Race Car Vehicle Dynamics & Design Applied to Formula Student

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1 Notation

Throughout this chapter the following notations are adopted:

Notation	*Unit*	*Definition*
l	m	Wheelbase
s	m	Distance from front axle centreline to CoG longitudinally
r	m	Distance from CoG to rear axle centreline longitudinally ($s+r=l$)
v	m	Vertical distance between CoG and Roll Axis
t	m	Track width
h_{SCoG}	m	Height of suspended mass CoG above ground
h_{UCoG}	m	Height of unsuspended mass CoG above ground
h_{rc}	m	Height of Roll Centre above ground
m_u	kg	Unsuspended mass
m_s	kg	Suspended mass
g	$9.\ 81ms^{-2}$	Gravity
a	g	Lateral acceleration in g units
W_{LatU}	N	Lateral weight transfer of unsuspended components
W_{latS}	N	Lateral weight transfer of suspended components $W_{LatS} = W_{LatGeom} + W_{LatElas}$
$W_{LatElas}$	N	Lateral weight transfer of suspended components (Elastic)
$W_{LatGeom}$	N	Lateral weight transfer of suspended components (Geometric)
F and R		F and R Prefixes Denote Front and Rear Respectively
S_{roll}	$N/degree$	Suspension roll stiffness (see Section 15.5)
γ	-	Grip factor

2 Index to FSAE Terms

This is a cross-referenced table of terms defined as essential by the SAE, see Section 4.2 for more details.

3 Synopsis

Part 1

This thesis is divided into two distinct parts. The first of these builds upon a list of terms commonly used in the discussion of vehicle dynamics. The list is supplied by the Society of Automobile Engineers (USA), and the Institute of Mechanical Engineers (UK) to teams participating in the Formula Student competition. These terms and the implications that their respective values have on the design and performance of a race car are considered to be essential. The objective of the first part of this thesis is to explain, discuss and evaluate the theoretical effects of the terms covered in the list.

The leading chapter begins with an introduction to the challenges associated with racing and the basic tools needed to understand and evaluate vehicle performance. The discussion then passes on to the tyre and tyre-road interface. The mechanisms by which the tyre generates friction are investigated and the subsequent consequence of load sensitivity is discussed. Analysis of the contact patch and an explanation of slip angles and generation of lateral and longitudinal forces then follow. The by-products and effects of slip and friction forces in the contact patch are further evaluated to explain tyre deformation, pneumatic trail and self aligning torque. Analysis of the tyre concludes with an explanation of tyre parameters, practical testing methods and results, as well the principles behind a detailed mathematical model of lateral force and self aligning torque curves.

A discussion on steady state balance is then entered with reference to slip angle, lateral force and instantaneous radius of turn. Theoretical states of understeer, neutral steer and oversteer are discussed along with consequences on driver input and ultimate limits of performance. Terms such as understeer gradient and the Ackermann steering angle are also related to cause and effect.

The next chapter covers terms related to basic vehicle geometry values of track and wheelbase. Firstly, defining them and then relating their effects on steady state balance, lateral weight transfer and longitudinal weight transfer.

Delving further into a vehicle's geometrical values, suspension kinematics are then covered. Roll centres and swing axle lengths are graphically defined and related to suspension performance in roll and bump motion. As a consequence, measures of roll and bump performance are then explained. Anti-squat and anti-dive parameters are also defined along with an evaluation of their value in a high performance suspension system, whilst considering the adverse effects that they might have.

Weight transfer and its effects are fundamental to vehicle performance. Consequently, the next section gives an in-depth explanation and breakdown of weight transfer as a result of lateral and longitudinal acceleration.

The next three chapters cover the definition of suspension rates and the respective necessary control motion of both suspended and unsuspended masses. Wheel rates are derived from motion ratios and spring rates. The suspension system is then represented as a simple mass spring system in order to explain oscillatory motion and the need for damping. The discussion then progresses with a definition of critical damping and a description of the ride versus handling compromise. Basic damping principles are covered along with the advantages and disadvantages of damper design for different applications. The notion of high and low speed damper motion is explained and related to system stimuli.

All of the previous investigations have been into individual systems that comprise a race car. Each system could be theoretically perfect but if incorrectly interfaced it will not function well. Hence, the chassis can be looked at as a system with a purpose of linking all other systems together in a manner that they are allowed to perform optimally. In this chapter important performance measures, chassis fabrication and chassis material are investigated and related to the performance increases they may give to other systems.

Part 2

The second part of this thesis looks at applying the theoretical information contained within the first part to the practical example of a Formula Student car. The process begins with some background on the design process and the numerical tools needed for analysis of the problem. A process of performance data collection is outlined along with a description of the hardware and analysis techniques needed to asses a current design. A mathematical simulation of a lap of the Formula Student circuit is developed.

Data from several test runs is then presented in varying graphical forms to best illustrate and quantify the performance of individual systems, a breakdown of the analysis technique is also covered with each data set. Closely associated with this analysis, the next chapter presents improvements and more advanced analysis techniques that could be developed or added to the data acquisition system to gather further vital performance information.

From the previous chapters conclusions are drawn about the performance capabilities of the current design based on a theoretical maximum. The track model is developed to include vehicle simulation and then used to quantify the effect to changes in performance characteristics. From these observations conclusions are then drawn on how the design can be improved to give maximum performance gains for limited design resources.

The next chapter uses both the theoretical principles presented in Part 1 and actual observations made in Part 2 to determine suitable values to be used as design parameters for future vehicle designs. All the concluded parameters are thoroughly discussed and related back to theoretical principles.

The final chapter in this thesis has been composed to be used as a tool for correctly setting up a new vehicle. It outlines a simple yet progressive testing program and result analysis technique for the setup engineer to use in order to tune the vehicle for that particular track.

4 Introduction

"The overall technical objective in racing is the achievement of a vehicle configuration, acceptable within the practical interpretation of the rules, which can traverse a given course in a minimum time (or at the highest average speed) when operated manually by a driver utilizing techniques within his/her capabilities. Suitable performance margins must be available for dealing with traffic, environmental factors such as wind and surface conditions, driver fatigue and emergencies." (Milliken and Milliken, 1995 p3).[1]

The winner will be the driver/car combination that can achieve the highest lateral and longitudinal accelerations. The exact combination of abilities that gives the optimum solution varies for different disciplines. A dragster will forfeit cornering capabilities for longitudinal acceleration, whilst an oval track car might opt for lateral acceleration over longitudinal acceleration. What remains true is that the competitor who has both better lateral and longitudinal acceleration capabilities will win.

4.1 Aims

The SAE[2] defines a list of essential terminology[3] that should be understood by students contributing to the design of a Formula SAE (Formula Student) car. The terms cover a broad basis to vehicle design and are mainly used to describe component and system characteristics. This thesis aims to define all of the terms presented in the list with respect to racing applications. It will highlight the importance of knowledge of the terms and the impact that their values have on a vehicle's performance characteristics. Where applicable, the design considerations required to optimise system performance to give the desired handling and vehicle performance characteristics will be discussed.

Outline the design process for a racing car highlighting the important considerations that must be made before detailed design can begin. Develop a set of performance characteristics suited to the formula student event.

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[1] Quote taken from Milliken's and Milliken's (1995) introduction, "The Problem Imposed by Racing."

[2] The Society of Automotive Engineers.

3 SAE's Vehicle Dynamics Terminology List – See Appendix A for full source.

Analyse data captured during track testing of the 2005 car. Apply the knowledge contained in the discussion of the terms to comparisons of theoretical and current car performance values. Draw conclusions from these comparisons as to the validity and capability of current designs. Suggest areas that will benefit most from theoretical analysis and design development, along with improved design specifications to reduce the gap between the ideal theoretical performance and the actual track performance.

4.2 Indexing and Order

The FSAE terminology list is included as additional contents to Part 2 of this report, allowing relevant information to be found easily. The sequence in which terms and discussions are ordered is not consistent with the terminology list, but with a more logical approach to vehicle dynamics and design. The first section covers theoretical discussion without reference to any particular racing discipline. The second section looks at the application of theories presented in the first section with respect to the design and analysis of a Formula Student car.

4.3 Measuring Performance

The capabilities of any car can be displayed on a g-g diagram (g circle), where lateral and longitudinal values are plotted against each other. The resulting ellipses can be compared to output from a data acquisition system logging the car's actual acceleration around a track. The best drivers will be able to operate at accelerations around the perimeter of the g circle for as much time as possible. The designer's main aim is to expand the dimensions of the ellipse on the g-circle graph, such that the driver has more potential acceleration at their disposal.

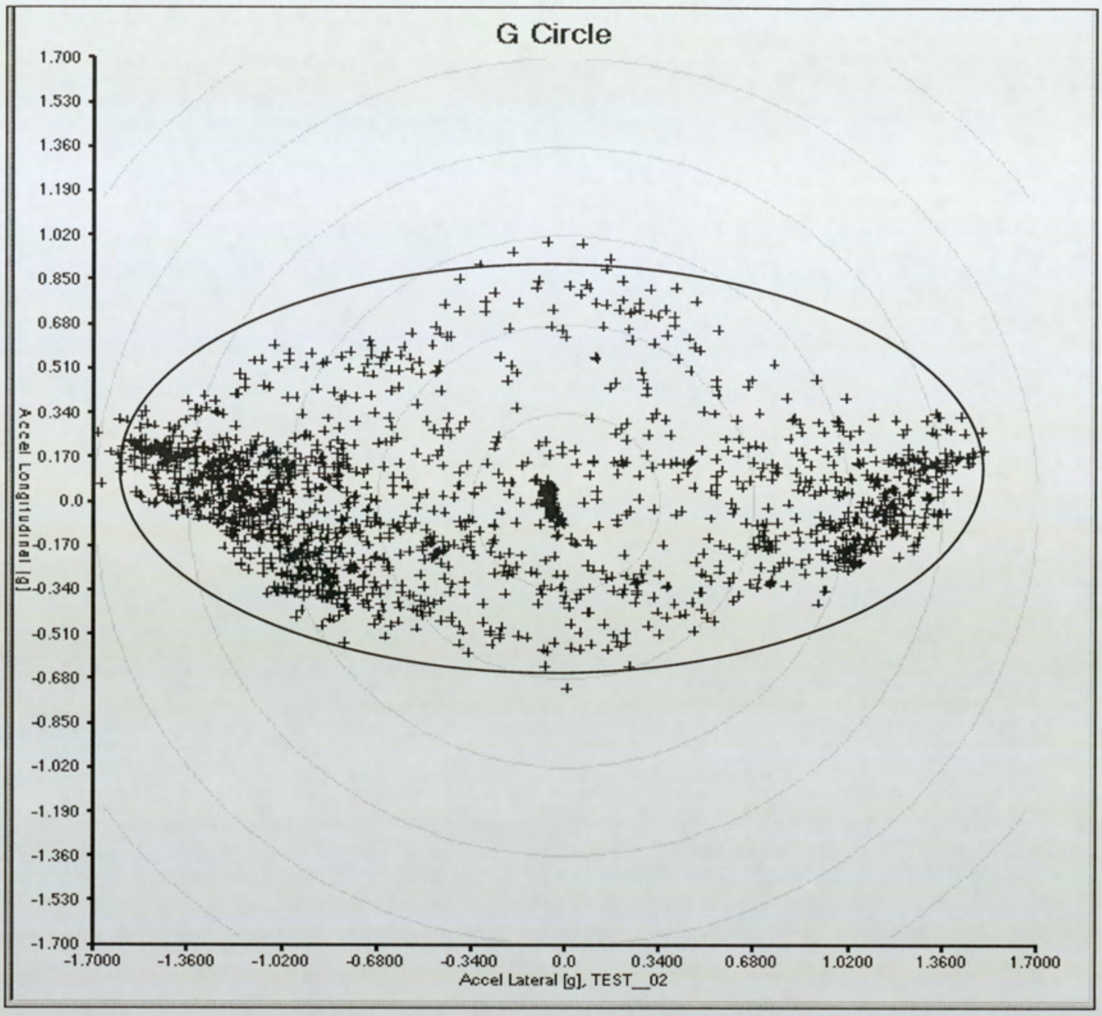

Figure 4-1 - G - Circle

(Created Using Race Technology Analysis Software [4])

G-circle analysis gives information about the limits of performance in a concise and easily interpretable manner. The population of the inner region of the circle can be used to determine the time spent not at maximum acceleration or, driver omitted, the response characteristics of the car to input. The less populated the inner circle of the circle the better.

[4] Race Technology Analysis Software (See Appendix B for information on Race Technology Ltd)

5 Axis and Motion Convention

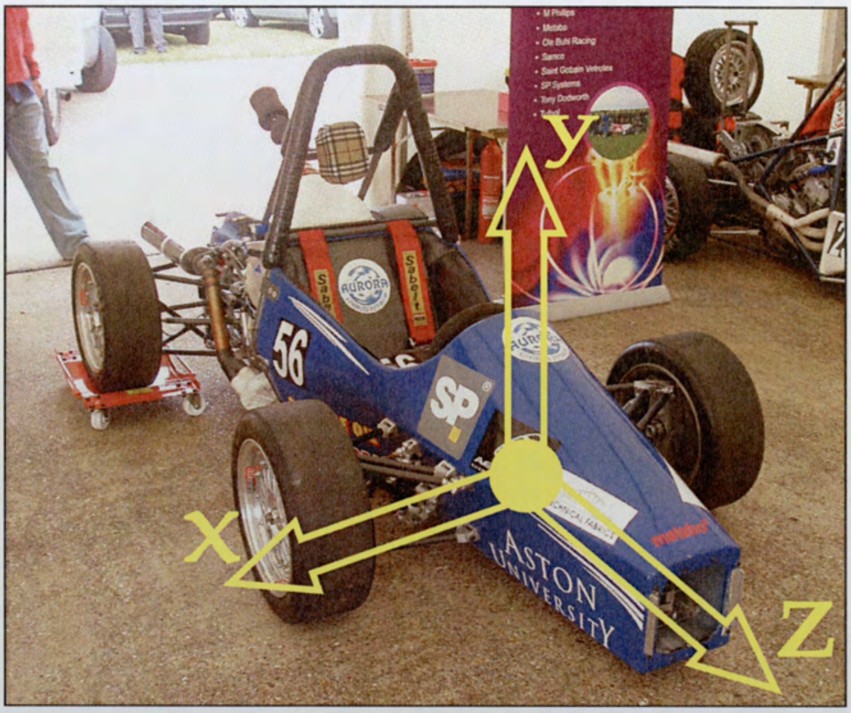

Figure 5-1 - Axis Convention

Datum – The datum point is on the floor directly below the centre of the front axle.

Axis[5]

- x – Horizontal from centreline toward sides of car.
- y – Vertical upwards from ground.
- z – Longitudinally from front axle.

Motion

- Pitch – Rotational motion around the x axis.
- Yaw – Rotational motion around the y axis.
- Roll – Rotational motion around the z axis.

[5] There are other recognised axis conventions used in the automotive industry (with y and z axis transposed), for the purpose of this thesis these axis have been adopted in order to complement SUSPROG suspension design software's suggested settings. See Appendix C for Information on SUSPROG.

- Heave – Vertical displacement of the suspended mass in the direction of the y axis, chassis remains parallel to the ground.

Part 1

6 Analysis of the Problem

Before any design can begin the problem must be analysed to derive a set of values that describe the expected performance characteristics of the final product. Inevitably at the initial stage some assumptions will have to be made in order to produce the design criteria. In time the individual criterion will become more descriptive and accurate. These criteria will include:

- Maximum speed
- Minimum speed
- Expected weight
- Expected engine power
- Maximum coefficient of friction of the tyre to surface
- Position of the CoG (Weight Distribution)
- Required suspension movement
- Braking requirements
- Corner radii
- Down force
- Aerodynamic drag

From these the following can be calculated:

- Expected accelerations
    - Lateral
    - Longitudinal
- Expected weight transfer (Estimation as final dimensions are yet to be decided)
- Maximum body roll at expected accelerations

6.1 Calculations

In order to generate some of the basic performance values for the vehicle the situation is greatly simplified, admittedly the values used and generated are not highly accurate to reality, but they provide the basis from which a refined model can be developed. In the following calculations the lateral and longitudinal accelerations are considered to be identical and defined by a static friction value for the road tyre interface. Hence, in the subsequent equations 'a' is not related to the co-ordinate system.

6.1.1 Acceleration

$$F_{friction} = ma \qquad \text{...(1)}$$

Where 'F' is the force generated at the road-tyre interface, 'm' is the mass of the car and 'a' is the acceleration.

$$F_{friction} = \mu(mg + F_{downforce}) \qquad \text{...(2)}$$

Where μ is the tyre's coefficient of friction[6] and $F_{downforce}$ is any aerodynamically produced force.

Equating (1) and (2) gives:

$$ma = mg\mu + F_{downforce}\mu$$

$$a = \frac{mg\mu}{m}$$

$$a = \mu g + \frac{F_{downforce}\mu}{m}$$

[6] See section 7.3 for further details

6.1.2 Weight Transfer – The Basics

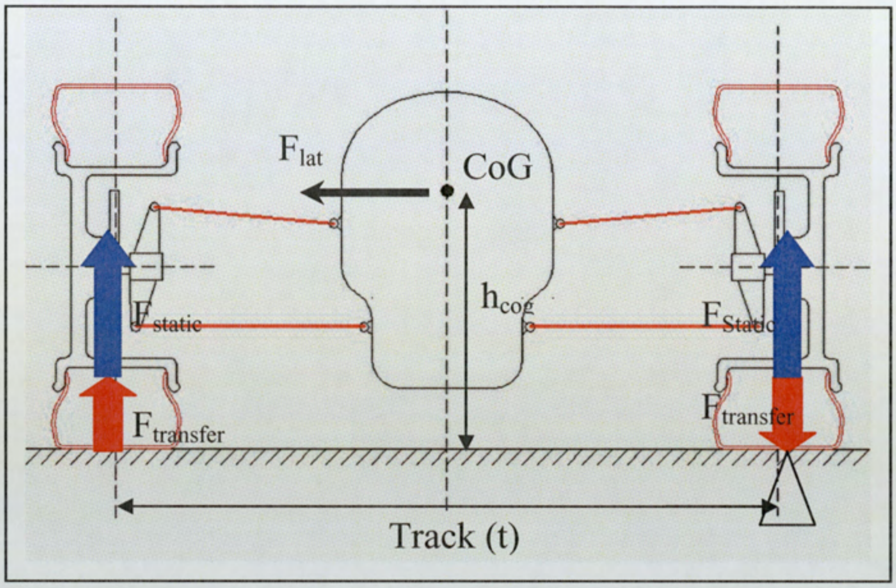

Figure 6-1 - Basic Weight Transfer Diagram

Using the acceleration previously described.

$$F_{Lat} = ma \qquad ..(1)$$

From acceleration above

$$F_{Lat} = m\mu g \qquad ..(2)$$

By taking moments about right hand wheel:

$$F_{transfer}t = F_{lat}h_{Cog}$$

$$F_{transfer} = \frac{F_{lat}h_{Cog}}{t}$$

The total weight[7] on each wheel is then given by:

$$F_{reactiuon} = F_{static} \pm F_{transfer}$$

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[7] This is a simplified model without the effects of suspension freedom. A more detailed breakdown is given in section 14.

7 The Tyre

The starting point for the design of any vehicle must be the point at which it interfaces with the environment in which it will move. All the forces that propel, stop and turn a vehicle are generated at the point where the tyre touches the ground. It is therefore important to understand what happens at this interface and to the tyre when under load.

The vehicle then needs to be designed to present the tyre to the ground at the best angle and with an optimum vertical reaction to maximise the forces the tyre generates. In the same way, an off-road vehicle designer will ensure that the chassis has enough ground clearance to make sure that the vehicle's weight is always on the tyres, a race car designer must propose suspension geometry that presents the tyre at the best attitude to the road surface.

7.1 Tyre Model

In order to help with visualisation of tyre deformation and lateral force mechanisms presented throughout this section, the manufacture of Milliken and Milliken's (1995)[8] tyre models is suggested. It consists of a thin rubber disk clamped between two stiff metal washers of smaller diameter. The diameter of the disks can be changed to represent different tyre cornering stiffness[9]. This model can be used to effectively represent a thin slice of a real tyre.

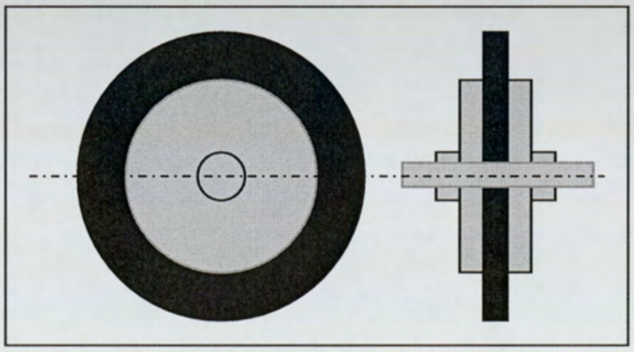

Figure 7-1 - Milliken and Milliken's Tyre Model

[8] Milliken and Milliken, 1995 p16

[9] Further descriptions and explanations of tyre cornering stiffness can be found in section 8.

7.2 Forces on the Tyre

At low speeds and neglecting aerodynamic force, there are four pure load states that the tyre is subjected to. At high speeds a vehicle will be also be affected by air resistance and may make use of aerofoil for added down-force or ground effect. These aero forces also cause added reaction at the road tyre interface, however, due to the complexity involved with calculating their magnitude they are not being considered in this study.

The states shown in Figure 7-2 are not mutually exclusive and at any one time there may be a combination of loads. For the purpose of steady state analysis each state is considered independently.

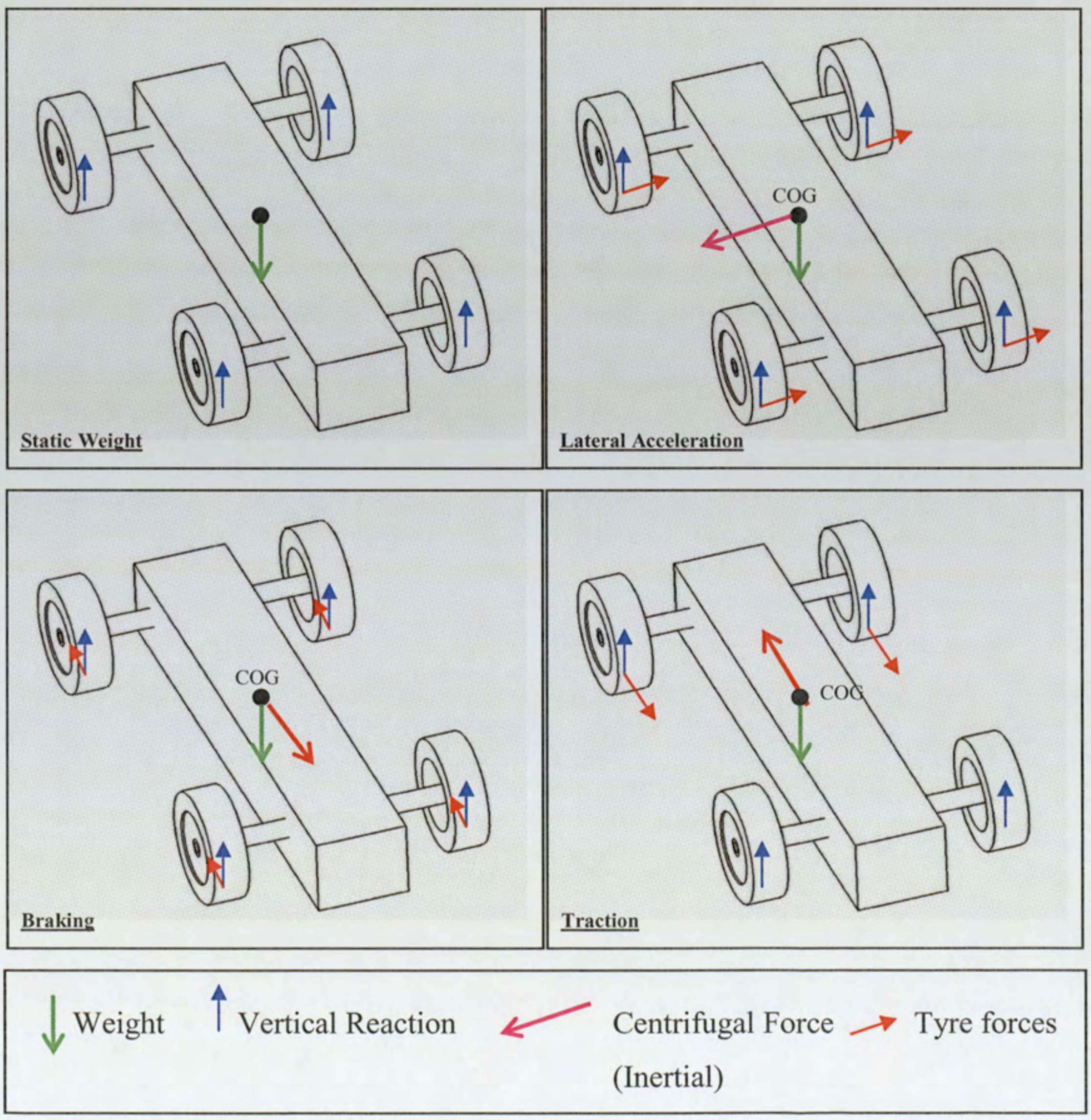

Figure 7-2 - Pure Tyre Loading States

7.3 Coefficient of Friction and Grip Factor

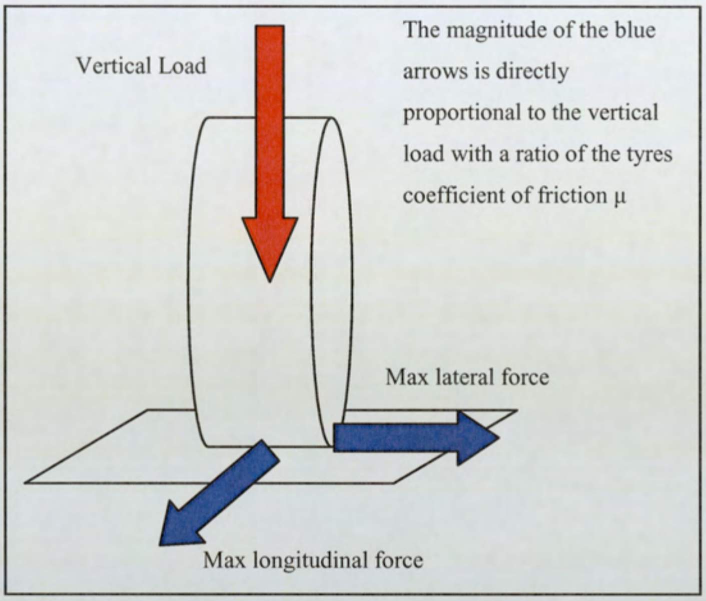

Figure 7-3 - Simple Coefficient of Friction

The classical static friction model describes a linear relation with a factor of μ (coefficient of friction) between the contact force between two surfaces and the force required to slide the two surfaces over each other.

$$F_{friction} = \mu F_{reaction}$$

For a tyre, the value of μ gives a clue as to the maximum possible grip available, and an indication of the performance that could be achievable for the design in question. Unfortunately, tyres do not conform to the basic theory of friction. There is no constant value that relates the reaction force to the driving or cornering forces available. Hence, μ is not a suitable concept to use past this point in the analysis. Instead we will introduce a new factor γ. The magnitude of γ is related by an inverse function to the vertical load on the tyre. Hence, γ will approximate to μ for low loads but then will reduce as the load on the tyre increases.

It is important to consider that 'γ' is not constant from tyre to tyre and also varies in different directions. These variations are cause by the tyre's construction, such as reinforcement direction and tread pattern. The geometrical shape of the contact patch also plays a part in the load sensitivity characteristic, for more information see Sections 7.4 and 7.8.

Grip Factor Ellipse

For any given vertical load the grip factor, γ, can be plotted from an origin in the relevant direction, creating a friction ellipse.

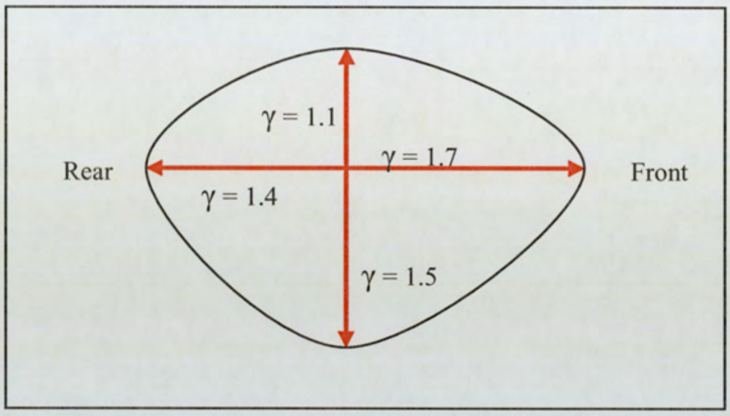

Figure 7-4 - Generalised Grip Factor Ellipse (Given Tyre, Given Load)

The shape of this ellipse varies depending on the application the tyre has been designed for.

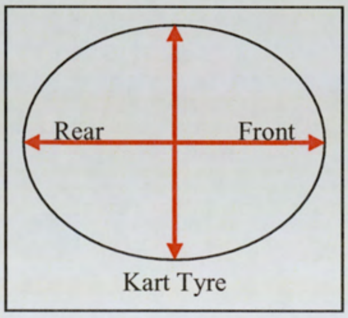

Figure 7-5 - Specific Grip Factor (γ) Ellipse (Kart Tyre)

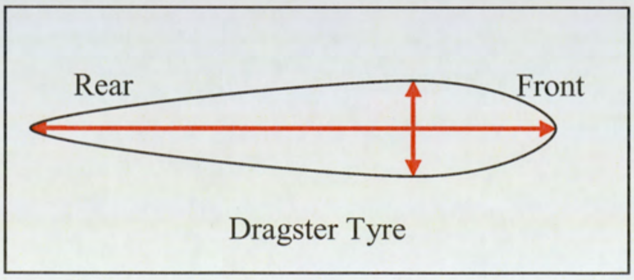

Figure 7-6 - Specific Grip Factor (γ) Ellipse (Drag Tyre)

The kart tyre is designed to have a similar grip factor in all directions, as longitudinal and lateral acceleration are equally important. In contrast, the dragster tyre will have very thin sidewalls to allow deformation, thus improving longitudinal grip at the cost of lateral stiffness and ultimate grip, as cornering ability is not required.

The above is a simplification, the values of γ may be as stated, but only for a given load. As discussed previously, the value of γ falls with increases vertical load on the tyre.

7.4 Load Sensitivity

The term load sensitivity refers to the diminishing nature of γ with increasing vertical forces on the tyre. For example a given a tyre with 1kN vertical load it is capable of generating 1.5kN of lateral force, however when loaded at 2kN it can produce only 2.5kN of lateral force, not the expected 3kN that would be produced if γ had remained constant.

The exact causes of tyre load sensitivity are numerous. However, the two most contributory factors are the tyre-road friction mechanism and tyre deformation.

7.4.1 Tyre Grip Mechanism

Tyre companies invest considerable resources in the development of the racing tyre, so that they might come to better understand the exact sciences involved and be able to push tyre technology forwards. It is accepted though that there are three main mechanisms; adhesion, deformation and wear. Although it is not so necessary for the vehicle designer to have an in depth knowledge of tyre-road friction mechanisms, it is important that they have an understanding of the consequences that they have. Haney (2003) Chapter 3, discusses rubber friction methods in some depth. The following three sections are summarised from his writings.

7.4.1.1 Adhesion

Adhesion comes from the adhesive property of the tyre compound and of the three mechanisms it is the least understood. Molecules on the surface of tyre and road form temporary bonds for the time that they are in contact. The magnitude of the total frictional force is proportional to the number of bonds formed. Hence more surface area equals more bonds formed and in turn greater forces are required to move surfaces relative to each other.

If tyre and road surfaces were perfectly smooth the surface area in contact would be maximised and the friction coefficient large. However, on a microscopic level both road and tyre are rough.

Under normal loading not all of the tyre in the contact patch will be in contact with the road surface. As vertical load on the tyre increases, the tyre deforms into the roughness of the road surface, increasing the surface area in contact and total adhesive force. At some given load the tyre will have deformed into all of the available spaces in the road surface, now any increase in vertical load will not lead to any increase in contact area. At this point grip will stop increasing and will stay constant.

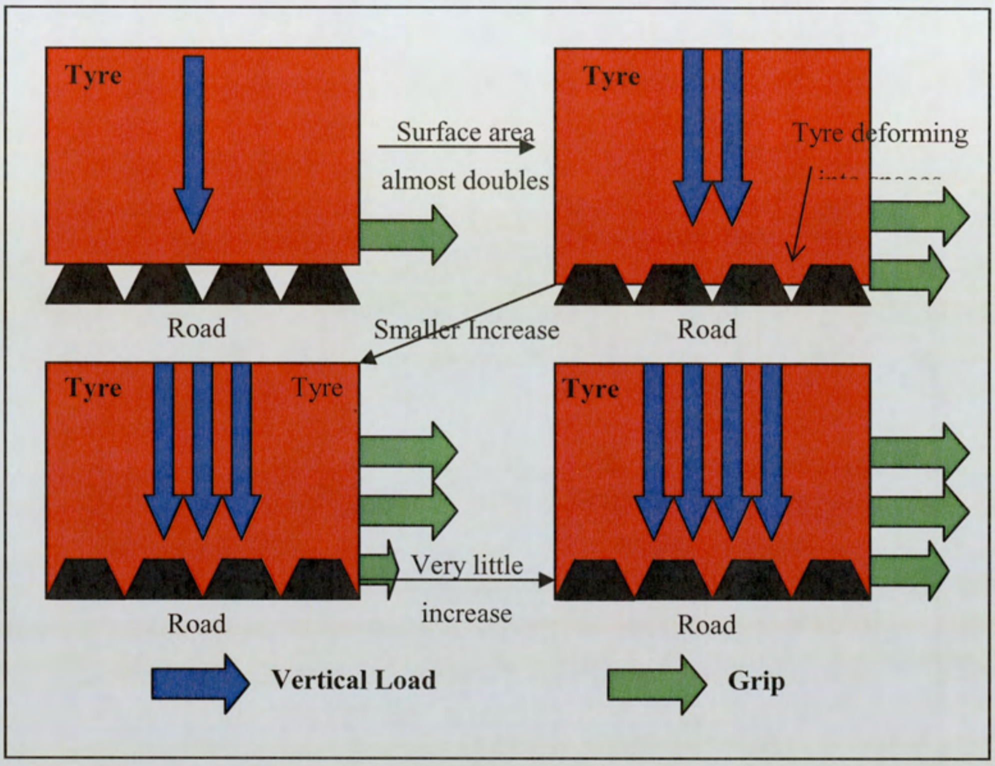

Figure 7-7- Adhesion and Deformation Components of Friction

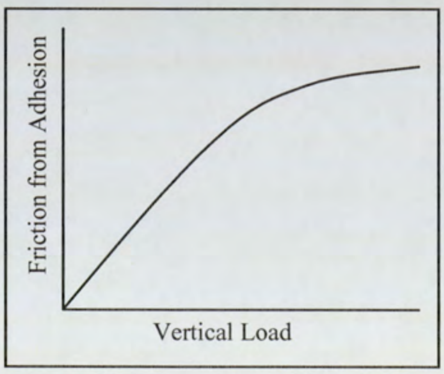

Figure 7-8 - Adhesive Friction Curve

As the adhesive component of friction can only occur if the surfaces are in contact this component is lost when the surfaces are lubricated with a fluid or any particle.

7.4.1.2 Deformation

As discussed above, the road surface is not smooth and vertical loads cause the tyre to deform into the road surface. As particles penetrate the tyre surface a mechanical key is generated. Deformation of the rubber creates a rough surface on the tyre and as expected, rougher surfaces are harder to slide over each other. Unfortunately, it is not quite simple enough to imagine the tyre surface as becoming rough like the sandpaper model shown in Figure 7-9. The particles on sandpaper are permanently of this shape, whilst the tyre has been temporarily deformed into that shape. The elastic nature of the tyre compound tries to spring back its original shape. In doing so, it applies a force to both sides of the key site.

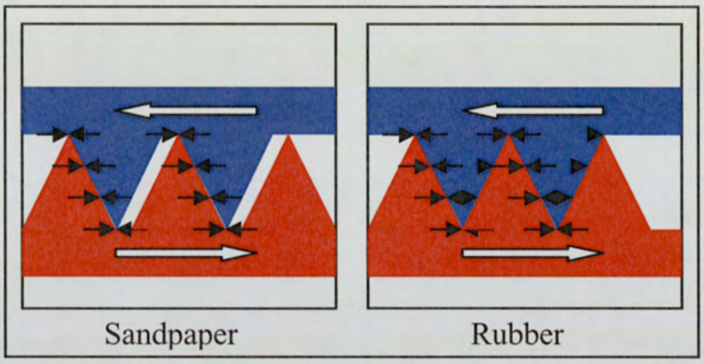

Figure 7-9 - Non-Elastic and Elastic Deformation Model

With the sandpaper model there is only pressure on one side of the contour. In contrast, the rubber has filled the space and is applying pressure to both sides of the cavity. As a result the useable net force in the required direction is less than that of the sandpaper.

By choosing a rubber compound that does not instantly spring back, but which slowly recovers, the pressure on the back of the cavity is reduced and the net grip is increased. The slower recovering compound deforms into the spaces but takes more time to recover. So for that instance the tread material only applies pressure to the one desired side of the cavity. Tyre manufacturers strive to make compounds with this visco-plastic characteristic that are still hard enough to give good wear, characteristics. For the tread compound to be visco-plastic the bonds in it have to be weak making the compound softer and more susceptible to wear. As deformation is the main friction mechanism in the wet, rain tyres are the ultimate in softness, but wear quickly when the track dries out.

The shape of the deformation-friction curve will be similar to that of adhesion but with less of a plateau. When all of the cavities in the road surface are filled the tyre is not able to generate a better mechanical key. However, more friction will come from the extra force required to slide the tyre over the asperities against the increased reaction force.

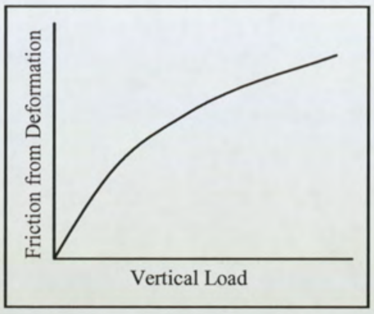

Figure 7-10 - Deformation Friction Curve

7.4.1.3 Wear

Deformation under high loads can raise local stress levels in the contact area to more than the compound's capabilities. As a result, the tyre surface will tear slightly as bonds within the structure are broken by the penetrating asperities of the road surface. Breaking these bonds consumes energy and hence increases the force required to slide the tyre a given distance at speed. Eventually, micro-cracks join and pieces of rubber will be removed from the surface of the tyre, this action constitutes wear, and can be commonly seen in racing with the formation of balls of rubber at the side of the track as the particles tack together.

7.4.2 Total Friction

The level of total grip will be a sum of all three components; adhesion, deformation and wear. The proportion that each mechanism contributes depends on the tyre structure and rubber compound. The texture, material and cleanliness of the road surface also play a large part in affecting the proportion of each contribution.

Regardless of the type of tyre or condition of the road surface, the sum of all the components do not equate to a constant value for γ. Every tyre will demonstrate a falling value of γ as the vertical load on it is increased.

7.5 Slip Angle

Other References:

- N/A

Units = degrees

Nomenclature = α

Definition:

Slip Angle – The mechanism by which a tyre produces lateral forces. It is measured as the angle between the direction defined by the wheel plane and the actual tyre direction.

Explanation:

<u>Without Slip</u>

At the instance the wheel is turned about the steering axis (steered), the wheel velocity vector and ground velocity vector diverge by the steering angle. By considering a car with no mass and tyres that are infinitely stiff with a very high coefficient of friction, the mechanism by which lateral motion is generated can be better explained. When two stationary points, one on the road and the other on the tyre meet at the front of the contact patch, they bond together through friction. In order for them to stay bonded as the tyre rolls the wheel must accelerate laterally, giving the vehicle a lateral speed and realigning the two velocity vectors.

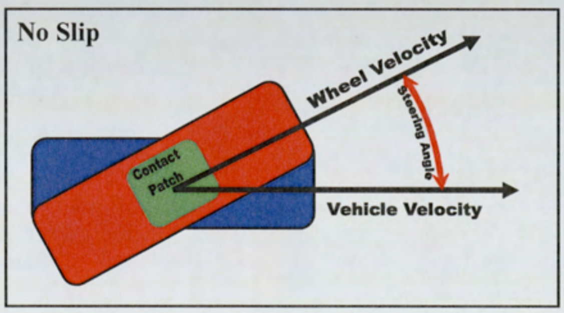

Figure 7-11- Steering Angle Definition

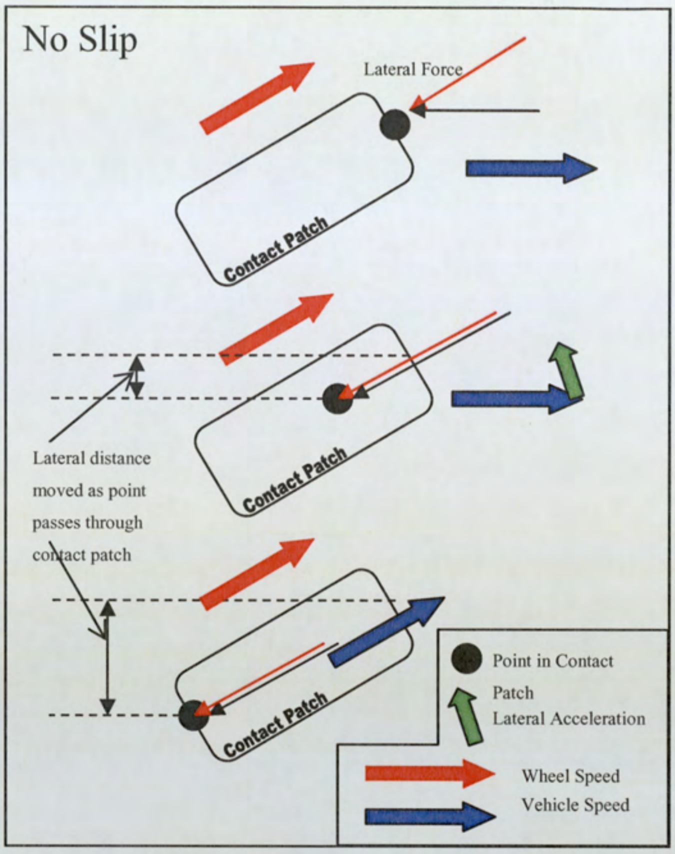

Figure 7-12 - Tyre Lateral Force Model (No Slip)

In reality tyres are flexible and the car will have considerable mass. Consequently, the tyres will deform as a result of the force required to accelerate the car laterally.

<u>With Slip</u>

Under the lateral loads associated with passenger vehicles the amount of slip that takes place in the contact patch is very small and as such the term slip angle could be considered a misguiding. The levels of slip in the contact patch increase considerably in racing conditions as the tyres are used much closer to their ultimate limits. The term slip angle refers to the difference between the steering angle and the actual angle of as result tyre distortion, not to the condition of the contact patch.

With slip, points still meet at the front of the contact patch and wheel is required to move laterally so the points stay in contact. The lateral movement of the points creates a force that that to laterally accelerates the mass of vehicle. The force generated in the contact patch is transferred to the wheel and in turn to the mass of the vehicle. The force creates considerable deformation in the tyre. The force bends the tyre carcass and the velocity of the leading edge of the contact patch is rotated away from the wheel velocity by the slip angle. The direction of the vehicle no longer follows the wheel velocity, but instead the tyre velocity.

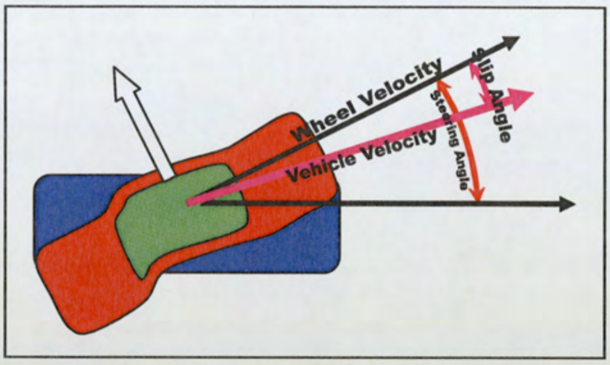

Figure 7-13 - Slip Angle Basic Definition

Reversely, slip angle can be looked at as a deformation in the tyre since it is the angular difference of the tyre velocity and the wheel velocity. As the tyre acts as a spring, the lateral force and slip angle are linked (increasing slip angle increases lateral force, to a point where the whole of the contact patch is sliding and then the lateral force is dependent on the coefficient of friction of the tyre to the road). It may be helpful to consider a tyre on ice, as the friction coefficient is low between the road and tyre. Only very small lateral forces can be sustained at the contact patch before it starts to slide. Small forces mean small deflections in the tyre carcass and hence small slip angles. Thus, a car on ice cannot develop big slip angles, and hence, has 'poor grip'.

"When a rolling pneumatic tire is subjected to a lateral force, the tire will drift to the side. An angle will be created between the direction of tire heading and the direction of travel. This angle is known as slip angle." Gillespie (1992)

In summary, for a tyre to produce a lateral force a slip angle has to be generated. This is because the slip angle represents deformation in the tyre. As the tyre carcass and the pneumatic capsule behave as a spring, a force must be present.

7.5.1 Effects of Slip Angles

Tyres can be designed to produce maximum lateral thrust at different slip angles. The magnitude of slip angle at which peak lateral force occurs affects the performance of both car and driver.

Tyres that produce maximum lateral thrust at large slip angles are normally narrow with a deep profile, allowing deformation in the tread face. As expected, a wide low profile tyre will produce maximum lateral thrust at lower slip angles.

7.5.2 Driver Feedback

The following table shows a comparison between two theoretical tyres at the extremes of lateral force and slip angle range.

Max F_{lat} Slip Angle	**25 degrees**	**3 degrees**
Max F_{lat} @ 2500N Normal load	3750N	3750N
Profile	High	Low
Width	Narrow	Wide
Driver Feel	Good	Bad
Power Consumption	High	Low
Applications	Rallying	Kart
Driver Feedback	The driver has a much larger range of slip angle over which he can obtain maximum grip. For Rallying where the car is sliding and on poor surfaces this would be desirable.	The driver has to be very precise with the steering as a small error will cause the driver to be a long way from maximum lateral forces and corner speeds.

Table 7-14 - Tyre Slip Angle Characteristics

7.5.3 Power Consumption (Slip Angle Induced Drag)

Even though the different tyres in Table 7-14 are capable of producing the same amount of lateral force the amount of drag that they produce in doing so varies. In karting where all drivers use the same engine and the field is very close, using a tyre that drags more could be the difference between winning and losing. Consequently, very stiff tyres are used and driver skill is required to get the most out of the tyre. However, in rallying most time can be made by cornering quickly on an unstable surface, so the driver needs good feel and a larger margin for error.

Example:

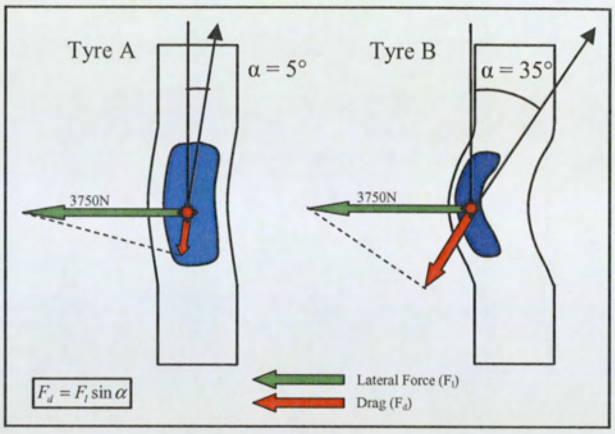

Figure 7-15 - Slip Angle Induced Drag

	Tyre A	Tyre B
Lateral Force (F_{lat})	3750N	3750N
Slip Angle	5°	35°
Drag	326N	2150N
Vehicle Speed 100mph	44.7 m/s	44.7 m/s
Power to overcome drag	**14.5kW 19 Hp**	**96kW 130 Hp**

Table 7-16 - Calculation of Drag for two Tyres

7.6 Longitudinal Slip Ratio

Other References:

- Percent Slip

Units = Percent (%)

Definition:

Slip Ratio – The mechanism by which a tyre produces longitudinal forces.

Explanation:

For a tyre to be producing some resultant force there must be some deformation between the contact patch and the wheel mounting bead. At that instance the contact patch must be moving around the tyre at a different speed than it is moving along the ground. Therefore, the slip ratio refers to the difference in the speed of the wheel described by the angular velocity, multiplied by the rolling radius and the actual velocity of the wheel centre in the direction of movement.

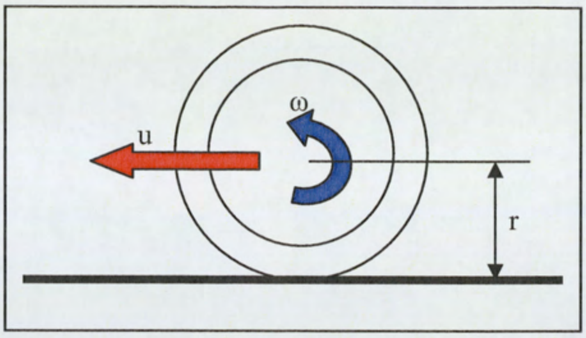

Figure 7-17 - Slip Ratio

Calculation:

$$SlipRatio = \frac{(u - \omega r)}{u} \times 100$$

Negative Slip Ratios represent traction forces and positive ratios represent braking forces.

7.7 Pneumatic Trail

Forces generated in the contact patch are not evenly distributed, so cannot be said to act through a force centre located in the middle of the contact patch. The location of the force centre is dependent on tyre loading conditions. When a tyre is producing some lateral thrust the force centre is located to the rear of the contact patch. The rearward force distribution also creates a moment about the steering axis known as self-aligning torque, more details of which can be found in Sections 7.7.1 and 8.3.1.

Force within the contact patch is related to deformation of the tyre. The tyre model described in Section 7.1 can be used to illustrate this. If the tyre is rolled across a flat surface with a vertical load and a slip angle the shape of the contact patch can be seen.

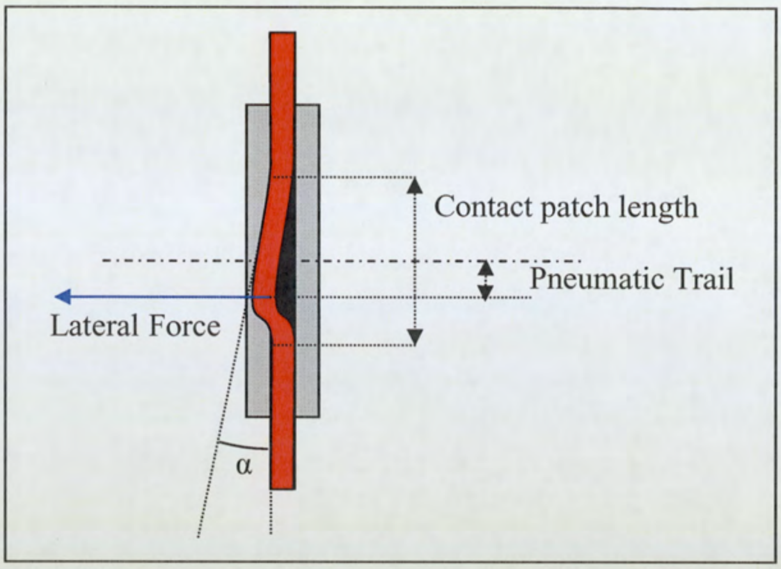

Figure 7-18 - Pneumatic Trail (Tyre Model)

The deformed shape is not symmetrical about the wheel's rotational axis, with the largest lateral deformation trailing. Pneumatic trail is the distance between the force centre and the axle centreline, see Figure 7-18. The force centre can also be associated with the sliding front. At small slip angles the sliding front is located at the rear of the contact patch and the area of slip is small, as the slip angle increases the sliding front progresses forward until the whole of the contact patch starts to slide. Generally pneumatic trail decreases as slip angle increases and could feasibly become pneumatic lead.

7.7.1 Self-Aligning Torque

A torque about the steering axis results from pneumatic trail. The magnitude of the moment is given by lateral force multiplied by pneumatic trail. However, we have seen that the pneumatic trail falls as lateral force increases. Consequently, peak torque might not coincide with peak lateral force. At any instance self aligning torque can be thought of as:

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7.8 Tyre Deformation

7.8.1 Vertical Load

A tyre deforms with vertical load, displaying a reasonable linear spring rate within the tyres working load range. A certain amount of hysteresis is shown during unloading of the tyre (see Section 8.2 for more information).

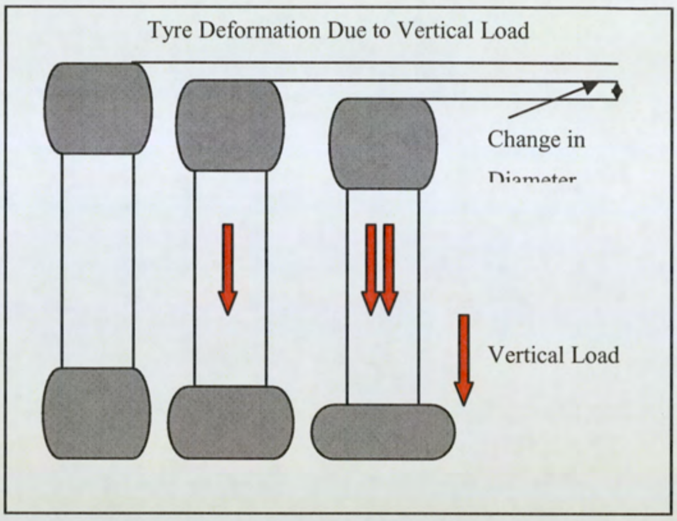


Figure 7-19 - Vertical Tyre Deformation

7.8.2 Lateral Load

Under lateral load the tyre and the contact patch deform sideways, as a result there is also a vertical deflection. As the slip angle α is increased, the increasing lateral load causes greater deformation. Lateral deflection causes a coning in the tyre shape, the outer diameter being smaller than the inside. The coning has a steer effect that can affect the balance of the car (see Section 12.5).

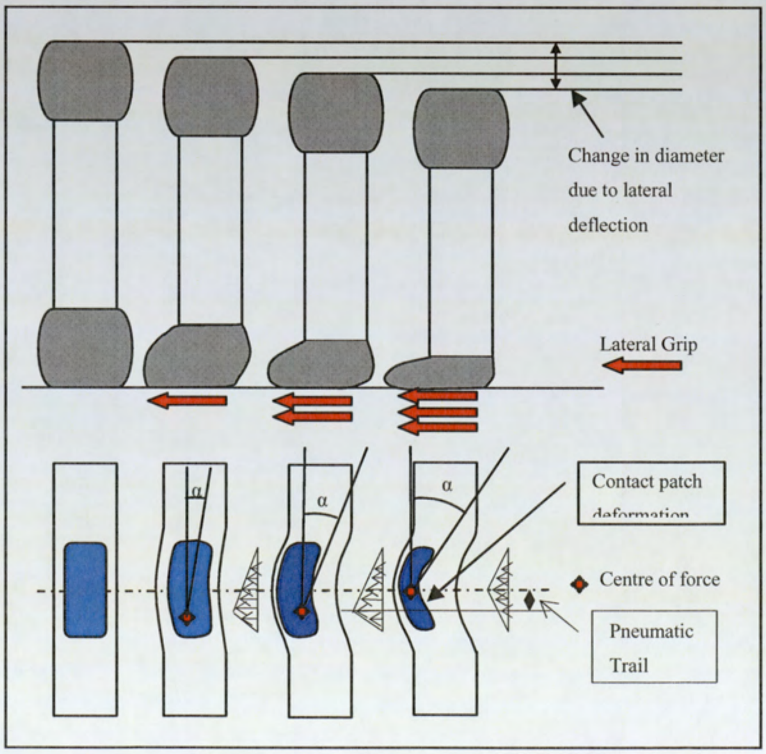


Figure 7-20 - Lateral Tyre Deflection with Pneumatic Trail

Diagram adapted form Rouelle (2003)

7.8.3 Traction Load

The deformation associated with a slip ratio causes a variation in pneumatic trail, rolling radius and the location of the contact patch.

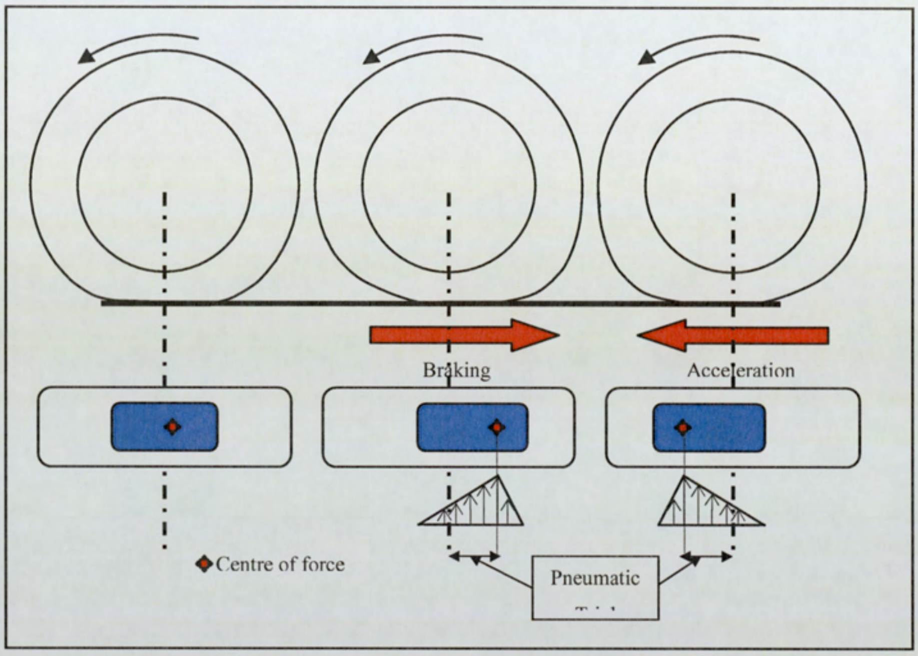


Figure 7-21 - Traction Deformation

Diagram adapted form Rouelle (2003)

8 Tyre Parameters –Tests and Data

Tyre manufacturers test tyres in order to sustain continual development and to produce data to aid in the vehicle design process. Two main tests are carried out to give basic tyre data covering three variables. In the absence of these tests the engineer can carry out some simple tests to gain basic data.

This section looks at the ways that tyres are tested, the data that comes out of the tests and the tyre parameters that can be extrapolated from the data.

8.1 Mathematical Models

Since the emergence of the first tyre data, designers have been trying to accurately model tyres with mathematical functions. The aim being to minimise the data needed from expensive physical tests without compromising knowledge about the tyre's performance. Various numerical methods have been used for fitting curves to test data. One of the most recent and accurate methods has been termed 'Pacejka's magic formula'. This formula can use up to 40 or so constants, measurable from the tyre, to predict the tyre's behaviour with impressive accuracy. This method is now commonly used across the vehicle simulation market, from arcade driving games to creditable vehicle analysis packages. It is becoming common practice for tyre manufacturers to give Pacejka constants for a tyre, allowing the designer to generate their own graphs and data for precisely the load range that they are interested in. Pacejka has produced an impressive text explaining many of the anomalies in tyre dynamics (Pacejka 2002)

8.2 Static Loading Tests

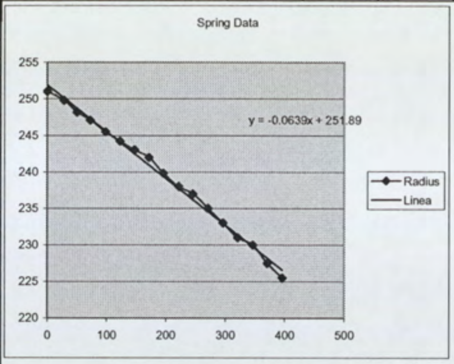
The tyre is mounted on a rim and set to a given pressure, load is incrementally applied to the tyre and the vertical displacement measured. The load is also incrementally removed to give an indication of any hysteresis. The test is repeated at different camber angles, since tyre spring rates fall as camber increases as a result of unequal compression of the tyre walls.

8.2.1.1 Spring Rates

The tyre spring rate graph shows the deflection of the tyre under vertical loads.

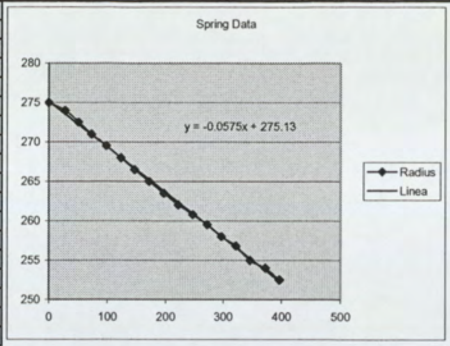
Tyre	7610	Size	6.2/20.0 - 1	inches	Camber	0	degrees
		Rim	6 x13	inches	Pressure	14	psi

Load	Radius
0	251
28	249.8
50	248.2
73	247.1
98	245.5
123	244.2
147	243
172	242
196	239.8
222	238
246	237
271	235
296	233
321	231
346	230
371	227.5
396	225.5
396	225



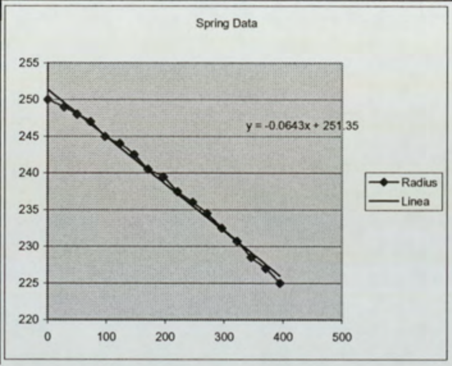
Tyre	7620	Size	8.2/20.0 - 1	inches	Camber	0	degrees
		Rim	8 x13	inches	Pressure	14	psi

Load	Radius
0	275
28	274
51	272.5
73	271
98	269.5
123	268
147	266.5
172	265
197	263.5
222	262
247	260.8
272	259.5
297	258
322	256.8
346	255
372	254
396	252.5
397	252.5



Tyre	7610	Size	6.2/20.0 - 1	inches	Camber	1	degrees
		Rim	6 x13	inches	Pressure	14	psi

Load	Radius
0	250
28	249
50	248
73	247
98	245
123	244
147	242.5
171	240.5
197	239.5
221	237.5
247	236
271	234.5
296	232.5
322	230.7
346	228.5
371	227
395	225
396	225



Tyre	7620	Size	8.2/20.0 - 1	inches	Camber	1	degrees
		Rim	8 x13	inches	Pressure	14	psi

Load	Radius
0	275.2
28	273.8
50	272.5
74	271
98	269.5
123	267.8
147	266.2
172	265
197	263.5
222	262
247	260.8
272	259.5
297	258
322	256
347	254.5
372	253.8
396	252.2
397	252.2

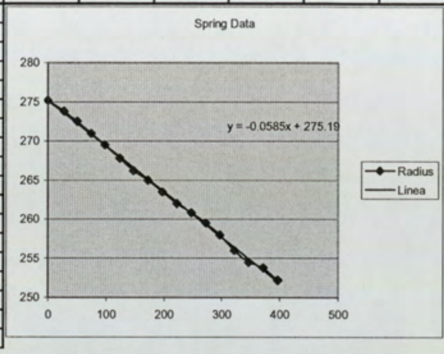


Figure 8-1 - Tyre Spring Rate Data

Source Data Avon Tyres (2005)

The gradient of the line is the tyre spring rate and is normally quoted in N/mm. The above chart shows the spring rates for two different sized Avon Tyres, both tyres are of the same construction and compound. The wider tyre has a higher spring rate, but both tyres show reducing spring rate as camber increases.

8.3 Dynamic Testing

The tyre is tested on what is known as a stability rig. These test rigs come in two common formats and are both accepted as a reliable method for producing tyre data. However, the quality of the drum rig data can depend on the ratio of the tyre to drum diameter.

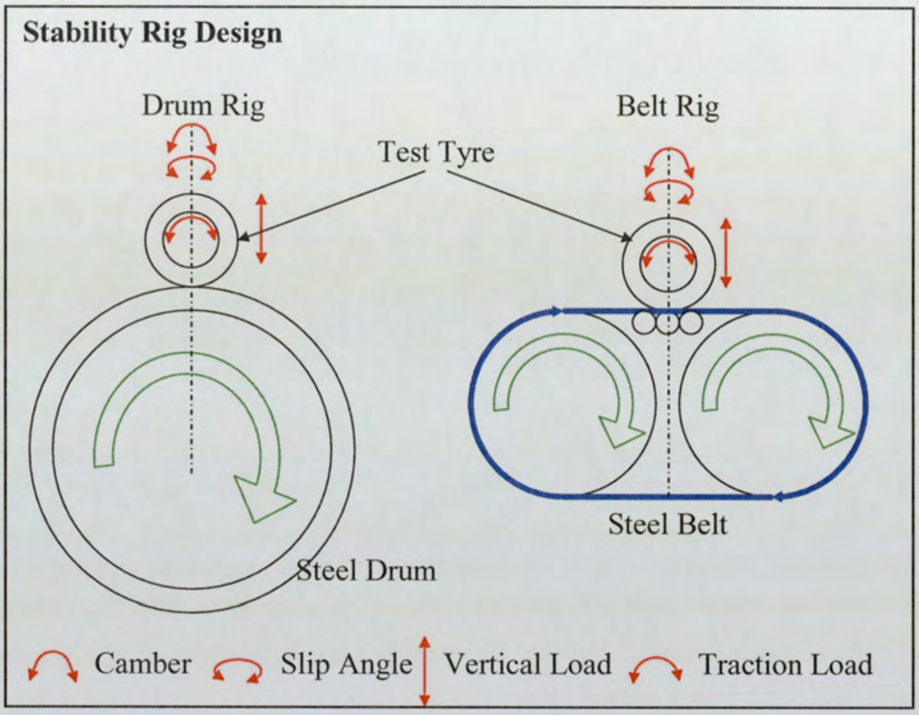


Figure 8-2 - Tyre Stability Test Rig Principles

The drum or belt surface is coated in order to have a surface finish similar to that of a road. The forces produced by the tyre are measured on both rigs by strain gauges fitted to the arms that position the tyre onto the contact surface. Forces are recorded as the speed of the belt, vertical load, traction load and camber and slip angle are altered. As both the arm on the test rig and the contact surface are fixed in relation to each other slip angle and steering angle become identical, allowing measurement of forces for given slip angles.

Other machines have been devised and have proved successful at recording tyre forces. These have included rigs towed behind or beside large road or rail trailers. Obviously conditions are a lot more variable with these kinds of test rigs, but useful data has come from them. Other low speed test machines such as Eric Gough's 1950 machine[10] have been used to assess tyre

[10] Eric Gough's tyre testing machine - Milliken and Milliken (1995) page 20

performance. In this machine the wheel and tyre are fixed on a rotating axle, a bogie on rails is then dragged under the tyre creating deformation and lateral forces. Levers and pads on the bogie measure forces and deflections as the bogie rolls under the tyre. This was one of the first machines that quantified pneumatic trail and lateral force distribution.

Interestingly, dynamic tyre data shows that lateral load generation is not greatly affected by speed. Consequently, most stability rig tests are carried out at relatively low road speeds, around 20 kph.

8.3.1 Self Aligning Torque

In Section 7.7 it was shown that as a result of pneumatic trail the tyre, when given a slip angle, produces a moment about the steering axis that tries to re-align the wheel with the straight ahead direction. The magnitude of this force is measured on the stability rig at given vertical loads and camber angles for a range of slip angles. Typically the information is given as per Figure 8-4.

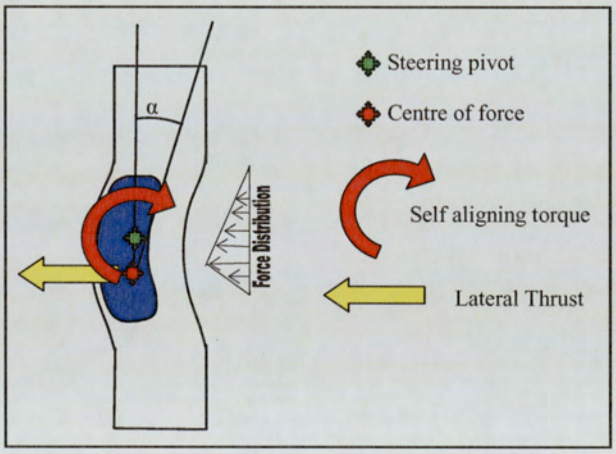


Figure 8-3 - Self Aligning Torque Principle
Adapted form Rouelle (2003)

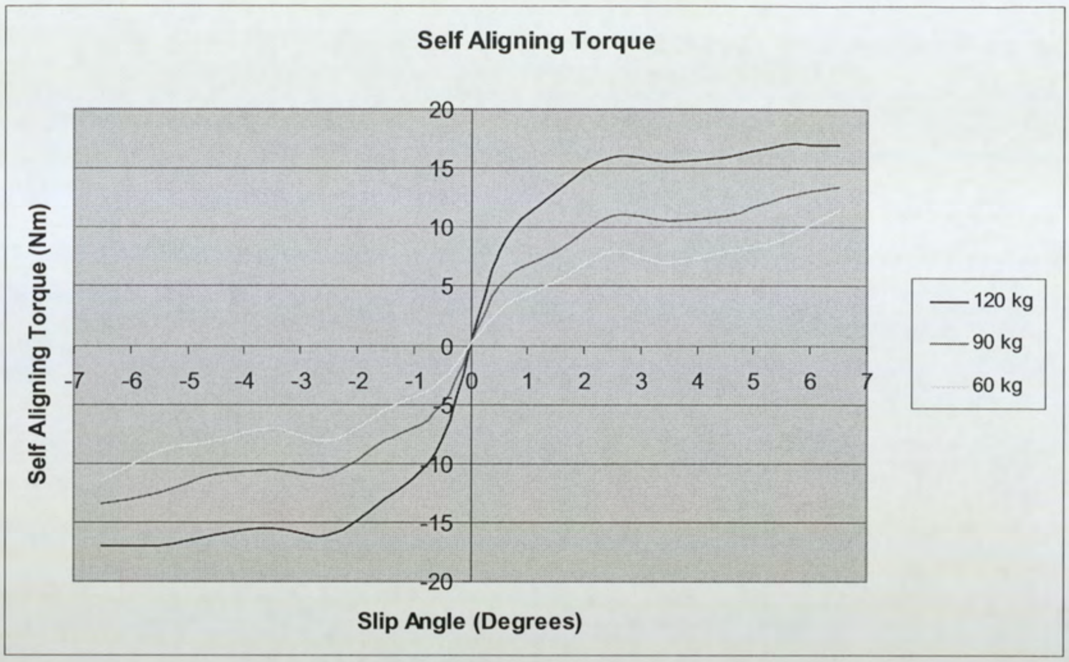


Figure 8-4 - Sample Self Aligning Torque Data

Avon Tyres (2005)

8.3.2 Cornering Force

The magnitude of lateral thrust is again measured on the stability rig at given loads and camber angles for varying slip angles.

Typically the data would be presented according to Figure 8-5. The test shown has been carried out to ± 7° and does not quite show the peaks of the tyres performance so is unfortunately of limited use, but it does show the nature of tyre load sensitivity.

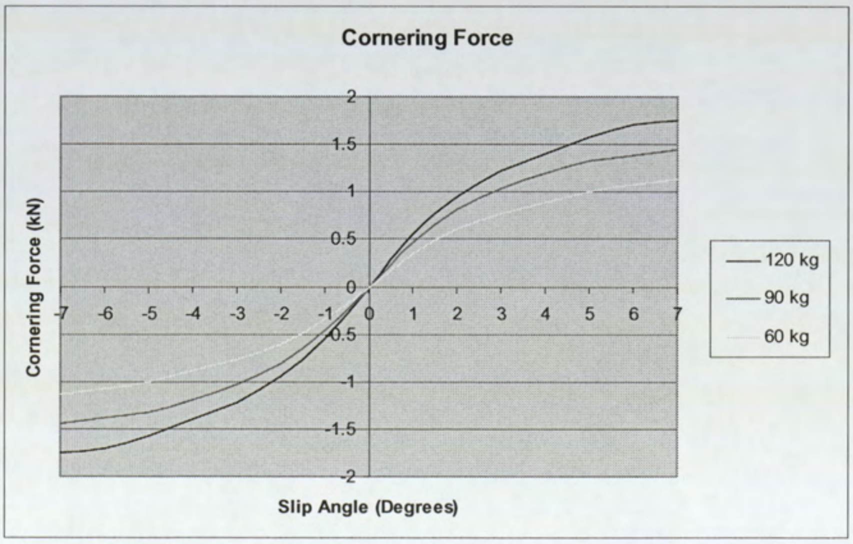


Figure 8-5 - Sample Cornering Force Data

Avon Tyres (2005)

It is important to appreciate that the lateral thrust produced by the tyre increases with slip angle to a maximum and then starts to fall away after the maximum is passed, the steepness with which it falls away depends on the tyre. Figure 8-6 shows an idealised tyre curve with different performance regions shown on it, these regions are important in the explanation of balance and limit handling. The cornering stiffness of the tyre can be calculated from the gradient of the curve in the elastic region.

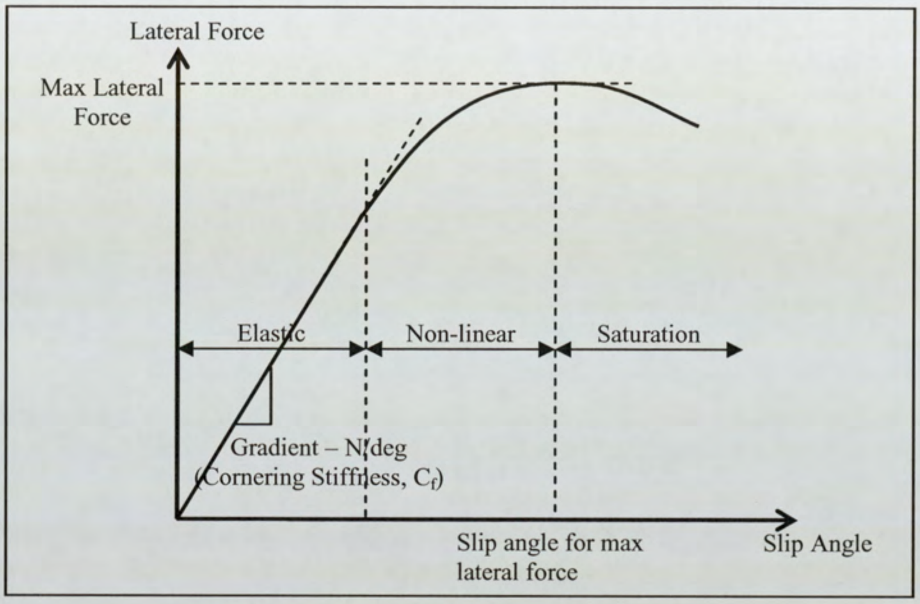


Figure 8-6 - Idealised Tyre Lateral Force Curve

Adapted from Rouelle (2003)

8.4 Size

It is a common belief that simply increasing the width of a vehicle's tyres will increase the performance. From the basic description of friction this would be incorrect, since friction is independent of contact area. However, we have seen that as the coefficient of friction depends on three mechanisms, all with diminishing returns as load is increased, tyres are load sensitive. By increasing the amount of rubber that can adhere or deflect into the surface of the road, more grip can be achieved. Theoretically then, a wider tyre will increase the amount of grip available and therefore the car would be fitted with the widest and largest diameter tyre possible. In reality though there are other performance limiting factors that determine the optimum tyre size for performance. These include the following:

- Stiffness and Deflection – The lower and wider the profile of a tyre, the stiffer the contact patch becomes laterally. If the contact patch is laterally stiff, maximum cornering forces will come with small slip angles. For any given racing discipline there will be a window in which the optimum slip angle will lie to give the best compromise of drag and driver control. If the chosen tyre is too far away from this, optimum performance will be forfeited.
- Compliance - As the profile of the tyre is reduced, the tyre walls become stiffer, reducing the tyre's compliance to bumps and features in the road surface.
- Housing - Normally there is a minimum diameter that the wheel can be in order to house the uprights and brakes and a maximum outer diameter of the tyre set by other constraints. Therefore, the profile must be specified to give the correct slip angle performance and tyre compliance sets the width.
- Camber – As a tyre becomes wider it becomes more sensitive to camber change. This means that the suspension must be capable of tight camber control in order to get the most out of the tyre. If tyres are being retrofitted to an existing suspension system it might be impossible to get the camber control required for wider tyres.
- Weight and Inertia – For more rubber to be in contact with the ground the tyre must be physically bigger. A bigger tyre and wheel will have more mass, requiring more force to accelerate it both forward and rotationally. Extra rotational inertia adds to the gyroscopic effect of the wheel and tyre, increasing the force required to change its direction. As the wheel and tyre's CoG is along its rotational axis, any increase in diameter will raise the CoG, increasing weight transfer under lateral acceleration.
- Temperature - As the size of the tyre increases so does the energy required to heat it. The area over which heat is lost will also increase with tyre size. Any racing tyre has an optimum temperature at which it will produce most grip, normally between 70 and 90 °C. The heat is generated as the tyre deflects and scuffs on the ground. If the tyre is too stiff to deflect and deform under the loads being imparted on it, less heat will be generated. Similarly, if the force required to slide the tyre cannot be generated by the car's mass and speed capabilities little heat will be generated. The car may have good grip, but no more than it might have with a smaller tyre running at its optimum temperature, without all the undesirable effects of large tyres.
- Scrub – When cornering, the inner edge of the tyre is turning about a smaller radius than the outer, in the same way the inside wheel is compared to the outside wheel and a differential is needed. With the tyre however, both edges are linked by the tread, so at some point in the contact patch slip must occur. The slip contributes nothing to lateral

grip, having only the effect of adding a rolling resistance to the wheel, slowing the car and heating and wearing the tyre. Obviously the wider the tyre, the greater the difference in the radius of turn and worse the effects will be.

- Frontal Area – In an open wheel formula the wheels and tyres contribute significantly to the total amount of aerodynamic drag.
- Drag – As the area of rubber in contact with the road surface increases for a given load the amount of adhesion will also increase. In order for the wheel to roll, the bonds must be made and broken, this action consumes energy. If grip is not required at that point in time this is wasted energy that could be better used.

8.5 Temperature

The tyre is not a perfect machine, some of the energy put into it to create deformation and slip is lost as heat. A portion of this heat energy will be absorbed by the tyres, increasing the temperature of the tread surface and the tyre carcass. Thermodynamics govern that as the difference between the temperature of the environment and the tyre increases, so will the rate of energy transfer from the tyre to the surrounding environment. Consequently, the tyre will reach equilibrium at some given temperature. However, the tyre is not constantly working at the same rate, so the temperature of the tread face will vary considerably over the course of a lap.

The tread compound is designed to become more compliant and chemically reactive at elevated temperatures, promoting mechanical keying and adhesion. Consequently, the coefficient of friction between the road and tyre increases with tyre temperature. Tyre compounds have a temperature limit at which they will start to delaminate, to the detriment of grip and integrity. The tyre carcass will also have temperature constraints, above which the tread compound and the carcass might start to delaminate. The initial signs of this are blisters on the tread face.

Tyre compounds will have optimum temperature ranges. Smith (1978, page 16) gives an indication that slick racing tyres are commonly designed to perform best in the range of 80-100 °C. The optimum operating temperature for any tyre should be sought from the tyre manufacturers or dealers, in order to ascertain whether the tyre is being used optimally.

Measuring tyre temperature accurately is somewhat of a challenge since contact is unacceptable whilst the car is moving and tyres cool quickly on the in-lap and pit lane. Infrared non-contact sensors provide the best method for recording surface temperatures whilst on track. The temperature at a series of points across the tyre face can be recorded to tell the engineer a great deal about vehicle performance and setup at specific parts of the track. Tyre temperature can be recorded manually in the pits with a temperature probe. Some allowance should be made for cooling that will have occurred on the in-lap and through the pits.

Tyre temperature is affected by:

- Inflation Pressure – Increased tyre heating is seen at lower tyre pressures, since the tyre deflects more, increasing the heat generated.
- Static Camber – Excessive camber can cause either the inside or outside of the tyre to be overworked, raising local temperatures and damaging the tyre.
- Static Wheel Alignment – Artificial creation of slip angles will increase the work that the tyre is doing whilst the car is travelling in a straight line, increasing the amount of heat generated over the whole lap.

8.6 Pressure

Increasing tyre pressure increases cornering stiffness. However, the range of acceptable tyre pressures is limited by the importance of having the tread face assuming the correct profile. Consequently, only minor adjustments to car balance can be achieve with tyre pressure since the range of adjustment is small. Correct tyre pressure is achieved when the tyre profile is suitable and the tread temperature profile range is acceptable.

The tyre manufacturers will have a range of acceptable pressures for the tyre, the minimum being based on the pressure required to keep the tyre on the rim during lateral loads. The maximum pressure is limited more by the strength of the carcass and the bonding between the carcass and tread compound. Smith [11] (1978) comments that race tyres and wheels are prone to air leakage, split rims are an invitation to leaks and race tyre walls are only just thick enough. The pressure in a tyre should be measured regularly to check that it does not fall so quickly as to have an effect during the race. In addition, the pressure should be set just before the car leaves for the track.

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[11] Smith draws form his experience of race tyres and wheels to comment on their susceptibility to leakage and design.

The pressure that the engineer is interested in is the hot pressure. The cold pressure obviously affects the hot pressure and hence an adjusted cold pressure can be set to give the correct running pressure. Race tyres commonly operate between 18 and 22 p.s.i. hot, the exact value will be dependent on loads and sizes.

Tyres can be inflated with nitrogen, as nitrogen expands less than air when heated, reducing the difference between hot and cold pressures.

The effects of tyre pressure are:

- Tyre Temperature – If the tyre pressure is too low the tyre temperature is likely to be too high.
- Response – Low tyre pressures give slow sloppy response to steering inputs since cornering stiffness is low.
- Tyre Profile – If the tyre pressure is too low the outsides of the tread section will take more load than the middle, causing increased local wear and heating. On the contrary too high a tyre pressure and the middle of the tyre will take increased load and wear unevenly.
- Tyre Compliance – As the pressure is increased the tyre compliance decreases, at some point the benefits of increased cornering stiffness will be outweighed by the loss of grip as a result of diminished tyre compliance to the road surface.

9 Balance – Steady State

Lateral forces generated by the tyres act in turn upon the vehicle's suspension and chassis causing the mass of the vehicle to accelerate laterally. The term balance is used when describing the distribution of the tyre forces generated by each tyre or axle with respect to tyre slip angle.

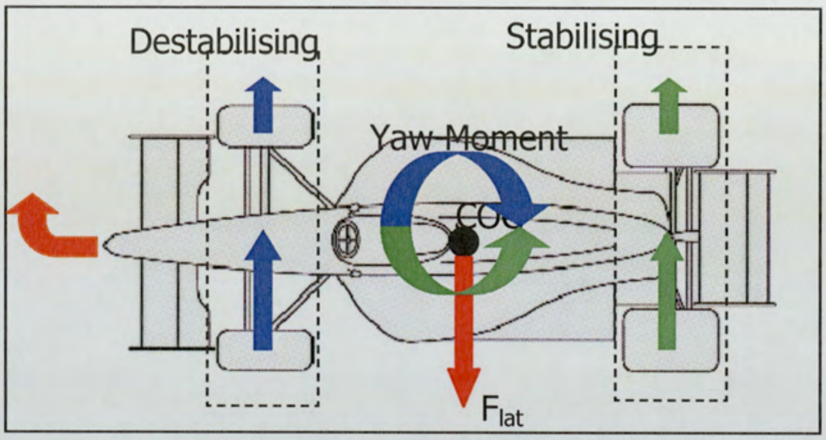

Figure 9-1 - Lateral Tyre Force and Yaw Moments

Figure 9-1 shows the forces acting on the car when in a steady state turn, both front and rear tyres produce a moment about the CoG. The moment from the front tyres is referred to as a destabilising moment as it is trying to turn the car away from its original direction. Meanwhile, the moment created by the rear tyres is trying to realign the car with its original direction, hence stabilising.

For steady state i.e. no yaw rotation, the moments must be equal and opposite so that they cancel each other out and there is no resultant moment and acceleration. The car will always try to assume a state of balance. If the moment from the front axle is greater than the rear, the resultant destabilising moment initiates yaw rotation, increasing the rear steer angle. If the opposite situation arises and the rear produces more stabilising moment, a yaw movement will reduce the front steer angle. As the magnitude of lateral thrust developed by the tyre varies with slip angle, the yaw movement causes change in the axle thrusts, changing the moment. Consequently, the vehicle will align itself to become stable and in a steady state.

To clarify, the car will always try to balance the yaw forces. The sum of the moments around the COG in steady state cornering will be zero, as the chassis will align itself so that the slip angles produce forces that give a resultant yaw moment of zero. The ratio of the slip angles at this state of balance affect how much the car will turn for a given steering input. The relation of the actual steering angle required to turn a given radius compared to the Ackermann angle leads rise to the terms of neutral steer, oversteer and understeer.

9.1 Ackermann Steering Angle

Not to be confused with Ackermann Steering Geometry, the Ackerman Steering Angle defines the steering angle required for a given car to turn a corner of a given radius. Ackermann Steering Geometry is included in the steering system, to alter the rate of change of actual steering angle at the road wheel to steering wheel angle. Pro-Ackermann Geometry allows the inside wheel to turn on a different radius to the outside to achieve rail-like steering.

By ignoring the effects of weight transfer and the width of the track the car can be treated as a bicycle. At very low speeds, tyre slip angle can also be ignored, allowing the Ackermann Steering Angle to be easily defined.

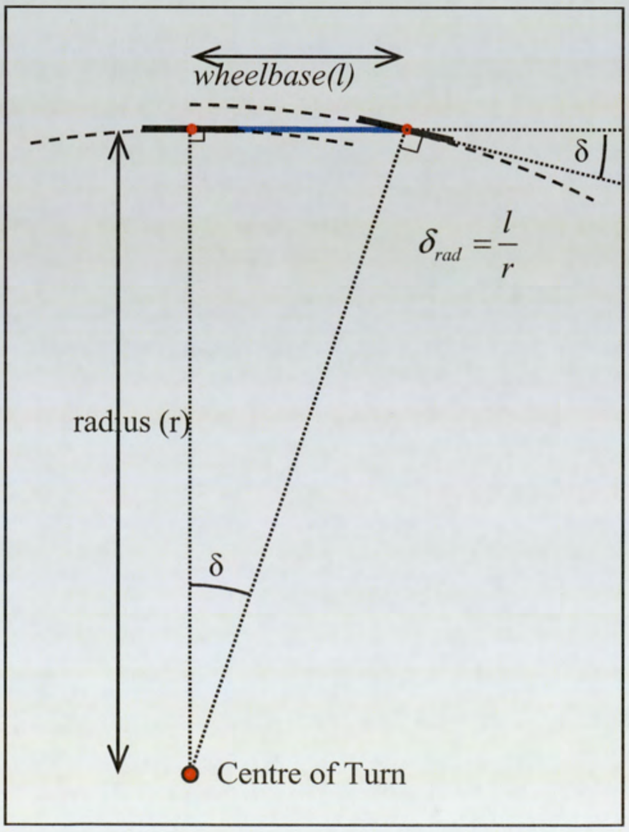

Figure 9-2 - Ackermann Steering Angle
Adapted from Pacejka (2002) page 27

The Ackermann steering angle in radians δ is a ratio of the wheelbase to the turn radius. Consequently, the longer the wheelbase, the more steering angle is required to turn a given radius.

This bicycle theory is explored and explained in various term in Milliken and Milliken (1995), Pacejka (2002) and Gillespie (1992).

9.2 The Three States of Elastic Range Balance

Understeer, neutral steer and oversteer cover the range of handling characteristics that are concerned with steering input to turn radius within the elastic portion of the tyre's performance curve. Neutral steer divides understeer and oversteer. Understeer and oversteer do not refer to the limits of the tyre grip, and should not be confused. They also have no effect on what the final breakaway state will be. It is more than possible for the driver of an understeer car to find that the rear wheels reach the limit of grip before the front and consequently spin.

Firstly, by defining neutral balance, the other two states of balance, understeer and oversteer can be derived.

9.2.1 Neutral

It is easiest to describe neural steer for a car with 50:50 weight distribution that is fitted with identical tyres all round, that are all set to the same pressure.

The neutral line is the path that the instantaneous corner centre traces as the front and rear slip angles are increased, with a ratio of 1:1.

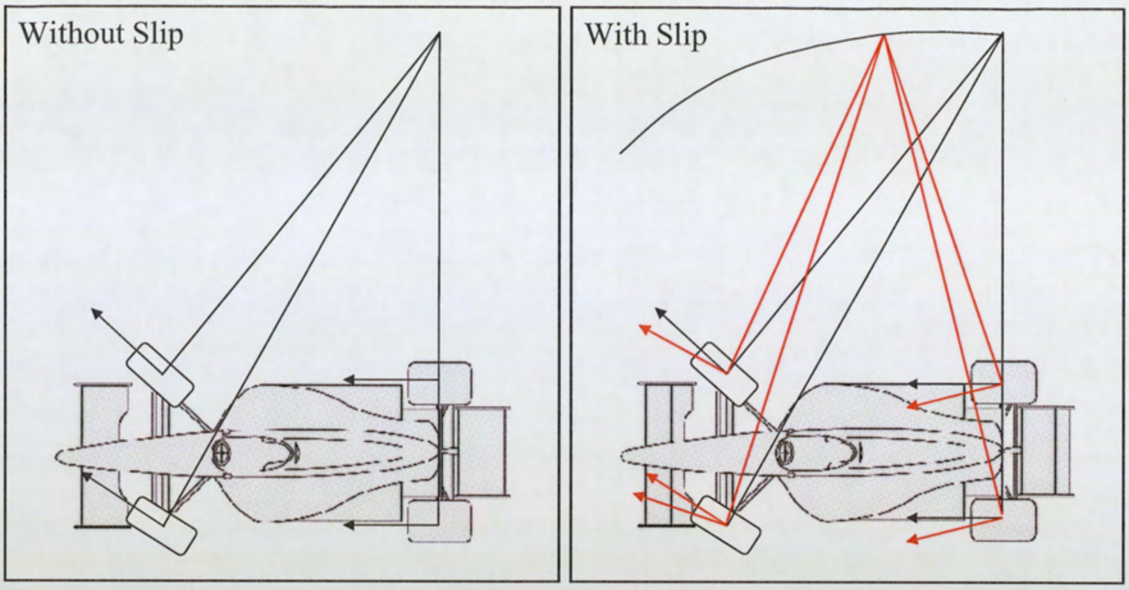

Figure 9-3 - Neutral Steer (Instantaneous Corner Centre)

Adapted from Rouelle (2003)

Thus, it can be said that a neutrally balanced car will have no resultant yaw moment when the front and rear slip angles are equal. The car always steers the same radius turn, irrespective of speed, within the range of lateral accelerations determined by the tyre's linear proportion, for a given steering input. The radius of that turn is defined by the Ackermann angle.

9.2.2 Oversteer

A car with oversteer requires a larger slip angle on the rear tyres to counteract the destabilising moment from the front tyres with a smaller slip angle. As a result, the instantaneous centre moves closer to the car, reducing the radius of the instantaneous turning circle and making the car turn faster, hence oversteer. It can also be said that the oversteer car will, for a given steering angle, turn a smaller radius than the neutral steer car or Ackerman radius.

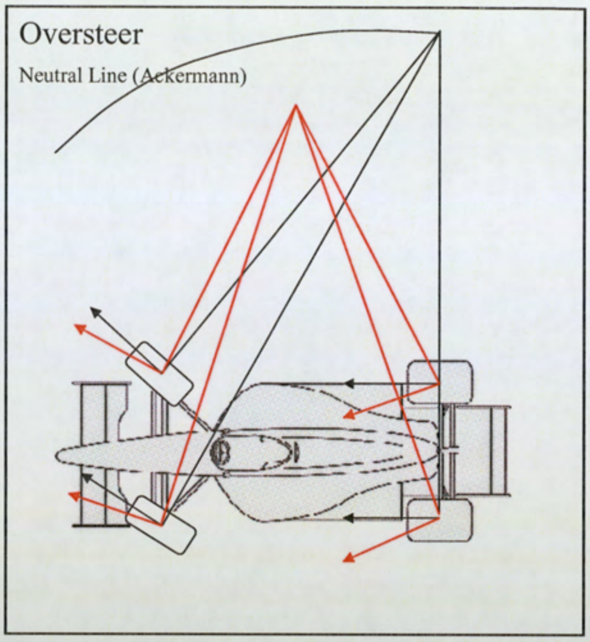

Figure 9-4 - Oversteer and Neutral Steer Comparison

Adapted from Rouelle (2003)

The steering angle required to keep the car on a constant radius decreases as lateral acceleration increases, again within the linear tyre region.

9.2.3 Understeer

A car with understeer requires a smaller slip angle on the rear tyres to counteract the destabilising moment from the front tyres with a larger slip angle. As a result, the instantaneous centre moves away from the car, increasing the radius of the instantaneous turning circle, hence understeer. It can also be said that the understeer car will, for a given steering angle, turn a larger radius than the neutral car or Ackerman radius. The steering angle required to keep the car on a constant radius increases as lateral acceleration increases, within the tyre's linear region.

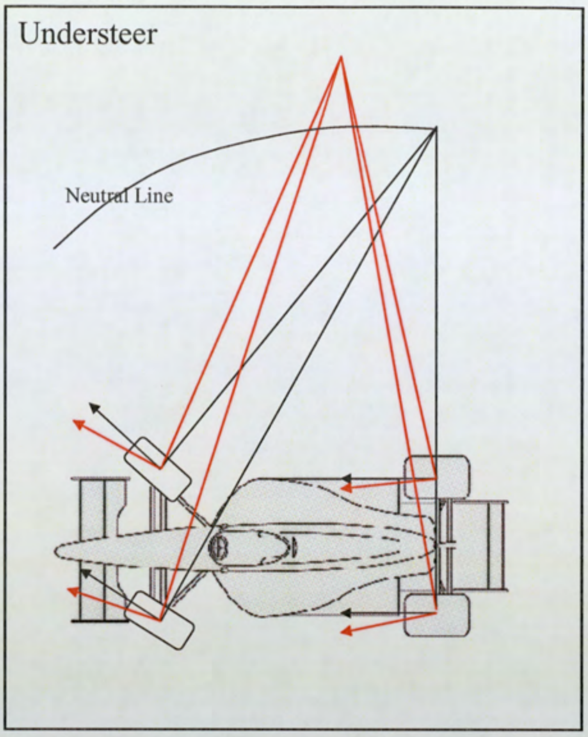

Figure 9-5 - Understeer and Neutral Steer Comparison

Adapted from Rouelle (2003)

9.3 Understeer Gradient

Units = Degrees/g. (Degrees of steering angle change per g of lateral acceleration)

Definition: The change in steer angle required for a car to remain on the same radius turn for a unit increase of lateral acceleration. The angle can be quoted as change in steering wheel angle, taking into account the steering ratio, or change in angle at the wheel flange. Similar to understeer and oversteer this is only valid for the linear portion of the tyre's grip curve, normally to 0.5 g on road cars and possibly higher on race cars due to higher ultimate grip levels.

Calculation:

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Where δ is the steering angle, α_f and α_r are front and rear slip angles respectively and A_y is the lateral acceleration in g.

The understeer gradient can either be positive (understeer) or negative (oversteer).

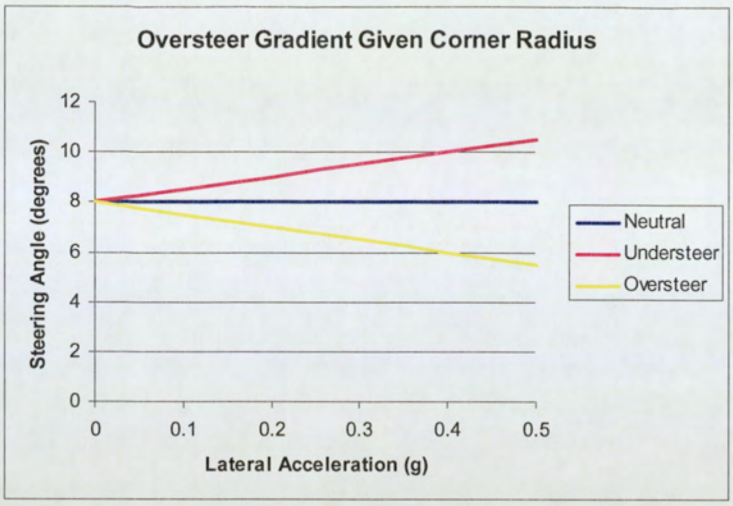


Figure 9-6 - Understeer Gradient Plot

Figure 9-6 shows how the steering angle for an understeer, oversteer and neutral steer car would change to maintain a constant radius turn as lateral acceleration increases.

9.4 Understeer, Oversteer and Stability

When an external lateral force, i.e. banked track or wind, is applied to the CoG, the tyres will deform, generating slip angles and lateral thrust that act against the disturbance. As there is no driver created steering angle, the slip angles become steer angles and their magnitude affects the direction of the car.

When the car is on a banked road a component of the car's weight acts as a lateral force on the CoG.

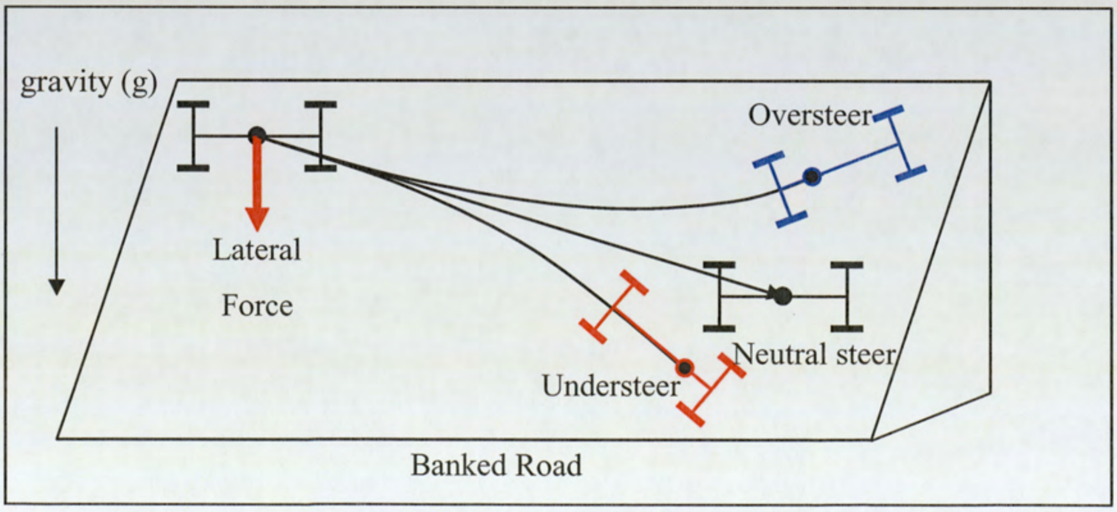


Figure 9-7 - Stability and Steer Characteristics

On the oversteer car the larger rear slip angles cause the rear of the car to run down the bank faster than the front causing the car to yaw and point up the slope. On the understeer car the larger front slip angles cause the opposite and the car yaws to face down the slope. In the case of the neutral steer car both front and rear axles have the same slip angles and the car drifts down the slope with a lateral velocity but no yaw velocity.

Pacejka concisely expresses the effects of understeer and oversteer.

> The meaning of understeer versus oversteer becomes clear when the steer angle is plotted against the centripetal acceleration while the radius *R* is kept constant. In Fig. 1.10 (left-hand diagram) this is done for three types of vehicles showing understeer, neutral steer and oversteer. Apparently, for an understeered vehicle, the steer angle needs to be increased when the vehicle is going to run at a higher speed. At neutral steer the steer angle can be kept constant while at oversteer a reduction in steer angle is needed when the speed of travel is increased and at the same time a constant turning radius is maintained.

Excerpt from Pacejka (2002, page 25)

9.4.1 Neutral Steer Point

The neutral steer point (NSP) can now be described as a point at which a lateral force can act without causing any yaw moment. If the NSP is in front of the CoG, the car will oversteer and if behind the CoG, the car will understeer. If the NSP and the COG coincide then the car has neutral steer.

"As noted above, the neutral steer point identifies that forelaft point on the vehicle where an external side force will not cause the vehicle to yaw." Gillespie, (1992) page 109

9.5 Final States

To re-clarifying, understeer and oversteer are terms used to describe the handling characteristics of a vehicle, but only within the performance envelope defined by the tyre's linear performance for lateral grip versus slip angle. Typically 0 – 0.5 g (Milliken and Milliken, 1995) for passenger vehicles but possibly more for race cars. Understeer and oversteer are not used to describe the cars performance close to the limits of performance. Terms used to describe the limits of traction relate more to the tyre's ultimate grip level, which is the value of μ.

In racing it is important to consider the car's performance close to the limit of traction and grip, i.e. in the performance envelope defined by the tyres transitional and frictional performance characteristics. This is because this corresponds to higher accelerations and as such a higher average speed and faster lap times.

9.5.1 Push (Plough)

Push is the term that a driver would use to describe a car in which the front axle reaches maximum grip before the rear axle. At this point the front tyres cannot produce any more lateral force even with an increase in slip angle. In the worst case the car will plough off the track head first.

The only option that the driver has in a car that is pushing is to slow down or to reduce rear grip with the use of power. Even if the front of the car is right on the limit of traction it will continue to carry along its present line. In this pushing state the car can be considered to be stable, although the driver has very little power to turn any tighter. In a car set up to push at the limit, small variations in front lateral force will slightly adjust the radius of turn and should not cause catastrophic loss of control.

9.5.2 Lose (Spin)

Lose is the term that a driver would use to describe a car in which the rear axle reaches maximum grip before the front axle. At this point the rear tyres cannot produce any more lateral force even with an increase in slip angle. In the worst case the car will spin off the track, usually facing backwards.

The only option that the driver has in a lose car is to increase the radius of the turn by reducing the front slip angle and reducing lateral force from the front axle. Again, as with push, if the rear of the car reaches the limit of traction it will continue on a pre-described line. However, the car is now unstable as any small variation in lateral force could cause the car to spin.

9.5.3 Drift at the Limit

A car that is set so that the front and rear axles meet the limit of traction at the same lateral acceleration will drift outwards on the corner without push or spin. Theoretically this setup will achieve the maximum lateral acceleration since all tyres are working at their limits, but it is not always preferred by the driver. In drift the car has lost stability and the ability to steer.

9.5.4 Driver Feel

The optimum balance setup would be neutral steer with drift at the limit as it would mean that all the tyres are producing as much lateral force as possible. However, drivers like some feedback from the car to tell them how close they are to the ultimate grip, allowing them to get as close to the limit as possible. A neutrally balanced car, although theoretically the quickest, will lack feedback and the driver may not have confidence to push as they are unaware of the limits, saying the car is unpredictable. Road cars are normally set up to exhibit mild understeer and push at the limit, this it deemed safer as the correct reaction to back off the accelerator and turn harder is more instinctive. A race engineer will aim to balance the car as close as possible to neutral steer with drift at the limit, for maximum performance, but with enough bias to suit the driver's requirements, commonly this is a small amount of push for stability.

10 Vehicle Geometry

10.1 Footprint

The footprint can be looked at as the box defined by the four wheels of the car. The ratio of width to length and dimensions play an important part in determining the handling characteristics of the car and its suitability for a particular event.

10.1.1 Track

Other References

N/A

Units = mm.

The term track refers to the distance between the centrelines of the wheels on either the front or rear axle. The term half track is the distance of a wheel centreline to the vehicle's centreline. The size of the track controls the amount of lateral weight transfer for a given lateral acceleration. The half-track is the length of the virtual lever used to resist the roll moment generated under lateral acceleration. Thus, the greater the half track the less force required at the wheel-ground interface to resist the lateral force generated at the CoG and hence, less weight transfer.

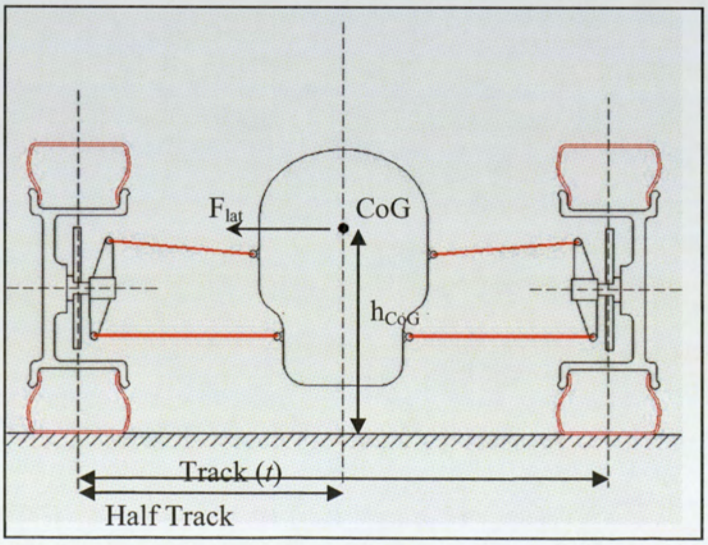


Figure 10-1 - Track Width Definition

When specifying the track width the following must be considered. (Calculations can be found in Section 6.1.)

- Expected height of the COG
- Expected lateral acceleration
- Lateral grip vs. load of the chosen tyre
- Cornering speeds
- Desired handling characteristics i.e. quick responsive turn in.
- Nature of the intended track
- Any rules for that particular class

Some rules stipulate a maximum footprint size for the car, this generally occurs when the ideal track width would be large, so for safety and in order to control track speeds the rule is imposed. Others, like Formula Student, outline a minimum track width to prevent unstable vehicles from running, when the tight circuit imposes a restriction on track width as it is harder to guide a wide car through a tight slalom.

It is always the aim of the designer to minimise weight transfer for the benefit of the tyre. Tyre load sensitivity is discussed in Section 7.4. As a consequence of load sensitivity, any weight transferred from the inside tyre to the outside tyre represents a reduction in the total amount of lateral thrust available.

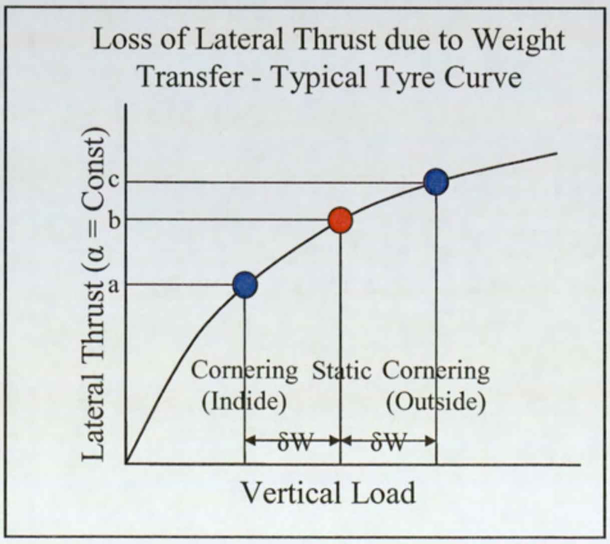


Figure 10-2 - Tyre Load Sensitivity and Weight Transfer

For a complete car:

$$4b > 2a + 2c$$

The only remedy to this problem would be to oversize tyres so that they operated more in their linear area, this solution however, has many more negative effects, as discussed in Section 8.4.

10.1.2 Wheelbase

Other References

N/A

Units = mm.

The term wheelbase refers to the distance between the centrelines of the front and rear axles. The wheelbase length plays an important role by defining the amount of longitudinal weight transfer under acceleration and braking (longitudinal acceleration).

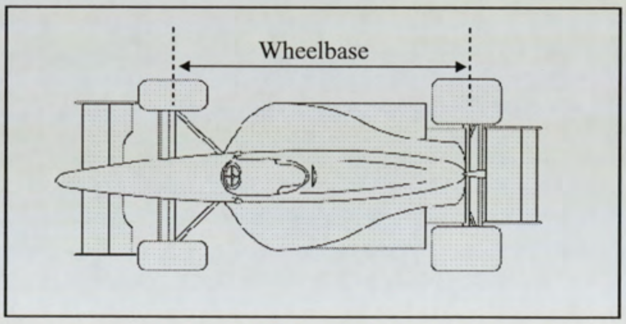


Figure 10-3 - Wheelbase Definition

Longitudinal weight transfer affects the car's stability and performance mainly on the entry and exit of corners when longitudinal acceleration is greatest. Ideally all longitudinal speed changes should have occurred before the corner is entered. However in reality, the driver will be setting the car up for the corner whilst braking or the suspension will still be responding to load changes as the corner is entered. To achieve the fastest possible lap times the tightest part of a corner, the apex, should be the slowest, implying that there will be some longitudinal acceleration right up to each side of it. Simply lifting off the accelerator will allow retarding forces to act on the vehicle, causing significant longitudinal weight transfer.

10.1.3 Effects of Wheelbase on Lateral Acceleration

In order to better explain the effects of longitudinal weight transfer, the examples of two rear wheel drive cars, one with a short wheelbase and one with a long wheelbase can be considered.

Short wheelbase

Entry – On entry to the corner the front wheels will be highly loaded in comparison to the rear wheels. Consequently, the front axle pair will generate a large lateral thrust for a given slip angle and the front of the car will turn in quickly. In response, the yaw movement creates slip angle on the rear tyres, however, as they are lightly loaded the slip angle required to produce enough lateral force to balance the destabilising moment will be greater than the front. In this instance the car is oversteering and assuming similar maximum capabilities of front and rear tyres will be loose at the limit.

Mid Corner (Apex) – With no longitudinal weight transfer the wheel loadings return to a similar value to the static load. The front wheels have now lost load and produce less lateral force for the slip that was initiated at corner entry. The balance of the car now becomes more neutral and the driver must increase the steering and consequent slip angles for the car to remain on the same line.

Exit – Past the apex of the corner the driver starts to accelerate into the straight. Weight transfers to the rear wheels, further unloading the front tyres and reducing the lateral force they are producing. In contrast, the rear tyres produce more lateral force, shifting the balance to understeer. Assuming that the front tyres have not reached their peak lateral force for this load, the driver must further increase the steering angle. If the tyres have reached the limit of lateral force then push will occur.

Long wheelbase

In contrast to the short wheelbase, the longer wheelbase has less longitudinal weight transfer for the same acceleration, giving these contrasting characteristics.

Entry – On entry to the corner the front and rear wheels will be more equally loaded. The front tyres produce some lateral thrust causing the car to turn in, but not as briskly as before, as a greater slip angle is needed to produce the same destabilising moment. For the same front and rear slip angle the moments will be equal and opposite. Thus, the car is more stable and is neutrally balanced and will tend away from entry oversteer to entry understeer.

Mid Corner (Apex) – As the weight distribution was more even on the entry to the corner the weight distribution has not changed much. The balance of the car will not change a great deal and consequently the car will remain on the same line it was initially set on.

Exit – Again less weight transfer occurs and the balance of the car remains closer to neutral. The driver has to make fewer corrections during the exit of the corner to make the car stay on the desired line.

10.1.4 Effects of Wheelbase on Longitudinal Acceleration

To stop the car with maximum longitudinal deceleration all four wheels would be loaded and retarded equally, utilising each tyre at its maximum efficiency. To do this the COG would have to be placed close to the rear of the car. Of course this would be undesirable for other aspects of the car's performance. Instead, the weight transfer from front to rear needs to be limited by using a long wheelbase so that all four wheels can be stopped equally.

On a rear wheel drive car in order to achieve the highest possible longitudinal traction the rear wheels must be loaded as much as possible. To achieve this' a short wheelbase would be opted for. On a four wheel drive or front wheel drive a long wheelbase would give the least weight reduction on the front wheels.

11 Suspension Kinematics

The aim of a suspension system is to restrain the wheel in relation to the chassis so that it sweeps a desired path, allowing a spring medium to absorb the effects of features in the road surface, thus improving tyre loading conditions. Suspension kinematics is often referred to as suspension geometry, and describes the arrangement of suspension links and their relative motion in order to give the wheel the desired pre-defined movement.

11.1 Double Wishbone Suspension (A-Arm)

Throughout the years many different mechanisms have been used to allow wheel movement relative to the chassis. Each different system has its own merits from cost of manufacture to reliability. Designs are far too numerous to consider each one individually, therefore the main focus of this section is the double wishbone arrangement. This variant is most favourable to the race car designer as it has the best combination of camber control in bump, droop and roll. The arrangement is highly versatile and the designer can tune the geometry for their particular application.

11.1.1 Components

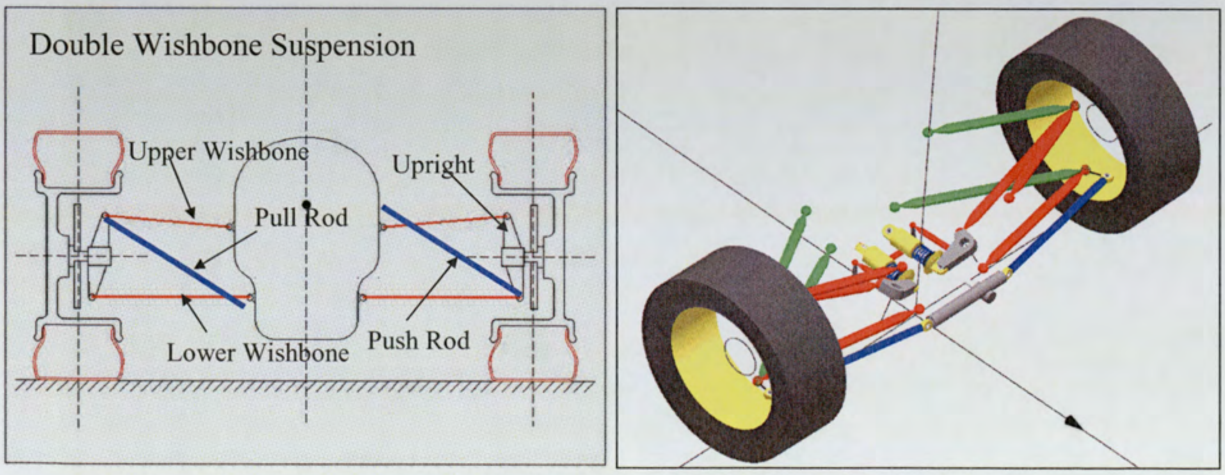


Figure 11-1 - Double Wishbone Suspension Component

Right Image Captured From SUSPROG 3d

Double wishbones can be used with a variety of different spring and shock absorber arrangements. Presently the most frequently used variant uses a push or pull rod to operate a rocker that acts on a spring coil over damper unit. The use of push/pull rods allows the damper and spring to be packaged inside the vehicle, thus reducing aerodynamic drag. The rocker can also be designed to give rising rate suspension characteristics, see Section 15.6 for more details. Historically the spring and damper were mounted in place of the push rod, simplifying the design but only allowing near linear wheel rates, this is still used in some enclosed wheel applications where the aerodynamics of the suspension are irrelevant.

When correctly designed, a double wishbone system can be extremely lightweight. Triangular A-arms are inherently strong and link angles can be aligned to reduce load. With the correct placing of pull or push rod connections, links are only ever in pure tension or compression, allowing them to be deceptively thin, yet still strong and stiff.

11.2 Instantaneous Centre (IC)

Defined as the point about which the wheel will pivot at a given instance. For any given independent suspension unit there are two instant centres. One point lies on a vertical plane that passes through the centre point of both wheels on the same axle. The other point is again on a vertical plane, but instead, parallel with the car centreline and passing through the wheel centre.

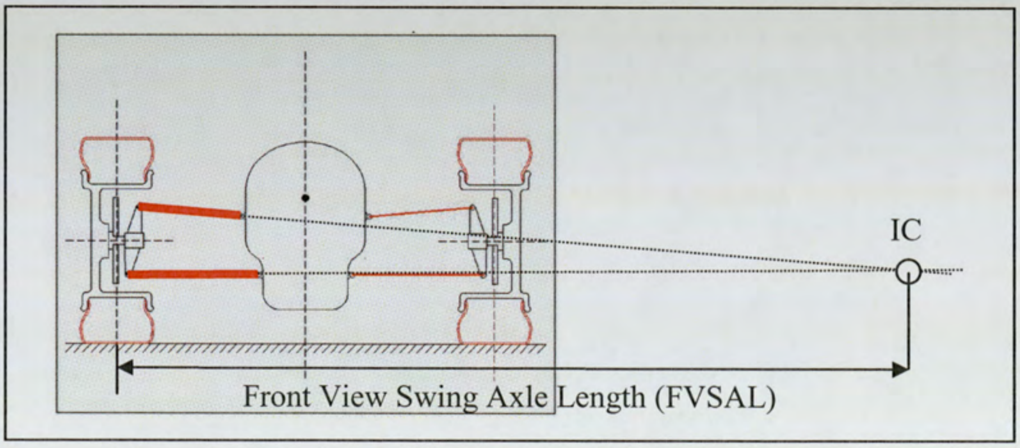


Figure 11-2 - Front View IC and FVSAL Definition

The front view instantaneous centre is found at the point where the extended wishbone lines intersect.

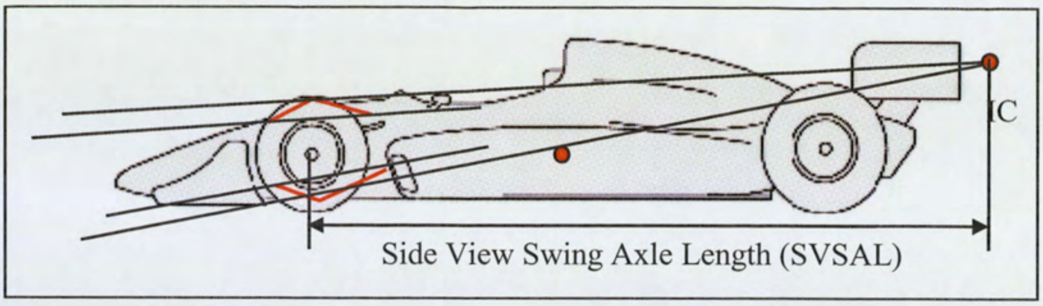


Figure 11-3 - Side View IC and SVSAL Definition

The side view instantaneous centre is found by drawing lines parallel to lines that bisect the top and bottom mounts at a point that passes through the outer steering pivot joints. Again the instantaneous centre is found where the extended lines intersect.

11.2.1 Front View Swing Axle Length (FVSAL)

The distance measured between the front view IC and the centreline of the wheel is the front view swing axle length. Swing axle lengths directly affect how the suspension geometry performs in roll and bump with respect to camber change. The FVSAL also plays a part in determining the amount of scrub (track width change) as the suspension moves. A long swing axle, >2.5 m, performs well in bump showing little camber change. A short swing axle, <1m, will perform well in roll but poorly in bump. Lengths between 1 and 2.5m are considered transitional and display a degree of both characteristics. There is always a trade off between roll and bump performance when specifying the length of the FVSAL.

11.2.2 Moving Front View IC

As its name would suggest, the location of the IC is only for a given instance and therefore subject to movement. As the suspension travels, the IC location can change, altering the FVSAL accordingly. The lengths of the top and bottom links affect how the FVSAL changes during suspension travel. If the designer can accommodate a bottom link that is longer than the top link, the difference in the arcs that the ends of the links sweep causes the IC to move closer to the centreline as the suspension compresses. Thus, the designer has a geometry that, when static, has a long FVSAL, but when the suspension is in compression the FVSAL rapidly shortens and may benefit from the characteristics described in Section 11.2.1. See Figure 11-4.

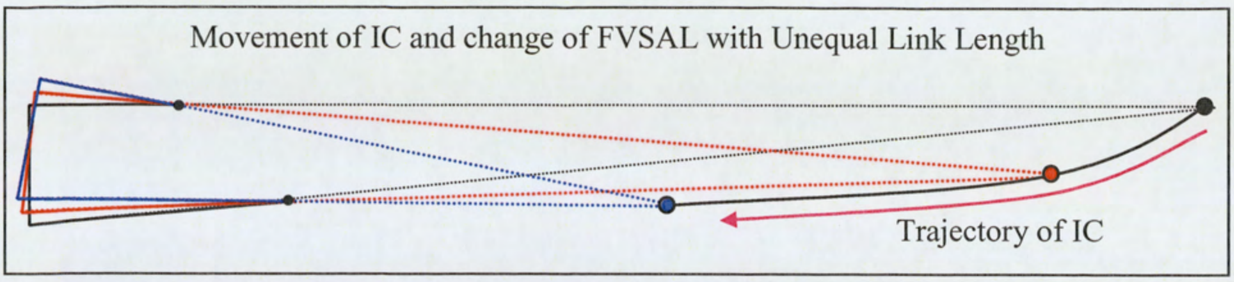


Figure 11-4 - Front View IC Movement

The performance suspension in bump (ride) and roll can be numerically described or specified to aid in the selection of a suitable FVSAL.

11.2.3 Camber Change in Ride

Other References

- Camber control in bump

Units = degrees/mm. (Degrees of camber change per mm of suspension travel)

Definition: The angle of camber change at the wheel as a result of a vertical displacement of the wheel.

Calculation:

$$\Delta C = \tan^{-1}(h / FVSAL)$$

Where 'h' is the suspension travel over which camber change is desired to be known.

11.2.4 Camber Change in Roll

Other References

- camber control in roll

Units = degrees/degree. (Degrees of camber change per degree of roll)

Definition: The angle of camber change of the wheel as a result of a rotational movement of the suspended mass about the roll axis.

Calculation:

$$\Delta C = r(1-(track/2\times FVSAL))$$

Where 'r' is the roll angle over which the camber change is desired to be known.

11.2.5 Side View Swing Axle Length (SVSAL)

The distance measured between the side view IC and the centreline of the wheel is the side view swing axle length. The SVSAL directly affect how the loads associated with longitudinal weight transfer are distributed. The SVSAL also plays a part in determining the amount of castor change as the suspension moves.

11.3 Roll Centre

The Roll Centre (RC) is a virtual point about which the suspended mass will roll as a result of any roll moment about it. Consequently, the RC can also be described as a point at which a force can be applied to the suspended mass without creating any roll angle. The location of the point is determined by the lateral and vertical position of the IC of each wheel. Diagrammatically it is found by linking the IC of each wheel back to the point where the centreline of the wheel intersects the ground.

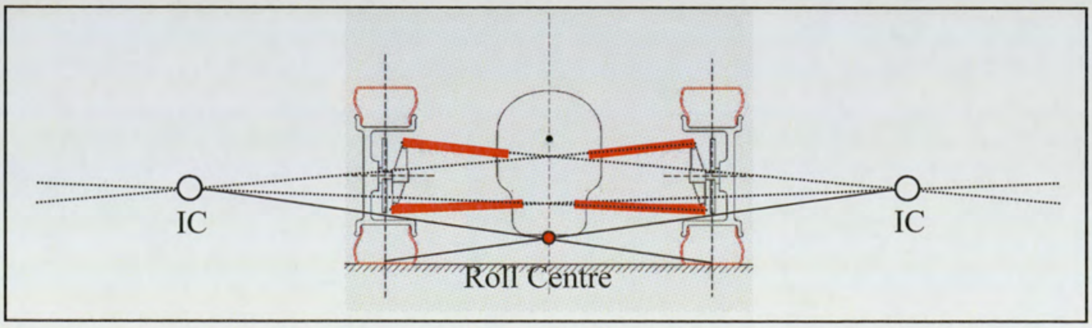


Figure 11-5 - Roll Centre Definition

"One very important property of a suspension relates to the location at which Lateral forces developed by the wheels are transmitted to the sprung mass. This point, which has been referred to as the roll center, affects the behavior of both the sprung and unsprung masses, and thus directly influences cornering." Gillespie, (1992) page 257.

11.3.1 Roll Centre Height and Lateral Location.

The location of the roll centre may be above or below the ground and not necessarily lie on the centreline of the car if an asymmetric geometry has been chosen. Its location is measured from the point where the car centreline intersects the ground. The location of the roll centre determines the following.

- The magnitude of the roll moment to be taken by the springs (Section 14.1.2)
- The magnitude and direction of geometric weight transfer (Section 14.1.4))
- The magnitude of any jacking or lowering forces (Section 11.3.2)
- The roll axis angle and location (Section 11.4)
- Steering response (Section 14.1.3)
- High-speed stability (Section 14.1.3)

11.3.2 Jacking Forces

In Section 11.2 the location of the front view IC is determined through the extension of the wishbone action lines. The IC can also be thought of as a point through which the lateral tyre forces act, since all the forces act through the suspension arms.

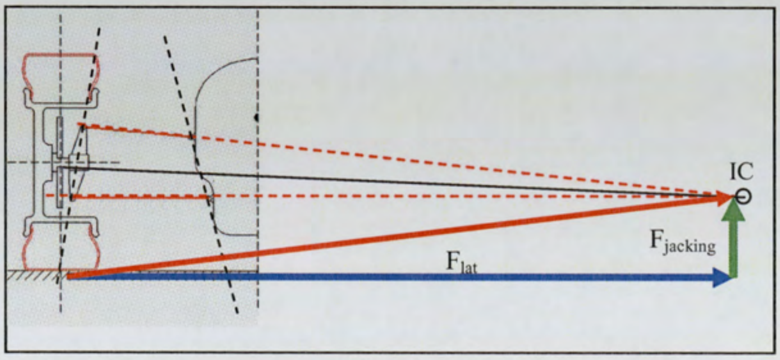


Figure 11-6 - Jacking Force diagram

Adapted from Rouelle (2003)

The result is the generation of a jacking force, the force can also be related to the height of the RC, since the RC and the IC are interlinked. If the IC is above the ground so must be the RC, similarly if one is below the surface. However, if the IC is below the ground the jacking force is negative and a lateral force will introduce a compressive force, lowering the suspended mass.

11.3.3 Roll Centre Migration (Vertical and Lateral)

We have seen that the designer can use wishbones of unequal length to move the IC closer to the centreline of the car during roll. The line that the IC moves along inevitably has an effect on the location of the roll centre.

If the link lengths are such that the roll centre is not adequately constrained and moves excessively during suspension movement, roll stiffness will change. Any jacking moments created by the roll centre will also be altered. Therefore, it is the designers aim to minimise roll centre movement during the expected movements of the suspension.

11.4 Roll Axis

Both front and rear suspension systems will have a roll centre. The roll axis is the virtual line that joins the two points together. The roll axis is therefore the axis of rotation for the suspended mass as the result of a roll moment. Inclining the roll axis to the rear of the car reduces the roll moment generated (Section 14.1.4) and in turn reduces the weight transfer taken by the rear springs. Softer rear suspension springs then allow better compliance and improved traction for acceleration out of the corner.

11.5 Pitch Centre

The Pitch Centre (PC) is a virtual point about which the suspended mass will pitch as a result of any pitch moment about it. Consequently, the PC can also be described as a point at which a force can be applied to the suspended mass without creating any pitch angle. The location of the point is determined by the longitudinal and vertical position of the IC of each wheel. Diagrammatically it is found by linking the IC of each wheel back to the point where the centreline of the wheel touches the ground.

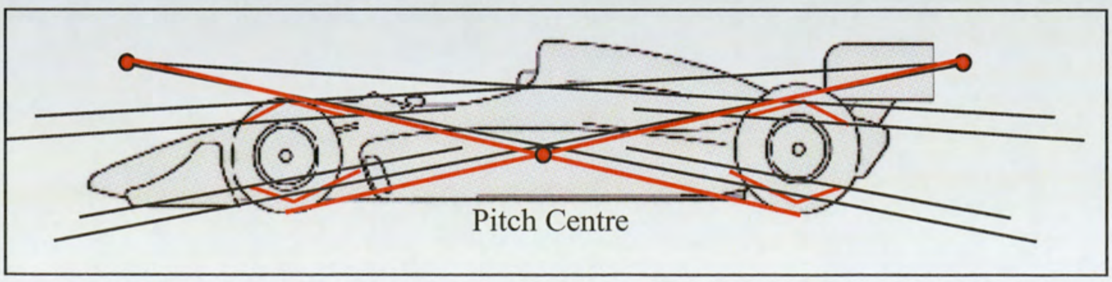


Figure 11-7 - Pitch Centre Definition

11.5.1 Anti-dive and Anti Squat

If the front axle side view IC is above the ground and behind the axle centreline the suspension will have some anti-dive characteristics. If the rear axle side view IC is above the ground and infront of the axle centreline the suspension will have some anti-squat characteristics.

Anti-dive and anti-squat parameters are measured as a percentage. Their value corresponds to the proportion of geometric and elastic weight transfer under longitudinal acceleration. See Section 14.2 for more details. The percentage anti-dive and anti-squat incorporated into a suspension system is normally quite low due to other geometric effects associated with the link locations, Section 14.2.2 has more discussion on this.

12 Steering Geometry

The front suspension in particular must have a steering system incorporated into it. The geometry discussed in Section 11 is used to give desirable camber control in bump and roll, but also has an effect on the wheel positioning when a steering angle is present. Associated with the steering axis, defined by the top and bottom wishbone to upright joints, are castor and kingpin angles.

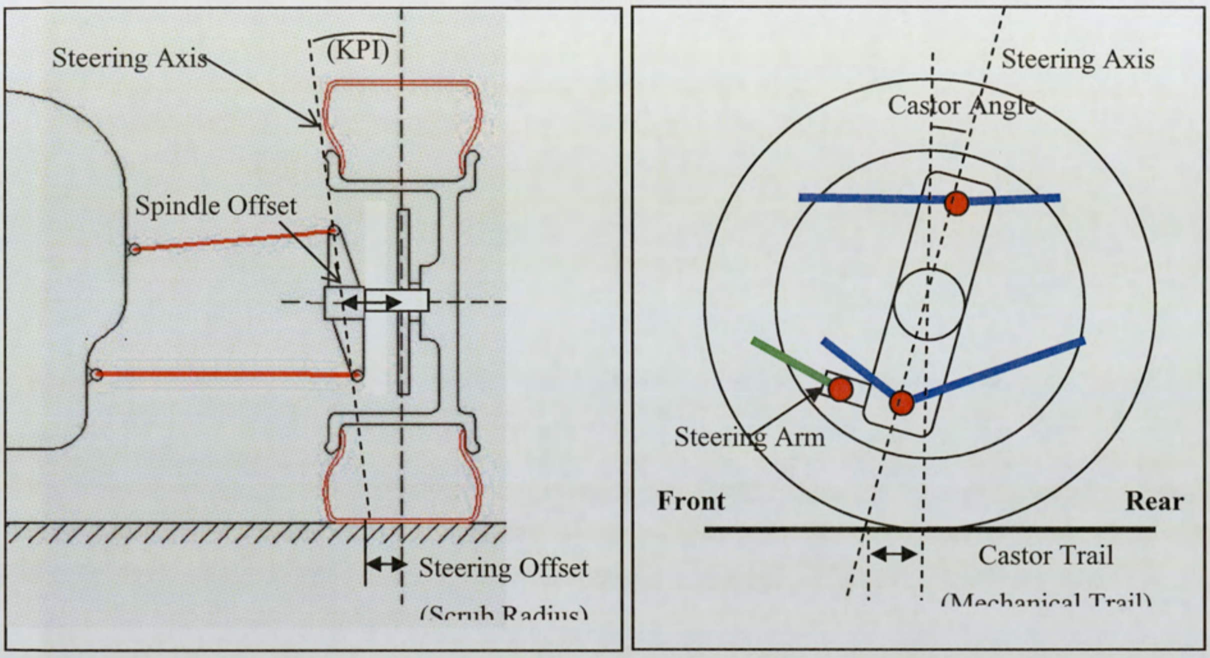


Figure 12-1- Kingpin and Castor Definitions

12.1 Kingpin

The term kingpin dates back to primitive hinge style joints used to pivot a wheel, the kingpin its elf is the bar that hinged sections pivoted on.

12.1.1 Kingpin Inclination (KPI)

Other References

- Kingpin Angle

Units = degrees (Positive shown in Figure 12-1)

Definition: The angle between the steering axis and the wheel centreline when viewing from the front. (Figure 12-1)

12.1.2 Steering Offset

Other References

- Scrub Radius
- KPI Trail
- KPI Offset

Units = mm (Positive to inside of wheel centreline- positive shown in Figure 12-1))

Definition: The distance between the point where the steering axis intersects the ground and the wheel centreline, when viewing from the front. (Figure 12-1)

12.1.3 Effects of KPI and Scrub Radius

KPI is normally incorporated into the front suspension to reduce the steering (scrub radius). Packaging within the wheel is normally difficult and it is unfeasible to install the top and bottom upright pivots on or close to the centreline of the wheel and vertically above each other. KPI can also be used to further reduce the length of the top wishbone, increasing the rate of reduction of the FVSAL during roll. KPI and Scrub radius have several affects, some desirable and some not.

- Jacking Effects – As a result of the KPI any movement of the wheel about the steering axis causes a lowering of the wheel or rising of the body. This effect is symmetrical from side to side without the presence of any castor angle.
- Camber Change During Steer – Steering movement will introduce positive camber to both inside and outer wheels. Spindle offset will affect the magnitude of the camber change, the longer the offset the more camber change.
- Steering Force – Tyre forces and scrub radius act together, creating a moment about the steering axis. Increasing the scrub radius increases the force that the driver feels as a result of any imbalance in forces between the two wheels. Wide tyres commonly used in racing are more likely to have an unequal force distribution within the contact patch as they are more sensitive to camber change. Consequently, reducing the scrub radius may

be less effective at eliminating excessive steering feedback, allowing packaging constraints to take precedence.

- Low Speed Steering Force – Forces required during low speed manoeuvring are increased by a large scrub radius.
- Stabilising – On front wheel drive vehicles, a negative offset is used to give a stabilising effect. If one wheel breaks traction the resultant steering torque generate but the wheel with traction causes it to pivot in a direction to counteract the yawing moment generated by the unequal driving force, encouraging the car to steer a straight line.

12.2 Castor

The castor angle included in steering mechanisms generates self aligning forces. The castor effect is generated by lag between the steering axis and the force centre of the contact patch (when viewed from the side).

12.2.1 Castor Angle

Other References:

- N/A

Units = degrees (Positive when the top mount is to the rear of car - positive shown in Figure 12-1).

Definition: the angle between the steering axis and a vertical line when viewing from the side. (Figure 12-1)

12.2.2 Castor Trail

Other References:

- Castor Offset
- Mechanical Trail (Opposed to Pneumatic Trail see Section 7.7)

Units = mm (Positive to inside of wheel centreline - positive shown in Figure 12-1).

Definition: The distance between the point at which the steering axis intersects the ground and the wheel centreline, when viewing from the side. As the name suggests the steering axis always intersects the ground in front of the centreline, hence the term trail. (Figure 12-1)

12.2.3 Effects of Castor Angle and Castor Trail

Castor trail is normally incorporated into the front suspension to introduce self aligning properties to the steering. Having the contact patch located behind the steering axis creates a moment about the steering axis as a result of any lateral tyre force. The handling effects associated with the size of castor trail and angle are:

- Aligning Torque – Increasing trail increases the moment created by lateral tyre forces about the steering axis and the consequential aligning force. This is the reason the steering axis always precedes the wheel centreline, as otherwise lateral forces would produce a destabilising moment. Castor trail normally provides the majority of aligning forces at speed and the associated high speed stability. Mechanical trial can be reduced to a very small distance to reduce steering input forces, at the cost of high speed self alignment.

- Camber Change During Steer – Like KPI, castor angle introduces camber change with steer angle, but contrary to KPI, it can be beneficial. With castor angle a steering angle induces negative camber on the outside wheel and introduces positive camber on the inside[12].
- Jacking Effects – Similar to KPI, any inclination of the steering axis has jacking effects. However, castor causes the inside wheel to raise and the outer to fall. The weight transfer is to the outside wheel and is therefore undesirable. The total weight transfer is a function of jacking height and roll stiffness.

- Driver Feedback – When cornering it is imperative that the driver can get the tyres as close to the limit as possible. A key force from the tyres that the driver feels is the self aligning torque acting though the steering mechanism. The total steering force felt by the driver is a result of the moment created by the tyre cornering force and the sum of both mechanical and pneumatic trail.

---

[12] Positive camber on the inside wheel is equivalent to negative camber on the outside. i.e. a good thing.

If mechanical trail is very much larger than pneumatic then the shape of the self aligning curve can be masked hiding, the telltale peak from the driver.

For Example:

With a given set of tyres, the pneumatic trial varies from 20 to 0mm between 80% 100% of maximum lateral force. Fitted to a car with 80 mm mechanical trail, as the limit is approached the driver will only see a reduction of approximately 20% in the steering force as the lever is now reduced from 100 mm to 80 mm long. On a car with 20 mm mechanical trial the driver would see approximately 50 % reduction in steering force.

12.3 Combined Effects of KPI and Castor Angle

KPI and Castor angle are usually used in some combination. The camber change with steer angle characteristics of each combine and the designer can to some extent remove the negative effects of KPI. The correct combination of KPI and castor can introduce camber on the outside wheel that is beneficial to the tyre. However, both KPI and castor angle have jacking effects. On the outside wheel it could be possible that they cancel each other out and no jacking occurs, but on the inside wheel they will add together and jacking will occur, transferring weight to the outside wheel.

12.4 Bump Steer

Both front and rear suspension systems have a steering system. On the front it is controlled by the steering rack, allowing the driver to steer the car. On the rear the steering is fixed, unless there is a four wheel steer system, and is set by the length of the toe arm. It is the aim of the designer to retain the steering angle pre-described by the driver or toe arm throughout the suspension movement. If the steering angle changes as the suspension moves through its travel, the car is said to bump steer. On a circuit race car where the suspension travel will be small, the presence of a small amount of bump steer might be acceptable, in fact it may be beneficial to tyre warming, on the other hand on an off-road car with long suspension travel the tolerances will be smaller.

The designer can change the bump steer characteristics by positioning the steering link joints. If the steering link is set to lie on a radial line from the front view IC and the joins are placed on lines that bisect the inner and outer wishbone joints, bump steer will be minimised. Designers often run the steering links on the plane defined by the three pivots of a wishbone, either directly in front or behind the wishbone, or in the case of some F1 teams, through the wishbone. This inline arrangement reduces both bump steer and frontal area.

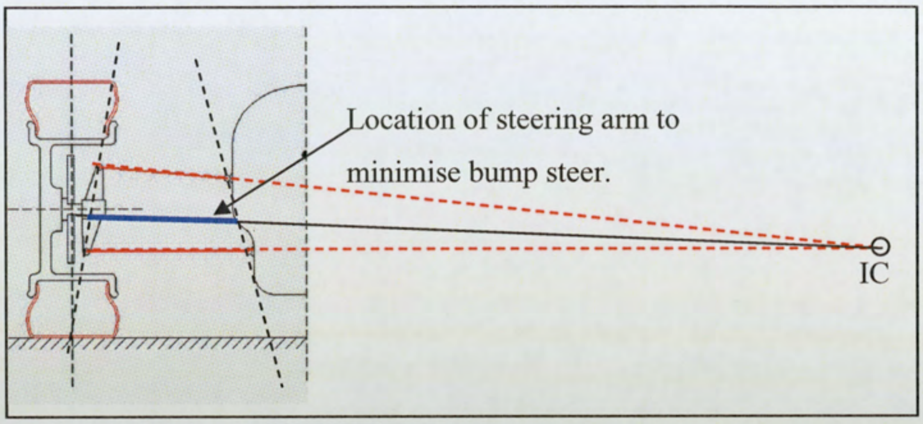


Figure 12-2 - Minimising Bump Steer

12.5 Roll Steer

Camber on a wheel has a steering effect, Milliken and Milliken (1995) suggest that as a rule of thumb, the ratio of camber to steer effect is about 10 % for radial tyres and 20 % for cross ply tyres. For example a cross ply tyre running at 1 degree camber will be equivalent to a steer angle of 0.2 degrees.

A car that has more camber gain in roll on the rear suspension than the front will loose camber on the front tyres during a corner. This will reduce the steer effect of the camber and the car might display understeer characteristics.

Another form of roll steer might result as a side effect of bump steer, in this case the positioning of steering components make the steering angle change as the suspension compresses in roll.

13 Wheel Alignment

The static alignment of the wheels can be used to give desired handling characteristics, firstly the definition of toe in and out shall be clarified.

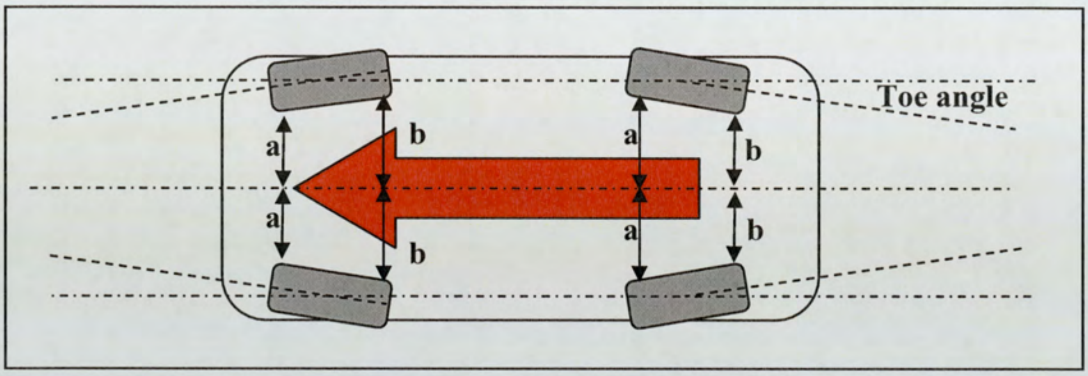


Figure 13-1 - Wheel Alignment Definition

- Toe In – The front of the wheel is closer to the vehicle centreline than the rear of the wheel. Distance ‘a’ is less than distance ‘b’. In the diagram the front wheels are toeing in.
- Toe Out – The front of the wheel is further from the vehicle centreline than the rear of the wheel. Distance ‘a’ is greater than distance ‘b’. In the diagram the rear wheels are toeing out.

13.1 Measuring Wheel Alignment

Wheel alignment (tracking), should be measured at wheel centre height. The easiest way to measure wheel alignment is to project the planes of the wheel fore or aft of the car and measure the distance between the lines at two points a known distance apart. The problem with this method is that the centreline has not been taken into account. On the front this is not such a problem as the steering can be corrected to set the wheels symmetrically to the centreline. On the rear there is no steering and this may be a problem.

The best way to measure wheel alignment is individually for each wheel, back to the centreline. Two frames can be made that attach to the front and rear of the car, extending a known distance outside of the widest track. If the ends are joined with two pieces of wire of the same length and the centre of the frames are on the vehicle centreline, the wires will be parallel to the centreline at a known distance. The distance from the wire to the wheel rim can be measured for each and the alignment set accurately to the centreline.

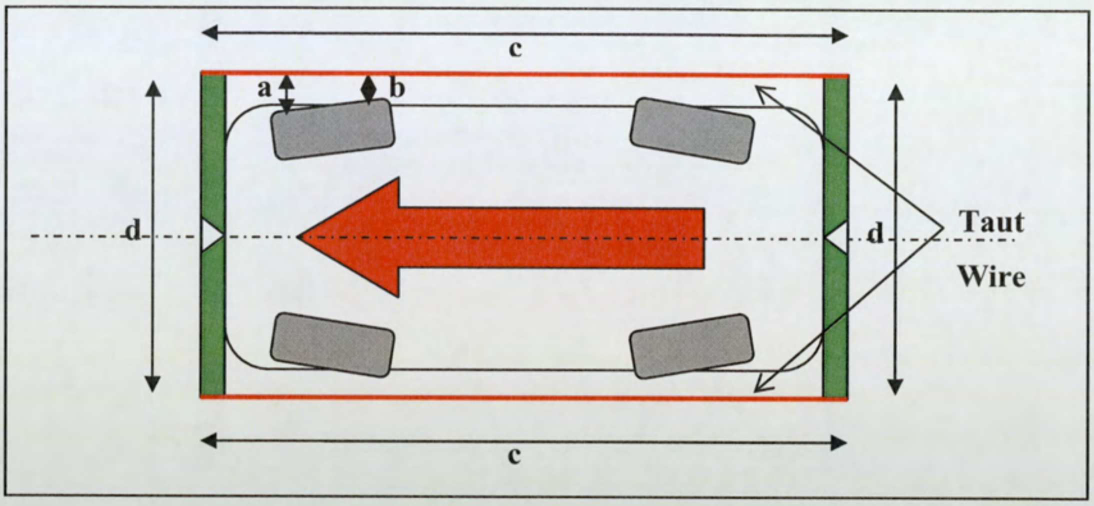


Figure 13-2 - Measuring Wheel Alignment

13.2 Effects of Wheel Alignment

Any wheel alignment out of plane with the vehicle direction will in effect create a slip angle and associated lateral force. This lateral force might have some stabilising or destabilising effect in a straight line, or it may aid or hinder corner entry or exit.

13.2.1 Front Wheels

The following effects of wheel camber are discussed by Smith (1978) in a questions and answers section starting on page 137. The following is an adaptation and elaboration on Smith's writing.

Toe in

- Too Much – The car is unstable under braking and is twitchy over bumps, as both tyres are producing a significant lateral force. As soon as one wheel bumps or the vertical load changes on it the resultant lateral force is high and yaw occurs.
- Too Little – Rear wheel drive cars are normally set with a little toe in or dead ahead since the lateral thrust gives stability to external inputs like cross wind. So not enough toe in will reduce stability to external input.
- Any – Ultimate cornering force may be compromised. The inside tyre has to go through a neutral lateral grip instance as the initial thrust is in the wrong direction. There is a

possibility that the inside tyre might not generate any thrust at all, if the steering angle is such that it is not creating a slip angle.

Toe Out

- Too Much – Instability in a straight line, weight transfer from external inputs causes the outside tyre to steer with the force, more wandering opposed to twitching. Initial turn in is good as outside tyre stops producing a lateral force that is opposing the inside wheel. The consequent large steering angle on inside tyre causes the initial response. Weight transfer to the outside wheel then occurs, where the outside wheel has only a small steering angle and the car does not take the initial line.
- Too little – Front wheel drive cars are normally set with mild toe out to improve stability and reduce torque steer. If a wheel breaks traction a yaw moment is created by the force from the other tyre still with traction. Toe out produces a lateral force that creates a moment opposed to the initial yaw moment, naturally steering against the torque steer.

13.2.2 Rear Wheels

Toe In

- Too Much – Straight line instability, similar reason to front wheels, lateral forces are too high and any fluctuation in a wheel loading causes a large imbalance and yaw effect.

- Too Little – Car is loose on corner exit, longitudinal weight transfer reduces rear tyre effectiveness. Added traction loads combined with a reduced steer angle stop the tyre producing enough lateral grip.

Toe Out

- Any – Toe out on the rear of the car is associated with a loose car on the corner exit. The reduced steer angle on the rear wheel and the added weight reduce the stabilising moment, normally the car oversteers progressing to spin. Toe out on the rear tyres would enhance turn in, for the same reasons it makes the car over steer.

14 Weight Transfer

Weight transfer must be looked at in more detail before the suspension rates can be calculated. In Section 6.1.2 a simplified model of the car was made that took no account of suspension characteristics. Although reasonably accurate for calculating the total weight transfer from inside to outside, it does not allow the designer to take account of the effects of roll stiffness.

14.1 Lateral Weight Transfer

A more detailed model must take into account the weight of front and rear unsuspended and suspended components, as well as the effects of the roll centre height.

Suspended (Sprung) Mass – The mass components of the car whose weight acts through the springs and dampers. I.e. frame, engine, driver, transmission and body shell.

Unsuspended (Unsprung) Mass – The mass of the components of the suspension and driveline that are not supported by the springs and dampers. (Wheels, tyres, uprights, driveshaft and brakes)

The order in which this problem will be addressed is as follows:

- Total weight transfer (Section 6.1.2)
- Suspended and unsuspended weight transfer single axle
- Suspended weight transfer breakdown
- Weight transfer all wheels together

During these calculations some assumptions will be made. These assumptions only create small errors in the final results but greatly simplify the calculation. They are:

- The chassis is stiff and the total of the roll moment is distributed according to the front and rear roll stiffness.

- The CoG of unsuspended mass is approximately located at the centre of the wheels and is symmetrical. Consequently all forces associated with it act through a point on the centreline of the car at the wheel centre height.
- Roll angles will be small and as such do not create significant change in the height of the CoG.
- The CoG of the suspended mass is on the centreline of the vehicle.
- The vehicle is in steady state lateral acceleration.
- The roll centre is stationary and is not affected by suspension roll.

14.1.1 Suspended and Unsuspended Weight Transfer Single Axle

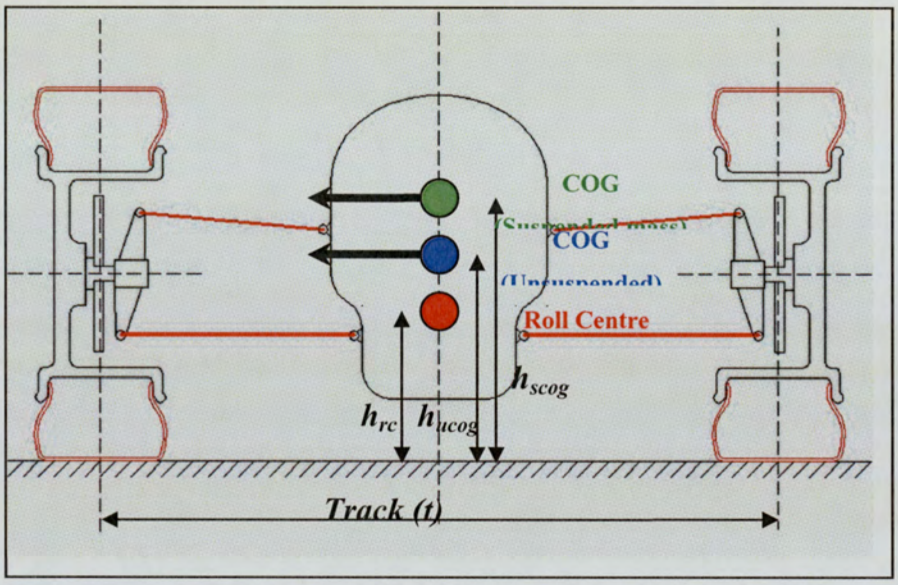


Figure 14-1 - Lateral Weight Transfer Components

The weight of both suspended and unsuspended components contribute to weight transfer, however the height of the COG of the two masses might be significantly different and as the transfer of unsuspended weight does not contribute to body roll they must be distinguished.

$$W_{latU} = \frac{m_u \times a \times h_{Ucog}}{t}$$

$$W_{latS} = \frac{m_s \times a \times h_{Scog}}{t}$$

Weight transfer is fixed for a given lateral acceleration, track and COG height, it is important to remember that anti-roll and anti-squat/dive devices do not reduce the amount of weight transfer. These devices or geometry simply change the effects that the weight transfer has on the position of the body and the distribution of weight transfer.

Weight transfer from the unsuspended components acts through the suspension arms and therefore does not cause any body roll or extra load on the springs.

Transfer of suspended weight is affected by the height of the roll centre and can be broken down further.

14.1.2 Suspended Weight Transfer Breakdown

A force on a body at a known point can be represented at any other point on the body by a force of the same magnitude and direction and a moment that is equal to the force times the perpendicular distance between the two points. Looking back to the definition of the roll centre (Section 11.3) it is a point at which a lateral force can be applied to the suspended mass without the creation of a roll moment, i.e. it is the pivot of the body rotation. By representing the lateral force at the CoG as a moment and force at the roll centre it is possible to differentiate a force at the roll centre that causes no roll and a moment about the roll centre.

Claude Rouelle (2003) suggests that the weight transfer that results from the force on the roll centre should be referred to as geometric transfer, since it is a function of the car's suspension geometry. He also suggests that the weight transfer caused by the roll moment should be called elastic transfer, since it acts through the dampers and springs, causing roll. Milliken and Milliken (1995) simply refer to roll moments and transfer forces. It is not important how the forces are referred to, just that they are fundamentally different. More important, are the effects of the loads and the paths through which weight is transferred. In this thesis Rouelle's concept of elastic and geometric weight transfer will be adopted.

The sum of the two components will be the same as the total calculated for suspended components in Section 14.1.1.

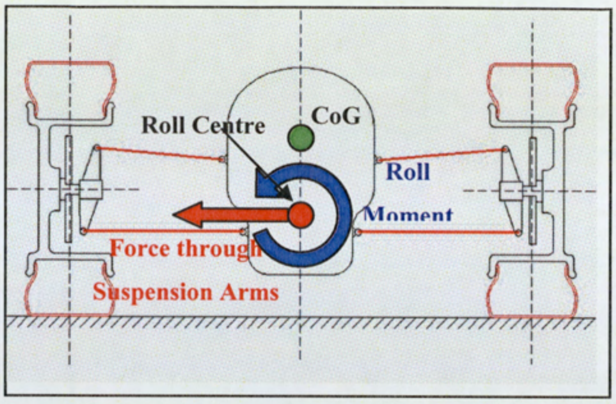


Figure 14-2 - Roll Moment and Geometric Force

$$W_{latGeom} = \frac{m_s \times ag \times h_{rc}}{t}$$

$$W_{latElas} = \frac{m_s \times ag \times (h_{Scog} - h_{rc})}{t}$$

14.1.3 Suspended Mass Weight Transfer and Roll Centres

Apart from the height of the CoG the roll centre is the only other variable that affects the proportion of elastic to geometric weight transfer. The table below summarises the effects of different roll centre heights.

	Moment (Elastic)	Force (Geometric)	Summary
RC above ground	Small	Large to outside wheel	Small roll moment so little weight transfer taken by the springs. Majority of transfer is taken by suspension arms as roll centre is above ground.
RC on ground	All	None	The force is acting at the same height as the road tyre interface, pivot point in the system, so no weight transfer results from it. Consequently, all weight transfer must be as a result of the roll moment and act through the springs.
RC below ground	V large	To inside wheel	Geometric weight transfer causes geometric weight transfer to the inside wheel. The roll moment is now even larger since the RC is a long way from the CoG, counteracting the geometric transfer to the inside wheel.

Elastic weight transfer has the added complication of acting as a result of movement. The suspension springs compress whilst resisting the force generated, the movement associated with this compression is time dependent. The time taken for the body to roll to the given angle at which the force in the springs resists all of the roll moment depends on the suspended masses' roll inertia.

14.1.4 All four wheels together

Similarity is still retained with the single axle scenario, however, the CoG is now between the axles and the proportioning of geometric loads becomes a ratio of the dimensions x and y. As the roll moment (elastic component) acts along the roll axis, the load taken at the front and rear becomes a function of the front and rear roll stiffness (see Section 15.5). The unsuspended weight transfer is calculated individually for front and rear and applied respectively.

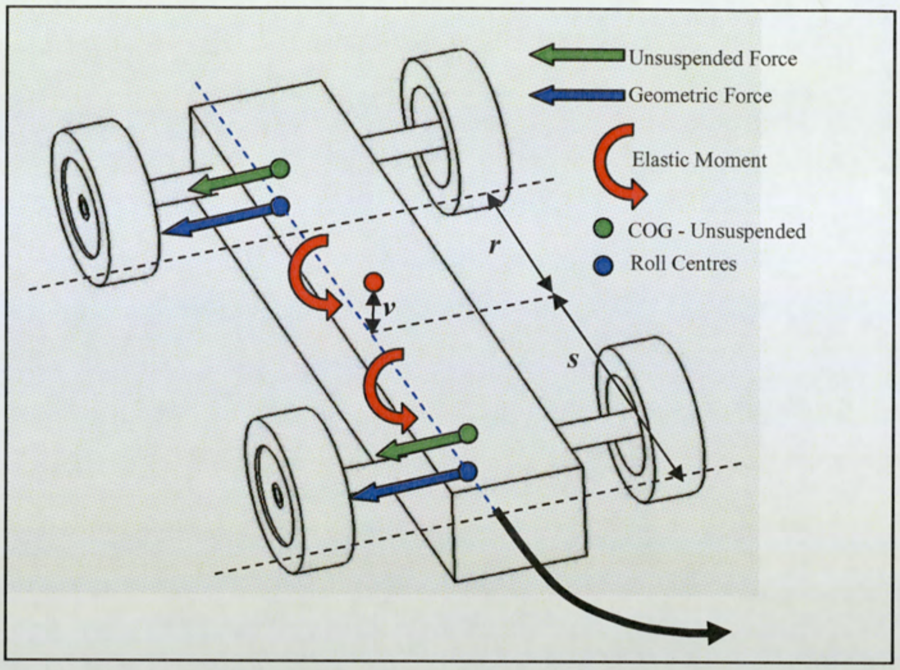


Figure 14-3 - Weight Transfer - All Four Wheels

Giving:

Unsuspended lateral weight transfer:

$$FW_{LatU} = \frac{Fm_U \times ag \times Fh_{UCoG}}{Ft}$$

$$RW_{LatU} = \frac{Rm_U \times ag \times Rh_{UCoG}}{Rt}$$

Geometric weight transfer:

$$FW_{LatGeom} = \frac{m_S \times ag \times \left(\frac{s}{r+s}\right) \times Fh_{RC}}{Ft}$$

$$RW_{LatGeom} = \frac{m_S \times ag \times \left(\frac{r}{r+s}\right) \times Rh_{RC}}{Rt}$$

Elastic Weight Transfer:

$$v = h_{CoG} - ((Fh_{RC} - Rh_{RC})\frac{r}{r+s} + Fh_{RC})$$

$$FW_{LatElas} = \frac{m_S \times ag \times v \times \left(\frac{FS_{roll}}{FS_{roll} + RS_{roll}} \right)}{Ft}$$

$$RW_{LatElas} = \frac{m_S \times ag \times v \times \left(\frac{RS_{roll}}{FS_{roll} + RS_{roll}} \right)}{Rt}$$

Adjusting any of the variables on a race car that affect the distribution of geometric weight transfer is normally difficult. On the other hand, roll stiffness is readily adjustable and is consequently the focus of the engineer's attention when trying to balance the car. Changing front or rear roll stiffness changes the weight transfer seen at each end. The effects of reducing the load transferred on the front axle during cornering, when combined with tyre load sensitivity, could be a net gain in lateral force at the front axle, curing a push problem.

14.1.5 Weight Transfer due to Steering Geometry

In Section 12 the relative merits and benefits of castor and kingpin inclination were discussed, as were their jacking effects. As a steering angle is created the front of the car will roll. This static weight transfer has the same effects as jacking or wedging the car (raising or lowering diagonally opposed wheels) and can be measured. The total weight transfer will be a function of the roll angle created and the ratio of front to rear roll stiffness. The easiest way to measure weight transfer through steering geometry is with corner scales and turn plates.

14.1.6 Lateral Weight Transfer per g

Other References

- n/a

Units = N/g

Definition: The weight transfer due to the physical dimensions of the car and the height of the CoG, normalised by gravity.

14.2 Longitudinal Transfer

In a similar way to lateral weight transfer, longitudinal weight transfer can be broken down into an elastic and geometric force. The geometric force is generated as a result of the location of the pitch centre relative to the ground and the CoG. If the pitch centre is located on the ground all of the pitch moment will be absorbed by the springs and dampers.

Anti-dive and anti-squat parameters are measured as a percentage. The percentage relates to the amount of longitudinal weight transfer that will be taken as a geometric force. For example if the suspension has 5 % anti-dive, 95% of the total longitudinal weight transfer will be taken by the springs whilst 5% will be taken by the suspension arms.

Longitudinal weight transfer can be easily calculated with the same basic methods that were used in lateral weight transfer.

14.2.1 Longitudinal Transfer per g

Other References

- n/a

Units = N/g

Definition: The weight transfer due to the physical dimensions of the car and the height of the CoG, normalised by gravity.

14.2.2 Effects of Anti-Squat and Anti Dive Geometry

There are advantages and disadvantages associated with anti-dive and anti-squat geometry.

Advantages:

- Instantaneous weight transfer – Load is transferred geometrically and instantaneously, so rear tyres get maximum traction faster.
- Limited suspension travel – Better aerodynamic stability is achievable as the attitude of the car to the ground remains more constant.
- Less load on springs – Springs can be softer for better road contact as the load seen under longitudinal acceleration is less

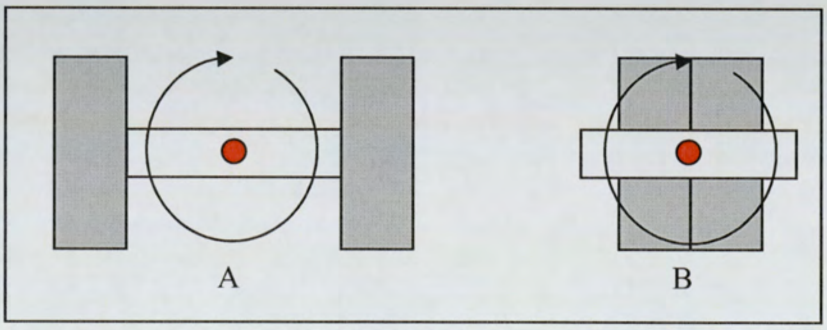
- Limited suspension travel – Transitions into and out of corner are smoother since the attitude of the car changes less, reducing the effects of camber change with suspension movement.

Disadvantages:

- Castor angle change – The wishbone pivot lines required to generate anti-dive/squat geometries dictate that the castor angles will change as the suspension travels. The effects of castor angle are discussed in Section 12.2.3.
- Roll Centre Migration – Associated with a change in castor angle must be a change in the angles between the wishbones when looking from the front view. Inevitably, the angle change affects the location of the IC and roll centre.

14.3 Moments of Inertia

Distribution of components within a vehicle will affect its moments of inertia about all three axes (origin at the CoG). The magnitude of the moments of inertia are not directly affected by the vehicle mass but are affected by the location of the mass. For example two cars of the same mass might have different polar moments of inertia. Similarly, two cars of different mass might have the same polar moments of inertia. The dumbbell analogy demonstrates this:



Although arrangements A and B have the same mass and CoG location the moment of inertia about the axes of theCoG (direction indicated by the arrow) will be greater for A than B.

14.3.1 Consequences of Moments of Inertia

Inertia should not affect steady state handling, since it does not alter weight transfer. However, moments of inertia have a considerable effect on transients, the response time of the car to a driver input. Inertia acts against any moments generated by the tyres or weight transfer, reducing peak roll, yaw and pitch accelerations and velocities. Consequently, the time to reach a steady state is affected.

Roll Inertia

Lowering the roll inertia has similar effects to raising the roll centre towards the CoG, in that weight transfer will be instantaneous and the time to reach steady state will be reduced. As roll inertia increases, lateral weight transfer and steady state will be delayed. This may continue to the point where peak roll might be seen after peak lateral acceleration. The effects of the roll will then be seen after the corner has been exited. Some degree of roll inertia will increase stability since small or rapid inputs will not cause excessive roll or rapid weight transfer.

Yaw Inertia

For a slip angle to be generated on the rear tyres the chassis must yaw. Until this slip angle and associated force have been generated the car cannot achieve maximum lateral acceleration. Thus, the time taken for the car to reach its maximum lateral acceleration can be considered undesirable. As such, the yaw acceleration achieved for a given destabilising force from the front tyres can be considered performance related and should be maximised.

Yaw inertia is greatly affected by engine position. The plan layout of the chassis should be a compromise of weight distribution and inertia, hence the desire to place the heaviest components close to the desired CoG location. This may not always be possible due to other packaging constraints. A 50:50 weight distribution is more obtainable with a front mounted engine rearward driver and fuel tank. However, this arrangement is long and yaw inertia will be high. A mid-mounted engine with a low fuel cell and driver in front is more compact and the moment of inertia will be lower. Often rules about the position of the driver's feet relative to the front axle usually cause this arrangement to have a weight distribution biased to the rear.

Pitch Inertia

For maximum longitudinal acceleration, pitch inertia would be reduced, increasing the speed of weight transfer to the rear wheels and decreasing the time before maximum traction force is available. On the other hand for stability and turn in it would be desirable to have larger pitch inertia, reducing sensitivity to sudden inputs by increasing the time taken for longitudinal weight transfer to occur.

15 Suspension Rates

15.1 Spring Rate

Other References:

- N/A

Units = Coil Spring – N/mm, pounds/inch or kg/cm
Torsion Bar Spring – Nm/degree or N/mm (displacement measured at arm)

Definition: 'Force per unit displacement for a suspension spring alone. For coils springs this is measured axially along the centreline. For torsion bar springs it is measured at the attachment arm.' Milliken and Milliken (1995) p580.

15.2 Wheel Rate

Other References:

- Wheel Centre Rate

Units = N/mm

Definition: 'Vertical force per unit vertical displacement at the location along the wheel spindle corresponding to the wheel centreline, measured relative to the chassis.' Milliken and Milliken (1995) p581.

15.3 Tyre Spring Rate

Other References:

- See Section 8.2.1.1 for more details and sample data

Units = N/mm

Definition: 'Vertical force per unit vertical displacement of the tyre at its operating load.' Milliken and Milliken (1995) p581.

15.4 Ride Rate

Other References:

- N/A

Units = N/mm

Definition: 'Vertical force per unit vertical displacement of the tyre ground contact reference point relative to the chassis. This is equal to the wheel centre rate modified by the tyre vertical rate. For an infinitely stiff tyre, the ride and wheel centre rate would be equal. For a real tyre (with vertical stiffness) the ride rate is always less than the wheel centre rate.' Milliken and Milliken (1995) p581.

15.5 Roll Rate

Other References:

- Roll Stiffness

Units = Nm/degree of body roll

Definition: 'moment resisting body roll per degree of body roll. The term can be applied to either an individual axle or to a complete car. The resistance to body roll is provided by the ride rates, axle track width and anti-roll bar.' Milliken and Milliken (1995) p581. The total roll stiffness being the sum of front and rear roll rates.

As mentioned above, the roll rate consists of two components, that arising from the springs and that from the anti roll bars. In the absence of anti-roll bars all the roll resistance comes from the springs. It is common to express the contribution of each component to the total roll stiffness as a percentage of the total roll stiffness.

Roll rates can then be normalised to give roll per unit of lateral acceleration, commonly degrees/g and can be quoted individually for front and rear axles. This value then neatly takes into account weight, weight distribution, roll stiffness distribution and roll centre heights. According to Milliken and Milliken (1995) p584 the value for race cars typically is 1.5 deg/g or less.

15.6 Motion Ratio

Other References:

- Installation Ratio

Units = dimensionless

Definition: Motion ratio relates displacement in the spring medium to displacement at the wheel centre. If the length of the spring is given by x and the distance between the wheel centre and a point on the chassis, directly above the wheel centre is y, then motion ratio is $\Delta x/\Delta y$.

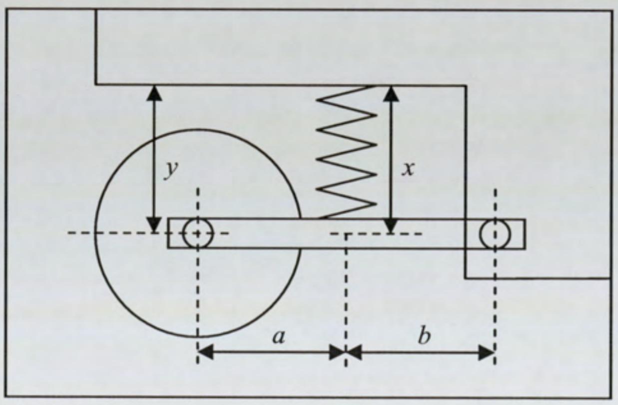


Figure 15-1 - Motion Ratio Definition

If a = b, for a 10 mm change in '*x*' a 20 mm change in '*y*' will occur, making the motion ratio 10/20 or 0.5. If the spring rate is 100N/mm then the spring has a force of 1000N on it and the force at the wheel is 500 N. The wheel rate is given by 500 N / 20 mm = 25 N/mm. The motion ratio (MR) causes a reduction in spring movement and a reduction in wheel rate, consequently the relation between spring rate (SR) and wheel rate (WR) is $WR = SR(MR)^2$

15.6.1 Rising Rate Suspension

The above is true assuming that the motion ratio is constant over the complete suspension range. The likelihood is that the geometry of the rocker system or the positioning of the spring will have been chosen to not give a linear motion ratio. The reasoning behind this is to give sufficient suspension compression in a static state such that there is ample rebound movement, but limited compression with weight transfer and down force loads. If this is the case the easiest method of analysing the motion ratio is with computer software or a spreadsheet, calculating for a series of equal wheel movements.

For all the benefits of non-linear motion ratios there is one major drawback. This is the adjustability of the suspension during setup. When setting the ride height, all of the rockers must be in the same position and the pull/push rods should be set to the desired length. Corner weights can only be adjusted using the spring platforms and not the pull/push rod lengths, otherwise the motion ratios will change from wheel to wheel, allowing the car to have dissimilar roll rates from right to left. In the worst case, the rate of change of motion ratio for each wheel will differ, making setup a very complicated affair.

16 Damping

In order to achieve maximum grip from the tyres they need to be in contact with the road and with a constant reaction. If the load fluctuates too much, tyre load sensitivity will affect the available grip. As the suspension system consists of a mass and spring it therefore has a natural frequency. Both unsuspended mass and suspended masses have different natural frequencies. Any mass suspended by a spring will, when given a displacement from its stationary position will oscillate at the natural frequency about the stationary position. The components of the car are no different and oscillate as the result of inputs from the road. Theoretically, the mass would continue to oscillate at that frequency indefinitely if no external forces were acting. The purpose of the damper is therefore to provide external resistance, reducing the oscillations and returning the wheel to the ground.

The spring-mass-damper system can be used to simply represent the vehicle and the suspension system. In this basic analysis the sprung mass 'm_s' is equal to a quarter of the suspended mass of the actual car. Hence, this is also know as the quarter-car model.

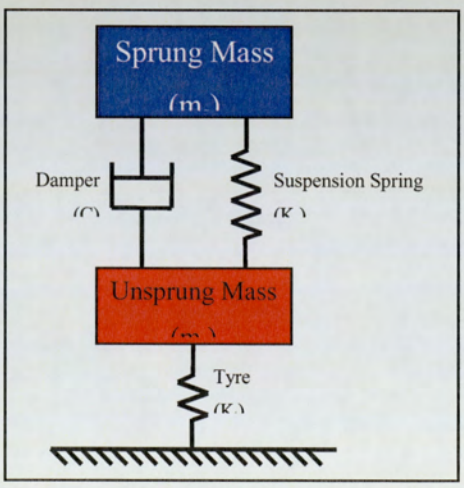


Figure 16-1 - The Mass Spring Damper Model

16.1 Suspension Motion

At this point it is necessary to define the motions that the suspension is likely to move in.

Bump (Compression) – A vertical displacement of the unsuspended mass relative to the suspended mass that causes a compression in the spring medium. Named such as it is the movement caused by driving over a bump.

Droop (Extension) – A vertical displacement of the unsuspended mass relative to the suspended mass that causes an extension in the spring medium. The downwards motion of a wheel relative to the chassis.

16.2 Chassis Motion

In addition to roll, pitch and yaw the independent suspension allows for further motions.

Heave (Bounce)- Movement of the suspended mass up and down, relative to the unsuspended mass, when all four wheels are in phase. I.e. all four suspension systems are in compression or rebound at the same time. The chassis remains parallel to the ground but the distance between the two changes.

16.3 Chassis Natural Frequency

Other References:

- N/A

Units = Hz

Definition: The frequency at which the suspended mass will oscillate as a result of a displacement in the suspension spring, if the wheel is stationary.

Calculation

$$\omega_c = \frac{\sqrt{\frac{K_s}{m_s}}}{2\pi}$$

Where m_s is the calculated quarter-car mass acting on that wheel in a static condition.

16.4 Wheel Natural Frequency

Other References:

- Wheel hop rate

Units = Hz

Definition: The frequency at which the unsuspended mass will oscillate as a result of a displacement in the suspension spring or tyre, if the chassis is stationary. According to Milliken and Milliken (1995) p792, wheel hop generally happens and becomes a problem when transmissibility[13] exceeds 2.5.

Calculation:

$$\omega_c = \frac{1}{2\pi}\sqrt{\frac{K_s + K_t}{m_u}}$$

16.5 What Does It All Mean?

When designing a race car suspension system, the aim is to devise a system that responds quickly and keeps the tyre on the ground and with the best possible vertical load. In Section 11-Suspension Kinematics, geometries to keep the wheel on the road at the right angle are discussed. The damper and suspension spring then try to keep best possible force between the tyre and the road.

Another suspension design concern is reducing the time that the car takes to reach steady state as result of lateral or longitudinal acceleration. Reducing suspension movement and unsuspended mass helps by reducing displacement and the force needed to accelerate the mass. Limiting

[13] Transmissibility – The ratio of input amplitude to output amplitude (see section 16.6.3)

suspension travel is also of benefit as roll angles are reduced, improving camber control. As a result spring rates increase to limit travel under the expected loads, increasing the natural frequency of the suspended and unsuspended masses.

The natural frequency of the suspended mass front and rear are in effect a measure of the suspension stiffness in comparison to the mass of the vehicle. Specifying wheel natural frequencies gives an indication of suspension movement, mass and dynamic performance. Taking it further, the designer can specify damping constants based on natural frequencies so that the car can extract as much performance from the tyres as possible.

16.6 Damping the System

The damper applies a force against the motion of the suspension in order to reduce oscillation. The force that the damper provides is proportional to the damper constant 'C', the units for 'C' are Newton/metre/second (Ns/m). When C = 0 the system will oscillate at its natural frequency indefinitely. A damper provides a force proportional to the speed that it is moved at.

The driving force for the oscillations comes from the road surface or roughness. The frequency of the input from the road surface is broad, due to the car's varying speed and the wavelength of the features in the road surface.

A basic analysis of a standard road surface gives insight into the nature of the stimuli to the suspension system. According to Dixon (1999) p118 the relation of wavelength to displacement is fairly proportional when considering the road surface. Thus the input from the road surface can be classed as constant speed with varying frequency or white noise.

Inputs from the road surface i.e. bumps and kerbs are grouped as high speed inputs and normally greater than 100 mm/second. High speed inputs affect vertical load consistency on the tyre. It is worth noting that the spectral distribution of the stimuli is not linear. The amount of longer wavelength disturbances is greater than short, so as vehicle velocity increases, the amount of low frequency input increases. Dixon (1999)p119 discusses this with reference to handling and the ISO defined standard equation to describe the spectral distribution.

The second input to the suspension system comes from weight transfer, be it lateral or longitudinal. The speed with which the suspension responds to these inputs affects the rate of weight transfer and transient control. The speeds of these displacements are considerably lower, they can be grouped as low speed and are typically less than 25 mm/sec.

16.6.1 Damping Ratio

Other References:

- n/a

Units = Dimensionless
Nomenclature = ζ

Definition: "The damping ratio is a convenient mathematical tool that shows the influence of the damping constant, 'C', on transient response." Milliken and Milliken (1995) p236. When 'C' is nonzero the transient response to an input in the time period between disturbance and coming to rest can be described with ζ.

Calculation:

$$\zeta = \frac{1}{2}\left(\frac{C}{m\omega_n}\right)$$

Or

$$\zeta = \frac{1}{2}\left(\frac{C}{\sqrt{Km}}\right)$$

16.6.2 Critical Damping

Other References:

- n/a

Definition: In a critically damped system the damping constant 'C' is such that the system will return to the static state in the minimal time without any overshoot or oscillation. For this system ζ is equal to 1.

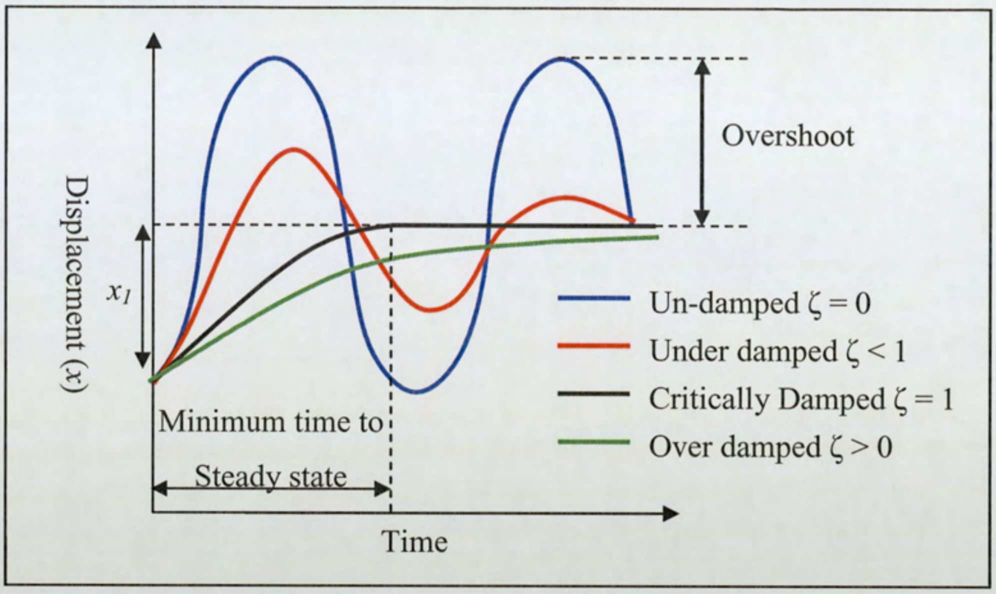


Figure 16-2 - Critical Damping Definition

The above figure shows a generalised frequency response plot to a step input of x_1 for a range of vales of ζ.

The critical value of C (C_{crit}) can then be found by solving when ζ is 1.

$$C_{CritSuspended} = 2\sqrt{K_s m_s}$$
$$C_{CritUnsuspended} = 2\sqrt{(Ks + Kt) m_u}$$

The damping ratio can also be expressed as:

$$\zeta = \frac{C}{C_{crit}}$$

If we firstly look at the natural frequency of the suspended mass we see that it is quite low typically 1-2 Hz. There is input to the wheel at or around this frequency from the road surface, the amount of which increases as vehicle speed increases. If left un-damped, through resonance, the amplitude of the oscillations of the suspended mass will tend to infinity. This must be reduced or damped, to stop the system becoming unstable. As the damping ratio of the suspension is increased, C increases towards C_{crit} and for the suspended mass, oscillations at the natural frequency will cease. As an inverse effect of this, the suspension will transmit more vibration to the unsuspended components across the rest of the frequency range, reducing the ride quality.

In order for passenger cars to have comfortable rides damping is kept to a minimum to suppress major oscillations at the natural frequency. Installing more suspension travel and softer springs and consequently lowering the natural frequency of the suspended mass reduces the passenger's awareness to the oscillation. Light damping preserves ride quality across the rest of the frequency range reducing harshness. However, this kind of setup is soft and is said to float, steering input is sluggish and roll is severe. On a race car where performance is paramount over ride quality the damping ratio is increased at the cost of ride quality improving steering response and high speed stability. Rubber pivot bushes are also replaced with solid bearings, further improving response.

16.6.3 Damping for Handling

The ideal path for the wheel centre would match exactly that of the road surface. If the wheel traced the same profile as the ground there would be no deflection in the tyre and hence no change in vertical load applied to it. The term transmissibility refers to the ratio of the wheel displacement to road displacement measured from some stationary axis (ratio of input amplitude to response amplitude). If transmissibility is 1 then the wheel is exactly following the road and the tyre is not subjected to any change in vertical force. Consequently, the transmissibility of the system can be used as a measure of performance with respect to tyre conditions.

The major concern in racing is obviously tyre performance, not ride comfort. The designer tries to specify or design a damper that can effectively deal with both high and low speed inputs, with the focus of its design on the performance of road contact.

The level of high-speed damping required to approach a transmissibility of 1 is subject to spectral analysis of the particular race track, mass of vehicle components, tyre and spring stiffness, motion ratios and vehicle speed. It may not be feasible to carry out a detailed study of each race track, analyse the data and then specify a damper for optimum performance. However, it is possible to specify a general damper that should perform well under most conditions whilst having a certain degree of adjustability.

As expected and confirmed by Dixon (1999) p54 road holding reaches an optimum as the damping ratio approaches unity. Whist the optimum for comfort zeta would be closer to 0.2.

16.6.4 Shock Absorber Bump and Rebound

The damper must operate in both directions, double acting. However, it stands to reason that optimum damping might not be the same in both directions. In compression, bump, the damper is mainly controlling the unsuspended mass. Whilst in rebound, droop, it is controlling the motion of the suspended mass. We know that both of these components have different mass and as such require different levels of damping. Damper valves are set to give more damping in droop to control the larger suspended mass.

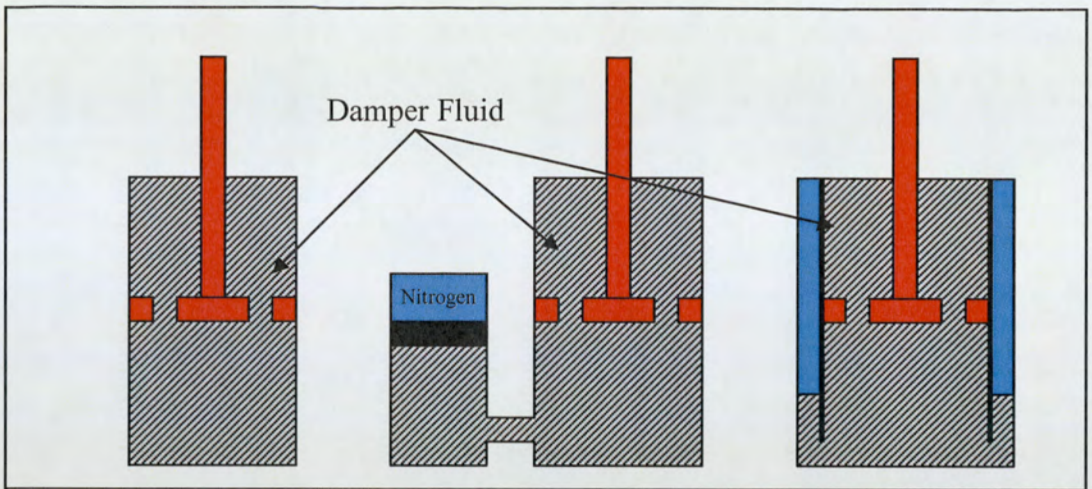
The exact ratio of droop to bump damping required for optimum handling depends on the track, vehicle and driver. It commonly varies from 3:1 for ride quality to possibly 5:3 for low mass handling orientated racing car. Milliken and Milliken (1995) suggest that for a racing car the ratio is normally in the order of 2:1 droop to bump. Whilst, Dixon (1999) p135, proposes that a ride optimised configuration would be around 20/80 bump to rebound damping, shifting to a more equal 40:60 for handling.

16.7 Damper Construction

16.7.1 Telescopic Hydraulic Dampers

The telescopic hydraulic damper uses a piston with some form of fluid either side of it, normally incompressible. As the piston is moved fluid is forced from one side of the piston to the other. As the damper compresses, more of the shaft enters the pressure chamber, reducing its volume. Consequently, some form of expansion chamber is needed to accommodate the excess fluid. Commonly, expansion chambers on racing dampers have a floating piston or bladder to separate the gas and fluid. Dampers for passenger vehicles often rely on gravity to separate gas and fluid, however this design is sensitive to mounting orientation and under severe duty the gas and fluid can be come emulsified, reducing the damping effect.

The passage that the fluid takes can be sized such that the force required to create movement is considerable, this force is the damping force. With the addition of one-way valves the path can be controlled to change the damping force in both directions of travel.



LOCKED	SINGLE TUBE	TWIN TUBE
The volume difference between the top and bottom chambers stops the piston from moving	The volume difference between the top and bottom chambers is compensated by compression or expansion of gas in the reservoir.	The volume difference between the top and bottom chambers is compensated by compression or expansion of gas in the reservoir, located

Figure 16-3 - Damper Types

16.7.2 Single Tube

More commonly used in motorsport, the single tube can be easily cooled and the addition of valves between the reservoir and the piston chamber allow external adjustment of low speed damping. The main disadvantage is poor damage resistance, as the piston chamber is not well protected. The external reservoir may either be fitted with a bladder or floating piston. The gas used can also be pressurised to reduce the risk of cavitation in the damping fluid. Commonly, the gas used to pressurise the damper is nitrogen, reducing the effects of heating. As the gas and fluid are separated the damper can be installed at any attitude.

16.7.3 Twin Tube

More commonly found on passenger vehicles, due to its compact design and durability. The top shaft seal can be located in the gas chamber, reducing the pressure that it has to withstand. Cooling is poor with the twin tube due to the insulating gas pocket around the fluid. Under extreme conditions the fluid can become emulsified and damping lost. The twin tube design is also sensitive to its installation angle since it relies on gravity to separate the gas and liquid.

16.7.4 Piston Nose Pressure

If the gas reservoir fitted is pressurised, the damper piston will be subject to some force trying to extend the damper. This is a result of the difference in area of the two sides of the piston.

The resulting force that extends the shaft is a function of the reservoir pressure and the cross-sectional area of the damper shaft. It is important to consider this in the calculation of the total suspension spring rate as some dampers may be pressurised to 10 bar. If the shaft on such a damper had a diameter of 12 mm there would be approximately 0.45 kN of force extending the damper which is not an insignificant figure.

16.8 Damper Valves

Racing dampers are normally fitted with at least four valves, if scaled correctly these can be used to control the two different speed ranges in both bump and rebound. Fluid dynamics affect the force required to move the piston at different speeds, and hence the opening of the valves.

When looking at fluid passing through a simple orifice it is found that the relationship of volume flow rate and pressure is not linear, see Figure 16-4. Consequently, one valve cannot be used to provide linear damping across the range of expected piston speed. Instead a simple orifice is used for low speed damping, the size of which can be modified with an adjustable needle style valve.

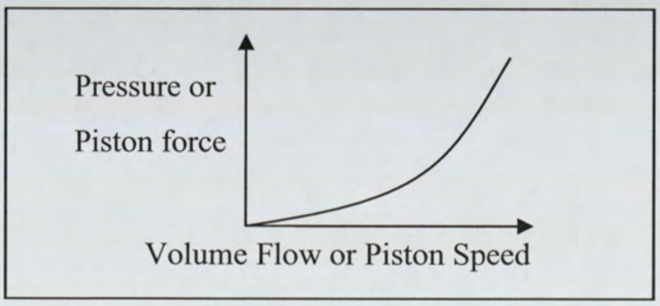


Figure 16-4 - Fluid Flow through an Orifice

High speed damping, associated with higher pressures, is controlled with a proportional valve installed with some pre-load. The high speed valve will show a similar characteristic to the plain orifice, however it can be larger to cope with higher flow rates whist remaining ineffective at low speed.

The high speed valve is characterised by the pressure at which it opens and the pressure at which it becomes fully open, then acting like a plain orifice. Damper characteristics are general described with FV curves in which the damping force (F) is plotted against shaft velocity (V).The pressure at which the high speed valve opens gives a 'knee' characteristic to the damper FV curve.

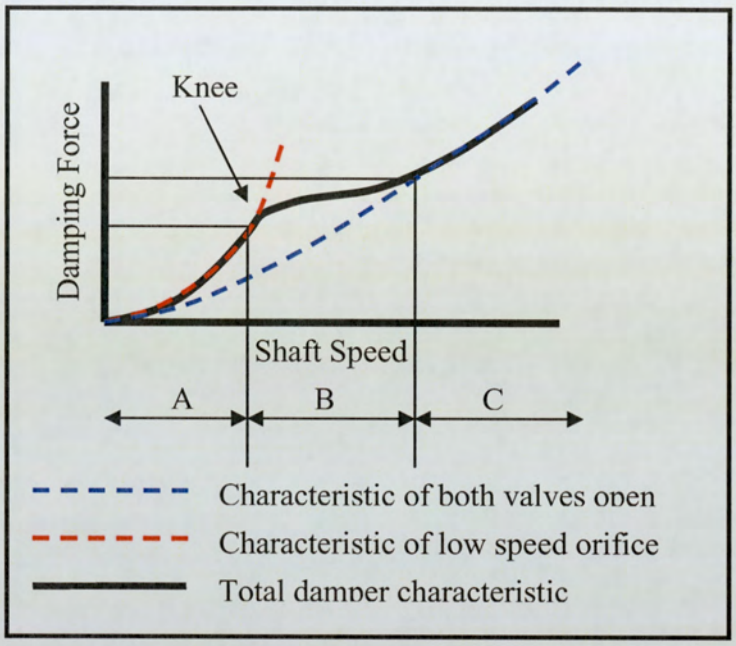


Figure 16-5 - Damper Characteristics (FV Graph)

The total damping characteristic curve is a combination of the high and low speed valves. Region A shows the damper in low speed motion with only the open low speed orifice having effect. At the lower end of region B the high speed valve is beginning to open and by the end of B the valve is fully open. Region B is known as the transitional region and is characterised by the knee. Region C represents the characteristic of both valves being open at the same time.

16.8.1 Valve Designs

16.8.2 Low Speed

It is beneficial to have the low speed valve adjustable from the outside of the damper, as it is used to adjust roll speed and the characteristics of the car when entering and exiting a corner.

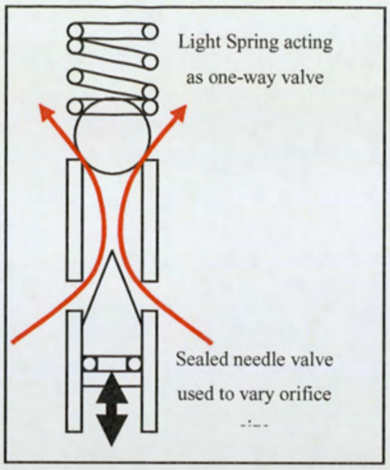


Figure 16-6 - Low Speed Adjuster Valve

The simplest design is a needle valve connected to an external thumb wheel. The one way valve makes it effective in only one direction, so a pair is installed in parallel but with opposed flow, giving independent bump and rebound control.

16.8.2.1 High Speed

The high speed valve is not normally adjustable without dismantling the damper. The valve mechanism can vary slightly between applications.

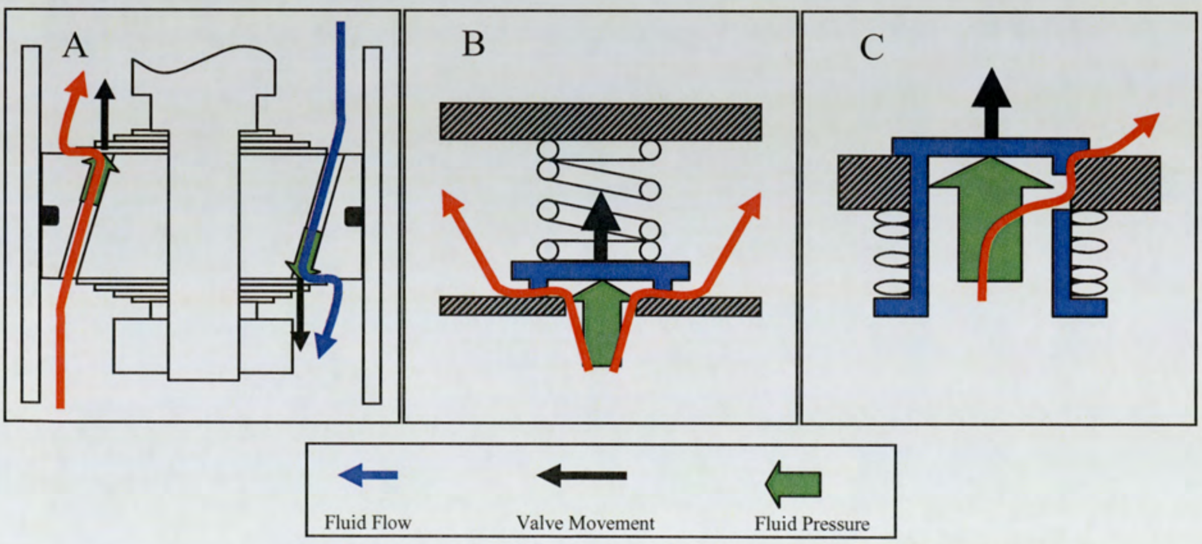


Figure 16-7 - High Speed Valve Designs

Adapted from Dixon (1

999)

a) Shim disk valve – This is the valve often chosen for racing purposes since is it the easiest valve to adjust. The thickness and quantity of spring washers can be changed as well as preload on them. The design is compact but the costs associated with setup and calibrations are greater.

b) Loaded disk – simple in design and cheap to manufacture. Problems are associated with guiding the disk and stopping flutter.

c) Spool Valve – The spool valve is inherently guided and conditioned by its mounting. The components are cheap and reliable making it the valve commonly found in passenger vehicles. The shape of the spool exit can be profiled to give varying discharge rates with pressure. However, the orifices should be symmetrically located to avoid a resultant force on the spool cap.

16.8.3 Friction and Stiction

Any suspension system will have a certain amount of friction, inhibiting free movement. Stiction is also a common occurrence, when normally free running low friction joints require a considerable force to initiate movement. The effect of friction and stiction can be seen when analysing the ratio of chassis vertical acceleration to wheel an input velocity to the wheel. At low velocities there is the presence of a dead band, where the suspended mass movement matches the wheel movement. In effect, the system is displaying a high damping coefficient at low velocity inputs. Although detrimental to ride quality, the presence of some friction can be advantageous as it damps low velocity movement, improving transient response and stability.

16.9 Adjustment Characteristics

Dixon (1999) dedicates a considerable chapter to the analysis of the adjustable valves. He draws conclusions from the analysis as to the results of changing valve characteristics. It is suggested that this source is consulted in order to appreciate the effects of adjustable valves on the damping constant and FV diagram.

16.9.1 Effects of Motion Ratio

Locating the damper away from the wheel has a similar effect to locating the spring away from the wheel with a given ratio. If the ratio of damper speed to wheel speed is R, the force at the wheel is also reduced by R. Consequently, the relation between damping the coefficient at the wheel and at the damper is:

$$C_{wheel} = C_{damper} R^2$$

17 Chassis

Up to this point most of the calculations concerning weight transfer have assumed that the chassis is infinitely stiff and consequently has no effect on weight transfer distribution. It must be considered how a chassis can be designed and tested to appreciate whether the rigid assumption is correct.

17.1 Stiffness and Strength

Flexibility in any system causes errors and the racing car is no exception. If the steering and suspension system have been designed to place the tyres down to within 0.1 degree of their optimum value, but the chassis twists 2° at that lateral acceleration, the suspension design becomes irrelevant. In the above case, the chassis might be perfectly strong enough to take the loads but it is not stiff enough. A flexible chassis might also be prone to fatigue and structural failure, which is not something to be encouraged at speed.

In controlling balance, the designer wishes to redistribute the roll moment between the front and rear. If the chassis is flexible, the effects if increasing the roll resistance at the front or rear might not be seen, since the chassis flexes as the extra load is applied. From this point of view the chassis could never be considered too stiff.

There is always going to be some compromise between weight and stiffness. However, the chassis should be designed for maximum stiffness at minimum weight; if it is stiff enough it is likely to be strong enough, however this too must be evaluated. The same compromise will occur in the suspension components, a reduction in unsuspended weight is immaterial if the components flex so much that they introduce steering and camber changes.

17.2 Chassis Material

The classic motorcar chassis is made from steel, but built with differing techniques. The first chassis were simple ladder shapes allowing individual fitting of custom built coach bodies. Understanding the importance of rigidity, these designs were replaced with a stressed steel monocoque, with the extra benefit of weight reduction. Within racing, designers looked towards space frame designs. These frames were made from either round or square tubular steel sections welded together. Round sections are preferred for symmetric stiffness properties, whilst square for ease of manufacture. Frames can be designed so that the members are only ever in tension or compression, and as such will have a high specific stiffness.

The next major development to come was the use of materials less dense than steel. Sheets of aluminium bonded either side of a core section were found to have high specific stiffness. Sheets were used to form extremely lightweight boxes or tubs. However the high cost of manufacture, labour intensive building techniques and form limitations restricted the use of this technology.

Most recent developments have come from the use of carbon reinforced plastics. Ultra fine carbon fibres supported in an epoxy resin matrix can be shaped and cured into complex forms. When two layers of carbon fibre are separated by a core section the structure can be exceptionally stiff.

17.2.1 Material Properties

The chassis designer must consider the properties of the materials that they are using for components of the vehicle and the effects that these might have.

Ultimate Tensile Strength - The UTS of a material is normally found from a simple tensile test or estimated from other tests such as hardness. The UTS of a material, usually in MPa, gives the designer a clue as to how much load they may put through a component. This is normally considered in design of components for which compliance is not important or a component that will see rare sees extreme load cases. It is also a consideration when evaluating the strength of a structure under extreme loading such as a crash or a spin on rough ground.

Yield Stress – The designer probably does not want to exceed this stress in any component under normal operation since permanent deformation will result in the component. Not all materials display a yield point and these are labelled brittle, carbon fibre being the obvious example.

Strain – This is not a material property, but a measure of how a force affects a material. It is calculated as:

$$\varepsilon = \frac{\delta l}{l}$$

Young's Modulus (Modulus of Elasticity) – Young's modulus for a given material is a constant. It is defined as:

$$E = \frac{\sigma}{\varepsilon}$$

Young's modulus, 'E', can be looked at as a measure of how elastic a material is since it is the ratio of stress to extension. Young's modulus does not give an indication of the material's ultimate strength. The value of 'E' for a material is important to a designer aiming for stiffness, the higher the better.

Fatigue Stress – Most materials suffer from fatigue, it is the slow failure of a material as a result of a cyclic load. However, in materials like steel there is a fatigue stress, at stress levels less than this, the material should be able to withstand an infinite number of cycles without failure. At stresses above this the material will start to show fatigue, the number of cycles to failure reduces the higher above the fatigue stress. Materials such as aluminium do not have a fatigue stress, meaning that any cyclic load will eventually cause failure. Consequently, any aluminium part will have a finite life expectancy, although it may be very long. As far as the designer is concerned if they are using a material above its fatigue stress they must consider the expected frequency of the cycle and the required life expectancy.

17.2.2 Carbon Fibre Reinforced Plastics

Carbon fibre, for short, is increasingly becoming the choice material for the motorsport industry. Carbon fibre is extremely versatile with excellent strength properties and when combined with a core good stiffness. There is not even any weight disadvantage since its density is approximately 4.5 times less than steel.

Carbon fibre is however anisotropic, the orientation of the fibres greatly affect the material's properties. Therefore, the designer must still consider load paths and ensure that there is sufficient fibre in the directions of the expected forces. The nature in which composite structures are manufactures allows the designer to orientate fibres to resist the forces, thus optimising the structure.

The weave of the raw carbon fabric can vary, the most versatile fabrics tend to be bundles of fibres woven into a twill. Twill fabric is very drape able and compliant prior to curing.

Unfortunately, when loaded the fibres can straighten slightly allowing the composite to stretch, as a result the modulus of elasticity for the composite is considerably less than the raw fibres. Tensile and compressive forces also create shear forces where opposed bundles interweave, significantly reducing the UTS of the composite compared to the fibres. The pressure of the cure also affects the consolidation of the fibres, higher pressures can give thinner composites with less elasticity and a higher UTS.

The main benefit of carbon is the specific stiffness that can be achieved. A structure of intricate shape can be designed that neatly packages all components. A continuous skin and complex shape can lead to a very stiff structure, also to the benefit of driver safety.

Carbon fibre can be used as an excellent lightweight energy absorber for crash structures. If a very fine weave is used the impact structure will fracture at intervals determined by the size of the fibre bundles. The sum of energy absorbed can be very large since the number of fractures created will be high.

Carbon reinforced plastics also have good damping properties to high frequency vibrations from engine and road, limiting stresses on sensitive electronics and the driver.

17.2.3 Steel Tubing

Most racing series have minimum specifications for critical members such a roll hoops, whilst the choice for the rest of the chassis members goes to the designer.

Round tubing is considered to be the ultimate tube for chassis construction since its stiffness properties are constant, and stress paths in the section are continuous. On the down side, round tubes, are significantly harder to join at the ends. If the load path to the tube is poor the properties of the round tube are negated. Therefore, box sections provide a good alternative especially to the amateur racer without the facilities to machine round tubes to accurately fit.

T45 steel tubing is considered to be the best for motorsport. It is a carbon manganese based steel that is formed into a seamless tube. Originally developed for the aircraft industry the tube is highly suitable to motorsport application. The tube can be cold worked and loses very little strength in the welding process, negating the need for any post manufacture treatment.

17.2.4 Required Chassis Stiffness

The main concern of the vehicle designer is the stiffness of the chassis in torsion, since it has most effect on lateral weight transfer. The shapes required to design a frame that is stiff in torsion encourage good bending stiffness. In the words of Milliken and Milliken (1995) p676, 'Design for High Torsional Stiffness'.

There is no magic number to aim for with chassis stiffness since the torsion loads in the chassis are dependant on the mass of the vehicle, the roll centre positions and lateral acceleration. However, Milliken and Milliken (1995) p676, suggest that a small formula car might have chassis stiffness of around 3000 lb/ft/degree, whilst a steel bodied road car would have 4000-10,000 lb/ft/degree stiffness. They also suggest that the key is to consider the magnitude of the roll moment that needs to be distributed either fully front or fully rear, and then consider how much deflection is acceptable for this moment, such that nearly all of the moment is transferred to the desired end.

17.2.5 Measuring the Stiffness

An acceptable method for measuring the stiffness of the chassis can be used to create a profile of deflection along the chassis, such a method is suggested by Milliken and Milliken (1995) p674. From the point of development, this will indicate to the designer where it would be most effective to place additional stiffening.

17.2.6 Chassis Stiffness by FEA

Chassis stiffness can be simply and reasonably modelled during the design stages. In the case of steel space frames bar elements can be used to model tubes in a pin jointed frame. As long as all of the frame members are only used in compression, in tension the results should be reasonably accurate. The figure yielded from the FEA models is likely to differ from that of the finished product since the joints on the frame will not be as good as the pin joint where all the loads meet precisely.

For composite chassis a specialised FEA package will probably need to be used, in order to accurately simulate all the properties of the constituent parts and the effects of fibre orientation. Before the development of these packages designers could test composite samples in order to estimate bulk properties of the composite, these could then be used to simulate a monocoque with a more traditional method. The results from this process might be useful regards deformed shape, highlighting high stress areas as a result of shape. However, the figures for actual stiffness are probably unreliable.

Any FEA simulation should be backed up with some kind of manual calculations to try to verify the results. Once completed the frame should be physically tested to further verify the data and highlight areas that could have been better simulated.

17.2.7 Installed Stiffness

The most important stiffness values need to be taken from the completed vehicle. Justification for a stiff chassis connected to flexible suspension with poor toe control is always going to be impossible. The installed stiffness tests will give the most accurate data on how stiff the whole package is, as every a-arm, rose joint and frame member is being tested at the same time. A profile of the whole car can be made, highlighting areas that might be easily modified to significantly improve the overall stiffness.

To highlight that the stiffness of the package and not the parts is important, the system can be likened to a series of springs. A basic model might only have three springs, front suspension, chassis and rear suspension, the reality is that every joint, nut and bolt will act like a spring of some kind.

The formula for the total stiffness of springs in series is:

$$\frac{1}{K_{total}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} \ldots \frac{1}{K_n}$$

As an example let us assume some arbitrary stiffness values for the three components:

	Scenario			
	1	**2**	**3**	**4**
Front suspension arms – K_1 (N/m)	100	200	1000	1000
Chassis – K_2 (N/m)	1000	200	100	1000
Rear suspension arms – K_3 (N/m)	100	200	1000	1000
Total Stiffness – K_{total} (N/m)	**47.6**	**66**	**83**	**333**

What we see, although highly simplified, is that the as a result of one or two weak components the complete stiffness can be severely affected. It becomes very important to look at the package as a whole and not just the individual parts. For example if car 1 was being developed, the designer could afford to not just add stiffness to the whole suspension system, with little weight penalty as they are relatively small components. But also, severely reduce the stiffness of the chassis saving weight whilst generating a better overall package.

In reality, the possibility is that the chassis will always be the most flexible part as a result of its scale in comparison to a suspension arm. The example however, does highlight how important the whole package is and that getting caught up in the detail of a specific part can be fruitless. The phrase ‘you are only as strong as your weakest link’ is very applicable to the racing car, if the designer identifies the ‘weak’ link and makes it ‘stronger’ the car will keep improving.

18 Brake System

It is the sole job of the brake system to apply a retarding force to the rotating wheel and in doing so create a slip ratio and longitudinal force that acts to reduce the vehicle's speed. The friction components of the brake system must dissipate the associated energy as heat.

The braking system must be controllable and predictable. Responses to driver input must be fast and consistent, since the driver will be applying the brakes as late as possible before the corner. If their response is varied the driver has to brake earlier in order to mitigate risks or will simply not make it around the corner. The input force required should be large enough that the driver can distinguish the load needed yet not so large that they are overstraining.

18.1 Brake Pedal Considerations

From the driver's point of view the starting point in the system is the brake pedal and its importance is often underestimated. Just as the tyres are the only interface the vehicle has with the ground, the brake pedal is the only interface that the driver has with the brake system. Consideration during the early design stages should be made for all of the pedals as they are an important part of the whole package. The driver can only excel if their environment is suitable.

Considerations:

Positioning – The driver should be able to comfortably operate the pedal over the whole range of movement. If during a long distance race the pedal stroke increases the driver must be able to adapt.

Adjustability – Unless the car is only being designed for one person some provisions for adjustment must be made.

Stiffness – A flexible brake pedal will mask the feedback through the driver's foot, telling them how the brakes are performing.

Freeness – The pedal pivots need to be free, stiction in the pivot could stop the master cylinder returning fully, partially locking the brakes on or stopping fluid from the reservoirs entering the system to compensate for pad wear. The damping effect of stiff pivots again reduces the feedback to the driver.

18.2 Pedal Ratio

The pedal is the first point in determining the forces in the braking system. The pedal has built into it a mechanical advantage over the master cylinders amplifying the force delivered by the driver's foot.

The pedal to master cylinder ratio is normally in the range of 3:1 to 5:1, but can feasibly be well outside this range as long as the desired hydraulic pressure can be achieved.

In passenger vehicles the braking system is normally split into two separate hydraulic systems with a singular master cylinder that has a floating piston to divide the circuits. The circuits are normally arranged such that diagonal wheels are on the same circuit, this arrangement ensures that a failure in one circuit still leaves the best possible balance. The rear brakes often have a pressure reduction valve inline or are sized such that they operate on the same pressure as the front brakes.

For a racing application greater control is needed over the balance of front to rear braking. The system is divided front to rear and is supplied by two master cylinders. This arrangement allows the ratio of front to rear master cylinder area to be varied proportioning clamping forces fore and aft. The inclusion of a driver adjustable pressure reduction valve in either circuit allows then allows the balance to be adjusted with a greater degree of precision.

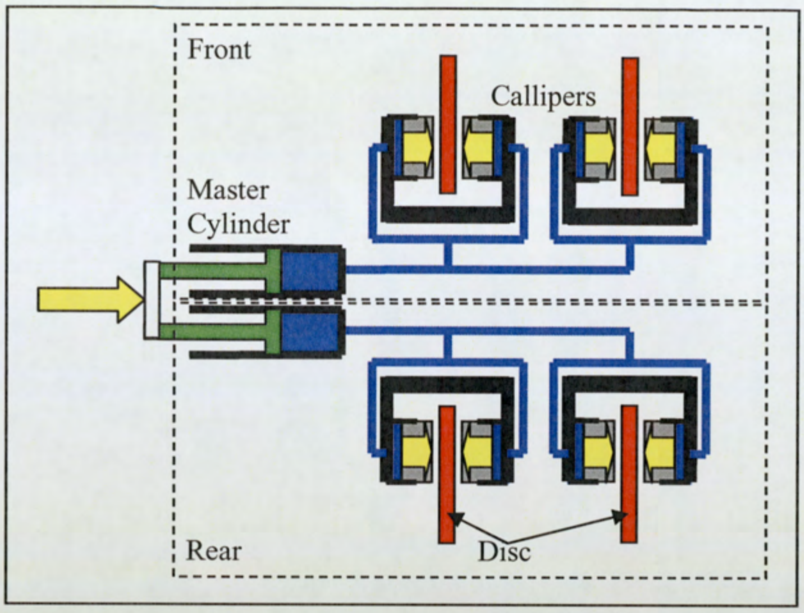


Figure 18-1 - Brake System Schematic

18.3 Forces in the Brake System

18.3.1 System Pressure

The pressure in the hydraulic system can be calculated from the pedal force and the mechanical ratio of the pedal. Note that the pedal force is distributed between both master cylinders by a ratio dictated by any balance bar assembly.

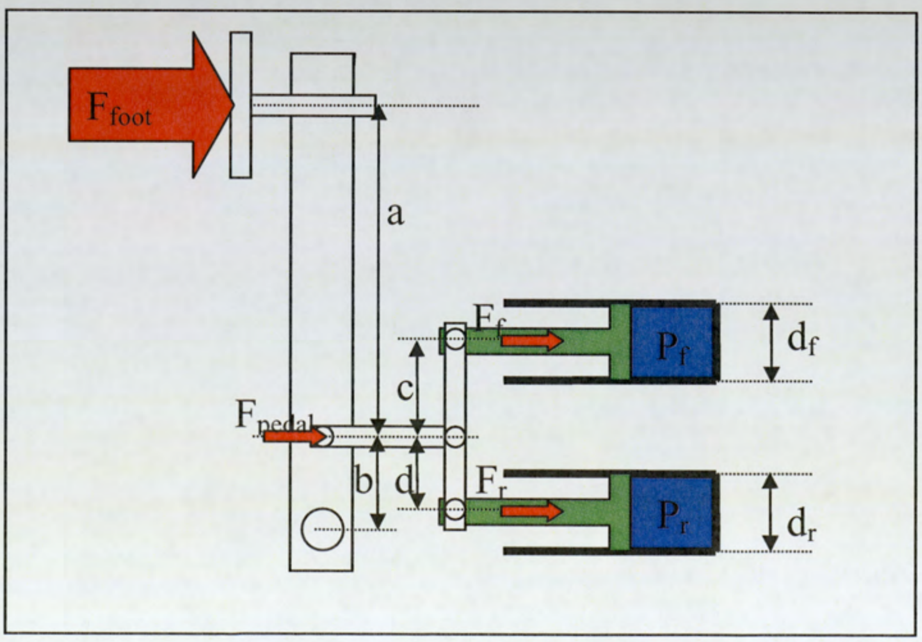


Figure 18-2 - Brake Pedal Mechanism

If all distances are in metres and forces are in Newtons the pressure in the front and rear systems, in N/m^2 (Pascal) is given by:

$$F_{pedal} = F_{foot} \frac{a+b}{b}$$

$$F_f = F_{pedal} \frac{d}{c+d}$$

$$F_r = F_{pedal} \frac{c}{c+d}$$

$$P_f = \frac{F_f}{\pi \left(\frac{d_f}{2} \right)^2}$$

$$P_r = \frac{F_r}{\pi \left(\frac{d_r}{2} \right)^2}$$

$$\therefore$$

$$P_f = \frac{\left(F_{foot} \frac{a}{b} \right) \left(\frac{c}{c+d} \right)}{\pi \left(\frac{d_f}{2} \right)^2}$$

$$P_r = \frac{\left(F_{foot} \frac{a}{b} \right) \left(\frac{d}{c+d} \right)}{\pi \left(\frac{d_r}{2} \right)^2}$$

100000 Pascal = 1 bar = 14.5 p.s.i.

18.3.2 Clamping Force

The clamping force that the disk and pads are subjected to is calculated from the calliper piston area and the line pressure. Where n is the number of pistons per calliper, for floating callipers the number of pistons should be doubled.

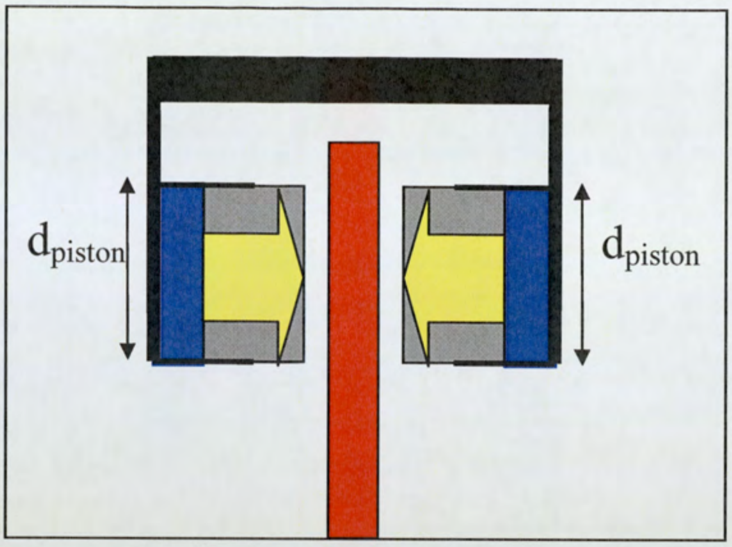


Figure 18-3 - Calliper Force Generation

$$F_{clamping} = nP\pi\left(\frac{d_{piston}}{2}\right)^2$$

The clamping force is commonly quoted per amount of line pressure, e.g. clamping force per 100 psi.

18.3.3 Pad Disk to Coefficient of Friction

The frictional force generated is dependent on the coefficient of friction between the pad and disk and the clamping force. Dynamic friction generates heat that in turn affects the coefficient, thus it is useful to know how the coefficient varies over a given temperature range.

The optimum operating temperature of the pad and disk should be decided from this curve, bearing in mind that the temperature is anything but constant during operation. Cooling for the brakes should provide enough heat loss so that the boiling point of the brake fluid or degradation temperature of the pad are not exceeded. The pad will probably have an optimum temperature range, which specifies the minimum temperature that the brakes should fall to.

18.4 Pad and Calliper Choice

The pad selection is normally based on durability, for short events where temperatures may not have time to build, i.e. qualifying, a soft pad may be chosen. For longer events where there will be time for temperatures to reach equilibrium and wear rates become a factor, harder pads will be chosen and the cooling will be specified accordingly.

Generally, as the compound hardness increases the coefficient of friction and wear rates fall, but the operating temperature rises. The main benefit for racers using soft pads is the initial bite and response of the brakes, the lower clamping forces required to produce the same braking force mean a lighter calliper may be used.

Obviously the mass of the vehicle and the operating speed play a part in disk and calliper selection as the kinetic energy to be turned into heat by the brakes is linked to the square of the vehicle's speed. The designer must estimate the forces required to lock the wheel under the ideal traction and acceleration situation. Based on this, the piston area and friction coefficient required at a reasonable line pressure can be specified. The chosen calliper should be capable of withstanding the system pressure without unreasonable deflection. The amount of heat to be dissipated can also be estimated, the pad disk and calliper should as a unit be capable of dissipating this much energy.

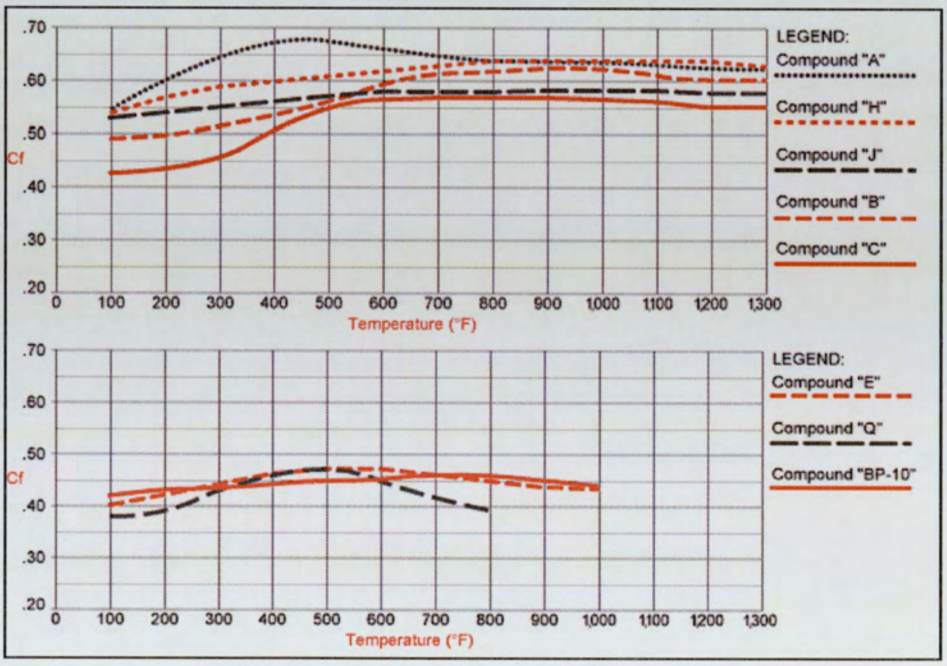


Figure 18-4 - Brake Pad Coefficient of Friction

Willwood Performance Brakes

Part 2 – A Practical Design Example

19 Design Procedure

The design for any complicated machine is going to be an iterative process, there are also going to be many compromises along the way. It is impossible to merge theoretically perfect systems together into a single package and as previously noted, it is the package that is important, not just the individual systems. The relationship between the quality of the overall package and the individual systems is governed more by an inverse law rather than a summation. Consequently, one weak system can cause the failure or poor performance of the whole package.

The designer/design team must look at systems individually, decide what is ideal and then look at the integration of them. At this point decisions on what compromises are acceptable will be made and another design iteration will commence. The number of iterations and compromises made will depend on resources available, but the quality of the car as a package depends on it.

Any rules for the particular class may be there for the driver's safety, or to make the designer think. Either way, it is of the utmost importance that the rules have been well digested and are followed. It is often true that the best teams are the ones that are always pushing the rules. Whether they are the best because they know the rules or the rules hold the best teams back remains to be seen. The fact is, the best teams are always in discussion with the governing bodies just to get that last little edge.

Some of the best design developments can come from previous experience and cars, often if something has been done before there was a good reason for doing it like that, similarly there will be equally good reasons for not doing it like that again. Sometimes the expression 'if it aint broke don't fix it' can be applied to save a great deal of resources. Equally, designs should be questioned sufficiently, there is nothing more annoying than accepting something that is fundamentally flawed, as it was the way that it was done before, only to pay the price when it fails. Along similar lines, a lot can be learnt from other teams and the way that they do things, but beware, just because they do it like that does not mean that it is the right way and you should do it like that. It is easy to make the common mistake of 'monkey see monkey do' when designing a race car, adopting other peoples designs and theories without significant questioning. However, the design process should involve a careful inspection of what has been done and what is accepted and proved as a good method or design. After all the main aim in racing is to win, if you know your competitor's designs you know their weaknesses!

19.1 Setting the Constraints

The first step is to lay down all of the restrictions that the design must be limited by, these may include:

Resources: A realistic assessment of the available resources needs to be made. These will include time, money, machining facilities, experience and design resources. From these a good prediction can be made about the level of achievement possible from the project.

Performance Characteristics: These may include; g-circle performance, top speed and economy.

Handling Characteristics: The features needed to give the driver enough control to operate the car around the perimeter of the g-circle. The details that need to be specified to achieve the handling characteristics are; suspension geometry, steering geometry and system, chassis rigidity, control systems, aerodynamics, natural frequencies and moments of inertia.

Chassis – A reasonably accurate load scenario from the start will help in the scaling and specification of the chassis and load paths. The loads can be derived manually from theoretical values or taken from previous data taken from the car. Some idea of the factors of safety to be used in critical areas is also essential.

Driver Environment – The performance of the driver is critical, they must have a good environment in which to operate. The space, temperature, controls and safety requirements should be set out from the beginning of the design process so that they can be included from initial concepts and not added as afterthoughts.

Tyres – Tyres are the lynchpin of a good package. The car should be designed to get the most out of the tyres. Therefore, they have to be specified from the start. Unless you are an F1 team the chances are that the tyre manufactures are not going to strive to give you a better tyre, you will have to do better for their tyre.

Adjustment – The kind and range of adjustment needed to setup the car at the track must be included in the constraints. Careful consideration should be given to the exact requirements; it is unlikely that a weekend racing team will want to adjust the types of things that an F1 team consider essential. Remember that the more there is to adjust the more confusing it is to know what to adjust, if the basic design is well considered for that application, adjustability of a certain parameter might not be essential. Some parameters are better to adjust using shims whilst others with threaded sections, generally shims are more repeatable and more accurate, a measurement can be taken, the error calculated and the appropriate shim made. Threaded adjustments are a little more subjective.

Miscellaneous – There are other parameters that need to be set out in the initial stages, they will include; fuel cell size, engine, transmission, fire system, roll cages, wheels and fixings and body shell.

19.2 Global Design

The aim of the first design iterations is to decide on an acceptable layout of the car. The layout will be based around packaging all of the components in suitable positions to best fulfil the design requirements. The CoG of the individual components can be estimated and hence the overall CoG. Locations of strategic components can then be revised such that the required CoG position can be achieved. Just as importantly the vehicle's moments of inertia can be estimated at the same time as weight distribution. At this stage the basic dimensions need to be set that achieve the performance criterion, these are; ride height, track, and wheelbase.

At this point adequate space needs to be allocated for ancillaries such as tanks, coolers, radiators and ducting. Consideration should also be given to access to these parts for maintenance and inspection.

19.3 Detailed Design

Once content that the global design fits the set of requirements set out in the specification stages the design can progress to a more detailed level. This is the stage where individual parts start to evolve and component analysis takes place. Final values are defined for all of the suspension rates, suspension dimensions and mounting positions of all the ancillaries.

19.4 Design Tools

The mathematical functions described in Section 1 that describe or analyse performance characteristics are easily computed using spreadsheets or relatively simple computer programs. The use of a computer allows many calculations to be done quickly, reducing design time and increasing the possibly of exploring different options. The accuracy and value of calculations carried out using small time steps on a simple spreadsheet should never be underestimated. These simulations are fast to generate and often reveal the importance of basic parameters that are too easily overlooked.

Accompanying this document are some electronic resources that can be used to explore the effects of changing one or many variables at a time. There then follows a description of commercial software available to analyse performance and/or design verification.

There are commercially available software packages that simulate the vehicle lap time and performance to a high level of detail. However, the downfall of all these packages is their complexity. In order to increase accuracy they require increasingly involved vehicle data. This software could undoubtedly be used in the Formula Student design process. However, the time involved with learning and implementing these packages will probably make it unfeasible in a short project. The advantages of these packages would be seen by a professional team that could dedicate a person or small team to the continual development of models and simulations. The output of these advanced programs can be likened to finite element analysis, in that the figures still need backing up with manual calculations and some form of convergence study should accompany them.

At the initial stage of design development the focus of analysis should be to identify areas of the design that can be easily improved to make the greatest performance increase.

19.4.1 Design Resources Spreadsheet

The resource spreadsheet has many interlinked sheets, each worksheet with a specific function. The first sheet contains global information needed in all of the subsequent sheets, and as such must be completed fully before other sheet will function correctly.

Data Input

The data input sheet contains generic information about the vehicle that will be used through out subsequent calculation.

This acceleration field is populated with a desired value for lateral acceleration for which weight transfer characteristic and suspension load will be calculated

This array contains information on the spatial characteristics of the car. These values are used in the simulation of weight transfer. The red cells contain calculated data and are for reference.

The population of the mass table is used mainly in the calculation of lateral weight transfer. Again red cells are for reference.

Input Data

Accelerations		
Lateral	1.5	g
Value for weight transfer calcs and suspension loading		

Tyre/Wheel Data		
Wheel Diameter	13	in
	330.2	mm
Tyre Diameter		
Front	20	in
	0.508	m
Rear	20	in
	0.508	m
Tyre Spring Rate		
Front	150	N/mm
Rear	150	N/mm
Friction (Max)		
	Lat	Long
	1.5	1.5
Value for braking calcs		

Dimensions		
Footprint		
Wheelbase	1.65	m
Front Track	1.3	m
Rear Track	1.3	m

Suspension		
Spring Rate		
Front	26	N/mm
Rear	26	N/mm
Motion Ratio		
Front	1	
Rear	1	
Roll Centres		
Front	65	mm
Rear	65	mm
CoG total		
Height	0.3	m
Distace from front axle	0.825	m
Distace from rear axle	0.825	m
CoG Suspended		
Height	0.312	m
Distace from front axle	0.825	m
Distace from rear axle	0.825	m

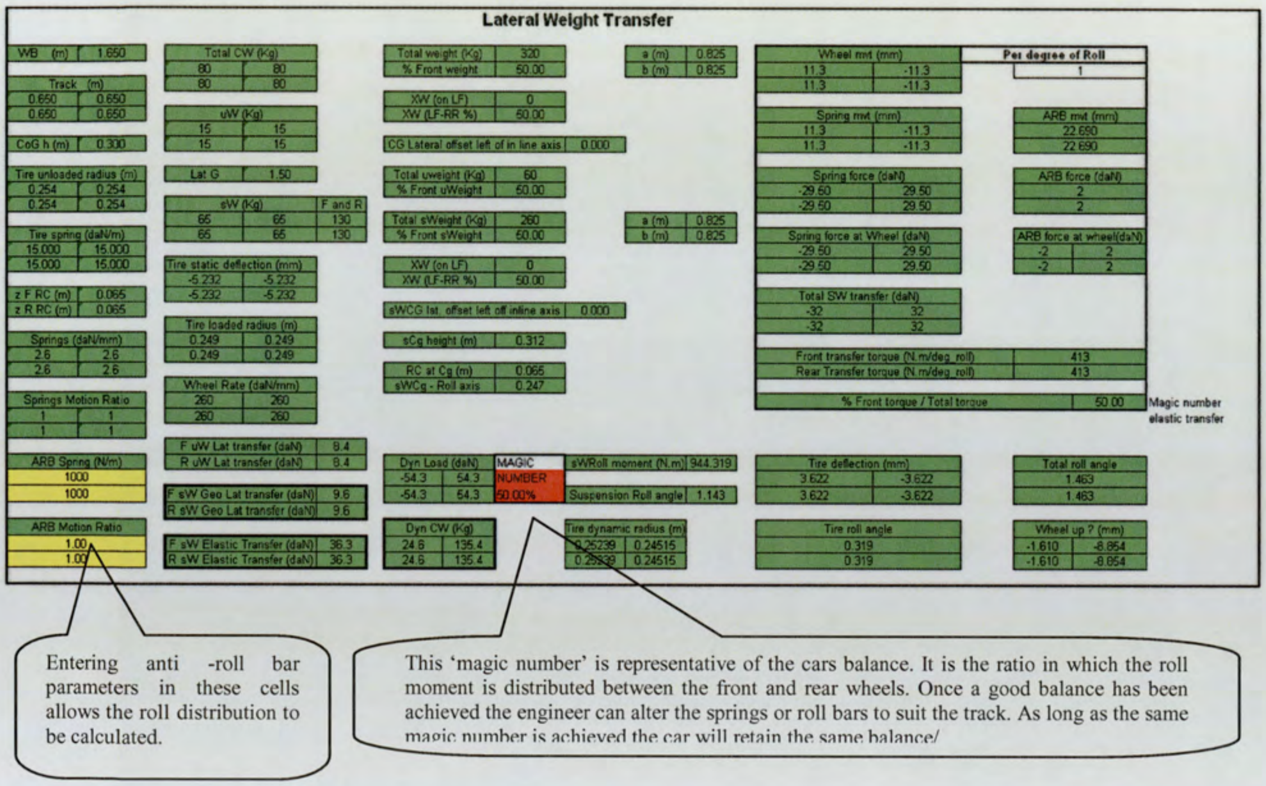
Mass (kg)		
Unsprung		
	L	R
Front	15	15
Rear	15	15
Cornerweights		
	L	R
Front	80	80
Rear	80	80
Suspended Mass		
total	260	kg
bias	50.00	% front
Total Mass		
Total	320	Kg

Tyre and wheel data is a common necessity for weight transfer and braking load calculations. Most of the values come from simple tests or tyre data.

CoG information is calculated form corner weights.

Lateral Weight Transfer

Lateral Weight Transfer (Steady State) – Adapted from a Claude Rouelle (2003) spreadsheet, this tool takes the basic vehicle geometry and calculates the individual components of weight transfer. Using calculated roll stiffness values the roll distribution is calculated and hence the balance. The effects of anti-roll bars and spring stiffness on roll moment distribution can be evaluated. The consequences of moving the roll centre can also be demonstrated. The calculated wheel loadings can also be used to resolve the forces imparted on the suspension.



The rest of the cells on this spreadsheet are calculated values that cover the state of the suspension at the simulated lateral acceleration. It is worth noting that these values used simplified calculations. No account is made for roll centre migration or rising rate suspension. However, this is a very useful tool that can be used to quickly iterate suspension design. SusProg 3d (see section 19.4.4)can then be used to generate suspension geometry that fits the desired criteria. SusProg 3d can the be used to asses the afore mentioned complications.

Suspension Loads

Using the wheel reactions loads calculated in the Lateral Weight Transfer and basic suspension geometry the magnitude and direction of loading on the suspension can be determined.

Suspension Natural Frequencies

The basic quarter car model natural frequencies are calculated. The effects that changing the damping ratio has on damping the unsuspended and suspended masses can be demonstrated. The calculations on this tab are as per section 16.6.

Braking Calculations

The brake calculations are driven by the desired pedal input required to lock all four wheels at the same time. Based on the dimensions of the pedal assembly, balance bar, callipers, brake disks and pad coefficient of friction the dimensions for the master cylinders are calculated. Further calculations determine the range of adjustment available based on the distance the pivot of the balance bar can be moved.

19.4.2 Gearing Spreadsheet

Written by D.I. Smith[14] this spreadsheet predicts the performance of the drivetrain based on CVT, engine and chassis parameters. It can be used to evaluate the effects of wheelbase, CoG, gearing and engine performance on longitudinal acceleration over a given speed or distance range.

19.4.3 Race Technology Data Analysis Software

The combination of a data logger installed in the car to measure various given parameters and software designed to process the data can be a very powerful tool for the designer. The Race Technology package is powerful yet reasonably priced and capable of complicated mathematical data manipulation.

The software is used throughout this report in order to analyse the performance of current designs. Suggestions, based on the data, can be made on the most suitable adjustments required to improve performance. Obviously the data recorded needs to be appropriate and as accurate as possible, so that the most valid conclusions can be drawn see Section 24.

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[14] D.I.Smith (2005) of Aston University Birmingham UK.

19.4.4 SusProg 3D

SusProg 3D is a dimension and co-ordinate driven suspension design package. Basic dimensions and masses can be set for the vehicle to give a quick analysis of suspension performance. The designer can use this tool and see the effects of moving roll centres and changing FVSAL[15]. When used to its full potential SusProg 3D can be used to design suspension kinematics, steering geometry, drive line positions and spring rates. Once all parameters are specified dynamic calculations can also be carried our to asses steady state balance at given lateral accelerations. The data can be output to Excel or MATLAB, where it could be combined with mathematical tyre models to generate reasonably accurate simulations.

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[15] FVSAL and Roll Centre – See section 11.2 onwards

20 The Formula Student Problem

The Formula Student competition requires a fairly unique set of performance characteristics from a car. The track is primarily low speed, with frequent tight corners and slaloms. As a result the steady state conditions are rare. Thus, the key to success lies in producing a vehicle that responds quickly to driver input and is able to achieve high acceleration both laterally and longitudinally, possibly at the cost of high speed stability. Aerodynamic down force at the expected speed is hard to achieve, the large wings needed to do so might be considered a weight penalty. Driver confidence will be generated by predictable handling as the car is thrown into corners at low speeds, not by being sure footed on the entry to a high speed corner.

The rules that dictate entry to the competition are comprehensive, yet thought provoking and as yet there does not seem to be generally accepted recipe for success. This is probably due to the wide background from which the teams come and the rewards for ingenuity and uniqueness. The way in which many teams function, producing a new car every year, with very limited resources and experience stretches the time budget to the extreme. Testing is an essential part of any motorsport, be it for reliability or tuning, but for formula student teams it is often a luxury that many never get. It becomes of the utmost importance that the car is designed well in order for it to perform on limited testing.

The following example looks at the global design of a suspension system suitable for a car to compete in the Formula Student event. Certain design aspects will be covered past global design. Certain parameters for the drivetrain and packaging have been assumed, such as power output and CoG location.

20.1 Outline of Design Process

The design process detailed below is a combination of personal experience and a design process map described by Milliken and Milliken (1995). The following steps will produce the foundations of a detailed design specification allowing detail of layouts and components to be created.

1. Analysis of problem – Calculation of basic facts and figures appropriate for FS.
2. Analysis of current performance – Examination of data for current FS car.

3. Focus of design – Comparison of theoretical performance and actual performance to determine area that can be most easily improved to give substantial performance returns.
4. Tyre selection – Tyre choice based on desired parameters.
5. Footprint – Sizing the vehicle footprint to optimum for competition.
6. Weight distribution – Suitable positioning of CoG and values for moments of inertia.
7. Suspension – General parameters and adjustability.
    a. FVSAL length – Roll bump compromise.
    b. SVSAL – Detraining anti-dive and anti-squat geometry.
8. Roll centres.
    a. Positioning – Optimising height of roll centre for competition.
    b. Control – Specifying maximum roll centre movement.
9. Wheel alignment – Suggestion of static wheel setting.
10. Suspension rates – Controlling suspension movement and natural frequencies for optimum tyre contact.
11. Roll Rates – Balancing the car and limiting roll for good tyre performance.
12. Damping – Specifying damping constant to optimise tyre contact.
13. Chassis Stiffness – Suggesting a minimum stiffness so the chassis can be considered stiff.
14. Braking system – Calculation of brake balance and pressures for maximum stopping force.

15.

21 Data Collection

21.1 Methodology

Data used throughout this report has been collected with an onboard Data Logging device. There are many commercially available devices capable of such data capture, each with their own unique characteristics. However, all devices are used to achieve a common goal so consequently an informal motorsport standard has evolved. Thus, most data logging systems are similar and have been designed to allow a common set of parameters to be easily recorded. Logging units commonly have 0-5v input for auxiliary sensors, 0-12v for recording working voltages and frequency inputs for the output of Hall Effect sensors and ignition pulses.

More advanced logging systems have the ability to interface with other devices either by CAN or serial communication protocols. This gives the ability to simultaneously record data output from other devices without the installation of additional sensors or making time based links between data sets.

For motorsport applications it has become standard to include a GPS receiver in the system. Position, direction and acceleration information can be extrapolated from the GPS data allowing for accelerometer verification data interpolation, to increase the resolution and frequency of data points.

21.2 Equipment

The following sections describe the equipment installed on the test car for the purpose of data acquisitions. Following on from this is a discussion on suitable logging frequencies and resolutions.

21.2.1 Data Logging Device:

Race Technology's DL1 unit

Specifications:

GPS- position and speed @ 5 Hz

GPS - External antenna

Analogue Inputs

7 external - 0-12v, 4 0-5v full scale (12 bit)

1 internal - power supply voltage (12 bit)

Frequency Input - 4 external max frequency > 2kHz.

Lap Beacon Input

+5v Reference Out

Serial Ports – 2 separate ports

Accelerometers – 2 dimensional to ±2g

Logging Frequency – 100 Hz

21.2.2 External Sensors

The following sensors were installed and provided the data analysed in Section 22. Section 24 contains recommendations about the sensors that should be installed to increase the level of analysis possible.

- Suspension Position – 4 Linear Potentiometers connected in parallel with spring damper units.
- Wheel Speed – 4 Hall Effect sensors counting notches on wheel hubs.
- Throttle Position – Potentiometer on throttle body.

21.2.3 Logging Frequencies

It is necessary to ensure that data is logged at suitable time intervals, in order that the data contains enough information about the measurand but is not excessively large. Variables such as oil and water temperature have reasonably long response times and consequently do not need logging at any frequency over 2Hz. Oil pressure will have a faster response time and should be logged at around 5-10 Hz.

The frequency at which the suspension position should be measured depends on the kind of tuning desired. Low speed movement of the suspension can be determined from a low logging frequency, whilst to appreciate the high speed performance smaller sampling periods are required.

The suspension potentiometers fitted to the FS car have an approximate travel of 80 mm for a full scale deflection on 0-5v. If connected to a 12 bit A-D converter the position resolution 'r' in mm is given by:

$$
\begin{aligned}
r &= \frac{80}{2^{12}} \\
&\approx 0.02\,mm
\end{aligned}
$$

From Section 16.6 we know that high speed damper movements are normally greater than 100 mm/second. Therefore, to track all discernable positions whilst the damper is in high speed motion a logging frequency of 100/0.02 = 5 kHz is required. At this frequency the volume of data recorded would be large and require excessive post processing. However, it does represent the maximum required logging frequency.

The DL1 logger being used in the FS car logs all channels at 100Hz and cannot be reconfigured. At this frequency, when travelling at 100 mm/second the position would be logged to accuracy of 0.02 mm at approximately 1 mm intervals. Therefore it should be questioned whether every high-speed input creates a disturbance of 1 mm in the spring, if it does not the logging frequency is insufficient and should be readdressed.

22 Track Data Analysis

In the introduction to this thesis it was discussed that exact performance characteristics required to give optimum performance depend mainly on the track. Therefore, it is necessary to carefully analyse data about the track in order to better understand the design requirements and objectives of the vehicle. When the track is understood, simulations of the vehicle on the track can be made to help determine the performance requirements. An accurate track model is as important as a car model, as even the most basic simulations depend on it. The following method describes a simple method for producing a track model in order to generate a basic track simulation.

22.1 Track Map

Taken from the Race Technology Data Analysis Software, this track map has been generated using GPS data interpolated with information from on-board accelerometers. This data was recorded during a testing session on the actual track used for the sprint lap at the FS competition.

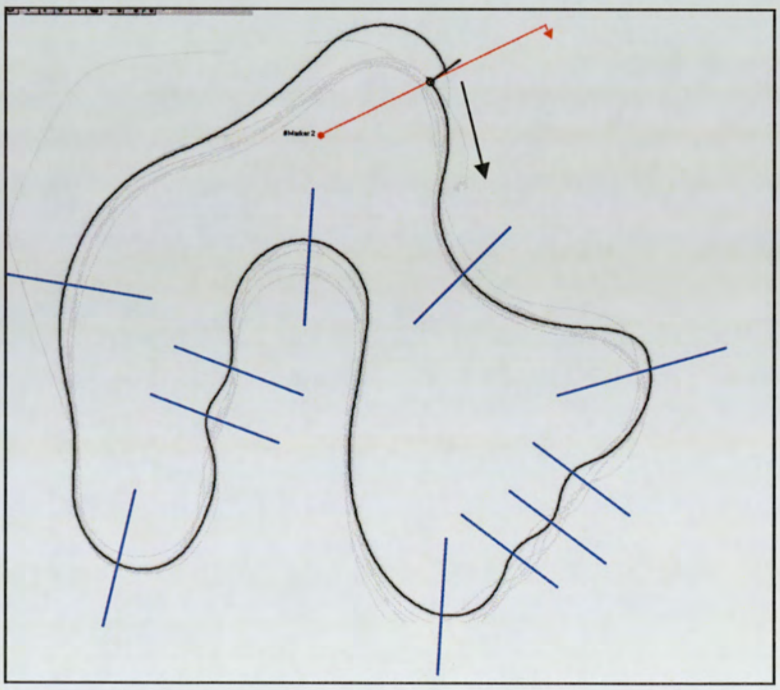

Figure 22-1 - Track Map

In Figure 22-1 the straight red line is used as a marker to represent the start/finish line, whilst the blue lines represent the corner apices and the slowest points on the track. In such a way the lap is broken down into twelve sectors with thirteen gates on a full lap.

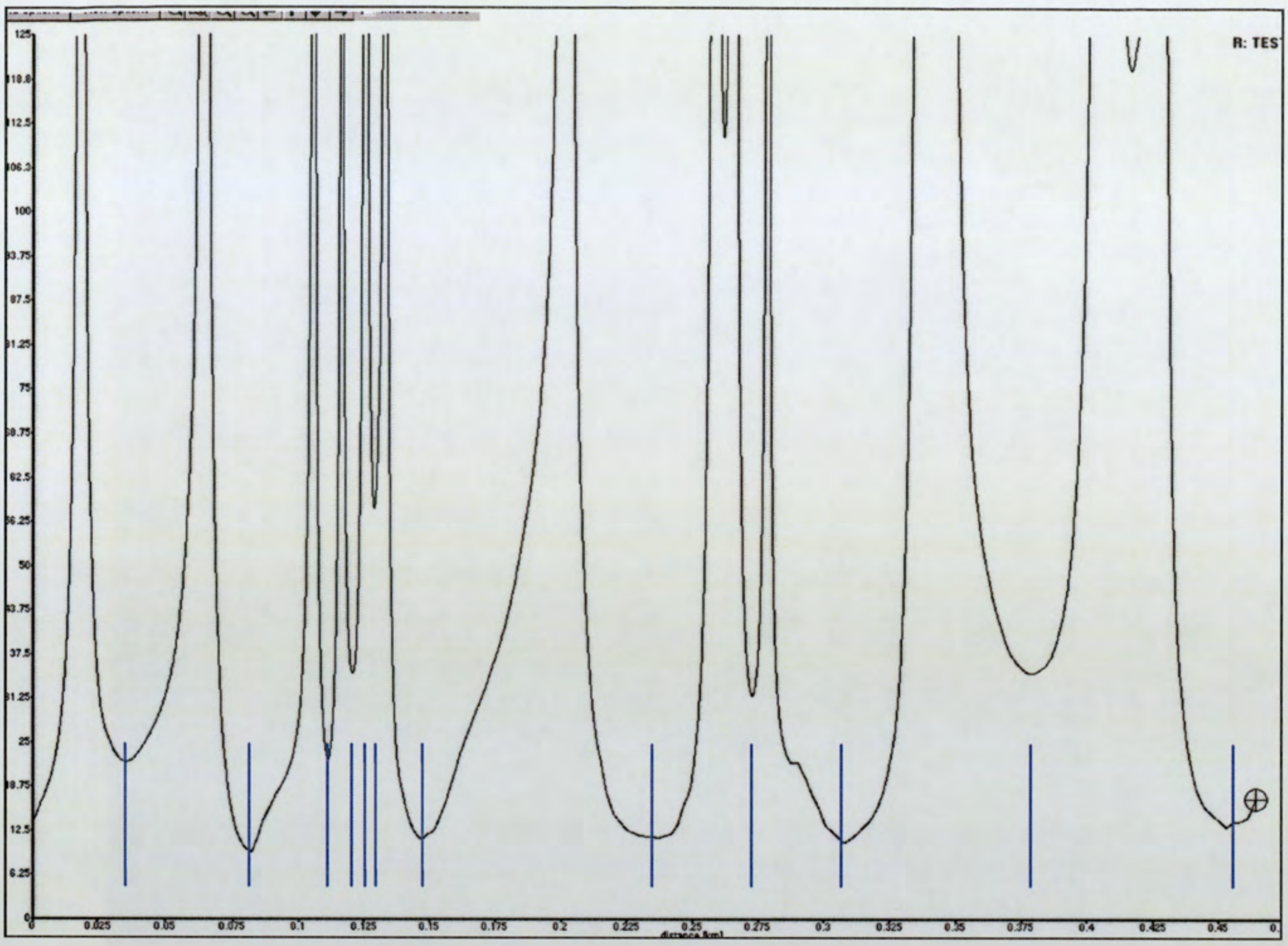

Figure 22-2 - Track Corner Radius

Figure 22-2 is a plot of the instantaneous radius of turn for the car against distance from start/finish line for a complete lap. The blue lines correspond to the positions of the gates on the track map and are measured as a distance from the start/finish line. The distance between gates and corner radius can be read From Figure 22-2 and used for further calculations.

22.2 Generating a Track Model

In order to quantify the net change in lap time caused as result of modification to any of the vehicle's acceleration capabilities a suitable mathematical model needs to be developed. In its simplest form the track can be modelled as a single straight line with a series of gate dividing it, these gates replacing the corners. The car must pass though each gate at a speed determined by lateral acceleration capabilities and the radius of the corner that the gate represents. At all other times the car must have maximum longitudinal acceleration, either positive or negative.

By splitting the track down into these smaller sections, where the start and end conditions are known, along with the distance between them, simple calculations can be made to simulate the speed of the vehicle over the full distance. The theoretical calculations are:

- Maximum theoretical corner speeds
- Maximum theoretical speed between gates
- Theoretical optimum braking point
- Theoretical optimum acceleration time
- Theoretical optimum braking time

The easiest way to make the calculation assumes constant longitudinal acceleration, introducing some error. However, the low speed and short straight length limits the speed to mainly the range where the engine develops enough power to produce constant acceleration. Thus, the error created is less significant. For the braking system, is not unreasonable to assume constant acceleration, as it is capable of producing constant torque, irrespective of speed. In any faster discipline where the aero effects are significant and the engine incapable of producing enough power to sustain constant acceleration this model will become far less accurate. However, nonlinear functions can be written to take into account engine power curves, drag effects and added traction from down force.

Another assumption is that the vehicle and driver are capable of achieving peak lateral acceleration on every corner. Neglecting the effect of the driver, it is reasonable to assume that on the larger radius corners the car is capable of regularly achieving and sustaining maximum lateral acceleration. However on the short sharp corners of the slalom sections, the dynamic response of the vehicle is going to have a larger impact and the peak acceleration seen will not be as high as the vehicle's steady state maximum.

The third and final assumption is that the driver is perfect, being able to control the car at the limit for all of the time and assessing braking points accurately. No driver is capable of this on every corner of every lap. However, their ability to get closer to the optimum improves with practice and guidance. Thus, development and use of this tool can be extended past the design stages onto driver training, giving the driver an indication of where to push harder to reduce lap times. The development of this model led to the inclusion of a driver factor, which adds a constant time period into the simulation that represents driver and vehicle reaction time.

22.3 A Track Model

The virtual gates described previously were examined and the relevant data recorded in order to produce a linear lap model. Some of the most advanced vehicle simulators build track maps in similar ways. More detail about each curve and straight will be used, but a similar segmented approach is still common.

Gate	Distance (m)	Corner Radius(m)	Sector	Distance (m)
0	0/453	13.5		
1	35	22.5	0-1	35
2	80	13	1-2	45
3	111	10	2-3	31
4	120	10	3-4	9
5	129	10	4-5	9
6	147	1	5-6	18
7	234	20	6-7	87
8	260	10	7-8	26
9	271	10	8-9	11
10	304	12	9-10	33
11	387	35	10-11	83
12	414	40	11-12	27
13	453	13.5	12-13	39

This track model along with the calculations described is used to further asses the vehicle's performance in Section 25.

23 Performance Data Analysis

23.1 Acceleration – G-Circle

Figure 23-1 is a G-Circle plot of 3 laps of the Formula Student track, similar to the sprint event at the competition. The concentration of points at the centre is due to the car being static before the start of the run.

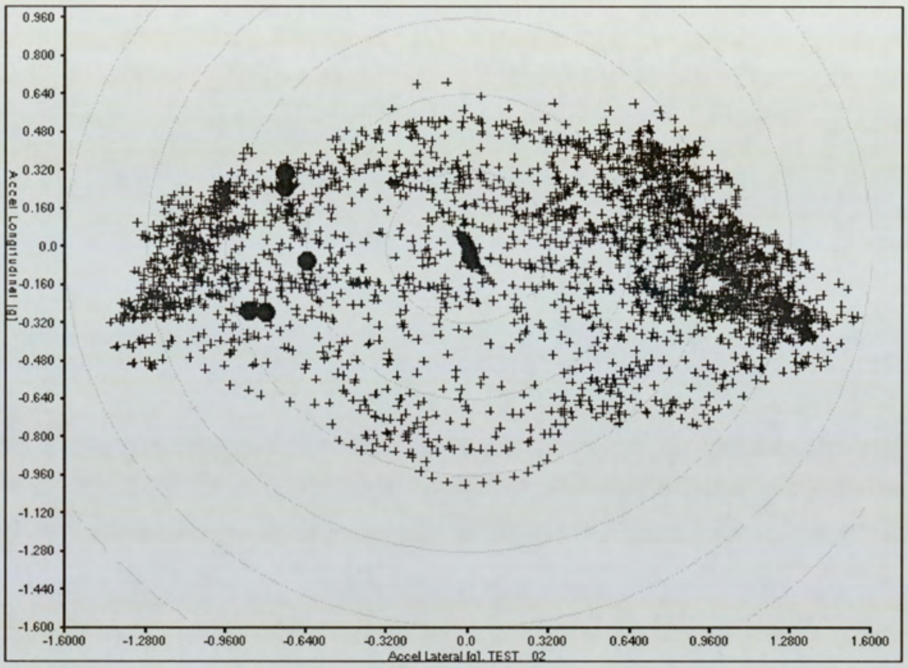

Figure 23-1 - 2005 Car G -Circle FS track

We can see that the car has reached 1.3 g lateral acceleration, which might possibly increase slightly when the tyres are fully warm. This short 1500m run is insufficient to bring the tyres to their full working temperature. The braking system, at maximum, achieves approximately 1g and the drivetrain 0.7 g. However, it is instantly apparent that a substantial amount of the population does not lay on the perimeter of the ellipse and thus maximum performance is not being achieved.

23.2 Performance Distribution

To assess the state of the formula student car over the course of a lap the frequency distribution of lateral and longitudinal accelerations can be examined.

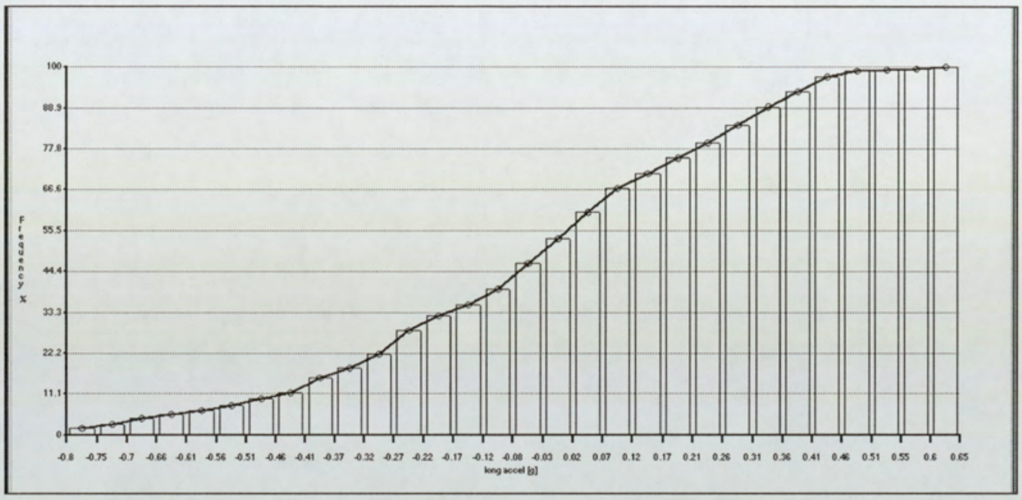

Figure 23-2 - Cumulative Distribution of Longitudinal Acceleration[16]

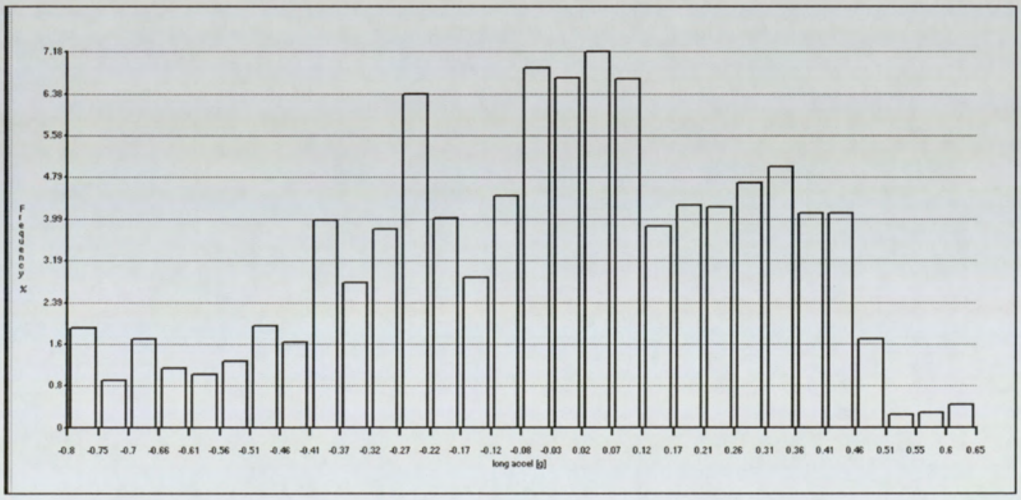

Figure 23-3 - Distribution of Longitudinal Acceleration

[16] Taken from Race Technology Analysis Software

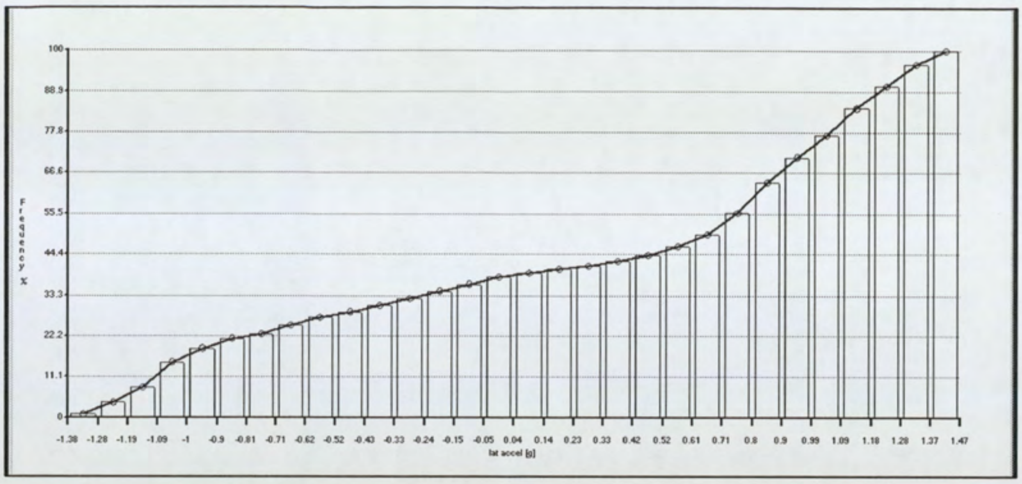

Figure 23-4 - Cumulative Distribution of Lateral Acceleration

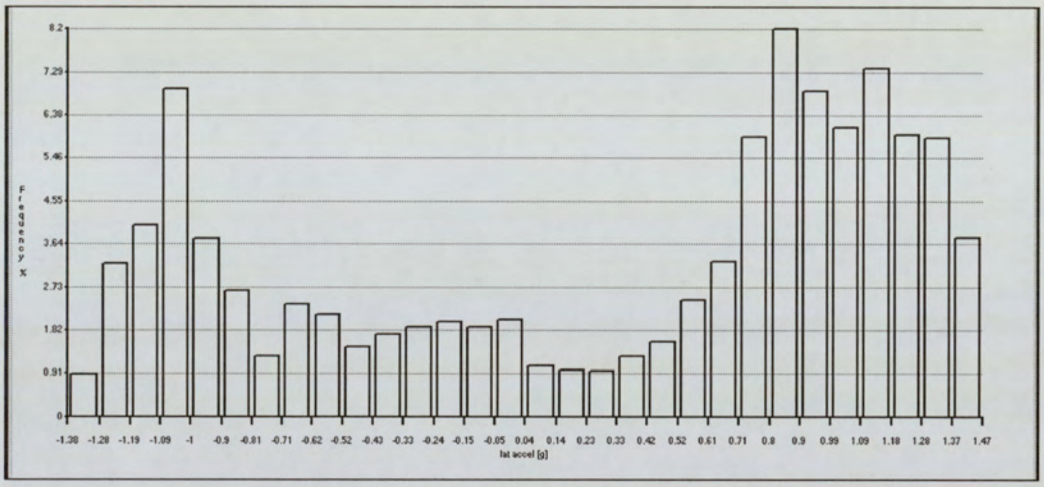

Figure 23-5 - Distribution of Lateral Acceleration

Analysis of these graphs show that 10 % of time spent in one lap is within 80 % of the maximum longitudinal acceleration, whilst 30% of lap time is spent within 80 % of maximum lateral acceleration. It could be questioned that the time spent at maximum longitudinal acceleration is affected by transient response to the throttle on the short straight sections. Consequently, the performance of the drivetrain should not be disregarded and lateral acceleration performance be concentrated on. Instead, the response time before maximum longitudinal acceleration is achieved should be reduced.

23.3 Suspension Motion

With the correct manipulation, data from the suspension position transducers can be used to give information on a wide variety of suspension performance characteristics, including:

Ride height – With correct calibration and information on the motion ratio[17] the logged voltage returned by the linear potentiometer can be converted to a wheel position.

Damper Speed - Differentiation of suspension position with respect to time will give an instantaneous velocity of the damper.

Roll Angle – Suspension deflection can be used to calculate roll angles for the suspended mass.

23.4 Damper Speed Time Analysis

In Sections 16.8.1 and 16.8.2.1 damper speeds and movement are discussed along with the associated stimuli. Damper motions are categorised into those considered to be high speed and those that are low speed. Dampers are designed in a way that allows the valves in them to act as independently as possible in order that each valve controls a specific speed range and direction of motion. Consequently, it is necessary to identify a method for examining the performance of each of these valves, in order to guide the engineer on the adjustments ultimately required to improve tyre loading.

Speed time analysis of damper motion can give visually satisfying information on damper performance. A frequency histogram of damper speed displays the performance of all four valves simultaneously.

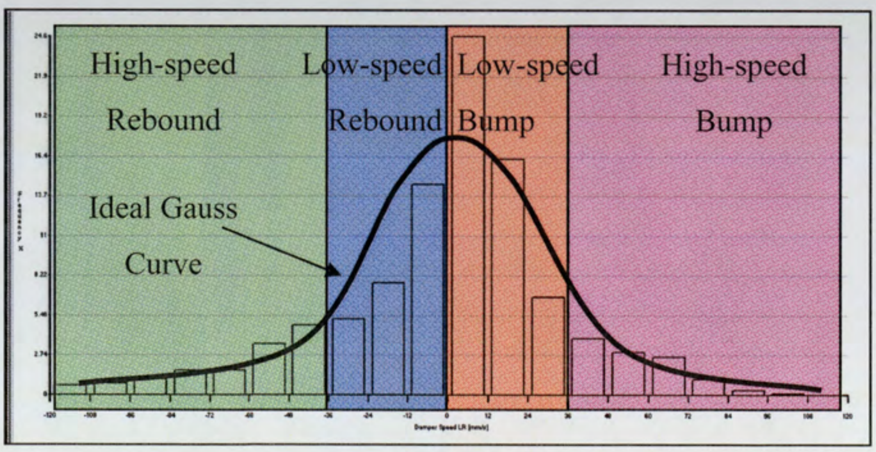

Figure 23-6 - Damper Speed Time Histogram

The ideal distribution would fit a normal Gauss curve, with the dampers spending a majority of time at low speed. The distribution should be symmetrical about zero displacement, showing that

[17] See section 15.6 for more information on motion ratio

the dampers spend equal amounts of time in compression and rebound. Interpretation of the data shown in the speed time histogram will enable the engineer to draw conclusions on the ratios of bump to rebound damping needed for maximum performance. However, it will have limited use towards quantifying the amount of damping required for optimal contact patch loading. The level of damping such that zeta is close to unity should be calculated and this technique used to refine the balance of damping.

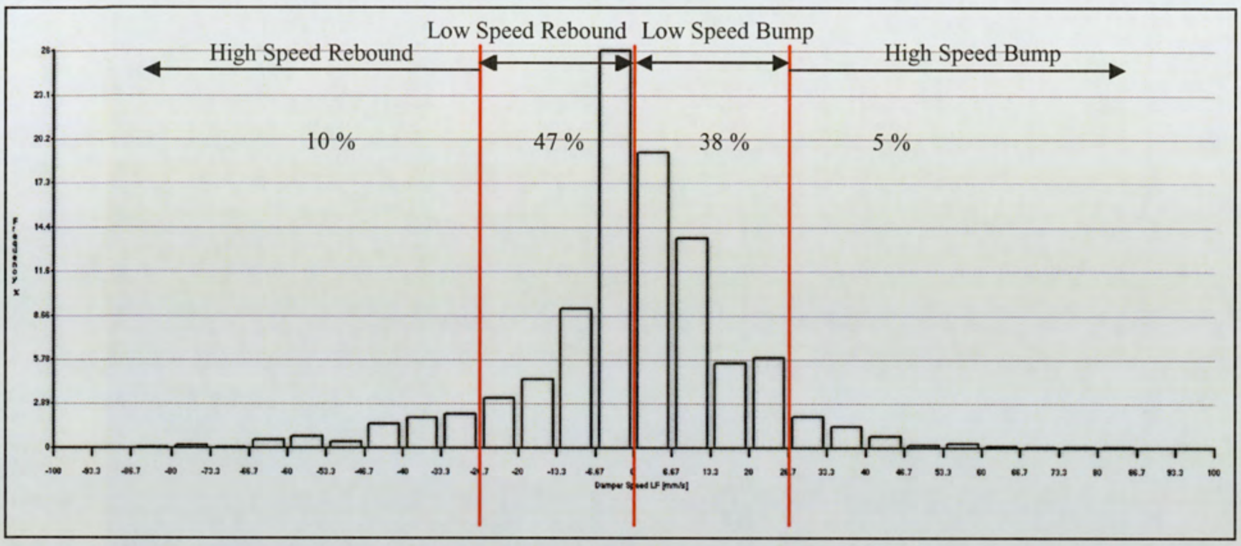

Figure 23-7 - LF Damper Speed-Time Histogram

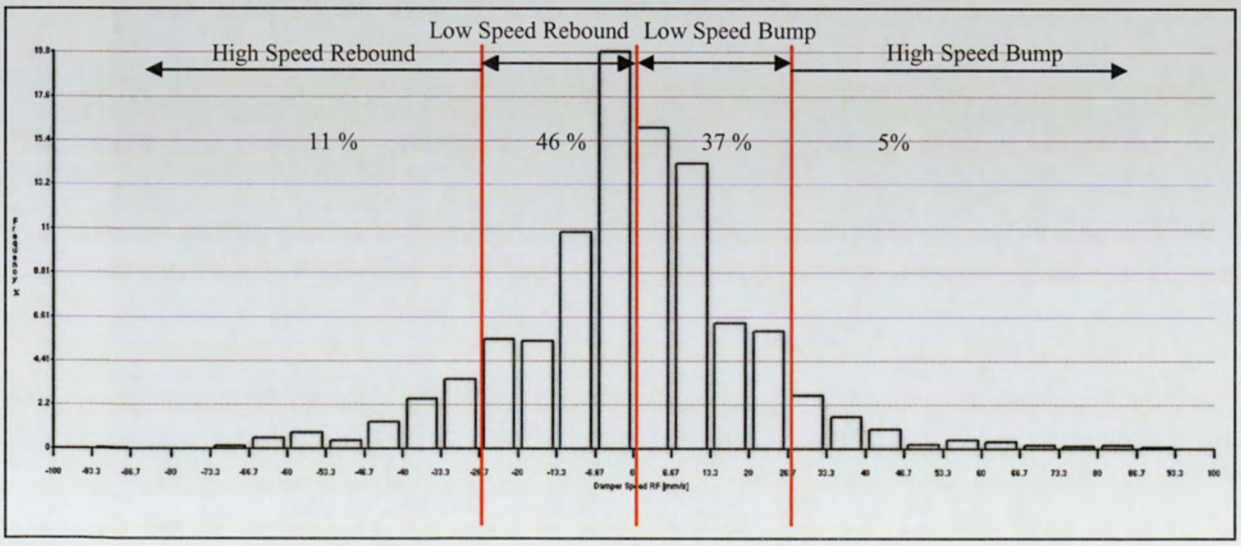

Figure 23-8 - RF Damper Speed-Time Histogram

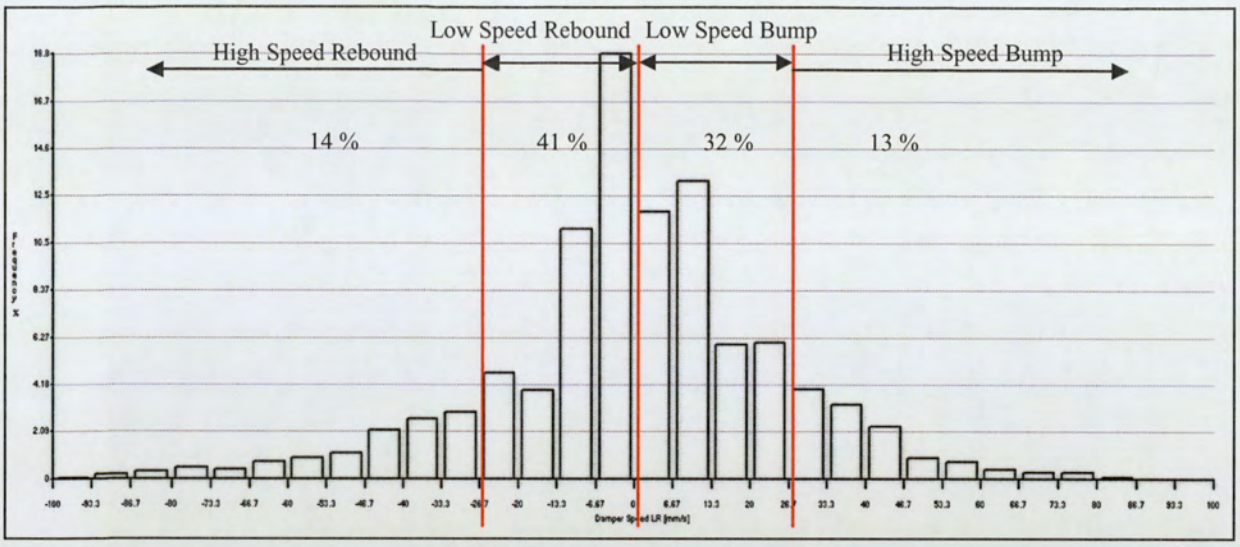

Figure 23-9 - LR Damper Speed-Time Histogram

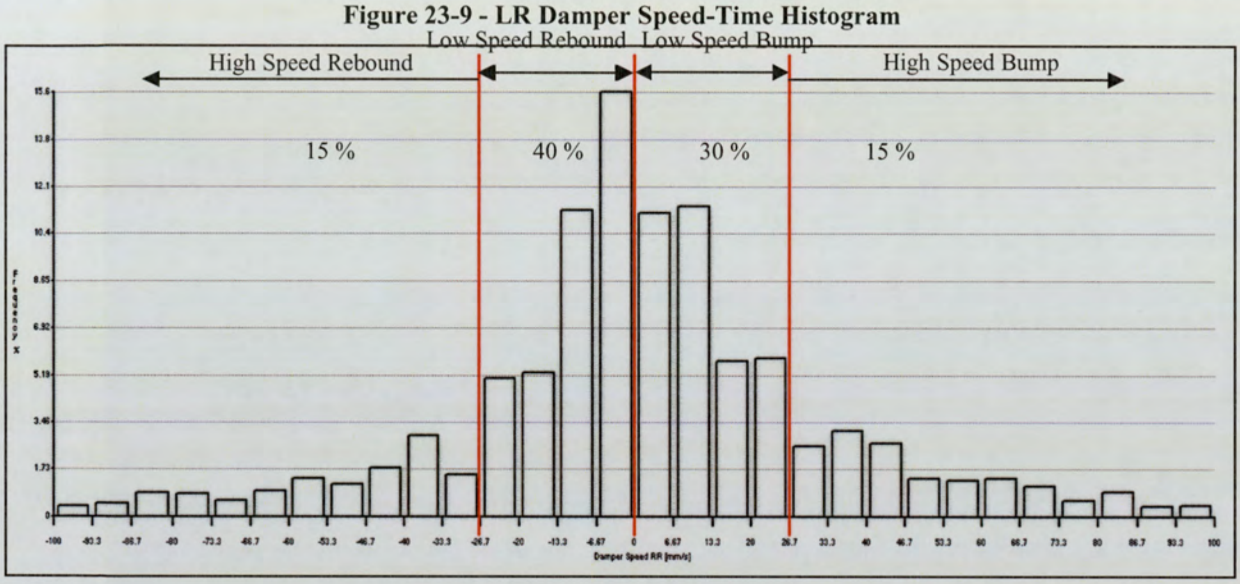

Figure 23-10 - RR Damper Speed-Time Histogram

Figure 23-7 through to Figure 23-10 show the distribution of damper speeds for a lap of the formula student sprint track. Preliminary inspection shows that the distribution of the damper speed approximates to a normal distribution curve as proposed above.

Firstly looking at the low speed motion of the front wheels, both left and right wheels spend significantly longer in low speed rebound than bump. This would suggest that either rebound is too stiff or bump is too soft. Either adjustment could be made to balance the distribution curve however the effects on the shape of the curve would be different. Softening rebound would reduce the very low speed peak, flattening the curve. Whilst, stiffening low speed bump would introduce a very low speed bump peak. On the contrary, when looking at high speed damper motion the time spent in high speed rebound is less than bump motion.

At this point it is worth considering that all four of the dampers are identically valved and hence have similar damping coefficients. As expected, the rear dampers spent more time in high speed motion, due to the rear bias weight distribution. The increased inertia at the rear of the car causes the rear suspension to deflect further than the front in response to a given stimulus. Looking at Figure 23-9 and Figure 23-10 it can be seen that the rear dampers spent similar amounts of time in high speed bump and rebound, implying that the balance of high speed damping is good. For the front wheels however the high speed damping is not ideal and possibly high speed bump damping should be softened to increase its occurrence.

Looking at the rear dampers now, there appears to be a similar trend to the front dampers where more time is spent in low speed rebound than in bump motion. Again there is more than one possible solution to balance the histogram, each with different consequences.

In order to predict the best adjustment to improve tyre loading, the test data must be given further evaluation. A good method for choosing between increasing damping in one direction or reducing it in the other is to consider the motion of the chassis.

23.5 Roll and Pitch Timing

In the previous section (23.4), it has been identified that the dampers installed on the car are not providing the correct ratio of bump to rebound damping. Consequently, the suspension is not cycling about its static deflection point. A ratio closer to ideal could be achieved by either reducing one value or increasing the other. However, increasing one will increase the total level of damping whilst reducing one value will have the opposite effect. With respect to low damping, assessment of the vehicle's motion will help to guide to the correct adjustment. Further calculations can be made from the damper position to calculate suspended mass roll angles. It is useful to compare front and rear roll angles in response to entering a corner. On a simpler level, subtracting left and right damper displacement from each other can be used as a dimensionless measurement of roll, which is good enough when only considering peak roll timing. The latter evaluation is considered below.

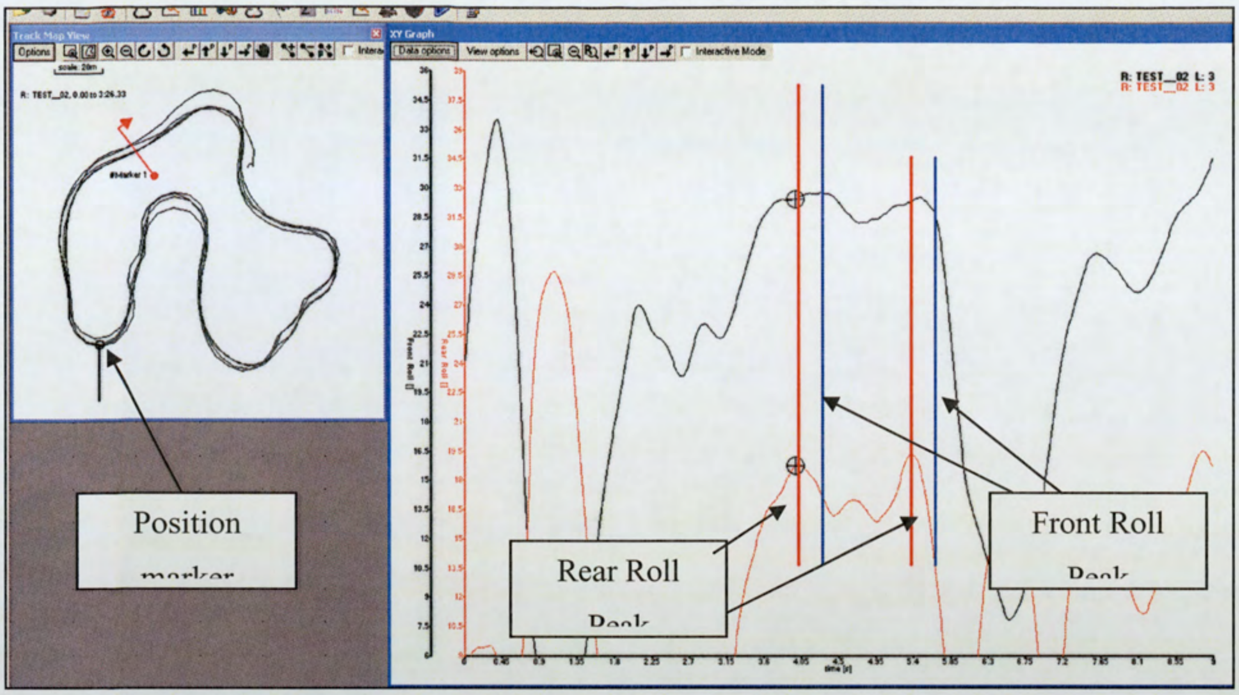

Figure 23-11 - Peak Roll Analysis

The above figure shows the magnitude of front and rear roll during a typical corner on the formula student track. It can clearly be seen that the peaks in front and rear roll are not coincidental and there is a lag time between them. As expected, and consistent with equal damping and spring rates front and rear, rear roll leads the front as a result of its higher mass. On the front, maximum roll is seen after the apex of the corner where lateral acceleration should be the highest. In this situation front roll needs to be advanced and rear roll delayed in order that peak front and rear roll coincide. Therefore, in order to balance the damper speed time histogram the low speed rebound damping on the front should be reduced, whilst rear low speed bump is increased.

24 Analysis Recommendations

In order to further increase the value of data taken and evaluated, the following methods and analysis methods should be considered. These simple sensors and or calculations could be combined with some of the previous techniques to expand our understanding of how the car is performing, or what areas would benefit from development.

24.1.1 Spring and Damper Loads

The force across the damper and suspension spring is a summation of the load in the spring and the load in the damper. The load in the spring is proportional to the spring constant and displacement, both of which are known. The force in the damper is a combination of the piston nose pressure[18] and the damping constant[19] multiplied by the damper speed. The damping constant, if not known, can be found by testing on a damper dynamometer. Differentiating the damper position with respect to time gives damper velocity.

24.1.2 Wheel Loads

Wheel loads are the spring and damper loads modified by the mechanical suspension's motion ratio (Section 15.6), both of which are known or can be calculated. Consequently, the wheel loading can be derived. However, it should be noted that this loading is only accurate in the absence of a lateral acceleration, since not all lateral forces generate a roll moment resisted by the spring. The error between the calculated wheel load and the actual wheel load will be a function of lateral acceleration and the height of the roll centre in relation to the ground. If the roll centre is located on the ground all weight transfer occurs through the roll moment and acts through the spring and damper. The designer would be able to use this information to their benefit, to accurately scale suspension components to withstand the actual loads imparted.

24.1.3 Steering Angle

The inclusion of a steering angle sensor would be beneficial by allowing the analysis of the vehicle's balance. If the car were to complete several laps of the circuit at very low speed a trace

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[18] See section 16.7.4 for more information on piston nose pressure.

[19] See section 16.6 for more information on the damping constant.

of steering angle against distance can be superimposed over the same trace of a high speed lap. The difference between the traces can be used to indicate the vehicle's state of balance.

At very low speeds lateral acceleration and forces are low. Consequently, only small slip angles are generated. If the speed is low enough, slip angles are small enough to be ignored, or considered to be equal front and rear. Thus, at low speeds the car can be considered to be neutrally balanced (Figure 9-6). As vehicle speeds increase the understeer gradient starts to take effect and the driver might need more or less steering angle to turn a given radius corner. Consequently, when the steering angle is plotted against distance for a high and low speed lap the difference between the required steering angles is a guide to the car's balance. In a similar manner this method also identifies the limits of grip, however if drift occurs the trajectory may change without change in steering angle, creating a reduction in accuracy.

With respect to sensor complication and data analysis, this is the simplest method for assessing the balance of a vehicle, but is probably the least accurate and reliable. However, it is a reasonable first-step towards quantifying an otherwise difficult to measure characteristic.

24.1.4 Constantly Variable Transmission (CVT) - Secondary Speed

The performance of the CVT drivetrain depends on the shifting characteristics of the mechanism. These characteristics can be adjusted to suit a particular installation with profiled cams and springs. Thus, it is necessary to examine the shifting and gear ratio at any point. The speed of the primary CVT pulley is directly linked to the engine speed, whilst the secondary pulley speed is controlled by the shifting mechanism. The effects of the differential make it difficult and inaccurate to calculate the speed of the secondary pulley from the wheel speed sensors, therefore a separate sensor is required on the secondary pulley or shaft.

24.1.5 Brake Pressures

Monitoring front and rear brake pressure individually, along with wheel speed, would allow the braking balance to be adjusted accurately so that maximum braking performance can be achieved. The data could also be used to identify the best braking point for a particular corner.

Combining pressure data with wheel loads could lead to a better understanding of the coefficients of friction between the tyre and road and the brake pad and brake disk.

A ratio of the two system pressures will indicate the presence and effects on balance of any compliance or component deformation. If the balance bar performs perfectly it should counteract the effects of compliance and the pressure ratio should remain constant. If the ratio is seen to change greatly the difference in compliance between the front and rear systems could be too great for the balance bar to overcome.

GPS speed and wheel speed can be used to calculate slip ratios[20]. It is possible also to combine tyre data and calculated tyre loadings[21] to read off the optimum slip ratio for the tyre at that load. This can then be compared to the actual slip ratio, giving an indication of over or under braking. Adjustment to the brake balance will in turn change the load on the tyre and alter the optimum slip ratio, but the process can be made iterative until a satisfactory balance is achieved.

24.1.6 Tyre Temperature

The effects of average tyre temperature and temperature fluctuation were discussed in Section 8.5. It is possible to continually measure tyre temperatures whist the car is on track. The nature of the measurand dictates that non-contact sensing must be used. Consequently, infra-red pyrometers are the best sensor for this application. However, these sensors can be prohibitively expensive for low budget racing and consequently, tyre temperature logging may not be a viable option. The sensors often need some form of auxiliary circuitry in order to produce a useful output, further complicating the procedure.

Tyre temperature data however, could be very useful to the designer and setup engineer. Ideally, a wide tyre surface would be covered with an array of sensors, giving a temperature profile across the face. Consequently, the fashion in which the suspension loads the tyre can be evaluated. Suspension parameters can then be tuned for the tyre's benefit. As it is then known how the tyres are performing at specific parts of the track, it becomes possible to tune the car to perform exceptionally well at critical points, to the detriment of other less critical areas.

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[20] For more information on slip ratio see section 7.6.

[21] For more information on calculated wheel loads see section 24.1.1

25 Where to Focus the Design

25.1 Design Objectives

In order to push the design in the most beneficial direction a clear set of design objectives must be set out. For the development of the FS and many other racing disciplines, these will always include.

- Improve dynamic performance and reduce lap times.
- Reduce manufacturing time.
- Reduce manufacturing costs.
- Improve reliability.
- Simplify designs.

The focus of this thesis is vehicle performance so we shall look at the design process and parameter specification with respect to this. The next step is to look at the methods for achieving the objective and the design and engineering consequences of each. The methods then need to be assessed with respect to the constraints on the project, set out in Section 19.1. A hierarchy of development areas can then be created. The areas at the top of the hierarchy would benefit the design objectives most whilst imposing least on the constraints.

The rest of this section looks at a method for assessing net performance gains i.e. lap time reduction for specific performance increases. This can be used as a tool to assess the relative merits of a performance gain.

This section concludes with a list of the components and systems that can be developed to satisfy the design objectives. They are ordered by the net performance gains weighted by the resources involved in development.

25.2 Gate Based Calculations

In Section 22 a track model was discussed. The model can be used to create a basic track simulation. From this the effects of performance changes can be seen on the net lap time. The first step is to calculate the optimum speed of the car at the gates and during the straights.

25.2.1 Maximum Corner Speed

The maximum theoretical corner speed can be calculated since the corner radius and maximum lateral acceleration of the car are known values. The tangential velocity 'v' of an object travelling on a circular path with radius 'r' and with centripetal acceleration 'a' is given by:

$$v = \sqrt{ar}$$

25.2.2 Maximum Speed

Assuming that the car can maintain constant acceleration, Figure 25-1 shows how the speed of the car varies between two gates positioned at 1 and 3. Point 2 represents the braking point at which maximum speed is seen.

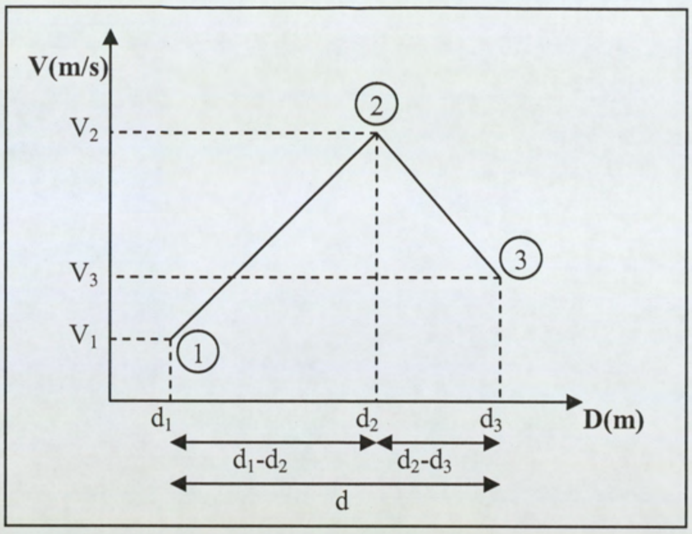


Figure 25-1 - Gate Speed Graph

$$d = d_{1-2} + d_{2-1}$$
$$d_{1-2} = \frac{v_2^2 - v_1^2}{2a}$$
$$d_{2-3} = \frac{v_2^2 - v_3^2}{2b}$$

Where 'a' is the vehicle's average traction acceleration and 'b' is the average braking acceleration, both are measured in m/s^2 and are positive.

Therefore:

$$d = \frac{v_2^2 - v_1^2}{2a} + \frac{v_2^2 - v_3^2}{2b}$$
$$b(v_2^2 - v_1^2) + a(v_2^2 - v_3^2) = 2abd$$
$$bv_2^2 - bv_1^2 + av_2^2 - av_3^2 = 2abd$$
$$bv_2^2 + av_2^2 = 2abd + bv_1^2 + av_3^2$$
$$v_2^2(b+a) = 2abd + bv_1^2 + av_3^2$$
$$v_2 = \sqrt{\frac{2abd + bv_1^2 + av_3^2}{(b+a)}}$$

25.2.3 Time and Braking Point

Now that the peak speed 'v_2' and time between the gates has been determined, the distance of the optimal braking point can be found:

$$t_3 = \left(\frac{v_2 - v_3}{b}\right) + \left(\frac{v_2 - v_1}{a}\right) + t_1$$

and

$$d_{1-2} = v_1\left(\frac{v_2 - v_1}{a}\right) + \frac{1}{2}a\left(\frac{v_2 - v_1}{a}\right)^2$$

25.3 Track Simulation

The previous calculations were incorporated into a spreadsheet in order to simulate a complete lap. As the calculations are driven by lateral and longitudinal acceleration, the values assumed can be changed to look at the net lap time reduction.

25.3.1 Simulation Refinements

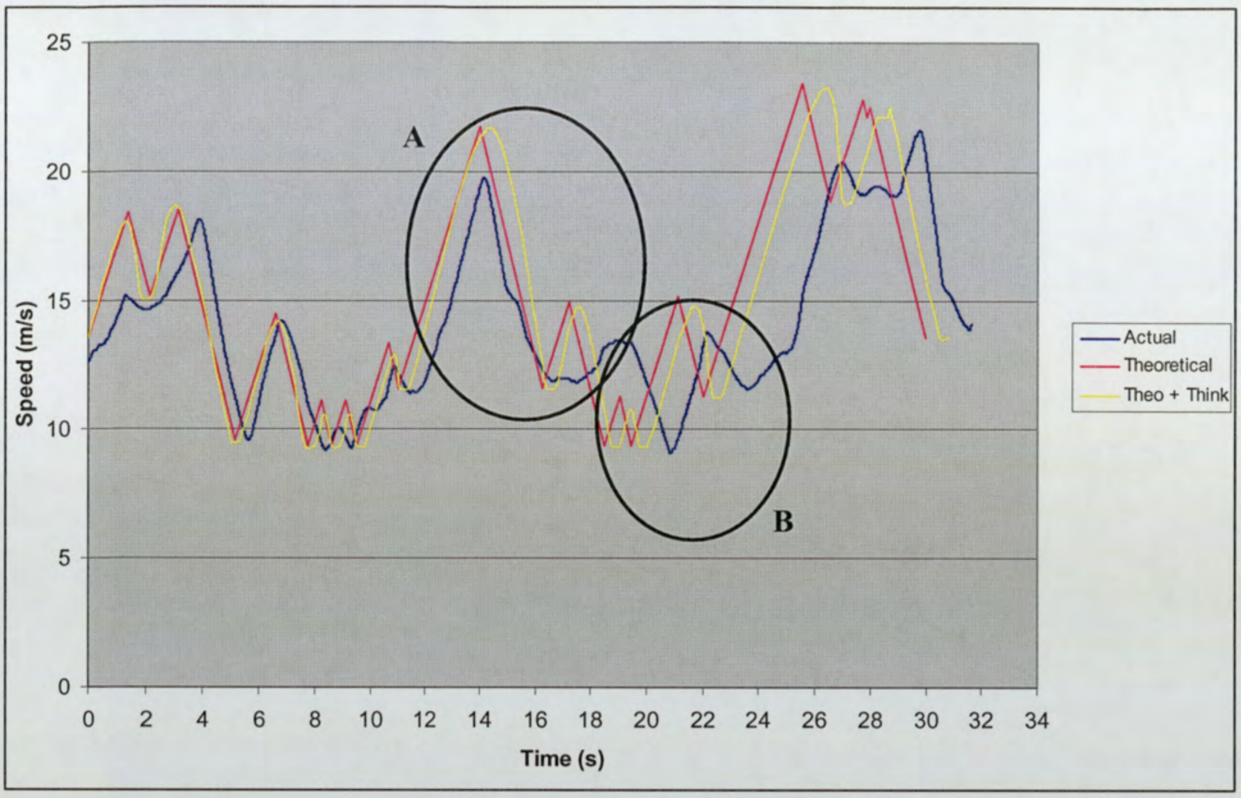


Figure 25-2 - Lap Time Simulation

Figure 25-2 is a plot of simulated speeds and overall lap time overlaid on data from an actual lap of the track. In order to compare the simulation with a real lap and thus judge its merits as an accurate tool, the target speeds for each gate were not calculated from maximum lateral acceleration but were taken to be the same as the actual speed at point of the gate.

The pink line is from an early simulation which was further refined to include a time period to represent driver control. This new function is shown by the yellow line. The 'thinking' period is a fixed time intended to represent the time taken by the driver and vehicle in reacting to inputs. The simulated line follows the actual line with remarkable accuracy and the points at which it deviates can be explained and are normally a result of driver error. The thinking time for the simulation shown is 0.3s.

In region 'A' the deviation is caused by two factors. Firstly, the driver started braking too early for the corner. As a consequence the minimum corner speed was held for a period of time rather than being momentarily. Secondly, the corner at the end of region 'A' is long requiring a more constant speed and consequently the simulation method becomes less accurate.

The track highlighted in region ‘B’ contains as slalom, the line the driver took enabled the first two corners to be taken at a higher speed than the corner radius would suggest. However, as a result of this line the driver had to slow the car for the final slalom, consequently delaying the point at which they could accelerate into the next straight.

The assessment of the accuracy of this simulation had revealed that it has uses as both a driver training tool as well as a design evaluation tool.

With a ‘thinking’ time of 0.3s included the simulated lap time is 30.82 seconds, whilst the actual lap was 31.57 seconds. It is also apparent from the simulation that the average longitudinal acceleration was approximately 0.35 g and 0.5 g in acceleration and braking respectively, figures that are significantly lower than the maximums that the tyre can be expected to generate.

The simulation gives the designer a realistic and representative indication of the effects that the most basic performance characteristics would have on achievable lap times. This can now be used as a tool to quantify the effects of any theoretical improvement to the fundamental performance characteristics of the car. From this point onwards the 30.82 second simulated lap will become the benchmark for comparing performance gains.

25.4 Performance Assessment

25.4.1 Lateral Acceleration

The G-circle analysis in Section 23.1 shows that the car is capable of reaching and sustaining lateral acceleration in the order of 1.5g. Tyre data for the selected Avon tyres suggest that at the expected vertical loads more acceleration might be achievable. A modification to the track simulation in Section 25.3.1 uses actual corner radii and lateral acceleration capabilities to estimate the target speed for each gate.

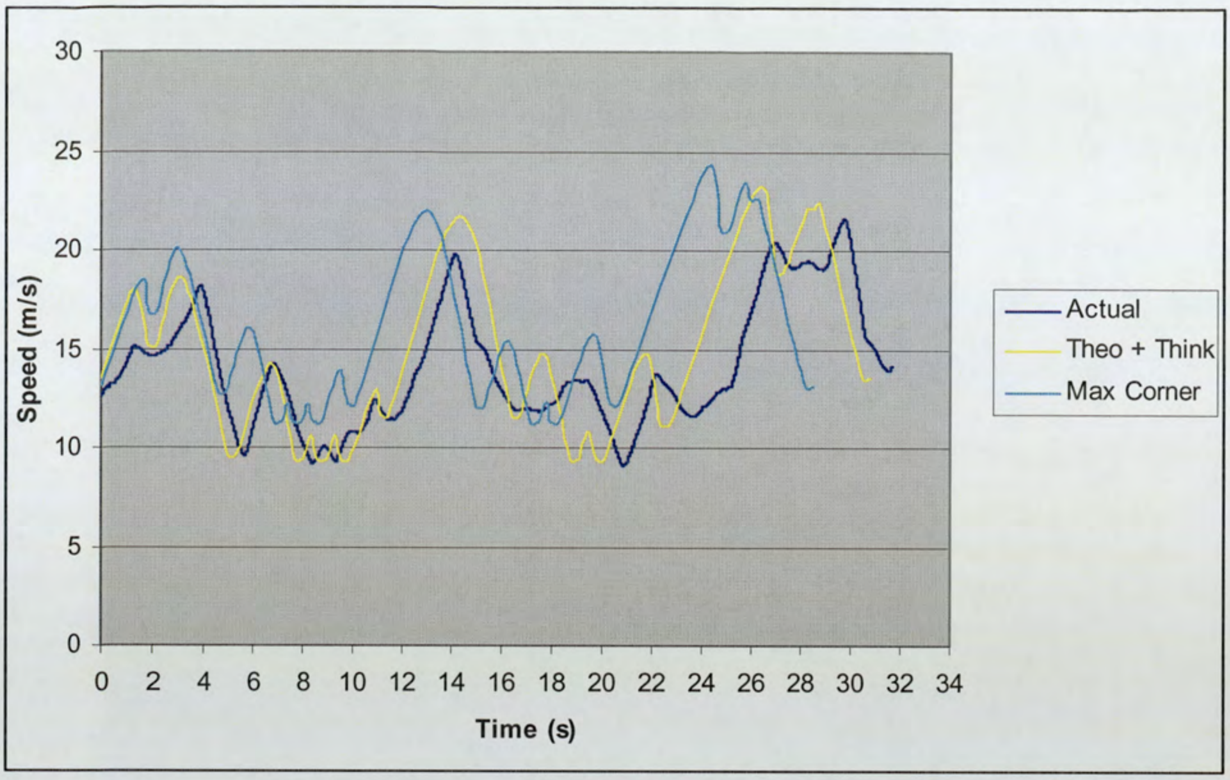


Figure 25-3 - Utilising Lateral Capabilities Simulation

Figure 25-3 shows the simulated lap assuming that 1.3g lateral acceleration, a figure that is achievable by the current design, is achieved on all corners. The simulated lap time is 28.48s approximately 2.4s less than the simulation based on actual current corner speeds. If 1.5g could be realised on every corner, the lap time could be expected to fall by a further 1.3s. However, to realize this gain in performance the resource consumption would be high since performance is already close to limits of lateral grip.

It is unlikely that any major improvements in ultimate lateral acceleration capabilities would be achievable without significant investment of time and resources during the design stages. However, a significant reduction in lap times could be seen from simple developments to improve drivability lateral acceleration capabilities should therefore be about making high acceleration consistently achievable and a reduction of response time. The driver then needs to be guided as to where the limits are how to reach them.

25.4.2 Longitudinal Acceleration – Braking

Theoretical analysis of the braking system and tyre grip curve indicates that the car should be capable of stopping with accelerations in the order of 1.5g. The average seen on track was about 0.5 g with peaks of 0.9 g. If the braking system were improved such that the driver could maintain 0.9 g continually when braking, the lap time could be reduced by 1 second over the benchmark lap.

The braking system is simple, easily monitored and quick to adjust. Consequently significant improvements should be possible in the system, with relatively little development, since the theoretical maximum is still a long way from being achieved. The braking systems fitted to FS cars cannot be fundamentally flawed since they must be capable of locking all four wheels. Thus probably all that is required is more time spent setting the front to rear biasing.

25.4.3 Longitudinal Acceleration – Traction

Calculations and practical tests have shown that the CVT and the engine should be able to deliver maximum torque to the wheels up to around 40mph. This torque should and will produce longitudinal accelerations of around $10 m/s^2$ or 1g, possibly more if the mass of the car is reduced.

The analysis of performance is not detailed enough to determine if it is the track limiting the maximum longitudinal acceleration or the drivetrain, further straight line tests should be analysed to show if the car is capable of higher longitudinal acceleration than achieved in the laps of the sprint track. Previous tests have however shown accelerations in the order of 1g. Analysis of the lag involved with the CVT should be carried out to ascertain if changing the gearing ratios might reduce response time, increasing the maximum acceleration achieved in the short straights.

Since the traction acceleration of the vehicle is the lowest of its acceleration capabilities, improvement to this alone should yield significant reductions in lap time. The simulation concurs with this hypothesis indicating that if the average acceleration were increased to 0.5g from 0.35g the lap time might reduce by 0.9s. A larger increase to 0.9g constant acceleration would reduce the lap time by over 2s over the benchmark lap.

25.4.4 Balance

More in depth analysis is necessary to ensure that the car is designed to be as neutral as possible so that the adjustment range included in the design is appropriate. The addition of a steering position sensor will aid in diagnosing the current steer characteristics, see Section 24.1.3 for more detail. A more accurate computer simulation could be carried out by linking the output of

SusProg 3d with a tyre data model. Then the effects of changes to the vehicle may be better understood, allowing for more off-track setup decisions to be made.

25.4.5 Damping

Very little development has gone into understanding, specifying and analysing the suspension damping involved with Formula Student. Significant steady state and transient performance gains might be made by improving the tyre road contact condition and roll speeds. Section 23.4 has highlighted that the current damper settings are not optimum. However, it is hard to quantify the net gains that might be made through improvements to the damping system. The fact remains though that a problem has been identified along with suitable remedies. The low speed characteristics are externally adjustable and their performance is easily monitored, thus their tuning should be considered and adjusted in order to appreciate the performance implications.

25.4.6 Transient Response

Reduction in the time taken for the car to respond to the driver input and reach a steady state is always going to be beneficial in such an event. The reduction of car mass and moments of inertia will reduce cutting response time. However, some study of the consequences of moments of inertia needs to be made to further quantify the rewards of any efforts made with respect to reducing them.

25.5 Design Focus Hierarchy

As set out in the introduction to Section 25 an ordered list can be constructed of the components and systems that can be developed to satisfy the design objectives. These will be placed into a hierarchy ordered by the resources consumed in developing them with respect to net performance gains.

1) Brake development/tuning
    a) Refinement of braking bias
    b) Improvement of pedal assembly and adjustability
2) Drivetrain development/tuning
    a) CVT tuning
    b) Gearing tuning
    c) Transient engine response
    d) Engine power output

e) Differential development

3) Weight reduction/distribution

   a) Overall design for loading

   b) Weight distribution to reduce inertia

4) Damper development/tuning

   a) Damper specification

   b) Damper testing

5) Suspension

   a) Kinematics refinement

   b) Spring/roll rates

25.6 Performance Targets

The lap simulation can be used to project what the minimum possible lap times might be if the car were developed to deliver the performance characteristics that are known to be achievable.

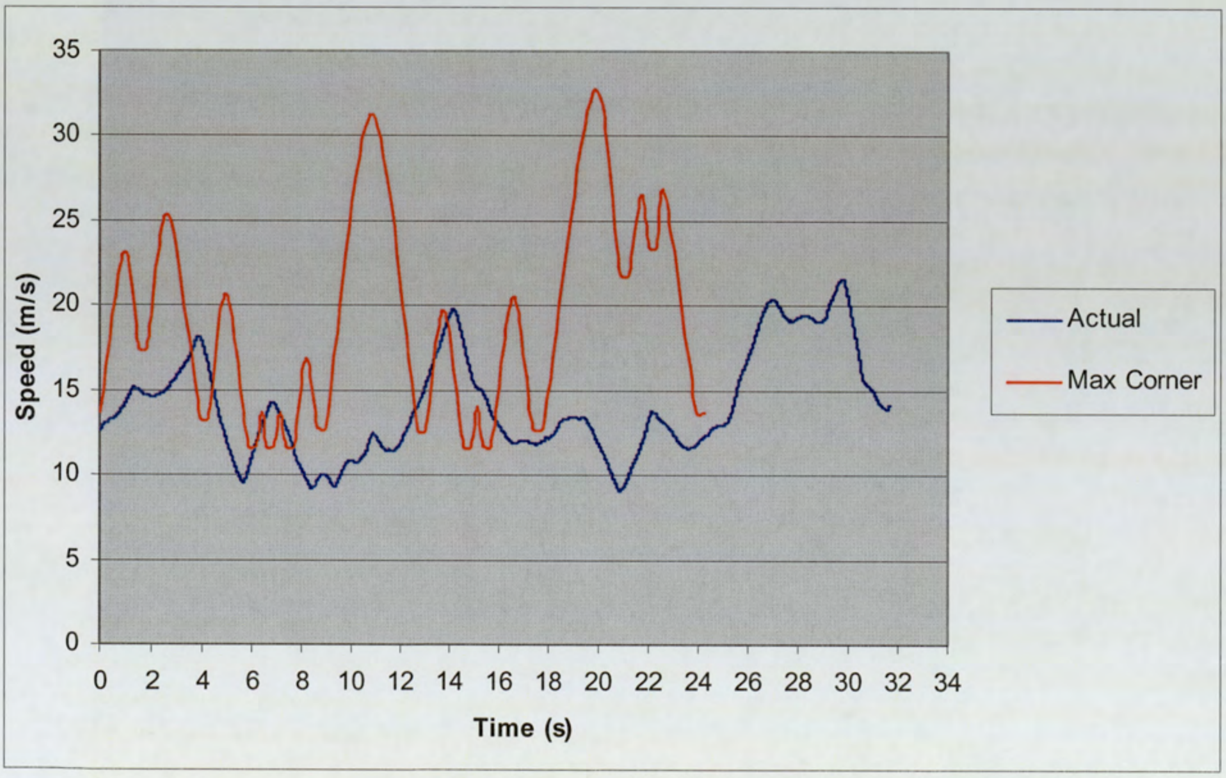


Figure 25-4 - Optimum Lap Time Simulation

Figure 25-4 is a lap time simulated using values of 1g for acceleration and braking and 1.4g for lateral acceleration. The simulated lap is displayed against the benchmark lap and is 24.25 seconds long.

Although the simulation and lap model are simple the effects of combining good all round performance can be demonstrated. It is unlikely that a driver or vehicle can be consistent enough to achieve this level of performance and thus the projected lap time will be inaccurate. However, the simulation has proved useful for assessing the merits of design development in different areas.

26 Vehicle Design Parameters

Based on the design objectives set out in Section 25.1 and the 'Design Focus Hierarchy' created in Section 25.5 an achievable set of vehicle parameters can be specified.

26.1 Tyre Selection

The choice of tyres suitable for the Formula Student application is limited. Low vehicle mass demands a lightweight tyre that will comply with the road surface under minimal vertical load. Minimum tyre size rules stop the use of kart style tyres and the selection of tyres to fit ten inch rims is limited. The range of tyres commercially available becomes broad enough to find a suitable tyre when the wheel size is increased to 13 inches. This wheel size also allows good space for packaging the brakes and upright whilst minimising the KPI needed to reduce the scrub radius.

Avon Tyres specialise in producing tyres for lightweight open wheel formula cars, as a result they offer a comprehensive range of soft compounds coupled with lightweight carcasses. Stability rig data for these tyres shown in Appendix C show the 13x20x6.2 inch tyre to have excellent lateral force development over a controllable range of slip angles. The 8.2 inch version of the tyre only generates a small amount of extra lateral force within the expected load range, and the excess weight and camber control required for the increase in grip negates its use.

Simple calculations show that the coefficient of grip for these tyres is 1.7 at a vertical load of 1.17kN, a similar load that to that expected on the outside tyre of a 280kg car at 1.7g lateral acceleration. This can even be considered as worse case since the load sensitivity has not been taken into account and the expected coefficient on the inside tyres could be higher. Lateral accelerations of greater than 1.44g were not seen in the 2005 skidpan competition. Thus a theoretical maximum of 1.7g given by the tyres is acceptable for competition performance.

26.2 Design Parameters

The following list is taken from Section 6, it covers the essential things to know before design can begin.

- Maximum speed – 50 mph sprint and endurance course (80 mph acceleration test)

- Minimum speed – 20 mph and endurance course
- Expected weight – 260 kg with driver
- Expected engine power – 52 kW
- Maximum coefficient of grip of the tyre to surface – 1.7
- Position of the COG (Weight Distribution)- x = 0 y = 300 z = -Wheelbase/2
- Required suspension movement - ±25 mm
- Braking requirements – Lock all four wheels
- Corner radii – minimum radius of 9 m
- Down force – None Expected
- Aerodynamic drag – Minimal Contribution

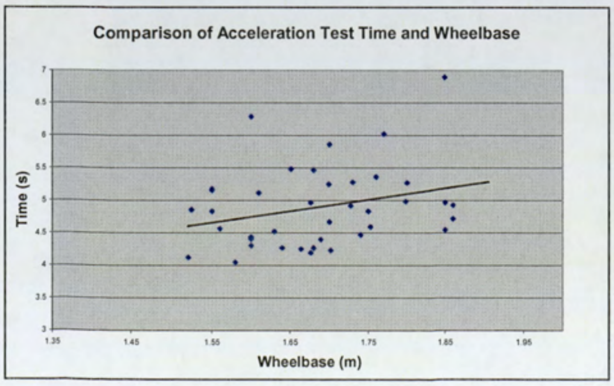
From these the following can be calculated:

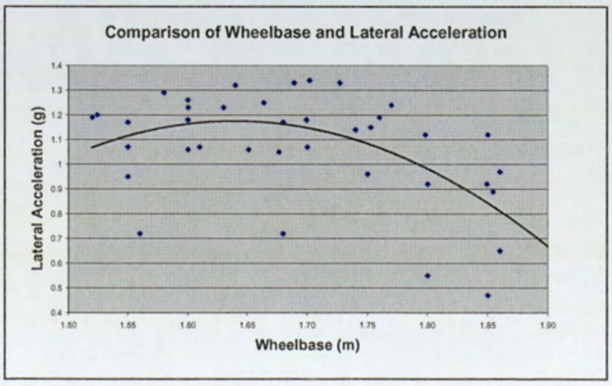
- Expected maximum accelerations
    - Lateral – 1.7 g
    - Longitudinal - 1.7 g
- Expected weight transfer – Approx 44% (1.2 m wheelbase)

26.3 Sizing the Footprint

26.3.1 Parametric Analysis of Competitor Performance

Data of competitors' performance can be easily interpreted and evaluated to back up the theories, aiding in decision making. The following series of graphs show the results of the 2004 competition against track and wheelbase dimensions[22].





[22] The data used in this analysis has been taken form IMeche Formula Student Programme 2005

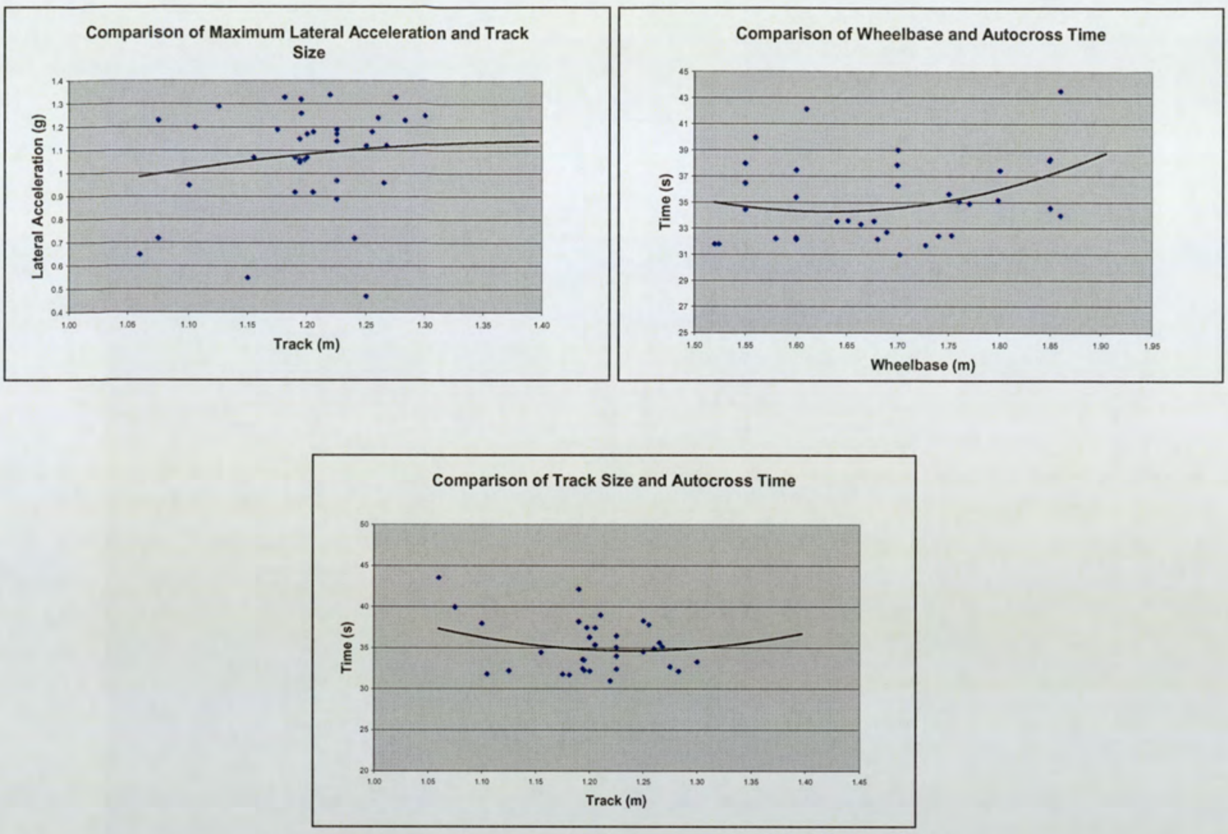


Figure 26-1 - Parametric Study of Footprint to Performance

The first of the graphs shows the relation between acceleration test times and wheelbase, as expected the shorter wheelbase cars achieve higher longitudinal acceleration. The reason is undeterminable from this data. However, it may be a result of more weight transfer to the rear wheels or as a consequence of an overall smaller and lighter package.

The next graph looks at the relation between wheelbase and maximum lateral acceleration achieved on the skidpan. Although the skidpan should test steady state cornering and not be affected by wheelbase, the extra stability of the longer wheelbase does have an effect. The trend shows that the cars with a wheelbase between 1.6 m and 1.7 m achieve the highest lateral acceleration.

The third graph looks at the same test as the second but with respect to the track width instead. Again as expected the trend implies that the wider the track is the higher the achievable lateral acceleration would be. It should be noted that there were cars achieving similar maximums with a wheelbase within the range of 1.12 m to 1.27 m.

The fourth and fifth graphs look at the autocross test results with respect, firstly, to wheelbase and secondly, to track size. These further confirm that there appears to be a 'sweet spot' for both track and wheel base with regard to good dynamic performance in this event. The trends appear to show that the optimum track width and wheelbase would be in the order 1.2 m to 1.25 m and 1.6 m to 1.65 m respectively.

26.3.2 Track

Using the simplified weight transfer model described in Section 6.1.2, the weight transfer function can be plotted, Figure 26-2. The function is asymptotic to zero at infinite track width, meaning that increases in track width produce diminishing returns with respect to weight transfer reduction. Any reduction that is achieved comes at the cost of long, flexible and heavy suspension arms and an increase in unsuspended mass.

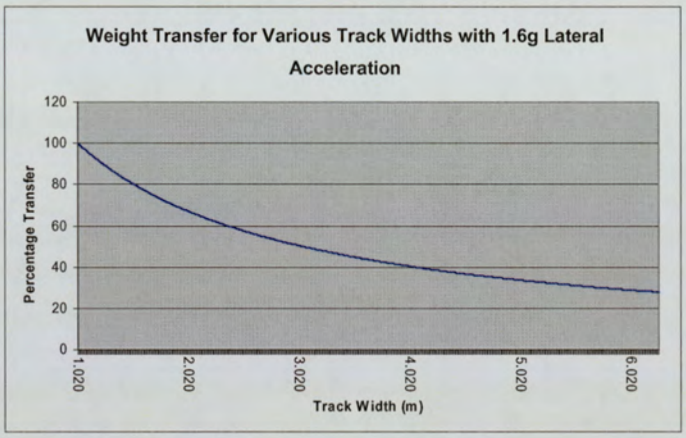


Figure 26-2 - Weight Transfer for Various Track Widths

For the Formula Student car where safety is paramount to performance a compromise has to be made on the track of the car. The track must have sufficient width to resist the expected lateral acceleration expected without being unstable and lifting inside wheels, whilst not being wider than necessary. The tyres used are capable of producing lateral thrust that will sustain lateral accelerations of 1.7g whilst the track is very tight and twisty and the ideal width for passing slaloms would be narrow. The result is a small vehicle packaged with a relatively large engine and with the driver affecting the height of the centre of gravity. The competition rules also state that the car must pass a tilt test to 60° representing a lateral acceleration of 1.7g.

The expected weight transfer against track width can be plotted for a car with a COG height of 0.3m and with lateral acceleration of 1.7g for track widths between 1m and 1.5m.

The minimum track width needed to withstand this lateral acceleration would be around a metre but by increasing further to 1.2 m the weight transfer can be reduced by 10 % more without much physical increase in vehicle size, but reducing the negative effects of tyre load sensitivity.

As stated before this is a simplified model, in reality the pivot is the outside corner of the tyre not the centreline of the wheel. Suspension geometry will also affect weight transfer as the unsuspended components do not transfer weight in quite the same manner as the suspended ones. Consequently, this is a worst case scenario, and the capabilities of this track width should exceed the above calculations.

The 2005 car was designed with front and rear tracks of 1.1m and 1.05m respectively, without fully understanding the capabilities of the tyres. Questions have been asked of the car's stability when under high lateral acceleration. These calculations demonstrate that the tyres demand a wider track in order to be stable at the limit of grip, therefore the final decision is that the car will have front and rear tracks of 1.2m.

26.3.3 Wheelbase

Limiting longitudinal weight transfer will be beneficial in the Formula Student competition, increasing stability on entrance and exit to the corners. In order to benefit stability the current 1.56m wheel base can extend to 1.65m. Cars with this wheelbase have shown better performance at the competition. The increased wheelbase also eases packaging constraints, allowing for better static weight distribution.

Weight transfer aside, the wheelbase will affect the Ackermann steering angle required to turn a given radius. The increase in steering angle required to turn the minimum radius corner of 9 m is given by:

$$\delta_{rad} = \frac{l}{R}$$

$$\delta_{\deg} = \frac{180}{\pi}\frac{1.56}{9} = 9.9^\circ$$

$$or$$

$$\delta_{\deg} = \frac{180}{\pi}\frac{1.65}{9} = 10.5^\circ$$

The increase in steering angle required therefore approximates to 6%, a minimal increase for the expected stability increase.

As explained in Section 25.5 longitudinal acceleration is of slightly less importance than lateral. The reduction in weight transfer to the rear wheels in acceleration is acceptable since benefits will be seen in handling and braking. The consequences of this increase in wheelbase are a reduction in weight transfer of 5%, and a reduction in predicted maximum longitudinal acceleration of $0.3 m/s^2$. However, since the maximum acceleration seen on the track was considerably less than the maximum predicted, it is clear that traction is not limiting performance, and the increase will have no negative effects on track performance.

26.4 Weight Distribution

26.4.1 Longitudinal

Since both front and rear tyres are identical, assuming that they run at the same pressure, they will have the same cornering stiffness. Thus to be neutrally balanced the CoG must be located mid wheelbase giving a 50:50 front rear distribution. Ideal for steady state cornering, it is not ideal for corner exit at the limit. The combination of increased load and traction forces will reduce the lateral coefficient of grip, promoting oversteer. Consequently for cornering the optimum weight distribution might be more like 60:40 to give steady state understeer that approaches drift at the limit. The problem with this configuration is that the low speed nature of the event dictates that the engine always has enough power to spin the rear wheels in a straight line, in which case the CoG needs to be moved towards the rear of the car for maximum traction.

Possibly experimenting with movable ballast to change the CoG might reveal the optimum solution to the problem. However, for the purpose of design a 55:45 front rear weight distribution will lend its self to adjustment and balancing by altering roll distribution.

26.4.1.1 Lateral

The CoG should be positioned on the centreline of the car. Acceleration capabilities are measured in both directions and the endurance track has a similar number of right and left turns, negating any benefit of an asymmetric setup.

26.5 Suspension Kinematics

26.5.1 Adjustability

Designing adjustability into a race car poses a lot of problems in itself. It is often very difficult to locate the adjuster such that it changes only the intended. On occasions there is no alternative and a single adjustment will alter two variables. With respect to adjustability, other designers have pondered and experimented with suitable methods, these should be evaluated and adopted where necessary.

Wherever possible it is more desirable to adjust something with a shim or packing. The exact size of shim can be calculated to give the right effect, similarly the original setup can be recorded and returned to quickly. If threaded adjustment is to be used, for accuracy it is better to use threads of the same direction but different pitch, as opposed to a left and right hand thread at each end of the adjuster. This way one turn of the adjuster could give a quarter less adjustment.

The table below lists the parameters that need to be adjustable on a Formula Student car, where appropriate a range and method is suggested.

Variable	**Method**	**Range**
Camber	Shims	±1.5°
Wheel Alignment	Toe and steering arm length	±1.5°
Ride Height /Corner Weights	Adjustable spring platform	
Ride Height	Pull rod length	
Wheel rate	Range of springs	
	Adjustable motion ratio	
Roll Rates	Range of springs	
	Adjustable motion ratio	

26.5.2 Front View Swing Axle Length

The simple functions can be plotted on the same axes to asses the effect that FVSAL has on them. It must be noted that bump compression will remove camber and extension will add camber. Figure 26-3 is for a vehicle of 1.2 m track with a predicted maximum of 1.5 degrees roll or 25 mm of bump travel. A sensible range of FVSAL for the low speed Formula Student car would be in the range of 1.5 to 3m.

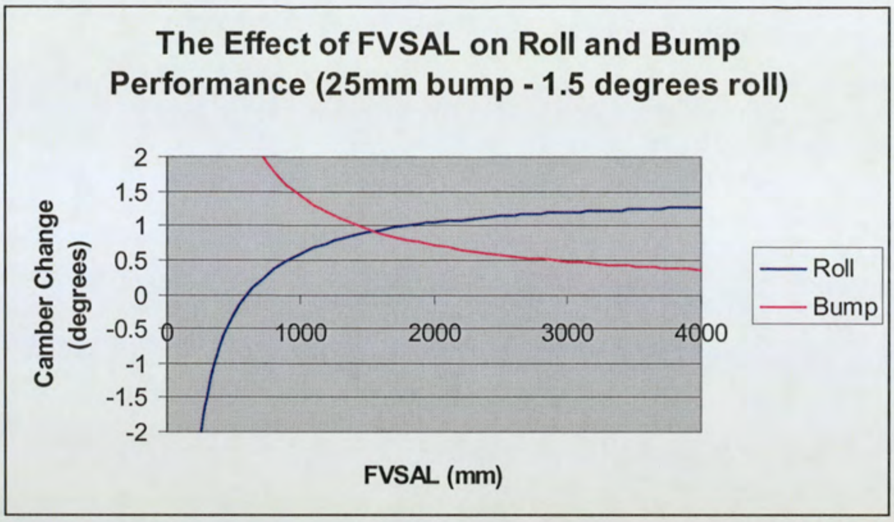


Figure 26-3 - Effect of FVSAL on Camber in Roll and Bump

The functions are simplified and do not take into account the possibility of reducing FVSAL with the onset of roll. However it is useful to visualise what happens as FVSAL is reduced and a useable FVSAL range can be determined for further suspension analysis. A more complete analysis of suspension performance is carried out in SusProg 3d[23].

It is common to use shorter FVSAL on the rear of the car to increase camber gain on the rear, improving corner exit and balancing the front camber gain from steering geometry.

26.5.3 Roll Centre

In Section 11.3 the effects of roll centre height on weight transfer are explored. Lifting the roll centre not only decreases the roll moment but increases the load transferred through the suspension arm, the geometric transfer. This is beneficial to response to steering input since the weight transfer is instantaneous and not affected by the dampers. This handling characteristic is desired for the Formula Student competition that requires constant direction change.

Limiting the roll moment also allows for the use of softer springs, improving suspension and tyre conditions. Jacking forces are also increased with a high roll centre. However, they are more detrimental to high speed stability where ride height is critical for aerodynamic performance.

An expected range would being the order of 60 – 120mm above the ground, the rear roll centre15-20 mm higher than the front, reducing the roll moment at the rear, biasing weight

[23] Appendix D contains sample SusProg 3d output for the 2006 design.

transfer to the front, freeing grip for rear traction. Roll centres any higher than this are hard to achieve because of packaging constraints and limit the adjustability of weight transfer.

By keeping the wishbones as parallel as possible and making the bottom wishbones as long as possible it is easier restrain the roll centre. For the purpose of Formula Student, movement of the roll centre should be minimised by experimenting with the geometry.

26.5.4 Wheel Alignment

Static negative camber is necessary on all four wheels to offset the effects of body roll. The level of acceptable static camber is determined such that the tyres are not over heated on the inside edge or excess lateral force is produced in a straight line. Straight line conditions are not common on the FS track, encouraging the use of large amounts of static negative camber. The reverse argument is that negative camber will reduce forces from the inside tyre and thus the net grip will be reduced. The analysis of performance showed that there was not a large amount of improvement to make in respect to lateral acceleration, consequently the static figures of 1 and 1.5 degrees front and rear respectively should be retained.

26.6 Suspension Rates

Specifying suspension rates is an iterative process. Packaging and weight constraints will determine the limits of the damper size. The range of damping constants that the damper can be adjusted to might depend on its size, whilst the motion ratio should be chosen to give the correct desired natural frequencies. All of this needs to be done whilst using as much of the damper stroke as possible and if possible with standardised spring rates on the front and rear.

26.6.1 Wheel Rates

The first step is to look at the minimum suspension movements required by the competition rules and use this as a basis for calculating spring rates. For Formula Student the rules state that there must be 25 mm of both compression and extension. Implying that in a static situation there must be at least 25 mm of compression in the springs.

From the chosen weight distribution and estimated vehicle mass we know that the front and rear wheels will support 71.5 kg and 58.5 kg respectively. Approximately 10 kg of this will be unsuspended mass, reducing the mass supported by the springs to 61.5 kg and 48.5 kg. Thus a starting point for the wheel rate is given by 61.5/22.5 = 2.7 kg/mm = 26.64kN/m.

26.6.2 Motion Ratio

There are a limited number of damper manufacturers that will supply dampers in the correct scale for the low mass FS car. The most viable option is to use a damper initially designed for competition bicycles that has been modified to be more suitable. Risse Racing[24] supply dampers to many other FS teams and are competitively priced and hence have been selected as a suitable unit.

The motion ratio needs to be specified so the damper operates over the maximum distance possible. However, some allowance should be made for the possible substitution of less stiff springs by not completely using all of the damper's motion.

The motion ration can be calculated knowing the allowable damper movement. The middle-sized damper has a movement of 57.15 mm, allowing for bump stop and softer springs we can assume that it would have a useable stroke of 50 mm. Therefore, an approximate value is given by 50/50 = 1.

26.6.3 Spring Rate

The dampers can be supplied in a range of three sizes with springs ranging from 150 to 400 lb/inch. If the proposed motion ratio of 1 is used then the wheel rate is equal to the spring rate. And consequently a spring of 2.7 kg/mm is required. This is equal to 150.9 lb/ inch which is within the supplied spring range.

26.6.4 Roll Rates

The spring rates of the anti-roll bars now need to be specified in order that roll is kept within specific limits and vehicle balance is preserved. Since the roll moment is dependant on the height of the roll centre and this may be subject to movement the analysis becomes complicated. Thus, roll stiffness is easiest specified with the use of SUSPROG. The main criterion is to limit the roll to 1 degree per g of lateral acceleration. See Appendix Dfor detailed output of SUSPROG design.

Analysis using SUSPROG has shown that the most suitable damper would be the middle size with 57 mm of travel and fitted with a 150 lb/inch spring. The motion ratio is then designed to

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[24] Risse Racing Technology, 1240 Redwood Blvd, Redding CA 96003

give linear rates and a wheel to spring ratio of around 1.1. The wheel rates will then be in the order of 2.2 kg/mm.

For the reasons suggested in Section 15.6.1 linear spring rates are suggested for the Formula Student application. As discussed the exact calculation of rates is best suited to a suspension analysis package.

26.6.5 Damping

The final thing to specify with respect to the suspension is the required damping. The racing application is heavily biased toward damping for handling and not for ride comfort [25]. Now the spring rates and motion ratio are known, the damping ratio 'C' can be calculated in order to give critical damping for the motion of the wheel as per Section 16.6.2.

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[25] For more on the handling/comfort compromise refer to section 16.6.3

27 Track Tuning

In Section 25.4 the performance analysis was looked at with a view to making recommendations about areas of the vehicle design that could benefit from development. In this section suitable test procedures will be described such that the most applicable data can be recorded. The analysis of this data will focus on what setup changes are necessary to improve performance.

The purpose of track testing is to improve the car's performance; this can be done in one of two ways. The first way would involve adjustment of settings one at a time and recording the result, with little or no thought process involved. Eventually a suitably performing combination would be stumbled upon. The second way, and by far the most feasible method involves thought and experience.

Analysing data from test and comparing it to the theoretical ideals will give an indication of the things to be improved. The next process is far more involved and complex. The test engineer must specify what needs adjustment and how far to adjust it. The test engineer will use theoretical knowledge and some experience to suggest what need adjustment, whilst how far to adjust it will be based more on experience.

The term 'experience' does not solely apply to the personal experience of the test team but also to careful documentation of previous testing. The use of 'magic numbers' that describe key setup parameters of the car can be used to help when determining how far things should be adjusted. For example, the car performed excellently on a previous track and the driver was happy with the balance of the car. However, the new track has a lot more bumps and the driver is complaining that the car is bouncing over the bumps and losing grip. Obviously the suspension needs softening to allow it to comply to the bumps a lot more. If the engineer has recorded the ratio of roll distribution front to rear from the previous car they can suggest a new set of spring and anti-roll bars to retain the same balance.

27.1 Testing Schedule

Track testing has been and always will be a necessary part of racing. However, it is expensive and there maybe limitations placed on it. Thus it is necessary to make the most out of the track time that is available. Preparation is one of the keys to this success, any time spent preparing is cheap in comparison to track time.

A well planed testing schedule known by all is a crucial part of track testing. Many experienced and successful engineers have extensively written about this topic.

27.2 Adjustment Table

Keeping a clear record of how the car is currently set up and any changes that have been made allow the engineer to assess why these adjustments are having an effect or if they are over compensating for something else that is wrong. In addition, if nothing is working to solve a problem the setup of the car can be easily returned to its original state and another route can be taken.

The adjustment table should also contain information about the adjustments, for example 1 turn on the tie rod affects the tracking by 0.2 degrees. This way the car can be taken to the track having been accurately set up in the garage and be confidently and quickly set to any desired state. Thus the need for complicated gauges being used at an inappropriate time is removed.

27.3 Preparation Adjustments

This is not an exhaustive list since any list of adjustments depends on a particular car. However, the following list includes the major adjustments that should be made in preparation for the car being taken the track for testing.

- Wheel Alignment
- Ride Height
- Corner Weights
- Wheel Camber
- Springs and shocks

With these accurately set in the garage the car can be taken to the track knowing that any setup can be achieved by a known alteration from this setup.

27.4 Trackside Adjustments

All of the settings mentioned in the previous section can be altered trackside however they may be difficult to set to a given value without delicate measuring, hence the value of knowing what a given adjustment means in terms of the actual variable.

Tyre pressures are uncommon in that there is no point setting them before getting to the track due to their instabilities. As the tyre is likely to loos pressure when left for any period of time pressures should only be set immediately before taking to the track.

27.5 Testing Schedule/ data analysis

The following sections describe suitable tests to aid in the collection of data such that a new car can be quickly set up to achieve a good performance level.

27.5.1 Braking

For obvious safety concerns it is sensible to commission the braking system early on in this process and in doing so achieve a suitable brake balance.

Logged Variables:

- Wheel speed
- GPS speed
- Brake hydraulic pressure
- Suspension position
- Longitudinal acceleration

Possible Tuning:

- Brake balance
- Low speed damper adjustment

Adjustment Table Value

- Balance bar adjustment effects (e.g. percent balance change per turn)
- Optimum brake balance (e.g. percent front/rear)

Test Procedure

Note that the brakes should be fully bedded in and up to working temperature otherwise balance could be affected.

- Perform a series of stops from a constant speed to standstill with increasing pedal pressure until one or a pair of wheels lock.
- Analyse data and make adjustments according.

Data Analysis

Simple analysis to look at which wheel stops turning first should indicate which way the balance need to be adjusted. A more complex analysis of slip ratio can be carried out, see Section 24.1.5 for more information. The low speed damper performance can be looked at by comparing roll angles left to right and pitch angles front to rear, see Section 0 for more details.

27.5.2 Skid Pan

The test can be carried out at low speed thus offering some protection to the driver of an untested vehicle. The exercise should be carried out on small radius circular skid pan (7-12m radius). The purpose of this test is to asses the balance of the vehicle through the elastic range and then towards the limit of grip in the tyres transitional and frictional performance range. The test uses a circular path to emulate steady state corning, therefore negating the effects of low speed damping and high polar moments of inertia. Thus, the driver should try to be as smooth as possible and keep the vehicle on a constant line at a constant speed long enough for equilibrium to be reached.

Logged Variables

- GPS speed
- Suspension position
- Steering angle
- Lateral acceleration

Possible Tuning

- Vehicle balance
- High-speed damping

Adjustment Table Value

- Ideal roll moment distribution (e.g. percent front/rear)

Test Procedure

Tyre pressures should initially be set to manufacturer's recommended pressure.

- Perform several low speed laps in both directions steering a constant line
- Gradually increase speed steering, on the same line as at low speed
- Stop when the limits of grip are approached front or rear
- Analyse data and make according adjustments

Data Analysis

Projecting steering traces for each completed lap on top of the low speed lap as per Section 24.1.3 will give an indication of the balance of the vehicle. Based on the data the distribution of the roll moment can be changed with adjustments of the front and rear roll rates. With the balance of the vehicle set for the elastic range the test can be carried out again to see how the car performs close to the limits of grip.

If the performance near the limit of grip is not suitable, the tester may choose to alter other parameters to improve the performance at the limit. Hopefully the ultimate grip levels of the end of the car that is running out of grip can be increased by altering variables that will not have a large effect on the elastic balance. These variables might include camber, Ackermann geometry, static wheel alignment and high speed damping. However it may be necessary to change the roll moment distribution to increase the grip on a given end, at the cost of elastic balance.
It is generally accepted that reducing the roll stiffness at any given end will increase that end's net lateral grip. This is as a result of the tyres' load sensitivity attributes. The net gain in grip

from reducing the load on the outside tyre and increasing the load on the inside tyre outweighs the grip lost to the axle at the other end of the vehicle that has an increased vertical load.

High speed damping can also be evaluated during this test as per Section 23.4.

27.5.3 Acceleration

The purpose of this test is to assess the straight line performance of the vehicle for the 75 m competition test. During this test the gearing configuration can be examined and tuned. In the basic test the vehicle will accelerate from a standing start over a distance of 75 m. An adaptation to this test would use a rolling start and shorter straight to assess response times and engine pickup.

<u>Logged Variables</u>

- Wheel speeds
- Suspension position
- Longitudinal acceleration
- Throttle position
- Engine speed

<u>Possible Tuning</u>

- Gearing
- High-speed damping

<u>Test Procedure</u>

Tyre pressures should initially be set to manufacturer's recommended pressure.

- Perform several acceleration tests
- Analyse data and make according adjustments

<u>Data Analysis</u>

Because of the CVT, adjustment of the gearing is not comparable to other racing applications. The CVT enables the engine to perform at its peak power speed. The CVT then automatically alters the gearing to maintain maximum torque for maximum acceleration. Thus tuning of this device is carried out to ensure the following:

- The CVT up-shifts at the correct engine speed
- The CVT operates within its recommended speed range
- Required gear ratios are not outside of the CVT's range
- The CVT engage speed

See Section 24.1.4 for more on this topic.

27.5.4 Sprint Track

The culmination of performance testing should be carried out on the actual track or a replica track as similar as possible. In this test the test team are looking to further refine all previous tuning. Earlier tests have tried, wherever possible, to remove the effects of a combination of lateral and longitudinal acceleration and transients. In this test all will be present and consequently the previous settings will not be optimal. However, the car will already be in a reasonable state of setup to tackle the track.

Logged Variables:

- Wheel speeds
- Suspension position
- Longitudinal acceleration
- Throttle position
- Brake pressures
- GPS data
- Engine speed

Possible Tuning:

- Gearing
- Balance
- Tyre pressures
- Low-speed damping
- High-speed damping
- Brake balance

Data Analysis

Analysis of data for complete laps becomes more complicated as combinations of actions can confuse the appearance of results. It therefore becomes important to develop good analysis methods that can extract relevant information.

Particularly useful analysis techniques to use at this stage of tuning are:

Wheels speeds – Use a plot of all four wheel speeds overlaid on each other. This alone will give an indication of brake and transmission performance. Calculating the difference between from and rear wheel speeds can be used to determine the slip ratio during acceleration.

Steering trace - As with the skidpan, steering traces of fast and slow laps can be used to indicate the balance of the car.

Suspension Position – Suspension position can be used extensively during testing on the sprint track. Damper speed time histograms and roll timing analysis, see sections 23.4 and 23.5, can be used to assess damping coefficients and the ratio of bump to droop damping.

28 Thesis Conclusions

The first of the two main objectives of this thesis was to present the background knowledge required in order to define terms included on the SAE's essential terminology list. In this section basic principles behind tyre force, grip and tyre deformation were explained. Terms covering vehicle and suspension geometry have been explained and related to vehicle performance. The necessity for suspension damping was also explored and the performance characteristics and design of suspension dampers explained.

The general aim for the second section was to apply the principles explained in Part 1 to the practical design problem of a formula student car. A general process map for the design of any racing vehicle was created to allow consideration of all important factors and performance characteristics.

Data from the acquisition system described was used to generate a mathematical model of the formula student track and quantify the performance capabilities of the car. Peak values for lateral acceleration were shown to be in the order of 1.3 g. Whilst, braking and traction forces could achieve accelerations of 1 g and 0.7 g respectively. Values that are significantly less than the proposed maximums. Further analysis showed that 30% of a lap time was spent at greater than 80% of maximum lateral and longitudinal accelerations. This 30% was bias 2:1 to lateral acceleration, implying that lateral acceleration capabilities are more critical for the formula student discipline.

Analysis of data from suspension position transducers was used to calculate damper speeds. Graphically displayed as a speed-time histogram this information was compared to a normal distribution curve. It was seen that both front dampers spent too long in low speed rebound compared to low speed bump, at 47 % to 38 %. The same trend was also seen for high speed motion with a distribution of 10 % and 5 % rebound and bump respectively. At the rear high speed damping was more equally balanced at 14 % and 13 % rebound and bump. For low speed damping the rear showed similar characteristics to the front with more time being spent in rebound than bump. Inspection of roll timing in the following section showed that front roll lagged rear roll and both ends lagging the apex of the corner. Thus, both front and rear roll needed promoting but with the emphasis on front end roll. In conclusion, the level of high-speed bump damping provided by the Risse Racing dampers was too great for the front unsuspended mass. Whilst suitable adjustments should be made to the low speed damping to reduce the amount of low speed rebound damping on both the front and rear dampers.

After considering all of the data collected and how different performance gains affect the overall lap time using the mathematical lap model the following hierarchy for design focus was drawn up.

1) Brake development/tuning
   a) Refinement of braking bias
   b) Improvement of pedal assembly and adjustability
2) Drivetrain development/tuning
   a) CVT tuning
   b) Gearing tuning
   c) Transient engine response
   d) Engine power output
   e) Differential development
3) Weight reduction/distribution
   a) Overall design for loading
   b) Weight distribution to reduce inertia
4) Damper development/tuning
   a) Damper specification
   b) Damper testing
5) Suspension
   a) Kinematics refinement
   b) Spring/roll rates

Analysis of tyre data supplied by Avon Tyres showed that the current selection of tyres, although possible not the best available, are capable of producing performance far in excess of the levels currently being achieved and winning dynamic events at the formula student competition.

Parametric analysis of competitor performance and vehicle footprint showed that previously chosen dimensions might not be ideal. This theory was further reinforced by weight transfer calculations at the accelerations that the tyres are capable of. Consequently it was recommended that the track be increased from 1.05 m to 1.2 m, reducing lateral weight transfer by approximately 8% and improving stability at high lateral acceleration. It was also recommended that the wheelbase be extended by 100 mm to 1.65 m in order to improve stability on entry and exit of corners.

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Appendix A – SAE Terminology List

Carroll Smith Consulting Inc (2002) Available from: http://www.sae.org/students/fsaeterms.doc
Accesses: 04/02/07

Formula SAE

Vehicle Dynamics Terminology List

The design judges in the USA, the UK and Australia have noted that many of the students are not familiar with standard vehicle dynamics terminology. This has caused confusion and wasted time during design judging. The following is a list of common terms with which we feel that the presenters during the design judging phase should be familiar. Definitions can be found in Milliken, Smith, Staniforth and Wright.

BALANCE

- Oversteer
- Understeer
- Understeer gradient in deg/g.

CHASSIS

- Ultimate tensile strength of materials used in chassis and suspension construction
- Modulus of elasticity for the same materials
- The difference between strength and stiffness
- Torsional rigidity by FEA and by physical measurement
- Torsional rigidity profile or slope
- Installed stiffness
- Bump, droop, roll, heave, warp

SUSPENSION RATES

- Ride rate, wheel rate, spring rate, motion ratio (installed ratio)

Body un-damped natural frequency

Suspension un-damped natural frequency (hop and tramp

The effect of "stiction" or friction in suspension pivots

Damping ratio

Single tube and twin tube shock absorbers (dampers)

Shock absorber bump and droop

Shock absorber piston speed

Shock absorber nose pressure

Shock absorber hysteresis

Critical damping

Roll stiffness

Percentage of roll stiffness from springs/anti roll bars

Roll stiffness per transverse g., front and rear

Static load distribution

Lateral load transfer per transverse g.

Longitudinal load transfer per g.

Load transfer due to steering geometry

Load transfer due to steering geometry

Anti squat and anti dive coefficients and in percentages

Advantages and disadvantages of anti squat and anti dive geometry

SUSPENSION GEOMETRY

Roll center (definition)

Roll center height and lateral location

Roll center migration (vertical and lateral

Roll axis

Camber

Camber change in ride (ride camber coefficient in degrees/inch)

Camber change in roll (roll camber coefficient in degrees/degree)

Caster

Camber change with steer angle due to caster

King pin inclination (steering axis)

Camber change with steer angle due to king pin inclination

Scrub radius (steering offset)

Mechanical trail (caster trail)

Spindle offset (wheel center to steering axis at spindle height)

Toe in and toe out

Ackerman, modified Ackerman and anti-Ackerman steering geometry

Bump Steer

- Ride steer
- Roll steer

MOMENTS

- Polar moment of inertia
- Moments of rotational inertia
- Yaw Moment
- Roll moment
- Pitch moment

BRAKE SYSTEM

- Pedal mechanical ratio
- Hydraulic ratio, front and rear
- Clamping load front and rear per 100 psi line pressure
- Coefficient of friction between disc and pad friction material at operating temperature
- Hysteresis

DRIVE LINE

- Differential bias ratio
- Angular capacities of joints used in drive shafts
- Drive line angles
- Open differential operation
- Salisbury or "plate" differential operation
- Cam and pawl differential operation
- Zexel/Gleason differential operation
- Spool operation

TIRES

- Coefficient of friction
- Slip angle
- Percent slip (slip ratio)

Cornering stiffness

Camber stiffness

Self aligning torque

Normal load sensitivity

Load transfer sensibility

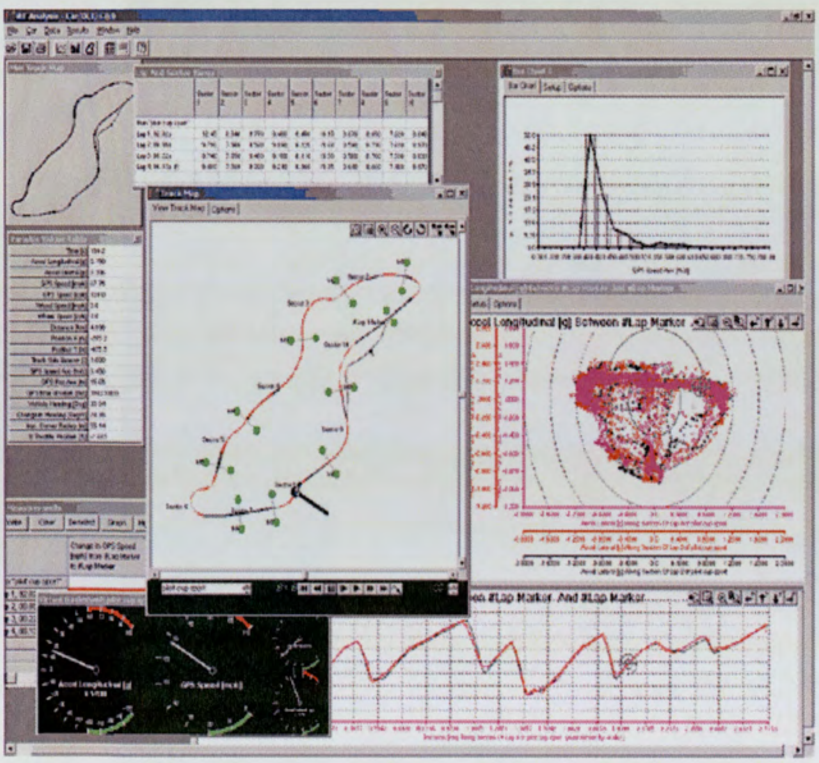
Pneumatic trail

Appendix B – Race Technology Software

"Race Technology is a Nottingham (UK) based company, developing high technology automotive electronic solutions. In particular, we are leaders in the field of GPS data loggers, CAN communications, instrumentation, and data acquisition for auto sport and industrial applications. We supply systems throughout the world for applications as diverse as accident reconstruction to drag racing, F1 boats to delivery vans."

"As well as having our own range of highly successful products, we also do custom work for many clients and a wide range of applications - If you have an automotive electronics project, then we can help."

<u>Race Technology Software</u>



The powerful and feature-packed Data Analysis software provided with every Race Technology data logger and performance meter continues to lead the industry when it comes to data processing. Alongside this we also offer easy to use configuration and on-line monitor software for each product, as appropriate.

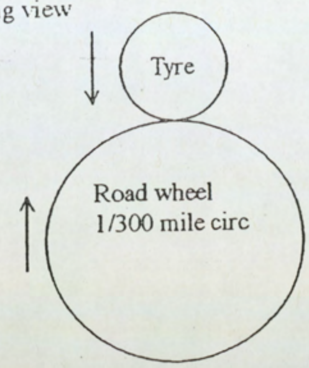
The above information has been taken from Race Technology's Website (26/04/07) accessible at http://www.race-technology.com/.

Appendix C – Tyre Data

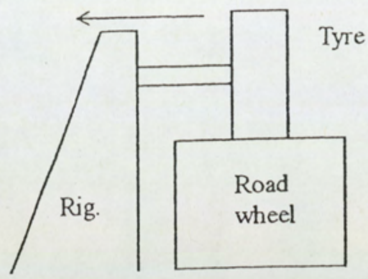
TYRE INFORMATION - INTRODUCTION.

Schematic diagram of the Avon Stability rig used to generate the cornering force / slip angle, and self aligning torque / slip angle graphs.

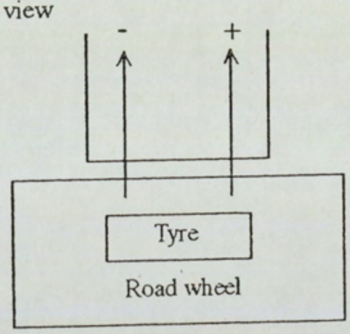
Rotation
Facing view



Camber.
Side view



Slip angle
Plan view



FROM : M.J Lynch

TO : M.Lynch

CC : F.P.Coates

24 August 1998 *PROJECT RC213 STB.*

This project is to generate stab and spring rate data on FSAE tyres.

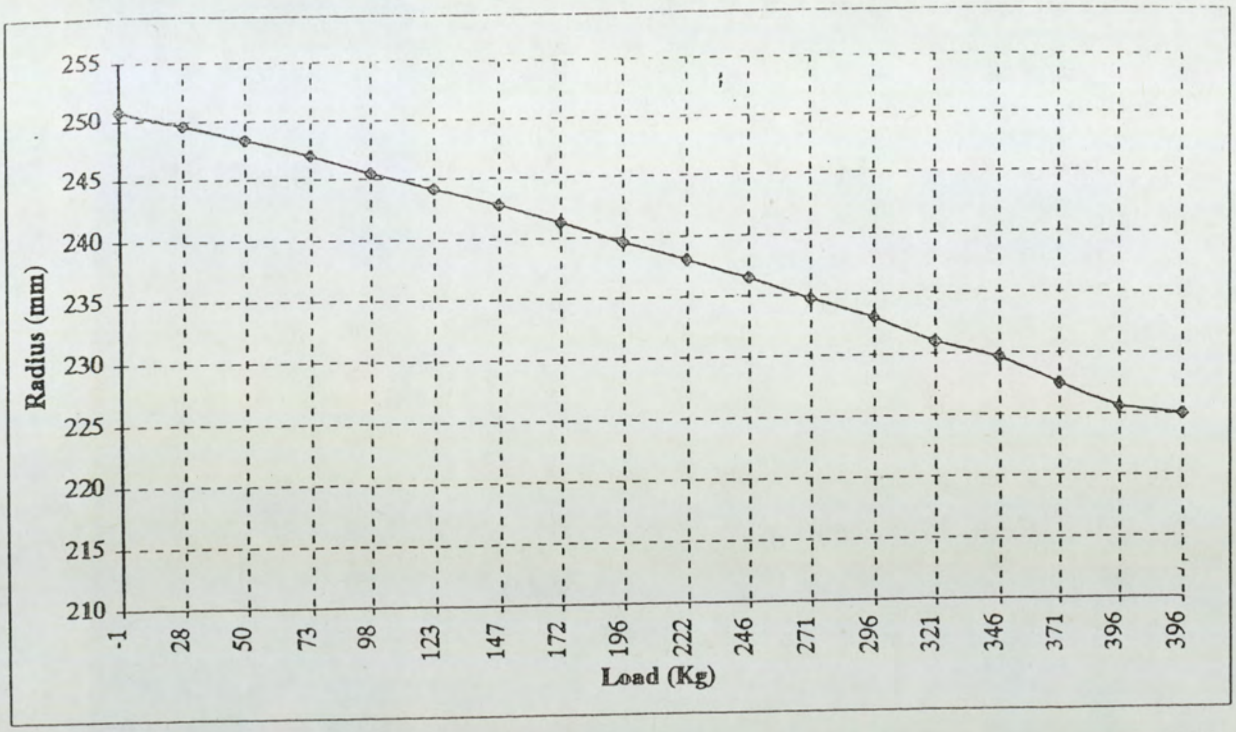
Tyres :	**A**	**6.2/20.0-13 A41 7610**
	B	**8.2/20.0-13 A41 7611**
Rims :	**A:**	**6J x 13**
	B:	**8J x 13**
Cambers:	***1,2***	
Loads :	**60, 90,120Kg**	
Slip Angle:	**-7° to +7°**	
Pressures:	**14, 16 psi**	

Measure pressures and temperatures after each run. Running pressures should be adjusted before each run.

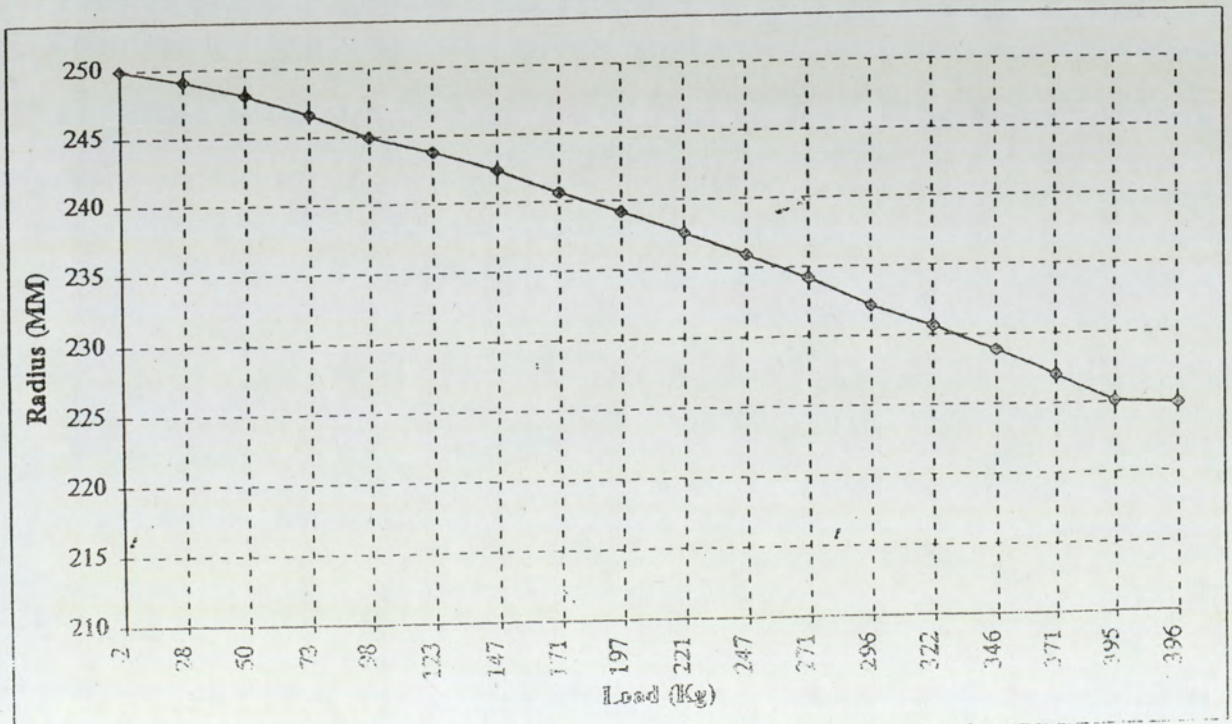
Please supply graphs and raw data.

Many thanks,

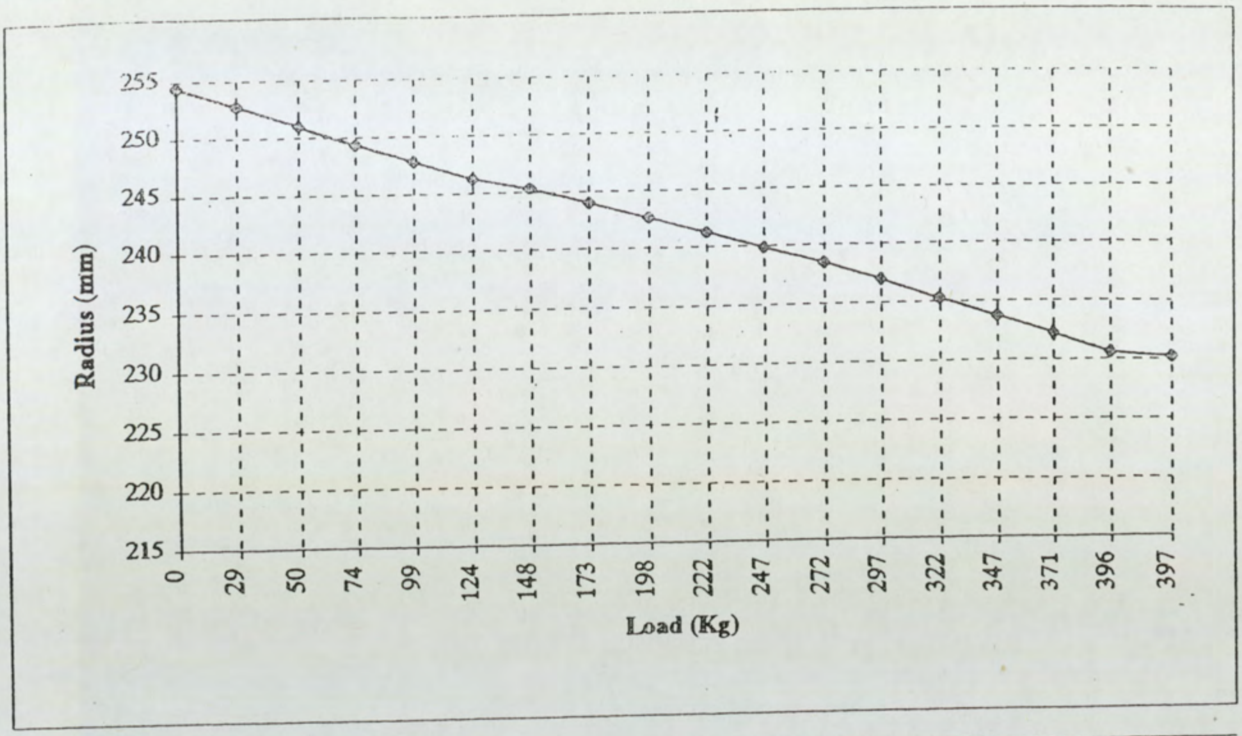
Project:	**RC213STB**	**Size:**	**6.2/20.0-13**	**Camber:**	**0°**
Part:		**Tyre:**	**7610**	**Pressure:**	**14psi**
Tested:	**#REF!**	**Rim:**	**6 x 13**	**Speed:**	**0**



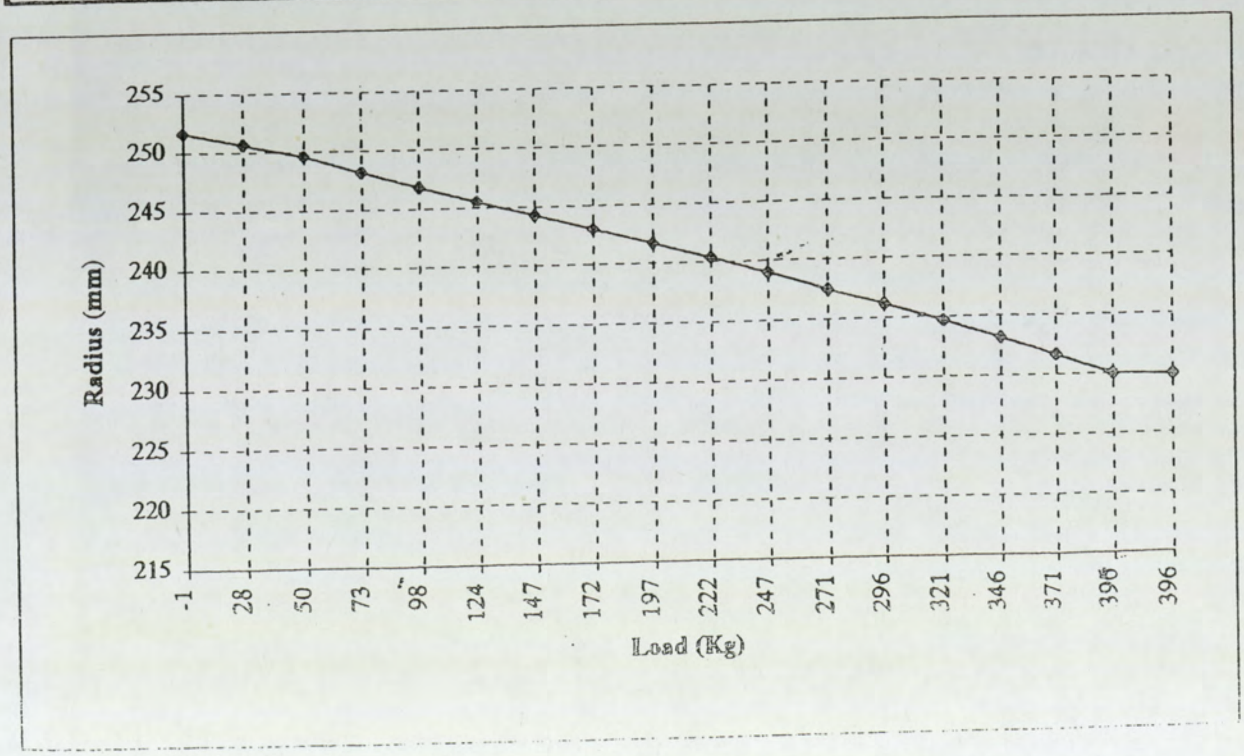
Project:	**RC213STB**	**Size:**	**6.2/20.0-13**	**Camber:**	**1°**
Part:		**Tyre:**	**7610**	**Pressure:**	**14psi**
Tested:	**#REF!**	**Rim:**	**6 x 13**	**Speed:**	**0**



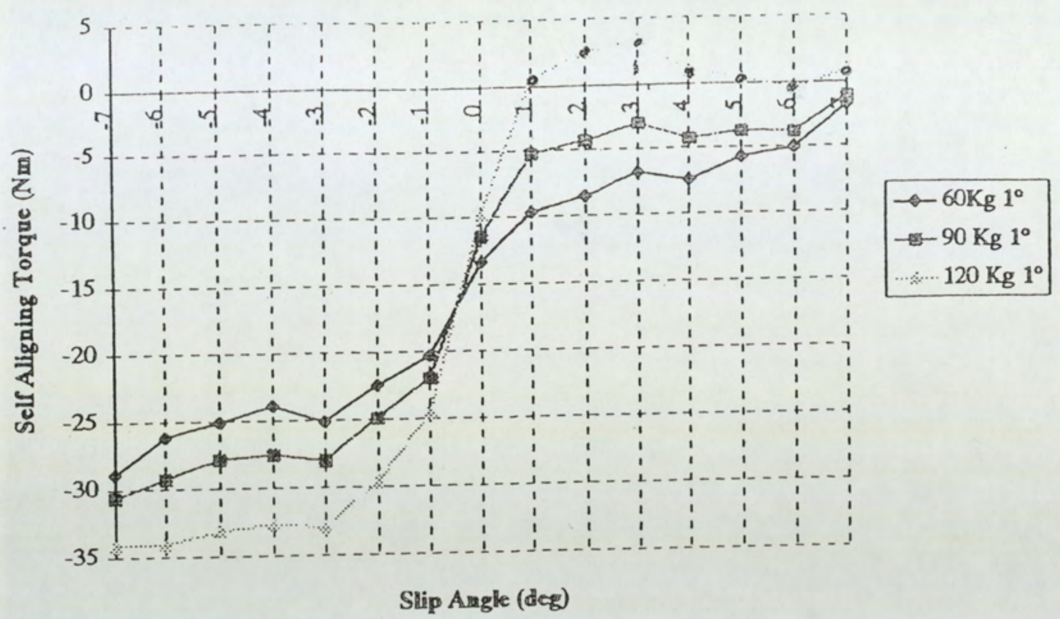
Project:	RC213STB	Size:	6.2/20.0-13	Camber:	0°
Part:		Tyre:	7610	Pressure:	18psi
Tested:	#REF!	Rim:	6 x 13	Speed:	0

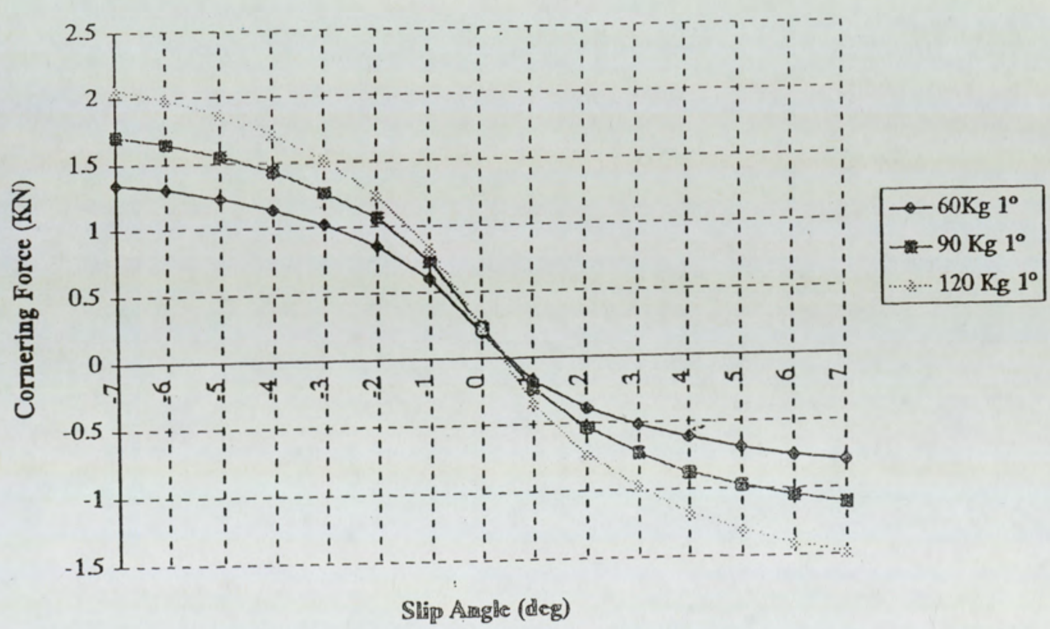


Project:	RC213STB	Size:	6.2/20.0-13	Camber:	1°
Part:		Tyre:	7610	Pressure:	18psi
Tested:	#REF!	Rim:	6 x 13	Speed:	0

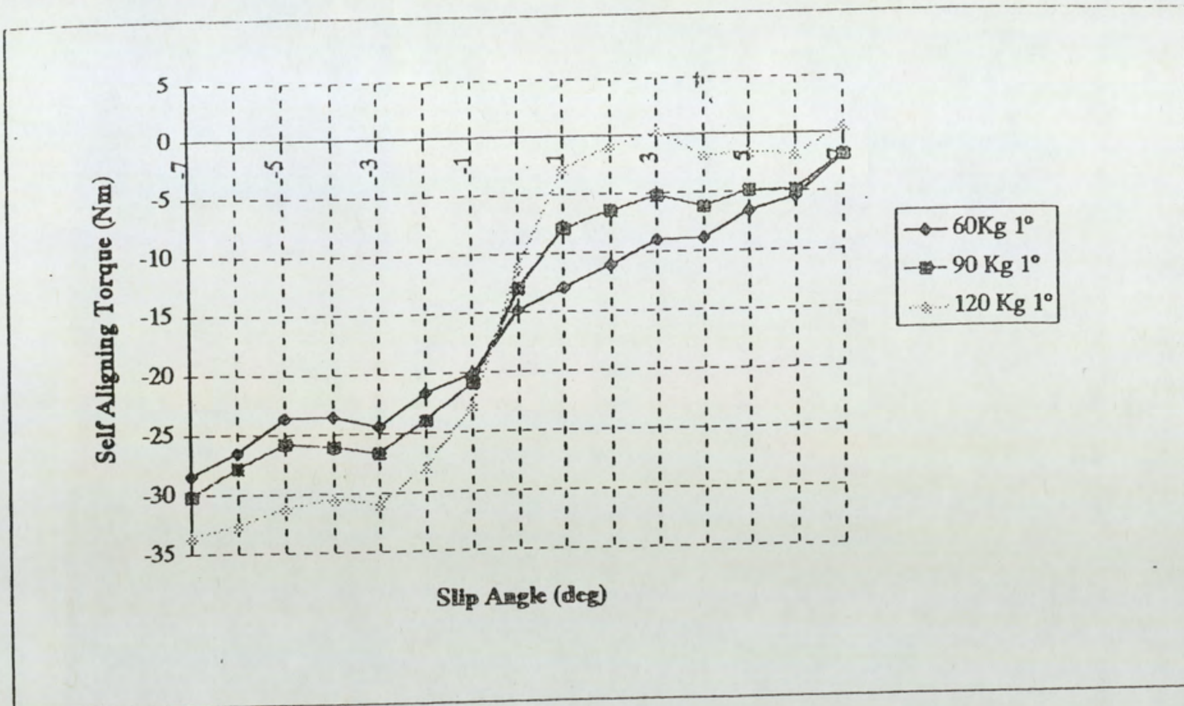


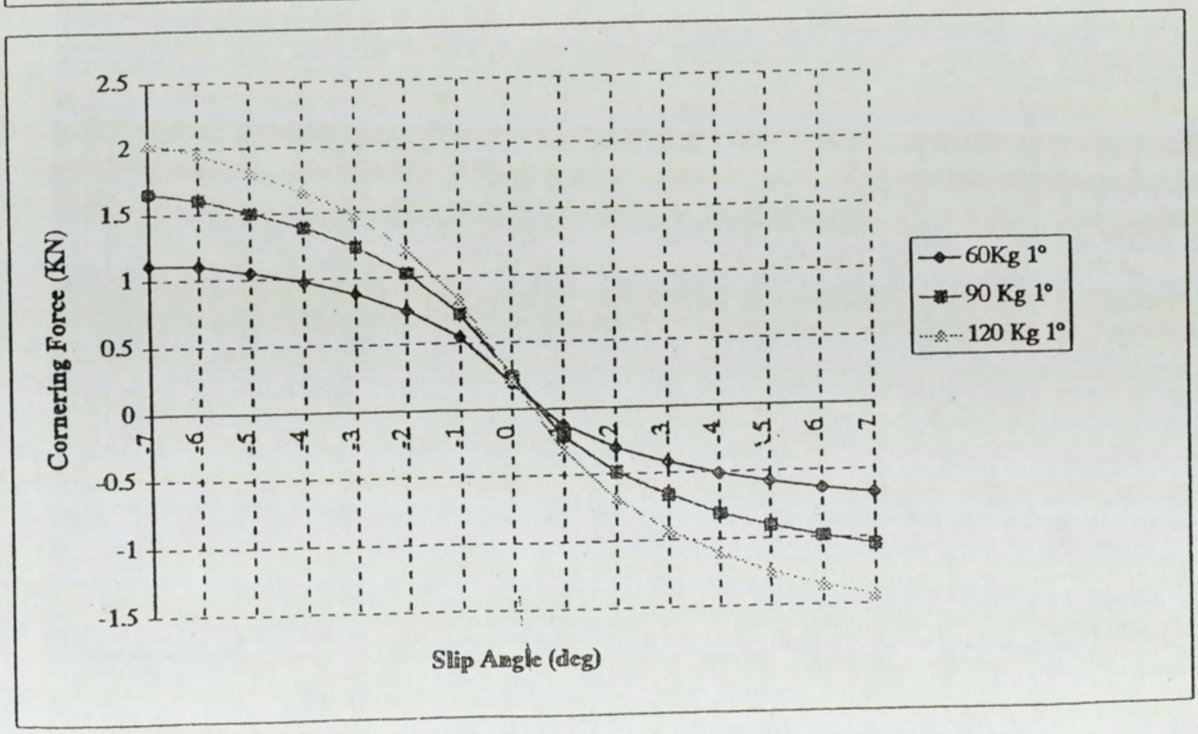
Project:	RC213STB	Size:	6.2/20.0-13	Camber:	1°
Part:		Tyre:	Slick	Pressure:	14psi
Tested:	#REF!	Rim:	6 x 13	Speed:	20Kph



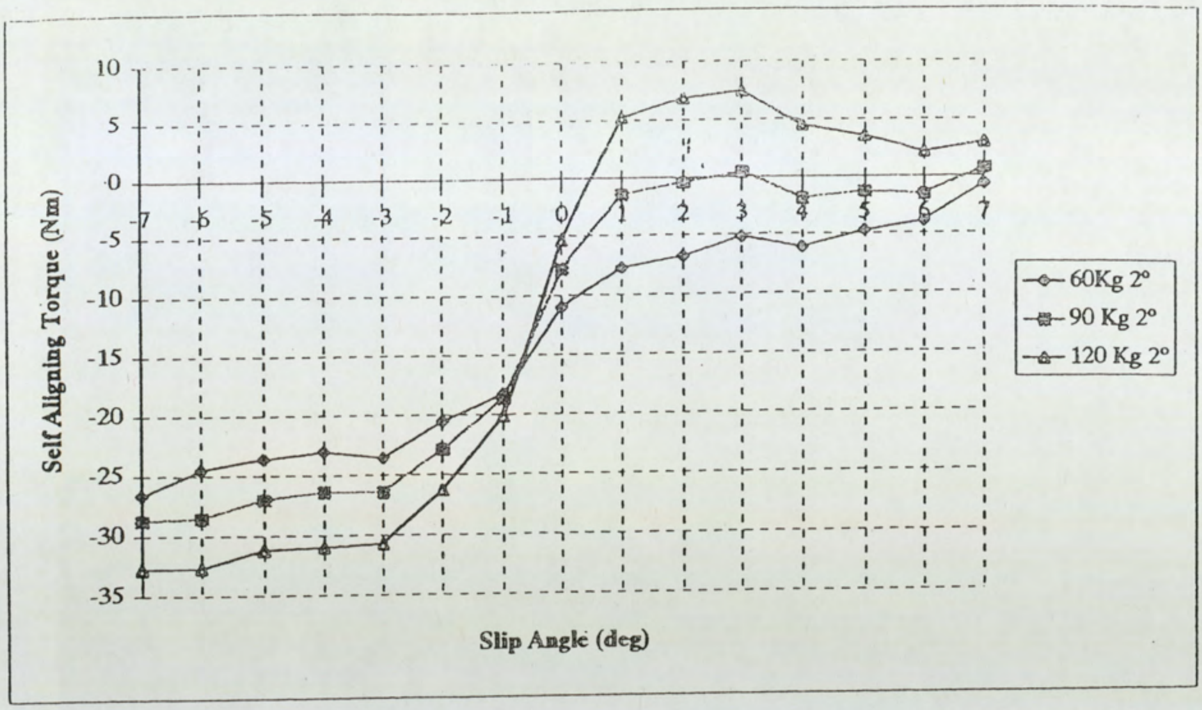


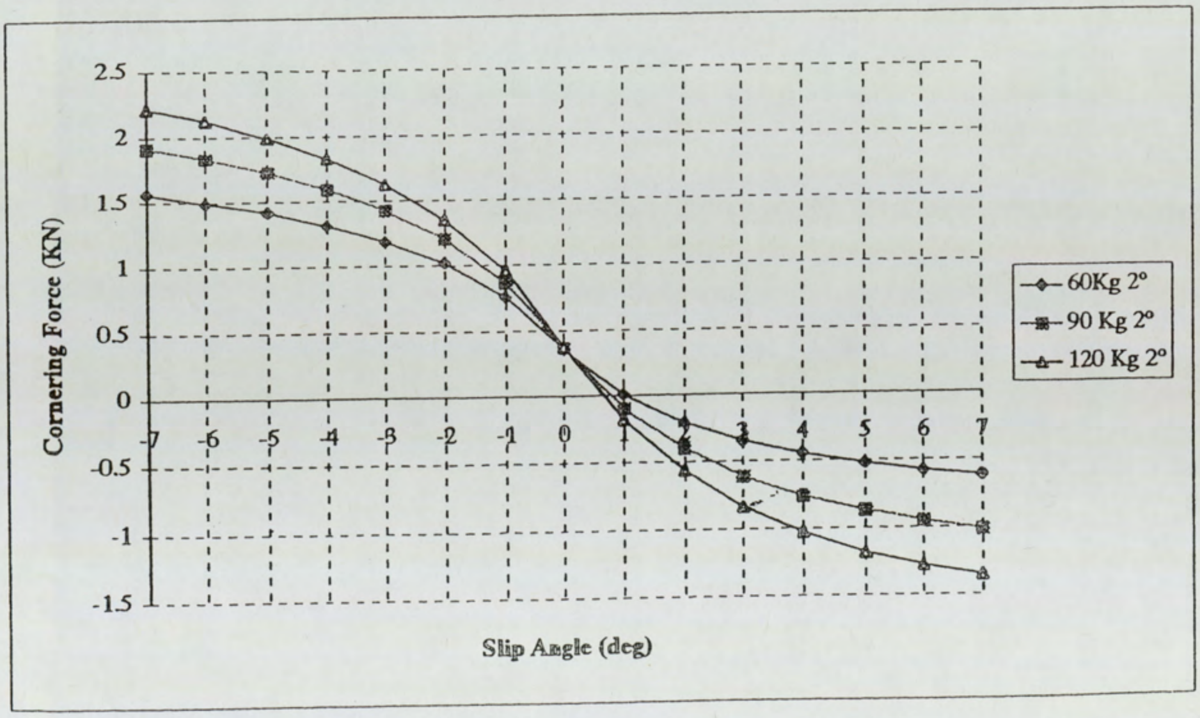
Project:	RC213STB	Size:	6.2/20.0-13	Camber:	1°
Part:		Tyre:	Slick	Pressure:	18psi
Tested:	#REF!	Rim:	6 x 13	Speed:	20Kph



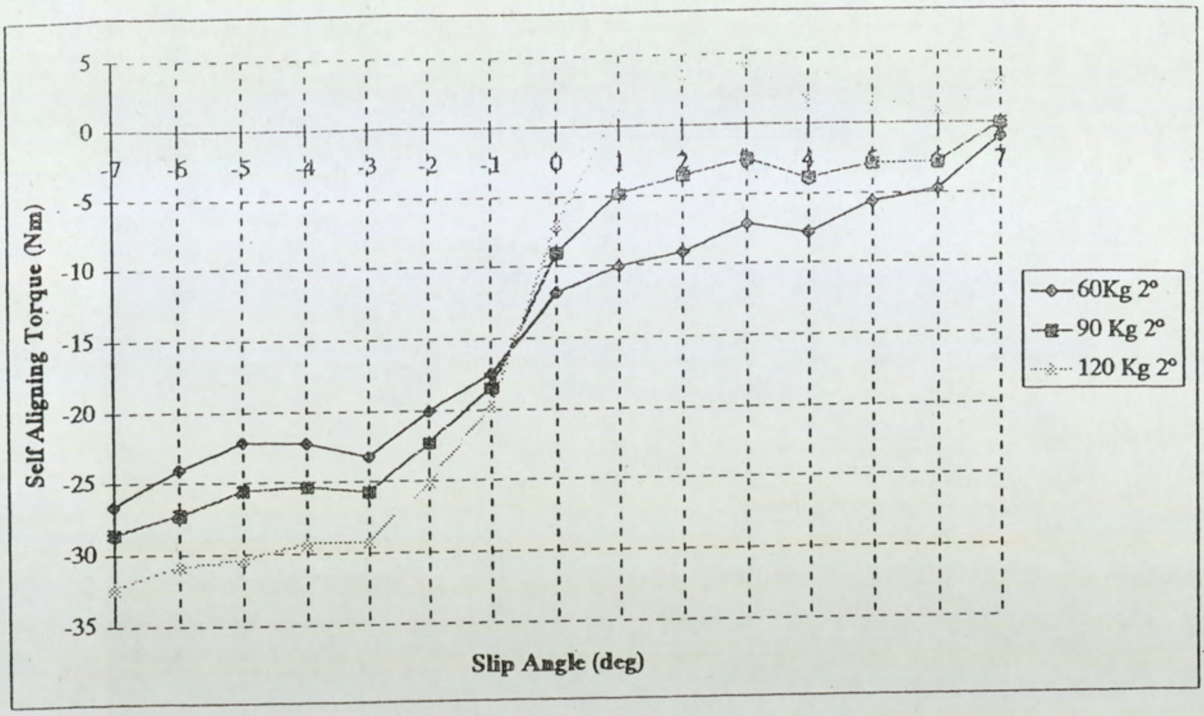


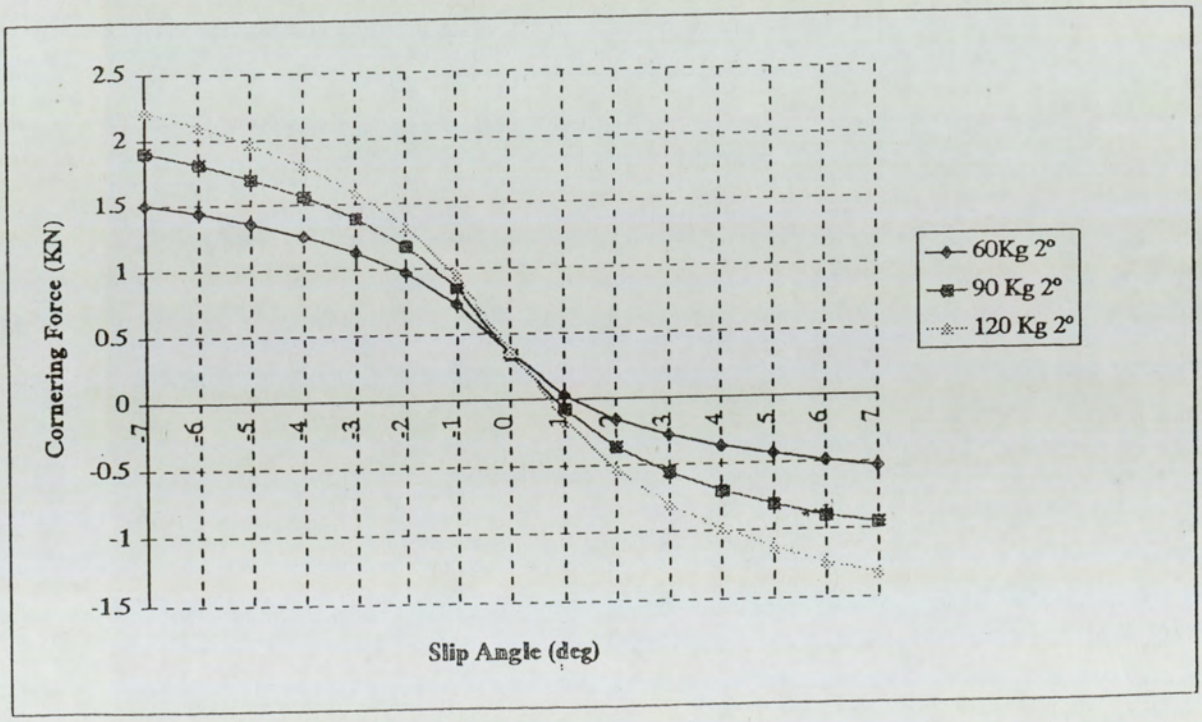
Project:	RC213STB	**Size:**	6.2/20.0-13	**Camber:**	2°
Part:		**Tyre:**	Slick	**Pressure:**	14psi
Tested:	#REF!	**Rim:**	6 x 13	**Speed:**	20



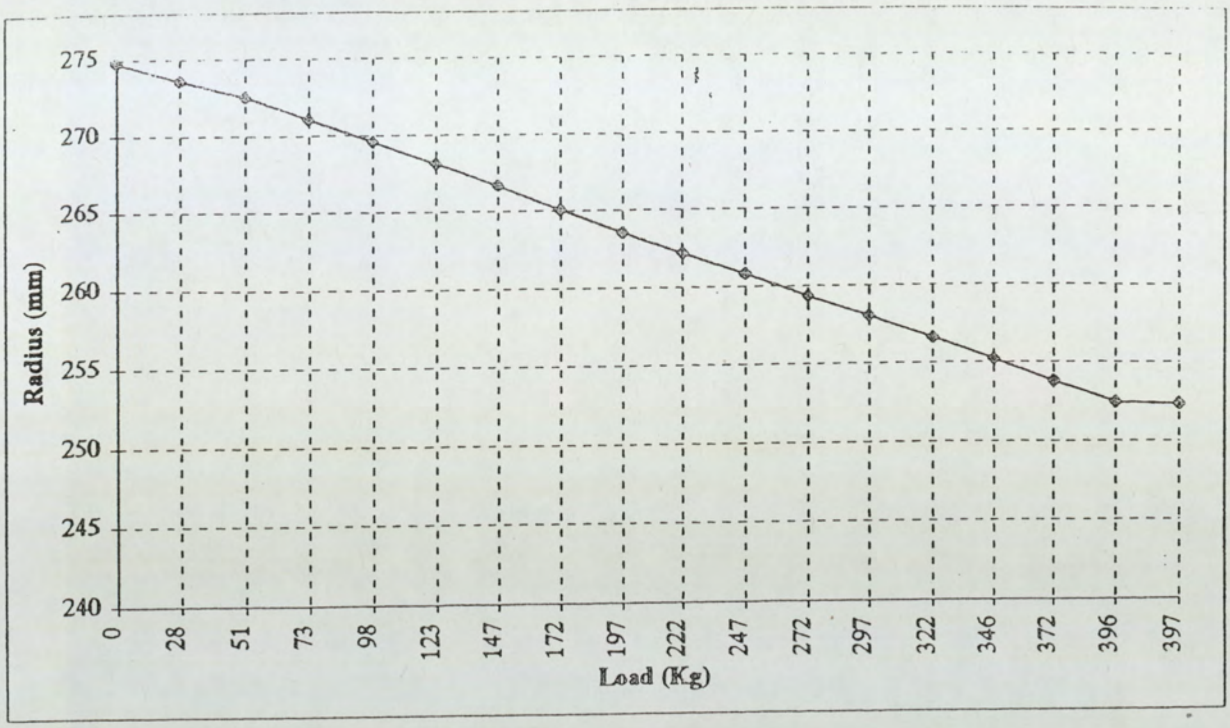


Project:	RC2133TB	Size:	6.2/20.0-13	Camber:	2°
Part:		Tyre:	Slick	Pressure:	18psi
Tested:	#REF!	Rim:	6 x 13	Speed:	20Kph

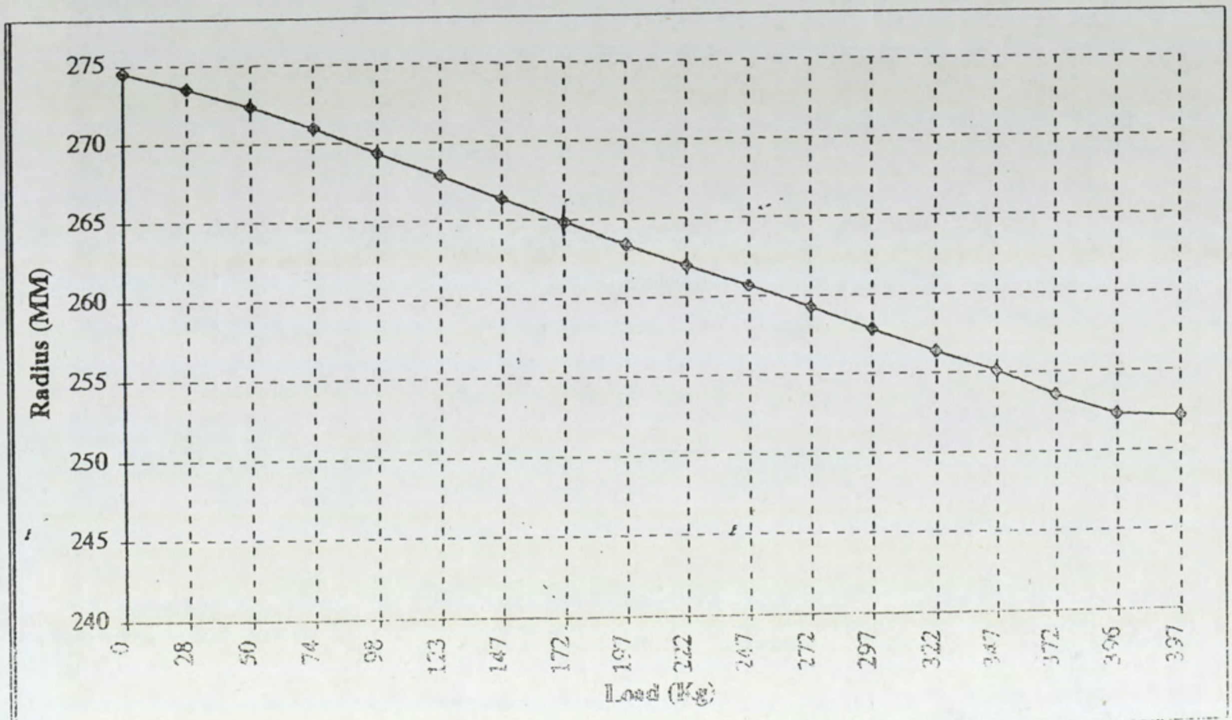




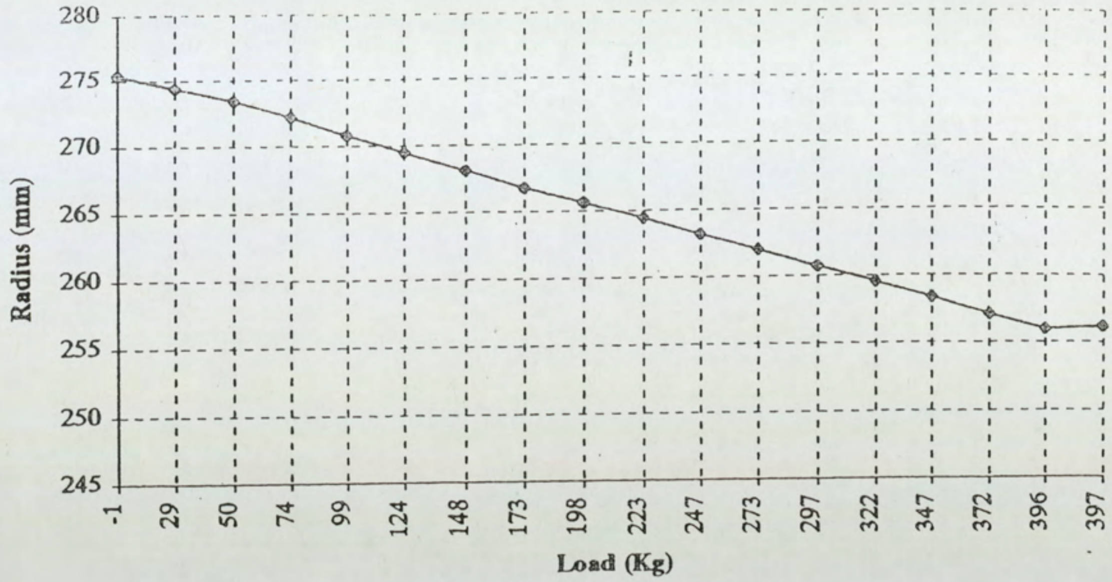
Project:	RC213STB	Size:	8.2/20.0-13	Camber:	0°
Part:		Tyre:	7611	Pressure:	14psi
Tested:	#REF!	Rim:	8 x 13	Speed:	0



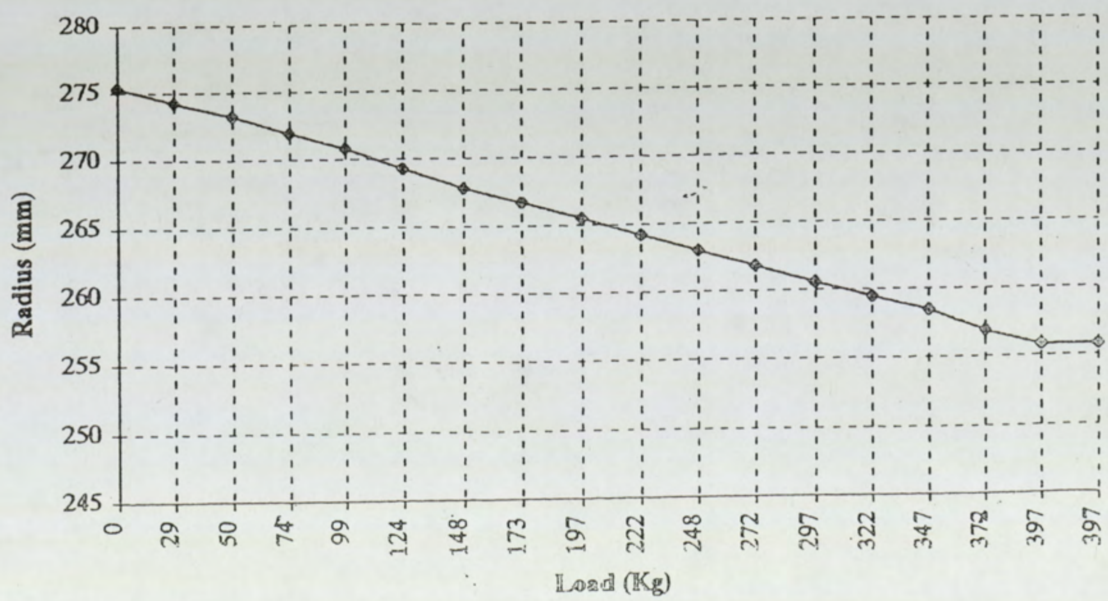
Project:	RC213STB	Size:	8.2/20.0-13	Camber:	1°
Part:		Tyre:	7611	Pressure:	14psi
Tested:	#REF!	Rim:	8 x 13	Speed:	0



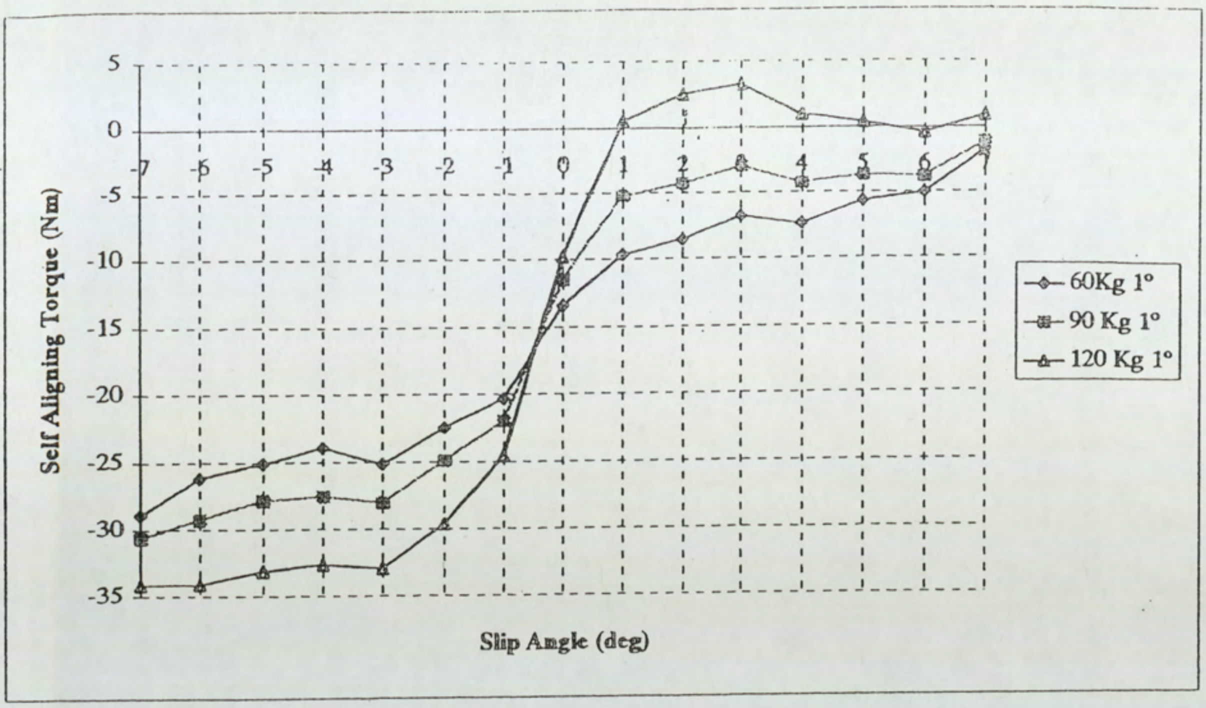
Project:	RC213STB	Size:	8.2/20.0-13	Camber:	0°
Part:		Tyre:	7611	Pressure:	18psi
Tested:	#REF!	Rim:	8 x 13	Speed:	0

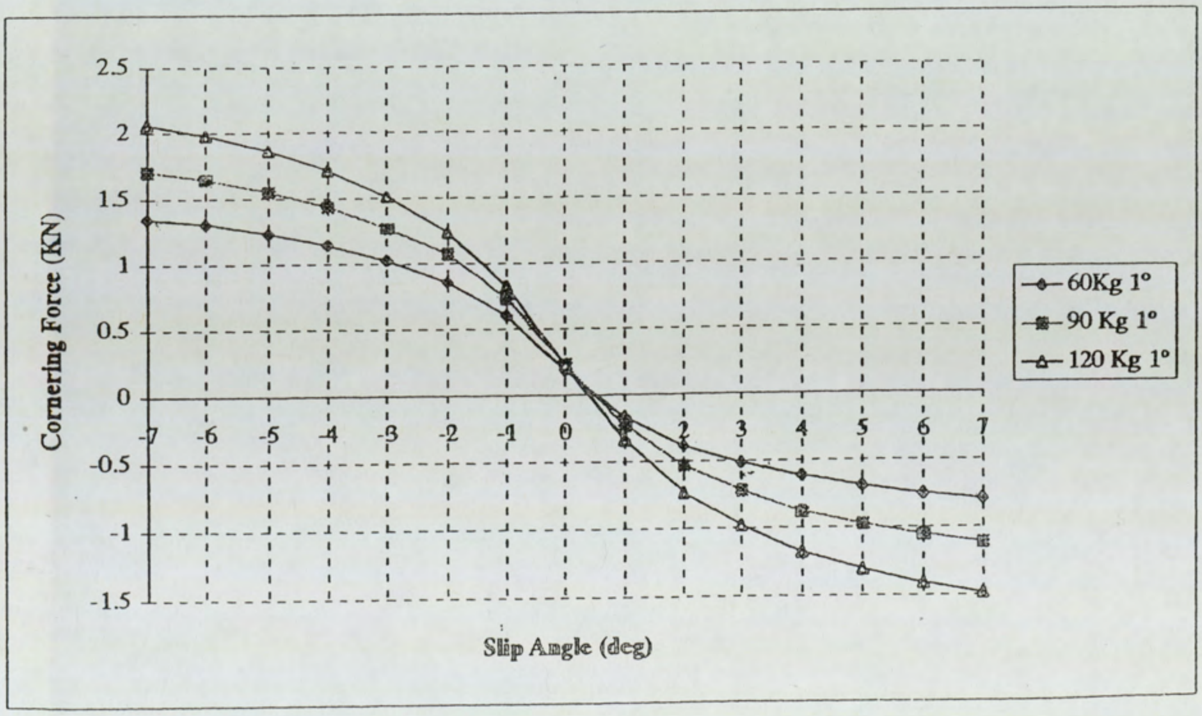


Project:	RC213STB	Size:	8.2/20.0-13	Camber:	1°
Part:		Tyre:	7611	Pressure:	18psi
Tested:	#REF!	Rim:	8 x 13	Speed:	0

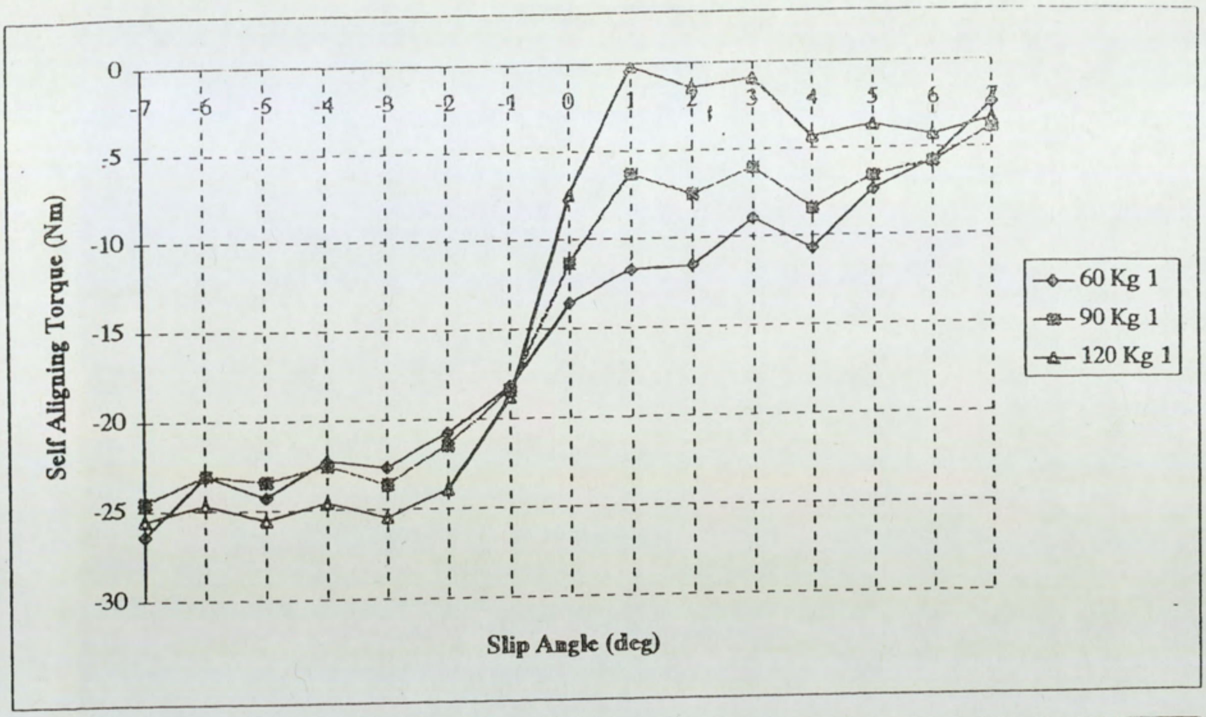


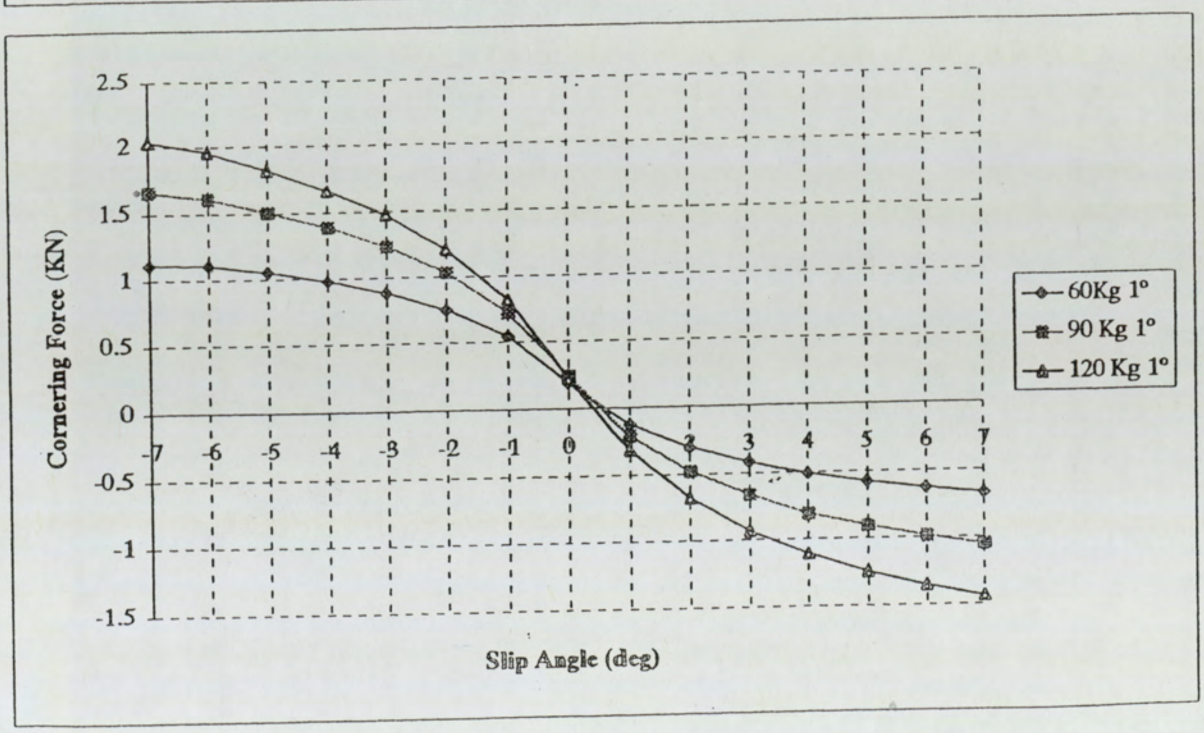
Project:	RC213STB	**Size:**	8.2/20.0-13	**Camber:**	1
Part:		**Tyre:**	Slick	**Pressure:**	14psi
Tested:	#REF!	**Rim:**	8 x 13	**Speed:**	20



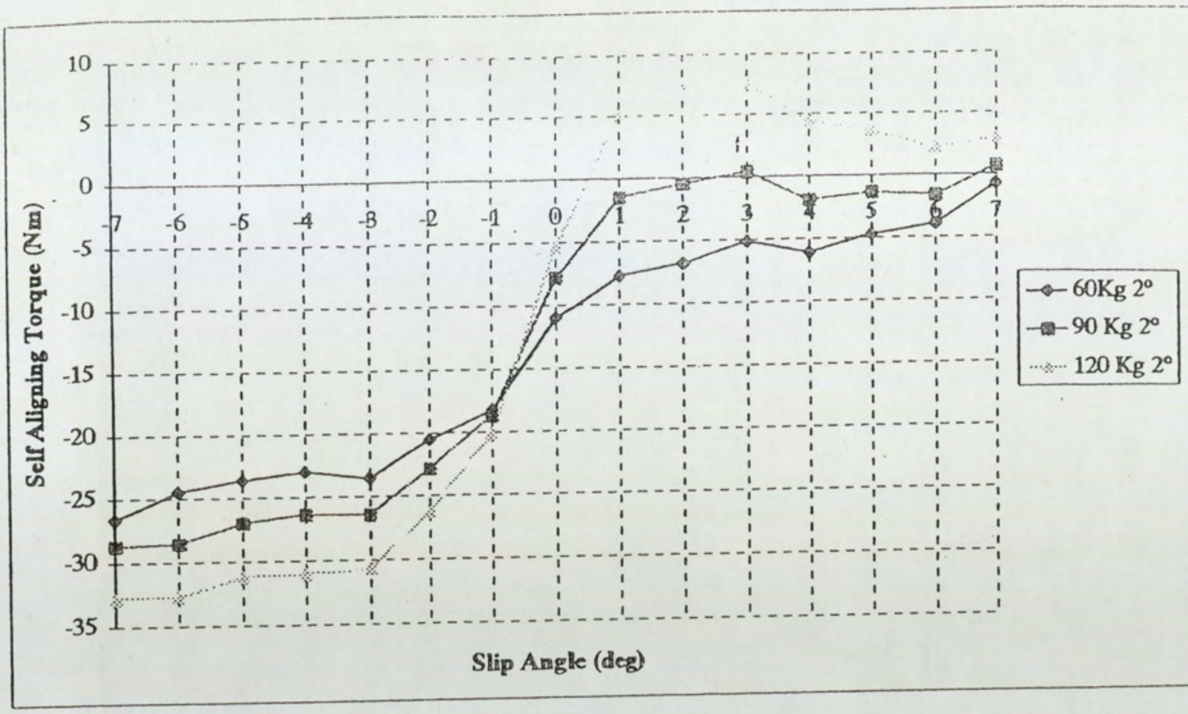


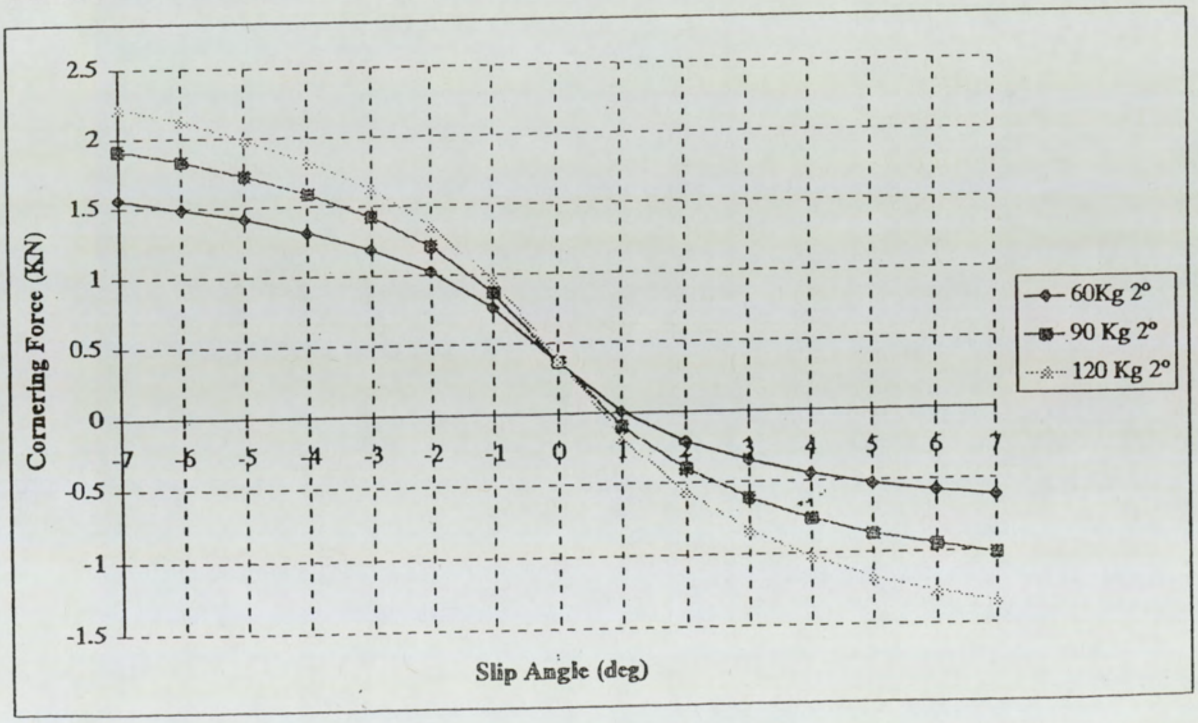
Project:	RC213STB	Size:	8.2/20.0-13	Camber:	1
Part:		Tyre:	Slick	Pressure:	18psi
Tested:	#REF!	Rim:	8 x 13	Speed:	20



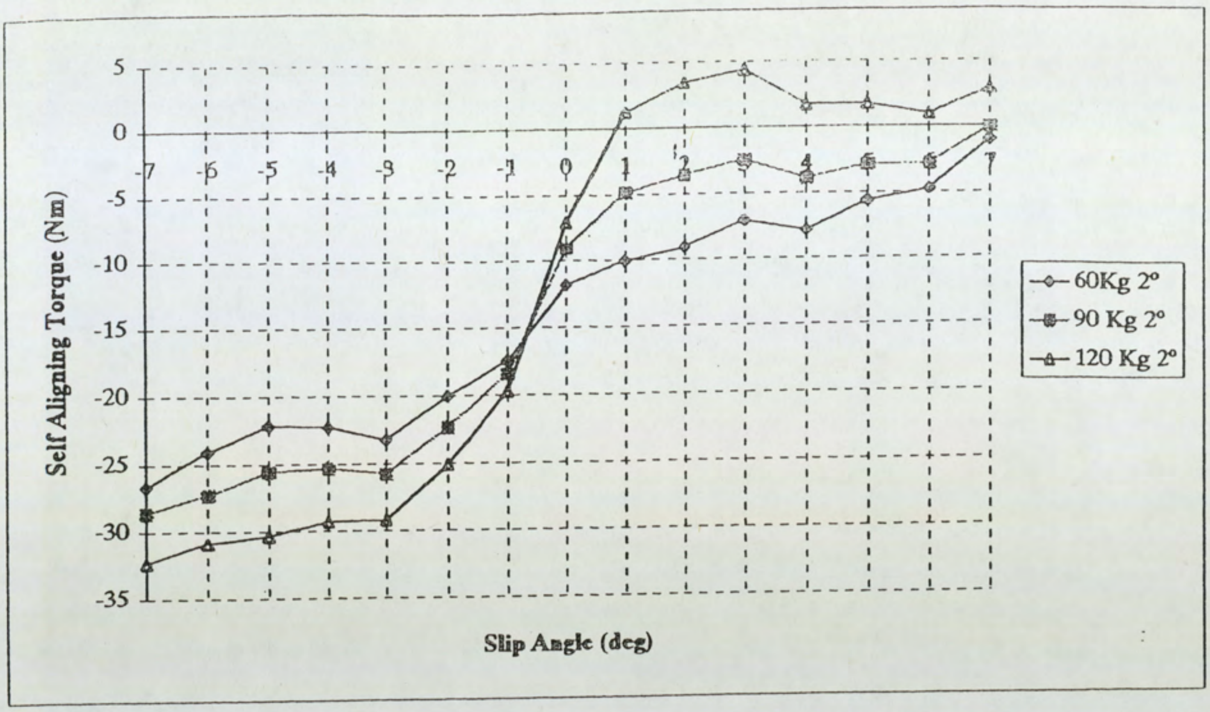


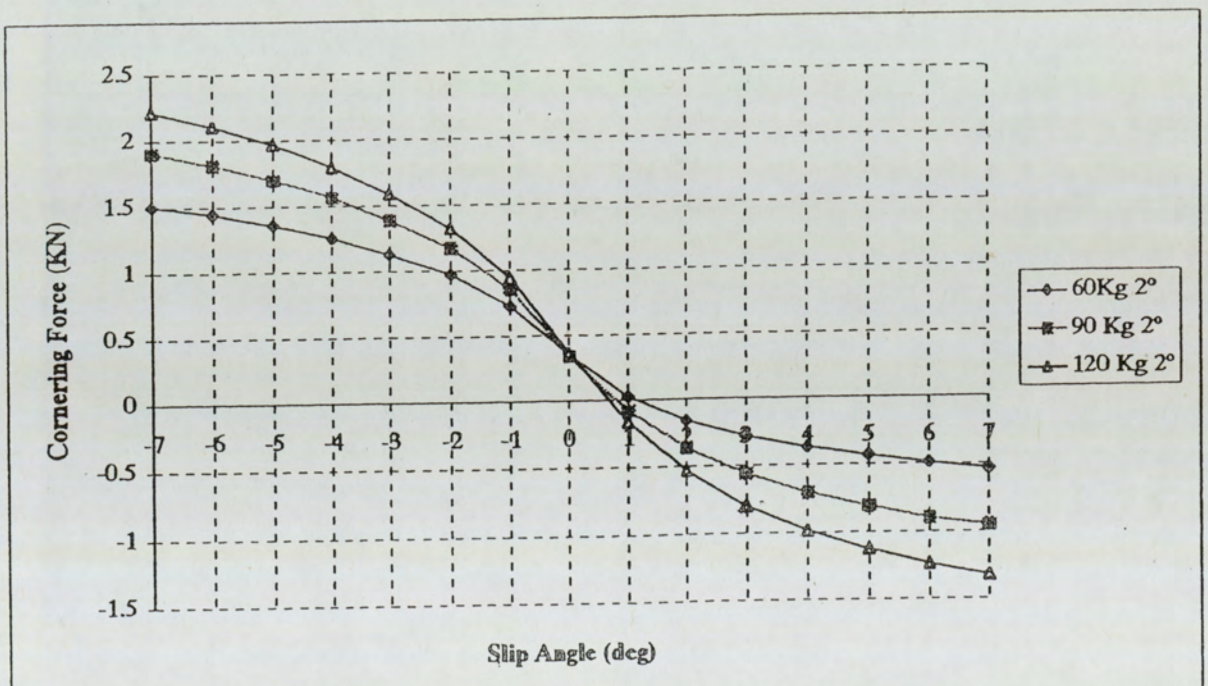
Project: RC213STB		Size:	6.2/20.0-13	Camber:	2°
Part:		Tyre:	Slick	Pressure:	14psi
Tested:	#REF!	Rim:	6 x 13	Speed:	20





Project:	RC213STB	Size:	8.2/20.0-13	Camber:	2°
Part:		Tyre:	Slick	Pressure:	18psi
Tested:	#REF!	Rim:	8 x 13	Speed:	20Kph





FROM : G.HEEKS

TO : N.CROOKES

CC : F.P.COATES

June 26th, 1995

PROJECT RC062 STB.

This project is for Stability Rig testing of FFord Zetec Tyres.

Tyres :	**A**	6.0/21.0-13	spec 8806
	B	7.0/22.0-13	spec 8807

Rims : **6.0"** for tyres **A** Set with **0, 0.5, 1, 1.5, and 2** degrees of camber
7.0" for tyres **B** Set with **0, 0.5, 1, 1.5, and 2** degrees of camber

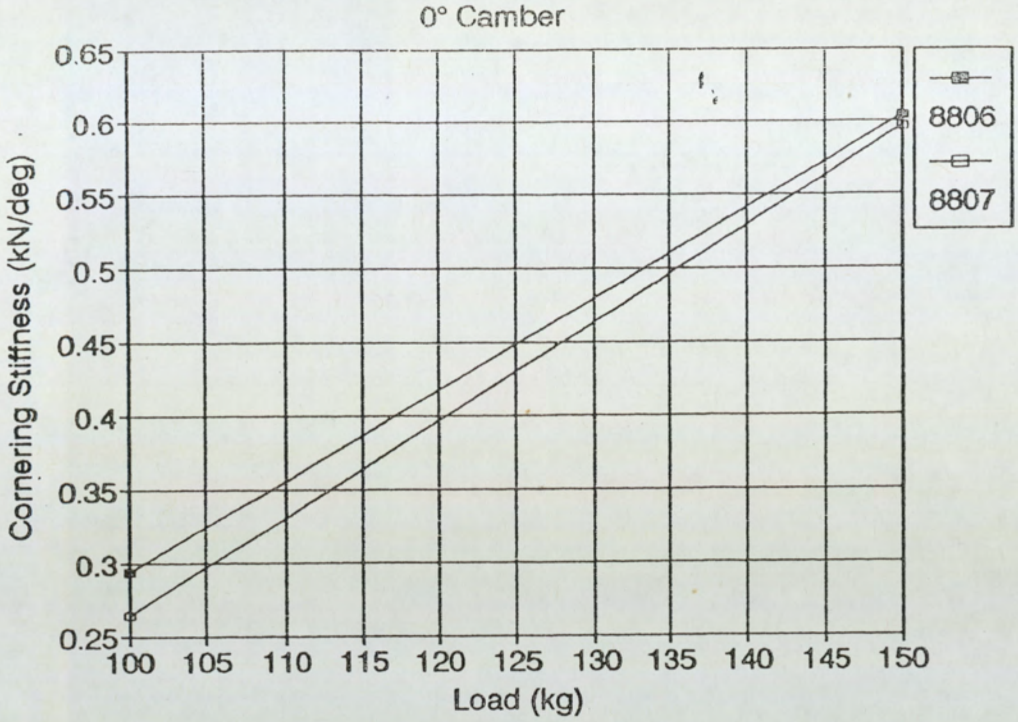
Loads : **100 & 150kg vertical** for tyres **A**
100 & 150kg vertical for tyres **B**

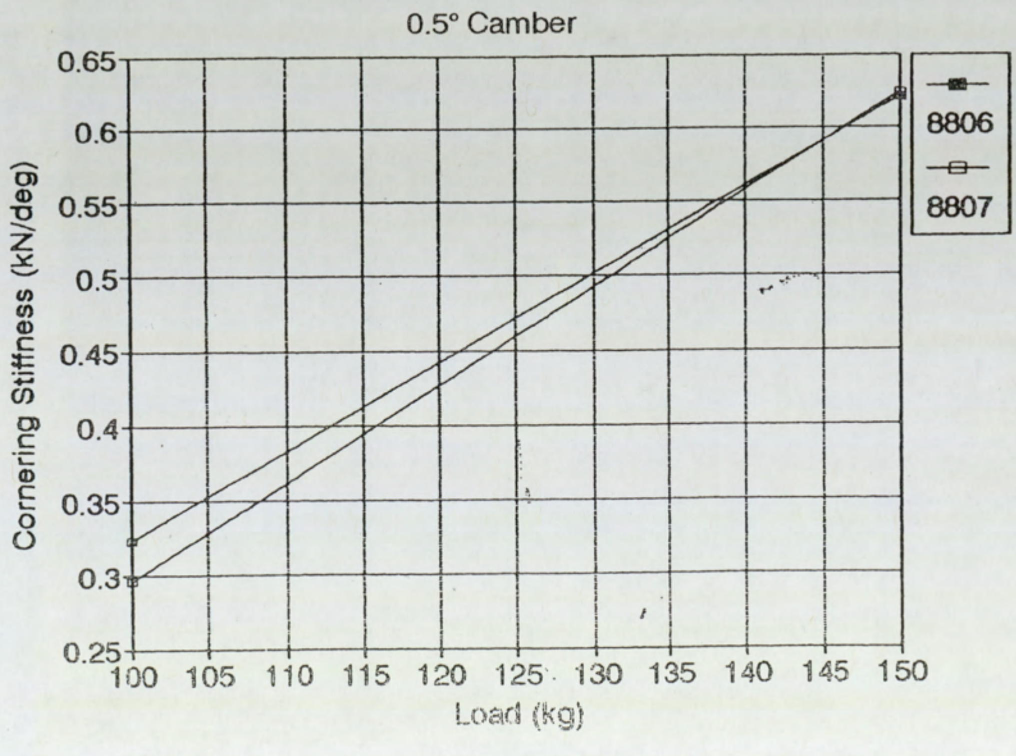
Measure pressures and temperatures after each run. Running pressures should be 20psi ± 1 psi for all tyres. Adjust before each run.

Please supply graphs and raw data for report to be written by G.Heeks.

Thanks.

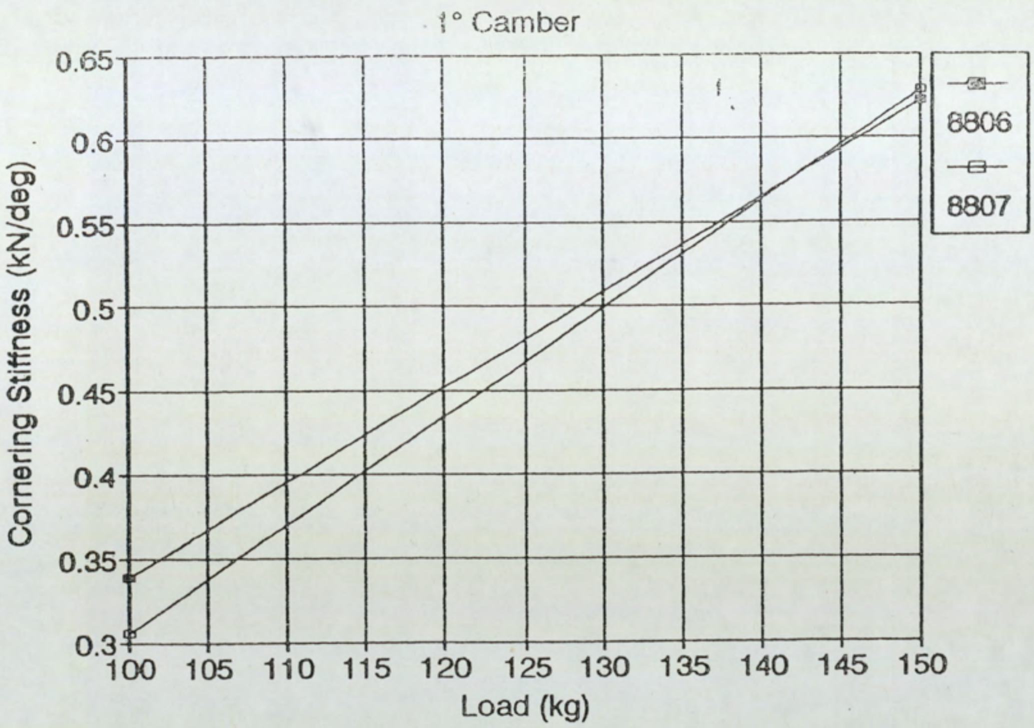
Project RC060STB Tested 3/7/95

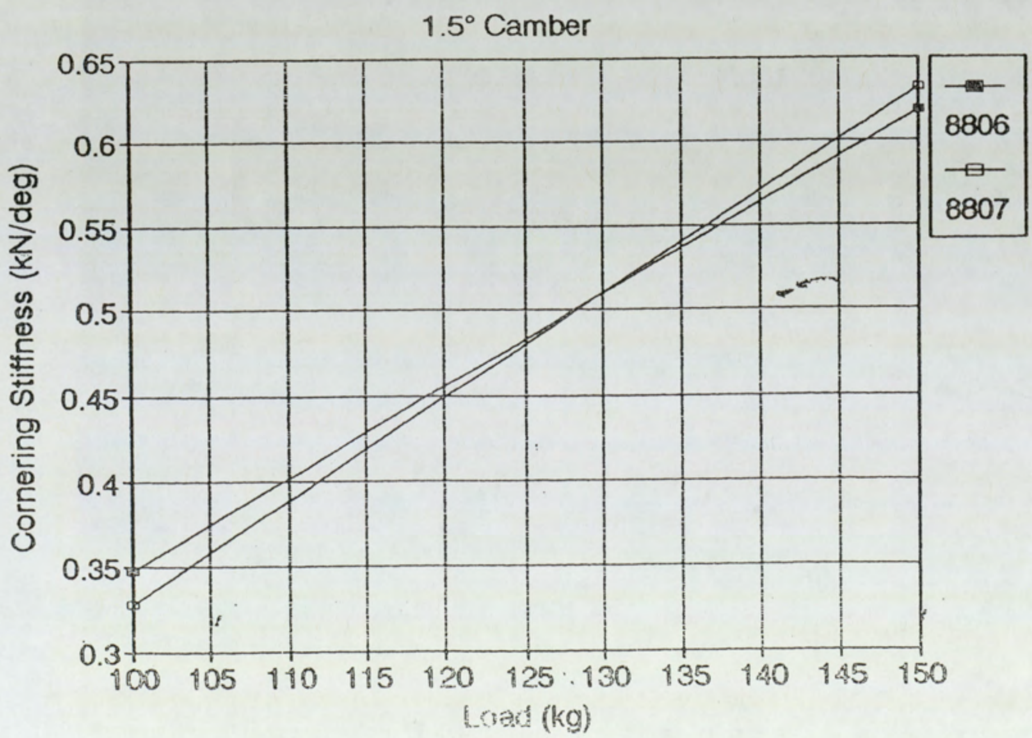


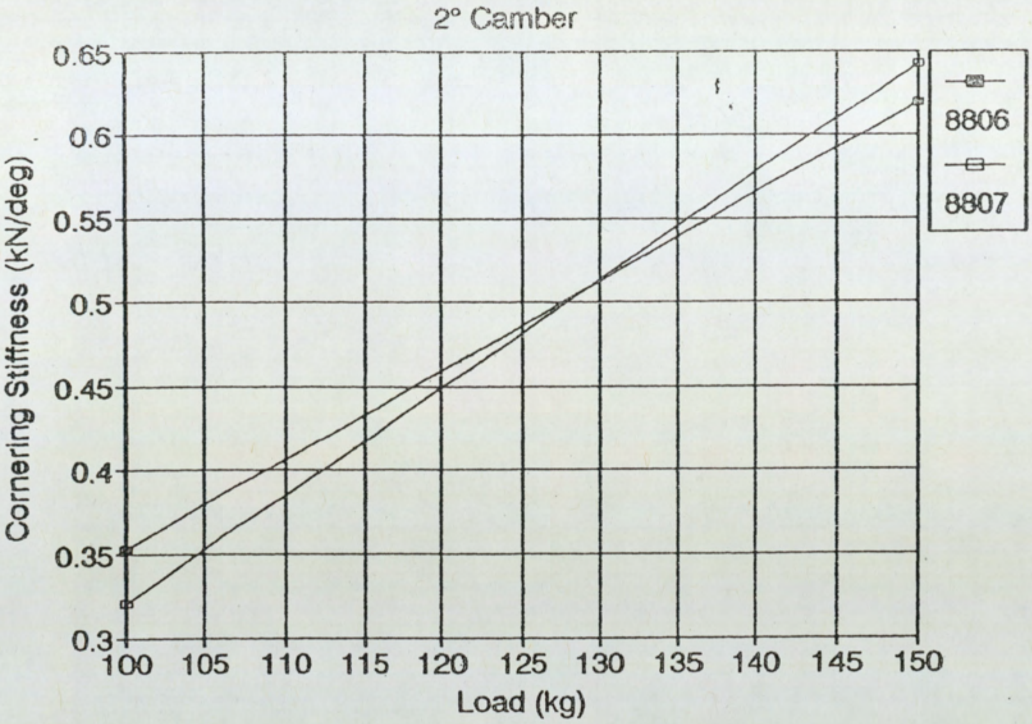


Project RC060STB

Tested 3/7/95






2° Camber
Cornering Stiffness (kN/deg)
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
100
105
110
115
120
125
130
135
140
145
150
Load (kg)
8806
8807

Project RC062STB

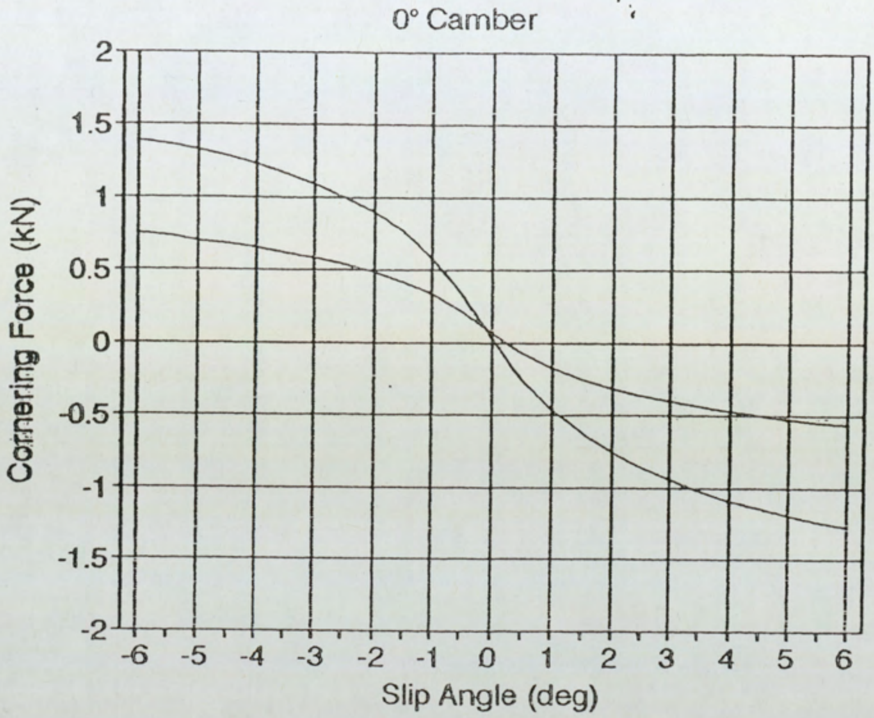
Tested 19/7/95

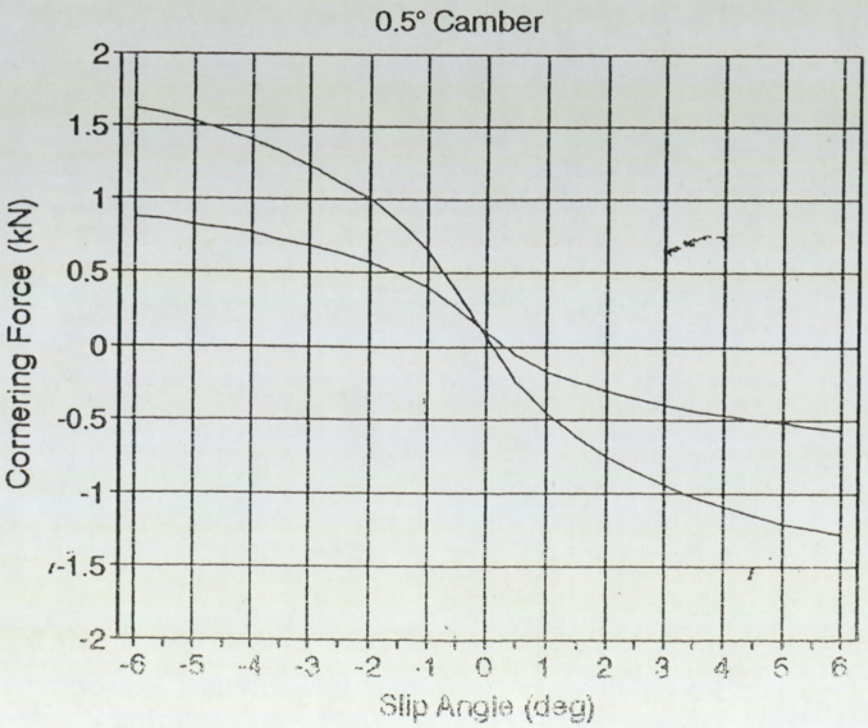
Size : 6.0/21.0-13 88ʋ6 on 6Jx13

Tyre : Formula Ford Zetec

Test : 20psi, 10kph

Load : 100kg , 150kg





Project RC062STB

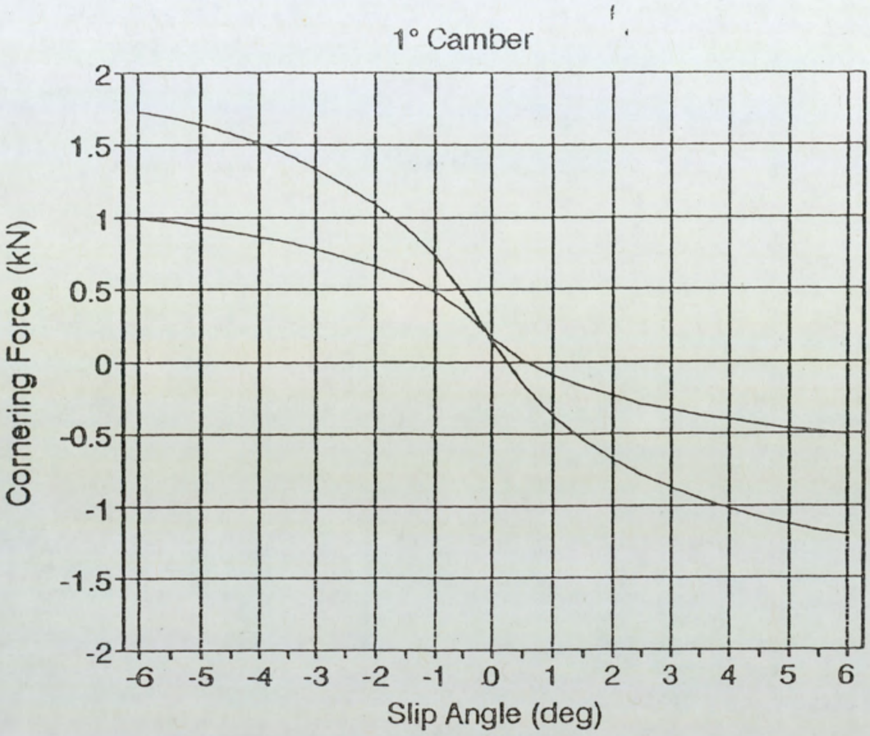
Tested 19/7/95

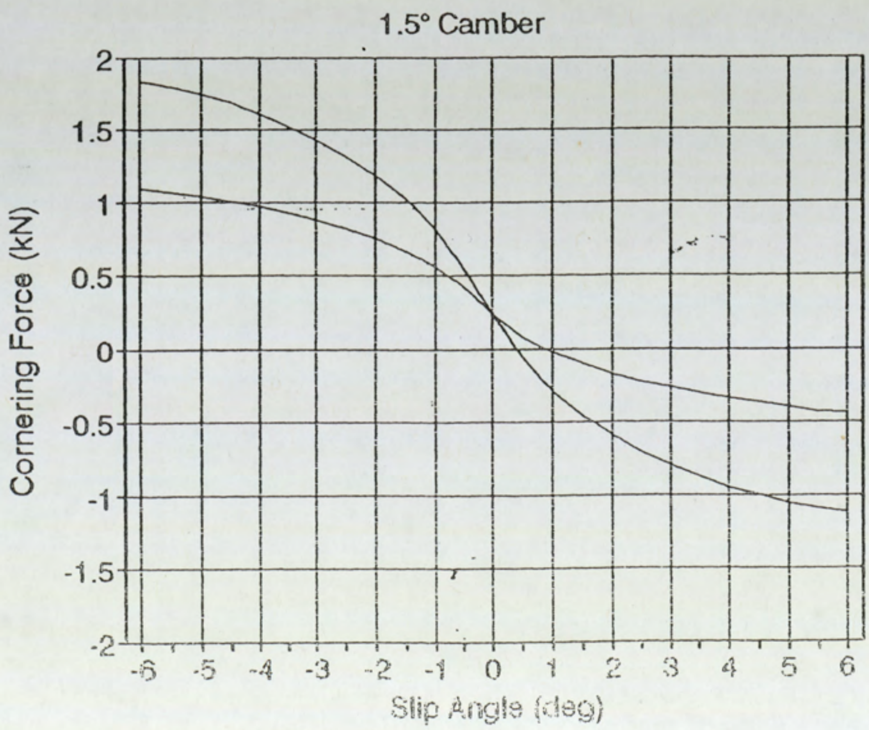
Size : 6.0/21.0-13 8806 on 6Jx13

Tyre : Formula Ford Zetec

Test : 20psi, 10kph

Load : 100kg , 150kg





Project RC062STB

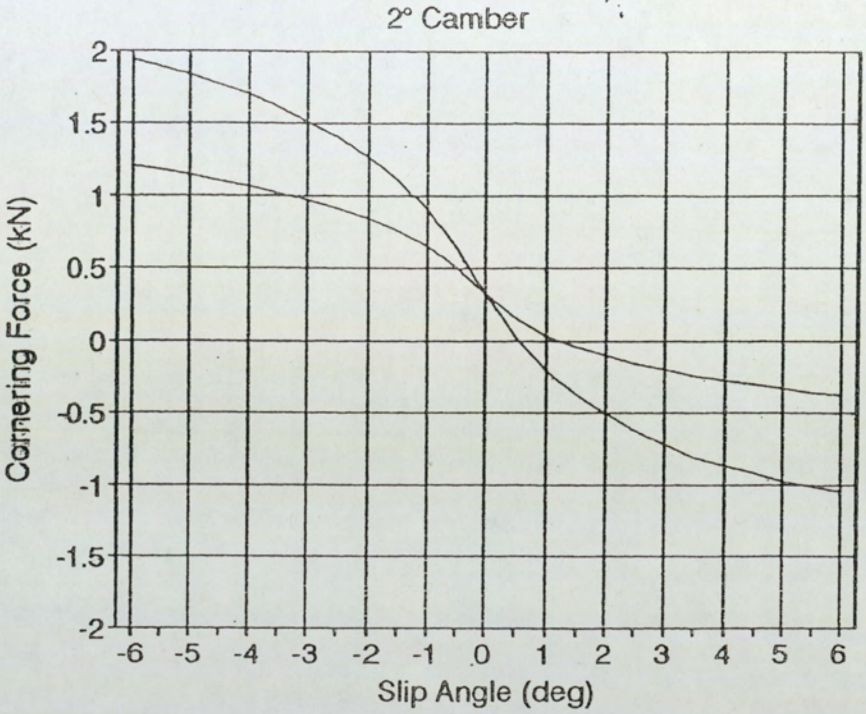
Tested 19/7/95

Size : 6.0/21.0-13 8806 on 6Jx13

Tyre : Formula Ford Zetec

Test : 20psi, 10kph

Load : 100kg , 150kg



Project RC062STB

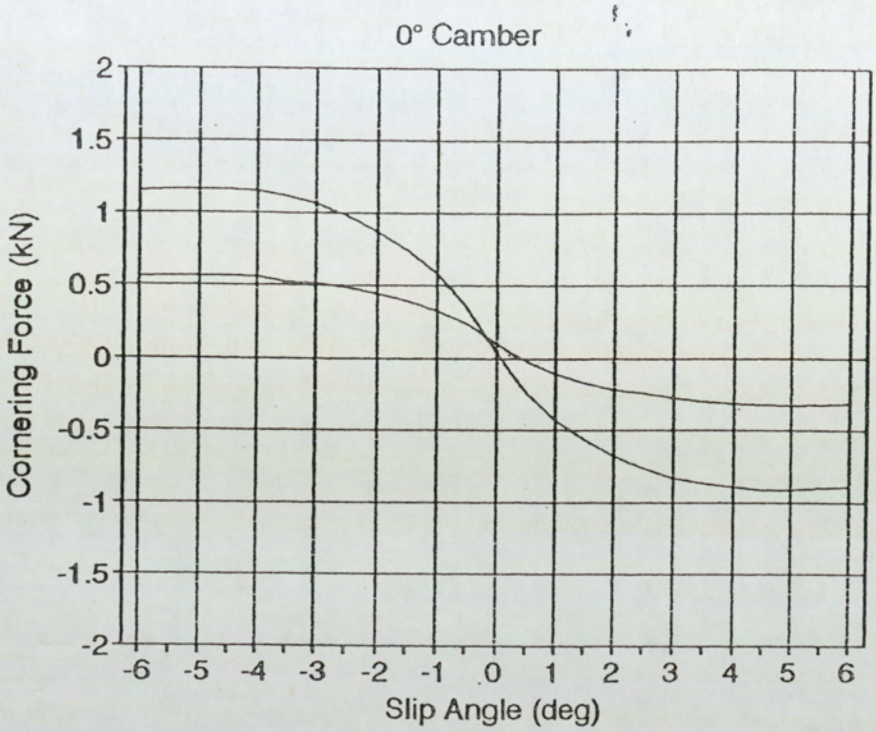
Tested 19/7/95

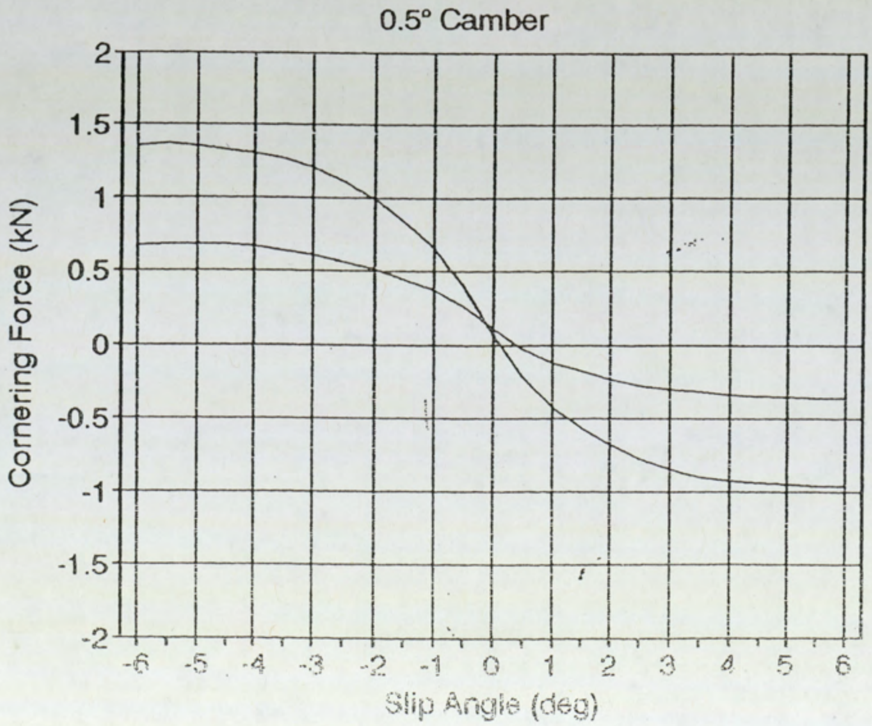
Size : 7.0/22.0-13 8807 on 7Jx13

Tyre : Formula Ford Zetec

Test : 20psi, 10kph

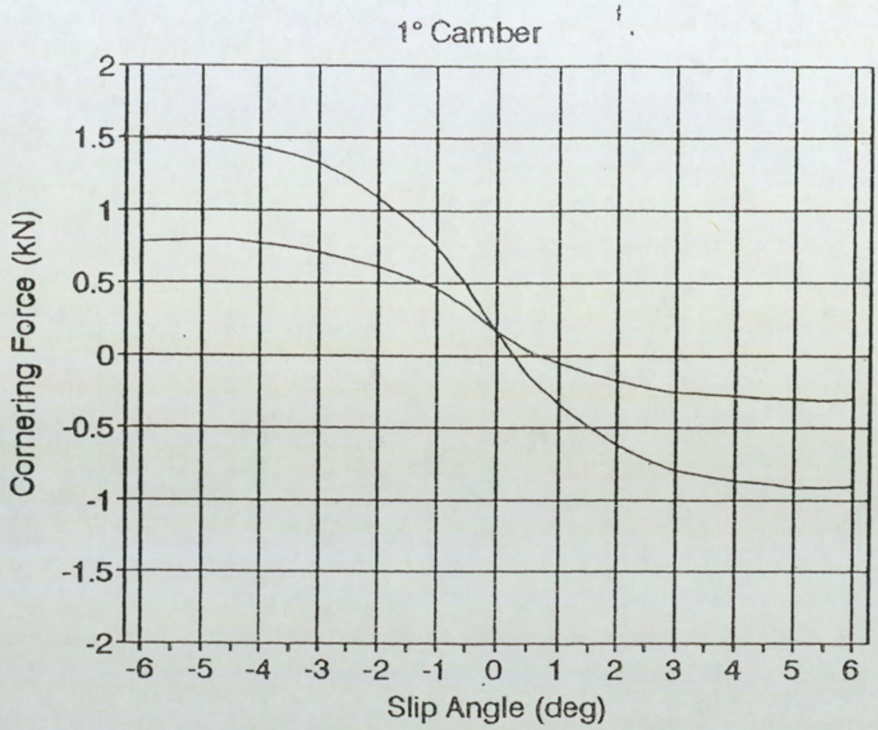
Load : 100kg , 150kg

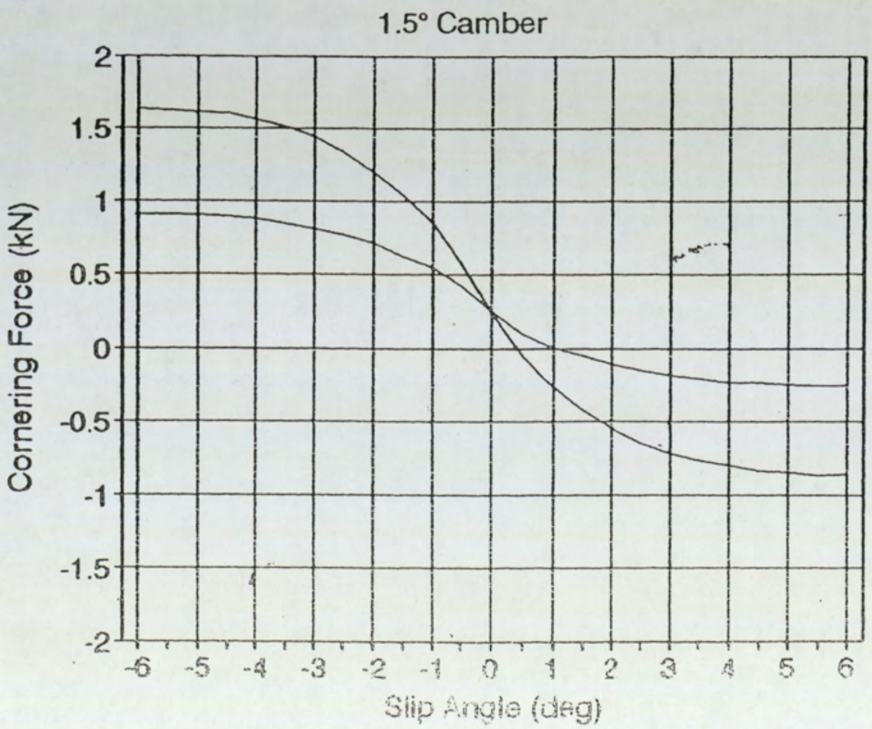




Project RC062STB Tested 19/7/95

Size : 7.0/22.0-13 8807 on 7Jx13
Tyre : Formula Ford Zetec
Test : 20psi, 10kph
Load : 100kg , 150kg





Project RC062STB

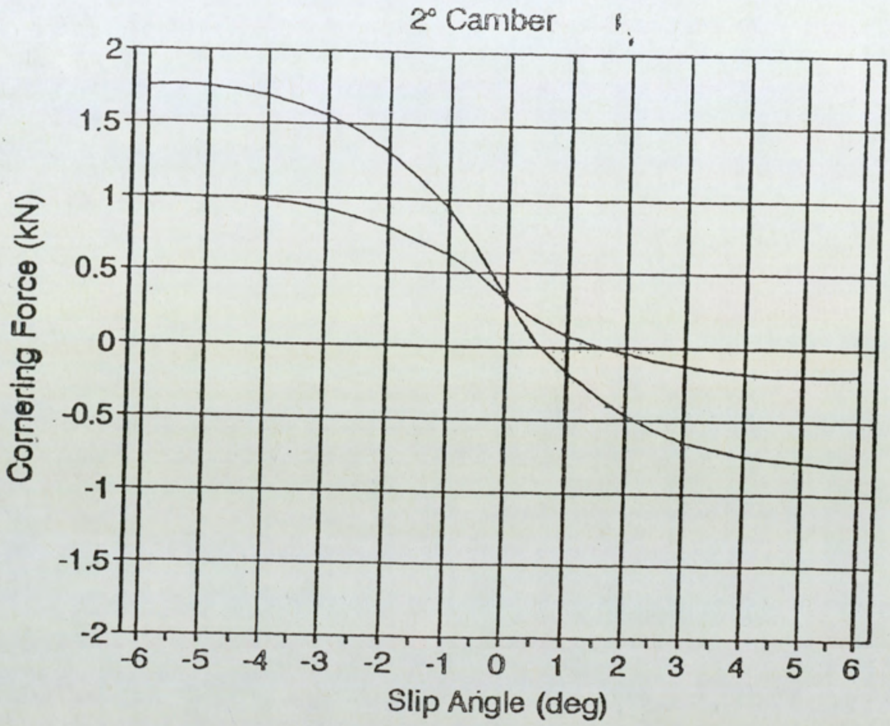
Tested 19/7/95

Size : 7.0/22.0-13 8807 on 7Jx13

Tyre : Formula Ford Zetec

Test : 20psi, 10kph

Load : 100kg , 150kg



FROM : G.HEEKS

TO : N.CROOKES

CC : F.P.COATES

June 26th, 1995

PROJECT RC063 SPG.

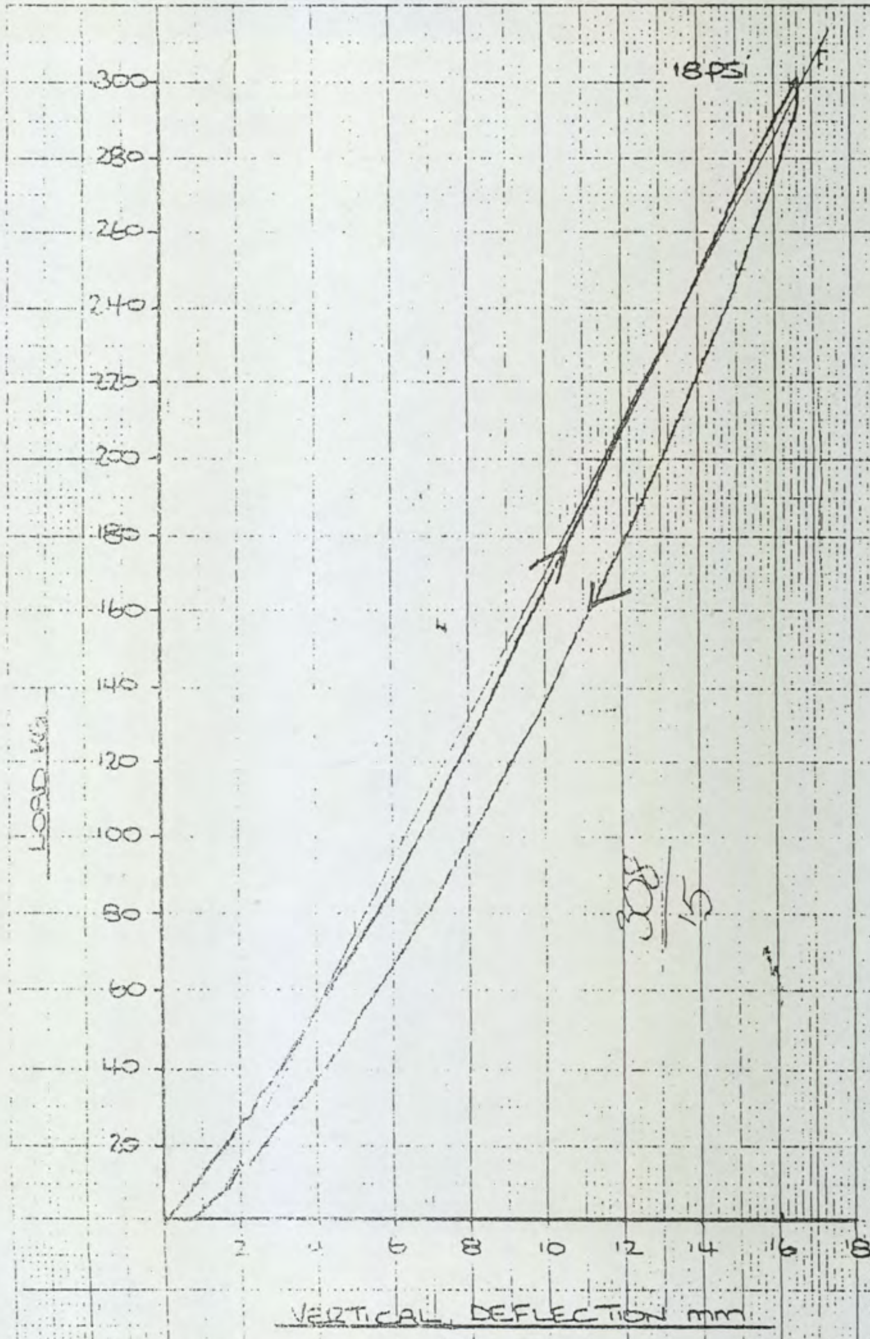
This project is for Spring Rate testing of current FFord Zetec Tyres. This data is to be distributed to the teams.

Tyres :	**A**	6.0/21.0-13	spec 8806
	B	7.0/22.0-13	spec 8807

Rims : **6.0"** for tyres **A** with 18, 20 & 22psi
7.0" for tyres **B** with 18, 20 & 22psi

Loads : 0-300kg Vertical Load
250kg vertical for A lateral deflections
300kg vertical for B lateral deflections

Thanks,



PROJECT RC063 SPG

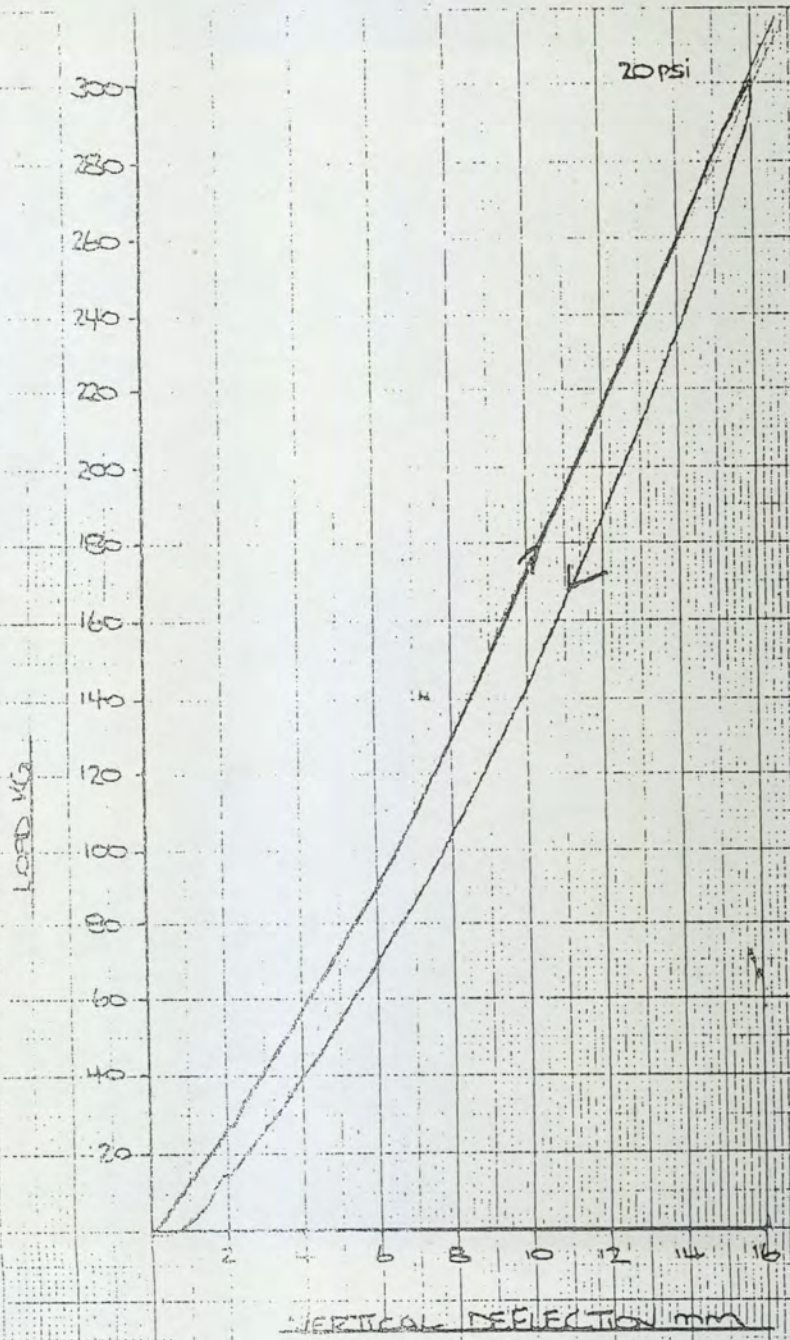
6·0/21·0-13 8806 AVON FF ZETEC AC80

(8806 ES'D) (A08428) 0165HM

Rim WIDTH : 6"

LOAD : 300 KG

DEFLECTION : 16·5mm



PROJECT RC063SPG

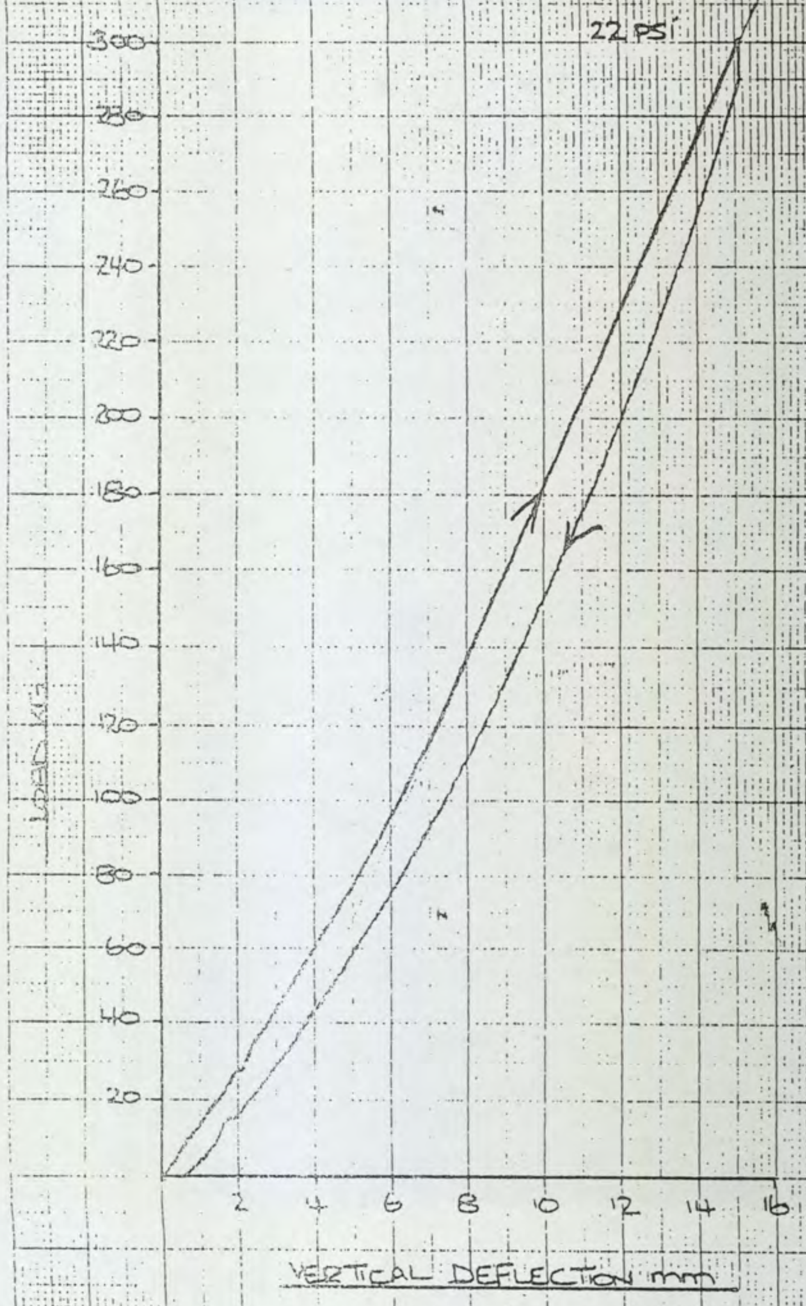
6.0/21.0-13 8806 AVON FF ZETEC AC80

(8806 ESID) (A08428) 01654m

RIM WIDTH : 6'

LOAD : 300 KG

DEFLECTION : 15.9mm



PROJECT RC963 SDG

6.0/21.0-13 8806 ANON FF ZETEC ACBO

(8806 ESID) (A08428) 01654m

Rim WIDTH : 6"

LOAD : 300 KG

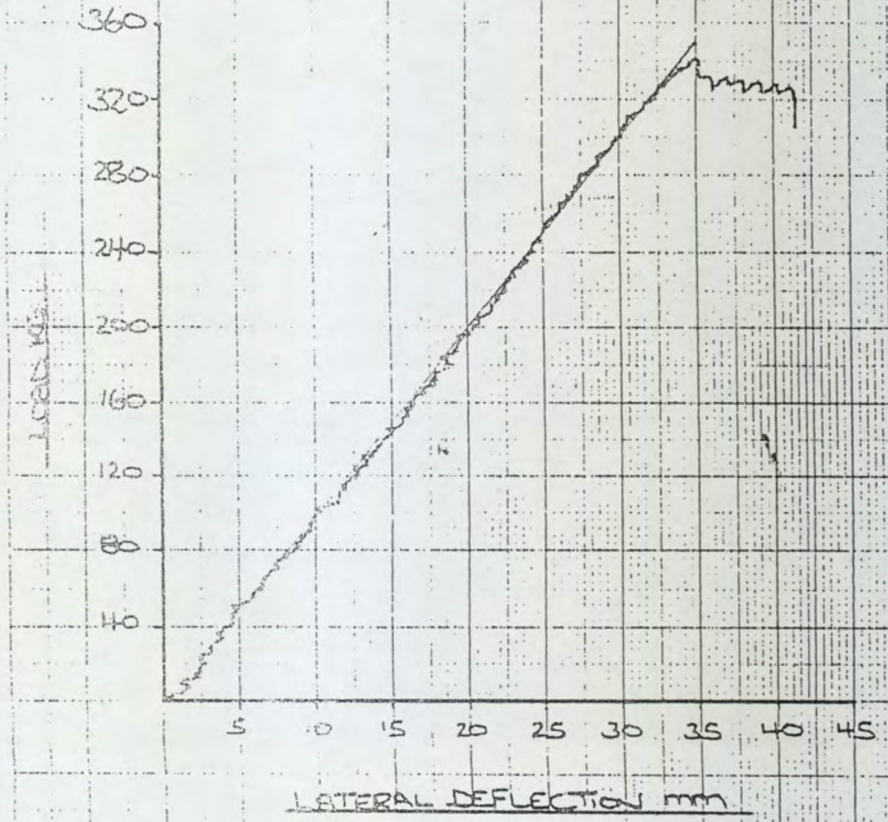
DEFLECTION : 15.1 mm

PROJECT RC063 SPG

60/21.0-13 8806 AVON FF ZETEC AC80

VERTICAL LOAD : 250 KG

PRESSURE : 18 PSI

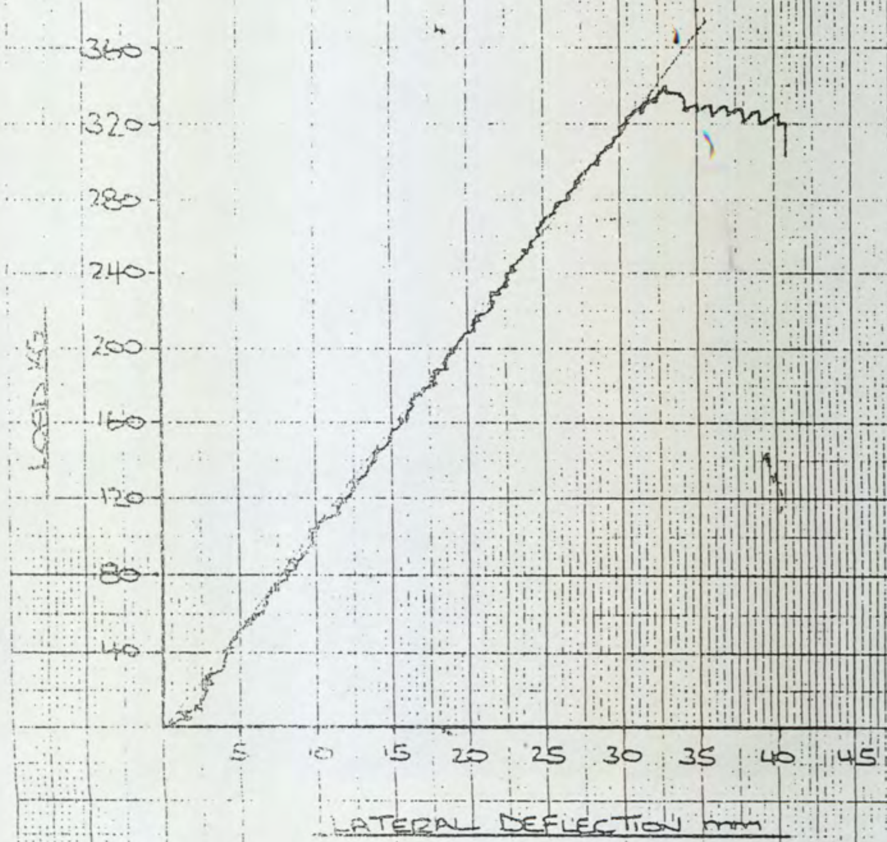


PROJECT RC063 SPG

6.0/21.0-13 8806 AVON FF ZETEC ACRO

VERTICAL LOAD : 250 KG

PRESSURE : 20PSI

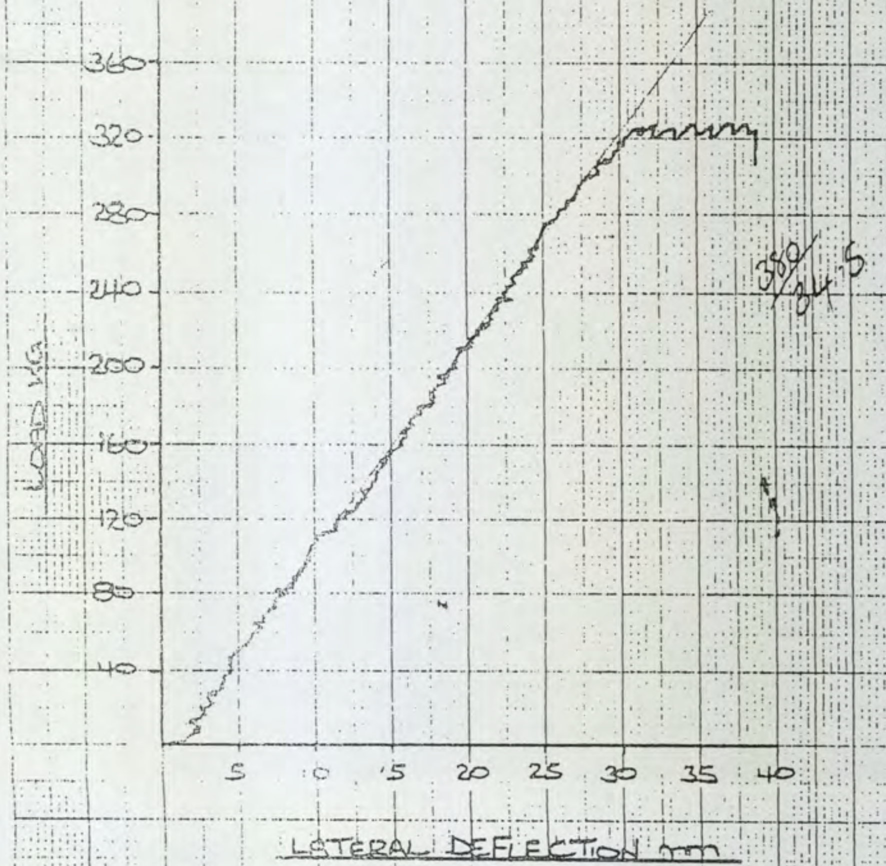


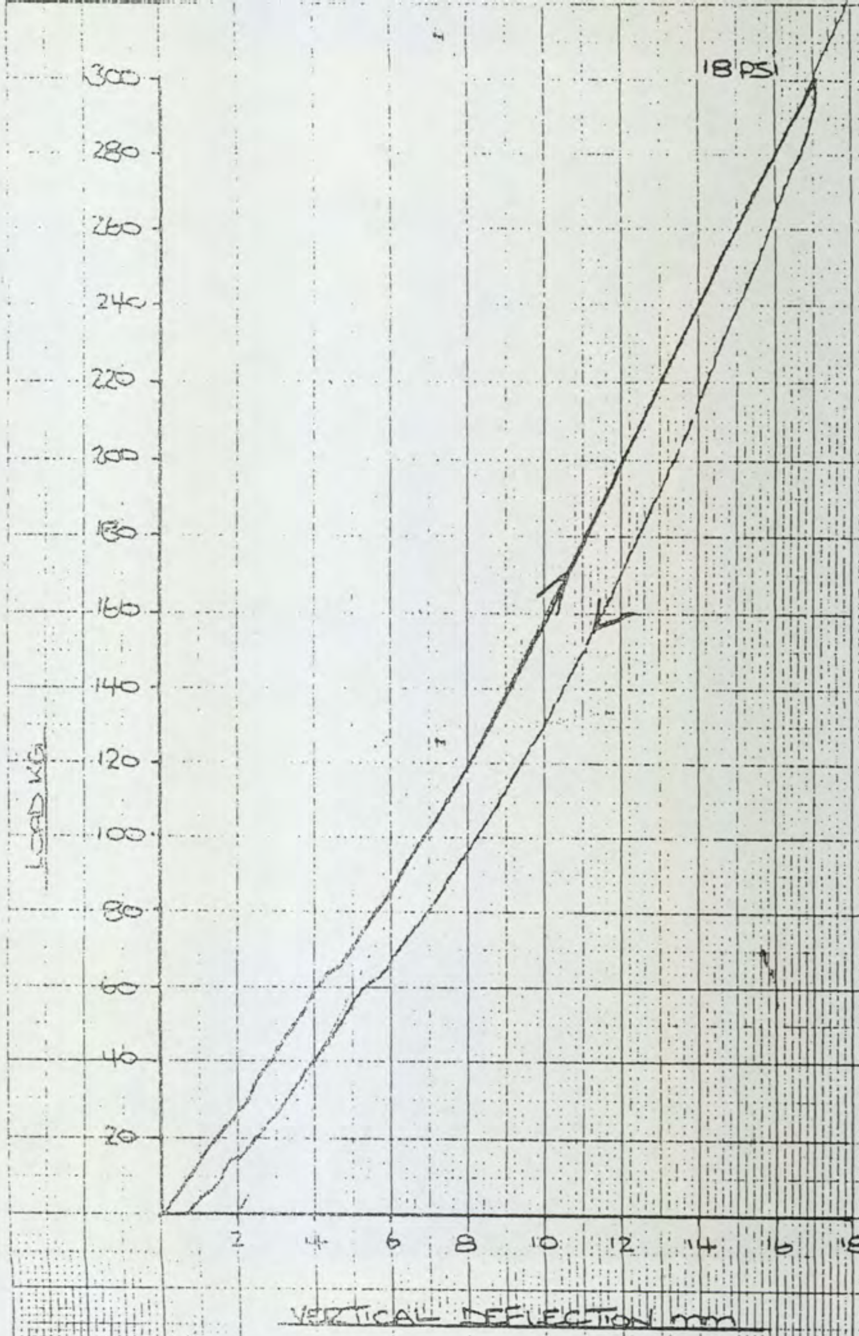
PROJECT RC0063 SPG

60/210-13 8806 AVON FF ZETEC ACBO

VERTICAL LOAD : 250 KG

PRESSURE 22 PSI





PROJECT RC063 SPG

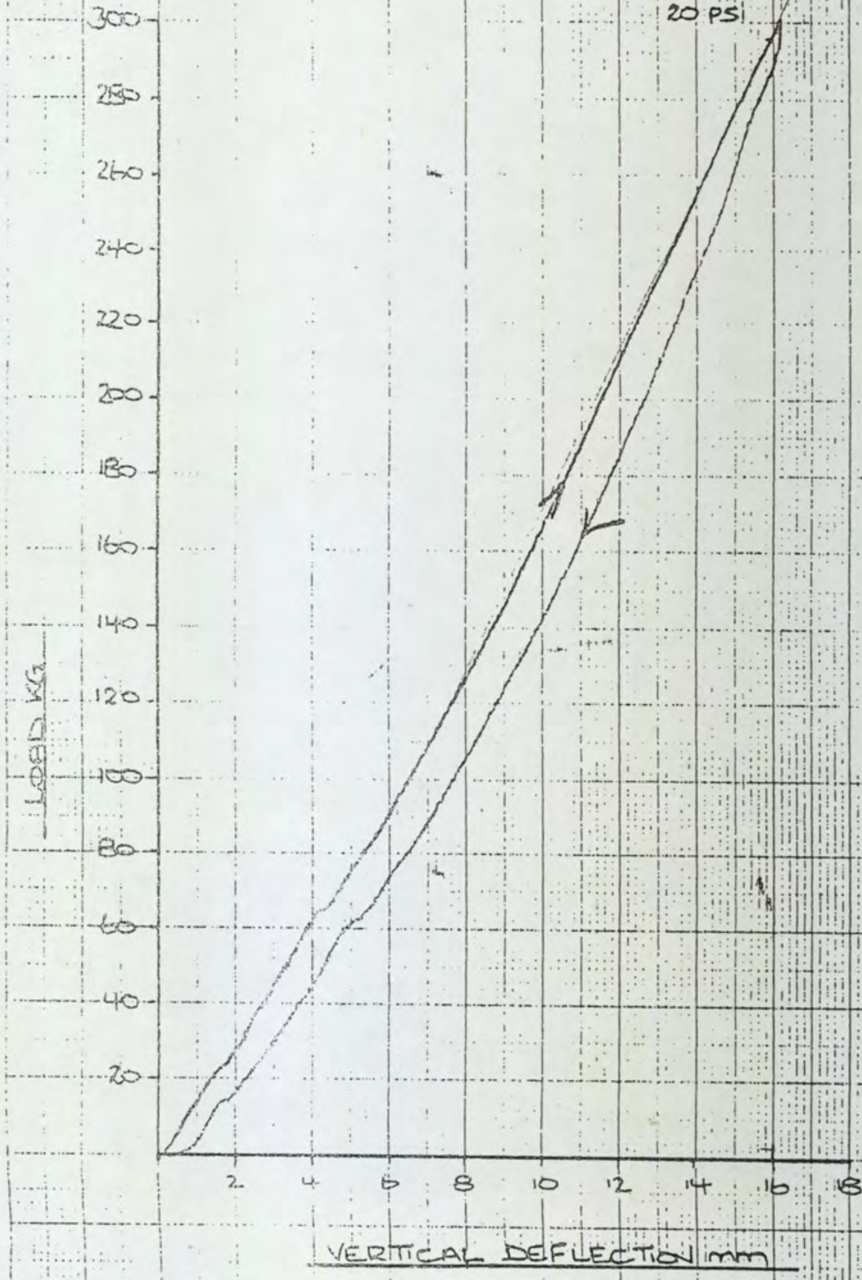
7.0/22.0-13 8807 AVON FF ZETEC A C80

(8807 TSID) (A09027) 01368M

Rim WIDTH : 7"

LOAD : 300 KG

DEFLECTION 17.1mm



PROJECT RC0635XG

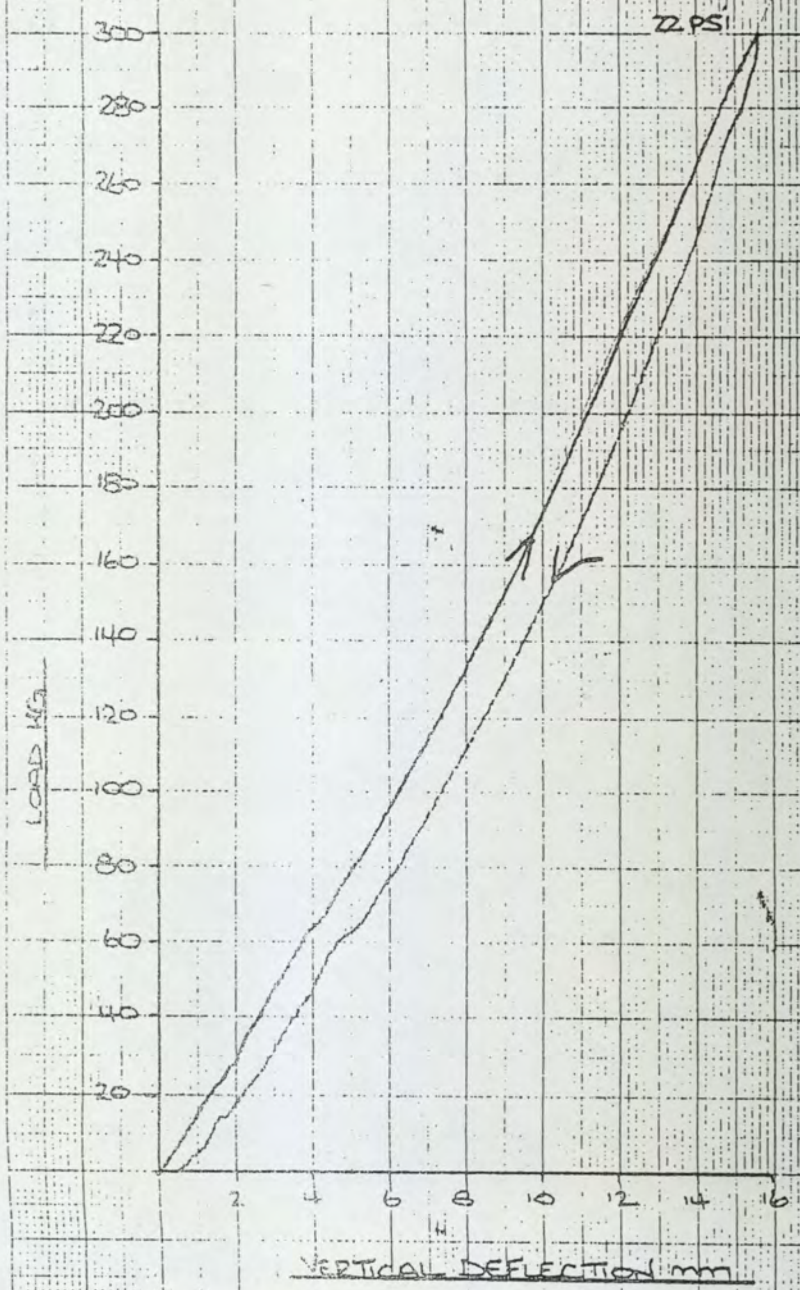
7.0/220-13 8807 AVON FF ZETEC AC80

(8807 TS1D) (A09027) 0136BM

RIM WIDTH : 7"

LOAD : 300 KG

DEFLECTION : 16.1 mm



PROJECT RC063.SPG

7.0/22.0-13 8807 AVON FF ZETEC AC80

(8807 TSID) (A09027) 01368M

RIM WIDTH : 4"

LOAD 300 KG

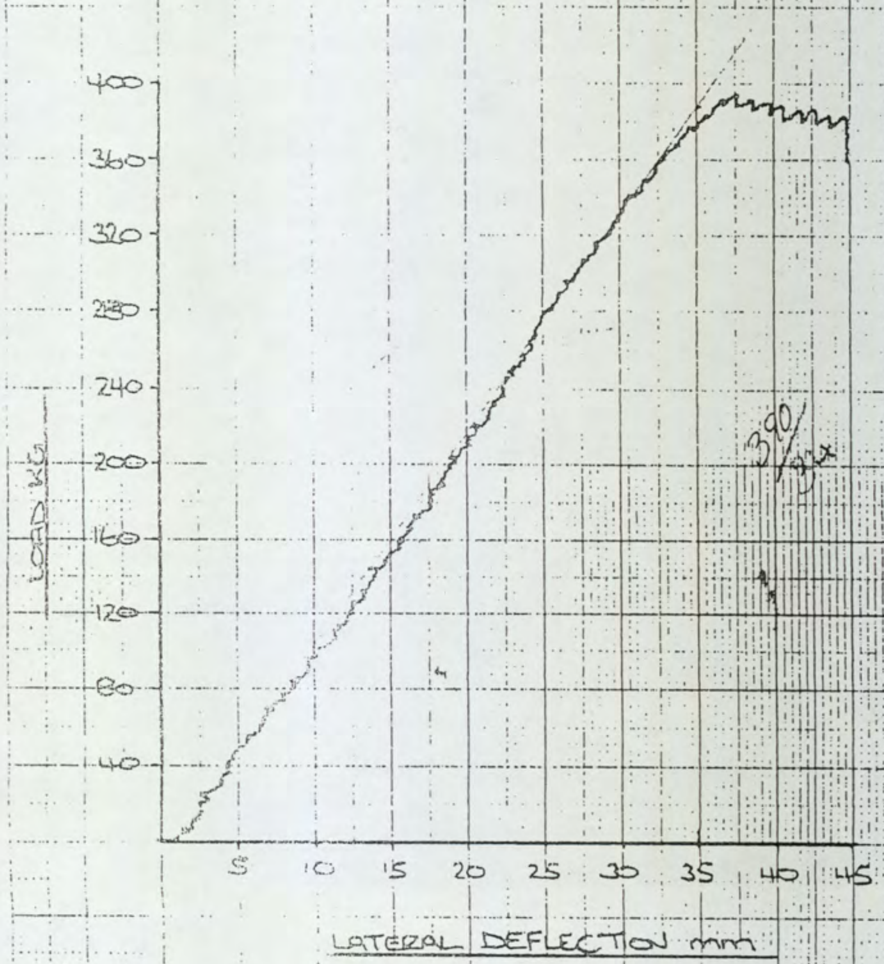
DEFLECTION 15.6 mm

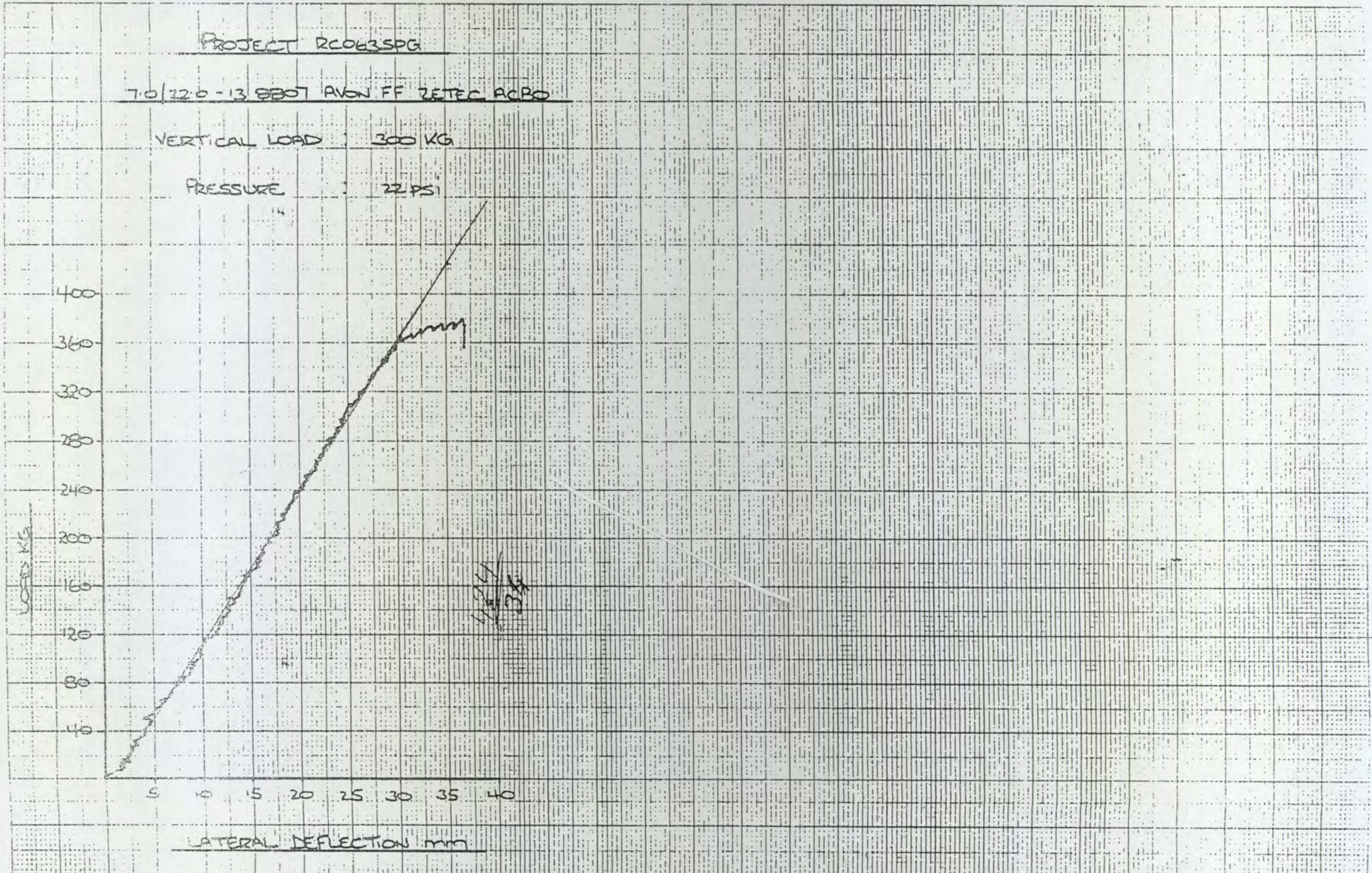
PROJECT RC063SPG

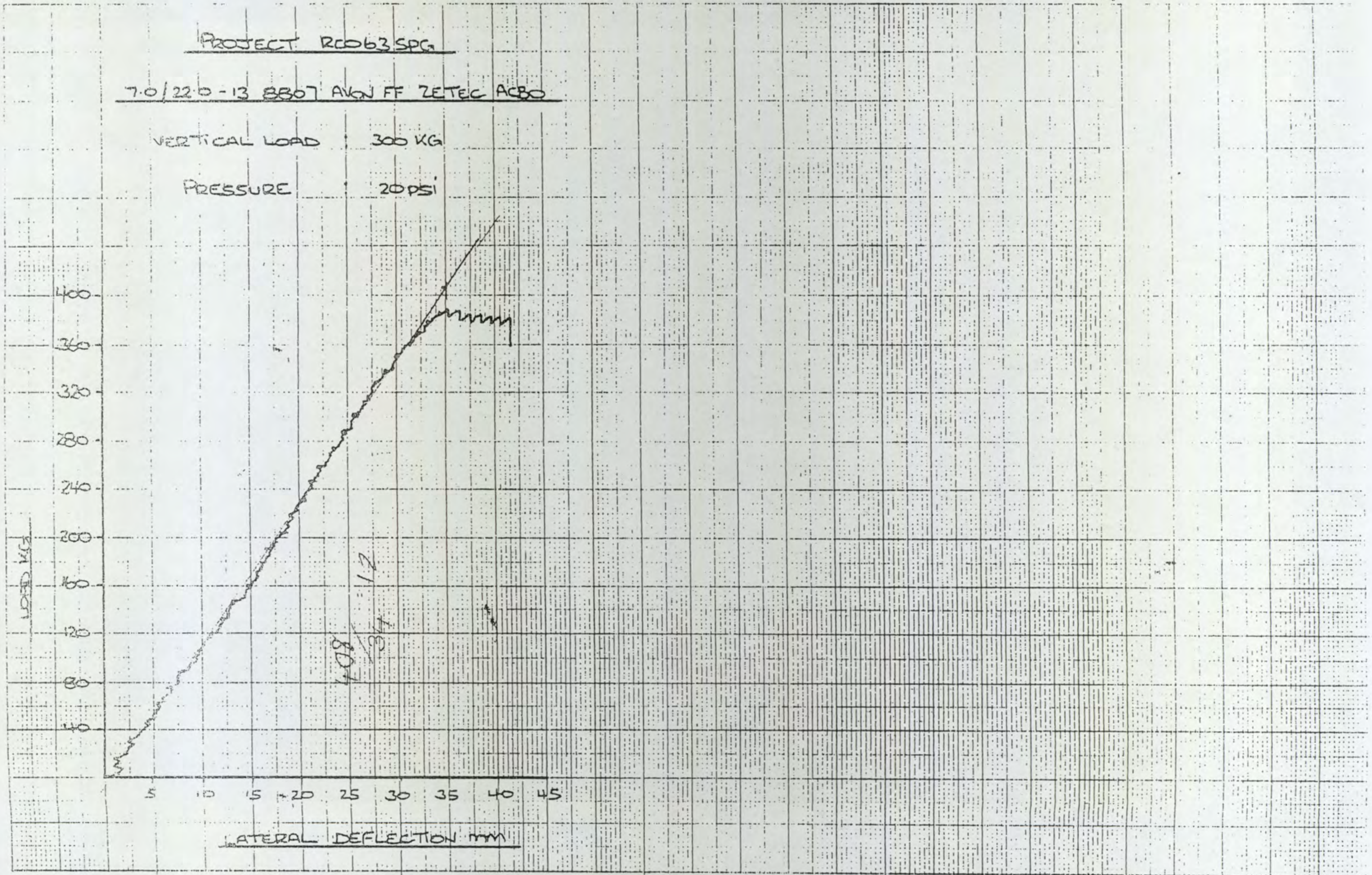
7.0/22.0-13 8807 AVON FF ZETEC AC80

VERTICAL LOAD 300 KG

PRESSURE 18 PSI




PROJECT RC063SPG
7·0/22·0 - 13 8807 AVON FF ZETEC ACBO
VERTICAL LOAD : 300 KG
PRESSURE : 22 PSI
LOAD KG
40
80
120
160
200
240
280
320
360
400
5
10
15
20
25
30
35
40
LATERAL DEFLECTION mm



PROJECT RC063SPG
7.0/22.0 - 13 8807 AVON FF ZETEC ACBO
VERTICAL LOAD : 300 KG
PRESSURE : 20 PSI
LOAD KG
400
360
320
280
240
200
160
120
80
40
5
10
15
20
25
30
35
40
45
LATERAL DEFLECTION mm
408/34
1.2

Appendix D – Electronic Resources

Included on this CD are the following:

- Design Resources Spreadsheet
- Lap Simulation Spreadsheet
- Race Technology Data Analysis Packages
- Susprog Design Files