To my MOTHER and FATHER

with Love and Affection

OPTIMUM DESIGN OF STRUCTURES

by

SHIRDEL NAJI AL-PASHA

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S U M M A R Y

The work presented in this Thesis aims at producing a computer method that uses mathematical programming to obtain optimum design of different types of structures. The optimisation of elastic structures usually turn out to be that of non-linear programming, thus the formulation of such problems is done in the form of sequentially approximating linear programming. The two-phase simplex method is then employed to obtain the solutions. The matrix displacement method is used in formulating the design problems. The method for optimum design is general and can be applied to minimise the weight or the total cost of the structure. The total cost is assessed realistically, and this includes the material and the construction costs of the members, plus the cost of constructing the foundations. The reduction of the weight does not include any variation in the configuration of the structure. But, on the other hand, minimising the total cost is achieved by altering the topology of the structure, depending on economical and structural requirements. This method of optimisation is applied for the design of three distinct types of large structures.

The first type includes plane rigidly jointed multi-storey steel sway frames. The method is applied to obtain minimum weight or minimum cost topological design that satisfies the stiffness, the sway deflection and the practical constraints. The stress constraints are not included in this case.

The second type is described as complete structures consisting of arbitrary parallel systems of reinforced concrete shear walls and floor slabs, with additional restraining frames made from steel or reinforced concrete. The structures are assumed to be subjected to the effect of static wind loads only. The optimisation method is employed to obtain a topological design of minimum cost that satisfies the stiffness, the differential sway deflection and the practical constraints.

The third type is represented by reinforced concrete horizontal grillages made from straight orthogonal rectangular beams, with or without supporting columns. The optimisation method is applied to obtain a minimum weight or a minimum cost topological design that satisfies stiffness, stress and deflection constraints. The stress constraints include that of bending moment and that of combined shear and torsion.

Key Words: OPTIMUM TOPOLOGY SWAY FRAME COMPLETE STRUCTURE GRILLAGE

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LIST OF CONTENTS

Page No

Page No

Page No

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION AND REVIEW OF PUBLISHED WORK

1.1 INTRODUCTION

A general design of structures can be regarded as a decision making process in which a certain goal has to be achieved while some design requirements have to be satisfied. The first decision, in general, concerns establishing the functional requirements which determine the shape of the structure. Then, the sectional properties of the members are determined so that the structure can safely withstand the external loads. Iterative methods for reanalysis are traditionally employed for this purpose and reasonably feasible designs are obtained.

After World War II, a relatively new mathematical science known as operational research was developed. A particular part of this science called mathematical programming made it possible to obtain a feasible design which is also optimal.

Structural optimisation seeks the selection of "design variables" to achieve, within the limits (constraints) placed on the behaviour of the structure, on its shape and on other factors the goal of optimality defined by the "objective function". The aim of the optimisation is to obtain the lowest weight or cost for the structure. This used to be achieved, for a structure of pre-selected shape, by determining the sectional properties of the members. However, it was later found possible to obtain greater weight or cost reduction with changes in the shape than with changes only in the member properties. Therefore, optimising the shape of a structure, which is very significant, has attracted much attention in recent years.

This thesis deals with three distinct types of optimisation problem:

(1) Optimum design of plane multi-storey steel sway frames.

(2) Minimum cost topological design of laterally loaded complete structures consisting of reinforced concrete shear walls and floor slabs, and additional restraining steel or reinforced concrete frames.

(3) Optimum design of reinforced concrete horizontal grillages made out of in-plane straight orthogonal beams with or without supporting columns

The review given in this Chapter is generally divided into two main parts. In the first part a review is given of the algorithms for mathematical programming which are utilised in the formulation and the solution of the design problems in this thesis. This part also includes a brief review of other practical optimisation algorithms. The second part deals with the published work on structural design and optimisation, considering the three different structures mentioned above.

1.2 FEATURES OF STRUCTURAL OPTIMISATION

Each problem of optimum design contains three basic features. These are the design variables, the objective function, and the design constraints which contrive to form the geometry of the design space. Each of these features will be described briefly below.

(a) The Design Variables

These may consist of the member sizes, the configuration of the structure, and the mechanical or physical properties of the material. Optimisation of the member sizes is widely used because of the relative simplicity of the problem, and because many practical structures have fixed shape and material properties. Configuration (shape) variables, often represented by the geometry, i.e. coordinate of element joints, and by the topology which is usually the number and the position of the members. Optimising the configuration is a difficult matter and is often treated to a limited extent. The full set of design variables in a problem is listed as vector $\{x_j\}$, $j = 1 \dots n$, where n is the total number of design variables. Each design variable represents an axis in the "design space" which can be described as an n-dimensional cartesian space. A surface in the design space represents a constraint, whereas a point in the design space represents a particular design. A point which does not violate any of the constraints is called a feasible design; the space for such points is known as the "feasible region".

(b) The Objective Function

This function constitutes a basis for the selection of one of several alternative feasible designs. The function is scalar with n design variables, and it can be of a linear or non-linear form. The weight of the structure is most frequently used as the objective function to be minimised in spite of the fact that the cost of the structure includes many other aspects other than the material weight. These aspects are often difficult to be usefully quantified. In most mathematical programming methods, the objective function is maximised. In such cases a minimum is obtained by multiplying the objective function by -1 and maximising its negative.

(c) The Design Constraints

A constraint, in any class of problem is a restriction to be satisfied in order to obtain a feasible design. It may take the form of an explicit limitation imposed directly on a variable or a group of variables. On the other hand, it may represent a limitation on quantities whose dependence on the design variables cannot be stated directly; such a constraint is an implicit one. The main types of design constraints are the equality constraints, such as the stiffness equations, and the inequality constraints of the form $\geq$ or $\leq$, such as the sway deflection or the stress limitation. Other important types of constraints are the practical and the behaviour constraints. A practical constraint is a specified limitation on a design variable, such as lower and upper bound on a sectional variable of a member, and it is therefore explicit in form. A behaviour constraint is usually a limitation on a joint displacement or a member stress. This can be an explicit or an implicit constraint.

1.3 LINEAR PROGRAMMING

In linear programming, the objective function and all the constraints are linear of the form:

$$\text{Max. } Z = \sum_{j=1}^{n} c_j \, x_j \tag{1.1}$$

subjected to the constraints:

$$\sum_{j=1}^{n} a_{ij} \, x_j \; \{\leq \text{ or } = \text{ or } \geq\} \; b_i \tag{1.2}$$

$$x_j \geq 0; \quad b_i \geq 0; \quad i = 1, \ldots m$$

where c_j, a_{ij} and b_i are constant coefficients, n is the number of variables and m is the number of constraints.

From computational considerations, all the variables x_j are required to be limited to the non-negative range, and the coefficients b_i are also required to be non-negative. The last requirement can always be satisfied since in case of $b_i < 0$, the corresponding constraint may be multiplied by -1 so that b_i becomes positive. In such a case it is also necessary to change the inequality sign from $\leq$ to $\geq$ and vice versa.

To solve a linear programming problem by the simplex method, (Dantzig, 1963) and (Hadley, 1962), the inequality constraints of the form $\leq$ are transformed to equalities by adding new non-negative unknown "slack" variables. Inequality constraints of the form $\geq$ can be converted into equalities by subtracting "surplus" variables. However, unlike slack variables, these surplus variables are treated in the same manner as the actual variables x_j, and are also added to the objective function with zero coefficients. In this way the actual value of the objective function is not affected by the presence of any surplus variable in the final solution.

Equality constraints must be fully satisfied, otherwise the solution is infeasible, thus slack variables cannot be added to these constraints. For this reason, artificial variables are added to all equalities and also to the inequality constraints of $\geq$ form.

The reason for adding the slack and the artificial variables is to obtain an initial (basic) feasible solution for the problem. However, the final solution cannot contain an artificial variable because such a solution violates the equality requirement of the problem. Thus effort is directed towards eliminating these artificial variables from the final solution.

1.4 THE SIMPLEX METHOD

This is a well known systematic method of solving a general linear programming problem. It is also known to be an efficient method used for structural design problems. The method manipulates the constant coefficients of the design variables. These coefficients should be arranged in matrix form. Thus, for a general linear programming problem, such as that expressed by function (1.1) and formula (1.2), the elements c_j and a_{ij} can be written as a two-dimensional matrix $\underline{D}$. While the right-hand side elements b_i can be written as a one-dimensional matrix $\underline{B}$. These matrices contrive to form the "simplex table" which can be written as:

$$\underline{D}\ \underline{C} = \underline{B} \tag{1.3}$$

where $\underline{C}$ is the vector that contains all the variables, including the surplus variables.

To begin with, all the design variables $\{x_j\}$ and the value of the objective function (Z) are equal to zero. While the elements of matrix $\underline{B}$ are set equal to the values of the slack and the artificial variables, which represent the basic feasible solution.

Once any basic feasible solution is determined, the simplex method can be used to obtain a minimum feasible solution in a finite number of iterations. At each iteration, a pivot element is selected which is on the row of the variables to be removed, and it is on the column of the variable to be entered in the solution. Once a pivot is selected, a new table, belonging to a new solution, is obtained.

Selection of the pivotal column in matrix $\underline{D}$ can be difficult, and in this thesis the algorithm "two-phase technique", (Garvin, 1960), is used.

1.4.1 The Two-Phase Technique

Using this technique, the linear programming problem is solved in two phases. The first phase derives all the artificial variables out, and the second phase minimises the actual objective function starting from a basic feasible solution which either contains no artificial variables or some with zero values.

In this two phased method, the removal of the artificial variables is accomplished not by considering the original objective function, but by minimising the infeasibility using a secondary function $\underline{Z}'$ defined by:

$$Z' = \sum_{j=1}^{n} d_j \; x_j \tag{1.4}$$

where n is the total number of variables, d_j is the sum of the elements at the rows with artificial variables in column j of matrix $\underline{D}$; d_j is also known as the relative cost coefficient, and Z' is the sum of the elements at these rows in the right-hand side matrix $\underline{B}$.

The simple method can now be applied to minimise Z', which is Phase I. This is accomplished if, for each iteration, a pivot is selected according to the "θ" rule on a column with the largest positive element in the Z' row. The calculations are then carried out to remove the artificial variables. After achieving this, in Phase II the actual objective function is minimised similarly without having artificial variables or having some of them at zero level.

Phase I is terminated if one of three situations is reached.

(1) The value of Z' is zero, and all the artificiaal variables are non-basic, i.e. removed from the solution. In this case a basic feasible solution has been obtained with all $d_j \geq 0$. The variables whose $d_j > 0$ are in fact dropped from further consideraton. Phase II then proceeds normally.

(2) The value of Z' is zero, and one or more artificial variables are basic, i.e. not removed, with a value of zero. This means that a degenerate basic feasible solution has been obtained which probably causes the problem to enter into cycling.

(3) The value of Z' is greater than zero and one or more artificial variables are basic. This shows that there is no feasible solution to the original problem.

There are some disadvantages in using this method. One of these lies in that many numbers are unnecessarily computed and recorded, which slow down the operation. To speed up the calculations, a modified method known as the "Revised Simplex Method" can be used. This method was developed by Dantzig (1953), using the same principles as the two-phase simplex method. In each of its iterations only one basic variable changes, and as a result only the essential quantities are computed which saves computer time and storage. The revised simplex method also requires fewer arithmetic operations when the original problem contains a large number of zeros such as in structural optimisation. Another disadvantage of the two-phase technique is that it cannot solve degenerate problems. However, it is rare for structural design problems to be degenerate or to end up in cycling, especially if they are fomulated correctly. To prevent cycling some procedures described by Garvin (1960), Hadley (1962) and others can be used.

When using the simplex method, it is assumed that the variables in the programming problems can have continuous values between specific limits. In practice, however, variables can only have discrete values. For instance, in the case of steel structures, the designer has to select available discrete sections for the members. Such a problem can be solved using integer programming. This type of programming can be applied only when both the constraints and the objective function are linear, and the variables in the final solution are required to be integers. An integer programming problem can also be solved using the simplex method, and to do this the concept of Gomory's cut (1958) was introduced. This makes use of the fractional part of the numbers in simplex table, as illustrated by Majid (1974a).

1.5 METHODS OF OPTIMISING NON-LINEAR PROBLEMS

Most of the structural design problems are mathematically non-linear, as they involve the product, the reciprocal or the higher powers of some or all the variables appearing either in the constraints or in the objective function or in both. The design problems thus turn out to be non-linear programming problems, particularly if the elastic theory is used for the formulation.

There are several different non-linear programming (NP) methods available. Zoutendijk (1966) has made an interesting comparison between some of the most popular methods which may be classified in three groups:

(1) Methods where the NP problem is solved by means of transformations to linearised problems. Most well known is Kelley's (1960) cutting plane methods and approximating programming methods developed by Griffith and Stewart (1961). Moses (1964) demonstrated the use of the cutting plane method, and Cornell, et al (1966a, b) applied a technique closely related to the method of approximating programming for structural optimisation.

(2) Methods of feasible directions. These methods solve the optimisation problems by directly considering the linear and the non-linear constraints as the limiting surfaces. The process is to start at some feasible point and then find a direction along

which the objective function can be improved while all the constraints are satisfied. Zoutendijk (1960) has described several variants of the method. Rosen's (1960) gradient projection method, and a method by Goldfarb and Lapidus (1968) belong to this group. Brown and Ang (1966) applied the gradient projection method to find optimum designs of frames and trusses.

(3) The Penalty-Function Method. The ideal behind this method is to transform the constrained non-linear problem into an unconstrained one by multiplying the constraints by a factor and adding them together to the objective function. By this means, a new unconstrained function is formed which may be minimised by one of the unconstrained minimisation techniques, such as the Direct Search, the Gradient and the Newtonlike techniques. The use of a penalty function was suggested first by Courant (1943), and then developed considerably by Carroll (1961). Fiacco and McCormick (1964) introduced the term "sequential unconstrained minimisation technique" SUMT, which is now widely used. Kavlie, et al (1966) applied this method for the optimisation of ship structures, and Kavlie and Moe (1969) applied it for the optimisation of grillage structures.

Generally methods of types (1) and (2) are most attractive for problems in which the constraints may be approximated closely by means of linearisations, while type (3) is of special interest when the objective function as well as the constraints are strongly non-linear. The amount of numerical work involved may, however, be considerably higher for methods of types (2) and (3). This makes methods of type (1) which depend on the idea of approximating non-linear programming problems by a sequence of linear programming problems attractive. After a finite number of design iterations such a procedure converges to an optimum solution. The simplex method is used for solving the linear programming problem during each design iteration.

1.5.1 Sequence of Linear Programs

Sequential linear programming methods are based on successive linearisation of the constraints and the objective function. There are two methods of linearisation. The first makes use of Taylor's series and takes the first order terms to approximate a non-linear function into a linear form. The second method is to approximate through replacing a non-linear function by a series of linear segments. This is known as the piecewise linearisation, and it is thoroughly explained by Hadley (1964).

The first method of linearisation is widely utilised, and because it is adopted in this thesis it will be briefly described here. A general non-linear programming problem which has a number of n variables and m constraints can be expressed as:

$$\text{Min. } Z = f(x_j) \tag{1.5}$$

subject to:

$$g_i\ (x_j)\ \{\leq \text{ or } = \text{ or } \geq\}\ 0 \tag{1.6}$$

$$x_j \geq 0; \quad j = 1, \ldots n; \quad i = 1, \ldots m$$

This problem can be linearised at any arbitrary point $\{x_j\}_o$ by using Taylor's series to become:

$$\text{Min. } Z = f\ (x_j)_o + \nabla f\ (x_j)_o \cdot [\{x_j\}_1 - \{x_j\}_o] \tag{1.7}$$

subject to:

$$g_i\,(x_j)_o + \nabla g_i\,(x_j)_o \cdot [\{x_j\}_1 - \{x_j\}_o] \leq \text{ or } = \text{ or } \geq 0 \qquad (1.8)$$

$$x_j \geq 0; \quad j = 1, \ldots n; \quad i = 1, \ldots m$$

where:

$$\nabla f(x_j)_o = [\frac{\partial f}{\partial x_{1,o}} \; \frac{\partial f}{\partial x_{2,o}} \quad \ldots \quad \frac{\partial f}{\partial x_{j,o}} \ldots \frac{\partial f}{\partial x_{n,o}}] \qquad (1.9)$$

and:

$$\nabla g_i\,(x_j)_o = [\frac{\partial g}{\partial x_{1,o}} \; \frac{\partial g}{\partial x_{2,o}} \quad \ldots \quad \frac{\partial g}{\partial x_{j,o}} \ldots \frac{\partial g}{\partial x_{n,o}}]_i \qquad (1.10)$$

are known as the gradient vectors. The value of every function is known at the initial point $\{x_j\}_o$. The problem in now linear with $\{x_j\}_1$ being the unknowns.

This method of linearisation was used by Kelley (1960) to develop an algorithm known as the cutting plane method. This was based on the useful property that the linearised constraints of the problems were convex. The method was widely applied, but some undesirable points of it were outlined by many researchers, (Moses, 1964), and others. Such as, the method is not applicable to every non-linear problem especially when the solution happens to be a non-vertex. Furthermore, this method cannot guarantee convergence for non-convex practical problems, and it cannot deal with the oscillation of the solution.

Griffith and Stewart (1961) also used Taylor's series to linearise the non-linear problem. Their approach, which is known as either the approximating programming method or the move limit method, does not suffer from the difficulties which arise in the cutting plane method.

This method is used to approximate all the non-linear structural problems in this thesis, and for this reason it will be described in some detail in the next sub-section.

1.5.1.1 The Approximating Programming - Move Limit - Method

A complete relinearisation is applied at each design iteration and move limits are imposed on the variables which do not allow them to move very far. This method proceeds as follows:

(1) The objective function and the constraints are linearised at any arbitrary point $\{x_j\}_o$, as in equations (1.7) and (1.8). Then, additional constraints are imposed by applying move limits, such as:

$$(1 - ML) \cdot \{x_j\}_o \leqq \{x_j\}_1 \leqq (1 + ML) \cdot \{x_j\}_o \tag{1.11}$$

where ML is the move limit.

(2) By solving the linear problem, the resulting unknowns $\{x_j\}_1$ are taken as the optimum solution. The process is repeated until convergence is obtained.

The approximating programming method has several properties which made its application in structural design popular. Some of these properties are:

(a) The constraints do not have to be convex.

(b) There is no restriction on the initial design point which can be feasible or infeasible.

(c) The number of constraints which are linearised at each iteration does not increase as is the case with the cutting plane method.

(d) In some problems the solution may oscillate. This is overcome by terminating the procedure with the predetermined value of the move limit, i.e. the value just before the current one.

The move limit is a positive constant factor less than one, e.g. ML = 0.6 . It can be arrangd arbitrarily, but usually it can be based on a certain preselected percentage of the current values of the design variables. Saka (1975) showed that, in the optimum design of rigidly connected frames, it is only necessary to impose move limits on the main design variables. These were the section areas of the members in a frame. He also suggested that the convenient value of the move limit can be chosen as ML = 0.9 and then reduced by 0.1 at each design iteration. Although this arrangement may require a great number of iterations, it provides large move limits during the first iterations and small move limits during the last ones. Thus:

(1) If the initial design point is chosen far from the true optimum, then it is necessary to employ large move limits in order to reduce the number of iterations to reach the optimum point.

(2) Tight move limits are required to achieve convergence in case the optimum point is not fully obtained. In case the convergence is not achieved when the value of ML becomes 0.1, then the design cycles are continued with this particular value of the move limits until convergence is achieved.

Another point which also should be taken into consideration, is that the bounds of move limits should not be less than the lower, or exceed the upper, practical design bounds specified by the engineer or by the code of practice. Otherwise, the practical bounds are emphasised to be used for the optimum design.

In the design of complete structures for minimum cost, (Chapter 4), it was found that move limits are unnecessary because the degree of the non-linearity is not high. Because of this, the use of lower and upper practical bounds on the main design variables is found sufficient.

1.6 SWAY FRAMES

A rigidly jointed sway frame is defined as one that resists lateral deflections in its own plane through the bending stiffness of its members. Such a frame has no other bracing to limit these deflections. Consequently, the combined vertical and wind load action is the determining factor which causes failure by overall lateral sway instability. The design of these frames is therefore difficult by any theory. This is particularly so because, unless limited, the deflections can become the only cause of failure.

Several methods have been used in an attempt to design these frames so that the strength and the stability requirements are satisfied. For instance, Horne and Majid (1966) and Majid and Anderson (1968b) used the elastic-plastic theory to ensure that the successive effects of gradual plasticity and instability do not lead to a premature failure. Having succeeded in fulfilling this aim, it was discovered that the elastic sway deflections of the resulting frames were unacceptable from various constructional points of view. It was thus established that the sway deflections, not strength or instability, dominated the design of these frames.

It follows that, if the sway criterion is satisfied first, the sections selected will then be large enough to satisfy instability and strength criteria. A final analysis will either confirm this claim or reveal the necessity of some minor alterations in the member sizes.

1.6.1 Design Methods of Sway Frames

Most of these design methods utilise the matrix displacement method for structural analysis. In this method, the unknown joint displacements represented by vector $\underline{X}$, are obtained by solving the matrix equation:

$$\underline{K}\ \underline{X} = \underline{L} \tag{1.12}$$

where $\underline{L}$ is the external load vector, and $\underline{K}$ is the overall stiffness matrix. Member forces are then calculated using the joint displacements.

The tendency for deflection to dominate the design is necessarily increased by the introduction of more refined design methods, and the introduction of higher-strength building material. High-yield steel provides the frame with more strength than mild steel, and with less expense. Needham (1977) has demonstrated that, if buckling or deflection are not the design criteria, then Grade 50 steel is more economical to use in multi-storey structures than Grade 43 steel. However, Okdeh (1980) claimed that the deflection becomes more critical when a higher grade steel is used.

In the design of concrete elements, when the permissible stress design was being used with traditional conservative design methods, problems due to excessive deflection were practically unknown. However, cracking in service has recently become more common.

Up to now few methods exist for the design of sway frames subject to deflection limitations. One of these methods is due to Stevens (1964) who suggested that the real basis for the design of sway frames should be the prevention of unacceptable deformations under the working loads. A design to satisfy such a criterion was obtained by selecting curvature pattern that would produce specified deformations, and was compatible with a bending moment distribution in equilibrium with the external loads. Sections were then selected by using moment-curvature charts.

Later Moy (1974 and 1976) proposed a design method used for sway frames in which an initial design with adequate strength but inadequate stiffness was corrected to satisfy permissible limits on horizontal deflections. The frame was divided into sub-assemblages on the basis of the portal method. In this method points of inflection were considered to develop at the mid-lengths of the members. Expression for the storey stiffnesses were obtained in terms of the member second moments of area of the storey, and in terms of the permissible sway deflections at working load. Two assumptions were made:

(1) The vertical loads were assumed to have a negligible effect on horizontal deflection.

(2) A point of contraflexure was taken to exist under horizontal loading at the mid-height of each column (except in the bottom storey), and at the mid-span of each beam. Thus, the frame was made statically determinate above the bottom storey, and each storey was considered in isolation.

The method is based on hand calculation, but it requires a preliminary design and repeated calculation to arrive at a feasible solution.

Anderson and Islam (1979), and Islam (1978) suggested a method for the design of multi-storey frames to sway deflection limitations. The assumptions used are those adopted by Moy, stated previously. Expressions relating the sway deflection over a storey height to the second moments of area of the corresponding columns and the surrounding beams were derived. These expressions were linked to a cost function and used to obtain an economical solution. Restrictions on section size and change of section, and the lack of continuous range of available sections were taken into consideration. Extra expressions were derived to deal with boundary regions of the frame, such as the top-most storey, the bottom storey and the external columns. However, they suggested that a quick design could be obtained if the design of the top storey was considered as the storey below it, and the design of the bottom storey was taken as the one above it. Furthermore, as the sway in each storey was assumed to be equal to a specified value, Anderson and Islam took it as being equal to the maximum allowable deflection of h/300, where h is the height of the storey under consideration. However, such ratio is difficult to be maintained for the ground floor columns, because these are connected to the foundations and therefore deflect less than the others.

The methods of Moy and Anderson and Islam are approximate, as hinges were assumed in the mid-length of each member to avoid designing the frame in its entirety. Majid and Okdeh (1982) and Okdeh (1980) proposed a direct design method for multi-storey plane frames subjected to sway limitations. The stiffness equations were modified so that the sway in each storey became equal to some specified values. The method starts by selecting initial values for the beam sections and the sway limitations. The modified stiffness equations are then solved by an iteration technique to calculate the cross-sectional properties of the columns as well as the other joint displacements. After selecting the initial beam sections, the method alters them in an attempt to reduce the total material cost of the frame. In this design, stability functions were used to include the effect of axial loads in the members. The final design of reduced cost was checked for strength requirements and the members were altered accordingly. The method was extended to design reinforced concrete frames in which the sway in the columns played an active part in the design criteria. Computer economy was achieved by avoiding the solution of the stiffness equations simultaneously.

1.6.2 Optimisation of Fixed Shape Frames

In the elastic design of structures, the design criteria which are commonly used are that the stresses in the members and the deflections at the joints should not exceed certain specified permissible values. It then becomes necessary to express the deflections and the stresses in the structure in terms of the design variables. Using the matrix displacement method, the optimum design problems becomes one of finding the sectional properties of the members so that three main constraints are satisfied. These are the stiffness equalities, the deflection and the stress inequalities.

Moses (1964) was one of the first researchers who applied the linearisation technique to the structural design problem. This was done by transforming the non-linear problem into a sequence of linear programming problems. The optimal design of a three bar truss and a one storey rigid frame, each subjected to two distinct load conditions, was obtained using the cutting plane method. Substantial savings were achieved in the weight of the structure with only a single iteration of linearisation.

Cornell, et al (1966a) formulated the design problem by the matrix displacement method. The sequential linear programming method was used. Stress and deflection constraints were both considered. Various convergence aids have been employed, such as move limits, constraint accumulation and second order corrections. The application of each of these has been done in several ways. Adoptive move limits were utilised to prevent oscillation. The method of constraint accumulation was found successful when the problem was strictly convex. It was stated that the best compromise of all would always remain dependent on the type of problem. In their later work (1966b) detailed explanation and comparison between a structural optimisation and an iterative design was given. The iterative design imposed no limitation on displacements and assumed that the best structure is a fully stress one. They have also shown that the use of reciprocal areas as a design variable reduces linearisation errors because the stresses in the members are linearly related to their reciprocal areas.

Toakley (1968) used the piecewise linearisation technique to solve the minimum weight design problem of statically determinate pin-jointed frames subjected to deflection and stress limitations. The unit load method was employed to formulate the design problem. The reciprocals of the member areas were introduced as the design variables. As a result the deflection constraints were in linear form and the objective function was non-linear but strictly convex. Hence this procedure gave the globally optimum solution.

Majid and Anderson (1972) used the matrix force method to formulate the design problem of statically indeterminate elastic structure subject to non-linear deflection and stress constraints. The piecewise linearisation technique was employed in conjunction with the simplex method. Due to the fact that the members in pin-jointed structures were subject to axial forces, the design variables considered were only the areas of the members and the axial forces in the redundants. In sway frames, the design variables considered were the second moments of area of the members but the axial deformations were neglected. The design problem turned out to be non-convex and gave local optima only. Furthermore, it was found that this procedure was only applicable to small bare frames. The reason for this was that piecewise linearisation required considerable computer storage and time to obtain an optimum solution.

Later, Majid (1974a) proposed a linear elastic, minimum weight method. A non-linear programming algorithm was adopted in which either the force or the displacement method was employed to formulate the problem which was then linearised by Taylor's series. The optimum solution was obtained using the simplex method. This method imposed an upper bound limitation on the absolute values of the joint deflections. It also imposed limit-state stress constraints obtained by combining the axial and the bending stresses in each member of the structure. The constraints take the form:

$$\pm P_i / A_i \pm M_i / Z_i \leq \sigma_t \text{ or } \sigma_c \qquad (1.13)$$

where P_i is the axial force, M_i is the bending moment at the end of member i, A_i and Z_i are the area and the section modulus of member i, σ_t and σ_c are the permissible design stress in tension and compression. Majid examined all the possible combinations of stresses in a member, and revealed that, for a section with two axes of symmetry, there are in fact eight stress constraints; four at each end.

The structural design problem could be enlarged unnecessarily if the non-linear inequalities in equation (1.13) are included in the design exercise. This is particularly so because most of these constraints are inactive, and not involved in deciding the properties of the members of a sway frame.

Saka (1975) used the above method to present a general computer program for the automatic optimum design of rigidly jointed steel plane frames. He employed the approximating programming-move limit-method for the minimum weight design of structures. The matrix displacement method was used to formulate the design problem. He found that the choice of the sections was governed by deflection requirements and not by the stress criteria given by equation (1.13). Saka also arranged the move limits so that the number of design iterations required to obtain the final solution was kept to a minimum. Although this method achieved a design in which all the design criteria were satisfied, it was not applicable to large frames, as it was essentially a procedure that required extensive storage space and consumed a considerable amount of computer time. This method will be extended in this thesis to cover the design of multi-storey steel sway frames, and it will be further discussed in Chapter 2.

A comprehensive review of developments in frame optimisation, including the application of the plastic theory or different mathematical methods of optimisation, is given by Sheu and Prager (1968), Majid (1973), Gallaghar and Zienkiewicz (1974), Saka (1975), Krishnamoorthy and Mosi (1979), Kirsch (1981) and others.

1.6.3 Shape Optimisation of Frames

The shape of a structure, both its geometry and topology, is of major significance. However, its determination is difficult, involving all the design requirements. In an optimum geometrical design, the joint coordinates are treated as independent design variables. Lengths and direction cosines of the members are expressed in terms of these variables. In general, the geometrical design procedure varies the joint coordinates, thus altering the lengths and the directions of the members. The design requirements are represented mainly by stress and buckling constraints, as in the case of trusses, but complications are encountered in the design of rigidly jointed frames, (Pedersen, 1973).

In an optimum topological design, the number and the position of the members are considered as the design variables. The aim of such a design is to reduce the number of the members, to minimise the objective function. The members to be removed are selected by the adopted design method which considers structural and economical factors for such a selection.

Dorn, Gomory and Greenberg (1964) were probably the first who introduced a method of shape optimisation which made use of the concept of "ground structures". The design space was covered by a set of admissible joints from which the joints of the final design were selected. The ground structure was obtained by linking each admissible joint to others in the design space. The minimum weight design of this ground structure was formulated as:

$$\text{Min. } W = \frac{\gamma}{\sigma} \sum_{i=1}^{m} | P_i | L_i \tag{1.14}$$

subject to:

$$\sum_{i=1}^{m} a_{ij} P_i = F_j \tag{1.15}$$

$$j = 1 \ldots n$$

where n is the number of admissible joints, m is the number of admissible bars, P_i is the force in bar i; this force was taken as a design variable, γ and σ denote the density and the yield stress of the given material respectively, L_i is the length of the bar, a_{ij} is its direction cosines, and F_j is the component of the external force at joint j. This linear programming problem was solved for P_i and bars with zero forces were removed. The removal of these members and the unloaded joints made it possible to obtain a structure which had a new topology as well as a new geometry. Included in the examples were planar trusses under one loading condition, which were optimised and found to be statically determinate and therefore fully stressed.

Dobbs and Felton (1969) extended this approach to deal with multiple loading conditions. This made the design problem non-linear and the steepest descent alternate algorithm was utilised for its solution. They also made the approach iterative so that the process might be repeated until no further topological changes were possible. The method was proved successful and promising. However, it covered only stress constraints and gave no justification for the deletion of the members.

Lipson and Agrawall (1974) optimised the shape (topology as well as geometry) of indeterminate trusses subject to multiple load conditions. The independent variables, which were taken as the joint coordinates and the sectional areas, were selected from discrete member spectrum. In the examples solved, only the stress constraints were considered. During the design process members which had zero areas and joints which had zero coordinates were deleted and the relevant stress constraints were omitted automatically. The examples illustrated showed that a non-convex feasible space only increased the number of iterations but presented no difficulty.

Majid and Elliott (1973a) stated the theorems of structural variation which made it possible to predict exactly how the forces and the deflections throughout the structure changed when some of its members were either altered or totally removed. Later (1973b) this was used in conjunction with the topological design of pin jointed structures. The matrix displacement method was used to formulate the design problem where stress and deflection requirements were considered. The problem turned out to be one of non-convex, non-linear programming. The feasible direction method was used to obtain its local optima. A ground structure was initially developed and then the members were removed until no further topological changes were possible. The self weight of the members were also included as design variables and it was found that this changed the shape of the final design and speeded up the search for the optimum shape.

Majid and Saka (1977) and Saka (1975) extended the above theorems to cover rigidly jointed frames. The approximating programming-move limit-method was used to obtain an optimum set of sections for the structure. The theorems were then used to remove members from the ground structure until the final shape, with minimum weight or cost, of the frame was obtained. Both stress and deflection limitations were considered.

Recently, Majid, Stojanovski and Saka (1980) proposed a method for the minimum cost design of rigidly jointed steel sway frames in which the final topology was determined by some economical and architectural requirements. Appropriate "differential" deflection constraints were imposed to control the sway in each storey. The stress constraints, equation (1.13) were excluded. The aim was to minimise the total cost, including material and any other handling (or construction) costs. The latter was expressed as a "fixed charge" to penalise a member retained in the final topology, (Hadley, 1964). The problem turned out to be that of non-linear programming with mixed integer-continuous variables. Suggestions were made to simplify the mathematical approach and to reduce the computation. Examples were given to demonstrate the flexibility of the method in varying the topology of the structure while minimising the cost. However, this method had some deficiencies which will be dealt with in part of the work presented in this thesis. The main deficiencies were (a) the method was only applied on small to medium frames where the sway does not play its full part, (b) the fixed charge on the member was assumed rather than assessed and this might have affected the member removal. Other improvements on this method will be discussed in Chapters 2 and 3 of this thesis.

1.7 COMPLETE STRUCTURES

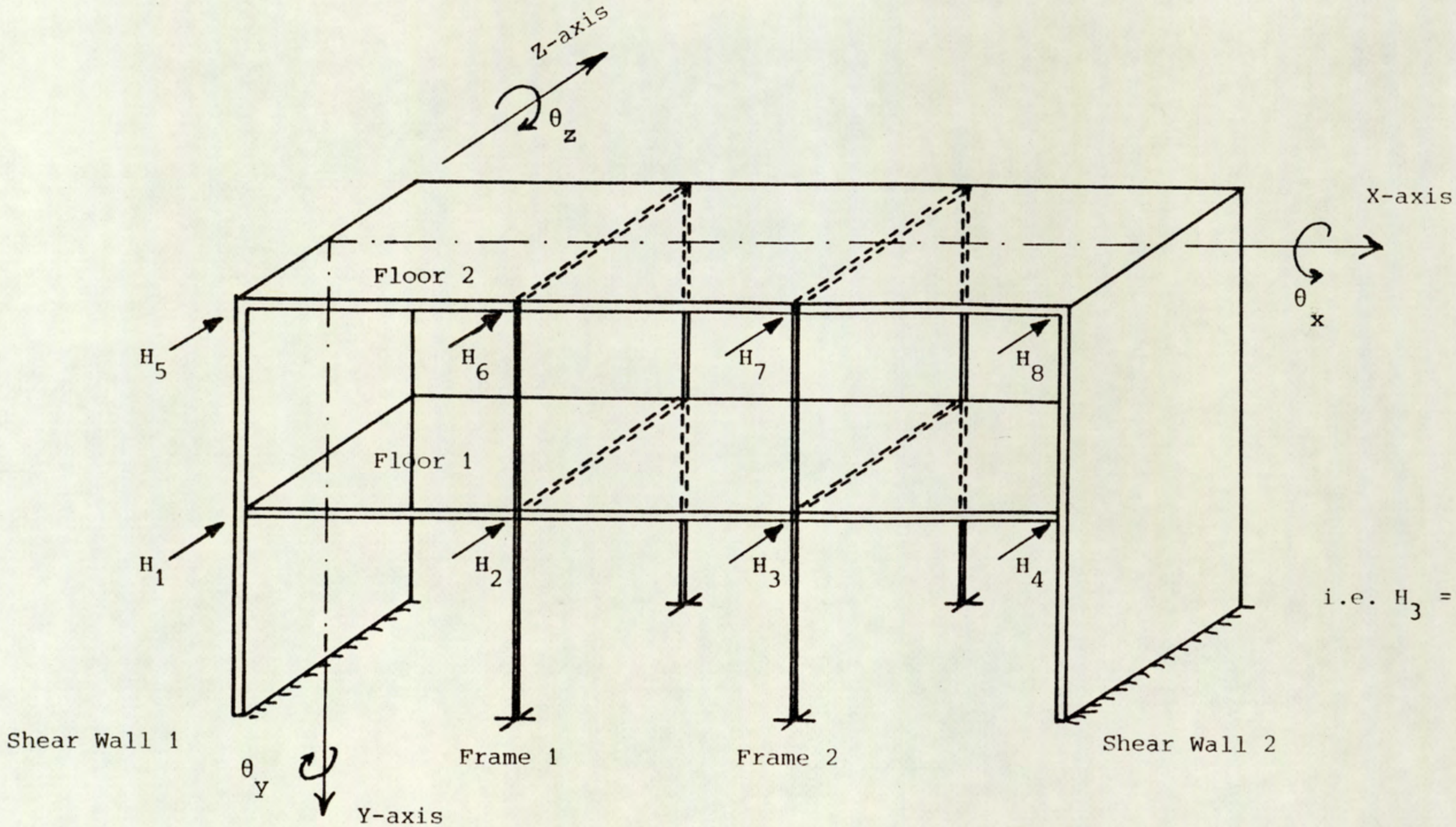
A complete building structure consists of a number of parallel shear walls and intermediate frames connected by floor slabs. The horizontal components (floor slabs) serve not only to collect and distribute the lateral forces to the walls and frames, but by structural interaction with them, increase the lateral stiffness of the building.

In the field of analysing the behaviour of complete structures, significant progress has been made since the early 1960's, mainly because of matrix and computer techniques. These facilitated the development of general analysis methods applicable to quite complex problems. The problem has been the object of considerable research during the last two decades and a number of reviews have been published, (Coull and Stafford, 1967, 1973), (Fintel, et al, 1971) and others.

Optimisation of complete structures has not had the necessary attention that it requires. However, optimising individual elements like shear walls, floor slabs, columns, beams and frames alone have been extensively investigated. Therefore, more research needs to be directed towards optimising structures, created from the interaction of such elements, and their overall behaviour. The review given below is mainly on the methods of analysing complete structures of the type considered in this thesis, Figure 1.1.

Theoretically, the finite element method, such as that suggested by Majid and Williamson (1967), was considered as a good tool to investigate the overall behaviour of a complete structure of any shape. However, this method is computationally expensive. To reduce the cost, some simplified methods have also been proposed for various specific types of structures.

Clough, King and Wilson (1964) used the wide-column frame analogy for structures consisting of skeletal frames and wall-frame systems. In the wide-column frame analogy, the finite width of the wall is simulated by connecting the joints on the centre line of the wall to the ends of the beams by rigid arms which rotate with the joints. However, this precludes the determination of stresses in these regions. This method ignored the in-plane bending of the slabs and the overall rotational stiffness of a structure. The stiffness matrix of each frame was reduced to a condensed form to cater for lateral displacements only. These condensed stiffness matrices were then superposed to form the overall stiffness matrix of the structure. The load vector consisted of the applied wind forces and the lateral equivalent of the vertical and rotational forces. Having calculated the common lateral displacements, the vertical and the rotational displacements of the frames were obtained by back substitution. An



i.e. H_3 = Static Concentrated Wind Load on Junction 3.

FIGURE 1.1: A LATERALLY LOADED COMPLETE STRUCTURE

analysis of a 20 storey structure showed that neglecting axial deformations led to errors of about 20 per cent in some of the columns. Because the overall rotation of the structure was ignored, this method was only applicable to symmetrical structures under symmetrical loads.

Goldberg (1966) proposed a method for the analysis of a type of complete structure consisting of parallel shear walls and frames. The floor slabs were assumed to behave as deep beams subject to shear distortion as well as in-plane flexture. Out of-plane bending was ignored. Axial deformations in the beams and columns of the frames were neglected and no provision was made for the inclusion of wide column effects. The equilibrium of the lateral forces at the slab-frame and the slab-shear walls was assumed, but moment equilibrium between the vertical bracing components and the slabs was ignored. The latter assumption can be simulated as the slabs pinned to the shear walls and the frames. This is a reasonable assumption as it has been shown that the torsional stiffness of walls and frames about their own vertical axes is relatively unimportant. Two symmetrical ten storey and 20 storey structures, with side walls and seven intermediate frames, were analysed by Goldberg. These showed that the bending of the slabs had an insignificant effect on the lateral deflections. It was found that shear in the bottom storey of the centre frame was about 50 per cent greater than that of the outer frame, while at the top the shear was almost the same. The effect of shear distortions in the walls and the slabs was shown to be significant and had to be taken into consideration.

Majid and Williamson (1967) used a sparse matrix method (Jennings and Majid, 1966) to develop a finite element analysis for structures consisting of prismatic members and plate elements subject to in-plane and out of-plane forces. The effects of bending and torsion were taken into consideration. A series of experiments were carried out on two and three storey frames with shear wall cladding to study the effect of shear walls on the stiffness of bare frames.

Majid and Croxton (1970) developed a method for the elastic analysis of complete structures consisting of a grillage of solid walls and floor slabs, stiffened against the horizontal displacements by the action of parallel frames, Figure 1.1. The structures were subjected to the effect of static wind forces and imposed vertical loads. The effects on the sidesways resulting from the action of eccentric vertical loads or non-symmetrical configuration of frames and walls were considered. The grillage and the frames were analysed separately under the action of a system of unit horizontal forces, and their influence coefficients were determined using the matrix displacement method. These components were then reassembled and, by using the matrix force method, horizontal equilibrium and compatibility conditions were satisfied at the slab-frame junctions. The parts of the horizontal forces transmitted to the slabs of the grillage and to the frames were thus calculated. By using the matrix displacement method again, each frame and the grillage were then analysed under their own share of the loads, and the forces and the deflections of each sub-structure were determined. In this approach the shear walls and the slabs were assumed to be deep beams under the effect of in-plane shear and in-plane bending. The stiffness coefficients of such beams were modified to take into account the effect of shear distortion. Each grillage joint was assumed to have three degrees of freedom. These were the sway in the wind direction and rotations about the vertical and the horizontal axes normal to the direction of the wind. The results of the investigation of the effects of various factors on the behaviour of a ten storey structure was reported. One purpose of the investigation was to study the variation of the bending moments and the deflections due to treating the slabs as rigid diaphragms. This was found to be leading to ill conditioned equations for large structures.

Croxton (1974) later modified this method and considered the complete structure as a grillage, laterally restrained by the frames. Matrix displacement method was used to determine the lateral stiffness of the individual frames for partitioning and condensation of their overall stiffness matrices. The stiffness matrix of the complete structure was then formed by superimposing the lateral frame stiffnesses on the stiffness matrix of the grillage. The stiffness matrices, of the shear walls and the floor slabs, produced by this method will be used for optimising the complete structures in this thesis.

Majid and Onen (1973), Onen (1973) and Majid (1974b) developed the approach of Majid and Croxton to analyse complete structures up to and including failure. The elastic-plastic analysis method of Majid and Anderson (1968b) was used in the individual analysis of each steel frame. The grillage system was assumed to be sufficiently strong to maintain its initial stiffness throughout the loading procedure. The effect of composite action between the floors and the beams of the frames was considered.

Majid and Celik (1979) and Celik (1977) continued the above method of analysis to include structures which may be made out of reinforced concrete or steel frames together with shear walls and slabs made out of reinforced concrete or any homogenous material. Lateral buckling prior to failure and cracking of panels were taken into consideration. Plastic hinges in the steel frames and critical stiffness changes in the reinforced concrete frames were also included. A method was given for calculating the failure load of reinforced concrete panels under the combined effects of bending shear and torsion.

1.8 HORIZONTAL GRILLAGE STRUCTURES

A horizontal grillage is identified as a class of structure in which the members all lie in one plane, but the loads are applied normal to this plane. This type of structure is frequently used in roofs, floors, foundations, bridges, and in ship building. The members of the grillage are usually beams spanned in two orthogonal directions. The torsional stresses developed by the twisting action help transfer an additional part of the load from one direction system to the other and increase the stiffness of the two-way system. This increase in the stiffness is virtually negligible for I-beams, but not entirely unimportant for beams of rectangular cross-section.

The horizontal grillage may either be simply supported or fixed at its ends. It may also be supported by columns connected to the intersection of the beams.

The horizontal grillage with its supporting boundary can be dealt with as a whole, and designed as a complete structure without separating it into individual systems or units.

1.8.1 Analysis of Grillages

Clarkson (1965) discussed different methods to analyse grillages. The elastic method of analysis was advocated and used to ensure that the range of stress due to the load was kept to some fraction of the yield stress, and in that way all noticeable plastic deformation was prevented.

With the availability of computers, the matrix methods for elastic analysis of flat grillages have been widely utilised. These methods could be either the matrix force or the matrix displacement method. The force method requires only one redundant force for each point of intersection between beams, while the displacement method leads to three degrees of freedom (two rotations and one deflection) for each intersection. Thus, the number of numerical operations during the analysis are higher in the displacement method than the force method. However, for more general problems, including skew and non-orthogonal grillage, the torsional stiffness and stepwise-varying cross-sections of members etc, the displacement method would be the obvious choice.

Lazarides (1952) was probably the first one who set up the slope deflection equations for compatibility at the grillage intersection points, and thus obtained the solution for the grillage. Lightfoot and Sawko (1959) demonstrated how a computer can be utilised to set up and solve the stiffness equations to produce a solution that gives the deflected profile, the forces and the moments in each member of the grillage. The effect of torsion in the members was included in the stiffness equations, and this meant that the method was applicable to the analysis of slabs and similar structures. Indeed, Sawko (1965) successfully employed this grillage approach to the analysis of bridge deck slabs. This was done by analysing an equivalent grillage created from dividing the slab into strips in the longitudinal and transverse directions, and an equivalent grillage member was taken in the centre of each strip. The member properties of the equivalent grillage were established. Later Sawko and Willcock (1967) considered the analysis of bridges with box girders of varying sectional properties.

The above approach was then used by Sawko and Mosley (1969) to investigate the lateral distribution of live load in simply supported span of composite trapezoidal box girder bridge decks. The box girder was wide and possessed two shear carrying webs. Two types of idealising the equivalent member properties were investigated. These were (a) a single equivalent member along the box centre line, or (b) equivalent grillage member at each web/slab intersection. The agreement between experimental results and grillage analysis was demonstrated to be remarkably good.

Chang and Pilkey (1971) developed a method for the analysis of grillages under general conditions. The method was exact within the framework of the classical theory of bending and torsion of bars. The technique, which was applied on ship structures, accepted arbitrary loading, nonprismatic beams, arbitrary spacing, inspan conditions such as support and releases, and arbitrary boundary conditions. The special features in ship structures such as deck openings, stanchion support, and various brackets were also handled.

Harris (1972) presented a method for static analysis of elastic orthogonal beam grillages. The method was particularly suited to the analysis and design of regular two-way concrete joist floors.

Hambly and Pennells (1975) used the grillage analysis approach in the design of cellular bridge decks. Differences between the physical behaviour of cellular decks and beams were identified, leading to the evaluation of stiffness parameters for an idealised grillage. Some guidance was given on the interpretation of results for design calculations.

Evans and Shanmugam (1979) idealised a continuous plated structure by discrete, skeletal grillage members, and they used the simplified grillage technique for the elastic analysis of multi-cellular structures. The results were then compared with finite element results and proved accurate.

1.8.2 Optimisation of Grillages

The optimisation of grillage structures has also had a good share of research work. Both the elastic and the plastic theories were employed by many researchers to create the features of the grillage design problems. Moses and Onoda (1969) used the matrix displacement method to formulate the minimum weight design of elastic grillages made of straight orthogonal steel beams normally loaded. Section properties of the beams were related by an empirical relationship which expressed the design variables in terms of the areas of each beam. Only stress constraints were considered. The optimisation results were obtained by employing three algorithms. These were the stress-ratio, the cutting plane and the useable-feasible gradient directions. A detailed comparison of these algorithms showed that the cutting plane method required fewer structural analysis cycles for convergence than others. In order to reduce the analysis cycles in the use of the useable-feasible method, a technique was utilised which first found a fully stressed design by the stress ratio and then began moving in the useable-feasible vector direction. It was stated that the stress-ratio method could be useful to find a good initial design point if the constraints were non-convex.

Kavlie and Moe (1969) have described the application of SUMT (sequential unconstrained minimisation technique) to the design of elastic grillages loaded laterally and in-plane. The matrix force method was used to formulate the design problems. Both deflection and stress constraints were considered. A comparison between two different search methods was given. It was found that SUMT could be used for non-convex sets of design variables. It was also shown that the initial design point and initial response factor had decisive influence on the results. It was verified that a fully stressed design may not necessarily correspond to the minimum weight design.

Rozvany and Adidam (1972) proposed a method to obtain the least weight of rectangular and square grillages by assuming that the depths of the beams were limited. Sufficient bracing was provided to avoid instability. The grillages had non-preassigned beam directions. It was shown that the minimum weight of both perfectly plastic and elastic grillages of given strength, as well as elastic grillages of given stiffness, can be determined by considering the moments and displacements of a perfectly plastic-rigid plate having a square locking surface. The beams of the grillages were placed in the direction of the principal moments to give the absolute minimum volume. The method made use of the Prager-Shield theory of optimal plastic design, (Prager and Shield, 1967).

Faulkner, et al (1973) described procedures which were used in a minimum weight grillage synthesis program based on the SUMT. All anticipated ductile failure modes for typical welded ship type deck and bottom grillages under uniform normal and biaxial in-plane loading were considered. Emphasis was specifically placed on compression modes of failure. General instability or grillage collapse was considered. In contrast to the more usual "stress" analysis, emphasis was on "strength" or the inability of the structure to carry further load.

Rozvany (1973) has applied the optimal plastic design for partially preassigned strength distribution to an idealised grillage. The design of this grillage was defined by two variables, and these are the plastic moments. The cost of the grillage was assumed to be proportional to the variables.

Reinschmidt and Norabhoompipat (1975) developed a technique for the preliminary structural design based upon the logic used by practicing designers. A linear structural problem was constructed by considering only the conditions of static equilibrium and stress admissibility, while the elastic compatibility was ignored. The resulting linear programming problem was then solved in the dual form by the revised simplex method. The solution to this problem gave an initial design that was generally close to the final optimum solution. Computer results presented for steel grillage design problems, previously solved by Moses and Onoda (1969), indicated that the proposed method can give the global optimum.

Rozvany, et al (1975) used extensions of Prager's theories of optimal plastic design to optimise grillages and slabs within various geometrical constraints ensuring simplicity and practicality. The two classes of problems considered were, (1) grillages consisting of prismatic beams of preassigned directions and length but variable spacing, and (2) slabs with curtailed negative reinforcement of preassigned length. The optimal grillages and slabs of constrained geometry were obtained, when the positive and negative bending moment capacities were equal and were required to be constant over the length in a given sub-domain. The most economic length of negative reinforcement was calculated for rectangular slabs.

Later, Prager and Rozvany (1977) dealt with the minimum weight design of elastic grillages in which the absolute value of the axial stresses did not exceed a prescribed value. After the optimality conditions for this problem were discussed, a geometrical method for obtaining the optimal beam directions was presented. Furthermore, Rozvany (1979) extended the theory of optimal grillages to include specific cost functions which were dependent on both bending moments and shear forces. A computer algorithm for driving analytically and plotting optimal structural layout for grillages was described by Rozvany and Hill (1979).

1.9 THE SCOPE OF THE PRESENT WORK

The aim of the work presented in this thesis is to obtain structural optimisation methods for three different types of large and practical structures. These are plane multi-storey steel sway frames, complete structures and horizontal grillages. The optimum design problems turn out to be that of non-linear programming. These are formulated in the form of sequential approximating linear programming problems, and then solved by using the two-phase simplex technique. The matrix displacement method is employed.

The methods can be applied to problems of either a minimum weight design, or a minimum cost topological design in which economic and some structural requirements determine the final topology of the structure. For problems of minimum cost topological design, the variables are a mixture of integer and continuous values. This requires some assumptions to be made for simplifying the mathematical approach. Apart from the weight, the total cost of the structure is assessed realistically. This cost includes the material cost, and the construction cost such as formwork, material provision, placing, etc, for the members, plus the costs of constructing the foundations for the columns and the walls.

Chapter 2 contains a method for an optimum design of plane multi-storey steel sway frames. The effect of the sway on optimising tall and slender frames was not considered previously. Therefore, this optimisation method is intended to be for either a minimum weight or a minimum cost topological design that satisfies the stiffness, the sway deflection and the practical constraints. The stress constraints are not considered. The move limits are imposed on the section variables only. The examples solved by this method are given in Chapter 3. For a minimum weight design, an investigation is carried out on the effect of the move limit arrangements, the selection of initial design point, and the design under different loading cases. For a minimum cost design, the effects of realistic cost assessment on the topology of the frames are examined.

The method presented in Chapter 4 is for a minimum cost topological design of laterally loaded complete structures. In this method, the complete structure is treated as a grillage of shear walls and floor slabs which act as deep beams, bending in their own planes, and braced against lateral displacement by the frames. The grillage is built from reinforced concrete, and the frames are made out of either fabricated steel or reinforced concrete. The method alters the topology of the structure by allowing the removal of some of the shear walls and the intermediate frames from the ground structure. This is obtained while the stiffness of the structure is maintained to withstand the action of the static wind loads and to satisfy the lateral sway deflection and the practical constraints. The move limits and the stress constraints are not considered. Chapter 5 contains design examples of the laterally loaded complete structures. Several design cases are investigated and the verification of one of them is given.

Chapter 6 contains a method for an optimum design of a reinforced concrete horizontal grillage structure, with or without the supporting columns. This is considered as a complete structure made out of a network of in-plane straight orthogonal beams, with the supporting columns and the applied loading being normal to this plane. From the review previously given, it was concluded that a method is required to optimise realistic reinforced concrete grillages when both the stress and the deflection limitations are considered at the same time. Therefore, this optimisation method is meant to be for either a minimum weight or a minimum cost topological design that satisfies the stiffness, the stress, the deflection and the practical constraints. The stress constraints include bending stress, and also combined shear and torsional stresses. The topology of the grillage is altered by removing beams selected by the design method as uneconomical or structurally insignificant. The move limits are imposed on the section variables only. The design examples on the flat grillages are given in Chapter 7. The loads are either applied at the intersection of the beams, or uniformly distributed on the whole grillage. The self weight of the grillage is included in the design, and the effect of considering such a weight as a variable is examined.

The procedure of formulating the design problem for each of the three types of structures was computerised. Furthermore, a program was written for solving the design problems by the simplex method. The explanation of the computer programs with the data preparation is given in detail in Chapter 8. All the programs were designed to use the direct access disc backing storage of the computer, so that large structures can be solved by using a small computer core.

CHAPTER 2

OPTIMUM DESIGN OF MULTI-STOREY STEEL SWAY FRAMES

2.1 INTRODUCTION

In this Chapter, a method is proposed for an optimum design of plane, rigidly jointed, multi-storey, steel sway frames. The main feature of the frames designed is that they are large and practical. They therefore demonstrate that the optimisation procedure is generalised to cover not only simple frames, but also those encountered by practical engineers.

Almost all previous optimisation methods either neglected the deflection limit state or imposed an upper bound on the joint deflections of the frames. None of them included the actual sway in each storey as a design criterion. However, Majid, Stojanovski and Saka (1980) used such a criterion in their proposed method for minimum cost topological design of steel frames. In their method, the frames designed were small and short, and consequently the sway in each storey did not play its full part in deciding the sections of the members. The main feature of the frames to be designed in the next Chapter, is that they are tall and slender.

Many attempts have been made to produce economic designs, but the actual cost of the frames has in fact either been excluded from the design objective, or assessed in a complicated or unrealistic way. Here, it is simply pointed out that, apart from the material cost, there are a large number of other construction costs which must be included in the economic objective of the design. These costs will be listed and quantified realistically. A simple way of including the construction costs is by treating them as a "fixed charge" imposed on the inclusion of any member in the final design. The value of this fixed charge varies from one member to another and from one frame to another. It may also be assessed differently by different engineers.

The matrix displacement method is used to formulate the optimum design problems of the frames. The problems are all non-linear and are linearised by means of the approximating programming method, and then solved using the two-phase simplex method. The objective is to minimise either the weight or the cost of the frames while the stiffness, deflection, strength and practical requirements are satisfied. For a minimum weight the main design variables are assumed to be available in a continuous range, while for a minimum cost design the topology of the frame is included as an extra unknown in the process, causing some variables to be of integer and some of continuous nature. Suggestions are made to simplify the mathematical approach. An economical computer program for the optimum design of frames will be described in detail in Chapter 8.

2.2 THE STIFFNESS MATRIX OF A FRAME

The optimum design problems of many elastic structures can be formulated by employing two main matrix methods that are well established. These are the force (flexibility) method and the displacement (stiffness) method. In the force method, the redundant forces are the unknowns and the number of equations to be solved is therefore equal to the number of redundancies. It is not so easily generalised for computer solutions as the displacement method because a degree of intuition is required in the selection of redundancies.

In multi-storey, multi-bay frames, if compatibility of horizontal displacements at storey levels is assumed, the number of redundancies can exceed the number of degrees of freedom; in which case the force method has no advantage. It also leads to ill-conditioned equations unless the redundancies are carefully chosen, (Bray, Croxton and Martin, 1976).

In the matrix displacement method, the member forces (vector $\underline{P}$) are related to its deformations (vector $\underline{U}$). The corresponding relationship for the whole structure is given in terms of the local member coordinates as:

$$\underline{P} = \underline{k}\ \underline{U} \tag{2.1}$$

where $\underline{k}$ is a two-dimensional matrix that represents a diagonal assemblage of member stiffnesses. In general system coordinates, the unknown joint displacements (vector $\underline{X}$) of the structure are related to the unknown external loads (vector $\underline{L}$) by the equation:

$$\underline{L} = \underline{K}\ \underline{X} \tag{2.2}$$

where $\underline{K} = \underline{A}^T \cdot \underline{k} \cdot \underline{A}$, and known as the overall stiffness matrix of the structure; $\underline{A}$ is known as the displacement transformation matrix and $\underline{A}^T$ is its transpose.

For a general frame member connected to joint i at end 1 and joint j at end 2, the stiffness equation (2.2) can be partitioned as:

$$\begin{bmatrix} \underline{L}_i \\ \underline{L}_j \end{bmatrix} = \begin{bmatrix} \underline{K}_{ii} & \underline{K}_{ij} \\ \underline{K}_{ji} & \underline{K}_{jj} \end{bmatrix} \begin{bmatrix} \underline{X}_i \\ \underline{X}_j \end{bmatrix} \tag{2.3}$$

where the submatrices $\underline{K}_{ii}$, $\underline{K}_{ij}$, $\underline{K}_{ji}$ and $\underline{K}_{jj}$ represent the contributions of a single member to the overall stiffness matrix of the frame. These contributions, and the vectors in equation (2.3), are expressed as:

$$
\begin{array}{ccccc}
 & & \begin{array}{cc} \text{Joint } i \qquad\qquad & \qquad\qquad \text{Joint } j \end{array} & & \\
\begin{bmatrix} H_i \\ V_i \\ M_i \\ \\ H_j \\ V_i \\ M_j \end{bmatrix} & = & \left[\begin{array}{ccc|ccc} D & B & -C & -D & -B & -C \\ B & F & -T & -B & -F & -T \\ -C & -T & e & C & T & f \\ \hline -D & -B & C & D & B & C \\ -B & -F & T & B & F & T \\ -C & -T & f & C & T & e \end{array}\right] & \begin{bmatrix} x_i \\ y_i \\ \theta_i \\ \\ x_j \\ y_j \\ \theta_j \end{bmatrix} & \begin{array}{c} \text{Joint } i \\ \\ \\ \\ \\ \text{Joint } j \end{array}
\end{array}
\tag{2.4}
$$

where $D = a\ \ell_P^2 + b\ m_P^2$

$B = (a - b)\ \ell_P\ m_P$

$C = -d\ m_P$

$F = a\ m_P^2 + b\ \ell_P^2$

$T = d\ \ell_P$

$e = 4\ E\ I\ /\ L$

$f = 2\ E\ I\ /\ L$

$a = E\ A\ /\ L$

$b = 12\ E\ I\ /\ L^3$

$d = -6\ E\ I\ /\ L^2$

$\ell_P = (x_j - x_i)\ /\ L$

$m_P = (y_j - y_i)\ /\ L$

in which A is the section area, I is the second moment of area for the section, E is the Young's modulus of elasticity, L is the length of that member, ℓ_P and m_P are the direction cosines for the longitudinal local axis P which is indicated by an arrow pointing to the second end of the member, Figure 2.1. It is noticed here that the axial stiffness

(EA/L) is included in the coefficients of (2.4). However, the stability functions are excluded which means that the effects of axial forces in the members are not taken into consideration.

The symbols H_i, V_i, M_i and H_j, V_j, M_j in equation (2.4) are the external horizontal, vertical and moment loads applied at joints i and j respectively. These loads are vectorially equivalent to the joint displacements x_i, y_i, θ_i and x_j, y_j, θ_j.

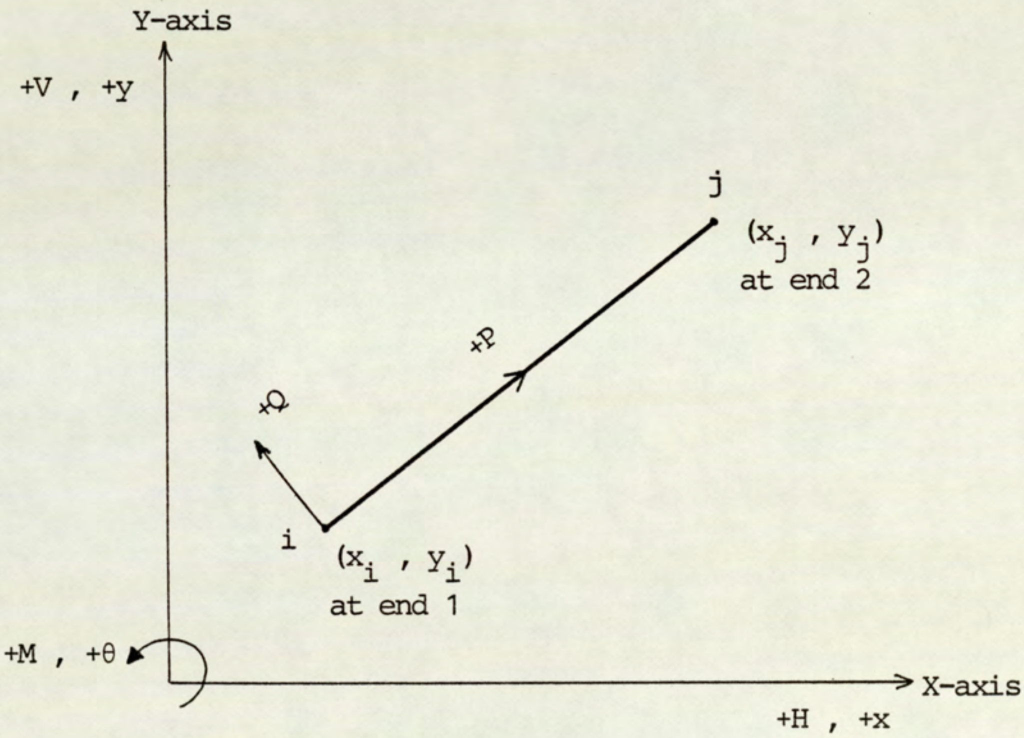


FIGURE 2.1: MEMBER COORDINATES AND POSITIVE SIGN CONVENTION

2.3 DESIGN ASSUMPTIONS

The approach adopted in this thesis to the optimum design of steel frames makes two assumptions. The first is that there is a continuous set of sections available from which to select. This is necessary for the application of the approximating programming method and does not cause serious errors. If a discrete set of sections is required to be used, then the Integer or the Dynamic programming can be employed, (Toakley, 1968) and (Bellman, 1957). However, these methods can complicate the design process and optimality cannot always be guaranteed.

The area (A) and the second moment of area (I) of the section are considered as the main unknown variables in the design of rigid frames. It is preferred to employ only one of these variables to derive the objective function and the constraints. Therefore, a second assumption is made by relating A and I to each other. Although these sectional properties do not have any direct linear relationship with each other, it is possible by fitting a curve to the discrete points to obtain reasonable relationships. Clarkson (1965) gave such relationships for British Standard joists. However, throughout this study the relationship given by Templeman (1971) for the "Universal" Beams are used

$$I = 3.2\,A^{2} \quad \text{i.e.} \quad A = 0.559\,(I)^{1/2} \tag{2.5}$$

This equation is employed only for the beams of the frame. No similar equation can be found for the columns. This is overcome herein by plotting log I against log A for all the "Universal" Columns available, as shown in Figure 2.2, and from this an equation can be found. The nearest linear curve that fits the discrete points in the Figure has the formula:

$$\log I = \phi \,.\, (\log A) + q \tag{2.6}$$

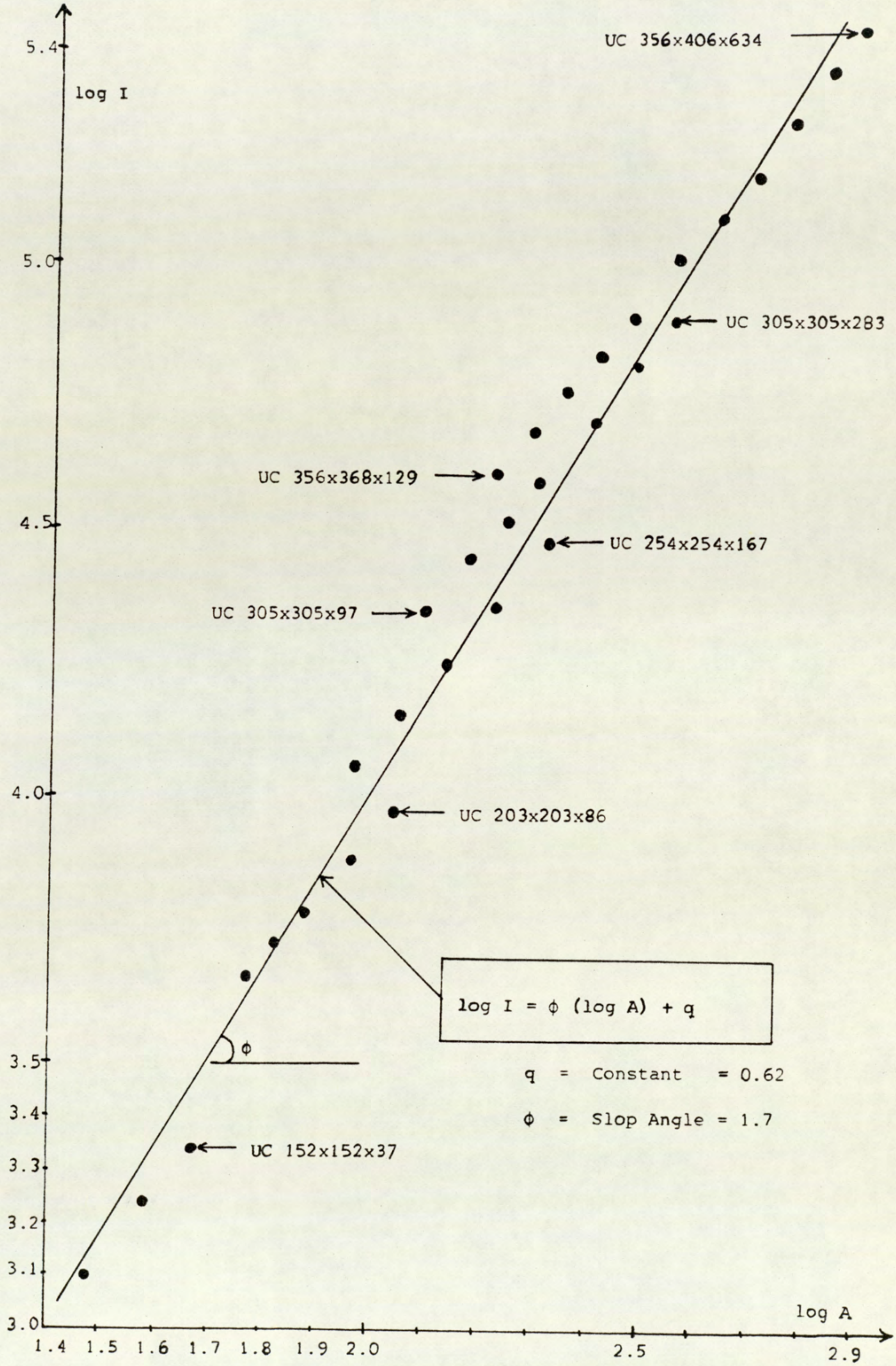


FIGURE 2.2: A GRAPH OF (LOG I) AGAINST (LOG A) PLOTTED FOR ALL THE UNIVERSAL COLUMNS AVAILABLE

with $\phi = 1.7$ and $q = 0.62$. Thus, after simple mathematical calculations, the following relationships between A and I of the columns can be deduced:

$$I = 4.17\,(A)^{1.7} \quad \text{i.e.} \quad A = 0.24\,(I)^{0.59} \tag{2.7}$$

2.4 THE DESIGN PROBLEM

The design problem described here is the optimisation of multi-storey, rigidly jointed, steel sway frames. The problem is divided into two types. The first type is minimisation of the total weight of a fixed shape frame. The second is minimisation of the total cost of the frame in which the number and the position of the columns are considered as variables. The objective function represents either the total weight or the total cost of the frame. The total cost includes that of the material and other handling costs. The latter are treated as a lump sum against each member.

The main design variables are the cross-sectional properties of the members which are determined by the design requirements, not by the designer. Using the matrix displacement method, there are three sets of design requirements which should be satisfied to obtain an optimum design. These are:

(1) The stiffness constraints which ensure that the frame is strong enough to carry the applied external loads.

(2) The sway deflection constraints which keep the relative deflection between the storeys below their specified allowable values.

(3) The practical constraints which ensure that the sections obtained can be supplied from the universal beams or columns.

In the design of rigidly jointed steel sway frames, the deflections are more critical than the strength requirements. This conclusion was confirmed by Saka (1975) who found that the choice of sections was governed by the deflection requirements, not by the stress criteria given by equation (1.13). Therefore, to save computer time and storage, no stress constraints will be considered here as they were proven to be inactive. Furthermore, the design herein assumes lateral support for beams and columns against buckling about minor axes. Therefore, buckling constraints are not considered.

2.5 THE OBJECTIVE FUNCTION

For minimum weight design, the objective function is calculated from:

$$Z_w = \sum_{i=1}^{M} A_i \ L_i \ \gamma_i \tag{2.8}$$

where M is the total number of members, and for member i:

A_i is its section area,
L_i is its length, and
γ_i is its density.

Equation (2.8) allows for variation in density from one member to another. However, for simplicity, γ is assumed constant in the present design problem. Furthermore, the members are often grouped together for practical reasons, thus the objective function (2.8) becomes:

$$Z_w = \sum_{g=1}^{NOG} A_g \ L_g \ \gamma \tag{2.9}$$

where g is a typical group of members, and NOG is the total number of groups.

In order to minimise the total cost, then, for a typical member i, let z_i be the total cost, c_i be the cost per unit weight of the material and R_i be all the other costs involved in constructing member i, such as fabrication, erection, surface treatment, etc. The total cost can then be expressed as:

$$z_i = A_i L_i \gamma_i c_i + R_i \qquad (2.10)$$

In this equation, when $A_i > 0$, z_i exceeds R_i. The value of c_i is allowed to vary from one member to another. In fact, by sub-division and insertion of extra joints along a member, it is possible for c to be varied even along each member. Equation (2.10) can cope with this variation. However, c is considered to have two different constant values, one for the beams and another for the columns (Section 2.11). In general, the total cost of a frame is minimum when:

$$Z_c = \sum_{i=1}^{M} (A_i L_i \gamma c + R_i) \qquad (2.11)$$

is minimum.

The objective weight and cost functions, equations (2.9) and (2.11) respectively, are linearly related to the design variable A_i. Suffix i in R_i indicates that the non-material costs vary from one member to another. A method is proposed for the assessment of R_i, and this will be explained in Section 2.11.

2.6 THE STIFFNESS CONSTRAINTS

It is necessary to select a set of sections for the members of a frame so that it becomes sufficiently stiff to carry the applied loads while the design requirements are satisfied. This necessitates the inclusion of the stiffness constraints which are in the form:

$$\underline{H} = \underline{K}\ \underline{X} - \underline{L} = 0 \tag{2.12}$$

where $\underline{H}$ represents these constraints.

If the sectional properties are known and specified, then the overall stiffness matrix $\underline{K}$ will contain three rows and three columns for each joint of the frame. This matrix can be constructed by finding the members connected to each joint and adding their contributions together at the locations corresponding to that joint. The process is repeated for all the joints in the frame. The matrix produced is symmetric.

However, during the design of a frame, this symmetry is lost because the sectional properties of the members are the design variables. In particular, the section areas of the members are considered to be the main design variables for the programming problem. Since the stiffness matrix contain both the area and the second moment of area of a member, it becomes necessary to separate the elements of expression (2.4).

Saka (1975) has shown that the submatrices $\underline{K}_{ii}$, $\underline{K}_{ij}$, $\underline{K}_{ji}$ and $\underline{K}_{jj}$ can be separated as follows:

$$\underline{K}_{ii} = \begin{array}{c} \begin{array}{ccc} \qquad\qquad x_i \qquad\qquad & \qquad\qquad y_i \qquad\qquad & \theta_i \end{array} \\ \left[\begin{array}{cc|cc|c} D_{11} \cdot A & D_{12} \cdot I & B_{11} \cdot A & B_{12} \cdot I & -C \cdot I \\ B_{11} \cdot A & B_{12} \cdot I & F_{11} \cdot A & F_{12} \cdot I & -T \cdot I \\ 0 & -C \cdot I & 0 & -T \cdot I & e \cdot I \end{array}\right] \end{array} \begin{array}{l} \\ \leftarrow H_i \\ \leftarrow V_i \\ \leftarrow M_i \end{array}$$

$$\underline{K}_{ij} = \begin{array}{c} \begin{array}{ccc} \qquad\qquad x_j \qquad\qquad & \qquad\qquad y_j \qquad\qquad & \theta_j \end{array} \\ \left[\begin{array}{cc|cc|c} -D_{11} \cdot A & -D_{12} \cdot I & -B_{11} \cdot A & -B_{12} \cdot I & -C \cdot I \\ -B_{11} \cdot A & -B_{12} \cdot I & -F_{11} \cdot A & -F_{12} \cdot I & -T \cdot I \\ 0 & C \cdot I & 0 & T \cdot I & f \cdot I \end{array}\right] \end{array} \begin{array}{l} \\ \leftarrow H_i \\ \leftarrow V_i \\ \leftarrow M_i \end{array}$$

$$\underline{K}_{ji} = \begin{array}{c} \begin{array}{ccc} \qquad\qquad x_i \qquad\qquad & \qquad\qquad y_i \qquad\qquad & \theta_i \end{array} \\ \left[\begin{array}{cc|cc|c} -D_{11} \cdot A & -D_{12} \cdot I & -B_{11} \cdot A & -B_{12} \cdot I & C \cdot I \\ -B_{11} \cdot A & -B_{12} \cdot I & -F_{11} \cdot A & -F_{12} \cdot I & T \cdot I \\ 0 & -C \cdot I & 0 & -T \cdot I & f \cdot I \end{array}\right] \end{array} \begin{array}{l} \\ \leftarrow H_j \\ \leftarrow V_j \\ \leftarrow M_j \end{array}$$

$$\underline{K}_{jj} = \begin{array}{c} \begin{array}{ccc} \qquad\qquad x_j \qquad\qquad & \qquad\qquad y_j \qquad\qquad & \theta_j \end{array} \\ \left[\begin{array}{cc|cc|c} D_{11} \cdot A & D_{12} \cdot I & B_{11} \cdot A & B_{12} \cdot I & C \cdot I \\ B_{11} \cdot A & B_{12} \cdot I & F_{11} \cdot A & F_{12} \cdot I & T \cdot I \\ 0 & C \cdot I & 0 & T \cdot I & e \cdot I \end{array}\right] \end{array} \begin{array}{l} \\ \leftarrow H_j \\ \leftarrow V_j \\ \leftarrow M_j \end{array}$$

(2.13)

where

$$D_{11} = E\ \ell_P\ /\ L,\ B_{11} = E\ \ell_P\ m_P\ /\ L,\ F_{11} = E\ m_P^2\ /\ L$$

are the coefficients of the area variables (A), and

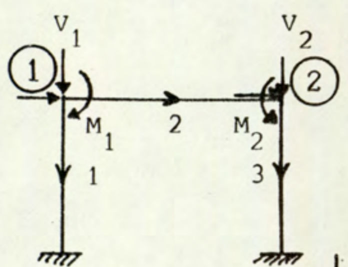
$$D_{12} = 12\ E\ m_P^2\ /\ L^3,\ \ B_{12} = 12\ E\ \ell_P\ m_P^2\ /\ L^3,\ \ C = -6\ E\ m_P\ /\ L^2$$

$$T = -6\ E\ \ell_P\ /\ L^2,\ e = 4\ E\ /\ L,\ f = 2\ E\ /\ L$$

are the coefficients of the second moment of area (I). The relationships (2.5) and (2.7) can now be used to express the second moments of area of the sections in terms of the areas alone. In this way, the first and the third columns of the sub-matrices in (2.13) are linear functions of the member area, while the rest are non-linear. This approach of separating the sub-matrices is adopted in the present design problem.

To construct the design stiffness matrix $\underline{K}$ for the constraints in (2.12), it is necessary to keep the contribution of each member separate. This means that the stiffness matrix will have three rows for each joint but five columns for each member connected to that joint. Generally, for a total of N joints in a frame, the overall stiffness matrix has 3N rows and $5\ (\sum_{j=1}^{N} M_j)$ columns. In other words, the matrix is not symmetrical and its order is $[3N,\ 5\ (\sum_{j=1}^{N} M_j)]$. Here M_j is the total number of members connected to a typical joint j. If, for practical reasons, members are grouped, then M_j will be defined as the total number of different groups connected to joint j.

Using Saka's (1975) approach, Figure 2.3 shows a layout of the overall stiffness matrix for a portal frame of two joints. The order of this matrix is as given above. It is assumed that each member belongs to a single group so that its coefficients have their own columns in the matrix. It is also noticed that member 3 does not contribute to rows of joint 1, and member 1 does not contribute to rows



		Joint 1										Joint 2										
		x_1				y_1				θ_1		x_2				y_2				θ_2		
		Mem. 1		Mem. 2		Mem. 1		Mem. 2		Mem 1	Mem 2	Mem. 2		Mem. 3		Mem. 2		Mem. 3		Mem 2	Mem 3	
Rows of Joint 1	Row 1	D_{11}	D_{12}	D_{11}	D_{12}	B_{11}	B_{12}	B_{11}	B_{12}	-C	-C	$-D_{11}$	$-D_{12}$	0	0	$-B_{11}$	$-B_{12}$	0	0	-C	0	← H_1
	Row 2	B_{11}	B_{12}	B_{11}	B_{12}	F_{11}	F_{12}	F_{11}	F_{12}	-T	-T	$-B_{11}$	$-B_{12}$	0	0	$-F_{11}$	$-F_{12}$	0	0	-T	0	← V_1
	Row 3	0	-C	0	-C	0	-T	0	-T	e	e	0	C	0	0	0	T	0	0	f	0	← M_1
Rows of Joint 2	Row 4	0	0	$-D_{11}$	$-D_{12}$	0	0	$-B_{11}$	$-B_{12}$	0	C	D_{11}	D_{12}	D_{11}	D_{12}	B_{11}	B_{12}	B_{11}	B_{12}	C	-C	← H_2
	Row 5	0	0	$-B_{11}$	$-B_{12}$	0	0	$-F_{11}$	$-F_{12}$	0	T	B_{11}	B_{12}	B_{11}	B_{12}	F_{11}	F_{12}	F_{11}	F_{12}	T	-T	← V_2
	Row 6	0	0	0	-C	0	0	0	-T	0	f	0	C	0	-C	0	T	0	-T	e	e	← M_2
	Section Variables	A_1	I_1	A_2	I_2	A_1	I_1	A_2	I_2	I_1	I_2	A_2	I_2	A_3	I_3	A_2	I_2	A_3	I_3	I_2	I_3	

FIGURE 2.3: THE LAYOUT OF THE OVERALL STIFFNESS COEFFICIENT MATRIX FOR THE PORTAL FRAME

of joint 2. The last line in Figure 2.3 specifies the properties of the members and the manner they contribute to the joints.

2.6.1 Linearisation of the Stiffness Constraints

As shown in the previous Section, the formulation of the stiffness constraints by the matrix displacement method reduces the structural design problem to a non-linear programming problem. The approximating programming method has been found by many research workers to be effective for linearising many problems, (Cornell, et al, 1966), (Saka, 1975) and others.

The vector for the design variables has the form:

$$\underline{V} = \{v_1 \ v_2 \ \ldots \ v_m \ , v_{m+1} \ \ldots \ v_{m+3N}\} \qquad (2.14)$$

where the first m variables represent the areas of the groups and the rest represent the displacements of the joints. Here, N is the number of joints in the frame. Vector $\underline{V}$ can be partitioned in the following manner:

$$\underline{V} = \{\underline{A} \ : \ \underline{X}\} \qquad (2.15)$$

where the sub-matrix $\underline{A} = \{A_1 \ \ldots . \ A_m\}$ contains the areas and $\underline{X} = \{x_1 \ y_1 \ \theta_1 \ \ldots . \ x_N \ y_N \ \theta_N\}$ contains the joint displacements. The stiffness constraints are functions of $\underline{A}$ and $\underline{X}$, and therefore they have the form:

$$\underline{H}\,(A, X) = \underline{K}\,(A) \cdot \underline{X} - \underline{L} = 0 \qquad (2.16)$$

The linearisation by Taylor's series requires the gradient vector

$\underline{\nabla H}$, which consists of the derivatives of a single constraint with respect to each design variable at given values of these variables, such as:

$$\underline{\nabla H} = [\ \frac{\partial H}{\partial A_1}\ \frac{\partial H}{\partial A_2}\ \cdots\ \frac{\partial H}{\partial A_m}\ \frac{\partial H}{\partial x_1}\ \frac{\partial H}{\partial y_1}\ \cdots\ \frac{\partial H}{\partial \theta_N}\ ] \tag{2.17}$$

Thus, the derivatives of the stiffness constraints with respect to the member areas are:

$$\frac{\partial H}{\partial A_j} = \frac{\partial K(A)}{\partial A_j} \tag{2.18}$$

$$j = 1 \ldots m$$

The relationship between the area A, and the second moment of area I is in the form:

$$I = p \cdot A^r \tag{2.19}$$

where p and r are constants given by the relationships (2.5) and (2.7). When the computer is used to carry out the calculations, the derivative of I with respect to A is done in the form:

$$\frac{dI}{dA} = p \cdot r \cdot A^{r-1} \tag{2.20}$$

The derivatives of the stiffness constraints with respect to the displacement variables are:

$$\frac{\partial H}{\partial x_j} = \underline{K}\ (A) \tag{2.21}$$

$$j = 1 \ldots 3N$$

2.7 THE DEFLECTION CONSTRAINTS

In frame optimisation, two types of deflection constraints are commonly specified. These are the sway deflection and the joint deflection constraints, which will be explained in some detail below.

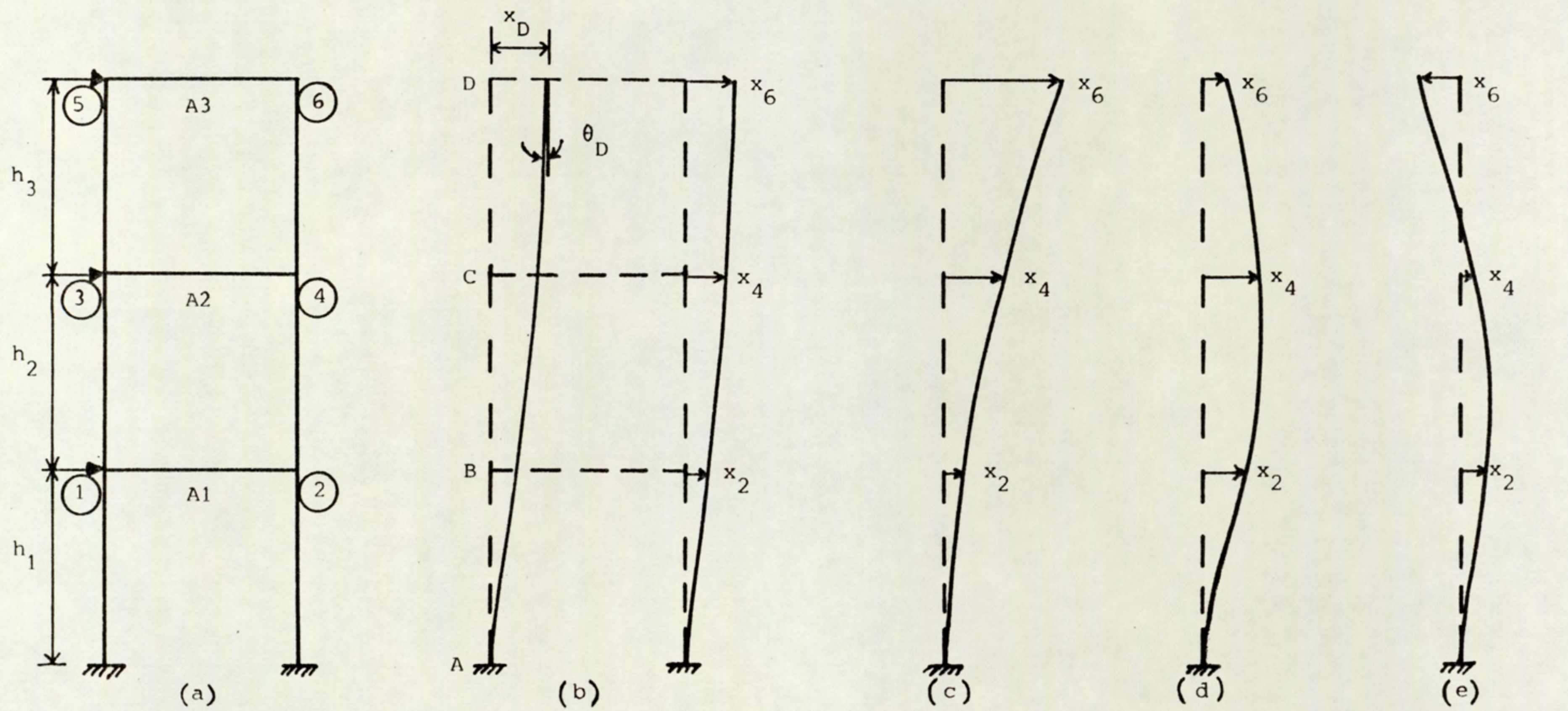
2.7.1 The Sway Deflections

The sway deflection constraints are often confused with those of the joint deflections, (Majid, 1974a) and (Saka, 1975). The BSI B/20 document 77/13908 DC (1977) imposes an upper bound not on the joint deflections but on the actual sway in a member. For instance, in the frame shown in Figure 2.4a, the sway in each storey should not exceed an upper bound of h/ℓ where h is the storey's height, and ℓ is a constant chosen by the designer such as 300, 350, 400 or any other value. In this manner, the sway deflection constraints that cover all the possibilities of the sway deflections for this frame become:

$$
\begin{aligned}
|x_2| &\leqq h_1 / \ell \\
x_4 - x_2 &\leqq h_2 / \ell \\
x_2 - x_4 &\leqq h_2 / \ell \\
x_6 - x_4 &\leqq h_3 / \ell \\
x_4 - x_6 &\leqq h_3 / \ell
\end{aligned}
\tag{2.22}
$$

In the general case, if x_i and x_{i+1} are the horizontal deflections at floors i and i+1 respectively, then the deflection constraints for the connecting columns are:

$$
\begin{aligned}
x_i - x_{i+1} &\leqq h_{i+1} / \ell \\
x_{i+1} - x_i &\leqq h_{i+1} / \ell
\end{aligned}
\tag{2.23}
$$



(a)	Joint numbers.
(b)	A typical joint displacement.
(c,d,e)	Modes of sway deformations.

FIGURE 2.4: HORIZONTAL DEFLECTION OF A FRAME

These two sway constraints are specified for each storey to cover the possibilities shown in Figure 2.4.

It is noticed that the sway requirement for the ground floor columns limits the actual deflection x_2. For each leeward column, above this floor, only the relative deflections are controlled. If the extension of the beams is neglected, sway constraints similar to equation (2.23) need not be specified elsewhere.

Reverse column taper is often avoided by structural engineers through making the section of a column larger than the one it supports. For instance, in Figure 2.4b column AB is made larger than BC which itself is made larger than CD. This requirement is not specified in any code and need not be implemented. However, if required by the engineer, it can be achieved by specifying deflection constraints of the type:

$$| x_4 - x_2 | \leq | x_6 - x_4 |$$

$$| x_2 | \leq | x_4 - x_2 |$$

$$\text{with} \quad A_1 \geq A_2 \geq A_3 \tag{2.24}$$

and in general:

$$| x_i - x_{i-1} | \leq | x_{i+1} - x_i | \tag{2.25}$$

where i is the floor number, and x_{i-1} is the deflection at the bottom of the lower column. These constraints ensure that whatever the magnitude of these deflections, the relative sway in the upper column will not be less than that in the column below. In this manner the upper is prevented from being stiffer than the one below it. The deflection profiles shown in Figure 2.4d and e, can still occur because the upper beams may be larger than required by the sway criterion. To avoid this, the last of the equation (2.24) is introduced, where A_1, A_2 and A_3 are the area of the beams as shown in Figure 2.4a.

2.7.2 The Joint Deflections

Since the joint displacements are introduced as the design variables, the actual value of the deflection x_j at a joint j may be limited by an upper bound U_j. The deflection constraint is then:

$$x_j \leq U_j \tag{2.26}$$

Mathematical programming problems are formulated subject to the condition that all the variables appearing in the solution are non-negative. However, in an actual structure it is difficult to predict the direction of each joint deflection as this is dependent on the unknown sectional properties of the members. To cater for negative deflections, it is necessary to alter inequality (2.26) by substituting:

$$x_j = X_j - e_j \tag{2.27}$$

where X_j is a new non-negative unknown variable and e_j is a constant. When X_j is not in the optimum solution, i.e. $X_j = 0$, equation (2.27) gives $x_j = - e_j$ and thus $- e_j$ is selected as the most negative value that x_j can possibly take. If, in addition, the permissible value of $+ x_j$ is limited to U_j, then the deflection constraint for x_j becomes:

$$x_j = X_j - e_j \leq U_j$$

i.e.

$$X_j \leq U_j + e_j \tag{2.28}$$

$$\text{for } X_j \geq 0$$

Equation (2.27) and inequality (2.28) ensure that $-e_j \leq x_j \leq U_j$. In the case where the most negative and positive values x_j can take are equal, inequality (2.28) becomes:

$$x_j = X_j - U_j \leq U_j$$

i.e.

$$X_j \leq 2\,U_j \tag{2.29}$$

This is the type of deflection constraint which is used herein for the design of frames. Thus, whenever x_j appears in the design formulation, it must be replaced by $X_j - U_j$, and further the deflection constraint (2.26) must be replaced by constraint (2.29).

Each joint of a plane frame introduces three variables to the design problem. These are the horizontal displacement x_j, the vertical displacement y_j and the rotation of that joint θ_j. The bounds on these variables are U_{xj}, U_{yj} and $U_{\theta j}$ respectively, and since the sway deflection constraints are the most significant for the design of multi-storey frames, these bounds are taken considerably large so that they do not play an important part in the design problem. However, there are some exceptions. For instance, BS 449 considers a structure safe when midspan deflection of its beams does not exceed L/360, where L is the length of a beam. In addition to that the sway deflection should be satisfied.

There are many points where the displacements are not restricted by the design requirements. For instance, the rotation, such as θ_D at joint D in Figure 2.4b, is not limited by any design code, except that it should be small. In this case, the value of $U_{\theta j} = 0.08$ radians is convenient, because linear structural theories are only applicable for small deflections with $|\theta| \leqq 0.08$ radians, (Majid, 1974a, p. 174).

In the design of sway frames, the joint displacements are not as significant as the sway deflections. However, these displacements must all be limited by some boundaries, otherwise, the linear programming process reduces them to zeros and excludes them from the solution process.

2.8 THE PRACTICAL CONSTRAINTS

The cross-section of any member should not be less than that dictated by the strength criteria. In a steel sway frame, the beams under dead load and vertical super load should not collapse by a beam-type mechanism, below a load factor $\lambda_1 = 1.75$, according to many codes of practice. The plastic hinge moment of any beam j can be calculated in advance using the simple plastic theory, and a section is assigned to it. The area of this section A_{pj} is then considered to be a lower bound for beam j. The upper bound being given by the largest available section in the "Universal" beams. For a column the upper and the lower bounds are decided by the largest and the smallest available sections in the "Universal" columns.

When the members are grouped together, the group areas are considered as the variables in the programming problem. Hence, it is necessary to impose a limitation of the form:

$$A_{pj} \leqq A_j \leqq A_{bmax} ;$$

$$A_{cmin} \leqq A_i \leqq A_{cmax} \qquad (2.30)$$

$$j = 1, \ldots .\ NOGB \quad i = 1, \ldots .\ NOGC$$

where NOGB is the number of beam groups, NOGC is the number of column groups, i.e. NOG = NOGB + NOGC, A_{pj} is given by the beam mechanism for beam group j, A_{bmax} is the largest beam section available, A_{cmin} and A_{cmax} are the smallest and the largest column sections available.

Through the design operation, the section areas are bounded by applying the move limits (ML) in the following way:

$$(1 - ML) \cdot A_\ell \leqq A_\ell \leqq (1 + ML) \cdot A_\ell \qquad (2.31)$$

$$\ell = 1, \ldots .\ NOG$$

The values of these boundaries should not be less than, and should not exceed, the boundaries described by (2.30), such as:

$$(1 - ML) \cdot A_\ell \geqq A_{p\ell} \quad \ldots . \quad \text{for Beams;}$$

$$(1 - ML) \cdot A_\ell \geqq A_{cmin} \quad \ldots . \quad \text{for Columns;}$$

$$(1 + ML) \cdot A_\ell \leqq A_{bmax} \quad \ldots . \quad \text{for Beams;}$$

$$(1 + ML) \cdot A_\ell \leqq A_{cmax} \quad \ldots . \quad \text{for Columns}$$

$$(2.32)$$

2.9 TOPOLOGICAL DESIGN OF MINIMUM COST

A significant feature of the design problem is that in equation (2.11) the handling cost R_i should be included only when it is economically advantageous to keep member i in the final topology of the frame. If not, then A_i should reduce to zero, as should R_i. Thus, there is a discrete problem of an either/or nature. In mathematical programming this gives rise to integer programming (Hadley, 1964).

The objective cost function (2.11) does not, as yet, cater for the fact that the cost R_i has a value only when member i is retained in the design. It is therefore necessary to define another new variable δ_i to be associated with each member. This must explicitly express two facts:

(1) Each δ_i must be equal to unity when member i is retained in the final design.

(2) Each δ_i must be equal to zero when it is economical to remove member i.

The objective cost function (2.11) is thus altered to become:

$$Z_c = \sum_{i=1}^{M} (A_i \; L_i \; \gamma \; c + R_i \; \delta_i) \tag{2.33}$$

Here for each i, $\delta_i = 0$ if $A_i = 0$, or else when $A_i > 0$, then $\delta_i = 1$. It is to be noticed that when $\delta_i = 0$, $R_i \; \delta_i$ vanishes and the handling cost of member i is saved because the member is excluded.

To guarantee that δ_i can take only the values of zero or one, (Hadley, 1964), a new constraint should be introduced, such as:

$$0 \leqq \delta_i \leqq 1 \tag{2.34}$$

$$i = 1 \;....\; M'$$

where M' is the total number of the members required to be removed. Notice that a value of $\delta_i = 1$ is given for all the members required to be kept. It is also necessary to specify an upper bound A_{imax} on each area A_i. This can simply be selected by the engineer as the largest available section. However, through the design operation, A_{imax} is decided by applying the move limit value on the section area, and this should not exceed the selected largest available section.

The mathematical programming problem now becomes that of minimising the objective cost function (2.33), subject to the design contraints stated earlier in Sections 2.6 and 2.7, and also subject to constraints of the type (2.34) and:

$$A_i - A_{imax}\,\delta_i \leqq 0 \qquad (2.35)$$

δ_i is integer

$i = 1, \ldots M'$

Constraints (2.34) and (2.35) ensure that the area of member i cannot be positive unless $\delta_i = 1$. This is because the only other value δ_i can take is zero and in this case $A_i = 0$, and the member is removed from the final design. If $\delta_i = 1$, then $A_i \leqq A_{imax}$ which is permissible. Furthermore, an optimal solution will not have $\delta_i = 1$ if $A_i = 0$, because the simplex method produces a cheaper design simply by reducing δ_i also to zero. This reduces the total cost specified in formula (2.33).

It was stated earlier that before using the simplex method, it is necessary to linearise the constraints. To encourage the convergence of the original non-linear problem, "move limits" may be needed to narrow the feasible region for each linearised substitute. The design problem presented here aims at altering the topology of the sway frame by removing column members from the original structure. Thus, for members allowed to be removed, there are no lower bounds imposed on their section areas. On the other hand, some members, e.g. the beams and the outer columns, have to be retained in the final topology. To keep these, lower bounds are imposed on their section areas, using move limits. The upper bounds imposed on all the members are also decided by using move limits, as described in Section 2.8.

It is an acceptable engineering opinion that the cost of a structure is reduced by grouping the members together, so that members in one group are manufactured out of the same section. The objective function (2.33) is therefore modified to become:

$$Z_c = \sum_{g=1}^{NOG} (A_g \; L_g \; \gamma \; c \; + \; R_g \; \delta_g) \tag{2.36}$$

Thus, a single δ decides the existence or the removal of a whole group. This saves cost and simplifies the problem.

2.9.1 Computational Economy

In the sort of problem formulated in the previous Section, there are two distinct types of variables. These are the areas, each of which has an associated fixed charge, and other free variables. These two types make the problem that of mixed integer-continuous case (Hadley, 1964). It is possible to use the integer programming algorithm to solve a general problem either for the all integer case or for the mixed integer-continuous variable case.

As it stands the integer programming problem is costly from the computation point of view. Toakley (1968) carried out an investigation into the minimum weight design with discrete sections which also gave rise to integer programming. His conclusion, also confirmed by Hadley (1964), was that integer programming enlarged the problem to the extent that it overshadowed its advantages. To avoid it in the present work, therefore, the following decisions are taken:

(a) At the end of each simplex solution, if any δ_i is reduced to zero, member i is removed from the problem. Thus, the problem itself is reduced before the next round of linearisation.

(b) If any δ_i is very small compared to the others, or it is smaller than a specified tolerance, it indicates that member i is not really significant from the structural engineer's point of view. Therefore, this member can be removed, provided that feasibility can be restored in the next design iteration.

In this manner, no attempt is made to use an integer programming algorithm. The simple steps which will be listed in the design procedure are thus adopted instead. Designing a large number of frames showed that this simplification always gave acceptable results.

2.10 COST ASSESSMENT OF A STEEL FRAME

In this Section, an investigation is carried out into the manner in which the cost of a plane rigidly connected steel frame is assessed. This assessment depends on the costs of the material and on other construction costs.

In a report written by Davis, Belfield and Everest (1980), sufficient information was presented about the rates for labour, materials and measured items of construction which were intended to be an indication of the pricing levels, at that time, for reasonable quantities of work. The hourly labour rate, on which the measured rates of construction have generally been based, was £2. This depended on the authorised rates plus certain allowances. Therefore, according to the report, the cost of a frame can be computed, using Appendix A, as follows:

(I) Material Cost

Market prices for structural steel materials are given below. They include delivery charges; they do not include any allowance for overheads and profit and they exclude any payments in respect of VAT.

- Universal Beams (Average) £225 / ton
- Universal Columns (Average) £235 / ton

(II) Construction Costs

The measured rates, given below, have been prepared on the basis of the labour rates and material prices indicated above or on sub-contractors' quotations. The rates include ten per cent for overhead charges and profit; they exclude any allowance for VAT. The items presented are generally in accordance with the Civil Engineering Standard Method of Measurement issued in 1976 by the Institution of Civil Engineers and the Federation of Civil Engineering Contractors. Some adjustments to this have inevitably been made to meet the needs of a price feature. The items considered are:

(a) Fabrication of members

- Columns for frames £500 / ton
- Beams for frames £470 / ton

(b) Erection of members

- For rigidly connected frames £ 70 / ton

(c) Surface treatment

- Shot blast and one coat primer at work £135 / ton

(d) Construction cost of an independent foundation for each ground column. (This will be discussed in the next Section.)

The main conclusion achieved here is that it is possible to express the construction cost in terms of member area, i.e. per unit weight. Thus, for a typical member i with cross-sectional area A_i and length L_i, the construction cost R_i is expressed as:

$$R_i = A_i \; L_i \; \gamma \; D_i \tag{2.37}$$

where γ is the steel density, and D_i represents the total cost of the construction items mentioned above. For beam i, the numerical value of D_i = £470 + £70 + £135 = £675/ton = £68.8 / KN. The cost of erecting the foundation, which is item (d) of the construction rates, is only included in assessing the cost of the ground columns of a frame.

2.10.1 Construction Cost of an Independent Reinforced Concrete Foundation

The estimate of the total cost of a structure should include the cost of its foundation. This fact has been ignored by many engineers who have tried to obtain minimum cost design. It is well known that for a particular structure there are different types of foundation that could be used, and choosing a suitable one depends on certain factors which were mentioned in many books, such as (Astill and Martin, 1975), and others. However, the type of foundation for all steel frames considered in this thesis is specified as an independent or "pad" foundation. In particular, a reinforced concrete pad foundation is considered where each pad carries one steel column only.

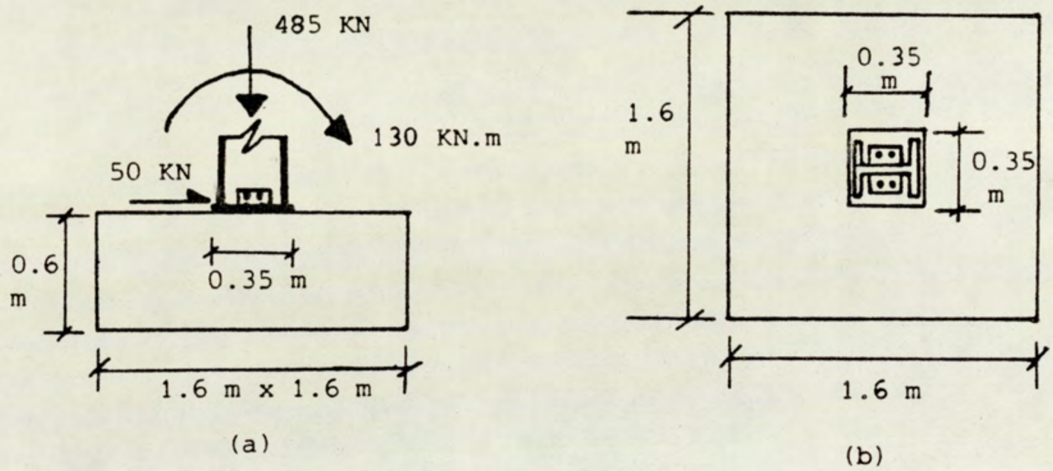
The measured items of construction presented by Davis, Belfield and Everest (1980) are used to assess the cost of these foundations. For the purposes of illustration, a typical simple example of a pad foundation is considered. This was designed to withstand the effect of the imposed loading shown in Figure 2.5a. The design methods used have been described by Allwood, et al, (1972), Allen (1974), Faber and Johnson (1976) and many others. The steel column, which transfers the loads, sits on a square steel slab with dimensions of 0.35 x 0.35 m, and is supported by a reinforced concrete base. The size of the base was determined by the design method used. It would have been possible to choose a different shape for the base but, for simplicity, it was decided to use a square base, as shown in Figure 2.5b. The allowable concrete compression was taken as 6100 KN/m^2, and the soil bearing capacity was considered to be 300 KN/m^2. Grade 25 concrete was used for the base. The reinforcement required was supplied by high yield bars of 16 mm diameter, the total mass of which was 0.11 ton/base.

The items of construction, Figure 2.5c, and the measured rates are all shown in the self explanatory Table 2.1. Amount means the price of an item, and is calculated as:

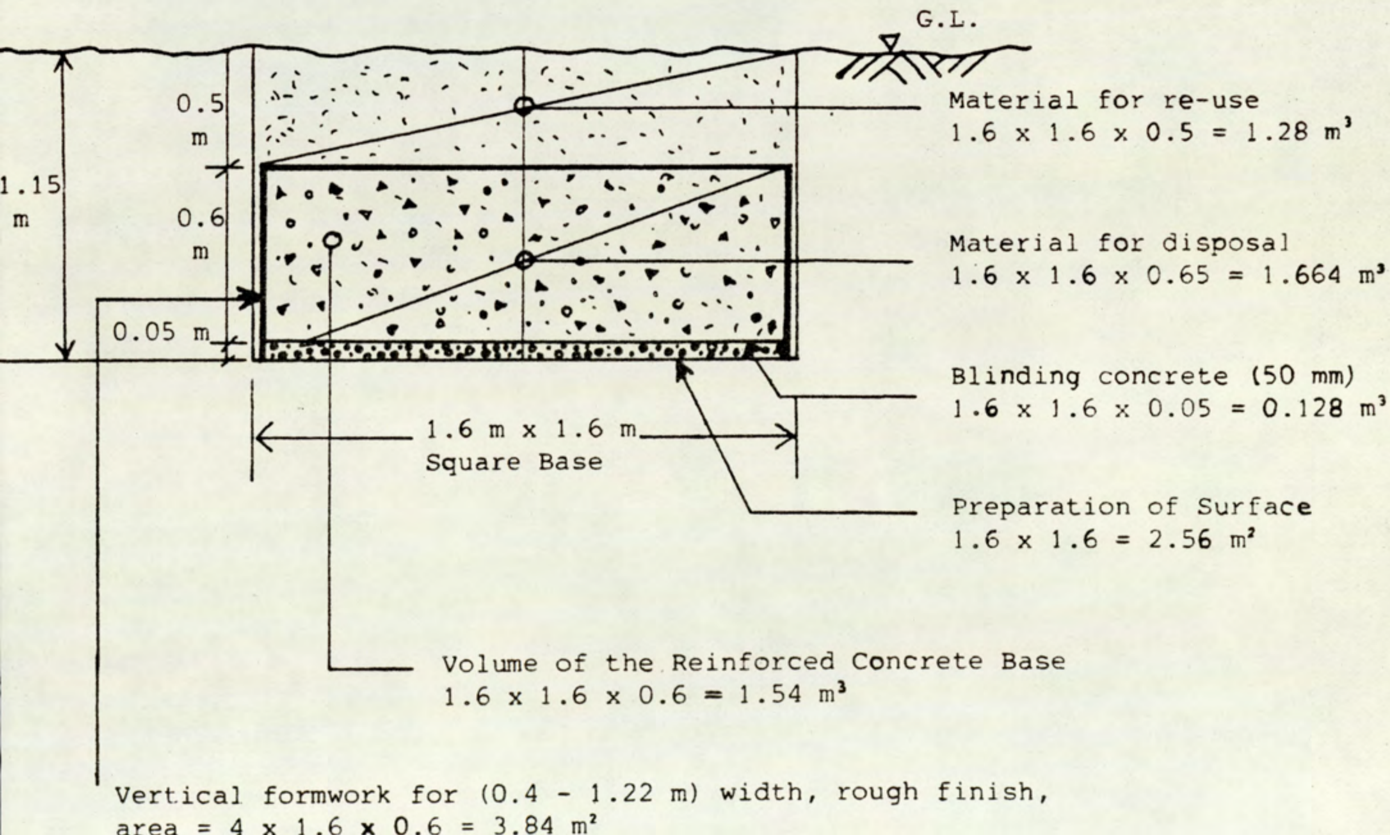
$$\text{Amount} = \text{Quantity of units} \times \text{Rate of one unit} \tag{2.38}$$

The measured rates are used under the same conditions mentioned in the previous Section. The main items of construction are earth work, in-situ concrete work, and concrete ancillaries. The costs of these are summarised at the bottom of Table 2.1, and the total construction cost of the pad footing is estimated to be £175.

The cost of the foundation, as assessed above, excludes the cost of all materials in the permanent work. This is because the material cost is considered to be irrelevant, and does not really effect the overall cost of the frame.



(a), (b) Dimension and loading.



(c) Construction items.

FIGURE 2.5: AN INDEPENDENT CONCRETE FOUNDATION FOR A STEEL COLUMN

Number	Item Description	Unit	Quantity	Rate £'s	Amount £'s
(1)	Earth Work: (In firm soil)				
	(a) Excavation, Maximum Depth (1 - 2 m).				
	- Material for re-use	m^3	1.28	2.88	3.69
	- Material for disposal	m^3	1.664	4.08	6.79
	(b) Excavation Ancillaries.				
	- Preparation of surface for natural materials	m^2	2.56	0.11	0.28
	(c) Filling and Compaction.				
	- To structures selected excavated material	m^3	1.28	0.65	0.83
	Total Earth Work Cost				11.59
(2)	In-Situ Concrete Work:				
	(a) Provision of concrete.				
	- Design mixture Grade 10 cement to BS4027, Sulphate resisting, 20 mm aggregate, for blinding the bottom of the base	m^3	0.128	28.41	3.64
	- Design mixture Grade 25 cement for the base	m^3	1.54	33.11	51.00
	(b) Placing of mass concrete.				
	- Blinding thickness not exceeding 150 mm	m^3	0.128	5.72	0.73
	(c) Placing of reinforced concrete.				
	- Bases thickness over 500 mm	m^3	1.54	4.00	6.16
	Total In-Situ Work Cost				61.53

TABLE 2.1: CONSTRUCTION COST OF A CONCRETE PAD FOOTING THAT CARRIES A STEEL COLUMN

(CONTINUED)/

Number	Item Description	Unit	Quantity	Rate £'s	Amount £'s
(3)	Concrete Ancillaries:				
	(a) Formwork rough finish.				
	- Plane vertical width (0.4 - 1.22 m)	m^2	3.84	8.99	34.52
	(b) High yield bar reinforcement.				
	- Bars to BS 449, 16 mm diameter	ton	0.11	394.17	43.36
	(c) Form pockets and grout.				
	- In holding down bolts	Nr*	4	6.0	24.00
	Total cost of Concrete Ancillaries				101.88
	Summary:				
	(1) Earth Work				11.59
	(2) In-Situ Concrete Work				61.53
	(3) Concrete Ancillaries				101.88
	∴ The total construction cost of a pad footing foundation				175.00

* Nr = Number

TABLE 2.1: (CONTINUED)

CHAPTER 3

EXAMPLES ON THE OPTIMUM DESIGN OF FRAMES

3.1 THE PRINCIPLE OF DESIGN

A multi-storey or a high-rise building is defined as one affected structurally by the lateral forces, such as wind, earthquake and blast forces. This type of building usually resists the lateral forces by using stiff core, shear walls, or specially designed bracing frames. If traditional beam-columns are included in such buildings, they may be designed to carry vertical loads only. However, such a construction may, in some cases, be found undesirable in order to satisfy other functional requirements of the building. In these cases the traditional frames are designed separately as plane sway frames that, by their beam-column combinations, resist some of the overall lateral forces applied to the building. The multi-storey frames designed here belong to this type of building, and the lateral forces are assumed to be only that of the wind.

3.2 THE DESIGN CRITERIA

As specified in British Standard B/20, document 77/13908 (1977), the design method controls the actual sway in the columns as opposed to the joint deflections. Only multi-storey, rigidly jointed steel frames, in which the sway of the columns governs the selection of the member sections, are considered here. The resulting design can then be checked, by an analysis, to ensure that the strength requirements are satisfied.

The design criteria adopted here are:

(a) Under dead load and vertical super load, the beams should not collapse by a beam-type mechanism below a load factor λ_1.

(b) Under specified unfactored combined dead, super, and wind loads, the elastic sway in any column should not exceed h/ℓ where h is the column height, and ℓ is a constant, such as 300, 350, or whatever value is required by the engineer.

(c) Under the combined dead, super, and wind loads, no plastic hinge should develop in a column below a load factor of λ_2.

In this chapter the factor λ_1 and λ_2 are taken as 1.75 and 1.4, respectively. Any other values can be given to these, depending on the code of practice used.

All the members of the frames are made out of fabricated steel material, with density 77 KN/m^3, Young's Modulus 207 KN/mm^2, and Yield Stress 0.25 KN/mm^2. Steel Grade 43 is used where the permissible stress in compression due to bending is 0.165 KN/mm^2 for web thickness up to 40 mm, and 0.15 KN/mm^2 for over 40 mm.

3.3 MINIMUM WEIGHT DESIGN

The examples solved for the minimum weight design of rigidly jointed multi-storey steel sway frames are related to the first part of this Chapter. Generally, the frames designed are practically large and tall, with realistic wind and vertical dead and live loadings. The frames can be governed by the limitations specified by the codes on the permissible horizontal sway of each storey.

By solving the design examples, an investigation is carried out on the effect of the move limits on obtaining the final design. The importance of selecting the initial design point, and whether this point could be found by using other methods, are also discussed. The reliability of the optimum design under different loading cases is also examined.

3.3.1 An Outline of the Design Procedure

The minimum weight design procedure of frames consists of the following steps:

(1) Develop the ground structure from a network of nodes or joints, and give a number to each joint and each member starting from the first floor.

(2) Group the members together, if required. This approach not only reduces the computation, but also can make use of engineering experience to advantage.

(3) Calculate the dead loads which are assumed as concentrated loads applied vertically on the joints. Each of these loads is taken as equal to half the weight of all the members connected to the joint. Introduce the life super loads, and specify whether they are concentrated or uniform or the combination of both. Furthermore, calculate the horizontal wind loads according to the code of practice (CP3, 1972). Allocate the values of the computed external loads on the joints of the frame.

(4) Select lower bound and upper bound areas for the columns. These bounds should not be exceeded by the ones imposed by the move limits.

(5) Select the upper bound areas for the beams, and also do the necessary computation to select the lower bounds so that criterion (a) is satisfied. Similarly to the columns, these bounds must not be exceeded by those imposed by the move limits.

(6) Select a set of initial section areas for the members. An arbitrary infeasible set, with no calculation, will suffice. However, engineering judgement in selecting these may reduce the computer time. Other iterative techniques may be used here to compute a better set of initial sections which may be nearer to

the optimum design and thus save computer time, as will be shown later. Notice that the dead load, in step (3) above, can be computed now.

(7) Use the section areas given to convert (I), by employing relationships (2.5) and (2.7), and analyse the structure by solving the stiffness equations ($\underline{K}\ \underline{X} = \underline{L}$), and find the exact displacements. Then use the area and the displacement variables as the starting point for the next design iteration.

(8) Formulate the objective function as the weight of the frame, and send the coefficients to the backing store, (Chapter 8).

(9) One row at a time, construct the linear forms of the design constraints and transfer them to the computer backing store. In this way all the coefficients of the design problem are stored on a disk, (Chapter 8).

(10) Use the simplex method to minimise the weight function.

(11) Use the areas obtained and repeat the process from step (7) until convergence is achieved and the optimum solution is determined. The convergence limit utilised is that the change of the objective function (Z) on two successive design iterations will be less than ε of its current value, i.e. $((Z^{i+1} - Z^{i}) / Z^{i}) \leqq \varepsilon$, where ε is a selected tolerance and i represents the present design iteration. In this thesis it is found reasonable to take the value of ε as about 0.1%. The flow diagram of the design procedure up to this step is shown in Figure 3.1.

(12) The above results are for unfactored combined loads and criterion (b) is fully satisfied. If required, analyse the structure, by solving the stiffness equations separately using the program written by Celik (1977). The section area (A) and the value of (I) for the universal sections, which are equivalent to the

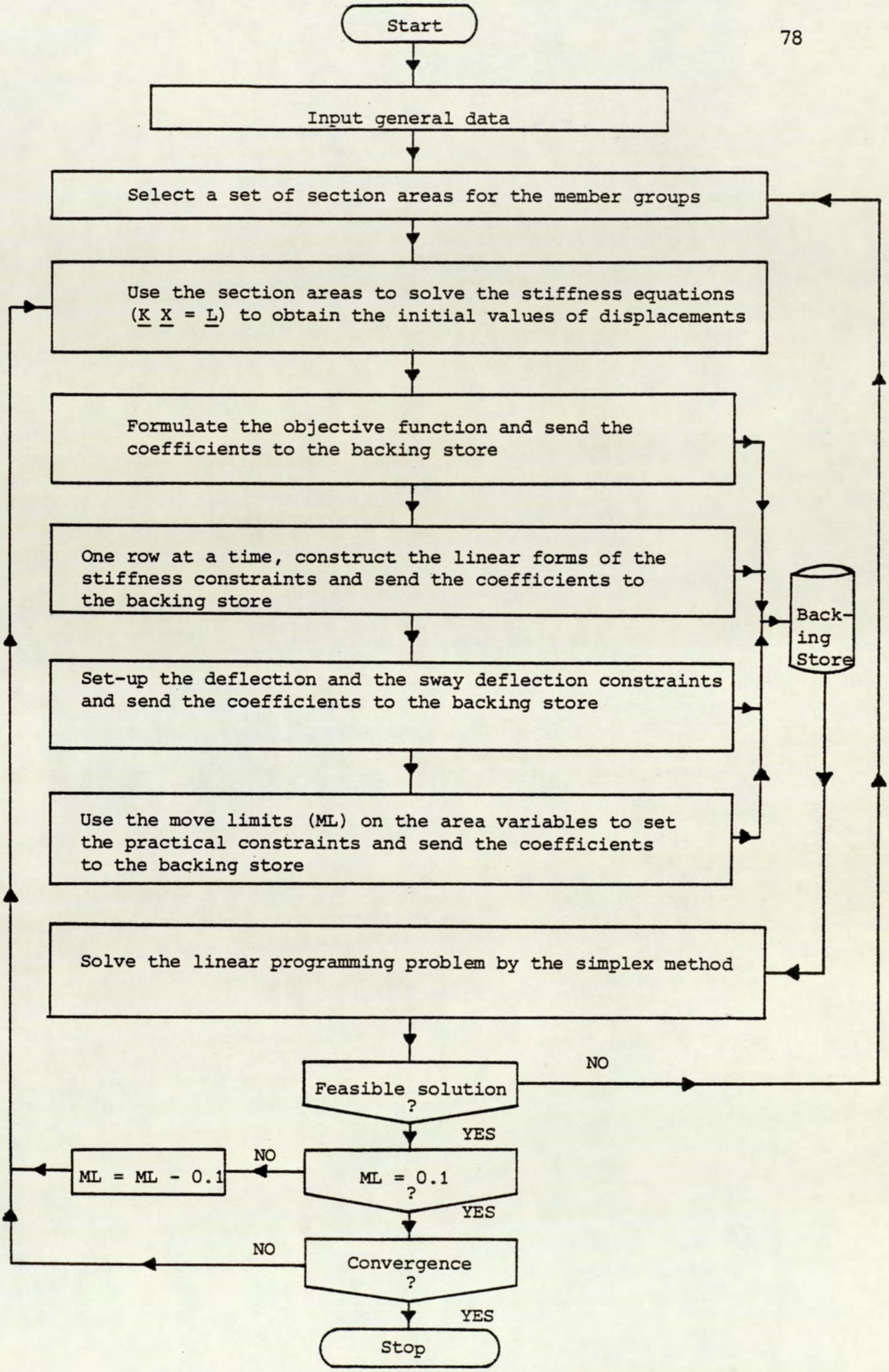


FIGURE 3.1: THE FLOW DIAGRAM OF THE MINIMUM WEIGHT DESIGN PROCEDURE FOR THE FRAMES

optimum sections obtained, are used in this analysis. Find the exact values of the bending moments, the shearing forces, the axial forces and the deflections. Multiply the maximum bending moment in each column by λ_2 . If the result is less than the reduced plastic hinge moment M_p, (Majid, 1978), the strength criterion (c) is satisfied for that column. If not, increase the column sections so that M_p is more than the factored maximum bending moment.

(13) Check if all the stress requirements are satisfied and carry out minor changes to the sections where these are violated. As stated the stress constraints were excluded for simplicity and speed of operation. In the case of sway frames, the wisdom of this decision was confirmed by the numerous examples solved.

3.3.2 The Effects of the Move Limits

When the non-linear programming problem of a steel sway frame is transferred to a linear programming one, it is obvious that this linearisation will introduce errors in the solution of the problem. These may be controlled by imposing some bounds on the design variables which are known as move limits (Chapter 1, Section 1.5.1.1). Saka (1975) found that it is only necessary to impose move limits on the main design variables. These are the areas of the members. The move limit is a positive constant factor less than one, e.g. ML = 0.6, and it is preselected but gradually reduced after each design iteration in steps of 0.1.

When the value of ML becomes 0.1 and convergence is not achieved, then the iterations are continued with these particular values of move limits until the optimum design point is reached. As was stated earlier, the boundaries of the move limits should not exceed the upper or lower bounds imposed on the variables.

3.3.2.1 An 11-Storey Frame with Four Equal Bays

This frame belongs to a structure consisting of parallel frames which are 5 m apart. The details of numbering the joints and the members of this frame are shown in Figure 3.2, while Figure 3.3 shows the way the members are grouped. The grouping of the columns in this frame was chosen to be symmetrical. This was because the horizontal wind load might act from either side. Figure 3.3 also shows the total amount of the horizontal wind load acting at each floor level. This is calculated according to CP3, Part 2. The wind load was divided equally on the joints of the floor to represent the horizontal load on each of them. The external moment and the vertical load applied on each joint was computed by assuming the intensity of the uniform vertical super load (ω) as 30 KN/m on the roof and 35 KN/m on each floor.

As the relative sway deflections between storeys are considered to be the main governing limit-states for the design of this frame, the upper bound on the relative sway in a storey was taken as h/350, where h is the height of the storey. When solving a linear programming problem, it is necessary to put upper bounds on all the joint deflections and rotations. For this reason, the upper bounds on the horizontal deflections were chosen arbitrarily but larger than the sway limitations. The upper bound imposed on each vertical deflection was 60 mm, and on each joint rotation it was 0.08 radian.

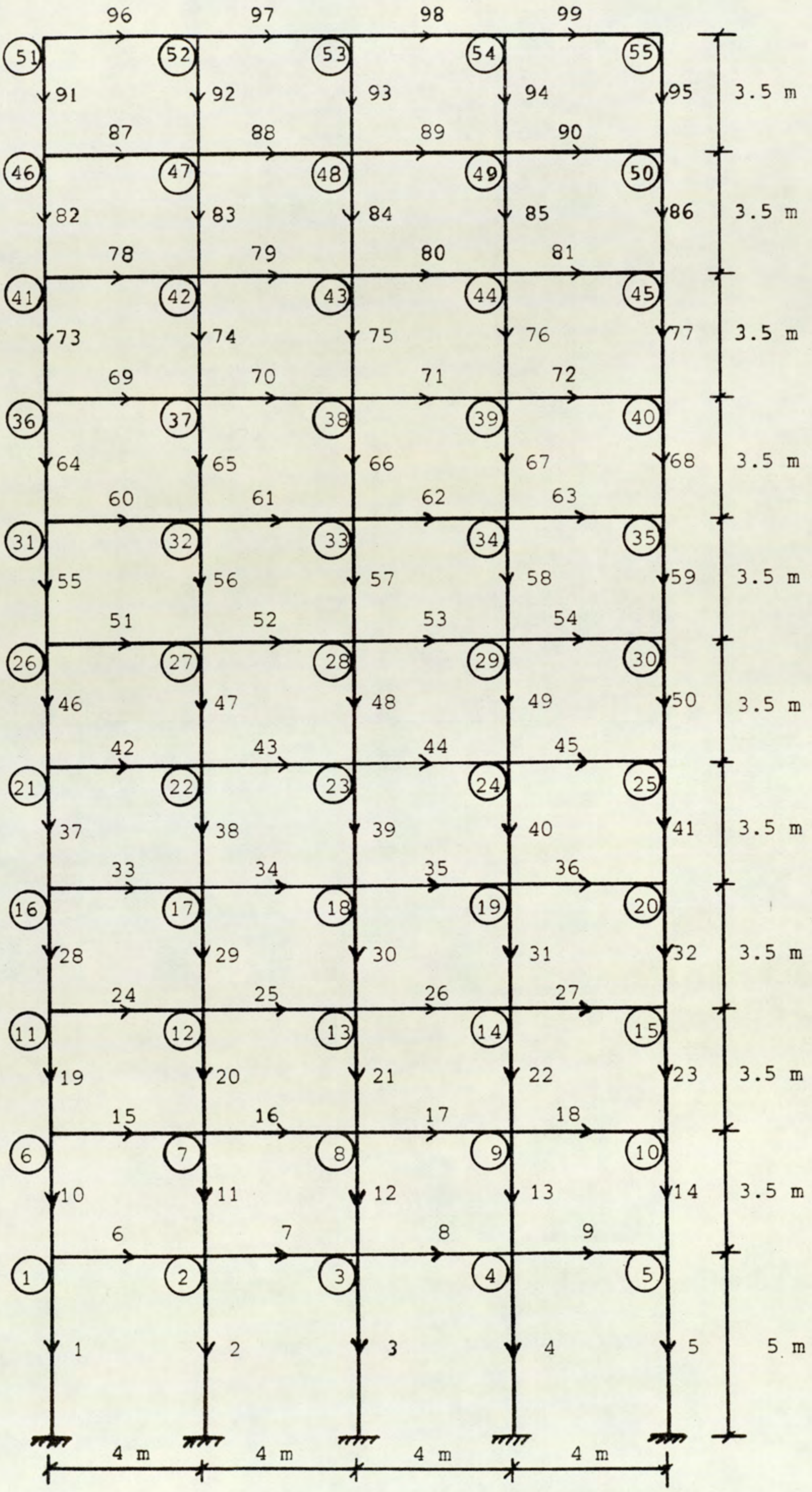


FIGURE 3.2: AN 11-STOREY 4 EQUAL BAY FRAME, DIMENSIONS, NUMBERING OF MEMBERS AND JOINTS

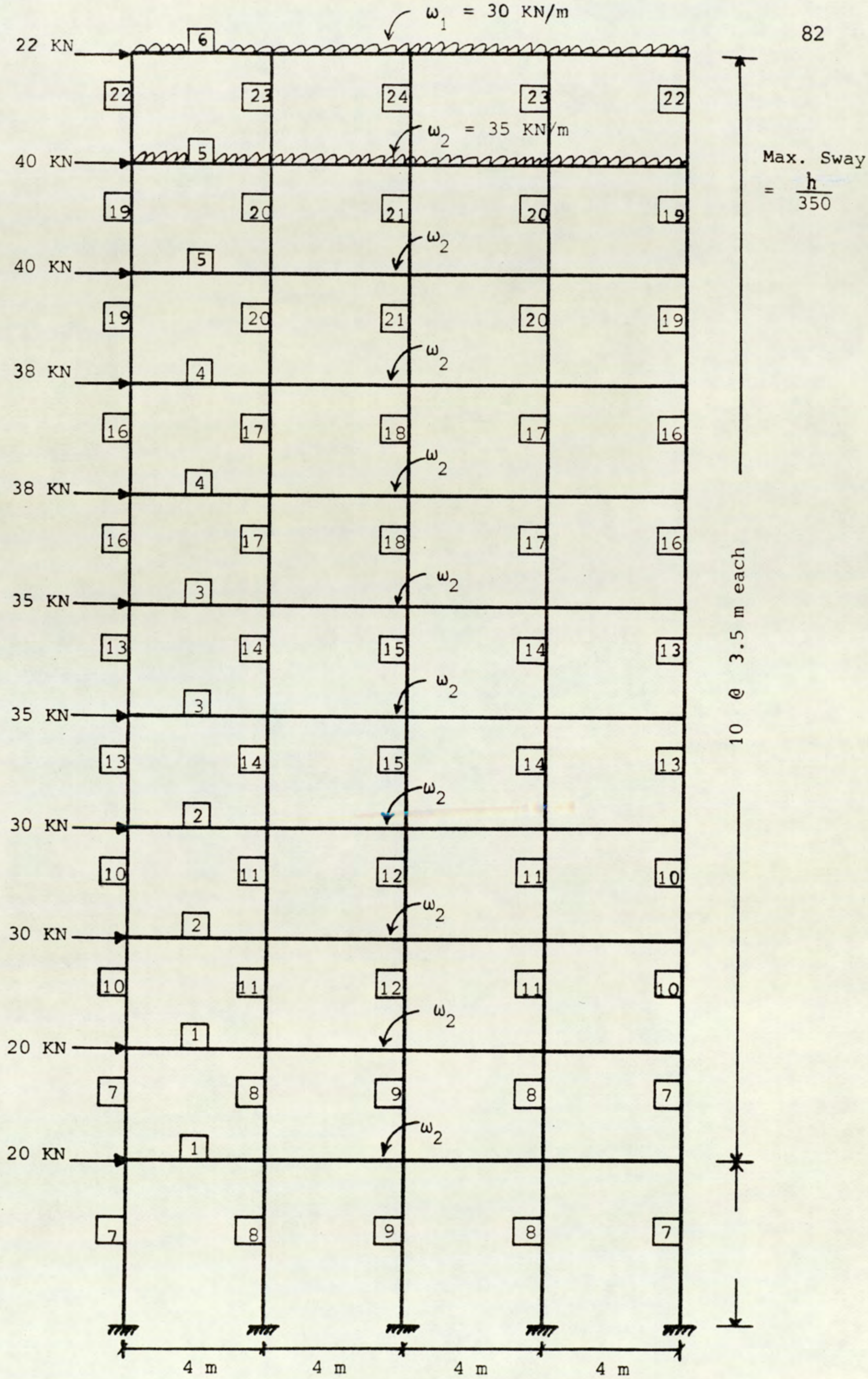


FIGURE 3.3: THE 11-STOREY 4 EQUAL BAY FRAME, LOADING AND NUMBERING OF GROUPS

The lower bound on the section areas of the columns was chosen to be the area of the smallest universal column section available, and the upper bound was that of the largest section. The lower bound on a beam was selected from a beam type mechanism, while the upper bound was chosen to be that of the largest available universal beam section.

The frame had a total of 55 joints, and 99 members grouped into 24 area groups. The first six were beams and the rest were columns, as shown in Figure 3.3. The design problem had 189 variables, 24 of which were areas and the rest were displacements with three variables for each joint. The problem had 388 constraints, 165 of which were stiffness constraints and 48 were constraints due to the application of move limits on the areas. There were also 165 deflection constraints and ten relative sway deflection constraints of the form:

$$x_{i+1} - x_i \leq h_{i+1} / 350 \qquad (3.1)$$

where x_i is the horizontal deflection at floor i, and x_{i+1} is that at floor i+1.

The purpose of choosing this example is to demonstrate the use of the move limits. It is shown here that by choosing suitable values for the move limits, convergence can be obtained regardless of the position of the starting design point. Three design cases were considered, and they are reported below.

Figure 3.4 shows the values of the weight function of the frame for two design Cases, 1 and 2. The initial design point for Case 1 was infeasible and the member sections were chosen as the smallest ones available for beams and columns. The beam sections were selected to withstand a beam-type mechanism. The infeasibility occurred due to the fact that when the frame was analysed using these areas, the deflection

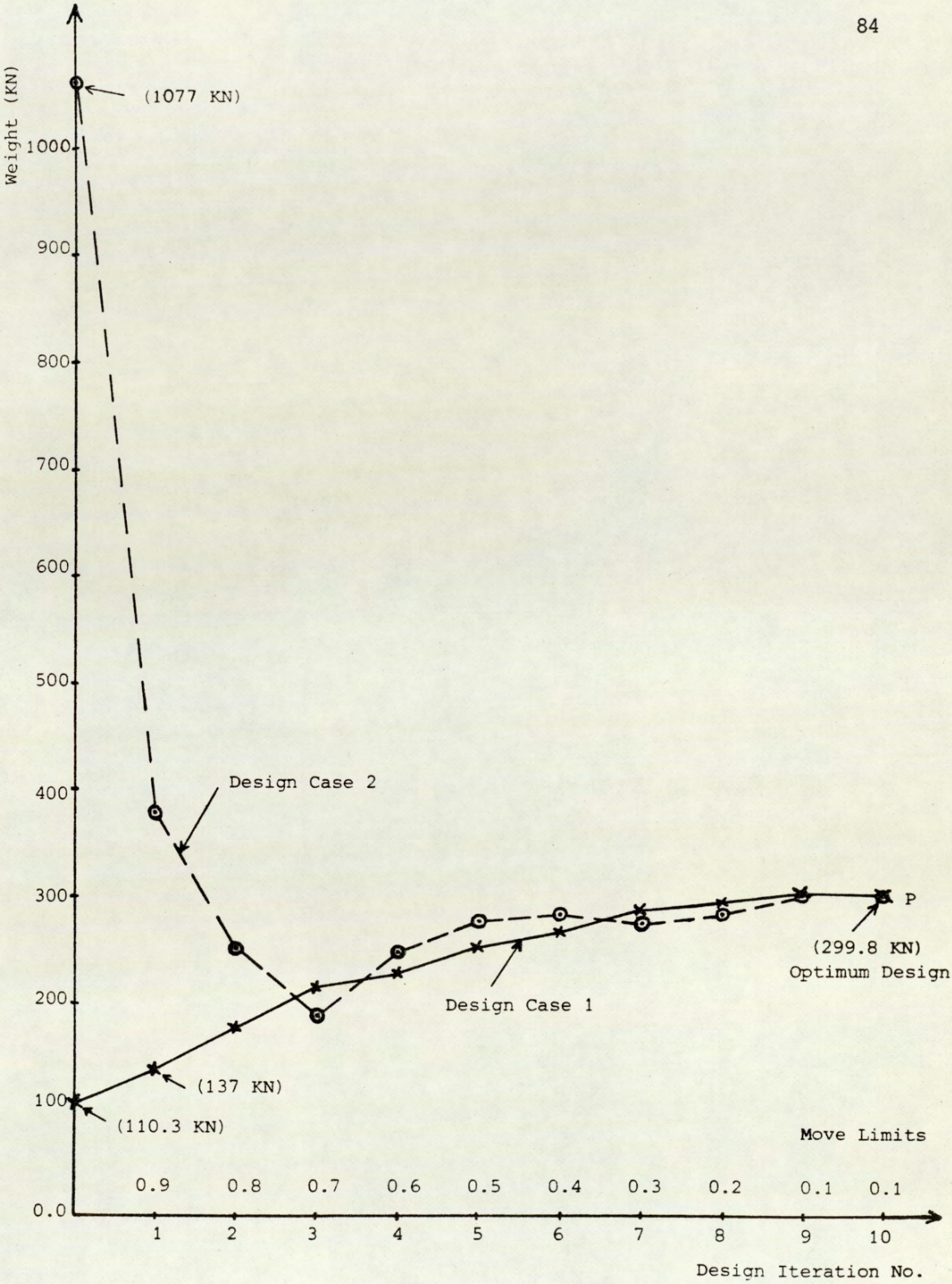


FIGURE 3.4: THE 11-STOREY 4 BAY FRAME - WEIGHT VARIATION IN DESIGN CASES 1 AND 2

and stress requirements were not satisfied. The total weight of the structure at the starting point was equal to 110.3 KN. The initial move limit was ML = 0.9 and the first design increased the weight to 137 KN. That completed one iteration toward the optimum design which was achieved in ten iterations. Point P in Figure 3.4 represents the optimum design with a weight of 299.8 KN.

In Case 2 of the optimum design, Figure 3.4, the starting point was chosen to be feasible. The initial design areas were given the largest sections available. The weight of the structure at the starting point was 1077 KN, and at the optimum design the weight became 299.6 KN.

The results of the two design cases are shown in Table 3.1, where the second and the fourth columns contain the initial sections. The third and the fifth columns are the optimum design achieved for each Case. It is noticed that, although the weight is the same in both cases, the areas of the groups in one design are slightly different from those in the other. The main conclusion obtained from this example is that the optimum design is independent of the starting point.

In Case 3, two design processes were considered. The starting point for the first process was chosen arbitrarily, and this is represented by point A in Figure 3.5 The initial sections for this point are shown in Table 3.2, where the weight of the frame was equal to 649 KN. The initial move limit was 90%, i.e. ML = 0.9, and then reduced by 0.1 at each iteration until convergence was achieved at point B with a total weight of 299 KN. The section areas are shown in Table 3.2. The process was repeated for a second design with the initial sections corresponding to point B. The optimal solution was achieved at point C with a total weight of 298.7 KN. This is virtually

Group Number		Design Case 1		Design Case 2	
		Initial Design Areas X 10^2 mm^2	Optimum Design Areas X 10^2 mm^2	Initial Design Areas X 10^2 mm^2	Optimum Design Areas X 10^2 mm^2
Beams	1	38.00	121.56	303.5	125.71
	2	38.00	97.21	303.5	109.63
	3	38.00	91.75	303.5	102.74
	4	38.00	83.82	303.5	74.99
	5	38.00	48.14	303.5	52.32
	6	38.00	38.00	303.5	38.00
Columns	7	38.20	127.37	432.7	116.64
	8	38.20	212.55	432.7	202.60
	9	38.20	247.54	432.7	275.31
	10	38.20	101.36	432.7	95.66
	11	38.20	163.56	432.7	153.94
	12	38.20	176.07	432.7	163.98
	13	38.20	68.81	432.7	86.14
	14	38.20	165.92	432.7	141.07
	15	38.20	177.26	432.7	144.02
	16	38.20	53.32	432.7	46.11
	17	38.20	125.44	432.7	132.73
	18	38.20	148.83	432.7	150.73
	19	38.20	43.97	432.7	38.28
	20	38.20	111.72	432.7	101.02
	21	38.20	76.82	432.7	111.20
	22	38.20	38.20	432.7	38.20
	23	38.20	38.20	432.7	38.20
	24	38.20	38.20	432.7	38.20
Total Weight		110.3 KN	299.8 KN	1077.0 KN	299.6 KN

TABLE 3.1: OPTIMUM SECTION AREAS OBTAINED AT DESIGN CASES 1 AND 2 FOR THE 11-STOREY 4 BAY FRAME

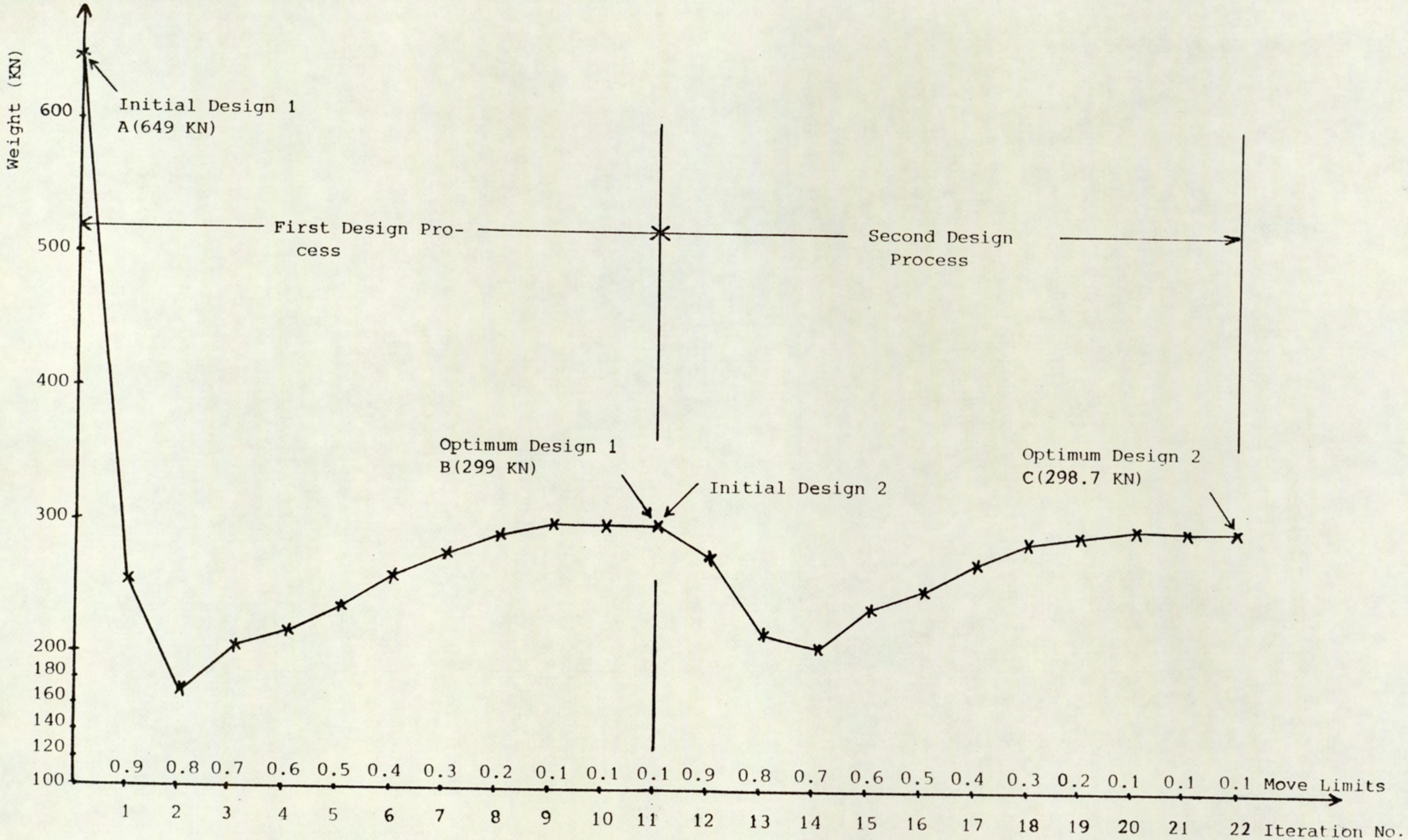


FIGURE 3.5: DESIGN CASE 3 FOR THE 11-STOREY 4 BAY FRAME - TWO DESIGN PROCESSES SHOWING THE WEIGHT VARIATION

Group Number		Initial Design Areas X 10^2 mm^2	The Optimum Design	
			For first process at B X 10^2 mm^2	For second process at C X 10^2 mm^2
Beams	1	303.50	120.24	121.10
	2	303.50	107.91	108.76
	3	212.70	90.94	95.91
	4	212.70	82.86	76.18
	5	155.60	49.31	45.37
	6	138.40	38.00	38.00
Columns	7	305.60	125.33	138.18
	8	305.60	225.55	211.28
	9	432.70	227.44	234.65
	10	252.30	102.45	104.29
	11	252.30	152.63	148.70
	12	305.60	165.29	165.90
	13	201.20	72.15	77.85
	14	201.20	159.92	144.29
	15	252.30	160.16	158.45
	16	174.60	56.17	54.15
	17	174.60	121.03	131.00
	18	201.20	156.61	143.82
	19	136.60	41.24	44.27
	20	136.60	93.47	101.27
	21	167.70	109.77	118.08
	22	114.00	38.20	38.20
	23	114.00	38.20	38.20
	24	136.60	38.20	38.20
Total Weight		649 KN	299 KN	298.7 KN

TABLE 3.2: OPTIMUM SECTION AREAS OBTAINED AT DESIGN CASE 3 FOR THE 11-STOREY 4 BAY FRAME

the same as point B but with a slightly different value for the area of each individual group. This confirms the conclusion that for this frame there is only one global optimum design. It is noticed that in the design tables 3.1 and 3.2, some of the group areas obtained, at the optimum design points, are at their lower bounds.

By analysing the optimum solutions of the three design cases above, as in step (12) of the design procedure, it was found that the member stresses were within the permissible stress limits specified in section 3.2.

3.3.3 The Significance of the Initial Design Point

In the previous Section, the initial trial areas for the members of the 11-storey frame were selected arbitrarily. The move limits were decided to be ML = 0.9, to start with, and then reduced by steps of 0.1. This approach used a considerable amount of computer time. However, engineering conception in choosing the set of initial trial sections for the structural members can reduce the number of design iterations and thus less computer time is needed. A method, presented by Okdeh (1980), can be used here to initiate an economical process that leads to an optimum design.

Okdeh proposed a direct procedure for designing plane steel frames subject to sway limitations. The stiffness equations were modified so that the sway in each storey was equal to some specified values. The modified equations were then solved by iteration to calculate the cross-sectional properties of the columns as well as the other joint displacements. The beam sections were selected initially and then altered in an effort to reduce the weight and the material cost of the frame. Okdeh employed stability functions to take the effect of axial loads in the members into consideration. The final design was checked for strength requirements and the members were altered accordingly.

The resulting sections for the members of a frame, obtained by using Okdeh's method, were claimed to be feasible and very near the optimum design. Using these sections at the starting point here can therefore reduce the number of iterations required to reach an optimum design. Two examples are given to demonstrate this.

3.3.3.1 A 5-Storey Frame with three Unequal Bays

This frame was designed by Okdeh using a computer program which employs the method described in the last Section. The details of member grouping, dimension and loading used for such a design are shown on the ground structure in Figure 3.6. The frame had 35 joints and 50 members grouped together into 25 groups. The final design obtained by Okdeh is shown in Table 3.3. The area and the second moments of area of the sections selected are shown in the table. The total weight of this design was 120.5 KN. The design was achieved by considering the axial load, but it should be emphasised here that the stability function effects were small in this frame and including them increased the weight by four per cent only.

The 5-storey, three unequal-bay frame was considered for an optimum weight design. As stated in Chapter 2, the axial stiffness EA/L of the member was included in the stiffness equations, but the stability functions were not. The loading and member groups were slightly altered. For instance, a vertically imposed joint load P at the midspan of a beam, was changed to a uniformly distributed load of an intensity equal to P/L where L, here, is the beam span. In this

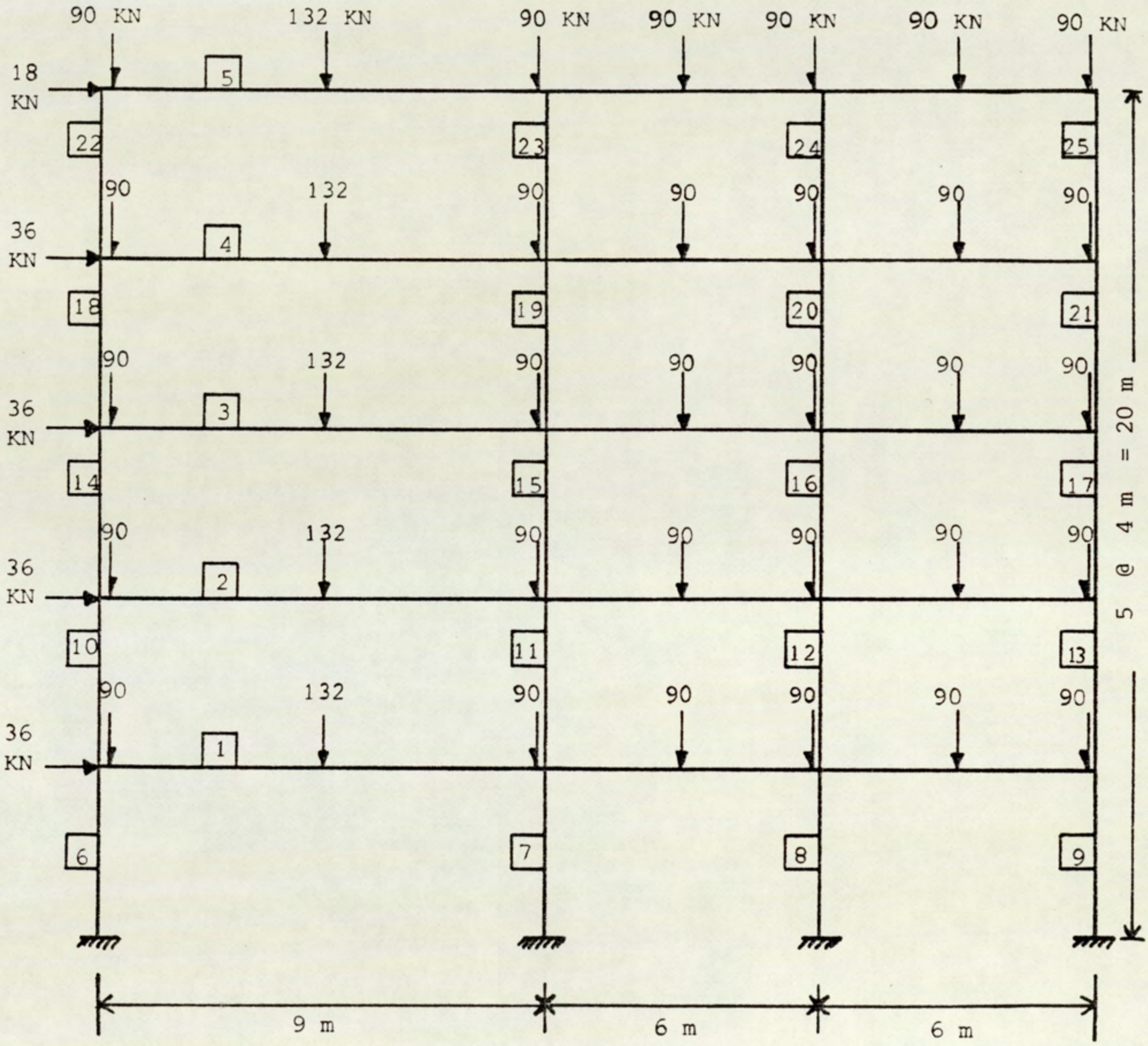


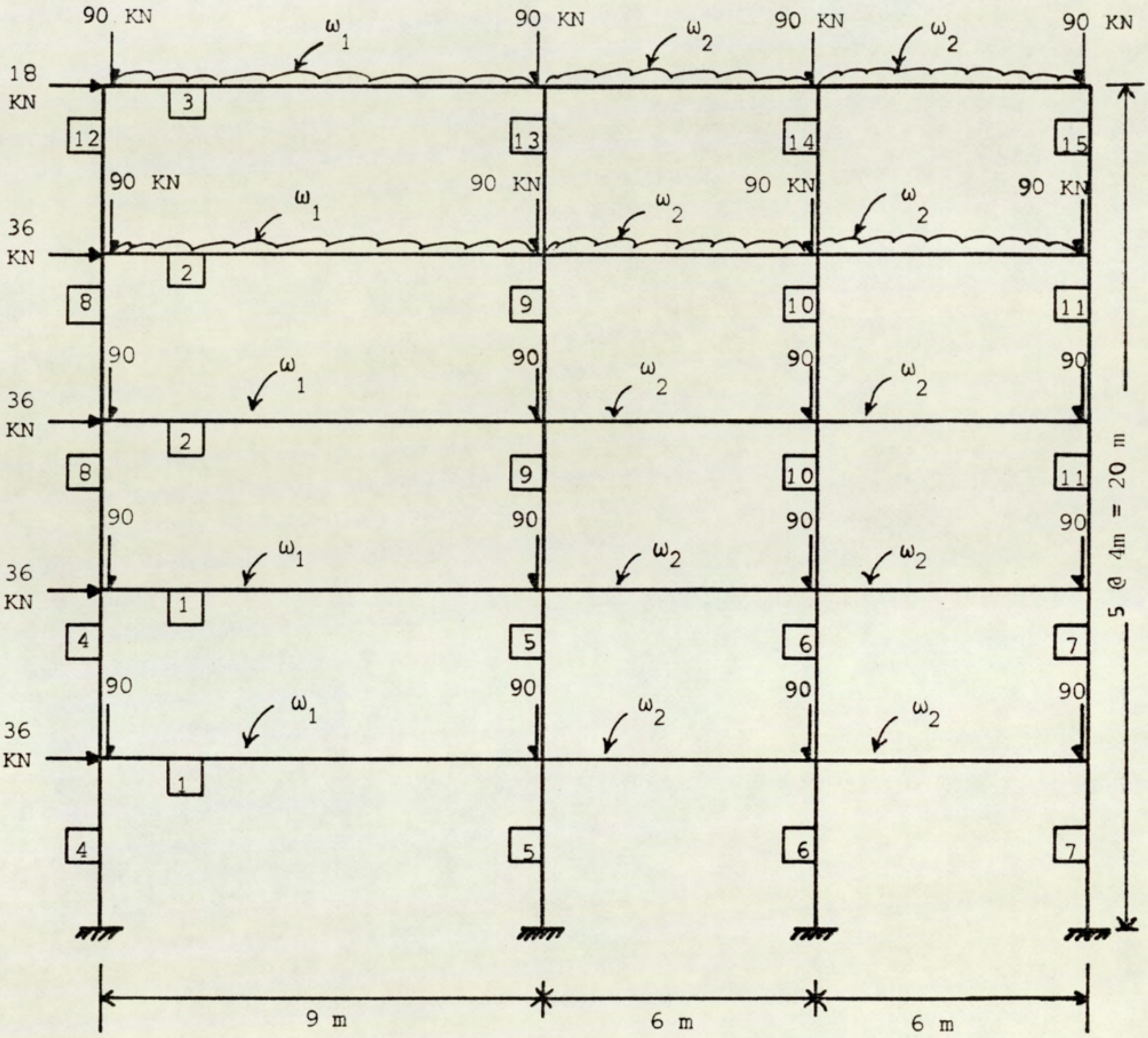
FIGURE 3.6: A 5-STOREY 3 UNEQUAL BAY FRAME - DIMENSIONS WITH OKDEH'S LOADING AND MEMBER GROUPING

Okdeh's Group Number		Final Design, Axial Load Effect is considered	
		(I) X 10^4 mm^4	Areas from Design Table (A) X 10^2 mm^2
Beams	1	47363	104.3
	2	47363	104.3
	3	21345	66.5
	4	21345	66.5
	5	18576	68.3
Columns	6	11360	92.9
	7	17510	136.6
	8	14307	114.0
	9	7647	91.1
	10	11360	92.9
	11	17510	136.6
	12	14307	114.0
	13	6088	75.8
	14	6088	75.8
	15	14307	114.0
	16	6088	75.8
	17	6088	75.8
	18	6088	75.8
	19	7647	91.1
	20	6088	75.8
	21	5263	66.4
	22	5263	66.4
	23	5263	66.4
	24	5263	66.4
	25	5263	66.4
Total Exact Weight			120.5 KN

TABLE 3.3: OKDEH'S DESIGN OF THE 5-STOREY 3 UNEQUAL BAY FRAME

manner a joint was omitted from the midspan of each beam. The grouping of the members was arranged so that more structural members were made out of the same section, and less groups were used. By making the two alterations, of loading and grouping, a considerable amount of saving in the size of the design problem was achieved. The effect of these alterations, on the results of a design, was trivial because here we are considering that the lateral sway in the columns decides the design outcome.

After the slight alterations mentioned above, the frame then had 20 joints and 35 members which were collected together into 165 area groups, three for beams and 12 for columns, as shown in Figure 3.7. The dimensions and loadings are also shown in the Figure. The intensity of the uniform vertical load on the 9 m beams was ω_1 = 14.7 KN/m, and on the 6 m beams was ω_2 = 15 KN/m. The upper bound on the relative sway was h/300. The limitation on the horizontal deflections of the joints were taken arbitrarily, but larger than the sway bounds. The upper bound on the vertical deflections of the beams was 25 mm, which was equal to 9000/360, and for all the joints it was 40 mm. The joint rotations were not allowed to exceed 0.08 radian. The upper bounds on the section areas of the beams and the columns was chosen to be the largest universal section available. The lower bounds on the areas were selected to satisfy criteria (a) and (c).



$$\omega_1 = \frac{132 \text{ KN}}{9 \text{ m}} = 14.7 \text{ KN/m}$$

$$\omega_2 = \frac{90 \text{ KN}}{6 \text{ m}} = 15 \text{ KN/m}$$

Max. Sway = h/300

FIGURE 3.7: THE 5-STOREY 3 UNEQUAL BAY FRAME - DIMENSIONS, LOADING AND REGROUPING OF THE MEMBERS FOR THE OPTIMUM DESIGN

Four design cases were considered. They were started with Okdeh's results as the initial design point. The exact weight of the frame at this point was 121 KN due to the regrouping of the members. The first design case was started with a move limit value of ML = 0.1 which kept unchanged for all the iterations until convergence was achieved. For the other three design cases, each one began with a certain value of move limit which was then reduced by 0.1 at each iteration until convergence was obtained. Figure 3.8 shows four curves which represent the design cases. Each curve gives the weight function for a certain case. This weight was calculated after the member sections were selected from a list of available sections.

It was noticed that the case which used an intial value of ML = 0.3 required only three iterations to reach the optimum. While the two cases with ML = 0.1 and ML = 0.2 required four iterations, and one with ML = 0.4 needed five iterations. This concluded that if a suitable starting point was selected with ML = 0.3 then a minimum number of iterations was required to reach convergence. Point P in Figure 3.8 represents the optimum design with an exact minimum weight of the frame equal to 107 KN. This was 11.6% less than that obtained by Okdeh.

The section properties at the initial and the optimum design are shown in Table 3.4. The second and the third columns are the values of the universal I and A selected by Okdeh. The section areas A are the values of the variables that were taken into consideration as a starting point. Notice that the initial values of some groups in this table were taken as the average of two different groups chosen by Okdeh, and these are marked with asterisks in the second column of Table 3.4. For instance, the initial values of the section properties for group 7 were taken as the average of groups 9 and 13 of Okdeh's design. The fourth column, in Table 3.4, represents the optimum

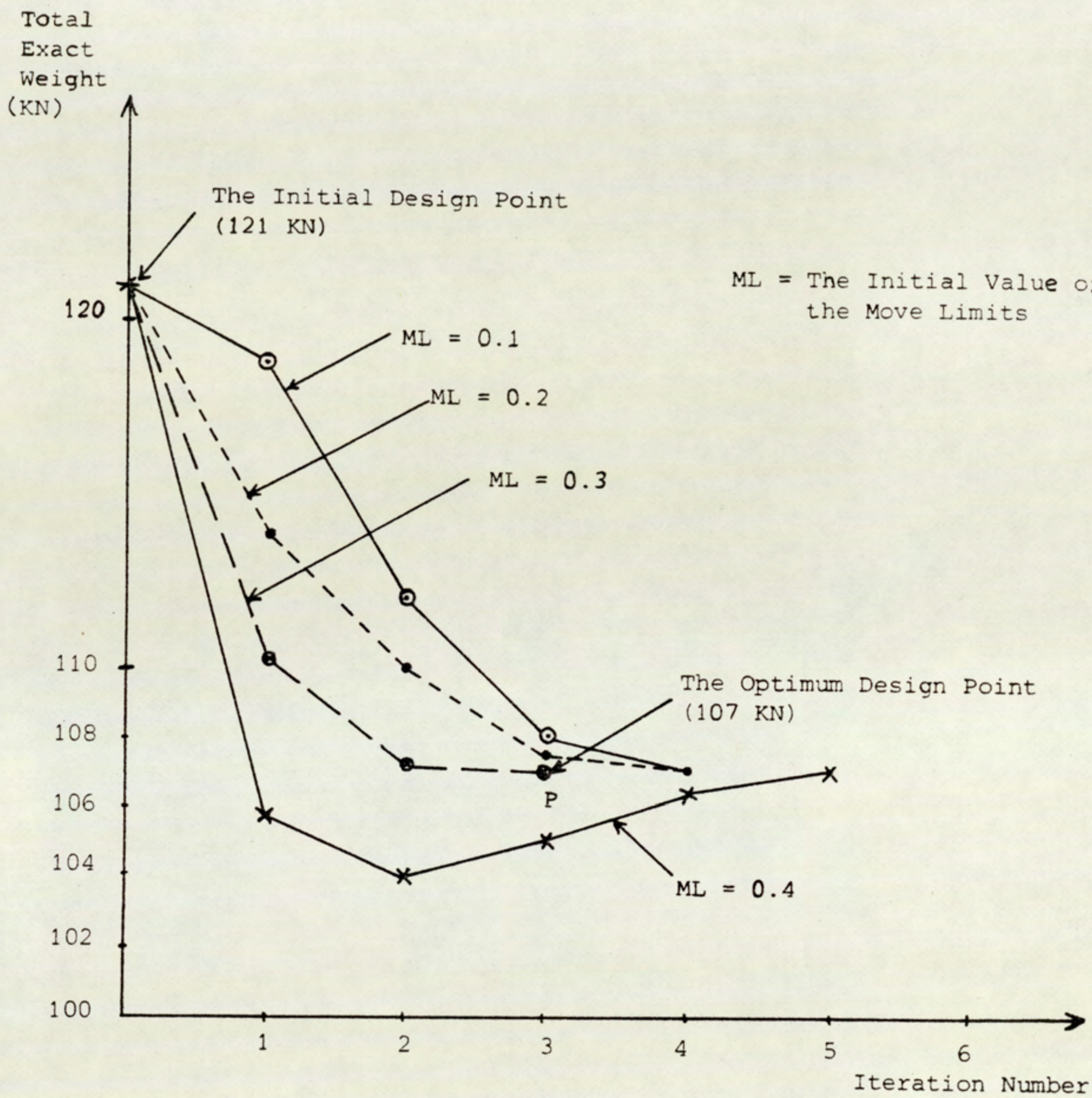


FIGURE 3.8: THE 5-STOREY 3 UNEQUAL BAY FRAME - VARIATION OF WEIGHT WITH DIFFERENT STARTING MOVE LIMITS

Group Number		Initial Design Using Okdeh's Results		Optimum Design (at point p)			
		$I \times 10^4$ mm⁴	Corresponding Areas from Design Tables (A) $\times 10^2$ mm²	Optimum Section Areas achieved $(A_{op}) \times 10^2$ mm²	$I_{op} = f(A_{op})$ $(I_{op}) \times 10^4$ mm⁴	I from Design Table $(I_{univ}) \times 10^4$ mm⁴	Corresponding Areas from Design Tables $(A_{univ}) \times 10^2$ mm²
Beams	1	47363	104.3	83.62	22375.4	25464	75.9
	2	21345	66.5	55.70	9924.0	9924	58.8
	3	18576	68.3	76.19	18576.0	18576	68.3
Columns	4	11360	92.9	38.2	1742.0	1742	38.2
	5	17510	136.6	99.24	10342.3	11360	92.9
	6	14307	114.0	180.68	28641.6	32838	174.6
	7	6867.5*	83.45	106.52	11665.0	14307	114.0
	8	6088	75.8	38.2	1742.0	1742	38.2
	9	10977.0*	102.55	116.81	13644.7	14307	114.0
	10	6088	75.8	100.32	10534.4	11360	92.9
	11	5675.5*	71.11	89.29	8642.0	9462	110.0
	12	5263	66.4	38.2	1742.0	1742	38.2
	13	5263	66.4	38.2	1742.0	1742	38.2
	14	5263	66.4	38.2	1742.0	1742	38.2
	15	5263	66.4	38.2	1742.0	1742	38.2
Total Exact Weight			121 KN	108.6 KN			107.0 KN

* Means an average section is selected from two different sections chosen by Okdeh.

TABLE 3.4: SECTION PROPERTIES AT THE INITIAL AND THE OPTIMUM DESIGN POINTS FOR THE 5-STOREY 3 UNEQUAL BAY FRAME

section areas A_{op} achieved, assuming that continuous sections were available. These areas were then used to compute the second moments of area I_{op}, using the relationships (2.5) and (2.7), and the results are shown in the fifth column. The real sections were selected, from the universal sections available (I_{univ}) and shown in the sixth column. The corresponding values of areas (A_{univ}) are listed in the seventh column, and these made the optimum design of the frame with a minimum weight of 107 KN. It should be emphasised here, that the values of the second moments of area were used to decide the selection of the sections. This was because these, not the areas, dominate the stiffness equations.

3.3.3.2 A 24-Storey Frame with three Equal Bays

This frame was also designed by Okdeh (1980) according to his method which was briefly described in Section 3.3.3. This rectangular frame belonged to a multi-storey structure consisting of parallel frames. Some of these frames assumed to have special bracing components. The frames were 4.5 m apart from each other. The vertical and the wind loads were calculated according to CP3 Parts 1 and 2. The wind loads were calculated for a frame to be built on a surface with a large and frequent obstruction. Each frame was symmetrical and loaded symmetrically.

The vertical load and the grouping of members, originally arranged by Okdeh, are altered for the sake of minimising the size of the design problem. The alteration is similar to the one which took place on the 5-storey frame. Figure 3.9 shows the details of member grouping, loading and dimension for the frame. The groups number for the internal and the external columns of the frame are listed on the right of the Figure. The vertical load is shown after uniformly distributing

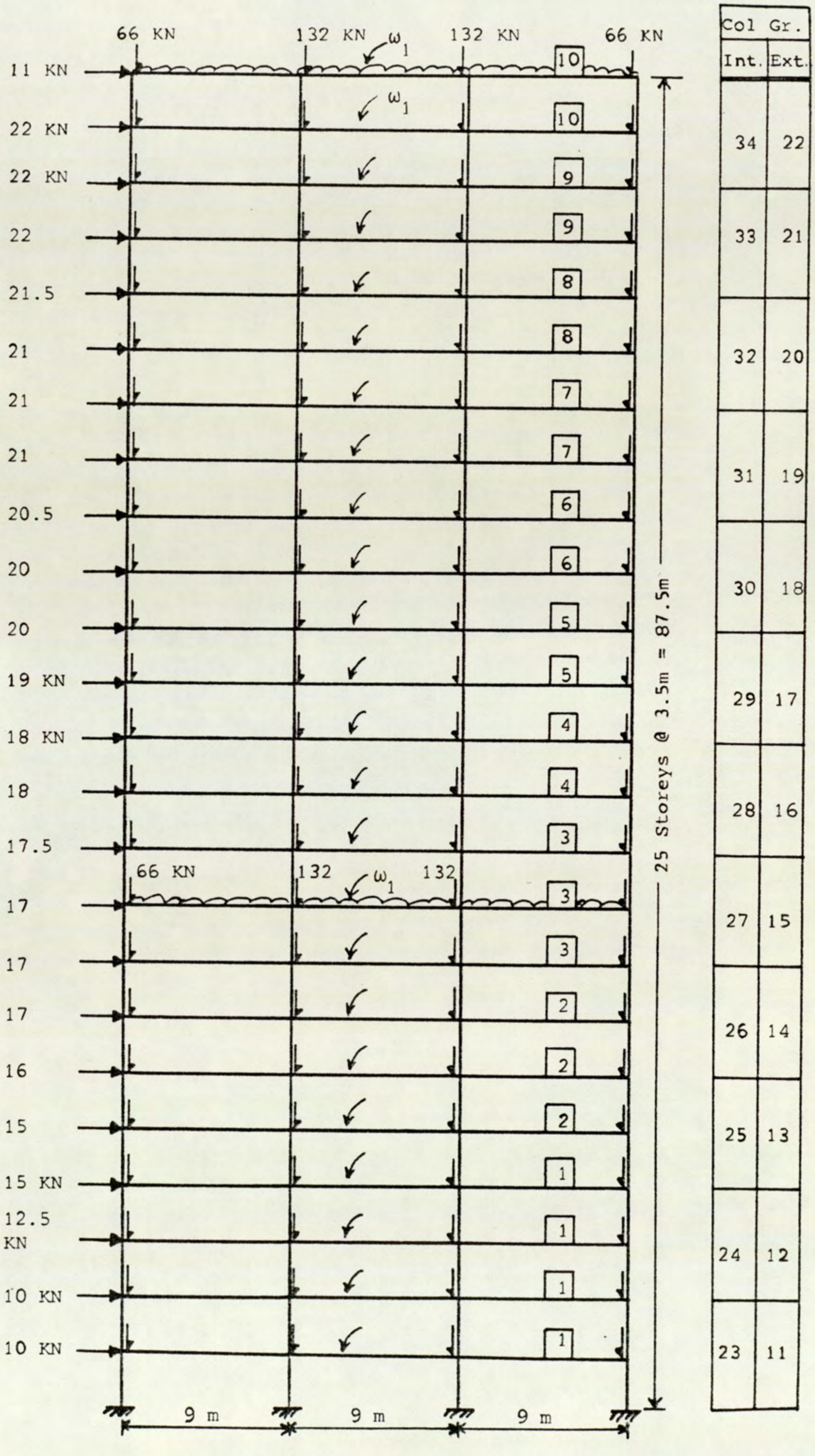


FIGURE 3.9: A 24-STOREY 3 EQUAL BAY FRAME - DIMENSIONS, LOADING AND MEMBER GROUPINGS

the midspan pointed load. The intensity of the new load is 14.7 KN/m.

The frame contained 96 joints and 168 members which were collected together into 34 area groups, ten for beams and 24 for columns. The upper bound on the relative sway was h/300. All the other bounds on the joint deflections and the section areas are taken as those given for the 5-storey frame. The optimum design problem for this frame consisted of 322 variables, 34 of which were areas and the rest were displacements; three for each joint. The problem also consisted of a total of 667 constraints, 288 of which were for the frame overall stiffness equations. A further 288 constraints were upper bounds for joint deflections. The move limits introduced 34 upper and 34 lower bound constraints. The application of inequality (3.2), of Section 3.3.2.1, introduced 23 sway deflection contraints.

Four design cases were taken into consideration. These are shown in Figure 3.10, where each case is represented by a curve which is the function of the exact weight, i.e. the weight after selecting universal members, of the frame. All these cases started with areas taken from Okdeh's design. Each case began with a different value of move limits, similar to the 5-storey frame.

It is noticed in Figure 3.10 that initial tight move limits are not necessarily useful to achieve convergence. This was proved in the design case where ML = 0.1 was taken as fixed value for all the design iterations. This case required six iterations to reach the optimum, and was clearly not an economical procedure. The exact weight at the starting point was 1096 KN, which represented Okdeh's design. The optimum design was at point C, Figure 3.10, with an exact weight of 891 KN which was less by 19% from the initial design. Point C was chosen because it required the least number of iterations, which was only

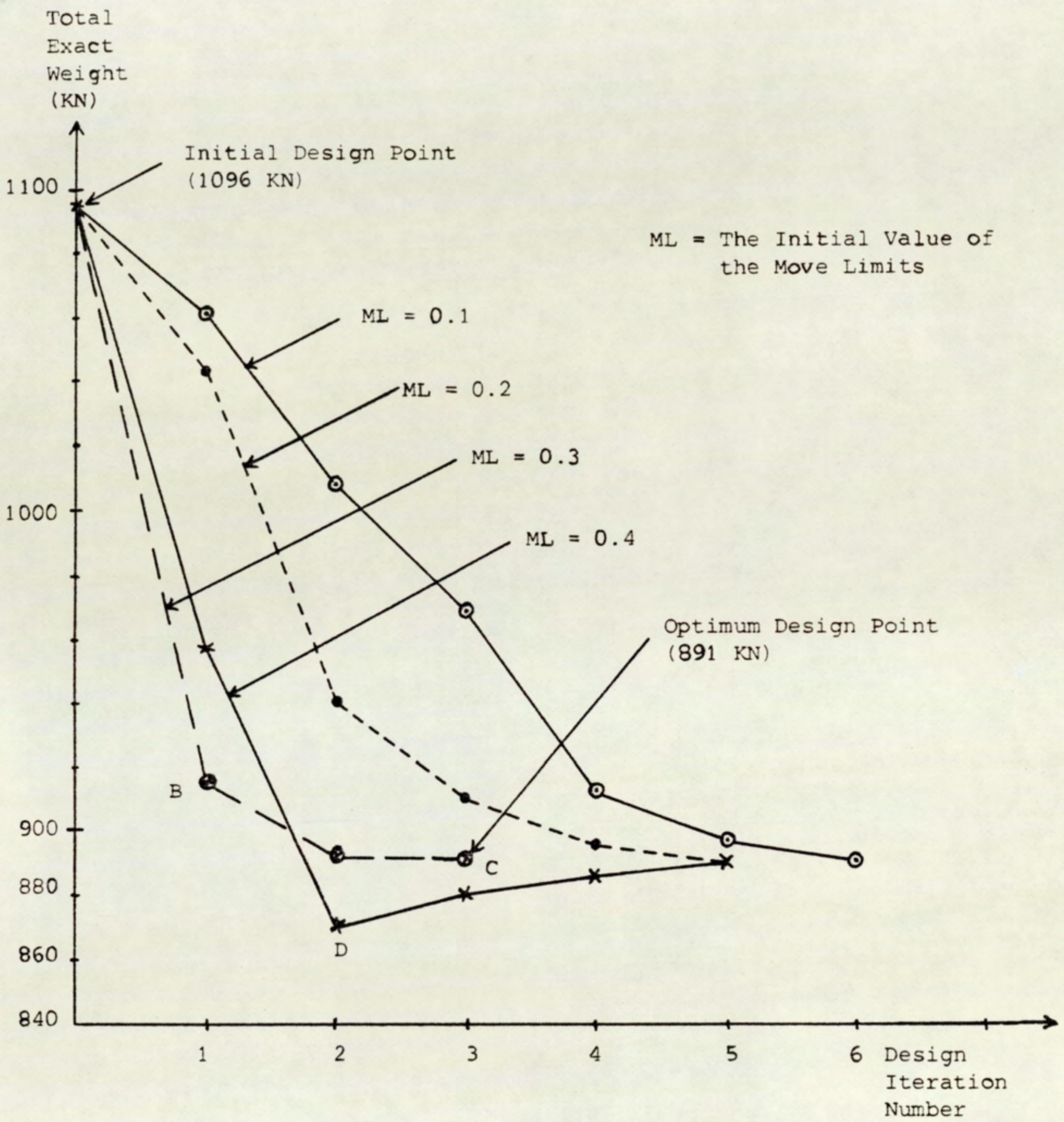


FIGURE 3.10: THE 24-STOREY 3 EQUAL BAY FRAME - VARIATION OF WEIGHT WITH DIFFERENT STARTING MOVE LIMITS

three. Thus less computations are needed and a great saving is obtained in the computer time.

Table 3.5 shows the section properties, i.e. I and A, for the groups of the members at the starting point, taken from Okdeh, and at the optimum design. Okdeh's design included the axial stiffness and the axial load effect, i.e. stability function. If those two effects were excluded, then the total weight of Okdeh's design would have been reduced by approximately 11%. However, in the optimum design here, only the axial stiffness was included, and the weight was reduced by 19% from the original design, as mentioned before.

To examine the efficiency of the optimum design, the frame was analysed using an independent existing computer program (Celik, 1977). The values of I_{univ} and A_{univ} were used for the analysis. These values were selected from several design points, such as B, D and C in Figure 3.10. The differential sway between the storeys of the frame, obtained by the analysis at each design point, is shown in Figure 3.11. The vertical line represents the limit h/300 on differential sway. Curve 1 shows the sway at each storey level for the optimum design at point C in Figure 3.10. None of the points of this curve are to the right of the vertical line. Curves 2 and 3 show the differential sway for the design obtained at points B and D respectively. Both violate the sway criterion.

The optimum design shown in Table 3.5 was obtained by applying the deflection requirements only. However, the design was checked for strength requirements, and this showed that all the sections were satisfactory. It is worth stating here that reverse column taper occurred in the lower five groups of the internal columns. This was because the deflection constraints of the type specified by equation (2.25), Chapter 2, was not included in this design. The reverse column

Group Number		Initial Design Using Okdeh's Results		Optimum Design (at point C)			
		$I \times 10^4$ mm^4	Corresponding Areas from Design Table (A) $\times 10^2$ mm^2	Optimum Section Areas achieved $(A_{op}) \times 10^2$ mm^2	$I_{op} = f(A_{op})$ $(I_{op}) \times 10^4$ mm^4	I from Design Table $(I_{univ}) \times 10^4$ mm^4	Corresponding Areas from Design Table $(A_{univ}) \times 10^2$ mm^2
Beams	1	87260	144.3	149.71	71721.8	75549	129.0
	2	87260	144.3	151.22	73176.0	75549	129.0
	3	87260	144.3	139.97	62693.1	63970	115.9
	4	66610	138.4	134.54	57923.2	63970	115.9
	5	55225	117.6	116.18	43193.0	47363	104.3
	6	40956	113.8	131.60	55419.4	55225	117.6
	7	31330.5*	90.15	103.51	34286.0	36160	104.4
	8	21345	66.5	91.01	26505.0	28522	85.3
	9	12828.5*	54.1	49.3	10054.0	10054	49.3
	10	10054.0*	49.3	49.3	10054.0	10054	49.3
External Columns	11	83150.5*	311.95	189.38	31025.5	40246	164.9
	12	44786.0*	226.75	197.41	33295.0	40246	164.9
	13	32838	174.6	215.59	38674.0	40246	164.9
	14	32838	174.6	164.47	24411.7	27601	149.8
	15	22416	167.7	131.38	16662.8	22202	123.3
	16	22416	167.7	131.38	16662.8	22202	123.3
	17	22416	167.7	131.38	16662.8	22202	123.3

* Means an average section is selected from two different sections chosen by Okdeh.

(CONTINUED)

TABLE 3.5: SECTION PROPERTIES AT THE INITIAL AND THE OPTIMUM DESIGN POINTS FOR THE 24-STOREY 3 EQUAL BAY FRAME

Group Number		Initial Design Using Okdeh's Results		Optimum Design (at point C)			
		$I \times 10^4$ mm^4	Corresponding Areas from Design Table (A) $\times 10^2$ mm^2	Optimum Section Areas achieved $(A_{op}) \times 10^2$ mm^2	$I_{op} = f(A_{op})$ $(I_{op}) \times 10^4$ mm^4	I from Design Table $(I_{univ}) \times 10^4$ mm^4	Corresponding Areas from Design Table $(A_{univ}) \times 10^2$ mm^2
External Columns	18	22416	167.7	87.59	8364.1	11360	92.9
	19	19963.0*	152.15	81.82	7449.2	7647	91.1
	20	15908.5*	125.30	71.59	5936.1	6088	75.8
	21	7775.0*	92.95	46.98	2900.7	4564	58.8
	22	6088.0*	75.80	40.69	2271.8	2218	47.4
Internal Columns	23	152796.0*	514.10	270.89	57016.4	57153	225.7
	24	89552.0*	332.90	295.39	66057.7	66307	257.9
	25	66307	257.9	248.66	49292.4	57153	225.7
	26	58569.5*	225.10	231.16	43541.3	48525	195.2
	27	50832	252.3	253.71	51006.3	57153	225.7
	28	50832	252.3	212.68	37790.8	40246	164.9
	29	50832	252.3	212.68	37790.8	40246	164.9
	30	50832	252.3	194.65	32507.5	40246	164.9
	31	38740	201.2	145.34	19783.6	22202	123.3
	32	32838	174.6	130.36	16443.5	22202	123.3
	33	15908.5*	125.30	132.95	17002.8	22202	123.3
	34	6088	75.80	50.67	3298.6	4564	58.8
Total Exact Weight			1096 KN				891 KN

*Means an average section is selected from two different sections chosen by Okdeh.

TABLE 3.5: (CONTINUED)

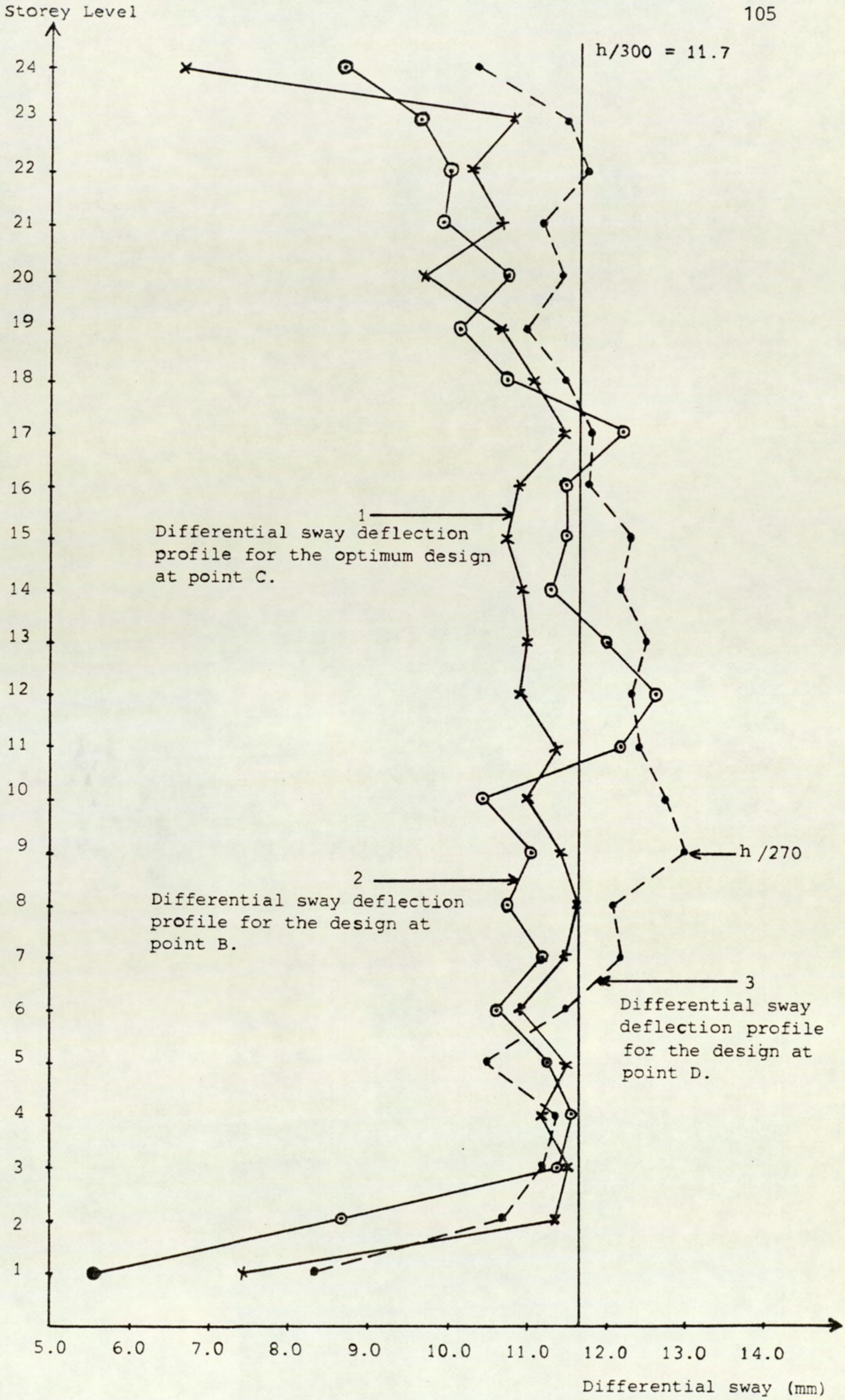


FIGURE 3.11: ANALYSIS OF THE 24-STOREY 3 BAY FRAME, USING THE SECTION PROPERTIES OBTAINED AT DIFFERENT DESIGN POINTS IN FIGURE 3.10

taper also occurred in the 5-storey frame for the same reason. In the next design example, an attempt will be made to avoid the reverse column taper.

3.3.4 Minimum Weight Design under Different Load Conditions

It is unrealistic to design a structure under the effect of one load condition. This is because the vertical and the horizontal design loads might be altered continuously. However, in order to ensure that the section of a structural member is designed for the worst condition which can be reasonably expected, a number of different load conditions must be considered for such a structure.

As stated before, the sway deflection constraints are considered as the governing limit-states for the design of tall sway frames. Thus, any changes in the direction or the value of the horizontal load might cause a considerable effect on the optimum design. The variation of the vertical load does play a small part in such a design, when this load or the frame itself is eccentric. An investigation will be carried out in the next Section on the design of an irregular multi-storey sway frame subjected to two different horizontal load conditions.

3.3.4.1 A 15-Storey, Irregular Frame

This frame was selected from a structure which was after encountered in practice. The structure, shown in Figure 3.12, consists of two non-similar shear walls and three irregular frames of which frame 1 is considered here. The vertical loads for this frame were calculated according to CP110, assuming one of the most critical loading arrangements where all the spans were carrying maximum uniform load. The horizontal wind load was calculated, using CP3 Part 2, for a

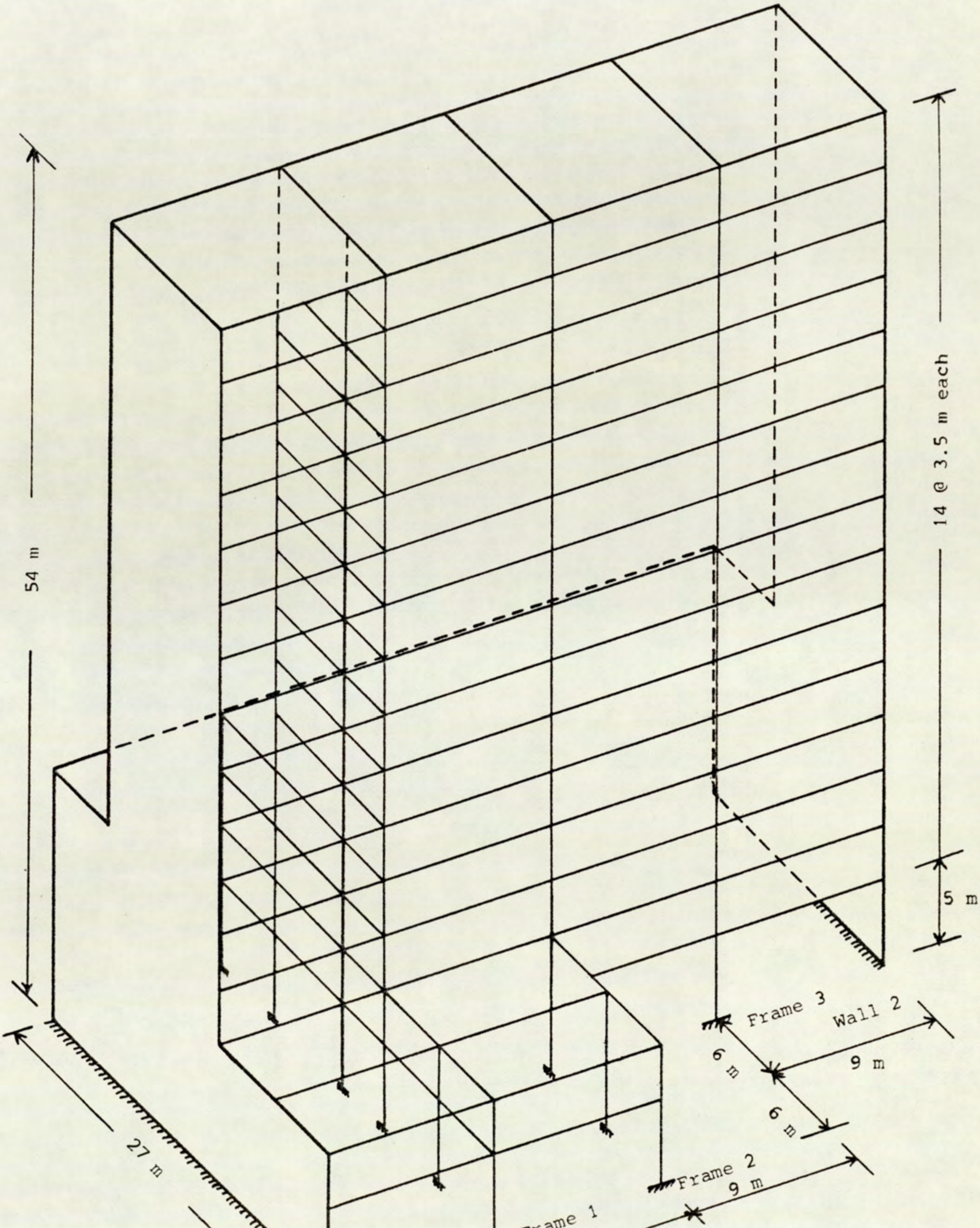


FIGURE 3.12: AN IRREGULAR 15-STOREY STRUCTURE

structure to be built in an open country with scattered wind breaks.

Figure 3.13 shows the way the members of the frame were grouped. The dimensions, the horizontal and the vertical loads are also shown. The different values of intensity for the uniform vertical loads are listed at the top right of the Figure. The frame has 53 joints and 91 members which were grouped into six for beams and 18 for columns. There were a total of 394 constraints and 183 design variables in the design problem. Since it was only necessary to apply move limits on areas, there were consequently 48 constraints. The rest consist of 159 stiffness, 159 deflection, and 15 sway limitation constraints. Another 14 deflection constraints were included, using the inequality (2.25), to prevent reverse column taper. The relative sway limitation was h/350. All the joints had an upper bound on vertical deflections of 40 mm, and on rotations it was 0.08 radian. The boundaries on the section areas were selected similar to the previous examples.

Two optimum design cases were considered. Both of them started with the same initial design point. The first design case was carried under load condition one, where the wind load was acting from the left side, as shown in Figure 3.13. In the second design case the wind load was assumed to be acting reversely, but with the same values, from the right side.

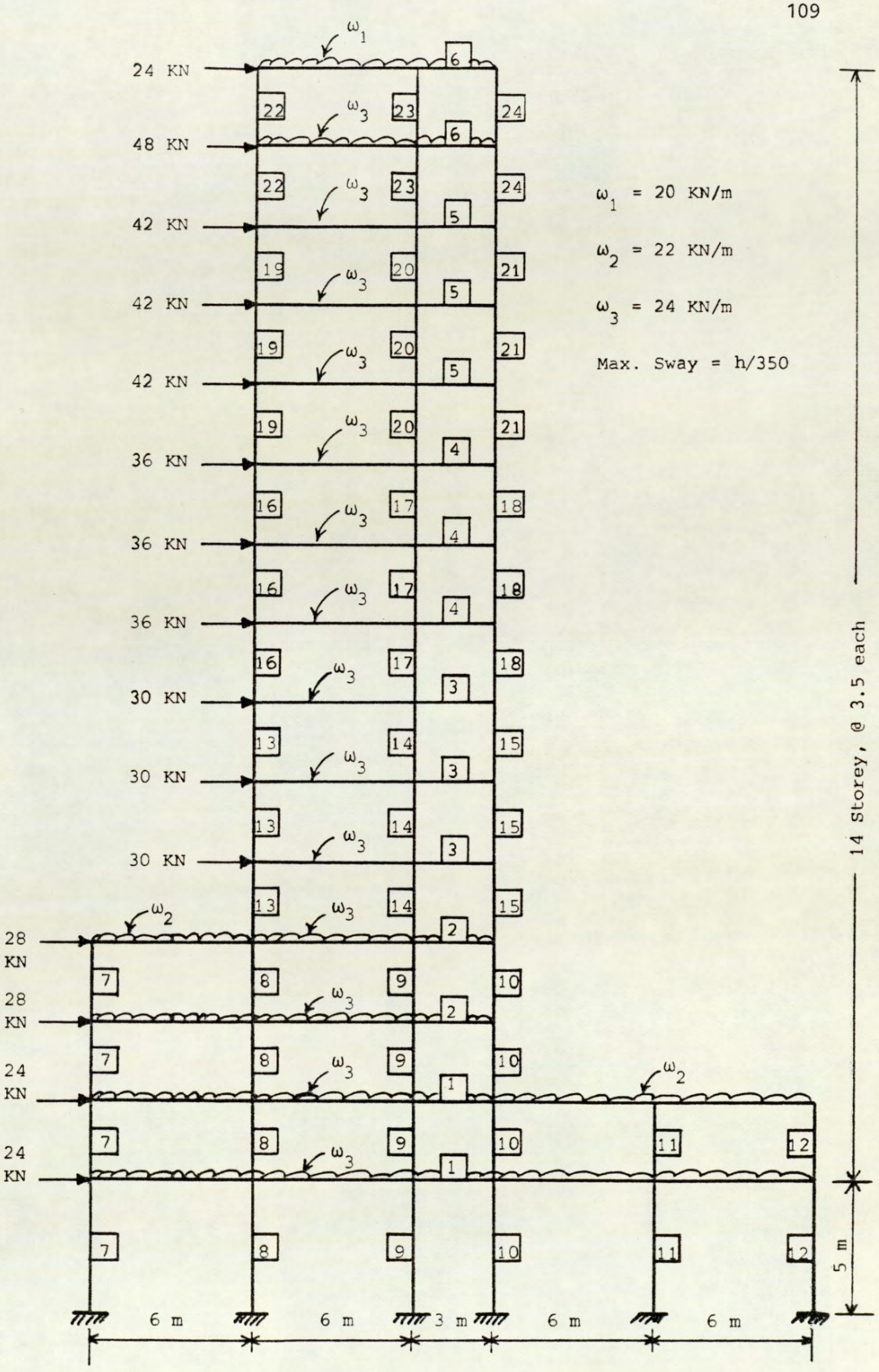


FIGURE 3.13: A 15-STOREY IRREGULAR FRAME - LOADS, DIMENSIONS AND MEMBER GROUPINGS

Figure 3.14 shows two curves which represent the design cases. The points on the curves are equal to the weight of the frame. The initial design was selected arbitrarily. Following the investigation which was carried out earlier, on the effects of move limits and initial design points, it was decided that the move limit should start with ML = 0.5. This was then reduced by 0.1 at each design iteration until convergence was achieved. The initial weight of the frame was 424 KN. The optimum design weight for Case 1 was 528 KN and for Case 2 it was 540 KN. Both cases reached the optimum within six design iterations.

In Table 3.6, the second column shows the initial section areas used for both design cases. The third column contains the areas obtained at the optimum design of Case 1. Notice that column groups 10, 14, 15, 17 and 20 have reached their upper bounds, while group 22 was at the lower bound. The fourth column of Table 3.6 contains the areas obtained at the optimum design of Case 2. Beam groups 3 and 4, and column groups 8, 13, 14, 16, 17 and 19 have reached their upper bounds. The section areas were obtained by the design process assuming continuous sections were available.

The values of the areas in the fifth column of Table 3.6 were selected as the largest areas obtained from either of the two designs. The collection of these areas meant that this design was reliable and that the structure was sufficiently stiff to withstand either of the two load cases. This caused the weight of the frame to be 600 KN which was larger than the optimum weight obtained by either of the two design cases.

The main conclusion achieved from this example is that the optimum design should be carried out under more than one load condition, and the maximum area for each group of members, obtained under the effect

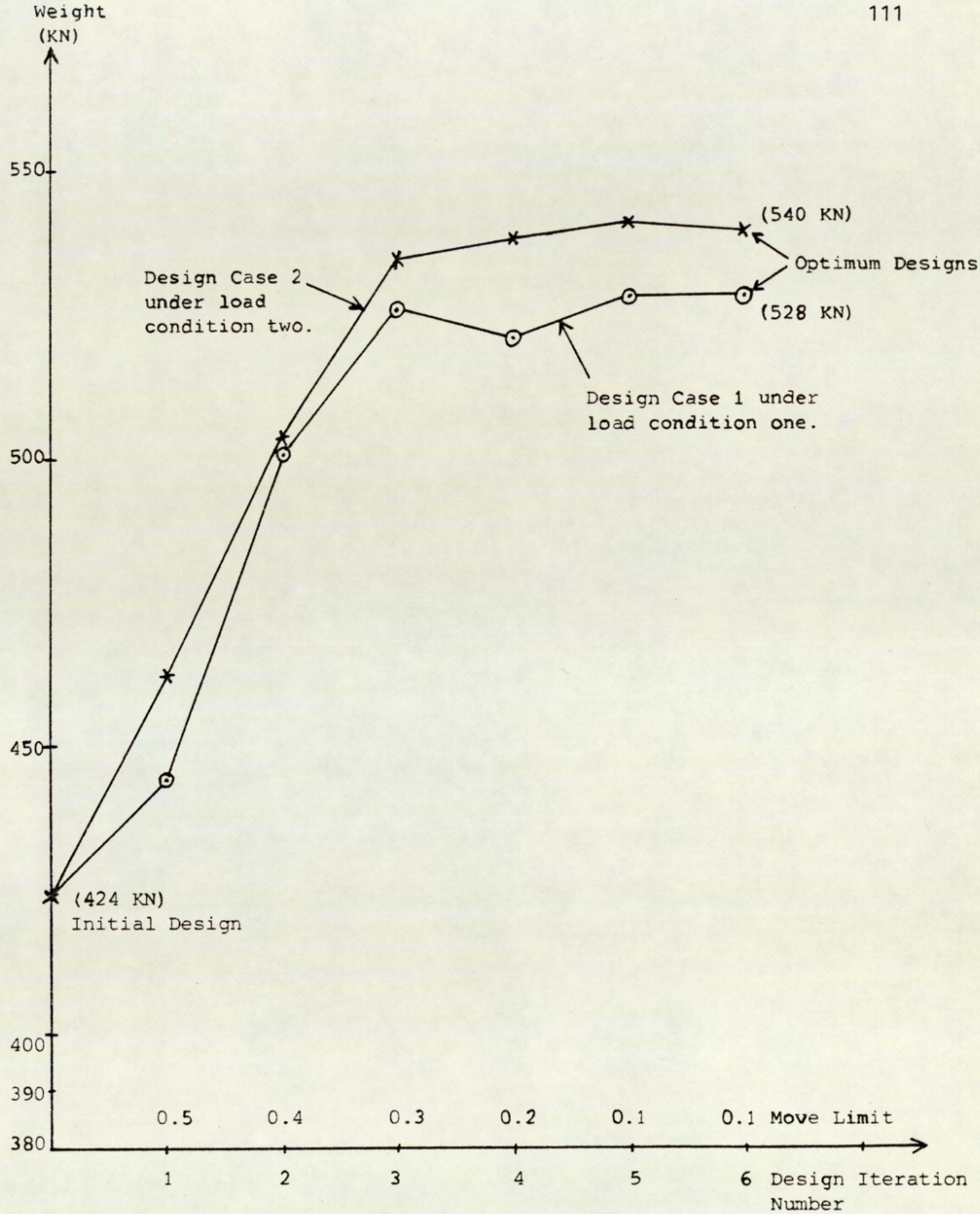


FIGURE 3.14: THE 15-STOREY IRREGULAR FRAME - VARIATION OF WEIGHT IN TWO DESIGN CASES

Group Number		Initial Design A X 10^2 mm²	Optimum Design		Average Design A X 10^2 m²
			Load Condition 1 A X 10^2 X mm²	Load Condition 2 A X 10^2 mm²	
Beams	1	113.8	140.6	143.3	143.3
	2	113.8	185.6	204.5	204.5
	3	113.8	214.8	250.5	250.5
	4	113.8	216.7	250.5	250.5
	5	113.8	137.0	175.4	175.4
	6	113.8	58.5	54.4	58.5
Columns	7	174.6	170.8	253.8	253.8
	8	252.3	232.2	305.6	305.6
	9	252.3	260.0	134.0	260.0
	10	252.3	305.6	228.7	305.6
	11	136.6	275.0	301.9	301.9
	12	136.6	70.4	38.2	70.4
	13	174.6	230.7	252.3	252.3
	14	174.6	252.3	252.3	252.3
	15	174.6	252.3	169.9	252.3
	16	167.7	181.3	252.3	252.3
	17	167.7	252.3	252.3	252.3
	18	167.7	205.7	71.0	205.7
	19	136.6	147.7	201.3	201.3
	20	136.6	201.3	198.5	201.3
	21	136.6	180.5	68.0	180.5
	22	114.0	38.2	136.2	136.2
	23	114.0	176.7	72.8	176.7
	24	114.0	48.7	60.5	60.5
Weight		424 KN	528 KN	540 KN	600 KN

TABLE 3.6: SECTION AREAS OBTAINED FROM DESIGNING THE 15-STOREY FRAME UNDER TWO DIFFERENT LOAD CONDITIONS

of any of these load conditions, should be used at the end. Another thing to be noticed is the disappearance of the reverse column taper. This is due to the inclusion of constraints of the type (2.25) in the design problem.

3.4 TOPOLOGICAL DESIGN OF MINIMUM COST

The topology is defined as the number and the position of the members relative to each other and the manner in which these are linked together to form a stable structure. The topology of a frame is a significant matter which needs to be included as an extra unknown in the design problem. Such topology is decided upon during the design process, purely by structural and economic factors. It should be pointed out that the functional requirements of a multi-storey frame often decide the number and the position of the beams and the external columns. The engineer, however, is left to decide the number and the position of the internal columns which are economical problems. Indeed, architects frequently require to minimise the number of these columns.

The minimum cost topological design method can be applied on a frame with a fixed topology, but the two examples selected both can have variable topologies.

3.4.1 An Outline of the Design Procedure

The design problem introduced intends to alter the topology of the sway frame, and at the same time tries to minimise its cost. The alteration could be done by removing members and joints from the original structure. The members required to be removed are mainly the internal columns. This is because the number and the position of the external columns and the beams are decided beforehand by the function of the frame.

To improve the ability of arriving at a globally optimum design, it is advised not to remove members at the early stages because the areas of the members may bear false relationships to each other. Instead attention is directed to reduce the weight without altering the shape of the structure, thus improving the relationships between the member areas. For this reason the design procedure is decided to consist of the following steps:

(1) Carry out a minimum weight design from step 1 to step 11 as described in Section 3.3.1.

(2) For the frame design achieved, calculate the fixed charge R_i as the cost of retaining each member, or each group of members. The way the fixed charge is computed was explained in Chapter 2.

(3) The lower bounds on the sections of the beams are selected so that criterion (a), Section 3.2, is satisfied. These lower bounds also retain the beams in the final topology. Since the topology of the frame is continuously changing during the design process, and as columns are removed, the lengths of the beams are changing and consequently the lower bounds on the beams may alter continuously.

(4) The section areas obtained at the end of the mimimum weight design are used to solve the stiffness equations of the frame. These areas and the resulting displacements are used as an initial design point for the minimum cost topological design.

(5) Derive the objective cost function and the linear forms of the design constraints, and transfer them, one row at a time, to the backing store. Since the topology of the frame is continuously changing during the design process, the constraints and the cost function are also changing.

(6) Use the simplex method and minimise the objective cost function.

(7) Remove all members or groups with $\delta = 0$ and, if desired, remove members with $\delta \cong 0$. If members with $\delta \cong 0$ are not removed at this stage, their δ value may change later, during the design process. This will improve the outcome but may increase the computer time.

(8) Repeat the process from step 2 until no further topological change is obtained. Notice that the steps are somewhat similar to those of the minimum weight design.

(9) Carry out the minimum cost design of the frame with its final topology until an optimum cost is achieved. The feasible frame obtained before this step was for the linearised problem and may not be feasible when the non-linear constraints are checked.

(10) At the end of the last step, criteria (b) and (c) should be satisfied. The way to achieve this is by using step 12 and step 13 of the minimum weight design procedure.

3.4.2 A 4-Storey Frame with Thirteen Equal Bays

This frame is to cover a span of 26 m and to have four storeys that make the total height of 17 m. It consists of 56 joints and 108 members. These members are gathered together into 18 groups, four for beams and 14 for columns. The details of dimension, grouping of members and loading are all shown on the ground structure in Figure 3.15. The large number of columns presented may reduce the sway in this ground structure to the extent that this initial frame may be disqualified from being classified as a sway frame. The beams of each storey belong to one group, and the columns are grouped symmetrically. It is required to retain the outer columns. Beams can have any span but, for structural reasons, it is considered (but not strictly specified) that in this example the beam should not exceed a clear span of 10 m. This means some inner columns should be retained as well.

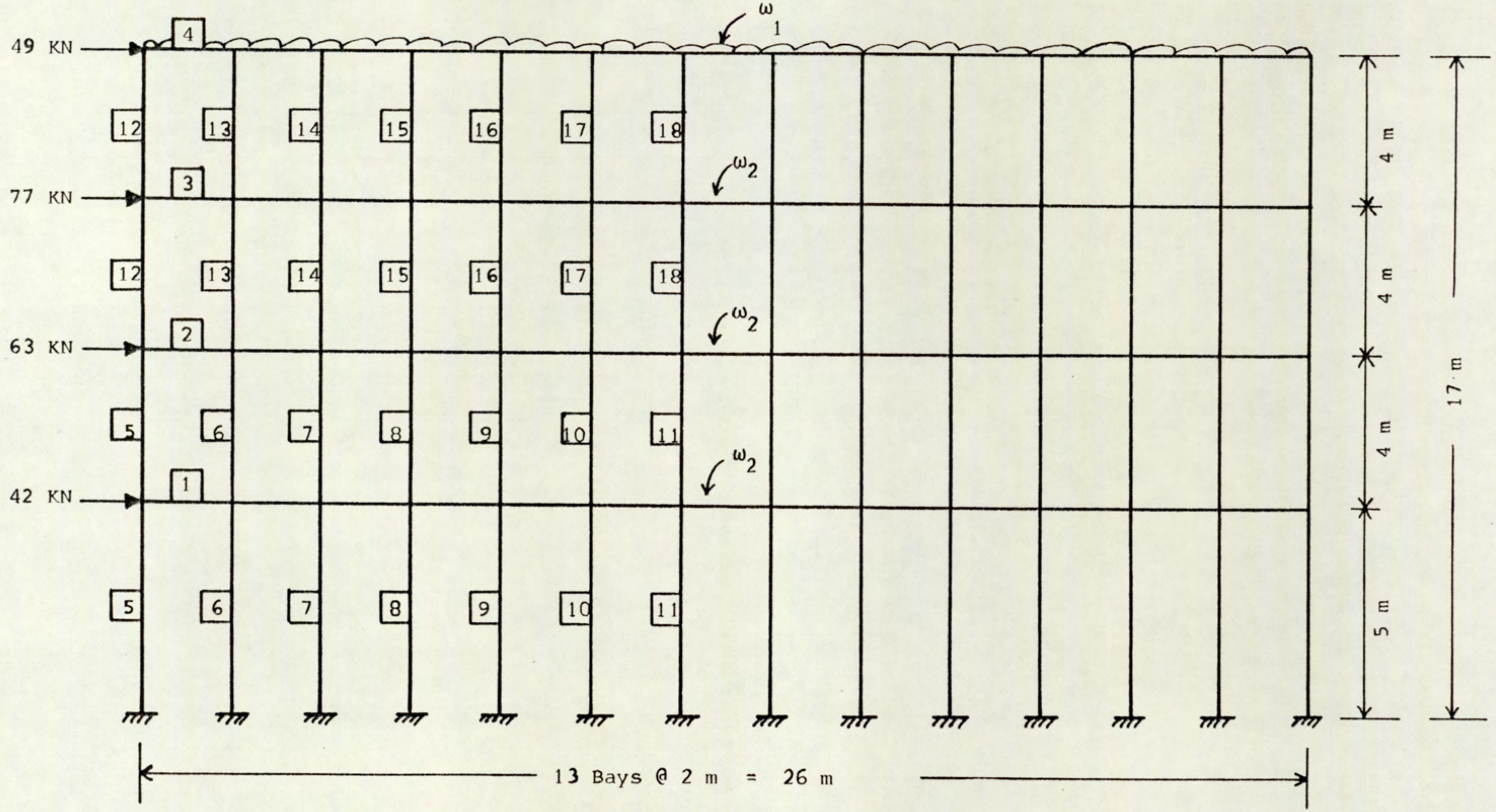


FIGURE 3.15: A 4-STOREY 13 EQUAL BAY FRAME - DIMENSIONS, LOADING AND MEMBER GROUPING

The vertical live load is chosen to be uniformly distributed with intensities of 30 KN/m on the roof and 36 KN/m on the other floors.

The upper bound on the relative sway between the storeys was h/400. The upper bounds on the joint deflections were taken as in the previous examples. To retain the beams and the outer columns, a lower bound was imposed on their areas. For the beams, the lower bound was considered to satisfy criterion (a), and this was 28.4×10^2 mm^2. The lower bound for the columns was chosen to be the smallest column section available, and that was 38.2×10^2 mm^2. The upper bound on the beams was chosen to be 113.8×10^2 mm^2, and for the columns of the first two storeys it was 167.7×10^2 mm^2, while for the columns of the third and the fourth storeys it was 122.0×10^2 mm^2.

The design procedure started with an arbitrary set of sections, these are listed in the second column of Table 3.7. The frame was analysed using these sections, and the loads applied on foundation were computed. Each of the independent foundations (i.e. pad footing) was then designed individually and its construction cost was assessed. Such cost was kept constant throughout the entire design operation.

The total weight of the frame at the starting point is 368 KN which is represented by point A in Figure 3.16. The assessment method described before was used to obtain point A' in the Figure. This point represents the total cost which includes the construction cost of foundation, the cost of the material and the erection of the frame. The overall value of the cost at the starting point was £29,200. Figure 3.16 also shows the manner in which the total weight and the corresponding total cost of the frame varied during the entire operation. Both of them were computed automatically at each iteration. During the minimum weight design, lower bounds were imposed on all the sections to prevent the removal of any member. The design with an

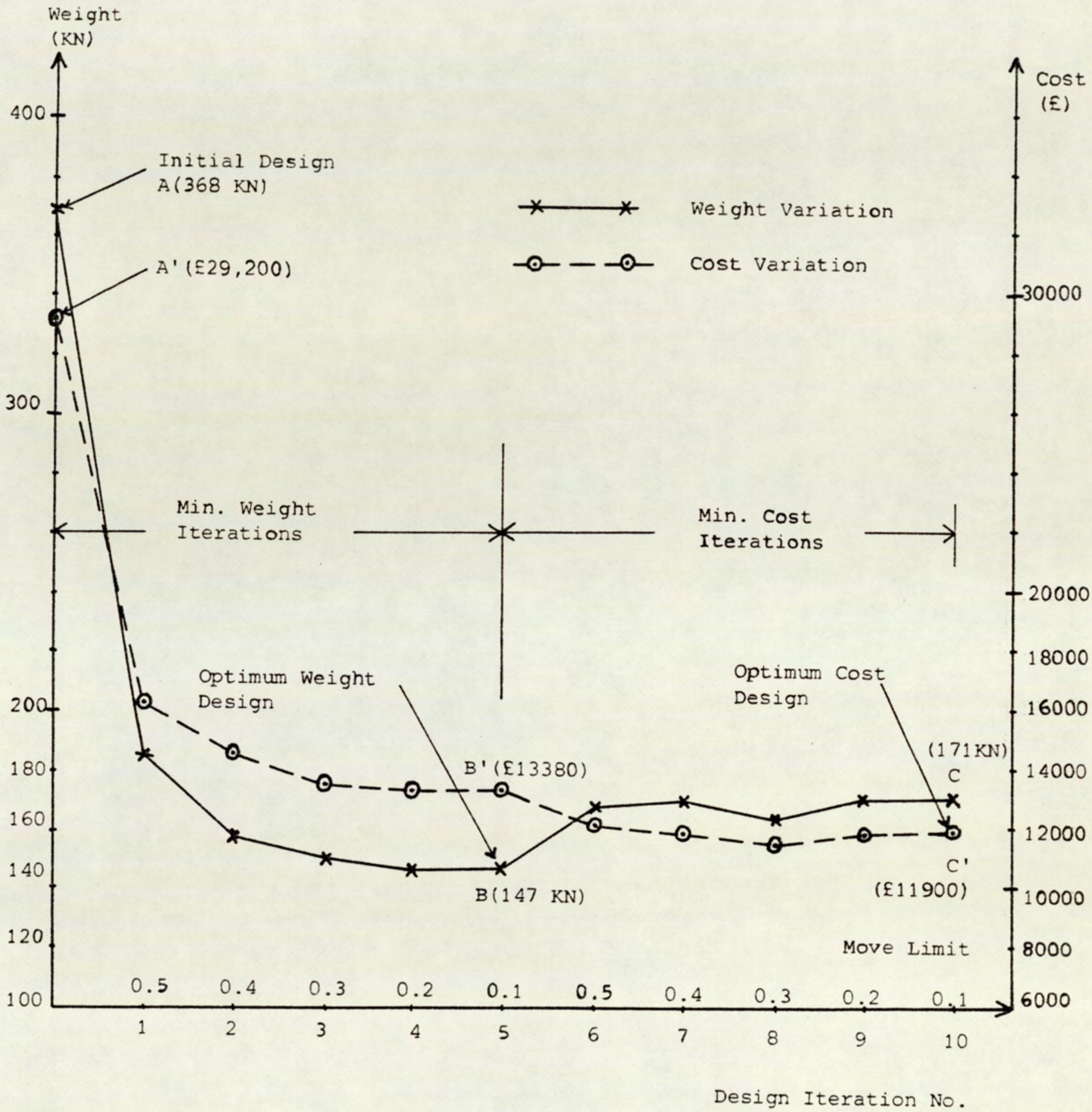


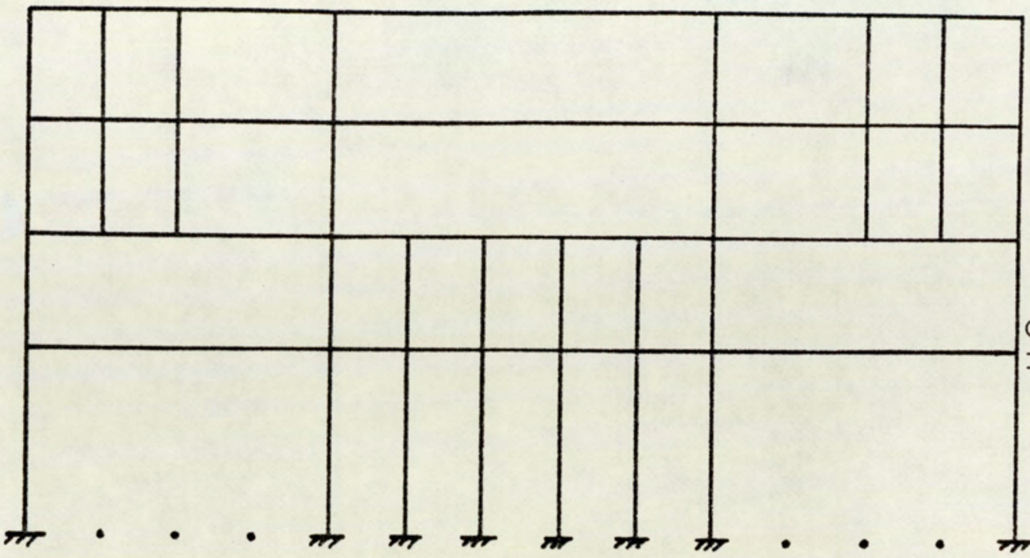
FIGURE 3.16: VARIATION OF TOTAL COST AND TOTAL WEIGHT FOR THE 4-STOREY 13 BAY FRAME

optimum weight of 147 KN was obtained at point B in Figure 3.16. The total cost at this point was £13,380 which is represented by point B'. The initial value of the move limits was 0.5 and then reduced by 0.1 at each design iteration.

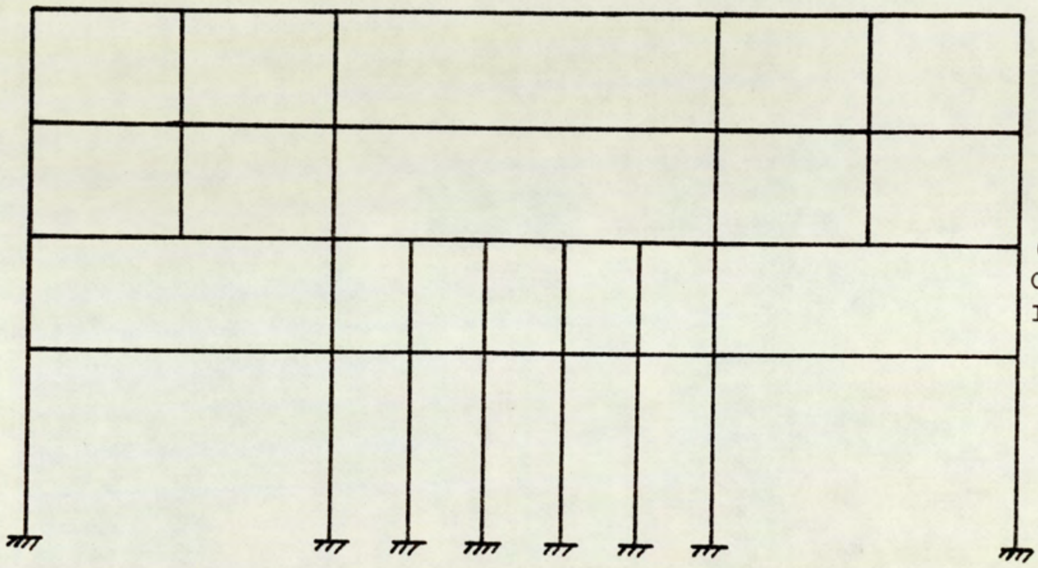
The minimum cost design was then started with the set of sections achieved at point B. Because it was not recommended to exceed a span of 10 m, columns in groups 5, 9, 12 and 16 had lower bounds imposed on them and on the beams so that they could be retained. The move limit was arranged similar to that in the minimum weight design.

Various topologies, obtained during the minimum cost design, are shown in Figure 3.17. The shape of the structure shown in Figure 3.17a was obtained at iteration six of the design. At this iteration, members belonging to column groups 6, 7, 8, 15, 17 and 18 are all removed. The total cost was decreased due to the elimination of the costs of foundation, material and construction of some columns. But the decrease was small because the erection cost of other members was increased. The overall weight of this shape of the frame was increased. This was due to the fact that new large lower bounds on the beams were used to satisfy criterion (a).

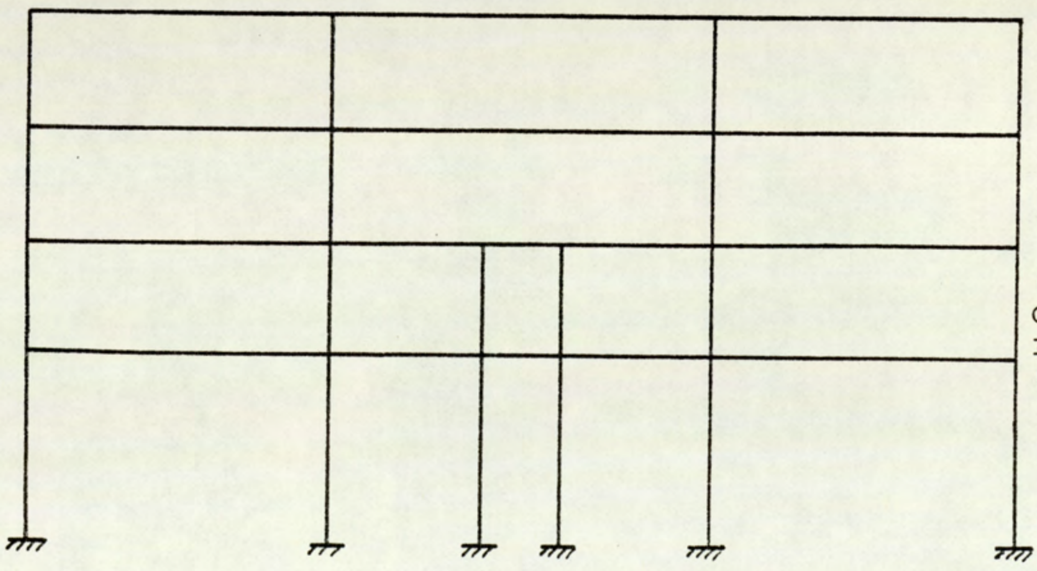
The shape of the frame shown in Figure 3.17b was obtained at iteration seven. Here, group 13 was removed and the cost reduced, but the weight slightly increased. The process continued at iteration eight with a decrease in both the weight and the cost when groups 10 and 14 were removed, Figure 3.17c. The deflection requirements here were not all satisfied. The topology of the frame did not change after this iteration. The minimum cost design was continued until an optimum cost of £11,900 was obtained after two more iterations. The value of this cost is represented by point C' in Figure 3.16, and the weight here was 171 KN at point C. The final shape derived is shown in Figure



(a)
Obtained at
Iteration 6



(b)
Obtained at
Iteration 7



(c)
Obtained at
Iteration 8
(Final Shape)

FIGURE 3.17: SHAPES OBTAINED FOR THE 4-STOREY 13 BAY FRAME AT DIFFERENT ITERATIONS OF THE MINIMUM COST DESIGN SHOWN IN FIGURE 3.16

Group Number		Initial Design at (A) A X 10^2 mm^2		Optimum Weight Design at (B) A X 10^2 mm^2		Optimum Cost Design at (C) A X 10^2 mm^2	
Beams	1	189.9		71.69		105.10	
	2	159.4		39.77		95.20	
	3	144.3		36.00		104.40	
	4	129.1		28.4		94.9	
Columns	5	92.9		38.2		137.70	
	6	136.6		38.2		—	
	7	167.7		38.2		—	
	8	174.6		38.2		—	
	9	201.2		38.2		137.70	
	10	212.4		148.00		—	
	11	225.7		167.7		167.7	
	12	58.8		38.2		122.0	
	13	66.4		42.00		—	
	14	75.8		52.10		—	
	15	91.1		65.80		—	
	16	91.1		59.00		122.0	
	17	114.0		38.2		—	
	18	114.0		39.77		—	
Weight/ Cost		368 KN	£29,200	147 KN	£13,380	171 KN	£11,900

TABLE 3.7: OPTIMUM SECTION AREAS OBTAINED AT THE DESIGN OPERATION SHOWN IN FIGURE 3.16, FOR THE 4-STOREY 13 EQUAL BAY FRAME

3.17c. The values of the section areas obtained are listed in the fourth column of Table 3.7. All the deflection requirements and the design criteria were satisfied.

3.4.3 A 9-Storey, Irregular Frame

This frame is selected for the purpose of investigating two design aspects. The first one is a topological design of minimum cost, and the second aspect is a topological design of minimum weight. For the latter one, the topology of the frame is allowed to be altered, by removing columns, for the sake of minimising the total weight.

The details of loading and dimension of the ground structure are shown in Figure 3.18, while the members grouping is illustrated in Figure 3.19. The frame consists of 56 joints and 103 members which collected together into 22 groups, four for beams and 18 for columns. In addition to the beams, six column groups are required to be retained in the final shape, and these are groups 5, 6, 7, 8, 9 and 10.

The design problem of this frame consists of 202 variables, 22 of which are areas, 168 of which are displacements and the last 12 of which represent the δ variables for the 12 removable column groups. The problem also contains a total of 388 constraints, 168 of which are for stiffness and another 168 are for deflection. The application of move limits on the retained groups, i.e. four beams and six columns, creates ten upper bound and another ten lower bound constraints. There are eight relative sway deflection constraints. The removable groups have 12 constraints of type (2.34), Section 2.9; and another 12 constraints represent type (2.35).

The upper bound on the relative sway was h/350. The limitations on the joint deflections were taken similar to the previous examples. Lower bounds on the group areas were represented by the smallest

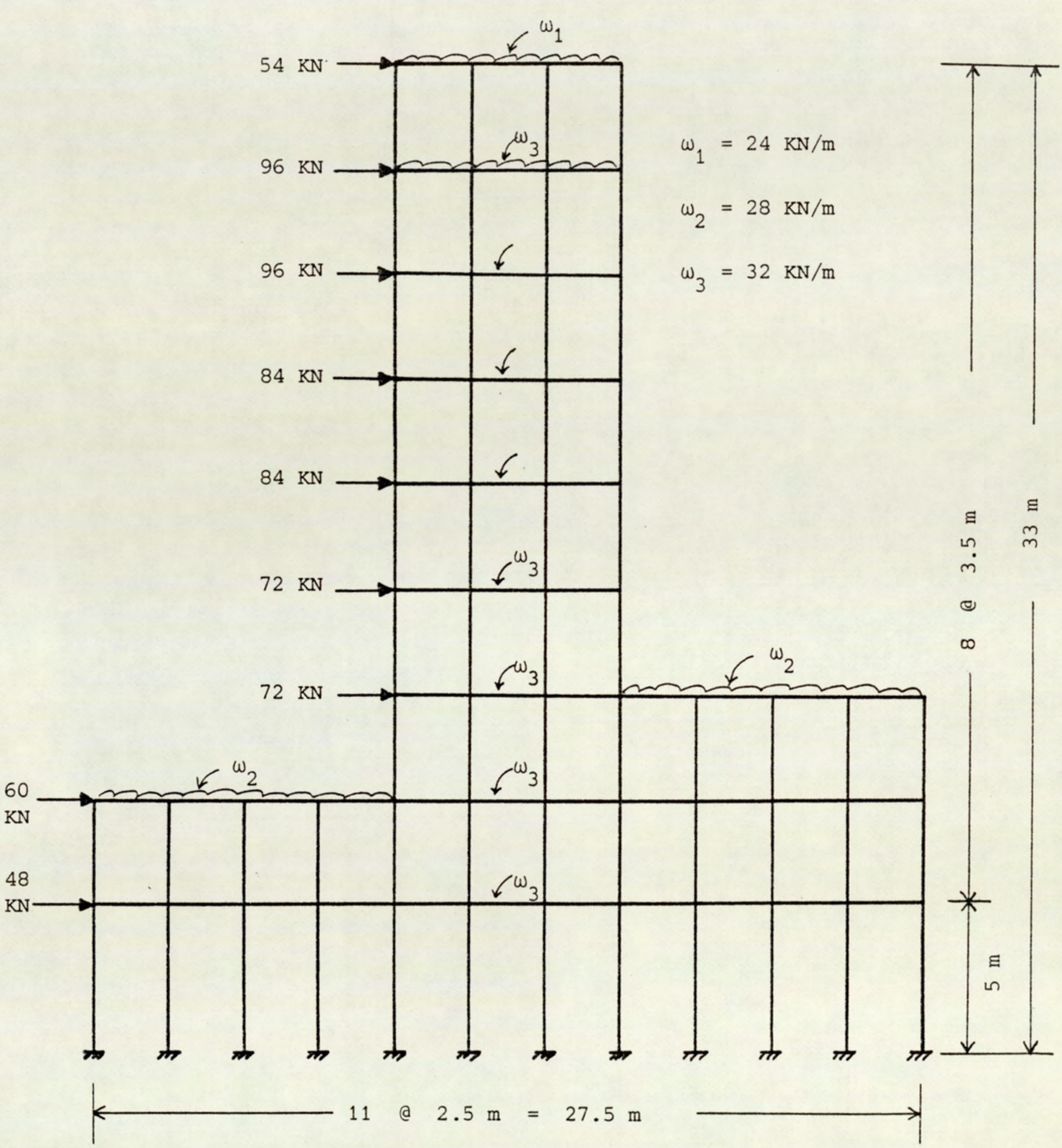


FIGURE 3.18: A 9-STOREY IRREGULAR FRAME, LOADS AND DIMENSIONS

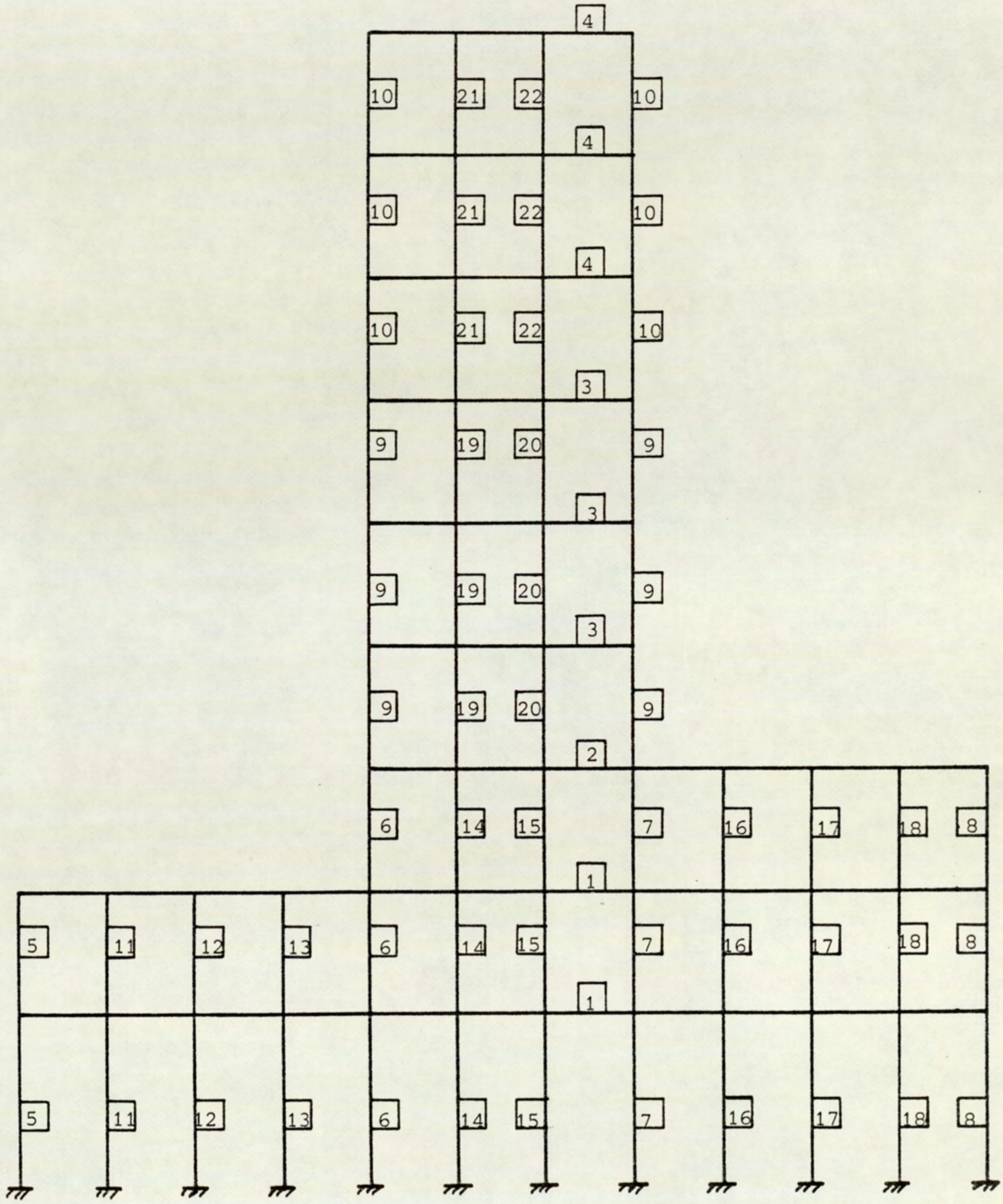


FIGURE 3.19: GROUPING OF MEMBERS FOR THE 9-STOREY IRREGULAR FRAME

available section that satisfies the design criteria (a) for beams, or (c) for columns. For either types of members, the largest available sections were taken as the upper bounds.

Three design cases were considered, all of them started with the same set of sections. This set was chosen arbitrarily, and it was listed in the second column of Table 3.8. In the first case, no fixed charge was involved, the frame was designed for a minimum weight and its topology was taken as a variable. In other words, the topology of the frame was permitted to change during the design operation. This was done by removing the member which had an area less than the declared lower bounds. Such a procedure applied in all the design iterations. All the areas of the member groups were bounded, except the removable column groups, i.e. 12 groups, where there were no lower bounds imposed on them. Thus, only these groups were allowed to be removed. It was true, in this design case, that some of the members were removed at the early stages of the operation, but there was almost always a non-feasible solution obtained later. Although different initial values of move limits were used, the problem never converged into a feasible design. This indicates that a minimum weight philosophy is unrealistic when the topology of the frame is a design variable. This case also disappoints advocates of topological design (Majid, Saka, 1977) and (Dorn, et al, 1964) who expected that the minimum weight topological design of the ground structure should by itself eliminate some members and obtain an optimum solution.

In Case 2, a minimum cost topological design of the frame was investigated. The first part of the operation is a minimum weight design where there is no member allowed to be removed. The initial section areas, shown in Table 3.8, were used to assess the cost of foundation. The weight of the frame at the starting point was 349 KN and the assessed total cost is £28,718. At the end of the minimum weight design, the optimum section areas obtained are listed in the third column of Table 3.8. The weight here was 345 KN and the cost was £28,341. After that, the minimum cost design was started. Some of the member groups were forced to be retained by imposing lower bounds on their section areas. These groups were classified in the beginning of this Section. Different topologies of the frame were obtained during this part of the operation. The final one, shown in Figure 3.20a, was derived when the total cost converged to a minimum value of £26,600. The last topology was obtained by computer, using the design specifications mentioned before. This shape of the frame was obviously unacceptable to the engineers, because column group 15 should have been retained if column group 21 was kept.

In Case 3, the frame was also designed for a minimum cost. The first part of the operation, which was a minimum weight design, was exactly the same as Case 2; the values obtained were all plotted in Figure 3.21. The minimum cost design started from point B in the Figure. Group 15 was decided to be retained, and this was done by imposing a lower bound on its area. This design case gave the frame in Figure 3.19b by the end of the operation. Such a shape is considered to be logical and acceptable to engineers. The optimum cost design was obtained at point C', Figure 3.21, where the cost was £27,601, and the weight of 363 KN is represented by point C. The section areas obtained are listed in the fourth column of Table 3.8. The cost of the material and the construction of each group of members is listed at the fifth column of the Table. The section areas of beam group two and column group seven have reached their upper bounds. All the deflection and strength requirements were satisfied.

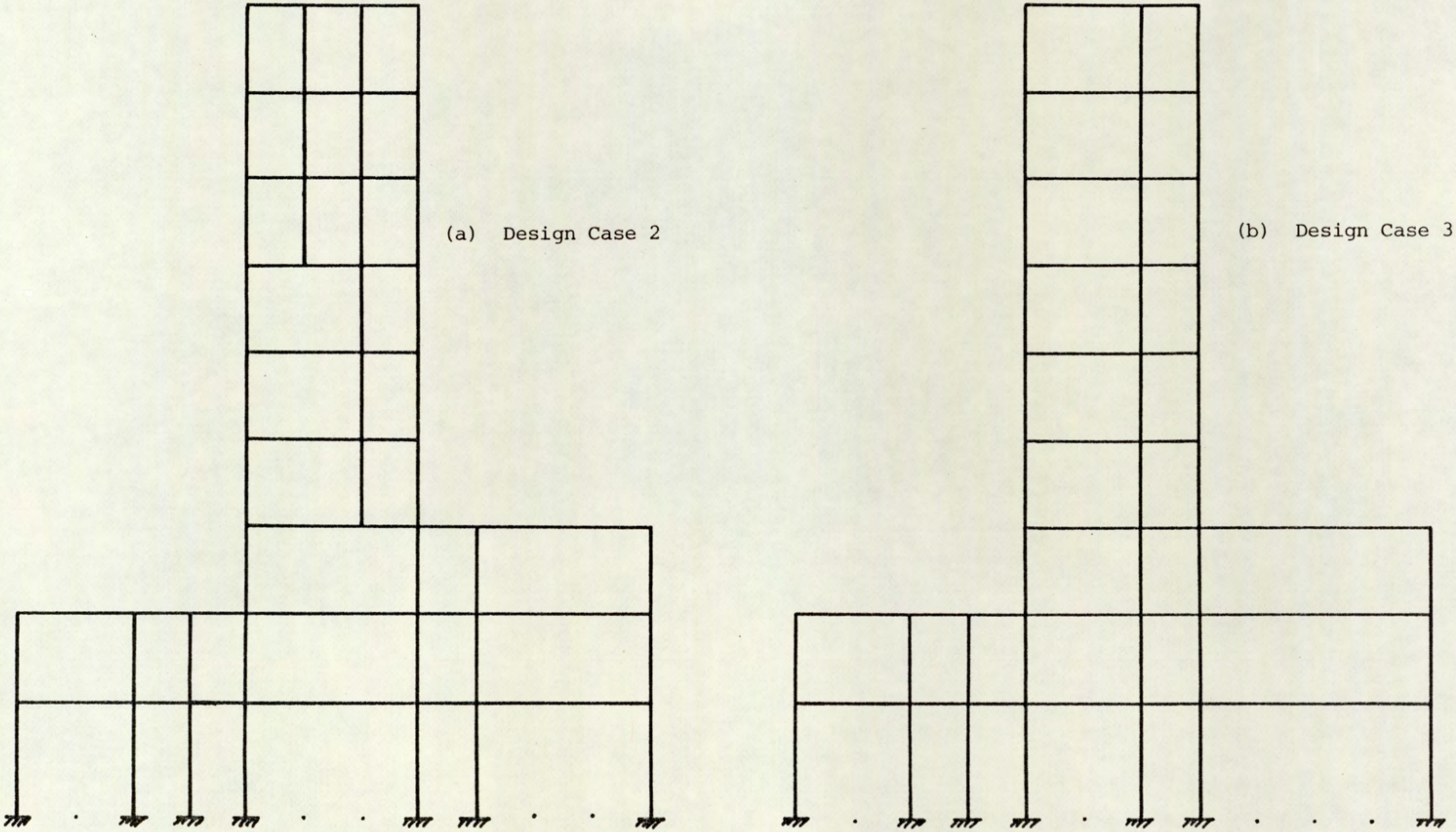


FIGURE 3.20: FINAL SHAPES OBTAINED FOR THE 9-STOREY IRREGULAR FRAME AT DESIGN CASES 2 AND 3

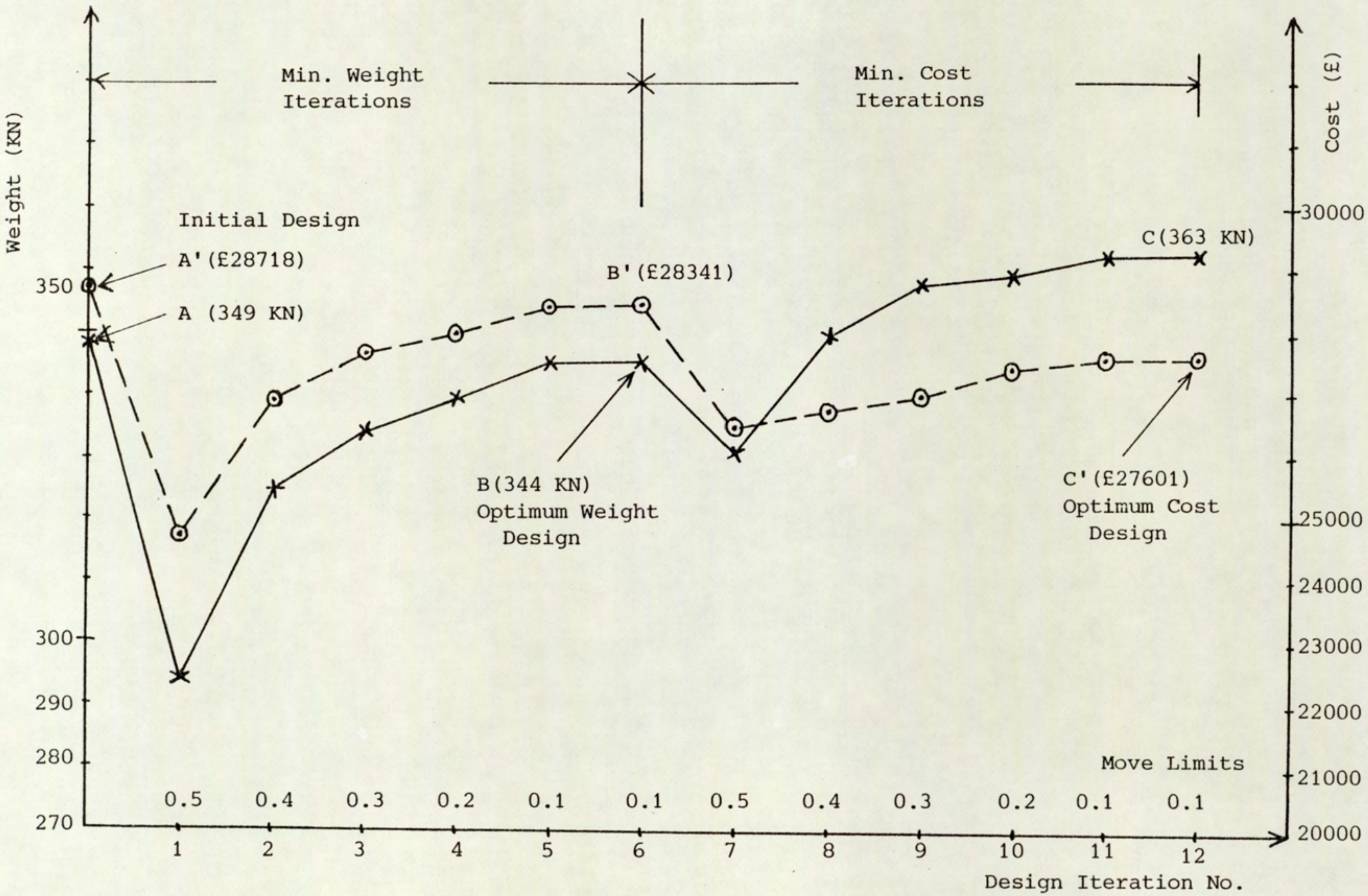


FIGURE 3.21: VARIATION OF TOTAL COST AND TOTAL WEIGHT FOR THE 9-STOREY IRREGULAR FRAME AT DESIGN CASE 3

Group Number		Initial Design at point (A) A X 10² mm²		Optimum Weight Design at (B) A X 10² mm²		Optimum Cost Design at point (C)	
						A X 10² mm²	Cost (£)
Beams	1	155.6		101.97		114.72	3,495
	2	138.4		140.85		303.5	2,942
	3	129.1		135.65		199.41	2,485
	4	114.6		81.46		163.54	2,038
Columns	5	92.9		34.8		95.00	647
	6	201.2		77.50		211.59	1,806
	7	201.2		354.72		432.7	3,276
	8	114.0		34.8		68.92	758
	9	136.6		158.38		256.59	2,985
	10	114.0		70.56		192.59	2,240
	11	92.9		34.8		—	—
	12	92.9		223.31		60.20	483
	13	92.9		432.7		169.00	796
	14	201.2		34.8		—	—
	15	201.2		34.8		274.20	1,823
	16	114.0		343.55		—	—
	17	114.0		34.8		—	—
	18	114.0		34.8		—	—
	19	136.6		186.52		—	—
	20	136.6		281.39		211.03	1,227
	21	114.0		166.22		—	—
	22	114.0		254.77		102.95	599
Weight/ Cost		349 KN	£28,718	345 KN	£28,341	363 KN	£27,601

TABLE 3.8: OPTIMUM SECTION AREAS OBTAINED AT THE DESIGN OPERATION SHOWN IN FIGURE 3.21, FOR THE 9-STOREY IRREGULAR FRAME

3.4.4 Discussion on the Minimum Cost Design

The method proposed in the previous Sections for the topological design of minimum cost of steel sway frames appears to be similar to the method proposed by Majid, Stojanovski and Saka (1980). Nevertheless, fundamental differences exist between the two methods. These are:

(1) The costs of the material, foundation, erection, etc., were assessed more realistically and correctly, while Majid, et al, assumed an arbitrary lump sum cost which can affect the topology of the structure.

(2) Majid, et al, suggested that higher charges speed up member removals, and higher initial areas for the beams were also encouraging the removal of the columns. Instead the author started his topological design by first obtaining a minimum weight design and then proceeded to obtain the minimum cost frame. The consideration of a minimum weight design as the first part of the whole operation was proved to be essential for a better relationship between the member areas. By following such a policy, the minimum cost topological design will not depend on an arbitrary set of initial section areas.

(3) Majid, et al, did not impose move limits during the solution of the linearised problem. The author found that in many cases this gave infeasible solutions. For this reason the author imposed upper bound move limits on all the members, even on those required to be removed from the final design. The lower bounds were imposed only on the members required to be retained.

CHAPTER 4

TOPOLOGICAL DESIGN OF MINIMUM COST FOR LATERALLY LOADED COMPLETE STRUCTURES

4.1 INTRODUCTION

In this Chapter a method is proposed for a minimum cost design of complete building structures consisting of skeletal frames, together with a grillage of shear walls and floor slabs. The efforts are focused on minimising the material and the construction costs of such structures. This could be achieved, for instance, by reducing the number of the intermediate bracing frames and the shear walls required for a given grillage system.

The structures are designed to resist the separate action of wind loads. The horizontal equilibrium and deflection requirements should be satisfied at the junctions of the floors with the vertical components. The lateral deformation of a complete structure is not affected by the vertical loading unless this loading is eccentric. The design method presented here does not include the effect of eccentric walls and frames and eccentric vertical loading. The vertical loading is not considered in this design method.

The matrix displacement method is employed to form the overall stiffness matrix of the structure. It is assumed that the grillage members act as deep beams under the action of bending moments and shear forces in their own plane. For the frames, only the lateral "shear" stiffness coefficients are considered, which are added to those of the grillage to form the overall stiffness matrix. The effect of axial strains in the columns of the frames is excluded.

The sequential approximating linear programming is used to construct the mathematical model for the design problem. In this model the objective function, which is minimised, represents the total cost of the structure. The design requirements are expressed by stiffness, deflection and other practical constraints. For a symmetrical structure, use is made of symmetry to reduce computer time and storage.

4.2 THE EFFECT OF SHEAR STRESS ON THE STIFFNESS OF A RIGIDLY CONNECTED MEMBER

The displacements of a rigidly connected member are the result of the combined action of bending and shear stresses. When the member is relatively slender the effect of the shear stress can be assumed to be insignificant and may be neglected. The member stiffnesses of equation (2.4), Chapter 2, are based on this assumption. It is well established that in the case of a deep member such as a shear wall panel, however, the distortion caused by the action of the shear stress may be important.

For a deep member rigidly connected at ends 1 and 2, the forces and the displacements at the ends of the member are related by the equation:

$$\underline{P} = \underline{k}\ \underline{U} \tag{4.1}$$

where $\underline{P}$ is the vector of member forces ($S\ M_1\ M_2\ T$), $\underline{U}$ is the corresponding vector of member displacements ($v\ \theta_1\ \theta_2\ \theta_T$) and $\underline{k}$ is the matrix of member stiffnesses. The elements of the vectors are defined in Figure 4.1, where S is the shear force, M_1 and M_2 are the moments at ends 1 and 2 respectively, T is the torque, v is the lateral deformation, θ_1 and θ_2 are the rotations, and θ_T is the angle of twist for the member.

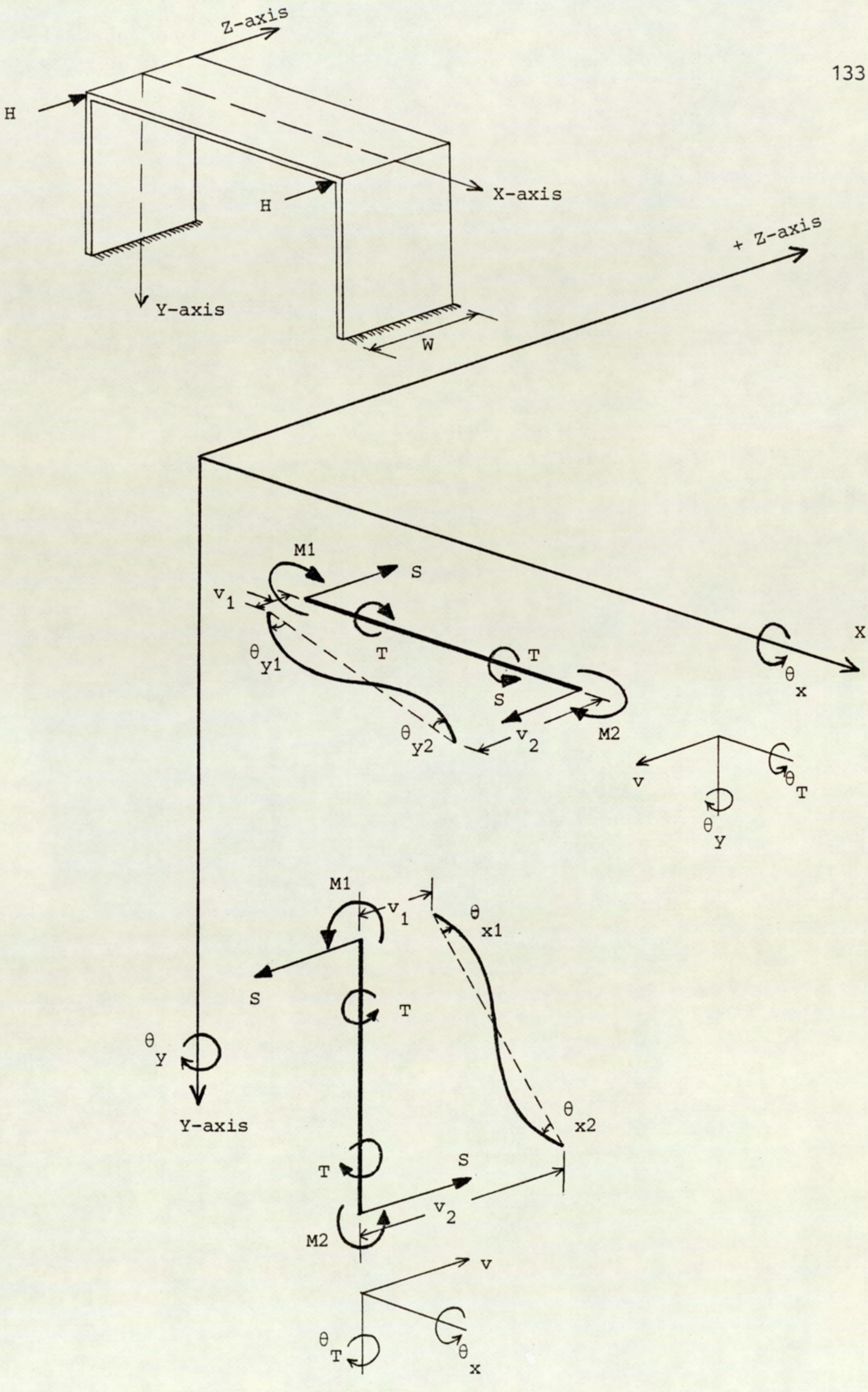


FIGURE 4.1: SIGN CONVENTION FOR GRILLAGE

Croxton (1974) has shown that if shear distortion is taken into account, the stiffness coefficients contributed by a deep member of rectangular section is given as:

$$\begin{bmatrix} S \\ M_1 \\ M_2 \\ T \end{bmatrix} = \begin{bmatrix} b & d & d & o \\ d & e & f & o \\ d & f & e & o \\ o & o & o & q \end{bmatrix} \begin{bmatrix} v \\ \theta_1 \\ \theta_2 \\ \theta_T \end{bmatrix} \tag{4.2}$$

where:

$$b = \frac{12EI}{L^3}\left(\frac{1}{1+2\psi}\right)$$

$$d = \frac{-6EI}{L^2}\left(\frac{1}{1+2\psi}\right)$$

$$e = \frac{4EI}{L}\left(\frac{1+0.5\psi}{1+2\psi}\right)$$

$$f = \frac{2EI}{L}\left(\frac{1-\psi}{1+2\psi}\right)$$

$$q = \frac{G t^3 W}{3L}$$

in which E is the Young's Modulus, L is the length of the member, I is the second moment of area, t is the thickness of the rectangular section and **W** is its width. The values in brackets are shear distortion factors based on a parabolic shear stress distribution. The value of the constant ψ (Psi), in equation (4.2), depends on the geometrical proportions of the member (Majid and Croxton, 1970), and is given as:

$$\psi = 7.2EI/L^2AG \tag{4.3}$$

where A is the section area, and G is the shear modulus which can be expressed in terms of the Young's Modulus as:

$$G = E / 2 . (1 + \nu) \tag{4.4}$$

where ν is Poisson's ratio.

In the design method proposed in this Chapter, the thickness (t) of a concrete grillage panel is considered as one of the main design variables. Therefore it is preferred to formulate the coefficients of equation (4.2) in terms of (t).

The panels of the grillage, i.e. the shear walls and the floor slabs, are considered as deep beams of rectangular sections with thickness t, width W and area A = W t. The second moment of area (I) of the cross section about the centroidal axis, for any of these panels is:

$$I = t \; W^3 \; / \; 12 \tag{4.5}$$

Using this, equation (4.3) becomes:

$$\psi = 7.2 \; E \; W^2 \; / \; 12 \; L^2 \; G \tag{4.6}$$

which shows that when the span L becomes large compared with the width of the member, ψ becomes very small and can be neglected. This reduces the stiffness equation (4.2) to that of a slender or prismatic member. It is noticed that ψ is not a function of t, i.e. does not depend on the thickness of the grillage panel. Using equation (4.5), the stiffness coefficients of equation (4.2), can be rewritten as:

$$b = \frac{E\,t\,W^3}{L^3} \left(\frac{1}{1 + 2\psi}\right)$$

$$d = -\frac{E\,t\,W^3}{2\,L^2} \left(\frac{1}{1 + 2\psi}\right)$$

$$e = \frac{E\,t\,W^3}{3L} \left(\frac{1 + 0.5\psi}{1 + 2\psi}\right)$$

$$f = \frac{E\,t\,W^3}{6L} \left(\frac{1 - \psi}{1 + 2\psi}\right)$$

$$q = \frac{G\,t^3\,W}{3L} \tag{4.7}$$

In which the coefficients are functions of a single design variable, and that is the thickness (t) of the grillage panel.

4.2.1 Stiffness Matrix of a Grillage

The floor slabs and the solid shear wall panels of the complete structure are considered to act as the horizontal and vertical members of a grillage loaded by the wind forces. It is assumed that the members are rigidly connected deep beams bending in their own plane and subjected to torsion about their longitudinal axes. The effect of shear distortion in these members is taken into account.

The walls in a grillage are assumed to be fixed (encastre') at their bases, and the junction of a wall and a floor constitutes a joint. The positive sign convention adopted for the forces and the displacements of grillage members is in accordance with the right-hand screw rule and is shown diagramatically in Figure 4.1. The axes of the members lie in the XY plane and the joints have degrees of freedom in the z, θ_x and θ_y directions. The out of plane bending, i.e. θ_z direction, under wind loading can be neglected, together with the deflection in X and Y directions.

Since the grillage members represent the floor slabs and walls of a building they are either horizontal or vertical. It is convenient therefore to construct two sets of the overall stiffness matrix $\underline{K}$ using the sign convention and notation of Figure 4.1. The local positive axis P for each structural member is directed from left to right for horizontal members and downward for vertical members. The axis is indicated by an arrow which points to the second end of the member.

Matrix $\underline{K}$ represents the contributions of the member to the overall stiffness matrix of the complete structure. These contributions are $\underline{K}_{ii}$ $\underline{K}_{ij}$, $\underline{K}_{ji}$ and $\underline{K}_{jj}$, where subscript i and j refer to the joints at ends 1 and 2 respectively of the member. Thus, for a horizontal member (parallel to X-axis), the contributions are, (Croxton, 1974):

$$\left[\begin{array}{c|c} K_{ii} & K_{ij} \\ \hline K_{ji} & K_{jj} \end{array}\right] = \begin{array}{c} \begin{array}{cccccc} z_i & \theta_{xi} & \theta_{yi} & z_j & \theta_{xj} & \theta_{yj} \end{array} \\ \left[\begin{array}{ccc|ccc} b & 0 & d & -b & 0 & d \\ 0 & q & 0 & 0 & -q & 0 \\ d & 0 & e & -d & 0 & f \\ \hline -b & 0 & -d & b & 0 & -d \\ 0 & -q & 0 & 0 & q & 0 \\ d & 0 & f & -d & 0 & e \end{array}\right] \end{array} \qquad (4.8a)$$

while for a vertical member (parallel to Y-axis), they are:

$$\left[\begin{array}{c|c} K_{ii} & K_{ij} \\ \hline K_{ji} & K_{jj} \end{array}\right] = \begin{array}{c} \begin{array}{cccccc} z_i & \theta_{xi} & \theta_{yi} & z_j & \theta_{xj} & \theta_{yj} \end{array} \\ \left[\begin{array}{ccc|ccc} b & -d & 0 & -b & -d & 0 \\ -d & e & 0 & d & f & 0 \\ 0 & 0 & q & 0 & 0 & -q \\ \hline -b & d & 0 & b & d & 0 \\ -d & f & 0 & d & e & 0 \\ 0 & 0 & -q & 0 & 0 & q \end{array}\right] \end{array} \qquad (4.8b)$$

General expressions for the determination of member forces $\underline{P}$ from the joint displacements $\underline{X}$ may also be obtained, (Croxton, 1974), using the following equation:

$$\underline{P} = \underline{k}\ \ \underline{A}\ \ \underline{X} \qquad (4.9)$$

where $\underline{A}$ is the displacement transformation matrix. Thus, for a horizontal member (parallel to X-axis):

$$
\begin{array}{c}
\\
\begin{bmatrix} S \\ M_1 \\ M_2 \\ T \end{bmatrix}
\end{array}
=
\begin{array}{c}
\begin{array}{cc} \text{joint i} & \text{joint j} \end{array} \\
\left[\begin{array}{ccc|ccc}
b & 0 & d & -b & 0 & d \\
d & 0 & e & -d & 0 & f \\
d & 0 & f & -d & 0 & e \\
0 & -q & 0 & 0 & q & 0
\end{array}\right]
\end{array}
\begin{array}{c}
\\
\begin{array}{cl}
\left[\begin{array}{c} z \\ \theta_x \\ \theta_y \\ \hline z \\ \theta_x \\ \theta_y \end{array}\right]
&
\begin{array}{l} \text{joint i} \\ \\ \\ \\ \text{joint j} \\ \\ \end{array}
\end{array}
\end{array}
\tag{4.10a}
$$

and for a vertical member (parallel to Y-axis):

$$
\begin{array}{c}
\\
\begin{bmatrix} S \\ M_1 \\ M_2 \\ T \end{bmatrix}
\end{array}
=
\begin{array}{c}
\begin{array}{cc} \text{joint i} & \text{joint j} \end{array} \\
\left[\begin{array}{ccc|ccc}
-b & d & 0 & b & d & 0 \\
-d & e & 0 & d & f & 0 \\
-d & f & 0 & d & e & 0 \\
0 & 0 & -q & 0 & 0 & q
\end{array}\right]
\end{array}
\begin{array}{c}
\\
\begin{array}{cl}
\left[\begin{array}{c} z \\ \theta_x \\ \theta_y \\ \hline z \\ \theta_x \\ \theta_y \end{array}\right]
&
\begin{array}{l} \text{joint i} \\ \\ \\ \\ \text{joint j} \\ \\ \end{array}
\end{array}
\end{array}
\tag{4.10b}
$$

The coefficients of equations (4.8) and (4.10) are the same as that of equation (4.7).

4.3 STIFFNESS MATRIX OF A RESTRAINING FRAME

Skeletal plane frames are used as parts of a complete structure to assist in restraining the grillage against lateral deformations caused by horizontal wind loading, Figure 4.2a. These frames are made up from prismatic slender members. The positive sign convention adopted for loads and deformations of each frame follows the same rule as that of the grillage. The axes of the members lie in the XZ plane, as shown in Figure 4.2.

The matrix displacement method is used to determine the lateral stiffnesses of the individual frames. To reduce the number of unknown in the optimisation, each frame can be regarded as having one degree of freedom only. For purposes of illustration, consider the frame shown in Figure 4.2b in which each column has a stiffness b'. The beam is constructed on the basis of composite action with the floor slab of the grillage. Therefore such a beam is assumed to be infinitely stiff to the extent that it can move horizontally without end rotations, (Majid, 1980). Thus, $\theta_B = \theta_C = 0$ and the shear force S' in each column is:

$$S' = 12\,E\,I\,z / L^3 = b'.\,z \qquad (4.11)$$

where $b' = 12\,E\,I / L^3$, the lateral shear stiffness of the column; Z is the horizontal movement of the beam.

The total lateral stiffness k' of this frame is therefore equal to the stiffness of the columns in the storey, that is, for Figure 4.2b:

$$k' = b'_{AB} + b'_{CD} \qquad (4.12)$$

In general, for a frame with several columns, say n, in each storey, the total lateral stiffness in that storey becomes:

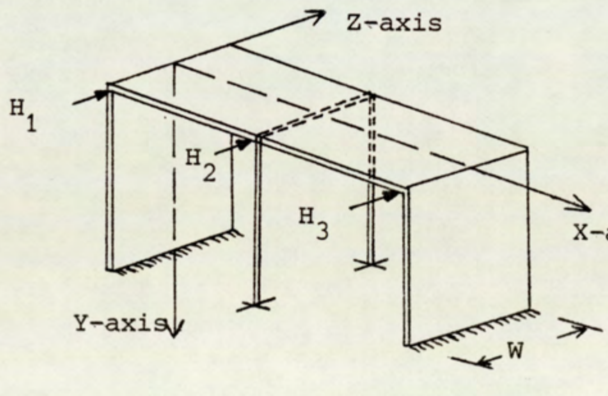
$$k' = \sum_{c=1}^{n} (12\,E\,I / L^3)_c \qquad (4.13)$$

It is assumed that the columns in a storey are connected, for instance, to joint i at the top end and joint j at the bottom end. The overall lateral stiffness matrix $\underline{K}'$ for the storey can therefore be derived, similar to equation (2.3), to relate the lateral stiffness of the column with the horizontal movements of the beams at the top and bottom ends of the storey. Thus:

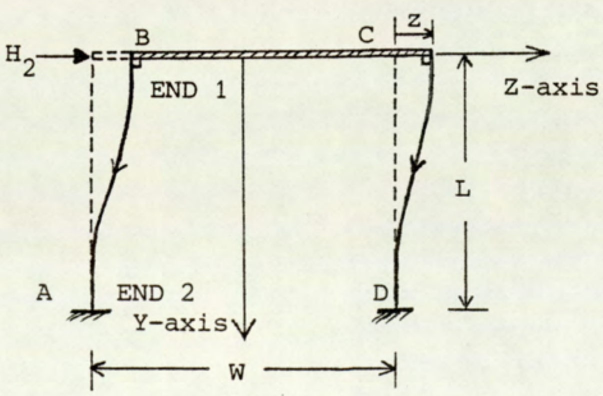
$$\underline{K}' = \begin{array}{cc} z_i & z_j \\ \end{array}$$
$$\underline{K}' = \left[\begin{array}{cc} k' & -k' \\ -k' & k' \end{array}\right] \qquad (4.14)$$

where z_i and z_j are the horizontal movements of the beams.

The overall stiffness matrix of a complete structure, such as that in Figure 4.2a, is formed by superimposing the lateral stiffness of the frame, equation (4.14) for each storey, on the stiffness matrix formed by the shear wall and floor slab panels. Solution of the load displacement equations for this restrained grillage yields the displacements at all the wall and frame junctions in the complete structure.



(a) A Simple Complete Structure



(b) Deflection of the Restraining Frame

FIGURE 4.2: SIGN CONVENTION FOR A FRAME

4.4 ANALYSIS FOR WIND LOADING

It is assumed that the wind load can be expressed as a system of concentrated loads acting at the junctions of the floors with the walls and the frames. Each of these junctions constitutes a joint in the grillage. Unsupported points in the floor slabs, such as on a vertical line of symmetry or at the free ends of cantilevered slabs, are also considered as joints. It is assumed however that all the joints have the same degrees of freedom which are in the z, θ_x and θ_y directions.

The matrix displacement method is used to analyse the restrained grillage structures. It is necessary to employ a system of joint numbering and member grouping. Figure 4.3 shows a simple one storey complete structure which has three joints (junctions) and is subjected to horizontal wind loads. The shear walls are numbered first, followed by the slabs and then the column members. The two shear walls are identical and thus belong to one group. The slabs also belong to one group. Since the shear stiffness of all the columns is the sum of the shear stiffnesses of the individual columns, it is possible to give one number to represent all the columns in a storey.

Figure 4.3 also shows the matrices required to analyse the complete structure. The overall stiffness matrix $\underline{K}$ is symmetrical and sparse. The subscript numbers, of the coefficients, refer to the members. The lateral stiffness coefficient ($k'_5 = \sum_{i=1}^{2} 12\, E\, I_i \,/\, L_i^3$) of the two columns, in the storey, is added to the horizontal stiffness of joint 2. The one-dimensional deflection matrix $\underline{X}$ contains all the unknown displacement variables of the joints. These joints are located in the central planes of the structure. Matrix $\underline{L}$ is the load vector in which the horizontal wind loads are the only non-zero elements.

$$
\underline{K} \qquad \underline{X} = \underline{L}
$$

$$
\begin{array}{ccccccccc}
z_i & \theta_{xi} & \theta_{yi} & z_j & \theta_{xj} & \theta_{yj} & z_\ell & \theta_{x\ell} & \theta_{y\ell} \\
\end{array}
$$

$$
\begin{bmatrix}
b_1 + b_3 & -d_1 & d_3 & -b_3 & 0 & -d_3 & 0 & 0 & 0 \\
-d_1 & e_1 + q_3 & 0 & 0 & -q_3 & 0 & 0 & 0 & 0 \\
d_3 & 0 & q_1 + e_3 & -d_3 & 0 & f_3 & 0 & 0 & 0 \\
-b_3 & 0 & -d_3 & b_3 + b_4 + k_5 & 0 & -d_3 + d_4 & -b_4 & 0 & d_4 \\
0 & -q_3 & 0 & 0 & q_3 + q_4 & 0 & 0 & -q_4 & 0 \\
d_3 & 0 & f_3 & -d_3 + d_4 & 0 & e_3 + e_4 & -d_4 & 0 & f_4 \\
0 & 0 & 0 & -b_4 & 0 & -d_4 & b_4 + b_2 & -d_2 & -d_4 \\
0 & 0 & 0 & 0 & -q_4 & 0 & -d_2 & q_4 + e_2 & 0 \\
0 & 0 & 0 & d_4 & 0 & f_4 & -d_4 & 0 & e_4 + q_2
\end{bmatrix}
\begin{bmatrix}
z_i \\ \theta_{xi} \\ \theta_{yi} \\ Z_j \\ \theta_{xj} \\ \theta_{yj} \\ Z_\ell \\ \theta_{x\ell} \\ \theta_{y\ell}
\end{bmatrix}
=
\begin{bmatrix}
H_i \\ 0 \\ 0 \\ H_j \\ 0 \\ 0 \\ H_\ell \\ 0 \\ 0
\end{bmatrix}
$$

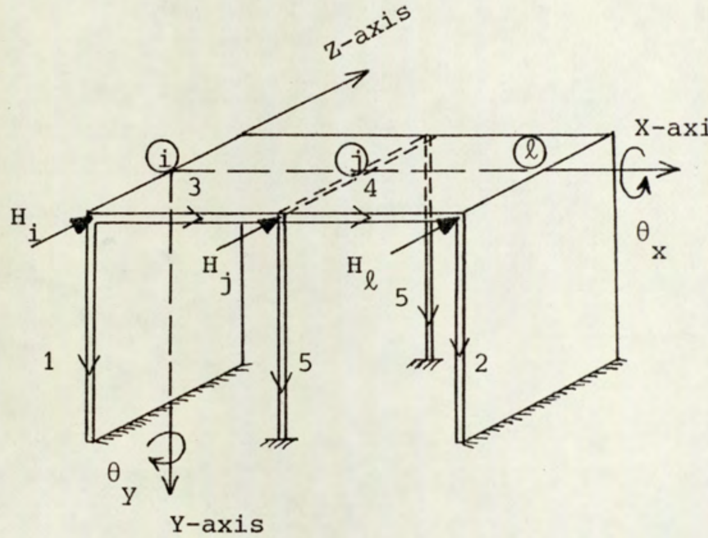


FIGURE 4.3: STIFFNESS EQUATIONS OF A COMPLETE STRUCTURE

The load displacement equations, which are also known as the stiffness equations, such as:

$$\underline{K}\ \underline{X} = \underline{L} \tag{4.15}$$

are then solved using the sub-routine described in Chapter 1. After finding the unknowns $\underline{X}$, the forces in the grillage panels can be computed by using equations (4.10a,b). The shear and the bending moments in each column of the frame are computed by using equations derived similar to equations (4.10a,b), such as:

$$\begin{bmatrix} S' \\ M'_1 \\ M'_2 \end{bmatrix} = \begin{bmatrix} -b' & b' \\ -d' & d' \\ -d' & d' \end{bmatrix} * \begin{bmatrix} z_i \\ z_j \end{bmatrix} \tag{4.16}$$

where $b' = 12EI/L^3$, $d' = -6EI/L^2$ and the subscripts i and j refer to the joints at the respective ends 1 and 2 of the column.

4.5 THE DESIGN PROBLEM

The problem dealt with here is that of designing complete structures that can withstand lateral wind loading. The shape (topology) of the structure is included in the design problem as an additional variable. This is decided by structural and economical requirements. The problem turns out to be that of non-linear programming which is linearised, using the approximating programming method, and then solved by the simplex method.

The aim is to obtain an acceptable shape structure, which has a minimum cost and satisfies the design requirements. The objective function represents the material and the other construction costs. The latter are treated as fixed prices assessed for each of the restraining frames and the grillage members. The design requirements are set as stiffness, deflection and other practical constraints which should be satisfied to achieve a laterally stable structure.

When the wind load is considered in isolation, the total cost of the complete structure is reduced mainly by removing frames or shear walls from the original topology of the structure. The number, the position and the sectional properties of the restraining frames are all design variables. Other variables are the thicknesses for the rectangular deep beam panels. The width of these panels is considered to be constant as decided by the functional requirements of the structure.

An economical computer program was written by the author to assess the cost of the structural members, and to construct the linear form of the design problem. The program was designed to make use of the backing storage, so that it can solve large problems with a moderate computer core store. The program and the data preparation will be given in Chapter 8.

4.6 THE OBJECTIVE FUNCTION

The main variables in a grillage are the thicknesses of the shear walls and the floor slabs. To compute the total cost b_i for a typical grillage member i, let t_i be its thickness, W_i its width, L_i its length, γ its density and c_i be the cost per unit weight of the material. In addition let R_i represent all the construction costs of member i. The total cost is then:

$$b_i = t_i W_i L_i \gamma_i c_i + R_i \qquad (4.17)$$

The function is linearly related to the variable t_i.

Sometimes t_i becomes equal to zero, i.e. $t_i = 0$, to indicate that member i does not exist, or removed, but when $t_i > 0$, b_i

exceeds R_i. The constants W_i and L_i are known dimensions of the member and the value of R_i is also constant for member i. The values of γ and c_i can vary from one member to another in the structure, and equation (4.17) can deal with this variation. However, all the grillage panels are in reinforced concrete with constant density γ = 24.5 KN/m^3. The cost per unit weight (c) has two values, one for the shear walls and the other for the slabs. For a general member, of either type, it could be considered as a constant. Thus, equation (4.17) becomes:

$$b_i = t_i W_i L_i \gamma c + R_i \tag{4.18}$$

The total cost (B) of a grillage structure, with M members (wall and slab panels), is then expressed as:

$$B = \sum_{i=1}^{M} (t_i W_i L_i \gamma c + R_i) \tag{4.19}$$

However, when these M members are grouped together into a number of NGR groups, equation (4.19) becomes:

$$B = \sum_{g=1}^{NGR} (t_g W_g L_g \gamma c + R_g) \tag{4.20}$$

where suffix g refers to a group of grillage members. Equation (4.20) is used as the objective cost function of the grillage, and it is linearly related to the variable t_g.

The objective cost function for a single skeletal prismatic frame, identified as j and consists of a total of m members (beams and columns), is expressed as:

$$b_j = \sum_{i=1}^{m} (A_i L_i \gamma c + R_i) \tag{4.21}$$

where the suffix i refers to member i, A_i is the member area, L_i its length and R_i is its construction costs. The constants γ and c are for the whole frame and they are respectively the density and the cost per unit weight. The frames, as parts of a complete structure, have beams with known cross section areas. These are built compositely with the slabs of the grillage. Thus, function (4.21) is minimum when:

$$b_j = \sum_{i=1}^{n} (A_i \; L_i \; \gamma \; c + R_i) \tag{4.22}$$

is minimum. Here, n is the total number of the column members only; all the beams are excluded from this formula.

It is occasionally assumed, by the designer, that all the columns in a frame are made out of the same section and belong to one group, which is referred to as j. Thus, function (4.22) becomes:

$$b_j = (n \,.\, A \,.\, L \,.\, \gamma \,.\, c)_j + R_j \tag{4.23}$$

where A, L, and c are properties of any of the column members in frame j; the value of R_i is computed independently and it includes the construction cost of beams and column in a frame. If the total cost of such a frame is calculated on the basis of one storey at a time, then n will be taken as the number of the columns in that storey.

The second moments of area (I) for the columns of the frames are adopted to be the design variables. The objective function (4.23) relates A and I through an exponential relationship. This causes a non-linear function. However, in order to avoid such non-linearity, the problem is considerably simplified, (Majid, 1974a), if function (4.23) is replaced by:

$$b_j = (n \,.\, I \,.\, L \,.\, \gamma \,.\, c)_j + R_j \tag{4.24}$$

By summing equation (4.24) for all the frames and combining it with equation (4.20), the objective cost function for the whole structure can then be expressed as:

$$Z = \sum_{g=1}^{NGR} (t_g\ W_g\ L_g\ \gamma\ c + R_g) + \sum_{j=1}^{NF} [(n.I.L.\gamma.c)_j + R_j] \tag{4.25}$$

where NGR is the total number of panel groups and NF is the total number of frames in the structure. This function is linearly related to the variables t_g and I_j.

The objective function (4.25) does not, so far, explain the fact that the non-material cost R_g (or R_j) has a value only when the grillage members group g (or frame j) is included in the design. Therefore, new variables δ_g or δ_j need to be defined and to be associated with each of the structural member concerned. Such that $\delta_g = 1$ when group g is kept in the final design while $\delta_g = 0$ when it is economical to remove members of group g. The objective function (4.25) is thus altered to become:

$$Z = \sum_{g=1}^{NGR} (t_g\ W_g\ L_g\ \gamma\ c + \delta_g\ R_g) + \sum_{j=1}^{NF} [(n.I.L.\gamma.c)_j + \delta_j\ R_j] \tag{4.26}$$

This function is used for the topological design of minimum cost for the complete structures.

4.7 THE STIFFNESS CONSTRAINTS

Using the matrix displacement method, it is necessary to select the stiffness matrix $\underline{K}$ so that the complete structure is capable of withstanding the lateral wind loads, represented by vector $\underline{L}$, and at the same time satisfies the deflection requirements. The stiffness constraints are equalities of the form:

$$\underline{H} = \underline{K}\ \underline{X} - \underline{L} = 0 \tag{4.27}$$

where $\underline{X}$ is vector of the joint displacements.

It is noticed that in the analysis of a complete structure, such as that shown in Figure 4.3, the section properties are known and specified while the displacements are unknown. This caused the overall stiffness matrix $\underline{K}$ to be symmetrical.

In the design of a complete structure, the symmetry of the stiffness matrix is lost because the sectional properties of the members are the unknown design variables. Therefore, it becomes necessary to keep the contribution of each member separate. This means that the stiffness matrix will have three rows for each joint, and three columns for each grillage member connected to that joint. In addition to that, if there is a frame linked to this joint, then an extra column will be added to the matrix.

Generally, for a total of N joints in a complete structure, the overall stiffness design matrix has 3N rows and $(3(\sum_{j=1}^{N} M_j) + \sum_{j=1}^{N} F_j)$ columns. Here M_j is the total number of grillage members connected to a typical joint j. The constant F_j is usually set equal to unity, i.e. $F_j = 1$, to represent a single frame at joint j, and if there is no frame at this joint then F_j is set equal to zero, i.e. $F_j = 0$.

If the grillage members are grouped, then M_j will be defined as the total number of different groups connected to joint j. Moreover, if it is assumed that each frame belongs to a single group then $F_j = 1$ at joint j, but if the columns of a frame belong to more than one group, then $F_j = 2$ at joint j where two different groups meet.

The design stiffness matrix of the structure shown in Figure 4.3 is displayed in Figure 4.4. The order of this matrix is

$$[3N\ ,\ (3(\sum_{j=1}^{N} M_j)\ +\ \sum_{j=1}^{N} F_j)]\ ;$$

where N is the total number of joints. It is assumed that each grillage member belongs to a single group, so that its coefficients have their own columns in the matrix. The bracing frame is considered as one member, and therefore it has one column in the matrix. The stiffness coefficients in each column of this matrix are functions of the unknown sectional properties of the members. For instance, $b_1 = E\ t_1\ W^3\ /\ L^3\ (1 + 2\ ) = f(t_1)$ and $q_1 = G\ t_1^3\ W\ /\ 3\ L = f(t_1^3)$ etc. The last line in Figure 4.4 shows these properties and the way they contribute to the joints.

The overall stiffness matrix $\underline{K}$ of the form similar to the one in Figure 4.4 is used in equation (4.27) to formulate the constraints. The stiffness coefficients of the grillage are all linear functions of the panel thickness (t) except for q, as shown in equation (4.7). The lateral stiffness coefficients of the bracing frames, equation (4.13), are linearly related to the second moment of area (I) of the column. Nevertheless, the stiffness constraints turn out to be mathematically non-linear because they involve the product z t, z I, θt^3, ... etc. The linearisation of these constraints will be explained in the next sub-section.

4.7.1 Linearisation of the Stiffness Constraints

As specified earlier, the main design variables for the optimisation of laterally loaded complete structures are the thicknesses of the grillage panels, the second moments of area for the columns of the restraining frames, and the displacements of the joints.

		Joint i						Joint j							Joint ℓ						
		z_i		θ_{xi}		θ_{yi}		z_j			θ_{xj}		θ_{yj}		z_ℓ		$\theta_{x\ell}$		$\theta_{y\ell}$		
		m1	m3	m1	m3	m1	m3	m3	m4	F_5	m3	m4	m3	m4	m4	m2	m4	m2	m4	m2	
Joint i	Row 1	b_1	b_3	$-d_1$			d_3	$-b_3$					$-d_3$								$\leftarrow H_i$
	2	$-d_1$		e_1	q_3						$-q_3$										$\leftarrow M_{xi}$
	3		d_3			q_1	e_3	$-d_3$					f_3								$\leftarrow M_{yi}$
Joint j	4		$-b_3$				$-d_3$	b_3	b_4	k_5			$-d_3$	d_4	$-b_4$				d_4		$\leftarrow H_j$
	5				$-q_3$						q_3	q_4					$-q_4$				$\leftarrow M_{xj}$
	6		d_3				f_3	$-d_3$	d_4				e_3	e_4	$-d_4$				f_4		$\leftarrow M_{yj}$
Joint ℓ	7								$-b_4$					$-d_4$	$-b_4$	b_2		$-d_2$	$-d_4$		$\leftarrow H_\ell$
	8											$-q_4$				$-d_2$	q_4	e_2			$\leftarrow M_{x\ell}$
	9								d_4					f_4	$-d_4$				e_4	q_2	$\leftarrow M_{y\ell}$
The stiff. coeff are functions of these unknowns		t_1	t_3	t_1	t_3^3	t_1^3	t_3	t_3	t_4	I_5	t_3^3	t_4^3	t_3	t_4	t_4	t_2	t_4^3	t_2	t_4	t_2^3	

m1 = Member Number 1, F_5 = Number 5 is given to the columns of the frame

FIGURE 4.4: THE DESIGN STIFFNESS MATRIX OF THE COMPLETE STRUCTURE SHOWN IN FIGURE 4.3

These variables can be arranged in a vector form, such as:

$$\underline{V} = \{v_1\ v_2\ \dots\ v_g\ \ v_{g+1}\ \ \dots\ v_{g+r}\ \ \dots\ v_{g+r+3N}\} \tag{4.28}$$

where the first g variables represent the thicknesses of the grillage members (or groups), r variables represent the second moments of area for the columns of the frames, and 3N variables represent the displacements of all the N joints; three at each joint. Vector $\underline{V}$ can be partitioned in a matrix form:

$$\underline{V} = \{\underline{t}\ :\ \underline{I}\ :\ \underline{X}\} \tag{4.29}$$

where the contents of the sub-matrix $\underline{t} = \{t_1\ t_2\ \dots\ t_g\}$, $\underline{I} = \{I_1\ I_2\ \dots\ I_r\}$, and $\underline{X} = \{z_1\ \theta_{x1}\ \theta_{y1}\ \dots\ z_N\ \theta_{xN}\ \theta_{yN}\}$.

With the above design variables, the stiffness constraints can be expressed as:

$$H\ (t,\ I,\ X) = \underline{k}\ (t,\ I)\ .\ \underline{X} - \underline{L} = 0 \tag{4.30}$$

The approximating programming method is used to linearise these constraints. This requires the gradient vector ∇H, i.e. $\nabla h\ (x_j)$, for each of these constraints, such vector has the form:

$$\nabla H = [\frac{\partial H}{\partial v_1}\quad \frac{\partial H}{\partial v_2}\quad \dots\quad \frac{\partial H}{\partial v_{g+r+3N}}] \tag{4.31}$$

which is the same as:

$$\nabla H = [\frac{\partial H}{\partial t_1}\ \dots\ \frac{\partial H}{\partial t_g}\ \ \frac{\partial H}{\partial I_1}\ \dots\ \frac{\partial H}{\partial I_r}\ \ \frac{\partial H}{\partial z_1}\ \dots\ \frac{\partial H}{\partial \theta_{yN}}] \tag{4.32}$$

at the known variables $\{x_j\}_o$.

The derivatives of the stiffness constraints with respect to the thicknesses of the grillage members are:

$$\frac{\partial H}{\partial t_j} = \frac{\partial K(t,I)}{\partial t_j} \cdot \underline{X} \tag{4.33}$$

$j = 1 \ldots g$

similarly, with respect to the second moments of area, thus:

$$\frac{\partial H}{\partial I_j} = \frac{\partial K(t,I)}{\partial I_j} \cdot \underline{X} \tag{4.34}$$

$j = 1, \ldots r$

and with resepct to the joint displacements, the derivatives are:

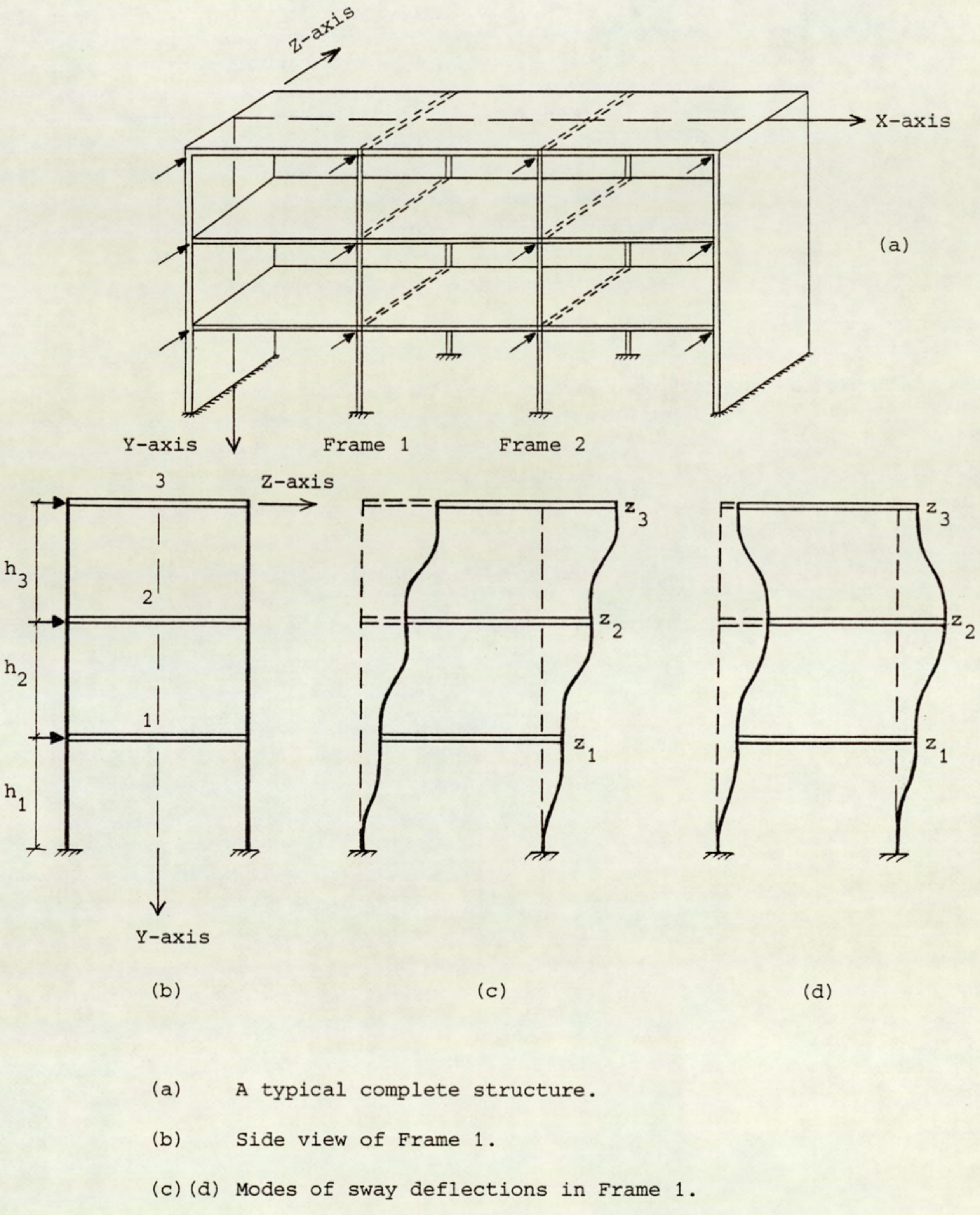
$$\frac{\partial H}{\partial X_j} = K(t,I) \tag{4.35}$$

$j = 1 \ldots 3N$

4.8 THE DEFLECTION CONSTRAINTS

Two types of deflection constraints are considered. These are:

(1) The sway deflection constraints which are used to govern the differential horizontal deflection between the storeys. For instance, in the structure shown in Figure 4.5a, the sway in each storey may be limited to an upper bound of h/ℓ where h is the storey's height, and ℓ is a constant such as 350, 400, or any other value specified by the engineer. In Figure 4.5b, a side view of one of the bracing frames is shown. Each junction is considered as a joint and given a number. The different possible horizontal deflections in the Z - direction are demonstrated by Figures 4.5c and d. For these the sway deflection constraints become:



(a) A typical complete structure.

(b) Side view of Frame 1.

(c)(d) Modes of sway deflections in Frame 1.

FIGURE 4.5: HORIZONTAL DEFLECTION IN A BRACING FRAME

$$
\begin{aligned}
z_1 &\leq h_1 / \ell ; \\
z_2 - z_1 &\leq h_2 / \ell ; \\
z_1 - z_2 &\leq h_2 / \ell ; \\
z_3 - z_2 &\leq h_3 / \ell ; \\
z_2 - z_3 &\leq h_3 / \ell
\end{aligned}
\tag{4.36}
$$

In the general case, if at floor i the horizontal deflection is z_i, and at floor i+1 it is z_{i+1}, then the deflection constraints in equation (4.36) will be in the form:

$$
\begin{aligned}
z_{i+1} - z_i &\leq h_{i+1} / \ell ; \\
z_i - z_{i+1} &\leq h_{i+1} / \ell
\end{aligned}
\tag{4.37}
$$

These two constraints are specified for each storey of the frame to cover all the possible modes of deformation. Furthermore, such constraints are imposed on all the vertical components including the shear walls. However, the engineer can select not all but some of the suitable components on which such sway constraints are imposed.

(2) The joint deflection constraints. Each junction of a complete structure introduces three variables to the design problem. These are the horizontal displacement (z), the rotation of the joint about X-axis (θ_x) and the rotation of the joint about Y-axis (θ_y). These displacements are considered as design variables which must all be bound by an upper limit, otherwise the linear programming process excludes them from the solution. The actual value of the deflection x_j at a joint j may be limited by an upper bound U_j. The deflection constraint is then:

$$x_j \leq U_j \tag{4.38}$$

Since the sway deflection constraints are the most important for the design of laterally loaded complete structures, the upper bounds on the deflections are taken to be large so that they do not play a significant part in the design problem. Furthermore, when solving the linear programming problem by the simplex method, all the design variables should be non-negative. Thus, because of the difficulty in predicting the direction of each joint deflection, especially the rotations, it is necessary to modify the inequality (4.38) as explained in Chapter 2.

4.9 OTHER PRACTICAL CONSTRAINTS

In a mathematical programming problem all the design variables should be bounded so that they can be included in the solution. The way the joint displacements are bounded was shown in the previous Section. The boundaries of the other design variables, such as the thicknesses of the grillage members and the second moments of area for the columns of the frames, are decided by the engineer or by the code of practice used.

The design problem presented here aims at altering the topology of the complete structure by removing shear walls and frames from it. Certain members, however, often have to be retained in the final topology, e.g. the floor slabs and any other member chosen by the engineer. To retain such members, lower bounds are imposed on their sections. Upper bounds are also specified on these variables. Therefore, the constraints created are in the form:

$$t_{min} \leq t_i \leq t_{max} ;$$

$$I_{min} \leq I_j \leq I_{max}$$

(4.39)

where t_i is the thickness of grillage panel i, t_{min} and t_{max} are the smallest and the largest thicknesses allowed for this panel. The variable I_j is the second moment of area for the columns of frame j and I_{min} and I_{max} are specified as the smallest and the largest available sections. Subscripts i and j here represent the retained components.

For components allowed to be removed, there are no lower bounds imposed on their sections. It is only necessary to specify upper bounds for these. Each one of these components is associated with its own variable δ_i which was included in the objective function (4.26). These types of constraints have the form:

$$t_i \leq t_{max} \cdot \delta_i ;$$

$$I_j \leq I_{max} \cdot \delta_j$$

i.e.

$$t_i - t_{max} \delta_i \leq 0 ;$$

$$I_j - I_{max} \delta_j \leq 0$$

(4.40)

Subscripts i and j here refer to the removable components of the structure. Each variable δ_i or δ_j can be either zero or unity, i.e.:

$$0 \leq \delta_i \leq 1 ;$$

$$0 \leq \delta_j \leq 1$$

δ_i and δ_j are integers (4.41)

The constraints (4.40) and (4.41) guarantee that t_i cannot be positive unless $\delta_i = 1$. This is because the only other value δ_i can take is zero, in which case $t_i = 0$ and the panel i is removed from the structure. Similarly for member j.

4.10 COST ASSESSMENT OF COMPLETE STRUCTURES

A method of assessing the cost of a complete structure is to be discussed in this Section. As previously stated, such cost includes that of the material and the construction. The assessment is carried out according to the information presented in a report by Davis, Belfield and Everest (1980). The report which gives details about the rates of labour, materials and measured items of constructions was discussed in Chapter 2.

It has been decided that all the examples of complete structures, which will be discussed in the next Chapter, are built from two different materials. These are reinforced concrete for the grillage, and fabricated steel or reinforced concrete for the frames. Figure 4.6 shows some details that are used to assess the cost of a complete structure consisting of steel frames. It is assumed that minimum percentages of reinforcement are specified for all the reinforced concrete grillage panels of the design examples.

Estimating the cost of the bracing fabricated steel frames was described in Chapter 2 and need not be repeated here. In this Section it is only required to demonstrate the cost assessment of the grillage structure. Thus, according to the report mentioned above, the cost of a grillage can be computed as follows:

(I) The Material Cost

The cost of reinforced concrete material is computed after assuming a minimum reinforcement for the grillage members. Table 4.1 shows the amount of the minimum specified reinforcements and the costs of material obtained. For a shear wall panel of thickness t_w, width W and height H, the area of the vertical reinforcement (ρ) used is 1 %, i.e. 0.01 t_w W, and the area of the horizontal reinforcement is 0.25%, i.e. 0.0025 t_w H. For a floor slab panel of thickness t_s, width W and span L, the area of the main reinforcement used is 0.25%, i.e. 0.0025 t_s W, and that of secondary reinforcement is 0.15%, i.e. 0.0015 t_s L. By considering one cubic meter of reinforced concrete, the cost of material is calculated as explained in Appendix **B**, thus:

Shear walls, c = £2.20/KN

Floor slabs, c = £1.62/KN

Min. specified Reinforcement	Shear Walls	Floor slabs
Main Reinforcement (ρ)	1%	0.25%
Secondary Reinforcement	0.25%	0.15%
Cost per unit Weight (c)	£2./20KN	£1.62/KN

Table 4.1: Cost per unit weight depending on the minimum specified reinforcement.

(II) <u>The Construction Cost</u>

The cost of erecting a grillage panel is done by using the measured rates of constructing items given in Appendix ~~D~~ A. The construction cost of a shear wall includes the cost of its continuous strip foundation. The design of this type of foundation is carried out per meter run, as described by Faber (1976) and many others, and its whole cost was assessed as demonstrated in Chapter 2.

For purposes of illustration, the method of assessing the construction cost of a typical grillage member is explained by considering an example of a shear wall and a floor slab. Thus, using the specifications shown in Figure 4.6, consider a single reinforced concrete shear wall member constructed to link two consecutive floor slabs.

The dimensions of this member are $t_w = 0.2$ m, H = 5 m and W = 4 m. Assuming minimum reinforcement, of density $\gamma = 77$ KN/m^3, is used. The calculations proceed as follows:

(1) Area of vertical reinforcement

$= 0.01 \times 0.2 \text{ m} \times 4 \text{ m}$

$= 0.008 \text{ m}^2$

$- \text{Volume} = 0.008 \text{ m}^2 \times 5 \text{ m} \qquad = 0.04 \text{ m}^3$

$- \text{Weight} = 0.04 \text{ m}^3 \times 77 \text{ KN/m}^3 \qquad = 3.08 \text{ KN/one storey}$

$= 0.314$ ton/one storey

(2) Area of horizontal reinforcement

$= 0.0025 \times 0.2 \text{ m} \times 5 \text{ m}$

$= 0.0025 \text{ m}^2$

$- \text{Volume} = 0.0025 \text{ m}^2 \times 4 \text{ m} \qquad = 0.01 \text{ m}^3$

$- \text{Weight} = 0.01 \text{ m}^3 \times 77 \text{ KN/m}^3 \qquad = 0.77 \text{ KN/one storey}$

$= 0.078$ ton/one storey

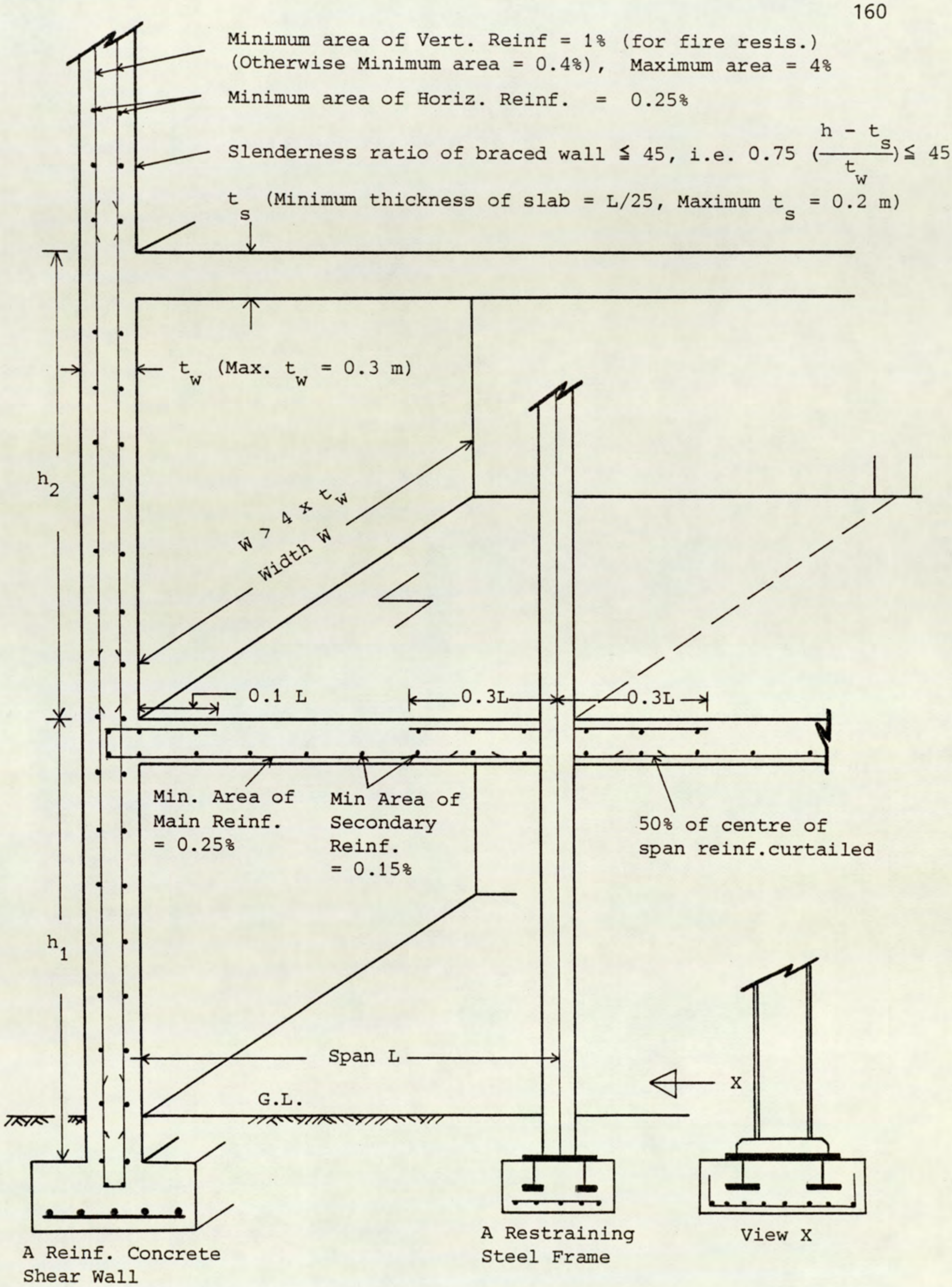


FIGURE 4.6: SOME SPECIFICATIONS REQUIRED TO ASSESS THE COST OF A COMPLETE STRUCTURE

(3) Volume of concrete used for the shear wall member

$= 0.2 \text{ m} \times 4 \text{ m} \times 5 \text{ m} \qquad = 4 \text{ m}^3$

(4) The surface area of the member which needs a fair finish formwork

$= W\,(2\,H - t_s) + 2\,t_w\,H$

Assume $t_s = 0.2$ m, thus:

$= 4 \text{ m} \times (10 \text{ m} - 0.2 \text{ m}) + 2 \text{ m} \times 0.2 \text{ m} \times 5 \text{ m}$

$= 41.2 \text{ m}^2$

If this shear wall is indicated as i, then the value of the construction cost for it is assessed to be R_i = £744.63, as shown in the self-explanatory Table 4.2. This computed value of R_i is used in the objective function, Section 4.6.

Similarly, for a single bay of a floor slab, the construction cost is assessed as shown in Table 4.3. The dimensions of this member are assumed to be $t_s = 0.2$ m, L = 4 m and W = 4 m. It is worth mentioning that the surface areas of the panel which need a fair finish formwork are:

Area of soffits $= 2 \times L \times W$

Area of edges $= 2 \times L \times t_s$

The cost of erecting this member is found to be R = £491.93.

The above method of assessing the construction costs of the grillage members is organised and written as part of the main computer program, which will be described in Chapter 8.

Number	Item Description	Unit	Quantity	Rate £'s	Amount £'s
(1)	<u>In-Situ Concrete Work</u>:				
	(a) Provision of concrete. - Design mixture Grade 28 cement, (30N/mm²).	m³	4	33.11	132.44
	(b) Placing of reinforced concrete. - Walls thickness 150 - 300 mm.	m³	4	12.87	51.48
					183.92
(2)	<u>Concrete Ancillaries</u>:				
	(a) Formwork, fair finish. - Vertical width over 1.22 m.	m²	41.2	10.03	413.24
	(b) High yield bar steel reinforcement to BS 449.				
	- 20 mm diameter bars	ton	0.314	363.47	114.12
	- 12 mm diameter bars	ton	0.078	424.88	33.35
					560.71
	<u>Summary</u>:				
	(1) The In-Situ cost				183.92
	(2) Concrete Ancillaries cost				560.71
	∴ The total construction cost of the shear wall member				744.63

TABLE 4.2: CONSTRUCTION COST (R) OF A TYPICAL SHEAR WALL MEMBER

Number	Item Description	Unit	Quantity	Rate £'s	Amount £'s
(1)	<u>In-Situ Concrete Work:</u>				
	(a) Provision of concrete. - Design mixture Grade 28 cement, ($30N/mm^2$).	m^3	3.2	33.11	105.95
	(b) Placing of reinforced concrete. - Slabs thickness 150 - 300 mm.	m^3	3.2	4.29	13.73
					119.68
(2)	<u>Concrete Ancillaries:</u>				
	(a) Formwork, fair finish.				
	- Horizontal width over 1.22 m.	m^2	32	9.87	315.84
	- Vertical width 0.2 - 0.4 m.	m^2	1.6	10.99	17.58
	(b) High yield bar steel reinforcement to BS 449.				
	- 20 mm diameter bars	ton	0.0621	363.47	22.82
	- 12 mm diameter bars	ton	0.0372	424.88	16.01
					372.25
	<u>Summary:</u>				
	(1) In-Situ cost				119.68
	(2) Concrete Ancillaries cost				372.25
	∴ The total construction cost of the floor slab member				491.93

TABLE 4.3: CONSTRUCTION COST (R) OF A TYPICAL FLOOR SLAB MEMBER

CHAPTER 5

EXAMPLES ON THE DESIGN OF LATERALLY LOADED COMPLETE STRUCTURES

5.1 THE PRINCIPLES OF DESIGN

The design method described in the previous Chapter is applied here to obtain a minimum total cost design for complete structures subjected to wind loads. This can be achieved, for instance, by reducing the number of vertical components that restrain the structure. The design method removes unwanted components automatically.

The main design criterion adopted here is that under the effect of specified unfactored wind loads, the elastic sway at any storey level should not exceed h/ℓ, where h is the storey height, and ℓ is a constant, such as 350 or 500 etc, specified by the engineer or by the code of practice used. The design method controls the differential sway between the storeys as opposed to the horizontal deflection of the junctions. The sway of the storey level governs the selection of the column sections of the frames and the thicknesses of the grillage panels.

The sections obtained from the final deflection limitation design are checked to ensure that the lateral strength requirements are satisfied. However, if it is required to produce a design which satisfies the wind and the vertical load cases, the sections obtained, when designing for the wind loading case, are used as lower bounds for the vertical dead and imposed live loading case. It is also possible that a column, of a frame, is removed during the design process because it is more economical to sustain the wind loads by other components. If, however, this column is needed to carry the vertical load, then it is constructed as a prop and treated as a pin ended compression member.

5.2 THE DESIGN PROCEDURE

The procedure for a topological design of minimum cost for laterally loaded complete structures consists of the following steps:

(1) Select a "ground complete structure". This is done by deciding the number and the position of the floor slabs and the shear walls. While the ground structure can be asymmetrical, the examples given are all symmetrical.

(2) Cover the structure area by a large number of bracing frames parallel to the shear walls, and specify the number of columns in each frame. In the design examples solved, it was decided that all the frames have the same number of columns.

(3) Give a joint number to each junction of the ground complete structure. Free ends of the floor slabs are also considered as joints. Usually the numbering starts from the first floor.

(4) Give a number to each member of the ground structure. The way to identify each single member will be clarified when dealing with the examples. The numbering starts with the members of the shear walls followed by the floor slabs then the frames.

(5) Group the members together. The grouping is carried out in the same sequence as the previous step. Usually, the parts of a shear wall spanning from the floor to another are identical and thus belong to one group. The members of a floor slab, at one or sometimes more storeys, are built out of the same section and belong to the same group. Furthermore, in this design method, it is assumed that each bracing frame belongs to one or more groups.

(6) Calculate the amount of wind loads according to the code of practice CP3. These are considered as concentrated lateral loads acting at the junctions of the floor slabs with the vertical components.

(7) Select upper bounds on the design variables. These variables are the thicknesses of the grillage members or groups, the second moments of area for the columns of the frames and the displacements of the joints. Also select lower bounds which will be imposed on members required to be retained.

(8) Select initial sectional properties for the members. Then analyse the structure to obtain the displacements of the joints. Thus, the set of starting design variables is now introduced.

(9) Use the sectional properties to calculate the material and the construction costs for each member or group of members. Therefore, the total cost of the structure can be assessed.

(10) Derive the objective cost function which is, as shown before, linearly related with the member properties. Send the constant coefficients of the cost function to the backing store, (Chapter 8).

(11) Use the row-by-row technique to construct and transfer all the linearised design constraints to the backing store. In this manner, all the coefficients of the design problem are stored on a computer disk, (Chapter 8).

(12) Use the simplex method to minimise the objective cost function.

(13) Remove all members, or groups, with $\delta = 0$. However, it is recommended, as it will be shown in the design examples, that these members should not be removed at the early stages. This is because the properties of the members may have unreal relationships to each other. Therefore, it was decided that, after about three iterations, if a member still has its δ equal to zero then it must be removed from the design. Members with $\delta \cong 0$ can be removed only if feasibility is restored in the next iteration.

(14) Use the sectional properties obtained to repeat the process from step (9) until a final topology of the structure is obtained.

(15) Continue the minimum cost design of the structure with its final topology until an optimum cost is reached.

(16) If required, check the lateral shear strength in the vertical components, examine the lateral stability of the frame and test if the sway deflection is within the specified limit.

It is worth mentioning that the operations from step (8) to the end were done automatically by using the computer program which will be described in Chapter 8.

5.3 A 5-STOREY, 21-BAY SYMMETRICAL STRUCTURE

An investigation is to be carried out on the optimum cost design of a hypothetical 5-storey complete building structure. This structure, which is illustrated in Figure 5.1, consists of reinforced concrete walls and slabs for which Young's Modulus (E) is 28 KN/mm^2, and Poisson's ratio (ν) is 0.2 . The frames are of steel sections with Young's Modulus equal to 207 KN/mm^2. The structure is restrained laterally by two walls at the ends and 20 intermediate frames. The parallel vertical restraining components are 4 m apart. Each frame was considered as a one bay frame that contains two columns at any particular floor. The width of the building was assumed to be 4 m. The first floor is 5 m high, while the rest are 3.5 m.

The lateral sway at each storey was limited to the value h/350. The upper bound imposed on the horizontal deflection of each joint (junction) was chosen to be 20 mm x f, where f is the floor number. Also, the joint rotations were limited by 0.08 radians. The upper bounds on the thicknesses of the grillage members were decided to be 200 mm for floor slabs and 300 mm for shear walls. The lower bounds

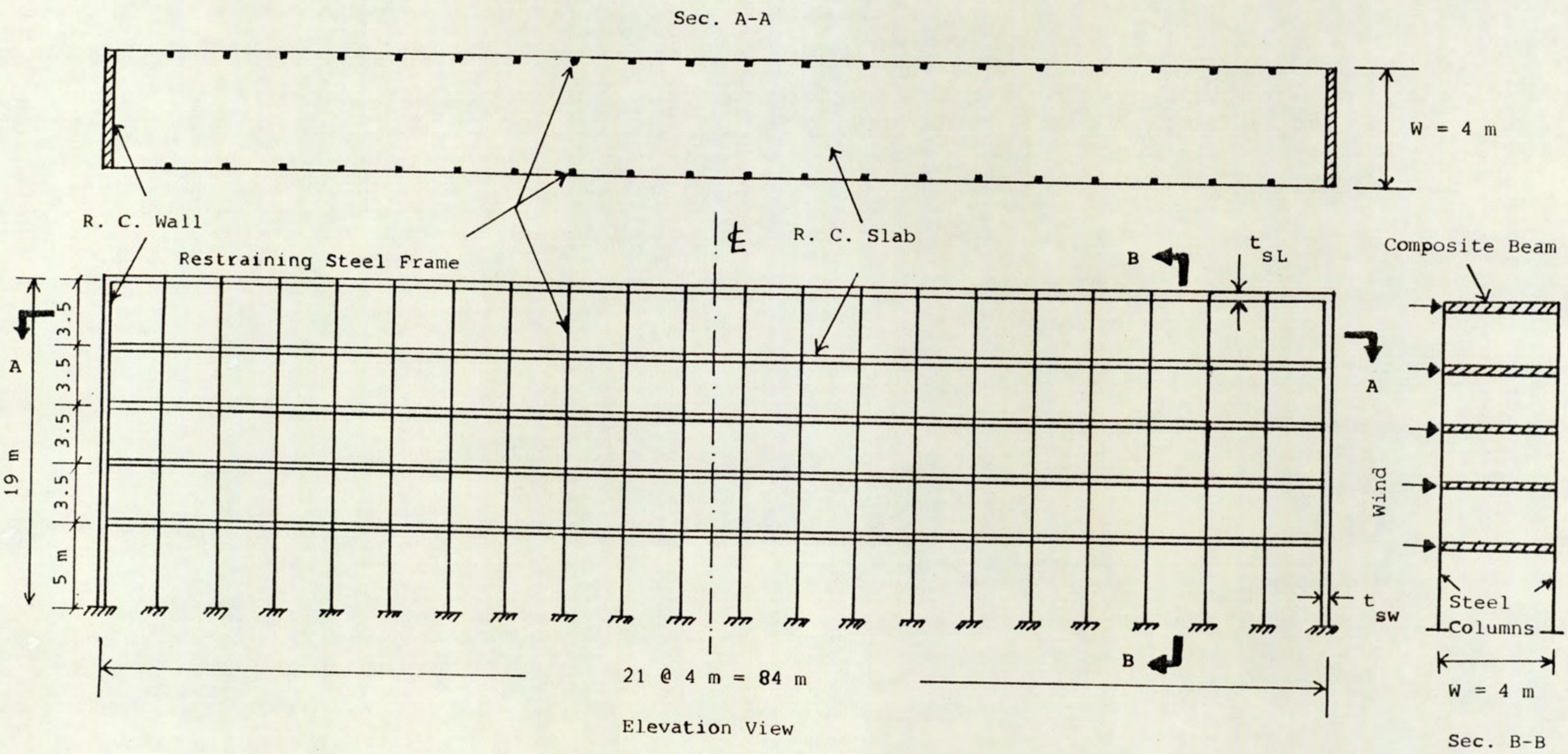


FIGURE 5.1: A5-STOREY, 21-BAY SYMMETRICAL COMPLETE STRUCTURE

t_{sw} = Shear Wall Thickness,

t_{sL} = Slab Floor Thickness,

W = Building Width (Breadth).

were selected according to the code of practice CP110, which specifies in clause 3.3.8.1 that the minimum thickness of a continuous slab, with a span L between two supports, is L/25. Thus, for a slab with a span of L = 4 m, the minimum thickness would be 160 mm. A smaller value of the allowable thickness is specified by CP114, in clause 309, but this is not used here. The smallest allowable thickness of a shear wall was assumed as 120 mm.

The lower and the upper bounds imposed on the second moments of area (I) for the columns of the frames were selected as the smallest and the largest I in the available universal column sections. A steel beam, in a frame, was assumed to have complete interaction with the concrete slab. Thus, a composite beam was formed by a concrete slab supported on, and connected to, the top flange of an uncased beam, (Davies, 1975). The sectional properties (I) of all the steel beams were assumed to have a single known value. This was given as data and was kept unchanged throughout the design process.

The structure is symmetrical about the vertical central line, as shown in Figure 5.1. Thus, only the left hand half was considered. This is shown in Figure 5.2 in which the joint numbers are circled. The unsupported points in the floor slabs on the vertical line of symmetry were also considered as joints. The displacements represented by the rotations about Y-axis (θ_y), for all the joints at the line of symmetry, were suppressed. This is because, at this line, the floor slabs do not rotate about the Y-axis.

Figure 5.2 also shows the numbering of the members. Each member has an arrow, which indicates the direction of the local axis. In each floor the numbering starts with the shear walls followed by the floor slabs, and then the frames. Grouping of the members is given by the number in squares. The members of the shear wall are built identical,

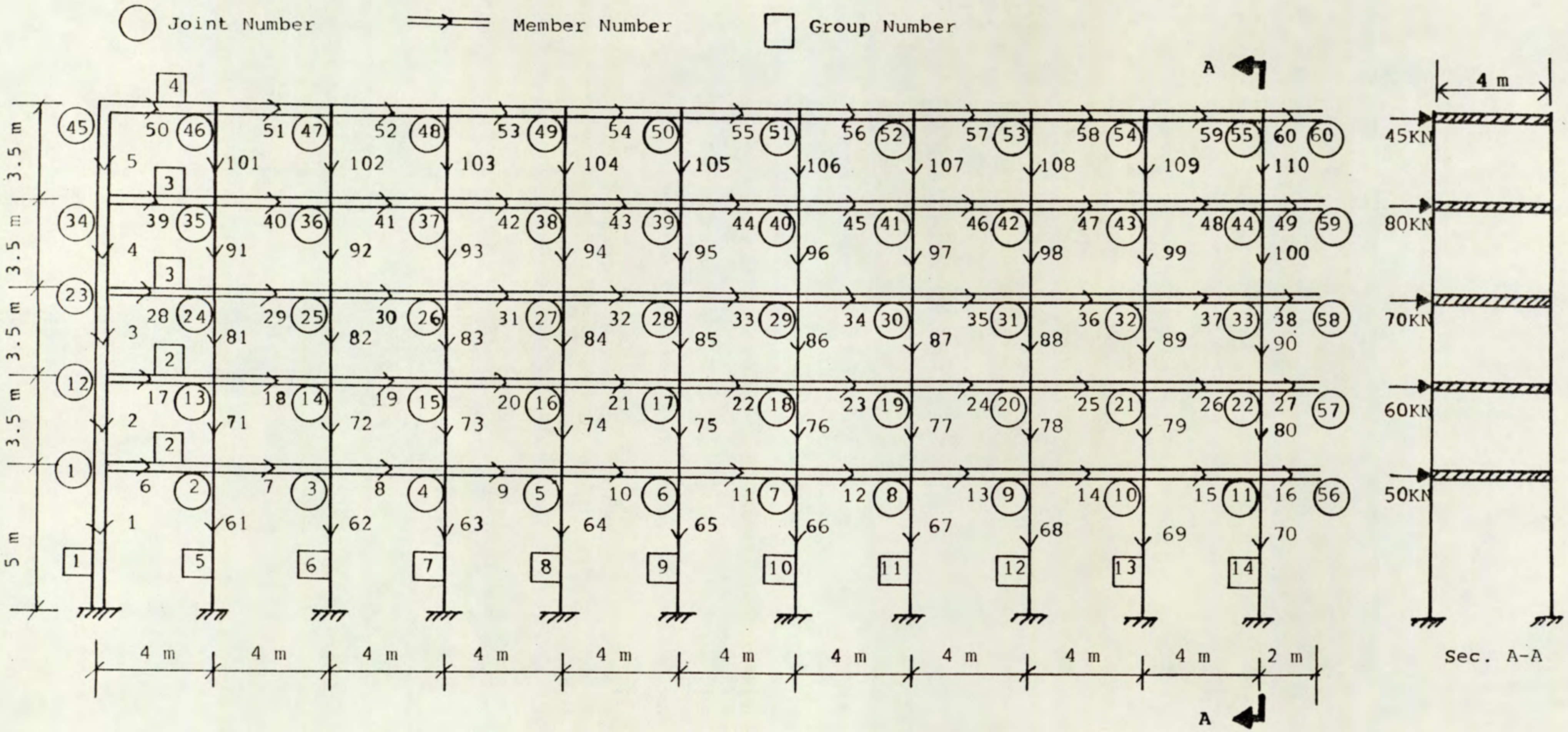


FIGURE 5.2: LEFT-HAND HALF OF THE 5-STOREY COMPLETE STRUCTURE THE WIND LOADING, AND THE NUMBERING OF JOINTS, MEMBERS AND GROUPS

i.e. one thickness, therefore they belong to a single group. Also, it was decided that slabs of more than one floor can be built out of the same group, e.g. the slabs in group 2 . Furthermore, the columns of each whole frame were assumed to be built from a uniform section belonging to one group.

The left hand half of the structure consists of 60 joints and 110 members which include five members for the shear wall, 55 for the floor slabs and 50 for the frames, Figure 5.2. All the members were collected into 14 groups one of which is for the shear wall, three for the slabs and ten for the frames. The design problem has 200 variables, four of which are the thicknesses of the grillage groups, and ten represent the second moments of area of the columns. There are also 175 displacements with three variables for each joint at the junctions of vertical and horizontal components, and with two variables (z, θ_x) for each joint at the line of symmetry. The last 11 variables, from the total, represent the δ variables for the 11 removable groups of shear walls and frames. The problem has a total of 422 constraints, of which 175 are stiffness constraints, 175 are deflections and 44 are relative sway deflection constraints. There are six boundary constraints imposed on the thicknesses of the floor slabs, i.e. the first part of the inequality (4.39). Also the removable groups have 11 constraints of the type (4.40) and another 11 of the type (4.41).

The wind loading shown in Figure 5.2, was applied on the junctions of the central frame. The same values imposed similarly on all the intermediate junctions created from the horizontal and the vertical components. The only exceptions are the junctions of the shear wall where the loading values shown in the Figure were halved.

5.3.1 The Design Cases

To begin with, the foundations of the shear walls and the frames are designed and costed. This was done by assuming that a uniformly distributed vertical live load of 5 KN/m^2 and 3.75 KN/m^2 are applied on the floors and the roof respectively. These loads were added to the self weight of the structure, which was computed assuming the largest possible section are used. With the same assumed sections and under the effect of wind loading only, the structure was analysed to find the displacements and to compute the shear forces and the bending moments which were acting on the foundations. The foundations were then designed and costed. It was assumed that the shear walls are supported by strip footing which was costed at £350. The cost of an independent footing for each steel column came to £360. Therefore, the foundation cost of each frame is £760. These costs were kept constant.

Because of symmetry, only the left hand half of the structure was designed. The procedure started with all grillage members having a thickness of 160 mm, and all steel frames having a second moment of area of 17510×10^4 mm^4 for their column sections. A value of I = 29337×10^4 mm^4 was specified for all the beams, and was kept unaltered. With these known starting values of the variables, the cost of the complete structure can therefore be assessed as explained in the previous Chapter . The mass per unit length of the universal steel members were used in calculating the material and the construction costs of the frames.

To demonstrate the versatility and the flexibility of the method, five design cases are now reported.

5.3.1.1 Design Case 1

In this case, the shear walls and all the frames were allowed to be removed. Thus, no lower bounds were imposed on them. The outputs of the design cycles are displayed in Table 5.1. At each cycle, the sectional properties of each group, their construction costs, i.e. R_i in objective function (4.26), and their δ variables were obtained. The last column in the Table gives the total cost calculated at each cycle. This includes the material and the construction costs of all the members. The costs of the foundation were included in the construction costs of the vertical components.

At some early stages of the design some member groups were obtained with $\delta = 0$ and their sectional properties were also reduced to zero. However, these were retained at this stage at their lower values of t and I. Such groups are marked by asterisks in Table 5.1. Nevertheless, if these groups persist to give zero values after two or three design iterations, then they must be removed from the design and a zero value should be given to their variables, as can be seen in the Table. The slab groups had no δ variables because they are the original retained member groups of the complete structure.

When the value of I for a column was obtained at the end of each design iteration, the mass per unit length of the nearest and the lightest "universal" column section available was used to assess its cost.

The topological design of minimum cost for the 5-storey symmetrical complete structure was obtained at the sixth design iteration, as shown in Table 5.1. The final shape achieved for the whole structure is shown in Figure 5.3. The Table in the Figure shows the optimum section variables which were required to restrain the structure against the lateral wind loading. The width W given in brackets for each group are constants selected before the design procedure was started. The I values in the Table were obtained by the

Iter. Number	Group Number	1 t (mm)	2 t (mm)	3 t (mm)	4 t (mm)	5 $Ix10^4 mm^4$	6 $Ix10^4 mm^4$	7 $Ix10^4 mm^4$	8 $Ix10^4 mm^4$	9 $Ix10^4 mm^4$	10 $Ix10^4 mm^4$	11 $Ix10^4 mm^4$	12 $Ix10^4 mm^4$	13 $Ix10^4 mm^4$	14 $Ix10^4 mm^4$	Total Cost
Initial Des.	t, I Cons.Cost δ	160 £2,857 1	160 £19,410	160 £19,410	160 £9,705	17500 £4,490 1	17500 £4,490 1	17500 £4,490 1	17500 £4,490 1	17500 £4,490 1	17500 £4,490 1	17500 £4,490 1	17500 £4,490 1	17500 £4,490 1	17500 £4,490 1	£114,657.0
Iter. (1)	t, I Cons.Cost δ	300 £3,591 1	200 £20,950	200 £20,950	160 £9,705	1263* £2,288 0	1263* £2,288 0	1263* £2,288 0	1263* £2,288 0	1263* £2,288 0	1263* £2,288 0	1263* £2,288 0	1263* £2,288 0	1263* £2,288 0	195433 £16,132 0.71	£109.188
Iter. (2)	t, I Cons.Cost δ	300 £3,591 1	200 £20,950	200 £20,950	160 £9,705	1263* £2,288 0	1263* £2,288 0	1263* £2,288 0	1263* £2,288 0	1263* £2,288 0	1263* £2,288 0	1263* £2,365 0	237368 £18,308 0.86	57893 £6,980 0.21	1263* £2,288 0	£118,350
Iter. (3)	t, I Cons.Cost δ	300 £3,591 1	200 £20,950	200 £20,950	160 £9,705	1263* £2,288 0	1263* £2,288 0	1263* £2,288 0	1263* £2,288 0	1263* £2,288 0	1263* £2,288 0	201464 £16,132 0.73	80438 £9,209 0.29	48485 £5,696 0.18	38102 £5,066 0.14	£126,669
Iter. (4)	t, I Cons.Cost δ	300 £3,591 1	200 £20,950	200 £20,950	180 £10,090	0.0	0.0	0.0	0.0	0.0	0.0	245533 £18,308 0.89	1263* £2,288 0	138223 £11,988 0.5	1263* £2,288 0	£108,625
Iter. (5)	t, I Cons.Cost δ	300 £3,591 1	200 £20,950	200 £20,950	160 £9,705	0.0	0.0	0.0	0.0	0.0	0.0	241501 £18,308 0.88	1263* £2,288 0	143487 £11,988 0.52	1263* £2,288 0	£108.240
Iter. (6)	t, I Cons.Cost δ	300 £3,591 1	200 £20,950	200 £20,950	160 £9,705	0.0	0.0	0.0	0.0	0.0	0.0	248308 £18,308 0.9	0.0	140242 £11,988 0.51	0.0	£102,644

* Frame supposed to be removed.

TABLE 5.1: THE 5-STOREY SYMMETRICAL COMPLETE STRUCTURE - RESULTS OF THE DESIGN ITERATIONS FOR THE LEFT-HAND-HALF OF THE STRUCTURE

computer. However, the values of I in the brackets are the nearest available "universal" column sections which were selected to be used.

The material and the construction costs, shown in the Table with Figure 5.3, were computed for the whole structure, i.e. for both halves. The optimum total cost obtained is £205,288, which is double that obtained at the sixth design iteration, in Table 5.1 Frame groups 11 and 13 remained with their δ variables at 0.9 and 0.51 respectively. The removable shear wall (group one) also remained and its thickness reached the upper bound. This caused its δ variable to be exactly one.

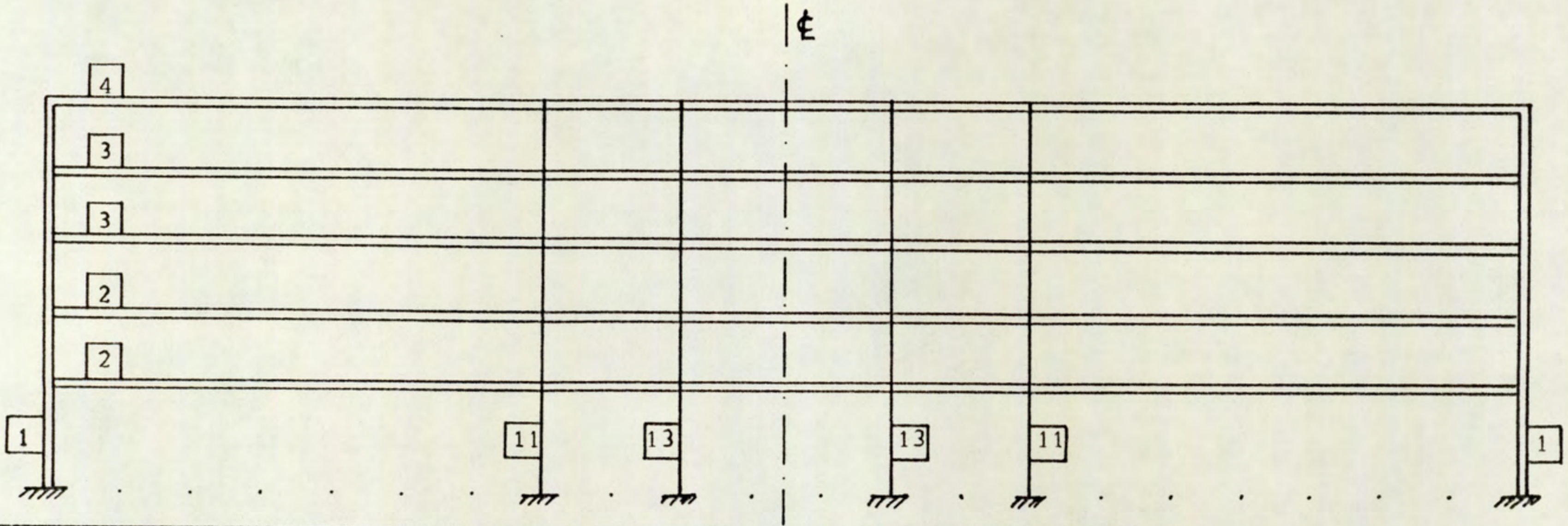
5.3.1.2 Design Case 2

In this case, it was required that frame group 11 must be retained in the final topology of the structure. Thus, a lower bound was imposed on the columns of this frame. The bound was chosen to be the smallest universal column section available with $I = 1263 \times 10^4 \ mm^4$.

All the other frames and shear walls were allowed to be removed. Similar to Case 1, the optimum cost design was obtained after six design iterations. The final topology achieved is shown in Figure 5.4, where it is noticed that group 12 is also retained at its upper bound with $I = 275140 \times 10^4 \ mm^4$. The group had $\delta = 1$. The thickness of the shear walls (group one) had reached its maximum at t = 300 mm and $\delta = 1$. The results prove that this case is very much similar to Design Case 1. The optimum total cost of the complete structure was £200067 which is only 2.5% less than that for the design in Case 1.

5.3.1.3 Design Case 3

In this case, it was decided to reduce the width of the shear



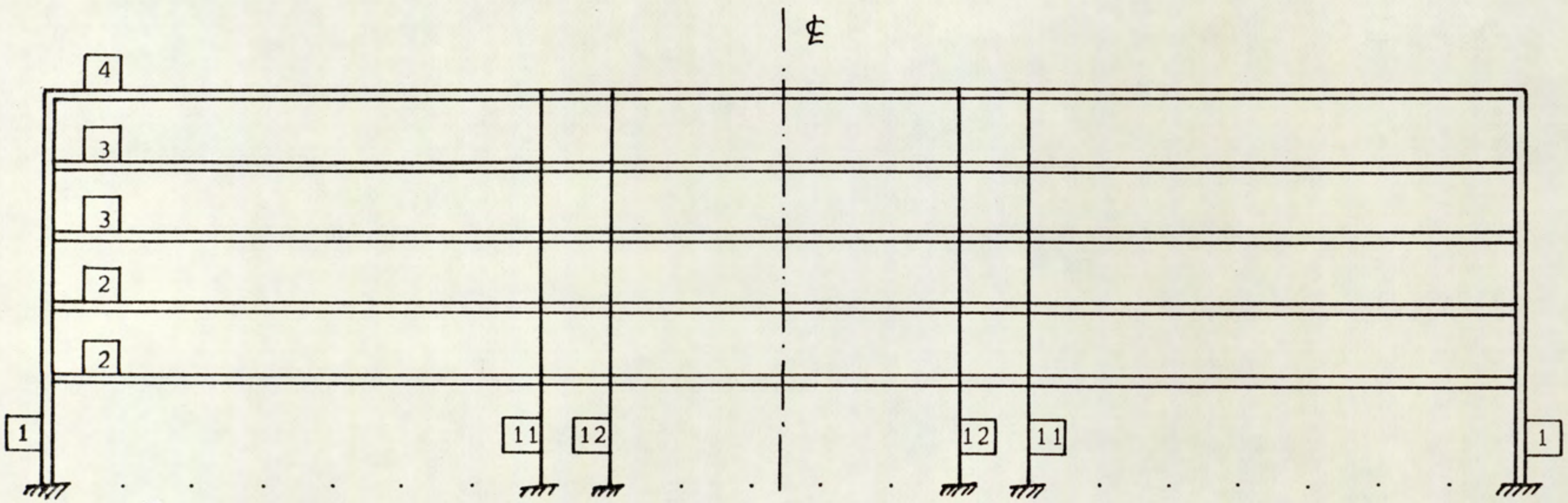
Group Number	Section Variables (mm)	Material Cost (£)	Construction Cost (£)	δ
1	t = 300 (W = 4000)	2,458	7,182	1
2	t = 200 (W = 4000)	5,268	41,900	-
3	t = 200 (W = 4000)	5,268	41,900	-
4	t = 160 (W = 4000)	2,107	19,410	-
11	$I = 248308 \times 10^4$ (Use 275140×10^4)	11,714	36,616	0.9
13	$I = 140242 \times 10^4$ (Use 146765×10^4)	7,496	23,976	0.51
	Total Cost = £205,288			

FIGURE 5.3: THE 5-STOREY STRUCTURE - OPTIMUM DESIGN IN CASE 1

t = Thickness of Panel

W = Width of Panel

I = Second Mom. of Area for a Steel Column in a Frame



Group Number	Section Variables (mm)	Material Cost (£)	Construction Cost (£)	δ
1	t = 300 (W = 4000)	2,458	7,182	1
2	t = 200 (W = 4000)	5,268	41,900	-
3	t = 200 (W = 4000)	5,268	41,900	-
4	t = 160 (W = 4000)	2,107	19,410	-
11	$I = 103488 \times 10^4$ (Use 122474×10^4)	6,568	19,676	-
12	$I = 275140 \times 10^4$	11,714	36,616	1
	Total Cost = £200,067			

FIGURE 5.4: THE 5-STOREY STRUCTURE - OPTIMUM DESIGN IN CASE 2

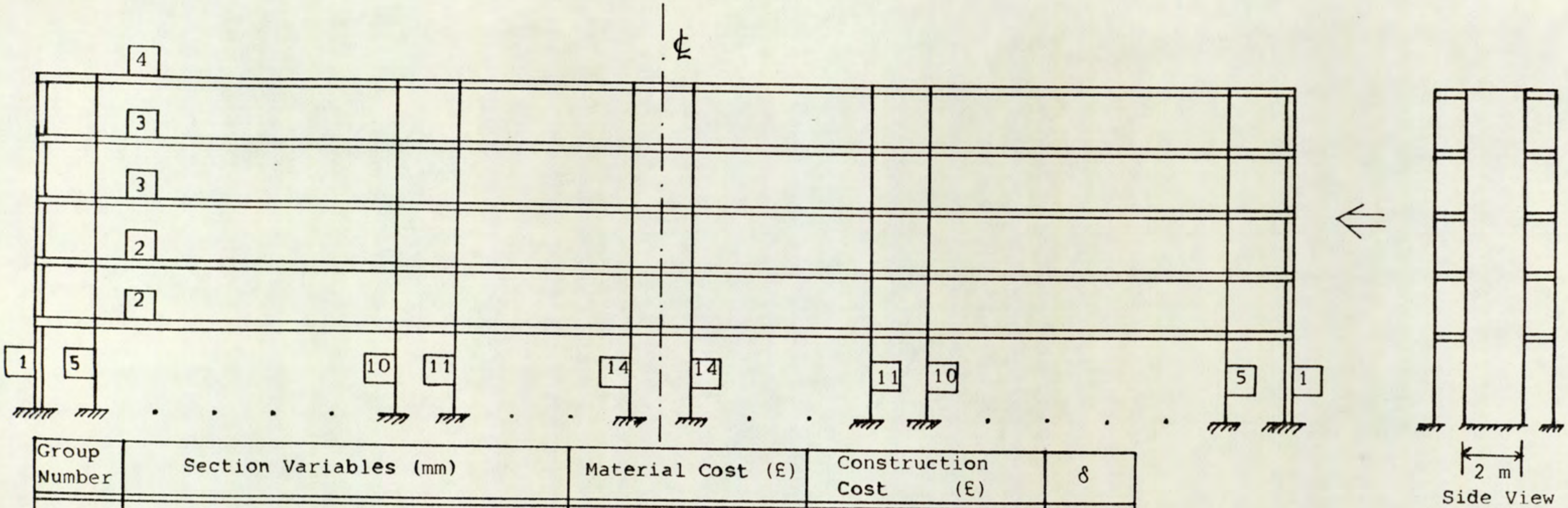
walls from 4 m to 2 m. The width of the slabs was kept unchanged at 4 m. The foundation cost of the new wall was reduced to £190. These edge shear walls and all the frames were allowed to be removed. After seven design iteration, the optimum cost design was obtained, and the final topology achieved for the complete structure is shown in Figure 5.5, where it is noticed that the number of frames retained has increased to eight. The frames adjacent to the walls were retained which indicate that the slender walls are insufficient to transmit the wind loads to the foundations. Therefore, more restraining frames were needed.

The sectional properties of the shear walls and those of the frames in group ten reached their upper bounds. The value of δ for these reached unity, i.e. $\delta = 1$. The thickness of the slabs in group two reached the upper bound, while that of group four was reached to its lower bound, Figure 5.5. Frames in groups five, 11 and 14 were all retained, but their δ values became small, nearer zero than one. It was therefore considered that there was a possibility of removing some or all of these frames. However, further designs without these frames proved infeasible.

The optimum total cost achieved in this case was £214079, which is 4.1% more than that for the design in Case 1. This increase is the result of increasing the number of frames.

5.3.1.4 Design Case 4

In this case, it was decided to insert another pair of shear walls into the structure. These replaced the frames in group 11. The width of all the walls and the slabs was set as 4 m. The foundation cost of each of the new walls was £430, while that of the edge walls was £350 each. The shear walls and all the frames were allowed to be removed.



Group Number	Section Variables (mm)	Material Cost (£)	Construction Cost (£)	δ
1	t = 300 (W = 2000)	1,229	3,750	1
2	t = 200 (W = 4000)	5,268	41,900	-
3	t = 180 (W = 4000)	4,742	40,360	-
4	t = 160 (W = 4000)	2,107	19,410	-
5	$I = 35068 \times 10^4$ (Use 40246×10^4)	2,875	8,613	0.23
10	$I = 275140 \times 10^4$	11,714	36,616	1
11	$I = 76487 \times 10^4$ (Use 79110×10^4)	4,730	14,171	0.38
14	$I = 60716 \times 10^4$ (Use 66307×10^4)	4,153	12,441	0.32
	Total Cost = £214,079			

FIGURE 5.5: THE 5-STOREY STRUCTURE - OPTIMUM DESIGN IN CASE 3

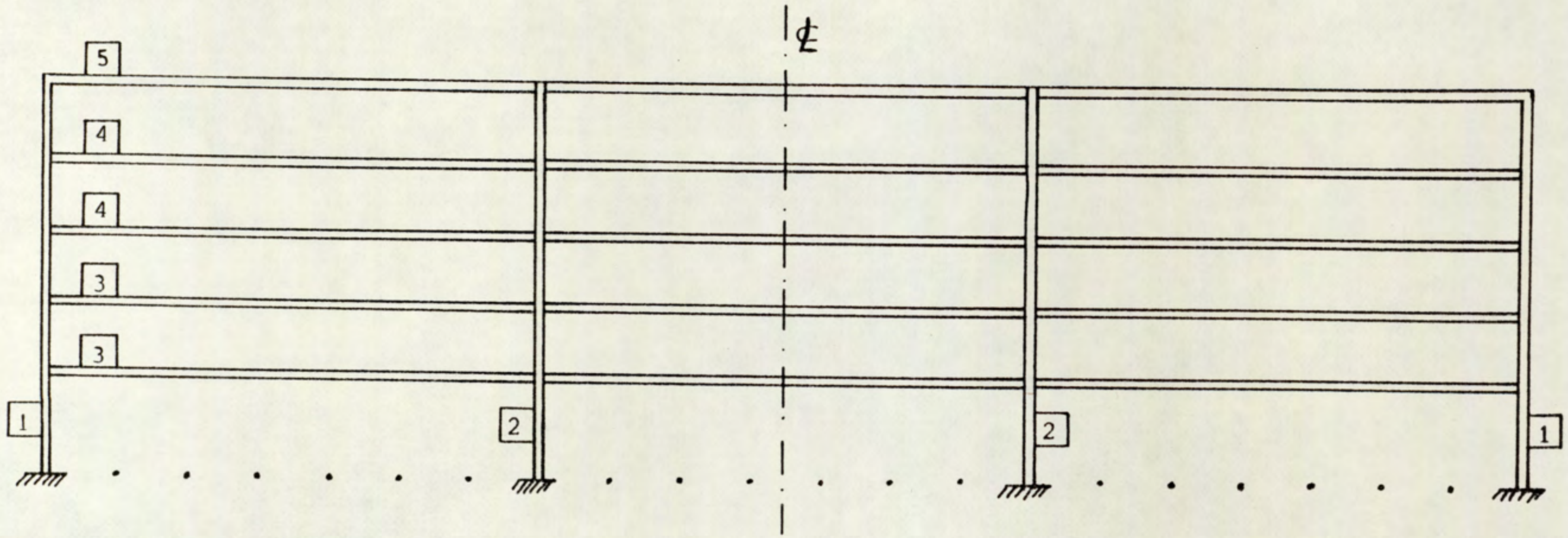
After five design iterations, only the shear walls and the central frame were retained. At this stage, shear wall in group two had a thickness of 270 mm, and floor slabs in group four had a thickness of 160 mm. It was decided to remove the central frame because its $\delta \cong 0$. As a result, after two more design iterations, the optimum design was obtained with the final topology shown in Figure 5.6.

The optimum values of the section variables are shown in the Table of Figure 5.6. The shear walls, with their maximum thicknesses, proved to be stiff and well capable of restraining the structure against the lateral wind loading.

The material costs of the shear walls in groups one and two were the same, but the construction costs were slightly different because of the variation in the foundation and the formwork costs. The optimum total cost achieved was £130140, which is 36.6% less than that for Case 1. This design indicated that it is by far more economical to use shear walls in place of the frames in group 11.

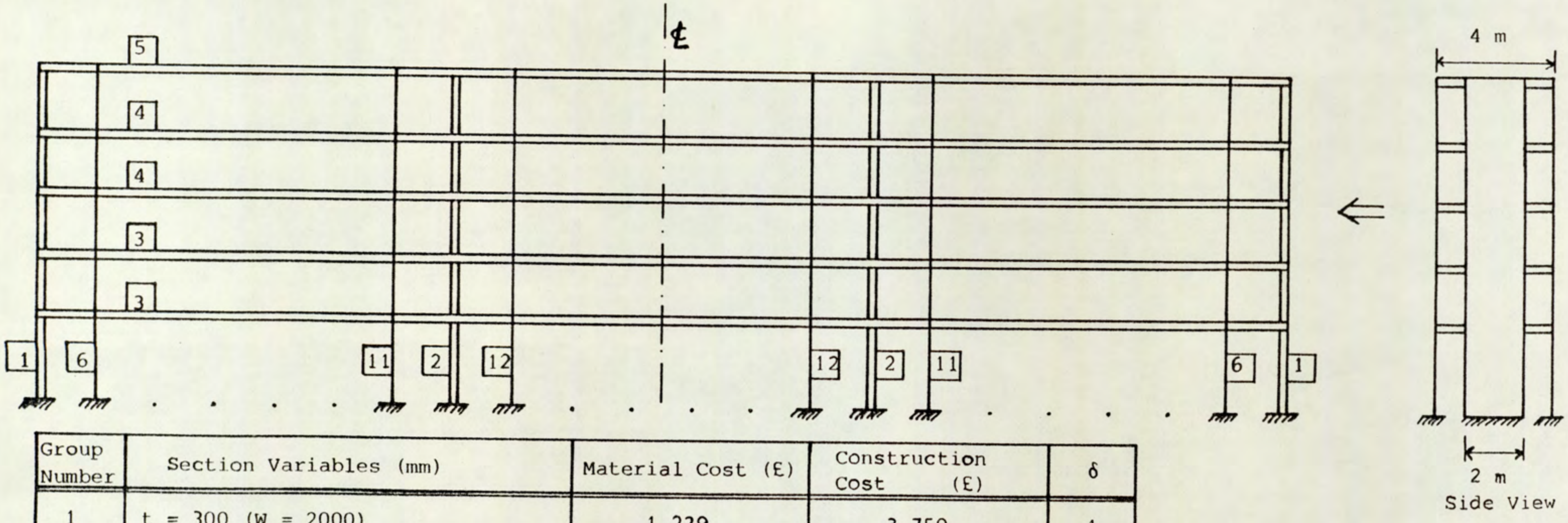
5.3.1.5 Design Case 5

In this case, the width of the shear walls, described in Design Case 4, reduced to 2 m. The optimum design was obtained after six design iterations with the final topology shown in Figure 5.7. It can be seen from the results that frame groups six, 11 and 12 were retained particularly to strengthen the shear walls against the wind loading. A non-feasible solution was always recorded whenever a frame was removed. The optimum total cost obtained is £168580 which is 17.8% less than that for Case 1. The foundation cost for the edge wall was £190, and for the inner wall was £240.



Group Number	Section Variables (mm)	Material Cost (£)	Construction Cost (£)	δ
1	t = 300 (W = 4000)	2,458	7,182	1
2	t = 300 (W = 4000)	2,458	7,359	1
3	t = 160 (W = 4000)	4,214	38,820	-
4	t = 170 (W = 4000)	4,478	39,590	-
5	t = 200 (W = 4000)	2,634	20,950	-
Total Cost = £130,143				

FIGURE 5.6: THE 5-STOREY STRUCTURE - OPTIMUM DESIGN IN CASE 4



Group Number	Section Variables (mm)	Material Cost (£)	Construction Cost (£)	δ
1	t = 300 (W = 2000)	1,229	3,750	1
2	t = 300 (W = 2000)	1,229	3,860	1
3	t = 200 (W = 4000)	5,268	41,900	-
4	t = 160 (W = 4000)	4,214	38,820	-
5	t = 160 (W = 4000)	2,107	19,410	-
6	$I = 41556 \times 10^4$ (Use 48525×10^4)	3,295	5,696	0.25
11	$I = 79017 \times 10^4$ (Use 79110×10^4)	4,730	14,171	0.39
12	$I = 76119 \times 10^4$ (Use 79110×10^4)	4,730	14,171	0.37
Total Cost = £168,580				

FIGURE 5.7: THE 5-STOREY STRUCTURE - OPTIMUM DESIGN IN CASE 5

5.4 A 9-STOREY, 11-BAY SYMMETRICAL STRUCTURE

Further investigation is to be carried out on the optimum cost topological design of a 9-storey structure of a type commonly encountered in practice. The investigation was extended to consider the effect of using restraining frames made out of reinforced concrete instead of steel, and the effect of using shear walls as the only restraining components. The ground structure, which is illustrated in Figure 5.8, consists of reinforced concrete walls and slabs for which Young's Modulus (E) is 28 KN/mm^2 and Poisson's ratio (ν) is 0.2. The intermediate frames can be built of steel sections for which E = 207 KN/mm^2, or of concrete sections for which E and ν values are as above.

Originally the structure was restrained laterally by two walls at the ends and ten intermediate frames. Each single bay frame consisted of two columns at any particular level. However, other possible topologies of the structure will be examined, and these demand slight alterations in numbering and grouping of the members. The restraining components were parallel and were 4 m apart. The width of the building was assumed to be 5 m. The height of the first storey was set as 5 m, while each of the other eight storeys was at 3.5 height, as shown in Figure 5.8.

The lateral sway at each storey, of height h, was limited to h/500. The horizontal deflection of each joint had an upper bound of 20 mm x f, where f is the storey number. The joint rotations were limited by an upper bound of 0.08 radians. The upper bounds imposed on the thicknesses of the grillage members were taken as 300 mm for the walls, and 200 mm for the slabs. While the lower bounds imposed were 120 mm and 160 mm on the walls and the slabs respectively. The value of I for the largest and the smallest universal column sections

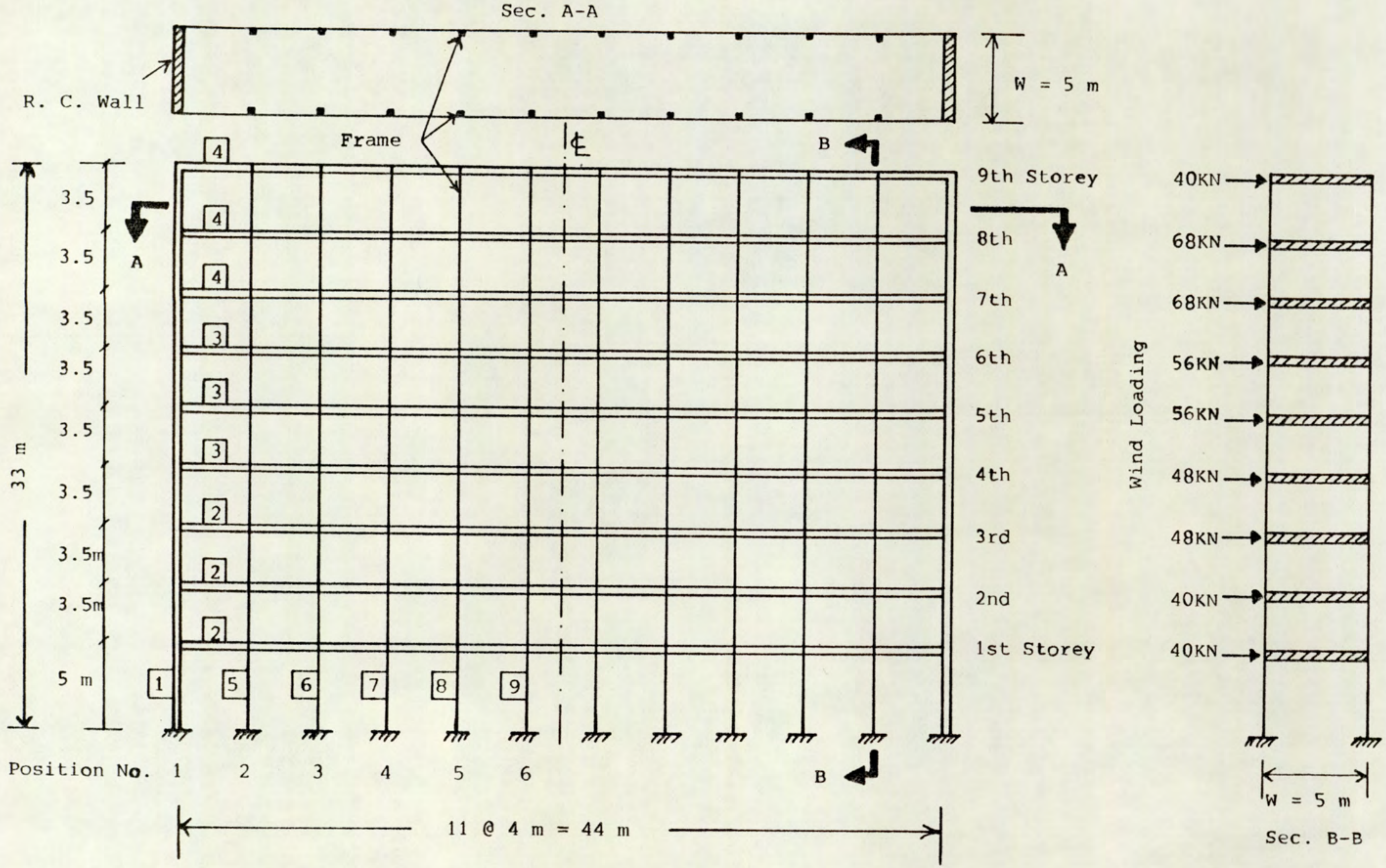


FIGURE 5.8: A9-STOREY, 11-BAY SYMMETRICAL COMPLETE STRUCTURE - WIND LOADING AND MEMBER GROUPING

available were set as bounds for the steel columns of the restraining frames. For concrete frames, the bounds on the section variables will be explained when dealing with Design Case 5.

The structure is symmetrical about the vertical central line, as shown in Figure 5.8. As a result, the rotations in the θ_y direction of all the joints at the line of symmetry vanish. Also, only the left hand half was considered for the design. This had 63 joints and 108 members which were originally collected together into nine groups, three for the slabs and six for the vertical components.

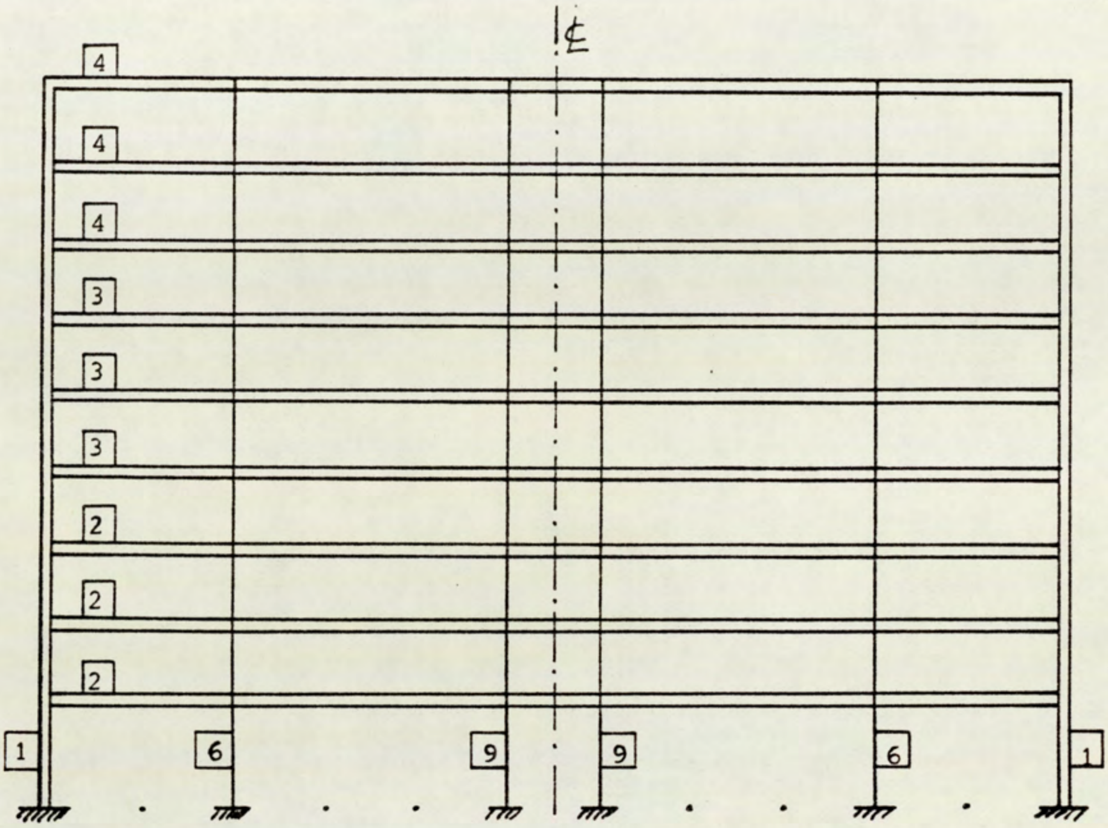
Figure 5.8 also shows the wind loads imposed on all the junctions except those of the edge walls where such loads became half the original values.

In all the design cases given below, the procedure started with the grillage members having a thickness of 160 mm, and all the steel columns having a second moment of area of $I = 17510 \times 10^4$ mm^4. The beams had a fixed value of $I = 29337 \times 10^4$ mm^4.

5.4.1 Design Case 1

In this design case, the shear walls and all the frames were allowed to be removed. Thus, no lower bounds were imposed on them. The shear walls were built from a uniform section, thus they belonged to a single group, e.g. group one. The slabs of every three consecutive storeys were assumed to belong to one group. Also, the columns of each steel frame were assumed to be built from one section, thus they belonged to a single group.

After six design iterations, for the left hand half of the structure, the optimum cost topological design was obtained. The final shape achieved for the whole structure is shown in Figure 5.9. The Table in this Figure shows the optimum section, for the member groups,



Group Number	Section Variables (mm)	Material Cost (£)	Construction Cost (£)	δ
1	t = 300 (W = 5000)	5,336	15,038	1
2	t = 200 (W = 5000)	5,174	20,147	-
3	t = 200 (W = 5000)	5,174	20,147	-
4	t = 200 (W = 5000)	5,174	20,147	-
6	$I = 90114 \times 10^4$ (Use 99994×10^4)	10,113	32,700	0.34
9	$I = 275140 \times 10^4$	20,662	64,305	1
Total Cost = £224,117				

t = Thickness of Panel

W = Width of Panel

I = Second Moment of Area for the Steel Columns of Frame

FIGURE 5.9: THE 9-STOREY STRUCTURE - OPTIMUM DESIGN IN CASE 1

which were required to stiffen the structure against wind loading. All the grillage members have reached their upper bounds. Frames in groups six and nine were retained with their δ variables at 0.34 and 1 respectively. The optimum total cost obtained is £224117 which includes the material and the construction costs of all the members in the complete structure, plus the foundation costs of the vertical components retained in the final topology.

5.4.1.1 Verification of the Design

Using the computer programme which will be described in Chapter 8, an investigation was carried out to examine the efficiency of the optimum design achieved in Case 1. The final design was checked to ensure that the lateral equilibrium and strength requirements were satisfied.

(a) The Sway Deflection

The final shape of the structure was analysed and the sway deflection profile of the end shear wall, i.e. group one, and of the central frame, i.e. group nine, were plotted as shown in Figure 5.10. The Figure also shows the allowable linear sway deflection profile in which the sway in a storey is restricted to h/500. It is noticed that the sway deflection at the first floor of the central frame has almost reached its limit. This showed that the sway deflection constraints dominated the design and it is apparent that the optimum result obtained without considering them will yield an unsafe solution. The sway deflection at all other storey levels were satisfied. The deflections at the first and the top level are plotted in Figure 5.11 for the left hand half of the structure.

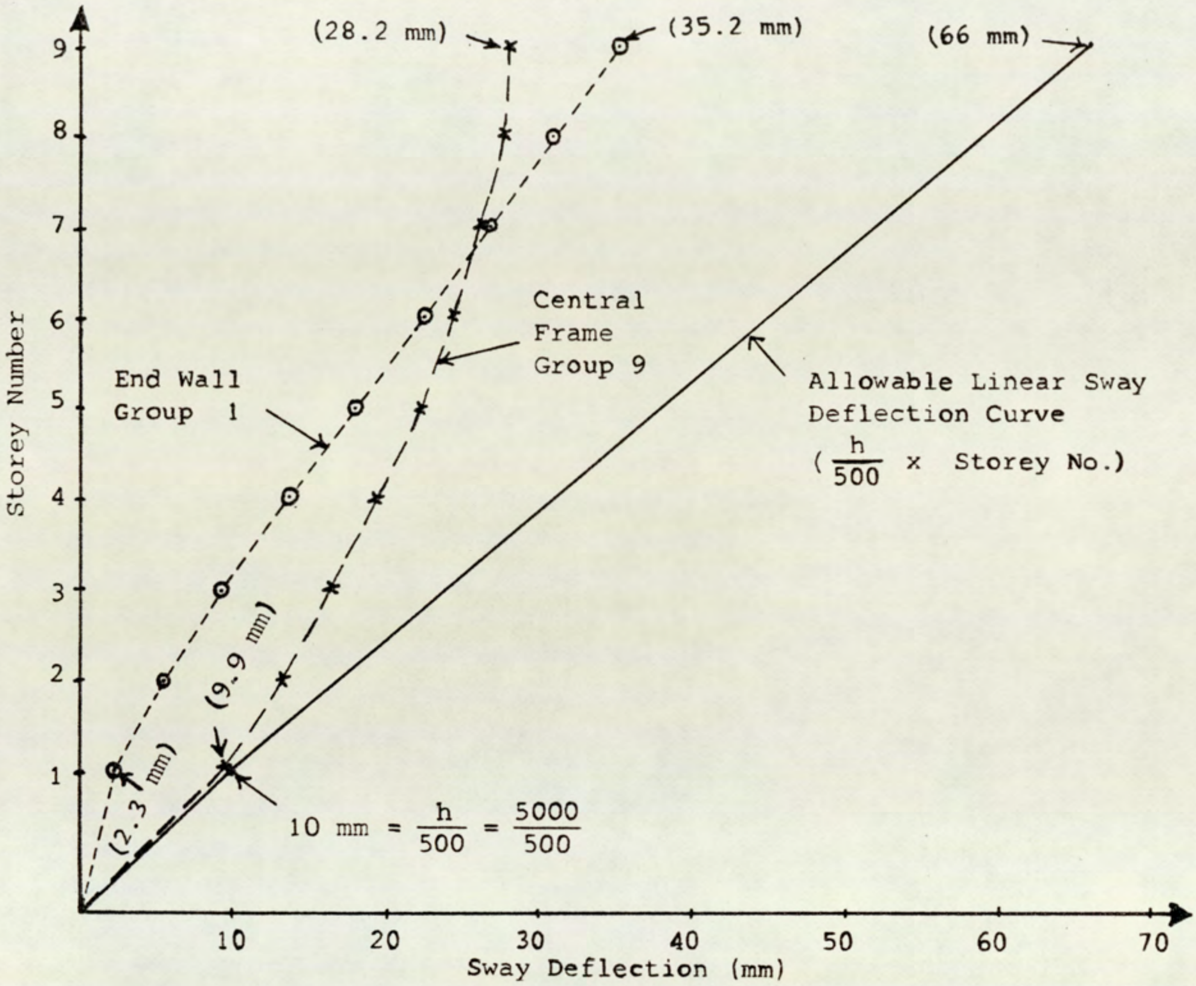
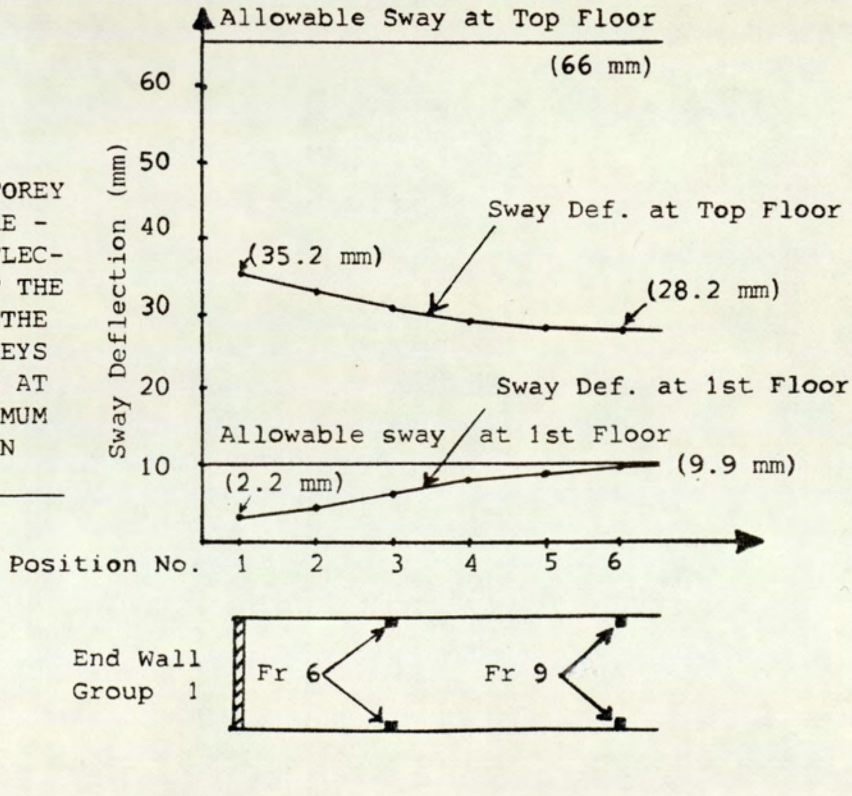


FIGURE 5.10: THE 9-STOREY STRUCTURE-SWAY DEFLECTION PROFILES FOR THE END WALL AND THE CENTRAL FRAME OBTAINED AT THE OPTIMUM DESIGN IN CASE 1

FIGURE 5.11: THE 9-STOREY STRUCTURE - SWAY DEFLECTIONS AT THE 1ST AND THE TOP STOREYS OBTAINED AT THE OPTIMUM DESIGN IN CASE 1



(b) Shear Forces

Figure 5.12 shows the distribution of lateral shear forces at the end wall and the central frame, obtained at the optimum design for Case 1. By following the sign convention, which was described in Figure 4.1, the direction of the lateral shear forces is designated as negative. The diagrams in Figure 5.12 show that the maximum shear force at the base of the end wall was about double that at the base of the central frame. This force reduced gradually for the first seven storeys of the wall, and then reverse in direction for the top two storeys.

The lateral shear force in the central frame was almost uniform throughout the first four storeys, and then it decreased steadily for the rest of the storeys. The shear force in the other frame (group six) was small and approximately uniform throughout its height.

As it was mentioned before, this design method does not include any vertical loading. Thus, the axial deformation in the columns of the frames is neglected. Because of this, and according to Croxton (1974), the shear forces in the frames are over-estimated by approximately 10% in all storeys. Nevertheless, to justify the design, the lateral shear strengths of the columns in the central frame, and of the end wall are checked below.

(c) Shear Stresses

According to BS 449 (1969), the average shear stress in a steel column, of web thickness greater than 40 mm, should not exceed 105 90 N/mm^2 for grade 43 steel. The universal column section used for the central frame had $I = 275140 \times 10^4$ mm^4 which has a web thickness of 47.6 mm, and section depth of 474.7 mm. The largest value of shear force carried by the two columns of the frame is

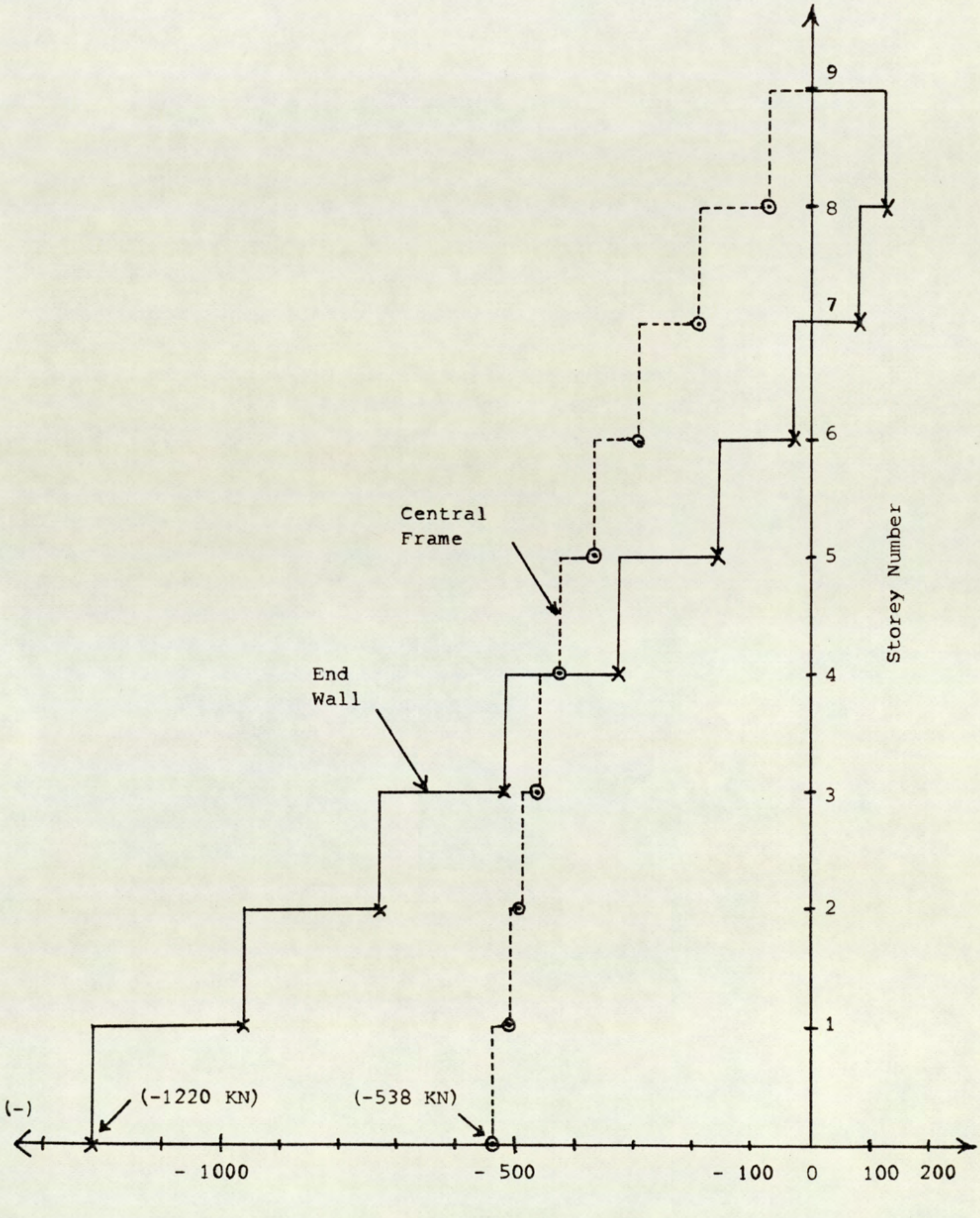


FIGURE 5.12: THE 9-STOREY STRUCTURE-DISTRIBUTION OF SHEAR FORCES IN THE END WALL AND THE CENTRAL FRAME OBTAINED AT THE OPTIMUM DESIGN IN CASE 1

538 KN, and thus each column carries 269 KN which is half the largest value. The shear stress is expressed as:

$$\text{Shear Stress} = \frac{\text{Shear Force}}{\text{Section Depth x Web Thickness}} \tag{5.1}$$

Therefore,

$$\text{Shear Stress} = \frac{269 \times 1000 \text{ N}}{474.7 \text{ mm} \times 47.6 \text{ mm}} = 11.9 \text{ N/mm}^2 \tag{5.2}$$

which is much less than the allowable 105 N/mm^2. The maximum shear force at the base of the concrete end walls is 1220 KN. Each wall had a width W = 5000 mm, and a thickness t = 300 mm. According to CP114: Part 2 (1969), clause 316, the shear stress which is calculated from:

$$\text{Shear Stress} = \frac{\text{Shear Force}}{t \,.\, W} \tag{5.3}$$

should not exceed the allowable 0.9 N/mm^2, (Table 6, CP114). However, the maximum shear stress at the base of the rectangular section of the wall is:

$$\text{Max.Shear Stress} = \frac{1.5 \times 1220 \times 1000 \text{ N}}{300 \text{ mm} \times 5000 \text{ mm}} = 1.215 \text{ N/mm}^2 \tag{5.4}$$

which is greater than the allowable. This means that the whole shearing force should be provided by shear reinforcement. As a result, the horizontal reinforcement must be increased by an amount slightly more than the minimum value of 0.25% specified in Section 4.10. However, as long as the maximum shear stress, equation (5.4), is less than four times the allowable, i.e.

$3.6\ N/mm^2$, the dimensions obtained for the wall section are acceptable and need not to be changed.

(d) Bending Moments

Figure 5.13 shows the bending moment diagrams for the end wall and the central frame obtained at the optimum design. The sum of the bending moments at both ends of the columns in a storey of the frame is equal, (equation 4.16). Nevertheless, the moments at the second, i.e. lower, end of each storey, of the frame and of the shear wall, were used for plotting the diagrams. It is noticed in Figure 5.12 that the maximum bending moment at the base of the central frame is about 1/10 of that at the base of the wall. This moment depended only on the differential sway, which was at its highest value at the first storey, Figure 5.10. The bending moment at the base of the wall, Figure 5.13, decreased gradually throughout the first five storeys, then it became negative for the rest of the four storeys, and ended up as zero value at the top of the ninth storey.

(e) Torsion

Torsion can be induced in the walls when the floor slabs bend in their own plane. The torques in the end wall of the 9-storey structure were found to be insignificant, amounting to a maximum of 1/250 of the bending moment at the base of the wall. A diagram representing the torques in the end wall is shown in Figure 5.14. The directions of torque in the Figure were plotted following the sign convention of Figure 4.1. Torques in the restraining frames were so small that they were ignored.

The maximum torque in the slabs was found to be only 1/500,000 of the bending moment at the base of the wall. It is evident therefore that the cumulative effect of the torques in the slabs,

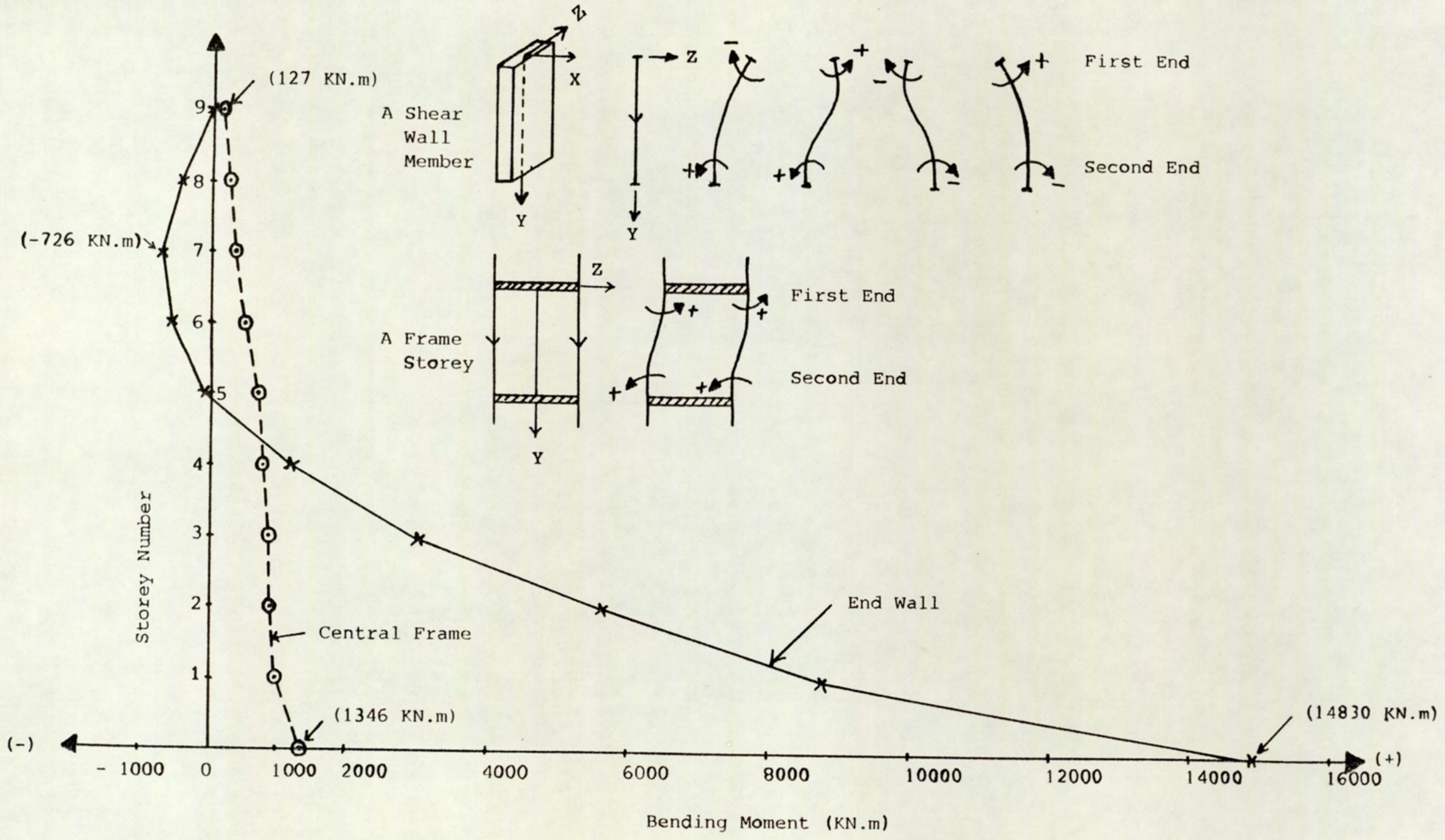


FIGURE 5.13: THE 9-STOREY STRUCTURE-BENDING MOMENTS AT THE SECOND ENDS OF A WALL AND A FRAME OBTAINED AT THE OPTIMUM DESIGN IN CASE 1

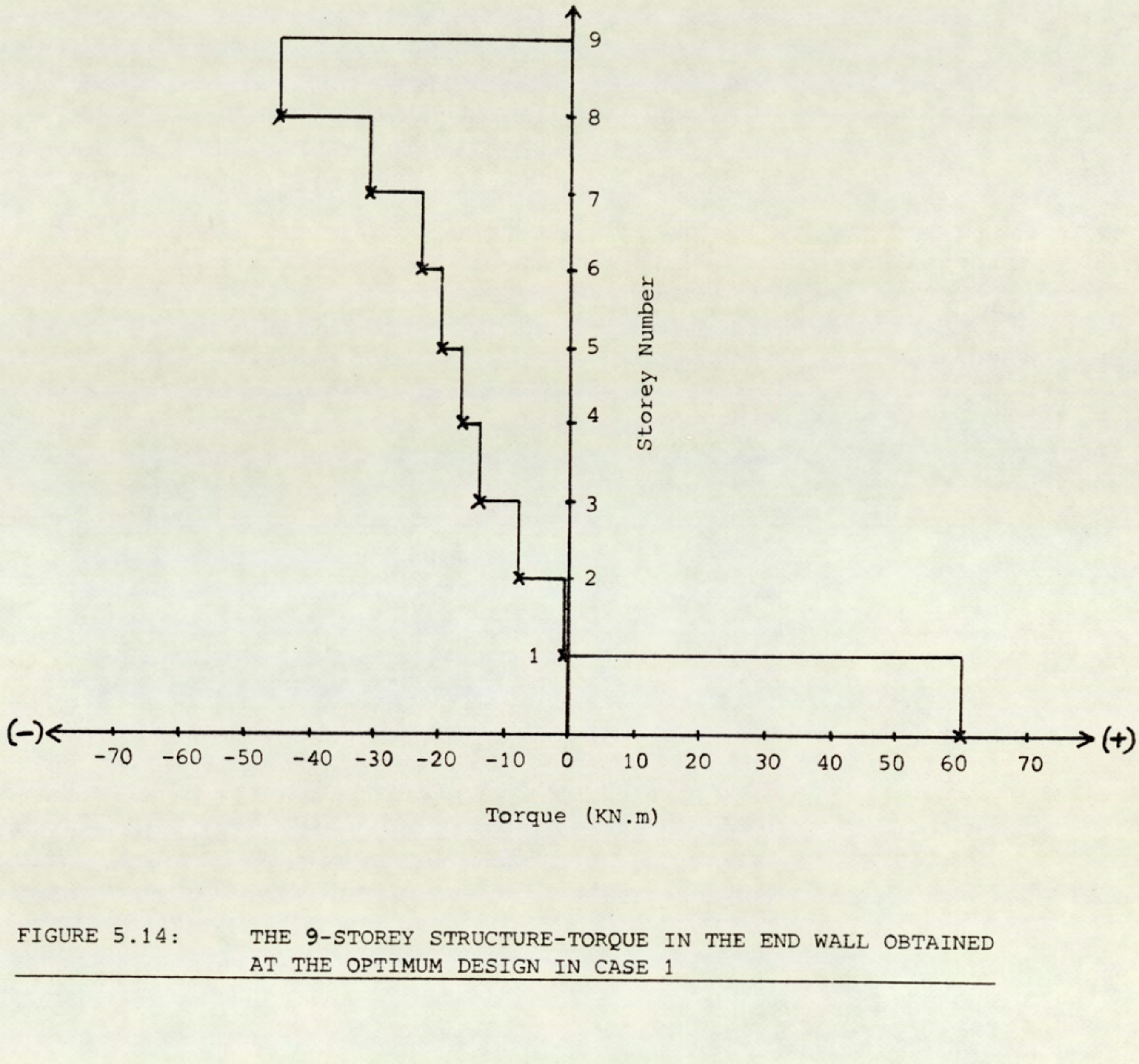


FIGURE 5.14: THE 9-STOREY STRUCTURE-TORQUE IN THE END WALL OBTAINED AT THE OPTIMUM DESIGN IN CASE 1

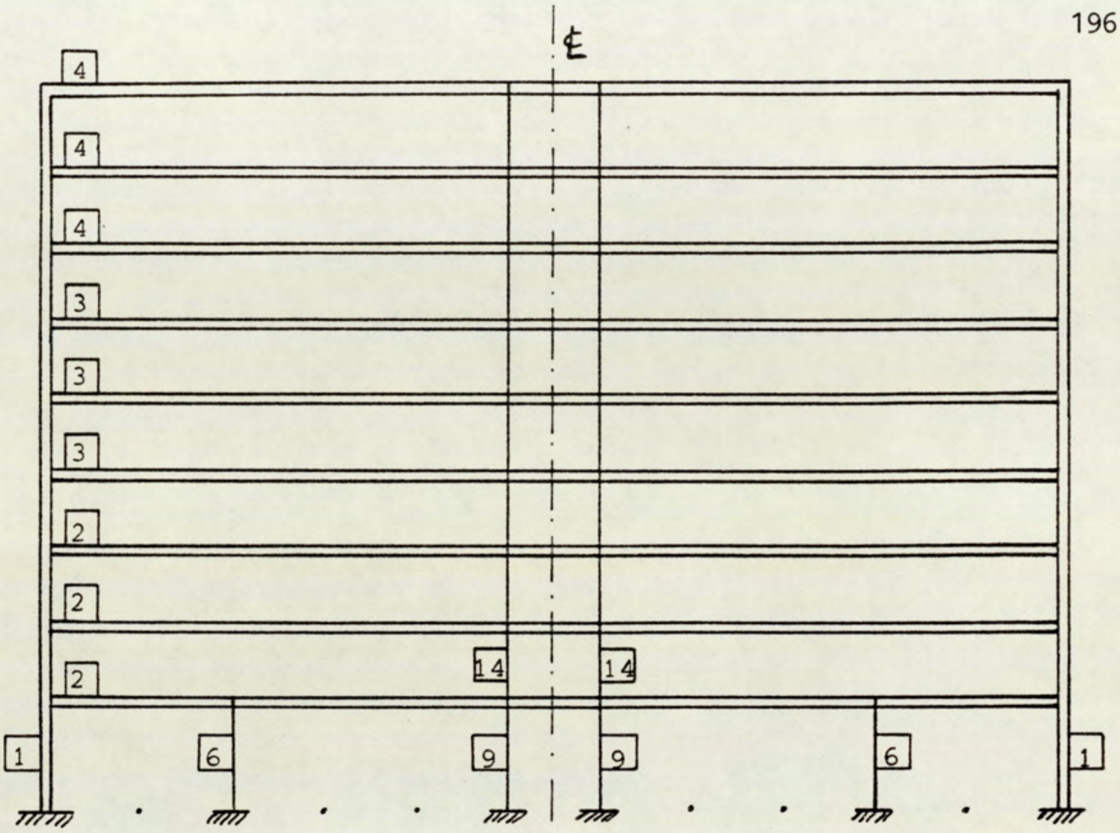
on the bending moment at the base of the walls, is almost unlikely to become significant in this structure. The same conclusion has been reached by Croxton (1974). This strengthens the assumption, made in Section 4.3, that the torsion of the floor slabs about their longitudinal axes should be ignored. Thus, by setting the torsional rotation of the slab to zero at the frame junctions, the column ends do not rotate. This causes the fact that these columns do not transfer the moments between the storeys of the frame. Therefore, such moments are created only from the differential sway between two consecutive storeys.

5.4.2 Design Case 2

The above arrangement of column grouping in Case 1 could be costly, from the designer's point of view. Therefore, in design Case 2, it was decided to construct each frame from two groups of columns. The advantage is to provide flexibility in removing unnecessary columns from the final topology.

The columns at the ground level belonged to a different group from the rest, and altogether there were ten groups of columns in the structure. This arrangement proved to be more economical as large column sections might be needed only at the ground floor.

After seven design iterations, for the left hand half of the structure, the optimum cost topological design was obtained and the final shape is shown in Figure 5.15. The grillage members reached their upper bounds. The columns at the ground level of the frame in position two, Figure 5.8, were retained. These columns belong to group six.



Group Number	Section Variables (mm)	Material Cost (£)	Construction Cost (£)	δ
1	t = 300 (W = 5000)	5,336	15,038	1
2	t = 200 (W = 5000)	5,174	20,147	-
3	t = 200 (W = 5000)	5,174	20,147	-
4	t = 200 (W = 5000)	5,174	20,147	-
6	$I = 23913 \times 10^4$ (Use 27601×10^4)	698	8,189	0.16
9	$I = 275140 \times 10^4$	3,074	15,310	1
14	$I = 150358 \times 10^4$ (Use 183118×10^4)	7,257	40,251	0.55
	Total Cost = £171,116			

FIGURE 5.15: THE 9-STOREY STRUCTURE - OPTIMUM DESIGN IN CASE 2

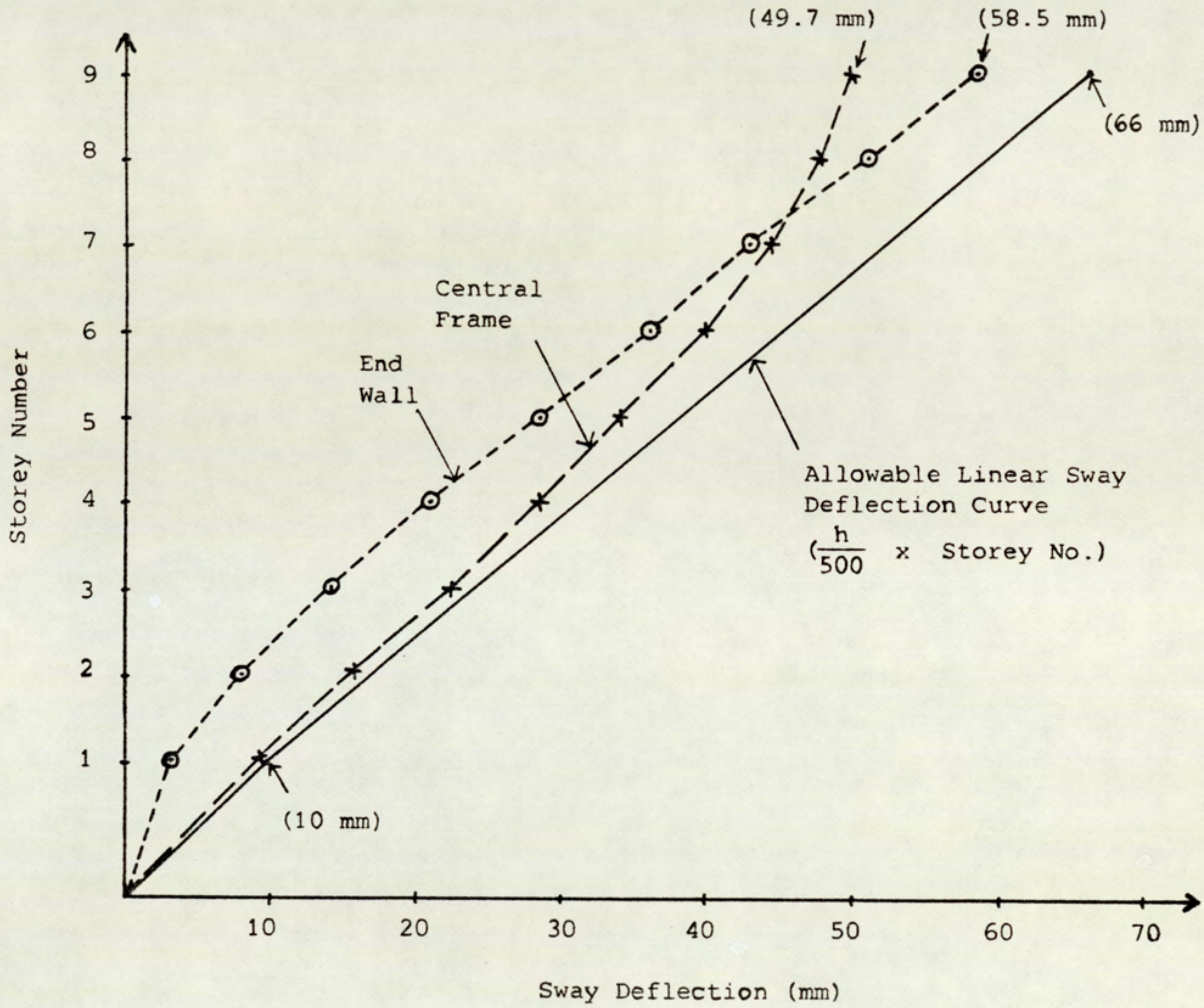


FIGURE 5.16: THE 9-STOREY STRUCTURE-SWAY DEFLECTION AT THE OPTIMUM DESIGN IN CASE 2

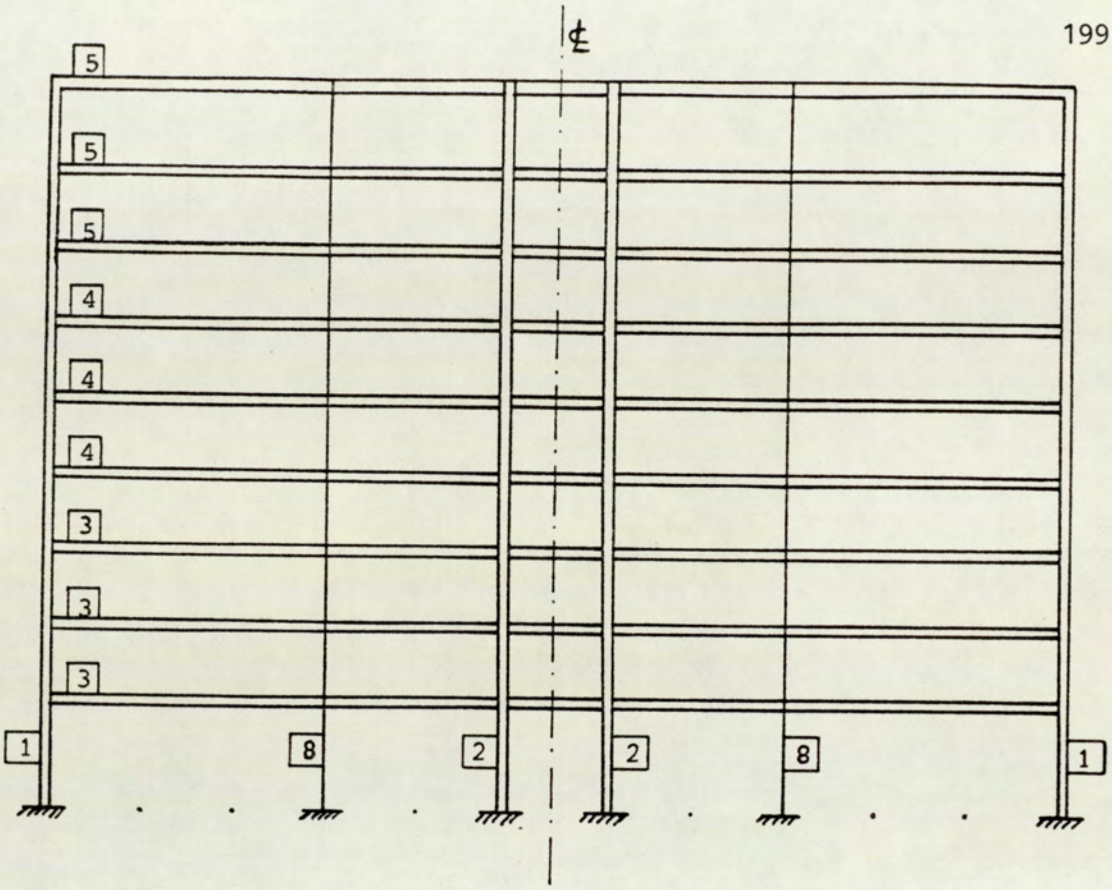
The central frame, in position six, was also retained. The columns in this frame belong to group nine at the ground level, and group 14 at the other storeys. Columns in group nine reached their upper bound with $\delta = 1$, while those in group 14 settled at a lesser value of the second moment of area with $\delta = 0.55$. Although the δ value for columns in group six is small ($\delta = 0.16$), no feasible solution was obtained when this group was removed. The optimum total cost achieved was £171116 which is 23.6% less than Case 1.

The sway deflection profiles for the end wall and the central frame, at the optimum design, are plotted in Figure 5.16. By comparing the deflection in this case and that in Case 1, Figure 5.10, it is noticed that the structure here is less rigid, although it satisfies the specified deflection. In this case, the deflection at the top storey of the central frame is 49.7 mm, while in Case 1 it is 28.2 mm. The deflection at the top storey of the end wall is 58.5 mm in this case, while it is 35.2 mm in Case 1. Notice that the allowable deflection at the top storey should not exceed 66 mm.

5.4.3 Design Case 3

In this design case, it was decided to replace the middle two frames by shear walls as shown in Figure 5.17. The width of all the walls and the slabs was kept at 5 m. The frames were steel and each pair belongs to a single group. The cost of foundation for each of the new walls was £680, while for each of the end walls it was £560.

The walls and all the frames were allowed to be removed. After six design iterations, for the left hand half of the structure, the optimum design was obtained. The walls and frames in group eight were retained in the final topology, which is shown in Figure 5.17. The Table in the Figure shows the section values. The optimum total cost



Group Number	Section Variables (mm)	Material Cost (£)	Construction Cost (£)	δ
1	t = 300 (W = 5000)	5,336	15,038	1
2	t = 300 (W = 5000)	5,336	15,302	1
3	t = 200 (W = 5000)	5,174	20,147	-
4	t = 160 (W = 5000)	4,139	18,724	-
5	t = 200 (W = 5000)	5,174	20,147	-
8	$I = 60776 \times 10^4$ (Use 66307×10^4)	7,529	24,959	0.25
	Total Cost = 147,005			

FIGURE 5.17: THE 9-STOREY STRUCTURE - OPTIMUM DESIGN IN CASE 3

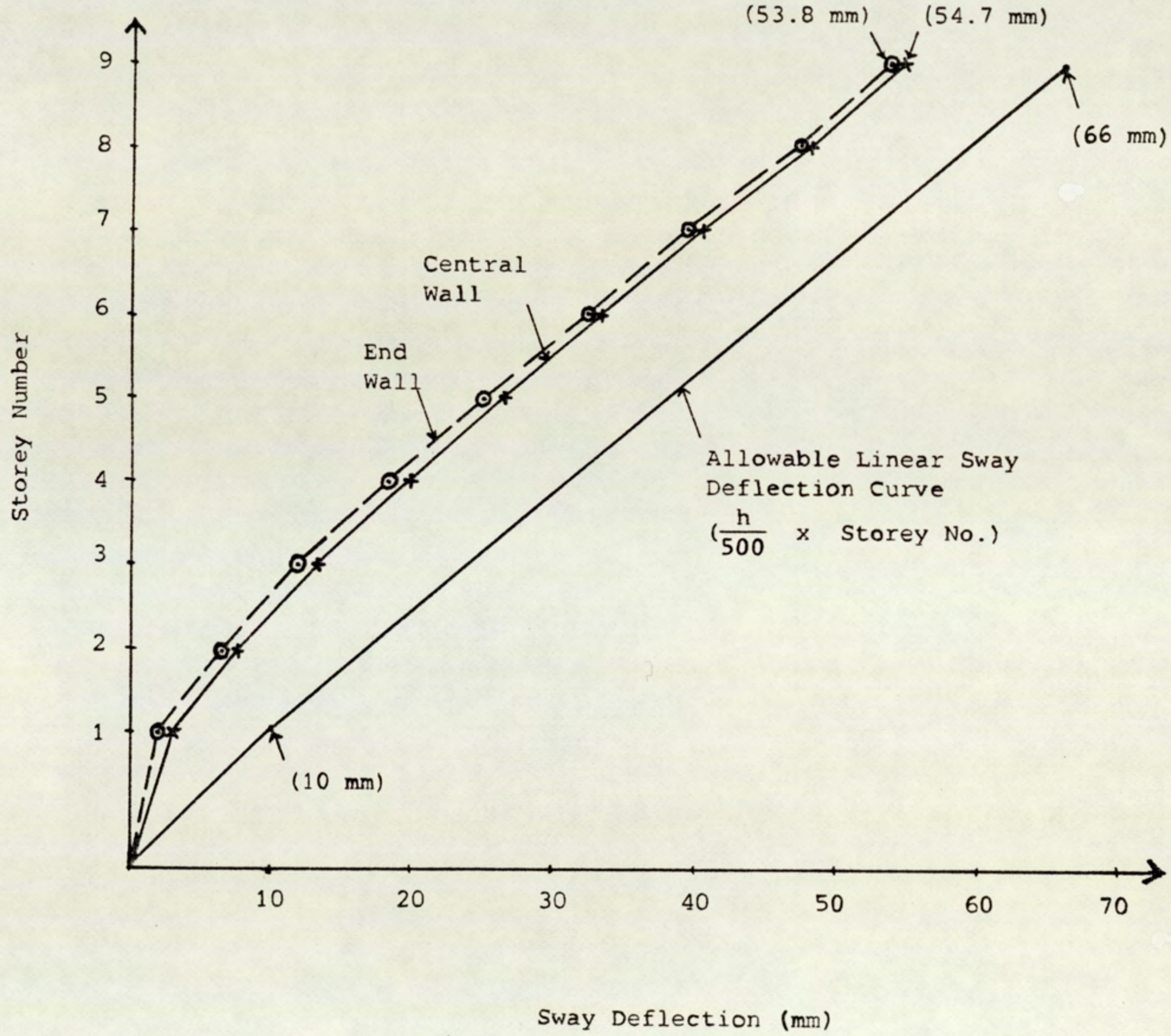


FIGURE 5.18: THE 9-STOREY STRUCTURE-SWAY DEFLECTION PROFILES AT THE OPTIMUM DESIGN IN CASE 3

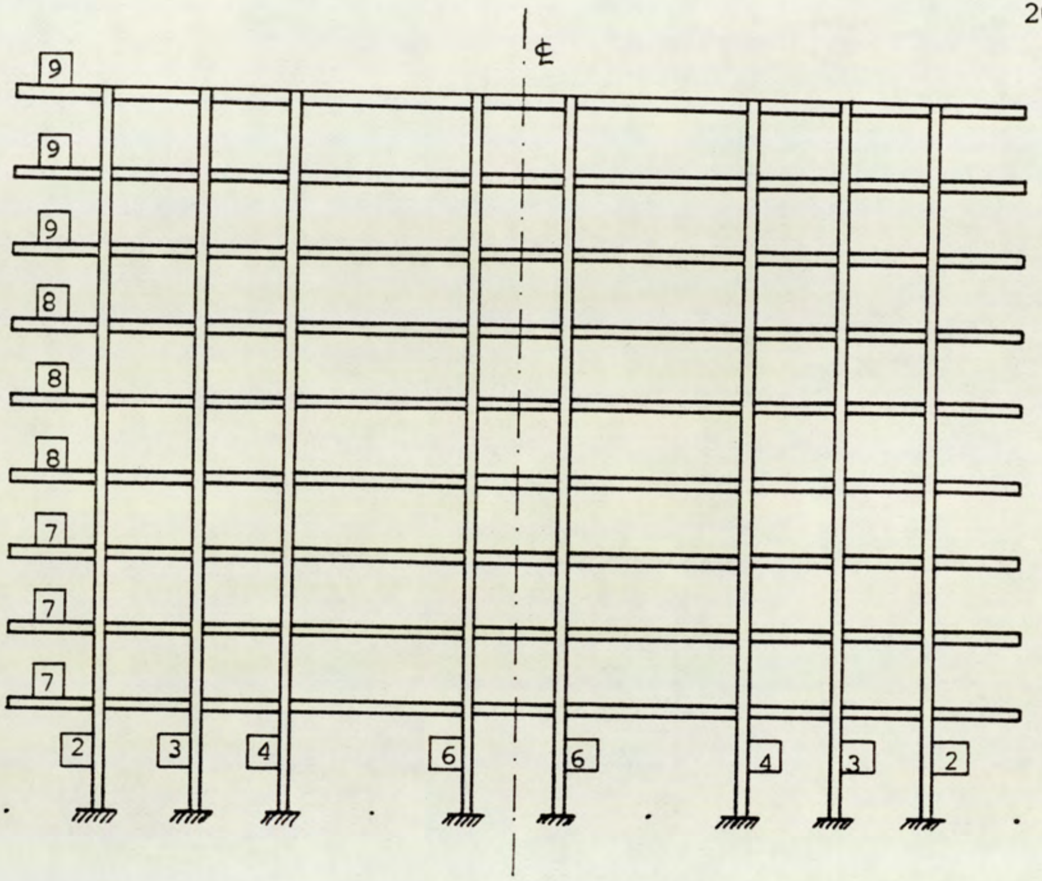
achieved is £147005 which is 34.4% less than that for Case 1. The sway deflection profiles for the central and the end walls are plotted in Figure 5.18. The lateral strength and stability requirements were checked and found satisfactory.

5.4.4 Design Case 4

In this case, it was decided to put all the vertical components as shear walls, i.e. no skeletal frames were used. The walls were placed in the same positions of the frames. The distances between the walls, and hence the wind loads at the junctions were kept at their original values. The aim of the design here was to find out the walls that are not effective in resisting the lateral wind loading. The widths of all the walls and the slabs were kept at 5 m. The upper and the lower bounds imposed on the thicknesses of the grillage members were taken as originally specified. The cost of the foundation for the end walls was £560 each, and for each of the other intermediate walls was £680.

The vertical components, of the 9-storey structure, were originally divided into six groups, (Section 5.4). Therefore, in this design case, the six groups represent the walls only. An additional three groups were specified for the slabs.

The design procedure was started with all the groups having a thickness of 160 mm. The policy of removing walls which were not required in the design is explained in Section 5.3.1.1, and need not be repeated here. Following such a policy, and after seven design iterations for the left hand half of the structure, the optimum design was obtained. The final topology achieved is shown in Figure 5.19. The final thicknesses of the walls are given in the Table below the Figure. It is shown that wall groups one and five were removed from the final topology while groups four and six have reached their upper



Group Number	Section Variables (mm)	Material Cost (£)	Construction Cost (£)	δ
2	t = 151 (W = 5000)	2,686	11,965	0.50
3	t = 285 (W = 5000)	5,069	14,961	0.95
4	t = 300 (W = 5000)	5,336	15,302	1
6	t = 300 (W = 5000)	5,336	15,302	1
7	t = 160 (W = 5000)	4,139	18,724	-
8	t = 160 (W = 5000)	4,139	18,724	-
9	t = 160 (W = 5000)	4,139	18,724	-
Total Cost = £144,546				

FIGURE 5.19: THE 9-STOREY STRUCTURE - OPTIMUM DESIGN IN CASE 4

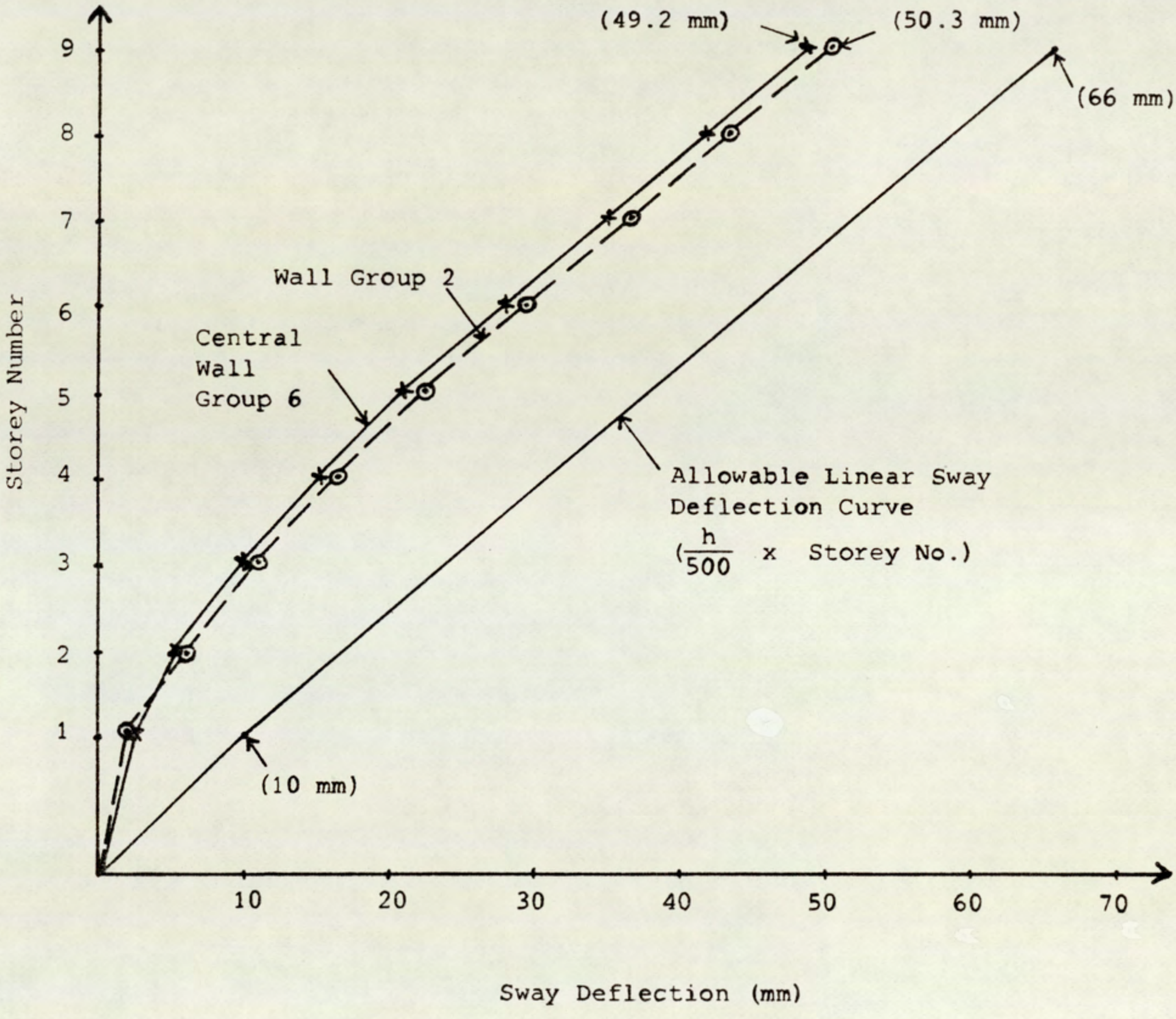


FIGURE 5.20: THE 9-STOREY STRUCTURE-SWAY DEFLECTION PROFILES AT THE OPTIMUM DESIGN IN CASE 4

bounds. The thicknesses of all the slab groups have settled on their lower bounds. The optimum total cost achieved is £144546 which is 35.5% less than Case 1.

The sway deflection profiles for wall groups two and six are plotted in Figure 5.20. It is noticed in this Figure that the sway in these two walls are very close. This gives the impression that the effect of bending of the floor slabs in their own plane is very small, and such slabs can be considered as rigid diaphragms.

5.4.5 Design Case 5

In this case, it was decided to use frames made out of reinforced concrete. The serviceability limit state method was used in the design of reinforced concrete columns of the frames. This method ensures satisfactory behaviour of the structure under service, i.e. working, loads. The principle criterion relating to serviceability is the prevention of excessive deflection.

The CP110 Code of Practice for the structural use of concrete outlines serviceability limit state calculations to ensure the avoidance of excessive deflection or cracking. As it stands, the code does not impose any limit on sway deflection of frames, but it suggests in clause 3.5.8 that if the depth of the cross section of a concrete column is more than its effective height/30 then the sway requirements may be satisfied without further calculation. However, the author found that this underestimates the column depth needed for the concrete restraining frames. A similar conclusion has been achieved by Okdeh (1980). Therefore, in the absence of a better limit, and as a large sway might produce excessive cracks, the limit of height/500 is adopted for the design of concrete columns of the sway frames, (Allen, 1974).

(a) Representation of Concrete Column Sections

One of the purposes of this design is to obtain the values of I for the columns so that the sway deflections are satisfied. The overall depth of each column is then obtained.

For a rectangular concrete column section, the second moment of area may be based on the assumption of using the gross uncracked section, where the entire cross-section, including the reinforcement on the basis of modular ratio, is considered.

For a rectangular section shown in Figure 5.21, the uncracked second moment of area is given by Reynolds and Steedman (1976) as:

$$I_{xx} = \frac{b\,d^3}{12} + 2\,A_s\,(\alpha - 1)\,\left(\frac{d}{2} - r\right)^2 \qquad (5.5)$$

This is assuming that the compression reinforcement is equal to the tensile reinforcement, and the distance of the neutral axis below the top edge is equal to d/2. Here d is the overall depth of the section, b is its width, r is the reinforcement cover, A_s is the area of tensile or compression reinforcements and α is the modular ratio which can be expressed as:

$$\alpha = E_s \;/\; E_c \qquad (5.6)$$

where E_s is the Young's Modulus of steel, and E_c is the Young's Modulus of concrete.

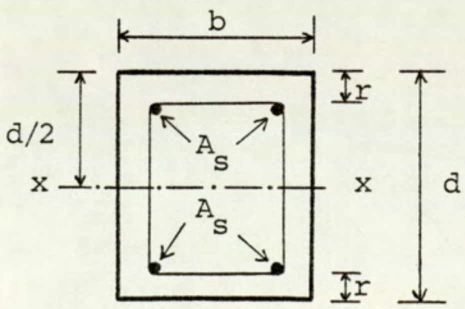


FIGURE 5.21: A TYPICAL SECTION OF A REINFORCED CONCRETE COLUMN

(b) Design Assumptions

If equation (5.5) is required to be written only in terms of thickness (b) and overall depth (d) of the section, then certain assumptions must be made. These are:

(i) Requirements regarding areas of reinforcement in the columns are defined in clauses 3.11.4.1 and 3.11.5 of CP110. The minimum area of main longitudinal bars is 1%, and the maximum should not exceed 6% of the gross cross-section area of the column. In this design problem, it was assumed that the percentage of reinforcement (ρ) is equal to 3%, i.e. $\rho = 2\, A_s / b\, d = 0.03$, thus:

$$2\, A_s = 0.03\, b\, d \tag{5.7}$$

The link reinforcement was assumed to be 0.15%.

(ii) It was assumed that:

$$\alpha = \frac{E_s}{E_c} = \frac{200\ KN/mm^2}{28\ KN/mm^2} = 7.14 \tag{5.8}$$

(iii) Different values of reinforcement cover (r) are given in clause 3.11.2 of CP110 code. However, this code always used fixed values of r to prepare its various design charts. One of these values, which was used here, is given as:

$$r = 0.1\, d \tag{5.9}$$

All the above assumptions were used in the design method described in this section, but other values can also be used by a designer.

These can be arranged as data given to the computer program which will be described in Chapter 8.

By substituting equations (5.7), (5.8) and (5.9) into equation (5.5), we obtain:

$$I_{xx} = \frac{b\,d^3}{12} + 0.03\,b\,d\,(7.14 - 1)\,(0.5\,d - 0.1\,d)^2 \qquad (5.10)$$

Thus:

$$I_{xx} = \frac{b\,d^3}{12} + \frac{0.354\,b\,d^3}{12} \qquad (5.11)$$

and:

$$I_{xx} = \frac{1.354\,b\,d^3}{12} = 0.1128\,b\,d^3 \qquad (5.12)$$

Thus by initially specifying the values of b and d, the second moment of area for an uncracked rectangular concrete column section can be computed from equation (5.12).

(c) Section Boundaries

The lower bound imposed on the second moments of area for the concrete columns was taken as $I = 18240 \times 10^4\ mm^4$, this is when $b = d = 200$ mm. The upper bound was taken as $I = 738720 \times 10^4\ mm^4$ when $b = 300$ mm and $d = 600$ mm. These bounds were computed using equation (5.12).

It is worth stating that the values of b and d were not involved whatsoever in the design process. This was because, only the second moment of area (I) for the column section was needed, but when an I value was obtained at the end of a design iteration, only the thickness (b) was required to be specified by the designer, and by using equation (5.12) the overall depth (d) can be computed. The assumed b and the calculated d were only used in assessing the material and the construction costs of the columns. The cost was calculated automatically by the computer program.

It was decided to make d ≧ b with d parallel to the Z-axis (Figure 4.1). As a result, a value of $I_i = 92340 \times 10^4\ mm^4$ for b = d = 300 mm, was used as an indicator. If any value of I for a column section, obtained from a design iteration, with $I \leq I_i$, then b was given the value of b = 200 mm, and if $I > I_i$ then b = 300 mm. This was done to ensure that the lower and the upper bounds on the column section were not violated. The bounds on the thickness of the grillage members were taken as originally specified in Section 5.4.

(d) Cost Assessment

In this design method the beam sections were specified before hand. One section was assumed for all the beams, and was kept unaltered during the design process. This hypothetical section was only used in the process of assessing the total cost of the structure. Thus, a beam section of thickness b = 200 mm and overall depth d = 450 mm was assumed. The percentage of the longitudinal reinforcement (ρ) was specified as 1.85%, and the link reinforcement was 0.15% of the longitudinal section.

Using the methods described in Appendix B and Section 4.10, the material cost for all the beams of a single 9-storey frame was found to be £263. While the construction cost was assessed as £1081. The material cost for the columns was found to be £3.4/kN. The foundation costs were estimated as £1400 per frame, i.e. £700 per footing. The cost of the end wall foundation was taken as £560, as in Case 1.

(e) <u>The Design Results</u>

The two edge shear walls and all the concrete frames were allowed to be removed. The design in Case 1 showed that the sway in the ground floor columns may decide the design outcome, it was therefore decided to group the members in the manner adopted in Case 2.

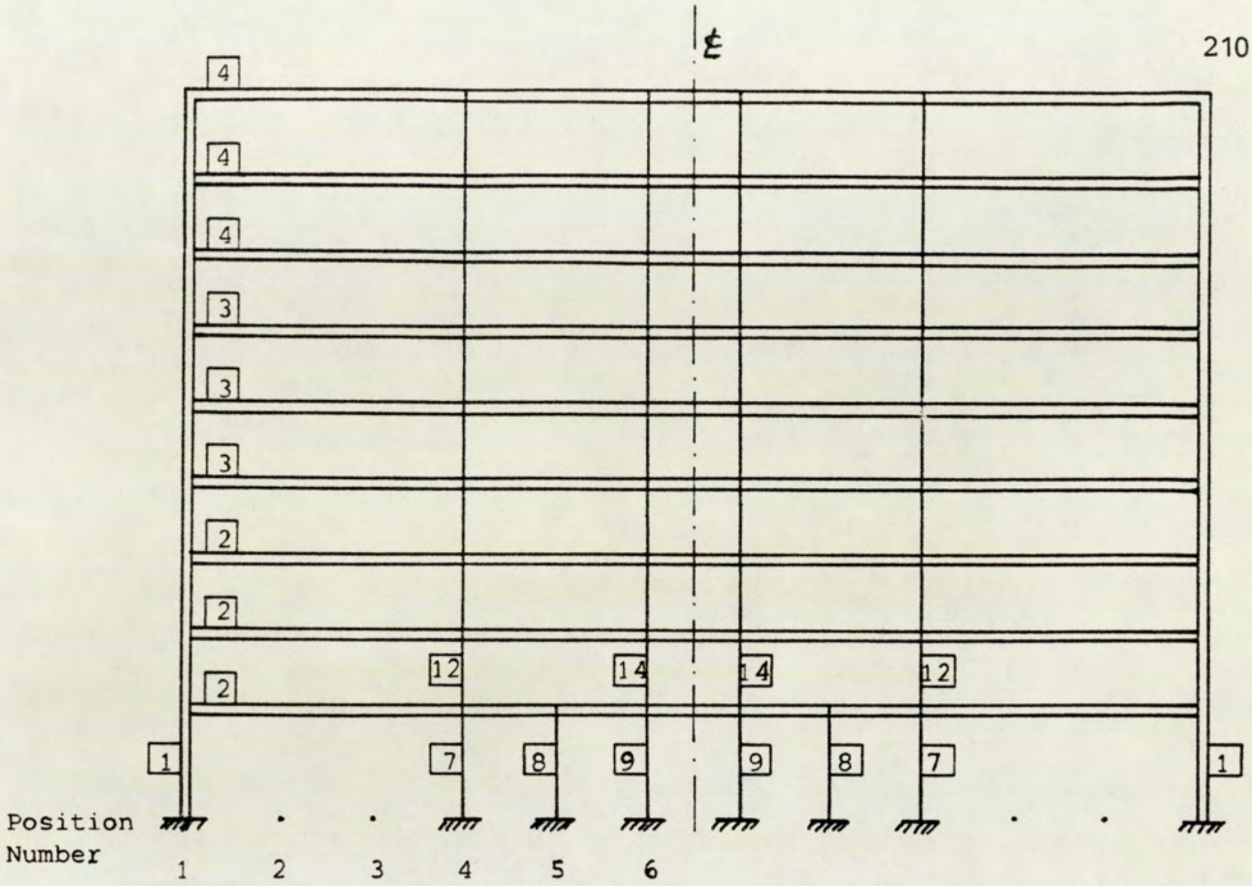
The design procedure started with the grillage members having a thickness of 160 mm. All the column groups had an initial second moment of area of $I = 92340 \times 10^4 \ mm^4$ which is the indicating value mentioned earlier in Section (c). The method that was used in removing members from the ground structure was explained in Section 5.3.1.1.

After eight design iterations, the optimum cost topological design was obtained, and the final shape achieved for the 9-storey structure is shown in Figure 5.22. The section variables of the grillage panels have reached their upper bounds.

The optimum second moments of area for the rectangular concrete sections of the retained columns are shown in the Table in Figure 5.22. The thickness (b) of each section was previously assumed by the designer, and the overall depth (d) of the section was computed using equation (5.12)

The frame in position two, which contains column groups five and ten, and the frame in position three, which contains groups six and 11, were removed. The columns in group 13 were also removed. The column groups eight and nine have reached their upper bound.

The optimum total cost achieved is £137138 which is 38.8% less than that for Case 1. One of the reasons for this substantial economical achievement is that the cost of concrete frames is much



Concrete Frames

Group Number	Section Variables (mm)	Material Cost (£)	Construction Cost (£)	δ
1	t = 300 (W = 5000)	5,336	15,038	1
2	t = 200 (W = 5000)	5,174	20,147	-
3	t = 200 (W = 5000)	5,174	20,147	-
4	t = 200 (W = 5000)	5,174	20,147	-
7	$I=547552\times10^4$ (b = 300, d = 543)	797	6,396	0.74
8	$I=738720\times10^4$ (b = 300, d = 600)	826	6,481	1
9	$I=738720\times10^4$ (b = 300, d = 600)	826	6,481	1
12	$I=660404\times10^4$ (b = 300, d = 578)	2,144	7,925	0.89
14	$I=366528\times10^4$ (b = 300, d = 475)	1,856	7,069	0.49
	Total Cost = £137,138			

I = Second moment of area for a rectangular concrete column section
b = Thickness of section
d = Overall depth of section

FIGURE 5.22: THE 9-STOREY STRUCTURE - OPTIMUM DESIGN IN CASE 5, WHERE REINFORCED CONCRETE FRAMES ARE USED

smaller than that for steel, although more concrete restraining frames were needed.

The lateral strength and stability requirements were checked as previously explained in design Case 1, and found to be satisfactory. The sway deflection profiles for the end wall and the central frame are plotted in Figure 5.23. It is shown in the Figure that the deflections at the first four storeys are very much near the limit.

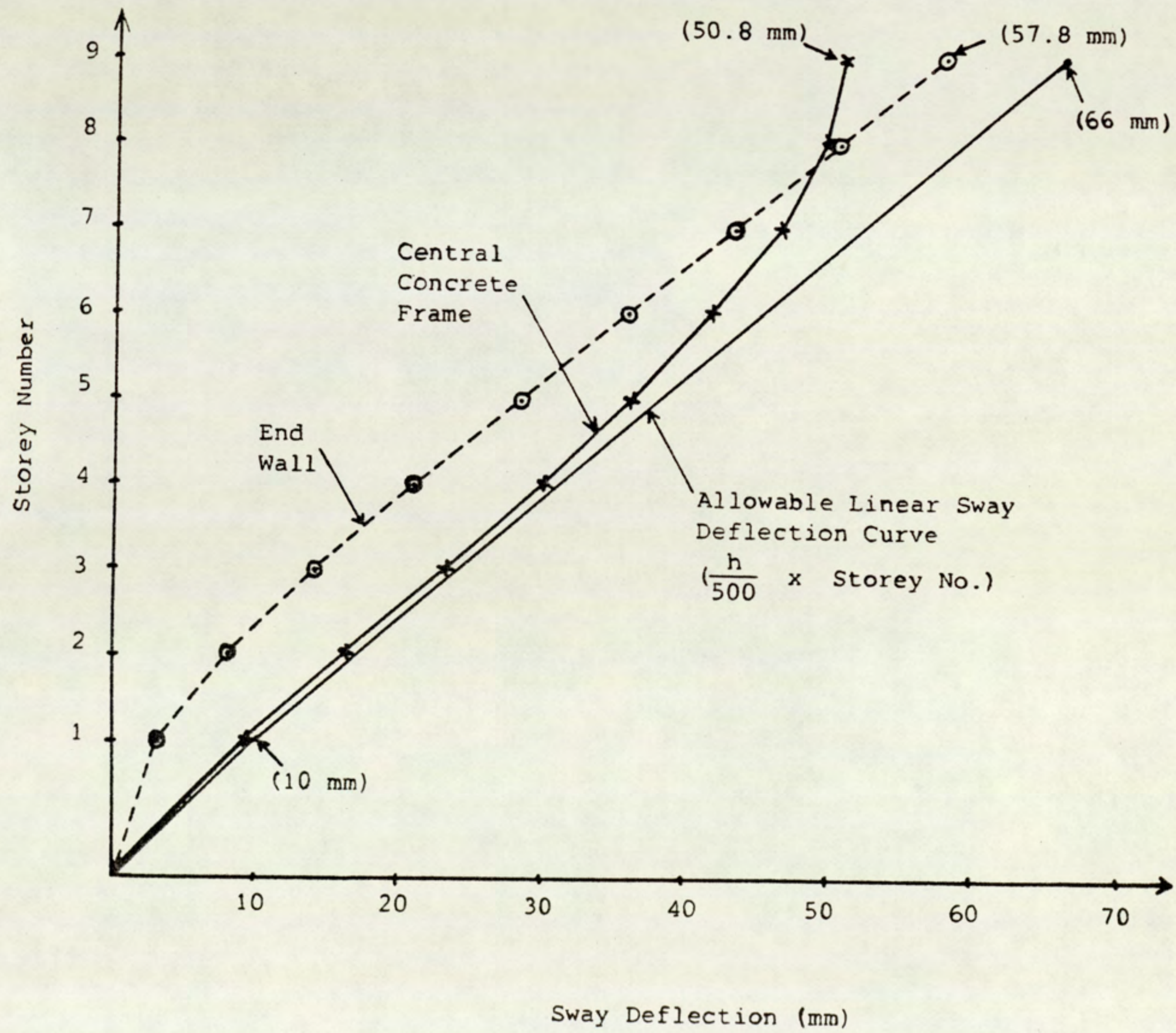


FIGURE 5.23: THE 9-STOREY STRUCTURE-SWAY DEFLECTION PROFILES AT THE OPTIMUM DESIGN IN CASE 5, WHERE REINFORCED CONCRETE FRAMES ARE USED

CHAPTER 6

OPTIMUM DESIGN OF REINFORCED CONCRETE FLAT GRILLAGES WITH SUPPORTING COLUMNS

6.1 INTRODUCTION

In this Chapter a method is proposed for a minimum weight or a minimum cost topological design of reinforced concrete flat grillages made of straight orthogonal members and subjected to normal dead and live loads. The members are considered as rigidly connected monolithic beams of rectangular sections. The minimisation is carried out subject to restrictions on structural behaviour, including stiffness, stress and deflection. The object of the optimisation is to find the concrete cross-sectional dimensions and the corresponding amount of reinforcing steel.

In each of the grillage structures considered here, the beams lie in one plane and they are free or rigidly connected at their ends. The grillage can be carried freely by a number of columns, or by columns and fixed end supports. The columns are built normal to the planes of the grillages as is the case of roof, foundation and bridge deck systems.

The matrix displacement method is used to construct the overall stiffness matrix of the grillage structures, including the supporting columns. The effect of shear distortion is included in the stiffness coefficients of the beams.

The approximating programming method is used to linearise the non-linear design problem by employing the first two terms of Taylor's series. This is then solved by the simplex method. An economical program, using the backing store, was written by the author to formulate the design problems. This program and the data preparation will be described in Chapter 8. For symmetrical structures, use is made of symmetry about one or two axes to reduce computer time and storage.

6.2 REPRESENTATION OF GRILLAGE SYSTEMS

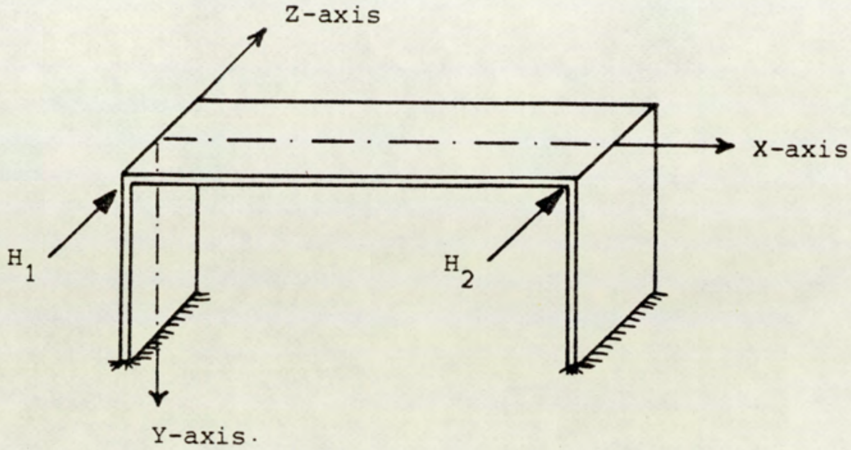
The grillage structural configuration is represented by discrete elements of beams, and supported, if required, by columns. The positive sign convention adopted for forces and displacements is in accordance with the right-hand screw rule. This was used in Chapter 4 for the design of deep beam vertical grillages. The same sign convention is adopted for the horizontal (flat) grillages, as illustrated in Figure 6.1. The axes of the beams lie in the XY plane, and if supporting columns are existing, then they will be erected in the Z-direction.

Joints are placed at all points where longitudinal and transverse beams intersect. Joints are also placed where there are more than one concentrated load or a discontinuity in the magnitude of the uniform load between intersection points. No joints are placed at the end points where the beams are fixedly supported (encastre'). Each joint has three degrees of freedom in z, θ_x and θ_y directions. The axial beam deformations, i.e. in-plane loads, are neglected here.

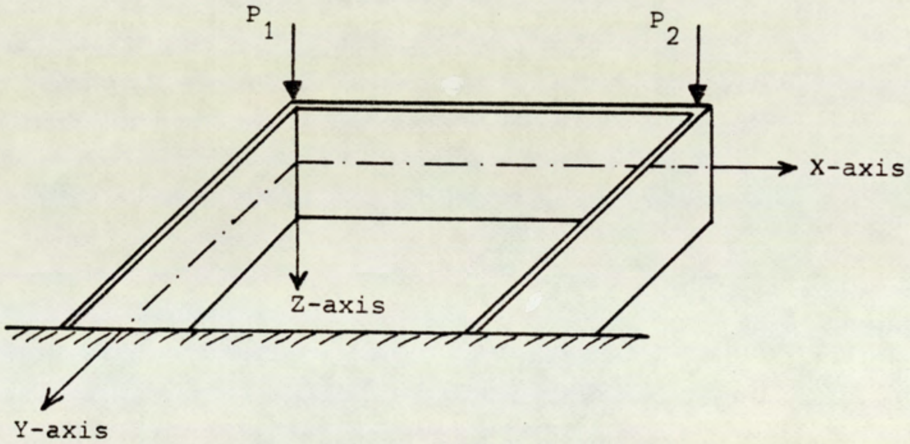
The external loads are applied in the vertical Z-direction. These loads are either concentrated, as pointed loads on the joints, or uniformly distributed on the beams.

6.2.1 The Stiffness Matrix of a Flat Grillage

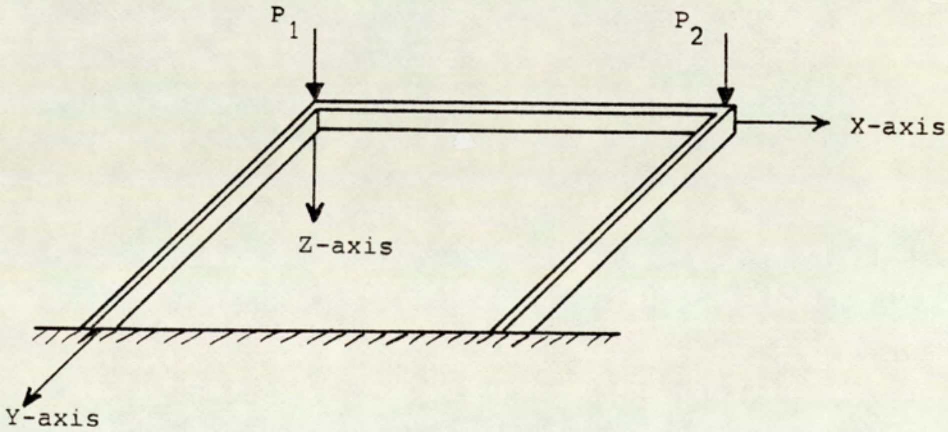
The thickness (t) and the overall depth (w) of the rectangular section of each concrete beam in the grillage are considered to be the main design variables. Since the grillage members are either transverse or longitudinal, it is convenient to construct two separate



(a) Vertical Deep Beam Grillage



(b) Horizontal Deep Beam Grillage



(c) Horizontal Grillage

FIGURE 6.1: SIGN CONVENTION FOR HORIZONTAL GRILLAGES

stiffness matrices. These were obtained in Chapter 4, Section 4.2.1. The contributions of a transverse (parallel to X-axis) member to the overall stiffness matrix are given by equation (4.8a), while the contributions of a longitudinal (parallel to Y-axis) member are given by equation (4.8b) The forces in a transverse member are expressed by equation (4.10a), and in a longitudinal member these forces are determined by equation (4.10b).

The only difference between the matrices used in this Chapter and that of Chapter 4 is that the stiffness coefficients here are functions of both t and w. Thus, by substituting the factor ψ, which was identified by equation (4.6) as a function of the variable w, into equation (4.7), the stiffness coefficients b, d, e and f will become:

$$b = \frac{B_1\, t\, w^3}{B_2 + B_3\, w^2} \tag{6.1a}$$

where $B_1 = GE$, $B_2 = GL^3$ and $B_3 = 1.2EL$ are constants,

$$d = -\frac{D_1\, t\, w^3}{D_2 + D_3\, w^2} \tag{6.1b}$$

where $D_1 = GE$, $D_2 = 2GL^2$ and $D_3 = 2.4E$ are constants,

$$e = \frac{E_1\, t\, w^3 + E_2\, t\, w^5}{E_3 + E_4\, w^2} \tag{6.1c}$$

where $E_1 = EGL^2$, $E_2 = 0.3E^2$, $E_3 = 3GL^3$ and $E_4 = 3.6EL$ are also constants, and

$$f = \frac{F_1\, t\, w^3 - F_2\, t\, w^5}{F_3 + F_4\, w^5} \tag{6.1d}$$

where $F_1 = EGL^2$, $F_2 = 0.6E^2$, $F_3 = 6GL^3$ and $F_4 = 7.2EL$ are constants. All the constants above are functions of the concrete elastic modulus E, the shear modulus G and the length L of the member.

The member torsional stiffness is expressed by the coefficient q in equation (4.7) for a deep member with w >> t, i.e. the ratio w/t > 10. However, due to the fact that the cross sectional dimensions (t and w) of the flat grillage member always vary, q is expressed here as:

$$q = G W t^3 / c_2 L \qquad (6.1e)$$

where c_2 is a constant factor depending upon the ratio of w/t, (Timoshenko, 1955) and (Shigley, 1976). Some of the values of c_2 are given in Table 6.1. In the design method used here, the member thickness (t) is always less than or equal to the overall depth (w), i.e. t ≦ w.

w/t	1.0	1.2	1.5	2.0	2.5	3.0	4	5	10	>10
c_2	7.11	6.02	5.10	4.37	4.02	3.80	3.56	3.44	3.20	3.00

TABLE 6.1: DIFFERENT VALUES OF TORSIONAL STIFFNESS CONSTANT c_2 FOR RECTANGULAR SECTIONS

6.2.2 The Stiffness Matrix of a Supporting Column

A concrete flat grillage can be built either with or without column supports. The numbers and the positions of the columns are decided upon by the designer to stabilise the whole structure. These columns have rectangular concrete sections in which the thickness

(t_c) and the overall depth (h_c) of each section are the unknown variables required to be obtained, see Figure 6.2.

The columns are only connected to the joints of the grillage where two horizontal orthogonal members intersect. In general, for a column rigidly connected at ends 1 and 2, Figure 6.2, the member forces $\underline{P}' = \{ P_c \; M_{x1} \; M_{x2} \; M_{y1} \; M_{y2} \}$ are related to member displacements $\underline{U}' = \{ u \; \theta_{x1} \; \theta_{x2} \; \theta_{y1} \; \theta_{y2} \}$ by the standard equation:

$$\underline{P}' = \underline{k}' \; \underline{U}' \tag{6.2}$$

where P_c is the column axial force, M_{x1} and M_{x2} are the moments about X-axis at ends 1 and 2 respectively, M_{y1} and M_{y2} are the moments about Y-axis, u is the axial distortion, θ_{x1} and θ_{x2} are the rotations of the member ends about X-axis, and θ_{y1} and θ_{y2} are the rotations about Y-axis. The $\underline{k}'$ matrix represents the member stiffnesses. Thus, equation (6.2) can be written in full as:

$$\begin{bmatrix} P_c \\ M_{x1} \\ M_{x2} \\ M_{y1} \\ M_{y2} \end{bmatrix} = \begin{bmatrix} a & 0 & 0 & 0 & 0 \\ 0 & e_x & f_x & 0 & 0 \\ 0 & f_x & e_x & 0 & 0 \\ 0 & 0 & 0 & e_y & f_y \\ 0 & 0 & 0 & f_y & e_y \end{bmatrix} \begin{bmatrix} u \\ \theta_{x1} \\ \theta_{x2} \\ \theta_{y1} \\ \theta_{y2} \end{bmatrix} \tag{6.3}$$

where:

$$a = E\,A / L_c \,, \quad e_x = 4\,E\,I_x / L_c \,, \quad f_x = 2\,E\,I_x / L_c$$

$$e_y = 4\,E\,I_y / L_c \,, \quad f_y = 2\,E\,I_y / L_c$$

in which E is the concrete Elastic Modulus, $A = t_c h_c$ is the cross sectional area, L_c is the length, I_x and I_y are the second moments of area of the section about the axes X and Y respectively.

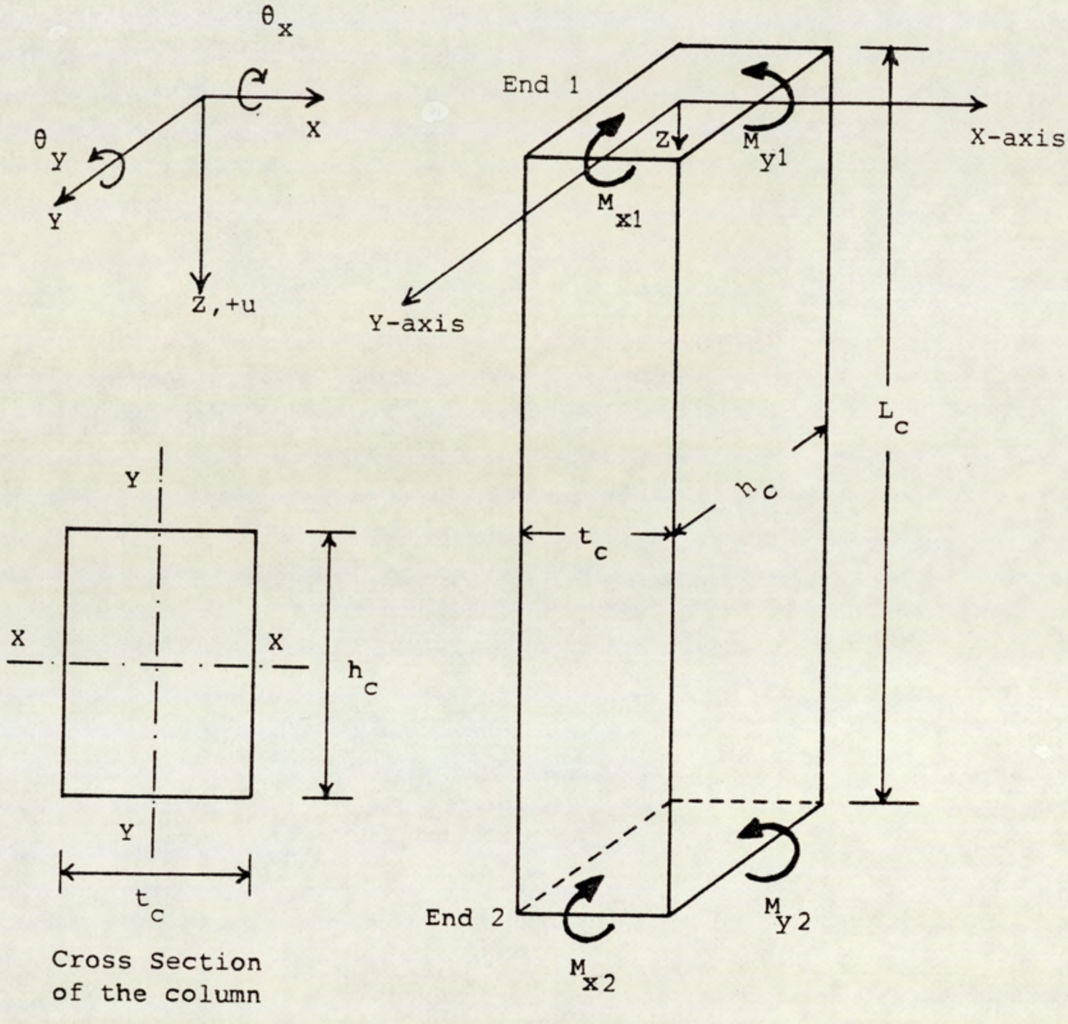


FIGURE 6.2: SIGN CONVENTION FOR A COLUMN SUPPORT

These can be expressed as:

$$I_x = t_c \; h_c^3 \; / \; 12 \tag{6.4a}$$

$$I_y = t_c^3 \; h_c \; / \; 12 \tag{6.4b}$$

The local positive axis P for the column shown in Figure 6.2 is indicated by an arrow pointing downward to the second end. For a column member connected to joint i at end 1 and joint j at end 2, the end displacements $\underline{U}'$ may be related to the joint displacements $\underline{X}$ by the relationship:

$$\underline{U}' = \underline{A} \; \underline{X}$$

i.e.

$$\begin{bmatrix} U \\ \theta_{x1} \\ \theta_{x2} \\ \theta_{y1} \\ \theta_{y2} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \begin{bmatrix} z \\ \theta_x \\ \theta_y \\ \hline z \\ \theta_x \\ \theta_y \end{bmatrix} \end{array} \begin{array}{l} \\ \text{joint i} \\ \\ \\ \text{joint j} \\ \\ \end{array} \tag{6.5}$$

where $\underline{A}$ is the displacement transformation matrix. The overall stiffness matrix $\underline{K}'$ is thus given by:

$$\underline{K}' = \underline{A}^T \; \underline{k}' \; \underline{A} \tag{6.6}$$

where $\underline{A}^T$ is the transpose of $\underline{A}$. Matrix $\underline{K}'$ represents the contributions of the column member to the overall stiffness matrix of the grillage, and it can be written as:

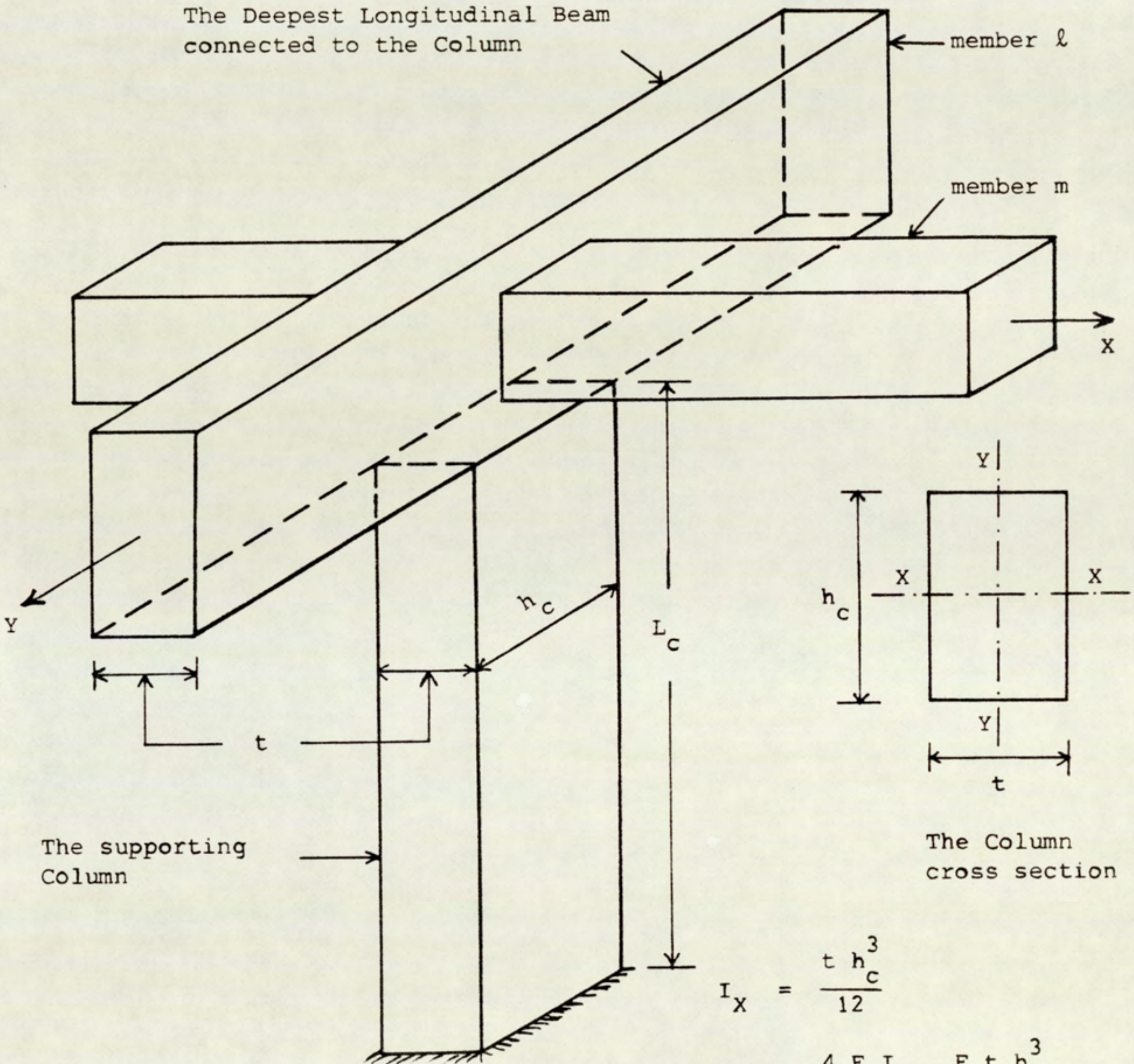
$$\begin{bmatrix} \underline{K}'_{ii} & \underline{K}'_{ij} \\ \underline{K}'_{ji} & \underline{K}'_{jj} \end{bmatrix} = \begin{array}{c} \begin{array}{cccccc} z_i & \theta_{xi} & \theta_{yi} & z_j & \theta_{xj} & \theta_{yj} \end{array} \\ \left[\begin{array}{ccc|ccc} a & 0 & 0 & -a & 0 & 0 \\ 0 & e_x & 0 & 0 & f_x & 0 \\ 0 & 0 & e_y & 0 & 0 & f_y \\ \hline -a & 0 & 0 & a & 0 & 0 \\ 0 & f_x & 0 & 0 & e_x & 0 \\ 0 & 0 & f_y & 0 & 0 & e_y \end{array}\right] \end{array} \tag{6.7}$$

In the design method described in this Chapter, it is assumed that the columns are rigidly connected to the grillage at their top ends, i.e. at joint i, and they are fixed at their bases, i.e. at joint j. Therefore, only the stiffness contributions relating to the displacements of joint i are considered. The $\underline{K}'$ matrix then becomes:

$$\underline{K}' = \begin{bmatrix} a & 0 & 0 \\ 0 & e_x & 0 \\ 0 & 0 & e_y \end{bmatrix} \tag{6.8}$$

which is the matrix used in the present work to represent the supporting column in the design of flat grillage structures. The coefficients of this matrix are identified in equations (6.3) and (6.4).

The thickness of a column section is taken equal to that of the deepest grillage member meeting at the column head, i.e. $t_c = t$. Therefore, the supporting column will be represented in the programming problem by its sectional overall depth (h_c) only. This is done to reduce the total number of the design variables. As a result, the stiffness coefficients of equation (6.8) will be functions of the thickness (t), of the deepest beam connected to the column, and h_c.



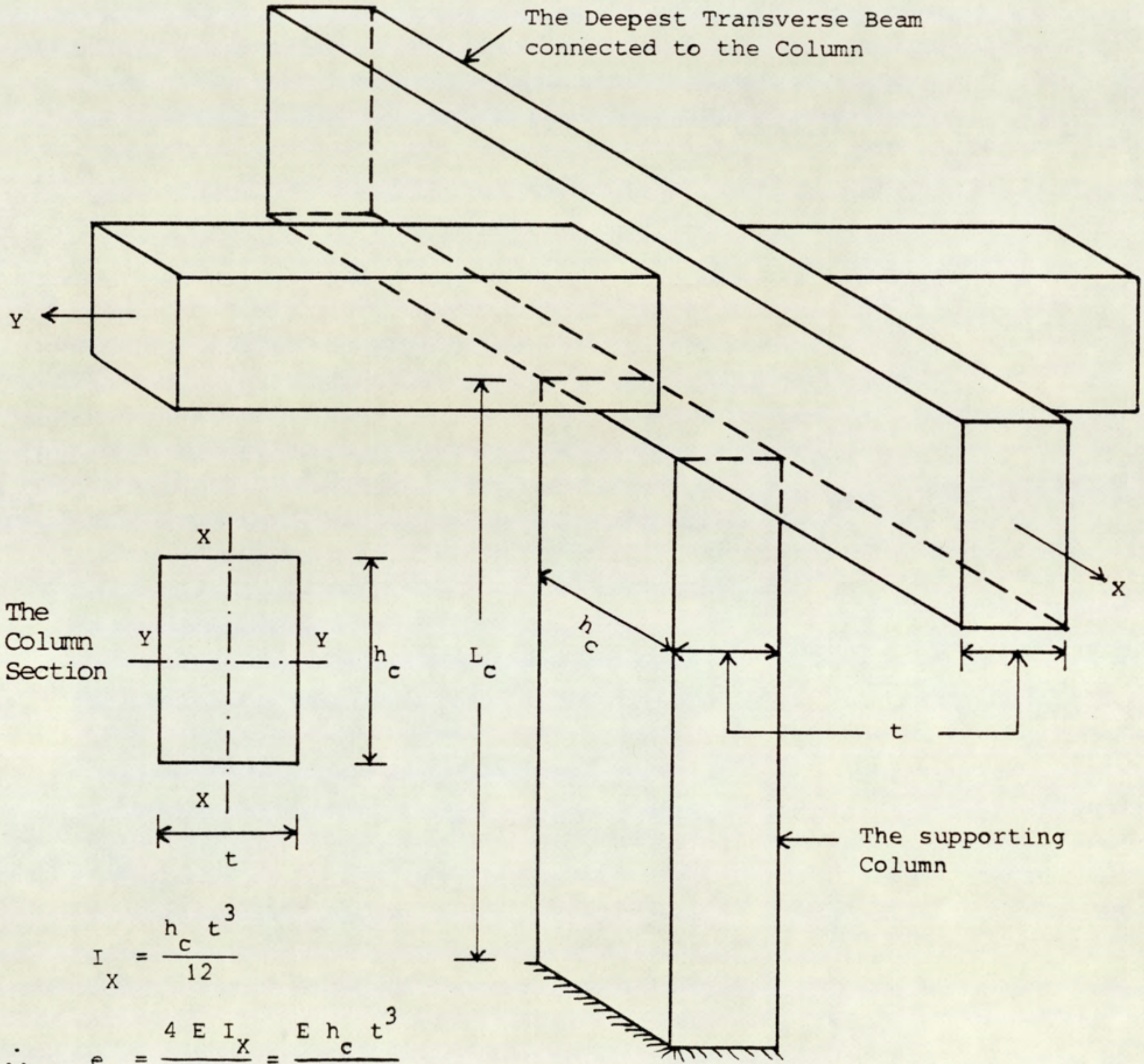
$$I_X = \frac{t\ h_c^3}{12}$$

$$\therefore\ e_X = \frac{4\ E\ I_X}{L_c} = \frac{E\ t\ h_c^3}{3\ L_c}$$

$$I_Y = \frac{h_c\ t^3}{12}$$

$$\therefore\ e_Y = \frac{4\ E\ I_Y}{L_c} = \frac{E\ h_c\ t^3}{3\ L_c}$$

FIGURE 6.3: FLEXURE COEFFICIENTS FOR A COLUMN CONNECTED TO A LONGITUDINAL (PARALLEL TO Y-AXIS) GRILLAGE MEMBER



$$I_X = \frac{h_c\, t^3}{12}$$

$$\therefore \quad e_X = \frac{4\, E\, I_X}{L_c} = \frac{E\, h_c\, t^3}{3\, L_c}$$

$$I_Y = \frac{t\, h_c^3}{12}$$

$$\therefore \quad e_Y = \frac{4\, E\, I_Y}{L_c} = \frac{E\, t\, h_c^3}{3\, L_c}$$

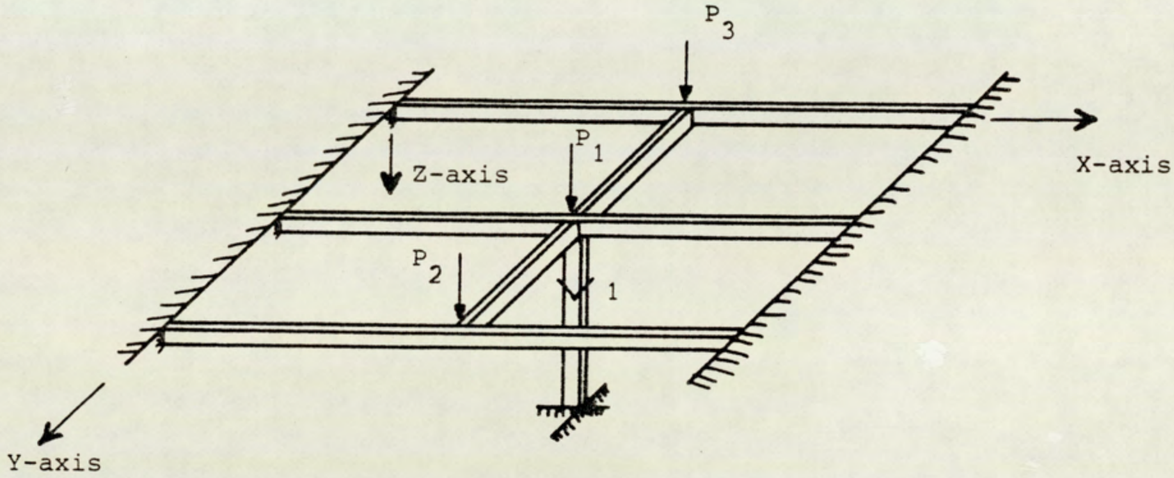
FIGURE 6.4: FLEXURE COEFFICIENTS FOR A COLUMN CONNECTED TO A TRANSVERSE (PARALLEL TO X-AXIS) GRILLAGE MEMBER

The beam-column connections are shown in Figures 6.3 and 6.4, where the flexural stiffness coefficients (e_x and e_y) of the columns are formulated in terms of the design variables t and h_c. Such formulation depends mainly on the direction of the grillage member to which the column is connected.

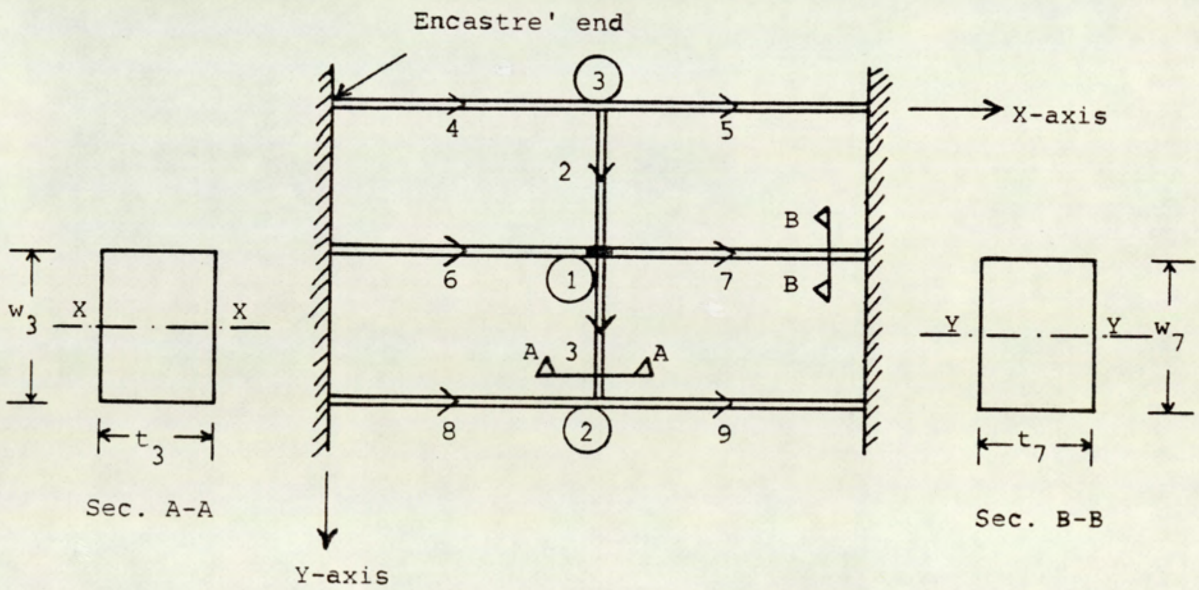
6.3 ANALYSIS OF FLAT GRILLAGE STRUCTURES

The matrix displacement method is used to analyse flat grillage structures which might contain column supports. The overall stiffness matrix of a structure is formed by superimposing the column stiffnesses, equation (6.8), on the stiffness matrix formed by the transverse and longitudinal grillage members. The displacements, z, θ_x and θ_y of all the joints are then found by solving the stiffness equations of the whole structure. An example is used to illustrate the formulation of the overall stiffness matrix.

Figure 6.5 shows a simple flat grillage made from eight members, which constitutes three joints, and supported by a column. The main topology is assumed to be known, including the number of beams, span lengths, loads, and supporting conditions. A system of joint numbering and member grouping is employed. The joints are numbered starting with the point where the column is connected to the grillage. The members of the structure are numbered starting with the column, which is given number 1, followed by the longitudinal and then the transverse grillage members. If there is any grouping of members, then it should follow the same order of the numbering. However, to distinguish the member stiffness coefficients from each other, it is assumed that each member is built from a single uniform section i.e. belong to one group.



(a) Three Dimensional View-Loading



(b) Top View-Numbering of Joints and Members

FIGURE 6.5: AN EXAMPLE OF A NORMALLY LOADED FLAT GRILLAGE WITH A COLUMN SUPPORT

	z_1	θ_{x1}	θ_{y1}	z_2	θ_{x2}	θ_{y2}	z_3	θ_{x3}	θ_{y3}	*		=	
Joint 1	$a_1 +b_2 -b_3 +b_6 +b_7$	$d_2 -d_3 +d_6 -d_7$		$-b_3$	$-d_3$		$-b_2$	d_2			z_1		P_1
	$d_2 -d_3 +d_6 -d_7$	$e_{x1} +e_2 +e_3 +e_6 +e_7$		d_3	f_3		$-d_2$	f_2			θ_{x1}		M_{x1}
			$e_{y1} +q_2 +q_3 +q_6 +q_7$			$-q_3$			$-q_2$		θ_{y1}		M_{y1}
Joint 2	$-b_3$	d_3		$b_3 +b_8 +b_9$	d_3	$-d_8 +d_9$					z_2		P_2
	$-d_3$	f_3		d_3	$e_3 +q_8 +q_9$						θ_{x2}		M_{x2}
			$-q_3$	$-d_8 +d_9$		$q_3 +e_8 +e_9$					θ_{y2}		M_{y2}
Joint 3	$-b_2$	$-d_2$					$+b_2 +b_4 +b_5$	$-d_2$	$-d_4 +d_5$		z_3		P_3
	d_2	f_2					$-d_2$	$e_2 +q_4 +q_5$			θ_{x3}		M_{x3}
			$-q_2$				$-d_4 +d_5$		$q_2 +e_4 +e_5$		θ_{y3}		M_{y3}

FIGURE 6.6: STIFFNESS EQUATIONS ($\underline{K}\ \underline{X} = \underline{L}$) OF THE FLAT GRILLAGE IN FIGURE 6.5

Figure 6.6 shows the stiffness equations of the flat grillage in Figure 6.5. The overall stiffness matrix $\underline{K}$ is symmetrical, in which the subscripts refer to the members. The stiffness coefficients of the column equation (6.8), are added to those of the grillage members at joint 1. All the unknown displacement variables of the joints are represented by the one-dimensional matrix $\underline{X}$. Matrix $\underline{L}$ represents the load vector. The vertically applied loads are either concentrated or uniformly distributed, and they include the live load and the dead weight of the structure.

The stiffness equations can be solved by using the sub-routine described in Chapter 8, and the unknowns $\underline{X}$ can be found. Therefore, the forces and the stresses in the grillage members can be computed by using equations (4.9a, b), in Chapter 4. The axial force and the bending moments at the top end of the column can be computed from:

$$\begin{bmatrix} P_c \\ M_x \\ M_y \end{bmatrix} = \begin{bmatrix} -a & 0 & 0 \\ 0 & e_x & 0 \\ 0 & 0 & e_y \end{bmatrix} \begin{bmatrix} z \\ \theta_x \\ \theta_y \end{bmatrix} \qquad (6.9)$$

6.4 THE DESIGN PROBLEM

The problem dealt with here is that of designing practical reinforced concrete flat grillage structures with column supports. The aim of the design is to produce optimum cross-sectional dimensions for the members. This is done to obtain a minimum weight or a minimum cost design for the structure. The cost includes that of the material, and of the construction such as provision, formwork, placing, etc. The topology of the flat grillage is included as a variable in the design problem. The design constraints used are the stiffness, the stress, the deflection, and the practical constraints such as the section size of the members. The stress constraints include that of the bending moment, and the combined action of shear and torsion.

The permissible stress method (CP114: Part 2: 1969) for the design of structural concrete members is used. In this method the moments and forces acting on a structure were calculated from the actual values of the applied loads, but the limiting permissible stresses in the concrete and the reinforcement were restricted to only a fraction of their true strengths, in order to provide an adequate safety factor. The method assumed a linear stress-strain relationship, a constant modular ratio of steel to concrete, and an uncracked concrete section.

The limiting stresses considered, are the permissible compressive stress for concrete in bending and the permissible concrete shear stress. The thickness (t) and the overall depth (w) of each concrete section alter continuously until the stresses are satisfied in each grillage member. The corresponding amount of tensile reinforcement is computed exactly to keep the member section elastically uncracked.

6.5 THE OBJECTIVE FUNCTION

6.5.1 The Weight Function

The objective weight function for the simple grillage structure shown in Figure 6.3 is given as:

$$Z_{weight} = t_\ell \, w_\ell \, L_\ell \, \gamma + t_m \, w_m \, L_m \, \gamma + t_\ell \, h_c \, L_c \, \gamma \tag{6.10}$$

where γ is the constant reinforced concrete density; L_ℓ, L_m and L_c are constants representing the lengths of members ℓ, m and the column respectively.

Formula (6.10) is a non-linear function which can be linearised by employing the first two terms of Taylor's series.

Thus:

$$Z_{weight} = Z\ (x_j)_o + \nabla\ Z(x_j)_o\ .\ [\{x_j\}_1 - \{x_j\}_o] \tag{6.11}$$

where $\{x_j\}_o$ is a vector of initially known section variables, i.e. $\{x_j\}_o = \{\ t_\ell^o\ \ w_\ell^o\ \ t_m^o\ \ w_m^o\ \ h_c^o\ \}$; $\{x_j\}_1$ is a new vector of the unknown section variables, i.e. $\{x_j\}_1 = \{t_\ell^1\ \ w_\ell^1\ \ t_m^1\ \ w_m^1\ \ h_c^1\}$; $Z(x_j)_o$ is the original function, equation (6.10), at the known variables, and $\nabla Z(x_j)_o$ is the gradient vector at the known variables.

The linear function (6.11) can be expressed in terms of the section variables as:

$$Z_{weight} = t_\ell^o\, w_\ell^o\, L_\ell\, \gamma + t_m^o\, w_m^o\, L_m\, \gamma + t_\ell^o\, h_c^o\, L_c\, \gamma +$$

$$[(w_\ell^o\, L_\ell\, \gamma + h_c^o\, L_c\, \gamma)\quad t_\ell^o\, L_\ell\gamma\quad w_m^o\, L_m\gamma\quad t_m^o\, L_m\gamma\quad t_\ell^o\, L_c\gamma]*\begin{bmatrix} t_\ell^1 - t_\ell^o \\ w_\ell^1 - w_\ell^o \\ t_m^1 - t_m^o \\ w_m^1 - w_m^o \\ h_c^1 - h_c^o \end{bmatrix} \tag{6.12}$$

which can be expressed in terms of the unknown variables, $\{x_j\}_1$, as:

$$Z_{weight} = t_\ell^1\ (w_\ell^o\, L_\ell\, \gamma + h_c^o\, L_c\, \gamma) + w_\ell^1\ (t_\ell^o\, L_\ell\, \gamma) + t_m^1\ (w_m^o\, L_m\, \gamma) +$$

$$w_m^1\ (t_m^o\, L_m\, \gamma) + h_c^1\ (t_\ell^o\, L_c\, \gamma) - W' \tag{6.13}$$

where the symbols in brackets are constants. The symbol W' is a constant that repesents the weight of the grillage structure at the initial design point $\{x_j\}_o$, and it can be expressed as:

$$W' = t_\ell^o\, w_\ell^o\, L_\ell\, \gamma + t_m^o\, w_m^o\, L_m\, \gamma + t_\ell^o\, h^o\, L_c\, \gamma \tag{6.14}$$

The general linear objective weight function used for a flat grillage made from NL longitudinal and NT transverse members, and supported by NC columns can be expressed as:

$$\begin{aligned} Z_{weight} = & \sum_{\ell=1}^{NL} [t_\ell^1 (w_\ell^o\, L_\ell\, \gamma + h_c^o\, L_c\, \gamma) + w_\ell^1\, (t_\ell\, L_\ell\, \gamma)] \\ & + \sum_{m=1}^{NT} [t_m^1\, (w_m^o\, L_m\, \gamma) + w_m^1\, (t_m^o\, L_m\, \gamma)] \\ & + \sum_{c=1}^{NC} [h_c^1\, (t_\ell^o\, L_c\, \gamma)] + W' \end{aligned} \tag{6.15}$$

If the grillage members are grouped, then NL and NT above represent the longitudinal and the transverse groups respectively. It is assumed that each supporting column belongs to a single group, i.e. built from one uniform section, therefore NC represents the number of the columns only.

6.5.2 The Cost Function

For a minimum cost design, the objective function represents the total material and construction costs of the structure. In this design method, the reinforcement always varies as it is computed to resist various bending moments occurred at the ends and at the middle of the grillage members, and as a result the cost per unit weight of the material ($c_{mat.}$) can be varied even along a single member. However, this variation is found to be small and does not greatly affect the assessment of cost. Therefore, to simplify the problem, it is assumed that $c_{mat.}$ is constant throughout the whole grillage structure. Sometimes a slightly different value of $c_{mat.}$ can be used for the columns. Nonetheless, the material cost of a grillage structure is generally computed, using equation (6.15), as:

$$Z_{mat.cos} = Z_{weight} \cdot c_{mat.} \tag{6.16}$$

In addition to the material cost, let R_ℓ be all the cost involved in constructing member ℓ, R_m is the construction cost of member m, and R_c is the construction cost of the column, (Figure 6.3). The method of assessing these costs will be discussed later in Section 6.10. Thus, the total cost of a grillage structure made from NL longitudinal and NT transverse members, and supported by NC columns will be formulated as:

$$Z_{cost} = Z_{mat.cos} + \sum_{\ell=1}^{NL} R_\ell + \sum_{m=1}^{NT} R_m + \sum_{c=1}^{NC} R_c \tag{6.17}$$

To explain the fact that the construction cost R_ℓ (or R_m) has a value only when the grillage member group ℓ (or m) is included in the design, a new variable δ_ℓ (or δ_m) is defined; such that, $\delta_\ell = 1$ when group ℓ is kept in the final design while $\delta_\ell = 0$ when it is cheaper to remove this group. The objective cost function (6.17) therefore becomes:

$$Z_{cost} = Z_{mat.cos} + \sum_{\ell=1}^{NL} \delta_\ell R_\ell + \sum_{m=1}^{NT} \delta_m R_m + \sum_{c=1}^{NC} R_c \tag{6.18}$$

As each grillage member is represented by its two unknown sectional dimensions (t and w), new δ variables should be defined to be associated with each of these dimensions. Such variables are δ_t and δ_w where:

$$\delta_t = \delta_w$$

i.e.

$$\delta_t - \delta_w = 0 \tag{6.19}$$

and when $\delta_t = 0$ then $\delta_w = 0$, and when $\delta_t = 1$ then $\delta_w = 1$. The construction cost (R) for each member should also be associated with the unknown sectional dimensions. The way to do that is by dividing R into two halves, one half linked with the t variable, and the other half with the w variable. Thus, function (6.18) is modified to become:

$$Z_{cost} = Z_{mat.cos} + \sum_{\ell=1}^{NL} (\delta_t \frac{R}{2} + \delta_w \frac{R}{2})_\ell + \sum_{m=1}^{NT} (\delta_t \frac{R}{2} + \delta_w \frac{R}{2})_m + \sum_{c=1}^{NC} R_c \tag{6.20}$$

This is the linear objective function which is used for the topological design of minimum cost for the flat grillage structures. Notice that there are no δ variables for the supporting columns.

6.6 THE STIFFNESS CONSTRAINTS

It is necessary to select the overall stiffness matrix $\underline{K}$ so that the grillage structure is capable of resisting the applied loads while the design requirements are satisfied. Thus, the stiffness constraints, matrix $\underline{H}$, are once again formulated as:

$$\underline{H} = \underline{K}\ \underline{X} - \underline{L} = 0 \tag{6.21}$$

As the sectional dimensions of the members are considered as the unknown design variables, it is necessary to keep the stiffness contributions of each member separate from the other, (Section 4.7, Chapter 4). Therefore, for a total of N joints in a grillage structure, the overall stiffness matrix $\underline{K}$ used in equation (6.21) will have an order of $[3 * N, 3 * \sum_{j=1}^{N} M_j]$, where M_j is the total number of the grillage members (or groups of members) connected to a typical joint j.

The design variables required to be computed in the design process can be arranged in a vector of the form:

$$\underline{V} = \{v_1 \; v_2 \ldots v_g \; v_{g+1} \ldots v_{g+2r} \ldots v_{g+2r+3N}\} \tag{6.22}$$

where the first g variables represent the section overall depths (h_c) for a number of g columns, 2r variables represent the sectional dimensions (t and w) for a total of r grillage members (or groups), and 3N variables represent the displacements of all the N joints. Vector $\underline{V}$ can be partitioned to:

$$\underline{V} = \{\underline{C} \; : \; \underline{M} \; : \; \underline{X}\} \tag{6.23}$$

where the contents of sub-matrix $\underline{C} = \{h_{c1} \; h_{c2} \ldots h_{c2}\}$,
$\underline{M} = \{t_1 \; w_1 \; t_2 \; w_2 \ldots t_r \; w_r\}$, and $\underline{X} = \{z_1 \; \theta_{x1} \; \theta_{y1} \ldots z_N \; \theta_{xN} \; \theta_{yN}\}$

With the design variables specified above, the stiffness constraints (6.21) can be expressed as:

$$\underline{H} \, (h_c, t, w, X) = \underline{K} \, (h_c, t, w) \, . \, \underline{X} - \underline{L} = 0 \tag{6.24}$$

These constraints are non-linear which can be linearised as explained in Section 4.7.1 of Chapter 4.

The stiffness coefficients (b, d, e, f), defined by equations (6.1a, b, c and d), for the grillage members are linear functions of t, while they are non-linear functions of w. But, the torsional stiffness coefficient q, equation (6.1e), is linearly related with w, and non-linearly related with t. The relationships of the column flexural coefficients, Figures 6.3 and 6.4, with the variables h_c and t depend on the direction of the grillage member with which the column is connected.

Using the computer, the derivation of the stiffness coefficients with respect to the sectional design variables is carried out in the manner:

$$y = \frac{u}{v} \tag{6.25}$$

$$\frac{dy}{dx} = \frac{v(du/dx) - u(dv/dx)}{v^2} \tag{6.26}$$

or in the manner:

$$y = x^s \tag{6.27}$$

$$\frac{dy}{dx} = s\,x^{s-1} \tag{6.28}$$

Thus, using equations (6.1a, b, c, d, e), the derivatives of the stiffness coefficients for the grillage members will be as follows:

$$\frac{\partial b}{\partial t} = \frac{b}{t} \tag{6.29}$$

$$\frac{\partial b}{\partial w} = \frac{3\, B_1\, B_2\, t\, w^2 + B_1\, B_3\, t\, w^4}{B_2^2 + 2\, B_2\, B_3\, w^2 + B_3^2\, w^4} \tag{6.30}$$

$$\frac{\partial d}{\partial t} = \frac{d}{t} \tag{6.31}$$

$$\frac{\partial d}{\partial w} = -\frac{3\, D_1\, D_2\, t\, w^2 + D_1\, D_3\, t\, w^4}{D_2^2 + 2\, D_2\, D_3\, w^2 + D_3^2\, w^4} \tag{6.32}$$

$$\frac{\partial e}{\partial t} = \frac{e}{t} \tag{6.33}$$

$$\frac{\partial e}{\partial w} = \frac{3\, E_1\, E_3\, t\, w^2 + E_1\, E_4\, t\, w^4 + 5\, E_2\, E_3\, t\, w^4 + 3\, E_2\, E_4\, t\, w^6}{E_3^2 + 2\, E_3\, E_4\, w^2 + E_4^2\, w^4} \tag{6.34}$$

$$\frac{\partial f}{\partial t} = \frac{f}{t} \tag{6.35}$$

$$\frac{\partial f}{\partial w} = \frac{3\, F_1\, F_3\, t\, w^2 + F_1\, F_4\, t\, w^4 - 5\, F_2\, F_3\, t\, w^4 - 3\, F_2\, F_4\, t\, w^6}{F_3^2 + 2\, F_3\, F_4\, w^2 + F_4^2\, w^4} \tag{6.36}$$

$$\frac{\partial q}{\partial t} = \frac{3\, G\, w\, t^2}{c_2\, L} \tag{6.37}$$

$$\frac{\partial q}{\partial w} = \frac{G\, t^3}{c_2\, L} \tag{6.38}$$

The derivatives of the stiffness coefficients of the supporting columns can be obtained similarly.

6.7 THE STRESS CONSTRAINTS

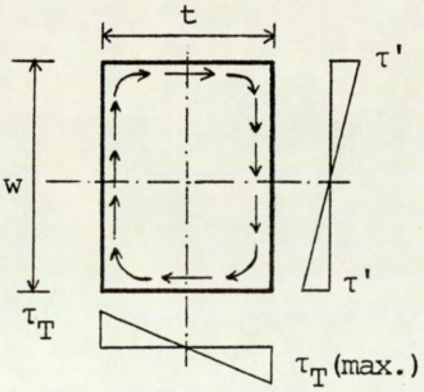
6.7.1 Shear and Torsion Stresses

The tangential distribution of the shearing stress across a rectangular section is known to be parabolic. In the design method proposed in this Chapter, only the maximum shear stress (τ_s) at an uncracked section of a reinforced concrete grillage member is required to be considered. For a grillage member j, this is calculated from:

$$\tau_{js} = 1.5 \; S_j \,/\, t_j \, w_j \tag{6.39}$$

where S_j is the total shearing force across the section, and it can be computed by using equations (4.10a, b); t_j is the thickness, and w_j is the overall depth of the uncracked section.

The torsion in a member, also creates shearing stress across the transverse plane of the rectangular concrete section. The torsional shearing stress varies along the sides of the cross section reaching a maximum value (τ_T) at the middle of the long edge w, and a value (τ') at the middle of the short edge t, as shown in Figure 6.7,



$$\frac{\tau'}{\tau_T} = \frac{t}{w}$$

FIGURE 6.7: TORSIONAL SHEARING STRESS DISTRIBUTION IN A RECTANGULAR SECTION

where:

$$\tau'/\tau_T = t / w \tag{6.40}$$

In this design method, only the maximum torsional shearing stress (τ_T) is considered. For a grillage member j, this can be calculated with satisfactory accuracy (Timoshenko, 1955) from:

$$\tau_{jT} = c_{1j} \; T_j / w_j \; t_j^2 \tag{6.41}$$

for $t_j \leq w_j$

where T is the twisting moment which can be computed by using equation (4.10a, b); c_{1j} is a constant factor depending upon the ratio t/w, and it can be computed from:

$$c_{1j} = 3 + 1.8 \; t_j / w_j \tag{6.42}$$

Equations (6.41) and (6.42) are used only when t_j is less than or equal to w_j.

The torsional shearing stress augments or diminishes the shearing stress associated with the lateral force. In other words, at any rectangular section there must be a point where the shear stress, caused by the torque and the lateral force, is at a maximum value. An examination of all the possible combinations of shear stresses in a rectangular section is shown in Figure 6.8. This shows that there is always a point in the section where a maximum shear stress develops irrespective of the direction of the shear force or the torsion. Thus, the critical shear stress, which is at the middle of the long edge, is

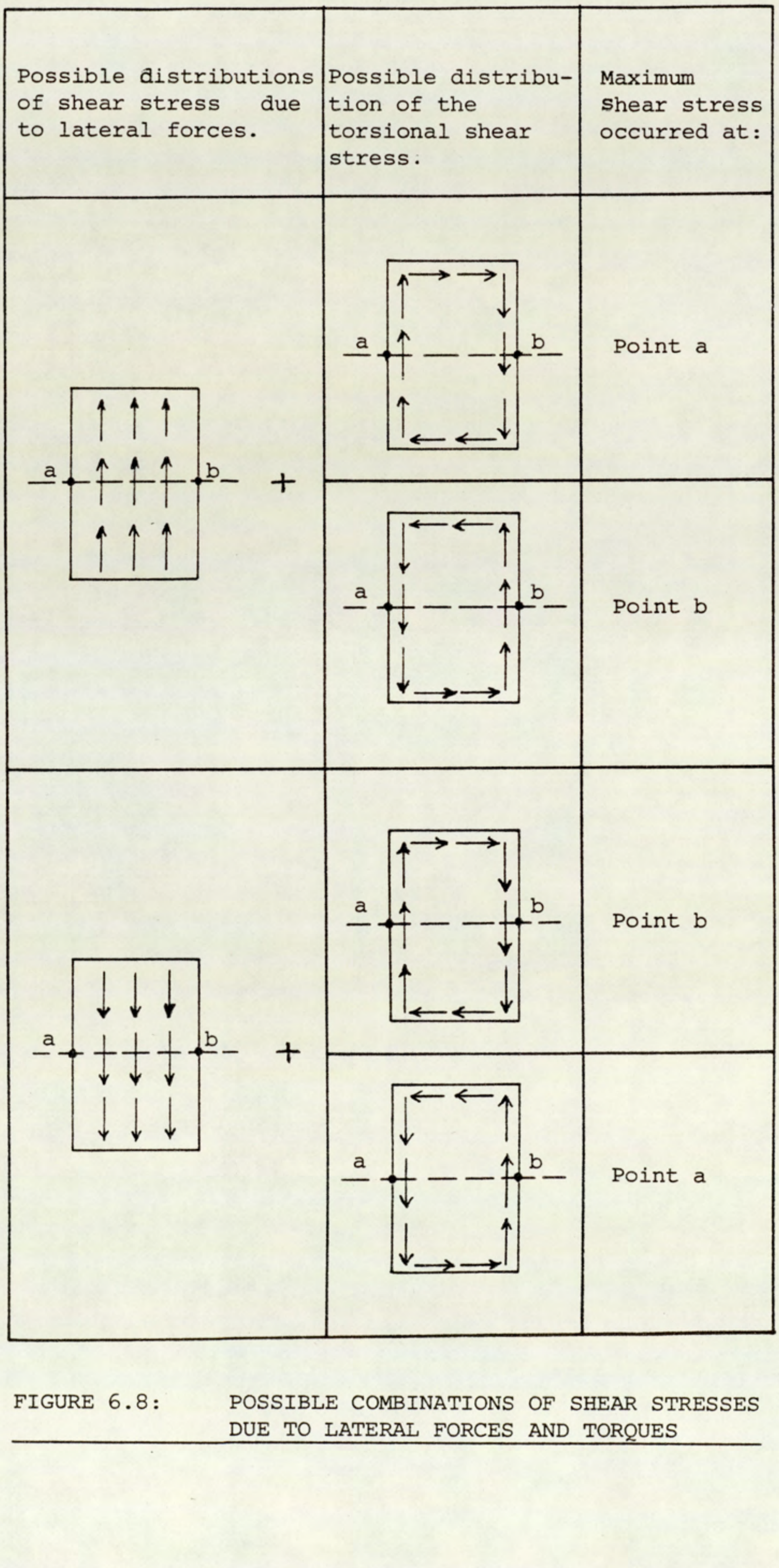


FIGURE 6.8: POSSIBLE COMBINATIONS OF SHEAR STRESSES DUE TO LATERAL FORCES AND TORQUES

obtained by combining the stresses of the lateral force and the twisting moment in the following way:

$$\tau_{jmax} = \tau_{js} + \tau_{jT}$$

i.e.

$$\text{jmax} = \frac{1.5 \mid S_j \mid}{t_j \ w_j} + \frac{c_{1j} \mid T_j \mid}{w_j \ t_j^2} \tag{6.43}$$

where the values of S_j and T_j are absolute, i.e. they do not depend on signs or directions.

The general formula of the combined shear and torsional stress constraint imposed on the grillage members can be expressed as:

$$\tau_{js} + \tau_{jT} \leq \tau_u$$

i.e.

$$\frac{1.5 \mid S_j \mid}{t_j \ w_j} + \frac{c_{1j} \mid T_j \mid}{w_j \ t_j^2} \leq \tau_u \tag{6.44}$$

$j = 1, \ldots r$

where r is the number of the grillage members, and τ_u is a constant representing the maximum allowed shear stress in concrete. Clause 316 in CP114 (Part 2: 1969) specifies τ_u to be equal to four times the permissible shear stress of concrete (τ_c) given in Clause 303, i.e. $\tau_u = 4 \ \tau_c$. If the total value of the combined shear stresses is greater than τ_u then t and w change until formula (6.44) will be satisfied.

The design problem could be enlarged unnecessarily if the inequality constraints (6.44) are imposed on each grillage member. In practice, the grillage members are usually grouped together. Therefore, it was decided that the shearing stress constraints should be imposed on each group. The critical section in each group was obtained by an analysis in which the location of the member with a maximum combined shear stresses was determined. By using the dimensions and the joint displacements of this member, the constraint which is imposed on the group will then be constructed.

During the course of redesign and optimisation, the located members of maximum combined stresses may change, and in the present approach this change is considered.

6.7.2 Bending Moment Stress

The bending moment (flexural) stresses in an uncracked reinforced concrete member may conveniently be calculated by the conventional bending formula, using the entire concrete cross section and ignoring the reinforcement. Thus, the maximum bending moment stress (σ_{jmax}) in a reinforced concrete grillage member, referred to as j, can be computed as:

$$\sigma_{jmax} = M_{jmax} / Z_j \tag{6.45}$$

where M_{jmax} is the maximum bending moment in the member calculated from the analysis results; Z_j is the member section modulus which is expressed in terms of the section thickness (t_j) and the overall depth (w_j) as:

$$Z_j = t_j \; w_j^2 \; / \; 6 \tag{6.46}$$

Hence, equation (6.45) can be written in terms of the design variables (t and w) as:

$$\sigma_{jmax} = \frac{M_{jmax}}{t_j \; w_j^2 / 6} \tag{6.47}$$

The bending moment stress constraint imposed on the reinforced concrete grillage members can generally be formulated as:

$$\frac{6 \mid M_{jmax} \mid}{t_j \, w_j^2} \leq \sigma_c \qquad (6.48)$$

$$j = 1, \ldots r$$

where r represents the total number of the grillage members, and σ_c is a constant representing the permissible compressive stress in concrete in bending. It should be emphasised that the absolute value of the maximum bending moment in the member is used in this constraint.

To reduce the size of the design problem the bending stress constraints are imposed on the groups of members and not on the individual members. By analysing the grillage structure, it is possible to locate the member with a critical bending stress in the group. The constraint, imposed on the group, is then formulated using the dimensions and the joint displacements of this member. The change in the location of such a member, during the design process, is also considered.

To assure finding a member's maximum bending stress the moment is computed at both ends of the member under concentrated load, using equation (4.10a, b), and at the maximum of a parabolic moment equation due to the uniform load. The largest moment is then employed to formulate constraint (6.48).

For a uniformly loaded member, a moment expression is required to compute the value and the location of the maximum moment. Using the positive sign convention for the grillage, Figure 4.1, the expression for a maximum moment in a uniformly loaded transverse (parallel to X-axis) member, shown in Figure 6.9, is determined as follows:

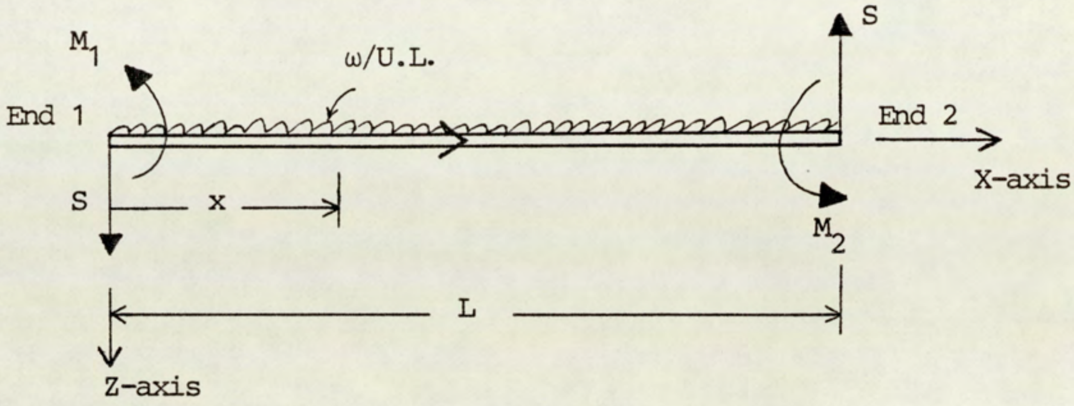


FIGURE 6.9: POSITIVE SIGN CONVENTION FOR A UNIFORMLY LOADED TRANSVERSE GRILLAGE MEMBER

$$M(x) = M_1 + S \cdot x + \omega x^2 / 2 \;; \tag{6.49}$$

$$\partial M/\partial x = S + \omega x = 0 \;; \tag{6.50}$$

$$x_{max.} = - S / \omega < \text{Length of Member} \tag{6.51}$$

$$\therefore \quad M_{max.} = M_1 - S^2 / 2\,\omega \tag{6.52}$$

where S is the shear force and ω is the uniform load intensity per unit length. Similarly, for a uniformly loaded longitudinal (parallel to Y-axis) member, the maximum moment expression is obtained as follows:

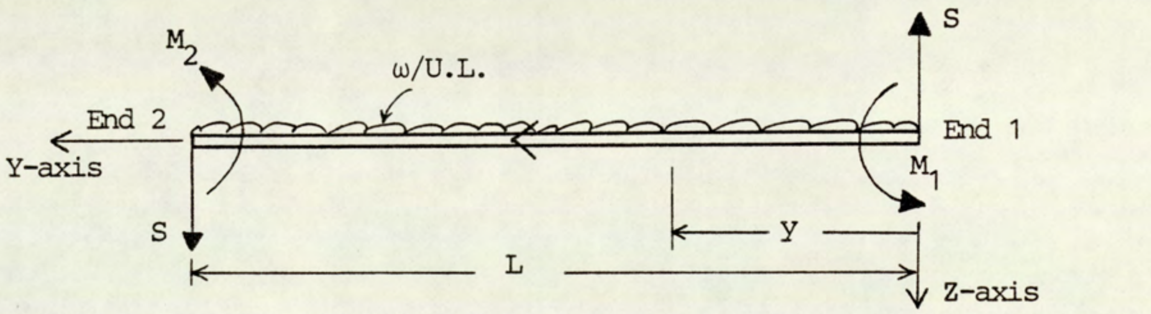


FIGURE 6.10: POSITIVE SIGN CONVENTION FOR A UNIFORMLY LOADED LONGITUDINAL GRILLAGE MEMBER

$$M(y) = M_1 + S \cdot y - \omega y^2 / 2 \tag{6.53}$$

$$\partial M / \partial y = S - \omega y = 0 \tag{6.54}$$

$$y_{max} = S / \omega < \text{Length of Member} \tag{6.55}$$

$$\therefore \quad M_{max.} = M_1 + S^2 / 2\omega \tag{6.56}$$

6.7.3 Linearisation of the Stress Constraints

The design variables that usually associate with the stress constraints are the sectional dimensions (t and w), and the joint displacements (z, θ_x and θ_y) at ends 1 and 2 of the grillage member with critical stress. These variables can be arranged in a vector form as:

$$\underline{x}_j = \{t \;\; w \;\; z_1 \;\; \theta_{x1} \;\; \theta_{y1} \;\; z_2 \;\; \theta_{x2} \;\; \theta_{y2}\} \tag{6.57}$$

$j = 1, \ldots r$

where r is the total number of the grillage groups.

The stress constraints, described earlier, are mathematically non-linear. They can be expressed as:

$$\tau_j(x_j) = \tau_{js} + \tau_{jT} - \tau_u \leq 0 \; ;$$

$$\sigma_j(x_j) = \sigma_{jmax} - \sigma_c \leq 0 \tag{6.58}$$

$$j = 1, \ldots r$$

The first two terms of Taylor's series are used to linearise the non-linear stress constraints. The linearisation process requires the gradient vector which, for instance for the combined stress constraints, is formulated as follows:

$$\nabla\tau_j(x_j) = \{\frac{\partial\tau}{\partial t} \ \frac{\partial\tau}{\partial w} \ \frac{\partial\tau}{\partial z_1} \ .... \ \frac{\partial\tau}{\partial\theta_{y2}}\} \tag{6.59}$$

The derivatives of $\tau_j(x_j)$ with respect to any of the associated design variables $\underline{x}_j$ can be computed as follows:

$$\frac{\partial\tau_j}{\partial x_j} = \frac{\partial\tau_{js}}{\partial x_j} + \frac{\partial\tau_{jT}}{\partial x_j} \tag{6.60}$$

Similar expressions can be obtained for $\sigma_j(x_j)$.

The terms S_j, T_j and M_{jmax} used in the stress constraints are obtained by collecting the elements of the rows in the product matrix $\underline{k}\ \underline{A}$ which correspond to the shear force, torsion and bending moments respectively. The derivatives of these terms with respect to $\underline{x}_j$, i.e. $\partial S / \partial x_j$, $\partial T_j / \partial x_j$ and $\partial M_{jmax} / \partial x_j$, are found by taking the derivatives of equation (4.9), as follows:

$$\frac{\partial(P)}{\partial x_j} = \underline{k}\ \underline{A}\ (\frac{\partial X}{\partial x_j}) + \underline{X}\ (\frac{\partial(k\ A)}{\partial x_j}) \tag{6.61}$$

For a uniformly loaded member the derivative $\partial M_{jmax} / \partial x_j$ is obtained by using equation (6.52) or (6.56). For instance, for a transverse member, such derivative is given as:

$$\frac{\partial M_{jmax}}{\partial x_j} = \frac{\partial M_1}{\partial x_j} - \frac{S}{\omega}\ (\frac{\partial S}{\partial x_j}) \tag{6.62}$$

6.8 THE DEFLECTION CONSTRAINTS

Since the joint displacements of the grillage are introduced as design variables, the actual deflection z_j and the rotations θ_{xj} and

θ_{yj} of joint j should be limited by upper bounds. In this Chapter, the deflection z_j and the permissible stresses of a member are considered as the governing limit-state for the design of horizontal grillages. Therefore, a practical upper bound on z_j is required to be specified.

None of the existing codes of practice specify an exact limit on the deflection of a reinforced concrete flat grillage. Therefore, in the absence of such a limit, and as a large deflection might produce excessive cracks, a value of L_j / ξ is used as an upper bound on deflection z_j. Here, L_j is a constant which represents the shortest distance of joint j from the nearest fixed end or column support. The value of L_j can easily be calculated by using the coordinates of joint j. The symbol ξ is a constant which can be chosen by the designer, such as 300, 400, 500, etc. The upper bounds on the rotations θ_{xj} and θ_{yj} are once again taken as 0.08. Thus, the deflection constraints imposed on a typical joint j are:

$$\begin{aligned} & z_j \leq L_j / \xi \\ & \theta_{xj} \leq 0.08 \\ & \theta_{yj} \leq 0.08 \\ & j = 1, \ldots N \end{aligned} \tag{6.63}$$

where N represents the total number of joints.

The above bounds on z_j are specified only for joints where the deflections are known to be critical, while less restricted bounds can be specified for the rest of the joints. This will be clarified when solving the design examples in the next Chapter.

6.9 THE PRACTICAL CONSTRAINTS

The practical constraints used here are meant to be the limitation on the cross-section dimensions t and w of the grillage members, and on the overall depth h_c of the columns. It was shown previously that the objective function, the stiffness and the stress constraints are all highly non-linear. These necessitate the use of move limits to be imposed on t, w and h_c.

For a minimum weight design of horizontal grillages, the practical constraints are:

$$\begin{aligned}
&(1 - ML)\ h_c^{(o)} \le h_c^{(1)} \le (1 + ML)\ h_c^{(o)} \ ; \\
&(1 - ML)\ t_j^{(o)} \le t_j^{(1)} \le (1 + ML)\ t_j^{(o)} \ ; \\
&(1 - ML)\ w_j^{(o)} \le w_j^{(1)} \le (1 + ML)\ w_j^{(o)} \\
&c = 1, \dots NC, \quad j = 1, \dots r
\end{aligned} \qquad (6.64)$$

In these $h_c^{(1)}$, $t_j^{(1)}$ and $w_j^{(1)}$ are the unknowns while $h_c^{(o)}$, $t_j^{(o)}$ and $w_j^{(o)}$ are known variables; ML is the move limit, NC is the number of supporting columns and r is the total number of grillage groups. The bounds specified in (6.64) should not exceed the maximum, or be less than the minimum, bounds specified by the engineer or by the code of practice used.

For a minimum cost design, the topology of the grillage structure could be altered to reduce the total cost. This alteration is done by removing grillage members that are proven by the design process to be structurally ineffective or uneconomical. However, some members, such as the edge grillage member and the columns, are often required to be retained in the final topology. To retain these members, lower and upper bounds are imposed. These are similar to (6.64).

It should be pointed out that members allowed to be removed have no lower bounds imposed on them. Each one of these members is associated with its own variables δ_t and δ_w which are included in the objective cost function (6.20). Thus, the constraints imposed on a number of r' removable grillage members are:

$$t_j^{(1)} \leq (1 + ML)\ t_j^{(o)}\ \delta_{tj} \ ;$$
$$w_j^{(1)} \leq (1 + ML)\ w_j^{(o)}\ \delta_{wj} \ ;$$

i.e.

$$t_j^{(1)} - (1 + ML)\ t_j^{(o)}\ \delta_{tj} \leq 0$$
$$w_j^{(1)} - (1 + ML)\ w_j^{(o)}\ \delta_{wj} \leq 0 \qquad (6.65)$$
$$j = 1, \ldots r'$$

Each of the associated variables δ_{tj} and δ_{wj} can be either zero or unity, thus:

$$0 \leq \delta_{tj} \leq 1$$
$$0 \leq \delta_{wj} \leq 1 \qquad (6.66)$$

and they must be equal to each other:

$$\delta_{tj} = \delta_{wj}$$

i.e.

$$\delta_{tj} - \delta_{wj} = 0 \qquad (6.67)$$

where δ_{tj} and δ_{wj} are integers.

6.10 COST ASSESSMENT

The total cost of a reinforced concrete flat grillage includes the material, and the construction costs. The cost of a column includes the construction cost of its foundation. The assessment in this Section is based on the information presented in a report by Davis, Belfield and Everest (1980). The report includes details about measured rates, as described in Chapter 2 and Appendix A. The costs are computed automatically by using sub-routine COST which is part of the main computer program. This sub-routine will be described in Chapter 8.

6.10.1 The Material Cost

As mentioned in Section 6.5.2, the reinforced concrete cost per unit weight $c_{mat.}$ used in the objective cost function depends on the reinforcement in that unit. This may vary from one member to another, but to simplify the problem, it is assumed that a fixed percentage of reinforcement is specified for all the grillage members and the columns. In this manner, a single constant value of $c_{mat.}$ can be computed and used in the design problem.

The specified fixed percentage of reinforcement, which includes the main and the secondary reinforcements, and the computed values of $c_{mat.}$ for the grillage members and the columns are arranged in Table 6.2; the computation of $c_{mat.}$ is done according to Appendix B.

Specified Reinforcement	Grillage Members	Supporting Columns
Main Reinforcement Percentage (ρ)	1.85%	1%
Secondary Reinforcement Percentage	0.15%	0.15%
Cost per unit weight ($c_{mat.}$)	£2.65 / KN	£2.00 / KN

TABLE 6.2 COST PER UNIT WEIGHT DEPENDING ON THE ASSUMED PERCENTAGE OF REINFORCEMENT

6.10.2 The Construction Cost

The construction cost is assessed separately for each member and each column in the grillage. This depends on the measured rates of construction items given in Appendix A. The costs of the joints are difficult to assess and are excluded. It is assumed that each grillage member contains tensile and shear reinforcements only. Since these reinforcements vary, the methods of computing them will be described here. The columns are assumed to have one per cent compression reinforcement together with 0.15% stirrups. Figures 6.11 and 6.12 illustrate these specifications.

The stirrups in a grillage member are required to resist the shear force. These depend on the value of the maximum combined shear and torsion stress (τ_{max}) computed by equation (6.43). Thus, (Clause 316, CP114: Part 2: 1969):

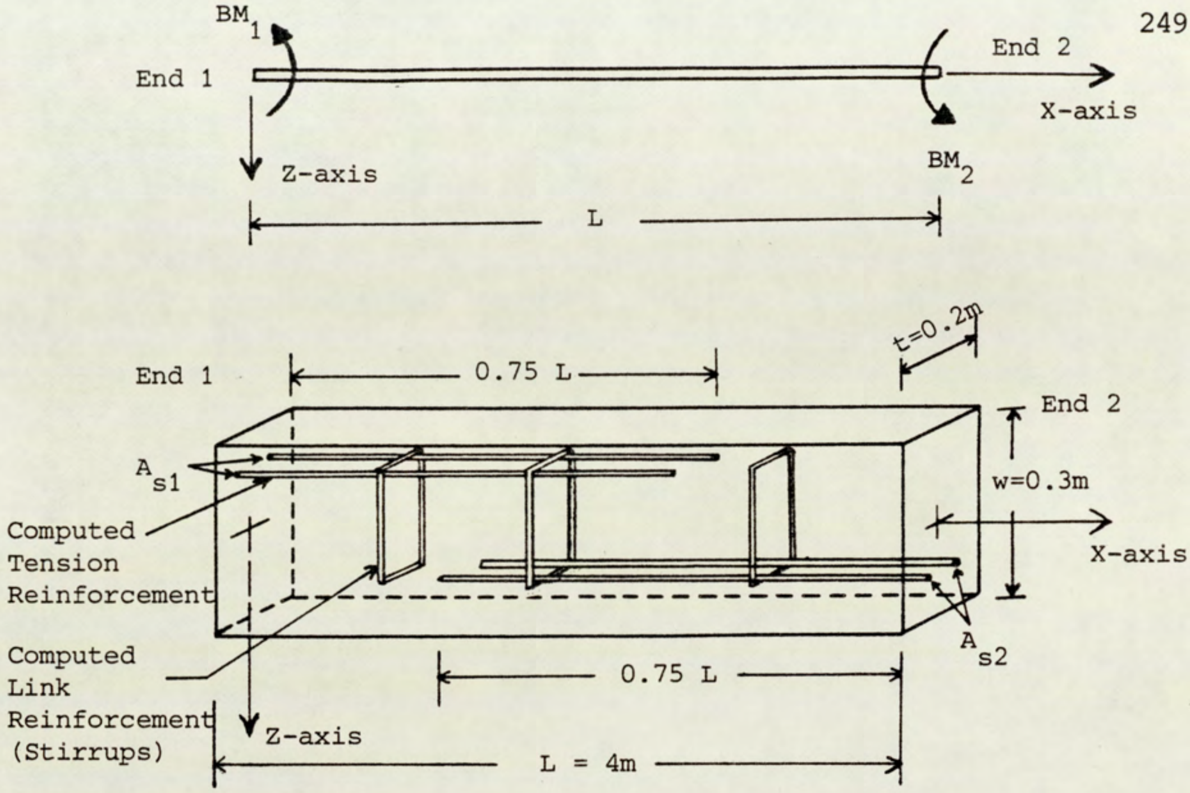


FIGURE 6.11: SPECIFICATIONS REQUIRED TO ASSESS THE COST OF A TRANSVERSE GRILLAGE MEMBER

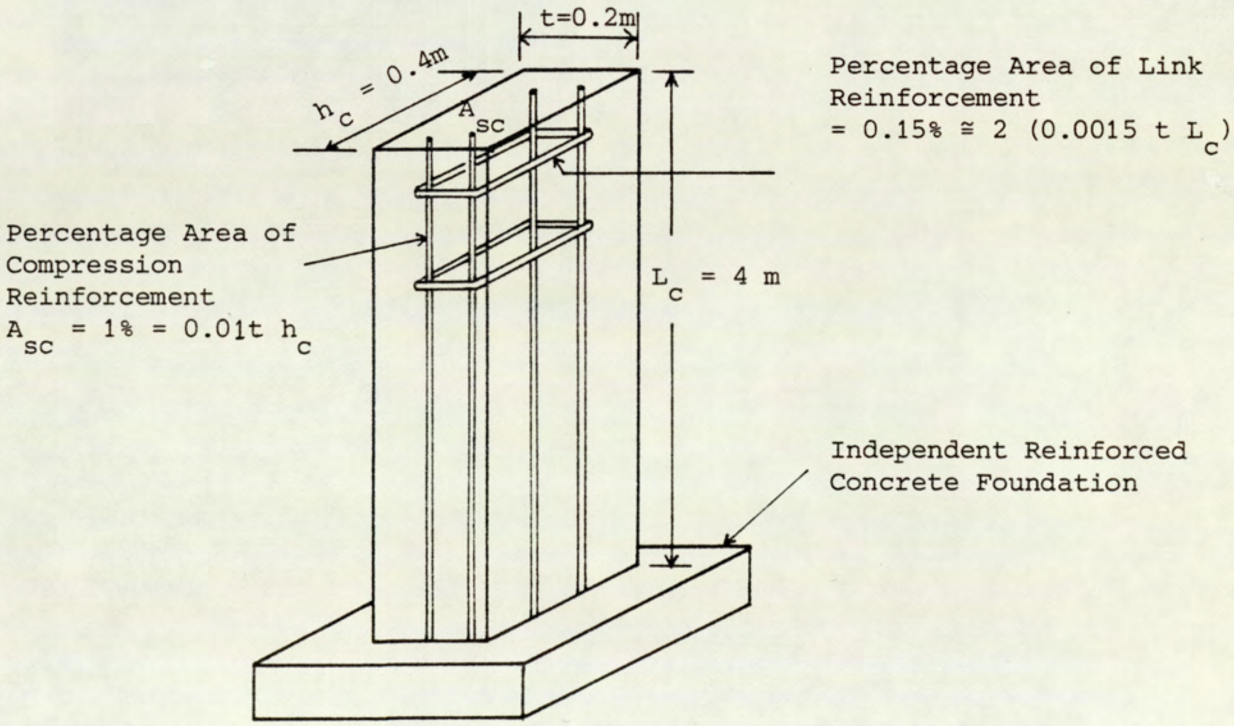


FIGURE 6.12: SPECIFICATIONS REQUIRED TO ASSESS THE COST OF A REINFORCED CONCRETE COLUMN

(a) If $\tau_{max} \leq \tau_c$, then the area of stirrups needed is equal to a minimum percentage of 0.15% of the horizontal section of the grillage member, i.e. 2(0.0015 x t x L).

(b) On the other hand, if $\tau_c < \tau_{max} \leq \tau_u$, then the whole shearing force at that cross-section should be provided for by the tensile resistance of the shear reinforcement acting in proper combination with compression in the concrete.

The reinforcement for torsional shear is ignored, while the reinforcement for the shearing force is calculated for the whole member as:

$$A_{sr} = S\,L \,/\, f_s\,\ell_a \qquad (6.68)$$

where A_{sr} is the cross-sectional area of the stirrups;

S is the shearing force across the reinforced concrete section;

L is the length of the member (Figure 6.11);

f_s is the permissible tensile stress in the shear reinforcement;

ℓ_a is the arm of resisting moment (Figure 6.13), which is assumed to be equal to the spacing of the stirrups.

The cross-sectional area of the main tension reinforcement in a grillage member is computed to resist the bending moment. The length of the computed tensile steel bar is assumed to extend for 3/4 of the member length as shown in Figure 6.11. On the other hand, if tension occurs within the beam length, the tensile steel is assumed to cover its whole length.

The permissible stress elastic method is used to compute the amounts of the tensile reinforcement in the grillage members. The method is based on the following assumptions, (CP114: Part 2: 1969):

(1) The steel and the concrete are elastic within the range of the permissible stresses.

(2) The stress is proportional to the strain.

(3) The variation of the strain over the section depth is linear.

(4) The whole of the tension is taken by the reinforcement.

(5) The modulus ratio E_s / E_c is assumed to be constant.

The following expressions can be deduced, assuming a balanced section in which both the concrete and the steel are stressed to the permissible limits.

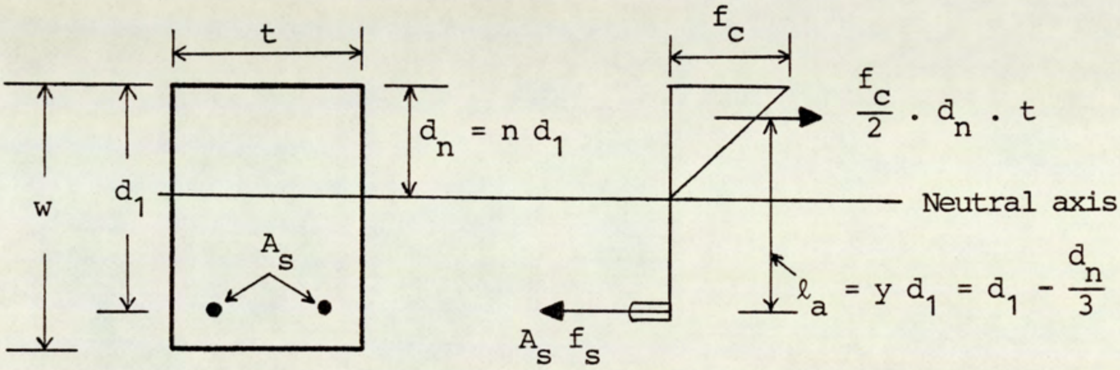


FIGURE 6.13: <u>CROSS-SECTION OF A TYPICAL GRILLAGE MEMBER</u>

$$n = \frac{f_c}{(f_c + f_s/\alpha)} \tag{6.69}$$

$$d_n = n\ d_1 \tag{6.70}$$

$$y = 1 - \frac{n}{3} \tag{6.71}$$

$$\ell_a = y \cdot d_1 \tag{6.72}$$

The effective depth (d_1) of the tensile reinforcement is assumed to be:

$$d_1 = 0.9\ w \tag{6.73}$$

$$\therefore \quad \ell_a = y\ (0.9\ w) = 0.9\ y\ w \tag{6.74}$$

The section moment of resistance = the applied bending moment = $A_s\ f_s\ \ell_a$ (6.75)

$$BM_1 = A_{s1}\ f_s\ \ell_a \tag{6.76}$$

$$A_{s1} = \frac{BM_1}{f_s\ \ell_a} = \frac{BM_1}{0.9\ (f_s\ .\ y\ .\ w)} \tag{6.77}$$

$$A_{s2} = \frac{BM_2}{f_s\ \ell_a} = \frac{BM_2}{0.9\ (f_s\ .\ y\ .\ w)} \tag{6.78}$$

where A_{s1} is the tensile reinforcement required to resist the calculated applied bending moment (BM_1) at end 1. Similarly for A_{s2} at end 2. In these equations f_c is the permissible compressive stress for concrete in bending and f_s is the permissible tensile stress for the steel. Notice that the minimum tensile reinforcement required is equal to 0.15%, i.e. 0.0015 x t x 0.9 x w.

As an illustration, the method of assessing the construction cost is explained by considering the grillage member shown in Figure 6.11 as an example. The dimensions of this member are t = 0.2 m, w = 0.3 m and L = 4 m. The shear reinforcement is assumed to be at its minimum value of 0.0015 x 2 x t x L. The following symbols are assumed to be known so that they could be used in the cost assessment.

$f_c = 10 \text{ N/mm}^2 = 10 \times 10^3 \text{ KN/m}^2$

$f_s = 225.5 \text{ N/mm}^2 = 225.5 \times 10^3 \text{ KN/m}^2$

$E_c = 28 \text{ KN/mm}^2 =$ Young's Modulus for Concrete

$E_s = 200 \text{ KN/mm}^2 =$ Young's Modulus for Steel Reinforcement

$\gamma_s =$ Density of steel reinforcement $= 77 \text{ KN/m}^3$

Assume $BM_1 = 250$ KN.m, $BM_2 = 400$ KN.m. The calculations proceed as follows:

$$(1) \quad \alpha = \frac{E_s}{E_c} = \frac{200}{28} = 7.143$$

$$(2) \quad n = \frac{10 \times 10^3}{(10 \times 10^3 + \frac{225.5 \times 10^3}{7.143})} = 0.24$$

$$(3) \quad y = 1 - \frac{n}{3} = 1 - \frac{0.24}{3} = 0.92$$

$$(4) \quad \ell_a = 0.9 \; y \; w = 0.2484$$

$$(5) \quad A_{s1} = \frac{BM_1}{f_s \times \ell_a} = \frac{250}{225.5 \times 10^3 \times 0.2484} = 4.463 \times 10^{-3} \text{ m}^2$$

- Volume $= 4.463 \times 10^{-3} \times 0.75 \times 4 = 13.39 \times 10^{-3} \text{ m}^3$
- Weight $= 13.39 \times 10^{-3} \times 77 = 1.03$ KN $= \underline{0.105}$ ton

$$(6) \quad A_{s2} = \frac{400}{225.5 \times 10^3 \times 0.2484} = 7.141 \times 10^{-3} \text{ m}^2$$

- Volume $= 7.141 \times 10^{-3} \times 0.75 \times 4 = 21.42 \times 10^{-3} \text{ m}^3$
- Weight $= 21.42 \times 10^{-3} \times 77 = 1.649$ KN $= \underline{0.168}$ ton

$\therefore$ Total tensile reinforcement = 0.105 + 0.168 = $\underline{0.273}$ ton

(7) $A_{s\ell}$ = Area of stirrups

$= 0.0015 \times t \times L \times 2$

$= 0.0015 \times 0.2 \times 4 \times 2 = 2.4 \times 10^{-3}\ m^2$

- Volume $= A_{s\ell} \times w \qquad = 2.4 \times 10^{-3} \times 0.3$

$= 2.4 \times 10^{-3} \times 0.3 = 0.72 \times 10^{-3}\ m^3$

- Weight $= 0.72 \times 10^{-3} \times 77 = 0.05544$ KN

$= \underline{0.0056}$ ton

(8) Volume of concrete $= t \times w \times L$

$= 0.2 \times 0.3 \times 4 \qquad = \underline{0.24}\ m^3$

(9) Horizontal area which needs fair finish

$= 2 \times t \times L = 2 \times 0.2 \times 4 = \underline{1.6}\ m^2$

(10) Vertical area which needs fair finish

$= 2 \times w \times L = 2 \times 0.3 \times 4 = \underline{2.4}\ m^2$

Using the values underlined the construction cost of the grillage member is computed as shown in the self-explanatory Table 6.3.

Number	Item Description	Unit	Quantity	Rate £'s	Amount £'s
(1)	In-Situ Concrete Work:				
	(a) Provision of concrete. - Design mixture Grade 28 cement, (f_c = 30 N/mm^2)	m³	0.24	33.11	7.95
	(b) Placing of reinforced concrete. - Slabs thickness 150 - 300 mm	m³	0.24	4.29	1.03
					8.98
(2)	Concrete Ancillaries:				
	(a) Formwork fair finish.				
	- Horizontal width 0.2 - 0.4 m	m²	1.6	10.83	17.33
	- Vertical width 0.2 - 0.4 m	m²	2.4	10.99	26.38
	(b) High-yield bar steel reinforcement to BS 449				
	- 20 mm diameter bars	ton	0.273	363.47	99.23
	- 6 mm diameter bars	ton	0.0045	537.19	2.43
					145.37
	Summary:				
	(1) In-Situ cost				8.98
	(2) Concrete Ancillaries cost				145.37
	∴ Total cost (R)				154.35

TABLE 6.3: CONSTRUCTION COST (R) OF A TYPICAL GRILLAGE MEMBER

CHAPTER 7

EXAMPLES ON THE DESIGN OF HORIZONTAL GRILLAGES

7.1 THE DESIGN SPECIFICATIONS

The optimum design method described in the previous Chapter is applied here to obtain a minimum weight or a minimum cost topological design of normally loaded reinforced concrete grillages. The method aims at obtaining the optimum cross-sectional dimensions of the concrete members and the corresponding amount of reinforcement needed. The design specifications concerning the stresses in the members of the grillage are taken from the CP114 code of practice. This code depends on the permissible stress method which is used here for the design of uncracked concrete members of the grillage. On the other hand, no design code of practice has specified clear limits on the joint deflections of the grillage. Therefore, such limits are left to the designer to decide, and this might affect the results.

Each grillage member is considered as a reinforced concrete member subjected to bending combined with shear and torsion. The effects of these forces on the strength of the member are mostly general and practical problems. There is a dearth of information on the theoretical development of this problem, primarily because of its extreme complexity. The design of a reinforced concrete member under the effect of combined bending and shear was tackled successfully by many design methods. For the effect of torsion, however, it was generally agreed (Zia, 1968) that in a reinforced concrete member subject to torsion the reinforcement has no appreciable effect on the stiffness before cracking. Therefore, as the grillage members are assumed to be uncracked, the elastic theory, which considers the concrete to be linearly elastic, is adopted for constructing the combined shear and torsional stress constraints. For this reason also, the reinforcement which might be needed for torsion is ignored in the grillage members.

The design specifications which are required to be satisfied are the following:

(1) The combined shear and torsional stress (τ_{max}) in any grillage member should not exceed the maximum allowed shear stress $\tau_u = 3.6$ N/mm^2. The permissible shear stress of concrete is taken here as $\tau_c = 0.9$ N/mm^2. As explained in the previous Chapter, τ_u governs the alteration of the sectional dimensions t and w, while τ_c decides the amount of stirrups required, (Clauses 303 and 316, CP114: Part 2, 1969).

(2) The bending moment compressive stress in any grillage member should not exceed $f_c = 10$ N/mm^2, (Table 6, CP114).

(3) The direct compressive stress in any supporting column must not be greater than $f_{cc} = 7.6$ N/mm^2, (Table 6, CP114).

(4) The deflection of any point in the grillage should be within the limit of L_j/ξ, (Section 6.8).

The permissible concrete stresses, specified above, are for nominal concrete of mix proportion 1:1:2, (Table 6, CP114). Other limiting stress values can be specified depending on the mix proportion, or on the code of practice used. The concrete used is Grade 30 for which the Young's Modulus of Elasticity (E_c) is 28 KN/mm^2, and poisson's ration (ν) is 0.2 . For the steel reinforcement, the Young's Modulus (E_s) is taken as 200 KN/mm^2 and the permissible tensile stress (f_s) is 225.5 N/mm^2.

7.2 THE DESIGN PROCEDURE

The procedure for the optimum design of a reinforced concrete grillage is to some extent similar to that of the frame and the complete structure, previously described. However, for a minimum weight design of a grillage, the procedure is as follows:

(1) Develop the ground grillage structure. This is done by defining the main topology of the structure which includes the length, the width and if necessary the height of the grillage. The number and the position of beams, and the boundary (support) conditions are predetermined and are not subject to variations during the search for an optimum design.

(2) Give a joint number to each point where transverse and longitudinal beams intersect. Joints are also placed where there is more than one concentrated load or a discontinuity in the magnitude of the uniform load between intersection points.

(3) Give numbers to the members of the structure, and if required group the members together. The numbering and the grouping start from the columns. In here, it is assumed that each column belongs to a single group.

(4) Specify the design load, which can be either concentrated or uniform, and allocate it on the joints of the grillage. This load includes the self-weight and the external load. The method of computing the grillage self-weight will be described in Section 7.3.1.

(5) Select the upper and the lower bounds on the section variables which are the thickness (t) and the width (w) of each beam and the overall depth (h_c) of each column. For all the design examples in this chapter, the upper bounds on t and w were taken as 800 mm and 1000 mm respectively, while the lower bound for them was taken as 200 mm. The upper bound on h_c was 1000 mm, while the lower bound was 200 mm. These bounds should not be exceeded by those of the move limits.

(6) Select initial sectional properties for the members, and then analyse the structure to obtain the joint displacements and the member forces.

(7) Use the member forces to compute the reinforcement required for each of the grillage members, Section 6.10, and calculate the material and the construction costs.

(8) Construct the linear form of the objective function and transfer its coefficients to the backing store.

(9) Use the row-by-row technique to construct the linear form of the design constraints, and transfer their coefficients to the backing store.

(10) Apply the Simplex Method to minimise the weight function.

(11) Use the section properties obtained to repeat the process from step (6) until convergence is achieved and the optimum design is determined.

(12) Occasionally some of the stress and the deflection requirements are satisfied at the optimum design, but are violated when the structure is finally analysed. This is because the optimisation is carried out on the linear model of a highly non-linear problem. Therefore, carry out minor changes to the sections where such violations occur.

For a minimum cost topological design, the aim is to reduce the total cost of the grillage structure. One way of achieving this is by trying to remove some of the beams from the original structure. The removable beams are selected automatically by the optimisation method which considers structural and economical factors for such selection. The procedure for the minimum cost design is as follows:

(1) Starts by minimising the weight, as described by the procedure above. The reason for such a start is to improve the relationships between the members before the process of removal begins.

(2) Calculate the fixed charge R_i as the cost of retaining each grillage member or group of members, Section 6.10, and introduce the associate variables δ_t and δ_w.

(3) Repeat the process of the minimum weight design from step 6 to step 10.

(4) Remove all members, or groups, with $\delta_t = \delta_w = 0$ or with $\delta_t = \delta_w \cong 0$. If members with $\delta_t = \delta_w \cong 0$ are not removed at this stage, their δ values may change later during the design procedure. Notice that members are removed on the condition that feasibility is maintained in the next design iteration.

(5) Repeat this procedure from step 2 until no further topological change is obtained.

(6) Continue the minimum cost design of the structure with its final topology until convergence is achieved and the optimum design determined.

7.3 DESIGN LOAD FOR GRILLAGE

In the grillage design, the total vertical load P imposed on a joint in the Z-direction is calculated as:

$$P = P_D + P_L \tag{7.1}$$

where P_D is the dead load (self-weight) and P_L is the live load imposed on the joint.

It is necessary to include an estimated self-weight of the grillage in the total load when calculating the dimensions of the sections. The self-weight constitutes a relatively greater proportion of the load as the member spans and sectional dimensions increase. In the present study, such weight is taken as a concentrated dead load which is estimated, for a typical joint j in a grillage, as equal to the sum of half the weight of each member connected to the joint. Such as:

$$P_{Dj} = \sum_{i=1}^{Mj} (\gamma \; (L_i/2) \; t_i \; w_i) \qquad (7.2)$$

where P_{Dj} is the dead load imposed on joint j,

M_j is the total number of members meeting at joint j,

γ is the density of reinforced concrete,

L_i, t_i and w_i are the dimensions of member i.

As an example, consider the grillage in Figure 7.1. The parts of the members included in estimating the dead load imposed on joint j are shown shaded.

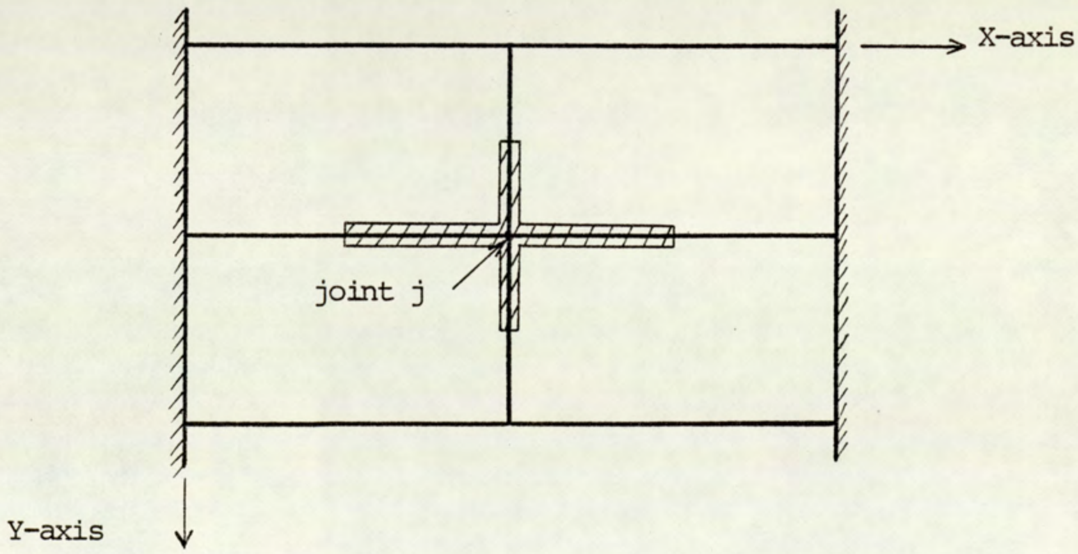


Figure 7.1: PARTS OF THE MEMBERS INCLUDED IN ESTIMATING THE DEAD LOAD IMPOSED ON JOINT j.

The live load P_L is usually specified as either concentrated or uniformly distributed over the face of the grillage. The method of apportioning the uniform load to the joints of a grillage is shown in Figure 7.2. By using the contributing areas of the uniform load, the imposed load and the moments acting on the joints can be calculated. As an illustration, assume that the uniform load imposed on the grillage of Figure 7.2 has an intensity of 2 KN/m^2. Thus, the load apportioned to joint j is 4 m x 4 m x 2 KN/m^2 = 32 KN. Similarly the load carried by joint d is 16 KN and by joint n is 8 KN.

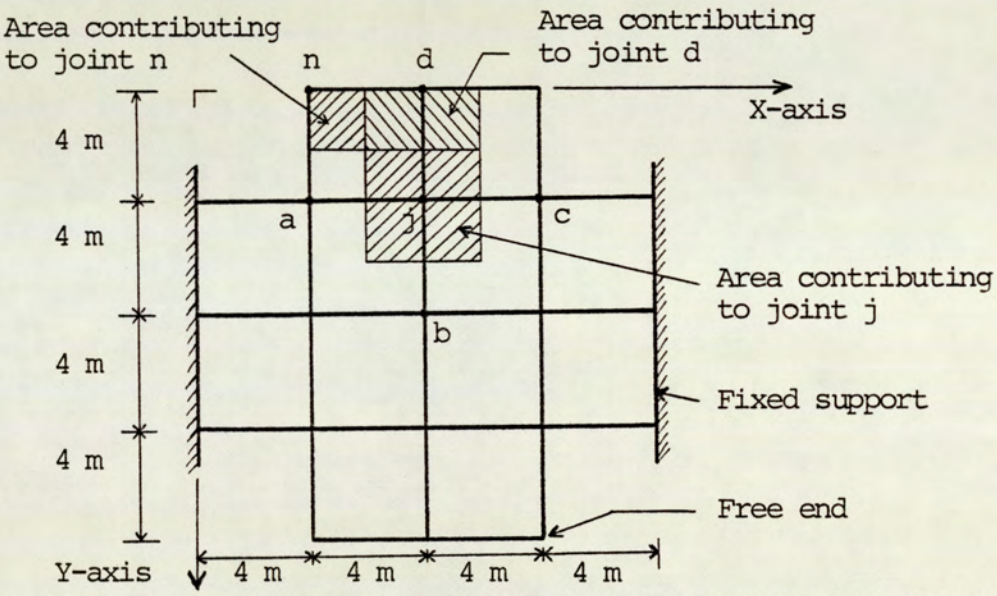


Figure 7.2: METHOD OF APPORTIONING UNIFORM LOAD TO THE JOINTS OF A GRILLAGE

The moments acting on a joint, usually known as the fixed end moments ($M_{F.E.}$), can be calculated by dividing the uniform load into strips. The width of each strip is considered to be of unit length. For instance, from Figure 7.2, consider the uniform load between joints d and j, as shown in Figure 7.3. The fixed end moments about X-axis acting on each of these two joints is calculated from the following equation:

$$M_{F.E.X.} = \omega \ L^2 / 12 \qquad (7.3)$$

where ω is the intensity of the uniform load strip, and here it is equal to 1 m x 4 m x 2 KN/m^2 = 8 KN/unit length. Thus, $M_{F.E.X.d} = - M_{F.E.X.j.} = \frac{8\ (4)^2}{12}$ = 10.67 KN.m. The fixed end moments, about X and Y-axes, acting on all the grillage joints can be calculated similarly.

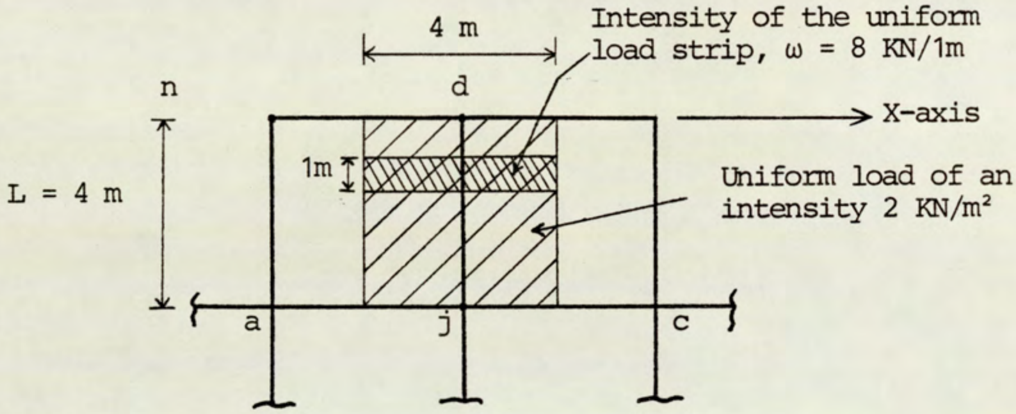


Figure 7.3: A STRIP OF A UNIFORM LOAD USED IN CALCULATING THE FIXED END MOMENTS ($M_{F.E.}$).

In the topological design of a grillage, some members, and consequently some joints, are removed. The load carried by one of these joints is allocated to the neighbouring joints. In the present work such allocation is done in an approximate way. As an illustration, assume that in the uniformly loaded grillage, shown in Figure 7.2, the member between joints d and b, and that between joints a and c are removed. Thus, joint j is also removed which means that a new uniform load apportioning is required to joints a, b, c and d. This is shown in Figure 7.4, where the new acting moments on joints d and b about X-axis will be calculated as $M_{xd}=-M_{xb}=\omega\ L\ (L/2)=\omega\ L^2/2$, where L is the distance of joint d (or b) from joint j; the moments about Y-axis for joints a and c can be computed similarly. Notice that each of such moments is calculated assuming that the uniform load apportioned to the joint is acting in a cantilever manner. It is also noticed that the contributing areas of the uniform load in Figure 7.4 overlap each other in the centre, i.e. at joint j. The effect of such overlapping on overestimating the new acting moments is ignored here. The imposed live load on the removed joint j, i.e. P_{Lj}, is divided between joints a, b, c and d. Such division depends on the distance between joint j and the other joints. For instance, in Figure 7.4, P_{Lj} is divided into four equal parts and added to the live loads of the other four joints.

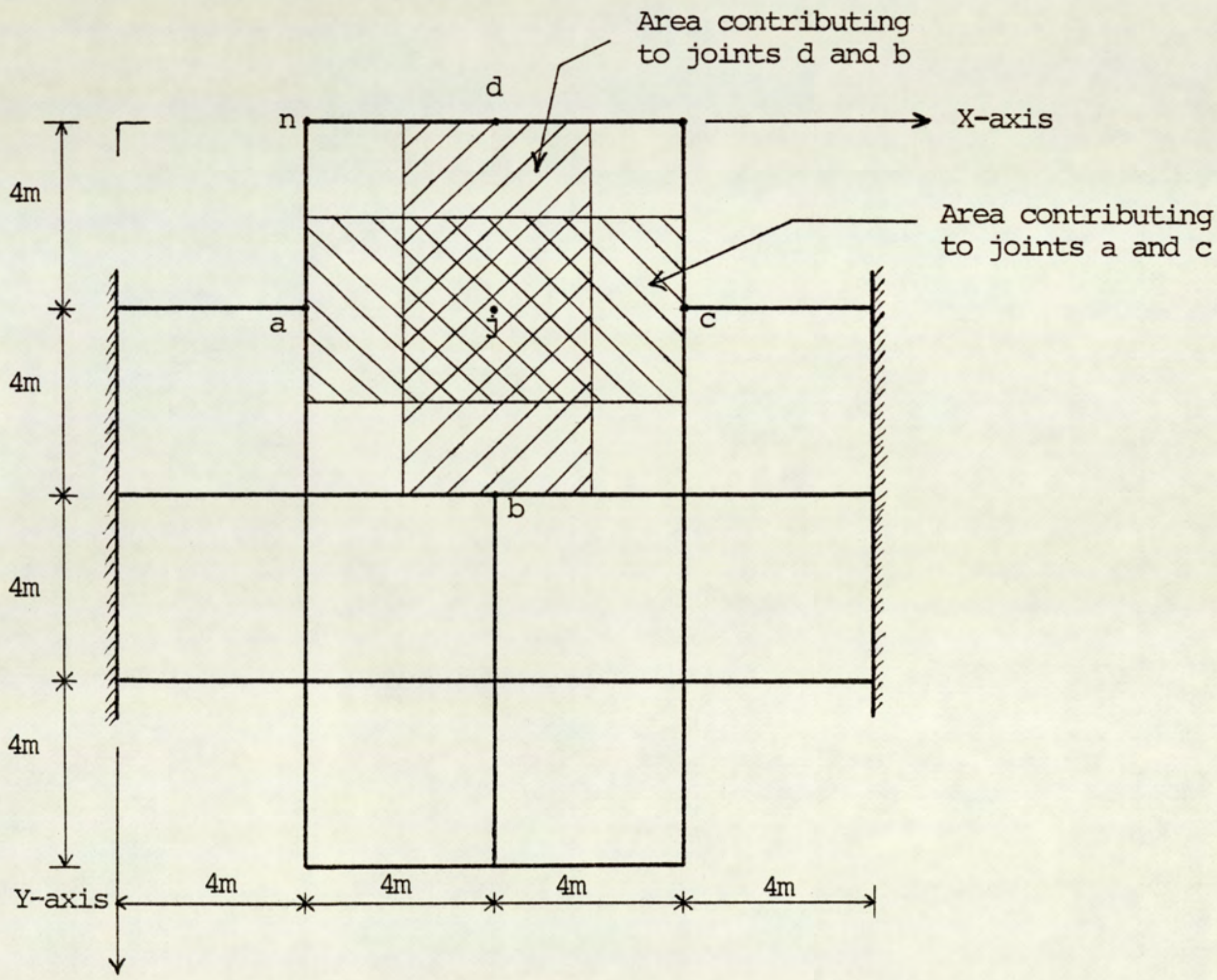


Figure 7.4 METHOD OF APPORTIONING UNIFORM LOAD AFTER JOINT j IS REMOVED.

7.3.1 <u>The Effect of Considering the Self-Weight as a Variable</u>

As shown by equation (7.2) in the previous section, the self-weight of the grillage depends upon the section variables t and w of the members. In the analysis of a grillage, these variables are already known and the self-weight is taken as a constant added to the live load, the total of which constitutes the design load, equation (7.1). On the other hand, in the optimisation process, the values of t and w are unknown and require calculation. Consequently in this process the self-weight is considered as a variable. This variable is included in the rows of the stiffness constraints which correspond to the displacements in the Z-direction. Thus, if the stiffness equations that correspond to such displacements can be expressed in a matrix form as:

$$\underline{K}\ \underline{X} = \underline{P}_D + \underline{P}_L \tag{7.3}$$

then the stiffness constraints for these rows become:

$$\underline{H}\ (h_c, t, w, X) = \underline{K}\ (h_c, t, w)\ \underline{X} - \underline{P}_D\ (t, w) - \underline{P}_L = 0 \tag{7.4}$$

The derivatives of the stiffness constraints with respect to the design variables are required for the linearisation process. Therefore, the derivatives of (7.4) with respect to the variable t are expressed as:

$$\frac{\partial H}{\partial t} = \frac{\partial\ \underline{K}\ (h_c, t, w)}{\partial t} \cdot \underline{X} - \frac{\partial\ \underline{P}_D\ (t, w)}{\partial t} \tag{7.5}$$

and similarly with respect to w:

$$\frac{\partial H}{\partial w} = \frac{\partial\ \underline{K}\ (h_c, t, w)}{\partial w} \cdot \underline{X} - \frac{\partial\ \underline{P}_D\ (t, w)}{\partial w} \tag{7.6}$$

In the present optimisation process, the self-weight can be considered as a constant or variable depending on the choice of the designer. However, it is proven here that there is an advantage in considering the self-weight as a variable, and this will be demonstrated when dealing with the example in the next section.

7.4 EXAMPLES ON THE MINIMUM WEIGHT DESIGN

7.4.1 An Ellipse Shaped Grillage

This is the type of structure which is usually encountered by the engineer to cover a space between buildings or to be used as part of a roof system. The grillage is considered to be fixed at ten points, as shown in Figure 7.5. The figure also shows the assumed dimensions of the grillage and the external loading, which consists of concentrated loads at the intersection points. The numerical values of the external loading are given for only one quarter of the grillage. The loading values on the other three quarters are just the same because the grillage is symmetrical about its central axes which are parallel to the X and the Y-axes.

Figure 7.6 shows the way in which the joints and the members of the grillage were numbered. Overall, there are 36 joints and 69 members which are gathered into seven groups. The members which are on one straight line between two supports are all considered to be built from one section, i.e. belong to a single group. The first four groups are for the transverse members, and the other three groups are for the longitudinal members. Notice that the grouping is symmetrical. However, the advantage of symmetry will not be demonstrated in this section, as this will be shown when dealing with the example in Section 7.5.

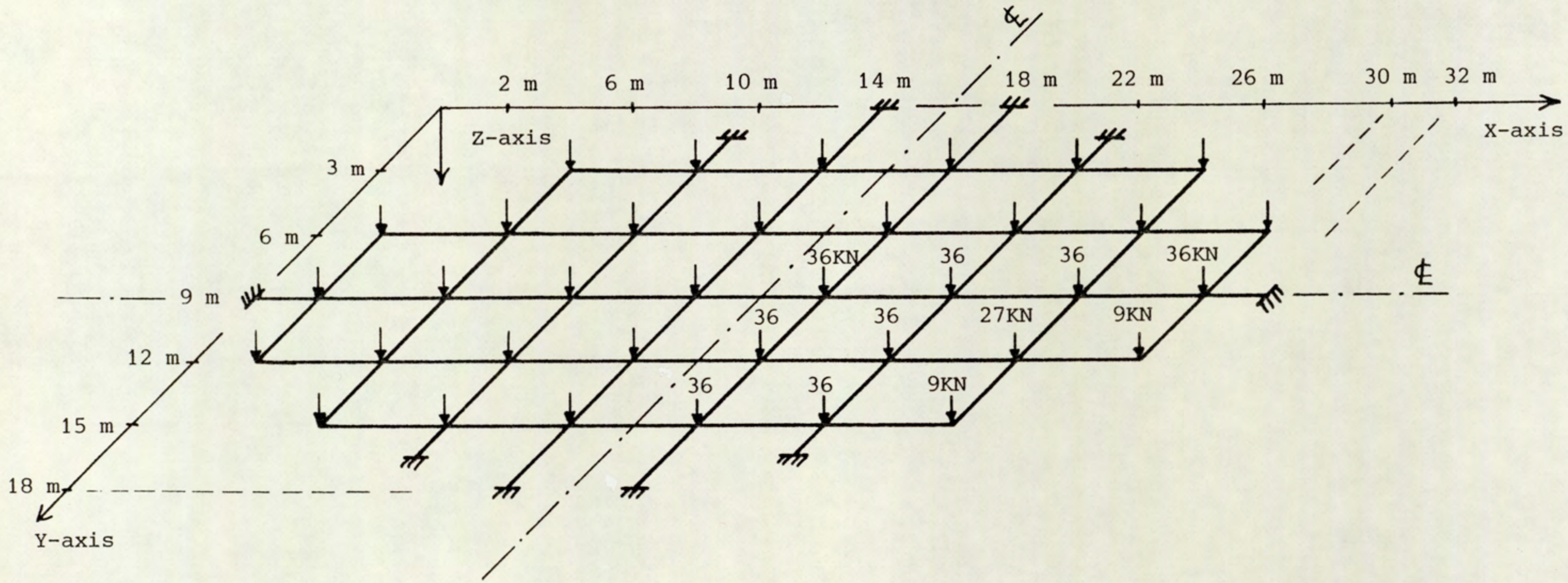


FIGURE 7.5: A SYMMETRICAL ELLIPSE SHAPED HORIZONTAL GRILLAGE - LOADING AND DIMENSIONS

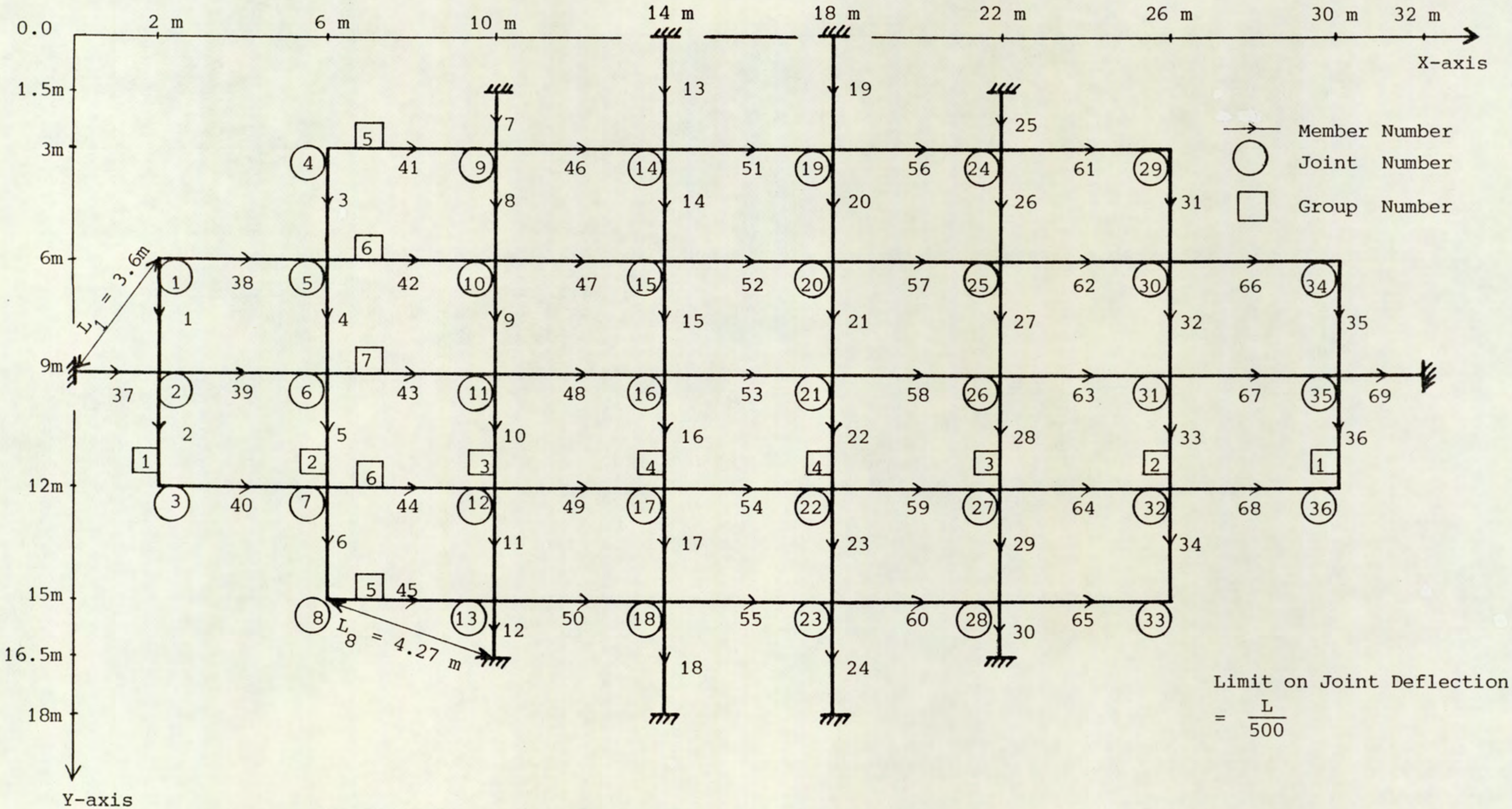


FIGURE 7.6: THE SYMMETRICAL ELLIPSE SHAPED HORIZONTAL GRILLAGE - NUMBERING OF JOINTS, MEMBERS AND GROUPS

The design problem has 122 variables, 14 of which are the section variables t and w of the seven grillage groups, and the rest are the displacement variables; three for each joint. The problem also has a total of 258 constraints, 108 of which are stiffnesses and another 108 are deflections. There are also 14 stress constraints which include the bending moment stress and the combined shear and torsional stress for each group of members. Furthermore, there are 14 upper and 14 lower bound constraints imposed on the section variables of the grillage groups.

The permissible stresses for the grillage concrete members were specified in Section 7.1. The limit on the joint deflection was taken as $L_j/500$, where L_j is the distance of joint j from the nearest fixed support. For instance, the limit on the vertical deflection of joint number 1, see Figure 7.6, was taken as $L_1/500 = 3600\text{mm}/500 = 7.2\text{mm}$, while for joint 8 it was $L_8 / 500 = 4270 \text{ mm} / 500 = 8.5 \text{ mm}$.
However, it should be emphasised here that such a deflection limitation was only specified for the joints where deflection was known to be critical. In this example, these joints with their deflection limitations are given in Table 7.1

Joint No.	1	3	4	8	11	15	16	17	20	21	22	26	29	33	34	36
Def.Limit (L_j/500)mm	7.2	7.2	8.5	8.5	15.0	12.0	18.0	12.0	12.0	18.0	12.0	15.0	8.5	8.5	7.2	7.2

Table 7.1: THE DEFLECTION LIMITATIONS ON THE CRITICAL JOINTS OF THE SYMMETRICAL ELLIPSE SHAPED GRILLAGE.

The limitations on the deflection of the other joints can be specified arbitrarily, but here it was taken as 20 mm for each joint. The limit on the rotations of the joints was once again taken as 0.08 radian.

There are two purposes for choosing this design example. The first one is to investigate the effect of considering the self-weight of the grillage as a variable included in the optimisation process. The second purpose is to examine the verification and the reliability of the optimum weight design achieved. For the first purpose, two design cases are reported here. In Case 1, the self-weight was calculated for each joint and was added as a constant to the live load to constitute the total imposed load. While in Case 2, the self-weight was considered as a constant when analysing the grillage, but it was included as a variable in the stiffness constraints when the grillage was optimised.

Both cases of the design started with equal sectional dimensions of t = w = 500 mm for all the groups of the grillage members. The initial weight was therefore equal to 1408 KN. The variation of the weight in the two design cases is shown in Figure 7.7. The initial move limit was taken as ML = 0.5 which was then reduced by steps of 0.1 at each design iteration until convergence was achieved. In the two cases each required seven design iterations to reach the optimum, and each of these iterations required an average number of about 200 simplex iterations.

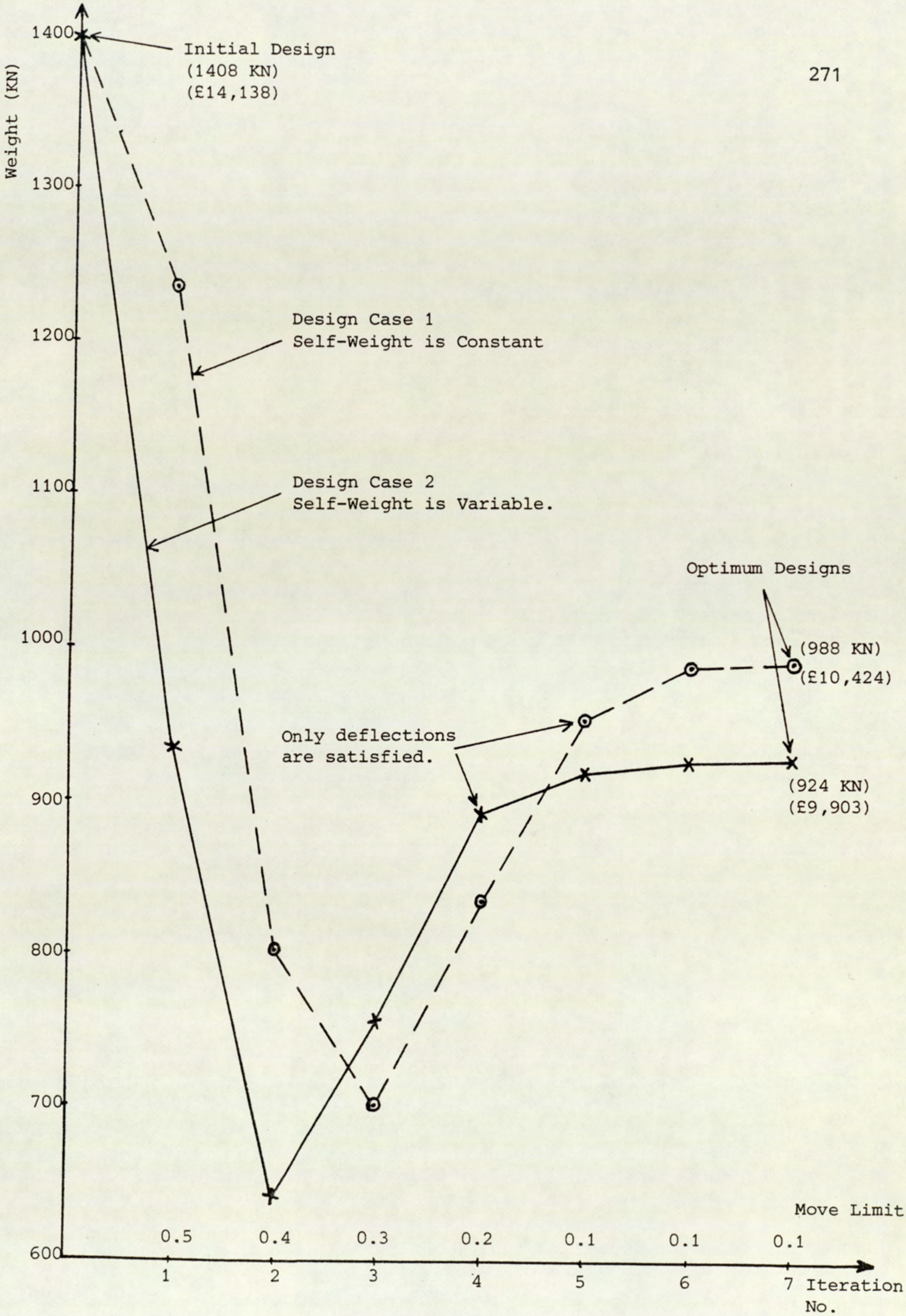


FIGURE 7.7: ELLIPSE SHAPED GRILLAGE - VARIATION OF WEIGHT IN TWO DESIGN CASES

The initial and the optimum sectional dimensions obtained at the two design cases are shown in Table 7.2. The optimum weight obtained for Case 1 is 988 KN, while for Case 2 it is 924 KN which is about 6.5% less than that for Case 1. Although this percentage is small, it proves the advantage of considering the self-weight as a variable for further reduction in the optimum weight of the grillage. By examining the optimum results in Table 7.2, it may be noticed that the grillage members tend to become thinner and deeper. This behaviour continues until the sectional overall depth w reaches its upper bound, and by then the sectional thickness t starts to increase. The lower bound of a sectional dimension is marked by an asterisk, while the upper bound is marked by two asterisks in the Table.

The computer program which was written for the purpose of optimising flat grillages is designed to compute the total cost of the structure at each design iteration. This cost, which includes the material and the construction costs of the members, is shown in Figure 7.7 and Table 7.2 at the initial and the optimum designs of the two cases.

As Case 2 of the optimum design has given a better result than that of Case 1, the steel reinforcements for the grillage members will be given below only for Design Case 2. This case is also used to examine the verification and the reliability of the design. The final steel areas obtained in Design Case 2 for uncracked sections of some grillage members are shown in Table 7.3. The steel areas for the rest of the members can be obtained due to symmetry. The area of stirrups, and the tensile steel areas at the two ends of each member were computed as explained in Section 6.10.2. Some of these steel areas are not required in some members, so a minimum amount of reinforcement had to be specified in their places. These are marked by an asterisk in

Group Number	Initial Design		Optimum Designs			
			Design Case 1 Self-Weight is Constant		Design Case 2 Self-Weight is Variable	
	t (mm)	w (mm)	t (mm)	w (mm)	t (mm)	w (mm)
1	500	500	200 *	466	200 *	358
2	500	500	200 *	200 *	200 *	361
3	500	500	319	1000 **	522	1000 **
4	500	500	341	1000 **	200 *	892
5	500	500	200 *	522	200 *	552
6	500	500	200 *	342	200 *	200 *
7	500	500	261	1000 **	200 *	1000 **
Total Weight	1408 KN		988 KN		924 KN	
Total Cost	£14,138		£10,424		£9,886	

* Indicates the sectional dimension is at the lower bound.

** Indicates the sectional dimension is at the upper bound.

TABLE 7.2: ELLIPSE SHAPED GRILLAGE - OPTIMUM SECTIONAL DIMENSIONS OBTAINED AT THE TWO DESIGN CASES.

Member Number	Area of Stirrup Reinf. for shear A_{sL} (mm^2)	Areas of Tensile Steel Reinf. for Bending		Material Cost £	Construction Cost £	Total Cost £
		Member End 1 A_{s1} (mm^2)	Member End 2 A_{s2} (mm^2)			
1	900*	96*	639	13.93	55.72	69.65
3	900*	97*	410	14.05	54.58	68.64
4	900*	412	644	14.05	58.11	72.16
7	2564	4663	2024	50.86	134.92	185.79
8	2350*	2011	1168	101.73	223.85	325.58
9	2350*	1161	3026	101.73	230.32	332.05
13	900*	1825	119	34.74	143.04	177.77
14	900*	241*	719	34.74	111.78	146.51
15	900*	719	638	34.74	114.33	149.07
37	1468	1785	306	25.97	94.19	120.16
38	1200*	54*	103	10.39	47.00	56.99
39	1200*	304	724	51.94	164.60	216.54
41	1200*	149*	986	28.68	103.51	132.20
42	1200*	92	147	10.39	47.70	58.09
43	1200*	689	604	51.94	166.88	218.82
46	1200*	495	246	28.68	100.14	128.82
47	1200*	77	54*	10.39	46.78	57.17
48	1200*	931	735	51.94	170.06	222.00
51	1200*	210	210	28.68	97.38	126.06
52	1200*	54*	54*	10.39	46.58	56.97
53	1200*	737	737	51.94	168.43	220.37

* Means Minimum Area of Steel Reinforcement is Specified.

TABLE 7.3: ELLIPSE SHAPE GRILLAGE - STEEL REINFORCEMENT AND COSTS OF SOME MEMBERS OBTAINED AT THE OPTIMUM DESIGN OF CASE 2

Table 7.3. The material and the construction costs are also given in the Table. The results in the Table show that most of the grillage members required the minimum amount of stirrups. The members that required additional stirrups are 7, 8 and 9, which belong to the shortest group, and members 13 and 37 which are connected to the fixed end supports. Notice that the tensile steel areas at both ends of members 51, 52 and 53 are equal due to symmetry.

The optimum designs in Case 1 and Case 2 satisfy all the design requirements which include the stiffness, the stress and the deflection. This will be verified for Design Case 2 in the next sub-section.

7.4.1 Verification of the Optimum Design

The computer program which was written for the purpose of optimising horizontal grillages has a sub-routine that analyses the structure at each design iteration. By employing this sub-routine, an analysis was carried out for the Ellipse Shaped Grillage using the sections obtained at the optimum design of Case 2. The forces and the stresses in all the members, and the deflections of the joints were calculated. However, due to symmetry the analysis results for only some of the members and the joints will be shown here. Table 7.4 shows the forces and the stresses for these members. In this Table the bending moments, at both ends of the members, the shear forces and the torques are given in their real directions depending on the sign convention shown in Figures 6.9 and 6.10 in Chapter 6, and Figure 4.1 in Chapter 4. The stresses, however, are given in the Table as absolute values.

Member Number	Member End 1		Member End 2		Shear		Torsion		Combined Shear and Torsion Stress (N/mm^2)*
	Bend Mom. (KN.mm)	B.M.Stress (N/mm^2) *	Bend Mom. (KN.mm)	B.M.Stress (N/mm^2) *	Force (KN)	Stress (N/mm^2) *	Torque (KN.mm)	Stress (N/mm^2) *	
1	1208	0.28	42648	10.00**	-15	0.31	271	0.07	0.38
3	-136	0.03	-27620	6.36	9	0.19	-3620	1.00	1.19
4	27746	6.39	43387	10.00**	-24	0.49	-3203	0.88	1.37
7	-870470	10.00**	377813	4.34	328	0.94	22691	0.33	1.27
8	-375437	4.31	-218007	2.50	198	0.57	-27891	0.40	0.97
9	216714	2.49	-564857	6.49	116	0.33	-30487	0.44	0.77
13	-303865	(11.46)	19832	0.75	95	0.79	-4289	0.41	1.20
14	-22072	0.83	-119794	4.52	47	0.39	-580	0.05	0.44
15	119754	4.52	-106220	4.01	-5	0.04	271	0.03	0.07
37	333333	10.00**	-57245	1.72	-138	1.04	0	0.00	1.04
38	271	0.20	3847	2.88	-1	0.04	1208	0.72	0.76
39	56702	1.70	135095	4.05	-48	0.36	0	0.00	0.36
41	-3620	0.36	-101641	10.00**	26	0.36	-136	0.02	0.38
42	-3429	2.57	-5495	4.12	2	0.08	1334	0.80	0.88
43	-128690	3.86	-112815	3.38	60	0.45	0	0.00	0.45
46	51058	5.02	25332	2.49	-19	0.26	2240	0.37	0.63
47	2898	2.17	2017	1.51	-1	0.05	41	0.02	0.07
48	173789	5.21	137135	4.11	-78	0.58	0	0.00	0.58
51	-21623	2.13	21623	2.13	0	0.00	0	0.00	0.00
52	-1166	0.87	1166	0.87	0	0.00	0	0.00	0.00
53	-137676	4.13	137676	4.13	0	0.00	0	0.00	0.00

* Means Absolute Values of Stresses are given.
** Means the stress reached to its permissible value.
( ) Means the stress exceeded its limit.

TABLE 7.4: ELLIPSE SHAPE GRILLAGE - FORCES AND STRESSES IN SOME MEMBERS OBTAINED AT THE OPTIMUM DESIGN OF CASE 2

It may be noticed in Table 7.4 that the maximum bending moment occurred at the first end of member 7 which belongs to Group 3. The bending stress at this end reached to the permissible value, i.e. 10 N/mm^2. This value is marked, for this member and some others, by two asterisks in the Table. By examining the bending stresses in the other members in Group 3, i.e. 8 and 9, it is obvious that member 7 is the one with the critical stress and was therefore used for constructing the bending stress constraint for this group. Member 7 was also used to construct the combined shear and torsional stress constraint for Group 3, because this member had the highest combined shear stress compared with members 8 and 9.

It may be further noticed that the bending moment stress at the first end of member 13, marked by brackets in Table 7.4, has exceeded the limit. The reason for this is that the optimisation was carried out on a linear model of highly non-linear problem. Therefore, while all the stresses were satisfied at the optimum design, some might not be when the structure was analysed.

It is also proved that the interaction of the two orthogonal systems of members helps in stiffening the structure and retain joint equilibrium. For instance, the bending moment at the first end of member 3 is equal to the torque in member 41.

From the analysis results in Table 7.4, it is shown that the combined shear and torsional stresses in the members are well below the permissible value of 3.6 N/mm^2. This shows that these stress constraints were non-effective in optimising the Ellipse Shaped Grillage. Notice also that the torques and the torsional stresses in members 37, 39, 43, 48, 51, 52 and 53 are equal to zero, while the shear forces and stresses in members 51, 52 and 53 are also equal to zero. This is due to symmetry.

Table 7.5 shows that, for each group, there is a member which has the critical stress. This member was used for constructing the stress constraint for the group. Both the critical bending stress and the critical combined shear and torsional stress do not necessarily occur at the same member in the group. This is proven in Group 5, where member 41 was used for constructing the bending stress constraint while member 46 was used for constructing the combined shear stress constraint for the group.

Group Number	Member used for constructing the B.M. stress constraint	Member used for constructing the combined shear and torsional stress constraints
1	1	1
2	4	4
3	7	7
4	13	13
5	41	46
6	42	42
7	37	37

Table 7.5: ELLIPSE SHAPED GRILLAGE - MEMBERS USED FOR CONSTRUCTING THE STRESS CONSTRAINTS FOR THEIR GROUPS AT THE FINAL DESIGN ITERATION.

The bending moment diagrams for the longitudinal members are shown in Figure 7.8, while for the transverse members the diagrams are shown in Figure 7.9. These diagrams are drawn on the tension side of the member. Notice that the diagrams are not continuous at the intersection point of the members. This is due to the fact that there

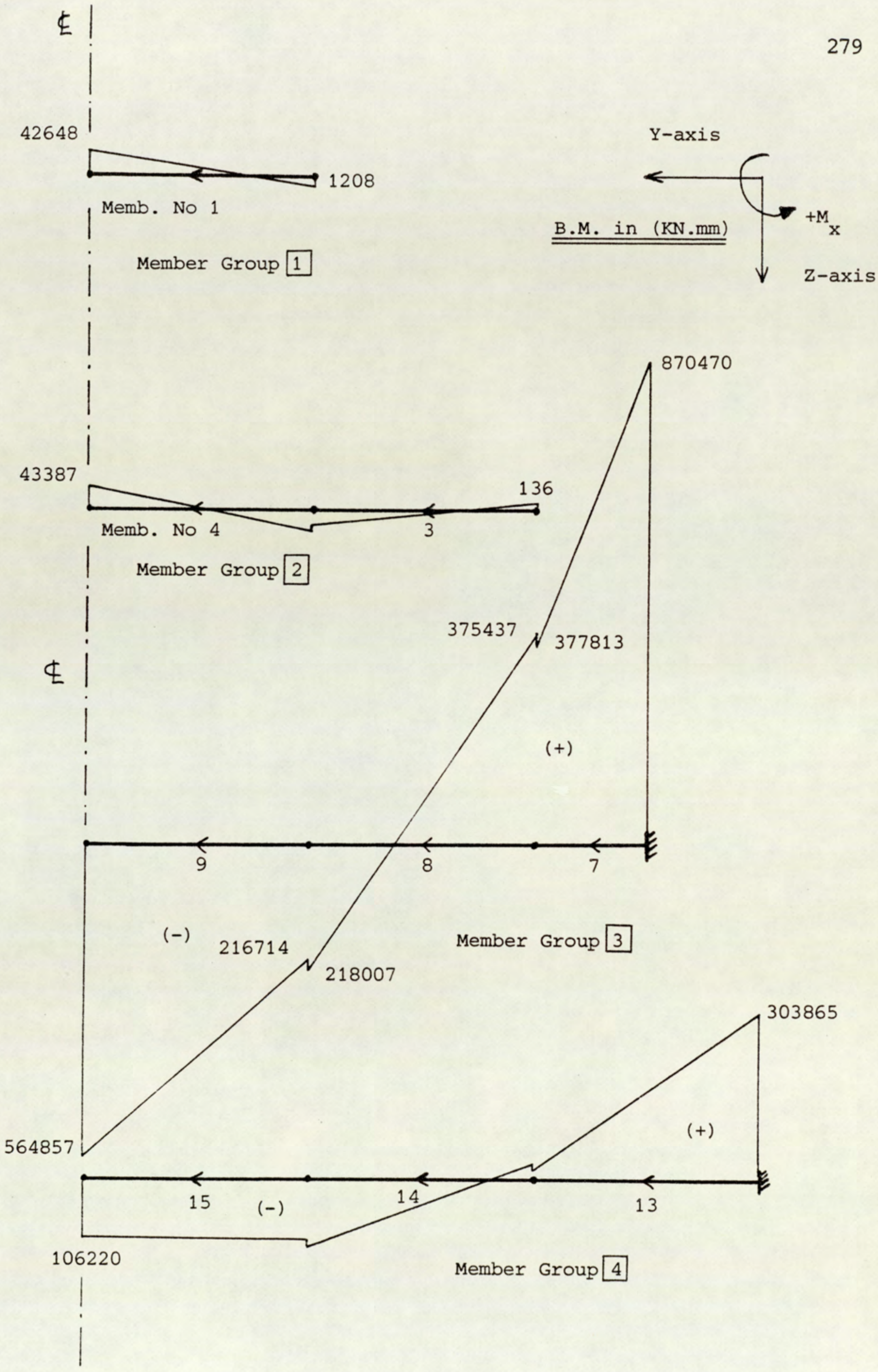


FIGURE 7.8: ELLIPSE SHAPED GRILLAGE - BENDING MOMENT DIAGRAMS FOR THE LONGITUDINAL MEMBERS OBTAINED AT THE OPTIMUM DESIGN OF CASE 2

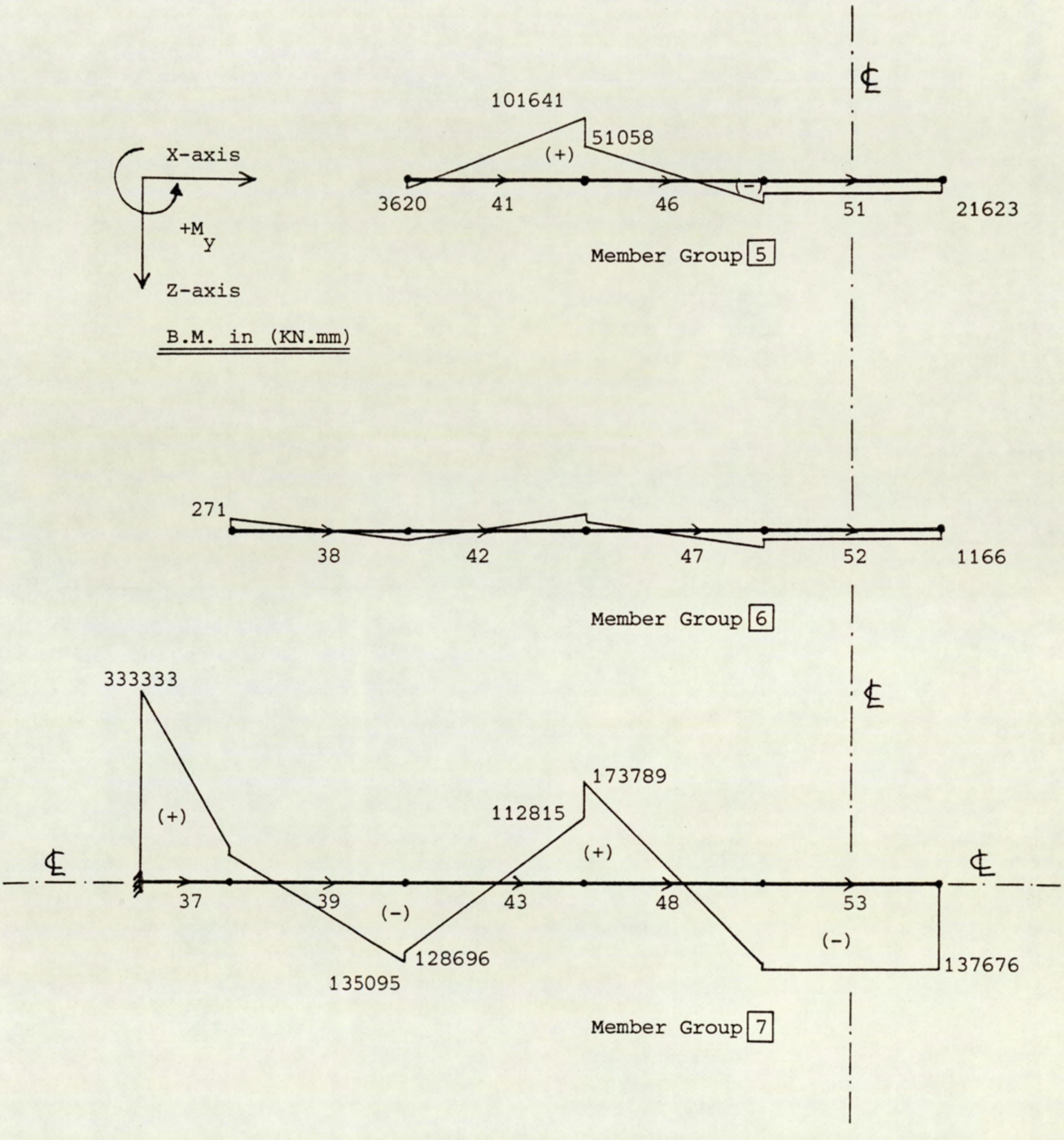
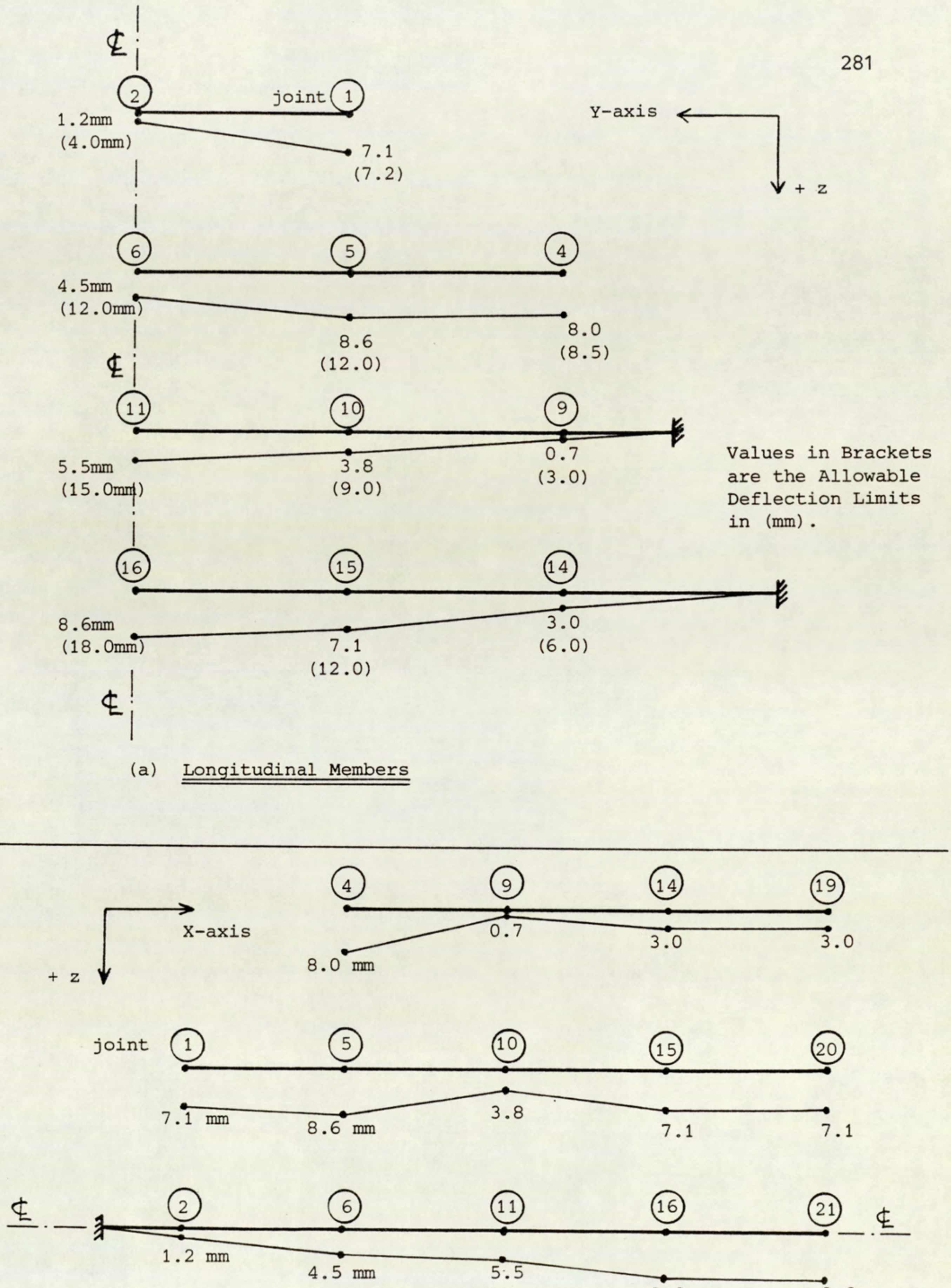


FIGURE 7.9: ELLIPSE SHAPED GRILLAGE - BENDING MOMENT DIAGRAMS FOR THE TRANSVERSE MEMBERS OBTAINED AT THE OPTIMUM DESIGN OF CASE 2



(a) Longitudinal Members

(b) Transverse Members

FIGURE 7.10: ELLIPSE SHAPED GRILLAGE - DEFLECTION PROFILES FOR THE GRILLAGE MEMBERS OBTAINED AT THE OPTIMUM DESIGN OF CASE 2

are two different value of bending moments at each of these points, the difference between such values being equal to the torques in the orthogonal members.

The vertical deflection profiles for the longitudinal members are shown in Figure 7.10a, while for the transverse members the deflection profiles are shown in Figure 7.10b. The values in brackets in Figure 7.10a are the allowable deflection limits for the joints. Although all the deflections are satisfied, it is noticed that none of them had reached the limit. The closest ones are joints 1 and 4. From observing the weight variation of Design Case 2 in Figure 7.7, it is shown that at iteration number 4 all the deflection requirements are satisfied but some stresses are not. From this it is concluded that the specified deflection requirements helped in converging the problem but they did not decide the final sections for the Ellipse Shaped Grillage, as this decision was left to the stress requirements, particularly those of the bending stress.

7.4.2 An Irregular Circular Flat Grillage Supported by Four Columns

This type of structure, shown in Figure 7.11, is commonly used as an independent roof system. The grillage, which was assumed to be uniformly loaded, is supported by four columns. The positions of these columns were chosen by the designer, as shown in the figure. Each column was assumed to be 4 m high, and to be fixed at both ends. The dimensions, and the numbering of joints, members and groups are shown in Figure 7.12. Notice that the numbering is started from the columns. As a whole, there are 47 joints and 73 members in this structure. Each column was assumed to be built from a single section, i.e. belong to a single group. Thus, there are four column groups. The grillage itself

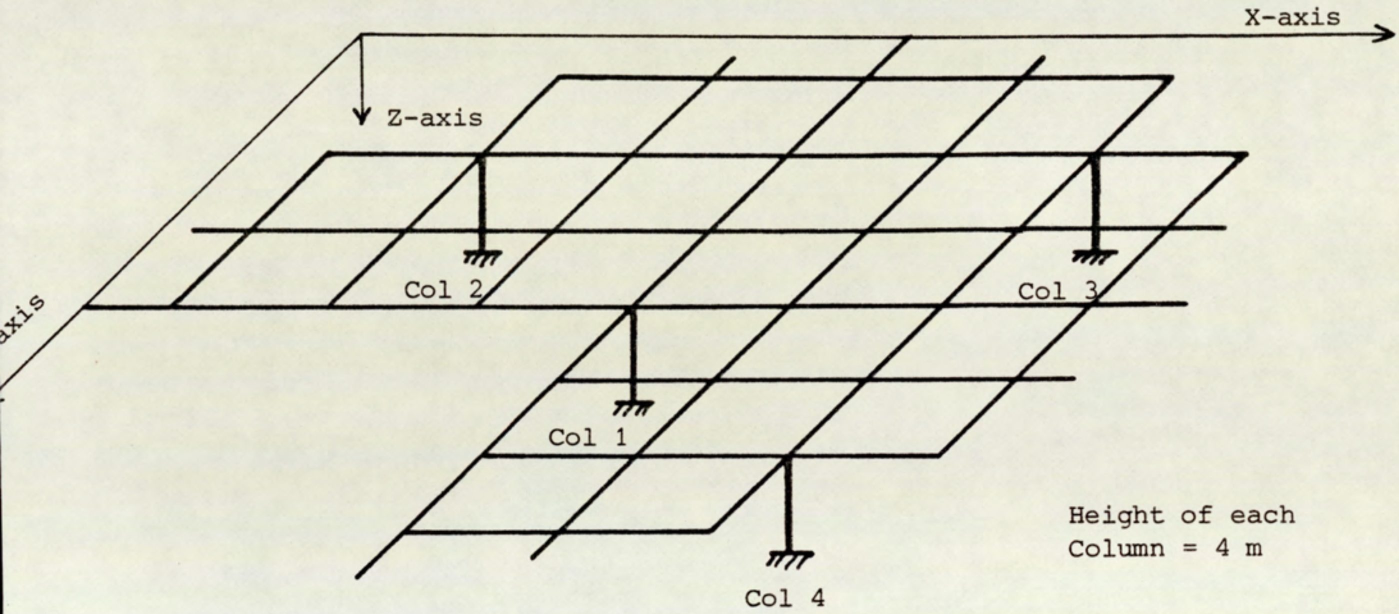
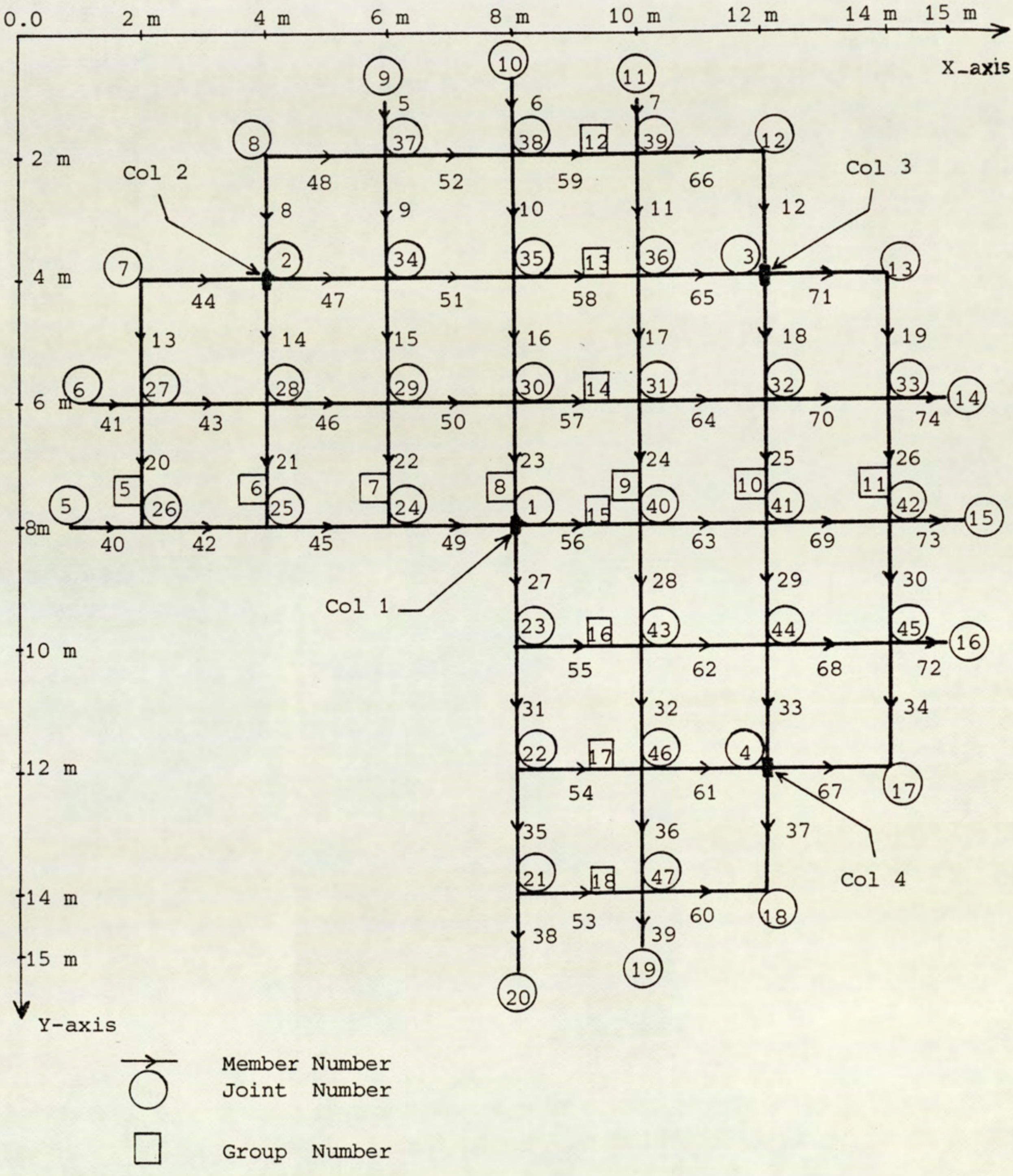


FIGURE 7.11: IRREGULAR CIRCULAR FLAT GRILLAGE SUPPORTED BY FOUR COLUMNS - THREE DIMENSIONAL VIEW SHOWING THE POSITIONS AND THE NUMBERING OF COLUMNS



Limit on Joint Deflection
= L/400

FIGURE 7.12: THE IRREGULAR CIRCULAR FLAT GRILLAGE - THE DIMENSIONS AND THE NUMBERING OF JOINTS, MEMBERS AND GROUPS

was made from 69 orthogonal members which belonged to 14 groups; seven as longitudinal and another seven as transverse groups.

The limit on the joint deflection was taken as $L_j/400$, where L_j is defined here as the distance of joint j from the nearest column. Such a limit was only specified for the joints where deflection was known to be critical. Other relaxed limits were specified for joints with non-critical deflection. The deflection limits specified for all the joints are given in the second column of Table 7.6. Each value of the relaxed deflection limit is marked by an asterisk in the Table. The joint rotations were limited once again to 0.08 radian.

As mentioned above, the grillage was assumed to be uniformly loaded, and the load intensity was taken as ω = 10 KN/m = 0.01 KN/mm. The live load and the moments acting on each joint, due to the uniform load, are given in Table 7.6. These were calculated as explained in Section 7.3. For some joints in the grillage, one of the acting moments was found to be equal to zero. The stiffness constraint that corresponds to this moment was also found to have a zero right-hand side. This in turn caused the optimisation problem to degenerate and to enter into cycling. To avoid such a problem, a small value of moment was assumed to act instead of the zero moment. This value of moment is marked by brackets in Table 7.6.

The design problem has a total of 173 variables, four of which represent the section overall depth (h_c) of the four columns, and another 28 variables represent the sectional dimensions t and w of the 14 grillage groups. The rest of the variables are joint displacements. The problem also has a total of 374 constraints, 141 of which are stiffnesses and another 141 are deflections. There are also 28 stress constraints, and 32 upper and another 32 lower bound constraints

Joint Number j	Specified Deflection Limit $L_j/400$ in (mm)	Imposed Live Load and Moments Caused by a Uniform Load of ω = 0.01 KN/mm		
		P_L (KN)	$M_{F.E.X.}$ (KN.mm)	$M_{F.E.Y.}$ (KN.mm)
1	1.0 *	15.0	-1670.0	-1670.0
2	1.0 *	15.0	1670.0	-1670.0
3	1.0 *	15.0	1670.0	1670.0
4	1.0 *	15.0	-1670.0	1670.0
5	12.8	3.0	(10.0)	- 600.0
6	8.8	4.5	(15.0)	- 675.0
7	5.0	5.0	1670.0	-1670.0
8	5.0	5.0	1670.0	-1670.0
9	8.8	4.5	675.0	(10.0)
10	12.8	6.0	1200.0	(15.0)
11	8.8	4.5	675.0	(10.0)
12	5.0	5.0	1670.0	1670.0
13	5.0	5.0	1670.0	1670.0
14	8.8	4.5	(10.0)	670.0
15	12.8	6.0	(15.0)	1200.0
16	8.8	4.5	(10.0)	670.0
17	5.0	5.0	-1670.0	1670.0
18	5.0	5.0	-1670.0	1670.0
19	8.8	4.5	-675.0	(10.0)
20	12.8	3.0	-600.0	(10.0)
21	11.2	8.0	-1070.0	-3340.0
22	10.0	10.0	0.0	-3340.0
23	5.0	10.0	0.0	-3340.0
24	5.0	10.0	-3340.0	0.0

* Means Relaxed Deflection Limit is specified.

TABLE 7.6: IRREGULAR CIRCULAR GRILLAGE - DEFLECTION LIMIT AND IMPOSED LIVE LOADS SPECIFIED FOR THE JOINTS

(Continued)

Joint Number j	Specified Deflection Limit $L_j/400$ in (mm)	Imposed Live Load and Moments Caused by a Uniform Load of ω = 0.01 KN/mm		
		P_L (KN)	$M_{F.E.X.}$ (KN.mm)	$M_{F.E.Y.}$ (KN.mm)
25	10.0	10.0	-3340.0	0.0
26	11.2	8.0	-3340.0	-1070.0
27	30.0 *	14.5	1670.0	-2670.0
28	30.0 *	20.0	0.0	0.0
29	7.1	20.0	0.0	0.0
30	30.0 *	20.0	0.0	0.0
31	7.1	20.0	0.0	0.0
32	30.0 *	20.0	0.0	0.0
33	30.0 *	14.5	-1670.0	2670.0
34	30.0 *	20.0	0.0	0.0
35	10.0	20.0	0.0	0.0
36	30.0 *	20.0	0.0	0.0
37	30.0 *	14.5	2670.0	-1670.0
38	30.0 *	16.0	2140.0	0.0
39	30.0 *	14.5	2670.0	1670.0
40	30.0 *	20.0	0.0	0.0
41	10.0	20.0	0.0	0.0
42	30.0 *	16.0	0.0	2140.0
43	7.1	20.0	0.0	0.0
44	30.0 *	20.0	0.0	0.0
45	30.0 *	14.5	-1670.0	2670.0
46	30.0 *	20.0	0.0	0.0
47	30.0 *	14.5	-2670.0	1670.0

* Means Relaxed Deflection Limit is specified.

TABLE 7.6: (CONTINUED)

imposed on the section variables of the column and the grillage groups.

The advantage of considering the self-weight of the grillage as a variable is demonstrated in Design Case 2 of the previous example. Therefore, the present design problem also considers the self-weight as a variable. The problem started with an arbitrary set of sectional dimensions, which made the total weight of the structure equal to 747 KN and its total cost became £7288. The weight variation accompanied by the cost, which was calculated at each design iteration, are shown in Figure 7.13. The initial move limit was taken as ML = 0.5. The optimum design was obtained after six design iterations. Each of these iterations required an average number of about 370 simplex iterations. The optimum weight otained for the structure is equal to 324 KN and the total cost at this point is £3944. The initial and the optimum sectional dimensions are shown in Table 7.7. It can be observed that the thicknesses of the grillage members always tend to decrease until they reach the lower bound, which is 200 mm. Furthermore, the overall section widths vary until the design requirements are satisfied.

Notice that at each column - grillage connecting point, the column is in fact connected to the deepest grillage member. Therefore, the thicknesses of the columns and the member are equal. In Table 7.7, the group of each of these members is marked by brackets near the column dimensions. In the initial design, the groups of the grillage members that are connected to the columns were chosen arbitrarily. However, during the design process these groups may change because at each design iteration the depths of the groups, and consequently the directions of the column sections, vary; see Figures 6.3 and 6.4 in Chapter 6. Notice that at the optimum design some of the grillage groups that are connected to the columns are different from those at the initial design.

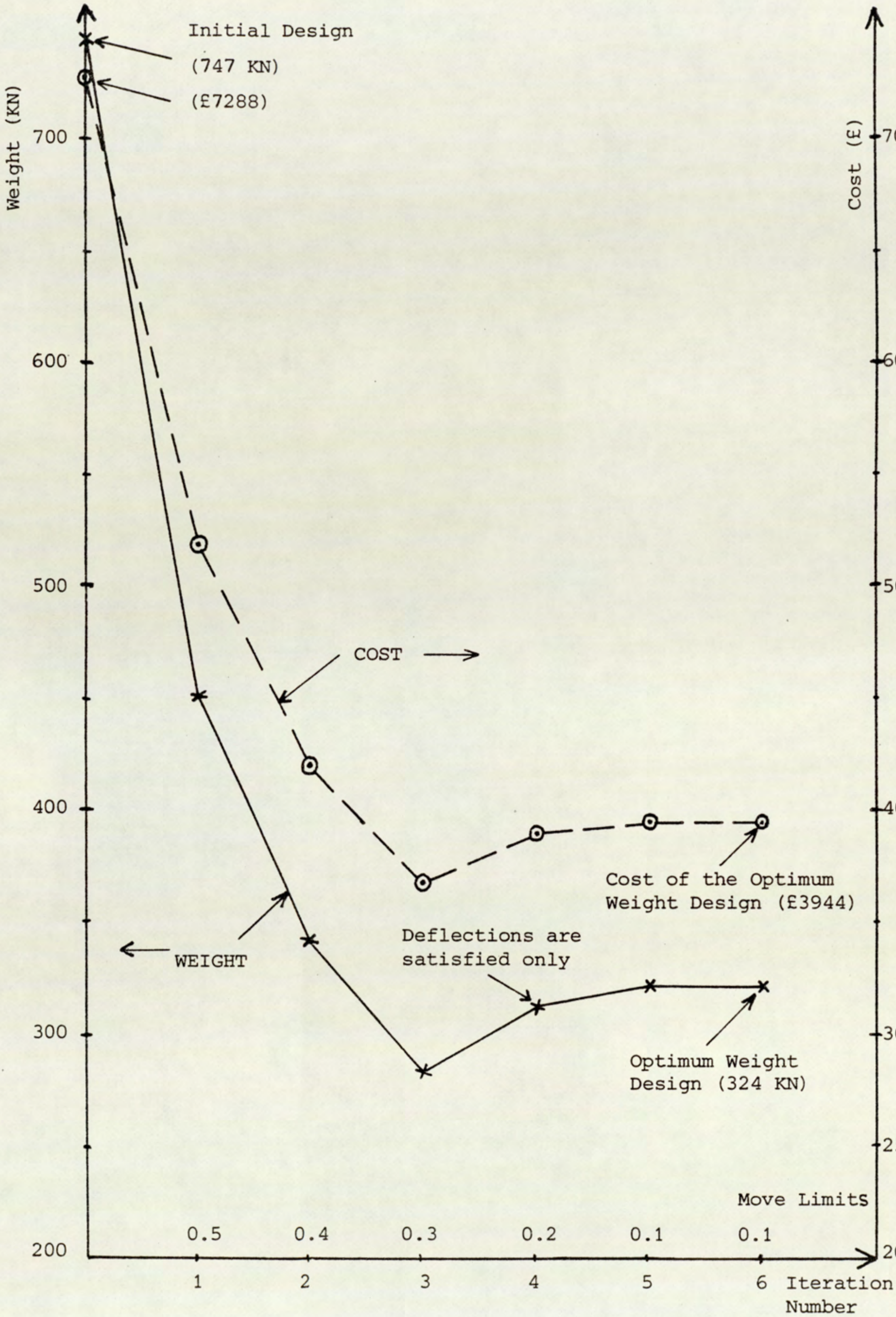


FIGURE 7.13: IRREGULAR CIRCULAR GRILLAGE - WEIGHT AND COST VARIATION

Group Number		Initial Design h$_c$ (mm)	Initial Design t (mm)	Initial Design w (mm)	Optimum Design h$_c$ (mm)	Optimum Design t (mm)	Optimum Design w (mm)
Columns	1	800	400 (Gr. 8)		280	200 (Gr. 15)	
	2	800	400 (Gr. 6)		230	200 (Gr. 6)	
	3	800	400 (Gr. 10)		250	200 (Gr. 13)	
	4	800	400 (Gr. 10)		220	200 (Gr. 17)	
Grillage Beams	5		300	600		200	200
	6		400	600		200	487
	7		300	600		200	200
	8		400	600		200	935
	9		300	600		200	200
	10		400	600		200	470
	11		300	600		200	474
	12		300	600		200	447
	13		300	600		200	483
	14		300	600		200	200
	15		300	600		200	943
	16		300	600		200	200
	17		300	600		200	514
	18		300	600		200	200
Total Weight		747 KN			324 KN		
Total Cost		£7288			£3944		

TABLE 7.7: IRREGULAR CIRCULAR GRILLAGE - SECTIONAL DIMENSIONS AT THE INITIAL AND THE OPTIMUM DESIGN

The areas of tensile steel for all the grillage members are shown in Table 7.8. Some members only required a minimum area of steel, given as 0.0015 x t x 0.9 x w, to be specified at one or both of their ends. Each of these areas of steel is marked by an asterisk in the table. It is noticed here that the computation of the tensile steel areas depended only upon the moments at the ends of the member; thereby ignoring the moments caused by the uniform load. This is because, in each grillage member of this structure, the moment at one of the ends is always greater than the moment between the ends. The shear reinforcement in all members was not required at the optimum design. However, a minimum stirrup area of 0.0015 x 2 x t x L was specified. Furthermore, throughout the design process, the columns were assumed to have fixed values of 1%, i.e. 0.01 x t x h_c, compressive steel together with 0.15% stirrups.

To verify the optimum design, the deflection profiles for all the grillage members are plotted in Figures 7.14 and 7.15. It can be noticed that all the joint deflections are satisfied; in other words, they are within the limits specified in Table 7.6. From observing the weight variation in Figure 7.13, it can be seen that at the fourth design iteration, only the deflections became satisfied. However, the sectional dimensions continued to alter and the weight increased slightly so that the stress requirements were met. This leads to the same conclusion as for the previous structure in that the stress requirements seem to dominate the optimum design. One of the possible reasons for such domination is that the deflection limit L/400 might not be sufficient to involve the deflection requirements as an active part in the design.

Member Number	Steel Area for Bending (mm^2)		Member Number	Steel Area for Bending (mm^2)		Member Number	Steel Area for Bending (mm^2)	
	Member End 1	Member End 2		Member End 1	Member End 2		Member End 1	Member End 2
5	54 *	119	29	471	346	53	139	191
6	252 *	252 *	30	635	509	54	162	139 *
7	54 *	119	31	878	333	55	267	54 *
8	131 *	819	32	75	289	56	1714	733
9	54 *	242	33	359	550	57	267	72
10	252 *	377	34	499	126	58	570	263
11	54 *	277	35	376	252 *	59	564	455
12	127 *	854	36	349	54 *	60	194	361
13	374	197	37	483	127 *	61	139 *	766
14	776	133	38	252 *	252 *	62	54 *	219
15	288	54 *	39	119	54 *	63	746	268
16	314	771	40	255 *	255 *	64	54 *	253
17	257	54 *	41	54 *	119	65	281	849
18	825	262	42	255 *	371	66	445	153
19	147	472	43	54 *	342	67	839	139 *
20	193	129	44	130 *	479	68	229	54 *
21	131 *	166	45	333	879	69	329	255 *
22	54	265	46	287	73	70	272	54 *
23	757	1720	47	590	378	71	892	130 *
24	71	272	48	125	473	72	119	54 *
25	244	492	49	871	1713	73	255 *	255 *
26	482	631	50	89	264	74	119	54 *
27	1715	870	51	364	553			
28	268	90	52	484	569			

* Means Miniumum Area of Tensile Steel is Specified.

TABLE 7.8: IRREGULAR CIRCULAR GRILLAGE - TENSILE STEEL AREAS FOR BENDING OBTAINED FOR THE MEMBERS AT THE OPTIMUM WEIGHT DESIGN

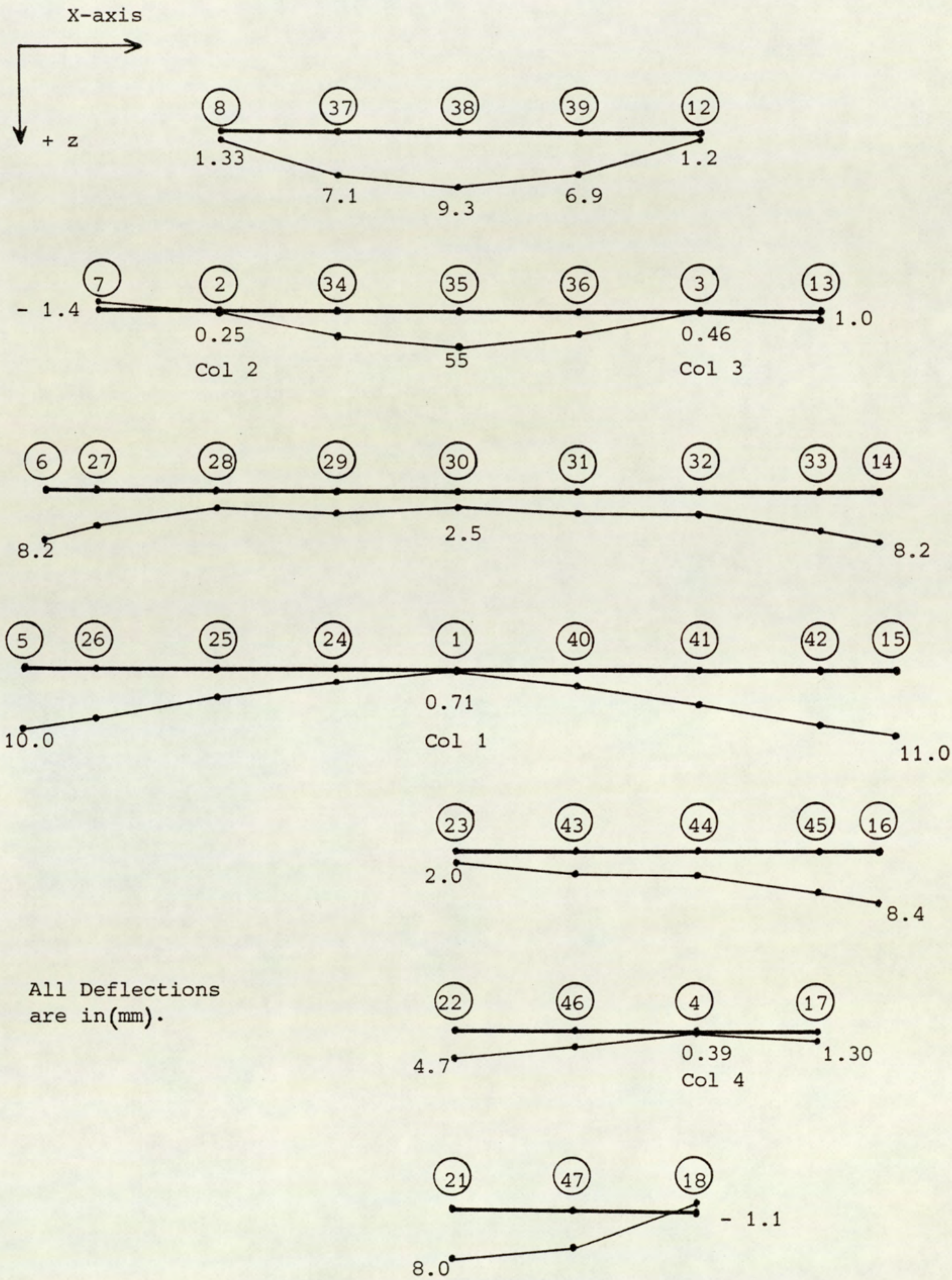


FIGURE 7.14: IRREGULAR CIRCULAR GRILLAGE - DEFLECTION PROFILES FOR THE TRANSVERSE MEMBERS OBTAINED AT THE OPTIMUM DESIGN

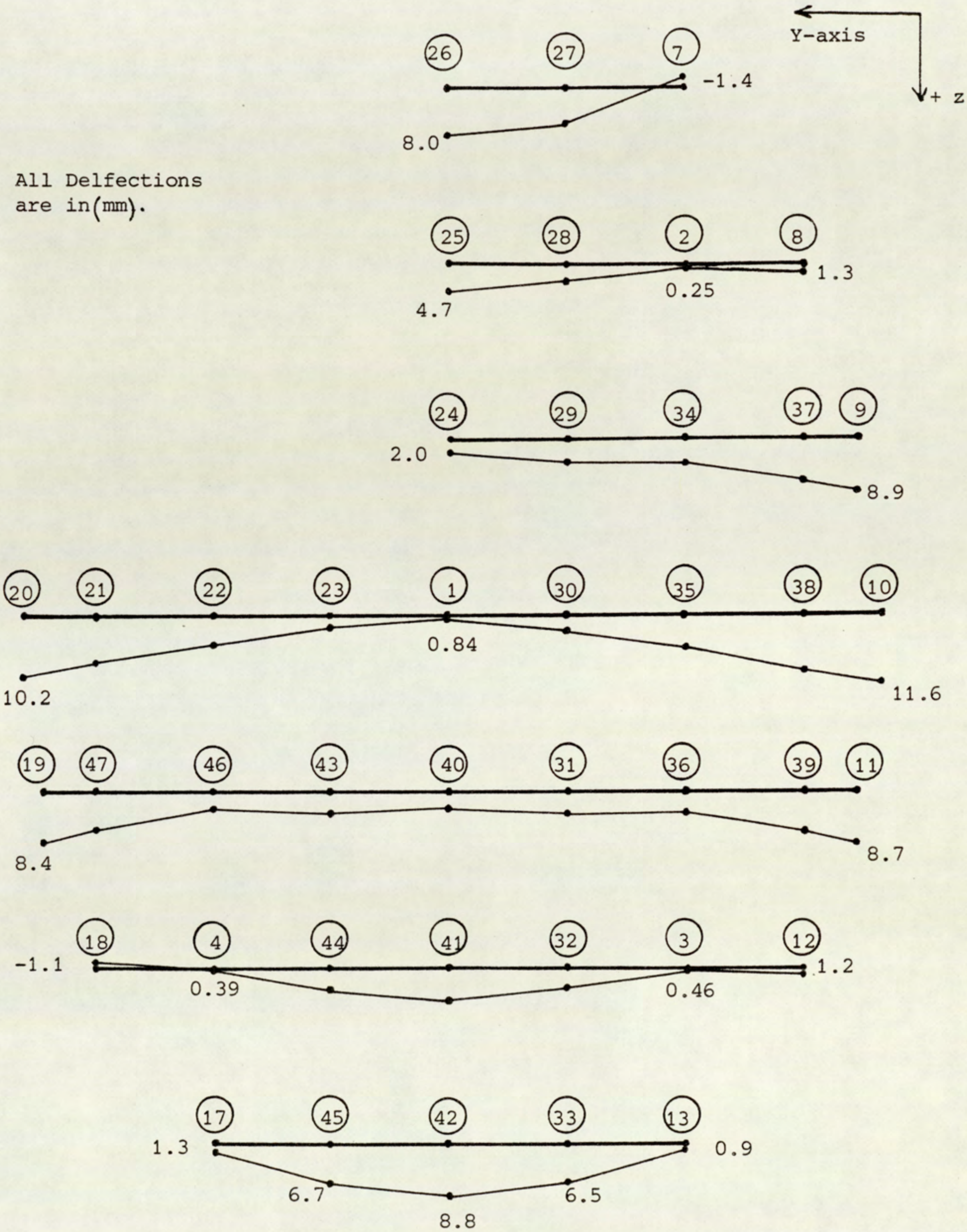


FIGURE 7.15: IRREGULAR CIRCULAR GRILLAGE - DEFLECTION PROFILES FOR THE LONGITUDINAL MEMBERS OBTAINED AT THE OPTIMUM DESIGN

To prove the stress domination, Table 7.9 shows the members with maximum stresses used for constructing the stress constraints for their groups at the final design iteration. The stress values give clear evidence that the bending moment stresses are more effective than the combined shear and torsional stresses in the structure considered. This is true as many grillage members reached the permissible bending stress which is marked by an asterisk in the Table.

In the present design problem, there are no specific design requirements which can be imposed on the column sections. These sections seem to be decided when the grillage members satisfy their requirements. The thickness of the column section is equal to the thickness of the grillage member connected to it. Therefore, only the overall depth (h_c) of the column needs to vary. However, the validity of the final column sections obtained at the otpimum design can be checked. This can be done according to Clause 322 (CP114: Part 2: 1969), or Clause 3.5.7.3 (CP110: 1972). For instance, consider column number one where t = 200 mm and h_c = 280 mm. From the final analysis results, the displacements of the joints at the top of this column, i.e. joint 1, are found to be z = 0.71 mm, $\theta_x = 0.137 \times 10^{-3}$ rad. and $\theta_y = 0.113 \times 10^{-3}$ rad. . Using equation (6.9), Chapter 6, the axial load and the moments are computed to give P_c = 280 KN, M_x = 716 KN.mm and M_y = 1157 KN.mm. The permissible combinations of direct load and bending moments, to which this column may be subjected, and the maximum stresses in the column were checked according to the clauses, mentioned above, and were found to be satisfactory. This was also found for the other three columns in this structure.

Group Number	Maximum B.M.Stress		Maximum Combined Shear and Torsional Stress	
	Member Number	Stress Value (N/mm^2)	Member Number	Stress Value (N/mm^2)
5	13 - End 1	10.0 *	13	2.5
6	8 - End 2	9.4	14	2.3
7	22 - End 2	7.4	15	1.4
8	23 - End 2	10.0 *	31	1.3
9	36 - End 1	9.7	17	1.6
10	12 - End 2	10.0 *	12	2.8
11	30 - End 1	7.5	19	1.3
12	52 - End 2	7.1	66	1.4
13	71 - End 1	10.0 *	71	2.8
14	43 - End 2	9.6	46	1.6
15	56 - End 1	10.0 *	45	1.3
16	55 - End 1	7.5	62	1.3
17	67 - End 1	9.1	61	2.4
18	60 - End 2	10.0 *	60	2.4

* Means the stress is at its permissible value.

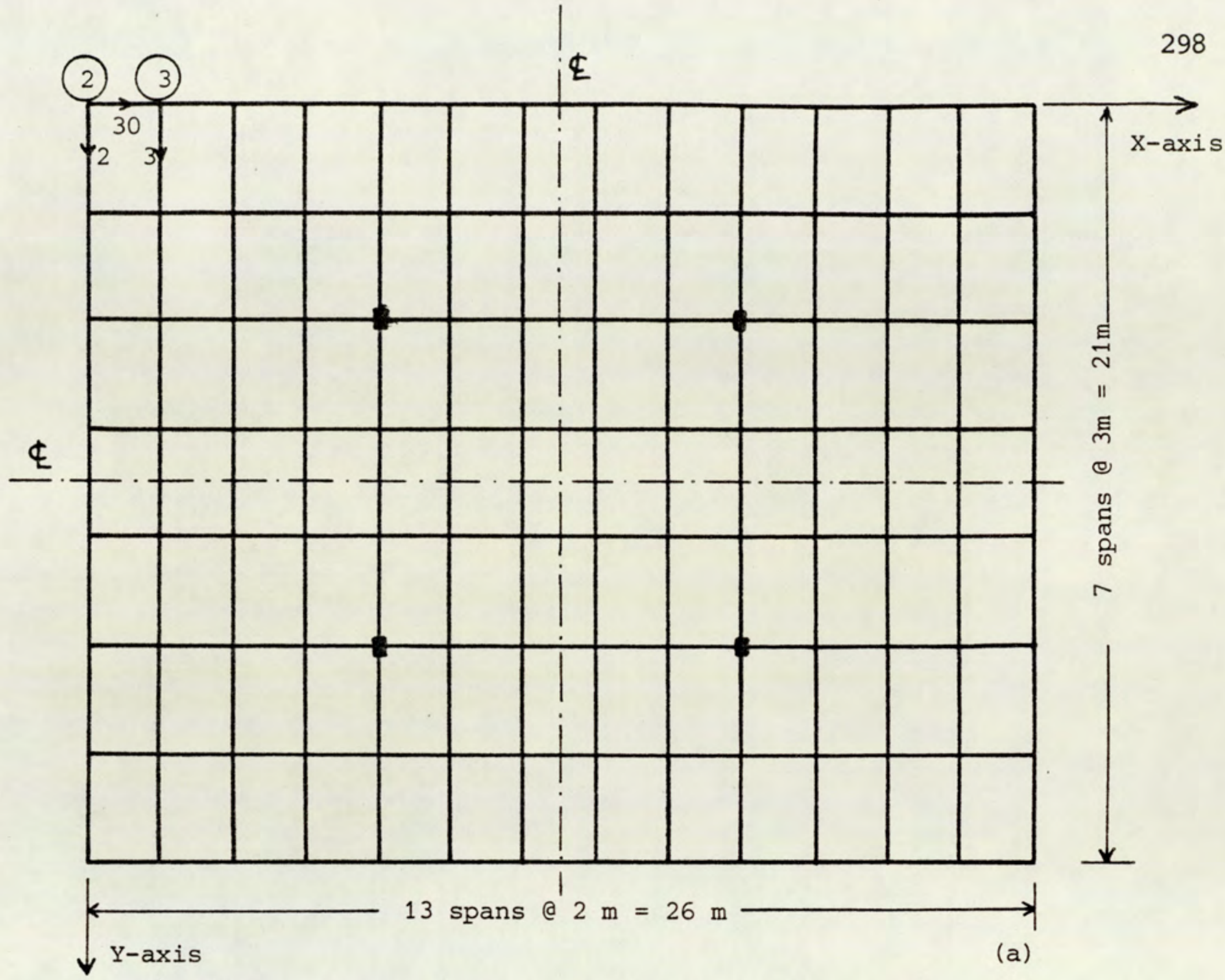
TABLE 7.9: IRREGULAR CIRCULAR GRILLAGE - MEMBERS WITH MAXIMUM STRESSES USED FOR CONSTRUCTING THE STRESS CONSTRAINTS FOR THEIR GROUPS AT THE FINAL DESIGN ITERATION

7.5 EXAMPLE ON THE MINIMUM COST DESIGN

7.5.1 A Symmetrical Rectangular Grillage Supported by Four Columns

Similarly to the structure in the previous Section, this one can also be used as an independent roof system. The grillage, shown in Figure 7.16a, is symmetrical and it is supported by four columns, each is 4 m high. The positions of the columns were chosen by the designer. The grillage was assumed to be uniformly loaded with a load intensity equal to ω = 5 KN/m. The purpose of choosing this example is to investigate the effect of topological alteration on the minimum cost design. Some members were allowed to be removed for the sake of reducing the total cost. This might decrease or increase the total weight as will be shown later. The removable members were selected by the design process which depends on economic and structural factors for such selections.

The structure is symmetrical about both of its central lines. Therefore, only the top-left-quarter of it will be considered for the design. The numbering of joints and members for this quarter is shown in Figure 7.16b. Notice that the ends of members which are on the central lines are considered as joints. Each of these joints has two degrees of freedom; the deflection in the Z-direction and the rotation about the axis which is parallel to the member local axis. This quarter of the rectangular grillage consists of a total of 39 joints and 57 members. The grouping of members is shown in Figure 7.17a. The column belongs to a single group, while the grillage members gathered into 18 groups; 12 longitudinal and 6 transverse groups. In the minimum cost topological design, some groups were required to be retained in the final design. These are the groups that represent the members on the edges, i.e. groups 2 and 4, and the members on the lines



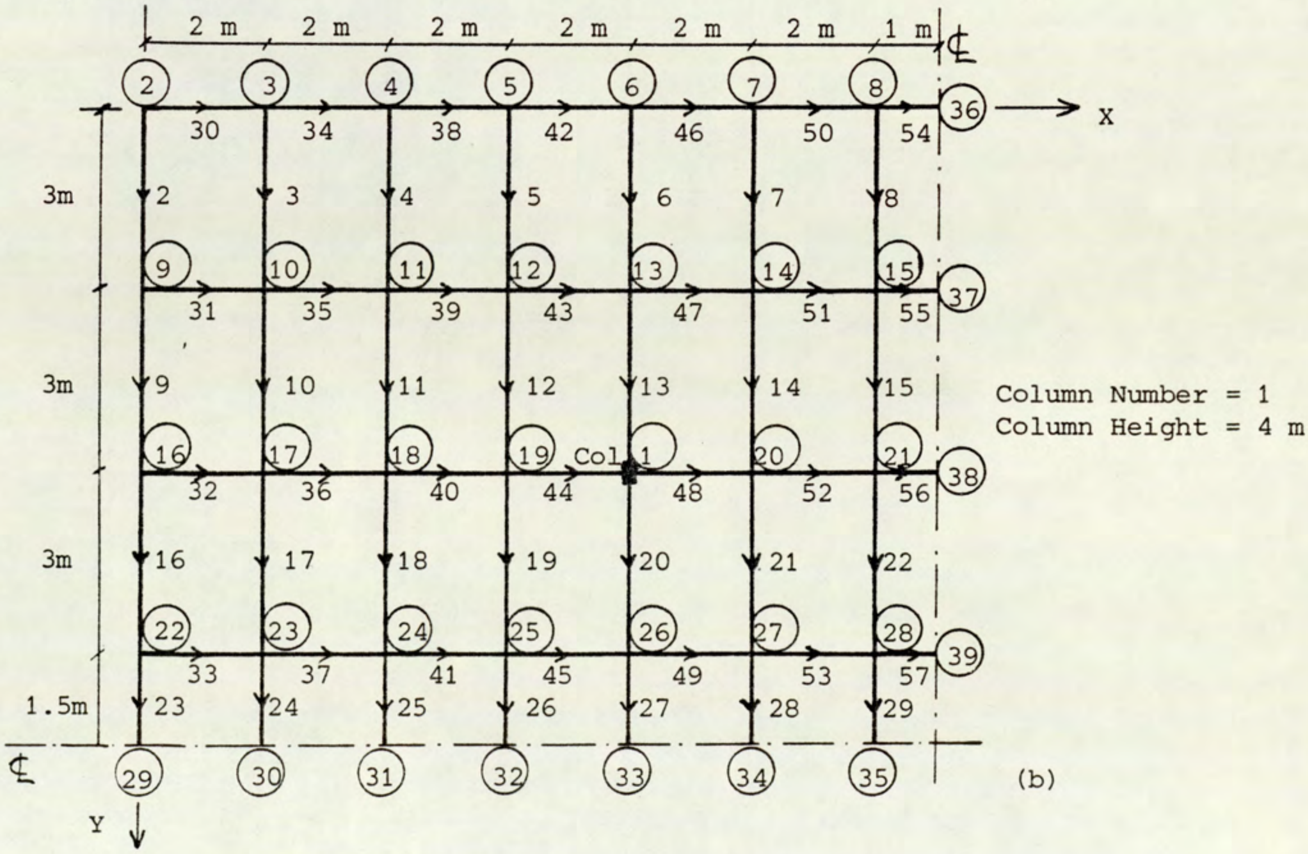
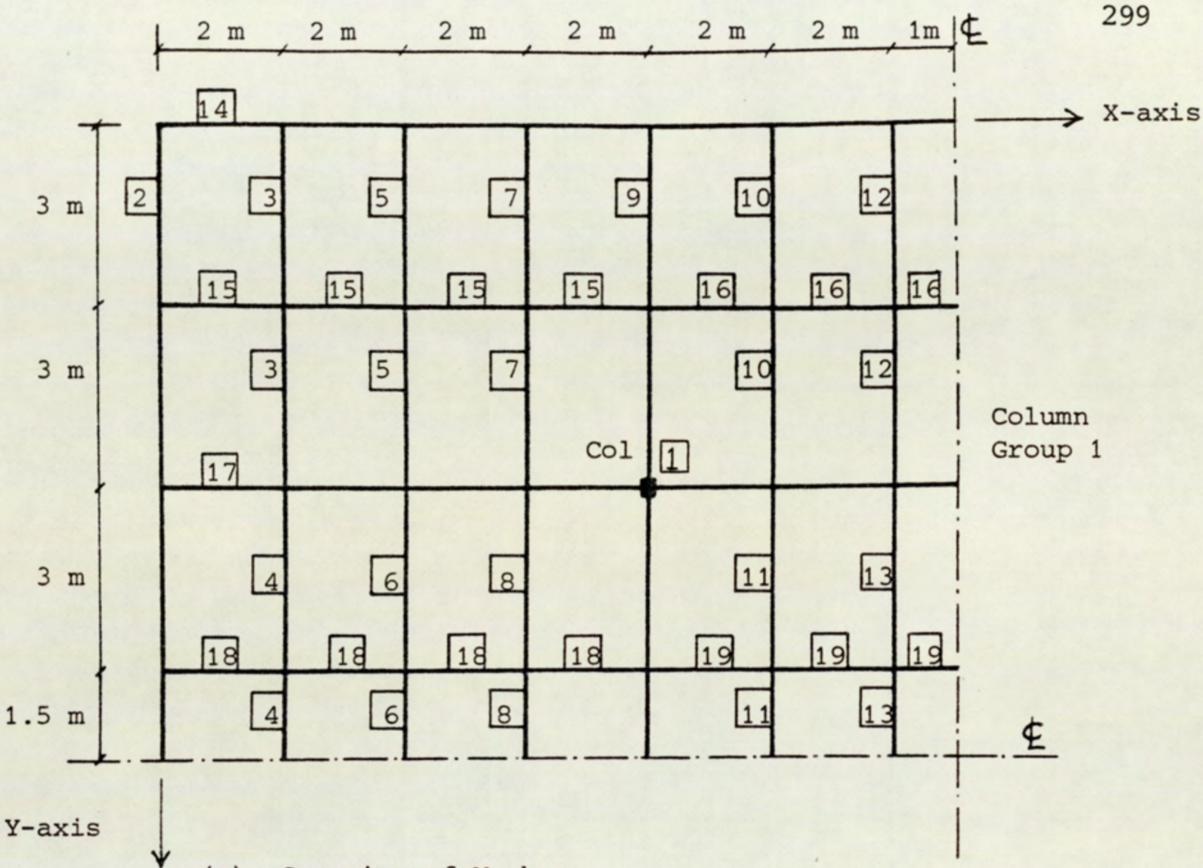
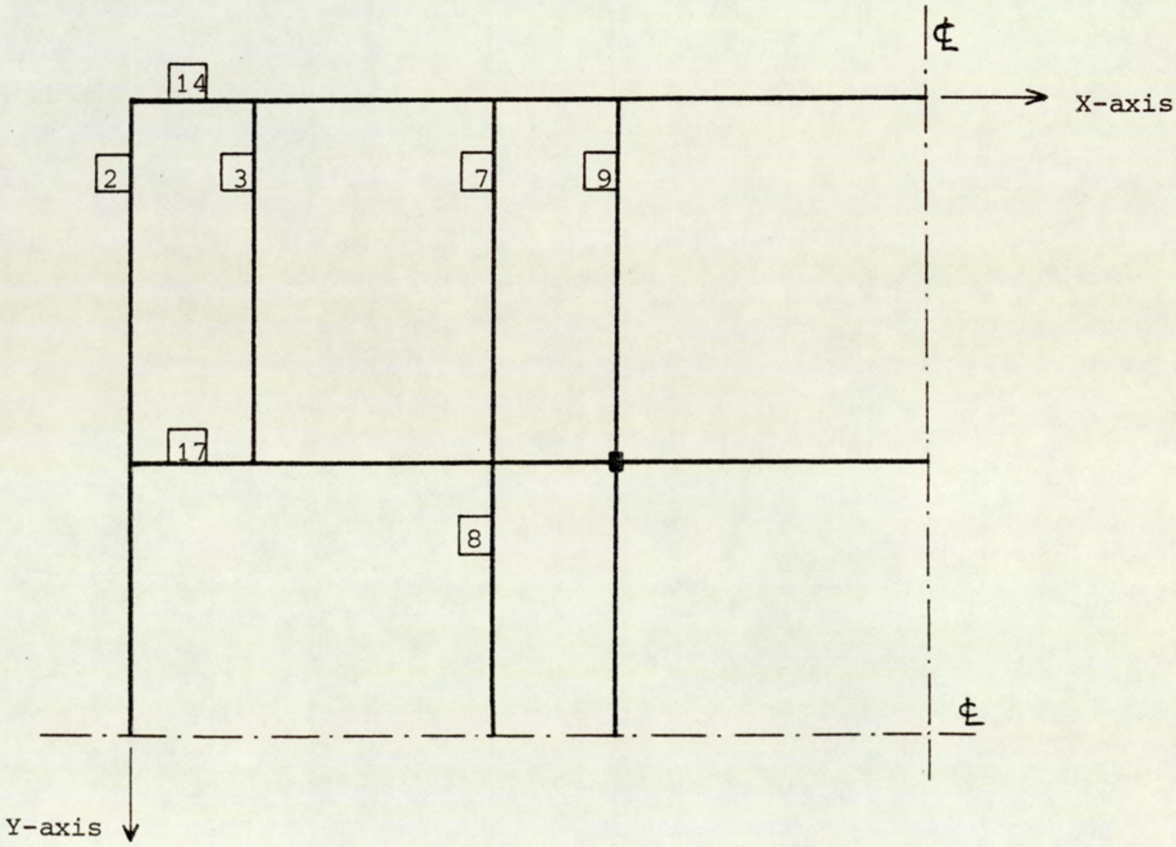


FIGURE 7.16: A SYMMETRICAL RECTANGULAR GRILLAGE SUPPORTED BY FOUR COLUMNS
(a) THE WHOLE STRUCTURE
(b) THE TOP-LEFT-QUARTER OF THE STRUCTURE - THE DIMENSIONS AND THE NUMBERING OF JOINTS AND MEMBERS



(a) Grouping of Members



(b) Optimum Topological Design of Minimum Cost

FIGURE 7.17: THE SYMMETRICAL QUARTER OF THE RECTANGULAR GRILLAGE

that pass through the column's top, i.e. groups 9 and 17. All the other groups were allowed to be removed.

The design problem for a minimum cost has a total of 171 variables, one of which is h_c of the column and another 36 variables are t and w of the grillage groups. There are also 106 joint displacement variables and 28 variables representing δ_t and δ_w of the removable groups. The problem also has a total number of 336 constraints, 106 of which are stiffnesses, 106 deflections, 36 stresses, 9 lower bounds and another 9 upper bounds for the sectional dimensions of the retained groups. There are also 14 constraints representing type (6.67), 28 constraints representing type (6.66) and another 28 representing type (6.65).

The limit on the joint deflection was taken as $L_j/400$, where L_j is once again defined as the shortest distance of joint j from the column. Such limit was specified for all the joints of the structure except the joint on the top of the column where a relaxed limit was given. The exact deflection limit, and the acting live load and moments, which were specified for all the joints, are given in Table 7.10. Notice that there are no loads acting on the joints of the central lines.

Similar to the previous problem, the present design problem also includes the self-weight of grillage as a variable. The initial set of sections was chosen arbitrarily, as shown in Table 7.11. The thickness of the column section is equal to the thickness of grillage group 17. These sections made the weight of the symmetrical quarter of the grillage equal to 1184 KN and the cost equal to £10,800. As explained in Section 7.2, the first stage of the design process is to minimise the total weight without topological alterations. This in fact gives a better relationship between the members before anyone of them can be

Joint Number (j)	Specified Deflection Limit, $L_j/400$ (mm)	Imposed Live Load and Moments Caused by a Uniform Load of ω = 0.005 KN/mm		
		P_L (KN)	$M_{F.E.X.}$ (KN.mm)	$M_{F.E.Y.}$ (KN.mm)
1	1.0 *	15.00	0.0	0.0
2	25.0 *	3.75	1875.0	-835.0
3	21.2	7.50	3750.0	0.0
4	18.0	7.50	3750.0	0.0
5	15.8	7.50	3750.0	0.0
6	15.0	7.50	3750.0	0.0
7	15.8	7.50	3750.0	0.0
8	18.0	7.50	3750.0	0.0
9	21.4	7.50	0.0	-1670.0
10	16.8	15.00	0.0	0.0
11	12.5	15.00	0.0	0.0
12	9.0	15.00	0.0	0.0
13	7.5	15.00	0.0	0.0
14	9.0	15.00	0.0	0.0
15	12.5	15.00	0.0	0.0
16	20.0	7.50	0.0	-1670.0
17	15.0	15.00	0.0	0.0
18	10.0	15.00	0.0	0.0
19	5.0	15.00	0.0	0.0
20	5.0	15.00	0.0	0.0
21	10.0	15.00	0.0	0.0
22	21.4	7.50	0.0	-1670.0
23	16.8	15.00	0.0	0.0

* Relaxed Deflection Limit is Specified.

TABLE 7.10: THE SYMMETRICAL QUARTER OF THE RECTANGULAR GRILLAGE - DEFLECTION LIMIT AND IMPOSED LIVE LOADS SPECIFIED FOR THE JOINTS

(Continued)

Joint Number (j)	Specified Deflection Limit, $L_j/400$ (mm)	Imposed Live Load and Moments Caused by a Uniform Load of ω = 0.005 KN/mm		
		P_L (KN)	$M_{F.E.X.}$ (KN.mm)	$M_{F.E.Y.}$ (KN.mm)
24	12.5	15.00	0.0	0.0
25	9.0	15.00	0.0	0.0
26	7.5	15.00	0.0	0.0
27	9.0	15.00	0.0	0.0
28	12.5	15.00	0.0	0.0
29	23.0	0.0	0.0	0.0
30	18.7	0.0	0.0	0.0
31	15.0	0.0	0.0	0.0
32	12.3	0.0	0.0	0.0
33	11.3	0.0	0.0	0.0
34	12.3	0.0	0.0	0.0
35	15.0	0.0	0.0	0.0
36	19.5	0.0	0.0	0.0
37	14.6	0.0	0.0	0.0
38	12.5	0.0	0.0	0.0
39	14.6	0.0	0.0	0.0

TABLE 7.10: (CONTINUED)

removed. In the second stage, the total cost, which represents the material and the construction costs, is minimised. Topological alterations are allowed in this stage.

The objective function at Stage 1 represents the total weight only. At each design iteration, the computer program calculates the total cost of the structure. The variations of the weight and the cost for Stage 1 are shown in Figure 7.18. The initial move limit was taken as ML = 0.5 and the optimum weight design was obtained after five design iterations. Each of these iterations required an average number of about 350 simplex iterations. The sections obtained at the optimum weight design are shown in Table 7.11. These sections made the weight equal to 866 KN and the cost equal to £7,930. At this design, the bending moment stress in groups 9 and 17 has reached to the permissible value, while the deflection at joints 2, 3, 9 and 10 has reached the limit. This gives the conclusion that a combined effect of deflection and stress constraints decided the optimum weight sections for Stage 1 of the design.

The design process continued into the second stage where the total cost, represented by the objective function, is minimised. The set of sections obtained at the optimum weight design in Stage 1 was used as the initial set in this stage. Once again the move limit was started with ML = 0.5. As shown in Figure 7.18, the process at Stage 2 required six iterations to reach the optimum cost design. At the first design iteration, which is numbered as 6 in Figure 7.18, groups 4, 5, 10, 11, 12 and 13 were removed. At the end of this iteration, the cost was reduced by about 9% while the weight decreased by about 2% only. At the second iteration, number 7 in the Figure, groups 15 and 16 were removed, while at the third iteration, number 8 in the Figure, groups 6, 18 and 19 were removed. The groups removed when their $\delta_t = \delta_w = 0$

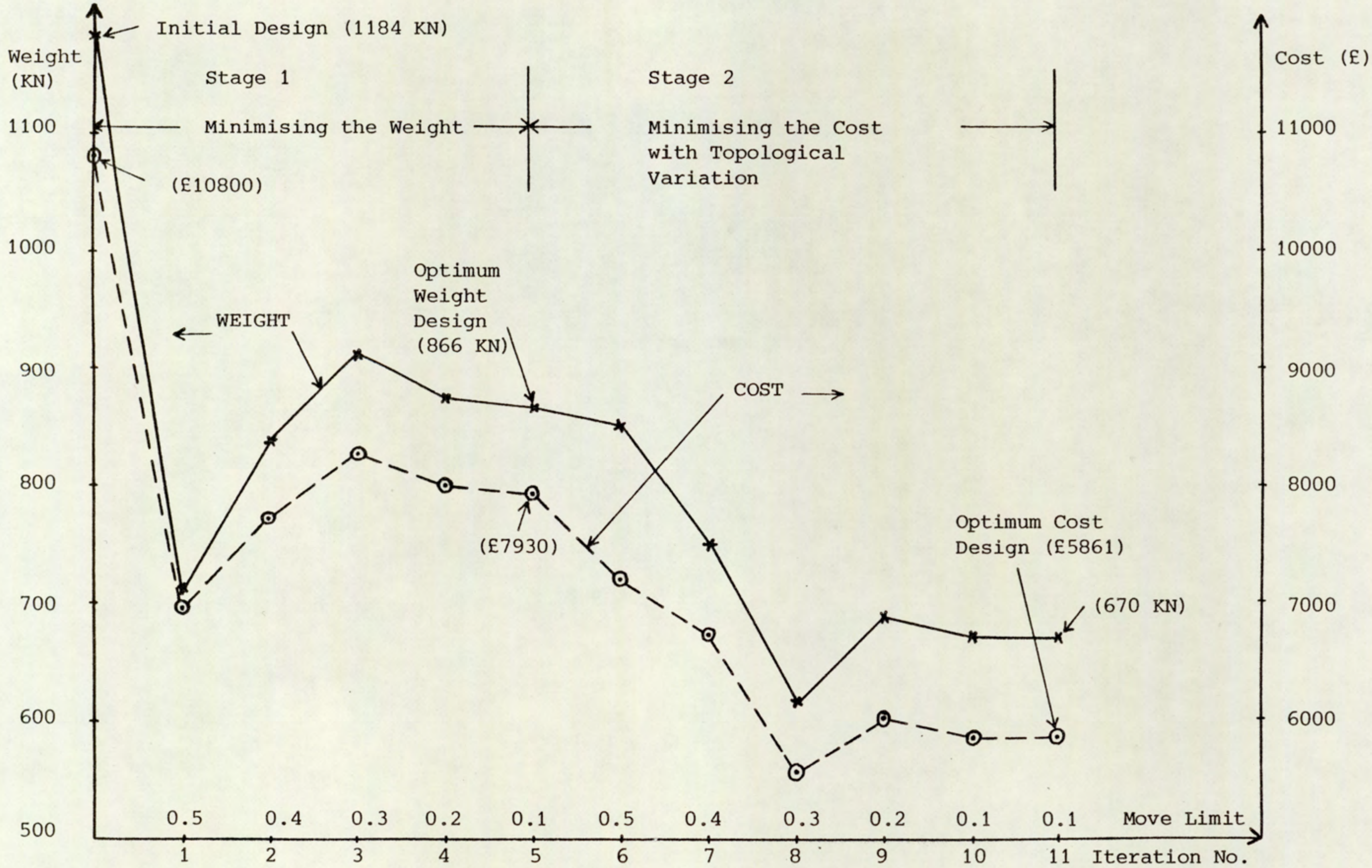


FIGURE 7.18: THE SYMMETRICAL QUARTER OF THE RECTANGULAR GRILLAGE - WEIGHT AND COST VARIATIONS

Grillage Group Number	Initial Design		Optimum Weight Design		Optimum Cost Design		
	t (mm)	w (mm)	t (mm)	w (mm)	t (mm)	w (mm)	$\delta_t = \delta_w$
2	400	900	200 *	200 *	200 *	500	-
3	400	900	200 *	509	200 *	839	0.87
4	400	900	200 *	200 *	0	0	0
5	400	900	200 *	227	0	0	0
6	400	900	200 *	334	0	0	0
7	400	900	265	852	254	600	0.59
8	400	900	200 *	815	334	1000 **	1
9	400	900	800 **	1000 **	800 **	1000 **	-
10	400	900	200 *	244	0	0	0
11	400	900	200 *	531	0	0	0
12	400	900	200 *	200 *	0	0	0
13	400	900	200 *	531	0	0	0
14	400	900	200 *	1000 **	275	1000 **	-
15	400	900	200 *	1000 **	0	0	0
16	400	900	257	1000 **	0	0	0
17	500	900	666	1000 **	618	1000 **	-
18	400	900	200 *	1000 **	0	0	0
19	400	900	525	1000 **			
Col. Group	t (mm)	h_c (mm)	t (mm)	h_c (mm)	t (mm)	h_c (mm)	
1	500 (Gr. 17)	1000	800** (Gr.9)	1000**	800** (Gr.9)	900	-
Total Weight	1,184 KN		866 KN		670 KN		
Total Cost	£10,800		£7,930		£5,861		

* Indicates that the sectional dimension is at the lower bound.

** Indicates that the sectional dimension is at the upper bound.

TABLE 7.11: THE SYMMETRICAL QUARTER OF THE RECTANGULAR GRILLAGE - SECTIONAL DIMENSIONS AT THE INITIAL AND THE OPTIMUM WEIGHT AND COST DESIGNS

or $\delta_t = \delta_w \cong 0$, i.e. $0 \leqq \delta < 0.5$. After the third iteration in Stage 2, no groups were allowed to be removed so that the feasibility of the solution could be maintained. The only groups left in the final design are numbers 3, 7 and 8 in addition to the originally specified groups 2, 9, 14 and 17. The groups of members retained in the optimum topological design of minimum cost, i.e. the final shape, are shown in Figure 7.17b. The sectional dimensions and the δ variables obtained for these groups are given in Table 7.11. The total weight at this design is 670 KN which is about 23% less than the optimum weight design of Stage 1, while the total cost is £5861 which is about 26% less than the cost of the optimum weight design.

The areas of stirrups and tensile steel obtained for the grillage members at the optimum cost design are given in Table 7.12. The material and the construction costs for these members are also given in the Table. All the members required the minimum area of stirrups. Some members also required the minimum area of tensile steel to be specified at one or both of their ends. Each value of minimum area of steel is marked by an asterisk in Table 7.12. Throughout the whole design process, the columns were assumed to have a fixed value of compressive steel area, which is 1%, and a minimum area of stirrups, which is 0.15%.

To verify the optimum cost design, Figure 7.19 shows the deflection profiles for the grillage members which were retained in the final shape shown in Figure 7.17b. The specified deflection limits are given in brackets underneath the exact deflection values. It can be noticed that many joint deflections have reached to their limits. However, due to non-linearity of the design problem, the analysis results show that some of these deflections have slightly exceeded the limit.

Group Number	Member Number	Area of Stirrup Reinf. A_{SL} (mm^2)	Areas of Tensile Steel Mem.End 1 A_{s1} (mm^2)	Areas of Tensile Steel Mem.End 2 A_{s2} (mm^2)	Material Cost (£)	Construction Cost (£)	Total Cost (£)
2	2	900 *	135 *	135 *	19.48	67.84	87.32
	9	900 *	135 *	356	19.48	69.27	88.75
	16	900 *	650	148	19.48	71.24	90.72
	23	450 *	178	178	9.74	34.20	43.94
3	3	900 *	226 *	226 *	32.68	103.04	135.72
	10	900 *	226 *	873	32.68	107.19	139.87
7	5	1143 *	205 *	205 *	29.66	87.99	117.65
	12	1143 *	205 *	1156	29.66	94.10	123.76
8	19	1503 *	858	450 *	64.98	161.14	226.12
	26	751 *	450 *	450 *	32.49	79.26	111.75
9	6	3600 *	1080 *	2598	155.82	301.66	457.48
	13	3600 *	2598	6884	155.82	338.96	494.78
	20	3600 *	1700	1080 *	155.82	295.89	451.71
	27	1800 *	1080 *	1080 *	77.91	145.95	223.86
14	30	825 *	371 *	371 *	35.70	94.49	130.19
	34	825 *	371 *	577	35.70	95.37	131.07
	38	825 *	577	1218	35.70	99.00	134.70
	42	825 *	1242	2085	35.70	105.56	141.26
	46	825 *	1375	838	35.70	100.79	136.49
	50	825 *	838	554	35.70	97.27	132.97
	54	412 *	547	547	17.85	48.00	65.85
17	32	1853 *	834 *	834 *	80.20	159.83	240.03
	36	1853 *	834 *	2291	80.20	166.07	246.27
	40	1853 *	2291	4607	80.20	182.23	262.43
	44	1853 *	4730	8669	80.20	210.09	290.29
	48	1853 *	2145	914	80.20	165.79	245.99
	52	1853 *	914	834 *	80.20	160.17	240.37
	56	926 *	834 *	834 *	40.10	79.91	120.01

* Means Minimum Reinforcement is Specified.

TABLE 7.12: THE SYMMETRICAL QUARTER OF THE RECTANGULAR GRILLAGE - STEEL REINFORCEMENTS AND COSTS OF MEMBERS OBTAINED AT THE OPTIMUM COST DESIGN

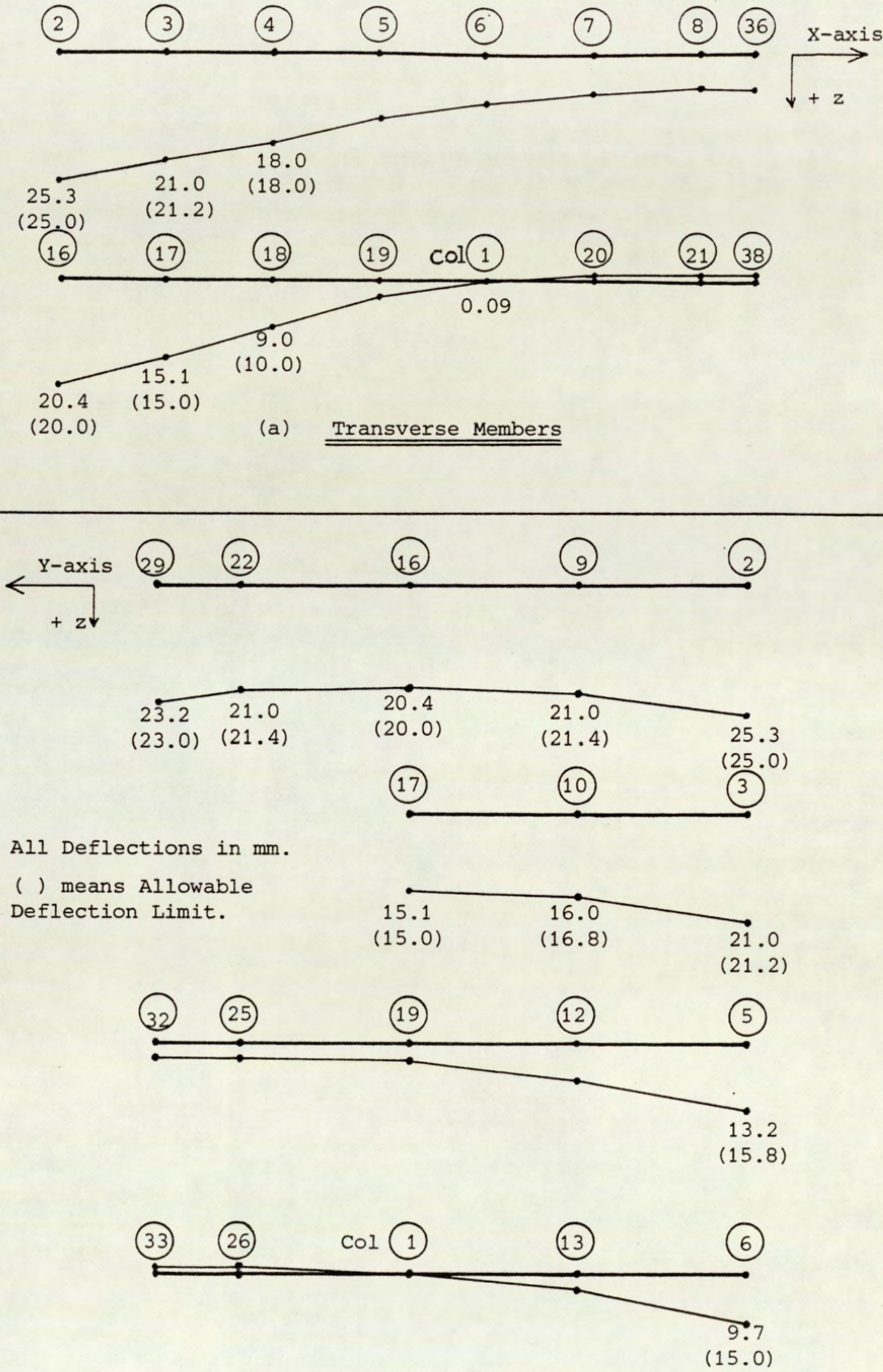


FIGURE 7.19: THE SYMMETRICAL QUARTER OF THE RECTANGULAR GRILLAGE - DEFLECTION PROFILES OBTAINED AT THE OPTIMUM COST DESIGN

Another verification of the optimum cost design is to check the stresses in the retained members. Table 7.13 shows the members with maximum stresses which were used for constructing the stress constraints for their groups at the final design iteration. The Table shows that the stresses in all the members are satisfied. However, it is noticed that the stresses in some members have reached the pemissible values. Each of these stresses is marked by an asterisk in Table 7.13.

Group Number	Maximum B.M. Stress		Maximum Combined Shear and Torsion Stress	
	Member No.	Stress Value (N/mm^2)	Member No.	Stress Value (N/mm^2)
2	16 - End 1	7.3	16	3.6 *
3	10 - End 2	5.8	10	0.1
7	12 - End 2	8.5	12	0.7
8	19 - End 1	2.9	19	0.9
9	13 - End 2	9.6	6	1.3
14	42 - End 2	8.5	46	2.3
17	44 - End 2	10.0 *	36	1.0

* Means the stress is at the permissible value.

TABLE 7.13: THE SYMMETRICAL QUARTER OF THE RECTANGULAR GRILLAGE - MEMBERS WITH MAXIMUM STRESSES USED FOR CONSTRUCTING THE STRESS CONSTRAINTS FOR THEIR GROUPS AT THE FINAL DESIGN ITERATION

The main conclusion that can be obtained from Figure 7.19 and Table 7.13 is that the deflection and the stress constraints are both involved in deciding the optimum sections of the final grillage shape shown in Figure 7.17b. However, in this structure it seems that the deflection requirements dominated the optimum design more than the stress requirements. This is true as many joint deflections have almost reached their limiting values, while only two member stresses have nearly reached the permissible values. Another point to be noticed here, is that the logic of including a design constraint that represents the torsional stress combined with the lateral shear stress is demonstrated in member 16 of group 2. The torsional stress in this member is 3.23 N/mm^2, while the lateral shear stress is only 0.37 N/mm^2.

CHAPTER 8

THE COMPUTER PROGRAMS

8.1 INTRODUCTION

The previous chapters contain all the essential features for the design problems of a sway frame, a complete structure and a horizontal grillage structure. The computer programs which were written for the automatic formulation of the design problems for these types of structure will be described in this Chapter. The three master programs, which make use of the design procedures, were written to utilise the computer backing store so that large structures can be designed within a small computer core. Each program constructs the problem one-row at a time which is then transferred to the backing store to make a space for the construction of the next row. However, as similar techniques were used in a number of instances, a full description of each program would involve undue repetition. Therefore, after describing a subroutine dealing with the general method of constructing the stiffness constraints, the description of each master program will be limited to brief discussions of the main features within the program. A fourth program was written for the purpose of solving the design problems by the simplex method. This program can be used independently or can be included as a subroutine in the master program. The backing store of the computer was also used in this program. Furthermore, a subroutine that solves the overall stiffness equations for analytical purposes is used in this thesis. This subroutine was developed by previous researchers, and thus only a brief description of it will be given.

All the programs were written in FORTRAN IV and run on the ICL 1904S computer at the University of Aston, and also on the CDC 7600 computer at the University of Manchester Regional Computer Centre. A set of subroutines, that may be incorporated in the FORTRAN of these computers, enable files on discs to be used for backing storage.

8.2 THE DISPLAY OF A TYPICAL DESIGN PROBLEM

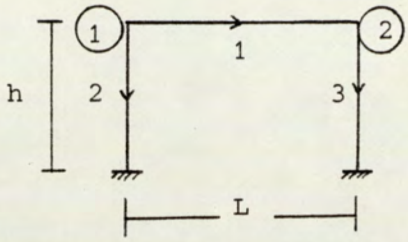
In Figure 8.1, a layout of the design problem, for minimum weight, is given for a portal frame having two joints, and three members of different grouping. Matrix $\underline{D}$ shown in the Figure, includes the coefficients of the objective function and all the constraints. The dimension of this matrix is [KB+1,NV], where KB+1 is the total number of the constraints and the objective function, and NV is the total number of the design variables which can be in the form:

$$NV = NOG + 3*NOJ \tag{8.1}$$

where NOG is the total number of groups, and NOJ is the number of joints.

In this Figure, the objective function has constant coefficients, see equation (2.14), where $G_g = L_g \gamma$ for g = 1, ... NOG. The factors $(r, s, t, u, v, w)_i$ for i = 1, ... NV, are the stiffness coefficients and they are either constants or zeros, depending upon the linearisation process. The constant 1, in matrix $\underline{D}$, represents the coefficient of any one of the variables listed at the top of the Figure, and it relates each variable to its lower or upper bound. The blank spaces in matrix $\underline{D}$ are all zeros.

The one dimensional matrix $\underline{B}$, in Figure 8.1, contains constant values, such as zero as the initial value of the objective function and the Right-Hand-Side (RHS) values of an NR number of linearised stiffness constraints. Matrix $\underline{B}$ also contains the upper bound values on the displacements of the two joints such as UB_1 and UB_2, and the



NV

C	A_1	A_2	A_3	x_1	y_1	θ_1	x_2	y_2	θ_2

	G_1	G_2	G_3	0	0	0	0	0	0	=	0	Objective Row		
1	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	=	RHS_1			+1
2	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	=	RHS_2			+2
3	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	=	RHS_3	NR		+3
4	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	=	RHS_4			+4
5	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	=	RHS_5			+5
6	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	=	RHS_6			+6
7				1						≦	UBx_1			0
8					1					≦	UBy_1	UB_1		0
9						1				≦	$UB\theta_1$		KB	0
10							1			≦	UBx_2			0
11								1		≦	UBy_2	UB_2		0
12									1	≦	$UB\theta_2$			0
13	1									≧	LBA_1			-13
14		1								≧	LBA_2	LBA		-14
15			1							≧	LBA_3			-15
16	1									≦	UBA_1			0
17		1								≦	UBA_2	UBA		0
18			1							≦	UBA_3			0
					D						B			RA

The number of the constraints

FIGURE 8.1: A LAYOUT OF A TYPICAL DESIGN PROBLEM

lower and upper bounds on the areas of the members, such as LBA and UBA. The dimension of $\underline{B}$ is KB+1.

The simplex method is used to solve the problem by first adding artificial, surplus and slack variables to the rows of matrix $\underline{D}$. All the unknowns of vector $\underline{C}$ are then calculated to give a design which is one step nearer to the optimum solution, see equation (1.3).

To identify the rows with artificial variables from those with slack variables, a new one-dimensional array was employed. This is shown as $\underline{RA}$, in Figure 8.1, and it contains constants that specify the type of each constraint. A zero indicates that the constraint has a slack variable. An equality constraint is identified by a positive number in $\underline{RA}$ and requires an artificial variable. Finally a negative number in $\underline{RA}$ indicates that the constraint requires a surplus as well as an artificial variable. The construction of array $\underline{RA}$ is done automatically, and its constant numbers correspond to the number of the rows. The dimension of $\underline{RA}$ is equal to KB.

8.3 THE CONSTRUCTION OF LINEAR STIFFNESS CONSTRAINTS

As shown in the previous chapters, the matrix displacement method was used in formulating the design problems for the three types of structure. Using this method, it was found that each structural member, whether it was a prismatic or a deep beam, required four submatrices, $\underline{K}_{ii}$, $\underline{K}_{ij}$, $\underline{K}_{ji}$ and $\underline{K}_{jj}$, to represent its contribution to the overall stiffness matrix of the structure. Furthermore, there were often three displacement variables (x, y and θ, or z, θ_x and θ_y) at each joint in a structure. Such common factors between the structures examined in the thesis made it possible to write a general subroutine that can be applied easily to construct the stiffness constraints for any of these structures. The subroutine which is called STFDRVRHS can also be applied to any other similar structure.

8.3.1 Sub-Routine STFDRVRHS

The construction of the stiffness constraints is done in a continuous joint-by-joint sequence. The major program for each type of structure uses two major nested loops for such construction. The outer loop takes each joint of the structure in turn. The inner loop cycles through the rows of the stiffness coefficients corresponding to the displacements at this joint. For each row, subroutine STFDRVRHS is called to construct a linear form of the constraint at this row. This subroutine consists of three parts. The first formulates the stiffness coefficients in the row. The second part computes the gradient vector and the third linearises the constraint. The flow diagrams for these parts will be discussed below, after considering one row, known as I, that corresponds to a displacement at a joint, called J.

8.3.1.1 Part 1 - Formulating the Stiffness Coefficients

The first part of subroutine STFDRVRHS consists of two nested loops, as shown in Figure 8.2. In the first loop the total number of different member groups at joint J are cycled. Such number is stored in an array called NG, when dealing with a frame, and in arrays called $\underline{NG1}$ and $\underline{NG2}$, when dealing with a complete structure or a grillage. These arrays are constructed in the master program of each type of structure, as will be shown later. In the second loop each of the members connected to that joint are taken and checked as to whether the group number of the member coincides with the group number of the first loop. If so, the contribution of it made to the submatrices $\underline{K}_{ii}$, $\underline{K}_{ij}$, $\underline{K}_{ji}$ and $\underline{K}_{jj}$ is computed.

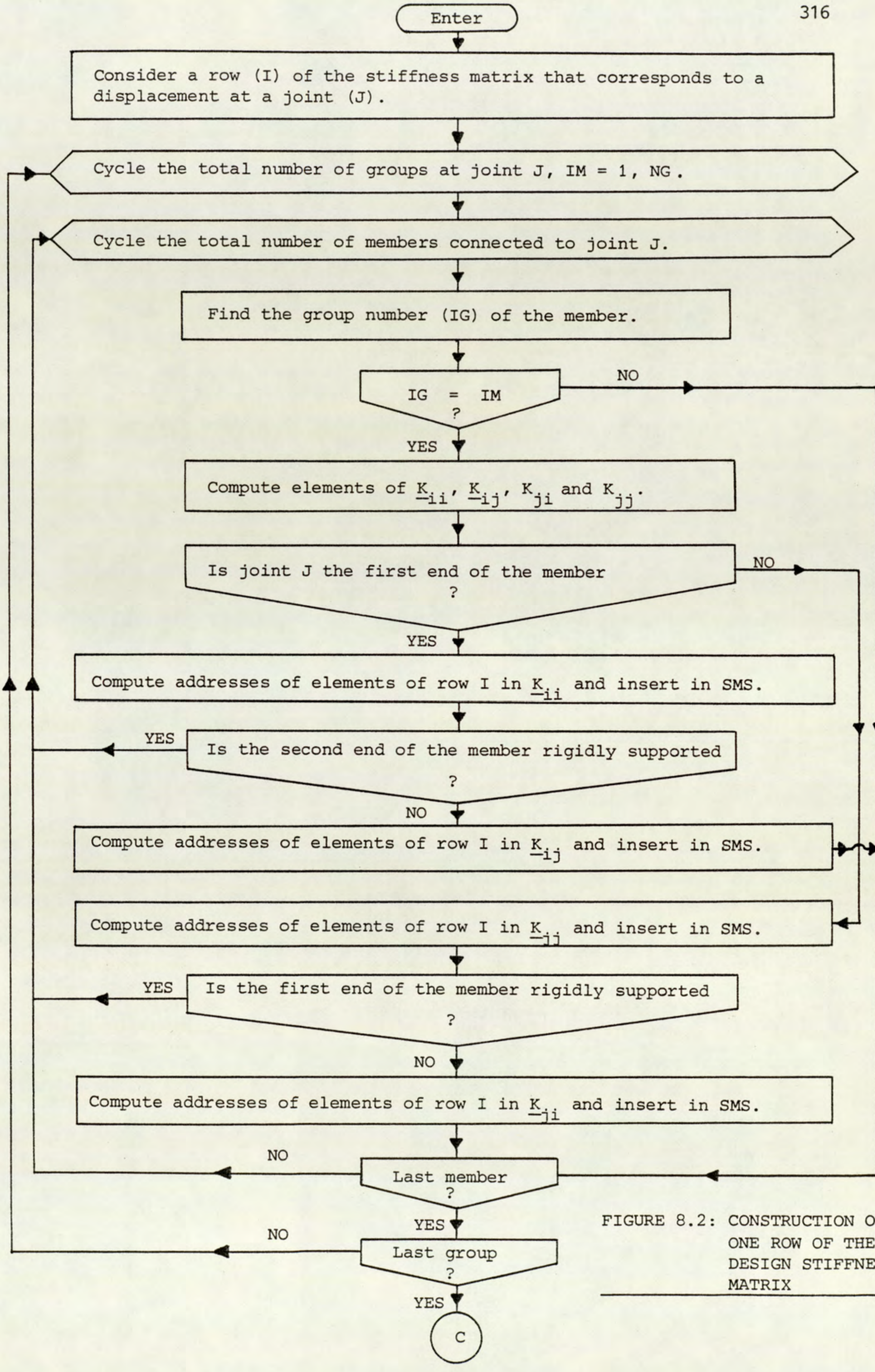


FIGURE 8.2: CONSTRUCTION OF ONE ROW OF THE DESIGN STIFFNESS MATRIX

The subroutine then proceeds to determine whether the joint taken in the first loop is the first end of the member. If so, array NI, in the case of a frame, or arrays NI1 and NI2, in the case of a complete structure or a grillage, are used to determine the addresses of the coefficients of row I in the submatrix $\underline{K}_{ii}$. These coefficients are then inserted, according to the addresses, in a one-dimensional array known as SMS, which represents row I of the design stiffness matrix. The addresses of the submatrix $\underline{K}_{ij}$ are found by using the array NIG. This array contains the number of different groups at each joint. The function of this array is to compute the number of groups, before the member group, at the joint corresponding to the second end of the member. This number is used to find the addresses of the coefficients of row I in submatrix $\underline{K}_{ij}$, and these coefficients are then inserted in array SMS. In the case when the second end of the member is supported by a fixed connection, the member will have no contribution from that end.

If joint J is the second end of the member, then the same procedure is applied to compute the addresses of the coefficients of row I in submatrices $\underline{K}_{jj}$ and $\underline{K}_{ji}$, and inserting these coefficients in array SMS. The size of this array is the number of columns NC which should be computed in advance by the master program. It should be noticed here that the SMS array contains only the constant coefficients, at row I, of submatrices $\underline{K}_{ii}$, $\underline{K}_{ij}$, $\underline{K}_{ji}$ and $\underline{K}_{jj}$. In other words, no unknown sectional properties are involved as they are considered as design variables. The SMS array is then used for constructing the elements of row I of the symmetrical overall stiffness matrix $\underline{K}$, which is employed in analysing a structure. Such construction does not introduce any difficulty as the only necessary computation is to multiply the coefficients of SMS array by the selected values of the sectional properties and add them together at each joint to obtain row I of matrix K. The elements of this row are stored in a one-dimensional array named as CM; the size of CM is 3*NOJ where NOJ is the total number of joints.

The SMS array is then used to calculate the gradient vector and to linearise the stiffness constraint at row I, as will be shown next.

8.3.1.2 Part 2 - Computing the Gradient Vector

The second part of subroutine STFDRVRHS deals with the derivatives of row I, of the design stiffness matrix, with respect to the design variables. These derivatives are computed and stored in a one-dimensional array called DK. The size of this array is NV, where NV = NOG+3*NOJ, as defined by equation (8.1). The first NOG elements of the DK array contain the derivatives with respect to the section variables. The rest, contain the derivatives with respect to the displacement variables. This part of the subroutine uses the array SMS. It cycles the group numbers, and at each joint determines whether there is an element corresponding to this group in row I of the design stiffness matrix. If there is, then the derivatives with respect to the design variables, are computed at this joint. The computation of the derivatives is explained by the flow diagram shown in Figure 8.3, which is the continuation of the flow diagram shown in Figure 8.2.

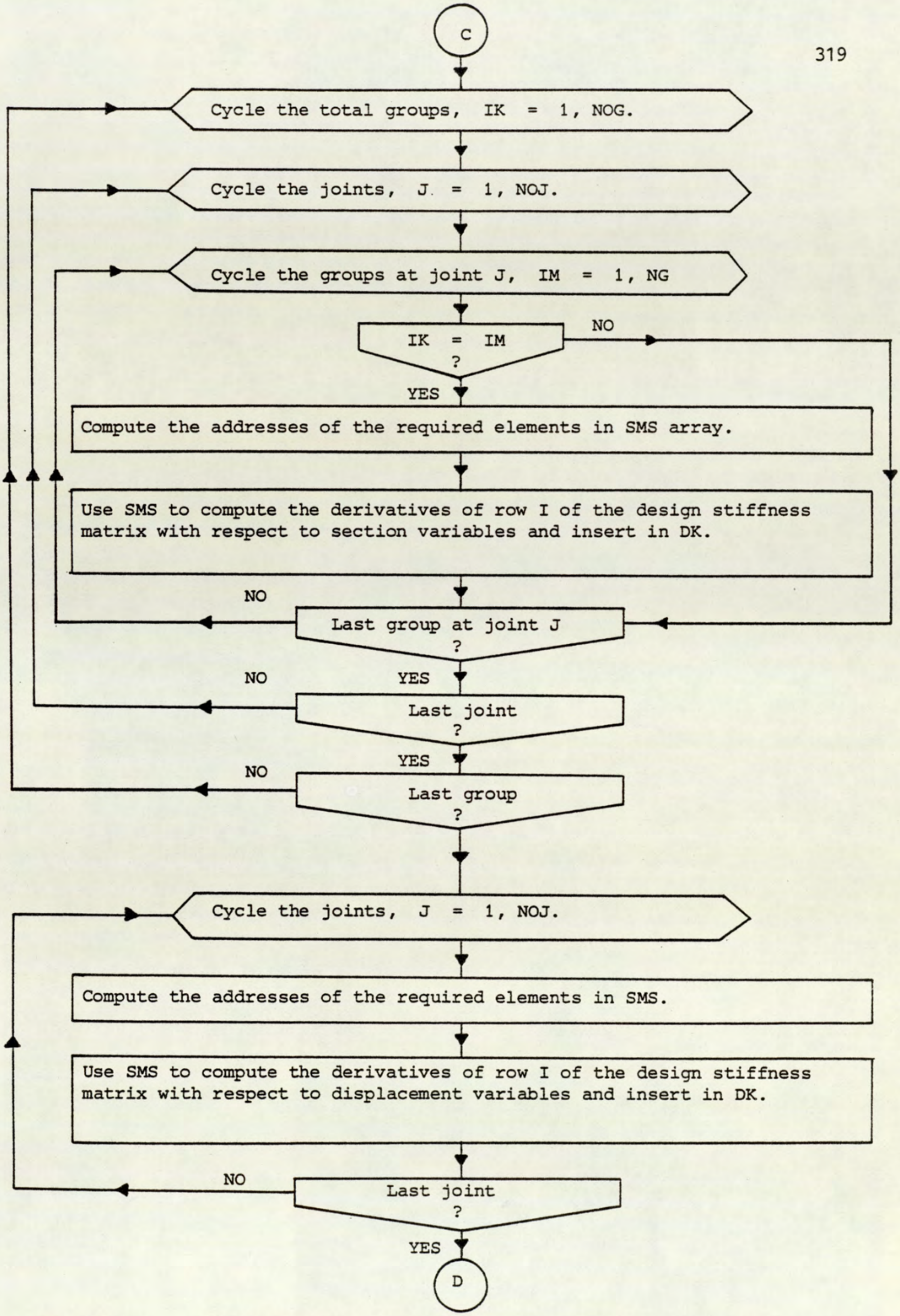


FIGURE 8.3: COMPUTATION OF THE GRADIENT VECTOR - CONTINUED FROM FIGURE 8.2

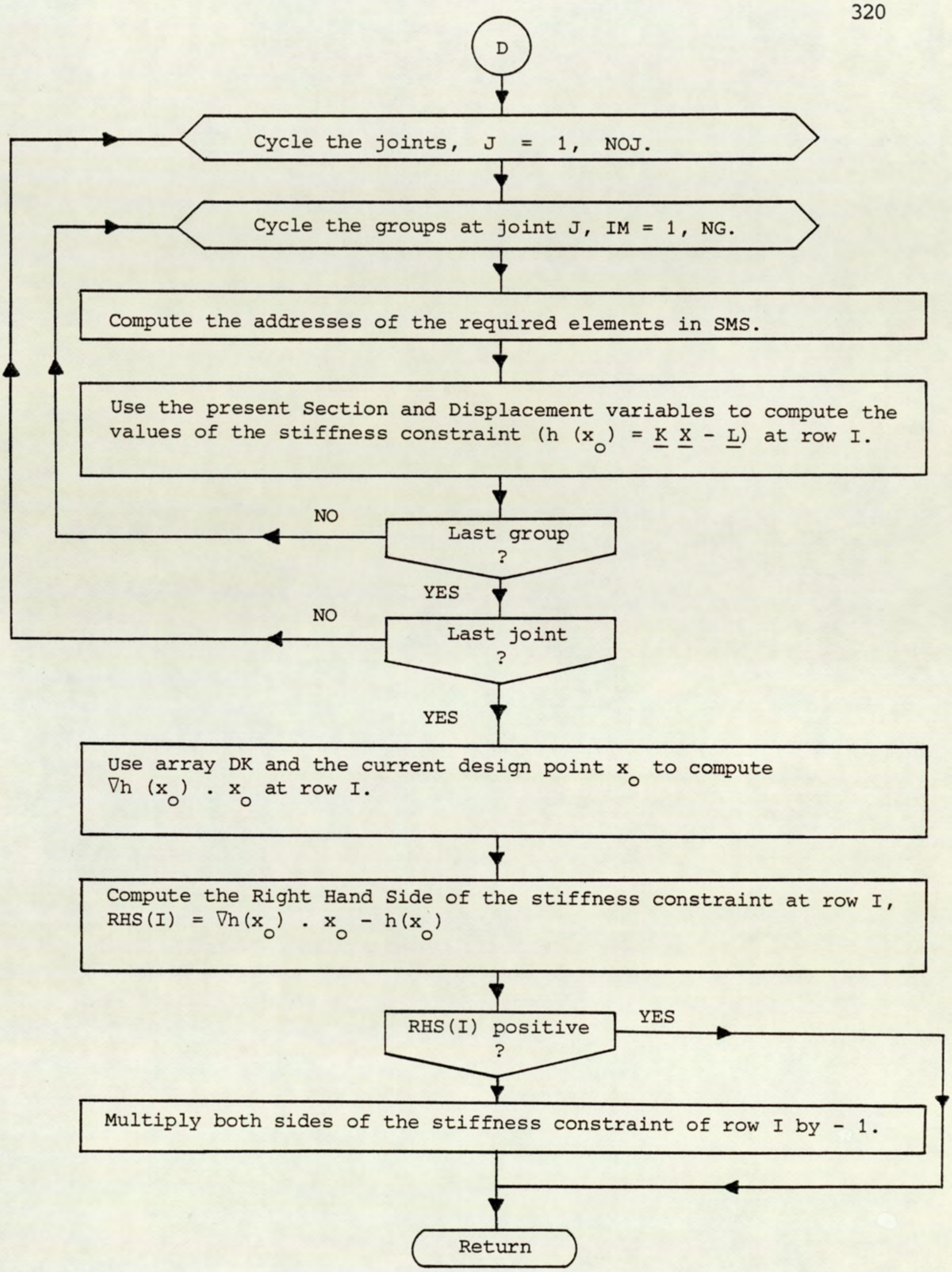


FIGURE 8.4: LINEARISATION OF ONE ROW OF THE STIFFNESS CONSTRAINTS

8.3.1.3 Part 3 - Linearising the Stiffness Constraints

The linearisation of the stiffness constraint at row I is carried out by the third part of subroutine STFDRVRHS. It can be seen from the flow diagram given in Figure 8.4 that, firstly the value of a stiffness constraint is computed at the current design point. Then using the gradient vector DK, constructed by the second part of the subroutine, the Right-Hand-Side of the constraint, RHS(I), is computed. In a linear programming problem, the Right-Hand-Side of a constraint should always be a non-negative. Hence, if RHS(I) is found to be negative, then both sides of the stiffness equality is multiplied by -1 to ensure that RHS(I) is positive.

8.4 THE FRAME PROGRAM

The computer program for an optimum design of a multi-storey, multi-bay frame requires simple data preparation. A format has been adopted which facilitates rapid transference of data from a frame diagram to the program. The data format is general and does not depend on the nature of the structure. Furthermore, with the introduction of random numbering of the joints and the members, the data format can deal with a frame of any shape and under any type of loading.

The input data for the program is generally divided into two parts. The first part is the data concerning the structural properties which include preliminary data about the frame, and detailed data about the members and the joints. The second part is the data which is relevant to the mathematical programming. This part contains the preselected area variables, the move limits, and the upper and lower bound values. It also contains integer constants which are the values of variables used by the program to define the type of the design problem. These variables are ID and ANLYS. ID=1 means a minimum weight design is required while ID=0 means a minimum cost topological design is required. On the other hand, ANLYS=0 means that the frame should be analysed with the current values of the member areas so that a new feasible starting point is established. ANLYS=1 means that an analysis is not required and the present values of areas and displacements are used to set the design problem for the next iteration.

The second part of the data also contains the information required for a topological design. This includes the total number of groups required to be retained in the final topology, the number of each of these groups and the lower bound imposed on its section area. It also includes the total number of the removable groups and the number of each one of them. A more detailed data format for the frame program will be given in Appendix C.

After entering the data, the program starts by constructing the objective function, and then begins the calculation of:

(i) The total number of members connecting to each joint;

(ii) The member, and the group, numbers of these members.

Joint number of the first and the second ends of each member are also checked at each joint, against the joint number, to determine whether that particular joint is the first end of the member. If so, the member number is multiplied by -1, so that the program will be able to identify which of the submatrices, $\underline{K}_{ii}$, $\underline{K}_{ij}$, $\underline{K}_{ji}$ and $\underline{K}_{jj}$, are to be used at that particular joint.

It is possible that some of the members connected to a joint belong to the same group. As a result their contributions to the design stiffness matrix can be added together at the rows and columns corresponding to that particular joint. It is therefore necessary to find the total number of different groups at each joint. This is carried out by checking the group numbers of members connected to that joint and storing the different ones in the array NG. The Jth element of this array gives the number of different groups at joint J. This is utilised in Sub-Section 8.3.1.1. As shown in Chapter 2, the contribution of each group is stored separately at each joint in the design stiffness matrix. The total number of "different groups at each joint" for the structure is obtained by adding the elements of array NG together, for all the joints, in array NI. The Jth element of this array gives the total number of "different groups at each joint" up to joint J.

The program then proceeds to compute the number of columns NC in the design stiffness matrix, and the total number of constraints KB. It also calculates the size NGC required for the lower half, including the diagonal elements, of the overall stiffness matrix K which is used to analyse the frame. After that it calculates the dead load at each joint and adds it to the vertical live load which is already imposed on that joint.

A flow diagram of the master program for the optimum design method of frames is given in Figure 8.5. The diagram is self-explanatory and will not be discussed in detail. However, the diagram shows that after the linear forms of the stiffness constraints are constructed and transferred in a row-by-row sequence to the backing store, the program proceeds to construct all the other constraints and transfers them in the same sequence, to the backing store. The program then uses the simplex subroutine until convergence is achieved. For each iteration the feasible values of areas for the selected groups are printed out with the displacements of the joints.

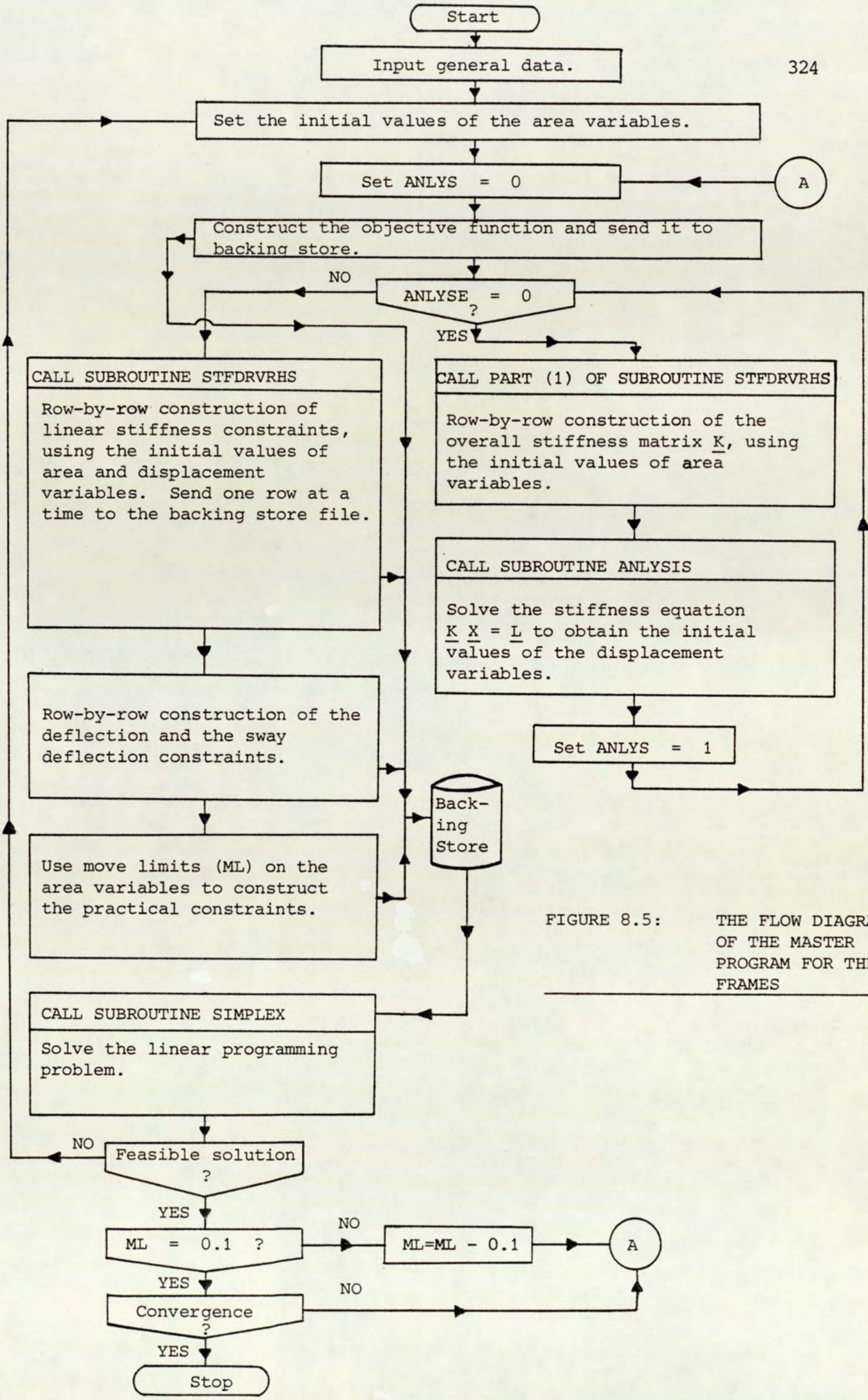


FIGURE 8.5: THE FLOW DIAGRAM OF THE MASTER PROGRAM FOR THE FRAMES

8.5 THE COMPLETE STRUCTURE PROGRAM

The main program for a minimum cost topological design of laterally loaded complete structures is similar to the frame program. The data format is flexible and can easily deal with symmetrical or non-symmetrical structures which contain arbitrary arrangements of parallel frames and walls and have fixed or different widths of slabs and walls. The format can also include steel or reinforced concrete frames with a constant or a variable number of columns in each frame. Furthermore, since the problem is for a minimum cost design, the format contains a substantial amount of data about the cost assessment of the structure. However, a detailed data format for the complete structure program will be given in Appendix D.

After entering the data, the program begins by formulating the objective function. This involves calculating the material and the construction costs for the components of the structure. The program then proceeds to compute the total number of members meeting at each joint, plus the member, and the group, numbers of these members. It also calculates the different groups that meet at each joint, and stores them in arrays NG1 and NG2. The Jth element of array NG1 represents the different grillage groups which connect to joint J, while that of NG2 represents the different column groups, of a frame, that meet at joint J. Consequently, two arrays, NI1 and NI2, are produced where the Jth element of either gives the total number of "different groups at each joint" up to joint J. These arrays are used for identifying the addresses of the coefficients which will be inserted in array SMS. The program continues to calculate the number of the design variables NV, the number of the design constraints KB and the number of columns NC in the design stiffness matrix. It also calculates the size NGC required for the lower half, including the diagonal elements, of the overall stiffness matrix K which is used in analysing the structure.

After such computations the program then starts to construct the stiffness constraints in a row-by-row sequence. This is done by calling subroutine STFDRVRHS for each row of the constraints. The construction of the other constraints, described in Chapter 4, are then followed. The flow diagram for the master program of complete structures is similar to that of frames shown in Figure 8.5. Thus, it is not necessary to plot such a diagram here.

8.6 THE HORIZONTAL GRILLAGE PROGRAM

The master program for an optimum design of reinforced concrete horizontal grillages is similar to those of the frames and the complete structures. The data format is versatile, general and able to deal with symmetrical or non-symmetrical grillages that can be supported by columns and fixed ends. A detailed data format for the grillage program will be described in Appendix E. Similar to the previous programs, the entry of data is followed by the calculation of the number of members that connect to each joint, plus the member number and the group number of each of these members. The program also calculates the different groups that meet at each joint and stores them in arrays NG1 and NG2 which are then used to produce arrays NI1 and NI2. The Jth element of array NG1 represents the number of the different groups of longitudinal members which connect to joint J, while that of NG2 represents the different groups of transverse members that meet at joint J. The program then continues in calculatig the number of the design variables, the number of the design constraints, the number of columns in the design stiffness matrix, and the size of the lower half including the diagonal elements of the overall stiffness matrix K which is required for analysing the grillage structure.

The master program then calls subroutine SELF-WEIGHT. This subroutine consists of two major loops, as shown by the flow diagram given in Figure 8.6. In the first loop, the elements of a two dimensional array, called BETA1(IK,J), are computed. Each element is a constant that represents the sum of the product (γ.L/2) of all the members that belong to group IK and meet at joint J, where, γ is the density of material and L is the length of member. The elements of array BETA1(IK,J) will later be used in the computation of the gradient vectors of the stiffness constraints. For instance, as shown by equation (7.5), the derivation of the dead load value at joint J, i.e. $P_{DJ}(t,w)$ of equation (7.2), with respect to t_{IK} can be expresed as $\partial P_{DJ}/\partial t_{IK}$=BETA1 (IK,J).$w_{IK}$. It should be mentioned here, however, that the elements of array BETA1 have values only when the dead load of the grillage is considered as a variable, otherwise these elements will be zero. In the second major loop of the subroutine, the total weight of half the grillage members that meet at each joint is computed and stored as constant values in a one-dimensional array BETA2(J). Each element in this array is equal to the dead load, i.e. P_{DJ} of equation (7.2), imposed on joint J. This value will be added to the vertical live load already imposed on joint J to form the total vertical load.

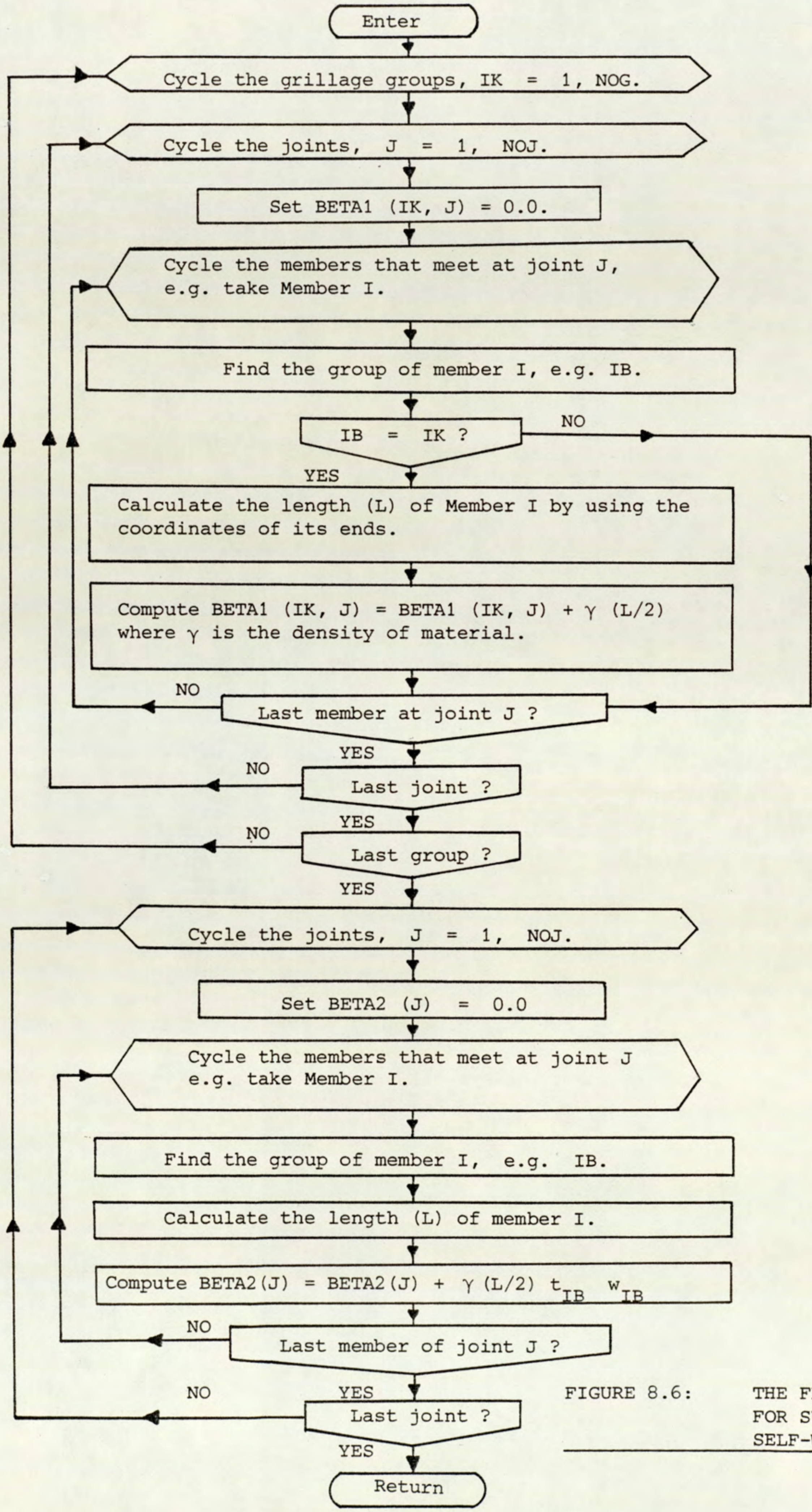


FIGURE 8.6: THE FLOW DIAGRAM FOR SUBROUTINE SELF-WEIGHT

After calling subroutine SELF-WEIGHT, the program proceeds to analyse the grillage structure and calculates the joint displacements. Following that, subroutine COST is called. The responsibility of this subroutine is to assess the total cost of each member of the structure. It starts by calculating the construction and the material cost of the supporting columns. Then it uses the joint displacements to compute the member forces and hence determines the amount of reinforcements for each member. Following that, subroutine COST continues in calculating the material and the construction costs for each member, as explained in Section 6.10. It then formulates either the objective weight on the objective cost function for the structure, depending on the type of problem.

After the formulation of the objective function, the master program then proceeds to construct the stiffness constraints, one row at a time, by using subroutine STFDRVRHS for each row. The construction of the stress, the deflection and the practical constraints, described in Chapter 6, are then followed. The stress constraints are also constructed in a sequence of one row at a time as will be shown in the next sub-section. It should be mentioned here, however, that in subroutine STFDRVRHS, array SMS contains constant values which are the derivatives of the stiffness coefficients with respect to the section thickness (t). On the other hand, the constants which represent the derivatives of the stiffness coefficients with respect to the section overall depth (w) are stored in another array named WSMS. Notice that these derivatives are given by equations (6.29) to (6.38).

The flow diagram for the master program of the horizontal grillages is almost similar to that of the frames given in Figure 8.5. The only differences, however, are the existence of a subroutine which deals with the self-weight, called before the analysis, and another subroutine for computing the cost, called after the analysis.

8.6.1 The Construction of Linear Stress Constraints for a Grillage

In the optimum design of a reinforced concrete grillage, two stress constraints are imposed on each group of members. The first one represents the combined shear and torsional stress, while the second constraint expresses the bending moment stress. Each constraint is constructed by using the joint displacements and the sectional properties of a single member in the group. This member is located and identified as to have the maximum stress among the rest of the members in the group. Sometimes, the combined shear and the bending moment stresses simultaneously reach their maximum values in the same member, in which case this member is used for constructing the two constraints.

The construction of a linear stress constraint is accomplished by using two subroutines. The first one, which is called STRESMEM, is used to locate the member which has the maximum stress in each group. This subroutine consists of two major nested loops, see the flow diagram in Figure 8.7. The outer loop takes each grillage group in turn. The inner loop cycles through all the grillage members, but considers only the members that belong to the group taken by the outer loop. The analysis results are utilised by the inner loop, where the joint displacements are used to calculate the member forces and hence the member stresses. After that, the stresses of each member are compared with those of the previous members in the group. The comparison process continues for all the members, and by the end of the inner loop the member with the maximum stress is located in the group. At this stage, the second subroutine, which actually constructs the

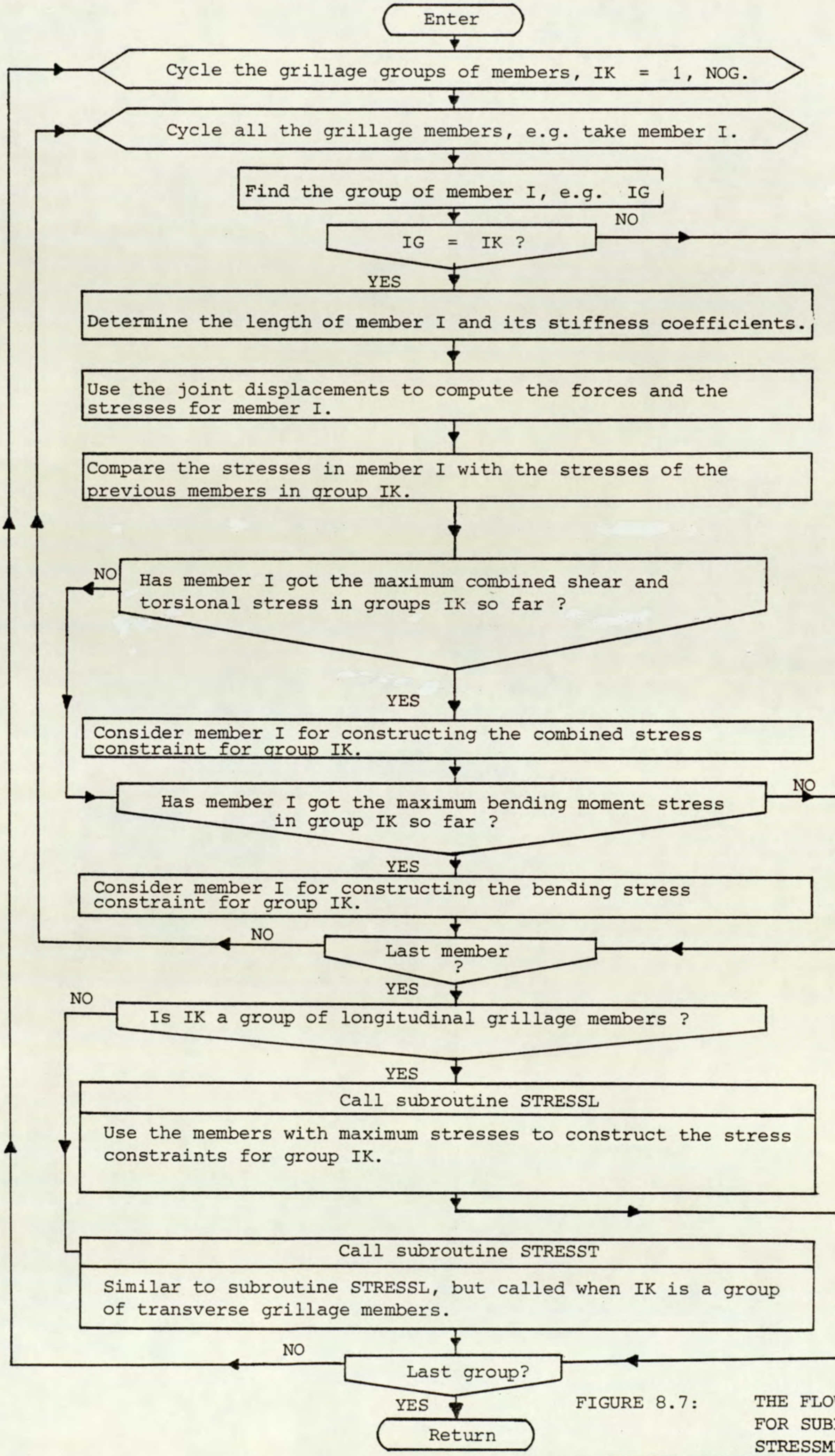


FIGURE 8.7: THE FLOW DIAGRAM FOR SUBROUTINE STRESSMEM

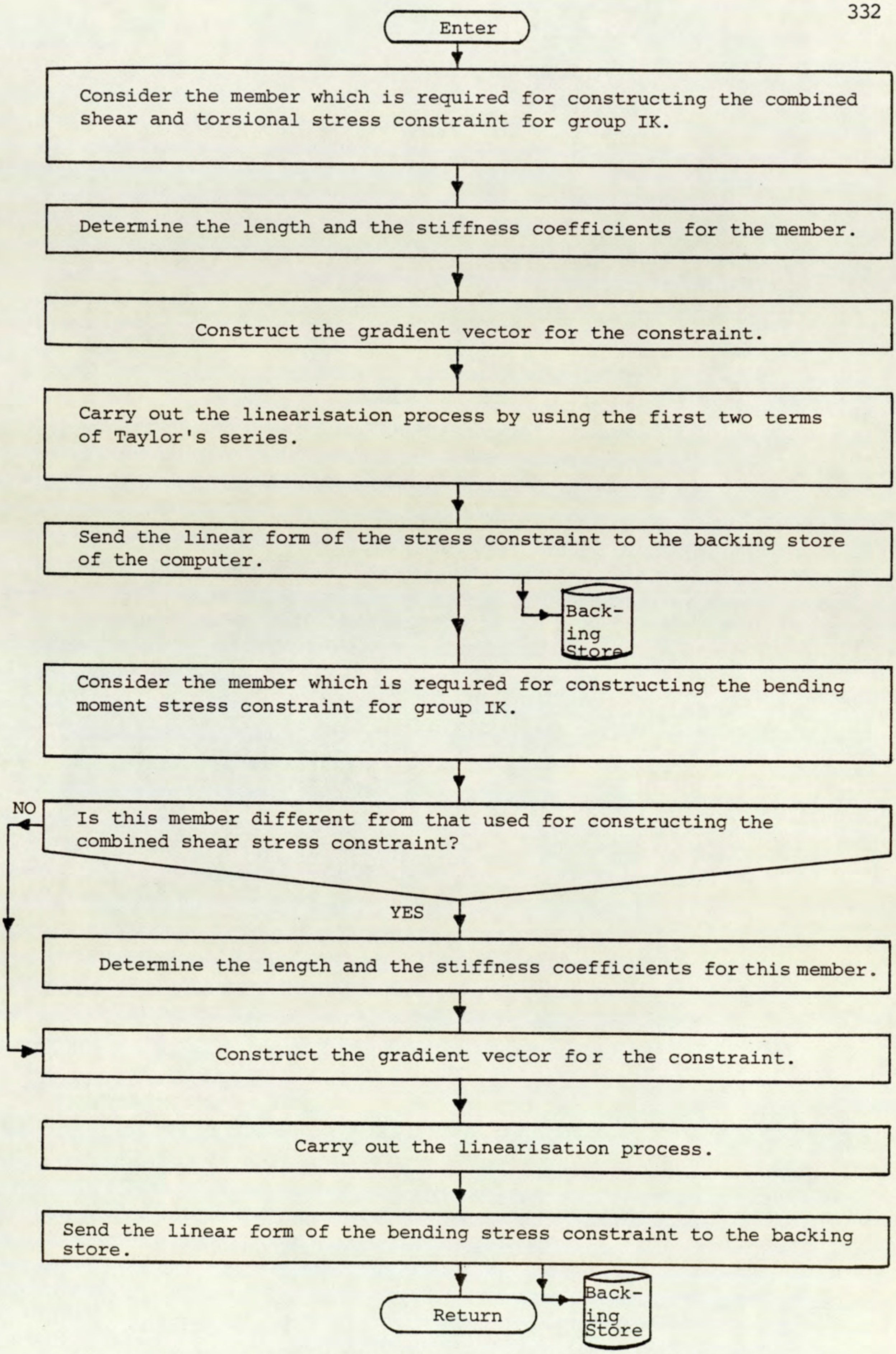


FIGURE 8.8: THE FLOW DIAGRAM FOR THE PROCEDURE OF CONSTRUCTING A LINEAR STRESS CONSTRAINT (SUBROUTINE STRESSL OR STRESST)

stress constraint, is called. Two versions of this subroutine are developed. These are known as STRESSL when dealing with a group of longitudinal members, and STRESST when dealing with a group of transverse members. The reason for developing two versions of the same subroutine is that the positions, i.e. the addresses, of the stiffness coefficients of a longitudinal member are different from those of a transverse member.

The construction procedure of a linear stress constraint, used by subroutines STRESSL and STRESST, is fairly straightforward. It starts by formulating the combined shear and torsional stress constraint, which is expressed by (6.44), and then the bending moment stress constraint which is given by (6.48). The flow diagram for the procedure is shown in Figure 8.8. The construction of the gradient vector for a constraint requires the derivativation of each item in the constraint with respect to the design variables. Such derivation can be done according to equations (6.25), (6.26), (6.60) and (6.61). The linear form of each stress constraint will then be sent to the backing store.

8.7 THE SIMPLEX PROGRAM

Two different simplex programs were employed to solve the structural linear programming problems which were discussed in the previous chapters. The first program was written by the author using The Regular Two-Phase Method described in Section 1.4.1. This program can be employed independently to solve a linear programming problem constructed by another program. On the other hand, it can also be included as a subroutine in the master program, in which case the entire operation of constructing the design problem and solving it is done automatically. This subroutine proved to be successful with a number of relatively large design problems, where very large two-dimensional matrix $\underline{D}$, similar to that shown in Figure 8.1, is stored in the computer backing store. The subroutine is written so that any column of matrix $\underline{D}$ envisaged for pivot selection is brought to the computer core at any time during the solution of the linear programming problem. Figures 8.9 gives a self-explanatory flow diagram of the simplex subroutine. The diagram depends on the information given in Sections 1.3, 1.4 and 1.4.1 in Chapter 1, and in Figure 8.1. The data parameters for the subroutine will be explained in Appendix F.

The second simplex program employed to solve the structural linear programming problem was written for ICL(LPMK2,1970) as a routine package kept in the computer centre. This routine uses The Revised Simplex Method which also depends on the principles of The Two-Phase Technique. The whole simplex table is stored as a data file in the backing store, and the routine package operates on such a file to give a new feasible solution. By then, one design iteration towards the optimum solution is accomplished. This method of producing a design iteration is found to consume a great amount of computer time. Consequently, because of the time limitation, each iteration requires a new job to be sent to the computer for operation, and manual intervention during such a process is inevitable. The procedure of obtaining the final solution by using this routine is very slow, although The Revised Simplex Method itself is powerful and requires less computation than The Regular Two-Phase Simplex Method. It should be mentioned here, however, that the routine package is only used for the problems of topological design of minimum cost. This is due to the fact that the Right-Hand-Side values of some constraints in these problems are equal to zero, which give rise to degenerate problems and cycling. The procedures of treating degeneracy and cycling are

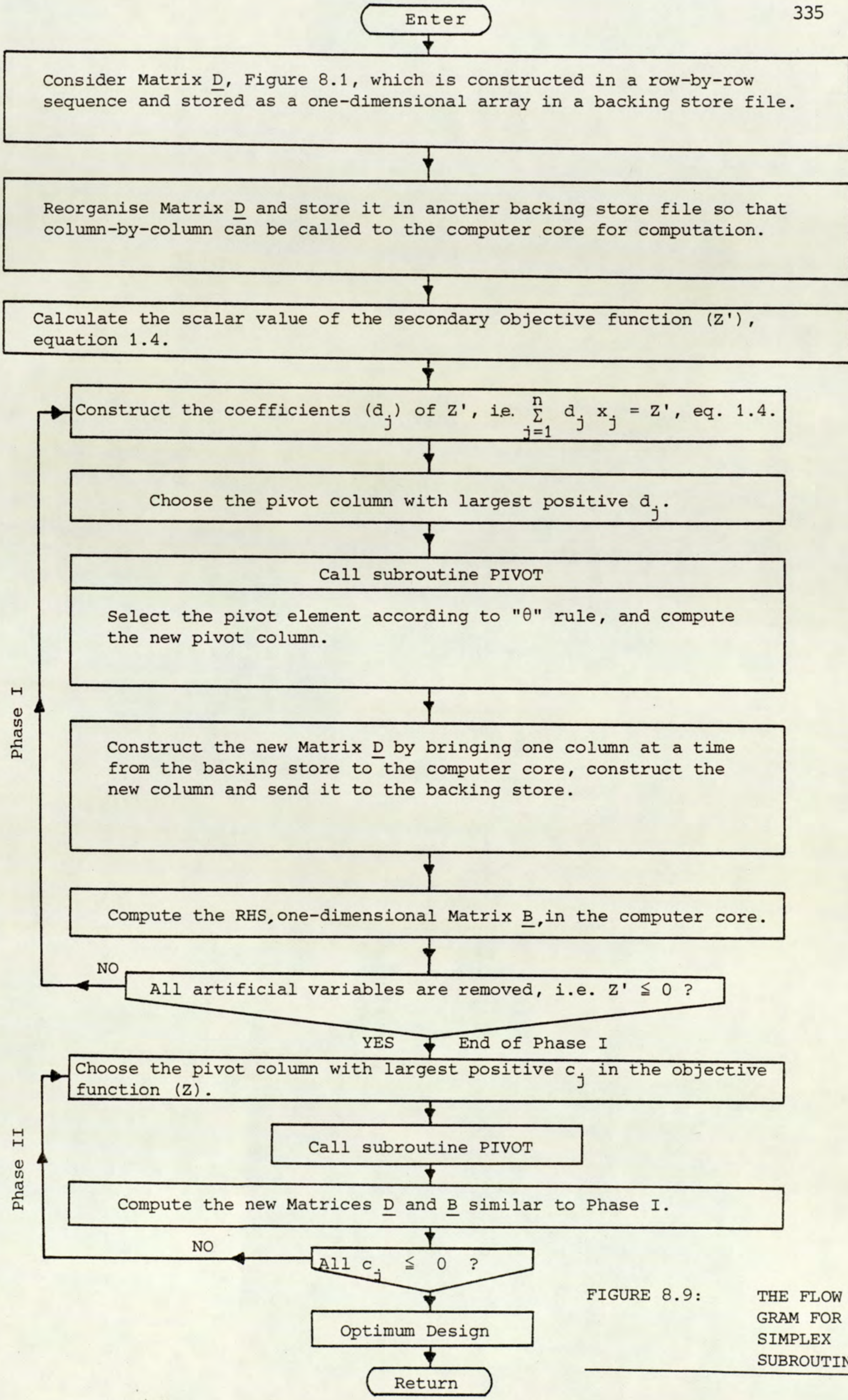


FIGURE 8.9: THE FLOW DIAGRAM FOR THE SIMPLEX SUBROUTINE

included in the routine package which proved to be efficient in dealing with such problems.

8.8 THE USE OF THE COMPUTER BACKING STORE

For both computers, the ICL1904S and the CDC7600, the backing store was used in a similar way. This was done in two stages. In the first stage, matrix $\underline{D}$, Section 8.2, is constructed and transferred to a temporary backing store disc file one row at a time. This matrix is stored as a one dimensional array of file elements. However, as explained in the previous Section, the simplex program needs to operate on matrix $\underline{D}$ by selecting one column of coefficients at a time. Therefore, in the second stage, matrix $\underline{D}$ is reorganised and transferred column-by-column to another temporary backing store disc file. From this file, the simplex program brings a column of coefficients to operate on, and then either returns it to the same position where it is brought from, or discards it completely. The latter case is considered when a pivot column is brought and an artificial variable is removed.

On the other hand, if a routine package of the simplex program (LPMK2,1970) is employed, then in the second stage of using the backing store, matrix $\underline{D}$ will be transferred column-by-column not to a temporary backing store file, but to a separate permanent data file. The RHS matrix $\underline{B}$ is also transferred to this file. A special organisation of the two matrices is required which will prepare such a file to be operated on by the routine package of the simplex program.

1 - File Specifying Subroutines		
ICL 1904S Computer	WORKFILE	Specifies that a scratch file is to be used as a temporary backing store file for both input and output. The file is closed and discarded automatically by the end of the master program.
CDC 7600 Computer	OPENWF	Similar to WORKFILE above, but the file does not close automatically.
	CLOSMS	Close the temporary backing store file.
2 - Array Handling Subroutines		
ICL 1904S	PUTPART	Transfers a specific part of an array to a backing store file.
	GETPART	Brings a specific part of an array from the backing store file to the computer core.
CDC 7600	PUTWF	Transfer an array to a backing store file.
	GETWF	Brings an array from a backing store file to the computer core.
	GETCOL	Rearranges matrix D from a row-by-row wise to a column-by-column.

TABLE 8.1: SUBROUTINES WHICH ENABLE THE USE OF THE COMPUTER BACKING STORAGE

A general description of the subroutines which enable files on discs to be used for backing storage is given in Table 8.1, (ICL FORTRAN, 1976) and (CDC FORTRAN, 1978).

8.9 THE ANALYSIS METHOD

In this method, the unknown joint displacements $\underline{X}$ are obtained by solving the matrix equation $\underline{K}\ \underline{X} = \underline{L}$. The member forces are then calculated using the joint displacements. This method was developed by, amongst others, Livesley (1956), and Jennings and Majid (1965) who prepared a computer program which can be used for elastic-plastic analysis of frames. The full matrix operation was used in order to solve the stiffness equations and obtain resulting joint displacements. However, because of the large storage required for their matrix, Jennings (1966) developed a compact storage scheme for storing the stiffness equations and solving them by using Gaussian elimination. This scheme was then used by Majid and Anderson (1968b) to develop a program that constructs the overall stiffness matrix in compact form, and a subroutine that solves it. The program and the subroutine were written in Atlas Autocode, which were then translated to FORTRAN and used by Celik (1977).

In this thesis, the structural analysis is used as part of the optimum design procedure to adjust the joint displacements at each iteration. The overall stiffness matrices for all the structures designed are large, sparse and symmetrical, but in which the elements do not usually form a uniform band width. For this reason the "compact storage scheme" (Jennings, 1966) was used. In this scheme, two "sequences" were introduced. The "main sequence" stored the elements which appeared between the first non-zero element and that on the leading diagonal, inclusive, in each row within the half band-width of the overall stiffness matrix. The second sequence was called the "address sequence" and was used to locate the positions of the leading diagonal elements within the main sequence. The zero elements existing within the irregular half band-width were also stored. In this way a large number of zeros were left outside the storage which resulted in a considerable reduction in the storage requirements.

The two sequences were used by all the master programs of the previously discussed structures. The irregular half band of the overall stiffness matrix was constructed row-by-row and stored as a one dimensional array in the computer core. Then, by using subroutine ANLYSIS (Celik, 1977), this array and the Right-Hand-Side array, which contains the applied loads, were solved by Gaussian elimination and back substituted to produce the joint displacements. The data parameters for subroutine ANLYSIS will be briefly described in Appendix G.

CHAPTER 9

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

9.1 THE DESIGN METHOD

The methods for optimum elastic design of rigid frames, complete structures and flat grillages, all used the matrix displacement technique for formulating the design problems. Such formulation was found effective and made it possible to produce general computer programs for automatic optimum design of realistic structures. Optimisation can be carried out using linear programming without need for an analysis. This was not the case with the problems solved in this thesis because all the programming problems turned out to be non-linear and repeated modifications were needed to obtain a set of linear constraints and a linear objective function. As a result of this, it was found that at the end of each linear solution, an analysis was necessary to ensure that errors introduced by the linearisation were excluded. This did not introduce significant difficulties as both computer time and storage were plentiful. The sequential approximating programming was proved to be very effective for obtaining the solution of a non-linear design problem. For the solution of each linearised problem, The Two-Phase Simplex Technique was found powerful.

The criteria defining the optimum design was either a minimum weight or a minimum cost of the structure. In this thesis, the topological designs were directed towards minimising the cost. However, topological designs for minimum weight were also investigated but found to be discouraging. The basic features of a topological design were that, once members in a structure were removed, the number of design variables and stress constraints were reduced. This caused the initial programming problem to be changed basically. In addition to that, due to the members removal, some joints may also be removed and hence reducing the number of stiffness and deflection constraints. The formation of the remaining constraints also changed during this operation. These factors contributed to modifying the objective function, the number of variables and the boundary of the feasible region.

The method of minimum cost topological design of a structure was not restricted by the limits of conventional techniques. It allowed the final shape of a structure to be decided not by intuition but by design requirements and economic factors. The total cost, which includes the material and the construction costs, was assessed realistically by using the rates of labour and measured items, and also by using the known sectional dimensions of members. Such method of cost assessment made the differences between the fixed charges of the removable members to be very small. This in turn caused the structural factors to be more dominant than the economic factors in selecting the members which should be removed. Such claim was proved in almost all the examples solved for minimum cost topological design in this thesis. Furthermore, it was found that, before starting topological changes, it is better to carry out a minimum weight design of the initial trial structure, so that the sectional dimensions given to the members are better related to each other. In the case of complete structures, a minimum weight design was avoided. It was therefore found necessary not to remove members in the first few iterations. This was to avoid the removal of vital structural components at an early stage.

In this thesis, all the design problems were constructed one row at a time. The computer programs which were written for constructing the problems in such manner, and then solving them, were designed to make use of the computer backing store facilities. This improved the economical use of the computer core. Furthermore, by writing the simplex program and including it as a subroutine in the master program, it was found unnecessary to use the routine package which belongs to the computer centre. Therefore, an economy in the computer time was achieved when the consecutive design iterations were carried out continuously without any interruption. The fact that an efficient simplex subroutine was produced, made the optimisation problem independent of library routines and also independent of the machine used for this purpose.

9.2 THE SWAY FRAMES

In a rigidly jointed steel frame, expressing the second moment of area in terms of the sectional area by approximating relationships made it possible to express all the main design variables by continuous functions of the variable areas. During the design process, the section areas were therefore assumed to be available in a continuous range. However, as shown in some design examples, the calculated areas were first converted to second moments of area which were then used in selecting the sections from a table for universal beams and columns.

In the design of multi-storey steel sway frames, the sections obtained were found to be governed by the limitations imposed on the relative sway deflection of the storeys. Frames designed in this manner were checked by an independent analysis and proved to satisfy the strength requirements. This made the stress constraints superfluous and were thus excluded from the design problem for simplicity, speed of operation and economy in computer time and storage. The wisdom of such an exclusion was confirmed by numerous examples.

The design of a number of frames for minimum weight or minimum cost, produced the following conclusions:

(1) The solution of the non-linear design problem always ended with the same design. This was verified by commencing with the solution of a particular problem from several widely different initial design points. Another verification was done by repeating the design process after considering the optimum solution of one stage as an initial design point for the second stage. This indicates that the design obtained was at least a local optimum design and may be even the global.

(2) A suitable starting value for the move limit can be taken as ML = 0.5 rather than ML = 0.9 which was used by previous researchers. The latter value of ML required unnecessary design iterations in the beginning of the process.

(3) The number of design iterations was further reduced considerably when it was decided to take the frames designed by Okdeh as the starting point for the optimisation.

(4) For irregular and asymmetrical frames, the optimum design should be carried out at least under two reversed wind loading conditions. In this way, each group of members will have two sections to select from, one from each condition. The largest of these may be used which consequently increases the weight or the cost obtained at the optimum design of the frame.

(5) The actual cost of foundation was found to be small compared with the cost of the columns it carried. This in turn did not significantly increase the "fixed charges" imposed on the inclusion of these columns in the final design. Columns were therefore removed for other reasons.

(6) A minimum weight design of a frame, excluding fixed charges, proved to be ineffective to be used to change the shape of a structure. A 9-storey irregular frame designed in Chapter 3,

Section 3.4.3, for minimum weight, while allowing member removals, by not imposing a lower bound on the sections, proved to give an infeasible result.

9.3 THE COMPLETE STRUCTURE

The optimisation method for a minimum cost topological design of laterally loaded complete structures was found to be flexible and versatile. The lateral loads were considered as the static wind forces. The vertical loads were not taken into account. The aim of the method was to reduce the total cost of the structure while satisfying the design requirements. One way of achieving such reduction was by trying to remove some of the vertical components which were selected by the method to be structurally and economically unimportant to keep.

For simplicity, the main design variables were taken as the thicknesses of the plate components and the second moments of area for the columns of the frames. In addition to that, the displacements (z, θ_x and θ_y) of each joint were also considered as variables. The widths of the plates and the second moments of area for the beams of the frames were assumed as constants and their values were chosen prior to the beginning of the design. The number of columns in each frame was also assumed. Such simplification of the problem was found effective in decreasing the number of the unknowns and thus reducing the level of non-linearity. As a result, it was proved that convergence can be achieved without the use of move limits. However, the only necessary limits required for the section variables were found to be the upper and the lower bounds taken from the available sections.

From the several design cases which were tried on two symmetrical complete structures, the following conclusions were drawn:

(1) In the examples designed, the limits on the lateral differential sway deflection between the storeys were taken as h/350 and h/500. For these, the sway was the governing limit state which decided the sections of the retained structural components. The strength requirements for these components were checked and found satisfactory. This might not be the case if the sway in a storey was restricted to h/300 or h/250.

(2) It was found that using shear walls reduced the need for intermediate frames, and the removal of these frames reduced the cost of the structure. Slender walls, however, were found insufficient to transmit the wind loads to the foundation. Such walls required more intermediate frames which consequently increased the cost of the structure. Furthermore, the actual costs of foundations were once again found to be small compared with the costs of the vertical components, and their effects in the components removal were negligible.

(3) Increasing the number of groups in each frame meant more columns could be removed and a further reduction in the cost could be achieved. Beams were not allowed to be removed unless the whole frame was to be removed.

(4) In Design Case 5 of the 9-storey structure, it was found that a more economical design was obtained when reinforced concrete frames were used, although the required number of such frames was greater than that of steel frames. This was because the cost of a reinforced concrete frame was found to be less than that of a fabricated steel frame.

(5) The design of laterally loaded complete structures can be taken as a preliminary design which should be followed by a design that considers a general load condition, i.e. lateral and vertical loading. However, the vertical components which were removed when designing under the case of lateral loading only, could be replaced by props if these were needed to carry the vertical loads.

The complete structures designed in Chapter 5 were symmetrical and the sidesways produced in the frames and the grillages were the result of the lateral wind loads only. It is well known, however, that sidesways also results from the action of vertical loads when they are asymmetrical applied, or when the shape of the frames or the walls are asymmetrical. It is desirable therefore to extend the present optimisation method to deal with generally loaded complete structures consisting of parallel vertical components that are of symmetrical or non-symmetrical shape. Further extension to the method can include complete structures of non-parallel bracing components, or structures consisting of shear walls with openings. These problems can be tackled, for instance, by either considering the grillage as a space frame with 6-degrees of freedom at each junction, or use the finite element approach.

It should be also pointed out that for the structures considered, the stress constraints can be more significant than the sway constraints. The method presented in Chapters 4 and 5 was limited to design structures for sway, but it is necessary to generalise the case to cover strength requirements. This is in spite of the fact that each member requires a large number of stress constraints which complicate the problem. One way of overcoming this difficulty is to impose stress constraints on critical members only when a trial analysis reveals that the stresses in these members are significant.

9.4 THE HORIZONTAL GRILLAGES

The unknown sectional dimensions for the horizontal grillages were assumed to be the thickness and the overall depth of each beam, and the overall depth of each column. The thickness of each column was taken as equal to the thickness of the beam connected to it. These sectional dimensions caused the design problem to be highly non-linear. Therefore, it was found necessary to use move limits to help to converge the approximated problem. It was found suitable to take the initial value of the move limits as ML =0.5, and then reduce it by steps of 0.1 at each design iteration until the optimum solution was obtained.

The design requirements were represented by the stiffness, the stress, the deflection and the practical constraints. The use of the stress and the deflection constraints together gave the design method a wide range of applications. In other words, the method is general and it could be applied on many types of horizontal grillages made from either reinforced concrete or fabricated steel where the deflection or the stress, or the combination of both, requirements dominate the design. Furthermore, a great reduction in the size of the problem, and consequently in the computer time and storage, was achieved when a particular stress constraint was applied only on one selected member in each group. The reason for such application was that, from the analysis results, the member concerned was found to have the critical stress among all the other members in the group. This meant that the number of stress constraints was in correspondence with the number of member groups in the grillage. Previous research workers applied a stress constraint on every member in a structure which obviously increased the size of the problem unnecessarily.

The design of a number of a grillages for minimum weight or minimum cost produced the following conclusions:

(1) The inclusion of the grillage self-weight as a variable in the design problem encouraged a further reduction in the total weight of structure. This was proved in the design of The Ellipse Shaped Grillage where the minimum weight obtained was reduced by about 6.5% when the self-weight was considered as a variable. Such percentage of weight reduction could have been more significant in a larger structure.

(2) In the design of The Ellipse Shaped Grillage and the Irregular Circular Grillage, the optimum sections obtained at the end of the process were decided mainly to satisfy the stress requirements. The deflection requirements, on the other hand, only helped in coverging the problem. However, in the topological design of the Symmetrical Rectangular Grillage, both the stress and the deflection requirements were involved in deciding the optimum sections of the final shape of the grillage. This showed that the domination of the stress or the deflection constraints depended mainly on the configuration of the grillage.

(3) It is possible that no design code of practice has clearly specified the limits that should be imposed on the joint deflections of a grillage. In this thesis, however, such limits were chosen by the designer as either $L_j/400$ or $L_j/500$, where L_j is the shortest distance between joint j and the support. Therefore, it was concluded that the effectiveness of the deflection constraints depended entirely on the values of the deflection limits chosen by the designer.

(4) From the design results it was shown that the grillage members tended to be thinner but deeper which gave a chance to the effects of shear distortions to play an important part in the design. Such behaviour continued until the sectional overall depth of a member reached its upper bound, and by then the sectional thickness started to increase.

(5) It was shown that in the Ellipse Shaped and the Irregular Circular Grillages, the bending moment stress has almost reached to the permissible limit, while the combined shear and torsional stress did not reach such limit. On the other hand, in the Symmetrical Rectangular Grillage, the topological alteration caused both stresses to reach the limit in some of the retained members. From this it can be concluded that the bending moment stress constraints were the more effective ones in deciding the optimum sections for the grillages considered.

(6) The minimum cost topological design applied on the Rectangular Symmetrical Grillage was successful. This was proved when the member removal from the original structure caused the weight and the cost of the optimum topological design to be about 24.5% less than those of the optimum non-topological design. However, the reallocation of the live load after a member or a joint removal was approximately tackled and might be overestimated.

The optimum design method of reinforced concrete grillage structures has a wide range of application. It can easily be used for the design of bridge decks which is simply supported at the columns and at some of the member ends, while tension and compression reinforcements are specified. The method can also be slightly modified to include beams of T or L-section, where the main variables in this case are the thicknesses of the web and the flange, while the overall depth of the section varies or can be kept as a constant. With the inclusion of the effect of torsion in the members, the method can be extended to cover the optimum design of building slabs or bridge slabs. In this case the design is carried out on equivalent grillage created from dividing the slabs into strips in the longitudinal and the transverse directions and an equivalent grillage member is taken in the centre of each strip. The method can also be extended to cover the design of bridges with box girders of varying sectional properties. Furthermore, the finite element approach can be utilised for formulating the design problems of plate components. These components can be assembled together to form a slab that covers a deck which is built as a fixed depth grillage.

APPENDIX A

MEASURED RATES OF MATERIALS AND CONSTRUCTING ITEMS

Market prices for the more important items of material and construction were presented in a report by Davis, Belfield and Everest (1980). A brief description about this report was given in Chapter 2. The items required are given below.

Item Description		Measuring Unit	Cost per unit (£)
I	MATERIAL:		
1	- Ready mixed concrete ($21 N/mm^2$)	m^3	28.24
2	- Ready mixed concrete ($30 N/mm^2$)	m^3	32.14
3	- Mild steel bar reinforcement (20mm)	ton	231.24
4	- High yield steel bar reinforcement (20 mm)	ton	239.24
5	- High yield steel bar reinforcement (12 mm)	ton	249.90
6	Universal beams (average)	ton	225.00
7	Universal columns (average)	ton	235.00
II	CONSTRUCTION		
1 —	Earthworks, (in firm soil):		
(a)	Excavation of foundations for maximum depth (1 - 2 m)		
	- Material for re-use	m^3	2.88
	- Material for disposal (500 m to tip)	m^3	3.94
	- Material for disposal (3 km to tip)	m^3	4.08
(b)	Excavation ancillaries		
	- Preparation of surfaces	m^2	0.11
(c)	Filling and compaction		
	- To structures selected excavated material	m^3	0.65
	- General selected excavated material	m^3	0.68
2 —	In-Situ Concrete Work		
(a)	Provision of concrete		
	- ($11.50 N/mm^2$) ready mixed (for blinding)	m^3	28.41
	- ($21 N/mm^2$) ready mixed	m^3	31.85
	- ($30 N/mm^2$) ready mixed	m^3	33.11
(b)	Placing of mass concrete		
	- Blinding thickness not exceeding 150 mm	m^3	5.72

Item Description	Measuring Unit	Cost per unit (£)
(c) Placing of reinforced concrete		
- Basses and slabs thickness not exceeding 150 mm	m^3	7.15
- Basses and slabs thickness 150 - 300 mm	m^3	4.29
- Basses and slabs thickness 300 - 500 mm	m^3	3.96
- Basses and slabs thickness over 500 mm	m^3	3.50
- Walls thickness not exceeding 150 mm	m^3	17.27
- Wall thickness 150 - 300 mm	m^3	12.87
- Columns cross sectional area 0.03 - 0.1 m^2	m^3	31.79
- Columns cross sectional area 0.1 - 0.25 m^2	m^3	26.51
- Beams cross sectional area 0.03 - 0.1 m^2	m^3	26.07
- Beams cross sectional area 0.1 - 0.25 m^2	m^3	22.11
3 — Concrete Ancillaries		
(a) Formwork rough finish		
- Horizontal width 0.1 - 0.2 m	m	1.87
- Horizontal width 0.2 - 0.4 m	m^2	9.30
- Horizontal width 0.4 - 1.22 m	m^2	8.82
- Horizontal width over 1.22 m	m^2	8.34
- Vertical width 0.1 - 0.2 m	m	1.95
- Vertical width 0.2 - 0.4 m	m^2	9.46
- Vertical width 0.4 - 1.22 m	m^2	8.99
- Vertical width over 1.22 m	m^2	8.50
(b) Formwork fair finish		
- Horizontal width 0.1 - 0.2 m	m	2.10
- Horizontal width 0.2 - 0.4 m	m^2	10.83
- Horizontal width 0.4 - 1.22 m	m^2	10.55
- Horizontal width over 1.22 m	m^2	9.87
- Vertical width 0.1 - 0.2 m	m	2.18
- Vertical width 0.2 - 0.4 m	m^2	10.99
- Vertical width 0.4 - 1.22 m	m^2	10.52
- Vertical width over 1.22 m	m^2	10.03
(c) Mild steel bar reinforcement		
- 6 mm diameter	ton	528.39
- 12 mm diameter	ton	416.08
- 20 mm diameter	ton	354.67
- 25 mm diameter	ton	347.23
(d) High yield bar reinforcement		
- 6 mm diameter	ton	537.19
- 12 mm diameter	ton	424.88
- 16 mm diameter	ton	394.17
- 20 mm diameter	ton	363.47
- 25 mm diameter	ton	356.03

Item Description	Measuring Unit	Cost per unit (£)
4 — Structural Steelwork		
(a) Fabrication of members		
- Columns for frames	ton	500.00
- Beams for frames	ton	470.00
- Roof trusses	ton	710.00
(b) Erection of members		
- Frames	ton	70.00
- Roof trusses	ton	140.00
(c) Surface treatment		
- Shot blast and one coat primer at works	ton	135.00

APPENDIX B

COSTING REINFORCED CONCRETE MATERIALS

Assume 1 m^3 of reinforced concrete
1 ton = 9.81 KN
Density of steel reinforcement = 7.85 ton/m^3 = 77 KN/m^3
Density of concrete = 2.5 ton/m^3 = 24.5 KN/m^3

Use Appendix A
Percentage of vertical reinforcement = 1%
HYS, 20 mm, cost = £239.24/ton = £24.39/KN

Percentage of horizontal reinforcement = 25%
HYS, 12 mm, cost = £249.90/ton = £25.47/KN

Ready mixed concrete (30N/mm^2), cost = £32.14/m^3

Volume of reinforcements = 0.01 m^3 + 0.0025 m^3 = 0.0125 m^3
Weight of the 20 mm reinforcements = 0.01 m^3 X 77 KN/m^3 = 0.770 KN
Weight of the 16 mm reinforcements = 0.0025 m^3 X 77 KN/m^3 =0.192 KN
Total weight of reinforcements = 0.77 + 0.1925 = 0.962 KN

Volume of concrete only = 1 m^3 - 0.0125 m^3 = 0.9875 m^3
Weight of concrete = 0.9875 m^3 X 24.5 KN/m^3 = 24.19 KN

Total weight = 0.9625 KN + 24.19 KN = 25.15 KN

Cost of 1 m^3 of reinforced concrete with 1.25% of reinforcement:

0.9875 m^3 X £32.14/m^3 = £31.74 Cost of concrete
0.77 KN X £24.39/KN = £18.78 Cost of 20 mm reinf.
0.1925 KN X £25.47/KN = £ 4.90 Cost of 12 mm reinf.

Total cost = £55.42

Cost of reinforced concrete material per unit weight
= £55.42/25.15 KN
= £ 2.20/KN

APPENDIX C

DATA FORMAT FOR THE FRAME PROGRAM

The data for the frame shown in Figure C1 are given below as an example:

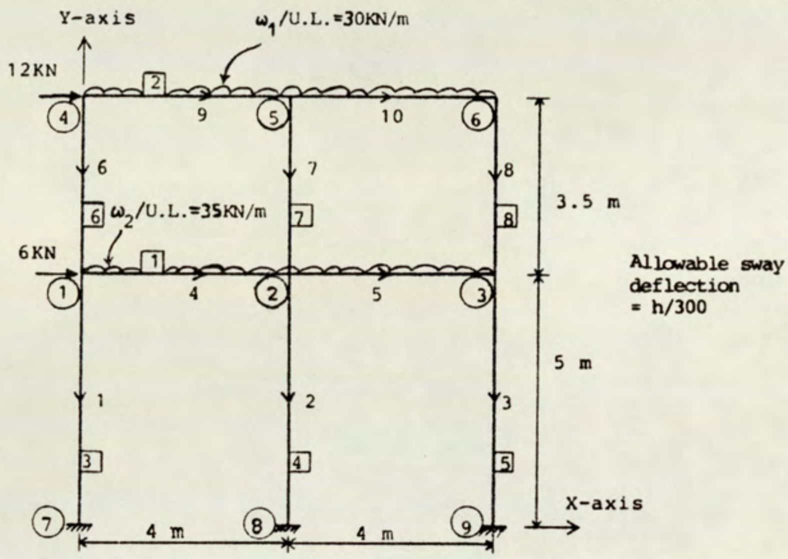


FIGURE C1: AN EXAMPLE USED FOR DATA PREPARATION OF THE FRAME PROGRAM

(a) Preliminary Data (2 Cards)

These are given in the following order:

(1) The total number of frame joints excluding the fixed supports. (For the frame shown in Figure C1, this is equal to 6).
(2) The total number of members (10).
(3) The total number of supports (3).
(4) The total number of member groups (8).
(5) The joint number of the first support (7).
(6) The maximum number of members connecting to a joint (4).
(7) The total number of storeys (2).
(8-11) The constants that provide the relationship between the section second moment of area I and the section area A. These are (3.20), (4.17), (2.0) and (1.7).
(12) Modules of Elasticity for fabricated steel in KN/cm^2 (20700).
(13) Steel Density in KN/cm^3 (0.000077).
(14) Average construction cost for a frame member in £/KN (70.4).
(15) Average material cost for a frame member in £/KN (23.5).

(b) Member Data (1 Card per member)

For each member of the frame, for instance member 1 in the frame of Figure C1, the following data is required:

(1) The member number (1).

(2) The joint number at the first end (1).
(3) The joint number at the second end (7).
(4) The group number of the member (3). Notice that the grouping of members starts with the beams.

(c) Joint Data

They include three types of data, where each type is demonstrated on joint 2 in the frame of Figure C1. These data are:

(1) Joint coordinates data - For every joint and support the X and

Y coordinates are given in cm, such as :

2 400.0 500.0 (1 Card per joint).

(2) Joint displacement data - For every joint the allowable limits on the x, y and θ displacements are given in cm and radian, such as:

2 3.0 4.0 0.08 (1 Card per joint).

(3) Joint loading data - For every joint the applied horizontal and vertical loading and the acting moment are given in KN and KN.cm, such as:

2 2.0 140.0 0.0 (1 Card per joint).

(d) Sway Deflection Data (1 Card per storey)

These include the number and the allowable differential sway limit in cm for the joints on the leeward columns. For the frame in Figure C1, such data would appear as follows:

3 1.67 (first storey)

6 1.16 (second storey)

(e) Section Data (1 Card per group of members)

For each group of members the following data is given:

(1) The group number.

(2) The section area of the group in cm^2.

(3) The move limit value.

(4) The upper bound on the section area; usually a universal section is used.

(5) The foundation cost for the ground column group, such cost is given as zero for all the other groups.

(f) Section Bounds Data

These are the following:

(1) One card that contains respectively the total number of beam groups, the total number of column groups, and the smallest universal column section available that can withstand a column-type mechanism in the frame.

(2) One card per beam group. This includes the number of the beam group and the smallest universal beam section that can withstand a beam-type mechanism in the frame.

(g) Type of Problem Data

One card that contains three integer values of the variables ANLYS, ID and SIMP, where:

ANLYS = 1 means analysis is not required.

ANLYS = 0 means analysis is required.

ID = 1 means the problem is for minimum weight design.

ID = 0 means the problem is for minimum cost topological design.

SIMP = 1 means the Simplex subroutine is used.

SIMP = 0 means the routine package of Simplex (LPMK2, 1970) is used.

Notice that in the case when ANLYS = 1, the joint displacements should be provided as data before the design starts.

(h) Topological Design Data

Such data is required when ID = 0, and they include the following:

(1) The total number of member groups to be kept in the final design (1 Card).

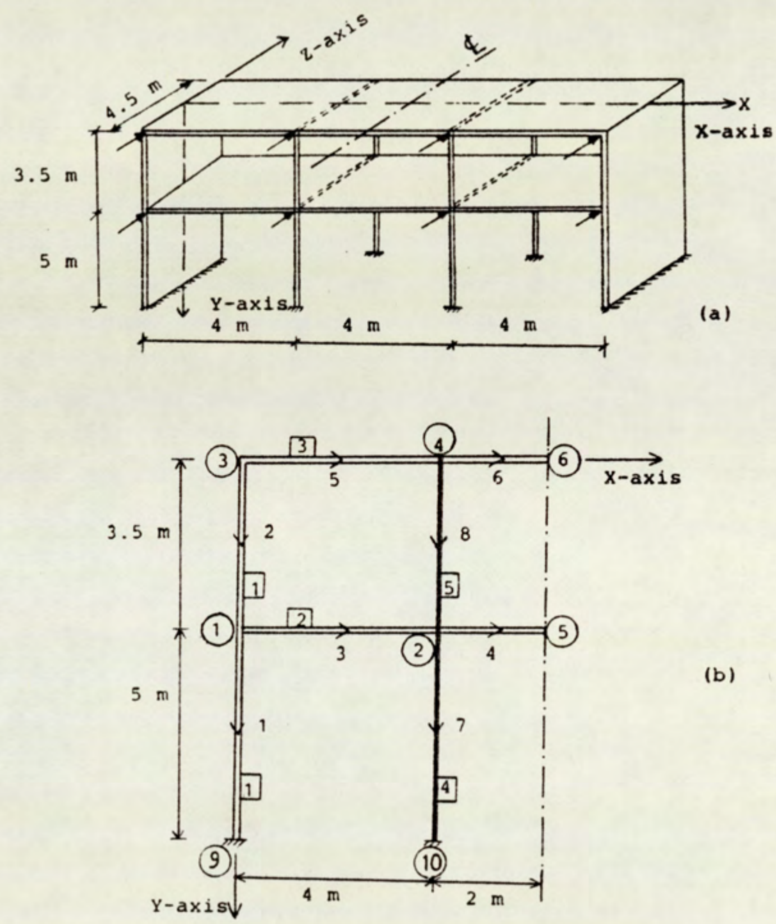
(2) For each group to be kept, the group number and the lower bound on its section area are given (1 Card per group).

(3) The total number of member groups to be removed, and the group number of each one of them (1 Card only).

APPENDIX D

DATA FORMAT FOR THE COMPLETE STRUCTURE PROGRAM

The data for the left half of the complete structure shown in Figure D1b is given below as an example. It should be noted that the member and the joint data are similar to those of the frame shown in Appendix C, and thus they will not be repeated here.



(a) A 2-storey 3 equal bay symmetrical complete structure.

(b) The left half of the structure.

FIGURE D1: AN EXAMPLE USED FOR DATA PREPARATION OF THE COMPLETE STRUCTURE PROGRAM

(a) Preliminary Data (2 Cards)

These should be provided in the following order:

(1) The total number of joints, excluding the supports. (This is equal to 6 for the structure shown in Figure D1b.)
(2) The total number of supports (2).
(3) The joint number of the first support (7).
(4) The maximum number of members meeting at any joint (4).
(5) The number of the storeys (2).
(6) The total number of the grillage groups of members (3).
(7) The total number of the column groups in the frames (2).

(8) The total number of the shear wall members (2).
(9) The total number of the floor slab members (4).
(10) The total number of the columns in the frames (2). Notice that, in a single frame, all the columns in a storey are represented by one number.
(11) The number of shear walls (1).
(12) The total number of joints excluding the supports and the joints on the central line (4). In case of a non-symmetrical structure, this number is equal to the total number of joints excluding the supports.
(13) The width of the shear walls in cm (450).
(14) The width of the floor slabs in cm (450).
(15) The total number of columns per one storey in each frame (2).

(b) Material Properties Data (1 Card)
These include the following:
(1) Modulus of Elasticity for concrete in KN/cm^2 (2800).
(2) Modulus of Elasticity for steel frames in KN/cm^2 (20700). In case reinforced concrete frames are used, this value will be equal to (2800).
(3) Poisson's ratio (0.2).
(4) Steel density in KN/cm^3 (0.000077).
(5) Reinforced concrete density in KN/cm^3 (0.0000245).
(6) Average material cost for a member of a steel frame in £/KN (23.5). This value is equal to (3.4) if reinforced concrete frames are used.
(7) Average material cost for a shear wall member in £/KN (2.2).
(8) Average material cost for a floor slab member in £/KN (1.6).

(c) Sway Deflection Data
These include the following data per storey:
(1) The total number of joints in the storey (1 Card).
(2) The number of each of these joints and the allowable sway deflection limit for it (1 Card).

(d) Section Data
These include three types which are as follows:
(1) For each shear wall group, the group number, the thickness of the section in cm, the upper bound on this thickness and the foundation cost if any, are given (1 Card per group).
(2) For each floor slab group, the group number and the thickness of the section in cm are given (1 Card per group).
(3) For each column group in a steel frame, the group number, the second moment of area for the column section in cm^4 selected from the universal columns, the upper bound on the second moment of area, the cost of foundation in £ if any and the mass per unit length for the column in kg/m, are given. If a reinforced concrete frame is used, the 1st item of these data would instead be the thickness of the column section in cm (1 Card per group).

(e) Type of Problem Data
One Card contains two integer values of the variables ANLYS and SIMP, as explained in Appendix C.

(f) Cost Assessment Data (2 Cards)
These depend on the items given in Appendix A. The first Card contains the following:
(1) Cost of concrete provision in $£/m^3$ (33.11).

(2) Cost of placing concrete in wall thickness not exceedig 150 mm in £/m^3 (17.27).
(3) Cost of placing concrete in wall thickness 150 mm - 300 m in £/m^3 (12.87).
(4) Cost of formwork fair finish in vertical width over 1.22 m in £/m^2 (10.03).
(5) Percentage of vertical reinforcement in a wall section (1%).
(6) Cost of high yield bars 20 mm diameter in £/ton (363.47).
(7) Percentage of horizontal reinforcement in a wall (0.25%).
(8) Cost of high yield bars 12 mm diameter in £/ton (424.88).

The second Card is included only when reinforced concrete frames are used, in which case it contains the following:

(1) Cost of placing concrete in column cross sectional area 0.03 - 0.1m^2 in £/m^3 (31.79).
(2) Cost of placing concrete in column cross sectional area 0.1 - 0.25m^2 in £/m^3 (26.51).
(3) Percentage of vertical reinforcement in a column (3%).
(4) Cost of high yield bars 6 mm diameter in £/ton (537.19).
(5) Total cost of material for all the beams of a frame in £ (Computed by assuming beam sections and using Appendix B).
(6) Total cost of construction for all the beams of a frame in £ (Computed by assuming beam sections and using Appendix A).

(g) Section Bounds Data (1 Card)

These include the following:

(1) The lower bound on the thickness of all the grillage members in cm (16.0).
(2) The upper bound on the thickness of the floor slab members in cm (20.0).
(3) The lower bound on the second moment of area for the columns of the frame in cm^4.
(4) The mass per unit length for the steel beams in kg/m.
(5) The total construction cost for all the steel beams of a frame in £. Notice that items (4) and (5) will be ignored or given as a unit in the case of using reinforced concrete frames.

(h) Topological Design Data

These include the data for the groups to be kept only, as explained for the frame in Appendix C.

APPENDIX E

DATA FORMAT FOR THE HORIZONTAL GRILLAGE PROGRAM

The data preparation for the reinforced concrete horizontal grillage shown in Figure E1 are discussed here as an example. Notice that due to symmetry, the data format will be given below only for the top left quarter of the grillage shown in Figure E1b. The member and the joint data are similar to those of the frame given in Appendix C.

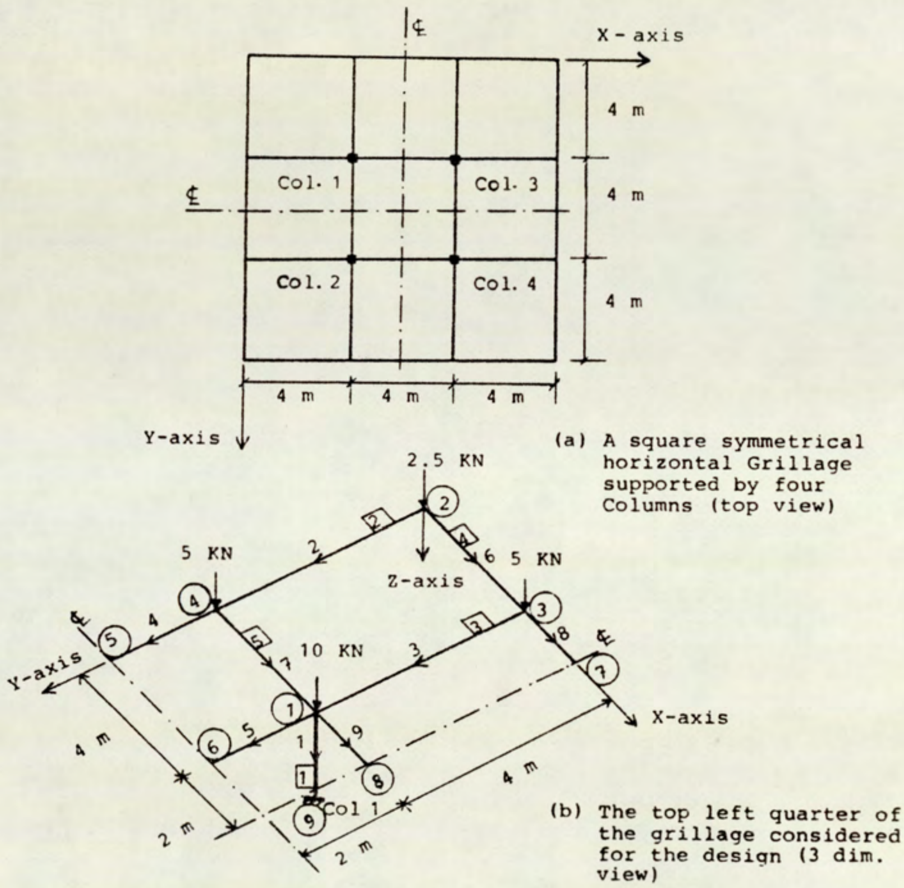


FIGURE E1: AN EXAMPLE USED FOR DATA PREPARATION OF THE HORIZONTAL GRILLAGE PROGRAM

(a) Preliminary Data (3 Cards)

These are given in the following order:

(1) The total number of joints excluding the fixed supports (8).
(2) The total number of supports (1).
(3) The joint number of the first support (9).
(4) The maximum number of members meeting at a joint (4).
(5) The total number of longitudinal (parallel to Y-axi) groups (2).
(6) The total number of transverse (parallel to X-axis) groups (2).
(7) The total number of longitudinal members (4).

(8) The total number of transverse members (4).
(9) The total number of joints on top of the columns (1). This value will be given as zero if there is no column support.
(10) The total number of joints excluding the fixed support and the joints on the central lines (4).
(11) The total number of joints on the central line which is parallel to the X-axis (2). These are joints 5 and 6.
(12) The total number of joints on the central line which is parallel to the Y-axis (2). These are joints 7 and 8.
(13) Modulus of Elasticity for concrete in KN/cm^2 (2800).
(14) Poisson's ratio (0.2).
(15) The permissible value of shear stress (τ_c) in KN/cm^2 (0.09).
(16) The maximum allowed value of shear stress (τ_u) in KN/cm^2 (0.36).
(17) The permissible bending moment compressive stress (f_c) in KN/cm^2 (1.0).
(18) The permissible tensile stress in the reinforcement in KN/cm^2 (22.55).
(19) Reinforced concrete density in KN/cm^3 (0.0000245).
(20) Steel density in KN/cm^3 (0.000077).
(21) Modulus of Elasticity for steel reinforcement in KN/cm^2 (20000).
(22) The intensity of the uniform load in KN/cm which is equal to (0.0) because the loads on the structure of Figure E1b are pointed on the joints.

(b) Section Data (1 Card per group)
These include two types of data which are as follows:
(1) For each column group, the group number, the overall section depth in cm, the length of column in cm and the grillage group which is connected to the column, are given (1 Card per group).
(2) For each grillage group, the group number, the thickness and the overall depth in cm, are given (1 Card per group).

(c) Type of Problem Data (1 Card)
These are similar to those of the frame in Appendix C, except that there is an additional integer value for the variable BETA, where:
BETA = 0 means the Self-Weight is included as a constant.
BETA = 1 means the Self-Weight is included as a variable.

(d) Cost Assessment Data
These can be arranged using Appendix A, as explained for the complete structure in Appendix D.

(e) Section Bounds Data (1 Card)
These are the following:
(1) The lower bound on the thicknesses of the grillage members.
(2) The lower bound on the overall depths of the grillage members.
(3) The lower bound on the overall depths of the column sections.
(4) The upper bound on the thicknesses of the grillage members.
(5) The upper bound on the overall depths of the grillage members.

(f) Topological Design Data
These are similar to those given for the frame in Appendix C, but an additional type of data concerning the removal of joints. These are as follows:
(1) The total number of joints removed (1 Card).
(2) The joint number of each of these joints (1 Card per joint).

APPENDIX F

DATA PARAMETERS FOR SUBROUTINE SIMPLEX

The subroutine can be specified as:

CALL SIMPLEX (NOJ, NOGGI, NV, NRR, KB, RHS, CLM, RA, XX)

where the input parameters denote the following:

NOJ - Integer specifies the total number of joints in a structure excluding the fixed supports.

NOGGI - Integer specifies the total number of section variables.

NV - Integer specifies the total number of columns in the simplex table. In other words, it is equal to the number of design and surplus variables.

NRR - Integer specifies the total number of design variables.

KB - Integer specifies the total number of constraints.

RHS - Real array of DIMENSION at least (KB). It contains the Right-Hand-Side of the simplex table, see Section 8.2.

CLM - Real array of DIMENSION at least (NV). It contains the constant coefficients of the objective function.

RA - Integer array of DIMENSION at least (KB). It contains constant values that indicate the type of the constraint. Such as:
RA(I), for I = 1, 2 ... KN, where:
RA(I) = 0 a "≦" constraint
RA(I) = + I an "=" constraint
RA(I) = - I a "≧" constraint

XX - Real array of DIMENSION at least (NRR). On exist, this array contains the optimal values of the design variables obtained at the end of a design iteration.

Notice that the main part of the simplex table, i.e. matrix $\underline{D}$ described in Section 8.2, is stored in the backing store. The simplex subroutine operates on the table by bringing one column at a time from the backing store to the computer core.

APPENDIX G

DATA PARAMETERS FOR SUBROUTINE ANLYSIS

This subroutine can be specified as:

CALL ANLYSIS (BA, NR1, AB, S, NGC)

where the input parameters indicate the following:

BA - Real array of DIMENSION at least (NR1). On entry BA contains the values of the joint loading, i.e. $\underline{L}$ of equation $\underline{K}\ \underline{X} = \underline{L}$. On exist BA will contain the values of the joint displacements, i.e. $\underline{X}$.

NR1 - Integer specifies the number of rows for the symmetrical overall stiffness matrix $\underline{K}$.

AB - Real array of DIMENSION at least (NGC). It contains the coefficients of matrix $\underline{K}$ as explained in Section 8.9.

S - Integer array of DIMENSION at least (NR1). It contains the positions of the leading diagonal elements of matrix $\underline{K}$.

NGC - Integer specifies the size of array AB above.

LIST OF REFERENCES

(1) ALLEN A H, 1974
"Reinforced Concrete Design to CP110 - Simply Explained", The Cement and Concrete Association, London.

(2) ALLWOOD B O, et al, 1972
"Steel Designer Manual", Crosby Lockwood, London.

(3) ANDERSON D, and ISLAM M A, 1979
"Design of Multi-Storey Frames to Sway Deflection Limitations", The Structural Engineer, Vol 57B, No 1, March.

(4) ASTILL A W, and MARTIN L H, 1975
"Elementary Structural Design in Concrete to CP110", Edward Arnold, London.

(5) BELLMAN R, 1957
"Dynamic Programming", University Press, Princeton, N J.

(6) BRAY K H M, CROXTON P C L and MARTIN L H, 1976
"Matrix Analysis of Structures", Edward Arnold, London.

(7) BRITISH STANDARD 449: Part 2: 1969
"Specification for the Use of Structural Steel in Building", British Standard Institution.

(8) BRITISH STANDARD CP114: Part 2: 1969
"The Structural Use of Reinforced Concrete in Buildings", British Standard Institution.

(9) BRITISH STANDARD CP110: 1972
"The Structural Use of Concrete", British Standard Institution.

(10) BRITISH STANDARD CP3: CHAPTER V: Part 2: 1972
"Basic data for the design of buildings, Part 2. Wind Loads." British Standards Institution.

(11) BRITISH STANDARD B/20, DOCUMENT 77/13908 DC, 1977
"Draft Standard Specification for the Structural Use of Steelwork in Building, Part 1: Simple Construction and Continuous Construction", British Standard Institution.

(12) BROWN D M, and ANG A, 1966
"Structural Optimisation by Non-Linear Programming", Proc ASCE, Jnl Struc Div, Vol 92, pp 319 - 340.

(13) CARROLL C W, 1961
"The Created Response Surface Technique for Optimising Non-Linear Restrained Systems", Operations Res, Vol 9, No 2, pp 169 - 184.

(14) CELIK T, 1977
"Elastic-Plastic Analysis of Complete Structures with Shear Walls and Frames", PhD Thesis, Univ of Aston in Birmingham.

(15) CDC FORTRAN, 1978
"Extended Version 4", Reference Manual, Chapter 8, Section 4, UMRCC.

(16) CHANG P Y and PILKEY W D, 1971
"Efficient, General Method for the Analysis of Grillages", J Ship Res, Vol 15, No 2, June, pp 153 - 163.

(17) CLARKSON J, 1965
"The Elastic Analysis of Flat Grillages", Cambridge University Press.

(18) CLOUGH R W, KING I P, and WILSON E L, 1964
"Structural Analysis of Multi-Storey Buildings", Proc ASCE, Vol 90, ST3, June, pp 16 - 34.

(19) CORNELL C A, REINSCHMIDT K F, and BROTCHIE J F, 1966a
"A Method for the Optimum Design of Structures", Int Symp on the Use of Computers in Structural Eng, Newcastle-Upon-Tyne.

(20) CORNELL C A, REINSCHMIDT K F, and BROTCHIE J F, 1966b
"Iterative Design and Structural Optimisation", Proc ASCE, Vol 92, No ST6.

(21) COULL A, and STAFFORD S B, (EDS), 1967
"Tall Buildings with Particular Reference to Shear Wall Structures", Proc Symp on Tall Buildings, Univ of Southampton, pp 139 - 155, Pergamon, London.

(22) COULL A, and STAFFORD S B, 1973
"Structural Analysis of Tall Concrete Buildings", Proc Inst Civ Engrs, Pt 2, Vol 55, March, pp 151 - 166.

(23) COURANT R, 1943
"Variational Methods for the Solution of Problems of Equilibrium and Vibrations", Bull Am Math Soc, Vol 49, pp 1 - 23.

(24) CROXTON P C L , 1974
"The Analysis of Complete Structures Consisting of Bare Frames, Shear Walls and Plate Components", PhD Thesis, The Univ of Aston in Birmingham.

(25) DANTZIG G B, 1953
"Computational Algorithm of the Revised Simplex Method", RM - 1266, The RAND Corp.

(26) DANTZIG G B, 1963
"Linear Programming and Extensions", Princeton University Press, Princeton, N J.

(27) DAVIES C, 1975
"Steel Concrete Composite Beams for Buildings", George Godwin, London.

(28) DAVIS, BELFIELD and EVEREST, 1980
"Measured Rates", Civil Engineering, London, April.

(29) DOBBS M W, and FELTON L P, 1969
"Optimisation of Truss Geometry", Proc ASCE, Vol 95, No ST10, pp 2105 - 2118.

(30) DORN W S, GOMORY R E, and GREENBERG H J, 1964
"Automatic Design of Optimal Structures", Journal de Mecanique, Paris, Vol 3, No 1, March.

(31) EVANS H R, and SHANMUGAM N E, 1979
"Approximate Grillage Approach to the Analysis of Cellular Structures", Proc Inst Civ Engr, Vol 67, Pt 2, March, pp 133 - 154.

(32) FABER J, and JOHNSON J, 1976
"Foundation Design Simply Explained", Eby House, London.

(33) FAULKNER D, et al, 1973
"Synthesis of Welded Grillages to Withstand Compression and Normal Loads", Comp and Struc, Vol 3, March, pp 221 - 246.

(34) FIACCO A V, and McCORMICK G P, 1964
"The SUMT Primal Dual Method", Management Science, Vol 10, No 4.

(35) FINTEL M, et al, 1971
"Response of Buildings to Lateral Forces", J Am Conc Inst, Vol 68, Feb, pp 81 - 106.

(36) GALLAGHAR R H, and ZIENKIEWICZ (EDS), 1974
"Optimum Structural Design: Theory and Application", John Wiley, London.

(37) GARVIN W W, 1960
"Introduction to Linear Programming", McGraw-Hill, New York.

(38) GOLDBERG J E, 1966
"Analysis of Multi-Storey Buildings Considering Shear Wall and Floor Deformations", Tall Building, Proc Symposium on Tall Buildings, Univ of Southampton, April, Pergamon Press, London 1967.

(39) GOLDFARB D, and LAPIDUS L, 1968
"Conjugate Gradient Method for Non-Linear Programming Problems with Linear Constraints", Ind Engng Chem Fundls, Vol 7, No 9.

(40) GOMORY R A, 1958
"An Algorithm for Integer Solutions to Linear Programs", Technical Report No 1, Princeton - IBM Mathematical Research Project.

(41) GRIFFITH R E, and STEWART R A, 1961
"A Non-Linear Programming Technique for the Optimisation of Continuous Processing Systems", Management Science, Vol 7, No 4, pp 379 - 392.

(42) HADLEY G, 1962
"Linear Programming", Addison-Wesley, Reading, Mass.

(43) HADLEY G, 1964
"Non-Linear and Dynamic Programming", Addison-Wesley, Reading, Mass.

(44) HAMBLY E C, and PENNELLS E, 1975
"Grillage Analysis Applied to Cellular Bridge Decks", Struct Engr, Vol 53, No 7, July.

(45) HARRIS P J, 1972
"Analysis of Rectangular Grillages", Eng Jr (Montreal), Vol 55, No 9, Sept.

(46) HORNE M R, and MAJID K I, 1966
"Elastic-Plastic Design of Rigid Jointed Sway Frames by Computers", Research Report No 1, Manchester Univ, Dept of Civil Eng, March.

(47) ICL FORTRAN, 1976
"Compiler Libraries, 1900 Series", Chapter 3, Fortran Direct Access Backing Store Package, The Computer Centre, University of Aston in Birmingham.

(48) ISLAM M A, 1978
"Optimum Design of Sway Frames", PhD Thesis, Univ of Warwick.

(49) JENNINGS A, and MAJID K I, 1965
"An Elastic-Plastic Analysis by Computer for Framed Structures Loaded up to Collapse", The Structural Engineer, Dec, No 12, Vol 43, pp 407 - 412.

(50) JENNINGS A, and MAJID K I, 1966
"The Computer Analysis of Space Frames Using Sparse Matrix Techniques", Paper C4, Int Conf on Space Structures, Univ of Surrey.

(51) JENNINGS A, 1966
"A Compact Storage Scheme for the Solution of Symmetric Linear Simultaneous Equations", The Comp Jn, Vol 9, pp 281 - 285.

(52) KAVLIE D, KOWALIK J, LUND S, and MOE J, 1966
"Design Optimisation Using a General Non-Linear Programming Method", Eur Shipbldg, Vol 15, No 4.

(53) KAVLIE D, and MOE J, 1969
"Application of Non-Linear Programming to Optimum Grillage Design with Non-Convex sets of Variables", Int J for Num Meth in Engr, Vol 1, pp 351 - 377.

(54) KELLEY J E, 1960
"The Cutting-Plane Method for Solving Complex Programs", J Soc Indus Appl Math, Vol 8, No 4, pp 703 - 712.

(55) KIRSCH U, 1981
"Optimum Structural Design", McGraw-Hill, N J.

(56) KRISHNAMOORTHY C S, and MOSI D R, 1979
"A Survey on Optimal Design of Civil Engineering Structural Systems", Engineering Optimisation, Vol 4, pp 73 - 88, London.

(57) LAZARIDES T O, 1952
"The Design and Analysis of Openwork Prestressed Concrete Beam Grillages", Civ Eng and Pubic Works Review, London, Vol 47, No 552, June, pp 471 - 473.

(58) LIGHTFOOT E, and SAWKO F, 1959
"Structural Frame Analysis by Electronic Computer: Grid Frameworks Resolved by Generalised Slop-Deflection", Engineering, Jan, pp 18 - 20.

(59) LIPSON S L, and AGRAWAL K M, 1974
"Weight Optimisation of Plane Trusses", Proc ASCE, Vol 100, No ST5.

(60) LIVESLEY R K, 1956
"The Application of an Electronic Computer to some problems of Structural Analysis", The Structural Engineer, Vol 34.

(61) LPMK2, 1970
"Linear Programming Mark 2", written for ICL 1900 Series, Computer Centre, University of Aston.

(62) MAJID K I, and WILLIAMSON M, 1967
"Linear Analysis of Complete Structures by Computers", Proc Inst Civ Engrs, Vol 38, Oct, pp 247 - 266.

(63) MAJID K I, and ANDERSON D, 1968a
"The Computer Analysis of Large Multi-Storey Structures", The Structural Engineer, Nov, No 11, Vol 46, pp 357 - 365.

(64) MAJID K I, and ANDERSON D, 1968b
"Elastic-Plastic Design of Sway Frames by Computer", Proc Inst Civil Engrs, Vol 41, Dec.

(65) MAJID K I, and CROXTON P C L, 1970
"Wind Load Analysis of Complete Building Structures by Influence Coefficients", Proc Inst Civ Engrs, Vol 47, Oct, pp 169 - 184.

(66) MAJID K I, and ANDERSON D, 1972
"Optimum Design of Hyperstatic Structures", Int J for Num Meth in Engr, Vol 4.

(67) MAJID K I, 1973
"Optimum Design of Structures (Review)", Proc International Symposium on Optimisation of Civil Eng, Liverpool, July.

(68) MAJID K I, and ELLIOTT D W C, 1973a
"Forces and Deflections in Changing Structures", The Struct Engr, Vol 51, No 3.

(69) MAJID K I, and ELLIOTT D W C, 1973b
"Topological Design of Pin Jointed Structures by Non-Linear Programming", Proc ICE (London), Vol 55, Part 2.

(70) MAJID K I, and ONEN Y H, 1973
"The Elasto-Plastic Failure Load Analysis of Complete Building Structures", Proc Inst Civ Engrs, Vol 55, Sept, pp 619 - 634.

(71) MAJID K I, 1974a
"Optimum Design of Structures", Newnes-Butterworths, London.

(72) MAJID K I, 1974b
"The Effect of Complete Composite Action on the Elasto-Plastic Analysis of Complete Building Structures", Finite Element Methods in Engineering, The Univ of New South Wales.

(73) MAJID K I, and SAKA M P, 1977
"Optimum Shape Design of Rigidly Jointed Frames", Proc of the Symp on Appl of Computer Meth in Engr, School of Engr, Univ of Southern California, Los Angeles, Aug.

(74) MAJID K I, 1978
"Theory of Structures with Matrix Notation", Newnes-Butterworths, London.

(75) MAJID K I, and CELIK T, 1979
"Bending, Shear and Torsion in Deep Rectangular Reinforced Concrete Panels", Proc Inst Civ Engrs, Part 2, Vol 67, Sept, pp 75 - 769.

(76) MAJID K I, STOJANOVSKI P, and SAKA M P, 1980
"Minimum Cost Topological Design of Steel Sway Frames", The Structural Engr, Vol 58B, No 1, March.

(77) MAJID K I, 1980
"Introduction to Matrix and Numerical Methods for Civil Engineers", Chapter 4, Woodstock, Oxford.

(78) MAJID K I, and OKDEH S, 1982
"Limit State Design of Sway Frames", The Structural Engineer, Dec, Vol 60B, No 4.

(79) MOSES F, 1964
"Optimum Structural Design Using Linear Programming", Proc ASCE, Vol 90, No ST6, Dec, pp 89 - 104.

(80) MOSES F, and ONODA S, 1969
"Minimum Weight Design of Structures with Application to Elastic Grillages", Int J for Num Meth in Engr, Vol 1, pp 311 - 331.

(81) MOY F C, 1974
"Control of Deflections in Unbraced Steel Frames", Proc Inst Civ Engrs, Vol 57, Dec.

(82) MOY F C, 1976
"Multi-Storey Frame Design Using Storey Stiffness Concepts", Proc ASCE, Vol 102, No ST6, June.

(83) NEEDHAM F H, 1977
"The Economics of Steelwork Design", The Structural Engineer, Vol 55, No 9, Sept.

(84) OKDEH S, 1980
"Design of Sway Frames", PhD Thesis, Univ of Aston in Birmingham.

(85) ONEN Y H, 1973
"Elastic-Plastic Analysis of Complete Building Structures", PhD Thesis, Univ of Aston in Birmingham.

(86) PEDERSEN P, 1973
"Optimal Joint Positions for Space Trusses", Proc ASCE, Vol 99, No ST12.

(87) PRAGER W, and SHIELD R T, 1967
"A General Theory of Optimal Plastic Design", J of Appl Mech, Vol 34, No 1, March, pp 184 - 186.

(88) PRAGER W, and ROZVANY G I N, 1977
"Optimal Layout of Grillages", J Struc Mech, Vol 5, No 1, pp 1 - 18.

(89) REINSCHMIDT K F, and NORABHOOMPIPAT T, 1975
"Structural Optimisation by Equilibrium Linear Programming", Proc ASCE, Vol 101, No ST4, April, pp 921 - 938.

(90) REYNOLDS C E, and STEEDMAN J C, 1976
"Reinforced Concrete Designer's Handbook", Cement and Concrete Association, London.

(91) ROSEN J B, 1960
"The Gradient Projection Method for Non-Linear Programming",
Part I - Linear Constraints, J Soc Indus App Math, Vol 8, 1960.
Part II - Non-Linear Constraints, J Soc Indus Appl Math, Vol 9, 1961.

(92) ROZVANY G I N, and ADIDAM S R, 1972
"Rectangular Grillages of Least Weight", Proc ASCE, Vol 98, No EM6, Dec, pp 1337 - 1351.

(93) ROZVANY G I N, 1973
"Optimal Plastic Design for Partially Preassigned Strength Distribution", JOTA, Vol 11, No 4, April, pp 421 - 436.

(94) ROZVANY G I N, GANGADHARAIAH C, and HILL R D, 1975
"Optimal Slabs and Grillages of Constrained Geometry", Proc ASCE, Vol 101, No EM6, Dec, pp 755 - 770.

(95) ROZVANY G I N, and HILL R D, 1979
"Computer Alogorithm for Deriving Analytically and Plotting Optimal Structural Layout", Computer and Structures, Vol 10, No 1 - 2, April, pp 295 - 300.

(96) ROZVANY G I N, 1979
"Optimal Beam Layouts: Allowance for Cost of Shear", Comp Meth Appl Mech Engr, Vol 19, No 1, June, pp 49 - 58.

(97) SAKA M P, 1975
"Optimum Design of Structures", PhD Thesis, University of Aston in Birmingham.

(98) SAWKO F, 1965
"Bridge Deck Analysis - Electronic Computers Versus Distribution Methods", Civ Eng and Publ Works Rev, Vol 60, No 705, April, pp 534 - 538.

(99) SAWKO F, and MOSLEY W H, 1969
"Grillage Analysis of Composite Box Girder Bridge Decks", Civil Engr and Publ Works Review, Vol 64, No 759, Oct, pp 979 - 981.

(100) SAWKO F, and WILLCOCK B, 1967
"Computer Analysis of Bridges Having Varying Section Properties", Struct Engr, Vol 45, No 11, Nov.

(101) SHEU C Y, and PRAGER W, 1968
"Recent Developments in Optimal Structural Design", Applied Mechanics Review, Vol 21, No 10, Oct.

(102) SHIGLEY J E, 1976
"Applied Mechanics of Materials", McGraw-Hill, New York.

(103) STEVENS L K, 1964
"Direct Design by Limiting Deformations", Proc Inst Civ Engrs, Vol 16, July.

(104) TEMPLEMAN A B, 1971
"Structural Design for Minimum Cost Using the Method of Geometric Programming", Proc ICE, Vol 46, pp 459 - 472.

(105) TIMOSHENKO S P, 1955
"Strength of Material Part 1, Elementary Theory and Problem", 3rd Ed, D Van Nostrand Co, Inc, New York, pp 289 - 290.

(106) TOAKLEY A R, 1968
"The Optimum Design of Triangulated Frameworks", Int J Mech Sci, Vol 10.

(107) ZIA P, 1968
"Torsion Theories for Concrete Members", Symposium on Torsion of Structural Concrete, Public SP-18, ACI, Michigan.

(108) ZOUTENDIJK G, 1960
"Methods of Feasible Directions", Elsevier, Amsterdam.

(109) ZOUTENDIJK G, 1966
"Non-Linear Programming: A Numerical Survey", J Soc Indus Appl Math, Vol 4, No 1.