ON THE MEASUREMENTS OF THE VELOCITY OF LONGITUDINAL WAVES IN WATER CONTAINED IN GLASS AND ALUMINIUM TUBES, AT LOW FREQUENCIES

awarded the degree of M.Phil

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CONTENTS

ABSTRACT

An experimental arrangement for measuring the velocity of sound in water by developing an acoustic standing wave in vertical water columns contained in glass and aluminium tubes had been established at low frequencies.

The excitation of the water columns into vibrations had been done using a diaphragm attached at the bottom of the tubes and driven by a loudspeaker moving coil.

In the case of the glass tube, the frequency developing half wave length as the height of a water column had been determined by detecting a maximum excess pressure amplitude at the midway of the water column, using a small-single piezoelectric ceramic disc-microphone (hydrophone) situated axially at midway of the water column.

In the case of the aluminium tubes, the same frequency had been determined by the same way as in the glass tube (with modifications of the hydrophone and diaphragm arrangements), and also by detecting the minimum current drawn by the sound source indicating resonance conditions for both the diaphragm and the water column.

At those frequencies, the law $C = \lambda_i . f$, had been applied as $C = \lambda_i . f_i$, for different heights of water columns and independent of the resonance frequency of the diaphragm.

In the case of the aluminium tubes, the diaphragm resonant frequency in air and in water had been obtained, also the height of water columns which resonated at the resonant frequency of the diaphragm in water had been determined.

Correction formulae had been applied to both tubes for the measured values of velocity of sound to reach the value of velocity in bulk water due to the yielding of the tube walls.

The water used in the glass tube was tap water while for the aluminium tubes, distilled water had been used.

CHAPTER 1

1.1 History of the Problem

The study of the problem began in 1847, when Wertheim experimentally found that the velocity of sound in an organ pipe immersed in water was reduced to 1173 m/s and he put forward the theory that the water in the pipe behaved as a quasi-elastic solid.

In 1848, Helmholtz by a general discussion, showed that this theory could not be correct and returned such a reduction due to yielding of the pipe walls.

In 1898, H. Lamb[(1)], obtained an expression for the correction of the velocity due to the give of the walls of the tube from the equations of the vibrations of the tube and the hydrodynamical equations of the liquid, on the assumption that the liquid fills the tube completely.

In 1923, H.G. Green[(2)], made a theoretical investigation for the problem based also on the give of the walls and obtained a formula which when he applied it on published well known experimental results had confirmed Helmholtz's theory.

In 1927, T.H. Gronwall[(3)] made another theoretical investigation based on Lamb's work and in connection with an experimental investigation made by L.G. Pooler[(4)] who applied the new formula obtained by Gronwall for the velocity correction in his experimental results.

All such works lead to a single equation for the velocity correction as:-

$$C^2 = \frac{C_o^{\,2}}{1 + \rho_o \cdot \alpha \cdot C_o^{\,2}} \quad \text{or} \quad C_o^{\,2} = \frac{C^2}{1 - \rho_o \cdot \alpha \cdot C^2} \text{, where:-}$$

ρ_o = density of the liquid

C = actual velocity measured through the liquid in the tube.

C_o = theoretical velocity in bulk liquid.

While the value of the correction factor (α) differs from one to another depending on how the problem had been tackled and the assumed boundary condition.

The main difference was in solving the equations for the axial and radial displacements of the tube, where the work of Green had limited the expansion of Bessel functions to single terms, giving the previous equation independent of frequency, while he stated that considering higher terms, the effect of frequency will be considered; while Gronwall had considered the series expansion in his formula.

(Later on in this work, it appeared that Gronwall's formula stopped at the fifth term, while the sixth term was not suitable for the velocity correction and hence by comparison with Green's formula the difference reported was 2% maximum).

The main technical aspects in this history are:-

(1) The waves propagated through the liquid in the tubes should be appreciably plane.

(2) The material of the tube used or directly the high sensitivity of the obtained formulae to the value of Poisson's ratio.

(3) The range of the thickness of the tube walls.

Also the form of solution for the radial and axial displacements of the tubes in the form of the direct and modified forms of solutions of Bessel functions had been both reported.

CHAPTER 2

Equipment used and the Block Diagram

a - The Glass Tube:-

1. The Sound Source

The sound source is a moving coil loudspeaker of 8Ω nominal impedance and of 6 in. advertised diameter.

It had been arranged for transmission under water by removing its paper cone and attaching instead a phosphor bronze diaphragm of 0.005 in. thick with a small aluminium mass load attached to its centre by a screw in order to couple it to the moving coil.

The following diagram shows a sketch of the arrangement:-

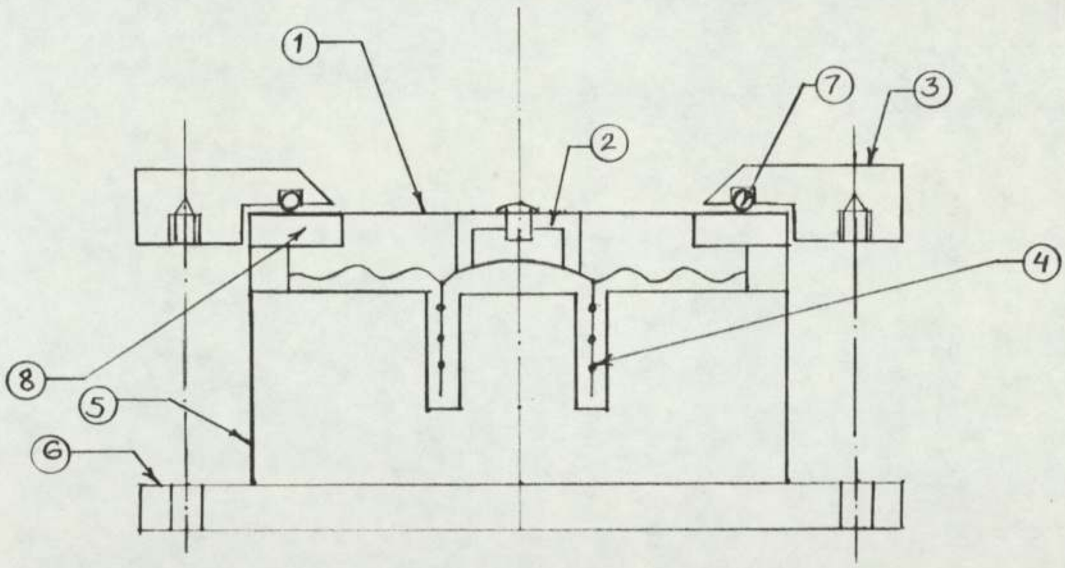


FIG. (II-1)

(1) Diaphragm.

(2) Small aluminium mass load.

(3) Aluminium triangular flange.

(4) Moving coil.

(5) Magnet casing.

(6) Plastic clamping triangular plate.

(7) An 'O' rubber ring for water seal.

(8) Aluminium bushing.

The diaphragm had been clamped at its edges between the bushing and the aluminium flange.

2 - The Microphone

The microphone used with the glass tube is of the ceramic type containing one PZT-4 disc of 20 mm. diameter and 5 mm. thickness. The casing is made of brass of an overall dimension of one inch in both diameter and height.

The arrangement had been made so that the axis of the test tube was parallel to the thickness of the ceramic disc, i.e. perpendicular to the major faces of the disc.

The microphone was tightly connected to the lower end of the central axial brass tube through which the signal cable passes.

This tube has an O.D. of $\frac{1}{8}$ of an inch and was held through a holding mechanism which was heavily weighted by a brass plate closing the upper end of the test tube.

The microphone has a resonant frequency of 500 k.Hz.

3 - The Audio Frequency Power Amplifier

The audio frequency power amplifier is a vacuum tube power amplifier having an output stage of EL-34 tubes in push-pull operation. Its rated output is 20 watts, and its output impedance was matched to the nominal impedance of the loudspeaker (8Ω).

For an input voltage of 3 m.v., an output of 210 m.v. was obtained under load conditions, while the open circuit voltage for the same input was 225 m.v.

This gives an average voltage amplification of 70.

The amplifier has a flat response through the audio range of frequency, under load conditions.

4 - The Wide Range Frequency Generator

It is the Marconi frequency generator of maximum attenuated output impedance of 600Ω and a frequency range of 10 Hz. up to 10 MHz.

The output voltmeter has two scales, $0 \rightarrow 3$ & $0 \rightarrow 10$. The attenuated voltage ranges are from $0 \longrightarrow 3$ m.v. up to $0 \longrightarrow 3$ volts, while the direct voltage ranges are $0 \longrightarrow 10$ and $0 \longrightarrow 30$ volts. Such ranges were available in both sine and square wave forms.

5 - The Digital Frequency Counter

It is the advance TC-11A frequency counter for measuring frequencies up to at least 15 MHz. on 4 digital display with maximum gate time of 1 second (1 Hz. resolution).

The instrument may also be used for counting and timing measurements using the start/stop gate facilities. An internal standard of 100 k.Hz is supplied and is accurate to 1 part in 10^5.

6 - The a.c. Wide Range Voltmeter

It is the Farnell TM-4 voltmeter having a voltage range from $0 \longrightarrow 0.3$ m.v. up to 100 volts, at various frequencies up to 25 MHz.

The meter has two scales $0 \longrightarrow 3$ and $0 \longrightarrow 10$, and both were calibrated in R.M.S. values of sinusoidal inputs, although the instrument is fundamentally mean rectified reading. For inputs which are not sinusoidal, the mean value can be calculated as the indicated reading divided by 1.111. The meter has an output from the rear panel of 1 volt and output impedance nominally of 50Ω for pen recorder. The instrument is fitted with high frequency square wave output via BNC socket used to assist on with compensating probe to TM-4 input. The output gives 300 m.v. deflection on the meter.

7 - The Double Beam C.R.O.

It is the Advance OS-250, 10 MHz oscilloscope. Two identical input channels. The bandwidth at - 3 dbs. is d.c. $\longrightarrow$ 10 MHz (2 Hz $\longrightarrow$ 10 MHz, on a.c.).

The sensitivity is 5 m.v./Cm. up to 20 v/Cm. The input impedance is 1 MΩ/approx. 28 P.f. and accuracy of $\pm$ 5%. The time base from 1 u.sec./Cm. up to 0.5 sec./Cm, and the instrument has an output of 1 volt of supply line frequency (50 Hz) from its socket at the front panel for calibration purposes.

8 - The Test Glass Tube

The tube is of $\frac{1}{4}$ in. thick walled glass tube on the average and of 3 in. I.D. It included three sections having lengths of 12,40 and 10 inches respectively.

The tube was assembled in two configurations, one of which included the 12 and 40 in. sections respectively from the brass base plate upon which the tube was vertically assembled and supported.

The other configuration included the three sections in the same order from the base plate.

The upper section of this configuration (10 in.) has a central horizontal aperture which can be used as water exit under water flow condition or connecting the tube to the atmosphere under standstill conditions, which was the case of the experiment.

The first configuration was used for the experimental verification of the water column boundary conditions, while the other one had been used for the investigation of the sound velocity measurement in tap water in the tube.

The Block Diagram

The following diagrams show the arrangements for each configuration:-

First tube configuration

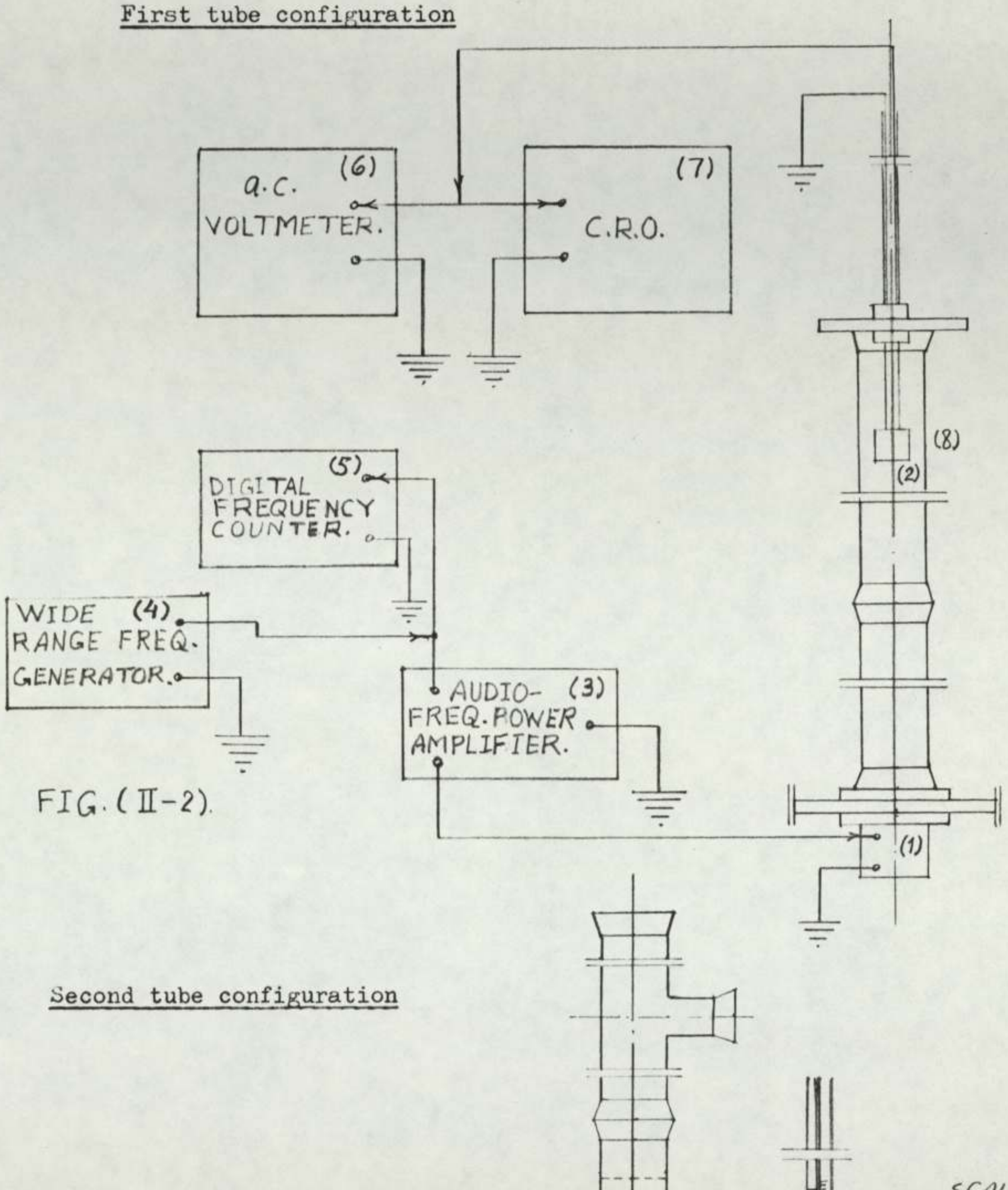


FIG. (II-2).

Second tube configuration

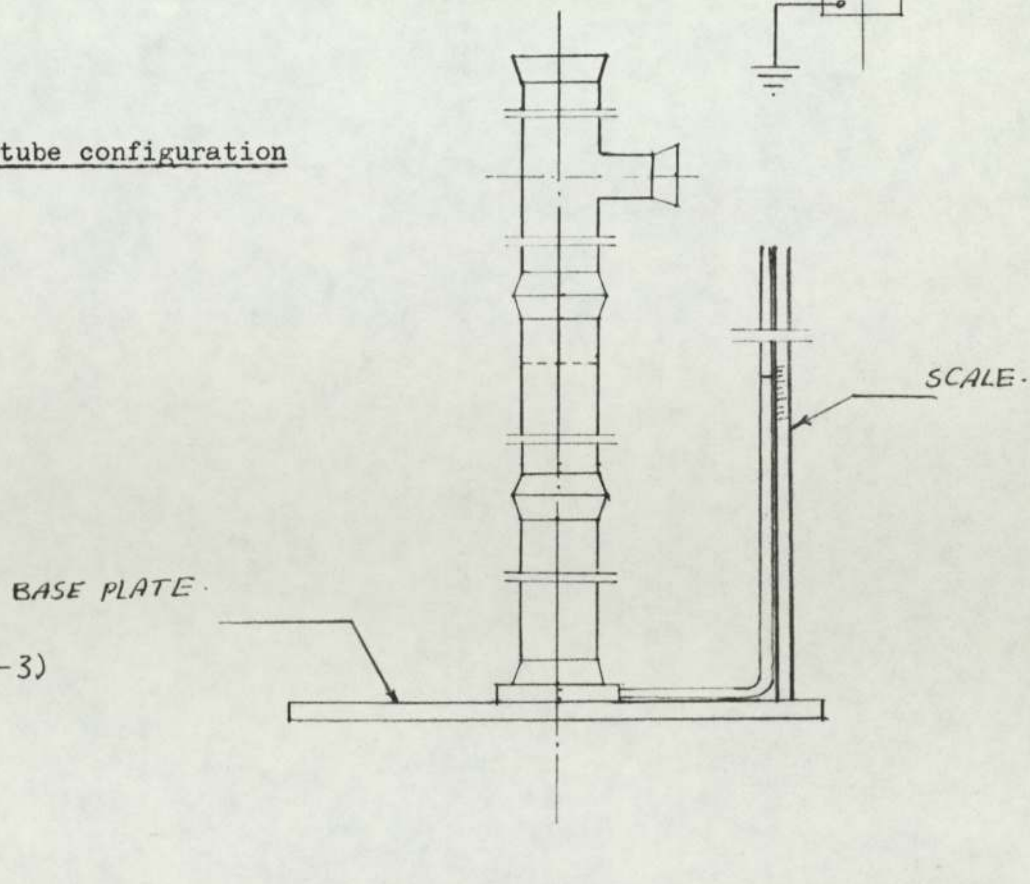


FIG. (II-3)

For both configurations, the water was fed through a plastic tube connected to a brass ring under the test tube, and acting at the same time as a simple manometer for determining the height of the water column inside the tube and the readings were taken on an attached scale the zero of which is the upper face of the brass base plate.

Each section is connected to the other through an $\frac{1}{8}$ in. thick rubber gasket and clamped together by means of bolted cast iron flanges. The detailed and scaled drawings will be obtained in the next section.

b - The Aluminium Tubes:-

Two aluminium tubes had been used having 0.065 and 0.13 in. wall thickness respectively. The length of each tube was 150 cms.

The same equipment used in the case of the glass tube had been applied, with the following modifications:-

1 - The diaphragm had been doubled in thickness i.e. 0.01 in. and also modifications in its attachment to both the loudspeaker and the test tubes had been done for obtaining new clamped boundary conditions.

2 - The hydrophone had been re-arranged without any casing and also prepared so that the axis of the test tube was parallel to the major faces of the ceramic disc.

3 - A 0.5Ω resistor had been inserted in series with the loudspeaker moving coil so that the current in the moving coil is picked up as voltage across this resistor.

4 - The hydrophone (microphone) holding mechanism had been removed and the hydrophone brass tube had been held by a moving slider on an outside stand so that the upper end of the test tube is clear and free.

5 - The brass base plate had been replaced by an aluminium one to suit the new modifications and equiped with three vertical screws for horizontal adjustment with respect to the ground.

6 - The a.c. voltmeter (6) had been used to measure both the output voltage of the microphone and the loudspeaker series resistor voltage (.5 Ω), alternatively.

The following block-diagram shows the arrangement:-

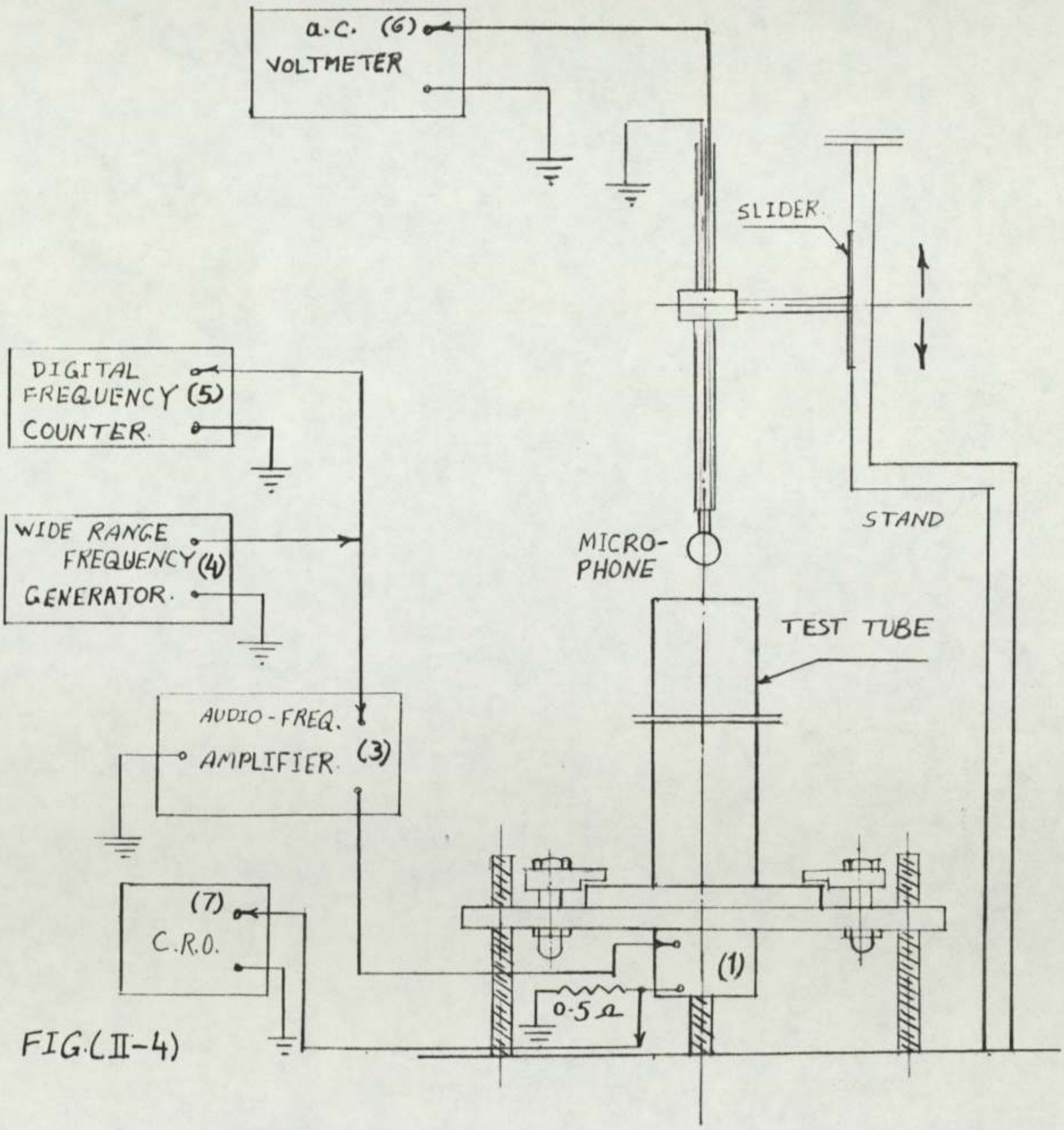


FIG.(II-4)

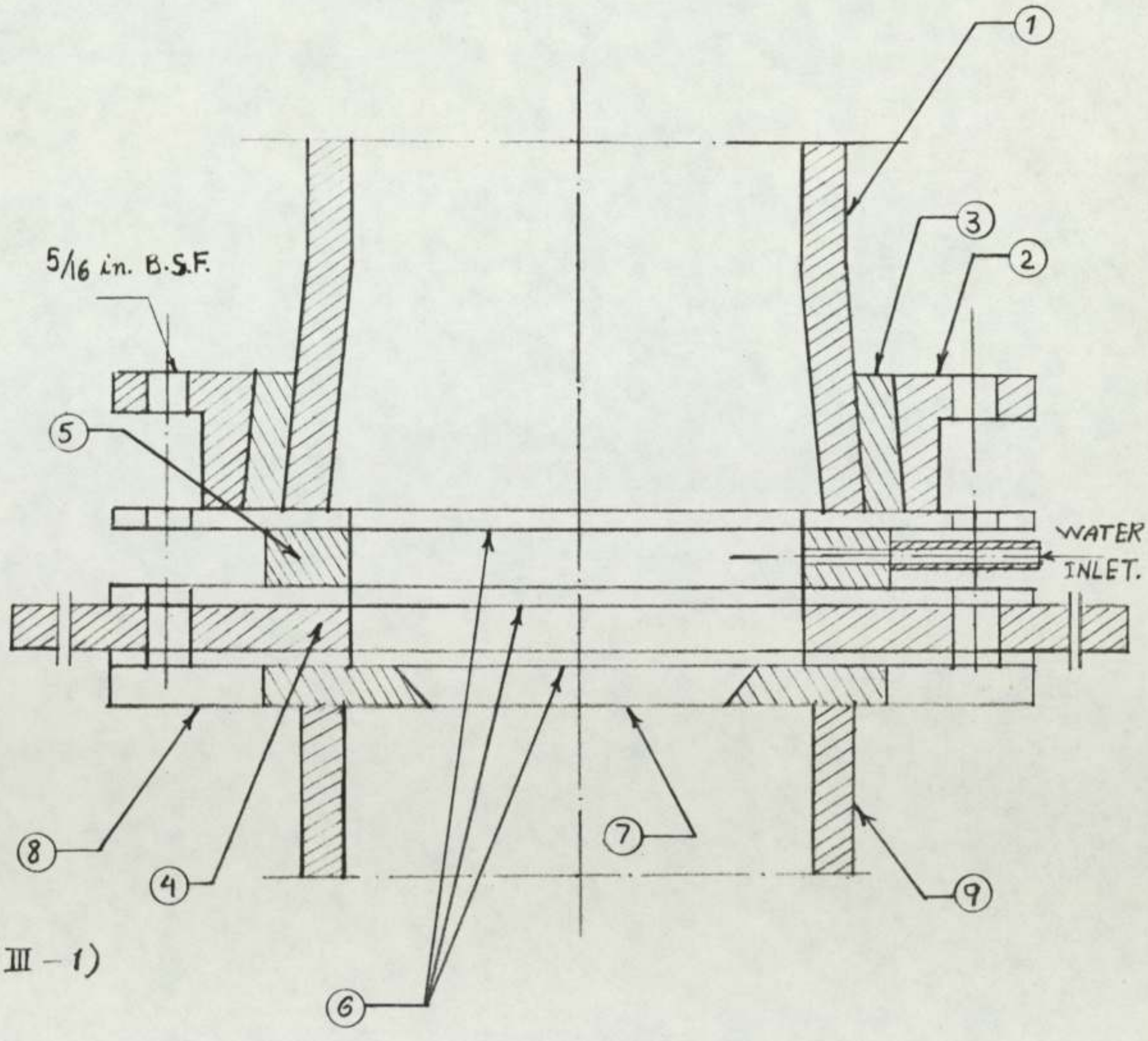
CHAPTER 3

III.1 The Experimental Verification of the Water Column Boundary Conditions

a - The Test Glass Tube:-

The following diagram shows a detailed and scaled section for the arrangement of the lower end of the glass tube with the diaphragm attached.

1. The test glass tube
2. The lower flange
3. Asbestos sheet for tightening
4. The brass-base plate



FIG(III - 1)

5. The brass ring for water inlet (or draining)
6. $\frac{1}{8}$" thick rubber gaskets
7. The diaphragm
8. The aluminium flange of the loudspeaker
9. The aluminium bushing of the magnetic casing extension

The figure will lead to a lower end of the water column as shown in the following diagram:-

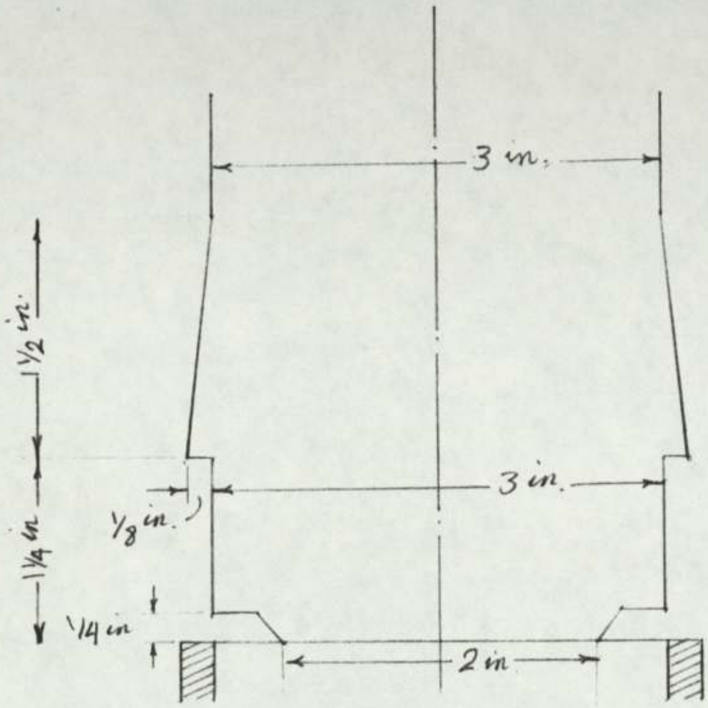


FIG.(III - 2)

Neglecting the small inclination of the lower end of the glass tube, this will lead to the following approximated figure:-

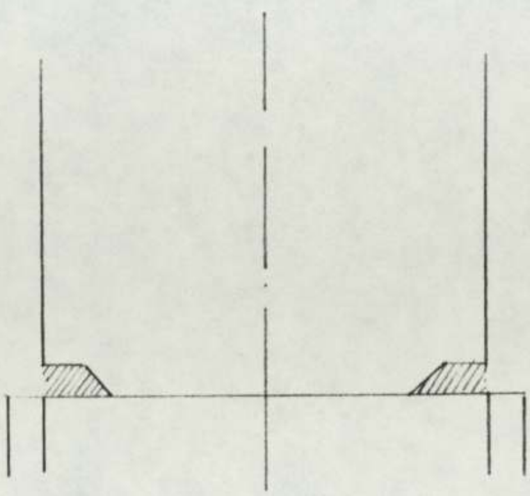


FIG.(III - 3)

The hatched cone of the aluminium flange had left an effective diameter for the diaphragm of 2".

Neglecting the depth of the water cone enclosed by the hatched aluminium cone w.r. to the long wavelength the lower end of the water column can be considered opened to the diaphragm which was actually flushing into the water at nearly Z = 0.

The detailed and scaled diagram for the upper end of the glass tube was as follows for both configurations:-

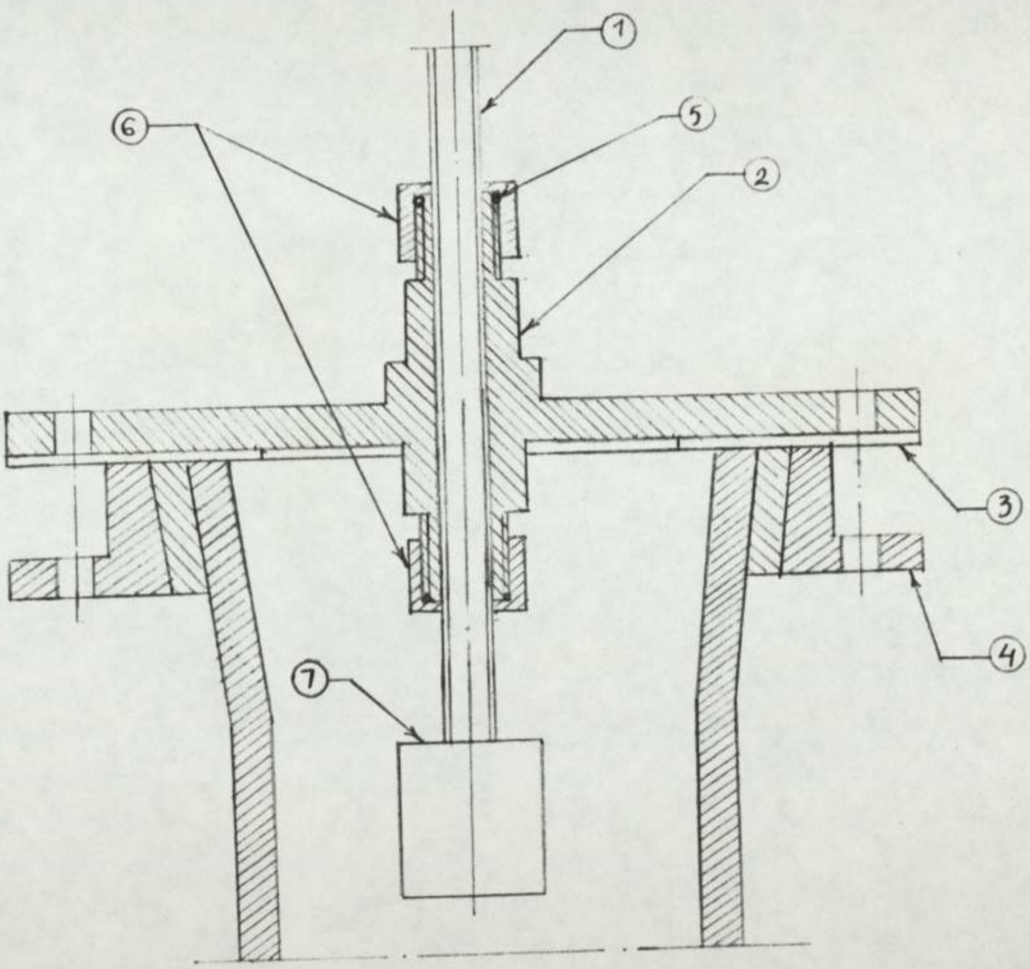


FIG.(III - 4)

1. The axial central brass-tube
2. The hydrophone holding mechanism
3. Rubber gasket
4. The upper-end flange
5. Rubber ring for holding the brass tube
6. Tightening nuts
7. The hydrophone

The first configuration had been assembled and the following experiment had been done as an experimental verification of the water column boundary conditions. The water was fed from the bottom till the top and then the tube was closed by the microphone holding mechanism.

There was no air region inside the tube and the water surface touched the lower surface of the brass plate, while the microphone was in upper position.

At first the microphone had been placed midway of the water column and a frequency scanning in the range of 100 ⟶ 1000Hz had been made. A frequency of 440 Hz at which the hydrophone in its midway position gave a maximum reading had been recorded.

Then a vertical scanning by the hydrophone (microphone) had been made at 440 Hz and its reading had been recorded at each vertical position down the tube. The unamplified input signal to the loudspeaker from the wide range frequency generator was 1 m.v.

The height of the water column obtained from this configuration was 135.5 cms. The results obtained were recorded in the following table and plotted in fig. (III-5).

TABLE III-1

Z	17	22	27	32	37	42.5	50	55.5	62	67
V_R	17	20.5	24.5	27.5	32.5	35.5	38.5	40.5	41.5	42
Z	75.5	86.8	98.5	108.5	117.5	125.5	130.5	132.5	138	-
V_R	42	40	36	31	22	14.25	10	6	1.8	-

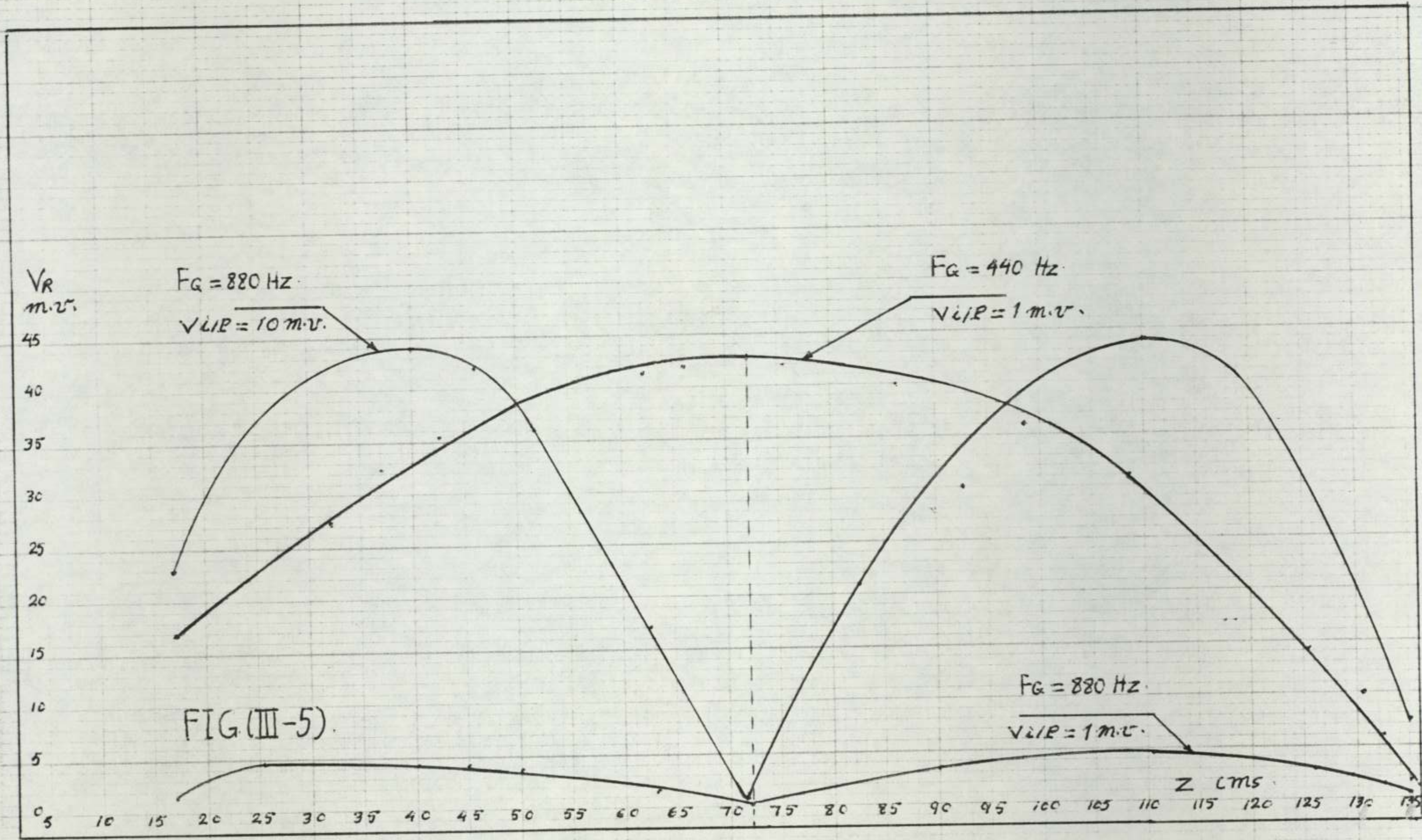
The experiment had been repeated for double the frequency (880 Hz) and at two un-amplified input signals of 1 and 10 m.v. respectively and the results were recorded in the following tables and plotted in figure (III-5).

TABLE III-2

Z	17	25.5	40	45	50	63	72
V_R	2.3	4.5	4.4	4.2	3.75	1.7	0.15
Z	90	110.5	126	129.5	132.5	135	--
V_R	3.5	4.7	2.9	2.1	1.4	0.6	--

TABLE III-3

Z	17	40	46	51.5	62.5
V_R	23	44	42	36	17
Z	71.5	82.5	92.5	110	135
V_R	0.4	21	30	44	7.3



V_R
m.v.
$F_G = 880$ Hz
$\overline{V_{i/P}} = 10$ m.v.
$F_G = 440$ Hz
$\overline{V_{i/P}} = 1$ m.v.
$F_G = 880$ Hz
$\overline{V_{i/P}} = 1$ m.v.
Z cms
45
40
35
30
25
20
15
10
5
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
105
110
115
120
125
130
135

FIG (III-5).

The chosen frequency determined a pressure node nearly at the midway of the water column instead of the fundamental pressure-antinode at the same point.

Also the suggested inputs were chosen to determine the relative node-antinode pressures with input power at the frequency of 880 Hz, and hence the following experiment had been done:-

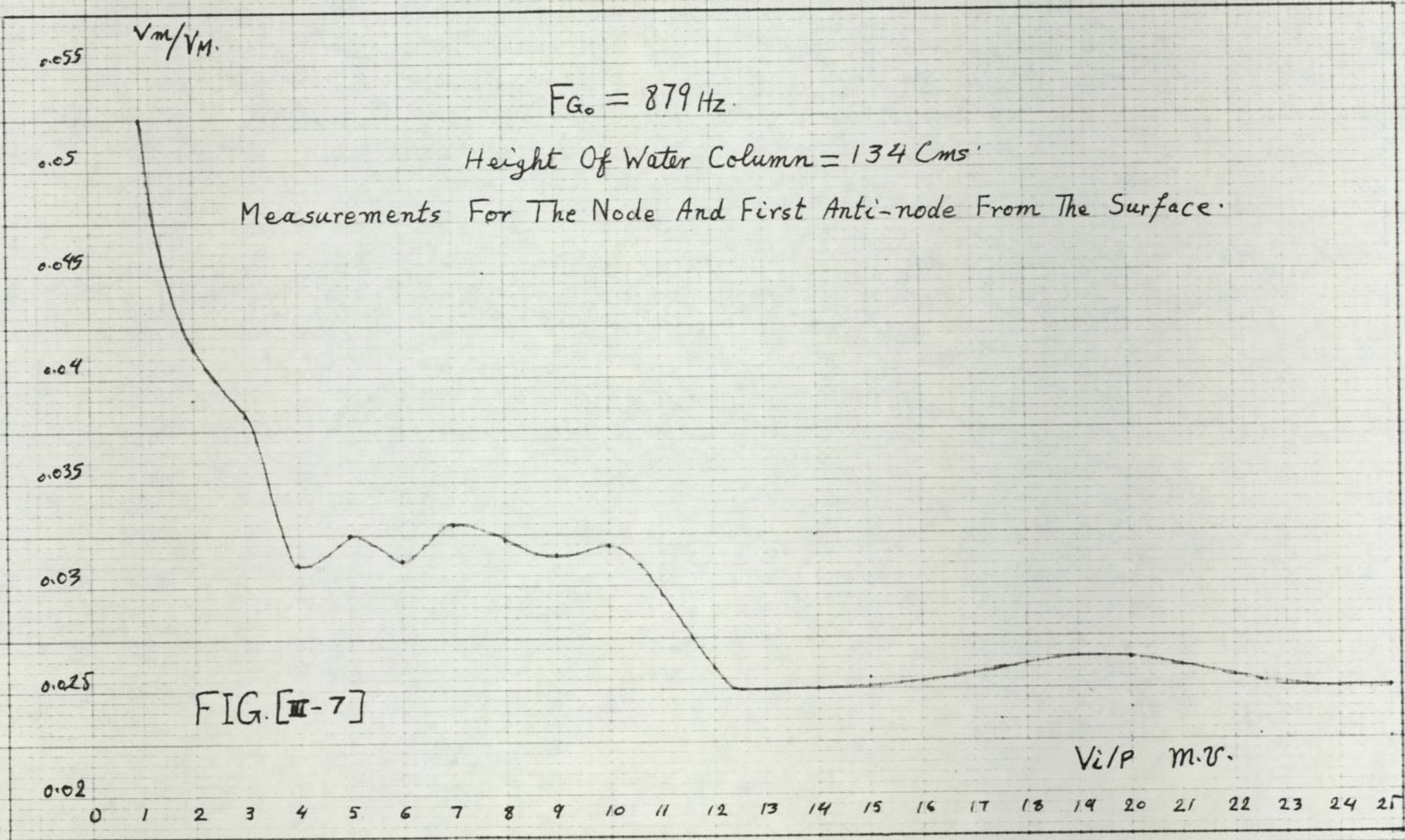
With the same test tube configuration, the hydrophone had been placed at the first antinode from the water surface and the midway pressure node respectively for each input voltage and the results were recorded in the following table and plotted in fig. (III-6).

TABLE III-4

$V_{i/p}$	1	2	3	4	5.3	7
$(V_R)m$	0.15	0.20	0.25	0.25	0.30	0.35
$(V_R)M$	5.0	7.4	11	16.5	18.5	24
$(V_R)m/(V_R)M$	0.03	0.027	0.02273	0.01515	0.01622	0.01458

Z	8.5	10	12.5	15	17.5	20
$(V_R)m$	0.4	0.55	0.6	0.75	0.8	1.0
$(V_R)M$	30	38	49	58	69	82
$(V_R)m/(V_R)M$	0.01333	0.01447	0.01225	0.01293	0.01159	0.0122

Z	22.5	25
$(V_R)m$	1.08	1.2
$(V_R)M$	93	105
$(V_R)m/(V_R)M$	0.01161	0.01143


V_m/V_M.
$F_{G_0} = 879$ Hz.
Height Of Water Column = 134 Cms.
Measurements For The Node And First Anti-node From The Surface.
0.055
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
Vi/P m.v.
FIG. [III-7]

The experiment had been repeated for the second tube configuration, where the water is in direct contact with air inside the tube with nearly the same height and frequency and the results were recorded in the following table and plotted in fig. (III-7).

TABLE III-5

$V_{i/p}$	1	2	3	4	5	6
$(V_R)m$	0.20	0.30	0.42	0.45	0.56	0.65
$(V_R)M$	3.8	7.2	11	14.5	17.5	20.5
$(V_R)m/(V_R)M$	0.052632	0.041666	0.038182	0.031038	0.0325	0.0317073

$V_{i/p}$	7	8	9	10	12.5	15
$(V_R)m$	0.8	0.9	1.0	1.15	1.30	1.60
$(V_R)M$	24.25	27.75	31.5	36	52	63
$(V_R)m/(V_R)M$	0.0333	0.03243	0.031746	0.031944	0.025	0.0254

$V_{i/p}$	17.5	20	22.5	25
$(V_R)m$	1.85	2.15	2.40	2.65
$(V_R)M$	71	81	95	105
$(V_R)m/(V_R)M$	0.026056	0.026543	0.025263	0.025238

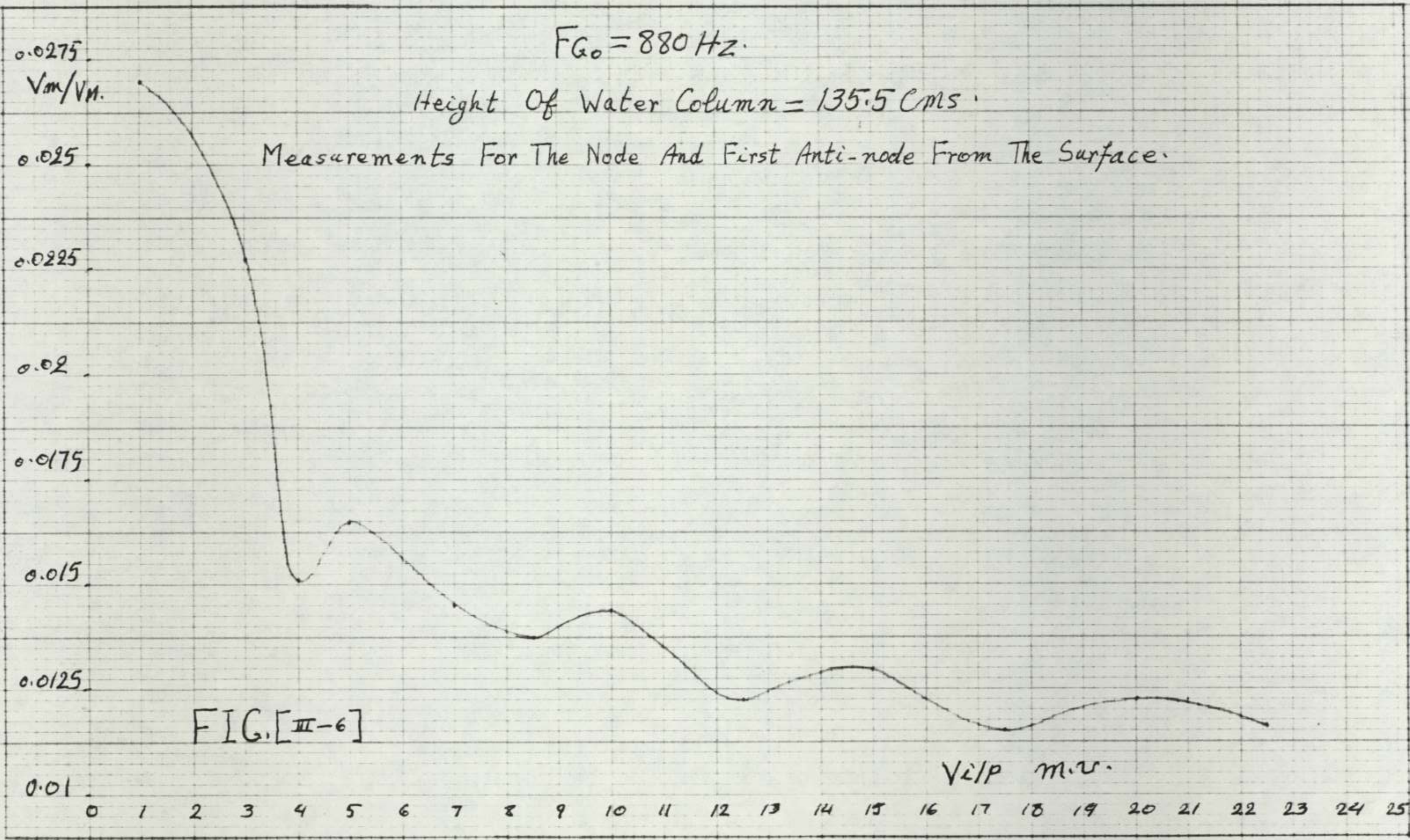

$F_{G_0} = 880$ Hz.
Height Of Water Column = 135.5 Cms.
Measurements For The Node And First Anti-node From The Surface.
Vm/Vn.
0.0275
0.025
0.0225
0.02
0.0175
0.015
0.0125
0.01
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
Vi/p m.v.

FIG. [III-6]

The data obtained from tables (III-4) and (III-5) can be used to determine experimentally the attenuation constant in the water, by using the formula(5):-

$\alpha_o = \frac{4f}{nc} \tanh^{-1} P\ min/P\ max$, where P min. and P max. are consecutive pressure node and antinode and (n) is the NO. of quarter of wave lengths from the water surface to the node or antinode in question.

Considering P min/P max. = $(V_R)m/(V_R)M$, then referring to fig. (III-6) for example, then:-

n = 2, f = 880 Hz. Thus for an input of 1 m.v.,

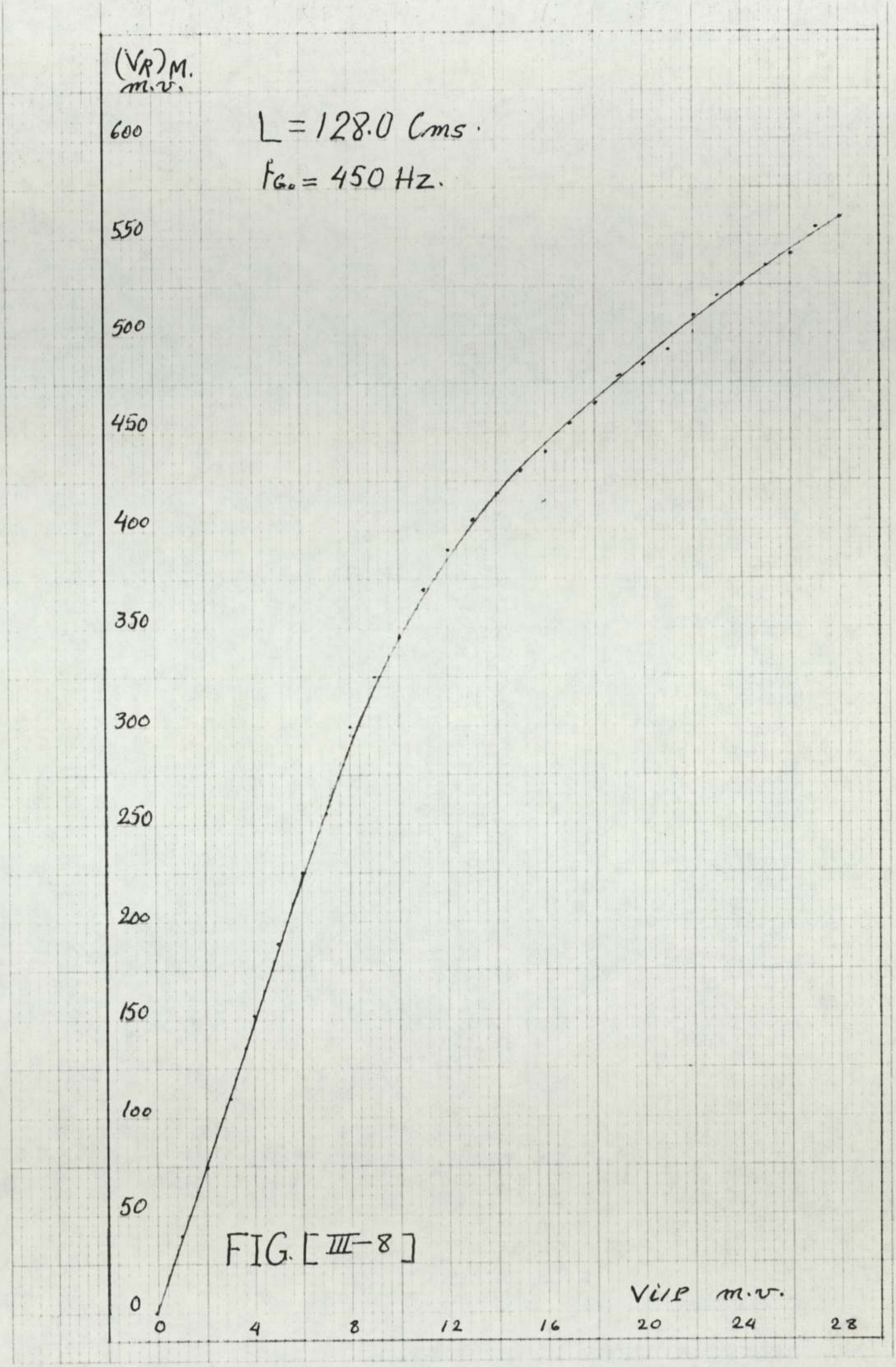
$$\alpha_o = \frac{4 \times 880}{2C} \tanh^{-1} 0.03. \text{ taking } C = 1.5 \times 10^5 \text{ cm/sec.}$$

$\therefore \alpha_o = 3.52 \times 10^{-4}$/cm. Converting its value to decibels, we multiply by 20 $\log_{10} e$.

$\alpha_o.L = 20 \times 3.52 \times 0.435 \times 135.5 \times 10^{-4} = 0.414$ dbs, which is a very small amount to affect the establishing of the standing wave in the tube, and $\alpha_o.L$ decreases with input voltage increase.

The variation of the maximum readings of the microphone from its midway position of 128 cms. height of a water column, with input voltage had been done and the results were recorded in the following table and plotted in fig. (III-8):-

$V_{i/p}$	1	2	3	4	5	6	7	8	9	10
$(V_R)_M$	39	74	110	152	188	225	256	300	325	345
$V_{i/p}$	11	12	13	14	15	16	17	18	19	20
$(V_R)_M$	370	390	405	418	430	440	455	465	478	485
$V_{i/p}$	21	22	23	24	25	26	27	28	29	30
$(V_R)_M$	492	510	520	525	535	542	555	560	570	580


(VR)M.
m.v.
L = 128.0 Cms.
FG0 = 450 Hz.
600
550
500
450
400
350
300
250
200
150
100
50
0
Vi/P m.v.
0
4
8
12
16
20
24
28
FIG. [III-8]

The resonance frequency of the water column was 450 Hz.

To check the presence of the brass plate at the boundary between the water surface and the extended air above the tubes, the problem can be related to sound propagation between water and air through a partition of finite thickness, which is in this case the brass plate and in case of normal incidence. The problem had been studied before by Rayleigh (6) for the case of a plate of finite thickness, totally immersed in water giving the following results:-

(1) For the thickness of the partition as $\frac{\lambda}{4}$, $\frac{3}{4}\lambda$, ...
the plate reflects most of the incident wave and transmission is minimum.

(2) For the thickness of the partition as o, $\frac{\lambda}{2}$, ...
the plate does not reflect any amplitude and the transmission is maximum.

In this case, the normal reflection coefficient had to be derived for water/brass plate/air respectively.

The following treatment had been based on two assumptions for plane-wave propagation:-

(1) The condensation at the boundary between two media is equal to zero.

(2) At the boundary between each two media, the excess pressure of the wave is continuous.

The following treatment gives the reflection coefficient for normal incidence:

Considering the displacement equations for the water, the brass plate partition and the extended air respectively we get:-

$$\xi_1 = A_1 e^{i(a_1 \cdot x + b_1 t)} + B_1 e^{i(-a_1 \cdot x + b_1 t)}$$

and $$\xi_2 = A_2 e^{i(a_2 x + b_2 t)} + B_2 e^{i(-a_2 x + b_2 t)}$$

Again considering the lower face of the brass plate as the partition plane between water and brass, then at this plane boundary:-

(a) $x = 0$

(b) The condensation $\frac{d\xi}{dx} = 0$

(c) For continuation of excess pressure at this plane, the difference in $\delta P = \rho h \frac{d^2\xi}{dt^2} = 0$, where h = thickness of the brassplate.

$$\therefore a_1 (A_1 - B_1) = a_2 (A_2 - B_2) \qquad \ldots\ldots (1)$$

and $$\rho_1 h b_1^2 (A_1 + B_1) = \rho_2 . h b_2^2 (A_2 + B_2)$$

since $b_1 = b_2$ = angular frequency ω, then:-

$$\rho_1 . (A_1 + B_1) = \rho_2 . (A_2 + B_2) \qquad \ldots\ldots (2)$$

Considering now the upper face of the brassplate as the partition plane, between water and air, then the same conditions were applied at this plane between brass and air for x = h, relative to the boundary x = 0, between water and brass. Hence:-

$$\xi_2 = A_2 . e^{i(a_2 h + b_2 t)} + B_2 . e^{i(-a_2 h + b_2 t)}$$

and $$\xi_3 = A_3 . e^{i(a_3 h + b_3 t)}$$

$$\therefore \frac{d\xi}{dx} = 0 \quad \text{and} \quad \rho . h . \frac{d^2\xi}{dt^2} = 0$$

$$\therefore a_2 (A_2 e^{ia_2 h} - B_2 e^{-ia_2 h}) = a_3 A_3 e^{ia_3 h} \qquad \ldots\ldots (3)$$

and $\rho_2 \, hb_2^2 \, (A_2 e^{ia_2h} + B_2 e^{-ia_2h}) = \rho_3 \, hb_3^2 \, A_3 e^{ia_3h}$ or

$$\rho_2 (A_2 e^{ia_2h} + B_2 e^{-ia_2h}) = \rho_3 A_3 e^{ia_3h} \qquad \ldots\ldots (4)$$

Eliminating A_3 by multiplying (3) by ρ_3, then:-

$$\rho_3 a_2 A_2 e^{ia_2h} - \rho_3 a_2 B_2 e^{-ia_2h} = \rho_3 a_3 A_3 e^{ia_3h}$$

From (3) and (4), we get:-

$$A_2 e^{ia_2h} (\rho_3 a_2 - \rho_2 a_3) = B_2 e^{-ia_2h} (\rho_3 a_2 + \rho_2 a_3).$$

$$\therefore \quad B_2 = A_2 e^{2ia_2h} \left[\frac{\rho_3 a_2 - \rho_2 a_3}{\rho_3 a_2 + \rho_2 a_3} \right]$$

$$\therefore \quad B_2 = A_2 e^{2ia_2h} \cdot R_2 \qquad \ldots\ldots (5)$$

where:-

$R_2 = \dfrac{\rho_3 a_2 - \rho_2 a_3}{\rho_3 a_2 + \rho_2 a_3}$, hence from (1), we get:-

$$a_1 (A_1 - B_1) = a_2 A_2 (1 - R_2 e^{2ia_2h}) = a_2 A_2 (1 - \alpha),$$

where $\alpha = R_2 e^{2ia_2h}$, and from (2), we get:-

$$\rho_1 (A_1 + B_1) = \rho_2 A_2 (a + R_2 e^{2ia_2h}) = \rho_2 A_2 (1 + \alpha)$$

$$\therefore \quad a_1 (A_1 - B_1) = a_2 A_2 (1 - \alpha) \qquad \ldots\ldots (6)$$

$$\text{and} \quad \rho_1 (A_1 + B_1) = \rho_2 A_2 (1 + \alpha) \qquad \ldots\ldots (7)$$

re-arranging (6) and (7), hence:-

$$\rho_2 a_1 (1+\alpha)(A_1 - B_1) = \rho_2 a_2 A_2 (1-\alpha)(1+\alpha)$$

and $$\rho_1 a_2 (1-\alpha)(A_1 + B_1) = \rho_2 a_2 A_2 (1-\alpha)(1+\alpha)$$

$$\therefore \rho_2 a_1 (1+\alpha)(A_1 - B_1) = \rho_1 a_2 (1-\alpha)(A_1 + B_1)$$

$$\therefore A_1 \left[\rho_2 a_1 (1+\alpha) - \rho_1 a_2 (1-\alpha)\right] = B_1 \left[\rho_2 a_1 (1+\alpha) + \rho_1 a_2 (1-\alpha)\right]$$

$$\frac{B1}{A1} = \frac{\rho_2 a_1 (1+\alpha) - \rho_1 a_2 (1-\alpha)}{\rho_2 a_1 (1+\alpha) + \rho_1 a_2 (1-\alpha)} = \frac{(\rho_2 c_2 - \rho_1 c_1) + \alpha(\rho_2 c_2 + \rho_1 c_1)}{(\rho_2 c_2 + \rho_1 c_1) + \alpha(\rho_2 c_2 - \rho_1 c_1)}$$

$$= \frac{\dfrac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1} + \alpha}{1 + \alpha\left[\dfrac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1}\right]} = \frac{R_1 + \alpha}{1 + \alpha R_1}$$, where:-

$$R_1 = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1} \qquad \frac{B1}{A1} = \frac{R_1 + R_2 e^{2ia_2h}}{1 + R_1 R_2 e^{2ia_2h}}$$. Let $2 a_2 h = \theta$

$$\frac{B1}{A1} = \frac{R_1 + R_2 (\cos\theta + i \sin\theta)}{1 + R_1 R_2 (\cos\theta + i\sin\theta)} = \frac{(R_1 + R_2 \cos\theta) + iR_2 \sin\theta}{(1 + R_1 R_2 \cos\theta) + iR_1 R_2 \sin\theta}$$

$$= \frac{(R_1 + R_2 \cos\theta) + iR_2 \sin\theta}{(1 + R_1 R_2 \cos\theta)^2 + R_1^2 R_2^2 \sin^2\theta} \left[(1 + R_1 R_2 \cos\theta) - iR_1 R_2 \sin\theta\right]$$

$$\frac{B1}{A1} = \frac{\left[(R_1 + R_2 \cos\theta)(1 + R_1 R_2 \cos\theta) + R_1 R_2^2 \sin^2\theta)\right]}{(1 + 2R_1 R_2 \cos\theta + R_1^2 R_2^2)}$$

$$+ i \frac{\left[R_2 \sin\theta (1 + R_1 R_2 \cos\theta) - R_1 R_2 \sin\theta (R_1 + R_2 \cos\theta)\right]}{(1 + 2R_1 R_2 \cos\theta + R_1^2 R_2^2)}$$

$$= (R_1+R_2\cos\theta+R_1^2R_2\cos\theta+R_1R_2^2\cos^2\theta+R_1R_2^2\sin^2\theta)/(1+2R_1R_2\cos\theta+R_1^2R_2^2)$$

$$+ i(R_2\sin\theta+R_1R_2^2\sin\theta\cos\theta-R_1R_2^2\sin\theta\cos\theta-R_1^2R_2\sin\theta)/(1+2R_1R_2\cos\theta+R_1^2R_2^2)$$

$$= R_1(1+R_2^2)+R_2(1+R_1^2)\cos\theta \;/(1+2R_1R_2\cos\theta + R_1^2R_2^2)$$

$$+ i\; R_2(1-R_1^2)\sin\theta \;/(1+2R_1R_2\cos\theta + R_1^2R_2^2)$$

For thin layer, $\theta = 2\,a_2h = \frac{4\pi \times 1.27}{\lambda_2} = \frac{4\pi \times 1.27}{1100}$

for a frequency of 500 c/s, $\theta = 0.0145$ rad.

$\therefore \cos\theta \cong 0.9999$ and $\sin\theta \cong 0.0145$

$$\therefore \left|\frac{B1}{A1}\right| \longrightarrow \frac{R_1(1+R_2^2) + R_2(1+R_1^2)}{(1 + R_1R_2)^2} = \frac{R_1+R_2}{1+R_1R_2}$$

ρc for water $= 1.43 \times 10^5$

ρc for brass $= 3.16 \times 10^5$

ρc for air $= 41.5$, Hence:-

$$R_2 = \frac{4.15 - 3.16 \times 10^5}{4.15 + 3.16 \times 10^5} = -0.99974$$

and $R_1 = \frac{3.16 - 1.43}{3.16 + 1.43} = 0.3769$

$$\therefore \left|\frac{B1}{A1}\right| = \frac{0.3769 - 0.99974}{1 - 0.3769 \times 0.99974} = 0.9942$$

which is actually equal to the reflection coefficient between water and air at normal incidence as if the brass plate is not present

$$\left(r = \frac{1.43 \times 10^5 - 43}{1.43 \times 10^5 + 93} = 0.99942\right)$$

If air is replaced by water, i.e. the brass is totally immersed in water, then $R_2 = -R_1$ and $\frac{B1}{A1} = 0$, agreeing with Rayleigh's result for reflection from a plate of finite thickness as:-

$$\frac{B1}{A1} = \left(\frac{\rho_1 C_1}{\rho_2 C_2} - \frac{\rho_2 C_2}{\rho_1 C_1}\right) / \sqrt{4\ \mathrm{Cot}^2\left(\frac{2\pi h}{\lambda}\right) + \left(\frac{R_1}{R_2} - \frac{R_2}{R_1}\right)^2}$$

where $\mathrm{Cot}\ \frac{2\pi h}{\lambda} \longrightarrow \infty$ and $\left|\frac{B1}{A1}\right| \longrightarrow 0$, between water and brass only, and no reflection occurs from the brass plate

By comparing figures (III-6) and (III-7), we notice that the presence of the brass plate at the boundary between water and air had caused a considerable end correction, with the result of reducing α_o.

(b) - The Aluminium Tubes:-

When the second tube configuration for the glass tube had been assembled and following the whole frequency range 100 $\rightarrow$ 1000 Hz, at different water heights, the hydrophone from its midway positions in the water columns, gave two appreciable maxima separated by 80 $\rightarrow$ 120 Hz on the frequency scale and their values differ from each other depending on water height.

This was due to the fact that the diaphragm had changed its mode of vibration under water to higher modes and thus the microphone gave another maximum when the frequency of the new mode of vibration was equal to the frequency of forced vibration.

This case had put the necessity of modifications when using the relatively thinner aluminium tubes to obtain new clamped boundary conditions for the diaphragm for obtaining more information about it in air and in water.

Also another method was needed more closer to the diaphragm rather than the water column to determine both frequencies.

Considering the following nomenclatures:-

$\mathcal{V}_c$ = Velocity of loudspeaker moving coil in m/sec. = $e/\beta l$.

β = Magnetic flux density in Weber/m^2 in the air gap of the permanent magnet.

e = Counter e.m.f. in volts corresponding to the motional electrical impedance.

l = Length of the moving coil in meters.

i = Current in the moving coil in amperes.

F_c = Mechanical force produced = βli in Newtons.

Zme = Motional effective electrical impedance

= $Zmm \cdot \beta^2 \cdot l^2$.

Zmm = Mechanical mobility of the loudspeaker

= $\mathcal{V}_c / F_c$.

Then the circuit of the loudspeaker can be represented as follows:-

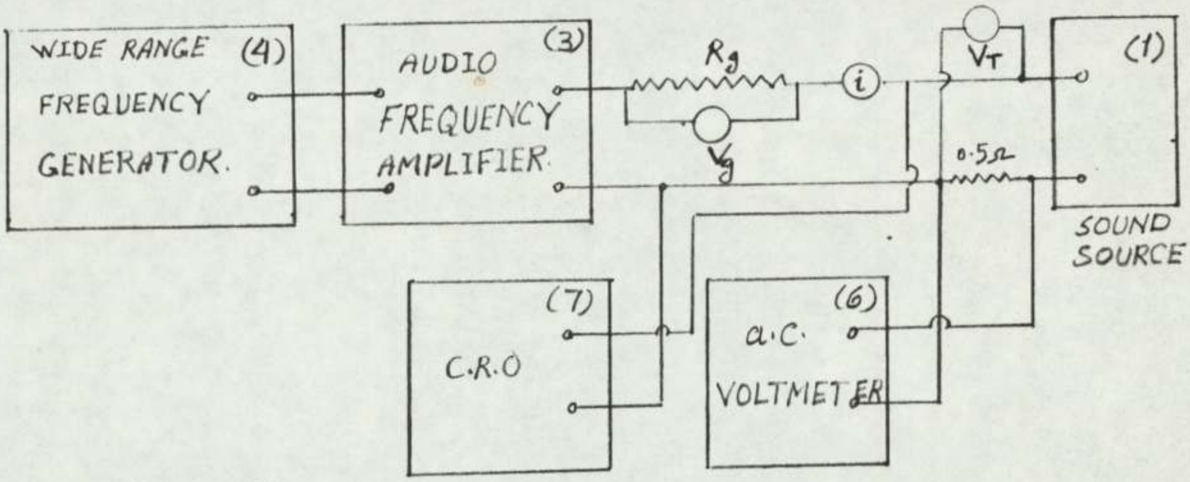


FIG.(III - 9)

R_g is the output resistance of the audio amplifier.

R_E is the resistance of the moving coil.

The resonance condition of the loudspeaker is defined by a maximum velocity of the moving coil $(\mathcal{V}_c)$ max. For the above circuit,

$$e = V_T - i\,(R_E + 0.5) = V_T - Vg\,\frac{(R_E + 0.5)}{Rg}.$$

$$\therefore \frac{e}{\beta l} = \frac{V_T}{\beta l} - i\,(R_E + 0.5)/\beta l$$

Thus for constant current i (and hence F_c), when $\mathcal{V}_c$ (or $e/\beta l$) is maximum, then V_T is maximum. Such condition had been observed on the C.R.O. In the same time, a decrease in the value of i to a minimum value of its recorded readings had been recorded.

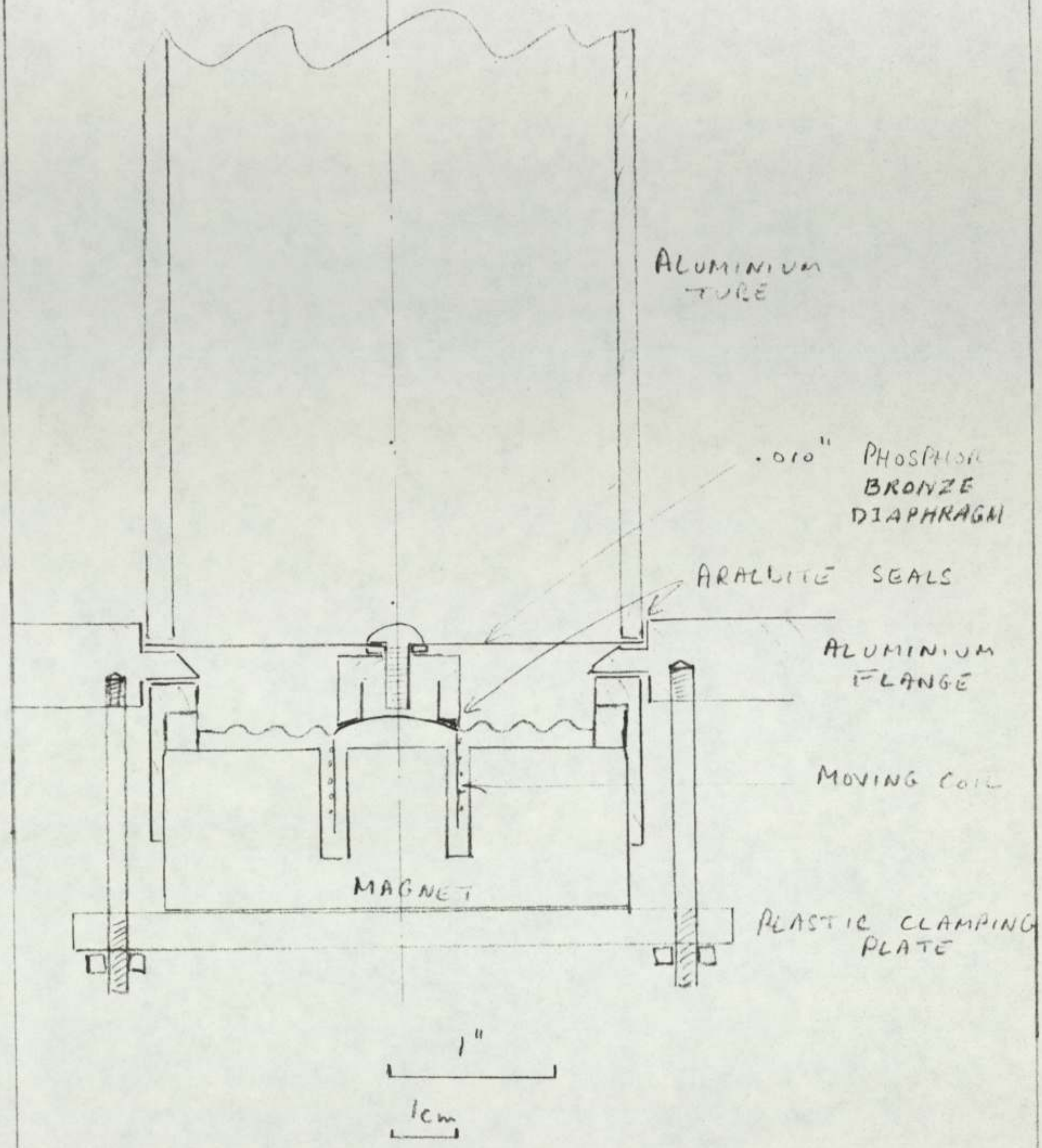
The decrease of the current i is due to the fact, that in case of resonance, the mechanical mobility and the electrical motional and hence the total electrical impedances were also at their maximum values.

The system had been used for loudspeakers in air and had been successfully applied to the case under water to determine the resonance modes.

The minimum current had been picked up at resonance as minimum voltage across the 0.5Ω resistor inserted in series with the moving coil.

The a.c. voltmeter gave two minimum voltages corresponding to the two maximum readings of the microphone in its midway position for any water column inside the tube.

Fig. (III-10) shows a detailed and scaled section of the lower end of the aluminium tube with the diaphragm attached.



GENERAL ARRANGEMENT OF ALUMINIUM TUBE, DIAPHRAGM AND DRIVING COIL

FIG III-10

CHAPTER 4

THE THEORETICAL TREATMENT

a - The Diaphragm:- In air:-

Referring to Rayleigh(6), the equation obtained for the vibration of a circular plate considering its centre as a velocity node is:-

$(\nabla^4 - K^4)w = 0$. Where w is the displacement perpendicular to the plane of the plate.

$$(\nabla^2 + K^2)(\nabla^2 - K^2)w = 0$$

and $\nabla^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + \frac{1}{r^2}\frac{d^2}{d\theta^2}$.

The Fourier expansion of w is:-

$$= w_0 + w_1 (\text{Cos}\ \theta + \alpha_1) + w_2 \text{Cos}\ (2(\theta + \alpha_2)) \ \ldots\ldots\ldots$$

Where $w_0, w_1, \ldots\ldots$ etc are functions of r only. Thus w_n has to satisfy the equation:-

$$\left(\frac{d^2}{dr^2} + \frac{1}{dr} - \frac{n^2}{r^2} \pm K^2\right) w_n = 0$$

The solution for (+) sign of K^2 is w_n prop.to Jn (Kr), while the solution of the (-) sign of K^2 is w_n prop.to Jn(iKr), irrespective of the boundary conditions.

$$\therefore w_n = \text{Cos}\ n\theta\,[\alpha J_n(Kr) + \beta\, Jn(iKr)] + \text{Sin}\ n\theta\,[\gamma\, Jn(Kr) + \delta\, Jn(iKr)].$$

From the form of solutions of these equations, it is evident that $\frac{\alpha}{\beta} = \frac{\gamma}{\delta} = \frac{1}{\lambda}$. Hence,

$$w_n = [Jn(Kr) + \lambda Jn(iKr)](\alpha\ \text{Cos}\ n\theta + \gamma\ \text{Sin}\ n\theta).$$

For symmetrical modes only n = 0, and $W_n = J_o(Kr) + \lambda \cdot I_o(Kr)$.

(independent of θ).

Since the plate is clamped at r = a, then both W and $\frac{dW}{dr} = 0$, then:-

$$J_o(Ka) + \lambda \cdot I_o(Ka) = 0.$$

and $J_1(Ka) - \lambda \cdot I_1(Ka) = 0.$

Eliminating λ from both equations, Rayleigh obtained his equation for the fundamental frequency of the plate as:-

$$\frac{J_1(Ka)}{J_0(Ka)} + \frac{I_1(Ka)}{I_0(Ka)} = 0.$$

Of which the lowest value verifying the above equation given by him is Ka = 3.2, giving:-

$\frac{\omega}{C_L} = \frac{K^2 \cdot h}{3} = \left(\frac{3.2}{a}\right)^2 \cdot \frac{h}{3}$, where C_L is the velocity of longitudinal wave in a thin plate as the diaphragm infinitely extended and h is the thickness of the diaphragm. The above equation gives f as:-

$$f = \frac{0.54303}{a^2} \cdot h \cdot C_L \quad \text{in vacuo.}$$

McLachlan(8), gave $C^2 = Eh^2/12\,(1 - \sigma^2)$ as the flexural rigidity/ρh. Corresponding to $h.C_L./\sqrt{12}$ and also gave (Ka) = 3.1955, thus:-

$\frac{\omega}{C_L} = \left(\frac{3.1955}{a}\right)^2 \cdot \frac{h}{\sqrt{12}}$, giving f as:-

$f = 0.469\, \frac{h.C_L}{a^2}$. closely agreeing with the value given by Wood according to Lamb as:-

$f = \frac{0.4745}{a^2} \cdot h \cdot C_L$, where for phosphor bronze,

E = Young's modulus = 11.7×10^{11}, σ = poisson's ratio = 0.364

ρ = density = 8.89 gm/cm^3. For the case of the glass tube,

a = 1", h = 5×10^{-3} inch, f = 364 Hz applying Lamb's formula.

For the aluminium tubes:-

a = 1.435", h = 10^{-2} inch, f = 353 Hz.

a = 1.37", h = 10^{-2} inch, f = 387 Hz.

Since the diaphragm had been loaded by a small aluminium mass load m attached at the centre, then according to Lamb, an unloaded diaphragm of total mass (M) is equivalent to a mass (M/5) concentrated at the centre. Consequently a small additional mass load attached at the centre of the diaphragm may be regarded as a uniformly distributed load of mass (5m) and the frequency of the diaphragm will diminish in the ratio of $(1 + 5m/M)^{-\frac{1}{2}}$ in air.

In the case of the glass tube the small aluminium mass load has nearly the following dimensions:-

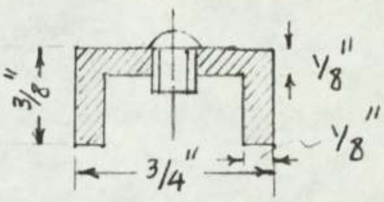


FIG.(IV-1)

$$m \cong \frac{\pi}{4}\left[\left(\frac{3}{4}\right)^2 \times \frac{3}{8} - \left(\frac{1}{2}\right)^2 \times \frac{1}{4}\right] \times 2.7 \times (2.54)^3 = \frac{\pi}{4}(2.54)^3\left[\frac{2.7 \times 19}{16 \times 8}\right]$$

and $M = \frac{\pi}{4}(2.54)^3 \times 5 \times 10^{-3} \times 8.89$

$$\therefore \frac{m}{M} = \frac{2.7 \times 19 \times 10^3}{8.89 \times 16 \times 8 \times 4 \times 5} = \frac{513000}{227584} = 2.2541$$

$$5m/M = 11.2705 \quad \therefore f_L = \frac{364}{\sqrt{12.2705}} = 104 \text{ Hz}$$

In the case of the aluminium tubes:-

For a = 1.435", $M = \frac{22}{7} \times (1.435)^2 (10^{-2}) (8.89) (2.54)^3$ gms.

$\therefore M = 9.428$ gms. The loaded frequency for the air filled tube had been obtained as $f_L = 262$ Hz.

$$f_L = \frac{f}{\sqrt{1 + 5\,m/M}} \quad \therefore \left(\frac{353}{262}\right)^2 = 1 + \frac{5m}{M} = 1.815$$

$$\therefore m = (0.815 \times 9.428)/5 = \underline{\underline{1.531 \text{ gms}}}$$

$$\text{For } a = 1.37", \; M = (9.428) \left[\frac{1.37}{1.435}\right]^2 = \underline{\underline{8.593 \text{ gms}}}$$

The loaded frequency for the air filled tube had been obtained as

$$f_L = \underline{\underline{222 \text{ Hz}}}.$$

$$\therefore \left(\frac{387}{222}\right)^2 - 1 = \frac{5m}{M} = \frac{m}{1.7186} = 2.038$$

giving m as $\underline{\underline{3.5 \text{ gms}}}$. It is clear that this tube gave a slightly and relatively lower value for the loaded frequency of the diaphragm in air.

The general equation for the clamped boundary conditions had been given by McLachlan as:-

$$J_n(Ka)\, I_{n+1}(Ka) + J_{n+1}(Ka)\, I_n(Ka) = 0$$

Giving $f_{n,m}$ according to Lamb's formula as:-

$$f_{n,m} = \frac{0.4745}{a^2} \cdot h \cdot C_L \cdot \left[\frac{(ka)_{n,m}}{3.1955}\right]^2 .$$

Where n is the order of Bessel function and m is the rank of the mode. n - determines the number of nodal diameters and (m-1), determines the number of internal nodal circles. McLachlan shows that when (Ka) is large enough, the asymptotic series for Bessel functions may be used to solve the above equation to an adequate approximation.

This gives:-

$$(ka)_{n,m} \cong \Theta - \frac{(4n^2 - 1)}{\Theta}\left[1 + \frac{1}{\Theta} + \frac{28n^2 + 17}{48\Theta^2} + \frac{3(4n^2 - 1)}{8\Theta^3} \;\ldots\ldots\right]$$

Where $\Theta = (m + \frac{n}{2})\pi$

The following table shows some of $(ka)_{n,m}$ values obtained:-

Table (IV-1)

m	Order n			
	0	1	2	3
1	3.1955	4.162	5.906	7.144
2	6.3064	7.799	9.197	10.536
3	9.4395	10.958	12.402	13.795
4	12.5771	14.109	15.579	17.005
5	15.7169	17.256	18.745	20.192

$$f_{1,1} = \frac{0.4745}{a^2} \cdot h.\, C_L \cdot \left[\frac{4.152}{3.1955}\right]^2$$

$$= 0.80489\, h.C_L/a^2$$

Table (IV-2)

m	n			
	0	1	2	3
1	0.4745	0.80489	1.62084	2.37155
2	1.8477	2.8264	3.9303	5.1583
3	4.1405	5.5798	7.1473	8.8428
4	7.3505	9.2964	11.2782	13.7211
5	11.4786	13.8369	16.3279	18.9458

For any value of n, the accuracy increases with increase of m, while it decreases for any m, with increase of n, using the dominant terms of the asymptotic expansion, the frequency equation takes the following form:-

$$\operatorname{Cos}\left(ka - \frac{\pi}{4} - \frac{n\pi}{2}\right) + \operatorname{Sin}\left(ka - \frac{\pi}{4} - \frac{n\pi}{2}\right) = 0.$$

$$\tan\left(ka - \frac{\pi}{4} - \frac{n\pi}{2}\right) = -1.$$

Hence $ka - \frac{\pi}{4} - \frac{n\pi}{2} = (m - \frac{1}{4})\pi$, from which

$$(ka)_{n,m} = (m + \frac{n}{2})\pi .$$

The following table shows the approximated roots:-

Table (IV=3)

m	n			
	0	1	2	3
1	3.14	4.7143	6.28	7.8571
2	6.28	7.8571	9.4285	11.000
3	9.4285	11.000	12.5714	14.1428
4	12.5714	14.1428	15.7142	17.2857
5	15.7142	17.2857	18.8571	20.4285

Applying the approximated root to Lamb's formula, then:-

$$f_{1,1} = 0.80489.\ h.\ C_L \left[\frac{4.7143}{4.162}\right]^2.$$

$$= 1.03267\ h.C_L/a^2.$$

Wood (9) gave the value of the numerical multiplier as 1.006 for $f_{1,1}$, from which for:-

a - The glass tube:-

$$f_{1,1} = \frac{1.006}{0.4745} \times 104 \cong 221 \text{ Hz.}$$

b - The aluminium tube of a = 1.435"

$$f_{1,1} = \frac{1.006}{0.4745} \times 262 = 555 \text{ Hz.}$$

c - The aluminium tube of a = 1.37"

$$f_{1,1} = \frac{1.006}{0.4795} \times 222 \cong 470 \text{ Hz.}$$ showing

again lower value than it ought to be.

For three nodal diameters:-

$$f_{3,1} = \frac{0.4745}{a^2}.\ h.\ C_L\left[\frac{7.144}{3.1955}\right]^2$$

$$= 5.267\ \left(\frac{0.4745}{a^2}\right).\ h.\ C_L,$$ thus for:-

a - The glass tube:-

$f_{3,1}$ = 5.267 x 104 547 Hz.

b - The aluminium tube of a = 1435"

$f_{3,1}$ = 5.267 x 262 1380 Hz.

c - The aluminium tube of a = 1.37"

$f_{3,1}$ = 5.267 x 222 = 1169 Hz.

The value of $f_{1,1}$ for this tube had been estimated as 550 Hz, from experimental considerations.

Considering the following nomenclatures:-

e_g. The open circuit voltage of the audio-frequency power amplifier and Rg, its output resistance.

M_D - Concentrated mass of the diaphragm = M/5.

M_A - Concentrated aluminium mass load m.

C_s - Mechanical compliance of the suspension = m/N.

r_s - Responsiveness of the suspension = 1/Rs = m/N.sec.

R_s - Mechanical resistance of the suspension = N.sec/m.

Z_R - Radiation mobility = 1/Radiation impedance.

i - Current through the moving coil.

U - Diaphragm velocity.

f_c and f_r - Mechanical and radiation forces.

Then the equivalent circuit of the loudspeaker of the mobility type(7) can be represented as follows:-

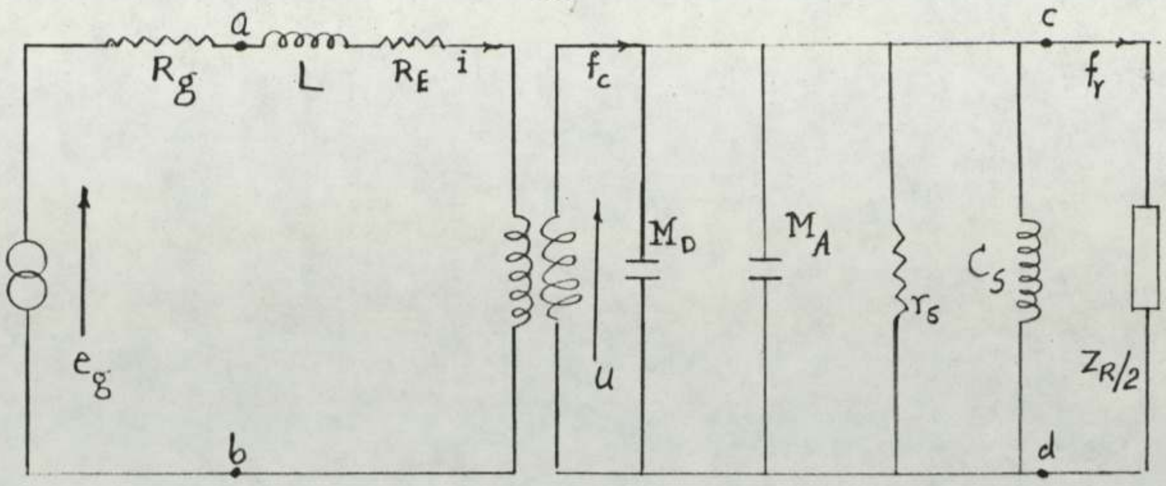


FIG.(IV-4)

The resonance condition in air is:-

$$\omega_o^2 = \frac{1}{(M_D + M_A)C_s} . \qquad \text{For a} = 1.435''$$

$M/5 = 1.88565 \times 10^{-3}$ Kgm and $M_A = 1.4368 \times 10^{-3}$ Kgm.

$$\omega_o = 2\pi(262) = \frac{11428}{7} \text{ and } \omega_o^2 = 2712138.45$$

$$\therefore 1/C_s = 2712.13845 \times 3.42245 = 9275.5188$$

$$\underline{\underline{C_s = 1.078 \times 10^{-4} \text{ m/N}}}$$

The quality Q for this tube is Q = 7.25 %

$$\therefore \quad r_s = \frac{Q}{\omega_o(M_D+M_A)} = \frac{0.0725 \times 7 \times 10^3}{11528 \times 3.42245} = \underline{\underline{0.0128 \text{ m/N.sec}}}.$$

For a = 1.37"

M 5 = 1.71868×10^{-3} Kgm. and $M_A = 3.50267 \times 10^{-3}$ Kgm.

$$\omega_o = 2\pi(222) = \frac{9768}{7} \text{ and } \omega_o^2 = 1947220.5$$

$$1/C_s = 10167.12 \qquad \underline{C_s = 0.9835 \times 10^{-4} \text{ m/N}}$$

The quality Q for this tube is Q = 9.46%

$$\therefore r_s = \frac{Q}{\omega_o(M_D+M_A)} = \frac{0.0946 \times 7}{9.768 \times 5.22135} = \underline{\underline{0.01298 \text{ m/N.sec}}}.$$

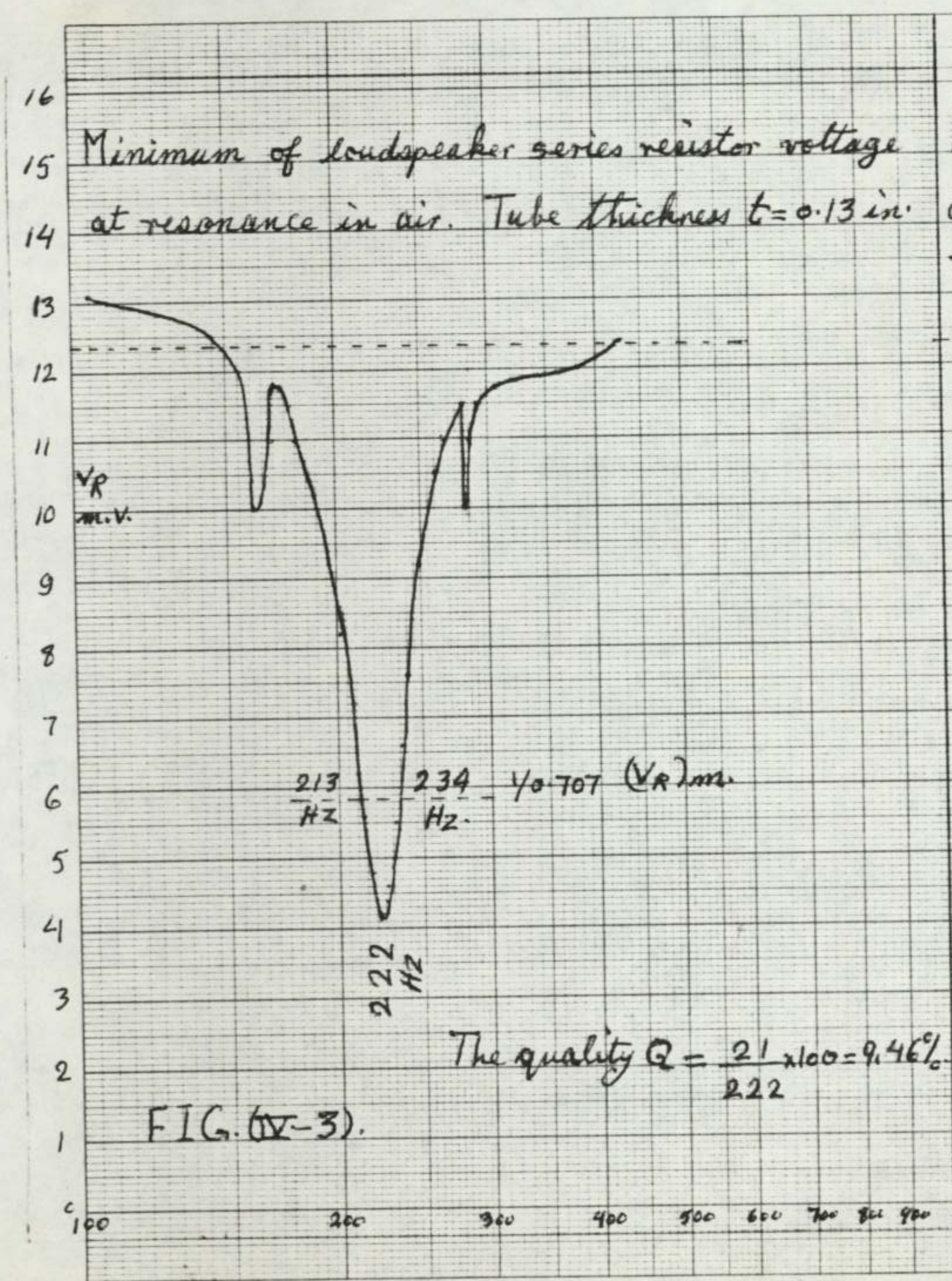


FIG. (IV-3).

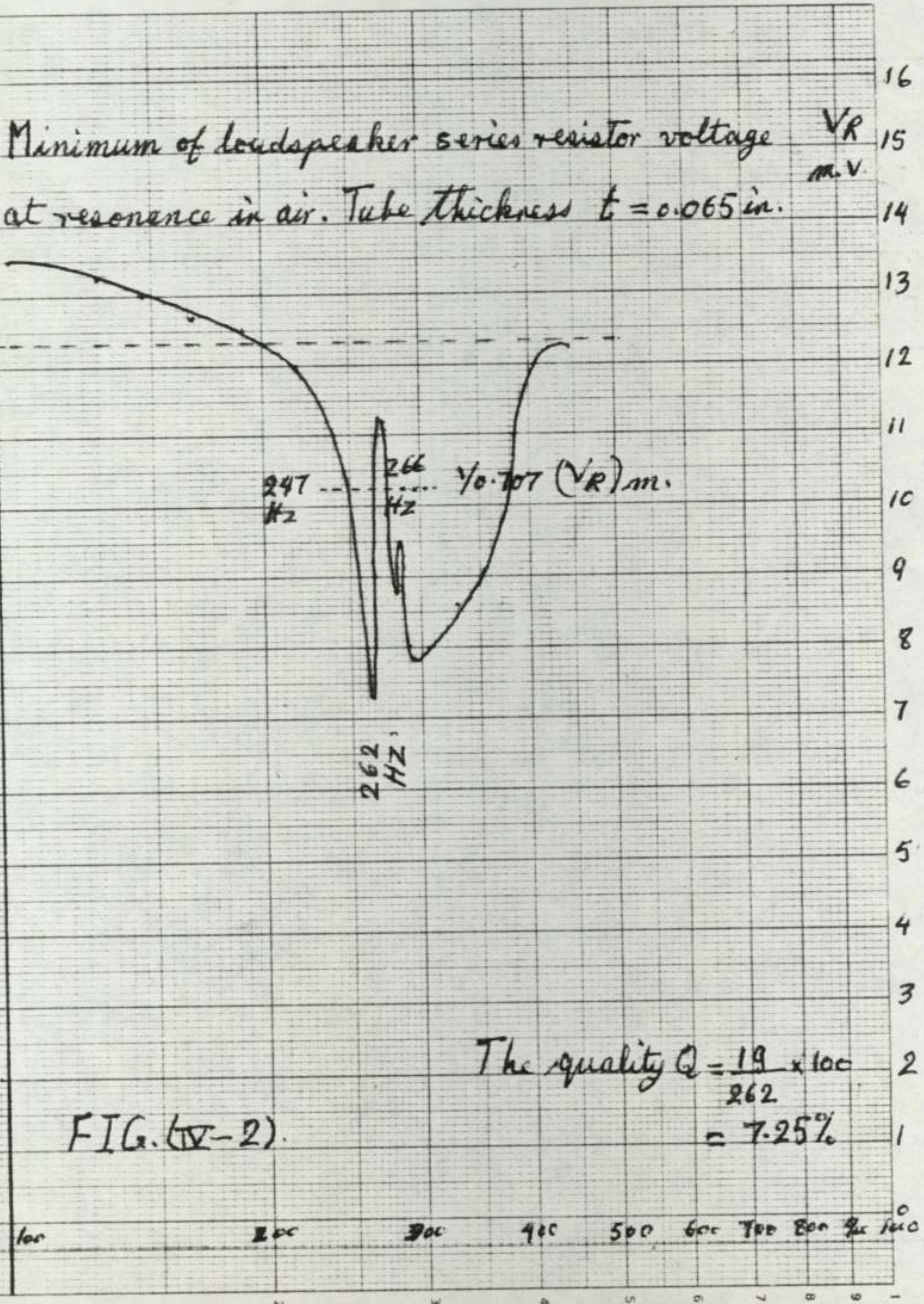


FIG. (IV-2).

Considering the electrical side, a 0.5Ω resistor was connected in series with the loudspeaker moving coil, so that the minimum loudspeaker current at resonance, is indicated as minimum voltage through the resistor.

The open circuit voltage of the audio frequency power amplifier for 3 m.v. input was 2 25 m.v. while the loaded voltage was 210 m.v.

Neglecting the effect of the inductance (L) at such low frequency, and considering $R_E \cong 8\Omega$ then $i = \frac{210}{8} = 26.25$ m.a. From which Rg can be obtained as $(225-210)/26.25 = 0.57\Omega$.

Again considering the 0.5Ω resistor inserted and no change in loaded voltage, we find that $i = 210/8.5 = 24.7$ m.a.

Referring to figs. (IV-2) and (IV-3), we see that 24.7 x 0.5 = 12.35 m.v., can be considered as an average value for the steady-state value of the loudspeaker series resistor voltage off resonance; and the load is still matched to the audio amplifier.

The input electrical power to the loudspeaker can be calculated as $i^2R = (29.7)^2 \times 8.5 \times 10^{-6} = 5.18 \times 10^{-3}$ watts = 5.18 m. watts.

The vibrations of the diaphragms under the tubes and in contact with water will be discussed in the following section.

b - <u>The Water Column</u>:- <u>Normal incidence</u>

According to the previous section, we shall assume that the excess pressure P = 0 at Z = 0 and Z = 1 in spite of the difficulties imposed on the measurements at the extreme ends of the water column, depending on obtaining a maximum excess pressure (antinode) and a minimum (node) at the same point midway of the water column for the fundamental and double frequency respectively.

Thus assuming:-

(1) u is the radial displacement $= U_{(r)} e^{i(wt+\frac{\pi z}{l})}$

(2) w is the axial displacement $= W_{(r)} e^{i(wt+\frac{\pi z}{l})}$

(3) P is the excess pressure $= P_{(r)} e^{i(wt+\frac{\pi z}{l})}$

where $U_{(r)}$, $W_{(r)}$ and $P_{(r)}$ are functions of r only, then by neglecting the dependance on Θ and taking the origin in the xy-plane ($Z = 0$), the polar differential equation, which p should satisfy is:-

$$\frac{d^2p}{dr^2} + \frac{1}{r}\frac{dp}{dr} + \frac{d^2p}{dZ^2} + K^2 p = 0 \cdot$$

where K = propagation constant $= \frac{\omega}{c} = \omega\sqrt{\frac{\rho_1}{K_1}}$, ρ_1 and K_1 being the density and bulk modulus of the water respectively. Substituting for p by its assumed value, we get:-

$$\frac{d^2P}{dr^2} + \frac{dP}{dr} + \left(\frac{\rho_1\omega^2}{K_1} - \frac{\pi^2}{l^2}\right) P = 0 \cdot$$

The hydrodynamical equations for the water column are:-

$$-\frac{dp}{dr} = \rho_1 \frac{d^2u}{dt^2} \qquad \therefore \rho_1\omega^2 U = \frac{dP}{dr}$$

and $$-\frac{dp}{dz} = \rho_1 \frac{d^2w}{dt^2} \qquad \therefore \rho_1\omega^2 W = i\frac{\pi}{l} P .$$

$\therefore (iK)^2 (iw) = \frac{\pi}{l}(\delta\rho)$. Hence the density disturbance $(\delta\rho)$ had caused both imaginary propagation constant and axial displacement verifying the clamped boundary conditions of the diaphragm at $Z = 0$.

Referring to Rayleigh for the solution of a rigidly closed cylinder, using the excess pressure P instead of the velocity potential ϕ, then P should satisfy the same polar equation as for aerial vibrations.

$$\therefore \frac{d^2P}{dr^2} + \frac{1}{r}\frac{dP}{dr} + \left(\frac{\rho_1\omega^2}{K_1} - \frac{\pi^2}{l^2}\right)P = 0$$

Re-arranging the equation, we write:-

$$\frac{d^2P}{dr^2} + \frac{1}{r}\frac{dP}{dr} - \left(\frac{\pi^2}{l^2} - \frac{\rho_1\omega^2}{K_1}\right)P = 0$$

By the previous arrangement, the solution becomes applicable to open ends of the water columns.

The Fourier expansion for P satisfying the boundary conditions is as follows:-

$$P = H_1 \sin\frac{\pi z}{l} + H_2 \sin\frac{2\pi z}{l} + \ldots\ldots + H_s \sin\frac{s\pi z}{l} + \ldots\ldots$$

of which each term has to satisfy both the boundary conditions and the polar equation by its own.

$$\therefore \frac{d^2H_s}{dr^2} + \frac{1}{r}\frac{dH_s}{dr} - \left(\frac{s^2\pi^2}{l^2} - \frac{\rho_1\omega^2}{K_1}\right)H_s = 0$$

Which can be re-written as

$$\frac{d^2H_s}{dr^2} + \frac{1}{r}\frac{dH_s}{dr} + \left[i^2\left(\frac{s^2\pi^2}{l^2} - \frac{\rho_1\omega^2}{K_1}\right)\right]H_s = 0$$

Of a particular solution of the form:-

$$P = \sum_{s=1}^{s=\infty} H_s \sin\frac{S\pi z}{l} \; J_n(i\delta_s r)\, e^{i\omega t}$$

Considering n = 0 then P is positive and real.

$$\therefore P = \sum_{s=1}^{s=\infty} H_s \sin \frac{s\pi z}{l} \; J_o(i\delta_s r)\, e^{i\omega t} \quad \text{or}$$

$$P = \sum_{s=1}^{s=\infty} H_s \sin \frac{s\pi z}{l} \; I_o(\delta_s r)\, e^{i\omega t}, \text{ where:-}$$

$$\delta_s^{\,2} = \frac{s^2\pi^2}{l^2} - \frac{\rho_1\omega^2}{K_1}.$$

For pure axial longitudinal vibrations $\delta_s = 0$.

Considering s = 1 for the fundamental frequency, then if l represents $\lambda/2$, the law $C = \lambda.f$ can be deduced.

Since the water column has sharp resonance with frequency, then the above assumption $\delta_s = 0$ can be applied. This reduces the equation of P to:-

$$P = \sum_{s=1}^{s=\infty} H_s . \sin \frac{s\pi z}{l} . e^{i\omega t}$$

Again $$-\frac{\partial P}{\partial z} = \rho_1 \frac{\partial^2 w}{\partial t^2} = \rho_1 \omega^2 w.$$

$$\therefore \rho_1\omega^2 w = \sum_{s=1}^{s=\infty} \frac{s\pi}{l} . H_s \cos \frac{s\pi z}{l} . e^{i\omega t} . I_o(\delta_s r), \text{ which reduces}$$

to:-

$$\rho_1\omega^2 w = \sum_{s=1}^{s=\infty} \frac{s\pi}{l} . H_s . \cos \frac{s\pi z}{l} . e^{i\omega t}, \text{ of which } w \text{ is maximum}$$

for both z = 0 and z = l, with (–ve) sign for s = 1.

It is clear from this sort of solution that both l and ω were arbitrary values, verifying the water column boundary conditions to yield the same value of sound velocity for different heights of water column, while relating such cases to the resonance frequency of the diaphragm is only a matter of extracting the maximum possible displacement yielding maximum possible sound energy radiated in the system.

This can be counteracted if the exciting system is of the high efficiency type as the one used and a considerable sound energy can be radiated in case of separate resonance conditions for the water column and the diaphragm, over a wide range of water column heights. This had been verified experimentally in the next section.

CHAPTER 5

Determination of the Velocity of Sound in the Tubes

a - The Glass Tube:-

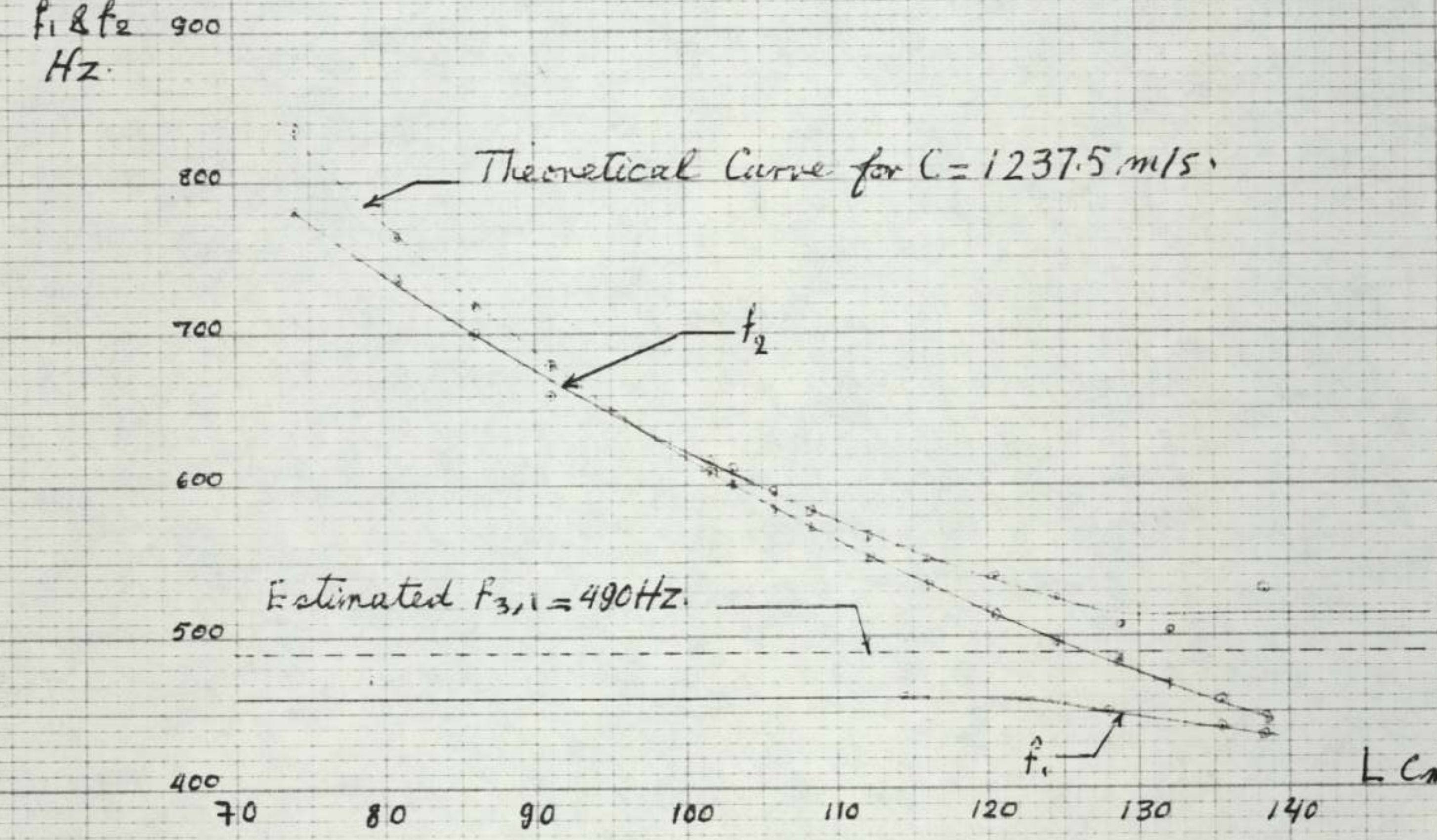
The second tube configuration was assembled and the tube had been filled with water to a level of 132 cms. and then the system was fed a 3 m.v. as an unemplified input voltage to the loudspeaker.

The microphone was placed midway of the water column, and a frequency scanning had been made in the range 100→1000 Hz. The microphone gave two maximum readings. A check had been made for the variation of each, with water height. The earlier maximum on the frequency scale gave no appreciable changes while the later one changed steadily with water height.

Hence draining the water from under the tube in steps of 2 →5 cms. level differences and adjusting the frequency of the generator for maximum readings of the microphone in its midway position of each new column and recording both the frequency and the corresponding readings V.S. water height, the following table had been obtained:-

TABLE (V-1)

L cms.	132	128.7	124.5	120.5	116	112	108.5
f_{G_o} Hz	501	504	525	539	551	567	582
$(V_R)_M$ m.v.	80	85	92	100	101	102	99
L cms.	105.8	103	101.5	91	86	80.8	74
f_{G_o} Hz	596	610	615	659	699	735	781
$(V_R)_M$ m.v.	95	97	99	108	40	17	5



f_1 & f_2 Hz.
900
800
700
600
500
400
Theoretical Curve for C = 1237.5 m/s.
f_2
Estimated $f_{3,1}$ = 490 Hz.
f_1
70
80
90
100
110
120
130
140
L Cms.

FIG.(V-1).

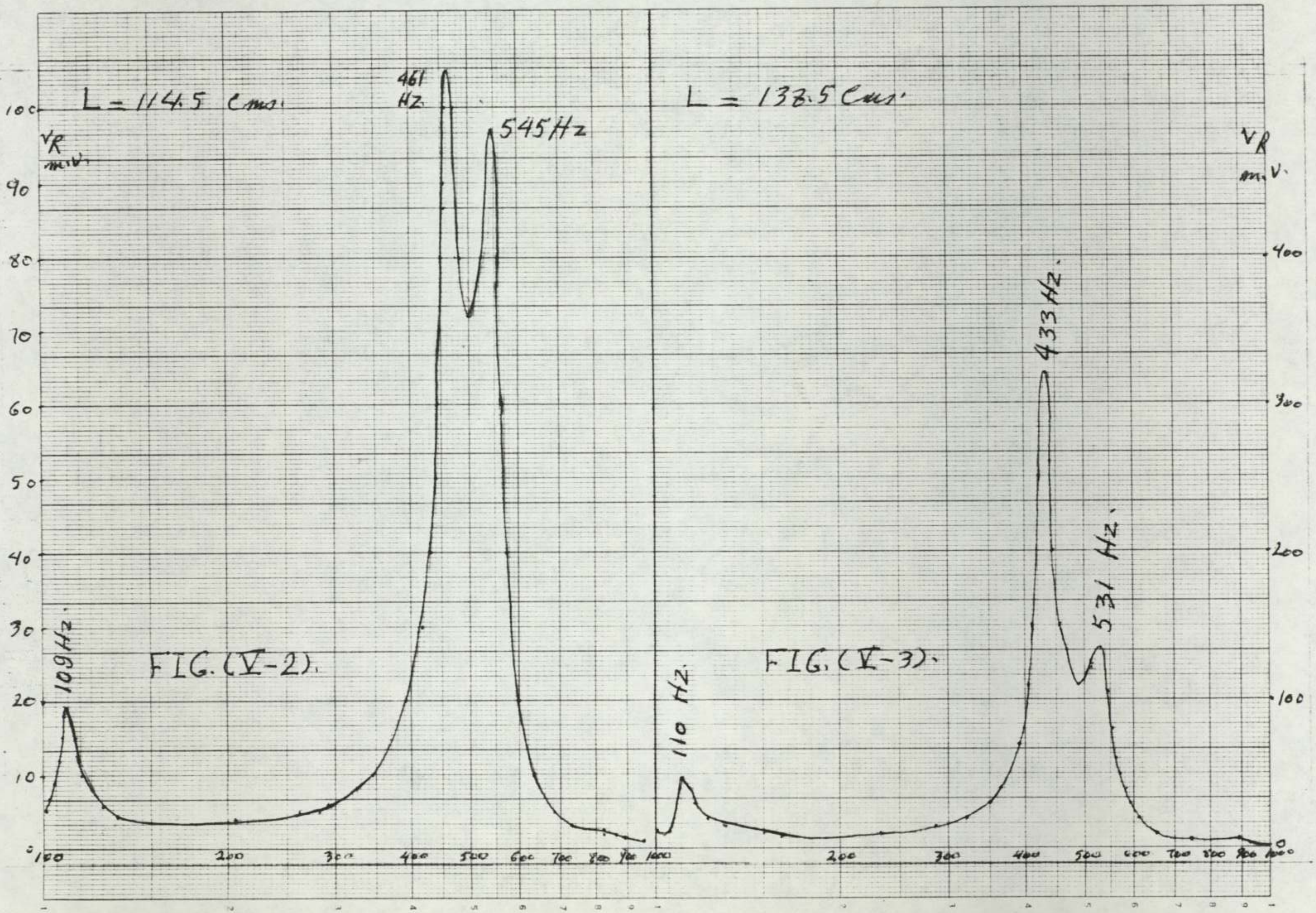
and the results were plotted in fig. (V-1). As a check for fig. (V-1), the microphone readings from the midway of 114.5 cms. and 138.5 cms. water columns had been recorded through the whole frequency range 100 ⟶ 1000 Hz. for unamplified input voltages of 3 and 6 volts respectively, and the results were recorded in the following tables:-

Table (V-2a) L = 114.5 cms.

f_G H_z	101	102	104	106	107	108	109	111	113	114	116	119
V_Rm.v.	5	7	9	12	15	18	19	17	15	12	10	8
f_c Hz	125	132	204	266	292	325	347	393	418	432	439	443
V_R m.v.	6	5	4	5	6	8	10	20	30	40	50	60
fG Hz	448	451	456	458	461	472	485	500	525	545	555	563
V_R m.v.	70	80	90	100	105	100	80	72	80	98	80	60
fG Hz	574	597	633	680	727	820	859	888	953	1039	1051	-
V_R m.v.	40	20	10	5	3	2.5	2	1.5	1	0.5	0.25	-

Table (V-2b) L = 138.5 cms.

fG Hz	100	105	108	110	113	116	122	128	150	159
V_R m.v.	13	11	40	48	40	30	20	15	10	8.9
fG Hz	233	285	319	350	366	392	406	411	423	433
V_R m.v.	10	15	20	30	40	70	110	150	250	320
fG Hz	440	446	456	487	512	531	545	553	562	571
V_R m.v.	260	200	150	110	125	135	105	80	62	50
fG Hz	580	593	609	655	744	892	923	940	950	983
V_R m.v.	40	30	20	10	5.5	6.0	3.8	1.0	0.88	1.5


L = 114.5 Cms.
L = 133.5 Cms.
VR mV.
VR mV.
461 Hz
545 Hz
109 Hz
110 Hz
433 Hz.
531 Hz.
FIG. (V-2).
FIG. (V-3).
100
90
80
70
60
50
40
30
20
10
0
400
300
200
100
0
100
200
300
400
500
600
700
800
900
1000
200
300
400
500
600
700
800
900
1000

and the results were plotted in figs. (V-2) and (V-3). Referring to figs. (III-5) and (III-8), we find that the earlier frequency mode had changed with water height according to the following table:-

Table (V-3a)

L cms	114.5	128	135.5	138.5
f_{G_o} Hz	461	450	440	433

and the results of those four points were gathered in fig. (V-1), as f_1.

Applying the law $C = \lambda . f = 2\ L\ f_{Go}$, we obtain the following table:-

Table (V-3b)

L cms	114.5	128	135.5	138.5
Cm/s	1055.69	1152	1188	1199.41

Fig. (V-1) showed two frequency modes f_1 and f_2 corresponding to the two maximum microphone readings on the frequency scale, of which f_1 is the resonance frequency of water column, while f_2 is the resonance frequency of the diaphragm v.s. water height. f_1 gave a steady state value of about 460 Hz, and f_2 gave a steady state value of about 515 Hz and the two steady state values behaved asymptotically around a frequency which is approximately equal to the three nodal diameter frequency of vibration for the loaded diaphragm in air, ($f_{3,1}$). $f_{3,1}$ had been estimated as 490 Hz.

Considering C = 1199.41 m/s, then the water height which would resonate at 490 Hz, will be L = 122.4 cms.

As it will be explained later, the only usable range for measuring the velocity in this tube was from 122.4 cm up to 138.5 cms due to the limited height of water column offered by the tube at such low frequency.

Also referring to figs. (V-2) and (V-3), we notice that the microphone gave an appreciable increase in its reading at 109 and 110 Hz, which is very close to the natural frequency of the loaded diaphragm in air calculated as 104 Hz. before.

Considering C_o for bulk water = 1485 m/s, then for a correction factor of 1.2, C = 1237.5 m/s. The frequency of which had been obtained v.s. height and drawn in fig. (V-1) for comparison.

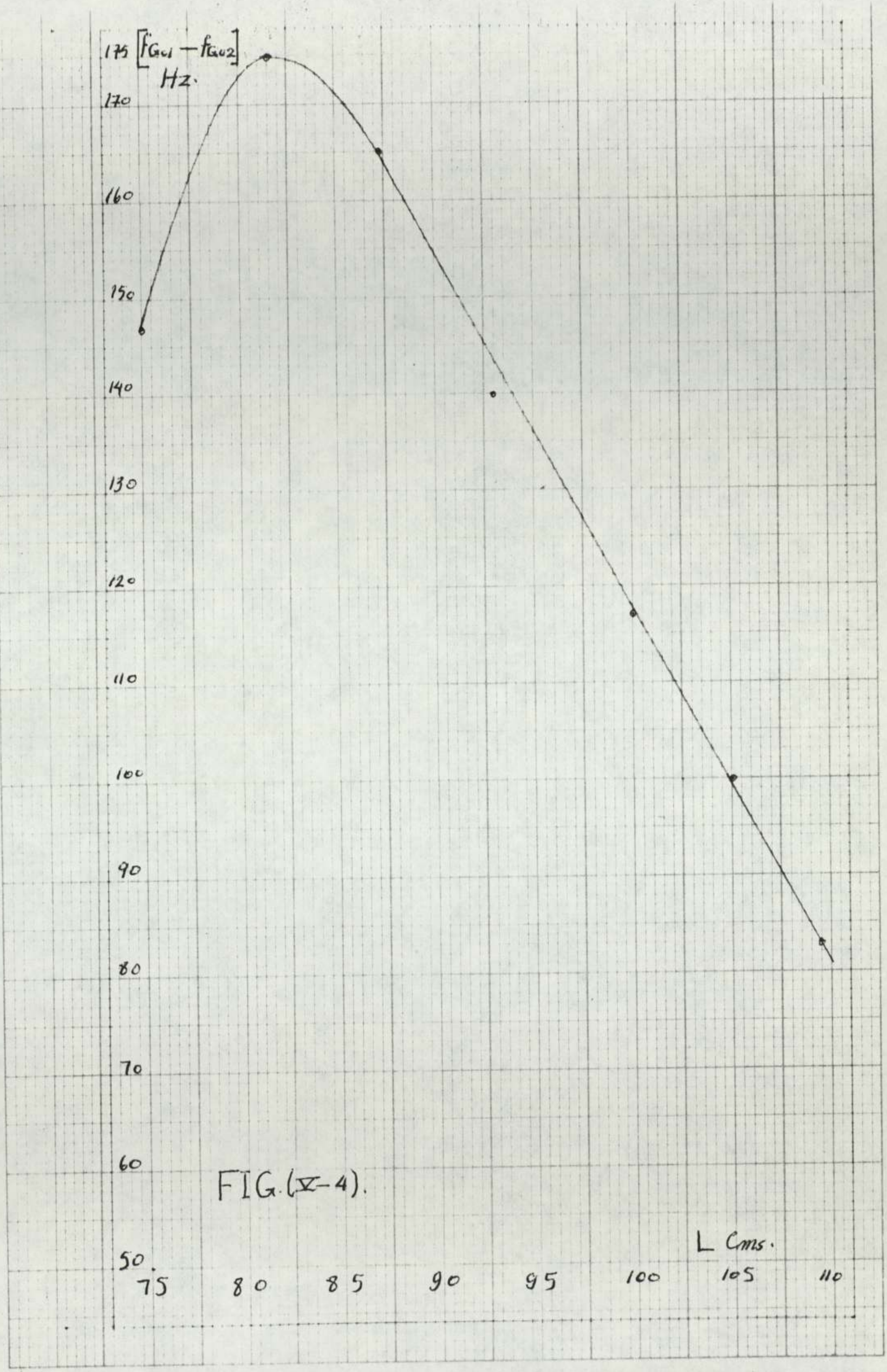
In some cases, the tap water had been oversaturated with air and the excess air diffuses through the water and collects on the inner side of the wall of the glass tube in the form of small uniform air bubbles.

Such condition had been observed and fig. (V-1) had been reproduced under observation of such bubbles on the inside wall, and the following table had been obtained:-

Table (V-4)

L cms	75	81.3	87.8	92.9	100	105	109.3	114.5
f_{G_o} Hz	627	555	523	510	503	498	497	494
(V_R) M m.v.	68	77.5	75.5	65	43	42	43	32

By comparing with fig. (V-1), the difference between the obtained frequencies for the same water height was drawn v.s. water levels in fig. (V-4), from which it is clear that the presence of such bubbles on the inner side of the glass tube due to air release had reduced both


[fG01 − fG02] Hz.
175
170
160
150
140
130
120
110
100
90
80
70
60
50
L Cms.
75
80
85
90
95
100
105
110
FIG.(V-4).

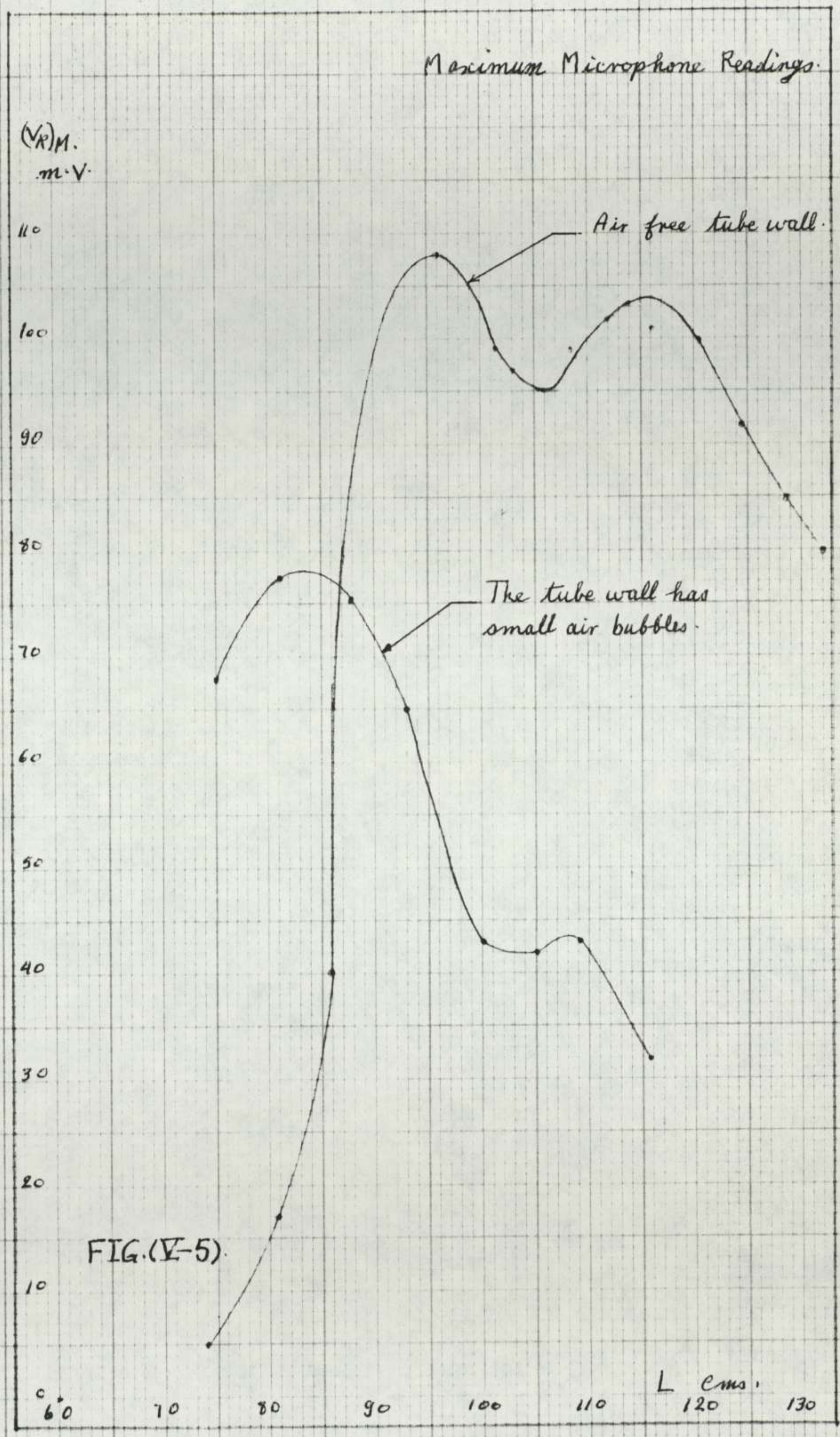

Maximum Microphone Readings.
$(V_R)_M$.
m.V.
110
100
90
80
70
60
50
40
30
20
10
0
Air free tube wall.
The tube wall has small air bubbles.
L cms.
60
70
80
90
100
110
120
130

FIG.(V-5).

frequency and velocity at resonance.

b - The Aluminium Tubes:-

The system for each tube had been assembled as in fig. (II-4) and since it appeared from earlier experiments on the subject[4] that having any hole in the wall of the tube will have an effect on the yielding of the tube and hence on the velocity measured, it was decided to fill or remove the distilled water from the tube, from its free top. For removing the water, a plastic tube attached to a graduated scale had been connected to the suction side of a 20 lbs per sq. in. small compressor (used also as vacuum pump) through an intermediate jar for collecting the water removed from the tube down to the required level which was read out by the attached scale.

The two resonance procedures previously mentioned for determining the resonant frequencies for both water column and the diaphragm had been applied to each tube. At first each tube had been filled to a level of about 143 cms. Then the microphone had been placed midway of the level and the slider was fixed to the external stand at that position. A frequency scanning in the range of 100 $\rightarrow$ 1000 Hz had been made at 3 m.v. input from the generator. The resonance frequencies had been recorded with their corresponding maximum readings of the microphone.

The steps had been repeated for each water height by removing the water from the tube in steps, till a level of water between 40 and 70 cms had been reached.

Then the microphone had been removed and by connecting the output of the 0.5 Ω series resistor with the loudspeaker moving coil, to the a.c. voltmeter instead, the experiment had been repeated.

The resonance frequencies had been recorded with their corresponding minimum readings of the meter for each water column. The results for both tubes were recorded in tables (V-5)⟶(V-12), and plotted in figs. (V-6)⟶(V-13).

Referring to those figs. we notice the following:-

1 - <u>For a = 1.435 in. and t = 0.065 in.</u>

a - <u>The maximum hydrophone (microphone) readings</u>:-

Referring to fig. (V-6), we see that f_2 had a steady state value of 588 Hz, at high water levels and starts to increase at a level of about 90 cms., and f_2 had a steady state value of 540Hz at low water levels and starts to decrease at the same level of 90 cms. The two steady state values had been asymptotically around $f_{1,1}$ for the loaded diaphragm in air.

b - <u>The minimum loudspeaker current</u>:-

Referring to fig. (V-8), the steady state value of f_2 is 584 Hz, and starts to increase at a level of about 90 cms. while f_1 had a steady state value of 535 Hz, and starts to decrease v.s. water height at the same level.

As in the case of the glass tube, the usable range for measuring the velocity of sound in water in the tube is the range where f_1 is changing v.s. water height, while f_2 is constant. This range exactly ceases on approaching the water height which resonates at the same frequency $f_{1,1}$ for the diaphragm. This range for this tube starts at 90 cms water level and proceeds up the tube.

To increase the usable range to lower levels of water, the developed mode of vibration of the diaphragm in water should be increased which can be obtained by increasing the thickness of the diaphragm. This had been experimentally proved as the following unscaled diagram shows:-

Table [V-5] Maximum Microphone Readings. t = 0.065 in.

L Cms.	40	45	50	55.5	60	64	68	72	76	80	84	87	90	93
f_1 Hz	535	538	540	541	542	542	540	541	536	529	516	505	492	480
f_2 Hz	1035	990	908	826	781	702	674	653	626	607	596	590	587	587
f_c Hz	1147	1020	918	830	765	717	675	637	603	573	546	527	510	493
L Cms	96	99	102.0	105	108.5	112	116.5	120	123	126.7	131	135	138.5	142
f_1 Hz	468	454	445	435	422	412	400	378	372	364	342	334	323	315
f_2 Hz	586	584	589	586	585	587	588	588	589	588	588	589	588	588
f_c Hz	474	463	450	437	423	410	394	383	373	362	350	340	331	323

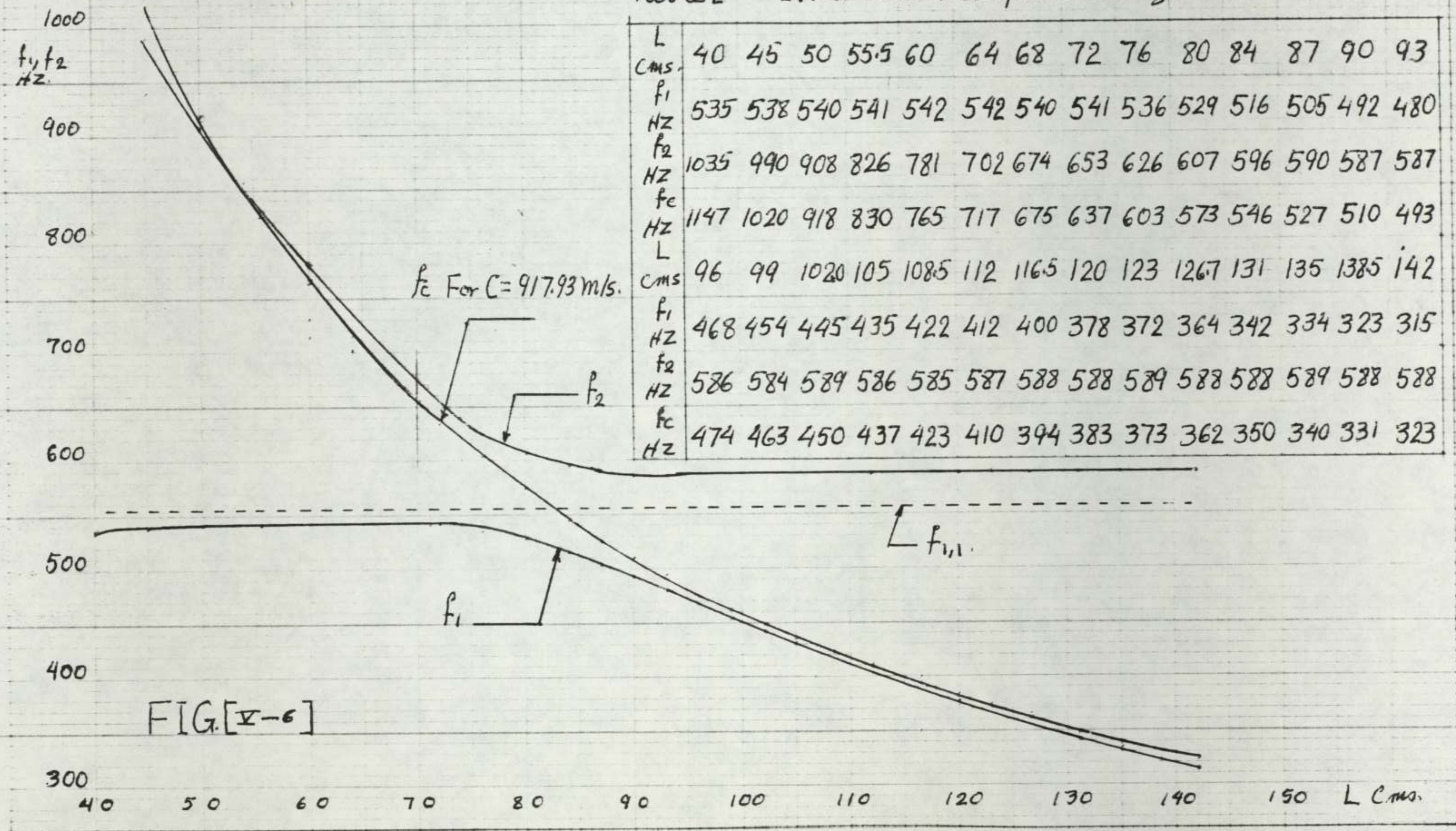


FIG. [V-6]

Table [V-6] Maximum Microphone Readings, t=0.065

L Cms.	40	45	50	55.5	60	64	68	72	76	80
V_{R1} m.v.	3.3	3.45	3.8	4.55	5.2	6.0	9.0	8.8	11.0	13.5
V_{R2} m.v.	1.5	3.7	4.2	5.2	6.2	6.9	12.5	13.75	14.25	14.0
L Cms.	84	87	90	93	96	99	102	105	108.5	112
V_{R1} m.v.	15.0	15.5	15.25	14.0	13.0	13.0	9.65	8.9	8.1	7.4
V_{R2} m.v.	12.5	10.5	8.85	7.3	6.7	5.6	5.15	4.7	4.1	3.8
L Cms.	116.5	120	123	126.7	131	135	138.5	142		
V_{R1} m.v.	6.2	4.9	4.8	4.8	3.5	3.2	3.05	3.0		
V_{R2} m.v.	3.45	3.2	3.0	2.85	2.42	2.35	2.15	2.10		

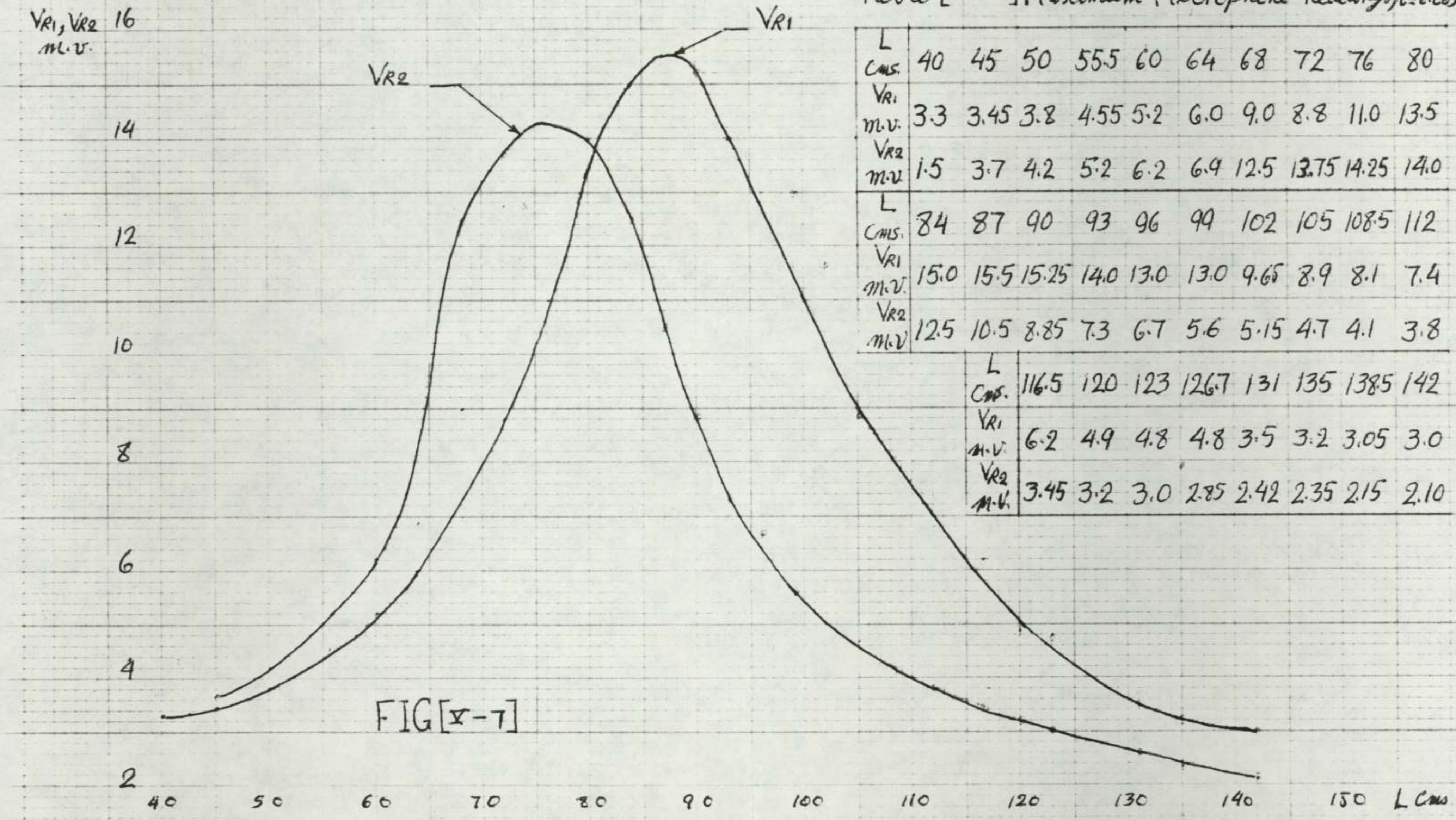


FIG [V-7]

Table [V-7] Minimum Loudspeaker Current Readings, $t = .065$

L cms	40	45	50	55	60	65	70	74	78	82	84.2	87.7
f_1 Hz	530	531	534	536	537	538	539	536	533	526	521	511
f_2 Hz	1082	1013	940	864	803	712	681	655	631	613	604	595
f_c Hz	1147	1020	918	835	765	706	656	620	588	560	545	523
L cms	90.1	92.2	97.2	102.4	107.5	112.5	117.5	122.6	127.6	132.8	137.8	143
f_1 Hz	502	495	475	454	436	419	404	394	370	358	347	334
f_2 Hz	591	588	583	584	584	584	586	586	587	587	586	587
f_c Hz	509	498	470	448	427	408	390	374	360	345	333	320

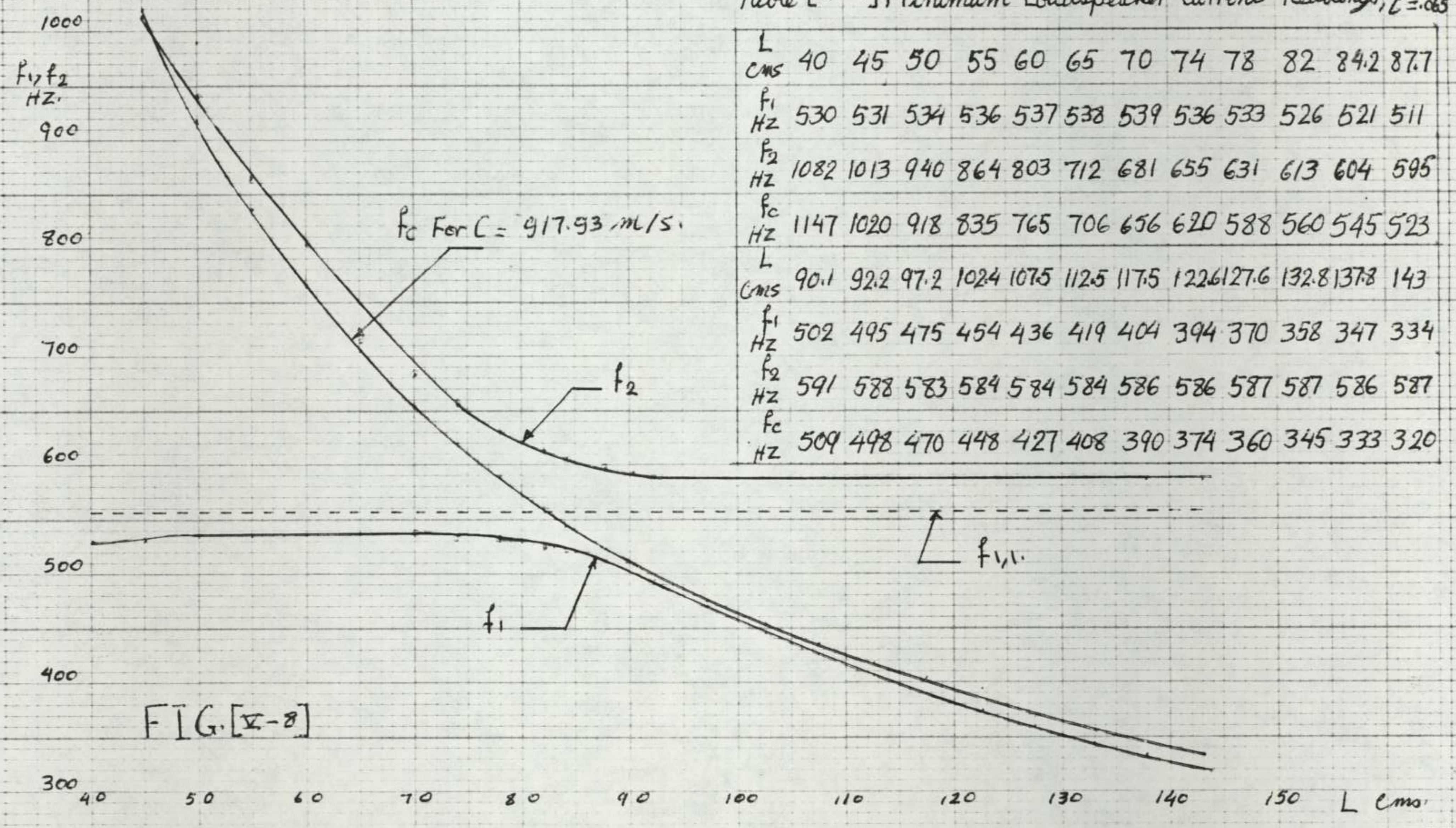
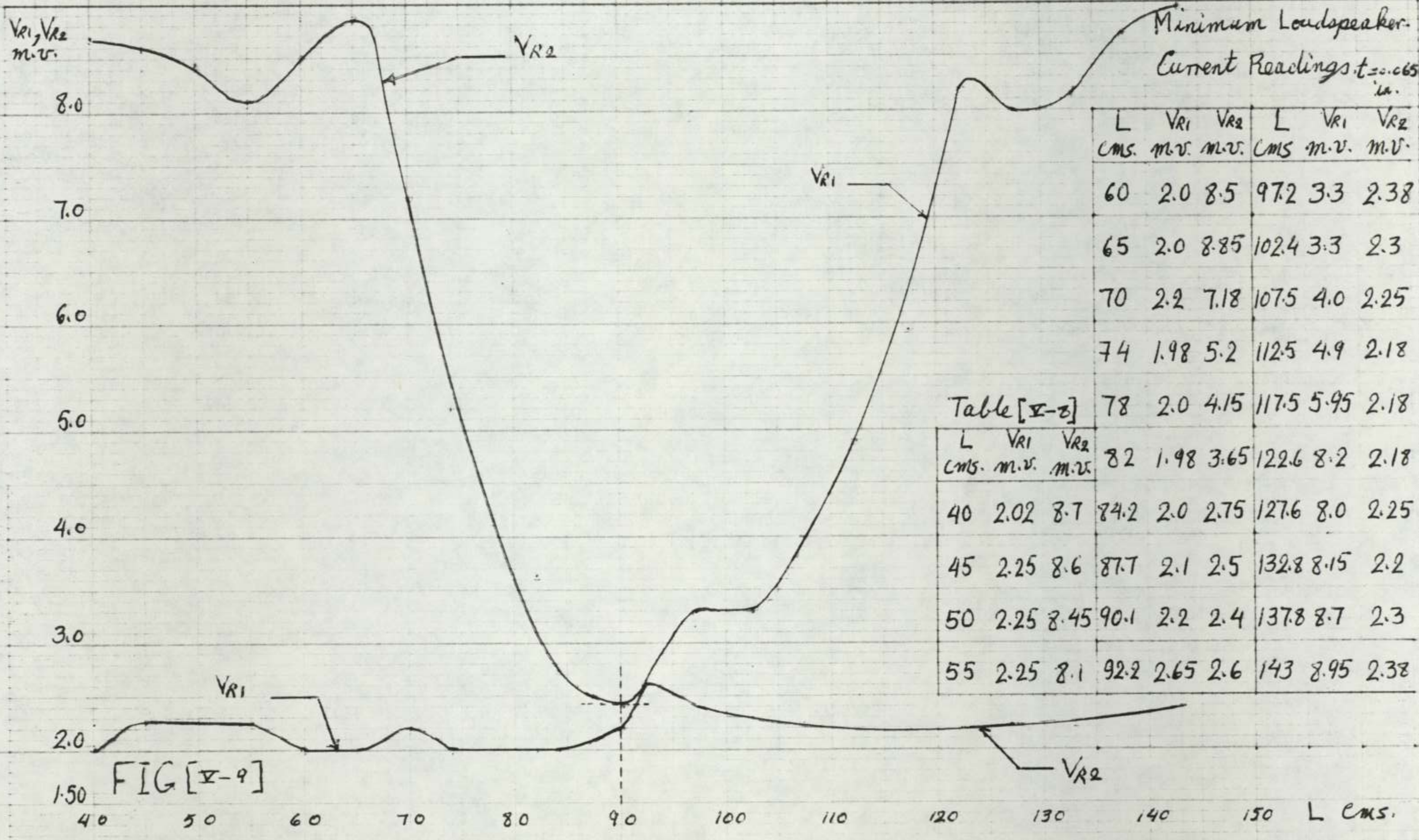


FIG. [V-8]

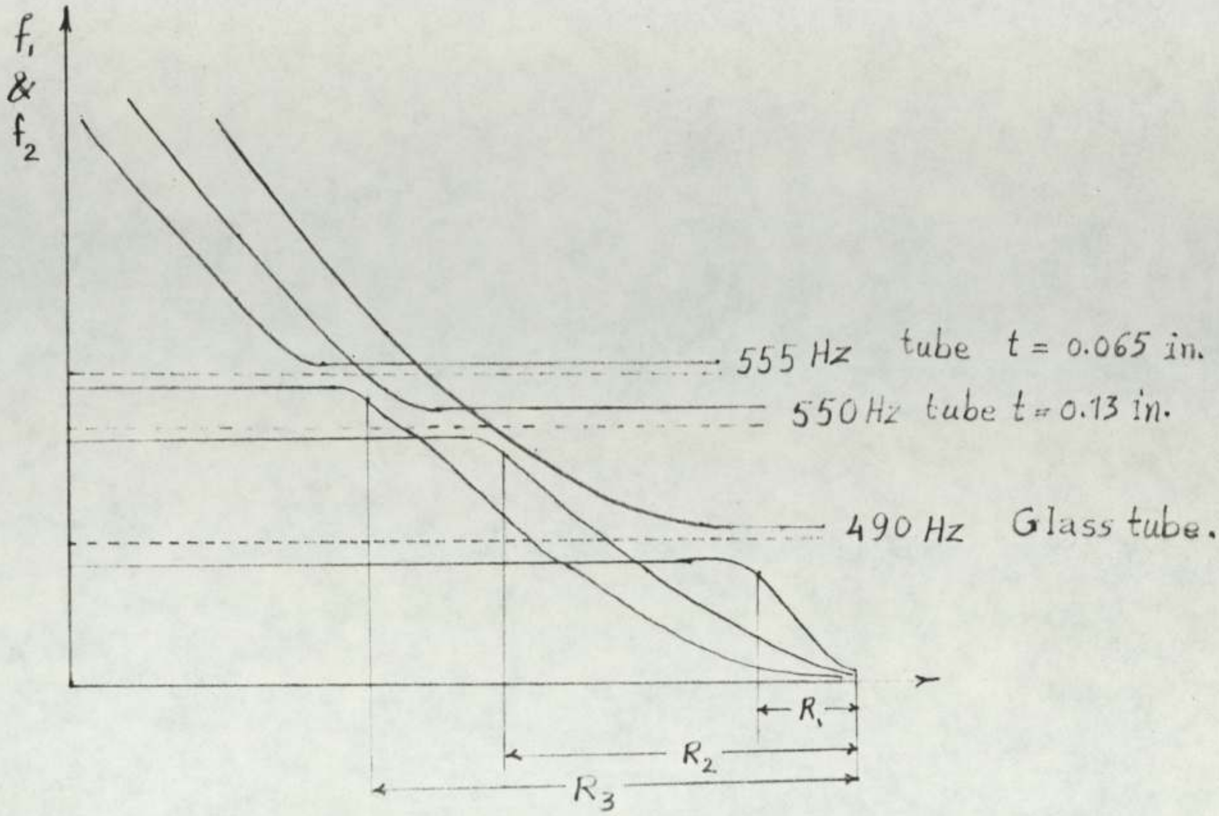


Minimum Loudspeaker Current Readings $t = 0.065$ in.

Table [V-8]

L cms.	V_{R1} m.v.	V_{R2} m.v.	L cms.	V_{R1} m.v.	V_{R2} m.v.	L cms.	V_{R1} m.v.	V_{R2} m.v.
			60	2.0	8.5	97.2	3.3	2.38
			65	2.0	8.85	102.4	3.3	2.3
			70	2.2	7.18	107.5	4.0	2.25
			74	1.98	5.2	112.5	4.9	2.18
			78	2.0	4.15	117.5	5.95	2.18
			82	1.98	3.65	122.6	8.2	2.18
40	2.02	8.7	84.2	2.0	2.75	127.6	8.0	2.25
45	2.25	8.6	87.7	2.1	2.5	132.8	8.15	2.2
50	2.25	8.45	90.1	2.2	2.4	137.8	8.7	2.3
55	2.25	8.1	92.2	2.65	2.6	143	8.95	2.38

FIG [V-9]



In previous publications(2, 3, 9) the determination of velocity of sound in water at such low heights, the frequency of the diaphragm had been chosen higher than the values used and also the measurement had been taken only at the height of water column to resonate at the resonance frequency of the diaphragm under water or in other words at the end of the usable range in this work.

The developed modes of vibration $f_{3,1}$ and $f_{1,1}$ for the diaphragms used, were due to the fact stated by Rayleigh before, that in spite of the easier calculations of the clamped boundary conditions for the diaphragm, yet the verification of the case experimentally was too difficult and in such case, possible modes of vibrations were likely to develop.

The following are the tables for the calculated velocity C v.s. water levels, for this tube:-

Table (V-5)a. Microphone readings.

L cms.	142	138.5	135	131	126.7	123	120	116.5
C m/s	894.6	894.71	901.8	896.09	921.376	915.12	908.20	932
L cms.	112	108.5	105	102	99	96	93	-
C m/s	924.88	915.74	913.5	907.8	908.92	898.56	892.8	-

Table (V-7)a. Loudspeaker current readings.

L cms.	142	137.8	132.8	127.6	122.6	117.5
C m/s	955.24	956.33	950.848	944.24	966.088	949.4
L cms.	112.5	107.5	102.4	97.2	92.2	-
C m/s	942.75	937.4	929.79	923.34	912.78	-

Considering $f_{1,1}$ = 555 Hz, for this tube, then $L_{1,1}$ as the height of water to resonate for C = 917.93 m/s, corresponding to C_o = 1485 m/s, is $L_{1,1}$ = 82.7 cms.

Referring to fig. (V-9), we see that V_{R2} gave a minimum at 90 cms.

2 - <u>For a = 1.37 in and t = 0.13 in</u>.

a - <u>The maximum microphone readings</u>:-

Referring to fig. (V-10), f_2 had a steady state value of 570 Hz, at high water levels and starts to increase at a level of about 110 cms, while f_1 had a steady state value of 522 Hz at low water levels and starts to decrease v.s. water heights up the tube at a level of about 100 cms.

Again the two steady state values were asymptotically around an estimated value of $f_{1,1}$. The estimation was based on the fact that in spite that this tube should theoretically give higher resonant frequency for the loaded diaphragm in air due to effective lower radius, yet experimentally it gave actually a lower value than the thinner tube.

b - <u>The minimum loudspeaker current</u>:-

Referring to fig. (V-12), f_2 had a steady state value of 575 Hz and starts to increase at a level of about 116 cms, while f_1 had a steady state value of 515 Hz and starts to decrease v.s. water heights up the tube at a level of about 100 cms., and the two steady state values were asymptotically around the previously estimated value of $f_{1,1}$.

Due to the relatively lower values of f_1, f_2 and $f_{1,1}$, the usable range for determining the velocity of sound in water in this tube had been decreased to confirm the unscaled diagram (page 47).

The following tables show the calculated velocity v.s. water heights for this tube:-

Table (V-9)a. Microphone readings.

L cms.	143	138	133	128	123	118	112.4	107.2	102.3
C m/s	1072.5	1070.88	1066.66	1062.4	1052.88	1036.04	1049.816	1033.408	1012.77

Table (V-11)a - Loudspeaker current readings.

L cms.	143	140	136	132	128	124
C m/s	1126.85	1125.6	1120.64	1106.72	1108.98	1098.69
L cms.	120	116	112	108	104	102
C m/s	1106.4	1095.04	1077.52	1069.88	1044.16	1030.2

Table[V-9] Maximum Microphone Readings. $t = 0.13$ in.

L cms	66.8	72	77	82	88.6	92	94	97	98.7	102.3	107.2	112.4	118	123	128	133	138	143
f_1 Hz.	521	523	523	522	522	519	517	511	508	495	482	467	439	428	415	401	388	375
f_2 Hz	808	761	719	678	641	621	612	600	595	588	575	571	571	570	570	574	574	576
f_c Hz	827	767	717	674	624	600	588	569	560	540	515	491	468	449	431	415	400	386

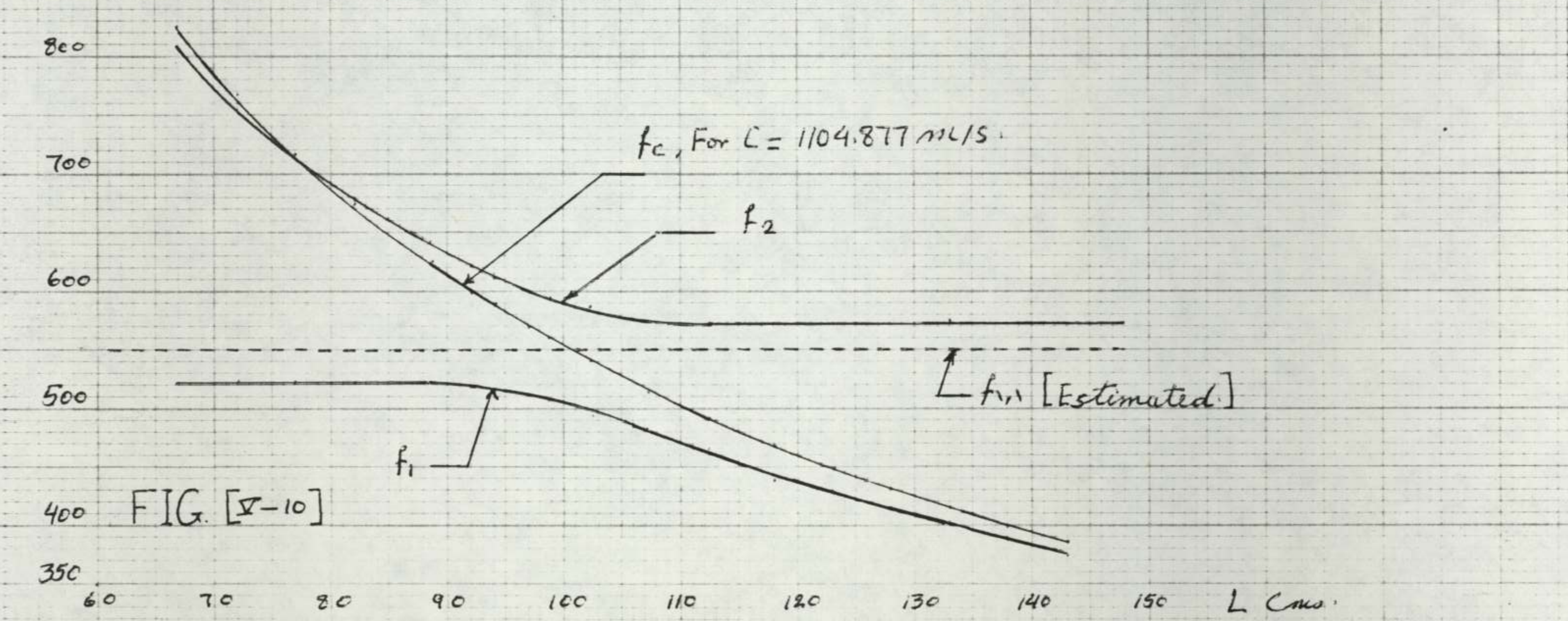


FIG. [V-10]

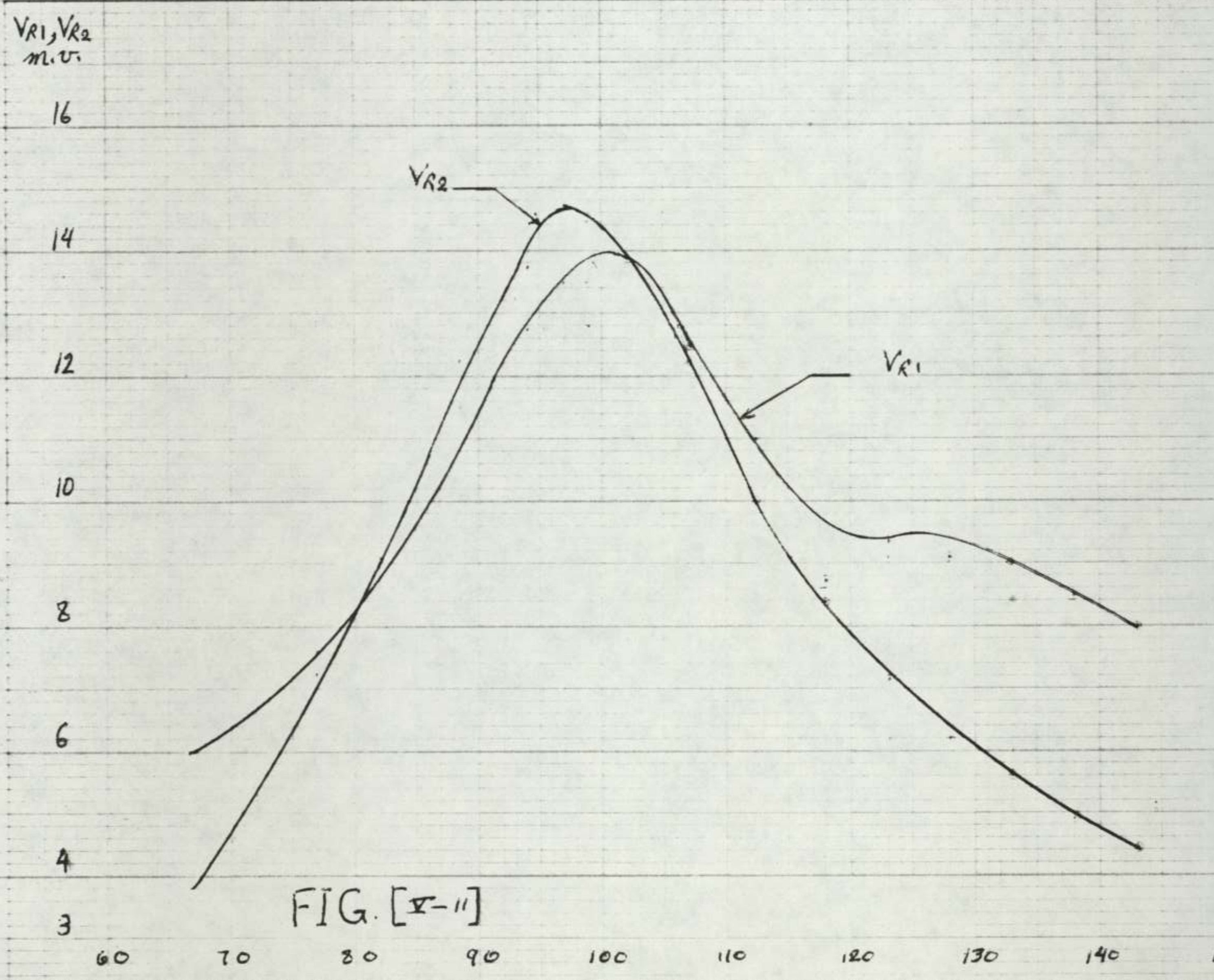


FIG. [V-11]

Table [V-10] $t = 0.13$ in

L Cms.	V_{R1} m.v.	V_{R2} m.v.	L Cms.	V_{R1} m.v.	V_{R2} m.v.
66.8	6	3.8	118	8.75	8.4
72	6.7	5.3	123	9.35	7.15
77	7.6	7.25	128	9.1	6.2
82	8.8	9.1	133	9	5.65
88.6	11.5	12.25	138	8.5	4.9
92	12.25	13.25	143	8	4.45
94	12.75	13.75			
97	13.5	14.75			
98.7	13.5	14.5			
102.3	13.9	14.25			
107.2	12.5	12.25			
112.4	11	10			

Maximum Microphone Readings.

Table [V-11] Minimum Loudspeaker Current readings. $t = 0.13$ in.

L Cms.	56	60	64	68	72	76	80	84	88	92	96	100	102	104	108	112	116	120	124
f_1 Hz.	515	516	517	520	520	520	520	520	519	517	515	511	505	502	493	483	472	461	443
f_2 Hz	1018	954	903	853	795	765	717	694	667	644	625	610	601	590	587	582	578	576	575
f_c Hz	987	921	863	812	767	727	690	658	627	600	575	552	541	531	511	497	476	460	446

L Cms	128	132	136	140	143
f_1 Hz	433	423	412	402	394
f_2 Hz	575	575	575	575	575
f_c Hz	431	418	406	394	386

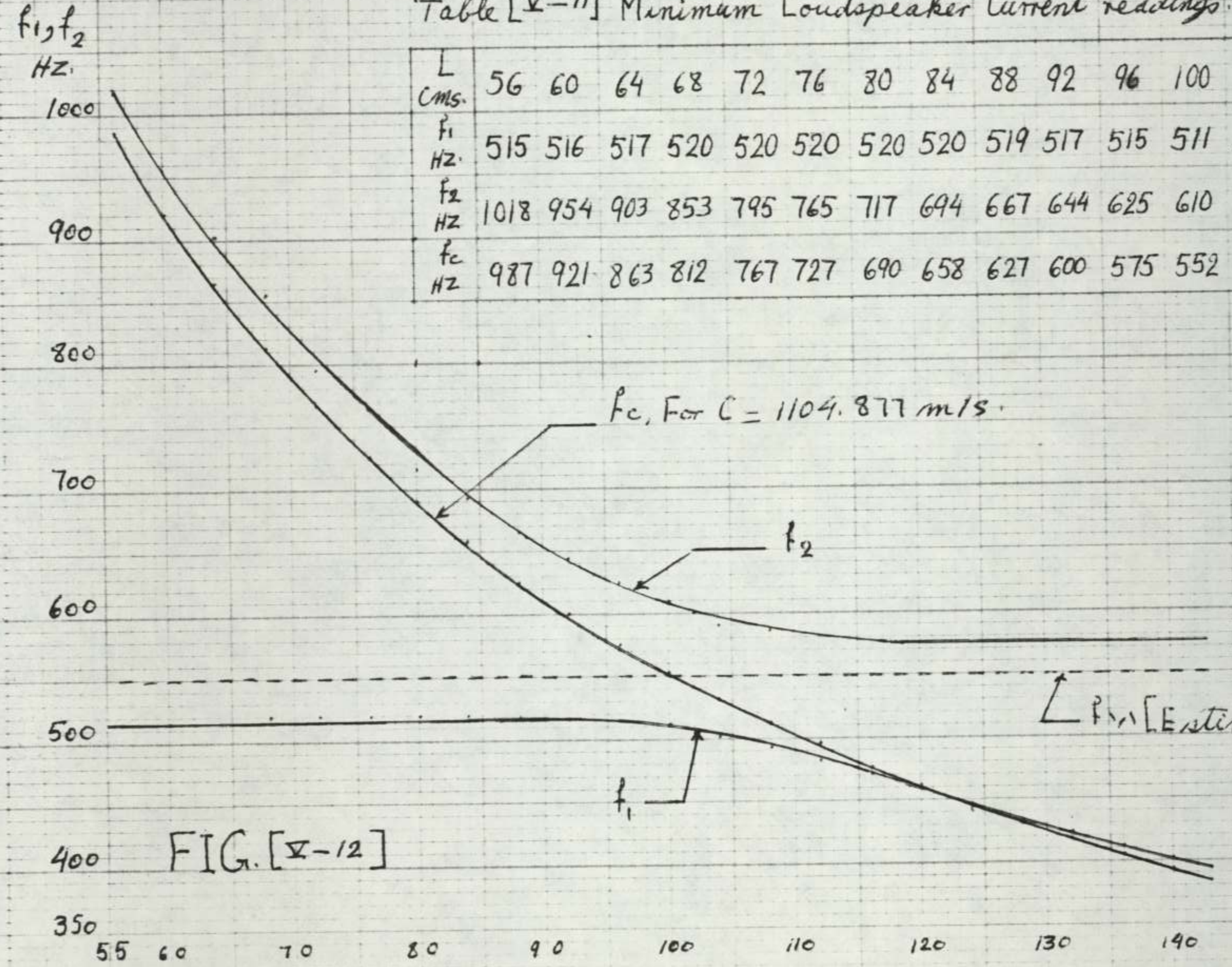


FIG. [V-12]

Table [V-12] Minimum Loudspeaker Current Readings. $t = 0.13$ in.

L cms.	56	60	64	68	72	76	80	84	88	92	96	100	102	104	108	112	116	120	124	128	132	136	140	143
V_{R1} m.v.	2.65	2.65	2.65	2.6	2.6	2.65	2.7	2.75	2.85	2.9	2.95	3.05	3.35	3.2	3.1	3.425	4.25	5.9	7.0	5.05	5.2	4.9	5.3	5.6
V_{R2} m.v.	9.6	9.65	9.65	10.25	10.4	10	9.5	7.25	7.2	5.15	4.3	3.65	3.55	3.4	3.45	3.225	3.2	3.15	3.18	3.18	3.18	3.18	3.18	3.18

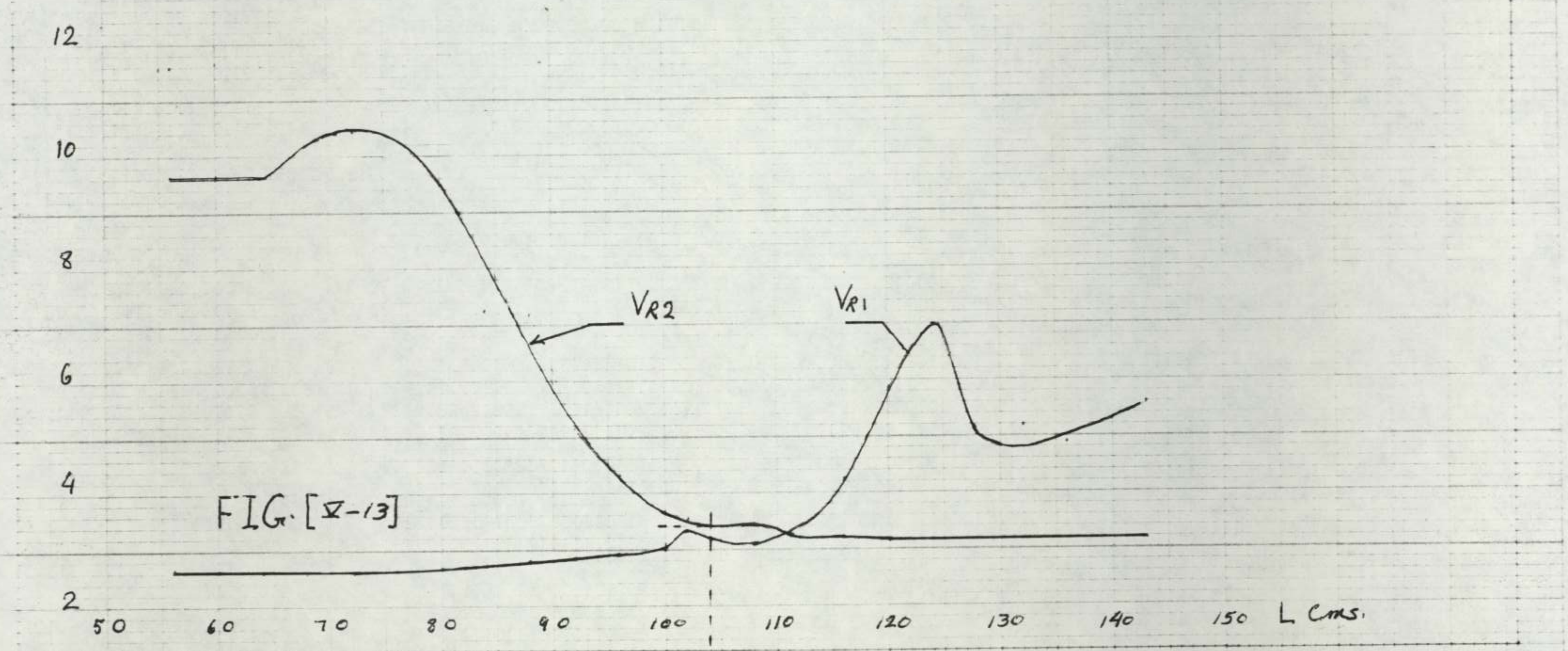


FIG. [V-13]

Considering $f_{1,1}$ for this tube = 550 Hz, then $L_{1,1}$ = C/1100. If C = 1104.877 m/s, for C_o = 1485 m/s, $\therefore$ $L_{1,1} \cong$ 100 cms. Referring to fig. (V-13), we see that V_{R2}, gave a minimum value at 104 cms.

Again referring to fig. (V-7) and fig. (V-11), we notice that the value of $L_{1,1}$ for both tubes, lie within the range of maximum sound energy radiated in the tube indicated by V_{R1}.

As in the case of the glass tube, the escape of air bubbles from an over — saturated water with air to the inner side of the tube wall had occurred and had been detected for the thicker tube by the minimum loudspeaker current resonance procedure. The results were recorded in tables (V-13) and (V-14), and plotted in figs. (V-14) and (V-15).

By comparing figs. (V-12) and (V-14), we find that both f_1 and f_2 had been reduced, with the result of reducing the value of C.

As in the case of the glass tube, the frequency of a theoretical curve having C = 917.93 m/s had been drawn v.s. water height in figs. (V-6) and (V-8), and another one, having C = 1104.877 m/s in figs. (V-10) and (V-12), for comparison.

Table [V-13] Minimum Loudspeaker Current Readings, $t = 0.13$ in.

L cms.	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82
f_1 Hz	507	507	507	506	506	506	506	503	500	496	492	488	480	474	468	456
f_2 Hz	877	823	802	760	741	699	660	650	627	609	594	584	574	566	563	559
L cms.	84	86	88	90	92	94	96	98	100	102	104	106	108	110	112	—
f_1 Hz	450	439	431	424	415	409	399	394	388	382	377	371	366	361	358	—
f_2 Hz	556	554	554	554	553	553	553	553	553	553	553	555	555	556	556	—

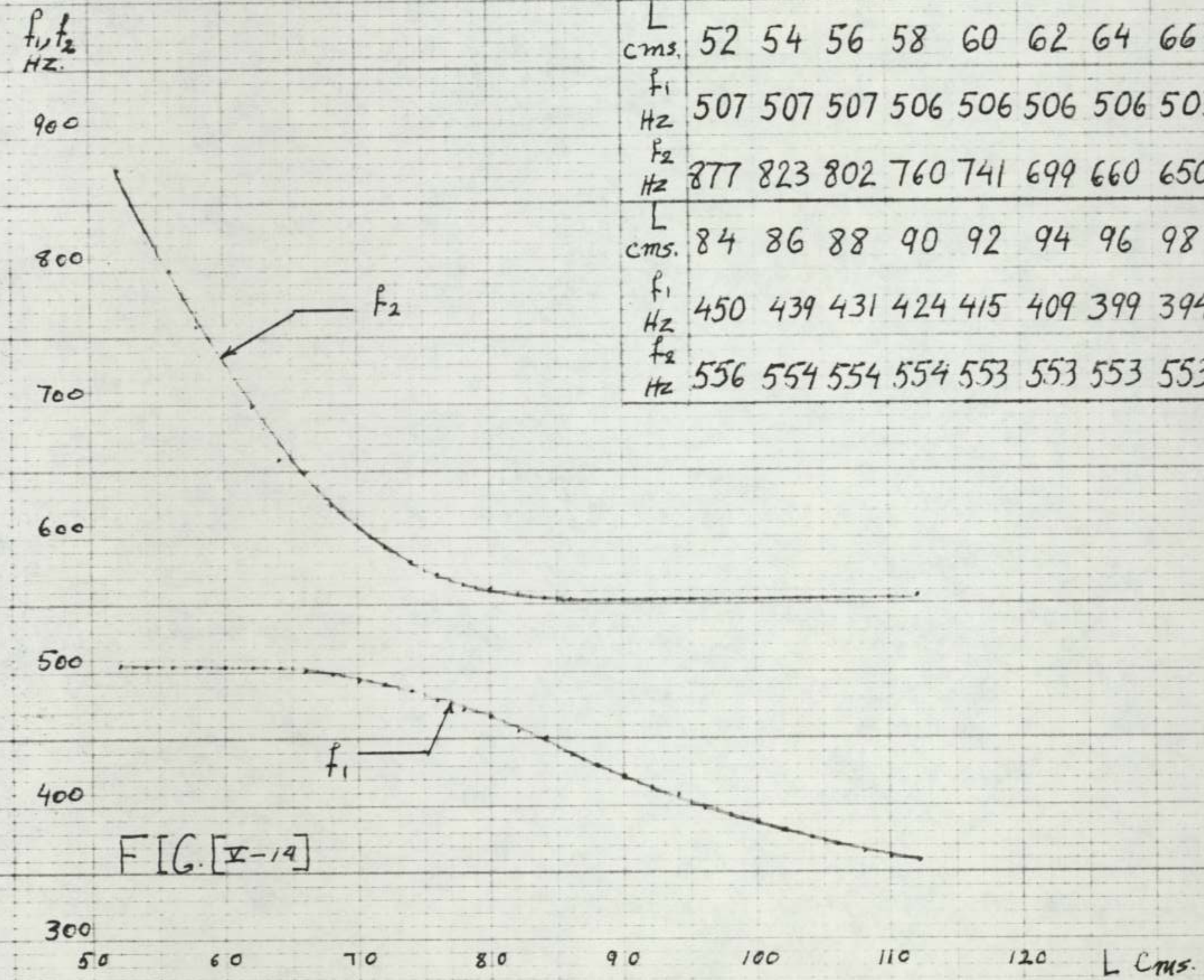


FIG. [V-14]

Table [V-14], Minimum Loudspeaker Current Readings, $t = 0.13$ in.

L cms	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94
V_{R1} m.v.	2.675	2.7	2.75	2.8	2.85	2.95	3.2	3.3	3.625	4.0	4.6	5.1	5.5	6.1	6.85	7.65	8.1	8.65	8.85	9.05	9.3	9.75
V_{R2} m.v.	10.6	10.7	10.8	11.0	11.1	10.5	10.6	10.25	9.6	8.95	8.15	7.25	6.45	5.7	5.0	4.55	4.1	3.9	3.75	3.55	3.45	3.35

L cms	96	98	100	102	104	106	108	110	112	—
V_{R1} m.v.	10.0	10.0	10.2	10.3	10.4	10.5	10.6	10.6	10.6	—
V_{R2} m.v.	3.25	3.2	3.18	3.15	3.175	3.1	3.10	3.08	3.15	—

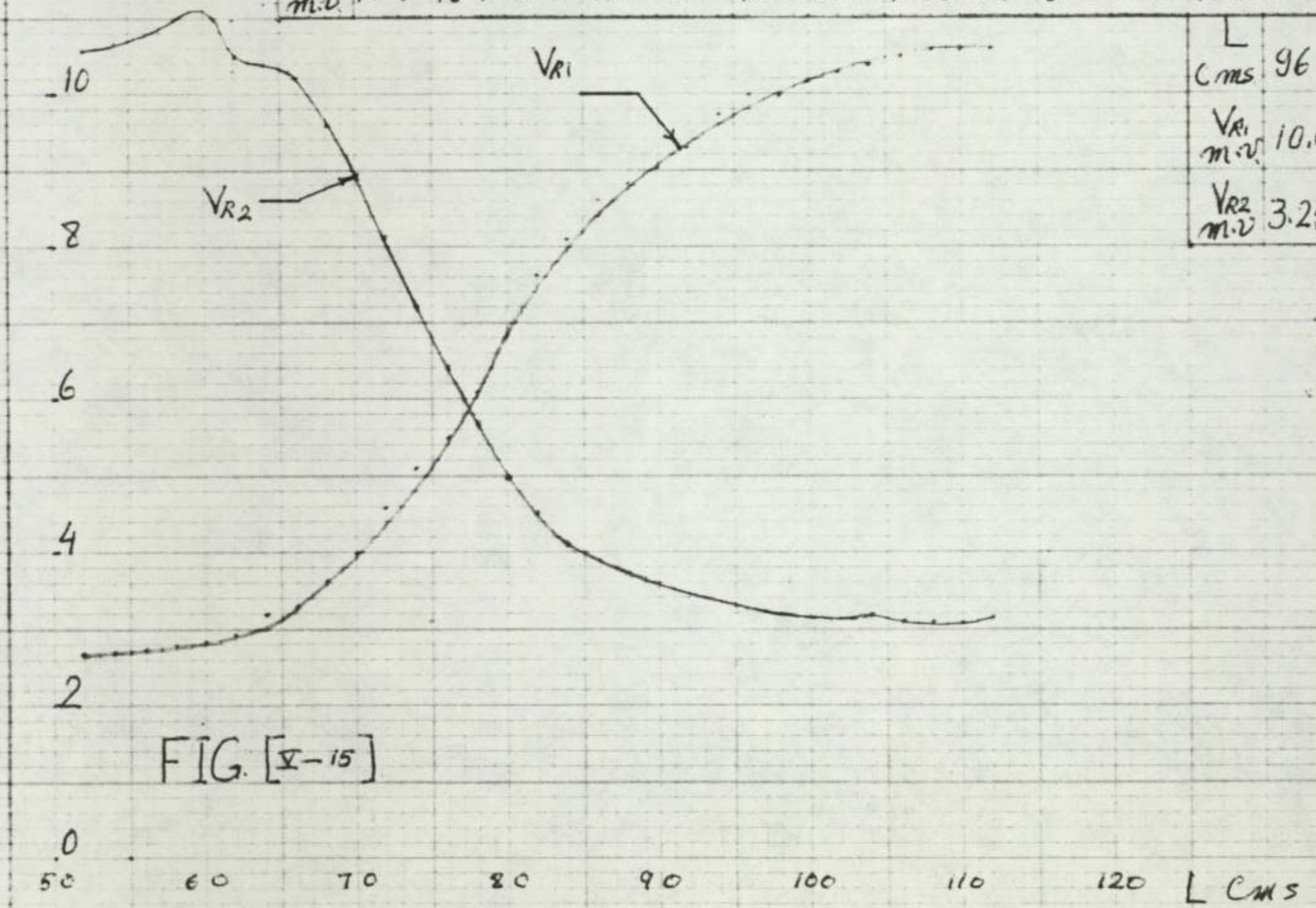


FIG. [V-15]

CHAPTER 6

The Velocity Correction:-

a - Elementary theory:-

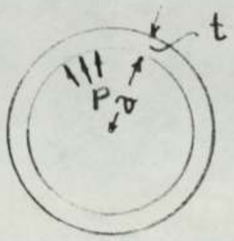


FIG. (VI-1)

Consider the following cross-section of a tube of internal radius (a) and wall thickness (t).

Due to the excess pressure of the wave through the liquid in the tube, a tension force is exerted upon the inside wall.

If (T) is the circumferential tension force per unit length then the pressure exerted on the wall is:-

$$P = T/a$$

If (δ_r) is the yield of the radius due to p, then the resulting increase in circumference is $2\pi\delta_r$. Hence the circumferential strain s is:-

$$S = 2\pi\delta_r/2\pi a = T/E.t = Pa/E.t, \text{ where}$$

E.t is the reaction force due to the elasticity of the walls, per unit length.

$\therefore \delta_r = Pa^2/E.t$, giving an increase of the tube cross-sectional area of $2\pi a\delta r = 2\pi Pa^3/E.t$, which is equal to the increase of volume of liquid per unit length of the tube having the above area as base. Hence the fractional increase of the volume of the liquid $= \delta v/V_o = 2\pi a\delta r/\pi a^2$

$$\therefore \delta V/V_o = 2\pi Pa^3/\pi a^2 E.t., \text{ from which:-}$$

$$(^1/p)(\delta V/V_o) = 2a/E.t = \text{increase in compressibility} =$$

increase of (^{1}/bulk modulus).

$$\frac{1}{K'} = \frac{1}{K} + \frac{2a}{E.t}$$, where K' is the apparent bulk modulus.

$$\frac{1}{K'} = \frac{1}{K}\left(1 + \frac{2Ka}{E.t}\right).$$ Since $C_o = \sqrt{K/\rho}$ and $C \cong \sqrt{K'/\rho}$

$$\therefore \frac{C_o}{C} = \sqrt{1 + 2aK/E.t},$$ which is the formula given by A.B. WOOD(9), according to Lamb. Hence for glass, $E = 6 \times 10^{11}$ dynes/cm^2, $K = 2.2 \times 10^{10}$ dynes/cm^2 for water.

If the tube has $t = \frac{1}{4}$ in., and internal radius $a = 1.5$ in then:-

$$Co/C = \left[1 + \frac{2 \times 2.2 \times 10^{10} \times 1.5}{0.25 \times 6 \times 10^{11}}\right]^{\frac{1}{2}} = \sqrt{1.44} = 1.2$$

Referring to table (V-3b), then for L = 138.5 cms., C = 1199.41 m/s and hence C_o = 1439.292 m/s.

For the aluminium tube having a = 1.435 in and t = 0.065 in, then if $E = 5.29 \times 10^{11}$ dynes/cm^2 and considering K is the same, then Co/C, will be:-

$$Co/C = \left[1 + \frac{2 \times 2.2 \times 10^{10} \times 1.435}{0.065 \times 6.29 \times 10^{11}}\right]^{\frac{1}{2}} = \sqrt{2.5443}$$

$$Co/C = 1.595$$

Considering the average value of C = Cav. = 908.47 m/s, for maximum microphone readings, then C_o is:-

$$C_o = 1449 \text{ m/s.}$$

If C = Cav. = 942.56 m/s, for minimum loudspeaker current readings, then C_o = 1503.38 m/s.

For the other tube having a = 1.37 in. and t = 0.13 in., then:-

$$C_o/C = \left[1 + \frac{2 \times 2.2 \times 10^{10} \times 137}{0.13 \times 6.29 \times 10^{11}}\right]^{\frac{1}{2}} = \sqrt{1.7371}$$

$$C_o/C = 1.318$$

For maximum microphone readings, Cav. = 1050.817 m/s, hence C_o = 1384.9768 m/s.

If C = Cav. = 1092.92 m/s for minimum loudspeaker current readings, then C_o = 1440.468 m/s.

Such values of C_o could be taken as an approximate estimation for the velocity of sound in bulk water, while the interpretation with more exact formulae, will follow.

b - <u>Green's[2] Formula</u>:-

Green's formula for the velocity correction had been given as:-

$$\alpha = \frac{1}{\mu(b^2-a^2)} \left[\frac{a^2\left(1 - \frac{\rho C^2}{(\lambda+2u)}\right)(\lambda+2\mu)}{\left(1 - \frac{\rho C^2}{E}\right)(3\lambda+2\mu)} + b^2\right] \quad \ldots\ldots\ (VI-1)$$

in λ and μ notation, where:-

a = inner radius of the tube

b = outer radius of the tube

ρ = density of the material of the tube

E = Young's modulus.

The normal stiffness $C_{11} = \lambda + 2\mu$ and the axial stiffness $C_{12} = \lambda$. Relating the stiffnesses to the corresponding compliances for aluminium as cubic system, we get according to J.F. Nye[10]:-

$$C11 = \frac{S11 + S12}{(S11=S12)(S11+2S12)} \quad \text{and } C12 = \frac{-S12}{(S11-S12)(S11+2S12)}$$

Nye[10] gave for aluminium, $S11 = 1.59 \times 10^{-12}$ cm^2/dyne and $S12 = -0.58 \times 10^{-12}$ cm^2/dyne. Also $E = {}^1/S11$ and Poisson's ratio $\sigma = -S12/S11 = -(S12)E$.

Hence changing the λ and u notation to σ and E notation, we find that:-

$$C11 = \frac{(1+S12/S11)}{S11(1-S12/S11)(1+2S12/S11)} = \frac{E(1-\sigma)}{(1+\sigma)(1-2\sigma)}$$

and $$C12 = \frac{E\sigma}{(1+\sigma)(1-2\sigma)}$$

Also $$3\lambda + 2u = C11 + 2\,C12 = \frac{E}{(1-2\sigma)}$$

and $$u = (C11 - C12)/2 = \frac{E}{2(1+\sigma)}$$

Thus the formula (VI-1) can be re-written as:-

$$= \frac{2}{E}\left[\frac{a^2 + b^2}{b^2 - a^2} + \sigma + \frac{a^2}{(b^2 - a^2)}\cdot\frac{2\sigma^2 \rho C^2}{(E - \rho C^2)}\right] \qquad \ldots\ldots \quad \text{(VI-2)}$$

Multiply both sides by $\rho_o C^2$, where ρ_o is the density of the water (or liquid), then:-

$$_oC^2 = \frac{2\rho_o C^2}{E}\left[\frac{a^2 + b^2}{b^2 - a^2} + \sigma + \frac{2\sigma^2 a^2 \rho C^2}{(b^2-a^2)(E-\rho C^2)}\right].$$

$$= \frac{2\rho_o C^2}{E}\left[\frac{a^2 + b^2}{b^2 - a^2} + \sigma\right] + \left[\frac{4\rho_o \rho \sigma^2 a^2 C^4}{E(b^2-a^2)(E-\rho C^2)}\right] \qquad \ldots\ldots \quad \text{(VI-3)}$$

Let $\rho_o C^2 \alpha = 2Z$, and since $C_o^2 = \dfrac{C^2}{1-2Z}$ $\qquad \ldots\ldots \quad$ (VI-4)

$C_o/C = 1/\sqrt{1-2Z}$, from which:-

$$C_o/C = 1 + Z + \frac{3}{2}Z^2 + \frac{5}{2}Z^3 + \frac{35}{8}Z^4 + \quad \ldots\ldots \quad \text{(VI-5)}$$

Which was the expansion obtained by Lamb before.

Equations (VI-3, 4 and 5) give a quadratic in C^2 which can be obtained if C_o had been assumed.

c - <u>Gronwall's[3] Formulae</u>:-

The formulae given by Gronwall in 1927 for the velocity correction were:-

$$y_N = \frac{\rho_o C^2}{E}\left[\left[\frac{2}{3}\ \frac{b^2+a^2}{b^2-a^2} + \sigma\right] + \frac{1}{3}\left(\frac{1}{2\sigma} - 1\right)\frac{a}{b-a}\right], \text{ for } \frac{b}{a} < 1.15 \quad \ldots\ldots \quad \text{(VI-6)}$$

and $y_K = \dfrac{\rho_o C^2}{E}\left[\dfrac{b^2+a^2}{b^2-a^2} + \sigma\right]$, for $\dfrac{b}{a} > 1.15$, while the expansion had been given as:-

$$C_o/C = 1 + y + 3y^2 + \left(12 - \frac{\pi^2}{3}\right)y^3 + (55 - 3\pi^2)\,y^4$$

$$+ \left(1428 - 455\frac{\pi^2}{3} + 137\frac{\pi^4}{15}\right)y^5 + \quad \ldots\ldots \quad \text{(VI-7)}$$

The two aluminium tubes used lie within $\frac{b}{a} < 1.15$.

Substituting for the coefficients of y and considering the tube having a = 1.435 in., then by assuming a value of C = 917.93 m/s for this tube (calculated from Green's formula for C_o = 1485 m/s), then:-

$$C_o/C = 1 + 0.241489 + 0.17495 + 0.122748 + 0.08619 + 0.6733 + \ldots$$

From which the sixth term is higher than any other term in the descending series and is not suitable for the velocity correction. The same had been noticed for the other tube, and the series stopped at the fifth term as:-

$$C_o/C = 1 + y + 3y^2 + (12 - \frac{\pi^2}{3})y^3 + (55 - 3\pi^2)y^4 \qquad \text{..... (VI-8)}$$

d - Comparison between Green's and Gronwall's Formulae:-

Referring to equation (VI-3) due to Green, we see that the first bracketed term in the R.H.S. is equal to 2 y_K due to Gronwall for $\frac{b}{a} > 1.15$.

To check the other term in the R.H.S. of equation (VI-3) we find that for the same tube and the value of C assumed before, the term

$$-\frac{a^2}{(b^2-a^2)} \cdot \frac{2\sigma^2 \rho C^2}{(E-\rho C^2)} = \frac{10.78 \times 0.266 \times 2.2777 \times 10^{10}}{60.6223 \times 10^{10}} = 0.1077,$$

which is comparable to the value of σ (see equation (VI-2)).

A computer program PET had been made based on the following:-

1. A range of values for C_o had been assumed to Green's formula from 1445 m/s up to 1495 m/s, and the corresponding values of C had been obtained for a range of thickness from 0.1651 up to 0.9906 cms in steps of 0.1651 cms., which is the thickness of the thinner tube.
2. The values of C obtained from Green's formula had been applied to Gronwall's formulae to calculate C_o^1 for both $\frac{b}{a} < 1.15$ and $\frac{b}{a} > 1.15$.
3. The ratio Co/C_o^1 had been obtained for the same range of Co and thickness assumed to Green's formula.
   For this, equation (VI-8) was used. The following diagram shows the arrangement:-

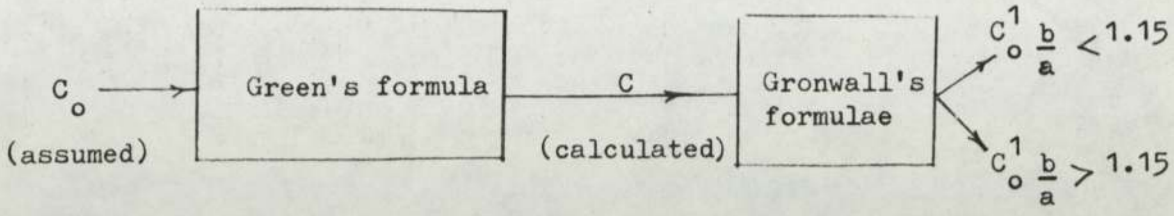


FIG. (VI-2)

The program PLT has shown that the maximum ratio for the range of tube thickness used and for the range of C_o assumed was $C_o/C_o^1 = 1.020353$.

Green's formula is continuous with thickness and in spite of its approximation, yet by applying the proper value for Poisson's ratio σ, it could be exact in determining α , and independent of frequency.

e - The Experimental Data Obtained:-

The following table shows the deviation of the average experimental values obtained for C from the values obtained by Green's formula at the range of C_o assumed, for the two aluminium tubes used:-

Table (VI-1)

Table No.	Average of Cm/s	Cav./C, for the following C_o in m/s					
		1445	1455	1465	1475	1485	1495
(V-5)a	908.47	1.0002	0.9975	0.99484	0.99224	0.98968	0.9871
(V-6)	942.56	1.0377	1.03495	1.0321	1.02949	1.0268	1.0242
(V-9)a	1050.817	0.96569	0.96194	0.95825	0.95463	0.95107	0.94756
(V-8)	1092.92	1.00438	1.000986	0.99665	0.99288	0.98917	0.98552

Table (VI-2)

	Maxm.deviation from Ca .
(V-5)a	1.8%
(V-7)a	3.1%
(V-9)a	3.7%
(V-11)a	5.8%

The recorded maximum deviation from the average had been always due to the points approaching the resonant frequency of the diaphragm, i.e. the end of the usable range in the tubes, while it largely decreases away from those points up the tube. In this work the value of C at C_o = 1485 m/s had been always chosen which was nearly the value determined by Pooler[4] before, so that the comparison between all formulae and this experimental work would be complete.

Also the difference in the values of Car/C between table (V-5)a and (V-6)a and table (V-7)a and (V-8)a decreases with increase in C_o assumed.

To check the stability of Gronwall's formula (after modification) for $\frac{b}{a} < 1.15$ at C_o = 1485 m/s, a trial and error method had been applied by assuming a value of C to determine the value of y. Then by the expansion (VI-8) the final value of C had been obtained. The new values of C had been assumed, until the assumed value of C to determine y was equal to the final value obtained from the expansion.

For the tube having a = 1.435 in., C = 916.2 m/s, and for the tube having a = 1.37 in, C = 1110.6 m/s, and hence the following table shows the final comparison:-

Table (VI-3)

Formula used	C, for a=1.435 in m/s	C, for a=1.37in
Green (or Lamb)	917.9354	1104.8771
Gronwall	916.2	1110.6
Experimental (average)	{908.477 942.56}	{1050.817 1092.92}

It is clear from the tables that the maximum microphone readings were better for the thinner tube, while on increasing the thickness, the minimum loudspeaker current readings were better, for the same outer diameter.

DISCUSSION

In this work, simple and direct methods had been applied to determine the velocity of sound in water in glass and aluminium tubes and gave values almost as predicted from the point of view of the tube material and thickness.

Also the project had shown that the water column resonance conditions had been achieved regarding the verification of its boundary conditions for better accuracy in the measurements.

The diaphragm had changed the mode of frequency of the vibrations under water and its variation with unbalanced water loads had been studied.

The project had shown that the usable range of water height had ceased when the length of water column, which resonated with the same frequency of the diaphragm had been reached, and such range could be increased for higher values of resonance frequencies of the diaphragm under water.

By comparing the results with previous publications in the subject, we see that the present arrangement obtained values within the range of the limited accuracy previously obtained.

Also it appeared from the project that some of both experimental and theoretical results need more careful consideration and investigation.

APPENDIX

Program 'PET'

1st Division

Green's Formula

```
'BEGIN' 'REAL' Z, B, D, E, M, F, U, T, R, S, Q, N, A, G, P, L, K, W,
               V, Y, C;
        'REAL' 'ARRAY' X [1:6];
        'FOR' I: = 1 'STEP' 1 'UNTIL' 6 'DO'
        X [I] : = READ;  B: = READ;  D: = READ;  E: = READ;  M: = READ;
        'FOR' Z: = 3.6449 'STEP' - 0.1651 'UNTIL' 2.7 'DO'
        'BEGIN'
               F: = (B+D)/((B-D)*(B+(D/0.5)));
               U: = (B-D)/((B-D)*(B+(D/0.5)));
               T: = 1/B;
               R: = - D/B;
               S: = F/E;
               Q: = T/E;
               N: = 0.5/(B-D);
               A: = ((M↑2)*(1+R)+(Z↑2)*(1-R))/((M↑2)-(Z↑2))/T;
               G: = (2/3)*A+((1/6)*(1-(R/0.5))*Z)/R/T/(M-Z);
               P: = (Z↑2)*F/(N*U*((M↑2)-(Z↑2)));
               L: = (M↑2)/(N*((M↑2)-(Z↑2)));
               NEWLINE (1);
               PRINT (Z, 2, 4);
               PRINT (P, 2, 15);
               PRINT (L, 2, 15);
```

```
'BEGIN'
        K: = (Q*P+S*L)*X [I]↑2;
        W: = (S+Q*S*P+Q*S*L)*X [I] ↑2;
        V: = Q*S*X [I] ↑ 2;
        Y: = ((W+Q*S)-SQRT((W+Q*S)↑2-4*V*(K+S)))/(2*(K+S));
        C: = SQRT(Y);
        NEWLINE (1);
        PRINT (C, 7, 2);
        PRINT (Y*G, 3, 15);
        PRINT (Y*A, 3, 15);
      'END';
   'END';
'END';
```

Six thickness conditions were obtained, from this program.

2nd Division

Gronwall's Formula (Modified)

```
'BEGIN' 'REAL' A, B, C, D, E, F, G, H, U, V, W, J;
        'INTEGER' I;
        'REAL' 'ARRAY' X [1:7] , Y [1:7], Z [1:7] , K[ 1:7] , L [1:7] ,
                                    P [1:7] ;
        'REAL' 'ARRAY' M [ 1:7 ], N [1:7] , Q [ 1:7] , S[ 1:7 ], R[ 1:7],
                                    T [1:7] ;
        'FOR' I: = 1 'STEP' 1 'UNTIL' 7 'DO'
        X [I] : = READ;
        'FOR' I: = 1 'STEP' 1 'UNTIL' 7 'DO'
        Y [I] : = READ;
```

```
'FOR' I: = ------------------
Z [I] : = READ;
'FOR' I: = ------------------
K [I] : = READ;
'FOR' I: = ------------------
L [I] : = READ;
'FOR' I: = ------------------
P [I] : = READ;
'FOR' I: = ------------------
M [I] : = READ;
'FOR' I: = ------------------
N [I] : = READ;
'FOR' I: = ------------------
Q [I] : = READ;
'FOR' I: = ------------------
S [I] : = READ;
'FOR' I: = ------------------
R [I] : = READ;
'FOR' I: = ------------------
T [I] : = READ;
'FOR' I: = ------------------
'BEGIN'
A: = M[I]*(1+X[I]+3*X[I]↑2+X[I]↑3/0.1148438+X[I]↑4/0.0394208);
B: = N[I]*(1+Y[I]+3*Y[I]↑2+Y[I]↑3/0.1148438+Y[I]↑4/0.0394208);
C: = Q[I]*(1+Z[I]+3*Z[I]↑2+Z[I]↑3/0.1148438+Z[I]↑4/0.0394208);
D: = S[I]*(1+K[I]+3*K[I]↑2+K[I]↑3/0.1148438+K[I]↑4/0.0394208);
E: = R[I]*(1+L[I]+3*L[I]↑2+L[I]↑3/0.1148438+L[I]↑4/0.0394208);
F: = T[I]*(1+P[I]+3*P[I]↑2+P[I]↑3/0.1148438+P[I]↑4/0.0394208);
```

```
NEWLINE (1);
PRINT (A, 7, 3);  PRINT (B, 7, 3);  PRINT (C, 7, 3);
PRINT (D, 7, 3);  PRINT (E, 7, 3);  PRINT (F, 7, 3);
G: = 144500/A;  H: = 145500/B;  U: = 146500/C;
V: = 147500/D;  W: = 148500/E;  J: = 149500/F;
NEWLINE (1);
PRINT (G, 2, 6);  PRINT (H, 2, 6);  PRINT (U, 2, 6);
PRINT (V, 2, 6);  PRINT (W, 2, 6);  PRINT (J, 2, 6);
    'END';
'END';
```

The Data of the Seventh Condition had been taken from Earlier Programs on the System, and had been put in order with the obtained values.

According to Gronwall's Formula, the product Y*G had been used for $(M-Z)/Z < 0.15$, while Y*A for $(M-Z)/Z > 0.15$.

The two products had been calculated for the whole range of thickness to visualize the difference.

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