transformation we can put $u(\mu) = \mu$ and $E(R-\mu)^2 = \sigma^2$, since $E(R-\mu) = 0$]

$$\text{putting } u''(\mu) = -\alpha$$

and $E[u(R)] = M$ (say)

$$\text{we have } M = \mu - \alpha \frac{\sigma^2}{2}.$$

The value of the constant α reflects the individual's attitude to risk.(i.e. The rate of change of marginal utility). Determining the form of the objective function and assuming α to be known the task is to find expression for μ and σ^2 .

Let the means and variances of the n rates of return, R_i ($i = 1, 2, \dots, n$) be μ_i and σ_{ii}^2 and the covariances are σ_{ij} . The rate of return R_i of the i th share is defined as

$$R_i = \frac{P_{t+1} - P_t}{P_t}$$

where

P_t = price at time t

$$\mu = E(R) = E\left(\sum_{i=1}^n X_i R_i\right)$$

$$= \sum_{i=1}^n X_i \mu_i$$

$$\sigma^2 = V\left(\sum_{i=1}^n X_i R_i\right)$$

4.1.1) contd.

$$= \sum_{\substack{i=1 \\ i \neq j}}^n \sum_{j=1}^n X_i X_j \sigma_{ij}$$

$$M(X) = \sum_{i=1}^n X_i \mu_i - \alpha \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n X_i X_j \sigma_{ij}^2$$

The problem therefore reduces to one of quadratic programming, i.e. to maximise $M(X)$ subject to the constraints

$$\sum_{i=1}^n X_i = 1 \quad (\text{Since the total investment is unity})$$

and $X_i \geq 0$ (One cannot invest a negative amount)

The above maximization of expected utility was presented by D.E.Farrar⁽⁹⁾.

This is the same formula that Markowitz derived independently.

He assumed that an investor wanted to have maximum return with minimum risk.

Let μ_i be the expected rate of return, for each share i , where $i = 1, 2, \dots, n$ and the covariances of rate of return, σ_{ij} , for each pair of shares i and j . ($i, j = 1, 2, \dots, n$).

The rate of return of a share is the same as defined above. The expected rate of return for the portfolio is the weighted sum of the expected rates of return for each share comprising the portfolio, the weight for the i th

4.1.1) contd.

share is the proportion of the total investment invested in that share. Thus, if X_i is invested in the i th share and μ_i is the expected rate of return for that share and if R_i is the true rate of return we have that

$$E[R_i] = \mu_i \text{ and hence } R = \sum_{i=1}^n R_i X_i$$

Thus the expected value of the true rate of return is

$$\begin{aligned} E[R] &= E \left[\sum_{i=1}^n R_i X_i \right] \\ &= \sum_{i=1}^n X_i E[R_i] \\ &= \sum_{i=1}^n X_i \mu_i \end{aligned}$$

The variance of rate of return, denoted by

V is

$$\begin{aligned} V &= E \left[\left(\sum_{i=1}^n R_i X_i - \sum_{j=1}^n X_j \mu_j \right)^2 \right] \\ &= \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n X_i X_j \sigma_{ij}^2 \end{aligned}$$

Now the efficient portfolios [the expected rate of return of any portfolio with the lowest possible standard deviation for its return is said to be efficient]

4.1.1) contd.

which provide the maximum expected return for each level of variance can be obtained by maximising

$$M(X) = \sum_{i=1}^n X_i \mu_i - \alpha \sum_{i=1}^n \sum_{j=1}^n X_i X_j \sigma_{ij}^2$$

$$\text{Subject to } \sum_{i=1}^n X_i = 1 \text{ and } X_i \geq 0 \quad (i = 1, 2, \dots, n)$$

which is obviously a quadratic programming problem.

J.Lintner⁽¹⁴⁾ suggested an alternative

$$M(X) = \frac{\mu}{V^2}$$

where μ = Expected rate of return

V^2 = Standard deviation

whereas the Markowitz function is of the form $M(X) = \mu - \alpha V$.

4.1.2) Optimization Methods.

The optimizations can be done by the following methods:

- (i) Efficient sets
 - (ii) Quadratic Programming
- (i) Efficient Set.

For the two variable case the relationship between the expected rate of return and the variances may be plotted by considering various values of ρ , where we have that the covariance between share 1 and 2, say, are as follows:

$$\sigma_{12}^2 = (\sigma_1^2 \cdot \sigma_2^2) \rho_{12}$$

where ρ_{12} is the coefficient of correlation, and σ_1^2

4.1.2) contd.

(i) contd.

and σ_2^2 are known.

To establish the method, from two variable case

$$\mu = X_1\mu_1 + (1-X_1)\mu_2$$

$$\text{and } V = X_1^2\sigma_1^2 + 2X_1(1-X_1)\sigma_1\sigma_2\rho_{12} + (1-X_1)^2\sigma_2^2$$

a) For the case when $\rho_{12} = 1$ we get

$$\begin{aligned} V &= X_1^2\sigma_1^2 + 2X_1(1-X_1)\sigma_1\sigma_2 + (1-X_1)^2\sigma_2^2 \\ &= [X_1\sigma_1 + (1-X_1)\sigma_2]^2 \end{aligned}$$

Hence

$$V^{\frac{1}{2}} = \sigma_a = \bar{+} [X_1\sigma_1 + (1-X_1)\sigma_2]$$

and represents a straight line.

b) Also, for $\rho_{12} = -1$,

$$\begin{aligned} V &= X_1^2\sigma_1^2 - 2X_1(1-X_1)\sigma_1\sigma_2 + (1-X_1)^2\sigma_2^2 \\ &= [X_1\sigma_1 - (1-X_1)\sigma_2]^2 \end{aligned}$$

$$\text{or } V^{\frac{1}{2}} = \sigma_b = \bar{+} [X_1\sigma_1 - (1-X_1)\sigma_2]$$

which also represents a straight line.

c) For $\rho_{12} = 0$,

$$V = X_1^2\sigma_1^2 + (1-X_1)^2\sigma_2^2$$

For the minimum risk (variance) therefore, we may differentiate with respect to X_1 to give

$$\frac{dV}{dX_1} = 2X_1\sigma_1^2 + 2(X_1-1)\sigma_2^2 = 0$$

$$X_1 = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} \left[\frac{d^2V}{dX_1^2} > 0 \therefore \text{Minimum} \right]$$

d) For $\rho_{12} = 0.5$

$$V = X_1^2\sigma_1^2 + X_1(1-X_1)\sigma_1^2\sigma_2^2 + (1-X_1)^2\sigma_2^2$$

Now, supposing we allocate the following hypothetical values for $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2$

4.1.2) contd.

(i) contd.

$$\mu_1 = m, \quad \mu_2 = 4m$$

$$\sigma_1 = 2S, \quad \sigma_2 = 4S$$

Then in the case (a) to (d) we are able to plot the following graph Figure (2) from $\mu = X_1m + 4(1-X_1)m$

$$\text{Hence } \frac{\mu}{m} = 4 - 3X_1$$

and from

$$\text{a) } V^{\frac{1}{2}}(\rho_{12}=1) = \sigma_a = \bar{+} (4S - 2X_1S)$$

$$\frac{V^{\frac{1}{2}}}{S} = \bar{+} (4 - 2X_1)$$

As $0 \leq X_1 \leq 1$ then

$$\frac{V^{\frac{1}{2}}}{S} = 4 - 2X_1$$

$$\text{b) } \frac{V^{\frac{1}{2}}}{S}(\rho_{12} = -1) = \bar{+} (6X_1 - 4) \geq 0$$

$$\text{c) } \frac{V^{\frac{1}{2}}}{S}(\rho_{12} = 0) = (20 X_1^2 - 32 X_1 + 16)^{\frac{1}{2}}$$

$$\text{d) } \frac{V^{\frac{1}{2}}}{S}(\rho_{12} = 0.5) = (12 X_1^2 - 24 X_1 + 16)^{\frac{1}{2}}$$

From Figure (2) we see that the efficient sets are bounded, that triangle with boundaries $\rho_{12} = 1$ and $\rho_{12} = -1$ i.e. the triangle ABC.

(ii) Markowitz⁽¹²⁾ gave a method called the critical line method to solve his quadratic programming, by which the set of efficient portfolio could be calculated.

If α is zero, the programming will select the portfolio with the largest rate of return. Intermediate values of α will return in portfolios that have lower than maximum expected return and higher than minimum variance. The objective function permits the selection

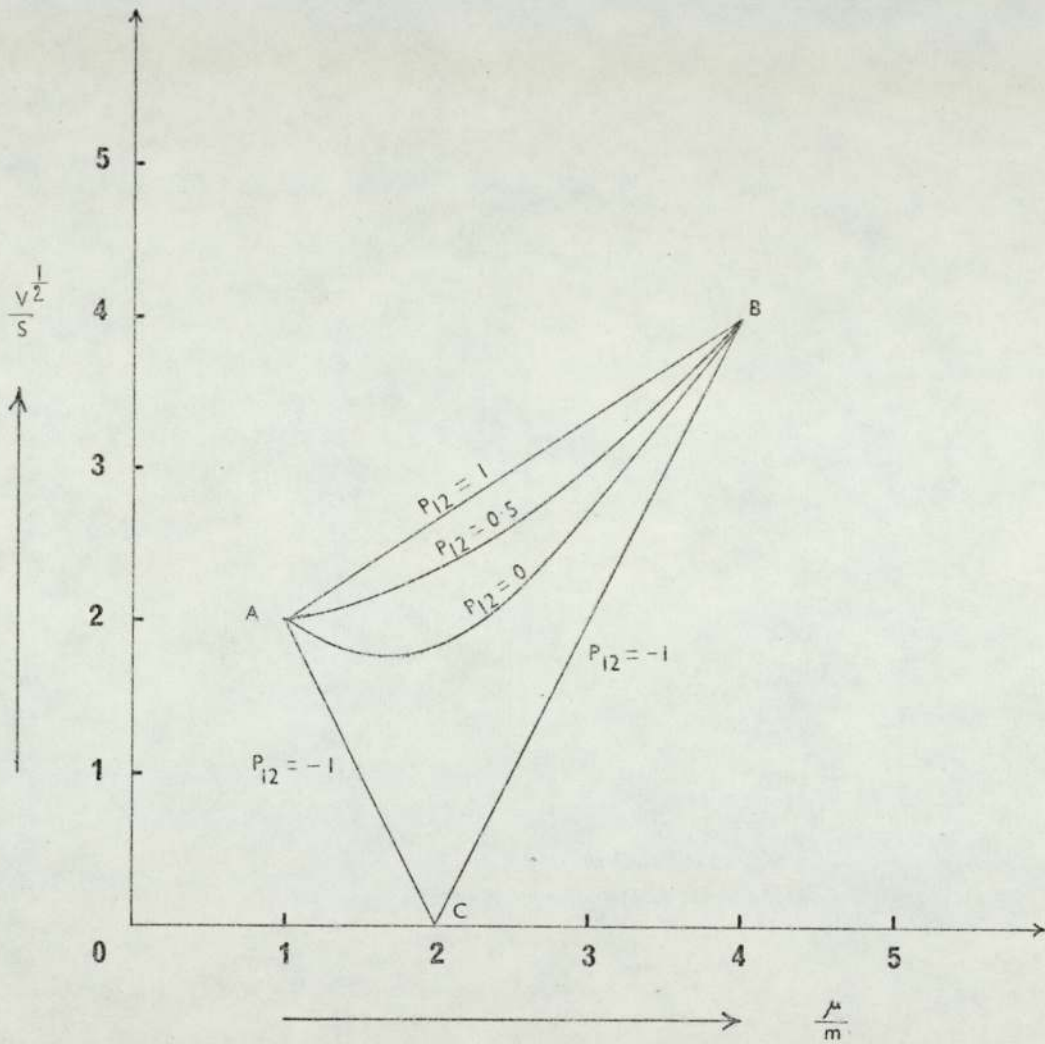


Figure - 2

4.1.2) contd.

(ii) contd.

of efficient portfolios only. The complete set of efficient portfolios is obtained, therefore, simply by solving quadratic programming problems for all values of α between zero and infinity. The selection of α by the investor is equivalent to specifying the form of his utility function i.e. his attitude towards risk.

For the analysis of the developed Markowitz's model, different sets of data are used and results obtained will be discussed in Chapter VI.

4.2) The Sharpe Diagonal Portfolio Selection Model.

4.2.1) Introduction.

Markowitz⁽¹⁷⁾ himself was well aware of the impracticality of trying to collect data directly on all the $\frac{1}{2}n(n+1)$ distinct entries in the covariance matrix. He (1959 p.56) acknowledged these difficulties and proposed an alternative simplified procedure which relates to the return and risk of individual shares to an overall market index. His sketch of this new model was developed by Sharpe^(22,23) which is known as the "Simplified diagonal model".

The "Simplified diagonal model" is so called because its covariance matrix is diagonal. Sharpe showed that this model simplifies the computational process leading to solution.

4.2.2) Formulation of the model for n Shares.

Let the rate of return of the i th share be R_i

$$\text{where } R_i = a_i + b_i I + \tilde{u}_i \quad (4.2.2.1)$$

4.2.2) contd.

a_i and b_i = Constants of regression

I = Market index

\tilde{u}_i = Random variables $\Omega N(0, \sigma_i^2)$

Let the expected value of $I = a_{n+1}$

and variance of $I = \sigma_{n+1}^2$.

The diagonal model assumes that shares are unrelated except through their common dependence on the market index (I); further, the random elements (\tilde{u}_i) are not related to the level of the index (I).

This is expressed by

Covariance (u_i, u_j) = 0 for all i and j $i, j = 1, 2, \dots, n$

Covariance (u_i, I) = 0

Taking the expected value of both sides of (4.2.2.1) we obtain

$$E(R_i) = a_i + b_i \hat{I}$$

The return of a portfolio is

$$R = \sum_{i=1}^n R_i x_i$$

where x_i are proportion.

The expected return of the portfolio R may be obtained from

$$\begin{aligned} E(R) &= \sum_{i=1}^n E(R_i) x_i \\ &= \sum_{i=1}^n (a_i + b_i \hat{I}) x_i \end{aligned}$$

The variance of the rate of return of share i is

4.2.2) contd.

$$V(R_i) = \sigma_i^2 + b_i^2 \sigma_{n+1}^2 \quad [A-2]$$

Thus the portfolio selection problem is now to maximise

$$M(\underline{X}) = E(R) - \alpha V(R)$$

where α = The rate of change of marginal utility or risk measure

Subject to the constraints

$$\sum_{i=1}^n x_i = 1$$

$$x_i \geq 0 \quad [A-2] \quad \text{for all } i$$

$$\sum_{i=1}^n x_i b_i - x_{n+1} = 0 \quad [A-3] \quad i = 1, 2, \dots, n$$

Thus the only data required are the n values for each a_i , b_i and σ_{n+1}^2 . For the analysis of n shares requires $(n+1)m+2$ items of input data, where m is the number of periods.

The diagonal model is particularly useful for working with large numbers of shares and hence for practical applications Sharpe performed initial test of both performance and efficiency.

4.2.3) Graphical representation of the model.

In Figure (3) a_i and b_i serve to locate the line relating the expected value of R_i to the market index (I). σ_i^2 indicates the variance of R_i around the expected relationship (this variance is assumed to be the same at each point along the line).

Finally, a_{n+1} indicates the expected value of I

4.2.3) contd.

and σ_{n+1}^2 the variance around the expected value.

Two sets of data are used for this analysis, the results will be discussed in Chapter VI.

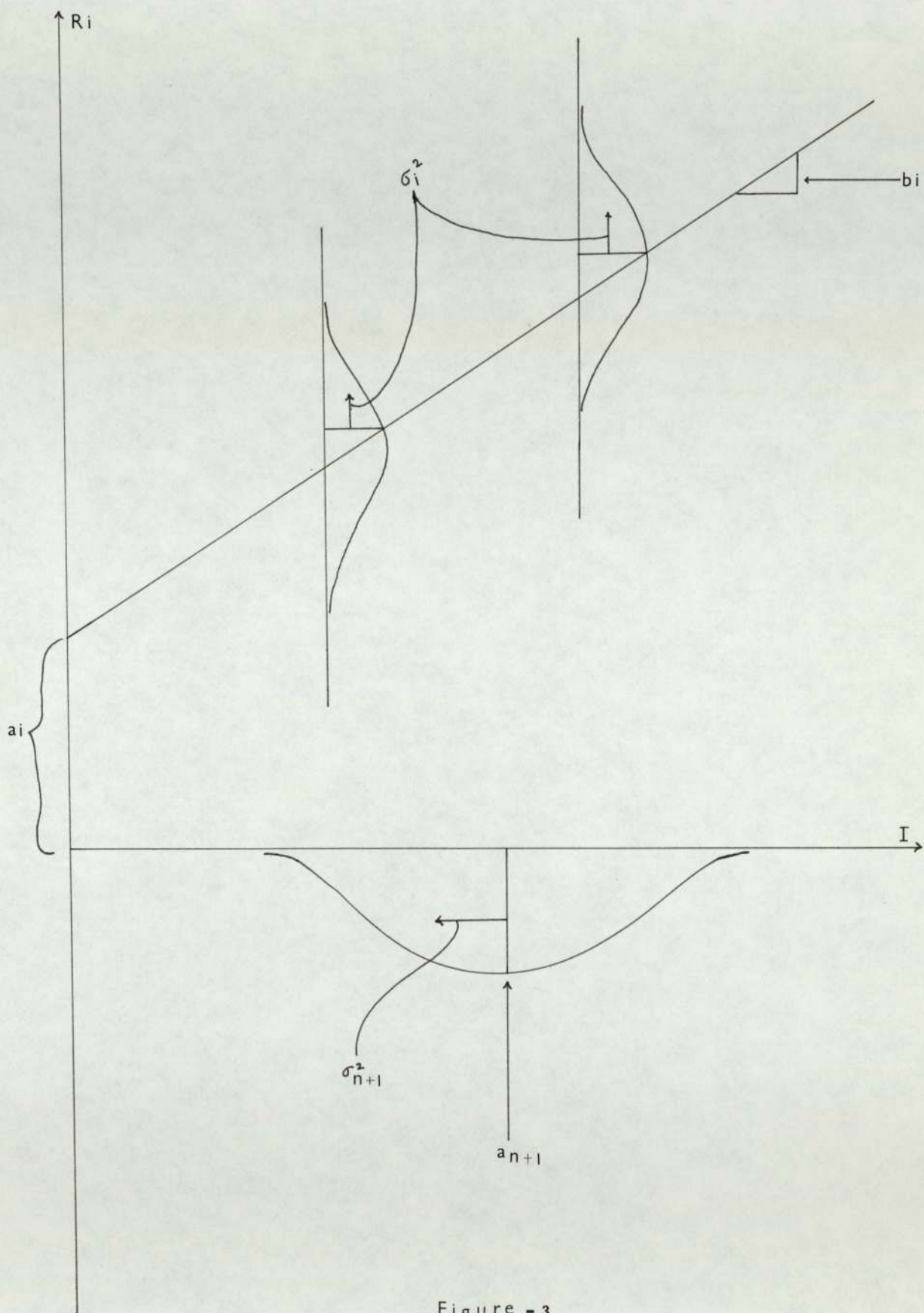


Figure - 3.

CHAPTER V

A MONTE CARLO TECHNIQUE.

5) Introduction.

The simulation model is basically an extension of Sharpe's model. The model purports to make future predictions (other models rely on historic estimates). Simulation models are particularly interesting because of their basic appeal to management.

5.1) The Share price regression model.

The model assumes that the rate of return of Share prices and the rate of return of Financial Times Index (RFTI) for the same period are related by linear relationship of the form:

$$P_j = a_j + b_j I_j + \tilde{u}_j \quad (5.1.1)$$

where P_j = Rate of return of Share prices

I_j = Rate of return of FTI

a_j and b_j are regression constants and \tilde{u}_j is a random variable which is distributed normally with mean zero and variance σ^2 where σ^2 is the variance obtained from the regression analysis.

The rate of return of share prices and RFTI are calculated as defined in Chapter IV Section 4.1.1. In this model 50 shares are selected. By using programme number 6 the rate of return of 50 shares and FTI for the years 1968 to 1970 are calculated on the basis of quarterly data.

The correlation coefficients (r) between the rate of return of share prices and RFTI are found to have a range of -0.029 to 0.96 (with 11 degrees of freedom). Out of the 50 correlation coefficients about two-thirds of the shares showed 'good' correlation (above 0.65) with

5.1) contd.

the corresponding RFTI as shown in Table 37. It seemed to indicate that the assumption of linearity is not unreasonable.

5.2) Regression line for the FTI against time lagged U.K. car production.

"For some years econometricians have been aware of the critical dependence of economic systems on the lag between cause and effect"⁽⁴⁾. Thus, in this model we assumed the FTI is linearly related to U.K. car production lagged by six quarters. Quarterly data for FTI and U.K. car production for the years 1968 to 1970 were used to fit a regression line of the form

$$I_t = a + b K_{t-6} + \tilde{u} \quad (5.2.1)$$

where a and b = are regression constants

K_t = U.K. car production at the end of t th period

I_t = FTI at the end of t th period

$\tilde{u} = \Omega N(0, S_0^2)$

Using application programme⁽²⁶⁾ the following

$$a = 250.92$$

$$b = 0.0004$$

$$r(\text{correlation coefficient}) = 0.673 \text{ (9 degrees of freedom)}$$

$$S_0^2 \text{ (variance)} = .0065.$$

5.3) Simulation Model.

A value of quarterly U.K. car production six quarters before the end of the simulation year (two quarters before beginning of the simulation year) is obtained. This is used to simulate a first value of FTI by using equation (5.2.1)

5.3) contd.

\tilde{u} is assumed to be distributed normally with mean zero and constant variance. The variance (ϵ^2) of a further reading about the line is estimated as:

$$\epsilon^2 = S_0^2 \left[1 + \frac{1}{n} + \frac{(x_i - \bar{x})^2}{\sum_{i=1}^n (x_i - \bar{x})^2} \right] \quad (5.4.1)$$

where

$$S_0^2 = \frac{\sum_{i=1}^n (\bar{y}_i - \bar{y})^2 - \frac{[\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})]^2}{\sum_{i=1}^n (x_i - \bar{x})^2}}{n - 2}$$

where n = number of points taken to calculate the regression coefficients

y_i = Value of FTI at time i

x_i = Value quarterly U.K. car production between periods $i-7$ and $i-6$.

The random disturbance term \tilde{u}_i was simulated in the first instance, by generating a uniform random variate in the interval generation $[0,1]$. (A standard I.C.L. facility was used for this purpose). This uniform variate (R_N say) was then 'normalised' by using the transformations:

$$\left. \begin{aligned} N_1 &= u \sin T \\ N_2 &= u \cos T \end{aligned} \right\} \quad (5.4.2)$$

where $u = \sqrt{-2 \log R_N}$

$$T = 2 \pi R_N$$

N_1 and N_2 are the required random unit normal variates.

Thus for each uniform random variate generated

5.3) contd.

two 'normal' variates are obtained. The values of N_1 and N_2 are multiplied by ϵ and the simulated values of FTTI are given by

$$\left. \begin{aligned} I_t' &= a + b K_{t-\epsilon} + \epsilon N_1 \\ I_t'' &= a + b K_{t-\epsilon} + \epsilon N_2 \end{aligned} \right\} \quad (5.4.3)$$

as shown in Figure (4).

Two values of RFTI say RFTI' and RFTI'' are then calculated.

Then another uniform random number is generated to give two more unit random normal variates. These values are multiplied by the standard error σ_j of \tilde{u}_j and the results are substituted for \tilde{u}_j in the following equation

$$R_j = a_j + b_j \text{RFTI} + \tilde{u}_j \quad (5.4.4)$$

$$j = 1, 2 \dots, 50$$

R_j = Rate of return of share j

\tilde{u}_j is random normal variate with mean 0 variance σ_j^2

$$\sigma_j^2 = \sigma_{0i}^2 \left[1 + \frac{1}{2} \frac{(x_i - \bar{x})^2}{\sum_{i=1}^n (x_i - \bar{x})^2} \right] \quad (5.4.5)$$

where

$$\sigma_{0i}^2 = \frac{\left[\sum_{j=1}^n (y_{ij} - \bar{y}_i)^2 - \frac{\left[\sum_{j=1}^n (x_j - \bar{x})(y_{ij} - \bar{y}_i) \right]^2}{\sum_{j=1}^n (x_j - \bar{x})^2} \right]}{n - 2}$$

where n = number of points taken in calculating the regression coefficients

y_{ij} = value of rate of return of share i for j th quarter

5.3) contd.

x_j = value of rate of return of FTI for j th quarter

x_i = value of rate of return of FTI predicted

this gives R_{N_1} and R_{N_2} as in Figure (5). Similarly 49 more random numbers are generated and used to simulate rates of returns for other 49 Shares. These give two sets of rates of returns for the 50 Shares analysed.

Each set is ranked and the highest (Best) and Lowest (Worst) shares noted.

This whole procedure is repeated a thousand times, after which the procedure terminates. The number of times each share is highest H_j ($j = 1, 2 \dots, 50$) and the number of times is lowest L_j ($j = 1, 2 \dots 50$) is counted.

Then the proportion invested in the i th shares P_i is then given by:-

$$P_i = \frac{K_i}{\sum_{j=1}^{50} K_j} \quad (5.4.6)$$

where

$$K_i = \frac{H_i}{1000} \left(1 - \frac{L_i}{1000} \right) \quad (5.4.7)$$

The formula (5.4.7) is just one of many strategies that can be considered for ranking the shares. It is sensible because it gives a portfolio with most funds invested in a share which has most recorded highest and least recorded lowest in the simulation.

Block diagram Figure (6) explains the structure of the above simulation. The computer programmes and their

5.3) contd.

specimen outputs can be found in Appendix A7.

Using programmes numbers 6 and 7 the results are presented in Tables 38 and 39.

A discussion of the results, together with a comparison against other models can be found in Chapter VI.

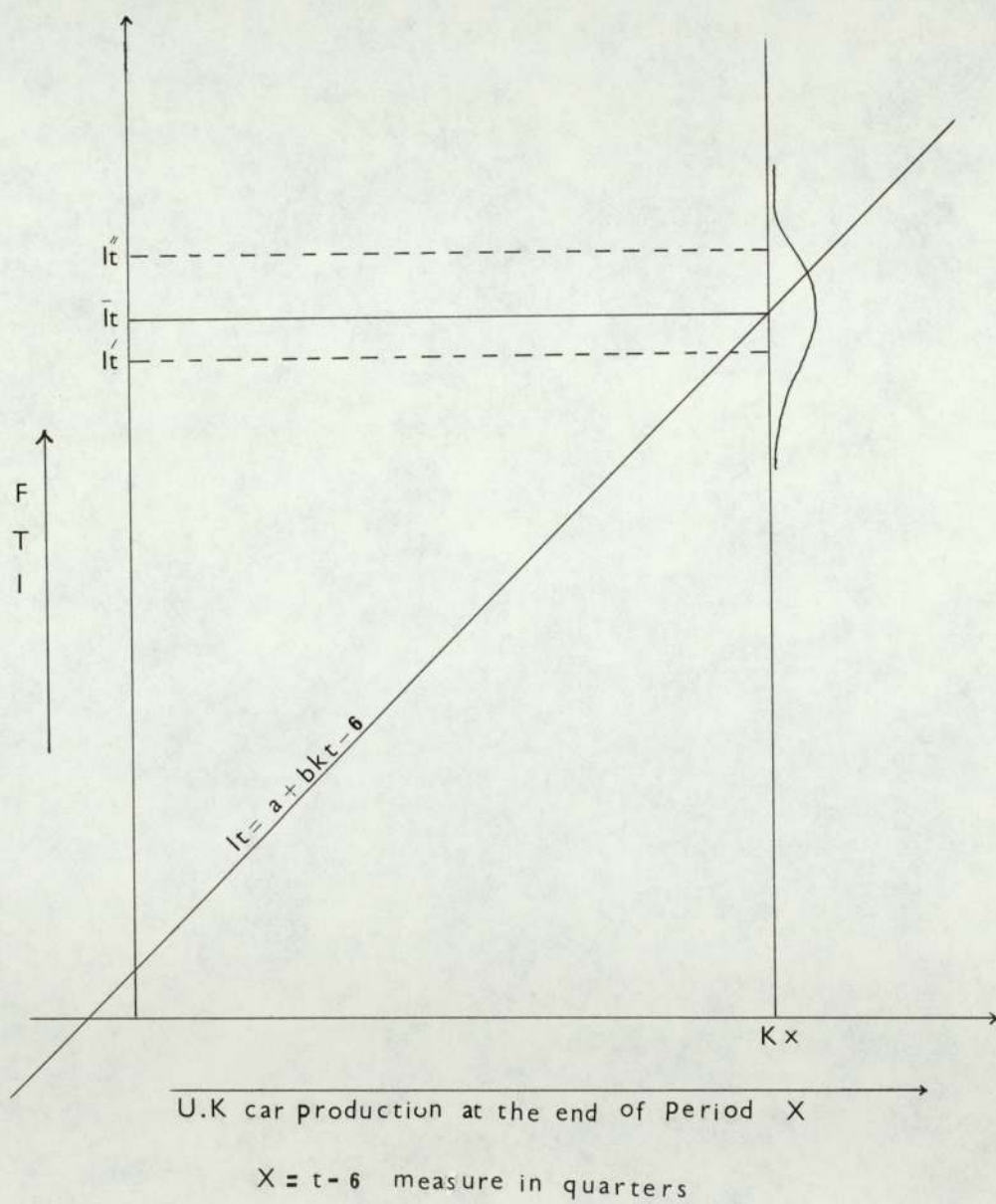


Figure - 4.

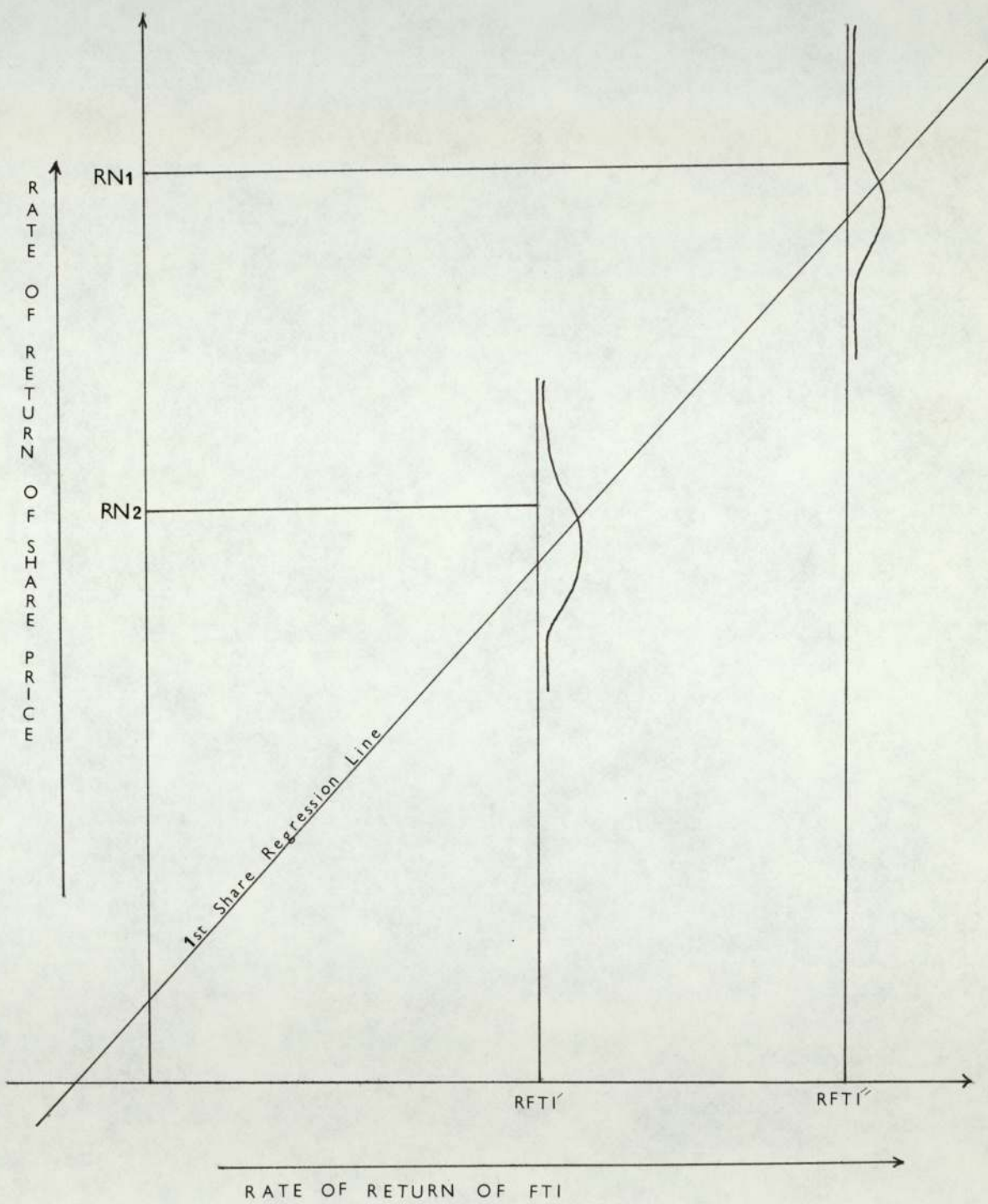
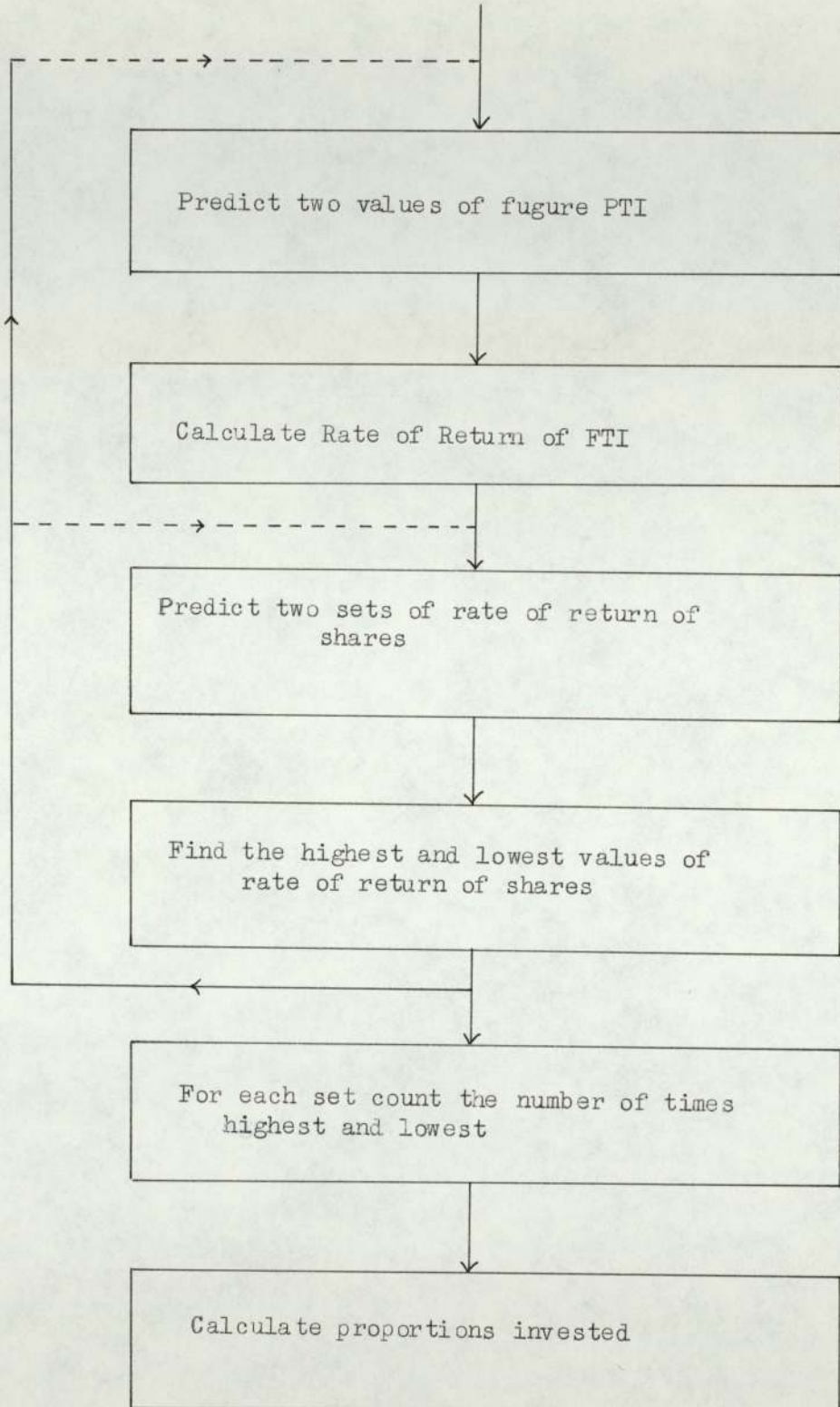


Figure - 5.

BLOCK DIAGRAM FOR MONTE CARLO SIMULATIONFIGURE 6.

C H A P T E R V I

DATA COLLECTION, RESULTS AND DISCUSSION.

6.1) Data Collection.

For this study two samples of closing share prices were selected. Sample 'X' consists of a detailed daily price study of nine equity shares over the period 1959 to 1968. Sample 'Y' consists of a wider study of quarterly price data relating 50 equity shares for the years 1968 to 1971. The Financial Times Index (F.T.I) was recorded quarterly for the period 1968 to 1971. The shares in samples 'X' and 'Y' were chosen randomly. A list of the sample members is given below.

Sample 'X'.

<u>Serial number</u>	<u>Name of Company</u>
1	Barclays Bank Limited
2	Distillers Co.,Ltd.
3	Imperial Chemical Industries Ltd.,
4	Marks & Spencer Limited.
5	British Insulated Callender Cables Ltd.
6	Prudential Assurance Co.Ltd. 'A'
7	Rolls-Royce Limited.
8	W.H.Smith & Son Ltd.,
9	Alfred Dunhill Ltd.,

Sample 'Y'

<u>Serial number</u>	<u>Name of Company</u>
1	Glaxo Group
2	Hoover Ltd.,
3	Robert Moss Ltd.,
4	Wilkinson Sword Ltd.,
5	Economic Insurance Co.,Ltd.
6	Guardian Royal Exchange Assurance Ltd.,
7	Pearl Assurance Co.,Ltd.
8	Royal Insurance Co.,Ltd.

6.1) contd.

Sample 'Y' (contd)

9	Sun Life Assurance Society Ltd.,
10	Prudential Assurance Co.Ltd., 'A'
11	Victory Insurance Co.,Ltd.
12	The British Motor Corporation Ltd.
13	Ford Motor Co.,
14	Dunlop Co.,Ltd.
15	The Zenith Carburetter Co.,Ltd. 'A'
16	Bristol Street Group Ltd.,
17	Penguin Publishing Co.,Ltd.
18	W.H.Smith & Son.,Ltd.
19	Dixon Peter & Son.,Ltd.
20	University Printers
21	Brixton Estate Ltd.,
22	Great Portland Estates Ltd.,
23	St.Martin's Property Corporation Ltd.,
24	James Fisher & Sons Ltd.,
25	Church & Co.,Ltd.
26	'K' Shoes Ltd.,
27	Thos.Frith & John Brown Ltd.,
28	Bear Brand Ltd.,
29	G.H.Heath & Co.,Ltd.
30	Shaw Carpets Ltd.,
31	Alfred Dunhill Ltd.,
32	Derby Trust Ltd.,
33	British Investment Trust Ltd.,
34	The Calcutta Electric Supply Corporation Ltd.
35	British Petroleum Co.,Ltd.
36	The Darjeeling Co.,Ltd.
37	The Ceylon Tea Plantation Holdings Ltd.,

6.1) contd.

Sample 'Y' (contd).

38	Barclays Bank Limited.
39	Bank of Ireland.
40	Bank of Scotland
41	Lloyds Bank Limited.
42	Midland Bank Limited.
43	The Distillers Co., Limited.
44	Arthur Guinness Son & Co., Ltd.
45	Imperial Chemical Industries Ltd.,
46	Marks & Spencer Ltd.,
47	British Insulated Callender Cables Ltd.,
48	Decca Ltd.,
49	Telefusion Ltd.,
50	Alfred Herbert Ltd.,

In subsequent tables shares are noted by their respective serial numbers. For consistency we have collected the data at the middle of each quarter, when data is not available on this date, the price on the 'nearest' day is used.

In order to reduce data collection to a minimum when applying Markowitz's and Sharpe's Models to the data from sample 'Y', the sample of 50 shares was 'censored' down to 10 shares.

The 50 shares were screened according to the following criteria:

- 1) Average total capital employed.
- 2) Average pre-tax profit.
- 3) Average earnings.

Data obtained from the Moody cards⁽²⁶⁾ was used as a basis for finding the ten most promising shares. It was interesting

6.1) contd.

to note that they were all well-known companies which, in a way, justified the method of selection employed. The monthly share prices and the F.T.I. were subsequently collected from the Financial Times for the years 1968 to 1971.

The 10 companies left after completing the censor exercise were:

<u>Serial Number</u>	<u>Name of Company</u>
1	Glaxo Group.
2	Hoover Ltd.,
3	The British Motor Corporation Ltd.,
4	Dunlop Co.,Ltd.
5	British Petroleum Co.,Ltd.
6	The Distillers Co.,Ltd.
7	Arthur Guinness Son & Co.,Ltd.
8	Imperial Chemical Industries Ltd.,
9	Marks & Spencer Ltd.,
10	British Insulated Callender Cables Ltd.

For the multiple regression analysis, a special share index (the industrial share price index) was used. The name of the various economic indicators used are given in Chapter II Section 2.1.2. Data were collected for the years 1968 to 1970. The data of Sample 'X' for Price/Earnings (P/E) ratio was obtained for the years 1968 to 1970.

The data for United Kingdom car production, Gross National Product (G.N.P) Bank rate, and the unemployment rate was obtained from the Monthly Digest of Statistics⁽²⁷⁾ for the years 1968 to 1971.

Minor adjustments to the share prices were

6.1) contd.

made when there was a "scrip issue" or a "rights issue". All the above share prices are recorded in units of 5p.

For the ease of analysis the different share data are lettered as given below:

Nine Shares daily data	-	'A'
Nine Shares quarterly data	-	'B'
Ten Shares monthly data	-	'C'
Fifty Shares quarterly data	-	'D'
Various economic indicators	-	'E'
Price changes of nine shares daily data	-	'F'

6.2) Results and Discussion.6.2.1) Multiple Regression Analysis.

The ICL statistical analysis package XDL3 is used to calculate the multiple regression lines for different sets of economic indicators (as described in Section 2.1.2 Chapter II)

The results obtained from the multiple regression analysis of the 'D' and 'E' data from the periods 1968 to 1970 are given in Table 1.

The prediction of share prices using ordinary economic indicators, is not satisfactory because the tests of significance of the correlation coefficients by 't' statistics indicated that they were insignificant at both the 5% and 1% levels (with 9 degrees of freedom correlation should be greater than or equal to 0.65 in order to be significant but they are in fact much below 0.65 whereas using share indices the predictions improved considerably, the correlation being above 0.65.)

The '*' marks shares are thus found to be

6.2.1) contd.

good predictors and as such can be used as a basis for next or subsequent years investment performance.

6.2.2) The Random Walk Hypothesis.

The hypothesis developed in Section 2.2 Chapter II are tested with respect to

- (a) Serial correlation
- (b) The normality assumption.

- a) For the serial correlation, data 'A' was analysed by using programme number 3, for three different years 1959, 1964 and 1968. The average value of correlations are shown in Table 2. None of the correlations are significant.
- b) Data 'A' was analysed for the years 1959 to 1968. By using programme number 4 empirical distributions of daily changes in logarithms of share prices for the year 1959 were obtained. One share (I.C.I) was investigated on the basis of 10 years data.

An attempt was made to fit a normal distribution to the share price fluctuations, but the fit was not good. Then logarithm of share price fluctuations are used for the year 1959. The Chi-square test was then used to check the goodness of it. The results are given in Table 3. The fit was improved by using logarithms of share price fluctuations. In general the fit was not satisfactory.

An analysis over an extended period (10 years) was carried out for one share (I.C.I). This again did not improve the fit. Results are shown in Table 4. Histogram of logarithm share price fluctuations for the year 1959 using data 'A' are shown in Figures (7) to (15). The logarithms of price fluctuations are found not to be normally distributed.

BARCLAYS SHARE

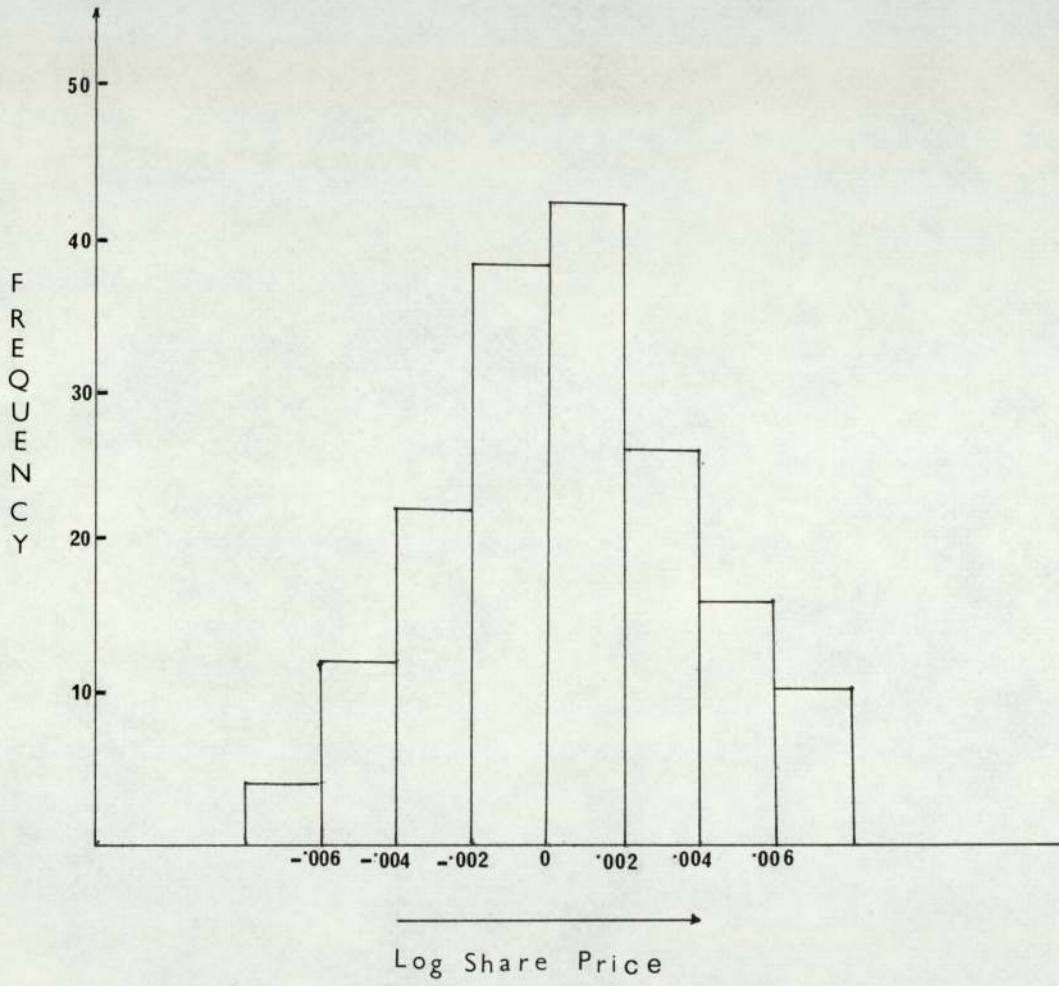


Figure-7.

DISTILLERS SHARE

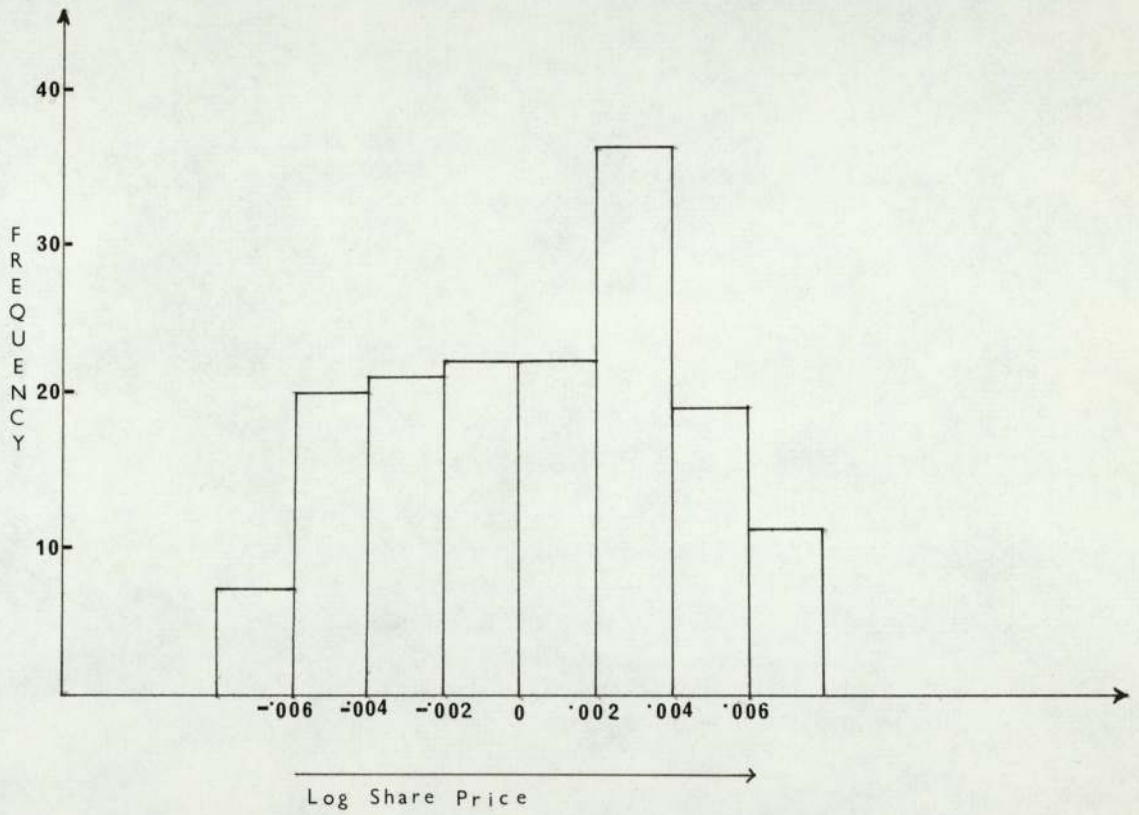


Figure - 8.

I.C.I SHARE

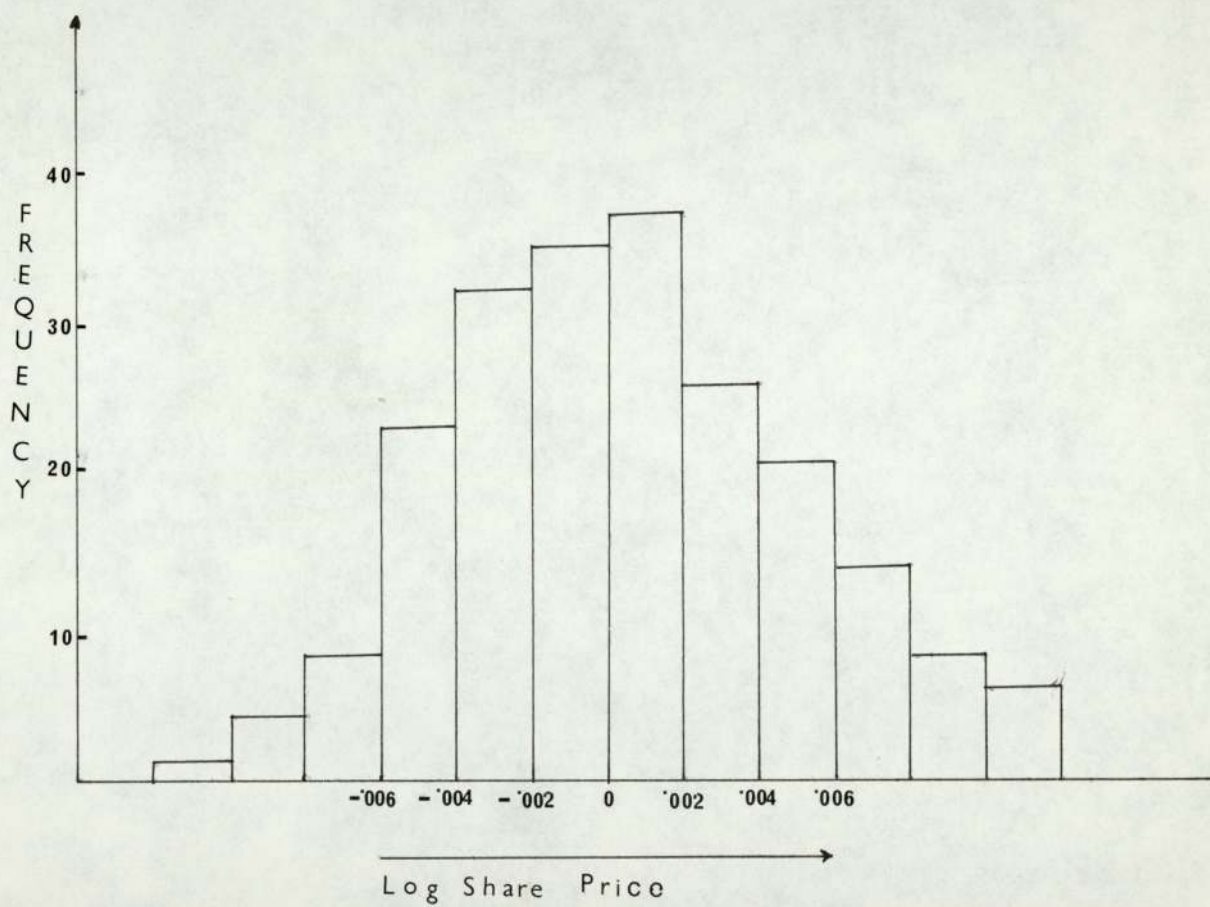


Figure - 9.

MARKS & SPENCER SHARE

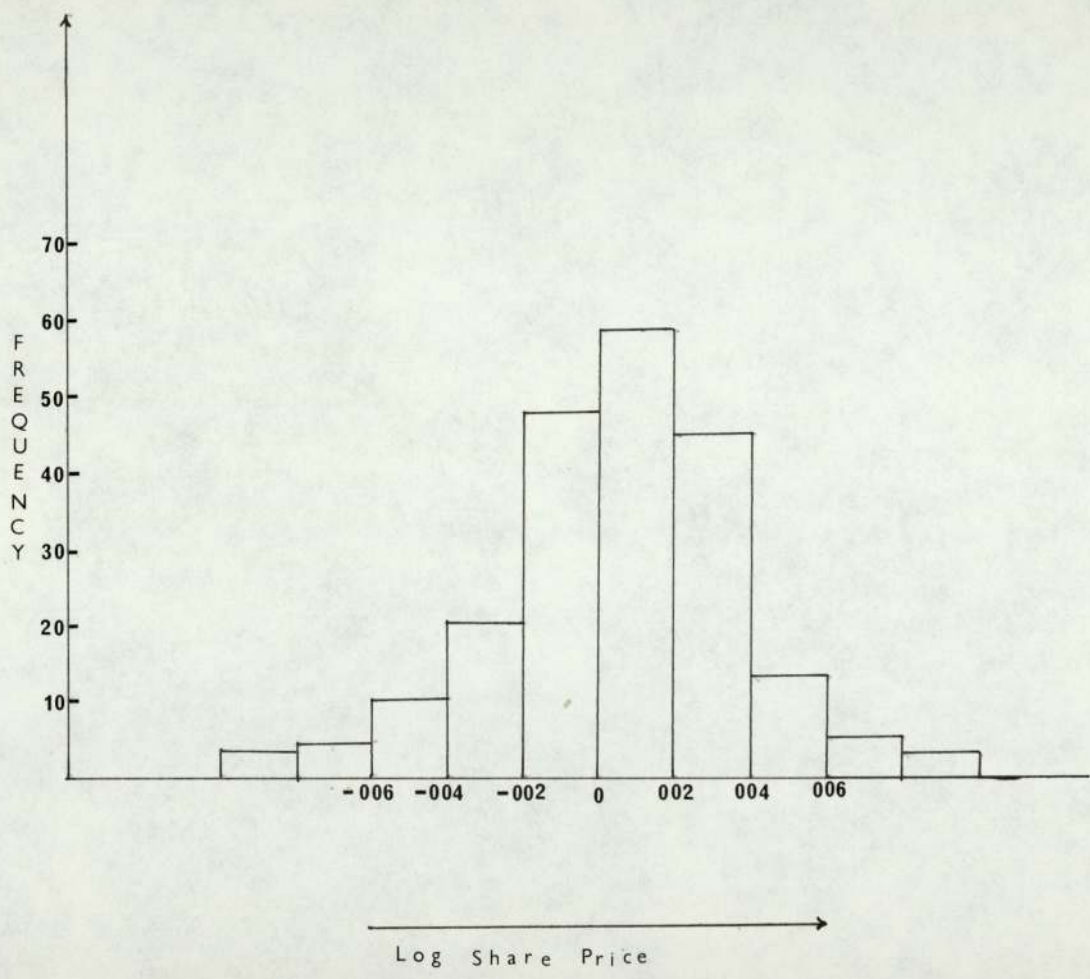


Figure - 10.

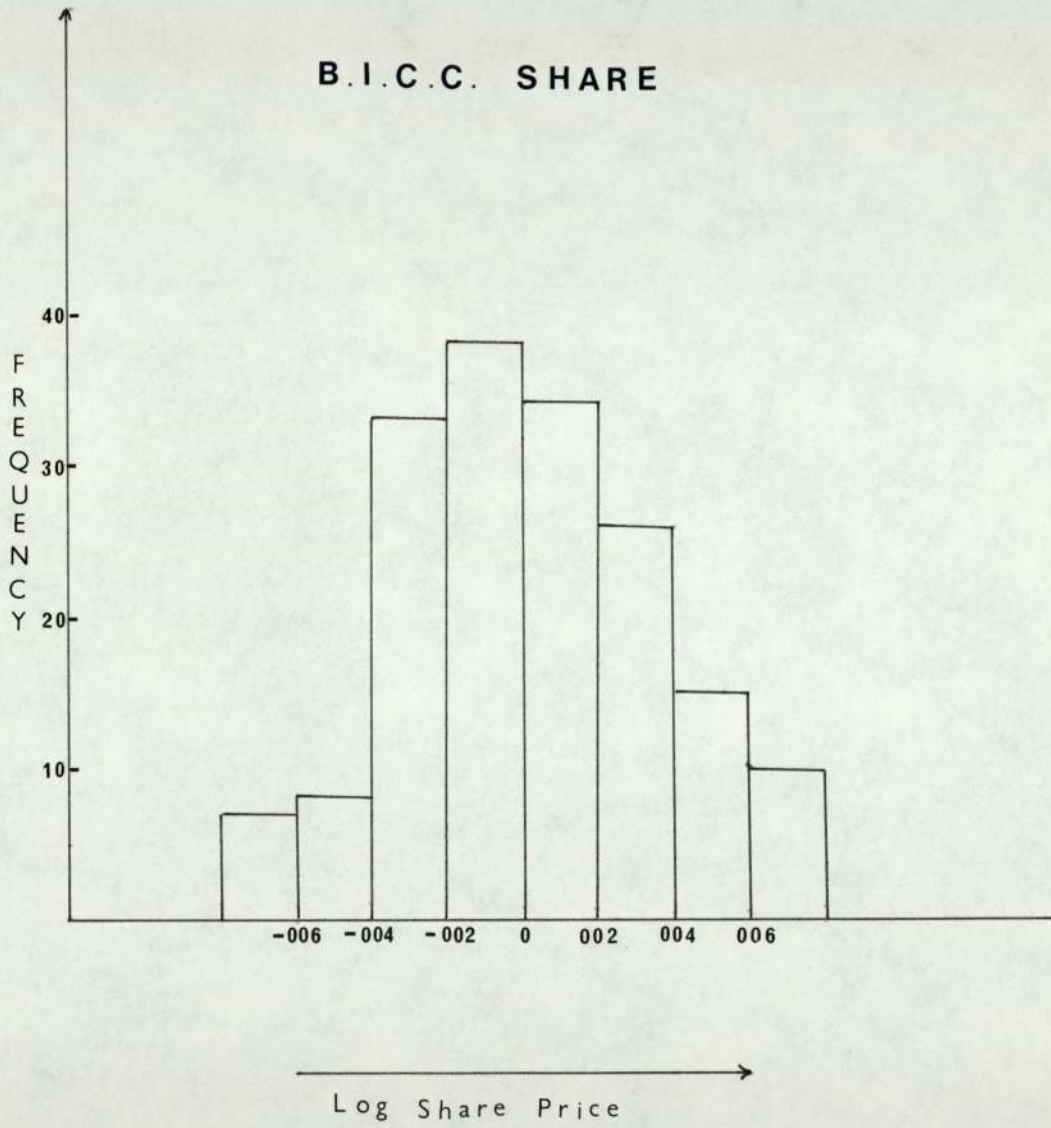


Figure -11.

PRUDENTIAL 'A' SHARE

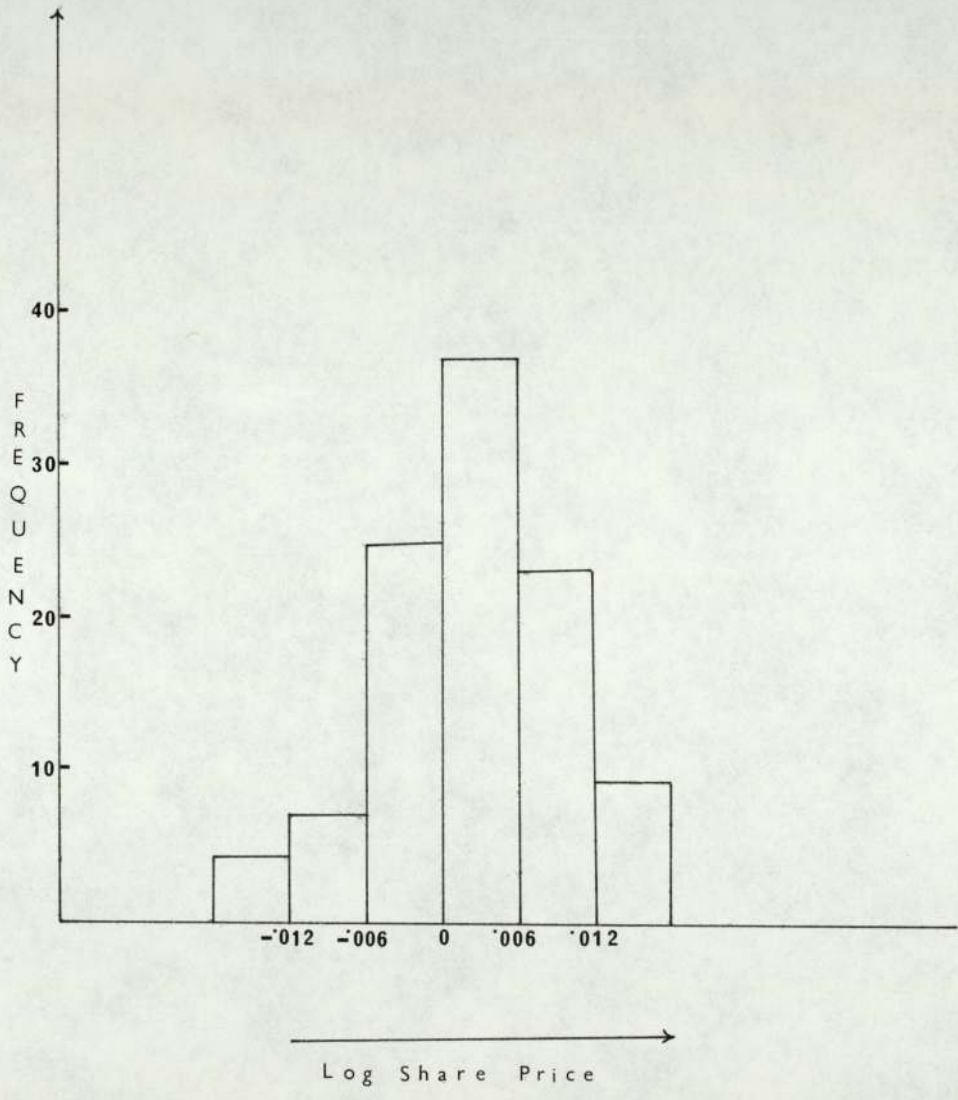


Figure - 12.

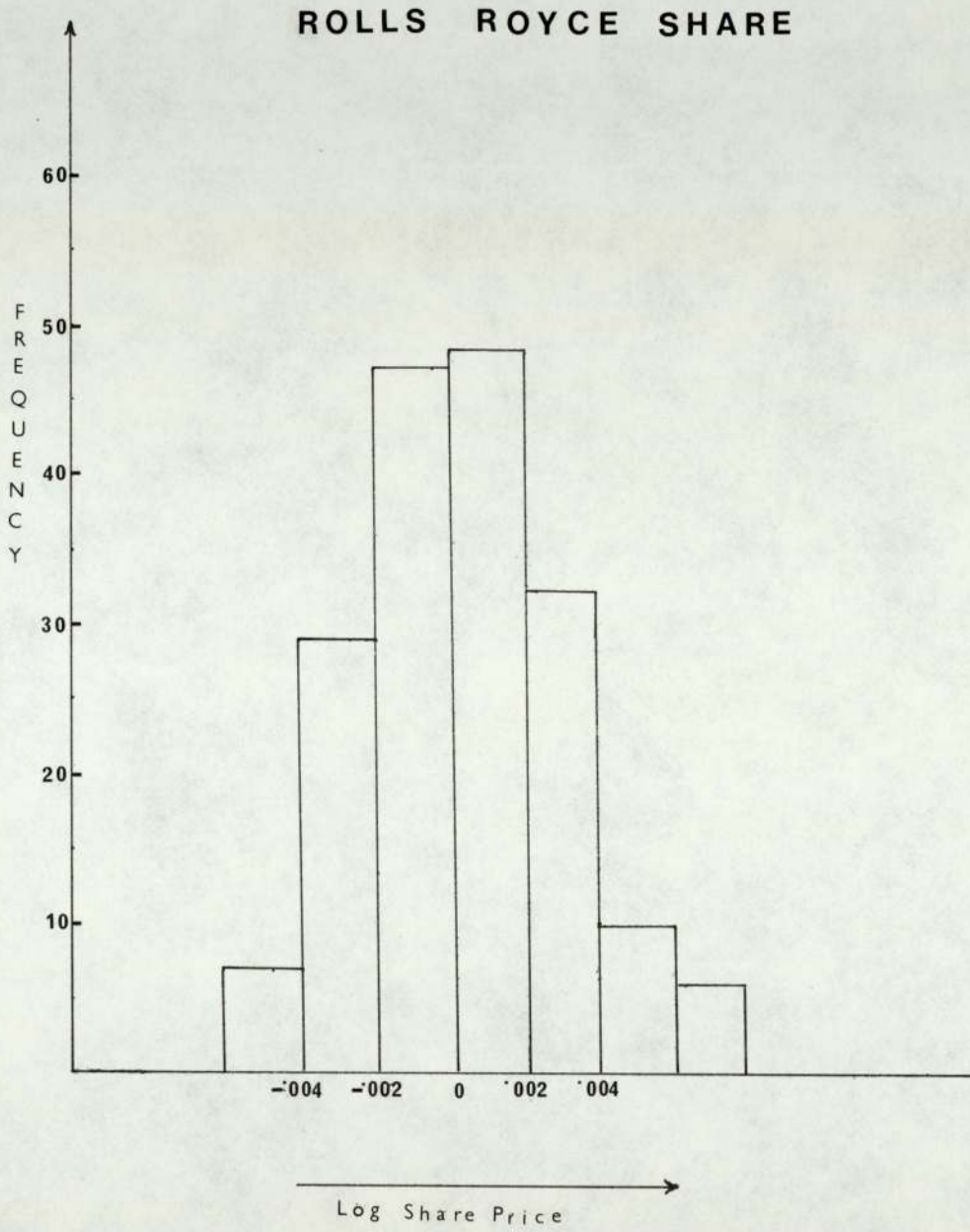


Figure - 13.

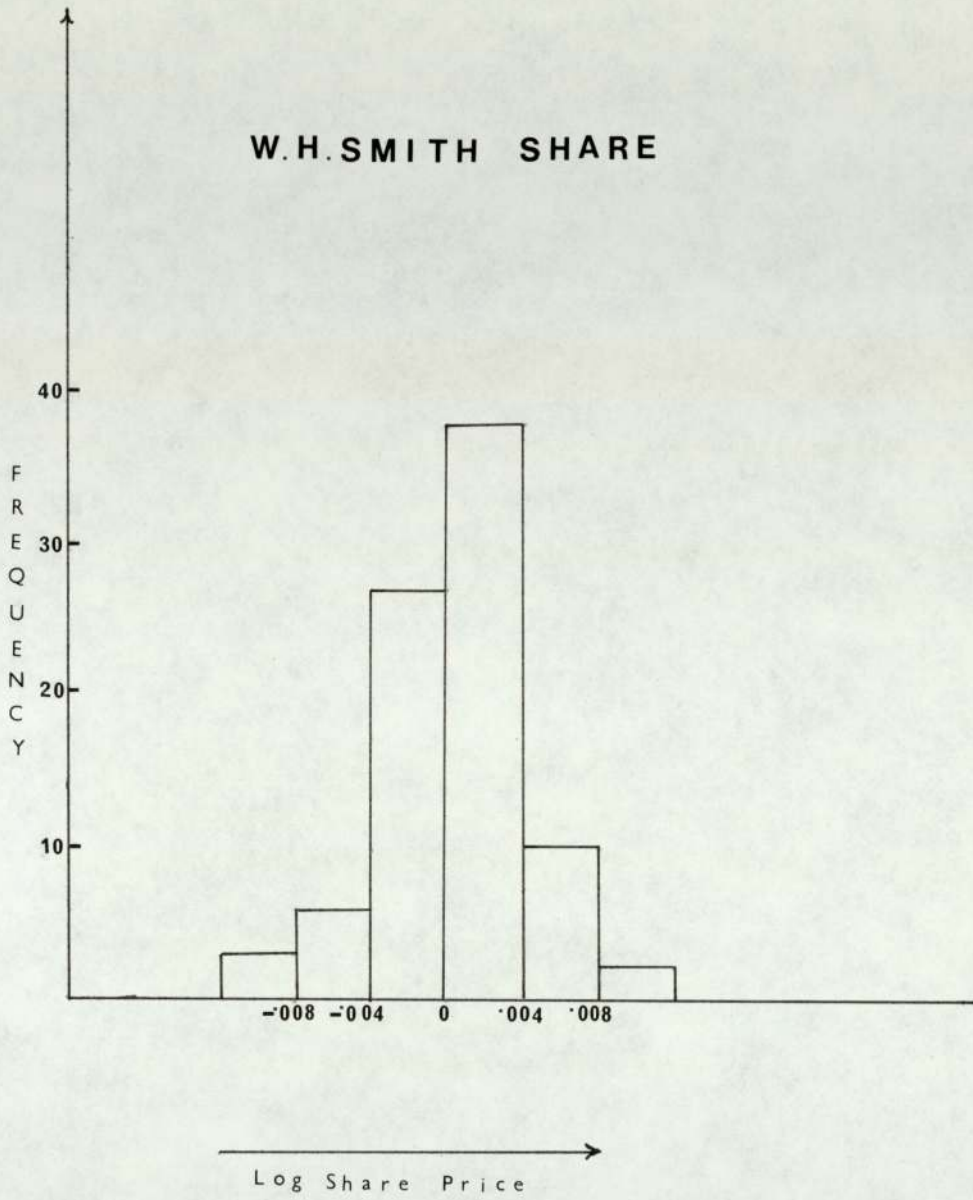


Figure -14.

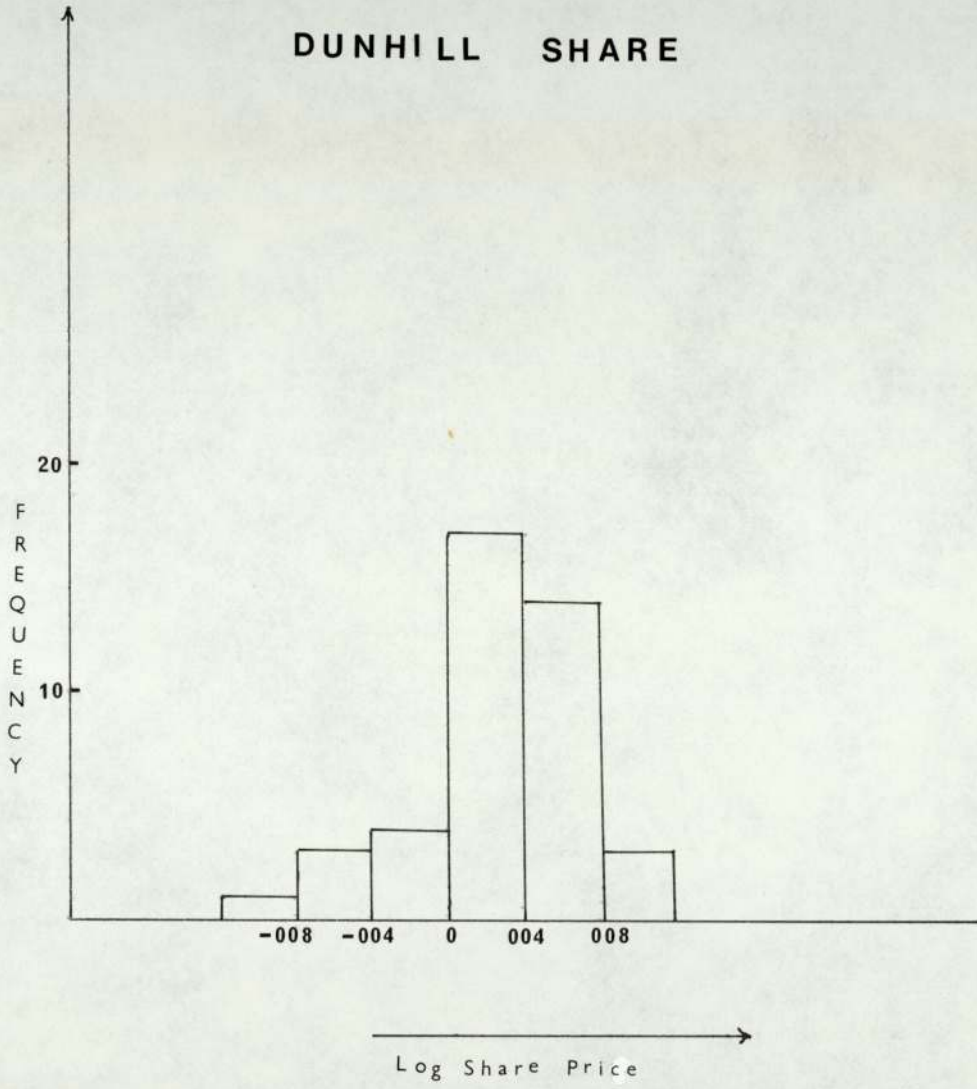


Figure - 15.

6.2.2) contd.

Results for the above analysis are believed to be more extensive than any previous study.

6.2.3) Ad-hoc methods.

Using the rules suggested in Section 3.1 Chapter III the following results are obtained with data 'B' and 'D' for the years 1968 to 1971.

Proportion of investment in each share is obtained from the data 'D' for the years 1968 to 1970. Using the same proportion in 1971 the actual rate of return obtained is given in the following tables.

Using rules 3.1.1.1 results obtained are given in Table 41. Using rules 3.1.1.2 the results are given in Tables 40 to 45.

Comparison with the Markowitz, Sharpe and Monte Carlo models are shown in Table 52.

A comparison between the rate of return of share prices using data 'B' for the periods (1968 to 1970) and 1971 are given in Figure 16.

It is observed that rule (e), Section 3.1.1.2 gives the best result followed by rule (b) in the same section.

6.2.4) Optimum Filter Rule.

This is a method of hindsight and it is simply a buying and holding policy. Data 'A' is used to calculate the optimum filter sizes as described in Section 3.2 Chapter III.

The results obtained using programme number 5

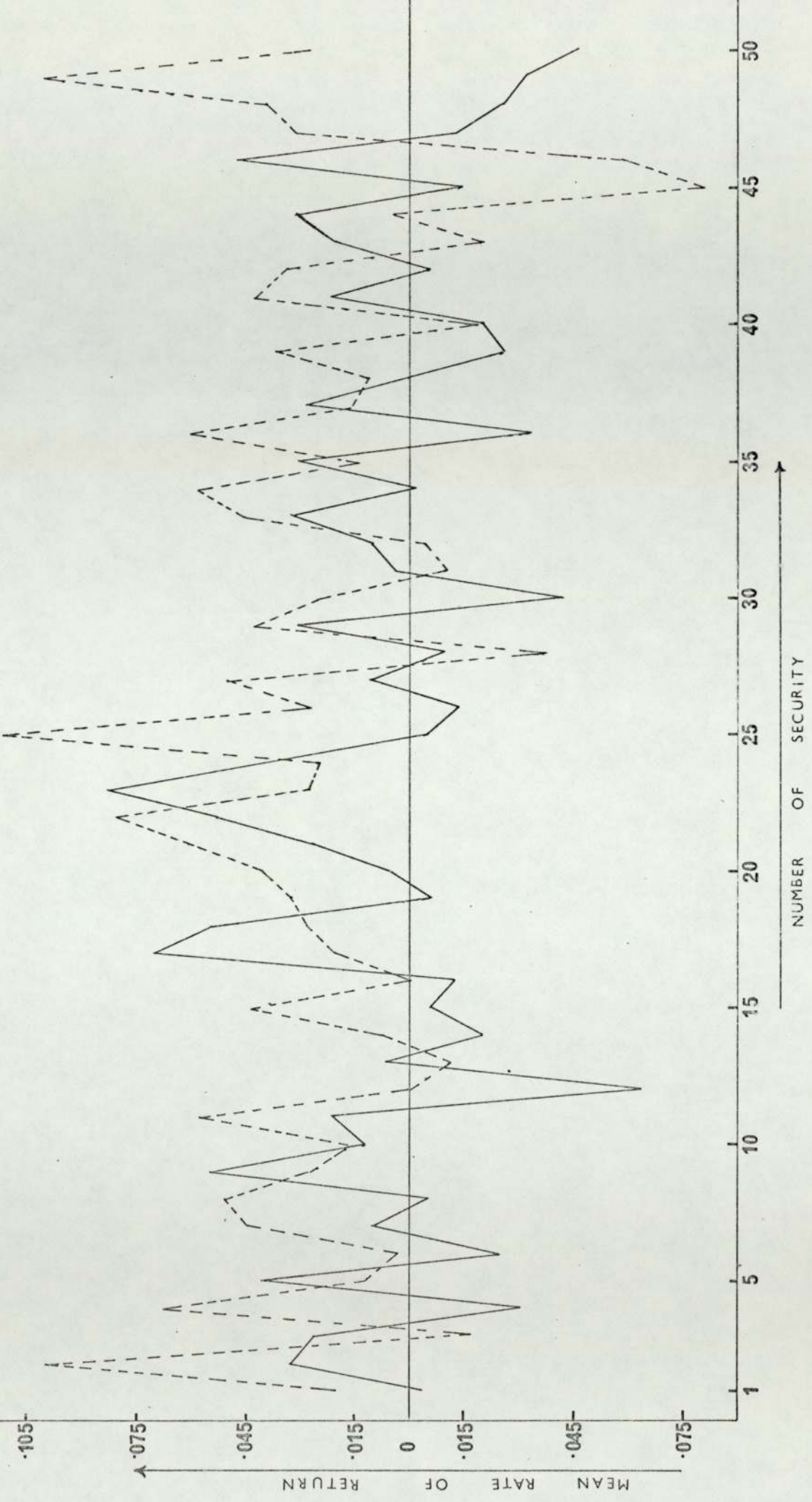


Figure - 16.

— 1968-1970
- - - 1971

6.2.4) contd.

are given in Tables (5) to (14) and the Tables are self-explanatory. Programme code and their output are given in Appendix A7.

The above method could be used as a predictor if the transaction cost is less than 2%.

6.2.5) Valuation Method.

Using data 'B' for the period 1968 to 1971 and ranking the corresponding products of risk and growth factors according to their ranks proportion invested, and the actual rate of return obtained on the basis of predictions from the previous years are shown in Tables 50 and 51.

As an empirical yardstick ad-hoc methods are used for purposes of comparison. Results obtained using data 'B' and ad-hoc rules suggested in Section 3.1 Chapter III are shown in Tables 46 to 49. The results by the valuation method are not satisfactory in comparison with ad-hoc methods.

6.2.6) The Markowitz Model.

The results obtained from the Markowitz analysis of the 'A', 'B', 'C' and 'D' sets of data by using programme number 1 are given in Tables 15 to 26, 27 to 28, 29 to 30 and 31 to 32 respectively.

In Tables 15 to 24 results are given for data 'A' computed year by year and compared each year for different values of α . The expected rate of return obtained on the basis of predictions from the previous year per day from 1959 to 1968 are shown in Figure (17). The graph shows that the degree of predictability will go up when α increases.

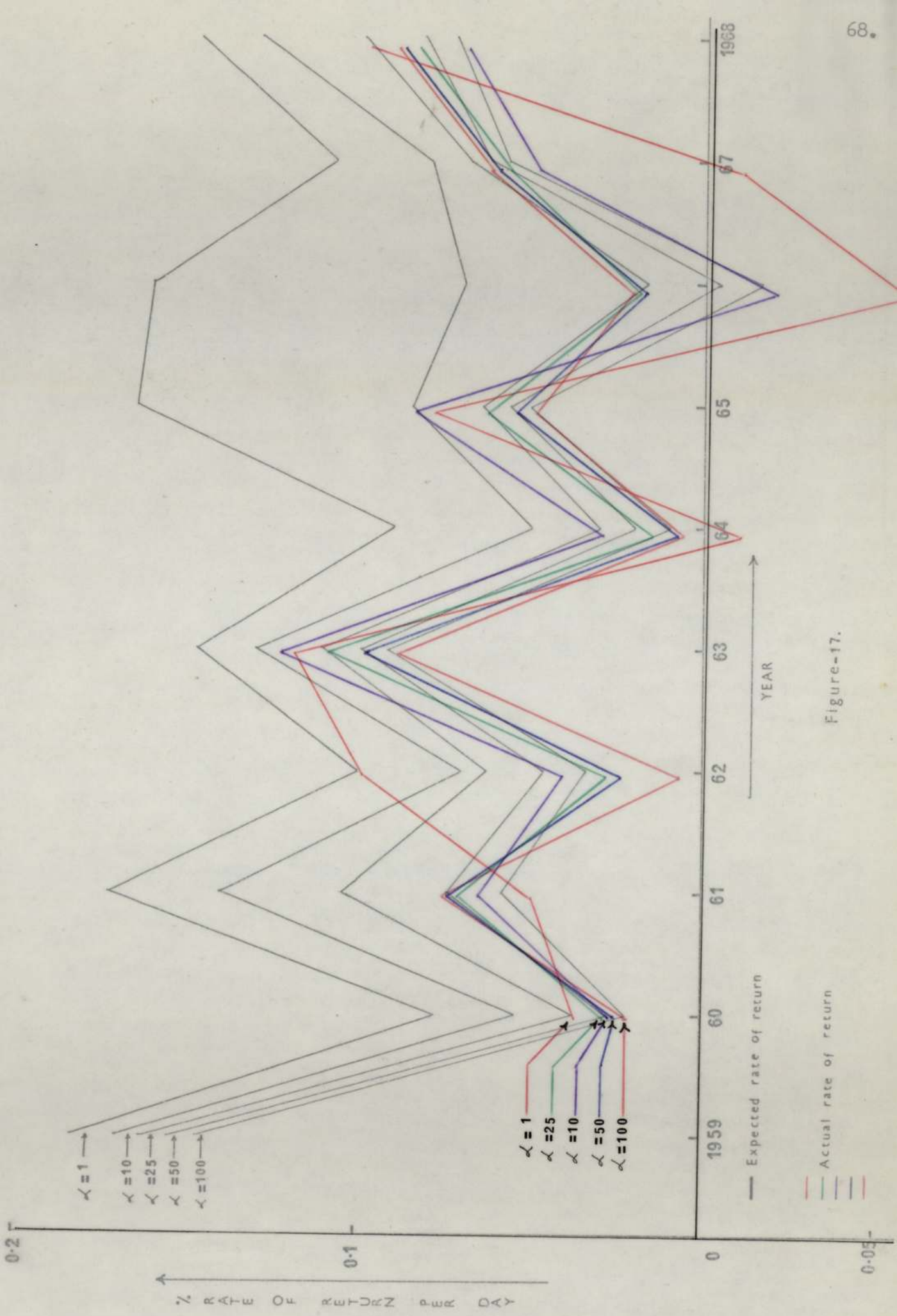


Figure-17.

6.2.6) contd.

A comparison is made in Table 25-26 for different values of α between the expected rate of return and the actual rate of return per day using data 'A' for the period 1959 to 1967 and the year 1968. The expected and actual rates of return obtained on the basis of predictions from the previous years per day are nearly the same even for the small values of α .

In Tables 27-28 a comparison is made using data 'B' for different values of α between the quarterly expected rate of return and actual rate of return for the period 1959 to 1970 and the year 1968.

The proportion investment figures for different values of α for the years 1968-1970 using data 'C' are used in Table 30 to calculate the actual rate of return per month for the year 1971. The above results (Table 30) are compared with the Sharpe model in Table 53.

Tables 31-32 are processed likewise using data 'D'. The results are compared with Ad-hoc, Sharpe and Monte Carlo methods in Table 52.

Table 54 shows the corresponding computer mill times for the different methods.

From the empirical results it may be concluded that in using a long time period and more detailed data prediction improves. This model gives the most satisfactory results of all the methods considered.

6.2.7) Sharpe model.

The results obtained from the Sharpe analysis of the 'C' and 'D' sets of data by using programme number 2, are given in Tables 33-36.

6.2.7) contd.

A comparison between the Sharpe and Markowitz models is shown in Table 53 and with other models in Table 52. From the empirical evidence it seems that more accurate predictions are obtained from higher values of α .

This model also gives the satisfactory results of all the methods considered. It takes less time in computing than Markowitz's, shown in Table 54.

6.2.8) The Monte Carlo Model.

The Monte Carlo model is tested using data 'D' for the periods 1968 to 1971.

The results obtained by using programmes numbers 6 and 7 are shown in Tables 38 and 39 respectively. Comparison with other methods are shown in Table 52.

From the empirical results it seems that the Monte Carlo model is a good predictor of rate of return of the shares, but it is costly in comparison with other methods as shown in Table 54.

6.2.9) Conclusion.

Of the models discussed in this investigation, it may be concluded that the following three stand out as the most powerful ones:

- 1) The Sharpe model
- 2) The Markowitz model
- 3) A Monte Carlo Technique

Empirical comparisons suggested that the other methods were either inefficient or too 'sensitive' for use.

6.2.9) contd.

The advantage of the ad-hoc methods is their simplicity. Originally suggested as a crude strategy against which the more sophisticated methods might be measured, empirical comparisons showed them to be as good as a number of the other methods.

The random walk hypothesis is valid only in the short term. An attempt was made to fit a normal distribution to logarithms of share price fluctuations, but in general the fit was not satisfactory.

The Monte Carlo model is found to be a good predictor of the rate of return of the shares, whereas the Sharpe and the Markowitz process only showed historical data. The advantage of simulation is that some attempt is made to predict the future behaviour of the share market.

Moreover, the simulation approach has a wide appeal to the managers. The major disadvantage of this model is its high cost.

The modified Markowitz model gives satisfactory results. In this model data input requires:

- (1) Risk measures (one value)
- (2) Share prices (m periods)

The difficulty is that the running time is increased since this model requires the complete covariance matrix as described in Appendix A1.

The Sharpe model helps to alleviate this difficulty. It reduces computing time by a half. In this model data input requires:

- (1) Risk measure (one value)
- (2) Share prices (m periods)
- (3) Market index (m periods)

6.2.9) contd.

The storage requirement for the Sharpe model is considerably less than that for the Markowitz model.

The Markowitz and Sharpe models are modified in such a fashion that any one wanting to use the programmes as a share market predictor simply has to input the official share price list (together with risk coefficient) to get the investment programmes straight away as output.

The Markowitz and the Sharpe models are comparable and show an improvement over the ad-hoc methods. Of the two the Sharpe is preferred because of its smaller computing cost. The Monte Carlo method is useful though expensive. However, it is believed that the approaches developed in the present dissertation for:

(i) The Monte Carlo Method

(ii) The Valuation Method

will open up a new area of research for producing more reliable methods of portfolio selection.

SECTION 6.3

TABLES 1 - 54

F* denotes only ore share (I.C.I)

Table Number	Data	Periods	Type of Analysis
1	D,E	1968-1970	Multiple regression analysis
2	A	1959 1964 & 1968	Serial Correlation
3	F	1959	Testing the fit of a Normal Distribution
4	F*	1960-1968	
5-14	A	1959-1968	Filter rule
15 - 26	A	1959-1968	Markowitz
27 - 28	B	1959-1968	
29 - 30	C	1968-1971	
31- 32	D	1968-1971	
33 - 34	C	1968-1971	Sharpe
35 - 36	D	1968-1971	
37	D	1968-1970	Correlation (r)
38- 39	D	1968-1971	Monte Carlo
40 - 45	D	1968-1971	Ad-Hoc
46 - 49	B	1968-1971	
50 - 51	B	1968-1971	Valuation
52	D	1968-1971	Comparison between Ad-Hoc, Markowitz's Markowitz's and Sharpe's
53	C	1968-1971	Comparison between Markowitz's and Sharpe's
54	A,B,C	1959-1968 1968-1971	Computer Time

TABLE 1

Degree of freedom 9 Independent Variables at significant level 99.00%

Share No.	Variables				Regression Constants					Multiple Correlation	t Statistics
	X ₁	X ₂	X ₃	X ₄	a ₀	a ₁ 10 ⁻⁴	a ₂ 10 ⁻⁴	a ₃ 10 ⁻⁶	a ₄		
1	1	2	3	4	113.85	-40	-318	10	2.24	0.568	0.43
	5	6(a)			46.95	1109	-1716			0.715	0.33
	1	2			135.33	-43	-268			0.483	0.35
2	1	2	3	4	99.06	21	-211	-2	8.07	0.860	0.17
	5	6(b)			48.37	-643	2910			0.211	0.55
	1	2			10.21	42	-85			0.583	0.16
3	1	2	3	4	0.49	13	-301	-6	3.50	0.814	1.04
	5	6(b)			20.09	27	-217			0.048	0.08
	1	2			26.83	12	-334			0.232	1.41
* 4	1	2	3	4	19.45	-17	-352	-10	3.87	0.925	0.21
	5	6(b)			0.14	837	-2716			0.662	1.11
	1	2			58.35	-24	-406			0.700	1.68
5	1	2	3	4	-30.42	52	538	20	-2.40	0.628	0.49
	5	6(c)			45.22	-509	1806			0.356	0.75
	1	2			-22.68	34	514			0.509	0.81
6	1	2	3	4	132.06	-118	1050	-30	-401	0.911	1.48
	5	6(c)			9.39	463	705			0.527	0.30
	1	2			38.51	-74	1200			0.611	2.37
7	1	2	3	4	131.97	-59	270	5	-2.91	0.445	0.36
	5	6(c)			-1.30	545	5149			0.915	3.26
	1	2			109.87	-58	298			0.418	0.28
8	1	2	3	4	55.72	-53	1201	20	-2.79	0.780	0.81
	5	6(c)			35.80	-323	2254			0.339	1.08
	1	2			-10.99	-21	1308			0.604	1.09
9	1	2	3	4	253.51	43	-1662	-50	-6.44	0.585	0.47
	5	6(c)			93.68	-1415	10049			0.479	1.63
	1	2			91.12	121	-1399			0.433	0.77
10	1	2	3	4	162.16	-73	-55	-10	-3.50	0.522	0.06
	5	6(c)			-4.93	557	4094			0.944	3.72
	1	2			105.97	-52	31			0.474	0.061

TABLE 1 (contd)

Degree of freedom 9 Independent Variables at significant level 99.00%

Share No.	Variables				Regression Constants					Multiple Correlation	t Statistics
	X ₁	X ₂	X ₃	X ₄	a ₀	a ₁ 10 ⁻⁴	a ₂ 10 ⁻⁴	a ₃ 10 ⁻⁶	a ₄		
11	1	2	3	4	98.76	-23	-189	-30	-0.18	0.700	0.08
	5	6(c)			43.75	-173	1819			0.464	1.52
	1	2			51.33	9	106			0.156	0.27
* 12	1	2	3	4	41.11	-37	-18	2	0.94	0.907	0.09
	5	6(d)			8.61	143	1702			0.964	4.12
	1	2			55.09	-42	-39			0.898	0.21
* 13	1	2	3	4	162.95	-63	-733	-18	-3.21	0.843	1.16
	5	6(d)			2.11	664	-185			0.788	0.14
	1	2			98.02	-34	631			0.651	1.51
* 14	1	2	3	4	225.01	-112	-489	-14	-5.65	0.902	1.12
	5	6(d)			-12.14	1289	-322			0.956	1.61
	1	2			141.18	-83	-363			0.828	0.68
* 15	1	2	3	4	138.87	-68	-391	-20	-4.06	0.926	2.44
	5	6(d)			11.79	753	-859			0.840	0.83
	1	2			66.05	-38	-278			0.738	0.81
* 16	1	2	3	4	143.81	-78	30	-20	-6.07	0.908	0.11
	5	6(e)			-9.26	932	-1441			0.855	1.32
	1	2			51.46	-45	170			0.660	0.40
17	1	2	3	4	-56.26	83	7	4	1.04	0.853	0.02
	5	6(e)			55.77	-25	-2362			0.490	0.51
	1	2			-38.54	76	-20			0.849	0.05
18	1	2	3	4	-64.64	84	610	9	-1.07	0.826	0.23
	5	6(e)			70.89	21	-3758			0.469	0.67
	1	2			-60.09	76	600			0.793	1.06
* 19	1	2	3	4	228.20	-80		-1	-1.77	0.798	0.02
	5	6(e)			-3.77	-822				0.710	1.44
	1	2			215.94	-77				0.692	1.96
20	1	2	3	4	-50	-124		-16	-4.53	0.616	0.23
	5	6(e)			681	-1454				0.539	0.47
	1	2			-21	-8				0.347	0.02

TABLE 1 (contd)

Degree of freedom 9 Independent Variables at significant level 99.00%

Share No.	Variables				Regression Constants					Multiple Correlation	t Statistics
	X ₁	X ₂	X ₃	X ₄	a ₀	a ₁ 10 ⁻⁴	a ₂ 10 ⁻⁴	a ₃ 10 ⁻⁶	a ₄		
21	1	2	3	4	49.06	-13	-103	-4	-1.44	0.410	0.35
	5	6(i)			5.77	-40	1764			0.789	1.93
	1	2			26.03	-4	-67			0.187	0.28
22	1	2	3	4	-4.32	52	-388	-4	1.39	0.645	0.21
	5	6(i)			32.88	-827	5109			0.515	1.60
	1	2			16.71	44	-420			0.583	0.92
* 23	1	2	3	4	-134.31	116	89	10	5.47	0.906	0.20
	5	6(i)			48.58	-1680	6662			0.700	1.70
	1	2			-49.04	85	-41			0.832	0.08
24	1	2	3	4	-34.67	8	175	-1	4.73	0.550	0.05
	5	6(f)			10.42	-120	1443			0.256	0.49
	1	2			14.39	-2	106			0.071	0.13
25	1	2	3	4	-10.21	10	27	7	2.15	0.785	0.21
	5	6(f)			17.29	-19	384			0.209	0.31
	1	2			25.40	-4	-27			0.219	0.16
* 26	1	2	3	4	82.42	-51	-440	-10	3.54	0.888	1.12
	5	6(f)			4.82	755	-811			0.789	0.31
	1	2			99.27	-45	-456			0.782	1.33
27	1	2	3	4	8.30	4	420	10	-1.96	0.608	0.20
	5	6(b)			20.79	-526	3209			0.420	1.24
	1	2			8.81	-6	412			0.406	0.82
28	1	2	3	4	18.73	-10	-31	0	-0.34	0.696	0.02
	5	6(g)			0.95	90	26			0.579	0.03
	1	2			15.25	-9	-26			0.680	0.28
29	1	2	3	4	-39.08	12	-39	10	5.38	0.565	0.06
	5	6(g)			-2.85	-169	3267			0.800	1.37
	1	2			41.33	-16	-160			0.282	0.45
* 30	1	2	3	4	194-14	-97	-207	-10	-949	0.953	0.74
	5	6(g)			-25.55	1311	-2095			0.818	0.57
	1	2			73.84	-62	-31			0.719	0.06

TABLE 1 (oontd)

Degree of freedom 9 Independent Variables at significant level 99.00%

Share No.	Variables				Regression Constants					Multiple Correlation	t Statistics
	X ₁	X ₂	X ₃	X ₄	a ₀	a ₁ 10 ⁻⁴	a ₂ 10 ⁻⁴	a ₃ 10 ⁻⁶	a ₄		
31	1	2	3	4	99.31	-55	1	-10	-3.00	0.809	0.00
	5	6(h)			-2.81	539	-284			0.879	0.17
	1	2			41.57	-30	91			0.613	0.27
32	1	2	3	4	18.4	20	-73	10	-1.59	0.543	0.23
	5	6(i)			28.56	389	-2100			0.500	1.55
	1	2			24.49	8	-90			0.201	0.30
33	1	2	3	4	65.35	-15	-393	-6	0.15	0.421	0.05
	5	6(i)			1.84	-124	4046			0.865	2.99
	1	2			56.59	-8	-377			0.289	0.90
34	1	2	3	4	37.53	-17	121	-2	-1.25	0.465	0.26
	5	6(j)			12.31	104	171			0.403	0.12
	1	2			20.33	-11	147			0.320	0.64
35	1	2	3	4	-79.18	13	-2186	10	38.8	0.738	0.09
	5	6(o)			-23.24	-1205	22845			0.916	5.63
	1	2			355.76	-92	-2805			0.369	0.94
* 36	1	2	3	4	78.36	-57	152	-7	-1.52	0.900	0.67
	5	6(k)			0.89	307	392			0.654	0.17
	1	2			50.21	-45	196			0.855	0.88
37	1	2	3	4	-18.10	38	-77	20	-2.66	0.667	0.13
	5	6(k)			29.71	-113	-556			0.483	0.26
	1	2			8.00	20	124			0.451	0.39
38	1	2	3	4	173.13	-111	517	-10	-1.85	0.640	0.24
	5	6(e)			-1.94	734	5389			0.898	1.89
	1	2			127.84	-90	590			0.600	0.62
* 39	1	2	3	4	794.35	-464	669	-40	-33.87	0.851	0.28
	5	6(e)			-144.93	3214	13978			0.959	2.32
	1	2			358.43	-334	1310			0.718	0.48
* 40	1	2	3	4	390.22	-237	340	-40	-10.51	0.911	0.40
	5	6(e)			-29.94	1567	5496			0.927	1.75
	1	2			202.77	-161	631			0.800	0.62

TABLE 1 (contd)

Degree of freedom 9 Independent Variables at significant level 99.00%

Share No.	Variables				Regression Constants					Multiple Correlation	t Statistics
	X ₁	X ₂	X ₃	X ₄	a ₀	a ₁ 10 ⁻⁴	a ₂ 10 ⁻⁴	a ₃ 10 ⁻⁶	a ₄		
41	1	2	3	4	93.98	-19	430	-9	-4.94	0.470	0.36
	5	6(l)			26.3	2389	5153			0.613	1.97
	1	2			26.45	3	530			0.145	0.77
* 42	1	2	3	4	232.66	-137	421	-20	-6.37	0.787	0.42
	5	6(l)			-11.14	674	5799			0.948	2.96
	1	2			125.84	-96	585			0.682	0.71
43	1	2	3	4	75.04	-13	-354	-6	-2.09	0.551	0.59
	5	6(m)			-5.21	109	2765			0.901	4.60
	1	2			42.97	-1	-306			0.375	0.10
44	1	2	3	4	-12.66	20	-205	-6	4.59	0.790	0.72
		5	6(m)		13.52	-192	2424			0.550	1.89
	1	2			25.79	17	-255			0.334	0.82
* 45	1	2	3	4	197.60	-108	-334	-9	-1.64	0.791	0.28
	5	6(a)			-11.76	454	5674			0.951	1.91
	1	2			165.37	-94	-283			0.779	0.40
* 46	1	2	3	4	95.98	43	-640	-3	-6.50	0.783	0.18
	5	6(n)			0.05	-746	8970			0.895	5.97
	1	2			46.73	26	-539			0.693	1.75
* 47	1	2	3	4	185.63	-86	-272	-5	-0.36	0.897	0.13
	5	6(j)			49.48	356	2698			0.728	0.73
	1	2			172.52	-79	-250			0.889	0.66
48	1	2	3	4	377.08	-172	-906	-5	-11.30	0.924	0.22
	5	6(p)			-14.18	2137	-609			0.960	0.27
					248.49	-140	-722			0.440	0.86
49	1	2	3	4	102.47	-71	830	10	-6.79	0.846	0.85
	5	6(p)			-6.92	142	4173			0.726	1.49
	1	2			54.89	-72	888			0.637	1.57
* 50	1	2	3	4	71.93	75	1429	20	-6.28	0.972	3.28
	5	6(q)			40.76	-684	4010			0.690	1.73
	1	2			47.34	-89	1449			0.676	3.35

TABLE 2

Time interval one day			
Share Number	Average correlations for the Year		
	1959	1964	1968
1	-0.50×10^{-2}	-0.50×10^{-2}	-0.86×10^{-3}
2	0.85×10^{-2}	-0.57×10^{-2}	-0.41×10^{-2}
3	0.40×10^{-2}	-0.59×10^{-2}	-0.36×10^{-2}
4	-0.84×10^{-2}	-0.33×10^{-2}	-0.16×10^{-2}
5	-0.45×10^{-2}	-0.33×10^{-2}	-0.22×10^{-2}
6	0.29×10^{-2}	-0.70×10^{-2}	-0.34×10^{-2}
7	-0.18×10^{-2}	-0.12×10^{-2}	-0.35×10^{-2}
8	-0.32×10^{-2}	0.66×10^{-2}	0.83×10^{-3}
9	0.15×10^{-1}	-0.46×10^{-3}	-0.16×10^{-2}

TABLE 3

Year 1959	Price changes number of days	χ^2	Degree of freedom	Level of Significance	
Share Number				5%	1%
1	170	17.19	4	0.711	0.297
2	158	16.63	5	1.15	0.554
3	215	16.41	7	2.17	1.24
4	210	23.16	7	2.17	1.24
5	171	14.21	5	1.15	0.554
6	105	4.61	2	0.103	0.0201
7	192	22.55	5	1.15	0.554
8	86	19.60	1	0.0039	0.002
9	42	8.74	3	0.352	0.115

TABLE 4

"I.C.I. Share"	Price changes number of days	χ^2	Degree of freedom	Level of Significance	
Year				5%	1%
1960	226	2.92	7	2.12	1.24
61	219	2.82	5	1.15	0.554
62	242	10.17	7	2.12	1.24
63	199	11.93	5	1.15	0.554
64	220	11.11	6	1.64	0.872
65	228	12.73	7	2.12	1.24
66	195	14.54	6	1.64	0.872
67	208	21.16	7	2.12	1.24
68	237	24.28	8	2.73	1.65
59-68	2189	11.42	9	3.33	2.09

TABLE 5

T denotes "Number of Transactions"

Year 1959	Share Statistics						% Maximum Profit Achieved
Share No.	Rises	Falls	No change	% Falls	% Rises	T	
1	103	87	116	1	7	9	90.12
2	95	75	136	1	10	3	23.33
3	110	110	86	2	8	4	36.48
4	128	88	90	6	1	5	62.85
5	89	92	125	8	9	5	45.44
6	71	22	183	2	8	5	45.62
7	94	107	105	5	5	2	- 2.26
8	55	36	215	2	6	3	15.89
9	35	7	264	1	10	3	24.86

TABLE 6.

T denotes "Number of Transactions"

Year 1960	Share Statistics						% Maximum Profit Achieved	% Forecasted profit achieved from previous year
Share No	Rises	Falls	No change	% Falls	% Rises	T		
1	101	107	97	1	1	6	7.42	1.52
2	94	102	109	5	9	3	10.96	1.83
3	111	123	71	1	2	8	17.86	8.44
4	127	120	58	1	4	3	11.57	1.16
5	83	98	124	3	1	12	18.96	10.03
6	63	63	179	2	2	6	10.97	-0.34
7	99	123	83	5	2	6	20.87	-10.63
8	44	41	220	1	10	3	12.71	6.92
9	14	15	276	4	6	3	14.79	11.54

TABLE 7.

T denotes "Number of Transactions"

Year 1961	Share Statistics						% Maximum Profit Achieved	% Forecasted profit achieved from previous year
	Share No.	Rises	Falls	No change	% Falls	% Rises		
1	114	102	87	2	10	5	51.74	9.41
2	104	98	101	6	10	5	38.94	37.97
3	109	119	75	10	3	4	15.52	-7.93
4	121	107	75	1	6	5	20.92	10.41
5	102	86	115	5	9	2	10.90	1.52
6	99	85	119	2	8	6	56.78	2.16
7	87	107	109	10	9	5	22.59	-5.51
8	71	52	108	7	3	4	10.07	-0.38
9	31	15	257	5	8	2	10.71	10.71

TABLE 8.

T denotes "Number of Transactions"

Year 1962	Share Statistics						% Maximum Profit Achieved	% Forecasted profit achieved from previous year
	Share No.	Rises	Falls	No change	% Falls	% Rises		
1	101	111	95	9	4	4	22.14	2.70
2	87	96	124	10	5	5	27.09	-6.23
3	109	119	79	4	4	5	20.44	8.41
4	100	122	85	5	5	2	-16.71	-16.71
5	96	90	121	1	8	5	29.87	22.32
6	69	79	159	7	6	5	33.77	0.18
7	80	105	122	7	1	10	37.35	-20.98
4	44	53	210	2	3	7	28.07	4.78
9	30	24	253	5	3	3	14.73	8.70

TABLE 9.

T denotes "Number of Transactions"

Year 1963	Share Statistics						% Maximum Profit Achieved	% Forecasted profit achieved from previous year
	Share No.	Rises	Falls	No change	% Falls	% Rises		
1	80	105	122	3	2	7	14.77	8.73
2	86	106	115	2	7	6	45.38	6.50
3	109	104	94	2	10	5	32.90	11.36
4	115	100	92	3	9	6	46.05	5.13
5	110	80	117	1	7	7	52.73	29.24
6	46	55	206	10	4	5	27.38	-3.65
7	73	94	140	6	10	4	35.39	3.37
8	50	42	215	4	8	4	27.91	23.27
9	37	9	261	5	9	2	13.58	3.70

TABLE 10.

T denotes "Number of Transactions"

Year 1964	Share Statistics						% Maximum Profit Achieved	% Forecasted profit achieved from previous year
	Share No.	Rises	Falls	No change	% Falls	% Rises		
1	88	106	112	2	4	6	26.13	14.87
2	89	113	104	6	3	7	22.76	-14.50
3	99	121	86	9	2	4	11.70	-12.26
4	89	123	94	9	2	6	12.77	-24.05
5	85	107	114	1	2	8	13.86	- 5.24
6	70	73	163	5	5	7	36.67	7.28
7	95	96	115	2	8	4	33.48	11.30
8	50	56	200	7	2	4	11.62	-2.31
9	39	40	227	4	10	4	30.61	9.24

TABLE 11.

T denotes "Number of Transactions"

Year 1965	Share Statistics						% Maximum Profit Achieved	% Forecasted profit achieved from previous year
	Share No.	Rises	Falls	No change	% Falls	% Rises		
1	93	86	129	1	5	6	30.79	26.01
2	90	98	120	7	3	5	18.43	17.01
3	104	100	104	1	3	8	16.91	10.09
4	97	92	119	3	5	4	15.24	2.47
5	100	106	102	9	7	5	32.34	22.15
6	81	61	166	1	7	7	38.95	9.05
7	93	96	119	5	10	5	42.46	9.20
8	60	49	199	2	4	5	18.58	5.46
9	55	43	210	5	10	2	11.76	11.76

TABLE 12.

T denotes "Number of Transactions"

Year 1966	Share Statistics						% Maximum Profit Achieved	% Forecasted profit achieved from previous year
	Share No.	Rises	Falls	No change	% Falls	% Rises		
1	80	94	128	10	2	3	4.81	2.79
2	85	114	103	10	1	3	-20.31	-28.87
3	93	106	103	4	2	12	93.98	-14.68
4	97	99	106	4	1	8	11.96	-10.20
5	104	89	109	1	4	3	25.46	9.17
6	81	81	104	7	6	5	30.05	4.57
7	115	103	84	1	8	8	89.57	10.05
9	25	31	246	10	2	4	7.53	-17.28

TABLE 13

T denotes "Number of Transactions"

Year 1967	Share Statistics						% Maximum Profit Achieved	% Forecasted profit achieved from previous year
	Share No.	Rises	Falls	No change	% Falls	% Rises		
1	108	94	105	1	5	6	33.06	4.76
2	93	91	123	1	5	10	56.91	34.12
3	123	98	86	1	10	6	48.83	8.32
4	108	105	94	2	9	6	37.86	3.53
5	99	101	107	1	5	7	33.74	23.70
6	92	87	128	1	4	8	23.61	6.37
7	108	110	89	10	9	5	35.30	24.15
8	68	43	196	5	8	5	33.35	5.16
9	33	32	242	2	2	8	23.24	2.99

TABLE 14.

T denotes "Number of Transactions"

Year 1968	Share Statistics						% Maximum Profit Achieved	% Forecasted profit achieved from previous year
	Share No.	Rises	Falls	No change	% Falls	% Rises		
1	103	112	89	5	10	4	36.51	30.43
2	102	111	91	4	10	6	49.19	37.06
3	127	113	64	1	4	13	69.82	50.54
4	126	106	72	1	8	4	30.54	12.62
5	102	94	108	9	7	4	22.16	3.62
6	106	90	108	5	10	2	11.30	4.47
7	108	104	92	2	3	7	30.26	9.50
8	82	65	157	6	9	6	66.30	12.50
9	53	53	198	1	9	5	28.79	5.52

TABLE 15.

Year 1959	%	Percentage Invested				
Share No.		Mean rate of return	$\alpha = 1$	$\alpha = 10$	$\alpha = 25$	$\alpha = 50$
1	0.17	10.92	15.84	11.86	9.80	8.34
2	0.13	0	0	0.91	3.90	3.98
3	0.17	5.84	13.08	13.51	12.24	11.13
4	0.23	21.16	2.69	1.17	0.60	0.35
5	0.05	0	0	0	2.93	5.32
6	0.17	4.18	7.71	5.80	5.17	4.61
7	0.004	0	0	0	0	2.62
8	0.12	0	5.64	19.55	23.55	25.35
9	0.17	57.90	55.05	47.20	41.78	38.29
% Expected rate of return per day		1.183	0.17	0.163	0.155	0.147

TABLE 16

Year 1960	%	Percentage Invested				
Share No.		Mean rate of return	$\alpha = 1$	$\alpha = 10$	$\alpha = 25$	$\alpha = 50$
1	0.01		0	4.10	4.82	4.93
2	0.01		0	0.13	1.41	2.05
3	0.03		0.51	3.64	3.96	4.03
4	0.08	100	26.53	14.83	9.37	5.87
5	-0.06		0	0	4.35	8.90
6	0.004		0	0	3.87	5.63
7	-0.02		0	1.47	3.35	4.20
8	0.04		47.44	45.56	40.44	37.11
9	0.03		25.52	30.27	28.73	27.25
% Expected rate of return per day		0.077	0.054	0.038	0.028	0.022
% Actual rate of re- turn per day from previous year.		0.037	0.027	0.028	0.026	0.023
% Annual rate of return		11.21	8.12	8.48	7.88	6.97

TABLE 17

Year 1961	% Mean rate of return	Percentage Invested				
Share No.		$\alpha = 1$	$\alpha = 10$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	0.13	0.98	20.15	10.03	5.06	2.57
2	0.05	0	0	6.33	6.27	6.24
3	-0.03	0	0	0	0	0
4	0.05	0	4.93	21.12	21.04	21.01
5	0.08	0	10.75	17.90	17.18	16.82
6	0.14	0	19.94	9.39	4.74	2.42
7	-0.05	0	0	1.05	10.52	15.25
8	0.02	0	0	4.28	12.19	16.14
9	0.17	99.02	44.23	29.89	22.99	19.54
% Expected rate of return per day		0.173	0.14	0.105	0.074	0.058
% Actual rate of return per day from previous year.		0.050	0.066	0.072	0.075	0.076
% Annual rate of return		15.20	20.06	21.89	22.80	23.10

TABLE 18

Year 1962	% Mean rate of return	Percentage Invested				
Share No.		$\alpha = 1$	$\alpha = 10$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	-0.004		0	0.64	2.20	2.67
2	-0.05		0	0	0	0
3	0.02		0	6.68	4.61	2.79
4	-0.05		0	0.01	12.61	19.44
5	0.07		43.57	36.10	27.76	22.92
6	-0.004		0	0	0	0
7	-0.10		0	0	0	2.14
8	0.03		12.81	22.95	24.17	24.22
9	0.10	100	43.62	33.62	28.37	25.81
% Expected rate of return per day		0.100	0.07	0.063	0.046	0.034
% Actual rate of return per day from previous year.		0.099	0.041	0.028	0.024	0.007
% Annual rate of return		30.10	12.46	8.51	7.30	2.13

TABLE 19.

Year 1963	% Mean rate of return	Percentage Invested				
Share No.		$\alpha = 1$	$\alpha = 10$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	-0.004		0	0	0	2.19
2	0.09			4.52	4.06	2.91
3	0.08			0	1.31	3.64
4	0.08			7.04	10.93	11.84
5	0.15	100	47.84	28.17	19.56	14.18
6	-0.03			0	0	0
7	0.06			3.15	4.56	5.08
8	0.05			13.68	22.53	26.61
9	0.12		52.16	43.43	37.06	33.54
% Expected rate of return per day		0.147	0.130	0.111	0.100	0.092
% Actual rate of return per day from previous year.		0.120	0.124	0.110	0.100	0.090
% Annual rate of return		36.60	37.82	33.55	30.50	27.45

TABLE 20

Year 1964	% Mean rate of return	Percentage Invested				
Share No.		$\alpha = 1$	$\alpha = 10$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	0.03		21.32	17.73	16.12	14.08
2	-0.04		0	0	0	0
3	-0.04		0	0	0.85	3.35
4	-0.08		0	0	0.07	3.15
5	-0.01		13.50	23.71	26.46	26.71
6	0.02		7.06	6.16	5.47	4.48
7	0.10	86.32	29.40	17.34	13.01	10.32
8	-0.02		0	14.51	20.54	22.67
9	0.06	13.68	28.72	20.55	17.47	15.23
% Expected rate of return per day		0.09	0.05	0.03	0.02	0.01
% Actual rate of return per day from previous year.		-0.01	0.03	0.016	0.009	0.007
% Annual rate of return		-3.06	9.18	4.90	2.75	2.14

TABLE 21

Year 1965	% Mean rate of return	Percentage Invested				
Share No.		$\alpha = 1$	$\alpha = 10$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	0.06		19.98	18.30	16.25	14.91
2	0.02		0	0	0	0
3	0.004		0	0	0.27	2.60
4	0.03		0	2.01	4.75	5.43
5	-0.01		0	1.72	4.94	6.28
6	0.07		25.00	19.27	16.84	15.51
7	0.07		6.76	7.16	6.35	5.73
8	0.03		24.48	38.54	41.45	42.36
9	0.07	100	23.79	12.99	9.14	7.16
% Expected rate of return per day		0.165	0.085	0.065	0.056	0.051
% Actual rate of return per day from previous year.		0.08	0.086	0.064	0.056	0.050
% Annual Rate of Return		24.32	22.14	19.46	17.02	15.20

TABLE 22

Year 1966	% Mean rate of return	Percentage Invested				
Share No.		$\alpha = 1$	$\alpha = 10$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	-0.03		4.08	23.42	26.32	26.96
2	-0.10		0	0	0	1.68
3	-0.03		4.20	3.41	3.04	2.80
4	-0.04		0	0	3.43	5.72
5	0.05		51.20	30.93	21.90	17.19
6	0.02		14.22	12.45	9.91	8.16
7	0.16	100	26.29	11.97	6.90	4.25
8	-0.06		0	8.75	14.34	16.80
9	-0.06		0	9.06	14.13	16.42
% Expected rate of return per day		0.161	0.07	0.017	-0.004	-0.016
% Actual rate of return per day from previous year.		-0.06	-0.002	-0.02	-0.019	-0.022
% Annual rate of return		-18.35	-0.61	-0.61	-0.59	-0.67

TABLE 23.

Year 1967	% Mean rate of return	Percentage Invested				
Share No.		$\alpha = 1$	$\alpha = 10$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	0.10	25.31	29.79	16.55	12.43	10.56
2	0.08	0	5.02	6.15	6.39	6.55
3	0.11	74.67	22.21	10.01	6.82	4.28
4	0.07		4.67	5.40	5.39	4.52
5	0.07		22.46	20.04	18.95	17.52
6	0.04		0.09	14.07	18.89	20.95
7	-0.01		0	0	0	3.50
8	0.04		9.88	16.96	19.03	19.72
9	0.04		5.89	10.22	12.10	12.90
% Expected rate of return per day		0.108	0.08	0.069	0.063	0.058
% Actual rate of return per day from previous year.		-0.10	0.05	0.059	0.062	0.064
% Annual rate of return		-30.7	15.3	18.1	19.0	19.4

TABLE 24.

Year 1968	% Mean rate of return	Percentage Invested				
Share No.		$\alpha = 1$	$\alpha = 10$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	0.06	0	0.91	8.11	9.15	9.68
2	0.08	0	0	0	0	0
3	0.11	0	19.80	13.53	9.55	7.56
4	0.11	0	10.75	9.05	7.95	7.39
5	0.004	0	0	12.12	24.31	30.40
6	0.14	27.56	24.24	14.50	9.75	7.37
7	0.04	0	0	5.67	7.28	8.09
8	0.15	72.44	36.30	26.71	21.95	19.57
9	0.07	0	8.00	10.30	10.05	9.93
% Expected rate of return per day		0.147	0.13	0.100	0.082	0.073
% Actual rate of return per day from previous year.		0.10	0.71	0.085	0.09	0.091
% Annual rate of return		30.5	21.6	26.0	27.4	27.8

TABLE 25.

Year 1959-67	% Mean rate of return	Percentage Invested			
Share No.		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	0.02	0	5.01	4.29	3.91
2	0.00	0	0	1.24	1.93
3	0.02	0	4.68	4.38	4.21
4	0.01	0	0.24	0.75	1.00
5	0.02	0	24.06	23.49	23.21
6	0.06	78.85	22.40	20.17	19.04
7	0.01	0	12.88	13.77	14.13
8	-0.00	0	12.64	14.59	15.57
9	0.04	21.15	18.09	17.37	17.00
% Expected rate of return per day		0.06	0.03	0.03	0.03

TABLE 26.

Year 1968	Mean rate of return	Percentage Invested			
Share No.		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	0.06	0	8.11	9.15	9.68
2	0.08	0	0	0	0
3	0.11	0	13.53	9.55	7.56
4	0.11	0	9.05	7.95	7.39
5	0.004	0	12.12	24.31	30.40
6	0.14	27.56	14.50	9.75	7.37
7	0.04	0	5.67	7.78	8.09
8	0.15	72.44	26.71	21.95	19.57
9	0.07	0	10.30	10.05	9.93
% Expected rate of return per day		0.15	0.10	0.08	0.07
% Actual rate of return per day from previous years		0.13	0.08	0.08	0.08
% Annual rate of return		39.7	24.4	24.4	24.4

TABLE 27.

Year 1959-67	% Mean rate of return	Percentage Invested			
Share No.		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	2.97	0	0	0	0
2	1.39	0	0	0	0
3	1.37	0	26.36	27.30	27.78
4	3.12	0	0	0	0
5	1.85	0	16.54	17.11	17.40
6	5.27	86.16	17.08	13.89	12.30
7	0.52	0	19.44	20.40	20.87
8	1.63	0	5.10	7.10	8.10
9	3.76	13.84	15.48	14.19	13.55
% Expected rate of return per quarter		5.06	2.33	2.17	2.09

TABLE 28.

Year 1968	Mean rate of return	Percentage Invested			
Share No.		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	2.65	0	0	0	0
2	1.21	0	0	0	0.65
3	1.77	0	25.54	25.97	26.02
4	2.59	0	0	0	0
5	2.07	0	15.93	16.10	16.10
6	5.03	78.71	17.89	15.06	13.39
7	0.40	0	19.82	21.11	21.77
8	1.58	0	55.29	7.99	9.08
9	3.95	21.29	15.29	13.78	12.99
% Expected rate of return per quarter		4.80	2.45	2.30	2.22
% Actual rate of return per quarter from previous years		4.65	2.35	2.29	2.22
% Annual rate of return		19.93	9.97	9.50	9.10

TABLE 29.

Year 1968-70	% Mean rate of return	Percentage Invested			
Share No.		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	0.66	0	13.92	14.17	14.30
2	1.25	39.59	16.13	14.70	13.98
3	-0.23	0	0	0	0
4	-0.24	0	0	0	0
5	0.02	0	10.13	10.73	11.03
6	0.60	0	0	0	0
7	1.03	19.03	20.62	19.61	19.10
8	0.20	0	0	0	0
9	1.38	41.38	17.33	16.78	16.51
10	-0.19	0	21.87	24.01	25.08

TABLE 30.

Year 1971	% Mean rate of return	Percentage Invested			
Share No.		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	2.91	0	13.92	14.17	14.30
2	5.44	39.59	16.13	14.70	13.98
3	5.22	0	0	0	0
4	2.03	0	0	0	0
5	-0.10	0	10.13	10.73	11.03
6	2.38	0	0	0	0
7	2.05	19.03	20.62	19.61	19.10
8	1.58	0	0	0	0
9	0.04	41.38	17.33	16.78	16.51
10	3.43	0	21.87	24.01	25.08
% Actual rate of return per month.		2.56	2.45	2.43	2.42
% Annual rate of return		30.5	33.7	33.4	33.2

TABLE 31

Year 1968-70	% Mean rate of return	Percentage Invested			
Share No.		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	-0.35	0	0	0	0
2	3.24	0	0	0	38.75
3	2.70	0	0	0	0
4	-0.31	0	0	0	0
5	4.32	0	0	0	0
6	-2.46	0	0	44.47	0
7	1.09	0	0	0	0
8	-0.43	0	0	0	0
9	5.62	0	0	0	0
10	1.36	0	0	0	0
11	2.14	0	0	0	0
12	-6.73	0	0	0	0
13	0.73	0	0	14.68	0
14	-2.07	0	0	0	0
15	-0.67	0	0	0	0
16	-1.18	0	0	0	0
17	6.97	0	0	0	0
18	5.40	0	0	0	0
19	-0.73	0	0	36.07	0
20	0.56	0	0	0	0
21	2.85	0	10.50	0	0
22	5.22	0	0	0	0
23	8.22	0	0	0	0
24	4.01	0	0	0	0
25	-0.48	0	0	0	44.25
26	-1.38	0	0	0	0
27	0.83	0	0	0	0
28	-1.09	0	0	0	0
29	3.08	0	0	4.70	0
30	-4.38	0	0	0	0
31	0.39	0	0	0	0
32	0.92	0	0	0	0

TABLE 31 (contd)

Year 1968-70	% Mean rate of return	Percentage Invested			
Share No.		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
33	3.25	0	0	0	0
34	-0.15	0	0	0	0
35	1.54	0	11.60	0	0
36	3.39	0	0	0	16.97
37	2.87	21.15	34.68	0	0
38	0.35	0	0	0	0
39	-2.61	0	0	0	0
40	-2.07	0	0	0	0
41	2.13	0	43.22	0	0
42	-0.51	0	0	0	0
43	1.82	0	0	0	0
44	3.09	0	0	0	0
45	-1.46	0	0	0	0
46	4.67	78.85	0	0	0
47	-1.22	0	0	0	0
48	-2.62	0	0	0	0
49	-3.21	0	0	0	0
50	-4.60	0	0	0	0

TABLE 32

Year 1971	% Mean rate of return	Percentage Invested			
Share No.		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	1.91	0	0	0	0
2	9.97	0	0	0	38.75
3	-1.61	0	0	0	0
4	6.74	0	0	0	0
5	1.40	0	0	0	0
6	0.26	0	0	44.47	0
7	4.45	0	0	0	0
8	5.03	0	0	0	0
9	2.77	0	0	0	0
10	1.47	0	0	0	0
11	5.79	0	0	0	0
12	0.01	0	0	0	0
13	-1.12	0	0	14.68	0
14	0.71	0	0	0	0
15	4.31	0	0	0	0
16	-0.004	0	0	0	0
17	2.04	0	0	0	0
18	2.69	0	0	0	0
19	3.25	0	0	36.07	0
20	4.03	0	0	0	0
21	6.07	0	10.50	0	0
22	7.98	0	0	0	0
23	2.72	0	0	0	0
24	2.42	0	0	0	0
25	11.33	0	0	0	44.25
26	2.71	0	0	0	0
27	4.77	0	0	0	0
28	-3.84	0	0	0	0
29	4.27	0	0	4.70	0
30	2.53	0	0	0	0
31	1.09	0	0	0	0
32	-0.20	0	0	0	0
33	4.55	0	0	0	0

TABLE 32 (contd)

Year 1971	% Mean rate of return	Percentage Invested			
Share No.		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
34	5.78	0	0	0	0
35	1.29	0	11.60	0	0
36	6.03	0	0	0	16.97
37	1.62	21.15	34.68	0	0
38	1.01	0	0	0	0
39	3.73	0	0	0	0
40	-1.86	0	0	0	0
41	4.22	0	43.22	0	0
42	3.41	0	0	0	0
43	-2.02	0	0	0	0
44	0.40	0	0	0	0
45	-8.01	0	0	0	0
46	5.41	78.85	0	0	0
47	3.12	0	0	0	0
48	3.87	0	0	0	0
49	9.94	0	0	0	0
50	2.78	0	0	0	0
% Actual return per quarter		4.37	3.17	1.32	6.04
% Annual rate of return		18.6	13.3	5.4	26.4

TABLE 33

Year 1968-70	% Mean rate of return	Percentage Invested			
Share No.		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	0.66	0	9.98	9.87	9.82
2	1.25	37.94	15.61	14.17	13.45
3	-0.23	0	0.51	0.66	0.73
4	-0.24	0	3.80	4.64	5.06
5	0.02	0	8.28	9.00	9.36
6	0.60	0	11.03	11.02	11.01
7	1.03	18.70	18.83	17.71	17.15
8	0.20	0	7.79	8.41	8.72
9	1.38	43.36	13.46	12.05	11.34
10	-0.19	0	10.71	12.48	13.45

TABLE 34

Year 1971	% Mean rate of return	Percentage Invested			
Share No.		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	2.91	0	9.98	5.61	5.33
2	5.44	37.94	15.61	9.36	7.65
3	5.22	0	0.51	3.21	2.61
4	2.03	0	3.80	8.52	8.60
5	-0.10	0	8.28	8.72	11.33
6	2.38	0	11.03	2.45	2.36
7	2.05	18.70	18.83	7.96	8.02
8	1.58	0	7.79	4.91	5.22
9	0.04	43.36	13.46	10.75	13.47
10	3.43	0	10.71	38.52	35.41
% Actual rate of return per month.		2.46	2.38	2.37	2.37
% Annual rate of return		29.52	28.56	28.44	28.44

TABLE 35

Year 1968-70	Mean rate of return %	Percentage Invested			
		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	-0.35	0	1.08	3.25	3.88
2	3.24	0	3.84	3.49	2.78
3	2.70	0	9.67	7.87	6.49
4	-0.31	0	0	0.88	1.54
5	4.32	0	1.56	1.22	0.92
6	-2.46	0	0	0	0.27
7	1.09	0	0.82	1.37	1.42
8	-0.43	0	0	0.69	1.32
9	5.62	0	3.12	2.20	1.56
10	1.36	0	1.26	1.71	1.68
11	2.14	0	4.28	4.58	4.13
12	-6.73	0	0	0	0.41
13	0.73	0	0.22	1.11	1.29
14	-2.07	0	0	0	1.07
15	-0.67	0	0	0.05	0.32
16	-1.18	0	0	0	0.21
17	6.97	22.01	6.21	4.06	2.76
18	5.40	0	4.86	3.41	2.45
19	-0.73	0	0.72	1.46	1.69
20	0.56	0	0.29	1.38	1.63
21	2.85	0	1.78	1.80	1.54
22	5.22	0	11.14	7.74	5.62
23	8.22	77.99	12.16	7.62	5.05
24	4.01	0	2.30	1.74	1.33
25	-0.48	0	3.11	6.14	7.02
26	-1.38	0	0	1.78	2.78
27	0.83	0	0.52	0.96	1.02
28	-1.09	0	0	0.08	0.27
29	3.08	0	1.25	0.98	0.79
30	-4.38	0	0	0	0
31	0.39	0	0	0.51	0.78
32	0.92	0	2.47	3.53	3.59

TABLE 35 (contd)

Year 1968-70	Mean rate of return %	Percentage Invested			
		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
33	3.25	0	2.99	2.51	2.03
34	-0.15	0	0.65	1.58	1.84
35	1.54	0	2.90	2.32	1.98
36	3.39	0	0	0	0.44
37	2.87	0	2.30	1.77	1.43
38	0.35	0	0	0.43	0.64
39	-2.61	0	0	0	0.31
40	-2.07	0	0	0	1.04
41	2.13	0	1.08	1.44	1.35
42	-0.51	0	0	0.19	0.63
43	1.89	0	4.83	4.88	4.40
44	3.09	0	6.74	5.37	4.34
45	-1.46	0	0	0.92	1.63
46	4.67	0	5.85	4.34	3.22
47	-1.22	0	0	2.62	5.25
48	-2.62	0	0	0	1.33
49	-3.21	0	0	0	0.07
50	-4.60	0	0	0	0.34

TABLE 36

Year 1971	% Mean rate of return	Percentage Invested			
Share No.		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
1	1.91	0	1.08	3.25	3.88
2	9.97	0	3.84	3.49	2.78
3	-1.61	0	9.67	7.87	6.49
4	6.74	0	0	0.88	1.54
5	1.40	0	1.56	1.22	0.92
6	0.26	0	0	0	0.27
7	4.45	0	0.82	1.37	1.42
8	5.03	0	0	0.69	1.32
9	2.77	0	3.12	2.20	1.56
10	1.47	0	1.26	1.71	1.68
11	5.79	0	4.28	4.58	4.13
12	0.01	0	0	0	0.41
13	-1.12	0	0.22	1.11	1.29
14	0.71	0	0	0	1.07
15	4.31	0	0	0.05	0.32
16	-0.004	0	0	0	0.21
17	2.04	22.01	6.21	4.06	2.76
18	2.69	0	4.86	3.41	2.45
19	3.25	0	0.72	1.46	1.69
20	4.03	0	0.29	1.38	1.63
21	6.07	0	1.78	1.80	1.54
22	7.98	0	11.14	7.74	5.62
23	2.72	77.99	12.16	7.62	5.05
24	2.42	0	2.30	1.74	1.33
25	11.33	0	3.11	6.14	7.02
26	2.71	0	0	1.78	2.78
27	4.77	0	0.52	0.96	1.02
28	-3.84	0	0	0.08	0.27
29	4.27	0	1.25	0.98	0.79
30	2.53	0	0	0	0
31	1.09	0	0	0.51	0.78
32	-0.20	0	2.47	3.53	3.59

TABLE 36 (contd)

Year 1971	% Mean rate of return	Percentage Invested			
Share No.		$\alpha = 1$	$\alpha = 25$	$\alpha = 50$	$\alpha = 100$
33	4.55	0	2.99	2.51	2.03
34	5.78	0	0.65	1.58	1.84
35	1.29	0	2.90	2.32	1.98
36	6.03	0	0	0	0.44
37	1.62	0	2.30	1.77	1.43
38	1.01	0	0	0.43	0.64
39	3.73	0	0	0	0.31
40	-1.86	0	0	0	1.04
41	4.22	0	1.08	1.44	1.35
42	3.41	0	0	0.19	0.63
43	-2.02	0	4.83	4.88	4.40
44	0.40	0	6.74	5.37	4.34
45	-8.01	0	0	0.92	1.63
46	5.41	0	5.85	4.34	3.22
47	3.12	0	0	2.62	5.25
48	3.87	0	0	0	1.33
49	9.94	0	0	0	0.07
50	2.78	0	0	0	0.34
% Actual rate of return per quarter		2.50	3.30	3.39	3.25
% Annual rate of return		10.00	13.20	13.56	13.00

TABLE 37

Confidence Limit 95%

Share Number	Regression Constants		Correlation Coefficients r
	a_i	b_i	
1	0.0063	0.6199	0.711
2	0.0422	0.6165	-0.110
3	0.0301	0.1983	-0.151
4	-0.0229	0.5079	0.608
5	0.0649	1.3674	-0.269
6	-0.0191	0.3410	0.720
7	0.0233	0.7820	0.840
8	-0.0030	0.0764	-0.027
9	0.0777	1.3536	-0.029
10	0.0324	1.1834	0.850
11	0.0270	0.3534	0.129
12	-0.0510	1.0267	0.893
13	0.0202	0.8111	0.820
14	-0.0032	1.1006	0.955
15	-0.0023	0.2784	0.826
16	0.0079	1.2385	0.824
17	0.0870	1.0925	-0.466
18	0.0678	0.8673	-0.425
19	0.0053	0.7903	0.854
20	-0.0358	0.1192	0.523
21	0.0511	1.4233	0.683
22	0.0674	0.9294	-0.156
23	0.0971	1.2006	-0.570
24	0.0597	1.2341	0.194
25	-0.0023	0.1574	0.184
26	-0.0072	0.4130	0.787
27	0.0227	0.9069	0.187
28	-0.0168	-0.3714	0.679
29	0.0551	1.5333	0.765
30	-0.0085	2.2147	0.811
31	0.0231	1.0809	0.878
32	0.0089	-0.0189	0.260

TABLE 37 (contd)

Share Number	Regression Constants		Correlation Coefficients r
	a_i	b_i	
33	0.0556	1.4551	0.706
34	-0.0012	0.0173	0.482
35	0.0281	0.8037	0.633
36	-0.0339	-0.0116	0.653
37	0.0420	-0.0002	-0.462
38	0.0252	1.3672	0.855
39	-0.0033	1.4377	0.933
40	-0.0077	0.8193	0.901
41	0.0378	1.0396	0.328
42	0.0148	1.2507	0.895
43	0.0292	0.6497	0.607
44	0.0389	0.5007	-0.164
45	-0.0010	0.8540	0.930
46	0.0509	0.6124	-0.129
47	-0.0073	0.3122	0.707
48	-0.0118	0.9041	0.960
49	0.0478	1.4304	0.642
50	-0.0463	0.0447	0.576

TABLE 38

Share Number	Number of times in thousand simulations using stream X.		Percentage Invested	% Mean rate of return for the year 1971	Weighted rate of return
	Best	Worst			
5	18	0	2.16	1.40	3.02
29	47	0	5.63	4.27	24.04
30	193	0	23.12	2.53	58.49
49	232	0	27.79	9.94	276.23
50	510	324	41.30	2.78	114.81
% Actual rate of return per quarter					4.77
% Annual rate of return					15.08

TABLE 39

Share Number	Number of times in thousand simulations using stream Y		Percentage Invested	% Mean rate of return for the year 1971	Weighted rate of return
	Best	Worst			
5	36	0	4.31	1.41	6.08
7	2	0	0.24	4.45	1.07
9	11	0	1.32	2.77	3.66
12	1	0	0.12	0.01	0.00
13	1	0	0.12	-1.12	-0.13
15	9	0	1.08	4.31	4.65
16	8	0	0.96	-0.004	-0.00
17	2	0	0.24	2.04	0.49
24	9	0	1.08	2.42	2.61
27	5	2	0.60	4.77	2.86
28	17	27	1.98	-3.84	-7.60
29	53	0	6.34	4.27	27.07
30	101	0	12.08	2.53	30.56
31	2	0	0.24	1.09	0.26
35	5	19	0.59	1.29	0.76
36	6	133	0.62	6.03	3.74
37	13	223	1.21	1.62	1.96
38	7	0	0.84	1.01	1.84
39	16	0	1.91	3.73	7.12
41	2	0	0.24	4.22	1.01
42	4	0	0.48	3.41	1.63
49	302	0	36.13	9.94	359.13
50	388	412	27.29	2.78	75.87
% Actual rate of return of the portfolio per quarter					5.24
% Annual rate of return					22.20

TABLE 40

Rank Number	Share Number	% Mean rate of return	Share Number	Variance(10^{-4})
1	23	8.22	33	21
2	17	6.97	22	26
3	9	5.62	23	30
4	18	5.40	21	36
5	22	5.22	47	36
6	46	4.67	40	48
7	5	4.32	14	48
8	24	4.01	48	50
9	33	3.25	25	50
10	2	3.24	1	51
11	44	3.09	43	52
12	29	3.08	3	73
13	37	2.87	10	73
14	21	2.85	11	74
15	3	2.70	26	88
16	11	2.14	44	91
17	41	2.13	2	92
18	43	1.89	32	96
19	35	1.54	45	96
20	10	1.36	46	106
21	7	1.09	17	111
22	32	0.92	42	119
23	27	0.83	41	122
24	13	0.73	18	144
25	20	0.56	9	145
26	31	0.39	39	148
27	38	0.35	13	148
28	34	-0.15	8	156
29	4	-0.31	31	158
30	1	-0.35	20	166
31	8	-0.43	38	166

TABLE 40 (contd)

Rank Number	Share Number	% Mean rate of return	Share Number	Variance (10^{-4})
32	25	-0.48	7	166
33	42	-0.51	12	167
34	15	-0.67	19	167
35	19	-0.73	34	181
36	28	-1.09	50	193
37	16	-1.18	4	198
38	47	-1.22	6	203
39	26	-1.38	16	223
40	45	-1.46	27	233
41	14	-2.07	24	237
42	40	-2.07	35	253
43	6	-2.46	5	284
44	39	-2.61	49	318
45	48	-2.62	30	332
46	49	-3.21	37	381
47	36	-3.39	36	385
48	28	-4.38	15	449
49	50	-4.60	29	461
50	12	-6.73	28	672

TABLE 41

Rules	Share No.	% Actual rate of return obtained on the basis of predictions from the previous years.
3.1.1.1 (a)	23	2.72
(b)	20	4.03
(c)	33	4.55
(d)	9	2.77
3.1.1.2 (a)	1 to 50	2.74

TABLE 42

Rule 3.1.1.2(b)	Share No.	% Mean Rate of return 1971	Weighted mean rate of return
Rank No.			
1	23	2.72	90.66
2	17	2.04	67.99
3	9	2.77	92.33
% Actual rate of return obtained on the basis of predictions from the previous years			3.43
% Annual rate of return			13.72

TABLE 43

Rule 3.1.1.2(c)

Rank Number	Share Number	Proportion Invested	% Mean rate of return of 1971	Weighted rate of return
1	23	10.35	2.72	28.15
2	17	8.77	2.04	17.89
3	9	7.07	2.77	19.58
4	18	6.80	2.69	18.27
5	22	6.57	7.78	52.43
6	46	5.88	5.41	31.81
7	5	5.44	1.40	7.62
8	24	5.05	2.42	12.22
9	33	4.09	4.55	18.61
10	2	4.08	9.97	40.68
11	44	3.89	0.40	1.56
12	29	3.88	4.27	16.57
13	37	3.61	1.62	5.85
14	21	3.59	6.07	21.79
15	3	3.40	-1.61	- 5.47
16	11	2.69	5.79	15.58
17	41	2.68	4.22	11.31
18	43	2.38	-2.02	- 4.81
19	35	1.94	1.29	2.50
20	10	1.71	1.47	2.51
21	7	1.37	4.45	6.10
22	32	1.16	-0.20	- 0.32
23	27	1.04	4.77	4.96
24	13	0.92	-1.12	- 1.03
25	20	0.70	4.03	2.82
26	31	0.49	1.09	0.53
27	38	0.44	1.01	0.44
% Actual rate of return per quarter				3.26
% Annual rate of return				13.04

TABLE 44

Rule 3.1.1.2 (d)

Rank Number	Share Number	Proportion Invested	% Mean rate of return 1971	Weighted rate of return
1	33	2.0296	5.55	9.23
2	22	2.0286	7.98	16.19
3	23	2.0278	2.72	5.52
4	21	2.0265	6.07	12.30
5	47	2.0265	3.12	6.32
6	40	2.0241	-1.86	-3.76
7	14	2.0241	0.71	1.44
8	48	2.0237	3.87	7.83
9	25	2.0237	11.33	22.93
10	1	2.0235	1.91	3.86
11	43	2.0233	-2.02	-4.09
12	3	2.0190	-1.61	-3.25
13	10	2.0190	1.47	2.97
14	11	2.0188	0.01	0.02
15	26	2.0160	2.71	5.46
16	44	2.0153	0.40	0.81
17	2	2.0151	9.97	20.09
18	32	2.0143	-0.20	-0.40
19	45	2.0143	-8.01	-16.13
20	46	2.0123	5.41	10.87
21	17	2.0113	2.04	4.10
22	42	2.0100	3.41	6.85
23	41	2.0090	4.22	8.48
24	18	2.0046	2.68	5.37
25	9	2.0043	2.77	5.55
26	39	2.0038	3.73	7.47
27	13	2.0038	-1.12	-2.24
28	8	2.0021	5.03	10.07
29	31	2.0017	1.09	2.18
30	20	2.0001	4.03	8.06
31	38	2.0001	1.01	2.02

TABLE 44 (contd)

Rank Number	Share Number	Proportion Invested	% Mean rate of return 1971	Weighted rate of return
32	7	2.0001	4.45	8.90
33	12	1.9999	0.01	0.02
34	19	1.9999	3.25	6.50
35	34	1.9970	5.78	11.54
36	50	1.9946	2.78	5.54
37	4	1.9936	6.74	13.44
38	6	1.9926	0.26	0.52
39	16	1.9885	-0.004	- 0.01
40	27	1.9865	4.77	9.48
41	24	1.9857	2.42	4.81
42	35	1.9824	1.40	2.56
43	5	1.9761	9.94	19.57
44	49	1.9691	2.53	4.97
45	30	1.9663	1.62	3.17
46	37	1.9564	6.03	11.79
47	36	1.9555	4.31	8.02
48	15	1.9425	3.87	7.51
49	29	1.9401	4.27	8.28
50	28	1.8972	-3.84	-7.29
% Actual rate of return obtained on the basis of predictors from the previous years				2.72
% Annual rate of return				10.88

TABLE 45

Rule 3.4.2(e)

Rank Number	Share Number	$\frac{\bar{X}}{\sigma}$	Proportion Invested	% Mean rate of return 1971	Weighted rate of return
1	23	1.50	16.91	2.72	46.00
2	22	1.02	11.50	7.98	91.71
3	33	0.71	8.00	4.55	36.40
4	17	0.66	7.44	2.04	15.18
5	21	0.48	5.41	6.07	32.84
6	9	0.47	5.30	2.77	14.68
7	18	0.45	5.07	2.69	13.64
8	46	0.45	5.07	5.41	27.43
9	2	0.34	3.83	9.97	38.19
10	44	0.32	3.61	0.40	1.44
11	3	0.32	3.61	-1.61	- 5.81
12	5	0.26	2.93	1.40	4.10
13	24	0.26	2.93	2.42	7.09
14	43	0.26	2.93	-2.02	- 5.92
15	11	0.25	2.82	5.79	16.33
16	41	0.19	2.14	4.22	10.17
17	10	0.16	1.80	1.47	2.65
18	37	0.15	1.69	1.62	2.74
19	29	0.14	1.59	4.27	6.79
20	35	0.10	1.13	1.29	1.46
21	32	0.09	1.01	- .20	-2.02
22	7	0.08	0.90	4.45	4.01
23	13	0.06	0.68	-1.12	-0.76
24	27	0.05	0.56	4.77	2.67
25	20	0.04	0.45	4.03	1.83
26	31	0.03	0.34	1.09	0.37
27	38	0.03	0.34	1.01	0.12
% Actual rate of return obtained on the basis of prediction from the previous years					3.63
% Annual rate of return					14.52

TABLE 46

Rule: 3.1.1.2

- a) Invested equal proportion in all shares = -0.97
- b) Invested equally in three shares with
highest rate of return = -1.73

TABLE 47

Rule 3.1.1.2(c)

Share Number	% Mean rate of return of 1968-70	Rank Number	Proportion Invested	% Mean rate of return of 1971	Weighted rate of return
1	0.35	5	2.50	1.01	2.53
2	1.89	3	13.52	-2.02	-27.31
3	-1.46	8	0	-8.01	0
4	4.67	2	33.40	-5.93	-198.06
5	-1.22	7	0	3.12	0
6	1.36	4	9.73	2.77	26.95
7	-1.78	9	0	-1.32	0
8	5.40	1	38.63	2.69	103.91
9	0.31	6	2.22	-1.09	-2.93
% Actual rate of return obtained on the basis of prediction from the previous years.					-0.95
% Annual rate of return					-3.8

TABLE 48

Rule 3.1.1.2 (d)

Share Number	Variance 1968-70	Rank Number	Proportion Invested	% Mean rate of return of 1971	Weighted rate of return
1	0.065	8	10.09	1.01	10.19
2	0.017	2	11.87	-2.02	-23.98
3	0.031	4	11.35	-8.01	-90.91
4	0.028	3	11.46	-5.93	-67.96
5	0.009	1	12.17	3.12	37.97
6	0.039	6	11.05	2.77	30.61
7	0.072	9	9.87	-1.32	-13.03
8	0.041	7	10.98	2.69	29.54
9	0.036	5	11.16	-1.09	-12.16
% Actual rate of return obtained on the basis of prediction from the previous years.					- 1.00
% Annual rate of return					- 4.00

TABLE 49

Rule 3.1.1.2 (e)

Share Number	$\frac{\bar{X}}{\sigma}$	Rank Number	Proportion Invested	% Mean rate of return	Weighted rate of return
1	1.37	6	1.74	1.01	1.76
2	14.49	3	18.35	-2.02	-37.07
3	-8.29	8	0	-0.01	0
4	27.91	1	35.35	-5.93	-209.63
5	-12.86	9	0	3.12	0
6	6.89	4	8.73	2.77	24.18
7	-6.63	7	0	-1.32	0
8	26.67	2	33.77	2.69	90.84
9	1.63	5	2.06	-1.09	-2.25
% Actual rate of return obtained on the basis of prediction from the previous years.					-1.32
% Annual rate of return					- 5.32

TABLE 50

Share Number	P/E Ratio	$V = 1 - \frac{1}{P/E}$	Growth rate (g) $(1+ig)^{\frac{1}{2}}$	gV	Weighted rate
1	24.3	0.95885	1.2226	1.4372	0.1702
2	16.6	0.93976	1.2392	1.1645	0.1379
3	14.6	0.93151	1.1261	1.0490	0.1242
4	24.4	0.95902	1.3548	1.2993	0.1539
5	15.6	0.93590	1.0774	1.0083	0.1194
6					
7					
8	16.6	0.93976	1.5245	1.4326	0.1696
9	11.5	0.91304	1.1547	1.0543	0.1248
Total				8.4452	1.0000

TABLE 51

Rank Number	Share Number	Growth and Risk	Proportion Invested	% Mean rate of return 1971	Weighted rate of return
1	1	1.437	17.02	1.01	17.19
2	8	1.433	16.96	2.69	45.62
3	4	1.299	15.39	-5.93	-91.26
4	2	1.165	13.79	-2.02	-27.86
5	9	1.054	12.48	-1.09	-13.60
6	3	1.049	12.42	-8.01	-99.48
7	5	1.008	11.94	3.12	37.25
8	6			2.77	
9	7			-1.32	
% Actual rate of return obtained on the basis of predictions from the previous years					-1.32
% Annual rate of return					-5.32

'ARR' denotes Actual rate of return obtained on the basis of predictions from the previous years.

Ad-Hoc Methods		Values of α	Models			
Rule 3.1.1.2	% ARR per quarter		% ARR per quarter			
			Markowitz	Sharpe	Monte Carlo Stream	
		X			Y	
(a)	2.74	1	4.37	2.50	4.77	5.24
(b)	3.43	25	3.17	3.30		
(c)	3.26	50	1.32	3.39		
(d)	2.72	100	6.04	3.25		
(e)	3.63					

TABLE 53

Values of α	Models	
	% ARR per month	
	Markowitz	Sharpe
1	2.56	2.46
25	2.45	2.38
50	2.43	2.37
100	2.42	2.37

TABLE 54

Programme Number	Name of the Model	Data Used	Mill time Seconds
1	Markowitz's	'A'	45
		'B'	76
		'C'	29
2	Sharpe's	'B'	35
		'C'	18
5	Filter	'A'	211
7	Monte Carlo	'B'	350

A P P E N D I X A 1

MARKOWITZ MODEL

Markowitz's model of the portfolio problem as developed in Section 4.1.1 Chapter IV reduces to the problem of optimising the quadratic form

$$M(\underline{X}) = \underline{p}'\underline{X} - \frac{\alpha}{2} \underline{X}' C \underline{X} \quad (A1-1)$$

Subject to the constraints

$$\sum_{i=1}^n x_i = 1 \quad (A1-2)$$

and $x_i \geq 0$ for all i

$$\alpha > 0 \quad i = 1, 2, \dots, n$$

In equation (A1-1)

\underline{p} is a $(1 \times n)$ column vector (the expected rate of return of the share)

\underline{X} is a $(1 \times n)$ column vector (proportion invested in each share)

C is a $(n \times n)$ matrix (the covariance matrix of the rate of return of shares)

α is a risk measure (the rate of change of marginal utility)

The dash represents the matrix transpose. The sum of the proportions x_i invested in each share must equal unity and only positive amounts can be invested as expressed in (A1-2).

1) Existence of Optimum.

We need to show that a maximum exists at a point \underline{X} say, and to examine what constraints, if any, there are on the form of \underline{p} and C in equation (A1-1).

Consider a neighbourhood point of the point \underline{X} , $\underline{X} + \delta\underline{X}$ in equation (A1-1)

$$M(\underline{X} + \delta\underline{X}) = \underline{p}'(\underline{X} + \delta\underline{X}) - \frac{\alpha}{2} (\underline{X} + \delta\underline{X})' C (\underline{X} + \delta\underline{X}) \quad (A1-1.1)$$

$$= M(\underline{X}) + \underline{p}' \delta\underline{X} - \frac{\alpha}{2} \underline{X}' C \delta\underline{X} - \frac{\alpha}{2} \delta\underline{X}' C \delta\underline{X} \quad (A1-1.2)$$

[since C symmetric]

for a stationary point at \underline{X} the coefficient of $\delta\underline{X}$ must vanish

1) contd.

at \underline{X}

$$M(\underline{X} + \delta\underline{X}) = M(\underline{X}) - \frac{\alpha}{2} \delta\underline{X}' C \delta\underline{X} \quad (A1-1.3)$$

Condition for maximum is

$$M(\underline{X} + \delta\underline{X}) < M(\underline{X}) \text{ for all } \delta\underline{X} \quad (A1-1.4)$$

in neighbourhood and hence

$$\frac{\alpha}{2} \delta\underline{X}' C \delta\underline{X} > 0 \text{ for all } \delta\underline{X} \quad (A1-1.5)$$

A matrix C for which this is true and is called positive definite. [Note that $\alpha > 0$].

1.1) Theorem.

Consider the symmetric matrix $C = \theta A'A$ $\theta > 0$

where A is a $m \times n$ matrix with column rank n. Then if $m \geq n$ C is positive definite otherwise it is positive semidefinite.

We need to show that $A'A$ is positive definite if $m \geq n$. We give only a heuristic discussion of the properties of $A'A$. A matrix B is said to be positive definite if for all \underline{X} in R_n ($\underline{X} \neq 0$) where R_n is the n dimension Euclidian space

$$\underline{X}' B \underline{X} > 0 \text{ for all } \underline{X} \in R_n \quad (A1-1.1.1)$$

case when $B = \theta A'A$

$$\text{Let } p = \theta \underline{X}' A'A \underline{X} = \theta (A \underline{X})' (A \underline{X}) \quad (A1-1.1.2)$$

$$\text{Let } \underline{Y} = \frac{1}{\theta} A \underline{X} \quad (A1-1.1.3)$$

$$\therefore p = \underline{Y}' \underline{Y} \geq 0 \quad (A1-1.1.4)$$

The size of \underline{X} in (A1-1.1.2) is $n \times 1$ and the column vector $A \underline{X}$ will be $m \times 1$ and each element will be a vector of dimension mn with the basis (x_1, x_2, \dots, x_n) .

If $m \geq n$ it is impossible to choose the n x_i 's so that $A \underline{X} = 0$ and hence from equation (A1-1.1.4)

$p > 0$ and $A'A$ is positive definite.

If $m < n$ it is possible to choose at least one

1) contd.

set of x_i 's so that $\underline{AX} = 0$ and so

$$p = 0 \text{ for some } x$$

(Note $p \neq 0$ for all x)

Then

(1) if $m \geq n$ $A'A$ is positive definite

(2) if $m < n$ $A'A$ is positive semi definite.

Example

$$A = \begin{pmatrix} 3 & 1 \\ 1 & 1 \\ 2 & 2 \end{pmatrix} \begin{matrix} n = 2 \\ m = 3 \end{matrix} \quad m > n$$

$$\underline{AX} = \begin{pmatrix} 3x_1 + x_2 \\ x_1 + x_2 \\ 2x_1 + 2x_2 \end{pmatrix} \quad \text{There exist no vector } \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ so that } \underline{AX} = 0$$

and hence

$$\underline{X}'A'\underline{AX} = (3x_1+x_2)^2 + (x_1+x_2)^2 + (2x_1+2x_2)^2 > 0$$

$\therefore A'A$ is positive definite

In this problem m and n are the number of periods and shares respectively. Thus providing we ensure that $m \geq n$ the matrix C is positive definite, the maximum exists and is unique.

2) Setting up the equations:

$$\text{Incorporating the constraint } \sum_{i=1}^n x_i = 1$$

we can write the equation (A1-1) in the more convenient form

$$M(\underline{X}) = \sum_{i=1}^n p_i x_i - \frac{\alpha}{2} \sum_{i=1}^n C_{ii} x_i^2 - \alpha \sum_{\substack{k=1 \\ i \neq j}}^n \sum_{j=1}^n C_{ij} x_i x_j \quad (\text{A1-2.1})$$

If we consider a delta variation of equation

(A1-2.1) we obtain

2) contd.

$$\delta M(\underline{X}) = \sum_{i=1}^n p_i \delta x_i - \frac{\alpha}{2} \left\{ \sum_{i=1}^n C_{ii} x_i^2 \delta x_i + \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n C_{ij} x_j \delta x_i \right\} \quad (\text{A1-2.2})$$

Consider a delta variation of (A1-2.2) such that

$$0 = \sum_{i=1}^n \delta x_i \quad (\text{A1-2.3})$$

Equation (A1-2.2) and (A1-2.3) are two equations relating the $\delta x_i = 1, 2, \dots, n$ and hence they are linearly dependent.

In other words at least one δx_i can be formed by a linear combination of the rest.

Let there exist a constant λ called a Lagrange multiplier such that equation (A1-2.2) and (A1-2.3) can be combined in a symmetric manner to form one equation in which the δx_i 's can be considered linearly independent. The justification for the process is that we should be able to determine a "unique value" of λ .

Thus equation (A1-2.2) and (A1-2.3) are combined to form the equation

$$\delta M(\underline{X}) = \sum_{i=1}^n \left[p_i - \frac{\alpha}{2} \left\{ C_{ii} x_i + \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n C_{ij} x_j \right\} \right] \delta x_i \quad (\text{A1-2.4})$$

The stationary point is given by $\delta M(\underline{X}) = 0$ and so equation (A1-2.4)

becomes

$$0 = \sum_{i=1}^n \left[p_i - \frac{\alpha}{2} \left\{ C_{ii} x_i + \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n C_{ij} x_j \right\} + \lambda \right] \delta x_i \quad (\text{A1-2.5})$$

Hence with the assume λ the δx_i can now be assumed linearly independent.

2) contd.

Thus from equation (A1-2.5) we say that

$$p_i - \frac{\alpha}{2} \left\{ C_{ii}x_i + \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n C_{ij}x_j \right\} + \lambda = 0 \text{ for all } i \quad (\text{A1-2.6})$$

$$i = 1, 2, \dots, n$$

If we combine the n equations of (A1-2.6)

with the constraint $\sum_{i=1}^n x_i = 1$, we have $(n+1)$ equations

for the n x_i 's and one λ .

Rewriting (A1-2.6) and (A1-2.1) in a more convenient form we obtain

$$2\alpha \left[C_{ii}x_i + \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n C_{ij}x_j \right] - \lambda = p_i \quad (\text{A1-2.7})$$

for all i

and $-\sum_{i=1}^n x_i = -1 \quad i = 1, 2, \dots, n$

Equations (A1-2.7) are set up in the form

$$A \underline{X}^* = \underline{p}^* \quad (\text{A1-2.8})$$

where \underline{X}^* is \underline{X} augmented by λ and \underline{p}^* is \underline{p} augmented by -1

A is of the form

$$\begin{array}{c} \xrightarrow{\quad n+1 \quad} \\ \left[\begin{array}{cccc} & & & -1 \\ & & & -1 \\ & & & \vdots \\ & & & -1 \\ & & & -1 \\ & & & \vdots \\ & & & -1 \\ & & & -1 \\ -1 & -1 & \dots & -1 & -1 & 0 \end{array} \right] \end{array}$$

$\mathcal{A}C$

2) contd.

where α is the risk measure

and C is the covariance matrix.

3) Iterative process.

a) Solve the matrix equation (A1-2.8) for \underline{X}^*

b) As we have not yet imposed the constraint that all the $x_i \geq 0$ for all $i \quad i = 1, 2, \dots, n$ we may (and probably will) get some $x_i < 0$.

The algorithm states that any $x_i \leq 0$ is set to zero and plays no further part in the problem. The justification can be easily seen in the case of the two shares X_1 and X_2 .

In Figure [18] PQ is the maximum of M with X_1 and X_2 . It is seen that $X_1 < 0$, since the shortest distance between P and valid domain is PS we set $X_1 = 0$.

c) Set up the reduced matrix system

$$A^{*R} \underline{X}^{*R} = \underline{p}^{*R} \quad (A1-3.1)$$

where information regarding those x_i 's set to zero is ignored.

d) We then return to (a) and resolve continuing the process until all the x_i 's in a particular iteration satisfying the constraint $x_i \geq 0$.

The two constraints impose $0 \leq x_i \leq 1$ for all i
 $i = 1, 2, \dots, n$

e) When the iteration is complete the optimum value can be calculated from (i.e. the utility). Utility

$$M(\underline{X}^R) = \underline{p}^R \underline{X}^R - \frac{1}{2} \alpha \underline{X}^R C^R \underline{X}^R \quad (A1-3.2)$$

where $\underline{p}^R \underline{X}^R$ = the expected rate of return.

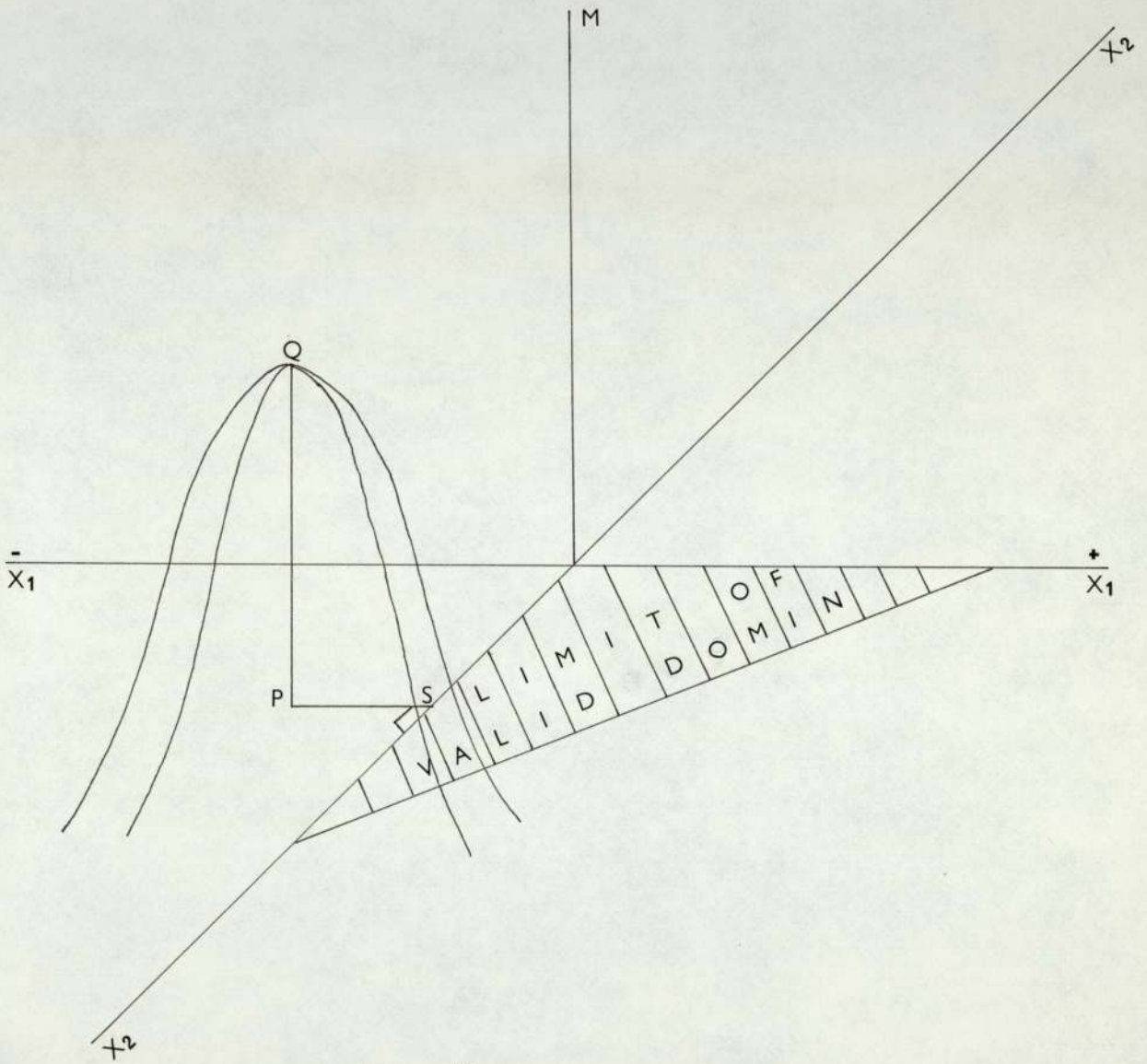


Figure - 18.

3) contd.

f) If at any stage of the process all the x_i 's are found to be ≤ 0 then no solution exists, though in this application it probably means that the data is incorrect.

A numerical example:

Optimize

$$M(\underline{X}) = p' \underline{X} - \frac{1}{2} \alpha \underline{X}' C \underline{X} \quad (A1-1)^*$$

subject to

$$\sum_{i=1}^3 x_i = 1 \quad (A1-2)^*$$

$$x_i \geq 0 \quad i = 1, \dots, 3$$

$$\alpha > 0$$

$$\text{Let } C = \begin{bmatrix} 20 & 1 & 2 \\ 1 & 10 & 4 \\ 2 & 4 & 30 \end{bmatrix} \quad \underline{p} = \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}$$

and $\alpha = 1$

from (A1-2.8) we get

$$\begin{bmatrix} 20 & 1 & 2 & -1 \\ 1 & 10 & 4 & -1 \\ 2 & 4 & 30 & -1 \\ -1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \lambda \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 1 \\ -1 \end{bmatrix}$$

$A \qquad \underline{X}^* \qquad \underline{p}^*$

i.e. $A \underline{X}^* = \underline{p}^*$

$$\underline{X}^* = A^{-1} \underline{p}^*$$

$$\underline{X}^* = A^{-1} \underline{p}^* = -\frac{1}{847} \begin{bmatrix} -32 & 25 & 7 & 246 \\ 25 & -46 & 21 & 496 \\ 7 & 21 & -28 & 105 \\ 246 & 496 & 105 & 5626 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 1 \\ -1 \end{bmatrix}$$

$$\therefore \underline{X}^* = \begin{bmatrix} 249/847 \\ 626/847 \\ -28/847 \\ 1415/847 \end{bmatrix}$$

Since $x_3 \leq 0$ it is set to zero

3) contd.

$$A = \begin{bmatrix} 20 & 1 & 2 & -1 \\ 1 & 10 & 4 & -1 \\ 2 & 4 & 30 & -1 \\ -1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 1 \\ -1 \end{bmatrix}$$

$$A^R = \begin{bmatrix} 20 & 1 & -1 \\ 1 & 10 & -1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ -1 \end{bmatrix}$$

$$\text{Then } \underline{X}^* = A^{-1R} \underline{p}^*$$

$$\underline{X}^* = -\frac{1}{28} \begin{bmatrix} -1 & 1 & 9 \\ 1 & -1 & 19 \\ 9 & 19 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ -1 \end{bmatrix} = \begin{bmatrix} 2/7 \\ 5/7 \\ 10/7 \end{bmatrix}$$

Then iterative process ends.

Here the Lagrange multiplier is $10/7$. Therefore in this example our share investment policy is to invest $2/7$ of our capital in share one, $5/7$ in share two and none in share three. With the policy our expected rate of return, expected utility are given by

$$\begin{aligned} M(\underline{X}) &= \underline{p}^R \underline{X} - \frac{1}{2} \alpha \underline{X}^R C^R \underline{X} \\ &= [5 \quad 6] \begin{bmatrix} 2/7 \\ 5/7 \end{bmatrix} - \frac{1}{2} [2/7 \quad 5/7] \begin{bmatrix} 45/7 \\ 52/7 \end{bmatrix} \\ &= \frac{40}{7} - \frac{1}{2} \cdot \frac{350}{49} = \frac{105}{49} \end{aligned}$$

The expected rate of return is $\frac{40}{7}$ and the expected utility is $\frac{105}{49}$.

4) Method used to solve the linear matrix equation

$$A\underline{X} = \underline{b} \quad (A1-4.1)$$

where A, X and b are real

A is an $n \times n$ matrix

\underline{X} is an $n \times 1$ vector

\underline{b} is an $n \times 1$ vector

The method used is similar to Crout factorisation (10)

4) contd.

using partial pivoting.

We split the matrix into the form

$$A = LU$$

where L is lower triangular and U is upper triangular.

Hence we can write (A1-4.1) in the form

$$L \underline{UX} = \underline{b} \quad (\text{A1-4.2})$$

$$\text{Let} \quad \underline{UX} = \underline{Y} \quad (\text{A1-4.3})$$

Thus (A1-4.2) reduces to

$$\underline{LY} = \underline{b} \quad (\text{A1-4.4})$$

$$\underline{UX} = \underline{Y} \quad (\text{A1-4.5})$$

We can now solve (A1-4.4) for \underline{Y} from L and \underline{b} using forward substitution. Then we solve (A1-4.5) for \underline{X} from U and \underline{Y} using backward substitution.

The splitting of A into L and U is not unique. The method of solving (A1-4.4) and (A1-4.5) suggests that we may reduce the rounding errors if we form L and U so that u_{ii} are as large as possible and $l_{ii} = \text{unity}$. These conditions imply that $l_{ij} < \text{unity}$ $i \neq j$. U is formed such that u_{ii} are as large as possible. Since l_{ii} are assumed unity, we do not store them and so it is possible to store a representation of LU on A. Computation uses double precision in the scalar products to help prevent a growth of rounding errors. The total matrix storage space is $n^2 + n$ where n is the size of A.

A P P E N D I X A2

THE SHARPE DIAGONAL MODEL.

This Appendix describes in detail the portfolio selection procedure which may be used when "Sharpe's" assumptions are acceptable.

The solution technique is developed as follows:-

1) Formulation of Problem:

$$\text{Let } R = \sum_{i=1}^n x_i R_i \quad (\text{A2-1.1})$$

where

R = The total return of portfolio

x_i = The proportion invested in each share

R_i = The average return of share i

n = The number of shares considered.

Suppose also that

$$R_i = a_i + b_i I + u_i \quad (\text{A2-1.2})$$

$$i = 1, 2, \dots, n$$

where

a_i and b_i = Constants of regression [see Appendix - A3]

I = Market index (e.g. Financial Times Index)

u_i = Error term $\sim N(0, \sigma_i^2)$

Where it is assumed that u_i is independent of u_j and I this is expressed as

$$\left. \begin{aligned} \text{Cov}(u_i, u_j) &= 0 \\ \text{Cov}(u_i, I) &= 0 \end{aligned} \right\} \quad (\text{A2-1.3})$$

Taking the expected value of both sides of (A2-1.2) we obtain

$$E(R_i) = \sum_{i=1}^n a_i + \sum_{i=1}^n b_i \hat{J}$$

The expected return of the portfolio may be obtained as

1) contd.

$$\begin{aligned} E &= E(R) = \sum_{i=1}^n E(R_i) \\ &= \sum_{i=1}^n (a_i + b_i \hat{I}) x_i \\ &= \underline{A}' \underline{X} + \underline{B}' \underline{X} \hat{I} \end{aligned}$$

where the matrices A, B, Q and \underline{X} are given by

$$\underline{A} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad \underline{B} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$\underline{Q} = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_n^2 \end{bmatrix} \quad \underline{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

The variance of the rate of return of share i is

$$V(R_i) = E(R_i^2) - [E(R_i)]^2$$

From equation (A2-1.2)

$$R_i^2 = a_i^2 + b_i^2 I^2 + u_i^2 + 2a_i b_i I + 2a_i u_i + 2b_i I u_i$$

$$E(R_i^2) = \sigma_i^2 + a_i^2 + 2a_i b_i \hat{I} + b_i^2 E(I^2)$$

$$\therefore V(R_i) = \sigma_i^2 + b_i^2 \sigma_{n+1}^2 \quad (\text{A2-1.4})$$

[Note that there is no covariance term in this expression owing to the independence assumptions of the diagonal model expressed in (A2-1.3)].

The covariances between the rates of return for the portfolio may be obtained as follows:

We have

$$\text{Cov}(R_i, R_j) = E(R_i R_j) - E(R_i) E(R_j)$$

Thus from (A2-1.2)

1) contd.

$$\begin{aligned} \text{Cov}(R_i R_j) &= E[(a_i + b_i I + u_i)(a_j + b_j I + u_j) \\ &\quad - (a_i + b_i \hat{I})(a_j + b_j \hat{I})] \\ &= a_i a_j + a_i b_j \hat{I} + b_i b_j \hat{I} + b_i b_j E(\hat{I})^2 \\ &\quad - a_i a_j - a_i b_j \hat{I} - b_i b_j \hat{I} - b_i b_j \hat{I}^2 \\ \text{Cov}(R_i R_j) &= b_i b_j; \quad \text{var}(I) = b_i b_j \sigma_I^2 \end{aligned}$$

Then from equation (A2-1.1)

$$V = V\left(\sum_{i=1}^n R_i x_i\right) = \sum_{i=1}^n x_i^2 V(R_i) + \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n x_i x_j \text{Cov}(R_i R_j)$$

Using (A2-1.4), this gives

$$V\left(\sum_{i=1}^n R_i x_i\right) = \sum_{i=1}^n x_i^2 (\sigma_i^2 + b_i^2 \sigma_{n+1}^2) + \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n x_i x_j b_i b_j \sigma_{n+1}^2$$

$$V = \underline{X}' Q \underline{X} + (\underline{X}' \underline{B})^2 \sigma_{n+1}^2$$

where Q is an $(n \times n)$ diagonal matrix with element σ_i^2 along the diagonal.

Following Sharpe's suggestion we may obtain a more compact expression for E and V by defining a hypothetical $(n+1)^{\text{st}}$ share such that

$$\begin{aligned} a_{n+1} &= \hat{I} \\ b_{n+1} &= -1 \\ \sigma_{n+1}^2 &= \sigma_I^2 \\ x_{n+1} &= \underline{B}' x \end{aligned}$$

Then

1) contd.

$$E = \underline{A}' \underline{X} + a_{n+1} x_{n+1} = \underline{A}'^* \underline{X}^*$$

$$V = \underline{X}' \underline{QX} + \sigma_{n+1}^2 x_{n+1}^2 = \underline{X}'^* \underline{QX}^*$$

from Section 4.2.2 Chapter IV.

Now the problem is to maximise

$$M(\underline{X}^*) = E(R) - \alpha V(R) \quad (\text{A2-1.5})$$

or
$$M(\underline{X}^*) = \underline{A}'^* \underline{X}^* - \alpha \underline{X}'^* \underline{QX}^*$$

subject to the constraints

$$\sum_{i=1}^n x_i = 1$$

$$x_i \geq 0$$

$$\sum_{i=1}^n x_i b_i - x_{n+1} = 0.$$

2) Solution of problem.

In this Section '*' has been omitted for the sake of clarity.

We seek to maximize the objective function

$$M(\underline{X}) = E(R) - \alpha V(R) \quad (\text{A2-2.1})$$

or
$$M(\underline{X}) = \underline{A}' \underline{X} - \alpha \underline{X}' \underline{QX} \quad (\text{A2-2.2})$$

subject to the constraints

$$(i) \sum_{i=1}^n x_i = 1$$

$$(ii) x_i \geq 0 \quad (\text{A2-2.3})$$

$$(iii) \sum_{i=1}^n x_i b_i - x_{n+1} = 0 \quad \text{for all } i$$

$$i = 1, 2, \dots, n$$

Does a maximum exist at the point \underline{X} ?

2) contd.

Consider a neighbourhood point of \underline{X} , $\underline{X} + \delta\underline{X}$

$$M(\underline{X} + \delta\underline{X}) = \underline{A}'(\underline{X} + \delta\underline{X}) - \alpha(\underline{X} + \delta\underline{X})' Q(\underline{X} + \delta\underline{X}) \quad (\text{A2-2.4})$$

$$= M(\underline{X}) + \underline{A}'\delta\underline{X} - \alpha\underline{X}'Q\delta\underline{X} - \frac{\alpha}{2}\delta\underline{X}'Q\delta\underline{X} \quad (\text{A2-2.5})$$

for a stationary point at \underline{X} the coefficient of $\delta\underline{X}$

$(\underline{A}' - \alpha\underline{X}'Q)\delta\underline{X}$ must vanish at \underline{X} and hence

$$M(\underline{X} + \delta\underline{X}) = M(\underline{X}) - \alpha\delta\underline{X}'Q\delta\underline{X} + O(\delta\underline{X})^2 \quad (\text{A2-2.6})$$

Condition for maximum is

$$M(\underline{X} + \delta\underline{X}) < M(\underline{X}) \text{ for all } \delta\underline{X} \text{ in the neighbourhood} \quad (\text{A2-2.7})$$

from equations (A2-2.6) and (A2-2.7) a maximum requires

$$\alpha\delta\underline{X}'Q\delta\underline{X} > 0 \quad (\text{A2-2.8})$$

for all $\delta\underline{X}$ in the neighbourhood.

The condition for a maximum, since $\alpha > 0$ is that Q is positive definite. The matrix Q is positive definite because it is a diagonal matrix whose diagonal elements are all positive hence for our problem a unique maximum exists.

Let us define the Lagrangian function to incorporate our constraints

$$M(\underline{X}) = \sum_{i=1}^{n+1} A_i x_i - \alpha \sum_{i=1}^{n+1} \sigma_i^2 x_i^2 + \lambda \sum_{i=1}^n x_i + \mu \left(\sum_{i=1}^n x_i b_i - x_{n+1} \right) \quad (\text{A2-2.9})$$

where λ and μ are two Lagrangian multipliers at a stationary

point $\frac{\partial \phi}{\partial x_i} = 0$ for all $i \quad i = 1, 2, \dots, n$

$$\frac{\partial \phi}{\partial x_i} = a_i - \alpha 2\sigma_i^2 x_i + \lambda + \mu b_i \quad (\text{A2-2.10})$$

$$\frac{\partial \phi}{\partial x_{n+1}} = a_{n+1} - \alpha 2\sigma_{n+1}^2 x_{n+1} - \mu \quad (\text{A2-2.11})$$

We may reformulate the above equations (i,ii) of (A2-2.3), (A2-1.10) and (A2-1.11) in matrix form as:

$$T \cdot \underline{Y} = \underline{C}$$

where

2) contd.

$$\begin{array}{c} \text{T} \\ \left[\begin{array}{cccccc} 2\alpha\sigma_1^2 & & & & & \\ 0 & 2\alpha\sigma_2^2 & & & & \\ \vdots & & & & & \\ 0 & & 2\alpha\sigma_n^2 & & & \\ 0 & \dots\dots\dots & 0 & 2\alpha\sigma_{n+1}^2 & 0 & \\ 1 & 1 & 1 & 0 & 0 & \\ B_1 & B_2 & B_n & -1 & 0 & \end{array} \right] \begin{array}{c} \underline{Y} \\ \left[\begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_n \\ x_{n+1} \\ \lambda \\ \mu \end{array} \right] \end{array} = \begin{array}{c} \underline{C} \\ \left[\begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_n \\ a_{n+1} \\ 1 \\ 0 \end{array} \right] \end{array} \end{array} \quad (\text{A2-2.12})$$

Equation (A2-2.12) may then be rewritten in partitioned form

$$\begin{array}{c} \left[\begin{array}{c|c} \xrightarrow{n+1} & \xrightarrow{2} \\ \uparrow n+1 & \left[\begin{array}{c} E \\ \hline E' \end{array} \right] \\ \downarrow 2 & \left[\begin{array}{c} \underline{X} \\ \underline{L} \end{array} \right] \end{array} \right] = \begin{array}{c} \left[\begin{array}{c} \underline{A} \\ \underline{1} \\ 0 \end{array} \right] \end{array} \quad (\text{A2-2.13})$$

where

$$E = (n+1) \times 2 = \text{matrix}$$

$$\underline{X} = (n+1) \times 1 = \text{vector}$$

$$\underline{L} = 2 \times 1 = \text{vector}$$

expanding the above system

$$E \underline{X} + E \underline{L} = \underline{A} \quad (\text{A2-2.14})$$

$$E' \underline{X} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (\text{A2-2.15})$$

We may rewrite equation (A2-2.14) as

$$\underline{X} = Q^{-1} \underline{A} - Q^{-1} E \underline{L} \quad (\text{A2-2.16})$$

Substituting (A2-2.15) into (A2-2.12) yields

$$E' Q^{-1} \underline{A} - E' Q^{-1} E \underline{L} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (\text{A2-2.17})$$

[Note $\underline{L} \neq E^{-1} \underline{A} - E^{-1} Q \underline{X}$ because E is not square]

Rearranging the equation (A2-2.17)

$$E' Q^{-1} \underline{A} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (E' Q^{-1} E) \underline{L} \quad (\text{A2-2.18})$$

solving for \underline{L} we obtain

2) contd.

$$\underline{L} = (E'Q^{-1}E)^{-1} \left\{ E'Q^{-1}\underline{A} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\} \quad (\text{A2-2.19})$$

$$Q = \begin{pmatrix} 2\alpha\sigma_1^2 & & & & 0 \\ & 2\alpha\sigma_2^2 & & & \\ & & & 2\alpha\sigma_n^2 & \\ & & & & 2\alpha\sigma_{n+1}^2 \\ 0 & & & & \end{pmatrix} \quad Q^{-1} = \begin{pmatrix} C_1 & & & & \\ & C_2 & & & \\ & & & & \\ & & & C_n & \\ 0 & & & & C_{n+1} \end{pmatrix}$$

$$\text{where } C_i = \frac{1}{2\alpha\sigma_i^2}$$

C_i are identified with Covar(I) in the subsequent program

$$E'Q^{-1}E = \begin{pmatrix} 1 & 1 & \dots & 1 & 0 \\ b_1 & b_2 & \dots & b_n & b_{n+1} \end{pmatrix} \begin{pmatrix} C_1 & & & & \\ & C_2 & & & \\ & & & & \\ 0 & & & C_n & \\ & & & & C_{n+1} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \\ b_{n+1} \end{pmatrix} \quad (\text{A2-2.20})$$

$$E'Q^{-1} = \begin{pmatrix} C_1 & C_2 & \dots & C_n & 0 \\ b_1 C_1 & b_2 C_2 & \dots & b_n & b_{n+1} C_{n+1} \end{pmatrix} \quad (\text{A2-2.21})$$

$$E'Q^{-1}E = \begin{pmatrix} \sum_{i=1}^n C_i & \sum_{i=1}^n C_i b_i \\ \sum_{i=1}^n C_i b_i & \sum_{i=1}^{n+1} C_i b_i^2 \end{pmatrix} \quad (\text{A2-2.22})$$

Using the notation of the program

Let

$$\text{SUMA} = \sum_{i=1}^n C_i$$

$$\text{SUMB} = \sum_{i=1}^n C_i b_i$$

2) contd.

$$\text{SUMC} = \sum_{i=1}^n C_i b_i^2$$

Now from equation (A2-2.21)

$$\begin{aligned} E'Q^{-1} \underline{A} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} C_1 & C_2 & \dots & C_n & 0 \\ b_1 C_1 & b_2 C_2 & & b_n C_n & b_{n+1} C_{n+1} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \\ a_{n+1} \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} \sum_{i=1}^n C_i a_i - 1 \\ \sum_{i=1}^n C_i b_i a_i \end{pmatrix} \end{aligned}$$

where

$$\sum_{i=1}^n b_i x_i - x_{n+1} = 0$$

$$\sum_{i=1}^n b_i x_i = 0$$

$$b_{n+1} = -1$$

Let

$$\text{SUMD} = \sum_{i=1}^n C_i a_i - 1$$

$$\text{SUME} = \sum_{i=1}^{n+1} C_i b_i a_i$$

2) contd.

$$\begin{pmatrix} \text{SUMA} & \text{SUMB} \\ \text{SUMB} & \text{SUMC} \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} \text{SUMD} \\ \text{SUME} \end{pmatrix}$$

$$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \frac{1}{\text{Determ}} \begin{pmatrix} \text{SUMC} & -\text{SUMB} \\ -\text{SUMB} & \text{SUMA} \end{pmatrix} \begin{pmatrix} \text{SUMD} \\ \text{SUME} \end{pmatrix}$$

where

$$\text{Determ} = \text{SUMC} \cdot \text{SUMA} - (\text{SUMB})^2$$

$$\lambda = \text{SUMC} \cdot \text{SUMD} - \text{SUME} \text{SUMD} / \text{Determ}$$

$$\mu = (\text{SUMA} \cdot \text{SUME} - \text{SUMB} \text{SUMD}) / \text{Determ}$$

We have solved for \underline{L} , now we are in a position to solve for \underline{X} using (A2-2.16) we have to form $\underline{E} \underline{L}$ where

$$\underline{E} \underline{L} = \begin{pmatrix} 1 & b_1 \\ 1 & b_2 \\ \vdots & \vdots \\ 1 & b_n \\ 0 & b_{n+1} \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} \lambda + \mu b_1 \\ \lambda + \mu b_2 \\ \vdots \\ \mu b_{n+1} \end{pmatrix}$$

We have to solve

$$\underline{X} = \underline{Q}^{-1}(\underline{A} - \underline{E} \underline{L}) \text{ from (A-2.16)}$$

$$\underline{X} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \\ a_{n+1} \end{pmatrix} - \begin{pmatrix} \lambda + \mu b_1 \\ \lambda + \mu b_2 \\ \vdots \\ \lambda + \mu b_n \\ \mu b_{n+1} \end{pmatrix} = \begin{pmatrix} a_1 - \lambda - \mu b_1 \\ a_2 - \lambda - \mu b_2 \\ \vdots \\ a_n - \lambda - \mu b_n \\ a_{n+1} - \mu b_{n+1} \end{pmatrix}$$

We have not yet incorporated the positivity constraints in (A2-2.16).

This can be effected in a similar manner to that used in the Markowitz model [A1].

At this point another advantage of the Sharpe model is highlighted in that matrix reduction is relatively

2) contd.

simple. As before we iterate until the reduced set of x 's are positive. When the iteration is complete the optimum value can be calculated from (A2-1.5) (i.e. the expected utility)

$$M(\underline{X}^*) = E(R) - \alpha V(R)$$

where

$E(R)$ = The expected rate of return of the portfolio

$\alpha V(R)$ = The variance of the portfolio

3) The Iterative process.

- a) Solve the matrix equation as described above.
- b) If the proportion invested in share (x_i) is found to be negative or zero, we shall remove the share from the portfolios. These shares are listed by the program as zero percentage invested.
- c) The matrix equation is reformulated omitting the shares eliminated by (b).
- d) We then return to (a) and continue the iterative cycle until no share proportions are found negative.

When the iterative is complete we calculate

- 1) The expected rate of return and
- 2) The utility

A P P E N D I X A 3

THE LEAST SQUARES LINEAR REGRESSION MODEL.

Consider the linear model

$$\underline{R} = X \underline{\beta} + \underline{\mu} \quad (\text{A3-1})$$

where

$$\underline{R} = \begin{pmatrix} R_1 \\ \vdots \\ R_n \end{pmatrix} \quad X = \begin{pmatrix} I_1 & 1 \\ \vdots & \vdots \\ I_n & 1 \end{pmatrix} \quad \underline{\beta} = \begin{pmatrix} b \\ a \end{pmatrix} \quad \text{and} \quad \underline{u} = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} \quad (\text{A3-2})$$

where $u_i \sim N(0, \sigma^2)$

The least squares criterion chooses the $\underline{\beta}$ such that $u'u$ is minimized with respect to $\underline{\beta}$

NOW

$$\begin{aligned} \underline{u}'\underline{u} &= (\underline{R} - X \underline{\beta})' (\underline{R} - X \underline{\beta}) \\ &= (\underline{R}' - \underline{\beta}' X') (\underline{R} - X \underline{\beta}) \\ &= \underline{R}'\underline{R} - \underline{R}' X \underline{\beta} - \underline{\beta}' X' \underline{R} + \underline{\beta}' X' X \underline{\beta} \end{aligned}$$

$$\frac{\partial(\underline{u}'\underline{u})}{\partial \underline{\beta}} = -(\underline{R}' X)' - X' \underline{R} + 2X' X \underline{\beta} = 0 \text{ for minimum}$$

$$\therefore X' X \underline{\beta} = X' \underline{R} \quad (\text{A3-3})$$

Substituting (A3-2) in (A3-3) we obtain in extenso

$$\begin{pmatrix} I_1 & \dots & I_n \\ 1 & \dots & 1 \end{pmatrix} \begin{pmatrix} I_1 & 1 \\ \vdots & \vdots \\ I_n & 1 \end{pmatrix} \begin{pmatrix} b \\ a \end{pmatrix} = \begin{pmatrix} I_1 & \dots & I_n \\ 1 & \dots & 1 \end{pmatrix} \begin{pmatrix} R_1 \\ \vdots \\ R_n \end{pmatrix} \quad (\text{A3-4})$$

For ease of printing the limits are omitted

$$\begin{pmatrix} \sum I_i^2 & \sum I \\ \sum I & n \end{pmatrix} \begin{pmatrix} b \\ a \end{pmatrix} = \begin{pmatrix} \sum I R \\ \sum R \end{pmatrix} \quad (\text{A3-5})$$

Then from (A3-5)

$$\begin{pmatrix} b \\ a \end{pmatrix} = \frac{1}{D} \begin{pmatrix} n & -\sum I \\ -\sum I & \sum I_i^2 \end{pmatrix} \begin{pmatrix} \sum I R \\ \sum R \end{pmatrix} \quad (\text{A3-6})$$

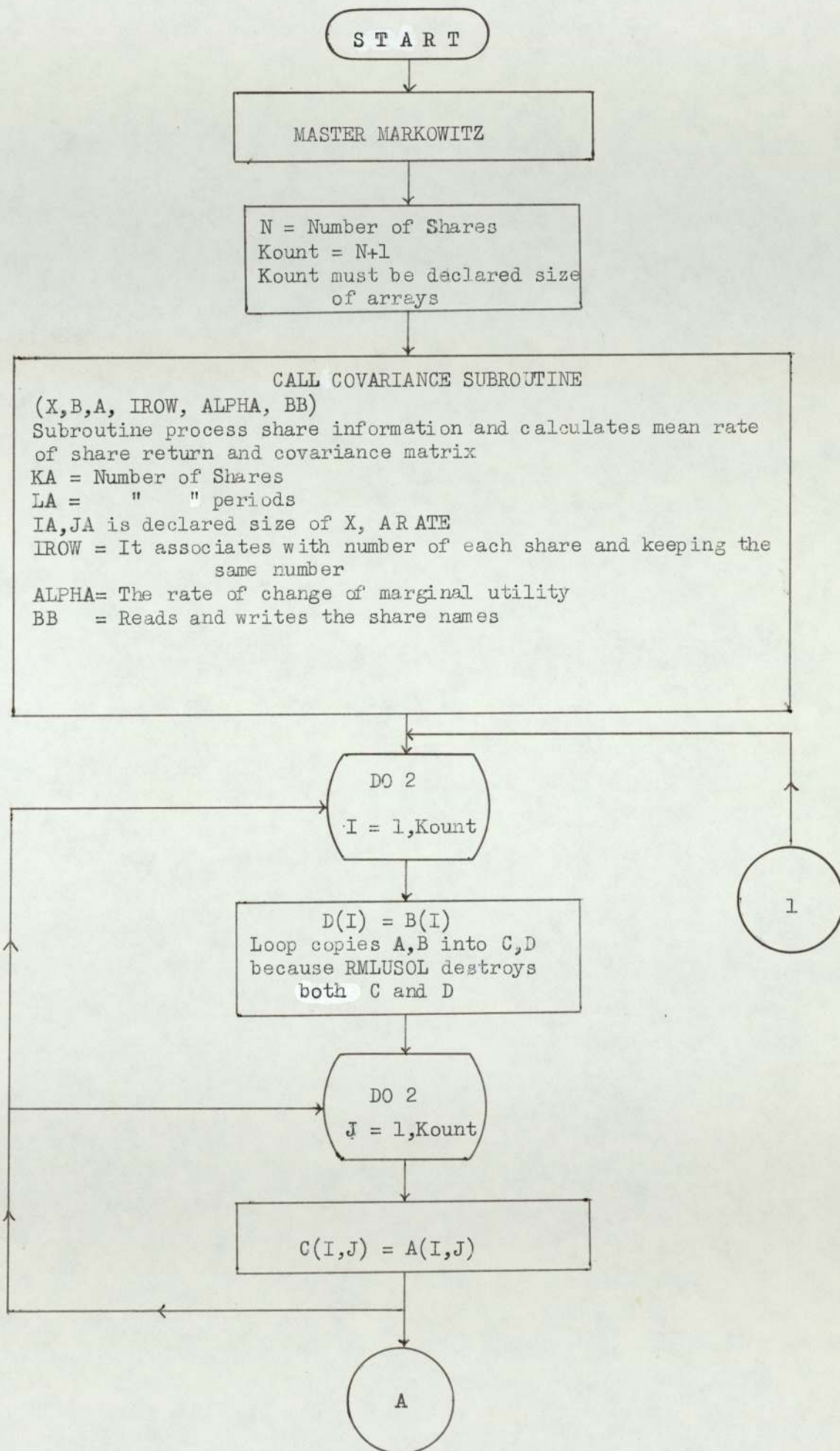
$$\text{where } D = n \sum_{i=1}^n I_i^2 - \left(\sum_{i=1}^n I \right)^2$$

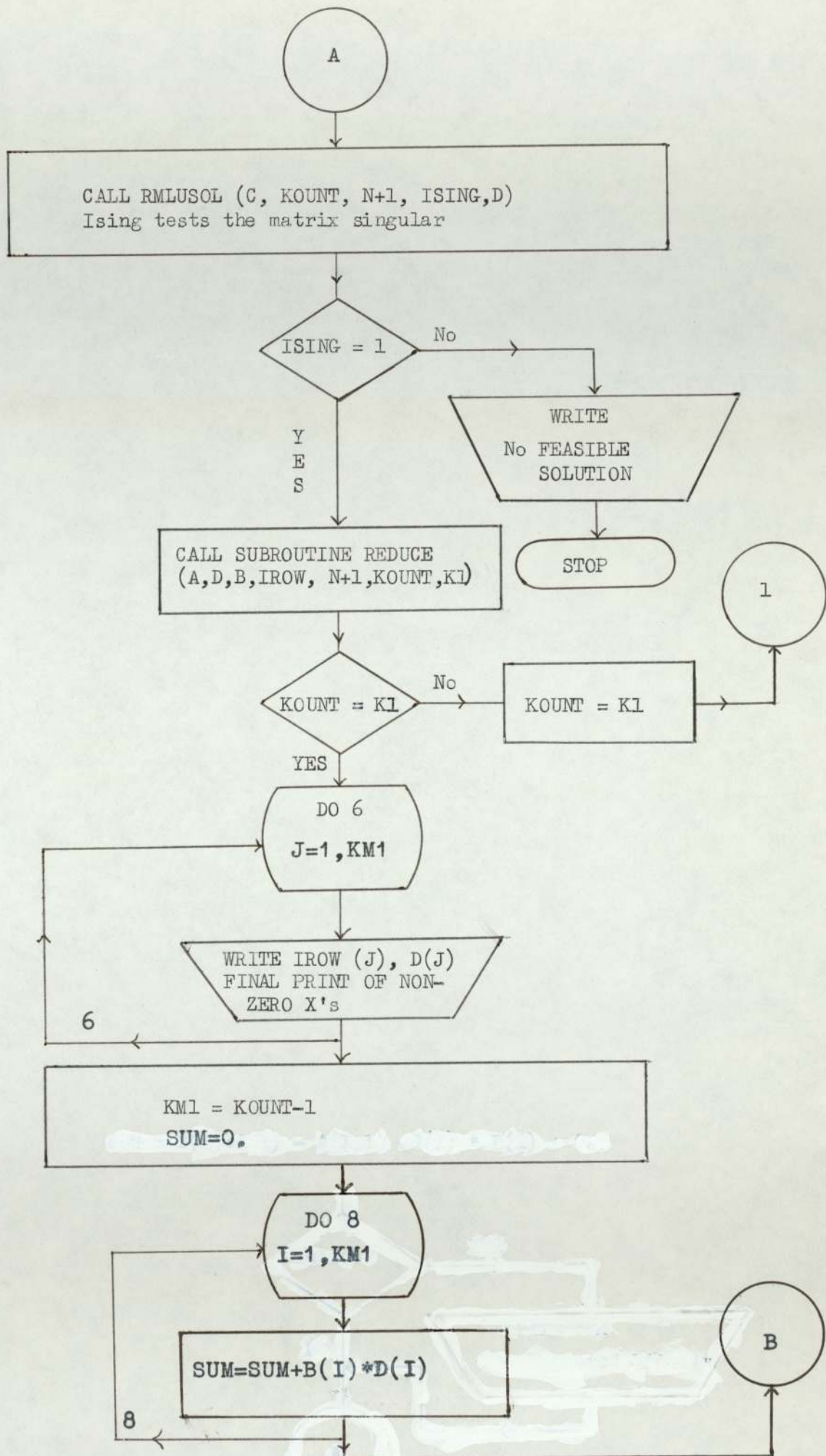
$$\therefore b = \left(n \sum_{i=1}^n I R - \sum_{i=1}^n R \sum_{i=1}^n I \right) / D \quad (\text{A3-7})$$

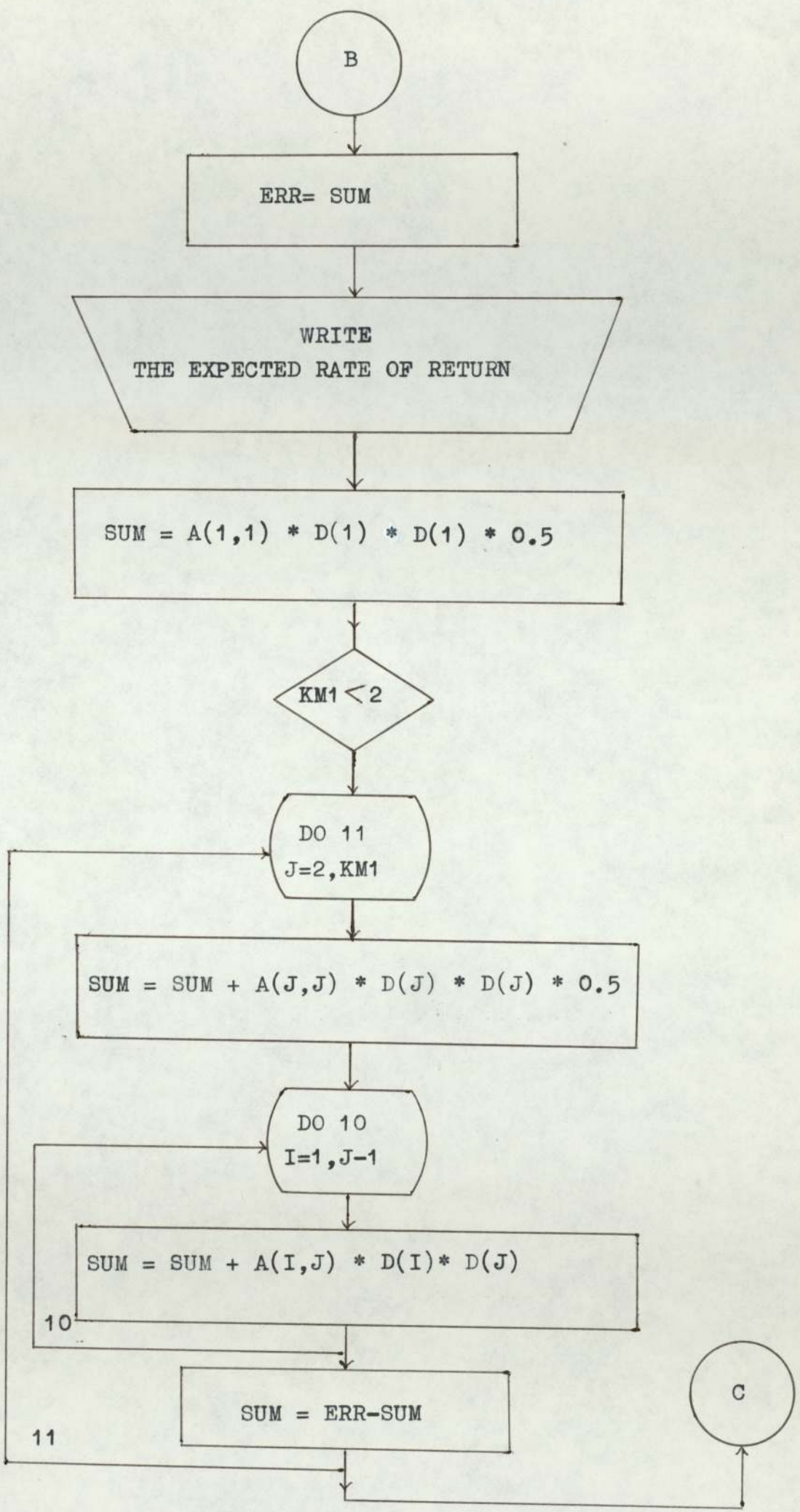
$$\text{and } a = \left(- \sum_{i=1}^n I \sum_{i=1}^n I R + \sum_{i=1}^n I_i^2 \sum_{i=1}^n R \right) / D \quad (\text{A3-8})$$

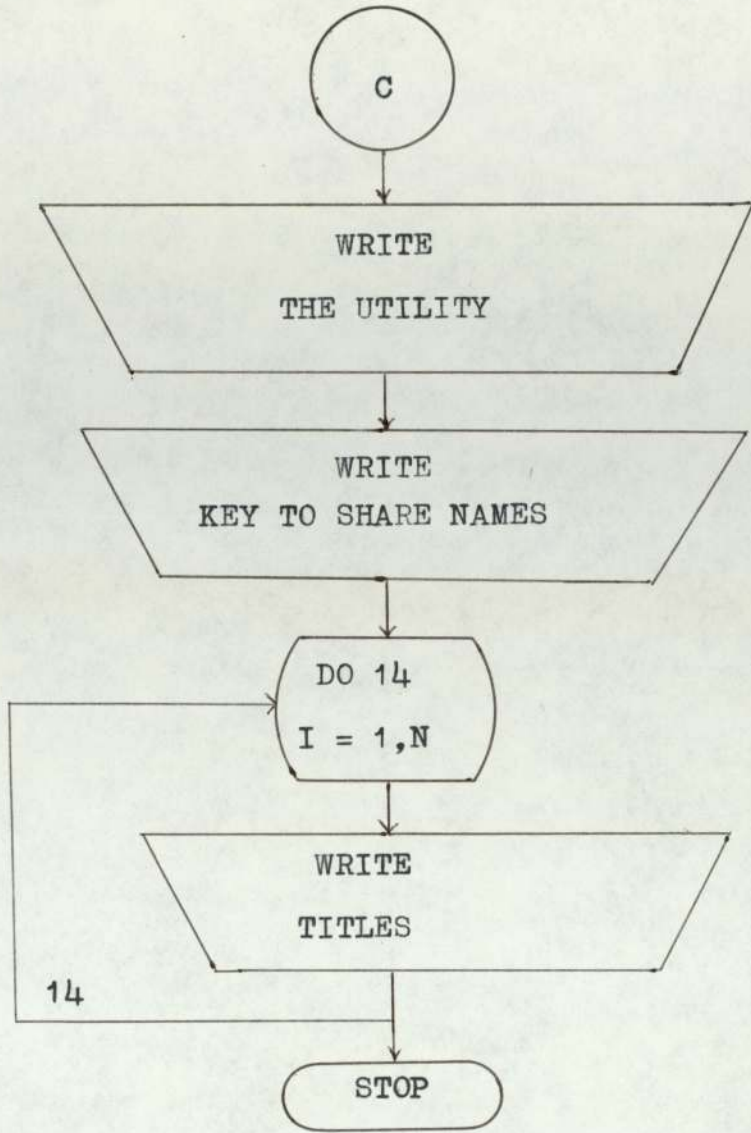
A P P E N D I X A4

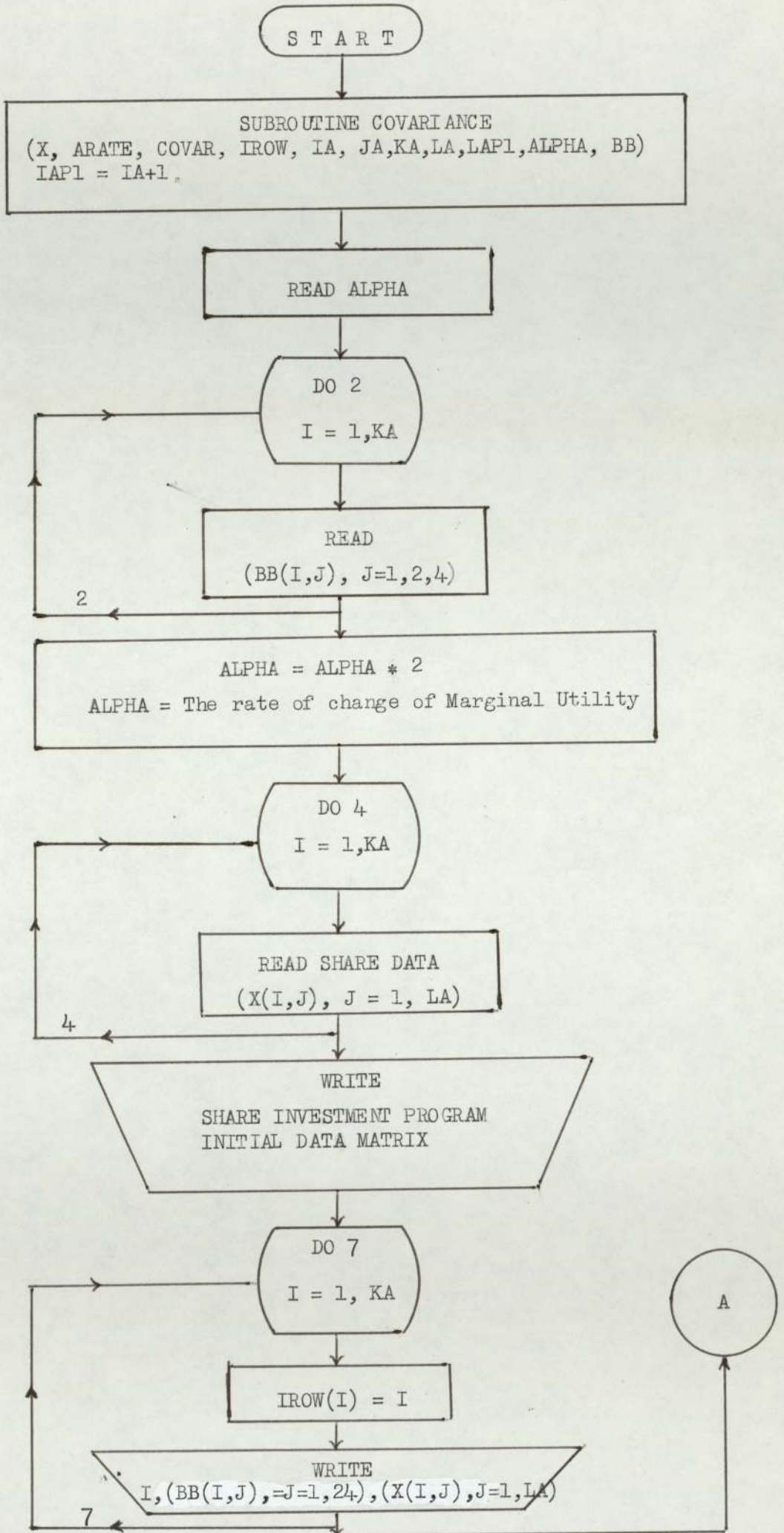
FLOW CHART OF MARKOWITZ'S MODEL

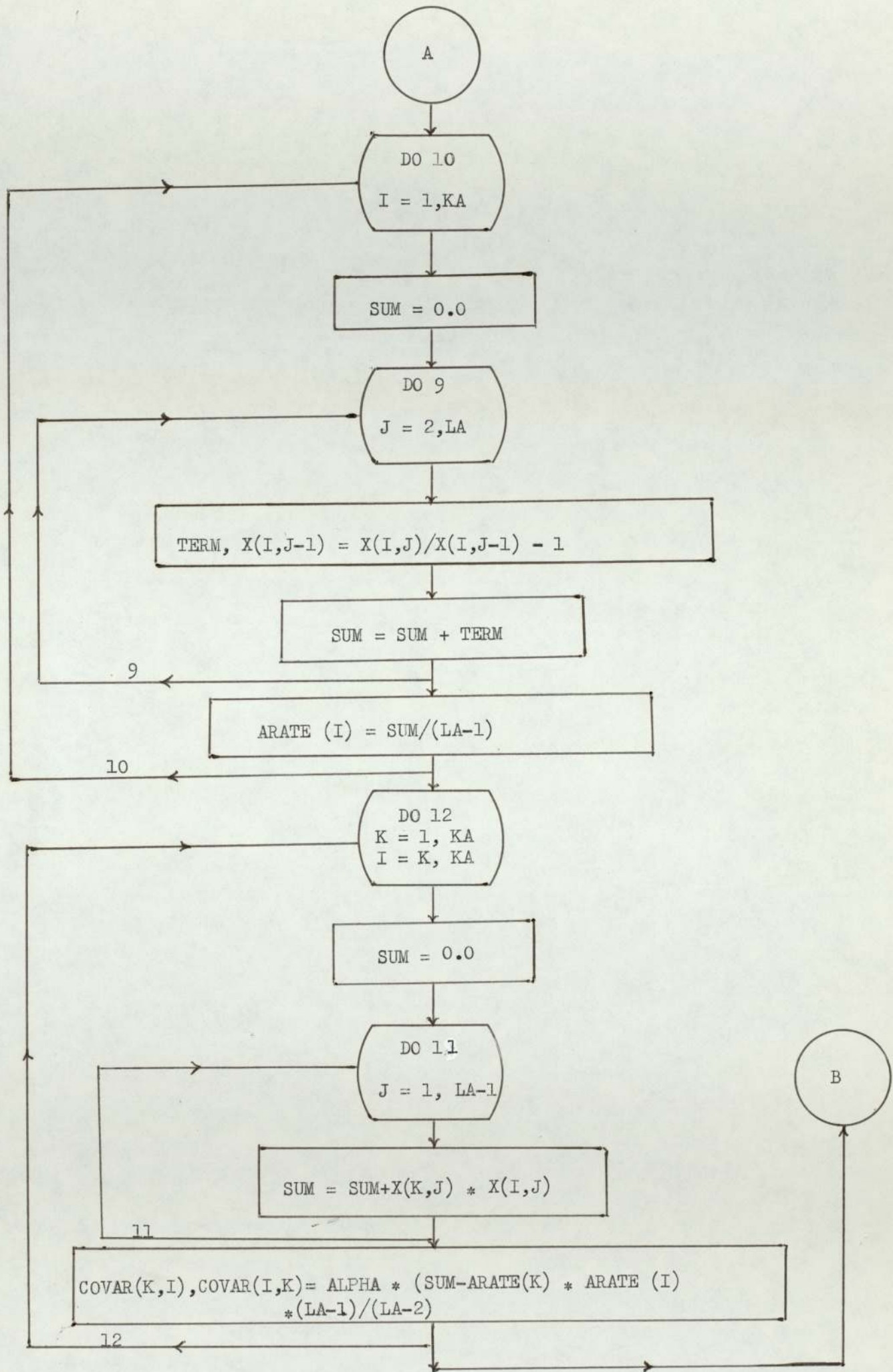


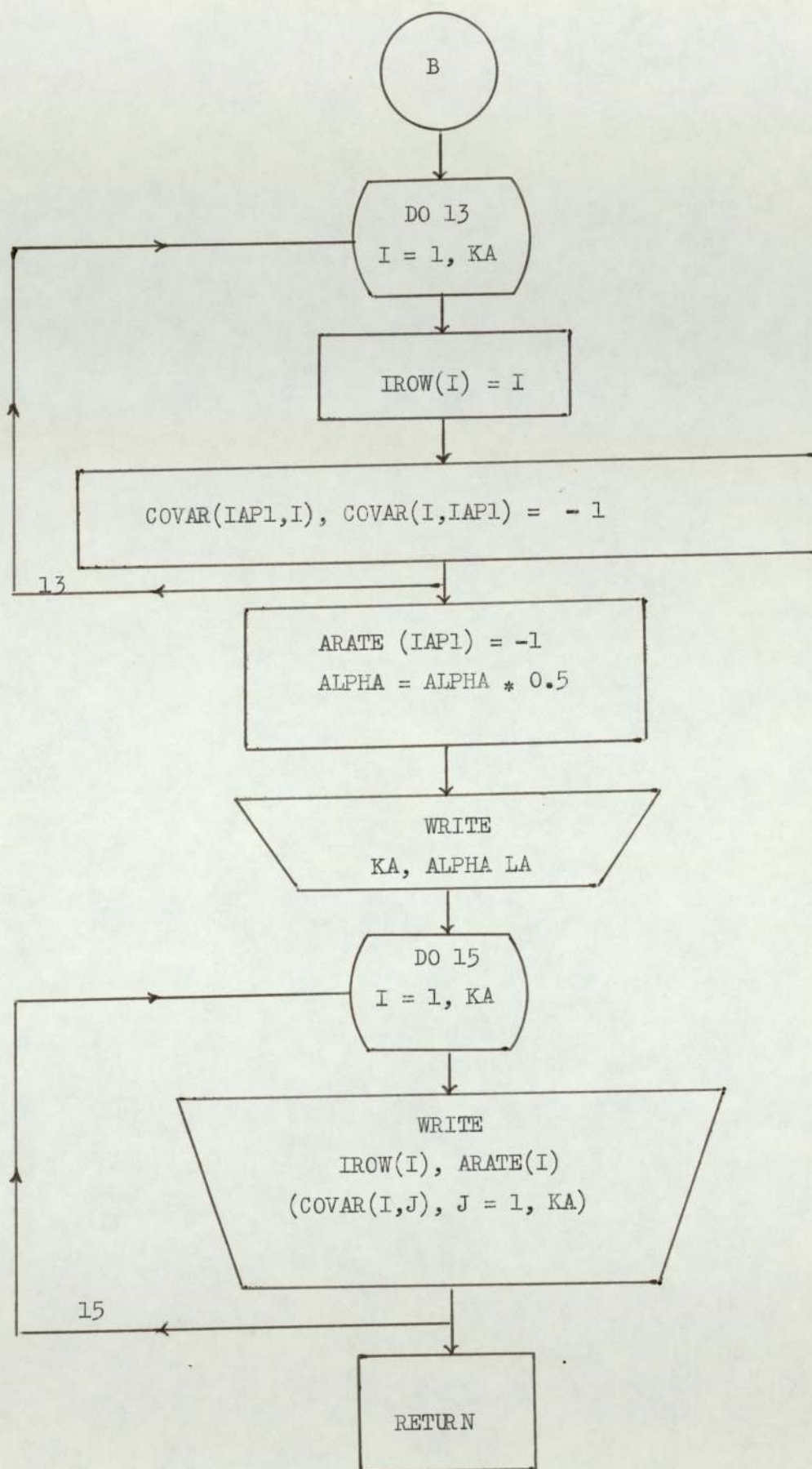


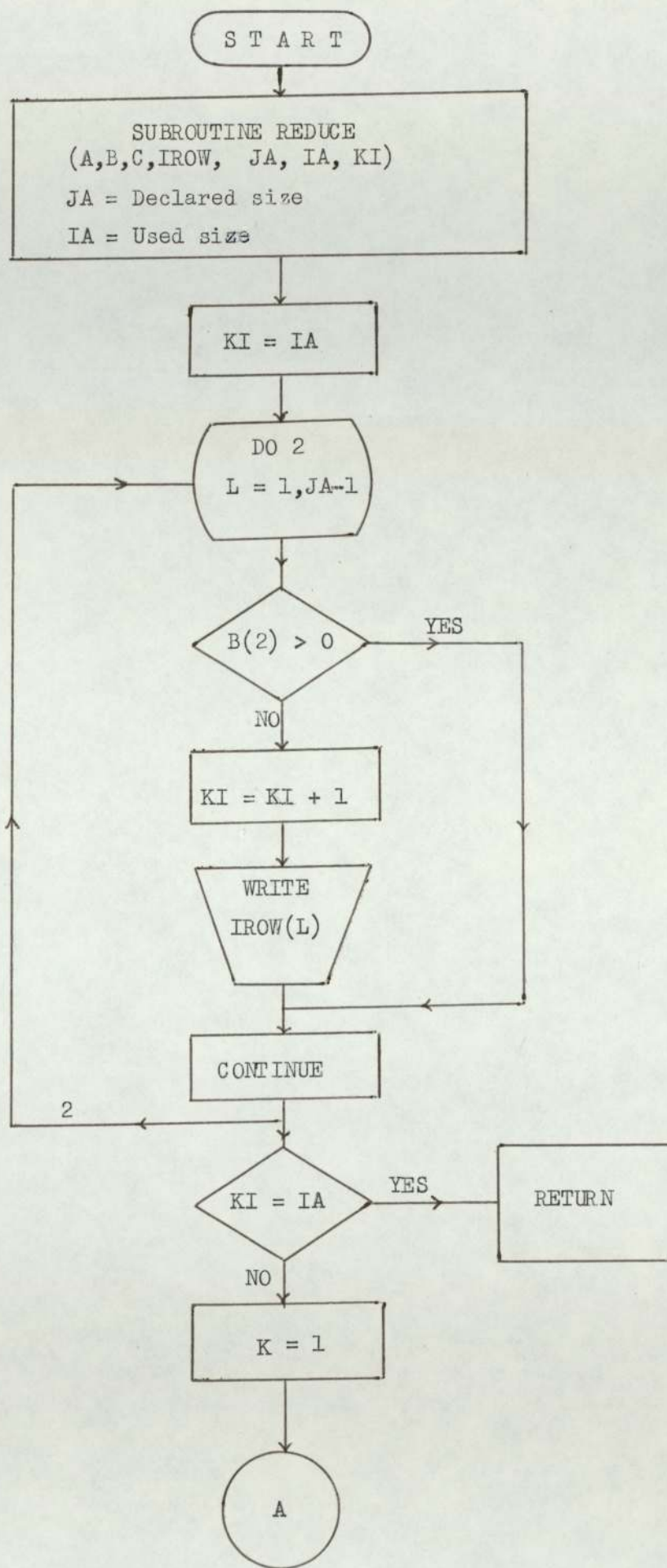


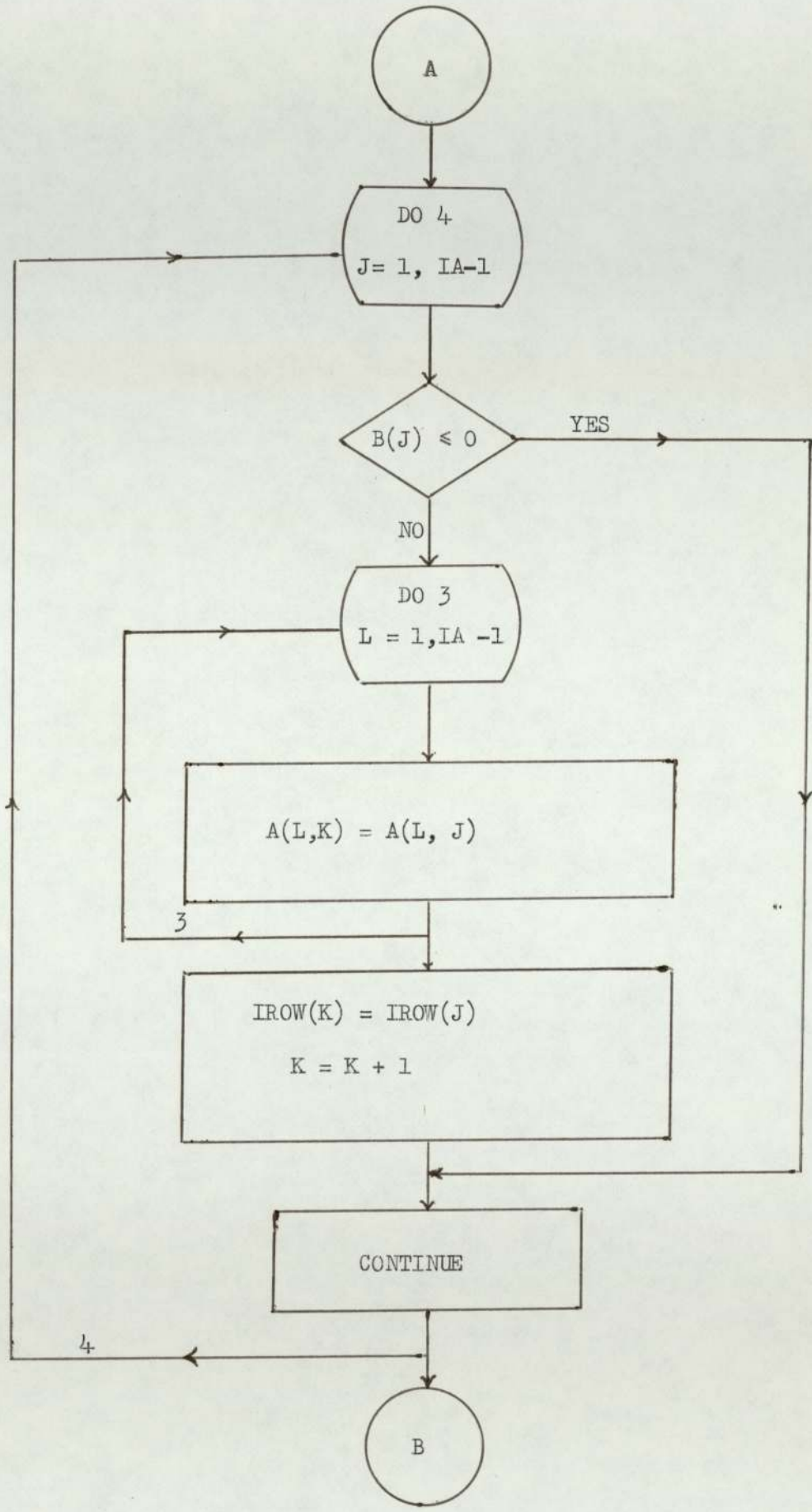


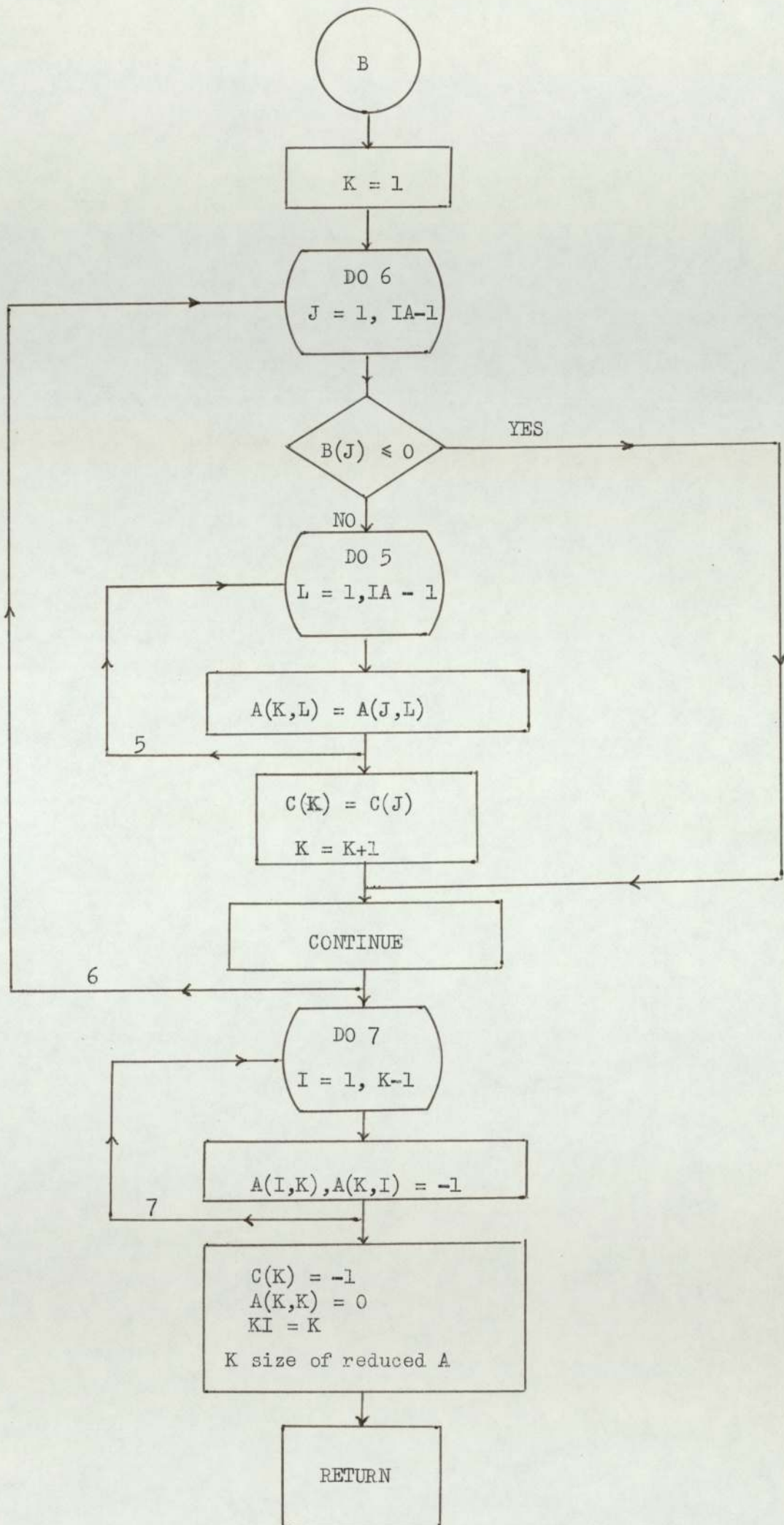


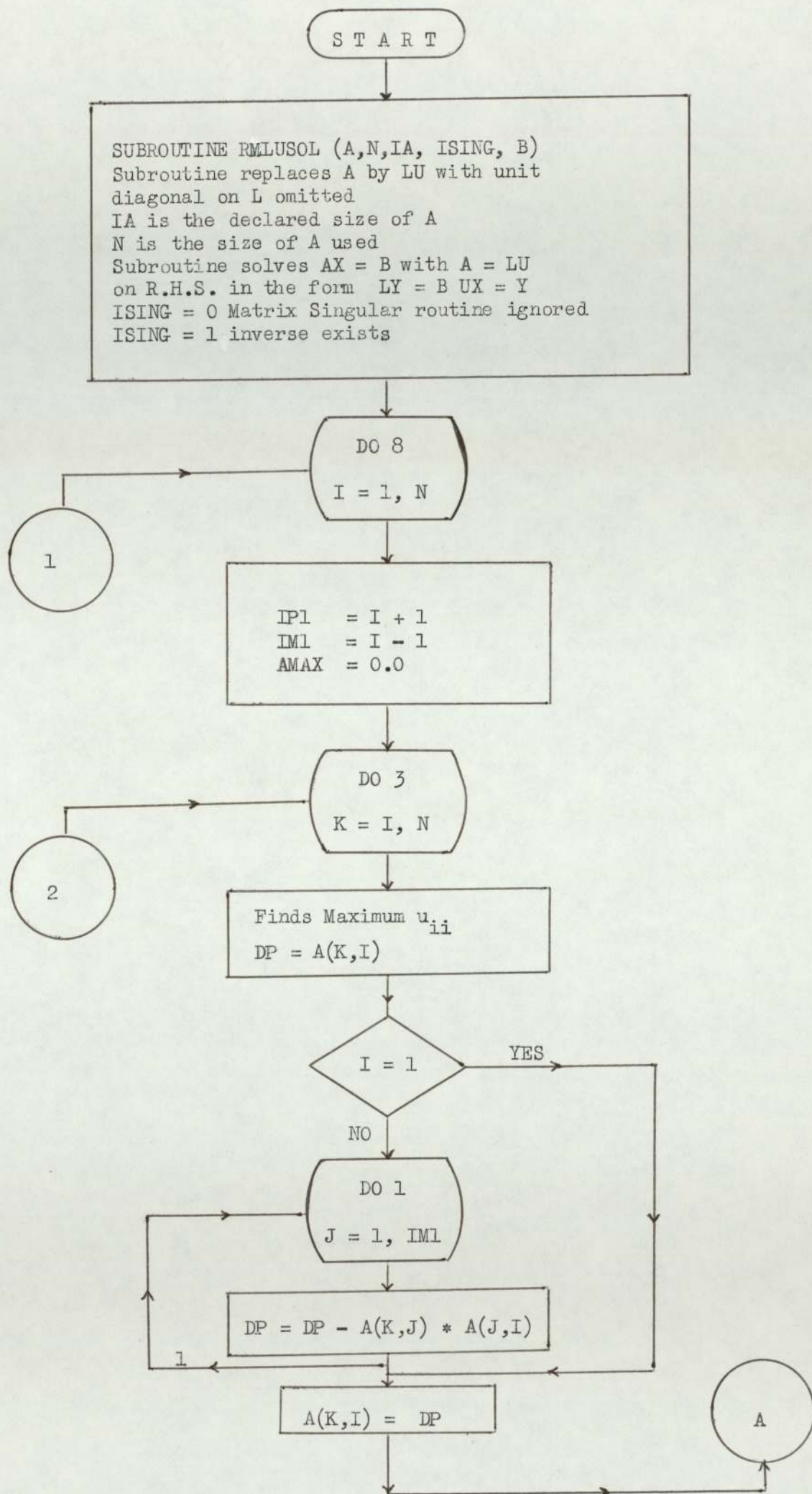


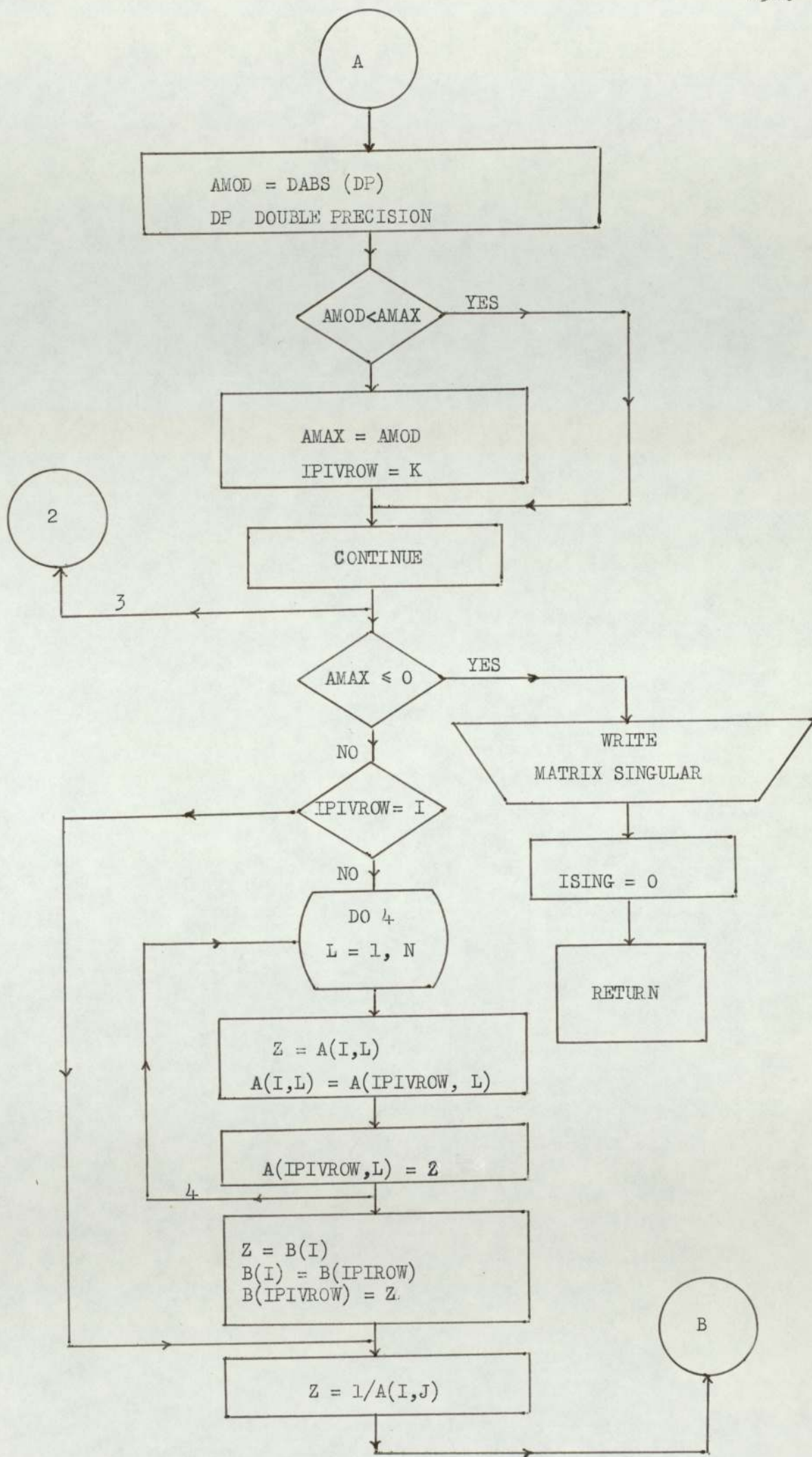


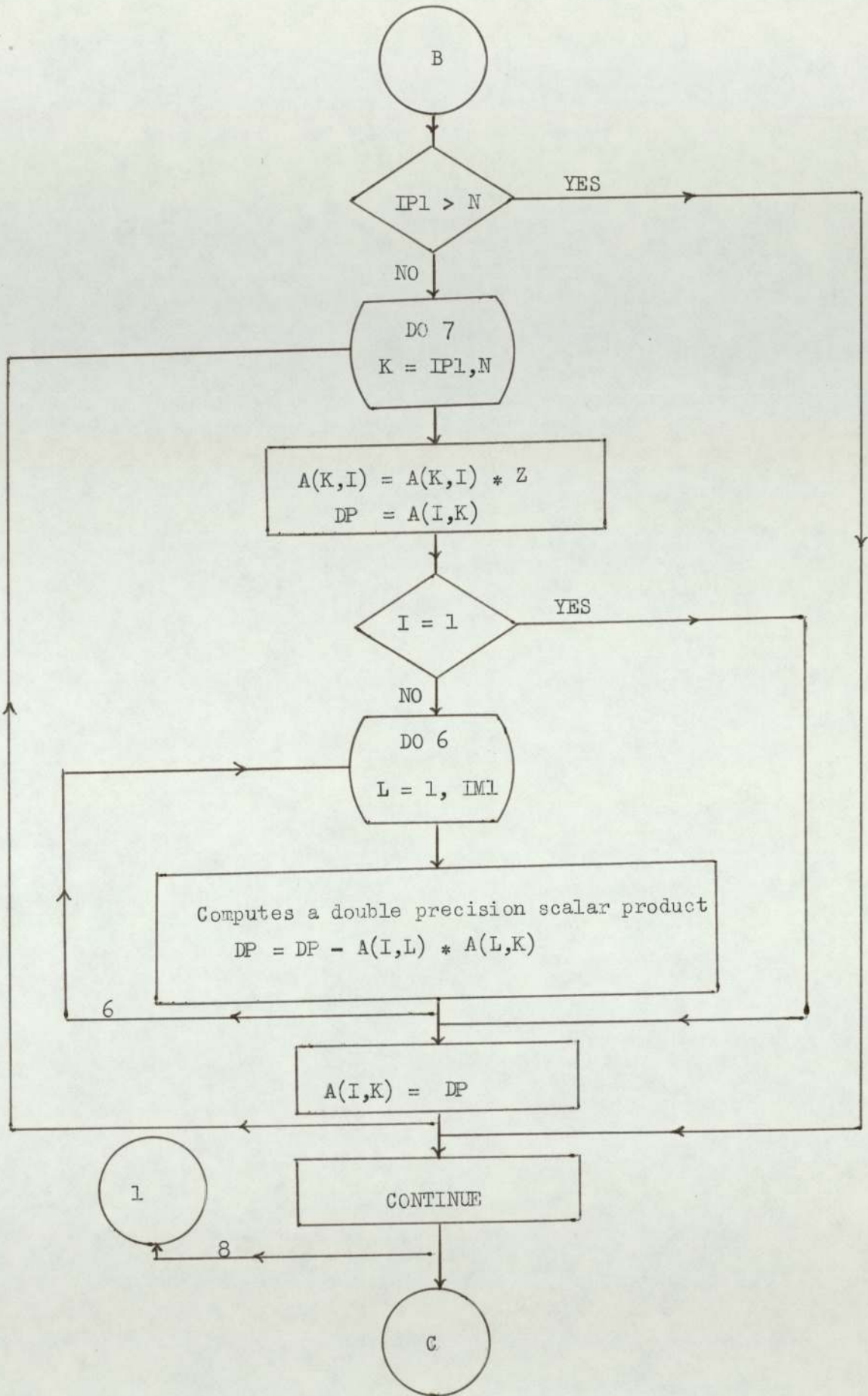


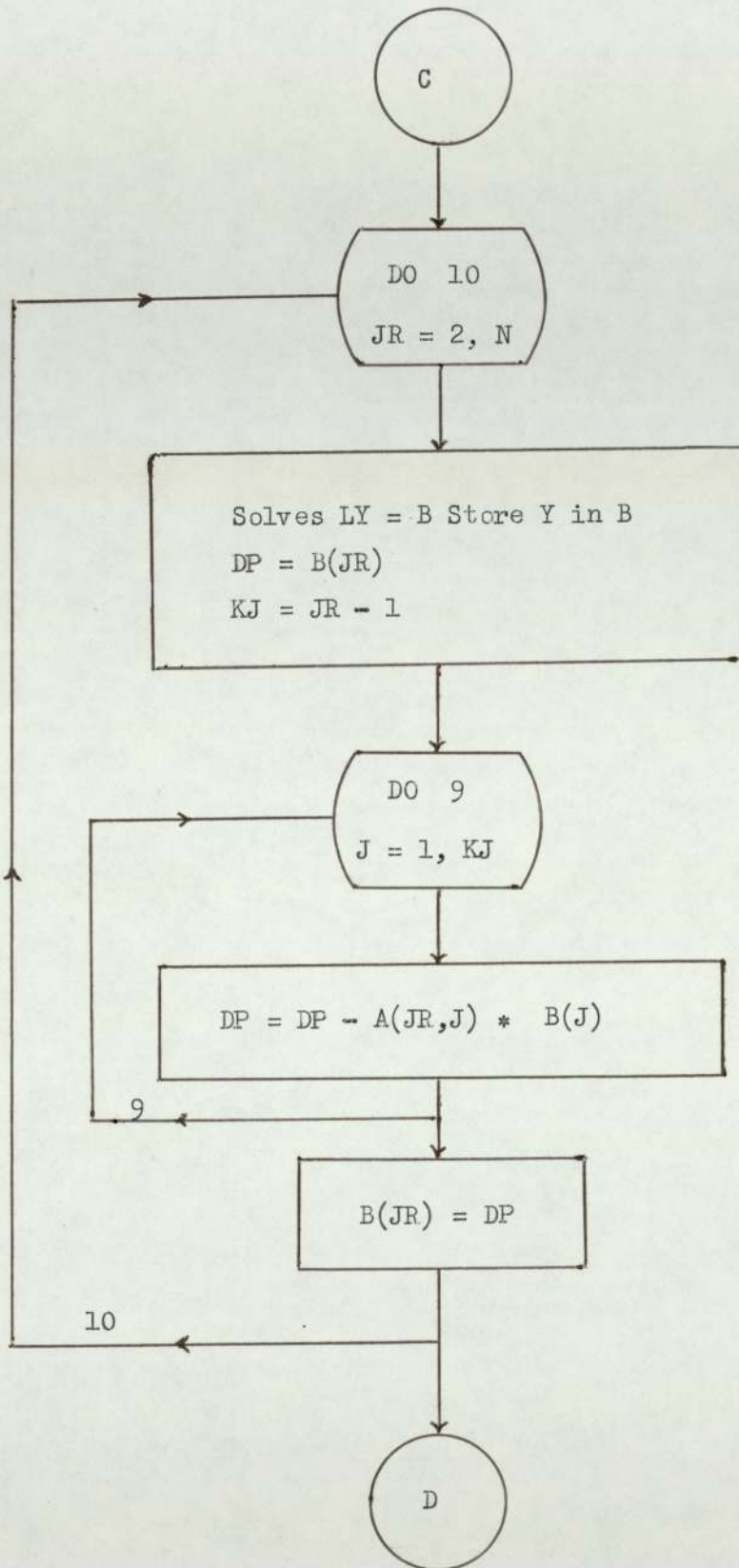


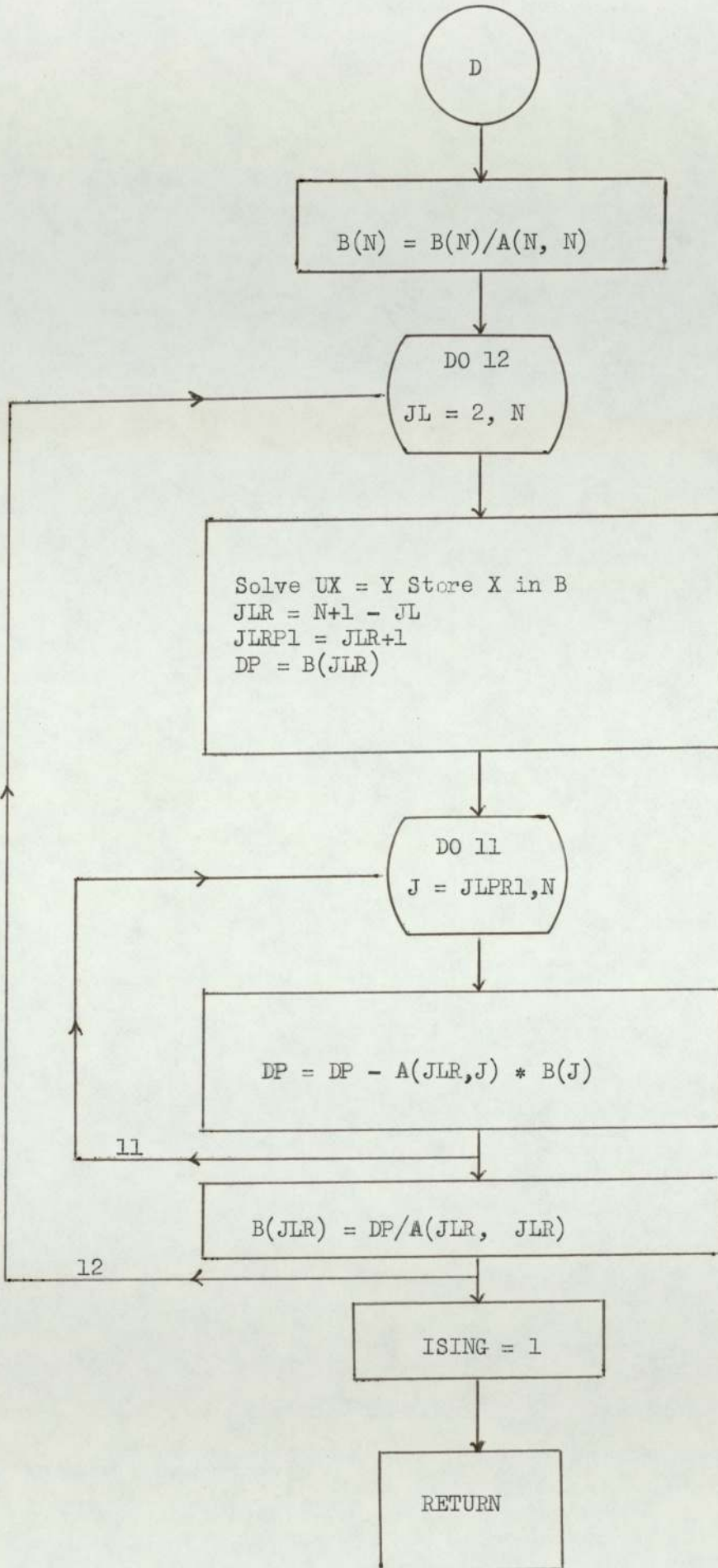






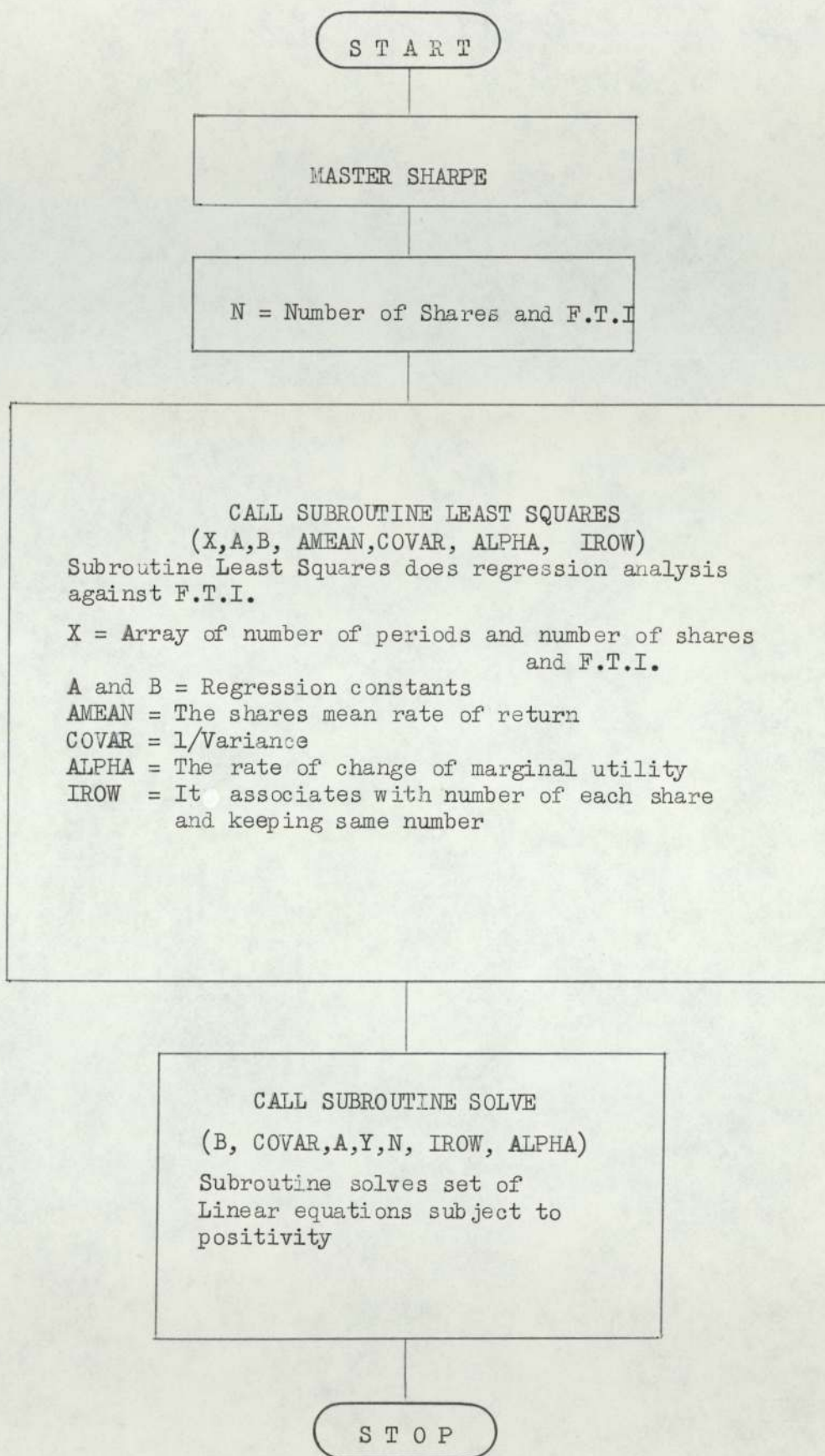


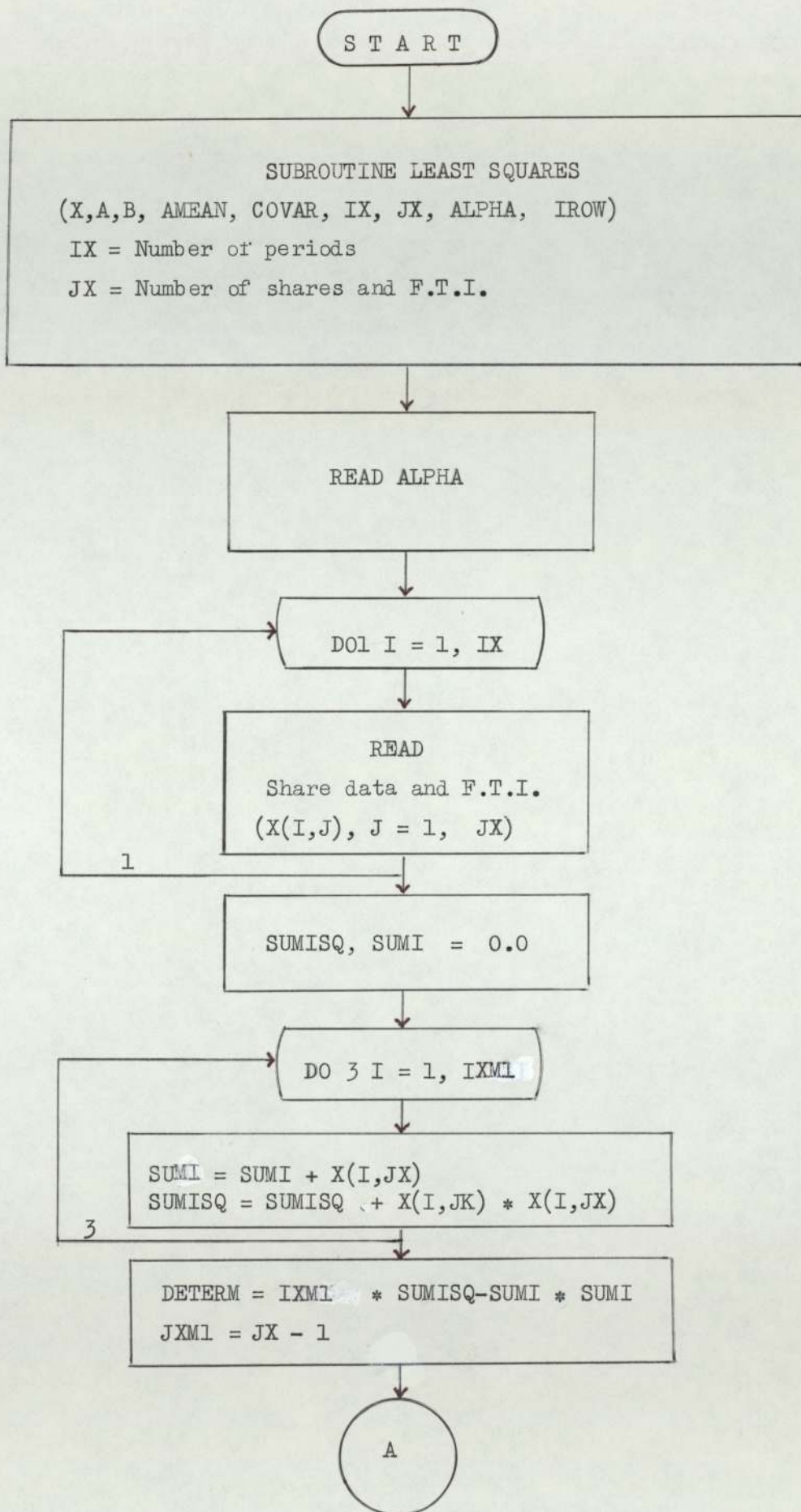


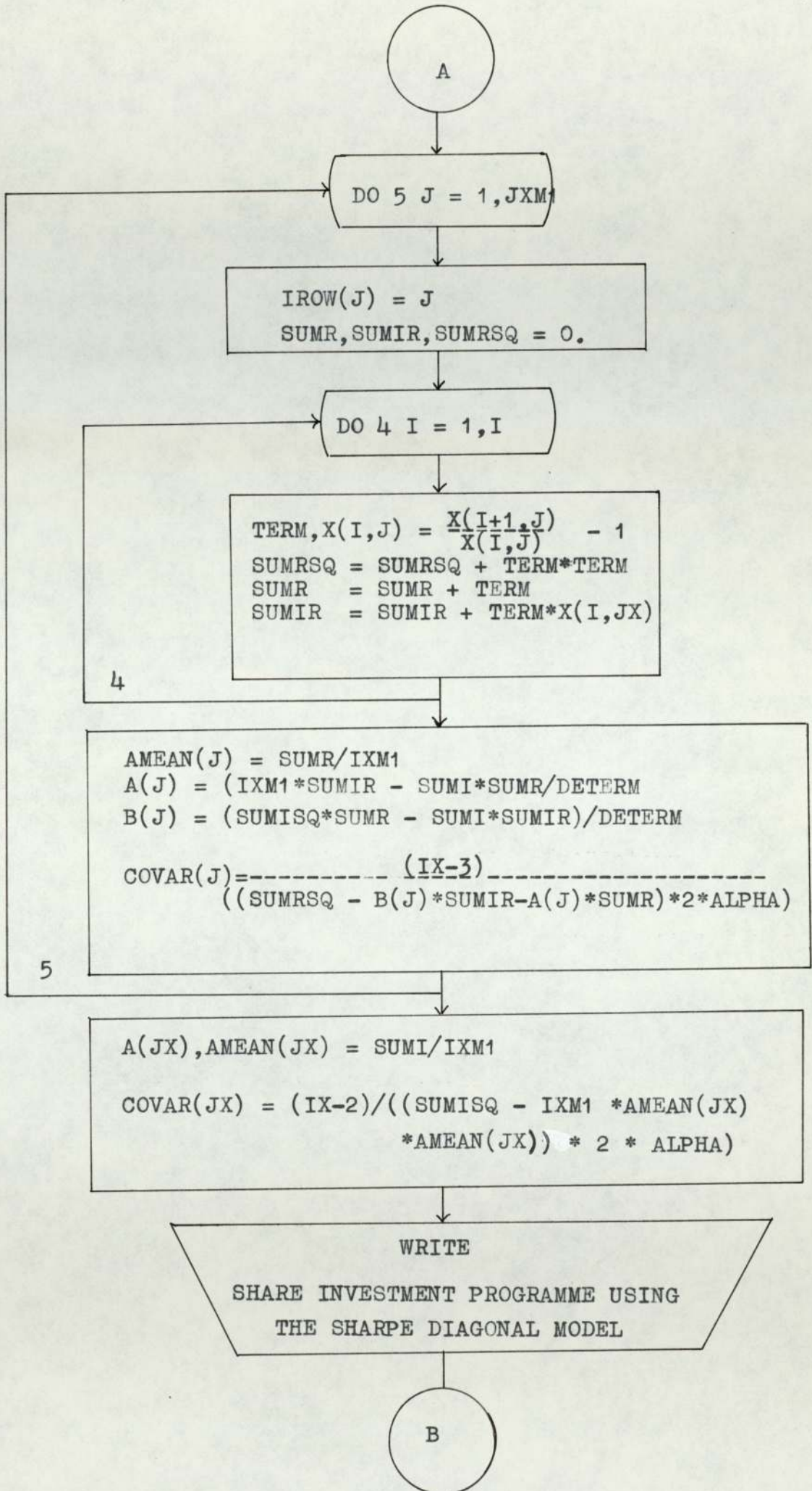


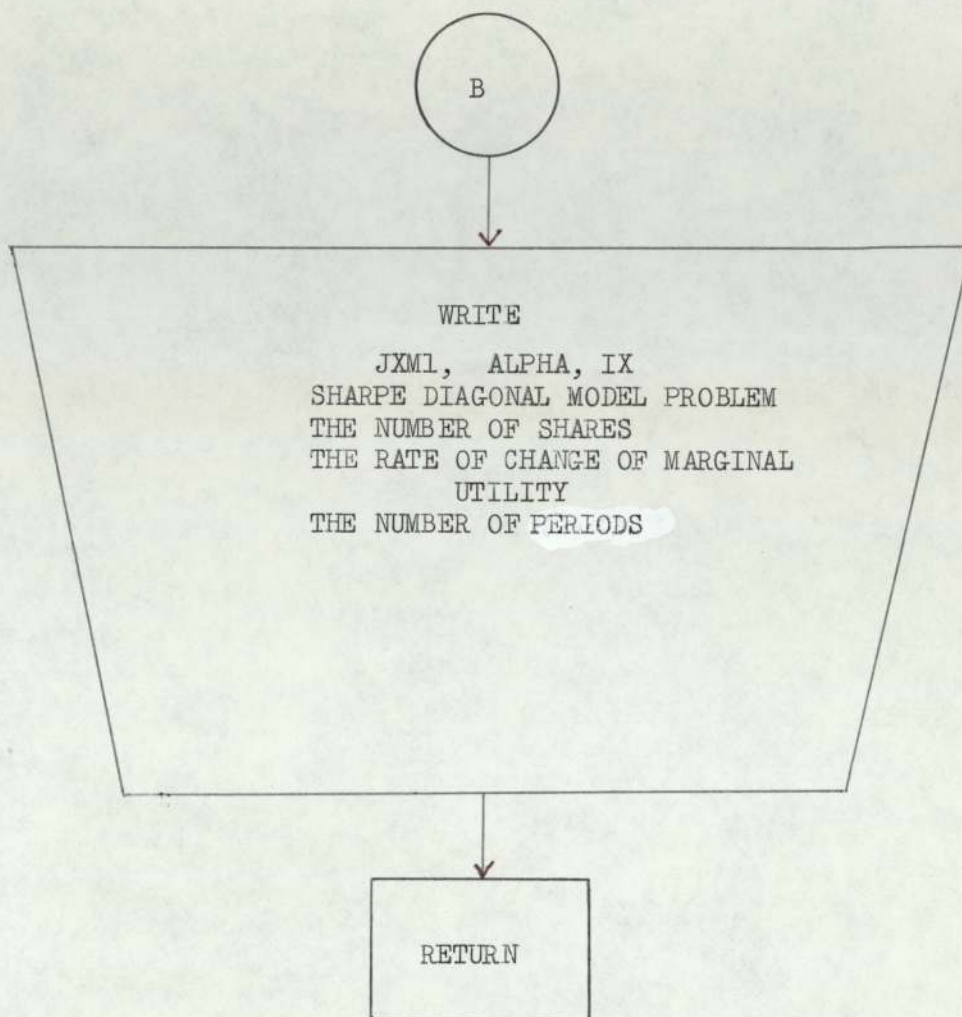
A P P E N D I X A5

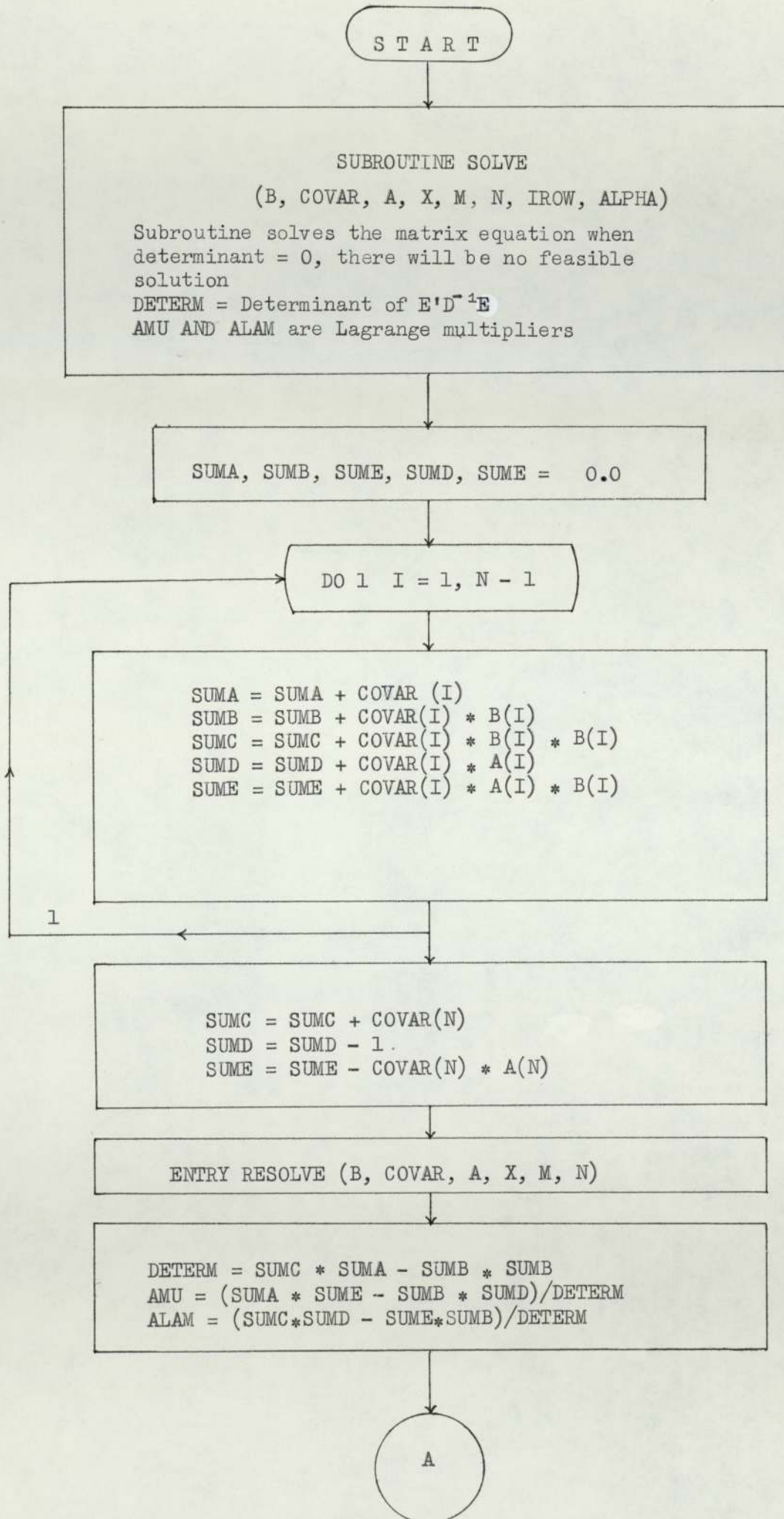
FLOW CHART OF SHARPE DIAGONAL MODEL.

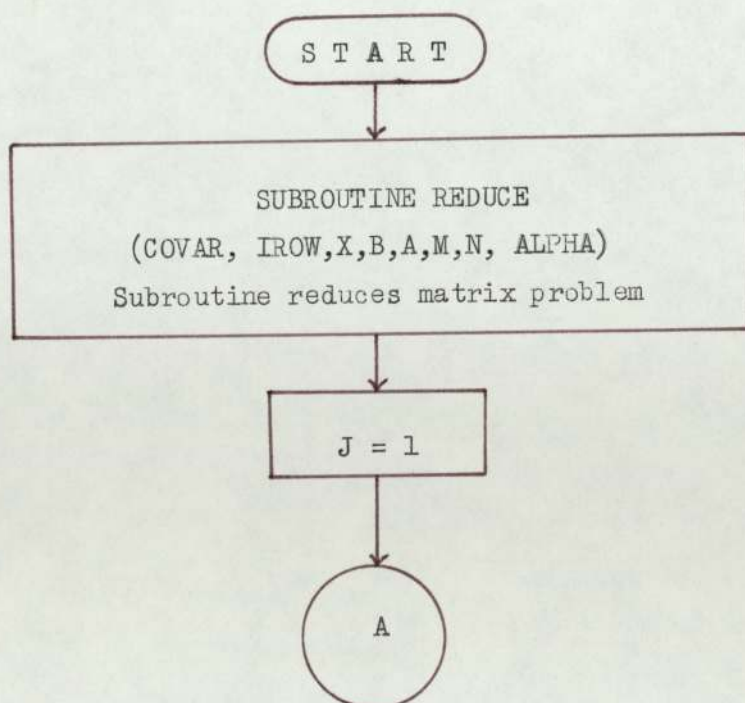
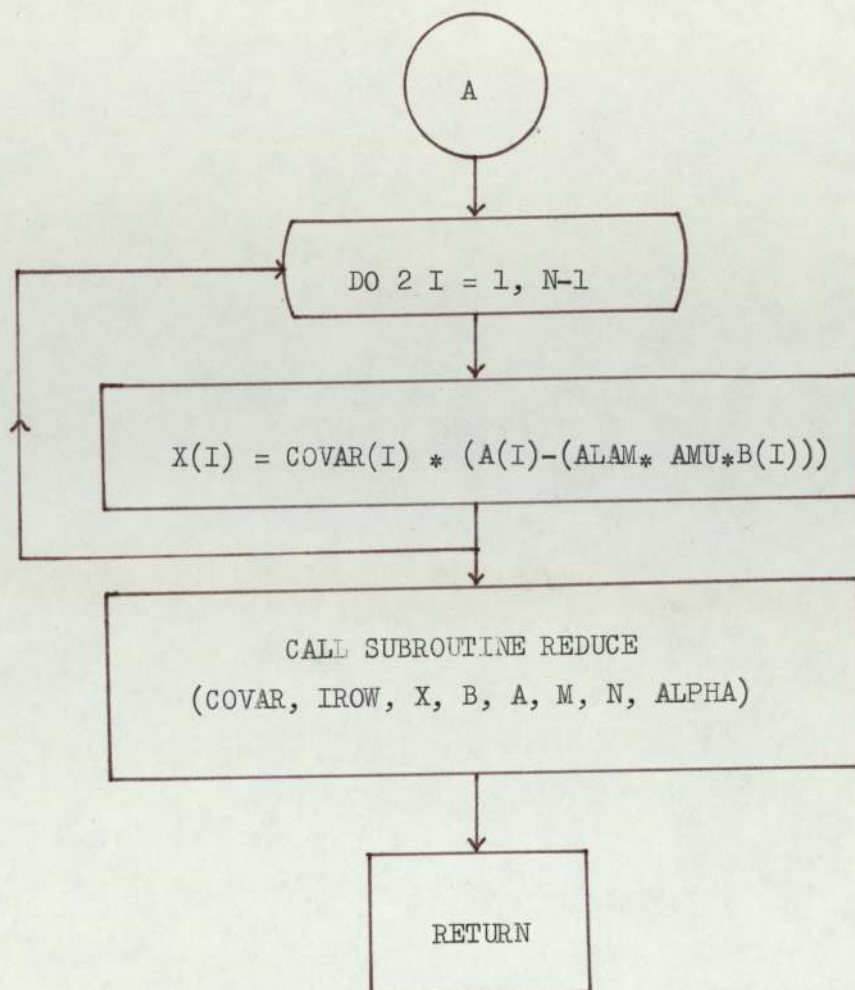


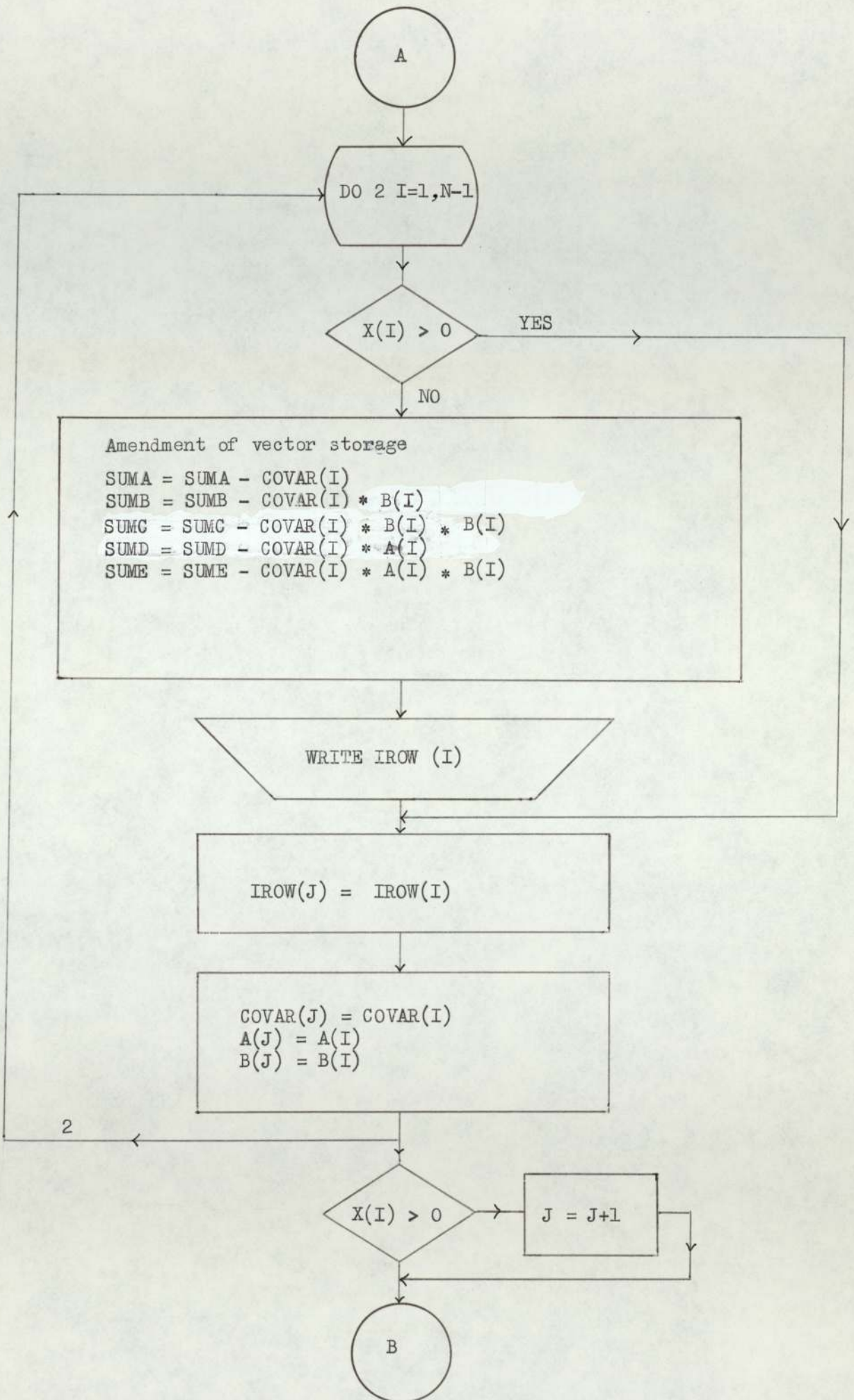


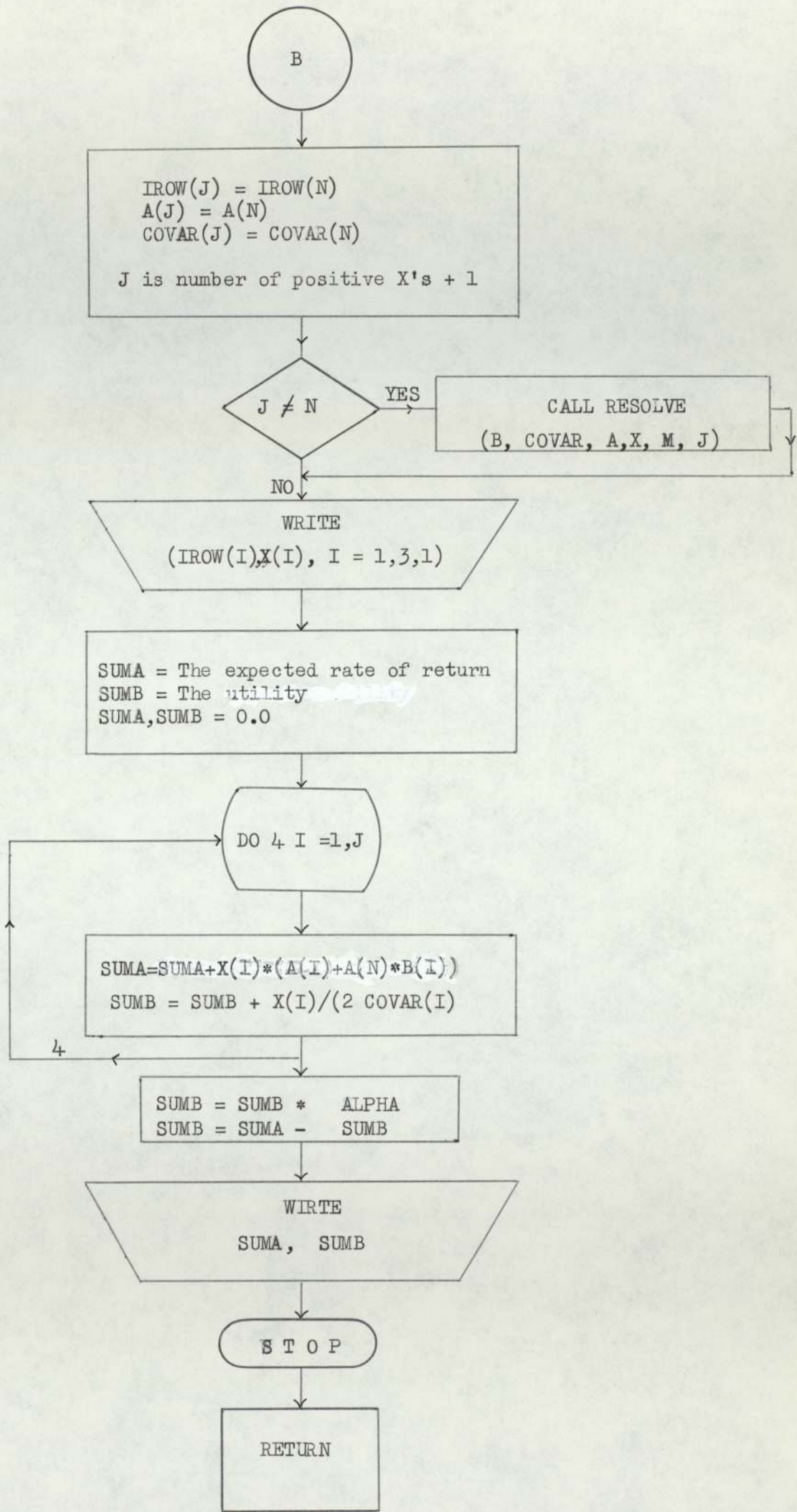






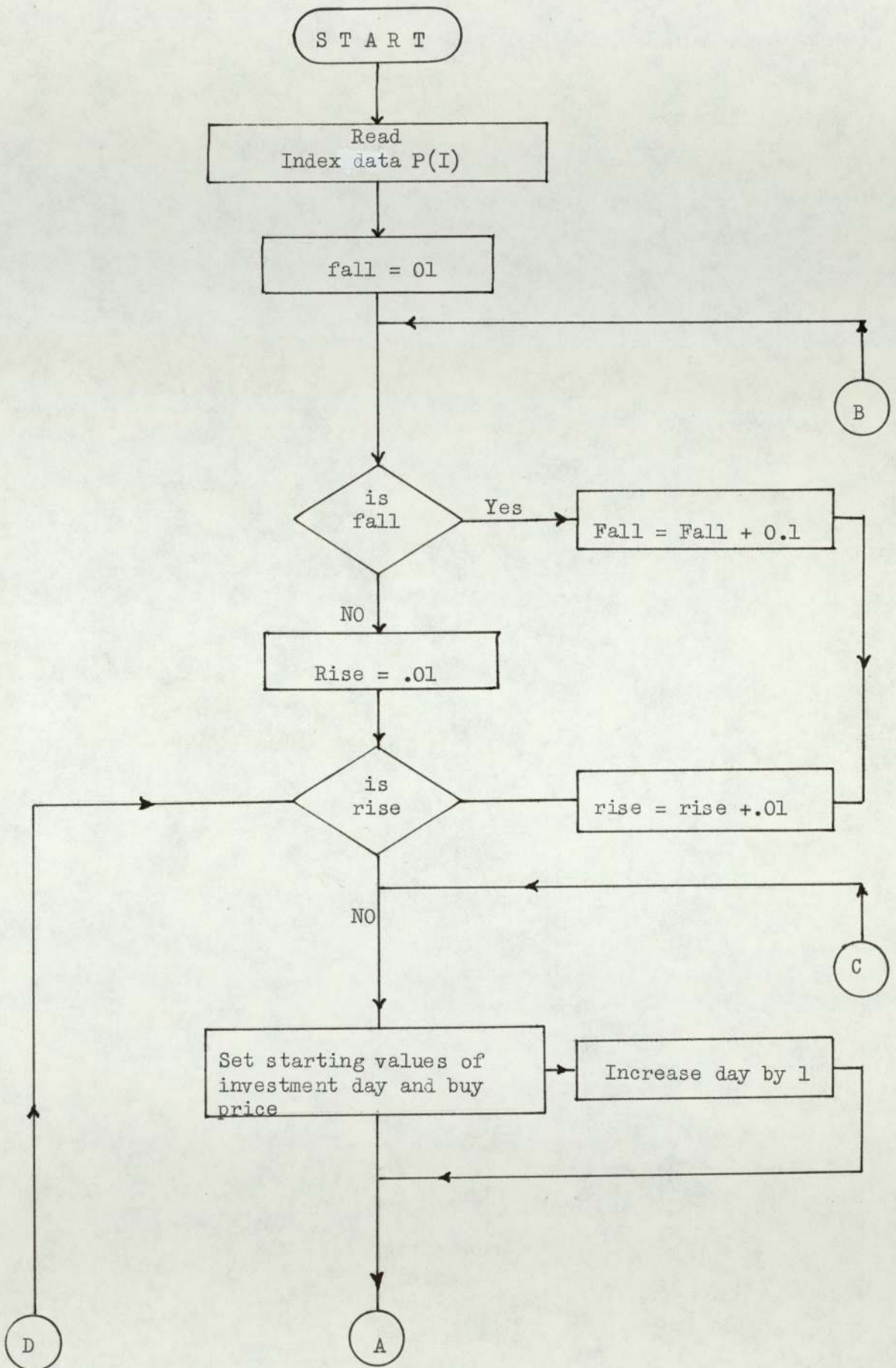


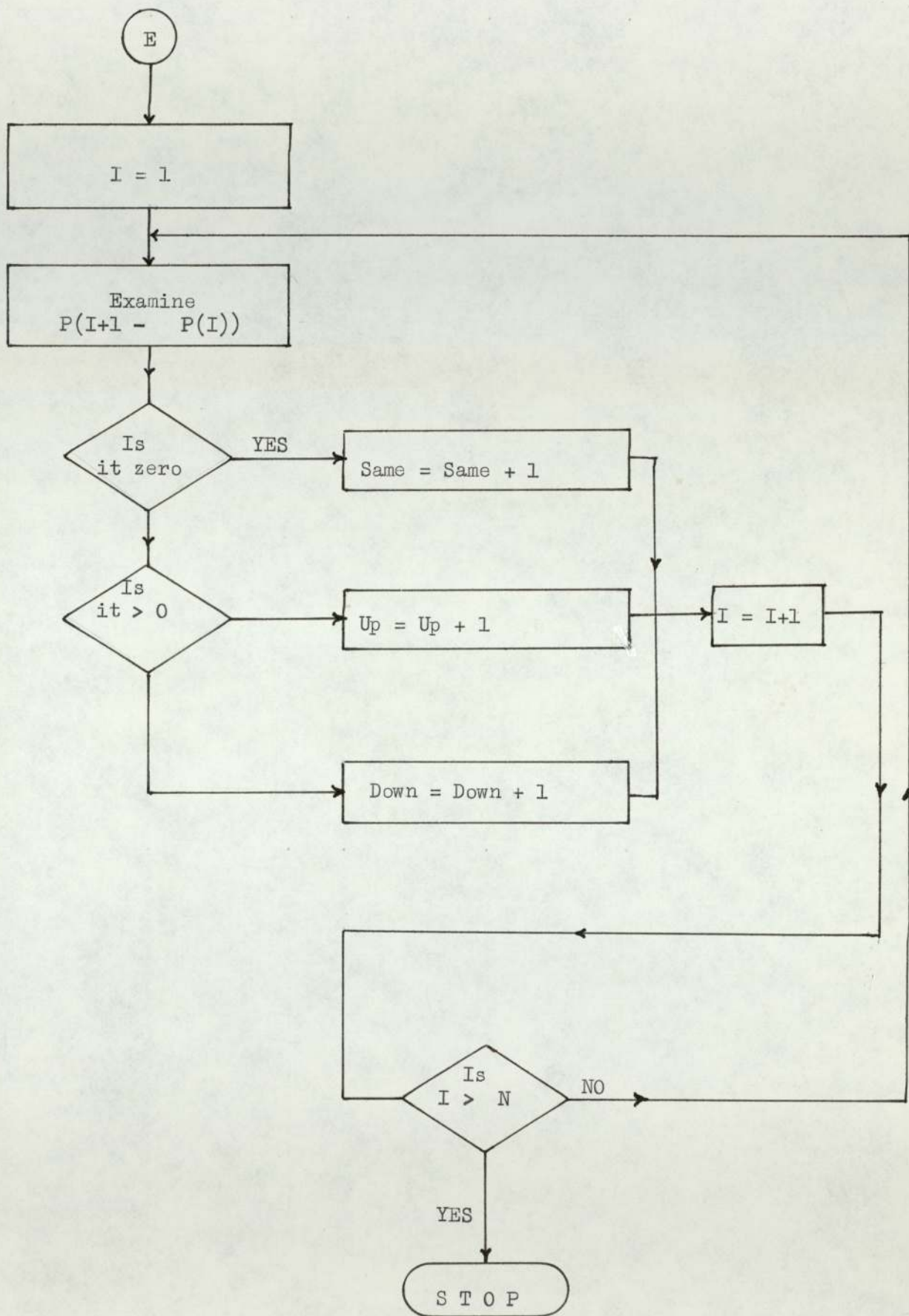




A P P E N D I X A6

FLOW CHART OF FILTER METHOD.





A P P E N D I X A7

CODES OF COMPUTER PROGRAMMES AND
THE SPECIMEN OUTPUTS

PROGRAMME NUMBER - 1.

```

MASTER MARKOWITZ
DIMENSION A(11,11),B(11),C(11,11),IROW(11),D(11),X(10,36),
1BR(10,24)
C N IS NUMBER OF SHARES
N=10
KOUNT=N+1
C KOUNT MUST BE DECLARED SIZE OF A,B,C,IROW,D
CALL COVARIANCE(X,B,A,IROW,10,36,10,36,11,ALPHA,BB)
1 DO 2 I=1,KOUNT
C LOOP COPIES A,B INTO C,D
C BECAUSE RMLUSOL DESTORYS BOTH C AND D
D(I)=B(I)
DO 2 J=1,KOUNT
2 C(I,J)=A(I,J)
C ISING TESTS THE MATRIX SINGULAR
CALL RMLUSOL (C,KOUNT,N+1,ISING,D)
IF(ISING.FO.1)GOTO 4
WRITE(2,3)
3 FORMAT(1H0,20HNO FEASIBLE SOLUTION)
STOP01
4 CALL REDUCE (A,D,B,IROW,N+1,KOUNT,KI)
IF (KOUNT.EQ.KI) GOTO 5
KOUNT=KI
GOTO 1
5 DO 6 J=1,KOUNT-1
6 WRITE(2,7)IROW(J),D(J)
C FINAL PRINT OF NON-ZERO X'S
7 FORMAT(1H0,'SHARE(',I2,2H)=,2PF6.2,'%')
C KM1 IS NUMBER OF SHARES WHICH ARE NOT NEGATIVE
KM1=KOUNT-1
SUM=0.
DO 8 I=1,KM1
8 SUM=SUM+B(I)*D(I)
ERR=SUM
WRITE(2,9)ERR
9 FORMAT(1H0,'THE EXPECTED RATE OF RETURN=',2PF5.2,'%')
SUM=A(1,1)*D(1)*D(1)*0.5
IF(KM1.LT.2) GO TO 11
DO 11 J=2,KM1
SUM=SUM+A(J,J)*D(J)*D(J)*0.5
C LOOP CALCULATES THE UTILITY
C UTILITY = F.R.R - 0.5*ALPHA*D'CD
DO 10 I=1,J-1
10 SUM=SUM+A(I,J)*D(I)*D(J)
11 SUM=ERR-SUM
WRITE(2,12)SUM
12 FORMAT(1H0,'THE UTILITY=',F9.4)
WRITE(2,13)
13 FORMAT(1H0,'KEY TO SHARE NAMES:--')
C N IS NUMBER OF SHARES
C LOOP WRITES THE TITLES
DO 14 I=1,N
14 WRITE(2,15)I,(BB(I,J),J=1,24)
15 FORMAT(1H0,'SHARE(',I2,2H)=,24A3)
STOP
END

```

PROGRAMME 1 (contd)

```

SUBROUTINE COVARIANCE(X,ARATE,COVAR,IROW,IA,JA,KA,LA,IAP1,ALPHA,
1BB)
C SUBROUTINE PROCESS SHARE INFORMATION
C OUTPUTS MEAN RATE OF SHARE RETURN
C OUTPUTS COVARIANCE MATRIX IN
C FORM SUITABLE FOR QUADRATIC PROGRAMMING
C KA NUMBER OF SHARES
C LA NUMBER OF PERIODS
C IA,JA IS DECLARED SIZE OF X,ARATE,COVAR IN CALLING SEGMENT
C KA,LA IS USED SIZE OF ABOVE
C IMPORTANT IAP1=IA+1
C DIMENSION X(IA,JA),ARATE(IAP1),COVAR(IAP1,IAP1),IROW(IA),BB(IA,24)
C READ(1,1)ALPHA
1 FORMAT(F0.0)
C LOOPS READS THE TITLE
DO 2 I=1,KA
2 READ(1,3)(BB(I,J),J=1,24)
3 FORMAT(24A3)
C ALPHA IS RISK MEASURE
ALPHA=ALPHA*2.
C READ IN SHARE DATA
DO 4 I=1,KA
4 READ(1,5)(X(I,J),J=1,LA)
5 FORMAT(10F0.0)
WRITE(2,6)
6 FORMAT(1H1,'SHARE INVESTMENT PROGRAM',/1H0,'INITIAL DATA MATRIX')
IROW(I)=I
7 WRITE(2,8)I,(BB(I,J),J=1,24),(X(I,J),J=1,LA)
8 FORMAT(1H0,'SHARE(',I2,2H) ,3X,24A3,/(1H0,10F7.2))
DO 10 I=1,KA
SUM=0.
DO 9 J=2,LA
TERM,X(I,J-1)=X(I,J)/X(I,J-1)-1.
9 SUM=SUM+TERM
10 ARATE(I)=SUM/(LA-1)
DO 12 K=1,KA
DO 12 I=K,KA
SUM=0.
DO 11 J=1,LA-1
11 SUM=SUM+X(K,J)*X(I,J)
12 COVAR(K,I),COVAR(I,K)=ALPHA*(SUM-ARATE(K)*ARATE(I)*(LA-1))/(LA-2)
DO 13 I=1,KA
IROW(I)=I
13 COVAR(IAP1,I),COVAR(I,IAP1)=-1.
ARATE(IAP1)=-1.
ALPHA=ALPHA*0.5
WRITE(2,14)KA,ALPHA,LA
14 FORMAT(///1H0,17HMARKOLITZ'S MODEL/
11H0,24HTHE NUMBER OF SHARES IS ,I2/
21H0,39HTHE RATE OF CHANGE OF MARGINAL UTILITY=,F5.1/
31H0,'THE NUMBER OF PERIODS IS ',I3/
41H0,'THE DATA USED FOR THE YEAR 1968 TO 70',/
51H0,'MEAN RATE OF RETURN AND COVARIANCE MATRIX')
DO 15 I=1,KA
15 WRITE(2,16)IROW(I),ARATE(I),(COVAR(I,J),J=1,KA)
16 FORMAT(1H0,'SHARE(',I2,3H) ,2PF5.2,'%'/((1H0,0PF7F9.4))
RETURN
END

```


PROGRAMME 1 (contd)

```

SUBROUTINE REDUCE(A,B,C,IROW,JA,IA,KI)
C  SUBROUTINE REDUCES MATRIX      JA=DECLARED SIZE IA=USED SIZE
  DIMENSION A(JA,JA),B(JA),IROW(JA),C(JA)
  KI=IA
  DO 2 L=1,IA-1
  IF (B(L).GT.0) GOTO2
  KI=KI+1
  WRITE(2,1) IROW(L)
1  FORMAT(1H0,'SHARE(',I2,3H)=0)
C  OUTPUT ALL VARIABLES SET TO ZERO
2  CONTINUE
  IF (KI.EQ. IA) RETURN
C  IF TRUE NO X WAS FOUND NEGATIVE
  K=1
  DO 4 J=1,IA-1
C  LOOP REMOVES ROWS OF A
  IF (B(J).LE.0) GOTO 4
  DO 3 L=1,IA-1
3  A(L,K)=A(L,J)
  IROW(K)=IROW(J)
  K=K+1
4  CONTINUE
  K=1
  DO 6 J=1,IA-1
C  LOOP REMOVES COLUMNS OF A
  IF (B(J).LE.0) GOTO 6
  DO 5 L=1,IA-1
5  A(K,L)=A(J,L)
  C(K)=C(J)
  K=K+1
6  CONTINUE
  DO 7 I=1,K-1
7  A(I,K),A(K,I)=-1
  C(K)=-1.
  A(K,K)=0.
  KI=K
C  K SIZE OF REDUCED A
  RETURN
  END

```


PROGRAMME 1 (contd)

```

SUBROUTINE RMLUSOL(A,N,IA,ISING,B)
C REAL MATRIX
C SUBROUTINE REPLACES A BY LU WITH UNIT DIAGONAL ON L OMITTED
C IA IS THE DECLARED SIZE OF A IN CALL ING SEGMENT
C N IS THE SIZE OF A USED
C SOLVES AX=B WITH A=LU ONE R.H.S.
C IN THE FORM LY=B UX=Y
C ISING=0 MATRIX SINGULAR ROUTINE IGNORED ISING=1 INVERSE EXISTS
DOUBLE PRECISION DP
DIMENSION A(IA,IA),B(IA)
DO 8 I=1,N
C ITH STAGE
IP1=I+1
IM1=I-1
AMAX=0.
DO 3 K=I,N
C FINDS MAXIMUM U(I,I)
DP=A(K,I)
IF (I.EQ.1) GOTO 2
DO 1 J=1,IM1
C COMPUTES A DOUBLE PRECISION SCALAR PRODUCT
1 DP=DP-A(K,J)*A(J,I)
2 A(K,I)=DP
AMOD=DABS(DP)
IF (AMOD.LT.AMAX) GOTO 3
AMAX=AMOD
IPIVROW=K
3 CONTINUE
IF (AMAX.LE.0) GOTO 13
C MATRIX SINGULAR IF TRUE
IF (IPIVROW.EQ.I) GOTO 5
DO 4 L=1,N
Z=A(I,L)
A(I,L)=A(IPIVROW,L)
4 A(IPIVROW,L)=Z
Z=B(I)
B(I)=B(IPIVROW)
B(IPIVROW)=Z
5 Z=1./A(I,I)
IF (IP1.GT.N) GOTO 8
DO 7 K=IP1,N
A(K,I)=A(K,I)*Z
DP=A(I,K)
IF (I.EQ.1) GOTO 7
DO 6 L=1,IM1
C COMPUTES A DOUBLE PRECISION SCALAR PRODUCT
6 DP=DP-A(I,L)*A(L,K)
7 A(I,K)=DP
8 CONTINUE
DO 10 JR=2,N
C SOLVE LY=B STORE Y IN B
DP=B(JR)
KJ=JR-1
DO 9 J=1,KJ
9 DP=DP-A(JR,J)*B(J)
10 B(JR)=DP
B(N)=B(N)/A(N,N)
DO 12 JL=2,N
C SOLVE UX=Y STORE X IN B
JLR=N+1-JL
JLRP1=JLR+1
DP=B(JLR)
DO 11 J=JLRP1,N
C COMPUTES A DOUBLE PRECISION SCALAR PRODUCT
11 DP=DP-A(JLR,J)*B(J)
12 B(JLR)=DP/A(JLR,JLR)
ISING=1
RETURN
13 WRITE(2,14)
14 FORMAT(1H1,15HMATRIX SINGULAR)
ISING=0
RETURN
END
FINISH

```

OUTPUT OF PROGRAMME NUMBER 1

MARKOVITZ'S MODEL

THE NUMBER OF SHARES IS 10

THE RATE OF CHANGE OF MARGINAL UTILITY= 1.0

THE NUMBER OF PERIODS IS 36

THE DATA USED FOR THE YEAR 1968 TO 70

MEAN RATE OF RETURN AND COVARIANCE MATRIX

SHARE(1)	0.66%						
	0.0124	0.0031	0.0268	0.0089	0.0030	0.0028	0.0023
	0.0089	-0.0002	0.0005				
SHARE(2)	1.25%						
	0.0031	0.0091	0.0058	0.0078	-0.0005	0.0038	0.0015
	0.0032	0.0044	0.0019				
SHARE(3)	-0.23%						
	0.0268	0.0038	0.1151	0.0324	0.0101	0.0031	0.0028
	0.0284	-0.0042	0.0060				
SHARE(4)	-0.24%						
	0.0089	0.0038	0.0324	0.0109	0.0040	0.0041	0.0044
	0.0115	0.0032	0.0056				
SHARE(5)	0.02%						
	0.0030	-0.0005	0.0101	0.0040	0.0153	0.0024	0.0030
	0.0056	-0.0005	0.0019				
SHARE(6)	0.60%						
	0.0028	0.0038	0.0031	0.0041	0.0024	0.0114	0.0043
	0.0016	0.0004	0.0019				
SHARE(7)	1.03%						
	0.0023	0.0015	0.0028	0.0044	0.0030	0.0043	0.0079
	0.0018	0.0034	0.0007				
SHARE(8)	0.20%						
	0.0089	0.0032	0.0284	0.0115	0.0056	0.0016	0.0018
	0.0130	0.0040	0.0034				
SHARE(9)	1.38%						
	-0.0002	0.0044	-0.0042	0.0052	-0.0005	0.0064	0.0034
	0.0020	0.0112	0.0005				
SHARE(10)	-0.19%						
	0.0005	0.0019	0.0060	0.0056	0.0019	0.0019	0.0007
	0.0034	0.0005	0.0089				

SHARE(4)=0%

SHARE(5)=0%

SHARE(6)=0%

SHARE(8)=0%

SHARE(3)=0%

SHARE(10)=0%

SHARE(1)=0%

SHARE(2)= 39.59%

SHARE(7)= 19.03%

SHARE(9)= 41.38%

THE EXPECTED RATE OF RETURN= 1.26%

THE UTILITY= 0.0097

KEY TO SHARE NAMES:-

SHARE(1)= 'GLAXO GROUP'

SHARE(2)= 'HOOPER LTD.'

SHARE(3)= 'BRITISH MOTOR'

SHARE(4)= 'DUNLAP CO.'

SHARE(5)= 'BRITISH PETROLEUM'

SHARE(6)= 'DISTILLERS'

SHARE(7)= 'GUINNESS(ARTHER)SON & CO.LTD.'

SHARE(8)= 'IMPERIAL CHEMICAL INDUSTRIES LTD.'

SHARE(9)= 'MARKS & SPENCERS'

SHARE(10)= 'BRITISH INSULATED CABLES'

PROGRAMME NUMBER 2

```

MASTER SHARPE
C PROGRAM USES THE SHARPE DIAGONAL MODEL
  DIMENSION Y(36,2),A(11),B(11),AMEAN(11),COVAR(11),IROW(11),X(11),
1 TITLES(11,24)
  N=11
C SUBROUTINE LEAST SQUARES DOES REGRESSION ANALYSIS AGAINST F.T.I
  CALL LEAST SQUARES(Y,A,B,AMEAN,COVAR,36,11,ALPHA,TITLES)
C SOLVES SET OF LINEAR EQUATIONS SUBJECT TO POSITIVITY
  CALL SOLVE(B,COVAR,A,X,11,N,IROW,ALPHA,TITLES)
  STOP
  END

SUBROUTINE LEAST SQUARES(Y,A,B,AMEAN,COVAR,IY,JY,ALPHA,TITLES)
C JY=NUMBER OF SHARES+INDEX
C IY=NUMBER OF PERIODS
  DIMENSION Y(IY,2),A(JY),B(JY),AMEAN(JY),COVAR(JY),TITLES(JY,24)
C ALPHA IS A RISK MEASURE
  READ(1,1)ALPHA
  1 FORMAT(F0.0)
  DO 2 I=1,JY
  2 READ(1,3)(TITLES(I,J),J=1,24)
  3 FORMAT(24A3)
  DO 4 I=1,IY
C READING INDEX
  4 READ(1,5)(Y(I,1),I=1,IY)
  5 FORMAT(36F0.0)
  SUMISO,SUMI=0.
  IYM1=IY-1
  DO 6 I=1,IYM1
  SUMI=SUMI+Y(I,1)
  6 SUMISO=SUMISO+Y(I,1)*Y(I,1)
  DETERM=IYM1*SUMISO-SUMI*SUMI
  JYM1=JY-1
  WRITE(2,7)
  7 FORMAT(1H1,'SHARE INVESTMENT PROGRAM',/1H0,'INITIAL DATA MATRIX')
  DO 10 J=1,JYM1
C READING SHARE DATA
  READ(1,5)(Y(I,2),I=1,IY)
  WRITE(2,8)J,(TITLES(J+1,K),K=1,24),(Y(I,2),I=1,IY)
  8 FORMAT(1H0,'SHARE(',I2,2H) ,3X,24A3/(1H0,9F7.2))
  SUMR,SUMIR,SUMRSO=0.
  DO 9 I=1,IYM1
C CALCULATING RATE OF RETURN
  TERM,Y(I,2)=Y(I+1,2)/Y(I,2)-1.
  SUMRSO=SUMRSO+TERM*TERM
  SUMR=SUMR+TERM
  9 SUMIR=SUMIR+TERM*Y(I,1)
  AMEAN(J)=SUMR/IYM1
C WHERE R(I)=A(I)+B(I)*INDEX
C A AND B ARE TWO REGRESSION CONSTANTS
  A(J)=(SUMISO*SUMR-SUMI*SUMIR)/DETERM
  B(J)=(IYM1*SUMIR-SUMI*SUMR)/DETERM
  10 COVAR(J)=(IY-3)/((SUMRSO-B(J)*SUMIR-A(J)*SUMR)*2.*ALPHA)
C VARIANCE ABOUT R(I)=A(I)+B(I)*INDEX
  WRITE(2,11)(TITLES(1,K),K=1,24),(Y(I,1),I=1,IY)
  11 FORMAT(1H0,'INDEX',6X,24A3/(1H0,9F7.2))
  A(JY),AMEAN(JY)=SUMI/IYM1
C WHERE COVAR=1/2*ALPHA*VARIANCE
  COVAR(JY)=(IY-2)/((SUMISO-IYM1*AMEAN(JY)*AMEAN(JY))*2.*ALPHA)
C VARIANCE ABOUT AMEAN
  WRITE(2,12)
  12 FORMAT(/1H0,'SHARE INVESTMENT PROGRAM USING THE SHARPE DIAGONAL',
-1X,'MODEL')
  WRITE(2,13)(J,A(J),B(J),AMEAN(J),COVAR(J),J=1,JYM1)
  13 FORMAT(1H0,1X,'I',7X,'A(I)',11X,'B(I)',7X,'MEAN RETURN',
-5X,'COVAR(I)'/((1H0,12,4E15.6))
  WRITE(2,14)JYM1,ALPHA,IY
  14 FORMAT(1H0,'SHARPE DIAGONAL MODEL PROBLEM'/
11H0,'THE NUMBER OF SHARES ',I2/
21H0,'THE RATE OF CHANGE OF MARGINAL UTILITY=',F5.1/
31H0,'THE NUMBER OF PERIODS ',I2/
41H0,'THE DATA USED FOR THE YEAR 1968 TO 70')
  RETURN
  END

```



```

SUBROUTINE SOLVE(B,COVAR,A,X,M,N,IROW,ALPHA,TITLES)
C SOLVES THE MATRIX EQUATION
C WHEN DETERMINANT=0 THERE WILL BE NO FEASIBLE SOLUTION
C DETERM=DETERMINANT OF E'(QM)E
C AMU AND ALAM ARE LAGRANGE MULTIPLIERS
C N IS THE CURRENT NUMBER OF SHARES + INDEX BEING PROCESSED
C M IS THE ORIGINAL NUMBER OF SHARES + INDEX
C DIMENSION B(M),COVAR(M),A(M),X(M),IROW(M),TITLES(M,24)
COMMON/SUM/SUMA,SUMB,SUMC,SUMD,SUME
SUMA,SUMB,SUMC,SUMD,SUME=0.
DO 1 I=1,N-1
IROW(I)=I
SUMA=SUMA+COVAR(I)
SUMB=SUMB+COVAR(I)*B(I)
SUMC=SUMC+COVAR(I)*B(I)*B(I)
SUMD=SUMD+COVAR(I)*A(I)
1 SUME=SUME+COVAR(I)*A(I)*B(I)
SUMC=SUMC+COVAR(N)
SUMD=SUMD-1.
SUME=SUME-COVAR(N)*A(N)
ENTRY RESOLVE(B,COVAR,A,X,M,N)
C INVERTING E'(QM)E
DETERM=SUMC*SUMA-SUMB*SUMB
AMU=(SUMA*SUME-SUMB*SUMD)/DETERM
ALAM=(SUMC*SUMD-SUME*SUMB)/DETERM
DO 2 I=1,N-1
2 X(I)=COVAR(I)*(A(I)-(ALAM+AMU*B(I)))
CALL REDUCE(COVAR,IROW,X,B,A,M,N,ALPHA,TITLES)
RETURN
END

```

```

SUBROUTINE REDUCE(COVAR,IROW,X,B,A,M,N,ALPHA,TITLES)
C SUBROUTINE REDUCES MATRIX PROBLEM
C IROW IS THE SHARE IDENTIFICATION VECTOR
C DIMENSION COVAR(M),A(M),B(M),X(M),IROW(M),TITLES(M,24)
COMMON/SUM/SUMA,SUMB,SUMC,SUMD,SUME
J=1
DO 3 I=1,N-1
C TEST FOR POSITIVITY
IF(X(I).GT.0.)GO TO 2
C AMMENDMENT OF VECTOR STORAGE
SUMA=SUMA-COVAR(I)
SUMB=SUMB-COVAR(I)*B(I)
SUMC=SUMC-COVAR(I)*B(I)*B(I)
SUMD=SUMD-COVAR(I)*A(I)
SUME=SUME-COVAR(I)*A(I)*B(I)
WRITE(2,1)IROW(I)
1 FORMAT(1H0,'SHARE(',I2,4H)=0%)
C REARRANGMENT OF VECTOR STORAGE
2 IROW(J)=IROW(I)
COVAR(J)=COVAR(I)
A(J)=A(I)
B(J)=B(I)
3 IF (X(I).GT.0.) J=J+1
IROW(J)=IROW(N)
A(J)=A(N)
COVAR(J)=COVAR(N)
C J IS NUMBER OF POSITIVE X'S+1
IF(J.NE.N)CALL RESOLVE(B,COVAR,A,X,M,J)
C IF TRUE SOME X'S NEGATIVE
C IF FALSE FINAL SOLUTION FOUND
SUMA,SUMB=0.
DO 6 I=1,J-1
SUMA=SUMA+X(I)*(A(I)+A(N)*B(I))
6 SUMB=SUMB+X(I)/(2.*COVAR(I))
SUMB=SUMB*ALPHA
SUMB=SUMA-SUMB
C SUMA=THE EXPECTED RATE OF RETURN
C SUMB=THE UTILITY
DO 4 K=1,J-2
DO 4 I=1+K,J-1
C LOOPS ORDERS X(I) INTO DESCENDING ORDER
IF(X(I).LE.X(K))GO TO 4
SWOP=X(I)
X(I)=X(K)
X(K)=SWOP
ISWOP=IROW(I)
IROW(I)=IROW(K)
IROW(K)=ISWOP
4 CONTINUE
WRITE(2,5)(IROW(I),X(I),I=1,J-1)
5 FORMAT(1H0,'SHARE(',I2,2H)=,2PF6.2,'%')
WRITE(2,7)SUMA,SUMB
7 FORMAT(1H0,'THE EXPECTED RATE OF RETURN= ',2PF5.2,'%'/
-1H0,'THE UTILITY= ',0PF9.4/)
WRITE(2,8)
8 FORMAT(1H0,'KEY TO SHARE NAMES:-')
DO 9 I=1,M-1
9 WRITE(2,10)I,(TITLES(I+1,J),J=1,24)
10 FORMAT(1H0,'SHARE(',I2,2H)=,24A3)
STOP 01
RETURN
END
FINISH

```


OUTPUT OF PROGRAMME 2

SHARE INVESTMENT PROGRAM USING THE SHARPE DIAGONAL MODEL

I	A(I)	B(I)	MEAN RETURN	COVAR(I)
1	0.694054E-01	-0.150488E-03	0.661518E-02	0.803934E 02
2	0.107252E 00	-0.227098E-03	0.124971E-01	0.110133E 03
3	0.219523E 00	-0.531632E-03	-0.229644E-02	0.868599E 01
4	0.127470E 00	-0.311202E-03	-0.237666E-02	0.501816E 02
5	-0.876964E-01	0.210697E-03	0.215526E-03	0.653044E 02
6	0.450491E-01	-0.936767E-04	0.596316E-02	0.875115E 02
7	0.218488E-01	-0.276982E-04	0.102919E-01	0.126842E 03
8	0.886852E-01	-0.207814E-03	0.197619E-02	0.770482E 02
9	0.886291E-01	-0.179205E-03	0.138569E-01	0.891967E 02
10	0.341444E-01	-0.864874E-04	-0.194187E-02	0.112854E 03

SHARPE DIAGONAL MODEL PROBLEM

THE NUMBER OF SHARES 10

THE RATE OF CHANGE OF MARGINAL UTILITY= 1.0

THE NUMBER OF PERIODS 36

THE DATA USED FOR THE YEAR 1968 TO 70

SHARE(3)=0%

SHARE(4)=0%

SHARE(5)=0%

SHARE(8)=0%

SHARE(10)=0%

SHARE(1)=0%

SHARE(6)=0%

SHARE(9)= 43.36%

SHARE(2)= 37.94%

SHARE(7)= 18.70%

THE EXPECTED RATE OF RETURN= 1.27%

THE UTILITY= 0.0078

KEY TO SHARE NAMES:-

SHARE(1)= 'GLAXO GROUP'

SHARE(2)= 'HOOVER LTD.'

SHARE(3)= 'BRITISH MOTOR'

SHARE(4)= 'DUNLAP CO.'

SHARE(5)= 'BRITISH PETROLEUM'

SHARE(6)= 'DISTILLERS'

SHARE(7)= 'GUINNESS(ARTHER)SON & CO.LTD.'

SHARE(8)= 'IMPERIAL CHEMICAL INDUSTRIES LTD.'

SHARE(9)= 'MARKS & SPENCERS'

SHARE(10)= 'BRITISH INSULATED CABLES'

PROGRAMME NUMBER 3

```

MASTER CORRELATION
INTEGER T, S
DIMENSION P(310), X(310), XDASH(310), R(310), B(24)
1 READ(1, 2, END=12) (B(I), I=1, 24)
2 FORMAT(24A3)
  READ(1, 3) N, (P(T), T=1, 307)
3 FORMAT(I3, 10F0.0)
  WRITE(2, 4) (B(I), I=1, 24)
4 FORMAT(24A3)
  SUMA, SUMB, SUMC=0.
  XBAR=(P(N)-P(1))/(N-1)
  DO 5 T=2, N
5 XDASH(T)=P(T)-P(T-1)-XBAR
  DO 8 S=1, N-2
  DO 6 T=1, N-1
  IF(T+S.GT.N-1) GO TO 7
  SUMA=SUMA+XDASH(T)*XDASH(T+S)
6 SUMB=SUMB+XDASH(T)*XDASH(T)
7 R(S)=SUMA/SUMB
8 SUMC=SUMC+R(S)
  RBAR=SUMC/(N-2)
  WRITE(2, 9) (S, R(S), S=2, 305)
9 FORMAT(/4(1X, 2HR(, I3, 2H)=, F7.3))
  WRITE(2, 10) (S, R(S), S=306, 307)
10 FORMAT(2(1X, 2HR(, I3, 2H)=, F7.3))
  WRITE(2, 11) RBAR
11 FORMAT(1H0, 'AVERAGE VALUE OF CORRELATION=', E17.8/)
  GO TO 1
12 CONTINUE
  STOP
  END
  FINISH

```

PROGRAMME NUMBER 4

```

MASTER DISTRIBUTION
DIMENSION P(400), C(400), R(400)
READ(1, 1) N
1 FORMAT(I3)
  READ(1, 2) (P(I), I=1, N)
2 FORMAT(10F0.0)
  DO 3 J=1, N-1
  C(J)=P(J+1)/P(J)-1.
3 R(J)=ALOG(P(J+1))/ALOG(P(J))-1.
  WRITE(2, 4) (I, P(I), C(I), R(I), I=1, N)
4 FORMAT(1H0, 2X, 'I', 17X, 'P(I)', 18X, 'DELTA', 10X, 'DELTA LOGP' /
-1H , I3, 3E17.8/)
  STOP
  END
  FINISH

```

OUTPUT OF PROGRAMME 3

'BARCLAYS' SHARE 1959, CALCULATION BASED ON DAILY SHARE INDEX

R(2)=	0.031	R(3)=	-0.009	R(4)=	-0.023	R(5)=	-0.034
R(6)=	-0.044	R(7)=	-0.037	R(8)=	-0.017	R(9)=	-0.013
R(10)=	-0.006	R(11)=	-0.011	R(12)=	-0.015	R(13)=	-0.017
R(14)=	-0.012	R(15)=	-0.008	R(16)=	-0.009	R(17)=	-0.005
R(18)=	-0.005	R(19)=	-0.007	R(20)=	-0.010	R(21)=	-0.008
R(22)=	-0.009	R(23)=	-0.010	R(24)=	-0.016	R(25)=	-0.015
R(26)=	-0.014	R(27)=	-0.012	R(28)=	-0.011	R(29)=	-0.012
R(30)=	-0.014	R(31)=	-0.017	R(32)=	-0.020	R(33)=	-0.020
R(34)=	-0.019	R(35)=	-0.012	R(36)=	-0.009	R(37)=	-0.004
R(38)=	-0.007	R(39)=	-0.011	R(40)=	-0.011	R(41)=	-0.011
R(42)=	-0.011	R(43)=	-0.011	R(44)=	-0.010	R(45)=	-0.008
R(46)=	-0.008	R(47)=	-0.009	R(48)=	-0.009	R(49)=	-0.006
R(50)=	-0.005	R(51)=	-0.005	R(52)=	-0.003	R(53)=	-0.002
R(54)=	-0.002	R(55)=	-0.001	R(56)=	-0.001	R(57)=	-0.003
R(58)=	-0.003	R(59)=	-0.005	R(60)=	-0.006	R(61)=	-0.006
R(62)=	-0.003	R(63)=	-0.000	R(64)=	-0.002	R(65)=	-0.002
R(66)=	-0.002	R(67)=	-0.004	R(68)=	-0.003	R(69)=	-0.003
R(70)=	-0.001	R(71)=	0.000	R(72)=	0.000	R(73)=	-0.000
R(74)=	-0.002	R(75)=	-0.002	R(76)=	-0.002	R(77)=	-0.002
R(78)=	-0.001	R(79)=	-0.001	R(80)=	-0.001	R(81)=	-0.002
R(82)=	-0.003	R(83)=	-0.003	R(84)=	-0.003	R(85)=	-0.002
R(86)=	-0.002	R(87)=	-0.003	R(88)=	-0.003	R(89)=	-0.003
R(90)=	-0.004	R(91)=	-0.003	R(92)=	-0.004	R(93)=	-0.004
R(94)=	-0.005	R(95)=	-0.006	R(96)=	-0.007	R(97)=	-0.006
R(98)=	-0.006	R(99)=	-0.004	R(100)=	-0.003	R(101)=	-0.003
R(102)=	-0.004	R(103)=	-0.003	R(104)=	-0.004	R(105)=	-0.003
R(106)=	-0.003	R(107)=	-0.003	R(108)=	-0.003	R(109)=	-0.003
R(110)=	-0.002	R(111)=	-0.002	R(112)=	-0.002	R(113)=	-0.002
R(114)=	-0.002	R(115)=	-0.002	R(116)=	-0.002	R(117)=	-0.003
R(118)=	-0.003	R(119)=	-0.002	R(120)=	-0.001	R(121)=	-0.001
R(122)=	-0.001	R(123)=	-0.001	R(124)=	-0.002	R(125)=	-0.002
R(126)=	-0.002	R(127)=	-0.002	R(128)=	-0.002	R(129)=	-0.002
R(130)=	-0.003	R(131)=	-0.003	R(132)=	-0.004	R(133)=	-0.003
R(134)=	-0.004	R(135)=	-0.005	R(136)=	-0.004	R(137)=	-0.005
R(138)=	-0.005	R(139)=	-0.006	R(140)=	-0.005	R(141)=	-0.005
R(142)=	-0.005	R(143)=	-0.006	R(144)=	-0.006	R(145)=	-0.005
R(146)=	-0.005	R(147)=	-0.005	R(148)=	-0.005	R(149)=	-0.004
R(150)=	-0.004	R(151)=	-0.004	R(152)=	-0.004	R(153)=	-0.004
R(154)=	-0.003	R(155)=	-0.003	R(156)=	-0.003	R(157)=	-0.003
R(158)=	-0.003	R(159)=	-0.003	R(160)=	-0.003	R(161)=	-0.003
R(162)=	-0.003	R(163)=	-0.002	R(164)=	-0.001	R(165)=	-0.002
R(166)=	-0.002	R(167)=	-0.003	R(168)=	-0.003	R(169)=	-0.003
R(170)=	-0.004	R(171)=	-0.004	R(172)=	-0.004	R(173)=	-0.004
R(174)=	-0.003	R(175)=	-0.003	R(176)=	-0.004	R(177)=	-0.003
R(178)=	-0.004	R(179)=	-0.005	R(180)=	-0.005	R(181)=	-0.004
R(182)=	-0.005	R(183)=	-0.005	R(184)=	-0.005	R(185)=	-0.005
R(186)=	-0.005	R(187)=	-0.005	R(188)=	-0.005	R(189)=	-0.004
R(190)=	-0.004	R(191)=	-0.004	R(192)=	-0.005	R(193)=	-0.005
R(194)=	-0.005	R(195)=	-0.006	R(196)=	-0.005	R(197)=	-0.005
R(198)=	-0.005	R(199)=	-0.004	R(200)=	-0.004	R(201)=	-0.004
R(202)=	-0.004	R(203)=	-0.004	R(204)=	-0.004	R(205)=	-0.004
R(206)=	-0.004	R(207)=	-0.005	R(208)=	-0.005	R(209)=	-0.005
R(210)=	-0.004	R(211)=	-0.004	R(212)=	-0.004	R(213)=	-0.005
R(214)=	-0.005	R(215)=	-0.005	R(216)=	-0.005	R(217)=	-0.005
R(218)=	-0.005	R(219)=	-0.005	R(220)=	-0.005	R(221)=	-0.005
R(222)=	-0.005	R(223)=	-0.005	R(224)=	-0.004	R(225)=	-0.004
R(226)=	-0.003	R(227)=	-0.004	R(228)=	-0.004	R(229)=	-0.004
R(230)=	-0.004	R(231)=	-0.004	R(232)=	-0.005	R(233)=	-0.005
R(234)=	-0.005	R(235)=	-0.005	R(236)=	-0.004	R(237)=	-0.005
R(238)=	-0.005	R(239)=	-0.005	R(240)=	-0.005	R(241)=	-0.005
R(242)=	-0.005	R(243)=	-0.005	R(244)=	-0.005	R(245)=	-0.005
R(246)=	-0.005	R(247)=	-0.005	R(248)=	-0.005	R(249)=	-0.005
R(250)=	-0.005	R(251)=	-0.005	R(252)=	-0.005	R(253)=	-0.005
R(254)=	-0.005	R(255)=	-0.005	R(256)=	-0.005	R(257)=	-0.005
R(258)=	-0.005	R(259)=	-0.005	R(260)=	-0.005	R(261)=	-0.005
R(262)=	-0.005	R(263)=	-0.005	R(264)=	-0.004	R(265)=	-0.005
R(266)=	-0.004	R(267)=	-0.004	R(268)=	-0.004	R(269)=	-0.004
R(270)=	-0.004	R(271)=	-0.004	R(272)=	-0.004	R(273)=	-0.004
R(274)=	-0.004	R(275)=	-0.005	R(276)=	-0.004	R(277)=	-0.005
R(278)=	-0.005	R(279)=	-0.005	R(280)=	-0.005	R(281)=	-0.005
R(282)=	-0.005	R(283)=	-0.005	R(284)=	-0.005	R(285)=	-0.005
R(286)=	-0.005	R(287)=	-0.005	R(288)=	-0.005	R(289)=	-0.005
R(290)=	-0.005	R(291)=	-0.005	R(292)=	-0.005	R(293)=	-0.005
R(294)=	-0.005	R(295)=	-0.005	R(296)=	-0.005	R(297)=	-0.005
R(298)=	-0.005	R(299)=	-0.005	R(300)=	-0.006	R(301)=	-0.005
R(302)=	-0.005	R(303)=	-0.005	R(304)=	-0.005	R(305)=	-0.005
R(306)=	0.000	R(307)=	0.000				

AVERAGE VALUE OF CORRELATION= -0.50477122E-02

PROGRAMME NUMBER 5

```

MASTER FILTER METHOD
REAL INDEX
INTEGER DAY,UP,DOWN,SAME
DIMENSION INDEX(1000),PROF(10,10),ITRAN(10,10),A(10),B(24)
DATA A(1)/3H /,A(2)/3H /,A(3)/3H /,A(4)/1H/,A(5)/1H/,
1A(6)/3HS /,A(7)/1H/,A(8)/1HS/,A(9)/3H /,A(10)/3H /
1 READ(1,2,END=20)(B(I),I=1,24)
2 FORMAT(24A3)
   SAME,UP,DOWN=0
   READ(1,3)N
3 FORMAT(I3)
   READ(1,4)(INDEX(J),J=1,N)
4 FORMAT(10F0.0)
   DO 9 I=1,10
   FALL=I/100.0
   DO 9 J=1,10
   RISE=J/100.0
   ITRANT=1.
   VEST=100.0
   DAY=1.
   BPRICE=INDEX(1)
5 DAY=DAY+1.
   IF(DAY.GT.(N-1))GO TO 7
   IF(INDEX(DAY).LT.(1.0+RISE)*BPRICE)GO TO 5
   ITRANT=ITRANT+1
   SPRICE=INDEX(DAY)
   VEST=VEST*SPRICE/BPRICE
6 DAY=DAY+1.
   IF(DAY.GT.(N-1))GO TO 8
   IF(INDEX(DAY).GT.(1.0-FALL)*SPRICE)GO TO 6
   BPRICE=INDEX(DAY)
   GO TO 5
7 VEST=VEST*INDEX(N)/BPRICE
   ITRANT=ITRANT+1
8 PROF(I,J)=VEST-100.0
   ITRAN(I,J)=ITRANT
9 CONTINUE
   WRITE(2,10)(B(I),I=1,24)
10 FORMAT(1H0,24A3)
   DO 14 J=1,N-1
   IF(INDEX(J+1)-INDEX(J))11,12,13
11 DOWN=DOWN+1
   GO TO 14
12 SAME=SAME+1
   GO TO 14
13 UP=UP+1
14 CONTINUE
   WRITE(2,15)UP,DOWN,SAME
15 FORMAT(1H0,12X,'SHARE STATISTICS'/1H0,12X,'RISES=',I3/1H0,
-12X,'FALLS=',I3/1H0,12X,'NO CHANGE=',I3)
   WRITE(2,16)
16 FORMAT(1H0,37X,'PROFIT ACHIEVED'/1H0,44X,'&'/1H0,38X,
-'TRANSACTIONS',///1H0,41X,'FALLSZ')
   WRITE(2,17)(I,I=1,10)
17 FORMAT(1H0,7X,10(I2,6X))
   DO 18 J=1,10
18 WRITE(2,19)(PROF(I,J),I=1,10),A(J),J,(ITRAN(I,J),I=1,10)
19 FORMAT(1H0,3X,10F8.2/1H ,A3,I2/1H ,7X,10(I2,6X))
   GO TO 5
20 CONTINUE
   STOP
   END
   FINISH

```

OUTPUT OF PROGRAMME 5

'DISTILLERS'SHARE 1968,CALCULATION BASED ON DAILY SHARE INDEX

SHARE STATISTICS

RISES=102

FALLS=111

NO CHANGE= 91

PROFIT ACHIEVED

8

TRANSACTIONS

FALLSX

	1	2	3	4	5	6	7	8	9	10
1	7.71	10.90	8.32	7.71	2.92	2.32	4.21	4.21	2.36	2.39
	6	7	5	4	3	3	3	3	3	3
2	21.28	15.92	10.14	9.51	9.51	4.64	5.30	5.37	5.37	6.66
	7	6	5	4	4	3	3	3	3	3
3	21.09	8.04	8.04	8.04	10.00	10.00	10.00	10.00	4.49	4.49
	6	3	3	3	3	3	3	3	2	2
R 4	42.30	14.66	9.83	9.83	10.00	10.00	10.00	10.00	4.49	4.49
	10	4	3	3	3	3	3	3	2	2
I 5	37.06	38.58	19.77	12.61	6.74	6.74	6.74	6.74	6.74	6.74
	8	6	4	3	2	2	2	2	2	2
S X 6	32.42	33.55	15.42	14.34	6.74	6.74	6.74	6.74	6.74	6.74
	7	5	3	3	2	2	2	2	2	2
E 7	34.10	37.65	17.22	17.22	17.88	16.79	8.40	8.40	8.40	8.40
	6	6	3	3	3	3	2	2	2	2
S 8	39.13	39.15	17.22	17.22	17.88	19.19	8.40	8.40	8.40	8.40
	6	6	3	3	3	3	2	2	2	2
9	43.10	43.33	45.28	47.97	22.05	22.05	20.96	21.66	10.65	10.65
	6	6	6	6	3	3	3	3	2	2
10	35.21	35.21	47.05	49.19	22.05	22.05	22.75	22.29	10.65	10.65
	5	5	6	6	3	3	3	3	2	2

PROGRAMME NUMBER 6

```

MASTER RATE I
DIMENSION X(51,12),XSUM(51),XBAR(51),XSUMSQ(51,51),
-COVAR(51,51),RATER(51,12),SIGMA(51)
M=12
N=51
WRITE (2,1)
1 FORMAT(1H1,15X,'RATE OF RETURN OF SECURITIES AND FTI'//1H0,17X,
-'DATA FOR 1968,69 AND 70 COMBINED')
DO 2 J=1,M
2 READ(1,3)(X(I,J),I=1,N)
3 FORMAT(51(8F0.0,1X),F6.2)
DO 6 J=2,M
DO 4 I=1,N
4 RATER(I,J-1) = (X(I,J)-X(I,J-1))/X(I,J-1)
WRITE (2,5) (RATER(I,J-1), I=1,N )
5 FORMAT(1H0/(1H0,8F8.4/))
6 CONTINUE
WRITE(2,7)
7 FORMAT(1H1,21X,'DATA FOR 1968,69 AND 70 COMBINED')
DO 8 J=1,M
8 WRITE (2,9) (X(I,J),I=1,N)
9 FORMAT(1H0,8F8.4/(1H ,8F8.4/))
WRITE (2,7)
DO 11 I=1,N
DO 10 J=1,M-1
X(I,J) = RATER(I,J)
10 XSUM(I)= XSUM(I) + X(I,J)
XBAR(I)= XSUM(I)/(M-1)
11 WRITE(2,12)I,XBAR(I)
12 FORMAT (1H0,10X,33HMEAN RATE OF RETURN OF SECURITY (,12,4H) = ,
-F7.4)
WRITE(2,15)
DO 16 K=1,N
DO 14 I =K,N
DO 13 J=1,M-1
13 XSUMSQ(I,K) = XSUMSQ(I,K) + X(K,J)* X(I,J)
COVAR(I,K) = XSUMSQ(I,K)/(M-2) - XBAR(K)*XBAR(I)*(M-1)/(M-2)
14 COVAR(K,I) = COVAR(I,K)
15 FORMAT(1H0,'COVARIANCE MATRIX C(I,J) FOR 1968,69, AND 70 COMBINED'
-/1H0,16X,'SECURITIES I,J=1,51'//1H3,20X,'F.T.I. I,J=51')
16 WRITE(2,17)(COVAR(I,K),I=1,N)
17 FORMAT(1H0/(1H0,8F8.4/))
WRITE(2,7)
DO 18 I=1,50
SIGMA(I) = (((N-1)*COVAR(I,I) -(((N-1)*COVAR(N,I))**2)/((N-1)*
-COVAR(N,N) ) ))/(N-2)
18 WRITE(2,19)I,SIGMA(I)
19 FORMAT(1H0,10X,35HVARIANCE ABOUT THE REGRESSION LINE ,12,5X,
-F8.4 )
STOP
END
FINISH

```

OUTPUT OF PROGRAMME 6

DATA FOR 1968,69 AND 70 COMBINED

MEAN RATE OF RETURN OF SECURITY (1) =	-0.0035
MEAN RATE OF RETURN OF SECURITY (2) =	0.0324
MEAN RATE OF RETURN OF SECURITY (3) =	0.0270
MEAN RATE OF RETURN OF SECURITY (4) =	-0.0310
MEAN RATE OF RETURN OF SECURITY (5) =	0.0432
MEAN RATE OF RETURN OF SECURITY (6) =	-0.0246
MEAN RATE OF RETURN OF SECURITY (7) =	0.0109
MEAN RATE OF RETURN OF SECURITY (8) =	-0.0043
MEAN RATE OF RETURN OF SECURITY (9) =	0.0562
MEAN RATE OF RETURN OF SECURITY (10) =	0.0136
MEAN RATE OF RETURN OF SECURITY (11) =	0.0214
MEAN RATE OF RETURN OF SECURITY (12) =	-0.0673
MEAN RATE OF RETURN OF SECURITY (13) =	0.0073
MEAN RATE OF RETURN OF SECURITY (14) =	-0.0207
MEAN RATE OF RETURN OF SECURITY (15) =	-0.0067
MEAN RATE OF RETURN OF SECURITY (16) =	-0.0118
MEAN RATE OF RETURN OF SECURITY (17) =	0.0697
MEAN RATE OF RETURN OF SECURITY (18) =	0.0540
MEAN RATE OF RETURN OF SECURITY (19) =	-0.0073
MEAN RATE OF RETURN OF SECURITY (20) =	0.0056
MEAN RATE OF RETURN OF SECURITY (21) =	0.0285
MEAN RATE OF RETURN OF SECURITY (22) =	0.0522
MEAN RATE OF RETURN OF SECURITY (23) =	0.0822
MEAN RATE OF RETURN OF SECURITY (24) =	0.0401
MEAN RATE OF RETURN OF SECURITY (25) =	-0.0048
MEAN RATE OF RETURN OF SECURITY (26) =	-0.0133
MEAN RATE OF RETURN OF SECURITY (27) =	0.0033
MEAN RATE OF RETURN OF SECURITY (28) =	-0.0109
MEAN RATE OF RETURN OF SECURITY (29) =	0.0308
MEAN RATE OF RETURN OF SECURITY (30) =	-0.0438
MEAN RATE OF RETURN OF SECURITY (31) =	0.0039
MEAN RATE OF RETURN OF SECURITY (32) =	0.0092
MEAN RATE OF RETURN OF SECURITY (33) =	0.0325
MEAN RATE OF RETURN OF SECURITY (34) =	-0.0015
MEAN RATE OF RETURN OF SECURITY (35) =	0.0154
MEAN RATE OF RETURN OF SECURITY (36) =	-0.0339
MEAN RATE OF RETURN OF SECURITY (37) =	0.0287
MEAN RATE OF RETURN OF SECURITY (38) =	0.0035
MEAN RATE OF RETURN OF SECURITY (39) =	-0.0261
MEAN RATE OF RETURN OF SECURITY (40) =	-0.0207
MEAN RATE OF RETURN OF SECURITY (41) =	0.0213
MEAN RATE OF RETURN OF SECURITY (42) =	-0.0051
MEAN RATE OF RETURN OF SECURITY (43) =	0.0189
MEAN RATE OF RETURN OF SECURITY (44) =	0.0309
MEAN RATE OF RETURN OF SECURITY (45) =	-0.0146
MEAN RATE OF RETURN OF SECURITY (46) =	0.0467
MEAN RATE OF RETURN OF SECURITY (47) =	-0.0122
MEAN RATE OF RETURN OF SECURITY (48) =	-0.0262
MEAN RATE OF RETURN OF SECURITY (49) =	-0.0321
MEAN RATE OF RETURN OF SECURITY (50) =	-0.0460
MEAN RATE OF RETURN OF SECURITY (51) =	-0.0159

OUTPUT OF PROGRAMME 6 (contd)

DATA FOR 1968,69 AND 70 COMBINED

VARIANCE ABOUT THE REGRESSION LINE	1	0.0051
VARIANCE ABOUT THE REGRESSION LINE	2	0.0092
VARIANCE ABOUT THE REGRESSION LINE	3	0.0073
VARIANCE ABOUT THE REGRESSION LINE	4	0.0198
VARIANCE ABOUT THE REGRESSION LINE	5	0.0284
VARIANCE ABOUT THE REGRESSION LINE	6	0.0203
VARIANCE ABOUT THE REGRESSION LINE	7	0.0166
VARIANCE ABOUT THE REGRESSION LINE	8	0.0156
VARIANCE ABOUT THE REGRESSION LINE	9	0.0145
VARIANCE ABOUT THE REGRESSION LINE	10	0.0073
VARIANCE ABOUT THE REGRESSION LINE	11	0.0074
VARIANCE ABOUT THE REGRESSION LINE	12	0.0167
VARIANCE ABOUT THE REGRESSION LINE	13	0.0148
VARIANCE ABOUT THE REGRESSION LINE	14	0.0048
VARIANCE ABOUT THE REGRESSION LINE	15	0.0449
VARIANCE ABOUT THE REGRESSION LINE	16	0.0223
VARIANCE ABOUT THE REGRESSION LINE	17	0.0111
VARIANCE ABOUT THE REGRESSION LINE	18	0.0144
VARIANCE ABOUT THE REGRESSION LINE	19	0.0167
VARIANCE ABOUT THE REGRESSION LINE	20	0.0166
VARIANCE ABOUT THE REGRESSION LINE	21	0.0036
VARIANCE ABOUT THE REGRESSION LINE	22	0.0026
VARIANCE ABOUT THE REGRESSION LINE	23	0.0030
VARIANCE ABOUT THE REGRESSION LINE	24	0.0237
VARIANCE ABOUT THE REGRESSION LINE	25	0.0050
VARIANCE ABOUT THE REGRESSION LINE	26	0.0088
VARIANCE ABOUT THE REGRESSION LINE	27	0.0233
VARIANCE ABOUT THE REGRESSION LINE	28	0.0672
VARIANCE ABOUT THE REGRESSION LINE	29	0.0461
VARIANCE ABOUT THE REGRESSION LINE	30	0.0332
VARIANCE ABOUT THE REGRESSION LINE	31	0.0158
VARIANCE ABOUT THE REGRESSION LINE	32	0.0096
VARIANCE ABOUT THE REGRESSION LINE	33	0.0021
VARIANCE ABOUT THE REGRESSION LINE	34	0.0181
VARIANCE ABOUT THE REGRESSION LINE	35	0.0253
VARIANCE ABOUT THE REGRESSION LINE	36	0.0385
VARIANCE ABOUT THE REGRESSION LINE	37	0.0381
VARIANCE ABOUT THE REGRESSION LINE	38	0.0166
VARIANCE ABOUT THE REGRESSION LINE	39	0.0148
VARIANCE ABOUT THE REGRESSION LINE	40	0.0048
VARIANCE ABOUT THE REGRESSION LINE	41	0.0122
VARIANCE ABOUT THE REGRESSION LINE	42	0.0119
VARIANCE ABOUT THE REGRESSION LINE	43	0.0052
VARIANCE ABOUT THE REGRESSION LINE	44	0.0091
VARIANCE ABOUT THE REGRESSION LINE	45	0.0096
VARIANCE ABOUT THE REGRESSION LINE	46	0.0106
VARIANCE ABOUT THE REGRESSION LINE	47	0.0036
VARIANCE ABOUT THE REGRESSION LINE	48	0.0050
VARIANCE ABOUT THE REGRESSION LINE	49	0.0318
VARIANCE ABOUT THE REGRESSION LINE	50	0.0193

PROGRAMME NUMBER 7

```

NO TRACE
MATHC MONTECARLO
REAL MAX,MIN
DIMENSION A(50),B(50),SIGMAFT(50),LX(50),LY(50),IGX(50),IGY(50),
-ANVESTX(50),ANVESTY(50),AICX(50),AICY(50),ALX(50),ALY(50),
-SIGMAX(50),SIGMAY(50),YX(50),YY(50)
READ(1,1)(A(I),I=1,50),(B(J),J=1,50),(SIGMAFT(K),K=1,50)
1 FORMAT(10F0.0)
READ(1,1)SIGMAF
TPI=8.*ATAN(1.)
FTI70=331.1
FTI71=423.18
XBAR=-.01589
COV=.0065
STREAM=0.7
N=50
DO 10 NI=1,1000
R=FPCRV(STREAM)
U=SQRT(-2.*ALOG(R))
T=TPI*(R)
X1=U*SIN(T)
Y1=U*COS(T)
FTIPX=FTI71+SIGMAF*(X1)
FTIPY=FTI71+SIGMAF*(Y1)
RFTIPX=(FTIPX-FTI70)/FTI70
RFTIPY=(FTIPY-FTI70)/FTI70
MAXX=-1.0
MAXY=-1.0
MINX=1.0
MINY=1.0
DO 9 I=1,50
YX(I)=A(I)+B(I)*RFTIPX
YY(I)=A(I)+B(I)*RFTIPY
R=FPCRV(STREAM)
U=SQRT(-2.*ALOG(R))
T=TPI*(R)
X1=U*SIN(T)
Y1=U*COS(T)
SIGMAX(I)=SQRT(SIGMAFT(I)*(1+1.0/FLOAT(N))+((RFTIPX-XBAR)**2)/
-(N-1)*COV))
SIGMAY(I)=SQRT(SIGMAFT(I)*(1+1.0/FLOAT(N))+((RFTIPY-XBAR)**2)/
-(N-1)*COV))
X1=SIGMAX(I)*(X1)
Y1=SIGMAY(I)*(Y1)
YX(I)=YX(I)+X1
YY(I)=YY(I)+Y1
IF(YX(I)-MAXX)2,3,3
2 IF(YY(I)-MINY)4,4,5
3 MAXX=YX(I)
JX=I
GO TO 2
4 MINY=YY(I)
KX=I
5 IF(YY(I)-MAXY)6,7,7
6 IF(YY(I)-MINY)8,8,9
7 MAXY=YY(I)
JY=I
GO TO 4
8 MINY=YY(I)
KY=I
9 CONTINUE
IGX(JX)=IGX(JX)+1
LX(YY)=LX(YY)+1
IGY(JY)=IGY(JY)+1
LY(YY)=LY(YY)+1
10 CONTINUE
WRITE(2,11)
11 FORMAT(1H1,2(20X,13HBEST  VORST))
DO 13 I=1,50
WRITE(2,12)I,IGX(I),LX(I),IGY(I),LY(I)
12 FORMAT(1H0,2HSECURITY( ,1D,1H),5X,210,18X,210)
AICX(I)=IGX(I)/1000.0
AICY(I)=IGY(I)/1000.0
ALX(I)=LX(I)/1000.0
ALY(I)=LY(I)/1000.0
ANVESTX(I)=AICX(I)*(1-ALX(I))
ANVESTY(I)=AICY(I)*(1-ALY(I))
TOTX=TOTX+ANVESTX(I)
13 TOTY=TOTY+ANVESTY(I)
WRITE(2,14)
14 FORMAT(1H1,10X,(20HTHE PORTFOLIO SELECTED USING MONTE CARLO SIMULA
TION TECHNIQUE)
DO 15 I=1,50
ALVESTX(I)=ANVESTX(I)/TOTX
ANVESTY(I)=ANVESTY(I)/TOTY
15 WRITE(2,15)I,ANVESTX(I),ANVESTY(I)
16 FORMAT(1H0,30HPORTFOLION INVESTED IN SECURITY( ,1D,2H)=,F6.4,15X,
-F6.4)
STOP
END
FINISH

```

OUTPUT OF PROGRAMME 7

	BEST	WORST	BEST	WORST
SECURITY(1)	0	1	0	0
SECURITY(2)	0	0	0	0
SECURITY(3)	0	0	0	0
SECURITY(4)	0	0	0	0
SECURITY(5)	18	0	36	0
SECURITY(6)	0	0	0	0
SECURITY(7)	0	0	2	0
SECURITY(8)	0	1	0	0
SECURITY(9)	0	0	11	0
SECURITY(10)	0	0	0	0
SECURITY(11)	0	0	0	0
SECURITY(12)	0	0	1	0
SECURITY(13)	0	0	1	0
SECURITY(14)	0	0	0	0
SECURITY(15)	0	3	9	0
SECURITY(16)	0	0	8	0
SECURITY(17)	0	0	2	0
SECURITY(18)	0	0	0	0
SECURITY(19)	0	0	0	0
SECURITY(20)	0	8	0	1
SECURITY(21)	0	0	0	0
SECURITY(22)	0	0	0	0
SECURITY(23)	0	0	0	0
SECURITY(24)	0	0	9	0
SECURITY(25)	0	7	0	5
SECURITY(26)	0	0	0	3
SECURITY(27)	0	0	5	2
SECURITY(28)	0	2	17	27
SECURITY(29)	47	0	53	0
SECURITY(30)	103	5	101	0
SECURITY(31)	0	0	2	0
SECURITY(32)	0	49	0	30
SECURITY(33)	0	0	0	0
SECURITY(34)	0	89	0	56
SECURITY(35)	0	1	5	19
SECURITY(36)	0	203	6	133
SECURITY(37)	0	234	13	271
SECURITY(38)	0	0	7	0
SECURITY(39)	0	0	14	0
SECURITY(40)	0	0	0	0
SECURITY(41)	0	0	2	0
SECURITY(42)	0	0	4	0
SECURITY(43)	0	0	0	0
SECURITY(44)	0	0	0	23
SECURITY(45)	0	0	0	0
SECURITY(46)	0	0	0	66
SECURITY(47)	0	0	0	0
SECURITY(48)	0	0	0	0
SECURITY(49)	232	0	302	0
SECURITY(50)	510	526	328	412

OUTPUT OF PROGRAMME 7 (contd)

	STREAM	X	Y
PROPORTION INVESTED IN SECURITY(1)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(2)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(3)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(4)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(5)	0.0216	0.0431	
PROPORTION INVESTED IN SECURITY(6)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(7)	0.0000	0.0024	
PROPORTION INVESTED IN SECURITY(8)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(9)	0.0000	0.0132	
PROPORTION INVESTED IN SECURITY(10)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(11)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(12)	0.0000	0.0012	
PROPORTION INVESTED IN SECURITY(13)	0.0000	0.0012	
PROPORTION INVESTED IN SECURITY(14)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(15)	0.0000	0.0108	
PROPORTION INVESTED IN SECURITY(16)	0.0000	0.0096	
PROPORTION INVESTED IN SECURITY(17)	0.0000	0.0024	
PROPORTION INVESTED IN SECURITY(18)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(19)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(20)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(21)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(22)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(23)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(24)	0.0000	0.0108	
PROPORTION INVESTED IN SECURITY(25)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(26)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(27)	0.0000	0.0660	
PROPORTION INVESTED IN SECURITY(28)	0.0000	0.0128	
PROPORTION INVESTED IN SECURITY(29)	0.0563	0.0634	
PROPORTION INVESTED IN SECURITY(30)	0.2312	0.1208	
PROPORTION INVESTED IN SECURITY(31)	0.0000	0.0024	
PROPORTION INVESTED IN SECURITY(32)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(33)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(34)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(35)	0.0000	0.0059	
PROPORTION INVESTED IN SECURITY(36)	0.0000	0.0062	
PROPORTION INVESTED IN SECURITY(37)	0.0000	0.0121	
PROPORTION INVESTED IN SECURITY(38)	0.0000	0.0084	
PROPORTION INVESTED IN SECURITY(39)	0.0000	0.0191	
PROPORTION INVESTED IN SECURITY(40)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(41)	0.0000	0.0024	
PROPORTION INVESTED IN SECURITY(42)	0.0000	0.0048	
PROPORTION INVESTED IN SECURITY(43)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(44)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(45)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(46)	1.0000	0.0000	
PROPORTION INVESTED IN SECURITY(47)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(48)	0.0000	0.0000	
PROPORTION INVESTED IN SECURITY(49)	0.2779	0.3413	
PROPORTION INVESTED IN SECURITY(50)	0.4130	0.2729	

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