# THE PRACTICAL ULTIMATE STRENGTH DESIGN OF REINFORCED CONCRETE FRAMEWORKS

by

VEDAT DEVECI



A Thesis submitted for the degree

of

MASTER OF PHILOSOPHY

NOVEMBER 1975

# CONTENTS

## ACKNOWLEDGEMENTS

The Author wishes to thank Mr. A. W. Astill for his help and advice during the supervision of this project. Grateful thanks are also to Professor M. Holmes, Head of Department of Civil Engineering, for his advice, and to Miss D. Drew for preparing the type script of this thesis.

SYNOPSIS

Practical ultimate design methods in finding bending moment envelopes are explained and examples are given. An attempt has been made to find the relation between the ratio of total load to dead load with respect to cracking lengths, hyper-plastic moments, support moments and efficencies of design. Graphs are plotted and tables are presented to verify such relations.

A new method is developed by using the relation of cracking length to span, hyper-plastic moment, support moment against the total load to dead load ratio. It is found that standard curves can be plotted from which bending moment values could be found easily for frameworks. These charts can be used for the reinforced concrete members in which redistribution up to 30% of moments can take place. This method of analysis is found satisfactory when compared with other methods in the analysis of five equal span continuous beam.

## NOTATIONS

### CP 110 Method

| | |
|---|---|
| $A_c$ | Area of Concrete |
| $A'_s$ | Area of Compression reinforcement |
| $A_s$ | Area of tension reinforcement |
| $A_s$ prov. | Area of tension reinforcement provided |
| $A_s$ req. | Area of tension reinforcement required |
| a | Deflection |
| a' | Distance from compression face to the point at which the crack width is being calculated |
| $a_{cr}$ | Distance from the point (crack) considered to the surface of the nearest longitudinal bar |
| b | Width of the section |
| d | Effective depth of tension reinforcement |
| d' | Depth to compression reinforcement |
| $E_c$ | Static secant modulus of elasticity of concrete |
| $E_s$ | Modulus of elasticity of steel |
| e | Eccentricity |
| F | Ultimate load |
| $f_{cu}$ | Characteristic concrete cube strength |
| $f_k$ | Characteristic strength |
| $f_y$ | Characteristic strength of reinforcement |
| G | Shear modulus |
| $G_k$ | Dead load |
| g | Distributed dead load |
| $h_f$ | thickness of flange |
| I | Second moment of area |
| h | Overall depth of section in plane of bending |
| M | Bending moment due to ultimate loads |
| $M_1$ | Maximum initial moment in a column due to ultimate loads |
| $M_t$ | Total moment in a column due to ultimate loads |
| $M_u$ | Ultimate resistance moment |
| N | Ultimate axial load at section considered |
| $N_{bal}$ | Axial load on a column corresponding to the balanced condition |

| | |
|---|---|
| $Q_k$ | Characteristic imposed load |
| $q$ | Distributed live load |
| $q_k$ | Characteristic live load per unit area |
| $S_v$ | Spacing of links along the member |
| $u$ | Perimeter |
| $V$ | Shear force due to ultimate loads |
| $v$ | Shear stress |
| $W_k$ | Characteristic wind load |
| $x$ | Neutral axis depth |
| $z$ | Lever arm |
| $\beta_b$ | Ratio of beam moments with respect of service stress in beams |
| $\beta_{red}$ | Ratio of reduction in bending moment |
| $\gamma_f$ | Partial safety factor for load |
| $\gamma_m$ | Partial safety factor for strength |
| $\varepsilon$ | Strain in concrete at the level considered |
| $\rho$ | $\rho = \frac{A_s}{bd}$ |
| $\Sigma Asv$ | Area of shear reinforcement |
| $\Sigma u_s$ | Sum of the effective parameters of the tension reinforcement |
| $\emptyset$ | Bar size |

A. L. L. Baker Method

| | |
|---|---|
| $A_c$ | Area of Concrete |
| $A_s$ | Area of tension steel |
| $A'_s$ | Area of compression steel |
| $b$ | Width of rectangular section |
| $br$ | Breadth of rib of T - section |
| $D$ | Overall depth of section |
| $d_1$ | Effective depth of section |
| $d_s$ | Depth of flange in T - section |
| $E$ | $\frac{\text{Stress}}{\text{Strain}}$ generally |
| $E_{c_1}$ | Value of E for concrete at limit-state of yield $L_1$ |
| $E_{s_1}$ | Value of E for steel at limit-state of yield $L_1$ |
| $f_s$ | Stress in steel |
| $f_c$ | Stress in concrete |

| | |
|---|---|
| $f_s'$ | Maximum stress in compression steel |
| $K$ | $K = M/bd_1^2 fc_2$ , in which $fc_2 = \sigma b^*$ in a balanced section |
| $K_1$ | $K_1$ = Average Compression stress/max compression stress |
| $K_2$ | $K$ = Depth of compression resultant/depth to neutral axis |
| $K_3$ | $K = fc\ /U_W$ |
| $K_4 d_1$ | $K_4 d_1$ = Depth to centre of compression steel when tension is developed across section |
| $K_5 D$ | Depth to centre of compression steel when section is subjected to compression only |
| $K_6$ | $K_6 = e_c/e_{c_1}$ |
| $K_7$ | $K_7 = fs/fs_1$ |
| $K^*$ | Adjustment factor to $\gamma k$ |
| $L_1$ | Limit - state of yield of material |
| $L_2$ | Ultimate limit - state of material |
| $M$ | Bending moment |
| $M_1, M_2$ | Bending moment at limit state $L_1$ and $L_2$ respectively |
| $M_i\ M_k$ | Ordinates of bending moment diagrams plotted along frame members |
| $n d_1$ | Depth to neutral axis |
| $n_1 d_1, n_2 d_1$ | Depth to neutral axis at limit $L_1$ and $L_2$ |
| $Q_k$ | Characteristic Load |
| $Q^*$ | Design load |
| $\sigma m$ | Mean unit strength of material |
| $\sigma k$ | Characteristic unit strength of material |
| $\sigma^*$ | Design unit strength of material $\sigma^* = \sigma k/ \gamma m$ |
| $\delta ik$ | Displacement in direction of action of Xi, when any other unknown Xk, assumed equal to unity, acts on frame made statically determinate by the insertion of sufficient number releases. |
| $\gamma_1 \ldots \gamma_9$ | Partial adjustment factors influencing $\gamma k$ |
| $\gamma s$ | Partial safety factors of lead. |
| $\gamma m$ | Partial safety factors for strength of material. |
| $\gamma k$ | $\gamma k = \gamma s\ \gamma m$. |
| $\theta$ | Rotation at any section between limit $L_1$ and $L_2$ |

Optimum Design Method

| | |
|---|---|
| $f$ | Degree of end fixity $f = o$ free ends, $f = 1$ fixed ends |
| $i$ | Subscript referring to possible mechanism $i = 1,2,....p$ |
| $j$ | Subscript referring to critical section |
| $k$ | Subscript referring to particular design solution |
| $l_j$ | Equivalent length over which Mpj = constant |
| $m$ | Number of independent mechanisms (equilibrium) equations |
| $Mj$ | Elastic envelope moment at section j |
| $M_{max}$ | Max elastic moment over all j |
| $M_{pj}$ | Plastic moment of section j |
| $n$ | Degree of static indeterminacy |
| $N$ | Number of spans of a continuous beam |
| $p$ | Total number of possible mechanisms |
| $q$ | Subscript referring to applied loads |
| $r$ | Total number of applied loads |
| $s$ | Number of critical sections |
| $Ui$ | Ultimate safety parameter of mechanism i, i/ o |
| $\vartheta_k$ | Efficency index of design k, $Vk/V_E$ |
| $Vk$ | Volume of flexural steel in limit design |
| $V_E$ | Volume of flexural steel in elastic design |
| $w$ | Dead to live load ratio, $W_d/W_L$ |
| $W_D W_L$ | Dead and live load, respectively |
| $W_q$ | Applied service load |
| $W_u$ | Ultimate load of the structure |
| $\alpha_j$ | Yield safety parameter of section j, $\lambda ij/\lambda o = \frac{Mpj}{\lambda oMj}$ |
| $\delta_{iq}$ | Kinematic displacement of loads Wg |
| $\lambda o$ | Specified overall load factor of the structure |
| $\theta ij$ | Relative notation of plastic hinge j |

# CHAPTER 1.

## HISTORICAL REVIEW

## 1.1. INTRODUCTION

In the last few years considerable developments have taken place in the field of structural concrete, particularly on limit state and optimum design. Inelastic behaviour of structural concrete has played an important part in recent design recommendations both in this country and abroad.

The purpose of design may, perhaps oversimply be stated as the provision of a structure complying with the clients requirements. In design appropriate attention must be paid to overall economy, the safety and aesthetics of the structure. The economic factor implies that the investment covering both first cost and maintanence should be the minimum consistent with the fulfilment of the clients requirements. The safety implies that the risk of failure of all parts of the structure should be sufficently small during its specified life. The aesthetic factor implies that the complete structure should be consonant with its environment and generally pleasing to the eye. In this case the economic aspect will be the most important one, but the design process entails finding the cheapest solution which is capable of satisfying the appropriate safety and serviceability considerations.

The application of plastic analysis in the design of redundant concrete structures is limited by the need for the fulfilment of safety conditions as well as serviceability condition. The main safety condition is the effective formation of the plastic mechanism which depends upon the rotation capacity of plastic hinges, (BAKER, A. L. L.; 1951,(40)) The classical methods of plastic analysis and design of structural frames assume moment-curvature relations of unlimited ductility. The limited ductility of reinforced concrete sections had led some authors to base ultimate load design methods on such limitations by imposing maximum values of plastic hinge angular discontinuites and has led others to emphasize the implied ductility without explicitly calculating discontinuites at or near failure. A more general approach has been developed in which the structure is reduced to a determinate form as a basis for both the statical and kinematical analyses but without implying that the selected hinges are the actual plastic hinges (COHN M.Z., 1962). The limit analysis methods, the ductility methods, and the flexibility methods of elastic analysis can all be regarded as special cases of this general method which is called optimum design.

Nowadays in this country, limit state design and its application by design charts as given in CP 110, part 2, is a very popular and useful method. The code of practice (CP 110, 1972) accepts a new limitation to redistribution design and new load factors depending on the nature of the load and the type of materials. Reasonably economical designs result.In this chapter papers on limit design methods and practical design methods are discussed.

## 1.2. PLASTIC DESIGN IN REINFORCED CONCRETE

The development of reinforced concrete design by plastic methods was based on the inelastic behaviour of redundant steel structures and the fundamental principle that a structure will not collapse until sufficient plastic hinges have developed to form a mechanism. Each hinge is permitted to develop its full plastic moment and any rotation of the members rotation of the members on either side of the hinge is assumed to have no effect on the development of any adjacent plastic hinges. Plastic steel design will be considered valid when it satisfies the following conditions.

a) Equilibrium Condition: Bending moment distribution must be in equilibrium with external loads.

b) Collapse Mechanism Condition: A sufficient number of plastic hinges must exist to transform either the whole or part of the structure into a mechanism.

c) The Yield Condition: Full plastic moment nowhere to be exceeded.

It is the angular rotation which differentiates between plastic theory applied to reinforced concrete and plastic theory applied to steel. The permissible rotation value ($\theta$) must be known in addition to maximum moment which can be carried by the section. The reinforced concrete designer when using plastic methods has therefore to restrict not only the number of plastic hinges but also the rotation at each. The ultimate strain in tension reinforcement varies from less than 0.5 to over 2 per cent. To avoid excessive flexural cracking, it is desirable to limit hinge rotation for structural concrete even when considerable rotation capacity is present after extensive cracking.

The other important respect in which limit design of structural concrete differs is the distribution of moment of resistance. By varying the amount and location of reinforcement, the positive and negative resistance of structural concrete members at ultimate load capacity will be reasonably close to the moment distribution corresponding to elastic behaviour. It is possible arbitrarilly to choose locations and plastic moments for a number of hinges required to form a mechanism in such a manner that the equilibrium conditions are satisfied. The yield condition may then be satisfied by proportioning reinforcement to avoid yielding between the chosen plastic hinges.

### 1.3.1. Early Investigations on Plastic Design

It is difficult to trace the origin of the concept of plastic design but as early as 1914 KAZINCZY G. V. (41) suggested the development of plastic hinges in continuous structure near ultimate load for steel structure. He also conducted in 1933 the first extensive test series demonstrating moment redistribution in reinforced concrete structures He tested ten two-span continuous beams loaded at third points (1) and he found that all beams failed when both span and support sections reached their maximum moment capacity as evaluated by the ultimate strength theory of that period.

Glanville and Thomas (1935, (35)) conducted a test series to verify and demonstrate the redistribution of moments in reinforced concrete beams and frames as a result of yield in either the concrete or steel. The beams tested were two-span continuous beams loaded with concentrated load in each span, no relationship was found between the amount of steel used and the degree of redistribution attained. For experiments on frames, pin-ended single bay portals were chosen. It was found for the former case that the columns fail first and it was seen that further analytical and experimental work was evidently necessary to enable design engineers to predict, with confidence, the safe degree of redistribution in any one particular structure.

In 1949 A. L. L. Baker (3) put forward a trial and error method of computing the amount of moment redistribution in continuous beams (1949, (3)). He showed that even in the elastic - plastic stage the slope of a beam could be expressed as $\int \frac{Mds}{EI}$ if EI values for the elastic and plastic stages are used appropriately. He also expressed the moment of inertia in terms of the deformation of concrete fig (1.1)

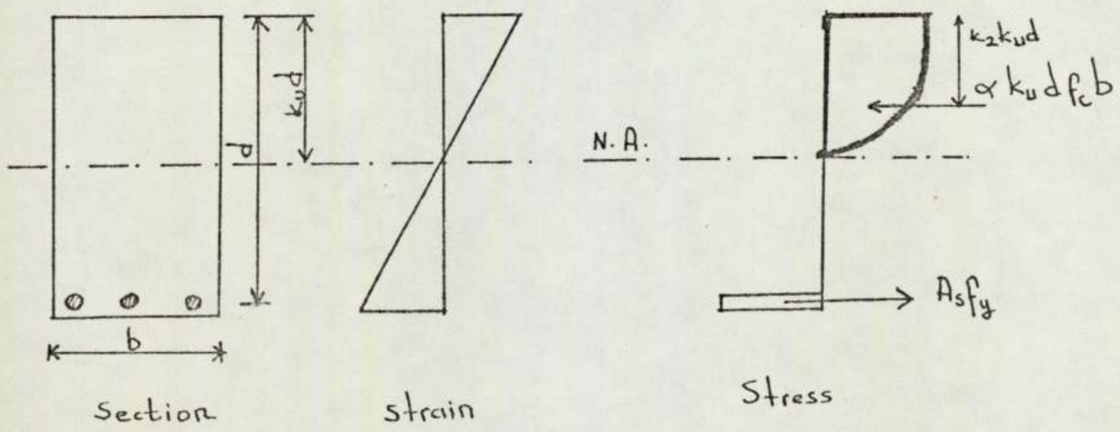



fig. 1.1.

$$I = \propto bd^3 \, (Ku^2 - K_2Ku^3) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

If the case of a two span continuous beam symmetrically loaded with a uniformly distributed load is considered, the moment diagram for the beam is as shown in fig. (1.2.) where $M_f$ and $M_F$ are the free and redundant moments respectively. Applying the moment area principle;

$$\text{Slope at B} = 0 = \frac{1}{\ell} \int \frac{M_{Fx}}{EI} \, ds - \frac{1}{\ell} \int \frac{M_{fx}}{EI} \, ds \;\ldots\ldots (2)$$

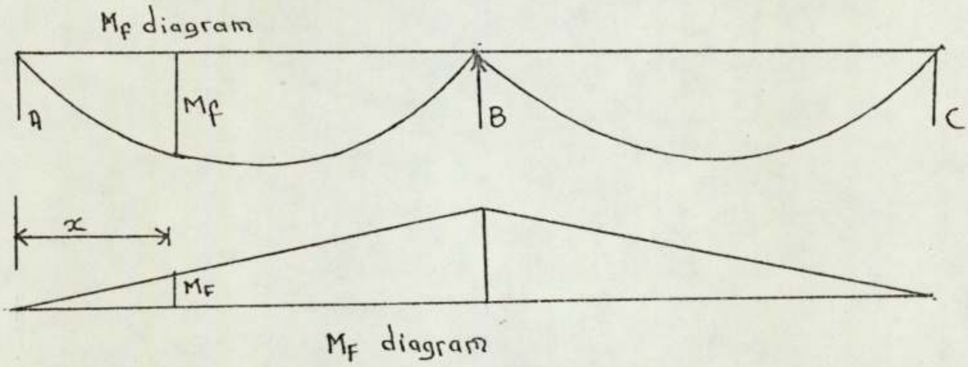



fig (1.2.)

For a particular percentage of steel in the support section, Ku could be computed for the various sections along the length of the beam between the support and first yielding sections. The appropriate EI value could then be determined. The correct $M_f$ value could be obtained by trial and error so that Equation (2) was satisfied. It was shown by this experiment that redistribution of moments due to primary crushing of concrete was not as effective as that due to primary yield of steel.

Following the introduction of the "Plastic Hinge Theory" of structural steel by J. F. Baker, A. L. L. Baker proposed the following equations which provided that strains at critical sections were checked (1951, (4)).

$$\delta_{01} + \overline{X}_1 \delta_{11} \quad + \overline{X}_2 \delta_{12} \quad .......... \quad + \overline{X}_n \ \delta_{in} = - \theta_1$$

$$\delta_{0n} + \overline{X}_1 \delta_{1n} \quad + \overline{X}_2 \delta_{2n} \quad .......... \quad + \overline{X}_n \ \delta_{nn} = - \theta_n \quad (3)$$

where, $\delta_{in}$. $\delta_{13}$ .......... $\delta_{in}$ are the influence coefficents for hinge rotations of the framework when unit moment is applied at hinge section 1,2, etc.

$\theta_1, \theta_2 \cdots\cdots \theta_n$ are the plastic rotations at hinge 1,2, ....n in rotations.

$\overline{X}_1, \overline{X}_2 \ldots\ldots \overline{X}_n$ are the unknown plastic moment magnitudes at plastic hinges.

In subsequent years A. L. L. Baker and his team developed much analytical and experimental data to verify the validity of the equations (5, 6, 7, 9, 17, 18, 56, 57). In 1953 A. L. L. Baker introduced the trial and adjustment method and at the same time he established some safe limiting $\theta$ values (1953 (5)). The fundamental principle of the method is to assign arbitrary values to $\overline{X}_1$ , $\overline{X}_2$ ..........$\overline{X}_n$ in Equ. (3) and evaluate the values. If the $\theta$ values so obtained are less than the safe limiting ones, then the chosen $\overline{X}_1$ , $\overline{X}_2$ ..........$\overline{X}_n$ values can be used, otherwise they are adjusted until the values are reduced to their permissible magnitudes. Baker's fundamental equation is general and applicable to all statically indeterminate reinforced concrete structures neglecting fatigue. The main difference between elastic and plastic analysis is that, elastic analysis in concerned with the behaviour of the structure before elastic breakdown of the materials whilst plastic analysis is concerned solely with the behaviour after elastic breakdown in certain critical sections .

$$\theta = \int^{l_p} \frac{M}{(EI)_p} dx - \int^{l_p} \frac{M}{(EI)_e} dx \qquad (4)$$

where, $l_p$ = length of spread of plasticity along the longitudinal axis of the member.

M = Moment at sections along yield length.

$(EI)_p$ (EI) value after yield.

$(EI)_e$ (EI) value before yield.

A. L. L. Baker derived and recommended the following expressions for $\theta$ (1956 (8)).

$$\theta = \frac{\varepsilon_u l_p}{kud} \quad \text{(tensile hinges)} \qquad (5a)$$

$$\theta = \frac{(\varepsilon_u - \varepsilon_s) l_p}{d} \quad \text{(compressive hinges)} \qquad (5b)$$

where $l_p$ = length of yield.

$\varepsilon_u$ = plastic strain of concrete

$\varepsilon_s$ = strain of reinforcement on least stressed edge

kud = the depth of neutral axis at the instant concrete is crushed.

$d$ = effective depth of the section.

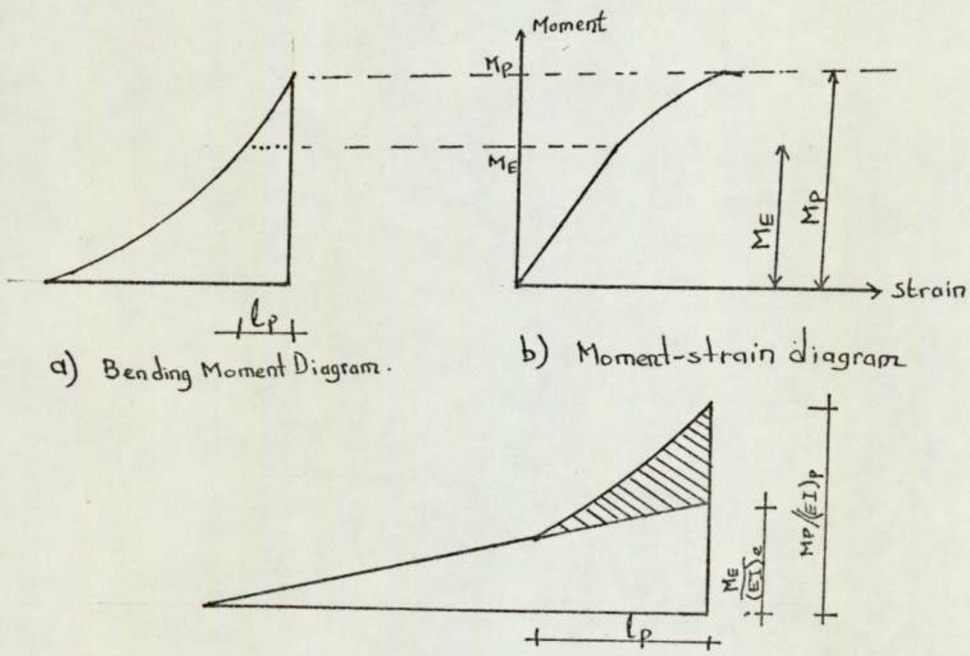
Baker recommended $\varepsilon_u = 0.001$, $(\varepsilon_u - \varepsilon_s) = 0.001$ and $l_p = d$ as safe limiting values to be used in design which is based on results obtained from tests of statically indeterminate members.

A. L. L. Baker and C. W. Yu demonstrated for rectangular frame structures that further simplification could be achieved and simple design formulas may also evolve (5, 6, 7, 57). A. L. L. Baker (1956 (20)) also suggested that hinges may conveniently be assumed at the intersections of beams and columns and he also has shown how graphs can be plotted to give $\theta$ values directly with respect to stiffness ratio between beams and columns. The method proposed by A. L. L. Baker is a very lengthy process and C. W. Yu (1954 (56, 57)) developed a "Block Relaxations" procedure that converges more rapidly.

The actual development of plasticity in the hinge sections as established by C. W. Yu was verified experimentally and generalized analytically by Chan (1954/55 (17, 18)). Referring to Equ. (4) it is evident that the length of yield is a function of

1) The moment - strain curve of the section.
2) The shape of the bending - moment diagram due to external load.

If the length of yield ($\ell p$) is determined from the above relationship, then $\Theta$ is represented by the shaded area of the curve as shown in fig. (1.3.c). Chan showed that the ultimate strain of the concrete could be controlled by placing an appropriate quantity of binders at the hinge section to increase the critical shaded area in fig. (1.3.c)




c) Curvature diagram

fig. (1.3.)

It has been shown that the concrete strain can be safely increased to as much as 0.01 when suitable binders are added. With this high strain it is possible to accommodate all practical and economical modes of moment distribution in a redundant structure. However cracks and deflections under working load conditions may often limit the permissible strains. This analysis is very doubtful, especially in a rigidly jointed structure where the sudden increase of rigidity near the junction causes a very complex distribution of localized stresses.

L.H.N. Lee (1955, (43)) suggested that, by assuming a stress - strain curve in concrete compression, a relationship could be established between moment and curvature which could be used as follows: By differentiating the general equation of equilibrium.

$$f_c = PE_s \left( e_s \frac{de_s}{de_c} + 2e_s \frac{de_s}{de_c} + e_s \right) \ldots\ldots\ldots \quad (6)$$

where

fc = strain in concrete

$e_s$ = tensile strain in steel

$e_c$ = compressive strain in concrete corresponding fc

By measuring $e_c$ and $e_s$ from beam tests a curve for fc can be traced with respect to $e_c$. Strain - stress relationship could be approximated by $fc = He_c - Be_c^2$, where H and B are constant and equal to $\left(2 \frac{fcm}{e_{cm}}\right)$ and $\frac{fcm}{(e_{cm})^2}$ respectively, fcm denotes the maximum compressive stress in concrete, and $e_{cm}$ its corresponding strain. The horizontal force equilibrium can be written

$$k^2 \left( \frac{H}{2} - \frac{B\chi kd}{3} \right) = pEs\ (1 - k) \quad (7)$$

The moment of resistance can be expressed as

$$Mr = bd^3 k^2 \chi \left[ \frac{H}{6}(3 - k) - \frac{B}{12} \chi kd(4 - k) \right] \quad (8)$$

where k = depth of neutral axis

$\chi$ = curvature.

The relationship between $\chi$ and M can be derived from Equ. (7) and (8). Then the distribution of moment due to plasticity for fixed-end beams, continuous beam and other simple structures can be determined in the conventional manner.

G. C. Ernst (1955 (32)) restated moment area theorems to include the behaviour of structures in inelastic range. Accordingly, a unit rotation diagram is used instead of the conventional M/EI diagram for the elastic case (fig. 1.4.)

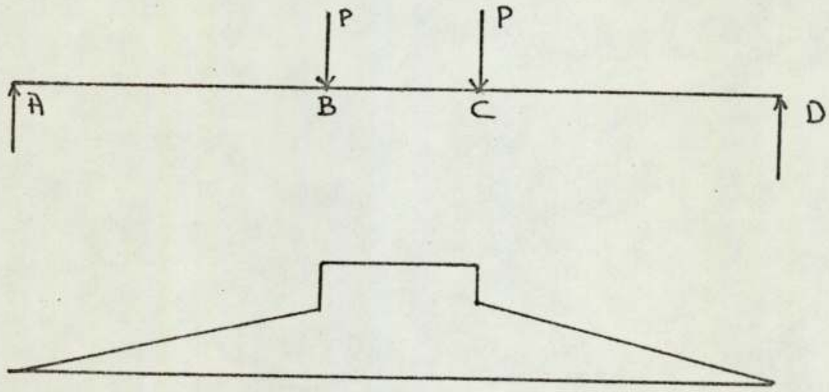



fig. (1.4.)

For the equilibrium of forces

$$p f_s = k\, f_{av} \qquad (9)$$

and for linear strain disbribution

$$\frac{e_c}{e_t} = \frac{k}{1-k} \qquad (10)$$

The unit rotations at any section = $\phi = (e_c + e_t)/d$ where d is the effective depth. From Equ. (9) and equ. (10)

$$\phi d = e_c + e_t = \frac{e_c f_{av}}{p f_s} \qquad (11)$$

where, $\theta e$ = Unit rotation at yield stage

$\theta u$ = Unit rotation at Ultimate stage

p = Percentage of steel

$f_s$ = Stress in steel

fav = Average concrete stress

k = Ratio of depth of neutral axis to the effective depth of beam

$e_c$ = Strain of concrete at extreme fibre

$e_t$ = Strain of steel.

Evidently $\phi o$ and $\phi u$ can be determined by substituting the appropriate values of $e_c$ .......etc., in equation (11) and the magnitudes of all the plastic rotations can be determined.

G. C. Ernst (1957) conducted a series of experiments to investigate the amount of plastic rotation in simulated beam and column connections for both slow and fast loading. The primary object of the tests was the study of plastic deformation available at failure. The principal conclusion derived was that the amount of plastic rotation increases with decreasing steel percentage confirming Baker's earlier finding that primary crushing of the concrete gave very little redistribution.

R. Gartner (1957 (34) recommended a more rigorous method of estimating $\theta$ values obtained by Baker's method. In the light of Chan's finding he also assumed that the length of plastic yield is a function of the external bending moment diagram ( $\ell p = \frac{Mp - Me}{V}$ ). He defined a steel hinge as one in which the concrete commenced to yield before the steel. The former $\theta = \mathcal{E}s\ell p/(1 - ku)d$ while for the latter $\theta = \mathcal{E}u\ell p/d$. For a section reinforced with particular percentage of steel maximum elastic moment ($M_E$) and Maximum plastic moment (Mp) evaluated by using the appropriate ku and $\alpha$ values (fig 1.1.) and hence the $\theta$ values can be checked. This evidently is a compromise between Baker's and Chan's method.

W. T. Marshall (1957, (44)) introduced a formula to evaluate $\theta$ involving the elastic and plastic moment of section. He suggested

$$\theta = \frac{A_x}{EI\ell}\left(1 - \frac{M_p}{M_E}\right) \qquad (12)$$

where

$A_x$ = Moment of the area of the free bending moment diagram about support B

E = Modulus of concrete, assumed equal to $1/_n$ times the modulus elasticity of steel.

Mp = Plastic moment assigned to the hinge section.

$\ell$ = The length of fixed end beam.

$M_E$ = Elastic moment

I = Moment of inertia of transformed section.

According to the formula the elastic distribution of moment must be known first before $\theta$ values can be assessed.

A. M. Mattock (1959 (45)) conducted two series of tests on two span continuous beams designed by an arbitrary redistribution method. He reported that, redistribution of design bending moments for reinforced concrete continuous beam by amounts up to 25% does not appear to affect adversely the performance of the beam either in the working-load range or at failure. Cracking and deflections of beams with redistributed design bending moments were not more severe than that of beams designed for the same load, but using distribution of bending moments predicted by elastic theory. The factor of safety against failure of reinforced concrete continuous beams is unaffected by redistribution of the design bending moments.

In the early 1960's Commission XI of the European Concrete Committee (C. E. B.) under Prof. A. L. L. Baker's chairmanship initiated and co-ordinated a fairly comprehensive experimental programme that was conducted at various research establishments in a number of countries. On the basis of this work Baker suggested that (25, 26)

$$\theta_p = \frac{(\varepsilon_{cu} - \varepsilon_{ce})}{n_{ud}} \ell_p \qquad (13)$$

where $\ell_p = \frac{k_1 k_2}{k_2 n} d$

Expression (13) was recommended in the report on "Ultimate Design of Reinforced Concrete Structures", published by Institution of Civil Engineers (1962,(52))

$$\theta_p = \frac{(\varepsilon_{cu} - \varepsilon_{ce})}{n_{ud}} \ell_p \quad \text{(tension occur at critical section)} \qquad (14a)$$

$$p = \frac{(\varepsilon_{cu} - \varepsilon_{ce})}{d} \ell_p \quad \text{(no tension)}$$

where $\varepsilon_{ce} = 0.002$

$\varepsilon_{cu} = 0.0035$ (unbound concrete)

$\varepsilon_{cu} = 0.012$ (well bound concrete)

$\ell_p = k_1 k_2 k_3 \left(\frac{z}{d}\right) \frac{1}{4} d$

$k_2 = (1 + 0.5 \frac{P}{Pu})$

Pu = ultimate capacity of the member for axial load when no bending moments act.

P = ultimate axial load for the member (allowing for the bending moment when present.)

$k_1 = 0.7$ (mild steel) ; $k_1 = 0.9$ (cold worked steel)

$k_3 = 0.6$ when $\varepsilon_{cu} = 41.1$ N/mm $k_3 = 0.9$ when $\varepsilon_{Cu} = 13.8$ N/mm$^2$

After two years Baker and Amarakone (1964 (13)) suggested the expression which gives a precise value for $\varepsilon_{cu}$, then the expression (14) becomes:

$$\theta_p = \frac{\varepsilon_{cu} - \varepsilon_{ce}}{d} \times 0.8 k_1 k_3 \quad (2) \qquad (15)$$

where $0.7 < k_1 < 0.9$ steel parameter

$0.6 < k_3 < 0.9$ concrete parameter

$k_2 = \left(1 + 0.5 \frac{P}{Pu}\right)$ Axial forces paramter

Amarakone in 1966 (14) gives a new expression for permissible rotation in his research report which is derived from expression (15). He proposed that;

$$k_1 k_3 = 0.5$$

$$\frac{z}{d} = 6$$

$$\frac{\ell_{pc}}{dn_2} = 0.8 k_1 k_3 \frac{z}{d} = 0.8 \times 0.5 \times 6 = 2.4$$

then,

$$\theta p = 0.8 \, ( \varepsilon c_2 - \varepsilon c_1 ) k_1 k_3 \, (\tfrac{z}{d_1})$$

$$\theta p = 2.4 \, ( \varepsilon c_2 - \varepsilon c_1 ) \qquad (16)$$

A. H. Mattock (1964 (33)pp. 143 - 183) tested thirty-seven beams involving the following variables; concrete strength, depth of beam, distance from point of maximum moment to point of zero moment, and amount, and yield point of reinforcement. He proposed a method whereby the rotational capacity of hinging region in a reinforced concrete beam may be calculated.

$$\text{Inelastic rotation} = \theta u = ( \phi m - \phi y \;\; Mu/My) \qquad (17a)$$

$$\frac{\theta tu}{\theta u} = 1 + (1.14 \;\; \frac{z}{d} - 1) \left[ 1 - \frac{q - q'}{q_b} \sqrt{\frac{d}{16.2}} \right] \qquad (17b)$$

where, $\theta u = \Psi u \;\; \frac{d}{z}$ ; $\theta y = \Psi y \;\; \frac{d}{z}$

$\Psi u, \Psi y$ being the curvatures at ultimate and at yield

$\theta tu$ = Total inelastic rotation at ultimate, occuring between the section of maximum moment and an adjacent section of zero moment.

$\theta u$ = Inelastic rotation at ultimate, occuring within a length d/2 to one side of the section of maximum moment.

q = tension reinforcement index = P $fy/f_c$

q' = Compression reinforcement index = $p' fy/fc'$

qb = tension reinforcement for balanced ultimate strength condition Pbfy/fc′

Mu = ultimate moment resistance

My = Moment at yield of tension reinforcement.

p,p′ = tension, compression steel ratio respectively $\frac{A_s}{bd}$, $\frac{As'}{bd}$

fy,fy′ = yield point stress of Compression and tension, reinforcement respectively, and

$$\varepsilon_{cu} = 0.003 + \frac{0.5}{z} \qquad (17c)$$

W. G. Corleys (1966 (30)) revised Mattock's original expression (Equ. 17a,b) in 1966. He conducted a test series of forty simply supported beams loaded at mid span at Portland Cement association (P.C.A.). He presented a new equation by changing Equ. (17c) into

$$\varepsilon_u = 0.003 + 0.02\,\frac{b}{z} + \left(\frac{P''\,fy}{20}\right)^2 \qquad (18a)$$

and total rotation $\phi$ occuring in length d/2 is given by the expression $\phi = \psi \frac{d}{2}$. He also revised the equation (17b) which was found by Mattock to give a relation between the ratio $\theta'_{tu}/\theta_u$ and the degree of reinforcement $(q - q')/q_b$

$$\frac{\theta_{tu}}{\theta_u} = 1 + \frac{0.4}{\sqrt{d}}\;\frac{z}{d} \qquad (18b)$$

This equation represents a family of curves that define the spread of yielding as a function of the geometry of the member.

F. N. Panell etal (1966 (48)) proposed a new expression from their test results:

$$\theta_{im} = \frac{0.0012}{q_u} + 0.0085 - 0.5q_u \quad \text{for } q < 0.17 \qquad (19a)$$

$$\theta_{im} = \frac{0.0012}{q_u} \qquad q > 0.17 \qquad (19b)$$

where $q_u = \frac{Asfy}{bdcu} = \frac{Asfy}{bdfc' \times 1.2} = 0.833\,q$ ; $Cu = 1.2\,fc'$

Cu = Compressive strength

q = pfy/fc′

qu = pfy/fcu

fc′ = Compressive strength

E. F. Burnett (1969 (29)) analysed the results which were obtained by Amarakone, Baker and Amarakone, Corley, Mattock and Panell etal. He drew fig (1.5) and gave the following comments:

1° For non dimensionalized moment shear ratios (m/Vd or z/d) of more than 3.0, at least two expressions those of Amarakone and Corley.

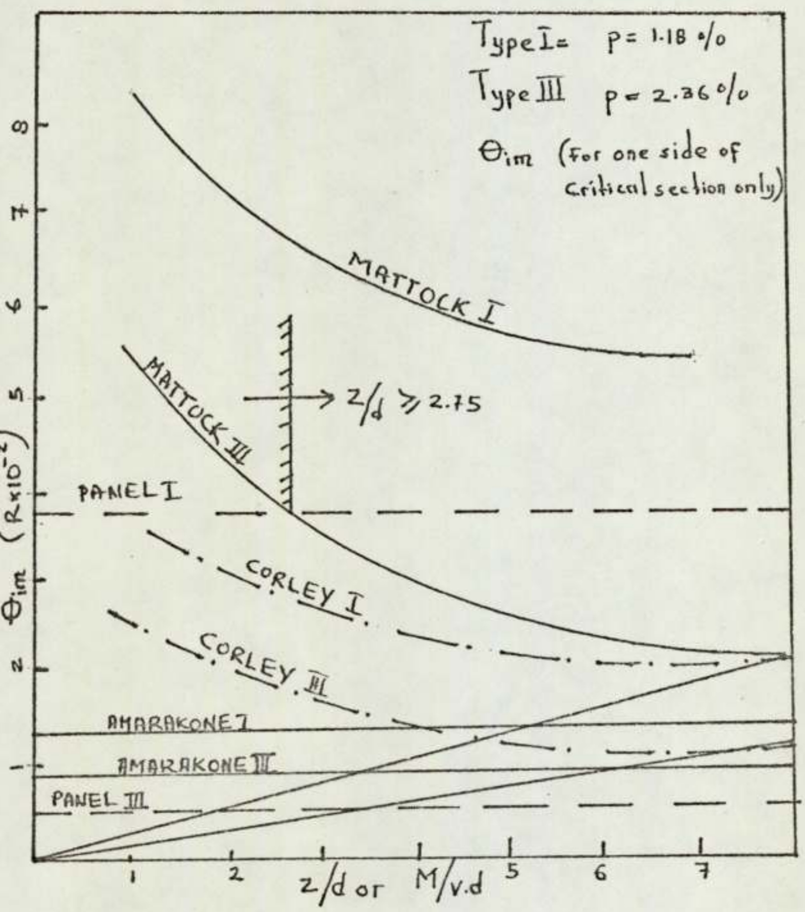



fig. (1.5.)

Influence of Amount of tension Reinforcement provide comparable values for $\theta$im.

$2^0$ Neither the volumetric proportion of lateral reinforcement, nor the spacing of ties has a significant influence on $\theta$ im.

$3^0$ On the basis of fig (1.5) it would appear that in a quantitative sense, Mattock's expression is either unacceptable or other expressions are extremely conservative.

$4^0$ For z/d > 3 all indicate that the inelastic rotation capacity is almost independant of the moment, shear force ratio.

$5^0$ For z/d < 3, it would appear that both Mattock's and Corley's result conflict with those of Baker and Amarakone as well as Chan and Bakers early theoretical expression.

The expression for values of z/d < 2.7 is therefore questionable, the diversity of these trends is indicative of the complications introduced by non-flexural effects such as shear and strut or arch action.

Cranston and Reynolds (1970) come to the conclusion, on the basis of their tests, that shear force has only a marginal influence on rotation capacity provided members were designed in accordance with the latest British code (CP 110) for structural concrete, and they also proposed new expressions for $\varepsilon$;

$$\varepsilon_{c_1} = \frac{k_c \sqrt{U_w}}{2250\,\gamma_{mc}} \quad ; \quad \varepsilon_{s_1} = 0.7 f_y = \frac{0.8 f_y}{\gamma_{ms}} \qquad (20)$$

The European Concrete Committee accepted limit state design in 1970 and also its relationship to the classical permissible working stress approach which is still likely to be found useful. In this concept, consideration is given to safety and serviceability at all stages of structural behaviour. Normally three limit states are considered; the limit state of ultimate strength, the serviceability limit state of deflections and cracking under service loads. The chance of reaching the limit state of ultimate strength is made very remote and much smaller than the chance of reaching the serviceability limit state of cracking. The aim in limit design is to ensure that the chance of each limit state being reached is substantially constant for all members in a structure and is appropriate to that limit state and consequently there is an adequate degree of safety against the structure becoming unfit for use.

In 1972 the structural use of Concrete (CP 110) was published by British Standard Institution which accepted limit state design, two partial safety factors and a new development to redistribution design. Two safety factors were introduced instead of one overall factor; enabling the uncertainties in assesing the loads and their influence on the structure to be considered separately in design from the uncertainties associated with the performance of the constructional materials. A detailed method of calculation for cracking and deflection was also given in CP 110. Limit state design is very easy to apply with the design charts given in (CP 110. part 2).

Experimental investigations of the flexural rigidity of T beams for frame conducted by C.S. Krishnamoorthy, and C. W. Yu, (1973). The moment curvature relationships for various sections and the distribution of flexural rigidity (EI) along the beam discussed equivalent EI value for T beams to be used in limit design of reinforced concrete frames are expressed in terms of critical section properties.

$$EI = \frac{Mu_1 x_1}{\varepsilon c_1} = \frac{Mu_1 \, (d - x_1)}{\varepsilon s_1} \qquad (21)$$

where $Mu_1$ = Moment of resistance at limit $L_1$

$x$ = depth of neutral axis

$x_1$ = depth of neutral axis at limit $L_1$

$\varepsilon c_1$ = strain at concrete at Limit $L_1$ , limiting value = 0.002

$\varepsilon s_1$ = strain at steel at Limit $L_1$ .

### 1.3.2. DEVELOPMENT ON OPTIMUM DESIGN

The optimum redistribution principle proposed by Valeriu Petcu (1961 (49)) using the factor of optimum redistribution. The idea of deriving such solutions in a simpler way has also been advanced by M. Z. Cohn (1962 (31)) using the concept of yield safety instead of redistribution factor. Early developments of the equilibrium methods were given by M. Z. Cohn (1965 (21)) along with the classification of limit design methods and full redistribution design (F.R.D.) and limited redistribution (L.R.D.) Solutions for continuous beams. A simple approach to the compatibility of such solutions was suggested also by M. Z. Cohn (1964 (33)). The effect of loading history and code definitions were studied in 1967 (22).

Apparently the first paper on optimal design of reinforced concrete beams and frames is due to Massonet and Save (1963 (45)). Initial attempts to cast the design problem as a set of linear equations and inequalities within the framework of the equilibrium (serviceability) methods (1965 (21)) resulted in mathematical programming formulations by M. Z. Cohn (1968 (23)). Cost, potential energy, material consumption were some merit functions adopted by Massonet and Save (1963 (45)), and by Kalisky (1965 (40)). The linearized merit function (single step variation of member resistances) adopted by M. Z. Cohn Etal (1970 (25)). Solutions by the kinematic approach were first given in (23). Computing techniques were developed by Grierson and combinations of (OLD) with (F.R.D.) and ultimate strength design (U.S.D.) solutions were studied in (1970 (25)). An extensive investigation on the application of equilibrium methods to continuous beams by M. Z. Cohn and Grierson (1968 (24)) extends previous results in (1965 (21)) clarifies the factor affecting optimal solutions and examines the relationship between OLD and FRD solutions. The kinematic approach was generalized in (1972 (47)) to allow for all major design criteria; including both elastic and plastic compatibility.

A procedure that avoids the difficulties of determining the complete set of active constraints in the kinematic approach by iteratively identifying the component constraints in the set is due to Ishikawa and Grierson (1972 (39)). The role of compatibility condition in limit design is still under discussion. An attempt to include it in design process under assumption somewhat similar to (1970 (36)) is presented by Munro, Krishnamoorthy and Yu (1972 (47)). Talwar and Cohn (1972 (55)) demonstrates that the plastic compatibility criterion is not a critical consideration for braced frames in current multi-storey buildings, and therefore it need not be included in the initial phase of the design.

More recently the techniques of mathematical programming have successfully been applied in investigating optimal solutions for continuous prestressed concrete structures, and including probabilistic considerations in single and multi-stage optimal designs (1972 (27)). Some possible code formulations and practical recommendations allowing for inelastic effects in structural concrete were suggested in (1970 (29)) by M. Z. Cohn Etal. Further efforts are required in order to give structural concrete designers full benefit of existing knowledge in optimal limit design.

# C H A P T E R 2.

## A.L.L. BAKER METHOD

## 2.1. INTRODUCTION

The limit design problem of reinforced concrete structures involves the derivation of design plastic moments for all its critical sections when the ultimate load is known. Plastic moments will be correct if they satisfy the basic condition of

a) Limit equilibrium

b) Rotation compatibility

c) Serviceability

Limit equilibrium and rotation compatibility assure the actual occurrence of the mechanism and therefore represent the necessary and sufficient conditions for plastic collapse of reinforced concrete structures. Limit design methods may be divided into two broad classes. Methods of the first class are based on limit equilibrium and compatibility considerations with the serviceability conditions to be investigated separately. These are called compatibility methods (A.L.L. Baker, Y. Guxon, G. Macchi).

Methods of the second class are essentially based on limit equilibrium with serviceability conditions and compatibility conditions being dealt with independently. These will be called serviceability or optimum limit design methods.

The process of deriving a design solution requires certain basic assumptions and a set of design criteria. Basic assumptions define a) the loading pattern, b) the ultimate load and loading history, c) plastic moment design d) the idealized behaviour of materials. Design criteria define a) limit criteria (nature of ultimate conditions; configuration of structure at ultimate stage and specific ultimate requirements), b) serviceability criteria.

In this chapter the most popular compatibility method which may be called the A.L.L. Baker Method is explained and examples on beam and frame design are also given.

## 2.2 FACTOR OF SAFETY.

"Load-factor" has been introduced to get over the difficulty which occurs when the load-stress relationship is not linear and is defined as the ratio of the ultimate load to the working load. It thus has a different value from the "stress factor of safety". A suitable factor of safety can be determined for structures having redundant members or for structures subjected to buckling forces by a process of judgement in which the influence of various factors on the probability of failure is considered. Basic values of the global factor $\gamma_k$ equal to ($\gamma_k = \gamma_L \gamma_M$) 2 for failure due to the concrete and 1.6 for failure due to steel may be assumed and adjusted to $K\gamma_k$, the value of "K" being obtained by estimating appropriate values for adjustment factors as given in Table 2.1. The values selected for the "weights" of the adjustment factors should be between a minimum unity and the maximum values given. Variations in the strengths of concrete

| Adjustment factor | Maximum weights | Description |
|---|---|---|
| $\gamma_1$ | 5 | Consequence of failure serious (human or economic) |
| $\gamma_2$ | 1 | High-grade quality-control |
| $\gamma_3$ | 2 | Medium-grade quality-control |
| $\gamma_4$ | 2.5 | No-warning of failure |
| $\gamma_5$ | 3 | No-transfer of load to stronger parts |
| $\gamma_6$ | 2 | Medium-grade maintenance |
| $\gamma_7$ | 2 | No load control |
| $\gamma_8$ | 5 | Support conditions uncertain |
| $\gamma_9$ | 2 | One simultaneous type of load |

Table 2.1 $\gamma_k$ values by "weighted" factors.

and steel can be fully taken into account in determining values for the characteristic strengths so that the value of $\gamma_m$ can be unity for both materials and that the values of $\gamma_s$ and $\gamma_k$ should be 2, unless a reduction can be justified by estimating the weights for influential factor and applying the expression of

$$\gamma_k = \frac{\Sigma \text{Estimated weights}}{32} + 1.25$$

The value of $\gamma_k$ for concrete or steel should be between 1.25 and 2.0 according to Table 2.2 in regard to safety. The full value of weighting factor must be used unless conditions are entirely favourable.

| Factor | Weighting factor for most unfavourable conditions |
|---|---|
| Seriousness of results of failure (human or economic) | 8 |
| Workmanship | 4 |
| Loading conditions | 4 |
| Importance of member in structure | 4 |
| Warning of failure | 3 |
| Loss of strength due to deterioration | 1 |

Table 2.2 Values of weight for different factors.

The following factors must be considered when estimating the values of safety factors.

1) Design strength = $\alpha = \alpha k/\gamma_m$.
2) The value of $\alpha$ must be reduced when necessary to allow for fatigue exceptional wear or corrosion.
3. The value of $Q_k$ must be increased, when necessary to allow for vibration or dynamic effects, unless special calculations are made.
4) Additional stresses, when significant must be calculated and allowed

for, such stresses may be due to

a) the ultimate eccentricity of loads or thrusts

b) the ultimate settlement of supports

c) internal displacements due to temperature, shrinkage, creep and residual strains due to non-recovery of creep or cracking.

5) The influence of age, temperature and biaxial or triaxial conditions of stress on the value of $\zeta$ for concrete must be considered.

6) Compatibility of stress and deformation must be satisfied in addition to the requirements of equilibrium.

## 2.3) THE IDEALIZED PLASTIC BEHAVIOUR OF MATERIALS

One span of a beam, loaded as shown in fig.2.1a., may be considered under increasing load until failure commences by a hinging action at one or both supports. The behaviour thereafter depends upon the rotational capacity of the section. If the rotational capacity of the section is adequate as in a steel beam then deflections increase until a third hinge occurs at or near the middle of the span. If it is not adequate extensive crushing of the concrete occurs accompanied by considerable loss of strength of the hinging section and consequent failure of the beam.

In order to calculate these rotations it is necessary to determine a suitable relationship between bending moment and curvature. It has been shown by A.L.L. Baker that a suitable relationship is the simple bilinear curve illustrated in fig 2.1d. The line $OL_1$ shows a linear relationship between bending moment and curvature $\phi$ which can be defined as the reciprocal of the radius $\left(\phi = \frac{1}{R}\right)$. The equation of line $OL_1$ may be written $\phi = KM$ and, in the simple theory of bending

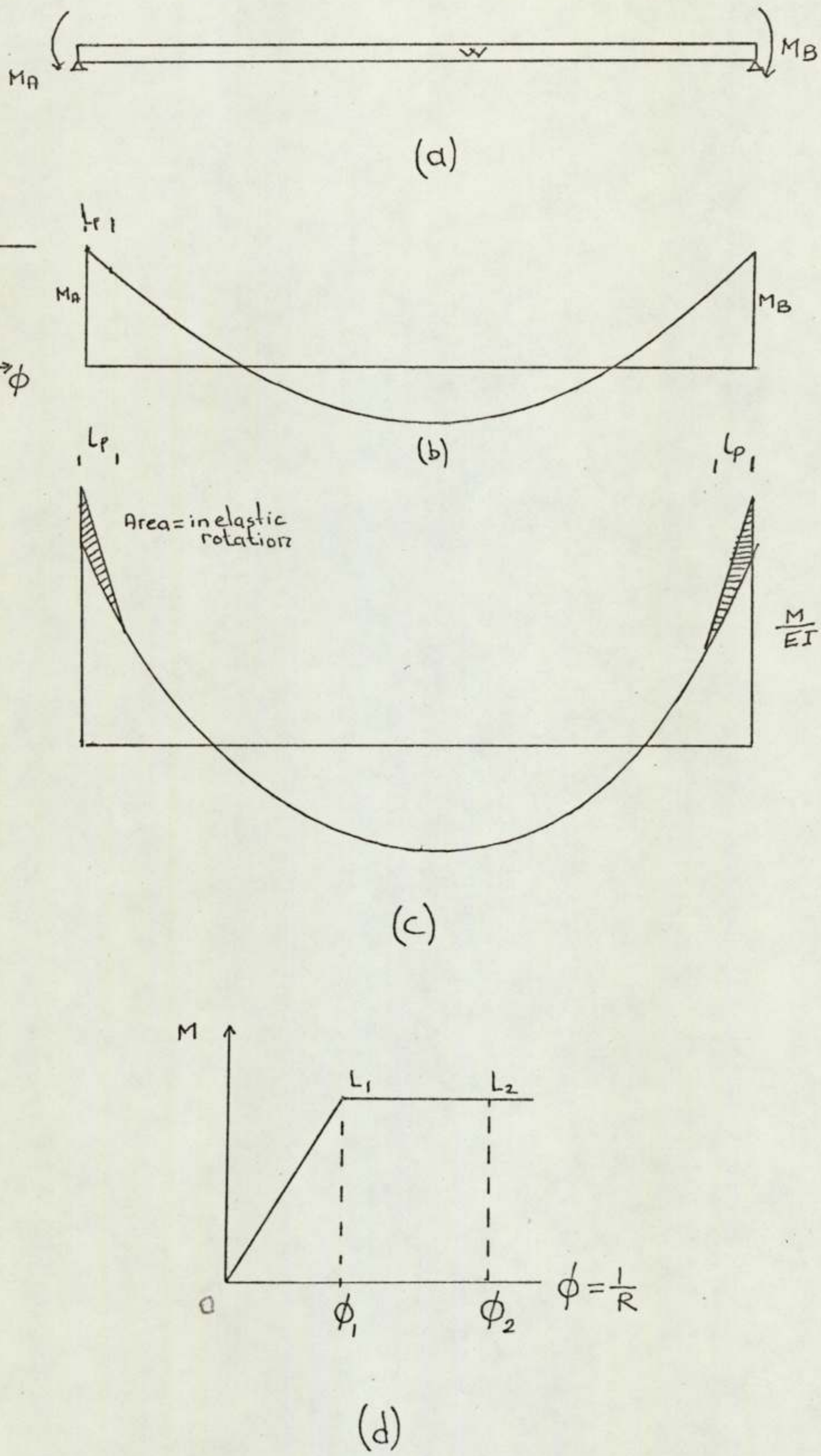

MA
W
MB
(a)
M
Mu
Mc
φ
MA
MB
(b)
Area = inelastic rotation
M/EI
(c)
M
L1
L2
O
φ1
φ2
φ = 1/R
(d)

fig. 2.1.

$$\frac{M}{EI} = \frac{1}{R} = \phi \qquad \text{and } \frac{M}{\phi} = EI \quad ; \quad \frac{M}{KM} = EI = \frac{1}{K}$$

The line $OL_1$ implies a constant value for EI up to point $L_1$. The value of EI for the member in compatibility calculations with respect to the ultimate limit state at $L_1$ is assumed;

$$EI = \frac{M_1 n_1 d_1}{e_{c_1}} \quad \text{or} \quad EI = \frac{M_1 (1-n_1) d_1}{e_{s_1}}$$

From which it is seen that, the term EI has a new meaning which is based not just on the properties of the materials and dimensions of the section, but on the integrated curvature at a particular condition, EI, therefore depends on the stress in the concrete.

## 2.3.1) RESISTANCE OF SECTION TO BENDING ONLY.

Assumptions:

1) The strain in the concrete and the steel is proportional to the distance from the neutral axis for all conditions.
2) The distribution of compressive stress in the concrete may be assumed to be parabolic, rectangular parabolic or linear according to the selected condition.
3) The tensile strength of concrete is zero.
4) When reinforcement is extended sufficiently beyond a section to develop full bond strength, no slip takes place between the concrete and steel. The same strain is therefore developed in the reinforcement and in the surrounding concrete.
5) The coefficients of thermal expansion for steel and concrete are equal for practical purposes.
6) Stress due tc shrinkage of the concrete during setting and hardening are neglected except in special cases such as arches and long members.

The section design for reinforced concrete framework is best carried out by a procedure of trial and adjustment with the aid of a suitable computation diagram which is given in A.L.L. Baker's book (1970). A practical method is to assume the dimensions of concrete sections and to check their resistance to bending and shear in terms of concrete strength or by reference to computation charts.

It is easy to obtain a safe limiting value of the lever arm from the limiting values of strain. A safe value of the required area of tension reinforcement can then be found.

Balanced sections have the limiting values of $n_1$ and $n_2$ and $l_1$ and $l_2$ . When a section contains less tension reinforcement than the value required for a balanced section at $l_1$ and $l_2$, it is said to be under-reinforced for $l_1$ or $l_2$ . When it contains tension reinforcement in excess of this value and which is stressed below the design values it is said to be over-reinforced. fig. (2.2)

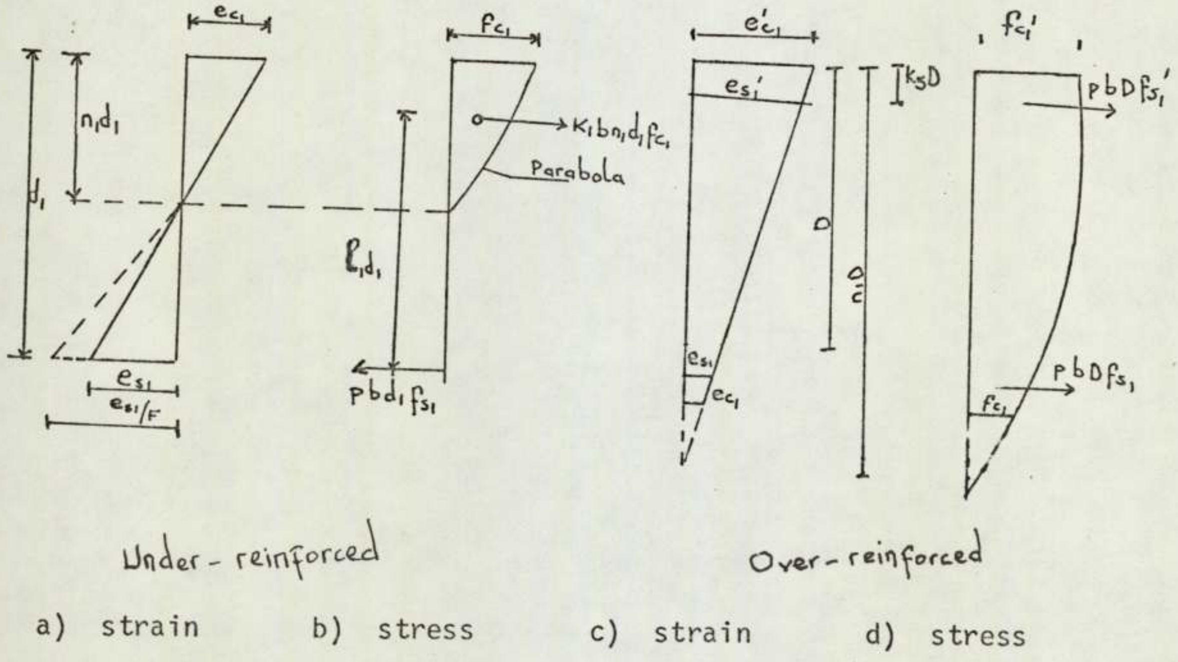


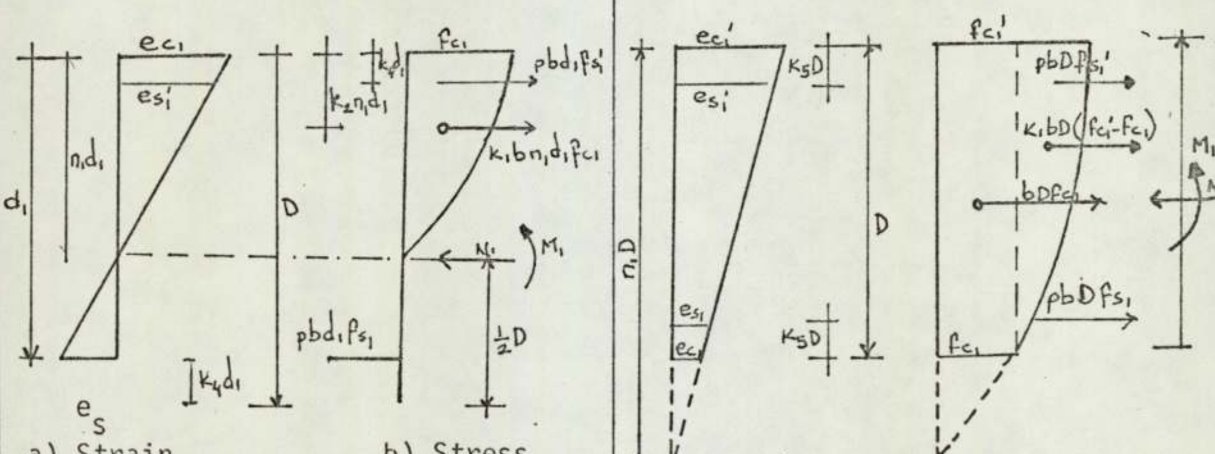

fig. 2.2.

The equations governing the preparation of computation charts such as those given in A.L.L. Baker's book (1970; fig 2.26 - 2.30 and fig. 2.33 - 2.48) are tabulated in table 2.3. for beam design and in table 2.4. for column design.

| Section | Formulae | No. |
|---|---|---|
| Rectangular beam | $n = \dfrac{1}{1 + \dfrac{e_s}{k6\, e_{c_1}}}$ where $k6 = \dfrac{e_c}{e_{c_1}}$ | (1) |
| | $\dfrac{EI}{bd_1^3 \sigma_b^*} = \dfrac{M}{bd_1^2 \sigma_b^*} \; \dfrac{n}{k_6\, e_{c_1}}$ | (2) |
| | $\dfrac{M}{bd_1^2 \sigma_b^*} = k_1 n(1 - k_2 n)$ | (3) |
| | $p\dfrac{f_{s_1}}{\sigma_b^*} = \dfrac{k_1 n}{k_7}$ | (4) |
| T Section | $\dfrac{M}{bd_1^2 \sigma_b} = k_1 n(1-k_2 n)-k_1' (n-s)\left(1-\dfrac{br}{b}\right)\left[(1-k_2' n)-s(1-k_2')\right]$ | (5) |
| | $p\dfrac{f_{s_1}}{\sigma_b} = \dfrac{1}{k_7}\left[k_1 n-k_1' (n-s) \left(1-\dfrac{br}{b}\right)\right]$ | (6) |
| | For $k_8$ between1.0 and 1.75 | |
| | $\dfrac{M}{bd_1^2 \sigma_b^*} = \left(1-\dfrac{br}{b}\right) s \left(1-\dfrac{s}{2}\right) + \dfrac{br}{b} k_1 n(1-k_2 n)$ | (7) |
| | $p\dfrac{f_{s_1}}{\sigma_b} = \dfrac{1}{k_7}\left[\left(1-\dfrac{br}{b}\right) s + \dfrac{br}{b} k_1 n\right]$ | (8) |

Table 2.3.

| | Formulae | No. |
|---|---|---|
| When tension developed across the section | Limit $L_1$ is similar to Limit $L_2$ by replacing $\leqslant a_2$ for $\leqslant a_1$ and $fs_2$ for $fs_1$ | |
| | $\frac{N_1}{bd_1 \leqslant b^*} = K_1 n_1 \frac{fc_1}{\leqslant b^*} + \frac{fs_1' - fs_1}{\leqslant a^*} p \frac{\leqslant a_1^*}{\leqslant b^*}$ | 9 |
| | $\frac{M_1}{bd_1^2 \leqslant b^*} = K_1 \frac{n_1 fc_1}{\leqslant b^*}\left(\frac{1}{2} + \frac{1}{2} k_4 - k_2 n_1\right) - \frac{(K_4 - 1)(fs_1' + fs_1)}{2 \leqslant a_1^*} p \frac{\leqslant a_1^*}{\leqslant b^*}$ | 10 |
| When no tension developed | $\frac{N_1}{bD \leqslant b^*} = \frac{K_1}{\leqslant b^*}(fc_1' - fc_1) + \frac{fc_1}{\leqslant b^*} + \frac{(fs_1' + fs_1)}{\leqslant a_1^*} p \frac{\leqslant a_1^*}{\leqslant b^*}$ | 11 |
| | $\frac{M_1}{bD^2 \leqslant b^*} = \frac{1}{\leqslant b^*}(fc_1' \; fc_1)(\tfrac{1}{2}K_1 - K_1 K_2) + p \frac{\leqslant a^*}{\leqslant b_1^*} fs_1' \frac{(\frac{1}{2} - K_5)}{\leqslant a_1^*} - p \frac{\leqslant a^*}{\leqslant b^*} fs \frac{(\frac{1}{2} - K_5)}{1 \leqslant a_1^*}$ | 12 |
| Values of EI | $EI = \frac{M_1 n_1 d_1}{e_{c_1}} = \frac{M_1 (1 - n_1) d_1}{e_{s_1}}$ | |




a) Strain b) Stress

when tension developed

strain

when no tension developed

Table 2.4.

## 2.3.2) BEAM DESIGN BY THE USE OF COMPUTATION CHARTS.

a) Specify value of $\lessdot b^*$ concrete stress

b) Calculate $M_1$ from structure

c) Assume b and $d_1$

d) Calculate $\frac{M_1}{bd_1^2 \lessdot_b^*}$

e) Find $p\frac{\lessdot a_1}{\lessdot b}$ from graphs given by A.L.L' Baker's book (1970; fig. 2.22 - 2.25.)

f) Calculate $A_{st}$ from values of $A_{st}$

g) Alternatively try $La/d_1 = 0.8$ and find $A_s$ from $M_1$ and $\lessdot b^*$ by using chart. If the resulting value of $\lessdot b^*$ is less than the specified value, the design is safe and the section is under-reinforced. Differing values of $\ell_a$ may be tried until the resulting value of $\lessdot b^*$ is equal to the specified value.

The same curves apply to "T" sections, but the limiting values of $ds/d_1$, must be observed or compression steel used.

## 2.3.3) COLUMN DESIGN BY THE USE OF COMPUTATION CHARTS

a) Assume b and $d_1$ (or b and D)

b) Specified $\lessdot b$

c) Calculate $\frac{M}{bd_1^2 \lessdot b^*}$ and $\frac{N}{bd_1 \lessdot b^*}$

d) Find $p\frac{\lessdot a_1^*}{\lessdot b^*}$ from charts given in A.L. Baker's book

e) Find $A_s$ from p

f) The reference strength $\lessdot a_1^*$ should be substituted in $p\frac{\lessdot a}{\lessdot b}$ to give $M_2$ at $l_2$ in the charts fig. 2.43. to fig. 2.46 which is published in A.L.L. Baker's book (1970)

Values of EI for rectangular and "T" beams may be Obtained by using fig 2.52 (A.L.L. Baker's book 1970). By reading off $EI/bd_1^3\varepsilon_b$ corresponding to the values of $p\frac{\varepsilon_{a_1}}{\varepsilon_b}$ obtained from fig 2.26 and 2.27 (A.L.L. Baker's book 1970).

Permissible crack-widths due to bending should be limited to $\ngtr$ 0.1 mm in corrosive working condition and to $\ngtr$ 0.3 mm in well protected condition.

## 2.4. DESIGN CRITERIA FOR FRAMEWORKS

The theory of plastic hinges may be applied for many times statically indeterminate structures by establishing simple design formulae for a wide range of structures subject to both vertical and horizontal loads. The procedure is first to assume the location of plastic hinges and values of plastic moments which appear to provide the best distribution of ultimate bending moments. A trial and adjustment process is then followed until conditions which indicate a correct choice of hinge positions is obtained and under working load excessive strain and deflection is avoided. The method provides a means of establishing simple design formulae for ultimate wind and vertical load bending moments for building frames.

### 2.4.1) SELECTION OF HINGE POSITION AND BENDING MOMENT VALUES.

When a set of hinges to be chosen for the calculation of ultimate load and hinge rotation; it is best to assume these n hinges to be situated at the junctions of members. In an ordinary frame, the hinges would be assumed with the experience of elastic theory at places shown in fig 2.3. Hinges should be assumed to develop at sections at which maximum bending moments occur under elastic conditions and, to have plastic bending moment values which produce an economic moment distribution. The assumed positions, plastic bending moments and deformability values of hinges are satisfactory if:

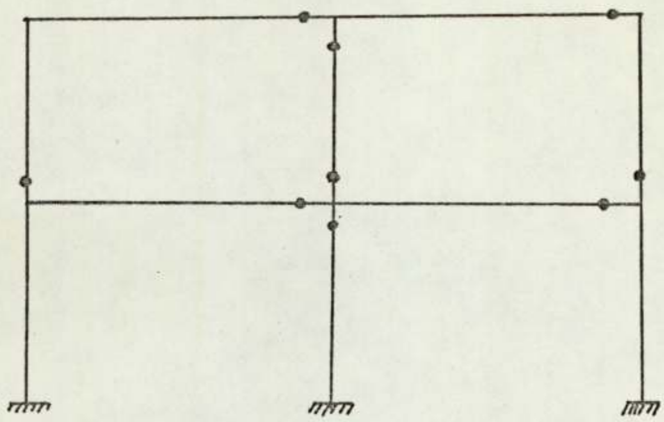


fig 2.3.

1) The sum of rotations at each hinge due to loads and all plastic hinge moments is negative in value.
2) Sections between the plastic hinges are within the elastic range.
3) The value of the rotation at each hinge does not exceed an appropriate safe value for that hinge in order to avoid premature crushing of the concrete.
4) At working load an elastic condition is obtained at all hinges and the strains are small enough to avoid wide cracks and large deflexions.

## 2.4.2) ANALYSIS OF INELASTIC HYPERSTATICAL FRAMES

The analysis of elastic hyperstatical frames may be carried out by the use of the Müller-Breslau general elastic equations

$$\delta_{io} + \sum X_k \delta_{ik} = 0$$

where $\delta_{ik} = \int \frac{M_i M_k}{EI} ds$ and $X_k$ is the kth redundant. If in a hyperstatic frame with n redundants are chosen n moments at the n hinge positions and the rest of frame stays elastic then;

$$\delta_{io} + \sum X_k \delta_{ik} + \theta_i = 0$$

when $\theta_i$ is the rotation at the $i_{th}$ hinge. Strictly in a frame of n redundants n+1 hinges should form for collapse. The Mo condition can be chosen to give the (n+1)th hinge. For example the beam with built-in ends shown in fig 2.4 has two redundants (three if axial force is taken into account). It may be made determinate by inserting plastic hinges at position 1 and 2. If the values of hinge moments are $X_1$ and $X_2$ the bending moment diagram will be as shown in fig 2.4.b. The third plastic hinge value $M_3$ forms at a distance $x$ from the support.

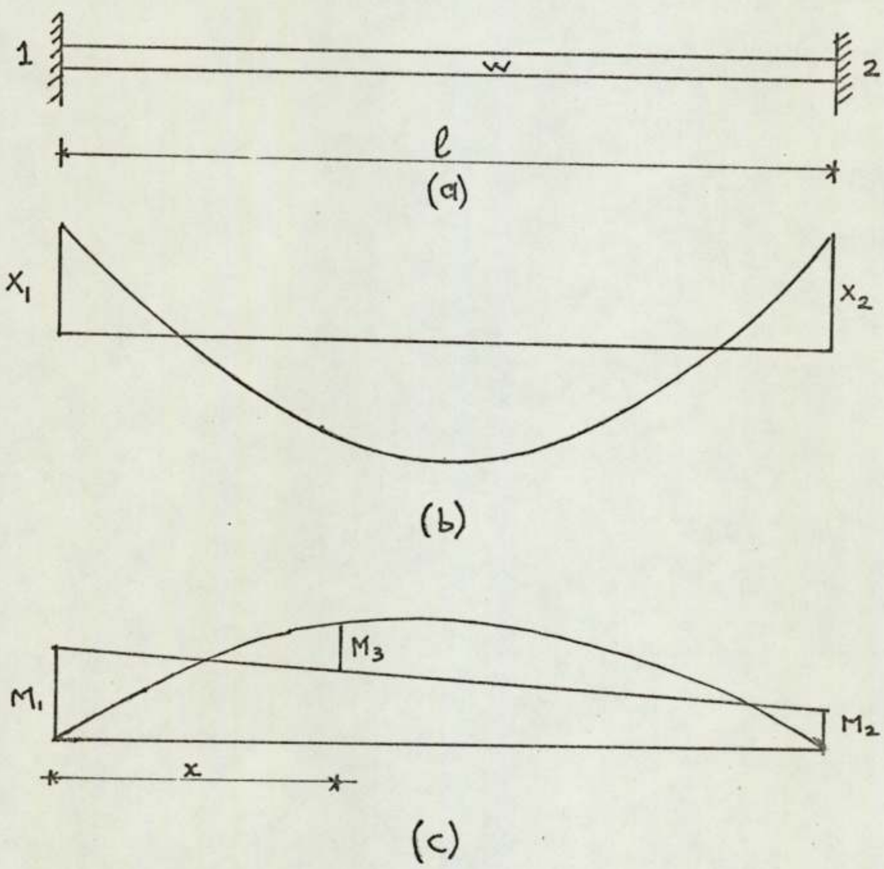



fig 2.4.

position as shown in fig (2.4c). Then

$$\frac{M_1(\ell - x)}{L} + \frac{M_2 x}{\ell} + M_3 = M_x$$

where $M_x$ is the free bending moment at point $x$.

In deriving the general elastic equations for a frame n times statically indeterminate, n frictionless hinges are assumed to be inserted in the frame and n unknown equal and opposite bending moments $X_1$.........Xn are assumed to act on members on either side of the hinges. For the elastic condition the rotation of each hinge due to external load and all unknown moments acting is zero. Hence for each of the hinges an equation is derived giving n equations from which the hinge rotation may be found. For example

$$\delta 10 + X_1 \delta_{11} + X_2 \delta_{12} + \dots\dots = 0$$
$$\delta 20 + X_1 \delta_{21} + X_2 \delta_{22} + \dots\dots = 0$$

where $\delta ik = \int \frac{MiMk\,ds}{EI}$, ds being a short increment of length along member of frame and $\delta ok = \int \frac{MoMk}{EI} ds$

In a frame n times statically indeterminate which has been loaded until n plastic hinges have formed; the rotations $\theta_1$, $\theta_2$ are the sum of the rotations due to external loads and plastic moments acting at each hinge, so that the general elastic equations are modified to

$$\delta 10 + X_1 \delta_{11} + X_2 \delta_{12} \dots\dots = -\theta_1$$
$$\delta 20 + X_1 \delta_{21} + X_2 \delta_{22} \dots\dots = -\theta_2$$

from which values of $\theta_1$ and $\theta_2$ ..........etc., may be determined.

For economical design the cross-section of the concrete would be uniform throughout the beam and the resultant ultimate bending moment at mid-span equal to the bending moment at the support, so that the area of reinforcement at the support is slightly greater than at mid-span in order to avoid excessive rotation of the hinge.

The required sign and safe value of $\theta$ for each hinge can also generally be obtained with a few adjustments if the following rules are observed

1) Adjust values of $\theta$ in order of magnitude error starting with the largest which is generally the maximum value of $\theta$
2) Adjust values of $\theta$ at each hinge by adjusting the assumed bending moment at that hinge, so causing the least disturbance to other hinge rotations.

3) Repeat the process until all values of $\theta$ are positive and sufficiently small.

4) If the frame has several stories it is best to complete the adjustments one storey at a time starting from the top.

The resultant bending moment of any section of the frame is obtained finally by superimposing the bending moments due to the external load and the adjusted values of the plastic hinge moment. That is the final value of X which is needed for bending moment diagrams of the structure.

## 2.5. DESIGN EXAMPLES

### EXAMPLE 1.

The dimensions of the beam considered, and the assumed sections are given in fig 2.5

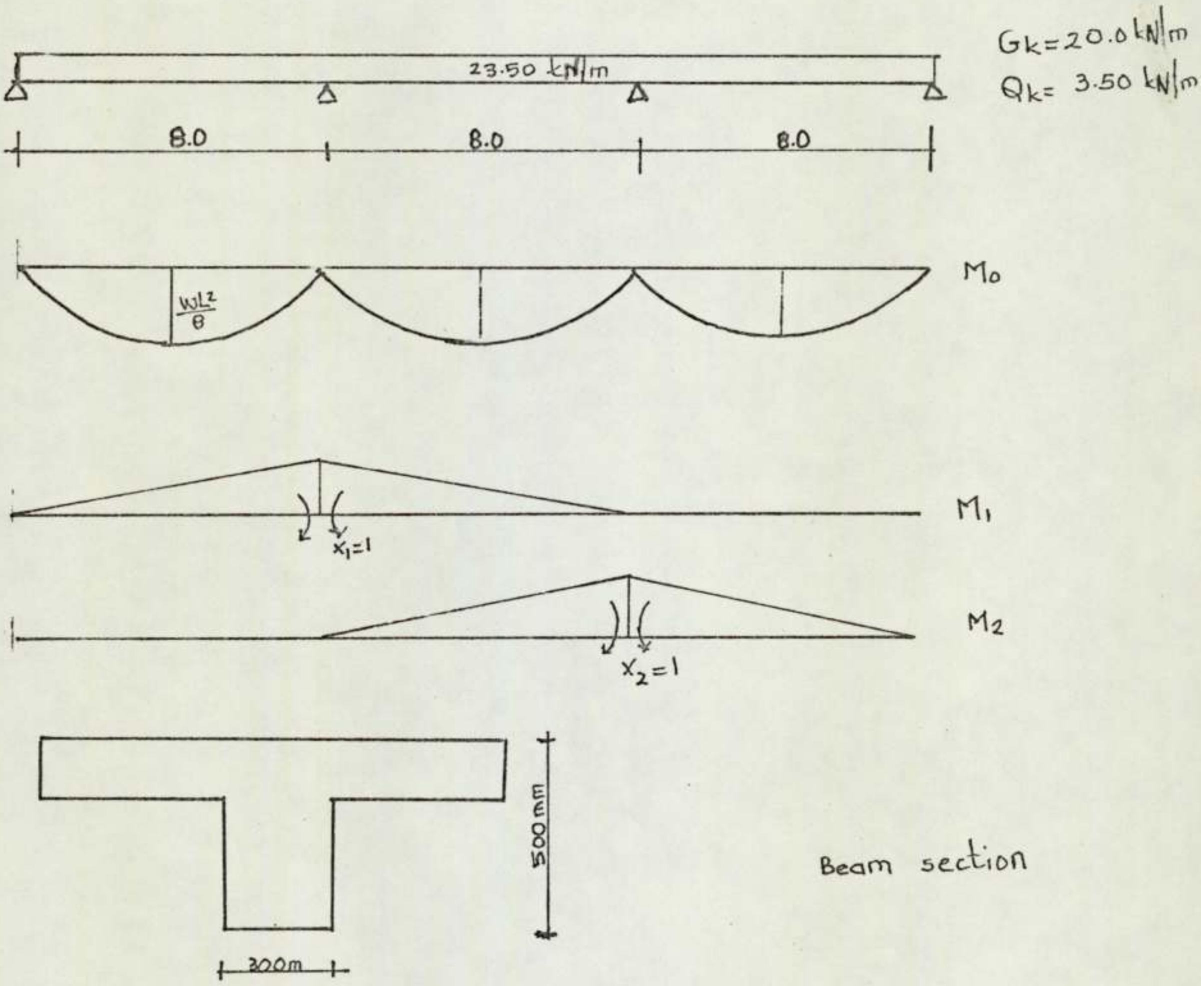



fig 2.5.

By using tables which gives the $\int_0^l$ MiMkds values, we can write Müller-Breslau equations for beam.

$$\delta_{01} + \bar{X}_1 \delta_{11} + \bar{X}_2 \delta_{12} = -\theta_1 \qquad (1)$$

$$-\frac{2}{3}\frac{Ml}{EI} + \frac{\bar{X}_1 2l}{3EI} + \bar{X}_2 \frac{l}{6EI} = -\theta_1$$

$$\delta_{02} + \bar{X}_1 \delta_{21} + \bar{X}_2 \delta_{22} = -\theta_2 \qquad (2)$$

$$-\frac{2}{3}\frac{Ml}{EI} + \frac{\bar{X}_1 l}{6EI} + \bar{X}_2 \frac{2l}{3EI} = -\theta_2$$

If it is desired, for economy, to use a uniform section throughout most of the beam, a suitable distribution of bending moments would be resultant inside moments at mid-span equal to moments at the support. So that $\bar{X}_1 = \bar{X}_2 = \frac{M_o}{2}$

Substituting in (1) and (2) respectively

$$\theta_1 = \frac{M\ell}{4EI} \quad \text{and} \quad \theta_2 = \frac{M\ell}{4EI}$$

All values of $\theta$ are positive, the position of the hinges has therefore, been correctly chosen. If the rotations are too great, the assumed values of $\bar{X}_1$, $\bar{X}_2$ must be increased, so that when the ultimate load acts the moments at the supports are slightly greater than the moments at mid-span

$Gk + Qk = 23.50$ kN/m and $Mo = \frac{23.5 \times 8^2}{8} = 188$ kNm.

$Ec = 25 \times 10^3$ N/mm$^2$

$Es = 200 \times 10^3$ N/mm$^2$

I concrete = $5458 \times 10^6$ mm$^4$ (for section chosen)

$$\theta = \frac{M\ell}{4EI} = \frac{188\times10^6\times8\times10^3}{4\times25\times10^3\times5458\times10^6} = 0.0027$$

Design strength of Materials:

Concrete: $\sigma b = 23.5$ N/mm$^2$

Steel: $\sigma a_1 = 410$ N/mm$^2$

$$\frac{M}{bd_1^2\sigma b^*} = \frac{188\times10}{300\times450\ \times23.5} = 0.131$$

from fig.(2.28)* (Given in A.L.L. Baker Books)*

$$p\frac{\sigma^* a_1}{\sigma b^*} = 0.138 \longrightarrow p = \frac{0.138}{17.5} = 0.078$$

from fig(2.52)* $\frac{EI}{bd_1^3\sigma b^*} = 72$

$bd_1^3\sigma b^* = 300\times450^3\times23.5 = 6423\times10^8$

$EI = 72\times6423\times10^8 = 4624\times10^{12}$ N.mm$^2$

(15)* A.L.L. Baker Limit Design of Reinforced Concrete (1970)

$\eta_2 = 0.192$ then from fig(3.1)* $p = 0.0162$

$\theta_p = 0.0162 > \theta_{required} = 0.0027$

Then Design will be satisfactory

$p = \frac{Ast}{bd_1}$ ; $Ast = p.bd_1 = 1053\ mm^2$

Ast provided $= 1257\ mm^2$ 4$\phi$20

EI design value $= 5458 \times 10^6 \times 25 \times 10^3 = 136.45 \times 10^{12}$ N-mm$^2$

EI calculated $= 46.24 \times 10^{12}$ N-mm$^2$

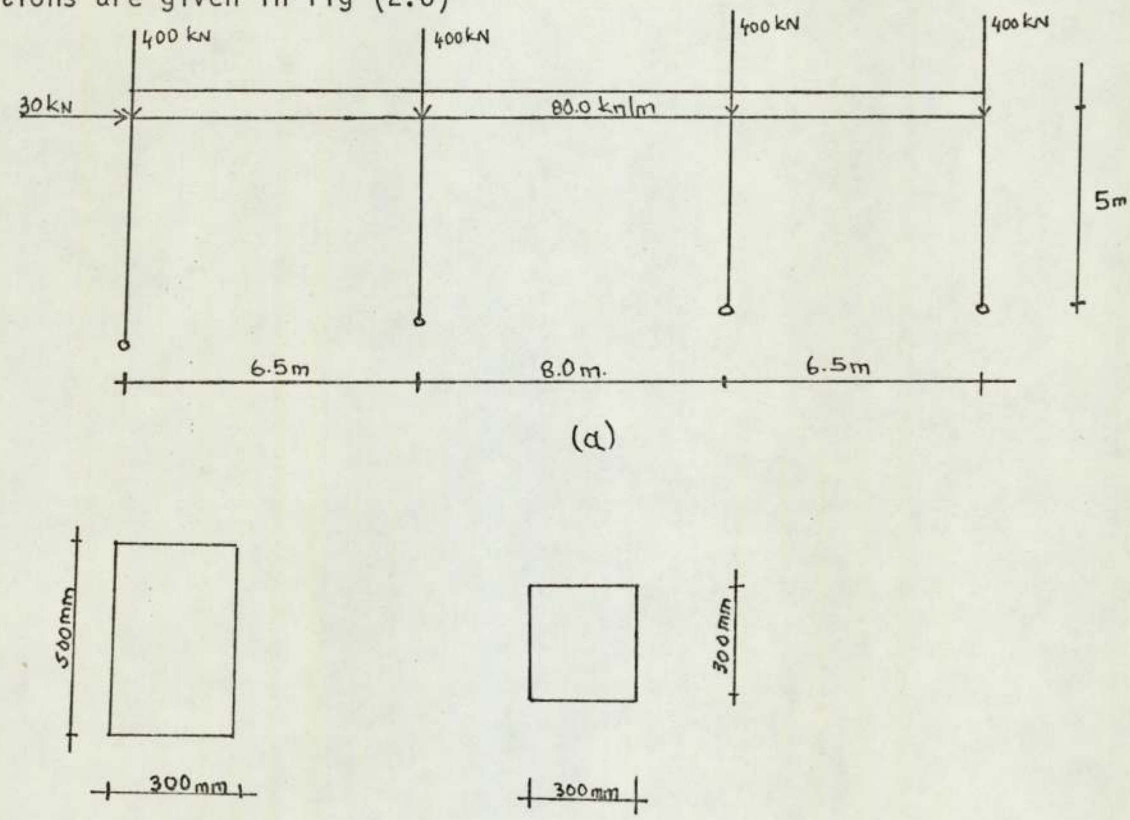
then EIprovided > EIcalculated

(15) * A.L.L. Baker Limit Design of reinforced concrete (1970)

EXAMPLE 2.

Frame design

The dimensions of the frame considered, and the assumed sections are given in fig (2.6)




Beam section

Column section

(b)

fig. 2.6. Design Values

Trial values of $\bar{X}$

$\bar{X}_1$ = 280.0 kNm

$\bar{X}_2$ = 320.0 kNm

$\bar{X}_3$ = 120.0 kNm

$\bar{X}_4$ = 37.5 kNm

$\bar{X}_5$ = 37.5 kNm

Vertical reactions on columns

Left-hand external column:

from fig 2.7.b

Total load = $400+260.0-2,310-0.154\bar{X}_1 + 0\bar{X}_2 + 0.154\bar{X}_3 + 0.154\bar{X}_4 + 0.154\bar{X}_5$

$= 400+260.0-2,31- 0.154(280,0) + 0.154(120 + 2 \times 37.5)$

$= 623.9$ kN

Left-hand internal column

from fig 2.7b

Total load = $400 + 580 + 23.1 + 0.279\bar{X}_1 - 0.125X_2 - 0.154\bar{X}_3 - 0.279\bar{X}_4 - 0.154\bar{X}_5$

$= 1003.1 + 0.279(280.0) - 0.125(320.0) - 0.154(120.0) +$

$37.5(-0.279 - 0.154)$

$= 1006.5$KN

Right-hand internal column

from fig 2.7b

Total load = $400.0 + 580.0 + 0 - 0.125\bar{X}_1 + 0.279\bar{X}_2 - 0.154\bar{X}_3 + 0.125\bar{X}_4 - 0.154\bar{X}_5$

$= 1014.6$KN

Right-hand external column

from fig 2.7.b.

Total load = $400 + 260 + 0 + 0\left(\bar{X}_1\right) - 0.154\bar{X}_2 + 0.154\bar{X}_3 + 0\bar{X}_4 + 0.154\bar{X}_5$

$= 63.5$KN

Total vertical reaction = 623,9 + 1006,5 +1014,6 +635.0 = 3280KN

Total vertical load acting = $\left[8 + (2\times6.5)\right] 80.0 + 4\times400 = 3280$KN

HORIZONTAL REACTIONS

Left-hand external column

$-30 + 0 + 0 - 0.2\bar{X}_3 - 0.2\bar{X}_4 - 0.2\bar{X}_5 = -9.0$KN = Total load

Left-hand internal column

total load = $0.2\bar{X}_4 = 7,5$KN

Right-hand internal column

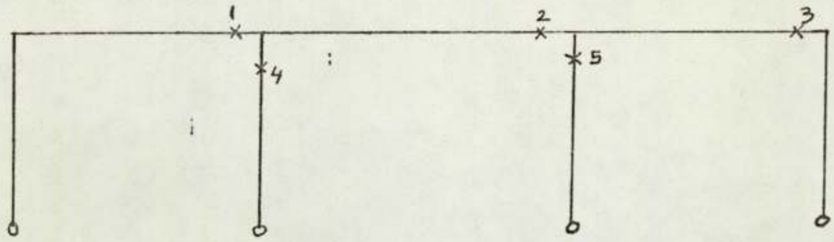
total load = $0.2\bar{X}_5 = 7,5$KN

Right-hand external column

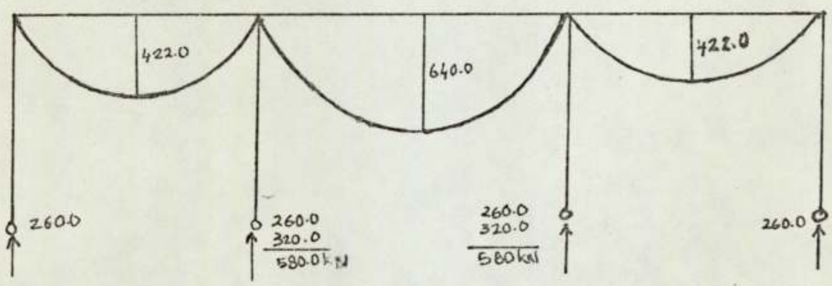
total load = $0.2\bar{X}_3 = 24.0$KN

Total vertical reaction = -9.0 + 7,5 + 7,5 + 24 = 30KN

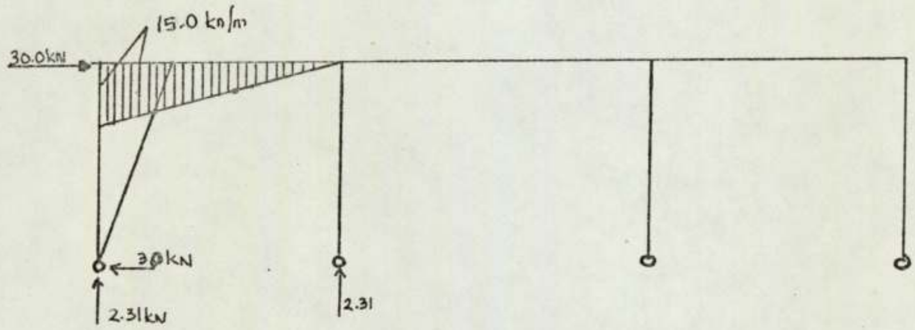
Total applied load = 30KN




a) Assumed positions of hinges




b) Free bending moments and reaction due to vertical loads




c) Free bending moments and reaction due to horizontal load.

fig(2.7)

Resultant final distribution of moments for first trial

Moment at junction of left-hand external column

Moment = $-150.0+\bar{X}_3+\bar{X}_4+\bar{X}_5$ = -150+120+37,5+37,5 = 45,0 kNm

Check from horizontal reaction:

Moment = -9x5 = -45,0 kNm.

Moment at junction of right-hand internal column

Moment = 320-37,5 = 282,5 kNm

Max'span moment (approximate) in left-hand span:

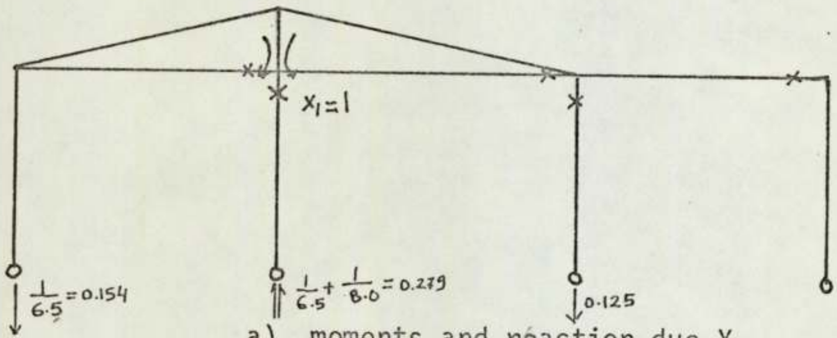
Moment = 422,0 - $\frac{1}{2}$(280 +45) = 422 - 162,5 = 259,5 kNm.

Max span moment (approximate) incentral span:

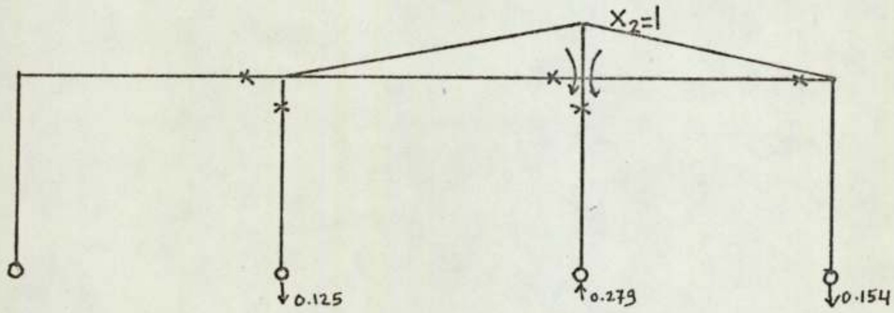
Moment = 640 - ½(320+242,50) = 358,75 kNm.

Max span moment (approximate) in right-hand span:

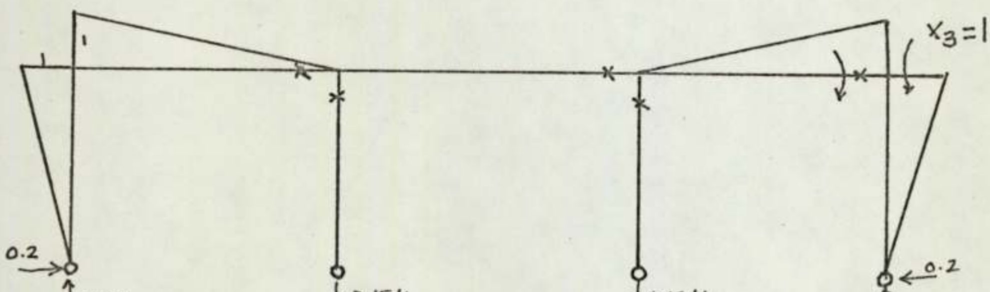
Moment = 422 - ½(282,50+120) = 220,75 kNm.



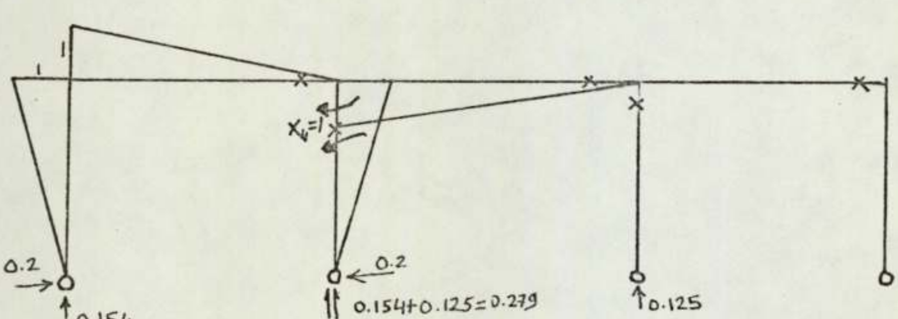

a) moments and reaction due $X_1$



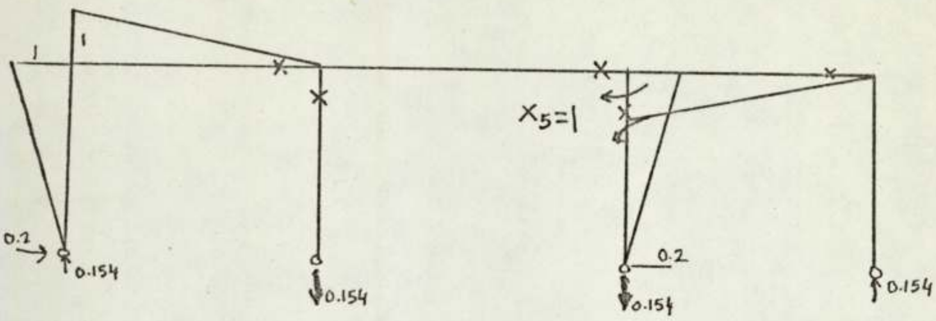

b) moments and reaction due $\bar{X}_2$




c) moments and reaction due $\bar{X}_3$




d) moments and reaction due $\bar{X}_4$




e) moments and reaction due $\bar{X}_5$

fig 2.8 Moments and reactions due $\bar{X}_1$, $\bar{X}_2$, $\bar{X}_3$, $\bar{X}_4$ and $\bar{X}_5$

Sections to be designed at limit $l_2$

Beam sections:

$\bar{X}_1$ moment = 280.0 kNm

$\bar{X}_2$ moment = 320.0 kNm

$\bar{X}_3$ moment = 120.0 kNm

Column sections:

$\bar{X}_4$ and $\bar{X}_5$ Moment = 37,5 kNm N = 101 4,60KN

Sections to be designed at limit $L_1$

Left-hand external column:

Bending moment = 120,0kNm N = 63,5 kN

Right-hand external column:

Bending moment = 45 kNm N = 623,9 kN

External spans:

Bending moment = 259,50 kNm

Internal span:

Bending moment = 358,75 kNm

The first trial result of bending moment are drawn in fig 2.9

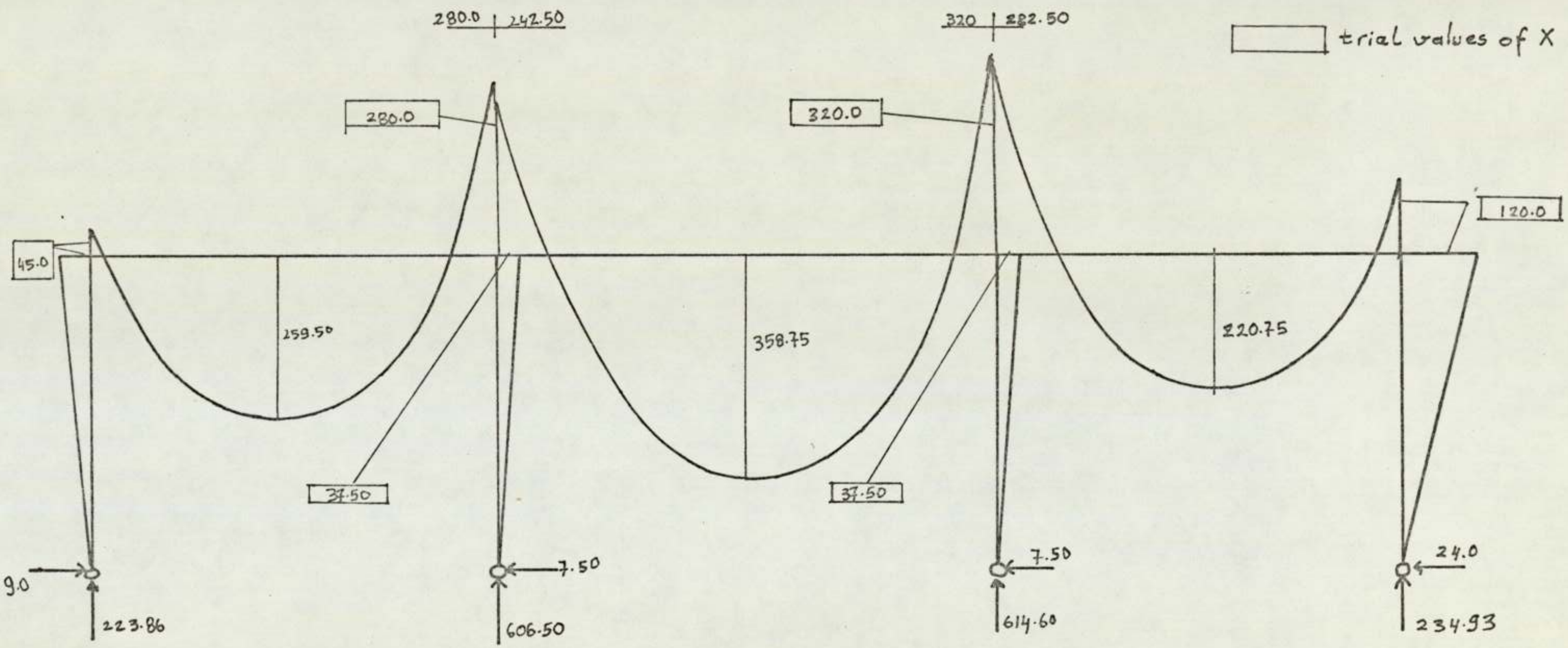



fig.(2.9) First trial values of moment distribution.

Strength of Material to be used

concrete for beams:

Design strength: $\sigma b^* = 23.5$ N/mm$^2$

concrete for columns:

$\sigma b^* = 14.7$ N/mm$^2$

Cold-worked reinforcement for beams

Design strength: $\sigma a_1^* = 410$ N/mm$^2$

Mild steel reinforcement for column:

Design strength: $\sigma a_1 = 210$ N/mm$^2$

Thus $\sigma a_1^* / \sigma b^* = 17.5$ for beams

$\frac{\sigma a_1^*}{\sigma b^*} = 14.0$ for columns

Design of sections at $l_2$ and calculation of EI values

Note: Design figures (15)* are taken from A.L.L. Bakers Book (1970)

Hinge $X_1$

$M = 280.0$ kNm $d_1 = 450$ mm $b = 300$ mm

$\frac{M_2}{bd_1^3 \sigma b^*} = 0.192$ then from fig(2.28)* $p\frac{\sigma a_1^*}{\sigma b^*} = 0.195$

therefore $p = 0.011$

with $p\frac{\sigma a_1^*}{\sigma b^*} = 0.195$ from fig(2.52)* $\frac{EI}{bd_1^3 \sigma b^*} = 43$

then $EI = 28{,}20 \times 10^{12}$ Nmm$^2$

and with $M_2/bd_1^2 \sigma b^* = 0.192$ from fig(2.29)* $n_2 = 0.28$

Hinge $\bar{X}_2$

$M = 320$ kNm $d_1 = 450$ mm $b = 300$ mm

$\frac{M_2}{b_1 d_1^3 \sigma b} = 0.219$ from fig(2.28)* $p\frac{\sigma a_1^*}{\sigma b^*} = 0.23$

therefore $p = 0.013$

with $p\frac{\sigma a_1^*}{\sigma b^*} = 0.23$ from fig(2.52)* $EI = 32{,}10 \times 10^{12}$ Nmm$^2$

and fig(2.29)* $n_2 = 0.3$

Hinge $\bar{X}_3$

$M = 120.$ kNm $\quad d_1 = 450$ mm $\quad b = 300$ mm

$\frac{M_2}{bd_1^3 \sigma b} = 0.0806$

from fig(2.28)* $\quad p\frac{\sigma a_1^*}{\sigma b^*} = 0.076$ and $p = 0.0043$

from fig(2.52)* $\quad EI = 13{,}10 \times 10^{12}$ Nmm$^2$

from fig(2.29)* $\quad n_2 = 0.14$

Internal columns:

$M = 3.75$ kNm $\quad N = 101.46$ KN $\quad b = 300$ mm $\quad D = 300$ mm

No tension to be likely developed.

Therefore $\sigma b^* = 0.8 \times 14.7 = 11.76$ N/mm$^2$

$\frac{N}{bd\sigma b^*} = 1.0 \quad ; \quad \frac{M}{bD^2\sigma b^*} = 0.116$

and from fig(2.43)* $\quad p\frac{\sigma a_1^*}{\sigma b^*} = 0.2$ and $n_2 = 1.025$

use fig(2.40)* which is corresponding chart for limit $L_1$, with a corresponding ratio of characteristic strength values.

$$p\frac{\sigma a_1^*}{\sigma b^*} = 0.2 \times \frac{1.15}{1.5} = 0.153$$

Then a line joining the origin and the point the co-ordinates of which are:

$\frac{N}{bD\sigma_1 b^*} = 1.0$ and $\frac{M}{bD^2\sigma b^*} = 0.116$ intersects the curve for $p\frac{\sigma a_1^*}{\sigma b^*} = 0.153$ at a value of $n_1 = 1.08$

At this intersection

$$M_1 = 0.105\ bD^2\sigma^* = 0.105 \times 300 \times 300^2 \times 14.7 = 510 \times 10^4 \text{Nmm}$$

$$EI = \frac{M_1 n_1 D}{e_{c_1}} = \frac{510\times10^4 \times 1.08\times300}{0{,}002} = 8{,}25\times10^{12} \text{ Nmm}^2$$

Design of sections at $L_1$ and calculation of EI values.

External columns:

$M = 120$ kNm $\quad N = 63.5$ KN $\quad b = 300$ mm $\quad D = 300$ mm ($d_1$=250mm)

$\sigma b^* = 14.7$ N/mm$^2$

$\frac{N_1}{bd_1\sigma b} = 0.563 \quad ; \quad \frac{M}{bd_1^2\sigma b^*} = 0.425$

and from fig(2.33)* $p\frac{\sigma a_1^*}{\sigma b^*} = 0.35$

To calculate the value of EI first adjust the

$p \frac{\sigma a_1^*}{\sigma b^*}$ ratio for characteristic strength values thus

$$p \frac{\sigma a_1^*}{\sigma b^*} = 0.35 \ \frac{1.15}{1.50} = 0.268$$

Now following the procedure described in the foregoing

$n_1 = 0.65 \qquad M_1 = 1555 \times 10^4$ Nmm

thus = $EI = \frac{M_1(1-n_1)d_1}{e_{s_1}} = \frac{1555 \times 10^4 \times (0.35) 250}{0.001} = 1360 \times 10^{10}$ Nmm$^2$

External span:

$M = 259{,}50$ kNm $\qquad b = 300$ mm $\qquad d_1 = 450$ mm

$$\frac{M}{bd_1^2 \sigma b} = 0.178$$

from fig(2.27)* $\qquad p \frac{\sigma a_1^*}{\sigma b^*} = 0.205$

from fig(2.25)* $\qquad EI/bd_1^3 \sigma b = 45$ therefore $EI = 29{,}50 \times 10^{12}$ Nmm$^2$

Internal span:

$M_1 = 358.75$ kNm $\qquad b = 300$ mm $\qquad d_1 = 450$ mm

$\frac{M}{bd_1^2 \sigma b^*} = 0.246 \qquad$ from fig(2.27)* $\quad p \frac{\sigma a_1^*}{\sigma b^*} = 0.31$

from fig(2.52)* $\quad \frac{EI}{bd_1^3 \sigma b} = 60$

therefore $EI = 39{,}20 \times 10^{12}$ Nmm$^2$.

<u>Values of EI for frame</u>

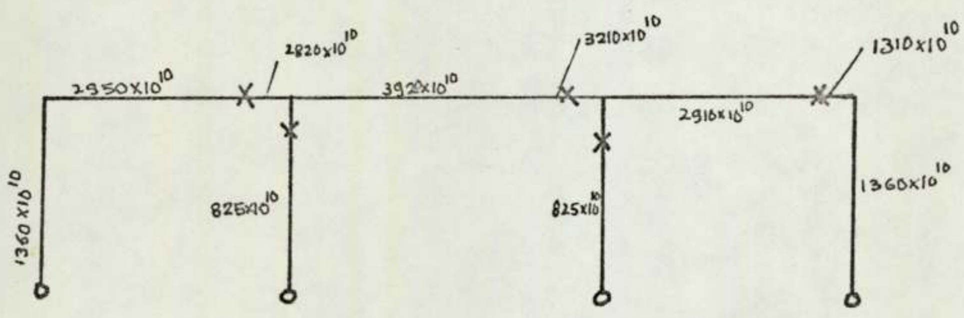



fig.2.10 EI values for frame in N.mm$^2$

Values of $\delta ik$

$(\delta 01)M = -(0.0313+0.0531)$

$(\delta 01)m = -\ 0.00575$

$\delta 01 = 0.0313 - 0.0531 - 0.00575 = 0.09015$

$\delta 02 = -0.1226$ | $\delta 13 = 3.83\times10^{-9}$ | $\delta 24 = 4.15\times10^{-9}$

$\delta 03 = -0.13165$ | $\delta 14 = 4.48\times10^{-9}$ | $\delta 25 = -16.3\times10^{-9}$

$\delta 04 = -0.00905$ | $\delta 15 = 3.83\times10^{-9}$ | $\delta 31 = \delta 13$

$\delta 05 = +0.00735$ | $\delta 21 = \delta 12$ | $\delta 32 = \delta 23$

$\delta 11 = 16.0\times10^{-9}$ | $\delta 22 = 24.8\times10^{-9}$ | $\delta 33 = 48.7\times10^{-9}$

$\delta 12 = 4.15\times10^{-9}$ | $\delta 23 = 8.3\times10^{-9}$ | $\delta 34 = 19.9\times10^{-9}$

$\delta 35 = 11.65\times10^{-9}$ | $\delta 51 = \delta 15$

$\delta 41 = \delta 14$ | $\delta 52 = \delta 25$

$\delta 42 = \delta 43$ | $\delta 53 = \delta 35$

$\delta 43 = \delta 43$ | $\delta 54 = \delta 54$

$\delta 44 = 48.5\times10^{-9}$ | $\delta 55 = 56.9\times10^{-9}$

$\delta 45 = 19.9\times10^{-9}$

General Comments regarding adjustment of values of $\theta$

The values of $\theta$ for the various hinges are shown in table 2.5. It is evident that for $\theta_4$ and $\theta_5$ to be positive, the values of $\bar{X}_4$ and $\bar{X}_5$ must be reduced. However, if only the values of $\bar{X}_4$ and $\bar{X}_5$ are adjusted, it is evident from the two internal junctions that the moment at $l_1$ would exceed that at $l_2$ at the same section.

If $X_4$ and $X_5$ are reduced to, say, 20 knm then according to the left-hand external span, the support section should be reinforced for a moment of 280 knm. at limit $L_2$ (fig.2.11 a) but according to the right-hand external span should be reinforced for a moment of 320 - 20.0 = 300 knm. at limit $L_1$ (fig.2.11 b) and this is not possible. Hence the following adjustments are made to the values of the moments.

$\Delta X_1 = 50$ kNm

$\Delta X_2 = 20$ kNm

$\Delta X_4 = 17.5$ kN m

$\Delta X_5 = 17.5$ kN m

The resultant values of $\theta$ are given in table 2.5 second trial

Checking Rotations:

for design purposes it is sufficient to use fig(3.1)*

The neutral-axis depth ratios at limit $L_2$ are as follows:

| | |
|---|---|
| section 1 | n = 0.28 |
| section 2 | n = 0.30 |
| section 3 | n = 0.14 |
| section 4 | n = 1.02 |
| section 5 | n = 1.02 |

On examining the rotations given in table $A_2$ it is evident that, the rotations at section 1, 2 and 3 are excessive. To reduce these values $X_1$ , $\bar{X}_4$ and $X_5$ must be increased. However as before $X_1$ and $X_2$ must be increased simultaneously and proportionately in order to avoid sections requiring higher values of M at limit $L_1$ than at limit $L_2$ . Therefore the following modifications are made

$\Delta \bar{X}_1 = +30$ kNm

$\Delta X_2 = +20$ kNm

$\Delta X_3 = +15$ kNm

The corresponding ajustments to the rotations are given in table 2.5 c as well as in the final rotations:

TABLE 2.5. a) FIRST TRIAL

| Sections | (1) $\delta_{ik}$ | (1) $X\delta_{ik}$ | (2) $\delta_{ik}$ | (2) $X\delta_{ik}$ | (3) $\delta_{ik}$ | (3) $X\delta_{ik}$ | (4) $\delta_{ik}$ | (4) $X\delta_{ik}$ | (5) $\delta_{ik}$ | (5) $X\delta_{ik}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $X_1 = 2.8 \times 10^6$ | $+16\times10^{-9}$ | +0.04500 | $+4.15\times10^{-9}$ | +0.001160 | $+3.83\times10^{-9}$ | +0.01075 | $-4.48\times10^{-9}$ | -0.0126 | $+3.83\times10^{-9}$ | +0.01075 |
| $X_2 = 3.2 \times 10^6$ | $+4.15\times10^{-9}$ | +0.01330 | $+24.82\times10^{-9}$ | +0.07960 | $8.3\times10^{-9}$ | 0.02655 | $-4.15\times10^{-9}$ | -0.0133 | $-16.3\times10^{-9}$ | -0.05230 |
| $X_3 = 1.2 \times 10^6$ | $+3.83\times10^{-9}$ | +0.00460 | $+8.30\times10^{-9}$ | +0.001000 | $48.7\times10^{-9}$ | 0.05850 | $+19.9\times10^{-9}$ | 0.0239 | $+11.65\times10^{-9}$ | +0.01100 |
| $X_4 = 0.375\times10^6$ | $-4.48\times10^{-9}$ | -0.00169 | $+4.15\times10^{-9}$ | -0.00156 | $19.9\times10^{-9}$ | 0.00747 | $48.5\times10^{-9}$ | 0.0182 | $+19.9\times10^{-9}$ | +0.00747 |
| $X_5 = 0.375\times10^6$ | $+3.83\times10^{-9}$ | +0.00143 | $-16.3\times10^{-9}$ | -0.00611 | $11.65\times10^{-9}$ | 0.00441 | $19.9\times10^{-9}$ | 0.00748 | $+56.9\times10^{-9}$ | +0.02140 |
| $\delta_{oi}$ | | -0.09015 | | -0.12260 | | -0.13165 | | -0.00905 | | +0.00735 |
| $-\theta = \Sigma X\delta_{ik} + \delta_{oi}$ | | -0.00275 | | -0.02907 | | -0.02397 | | +0.01462 | | +0.00867 |

b) SECOND TRIAL

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\Delta X_1 = 0.5 \times 10^6$ | $+16\times10^{-9}$ | +0.00800 | $+4.15\times10^{-9}$ | +0.00208 | $+3.83\times10^{-9}$ | 0.00192 | $-4.48\times10^{-9}$ | -0.00224 | $3.83\times10^{-9}$ | +0.00192 |
| $\Delta X_2 = 0.2 \times 10^6$ | $+4.15\times10^{-9}$ | +0.00083 | $+24.82\times10^{-9}$ | +0.00496 | $+8.3\times10^{-9}$ | 0.00166 | $-4.15\times10^{-9}$ | -0.00083 | $-16.3\times10^{-9}$ | -0.00326 |
| $\Delta X_4 = -0.175\times10^6$ | $-4.48\times10^{-9}$ | +0.00078 | $-4.15\times10^{-9}$ | +0.00073 | $+19.9\times10^{-9}$ | -0.00348 | $+48.5\times10^{-9}$ | -0.00850 | $+19.9\times10^{-9}$ | -0.00348 |
| $\Delta X_5 = -0.175\times10^6$ | $+3.83\times10^{-9}$ | -0.00067 | $-16.3\times10^{-9}$ | +0.00285 | $+11.65\times10^{-9}$ | -0.00204 | $+19.9\times10^{-9}$ | -0.00348 | $+56.9\times10^{-9}$ | -0.00995 |
| $\Sigma\Delta\theta$ | | +0.00894 | | +0.01062 | | -0.00194 | | -0.01505 | | -0.01477 |
| $-\theta$ from first trial | | -0.02751 | | -0.02907 | | -0.02397 | | +0.01462 | | +0.00867 |
| $-\theta = \Sigma\bar{X}\delta_{ik} + \delta_{oi}$ | | -0.01857 | | -0.01845 | | -0.02591 | | -0.00043 | | -0.00610 |

c) THIRD TRIAL

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\Delta X_1$ | $+16\times10^{-9}$ | +0.00480 | $+4.15\times10^{-9}$ | +0.00125 | $3.83\times10^{-9}$ | +0.00115 | $-4.48\times10^{-9}$ | -0.00135 | $+3.83\times10^{-9}$ | +0.00115 |
| $\Delta X_2$ | $+4.15\times10^{-9}$ | +0.00083 | $+24.82\times10^{-9}$ | +0.00497 | $+8.30\times10^{-9}$ | +0.00164 | $-4.15\times10^{-9}$ | -0.00083 | $-16.3\times10^{-9}$ | -0.00326 |
| $\Delta X_3$ | $+3.83\times10^{-9}$ | +0.00058 | $+8.3\times10^{-9}$ | +0.00125 | $+48.7\times10^{-9}$ | +0.00730 | $+19.9\times10^{-9}$ | +0.00298 | $+11.65\times10^{-9}$ | +0.00175 |
| $\Sigma\Delta\theta$ | | +0.00621 | | +0.00747 | | +0.01009 | | +0.00080 | | -0.00036 |
| $\theta$ from second trial | | -0.01857 | | -0.01845 | | -0.02591 | | -0.00043 | | -0.00610 |
| Resultant $\theta$ | | -0.01236 | | -0.01098 | | -0.01582 | | +0.00037 | | -0.00646 |

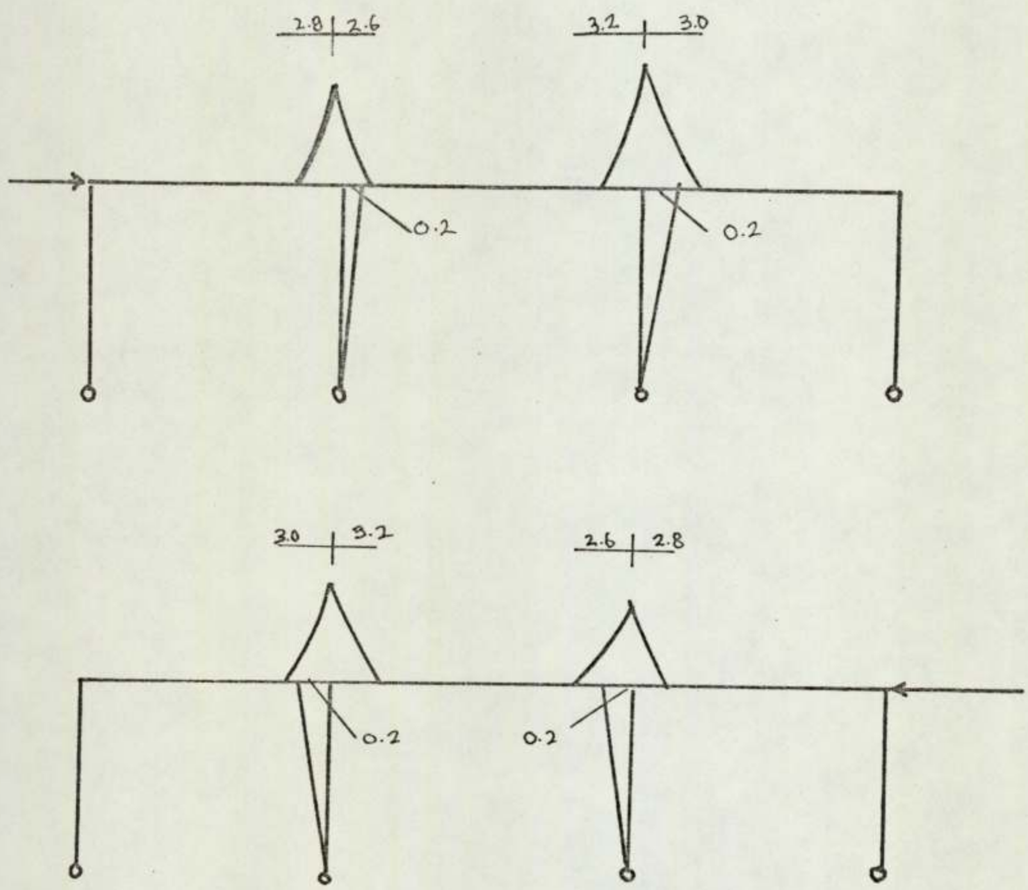



fig.(2.11) Adjustment to values of θ

Using fig(3.1)* the final results are as tabulated in the following:

| Section | $n_2$ | $\theta_p$ | $\theta$ req. |
|---|---|---|---|
| 1 | 0.28 | 0.0120 | +0.0123 |
| 2 | 0.30 | 0.0105 | +0.01098 |
| 3 | 0.14 | 0.0190 | +0.01582 |
| 4 | 1.02 | 0.0025 (without binders) | -0.00037 |
| 5 | 1.02 | 0.0025 (without binders) | +0.00646 |

Sections 1,2,3 and 4 are satisfactory; in the case of section 5 the provision of a small amount of binders will readily increase the $\theta_p$ to required value.

From table 2.5. the moments of various hinges are as follows:

X = 280 + 50 + 30 = 360 kNm

X = 320 + 20 + 20 = 360 kNm

X = 120 + 0 + 15 = 135 kNm

X = 37.5 - 17.5 + 0 = 20 kNm

The final distribution is shown in fig (2.12)

It is necessary to check the EI values to ensure that the values which have been used are approximately correct.

| Hinge Section | Assumed Value EI | Used Value EI | Comparis on |
|---|---|---|---|
| 1 | 35.5 x 10 | 28.2 x 10 | 25% smaller |
| 2 | 35.5 x 10 | 32.1 x 10 | 10% smaller |
| 3 | 14.0 x 10 | 13.1 x 10 | 7% smaller |
| 4,5 | 6.4 x 10 | 8.25 x 10 | 20% bigger |

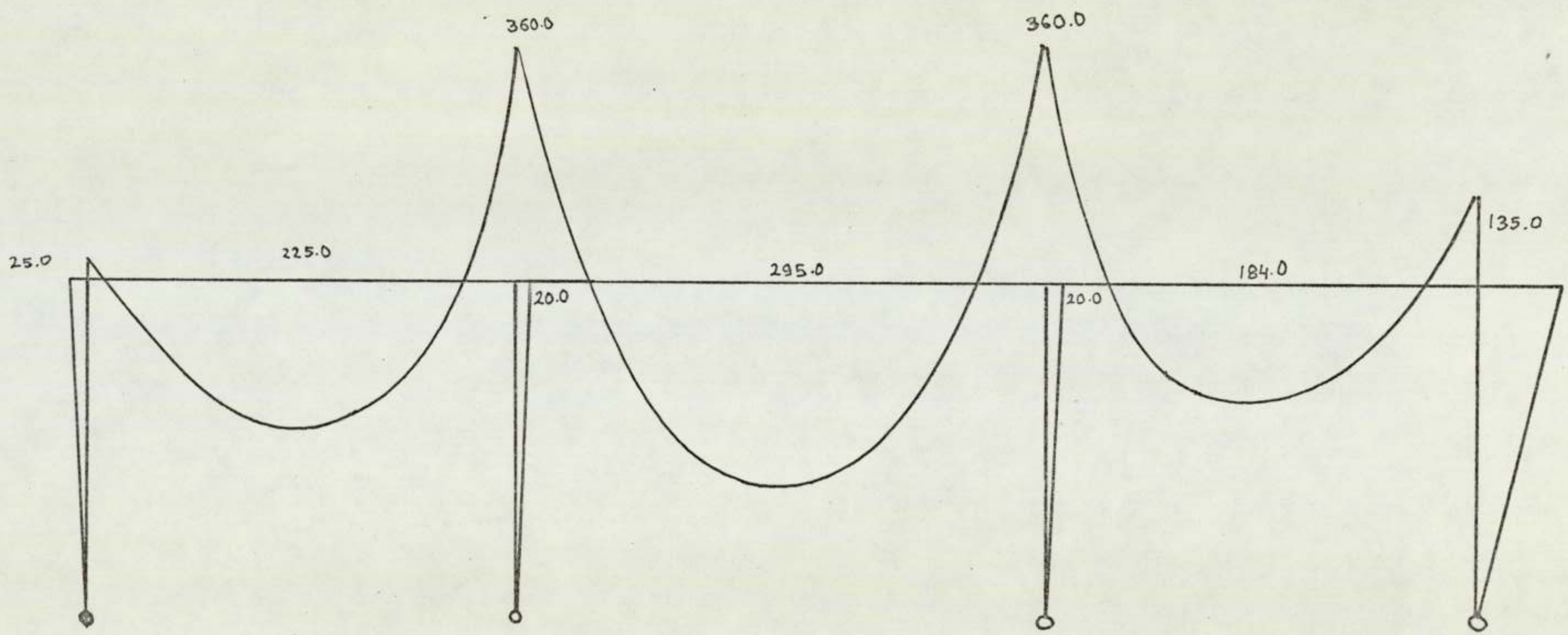



Fig.(2.12) Final distribution of moments (knm)

It was seen from result that EI values for columns are 20% larger. Because of large difference between the magnitudes of the beam and column moments the effect on the column rotation of a 20 per cent.

# C H A P T E R  3.

## CP 110 METHOD

## 3.1. INTRODUCTION

Structural design is largely controlled by regulations or codes but, even within such bounds, the designers must exercise judgement in their interpretation of the requirements, attempting to grasp the spirit of the requirements rather than to design to the minimum allowed by the letter of a clause. In the United Kingdom, the structural use of concrete is largely based on CP 110 (1972) which was prepared by a Code Drafting Committee of the B.S.I.

In the code of practice (CP 110), certain changes in the procedure for designing reinforced concrete were made from CP 114. New developments on limit state theory and adoption of safety factors and international recommendations, which were published by the European Concrete Committee (C.E.B. June 1970), were also accepted by the Code Drafting Committee.

In this chapter, limit state design methods, the concepts of partial safety factors, characteristic loads and strengths are considered and examples on beam and frame design are also given.

## 3.2. GENERAL PRINCIPLES OF CP 110 METHOD

Satisfactory design must ensure the achievement of an acceptable probability that the specified life of a structure is not curtailed prematurely due to the attainment of an unsatisfactory condition or limit state. For reinforced concrete structure the most critical limit state is often the ultimate limit state.

According to the code of practice every limit state should be considered in the design so as to ensure an adequate degree of safety and serviceability. The usual approach will be to design on the basis of the most critical limit state and then to check that the remaining limit states will not be reached.

3.2.1) LIMIT STATES

Limit state design sets out to achieve an acceptable probability that the structure which will not become unserviceable in its lifetime. The condition which causes a structure to become unserviceable is called a "limit state". The most important of these limit states are:-

1) Ultimate limit state: The usual collapse limit states, collapse due to fire explosive, pressure, etc.
2) Serviceability limit states: Local damage and deflection, durability, vibration, fire resistance, fatigue and lightning.

Limit state of collapse

The strength of the structure should be sufficient to resist the design loads taking due account of the possibility of overturning or buckling. The collapse may be caused by elastic or plastic instability, including the effects of sway. The structure should be designed in such a way that the probability of any limit state being attained is substantially constant, for all component members or the structure as a whole.

Limit state of impact resistance

It is necessary to consider the effects of impact, explosions or earthquakes (inertia forces) on the structure when considering the structural collapse. The exceptional events to be considered can vary considerably and include accidental impacts and accidental explosions.

Limit State of deflection

Certain deformation limits must not be exceeded to ensure normal performance under working loads. The safety margins clearly need not be as great as for the ultimate state. The designer must ensure that deflections are not excessive to preserve the appearance of the structure and to ensure that damage to finishes or partitions does not occur. As a guide suitable empirical procedures are explained in CP 110. To undertake a full theoretical analysis for every section would be time-consuming as well as being unnecessary.

Limit state of local damage

Cracks are caused not only be flexure but by shrinkage and temperature effects as well. Cracks due to shrinkage and temperature effects are more variable than those caused by flexure. All cracks allow the entry of water which causes corrosion of the reinforcement. In aggressive environments the attack can be rapid. It may be necessary to take special steps to limit these effects. CP110 gives a reasonable limit for cracking in clause (2. 2. 3. 2.)

Limit state of vibration

Excessive vibration causes discomfort, alarm or actual damage, or interferes with the proper function of the structure. Acceptable limits to the level of vibration vary according to usage. It may be necessary to isolate the source of vibration.

Limit state of fatigue

The effects of fatigue should be considered if the imposed load on a structure, or part of a structure is predominantly cyclic in character. Particular attention should be given to the deflections which would occur under repetions of load, to ensure that these are within acceptable limits.

CP 110 also gives rules to ensure adequate durability and fire resistance(clause 2.2.4.2 - 3)

3.2.2) SAFETY FACTORS

An acceptable probability that the structure will not reach an ultimate limit state throughout its specified life can only be provided by employing various partial safety factors for loads and strengths.

Some details of partial factors of safety specified in CP 110 and their application are set out in table 3.1. It will be seen from this table that two partial safety factors are involved for each limit state considered. The characteristic loads are multiplied by a partial safety factor $\gamma f$ to obtain the design loads, thus enabling the bending moments and shearing forces to be obtained which the members must be designed to carry. Thus if the characteristic loads are multiplied by the value of $\gamma f$ corresponding to the ultimate limit state, the moments and forces subsequently determined will represent those occuring at failure, and the section must be designed accordingly. Similarly, if the value of $\gamma f$ corresponding to the limit state of serviceability is used, the moment and forces under service loads will be obtained. In a similar manner, characteristic strengths of materials used are divided by partial safety factors for materials ($\gamma$ m) to obtain appropriate design strengths for each material. Although serviceability limit state calculations, to ensure the avoidance of excessive cracking or deflection may be undertaken, and suitable procedures are outlined in CP 110; it would be too time-consuming, and unnecessary to undertake

TABLE 3.1.

<table>
<tr><td colspan="8">SAFETY FACTORS</td></tr>
<tr><td rowspan="2">Partial Safety Factors</td><td rowspan="2">Condition of Loading</td><td colspan="3">ULTIMATE STATE</td><td colspan="3">SERVICEABILITY LIMIT STATE</td></tr>
<tr><td>Dead Load Factor</td><td>Imposed Load Factor</td><td>Wind Load Factor</td><td>Dead Load Factor</td><td>Imposed Load Factor</td><td>Wind Load Factor</td></tr>
<tr><td rowspan="3">partial safety factors for load $\gamma_f$</td><td>dead & imposed load</td><td>1.4</td><td>1.6</td><td>-</td><td>1.0</td><td>1.0</td><td>-</td></tr>
<tr><td>dead & wind load</td><td>0.9</td><td>-</td><td>1.4</td><td>1.0</td><td>-</td><td>1.0</td></tr>
<tr><td>dead & imposed & wind</td><td>1.2</td><td>1.2</td><td>1.2</td><td>1.0</td><td>1.0</td><td>0.8</td></tr>
<tr><td rowspan="3">Partial safety factors for materials $\gamma_m$</td><td>MATERIAL</td><td colspan="2">For effects of excessive loads or damage</td><td>otherwise</td><td colspan="2">For calculation of deflection</td><td>For calculations of stresses or cracks</td></tr>
<tr><td>CONCRETE</td><td colspan="2">1.3</td><td>1.5</td><td colspan="2">1.0</td><td>1.0</td></tr>
<tr><td>REINFORCEMENT</td><td colspan="2">1.0</td><td>1.15</td><td colspan="2">1.0</td><td>1.0</td></tr>
</table>

* Max loads of (1.4Gk + 1.6Qk) and min. loads of 1.0Gk should be arranged to give the most unfavourable arrangement of loading.

** To consider the probable effects of (i) excessive loading or (ii) localized damage take $\gamma_f = 1.05$

Design load = Characteristic load x partial safety factors for loads ($\gamma_f$)

$$\text{Design strength} = \frac{\text{Characteristic strength}}{\text{partial safety factors for materials } \gamma_m}$$

such a full analysis for every section. CP 110 specifies certain limits relating to bar spacing, slenderness, etc., and if these criteria are met, more detailed calculations are unnecessary.

Since, apart from the partial factor of safety for (dead + imposed + wind) load, all the partial safety factors relating to the serviceability limit state are equal to unity, the calculation of bending moments and shearing forces by using unfactored dead and imposed laods, as undertaken with modular-ratio and load factor design, may conveniently be thought of as an analysis under service loading, using limiting permissible service stresses that have been determined by applying overall safety factors to the materials strengths.

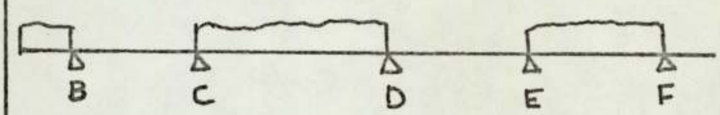
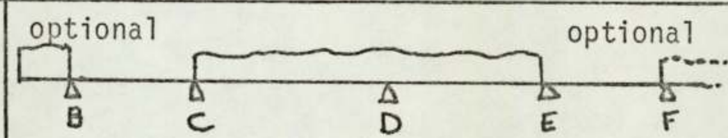
### 3.2.3) BENDING MOMENT AND FORCES FOR BEAMS

By considering the actual conditions at collapse, the distribution of moments throughout the structure and the moment of resistance of each section can be predicted. The structure must be designed so as not to collapse. The design resistance to bending shear, torsion and axial load at every section should not be less than the maximum at that section produced by the most severe arrangement of design loads on the structure. The values of the bending moments at the support and in the span depend upon the incidence of imposed load, and for equal spans or spans approximately equal the dispositions of imposed load illustrated in table 3.2. give the maximum positive moment at midspan and maximum negative moment at a support.

When undertaking limit state design according to CP 110, the spans carrying the maximum load to produce the critical condition at the section under consideration should support a total load of (1.4Gk + 1.6Qk), while the spans carrying the minimum load should support a load of only 1.0Gk overall spans and for imposed load of

(0.4Gk + 1.6Qk) acting only on those spans that will cause the maximum moment to be induced at the section being considered. As required by CP 110, for maximum support moments the spans on each side of the support only, and for maximum span moment, the span under consideration and all alternate spans will be loaded.

TABLE 3.2. Critical Loading

| incidence of imposed load | |
|---|---|
| To produce max. bending moment at span CD |   |
| To produce max. negative moment at support D |   |

* According to CP110 loads on spans CD and DE only need be taken into account for the second loading condition.

** For service - load design, consider a dead service load of $g_k$ and an imposed load $q_k$

*** For ultimate limit state design, consider a dead load of $G_k$ and and an "imposed ultimate load" of $(0.4G_k + 1.6Q_k)$

Table 3.2. Critical loading

## 3.2.4) CP 110 REQUIREMENTS FOR FRAMED STRUCTURE

It is unnecessary to carry out a full structural analysis of the entire frame as a single unit. Each floor may be considered as a separate sub-frame formed from the beams at that floor level together with the columns above and below, these columns being assumed to be fully fixed in position and direction at their further ends. The loading condition can be adopted by choosing a dead load of 1.0Gk and variable load of 0.4Gk + 1.6Qk. The individual beam may be considered separately by analysing a sub-frame consisting of the beam concerned together with the upper and lower columns and adjacent beams at each end. These beams and columns are assumed to be fixed at their further ends and the stiffness of two outer beams is taken to be only one-half of their true values. The sub-frame should be then analysed for the combination of loading previously described.

If the frame also provides lateral stability the following two-stage method of analysis is recommended by CP 110, unless the columns provided are slender. Firstly, each floor is considered as a separate sub-frame formed from the beams comprising that floor together with columns above and below these columns being assumed to be fixed at their further ends. Each sub-frame is subjected to a single vertical ultimate loading of 1.2(Gk + Qk) acting on all beams, simultaneously with no lateral load applied. Next, the complete structural frame should be analysed as a single structure when subjected to a separate ultimate lateral wind load of 1.2Wk only. In certain cases, the combination of dead and wind load should also be considered when lateral loading occurs. The code handbook suggests that this is only necessary where it is possible that the structure may overturn as for buildings that are tall and narrow or cantilevered.

### 3.2.5) DESIGN OF THE BEAM SECTION

After drawing the bending moment envelope for the ultimate limit state the section properties, namely breadth, depth and quantity of steel reinforcement have to be determined. CP 110 gives formulae and graphs from which beam and column sections may be designed. The values obtained by design graphs will be more accurate than by the simple method, since the parabolic stress distribution in the concrete has been used, and the stress in the

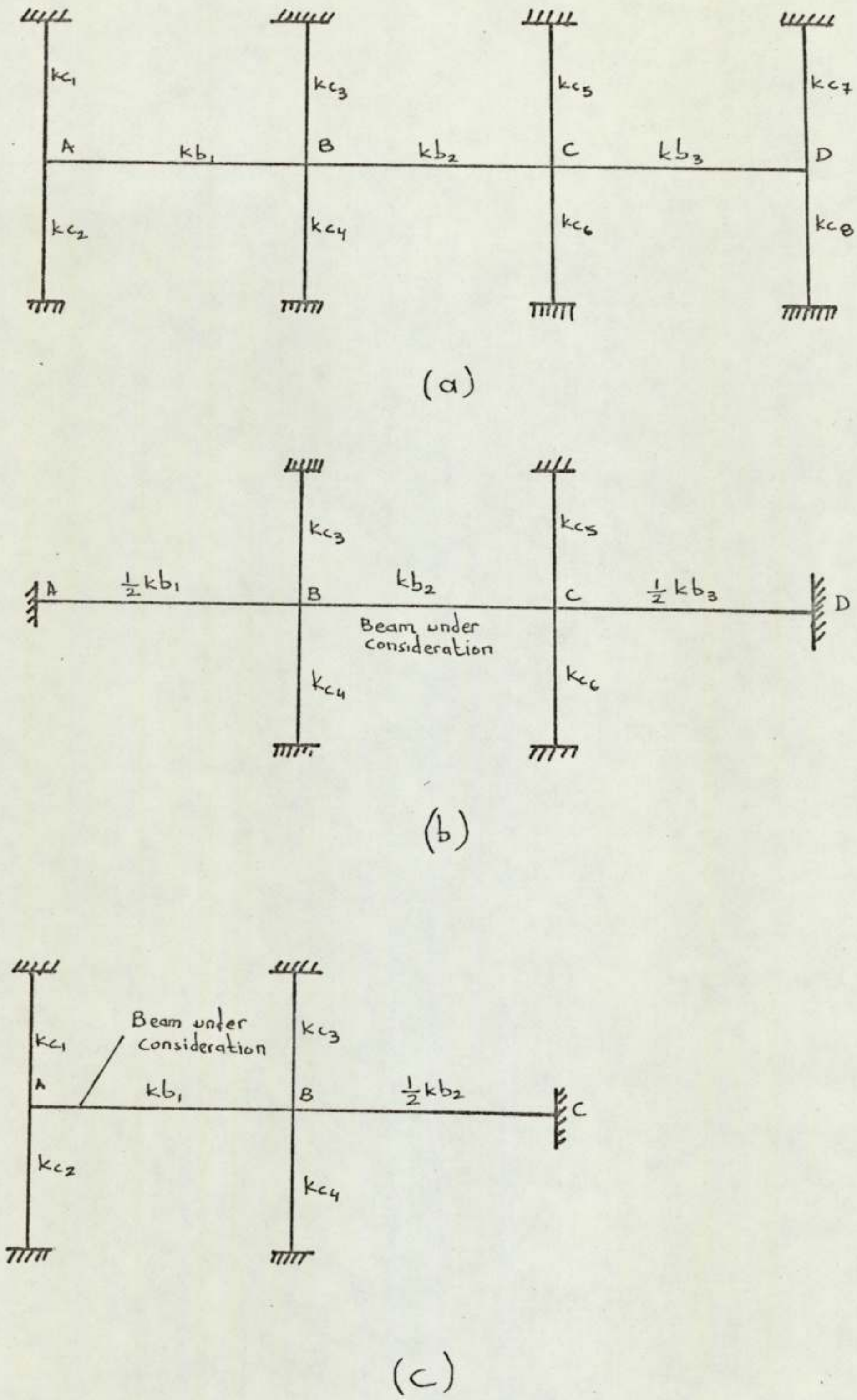
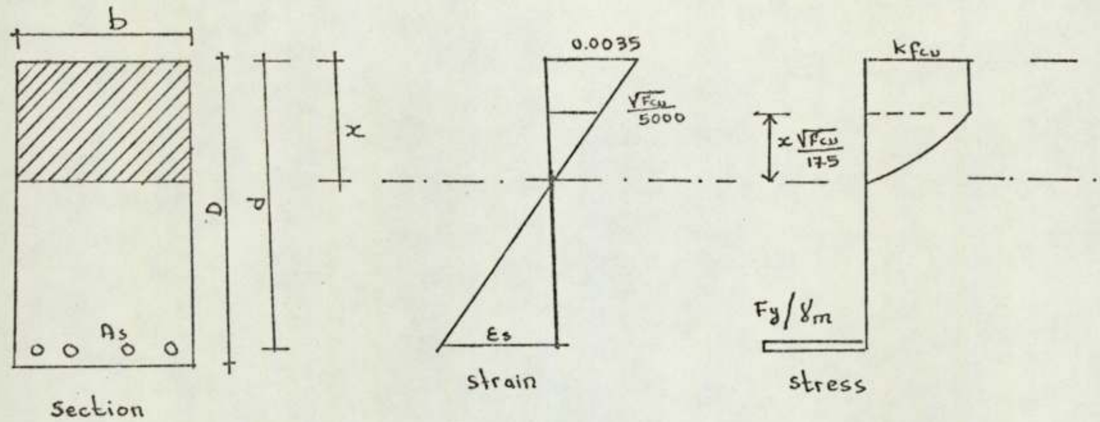
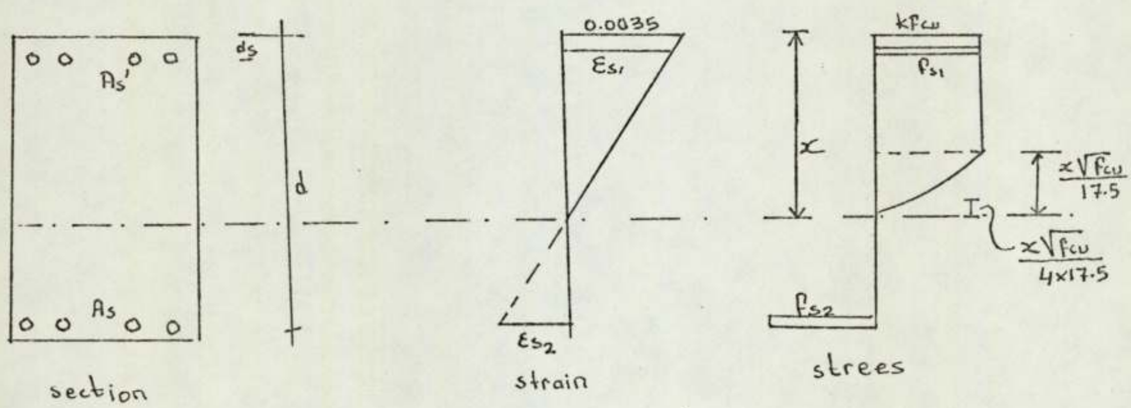

kc1
kc2
kc3
kc4
kc5
kc6
kc7
kc8
A
B
C
D
kb1
kb2
kb3
(a)
½kb1
½kb3
Beam under consideration
(b)
½kb2
Beam under consideration
(c)

Fig. 3.1.

compression reinforcement is $\frac{fy}{1.15+fy/2000}$. These graphs were developed from the relationship of $M/bd^2$ and $x/d$ to $\frac{100As}{bd}$




a) Singly Reinforced beam




b) Doubly Reinforced Beam

Fig 3.2.

To use the graphs it is necessary

1) Estimate (b) and (d) to determine $M/bd^2$

2) Choose fcu, fy and $d'/d$ and find the graph from CP 110 part II.

3) Read the total area of tension steel 100As/bd from the graph which depends on choice of 100As'/bd.

4) For (T) and (L) beam check that $x \leq h_f$

## 3.3) REDISTRIBUTION OF MOMENTS

An extension of the elastic analysis method which is permitted by CP 110 is the "Redistribution Method". It is permissible to redistribute the elastic bending moments provided certain conditions are satisfied. The arbitrary reduction of the elastic bending moments at the supports, initially calculated using the elastic theory leads to a reduction in the congestion of reinforcement at the support sections; this in turn makes better compaction of the concrete possible and enables detailing of reinforcement to be simplified.

Redistribution usually means "Reduction", so if the calculated elastic moments at the support are reduced by 10% - 30%, then this means providing a resistance moment at that position which will be capable of resisting less than the total elastic moment it can get. So at this position the member will become plastic and yield with resultant rotations. After reduction at the support, the other values of the bending moment diagram will be re-established according to the new support moments.

If we consider the behaviour of a beam which has fixed ends carrying a total uniformly distributed load W at the ultimate state. (fig.3.3)

It should be noted from the elastic analysis of the bending moment diagram that the total depth of the bending moment diagram is $\frac{wl}{8}$ and that the bending moment at the support is exactly twice the bending moment at the centre of the span. As we know the design would be better if the bending moment at the support and at the centre were more nearly equal. This is permitted and the effect of 30% redistribution is shown at fig(3.3.b) The support moment now becomes $0.7\frac{wl}{12}$ and moment at the centre of span becomes $\frac{wl}{8} - \frac{0.7wl}{12}$

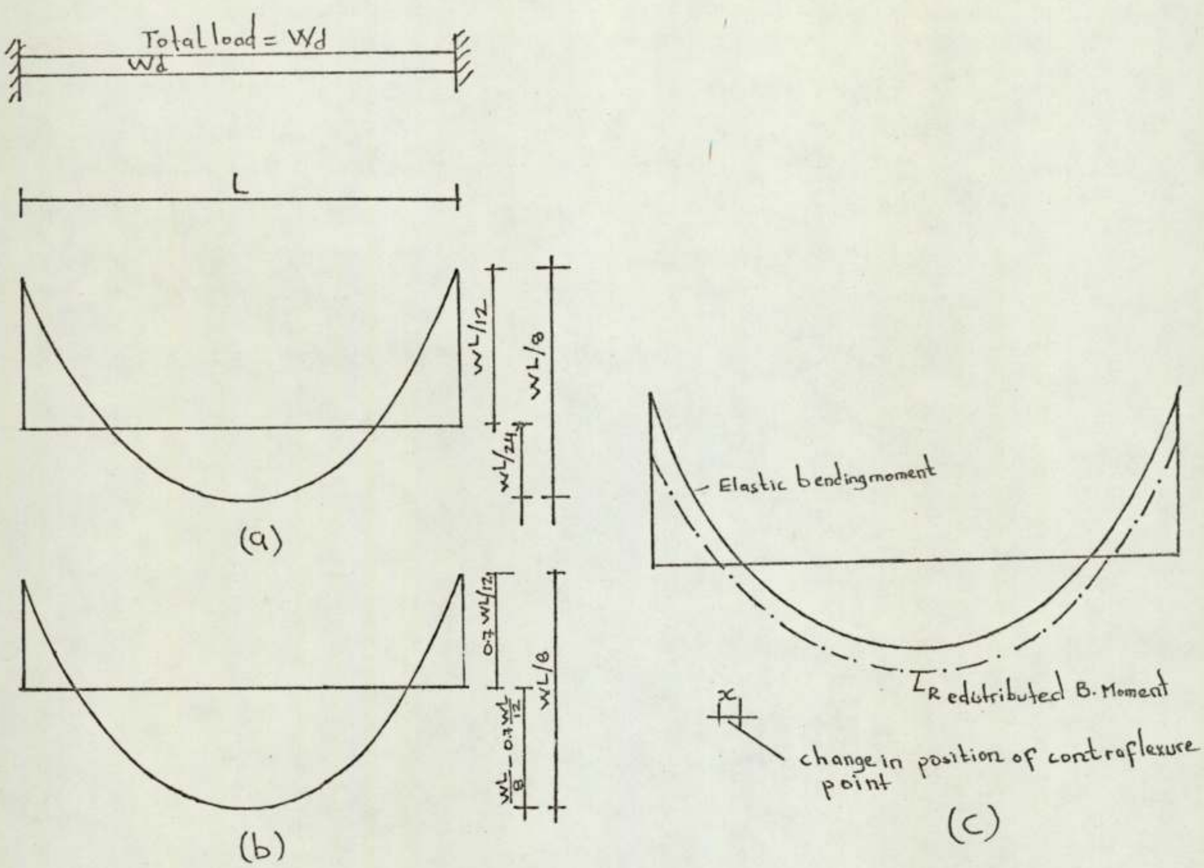



Fig 3.3.

The percentage by which a moment is reduced from the elastic value is a measure of the rotation of the hinge. When the elastic bending moments are reduced the points of contraflexure in the member change position fig(3.3.c). If the bending moment diagram after redistribution is used to curtail bars, there would be a sagging moment in length $x$ in the elastic stage, at the serviceability limit state length $x$ is a hogging moment. The design of the section must cover both eventualities. Ultimate load conditions require no reinforcement in this region and very wide cracks would develop here. Supplying reinforcement to carry at least 70% of the maximum elastic moment means that the structural response will remain roughly elastic at loads equal to or less than 70% of the total ultimate load. The loading corresponding to the serviceability limit state is always less than this and thus the possibility of wide cracking is ruled out.

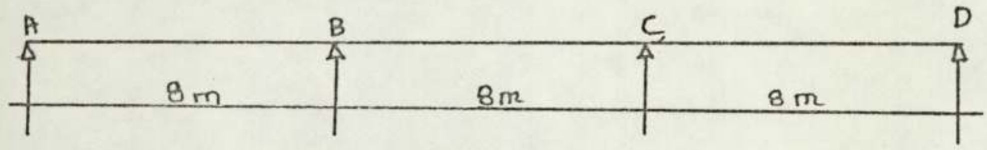
Earlier it was stated that if we make some reduction in moment at a section we will get rotation and the section design must cater for this. The amount of rotation which any section can undergo depends on how under-reinforced it is. If the reinforcement reaches its yield stress at the same time as the concrete reaches its ultimate strain, little rotation can take place. If the reinforcement reaches it yield stress long before the concrete fails, then considerable rotation can take place. The depth of neutral axis at failure gives a reasonable estimate of the rotation capacity. With a large neutral axis depth, the concrete will fail before the reinforcement yields, whereas with a small neutral axis depth, the reinforcement yields first. The code states that where the resistance moment at a section is reduced the neutral axis depth $x$, should not be greater than

$$x = (0.6 - \beta red)d$$

where $\beta$red is the ratio of the reduction in resistance moment to the numerically largest moment given anywhere by the elastic maximum moment diagram for that particular member, covering all appropriate combinations of loads.

The condition concerning the neutral axis depth will rule out the possibility of reduction in moments in a column unless the axial load is very small. The plasticity occurs in a beam rather than in the column. From that reason we have the singular position in frame structures that if we redistribute the beam moments at the junction with a column we can not adjust the column moments and in consequence we shall not get balance of moment at the junction. Where structural frames provides stability for a building, we are restricted to a 10% reduction in moments if the frame is more than four storeys in height.

EXAMPLE 1: DESIGN OF THREE SPAN BEAM USING CP 110 METHOD




Loadings:

Dead Load ($G_k$) = 20.0 kN/m

Imposed Load ($Q_k$) = 3.75 kN/m

Variable Load = $0.4\ G_k + 1.6Q_k = 8.0 + 6.0 = 14.0$ kNm

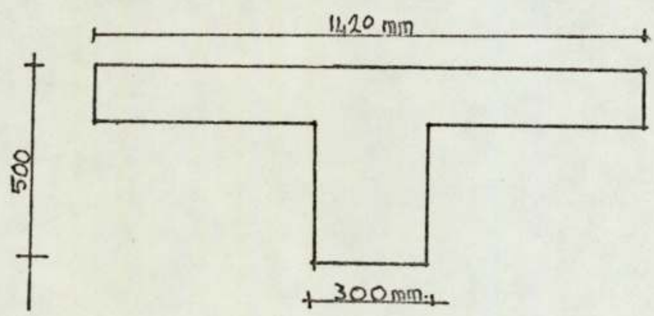
Total Load = 34.0 kN/m

Material Properties:

Concrete: Characteristic strength = ($f_{cu}$) = 20 N/mm$^2$

Reinforcement: Characteristic strength = ($f_y$) = 410 N/mm$^2$

Section of the beam chosen




Effective flange breadth = $\frac{1}{5} \times 0.7L + b = \frac{0.7 \times 8000}{5} + 300$

$= 1420$ mm

$d = 500 - 25 - 10 - 15 = 450$ mm

Elastic Design of the Beam

Structural analysis of the beam is done by use of a computer programme for different loading conditions and the results are

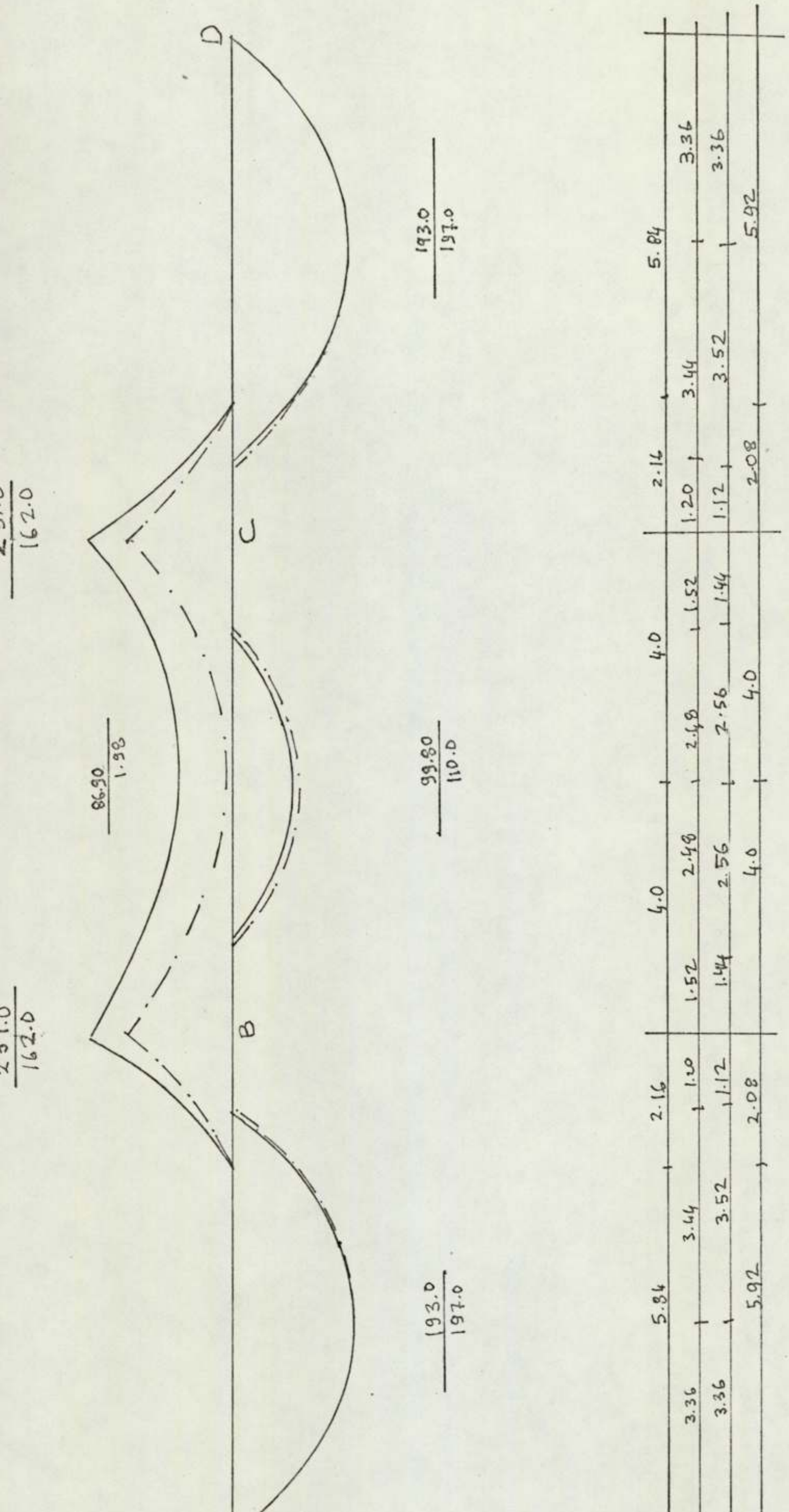

A
B
C
D
231.0
162.0
86.90
1.98
193.0
197.0
99.80
110.0
5.84
2.16
4.0
3.36
3.44
1.20
1.52
2.48
3.52
1.12
1.44
2.56
5.92
2.08

fig (3.1)

| | SPAN No | Dead Load (kn) | Variable Load (kn) | Total Load (kn) | Support Moment (knm) Left | Support Moment (knm) Right | $M_{max}$ (knm) | SHEAR FORCE (kn) Left | SHEAR FORCE (kn) Right | $n_{max}$ * |
|---|---|---|---|---|---|---|---|---|---|---|
| CASE 1 | AB | 20.0 | 14.0 | 34.0 | — | 172.41 | 192.75 | 114.44 | 157.56 | 3.36 |
| | BC | 20.0 | — | 20.0 | 172.41 | 172.41 | -12.41 | 80.0 | 80.0 | 4.0 |
| | CD | 20.0 | 14.0 | 34.0 | 172.41 | — | 192.75 | 157.56 | 114.44 | 4.64 |
| CASE 2 | AB | " | — | 20.0 | — | 172.28 | 85.53 | 58.47 | 101.53 | 2.92 |
| | BC | " | 14.0 | 34.0 | 172.28 | 172.28 | 99.77 | 136.0 | 136.0 | 4.0 |
| | CD | " | — | 20.0 | 172.28 | — | 85.53 | 101.53 | 58.47 | 5.08 |
| CASE 3 | AB | " | 14.0 | 34.0 | — | 231.40 | 168.83 | 107.07 | 162.93 | 3.15 |
| | BC | " | 14.0 | 34.0 | 231.40 | 157.82 | 78.65 | 145.19 | 126.81 | 4.27 |
| | CD | " | — | 20.0 | 157.82 | — | 90.78 | 99.72 | 60.27 | 4.98 |
| CASE 4 | AB | " | — | 20.0 | — | 157.82 | 90.78 | 60.27 | 99.72 | 4.98 |
| | BC | " | 14.0 | 34.0 | 157.82 | 231.40 | 78.65 | 126.81 | 145.19 | 4.27 |
| | CD | " | 14.0 | 34.0 | 231.41 | — | 168.83 | 162.93 | 107.07 | 3.15 |

Redistributed Design Values (30% at the Support)

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| CASE 1 | AB | 20.0 | 14.0 | 34.0 | — | 120.68 | 197.0 | 120.92 | 151.08 | 3.55 |
| | BC | 20.0 | — | 20.0 | 120.68 | 120.68 | 36.20 | 80.0 | 80.0 | 4.0 |
| | CD | " | 14.0 | 34.0 | 120.68 | — | 197.0 | 151.08 | 120.92 | 4.45 |
| CASE 2 | AB | " | — | 20.0 | — | 120.56 | 95.88 | 64.93 | 95.07 | 3.24 |
| | BC | " | 14.0 | 34.0 | 120.56 | 120.56 | 137.67 | 136.00 | 136.0 | 4.0 |
| | CD | " | — | 20.0 | 120.56 | — | 95.88 | 95.07 | 64.93 | 4.66 |
| CASE 3 | AB | " | 14.0 | 34.0 | — | 161.98 | 179.14 | 115.76 | 156.24 | 3.40 |
| | BC | " | 14.0 | 34.0 | 161.98 | 110.47 | 121.04 | 142.43 | 129.57 | 4.19 |
| | CD | " | — | 20.0 | 110.47 | — | 99.59 | 93.80 | 66.20 | 4.69 |
| CASE 4 | AB | " | — | 20.0 | | 110.47 | 99.59 | 93.80 | 66.20 | 4.69 |
| | BC | " | 14.0 | 34.0 | 110.47 | 161.98 | 124.04 | 142.42 | 129.57 | 4.19 |
| | CD | " | 14.0 | 34.0 | 161.98 | — | 179.14 | 115.76 | 156.24 | 3.40 |

TABLE - 3.3

tabulated on table (3.3), a graph of the elastic and redistributed bending moment envelope (10% at midspan, 30% at support) is drawn on graph (3.1)

BEAM DESIGN WITHOUT REDISTRIBUTION

SPAN AB

$M_{max} = M_{AB} = 193.0$ kNm (Load case 1)

$f_{cu} = 25.0$ N/mm$^2$

$d = 450$ mm

$b = 1420$ mm

$$M_u = 0.4 f_{cu} b h_f \left(d - \frac{h_f}{2}\right) = 0.4 \times 25 \times 1420 \times 150 \left(450 - \frac{150}{2}\right) \times 10^{-6}$$

$$= 798.75 \text{ kNm}$$

$M_u = 798.75 > 193.0$ kNm

∴ Comp. zone O.K.

$$\frac{M}{bd^2} = \frac{193.0 \times 10^6}{1420 \times 450} = 0.671$$

CP 110; design Chart 2.

$$\frac{100\,As}{bd} = 0.19 \longrightarrow As = \frac{0.19 \times 1420 \times 450}{100} = 1214 \text{ mm}^2$$

As provided = As = 1257 mm$^2$ 4Y20

by equating compressive force to tensile force

$$b\,x \times 0.4 f_{cu} = \frac{As f_y}{1.15}$$

$$x = \frac{1257 \times 410}{1420 \times 0.4 \times 25 \times 1.15} = 31.55$$

$$x = 31.55 < h_f = 150 \text{ mm.}$$

SPAN BC

$M_{max} = 99.8$ kNm; $\frac{M}{bd^2} = \frac{99.75 \times 10^6}{1420 \times 450^2} = 0.347$ from design chart 2

$\frac{100\,As}{bd} = 0.10$ ; $As = 639$ mm$^2$

and use 3Ø20 at mid span $\underline{\text{As prov} = 981.7 \text{ mm}} = \underline{3Ø20}$

Max. support Moment is at B in load case 3.

$M_B = 231.40$ kNm

$$\frac{M}{bd^2} = \frac{231.40 \times 10^6}{300 \times 450^2} = 3.80 \quad ; \quad \text{From chart 2 (CP 110)}$$

$$\frac{100\,As}{bd} = 1.36 \quad ; \quad As = \frac{1.36 \times 300 \times 450}{100} = 1836 \text{ mm}^2$$

<u>As provided = 1964 mm$^2$ - 4Ø25</u>

BAR CURTAILMENT (Cl.3.11.7.1)

SPAN AB : if we curtail 4Ø20 to 2Ø20

Max shear at A = 114.44 kN

$$\text{flexural bond stress} = \text{fbs} = \frac{V}{\sum \mu s.d} = \frac{114.44 \times 10^3}{(2 \times 62.83)\,450} = 2.02 \text{ N/mm}^2$$

Assuming deformed bars, type 2; allowable stress is = 2.8 x 1.2 = 3.36 N/mm$^2$

So 2Ø20 bars are satisfactory for local bond (2.02 < 3.36)

Moment of resistance of beam with 4Ø20 can be calculated by using Chart 2 (CP 110)

As = 1257 mm$^2$ (4Ø20)

$$\frac{100\,As}{bd} = \frac{100 \times 1257}{1420 \times 450} = 0.196 \quad ; \quad \frac{Mu}{bd^2} = 0.70$$

Mu = 201.28 kNm

Moment resistance provided by two bars

As = 628.3 mm$^2$ ; $\frac{100\,As}{bd} = 0.098$ from chart 2 $\frac{Mu}{bd^2} = 0.35$

Mu = 100.64 kNm.

The theoretical curtailment point occurs where the maximum elastic bending moment is 100.64 kNm. Examination of the elastic envelope shows this to occur at 1.05m from A and 2.2m from B. In both cases the curtailed bars must be continued beyond this point the absolute minimum continuation must be the greater of the effective depth, 0.45m, or twelve bar diameters, 12 x 20 mm = 0.24 m. Therefore the bars must be continued to 1.05 - 0.45 = 0.60 m. from A and 2.20 - 0.45 = 1.75 m. from B. If, however, these points fall within tension zone further check must be made.

These are:

(1) The bending moment capacity of the continuing bars must be twice the maximum bending moment which can occur at that section. The bending moment capacity of the section with continuing bars is 100.64 kNm. Therefore, the four bars should continue to the point at which the maximum bending moment is 50.32 knm which occurs at 0.45 from A and 1.60 m from B. This rule gives a longer extension than that above and this position will be used unless one of the following checks gives a more economical result.

(2) The shear capacity of the section with the continuing bars must be at least twice the maximum shear force where the bars are curtailed. Shear capacity is given by

$$V_c = V_c bd + \frac{0.87 \text{ fys } A_{sv} d}{S_u}$$

$$\frac{100 \text{ As}}{bd} = \frac{100 \times 628.3}{300 \times 450} = 0.505$$

from table 5 (CP 110) $V_c = 0.50$

stirrups are 10mm at 270 mm pitch ∴ $A_{sv} = 157 \text{ mm}^2$

$$V_c = \frac{0.50 \times 300 \times 450}{10^3} + \frac{0.87 \times 410 \times 157 \times 450}{270 \times 10^3} =$$

$V_c = 67.5 + 93.34 = 160.84$ kN. $V_c/2 = 80.42$ kN.

Max shear force at A = 114.44 kN. for load case 1

Max shear force at B = 162.93 kN. for load case 3.

for load case 1 shear force 80.42 occurs at

$$\frac{114.44 - 80.42}{34.0} = 1.00 \text{ m. from A.}$$

for load case (3) shear force 80.42 kN occurs at

$$\frac{162.93 - 80.42}{34.0} = 2.42 \text{ m. from B.}$$

This rule cannot improve economy because shear force increases towards the end of the bar.

(3) The bar may be extended by a bond length beyond the theoretical curtailment point.

$$\text{bond length} = \frac{0.87 \times 410 \times 20}{4 \times 1.9 \times 1.3} = 0.72 \text{ m.}$$

bars to end at 1.05 - 0.72 = 0.33m from A.

and at 2.20 - 0.72 = 1.48m from B.

The bending moment rule therefore gives the best answer i.e.

Curtail bars marked II at a distance 0.45 from A and 1.60 from B in AB Span.

From bending envelope extreme point of contraflexure is 1.20m from B and extend bars marked I Until the support centre.

SPAN AB - HOGGING SIDE

Curtail 4Ø25 to 2Ø25

Moment resistance provided by two bars

$$As = 982 \text{ mm}^2 \qquad \frac{100As}{bd} = \frac{100 \times 982}{300 \times 450} = 0.727$$

$$\therefore \text{ from chart 2 } \frac{Mu}{bd^2} = 2.25$$

$$Mu = 137.7 \text{ kNm.}$$

Examination of the elastic envelope shows this will occur at a distance 0.65 m from B. 12d = 12 x 25 $\leqslant$ 0.45 m.

Then the bars must continue 0.65 + 0.45 = 1.10 m from B.

(1) Apply bending moment rule.

$\frac{Mu}{2}$ = 68.85 kNm will occur at 1.25 m from B. This rule gives a longer extension than the above, and this position will be checked by applying the other two rules.

(2) Shear rule.

Shear capacity is given by

$$V_c = \upsilon cbd + \frac{0.87\ fy\upsilon.\ As\upsilon d}{s\upsilon}$$

$$\frac{100\ As}{bd} = \frac{100 \times 982}{300 \times 450} = 0.727 \quad \therefore \text{ from table 5 (CP 110)}$$

$\upsilon c = 0.57$, $s\upsilon = 270$, $As\upsilon = 157mm^2$ for 10mm stirrups

$$V_c = \frac{0.57 \times 300 \times 450}{10^3} + \frac{0.87 \times 410 \times 157 \times 450}{270 \times 10^3} = 170.0 \text{ kn.}$$

$\frac{Vc}{2}$ = 85 kn. which will occur at $\frac{162.93 - 85.0}{34}$ = 2.29 m.

from B. which is not an economic result.

(3) Bond rule.

$$\text{bond length} = \frac{0.87 \times 410 \times 25}{4 \times 1.9 \times 1.3} = 0.902 \text{ m.}$$

extend bars beyond the theoretical curtailment point

0.65 + 0.90 = 1.55 m.

Therefore bending moment rules give the best answer, and

curtail bars marked 5 at a distance 1.25 m from B.

Contraflexure point occur at 2.15 m from B and extend bars marked 4 at a distance 2.15 + 0.45 = 2.60 m from B.

If top tension steel is reduced to 2Ø16 and compression concrete is ignored, lever arm Z = 450.50 = 400 mm.

$$Mu = \frac{402 \times 0.87 \times 410 \times 400}{10} = 57.4 \text{ kNm.}$$

$Mu \simeq$ 57.4 kNm occurs at 1.40 m from B.

1.40 + 0.45 = 1.85 m $<$ 2.60 m.

(1) Moment rule.

$\frac{Mu}{2}$ = 28.7 kNm occurs at 1.70 m from B.

(2) Shear criterion

for 2Ø16 $\frac{100\ As}{bd} = \frac{100 \times 402}{300 \times 450} = 0.30 \quad \therefore$ from table 5(CP 110)

$\vartheta c = 0.37$ then $V_c = \dfrac{0.37 \times 300 \times 450}{10^3} + \dfrac{157 \times 0.87 \times 410 \times 450}{270} =$

$$V_c = 49.95 + 93.32 = 143.27 \text{ kN.}$$

$Vc/2 = 71.61$ kN occurs at $\dfrac{162.93 - 71.61}{34} = 2.67$ m from B.

then stop 2Ø25 at 2.60 m from B.

Lap length for 2Ø16 bars = 1.25 x 52.5 x 16 = 1050 mm

Lap Ø16 1.05 m.

SPAN BC - BOTTOM SIDE

Curtail 3Ø20 to 2Ø20

As = 628.3 $\qquad \dfrac{100\ As}{bd} = 0.098$ from chart 2 $\dfrac{Mu}{bd^2} = 0.35$

Mu = 100.64 kNm

From the elastic envelope 100.64 will not occur on bending moment envelope, but max moment will occur at centre of span, which is equal to 99.80 kNm. Therefore the bars must be continued to 4.0 - 0.45 = 3.65 m from B and C respectively.

(1) Bending moment rule:

$\dfrac{Mu}{2} = 50.32$ occur at 2.20 m from B. This rule gives a longer extension than above.

(2) The shear rule:

$$V_c = \frac{As\vartheta + 0.87\ fy\vartheta}{s\vartheta} + \vartheta cbd$$

$\dfrac{100\ As}{bd} = 0.505$ from table 5 (CP 110) $\vartheta c = 0.50$

10 mm stirrups at 270 mm pitch ∴ $As\vartheta = 157\ mm^2$

$V_c = 160.84$ kN.

$\dfrac{V_c}{2} = 80.42$

Vmax = 145.19 kN. (Load case 3)

$\dfrac{V_c}{2} = 80.42$ occurs at $\dfrac{145.19 - 80.42}{34} = 1.90$ m. from B.

(3) Bond rule:

Bond length = $\dfrac{0.87 \times 410 \times 20}{4 \times 1.9 \times 1.3}$ = 0.72 m

bars will be stopped at 4.0 - 0.72 = 3.28 m from B.

3.28 < 3.65 m and 3.65 not satisfactory, then curtail 3Ø20 to 2Ø20 at 2.20 m from B.

TOPSIDE - (HOGGING SIDE)

Mu = 137.7 kNm for 2Ø25 as calculated before.

Mu = 137.7 kNm occurs at 1.55 m from B. The bars must extend 0.45 m (0.45 > 12 x 0.025) from that point. Then bars must stop at 1.55 + 0.45 = 2.0 m from B.

(1) Apply bending moment rule.

$\dfrac{Mu}{2}$ = 68.85 kNm will not occur on diagram.

(2) Apply shear rule.

$V_c$ = 17.00 kNm as calculated for support B.

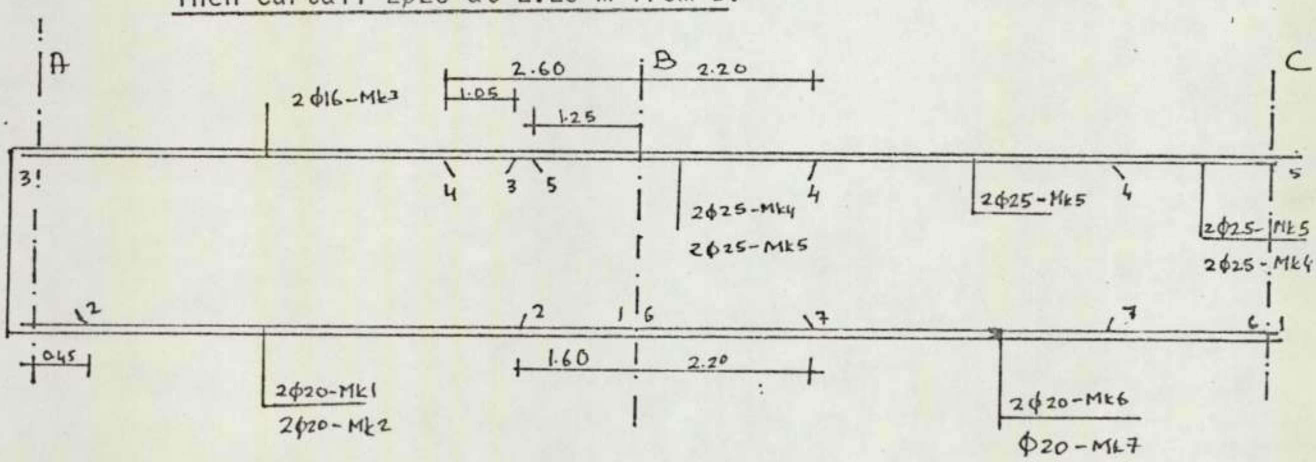
$\dfrac{Vc}{2}$ = 85.0 kN occur at $\dfrac{145.19 - 85.0}{34}$ = 1.77 m from B.

(3) Bond rule.

Bond length = $\dfrac{0.87 \times 410 \times 25}{4 \times 1.9 \times 1.3}$ = 0.902 m.

Curtail bars at 1.55 + 0.90 = 2.45 m from B.

<u>Then curtail 2Ø25 at 2.20 m from B.</u>




Curtail Diagram

SHEAR

Min. area of effective reinforcement is 2Ø20 at support A

$\frac{100A_s}{bd} = \frac{100x628.3}{300x450} = 0.465$ ; from table 5 Vc = 0.45

Minimum area of links: $\frac{Asv}{sv} = 0.0012\ bt = 0.0612 \times 300 = 0.36\ mm^2/mm$

Maximum spacing of links: 0.75 d = 0.75 x 450 = 337.5 mm

Says sv = 275 mm then, Asu = Sv x $\frac{Asv}{Sv}$ = 275 z 0.36 = 99.0 $mm^2$

Use Ø ($Asv = 10/mm^2$)

Shear stress resistance of links = $\frac{Asv}{Sv} \times \frac{0.87\ fyv}{b} = \frac{101}{275} \times \frac{0.87 \times 410}{300}$

= 0.436 $N/mm^2$

shear stress at which reinforcement needed is 0.436 + 0.45 = V

$v = 0.886\ N/mm^2$; so V = vbd = 0.986 x 300 x 450 = 119.61

SPAN AB.

Max V = 162.93 kN.

$x = \frac{162.93 - 119.61}{34} = 1.27$ m from B

SPAN BC

Max V = 145.19 KN

$x = \frac{145.19 - 119.61}{34.0} = 0.75$ m from B.

SPAN AB

Max shear stress at B = $\frac{162.83 \times 10^3}{300 \times 450} = 1.20\ N/mm^2$

2Ø25 As = 981.7 $mm^2$ $\frac{100\ As}{bd}$ = 0.727

from table 5 $v_c = 0.56$ $v - v_c = 1.20 - 0.56 = 0.64$

Using $\phi 8$ links $\quad S_v = \dfrac{A_{sv} \times 0.87 f_{yu}}{b(v-u_c)} = \dfrac{101 \times 0.97 \times 910}{300 \times 0.54} = 187.6$

$S_v = 198$ mm

Use $\phi 8$ at 190 mm pitch stirrups

SPAN BC

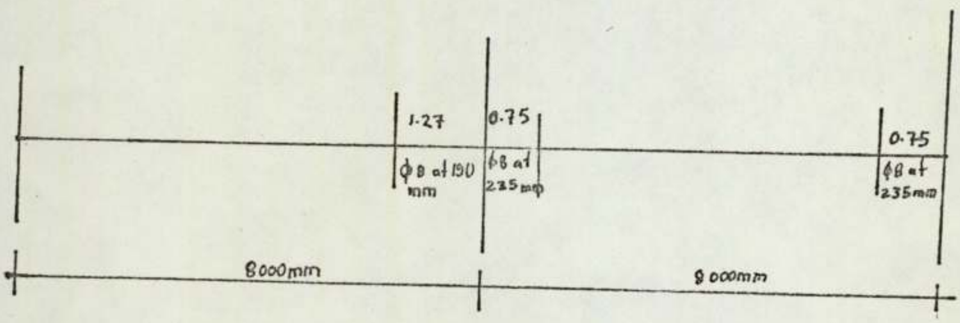
Max.shear stress at B = $\dfrac{145.19 \times 10^3}{300 \times 450} = 1.07\ N/mm^2$

$V_c = 0.56$ (same as above)

$V - V_c = 1.07 - 0.36 = 0.51$

$S_v = \dfrac{A_{sv} \times 0.87 f_{yv}}{b(V-V_c)} = \dfrac{101 \times 0.87 \times 410}{300 \times 0.51} = 2.35$ mm

$\phi 8$ of 235 mm centres stirrups




Serviceability limit state

$\dfrac{L}{d} = 26$ (basic)

$\dfrac{100A_s}{bd} = \dfrac{100 \times 1259}{1420 \times 450} = 0.196$

$f_s = \dfrac{0.58\ f_y\ A_{req.}}{A_s\ prov.}$ (Elastic)

$f_s = \dfrac{0.58 \times 910 \times 1182}{1257} = 223.62\ N/mm^2$

from table 10 multiplying factor = 1.67

$\dfrac{b_w}{b} = \dfrac{300}{1420} = 0.211$ ; multiplying factor = 0.8

Minimum depth = $\frac{8000}{26 \times 0.8 \times 1.67}$ = 230.3 mm

CRACKING

for fy = 410 and zero redistribution

Max. distance between bars = 185 mm

Max cover = $\frac{185}{2}$ = 92 mm (O.K.)

<u>Design of Redistributed beams</u> (30% at supports)

SPAN AB

Mmax = 197.0 kNm

$$\frac{M}{bd^2} = \frac{197 \times 10^6}{1420 \times 450^2} = 0.485$$ From chart 2 $A_s$ = 1257 $mm^2$ = 4Ø20

Support B

Max support moment = 162.0 kNm

$\frac{x}{d}$ = 0.3

$$\frac{M}{bd^2} = \frac{162.0 \times 10^6}{300 \times 450^2} = 2.55$$ From chart 2 $\frac{100As}{bd}$ = 0.82

As = 1107 $mm^2$ As provided = 1257$mm^2$ 4Ø20

SPAN BC

Mmax = 110.0 knm

$$\frac{M}{bd^2} = \frac{110 \times 10^6}{300 \times 450^2} = 0.382$$ from chart 2 $\frac{100As}{bd}$ = 0.105

As = 670.95 $mm^2$ As provided = 981.7 - 3Ø20

BAR CURTAILMENT (cl 3.11.7.1)

SPAN AB

If we curtail 4Ø20 to 2Ø20 we will have 2Ø20 at section A.

Max shear force at A = 120.92 kN.

flexural bond stress = fbs = $\frac{V}{\Sigma u_s d}$ = $\frac{120.92}{2(62.83)450}$ = 2.138N/$mm^2$

Assuming deformed bars, type 2, allowable stress is = 2.8x1.2=3.35 N/$mm^2$

So 2Ø20 is satisfactory for local bond (2.138<3.36 N/$mm^2$)

Moment resistance provided by two bars = 110.67 kNm (same as elastic curve) It will be seen from the bending envelope that there is not any difference between two diagrams. Curtail bars same as elastic case.

Hogging Side

Curtail 4Ø20 to 2Ø20

Moment of resistance of 2Ø20

$\frac{100\ As}{bd} = \frac{100x628.3}{300x450} = 0.415$ From chart 2 $\frac{Mu}{bd^2} = 1.55$

My = 1.55 x 300 x $450^2$ x $10^{-6}$ = 94.16 kNm.

Mu = 94.16 kNm will occur at 0.65m from B, extend bars 0.45m from B. Then stopped bars marked 4 of a distance 0.65 + 0.45 = 1.10m from B.

1. Bending moment rule

   Mu/2 = 47.16 knm occur at 1.20m from B.

   1.20>1.10m        1.20 satisfactory

2. Shear rule

   Vc = $v_c bd + \frac{0.87 x f_{yv} x A_{sv} . d}{S_v}$

   Vc = 0.50 from table 5 for $\frac{100\ Ast}{bd}$ = 0.505

   $S_v$ = 270, $A_{sv}$ = 257 $mm^2$ for 10mm stirrups,

   Vc = 160.84        Vmax = 156.24 for load case 3.

   $\frac{Vc}{2}$ = 80.42 will be occur at $\frac{156.24 - 80.42}{34}$ = 2.23m from B.

3. Bond rule

$$\text{bond length} = \frac{0.87 \times 410 \times 20}{4 \times 1.9 \times 1.3} = 0.72\text{m}$$

Then curtail 2Ø20 at 1.20m from B

Contra-flexure point occurs at 2.08m from B and extend bars marked 4 at a distance 2.08+0.45 = 2.53 2.55 m from B.

Lap length = 1.05m for 2Ø16 which will satisfy.

Moment resistance required on hogging side.

SPAN BC

Sagging Side:

Curtail 3Ø20 to 2Ø20

$As = 528.3$; $\frac{100\ As}{bd} = 0.098$ Mu = 100.64 from chart 2.

from bending envelope 100.64 will occur at 3.10m from B.

extend bars 0.45m = effective depth (0.457>12d)

3.10 - 0.45 = 2.65 m from B

1. Bending moment rule

$\frac{Mu}{2} = 50.32$ kNm occurs at 1.90m from B.

2. Shear rule.

$\frac{100\ As}{bd} = \frac{100 \times 628.3}{300 \times 450} = 0.505$ from table 5, $v_c = 0.50$

$As_v = 157\text{mm}^2$; $S_v = 270$ for 10mm stirrups at 270mm pitches.

$$Vc = v_c bd = \frac{0.87\ fy_v + As_v d}{S_v}$$

Vc = 160.84kN ; Vmax = 142.93 kN.

$\frac{Vc}{2} = 80.42$ kN will occur at $\frac{142.93-80.42}{34} = 1.84\text{m}$

3. Bond rule

$$\text{Bond length} = \frac{0.87x416x20}{4x1.9x1.3} = 0.72m$$

3.10 - 0.72 = 2.38m

then curtail bars at 1.90 m from B.

Hogging Side

Mu = 94.32 for 2Ø20 will occur at 0.85m from B.

0.85x0.45 = 1.30m. Curtail 2Ø20 at 1.30m from B.

1. Bending moment rule.

$\frac{Mu}{2}$ = 47.16kNm which occur at 1.80m from B. This rule gives a longer extension that above this position will be used unless one of the following checks give more economical results.

2. Shear rule.

Vc = 160.84 KN for 2Ø20 + $\phi$10mm stirrups at 2.70 mm pitches.

Vmax = 142.93 KN

$\frac{Vc}{2}$ = 80.42 will occur at $\frac{142.93-80.42}{34}$ = 1.84m from B.

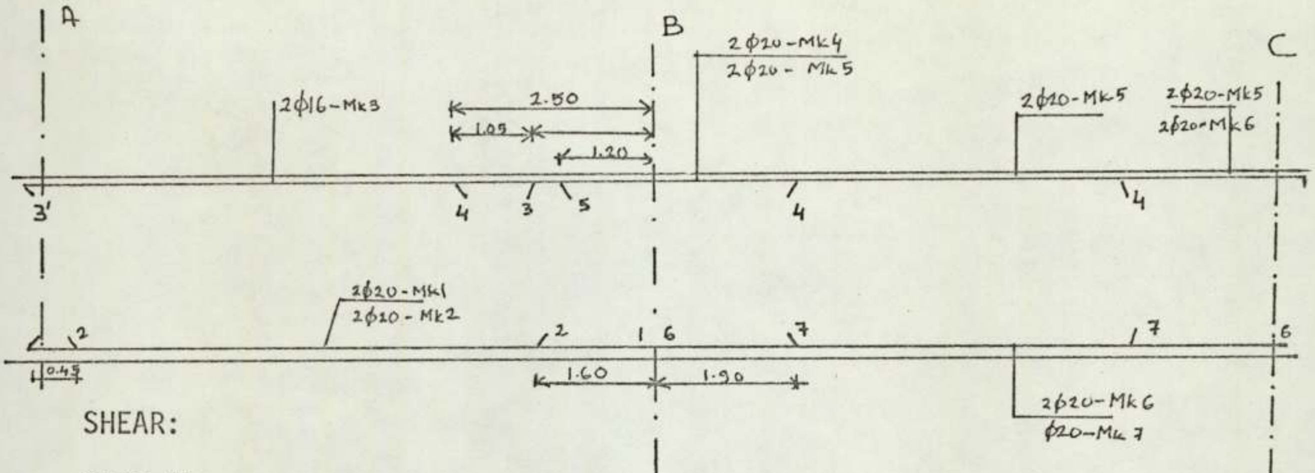
3. Bond rule

bond lengths = $\frac{0.87x410x20}{4x1.9x1.3}$ = 4.72 m

0.72 + 0.85 = 1.57m

Then curtail bars at 1.80m from B.

Curtailment Diagram




SHEAR:

SPAN AB:

Min.area of reinforcement 2Ø20; $\frac{100As}{bd} = 0.465$ $v_c = 0.465$ (from Table 5)

shear stress at B = $\frac{156.24 \times 10^3}{300 \times 450} = 1.14$ N/mm$^2$

$v - v_c = 0.675$ N/mm$^2$ using ϕ8 links $A_{sv} = 104.0$ mm$^2$.

$$S_v = \frac{0.87\ f_{yv}.A_{sv}}{b(v - v_c)} = \frac{101 \times 0.87 \times 910}{300(0.675)} = 177.90$$

shear resistance of links (ϕ8) = 0.436 N/mm$^2$

$0.436 + 0.45 = 0.886$

$V = vbd = 0.886 \times 300 \times 450 \times 10^{-3} = 119.61$ kN

$$x = \frac{156.24 - 119.61}{34} = 1.07\text{m}$$

in span BC

$V_{max} = 142.93$ KN

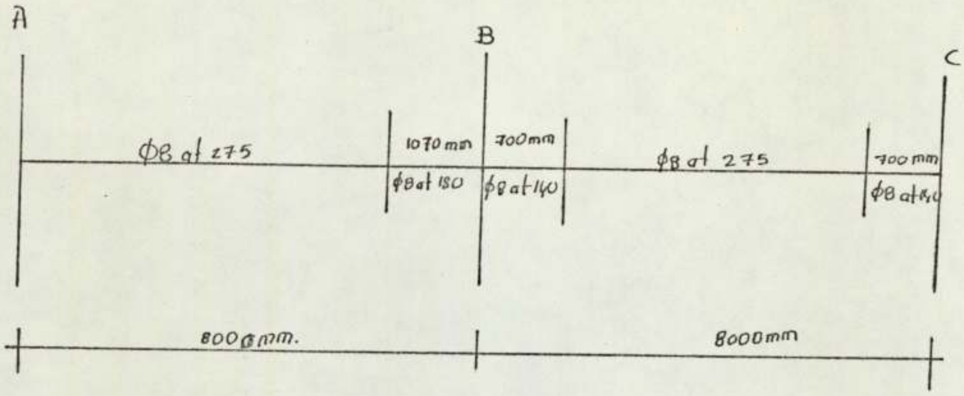
$$x = \frac{142.93 - 119.61}{34} = 0.685 \qquad 0.70\text{m}.$$

$$v = \frac{142.93 \times 10^3}{300 \times 450} = 1.05$$

$v_c = 1.05 - 0.465$ ($v_c = 0.465$ for 2Ø20)

$= 0.585$ N/mm$^2$

$$S_v = \frac{0.87 \times f_{yv} \times A_{sv}}{b(v - v_c)} = \frac{0.87 \times 101 \times 410}{450 \times 0.585} = 136.85 \text{ mm}$$




SERVICEABILITY LIMIT STATE

Deflection:

$$\frac{\text{Basic span}}{\text{Effective depth}} = 26$$

for mid span

$$fs = \frac{0.58 x fy x As req.}{As\ prov.} \times \frac{1}{\beta_b}$$

$$\beta_b = \frac{215}{230} = 0.93$$

As .req. = 1342 As .prov. 1443.8

$$fs = \frac{0.58 x 410 x 1342}{1443.8 x 0.93} = 237.66\ N/mm^2$$

by interpolation from table 10

$$\frac{100\ As}{bd} = \frac{100 x 1443.8}{1420 x 450} = 0.22$$

Modification factor for tension steel = 1.50

" " " flanged beam = 0.8

$$\text{Minimum effective depth} = \frac{8000}{1.5 x 26 x 0.8} = 256\ mm$$

CRACKING (3.3.9 and 3.11,8.2)

From table 24 max clear distance between bars = 185 mm.

Actual clear distance = $\frac{1}{2}$ (300-2x35-2x25-2x16) = 74 mm

Max.distance from corner = $\frac{185}{2}$ = 92 mm

Actual distance = $(35+12{,}5)\sqrt{2}-12{,}5$ = 55 mm

Note: In this design fire resistance has been ignored but, if a 4 hour resistance were required; the minimum concrete cover would be 65mm the actual distance for the corner is now $(65+12{,}5)\sqrt{2}-12{,}5$=96 mm. 96mm>92mm

check cracking using Appendix A3.2 for service load.

For service load

$Ms_{max}$ = 145.92 kNm

from table 1 $E_c = 26 kN/mm^2$; $E_s = 200x2\ kN/mm^2$

$$\alpha_e = \frac{E_s}{E_c} = \frac{200x2}{26} = 15.38$$

$$P = \frac{As}{bd} = \frac{1443.8}{1420x450} = 0.002$$

$$\alpha_e P = 0.0022x15.38 = 0.034$$

$$(\alpha_e P)^2 = 0.00115$$

$$\frac{X}{d} = -\alpha eP + \sqrt{(\alpha ep)^2+2\alpha ep}$$

$$= -0.034 + \sqrt{0.0011 + 0.068}$$

$$\frac{X}{d} = 0.228$$

$X = 102.6mm < 150mm$

$x \cong 103$ mm

$\frac{Z}{d} = 1 - \frac{1}{3}(\frac{x}{d}) = 1 - \frac{1}{3}(0.228)$

$= 1 - 0.074 = 0.926$

$Z = 450 x 0.926 = 416.7$ mm

$fs = \frac{M_s}{A_s z} = \frac{145.92 \times 10^6}{1443.8x416.7} = 242.54 < 0.87$ fy.

$e_s = \frac{fs}{E_s} = \frac{242.54}{200x10^3} = 0.0012$

at corner of beam

$\mathcal{E}_h = \frac{h-x}{d-x} \; \varepsilon_s = \frac{500-103.0}{450-1030} \; 0.0012$

$\varepsilon_h = 0.0013$

$\mathcal{E}mh = \epsilon_h - \frac{1.2b_t \cdot h}{As\ fy} = 0.0013 - \frac{1.2x300x5.60}{1443.8x410} \times 10^{-3}$

$= 0.0013 - 0.000304$

$mh = 0.0010$

crack width $= \frac{3ac\mathcal{E}mh}{1 + 2(\frac{acr-cmin}{h-x})} =$

$a_c = 96.5$

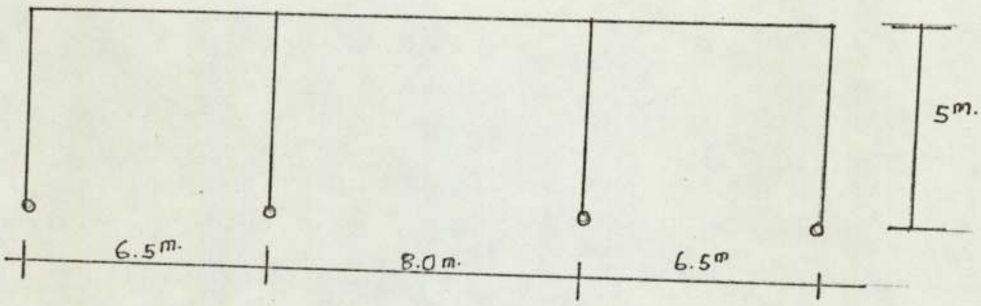
$c_{min} = 65$ mm

$W = \frac{3x96.5x0.0010}{1+2\left(\frac{96.5-65}{500-103}\right)} = \frac{0,289}{1.158} = 0.25$

$0.25 < 0.3$ mm

then cracking will be O.K.

Example 2 Design of frame by CP110 Method




Loadings

a) Limit state of collapse ($1.4G_k + 1.6Q_k$)

Dead load (Gk) = 36.65 kN/m

Imposed load ($Q_k$) = 30.00 kN/m

Variable load (F) = 62.66 kN/m ($0.4G_k+1.6Q_k$)

Total load = 99.32 kN/m

Crane load on each column = 533.28 kN

b) Serviceability Design 1.2(Qk+Gk+Wk)

1.2Gk + 1.2Qk = 80.0 kN/m

1.2 Wind load = 30.0 kN

Crane load on each column 400.0kN

Strength of Materials

a) Beams:

Steel design strength fy = 410 N/mm$^2$

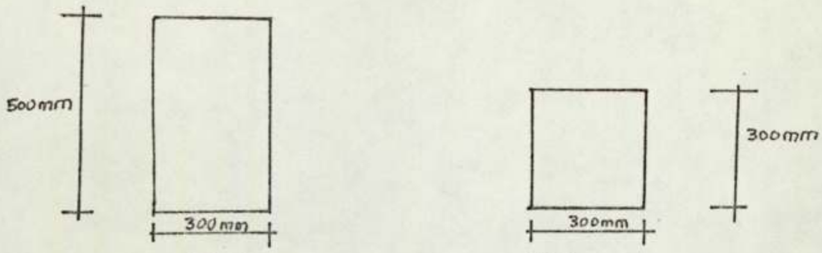
Concrete $fc_y$ = 25 N/mm$^2$

b) Columns:

Steel fy = 250 N/mm$^2$

Concrete fcu = 25 N/mm$^2$

Section chosen




BEAM DESIGN WITH 30% Redistribution at supports.

SPAN AB

Mmax = 319.0 kNm; = 500-=5-10-15=450mm; b = 300 mm

$\frac{M}{bd^2} = \frac{319.0x10^6}{300x450^2} = 5.25$

from chart 21

$\frac{100\ As}{bd} = 1.9;$ $\frac{100\ As}{bd} = 0.5$

As .req. = 2.565 $mm^2$ As = 3217 $mm^2$ - 4Ø32

As'.req. = 675 $mm^2$ As ' = 981.7 $mm^2$ - 2Ø25

SPAN BC

Mmax = 407 kNm

No redistribution at midspan

$\frac{M}{bd^2} = \frac{407x10^6}{300x450^2} = 6.70$ from chart 21, $\frac{100As}{bd} = 2.22;$

$\frac{100\ As\ '}{bd} = 1.50$

$As = \frac{2.22x300x450}{100}$ $As\ ' = \frac{1.5x300x450}{100} = 2025\ mm^2$

As .req. = 2997$mm^2$; $As\ '_{req.}$ = 2025 $mm^2$

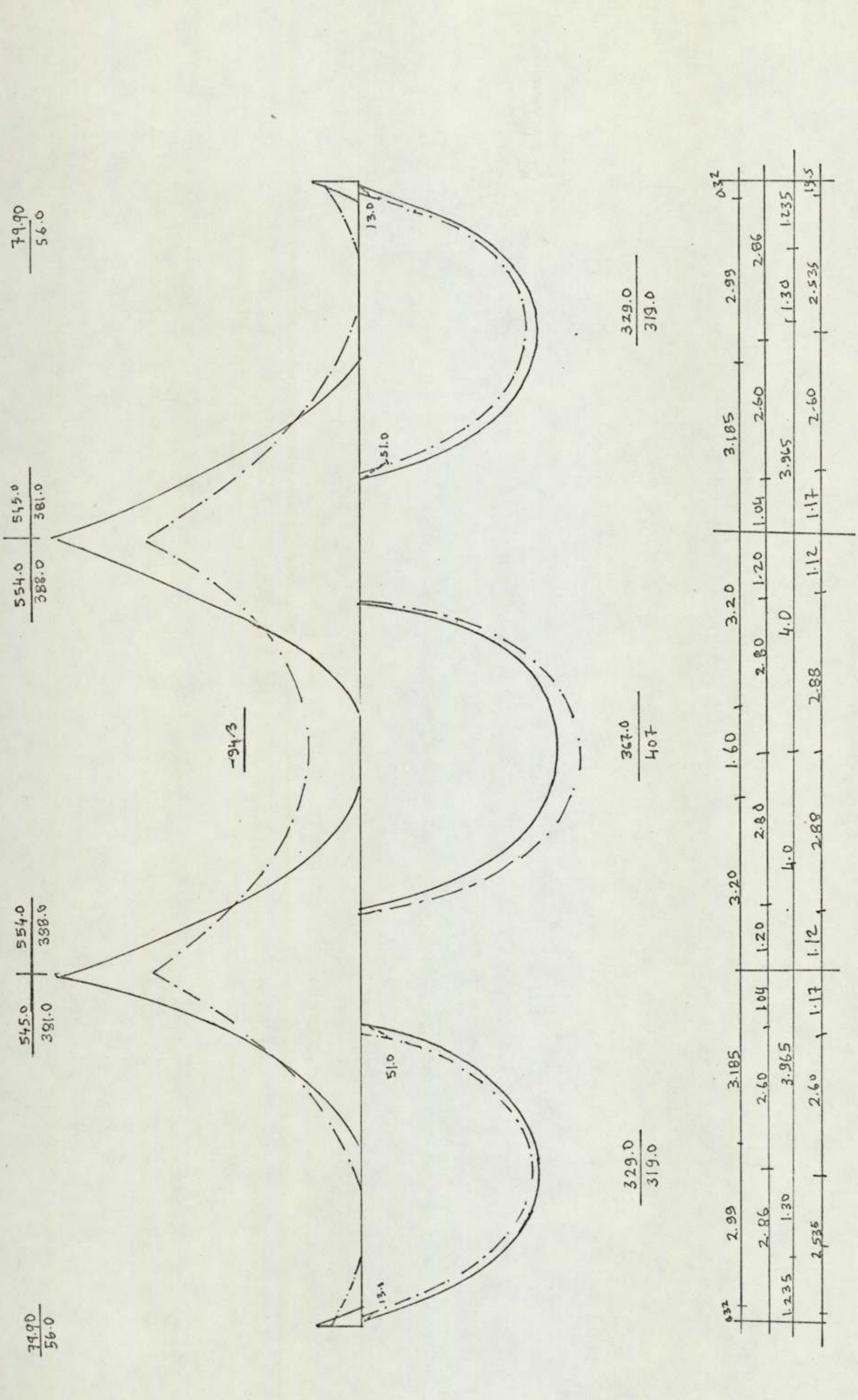
As provided = 3127 $mm^2$ - 4Ø32 As ' = 2Ø32+Ø25 = 2098.7 $mm^2$

# ELASTIC DESIGN OF FRAME

| | Span No | Dead Load kn/m | Variable Load kn/m | Total Load | Support Moment (knm) Left | Support Moment (knm) Right | $M_{max}$ knm | Shear Force (kn) Left | Shear Force (kn) Right | $x_{max}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| CASE 1 | AB | 36.66 | — | 36.66 | -2.75 | 374.57 | 49.52 | 61.94 | 176.35 | 1.69 |
| | BC | " | 62.66 | 99.32 | -427.52 | 427.52 | 367.04 | 397.28 | 397.28 | 4.0 |
| | CD | " | — | 36.62 | -374.57 | 2.75 | 49.52 | 176.35 | 61.94 | 4.81 |
| CASE 2 | AB | " | 62.66 | 99.32 | -79.94 | 324.73 | 329.25 | 285.13 | 360.45 | 2.87 |
| | BC | " | — | 36.62 | -280.61 | 280.61 | 12.57 | 146.64 | 146.64 | 4.0 |
| | CD | " | 62.66 | 99.32 | -324.73 | 79.94 | 329.25 | 360.45 | 285.13 | 3.63 |
| CASE 3 | AB | " | 62.66 | " | -52.22 | 544.56 | 253.51 | 247.04 | 398.53 | 2.48 |
| | BC | " | 62.66 | " | -553.61 | 391.13 | 222.67 | 417.59 | 376.97 | 4.20 |
| | CD | " | — | 36.66 | -340.78 | 10.92 | 50.57 | 169.89 | 68.39 | 4.63 |
| CASE 4 | AB | " | — | " | -10.92 | 340.78 | 50.57 | 68.39 | 169.89 | 1.87 |
| | BC | " | 62.66 | 99.32 | -391.13 | 553.61 | 322.67 | 376.97 | 417.59 | 3.80 |
| | CD | " | 62.66 | " | -544.56 | 52.22 | 253.51 | 398.53 | 247.04 | 4.02 |

## REDISTRIBUTED DESIGN (No red. at midspan 30 % at Supports)

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| CASE 1 | AB | " | — | 36.66 | -2.06 | 262.10 | | 76.26 | 162.03 | 2.08 |
| | BC | " | 62.66 | 99.32 | 299.20 | 299.20 | | 397.28 | 397.28 | 4.0 |
| | CD | " | — | 36.66 | 262.10 | 2.06 | | 162.03 | 76.26 | 4.42 |
| CASE 2 | AB | " | 62.66 | 99.32 | 55.95 | 227.31 | | 296.42 | 349.16 | 2.98 |
| | BC | " | — | 36.66 | 196.42 | 196.42 | | 146.64 | 146.64 | 4.0 |
| | CD | " | 62.66 | 99.32 | 227.31 | 55.95 | | 349.16 | 296.42 | 3.52 |
| CASE 3 | AB | " | " | " | 36.55 | 381.19 | | 269.71 | 375.82 | 2.71 |
| | BC | " | " | " | 387.52 | 273.79 | | 411.47 | 383.09 | 4.14 |
| | CD | " | — | 36.66 | 238.54 | 7.64 | | 154.66 | 83.63 | 4.21 |
| CASE 4 | AB | " | — | " | 7.64 | 238.54 | | 83.63 | 154.66 | 2.29 |
| | BC | " | 62.66 | 99.32 | 273.79 | 387.52 | | 383.09 | 411.47 | 3.86 |
| | CD | " | " | " | 381.19 | 36.55 | | 375.82 | 269.76 | 3.79 |



79.90
56.0
554.0
388.0
545.0
381.0
329.0
319.0
367.0
407
-94.3
13.0
51.0
13.1
0.32
1.235
2.86
2.99
1.30
2.535
3.185
2.60
3.965
1.04
1.17
1.20
1.12
3.20
2.80
4.0
2.88
1.60
19.5

SUPPORT B

$\frac{x}{d}$ = 0.3 (30% redistribution)

Mmax = 388.0 kNm

$\frac{M}{bd^2} = \frac{388x10^6}{300x450^2} = 6.38$ from chart 21

$\frac{100\ As}{bd} = 2.12$ As = 2862 $mm^2$; $\frac{100\ As'}{bd} = 1.50$ As ' = 1687.5$mm^2$

As .prov. = 3217$mm^2$ - 4Ø32; As $'_{prov.}$ = 2336 $mm^2$ (2Ø32+2Ø20)

2Ø32 + 2Ø32 2Ø32+ 2Ø20

BAR CURTAILMENT (CL 3.11.7.1)

SPAN AB

Bottom side

Curtail 4Ø32 to 2Ø32

Max shear at A = 296.42 kN (Load case 2)

flexural bond stress = fbs = $\frac{V}{\Sigma usd}$

$\Sigma$ us = 2x100.5 = 201 $mm^2$ (Bottom bars are in tension)

$fbs = \frac{296.42x10^3}{201x450} = 3.27\ N/mm^2$

Allowable shear stress for deformed bars type 2 = 2.8x1.2=3.36$N/mm^2$

3.27<3.36 so 2Ø32 are satisfactory for local bond.

Moment resistance of As = 1608$mm^2$ and As ' = 981.8 $mm^2$ can be calculated by using chart 21 (CP110)

$\frac{100\ As}{bd} = \frac{100x1608}{300x450} = 1.19$

$\frac{100\ As'}{bd} = \frac{100x981.8}{300x450} = 0.727$ $\frac{Mu}{bd^2} = 3.65$

$M_u$ = 221.73 knm and $Mu_{/2}$ = 110.86 knm, $M_u$ = 221.73 will occur at 1.20m from A and 2.00m from B, in redistributed envelope

12d = 12x32 = 384mm<450mm (effective depth)

extend bars 0.45m from theoretical cut-off point then curtail bars marked II at a distance 2.00 - 0.45 = 1.55 m from B. and 1.20 - 0.45 = 0.75 m from A.

1. Use full bond length rule:

$$\text{full bond length} = \frac{0.37 \times \emptyset fy}{4fbs \times 1.3} = \frac{32 \times 0.87 \times 410 \times 10^{-3}}{4 \times 1.9 \times 1.3} = 1.154$$

Bars to end at 2.00 - 1.154 = 0.845 m from B

1.25 - 1.154 = 0.096 m from A.

2. Use shear rule:

Shear resistance of 2Ø32 and 10mm stirrups with 270mm pitches can be calculated by using equation 3(CP110) to form

$$Vc = \upsilon cbd + \frac{Asv \times 0.87fyvd}{Sv}$$

$\upsilon c = 0.688$ for $\frac{100\ As}{bd} = 1.19$ (from table 5 CP110)

$Asv = 157\ mm^2$ $sv = 270$ mm

$$Vc = \frac{0.688 \times 300 \times 450}{10^3} + \frac{157 \times 0.87 \times 410}{270 \times 10^3} = 93.83 + 92.88$$

Vc = 186.21 kN; Vmax = 375.82 (Load case 2)

Vc = 93.10 kN will occur at $\frac{375.82-93.10}{99.32}$ = 2.84 from B

3. Bending moment rule

$\frac{Mu}{2}$ = 110.86 kNm will occur at 0.55m from A and at 1.3m from B.

Then curtail bars at 0.55m from A & 1.35m from B respectively

SPAN AB (Hogging side)

Curtail 4Ø32 to 2Ø32 and 2Ø32+2Ø20 to 2Ø32 respectively.

As = 1608.5$mm^2$ As ' = 1608.5 $mm^2$

$\frac{100\ As}{bd}$ = 1.19 and $\frac{x}{d}$ = 0.3

$\frac{100\ As'}{bd}$ = 1.19

from chart 21 ; $\frac{Mu}{bd^2}$ = 3.6 Mu = 218.70 kNm

Mu = 218.70 kNm occur at 1.00m from B in span AB

12d = 0.032x12 = 0.384m<0.45m. Then extend bars 1.0+0.45 = 1.45m from B.

1. Bending moment rule:

$\frac{Mu}{2}$ = 109.35 kNm occur at 1.95m from B. which gives a longer extension.

2. Bond rule

bond length = $\frac{0.87 \cdot \phi \cdot f_y}{4 f_{bs} \cdot 1.3}$ = 1.154 for $\phi$32.

stops bar at 1.10+1.154 = 2.254m from B.

3. Shear rule:

$\frac{100\ As}{bd}$ = 1.19 and from table 5 (CP110) Vc = 0.688 for 2Ø32

10mm stirrups with 270mm pitches arranged.

$$Vc = \frac{157x0.97x410x450}{270x10^3} + 0.688x300x450$$

Vc = 186.21 kN ; $V_{max}$ = 375.84 load case 2.

$\frac{Vc}{2}$ = 93.10 kN will occur at $\frac{375.82x83.10}{99.32}$ = 2.84m from B.

Then curtail bars at 1.95m from B.

Extreme point of contraflexure will occur at a distance 3.185m from B. and stopped bars marked (4,4') at 3.185+0.45   3.65m from B.

SPAN BC (Sagging side)

Curtail 4Ø32 to 2Ø32 and 2Ø32 + Ø25 to 2Ø32

$$\frac{100\ As}{bd} = \frac{100x1608}{300x450} = 1.19$$

$$\frac{100\ As'}{bd} = \frac{100x1608}{300x450} = 1.19$$

from chart 21 $\frac{Mu}{bd^2}$ = 3.6

Mu = 218.70 will occur at 1.70m from B and C respectively.

12d = 0.032 x 1= = 0.30m<0.45m (effective depth). Then curtail bars at 1.70 - 0.45 = 1.25 m from B.

1. Bending moment rule:

$\frac{Mu}{2}$ 109.35 kNm will occur at 1.30m from B and C respectively.

2. Shear rule:

$\frac{100\ As}{bd} = \frac{100x1603}{300x450}$ = 1.19 from table 5 $v_c$ = 0.688.

$S_v = 270$ and $A_{sv} \simeq 157.0\ mm^2$ for 10mm stirrups with 270mm pitches.

$$V_c = v_c bd + \frac{A_{sv}.f_{yv}.0.87}{S_v} = \frac{0.98x300x450}{10^3} + \frac{157x0.87x410x450}{270 \times 10^3}$$

$V_c = 186.21kN$

$V_{max} = 397.28\ kN$ $\qquad \frac{V_c}{2} = 93.10$ will occur at

$\frac{397.28 - 93.10}{99.32} = 3.06m$ from B.

3. Bond rule:

bond length = $\frac{6.87x410x32}{4x1.9x1.3}$ = 1,154m ≅ 1.15m

bars stopped at 1.70 - 1.15 = 0.55m from B and C respectively.

Then curtailed bars at 1.30m from B and C respectively

SPAN BC (Hogging side).

Mu = 218.70 kNm which calculated occur at 1.50m from B.

1.50+0.45 = 1.95m. Then curtail bars at 1.95m from B.

1. Bending moment rule:

$\frac{Mu}{2} =$ 109.35kNm will occur at 3.20m from B. and C respectively.

2. Shear rule:

Vc = 186.21 kN as calculated for 2Ø32 + 10mm stirrups with 270m pitches.

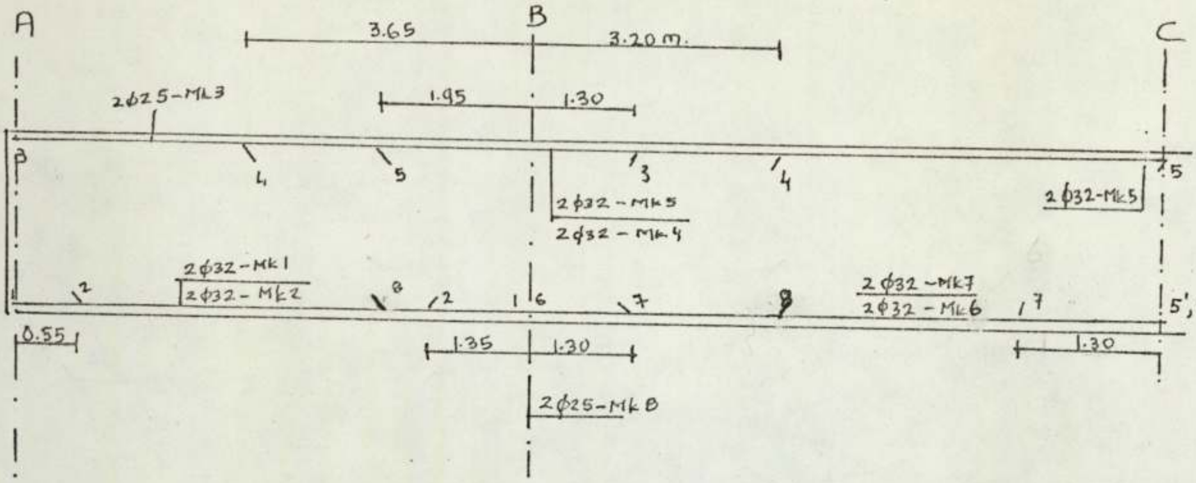
$V_{max} = 93.10$ kN occurs at $\frac{397.28 - 93.10}{99.32}$ = 3.06m from B.

3. Bond rule:

bond length = 1.154m for Ø32

curtailed bars at 1.154 + 1.50 = 2.654m from B

Curtailed bars at 3.20m from B.




Curtailment Diagram

SHEAR

Min.area of reinforcement in span AB is at Support A is

As = 1608mm (2Ø32).

$\frac{100\ As}{bd} = \frac{100x1608}{300x450} = 1.19$

from table 5 Vc = 0.70

Min.area of links = $\frac{Asv}{Sv}$ = 0.0012bt = 0.0012x300 = 0.36

Max. space of links = 0.75 d = 0.75x450 = 337.5 say 275 mm.

Asv = 0.86x275=99.0mm$^2$

Use ϕ10 (Asv = 157.8mm$^2$) stirrups.

shear stress resistance of links = $\frac{Asv}{Sv}$ x $\frac{0.87\ fyv}{b} = \frac{157x0.97x410}{275x300}$

= 0.68N/mm$^2$

= 0.70+0.68 = 1.38

V = vbd = 1.38x300x450x10$^{-3}$ = 186.3 kN

$V_{max}$ = 375.82 kN (in span AB Load case 3)

x = $\frac{375.82-186.3}{99.32}$ = 1.90m

in span BC

$$x = \frac{411.47 - 190.35}{99.32} = 2.22m$$

SPAN AB

Shear stress at B:

Max shear (Load case 3) = 375.82 kN

$$\text{Shear stress} = \frac{375.82x10^3}{300x450} = 2.70 \ N/mm^2$$

tension steel at B 4Ø32 As = $3217mm^2$; $\frac{100\ As}{bd} = \frac{100x3217}{300x450}$

$\frac{100\ As}{bd} = 2.38$ ; (table 5) $v_c = 0.87$

$v - v_c = 2.78-0.87=1.91$

using $\phi$10 links $S_v = \frac{157,28x0.87x410}{300x1.91} = 97.90$ 100 mm

Using $\phi$10 at 100m centres

SPAN BC:

Max shear at B = 411.47 kN (Load case 3)

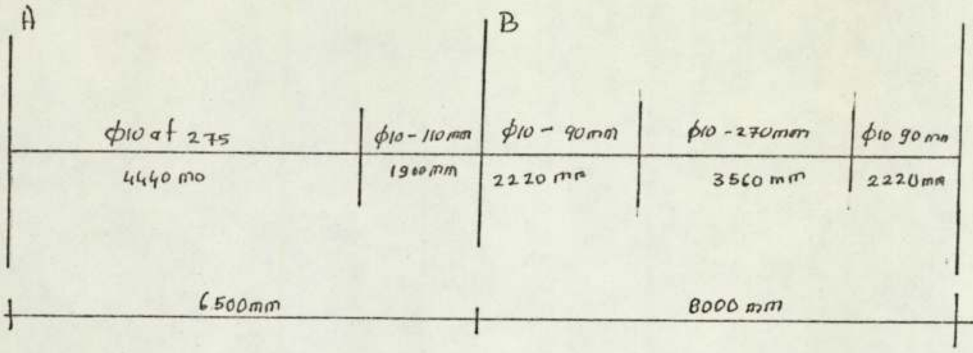
$$\text{shear stress} = v = \frac{411.47x10^3}{300x450} = 3.04 \ N/mm^2$$

$v_c = 0.87$ $v - v_c = 3.04-0.87 = 2.17N/mm^2$

$$S_v = \frac{A_{sv}x0.87\ fy}{bx(v-v_c)} = \frac{157.28x0.87x410}{300x2.17} = 86.17$$

$S_v \cong 90$ mm

Using $\phi$10 at 90 mm centers.




SERVICEABILITY DESIGN

1. Deflection (3.3.8)

$$\frac{\text{Basic span}}{\text{effective depth}} = 26$$

for mid span

$$fs = \frac{0.58x\ fy\ xAsreq.}{As\ prov.} \quad x \quad \frac{1}{\beta_b}$$

$$\beta_b = \frac{\text{max.moment}}{\text{resistance moment}} = \frac{475.30}{531.56} = 0.93$$

$$fs = \frac{0.58x410x3591}{3845x0.93} = 238.80\ N/mm^2$$

$$\frac{100\ As}{bd} = \frac{100x3845}{300x450} = 2.84$$

Modification factor for tension steel = 0.78 (CP110 table 10)

Modification factor compressed steel

$$\frac{100\ As'}{bd} = \frac{100x3217}{300x450} = 2.38$$

Modification factor = 1.44 (compression) (table 10)

$$\text{Min effective depth} = \frac{8000}{26x1.5x1.44} = 273.94$$

effective depth O.K.

CRACKING (3.3.9 and 3.11.8.2)

From table 24 max.clear distance between bars = 185 mm.

Actual clear distance = $\frac{1}{2}$ (300-2x35-4x32-2x30) = 31 mm

Max.distance from a corner = $\frac{185}{2}$ = 92 mm

Actual distance = $(351 \frac{32}{2})\sqrt{2} - \frac{32}{2}$ = 56 mm.

In this design fire resistance for four hours is required the min.cover ($c_{min}$) would be 65 mm, the actual distance to corner is now $(65+16)\sqrt{2}-16$ = 98.5 mm 92mm

Check cracking using appendix A32. in CP 110 for service load

$Ms_{max}$ = 285.43 kNm

from table 1 for fcu = 25 N/mm$^2$ Ec = 26.0 kN/mm$^2$

Es = 2x200kN/mm$^2$

$\alpha_e = \frac{E_s}{E_c} = \frac{2x200}{26} = 15.3$ ; $\frac{d'}{d} = \frac{50}{450} = 0.11$

$p = \frac{As}{bd} = \frac{3845}{300x450} = 0.028$ ; $p' = \frac{As'}{bd} = \frac{3217}{300x450} = 0.023$

For double reinforcement beam section

$$\frac{x}{d} = -[\alpha ep+(\alpha e-1)p'] + \sqrt{[\alpha ep+(\alpha e-1)p']^2 + 2[\alpha ep+(\alpha e-1)p' \frac{d'}{d}]}$$

$$\frac{x}{d} = -[o.43+0.324] + \sqrt{[0.43+0.329]^2 + 2[0.43+(0.329)0.11]}$$

$\frac{x}{d} = 0.465$, x = 0.465x450 209 mm

$\frac{z}{d} = 1.- \frac{1}{3}(\frac{x}{D}) = 1 - \frac{1}{3}(0.465) = 0.845$

z = 380.25

Average surface stress at bottom of the beam

$$fs = \frac{M_s}{A_s \cdot Z} = \frac{284.43 \times 10^6}{3845x380} = 195.35\ N/mm^2$$

$$es = \frac{fs}{E_s} = \frac{195.35}{200x10^3} = 0.00097$$

at corner of the beam

$$\varepsilon_h = \frac{h-x}{d-x} \varepsilon_s = \frac{500-209}{450-209}\ 0.00097 = 0.00117$$

$$\varepsilon_{mh} = \varepsilon_h - \frac{1.2bt \cdot h}{A_s\ f_y} = 0.00117 - \frac{1.2x300x500}{3845x410} \times 10^{-3}$$

$$\varepsilon_{mh} = 00.00105$$

at corner of the beam

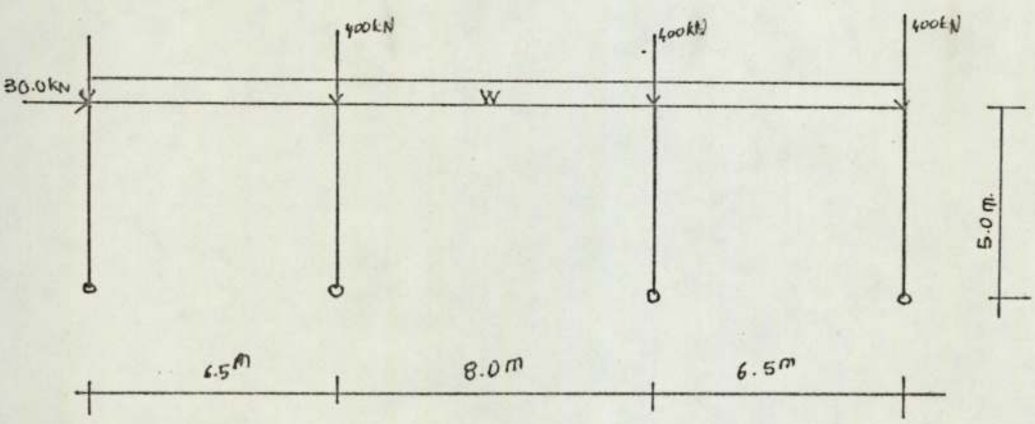
$$a_c = 96.5\ mm$$

$$\text{crack width } (W_{cr}) = \frac{3ac\ \varepsilon mh}{1+2\left(\frac{Acr-Cmin}{h-x}\right)} = \frac{3x98.5x0.00105}{1+2\left(\frac{98.5-65}{500-209}\right)}$$

$$= 0.252 < 0.3.$$

O.K.

Design of Columns :



Limit state of collapse

$b = 300$ mm

$d = 300$ mm

$$I_{c1} = \frac{bh^3}{12} = 67.5x10^6 mm^4$$

Column stiffness $= \frac{I}{L} = \frac{675x10^6}{5000} = 135x10^3$

beam stiffness

$$I = \frac{bh^3}{12} = \frac{300x500^3}{12} = 3125x10^6\ mm^4$$

beam stiffness

beam stiffness $= \frac{I}{\ell} = \frac{3125x10^6}{6.500} = 480,76923x10^3$ (external beams)

$$\frac{I}{\ell} = \frac{3125x10^6}{800} = 390.625x10^3$$

CP110 Equation 21 $\ell e = \ell o\ (0.85 + 0.05\ \ cmin) < \ell o$

$$\left.\begin{aligned} \alpha c_1 &= \frac{675}{3125} = 0.216 \\ \alpha c_2 &= 1.0 \end{aligned}\right\} \alpha cmin = 0.216$$

$$\ell e = \ell o\ (0.851+0.05x0.21)$$

$$= 0\text{-}8608\ell o = 0.8608x4500 = 3873mm$$

$\frac{3873}{300} = 12.91 > 12$ slender column.

$\ell e = 3878$ mm.

Column Reactions for Limit state of Collapse and Limit state of Serviceability

| | Load Case | Member No. | Support Reactions kN +X | Support Reactions kN +Z | End Moment kNm Left | End Moment kNm Right |
|---|---|---|---|---|---|---|
| Limit State Collapse (1.4Gk+1.6Qk) | CASE 1 | 2.1 | 0.55 | 595.17 | 2.74 | 0.0 |
| | | 4.3 | 10.59 | 1106.85 | 5.29 | 0.0 |
| | CASE 2 | 2.1 | 15.98 | 818.36 | 79.94 | 0.0 |
| | | 4.3 | 18.88 | 1040.32 | 44.12 | 0.0 |
| | CASE 3 | 2.1 | 10.44 | 780.27 | 52.22 | 0.0 |
| | | 4.3 | 1.18 | 1349.35 | 9.04 | 0.0 |
| Limit State Serviceability 1.2(Gk+Qk+Wt) | CASE 4 | 8.7 | -19.51 | 635.48 | -97.58 | 0.0 |
| | | 4.3 | -13.86 | 846.60 | -69.33 | 0.0 |
| | CASE 5 | 8.7 | -0.439 | 892.63 | -44.19 | 0.0 |
| | | 4.3 | -8.83 | 481.11 | -2.19 | 0.0 |
| | CASE 6 | 8.7 | -6.51 | 1060.35 | -32.55 | 0.0 |
| | | 4.3 | -9.97 | 485.57 | -49.84 | 0.0 |

External column

Max total axial load = N = 1060 kN (CASE 6)

Moment into column: Mmax = -97.58 kNm

a) Bending moment

Nominal eccentricity B.M. = $0.05\ h_{min}\ N = 0.05 \times 3.00 \times 1060 = 15.9$ kNm.

$15.9 < 97.58$

$M_1 = 0 \qquad M_2 = 97.58$

$M_i = 0.4M_1 + 0.6\ M_2 = 0.6 \times 97.58 = 58.54$

$$M_t = Mi + \frac{Nh}{1750} \left(\frac{\ell e}{h}\right)^2 \left(1 - 0.0035 \frac{\ell e}{h}\right)$$

$$M_t = 58.54 + \frac{1060 \times 300}{1750} \quad (12.91)^2 \; (1 - 0.0035 \times (12.91) \times 10^{-3}$$

$$M_t = 58.54 + 29.07 \quad = 87.61 \text{ kNm}$$

$$M_t = 87.61 \text{ KNm}$$

Reinforcement

$$\frac{d}{h} = \frac{240}{300} = 0.8$$

$fcu = 25.0$ N/mm$^2$ $\qquad \frac{N}{bh} = \frac{1060 \times 10^3}{300 \times 300} = 11.77$ N/mm$^2$

$fy = 250.0$ N/mm$^2$ $\qquad \frac{M}{bh^2} = \frac{87.61 \times 10^6}{300^3} = 3.24$

Adopt $100 \frac{Asc}{bh} = 4.75$ $\qquad Asc = 4.75 \times 300^2 \doteq 4275$ mm$^2$

Provided 6Y32 $\qquad Asc = 4825$ mm$^2$; $\qquad Asc_{/2} = 2412$ mm

$$K = \frac{Nuz - N}{Nuz - Nbal}$$

$$Nuz = 0.45 \text{ fcu Ac} + 0.75 \text{ fy As}'$$

$$= (0.45 \times 25 \times 300 \times 300 + 0.75 \times 250 \times 2412)/300 \times 300$$

$$Nuz = 11.25 + 5.02$$

$= 16.27$ N/mm$^2$ $\qquad$ N Balance $= 7.5$ N/mm$^2$

$$\frac{N}{bh} = 11.77$$

$$K = \frac{16.27 - 11.77}{16.27 - 7.5} = 0.51 < 1.0$$

Design of internal column:

$N_{max}$ = 1349.0 kN

$M_{max}$ = 69.33 KNm.

$\frac{d}{h} = \frac{240}{300} = 0.8$

$\frac{N}{bh} = \frac{1349.0 \times 10^3}{300^2} = 14.98$

$\frac{M}{bh^2} = \frac{63.33 \times 10^6}{300^3} = 2.56$ N/mm²

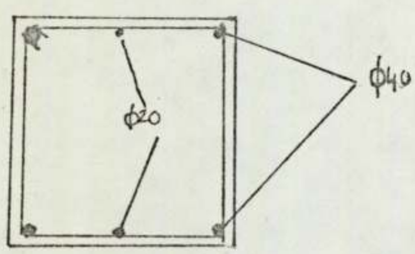
$f_{cu}$ = 2 5 N/mm²
fy = 250 } $\frac{100\ Asc}{bh}$ = 5.70

Asc = 5130 mm²

4Ø40 = 5026

2Ø20 = 628

Asc = 5.654



# C H A P T E R 4.

## OPTIMUM DESIGN OF REINFORCED CONCRETE

## 4.1. INTRODUCTION

Optimum limit design (OLD) methods aim at structural solutions that minimize or maximize a chosen merit function (the optimality criterion) and satisfy the criteria of limit equilibrium, compatability, and serviceability. The method is presented here by first deriving solutions that comply with equilibrium serviceability and optimal criteria and subsequently verifying the satisfaction of the compatibility requirements.

It has been shown that it is possible to formulate the limit design problem so that the solution can be truly optimal in a mathematical sense i.e., the solution minimises the chosen merit function which relates to the cost of the structure or the amount of material used.

It is now possible theoretically to formulate problems that satisfy simultaneously the condition of limit equilibrium serviceability and rotation compatability, along with an optimality criterion and elastic continuity conditions. However practical design applications appear simpler when limit equilibrium and serviceability conditions are only considered initially. Recent investigations already show that the economy of such solutions compares favourably with that of truly optimal design. In this chapter, optimum design solutions of reinforced concrete frameworks by M. Z. COHN etal (1968) are described and examples are also given.

## 4.2. ASSUMPTIONS.

a) Reinforced concrete can be idealized as an elastic-plastic material with limited ductility, fig. (4.1.)

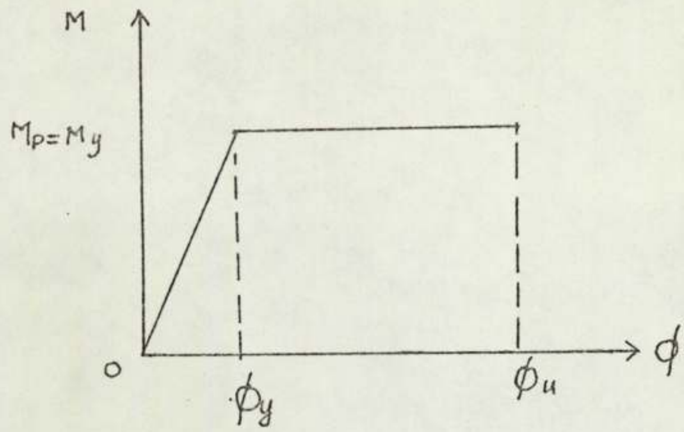



fig 4.1

b) Members resist forces by bending. Axial and shear forces as well as instability phenomena are of no concern.

c) Live loads may have any possible location, so that the worst combination could be taken into account for a particular section.

d) Each critical section may be the first to yield for a particular arrangement of the live loads.

e) Dead and live loads vary proportionally between first yield and collapse for any loading arrangement. fig.(4.2)

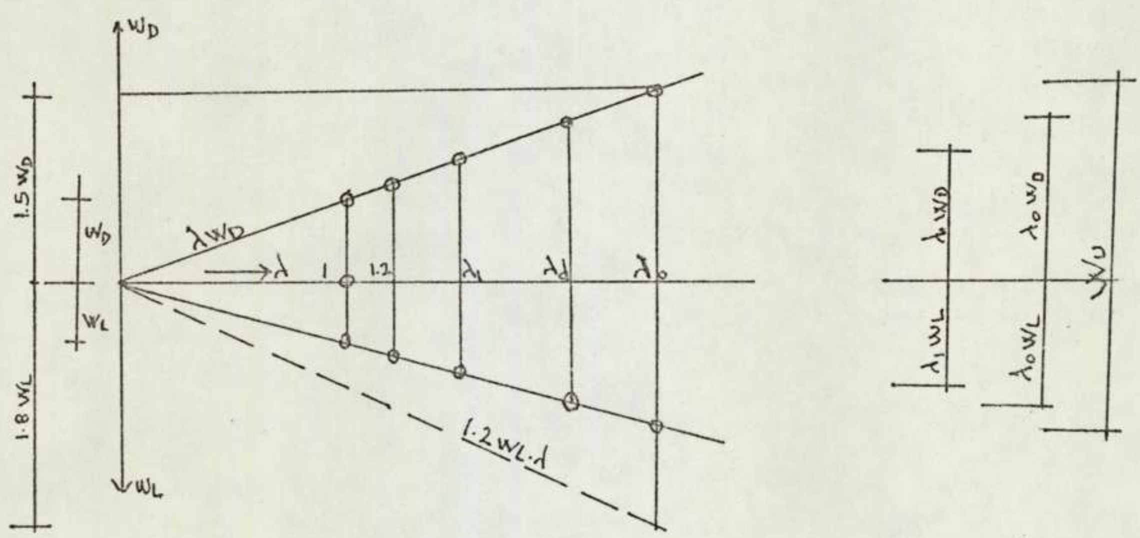


fig 4.2.

## 4.3. BASIC CONCEPTS OF OPTIMUM LIMIT DESIGN.

The complete design of a reinforced concrete structure involves the derivation of the plastic moment for all its critical sections. A critical section is located at each support and in each span. The total number of critical sections is s = m + n = N + n; where m is the number of the elementary mechanisms which is equal to the number of spans N. The number of the redundancies is n. The problem is to derive a distribution of plastic moments that fulfils simultanously the optimum criteria accepted. The limit criteria is expressed with m = N available. Limit equilibrium conditions and n additional serviceability criteria equations. The number of the equations to be determined (s) can be expressed in terms of N and the number of fixed ends (f), $s = 2N + f - 1$, for symetrical beams $s = N + f/2$.

### 4.3.1. GENERAL EXPRESSION OF PLASTIC MOMENTS.

If we consider bending moments of any critical section (j) under a particular arrangement of load causing the first yield of section j,

a) In the working range the elastic moment is

$$M_j = a_j GL + b_j PL \qquad (1)$$

b) At first yield section j, only the live load increasing from P to $\lambda_{1j}$ P

$$Mp_j = a_j GL + b_j \lambda_{ij} PL \qquad (2)$$

c) At the collapse of the span the ultimate live load being $\lambda o P$

$$\overline{Mp}_j = a_j GL + b_j \lambda o PL \qquad (3)$$

which

G and P = dead and live load respectively

L = length of span

$a_j, b_j$ = absolute values of constants defining the max elastic moment at section j under action of G and P respectively.

$\lambda ij$ = yield load factor.

$\lambda o$ = Ultimate load factor.

Since according to assumption (a) the plastic moment is invariable from first yield to collapse, from equation (2) and (3) it follows that:

$$b_j \lambda_{ij} = \bar{b}_j \lambda o \quad \text{or} \quad b_j = \bar{b}_j \frac{\lambda ij}{\lambda o} = b_j x_j \qquad (4)$$

$x_j$ being called yield safety parameter. Therefore the plastic moment at a critical section can be written as

$$Mp_j = \bar{X}_j = a_j GL + b_j x_j \lambda_o WL \qquad (5)$$

It can also be defined as

$$Mp_j = x_j \lambda o M_{p_j} \qquad (6)$$

where $M_{p_j}$ is the elastic envelope moment due to service load for section j.

4.3. OPTIMUM CRITERIA AND OLD PROCEDURES FOR CONTINOUS BEAMS.

The best limit criterion for continous beams is to ensure the same ultimate safety for all possible elementary mechanisms. Ideally the best serviceability criterion is that all critical sections of the beam are provided with the same yield safety.

Limit equilibrium condition.

In this condition no collapse mechanism may form prior to the specified ultimate load for the structure. For a structure with the possible modes of collapse and r applied loads, this condition may be expressed as:

$$\sum_j x_j \lambda_o M_j \Theta_{ij} \geq \sum_q \lambda_o \delta_{iq} W_q \quad (i = 1,2.....p; q = 1,2....r) \qquad (7)$$

where, $\Theta ij$ is the inelastic rotation of critical section j in the mechanism j in the direction of the collapse load $\lambda_o Wq$.

Serviceability.

Serviceability conditions require that, plasticity will not occur and deflections and crack widths will remain within allowable limitation. These requirements are satisfied if the yield load factor for each critical section is less than a specified lower limit $\lambda 1$

i.e. $\lambda ij > \lambda_1$ Therefore as $x_j = \frac{\lambda ij}{\lambda o}$ and this condition becomes

$$x_j \geq \frac{\lambda 1}{\lambda o} \qquad (j = 1,2 \ldots\ldots s) \qquad (8)$$

Assumption (e) places an upper limit of $\lambda o$ on the yield load factor for each critical section j or:

$$x_j \leq 1.0 \qquad (j = 1,2 \ldots\ldots s) \qquad (9)$$

FIRST OPTIMUM PROCEDURE (full redistribution design)

This particular approach to the equilibrium method yields designs for which all possible mechanisms may form at the specified ultimate load. The object of the equilibrium method is to find the set of $x_j$ values for the s critical sections of the structure that satisfy, equations (7) , (8) , and (9). To do this, s independant design conditions must be specified. The ultimate safety is the same for all mechanisms, the corresponding limit equilibrium equation having the general form:

$$\sum_i \gamma ik \, \bar{x}j = \phi_k \, (GL + \lambda oPL) \qquad (10)$$

where $\gamma ik$ is a parameter defining the location and contribution of the plastic hinge j of mechanism k, and $\phi k$ is a dimensionless parameter defining the max. free bending moment of the span corresponding to mechanism k. Substitution of $\bar{x}_j$ values from equation (5) result in

$$\sum_j (a_j \gamma ik \, GL + b_j x_j \; \gamma ik \lambda oPL) = \phi k(GL + \lambda oPL) \qquad (11)$$

Dead loads and live loads may be considered separately in any equilibrium relationship and considering the live loads only

$$\sum a_j \; \gamma ik \, GL = \phi_k GL \qquad (12)$$

and equation (11) simplifies finally to:

$$\sum b_j x_j \; \gamma ik = \phi k \qquad (13)$$

which is the basic limit equilibrium equation for $k^{th}$ mechanism. The limit equilibrium condition, equation 7 becomes:

$$\sum_j x_j \lambda_o M_j \theta_{ij} = \sum_q \lambda_o W_q \delta_{iq} \quad (j = 1,2 \ldots.n; q = 1,2 \ldots r) \quad (14)$$

Two full redistribution (FRD) approaches may be formulated depending on the serviceability criteria adopted to establish the remaining n = s - m conditions which are necessary for solution.

FRD - Equal Minimum Yield Safety for Support Critical Sections

This criterion results in designs for which all support critical sections have equal and minimum $x_j$. To determine this minimum value set the $x_j$ values equal in each of the equations (14) in turn. Take the minimum $x_j$ value thus obtained from the equations:

$$x_j = \min \left[ \frac{\phi_k}{\Sigma\, \gamma_{ikbj}} \right] \quad (15a)$$

or

$$x_j = \min \frac{\Sigma \lambda_o W_q \delta_{iq}}{\Sigma \lambda_o M_j \theta_{ij}} \quad (j = 1,2 \ldots.m) \quad (15b)$$

FRD - Equal Minimum Design Plastic Moment for Support Critical Sections

As $M_{pj} = x_j \lambda_o M_j$ = constant is provided for all support critical sections, the minimum $M_{pj}$ is to be found. To determine this value assign the minimum $x_j$ value permitted by serviceability to the support section having the largest elastic envelope moment $M_j$ max; i.e. for $x_j = \frac{\lambda_1}{\lambda_o}$

and $M_{pj} = x_j \lambda_o M_j$ then:

$$M_{pj} = \lambda_1 M_{j_{max}} \quad (16)$$

By assigning $M_{pj}$ values from expression (16) to support critical sections n additional conditions are provided. The resulting $x_j$ values are

$$x_j = \left(\frac{\lambda_1}{\lambda_o}\right) M_{j_{max}} / M_j \quad (17)$$

The following steps are involved in practical derivation of the design plastic moments based on the first OLD procedure.

a) Derive $a_j$, $b_j$ for all spans and support critical sections:

b) Select the support yield safety parameter $\bar{x}_o$ from tables according to the specific data of the problem.

c) Select a convenient span yield safety parameter.

d) Apply equation (7) to find required plastic moment for all critical sections.

SECOND OPTIMUM PROCEDURE (limited redistribution design)

In this procedure equal yield safety for all critical sections of the beam is accepted as the serviceability criterion.

$$x_j = x_o = \text{constant} \qquad (18)$$

Accordingly the limit equilibrium equation (13) becomes a set of inequalities of the form:

$$\sum_j b_j x_j \delta_{ik} \geq \phi \qquad (19)$$

Obviously the solution of the set (equation 19) verifying the serviceability conditions (Equation 20) is given by

$$x_o = \max \bar{x}_k = \max \left[ \frac{\phi_k}{\sum_j \delta_{ik} b_j} \right] \qquad (20)$$

Criterion (20) must be considered only ideally, since to have better service conditions which will be preferable to provide larger $x_j$ values for span than for support critical sections. It will be seen, however, that providing $x_j$ (Equ.18) separately for span and support sections along with limit condition (Equ.19) yields a most simple and efficient solution for design of reinforced concrete beams with equal spans.

OPTIMUM LIMIT DESIGN (OLD)

This particular approach to the equilibrium method results in designs for which the material consumption is minimized, subject to the limit equilibrium and serviceability constraints. A suitable economic criterion for design of reinforced concrete continous beams is to minimize the volume of flexural reinforcement. This criterion implies that the concrete cross-section is constant and that the longitudinal reinforcement alone is varied to provide the required moment capacities. Under this condition a linear objective function can be formulated as

$$Vk = \sum_{j} Mpj\,\ell j = \sum_{j} xj\,\lambda_o\,Mj\,\ell j \quad (j = 1,2 \ldots .s) \quad (21)$$

where Vk is the total volume of longitudinal reinforcement for the structure, $\ell j$ is the equivalent length over which the flexural reinforcement for section j prevails constant and subscript k refers to the particular design considered.

Relations (7), (8),(9) and (21) enable the (OLD) problem to be stated as follows:

Minimize:

$$Vk = \sum_{j} xj\,\lambda oMj\ell j \qquad (j = 1,2 \ldots .s) \quad (21a)$$

subject to

$$\sum_{j} xj\lambda_o Mj\,\theta ij \geq \sum_{q} \lambda_o Wq\,\delta iq \; (j = 1,2 \ldots .s; q = 1,2 \ldots .r) \quad (21b$$

and

$$\frac{\lambda 1}{\lambda o} \leq xj \leq 1.0 \qquad (j = 1,2 \ldots .s) \; (21c)$$

OLD - Equal yield safety for support critical sections

Because $xj = \lambda ij/\lambda o$, this criterion implies that equal $xj$ is to be provided for all support sections. This can be achieved by adding the following constraints to the general (OLD) formulation:

$$xj = xj + 2 \qquad (22)$$

where now subscript j refers to all support critical section.

OLD - Equal design, plastic moment for support critical sections

The criterion $Mpj = x_j \lambda_o M_j$ = constant for all critical sections can be satisfied by adding the following constraints to the general OLD formulation:

$$x_j \lambda_o M_j = X_{j+2} \left( \lambda_o M_{j+2} \right) \qquad (23)$$

Efficency of limit design solutions

A measure of the relative economy of a limit design and the elastic design is the efficency index $\vartheta_k$ which is defined as the ratio of the volume of the flexural reinforcement resulting for limit design and elastic design from equation (21)

$$\vartheta_k = \frac{Vk}{VE} = \frac{\sum x_j \lambda_o M_j l_j}{\sum \lambda_o M_j l_j} \qquad (j = 1,2 \ldots s) \qquad (25)$$

where Vk is the steel volume for the limit design by the approach k and $V_E$ is the steel volume for the elastic case.

## 4.5. DESIGN OF FRAMES

Reinforced concrete frame design for maximum economy depends on an initially selected elastic design. Against the plastic collapse of the structure and for the first yield of the section the same load factors are used. It is assumed that in bending action inelastic rotations are concentrated at critical sections as in simple plastic theory. Furthermore inelastic rotations remain within permissible limits to take the advantage of the inelastic strength to simplify the optimum design for concrete frames.

General Design

A design solution for the frame is found when a set of Ij, + Mpj, - Mpj is assigned to all critical sections of the frame such that the following conditions are satisfied.

A) LIMIT EQUILIBRIUM

The frame will resist any load combination of less intensity than the prescribed ultimate load Wu and will collapse plastically for a load $+W \geq W_u$

B) SERVICEABILITY

The critical sections of the frame will remain well within the elastic range for any combination of the service loads and hence will have a safety factor against first yield above a specified value .

C) OPTIMUM

The design will result in the largest possible overall moment reduction versus an initially selected elastic design.

A design solution is feasible when it is safe and serviceable and it is optimal when it is safe, serviceable and economical at the same time. The plastic moment for critical section j can be shown to be

$$Mpj = xj \lambda oMj = xjbjWuL \qquad (25)$$

The value of the yield safety parameter $xj$ implies better serviceability when it is large, and small $xj$ values correspond to better economy. Optimal solutions will place the design plastic moments and $xj$ values at levels consistent with both requirements.

The serviceability condition requiring that no plastic hinges form at working load implies $\lambda ij > 1$ or $xj > \frac{1}{\lambda}$ for any j = 1,2 ........s. Moreover, for the yield load factor or any critical section to be no less than the specified minimum value $\lambda_1$ . It is necessary that $\lambda ij > \lambda_1$ or $xj \geq \frac{\lambda_1}{\lambda_0}$ . Also because a structure can not collapse prior to yield at its critical sections $\lambda o > \lambda ij$ or $xj < 1$. Hence $xj$ is bounded from above and below

$$\frac{\lambda 1}{\lambda 0} \leq xj \leq 1 \qquad (26)$$

A limit equilibrium condition is associated with each possible mode of collapse of the frame. Energy dissipated by the plastic hinges (Ui) will be bigger than external work (Ei) done by the ultimate load corresponding to the mode of collapse

$$U_i > E_i \tag{27}$$

Let, angular displacement ($\theta j$) = $Mij\theta$, Linear displacement ($\delta ij$) = $\vartheta ij\theta \ell$ and loads doing active work in the ith mechanism $Wj = \eta jW$, where

$\theta_{ij}$ = an arbitrary rotation.

M, $\vartheta ij$ and nij are constants.

$U_i = \sum_j Mpj\,\theta j = \sum_j xjbj\lambda oWL\mu ij\theta$ and $E_i = \sum_j \lambda_o Wj\,\delta ij =$

$\sum_j \lambda_o \eta jW\ \vartheta ij\,\theta L$

so that after substitution and simplifications, Equation (27) becomes:

$$\sum_j aij\,x_j \geq c_i \ .......... \tag{28}$$

In which $aij = bj\ \mu ij$ and $c_i = \sum_j \eta j\,\vartheta ij$

The optimum criterion requires the over-all moment reductions over the elastic design to be maximum, i.e., the area between the elastic moment envelope and the design solution to be as large as possible. If $Y_1$ is this area it is required that:

$$Y_1 = \int_s (\lambda_o M_j - M_{Pj})ds = \lambda o \int_s M_j(1 - x_j)ds = \lambda_o WL \int_s b_j(1 - x_j)ds = Ma \tag{29}$$

or alternatively if $Y_2$ is the design plastic moment area.

$$Y_2 = \int_s M_{Pj}\,ds = \lambda o \int_s M_j x_j ds = \lambda_o WL \int_s b_j x_j ds = Min \tag{30}$$

in both expressions the integrals are extended over the whole frame.

If we summarize the optimal design problem within the accepted assumption and limitations consist of determining the $x_j$ values for s critical sections of the frame to satisfy the optimum criterion (Equ. 29 - 30), the limit equilibrium conditions (Equ. 28) and serviceability conditions (Equ. 26) with $q_{ij} > 0$ and $c_j > 0$.

This is a typical linear and non-linear programming problem depending on the nature of function $Y_1$ and $Y_2$. The general problem is complex because the merit function $Y_1(x_j)$ or $Y_2(x_j)$ must be expressed analytically.

Feasible solutions can be obtained by using the kinematic theorem of plastic analysis which states that the collapse mode corresponds to the smallest kinematically admissible multiplier (Equ. 26). Application of the theorem to equation (28) indicates that the actual collapse corresponds to the particular mode satisfying the condition

$$\text{Min} \sum_j \frac{x_j q_{ij}}{C_i} = 1 \qquad (31)$$

While the number p of the limit equilibrium conditions Equation (28) is usually very large, only a limited number of collapse modes are critical, in the sense that many modes can not occur for any combination of design values $x_j$ or $\sum_j x_j q_{ij}/C_i > 1$.

Frames with equal yield safety:

A particular but important solution can be obtained by assuming that $x_j = \bar{x}$ = Constant for all critical sections. In this case as $x_j = \frac{Mpj}{\lambda o Mj}$ = Constant a design is achieved in which the elastic moment envelope for the ultimate load is reduced by the same amount for all critical sections of the frame. The solution for this particular case follows from Equation (31)

$$\bar{x} = \text{Max } \bar{x}_i = \text{Max} \frac{Ci}{\sum_j q_{ij}} \qquad (32)$$

The design value (Equ. 32) is obtained by equalizing all $x_j$ (Equ.28) in turn to get $x_j = Ci/\sum_j q_{ij}$ and then by selecting the largest $\bar{x}_i$ thus obtained overall possible mechanism. Obviously this corresponds to a safe upper bound Mpj values because $U_i > E$ in all but the critical mechanism associated with Equation (32).

A more efficient solution may be found by trying to obtain $x_j < \overline{X}$ for critical sections not involved in the critical mechanism. Substitute wherever appropriate the value of $\overline{X}$ in the limit equilibrium equations and find again the largest $\overline{X}$ obtained by equalizing the yield safety parameters for sections not entering in the critical collapse mode. The process can be continued until each critical section of the frame enters at least once into the limit equilibrium equation.

Partially Elastic Design:

The design problem can be considerably simplified in cases in which $x_j$ values are specified for various classes of sections. If we assume same yield safety parameter at beam support sections and $Z_1$ = the corresponding yield safety parameter. Similarly the span sections have the same yield safety parameter and column sections are designed for the same yield safety parameter $Z_3$.

It was shown by M. Z. COHN (1968) that, the problem can be reduced to a bidimensional one for the particular conditions.

1) Elastic span sections $Z_1 = 1$. No redistribution of moments permitted for span sections.
2) Elastic column section $Z_2 = 1$. No redistribution of moments permitted for span sections.
3) Elastic column section $Z_3 = 1$. No redistribution is allowed for the column moment (similarly to the strong column-weak beam design in steel).

Design solutions based on the serviceability methods should be checked for the satisfaction of rotation compatability. This condition is satisfied and the design is correct if the inelastic rotations $\theta_{ij}$ at any critical section j under any loading conditions (collapse model) does not exceed the rotation capacity $\theta_{pj}$ of the section.

$\theta_{ij} \leq \theta_{pj}$

Determination of $\theta_{pj}$ is object of extensive experimental research and determination of $\theta_{ij}$ can be done by A.L.L. Baker approaches.

Example 1) A five-span reinforced concrete continuous beam with equal spans under the action of uniformly distributed load with reference to fig. (4.3.):

$s = 9$, $n = 4$, $m = s - n = 5$, i.e., number of span.

Because of symetry s reduces to 5 and n to 2 and thus m = 3.

1) ELASTIC BEHAVIOUR.

$$M = a_j GL^2 + b_j \lambda PL^2$$

Values of $a_j$ and $b_j$ can be found from handbooks or by calculation (fig. 3: ii - vii)

The uniform dead load = $W_D$

The uniform live load = $\lambda W_L$

and $V = G/P = W_D/W_L$

| Critical Sections | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $a_j$ fig.(4.3) | 0.078 (vi) | 0.105 (vi) | 0.035 (vi) | 0.079 (vi) | 0.046 (vi) |
| $b_j$ fig.(4.3) | 0.098 (ii) | 0.120 (iv) | 0.079 (iii) | 0.111 (v) | 0.079 (v) |

NOTE: a) for the live loads it is assumed that the critical section occurs at midspan for convenience finding values for $\gamma$

b) The actual load distribution for the first hinge to form at any critical section depends on the $W_D/W_L$ ratio, which is usually known, and also the moment of resistance of the

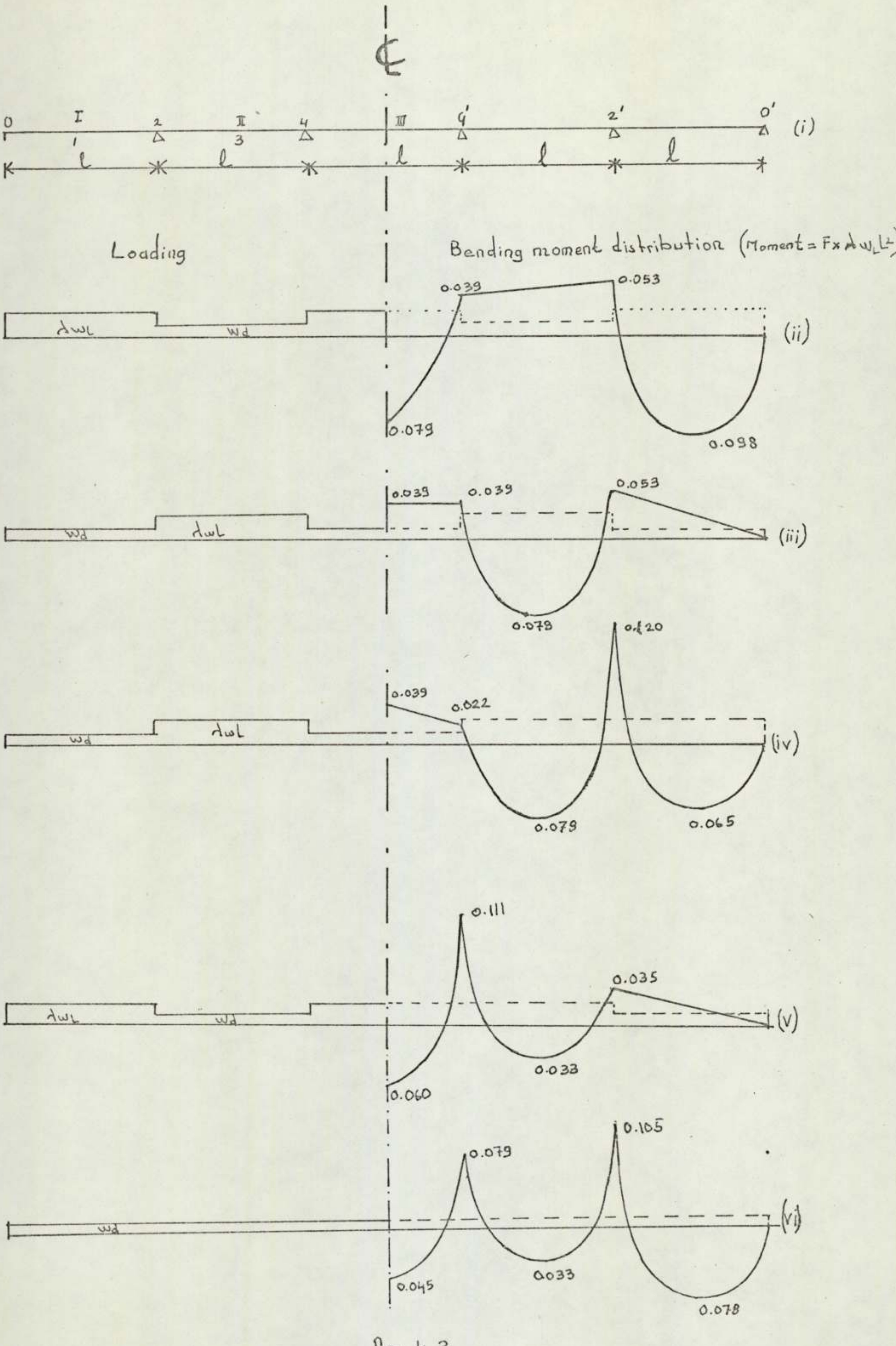

℄
0
I
1
2
II
3
4
III
4'
2'
0'
(i)
l
Loading
Bending moment distribution (Moment = F x λw_L l²)
λw_L
w_d
0.039
0.053
0.079
0.098
(ii)
0.039
0.039
0.053
0.079
(iii)
0.120
0.039
0.022
0.079
0.065
(iv)
0.111
0.035
0.060
0.033
(v)
0.105
0.079
0.045
0.033
0.078
(vi)

fig 4.3

various sections which are not known.

The plastic moments are (as per eqn. 5)

$$-\bar{X}_1 = (0.078\ W_D + 0.098\, x_1 \lambda_o W_L)\ell$$

$$\bar{X}_2 = (0.105\ W_D + 0.120\, x_2 \lambda_o W_L)\ell$$

$$-\bar{X}_3 = (0.033\ W_D + 0.079\, x_3 \lambda_o W_L)\ell$$

$$\bar{X}_4 = (0.079\ W_D + 0.111\, x_4 \lambda_o W_L)\ell$$

$$-\bar{X}_5 = (0.046\ W_D + 0.079\, x_5 \lambda_o W_L)\ell$$

COLLAPSE CONDITION:

Consider the equilibrium of each elementary mechanism, it being assumed that each mechanism has the same collapse bound applying the principle of virtual work to small motion of the appropriate mechanism.

I $\quad 2\bar{X}_1 + \bar{X}_2 = \frac{1}{4}(W_D + \lambda_o W_L)L$

II $\quad \bar{X}_2 + 2\bar{X}_3 + \bar{X}_4 = \frac{1}{4}(W_D + \lambda_o W_L)L$

III $\quad 2\bar{X}_4 + 2\bar{X}_5 = \frac{1}{4}(W_D + \lambda_o W_L)L$

and only considering the contribution from the live loads, the work equation will be

(note: in this work equation all the terms have positive sign)

I $\quad 2 \times 0.098\bar{x}_1 + 0.120\bar{x}_2 = 0.25$

II $\quad 0.120\bar{x}_2 + 2 \times 0.079\bar{x}_3 + 0.111\bar{x}_4 = 0.25$

III $\quad 2 \times 0.111\bar{x}_4 + 2 \times 0.079\bar{x}_5 = 0.25$

To find $\bar{x}_o$ consider the equalisation of all $x_i$ in each mechanism in turn

I $\quad \bar{x}_0^{I} = \frac{0.25}{0.316} = 0.791$

II $\quad \bar{x}_0^{II} = \frac{0.25}{0.389} = 0.643$

III $\quad \bar{x}_0^{III} = \frac{0.25}{0.380} = 0.658$

$\therefore$ The minimum value $\bar{x}_o = 0.643 = \bar{x}_2 = \bar{x}_3 = \bar{x}_4$. $n = 2$ therefore $n + 1 = 3$ and it so happens that for this particular case three equal values are obtained from mechanism II and the remaining s = (n+1) values for $x_j$ are obtained directly by substitution.

$$\bar{x}_1 = 0.885$$

$$\bar{x}_5 = 0.678$$

and $$\bar{x}_2 = \bar{x}_3 = \bar{x}_4 = 0.643$$

The values for $\bar{X}$ may be calculated using these values for $X_i$

$$-\bar{X}_1 = (0.078V + 0.098 \times 0.885\,\lambda o)W_L L$$

$$\bar{X}_2 = (0.105V + 0.120 \times 0.643\,\lambda o)W_L L$$

$$-\bar{X}_3 = (0{,}033V + 0.079 \times 0.643\,\lambda o)W_L L$$

$$\bar{X}_4 = (0.079V \div 0.111 \times 0.643\,\lambda o)W_L L$$

$$-\bar{X}_5 = (0.046V + 0.079 \times 0.678\,\lambda o)W_L L$$

Consider the case when at working loads the dead and live loads are equal and $\lambda_o = 2$:

$$-\bar{X}_1 = (0.078 + 0{,}0865 \times 2)W_L L = 0.251W_L L$$

$$\bar{X}_2 = (0.105 + 0.077 \times 2)W_L L = 0.259W_L L$$

$$-\bar{X}_3 = (0.033 + 0.051 \times 2)W_L L = 0.135W_L L$$

$$\bar{X}_4 = (0.079 + 0.071 \times 2)W_L L = 0.221W_L L$$

$$-\bar{X}_5 = (0.046 + 0.0535 \times 2)W_L L = 0.153W_L L$$

and the bending moment can be plotted (fig 4.4)

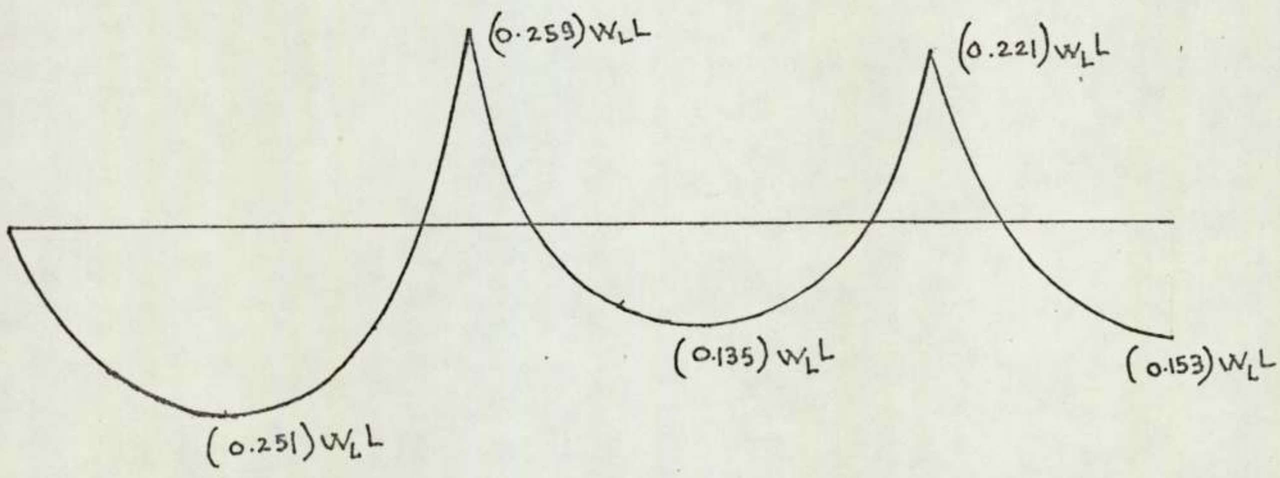



fig (4.4)

Note: For $\lambda_o = 2$ and $x_i = \frac{\lambda_{ij}}{\lambda_o}$, the lowest value for $\lambda_{1i}$ = 2 x 0.643 = 1.286 which is greater than 1 and the working load behaviour is always elastic. If a value of $\lambda < 1.56$ (i.e. $\frac{1}{0.643}$) is used the working load criteria will not be satisfied, which means that the total increase in load from working load, i.e., $2W_L$, to collapse load, which equals $2.56W_L$ or less, is too small to allow both the collapse mechanism to form and the working load conditions to be satisfied; to design a beam for these conditions would be an uneconomic or non-optimum solution.

USING SECOND OPTIMUM APPROACH

Substituting $x_j = \frac{\lambda_{ij}}{\lambda_o}$ in Eqn.19

$$\sum \gamma_{ij}\, b_j\, \lambda_{ij} > \phi_k \lambda_o \qquad \text{(a)}$$

Mechanism I $\quad 2 \times 0.098\lambda_{I1} + 0.120\lambda_{I2} \geq \frac{1}{4}\lambda_u$

Mechanism II $\quad 0.120\lambda_{I2} + 2 \times 0.079\lambda_{I3} + 0.111\lambda_{I4} \geq \frac{1}{4}\lambda_u$

Mechanism III $\quad 2 \times 0.111\lambda_{I4}$

$$\lambda_{I1} = \lambda_{I3} = \lambda_{I5} \quad \text{and} \quad \lambda_{I2} = \lambda_{I4} \quad \text{and} \quad \lambda_{I2} \leq \lambda_{I1}$$

for each mechanism Eqn (a)

$$\gamma_o^1 b_o \lambda_{10} + \gamma_1^1 b_1 \lambda_{11} + \gamma_2^1 b_2 \lambda_{12} \geq \phi^1 \lambda_o$$

$$\gamma_2^2 b_2 \lambda_{12} + \gamma_3^2 b_3 \lambda_{13} + \gamma_4^2 b_4 \lambda_{14} > \phi^2 \lambda_o$$

---------------------

---------------------

$$\gamma_{2j}^k b_{2j} \lambda_{12j} + \gamma_{2j+1}^k b_{2j+1} \lambda_{12j+1} + \lambda_{12j+1} + \gamma_{2j+2}^k b_{2j+2} \lambda_{12j+1} \geq \phi^k \lambda_o$$

Since $\lambda_{10} = \lambda_{12} = \lambda_{12j}$

and $\lambda_{11} = \lambda_{13} = \lambda_{12j+1}$ a safe solution is given by:

$$\lambda_{10} = \text{MAX} \left[ \frac{\phi^k \lambda_u - \gamma_{2j+1}^k b_{2j+1} \lambda_1}{\gamma_{2j}^k b_{2j} + \gamma_{2j+2}^k b_{2j+2}} \right] \qquad \text{(b)}$$

reference to equation (b)

$$I: \quad \lambda_{12} \geqslant \frac{0.25\,\lambda o - 0.196\lambda_{I1}}{0.120}$$

$$II: \quad \lambda_{12} \geqslant \frac{0.25\,\lambda o - 0.158\lambda_{I1}}{0.120+0.111}$$

$$III: \quad \lambda_{12} \geqslant \frac{0.25\,\lambda o - 0.158\,\lambda_{I1}}{0.222}$$

$\lambda_u = 2$ and choosing a value for $\lambda_{I1} = 1.766$ (which will give a value for $\lambda_{I1}/\lambda o = 0.883$, which was the value for $x_i$ in the other approach).

I: $\lambda_{12} \geqslant 1.286$ or $\lambda_{12}/\lambda o \geqslant 0.643$ i.e. maximum

II: $\lambda_{12} \geqslant 0.956$ or $\lambda_{12}/\lambda_o \geqslant 0.478$

III: $\lambda_{12} \geqslant 0.996$ or $\lambda_{12}/\lambda o \geqslant 0.498$

$\therefore \lambda_{12} = \lambda_{14} = 1.286$

$\lambda_{11} = \lambda_{13} = \lambda_{15} = 1.766$

Applying Equation 5 ($ajGL + bj x_j \lambda oPL$) and $V = \frac{G}{P} = 1$

$$-\overline{X}_1 = (0.078V + 0.098 \times 1.766)W_L \times L = 0.251W_L \times L$$

$$\overline{X}_2 = (0.105V + 0.120 \times 1.286)W_L \times L = 0.259W_L \times L$$

$$-\overline{X}_3 = (0.033V + 0.079 \times 1.766)W_L \times L = 0.172W_L \times L$$

$$\overline{X}_4 = (0.079V + 0.111 \times 1.286)W_L \times L = 0.221W_L \times L$$

$$-\overline{X}_5 = (0.046V + 0.079 \times 1.766)W_L \times L = 0.185W_L \times L$$

the results are drawn on fig (4.5)

Example: Optimum design of 3 span frame which was shown on fig (4.5)

For simplicity it will be accepted that the load factor applies to the total load as in A.L.L. Baker example. The optimum design applies as the dead load would be negligible versus the live load. However, all the possible partial loadings will be considered.

The maximum (minimum) elastic moments are first derived

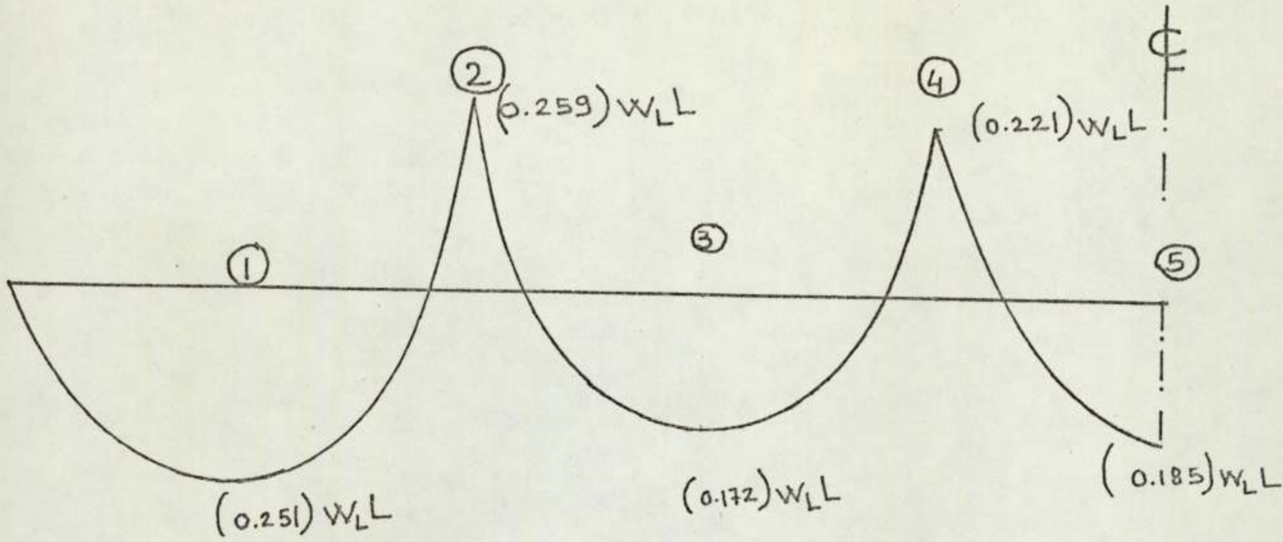



fig (4.4. a)
second optimum design

and the moment constants are tabulated on table (4.1) for five cases (fig 4.5. a,b,c,d,e) The elastic moment for the factored load are divided by $10^2$ kNm and from the loading combinations indicated in the parantheses the moments were found to be:

$M_1$ = -0.736 (b)
$M_2$ = 4.47 (b+e)
$M_3$ = 4.79 (d+e)
$M_4$ = 4.63 (d)
$M_5$ = 6.40 (c)
$M_6$ = 2.68 (d+e)
$M_7$ = 1.94 (d)
$M_8$ = 4.47 (b)
$M_9$ = 1.08 (b+e)
$M_{10}$ = 0.98 (b+e)
$M_{11}$ = 1.03 (c+e)

These are the envelope moments used in elastic design if selection are proportioned according to the ultimate strength theory and illustrated on fig (4.6)

| LOAD CASE | Moment /$10^2$ kNm. | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | AB | BC | | CB | CD | CE | | EC | EF | EG | | GE | GH |
| a | 0.486 | -0.486 | | 4.114 | 0.052 | -4.166 | | 4.166 | -0.052 | -4.114 | | 0.486 | -0.486 |
| b | 0.736 | -0.736 | | 1.738 | -0.593 | -1.145 | | 1.145 | 0.593 | -1.738 | | 0.736 | -0.736 |
| c | -0.249 | 0.249 | | 2.375 | 0.645 | -3.021 | | 3.021 | -0.645 | -2.375 | | -0.249 | 0.249 |
| d | 0.382 | -0.382 | | 4.545 | 0.085 | -4.630 | | 2.556 | -0.612 | -1.944 | | -0.124 | 0.124 |
| e | -0.355 | 0.355 | | 0.258 | -0.395 | 0.136 | | 0.136 | -0.394 | 0.258 | | 0.354 | -0.354 |

TABLE (4.1)

For limit equilibrium equations can be written, corresponding to the three beam mechanism and to the panel mechanism available. With $\gamma = 0.5$ (since span plastic hinged assumed at mid spans). Equation

$$\sum \gamma_{jk} x_j M_i = M_{ok} \quad \text{are:}$$

I $\quad 0.5 \times 0.735 x_1 + 4.47 x_2 + 0.5 \times 4.79 x_3 = 4.22$

II $\quad 0.5 \times 4.63 x_4 + 6.40 x_5 + 0.5 \times 2.68 x_6 = 6,40$

III $\quad 0.5 \times 1.94 x_7 + 4.47 x_8 + 0.5 \times 1.08 x_9 = 4.22$

IV $\quad -0.735 x_1 + 1.08 x_9 + 0.98 x_{10} + 1.03 x_{11} = 1.50$

Balancing in turns the unknown equation results in $x_{I} = 0.583$, $x_{II} = 0.631$, $x_{III} = 0.735$ and $x_{IV} = 0.636$. Substitution of minimum value $x_1 = x_3 = x_0 = 0.583$ result in

I $\quad x_2 = 0.585$

II $\quad x_5 = 0.667$

III $\quad x_8 = 0.747$

IV $\quad x_{10} = x_{11} = 0.646$

All $x$ values are smaller than 1 than design will be successful. The final plastic moments $x_i = x_j \bar{M}_j$ and

$\overline{X}_1 = 0.736 \times 0.583 = 0.429$

$\overline{X}_2 = 4.47 \times 0.585 = 2.627$

$\overline{X}_3 = 4.79 \times 0.583 = 2.792$

$\overline{X}_4 = 4.63 \times 0.583 = 2.699$

$\overline{X}_5 = 6.40 \times 0.667 = 4.268$

$\overline{X}_6 = 2.68 \times 0.583 = 1.562$

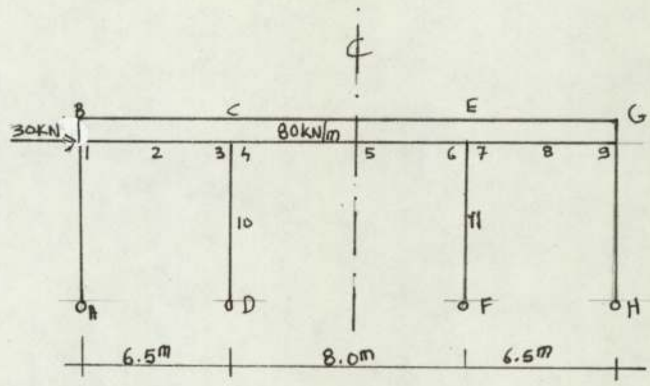
$\overline{X}_7 = 1.94 \times 0.583 = 1.131$

$\overline{X}_8 = 4.47 \times 0.747 = 3.339$

$\overline{X}_9 = 1.08 \times 0.583 = 0.629$

$\overline{X}_{10} = 0.98 \times 0.646 = 0.633$

$\overline{X}_{11} = 1.03 \times 0.646 = 0.665$




The final moments are also specified in fig.4.6. by dotted line.

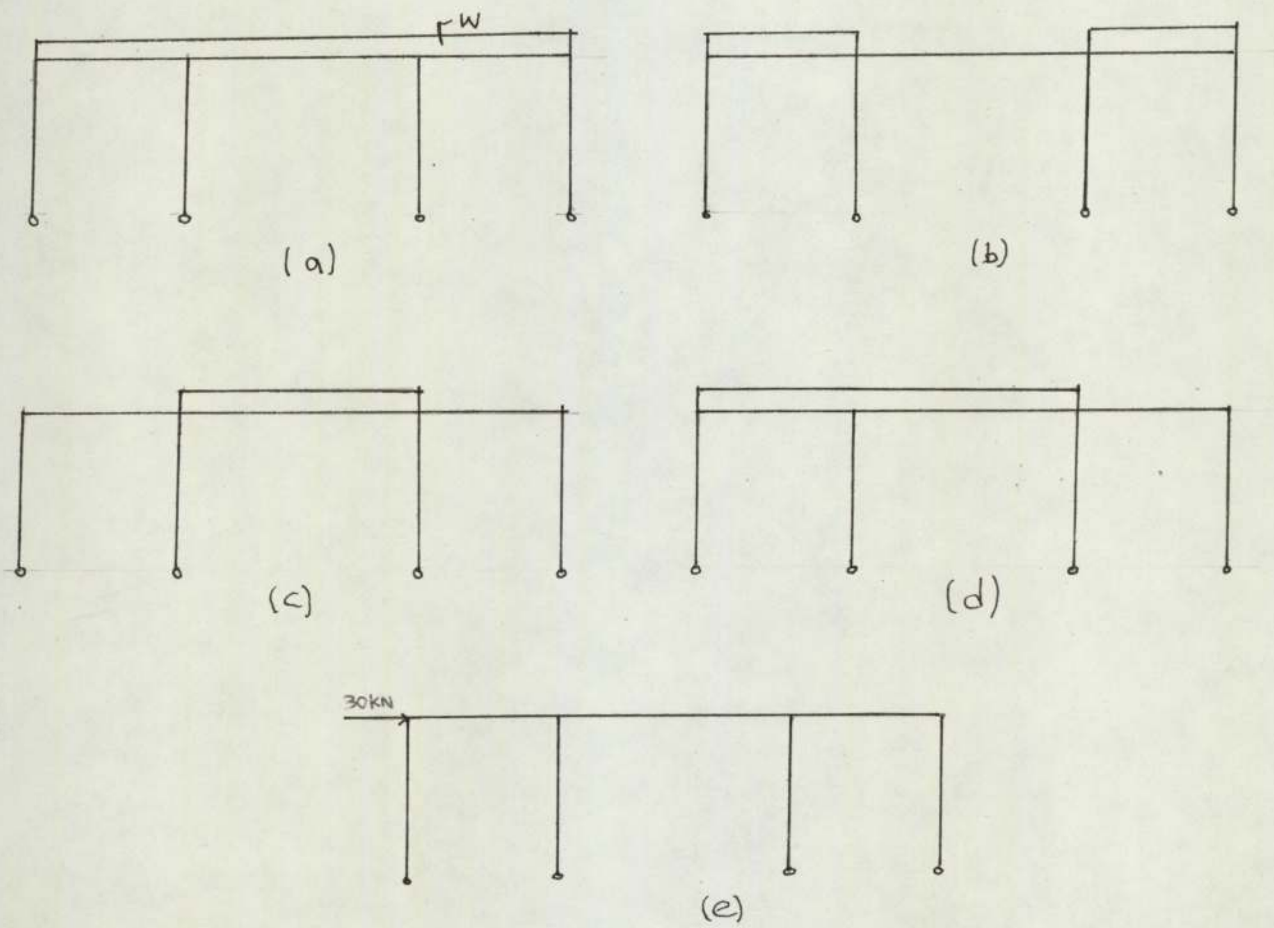



fig (4.5)

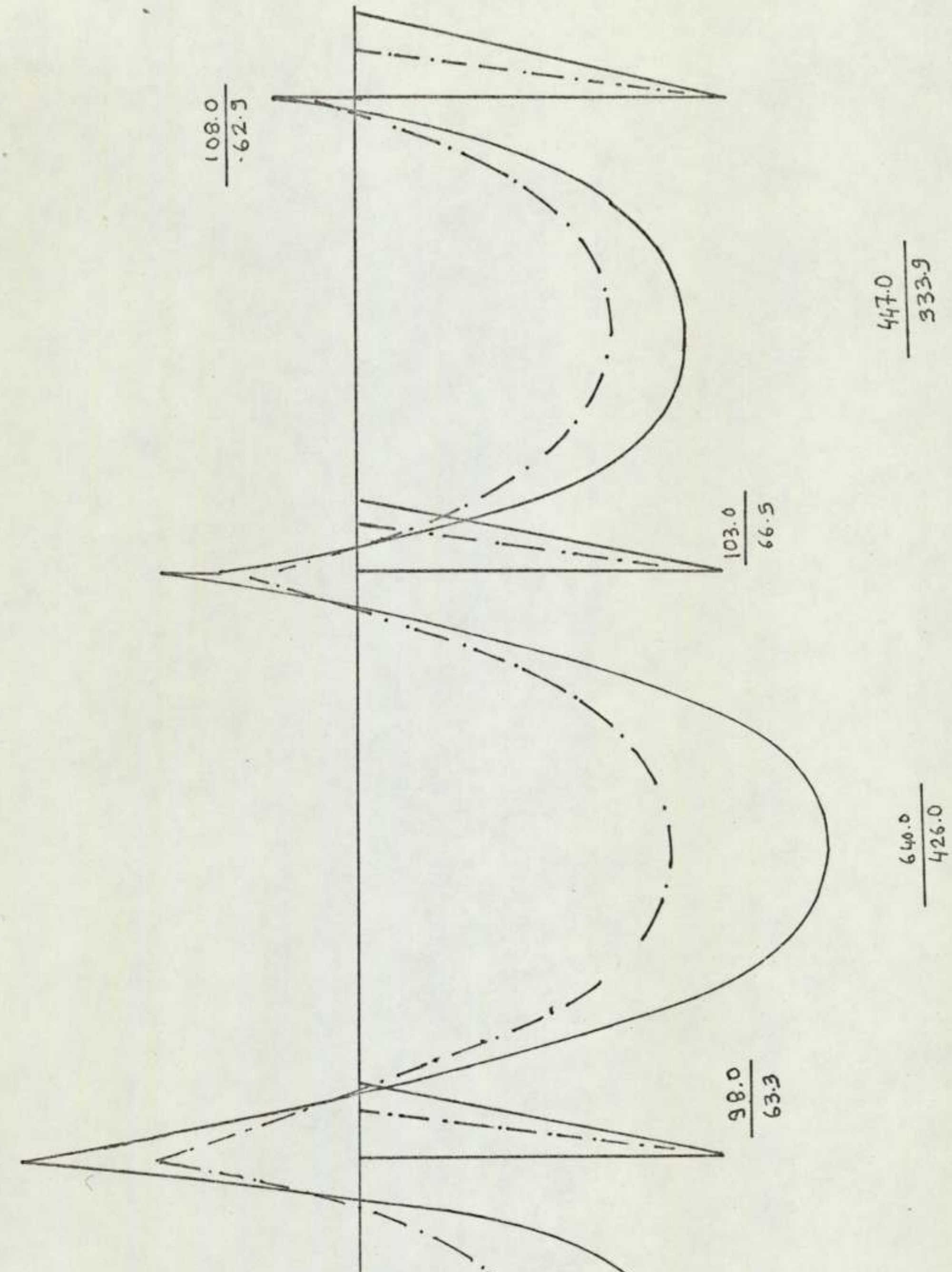

73.60
42.90
479.0 | 463.0
279.0 | 269.9
447.0
262.0
98.0
63.3
640.0
426.0
268.0 | 194.0
152.8 | 113.1
103.0
66.5
108.0
62.9
447.0
333.9

fig (4.6)

# C H A P T E R  5.

## INVESTIGATION OF THE EFFECTS OF VARIATION OF STRENGTH OF MATERIAL AND 30% RULE IN CP 110

INTRODUCTION

Redistribution of design bending moments in continous reinforced structures is widely recognized as a most useful tool in the hands of reinforced concrete designers. The arbitrary reduction of bending moments at supports, initially calculated using the elastic theory leads to a reduction in congestion of reinforcement at support sections. This in turn makes better compaction of the concrete possible and enables detailing of reinforcement to be simplified.

The code (CP 110) requires that the ultimate moment of resistance provided at any section shall be not less than 70% of the bending moment at that section taken from the elastic bending moment envelope. Reinforcement must therefore be checked at every place where the 30% redistributed bending moment envelope gives a lower value than 70% of the elastic bending moment envelope. This rule affects the curtailment of bars in some places. The difference between the redistributed bending moment and modified elastic bending moment is slight but, can be critical particularly near the points of contraflexure. It is obvious that if elastic behaviour continues beyond the loads at which the design method indicates plasticitiy; tension can occur in sections which are not designed for it and large cracks possibly leading to failure can result. The length over which cracks can occur is very critical in design and can be called the cracking length. In this chapter, cracking length, Hyper plastic moment and efficiency of design relations are discussed, and a new method for redistributed bending envelope is proposed and a comparison of various methods is done by giving examples.

Definitions:

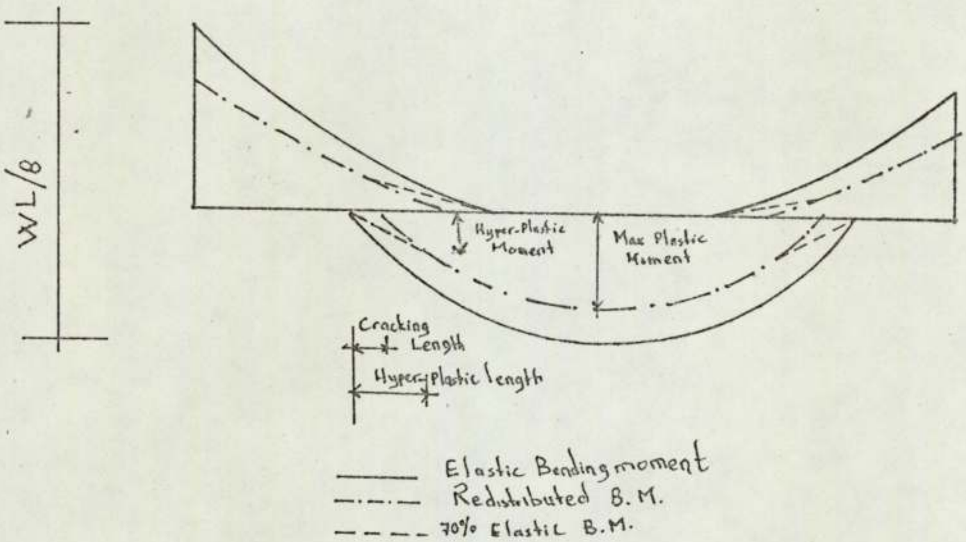



fig.5.1.

Hyper-plastic Moment: At the critical points 70% of elastic moment is numerically bigger than the plastic moment; the moment which occurs at this stage is called Hyper-plastic moment and the length over which that moment occurs is called the Hyper-plastic length.

Cracking length: The length between contraflexure points of the plastic and elastic moment. i.e., the length over which the hyper-plastic moment may have a sign opposite to that indicated by plastic analysis.

Notations:

A = Ratio of Max. hyper-plastic moment to max plastic moment

B = Rotio of Min. hyper-plastic moment to min plastic moment

C = Ratio of hyper-plastic length to span

D = Ratio of cracking length to span

STEPS OF THE ANALYSIS

Limit state design calculations were carried out for three, four and five equal 8m spans for continuous beams and frames which were subjected to uniform loading (fig. 2 - 3). Three ratios of imposed load to dead load namely $\frac{Qk}{Gk}$ = 0.75, 1.0 and 1.5 are considered.

1st. Step: Beams and frames were loaded by $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1.0 variable. Loads with dead loads 20 KN/m. and the ratio of total load to dead load ($F/G_k$) was taken 1.4, 1.5, 1.7, 1.8, 2.0, 2.2, 2.4, 2.5, 2.6 3.0, 3.1 and 3.8 respectively.

2nd. Step: Elastic end moments of the beams were calculated by using a computer programme for the likely critical loading conditions.

3rd. Step: Elastic moment, 70% Elastic reduced moment, and redistributed moment were calculated by using the computer programme written by A. W. Astill. This reduction was taken as 30% of the max. elastic support moment.

4th. Step: Elastic and plastic bending envelopes were drawn and effective points of 70% elastic bending moment envelope were plotted on graphs (Appendix 1 - 72)

5th. Step: From the drawn graph hyper-plastic moment/Max plastic moment, cracking length/span for sagging and hogging side and hyper-plastic length/span with design efficiencies are tabulated on table (1 - 3). Span moment/FL/ 8, support moment/FL/8 for both sides are tabulated on table (5 - 8).

## 5.3. ANALYSIS OF BEAMS

Investigation of the CP 110 method for beams and frames show that the differences between the redistributed bending moments and modified elastic bending moments are critical near points of contra-flexure. From the elastic and redistributed bending moment envelopes which are given in appendix I the following attempts were made to find a relationship.

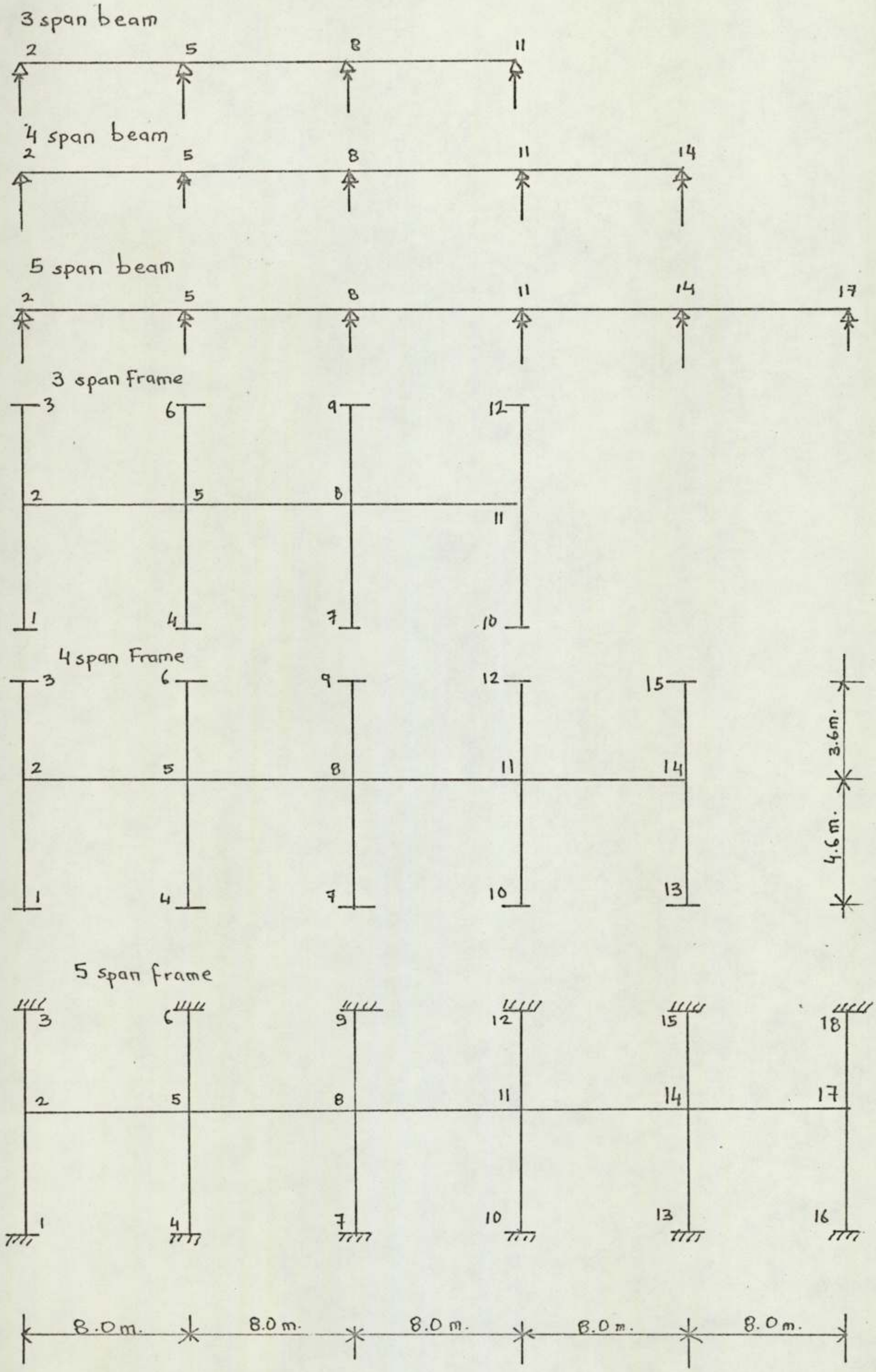

3 span beam
2
5
8
11
4 span beam
2
5
8
11
14
5 span beam
2
5
8
11
14
17
3 span frame
3
6
9
12
2
5
8
11
1
4
7
10
4 span Frame
3
6
9
12
15
2
5
8
11
14
1
4
7
10
13
3.6m.
4.6m.
5 span frame
3
6
9
12
15
18
2
5
8
11
14
17
1
4
7
10
13
16
8.0m.
8.0m.
8.0m.
8.0m.
8.0m.

fig 5(2-3)

Cracking Length:

Cracking length is tabulated from elastic and redistributed bending moment envelopes by dividing it by span length on tables (1 - 3) Column D. It can be seen from table 4 that, the critical cracking length/span values are within (0.0) to (-0.03) for 3,4 and 5 span beams at end span beam and (0.0) to (-0.08) for five span beam on sagging side. It can also be seen from table (4) that these values for the hogging side are within (0.0) to (-0.03) for three span (0.00) to (0.04) for four span and (0.0) to (0.045) for 5 span beams at end spans.

For interior span at hogging side (0.0) to (-0.10) for three span beams, (0.0) to (-0.07) for four span beams and (0.0) to (-0.07) for five span beam respectively.

In the second part of the study cracking length/span relation are traced against $F/G_k$ ratio on figures (5.4), (5.5), (5.6) for 3 spans, 4 spans and 5 span beams for sagging and hogging side. It was found that; there is a relation between cracking length/span ratio with $F/G_k$ ratio which gives a curve for sagging and hogging side as shown in figures (5.4) - (5.6). It will also be seen from the result that cracking length is bigger at inner spans than End span at sagging side. Cracking length increases with $F/G_k$ ratio on sagging side but decreases on hogging side. It will also be seen from figures that curve is flatter at high $F/G_k$ ratio for sagging side.

Hyper-plastic Moment

From the same bending moment envelopes, the values of the ratio of maximum hyper-plastic moment to maximum plastic moment are tabulated in columns A and B of tables 1 to 3. Column A contains the figures for sagging side and Column B contains the figures for the hogging side. Hyper-plastic moment/FL/8 values also tabulated on

TABLE 1. 3 Span Beam

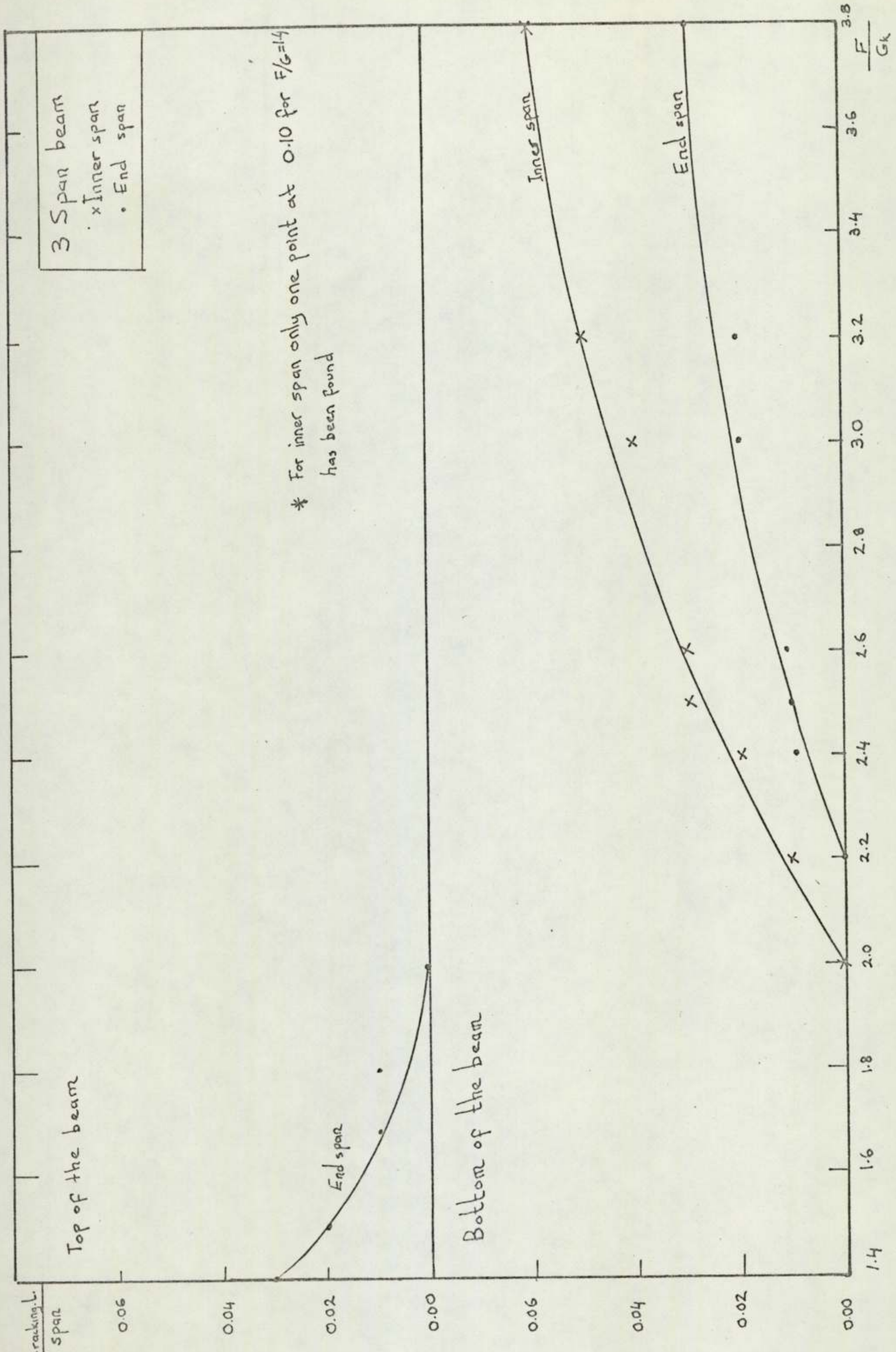
| SPAN NUMBER | DEAD LOAD | VARIABLE LOAD | TOTAL LOAD | TOTAL LD. / DEAD LD. | EFFICENCY | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|
| 2-5R | 20.0 | 8.00 | 28.0 | 1.4 | 0.862 | - | 0.786 | 0.23 | -0.03 |
| 5-8R | " | " | " | " | 0.797 | - | 0.356 | 0.40 | -0.10 |
| 2-5R | 20.0 | 10.0 | 30.0 | 1.5 | 0.856 | - | 0.420 | 0.18 | -0.02 |
| 5-8R | " | " | " | " | 0.786 | - | - | - | - |
| 2-5R | 20.0 | 14.0 | 34.0 | 1.7 | 0.846 | - | 0.106 | 0.05 | -0.01 |
| 5-8R | " | " | " | " | 0.772 | - | - | - | - |
| 2-5R | 20.0 | 16.0 | 36.0 | 1.8 | 0.340 | - | 0.56 | 0.03 | -0.01 |
| 5-8R | " | " | " | " | 0.764 | - | - | - | - |
| 2-5R | 20.0 | 20.0 | 40.0 | 2.0 | 0.834 | - | 0.00 | 0.00 | 0.00 |
| 5-8R | " | " | " | " | 0.754 | 0.0 | - | 0.00 | 0.00 |
| 2-5R | 20.0 | 24.0 | 44.0 | 2.2 | 0.829 | 0.0 | - | 0.0 | 0.0 |
| 5-8R | " | " | " | " | 0.744 | 0.150 | - | 0.04 | -0.01 |
| 2-5R | 20.0 | 28.0 | 48.0 | 2.4 | 0.325 | 0.090 | - | 0.03 | -0.01 |
| 5-8R | " | " | " | " | 0.739 | 0.231 | - | 0.06 | -0.02 |
| 2-5R | 20.0 | 30.0 | 50.0 | 2.5 | 0.822 | 0.125 | - | 0.04 | -0.01 |
| 5-8R | " | " | " | " | 0.729 | 0.243 | - | 0.06 | -0.03 |
| 2-5R | 20.0 | 32.0 | 52.0 | 2.6 | 0.823 | 0.133 | - | 0.03 | -0.01 |
| 5-8R | " | " | " | " | 0.730 | 0.261 | - | 0.06 | -0.03 |
| 2-5R | 20.0 | 40.0 | 60.0 | 3.0 | 0.813 | 0.207 | - | 0.07 | -0.02 |
| 5-8R | " | " | " | " | 0.722 | 0.355 | - | 0.09 | -0.04 |
| 2-5R | 20.0 | 42.0 | 62.0 | 3.1 | 0.811 | 0.213 | - | 0.07 | -0.02 |
| 5-8R | " | " | " | " | 0.721 | 0.544 | - | 0.14 | -0.05 |
| 2-5R | 20.0 | 56.0 | 76.0 | 3.8 | 0.817 | 0.284 | - | 0.10 | -0.03 |
| 5-8R | " | " | " | " | 0.711 | 0.736 | - | 0.14 | -0.06 |

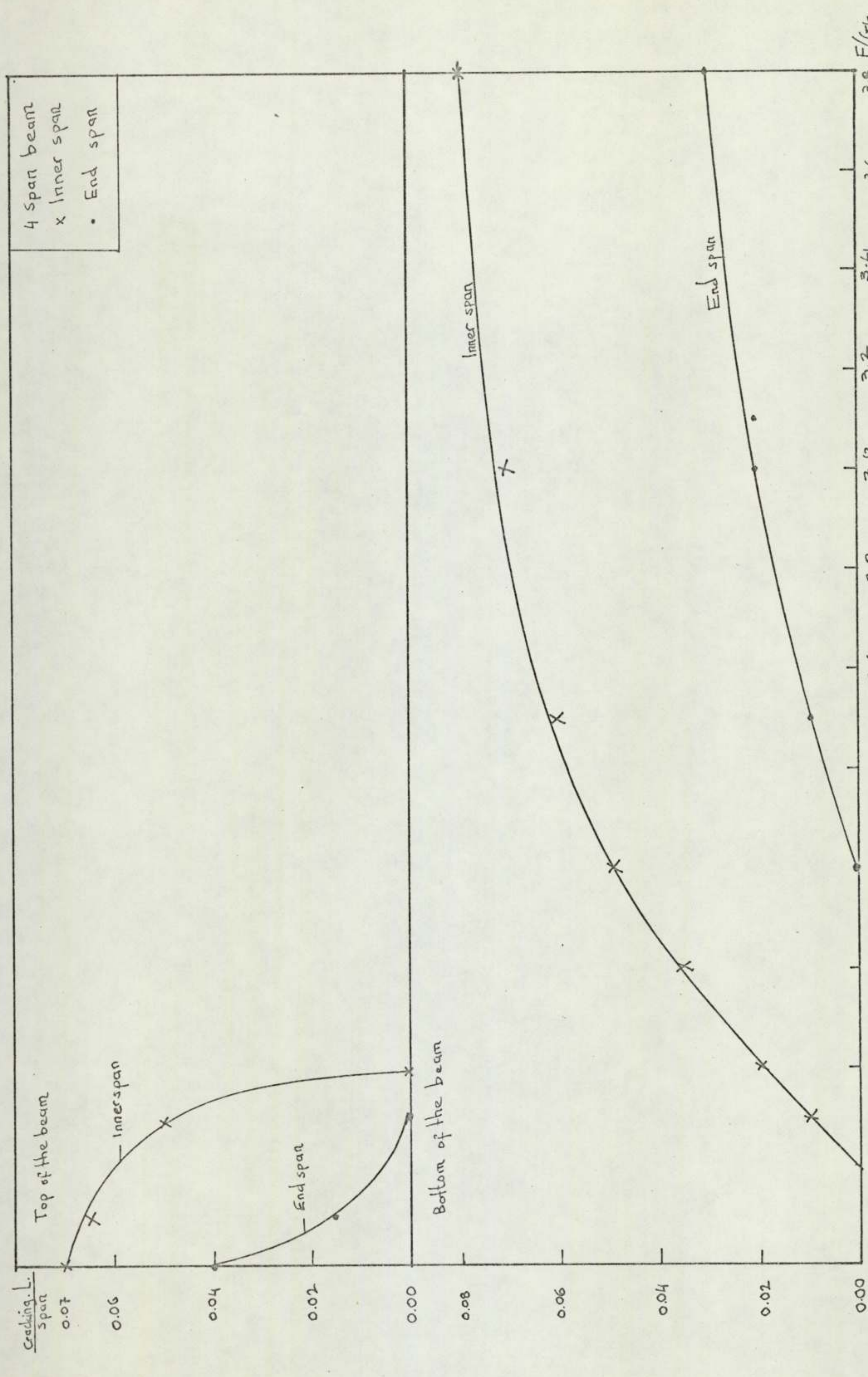
Table 2 4 Span Beam

| No. of span | Dead Load ($G_k$) | Variable Load (V) | Total Load (F) | Tot.load / d'd load | Efficiency | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|
| 2-5 R | 20.0 | 8.00 | 28.0 | 1.4 | 0.856 | - | 0.840 | 0.21 | -0.04 |
| 5-8 L | " | " | " | " | 0.815 | - | 0.376 | 0.20 | -0.07 |
| 2-5 R | 20.0 | 10.0 | 30.0 | 1.5 | 0.853 | - | 0.116 | 0.05 | -0.015 |
| 5-8 L | " | " | " | " | 0.812 | - | 0.267 | 0.17 | -0.065 |
| 2-5 R | 20.0 | 14.0 | 34.0 | 1.7 | 0.842 | - | 0.0 | 0.0 | 0.0 |
| 5-8 L | " | " | " | " | {0.798} | - | 0.094 | 0.11 | -0.05 |
| " R | " | " | " | " | | 0.091 | - | 0.13 | -0.015 |
| 2-5 R | 20.0 | 16.0 | 36.0 | 1.8 | 0.838 | - | - | - | - |
| 5-8 L | " | " | " | " | {0.787} | - | 0.00 | 0.00 | 0.00 |
| " R | " | " | " | " | | 0.177 | - | 0.04 | -0.01 |
| 2-5 R | 20.0 | 20.0 | 40.0 | 2.0 | 0.833 | - | - | - | - |
| 5.8 R | " | " | " | " | 0.782 | 0.272 | - | 0.08 | -0.03 |
| 2-5 R | 20.0 | 24.0 | 44.0 | 2.2 | 0.823 | 0.0 | - | 0.0 | 0.0 |
| 5-8 R | " | " | " | " | 0.752 | 0.421 | - | 0.12 | -0.05 |
| 2-5 R | 20.0 | 28.0 | 48.0 | 2.4 | 0.822 | 0.062 | - | 0.03 | -0.01 |
| 5-8 R | " | " | " | " | 0.772 | 0.456 | - | 0.14 | -0.05 |
| 2-5 R | 20.0 | 30.0 | 30.0 | 2.5 | 0.821 | 0.100 | - | 0.04 | -0.02 |
| 5-8 R | " | " | " | " | 0.776 | 0.442 | - | 0.14 | -0.06 |
| 2-5 R | 20.0 | 32.0 | 52.0 | 2.6 | 0.819 | 0.105 | - | 0.04 | -0.02 |
| 5-8 R | " | " | " | " | 0.765 | 0.475 | - | 0.15 | -0.06 |
| 2-5 R | 20.0 | 40.0 | 60.0 | 3.0 | 0.815 | 0.189 | - | 0.06 | -0.02 |
| 5-8 R | " | " | " | " | 0.757 | 0.555 | - | 0.18 | -0.07 |
| 2-5 R | 20.0 | 42.0 | 62.0 | 3.1 | 0.809 | 0.189 | - | 0.09 | -0.02 |
| 5-8 R | " | " | " | " | 0.753 | 0.558 | - | 0.21 | -0.07 |
| 2-5 R | 20.0 | 56.0 | 76.0 | 3.8 | 0.804 | 0.263 | - | 0.09 | -0.03 |
| 5-8 R | " | " | " | " | 0.746 | 0.657 | - | 0.22 | -0.08 |

TABLE 3. 5 Span Beam.

| NO. of Span | Dead Load ($G_k$) | Variable Load | Total Load F | $F/G_k$ | Efficiency | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|
| 2-5R | 20.0 | 8.0 | 28.0 | 1.4 | 0.894 | - | 0.786 | 0.26 | -0.045 |
| 5-8L | " | " | " | " | 0.818 | - | 0.377 | -0.14 | -0.07 |
| 8-11R | " | " | " | " | 0.797 | - | 0.221 | 0.09 | -0.03 |
| 2-5R | 20.0 | 10.0 | 30.0 | 1.5 | 0.855 | - | 0.260 | 0.10 | -0.04 |
| 5-8L | " | " | " | " | 0.784 | - | 0.272 | 0.13 | 0.065 |
| 8-11 | " | " | " | " | 0.787 | - | - | - | - |
| 2-5R | 20.0 | 14.0 | 34.0 | 1.7 | 0.843 | - | 0.130 | 0.07 | -0.025 |
| 5-8L | " | " | " | " | (0.768) | - | 0.033 | 0.07 | -0.04 |
| 5-8R | " | " | " | " | | | - | 0.02 | 0.005 |
| 8-11R | " | " | " | " | 0.770 | 0.102 | - | 0.03 | -0.01 |
| 2-5R | 20.0 | 16.0 | 26.0 | 1.8 | 0.838 | - | 0.081 | - | -0.020 |
| 5-8L | " | " | " | " | | - | 0.00 | 0.00 | -0.00 |
| 5-8R | 20.0 | " | " | " | 0.762 | 0.125 | - | 0.04 | -0.015 |
| 8-11R | 20.0 | " | " | " | 0.763 | 0.165 | - | 0.04 | -0.02 |
| 2-5R | 20.0 | 20.0 | 40.0 | 2.0 | 0.834 | - | 0.063 | 0.01 | 0.00 |
| 5-8R | " | " | " | " | 0.751 | 0.226 | - | 0.07 | -0.075 |
| 8-11R | " | " | " | " | 0.751 | 0.250 | - | 0.07 | -0.030 |
| 2-5R | 20.0 | 24.0 | 44.0 | 2.2 | 0.827 | 0.00 | - | 0.00 | 0.00 |
| 5-8R | " | " | " | " | 0.741 | 0.358 | - | 0.11 | -0.04 |
| 8-11R | " | " | " | " | 0.739 | 0.370 | - | 0.12 | -0.0425 |
| 2-5R | 20.0 | 28.0 | 48.0 | 2.4 | 0.824 | 0.070 | - | 0.03 | -0.01 |
| 5-8R | " | " | " | " | 0.734 | 0.451 | - | 0.13 | -0.04 |
| 8-11 | " | " | " | " | 0.732 | 0.471 | - | 0.13 | -0.04 |
| 2-5R | 20.0 | 30.0 | 50.0 | 2.5 | 0.821 | 0.107 | - | 0.04 | -0.01 |
| 5-8R | " | " | " | " | 0.730 | 0.495 | - | 0.13 | -0.05 |
| 8-11R | " | " | " | " | 0.729 | 0.510 | - | 0.14 | -0.055 |
| 2-5R | 20.0 | 32.0 | 52.0 | 2.6 | 0.820 | 0.119 | - | 0.04 | -0.01 |
| 5-8R | " | " | " | " | 0.729 | 0.531 | - | 0.16 | -0.06 |
| 8-11R | " | " | " | " | 0.726 | 0.572 | - | 0.16 | -0.05 |
| 2-5 | 20.0 | 40.0 | 60.0 | 3.0 | 0.813 | 0.190 | - | 0.07 | -0.02 |
| 5-8 | " | " | " | " | 0.719 | 0.684 | - | 0.20 | -0.065 |
| 8-11 | " | " | " | " | 0.716 | 0.720 | - | 0.21 | -0.07 |
| 2-5 | " | 42.0 | 62.0 | 3.1 | 0.812 | 0.194 | - | 0.07 | -0.02 |
| 5-8 | " | " | " | " | 0.718 | 0.714 | - | 0.22 | -0.070 |
| 8-11 | " | " | " | " | 0.715 | 0.742 | - | 0.22 | -0.075 |
| 2-5 | " | 56.00 | 76.0 | 3.8 | 0.806 | 0.276 | - | 0.09 | -0.03 |
| 5-8 | " | " | " | " | 0.707 | 0.884 | - | 0.28 | -0.075 |
| 5-11 | " | " | " | " | 0.704 | 0.927 | - | 0.28 | -0.08 |


Cracking.L / span
3 Span beam
x Inner span
• End span
* For inner span only one point at 0.10 for F/G=1.4 has been found
Top of the beam
End span
Bottom of the beam
Inner span
End span
0.06
0.04
0.02
0.00
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
F / Gk



4 span beam
x Inner span
· End span
Cracking L. / span
Top of the beam
Innerspan
End span
Bottom of the beam
Inner span
End span
0.07
0.06
0.04
0.02
0.00
0.08
0.06
0.04
0.02
0.00
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
F/Gk


Fig (5.5)

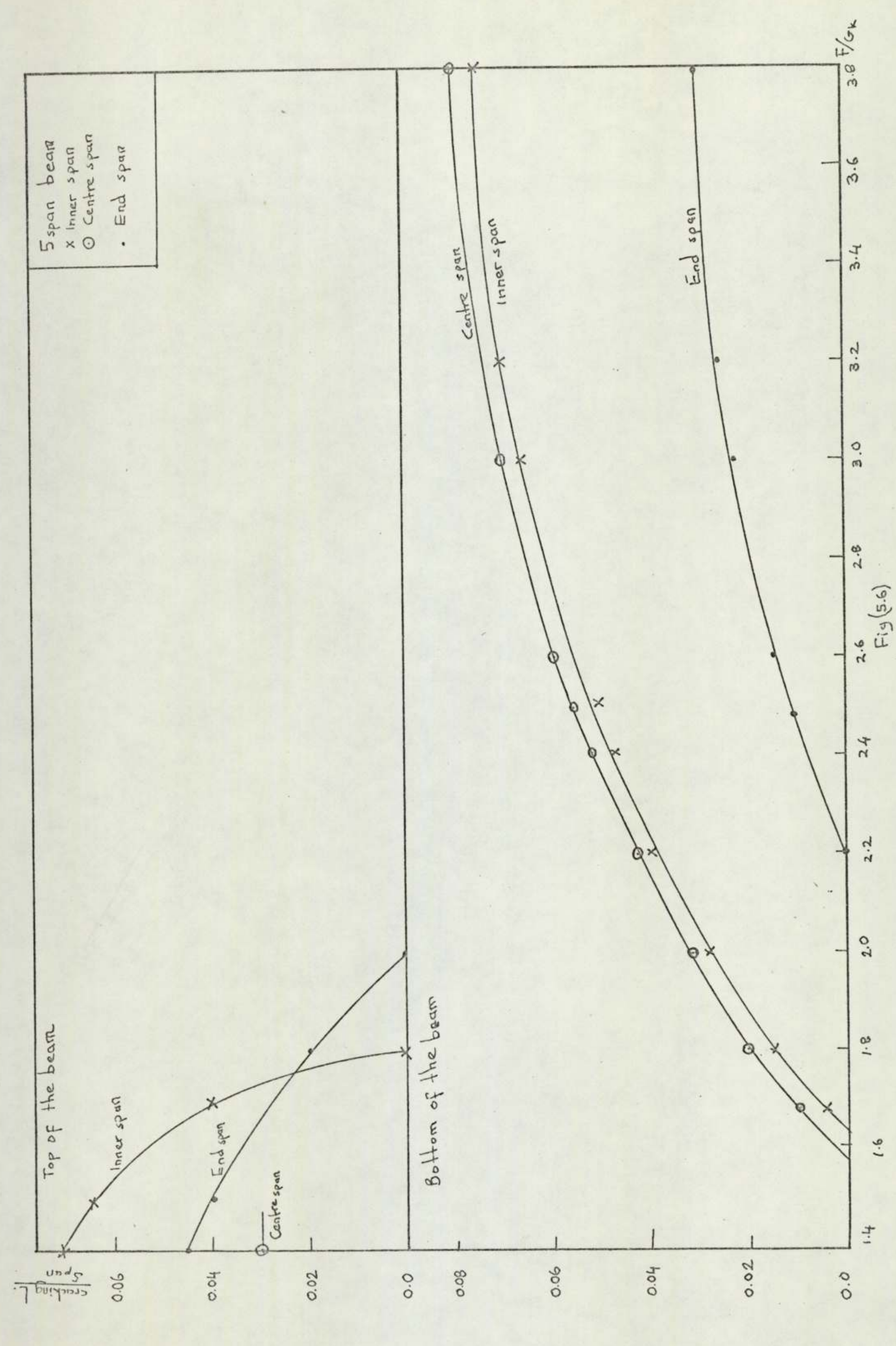

5 span beam
x Inner span
⊙ Centre span
· End span
Top of the beam
Inner span
End span
Centre span
Bottom of the beam
Centre span
Inner span
End span
F/Gk
0.06
0.04
0.02
0.0
0.08
0.06
0.04
0.02
0.0
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8

Fig (5.6)

TABLE 4

| NUMBER OF SPAN | SAGGING SIDE Critical Cracking length/ Span Values | | | | HOGGING SIDE Critical Cracking length/ Span Values. | | | |
|---|---|---|---|---|---|---|---|---|
| | END SPAN (2-5) | | INTERIOR SPAN (5-5) | | END SPAN (2-5) | | INTERIOR SPAN (5-5) | |
| | Min. | Max. | Min. | Max. | Min. | Max. | Min. | Max. |
| 3 Span | 0.0 | -0.03 | 0.0 | -0.06 | 0.0 | -0.03 | 0.0 | -0.10 |
| 4 Span | 0.0 | -0.03 | 0.0 | -0.08 | 0.0 | -0.04 | 0.0 | -0.07 |
| 5 Span | 0.0 | -0.03 | 0.0 | -0.07 | 0.0 | -0.045 | 0.0 | -0.07 |

tables 5 to 7. These results are then plotted against $F/G_k$ ratios in figures 5.7 to 5.12. The variation of the points at which the maximum and minimum values of hyper-plastic moment are also tabulated in table (8). It may be seen from these graphs that:

a) Hyper-plastic moment is bigger for inner span than end span at sagging side.
b) By increasing load hyper-plastic moment decrease at hogging side and increase at sagging side.
c) Maximum hyper-plastic moment at sagging side gives nearly the same result for end span. (0.205 FL/8 for 3 span, 0.185 FL/8 for 4 span and 0.195 FL/8 for 5 span.) and they will be equal to zero at $F/G_k = 2.2$

Efficency of Design:

Relative economy can be measured by an efficency index for elastic and redistributed design. Efficency can be defined as the ratio of the volume of flexural reinforcement resulting from elastic and redistributed design. It was suggested by M. Z. Cohn (1970) that the volume of flexural reinforcement for plastic case (Vk) can be formulated by Vk = MpjLj and similarly for elastic case $V_E = M_{Ej}L_j$. And efficency index will be $\vartheta k = \sum \frac{Vk}{V_E} = \sum \frac{MpjLj}{M_{Ej}L_j}$

where $M_E$ = Elastic bending moment

Mp Mp = Plastic moment

Lj = Span length

That equation may be applied by taking redistributed values of bending moments instead of plastic values. Then efficency index of redistributed design will be;

$$\vartheta_k = \frac{\sum M_{Rj}L_j}{\sum M_{Ej}L_j}$$

For example: Efficency index of the beam shown on figure (5.13) can be calculated by applying the equation above. Efficency index =

$$\sum \frac{287 \times 8 + 287.0 \times 8 + 218 \times 8.0}{397.0 \times 8 + 397.0 \times 8.0 + 289 \times 8.0} = 0.731$$

Efficencies of the redistributed bending moment designs are calculated by applying the same way for all loading conditions and tabulated on tables 1 to 3 and on table 9. Reciprocal of efficencies against $F/G_k$ are plotted in figure 14. It may be seen from these results that:

a) There is a relation between $\frac{F}{G_k}$ ratio and 1/Efficency. This is due to the fact that an increase in $F/G_k$ generates an increase in 1/Efficency and its maximum value occur at maximum $F/G_k$ ratio.

b) Reciprocal of efficencies is bigger for inner span than end span.

c) Maximum reciprocal of efficencies nearly equal at end spans for 3 spans (1.223), 4 span (1.243), 5 span (1.240) respectively.

d) For 5 span beam, reciprocal of efficencies are nearly the same for the centre span and inner span.

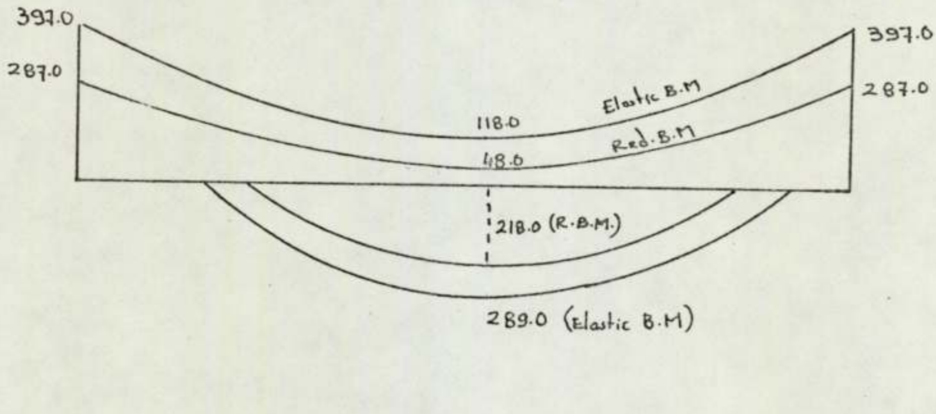



fig. 5.13

Table 5 3 span Beam

| Span No. | Total load | E $G_k$ | Bottom M.H.P.M. $Fl/8$ | Top M.H.P.M. $Fl/8$ | ELASTIC CASE | | REDISTRIBUTED | |
|---|---|---|---|---|---|---|---|---|
| | | | | | Support M $Fl/8$ | Span M $Fl/8$ | Support M $Fl/8$ | Span M $Fl/8$ |
| 2.5 R | 28.0 | 1.40 | - | 0.459 | 0.834 | 0.687 | 0.584 | 0.727 |
| 5-8 R | " | 1.40 | - | 0.356 | " | 0.316 | " | 0.415 |
| 2-5 R | 34.0 | 1.70 | - | 0.063 | 0.849 | 0.709 | 0.595 | 0.725 |
| 5-8 R | " | " | - | - | " | 0.366 | " | 0.404 |
| 2-5 R | 36.0 | 1.80 | - | 0.039 | 0.854 | 0.715 | 0.597 | 0.724 |
| 5-8 R | " | " | - | - | " | 0.379 | " | 0.402 |
| 2-5 R | 40.0 | 2.0 | - | 0.0 | 0.868 | 0.721 | 0.609 | 0.718 |
| 5-8 R | " | " | 0.00 | - | " | 0.396 | " | 0.390 |
| 2-5 R | 44.0 | 2.2 | 0.00 | - | 0.869 | 0.730 | 0.613 | 0.717 |
| 5-8 R | " | " | 0.058 | - | " | 0.420 | " | 0.389 |
| 2-5 R | 48.0 | 2.4 | 0.060 | - | - | 0.736 | 0.872 | 0.716 |
| 5-8 R | " | " | 0.087 | - | - | 0.435 | " | 0.388 |
| 2-5 R | 50.0 | 2.5 | 0.090 | - | 0.882 | 0.736 | 0.617 | 0.715 |
| 5-8 R | " | " | 0.120 | - | " | 0.437 | " | 0.382 |
| 2-5 R | 52.0 | 2.6 | 0.095 | - | 0.887 | 0.742 | 0.877 | 0.716 |
| 5-8 R | " | " | 0.140 | - | " | 0.448 | " | 0.381 |
| 2-5 R | 60.0 | 3.0 | 0.140 | - | 0.891 | 0.891 | 0.622 | 0.712 |
| 5-8 R | " | " | 0.190 | - | " | 0.464 | " | 0.377 |
| 2-5 R | 62.0 | 3.1 | 0.152 | - | 0.890 | 0.754 | 0.623 | 0.712 |
| 5-8 R | " | " | 0.207 | - | " | 0.472 | " | 0.376 |
| 2-5 R | 76.0 | 3.8 | 0.205 | - | 0.893 | 0.763 | 0.625 | 0.710 |
| 5-8 R | " | " | 0.276 | - | " | 0.496 | | 0.375 |

TABLE 6. 4 Span Beam

| SPAN NO. | Total Load | $\frac{F}{G_k}$ | Bottom Side M.H.P.M. / Fl /8 | Top Side M.H.P.M. / Fl /8 | Elastic Case | | Redistributed | |
|---|---|---|---|---|---|---|---|---|
| | | | | | Supp.Mom. / FL/8 | Span M. / FL/8 | Supp.M. / FL/8 | Span M. / FL/8 |
| 2-5R | 28.0 | 1.40 | - | 0.517 | 0.883 | 0.669 | 0.616 | 0.714 |
| 5-8R | " | " | - | 0.231 | 0.656 | 0.343 | 0.459 | 0.464 |
| 2-5R | 30.0 | 1.50 | - | 0.072 | 0.887 | 0.675 | 0.620 | 0.712 |
| 5-8R | " | " | 0.00 | 0.166 | 0.666 | 0.352 | 0.466 | 0.458 |
| 2-5R | 34.0 | 1.70 | - | 0.0 | 0.893 | 0.691 | 0.625 | 0.709 |
| 5-8R | " | " | 0.041 | 0.059 | 0.687 | 0.367 | 0.481 | 0.448 |
| 2-5R | 36.0 | 1.8 | - | - | 0.899 | 0.694 | 0.628 | 0.710 |
| 5-8R | " | " | 0.078 | 0.0 | 0.697 | 0.375 | 0.489 | 0.440 |
| 2-5R | 40.0 | 2.0 | - | - | 0.903 | 0.706 | 0.631 | 0.709 |
| 5-8R | " | " | 0.118 | - | 0.712 | 0.384 | 0.50 | 0.434 |
| 2-5R | 44.0 | 2.2 | 0.00 | - | 0.909 | 0.713 | 0.636 | 0.707 |
| 5-8R | " | " | 0.151 | | 0.724 | 0.390 | 0.508 | 0.427 |
| 2-5 | 50.0 | 2.5 | 0.07 | - | 0.915 | 0.722 | 0.640 | 0.705 |
| 5-8R | " | " | 0.186 | - | 0.740 | 0.405 | 0.517 | 0.422 |
| 2-5R | 52.0 | 2.6 | 0.074 | - | 0.918 | 0.729 | 0.642 | 0.705 |
| 5-8R | " | " | 0.200 | - | 0.749 | 0.491 | 0.523 | 0.418 |
| 2-5R | 60.0 | 3.0 | 0.129 | - | 0.920 | 0.735 | 0.645 | 0.704 |
| 5-8R | " | " | 0.229 | - | 0.758 | 0.418 | 0.531 | 0.412 |
| 2-5R | 62.0 | 3.1 | 0.136 | - | 0.921 | 0.737 | 0.645 | 0.699 |
| 5-8R | " | " | 0.230 | - | 0.762 | 0.419 | 0.534 | 0.411 |
| 2-5R | 76.0 | 3.8 | 0.185 | - | 0.929 | 0.748 | 0.649 | 0.700 |
| 5-8R | " | " | 0.264 | - | 0.777 | 0.432 | 0.544 | 0.402 |

TABLE 7. 5 Span Beam

| SPAN NO. | F | $F/G_k$ | Bottom M.H.P.M. FL /8 | Top M.H.P.M. FL /8 | ELASTIC | | REDISTRIBUTED | |
|---|---|---|---|---|---|---|---|---|
| | | | | | Supp.Mom. FL/8 | Span M FL/8 | Supp.Mom FL/8 | SpanM. FL/8 |
| 2-5 | 28.0 | 1.4 | - | 0.477 | 0.870 | 0.611 | 0.607 | 0.718 |
| 5-8L | " | " | - | 0.229 | 0.705 | 0.371 | 0.491 | 0.450 |
| 8-11 | " | " | - | 0.108 | - | 0.457 | - | 0.508 |
| 2-5 | 30.0 | 1.5 | - | 0.161 | 0.875 | 0.679 | 0.612 | 0.716 |
| 5-8 | " | " | - | 0.166 | 0.716 | 0.389 | 0.50 | 0.441 |
| 8-11 | " | " | - | - | - | 0.472 | - | 0.500 |
| 2-5 | 34.0 | 1.7 | - | 0.081 | 0.882 | 0.694 | 0.617 | 0.713 |
| 5-8 | " | " | 0.015 | 0.064 | 0.735 | 0.419 | 0.514 | 0.433 |
| 8-11 | " | " | 0.049 | - | - | 0.498 | - | 0.485 |
| 2-5 | 36.0 | 1.8 | - | 0.050 | 0.888 | 0.701 | 0.621 | 0.711 |
| 5-8 | " | " | 0.054 | 0.00 | 0.743 | 0.430 | 0.520 | 0.430 |
| 8-11 | " | " | 0.078 | - | - | 0.507 | - | 0.479 |
| 2-5 | 40.0 | 2.0 | - | 0.0052 | 0.893 | 0.709 | 0.625 | 0.712 |
| 5-8 | " | " | 0.090 | - | 0.756 | 0.450 | 0.531 | 0.421 |
| 8-11 | " | " | 0.129 | - | - | 0.525 | - | 0.468 |
| 2-5 | 44.0 | 2.2 | 0.0 | - | 0.897 | 0.718 | 0.627 | 0.710 |
| 5-8 | " | " | 0.149 | - | 0.769 | 0.465 | 0.536 | 0.417 |
| 8-11 | " | " | 0.161 | - | - | 0.539 | - | 0.463 |
| 2-5 | 48.0 | 2.4 | 0.049 | - | 0.903 | 0.723 | 0.632 | 0.708 |
| 5-8 | " | " | 0.185 | - | 0.778 | 0.481 | 0.544 | 0.411 |
| 8-11 | " | " | 0.205 | - | - | 0.551 | - | 0.455 |
| 2-5 | 50.0 | 2.5 | 0.063 | - | 0.905 | 0.727 | 0.633 | 0.707 |
| 5-8 | " | " | 0.198 | - | 0.782 | 0.487 | 0.547 | 0.410 |
| 8-11 | " | " | 0.231 | - | - | 0.557 | - | 0.452 |
| 2-5 | 52.0 | 2.6 | 0.084 | - | 0.906 | 0.730 | 0.634 | 0.709 |
| 5-8 | " | " | 0.218 | - | 0.786 | 0.492 | 0.550 | 0.408 |
| 8-11 | " | " | 0.257 | - | - | 0.562 | - | 0.449 |
| 2-5 | 60.0 | 3.0 | 0.134 | - | 0.912 | 0.739 | 0.637 | 0.706 |
| 5-8 | " | " | 0.275 | - | 0.797 | 0.510 | 0.558 | 0.402 |
| 8-11 | " | " | 0.290 | - | - | 0.579 | - | 0.441 |
| 2-5 | 62.0 | 3.1 | 0.137 | - | 0.914 | 0.743 | 0.639 | 0.709 |
| 5-8 | " | " | 0.286 | - | 0.803 | 0.517 | 0.564 | 0.400 |
| 8-11 | " | " | 0.300 | - | - | 0.580 | - | 0.439 |
| 2.5 | 76.0 | 3.8 | 0.194 | - | 0.919 | 0.751 | 0.643 | 0.703 |
| 5-8 | " | " | 0.345 | - | 0.815 | 0.536 | 0.570 | 0.393 |
| 8-11 | " | " | 0.398 | - | - | 0.600 | | 0.429 |

TABLE 8.

| | $F/G_k$ | End Span | | | | Inner Span | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Sagging Side | | Hogging Side | | Sagging Side | | Hogging Side | |
| | | H.P.M / M.P.M. | H.P.M. / FL/8 | H.P.M / M.P.M | H.P.M / FL/8 | H.P.M / M.P.M | H.P.M / FL/8 | H.P.M / M.P.M | H.P.M / FL/8 |
| 3 Span | 1.4 | - | - | 0.786 | 0.459 | - | - | 0.350 | 0.356 |
| | 2.0 | - | - | 0.0 | 0.0 | 0.0 | 0.0 | - | - |
| | 2.2 | 0.0 | 0.0 | - | - | 0.150 | 0.058 | - | - |
| | 3.8 | 0.284 | 0.205 | - | - | 0.736 | 0.276 | - | - |
| 4 Span | 1.4 | - | - | 0.840 | 0.517 | - | - | 0.376 | 0.231 |
| | 1.5 | - | - | 0.116 | 0.072 | - | - | 0.267 | 0.166 |
| | 1.7 | - | - | 0.0 | 0.0 | 0.091 | 0.040 | 0.094 | 0.059 |
| | 1.8 | - | - | - | - | 0.177 | 0.060 | 0.0 | 0.0 |
| | 2.2 | 0.0 | 0.0 | - | - | 0.421 | 0.151 | - | - |
| | 3.8 | 0.263 | 0.185 | - | - | 0.657 | 0.264 | - | - |
| 5 Span | 1.4 | - | - | 0.786 | 0.477 | - | - | 0.266 | 0.130 |
| | 1.7 | - | - | 0.130 | 0.081 | 0.035 | 0.015 | 0.033 | 0.017 |
| | 2.0 | - | - | 0.063 | 0.0052 | 0.226 | 0.095 | - | - |
| | 2.2 | 0.0 | 0.0 | - | - | 0.358 | 0.149 | - | - |
| | 3.8 | 0.276 | 0.194 | - | - | 0.884 | 0.345 | - | - |

TABLE 9

| | $\frac{F}{G_k}$ | 1/Efficency | | |
|---|---|---|---|---|
| | | End Span | Inner span | Centre span |
| 3 Span | 1.4 | 1.160 | 1.254 | —— |
| 4 Span | | 1.168 | 1.226 | —— |
| 5 Span | | 1.157 | 1.257 | 1.254 |
| 3 Span | 1.5 | 1.168 | 1.272 | —— |
| 4 Span | | 1.172 | 1.231 | —— |
| 5 Span | | 1.169 | 1.275 | 1.270 |
| 3 Span | 1.7 | 1.182 | 1.295 | —— |
| 4 Span | | 1.187 | 1.253 | —— |
| 5 Span | | 1.186 | 1.302 | 1.298 |
| 3 Span | 2.0 | 1.199 | 1.326 | —— |
| 4 Span | | 1.200 | 1.278 | —— |
| 5 Span | | 1.199 | 1.331 | 1.331 |
| 3 Span | 2.2 | 1.206 | 1.344 | —— |
| 4 Span | | 1.215 | 1.307 | —— |
| 5 Span | | 1.209 | 1.349 | 1.353 |
| 3 Span | 2.5 | 1.216 | 1.371 | —— |
| 4 Span | | 1.218 | 1.305 | —— |
| 5 Span | | 1.218 | 1.369 | 1.371 |
| 3 Span | 3.0 | 1.230 | 1.366 | —— |
| 4 Span | | 1.226 | 1.321 | —— |
| 5 Span | | 1.230 | 1.390 | 1.396 |
| 3 Span | 3.8 | 1.223 | 1.406 | —— |
| 4 Span | | 1.243 | 1.340 | —— |
| 5 Span | | 1.240 | 1.414 | 1.420 |

Span and Support Moments:

Span and support moments are obtained from elastic and redistributed bending moment envelopes (Appendix 1) and the ratios of $M_E$/FL/8 and $M_R$/FL/8 are tabulated in tables 5 to 7. These results are also plotted in figures (15 to 20) against $F/G_k$ ratio. It can be seen from these values that

a) Support moments are bigger at penultimate support than inner support.

b) Support moment increases when $F/G_k$ increase but there is not a big difference between minimum and maximum values.

c) Span moment is bigger at end span than inner span.

d) Elastic span moment increase and redistributed span moment decrease by increasing $F/G_k$ ratio, and they will also intersect each other.

PROPOSED METHOD

A new method can be developed by using the relation of cracking length/span, hyper-moment/FL/8, support moment/FL/8 against the $F/G_k$ ratio. It can be seen from figure 17 for the 5 span cases that redistributed support moments are within the range (0.610)FL/8 - 0.65 FL/8 for penultimate supports and within (0.49 )FL/8 - (0.570)FL/8 for inner supports respectively. If the fixed bending moment values of 0.65FL/8 is adopted for penultimate support and (0.57)FL/8 for inner supports so that 30% reduction from elastic moment is not exceeded for every value of $F/G_k$ from 1.4 to 3.8. Redistributed bending moment envelopes can be drawn using the following procedure based on the method outlined by A. W. Astill (1973).

First dealing with the maximum load case set up the fixed end moment values for the two supports of the span being considered i.e. for the penultimate span of the 5 span beam 0.65 and 0.57. From these two points draw the parabolic bending moment diagram for maximum

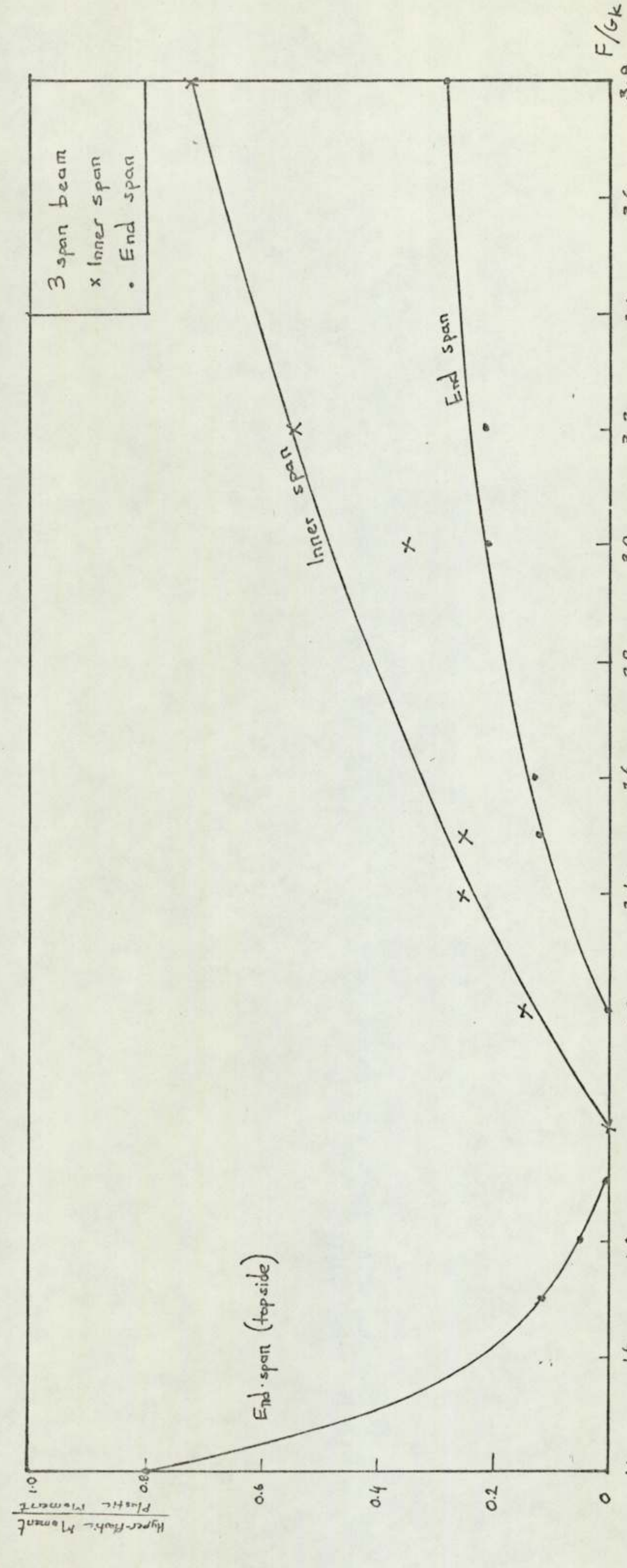

3 span beam
x Inner span
• End span
Inner span
End span
End·span (topside)
F/Gk
Hyper-Plastic Moment / Plastic Moment
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
0
0.2
0.4
0.6
0.8
1.0

fig (5.7)

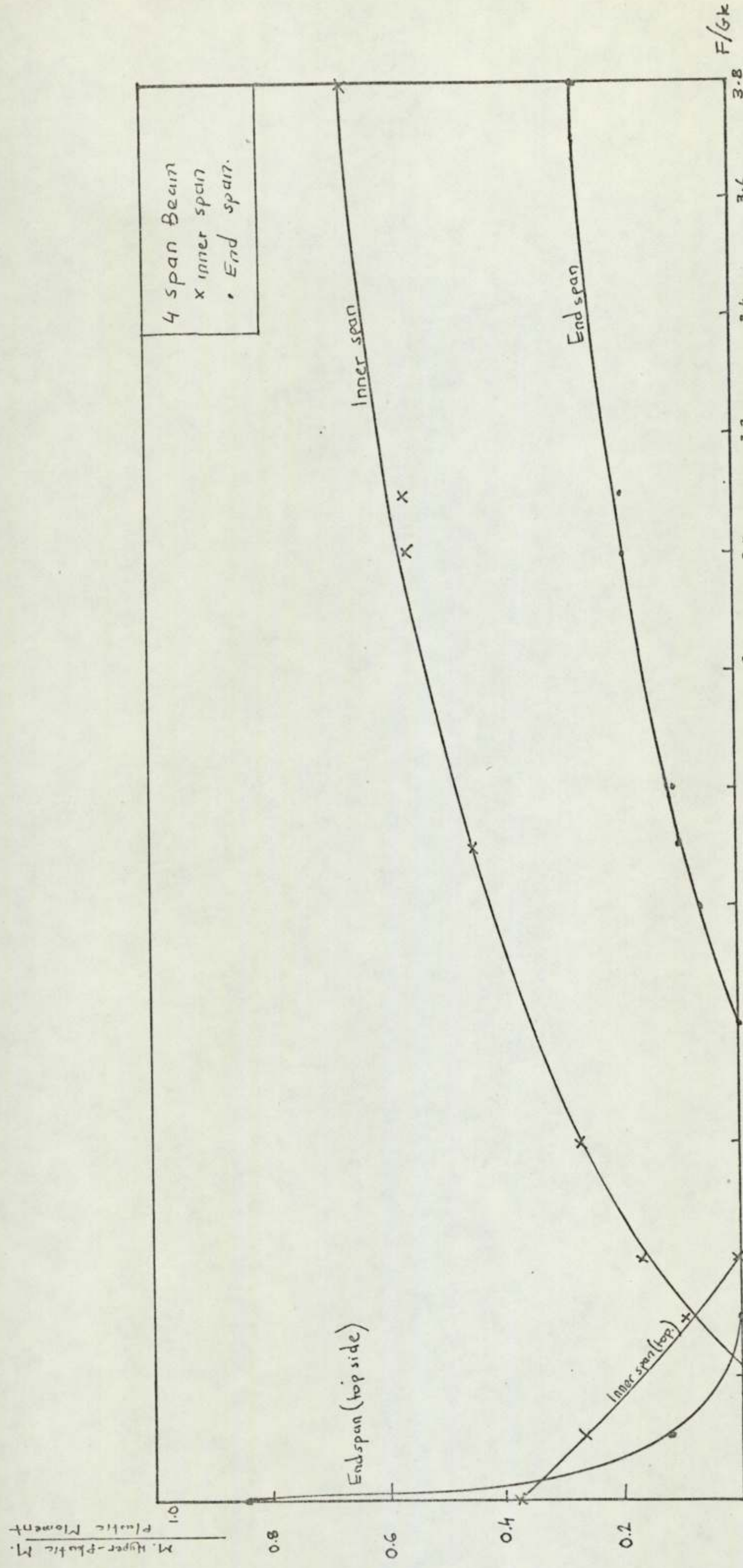
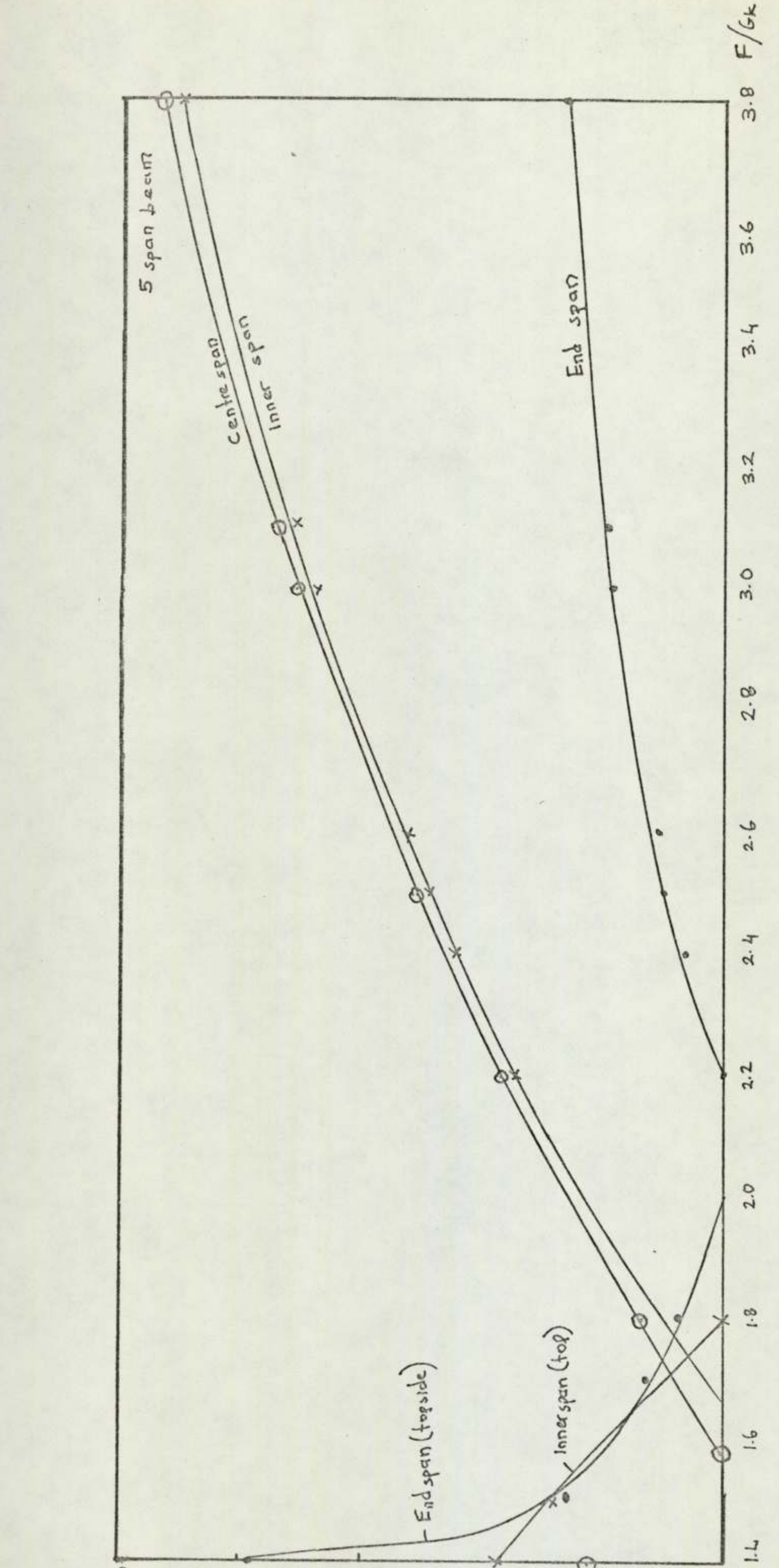
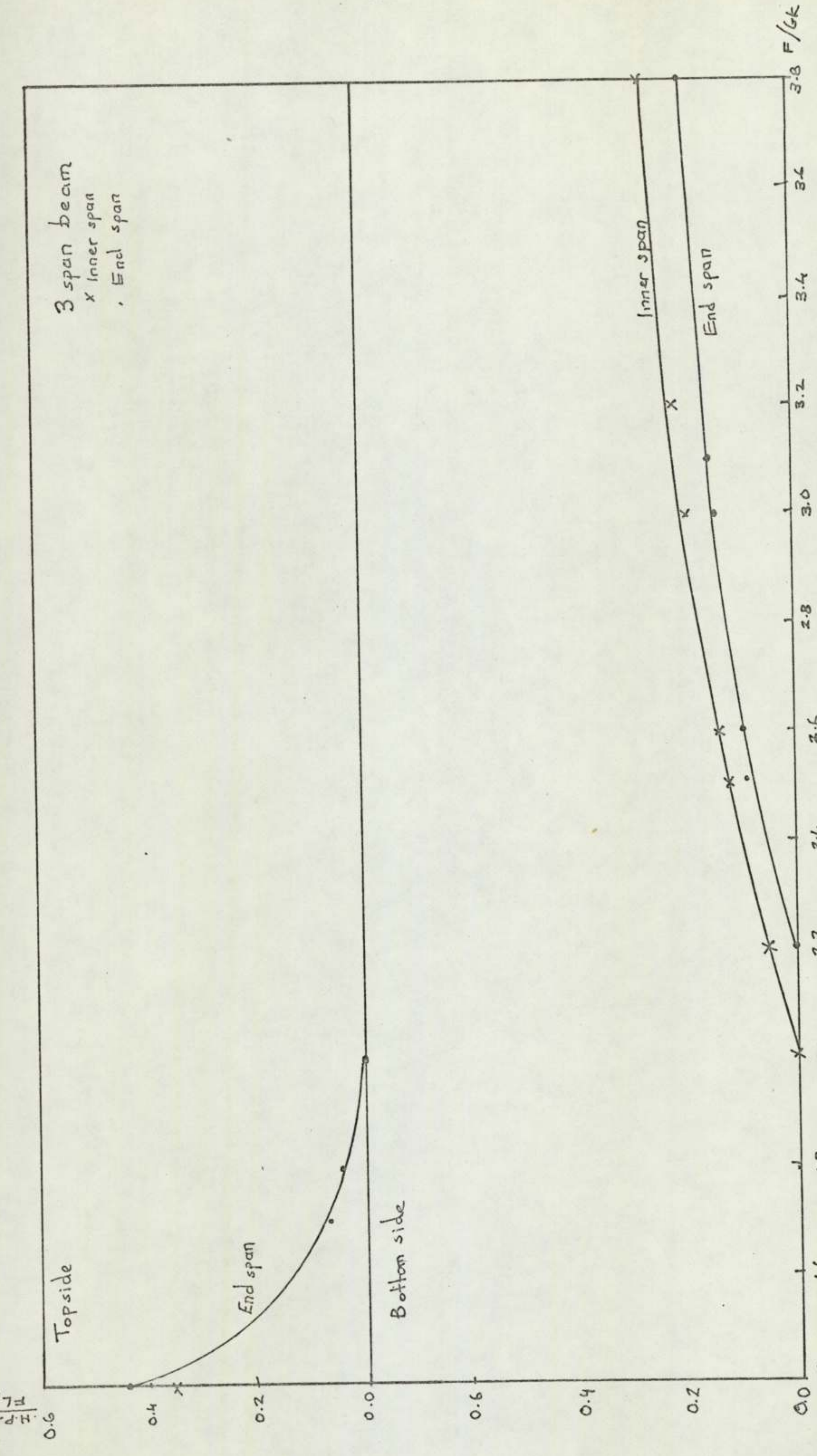

4 span Beam
x inner span
· End span.
Inner span
End span
Endspan (top side)
Inner span (top)
F/Gk
M. Hyper-Plastic M.
Plastic Moment
1.4
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
0.2
0.4
0.6
0.8
1.0

Fig. (5.8)


5 span beam
Centre span
Inner span
End span
End span (topside)
Inner span (top)
F/Gk
Hyper-plastic M. / Plastic M.
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
0.0
0.2
0.4
0.6
0.8


3 span beam
x Inner span
, End span
Topside
End span
Bottom side
Inner span
End span
F/Gk
0.6
0.4
0.2
0.0
0.6
0.4
0.2
0.0
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8

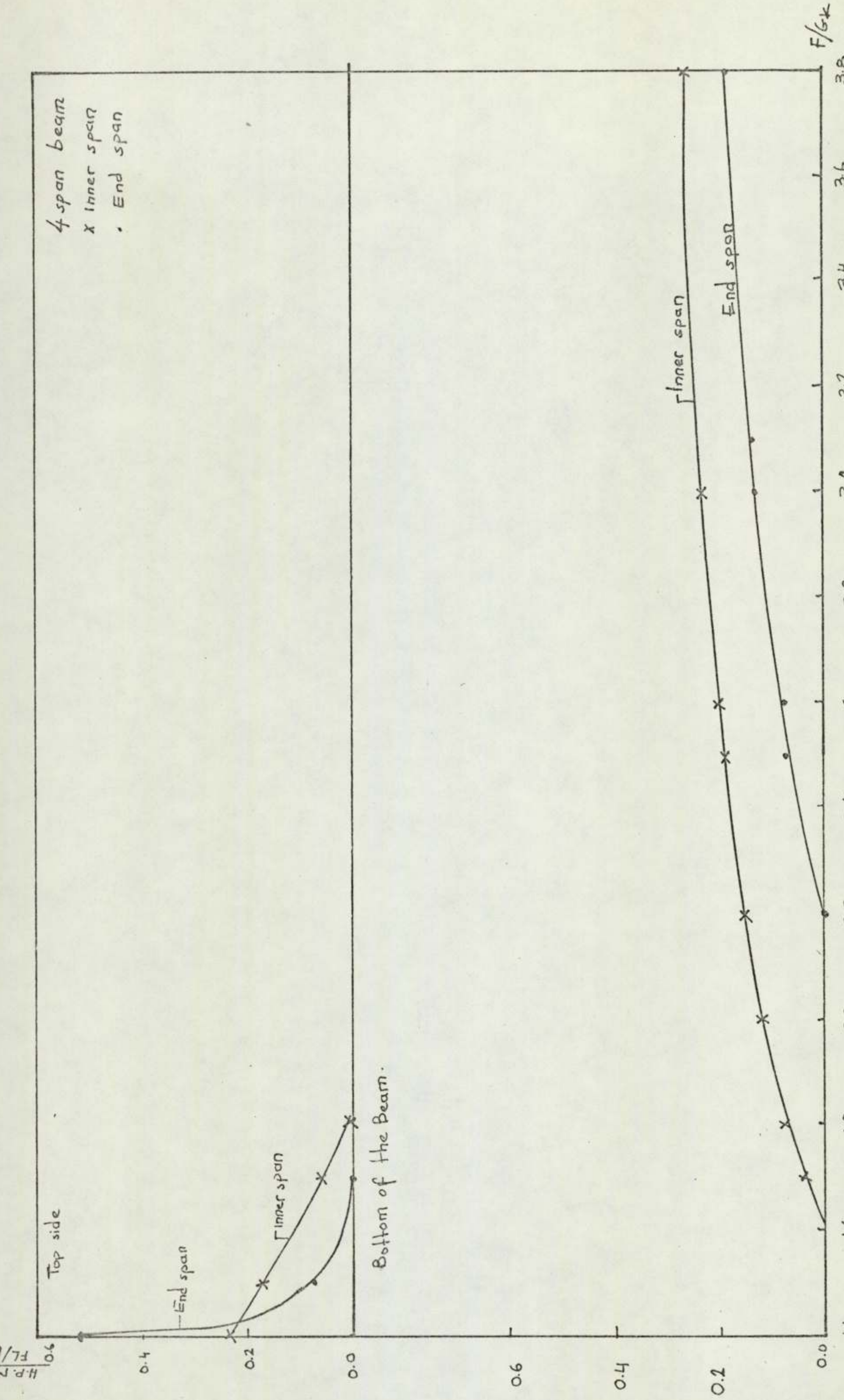

4 span beam
x Inner span
. End span
Top side
End span
Inner span
Bottom of the Beam.
Inner span
End span
H.P. Moment / FL/8
F/Gk
0.6
0.4
0.2
0.0
0.6
0.4
0.2
0.0
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8

fig 5.11

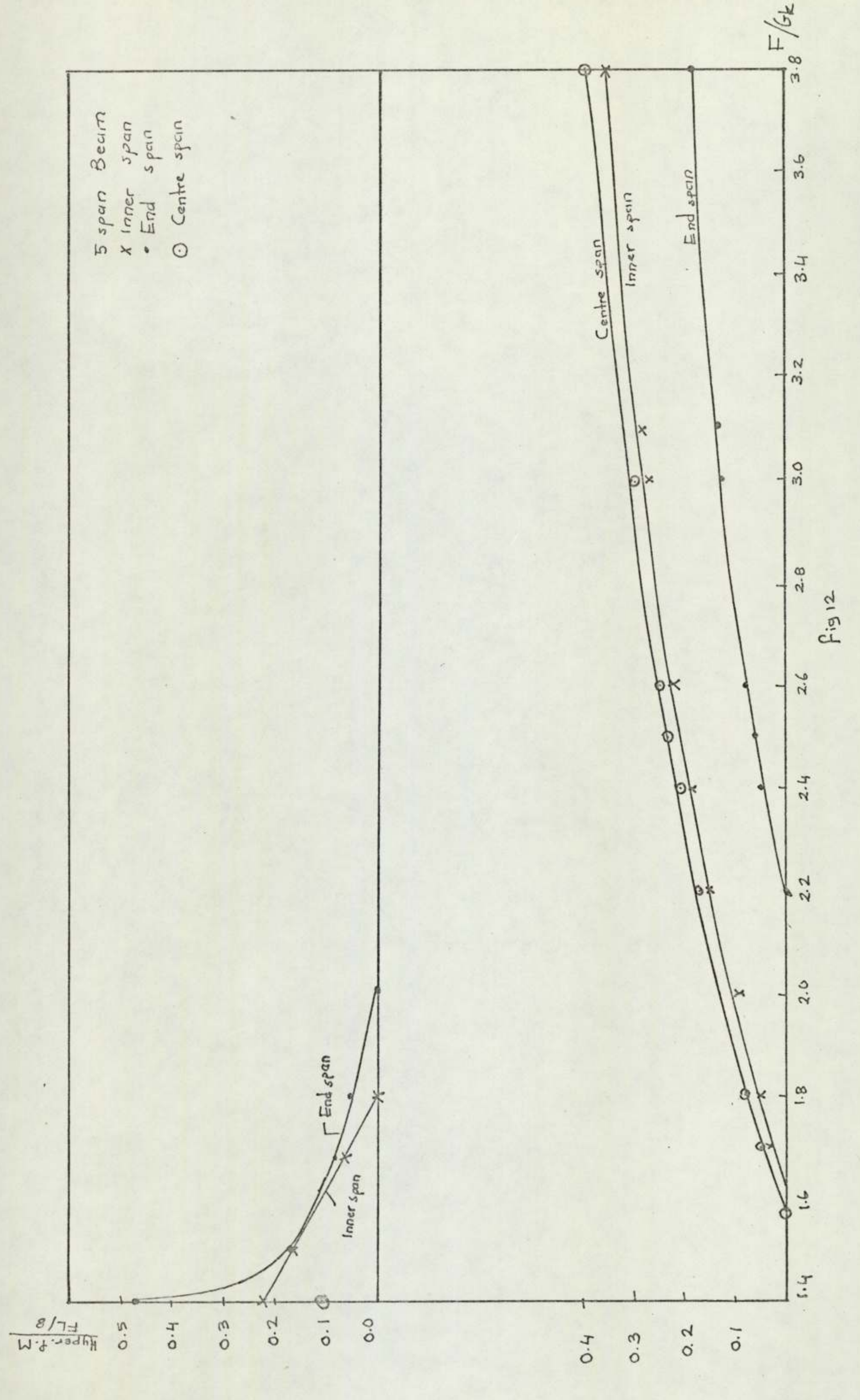
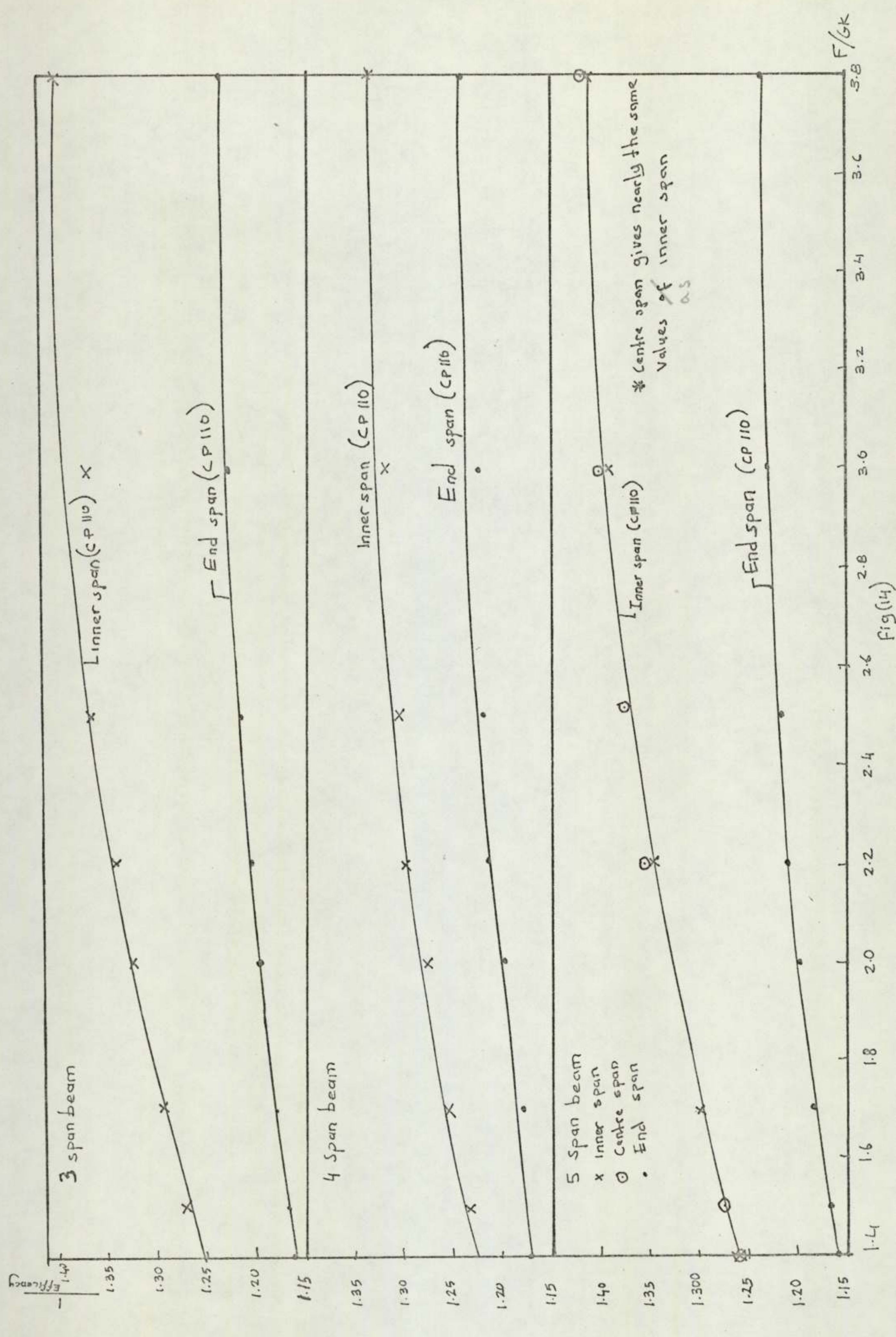

5 span Beam
x Inner span
• End span
⊙ Centre span
End span
Inner span
Centre span
Inner span
End span
Hyper. F.M / FL/8
0.5
0.4
0.3
0.2
0.1
0.0
0.4
0.3
0.2
0.1
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
F/Gk

Fig 12


Efficiency
1.40
1.35
1.30
1.25
1.20
1.15
3 span beam
Inner span (CP 110)
End span (CP 110)
4 Span beam
Inner span (CP 110)
End span (CP 110)
5 Span beam
x Inner span
⊙ Centre span
• End span
Inner span (CP110)
End span (CP 110)
* Centre span gives nearly the same values as of inner span
1.300
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
F/GK

Fig(14)

load F so that at the mid-span the sagging bending moment is $1.0 - \frac{1}{2}(0.65 + 0.57) = 0.39$ as shown on figure 5.26. From figure 5.6 the maximum cracking length is 0.075L and from fig. 5.12 maximum value of hyper-plastic moment/FL/8 is 0.35FL/8. Using these two values the limiting line for 70% elastic bending moment can be drawn for the sagging side as shown on figure 5.25. The outer line shown is for $F/G_k = 3.8$ and if it is used to determine the bending moment envelope for lesser values of $F/G_k$ a conservative, oversafe design will result. Other lines for other values of $F/G_k$ are also shown. It may be noted at this stage that, the 70% line does not occur for values of $F/G_k \leq 1.6$ approx. At this stage the design envelope for the sagging side is complete. The remainder of the diagram for the hogging side is drawn in a similar way but separate lines must be drawn for each value of $F/G_k$. In this case the 70% line does not occur for the values of $F/G_k \geq 1.8$. These envelopes are drawn for 3,4 and 5 span beam, by following the same procedure on figures 5.21 - to 5.27.

Example Using charts

A beam of five equal 8m spans is loaded with $G_k = 20.0$ and F = 60.kN/m. Draw the bending moment envelopes for end, inner and centre span respectively and design the reinforcement to CP 110.

Procedure of the Design

a) Trace the $G_k = \frac{F}{3.0}$ curve for end span, inner and centre span. Figures (5.25, 5.26, 5.27)

b) Calculate the end moments and span moments from formulated values

End moments:

$$0.65\,\frac{FL}{8} = 0.65 \times 60.0 \times \frac{8.0}{8} = 312.0 \text{ kN/m.}$$

$$0.57\,\frac{FL}{8} = 0.57 \times 60.0 \times \frac{8.0}{8.0} = 273.6 \text{ kN/m.}$$

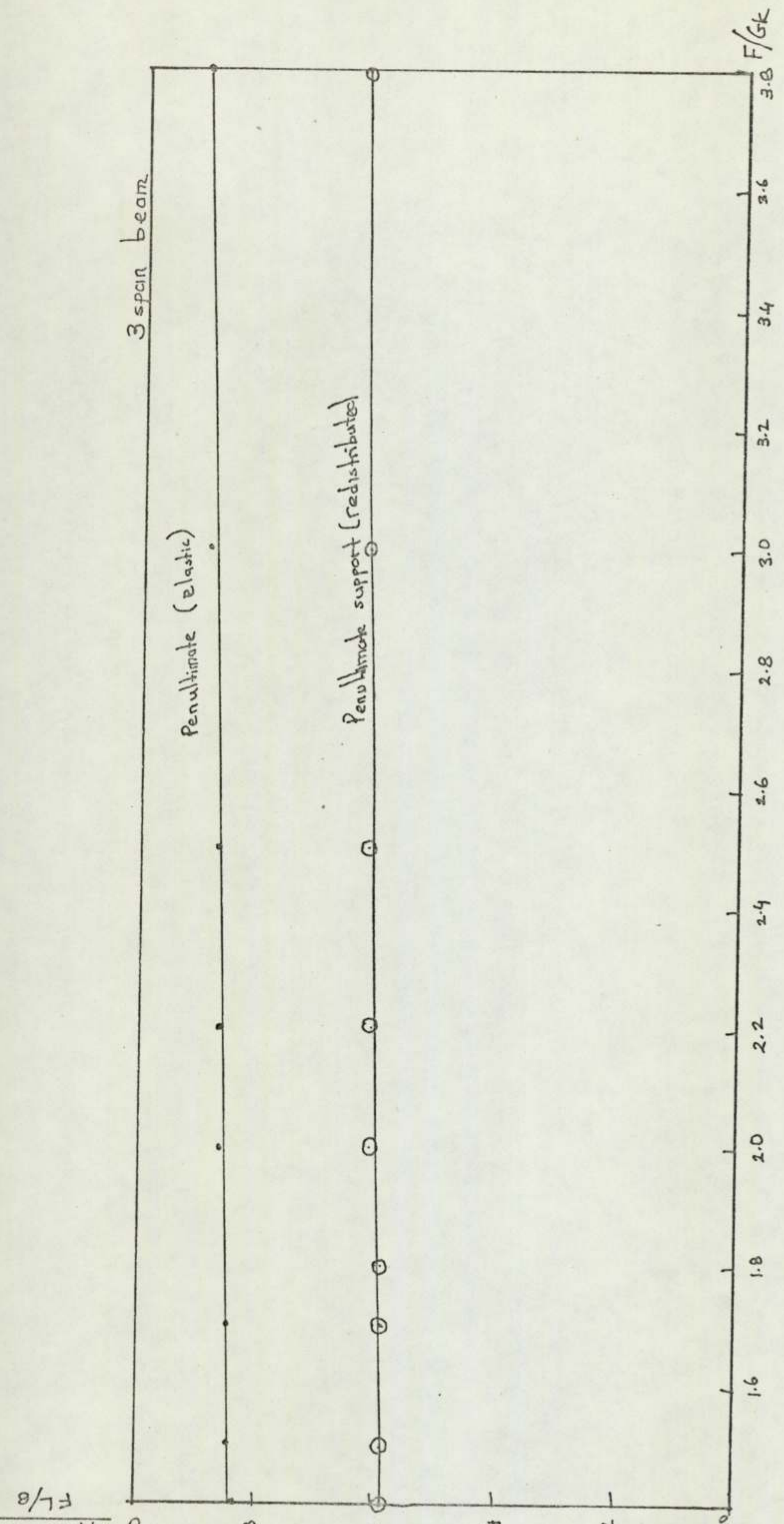

3span beam
Penultimate (elastic)
Penultimate support (redistributed)
Support Moment / FL/8
F/Gk
0.0
0.2
0.4
0.6
0.8
1.0
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8

fig (5.15)

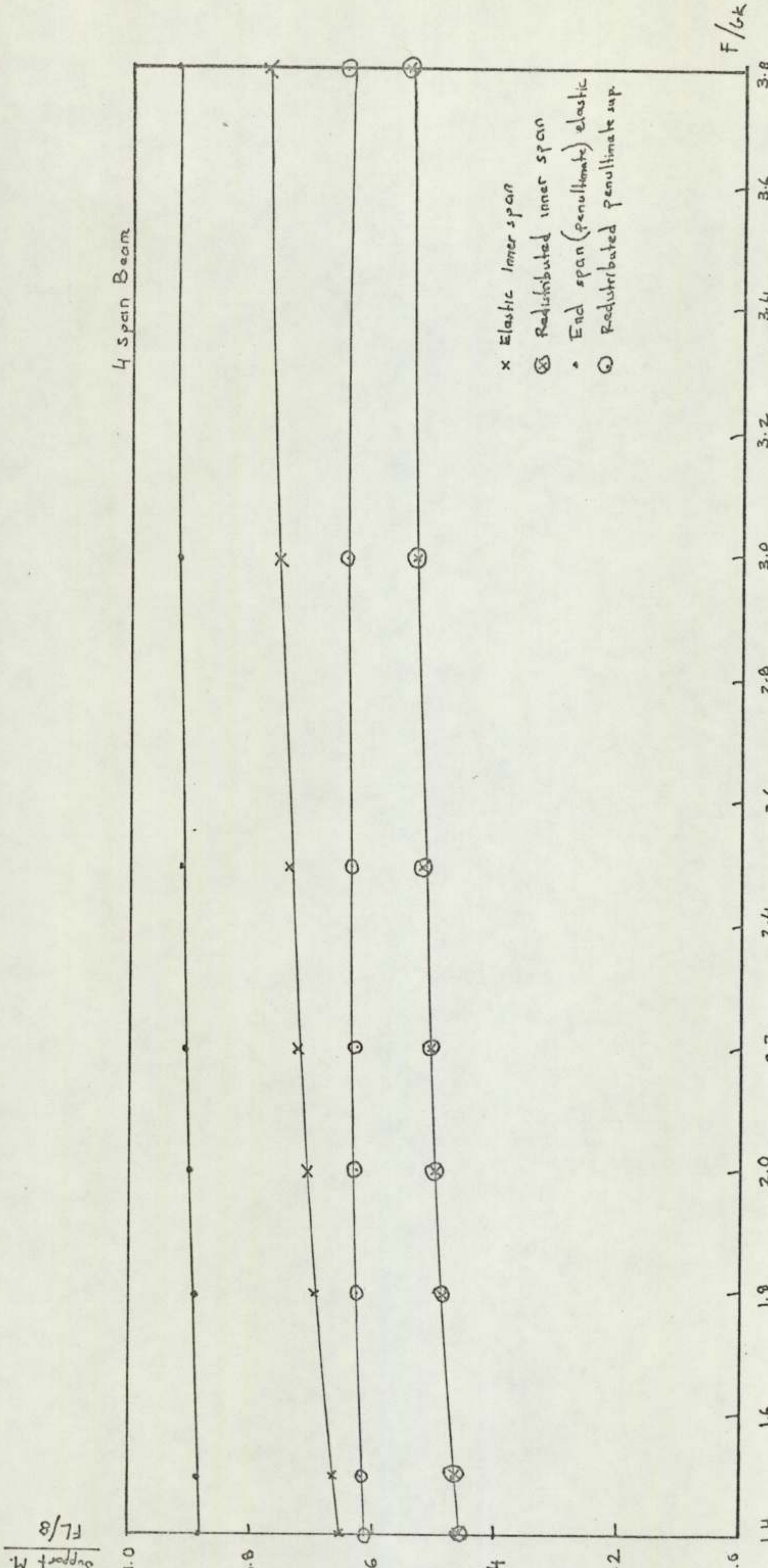

4 span Beam
Support M. / FL/8
F/Gk
1.0
0.8
0.6
0.4
0.2
0.0
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
× Elastic Inner span
⊗ Redistributed inner span
• End span (penultimate) elastic
⊙ Redistributed penultimate sup.

fig 5.16

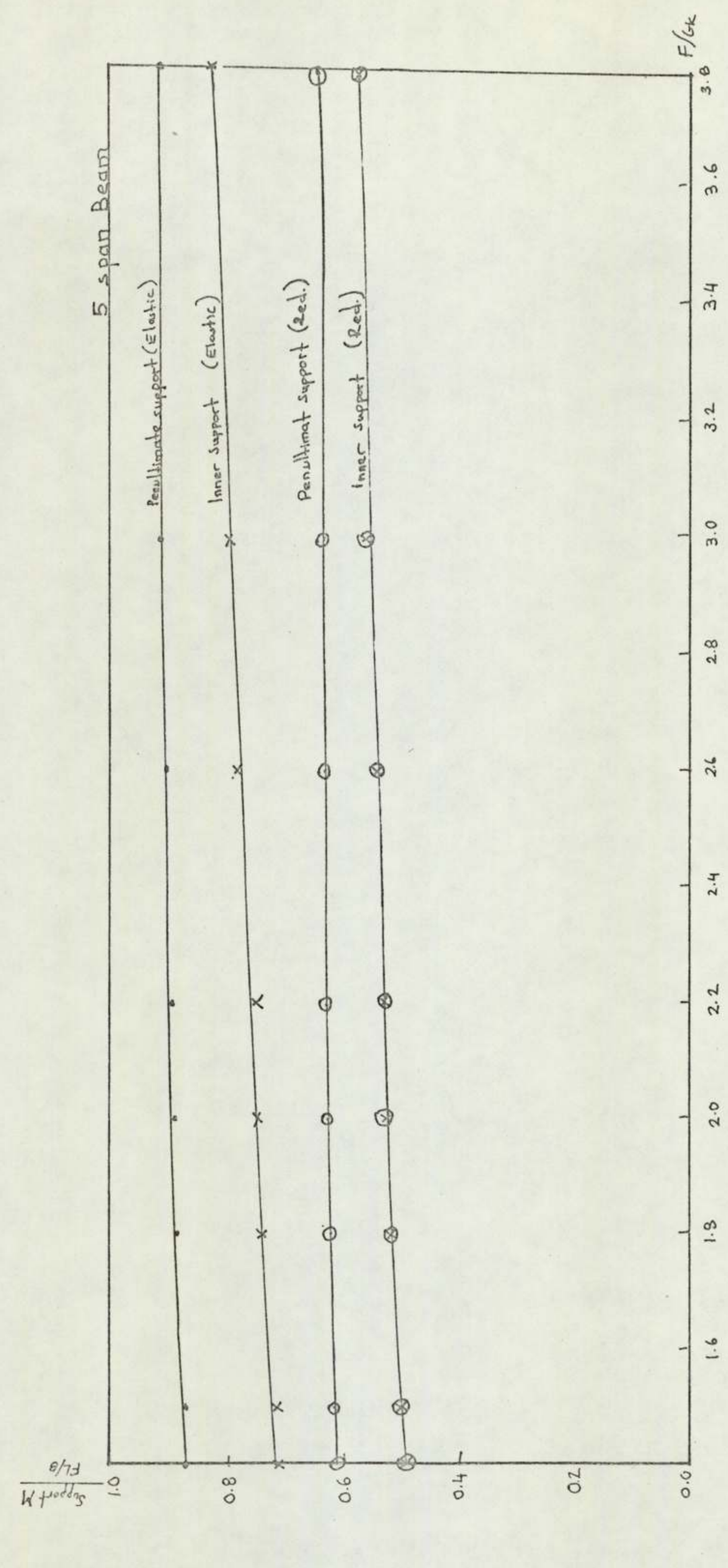

5 span Beam
Penultimate support (Elastic)
Inner support (Elastic)
Penultimat support (Red.)
Inner support (Red.)
Support M / FL/8
1.0
0.8
0.6
0.4
0.2
0.0
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
F/Gk

fig 5.17

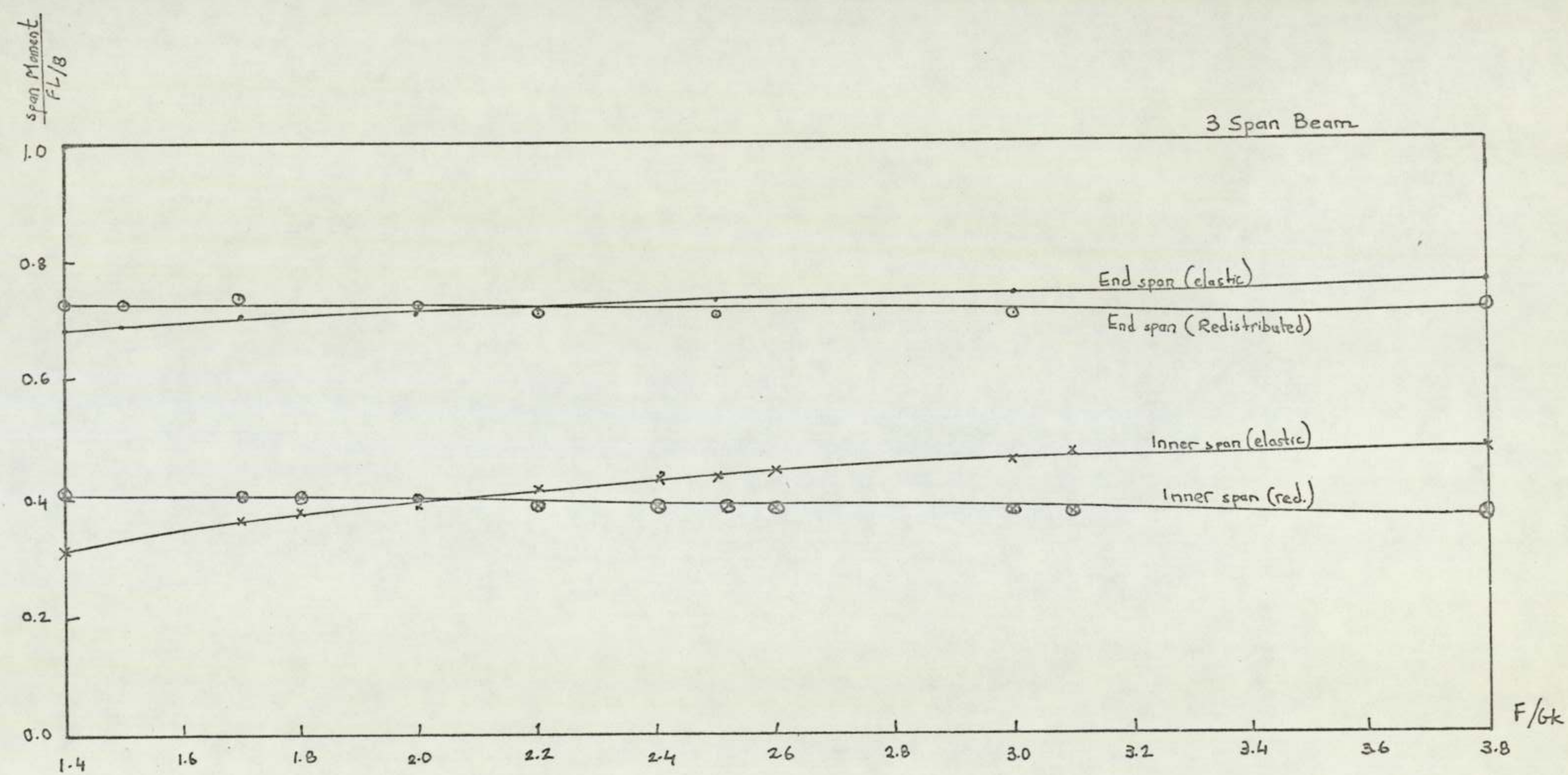

Span Moment / FL/8
3 Span Beam
End span (elastic)
End span (Redistributed)
Inner span (elastic)
Inner span (red.)
1.0
0.8
0.6
0.4
0.2
0.0
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
F/σk

figure (5.18)

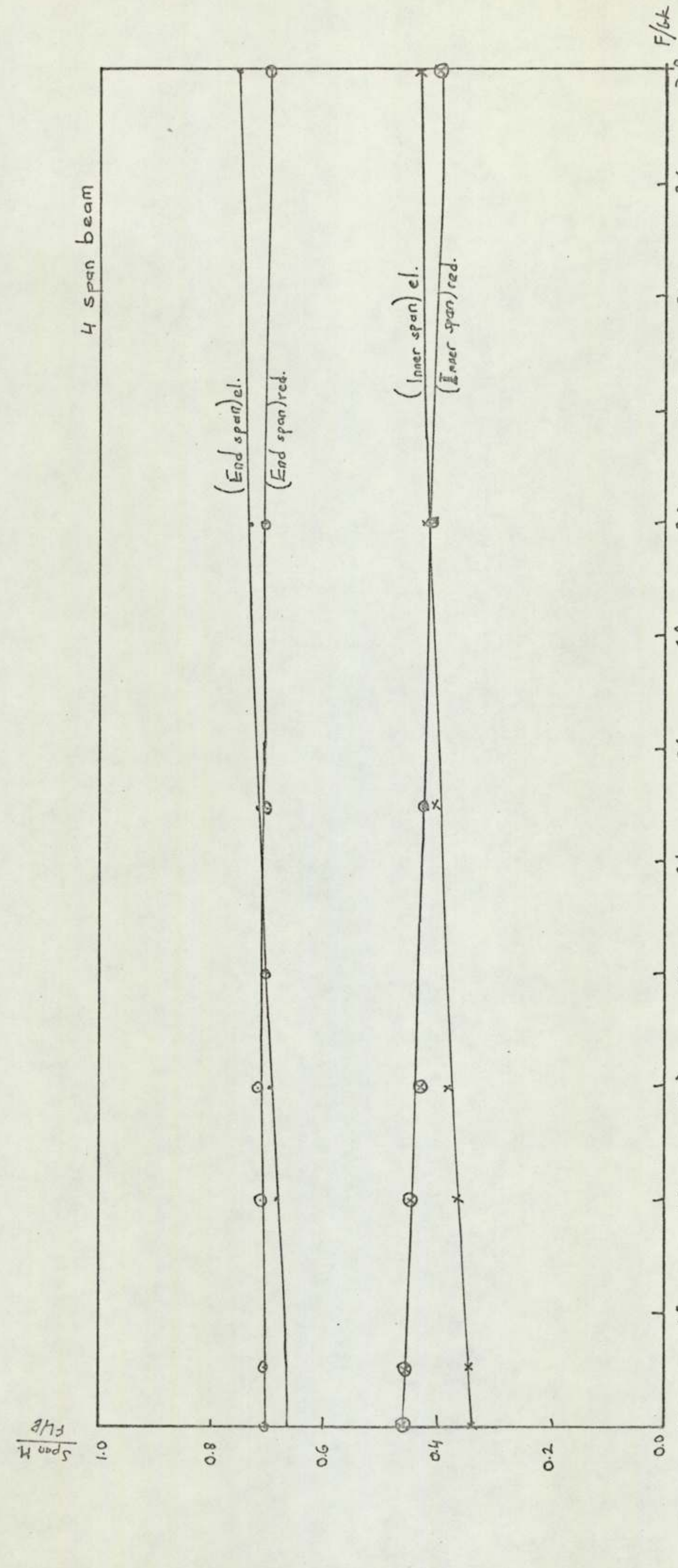

4 span beam
(End span)el.
(End span)red.
(Inner span) el.
(Inner span)red.
1.4
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.6
3.8
0.0
0.2
0.4
0.6
0.8
1.0

fig (5.19)

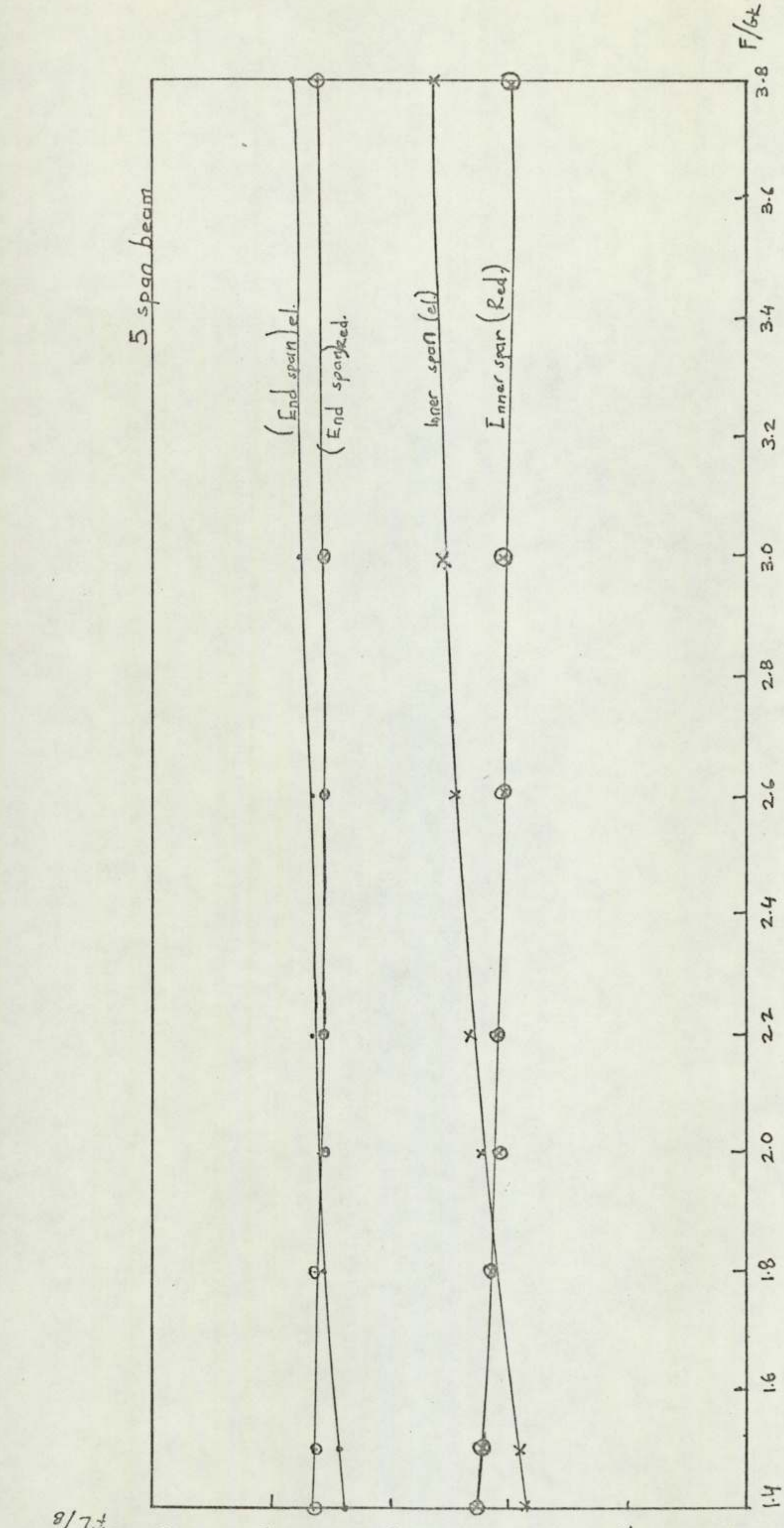

5 span beam
(End span) el.
(End span) Red.
Inner span (el.)
Inner span (Red.)
Span Moment / FL/8
1.0
0.8
0.6
0.4
0.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
F/bt

fig (5.20)

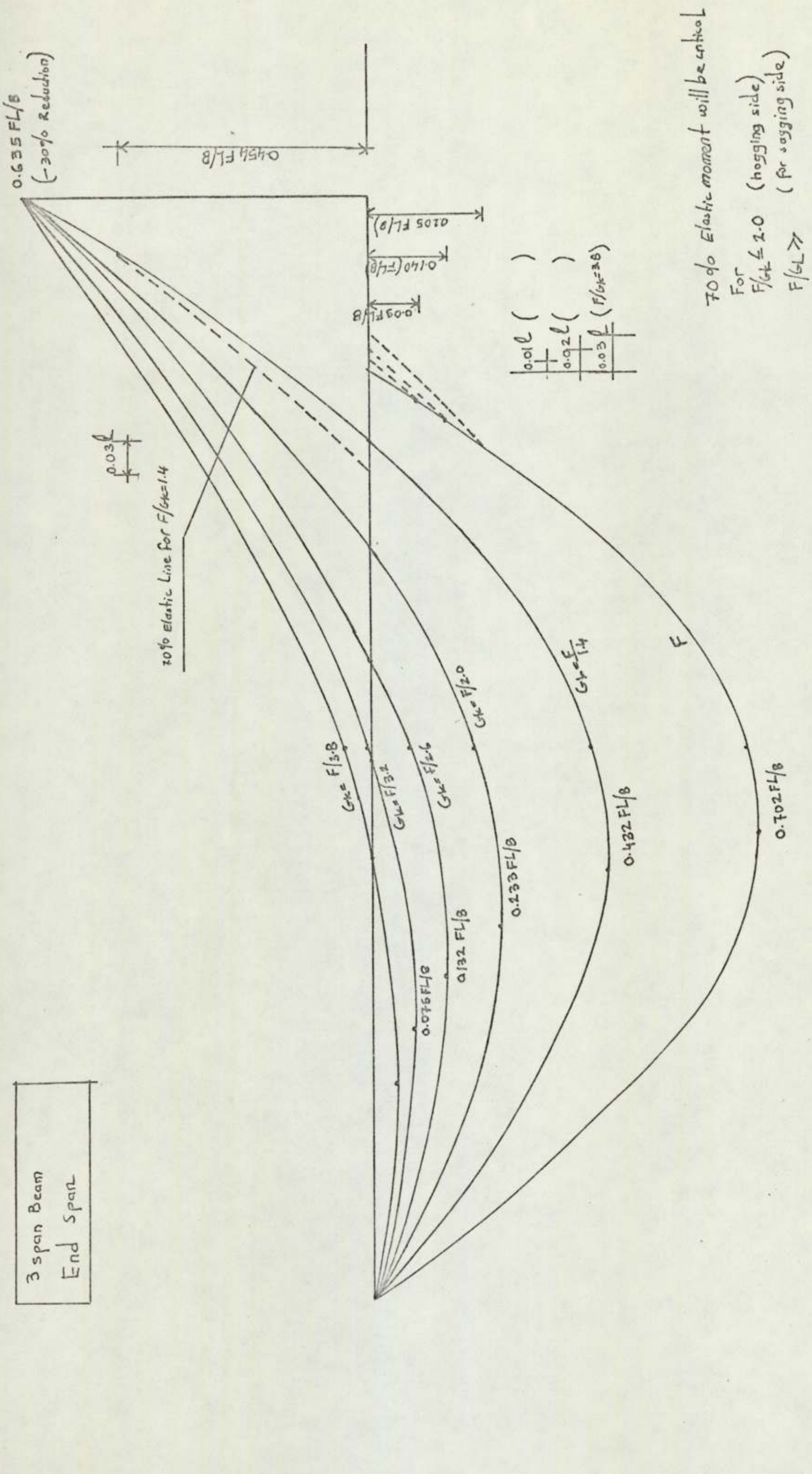

3 span Beam
End Span
0.635 FL/8
(-30% Reduction)
0.454 FL/8
0.205 FL/8
0.140 FL/8
0.09 FL/8
0.01 ℓ
0.02 ℓ
0.03 ℓ (F/GL=3.8)
0.03 ℓ
70% Elastic Line for F/GL=1.4
70% Elastic moment will be critical
For
F/GL ≤ 2.0 (hogging side)
F/GL > (for sagging side)
GL= F/3.8
GL= F/3.2
GL= F/2.6
GL= F/2.0
GL= F/1.4
F
0.075 FL/8
0.132 FL/8
0.233 FL/8
0.432 FL/8
0.702 FL/8

Fig 5.21

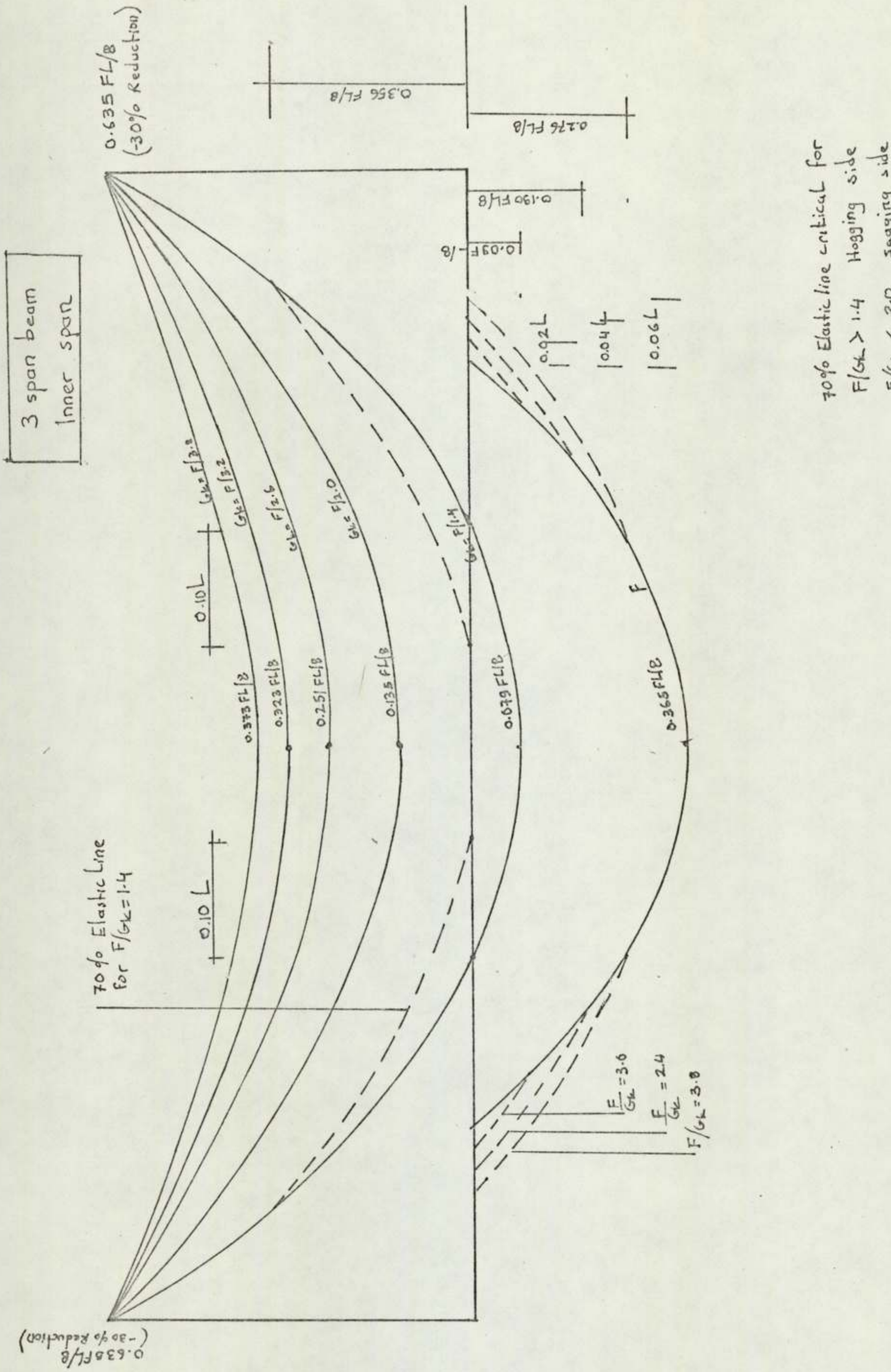

3 span beam
Inner span
0.635 FL/8
(-30% Reduction)
0.356 FL/8
0.276 FL/8
0.190 FL/8
0.09FL/8
0.02L
0.04L
0.06L
70% Elastic line critical for
F/GL > 1.4 Hogging side
F/GL < 2.0 sagging side
0.10L
GL = F/3.8
GL = F/3.2
GL = F/2.6
GL = F/2.0
GL = F/1.4
0.373 FL/8
0.323 FL/8
0.251 FL/8
0.135 FL/8
0.079 FL/8
0.365 FL/8
F
0.10 L
70% Elastic Line
for F/GL = 1.4
F/GL = 3.0
F/GL = 2.4
F/GL = 3.8
0.638 FL/8
(-30% Reduction)

Fig. (5.22)

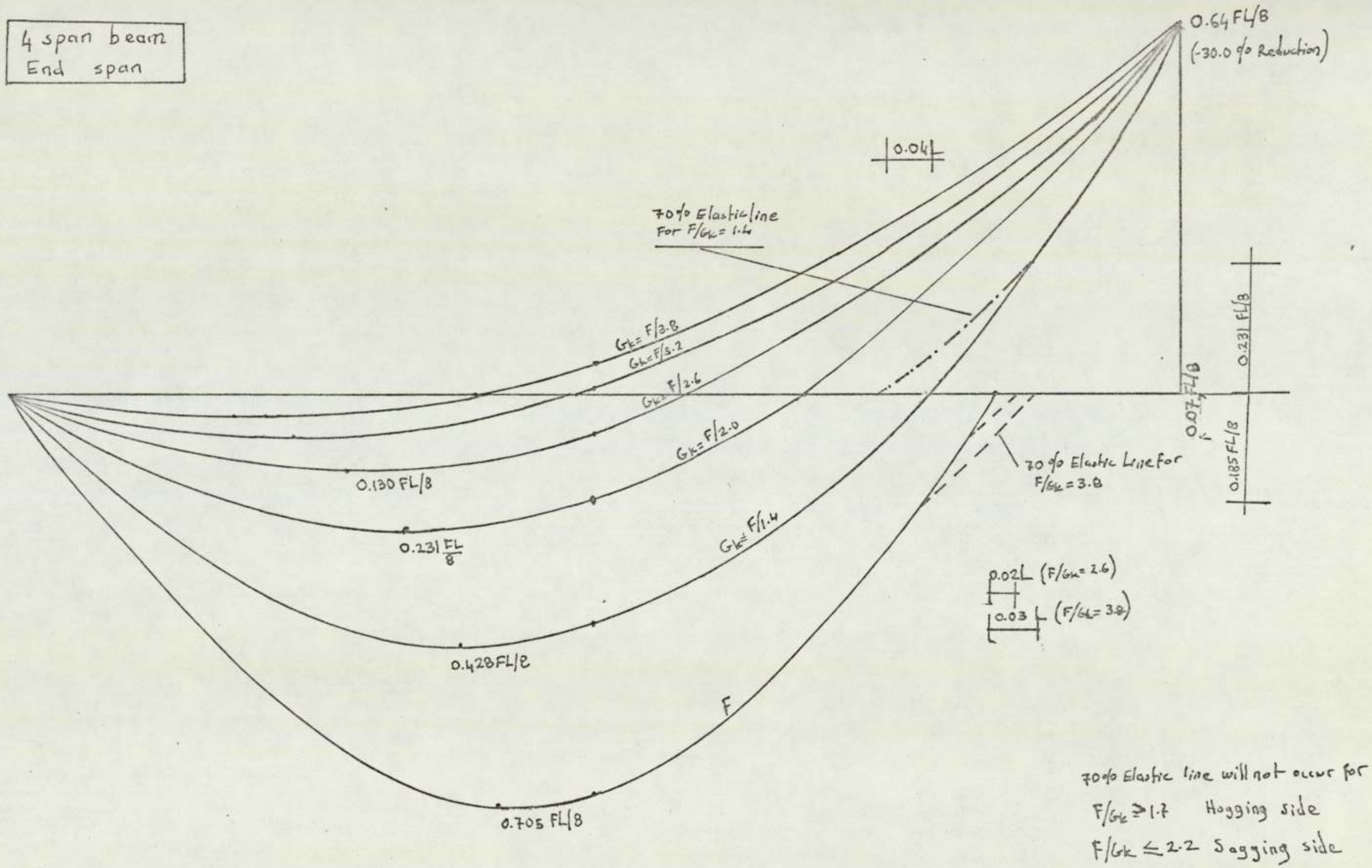

4 span beam
End span
0.64 FL/8
(-30.0 % Reduction)
0.04
70% Elastic line
For F/Gk = 1.4
Gk= F/3.8
Gk=F/3.2
Gk= F/2.6
Gk= F/2.0
Gk= F/1.4
F
0.130 FL/8
0.231 FL/8
0.428 FL/8
0.705 FL/8
0.07 FL/8
0.231 FL/8
0.185 FL/8
70% Elastic Line for
F/Gk = 3.8
0.02L (F/Gk= 2.6)
0.03 (F/Gk= 3.8)
70% Elastic line will not occur for
F/Gk ≥ 1.7 Hogging side
F/Gk ≤ 2.2 Sagging side

figura (5.23)

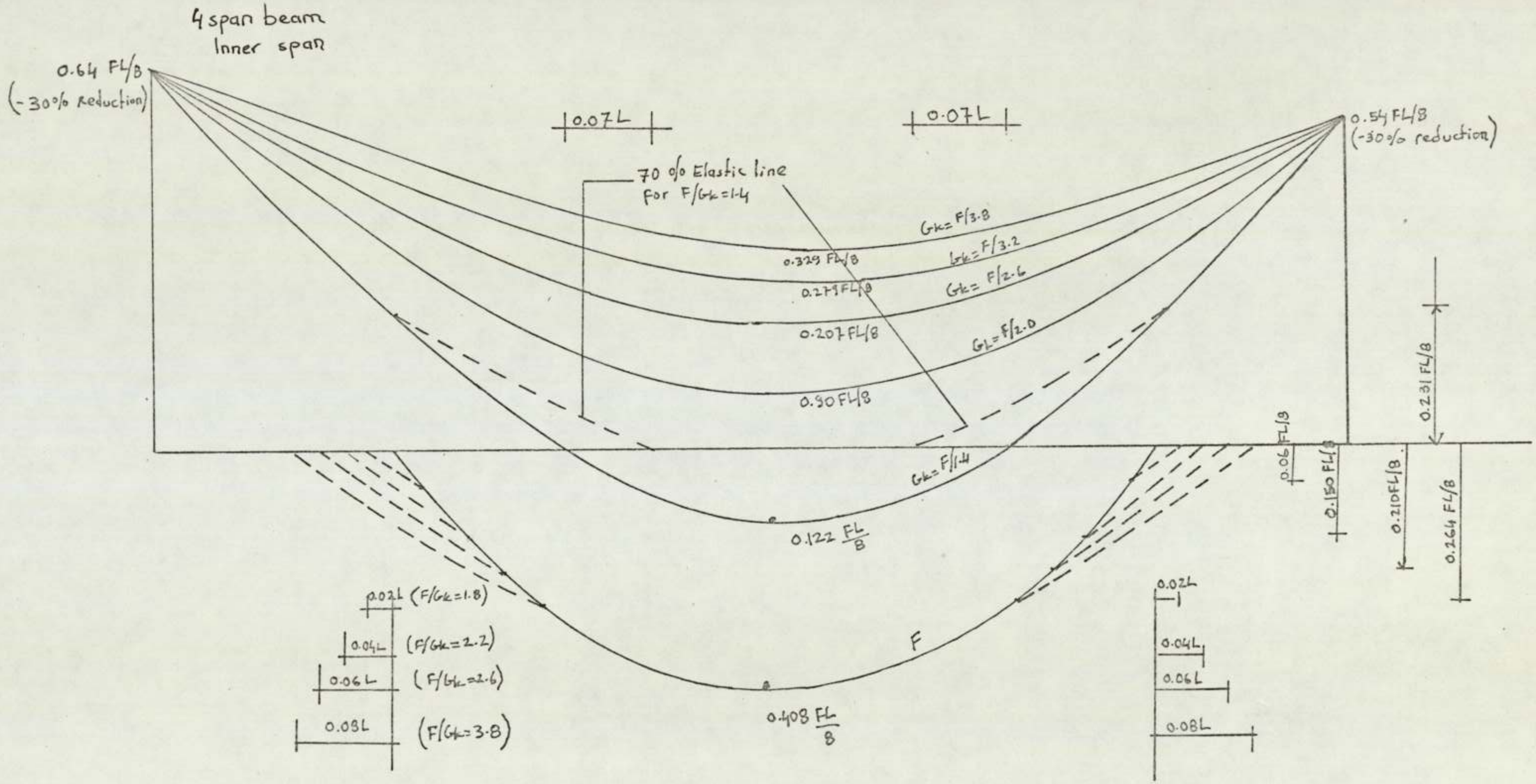



fig. 5.24

70% Elastic line will be critical for
$F/G_k \geq 1.8$ (Hogging side)
$F/G_k \leq 1.6$ (Sagging side)

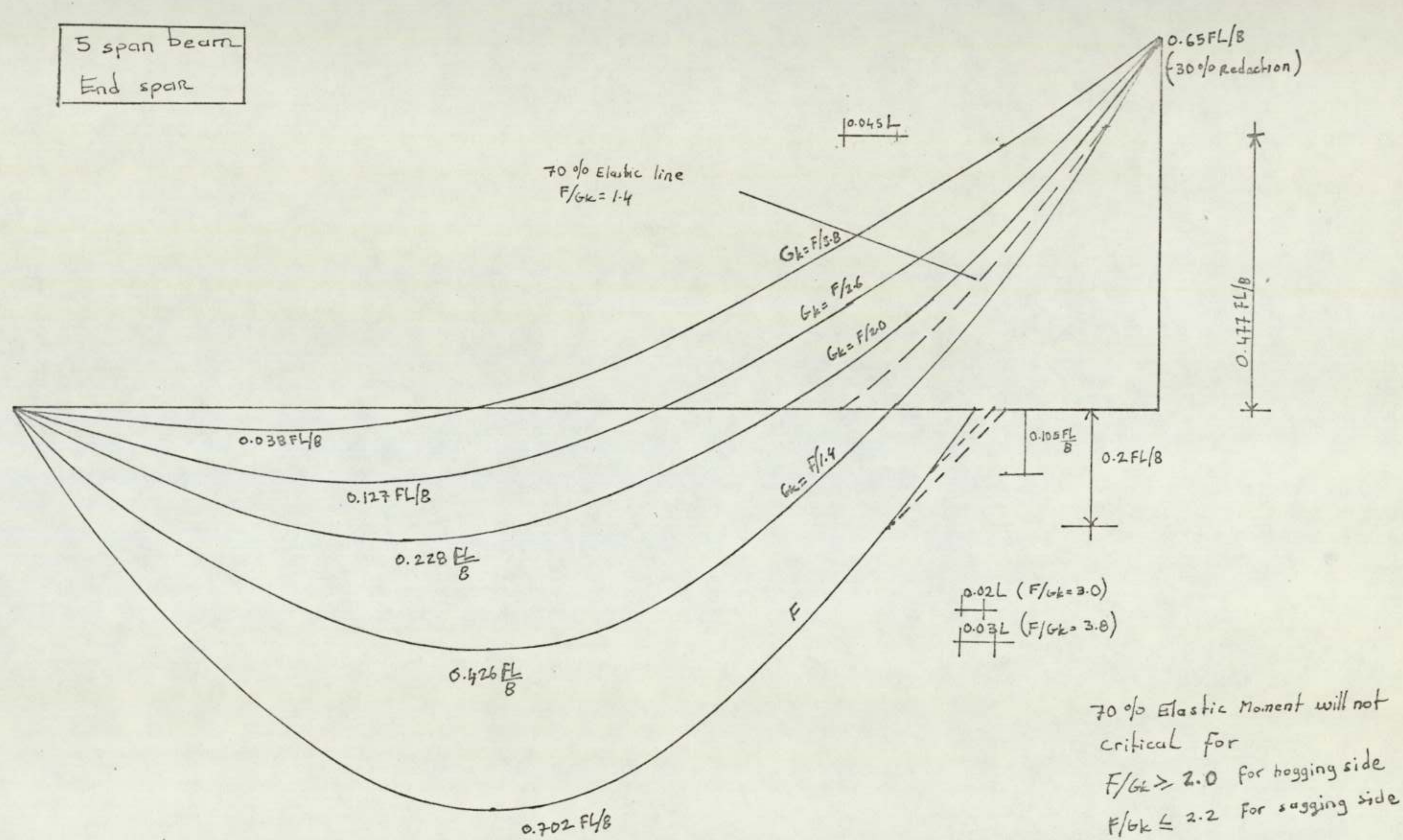
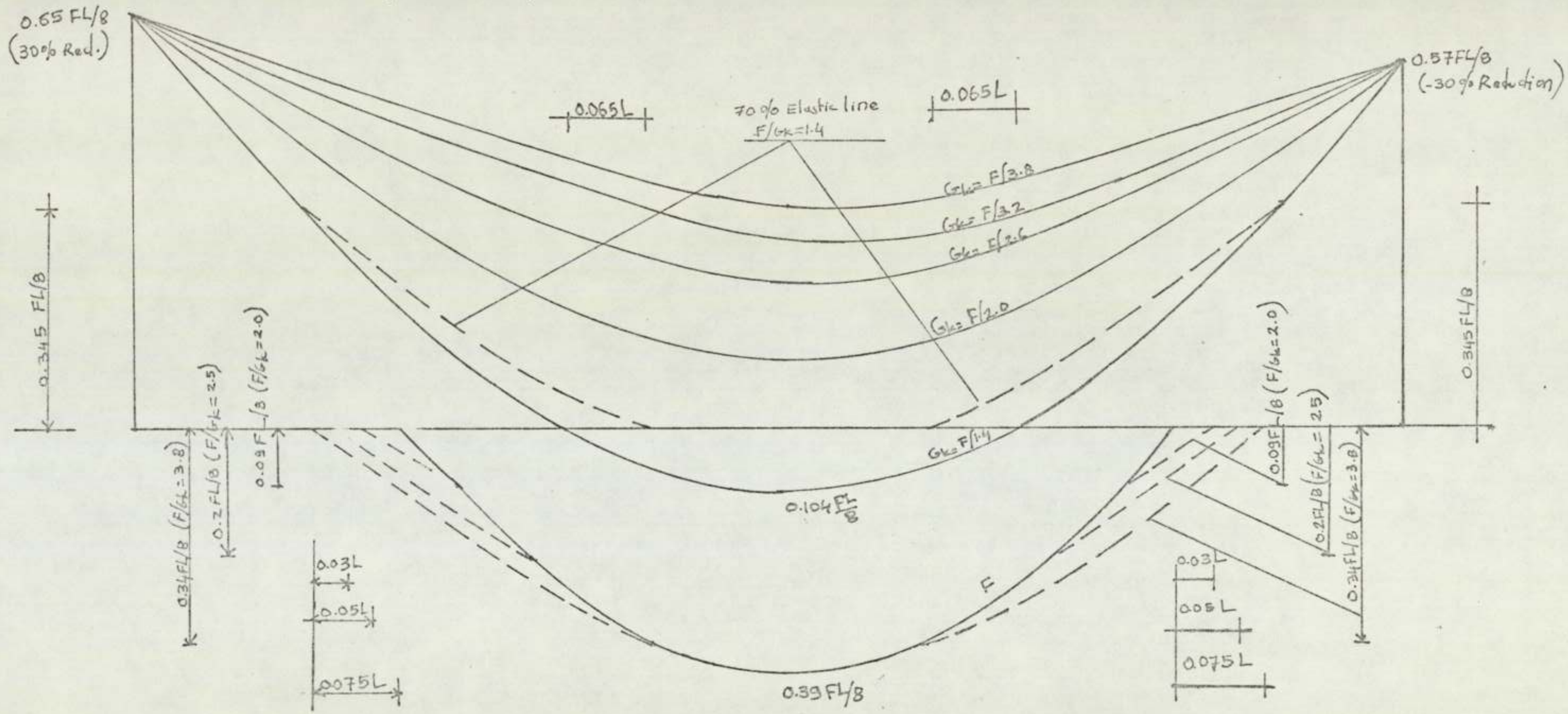

5 span beam
End span
0.65FL/8
(-30% Reduction)
0.045L
70 % Elastic line
F/Gk = 1.4
Gk=F/3.8
Gk= F/2.6
Gk= F/2.0
Gk= F/1.4
F
0.477 FL/8
0.105FL/8
0.2FL/8
0.038FL/8
0.127 FL/8
0.228 FL/8
0.426 FL/8
0.702 FL/8
0.02L (F/Gk=3.0)
0.03L (F/Gk=3.8)
70 % Elastic Moment will not
critical for
F/Gk ≥ 2.0 for hogging side
F/Gk ≤ 2.2 For sagging side

fig. (5.25)

5 span beam (inner span)




70% Elastic line will not occur for
$F/G_k \leq 1.69$ for sagging side
$F/G_k \geq 1.80$ for hogging side.

fig 5.26

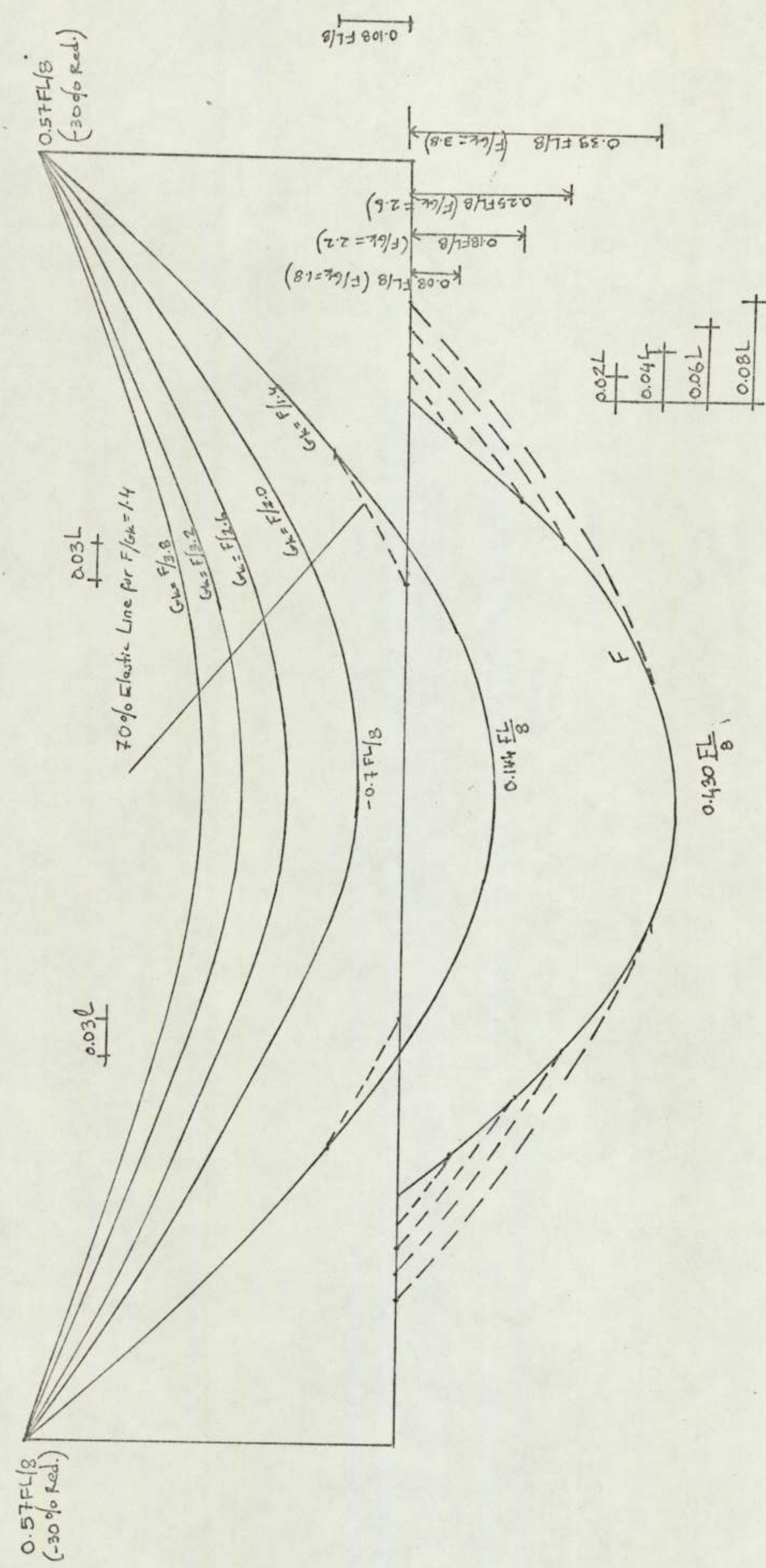



fig 5.27

70% Elastic line will not critical for
$F/G_k \geq 1.4$ Hogging side
$F/G_k \leq 1.7$ Sagging side

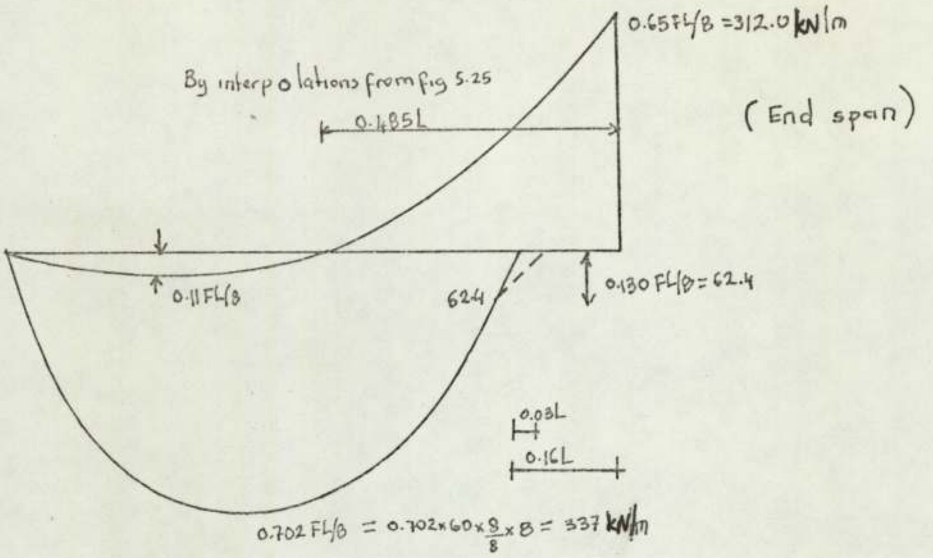



fig 5.28

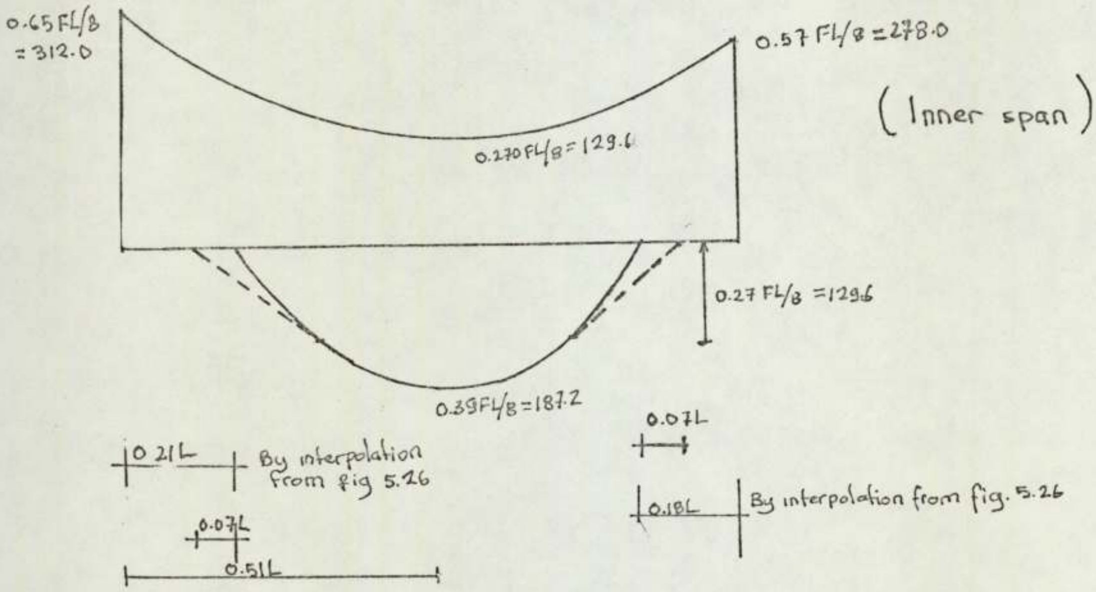



fig 5.29

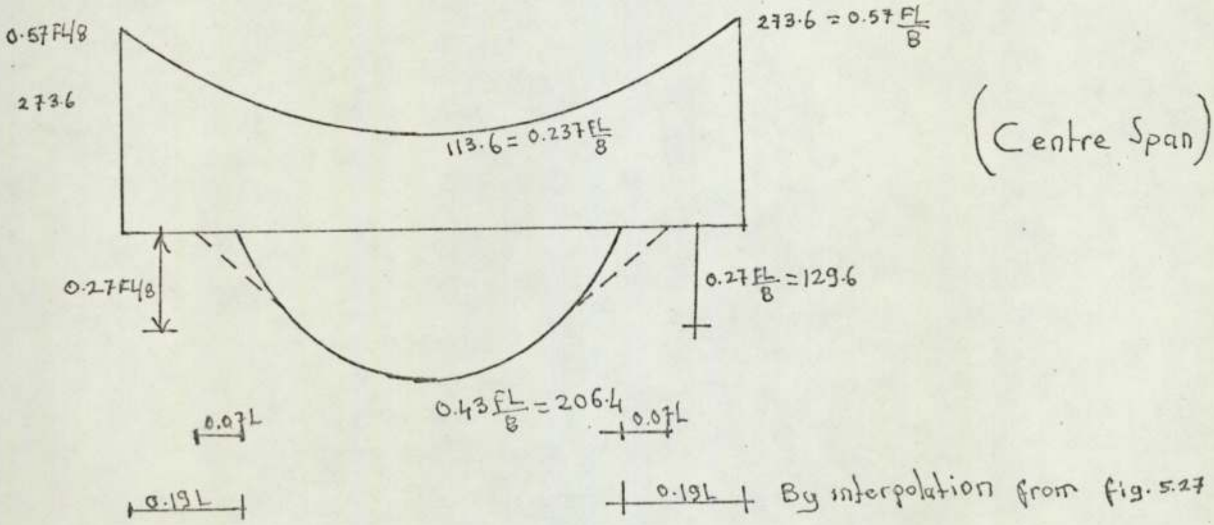



Fig 5.30

Span moments:

$0.702 \frac{FL}{8} = 0.702 \times 60 \times \frac{8}{8} = 337.$ kNm.

$0.390 \frac{FL}{8} = 0.390 \times 60 \times \frac{8}{8} = 187.2$ kNm.

$0.43 \frac{FL}{8} = 0.43 \times 60 \times \frac{8}{8} = 206.6$ kNm.

c) Trace the redistributed curve by using the ratio of the distance for contraflexure points from support to span, and maximum moment values (figures 5.28 to 5.30).

d) Plot the 70% length and figure 5.12 for hyper-plastic moment values for top and bottom of the beam.

## 5.4. ANALYSIS OF FRAMES

The same method which was used for beam analysis was extended to frameworks by including columns above and below the beams used. If we consider a 3 span frame which has end beams loaded with total load to collapse and middle span loaded with dead load only fig. 31,a). The unloaded span with its column may be considered as separately as shown on figure 31,b). It can now be seen that the tendency for the unloaded span to hog is reduced because the columns will assist in resisting the ultimate moments Mu and Mu from adjacent spans. When analysing the subframe shown on figure 30 b); it must be remembered that the beams at levels above and below may be loaded differently. The worst case for hogging of unloaded beam would be when the loads were placed so that the centre span of the three was carrying total load (F) and the two sides were carrying ($G_k$) as shown on figure (32.)

The difference between the redistributed bending moments and modified elastic bending moments are also critical near points of contraflexure. Cracking length, hyper-plastic moment, design efficiencies are tabulated from the elastic and redistributed bending moment diagrams

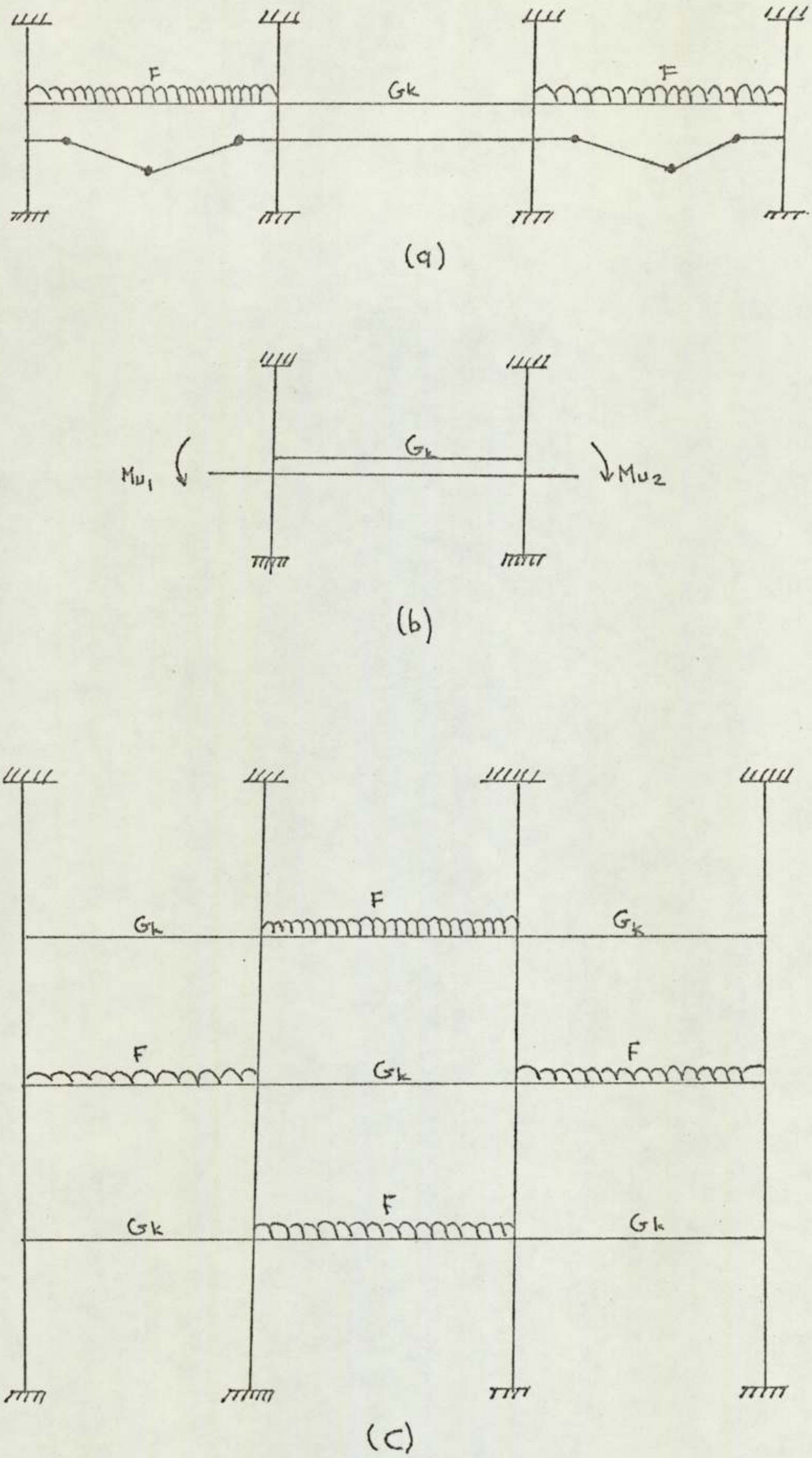



Fig 5.32

TABLE 10. 3 Span frame analysis

| NO. of Span | Dead Load $G_k$ | Vari-able Load$V_k$ | Total Load F | $\frac{F}{G_k}$ | Effic-ency | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|
| 2-5 | 20.0 | 8.0 | 28.0 | 1.4 | 0.869 | - | 1.0 | 0.24 | -0.02 |
| 5-8 | 20.0 | 8.0 | " | " | 0.842 | - | 1.0 | 0.25 | -0.03 |
| 2-5 | 20.0 | 10.0 | 30.0 | 1.5 | 0.865 | - | 0.131 | 0.04 | -0.01 |
| 5-8 | " | " | " | " | 0.839 | - | 0.069 | 0.06 | -0.02 |
| 2-5 | 20.0 | 14.0 | 34.0 | 1.7 | 0.860 | - | - | - | - |
| 5-8 | 20.0 | 14.0 | " | " | 0.832 | - | 0.024 | 0.02 | -0.01 |
| 2-5 | 20.0 | 16.0 | 36.0 | 1.8 | 0.857 | - | - | - | - |
| 5-8 | " | " | " | " | 0.825 | - | - | - | - |
| 2-5 | " | 20.0 | 40.0 | 2.0 | 0.856 | - | - | - | - |
| 5-8 | " $\frac{7}{8}$ | " | " | " | 0.820 | - | - | - | - |
| 2-5 | " | 24.0 | 44.0 | 2.2 | 0.852 | - | - | - | - |
| 5-8 | " | " | " | " | 0.816 | - | - | - | - |
| 2-5 | " | 28.0 | 48.0 | 2.4 | 0.851 | - | - | - | - |
| 5-8 | " | " | " | " | 0.813 | - | - | - | - |
| 2-5 | " | 30.0 | 50.0 | 2.5 | 0.850 | - | - | - | - |
| 5-8 | " | " | " | " | 0.807 | - | - | - | - |
| 2-5 | " | 32.0 | 52.0 | 2.6 | 0.849 | - | - | - | - |
| 5-8 | " | " | " | " | 0.806 | - | - | - | - |
| 2-5 | " | 40.0 | 60.0 | 3.0 | 0.847 | - | - | - | - |
| 5-8 | " | " | " | " | 0.803 | - | - | - | - |
| 2-5 | " | 42.0 | 62.0 | 3.1 | 0.845 | - | - | - | - |
| 5-8 | " | " | " | " | 0.802 | - | - | - | - |
| 2-5 | " | 56.0 | 76.0 | 3.8 | 0.842 | - | - | - | - |
| 5-8 | " | " | " | " | 0.797 | - | - | - | - |

TABLE 11. 4 Span frame.

| Span No. | Dead Load | Variable Load | Total Load | $\frac{F}{G_k}$ | Efficency | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|
| 2-5 | 20.0 | 8.0 | 28.0 | 1.4 | 0.861 | - | 1.00 | 0.19 | -0.01 |
| 5-8 | " | " | " | " | 0.844 | - | - | - | - |
| 2-5 | " | 10.0 | 30.0 | 1.5 | 0.860 | - | 0.004 | 0.01 | -0.01 |
| 5-8 | " | | " | | 0.842 | - | - | - | - |
| 2-5 | " | 14.0 | 34.0 | 1.7 | 0.859 | - | - | - | - |
| 5-8 | " | 14.0 | " | 1.7 | 0.841 | - | - | - | - |
| 2-5 | " | 16.0 | 36.0 | 1.8 | 0.848 | - | - | - | - |
| 5-8 | " | " | | " | 0.838 | - | - | - | - |
| 2-5 | " | | 40.0 | 2.0 | 0.856 | - | - | - | - |
| 5-8 | " | | | " | 0.833 | - | - | - | - |
| 2-5 | " | | 44.0 | 2.2 | 0.841 | - | - | - | - |
| 5-8 | " | | " | " | 0.817 | - | - | - | - |
| 2-5 | " | | 50.0 | 2.5 | 0.848 | - | - | - | - |
| 5-8 | " | | " | " | 0.825 | - | - | - | - |
| 2-5 | " | | 52.0 | 2.6 | 0.836 | - | - | - | - |
| 5-8 | " | | " | " | 0.809 | - | - | - | - |
| 2-5 | " | | 60.0 | 3.0 | 0.843 | - | - | - | - |
| 5-8 | " | | " | " | 0.819 | - | - | - | - |
| 2-5 | " | | 62.0 | 3.1 | 0.842 | - | - | - | - |
| 5-8 | " | | " | " | 0.818 | - | - | - | - |
| 2-5 | " | | 76.0 | 3.8 | 0.841 | - | - | - | - |
| 5-8 | " | | " | " | 0.817 | - | - | - | - |

TABLE 12. 5 Span Frame

| NO. of Span. | Dead Load | Variable Load | Total Load | $F/G_k$ | Efficency | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|
| 2-5 | 20.0 | 8.0 | 28.0 | 1.4 | 0.870 | - | 1.00 | 0.25 | -0.03 |
| 5-8 | " | " | " | " | 0.841 | | 0.106 | 0.05 | -0.02 |
| 2-5 | 20.0 | 10.0 | 30.0 | 1.5 | 0.864 | - | 0.337 | 0.11 | -0.03 |
| 5-8 | " | " | " | " | 0.836 | - | - | - | - |
| 2-5 | 20.0 | 14.0 | 34.0 | 1.7 | 0.860 | - | - | - | - |
| 5-8 | " | " | " | " | 0.829 | - | - | - | - |
| 2-5 | 20.0 | 16.0 | 36.0 | 1.8 | 0.859 | - | - | - | - |
| 5-8 | " | " | " | " | 0.826 | - | - | - | - |
| 2-5 | 20.0 | 20.0 | 40.0 | 2.0 | 0.856 | - | - | - | - |
| 5-8 | " | " | " | " | 0.820 | - | - | - | - |
| 2-5 | 20.0 | 24.0 | 44.0 | 2.2 | 0.852 | - | - | - | - |
| 5-8 | " | " | " | " | 0.813 | - | - | - | - |
| 2-5 | 20.0 | 28.0 | 48.0 | 2.4 | 0.850 | - | - | | |
| 5-8 | " | " | " | " | 0.805 | - | - | | |
| 2-5 | 20.0 | 32.0 | 52.0 | 2.6 | 0.848 | - | - | | |
| 5-8 | " | " | " | " | 0.808 | - | - | | |
| 2-5 | " | 40.0 | 60.0 | 3.0 | 0.844 | - | - | | |
| 5-8 | " | " | " | " | 0.800 | - | - | | |
| 2-5 | " | 42.0 | 62.0 | 3.1 | 0.843 | - | - | | |
| 5-8 | " | " | " | " | 0.800 | - | - | | |
| 2-5 | " | 56.0 | 76.0 | 3.8 | 0.840 | - | - | | |
| 5-8 | " | " | " | " | 0.793 | - | - | | |

TABLE 13

| Number of Span | $\frac{F}{G_k}$ | 1/Efficency | |
|---|---|---|---|
| | | End spans | Inner spans |
| 3 Span | 1.4 | 1.150 | 1.187 |
| 4 Span | " | 1.161 | 1.184 |
| 5 Span | " | 1.149 | 1.189 |
| 3 Span | 1.5 | 1.156 | 1.194 |
| 4 Span | " | 1.162 | 1.187 |
| 5 Span | " | 1.157 | 1.196 |
| 3 Span | 1.7 | 1.162 | 1.201 |
| 4 Span | " | 1.164 | 1.189 |
| 5 Span | " | 1.162 | 1.206 |
| 3 Span | 2.0 | 1.160 | 1.219 |
| 4 Span | " | 1.168 | 1.200 |
| 5 Span | " | 1.168 | 1.219 |
| 3 Span | 2.2 | 1.173 | 1.225 |
| 4 Span | " | 1.170 | 1.211 |
| 5 Span | " | 1.173 | 1.230 |
| 3 Span | 2.5 | 1.176 | 1.239 |
| 4 Span | " | 1.179 | 1.212 |
| 5 Span | " | 1.177 | 1.231 |
| 3 Span | 3.0 | 1.180 | 1.245 |
| 4 Span | " | 1.186 | 1.221 |
| 5 Span | " | 1.184 | 1.250 |
| 3 Span | 3.8 | 1.187 | 1.254 |
| 4 Span | " | 1.189 | 1.223 |
| 5 Span | " | 1.190 | 1.261 |

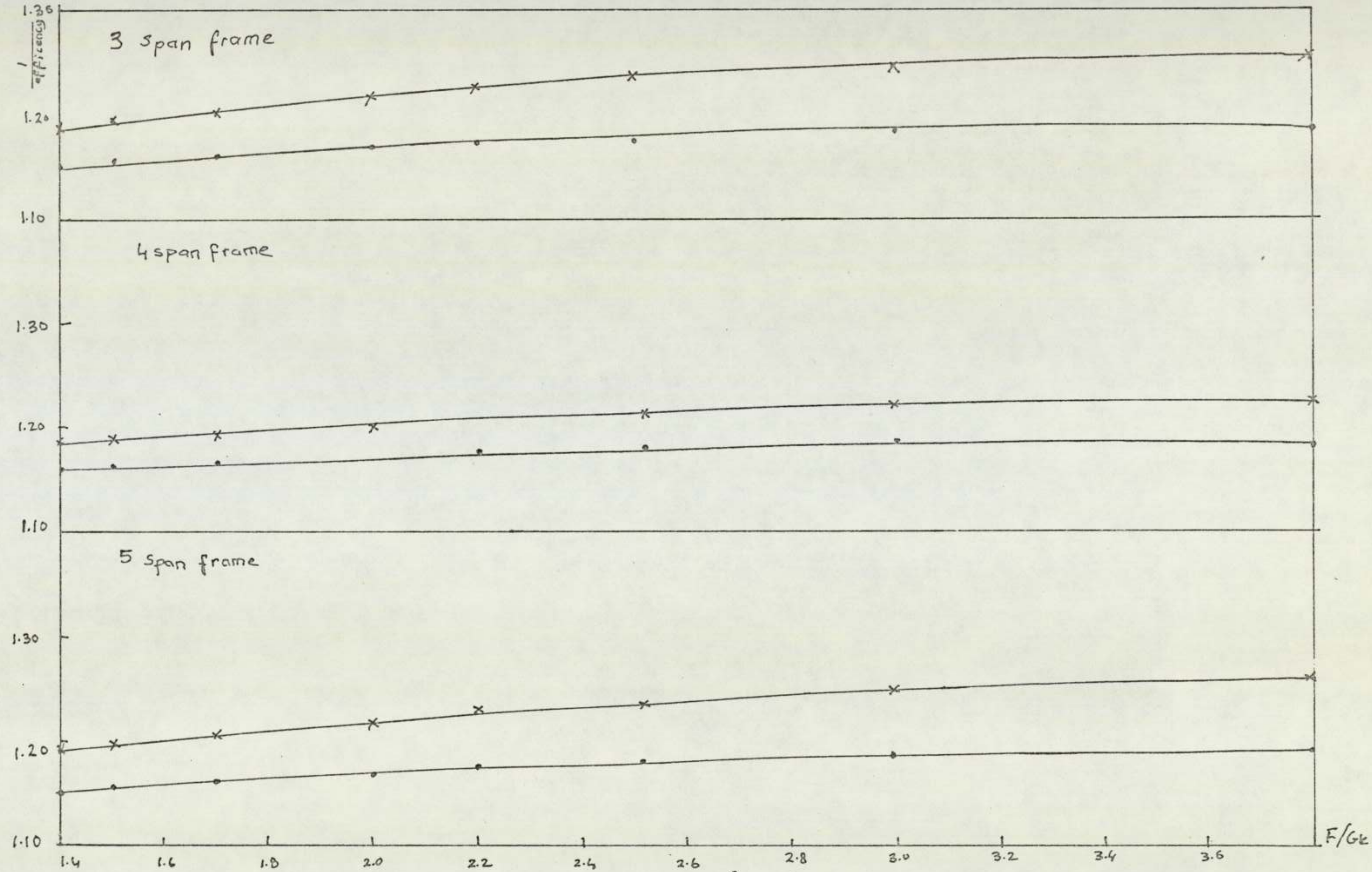

3 span frame
4 span frame
5 span frame
1/efficiency
1.30
1.20
1.10
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
F/Gk

fig 5.36

given in Appendix I figures (A33 - A72) on tables (10 - 12). It will be seen from figures and tables that:

a) Critical points appear only for small variable loads (i.e. $\frac{Vk}{Gk}$ = 0.5, 0.6, 0.7 for 3 span frame; $\frac{V_k}{Gk}$ = 0.4, 0.5 for 4 span and 5 span frame).

b) There are no critical points for the bottom side of the frame.

c) Maximum cracking length/span is equal to (-0.02) for 3 span frame, (-0.01) for 4 span frame and (-0.03) for 5 span frame on the top-side for end spans.

d) Maximum cracking length for inner span is equal to (-0.03) for 3 span and 5 span frame (-0.01) for 4 span frame.

e) Design efficency is bigger at the end span than the inner span. The number of spans also influence the efficency of the design.

g) Efficency has a relation with $F/G_k$ ratio because $\frac{1}{\text{efficency}}$ increase proportionally against $F/G_k$ ratio (fig. 3.6, table 13).

## 5.5. COMPARISON OF VARIOUS METHODS FOR THE DESIGN OF CONTINUOUS BEAMS

For the purposes of illustration the example of a 5 span continuous beam will be considered in Fig. (38a) but the discussion will be kept as general as possible.

a) Elastic Design (Limit State Design)

Limit state method was employed to find the bending moments of 5 span beam. Bending moment envelope (fig. 38b) as drawn from the bending moment values of elastic design (Table 14).

b) A. L. L. BAKER METHOD (Limit design method)

It is assumed that, the beam has constant flexural rigidity and the total load of $(G_k + \lambda Q_k)$ which equals $(W + \lambda u)p = 3W$

in the example. The compatibility equations at sections 2 and 4 are as follows:

At 2 : $-\frac{2}{3}Mo + \frac{2}{3}\bar{X} + \frac{1}{6}\bar{X} = -\theta\ EI$

At 4: $-\frac{2}{3}Mo + \frac{1}{6}\bar{X} + \frac{2}{3}\bar{X} + \frac{1}{6}\bar{X} = -\theta\ EI$

$\theta_i$ must have the same sign as $\bar{X}_i$ (i.e. positive here). In order to obtain small values for $\theta_i$ obviously large values of $\bar{X}i$ should be used and maximum allowable $\theta_i$ may be obtained by using the min. value of $X_i$

A. L. L. BAKER recommended the following rule which states that "in coninuous beams of approximately equal span supporting uniformly distributed load the support moments (except penultimate) may be assumed equal to the mid-span moments provided the permissible value of $\theta$ for the support section is greater than $\frac{0.16\ Mo}{EI}$ and for external span does not exceed $\frac{0.25\ Mo}{EI}$ ". i.e. at all support sections and internal span sections the moment of resistance equals $\frac{Mo}{2}$ and at external span the moments of resistance is equal to $\frac{3}{4}Mo$, and for these values:

$$\theta_2 = \frac{Mo\,\ell}{4EI} \quad \text{and} \quad \theta_4 = \frac{Mo\,\ell}{6EI} . \qquad \theta_2 \text{ and } \theta_4$$

are assumed to be $< \theta pi$

$$X_2 = \bar{X}_4 = \bar{X}_3 = \bar{X}_5 = \frac{Mo}{2} = \frac{3w\,\ell 2}{16}$$

$$X_1 = \frac{3}{4}Mo = \frac{9}{32}w\,\ell^2$$

Redistribution method: The example which is shown on fig. 38a also designed by this method and results are tabulated on table (14) by using the computer programming.

## Optimum Method

Values have been determined for both the moment of resistance at each critical section as well as the minimum working load moments possible at each section. These values are tabulated in table (13 - 14) with the results of the other methods.

COMPARISON

It is seen from table (13 - 14) that elastic solutions are very conservative. Apart from the external spans the values are at least 27% greater than redistributed values and 33% greater than the equivalent optimum vales. For the external spans this is down to 5% for redistributed vales and to 9% for optimum values. This elastic solution satisfies both working load and ultimate load conditions. However it is seen that the A. L. L. BAKER method solution, although satisfying all the ultimate load criteria does not comply with the condition that at working loads the behaviour should be elastic everywhere refering to fig. 38 not only at section 2 but also at section 4 (i.e. at all support sections). Plastic hinges will occur according to the plastic theory of structures.

The optimum solution satisfies all the criteria of both working and ultimate load conditions. The essential difference between those two methods is that:- the optimum solution considers all loading conditions (i.e. Working, first yield at all hinges and ultimate load) and the rotation compatibility is finally checked. The A. L. L. Baker method depends purely on ultimate load criteria and an independant rotation condition the working load criteria being checked finally. The A. L. L. Baker method has the advantage that the variation in maximum hogging and sagging moments may be reduced to a minimum so that a uniform section may be designed. In reinforced concrete this advantage is not very important, however, without varying the concrete section, quite a range of moments may be covered by varying the steel.

The disparity in sectional properties is such that at supports the moment of resistance varies between 10% to 21% less than

TABLE 14. Comparison with Redistributed Method

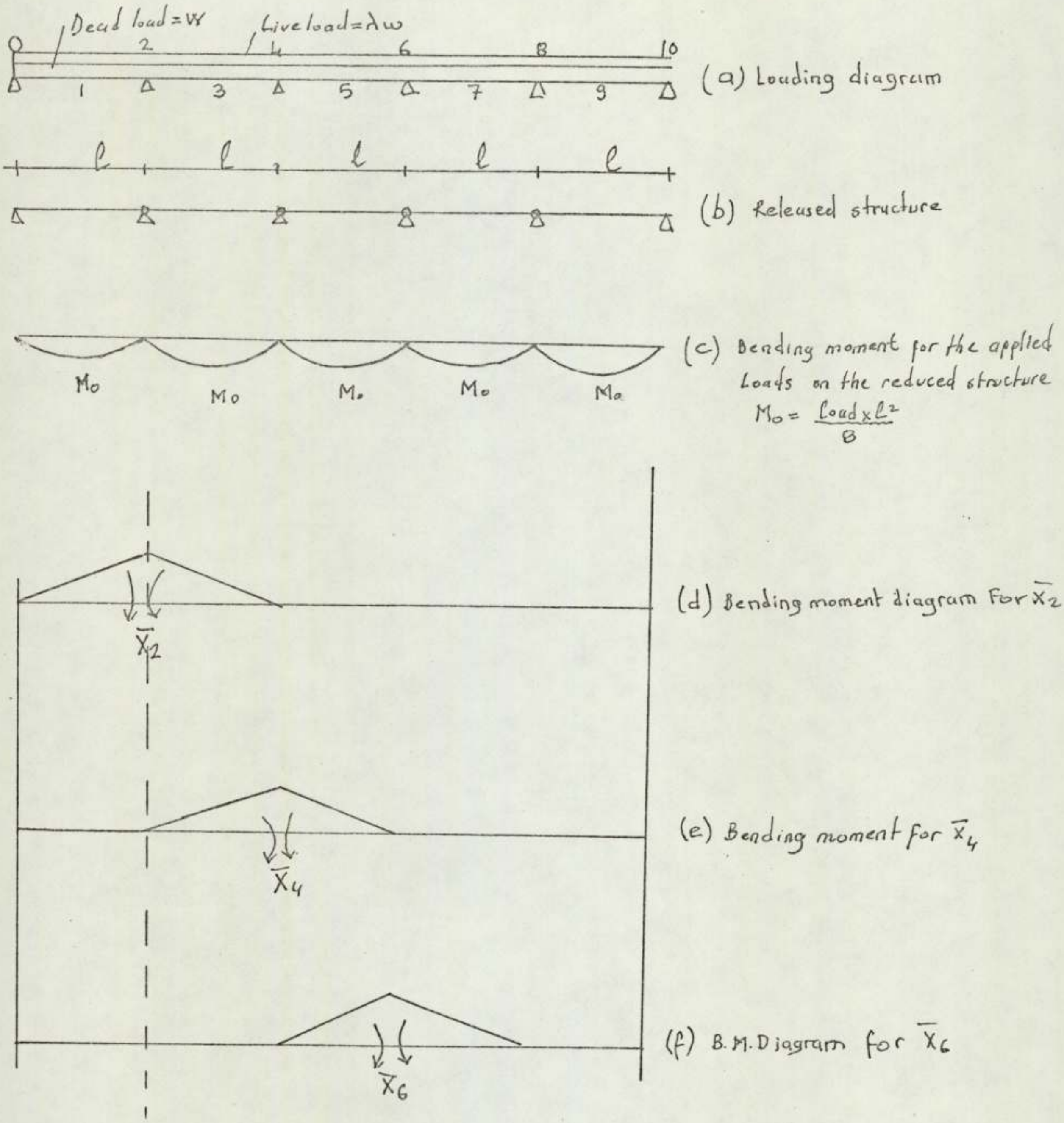
| Design Method | Sections | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| Limit State Design (Elastic Design) | 0.277 (5%) | 0.342 (43%) | 0.191 (27%) | 0.299 (43%) | 0.217 (31%) |
| A. L. L. BAKER METHOD | 0.281 (6%) | 0.1875 (-21%) | 0.1875 (24%) | 0.1875 (-10%) | 0.1875 (+13%) |
| OPTIMUM DESIGN | 0.251 (-4%) | 0.259 (8%) | 0.135 (-10%) | 0.221 (5%) | 0.153 (-4%) |
| Redistribution Method | 0.264 | 0.239 | 0.150 | 0.209 | 0.165 |
| Proposed Method | 0.263 (-0.3%) | 0.243 (+1.6%) | 0.146 (-2%) | 0.213 (+1.9%) | 0.161 (-2%) |

$G_k$ = 20 kN/m    L = 8.0 m

* (Moment values were divided by $GkL^2$ in tables)

TABLE 15. Comparison with optimum design method

| Design Method | Sections | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| Limit State Design (Elastic Design) | 0.277 (9%) | 0.342 (33%) | 0.191 (41%) | 0.299 (36%) | 0.217 33%) |
| A. L. L. BAKER METHOD | 0.281 (12%) | 0.1875 (-28%) | 0.1875 (39%) | 0.1875 (-15%) | 0.1875 (23%) |
| OPTIMUM DESIGN | 0.251 | 0.259 | 0.135 | 0.221 | 0.153 |
| Redistributed Method | 0.264 (5%) | 0.239 (-8%) | 0.150 (10%) | 0.209 (-5%) | 0.165 (+7.8%) |
| Proposed Method | 0.263 (+4.7%) | 0.243 (-6.1%) | 0.146 (+8%) | 0.213 (-3.6%) | 0.161 (+5.2%) |




$G_k = w = 20$ kN/m    $F/G_k = 3.0$

$F = 60.0$ kN/m

Figure 38

redistributed values and 12 to 39% greater than optimum values. However, the plastic rotation criterion for optimum design at the support section may be less than the maximum permissible value and it will also be much less than the A. L. L. Baker method values. As regards economy there will be little to choose between them. The fact that the greater bending moment values occur at the support section where as the change of moment is greater at mid-span is advantageous; overall rotations and deformations are less which is a point in favour of optimum design. It will be seen from table 14 that Redistributed support moment values are smaller than optimum design values (5 to 8%) and the redistributed span moment is 4 to 10% larger than optimum design values. This is a point in favour of redistributed design. It was also shown by A. W. Astill (1973) that redistributed design is quicker than others. It was also shown in tables (14 - 15) that there is a small difference between redistributed values and the proposed method values ((-2.0) to (+1.9) percent). This is reasonable in application as the proposed method may be quicker and simpler by using the given charts.

CONCLUSIONS

(i) Cracking length, hyper-plastic moment is dependant on the $F/G_k$ ratio. Cracking length and hyper-plastic moment increase at sagging side but decrease at hogging side by increasing $F/G_k$. It was also seen that cracking length and hyper-plastic moment is bigger for inner span than end span at sagging side.

(ii) Support moment/FL /8 values for different $F/G_k$ nearly gives a straight line, and there is not a big difference between the minimum and maximum value which can be accepted as constant depending on support section (penultimate, inner) and number of spans (3 span, 4 span, 5 span).

(iii) Elastic and redistributed span moment/FL /8 also gives a relation against $F/G_k$. Elastic moment/FL /8 increase and redistributed bending moment/FL /8 decrease by increasing $F/G_k$.

(iv) Reciprocal of efficency index will be increased when $F/G_k$ increase and it is give a curve against $F/G_k$

(v) Cracking length and hyper-plastic moment was not very critical for frame design for the ratio of beam to column stiffness considered. Reciprocal of efficencies gives a curve against $F/G_k$.

(vi) Using the information obtained the standard Bending moment curves of Fig. (5.21 to 5.27) can be plotted so that for normal values of $F/G_k$ ratios and 3 span or greater are full structural analysis is not necessary for each case. The standard charts can be used for the design of reinforced concrete beams with up to 30% redistribution of moments thus achieving the economies of materials without the eccessive costs of a full analysis.

REFERENCES

1. ALLEN A.H. "Reinforced Concrete Design to CP 110" Cement and Concrete Assoc. Pub. 1974

2. ASTILL A.W. "Residtribution of Moments" Concrete, Oct, 1973, p.g 46 - 48

3. BAKER A.L.L. "A Plastic Theory of Design for Ordinary Reinforced and Prestressed Concrete, including Moment Redistribution in Concrete Members". Magazin of Concrete Research (LONDON), V.1. No:2, June 1949 pp. 57 - 66

4. BAKER A.L.L. "Recent Research in Reinforced Concrete and Its application to Design" Journal of the Institution of Civil Engineers (LONDON), V.35, No:4, Feb. 1951, pp 262 - 298

5. BAKER A.L.L. "Further Research in Reinforced Concrete and Its Application to Ultimate Load Design" Proc. of Inst. Civ. Eng. (LONDON) V.2, Part 3 No:2, Aug. 1953, pp 269 - 310.

6. BAKER A.L.L. "Recent Research at Imperial College", The REinforced Concrete Review. (LONDON) V.3 (1954) pp 313 - 353

7. BAKER A.L.L. "High Building Frames and Foundations". Proc. Of Inst. Civ. Eng. V.4 Part 3 No:2, Aug. 1955, pp 228 - 245

8. BAKER A.L.L. "Ultimate Load Design of Reinforced and Prestressed Concrete Frames". Symp. on the Strength of Concrete Structures, Cement and Concrete Assoc. (1956)

9. BAKER A.L.L. "The Ultimate Load Theory Applied to Design of Reinforced and Prestressed Concrete Frames" Concrete Publication Limited (LONDON) 1956

10. BAKER A.L.L. "A General Analysis of Elasto-plastic Three Dimensional Frames", Publication of International Assoc. of Bridge and Structural Engineering (Zurich) 1960

11. BAKER A.L.L. "A General Equation for Frame Analysis" Concrete and Constructional Engineering (LONDON) V.56, No:3, March 1961, pp 262 - 298

12. BAKER A.L.L. "Maximum Errors in Elastic and Limit Theories" Bulletin d information CEB (Paris) No:34 Oct. 1961

13. BAKER A.L.L. and AMARAKONE A.M.N. "Inelastic Hyperplastic Frame Analysis" International Symposium on Flexural Mechanics of Reinforced Concrete, (Miami) Florida, Nov. 1964, ACJ special publication SP12

14. BAKER A.L.L. and AMARAKONE A.M.N. "Inelastic Hyperplastic Frame Analysis" C.E.B. Bulletin No:52 Nov. 1965

15. BAKER A.L.L. "Limit State Design of Reinforced Concrete" Concrete Publications (1970)

16. BECKETT D. "Limit State Design" Surrey University Press (1974)
17. CHAN W.W.L. "An Investigation of the Characteristics of Plastic Hinges in Reinforced Concrete" PhD. Thesis, University of London (1954)
18. CHAN W.W.L. "The Ultimate Strength and Deformation of Plastic (LONDON) V.7 No:21, Nov. 1955, pp 121 - 132
19. CHAN W.W.L. "The Rotation of Reinforced Concrete Plastic hinges at Ultimate Load" Magazine of Concrete Research Vo. 14 No:41 July 1962 pp 63 - 72
20. COHN M.Z. "Limit Design of Reinforced Concrete Structures for Max Yield Safety" Indian Concrete Journal Vol 36 pp 214 - 224 June 1962
21. COHN M.Z. "Optimum Design for Reinforced Concrete Continuous Beams" Proc. I.C.E. Vol 3o, April 1965, pp 675 - 707
22. COHN M.Z. "Limit Design Solutions for Concrete Structures" Journal Structural Div. A.S.C.E. Vol 93. No:ST1 Feb 1967 pp 37 - 58.
23. COHN M.Z. "Limit Design of Concrete Frames" Journal Str. Div.A.S.C.E. Vol. 94 No:ST10 Oct. 1968
24. COHN M.Z. and GRIERSON, D.E. "Optimal Design of Reinforced Concrete Beams and Frames" Final Report 8th I.A.B.S.E. Congress New York Sept. 1968 pp 215 - 226
25. COHN M.Z. and Etal "Application of Limit Design to Reinforced building structure" Proc. I.C.E. paper 7335S 1970, Supplement XVII pp 375 - 414
26. COHN M.Z. and GRIERSON D.E. "Further Results on the Equilibrium Methods of Limit Design" Proc. I.C.E. V. 46 June 1970 pp 143 - 168
27. COHN M.Z. PARIMI S.R. "Multi-Criteria Probabilistic Design of Reinforced Concrete Structures" Inelasticity and Non-Linearity in Structural Concrete Paper 19, SM Study No:8 Solid Mechanic Div. University of Waterloo Press 1972
28. COHN M.Z. "Optimal Limit Design for Reinforced Concrete Structures" Study No:8 paper 15 University of Waterloo Press 1972
29. COHN M.Z. "Opimum Limit Design for Concrete Frames" A.S.C.E. Annual Meeting, Chicago 1969 Preprint 1030 Proc. I.C.E. paper 7335S 1970, Supplement XVII pp 375 - 414
30. CORLEY W.G. "Rotational Capacity of Reinforced Concrete Beams" Proc. Amarican Soc. Civ. Eng. ST5 V.92 Oct. 1966
31. CRANSTON W.B. and REYNOLDS G.C. "The influence of shear on Rotation Capacity of Reinforced Concrete Beams" Cement and Concrete Assoc. Technical Report TRA 439, LONDON April 1970

32. ERNST G.C. "Ultimate Slopes and Deflections - A Brief for Limit Design" proc. of Inst. Civ. Eng. V.81 Jan 1955 pp 583 - 1 to 583 - 21
33. "Flexural Mechanic of Reinforced Concrete" proc. of International Symp. Miami, Florida Nov. 1964
34. GARTNER R. "Design of Indeterminate Structures by the Plastic Method" Proc. of I.C.E. V.48 No:1 Jan. 1953 pp 3 - 8
35. GLANVILLE W.H. and THOMAS F.G. "The Redistribution of moments in Reinforced Concrete Beams and Frames" Journal I.C.E. paper No:5061 V.3, pp 291 - 329 (1935 - 36)
36. GRIERSON D.E. and COHN M.Z. "A General Formulation of the Optimal Frame Problem" Journal Applied Mechanic Trans. A.S.M.E. Vol.70 pp 356 - 360
37. HOGNESTAD E. "A study of Combined Bending and Axial load in Reinforced Concrete Members". Bulletin University of Ulinois engineering experimental station No:399 Nov. 1951, pp 262 - 298
38. HUGHES B.P. "limit State Theory for Reinforced Concrete" Pitman Pub. 1971
39. ISHIKAWA N. GRIERSON D.E. "Inerative Optimal limit Design of Concrete Frames" Inelasticity and Non-Linearity in structural concrete, paper 16. SM study No:8 Solid Mech. Div. University of Waterloo 1972
40. KALISZKY S. "Optimum limit Design for Reinforced Concrete Structures" Research report, University of Southampton 1965 pp 114
41. KAZINCZY G.W. "Plasticity of RLinforced Concrete" (Die Plastizitat des Eiesenbetons) Beton u Eisen (Germany) V.32 pp 74 - 80, 1933
42. KRISHNAMOORTHY C.S. YU C.W. "An experimental Investigation of the Flexural Rigidity of T beams for frame Design" Magazine of Concrete Research Vol 25 No:84 Sept. 1973 pp 155 - 168
43. LEE L.H.N. "Inelasticity Behaviour of Reinforced Concrete Members" Transactions of the Amarican Soc. Civil Engineering. V.120 pp 181 - 207 (1955)
44. MARSHALL W.T. "Plastic Design in Reinforced Concrete" The Structural Engineer LONDON V.35. No:7 July 1957 pp 243 - 251
45. MASSONET C. and SAVE M. "Optimum Design of Beams and Frames in Reinforced Concrete" Progress in Applied Mech. Prager Anniversary Volume, MacMillan Comp. N.Y. 1963.
46. MATTOCK A.M. "Redistribution of Bending Moments in Reinforced Concrete Continuous Beams" Proc. I.C.E. paper No: 6314 May 1959

47. MUNRO J. KRISHNAMOORTHY C.S. YU. C W. "Optimal Design of Reinforced Concrete Frames" The Structural Engineer V.50 No:7 July 1972

48. PANELL F.N. LOWNDS P. KHAYAT M.S. "Investigation of the Behaviour at Rigid Joints in Rhinforced Concrete Frames" Final Report Phase I Oct. 1966. Manchester, The College of Science and Technology, Structural Engineering Dept. 1966 p154.

49. PETCU V. "The Optimum Redistribution Principle" The Indian Concrete Journal July 1961. pp 259 - 260.

50. PETCU V. "Optimum Plastic Moments of Reinforced Concrete Continuous Beams Indian Concrete Journal Vol. 36 Nō:3 March 1962 pp 110 - 113

51. POOLOGASOUNDRANYAGAM K. "An Analytical and Experimental Investigation of the formation and Behaviour of Plastic Hinges in Prestressed and Reinforced Concrete frames". Phd. Thesis, University of London 1960

52. REPORT OF INSTITUTION RESEARCH COMMITTEE "Ultimate load Design of Concrete Structures" Proc. I.C.E. (London) Vol. 21. Feb.1962 pp 339 - 442.

53. SAW C.B. "Optimum Moment Redistribution for Reinforced Concrete Continuous Beams" Civil Engineer Nov. 1973.

54. SAWYER H.A. "Elastic-plastic Design of Single-Span Beams and Frames". Proc. of American Soc. Civ. Eng. V.120. pp 181 - 207 (1955)

55. TALWAR S. COHN M.Z. "Compatibility analysis and design of braced reinforced Conçrete Frames". Inelasticity and Non-linearity in Structural Concrete". Paper 17 SM Study No:8 Solid Mech. Div. University of Waterloo press. 1972.

56. YU C.W. "Further Research in Reinforced Concrete and Its application to Ultimate Load Design" Proc. I.C.E. (London) V.3. Part 3, No:1. April 1954 pp 302 - 308.

57. YU C.W. "An Analytical Investigation of the Ultimate Load Design of Reinforced Concrete Framed Structures". Phd thesis University of London 1954.

58. YU C.W. Discussion on a Paper, "Ultimate Load Design of Reinforced and Prestressed Concrete Frames". Symposium on the Strength of Concrete Structures, Cement and Concrete Assoc. 1956.

59. YU C.W. and E. HOGNESTAD. "Review of Limit Design for Structural Concrete". J. Strut. Div. American Soc. Civ. Eng. Dec. 1958 paper No:1878, pp 1 - 28.

60. YU C.W. REGAN P.E. "Limit State Design of Structural Concrete" Chato - Windus Pub. 1973.

# APPENDIX ONE

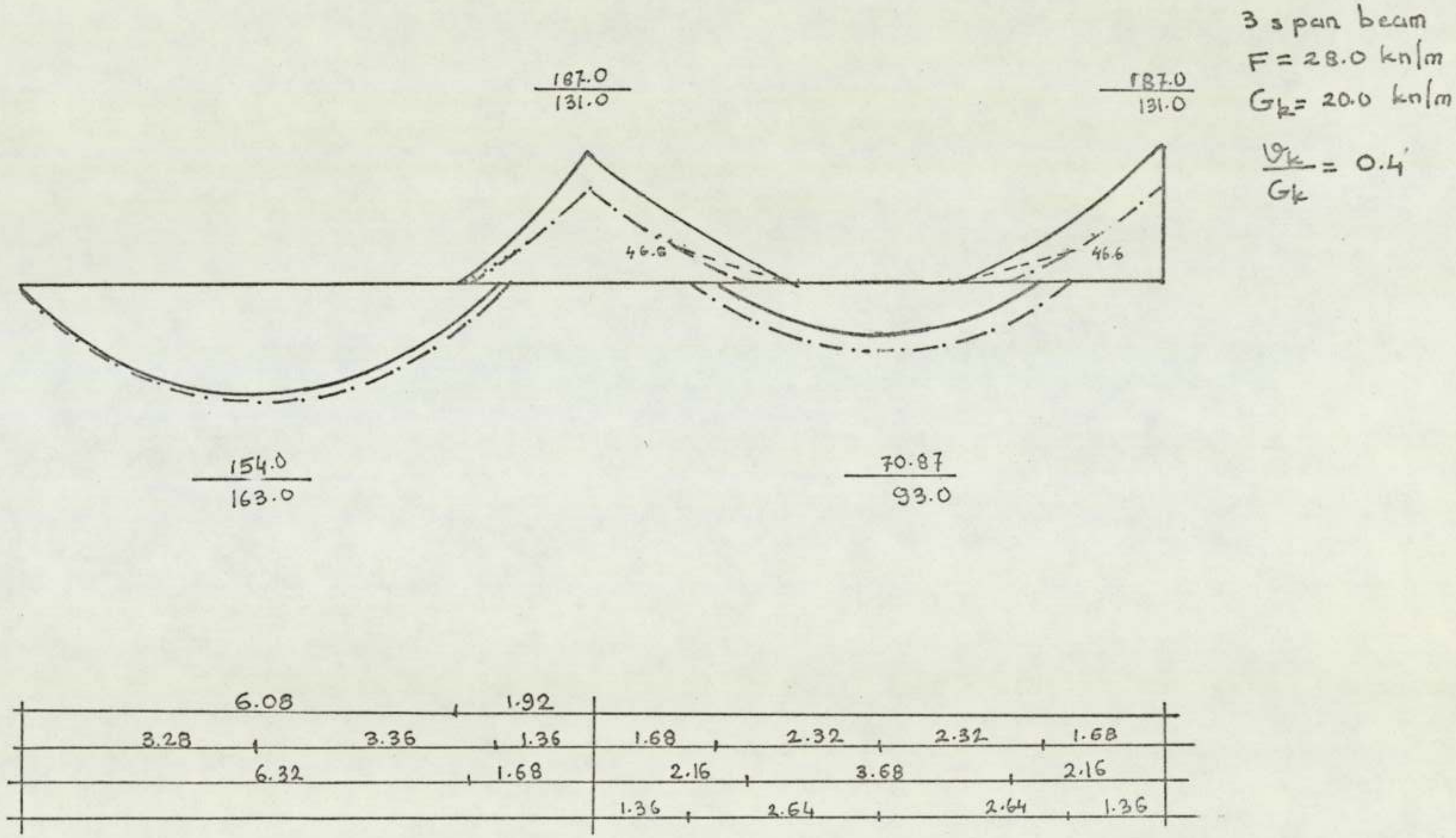

3 span beam
F = 28.0 kn/m
$G_k$ = 20.0 kn/m
$\frac{Q_k}{G_k}$ = 0.4
187.0
131.0
187.0
131.0
46.6
46.6
154.0
163.0
70.87
93.0
6.08
1.92
3.28
3.36
1.36
1.68
2.32
2.32
1.68
6.32
1.68
2.16
3.68
2.16
1.36
2.64
2.64
1.36

fig. (A.1).

F = 30.0 kn/m

$G_k$ = 20.0 kn/m

$\frac{Q_k}{G_k} = 0.5$

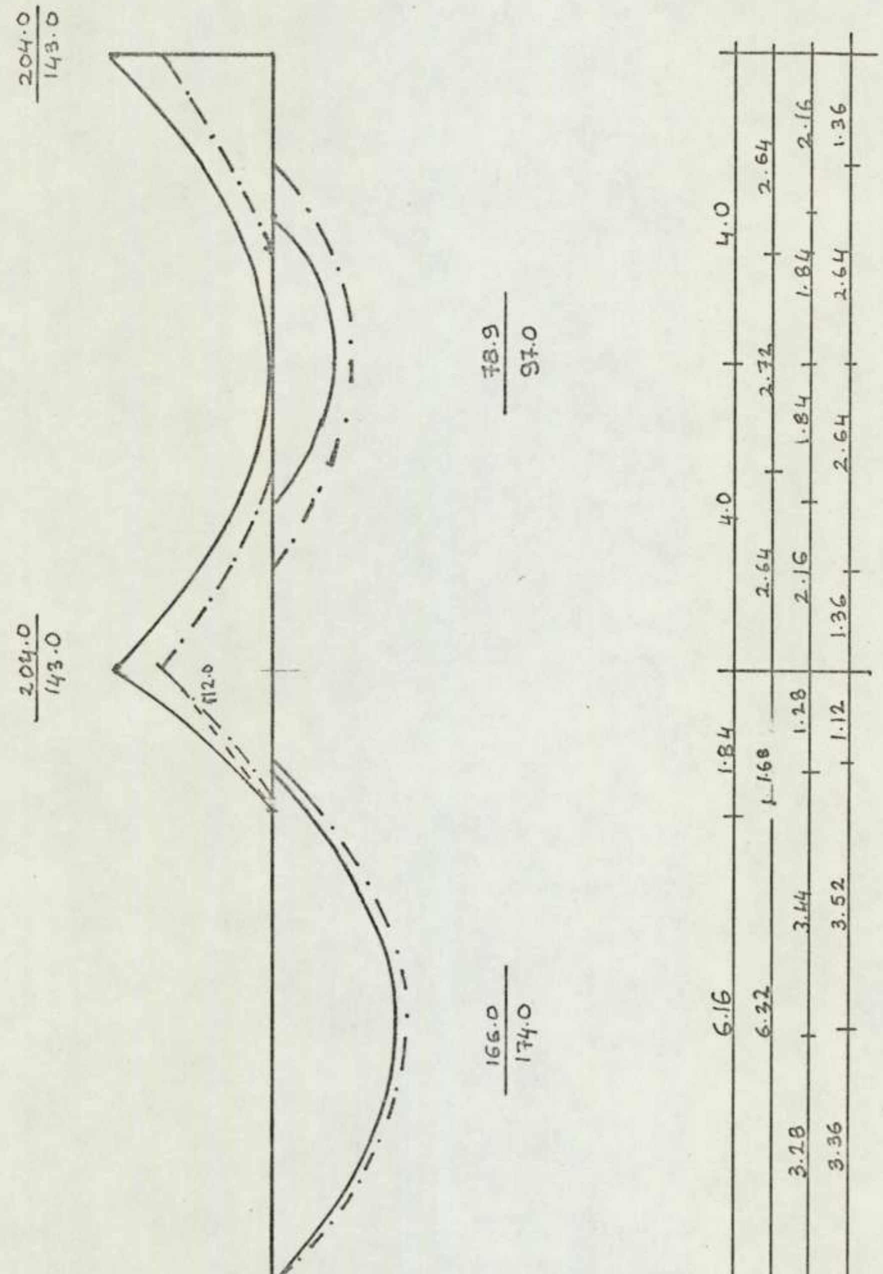



Fig.(A.2)

$F = 34.0$ kn/m

$G_k = 20.0$ kn/m

$\frac{Q_k}{G_k} = 0.7$

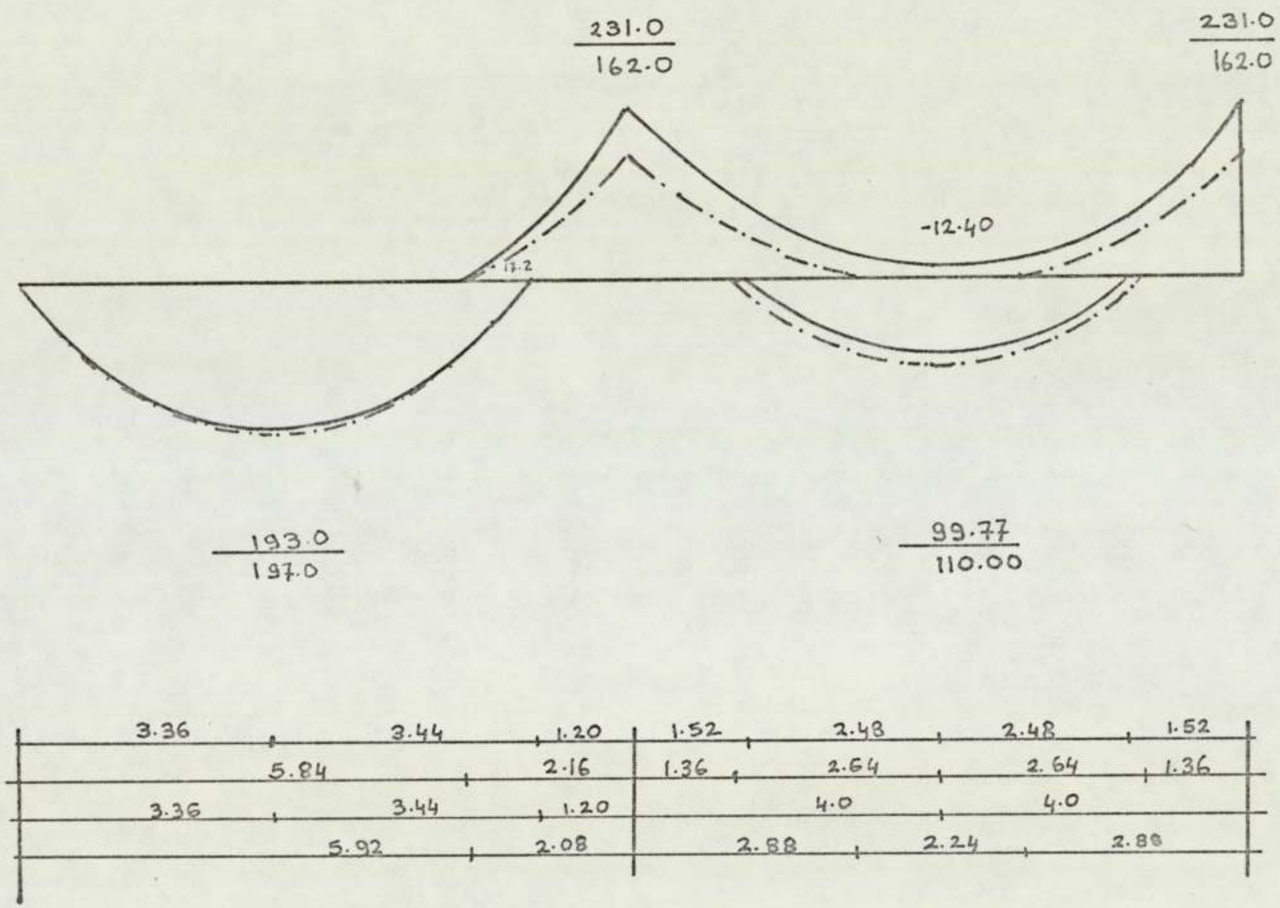



fig. (A.3).

3 span beam

$F = 36.0$ kn/m

$G_k = 20.0$ kn/m

$\frac{Q_k}{G_k} = 0.8$

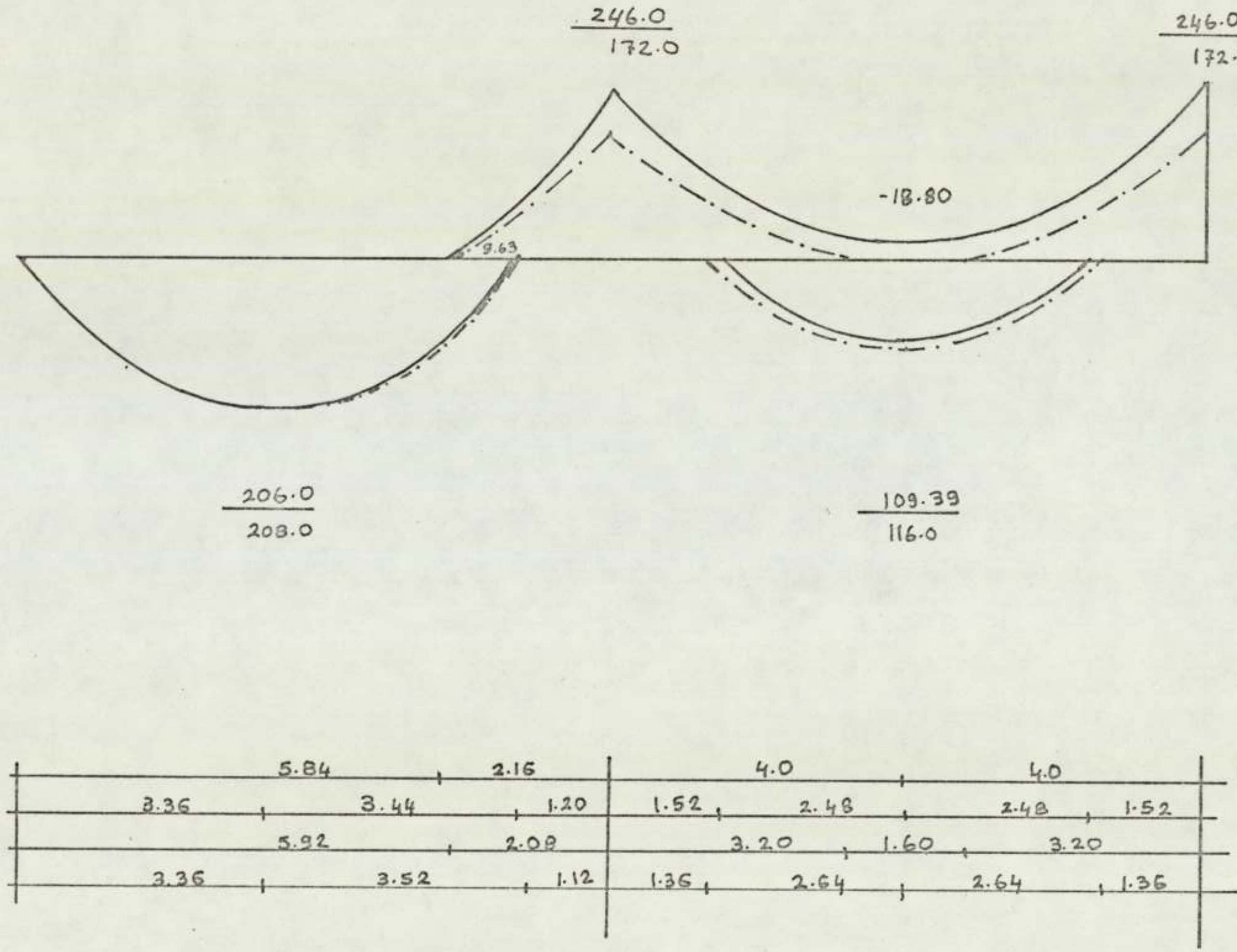



Fig. (A.4)

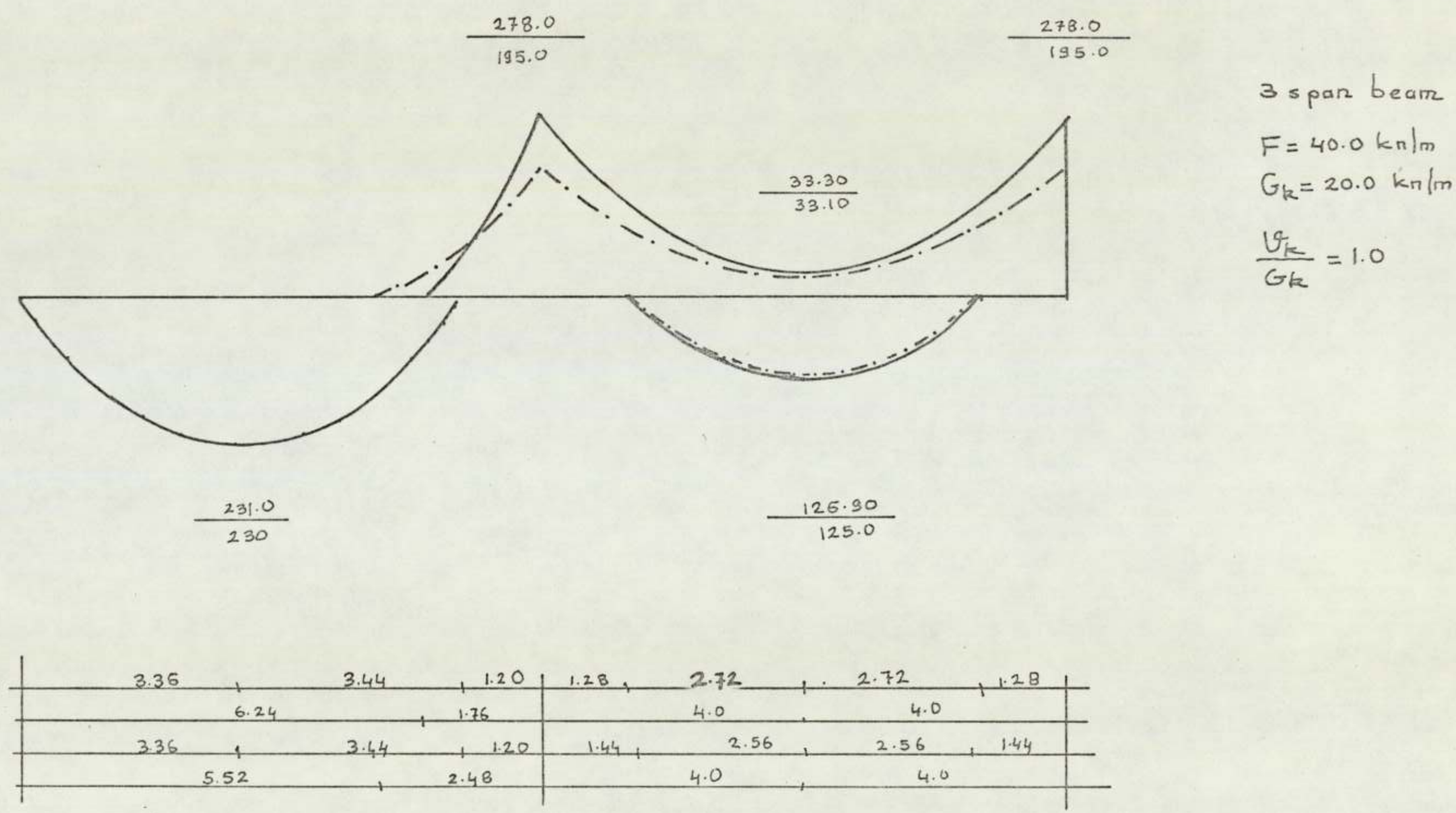

278.0
195.0
278.0
195.0
33.30
33.10
231.0
230
126.90
125.0
3.36
3.44
1.20
1.28
2.72
2.72
1.28
6.24
1.76
4.0
4.0
3.36
3.44
1.20
1.44
2.56
2.56
1.44
5.52
2.48
4.0
4.0
3 span beam
F = 40.0 kn/m
Gk = 20.0 kn/m
Qk/Gk = 1.0

Fig. (R. 5)

3 span beam

$F = 44.0$ kn/m

$G_k = 20.0$ kn/m

$\frac{V_k}{G_k} = 1.2$

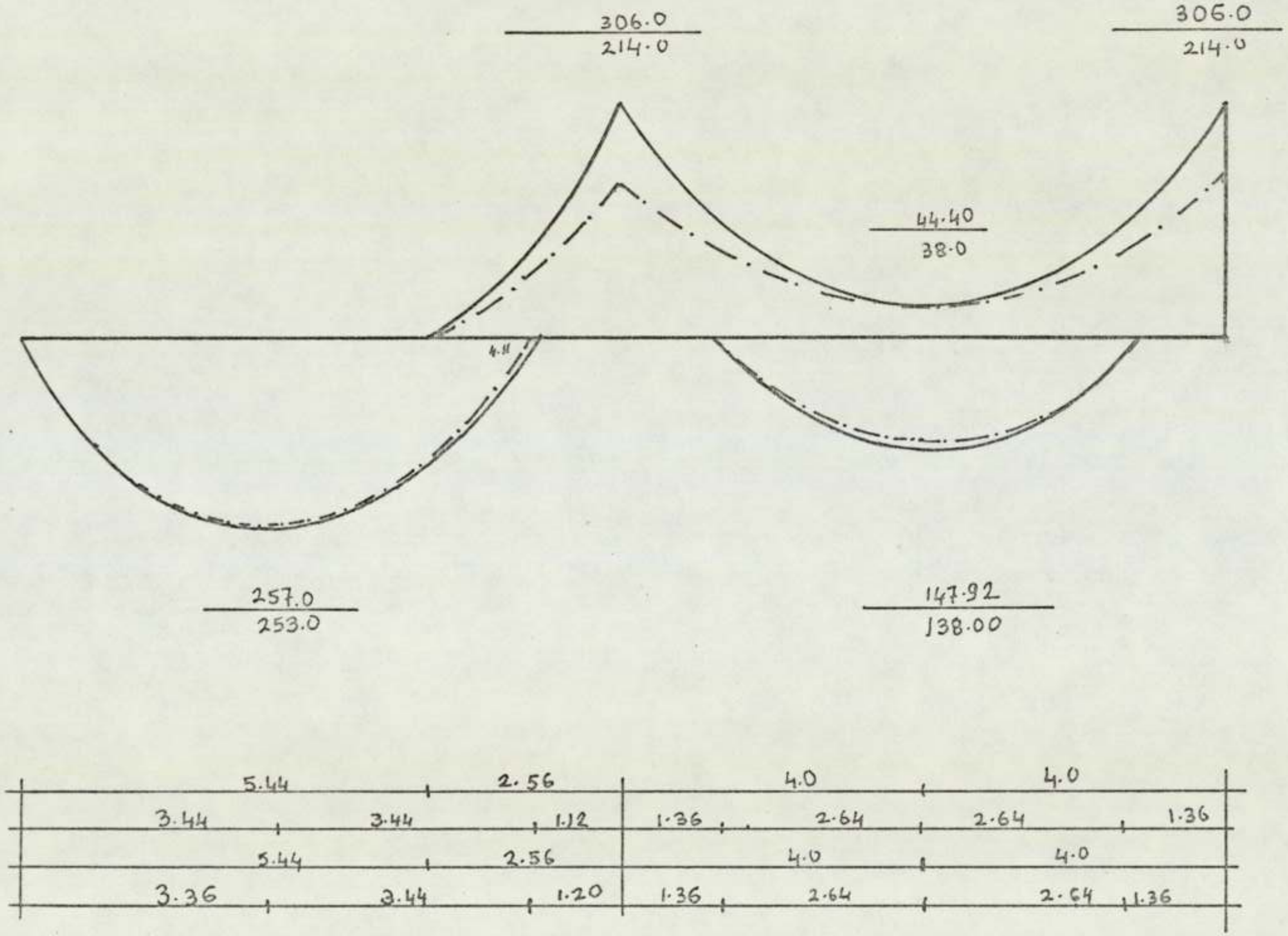



Fig.(R.6)

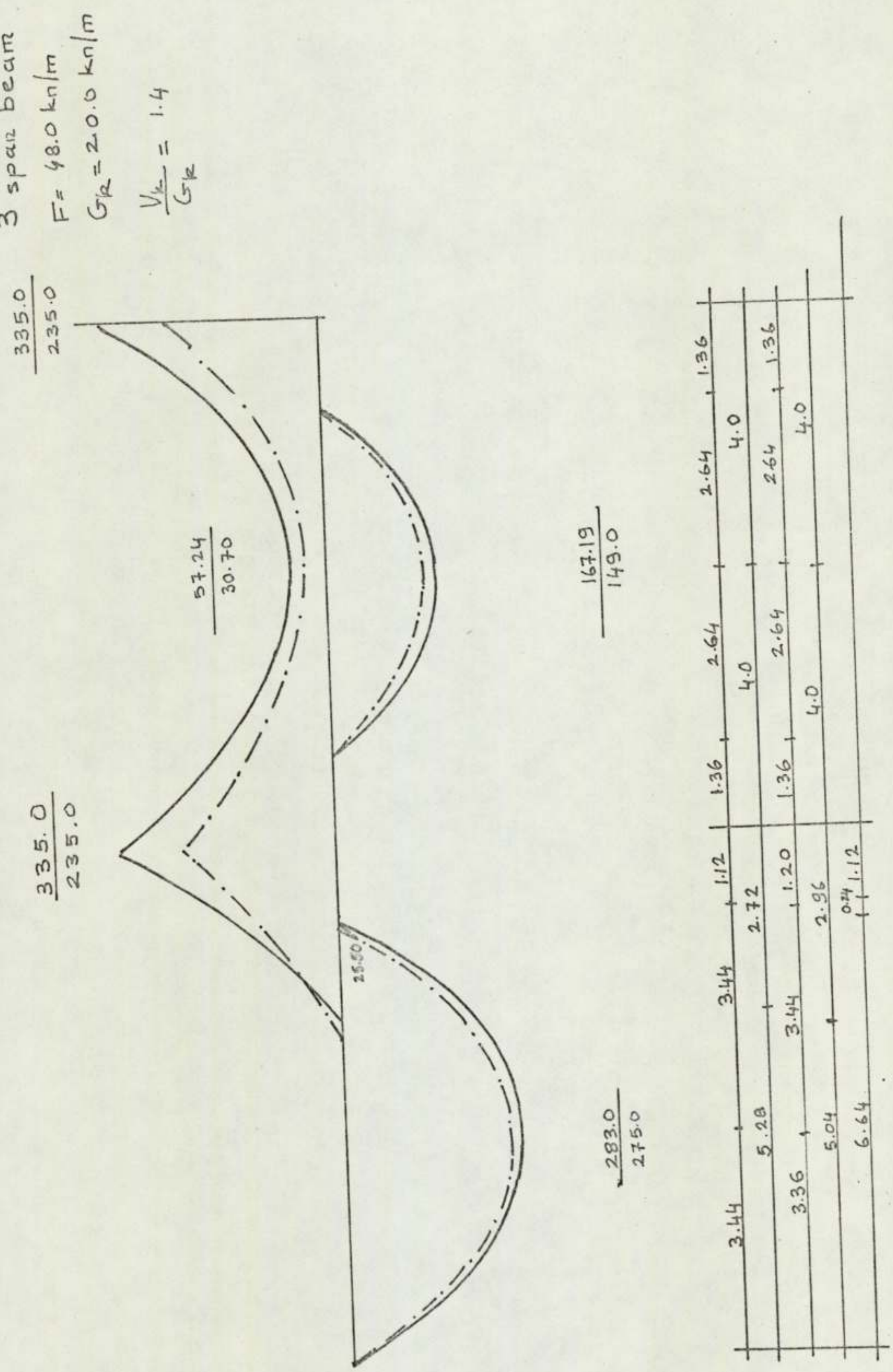

3 span beam
F= 48.0 kn/m
Gk = 20.0 kn/m
Vk/Gk = 1.4
335.0
235.0
57.24
30.70
335.0
235.0
167.19
149.0
25.50
283.0
275.0
3.44
5.28
3.44
1.12
1.36
4.0
2.64
2.64
4.0
1.36
2.72
3.36
3.44
1.20
1.36
4.0
2.64
2.64
4.0
1.36
5.04
2.96
0.24
1.12
4.0
4.0
6.64

fig. (A.7)

3 Spanbeam

$F = 50.0$ kn/m

$G_k = 20.0$ kn/m

$\frac{Q_k}{G_k} = 1.5$

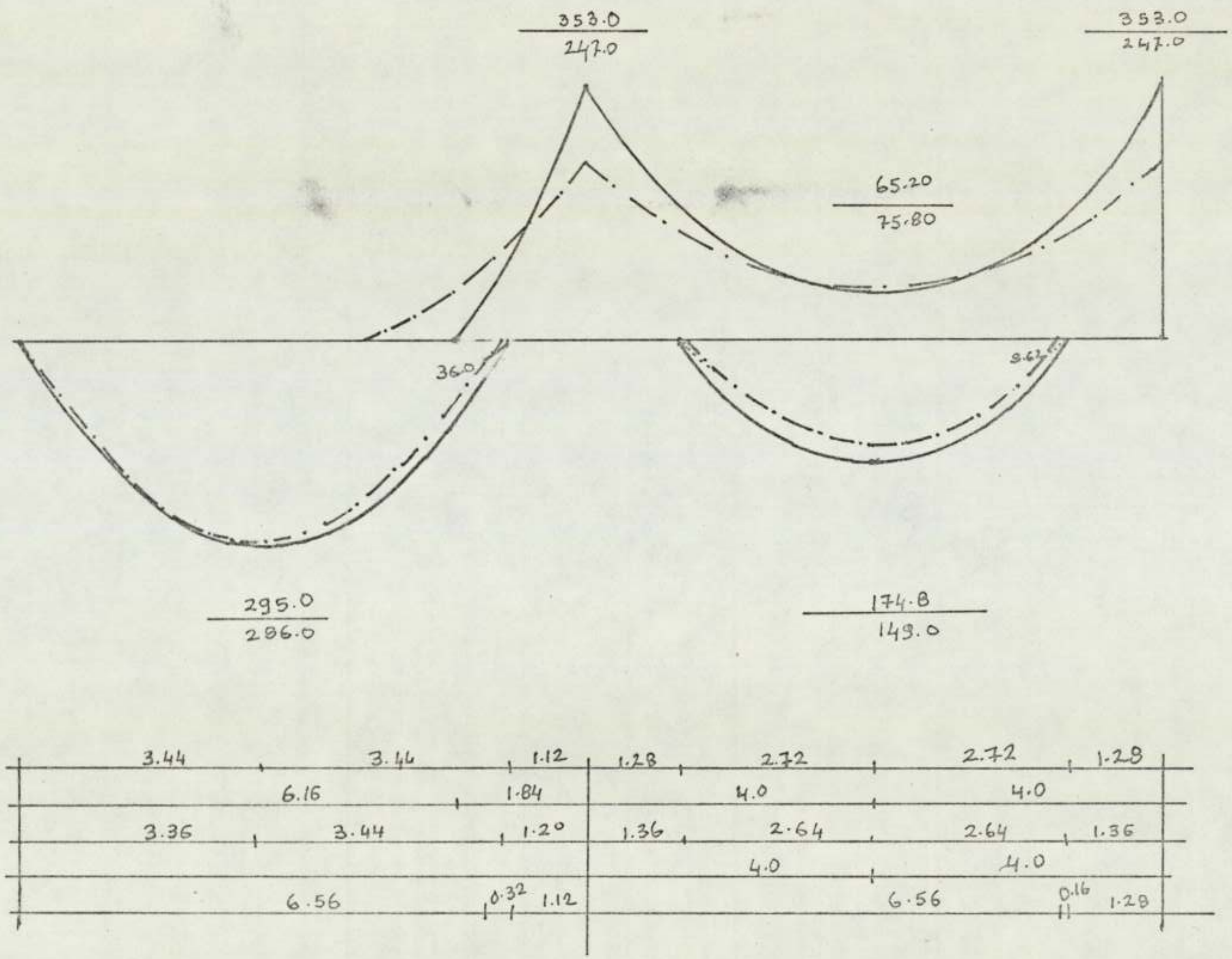
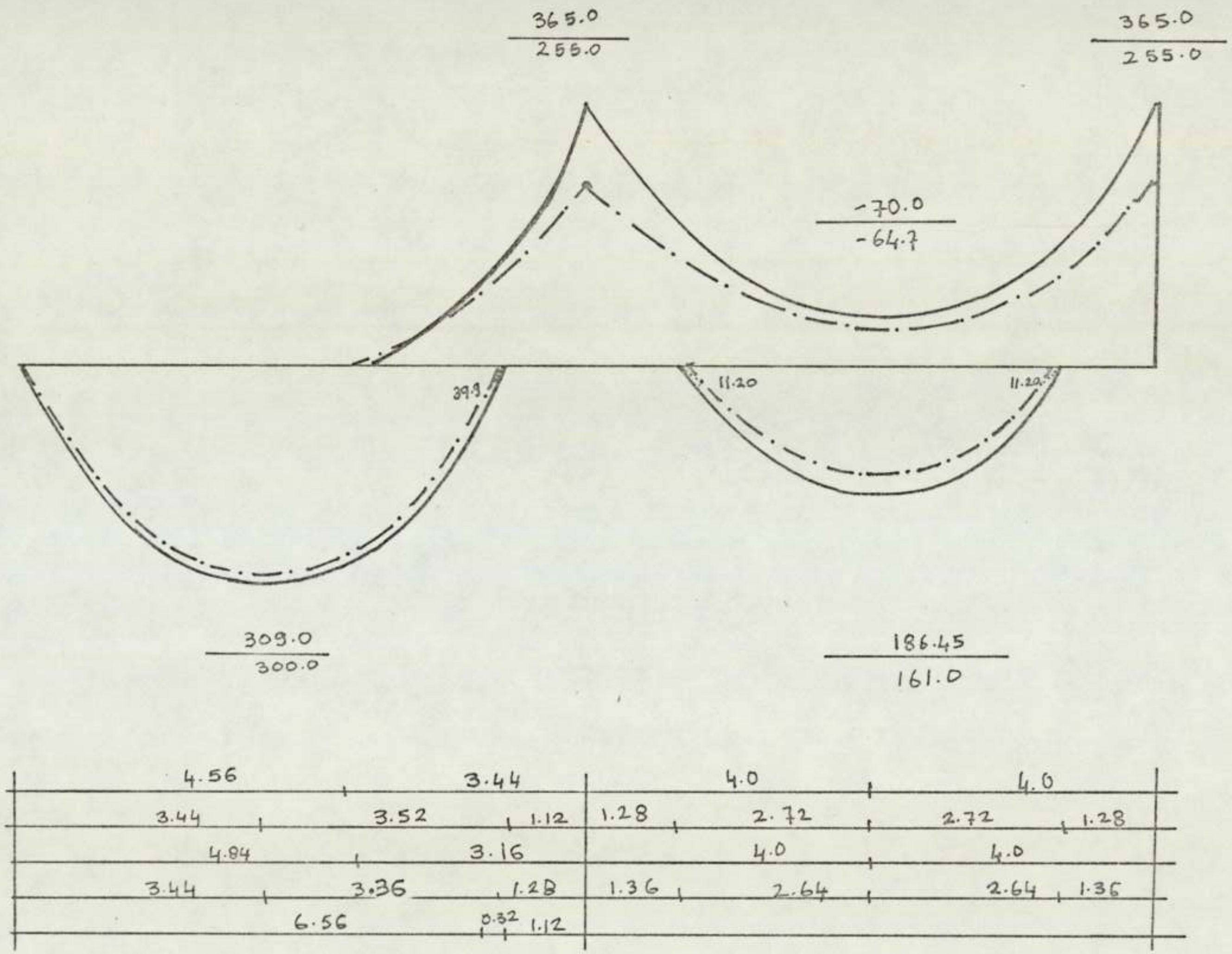



Fig. (A. 8)




3span beam

$F = 52.0$ kn/m

$G_k = 20.0$ kn/m

$\frac{Q_k}{G_k} = 1.6$

Fig.(A.9.)

3 span beam

$F = 60.0\ kn/m$

$G_k = 20.0\ kn/m$

$\frac{Q_k}{G_k} = 2.0$

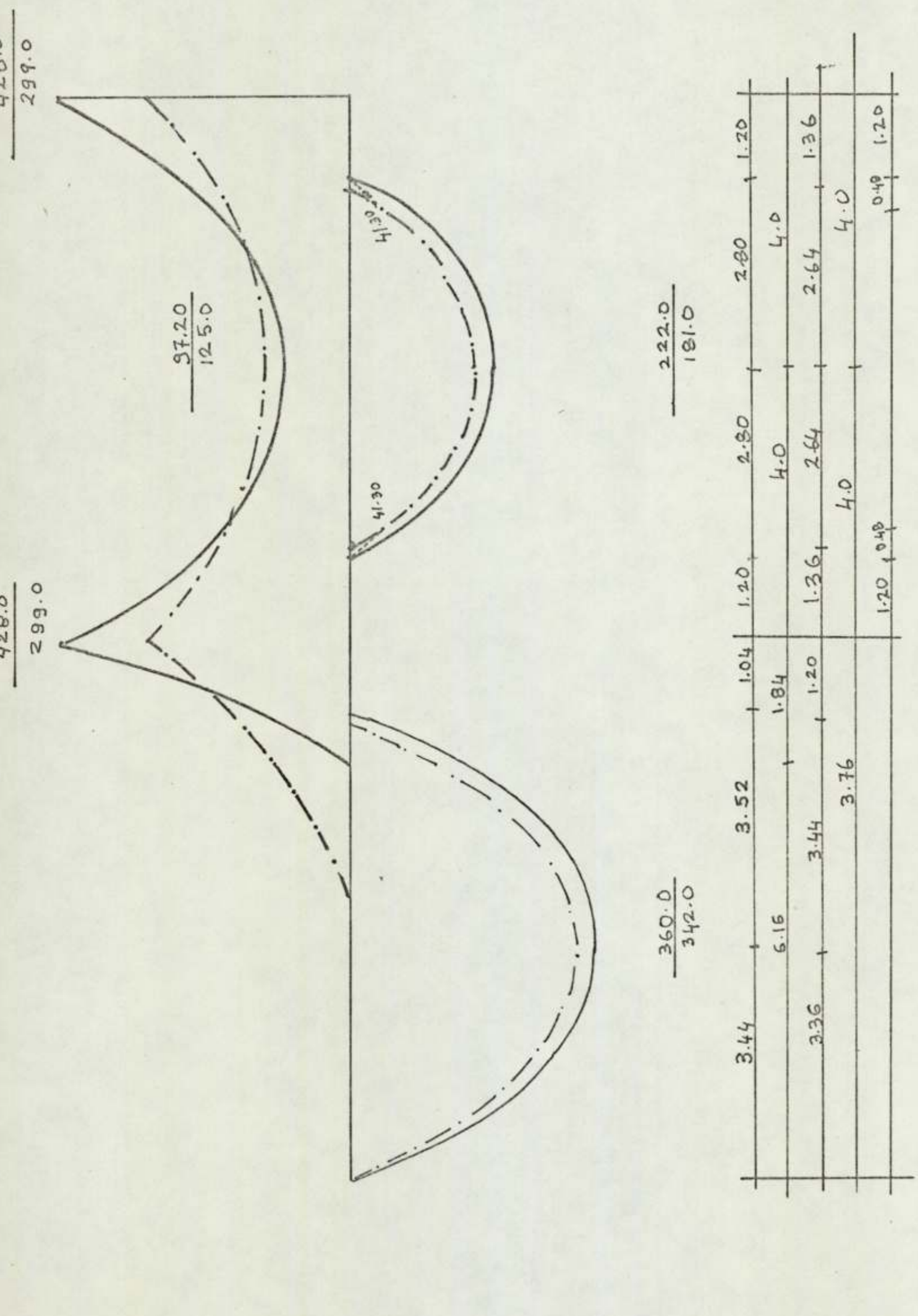
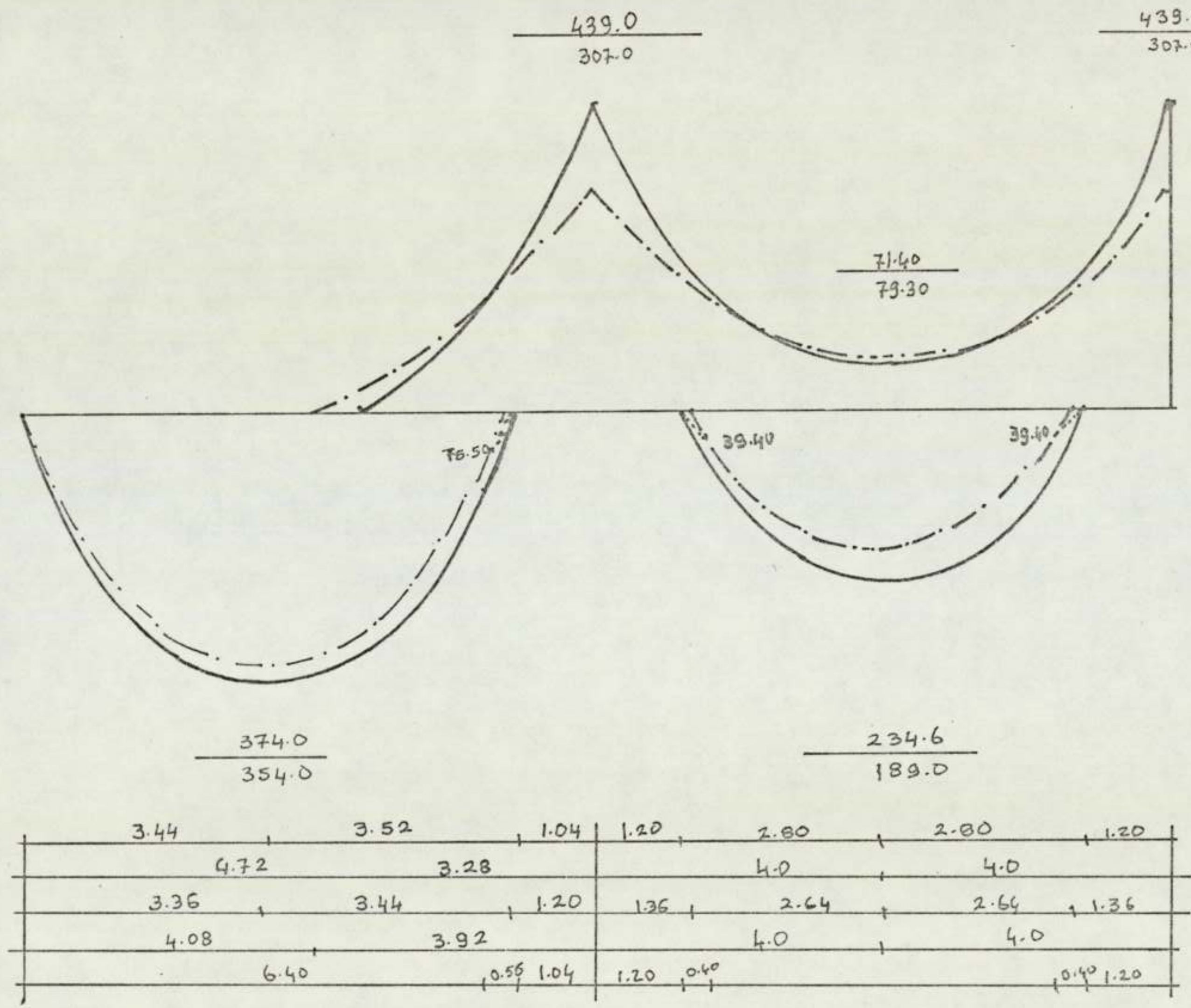



Fig. (A.10)




3 span beam

F = 62.0 kn/m

$G_k$ = 20.0 kn/m,

$\frac{Q_k}{G_k} = 2.1$

Fig. (A.11)

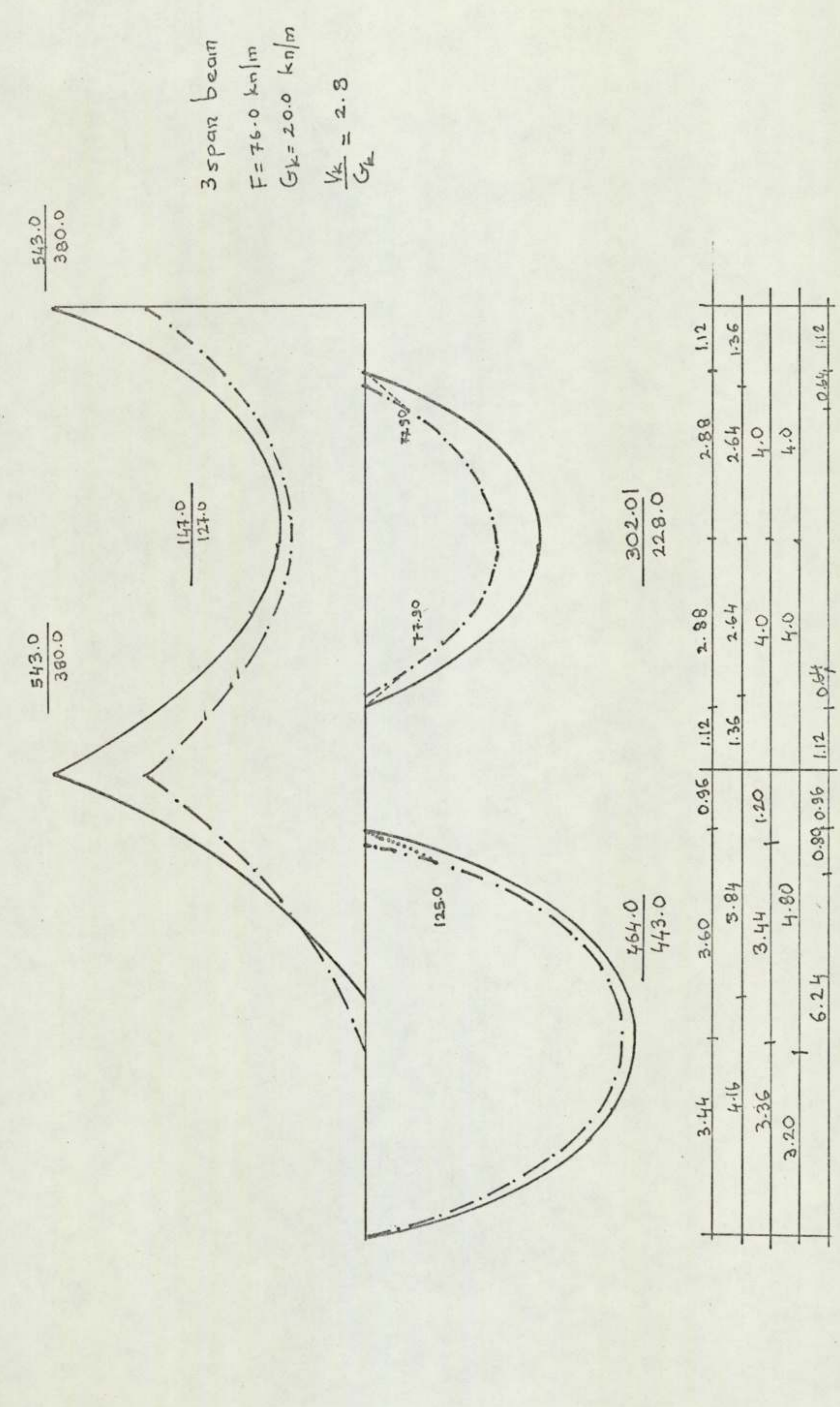
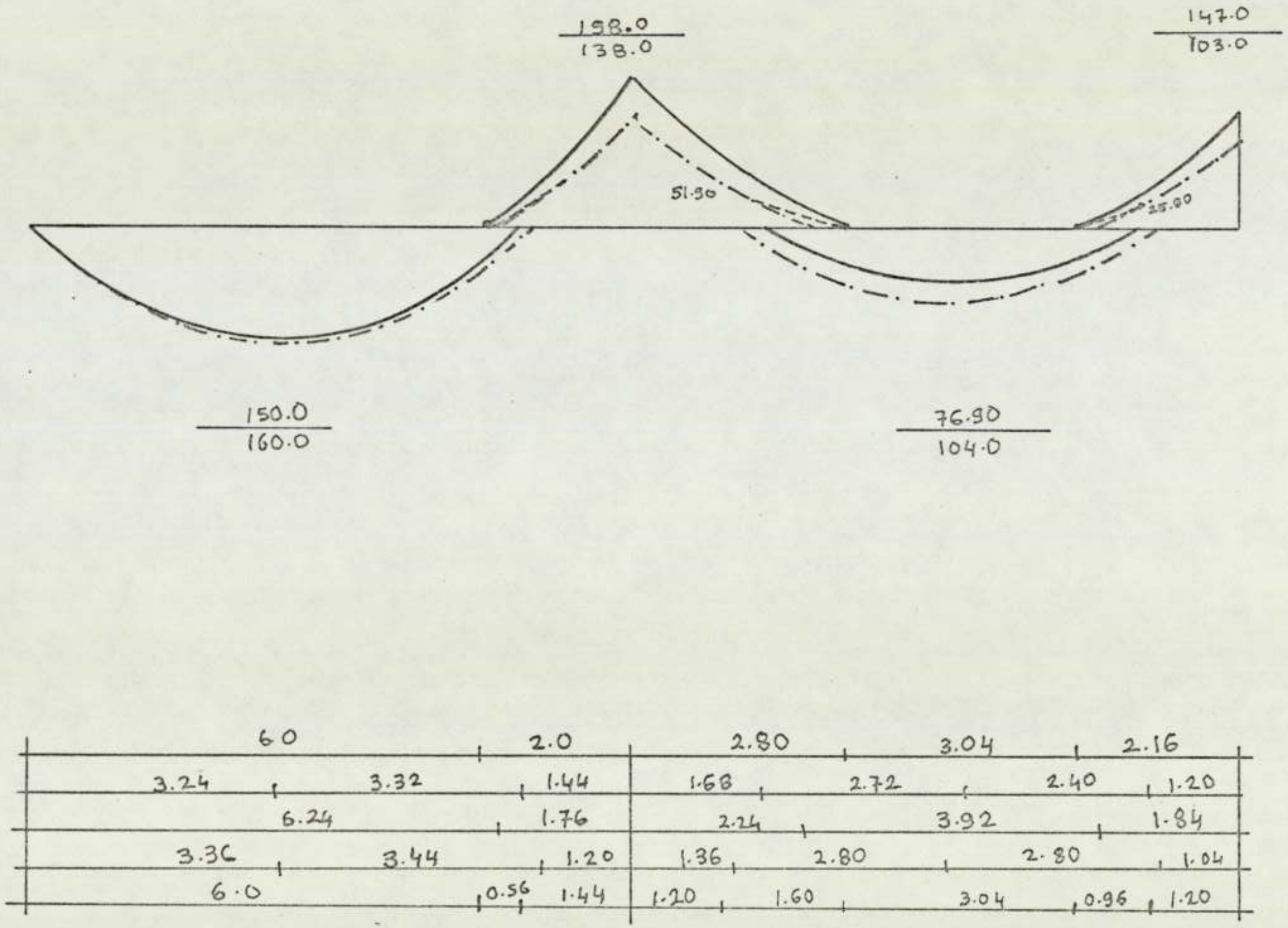

3 span beam
F = 76.0 kn/m
Gk = 20.0 kn/m
Vk/Gk = 2.8
543.0 / 380.0
543.0 / 380.0
147.0 / 127.0
302.01 / 228.0
464.0 / 443.0
125.0
77.90
77.90
3.44
3.60
0.96
1.12
2.88
2.88
1.12
4.16
3.84
1.36
2.64
2.64
1.36
3.36
3.44
1.20
4.0
4.0
3.20
6.24
4.80
0.80
0.96
1.12
0.64
4.0
4.0
0.64
1.12

Fig. (A.12.)



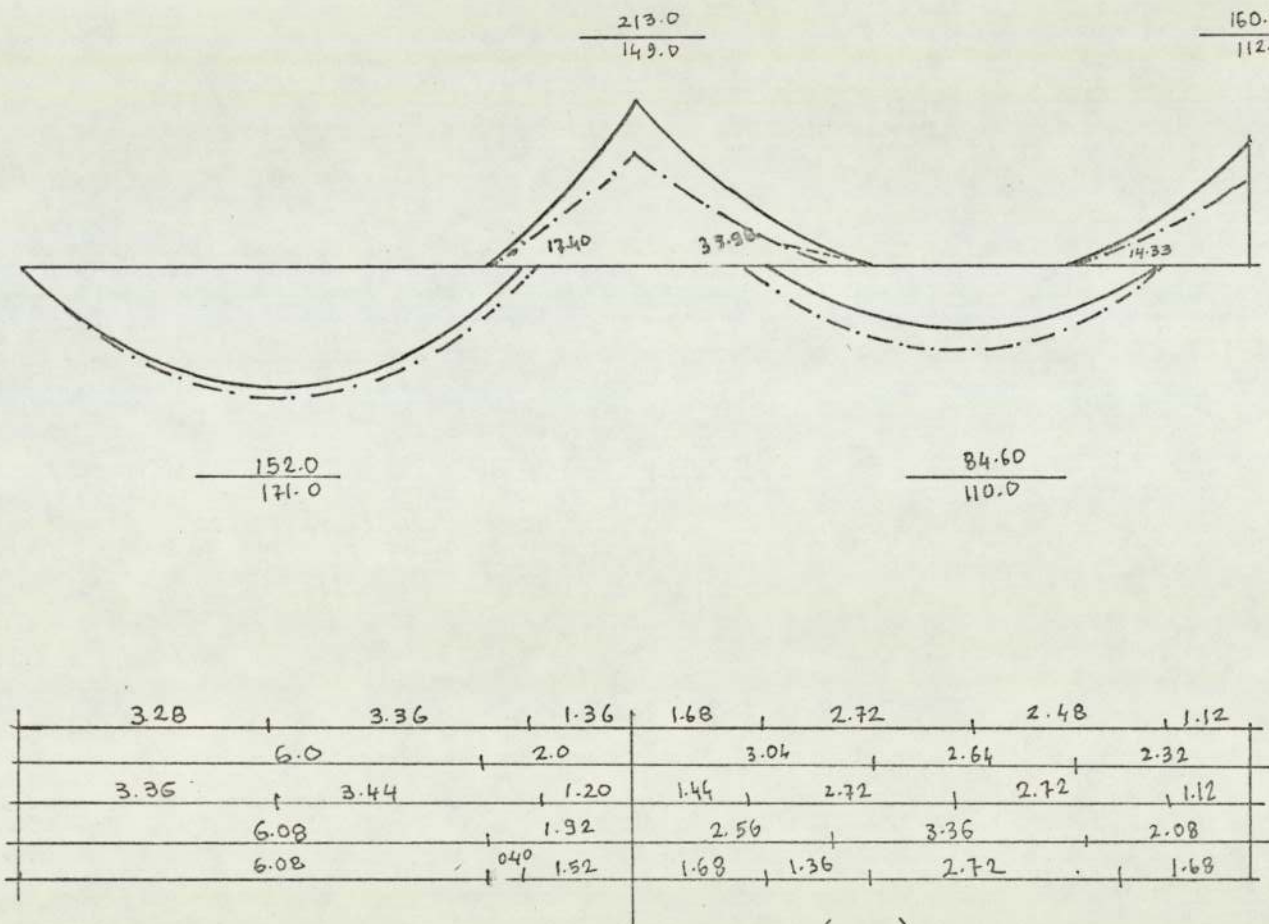

4 span beam

F = 28.0 kn/m

$G_k$ = 20.0 kn/m

$\frac{V_k}{G_k} = 0.4$

fig (A.13.)



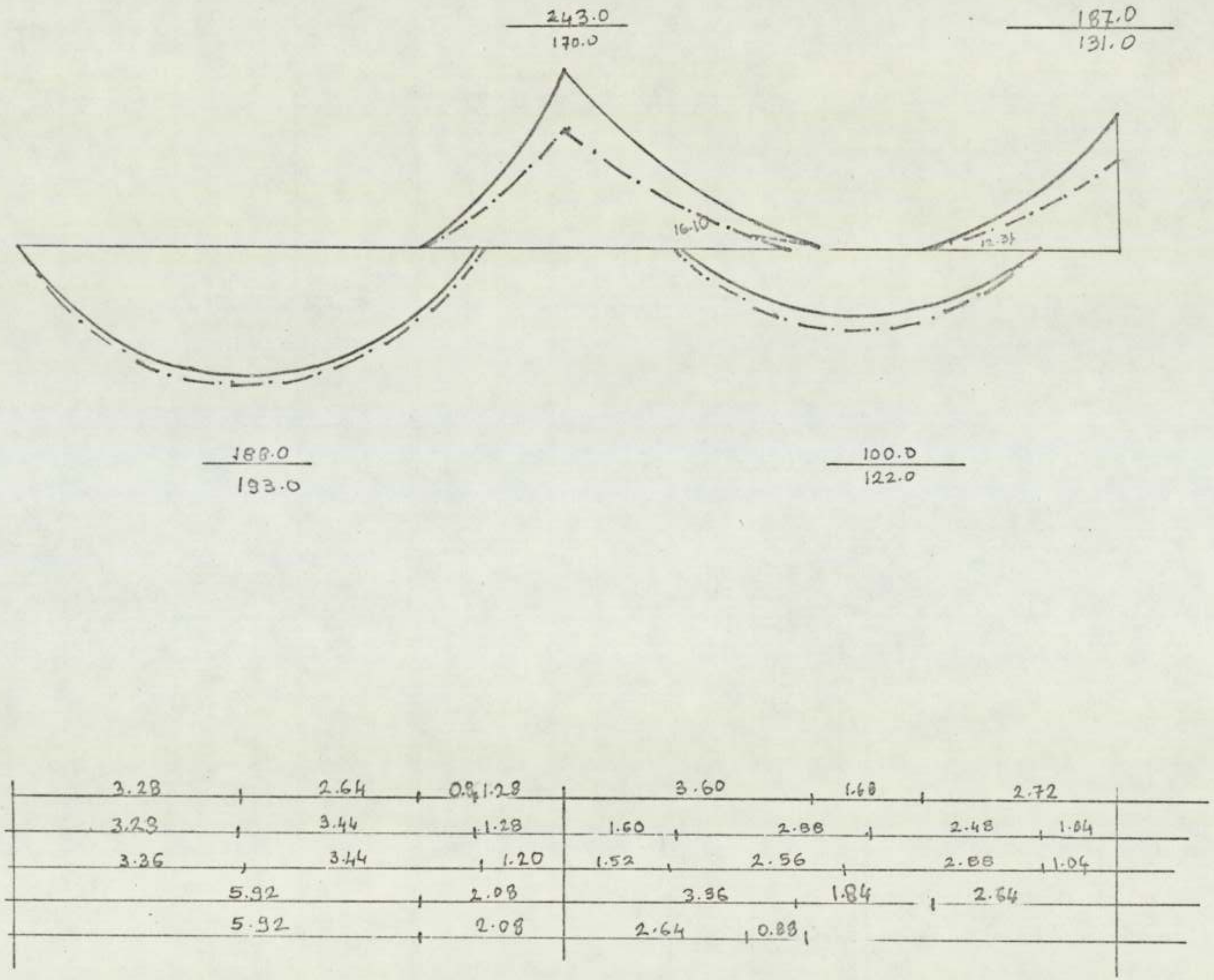

4 span beam

F = 30.0 kn/m

$G_k$ = 20.0 kn/m

$\frac{V_k}{G_k} = 0.5$

fig. (A.14)



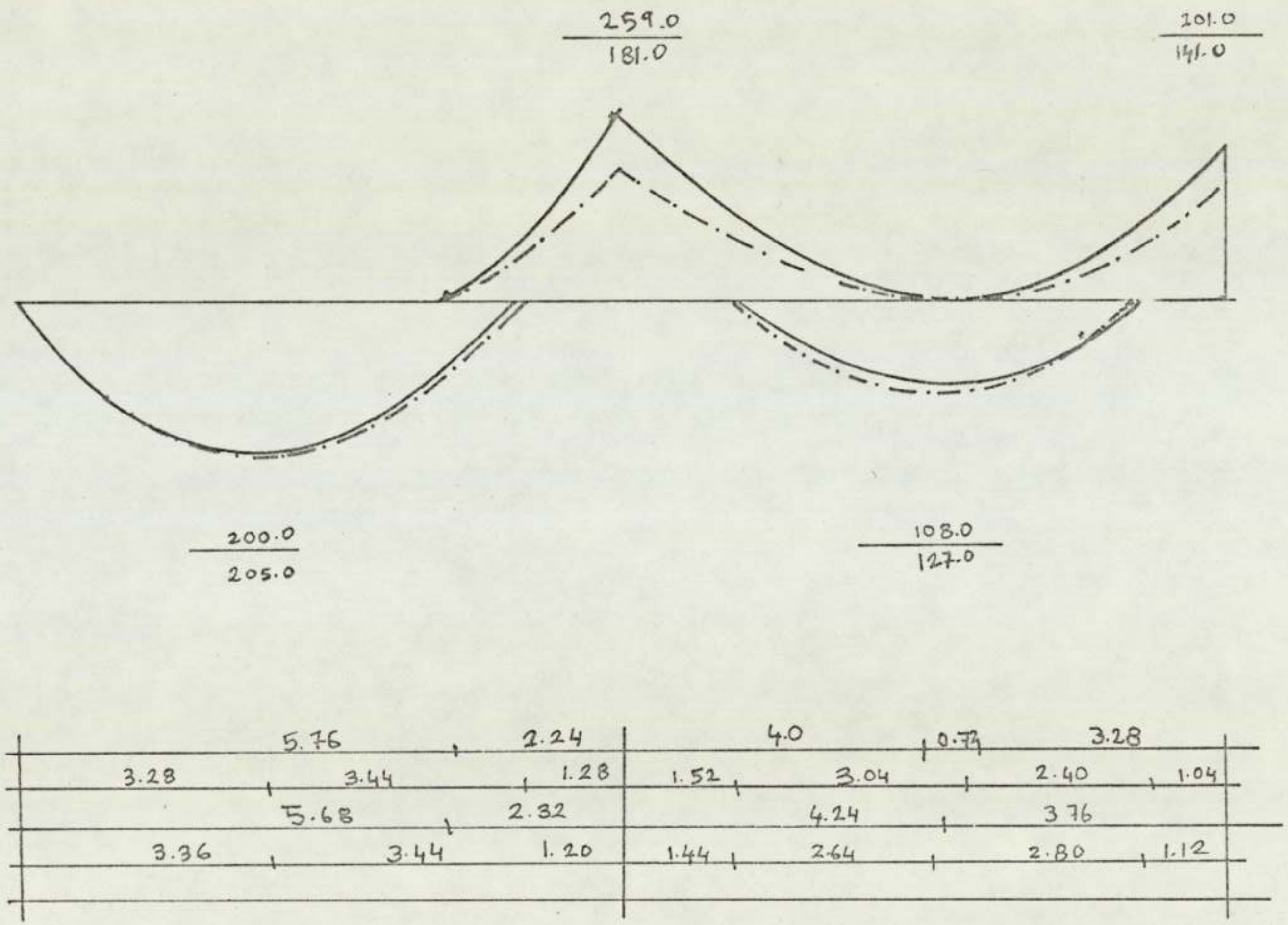

4 span beam

$F = 34.0$ kn/m

$G_k = 20.0$ kn/m

$\frac{V_k}{G_k} = 0.7$

Fig(A.15)



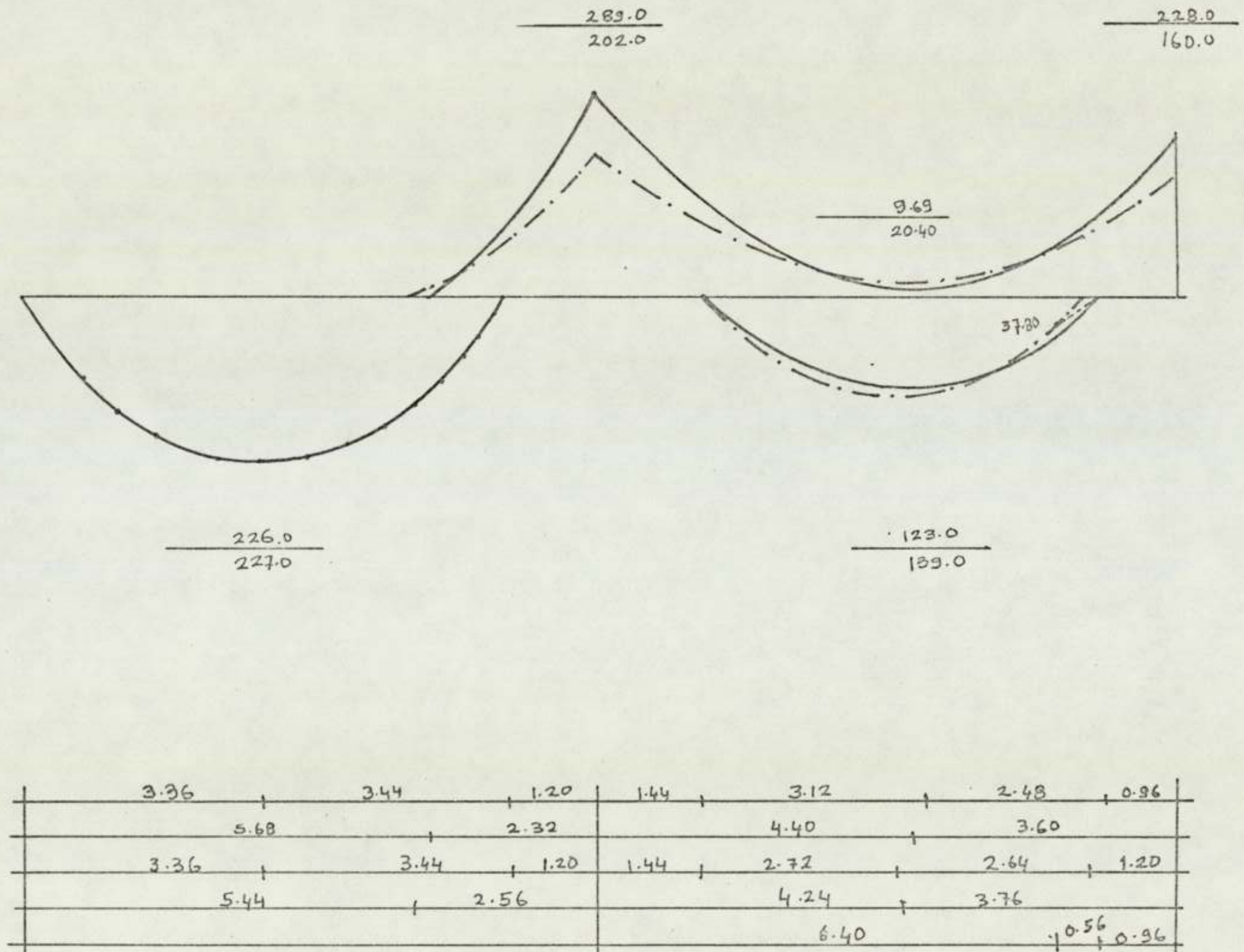

4 span beam

$F = 36.0$ kn/m

$G_k = 20.0$ kn/m

$\frac{V_k}{G_k} = 0.8$

fig. (A.16)




4 span beam

$F = 40.0$ kn/m

$G_k = 20.0$ kn/m

$\frac{V_k}{G_k} = 1.0$

Fig.(A. 17)

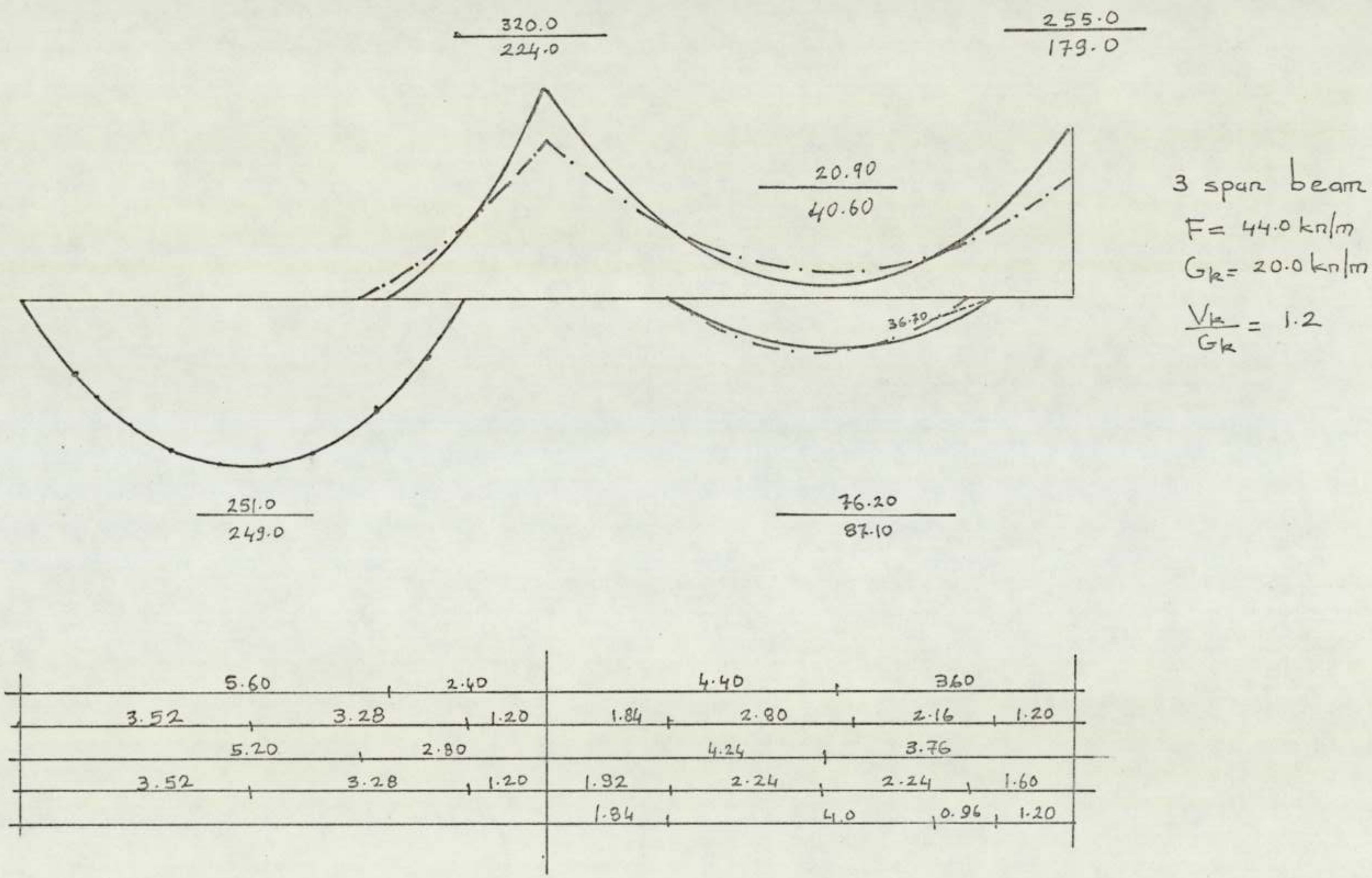

320.0
224.0
255.0
179.0
20.90
40.60
36.70
251.0
249.0
76.20
87.10
3 span beam
F= 44.0 kn/m
Gk= 20.0 kn/m
Vk/Gk = 1.2
5.60
2.40
4.40
360
3.52
3.28
1.20
1.84
2.80
2.16
1.20
5.20
2.80
4.24
3.76
3.52
3.28
1.20
1.92
2.24
2.24
1.60
1.84
4.0
0.96
1.20

Fig.(A.18)

4 span beam
F = 48.0 kn/m
$G_k$ = 20.0 kn/m
$\frac{Q_k}{G_k} = 1.4$

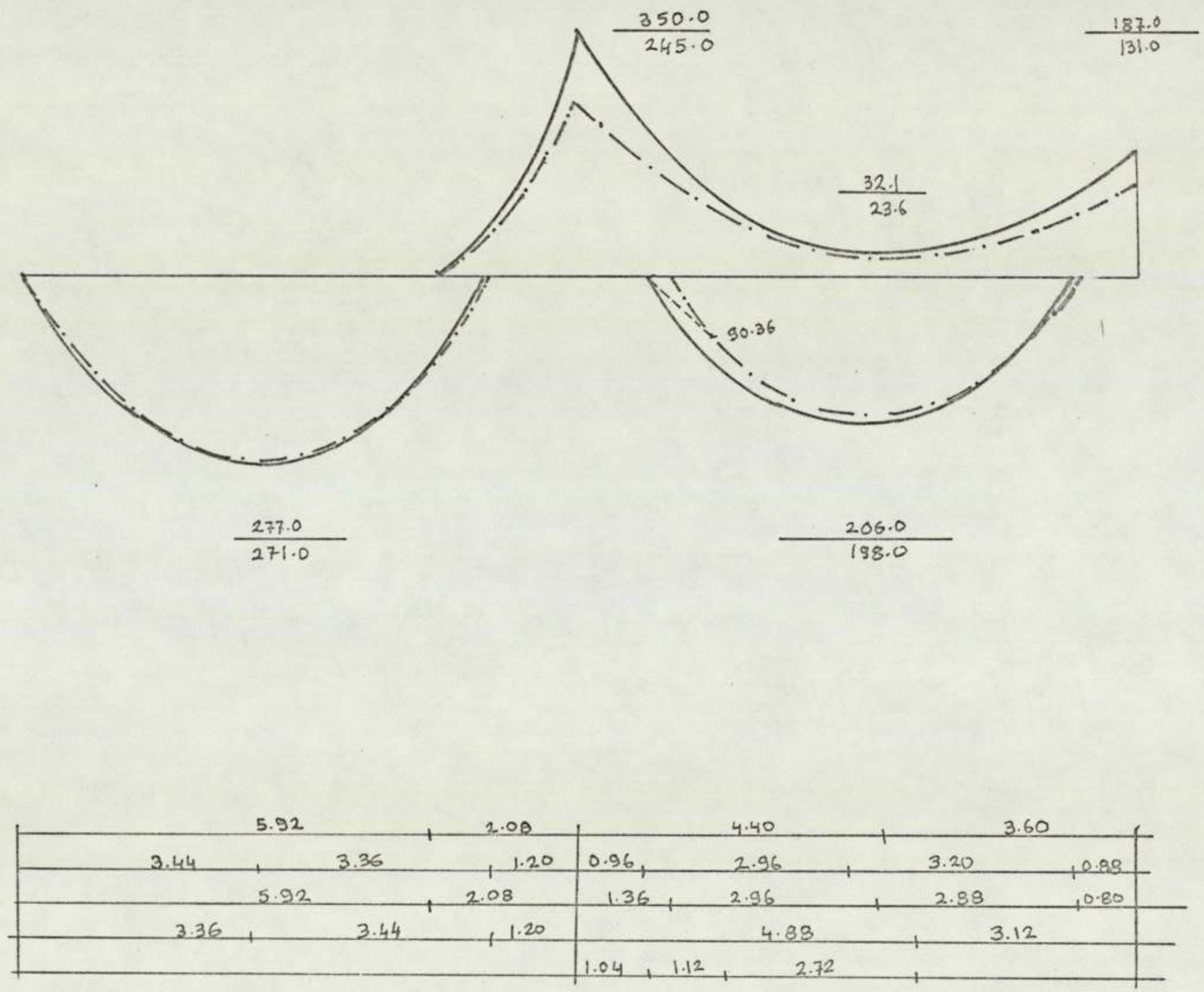
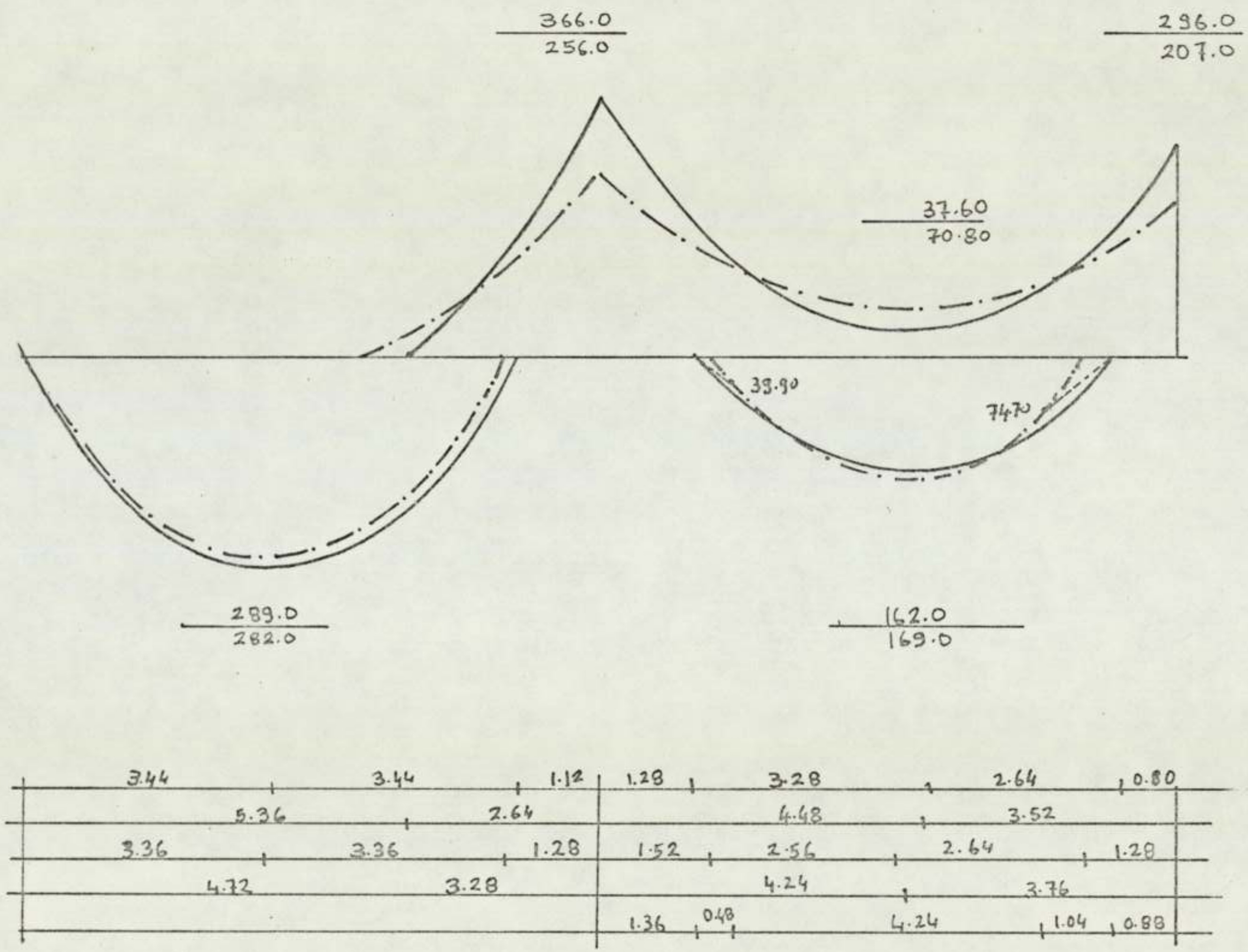



fig (A.19)



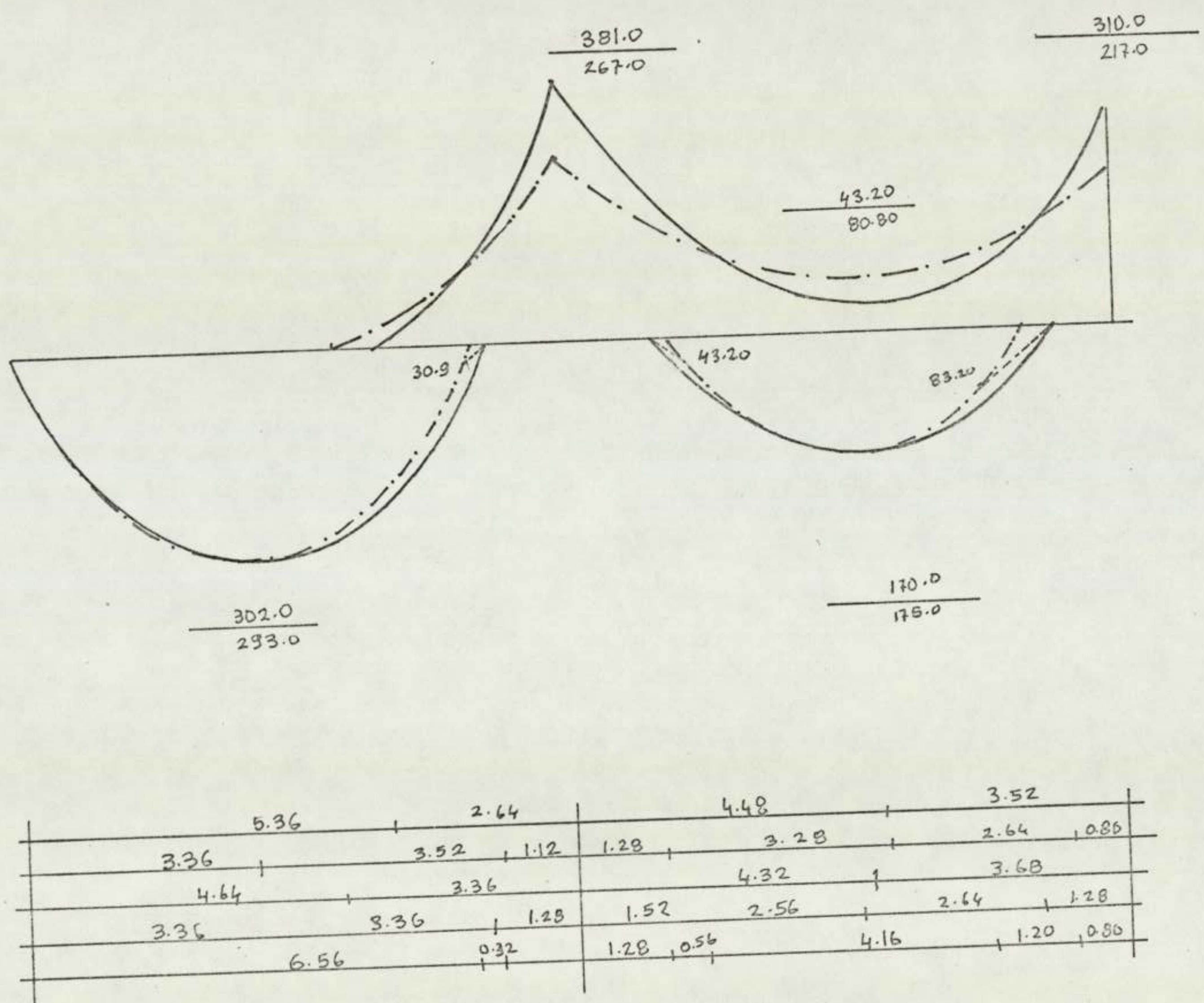

4 span beam

F= 50.0 kn/m

$G_k$= 20.0 kn/m

$\frac{V_k}{G_k} = 1.5$

Fig. (A.20)



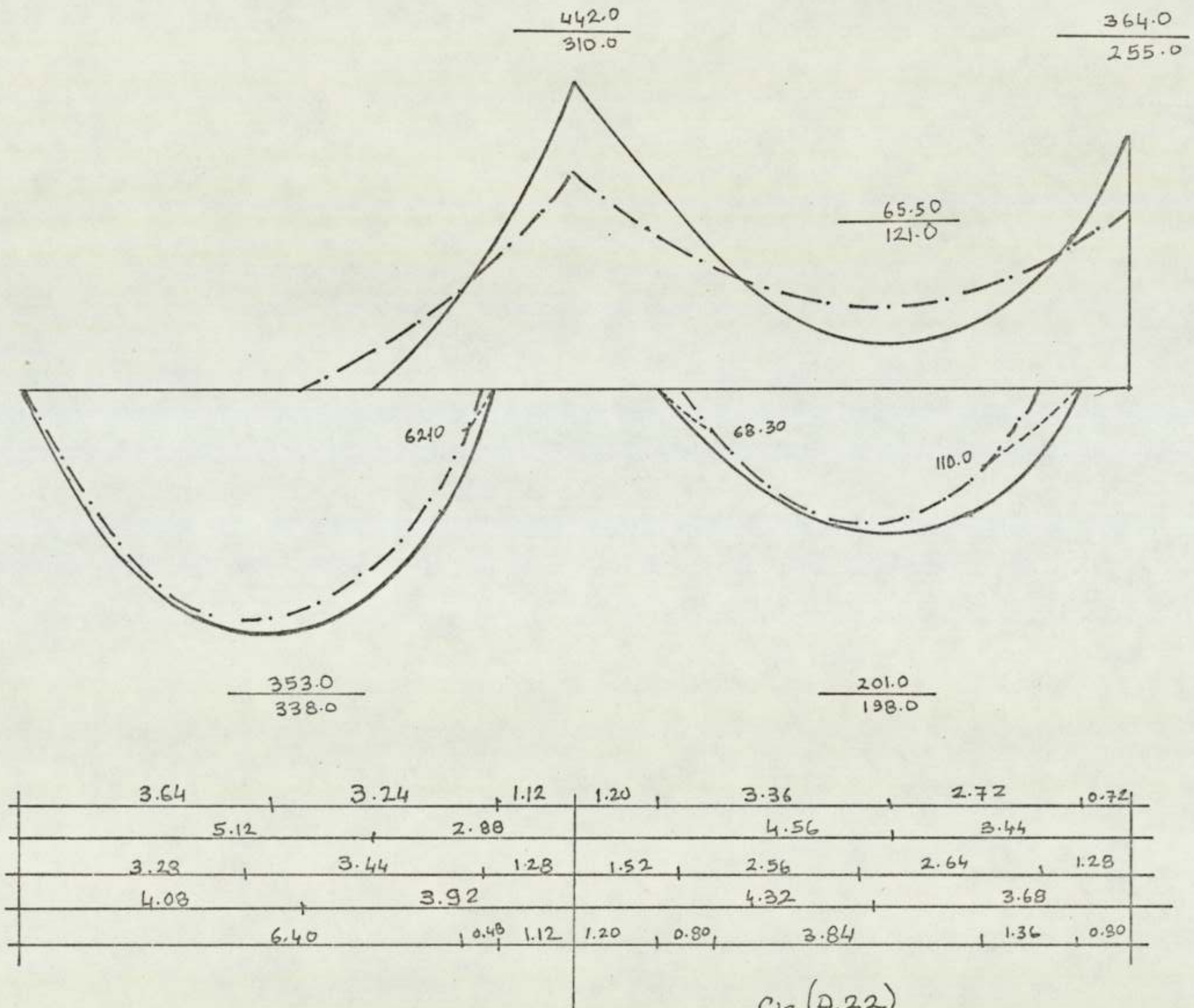

4 span beam

F = 52.0 kn/m

$G_k$ = 20.0 kn/m

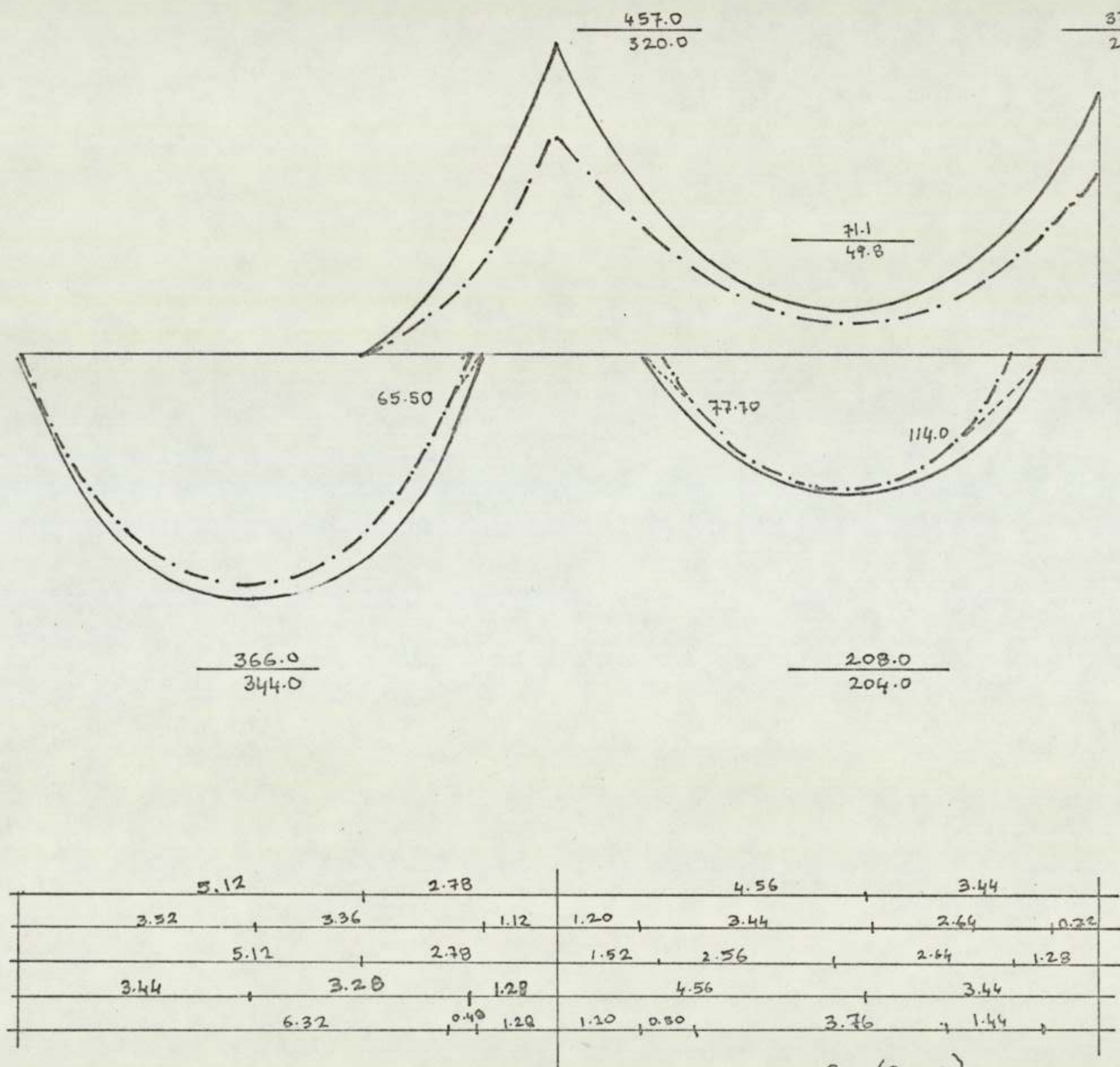
$\frac{Q_k}{G_k} = 1.6$

fig.(A.21.)




4 span beam
F = 60.0 kn/m
$G_k$ = 20.0 kn/m
$\frac{Q_k}{G_k} = 2.0$

Fig.(A.22)



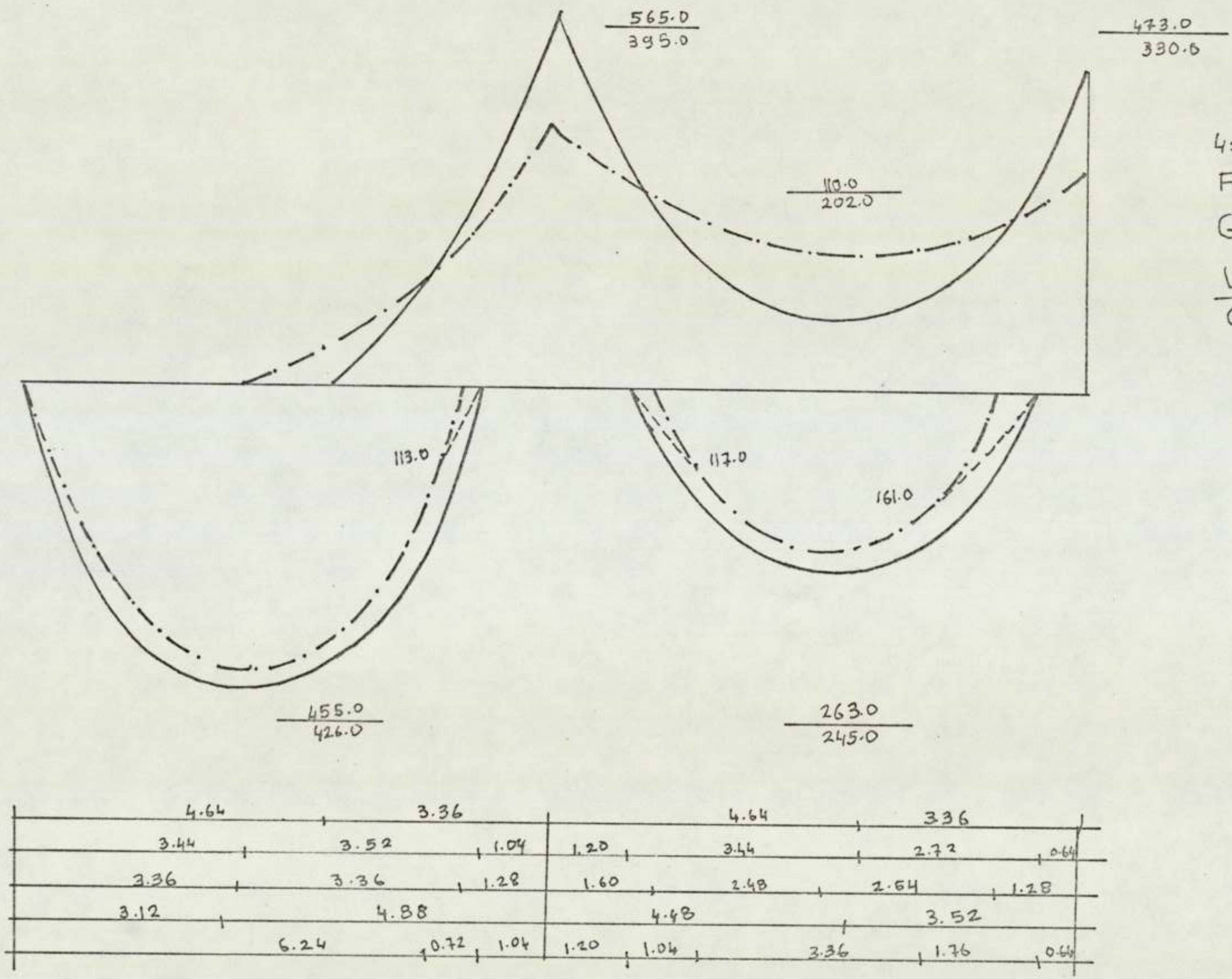

4 span beam

F = 62.0 kn/m

$G_k$ = 20.0 kn/m

$\frac{Q_k}{G_k} = 2.1$

Fig. (A.23)




4span beam

F = 76.0 kn/m

$G_k$ = 20.0 kn/m

$\frac{Q_k}{G_k} = 2.8$

fig. (A.24)

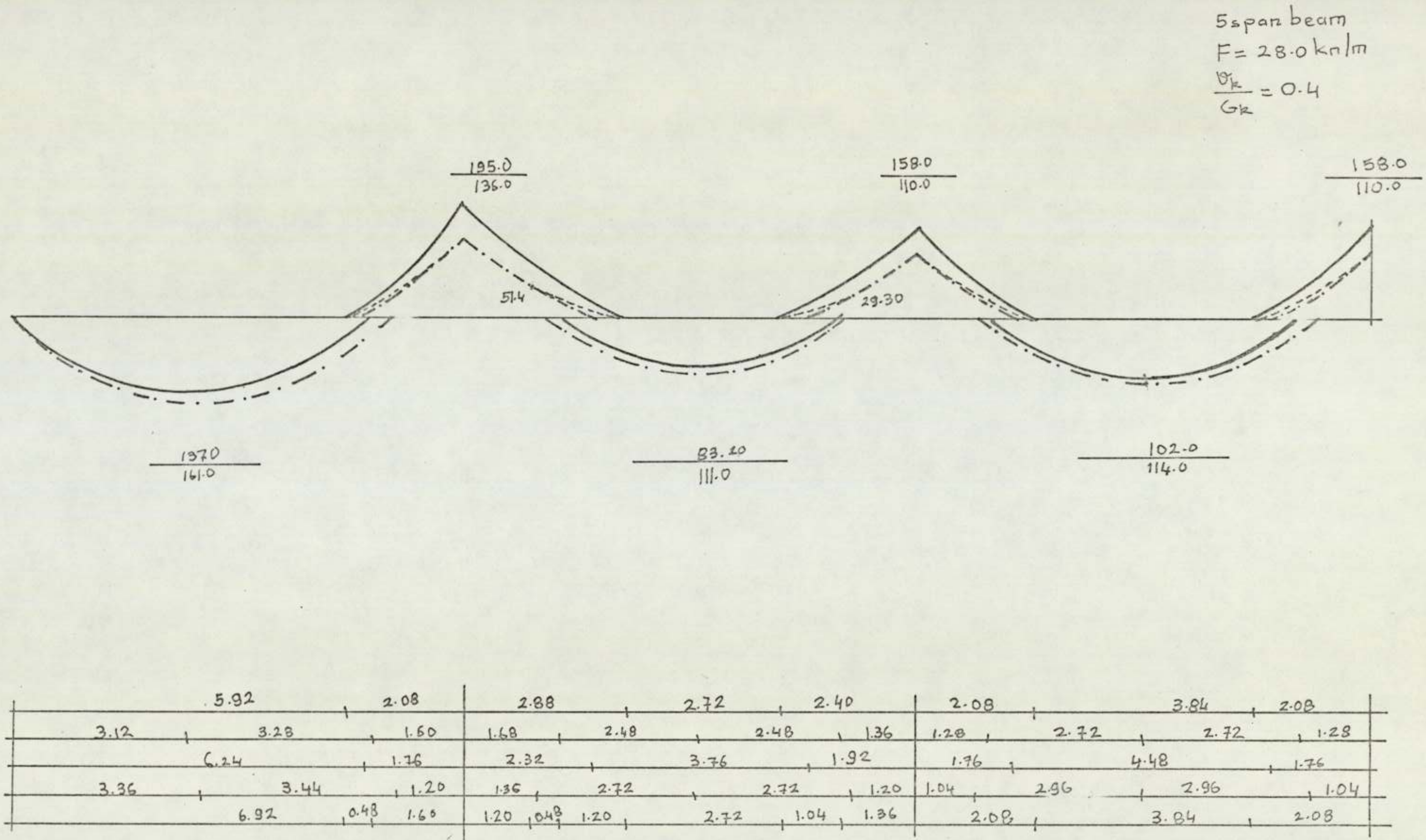

5span beam
F= 28.0 kn/m
Qk/Gk = 0.4
195.0 / 136.0
158.0 / 110.0
158.0 / 110.0
51.4
29.30
1370 / 161.0
83.20 / 111.0
102.0 / 114.0
5.92 2.08 2.88 2.72 2.40 2.08 3.84 2.08
3.12 3.28 1.60 1.68 2.48 2.48 1.36 1.28 2.72 2.72 1.28
6.24 1.76 2.32 3.76 1.92 1.76 4.48 1.76
3.36 3.44 1.20 1.36 2.72 2.72 1.20 1.04 2.96 2.96 1.04
6.92 0.48 1.60 1.20 0.48 1.20 2.72 1.04 1.36 2.08 3.84 2.08

Fig.(A.25)

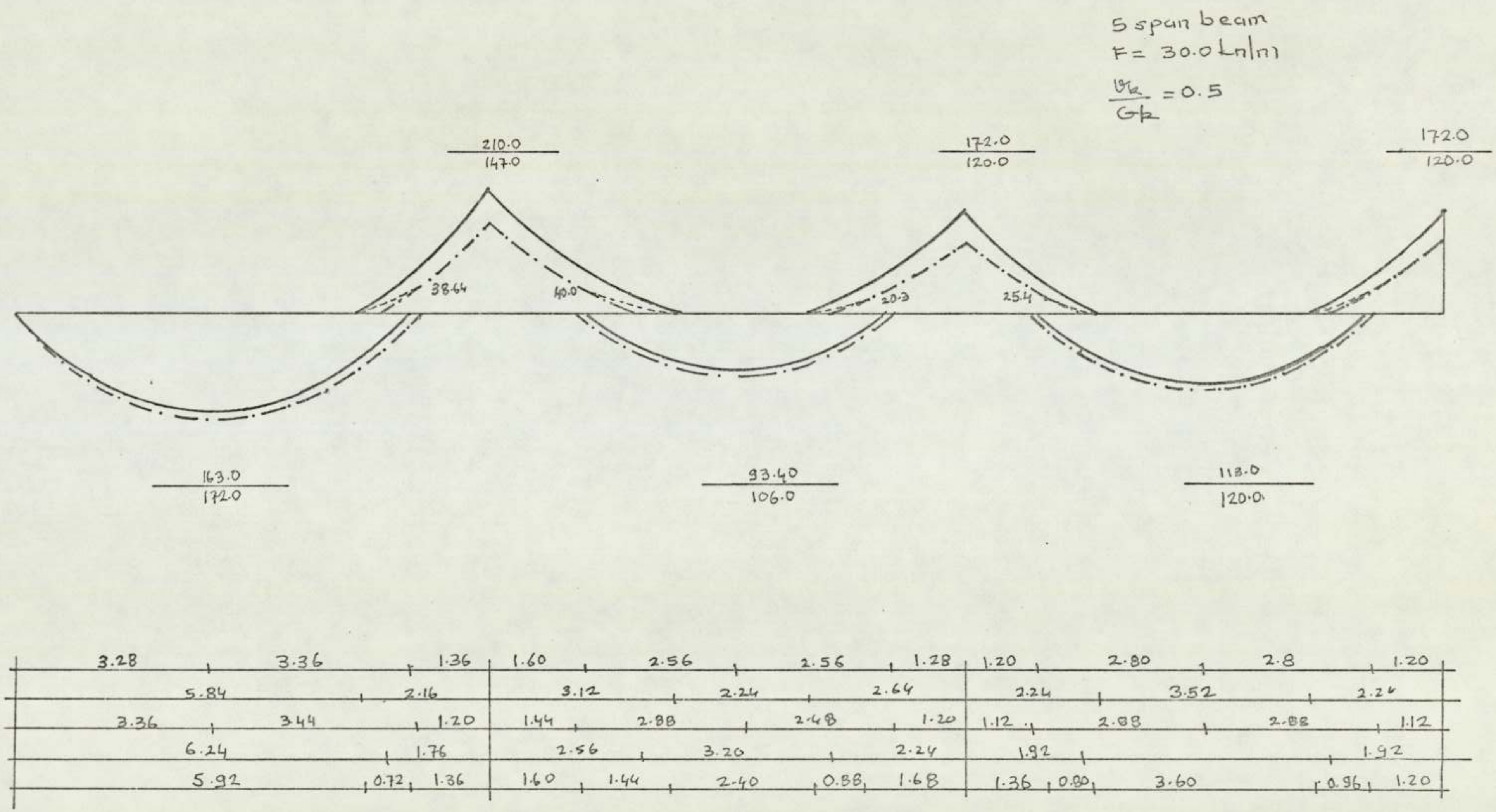

5 span beam
F= 30.0 kn/m
Vk/Gk = 0.5
210.0 / 147.0
172.0 / 120.0
172.0 / 120.0
38.64
40.0
20.3
25.4
163.0 / 172.0
93.40 / 106.0
113.0 / 120.0
3.28 3.36 1.36 1.60 2.56 2.56 1.28 1.20 2.80 2.8 1.20
5.84 2.16 3.12 2.24 2.64 2.24 3.52 2.24
3.36 3.44 1.20 1.44 2.88 2.48 1.20 1.12 2.88 2.88 1.12
6.24 1.76 2.56 3.20 2.24 1.92 1.92
5.92 0.72 1.36 1.60 1.44 2.40 0.58 1.68 1.36 0.80 3.60 0.96 1.20

Fig.(A.26)

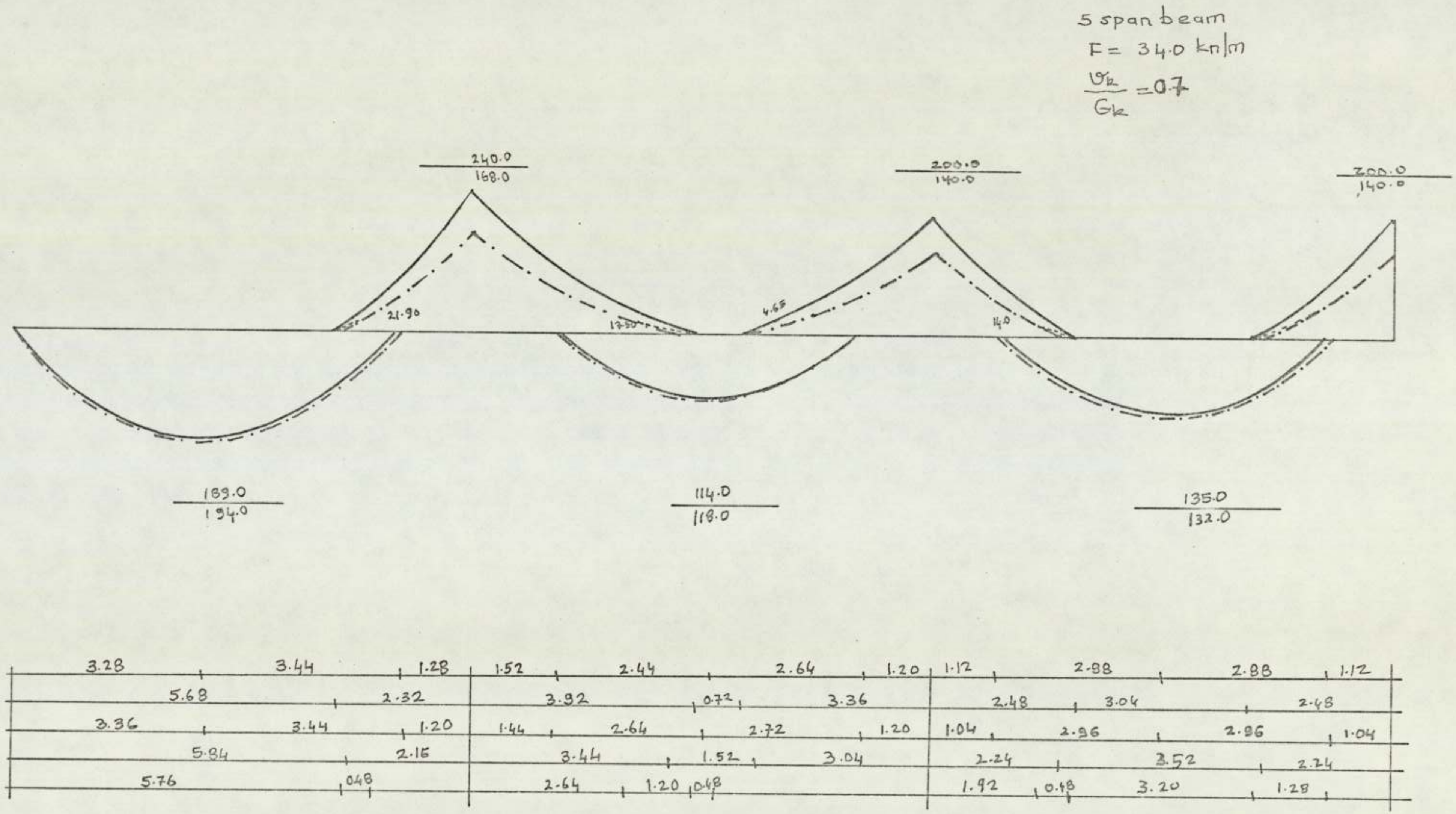

5 span beam
F = 34.0 kn/m
Qk/Gk = 0.7
240.0
168.0
200.0
140.0
200.0
140.0
21.90
13.50
4.65
14.0
189.0
194.0
114.0
118.0
135.0
132.0
3.28 3.44 1.28 1.52 2.44 2.64 1.20 1.12 2.88 2.88 1.12
5.68 2.32 3.92 0.72 3.36 2.48 3.04 2.48
3.36 3.44 1.20 1.44 2.64 2.72 1.20 1.04 2.96 2.96 1.04
5.84 2.16 3.44 1.52 3.04 2.24 3.52 2.24
5.76 0.48 2.64 1.20 0.48 1.92 0.48 3.20 1.28

Fig (A.27)

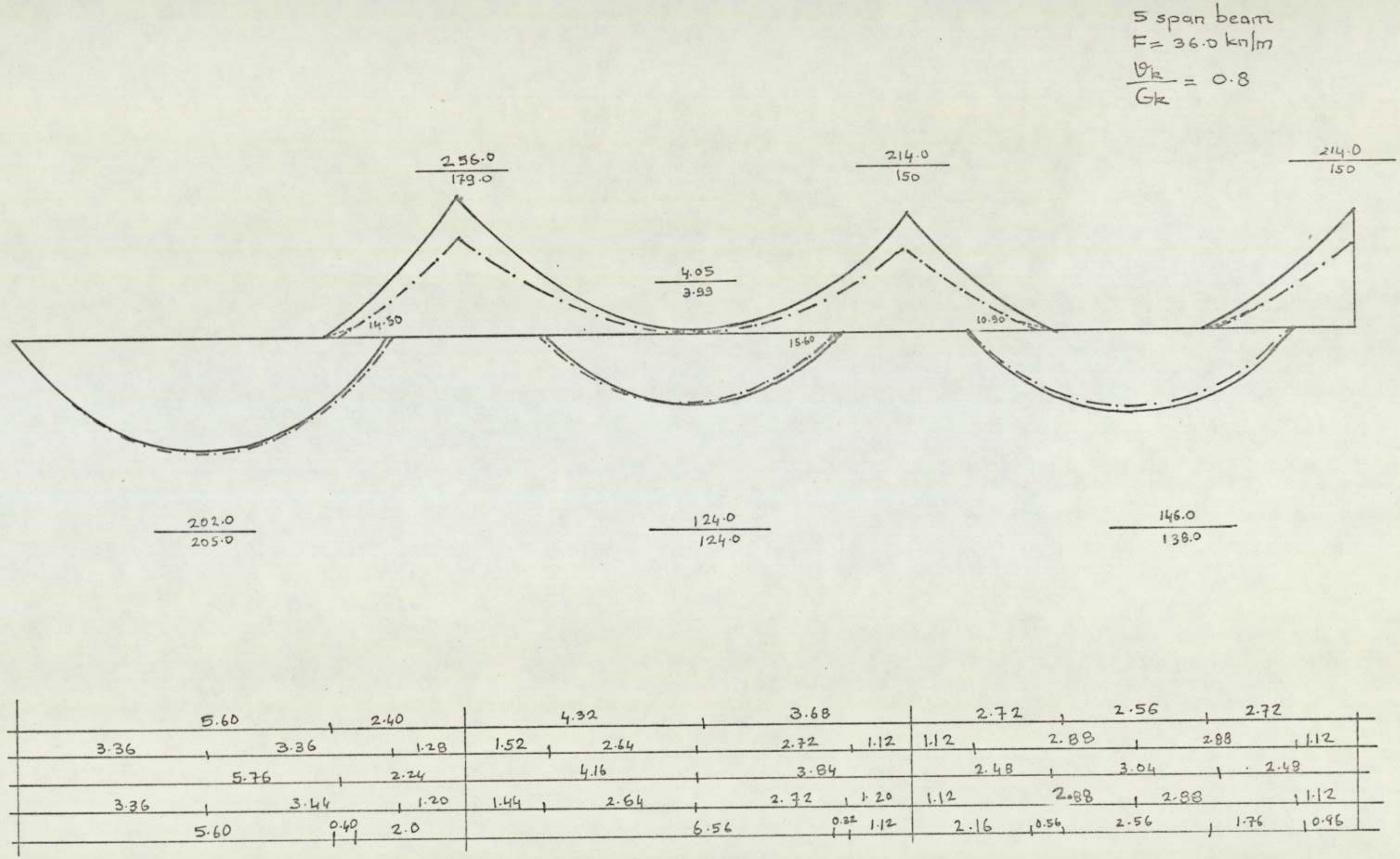

5 span beam
F = 36.0 kn/m
Qk/Gk = 0.8
256.0
179.0
214.0
150
214.0
150
4.05
3.99
14.50
15.60
10.90
202.0
205.0
124.0
124.0
146.0
138.0
5.60
2.40
4.32
3.68
2.72
2.56
2.72
3.36
3.36
1.28
1.52
2.64
2.72
1.12
1.12
2.88
2.88
1.12
5.76
2.24
4.16
3.84
2.48
3.04
2.48
3.36
3.44
1.20
1.44
2.64
2.72
1.20
1.12
2.88
2.88
1.12
5.60
0.40
2.0
6.56
0.32
1.12
2.16
0.56
2.56
1.76
0.96

fig. (A.28)

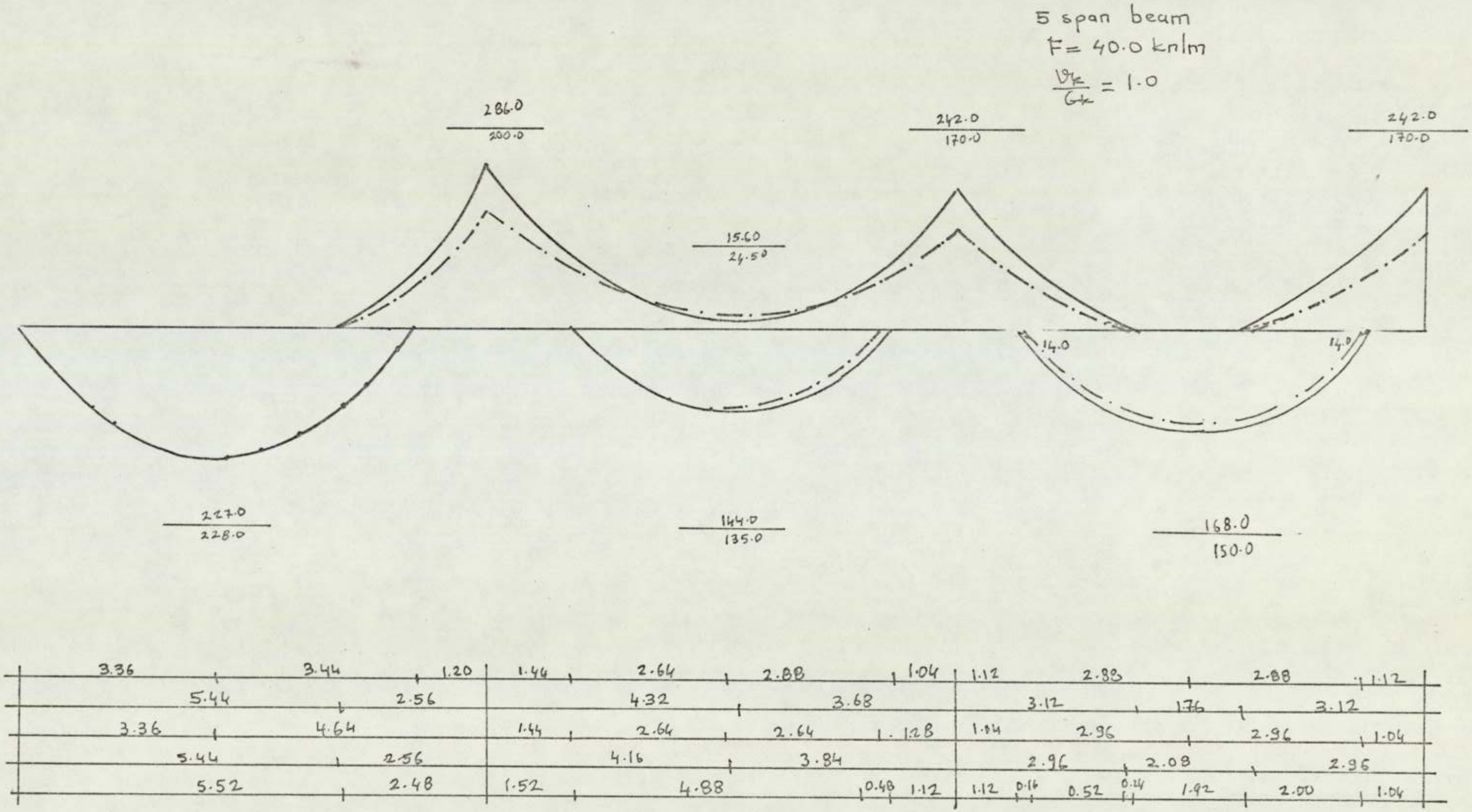
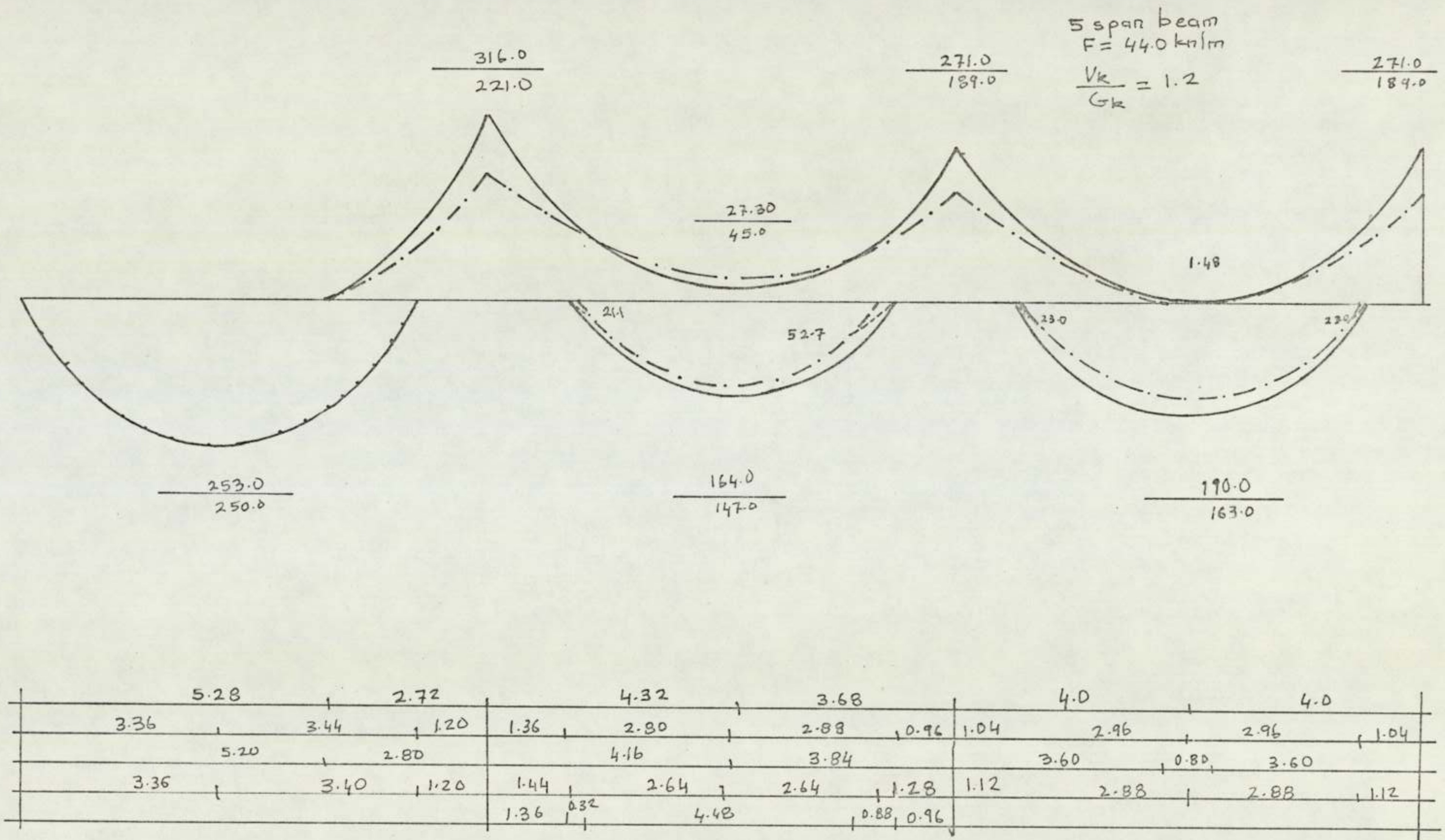

5 span beam
F= 40.0 kn/m
Vk/Gk = 1.0
286.0
200.0
15.60
24.50
242.0
170.0
242.0
170.0
14.0
14.0
227.0
228.0
144.0
135.0
168.0
150.0
3.36 3.44 1.20 1.44 2.64 2.88 1.04 1.12 2.88 2.88 1.12
5.44 2.56 4.32 3.68 3.12 1.76 3.12
3.36 4.64 1.44 2.64 2.64 1.28 1.04 2.96 2.96 1.04
5.44 2.56 4.16 3.84 2.96 2.08 2.96
5.52 2.48 1.52 4.88 0.48 1.12 1.12 0.16 0.52 0.14 1.92 2.00 1.04

fig.(A.29)



5 span beam
F= 44.0 kn/m
$\frac{V_k}{G_k} = 1.2$
316.0 / 221.0
271.0 / 189.0
271.0 / 189.0
27.30 / 45.0
1.48
21.1
52.7
23.0
23.0
253.0 / 250.0
164.0 / 147.0
190.0 / 163.0
5.28
2.72
4.32
3.68
4.0
4.0
3.36
3.44
1.20
1.36
2.80
2.88
0.96
1.04
2.96
2.96
1.04
5.20
2.80
4.16
3.84
3.60
0.80
3.60
3.36
3.40
1.20
1.44
2.64
2.64
1.28
1.12
2.88
2.88
1.12
1.36
0.32
4.48
0.88
0.96


Fig. (A.30)

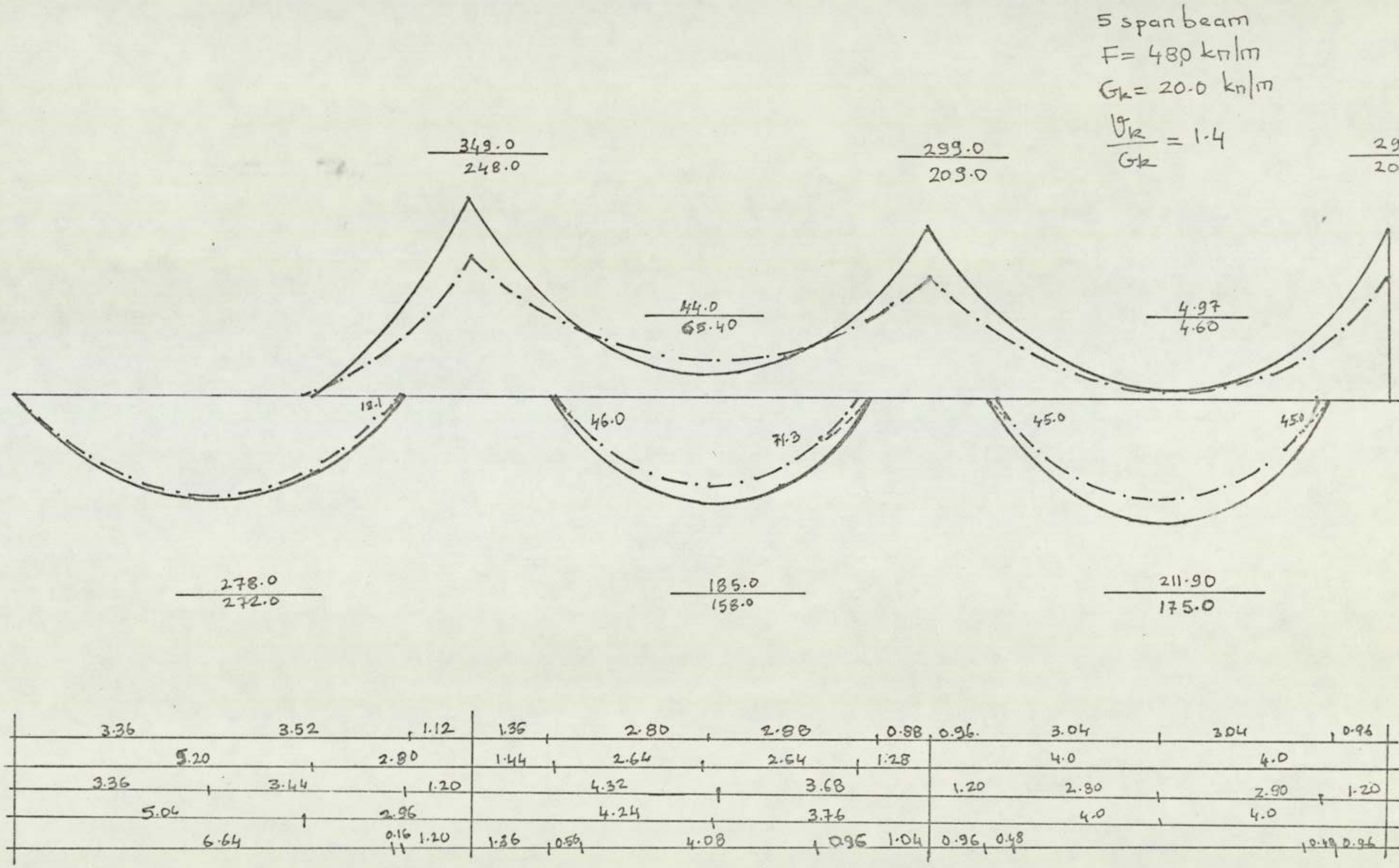
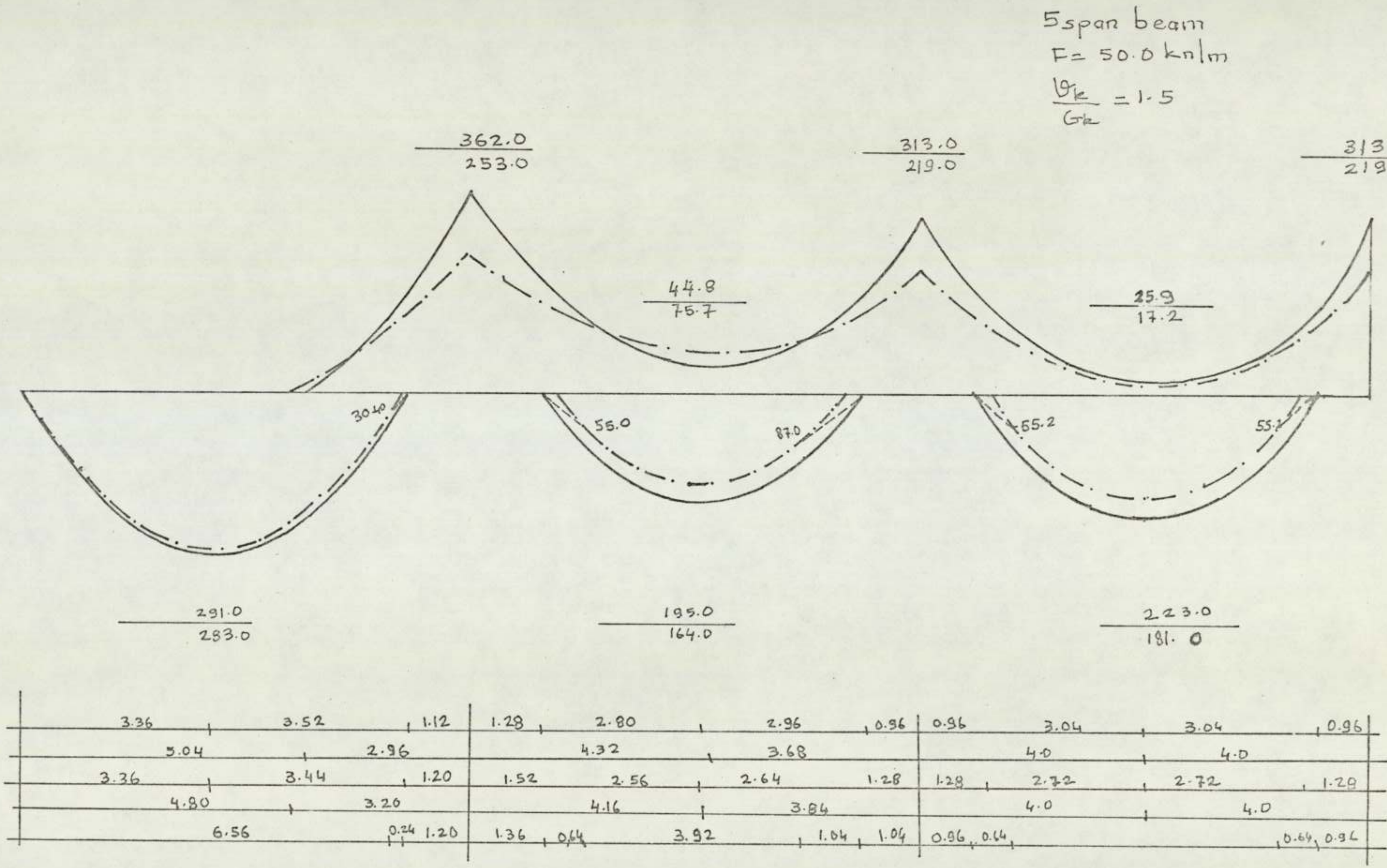
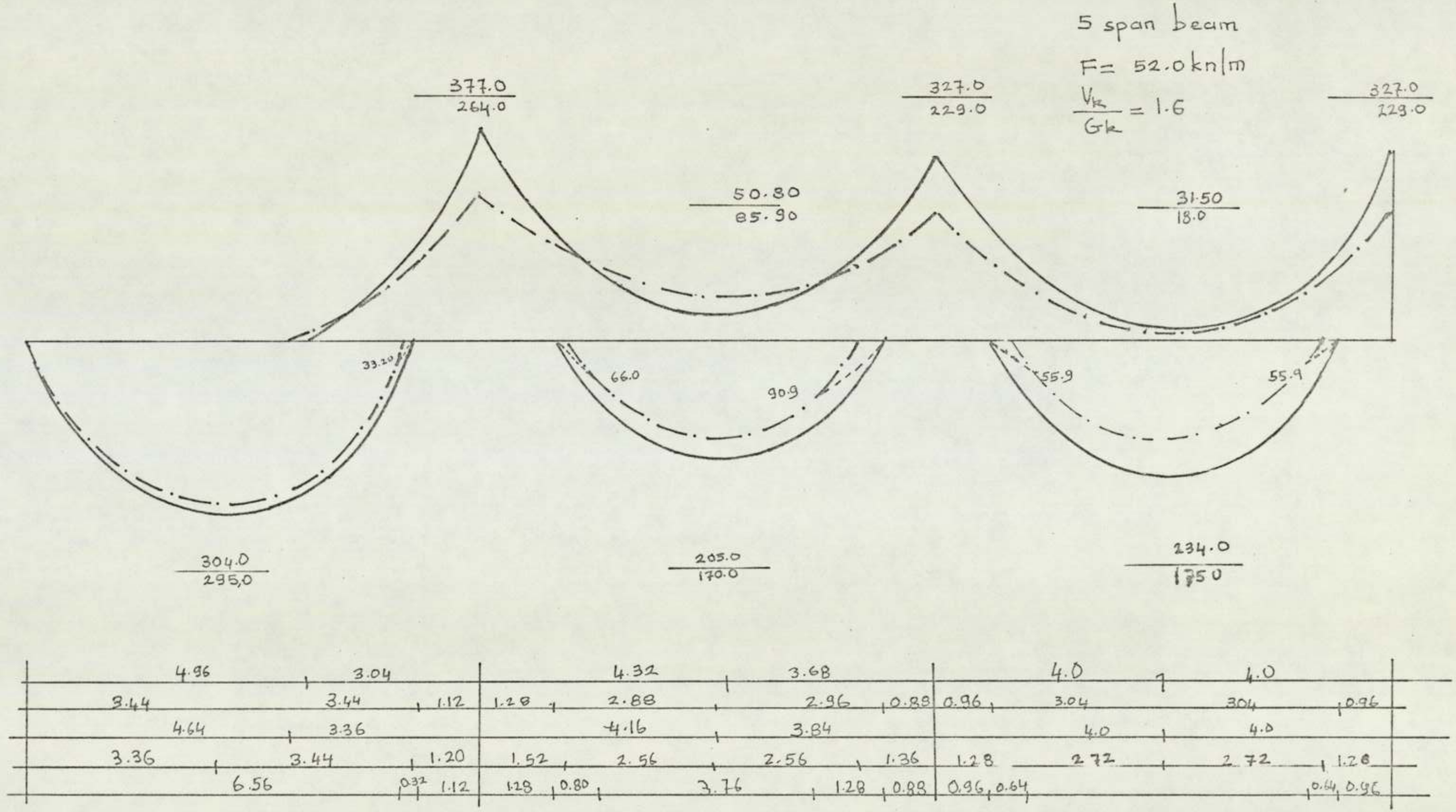

5 span beam
F= 480 kn/m
Gk= 20.0 kn/m
Vk/Gk = 1.4
349.0
248.0
299.0
209.0
299.0
209.0
44.0
65.40
4.97
4.60
18.1
46.0
71.3
45.0
45.0
278.0
272.0
185.0
158.0
211.90
175.0
3.36
3.52
1.12
1.36
2.80
2.88
0.88
0.96
3.04
3.04
0.96
5.20
2.80
1.44
2.64
2.64
1.28
4.0
4.0
3.36
3.44
1.20
4.32
3.68
1.20
2.90
2.90
1.20
5.04
2.96
4.24
3.76
4.0
4.0
6.64
0.16
1.20
1.36
0.56
4.08
0.96
1.04
0.96
0.48
0.48
0.96

Fig. (A.31)



5span beam
F= 50.0 kn/m
Qk/Gk = 1.5
362.0
253.0
313.0
219.0
313.0
219.0
44.8
75.7
25.9
17.2
30.0
55.0
87.0
55.2
55.2
291.0
283.0
195.0
164.0
223.0
181.0
3.36
3.52
1.12
1.28
2.80
2.96
0.96
0.96
3.04
3.04
0.96
5.04
2.96
4.32
3.68
4.0
4.0
3.36
3.44
1.20
1.52
2.56
2.64
1.28
1.28
2.72
2.72
1.28
4.80
3.20
4.16
3.84
4.0
4.0
6.56
0.24
1.20
1.36
0.64
3.92
1.04
1.04
0.96
0.64
0.64
0.96


fig. (A.32)



5 span beam
F= 52.0kn/m
$\frac{V_k}{G_k} = 1.6$
377.0 / 264.0
327.0 / 229.0
327.0 / 229.0
50.80 / 85.90
31.50 / 18.0
33.20
66.0
90.9
55.9
55.9
304.0 / 295.0
205.0 / 170.0
234.0 / 175.0
4.96 3.04 4.32 3.68 4.0 4.0
3.44 3.44 1.12 1.28 2.88 2.96 0.88 0.96 3.04 304 0.96
4.64 3.36 4.16 3.84 4.0 4.0
3.36 3.44 1.20 1.52 2.56 2.56 1.36 1.28 272 2.72 1.28
6.56 0.32 1.12 1.28 0.80 3.76 1.28 0.88 0.96 0.64 0.64 0.96

fig. (A.33)

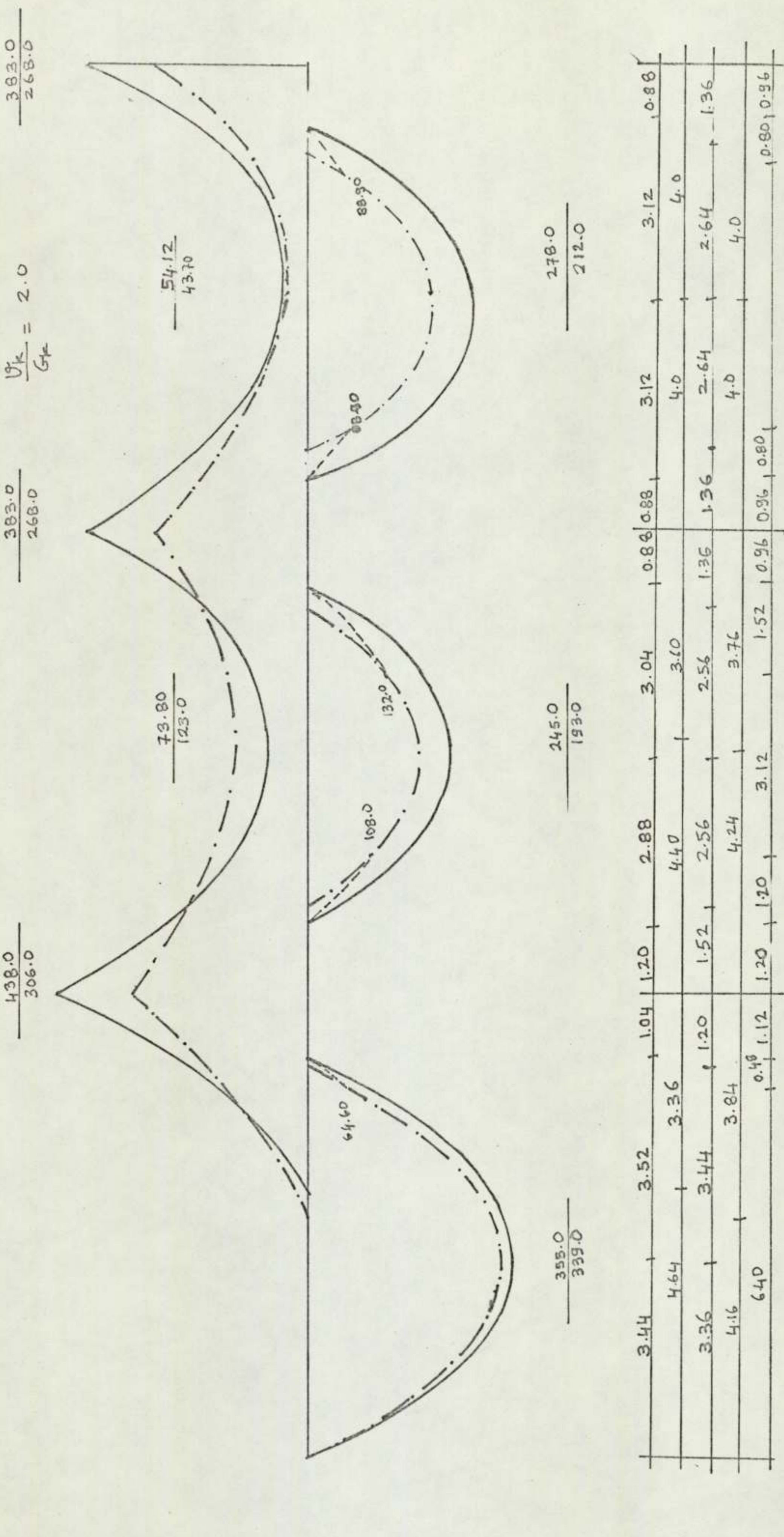

5 span beam
F = 60.0 kn/m
$\frac{Q_k}{G_k}$ = 2.0
383.0 / 268.0
54.12 / 43.70
383.0 / 268.0
73.80 / 123.0
438.0 / 306.0
278.0 / 212.0
245.0 / 193.0
355.0 / 339.0
88.90
88.80
132.0
108.0
64.60
3.44
4.64
3.52
3.36
1.04
1.20
2.88
4.40
3.04
3.10
0.88
0.88
3.12
4.0
3.12
4.0
0.88
3.36
3.44
1.20
1.52
2.56
2.56
1.36
1.36
2.64
2.64
1.36
4.16
3.84
4.24
3.76
4.0
4.0
640
0.48
1.12
1.20
1.20
3.12
1.52
0.96
0.96
0.80
0.80
0.96

Fig (A.34)

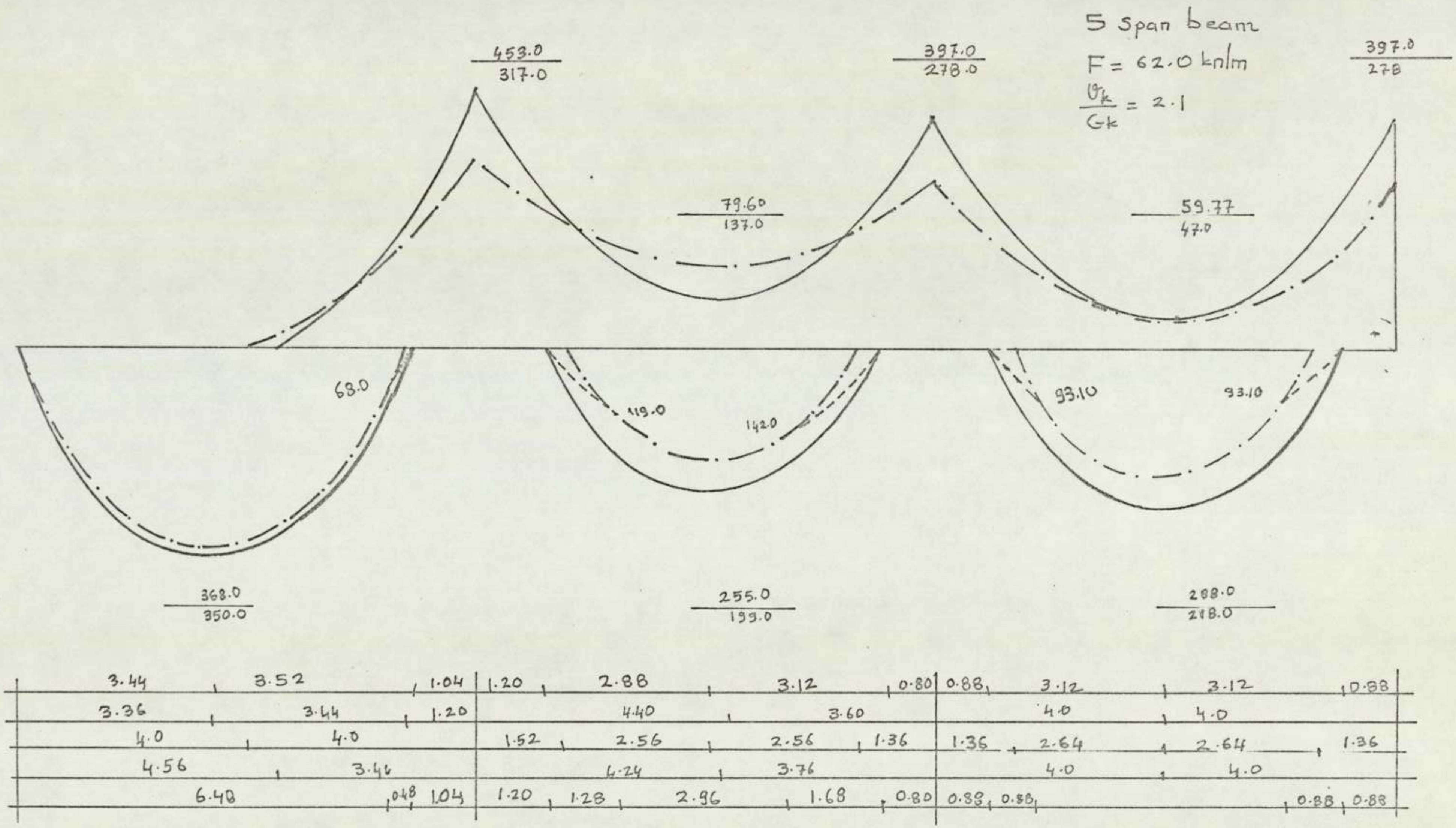

5 Span beam
F = 62.0 kn/m
$\frac{Q_k}{G_k} = 2.1$
453.0 / 317.0
397.0 / 278.0
397.0 / 278
79.60 / 137.0
59.77 / 47.0
68.0
119.0
142.0
93.10
93.10
368.0 / 350.0
255.0 / 199.0
288.0 / 218.0
3.44 3.52 1.04 1.20 2.88 3.12 0.80 0.88 3.12 3.12 0.88
3.36 3.44 1.20 4.40 3.60 4.0 4.0
4.0 4.0 1.52 2.56 2.56 1.36 1.36 2.64 2.64 1.36
4.56 3.46 4.24 3.76 4.0 4.0
6.48 0.48 1.04 1.20 1.28 2.96 1.68 0.80 0.88 0.88 0.88 0.88

fig (A.35)

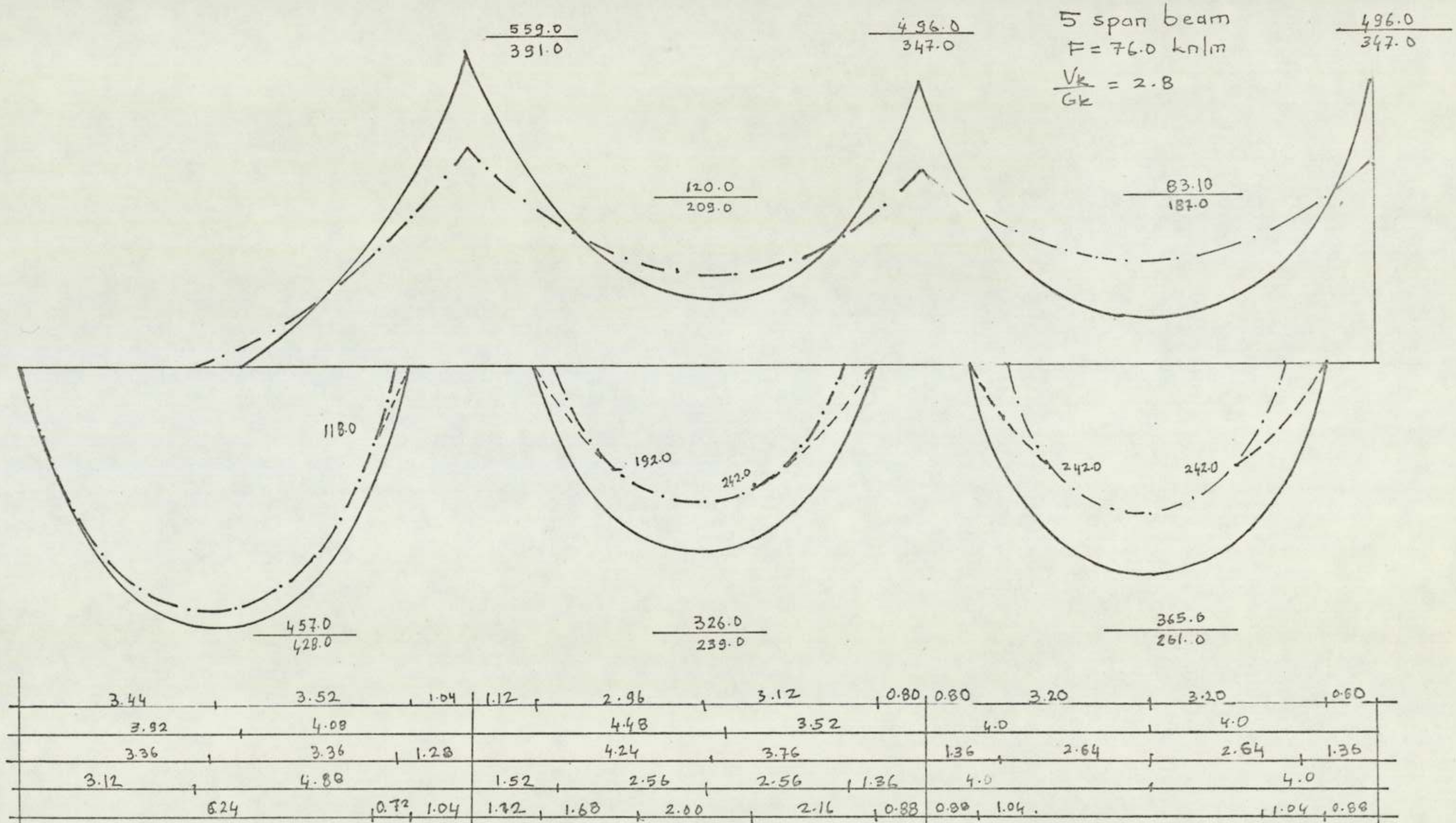

5 span beam
F = 76.0 kn/m
Vk/Gk = 2.8
559.0
391.0
496.0
347.0
496.0
347.0
120.0
209.0
83.10
187.0
118.0
192.0
242.0
242.0
242.0
457.0
428.0
326.0
239.0
365.0
261.0
3.44
3.52
1.04
1.12
2.96
3.12
0.80
0.80
3.20
3.20
0.80
3.92
4.08
4.48
3.52
4.0
4.0
3.36
3.36
1.28
4.24
3.76
1.36
2.64
2.64
1.36
3.12
4.88
1.52
2.56
2.56
1.36
4.0
4.0
6.24
0.72
1.04
1.12
1.68
2.00
2.16
0.88
0.88
1.04
1.04
0.88

fig.(A.36)

3 span frame

F = 28.0 kn/m

$\frac{V_k}{G_k} = 0.4$

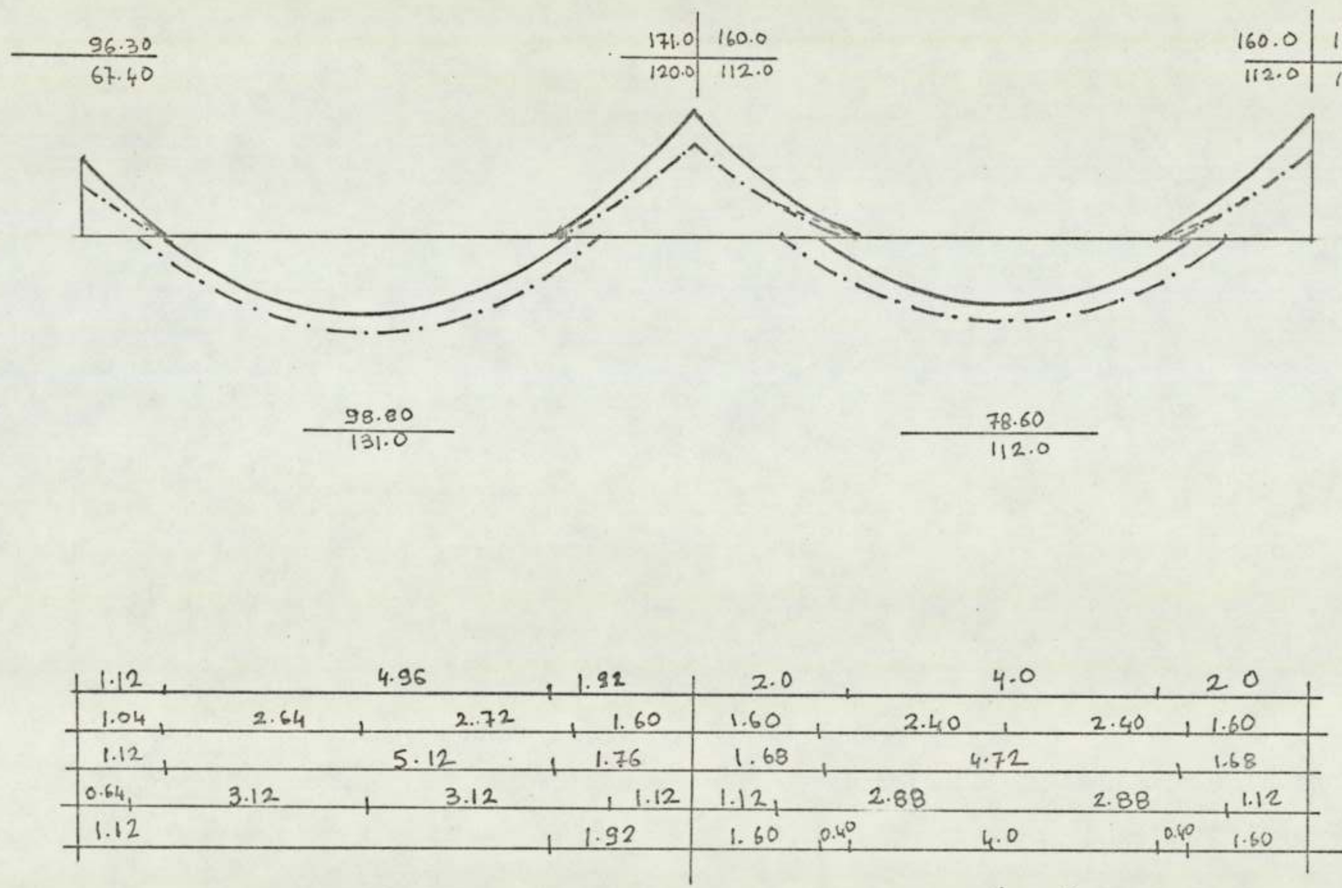



Fig.(A.37)

3 span frame

$F = 30.0$ kn/m

$\frac{V_K}{G_K} = 0.5$

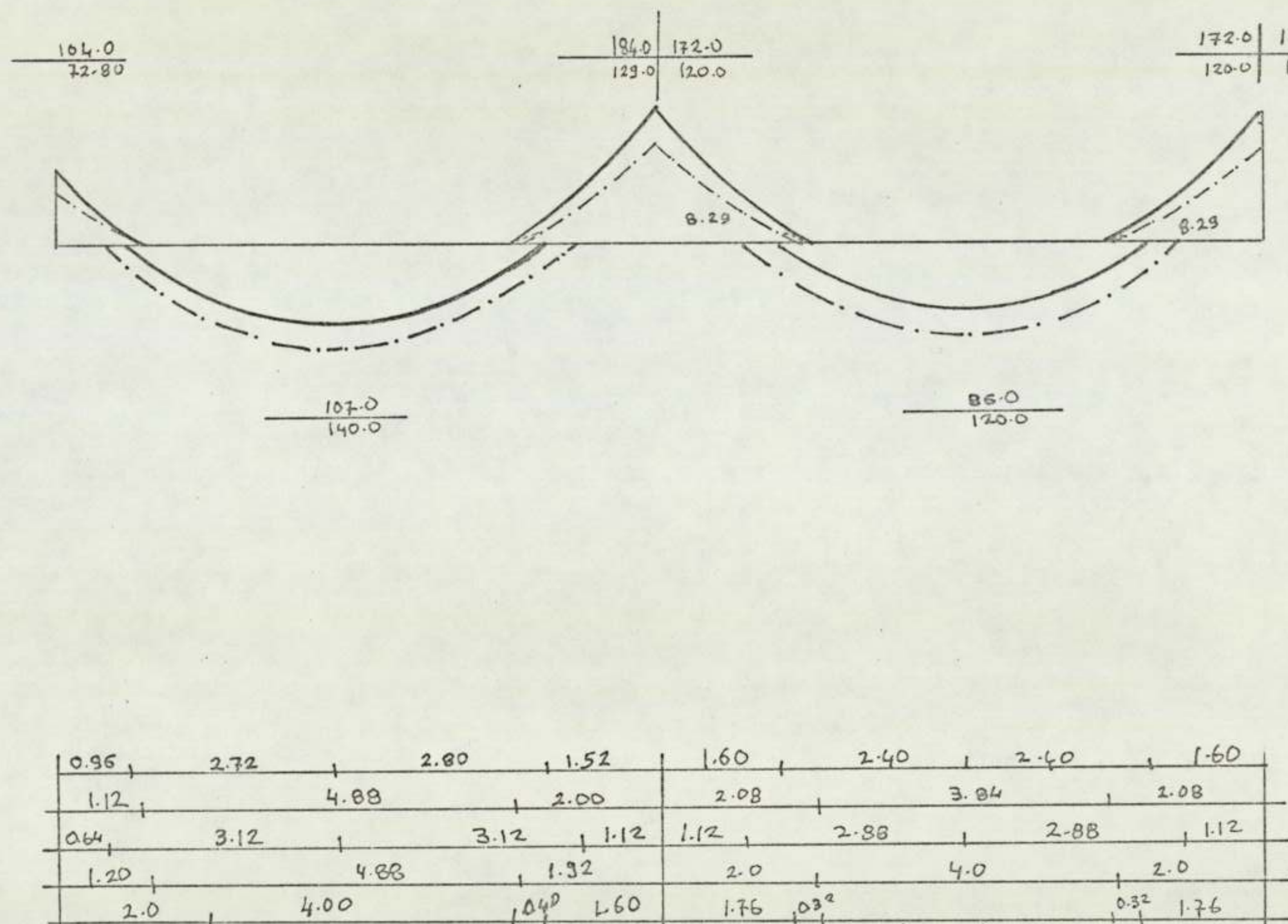



Fig (A.38)

3 span frame

$F = 34.0$ kn/m

$G_k = 20.0$ kn/m

$\frac{V_k}{G_k} = 0.7$

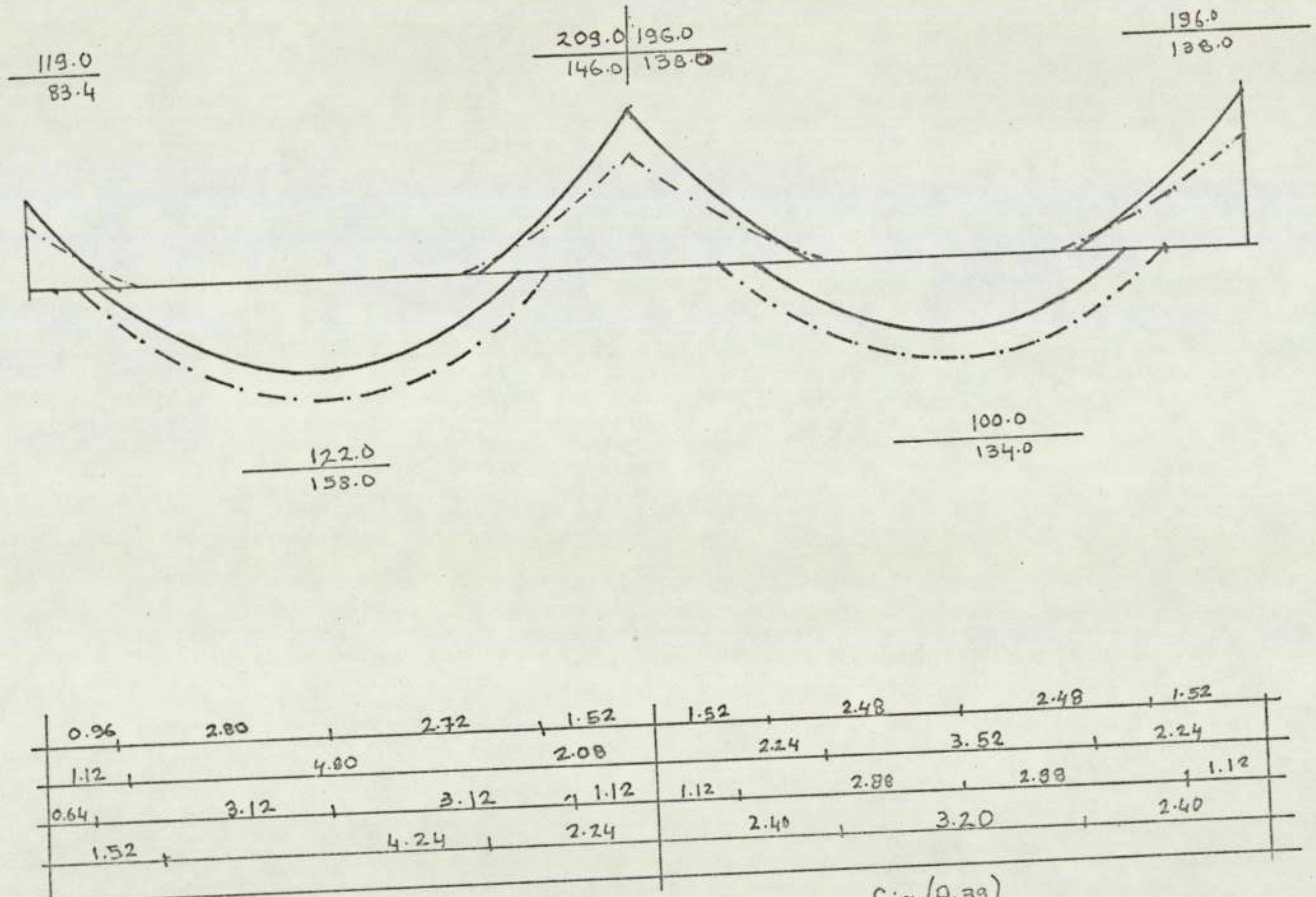
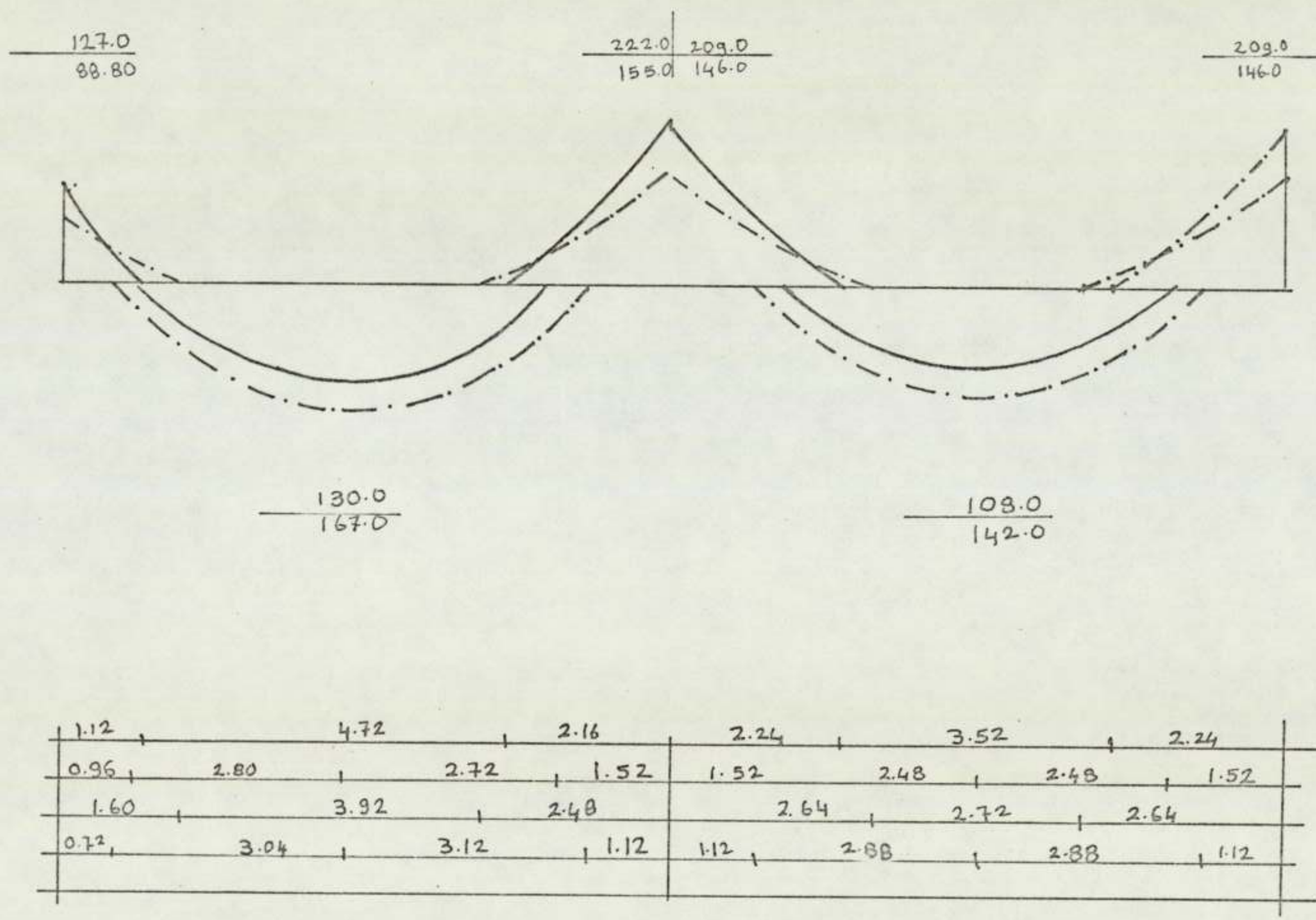



Fig.(A.39)




3 span frame

F = 36.0 kn/m,

$G_k$ = 20.0 kn/m

$\frac{Q_k}{G_k} = 0.8$

Fig (A.40)

3 span frame

F = 40.0 kn/m

$\frac{V_k}{G_k} = 1.0$

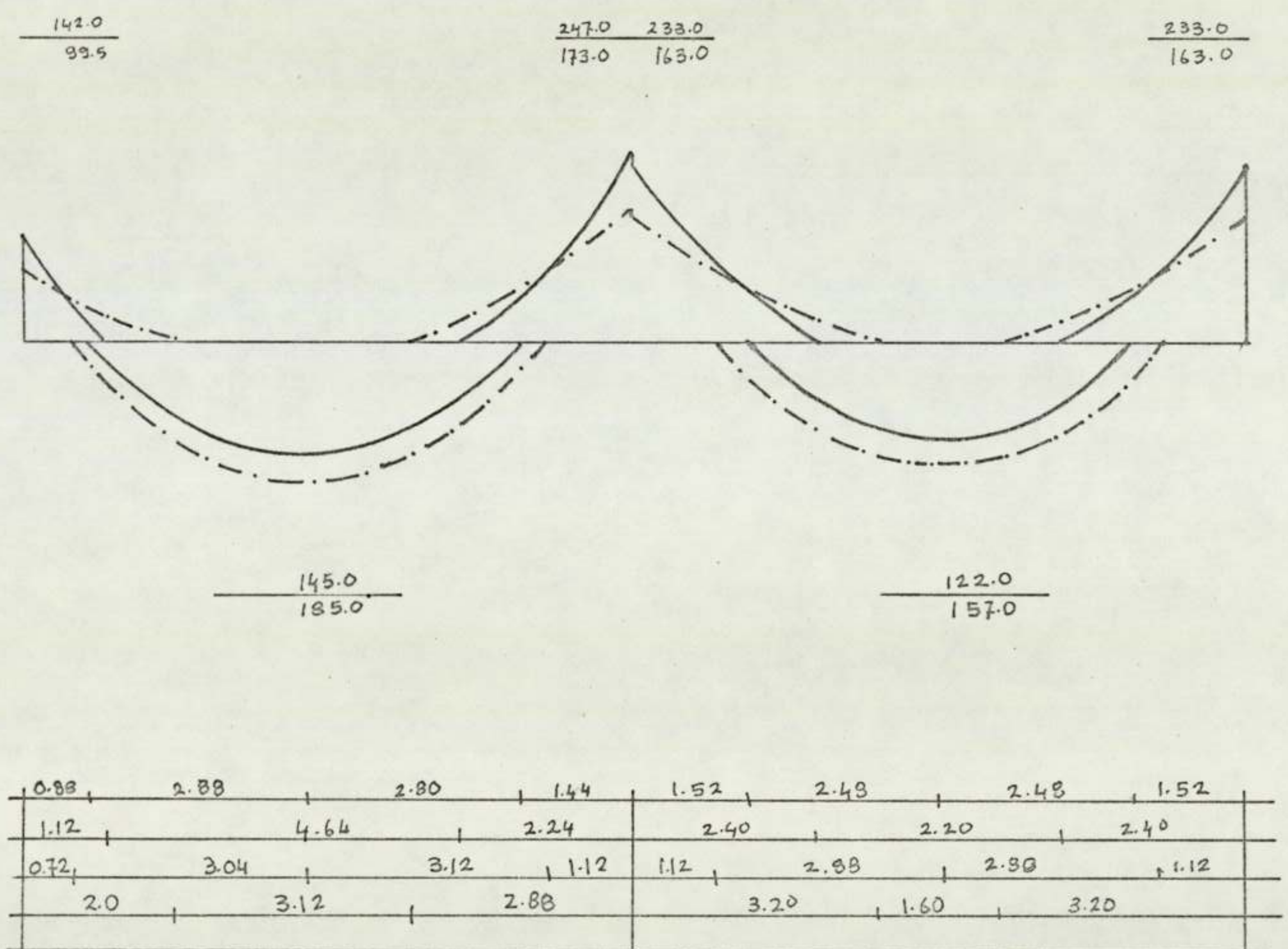
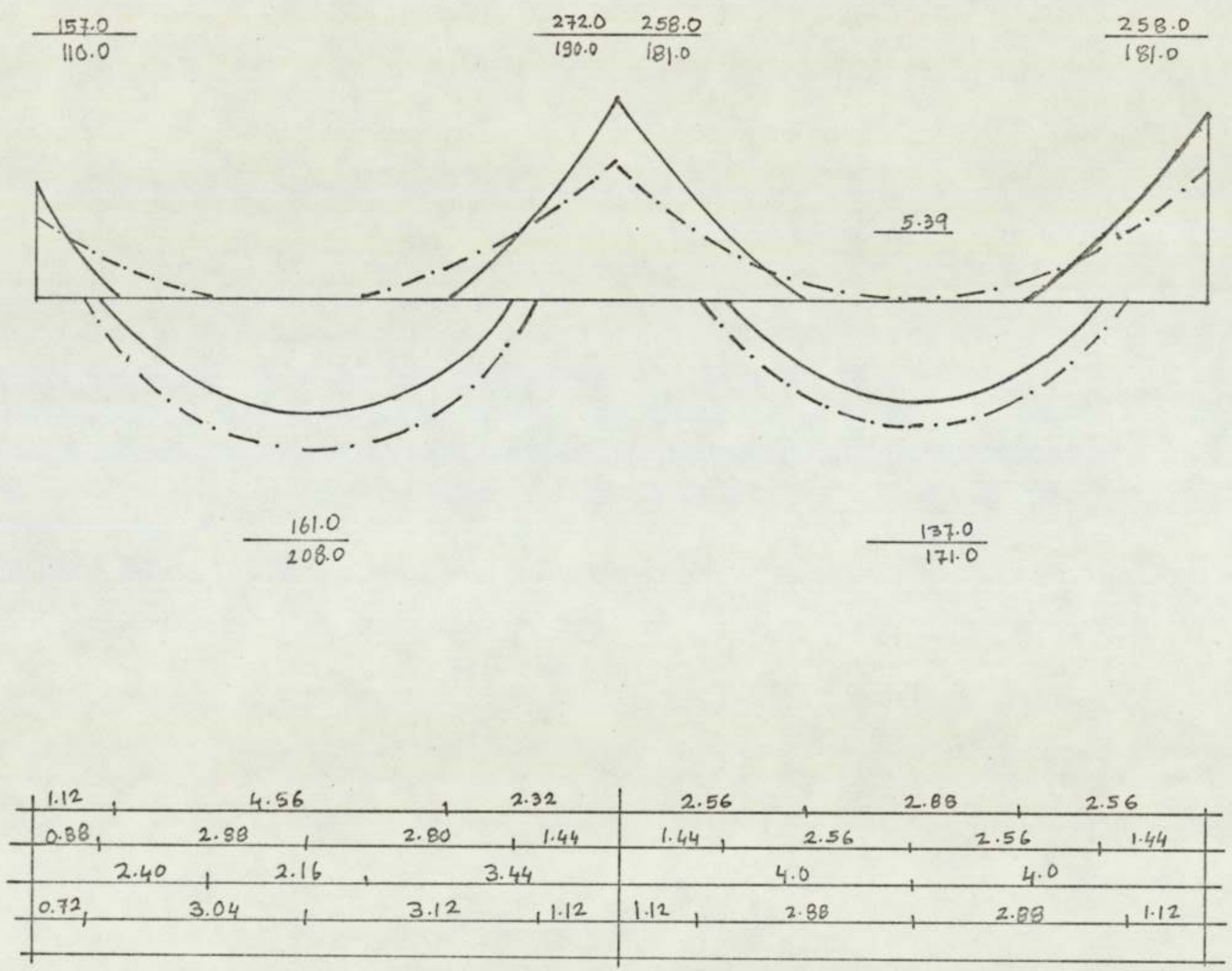



fig. (A.41)

3 span frame

$F = 44.0$ kn/m

$\frac{V_k}{G_k} = 1.2$




fig(A.42)

3 span frame

F= 48.0 kn/m

$\frac{V_k}{G_k} = 1.4$

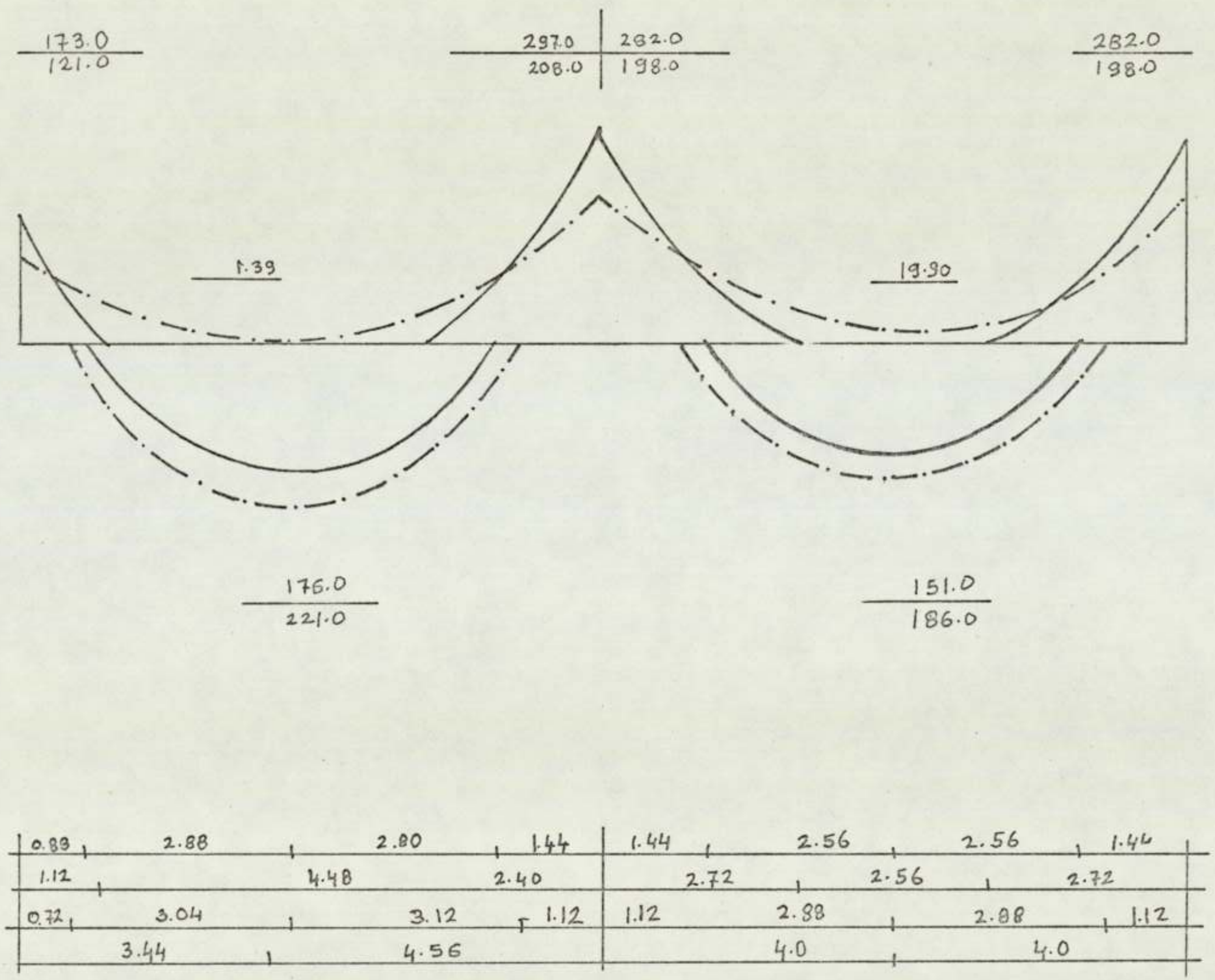



fig.(A.43)

3 span frame

F = 50.0 kn/m

$\frac{V_k}{G_k} = 1.5$

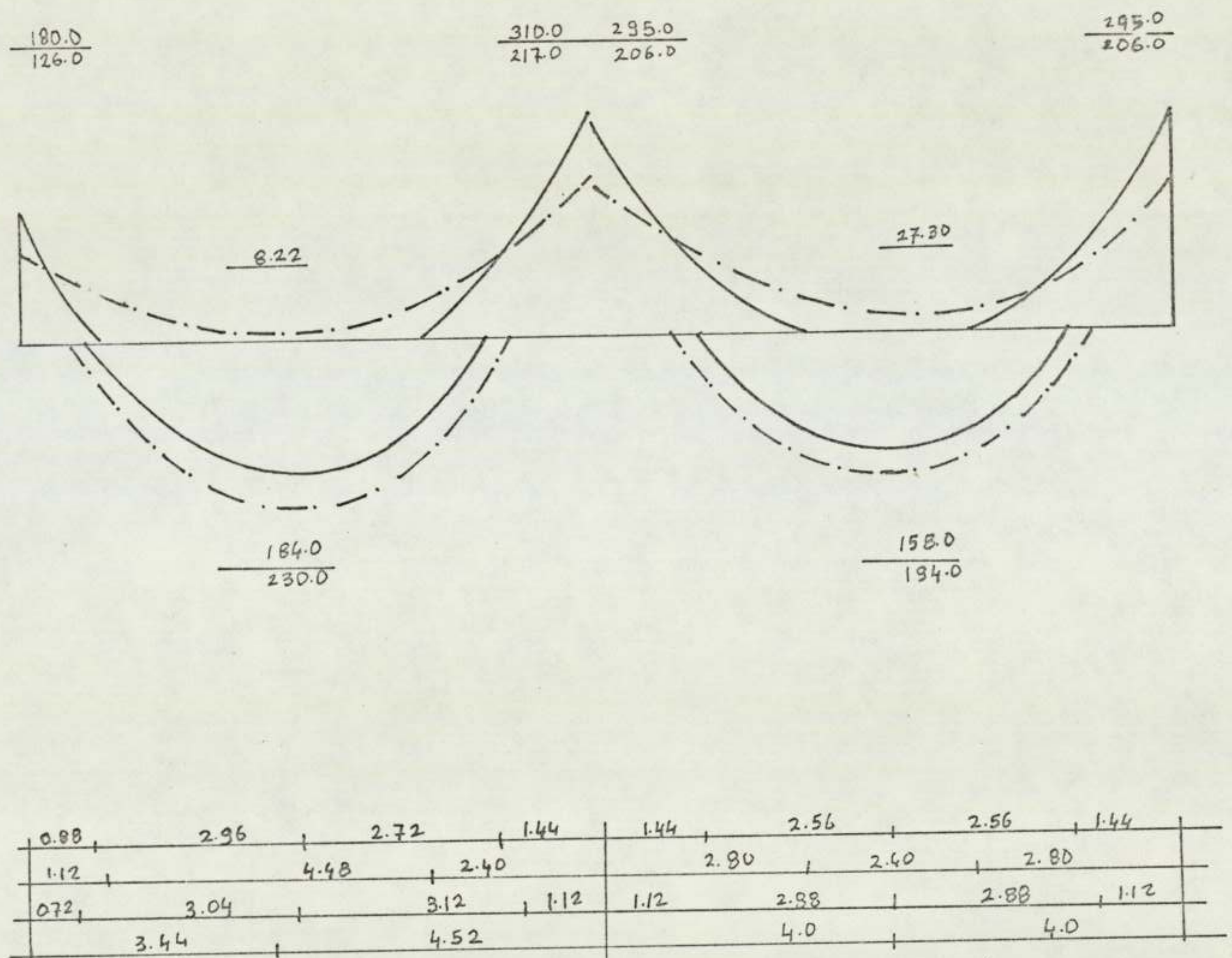



fig (A.44)

3 span frame

F = 52.0 kn/m

$\frac{V_k}{G_k} = 1.6$

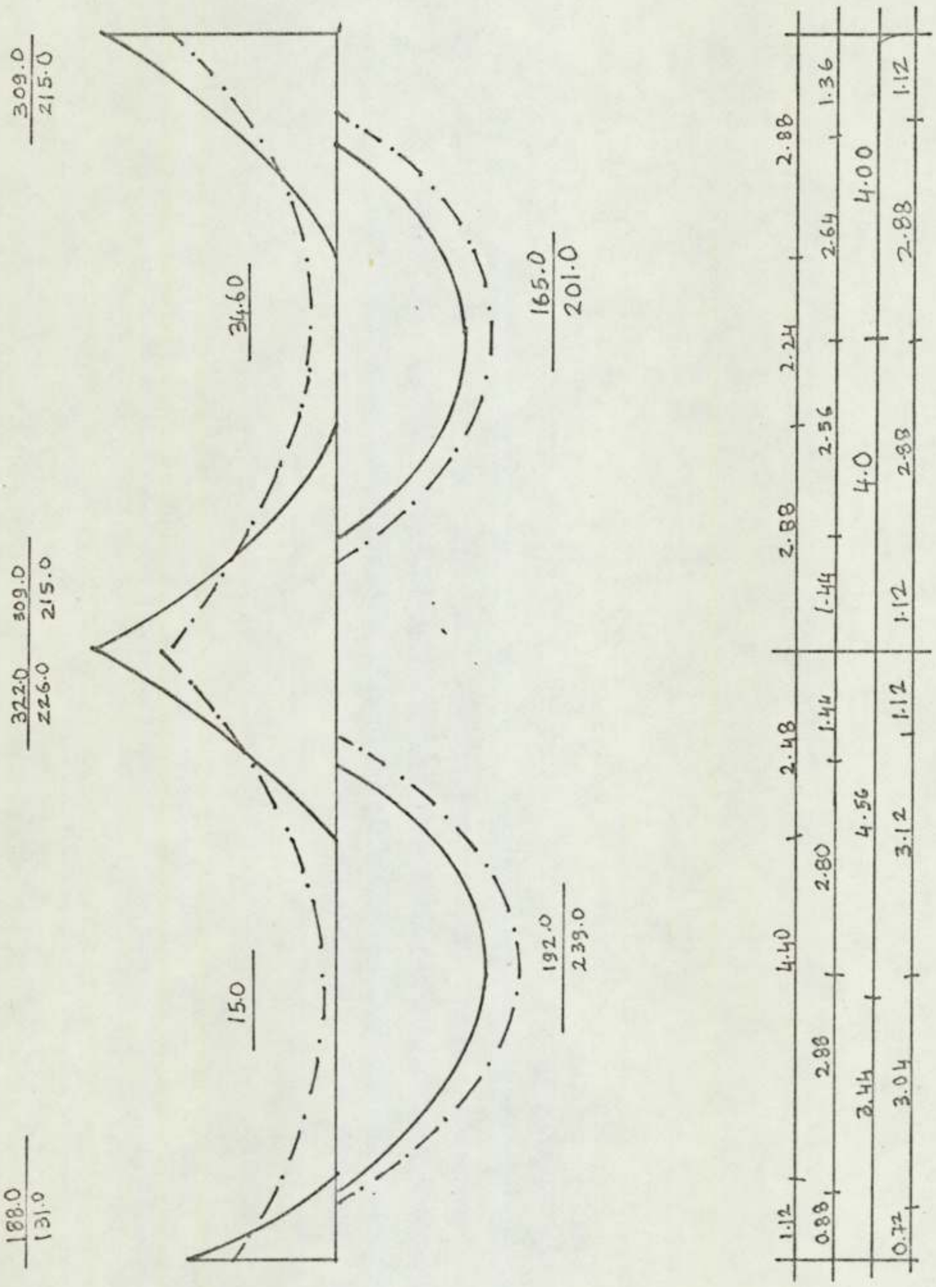



Fig.(A.45)

3 span frame

$F = 60.0$ kn/m,

$\frac{V_k}{G_k} = 2.0$

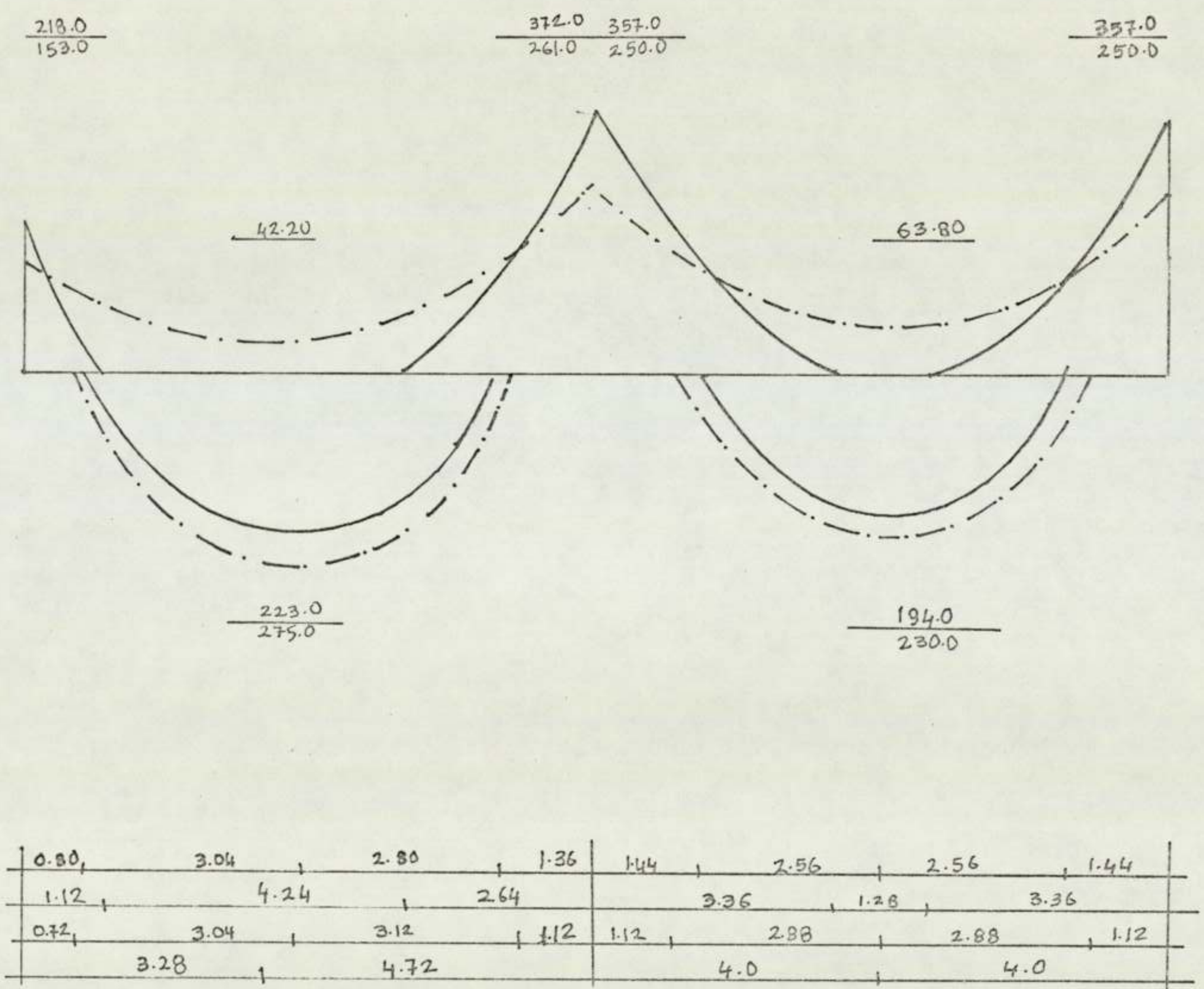



Fig.(A.46)

3 span frame

F= 62.0 kn/m

$\frac{V_k}{G_k} = 2.1$

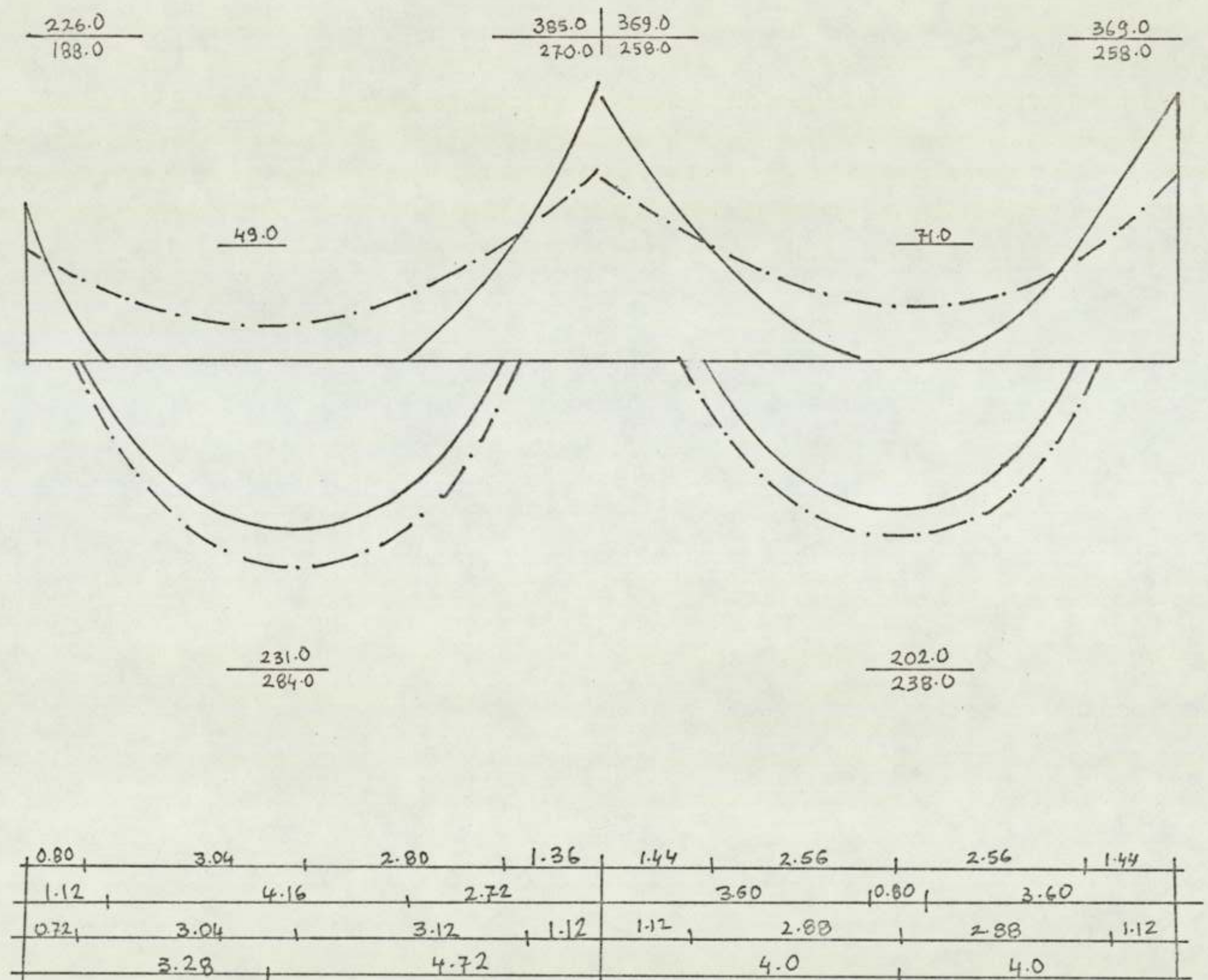
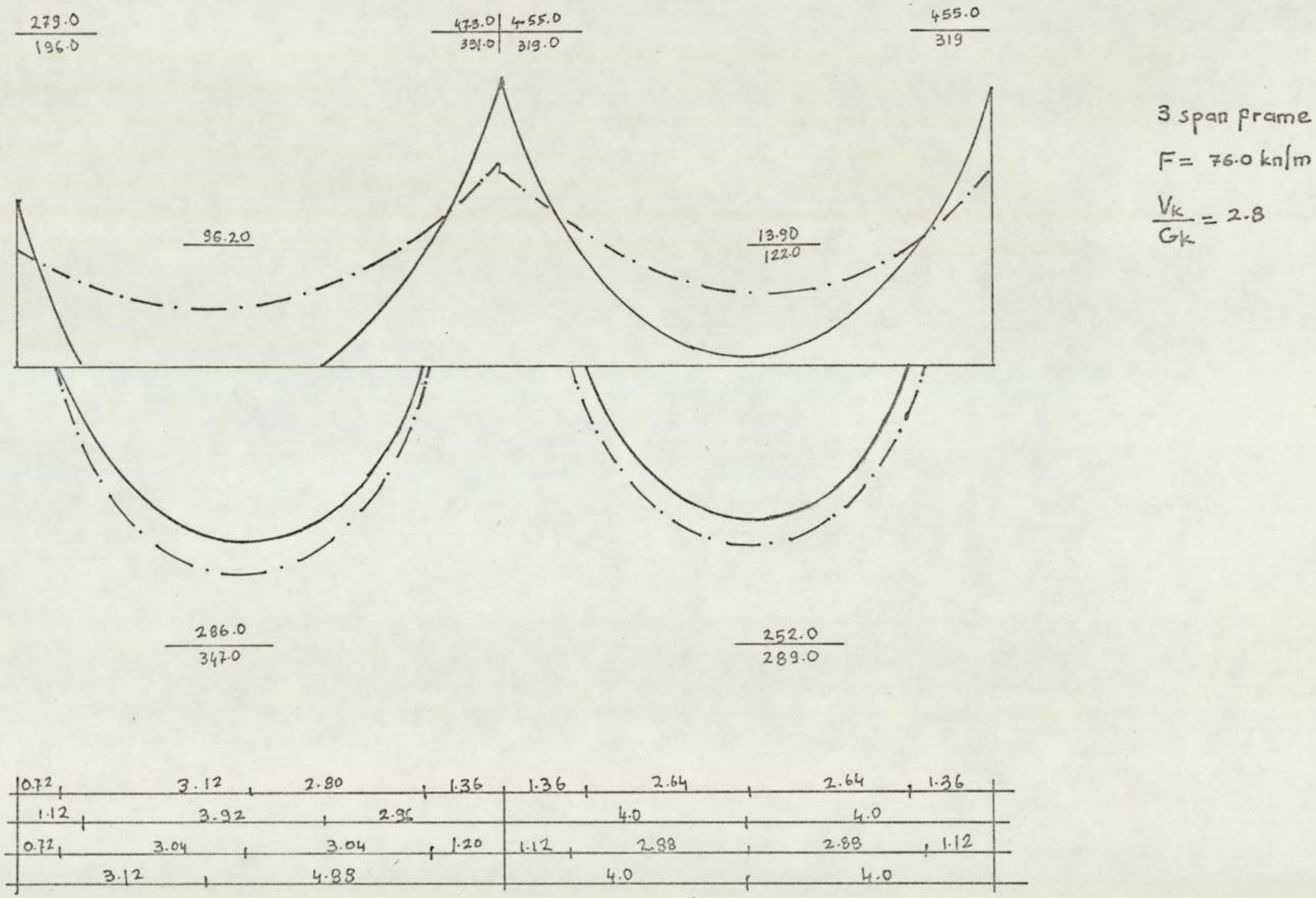



Fig. (A.47)



279.0 / 196.0
473.0 | 455.0
331.0 | 319.0
455.0 / 319
96.20
13.90 / 122.0
286.0 / 347.0
252.0 / 289.0
3 span frame
F = 76.0 kn/m
Vk/Gk = 2.8
0.72 3.12 2.80 1.36 1.36 2.64 2.64 1.36
1.12 3.92 2.96 4.0 4.0
0.72 3.04 3.04 1.20 1.12 2.88 2.88 1.12
3.12 4.88 4.0 4.0


Fig. (A.48)

4 span frame

F = 20.0 kn/m

$\frac{V_k}{G_k} = 0.4$

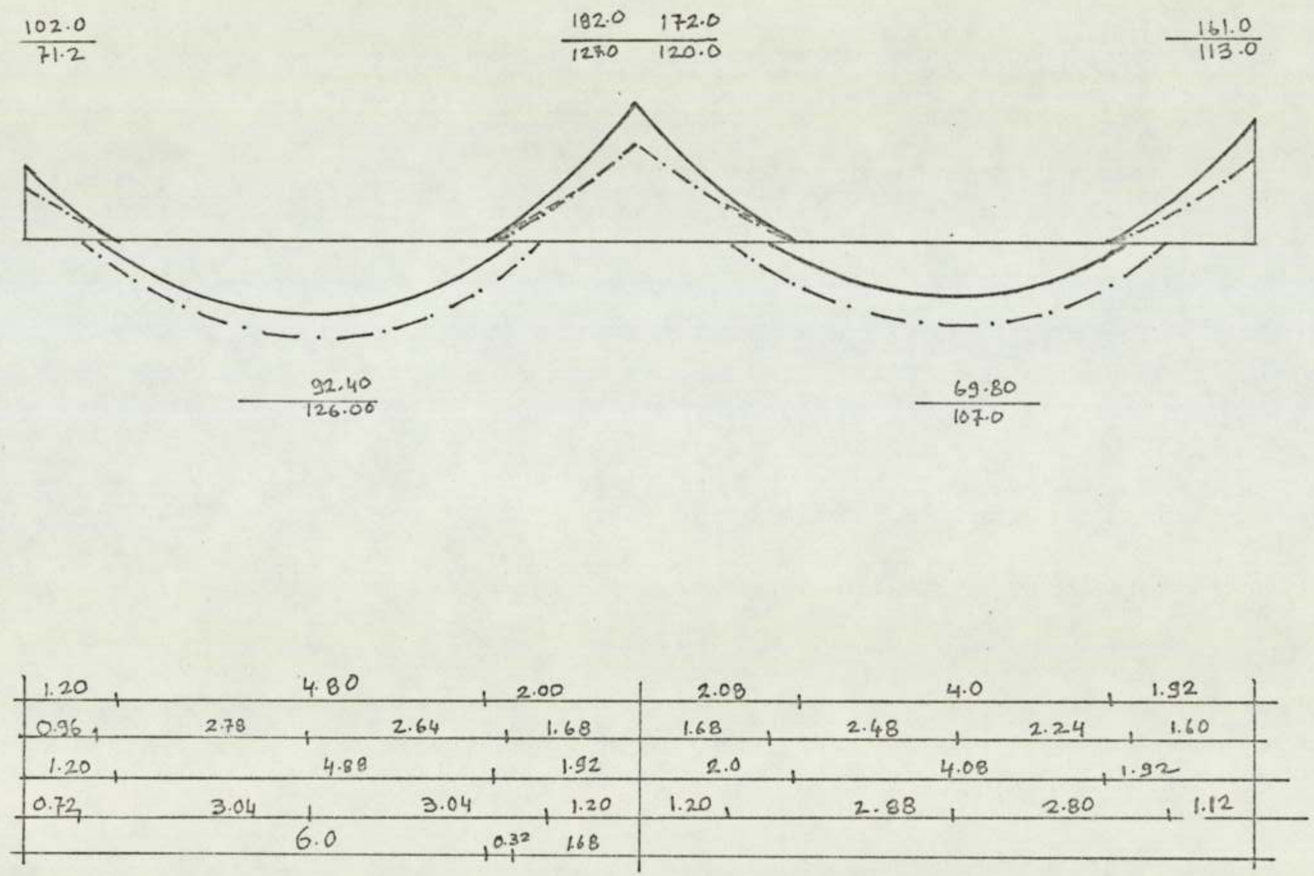



Fig. (A.49)

4 span frame
F= 30.0 kn/m
$\frac{V_k}{G_k} = 0.5$

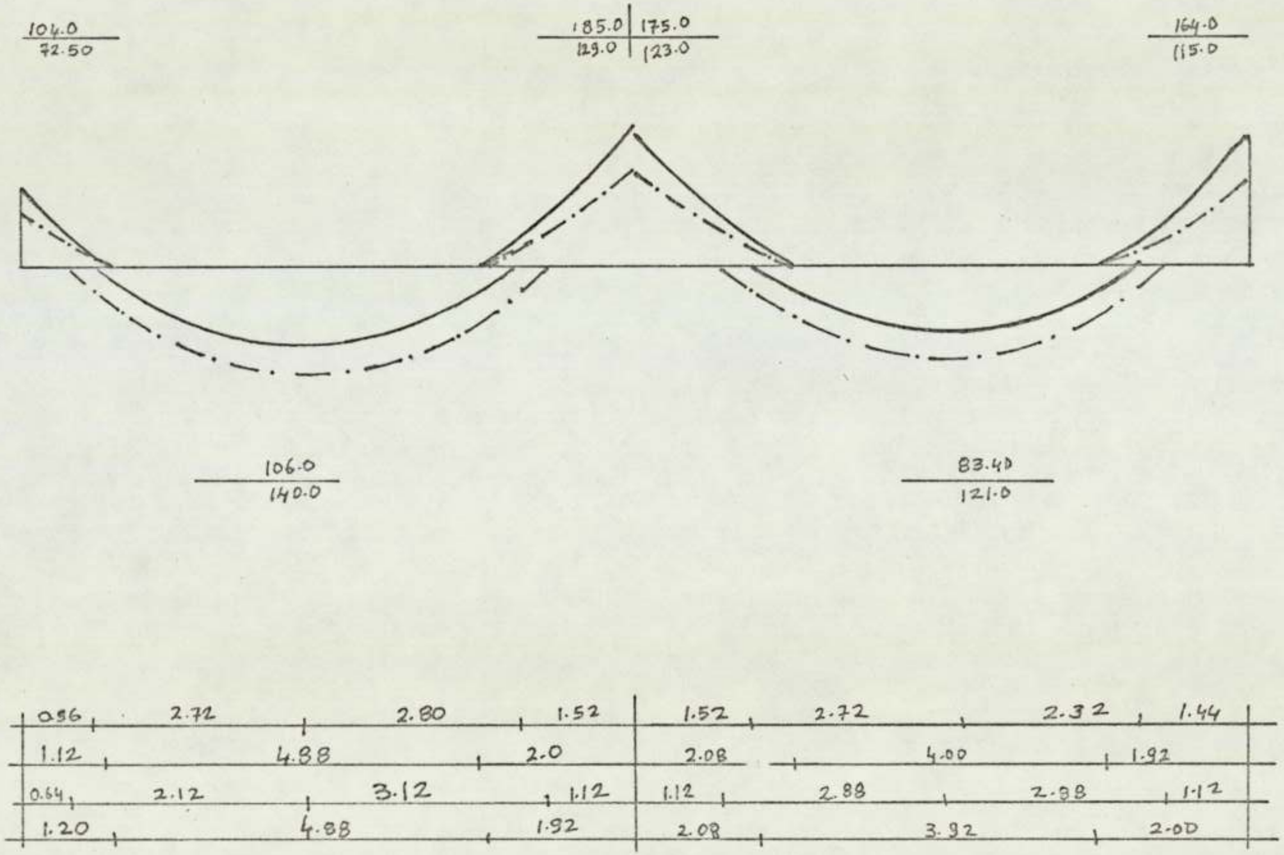



fig (A.50)

4 span frame

$F = 34.0$ kn/m

$\frac{V_k}{G_k} = 0.7$

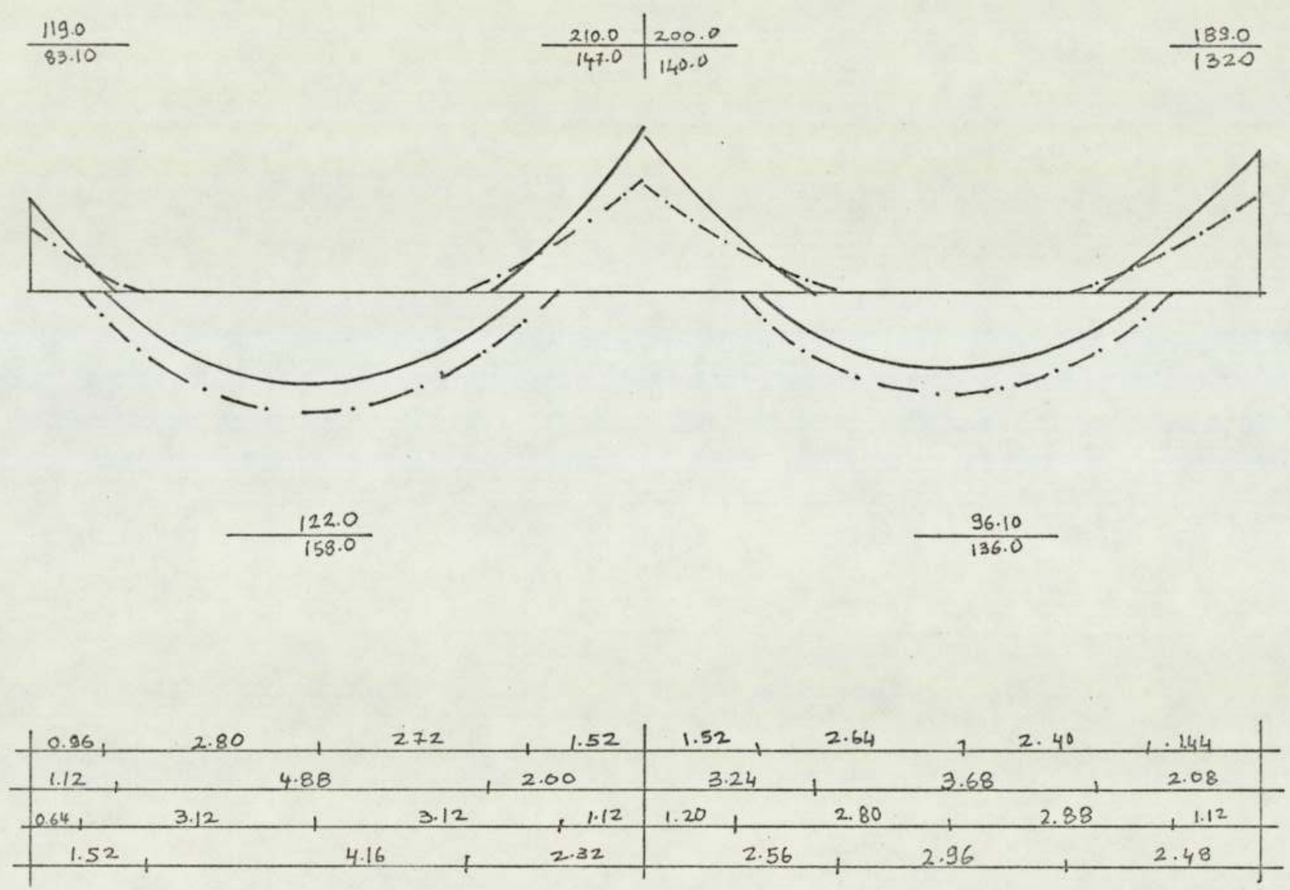



fig.(A.51)

4 span Frame

F= 36.0 kn/m

$\frac{V_k}{G_k} = 0.8$

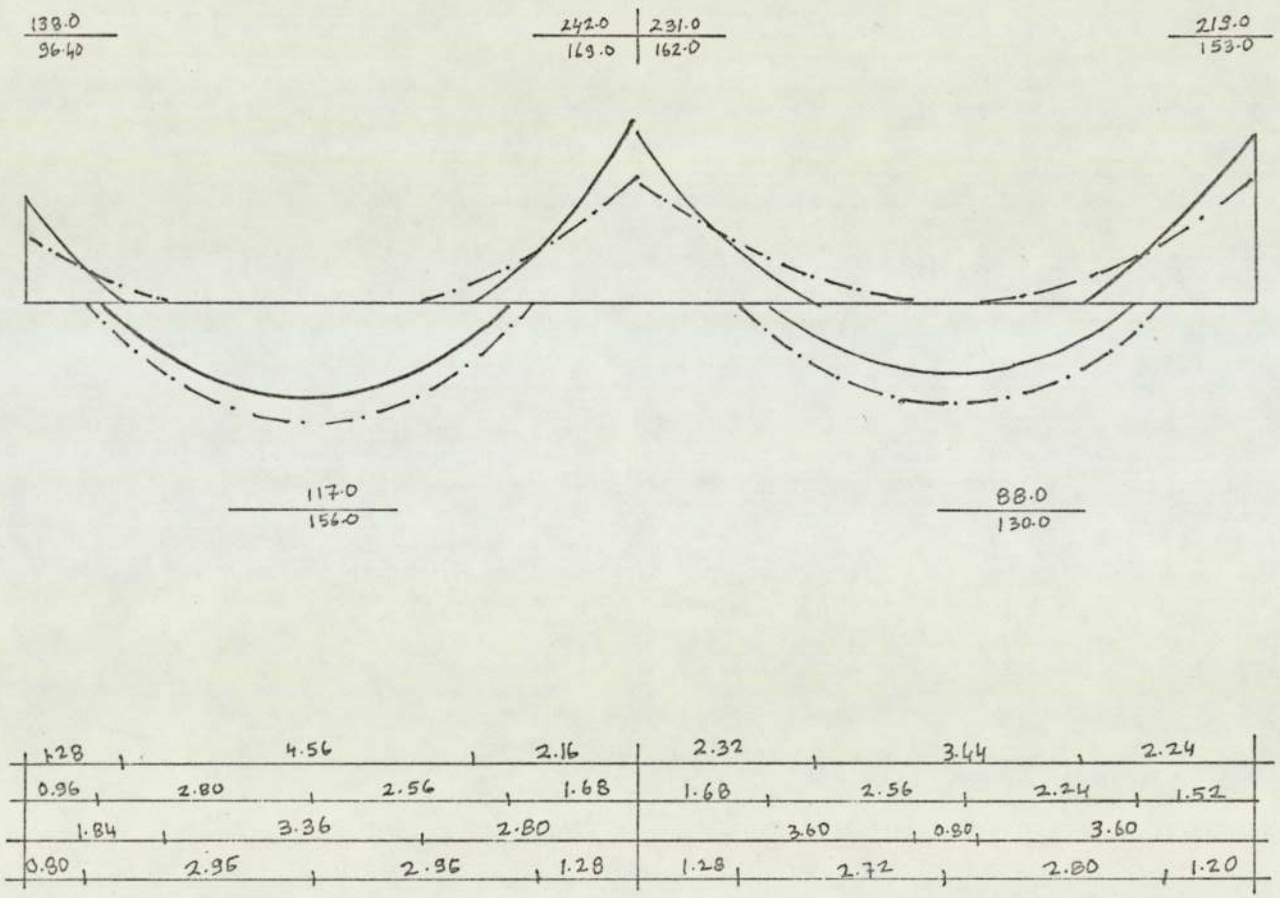



Fig.(A.52)

4 span frame
F= 40.0 kn/m
$\frac{V_k}{G_k} = 1.0$

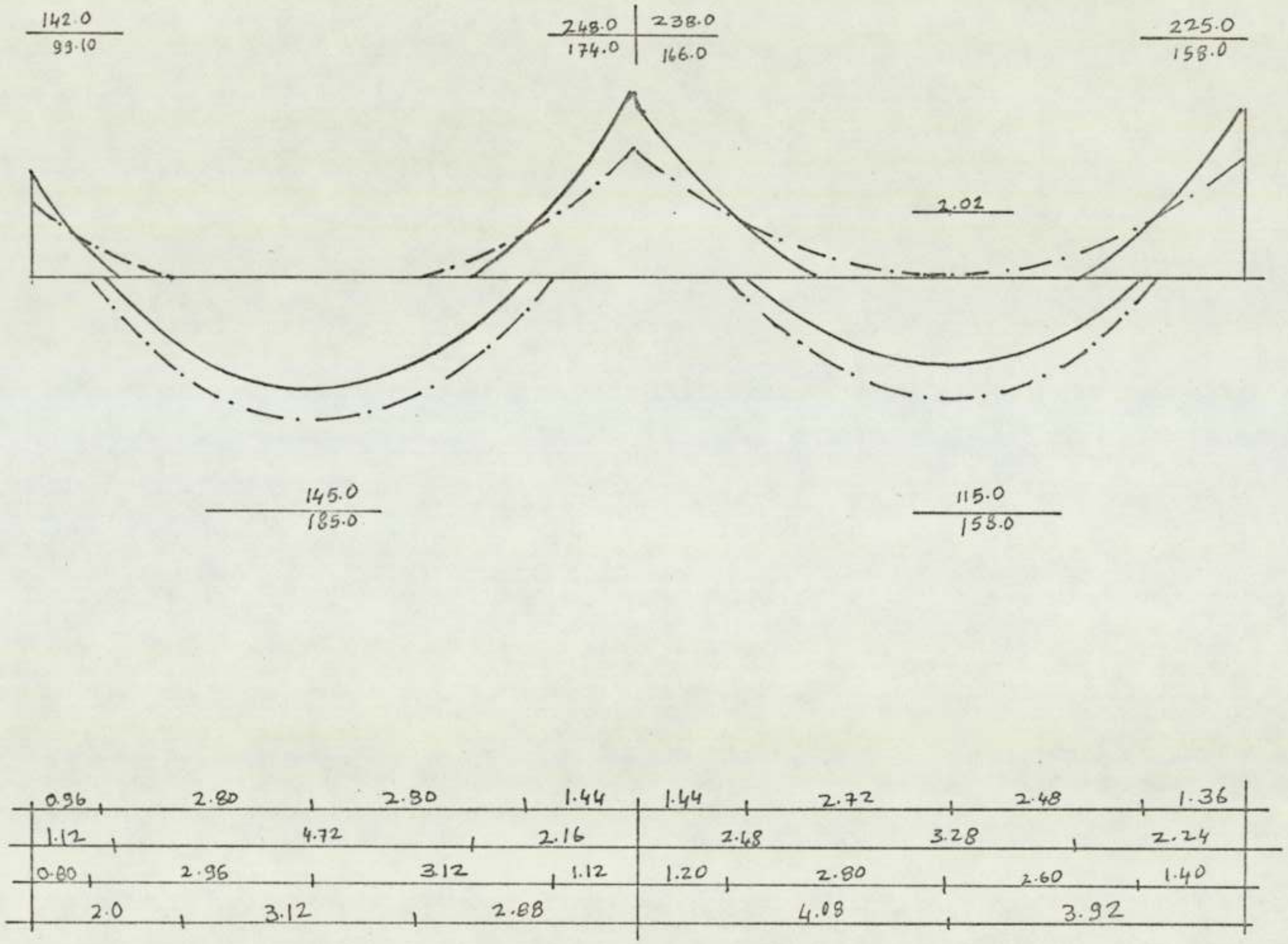



fig (A.53)

4 span frame

F= 44.0 kn/m

$\frac{V_k}{G_k} = 1.2$

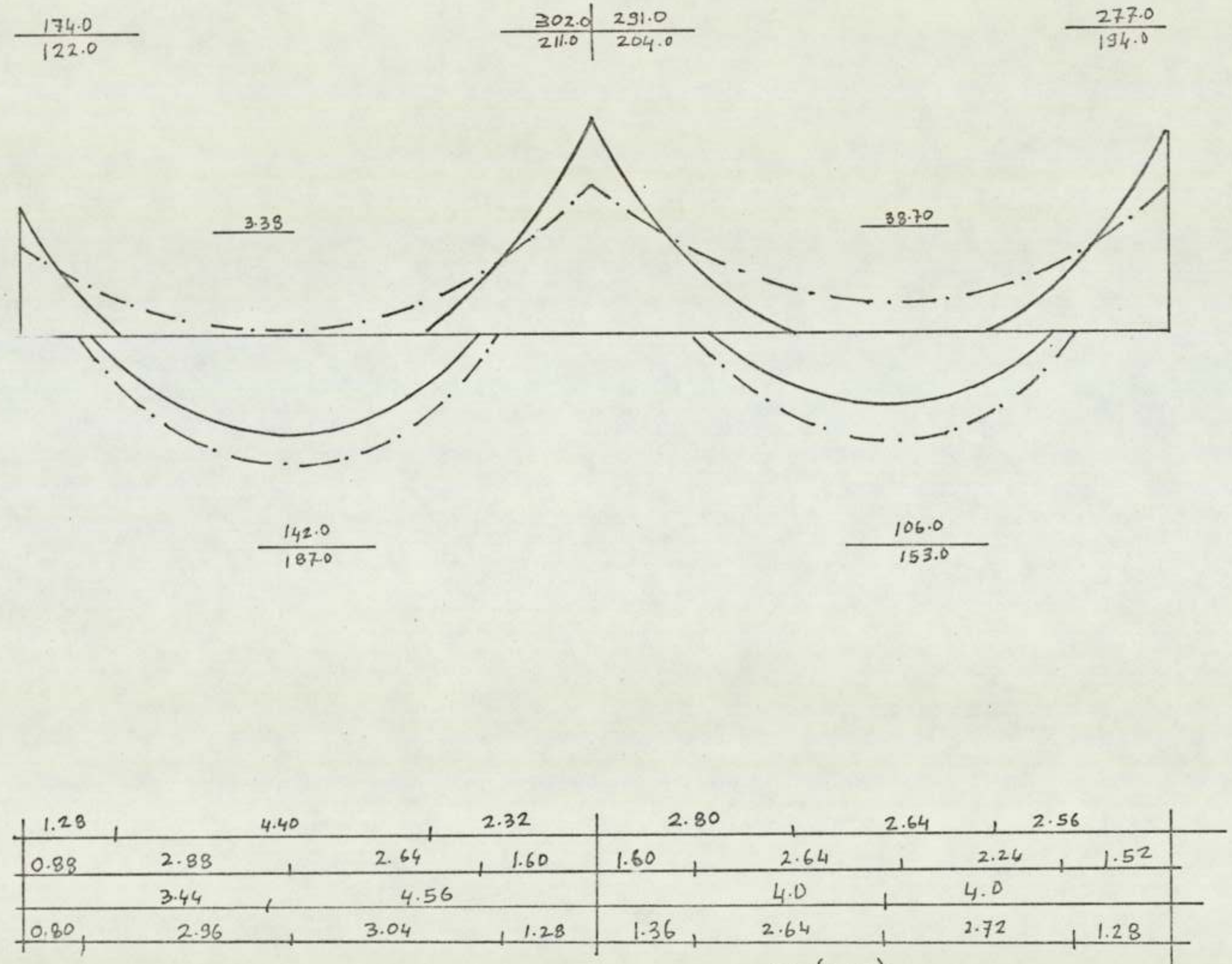



Fig.(A.54)

4span frame
F = 48.0 kn/m

$\frac{V_k}{G_k} = 1.4$

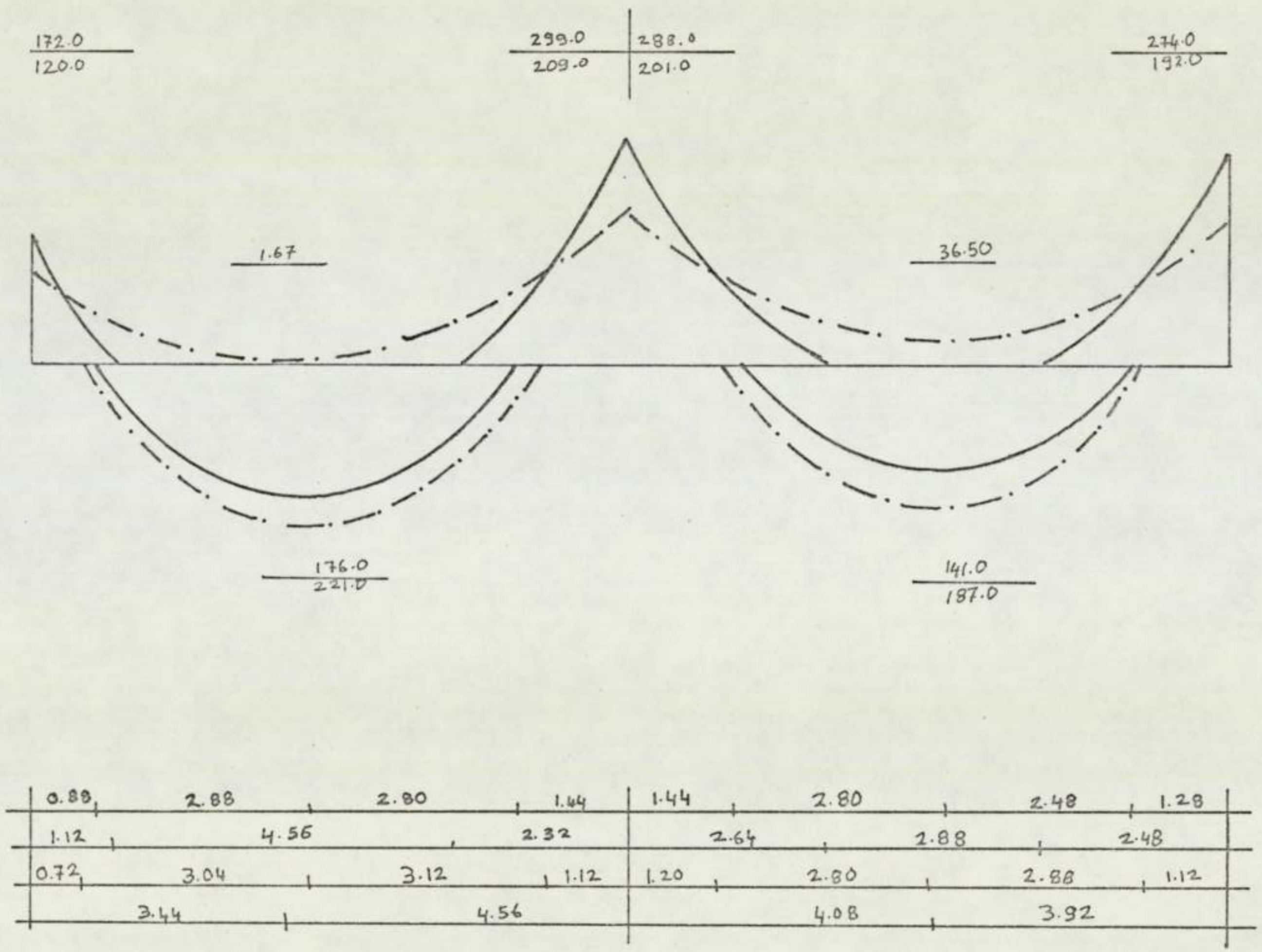
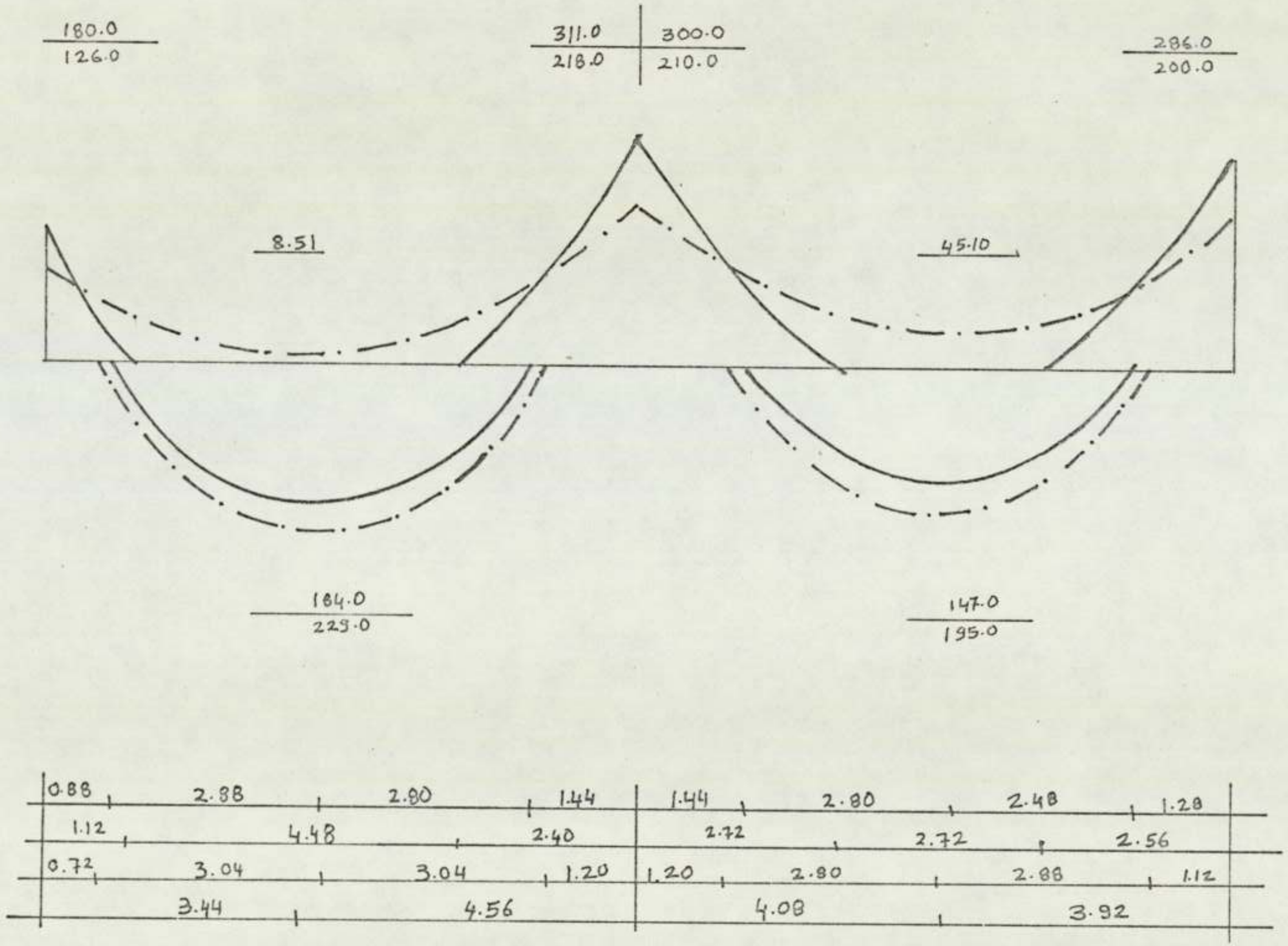



fig. (A.55)




4 span frame

$F = 50.0\ kn/m$

$\frac{V_k}{G_k} = 1.5$

Fig. (A.56)

4 span frame

F = 52.0 kn/m

$\frac{V_k}{G_k} = 1.6$

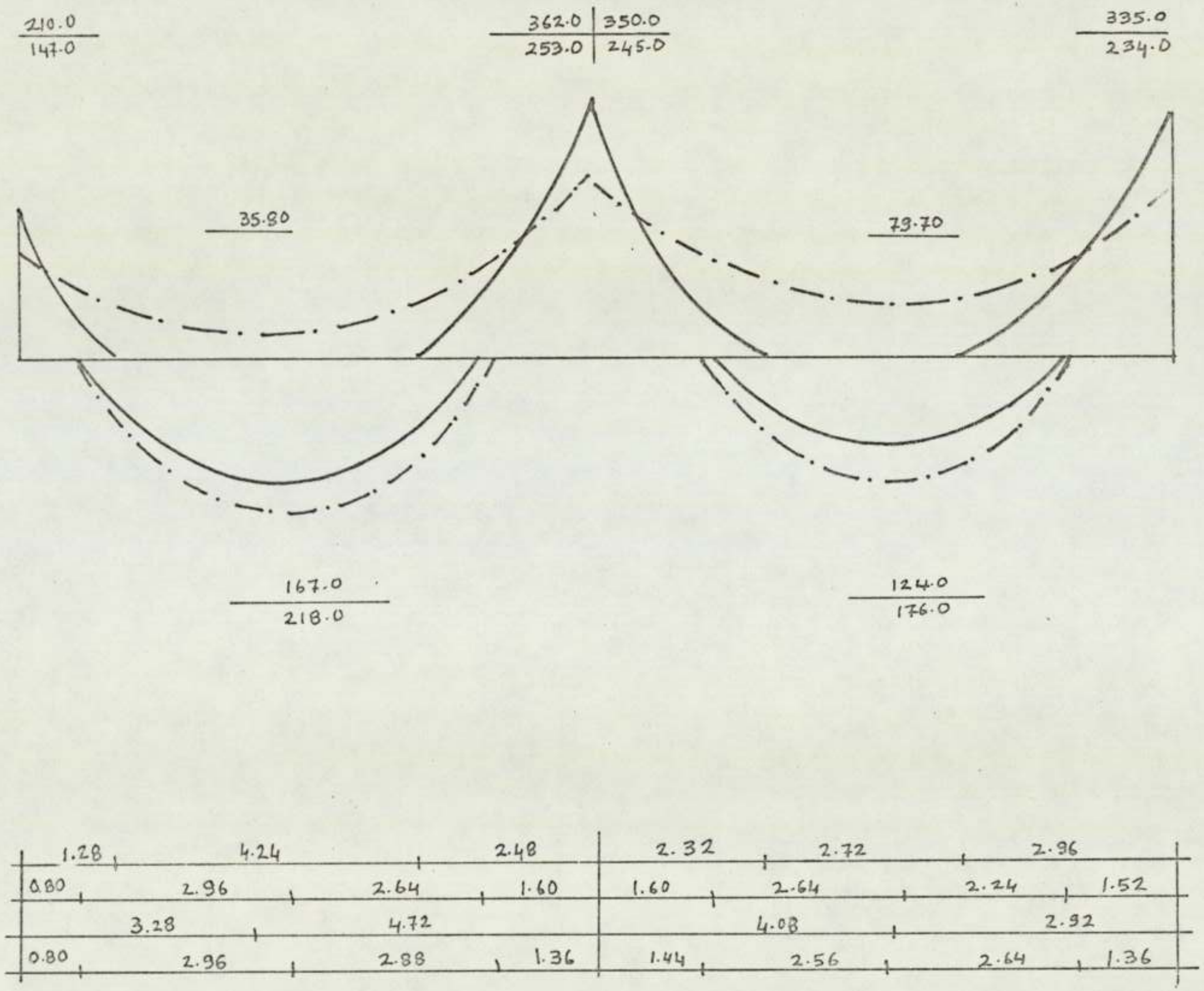
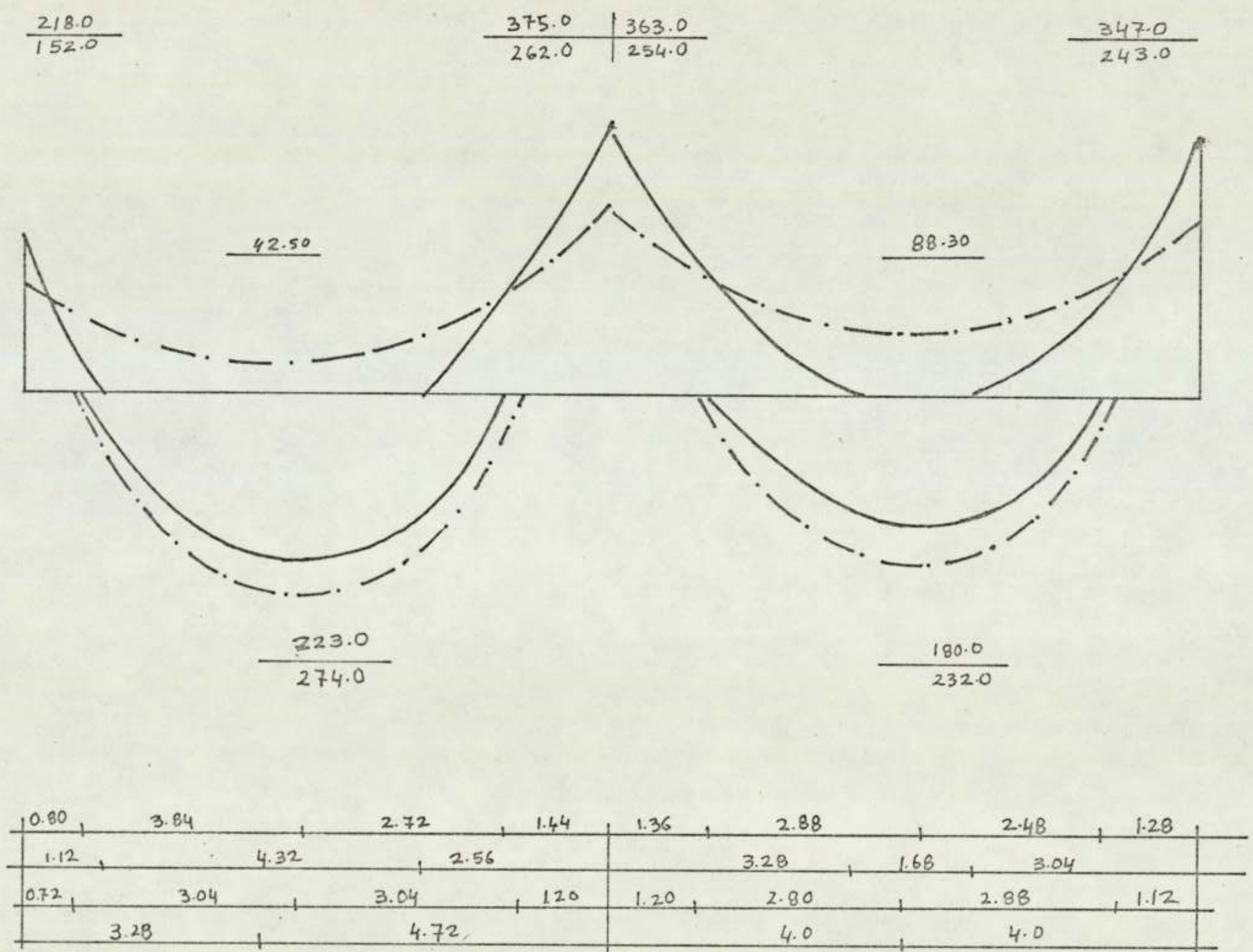



Fig. (A.57)




4 span frame

F = 60.0 kn/m

$\frac{V_k}{G_k} = 2.0$

fig.(A. 58)

4 span frame

$F = 62.0$ kn/m

$\frac{V_k}{G_k} = 2.1$

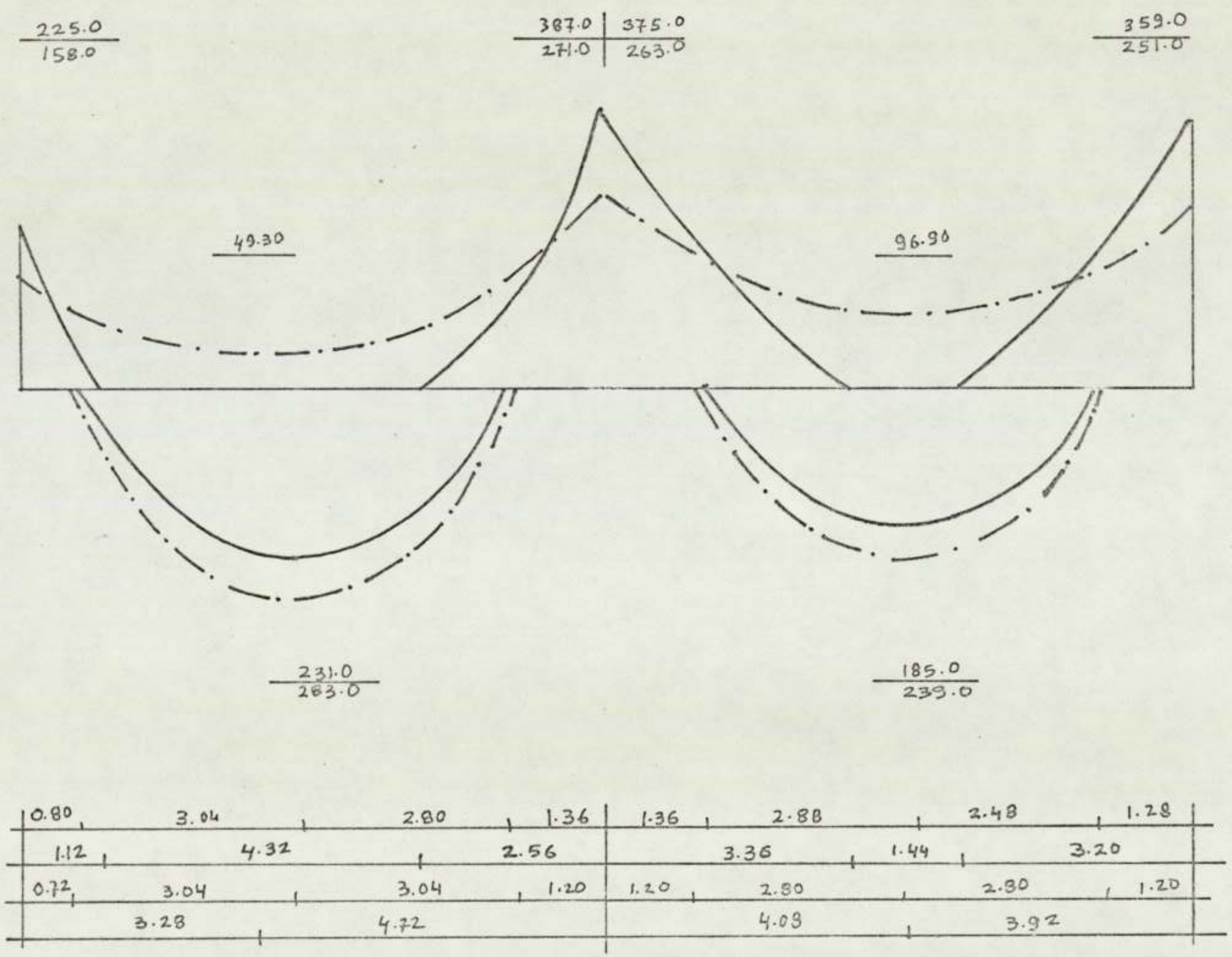



fig. (A.59)

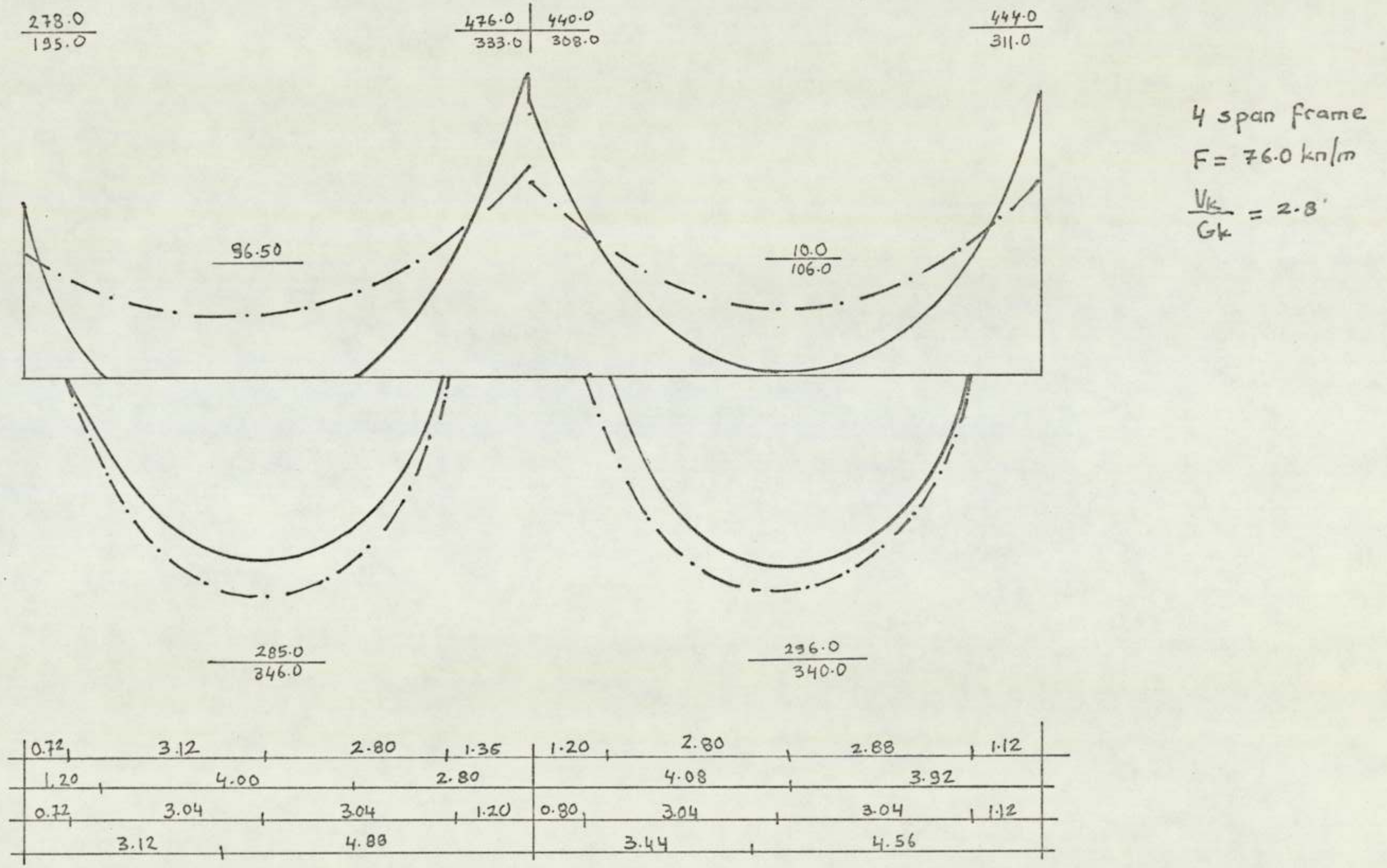

278.0
195.0
476.0 | 440.0
333.0 | 308.0
444.0
311.0
96.50
10.0
106.0
285.0
346.0
296.0
340.0
0.72 3.12 2.80 1.36 1.20 2.80 2.88 1.12
1.20 4.00 2.80 4.08 3.92
0.72 3.04 3.04 1.20 0.80 3.04 3.04 1.12
3.12 4.88 3.44 4.56
4 span frame
F = 76.0 kn/m
Vk/Gk = 2.8

Fig.(A.60)

5 span frame
F= 28.0 kn/m
$\frac{V_k}{G_k} = 0.4$

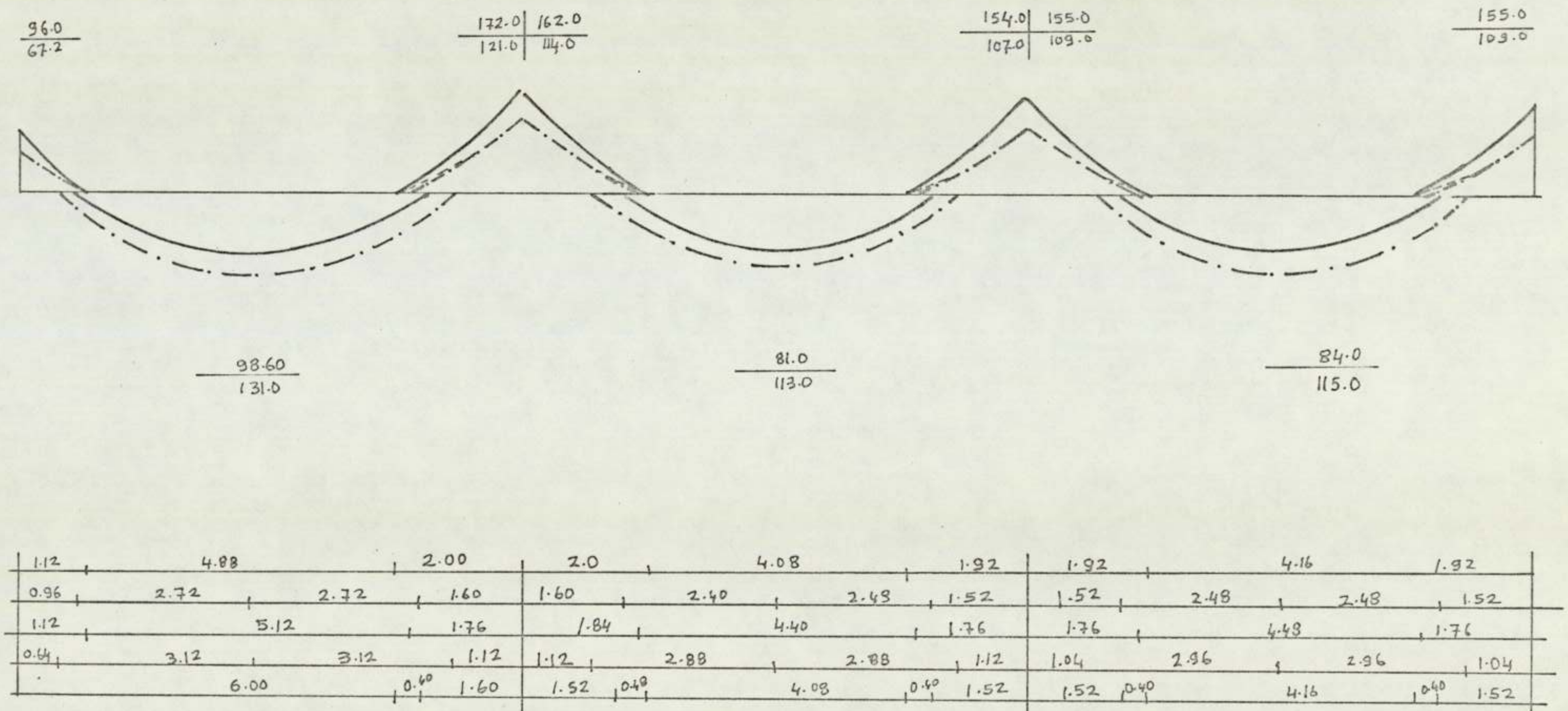
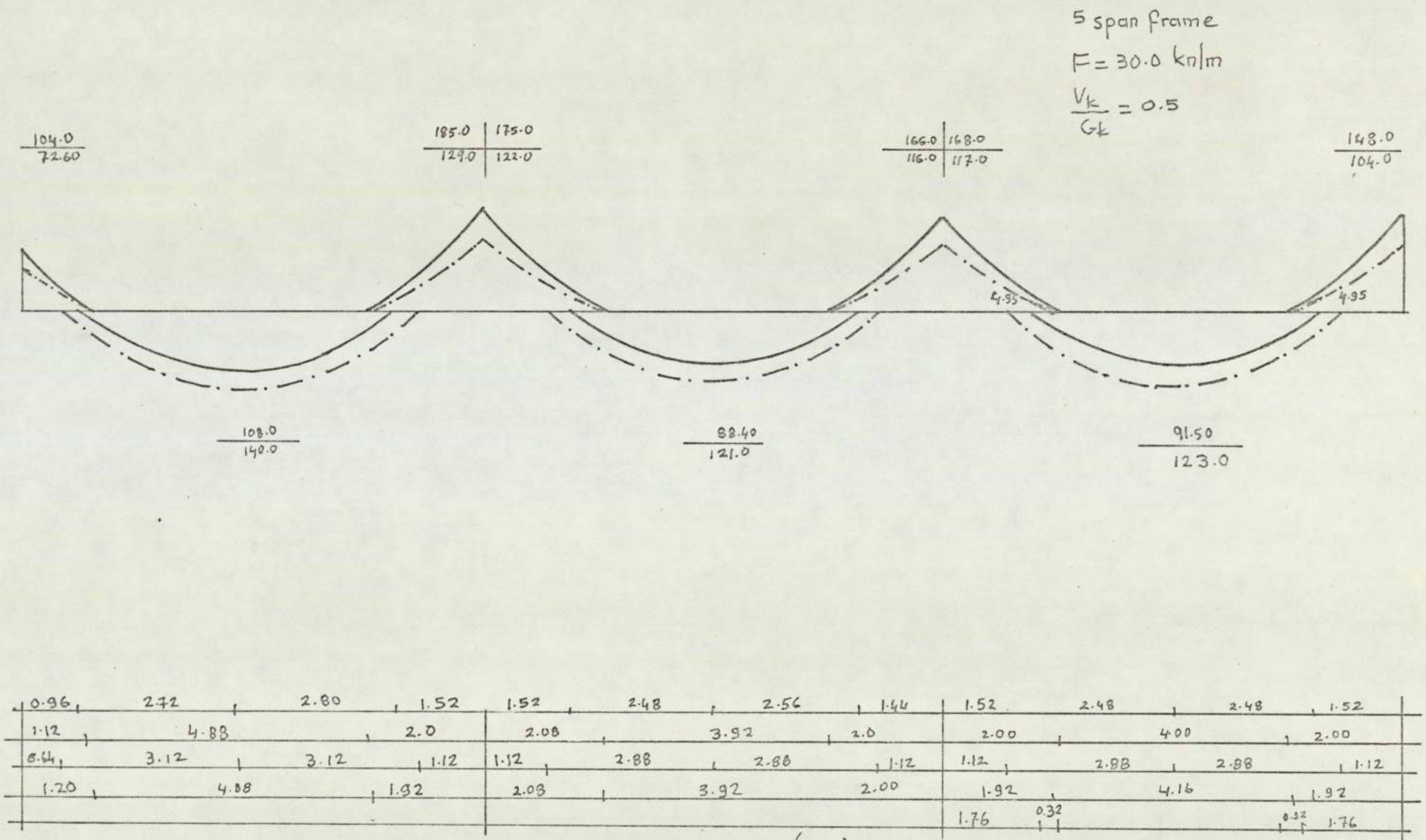



Fig.(A.61)


5 span frame
F = 30.0 kn/m
Vk/Gk = 0.5
104.0 / 72.60
185.0 | 175.0
129.0 | 122.0
166.0 | 168.0
116.0 | 117.0
148.0 / 104.0
4.95
4.95
108.0 / 140.0
88.40 / 121.0
91.50 / 123.0
0.96 2.72 2.80 1.52 1.52 2.48 2.56 1.44 1.52 2.48 2.48 1.52
1.12 4.88 2.0 2.08 3.92 2.0 2.00 4.00 2.00
0.64 3.12 3.12 1.12 1.12 2.88 2.88 1.12 1.12 2.88 2.88 1.12
1.20 4.88 1.92 2.08 3.92 2.00 1.92 4.16 1.92
1.76 0.32 0.32 1.76

Fig(A·62)

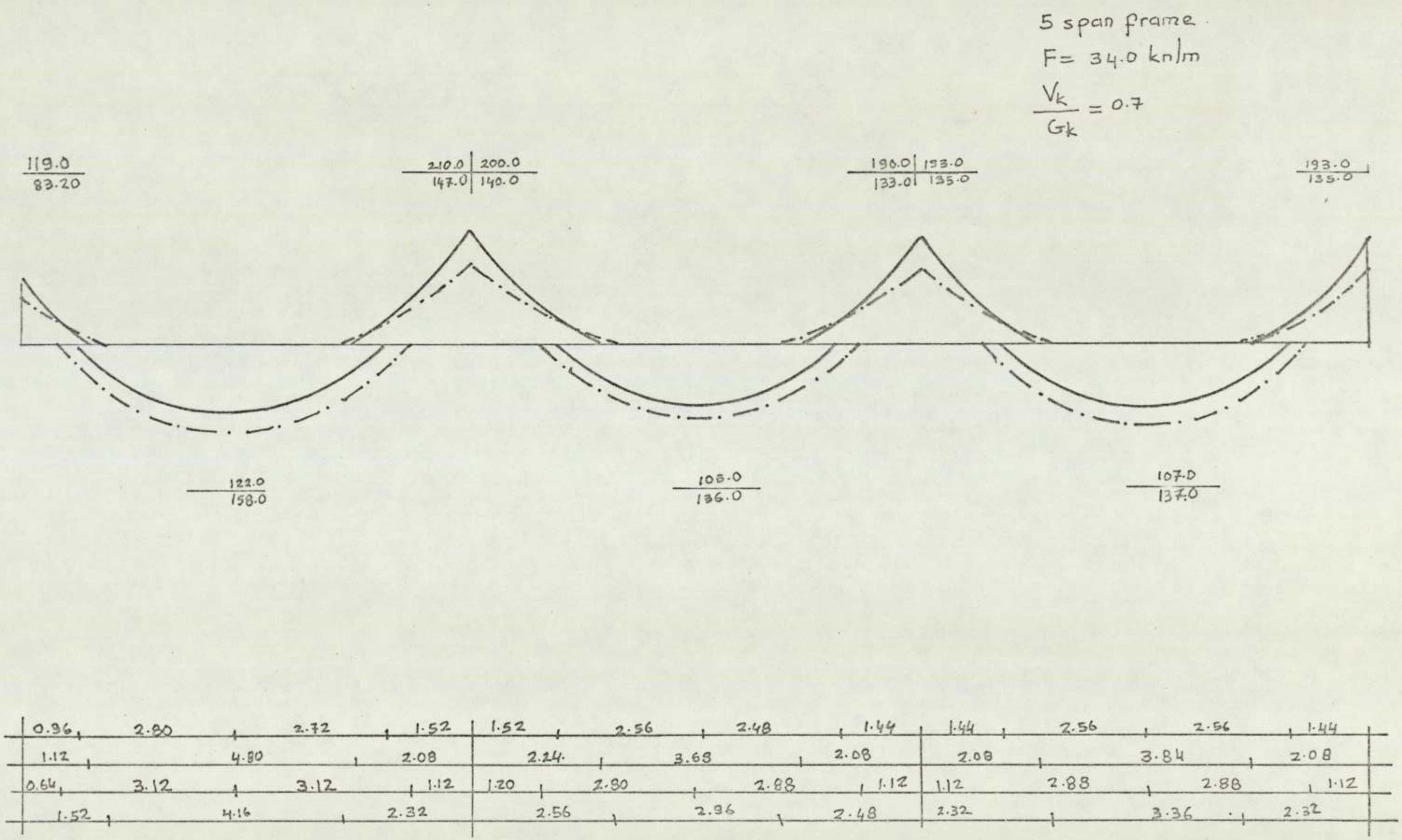

5 span frame
F= 34.0 kn/m
Vk/Gk = 0.7
119.0 / 83.20
210.0 | 200.0
147.0 | 140.0
190.0 | 153.0
133.0 | 135.0
193.0 / 135.0
122.0 / 158.0
108.0 / 136.0
107.0 / 137.0
0.96 2.80 2.72 1.52 1.52 2.56 2.48 1.44 1.44 2.56 2.56 1.44
1.12 4.80 2.08 2.24 3.68 2.08 2.08 3.84 2.08
0.64 3.12 3.12 1.12 1.20 2.80 2.88 1.12 1.12 2.88 2.88 1.12
1.52 4.16 2.32 2.56 2.96 2.48 2.32 3.36 2.32

Fig. (A.63)

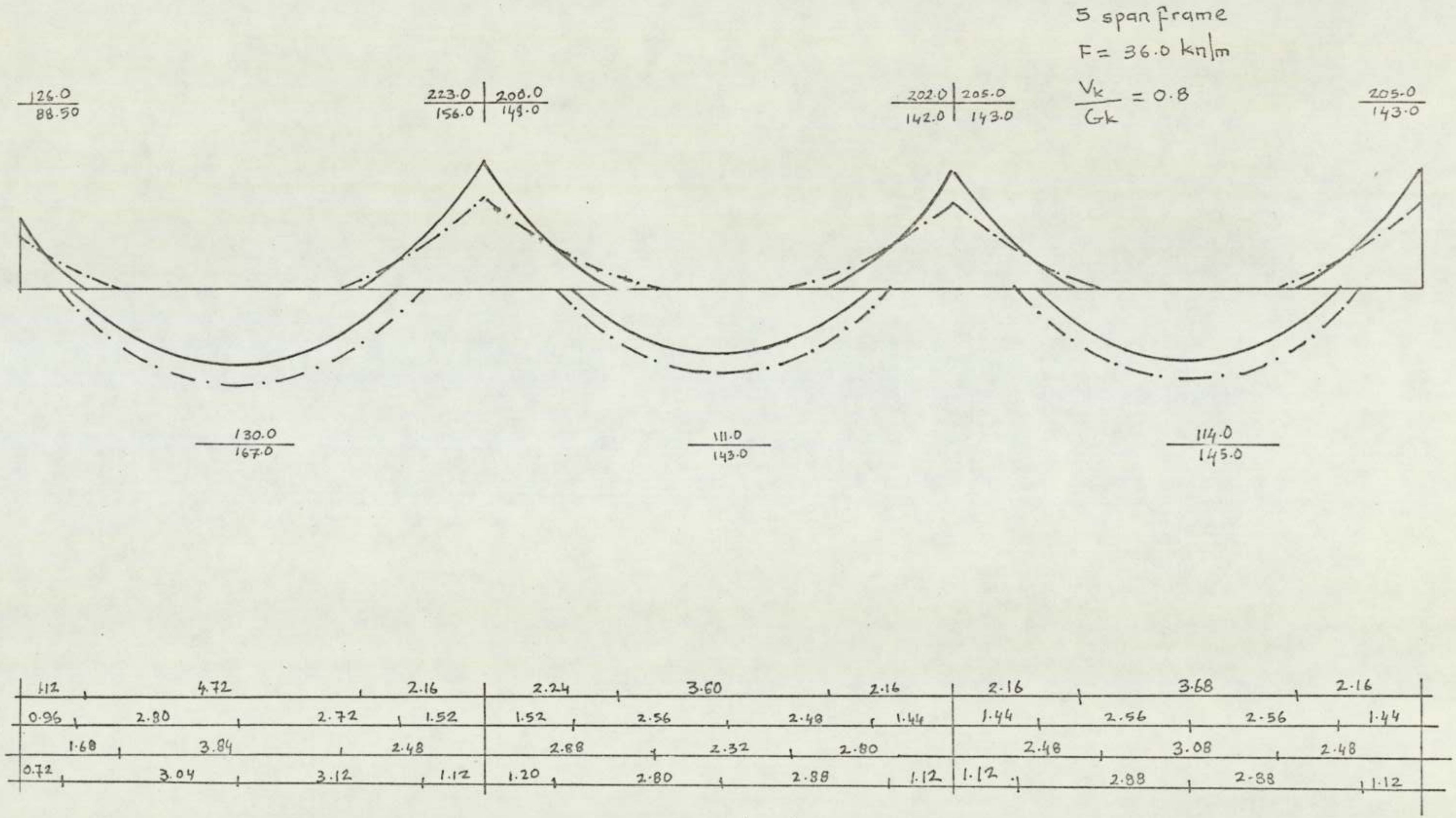
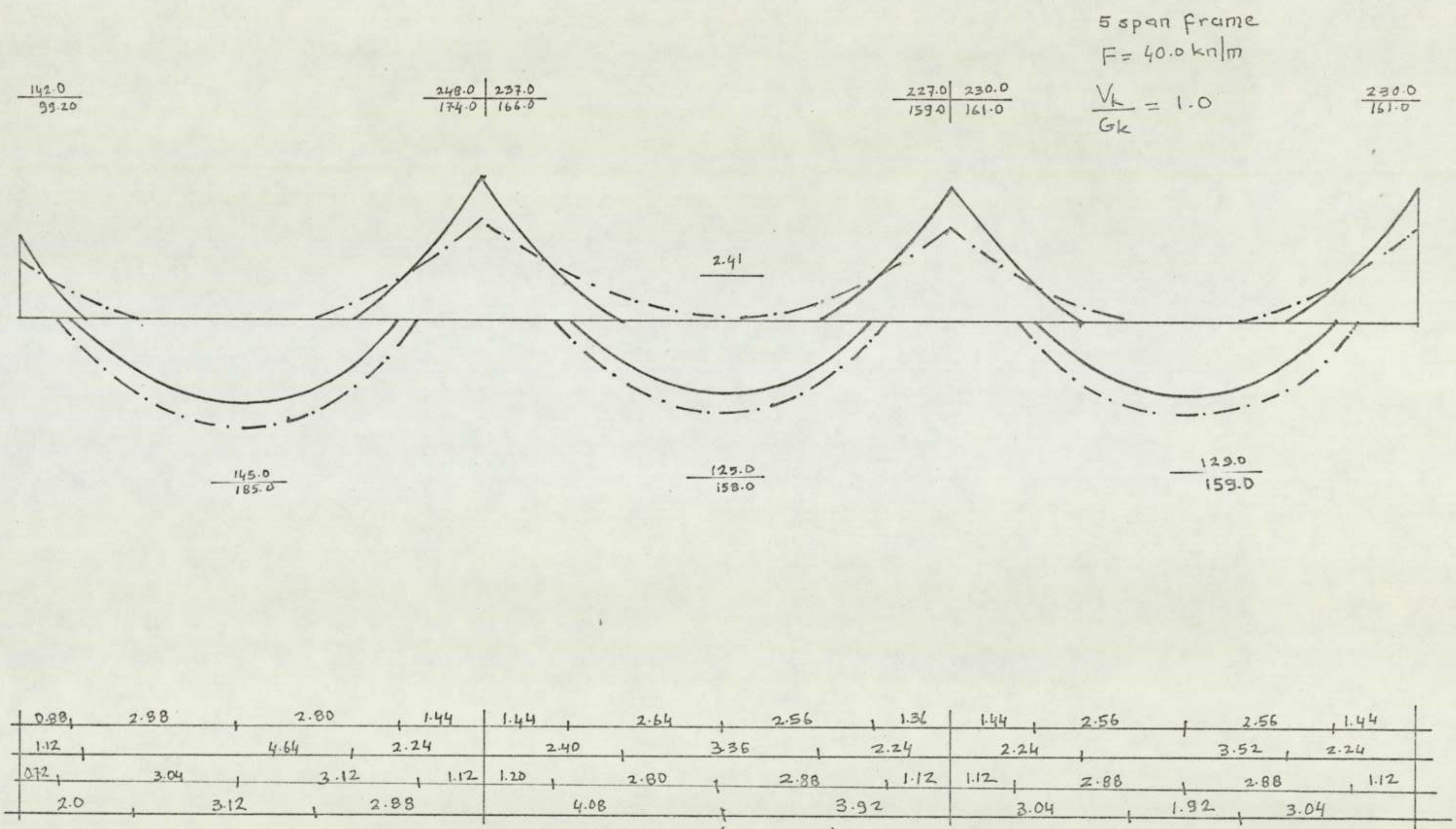

5 span frame
F = 36.0 kn/m
Vk / Gk = 0.8
126.0 / 88.50
223.0 | 200.0
156.0 | 149.0
202.0 | 205.0
142.0 | 143.0
205.0 / 143.0
130.0 / 167.0
111.0 / 143.0
114.0 / 145.0
1.12 4.72 2.16 2.24 3.60 2.16 2.16 3.68 2.16
0.96 2.80 2.72 1.52 1.52 2.56 2.48 1.44 1.44 2.56 2.56 1.44
1.68 3.84 2.48 2.88 2.32 2.80 2.48 3.08 2.48
0.72 3.04 3.12 1.12 1.20 2.80 2.88 1.12 1.12 2.88 2.88 1.12

Fig.(A-64)


5 span frame
F= 40.0 kn/m
Vk/Gk = 1.0
142.0 / 99.20
248.0 | 237.0 / 174.0 | 166.0
227.0 | 230.0 / 1590 | 161.0
230.0 / 161.0
2.41
145.0 / 185.0
125.0 / 158.0
129.0 / 159.0
0.88 2.88 2.80 1.44 1.44 2.64 2.56 1.36 1.44 2.56 2.56 1.44
1.12 4.64 2.24 2.40 3.36 2.24 2.24 3.52 2.24
0.72 3.04 3.12 1.12 1.20 2.80 2.88 1.12 1.12 2.88 2.88 1.12
2.0 3.12 2.88 4.08 3.92 3.04 1.92 3.04

(Fig. A.65)

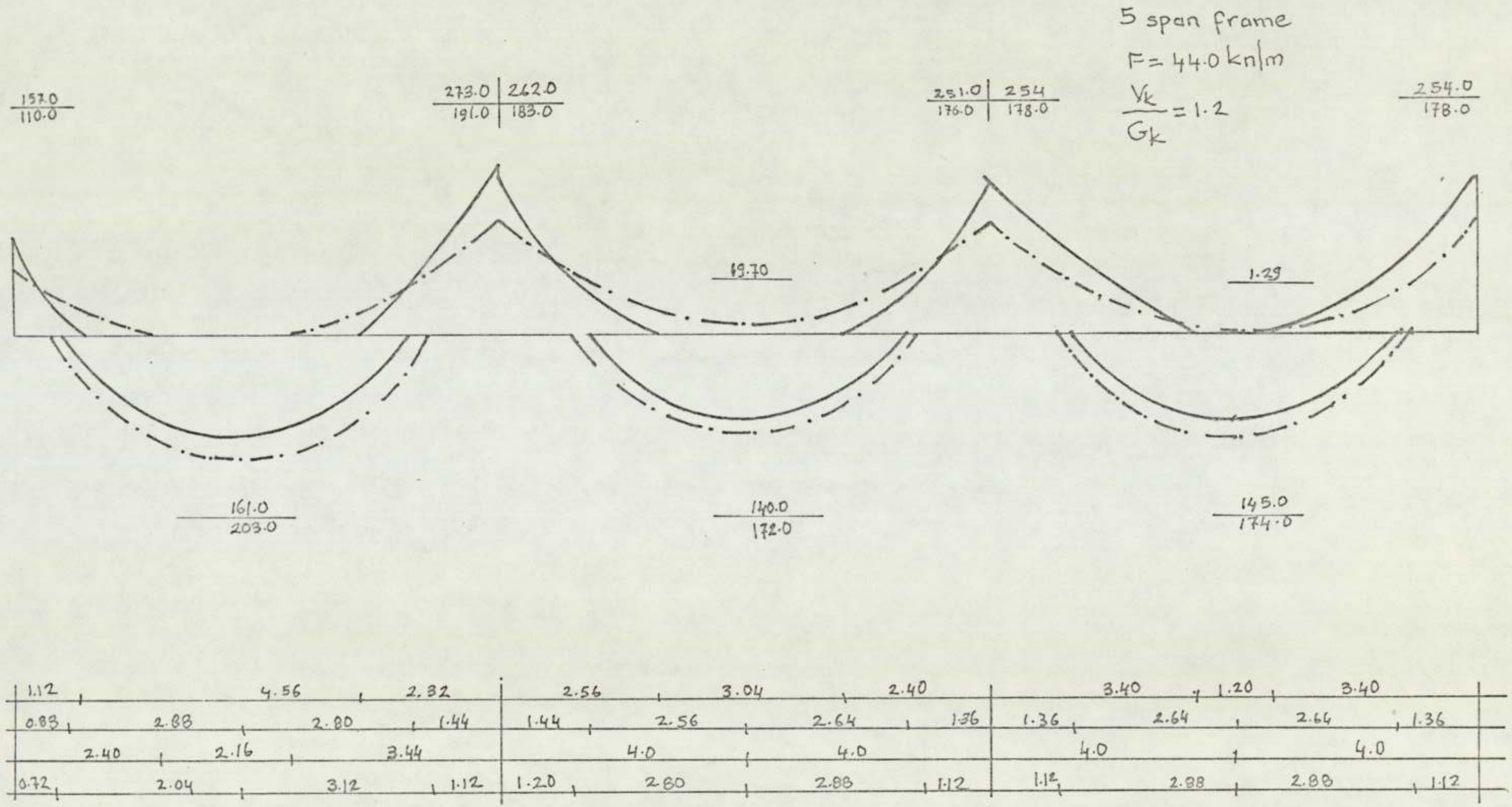

5 span frame
F= 44.0 kn/m
Vk / Gk = 1.2
157.0 / 110.0
273.0 / 191.0
262.0 / 183.0
251.0 / 176.0
254 / 178.0
254.0 / 178.0
69.70
1.29
161.0 / 203.0
140.0 / 172.0
145.0 / 174.0
1.12 4.56 2.32 2.56 3.04 2.40 3.40 1.20 3.40
0.83 2.88 2.80 1.44 1.44 2.56 2.64 1.36 1.36 2.64 2.64 1.36
2.40 2.16 3.44 4.0 4.0 4.0 4.0
0.72 2.04 3.12 1.12 1.20 2.80 2.88 1.12 1.12 2.88 2.88 1.12

Fig (A.66)

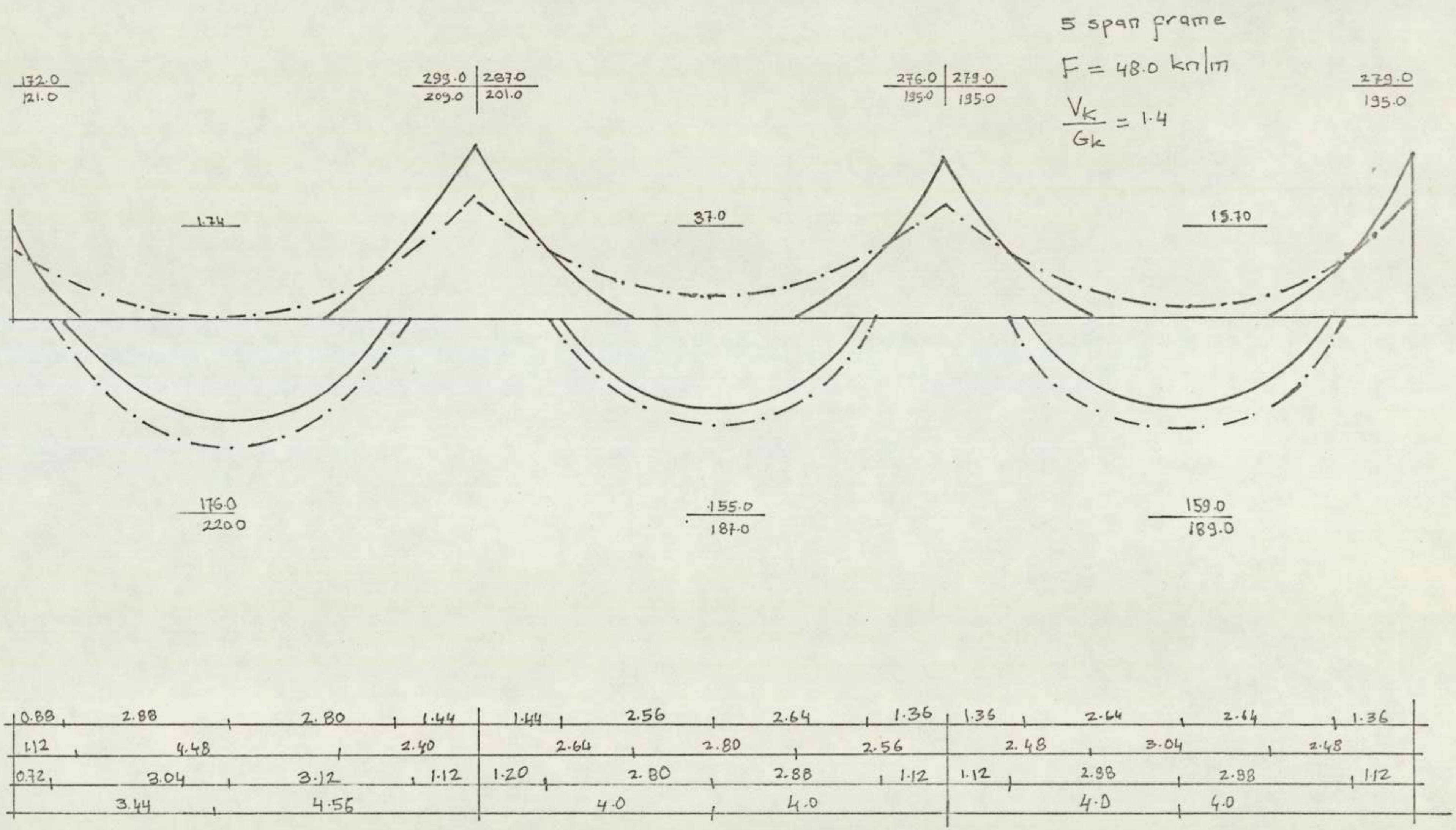

5 span frame
F = 48.0 kn/m
Vk/Gk = 1.4
172.0
121.0
299.0 | 287.0
209.0 | 201.0
276.0 | 279.0
195.0 | 195.0
279.0
195.0
1.74
37.0
15.70
176.0
220.0
155.0
187.0
159.0
189.0
0.88 2.88 2.80 1.44 1.44 2.56 2.64 1.36 1.36 2.64 2.64 1.36
1.12 4.48 2.40 2.64 2.80 2.56 2.48 3.04 2.48
0.72 3.04 3.12 1.12 1.20 2.80 2.88 1.12 1.12 2.88 2.88 1.12
3.44 4.56 4.0 4.0 4.0 4.0

Fig (A.67)

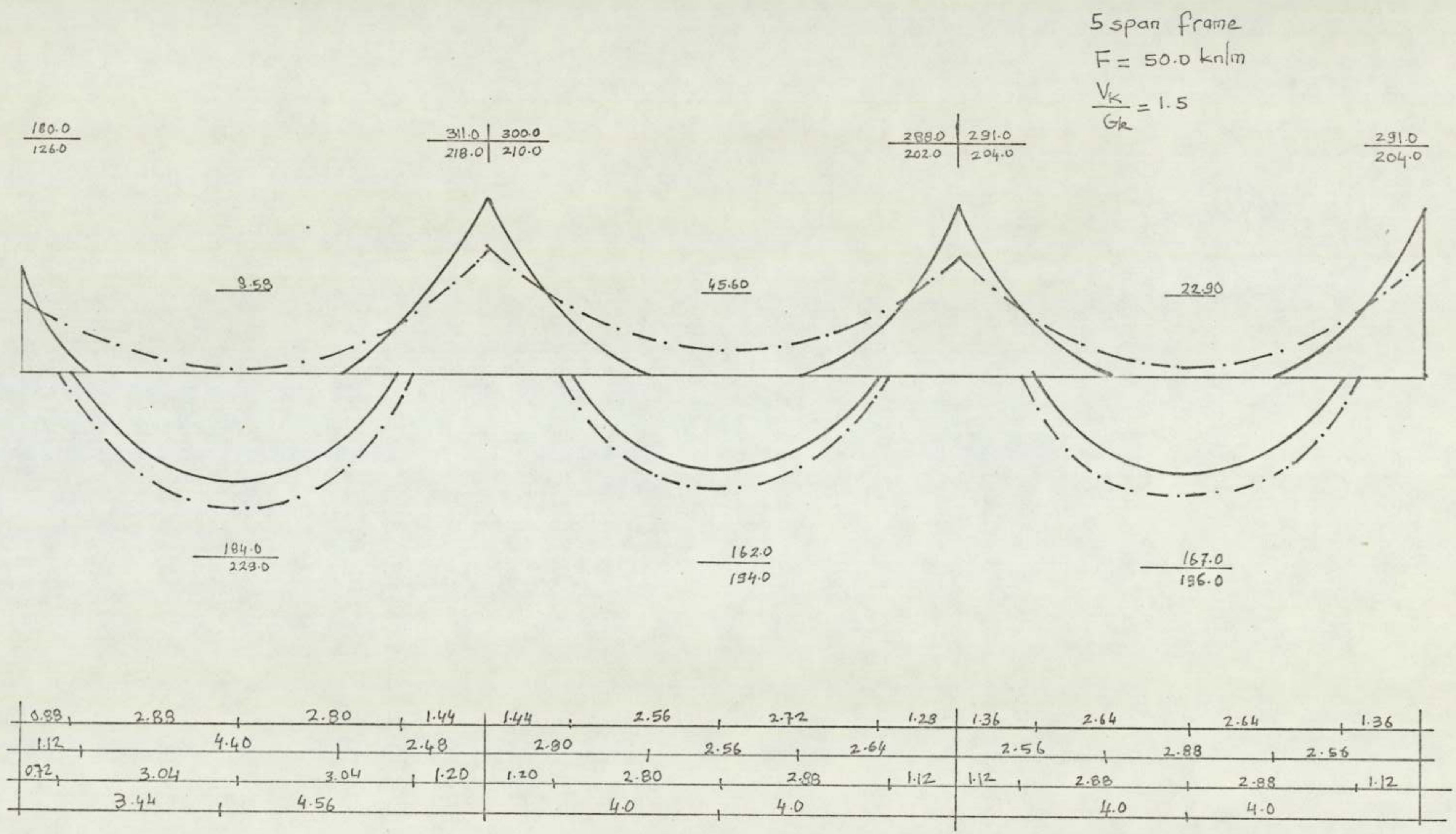
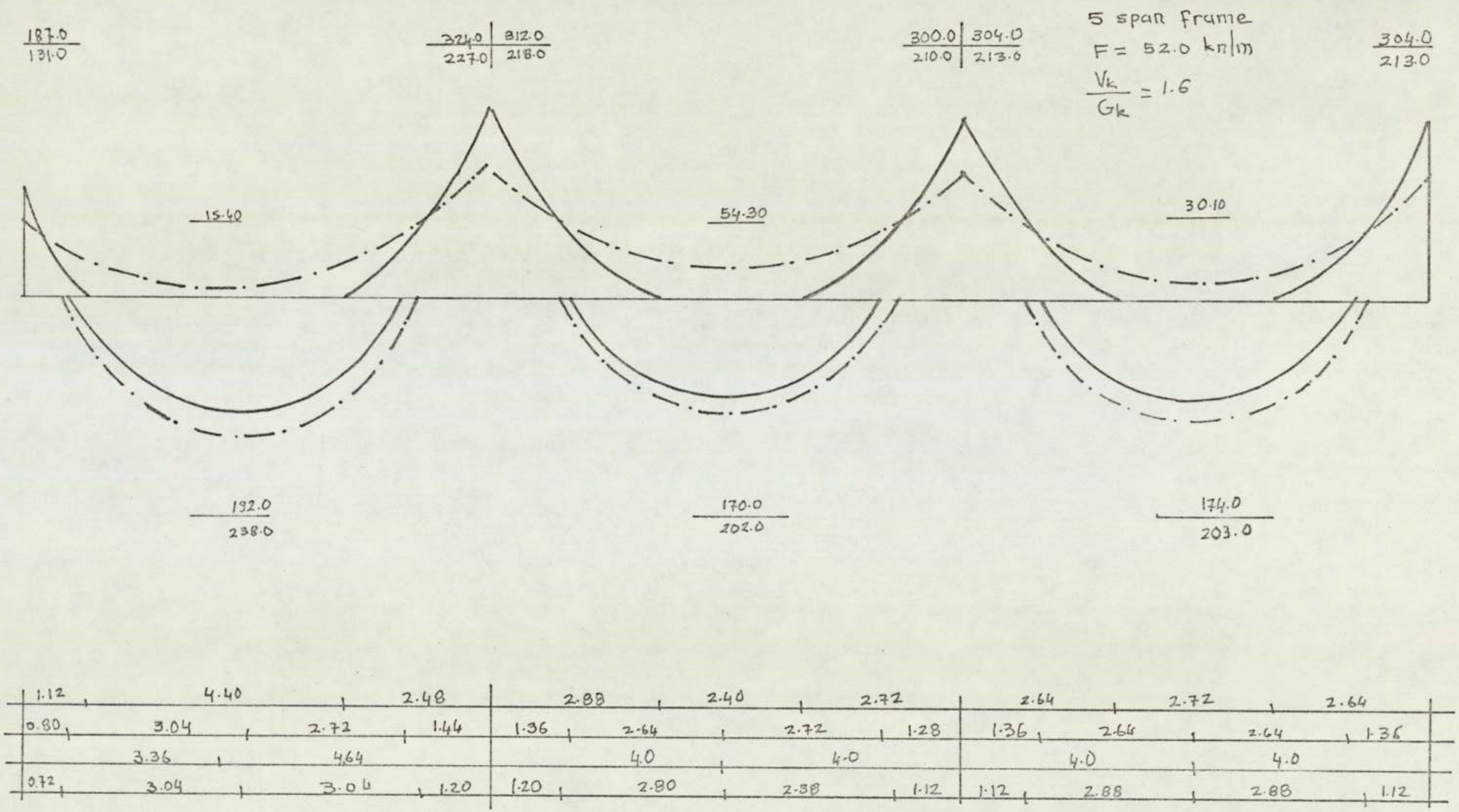

5 span frame
F = 50.0 kn/m
Vk/Gk = 1.5
180.0 / 126.0
311.0 | 300.0
218.0 | 210.0
288.0 | 291.0
202.0 | 204.0
291.0 / 204.0
8.58
45.60
22.90
184.0 / 229.0
162.0 / 194.0
167.0 / 196.0
0.88 2.88 2.80 1.44 1.44 2.56 2.72 1.28 1.36 2.64 2.64 1.36
1.12 4.40 2.48 2.80 2.56 2.64 2.56 2.88 2.56
0.72 3.04 3.04 1.20 1.20 2.80 2.88 1.12 1.12 2.88 2.88 1.12
3.44 4.56 4.0 4.0 4.0 4.0

Fig (A.68)


5 span frame
F = 52.0 kn/m
Vk/Gk = 1.6
187.0
131.0
324.0 | 312.0
227.0 | 218.0
300.0 | 304.0
210.0 | 213.0
304.0
213.0
15.40
54.30
30.10
132.0
238.0
170.0
202.0
174.0
203.0
1.12 4.40 2.48 2.88 2.40 2.72 2.64 2.72 2.64
0.80 3.04 2.72 1.44 1.36 2.64 2.72 1.28 1.36 2.64 2.64 1.36
3.36 4.64 4.0 4.0 4.0 4.0
0.72 3.04 3.04 1.20 1.20 2.80 2.38 1.12 1.12 2.88 2.88 1.12

Fig(A.69)

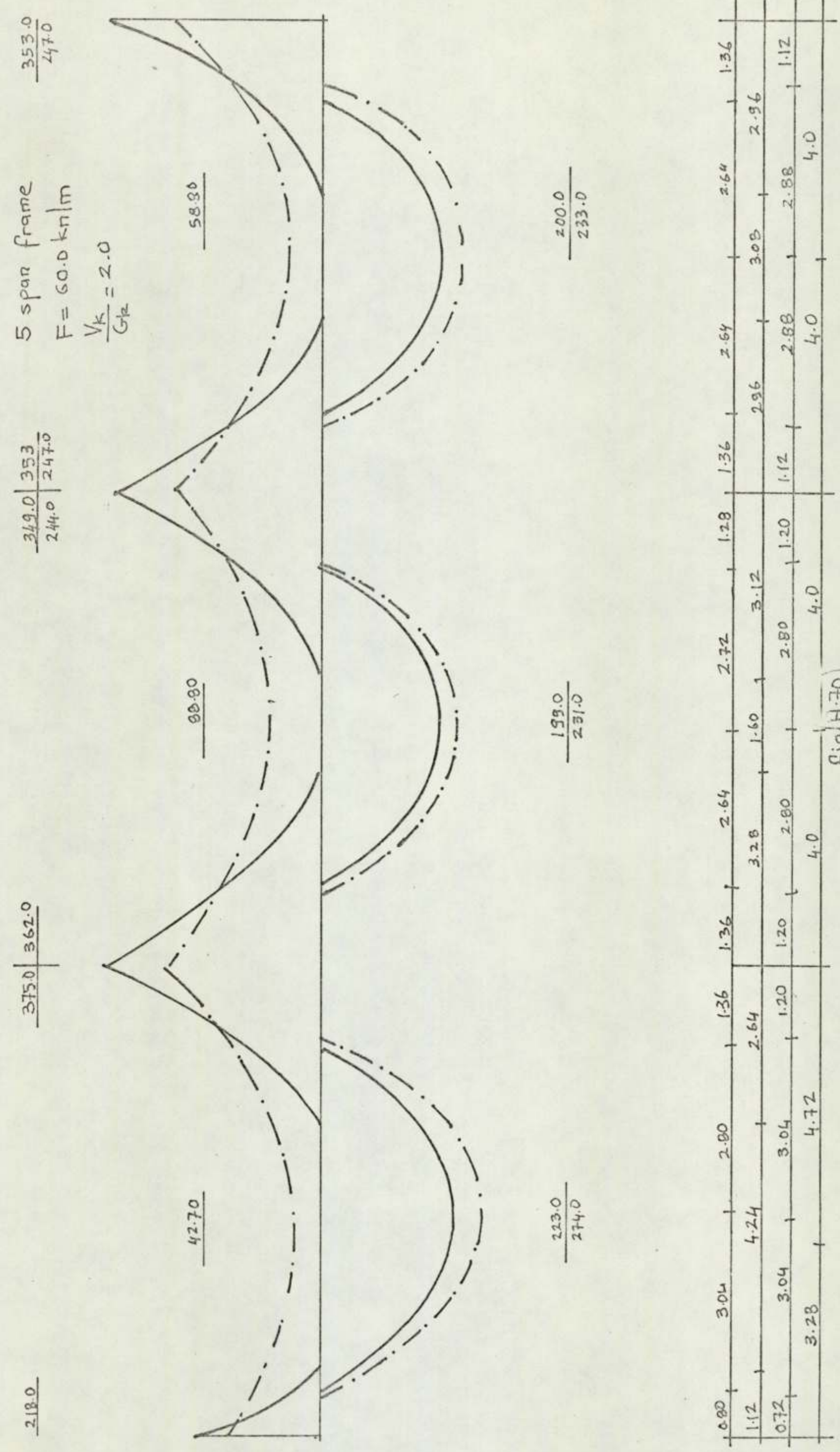
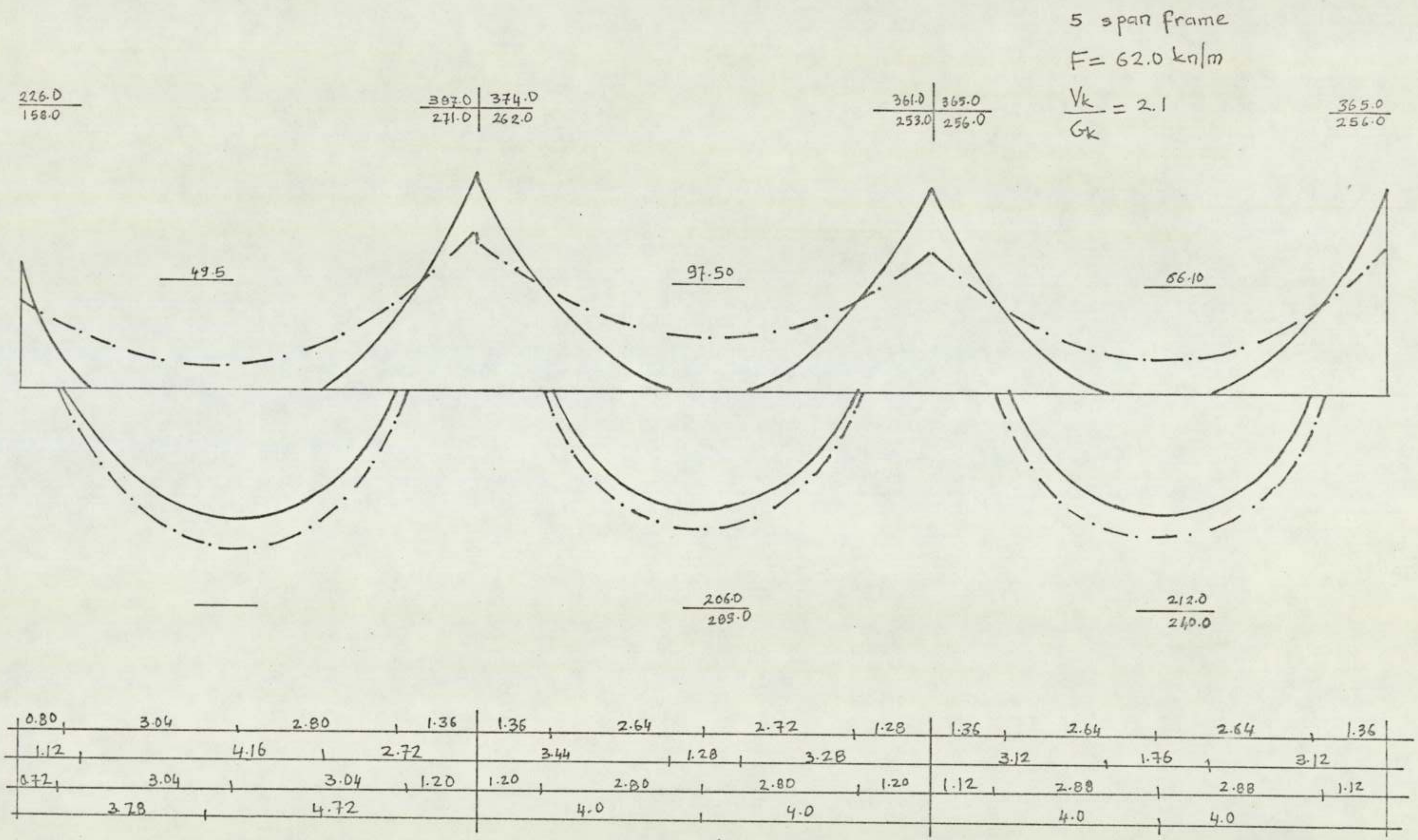
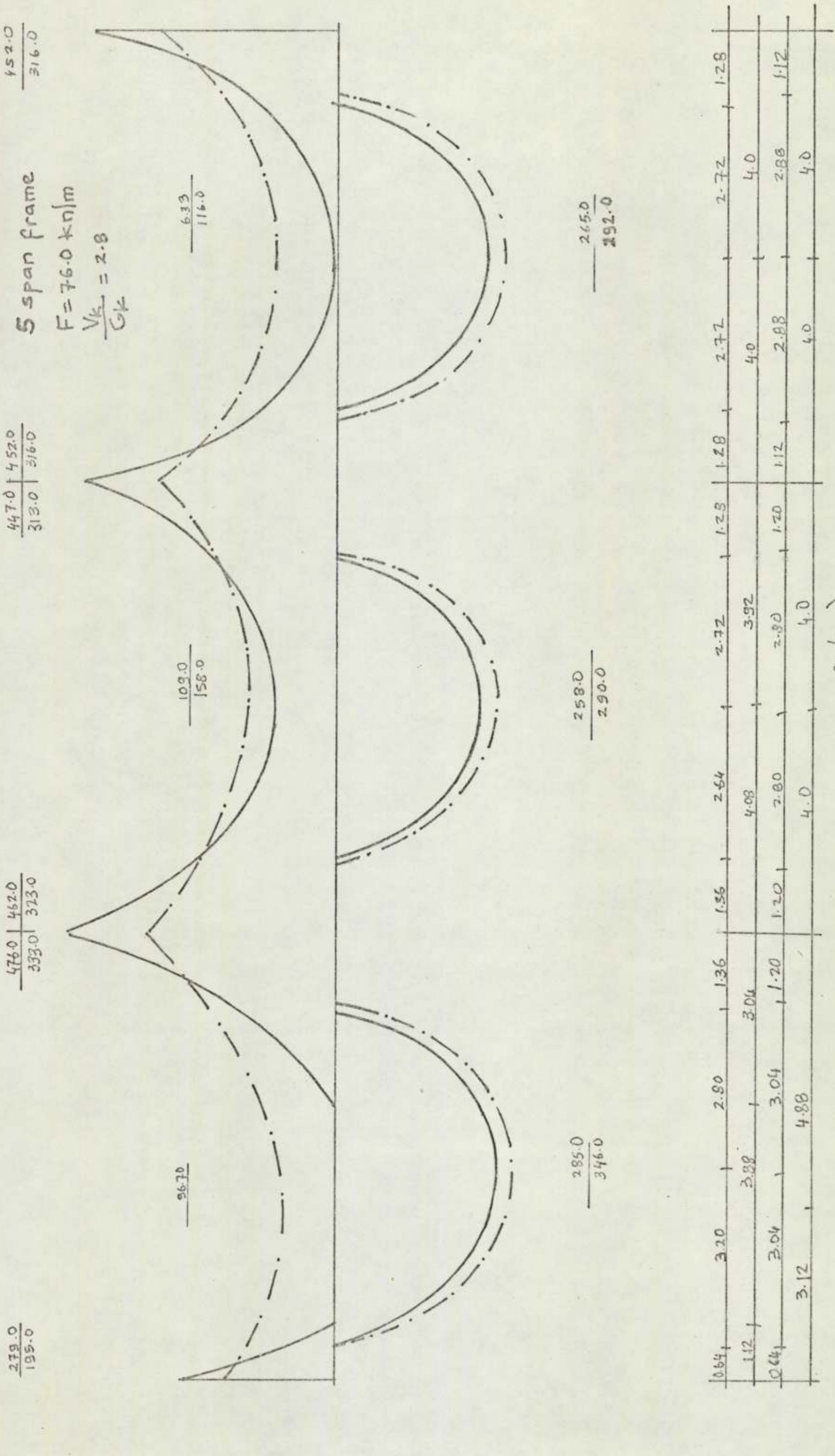

5 span frame
F = 60.0 kn/m
$\frac{V_k}{G_k} = 2.0$
218.0
375.0 | 362.0
349.0 | 353
244.0 | 247.0
353.0
247.0
42.70
88.80
58.80
223.0
274.0
199.0
231.0
200.0
233.0
0.80 3.04 4.24 2.80 1.36 1.36 2.64 1.60 2.72 1.28 1.36 2.64 3.08 2.64 1.36
1.12 3.04 3.04 2.64 1.20 3.28 2.80 3.12 1.20 1.12 2.96 2.88 2.88 2.96 1.12
0.72 4.72 1.20 2.80 2.80 1.20 4.0
3.28 4.0 4.0 4.0 4.0

fig (H·70)


5 span frame
F= 62.0 kn/m
$\frac{V_k}{G_k} = 2.1$
226.0
158.0
397.0 | 374.0
271.0 | 262.0
361.0 | 365.0
253.0 | 256.0
365.0
256.0
49.5
97.50
66.10
206.0
285.0
212.0
240.0
0.80 3.04 2.80 1.36 1.36 2.64 2.72 1.28 1.36 2.64 2.64 1.36
1.12 4.16 2.72 3.44 1.28 3.28 3.12 1.76 3.12
0.72 3.04 3.04 1.20 1.20 2.80 2.80 1.20 1.12 2.88 2.88 1.12
3.28 4.72 4.0 4.0 4.0 4.0

Fig(A.71)


5 span frame
F = 76.0 kn/m
Vk/Gk = 2.8
279.0/195.0
476.0/333.0
462.0/323.0
447.0/313.0
452.0/316.0
452.0/316.0
96.70
109.0/158.0
63.3/116.0
285.0/346.0
258.0/290.0
265.0/292.0
0.64 3.20 2.80 1.36 1.36 2.64 2.72 1.28 1.28 2.72 2.72 1.28
1.12 3.88 3.04 4.08 3.92 4.0 4.0
0.64 3.04 3.04 1.20 1.20 2.80 2.80 1.20 1.12 2.88 2.88 1.12
3.12 4.88 4.0 4.0 4.0 4.0

Fig(7.72)