ANALYSIS OF THE PROCEDURES ASSOCIATED WITH THE DECISION TO SUB-CONTRACT WORK.
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## SUMMARY

Press work is subcontracted when the domestic manufacturing facilities are insufficient to meet the demand. Planned subcontracting of press work occurs at peak demand periods, but in addition unplanned subcontracting of press work frequently occurs during peak and non-peak periods. The project can be split up into two distinct phases -

PHASE 1 - was concerned with identifying and analysing the major difficulties associated with the five week rota cycle system operating on the Large Press Section (selected as a pilot area) that gave rise to unplanned subcontracting measures. Nethods to overcome the difficulties associated with the system were proposed and successfully implemented. A model to allocate press work between the company and subcontractors when the domestic capacity was exceeded is described in Chapter 7. The model minimises the sum of the domestic manufacturing cost and the total subcontracting cost and also smooths the domestic weekly loads across the cycle.

PHASE II - was concerned with the study of more economical manufacturing rota cycles than the five week rota cycle, based on economic batch quantities and tailored to a geometric series. A study of the component storage levels required in rota cycle manufacture was made prior to the implementation of the multi-rota cycles.

## CHAPTER 1 - THE COMPANY

### 1.1. General Introduction:

The nature of the project originated through consultations with the National Computing Centre. It was one of five possible areas of Production Control suggested by them, and the one chosen out of the five at a meeting at the company, which included management, production and operational research staff, as being of particular interest to them.

The first three months of the project were spent in gaining a broad understanding of the whole company. The next three months in probing deeper into the areas of particular relevance. This initial period of time ensured good communication, and laid the ground work for the subsequent phases of the project.

### 1.2. Description of the Company and its Products:

Parkinson Cowan Appliances Limited is one of the divisions of Parkinson Cowan Limited. The Parent Company is situated in London and Parkinson Cowan Appliances Limited, where the project was carried out, is at Stechford, Birmingham.

Parkinson Cowan Appliances Limited are concerned with the manufacture of a wide range of domestic cookers, fires and conversion units for gas appliances already installed in areas that
are converting from 'Town' to 'Natural' gas.

They are one of six major companies that manufacture domestic gas appliances for sale mainly to the Regional Gas Boards.

## 1. 3. The Manufacturing Facilities of the Company:

The manufacturing facilities of the company consist of five main production areas -
(1) Press Shop.
(2) Finishing Shop (enamelling, plating etc.).
(3) Foundry.
(4) Assembly Shop.
(5) General Machine Shop.

On past work load levels, all of the production areas, with the exception of the Press Shop, were able to meet within their available capacities, the peak demands of a variable domestic market.

The capacity of the Press Shop, which had included a regular night shift together with overtime on both the day and night shifts, had been exceeded by up to $50 \%$ at peak demand periods with the subsequent need for short term subcontracting to meet these peak demands.

Loan capital repayment was two years at this time and expansion of the Press Shop to meet the peak demands of a fluctuating domestic market could not be justified on economic grounds.

The volume of work subcontracted from the Press Shop had varied disproportionately to the work load, and the companies concern with this situation, together with the nature of the project, made the Press Shop an ideal area for analysis.

## 1. 4. Summary of the First Phase of the Project:

To deal with peak demands, work was subcontracted from the Press Shop for periods of up to six months. During these periods, and also at non-peak demand periods, shortages built up and caused delays and stoppages in the Finishing Shop and Assembly Shop. To prevent further delays and stoppages, crash subcontracting measures occurred. It was therefore necessary to find out why shortages built up, and to determine whether the volume of shortages could be brought within the practical limits that were normally found in a production area, before meaningful rules were formulated on which jobs and the number of jobs to subcontract when the domestic capacity was exceeded.

It was therefore necessary to find out why shortages built up and this entailed an examination of the existing Press Shop's operating and control systems (Chap. 2) and an analysis of the difficulties associated with these systems (Chap. 3). The examination of the loading system i. e. how batch quantities were determined and the scheduling of jobs onto the Press Shop, was confined to the Large Press Section which had been chosen as the Pilot Area (Section 3. 4).

The loading system and the difficulties associated with this system are described in Chapter 4.

The analysis of the Control System showed that press utilisation was not known or being measured, and the data to determine it was not available in the control data collected in the existing system. Measurement of press utilisation is described in Chapter 5.

Since press utilisation was not known, the actual usable press capacity was not considered when loading the Large Press Section. Also the loading system did not include any mechanism for balancing the load across the five week rota cycle

Unbalanced loads and/or overloading caused shortages, and the volume of shortages was found to snowball over several five week cycles as the press utilisation and operators performance index decreased.

The existing loading system was replaced by a balanced load system (Chapter 6) and incorporated the usable capacity available calculated from the press utilisation and operators performance index. A new control card was introduced to facilitate the collection of more control data. When the new systems were implemented and run over several five week cycles the volume of shortages was drastically reduced from previous levels of up to $60 \%$ to less than $5 \%$. The press utilisation was raised from $40-45 \%$ to $55 \%$ plus and the operator performance index from approximately 90 to 110. The volume of work to subcontract could thus be
determined and continually revised by the up-to-date press utilisation and operators performance index. Thomson ( $\overline{2}$ ) indicated that one of the major difficulties in the implementation of his decision model on what jobs to subcontract would be 'in obtaining true capacity figures'.

The decision rules on what jobs to subcontract are described in Chapter 7.

### 2.1. Description of the Press Shop:

The Press Shop was made up of a guillotine section, welding section, rectification section, hand press section, trimmers, and two main press sections.

The two main press sections were -
(i) The Large Press Section with twelve presses
in the range 100-400 tons which were used to manufacture approximately 130 different items.

It operated on a five week rota cycle i. e. a batch quantity of all items was manufactured every five weeks in a fixed order in each cycle. This gave rise to a regular pattern of batch manufacture that, satisfied a given five week forward demand.

Shanks (1) defines rota cycling as -
'Rota cycle scheduling is a method of scheduling a production facility by reviewing all jobs regularly i. e. at fixed time periods, and determining manufacturing quantities. Often it is called the lot size problem, but this title covers the rota cycle and the general problem of scheduling 'lots' whether in a rota cycle or not. I am interested in the rota cycle which has a basic cycle, but parts can be made more
than once in that cycle if required, in a regular pattern. I will call this the multi-rota cycle problem, and the case when all parts are made to the same frequency, the simple rota cycle problem. '

The five week rota cycle operated in the Large Press Section conformed with Shanks' simple rota cycle problem.

Many of the items manufactured on this press section required from two to four Press operations. The separate tools for these operations were set up on one or more presses depending upon the number of operations and/or tool sizes, and was referred to as the 'ganging' of tools. The number of operators required for the different jobs could thus vary.
(ii) The 'Small Press Section' with twenty-one presses in the range $1 \frac{1}{2}$ to 100 tons were used to manufacture approximately 350 different items. The press section worked to a shortages list and not to a five week rota cycle as on the Large Press Section. A 'shortage' occurred when the quantity of an item reached a predetermined stock level. A batch quantity based on a function of a five week demand was then manufactured. The company were not satisfied with this re-order point system and intended to introduce a rota cycle system in the future.

The two main press sections were operator orientated rather than press orientated i. e. operators were assigned to presses as required rather than being permanently allocated to a particular press. Consequently operators and also setters had to be sequenced as well as jobs onto the presses. The sequencing was more difficult on the Large Press Section as the number of operators required for jobs varied.

## 2. 2 Documentation Flow and Operation of the Press Shop:

The existing documentation and materials flow tirrough the Press Shop and the feedback of information for costing and production control was examined, and represented in the schematic diagram given in Appendix A. Jobs were issued to the Press Shop Office on a two part card referred to as the 'Material Identity Card' (M. I. C. ) A sample card is given in Appendix A. The top portion of the card was used for materials control and the bottom portion for job and production control.

The allocation and sequencing of the jobs, operators and setters onto the presses was done by the Press Shop Office. The order of manufacture of jobs was taken as the date of issue of the cards by Production Control i. e. first in first out, with the exception of shortages that were urgently required. Normally the jobs were issued to the Press Shop Office up to a week prior to the date of manufacture, to enable them to inform the Materials Control Office, by sending to them the top portion of the card, of pending
material requirements. The timing of the release of the material from the Nital Stores to the Guillotine Section for the batch quantities of blanks to be prepared for a job before a press run, was verbally arranged between the Press Shop Office, Guillotine Section and the Metal Stores. This was to reduce the number of blanks stored in the Metal Park on the shop floor that were awaiting press runs. For large batch quantities for jobs on the Large Press Section, the Metal Park area was inadequate, and the Guillotine Section built up a buffer stock before the start of a press run, and continued to maintain a buffer stock throughout the press run. The Guillotine Section on completion of a batch quantity of blanks forwarded the top portion of the card to the Stocks Record Department for the up-dating of materials stock.

When the press run was ready to start, the bottom portion of the card was forwarded to the Inspection Department. The first pressing off the press was dimensionally checked by an Inspector before the run was allowed to continue.

During a press run, completed pressings were stored in pallets conveniently positioned near to the press. Thousands of pressings off a press in the Small Press Section can be stored per pallet, but on the Large Press Section, pallets held as little as 70 components and pallet changeover i. e. full for empty pallets, occurred at 20 minute intervals. The changeover time was two to twelve minutes and this, as will be seen later, is a fairly large factor in
the losses in press utilisation on the Large Press Section.

A patrol inspector regularly checked the pressings during a press run and recorded the number of pressings scrapped and the number that required rectification. The sum total of these numbers was subtracted from the batch quantity automatically recorded by a clock counter on the press set to zero at the beginning of the run by an inspector. The total number of acceptable pressings was recorded on the bottom portion of the card. Rectification and scrap notes were issued by the Inspection Department and a copy of each was sent to the Production Control Department and Quality Control. The bottom portion of the card was returned to the Production Control Department on completion of a job, to up-date their records on batch quantities completed. Finally the bottom portion of the card was forwarded to the Cost Department for the costing of stock.

CHAPTER 3 - DIFFICULTIES ASSOCIATED WITH THE DOCUMENTATION, CONTROL AND OPERATION OF THE PRESS SHOP.
3.1. Documentation:

The Materials Identity Card was designed for use in the General Machine Shop and had been adopted for use in the Press Shop. II/C Group on the card did not refer to a press group, as presses were not as yet classified into press groups but to machine groups in the General Machine Shop.

The control portion of the card (bottom portion) did not allow for the recording of information that may be used for effective production control. As an example, it would have to be re-designed to include space for recording the time lost during press runs due to the frequent occurrence of various factors such as tool failures. The information, if available, could then be used to -
(1) Analyse the reasons for the frequent occurrence of such factors and may lead to corrective measures being taken, where possible, to reduce lost time.
(2) Estimate the amount of press capacity available when loading the Press Shop.

Scrap and rectification notes were not used to either up date stock levels or analyse the reasons for the scrap or rectification. This was apparently due to the large number that were issued daily and the lack of man power to deal with them. Although they were not used, the procedure of issuing them continued.

## 3. 2. Recording of Batch Quantities:

Sheet metal was issued by the Metal Stores to the Press Shop on a weight basis. Since large tolerances were allowed on the thickness of the metal sheet when purchased, a batch issued of apparently 1,000 sheets could be for example, anywhere in the range of 800 - 1,200 sheets. More attention was paid when recording the batch quantity of an item manufactured to the amount issued rather than to the clock counter on the press. Hence the size of recorded batches were unreliable.

Current stocks of items were not used when determining the batch quantities that were loaded on the Press Shop, and together with the unreliab le batch quantities recorded, contributed towards the unexpected shortages arising on items.
3. 3. Difficulties associated with the Operation of the Press Shop: The frequent occurrence of shortages disrupted the pattern and rhythm of work in the Press Shop and also caused delays and stoppages in the Finishing Shop, which in turn, affected the Assembly Shop.

Notification of shortages originated from the Pallet Stores. The Pallet Stores worked to a daily pre-selection list of items that were required by the Finishing Shop. When stocks of items were low or zero, requests were made for those items in a shortage list to Production Control.

The shortages list was a part of the normal routine for scheduling the small press section, but this was not so with the Large Press Section, although it had apparently become so, due to the extensive shortages list.

The effect of shortages on the two press sections was very different. In the case of the Small Press Section a press could be set up in less than fifteen minutes and batch run times were relatively short compared with the Large Press Section. Set-up times on the Large Press Section however varied from two to six hours and in extreme cases up to two days when either presses broke down or tools had to be repaired or parts of tools such as punches had to be replaced. Since many of the jobs on the Large Press Section were multi-operation ones this also entailed having the required number of operators available to start the run. Consequently when shortages arose for immediate manufacture, the effect on the operation of the Large Press Section was frequently very disruptive. Normally all the presses and/or operators on the Large Press Section were employed on jobs and an urgent shortage requirement involved the stopping of a press, stripping and re-tooling the press. During this time the guillotine section were preparing the required blanks, and frequently the storage of blanks spread from the Metal Park to the Press Shop floor causing confusion with blanks cut for the jobs stopped for shortages, and blanks started for later jobs. Added to this were pallets in which the items were stores and were kept on the already limited spare area on the Press Shop floor, due to the Pallet Stores being overloaded. The result was that there
was no coherent pattern of work load, and this undoubtedly contributed to low operators performance index.

### 3.4. Selection of a Pilot Area:

It was felt at this stage in the project that the analysis of the two main press sections simultaneously would be too great a task and the decision to analyse the large press section was based on the following reasons -
(1) Of the 20 or so subcontractors used by the company, only four had press capacity similar to that of the Large Press Section. Difficulties had been experienced at times, in obtaining sufficient large press capacity at the four subcontractors when the Large Press Section was overloaded, and in particular when crash subcontracting measures occurred. The subcontractors were situated in the Birmingham area and consequently geared to the local car industry to whom they gave first priority. Hence when they were loaded at short notice for urgent delivery, the work had to be fitted in and delays in delivery frequently occurred.
(2) The build up of the shortages list which caused the crash subcontracting measures and delays and stoppages in the Finishing and Assembly Shops.
(3) The company intended to introduce a rota cycle system into the Small Press Section. Hence an analysis of the problems associated with the existing rota cycle
on the Large Press Section, and subsequently the decision rules on subcontracting may yield useful and relevant results that may be applied to the Small Press Section.

CHAPTER 4 - THE LOADING SYSTEM ON THE LARGE PRESS SECTION

## 4. 1 Calculation of Batch Quantities

A batch quantity was calculated for a five week forward demand for each component part of the appliances. The quantity of each appliance, as given in the Master Production Schedule, was specified weekly for the first eight weeks and thereafter in four weekly amounts for a further sixteen weeks. The quantities of each appliance were determined from a six month sales forecast based upon the monthly Gas Board total sales figures for all manufacturers, and the proportion of these sales attributed by the company's sales department to Parkinson Cowan.

The first eight weeks of the schedule were quoted as a firm production requirement, although the second four weeks were subject to minor alteration in the monthly up-dating of the schedule. The first eight weeks were normally covered by verbal and/or firm orders placed by the Regional Gas Boards. A very small percentage was added into the schedule to meet orders from other customers. Greater attention was paid to the verbal and/or firm orders in deciding upon the quantities for the first eight weeks in the schedule rather than to the forecast.

A $10 \%$ allowance was added to the batch quantities calculated from the schedule to cover rejects and scrap, but there was no means of checking this allowance. The current stock level of components was not used to modify the batch quantities.

## 4. 2 Scheduling of the load over the 5 week cycle

The scheduling was done by means of a strip index system. An example of a strip index is given in Figure 4. 21.

| Drawing <br> Number | Description | Appliance | Quantity | Date | Batch <br> Number |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Figure 4. 21
Each component to be manufactured in the cycle was recorded on a separate strip and allocated to a particular week in the cycle. The strips were kept in a folder in week order of the cycle. This was to ensure that items were manufactured in the same week of each cycle. The depletion period for a batch quantity of a component was five w'eeks.

Figure 4. 22 illustrates the depletion at a constant rate, of a batch quantity manufactured in week two of a cycle, over a five week period. If manufacture of the batch was delayed say to week three of the cycle, shortages would occur.


Figure 4.22

The strip index system of scheduling had been introduced about ten years ago, and since that time several appliances had been discontinued and replaced by others. There was no mechanism in the system to show the balance across the cycle, and changes to the weekly load had been carried out intuitively. Press utilisation was not known or being measured, and consequently the actual load that could be manufactured per week was not established.

## 4. 3 The Load Across the 5 Week Cycle

The scheduled weekly load across the current cycle was calculated using the strip index system, standard times and batch quantity for
each component. Besides the schedule load there was generally a shortages load that superseded the cycle. To obtain the total weekly load across the current cycle the shortages superseding the cycle were loaded into their particular week in the cycle and added to the scheduled load. Both scheduled and total loads showed considerable out-of-balance across the cycle. The weekly scheduled and total loads are given in histogram form in Figure 4. 31.


Figure 4.31

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The total scheduled load across the cycle, was 2180 standard hours and the shortages load 1429 standard hours. The estimated output, 1790 standard hours for the cycle, was 290 standard hours less than the scheduled load, giving rise to further shortages.

Under these conditions the batch quantities as calculated for the scheduled load (Section 3.1) gave way to the shortages list. To meet the demand of the Finishing Shop batch quantities were intuitively reduced, and many items were manufactured twice in the cycle. Batch quantities were found to be approximately halved but some were as low as $5 \%$ of the scheduled batch quantity.
4. $4 \frac{\text { Distribution of the Scheduled Load on the Presses in The Large }}{\text { Press Section }}$

The loading of jobs onto the presses was carried out by the Press Shop Office. There was no formal method available to either plan the job loading or show the load on the presses. The job loading was done intuitively.

The press or presses a component could be manufactured upon in the Large Press Section was determined by the tool size and the bed size of the presses. To analyse the distribution of the scheduled load on the presses in the Large Press Section, a list was made of all
the components manufactured in this section together with the press or presses on which they could be manufactured. Presses in the Large Press Section were numbered. The press numbers were used in the listing to identify the presses upon which a component could be manufactured, as illustrated in Figure 4.41 by a cross in the appropriate press number column.


Figure 4. 41

38 of the 130 components listed could be manufactured on any one of the four presses, 3, 32, 33 and 36 , five components on presses 32 or 33 , one on presses 3 or 36 , and one on presses 3,32 or 33 . This gave a total of 45 components that could be manufactured on all or some of the group of presses $3,32,33$, and 36 and not on any other presses. These four presses were designated as Press Group 1. Three other distinct press groups were formed in this way such that a component could be made on one or more presses in a press group
but not in any other press group. The number of components that could be manufactured in one press group and no other press group, as given in Table 4. 42, accounted for $80 \%$ of the 130 listed components.

| Press Group | Press Number <br> in Group | Number of <br> Components |
| :---: | :--- | :--- |
| 1 | $3,32,33,36$ | 45 |
| 2 | 6 | 6 |
| 3 | $27,28,29$ | 17 |
| 4 | $1,2,4,5$ | 36 |

Table 4. 42

The weekly scheduled load in each press group, calculated from the strip index scheduling system for the $80 \%$ of the components, showed considerable out-of-balance across the five week cycle. The remaining $20 \%$ of the components, referred to as press group variables, shared the press groups. For example five components could be made in press group 1 or 4,12 components in press group 3 or 4 and so on.

From the strip index scheduling system it was found that jobs that were press group variables occurred in every week of the five week cycle. The distribution of the press group variables load, in any week of the cycle, on the press groups depended upon the scheduling of jobs onto the presses by the Press Shop Office. The scheduling
of the jobs onto the presses was either intuitive or governed by the shortages list, and both resulted in a variable distribution of the press group variables load on the press groups. In the case where the Press Shop Office were working to a shortages list, the order of manufacture of jobs was generally taken as the order in which they occurred on the shortages list. Consequently jobs were manufactured on the first press to become available on which they could be made to meet the demands of the shortages list; and could result in the distribution of the press group variable load increasing the degree of unbalance and/or overload already found in the press groups for the $80 \%$ of the components.

## 4. 5 Press Group Utilisation

The current press groups utilisation over a five week cycle was estimated from the output in standard hours from each press group, the operators performance index and press group capacity in standard hours. The press group utilisation required to meet the current scheduled load in each press group balanced across the cycle was calculated using the Balance Load System described in Chapter 6. In the Balance Load System press group variables were allocated to one of the two press groups on which they could be manufactured.

A comparison of the current press group utilisation and the press group utilisation required to meet the balanced load in each press group across the five week cycle, as given in Table 4.51, showed that the estimated current utilisation in each group was too low to meet the current scheduled loads.

| PRESS | PRESS GROUP UTILISATION |  |
| :---: | :---: | :---: |
|  | CURRENT | REQUIRED TO INEET <br> THE BALANCED LOAD |
| 1 | 41.9 | 52.5 |
| 2 | 25.0 | 27.5 |
| 3 | 32.9 | 38.7 |
| 4 | 39.1 | 47.8 |

Table 4.51

To find out if press group utilisation could be improved, the factors effecting utilisation such as the time lost due to pallet change-overs were measured and analysed. The measurement and analysis of press utilisation together with a discussion on operators performance is given in Chapter 5.

## 4. 6 Discussion on Shortages

The unbalanced scheduled weekly load, excluding the press group variables, across the cycle as illustrated in Figure 4.61 for press group 1 was characteristic of all the press groups.


Figure 4.61

At the current estimated press group 1 utilisation of $41.9 \%$, weeks 1 to 4 of the cycle were overloaded for this press group, and shortages occurred to supersede the next cycle. The shortages occurring in the cycle were in addition to any shortages from the previous cycles. The allocation of press group variables by the Press Shop Office to this group would increase the overload and
hence shortages. Unexpected shortages also occurred in a cycle due to the unreliable batch quantities recorded in the previous cycles. To meet the demands of the shortages list, involved more tool change-overs which in turn increased for example the time lost on the presses due to the extra number of first off inspections. Consequently press utilisation was reduced and gave rise to further shortages.

The volume of shortages therefore built up from one cycle to the next and when the demands of the shortages list could not be met domestically, crash subcontracting measures were taken. The balanced load of 168 standard hours per week over the cycle for press group 1 could be achieved with a press group 1 utilisation of $52.5 \%$, but with the same press group 1 utilisation in weeks 2, 3 and 4, the unbalanced load would give rise to shortages to supersede the next cycle. The need for balancing the load across the cycle in each press group was evident, but at the current press group utilisation the balanced loads in each press group would not be achieved.

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CHAPTER 5 - INEASUREMENT AND ANALYSIS OF PRESS
    UTILISATION
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### 5.1. Neasurement of Idle Time Parameters

Graph paper with a pre-printed time scale and headings was used to record to the nearest minute press idle time together with the reason. Figure 5.11 illustrates one day's recordings. It was found physically impossible to record all press groups at the same time, and press group 1 was used as the pilot press group.

Daily recordings were taken for press group 1 over a five week cycle scheduled on a shortages list. The reasons causing idle time were analysed and classified into the following idle time parameters.
(A) Tool Change-over Time

The time taken to strip and re-tool a press.
(B) Awaiting Tool Change-over

The time a press stood idle waiting for setters to become available.
(C) Awaiting Operators

The time a set-up press stood idle waiting for operators to become available.
(D) Awaiting Material

The time waiting for cut blanks from the guillotine section.
(E) Tool Troubles

The time lost in tool repairs, replacement of parts in tools, and re-alignment of tools on the presses.
(F) Awaiting Maintenance

The time waiting for maintenance staff to become available.
(G) Maintenance

The time spent on press maintenance.
(H) Awaiting Inspection

The time waiting for inspectors to become available.
(I) Inspection

The time taken for inspection to dimensionally check the first pressing before a press run was allowed to continue.
(J) Pallet Change-over

The time waiting for empty pallets to replace full pallets.
(K) Instructions

The time taken in instructing operators before press runs.
(L) Awaiting Metal Stores

The time lost in waiting for material from the metal stores.
SHOP
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[^0]FIGURE $5 \cdot 11$

The time lost by meetings, accidents, delays in starting work etc.

The time incurred by each idle time parameter per press per day on press group 1 were summed. A form was designed to keep a record of the daily totals. An example of the form is given in A ppendix A. It was also envisaged that the form would be used, when an analysis of press utilisation was required in the future, to record the summed daily totals of the idle time parameters that would be recorded on the new control card when introduced.

## 5. 2 Calculation of Press Group Utilisation


(Total Press Group Time Available ) $T$
Where $T=$ period of time considered
$\mathrm{n}=$ number of presses in the press group
$A_{i}, B_{i}, C_{i}-\ldots-----$ - the idle time parameters, defined in section 5. 2, for press i in the press group.

The Work Study Department established standard times in minutes for the manufacture of a component. The time allowed for a batch quantity in standard hours was determined from
Size of Batch x Standard Time

If the operators achieved the time in standard hours for a batch quantity their operator performance index (P. I.) was at the ' 100 rating' : otherwise the P.I. rating was calculated from

$$
\frac{\text { (Standard hours allowed for batch quantity) }}{(\text { Actual hours taken for batch quantity })} \times 100
$$

A. P. I. was calculated to represent the overall weekly average operators performance on the large press section. This was done by a clerk from the Wages Department, permanently stationed in the Press Shop Office. Effective press group utilisation was determined from Calculated Press Group Utilisation $x \frac{\text { P.I. }}{100}$

Effective press group utilisation was subsequently used in estimating the true capacity available when loading the press groups.

### 5.3 Presentation of The Results To Nanagement

The objective was to demonstrate to management that the current scheduled load, when loaded by press groups, and each press group balance loaded across the cycle, could be met domestically by raising the press group utilisation by improvements to the idle time parameter as described in section 5. 4. To effect these improvements required the backing of management, and to this end the results were represented in the form as shown in figure 5.31 for press group 1, together with

the effect of varying the operators performance index. Operators performance was the concern of management and shop floor supervision, and the results presented highlighted the effect on press utilisation of improving the operators performance. The recorded results for the idle time parameters were given together with rough guide lines on the improvements possible described in section 5. 4 to tool changeovers, tool-troubles and pallet change-overs. The estimated press group 1 utilisation of $59.3 \%$ at 90 P. I., the current operators performance index, would give an effective press group 1 utilisation of $53.3 \%$. The current scheduled load on press group 1 when balanced across the cycle could be achieved with an effective press group 1 utilisation of $52.2 \%$. The implementation of the current scheduled loads on each press group balanced across the cyle would serve to establish if the press group utilisation necessary to meet the current press group loads were possible, as was indicated by the press group utilisation analysis and estimates given in section 5. 4. The implementation would also indicate the extent to which the balance load and new control systems were effective in reducing shortages and the build-up of shortages. It was the intention of management to sub-contract sufficient work to meet the shortages that currently superceded the cycles before the balanced load system was implemented.

The continued monitoring of press group utilisation, as described in section 5. 5 the feed back of information from the new control system, and up-todate P. I. would enable the Production Control Department to continuously estimate and update the effective press group utilisation
that could possibly be achieved and maintained. Thus the forward planning of the amount of work to sub-contract from press groups should be possible to avoid overloading the press groups.
5. 4 Analysis of, and estimated improvements to,the Idle Time Parameters The average time over the cyle in press group 1 for a tool change-over and an inspection of the first pressing, was 2.1 and 0.34 hours respectively. These times were within acceptable limits to management, and therefore the total time recorded for each was directly attributed to the number of tool change-overs thai occurred in press group 1 over the cycle. The capacity available on the press group, based on effective press utilisation together with a balance load across the cycle, and more accurate recording of batch quantities, should reduce shortages and hence the number of tool change-overs. A reduction in the number of tool change-overs should also reduce the idle time parameters awaiting tool change-over, awaiting inspection, inspection and instructions, but those were discounted in estimating press group utilisation that may be achieved to partly allow for the fluctuations to be expected in all parameters. The total number of hours available on press group 1 over the cycle was 1600 , and using, 2. 1 hours per tool change-over, gave a loss in utilisation of $0.13 \%$ per tool change-over. The scheduled number of 45 jobs for press group 1 would incur a loss in press group 1 utilisation of $0.13 \times 45=5.9 \%$, and this figure was used for tool change-over in the estimate of utilisation that may be achieved in press group 1.

A waiting operators was the largest idle time parameter recorded. The number of operators required on any press in the operator orientated large press section varied from 1 to 4 depending upon the job. The total number of operators employed on the large press section were utilised where needed on the presses in the large press section. Hence the number of operators required throughout a shift fluctuated as jobs were completed and others started. The total number of operators available per shift also varied due to absences. The total number employed on the day shift was 30 , and for the cycle analysed, the number present varied between 26 and 29 over the cycle. The number required varied between 25 and 34 , and was a function of the shortages list which generally governed the priority of jobs. Full advantage therefore could not be taken in utilising the total operator force present by sequencing the jobs to balance the number of operators made available by the completion of jobs, and the number required to start other jobs, to minimise delays in press runs. Delays occurred on press runs when the number of operators made available by the completion of jobs were less than the number required to start other jobs. Greater freedom to sequence jobs/operators to reduce the time lost in waiting for operators would depend upon the effectiveness of the press group balance load and new control systems in reducing shortages. There was no formal method available for sequencing and a scheme to sequence jobs/opera*ors to minimise time lost on the presses due toawaiting operators was proposed to the company, and is described in Appendix B. It was the
subject of a meeting at the company, and was accepted in principle by management. The implementation of the sequencing scheme would require an extra man in the press shop office. Management decided to defer the implementation of the sequencing scheme until the balance load and new control systems were functioning smoothly. Work on determining the optimal number of operators required on the large press section is proceeding.

The extent of an improvement to the recorded loss of $12.2 \%$ for awaiting operators due to a reduction in the number of tool change-overs was uncertain. This, together with no immediate improvement in the sequencing of jobs and the question about the optimum number of operators required on the large press section led to the recorded figure of $12.2 \%$ being used as a rough guide for the awaiting operators parameter when estimating the possible press group 1 utilisation that may be initially achieved.

Pallet change-over was the second largest idle time parameter recorded. Pallet change-over times recorded on the same press and job varied between 2 and 12 minutes. As pallets became full during a press run they were replaced by empty pallets before the run was continued. Frequently the empty pallets had to be obtained from the Pallet Stores, which was some distance from the Press Shop, or other parts of the factory. Fork lift trucks were used to transport pallets. The full pallets taken to the Pallet Stores were unloaded
and empty pallets were then sought from within the Pallet Stores or other parts of the factory. This procedure accounted for the excessive times recorded on pallet change-overs. The average pallet change-over time was 8 minutes over the cycle. When pallets were available on the shop floor the average pallet change-over time was reduced to 4 minutes. To achieve an overall average of 4 minutes in pallet change-overs, management subsequently introduced a buffer stock of empty pallets to be kept on the shop floor. With 350 pallet change-overs for the balanced load system on press group 1 over the cycle, estimated from the number of pallets required per batch size for each component, would incur a $2.9 \%$ loss in press group 1 utilisation compared with the $7.4 \%$ recorded.

The time lost on tool-troubles during press runs, the third largest idle time parameter recorded, was caused by minor tool splits, the necessity to re-align tools, and the frequent breakage of punches that formed parts of nearly all tools. Presses stuod idle while replacement punches were made by the toolroom, and to reduce this time management sanctioned the stocking of duplicate punches for tools on which punches frequently broke. Punch breakages accounted for approximately half the time lost in tool-troubles and $3 \%$ was used for this parameter in estimating press group I utilisation that may be
achieved. The cause of minor tool splits was to be investigated by management, and one avenue to be explored was the pressure required to manufacture each component.

The remaining idle time parameters awaiting material, maintenance, awaiting metal stores and miscellaneous were within acceptable limits to management. Further recordings were made on all press groups over a period of one cycle. The frequency of the recordings were subject to the availability of manpower in the Press Shop Office, but when recorded were for complete days, and served to confirm the analysis on press group 1. From the results the estimated press group utilisation given in Table 4.42 for the current load when balanced could be achieved. The balancing of the load in each press group using the estimated press groups utilisation, implemention and results achieved are described in Chapter 6.

The current low operator performance index of 90 was partly attributed to the small batch quantities, caused by shortages, being manufactured resulting in shorter press runs. Short press runs involved the frequent changing of operators from one job to another, thus disrupting their rhythm of work. With the introduction of the balanced load system and the new control system to reduce shortages and clear objectives, brought about by these systems, to all levels of supervision on the shop floor, it was envisaged by management
that the operators performance would improve.

## 5. 5. Mechanical Recorders

To reduce the occurrence of shortages, the load across a cycle that may be achieved, should be based upon the previous cycle or cycles press utilisation. This would involve the continuous monitoring of press utilisation. The method employed in the analysis of press utilisation would require 2 men on both the day and night shift for the large press section, and was not a practical or economic proposition.

The company had some time previous to the press utilisation analysis carried out a survey on mechanical recorders available on the market. One was selected that was capable of monitoring 6 out of 12 processes on the large press section, and became available during the latter stages of the analysis on press utilisation. The output from the mechanical recorder, as illustrated in figure 5.51, displayed each stroke of a press on a time axis, graduated in time intervals of 0.8 minutes, by drawing a line. The lines were spaced at fairly regular intervals when the press was running, and were generally easy to distinguish from the times when the press stopped, represented by much longer blank intervals. The periods of run-time were summed over the length of time considered and press utilisation determined. Six presses could be monitored simultaneously.

By counting the number of lines recorded, i. e. strokes of the press in a run period and dividing this number into the run period time, the

time of manufacture per pressing over the run time was determined.
P. I. was then obtained from the standard time allowed for the pressing and the time calculated from the output of the mechanical recorder.

It was realised by management that the output from the mechanical recorder was in an inconvenient form, and did not provide all the information required for effective production control such as the reasons for stoppages. More sophisticated output systems were necessary, and one such system being considered was a 'Digital LostTime Monitoring System', electric impulses are fed from the presses to the 'Digital lost time monitoring system'. The system includes a visual display unit at the presses, an alarm system and error code system for any stoppages. Information is fed to a date logger, and this is used for further analysis e. g. causes of stoppages, stock levels, operator payments and so on. Cost $£ 380$ per press.

## CHAPTER 6 - THE BALANCE LOAD SYSTEM AND REVISED CONTROL SYSTEMI

## 6. 1 The Balance Load System

A heuristic method was developed in co-operation with the Production Control Department, based on the two tables illustrated in Table 6.1A and 6.1B.

| Total Time |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Component | Std. time | $\begin{gathered} \text { Press } \\ 1 \end{gathered}$ | $\begin{gathered} \text { Gro } \\ 3 \end{gathered}$ | ${\underset{4}{\text { oup }}}^{2}$ | Trimmer | No. of operators | No. of pallets | Operator hours | Week No. |
| X | X |  | X |  | X | X | X | X | X |
| X | X | X |  |  |  | X | X | X | X |

TABLE 6.1A


Press group variables, section 4.4 , were initially allocated to press groups to bring the five week rota cycle press group loads to within the loads based on the press groups utilisation figures given in Table 4.51. The average weekly load across the five week rota cycle was then calculated for each press group and recorded in Table 6.1B.

The existing weekly loads in each press group governed by the weekly job allocation in the existing rota cycle scheduling system and recorded in Table 6.1A, were calculated and the results recorded in Table 6.1B. The weekly deviations from the average in each press group were smoothed by changing jobs around in Table 6.1A. During this time Table 6.1B was continually updated until the best weekly balance was achieved. Some press group variables were re-allocated to improve the balance and new press group averages calculated.

The process of changing jobs from one week to another in Table 6.1A and continually updating Table 6.1B was organised as follows five large sheets of Table 6.1A, representing weeks of the cycle, with Table 6.1B alongside, were pinned to a pin board and mounted on a wall at working height. A transparent plastic sheet was superimposed over the tables so that changes to the tables could be temporarily added on the plastic sheet. Jobs changed from one week to another had their existing rota schedule week number recorded in Table 6. 1A under the column labelled 'Week No'.

The final allocation of jobs to the press groups in each week of the cycle and the total weekly press group loads achieved were recorded on new blank copies of Tables 6.1 A and 6.1 B respectively. All press groups were balanced to within $5 \%$ of their weekly average load. A similar procedure for balancing would be followed when future press group loads changed to meet the demand.

### 6.2 Implementation

Implementation of the balanced weekly schedule in each press group was carried out over two five week cycles. This was to allow for jobs that had been allocated to different weeks of the cycle relative to the weeks that they occupied in the existing scheduling system to be brought into phase with the demand. For example, where a job was previously in week number five and was now in week number two required a decision on whether to extend the run to seven weeks (in the first five week cycle) or complete two runs, one of two weeks (in the first five week cycle) and another of five weeks (in the second five week cycle). Decisions of this nature were undertaken by the Production Control Department and based on the available capacity and planned overtime. The methods to improve the idle time parameters described in Chapter 5 for ' pallet change-overs' and 'tool troubles' had been implemented prior to the introduction of the balance load system.

After the two five week cycle change-over period the balance load system ran smoothly and the press groups utilisation (Table 4. 51) required to meet the current weekly balanced press groups loads were achieved. Shortages did occur in the system but, there were no build-ups of shortages as in the previous system. Of the shortages that did occur, many were for components required in their week of manufacture. This was attributed mainly to the random reject rate on enamel work in the Finishing Shop, and resulted in a more rapid depletion rate of stocks than allowed. This is being investigaged by the Production Control Department.

Higher effective press groups utilisation have been achieved, relative to those achieved with the current loads, during peak demand periods, for example 60\% in press group 1. Operator performance index has risen to 112 compared to the previous value of 90 recorded before the introduction of the balance load and new control system.

During peak demand periods, work is now subcontracted from overloaded press groups from each week of the rota cycle in order to retain a balanced load across the cycle. Smoothing press group loads together with the minimising of the sum of the domestic manufacturing costs and the total sub-contracting cost is discussed in Chapter 7.

Management have employed an extra man in the Production Control Department. His main function will be to prepare for the implementation of a sequencing scheme on the large press section based on the one proposed by the author in Appendix B. It will involve the sequencing of operators, jobs and setters onto the presses, and help in establishing the optimum number of operators required on the operator orientated large press section. The effect on the number of operators required with different work loads should also be investigated.

## 6. 5 The Revised Control System

A new Strip Index has been designed by the Production Control Department and is being implemented. The illustration in Table 6.5A shows the general form.

| Drg. <br> No. | Component Description | Quantity | Standard Time |  |  |  | Trimmer | Pallets | Operator <br> Hours |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 1 | 2 | 3 | 4 |  |  |  |

## TABLE 6.5A

A different colour strip will be used to indicate $\epsilon$ ch appliance. The strips will be kept in 'special purpose folders' as in the existing strip index system, but there will be one folder for each week of the cycle. At the bottom of each page the totals within each section will be determined and used to indicate whether the loads fall within the current available capacities. The current capacity in each press group will be
updated periodically to allow for changes in press group utilisation. Press group utilisation is now monitored daily on the mechanical recorder. A modification to the paperwork to load the Press Shop has been completed with the introduction of 'THE PRODUCTION CONTROL CARD' (Appendix A). This indicates the full job and run specification and provides space for the booking of work by the operators and space to record the reason for stoppages. A full set of Process Layat Sheets are being compiled to ensure a full record of all work.

To operate the system the controller will obtain a stock check on the components to be run. This will be subtracted from the quantity for a five week run, plus a $10 \%$ allowance for scrap, and the result is the figure added to the Production Control Card. All other data for the Production Control will be obtained from the layout sheet. The load is added to the strip index and the totals for the week are calculated and passed to the Production Manager for information.

The feedback of information recorded in the Production Control Cards indicating reasons for stoppages in press runs will be analysed periodically by the Production Control Department. In this way changes in idle time parameters will be monitored, and action taken where necessary.

An analysis of the information collected to date on the Production Control Cards show that the estimated improvements to the idle time parameters
(CHAPTER 5) - Pallet Change - Over, and Tool - Troubles are being achieved.

## 7. 1 Published Work

Subcontracting of press work occurs when the domestic press capacity is insufficient to meet the requirements of the manufacturing schedule. Although the study on subcontracting is primarily an allocation problem it can be easily classified as a production smoothing problem. The smoothing of a production load can be achieved by -
(i) Manufacturing work earlier than intended to reduce a peak on load, or
(ii) sending the work out to subcontractors.

There is a considerable amount of literature on (i) but only one published work by Thomson (2) could be found on the subcontracting aspect of production smoothing.

Thomson describes an allocation model which determines how the work load should be divided between the domestic manufacturing facilities and subcontractors when the domestic capacity is exceeded; whilst meeting a primary objective of minimising the cost of domestic manufacture and the cost of subcontracting. Thomson classified similar components into groups and used the groups as variables in a Linear Programming Formulation. The values of the variables were restricted to the range $0-1$. A value of 1 allocatated by the model to a variable indicated it was to be made domestically i. e. the group
of components represented by the variable and avalue of zero to be made at the subcontractors. Rational values allocated to variables by the model between 0 and 1 indicated a split group. For example if the model allocated a value of 0.61 to a variable (group of components) this meant that $61 \%$ of the group was allocated to the company for manufacture and $39 \%$ to the subcontractor for manufacture. To avoid the expense of providing duplicate tooling to cater for split groups, Thomson developed a 'rule of thumb' method to allocate split groups either completely to the company or completely to the subcontractor. However he says that 'the rule of thumb method in dealing with split groups was found to be inadequate in its present form'.

He assumed in his multi-period allocation model that both domestic and subcontractor machining capacities were known. In practice he found that true capacity figures were not known and says that 'this would be a major problem in the implementation of the model'. The model does not consider the selection of individual subcontractors and hence cannot take into account differences in subcontractors manufacturing costs or possible limitations in the individual subcontractors available capacities.

## 7. 2 The Existing Subcontracting System on the Large Press Section.

 Subcontractors component manufacturing costs were only known for about $20 \%$ of the total number of components made within the Large PressSection. The components subcontracted by the company were confined to all or a selection of these $20 \%$ of the components, depending upon the magnitude of the peak loads and/or extra loads due to shortages. The reason why this particular $20 \%$ of components had been originally selected for subcontracting was obscure, but some had been apparently chosen because of difficulties experienced in tool setting on the domestic presses.

Changes in component batch sizes, from one peak demand period to another and extra loads due to shortages produce variable domestic press load distributions. The existing subcontracting procedure did not consider the smoothing of the weekly domestic press loads across the rota cycles when work was being selected for subcontracting to meet the peak loads.

The lack of subcontractors component manufacturing costs for the majority of the components made in the Large Press Section precluded a full consideration being given to minimising the total domestic and subcontracting cost.

## 7. 3 Proposed Subcontracting System on the Large Press Section.

A model was required to allocate component batches between the company and the subcontractor, when one or more of the domestic press group capacities were exceeded, whilst meeting all the following conditions -
(i) does not exceed the available capacity per week across the rota cycle in each press group,
(ii) minimises the domestic component batches manufacturing costs and the total subcontracting cost (i. e. subcontractors component batches manufacturing costs, plus transportation costs of component batches between the subcontractors and the company),
(iii) utilises as much of the domestic press groups capacities as possible or an amount specified by management,
(iv) allocates at least a minimum amount of work to particular subcontractors. The minimum amount to be specified by management,
(v) selects the subcontractor and the component batches to be allocated to the subcontractor,
(vi) ensures that a component batch is made.
7. 4 Model - Formulation
$d_{j l w}=$ factory cost of component batch made on the day shift for the jth component in the lth press group in week w of the rota cycle.
$n_{j l w}=$ Factory cost of component batch made on the night shift for the jth component in the lth press group in week w of the rota cycle.
$s_{\text {ijlw }}=$ ith subcontractors cost (plus transportation cost) of a component batch for the $j$ th component of the lth domestic press group in week w of the domestic rota cycle.
$b_{j l w}=$ Factory processing time in standard hours for a batch of the jth component in the lth press group in week w of the rota cycle.
$v_{\text {ijlw }}=\cdot$ ith subcontractors processing time in standard hours for a batch of the jth component in the lth domestic press group in week w of the domestic rota cycle.
$\mathrm{A}_{1 \mathrm{w}}$
$=$ Capacity available on the factory day shift in standard hours on the lth press group in week w of the rota cycle.

A $_{\text {Iw }}=$ Minimum capacity to be utilised on the factory day shift in standard hours on the lth press group in week w of the rota cycle.
$B_{l w}=$ Capacity available on the factory night shift in standard hours on the lth press group in week w of the rota cycle.
$B^{\prime}{ }_{l w}=$ Minimum capacity to be utilised on the factory night shift in standard hours on the lth press group in week w of the rota cycle.
$C_{\text {il }}=$ Capacity available at the ith subcontractor for lth domestic press group.
$\mathrm{C}^{\prime}{ }_{\text {il }}=$ Minimum capacity to be utilised at the ith subcontractor for the lth domestic press group.


Let $\mathrm{W}=$ number of weeks in the rota cycle or number of weeks to be considered.

Let $L=$ number of domestic press groups.
Let I = number of subcontractors.

The objective is to minimise the sum of the domestic manufacturing costs and the total subcontracting cost subject to the capacity constraints in each press group in each week of the rota cycle i. e.
MINIMISE $\sum_{1=1}^{1=L} \sum_{w=1}^{w=W} \sum_{j=1}^{K}\left(d_{j l w} X_{j l w}+n_{j l w} Y_{j l w}+\sum_{i=1}^{i=1} S_{i j l w} Z_{i j l w}\right)$
Subject to

$$
A_{w l}^{\prime} \leqslant \sum_{j=1}^{K_{w l}} b_{j l w} X_{j l w} \leqslant_{w l}
$$

Weekly capacity constraints for the factory day shift throughout a rota cycle in each press group.

$$
B_{w l}^{\prime} \leqslant \sum_{j=1}^{K_{w l}} b_{j l w} Y_{j l w} \leqslant B_{w l}
$$

Weekly capacity constraints for the factory night shift throughout a rota cycle in each press group.

$$
\begin{equation*}
\mathrm{C}_{\mathrm{il}} \leqslant \sum_{j=1}^{\mathrm{K}_{\mathrm{wl}}} \mathrm{~V}_{\mathrm{ijlw}} \mathrm{Z}_{\mathrm{ijl} 1 \mathrm{w}} \leq \mathrm{C}_{\mathrm{il}} \tag{7.43}
\end{equation*}
$$

Capacity constraints for each subcontractor throughout the domestic rota cycle.
$X_{j l w}+Y_{j l w}+\sum_{i=1} Z_{i j l w}=1$
Ensures that a component batch is made.
Where $K_{w l}$ is the number of part numbers in week $w$ of the rota cycle in press group $\mathcal{R}$.
NOTE: $\quad X_{j l w}, Y_{j l w}$ and $Z_{i j l w}$ for all i represent the same component batch at the different locations.

In solving the model (the methods of solution are discussed in Section 7.5)., each capacity constraint represented by 7.41 would normally be written in the form -

$$
\begin{aligned}
& \sum_{j=1}^{K} w l b_{j l w} x_{j l w} \geqslant A_{w l}^{\prime} \\
& \sum_{j=1}^{K} w l b_{j l w} X_{j l w} \leq A_{w l}
\end{aligned}
$$

i. e. two inequalities. These may be replaced by one equation -

$$
\sum_{j=1}^{K} w l b_{j l w} X_{j l w}+Q_{w l}=A_{w l}
$$

Where $Q_{w l}$ is a slack variable such that

$$
0 \leqslant Q_{w l} \leqslant\left(A_{w l}-A_{w l}^{\prime}\right)
$$

and an upper bound of value ( $A_{w l}-A_{w l}^{\prime}$ ) standard hours imposed on the slack variable $Q_{w l}$.

This would be repeated for the capacity constraints represented by 7. 42 and 7. 43, and hence reduce the number of capacity constraints by a half. This is important in computer run time (and cost) in solving the model as the time of solution is dependent on the number of constraints.

In the case of subcontractor' $i$ ' the difference between the available capacity $\mathrm{C}_{\mathrm{il}}$ and the upper bound imposed on the slack variable is the minimum amount of work that must be allocated by the model to subcontractor ' i '. If a minimum amount of work does not have to be allocated to a subcontractor the upper bound on the slack variable is omitted.

Peak demand periods currently occur over about three five week cycles. The production load across the peak demand period is normally, smoothed. Therefore the decisions made on the allocation of work between the company and the subcontractors, by the model for one five week rota cycle in the peak demand period, will remain valid for the other rota cycles in the peak demand period.

The model will also be valid for the multi-rota manufacturing cycles proposed in Chapter 9. The longest cycle to be allowed by management in the multi-rota cycles is to be 16 weeks, which is also about the length of each of the peak demand periods. W in the model would therefore be 16 and the domestic component batch sizes and hence costs would be based on Economic Baṭch Quantities.

## 7. 5 Methods of Solution

The allocation model formulated in Section 7.4 may be solved by -
(i) The optimisation technique of linear programming using the Revised Simplex Algorithm with upper bounded variables. The solution will give rational values for the $X^{\prime} s, Y^{\prime} s$, and $Z^{\prime}$ 's with some $X^{\prime} s$, $Y^{\prime} s$, and $Z^{\prime}$ 's equal to $O$, other $X^{\prime} s, Y^{\prime} s$ and $Z^{\prime} s$ equal to 1 , and the
remainder of the $X^{\prime} s, Y^{\prime} s$ and $Z^{\prime} s$ equal to son fractional value in the range $O$ to 1 . Fractional values of the $X^{\prime} s, Y^{\prime}$ 's and $Z$ 's represent split batches between locations. Where a batch is split between the X's and Y's only, this means in practice a continuation of the batch manufacture from the factory day to night shift or vice versa. Other combinations of fractional values for the $X^{\prime} s, Y^{\prime} s$, and $Z^{\prime}$ 's represent split batches either between the factory and subcontractors or between subcontractors. Since there is only one tool available per job, a job must be completely made at one location. The problem of allocating split batches is discussed in Section 7.7.
(ii) Restrict the $X^{\prime} s, Y^{\prime} s$ and $Z s$ to 1 or $O$, and use integer programming. The solution by integer programming avoids the problem of split batches. Since the allocation model contains all zero-one variables it may be solved by the Implicit Enumeration Technique based on the original work of Le Garff and Malgrange (12), Balas (13) and others. Alternatively it may be solved by the Branch and Bound Method based on the original work of Land and Doig (14).

A survey on the many approaches to solving integer programming problems have been made for example by Beale (15). Beale says about the choice of Integer Programming Methods that ' none can be relied on to give the optimum solution to an arbitrary integer programming problem in a reasonable length of time - even if we restrict the number of integer variables to say 50. Some people will conclude from this that integer programming is an unreliable technique, but it seems fairer
to conclude merely that it has to be used intelligently. The branch and bound approach has an important property, that is shared to some extent by the implicit enumeration approach, that it will usually produce a good solution - which may even turn out to be the optimal solution - even if one does not feel justified in spending the time to explore all alternatives thoroughly enough to reach a guaranteed optimal solution.'

The computer software package (16) used by the author to obtain integer solutions to the allocation model was based on the branch and bound method of Dakin (17) and incorporating four variable selection strategies designed to reduce the tree search time. Which one of the four variable selection strategies to employ was left to the user. The best one to use for a particular integer problem would be a matter of experimentation.

The package first solves the integer problem as a linear programming problem (using the Revised Simplex Algorithm) i. e. it ignores the integer restrictions on variables during this stage. If it finds a continuous optimal solution, it then goes on to solve the integer problem using the branch and bound technique together with the variable selection strategy assigned by the user. The value of this objective function found for the continuous (L. P.) solution is used by the package as the aspiration (target) level for integer solution. As integer solutions are found by the branch and bound algorithm, the solutions are printed out. The package (i. e. program) may be terminated after
the first or any subsequent integer solution found. If not otherwise declared the package will proceed to complete the search of the tree or terminate prior to the completion of the tree search if the aspiration level has been achieved. The optimum integer solution is then printed out. Table 7. 5A gives the computer times taken by the package in obtaining the optimal L. P. solution and some of the I. P. solutions found before reaching the optimal I. P., for the allocation model using the test data given in Appendix C. The objective function in the model was minimised.

| Solution | Value in £s <br> of objective <br> function | \% above optimal <br> L. P. objective <br> function value | Solution found <br> after a computer <br> run time in <br> minutes of | Objective <br> cut-off <br> value <br> £s |
| :--- | :--- | :--- | :--- | :--- |
| Optimal L. P. | $4,352.78$ | - | 1 | - |
| 1st I. P. | 4,600 | 5.68 | 22 | $+\infty$ |
| 2nd I. P. | 4,590 | 5.45 | 31 | 4,600 |
| 3rd I. P. | 4,530 | 4.07 | 37 | 4,590 |
| Optimal I. P. | 4,430 | 1.77 | 101 | 4,460 |

## TABLE 7.5A

The objective cut off value, referred to in Table 7. 5A, is used in the branch and bound algorithm to reduce time in searching the tree for integer solutions. Any integer solution above the current objective cut-off value being ignored.

The package uses the objective function value of the previous I. P. solution as the objective cut-off value in searching for the next I. P. solution. Objective cut-off values may also be assigned by the user, normally after the first integer solution has been found by the package using the $+\infty$ cut-off value, in an attempt to accelerate the package search for a better or an optimal solution. Unfortunately there is no way at present, to pre-determine whether an integer solution exists at or below the cut-off value chosen. Consequently it is a matter of trial and error, and may prove just as long as with the normal cut-off method used by the package.

If the objective cut-off value chosen happens to be near the optimal solution considerable savings in the time of solution may result. This is illustrated for the test data (Appendix C) using an objective cut-off value of $£ 4,450$, and two different variable selection strategies. One solution of $£ 4,440$ was obtained after only four minutes, and the other (using a different variable selection strategy) of $£ 4,430$, an optimal solution, after twentysix minutes. Work is proceeding on testing other integer programming methods with the model in an attempt to reduce the solution time.

The cost justification of finding a cheaper I. P. solution relative to the lst I. P. solution will depend on the difference between the optimal L. P. and the lst I. P. If the difference is small relative to the objective value of the optimal L. P. the computer cost for a further search may not be justified. But if the difference is large, then a further search for a cheaper I. P. solution may be justified.

The cost of an I. P. solution to the allocation model would be generally much larger than the L. P. solution. In the case of the test data, for example, the time of solution by I. P. was 22 times greater than the L. P. solution in finding the lst I. P. solution. The ratio of the times of solution by L. P. and I. P. would also approximately represent the ratio of the costs for the two methods of solution.

In practice the L. P. solution may be adequate where a small number of split batches occur, since the manual allocation of a small number of split batches should be relatively straightforward. This is discussed in Section 7. 7. With a large number of split batches in the L. P. solution, a manual allocation would certainly be difficult, also possibly lengthy and costly. As a general rule I. P. should be used where a large number of split batches occur in the L. P. solution.

## 7. 6 Results with the Allocation Model using Test Data.

The results obtained with different sets of test data have shown that the model fulfils all the conditions set out in Section 7. 3. The results using the model for one set of test data and both the linear programming and integer programming solutions are summarised in Tables 7. 61 and 7. 62. The test data was designed to represent 20 different jobs with a total load of 102 standard hours compared with a total number of standard hours available in the factory (day and night shift) of 60 , on one press group for one week of the rota cycle. Four subcontractors $A, B, C$, and $D$ were used. Details of the standard hours required for the manufacture of each job, the cost of each job at each location are given in Appendix C.

Table 7.6A shows the maximum standard hours (upper bounds)
available at each location, the minimum standard hours (lower bounds) to be allocated to each location together with the total standard hours allocated by the model using linear programming (L. P.) and integer programming (I. P.)

|  | Factory |  |  | Subcontractors |  |  |  |
| :--- | :---: | :---: | ---: | ---: | ---: | ---: | :---: |
|  | DAY <br> SHIFT | NIGHT <br> SHIFT | A | B | C | D |  |
|  | 40 | 20 | 20 | 20 | 20 | 20 |  |
| Lower Bound | 38 | 18 | 6 | 12 | 6 | 6 |  |
| L. P. Allocation | 39 | 18 | 6 | 12 | 7 | 20 |  |
| I. P. Allocation | 39 | 19 | 6 | 14 | 7 | 17 |  |

## TABLE 7.6A

Of the subcontractors, subcontractor $D$ had the lowest costs per job, and referring to Table 7.6 A , the model allocated the largest amount of standard hours to subcontractor D in both the L. P. and I. P. solutions.

Subcontractor B with the highest minimum standard hours to be allocated also had the highest costs. The allocation of standard hours made by the model to subcontractor $B$ with the L. P. was equal to the minimum standard hours (lower bound) and slightly above the lower bound using the I. P.

Table 7. 6B shows the number of jobs allocated by the model to each location for both the L. P. and optimal I. P. solutions. The fractional values for the split batches between locations arising in the L. P. solution are also given.

|  |  | FACTO |  | SUBC | TRAC | RS |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { DAY } \\ & \text { SHIFT } \end{aligned}$ | NIGHT <br> SHIFT | A | B | C | D |
|  | Complete Batches | 7 | 1 | 1 | 1 | 2 | 5 |
| L. P. | Split Batch <br> P/N 12 <br> P/N 16 $\text { P/N } 18$ | $\begin{array}{r} .5000 \\ .1111 \end{array}$ | $.6250$ $\text { . } 8889$ |  | . 3750 |  | . 5000 |
| I. P. |  | 8 | 3 | 1 | 2 | 2 | 4 |

TABLE 7.6B

The total cost of the allocation made by the model using the L. P. was $£ 4,352.78$ and for the optimal I. P. solution $£ 4,430$. Detailed results of the allocation of part numbers for the test data for both the L. P. and I. P. solutions are given in Appendix C.

## 7. 7 Allocation of Split Batches

From the allocation made by the model when solving by L. P. any batches that are split between the factory and the subcontractor or between subcontractorsmust be allocated to one location for manufacture. Split batches between the factory day and night shift are acceptable since the work is a continuation from one shift to another at the same location.

Referring to Table 7.6B a manual decision may be to allocate
(i) Part Number 16 (processing time for batch $=2$ standard hours) to the factory day shift, which would bring the number of standard hours allocated to the factory day shift up to the maximum capacity available ( 40 standard hours).
(ii) Part Number 12 (processing time for batch $=8$ standard hours) to the factory night shift. However this would mean exceeding the maximum standard hours available on the night shift and underloading subcontractor $B$ relative to the minimum load allocated to $B$ in the model. In practice 'minimum' loads selected by management to be made at particular subcontractors, should be flexible enough to vary within acceptable limits when allocating split batches.

Overloading the domestic manufacturing capacities in allocating split batches will require planned overtime to accommodate the excess loads. Where a large number of split batches are involved integer programming should be used.
7. 8 Data Required for the Implementation of the Allocation Model in the Large Press Section.
(i) Domestic manufacturing costs per component batch.
(ii) Subcontractors costs per component batch.
(iii) Transportation costs per component batch between the company and each subcontractor.
(iv) Available capacity at each subcontractor in standard hours classified in terms of the domestic press groups.
(v) Available domestic capacity in each press group in standard hours.
(vi) Domestic manufacturing times per component batch in standard hours.
(vii) Subcontractors time per component batch in standard hours.

## 7. 9 Availability of Data.

7. 8(i) and (iv) are available.
8. 8(v) has been established (Chapter 5)
9. 8(ii) are only available for $20 \%$ of the components.

The collection of data for 7.8 (ii), (iii), (iv) and (vii) was authorised by management.

To obtain 7. 8 (iv), letters were sent to all subcontractors requesting information on their number of presses, together with press bed size and tonnage- from the data received back, and with the aid of the company, reasonable estimates of available capacities in terms of the domestic press groups have been established. Subcontractors manufacturing costs and manufacturing times per component are currently being collected by the company.

The company's Cost Department are currently establishing the transportation costs (incurred by the company) per component batch between the subcontractor and the company.

## 7. 10 Comments on the use of the Allocation Model

The week ly factory capacity constraints for a press group across the cycle, used for smoothing the load, may be replaced by one capacity constraint for the complete cycle, and smoothing of the domestic press group load carried out manually. Although with small amounts of work to subcontract, smoothing of the domestic press group loads may be unnecessary. The allocation model has been used on the Small Press Section with one subcontractor and one factory capacity constraint.

The consideration of the decision rules for subcontracting concludes the first phase of the project.

## 7. 11 Introduction to Phase II of the Project

A preliminary study of Economic Batch Quantities for components made on the Large Press Section showed that these were generally larger than the existing batch quantities manufactured in the five week rota cycle. Manufacturing Economic Batch Quantities would therefore involve the introduction of longer manufacturing rota cycles, which in turn, would increase the number of components to be stored. The current storage situation was already acute and any increase in storage requirements could not apparently be accommodated in the system.

However, management had suspected for some time that the current storage level was not in keeping with the amount of work being manufactured, but since there was no method available in the system to determine the storage level they accepted the current storage position. Although management were very enthusiastic about introducing rota cycles based on Economic Batch Quantities, they could not commit themselves without first know/the storage levels to be expected. A study of storage levels required in rota cycle manufacture was therefore undertaken and is described in Chapter 8. The study led to the storage level being reduced by approximately two thirds. It was also established that the storage space available was more than adequate to accommodate the storage requirements of the multi-rota manufacturing cycles described in Chapter 9.

# CHAPTER 8 - ANALYSIS OF THE STORAGE REQUIREMENTS IN ROTA CYCLE MANUFACTURING 

## 8. 1 Introduction

The Pallet Stores was used to store the components manufactured by the Press Shop until they were required by the Finishing Shop. The number of pallets containing components in the system was approximately 3,500 compared with the Pallet Stores capacity of 2,950 . Pallets that could not be accommodated in the Pallet Stores were stored on the Press Shop floor and on outside areas of the factory site. This resulted in difficulties in keeping a check on stock and in locating stock required by the Finishing Shop. The storage of excessive numbers of pallets on the Press Shop floor disrupted the flow of work through the Press Shop. Storage of pallets containing components on the outside areas of the factory site produced problems in protecting the components from the effects of the weather.

## 8. 2 Investigation into the Number of Pallets Stored

An examination of the pallets stored showed that approximately $13 \%$ contained 'dead' stock of obsolete components, and a high percentage of the remaining pallets were only partly filled with components. Partly filled pallets could occur when -
(1) There was an insufficient quantity of a component at the end of a press run to fill a pallet.
(2) The quantity of a component in a pallet sent to the Finishing Shop was greater than required. The partly filled pallet being returned to the Pallet Stores.
(3) Not enough attention was paid in the Press Shop to completely filling pallets during press runs. This caused more pallets to be used than was necessary.

Approximately $80 \%$ of the pallets stored were taken up by components from the large press section and the remaining $20 \%$ with components from the small press section. The quantity of a component that could be stored per pallet from the large press section ranged from 60 for the largest to 700 for the smallest component, and for the small press section from 1, 000 for the largest to 3,000 plus for the smallest. The quantity of a component in a partly filled pallet for the larger components on the large press section could be reasonably estimated, but for the majority of components, it was much more difficult. When the Finishing Shop required a quantity of a component that was less than normally accepted to be contained in a full pallet, the tendency was to supply a full pallet from the Pallet Stores in preference to a partly filled pallet, when it was intuitively felt, that the
quantity it contained might not meet the demand. This frequently resulted in the storage of two partly filled pallets, where only one pallet may have been necessary, if the partly filled pallet together with a full pallet, or another partly filled pallet, had been supplied. An accumulation of partly filled pallets thus occurred.

Tie on card labels, attached to the pallets, gave the component description, drawing number and quantity.

The frequent moving of pallets in the Pallet Stores in order to gain access to components that were required for the Finishing Shop resulted in the loss of many labels. This gave rise to difficulties in identifying some components, and efforts to were identify the components/sometimes delayed for periods of time longer than the normal depletion period. Valuable storage space was used up in this way, plus the fact that some of the components were reported as shortages, and caused unnecessary extra loads to be placed on the already over-loaded Press Shop.

From the investigation it was apparent that a significant reduction in the number of pallets currently being stored was possible. Scrapping of the obsolete components would almost bring the number of pallets to be stored to within the capacity of the Pallet Stores, and thus help to relieve the congestion in the Press Shop
caused by the excessive number of pallets stored on the Press Shop floor. Minimising the number of partly filled pallets would effect a significant reduction in the storage capacity required.

Management required guide lines on the weekly storage level, in terms of number of pallets to be expected throughout the current five week rota cycle. They also required to know the weekly storage level necessary if rota cycles based on economic batch quantities were introduced in place of the existing batch quantities manufactured in the current five week rota cycle. The economic batch quantities were found in a preliminary survey to be generally larger than the existing batch quantities for the majority of the components manufactured. Economic batch quantities would therefore lead to cycle lengths greater than the current five week cycle and to correspondingly longer periods of depletion, which would result in larger weekly storage requirements.

To determine the weekly storage requirements throughout a rota cycle and study the effect on the weekly storage level when the period of depletion and lead time were varied, a theoretical study was carried out. The theory and equations that were developed are given in Appendix D.

## 8. 3. Comparison of the Theoretical Number of Pallets to be stored and the Number Achieved in Practice.

The input of pallets to the Pallet Stores from the large press section in each week of the five week cycle was calculated for each component batch size and the quantity of a component that filled a pallet. The week of manufacture of a component was obtained from the weekly allocation made in the Balance Load System. The weekly input of pallets to the Pallet Stores from the large press section was found to be fairly constant across the five week cycle and approximately equal to 300 . Using equation 4, Appendix D, for a constant weekly input across the cycle with $T=5, T^{\prime}=5$, and $L=O$ gave 900 pallets to be stored per week.

For the small press section the input to the Pallet Stores was 60 pallets per week. On this section the cycle length and period of depletion varied. An average depletion period of five weeks was taken and from equation 4 with $T=5$ and $L=O$ gave 180 pallets to be stored per week.

The total theoretical number of pallets to be stored for both the large and small press sections should be 1,080 pallets compared with the current number of approximately 3,100 . The figure of 3, 100 was obtained from the total stored of 3,500 minus the 400 pallets holding obsolete components. A re-organisation of the

Pallet Stores together with methods to reduce the number of partly filled pallets, improvements in the labelling of pallets were undertaken by the Production Control Department. The nett result was that the number of pallets stored throughout a cycle was reduced from 3,100 to $1,200-1,300$ compared to the theoretical target of 1,080 pallets.

Allowing for practical considerations such as variations in the quantity of a component packed into a pallet, and the occurrence of partly filled pallets and assumptions in theory, of constant input and output of pallets from the system, the difference between the theoretical number of pallets to be stored and the number achieved was thought reasonable. Work is proceeding to determine guide lines on the number of partly filled pallets that may be expected in the system, and the variation to be expected in the packing density of components into a pallet.

The economic batch quantities for a large percentage of the components manufactured on the large press section for the current load would give rise to rota cycles of 8 to 10 weeks. Taking a 10 week rota cycle with the period of depletion $T^{\prime}=10$ weeks and the lead time $L=O$ weeks, from equation 4 Appendix $D$, for a constant weekly input of 300 pallets would result in 1,650 pallets tobe stored per week. Allowing 200 pallets per week for the small press section, the total to be stored of 1,850 pallets would be well within the capacity of 2,950 pallets of the Pallet Stores.

## 8. 4. Effect on the weekly storage requirements of varying the period of depletion and/or the lead time for constant weekly input of pallets.

The percentage change in the total number of pallets stored, when the period of depletion $T^{\prime}$ weeks and the lead time $L$ weeks are varied, as compared with the number to be stored, when the lead time is zero, and the period of depletion $T^{\prime}$ equals the rota cycle $T$ is obtained from equations 1 and 4 in Appendix D.
$\begin{aligned} \text { Percentage change in number } & =\frac{\mathrm{y} / 2^{\left(\mathrm{T}^{\prime}+2 \mathrm{~L}+1\right)-\mathrm{y} / 2(\mathrm{~T}+1)}}{\mathrm{y} / 2_{2}(\mathrm{~T}+1)} \times 100 \% \\ & \left.=\frac{\left(\mathrm{T}^{\prime}-\mathrm{T}+2 \mathrm{~L}\right)}{\mathrm{T}+1}\right) \times 100 \%\end{aligned}$

For the current five week rota cycle numerical values for the percentage change in the number of pallets to be stored was obtained from equation 8.41 by setting $L=-1$ and varying $T^{\prime}$ from 5 weeks in increments of 1 week up to 11 weeks and repeating for $L=O, 1$ and 2 weeks. The results are shown in Graph 1. Appendix D.

Graph 1. may be thought of as a model of the Pallet Stores, with constant input for a five week rota cycle, and makes the effect of changing the period of depletion and/or the lead time easily observed. For example if the constant input is 300 pallets per week from equation 1 with $T=5$ the number of pallets stored in any week of
the cycle is 900 . The effect of increasing the period of depletion by one week to six weeks and the lead time from $O$ to one week, will be to increase the number of pallets stored per week by $50 \%$ i. e. raise the number stored from 900 to 1, 350 pallets. Again if the lead time is changed to -1 , and the period of depletion remains at 5 weeks, the number of pallets to be stored is reduced by $33 \frac{1}{3} \%$ i. e. reduced from 900 to 600 pallets to be stored.

## 8. 5. Variable Weekly Input of Pallets

The weekly number of pallets to be stored in each week of a rota cycle with a variable weekly input of pallets is given by equation 3 in Appendix D.

A computer program, listed in Appendix E, was written to calculate the weekly storage level for both the variable and constant weekly input of pallets given byequation 3 and 4 in Appendix D respectively, for any rota cycle length $T$ weeks, depletion period $T$ ' weeks and lead time L weeks specified. The constant weekly input value, calculated in the program, represents the mean of the variable weekly inputs across the cycle.

The output from the program gives the weekly storage level, in terms of number of pallets, in each week of the rota cycle for a variable weekly input of pallets, and the deviation of these weekly storage levels from the storage level for the constant weekly input.

Figure 8.51 gives an example of the results from the program with a rota cycle of $T=5$ weeks, period of depletion $\mathrm{T}^{\prime}=5$, lead time $L=-1$; and inputs in weeks $1,2,3,4$ and 5 of the rota cycles of $400,400,300,200$, and 200 pallets respectively with a mean of 300 pallets per week across the cycle. The deviations of these weekly storage levels from the constant storage level of 600 pallets for the constant weekly input of 300 pallets, showed maximum deviations of 80 pallets above and 120 below. Changing the order of the above weekly input values,

| INPUT |  |  | OUTPUT |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Week of Rota Cycle | Variable <br> Pallet <br> Input | Constant Input | Number of Pallets Stored |  | Deviation |
|  |  |  | Variable Input | Constant Input |  |
| 1 | 400 | 个 | 580 | $\uparrow$ | - 20 |
| 2 | 400 | Mean $=$ | 680 |  | 80 |
| 3 | 300 | 300 | 680 | 600 | 80 |
| 4 | 200 |  | 580 |  | - 20 |
| 5 | 200 | $\downarrow$ | 480 |  | - 120 |

FIGURE 8.51
produce different maximum deviations above and below the constant input case. For example maximum deviations of 40 above and 60 below occur with the order of input 400, 300, 200, 400 and 200 in weeks $1,2,3,4$, and 5 of the rota cycles respectively.

In practice, with variable weekly input values across the rota 1
cycles the value of the maximum positive deviation above the constant storage level (calculated from the mean of the variable weekly input values across the cycle) may be of particular interest when, for example, the level of storage in the Pallet Stores is critical.

The value of the maximum positive deviation depends upon
(i) the order in which the variable weekly input values occur in the cycles.
(ii) the extent to which the variable weekly input values can be 'smoothed' across the cycle.

To illustrate the variation in the maximum positive deviation when the order of the weekly input values are varied, the 30 re-arrangements (5./2'.2!) of the weekly input values $400,400,300,200$ and 200 (all with a mean of 300 , and weekly input values in the range $300 \pm$ 100) for a 5 week rota were used. The results, obtained from the computer program are given in Appendix E (Table 1), for a period of depletion $\mathrm{T}^{\prime}=5$ weeks, lead time $\mathrm{L}=\mathrm{O}$ weeks and rota cycle length $\mathrm{T}=5$ weeks.

With reference to Table 1 Appendix E, consecutive weekly input values in ascending order of magnitude across the cycles gave rise to the highest maximum positive deviation i.e. the sequence 200,200 , 300, 400 and 400 occurring in Sample numbers 1 to 5 , across the rota cycles produced the highest maximum positive deviation of 120 above the constant storage level. Samples with alternate low and high input values (Sample number 21 to 25 ) and samples with the
high input values separated by one or more low input values (Sample numbers 26 to 30 ) gave lower maximum positive deviations than any other possible sequences of the input values.

The effect on the maximum positive deviation of varying the range of the input values about the mean across the cycle is illustrated in

Table 2 Appendix E. With reference to Table 2 Appendix E, the mean of each sample is 300 and the deviation about the mean of the input values range from $300 \pm 50$ (Sample numbers 1 and 2) in steps of 50 to $300 \pm 300$. Sample number 1 represents the 'worst' sequence (i.e. values in ascending order of magnitude across the cycle) giving the highest maximum positive deviation, and Sample number 2 the 'best' sequence (i.e. high input values separated by one or more low input values) giving the lowest maximum positive deviation, for input values in the range $300 \pm 50$. Similarly, for sample numbers 3 and 4 with input values in the range $300 \pm 100$, Sample 3 represents the 'worst' sequence and Sample 4 the 'best' sequence, and so on for the remaining pairs of samples with the range about the mean increasing in steps of 50 .

Large deviations in the input values about the mean can give relatively small maximum positive deviations above the constant storage level if the 'best' sequence of input values can be used. For example Sample number 6 with the range of input values $300 \pm 150$ and 'best' sequence gives a maximum positive devaition of 60 , which is comparable to Sample number 1 with the 'worst sequence but deviations about the mean of $300 \pm 50$.

In practice therefore, if the input values correspond for example to Sample number 5, lower maximum positive deviations may be obtained by either re-arranging the weekly input order to that of Sample number 6 i.e. the 'best' sequence for these input values or try to smooth the weekly input values to patterns similar to say Sample numbers 2 or 4, or ideally try to produce constant weekly input values across the cycle. Keeping balanced loads in press groups across the cycle will impose restrictions on what items that can be interchanged between manufacturing weeks of the cycle, to help to smooth and/or interchange variable weekly input values to achieve low maximum positive deviations above the constant storage level.

The effect of varying the period of depletion $T$ ' weeks and/or the lead time L weeks on the maximum positive deviation may be obtained for any sequence of input values occuring in practice from the computer program (Appendix E). An example of the results when $T$ ' was varied over the range 5(1)9weeks with $L=O$ weeks for the sample 200, 200, 300, 400 and 400 are given in Table 3 Appendix E. The maximum positive deviation varied for the sample between 100 and 137 when $T$ ' and $L$ were varied over the ranges $5(1) 9$ and $-1(1) 2$ respectively. The maximum positive deviation for the 'best' sequence of the sample $(300,200,400,200,400)$ varied between 33 and 50 over the same range of $T^{\prime}$ and $L$.

## CHAPTER 9 - ECONOMIC MANUFACTURING QUANTITIES AND MIULTI-ROTA CYCLING

## 9. 1 Introduction

The economic manufacturing quantities calculated for the individual components manufactured in the large press section gave rise to manufacturing cycle lengths in the range 4 to 40 weeks. To produce a production schedule to include all the different length cycles was impractical. The problem was to determine batch sizes, and hence cycle lengths, which, when woven together, gave a schedule with feasible smooth machine loads consistent with the level of demand and meet a primary objective of minimising costs. When the schedule was completed, it would be subsequently repeated.

This is called rota cycling and is a periodic system of productioninventory control.

A review on the published work on general rota cycling has been recently given by Shanks (1).

Several authors have suggested ways of approaching the problem but unfortunately the methods are only applicable to small scale problems e. g. ten parts on one machine centre. The most promising has been a dynamic programming formulation by Bomberger (3). Bomberger's initial paper has prompted several other papers (4), (5) and (6) suggesting improvements to the method. For large scale problems the most promising has been the method suggested by Shanks (1) of selecting manufacturing cycles in the form of a geometric series.

## 9. 2

 Bomberger's ProblemThe solution of Bomberger's problem, has appeared to have become the 'standard' by which other authors have measured the cost effectiveness of their own methods compared to Bombergers. The details of Bomberger's problem are as shown in Table 9. 2A.

| Part <br> No. | Set-Up Cost <br> $S_{i}(\$)$ | Piece Cost | Production <br> Rate <br> Pi (unit/day) | Demand | Set-Up <br> Time |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | 15 | .0065 | 30,000 | 400 | 1 |
| 2 | 20 | .1175 | 8,000 | 400 | 1 |
| 3 | 30 | .1275 | 9,500 | 800 | 2 |
| 4 | 10 | .1000 | 7,500 | 1600 | 1 |
| 5 | 110 | 2.7850 | 2,000 | 80 | 4 |
| 6 | 50 | .2675 | 6,000 | 80 | 2 |
| 7 | 310 | 1.5000 | 2,400 | 24 | 8 |
| 8 | 130 | 5.9000 | 1,300 | 340 | 4 |
| 9 | 200 | .9000 | 2,000 | 340 | 4 |
| 10 | 5 | .0400 | 15,000 | 400 | 6 |

Note: Bomberger uses a holding cost of $0.1 \mathrm{dol} / \mathrm{dol} \mathrm{yr}$. and 240 working days in a year at 8 hours per day.

## TABLE 9.2A

Shanks has given in his work the costs, in terms of the total variable cost per day, of other authors solutions to Bomberger's problem. These results are quoted in Table 9. 22 together with the results achieved by Shanks using his method and those obtained by the author using a slightly modified Magee's Method (7), Section 9. 3, for determining cycle lengths for multi-products.

| METHOD | Cost of Schedule <br> Total Variable cost in dollars <br> per day |
| :--- | :--- |
| Bomberger (1) | 36.65 |
| Stankard (6) | 36.24 |
| Madigan (5) | 33.94 |
| Hodgson (4) | 33.86 |
| Shanks (1) | 32.08 |
| Based on | 39.85 |
| Magee (7) | 31.62 |

TABLE 9.22

### 9.3 Magee's Method

Magee's method (formulae used are given in Appendix F) for determining a multi-product cycle is -
(i) Calculate the number of runs per year $N_{i}$ for each individual product $\mathrm{i} . \mathrm{i}=1,2,3 \ldots---\mathrm{m}$ where $\mathrm{m}=$ number of products.
(ii) Calculate the number of runs per year $N$ for the multi-product.
(iii) Compare each $N_{i}$ with $N / 2$ and extract the products that do not satisfy the condition $N_{i}<N / 2$.
(iv) Calculate a new N for the remaining products and again extract (if any) products that do not satisfy the condition $\mathrm{N}_{\mathrm{i}}<$ new $\mathrm{N} / 2$.
(v) Repeat (iv) until the condition $N_{i}<($ a new $N) / 2$ is satisfied and at this stage the newest calculated N is the multi-product cycle.
(vi) Treat all extracted items as single products.

Instead of step (vi) apply steps (ii) through to step (v) to the extracted products to obtain a second multi-product cycle which in turn may have products to be extracted. The extracted products from the second multi-product cycle may form a third multi-product cycle and so on.

Using the modified step (vi) to Magee's method ave the results quoted in Table 9. 31 for Bomberger's problem together with the total variable cost of the schedule per day.

| Part Number | Economic Manufacturing Cycle | Allocated to Cycles in Geometric Progression $2,4,8,16,32$ | Multi-Product Cycles 5. $44,13.04,34.02$ | Multi-Product Cycles adjusted to 6,12,36 | Multi-Product Cycles adjusted to $5,15,30$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 33. 4 | 32 | 34. 04 | 36 | 30 |
| 2 | 9. 2 | 8. | 5. 44 | 6 | 5 |
| 3 | 7. 9 | 8 | 5. 44 | 6 | 5 |
| 4 | 4 | 4 | 5. 44 | 6 | 5 |
| 5 | 10 | 8 | 5. 44 | 6 | 5 |
| 6 | 21. 4 | 16 | 13.04 | 12 | 15 |
| 7 | 40. 8 | 32 | 34.04 | 36 | 30 |
| 8 | 4. 2 | 4 | 5. 44 | 6 | 5 |
| 9 | 12. 2 | 8 | 13. 04 | 12 | 15 |
| 10 | 7. 8 | 8 | 5. 44 | 6 | 5 |
| Total |  |  |  |  |  |
| Day. | 31.62 | 32. 08 | 39. 45 | 43. 30 | 39. 85 |

[^1]For feasible multi-rota cycles production schedules the largest multi-product cycle was adjusted together with the smaller multiproduct cycles, so that the smaller multi-product cycles were multiples of the largest. Two multi-rota cycles were formed and these are given together with the total variable costs per day in Table 9. 31.

The modified Magee method was found to be feasible when applied to several other problems with the number of components greater than that in Bomberger's problem. But the cost of the schedules were greater than those using Shank's method.

Shank's method was therefore considered in detail for application to the large number of components on the large press section, both from the point of minimising costs and the fact that the load in fixed cycle lengths in the form of a geometric progression, may be conveniently arranged in complimentary load patterns between the fixed cycles for balancing (Section 9. 7).

Shank's method is given in Section 9. 4. 'Alternative methods to Shank's of allocating components and balance loading the fixed cycles are given in Sections 9.5 and 9.7 respectively.
9. 4 Shanks Method - The method has three stages :

Stage 1 (i) Pre-select set of batch manufacturing cycles. He presents evidence in his work to show that costs can be controlled if manufacturing cycles are selected in the form of a geometric progression e. g. $2,4,8,16 \ldots \ldots$ week cycle lengths.

## Stage 1.

This is achieved as follows -
(a) Calculate the dividing point between cycles by finding the geometric mean
Dividing point $C=\sqrt{C_{i} C_{i}+1}$
for $\mathrm{i}=1,2,3 \ldots \ldots \ldots . \mathrm{N}$ where N is the maximum allowable cycle.
(b) Let $C E B Q j=E B Q$ manufacturing cycle for the jth part. Now for $\mathrm{j}=1 . \ldots . \mathrm{M}^{2}$, where there are M parts, calculate CEBQj

If $C_{i} \leqslant C E B Q j<C \quad$ then $C P j=C_{i}$ If $\mathrm{C} \leqslant \mathrm{CEBQj}<\mathrm{C}_{\mathrm{i}}+1$ then $\mathrm{CPj}=\mathrm{C}_{\mathrm{i}}+1$ for $\mathrm{i}=1 \ldots \ldots . . \mathrm{N}$. $C P j=$ allocated manufacturing cycle to part $j$.
(c) Calculate time each machine is required for each complete rota cycle, using processing and setting time. If time required is greater than available, a decision has to be made whether to adapt batch sizes to make feasible loading, or to delete some parts from the manufacturing lists, or to increase machine capacity. The choice will depend on circumstances. Eilon (8) discusses the above situation in some detail.

Shank states that there is more than one heuristic approach and each scheduling problem requires a variation of the following theme -
(a) Divide the longest cycle length derived in Stage 2 into periodic time units which recognise both cycle lengths and typical loads imposed by work batches.
(b) Select minimum cycle length, maximum production load part and schedule into machines obeying the type of rules shown below -
(i) Maintain smooth periodic machine loads and/or
(ii) not exceeding periodic capacity
(c) Repeat (b) until all parts scheduled.
9. 5 Allocation of the Components to the fixed manufacturing cycles using the least unit cost method.

The economic batch quantity is based on the 'least unit cost method' (9) and (10). This simply goes along the schedule batching in the next requirement until the unit cost starts to increase -

Schedule


If the set-up cost of a batch is $£ S$ and the cost of carrying the inventory of one component for one period is $£ C$, initially the variable cost per part of just making $q_{1}$ is
$C_{1}=s / q_{1}=$ Set-Up Cost/Quantity to make
batching in $\mathrm{q}_{2}$ we get - le adding $q_{2}$ to $q$,
$\mathrm{C}_{2}=$ SET-UP COST + (QUANTITY BATCHED IN) (Number of Periods Held) $\times \mathrm{C}$ TOTAL BATCH QUANTITY

$$
=\frac{s+\left(q_{2}\right)(1)(C)}{q_{1}+q_{2}}
$$

and so on.
In general

$$
i=N
$$

$$
C_{N}=\frac{S+\sum_{i=1} q_{i}^{i}(i-1) C}{\sum_{i=1}^{i=N} q_{i}}
$$

and when $\mathrm{C}_{\mathrm{N}+1}>\mathrm{C}_{\mathrm{N}} \quad$ the least unit cost is $\mathrm{C}_{\mathrm{N}}$.

For constant rates of demand i. e. $q_{1}=q_{2}=\ldots-q_{N}$ equation 9.51 becomes

$$
C_{N}=S+N(N-1) \quad q C / 2
$$

Nq

The results of using equation 9.52 for constant rates of demand, with Part 9 of Bomberger's problem are given in Table 9. 5A.

| Batch Quantity | Cycle Length in weeks. | Variable Cost per unit (\$) |
| :---: | :---: | :---: |
| 1,700 | 1 | . 1176 |
| 3,400 | 2 | . 0597 |
| 5,100 | 3 | . 0494 |
| 6,800 | 4 | . 0320 |
| 8,500 | 5 | . 0270 |
| 10,200 | 6 | . 0239 |
| 11,900 | 7 | . 0220 |
| 13, 600 | 8 | . 0207 |
| 15,300 | 9 | . 0200 |
| 17,000 | 10 | . 0195 |
| 18,700 | 11 | . 0193 |
| 20,400 | 12 | . 0193 |
| 22,100 | 13 | . 0194 |
| 23,800 | 14 | . 0196 |
| 25,500 | 15 | . 0200 |
| 27,200 | 16 | . 0203 |

TABLE 9.5A

Referring to Table 9. 5A the least unit cost for Part 9 is for an 11 or 12 week cycle. Shanks obtained an 11.2 week cycle with the standard square root E. B. Q. formula and 12.2 week cycle allowing for production rates in the E. B. Q. formula. (The geometric mean between the 8 and 16 week cycle is 11.31 weeks). He produced a solution to Bomberger's problem for both the standard formula and the modified standard formula taking into account production rates. In both cases he allocated Part 9 to the eight week cycle, which is not strictly in accordance with the condition for allocation given in Stage 2 of his method for the 12.2 week cycle calculated using production rates in the standard formula. Referring to Table 9.5 A the cost of this decision is readily seen to be an increase in unit cost of 0.004 dollars relative to the sixteen week allocation. This amounts to 32 dollars per year increase in the total variable cost relative to the sixteen week cycle allocation, but the average stock level of the part is halved over the year.

The Least Unit Cost Method offers more flexibility than Shank's method in the allocation of a part to fixed cycles; particularly in marginal cases as with Part 9 where average stock level is also considered, as the cost of the decision is readily available. When the Least Unit Cost Method was applied to the other parts in Bombergen's problem, it produced the same allocation as with Shank's method.

## 9. 6. Least Unit Cost Look-Up Tables

Least Unit Cost Tables, using equation 9.52 for constant weekly rates of demand, were produced to allocate the components on the large press section to the fixed cycle lengths. The weekly rate of demand although constant over periods of approximately sixteen weeks, could change from one sixteen week period to the next. The tables were therefore extended to cover the range of weekly demand rates expected. A computer program was written to produce the tables. The computer time to calculate and print the complete tables to cover all the components made on the large press section was about 3 minutes. A part of the tables is given in Appendix G, for a component costing 20 p with a set-up cost of $£ 60$ and an annual holding rate of $18 \%$. Note: With variable weekly demand rates for items, equation 9.51 should be used.

## 9. 61 Sensitivity of the allocation of a component to the fixed cycles to changes in the weekly demand rate.

The sensitivity of the allocation of a component to a fixed cycle for a range of weekly demand rates may be observed directly from Least Unit Cost Tables. For example, referring to the table in Appendix G, the component would be allocated to an eight week cycle for weekly demand rates in the range 500 to 1350 , and to a sixteen week cycle in the range 1350 to 2,500 .

A graphical representation of these results is given in Graph 1 Appendix G. where the weekly demand rates were plotted against -
(i) The least unit cost.
(ii) The unit cost when allocated to an eight week cycle. The unit cost when allocated to a sixteen week cycle.

The figures in the circles are the cycle length in weeks for the least unit cost corresponding to the week ly rate of demand.

If the weekly demand rate alternates, say every sixteen weeks, between a value in the lower range $(500-1350)$ and a value in the higher range (1350-2500) a decision has to be made on whether to -
(i) Allocate the component to either an eight or sixteen week cycle over the year, or
(ii) allocate the component to a sixteen week cycle when the weekly demand rate is in the lower range and switch the allocation to an eight week cycle when the weekly demand rate is in the higher range.

The cost of either strategy i. e. (i) or (ii) may be determined from the Least Unit Cost Tables. For example Table 9.61A gives the total annual variable costs TVC (based on a fortyeight week working year) for weekly demand rates of 750,1500 , and 750 in each of three consecutive sixteen week periods respectively when the component is allocated to
(a) an 8 week cycle, or
(b) a 16 week cycle, or
(c) a 16 week cycle in the first sixteen week period, and
an 8 week cycle in the second sixteen week period, and
a 16 week cycle in the third sixteen week period.
former allocation．



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VI9＊ 6 田＇TGV
9. 7 An Heuristic Method for producing a balanced weekly load with multi-rota cycles in the form of a geometric progression.
9. 71 Introduction

The heuristic method, developed by the author, is best illustrated by an example. Table 9.71A gives the batch quantities, expressed in standard hours, of all the components made on one of the press groups in the Large Press Section. The figure in brackets is the part number of the component. The allocation of the components to the 8, and 16 week cycles were made using the Least Unit Cost Tables (Section 9.6).

| BATCH QUANTITIES <br> 16 EXPek cycle <br> Components |  | 8 week cycle components |  |
| :---: | ---: | :---: | :--- |
| $80(1)$ | $100(10)$ | $47(19)$ | $36(27)$ |
| $58(2)$ | $79(11)$ |  | $34(28)$ |
| $56(3)$ | $76(12)$ | $46(20)$ | $34(29)$ |
| $50(4)$ | $61(13)$ | $44(21)$ | $33(30)$ |
| $47(5)$ | $58(14)$ | $43(22)$ | $31(31)$ |
| $42(6)$ | $55(15)$ | $43(23)$ | $31(32)$ |
| $39(7)$ | $52(16)$ | $38(24)$ | $29(33)$ |
| $19(8)$ | $52(17)$ | $38(25)$ | $28(34)$ |
| $17(9)$ | $47(18)$ | $37(26)$ | $26(35)$ |

TABLE 9.71A

The stages in the heuristic are as follows -

## Stage 1

(i) Load the longest cycle components first (16 week in the example).

Arrange the components into two groups of similar weekly standard hours load patterns in descending order of magnitude. The number of weeks in each load pattern should be equal and not exceed the cycle length of the next longest cycle, i. e. 8 week cycle in the example. The two similar load patterns obtained for the example are given together with the week numbers in which they are loaded in Figure 9. 72A.


FIGURE 9. 72 A

It should be noted that the 81 standard hours loaded in week 1 is
a combination of 42 (6) and $39(7)$. The next longest cycle components will be loaded on each of the two load patterns formed. The difference in standard hours between corresponding weeks of the load patterns e. g. 1 standard hour for weeks 1 and 9, in the example, will effect the final load balance. To minimise the final out-of-balance due to the differences in the two load patterns, the average of the corresponding weeks in the two load patterns are found e. g. for weeks 1 and 9 the averageis $(81+80) / 2=80 \frac{1}{2}$ standard hours in the example. The average load pattern is used when loading the next longest cycle items i. e. the 8 week in the example.

## Stage 2

Load the components of the next longest cycle.
(i) Calculate the total standard hours loaded in Stage 1.
(ii) Calculate the total standard hours to be loaded in Stage 2.
(iii) Calculate the average weekly number of standard hours to be loaded from (i) and (ii). The week'v average for the example is 175.25 standard hours for the 16 and 8 week cycles.
(iv) Calculate the weekly number of standard hours across the cycle required to be loaded in Stage 2 to achieve the weekly average found in (iii) i. e. subtract the weekly number of standard hours in the average load pattern (from Stage 1) from the average found in (iii).
(v) Arrange the standard hours of the components to be loaded in Stage 2 in descending order of magnitude. Load the components, one at a time into the weeks of the Stage 2 cycle starting with the last week of the cycle, and continuing through to the first week - repeat until all the components have been allocated.
(vi) Calculate the weekly standard hours totals from (v).
(vii) Calculate the weekly out-of-balance in standard hours across the cycle from (iv) and (vi).

Table 9. 72 B shows the results in tableau form after applying steps (iii) to (vii) to the example.

## WEEK NUMBER

1 (and 9) $2($ and 10) $3($ and 11) $4($ and 12) $5($ and 13) $6($ and 14) $7($ and 15) $8($ and 16)

| 52 | 52 | 55 | 58 | 61 | 76 | 79 | 100 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 38 | 38 | 43 | 43 | 44 | 46 | 47 | 47 |
| 29 | 31 | 31 | 33 | 34 | 34 | 36 | 37 |
| 0 | 0 | 0 | 0 | 0 | 0 | 26 | 28 |


| Total | 119 | 121 | 129 | 134 | 139 | 156 | 188 | 212 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Reqd. | $94 \frac{3}{4}$ | $118 \frac{1}{4}$ | $126 \frac{3}{4}$ | $157 \frac{1}{4}$ | $175 \frac{1}{4}$ | $175 \frac{1}{4}$ | $175 \frac{1}{4}$ | $175 \frac{1}{4}$ |
| out of ) <br> balance) | $-24 \frac{1}{4}$ | $-2 \frac{3}{4}$ | $-2 \frac{1}{4}$ | $+23 \frac{1}{4}$ | $+36 \frac{1}{4}$ | $+19 \frac{1}{4}$ | $-12 \frac{3}{4}$ | $-36 \frac{3}{4}$ |

## TABLE. 9. 72B

## Stage 3

To improve the weekly out-of-balance:
(i) Select the weeks with the largest positive and the largest negative out-of-balance standard hours.
(ii) If the woek with the negative out-of-balance contains one or more larger standard hours value than those in the positive out-of-balance week an interchange of components between these weeks is possible. Otherwise proceed to step (V).
(iii) Interchange the components that give the greatest improvement in the out-of-balance in each of the two weeks.
(iv) Calculate the new out-of-balance values for the two weeks and update the tableau. Repeat steps from step (i).
(v) Select the next largest positive out-of-balance week and repeat steps from step (ii), or if all the positive out-of balance weeks have been tested against the negative out-ofbalance week (without an interchange of components taking place) proceed to step (vi).
(vi) Select the next largest negative out-of-balance week and - repeat steps from step (ii), or stop if there are no more negative out-of-balance weeks. The tableau then gives the best weekly balance.

In the example the first updating of the tableau (Table 9. 72 B ) by applying steps (i) to (iv) is given in Table 9. 72C. The quantities interchanged are shown in the squares.

## WEEK NUMBER

1 (and9) 2 (and 10) 3 (and 11) 4 (and 12) 5 (and 13) 6 (and 14) 7 (and 15) 8 (and 16)

| 52 | 52 | 55 | 58 | 61 | 76 | 79 | 100 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 38 | 38 | 43 | 43 | 44 | 46 | 47 | 47 |
| 29 | 31 | 31 | 33 | 34 | 34 | 36 | 0 |
| 0 | 0 | 0 | 0 | 37 | 0 | 26 | 28 |
| 119 | 121 | 129 | 134 | 176 | 156 | 188 | 175 |
| $94 \frac{3}{4}$ | $118 \frac{1}{4}$ | $126 \frac{3}{4}$ | $157 \frac{1}{4}$ | $175 \frac{1}{4}$ | $175 \frac{1}{4}$ | $175 \frac{1}{4}$ | $175 \frac{1}{4}$ |


| out of ) |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| balance ) | $-24 \frac{1}{4}$ | $-2 \frac{3}{4}$ | $-2 \frac{1}{4}$ | $+23 \frac{1}{4}$ | $-\frac{3}{4}$ | $+19 \frac{1}{4}$ | $-12 \frac{3}{4}$ |$+\frac{1}{4}$

TABLE 9.72C

A computer program was written for Stage 2 and Stage 3 of the heuristic. The listing of the program is given in Appendix H., together with the output from the program for the example. The output gives each updating of the tableau and the final balance achieved with both the 16 and 8 week cycle components. The input to the program consists of the standard hours of the components in the two load patterns obtained in Stage 1, and the standard hours in any order of the components in Stage 2. The program sorts the standard hours of the components in Stage 2 into descending order of magnitude. This is not strictly necessary for the computer solution but it does cut down computer run time. The average load pattern for Stage 1 is calculated in the program.

The weekly standard hours of the components, stored in the columns of a matrix, are kept in descending order of magnitude by the program. This reduces the number of combinations of the components to be tested for interchange between two selected weeks for the best improvement in balance.

The algorithm for smoothing the weekly production load has been tested for 16 and 8 week cycle loads using random samples of numbers taken from Kendall and Smith (11), to represent standard hours of components. Samples of 48 numbers in the ranges $0-100,20-80$, 30-70 and 40-60 with a mean of 50 were used. In all cases, including the example used in the heuristic, were balance loaded to within $1 \%$ of the average weekly load calculated for the samples.

The algorithm for smoothing the weekly load may be used for simple rota cycles e. g. could be used to balance load the current five week cycle in the Large Press Section.

## Stage 4

Load the next largest cycle (this does not apply to the example).
Find the average weekly load pattern, for this cycle length from the weekly loads found at the end of Stage 3 across the longest cycle. If the cycle length in this stage is four weeks, this is achieved by adding the standard hours loaded for the 16 and 8 week cycle components in week numbers.

1, 5, 9 and 13
(ii)
$2,6,10$ and 14
3, 7, 11 and 15
4, 8, 12 and 16
Find the average of (i), (ii), (iii) and (iv) to give the average 4 weekly load pattern over the 16 week cycle. Use the methods in Stage 2 and 3 to achieve balance.

Repeat this stage for 2 week cycle components, calculating the average 2 weekly load pattern in a similar way to the method used for the average 4 weekly cycle pattern.

## 9. 73 Discussion on Individual Press Loads.

In the press group used as the example, Section 9. 71, to illustrate the heuristic method for balance loading, there are four presses. 80 hours per press per week are, available through a day and regular night shift. 10 hours per press per week on both day and night shifts are also available if required.

A 60-65\% effective press group utilisation has been achieved and maintained in the five week rota cycle when required during peak demand periods. Using this range of effective press group utilisation, would give 48-52 standard hours on each press per week out of the total of 80 hours per week available per press on the day and night shift, and 60-65 standard hours with 20 hours overtime per press per week.

Table 9. 73A gives the component allocation together with the standard hours for the press group example for week numbers 8, 9 and 10 of the schedule obtained from the heuristic method. In week number 8

|  | WEEK NUMBER |  |  |
| :---: | :---: | :---: | :---: |
|  | 8 | 9 | 10 |
|  | $\begin{array}{r} 100(10) \\ 46(20) \\ 31(31) \end{array}$ | $\begin{aligned} & 80(1) \\ & 38(24) \\ & 29(33) \\ & 28(34) \end{aligned}$ | $\begin{aligned} & 56(3) \\ & 47(18) \\ & 38(25) \\ & 33(30) \end{aligned}$ |
| $\left\lvert\, \begin{aligned} & \text { Total weekly } \\ & \text { standard hours } \end{aligned}\right.$ | 177 | 175 | 174 |

TABLE 9.73A
and 9 of the schedule the standard hours for batches of Part Numbers 10 and 1 respectively exceed the maximum i. e. with overtime of 65 standard hours available on a press per week. Since there is only one tool available for the manufacture of each component part number, the batches cannot be completed in one week and therefore the manufacture of the batches must continue into another week.

The 100 standard hours for part number 10 and 80 standard hours for part number 1 may be split for example between week numbers 8 and 9 to give feasible weekly press loads, whilst still maintaining the balanced weekly loads and keeping overtime to a minimum. Table 9. 73B shows possible splits of the 100 and 80 standard hours between week numbers 8 and 9, that are within the minimum of 52 standard hours available on a press without overtime. Similarly the 57 standard hours in week number 9 and the 56 standard hours in week number 10 can be split to avoid overtime i. e. 5 standard hours of the 57 could run over into week 10 on Press D and 4 standard hours of the 56 transferred to week number 9 (i. e. start the batch in week number 9) on Press B.

| PRESS | WEEK NUMBER |  |  |
| :--- | :---: | :---: | :--- |
|  | 8 | 9 | 10 |
| A | $50(10)$ | $50(10)$ | $47(18)$ |
| B | $50(1)$ | $30(1)$ | $56(3)$ |
| C | $46(20)$ | $38(24)$ | $38(25)$ |
| D | $31(31)$ | $57(33) \&$ | $33(30)$ |
| Total Standard Hours per week | 177 | 175 | 174 |

TABLE 9. 73B

## 9. 8

 Discussion on the Heuristic Method and Subcontracting.The heuristic described in Section 9. 72 provides a more formal method of balanced loading fixed cycles than the method of Shank's Section 9. 4. The heuristic has been applied to a number of different schedules and balance has been achieved to within $1 \%$ of the weekly average load across the cycles. The balance achieved with Shank's method was generally larger than $1 \%$.

When the average weekly load exceeds the capacity, determined from the effective press group utilisation, work is subcontracted. The decision rules developed in Chapter 7 for subcontracting work from the current five week cycle, may also be used to maintain smooth weekly production loads across the fixed cycles. The cost of the components, in the Linear Program Formulation, compared to the subcontractors cost would include variable production costs based on the fixed cycle to which they have been allocated.

## 9. 9 Comments

The introduction of 4,8 and 16 week multi-rota cycles in the large press section would save at least $£ 15,000$ in the total variable production costs per annum relative to the current five week rota cycle. The allocation to the multi-rota cycles using the least unit cost method and the balance loading heuristic (section 9.7 ) have been
demonstrated to the company.

The introduction of the multi-rota cycles on the large press section would involve greater levels of stock than the curent 5 week rota cycle, and hence greater capital outlay. The company considered this together with -
(i) The savings on the total annual variable production costs.
(ii) The advantages of longer press runs that may lead to
(a) reductions in some of the idle time parameters, due to less tool change overs, and thus improvements to press utilisation.
(b) Improvements in operator performance and hence effective press group utilisation.
(c) Less tool wear.
(iii) The storage capacity required for the multi-rota cycles was within the capacity of the Pallet Stores (Chapter 8).

The company decided that the advantages out-weighed the disadvantages and have given financial backing to the introduction of multi-rota cycles on both the large and small press sections.

Work is proceeding on the conversion from the 5 week rota-cycle on the large press section to 4,8 and 16 week multi-rota cycles.

## 10. 1 Allocation Model

The subcontractors normally quote a fixed price per component irrespective of the component batch size. The company intend to consider negotiating price breaks for different component batch sizes. In this case, the objective function in the allocation model would be non-linear. Integer programming solutions of the model would allow non-linear objective functions and further work is proceeding in this direction together with the study of other integer programming techniques to reduce the time of solution.

### 10.2 Multi-Rota Cycling

Work is proceeding in preparing for the conversion from the five week rota to multi-rota cycles on the Large Priss Section.

Implementation of the system will be supervised by the author. In parallel with the implementation it is hoped to convert the balance load program (Appendix H) into a suitable form for use on the company's computer terminal. The program will be used to rebalance the multi-rota cycle system after changes to the production schedule and introduction of new appliances.

## 10. 3 Minimum Trim Loss

An investigation into Steel Trim Loss to minimise cost is proceeding. A study of the cutting patterns arising out of the investigation may predetermine the sequencing of some jobs. Consideration will
therefore have to be given to methods of incorporating sequences of jobs into the balance load heuristic (Chapter 9). The effect on subcontracting rules of using cutting patterns to minimise cost will also have to be studied.

## APPENDIX A



Figure 1


Figure 2



| Peek | Peek |  |  |
| :--- | :--- | :--- | :--- |
| Press | Press | Press | ff.Press |

## $\square$

|  |  |  |
| :--- | :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |





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## APPENDIX B - PROPOSED SEQUENCING SCHENIE

The principle of the Gantt chart could possibly be used to improve the sequencing of jobs, operators and setters onto the presses on the large press section to reduce idle time. The following scheme is therefore proposed.

The sum of the manufacturing time in standard hours for a component batch and the set-up time are entered on a strip of cardboard together with the number of operators required, the part number and press group. A different coloured strip is used for each week of the cycle, so that work carried over from one week to another may be easily distinguished. The length of the strip is proportional to the number of standard hours representing the batch quantity and set-up time. The strips are mounted onto a special wall board with transparent plastic pockets that run the length of the board and along which the strips may slide. The board is divided into five equal lengths (possibly by vertical lines painted on the board) to represent each week of the rota cycle. Each plastic pocket is labelled with a press number, and press numbers in a press group are kept adjacent to one another. Figure B illustrates the strips mounted on the board.

The board when loaded represents in histogram form the load on each press group and the individual presses within the press group.

By visually scanning the board vertically and re-arranging card positions it should be possible to sequence jobs, operators and setters, to minimise the idle time between press runs. Jobs should be arranged so that the number of operators released on the completion of jobs and the number required to start new jobs are equal. Press set-ups should be staggered to allow for the number of setters available.


Figure $B$

Due to factors such as tool troubles, press breakdowns, variation in operator efficiency and shortages the loading of the presses will be
continually in a dynamic state and hence require re-sequencing throughout the cycle.

The system described above caters for such a dynamic state. Changes made during the cycle to the loading schedule can be effectively represented and observed on the board, and enable measures to be taken to plan overtime to accommodate the differences between the schedule and the changes that occur.

The Production Control Department have purchased a 'double channel' board. One pocket in a double channel will be used to display the scheduled load in a press and the other channel for the changes made. Implementation of a sequencing scheme, based on the above sequencing scheme, is currently being undertaken by the company.

It should be noted that an allowance should be made in the processing time per component batch in standard hours for all the idle time parameters with the exception of -
(i) Awaiting Operators.
(ii) Awaiting Tool Change-over.

APPENDIX C - ALLOCATION MODEL - DATA

| Component <br> Part <br> Number | Component Batch Cost in 1.5 |  |  |  |  |  | Component Batch Manufacturing Time in std. hrs. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Factory Shift |  | Subcontractors |  |  |  |  |
|  | Day | Night | A | B | C | D |  |
| 1 | 250 | 270 | 230 | 260 | 330 | 190 | 2 |
| 2 | 170 | 190 | 270 | 280 | 280 | 160 | 7 |
| 3 | 470 | 510 | 530 | 610 | 610 | 410 | 9 |
| 4 | 330 | 360 | 400 | 360 | 330 | 270 | 2 |
| 5 | 280 | 300 | 340 | 310 | 360 | 220 | 9 |
| 6 | 170 | 190 | 270 | 280 | 280 | 160 | 4 |
| 7 | 180 | 190 | 210 | 220 | 130 | 120 | 4 |
| 8 | 370 | 410 | 520 | 540 | 420 | 360 | 9 |
| 9 | 120 | 130 | 200 | 210 | 210 | 110 | 3 |
| 10 | 180 | 190 | 160 | 220 | 220 | 120 | 6 |
| 11 | 310 | 340 | 440 | 460 | 430 | 300 | 6 |
| 12 | 140 | 150 | 220 | 230 | 210 | 130 | 8 |
| 13 | 120 | 130 | 200 | 210 | 170 | 110 | 2 |
| 14 | 170 | 190 | 270 | 280 | 170 | 160 | 4 |
| 15 | 310 | 340 | 380 | 390 | 390 | 250 | 4 |
| 16 | 150 | 160 | 230 | 240 | 220 | 140 | 2 |
| 17 | 250 | 270 | 360 | 380 | 330 | 240 | 4 |
| 18 | 200 | 220 | 300 | 310 | 280 | 190 | 9 |
| 19 | 330 | 360 | 460 | 480 | 300 | 320 | 3 |
| 20 | 110 | 120 | 190 | 200 | 200 | 100 | 5 |
|  |  |  |  | TOT |  |  | 102 |


| Component |  | Compone | t Ba | atch Al | atio |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Part | Factory | Shift |  | Subcon | ctor |  |
| Number | Day | Night | A | B | C | D |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 2 | 1 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 1 |
| 4 | 0 | 0 | 0 | 0 | 0 | 1 |
| 5 | 0 | 0 | 0 | 1 | 0 | 0 |
| 6 | 1 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 | 0 | 0 |
| 9 | 1 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 1 | 0 | 0 | 0 |
| 11 | 1 | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | . 6250 | 0 | . 3750 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 | 0 | 1 |
| 14 | 1 | 0 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 1 |
| 16 | 0. 5000 | 0 | 0 | 0 | 0 | 0. 5000 |
| 17 | 1 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0. 1111 | 0. 8889 | 0 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 1 | 0 |
| 20 | 0 | 1 | 0 | 0 | 0 | 0 |
| Maximum capacity available (upper bound)std. hrs | 40 | $20^{\circ}$ | 20 | 20 | 20 | 20 |
| Minimum capacity to be utilised(lower bound)st. hrs. 38 |  | 18 | $\varepsilon$ | 12 | 6 | 6 |
| Capacity allocated by model in std. hrs. | 38. 9999 | 18.0001 | 6 | 12 | 7 | 20 |

COMPUTER TIME FOR SOLUTION = 1 MINUTE
TOTAL COST (OBJECTIVE FUNCTION VALUE) $=84,352.78$

1st INTEGER SOLUTION USING - Objective cut-off value $+\infty$, variable selection strategy - Tomlin Penalty Selection (16)

| Component |  | Comp | nt B | Al | tion |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Part | Fact | Shift |  | Subc | ract |  |
| Number | Day | Night | A | B | C | D |
| 1 | 0 | 0 | 0 | 0 | 0 | 1* |
| 2 | 1* | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 1* |
| 4 | 0 | 1 | 0 | 0 | 0 | 0\% |
| 5 | 0 | 0 | 0 | 1* | 0 | 0 |
| 6 | 1* | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 1* | 0 |
| 8 | 0* | 1 | 0 | 0 | 0 | 0 |
| 9 | 1* | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 1* | 0 | 0 | 0 |
| 11 | 1* | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | 1* | 0 | 0* | 0 | 0 |
| 13 | 1 | 0 | 0 | 0 | 0 | 0* |
| 14 | 1* | 0 | 0 | 0 | 0 | 0 |
| 15 | 1 | 0 | 0 | 0 | 0 | 0\% |
| 16 | 0* | 0 | 0 | 0 | 0 | 1* |
| 17 | 1* | 0 | 0 | 0 | 0 | 0 |
| 18 | 0\% | 0\% | 0 | 1 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 1* | 0 |
| 20 | 1 | 0* | 0 | 0 | 0 | 0 |
| Maximum capacity available (upper bound)in std. hrs | $\text { s. } 40$ | 20 | 20 | 20 | 20 | 20 |
| Minimum capacity to be utilised(lower bound)in std. hrs. | 38 | 18 | 6 | 12 | 6 | 6 |
| Capacity allocated by model in std. hrs. | 39 | 19 | 6 | 18 | 7 | 13 |

*Corresponding L. P. solution. Two asterisks in a row indicate the split COMPUTER TIME FOR SOLUTION $=22$ MTNUTE
batches.

1 st INTEGER SOLUTION USING - Objective cut-off value $=£ 4,450$, variable selection strategy - Tomlin : Penalty Selection (16)

| Component |  | Comp | nt B | Allo | ion |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Part |  | ry Shift |  | bcon | ctor |  |
| Number | Day | Night | A | B | C | D |
| 1 | 0 | 0 | 0 | 0 | 0 | 1* |
| 2 | 1* | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 1* |
| 4 | 0 | 0 | 0 | 0 | 0 | 1* |
| 5 | 0 | 0 | 0 | 1* | 0 | 0 |
| 6 | 1* | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 1* | 0 |
| 8 | 0* | 1 | 0 | 0 | 0 | 0 |
| 9 | 0 * | 0 | 0 | 1 | 0 | 0 |
| 10 | 0 | 0 | 1* | 0 | 0 | 0 |
| 11 | 1* | 0 | 0 | 0 | 0 | 0 |
| 12 | 1 | 0* | 0 | 0* | 0 | 0 |
| 13 | 1 | 0 | 0 | 0 | 0 | 0* |
| 14 | 1* | 0 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 1* |
| 16 | 0* | 0 | 0 | 0 | 0 | 1* |
| 17 | 1* | 0 | 0 | 0 | 0 | 0 |
| 18 | 0* | 1* | 0 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 1* | 0 |
| 20 | 1 | 0* | 0 | 0 | 0 | 0 |
| Maximum capacity available(upper bound) in std. hares. | 40 | 20 | 20 | 20 | 20 | 20 |
| Ninimum capacity to be utilised(lower bound) in std. hares | 38 | 18 | 6 | 12 | 6 | 6 |
| Capacity allocated by model in std. hours. | 40 | 18 | 6 | 12 | 7 | 19 |

*L. P. Solution. Two asterisks in a row indicate the split batches. COMPUTER TIME FOR SOLUTION $=4$ minutes.
TOTAL COST (objective function value) $=£ 4,440$

1st INTEGER SOLUTION USING - Objective cut-off value $=£ 4,450$, variable selection strategy - Actual fraction selection (nearest to nearest integer) ( 16 )

| Component |  |  | nen | ch A | atio |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Part |  | ory Shi |  | con | tor |  |
| Number | Day | Night | A | B | C | D |
| 1 | 0 | 0 | 0 | 0 | 0 | 1* |
| 2 | 1* | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 1* |
| 4 | 0 | 0 | 0 | 0 | 0 | 1* |
| 5 | 0 | 0 | 0 | 1* | 0 | 0 |
| 6 | 1* | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 1* | 0 |
| 8 | 1* | 0 | 0 | 0 | 0 | 0 |
| 9 | 1* | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 1* | 0 | 0 | 0 |
| 11 | 1* | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | 1* | 0 | 0* | 0 | 0 |
| 13 | 1 | 0 | 0 | 0 | 0 | 0* |
| 14 | 1* | 0 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 1* |
| 16 | 0* | 1 | 0 | 0 | 0 | 0* |
| 17 | 1* | 0 | 0 | 0 | 0 | 0 |
| 18 | 0* | 1* | 0 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 1* | 0 |
| 20 | 0 | 0* | 0 | 1 | 0 | 0 |
| Maximum capacity available(upper bound) in std. hours. | 40 | 20 | 20 | 20 | 20 | 20 |
| Ninimum capacity to be utilised in std. hours. | 38 | 18 | 6 | 12 | 6 | 6 |
| Capacity allocated by model in std. hrs. | 39 | 19 | 6 | 14 | 7 | 17 |

*L. P. solution. Two asterisks in a row indicate the split batches.
COMPUTER TIIE FOR SOLUTION = 26 MINUTES
TOTAL COST (OBJECTIVE FUNCTION VALUE) $=£ 4,430$ (optimal)

## APPENDIX D - THEORETICAL STUDY OF THE PALLET STORES

The weekly input of Y pallets to the Pallet Stores from the Press Shop and the output of the Y pallets at a constant rate over 5 weeks to the Finishing Shop is illustrated in Figure 1 for a five week cycle. Each week of the five week cycle contributes to the total number of pallets $\left(\mathrm{S}_{5}\right)$ in the Pallet Stores at the end of week five of the cycle, and is given by the sum of the intercepts on the line XX.'


More generally if $T$ is the time cycle of the Press Shop in weeks and $T^{\prime}$ weeks the period of depletion of pallets, the total number of pallets stored (S) in the Pallet Stores in any week (i) in the cycle, with a constant weekly input of Y pallets, is given by
$\left.S=\sum_{i=1}^{i=T^{\prime}} \sum_{( }^{( } Y-\left(T^{\prime}-i\right) \quad \frac{Y}{T^{\prime}}\right)$

$$
=\frac{Y}{T^{\prime}} \sum_{i=1}^{i=T^{\prime}} i
$$

but

$$
i=T^{\prime}
$$

$\sum_{i=1} i$ is an Arithmetic Progression whose sum
is

$$
\frac{\left(\mathrm{T}^{\prime}+1\right) \mathrm{T}^{\prime}}{2}
$$

$$
\begin{equation*}
S=\frac{Y}{2} \quad\left(T^{\prime}+1\right) \tag{1}
\end{equation*}
$$

The equation for the total number of pallets stored Sj at the end of week j where the following conditions apply.

1. The input to the Pallet Stores each week varies over the T week cycle.
2. The period of depletion at a constant rate is $T^{\prime}$ weeks. In other words the finishing shop is working on a $T^{\prime}$ week rota cycle.
3. The lead time L weeks, defined as the time in weeks between the end of the week of production and the beginning of the supply of the pallets to the finishing shop at a constant rate.

The lead time $L=-1$ physically means inputting $\frac{\mathrm{Y}}{\mathrm{T}^{\prime}}$ pallets in the week of production to the finishing shop for processing, instead of to the pallet stores for storage, is given by
$S_{j}=\sum_{i=1}^{i=T^{\prime}} \sum_{\left(Y_{\left(j-T^{\prime}+i-L\right)}-\left(T^{\prime}-i\right) \frac{Y}{T^{\prime}}\left(j-T^{\prime}+i-L\right)\right)+\lambda, ~(2)}^{\left.L^{\prime}\right)}$
$\lambda=-Y_{\left(j-T^{\prime}+i-L\right)}=-Y_{(j+1)}$
for $L=-1$
$\lambda=o$
for $L=O$
for $L=1,2, \ldots \ldots$
. $K=L$
$\lambda=\sum_{K=1} Y_{(j-K+1)}$
(2. $x^{2}$

Equation 2 reduces to

$$
\begin{equation*}
S_{j}=\frac{1}{T}, \sum_{i=1}^{i=T^{\prime}}\left(i Y_{\left(j-T^{\prime}+i-L\right)}\right)+\lambda \tag{3}
\end{equation*}
$$

When the input to the Pallet Stores is kept constant and equal to $Y$ equation 3 for all L becomes

$$
\begin{align*}
& S_{j}=\left[\frac{Y}{T^{\prime}} \sum_{i=1}^{i=T^{\prime}} i\right]+L Y \\
& S=\frac{Y}{T^{\prime}} \cdot \frac{\left(1+T^{\prime}\right) T^{\prime}}{2}+L Y \\
& S=\frac{Y}{2}\left(T^{\prime}+2 L+1\right) \tag{4}
\end{align*}
$$

From equation 4 when $\mathrm{L}=\mathrm{O}$
$S=\frac{Y}{2}\left(T^{\prime}+1\right)$
which is equation 1.


FIGURE I


Graph 1.

C TI = DEPLETION IN WEEKS
C $\quad T=$ TIME CYCLE OF PRESS SHOP
$C \quad J=$ WEEK OF STORAGE
$L=L E A D$ TIME IN WEEKS
C $\quad M=$ NUMBER OF SAMPLES
C SS = NUMBER OF PALLETS STORED FOR CONSTANT I/P
C LAMDA = CORRECTION FACTOR FOR DIFFERENT L'S
C $\quad S(J)=$ NUMBER OF PALLETS STORED AT END OF WEEK $J$
$C \quad D E V=$ DIFFERENCE BETWEEN $S(J)$ AND $S$ - CAN BE -VE, O, OR +VE
C
INTEGER Y,TI,T,YY,SS
DIMENSION Y(50,20),S(20),OUT1(5),OUT2(5)
MC $=0$
C
C READ IN DATA
READ $(5,100)$ TI, T,J,L,M
$\operatorname{READ}(5,101)((Y(N, K K), K K=1,20), N=1, M)$
I $M=1$
$Y Y=0$
DO $1 \quad I=1, T$
$Y Y=Y Y+Y(1, I)$
$Y Y=Y Y / T$
$S S=Y Y *(T I+2 * L+1) / 2$
WRITE $(9,200) T, L, T!, S S$
I I =1
$S(J)=0$
DO $8 \quad \mathrm{I}=1, \mathrm{~T} \mathrm{I}$
$8 \quad S(J)=S(J)+I * Y(I M, J-T I+I-L)$
IF (L) $0,2,3$
LAMDA $=(-Y(I M, J+1))$
GO TO 99
LAMDA $=0$
GO TO 99
LAMDA $=0$
DO $6 \mathrm{~K}=1$, L
6
$L A M D A=L A M D A+Y(I M, J-K+1)$
$99 \quad S(J)=S(J) / T T+$ LAMDA
$D E V=S(J)-S S$
OUT1 (II) $=\mathrm{Y}(\mathrm{IM}, \mathrm{II})$
OUT2(II)=DEV
I I = I I + 1
$J=J+1$
IF(II-T)7,7,0
$M C=M C+1$
$J=\mathrm{J}-\mathrm{T}$
HIGH $=0.0$
DO $14 \mathrm{NN}=1,5$
IF (OUT2(NN), GT, HIGH)HIGH=OUT2(NN)
CONTINUE
WRITE $(9,202)$ MC, OUT1, OUT2,HIGH
IF (MC-M) 0,12,0
I $M=I M+1$
GO TO 5
I $M=1$
$M C=0$
$\mathrm{T} \mathrm{I}=\mathrm{T} \mathrm{I}+1$
IF (TI-10) $4,0,4$
$T \mathrm{I}=\mathrm{T}$
I $M=1$
$M C=0$
$\mathrm{L}=\mathrm{L}+1$
IF $(\mathrm{L}-4) 4,13,4$
STOP



| Week of <br> Rota Cycle | Variable <br> Pallet <br> Input | Period of Depletion $T^{\prime}$ Weeks |  |  |  |  |  |  |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: | :---: | :---: |
|  | 5 | 6 |  |  |  |  |  |  |
| 1 | 200 | 20 | 0 | 0 | 12.5 | 22.2 |  |  |
| 2 | 200 | -80 | -83.3 | -100 | -100 | -88.9 |  |  |
| 3 | 300 | -80 | -66.7 | -71.4 | -87.5 | -88.9 |  |  |
| 4 | 400 | 20 | 33.3 | 43.9 | 37.5 | 22.2 |  |  |
| 5 | 120 | 116.7 | 128.6 | 137.5 | 133.3 |  |  |  |
| Mean <br> Constant Storage Level | 900 | 1050 | 1200 | 1350 | 1500 |  |  |  |

Table 3

## APPENDIX $F$ - NAGEE'S FORMULAE


APPENDIX G - LEAST UNIT COST (L.U.C.) LOOK-UP TABLES

|  | $\begin{aligned} & \stackrel{0}{\sim} \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { g } \\ & \text { ష్0 } \\ & \text { g } \\ & \text { H } \end{aligned}$ |  | $\stackrel{9}{\square}$ | $\stackrel{\square}{\square}$ | $\stackrel{\infty}{-}$ | $\stackrel{\sim}{\sim}$ | $\because$ | $\bigcirc$ | $\infty$ | $\infty$ | $\infty$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $i$ <br> p <br> - |  |  | N L N - - |  | $\infty$ 0 0 $\infty$ 0 0 | $\begin{aligned} & \infty \\ & 0 \\ & \stackrel{\infty}{2} \\ & 0 \\ & \hline 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { H } \\ & \text { H } \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} \text { N } \\ \stackrel{\sim}{5} \\ \dot{0} \end{gathered}$ | ¢ ¢ै 0 |
|  |  |  | $\stackrel{\sim}{\infty}$ |  | $\begin{aligned} & \text { or } \\ & \text { N } \\ & \text { o } \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & N \\ & \sim \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { r- } \\ & \text { N } \\ & \text { N- } \end{aligned}$ | -1 0 - - - -1 | $\begin{aligned} & \text { O. } \\ & \text { O } \\ & \stackrel{1}{i} \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{0} \\ & \stackrel{+}{1} \\ & -i \end{aligned}$ | + |  |
|  |  |  | $\stackrel{\square}{\sim}$ | N e N $\sim$ -1 | N <br> $\substack{\text { - } \\ --- \\ \hline}$ | $\begin{aligned} & \text { Î } \\ & \text { © } \\ & \text { O } \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \infty \\ & -1 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \text { ob } \\ & \stackrel{0}{2} \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \\ & \stackrel{0}{0} \end{aligned}$ | $\begin{aligned} & \tilde{\circ} \\ & \stackrel{\rightharpoonup}{2} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \text { of } \\ & \text { in } \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \text { O } \\ & \text { © } \\ & 0 \end{aligned}$ |
|  |  |  | $\infty$ | N N - | $\begin{gathered} \underset{\sim}{N} \end{gathered}$ | $\begin{aligned} & \infty \\ & \text { N } \\ & \text { - } \\ & \dot{\circ} \end{aligned}$ | $\begin{aligned} & \infty \\ & \text { M } \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \text { ㅌ } \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & 0 \\ & \stackrel{0}{0} \\ & \stackrel{0}{6} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\sim}{0} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \circ \\ & \stackrel{0}{\circ} \\ & \stackrel{0}{\circ} \\ & \dot{0} \end{aligned}$ | ¢ ¢ै $\stackrel{0}{0}$ |
|  |  |  | + | $\begin{gathered} \infty \\ 0 \\ 0 \\ 0 \\ \infty \\ \infty \end{gathered}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & \vdots \\ & \dot{c} \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \\ & - \\ & -1 \end{aligned}$ | $\begin{aligned} & \infty \\ & \text { O } \\ & 0 \\ & \infty \\ & -1 \end{aligned}$ | $\infty$ 0 0 $\vdots$ - | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \infty \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { no } \\ & \stackrel{2}{2} \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & \stackrel{0}{2} \\ & 0 \end{aligned}$ |
|  |  |  | $\sim$ | $\begin{aligned} & \circ \\ & \text { ¢ } \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \circ \\ & \text { ๗/ } \\ & 0 \\ & + \end{aligned}$ | $\begin{aligned} & 0 \\ & \underset{\sim}{\infty} \\ & \circ \\ & \infty \end{aligned}$ | $\begin{aligned} & \text { o } \\ & \text { N } \\ & \underset{\sim}{1} \\ & \text { ion } \end{aligned}$ | $\begin{aligned} & \text { o } \\ & \text { N } \\ & 0 \\ & \text { i } \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \stackrel{0}{4} \\ & \text { - } \end{aligned}$ | $\begin{aligned} & \text { o } \\ & \text { è } \\ & \text { in } \\ & \text { ri } \end{aligned}$ | $\infty$ $\sim$ 0 $\sim$ $\sim$ | $\circ$ $\sim$ $\sim$ $\sim$ $\sim$ |
|  | H ¢ 0 is | $\begin{aligned} & \text { ๙ } \\ & \underset{\sim}{0} \end{aligned}$ | $\begin{gathered} \frac{0}{0} \\ 0 \\ 0 \end{gathered}$ | $\begin{aligned} & 10 \\ & \infty \\ & 0 \\ & 0 \\ & 0 \\ & i \\ & \hline \end{aligned}$ | $\begin{aligned} & \sim \\ & \infty \\ & \sim \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \infty \\ & - \\ & \hline \end{aligned}$ | $\begin{array}{r} 10 \\ 0 \\ 0 \\ 0 \\ -1 \\ \hline \end{array}$ | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \infty \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { N̈ } \\ & \text { N } \\ & \text { o. } \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{1}{5} \\ & 0 \\ & 0 \end{aligned}$ | 10 0 0 0 0 |
| ® तू ¢ A |  | $\begin{gathered} 8 \\ 0 \\ 0 \end{gathered}$ | $\begin{aligned} & \text { y } \\ & \text { む } \\ & \text { z } \end{aligned}$ | \% | \% | 8 | - | 8 0 $\sim$ | 읃 | $\begin{aligned} & \circ \\ & \stackrel{\circ}{\circ} \\ & \text { N } \end{aligned}$ | 이N | \% ¢ $\sim$ |



```
C READ IN 16 WEEK ITEMS IN PREDETERMINED ORDER
C
    \(\operatorname{READ}(5,100)((A 16(1, J), J=1,16), I=1,20)\)
    FORMAT (16F4.0)
C
C READ HEAUING, TEST FOR LAST SAMPLE
C
500 READ 5,700 ) HEAD
700 FORMAT (4AB)
    CALL COMPB(BLANK,HEAD(1), I)
    IF \((I-1) 600,600,0\)
C
C. READ ABIN MATRIX
C
    \(\operatorname{READ}(5,101) \operatorname{ABIN}\)
    FORMAT (8F4.0)
c
C READ NO. OF ITEMS
C
102 FORMAT (I \(厶\) )
C
C FIND AVEPAGE OF SAMPLE
C
    \(S U M=0,0\)
    DO \(800 \quad \mathrm{~J}=1,160\)
\(800 \quad\) SUM \(=\) SUM \(+A 8 I N(J)\)
    \(A V E R=S U M / F L O A T(N)\)
C
C SORT INTO DESCENDING ORDER OF MAGNITUDE
C
    DO \(801 K=1,159\)
    NK \(=160-K\)
    DO 801 LK=1,NK
    \(J K=L K+1\)
    IF (A8IN(LK)-A8IN(JK))0,801,801
    IX=A8IN(LK)
    \(48 I N(L K)=A 8 I N(J K)\)
    \(A 8 I N(J K)=I X\)
80? CONTINUE
C
C CALC NO OF ROWS IN 8 WEEK MATRIX
C
    \(\mathrm{N}=\mathrm{N}+16\)
    \(X N=N\)
    \(N=F L O A T(N) / 8 。 * 0.9\)
C
C TRANSFER SAMPLE TO 8 WEEK MATRIX
C
    \(k=1\)
    \(I=1\)
\(996 \quad \mathrm{~J}=8\)
    \(00997 \mathrm{~L}=1.8\)
    A8 \((I, J)=A 8 I N(K)\)
    \(K=K+1\)
\(997 \quad J=j-1\)
    \(I=I * 1\)
```

c
C FIND STAMDARD DEVIATION OF SAMPLE
c
$S U M=0.0$
DO $999 \quad \mathrm{I}=1, \mathrm{~N}$
DO $998 \mathrm{~J}=1.8$
IF $(A 8(I, j), E Q, 0) G 0$ TO 998
$D E V=(A V E R-A B(I, J)) * * 2$
$S U M=S U M+D E V$
998 CONTINUE
999 CONTINUE
$S D=S Q R T(S U M / X N)$
c
C PRINT HEADING, SAMPLE, AVERAGE, ST, DEV,
c
701 FORMAT (1H1.10X,4A8/1)
WRITE 9,555$)((A 8(I, J), j=1,8), I=1, N)$
WRITE (9,702)AVER,SD

C SUM 16 , 8 WEEK MATRICES, CALC LOAD
c
Sum1, SUM2 $=0.0$
DO $13 \mathrm{~J}=1,8$
$\operatorname{AV}(J)=0.0$
$\operatorname{TEST}(J)=0.0$
$\operatorname{TOTAL}(J)=0.0$
DO $1 \quad 1=1,20$
DO $1 \quad J=1,16$
SUM1 $=$ SUM1 + A16(I,J)
DO $2 \mathrm{I}=1, \mathrm{~N}$
DO $2 \mathrm{j}=1: 8$
SUM $2=\operatorname{SUMZ*A8(I,J)~}$
LOAD $=$ (SUM1/16*SUM2/8)
c
C
$C$
CALC AVERAGES OF A 16 COLS AND REQD FIGS.
D0 $3 I=1,20$
DO $3 \mathrm{~J}=1,8$
$\operatorname{AV}(J)=A V(j) *(A 16(1, J)+A 16(1, J+8)) / 2$.
DO $4 \mathrm{j}=1,8$
$\operatorname{REQD}(\mathrm{J})=\operatorname{LOAD}-\operatorname{AV}(\mathrm{J})$
SUM COLS OF AB, CALC OBAL
DO $5 \quad I=1, N$
DO $5 \mathrm{j}=1.8$
TOTAL(j) $=$ TOTAL $(J) \div A 8(I, j)$
DO $6 \mathrm{~J}=1,8$
OBAL(J)=REQD(J)-TOTAL(J)
WRITE $(9,666)$ TOTAL,REQD,OBAL
c
C TEST FOR ACCURACY ACHIEVED , HAS OBAL CHANGED?
c
99 DO $52 \mathrm{~J}=1.8$
IF(OBAL(J)-TEST(J)) 9,0,9
52 CONTINUE
GO TO 48
DO $53 \mathrm{~J}=1,8$
53
$\operatorname{TEST}(J)=O B A L(1)$
I,J=1
II,JJ=0
OLDTOEAL=999
TOTALOBAL=ABS (OBAL(NN)) +OBAL(NP)
DIFF=A8(J,NNS-AB(I,NP)
IF(AB(J,AN),EQ,O,O,AND,A8(I,NP),EQ,O.O)GO TO 50
IF(DIFF)0,0.14
IF(I-N)0,15,0
I=I+{
GO TO 33
15 IF (J-N)0.16,0
J=j+1
I=1
GOTO 33
CONTINUE
IF(DIFF-TOTALOBAL)O,15,15
C
C
C
NEWTOBAL=ABS(OBAL(NP) -DIFF)\divABS(OBAL(NN)\divDIFF)
IF(NEWTOBAL-OLDTOBAL)0,50.50
OLDTOBAL=NEWTOBAL
II=I
JJ= J
GO TO 50
IF(II)17,17,0
I=II
J=\J
GO TO 19
C NO INTERCHANGE POSSIBLE
C LOOKED AT ALL NEGATIVES
17 IF(NEG-1)0,49,0
KN(NN)=NN
GO TO 12

```
```

C
SELECT NEXT POSITIVE, RESET NEGATIVE COUNT
C
49 DO 51 J=7,8
51 KN(J)=0
KK(NP)=NP
C
C LOOKED AT AIL POSITIVES ?
IF(NPOS-{)12,43,12
48 WRITE(9,203)
203 FORMAT(1H1,20X,'ACCURACY ACHIEVED')
G0 TO 27
C
C INTERCHANGE OF ELEMENTS AND UPDATING OF OBAL.
19 ZW=A8(I,NP)
A8(I,NP)=AB(J,NN)
A8(J,NN) :=ZW
TOTAL(NP),TOTAL(NN)=0
D) 20 L=1.N
TOTAL(NP)=TOTAL(NP)+AB(L,NP)
TOTAL(NN)=TOTAL(NN)*AB(L,NN)
OBAL(NP)=REQD(NP)-TOTAL(NP)
OBAL(NN)==REQD(NN)-TOTAL(NN)
C
C SORT POSITIVE COLUMN
G
DO 21 K=1,N-1
NK=N-K
DO 21 LK=1,NK
JK=LK*1
IF(A8(LK,NP)-A8(JK,NP))0,21.21
IX=AB(LK,NP)
A8(LK,NP)=A8(JK,NP)
AB(JK,NP) =IX
CONTINUE
21
C
C
SORT NEGATIVE COLUMN
DO 22 K=1,N-1
NK=N-K
DO 22 LK=1,NK
JK=LK+1
IF(A8(LK,NN)-A8(JK,NN))0,22,22
IX=A8(LK,NN)
AB(LK,NN)=A8(JK,NN)
AB(JK,NN)=IX
22 CONTINUE
C
C
C
WRITE (9,555)((A8(K,J),J=1,8),K=1,N)
WRITE(9,666) TOTAL,REQD,OBAL
GOTO 99
C
C
C
10 WRITE (9.201)
201 GORMAT(1H1,20X, 'ACCURACY ACHIEVED')
C
C CALCULATE I6 CULUMN TOTALS
C
27 DO 29 K=1,16
29 TTOT(K)=0.0
DO 23 K=?,20
DO 23 J={.8

```
```

    TTOT(J)=TTOT(J)*A16(K,J)
    TTOT}(J+8)=T\operatorname{TOT}(J*8)*&.16(K,J*8
    CONTINUE
    DO 30 K={.8
    TTOT(K)=TTOT(K)+TOTAL(K)
    TTOT(K*8)=TTOT(K*8)*TOTAL(K)
    REQUIRED=LOAO: TOTAL.OBAL=LOAD-TOTALS
    DO 24 J={,16
    RTOT(J)=1.0AD
    OTOT(J)=LOAD-TTOT(J)
    DO 25 1=?,20
    IF(A16(J,1))25,25,0
    WRITE (9,2!2)(A16(J,K),K=1,16)
    202 FORMAT(1H0,1ÓF7.2)
25 CONTINUE
DO.26 J=1,N
WRITE(9,202)(A8(J,K),K=1,8),(A8(J,K),K=1,8)
WRITE(9.222)TTOT,RTOT.OTOT

```
WEEK NUMBER
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline I & 11 & III & IV & V & VI & VII & V111 \\
\hline 52.00 & 52.00 & 55.00 & 58.00 & 61.00 & 76.00 & 79.00 & 100.00 \\
\hline 38.00 & 38.00 & 43.00 & 43.00 & 44.00 & 46.00 & 47.00 & 47.00 \\
\hline 29.00 & 31.00 & 31.00 & 33.00 & 34.00 & 34.00 & 36.00 & 37.00 \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 26.00 & 28.00 \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
\hline 119.00 & 121.00 & 129.00 & 134.00 & 139.00 & 156.00 & 188.00 & 212.00 \\
\hline 94.75 & 118.25 & 126.75 & 157.25 & 175.25 & 175.25 & 175.25 & 175.25 \\
\hline \(-24.25\) & \(-2.75\) & -2.25 & 23.25 & 36.25 & 19.25 & \(-12.75\) & -35.75 \\
\hline
\end{tabular}

\section*{WEEK NUMBER}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline 1 & II & III & IV & V & VI & VII & VIII & \\
\hline 52.00 & 52.00 & 55.0 .0 & 58.00 & 61.00 & 76.00 & 79.00 & 100:00 & \\
\hline 38.00 & 38.00 & 43.00 & 43.00 & 44.00 & 46,00 & 47.00 & 47.00 & \\
\hline 29.00 & 31.00 & 31.00 & 33.00 & 37.00 & 34.00 & 36.00 & 28.00 & \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 34.00 & 0.00 & 26.00 & 0.00 & \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & \\
\hline 119.00 & 121.00 & 129.00 & 134.00 & 176.00 & 156.00 & 188.00 & 175.00 & TOTAL \\
\hline 94.75 & 118.25 & 126.75 & 157.25 & 175.25 & 175.25 & 175.25 & 175.25 & REQUIRED \\
\hline \(-24.25\) & \(-2.75\) & -2.25 & 23.25 & \(=0.75\) & 19.25 & \(=12.75\) & 0.25 & OUT OF BALANCE \\
\hline
\end{tabular}

WEEK NUMBER
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline 1 & 11 & 111 & IV & V & VI & VII & V1II & \\
\hline 38.00 & 52.00 & 55.00 & 58.00 & 61.00 & 76.00 & 79.00 & 100.00 & \\
\hline 33.00 & 38.00 & 43.00 & 52.00 & 44.00 & 46.00 & 47.00 & 47.00 & \\
\hline 29.00 & 31.00 & 31.00 & 43.00 & 37.00 & 34.00 & 36.00 & 28.00 & \\
\hline 0.00 & 0.00 & 0.00 & 0,00 & 34.00 & 0,00 & 26.00 & 0.00 & \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & \\
\hline 100.00 & 121.00 & 129.00 & 153.00 & 176.00 & 156.00 & 188,00 & 175.00 & TOTAL \\
\hline 94.75 & 198.25 & 126.75 & 157.25 & 175.25 & 175.25 & 175.25 & 175.25 & REQUIRED \\
\hline -5.25 & \(-2.75\) & -2.25 & 4.25 & \(-0.75\) & 19.25 & \(-12.75\) & 0.25 & OUT OF BALANCE \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline 1 & 11 & 111 & IV & V & \(V 1\) & V11 & VIII \\
\hline 38.00 & 52.00 & 55.00 & 58.00 & 61.00 & 76.00 & 79.00 & 100.00 \\
\hline 33.00 & 38.00 & 43.00 & 52.00 & 44.00 & 47.00 & 36.00 & 47.00 \\
\hline 29.00 & 31.00 & 31.00 & 43.00 & 37.00 & 46,00 & 34.00 & 28.00 \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 34,00 & 0.00 & 26.00 & 0.0 .0 \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
\hline 100.00 & 121.00 & 129.00 & 153.00 & 176.00 & 169.00 & 175.00 & 175.00 \\
\hline 94.75 & 118.25 & 126.75 & 157.25 & 175.25 & 175.25 & 175.25 & 175.25 \\
\hline \(-5.25\) & \(-2.75\) & \(-2.25\) & 4.25 & -0.75 & 6.25 & 0.25 & 0.25 \\
\hline
\end{tabular}
55.00
43.00
\(\begin{array}{llll}0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \cdots & 0 & 0 & 0\end{array}\) \(\begin{array}{llll}0 & 0 & \text { in } & i n \\ 0 & 0 & \sim & \sim \\ 0 & 0 & 0 & \sim \\ & \sim & \sim & \ddots\end{array}\)



II
WEEK NUMBER
\begin{tabular}{rrrrrrrrr} 
I & II & III & IV & VIII \\
\hline 38.00 & 47.00 & 55.00 & 58.00 & 61.00 & 76.00 & 79.00 & 100.00 & \\
33.00 & 38.00 & 43.00 & 52.00 & 44.00 & 52.00 & 36.00 & 47.00 & \\
29.00 & 31.00 & 31.00 & 43.00 & 37.00 & 46.00 & 34.00 & 28.00 & \\
0.00 & 0.00 & 0.00 & 0.00 & 34.00 & 0.00 & 26.00 & 0.00 & \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & \\
100.00 & 115.00 & 129.00 & 153.00 & 176.00 & 174.00 & 175.00 & 175.00 & TOTAL \\
94.75 & 118.25 & 126.75 & 157.25 & 175.25 & 175.25 & 175.25 & 175.25 & REQUIRED \\
-5.25 & 2.25 & -2.25 & 4.25 & -0.75 & 1.25 & 0.25 & 0.25 & OUT OF BALANCE
\end{tabular}
\[

\]
\[
\begin{array}{r}
\frac{1}{38.00} \\
33.00 \\
29.00 \\
0.00 \\
0.00 \\
0.00 \\
100.00 \\
94.75 \\
-5.25
\end{array}
\]
\[
\begin{array}{r}
11 \\
47.00 \\
38.00 \\
31.00 \\
0.00 \\
0.00 \\
0.00 \\
115.00 \\
118.25 \\
2.25
\end{array}
\]
\[
\begin{array}{r}
111 \\
\hline 52.00 \\
43.00 \\
31.00 \\
0.00 \\
0.00 \\
0.00 \\
126.00 \\
126.75 \\
0.75
\end{array}
\]
\[
\begin{array}{rr}
\text { VII } & \text { VIII } \\
\hline 79.00 & 100.00 \\
36.00 & 47.00 \\
34.00 & 28.00 \\
26.00 & 0.00 \\
0.00 & 0.00 \\
0.00 & 0.00 \\
175.00 & 175.00 \\
175.25 & 175.25 \\
0.25 & 0.25
\end{array}
\]

WEEK NUMBER


\section*{WEEK NUMBER}

WEEK NUMBL
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline 1 & 11 & 111 & IV & V & VI & VII & VIII \\
\hline 38.00 & 47.00 & 52.00 & 58.00 & 61.00 & 76.00 & 79.00 & 100.00 \\
\hline 29.00 & 38.00 & 43.00 & 55.00 & 43.00 & \(52,0.0\) & 36.00 & 47.00 \\
\hline 28.00 & 33.00 & 31.00 & 44.00 & 37.00 & 46.00 & 34.00 & 31.00 \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 34.00 & 0.00 & 26.00 & 0.00 \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
\hline 95.00 & 118.00 & 126.00 & 157.00 & 175.00 & 174.00 & 175.00 & 178.00 \\
\hline 94.75 & 118.25 & 126.75 & 157.25 & 175.25 & 175.25 & 175.25 & 175.25 \\
\hline 00.25 & 0.25 & 0.75 & 0.25 & 0.25 & 1.25 & 0.25 & \(-2.75\) \\
\hline
\end{tabular}

\section*{WEEK NUMBER}
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline I & 11 & 111 & IV & V & VI & - VII & VIII \\
\hline 38.00 & 4.7 .00 & 52.00 & 58.00 & 61.00 & 76.00 & 79.00 & 100.00 \\
\hline 29.00 & 38.00 & 43.00 & 55.00 & 43.00 & 52.00 & 36.00 & 46.00 \\
\hline 28.00 & 33.00 & 31.00 & 44.00 & 37.00 & 47.00 & 34.00 & 31.00 \\
\hline 0,00 & 0.00 & 0.00 & 0.00 & 34.00 & 0.00 & 26.00 & 0.00 \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
\hline 0.00 & 0.00 & 0.00 & 0.00 & 0,00 & 0.00 & 0.00 & 0.00 \\
\hline 95.00 & 118.00 & 126.00 & 157.00 & 175.00 & 175.00 & 175.00 & 177.00 \\
\hline 94.75 & 118.25 & 126.75 & 157.25 & 175.25 & 175.25 & 175.25 & 175.25 \\
\hline \(=0.25\) & 0.25 & 0.75 & 0.25 & 0.25 & 0.25 & 0.25 & \(-1.75\) \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{\begin{tabular}{l}
ITEM \\
CYCLE \\
LENGTH
\end{tabular}} & \multicolumn{8}{|c|}{WEEK NUMBER} \\
\hline & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline 16 WEEK & 81.00 & 58.00 & 50.00 & 19.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
\hline \multirow{4}{*}{\[
\begin{gathered}
8 \\
\text { WEEK }
\end{gathered}
\]} & 38.00 & 47.00 & 52.00 & 58.00 & 61.00 & 76.00 & 79.00 & 100.00 \\
\hline & 29.00 & 38.00 & 43.00 & 55.00 & 43.00. & 52.00 & 36.00 & 45.00 \\
\hline & 28.00 & 33.00 & 31.00 & 44.00 & 37.00 & 47.00 & 34.00 & 31.00 \\
\hline & 0.00 & 0.00 & 0.00 & 0.00 & 34.00 & 0.00 & 26.00 & 0.00 \\
\hline & \multicolumn{8}{|c|}{WEEKLY LOAD TOTALS IN STD. HOURS} \\
\hline ACHIEVED & 176.00 & 176.00 & 176.00 & 176.00 & 175.00 & 175.00 & 175.00 & 177.00 \\
\hline REQUIRED & 175.25 & 175.25 & 175.25 & 175.25 & 175.25 & 175.25 & 175.25 & 175.25 \\
\hline OUT OF BALANCE & -0.75 & -0.75 & \(-0.75\) & -0.75 & 0.25 & 0.25 & 0.25 & -1.75 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{ITEM CYCLE LENGTH} & \multicolumn{8}{|c|}{WEEK NUMBER} \\
\hline & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\
\hline 16 WEEK & 80.00 & 56.00 & 47.00 & 17.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
\hline \multirow{4}{*}{\begin{tabular}{l}
\[
8
\] \\
WEEK
\end{tabular}} & 38.00 & 47.00 & 52.00 & 58.00 & 61.00 & 76.00 & 79.00 & 100.00 \\
\hline & 29.00 & 38.00 & 43.00 & 55.00 & 43.00 & 52.00 & 36.00 & 46.00 \\
\hline & 28.00 & 33,00 & 31.00 & 44.00 & 37.00 & 47.00 & 34.00 & 31.00 \\
\hline & 0.00 & 0,00 & 0.00 & 0.00 & 34.00 & 0.00 & 26.00 & 0.00 \\
\hline & \multicolumn{8}{|c|}{WEEKLY LOAD TOTAIS IN STD. HOURS} \\
\hline ACHIEVED & 175.00 & 174,00 & 173.00 & 174.00 & 175.00 & 175.00 & 175.00 & 177.00 \\
\hline REQUIRED & 175.25 & 175.25 & 175.23 & 175.25 & 175.25 & 175.25 & 175.25 & 175.25 \\
\hline OUT OF BALANCE & 0.25 & 1.25 & 2.25 & 1.25 & 0.25 & 0.25 & 0.25 & -1.75 \\
\hline
\end{tabular}
1. SHANKS, R.W.

Control of Production Processes.
To be submitted later as a Ph. D Thesis, University of Warwick.
2. THOMSON, D.R.

An Analysis of Decisions to Subcontract
IV. Sc Thesis. University of Strathclyde 1968.
3. BOMBERGER, E. E.

A Dynamic Programming Approach to a Lot Size Scheduling Problem.
Management Science V12 N11 1966.
4. HODGSON, T.S.

Addendum to Stankard and Gupta's Note on Lot Size
Scheduling.
Nianagement Science V16 N7 1970.
5. MADIGAN, J. G.

Scheduling a Multi-product Single Machine for an Infinite Planning Period.
Management Science V 14 N11 1968.
6.

STANKARD, IM. F. and GUPTA, S.K.
A Note on Bomberger's Approach to Lot Size Scheduling. Heuristic Approach
Management Science V 15 N 71969.
7. MAGEE, J. F.

Production Planning and Inventory Control McGraw Hill 1958.
8. EILON, S.

Elements of Production Planning and Control Macmillan Co. 1962.
9. ROLIS-ROYCE Operational Research Group Operational Research in Production and Inventory Control (Batching Algorithms)
Internal paper 1969
10. INTERNATIONAL BUSINESS MLACHINES CORPORATION Systems/360 Inventory Control 1968.
12. LE GARFF, A. and MALGRANGE, Y
13. BALAS, E.
11.
KENDALL, IN. G. and SNITTH, B. B.
Tables of Random Sampling Numbers
Cambridge University Press 1946. Proceedings of the 3 rd International Conference on Operational Research
English Universities Press 1963

An Additive Algorithm for Solving Linear Programs with Zero-One Variables.
Operations Research 131965
14. LAND, A. II. and DOIG, A. G.

An Automatic Method of Solving Discrete Programming Problems.
Econometrica, \(28 \quad 1960\).
BEALE, E. IV. L.
Survey of Integer Programming
Operational Research Quarterly. 1965

INTERNATIONAL COMPUTERS LIMITTED Linear Programming Mark 31971.

DAKIN, R.J.
A Tree Search Algorithm for Mixed Integer Programming Problems.
Computer Journal, 81965.```


[^0]:    KEY: T/T-TOOL TROUBLES; G/O-PALLET CHANGE-OVER; WTG.OP. -AWAITING OPERATORS.

[^1]:    Note: Formulae used accounted for production rates.

