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INDUCTION HEATING OF MILD STEED, VESSELS

IN A PULSATING FIELD

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SUMMARY

This work examines the heating of mild steel vessels in a pulsating field. The vessel heater is a constant voltage mains frequency device which is closely related to a transformer with a short circuited secondary. The development of a design method is the objective of this thesis which is primarily concerned with unshielded heaters.

Chapter two discusses the electrical design problems: the number of heater coil turns required and the prediction of its powerfactor. The chief complication is magnetic saturation and it is worsened by the lack of a highly permeable magnetising flux path. Chapter three outlines the solutions of the latter problem, whilst Chapter four examines the literature and other authors' approaches to the total problem. It shows the necessity of further experimental work. Chapter five describes the author's experiments. Simple power index laws relating power input and magnetising flux are demonstrated and compared with the theories discussed in Chapter four. The permeances of the major flux paths are found to be unaffected by magnetic saturation. Chapter six's analysis rests on this work. A new theory of field distribution above a permeable and conductive surface is developed and used to describe the permeances and the loss distribution. This analysis combines with the vector diagram yielding a design method. It is presented in tabular form in Appendix D and is demonstrated in the design study of Chapter seven. The related problem of flux shielded heaters is discussed in Appendix A. It is felt that the major problems of design have been overcome in this work.

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LIST OF SYMBOLS

The list contains certain symbols specific to particular theories; a few symbols have been changed to avoid confusion but in general those of the original author have been kept. Wherever possible generally accepted characters have been used, though this does allow some overlap when discussing related electrical and thermal problems; this is explained in the relevant text.

A Potential function

	Coil resistance function (Vaughan and Williamson)	Ω
Ag	Area of coil-vessel gap	m ²
an	Harmonic solution constant	-
A2	z directed magnetic vector potential function	V/ms
	between coil and vessel	
A 26	Leakage component of A2	V/ms
A2m	Magnetising component of A2	V/m s
В	Flux density	Т
	Vessel resistance function	Ω
	(Vaughan and Williamson)	
B	Maximum flux density (Agarwal)	Т
Bn	Harmonic solution constant	-
B _n	Knee-point flux density	Ţ
B _{sat}	Saturation flux density	Г
С	Coil reactance function (Vaughan and Williamson)	Ω
	Specific heat	W/kg

Cn Harmonic solution constant

- v -

D	Vessel reactance function	Ω
	(Vaughan and Williamson)	
	Diameter of inverse ellipsoid	m
Dn	Harmonic solution constant	-
Е	Electromotive force	V
	Electric field strength	V/m
	Air gap reactance function	Ω
	(Vaughan and Williamson)	
F	Classical eddy-current loss function	-
G	Leakage flux function for an infinite vessel	-
Gr	Grashof number	-
Η	Magnetic field strength	At/m
Н	Heat input to the vessel walls	W
H _f	Heat loss to vessel contents	W
Hr	Reaction field set up by eddy-currents	At/m
Hn	Knee point value of H on the B-H curve	At/m
Hs	Vessel surface H (Dreyfus)	At/m
Hs	Heat loss from the outer surface of the	
	vessel	W
Ht	Tangential surface value of H	At/m
H,	Hankel function	-
H	Maximum field strength	At/m
I	R.m.s. current	A
Il	R.m.s leakage current	А
I _m	R.m.s. magnetising current	A
I _t	R.m.s. total heater current	A

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 A/m^2 Current density J Nagioka's constant relating field strength Kn to finite coil length effects Vaughan and Williamsons' modification of K Nagioka's constant for induction heating A/m² Peak value of current density z directed K, (Stoll and Hammond) Constants defined by Vaughan and Williamson K ... K Sum of exponential series M Number of coil turns N Demagnetising factor for an ellipsoid 0_f (Osborn) Reactance multiplying constant for a solid P cylinder (Baker) W Eddy-current loss P W Hysteresis loss Ph Power input to heater (Baker) W Po Prandtl number Pr Power input to heater (Vaughan and Williamson) W P+ W Power input to vessel (Baker) Pw Resistance multiplying constant for a solid Q cylinder Ω Heater coil resistance R Ω Reluctance of magnetising flux path (Baker) R

Re	Reynolds number	-
R _h	Equivalent hysteresis resistance	Ω
Rw	Vessel resistance	Ω
Rw	Vessel resistance refered to the coil	Ω
R,	Primary resistance of transformer	Ω
R2	Secondary resistane of transformer	Ω
S	Vessel current distribution factor	-
V	Voltage	V
	Velocity in main stream or mean velocity	
W	Loss density	W/m ²
Wn	Loss density at the knee point of the B-H curv	e W/m ²
Xc	Heater coil conductor reactance	Ω
Х _е	Magnetising reactance (Baker)	Ω
Xg	Reactance of the air gap path between coil and	
	vessel	Ω
Xm	Magnetising reactance	Ω
Xo	Total reactance refered to the supply	Ω
Xw	Vessel reactance	Ω
х,	Transformer primary reactance	Ω
X2	Transformer secondary reactance	Ω
Zo	Total heater impedance (Baker)	Ω
a	real constant of a bilinear transformation	-
b .	77 77 77 77	m
	steel saturation law constant	T/(At/m)
	bus-bar semi- thickness	m

с	Real constant of bilinear transformation	-
d	Real constant of bilinear transformation	-
dc	Diameter of coil	m
ds	Diameter of flux shield	m
đw	Diameter of vessel	m
f	Supply frequency	Hz
g	Mean coil-vessel gap	m
h	Bus-bar thickness	т
	Plate thickness	m
	Limiting penetration depth (Dreyfus)	т
^h eff	Effective power penetration depth	m
j	Complex operator √-1	-
k	Diffusion equation constant	
11	Diffusion equation constant	1810.
	Thermal conductivity	W/Cm
1	Thermal conductivity Base length for dimensional analysis	W/Cm m
l l [.] c	Thermal conductivity Base length for dimensional analysis Axial length of heater coil	W/Cm m m
l lc lw	Thermal conductivity Base length for dimensional analysis Axial length of heater coil Axial length of vessel	W/Cm m m
l lc lw m	Thermal conductivity Base length for dimensional analysis Axial length of heater coil Axial length of vessel Initial magnetisation curve index B=b H ^m	W/Cm m m
l lc lw m	Thermal conductivity Base length for dimensional analysis Axial length of heater coil Axial length of vessel Initial magnetisation curve index B=b H ^m Loss index	W/C°m m m
l lc lw m n p	Thermal conductivity Base length for dimensional analysis Axial length of heater coil Axial length of vessel Initial magnetisation curve index B=b H ^m Loss index Perimeter of heater coil	W/C°m m m - - m
l l _c l _w m n p	Thermal conductivity Base length for dimensional analysis Axial length of heater coil Axial length of vessel Initial magnetisation curve index B=b H ^m Loss index Perimeter of heater coil Reactance multiplying constant for a solid	W/Cm m m - - m
l lc lw m n p	Thermal conductivity Base length for dimensional analysis Axial length of heater coil Axial length of vessel Initial magnetisation curve index B=b H ^m Loss index Perimeter of heater coil Reactance multiplying constant for a solid cylinder (Vaughan and Williamson)	W/C°m m m - - m
l lc lw m n p	Thermal conductivity Base length for dimensional analysis Axial length of heater coil Axial length of vessel Initial magnetisation curve index B=b H ^m Loss index Perimeter of heater coil Reactance multiplying constant for a solid cylinder (Vaughan and Williamson) Skin effect factor (Stoll and Hammond)	W/C°m m m - - m -

a	Distance between points on coil and vessel	m
	Resistance multiplying constant for a solid	
	cylinder (Vaughan and Williamson)	-
	π / pole pitch of current sheet (Stoll and	m ⁻¹
	Hammond)	
	and the second and the second and the	m
r _h	Radius of vessel to limiting penetration depth	
	(Dreyfus)	~
rm	Mean of rh and rs	m
rs	Vessel radius (Dreyfus)	
S	Thickness of the vessel wall	m
t	Time	S
u	Potential function	-
	Distance from coil mid plane	m
		_
v	Stream function	
W	Angular frequency	Hz
	Bilinear transformation function	-
x	Cartesian co-ordinate	
xs	Decay depth of permeability (Ollendorf)	
у	Cartesian co-ordinate	m
Z	Complex number	m
	Cartesian co-ordinate	
Г	Coil-vessel gap field decay correction	
	factor (Lavers)	-
		_
α	Loss index	Tearees
	Phase angle total ilux to total current	acgrees.

- x -

β	Loss constant	
	Phase angle magnetising flux to total flu	ax degrees.
	Relative thickness to skin depth	-
	Coefficient of thermal expansion	m/C°
γ	Finite length leakage flux modifying func	tion -
δ	Classical eddy-current skin depth	m
0	Phase angle leakage flux to magnetising f	lux degrees.
θ,	Vessel wall temperature	C°
ę	Vessel wall temperature	C°
~	Pacistivity	Qm
P	Dengity (5 10 and B)	ka/m ³
,	Appendit pocistivity	Ωm
p	Apparent resistivity	
τ	Loss index, flux driven	-
μ	Magnetic permeability	Wb/At m
μ	Rolative permeability	-
μ _c	Surface value of permeability (Dreyfus) (01	lendorf)
μ _o	Free space permeability	Wb/At m Wb/At m
4	Magnetic flux	W5
ø	Haghette Hux	W/m^2
d.º	Magnetic flux per unit length of vessel a	perinhery WVm
ø	Laskage flux	Wb
øl	Magneticing flux	Wb
m d	Magnetic flux per unit length of vessel a	perinhery
^p n.	at the knee point of the R-H curve	Wb/m
d	Total heater magnetic flux	Wb
pt	TOTAL HEADET. MAGNETIC IIUX	110

CHAPTER 1 INTRODUCTION

CHAPTER SUMMARY

A brief account is given of the applications of this form of vessel heating, and the aims and objects of the thesis are stated.

1 Introduction

The chemical engineering industry frequently needs a flame-proof source of heat. Heat is required for certain reactions, and yet the reaction products may be highly inflammable. A typical example is to be found in the production of paint resins. Paint is produced by a batch process in steel retort vessels. The retort cannot be flame heated because of the fire hazard. It is in this type of application that Induction Heating comes into its own, being in practical terms the method with the simplest form of construction and lowest capital cost. Induction heating creates heat where it is most useful- directly in the vessel walls. Other methods of heat input require intermediate stages between the production of heat and the heat input to the vessel. Non-electrical methods of heating, e.g. by a heat transfer oil, require ancillary equipment and inevitably cost more. These other types of heating introduce an additional thermal mass which complicates the control of the reaction. Induction heaters are currently offered in a range of 9kW to 454 kW. However, each heater is an individual construction, and current design methods are insufficiently accurate to permit manufacture on other than a cut and try basis.

The aim of this thesis is to improve the methods of design and to enlarge the understanding of induction vessel heaters.

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CHAPTER 2 THE INDUCTION HEATING PROBLEM

CHAPTER SUMMARY

Induction heating is discussed by means of comparison with a transformer with a shorted secondary, and this is used to produce an equivalent circuit. In section two the eddy-current loss mechanism is discussed in non mathematical terms, and the discussion shows the importance of the steel properties and the tangential magnetic field strength. The equations controlling the external field are discussed, and the complication of magnetic saturation is introduced.

CHAPTER CONTENTS

- 2-1 Principles
- 2-2 Eddy-current loss

2-1 Principles

Induction Heating is based on the mechanism of induced ohmic loss. If a current is passed through a conducting body, in this case a steel retort vessel, it heats that body. If heating is done directly, this means that electrodes have to be fixed to the surface of the steel, a current being fed from a matching transformer through the vessel between these electrodes.

A matching transformer changes the supply voltage to a suitable secondary voltage, the transformer being a source of both loss and capital cost. By this arrangement (fig. 2-1) the vessel takes the place of two parts of the transformer; it has become both the secondary winding and the central limb of the flux path. Under certain conditions the rest of the magnetic circuit can be removed and this is the usual form of an Induction Heater ⁵⁸ (fig. 2-2). There is a point of interest as this machine corresponds closely to the air-gap transformer of reference 36: the chief difference is that the transformer has a laminated core; the important and encouraging aspect is that the machine is claimed to have a performance comparable with a conventional transformer.

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Vessel

Matching Transformer

_Fig. 2-1 Direct Conduction Heating.



Vessel

<u>Fig 2-2</u>

Induction Heating

Much may be learnt about the vessel heater by comparison with a conventional transformer on load. The physical differences show in the performance of the Induction Heater. A transformer has well defined flux and current paths, which in turn give rise to its lumped equivalent circuit (fig. 2-3). The circuit



Fig. 2-3

Transformer Equivalent Circuit

consists of lumped resistances and reactances which are not necessarily formed by discrete flux and current paths. The magnetising reactance X_m is set up by the flux which links with both primary and secondary windings, whereas the leakage reactances X_1 and X_2 are formed by the fluxes which link one coil only. The two resistances R_1 and R_2 are created by the primary and secondary windings. The hysteresis resistance R_h is included to account for the transformer core loss. This equivalent circuit may be simplified when representing an Induction Heater (fig. 2-4). The Induction Heater has no secondary terminals and it is



Fig. 2-4

Induction Heater Equivalent Circuit

reasonable to refer all quantities to the exciting winding. Furthermore, there cannot be loss in the non-existent transformer limbs and thus R_h must be infinite. The primary resistance is unaltered. The secondary resistance has become part of the load and its calculation is not as straightforward as might at first be supposed. The load resistance is bound up with the mechanism of eddy current heating, and this is both determined by and determines the field pattern outside the vessel.

2-2 Eddy current loss

The eddy current heating mechanism is the basis of the machine. Consider a ring from the vessel middle. The mild steel annulus carries a pulsating flux of r.m.s. magnitude ϕ . It is reasonable to assume that, as a first approximation, the flux-density is uniform. The pulsating flux induces an e.m.f. according to Faraday's Law; a progressively larger e.m.f. E is induced from the centre of the ring outwards. The induced e.m.f. drives an eddy current through the resistance of the ring creating heat. The eddy current sets up its own reaction field H_r . Thus H_r increases from the outer surface inwards and acts on the magnetic permeability of the material to give a reaction flux density. A second approximation to the flux density has been derived. This simple approach serves to illustrate a number of points. Firstly, the eddy currents screen the inner steel from flux, creating a skin where flux, current and losses exist. Secondly, the loss depends on frequency, resistivity and permeability. Thirdly, the tangential field strength is controlled by the eddy current loss.

In an Induction Heater it is the field pattern set up by the exciting coil that influences the distribution of loss over the vessel. The field pattern controls the overall performance of the heater, describing both the reluctances of magnetising and leakage flux.

It can be shown simply, ⁶ that the tangential component of H and the normal component of B are continuous across a metallic boundary. Thus the surface value of H in the steel is also the air-gap value just outside the steel. It is this component which sets the level of flux and loss. The tangential value of field strength is not uniform over the surface of the vessel; both H_t and the loss are greatest under the coil centre.

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The Induction Heating problem would be relatively straightforward if this were all that was involved. However the air-gap field is Laplacian, obeying the equation

 $\nabla^2 A = 0$

which when written in Cartesian coordinates in full is -

$$\frac{\partial^2 \mathbf{x}}{\partial \mathbf{x}^2} + \frac{\partial^2 \mathbf{x}}{\partial \mathbf{y}^2} + \frac{\partial^2 \mathbf{x}}{\partial \mathbf{z}^2} = 0$$

when A represents a scalar quantity such as magnetic scalar potential, and when A is a vector quantity i.e flux density B or field strength H

$$\frac{\partial^2}{\partial x^2} \frac{A_x}{x^2} + \frac{\partial^2}{\partial y^2} \frac{A_x}{x^2} + \frac{\partial^2}{\partial z^2} \frac{A_x}{x^2} = 0$$

$$\frac{\partial^2}{\partial x^2} \frac{A_y}{x^2} + \frac{\partial^2}{\partial y^2} \frac{A_y}{x^2} + \frac{\partial^2}{\partial z^2} \frac{A_y}{x^2} = 0$$

$$\frac{\partial^2}{\partial x^2} \frac{A_z}{x^2} + \frac{\partial^2}{\partial y^2} \frac{A_z}{x^2} + \frac{\partial^2}{\partial z^2} \frac{A_z}{x^2} = 0$$

under certain conditions the field quantities in the steel vessel obey the Diffusion equation -

 $\nabla^2 A = k^2 A$

which may be expanded in a similar manner to Laplace's equation. The diffusion equation constant k for steel is given by the relationship -

$$k^2 = \frac{jW\mu}{\rho}$$

where

 $j = \sqrt{-1}$

w = angular frequency

 μ = magnetic permeability

 $\rho = resistivity$

The field problem of this air and steel combination has

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Magnetisation and Permeability curves for Mild Steel

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been solved 38 and the solution is discussed in section 4-3-8. However, the tacit assumption has been made that the magnetic permeability is a constant whereas the B-H relationship is a non-analytic curve. The curve for mild steel is shown in fig. 2-5 together with the permeability curve. The permeability is defined as the slope of the chord passing through the origin on the B-H curve. The permeability rises to a marked knee point and decays with increasing H or B. Moreover, the variation of µ has two main effects. The first effect is that the flux in the steel is no longer linearly related to the field, so that the penetration pattern of flux and loss in the vessel is altered and the surface value of Ht is linked non-linearly to the flux within the vessel. The second difficulty arises from the first in that the field pattern changes shape with the level of excitation. Hence the changing field pattern alters the resistance and reactances of the machine.

Since the major problems arise from the non-linear B-H relationship it is felt that it would be worthwhile to elaborate on the magnetisation process. Weiss has shown that the increased flux density due to the presence of iron arises from magnetic domains, (Brailsford describes this theory $^{\mathcal{B}}$). Domains are small regions within the crystals of steel which have a self magnetic field. In a demagnetised state these domains are semi-randomly orientated through the steel. They have preferential directions in the crystal lattice but in most materials

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the crystals themselves have a random orientation. In any case the domains order themselves in the positions of minimum stored energy. They produce no net external field of their own. A small externally applied field causes a growth of favourably orientated domains at the expense of the other domains. The domains continue to increase with an increasing applied field until an instability sets in to the domain pattern. The pattern now changes in sudden small finite steps - Barkhausen jumps. This is the steepest part of the B-H curve. The Barkhausen process will continue until the domains are fully aligned in the crystal lattice directions closest to the direction of the applied field. Increasing H above this level pulls the domain alignment from the crystal axes, the steel is operating above the knee of the B-H curve. The above is a description of the initial magnetisation process which does not include the effect of eddy-currents. All that will be said at this point is that eddy-currents have a damping effect. Similarly the magnetisation process must influence the eddy-current mechanism. If the steel is excited with a cyclic flux the magnetisation curve follows the familiar B-H loop, where the peaks of the loop lie on the initial magnetisation curve. Similar processes occur within the domains in loop processes as in the initial curve.

Induction Vessel Heating requires the operation of the vessel steel at flux levels that are well into

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saturation. Moreover the steel is operated under conditions of elevated temperature and strong eddycurrent flow, all of which influence the interrelationship of B and H⁴⁷. An important point of discussion in future chapters will be the selection of a suitable mathematical substitution for this relationship. However this must not obscure the chief aim and object of this thesis, namely to provide a workable engineering solution for Induction Heater design. The major problems to be overcome are those of eddy-current loss in steel and the free field distribution surrounding the vessel and heater coils.

CHAPTER 3 METHODS OF SOLUTION OF THE EXTERNAL FIELD

CHAPTER SUMMARY

This chapter concerns the fields outside the vessel. It extends the discussion of Laplace's equation, begun in the previous chapter, to cover the possible means of solution. It outlines their weaknesses and strengths, and brings the study to a point where a selection of the most suitable method of analysis may be made in Chapter 6.

CHAPTER CONTENTS

- 3-1 Laplace's equation
- 3-2 Solution by empirical assumption
- 3-3 Harmonic solutions
- 3-4 Method of images
- 3-5 Complex function solutions
- 3-6 Analogues

3-1 Laplace's Equation

Half of the total Induction Heating problem is concerned with the field outside the vessel. The field in air outside the vessel is governed by Laplace's equation $\nabla^2 A=0$. This equation is the simplest form of partial differential equation; its solutions obey Duhamel's Theorem - they are super-imposable, and can be added. Moreover the solutions are closed and obey the Cauchy-Riemann conditions which are :

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$
$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$







As Laplace's equation is the simplest form of partial differential equation and boundary value problem it is also natural that it should also have the widest possible range of solutions. They range from empirical assumptions as used by Pohl⁵¹ for fields in salient pole machines to the elegant Schwartz-Christoffel transformation methods applied by Gibbs²⁸, Carter¹⁴ and others. It is the purpose of this chapter to discuss these methods and highlight their suitability and their weaknesses. Only certain methods can be applied to the problem of Induction Heating of vessels.

3-2 Solution by Empirical Assumption

Pohl⁵⁷ successfully applied this technique to salient pole machines. This method works in this particular case because the designer is able to control the field patterns. There is no additional magnetic field set up by eddy-currents and hence the field pattern is essentially magnetising. Moreover this field is two-dimensional in a long salient pole machine. Conditions are very different in the vessel heating problem. Firstly, it is not easy to visualise the full field pattern which is influenced by eddy-currents. Secondly, the magnetising flux effects are strongly three-dimensional and very sensitive to error.

3-3 Harmonic Solutions

Although this technique may be applied in three dimensions, for simplicity it will be discussed in two dimensions. In Cartesian coordinates Laplace's equation

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becomes

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = 0$$

and a general form of the solution may be written $A = [O_n e^{knx} + B_n e^{-knx}] [C_n Sin(kny) + D_n Cos(kny)]$

where $Q_m B_n C_n, D_n, k$ are constants. Applying the principle of superposition

 $A = \sum_{n=0}^{\infty} [Q_n e^{knx} + B_n e^{-knx}] [C_n Sin(kny) + D_n Cos(kny)]$ Now since any single value continuous periodic function may be written as a Fourier series of the form

$$S = \sum (M) [C_n Sin(kny) + D_n Cos(kny)]$$

it follows that Q, B, C, D, may be found by matching given boundary conditions and hence a solution for A may be obtained. This procedure may also be applied to cylindrical coordinates. Moreover if the period of y is sufficiently large the effect of repeated excitations are minimised and the solution becomes that for an isolated excitation pattern. This form of solution also applies to regions obeying the diffusion equation and as such it offers the possibility of a complete solution to the Induction Heating problem 38,39 . However, this is only applicable in as far as the diffusion equation truly describes the behaviour of steel. The second criticism is that the spacial harmonics introduce artificialities into the problem. For example, consider the unit stepfunction. When this is described in terms of a Fourier series the function over shoots at the discontinuities and it becomes impossible to fully represent the stepfunction.

Providing that geometries are simple and that some mechanical means are available to calculate the Fourier series sums the harmonic method provides an excellent means for handling the excitation and field problems. Nevertheless if the field solution is to be at all accurate the large number of terms in the solution masks the true nature of the field pattern and the method does not lead to a good physical understanding of the problem.

3-4 Method of Images

A third and interesting method of solution is the Method of Images. The method relies on the properties of material surfaces. It can be shown ⁶ that if the material beneath the surface has a very high magnetic permeability ($\mu_r > 100$) then the surface of that material behaves as an equipotential to the surrounding field. Similarly if the vessel material is highly conductive then it acts as a barrier to flux, and the surface becomes a line of force. The Method of Images may now be introduced



Fig. 3-2

High permeability image of a conductor

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Fig. 3-3

High conductivity image of a conductor

utilising this as a basis. Consider the field pattern produced by a pair of conductors carrying currents in the same direction (fig. 3-2). The straight line of equipotential passes midway between the two conductors. This equipotential could be the steel surface, in which case the second conductor can be seen as a reflection of the source current in the surface. Similarly fig. 3-3 shows the field pattern around positive and negative current flows and it is the field line that forms the effective reflective surface: this case corresponds to the high conductivity condition. Either case may be extended by the principle of super-position and field patterns can be calculated for complicated conductor arrangements surrounding simple material geometries. In this way the method is much more powerful than the . harmonic method previously discussed. Surface field strengths are easily calculated and conductor geometries may be modified to change heating patterns.

The actual vessel heating problem concerns surfaces which are conductive and permeable. Stoll and Hammond⁵⁵ alone apply the method to this case and they show that the image is out of phase with the coil current and that its relationship can only be determined from another type of solution. Chapter 6 extends and frees their work from this limitation.

3-5 Complex function solutions

It is well established that any complex function in its real and imaginary parts forms a solution to Laplace's equation⁵. Since only two variables are involved the solutions have to be two dimensional. Complex function solutions can, in general, only be applied to cases where the boundaries to the problem fit a known complex function and because of this its range of application is limited.

Any known solution may be modified by a bi-linear transformation - $w = \frac{az+b}{cz+d}$ where a,b,c,d are real constants, and w and z are complex variables. The bi-linear transformation effects a rotation magnification or inversion of the solution. It is the property of inversion which is most useful, as in this case geometries are radically altered. In the field of

aerodynamics the equations of fluid flow are similar to magnetic field equations. Bi-linear transformations are

Air Flow Stagnation Pattern Point

Fig. 3-4 Joukowsky Aerofoils

used to produce the flow pattern around certain aerofoil sections fig. 3-4. The lift and drag may be calculated from this field pattern.

If the geometry of the problem is simple and consists of straight uniform potential boundaries with a small number of right angles, the complex function may be built up using the Schwartz-Christoffel transformation. The method may be applied to gaps in magnetic circuits to calculate fringing effects and the results of this type of analysis will be applied to the problems of flux-guides in small induction heaters (App. A). However, this

method cannot be applied to a three dimensional problem such as a large induction heater where the flux paths are free.

3-6 Analogues

There remains one major avenue of attack in a Laplacean field problem, namely, that of modelling the problem. The most obvious model is a real vessel heater where measurements can be taken directly from the apparatus (Chap. 5). Otherwise the analogue may either be numerical or physical.

There is one major criticism which applies to all analogue methods. The model can only represent one problem in particular. Although the results may be partially generalised by the formation of dimensionless groups, the fact remains that general trends cannot be established from a single model. Only by building numerous models can the general behaviour be predicted. It is felt that apart from the real heater analogues should be seen as a check of analysis, or as a last resort if analysis proves to be impossible.

Numerical analogues are due to Southwell⁵⁴. The technique consists of considering the Laplacean field split into small regions where the equation may be linearised. The method may be applied in any number of dimensions. As a solution it is a long and slow process by hand and, in general, requires the use of digital computer.

Physical analogues may be used. The principle of the method is that the analogue quantity obeys Laplace's equation. Resistance analogues are commonly used³⁵. They may be either a resistance paper model or an electrolytic tank analogue; the former applies to two dimensional problems whilst the latter is used to represent three dimensional situations. Lozinskii⁴¹ describes the modelling of the induction heating problem by an electrolytic tank. A criticism of resistance analogues is that they model from one invisible system to another. A much more pleasing solution arises from the use of visible analogues:

CHAPTER 4 LITERATURE SURVEY

CHAPTER SUMMARY

The relevant literature is concerned with two broad categories the induction heating mechanism in the vessel wall, treated here in order of complexity, and the total circuit impedance, considered by reference to Baker's⁴ design theory.

CHAPTER CONTENTS

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4 Literature Survey

4-1 Previous Published Work

Chapter 2 has shown that there are two major interrelated problems: the effect of eddy-currents in steel and the prediction of the free air flux pattern surrounding the partially saturated steel vessel. There do not appear to be any references which deal specifically with both problems jointly and fully. Although certain papers deal in part with both questions, this survey discusses problems separately. The advantage of separate treatment is that it enables papers to be drawn upon which would otherwise be excluded. These papers relate to machines with associated problems - the eddy-current coupling, and from research specifically in eddy-currents and fields.

4-2-1 Eddy-currents in Steel

Many authors have attempted the problem of eddycurrent loss in steel. The chief difficulty of solution rests in the analytical representation of the B-H curve and the methods of solution may be typed by curve fit. The chosen curve and the method of inclusion into the solution naturally influences the resulting theoretical loss.

4-2-2 Linear Solutions

The simplest analytic relationship that can be given

to the B-H curve is a straight line i.e. B=µH. It is natural that this the simplest relationship should be the first used. J. J. Thomson published a paper in 1892; this classic work ⁵⁷ dealt with eddy-current loss in thin sheets of steel. He concerned himself with a uniform double-sided sinusoidal field excitation, and the solution is thus one dimensional. He showed that the loss in the sheet was :

$$W = \frac{h^{2} \rho m^{3} B^{2} (e^{2mh} - e^{-2mh} - 2Sin(2mh))}{\mu (e^{2mh} + e^{-2mh} - 2Cos(2mh))}$$
(4-1)
here

B = average flux density carried by the sheet m = $\sqrt{w\mu/\rho}$ (4-2)

- W = power loss per unit area
- h = plate thickness
- μ = magnetic permeability
- w = angular frequency

o = resistivity

W

The above is given in terms of M.K.S. units. Equation 4-1 is still the standard loss equation for packets of laminations. The paper demonstrates that at 100 Hz there is no greater loss in an infinite plate than a 2 mm thick plate. In one sense then this paper can act as a guide to the induction heating problem for it already shows that the thickness of steel used for the vessel walls has a diminishing effect on the heat produced in them.

The simplest eddy-current problem is the penetration

of a single electromagnetic wave into a semi-infinite constant permeability steel plate. Lammeraner and Stafl reproduce this theory in their excellent book ³⁷ "Eddy-currents" in Chapter 3. A tangential sinusoidal H wave with period w and peak value H is absorbed by the conducting material. It is proved that the induced current, flux density and e.m.f. decay exponentially and vary in phase with depth. The total eddy-current may be thought of as flowing in a skin layer δ , where

 $\delta = \sqrt{2\rho} / w\mu$ (4-3) The power loss density which is derived from an I² R integral is

$$W = \frac{H^2}{2\delta}$$
(4-4)

The solution like that of Thomson is based on Maxwell's equations, and it is hardly surprising that the results are similar. However, this reference gives a useful table of skin depths for copper, aluminium, and steel.

Vessels are naturally cylindrical and it might be expected that the most useful loss solutions would be those treating a cylindrical geometry. The first of these is due to Heaviside 33 . Again this solution is for a material of constant μ . The diffusion equation applies and a one dimensional solution is obtained. The cylindrical nature of the problem causes the diffusion equation to have a Bessel form whose solution is in terms of complex Bessel functions. Heaviside did not put his results in this form but left them as a series. McLachlan⁴⁵ has given this solution together with tables of the necessary functions. The power loss for a unit length of cylinder is given by:

$$P = \pi H^{2} \rho ma \left[\frac{M(ma)}{M(ma)} \right] Cos(\theta_{1} - \theta_{0} - (\pi/4))$$
(4-5)
here:

$$m = \sqrt{w \mu / \rho}$$

$$a = cylinder radius$$

$$M_{0} = magnitude of the complex Bessel function J_{0}(zi^{2})$$

79

 $\Theta_{o} = argument$

wł

99

99

$$W = \frac{H^2}{28} \rho \sqrt{\frac{\pi}{4}}$$
(4-6)

The equation is identical in form to the semi-infinite slab equation 4-4, with a multiplying factor of $\sqrt{\pi/4}$. Hence this is an indication of the error in taking a flat plane solution to a cylindrical problem. Dwight and Bagai ²² have applied Heaviside's solution to a coreless induction furnace. The coreless induction furnace has the same geometry as a vessel heater, but it contains a high temperature non-magnetic core in place of the vessel. Baker ⁴ has extended Dwight's work to account for induction heated bodies with walls of varying thickness. The results are plotted as two sets of design curves, P curves associated with the imaginary part of the complex volt-amps of the body, and Q curves with the real part. However the range of curves is limited to values of the ratio of work diameter to skin depth of 12.

The authors have shown in the papers discussed that the geometry of the loss member affects both the phase and magnitude of the loss. It is apparent that the power-factor for large cylindrical bodies is the same as for large flat bodies. The loss is also shown to be dependent on the magnetic permeability of the load. The chief difficulty in applying a linear solution to the eddy-current loss in steel lies in the choice of μ . It is known that the permeability reduces with saturation so that the loss cannot be simply related to H^2 .

4-2-3 Semi-linear Solutions

In general the semi-linear methods, as they might be termed, rely on a linear solution for the loss in steel in terms of H and μ ; then finding the substitution for μ at the level of H to give the required loss. Various equations are given relating μ and H or μ and B etc.^{4,9} However, these are laborious trial and error methods as the simultaneous equations are not usually soluable. The most successful method is that of Davies, applied to an eddy-current coupling, he derives a Gibbs type solution² for the loss in an eddy-current drum as : $(\mu\nu)^{\frac{1}{4}}$ H = $\sqrt[7]{8W^2/\rho W}$ (4-7)



Eddy current loss and relative permeability.

V

<u>Peak average flux density</u> <u>for Dynamo Steel at 50Hz</u> (<u>Curves taken from Brailsford ⁷and replotted</u>)

<u>Fig 4-1a</u>

He uses a subtile substitution of

 $(\mu\mu)^{\prime\prime}$ H = kHⁿ

which enables the loss to be written in terms of H alone. The substitution is shown to be accurate for H greater than 250 At/m. In a later paper Davies indicates that the substitution is equivalent to

 $B = bH^{m}$ (4-9)

4-2-4 The effects of non-linearity

Perhaps the most worrying piece of work with relevance to linear theories is the practical measurement of anomalous loss by Brailsford 7 . This work is an experimental comparison with Thomsons formula (4-1) for packets of laminations. The tests were carried out over a wide range of excitations, from well below the kneepoint of the B-H curve to full saturation. The results show that the measured loss is over one-and-a-half times the predicted loss even below the knee-point where µ is usually considered to be a constant. Brailsford's work is only relevant if the anomaly is not brought about by the special metallurgical conditions of lamination. Even so, his work is a little disturbing and forces the conclusion that it may not be possible to treat steel as a uniform material with constant properties.

A full non-linear solution for loss and reactive energy is required. A description of the results of nonlinearity will be helpful in a discussion of these types of solution. The chief effects are to cause :

(4-8)

a. distortion of the flux penetration pattern7

b. non-sinusoidal oscillation of flux density^{7,1} Now in a linear condition the phase changes with depth³⁷ Hence (a) alone causes an alteration of the resistance and power-factor of steel, whilst (b) has similar effects to (a). The differences between the various solutions depend on the weight placed on these two factors. Unlike linear solutions the results will vary with the type of excitation: generally these are either sinusoidal flux and voltage or sinusoidal field intensity and current. Although these conditions are seldom realised in practice, the majority of real applications fall close to one or the other.

The other question which has to be settled when a non-linear solution is being built up is to decide the process that the steel is actually going through. Most authors assume that the B-H law followed is the initial magnetisation curve. Although, it might be more accurate to assume some cyclic loop relationships, the usual argument runs that if the hysteresis loss is small then the B-H loops are thin and are not far removed from the initial magnetisation curve. Moreover, the peak values of B and H lie on the initial curve. However, there is no guarantee that the behaviour of steel in tests where eddy-currents are minimised, e.g. a magnetisation experiment, should bear a close relationship to the magnetisation process during eddy-current loss. With this in mind some authors have gone to a domain level approach ^{52,61} but the more microscopic the approach the harder it is to apply the theory to massive steel.

4-2-5 Equivalent sinusoidal solutions

The easiest way to deal with non-linear equations, apart from assuming them to be linear, is to find an equivalent set of linear differential equations to solve. This type of solution relies on the principles discussed in McLachan's book 44 . The principle is that of equivalent energy. He states the principle as follows : "the energy dissipated per cycle is equal in the nonlinear and equivalent linear systems.". He applies it to a series tuned L-C-R circuit with a saturated inductance. Panasenkov⁵⁰ has applied the principle of equivalent cyclic energy to the problem of eddy-current loss in massive solid iron. He has done this by using a complex magnetic permeability to account for the phase shifting effect of non-linearity. It is shown that the maximum value of magnetic energy (B.H) is twice that of the mean energy, in all cases irrespective of non-linearity. This fact is used to decide the phase shift between B and H, and thus gives the complex permeability. Panasenkov points out that B.H. versus H is a straight line over a wide range of values and that this may be used to give a fully analytic solution to the complex permeability. The method is applied to a thin steel ring excited by a uniform torroidal coil which is, of course, not directly

applicable to induction vessel heating. Subba Rao⁵⁶ has proposed a graphical form of the same method and applied it to the solid square section loss problem solved linearly by Bewley⁵. Subba Rao's treatment has the virtue of being simpler and more intelligible than that of Panasenkov. Both solutions apply to problems with uniform surface excitation.

The basis of the equivalent sinusoidal solutions is that the higher harmonics may be included in the fundamental. The solutions are only correct on this basis. Their chief strength is that they may be applied to any known solution with a uniform surface excitation with but one other reservation that although the loss and power factor of the steel will be nearly correct the field distribution must be in error.

4-2-6 Solutions with an assumed distribution of µ

A more rigorous form of solution is given when an approximation to the B-H curve is used to give the distortion of the flux pattern, and taking μ to be constant with position. It is the time variation of μ which generates the magnetising harmonics. This form of solution can be expected to give the phase-shifts of the fundamental correctly, but not the harmonics. There is thus no difference in solution between a sinusoidal flux and a sinusoidal current excitation.

4-2-7 B = bH^m type solutions

There is a one dimensional solution due to Nejman which is reproduced in reference (37 page 53) where a fit of $B = bH^{m}$ is made to the magnetisation curve. The magnetic permeability is shown to be spacially distributed with depth such that -

$$\mu = \frac{1}{(c-dx)^2} \tag{4-10}$$

where

x = depth from the steel surface

c,d = constants

The solution starts directly from Maxwell's equations rather than from the Diffusion equation and proceeds to find equations in terms of surface H. The form of μ distribution is assumed and the constants are found by matching equations. However, the complicated mathematics leads to straightforward results. Firstly, with strong fields current is confined to flow in a skin depth below the surface. Unlike the linear solution, all the current and flux are carried in this layer and it does not merely represent the exponential decay depth of the linear theory. The limited penetration depth arises because the B = bH^m law indicates that μ tends to infinity as H tends to zero. Secondly, the power loss is now proportional to Hⁿ

(4 - 11)

(4-12)

WαHⁿ

where

 $n = \frac{3+m}{2}$ m = 0.1 for mild steel

- 35 -

Equations 4-11 and 4-12 are identical to those of Davies¹⁶⁻¹⁷ Thirdly, the power factor is now better than the 0.7 of the linear theory.

Dreyfus 20 has extended this work to give a solution for a cylindrical bar heated by a longitudinal flux. Dreyfus works in units other than the M.K.S. system and a simplified reworking in this system is given in the Appendix (E) and it is hoped that this will be useful to the reader. This analysis shows that for the dimensions encountered in vessel heating a flat solution is generally adequate.

4-2-8 Complex Bessel function permeability

Ollendorf has shown that if -

 $\mu = \mu_{\rm s} e^{\rm X/X_{\rm s}}$

(4 - 13)

in the case of a steel slab semi-infinite in the x direction where

 μ_{e} = surface value of permeability

 $x_s = decay depth of permeability$

then Bessel equations are derived from Maxwells equations. He shows that it is necessary for B and H to be related by complex Bessel functions. The only suitable form is the Hankel function, and this can be made to fit the initial magnetisation curve closely. The derived expression for µ and H is

$$\frac{\mathrm{H}}{\mathrm{H}_{s}} = \frac{\mathrm{H}_{o}^{(\prime)}(\beta \sqrt{j\mu}/\mu_{s})}{\mathrm{H}_{o}^{(\prime)}(\beta \sqrt{j})}$$
(4-14)

where β is a constant. The major disadvantages of this theory is that the loss expression is even more complicated than equation 4-14 and that the Hankel functions are not well tabulated 34 .

Gonen and Stricker ³⁰ have applied this theory to the analysis of an eddy-current brake. The way in which they have had to use it shows that it is too cumbersome for design and only works in analysis if the errors are . artificially shared between the predicted results and the enforced B-H curve fit.

4-2-9 Step-function approximation to the B-H curve

There are a number of solutions based on a novel approach. The initial premise that the B-H curve may be approximated by a step-function can be derived from a $B = bH^{m}$ law by putting m = 0. Under these conditions magnetic permeability has no meaning. These theories are related to those of Dreyfus and Neiman. However, there is a major conceptual difference in field behaviour: field quantities only change at an infinitely thin moving boundary, rather than generally and diffusely. The moving separation layer is a direct result of the stepfunction approximation. This method is the only type of solution to account for all the affects of non linearity. The method over emphasises the distortion of the magnetisation curve and must inevitably exaggerate its influence. Thus these solutions provide an upper bound to the effects

- 37 -

of non-linearity.

The simplest solution is for a semi-infinite slab excited by a sinusoidal surface field H_t. This solution was first proposed by Vaughan and Williamson in 194660. It is shown that with these conditions the powerfactor is 0.894 and that the power is proportional to $H_t^{1.5}$. A skin effect is still present and the active effects are limited to this layer. This work was neglected for a number of years, presumably because this theory is treated by the authors as an approximation and is only a small part of their induction heating paper. Eight years later, in 1954, Maclean⁴² published a similar theory. His paper is not a derived theory but a proof that the results are sufficient. In the same year and quite independently McConnel⁴³ published a solution with the same results but with the merit of being necessary rather than sufficient. Moreover, he gives the solution to two problems, the problem of Maclean and a partial solution for the loss in a cylindrical iron bar heated by a sinusoidal voltage. McConnel goes to great lengths to show the conceptual similarities between the linear and fully non-linear solutions. The effects of flux distribution in both cases are best illustrated by his own diagrams reproduced here for clarity (fig. 4-1b). In the case of constant µ the total time variation of flux is the sum of similar flux waves whose phase changes and whose amplitude decreases with depth. Whereas, in the step function B-H



Linear theory

Step-function theory

Variation of Flux density with Time at various Depths below the surface of a flat slab for Sinusoidal Total Flux (McConnel⁴³)

	depth mm
a	0.0
Ь	0.176
С	0.352
d	0.528
е	0.704
f	0.880

Fig. 4 - 1b

case the total flux wave is the sum of equal square waves of flux density. The solution is also available for a sinusoidal e.m.f semi-infinite flat loss solution which is given on page 51 of reference 37. The behaviour of the system is almost identical to the m.m.f driven case, the chief point of difference is that the powerfactor is improved to 0.92. Loss is shown to be proportional to the voltage cubed. Davies and Bowden have extended McConnel's work, giving curves for the relationships of both power and reactive power flow for a cylindrical bar. There is another paper which merits discussion -P.D. Agarwal's treatment of m.m.f. induced loss in laminations. To say that Agarwal had merely extended the work of Maclean and McConnel would be to do his work a major disservice. The real strength of his work lies in the theoretical basis of the problem. He shows that the effect of the step-function law and the moving B sat boundary are equivalent to the domain model of Williams, Shockley and Kittel 61, which is based on the simple structure in a hollow rectangular crystal. Instead of basing his theory, as all other major theories are, on a fit to the initial magnetisation curve, the step-function theory is shown to be a magnification of the domain process; discussed in section 2-2. His results are singularly successful in predicting the flux-density waveforms in thin sheets, and they may be diluted to give the same semiinfinite solution as Maclean. However, he deals with another problem avoided by the other authors, namely, the choice of B_{gat} He shows graphically from curves



 $1 \hat{H} = 23;600 \text{ At/m} \\ 2 \hat{H} = 15,800 \text{ At/m} \\ 3 \hat{H} = 7,900 \text{ At/m} \\ 4 \hat{H} = 3,950 \text{ At/m} \\ (surface values.)$

Fig. 4-2

Measured curves of B with depth - Agarwal

of maximum flux density versus depth (fig.4-2) that a good choice for B_{sat} is 0.75 times the peak surface flux density. Unfortunately he leaves the matter there. If he had continued one step further he would have obtained a full and more precise solution. Consider the result of applying the initial magnetisation law of Davies $B=bH^m$ to Agarwals loss equation

$$W = \frac{8\rho\hat{H}^2}{3\pi\delta}$$

where

$$\delta = \sqrt{\frac{2 \rho}{W \left(\frac{0 \cdot 75 B}{H}\right)}}$$

this gives

$$W = \frac{1}{3} \sqrt{\frac{8wbo}{3}} \cdot \hat{H}^n$$

and as before

$$n = \frac{2+n}{2}$$

(4-15)

$$(4 - 16)$$

(4 - 17)

(4 - 18)

This equation bears a striking resemblance to the solutions of Davies, Dreyfus and Nejman. This theory is, perhaps, still an over emphasis of the effects of non-linearity. Firstly, there is the assumption that B changes direction instantaneously and that this change occurs at an infinitely thin boundary. Indeed, the walls, termed Bloch-walls, have a finite thickness (ref. 8 p 164) and the change of direction is not instantaneous. These theories must over predict the magnitudes of the harmonics. Secondly, the process of domain movement is assumed to be demonstrated on the macroscopic scale that the domains switch in discrete jumps (ref. 8 p 161) known as Barkhausen jumps. This suggests that the very highest harmonics may not be over predicted. However, their value is only a small percentage of the fundamental, and the Barkhausen effect may be thought of as the difference between sliding over glass or sandpaper.

4-2-10 Comparison of Theories

The steel loss theories contain an area of common ground for if the loss is sinusoidal m.m.f. driven then in most theories

 $W = \beta \sqrt{W} \beta H^{\alpha}$

(4-19)

where α and β are constants peculiar to the theory and w and ρ are the angular frequency and conductivity of the problem. Similarly, if the loss is e.m.f. or flux driven the loss may generally be written as

$$W = \sqrt{\frac{\varepsilon}{W\rho}} \phi^{\tau}$$
(4-20)

- 42 -

where ε and $\dot{\tau}$ are constants peculiar to the theory. a is (3+m)/2 for all the non-linear theories with the exception of Ollendorf's, this even applies to the semigraphical methods of Panasenkov and Subba Rao using a Davies type ($B=bH^m$) steel. For constant permeability m=1 and $\alpha=2$ its linear value. Similarly, in general $\tau = \frac{3+m}{1+m}$. Thus the loss behaviour of the steel is essentially very similar in all theories, and the form of the loss equations is the same throughout. The major difference between linear and non-linear theories is the value of the index. Whilst the chief difference in the non-linear theories lies in the results for powerfactor. At this stage it should be possible to narrow the range of choice by relating to previous experimental results directly applicable to induction vessel heating.

THEORY	pf	ANGLE	<u>a</u>
LINEAR	0.707	45°	2.0
SEMI-NON-LINEAR (DAVIES)	0.707	45 ⁰	1.54
DREYFUS n = 0.1	0.804	36.9 ⁰	1.55
STEP-FUNCTION H Sinusoidal	0.895	26.6 ⁰	1.5
ø Sinusoidal	0.92	23 ⁰	-

Table 4-1

- 43 -

4-2-11 Previous experimental measurements

The most important work with relevance to induction. vessel heating is that of M.A. Thornton⁵⁸. He undertook direct heating tests on mild steel pipe with a sinusoidal voltage supply. The tests were made under a range of supply conditions for four pipe sizes from $1\frac{11}{16}$ in.(43 mm) to $3\frac{1}{2}$ in. (89 mm) outside diameter. The pipes were standard steam pipe. An empirical law is deduced from the measured power loss in the pipe:

$$P = kT^{1.57} 0.5$$
 (4-21)

where

- k = arbitrary constant
- I = current in the pipe
- f = supply frequency

This result is well in keeping with the theoretical general power loss law (4-19), $P = \beta \sqrt{w/\gamma} H^{\alpha}$ with α taking the value 1.57. From equation (4-18) if $\alpha = 1.57$ then the steel index should 0.14, and the value of index of this order is found to work well . Thornton's experimental work proves that linear loss theories with a loss variation of I² cannot be applied to mild steel at vessel heating power densities. It is a great pity that more details are not given in the paper concerning the methods of measurement. No attempt was made to measure the harmonic components of current, and there is some difficulty concerning the powerfactor. The powerfactor results which, are only given for one pipe, $2\frac{2}{8}$ in. - 45 -



Steel loss power-factor (measured)

Fig 4 · 3

(60 mm) show a wide variance (fig. 4-3). It is impossible to tell from the paper whether readings were taken from the primary or secondary side of the supply transformer. In any case no mention is made of any attempt to eliminate stray inductance. Whilst stray inductance might easily have lowered the powerfactor it cannot raise it, and it is unlikely that there would be sufficient stray resistance to raise the pf. to 0.76 of one test. Thornton also gives the test results for large induction heated vessels. The results are shown to agree with the empirical law. He empirically predicts an overall powerfactor for these machines of 0.65. These particular tests are the only ones in the paper which exactly match the thesis problem where the currents flow circumferentially.

Thornton's work has been extended jointly by Haywood Knights Middleton and Thom³². Their apparatus formed part of the heating section for a study of steam flow conditions and, as such, it consisted of a 24 ft (7.3 m) pipe. The apparatus was arranged to induction heat pipes of various diameters with a fixed set of coils. General heating tests were conducted over lengths of 24 ft (7.3 m) or 16 ft (4.9 m), whilst the high power tests were conducted on an 8 ft (3.4 m) length. The high power tests prove that Thornton's empirical formula (equ 4-21) applies up to power densities of 400kW/m^2 which

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⊙ 57·2 mm o.d pipe
× 31·8 mm o.d pipe

_____Fig <u>4-4</u>____ Variation of loss with current³²

is several times that used in vessel heating. The curves obtained in these tests are replotted in MKS units in figure 4-4 (the curve for the copper coated tube has been omitted). The authors say "the falloff in power at the highest inputs is associated with the progressive saturation of the steel tube and is accompanied by a serious decrease in the powerfactor ". In this they have shown the importance of the leakage flux, for comparable loss densities must represent comparable electromagnetic conditions in the steel. Hence, the difference in the two curves must be due to effects outside the steel. It is felt that the discrepancy arises from the design of the apparatus. The induction coils were made to heat tubes of various diameters, and it follows that the smaller pipes must have had a large air gap. The flattening of the 32mm o.d. pipe curve must surely be looked on as a leakage flux effect.

The results of Middleton et al show the two major features of induction heating measurement, namely, the ease with which the power loss may be gauged and the great difficulty in determining the reactive portion due to the steel alone. This latter is most unfortunate since the differences in the steel theories lie chiefly in the prediction of powerfactor, apparently a quantity which is not easily measured.

Several authors^{1,50} have measured the powerfactor

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and none with any greater success. For example, Agarwal's measurements of the loss in a torpoid are shown in (fig.4-3). For eddy-current loss is characterised by very high currents and very low voltages, making direct phase measurement very difficult. Indirect current measurement with a coil introduces stray flux effects, whilst steel surface voltage measurements are only surface quantities and not bulk quantities.

The loss mechanism is also likely to be upset by the geometry of the loss member and by its change of physical properties with temperature (App E-4).

4-3-1 Solutions to the vessel heating problem

Chapter 2 has shown that induction vessel heating depends jointly on the eddy current loss in steel and the free flux flow around a conducting steel body and coil. There is a literature classification difficulty concerned with the latter problem. The effects of free flux flow show themselves in a number of ways and there are few attempts at a total solution. Hence, it is difficult to classify solutions by the effects described. Solutions are often made using a number of methods so that it is difficult to classify total solutions to the vessel heating problem by method.

The problem will be discussed by initial reference to Baker's theory, which is total and simple, and then the simple solutions to parts of the problem given by Lozinskii and others will be introduced in the discussion of this paper.

4-3-2 Baker's theory

The simplest treatment of the full vessel heating problem is due to Baker ⁴. His paper deals with the general induction heating problem, where he shows that the performance of a cylindrical workpiece may be represented by an equivalent circuit, (fig 4-5). Other authors have proposed similar circuits⁵⁹. However, the importance of an equivalent circuit lies in the way its components are derived. It is worthwhile to look at

- 50 -



Fig. 4-5

Baker's equivalent circuit for an induction heater

each element in turn, assessing its relevant importance and accuracy.

4-3-4 General circuit criticisms

If an equivalent circuit is to be meaningful, it must simulate the behaviour of the machine from its theoretical point of connection. It is not really possible to separate the reactances from a supply measurement alone into leakage and magnetising components. However, the position of R_w between the leakage and magnetising reactances implies such a split. It seems more precise to consider the vessel resistance R_w after the reactances in the leakage branch of the equivalent circuit.

4-3-5 Coil resistance and reactance

Baker was concerned to provide a comprehensive solution to the problem of induction heating and as such he has allowed for the skin effect in the coil conductor. The skin effect is generally present in all heating coils above mains frequency. Baker calculates the coil resistance on the basis of current penetration into the coil conductor. Re-arranged and in M.K.S. units this is

$$R_{c} = N^{2} k_{\mu} \rho \pi D / \delta_{c} l_{c}$$
 (4-22)

where

D = coil diameter k_r = coil space factor l_c = coil length N = number of turns R_c = coil resistance δ_c = skin depth coil conductor at heating frequency

It follows that the reactance of the coil should be numerically the same as the resistance. Baker holds that these relationships apply even to 50 Hz operation. Indeed Baker says "The reactance X_c is small in comparison to the total reactance in high frequency jobs and may be neglected." Thus he clearly intends this formula to be applied to low frequency heating. At this point it is interesting to compare Baker's result with that given by Stafl and Lammeraner³⁷(p 27) for the resistance and reactance of a pair of bus-bars. It can be shown that these form a model of the coil conductors when the method of images is applied to the coil surrounding a high permeability and resistivity vessel,



A pair of thin Bus-bars

(fig 4--6). Similar solutions can be derived for the other image condition of a low permeability and resistivity vessel. Stafl and Lammeraner ³⁷give the following results for bus-bars:

R	=	$\frac{L \rho}{4ht\beta}$	$\frac{\sinh\beta + \sin\beta}{\cosh\beta - \cos\beta}$	(4-23)
X	=	Lo 4htp	$\frac{\sinh\beta - \sin\beta}{\cosh\beta - \cos\beta}$	(4-24)

Where

$$\beta = \frac{2b}{\delta}$$

and h,t,L are defined in fig 4-6.

These equations are plotted in fig 4-7 both for a single bus-bar and for a pair. The true current distribution lies somewhere between the single bus-bar case and the paired case. The conditions described by Baker of current confined to a surface skin depth on the inner surface of the coil, and a ratio of resistance to reactance of 1, apply over a limited range. The theory



Fig. 4-7 Resistance and reactance functions for bus-bars

applies where the thickness of coil conductor is greater than 70% of the skin depth if the paired bar condition is assumed. However, the skin depth of copper at 50 Hz is approximately 1 cm so that the minimum depth of conductor, applicable to the theory, is 7 mm. If this is a square conductor, it represents a current carrying - 55 -



Fig. 4-8

Resistivity and current density distributions for square and circular conductors

capacity of 230 A. (at 4.65 A/mm^2). It seems more reasonable to assume a zero reactance and a coil resistance equal to its d.c. resistance for a practical vessel heater.

A further complication is introduced by the use of round wire. The coil can no longer be treated as a solid uniform bar cut by infinitely thin slits. Owing to the circular nature of the wire some allowance must be made for the loss of conductor area (fig 4-8). Baker makes the allowance by including an empirical coil space factor k, as a resistance multiplier. It is a little difficult to see what k, represents. It cannot be the coil space factor usually defined as "the ratio of active conductor section to coil section. "(ref53 p 84). A uniform surface skin effect implies that the resistance is decreased by increasing the surface area making k greater than unity for round wires whilst the more usual space factor is less than one. It is likely that the reduction in conductor area increases the effective resistivity by a factor of $4/\pi$ in single layer coils (fig 4-5). On this basis the skin depth increases by a factor of $\sqrt{4/\pi}$. The system could be analysed in terms of a rectangular bar whose resistivity increases with distance from its centreline according to -

$$\rho' = \frac{\rho}{\sqrt{1 - (x/r)^2}}$$
 Ω (4-25)

This has the effect of preventing current flow near the surface of the coil and, in turn, causes a still greater increase in apparent skin depth. Hence, the skin depth applicable to the case of circular conductors in a coil is likely to be of the order of 14 mm. The idea of the coil resistance being its d.c. resistance with negligible reactance applies to a wider range of round wires than square conductors.

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4-3-6 Leakage reactance

Moving inwards from the coil the next problem concerns the flux in the air gap and the associated leakage reactance. Baker makes the useful and simplifying assumption that the magnetic field intensity is uniform over the air gap. This has two beneficial effects. The first effect is that H at the surface of the coil is also H at the surface of the vessel, the consequences of this will be discussed in due course. The second result is that the calculation of the air gap flux becomes straightforward. The leakage flux is -

 $\Phi_1 = \mu_0 \cdot H \cdot A_g$ Wb (4-26)

and hence

 $X_{g} = w\mu N^{2} A_{g}/l_{c} \qquad \qquad \Omega \qquad (4-27)$

where

Ag	=	cross-sectional	area (of	gap	m ²
1 _c	=	length of coil				m

N = number of turns

Other authors have attempted to calculate the leakage reactance. The paper by Vaughan and Williamson⁵⁹, which is primarily concerned with induction heating in the 10 kHz range, uses a different method of computation. Their method is based on the work of Nagioka, who determined reactance factors for empty solenoidal coils. They modified Nagioka's work by assuming that the heated load carries a flux in proportion to the area of the load divided by the area of the coil. This is used as a modification of Nagioka's constant. On this basis the leakage reactance becomes -

where

$$K_{v} = K_{n} [1 - (d_{w}/d_{c})^{2}] + (d_{w}/d_{c})^{2}$$
(4-29)

and

 $K_n = Nagioka's constant (fig 4-9)$ $d_w = Outer diameter of the vessel m$ $d_c = Inner diameter of the coil m$



Nagioka's constant

Equation 4-28 is identical to Baker's formula 4-27 except that the reactance is reduced by K_v which is included to account for the decrease of H across the gap and at the coil ends. Reference will be made to K_v in connection with the discussion of load resistance. K_v is generally close to unity for the range of sizes of induction vessel heaters, which justifies Baker's theory.

4-3-7 Magnetising reactance

The calculation of magnetising reactance represents an acute difficulty. Most authors completely disregard it because at high frequencies the coefficient of coupling is approximately unity. Baker treats the magnetising reactance as a problem of a short coil. A decrease in coil length increases the impedance of the leakage branch of the equivalent circuit and thus increases the importance of the magnetising reactance. He assumes that little m.m.f. is used internally and that the major reluctance of the magnetising flux path is external to the coil. Baker states that this reluctance is substantially independent of coil length and inversely proportional to the perimeter.

 $R_{e} = \frac{1 \cdot 80 \text{ k}}{p} \begin{bmatrix} 10^{9} \\ 4 \pi \end{bmatrix} \qquad (4-30)$ This is an empirical expression where k is assumed as unity for lack of data. It will be shown (6-2-4) that this equation is approximately true but that k is not a constant.

Laithwaite³⁶ has also attempted a solution; in this case for the magnetising reactance of an open bar-type transformer. His method is to calculate an effective air gap. The air gap is based on the demagnetising factors for ellipsoids calculated by Osborn⁴⁹. Unfortunately, a prerequisite for these demagnetisation factors is a uniform applied field. The field in the case of the bar transformer and vessel heater is not uniform. Laithwaite has published experimental results which show errors in the region of 100%.

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4-3-8 Vessel resistance and reactance

The assumption that the field strength at the coil surface is also the field strength at the vessel surface leads to a limitation of uniform current under the coil. Baker⁴uses a semi-linear theory (4-2-3) to calculate the loss and derives -

$$R_{W} = \frac{W\mu N^{2}}{l_{c}} \frac{\pi d^{2}}{4} W \cdot Q \qquad \Omega \qquad (4-31)$$

$$R_{W} = \rho N^{2} \left(\frac{\pi d}{l_{c} \delta} W\right) \cdot \left(\frac{Q d}{2 \delta} W\right) \qquad \Omega \qquad (4-32)$$

or

where Q is given below and has been calculated for solid





and hollow loads. Whilst Vaughan and Williamson⁵⁹ assume that H is uniform and decreased by K_v (equ 4-29). Hence they derive the power loss in a linear material in terms of $(K_v.H)^2$. The work resistance becomes - $R_w = K_v^2 \rho N^2 \left(\frac{\pi d}{l \delta} w\right) \cdot q$ Ω (4-33)
where

a

$$I = \frac{Q d}{2 \delta} W$$

They also derive a resistance formula for use with a saturated steel 60

 $\delta_{\rm m}$ is the maximum penetration depth according to a step function theory. The penetration depth varies with excitation so that the equivalent circuit changes with supply conditions. Thornton⁵⁸uses the opposite assumption of current distribution to the preceding authors. He calculates the loss on the basis of a uniform current spread over the entire surface of the vessel. It will be shown that the truth lies somewhere between the two extremes.

Baker has used a linear theory to calculate the vessel reactance. If a linear theory is applied to a slab the resistance is equal to its reactance. He recognises that this is not true for small cylinders and he has derived curves of the necessary multiplying factor P in equation 4-36 below - these are plotted in (fig 4-10).

ing factor which arises from the assumption of a saturating steel. The relevant equation is -

$$X_{W} = 0.65 R_{W}$$
 (4-37)

(4 - 34)

Lozinskii⁴¹ has dealt with the problem of the effect of coil length on load resistance in great depth. He describes two approaches; the first is that of the method of images as used by Brown, Hoyler and Bierwirth⁹ and the second is the electrolytic tank. His figure 14 corresponds to figure 6-7 of this thesis and is derived from the same equations. Even at this stage in the thesis it has been shown that the power loss in the vessel can be written as -

 $P = \pi d_{W} \cdot \int W(x) \cdot dx$ (4-38) over the surface of the vessel. Applying equation 4-19 this becomes-

 $P = \pi d_{w} \cdot \beta \sqrt{w} \int H(x)^{\alpha} dx$





Distribution of surface current density over an infinite flat plate under a current carrying strip (Lozinskii).

Jozinskii has used an equation of the type 4-39 to derive resistance factors to account for the non-uniform current

W (4-39)

distribution shown in figure 4-8. Unfortunately, he has assumed linear conditions so that a is 2 whereas under non linear conditions α is approximately 1.5. Hence, the resistance factors calculated from the current density curves cannot be applied to saturated steel. Similar current distribution curves have been derived for varying diameters and varying coil lengths and vessel lengths, by using an electrolytic tank. Resistance factors have been calculated from these results. However, these curves are also based on the linear loss index. Moreover, the field system modelled in the tank neglects the magnetising flux and this may introduce some small error. Lozinskii was concerned to produce data for industrial high frequency hardening and annealing work. The non uniform heating effect is presented as a ratio of load resistance to inductor resistance where both are fashioned from the same material. This presentation makes the calculation of inductor efficiency easier than the resistance multiplying function of reference 39. However, Lozinskii's treatment is not as physically meaningful as a resistance multiplier.

Notable recent work in the subject of fields surrounding cylindrical coils and conductive bodies is due to J.D. Lavers ^{38,39,40}. This work is based on the harmonic model (Chapter 3-3). The subject treated is the coreless induction furnace which is similar to the induction vessel heater save that the core of the furnace consists of molten metal whose relative permeability is unity. This problem is in every way linear. All the field equations are linear partial differential equations, both inside and outside the core. Lavers' work is a linear counterpart to the thesis problem. The initial harmonic model of his problem is simplified (fig 4-12). The core is taken to be continuous and heated by a number of exciting coils.





Fig. 4-12

Lavers' model for the coreless induction furnace

The spacial period of the exciting coils is greater than thirty times the coil length which gives an approximation of an idealised isolated coil and an infinitely long conducting cylinder.

Lavers, like Lozinskii, has calculated factors to account for the action of coil geometry on the load resistance. Their results when plotted jointly can be seen to be in close agreement (fig 4-13). The equations have had to be manipulated in order to put them



<u>Fig. 4-13</u> <u>Resistance multiplying factors</u> <u>Lavers and Lozinskii</u>

on a common basis. The curves are in close agreement which is remarkable when the different approaches are considered. Lavers' curves represent a total solution to the field problem for a special condition of core length. Whilst the Lozinskii solution correctly models the geometry but fails to account for the effect of current penetration and magnetising flux upon the field patterns. These results tend to suggest that neither the magnetising field nor the penetration depth influences the resistance multiplier. Thus it seems possible that the resistance multiplier may be independent of the permeability of the load.

CHAPTER 5 EXPERIMENTAL WORK

CHAPTER SUMMARY

Two classes of experiment are described - experiments (5-2) to (5-8) with small diameter model heaters and experiments (5-9) to (5-13) with a full scale heater. The model experiments are used to establish the detailed behaviour of a heater. It is found that the power loss in the vessel is an eddy-current mechanism (5-3) accurately described by the theory of Dreyfus²⁰ (5-4), (5-6). The permeances of a vessel heater are unaffected by saturation (5-7), (5-9) and cannot be described by the theories of Chapter 4. An empirical law is derived for the external flux distribution.

The full scale experiments are used to complement the others and it is shown that the electrical power measurement method can be corroborated by a purely thermal measuring technique. The field surface strengths are measured and it is shown that their wave shapes and the supply waveforms can be best described by the step-function steel loss theory.

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Model experiments

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- 5-3 Variable frequency experiments
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- 5-5 The relationship of eddy-current loss to leakage flux and magnetising flux
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- 5-10 Experimental measurement of the power input to the vessel by a method similar to Gilbert's method
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Note

Graphs and tables of results are given in Appendix C where graph (3-2) is the second graph of experiment (5-3).

5 Experimental work

5-1 Aims and objects of the experiments

In any investigation there is always a problem concerning the order and merit of experimental and theoretical work. Had there been sufficient experimental work concerning induction vessel heating it would have been reasonable to have built a theoretical analysis on this and then to have tested this theory experimentally. However, Chapter 4 has shown that there is little experimental evidence on which to base a theory. Hence, an experimental foundation has to be built before any theory can be started. A number of points must be clarified. Firstly, the loss behaviour in the steel during the eddycurrent process must be investigated in order that the appropriate steel loss theory may be chosen. The suitability must be judged vis-a-vis the relationship of loss energy to flux, current and powerfactor. Secondly, measurements of the magnetising and leakage permeances are needed for the construction of a complete mathematical model. This in turn implies the measurement of the external field patterns. Thirdly, the loss distribution must be ascertained in order to judge the need for a loss distribution factor of the type described in section 4-3-8. However, the primary need is for a series of general measurements to establish the general behaviour of induction vessel heaters.

5-2-1 Underlying considerations of the initial experiments

These experiments are intended to show the performance of induction vessel heaters. As such they should deal with a general range of heater dimensions and supply conditions. The heater must be simplified so that the experiments can be easily understood. Under these conditions each variable can be changed in turn. A vessel heater can be reduced to a simple coil and vessel system: hence it becomes a device with only two elements. In this form there are only three supply variables at mains frequency, namely, current, voltage and power. This is a practical form of the machine. However, stray losses and stray field distortions must be avoided. The field requirements imply that there should be no other metal work near the heater, i.e. both the coil former and the vessel supports should, where possible, be non-metallic.

5-2-2 Apparatus used in the initial experiments

The majority of apparatus used in the preliminary tests is shown in plate 1 . The coil consists of 308 turns of 18 s.w.g wire wound on a wood and fibre former. The simulated vessels were made from lengths of 145 mm dia. and 160 mm dia. steam-pipe. This pipe was used for commercial and technical reasons. Commercially it represents a cheap and easily available vessel substitute. This pipe section also represents a commercial problem, e.g. when carrying an oil that needs trace heating. Technically, steam-pipe is made from the correct material - mild steel. It is manufactured to the correct standards. It must have a good



and uniform weld to withstand steam pressure, which is also a criterion for uniform induction heating. Moreover, if the wall thickness is greater than the classical skin depth then it is assumed that the vessel will behave as though infinitely thick. Now the linear skin depth of mild steel calculated for coil surface conditions when the test coil carries 10 Ar.m.s. is 2.5 mm whereas the wall thickness of the steam pipe is 3 mm. Hence, the steam pipe looks like a solid body to the coil and the variation of wall thickness does not appear as an experimental variable. In order to avoid stray losses the vessel supports are deal and beech.

5-2-3 Test method

The apparatus was supplied from a 'variac' and the current voltage, and power measured. Initial trials showed that the coil heated, rapidly changing its resistance. Experimental practice was to connect the coil to a D.C. supply both before and after taking an A.C. reading. Measurements of the direct current and voltage enabled the coil resistance at each operating point to be found. The heater was operated from the circuit as shown. (fig 5-2). Readings were taken by starting from low power inputs working up to a supply maximum and then the power was reduced. It was hoped that the method of measurement would at least average the temperature effects. A Digital PDP-9 computer was used for the routine calculation of power, reactive power and power factor, and the results plotted by hand.

5-2-4 Initial experiment results

In spite of the complications of magnetising and leakage reactances, it appears that the behaviour of each vessel is governed by a power index law. The power index laws are demonstrated by the log-log plots of power and reactive power versus supply current, voltage and coil flux. The resulting laws are shown in table 5-1. The indices of power loss in the vessel for current, voltage and flux excitation do not match their theoretical counterparts. Non-linear theories predict that power loss should be either $P = kI^{1.57}$ for a current driven loss, or $P = kV^{2.85}$ for a voltage driven loss, this is not found experimentally. The condition arises because, in the case of current driven loss, the magnetising current depends on the flux level and on the magnetising permeance. Under non-linear conditions the flux in the vessel is related to the current in it since:

 $P = kI^{1.57} = j\phi^{2.84}$ W (5-1)

whence

 $\phi = \left(\frac{k}{1}\right)^{0.35} \frac{1}{1.8}$ Thus the index connecting coil current to power loss in the vessel can be expected to be above the 1.57 of (5-1). Similarly, in the case of voltage and flux driven loss it must be remembered that the coil also provides the leakage flux. The leakage flux is proportional to the coil current; this has the effect of reducing the indices. It might be argued that an account of the steel behaviour which did not include saturation might be sufficient. However, this

Wb (5-2)

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can be ruled out by considering the effect of a linear μ flux loss law.

 $P = k\phi_m^2$ (5-3) The magnetising flux is the difference between the total flux and the leakage flux so that equation 5-3 can be written:

 $P = k\varphi_t^2 \Big[1 - (\varphi_1/\varphi_t)^2 + 2 \varphi_1/\varphi_t \Big]$ (5-4) but the ratio of leakage flux to total flux is constant in an unsaturated system. Hence, a linear theory can only give an index of two. This experiment has shown the need for a non-linear theory to account for the eddy-current effects in the vessel. It has shown that the behaviour is complicated by the magnetising and leakage reactances, and that the behaviour is controlled by the magnetising reactance when the vessel is short (graph 2-3).

INDEX LAWS OF THE FORM $P \alpha F^m$

	Index (m)		
Power		Reactive	e Power
F= { Coil Volts	Coil Current	Coil Volts	Coil Current
2.64	2.04	2.15	1.88
2.56	1.84	2.05	1.88
2.56	1.84	2.03	1.88
2.58	1.81	2.03	1.88
	$F = \begin{cases} Powe \\ Coil \\ Volts \end{cases}$ 2.64 2.56 2.56 2.58	Index (m) $F = \begin{cases} Power \\ Coil & Coil \\ Volts & Current \end{cases}$ $2.64 & 2.04$ $2.56 & 1.84$ $2.56 & 1.84$ $2.56 & 1.84$ $2.58 & 1.81$	$Index (m) \\ F = \begin{cases} Power & Reactive \\ Coil & Coil \\ Volts & Current & Volts \end{cases}$ $2.64 & 2.04 & 2.15$ $2.56 & 1.84 & 2.05$ $2.56 & 1.84 & 2.03$ $2.58 & 1.81 & 2.03 \end{cases}$

166 mm diameter vessels

Table 5-1

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5-3-1 Aims of the variable frequency experiments

There were three objects of these experiments. Firstly, to investigate the general effects of frequency on loss. It is expected that the power will vary as the square root of the frequency for a constant current in the vessel. Whilst the effect of frequency will be different under constant coil voltage conditions, this is due to the influence of frequency on the flux levels. Secondly, it has been tacitly assumed that the hysteresis loss component is insignificant. The classical method of separation of losses is based on the variation of loss, at a given flux level, with frequency. The eddy-current percycle loss is frequency sensitive whilst the hysteresis loss/is a constant. Hence a variable frequency experiment should show the relative magnitude of the losses. The third effect is that of scaling. It can be expected that the behaviour of the heater is influenced by its size. The critical dimension for induction heating is the skin depth, and all other dimensions may be measured relative to this. Now δ is inversely proportional to the square root of the frequency so that increasing the frequency by a factor of four, halves 6. A variable frequency experiment may be looked upon as a means of expanding the size range of the experimental apparatus.

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5-3-2 Method of test

The supply was obtained from a d.c. driven multiple motor alternator set and the method of test was identical to the previous experiment. The motor alternator gave a frequency range from 25 Hz to 100 Hz. This range more than covers the commercially available mains frequencies. It is also of sufficient breadth to enable the separation of hysteresis loss from eddy-current loss at 50 Hz. It also offers a scale range of two. Voltage variation was made by changing the excitation of the generator. However, the output of the alternator was limited at the extreme frequency ranges, being unable to supply the necessary voltage at low frequencies and the current at high frequencies.

5-3-3 Results of the variable frequency tests

The indices of power and reactive power are; $P \propto f^{1.15}$ $Q \propto f^{0.71} (at l = 9A)$, obtained from graphs 3-1 in the Appendix. If an over-simplified view is taken of the mechanisms in the induction vessel heater then the resulting indices cannot be explained. However, when considering the variation of power at constant coil current with supply frequency, it should be remembered that a component of the total current is made up by the magnetising current. The magnetising current drives the magnetising flux and this flux is related to the loss by \sqrt{f} . Thus, the leakage current is related to the loss by \sqrt{f} . Thus, the law relating power input to the vessel at constant total coil current to frequency is an approximate power law whose index lies somewhere between 0.5 and 1.5.

Turning to the measurement of hysteresis loss. The classical method of loss separation assumes that both flux and losses are uniformly distributed. It may be applied to laminations. Under the classical conditions the hysteresis loss is $P_h \alpha V/f$ W (5-5) and the eddy-current loss is $P_e \alpha f^2 (V/f)^2$ W (5-6) A plot of the total power against f^2 for constant flux and thus constant V/f should give a straight line whose intercept with the y axis is the hysteresis loss. Graph (3-2) shows that the hysteresis component of loss is of the order of 1% or less of the total loss.

The scaling effects show that a larger physical scale machine operates with a better powerfactor and efficiency, graph 3-3 . The prospect is that larger induction heaters are easier to build and that their per unit costs will be lower.

5-4-1 Steel eddy-current loss tests - Objects

The preliminary experiments (5-2) showed that the power input to the vessel was a power law of the driving current. The indices measured in this experiment were not those reported by other authors. It was assumed that these differences were due to the magnetising and leakage reactances of the heater. It was felt necessary to find the true loss to current index which could only be done with a machine having a high magnetising reactance.

5-4-2 Apparatus

Apparatus was specially constructed to give a heater with a high magnetising reactance. The equipment constructed is shown in plate 2 . The magnetising flux path was formed by four laminated C-shaped cores bored to give a sliding fit over a mild steel cylinder used as the test body. The cylinder had approximately 60 mm thick walls and was effectively a solid body. The C-cores were held in place by brass side-plates which were cut to eliminate eddycurrents. The C-cores enclosed the heating coil which consisted of 144 turns of 12 gauge wire wound in 12 layers.

5-4-3 Experimental tests and results

The experimental method was the same as that used in the previous experiments. The power input to the steel was measured as the net power, being the total power less the coil loss. The stray losses in this machine were reduced by design and neither the side plates nor the cores were found to be heated by eddy-currents.

The results are plotted in graphs 4-1 . The relationships from these graphs are -

$$P = kI^{1.6}$$
 (5-11)

and

$$P = kV^{2.85}$$
 (5-12)

where P is the power input to the steel, I is the coil current and V is the coil voltage. The coil resistance is low so that under the conditions of these tests the coil volt drop is less than 2% of the supply voltage. The voltage index of equation 5-12 can be taken as the index for flux variation. The indices measured are close to those predicted by other authors 1,16,20,32,58 . Moreover, the power densities used in this experiment represent a range of 15 to 1 with a maximum of 100 kW/m² which more than covers the usual range of vessel heaters. The measured powerfactor lies between 0.81 and 0.75. The results can be seen to be intermediate between the extremes of the semi-linear theories and the step function theories. On the basis of this experiment, the theories of Dreyfus and Nejman are closest to the conditions of induction vessel heating.

5-5-1 The relationship of eddy-current loss to leakage flux and magnetising flux

The previous experiment has established the relationships between voltage, current and power within the vessel walls. The powerfactor obtained was found to be approximately constant whilst the laws relating voltage and current agree with theory. However, the preliminary experiments show that the total behaviour of a vessel heater obeys more complicated laws. It was the purpose of this experiment to try to simplify the problem by establishing the relationships of magnetising flux and leakage flux to the power in the vessel.

5-5-2 Apparatus

The experimental model was constructed using the coil

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from the first experiment together the 240 mm long, 140 mm diameter specimen and positioned symmetrically. Two additional search coils were added. An outer search coil was wound just under the coil at its centreline: hence its induced e.m.f. was proportional to the total flux. A second search coil was wound tightly on the vessel directly beneath the outer search coil. The induced e.m.f. in the inner search coil was proportional to the magnetising flux. The leakage or air gap flux could be measured in terms of the difference voltage between the coils.

5-5-3 Method of test

The experimental method was identical to that of the preliminary experiments. Search coil readings were taken at each supply setting. These voltages were measured with a true r.m.s valve voltmeter. The temperature of the vessel was measured with a copper constantin thermocouple and the temperature at each reading was arranged to be at the same value.

5-5-4 Results

 $P \alpha \phi_m^{2.84}$

 $P \alpha \phi_1^{1.54}$

The results show that

and

(5-13) (5-14)

this compares with the measured value of index of power with voltage of 2.85 and power with current of 1.6 measured in the previous experiment. This confirms that the

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magnetising flux is associated with voltage induced in the vessel whilst the leakage flux is related to the current in the vessel. Thus it would seem reasonable to assume that a related powerfactor triangle holds. The angle between ϕ_1 and ϕ_m is 90+0 and like the powerfactor angle this may be assumed to be constant. It was hoped that the angle could be measured from this experiment but the search coil values were such that the equations were ill-conditioned.

5-6-1 Measurement of the flux triangle

The previous experiment failed to measure the angles in the flux triangle. Whilst the angular relationship could be assumed it was felt that it would be worthwhile to perform a simple experiment to measure the angle $\phi_1^{\frown} \phi_m$. The last experiment failed because of the low value of leakage flux. The logical solution was to use apparatus with a higher proportion of leakage flux to magnetising flux.

5-6-2 Apparatus

Similar apparatus was used to the last experiment. In this case a 6cm diameter pipe was used as the vessel, this gave an increased air gap. The enlarged air gap naturally led to much higher values of leakage flux. The search coil arrangement was as before consisting of an outer total flux coil and an inner magnetising flux coil.

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5-6-3 Method of test

The experimental method consisted of measuring the search coil voltages with a valve voltmeter for a range of supply voltages.

5-6-4 Results

The angle between leakage and magnetising flux was calculated by the cosine rule. It is plotted in graph 6-1 At first sight this result does not appear to agree with the previous measurement of powerfactor. However, the pipe has a much smaller diameter than any other load used experimentally. When the effects of curvature are calcu-(APPE) lated from Dreyfus' theory the results are found to be in close agreement. The variation in powerfactor angle is caused by a combination of saturation and curvature. Thus, if this result applied to larger vessels then the curvature effects can be neglected and the angle $\phi_1 - \phi_m$ is a constant and close to the powerfactor angle plus 90°. This experiment has shown that when the steel is saturated $\phi_1 - \phi_m = 129^\circ$ for the small cylinder and approximately 125° for a slab.

5-7-1 Magnetising permeance and leakage permeance experiment

The initial experiment has shown that the performance is influenced by the geometry of the heater. This is most probably due to the change of magnetising and leakage reactances. The following experiment was designed to measure the variation of magnetising and leakage permeances with vessel length. Baker ⁴ predicts that the magnetising permeance should be independent of the vessel length. On the other hand, Laithwaite predicts that the permeance should increase with vessel length. Both authors assume that the magnetising permeance is not affected by the level of excitation. Turning to the leakage reactance, it is generally assumed that this is independent of the vessel length ^{4,59,60}.

5-7-2 Apparatus

The apparatus consisted of four lengths of mild steel pipe 145 mm diameter and one length 160 mm diameter. These were heated with the coil used in the previous tests symmetrically placed. A total flux search coil was wound directly under the centreline of the heating coil and a magnetising flux search coil was wound tightly on each specimen directly under the total flux coil.

5-7-3 Method of test

The apparatus was connected as in the initial experiment. In this experiment the surface temperature of the vessel was monitored with a copper constantin thermocouple placed beside the magnetising flux coil. Readings of heater coil current, voltage and power were taken as before together with search coil readings for total, magnetising and leakage fluxes. D.c. coil resistance was measured with a test current. This procedure was repeated for a range of supply voltages at a constant vessel temperature.





The method of calculation used in this experiment is somewhat complicated. For this reason it is felt that an explanation is necessary. The phasor diagram above applies generally to induction heating, and this experiment is essentially a measurement of the phase relationships during heating. In this experiment the angle between ϕ_m and ϕ_1 , and I_m and $I_1: \Theta$ is assumed to be 125°. This is an assumption that the vessel powerfactor is 0.82, and is intermediate between experiments 5-4 and 5-6 implying a reduction of curvature compared with the latter. The angle between

$$\alpha = \operatorname{Arcsin}\left(\frac{P-IR}{VI}c\right)$$
(5-15)

where

P = total input power $R_{c} = \text{coil resistance}$ V = supply voltageNow the angle $\hat{p} - \hat{p}_{m}$ is given by the sine formula in the flux triangle. Hence $\beta = \operatorname{Arcsin}\left(\operatorname{Sin} \theta \cdot \frac{\hat{p}_{1}}{\hat{p}}\right) \qquad (5-16)$ Turning to the current triangle the angle

$$I-I_1 = \alpha + \beta \tag{5-17}$$

Again using the sine formula gives

$$I_{1} = I \cdot \frac{\operatorname{Sin}(\alpha + \beta)}{\operatorname{Sin}\Theta}$$
(5-18)

and

$$I_{m} = I \cdot \frac{\sin(\pi - \alpha - \beta - \theta)}{\sin \theta}$$
(5-19)

The magnetising and leakage permeances are easily computed from the ratio of flux to current.

5-7-5 Results

It is shown in graphs 7-1,7-2 that the relationships between power, leakage current and magnetising flux are in agreement with the non-linear theories for steel.

In accordance with theories previously discussed in 4-3-6 and 4-3-7 both the leakage and magnetising permeances should be independent of the field strengths. This is

borne out by the results plotted in graphs 7-3, 7-4, which are plots of the permeances against vessel power. Since vessel power is related to the field conditions the curves can be said to illustrate this independence. However, contrary to Baker's theory, both the magnetising permeance and the leakage permeance vary with vessel dimensions. Lengthening the vessel produces a marked increase in the magnetising permeance and a decrease in the leakage permeance. This is shown by graphs 7-5, 7-6. The former is a plot of magnetising permeance against work length in comparison with Baker's and Laithwaite's predictions. Although Laithwaite claims that his formula allows for the effect of length, Baker gives answers no closer to the truth, and in this case appears to give the value when the vessel length equals its diameter. The second graph shows that Baker over-predicts the value of leakage permeance. The high values arise from the assumption that the leakage flux is uniform across the gap. Vaughan and Williamson's correction factor reduces Baker's value to another constant value.

5-8-1 External flux measurements

The magnetising reactance can be a controlling factor in the design of an induction vessel heater. Since the theories for its prediction are in error a closer assessment of its path is clearly called for. This entails the measurement of the flux distribution outside the heater coil. The external flux is composed of both the magnetising and leakage fluxes, but the leakage flux is a small proportion of the total flux. Hence the total flux distribution is close to that of the magnetising flux.

5-8-2 Apparatus and experimental method

It was felt that the simplest way to measure the flux distribution was by using a fixed system of search coils, which would enable exactly the same experiment to be repeated over a range of vessel lengths. A sketch of the apparatus is shown below. The small diameter of load



Fig. 5-3

External flux distribution apparatus

used in the experiments, 140 mm, gave the opportunity of measuring the flux levels to over twice the diameter of the vessel.

The method adopted during the test was exactly similar to that used in the previous experiment.

5-8-3 Results

The experiment shows that for each coil vessel combination the shape of the flux distribution is unaltered by the level of coil currents (graph 8-1). Hence, this supports the results of the previous experiment that increasing saturation does not affect the magnetising reactance. The test also show that the external flux distribution conforms to a power law of the form

$$\frac{\phi_{\rm s}}{\phi} = \left(\frac{\rm d_{\rm s}}{\rm d_{\rm c}}\right)^{-n} \tag{5-20}$$

which is derived from graph 8-2. Moreover, in this experiment it was found that

$$h \neq l + k \left(\frac{d_W}{l_W} \right)^2$$
 (5-21)

where k = 1.25 from graph 8-3.

This serves as an indication that a simple inverse power law applies to an infinitely long vessel, whilst slightly higher indexes apply to shorter vessels, and n did not reach 1.5 in this test.

5-9-1 Full-sized experiments

The preceding experiments are felt to have explored the possible variations of performance with a fixed coil and varying vessel geometries. The experiments have necessarily been on a small scale. Moreover, they did not include the impressed temperature effects of a fluid load. For these reasons it was decided to build apparatus of commercial size. The apparatus would fill two roles, firstly a full-sized confirmation of earlier experiments, and secondly measure other quantities associated with the load and coil more easily monitored on full-sized apparatus. It was felt that it should be water loaded and be capable of measuring the effects of various coil geometries. It would also be equipped to measure surface current and temperature distributions.

The apparatus built consisted of a 1.2m high by 0.5m diameter mild steel cylinder. The cylinder was formed from a rolled sheet of 10mm thick steel plate welded along its seam. Unfortunately, because of the thickness of plate, it apparently proved impossible to produce a perfect cylinder and the resulting form had a 5mm oversize diameter along the axis of the weld. The cylinder was lagged with a domestic hot water cylinder lagging jacket, and was filled with water to 70mm below the top rim. The cylinder was heated by two sets of induction heating coils. In order to avoid the problems of coil temperature rise and its associated effect on the coil resistance and thus coil

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loss as had occurred in the earlier apparatus, it was decided to water cool the coils. The choice was either to use a water filled conductor or to use a normal conductor and to place this in a water carrying tube. For reasons of simplicity, and strength of construction, the former course was chosen. It was found that 6 mm 'micro-bore' central heating pipe was a suitable hollow form of conductor. It was of acceptable size and resistance and also readily available at low cost compared with specially drawn conductors. Each set of coils were wound as separate layers of 33 turns each. For mechanical strength they were wound interleaved with a spacer string on to a collapsible drum type former and then coated with an expoxide resin paste. The paste was allowed to cure before the next layer was wound. Finally, the former was removed and the resulting coils were bound longitudinally with glass fibre tape. Thus each set of four coils of 33 turns each became a solid block with great mechanical strength.

Glass tape	Paper	
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000000000000	030306090606060	a)a)a)a(a)a(a)a)a)a)a)a)
00000000000	X0000000000000000	
		02
0000000000	000000000000000000000000000000000000000	000000000000000000000
1		
/	/	/
Copper Tube	String_	<u>Epoxy Matrix</u>

Fig. 5-4

Coil cross-section

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The disadvantage of this form of construction was that in placing the coils so closely it became impossible to take tappings from the individual layer coils. However, it was felt that sufficient readings could be taken without need of additional tappings. There is one attendant problem of water cooled coils, namely, that the water system must not be allowed to short-circuit the electricity supply. At some point the electrical and water paths must split. This was done by taking the electrical tapping from the pipe conductor and inserting a length of nonconductive hose between the water supply and each coil conductor. Fortunately, Birmingham town main water, which was used to cool the coils, has a high resistivity and the leakage path through the cooling water was thus restricted. Measurements with a megger tester shows that the leakage resistance between two individual coils, set up by the cooling water input and output through 250 mm of 6 mm rubber tube is 1.6 Ma. Hence the leakage resistance effects can be safely discounted with the apparatus. The cooling water circuit is as shown in (fig 5-6) and coil system is shown below



Coil terminations

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<u>Fig 5-6</u> <u>Cooling water circuit</u>.

The coils can be connected in various layers, in series or in parallel, so that the effects of coil gap and coil length may be investigated. The coils were made equal and of such a length that at some future date a third set of coils could be made, thus forming a rudimentary form of three-phase heater.

It was decided that the current distribution on the surface of the vessel should be measured. This would give a correlation with the curves discussed in section (4-3-8) as these curves have not been checked with reference to a mild steel vessel. These currents could have been measured by current density probes, but it was felt that a search coil measurement would give the same information with a stronger signal (Section 5-10). It was also felt that this apparatus might be used as an opportunity to check the power input by the method of temperature rise proposed by Gilbert 29 , and for this reason thermocouples were attached to the vessel surface over a longitudinal spread. The couple leads were glued to the surface of the vessel and taken down its length, to eliminate conducted heat errors in measurement. Other thermocouples have been arranged to measure the internal water temperature.

5-9-2 Experiments to measure the effect of gap

Initial experiments established that the apparatus could not be operated with existing supply arrangements with less than 66 turns. The coil system had to be operated with two 'coils' in series, this enabled tests to be carried out on a 0.6m long single layer coil with four possible gaps and on a 0.28m long double layer coil with three possible effective gaps. The only means of supply variation capable of controlling the 200 Amp coil current was by using a number of matched transformers to buck the supply voltage, the last of these being supplied by a variac (fig 5-7).





The results of the experiment are shown in graphs 9-1 & 9-2. They show that simple power index laws are still obeyed, and that for this size of apparatus the index of power input versus current for any coil configuration is close to that predicted by Thornton. In turn, this suggests that the magnetising reactance for a large coil-vessel system is naturally high and, in these terms, the larger the vessel heater the simpler its design and the better its performance. The curves of work power density versus gap are thus an indication of the effect of the change of current density pattern caused by the prox-40.41 and the decay of H across the gap.

The curves of reactive power density are formed from two components: the reactive element in the steel and the reactive element in the air gap. Now experiments 5-4 and 5-6 have shown that the power factor is approximately constant with excitation and is 0.8 : taking this value enables graph 9.4 to be plotted from graphs 9.3 and These curves show that, for a given ratio of coil length to work length, the reactive power in the gap is as Baker ⁴ suggests, proportional to the area of the gap. However, the constant of proportionality varies with the ratio of coil length to work length.

It was also found that the resistance of a coil measured in a d.c. resistance test was the same as that measured from the temperature rise produced in a known flow of cooling water during the alternating current

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operation of the machine. Thus the proximity effect, alluded to by Baker in this general paper, does not appear relevant when discussing low frequency coils.

5-10-1 Experimental measurement of power input to the vessel by a method similar to Gilbert's method

All the experimental work of this chapter has been based on an implied measurement of loss in the vessel. There has been no direct measurement of the heating effect in the vessel. The assumption has been made throughout that the power input to the vessel is the total coil input power less the coil losses. No allowance has been made for stray losses. An alternative experimental method is required to check this assumption. One way of doing this is to use a thermometric method.

5-10-2 Literature

A.J. Gilbert²⁹ described a method of measuring loss distribution in machine cores. His technique is based on the thermal behaviour of steel during heating. It is concerned with the heat generated in a thermally insulated semi-infinite steel slab. He provides a one-dimensional solution to the heat flow problem of induced loss. The loss at a point is shown to be directly proportional to the true initial incremental temperature rise at that point. Under normal test conditions the heat loss from a steel surface reduces the initial rise of temperature. He shows that this can be compensated by adding the initial rate of fall of temperature at switch off.

However, Gilbert's method is unsuitable for vessel heaters as it assumes that the steel loss member is effect-
ively infinitely thick which is not true in this case. There is a further complication caused by the motion of fluid within the vessel. This has the surprising effect of causing a localised initial decrease of temperature at switch on. A parallel approach to Gilbert's method is needed if the electromagnetic measurements of loss are to be checked.

5-10-3 Alternative theory

Assumptions

- That the fluid has reached a fully developed flow condition which has a quasi steady flow velocity over the vessel surface.
- That the temperature drop across the vessel wall is small.
- 3. That the power flow along the vessel walls is small so that a one-dimensional analysis may be applied.



In time δt power δH is absorbed by the small element of vessel wall.

Therefore

$$\frac{\delta H}{\delta t} = \rho C \pi D s \delta x \left(\frac{\delta Q}{\delta t} \right) + \frac{\delta H}{\delta t} s + \frac{\delta H}{\delta t}$$
(5-22)

If the power is suddenly removed the vessel continues to lose heat to its surroundings and contents. Then

$$0 = \rho C \pi D s \delta x \left(\frac{\delta \Theta}{\delta t} \right) + \frac{\delta H}{\delta t} s + \frac{\delta H}{\delta t} f \qquad (5-23)$$

Subtracting these equations gives

$$\frac{\delta H}{\delta t} = \rho C \pi D s \delta x \left(\frac{\delta \Theta}{\delta t} - \frac{\delta \Theta}{\delta t^2} \right)$$
(5-24)

In the limit as $t \rightarrow 0$, the surface power density becomes

$$W = \rho C s \left(\frac{d\Theta}{dt} - \frac{d\Theta}{dt^2} \right)$$
 (5-25)

The power loss per unit surface area may be simply measured, by determining the sum of the rate of rise of temperature before switch off and the rate of fall after, and applying this to equation 5-25.

5-10-4 Apparatus and method

The temperatures were measured with a copper constantin thermocouples fed through a reed switch box via a Textronic 1 A7A differential amplifier into an x-y chart recorder. Every effort was made to screen leads. The amplifier gave an output of 0.25 V for 1mV input and the chart recorder used the 0.05 in/s and 0.05 V/in ranges. The temperature constant of copper constantin is $40 \,\mu\text{V/C}^{\circ}$. Hence the chart has a scale of 20°C/in .

The heater was connected with the two innermost coils

in series forming a single 600 mm coil, and operated at 236 V. As a number of similar experiments constitute the test, the interval and duration between each test was carefully timed at 2 minutes. The experiment had to be abandoned when the vessel containing 250 kg of water boiled over, but by this time more than half the vessel had been tested and the results could be calculated on the basis of a symmetrical power input.

5-10-5 Results

The temperature measurement curves show that there is an initial period of unsteady fluid flow in the vessel. This is followed by a steady flow region, readings taken in this period give graphs 10-1, 10-2 . Each graph shows a small instability at switch off, this is due to the equalising of temperature across the steel thickness and the response time of the amplifier. These were used to yield the graphs of total slopes which is directly proportional to the power distribution. The area under these graphs was used to give a mean rate of change of temperature over the vessel surface. Then the total input power is given by

W (5-26)

where

Hence

C D L w s d d t		435 J/kgK 500 mm 1.2 m 9 mm 0.595 C/s
p	=	7.85 kg/litre
P	=	34 kW

 $P = \rho C \pi DLs \frac{d\Theta}{dt}$

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This result compares with 34.17 kW of experiment 5-9. It is a justification of the experimental method used throughout the remainder of this chapter.

5-10-6 Surface temperature distribution

Measurements of the surface temperature pattern were taken using the same chart recorder apparatus. The resulting curve is shown in figure 5-9. Information concerning the flow pattern inside the vessel can be deduced from this temperature profile. If the flow pattern was that of figure 5-8 where a uniform surface layer of fluid is in motion, then the fluid temperature at a given height x



could be expected to be

$$\Theta_{f} = mC \int W dx$$

where m is the mass of fluid in the moving stream. This again must result in a temperature pattern of figure 5-9

which does not compare



Fig 5-9

Vessel surface temperature distributions

with the experimental results. This suggests that the flow pattern must be of the form (fig 5 - 10) below



where the moving layer of fluid is thickened by the increasing energy level in the water. This type of flow pattern adds a complication to the previous experiment, and to the prediction of heat transfer. The heat transfer mechanism is discussed in Appendix B.

5-11-1 Vessel surface field measurements

The distribution losses measured in experiment(5-10) can also be obtained from field measurements. If the current densities in the vessel are known and the relationship between current and loss is known, it follows that the loss distribution is known. This relationship depends on the electromagnetic properties of the vessel and can be calculated theoretically. The relative magnitude of loss along the surface of the vessel is generally given by

 $\frac{W_1}{W_2} = \begin{pmatrix} J_1 \\ - \\ J_2 \end{pmatrix}$ re τ is the

where τ is the index which varies between theories but is generally taken to be 2.84 or thereabouts and where J is the surface value of current density.



Probe Circuit

Twisted Weld

Probe details

<u>Fig. 5-11</u> <u>Current Density Probes</u>

Burke and Alden¹³ discuss current density probes which show that with the typical arrangement of figure 5-11 above, the current density along the line of the probe ends is given by $J = V/\rho$ where V is the valve voltmeter reading and ρ is the resistivity of the conductor. Hence, the valve voltmeter is in fact reading the value of ρJ which is E the electric field strength. Under certain conditions E may be measured with a search coil. These are that the direction of current flow is known and that its path may be followed by a search wire. This eliminates the need for welded probes inherent in the J measuring apparatus. For this reason 23 equally spaced circumferential search coils were wound on the 1.2 vessel. Tests were made of each coil arrangement at various supply voltages. Throughout the series of tests, readings of E were taken using a true R.M.S valve voltmeter, using screened leads and a specially constructed screened switchbox.

5-11-2 Results

It was found that the surface E distribution set up by each heating coil arrangement could be normalised into a single curve, independent of the supply level (graph 11-1) Comparison of the appropriate derived power curve with that of temperature test 5-10-2 show a degree of similarity (graph 11-2). The correlation is improved when allowance is made for temperature distribution (5-10-3). The measurements in this chapter have assumed that the currents are sinusoidal. It was felt that this assumption should be examined, and hence the following experiment was conducted.

5-12-2 Method

The large apparatus was used as this came closest to a commercial heater. The current drawn by this apparatus is too large to enable a shunt to be used to provide a current dependent voltage signal. For this reason the current waveform was taken across a low value resistor in a current transformer circuit. A supply voltage signal was taken with a potential divider and was used to give a reference for the current waveform. The waveforms were observed with a dual trace oscilloscope. The apparatus was supplied directly from the mains, and the vessel was heated by the fourth layer coil at 233V, 242A, 44kW supply conditions.

5-12-3 Results

As expected the supply voltage is sinusoidal, whilst the current waveform contains harmonics and is triangular in nature (fig 5-12). Chapter 4-2 has shown that the only analytical theories capable of predicting the waveform harmonics are the limiting non-linear theories which are based on a step-function law approximation to the B-H curve. The measured current waveform may be built up by adding a sinusoidal magnetising current to a leakage current defined in terms of the voltage driven limiting nonlinear loss solution

Whence

$$I_1 = \hat{I}_1(1.299) \text{ Sin wt (1-Cos wt)}$$
 (5-28)

and

 $I_t = I_m \cos wt + \hat{I}_1(1.299) \sin wt (1-\cos wt) (5-29)$

This theoretical curve is drawn in (fig 5-12) It does not allow for the phase shifting effect of the leakage flux on the magnetising current. This is a correction which would bring the theoretical and measured waveforms closer still but its magnitude is not known without measuring the flux triangle.

It is interesting to compare the powerfactor derived from this theory as defined by

 $\cos \phi = \frac{P}{VI}$

where P, V, I are r.m.s. quantities with the experimental value. This gives a theoretical value of 0.89 which contrasts with a measured value from experiment 5-4 of 0.78.

5-13-1 Measurement of surface E and magnetising flux waveforms

The result of the previous experiment implies that the steel loss mechanism is complicated and unexplained by analytical theory. Moreover, it has been assumed that the eddy-current mechanism is the same at all points on the vessel surface. These two points prompted the



Measured Waveforms





Theoretical Waveforms

Fig.5-12

measurement of the vessel field quantities in the hope that a better understanding of this mechanism might simplify the analysis of the problem.

5-13-2 Method

The method of measurement was to record the waveforms from search coils on the vessel surface. The 1.2m long vessel had coils for this purpose. The surface electric field intensity waves were measured directly from the individual search coils on the vessel surface. Whilst the magnetising flux waves were measured by taking readings from pairs of adjacent search coils. The vessel was heated as before, and supply voltage used as a reference waveform on a dual trace oscilloscope.

5-13-3 Results

The resulting waveforms are shown in (figs 5-13) together with the search coil positions and dimensions (table5-2). The waveforms were found to be symmetrical about the middle plane of the heater coil and for this reason only the lower half readings are given. All waveforms contain harmonics, and, as in the previous experiment, it is useful to compare the results with those predicted by the limiting non-linear theories shown in fig 5-14.

There are two types of surface E wave forms. That measured outside the coil shown in traces A, B, C and that within the coil shown in traces D, E, F. The

Search Coil	Distance from · Vessel top
No.	
А	50
В	150
С	250
D	350
E	450
F	550
Ę	600
Search Coil* No.	Distance of Coil Centre line from Vessel top min
а	75
Ь	175
С	275
d	375
d e	375 475

Table 5-2





Measured Waveforms



<u>Theoretical Waveforms</u>, <u>Fig. 5-14</u> latter can be thought of as a sinusoidal voltage driven effect.

The trace obtained at the coil mid-plane, as shown in fig (5 14), may be deduced by subtracting the leakage flux produced voltage, predicted by a step-function B-H law, from the total coil voltage.

The region outside the steel is one of current driven loss and may be seen by comparing the measured E trace with the theoretical curve figure (5-15) search coil

The magnetising/voltage waveforms are not as easily explained as the preceding E waveforms. However, certain points can be noted. Firstly, the waveforms are weakest within the coil and largest at the coil ends. Secondly, all the magnetising voltage waveforms contain spikes which correspond to the discontinuity in the associated E waves.

A consequence of the limiting non-linear theory is that the direction of flux density changes at moving separation layer. This separation layer progresses from the surface into the steel cyclically. The discontinuity of surface waveforms occur when this separation layer is at the surface. Assuming that the layer cuts the steel surface at a small constant angle, then this would produce search coil voltages of alternate positive and negative spikes. This is not the case except at the coil centre and the eddy-current process cannot be explained in these terms. Some rotation of the saturation direction must be allowed for. Hence the non-linear theory only partially explains the vessel behaviour.



Fig 5-15



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5-14-1 Experimental measurement of vessel resistivity

It will be shown in section 6-4-2 that the vessel resistivity is a critical quantity in the design of an induction heater. A specimen of the steel used in the construction of the 1.2 m vessel was machined to a thin flat bar and its resistance was measured with a Kelvin bridge. This gave the resistivity of the vessel steel as 222 nm.

5-15 -1 Comparative criticisms of the vessel models

There are a number of areas in which the experimental models differ from compercial induction vessel heater practice. Retort vessels are hemispherically closed--ended structures, whilst the vessel models have been formed from open-ended pipe sections. The full sized heater (5-9) has a non-metallic bottom and a 9mm thick mild steel removable flat/lid. Without the lid it is a scaled model of the small apparatus, whilst with the lid it is a step towards the more usual form of retort. Although the surface magnetising field pattern was slightly distorted by the lid (5-12) it was found that the performance was the same with and without it. It is felt that this validates the open ended form of model, with the reservation that the coils are away from the vessel ends.

All measurements have been made with the coil and vessel symmetrically placed. It has been noticed that this is the position of least reactive energy and is the best position for the coil relative to the vessel, this may not be practicable, in commercial applications.

A commercial vessel usually has a thin stainless steel inner cladding to prevent reaction with the vessel walls. The vessel models were plain and without any lining. Theoretically, the lining should have no electrical effect providing that the mild-steel outer layer is greater than the skin depth.

CHAPTER 6 THEORY OF AN INDUCTION VESSEL HEATER

CHAPTER SUMMARY

The problem of Chapter 2 is solved with reference to the experimental work of Chapter 5, and the method of images Chapter 3. The latter is extended to steels with constant properties and two sets of images are substantiated. The leakage permeance and loss distribution are calculated in a flat model, whilst a form of inverted ellipsoid is used to account for the magnetising permeance. A design method is built from the vector diagram; the tables and graphs derived are given in Appendix D. Finally, the theory is experimentally verified.

CHAPTER CONTENTS

- 6-1-1 Analysis
- 6-1-2 Steel theories
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- 6-2-2 Calculation of eddy current loss
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- 6-3-3 Analysis of the vector diagram

6-4-1 Discussion

NOTE - The definition of leakage flux used in this chapter is that given by Baker 4 and represents the leakage of the vessel.

6-1-1 Analysis

This chapter develops the theory for an induction vessel heater where the vessel is saturated. The object is to derive a design method at once both simple to use and wide in its application. It is essential that this method and theory should agree with the experimental work of Chapter 5.

6-1-2 Steel theories

It was shown in sections (5-4) and (5-5) that the Dreyfus theory for eddy-current loss in steel gave the closest fit to the experimental results. This is only true if the total current is assumed to be sinusoidal. But there is no reason why the Dreyfus theory cannot be used, and the harmonic components subtracted from the apparent total current. The Dreyfus theory has the advantage of being straightforward and simple which makes it suitable for design calculations. However, it is only a one-dimensional model and applies to vessel heating if the saturation depth is small relative to the coil length.

6-1-3 Reactances

The main problem lies in the calculation of machine reactances (section 2). Owing to the saturation of steel this is not as yet amenable to a true analytical solution. The possible methods of solution were outlined in section 3 and the unworkable approaches to the problem may now be

eliminated. Generality of solution must rule out analogues. The continuously variable surface field conditions prevent the use of complex functions. This leaves two main avenues of solution, the harmonic model and the method of images. Whilst non-uniform dimensional boundary conditions can be described by a harmonic model, the variation of permeability over the surface of the vessel inhibits this form of solution. Thus the method of images is the only promising solution. This can be applied to regions surrounding steel with µ greater than 100 with no provision that μ is constant ⁶ . A similar image method may be applied to air regions surrounding a highly conductive material. The methods have not been used together, however, physical reasoning leads in this direction. Consider the equivalent circuit (fig 2-4); it contains two main reactances, a magnetising reactance and leakage reactance. The directions of magnetising flux and leakage flux, at the surface of the vessel, are nearly at right angles to each other, and the total field pattern in air is their joint sum. The leakage flux is the component which cannot by definition enter the vessel and thus fulfils the conditions for a high conductivity image. Similarly, the magnetising flux leaves the vessel close to the normal and satisfies the requirements for a high permeability image.

There appears to be no previous proof that the method of images alone can be used to solve the field problems of a high $\mu\rho$ material, whilst reference⁵⁵ describes a method which requires prior knowledge of the

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<u>Fig_6-1</u> Stall and Hammond loss model

solution. Moreover, there is no proof of the idea that magnetising and leakage image patterns can be derived separately. The following is an attempt to justify and verify these ideas. The justification is based on the work of Stoll and Hammond. Their paper discusses a two-dimensional harmonic model. The model consists of a stationary, sinusoidal, alternating current sheet separated by a uniform gap from a magnetically permeable and conducting plate. Field solutions are derived in terms of the magnetic vector potential A, where curl A = B and div A = 0. The solution to a semi-infinite slab problem gives their equation (17) . $A_2 = \frac{\nu_1 \hat{K}_2}{2 \cdot q} \left\{ e^{q(y-y_i)} + \left[\frac{\mu_2 - \sqrt{1+ip^2}}{\mu_3 + \sqrt{1+ip^2}} \right] e^{-qb} e^{-q(y-y_i)} \right\} e^{-jqx}$

(6-1)

where $A_2 = \operatorname{Re}(\widehat{A}_2 e^{jwt})$ $q = \pi/g'$ $p = \sqrt{2}/q\delta$ $\delta = \sqrt{2p/\mu_3 \mu_3}$ Shifting the origin to y in equation 1 gives

$$A_{2} = \frac{\mu_{2} \widehat{K}_{z}}{2 q} \begin{bmatrix} e^{q(y-b)} + \frac{\mu_{3} - \sqrt{1+jp^{2}}}{\mu_{3} + \sqrt{1+jp^{2}}} & e^{-q(y+b)} \end{bmatrix} e^{-jqx}$$
(6-2)

Rearranging the second term -

$$A_{2} = \frac{\mu_{e}\hat{K}}{2q} \cdot \frac{1}{\mu_{3} + \sqrt{1+jp^{2}}} \left[e^{q(y-b)}(\mu_{3} + \sqrt{1+jp^{2}}) + e^{-q(y+b)}(\mu_{3} + \sqrt{1+jp^{2}}) \right] e^{-jqx}$$
(6-3)

The bracketed term may now be adjusted to give -

$$A_{2} = \frac{\mu_{e}\hat{K}}{2q} \cdot \frac{e^{-jqx}}{\mu_{3} + \sqrt{1+jp^{2}}} \left[\mu_{3}(e^{q(y-b)} + e^{-q(y+b)}) + \sqrt{1+jp^{2}}(e^{q(y-b)} - e^{-q(y+b)}) \right]$$

$$(6-4)$$

This is a pattern of two superimposed image fields. The magnetising field is given by -

$$A_{2m} = \left\{ \frac{\mu_{0}K}{2q} \frac{\mu_{3}}{\mu_{3} + \sqrt{1+jp^{2}}} e^{-q(jx+b)} \right\} (e^{qy} + e^{-qy})$$
(6-5)

and the leakage field by -

$$A_{2L} = \left(\frac{\mu_{0}K}{2 q} \frac{\sqrt{1+jp^{2}}}{\mu_{3} + \sqrt{1+jp^{2}}} e^{-q(jx+b)}\right) (e^{qy} - e^{-qy}) \quad (6-6)$$

But the field of the current sheet is

$$A_{2} = \frac{\mu k}{2q} e^{-q(jx + b)} e^{qy}$$
 (6-6a)

Hence it can be seen that the magnetising field is produced by a current sheet and its positive image, whilst the leakage field is given by a different coincident current sheet and its negative image. This result has been derived without approximation. Since linearity has been assumed in the Stoll and Hammond solution and any periodic function may be made up of a sum of sine waves, it follows that the equations 5 and 6 may be generalised. Hence, it has been proved that:

The field pattern above a steel slab with constant properties may be represented as the sum of two image fields whose magnitudes depend on the properties of the steel.

Yet, one problem remains. The vessel material is saturated and its value of permeability varies over the surface of the steel. However, the regions where it is small are also the regions of low magnetising flux density. The errors in this approximation of two super imposable fields are such that they are not greatly affected by saturation.



Magnetising flux distribution for 1.2 m Vessel (section 5-11)

Fig. 6-2

6-2-1 Assumptions

The basis has been laid for the development of a solution with the following assumptions:

 The influence of vessel curvature on the leakage reactance is so small that curvature effects may be disregarded and the vessel treated as a semi-infinite flat steel sheet.

> Even in small vessels the ratio of vessel radius to gap is greater than 10, and vessel radius to skin depth greater than 30. Baker has shown that these values give the same results as a slab.

2. Curvature effects influence the value of magnetising reactance and that a threedimensional model must be used in its calculation.

3. That a double field system may be used.

- 4. The permeability is large enough for the loss in steel to be controlled by and to control the tangential surface value of field strength.
- 5. The initial magnetisation curve may be approximated by $B = kH^{m}$.

$$W = H_{t}^{1.5+(m/2)} \sqrt{\frac{wk}{2p}} \sqrt{\frac{12}{m+1}} \sqrt{\frac{2}{m+3}} \qquad kW/m^{2} (6-7)$$

where

 H_t = tangential value of field strength At/m Since the tangential value of field strength is the only variable in equation 7, it is reasonable to start the analysis with a calculation of its distribution. The model based on assumptions 1 and 3 is shown below.



Tangential H distribution

The coil is represented by a current sheet of NI/l_c At/m Hence the total surface value of H_t at Q is given by

$$H_{t}(x,0) = 2 \frac{NI}{2\pi l_{c}} \int_{-l_{c}/2}^{l_{c}/2} \frac{\sin \theta}{q} du \qquad At/m \quad (6-8)$$

Substituting for Sin 0 and q and simplifying

$$H_{t}(x,0) = \frac{NI}{\pi l_{c}} \int_{-l_{c}/2}^{l_{c}/2} \frac{g}{g^{2} - (x-u)^{2}} du \quad At/m \quad (6-9)$$

Hence

$$H_{t}(x,0) = \frac{MI}{\pi l_{c}} \left(\operatorname{Arctan} \frac{x+l_{c}/2}{g} - \operatorname{Arctan} \frac{x-l_{c}/2}{g} \right)$$

At/m (6-10)

The solution may be further generalised to account for the length of the vessel. The end of the vessel can be treated as a corner, and the single current sheet becomes part of an infinitely repeated series of reflections.



The surface field strength becomes

$$H_{t}(x,0) = \frac{N I}{\pi l_{c}} \sum_{n=\infty}^{n=\infty} \operatorname{Arctan} \frac{x + \frac{l_{c}}{2} + \frac{l_{w}n}{g}}{g} - \operatorname{Arctan} \frac{x - \frac{l_{c}}{2} + \frac{l_{w}n}{g}}{g}$$

$$At/m \quad (6-11)$$

Unfortunately there is no sum readily available for this series. However, it rapidly converges and the series may be truncated after a few terms.

Whilst the magnetic field strength $H_t(x,0)$ has not been measured, the electric field strength E(x,0) was recorded in experiment 5-11. Now according to the Dreyfus theory E is related to H by

$$E(x,0) \propto H_t(x,0)^{0.5 + m/2}$$
 (6-12)

The value of H_t is a maximum at x = 0, and it is convenient to normalise the expression so that.

$$\frac{E(x,0)}{E(0,0)} = \left[\frac{H_{t}(x,0)}{H_{t}(0,0)}\right]^{0.5 + m/2}$$
(6-13)

Hence

$$E(x,0) = E(0,0) \left\{ \sum \arctan \frac{x + \frac{1}{2}c + \frac{1}{w}n}{g} - \arctan \frac{x - \frac{1}{2} + \frac{1}{w}n}{g} \right\}^{2}$$

1+m

These curves have been drawn for n=10 and are plotted a against measured values of E in groph(6-2p/149). The model seems to be a reasonable fit even in regions where the eddy-currents are weak.

Similarly the loss equation 7 may be normalised and the surface power density may be written

$$\frac{W(x,0)}{W(0,0)} = \left[\frac{H_{t}(x,0)}{H_{t}(0,0)}\right]^{\frac{3+m}{2}}$$
(6-15)

However the total power input may be written as

$$P = \pi D \int_{-\frac{1}{2}W}^{\frac{1}{2}W} dx \qquad kW \quad (6-16)$$

Combining this with equation 15 gives an expression in terms of $W(0,0)\pi Dl_c$. This is the power which

would be put into the vessel if the heat was uniformly confined to the area under the coil. The equation is -

$$P = W(0,0) \pi D l_{c} \int_{-\frac{1}{2l_{c}}} \left[\frac{H_{t}(x,0)}{H_{t}(0,0)} \right]^{\frac{2+m}{2}} d(x/l_{c})$$
(6-17)
$$- \frac{1}{2l_{c}} \frac{W}{2}$$

The integral part of equation 17 represents the effect of necessary the current distribution, and it becomes/to define this as a factor S. equation 17 becomes :

$$P = W(0,0) \pi D l_{c} S \qquad kW \qquad (6-18)$$

where

$$S = \int_{-1_{W}/21_{c}} \left[\frac{\sum_{n=0}^{\infty} \operatorname{Arctan}}{\sum_{n=0}^{\infty} \operatorname{Arctan}} \frac{\frac{2x+1+2n}{c}}{2g} - \operatorname{Arctan} \frac{2x-1+2ln}{2g}}{\sum_{g}^{\infty} \operatorname{Arctan}} \frac{\frac{2x+1+2n}{2g}}{2g} \right]^{d} d(x/1_{c})$$

(6-20)

S has been computed for various gap to coil length ratios for different values of vessel length to coil length. Twenty one image terms were taken and the computation was made with a one hundred step simpsons rule integration. This was done for m = 0.1 and the results are shown in table D-2 . This factor is independent of size but is affected by the angles of the coil vessel geometry. It is usualy less than one, and is worsened by increasing the ratio of gap to coil length.

6-2-3 Calculation of leakage flux

The leakage flux can be calculated from the same model as the eddy-current loss. Consider the case of an infinitely long vessel shown below. The total leakage flux



Leakage flux model.

passes through the coil centre line and thus

but

$$H(0,y) = \int \frac{NI}{\pi l_c} \frac{\sin \theta}{q} + \frac{\sin \theta}{q} du \qquad At/m \quad (6-22)$$

Solving

$$H(0,y) = \frac{NI}{\pi l_c} \left[\arctan \frac{l_c}{2(g-y)} + \arctan \frac{l_c}{2(g+y)} \right]$$

$$At/m \quad (6-23)$$



Substituting equation 23 into equation 21 and performing the integral for ϕ gives

$$\phi_{1} = \mu H(0,0) \pi D g \left\{ \begin{array}{c} \operatorname{Arctan} \frac{1_{c}}{4 g} - \frac{\pi}{2} \frac{1_{c}}{4 g} \operatorname{Log}_{e} \left[\begin{pmatrix} 4g \\ 1_{c} \end{pmatrix}^{2} + 1 \right] \\ \operatorname{Arctan} \frac{1_{c}}{2 g} \end{array} \right\}$$

Which is identical to Baker's expression for leakage flux with the addition of a non-dimensional multiplying factor Hence, equation 24 may be written

Wb

(6-24)



Reflected pair of images; part of an infinite series.

Turning to the effect of limited vessel length, the coil and vessel may be represented by an infinite series of images reflected at the boundary corners. In this case it is more convinent to look on this system of images as a series of positive and negative coils as shown above. Equation 25 may now be generalised as

$$\phi = \mu H(0,0) \pi D g \cdot \left\{ G(\frac{g}{l_c}) + \sum_{n=1}^{\infty} \left[G(\frac{g}{2nl_w+l_c}) - G(\frac{g}{2nl_w-l_c}) \right] \right\}$$

Wb (6-26)

Fortunatately a useful simplification may be applied to this equation. For

Arctan $\frac{1}{\Theta} = \frac{\pi}{2} - \Theta$ and $\log_{\Theta}(1 + \Theta^2) = \Theta^2$ where Θ is small. Then using these approximations -

$$G(\frac{g}{2nl_{w}+l_{c}}) = 1 - (\frac{\pi+l}{\pi}) \left(\frac{4g}{2nl_{w}+l_{c}-(4/\pi)g} \right)$$
(6-27)

It is convenient to define a function γ such that

$$\gamma = \frac{l_{c}}{g} \sum_{n=1}^{\infty} G(\frac{g}{2nl_{w}+l_{c}}) - G(\frac{g}{2nl_{w}-l_{c}})$$
(6-28)

Applying equation 27 to 28

$$\gamma = \sum_{n=1}^{\infty} 4 \left(\frac{\pi + 1}{\pi} \right) \left[\frac{l_c}{2\pi l_w - l_c - (4/\pi)g} - \frac{l_c}{2\pi l_w + l_c - (4/\pi)g} \right]$$
(6-29)

The function γ may be further simplified by reference to the relative sizes found in vessel heaters. Usually $(4/\pi)g\ll 2l_w-l_c$ so that equation 29 becomes

$$\gamma = \left(\frac{\pi + 1}{\pi}\right) \left(\frac{2 \, l_c^2}{l_w^2}\right) \sum_{n=1}^{\infty} -\frac{1}{n^2 - (l_c/2l_w)^2}$$
(6-30)

The series can now be summed using the result given in reference 46 page 79 no.(b.3).

$$\sum_{n=1}^{\infty} \frac{1}{z^2 - n^2} = \frac{\pi}{2z} \cdot \cot \pi z - \frac{1}{2z^2}$$
(6-31)

Then y becomes

$$\gamma = 4\left(\frac{\pi+1}{\pi}\right) \left[1 - \frac{\pi l_c}{2 l_w} \cdot \cot\left(\frac{\pi l_c}{2 l_w}\right)\right]$$
(6-32)

Equation 23 shows that γ is independent of the coil to vessel gap and has been plotted for ratios of coil length

to vessel length in graph D-3. The leakage flux may now be written as

$$\phi = \mu H(0,0) \pi D g \left[G + \left(\frac{g}{l_c}\right) \cdot \gamma \right] \quad Wb \quad (6-33)$$

where the geometrical effects have been reduced to seperate functions. The magnitudes of G and $(g/l_c) \cdot \gamma$ are such that the latter generally acts as a correction factor.

The multiple image caused by the finite vessel length (fig 6-6) influences the field strength at the vessel surface, and equation 10 becomes

$$H(0,0) = \frac{2 N I}{\pi l_c} \left[\operatorname{Arctan} \left(\frac{l_c}{2 g} \right) + \frac{1}{2} \sum_{n=1}^{n=\infty} \operatorname{Arctan} \left(\frac{2 n l_w + l_c}{2 g} \right) - \operatorname{Arctan} \left(\frac{2 n l_w - l_c}{2 g} \right) \right]$$

$$At/m \quad (6-34)$$

By similar reasoning to that used between equations 27 and 32

$$H(0,0) = \frac{2 N I}{\pi l_c} \left[\operatorname{Arctan} \frac{l_c}{2 g} + \frac{\pi}{4(1+\pi)} \frac{g}{l_c} \gamma \right] \quad At/m \quad (6-35)$$

6-2-4 Magnetising flux

It has been established that two currents (6-1-2) flow jointly in the coil. One current sets up the leakage field and controls the saturation, whilst the other is responsible for the magnetising flux. The steel acts as a link between the magnetising flux and the surface value of H. Although the method of images was used to calculate the leakage fluxes, there is no reason why it must be used to calculate the magnetising fluxes. The author had hoped that a semiinfinite flat image solution would be sufficient to describe the magnetising flux. However, whilst the relative flux densities are correctly described, the absolute magnitudes are reduced. The magnetising permeance is most sensitive to the topology since it is increased by radius effects. Another method of solution has had to be attempted.

Maxwell has shown that if a uniformly permeable ellipsoid is placed in a uniform field then it will not upset the field pattern. If this is true for case <u>a</u> of (fig 6-8) then it is certainly true for case <u>b</u> where the relative permeabilities have been reversed. Consider the effect of the inverse of <u>b</u> with respect to the centre of the ellipse. The field pattern is now radically altered and shapes are produced which closely resemble retort vessels (fig 6-9). The resulting field pattern is that produced by a single coil exciting a retort-shaped body. Now, Osborn has produced tables of demagnetisation factors for ellipsoids, and these






Fig. 6-8

Ellipsoid field patterns

can now be applied to give an equivalent uniform gap in the flux path in <u>b</u>. The equivalent inverse figure to the ellipses of same diameter has a length l_w and the ellipse semi-axis becomes D/l_w , and the Osborn curve for the demagnetisation of an oblate spheroid plotted for the semiaxis length c/a is equivalent to the curve plotted for D/l_w where these are the vessel dimensions.

This is shown in figure (6-10) over:

Moreover, it has been shown experimentally (5-8-3) that the flux outside the vessel obeys the law.

$$\frac{\phi_{\rm S}}{\phi_{\rm c}} = \left(\frac{\rm d_{\rm S}}{\rm d_{\rm c}}\right)^{-n} \tag{5-20}$$

where n = 1

the flux is that at a distance D and the subscript \underline{c} refers to the coil. Thus inverting the geometry and the problem becomes one of nearly uniform flux distribution. This again conforms to the idea of an inverse figure type



<u>Fig.6-10</u> Demagnetising Factors

solution. Thus the magnetising flux may be written as

$$\phi_{\rm m} = \pi^2 \,\mu \, {\rm DO}_{\rm f}(1_{\rm C}/1_{\rm W}) \, NI \qquad \qquad {\rm Wb} \quad (6-36)$$

where O_f is given above and is the Osborn demagnetising factor.

6-3-1 SYNTHESIS

The preceding analysis makes it possible to calculate the circuit quantities in terms of the heater geometry and physical constants. The remaining and more difficult problem is that of creating a design method. The designer will be confronted with a number of target quantities: power input, powerfactor and efficiency. The vessel dimensions will have been decided upon for mechanical considerations and the vessel material will, presumably, be mild steel by reasons of cost and performance. The designer will be faced with a maximum power-density constraint and a plant operating temperature range. Knowing the operating temperature, he can be expected to make a reasonable choice of gap between the coil and vessel to minimise thermal losses, and the length of coil will be dictated by the power input and permissible power density. Hence, all quantities are fixed with the exception of coil turns.

6-3-2 Normalisation of the Dreyfus theory

The calculation of power from the Dreyfus theory is complicated, and is simplified by a process of normalisation. This enables the constants to be calculated once and once only. The only problem is to decide what value to use as a normalising quantity. A unique point is required on the B-H curve, and the knee point is suitable (fig 2-5). It may be assumed that a saturated design method might fail below this value and that all normalised

Ъ	0.6872		
m	0.10777		Independent of
Cos O	0.80382		temperature
. θ	36° 30)
μ _r	2500		Vaco point
H _n	419.3	At/m	values
B _n	1.371	Wb/m ²	
p	250	nΩm	
Wn	49.28	W/m ²	Knee point
ø'n	7.812	m Wb/m	values
δ	0.7118	mm	(graph D-1)
h _n	1.032	mm	

TABLE 6-1 Mild Steel (Saturated) Define the quantities at the knee point by the suffix n.

Then

$$\frac{H}{H_n} = \left(\frac{W}{W_n}\right)^{\frac{2}{3+m}}$$
(6 - 37)

and

and

111

$$S_n = \left(\frac{W}{W_n}\right)^{2\frac{1-m}{3+m}}$$
(6 - 39)

these are the three quantities required in the power input calculation. W_n , \emptyset_n , b_n , vary with resistivity. Graph D-1 shows them plotted for mild steel against resistivity. The powerfactor is assumed constant and depends only on the index of the steel

$$\cos \theta = \sqrt{\frac{2}{3+m}} \qquad (6-40)$$

Given the previous table of values, it is now an easy matter to scale the loss problem to the level of excitation, and the major area of difficulty has now moved from the steel to the overall circuit problem.

6-3-3 Analysis of the Vector diagram

The complications of induction vessel heater designate are clearly demonstrated by the complexities of the vector diagram. It consists of three triangles, a flux triangle, a current triangle and a voltage triangle. Only two angles are known, both of which are equal to the complementary powerfactor angle of the steel (experiment 5-6).

These repeated angles lead to a repeated geometrical problem. In both the current and flux triangles two sides and an included angle are known, and the problem in each case is to find the other angle. Hence, it is useful to modify the sine and cosine rules for this case.



A Vector Triangle

From the sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
(6-41)

and from the cosine rule

$$b^2 = a^2 + c^2 - 2ac \cos B$$
 (6-42)

Combining these gives

$$\sin A = \frac{\sin B}{\sqrt{1 + (\frac{c}{a})^2 - 2(\frac{c}{a}) \cos B}}$$
(6-43)

All that is needed to find the angle is the ratio of the lengths of sides and the angle between them. It has been shown that for both flux and current triangles the ratio of the sides is a simple function independent of the number of turns in the coil (5-7). Equation 43 has been plotted for $B = 126 \cdot 6^{\circ}$ for values $(\frac{c}{a})$. Flux triangle

In section 6 it was shown that

$$P = W(0,0) \pi Dl_c S$$
 (6-18)

Dividing by Wn TDL S gives

$$\frac{W_c}{W_n S} = \frac{W(0,0)}{W_n}$$
(6-44)

applying equation 38 and simplifying

$$\phi_{\rm m} = \phi_{\rm n}' \pi D \left(\frac{W_{\rm c}}{W_{\rm n} S} \right)^{\frac{1+m}{3+m}}$$
 Wb (6-45)

Turning to the leakage flux: it has been shown that

Combining this with equations 44 and 38 gives

$$\phi_{1} = \mu H_{n} \left(\frac{W_{c}}{W_{n} S} \right)^{2(3+m)} \pi Dg \left[G + \left(\frac{g}{l_{c}} \right) \gamma \right]$$

$$Wb \quad (6-46)$$

Equations 45 and 46 are used as design equations for magnetising flux and leakage flux. The ratio between ϕ_1 and ϕ_m is used with equation 42 to predict the angle between the leakage flux and the back e.m.f. E Current triangle

This is the second of the three vector triangles

E

Voltage Triangle

construction enables a useful trigonometrical result to be derived connecting all three quantities.

First multiply the triangle by I_t. Then from P drop a perpendicular onto OK at Q cutting ON externally at R Now

OQ = P the power input to the vessel and $NP = I^2 R$ the coil loss

 $= (1 - \eta) P$

Then since NP is parallel to OQ

 $\frac{PQ}{RQ} = \eta$

that has to be predicted in a design. Fortunately, only the relative magnitudes of current are required to give the angles of the triangle. The relative currents are fixed by the ratio of magnetising and leakage fluxes. Combining 36, 33 and 35 gives

$$\frac{\phi_{1}}{\phi_{m}} = \frac{\frac{N I_{1}}{\nu_{c} I_{c}} \frac{2}{\pi} \left[\operatorname{Arctan} \frac{1_{c}}{2g} + \frac{\pi}{4\pi + 4} \frac{g}{1_{c}} \gamma \right] \pi Dg \left[G + \frac{g}{1_{c}} \gamma \right]}{\mu_{o} N I_{m} \pi^{2} D O_{f}}$$

(6-47)

Simplifying

$$\frac{I_{1}}{I_{m}} = \frac{\phi_{1}}{\phi_{m}} \frac{\pi l_{c}}{g} \frac{2}{\pi} \left[\operatorname{Arctan} \frac{l_{c}}{2g} + \frac{\pi}{4\pi + 4} \frac{g}{l_{c}} \gamma \right] \left[G + \frac{g}{l_{c}} \gamma \right]$$

$$(6-48)$$

This equation gives results which can be combined with equation 42 tobgive all the angles in the current triangle. The angle between the total current and the back e.m.f. is the sum of the angles between I_t and I_1 , and between E and ϕ_1 . The remaining phase shift occurs in the voltage triangle.

Voltage triangle

If the assumption can be made that the self reactance of the copper of the coil is small, then the voltage triangle is readily amenable to solution. The design has already derived the angle between E and I, and the efficiency and powerfactor are all that are required to complete the solution. The following geometrical But $\operatorname{Tan} \Theta_{1} = \frac{RO}{OQ}$ and $\operatorname{Tan} \Theta_{2} = \frac{PO}{OQ}$

Hence

$$\operatorname{Tan} \Theta_2 = \eta \operatorname{Tan} \Theta_1$$

(6-49)

The above equation may be used to determine the running powerfactor angle Θ from the electrical efficiency and the angle Θ .

Applying the Sine formula to the voltage triangle gives:

$$\frac{V}{\operatorname{Sin}(180-\theta_2)} = \frac{E}{\operatorname{Sin}\theta_1}$$
(6-50)

but

$$E = \frac{w N \phi_t}{\sqrt{2}}$$
(6-51)

Which is an assumption that the flux completely links the coil. Combination of equations 49, 50, and 51, yields an expression for the number of coil turns.

$$N = \frac{\sqrt{2} V K}{w \phi_{t}}$$
(6-52)

where

$$K = \frac{1}{\sin \theta_{1} \sqrt{1 + \frac{1}{\eta^{2} \operatorname{Tan}^{2} \theta_{1}}}}$$
(6-53)

K is a design function, and is given for a range of values of θ and η in Appendix D.

The equations derived in this section form the basis of a design method. Only the supply conditions, power input and electrical efficiency need be known to determine the number of heater coil turns for a given coil-vessel geometry.

6-4-1 Discussion

The results taken from the phasor diagram could have been derived from the equivalent circuit. However, the diagram is a less complicated approach. The equivalent circuit has the advantage of highlighting the heater behaviour. Naturally each triangle in the diagram represents a part of the equivalent circuit.

The flux triangle is formed from the vessel impedance and the air-gap reactance. Hence the phase angle between the leakage flux and the back e.m.f. is independent of the magnetising reactance. The current triangle represents the equations derived for the magnetising and leakage branches of the equivalent circuit. The relative values of the branches are such as to reduce errors arising from the magnetising reactance. The voltage triangle is the addition of the coil impedance to the remaining circuit impedance. This addition also reduces the relative size of errors in the estimation of e.m.f.

It is possible to use the previous experimental work of this thesis as a check of the theory. A useful starting point can be made in that if the magnetising search-coil voltage is constant then the loss conditions in the steel beneath it are constant. Thus the results of experiments 5-7 and 5-11 may be used as a check of S (equation 20). For a curve of power for different vessel lengths at constant gap to coil length, and $V_m(0,0)$ should be a curve of S. The curves come within $\pm 10^{\circ}$ of each other. The errors are felt to arise for two reasons. Firstly, the vessel outside the coil may well be unsaturated and the loss may well be proportional to H² rather than the non-linear index used in calculating S. Secondly, the magnetising flux is assumed to leave the steel surface normally, whereas in reality a refractive law is obeyed

$$\frac{\sin \theta}{\sin \theta} = \frac{\mu}{\mu}, \tag{6-54}$$

and thus there is a small tangential component of H_{m} in air, such that:

$$H_{tm} = \frac{H_{m}}{\mu_{r}}$$
(6-55)

Thus the total tangential value of H is (6-56)

$$H_t = H(leakage) + \frac{H(magnetising)}{\mu_r}$$

It is the tangential magnetising component of H that leads to the distortion of the measured surface E pattern. Fortunately the effect on S is small. This error results in an over-prediction of approximately $3^{\circ}/_{\circ}$ in the total flux and about a 1° under-prediction in the angle ϕ -E. These two errors tend to be self-correcting when applied to the calculation of turns.

The heater coils of experiment 11 are too close to enable curves of S for fixed work length to coil length to be drawn versus gap. However, the measured surface E patterns are consistant and provide a means of calculating S from the numerical integral



$$S = \frac{1}{n \frac{1}{c}} \sum_{k=0}^{n} \left[\frac{E(x)}{E(0)} \right]^{2.82}$$
(6-57)

Gap	Length	Length	S	S
	of Coil	of Work	Measured	Calculated
mm	m	m		
19.75	0.6	1.2	1.0134	0.967
28.5	0.6	1.2	1.0186	0.956
37.25	0.6	1.2	1.0267	0.947
46.0	0.6	1.2	1.0452	0.937

and the results are shown below

Table 6-2

The function $\begin{bmatrix} G + (\frac{E}{l_c})\gamma \end{bmatrix}$ of equation 33 is also required for the calculation of ϕ_1/ϕ_m . The expression γ is verified by holding W(0,0) constant in equation and using the results of experiment with constant $\frac{E}{l_c}$ which implies constant G, thus a curve of $V_1/V_1(\infty)$ versus l_w/l_c for constant V_m is an implied curve of γ (graph 6-1). This shows very close agreement with the theoretical curve. A deviation occurs when the coil length is very nearly equal to the vessel length, which is to be expected as this violates the theoretical assumption that $l_w-l_c > 2g$.

The magnetising permeances have been measured experimentally (5-7) and show an agreement within 20% of that calculated from equation 36.

Vessel	Vessel	Measured	Calculated
length	diameter	Am	A _m
mm	mm	xl0 ⁻⁶	x10 ⁻⁶
240	140	0.72	0.585
320	140	0.93	0.69
480	140	1.00	0.82
1800	140	1.25	1.02
480	165	1.20	0.78

Table 6-3

Similarly equation 35 has been checked graphically and shows a similar measure of agreement.



Surface E values at the steelsurface v Axial position.

gap = 2.375 cm

coil length= 60cm

work length=120cm

Graph 6-2

CHAPTER SUMMARY

The measured performance of a heater is used as a design specification for three design theories including the thesis method. The results are compared with the experimental machine to show the competence of the thesis method.

CHAPTER CONTENTS

- 7-1 Design studies
- 7-2 Baker's design method
- 7-3 Vaughan and Williamson's design method
- 7-4 Thesis design method

7-1 Design studies

The basic equations for a design method have been established in the previous chapter. The method and necessary functions are given in Appendix D.

Comparison has been made by relating the analytical expressions for each part of the machine performance but this gives no indication as to whether or not the errors are self-compensating. The only real test of a design method is one of practise. Does it produce a heater within the design specification and is it straightforward in application? Hence the method will be used to design heaters which have already been tested. The following are comparisons of the results of this method with Baker's, Vaughan and Williamson's method and the experimental work of this thesis.

Specification

Vessel	Material	Mild steel
	Length	1.2m
	Diameter	0.5m
	Resistivity	220 n.r.m
Coil	Length of coil	0.6m
	Coil vessel gap	20mm
	Conductor	Water-cooled tubing
Performance	20kW input to ves	ssel 71% oeff.
	from 187V 50Hz st	upply
Experimental	ly measured from	graph
-	Number of turns	66
	Powerfactor	0.83

7-2 Baker's design method

Baker gives a full induction heater design method which will now be used to design the machine specified. The special nature of coil conductors used in this machine makes it difficult to apply Baker's coil resistance and reactance formulae and hence the design will be slightly different in one equation only, being based in this case on the electrical efficiency. Moreover, as Baker's formulae are given in c.g.s units these will be used in this section with reference to his work. The first step of the design is the calculation of the permeability of the vessel steel. Baker says "determine H_0 and μ from equation 22" below

$$P_{\rm W} = 2.5 f H_0^2 l_0 10^{-8} (\mu A_{\rm W} Q)$$
(7-1)

However, μ is a function of H and Q is a function of μ . Baker suggests the relationship

$$\mu = \frac{32400}{H_0} + 1$$
(7-2)

for mild steel. Whilst for a solid cylindrical body when $d/\delta > 8$, which applies in this case

$$Q = \frac{2}{d/\delta + 1.23}$$
 (7-3)

where

$$\delta = 5040 \sqrt{\rho/f_{\mu}}$$
 cm (7-4)

and H_0 and μ can only be found by a cut and try procedure and, indeed, Baker assumes a reasonable value of H_0 . For this initial guess, equation 2 was approximated to

$$\mu = \frac{32400}{H_0}$$
(7-5)

and equation 3 approximated to

$$Q = 2\delta/d \tag{7-6}$$

whence

$$\mu^{2} = \frac{\pi D l}{P_{W}} c \sqrt{f} \phi 6.62$$
 (7-7)

This gave an initial value of μ of 475. The value of μ was applied to equations 2, 4, 3 and 1 to find H_o and then after two iterations the following was obtained:

$$H_0 = 724$$

 $\mu = 451$
 $\delta = 0.497 \text{ cm}$
 $Q = 9.94. 10^{-3}$

The full working is not given as this represents a long, repetitive, arithmetical calculation.

The remainder of Baker's method consists of calculating the resistances and reactances in the equivalent circuit.



Fig 7-1



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Baker gives

$$R_{W} = \frac{8\pi^{2} f N^{2} 10^{-9}}{l_{c}} (\mu A_{W} Q)$$

$$X_{W} = \frac{8\pi^{2} f N^{2} 10^{-9}}{l_{c}} (\mu A_{W} P)$$
(7-8)
(7-9)

$$X_{g} = \frac{8\pi^{2} f N^{2} 10^{-9}}{l_{c}} A_{g}$$
(7-10)

$$X_{e} = \frac{8\pi^{2} f N^{2} 10^{-9}}{1 \cdot 8} p_{c}$$
(7-11)

$$X_1 = X_c + X_w + X_g$$
 (7-12)

$$X_{0} = \frac{X_{e}(R_{W}^{2} + X_{1}^{2} + X_{e}X_{1})}{R_{W}^{2} + (X_{1} + X_{e})^{2}}$$
(7-13)

$$R'_{W} = \frac{R_{W}X_{e}}{R_{W}^{2} + (X_{1} + X_{e})^{2}}$$
(7-14)

$$R_{o} = R_{W} + R_{c}$$
(7-15)
$$Z_{o} = \sqrt{R_{o}^{2} + X_{o}^{2}}$$
(7-16)

On a one turn basis

Rw	=	57.5	mΩ					
Xw	=	58.2	mΩ					
Xg	=	21.5	mΩ					
Xe	=	372	mΩ					
Xc	+	0	This to	is because coil thick	of the ness.	ratio	of	diameter
X	=	79.7	mΩ					
Xo	=	72.8	mΩ					
R'u ¹	=	38.5	mΩ					
Ro	=	54.2	mΩ					

Then calculated from

$$\eta = \frac{R_{W}}{R_{c} + R_{W}} 100\%$$
(7-17)

Z = 91.6 m.

Now Baker gives the powerfactor as the ratio $\cos \phi = \frac{R_0}{Z_0}$ and in this case $\cos \phi = 0.650$ and

Volts/Turn =
$$Z_0 \sqrt{\frac{P_w}{R_w}}$$

Thus for a 187V supply 82 turns are required or, alternatively, the volts required applied to a 66-turn coil is 150V, which does not compare with test results (5-9-2). The solution is well in error, both in powerfactor and in the required volts/turn. It seems reasonable to apply the coil length correction factor given by Lavers³⁹ The loss equation becomes

(7 - 18)

$$P_{W} = 2.5 f(H_{0}\Gamma)^{2} l_{\mu} \mu_{W} Q 10^{-8} \qquad W \qquad (7-19)$$

and all other equations, except those derived from the above, are unaltered. On the basis of the curves given by Lavers, $\Gamma = 0.87$, which gives a powerfactor of 0.607 and a requirement for 72 turns at 187 V. Thus the Γ factor represents only a small improvement. Moreover, applying a correction in the steel powerfactor to 22.5° , keeping the total steel impedance constant improves the powerfactor to 0.82 but it still requires 82 turns to provide the necessary flux at 187V rms excitation. 7-3 Vaughan and Williamson's design method

Vaughan and Williamson^{59,60} give a non-linear steel design theory that neglects the effects of magnetising reactance and assumes that heating is confined to an area under the coil. Their equations are given in terms of inch units. In the following design, to Specification 1, the same assumptions concerning efficiency will be made here as in the design according to Baker's method. The design equations are as follows and are in inch units,

Rc	=	N ² A	(7-20)
Rw	=	N ² B	(7-21)
Xc		N ² C	(7-22)
Xw	=	N ² D	(7-23)
X _o .	=	N ² E	(7-24)
0			

where N = number of turns of coil

B

F

$$= \frac{3 \cdot 14d}{10} \frac{K_{1}^{2}}{10} \sqrt{f} K_{7} \times 10^{-3}$$
(7-25)

$$K_1 = K_5 \begin{pmatrix} 1 - \frac{d}{w_2}^2 \\ d_c^2 \end{pmatrix} + \left(\frac{d}{d_c}^W \right)^2$$
(7-26)

where $K_5 = Nagioka's$ constant for the unloaded coil (fig. 7-2) and where K_7 is given from an experimental curve, Vaughan and Williamson's curve (plotted over page) for K_7 versus W/\sqrt{f} . Whilst

$$C = \frac{3 \cdot 14}{s} \frac{\rho_c}{c} \frac{d}{c}$$
(7-27)

where s is the coil space factor. C arises from a skin effect on the coil, and in this case is taken to be zero, as δ_c is greater than the conductor thickness.

As a consequence of the limiting non-linear theory



Fig 7-2

Load resistance coefficient K7_

for steel loss $X_w = 0.65 R_w$ (7-28) hence D = 0.65 B (7-29) finally E = $15.7(d_c^2 - d_w^2)(K_1/l_c) \ge 10^{-8}$ (7-30) Now the inner coil diameter (d_c) is 536 mm and thus $K_5 = 0.700$ $K_1 = 0.963$ $K_7 = 0.0195$ whence B = $0.344 \ge 10^{-3}$ D = $0.224 \ge 10^{-3}$ E = $2.78 \ge 10^{-6}$ and by reason of the electrical efficiency

 $A+B = \frac{100}{\eta} B$ (7-31) $A+B = 0.499 \times 10^{-3}$ Now the number of turns is given by

$$N = V \sqrt{\frac{A+B}{P_{t}(A+B)^{2} + (C+D+E)^{2}}}$$
(7-32)

which makes

N = 45

Whilst the total powerfactor is given by

$$pf = \frac{A+B}{\sqrt{(A+B)^{2} + (C+D+E)^{2}}}$$
(7-33)
$$pf = 0.905$$

7-4 Thesis design method.

The following is a design to the procedure in Appendix D. For Mild Steel of any resistivity table 6-1 gives

 $H_{n} = 419.3$

At/m

and the index of the B-H curve is

m = 0.1077

Now for a resistivity of 220nnm graph D-1 (page 238) gives . the following:

 $W_n = 45.5$ $\phi'_n = 0.740$ W/m² mWb/m

Table 7-1 shows the general dimensions of the heater. These dimensions give the ratio of coil vessel gap to coil length as

 $g/1_{c} = 0.0333$

and the ratio of length of vessel to the length of coil as $l_w/l_c = 2.0$

whilst, the ratio of the diameter of the vessel to the length of vessel becomes-

 $D/1_{_{\rm W}} = 0.416$

Hence table D-2(page236) gives

S =0.92

In table D-3 (page 236)

G = 0.422

and the vessel length correction factor is

 $\gamma = 1.1$

from graph D-3 (page 240). Table 6-10 (page 240) gives

 $O_{f} = 0.53$ Since the coil output power density is defined as

 $W_{c} = \frac{Supply power - Coil loss}{Surface area of the vessel under the coil} W/m^{2}$

 $W_{\rm C} = 21.2 \qquad kW/m^2$

The parameters above are the major design constants for the vessel. The design continues by calculating the fluxes and their phase relationships. The leakage flux is given by equation 6-46

$$\phi_1 = \mu H_n (W_c/W_n S)^{2/(3+m)} \pi Dg \{G + (g/l_c)_{\gamma}\}$$
 Wb

which makes

 $\phi_1 = 0.4375$ mWb The magnetising flux is described by equation 6-45 as -

$$m = \phi_n \pi D (W_C / W_R S)$$

whence

 $\phi_{\rm m}$ = 10.02 mWb The total flux is calculated from the cosine rule in the flux triangle where-

$$\phi_{t}^{2} = \phi_{m}^{2} + \phi_{1}^{2} + 2\phi_{m}\phi_{1}Cos(36.5)$$
 Wb²
thus

$$= 10.04$$

Since

 $\phi_1/\phi_m = 0.0433$

the angle between the e.m.f. and the leakage flux is given as $\hat{\phi_1 - E} = 38.4$

mWb

from table D-1B (page 235). Now the ratio of leakage current to magnetising current is given by eqation 6-48.

$$\frac{\mathbf{I}_{1}}{\mathbf{I}_{m}} = \frac{\phi_{1}}{\phi_{m}} \frac{\pi^{1} \mathbf{c}}{g} \left(\frac{2/\pi}{2/\pi}\right) \left(\frac{\operatorname{Arctan}}{\frac{1}{2g}} + \frac{\pi}{\pi 4 + 4} \frac{g}{1_{c}} \gamma\right) \left(\frac{G + g\gamma/1_{c}}{g}\right)$$

Hence

 $I_{m}/I_{1} = 0.1321$

	Number of Heater Coil Turns	Minimum Resulting Error in the Power Supplied	Overall Power- Factor
Experimentally			
Measured	_66		0.83
Baker	82	-48%0	0.650
Baker/Lavers	72	-18%0	0.607
Baker/Agarwal	82	-48%	0.820
Vaughan and Williamson	45	62%	0.905
Thesis	66	_0_	0.825

Table 7-2

Results of the design study set out in Table 7-1

Then from table D-1A (page 235)

 $I_{+} - I_{1} = 5.6$

Now the phase angle between the e.m.f and the current is- $E-I_t = 44.0$

Making allowance for the resistance of the coil

 $Tan(V-I_+) = \eta Tan(E-I_+)$

Since the efficiency is 71%

 $V-I_t = 34.4$ from table D-34 (page 237). Hence the overall power factor is 0.825. Now from graph D-2(page239)

K = 0.812

and the required number of coil turns is given by equation 6-52 $N = VK/\sqrt{2}\pi f\phi_{+}$

Then N = 66 turns

At this point the design study ceases as the conductor sizes are known. However, it is interesting to calculate the coil resistance and to compare this with the measured value of 245 mQ. The coil resistance is given by-

 $R_{c} = (V \cos \phi)^{2} P_{v} (1-n) / n$

whence $R_c = 246m\Omega$. The design method of this thesis has been shown to be adequate both in the prediction of perform -ance and in the simplicity of its application.

CHAPTER 8 CONCLUSIONS

CHAPTER SUMMARY

Theoretical and practical conclusions are drawn from the results of this thesis (Chapters 5, 6, 7, A) concerning the choice of vessel materials, heater construction and performance.

CHAPTER CONTENTS

- 8-1 Scope of work
- 8-2 Steel loss theories
- 8-3 Field distribution
- 8-4 Vessel material
- 8-5 Heater power densities
- 8-6 Size and performances
- 8-7 Future prospects

8 Conclusions

8-1 Scope of work

The work of this thesis has been concerned with the selection of a suitable theory for loss in steel, and with the development of the analytical techniques necessary to describe the external field patterns. The work has involved the building and testing of a range of vessel heaters, with the aims of establishing their general behaviour and a suitable simple design method.

8-2 Steel loss theories

It is essential that any steel loss theory applied to vessel heating accounts for saturation and need not include hysteresis loss. Moreover, it has been found that as a consequence of steel behaviour the results of such theories are similar. Experiment has shown (5-4 and 5-5) that the powerfactor angle is of the order of 32°, and that the loss obeys power index relationships (5-4): on this basis the theory closest to the results was that of Dreyfus²⁰ and Nejman ³² p.53. Whilst the theory of Davies 16 represents the simplest approach which describes the behaviour of mild steel, although the powerfactor angle is in error by an angle of 13° . This semi non-linear approach is useful in obtaining a general understanding of the problem, in first order predictive calculations. The flux (5-13) and current waveforms (5-12) may be predicted with some confidence by the limiting non-linear theories 1,42,43. and these offer the

facility of predicting the harmonic content of the total current derived from the Dreyfus and Nejman theories.

8-3 Field distribution

The magnetising and leakage fields of the pulsating flux heater problem may be calculated separately. The decoupling of fields is only possible because of the permeability distribution in this problem, and need not apply, for instance, where the flux was a travelling wave. Moreover, the problem has been simplified by the straightforward nature of its boundary conditions which allow the use of the method of images. As a result of theoretical calculations and experimental measurements, it has been shown that the power input is limited to the area under the coil.

It has also been shown experimentally that the magnetising field is widely spread outside an unshielded vessel. A high permeability flux shield is required for safe operation in hazardous areas. The screen is simple, straightforward and relatively cheap. The best position for such a shield is directly adjacent to the coil, causing the minimum magnification of the leakage flux. Since all induction heater coils require protection from mechanical damage, it is natural that they should be provided with some kind of box or cage, and the flux shield is an extension of this. The example of section 7 would require a 6 mm flux shield manufactured as a slitted ring from a coil of lamination steel which would form the outer box of the heater. The box shield can be an acceptable form of construction. However, greatly increased magnetic loading of the heater would make the weight of a fully effective flux shield prohibitive.

8-4 Vessel material

The coil conductor size is fixed by the supply current and the electrical efficiency. The number of turns of this conductor in the coil is given by the supply voltage and the induced e.m.f. in the vessel to produce the loss. The number of turns should be minimised to minimise copper volume, which is a statement that the flux in the vessel must be increased. Hence, the cost of the heater coil depends upon the relative electrical and magnetic loadings of the heater. In an unshielded heater the magnetic circuit consists of the vessel and the free air flux path. A higher magnetic loading can be used with the unshielded heater than with a shielded heater and is determined by the heating power density and the material constants of the vessel steel.

Since the loss in the vessel is voltage driven, and the vessel e.m.f is to be maximised, it follows that the vessel resistance R_w must be maximised. For a given geometry R_w is proportional to $\sqrt{\rho\mu}$. In limiting nonlinear terms the maximising quantity is ρB_s^{-37p52} . Since the saturated penetration depth is E/wB_s^{-37p51} ,

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Material	/oB _s naTm	$\frac{\sqrt[3]{p/B_s}^2}{\sqrt[3]{n \Omega m/T^2}}$	
Wrought Iron	250	3.5	
Mild Steel	395	4.15	
Cast Iron	880	8.7	

Table 8-1

it follows that raising the magnetic loading increases the required vessel wall thickness. The wall thickness is proportional to $\sqrt[6]{P/B_s}^{2-}$. Table 9-1 appears to show that Cast Iron is the ideal vessel material requiring less than half the copper of a mild steel vessel. It will serve as a small open unlined vessel but being weak in tension it does not have the necessary mechanical properties for a sealed retort vessel. Mild steel has the necessary mechanical and electrical properties for a good vessel heating steel.

8-5 Heater power densities

The magnetic loading may be increased by raising the power density. So that reducing the coil length reduces the volume of copper in the heating coil but increases the necessary thickness of vessel walls.

Where flux shielding is employed in the magnetic circuit it will always be thinner than the vessel walls because it carries flux evenly without eddy-currents, and does not represent a difficulty in obtaining high magnetic loadings. The chief problems arise from the thickness and cost of the vessel itself. Where reduced coil lengths are employed there is the added difficulty of heat transfer from the vessel walls to the contents. Many chemicals processed in retort vessels have maximum critical temperatures; coupled with a given surface heat transfer coefficient and bulk vessel content temperature, this defines an upper limit to the power input density. If the surface heat transfer coefficient can be raised then the heater costs are correspondingly reduced. Doubling the heat transfer coefficient doubles the permissible power density increasing the vessel wall thickness by 20%.

8-6 Size and performance

If the surface power density is limited by thermal considerations it follows that the power input per unit volume is reduced with increasing vessel size. Vessel heating is slower with large vessels. However, there are advantages associated with large scale. The magnetising reactance is naturally increased with size and it becomes much easier to build a large vessel with good coupling.

Whilst the leakage reactance depends upon the gap, the gap is fixed by the surface power density, temperature gradient and proportion of thermal loss, all of which are independent of vessel size. Hence, the ratio of magnetising flux to leakage flux is improved with increasing size. Clearly the larger the machine the better its powerfactor.

Experiments (5-9) have shown that the induction vessel heater need not be looked upon as an inefficient low powerfactor device. There is no reason why machines of the size used in these experiments (1.2m long by 0.5m dia.) and larger should be provided with additional powerfactor correction equipment as the 0.75pf recorded is close to the generally accepted economic level.

The machine is very susceptible to voltage variation and a $\pm 10\%$ supply change introduces a $\pm 25\%$ difference in input. Hence, care must be taken in the siting of the heater to minimise supply variations as much as possible.

8-7 Future prospects

Electromagnetic induction is a low capital cost form of vessel heating. However, it uses electricity, a high quality fuel compared with fossil fuels, and its running costs are proportionally higher. The immediate prospect for these machines is for small pilot plant and for large retorts where the process requires accurate and rapid control. Looking to a future where fossil fuels are increasingly scarce and where nuclear generation is preponderant, it is likely that the comparative running costs will become more favourable to induction heating. Moreover, the qualities of simplicity, cleanliness, ease of control and economy of materials will have increasing relevance. Induction heating has an assured future.
CHAPTER SUMMARY

Future research is proposed using the existing full-scale apparatus as a multiphase heater, and as a basis for large flux shielding experiments. The thesis work has shown a need for an investigation into the effects of flux induced vibration on heat transfer rates. Work is also required to define the explosion initiating nature of stray electromagnetic flux.

CHAPTER CONTENTS

- 9-1 Multiphase operation
- 9-2 Full-scale flux shielding experiments
- 9-3 Vibration and heat transfer effects
- 9-4 Explosion hazard of stray flux

Suggestions for future work

9-1 Multiphase operation

Large single phase loads are undesirable because of their unbalancing effect on the supply system. Whilst single phase induction vessel heaters of 100 kW could be constructed, it seems much more reasonable to attempt three phase operation. A single phase load is essentially a pulsating flux system whilst a three phase load implies a rotating or travelling flux, so that a Scott or Leblane type transformer cannot be used to provide balanced three phase currents for a single phase load. The two coil groups of the 1.2 m vessel heater could be used to measure the effects of connection to red and yellow, and yellow and blue, phases and might be used to even the current distribution between phases, at least so that $I_R = I_Y/2 = I_R/2$. When operated in conjunction with a suitably designed flux guided base heater, in this case supplied between yellow and blue phases, it could produce a balanced three phase heater.

9-2 Full scale flux shielding experiments

The small scale experimental work concerning flux shields can be reproduced on the 1.2 m vessel as a justification and verification of this work.

9-3 Vibration and heat transfer effects

Appendix B shows that vibration promotes heat transfer from vessel wall to contents. Ultrasonic vibration is particularly beneficial. Experiment (5-13)showed that high frequency spikes were present in the magnetising flux search coil voltages (fig. 5-16); it is not clear whether or not this gives rise to a physical vibration of the vessel. Since there are clear financial advantages in increasing the vessel power density, and hence reducing the process time, it seems reasonable to carry out a two-fold investigation. Firstly, into the vibration harmonics induced by induction heating and, secondly, into the effects of induction heating vibration on heat transfer rates. The latter can be measured by a simple experiment, where a small vessel containing a liquid such as glycerol is heated by an induction coil. The power input pattern can be measured by a series of surface resistance coils, acting initially as search The vessel wall to liquid temperature difference coils. may be measured directly with a thermocouple. The same power input may also be achieved by surface resistance coils from a d.c. source, and adjusting the voltages across them to give the desired power input distribution. Any difference in the thermocouple reading must be due to a change in the heat transfer coefficient.

9-4 Explosion hazard of stray flux

Appendix A shows that there is a possible explosion hazard arising from a combination of steel plate flooring and stray fluxes. A definition of this hazard is required before steel plates are permitted in an area of stray fluxes.

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APPENDIX A FLUX GUIDES AND FLUX SHIELDS

APPENDIX SUMMARY

The reasons for flux shielding are discussed with relevance to the explosion producing hazard. Experimental work with a flux shielded heater is described. Theories are proposed to account for flux shielded coils and flux guided heaters.

APPENDIX CONTENTS

A-1	FLux	guides	and	shields	

- A-2 Acceptable levels of stray flux
- A-3-1 Flux shields
- A-3-2 Experiments with a flux shielded heater
- A-3-3 Method of test
- A-3-4 Results
- A-3-5 Theory
- A-3-6 Design consideration
- A-3-7 Aluminium flux shield experiments
- A-4-1 Flux guided heaters
- A-4-2 Magnetising permeance
- A-4-3 Leakage permeance

The operation of a free flux heater has already been described in section 2-1 in terms of a transformer where the main flux path passes outside the machine through the surrounding free space. The flux can set up circulating currents in any stray metal work and the interruption of these stray currents gives rise to sparking. In ordinary circumstances these circulating currents and the quite small sparks that they produce are no more than an inconvenient source of additional loss. However, in a fire hazard area this sparking may initiate an explosion. Under these circumstances the flux density outside the machine and its control and limitation is a matter of some importance.

A-2 Acceptable levels of Stray Flux

There are no British Standards or Codes of Practice which specify the acceptable level of stray flux outside a machine in a fire hazard area. Under certain conditions the stray flux can give rise to sparking. The stray flux sets up circulating currents in metal objects in contact, the breaking of the current path gives rise to an attendant spark. Not all sparks necessarily set off an explosion. Low level sparking is permissible in hazardous areas. This has led to the adoption of intrinsically safe apparatus ¹². Intrinsically safe apparatus is non flame proof equipment whose sparking, according to a break flash test will not cause an explosion in a specified class of dangerous atmosphere.

There are four classes of gasses:

Methane class. Tests based on an air methane mixture. 91.7% air, 8.3% methane/by volume.

Pentane cla	ass. Tes	ts on 96.1%	penta air,	ane and 3.% p	air i entane	n the	e ratio:
Ethylene c	lass. Tes	ts on 92.2%	ethyl air,	ene an 7.8% e	d air: thylen	e.	
Hydrogen c	lass. Tes	ts ba 79% a	sed on ir, 21	an ai % hydr	r-hydr ogen.	ogen	mixture:

Some guidance, as acceptable flux levels, can be obtained from the Electrical Research Association's work on sparks in these gas mixtures.

Maximum permissible spark energies.

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In	the	presence	of	Zinc,	Cadmium	or	Magnesium	n.
----	-----	----------	----	-------	---------	----	-----------	----

Methane	544 μW	1.3	A
Propane	338 µ₩	0.95	A
Ethylene	144 μW	0.700	A
Hydrogen	36.1 μW	0.280	A

In situations where cadmium, zinc or magnesium can be excluded.

Methane	1012	μW	3.2	A
Propane	648	μ₩	2.5	A
Ethylene	450	μ₩	1.9	A
Hydrogen	112	μW	1.0	A

Their work has been concerned with the break flash testing of a known inductance carrying a known current.

In each case the spark for explosion is set up by an approximately constant stored energy condition. The change of stored energy may be usefully exploited in the specification of the stray flux hazard. In a linear eddy-current loss system the stored energy per cycle is equal to the energy loss per cycle. Moreover this represents the worst case, for non linear systems the energy loss is more than the stored energy. The spark energy between two metal objects is the difference between their joint eddy-current loss when in contact and when separated.

Hence it is clear that the spark energy, and hence hazard, is not only a function of the stray flux exciting the bodies but also a function of the geometry and material properties of the materials in the region of stray flux.

e.g. The interaction of two thin steel plates carrying flux longitudinally. The classical eddy-current loss formulae gives:

$W = \frac{B^2 w^2 h^3 F(\beta)}{\sqrt{8}}$	W/m ² (A-5)
---	------------------------

Where h = the plate thickness

$\beta = \frac{h}{\delta}$	(A-6)
$F(\beta) = \frac{1}{\beta} \cdot \frac{\sinh\beta - \sin\beta}{\cosh\beta - \cos\beta}$	(A-7)

m

Then the total loss when two plates touch is

$$W = \frac{B^2 w^2 h^3 F(2\beta)}{\sqrt{2}} \qquad W/m^2 (A-8)$$

and the total loss when separated becomes

$$W = \frac{B^2 W^2 h^3}{\sqrt{2}} \left(\frac{F(2\beta) - F(\beta)}{4} \right) \qquad W/m^2 (A-10)$$

for large β

$$W = \frac{b^2 w^2 h^2 7\delta}{\sqrt{28}} \qquad W/m^2 (A-11)$$

When equation A-11 is applied to two steel plates such as might be used for flooring, in this case one metre square and 5 mm thick, with a resistivity of 220 n Ω m and a skin depth of 5 mm, very low levels of flux density are required to set off an explosion. The maximum safe level of r.m.s. flux density is:

> μ T in Methane μ T in Propane μ T in Ethylene μ T in Hydrogen

Moreover, if the sheets are zinc plated this value is reduced to approximately 70% of the above. Now the above applies to the case where the sheets are laid flat on top of one another and instantaneously parted. Not only is this an unusual situation but it may not represent the highest possible spark condition that can be obtained from these plates at the given levels of B. An alternative geometry may produce higher spark energies. However, the analysis makes it clear that the permissible flux level depends on the area and thickness, and upon the quality of the steel work in the stray flux area. There is clearly a need for a standard of stray flux and a definition of safe steel work.

E earth = $25-70 \ \mu T$.

A-3-1 Flux shields

There are two methods of reducing flux levels outside the heater. The first is to provide a high permeance magnetic path, in the form of a flux shield. The governing equations are such that every effort must be made to prevent the flow of eddy-currents in the shield. This usually is done by forming an incomplete ring.



Fig. A-1

Typical Flux Shield showing a sectional view

Alternatively, the flow of eddy-currents may be encouraged by providing a low permeability low resistivity shield. However, encouraging the flow of current in a shield does reduce the stray flux, at the expense of greatly increasing the magnetising ampere turns, for it restricts the flux paths and increases the reluctance seen by the magnetising flux.

An interesting combination may be formed from the two flux shields used jointly. The steel flux shield providing the first line of flux reduction and providing the main flux path, and a low resistance shield, of say aluminium, outside this as a final means of flux reduction.

A word of caution must be given here in that aluminium must not be used without a protective coating in a fire hazard area because of the danger of thermic flashing. It is suggested that the shield is nylon coated.

A-3-2 Experiments with a flux shielded heater

The following experiment was conducted to investigate the behaviour of a flux shielded heater.

A heater of the type shown in figure A-1 was constructed from the equipment of plate A-1. It consisted of a 240 mm long, 140 mm diameter pipe, heated by the 115 mm long 180 mm internal diameter coil. The shield consisted of a single sheet of transformer steel, 240 mm wide, 0.32 mm thick, wound into a 290 mm diameter cylinder around the vessel and coil. Search coils were wound on the heater coil midplane, on the vessel, the heater coil, flux shield and externally. Search coils were also provided on the inner surface of the shield.

A-3-3 Method of test

The method of test was the same as that used in experiment 5-2 except that the additional search coil readings were taken with a valve voltmeter. The coil loss was derived by measuring the d.c. resistance with a test current and then calculating the I²R loss. The power input to the vessel was taken to be the supplied power less the coil loss, which is an assumption that the shield loss was small. The apparatus was tested over a range of supply voltages.

A-3-4 Results

It was noted that the shield remained cool throughout the experiment. Since it has a very small thermal capacity, this indicates that the shield losses were small. When the total power is plotted against the supply voltage (graph A-1) it is clear that the shield has very little effect. It appears to reduce the current drawn from the supply (graph A-2) by increasing the reactance of the heater (graph A-3). The distribution of loss on the vessel surface does not appear to be influenced by the shield as it requires the same magnetising flux to produce a given vessel power (graph A-4). Moreover, the same relationship between magnetising flux and supply voltage appears to hold (graph A-5), whilst the leakage flux is clearly increased. The behaviour of a shielded heater may be explained in terms of an increase of leakage flux. In spite of the increase of leakage flux, flux levels are reduced outside the shield (graph A-7) and the shield carries a flux that is independent of the level of excitation. The distribution of flux at the shield can be deduced from graph A-8.

A-3-5 Theory

The shield may be considered in terms of the method of images and the leakage flux image for an infinitely long vessel becomes that of figure A-1a (over page). The strength of the shield image is increased by the proportion of flux carried by the shield. The power input is determined by the value of H at the surface of the vessel. The shield influences the distribution of H in the gap so that H at the surface of the coil is increased. When the shield is moved to the plane of the coil, the Heater coil and its Image coincide



Image effect of a flux shield

and there is no distortion of the air-gap field pattern.

A-3-6 Design consideration

The previous section has shown that the air-gap distribution of flux is only detrimentally influenced by the shield if the shield is spaced from the coil. The best position for a shield is thus at the coil, and under these conditions design may be undertaken as though the shield was absent. The thickness of shield material should be chosen to carry the total flux.

A-3-7 Aluminium flux shield experiments

A shielded heater was constructed using the same vessel and coil of the previous experiment, with an aluminium ring type shield, shown in plate 4 , of comparable dimensions to the slitted steel shield. Initial experiments showed that this form of flux shielding greatly increased the reactive power drawn by the heater and appeared to produce greater heat in the shield than in the vessel. The experimental work was abandoned at this point.



Shielded results ----- © +

<u>Vessel 240 mm 140 mm dia</u> <u>Shield 240 mm 290 mm dia. 0.32 mm thick</u> <u>Coil 115 mm 180 mm id 220 mm od.</u> <u>308 T 18 swg</u>









<u>Coil</u> <u>115 mm 180 mm.id. 220 mm.od</u>. <u>308 T 18 swg</u> Search Coil 140 mm dia at Heater Coil mid-plane









A-4-1 Flux guided heaters

The analytical problems associated with flux guided heaters are fewer than those of a free flux heater. The chief flux paths are formed by laminated cores looped over the heating coil, as shewn in figure A-2.



Fig. A-2

Typical construction of a flux guided heater

The reluctances of the magnetising and leakage flux paths are controlled by well defined air gaps. The fringing effects of the poles have already been studied in problems associated with machines.



Fig A 2a

Mirror Image Model of the flux guided heater of Fig A 2

A-4-2 Magnetising Permeance

The magnetising permeance is set by the permeance of the flux path which enters the vessel normally. As a first approximation

$$\Lambda_m(per \ pole) = \mu_b \frac{dt}{4g} \qquad (A - 12)$$

however this does not take account of the fringing at the poles. There are two fringing effects - where the fringing is associated with the pole only, and the additional permeance may be accounted for by the coefficient given by Bewley

Hence

$$\Lambda_{m}(perpole) = \mu_{o} a B\left(\frac{h}{2g}, \frac{t}{2g}\right) t \qquad (A-13)$$

This coefficient can only be used when the poles are substantially isolated. Whilst the permeance in view (A) is always likely to be set by this form of isolated fringing, that in view (B) is more likely to become that of (C) which is a Carter's coefficient problem. Hence

$$\Lambda_{\tilde{m}}(per \ pole) = \mu_o \frac{a B t C}{4g} \qquad (A - 14)$$

Suitable curves of the Carter's coefficient are generally available and can be found in Gibbs (Ref. 28 page 123).



Fig. A-3

A-4-3 Leakage Permeance

The leakage permeance is the permeance of the proportion of the total flux which links the winding but does not link the vessel.



Fig. A-4

Leakage flux paths

The permeance is like the magnetising permeance again controlled by the air gaps in the circuit. The surface of the vessel behaves as an impenetrable barrier for leakage flux - this is a consequence of section 6-1. Hence the system of yokes above the vessel surface. It may be replaced by a mirror image system. A view along the plane OZ, shows that this plane is an equipotential, and is shown in



Fig. A-5

Half section of leakage image

Here region A'B'C'D'O D C B A represents part of a Carter's tooth area, and Carter's coefficient C_{c1} may thus be used to calculate the effective permeance of this region. Where $\frac{\text{slot width}}{\text{tooth width}} = \frac{g}{d}$ and $\frac{\text{slot width}}{\text{gap}} = \frac{4g}{c}$

The fringing effect in the perpendicular plane is also given by an application of Carter's coefficient, along the plane. (fig. A-6).



Fig. A-6

Leakage field pattern at the vessel surface

Here the appropriate coefficient is that of Cc2

- Where $\frac{\text{slot width}}{\text{tooth width}} = \frac{s}{\xi}$
- and $\frac{\text{slot width}}{\text{gap}} = \frac{2s}{c}$

The leakage permeance of the flux which links all the winding is

$$\Lambda = \mu_0 \left(\frac{\mu_{\rm db}}{c} \right) \cdot {}^{\rm C} {}_{\rm c1} \cdot {}^{\rm C} {}_{\rm c2} \cdot {}^{\rm n}$$
(A-15)

There now remains the problem of the leakage flux which partially links the coil. The assumption is made that the flux passes straight across the winding.



Section of coil and core

Now, under the conditions of figure A-7 the m.m.f. at x is-

$$H = N I_1 x / m \qquad (A - 16)$$

Then the flux set up by the total m.m.f. is given by the integral

$$\phi_{L}^{"} = \left[\int_{0}^{m} \left(\frac{x}{m} N \mathbf{I}_{1} \mu_{o} \frac{\mathbf{b}}{\mathbf{c}}\right) \cdot \mathbf{d}x\right] \mathbf{n} \qquad (A-17)$$

$$\frac{b_{i}}{N I_{i}} = \frac{\mu_{om} b n}{2 c}$$

When allowance is made for the gaps between cores the permeance of this part of the magnetic circuit becomes -

$$\Lambda' = \frac{\mu_{\rm c} \,\mathrm{m} \,\mathrm{b} \,\mathrm{n} \,\mathrm{C}}{2 \,\mathrm{c}} \tag{A-18}$$

The total leakage permeance is the sum of the permeance outside the coil and the permeance of the flux path which partialy links with the coil. Hence

 $\Lambda = \Lambda' + \Lambda'' \qquad (A - 19)$

$$\Lambda = \frac{\mu b}{c} \left(\frac{\mu}{d} C_{c1} + \frac{m}{2} \right) C_{c2} n \qquad (A - 20)$$

APPENDIX B VESSEL HEAT TRANSFER

APPENDIX SUMMARY

This chapter is a theoretical discussion of the heat transfer processes within the vessel and reaches the conclusion that there may be beneficial side effects to induction heating.

APPENDIX CONTENTS

- B-1 The convection mechanism
- B-2 The influence of the thermal sub-layer
- B-3 The effect of vibration on the heat transfer coefficient
- B-4 Possible thermal side effects of induction heating

B-1- Convection mechanism

The heat transfer mechanism from vessel wall to liquid is one of convection. Convection is the transfer of thermal energy by fluid motion. It may be divided into two classes: forced convection where fluid motion is set up by an external agency, and free convection where fluid motion is caused by the changes of density with temperature. Convection is the most complex method of heat transfer and, unlike conduction and radiation, it is not easily analysed. Generally, convection is described by dimensionless parameters. These are groupings of the quantities involved in the problem and, as such, control the convection behaviour. There are four chief numbers. The Nusselt number

$$N_{\rm u} = \oint_{\Delta \Theta k} 1 \tag{B-1}$$

describes the ratio of actual heat transfer to plain thermal conduction of the fluid in a non-dimensional form. Whilst the Prandtl number

$$P_{r} = \frac{\eta Q_{p}}{k} p^{-}$$
(B-2)

can be thought of as a ratio of kinematic viscosity to temperature diffusivity, it provides a measure of the relative rate at which velocity and temperature disturbances are propagated through the fluid. The third number is the Reynolds number

$$R_e = \frac{V_{Ol}}{\eta}$$
(B-3)

which describes the fluid flow. It represents the ratio of fluid inertia forces to viscous fluid forces. Finally the Grashof number which is only applied to natural convection

$$G_{\mathbf{r}} = \frac{\beta g \rho^2 \mathbf{1}^2 \Delta \Theta}{n^2} \tag{B-4}$$

This number represents the product of the ratios of bouyancyforces to

viscous forces, and inertia forces to viscous forces.

The equations of fluid flow and heat flow are similar and with suitable simplifications the laminar problem may be solved analytically for flow over a semi-infinite flat plate,

$$Nu_x = 0.332 \text{ Re}_x^{\frac{1}{2}} P_{\frac{1}{3}}$$
 (B-5)

where x is the length of plate considered.

Whilst the above solution can be derived for a laminar flow condition, turbulent flow is not amenable to analysis. Reynolds pointed out that if the turbulence effects are so high that they mask the thermal conductivity and viscosity effects, the processes of heat and momentum transfer are identical, and for turbulent flow

$$Nu_x = 0.029 \text{ Re}_x^{-\frac{4}{5}} \text{ Pr}$$
 (B-6)

the analogy is only really reliable for P = 1. The Reynolds analogy has been extended by Prandtl and Taylor.



Fig. B-1

Heat and fluid flow conditions over a flat surface

This analogy is based on the observation that the degree of turbulence present in the boundary layer becomes less as the solid surface is approached. It is assumed that at the surface there is a layer of fluid (6 thick) in laminar flow, above this there is a turbulent flow region. On this basis the expression for heat transfer from a plate in turbulent flow becomes

$$Nu = \frac{0.029 \text{ Re}^{\frac{1}{5}} \text{Pr}}{1 + m (\text{Pr} - 1)}$$
(B-7)

where in general

0.4 < m < 0.6

Flow patterns vary between forced and free convection. Typical velocity profiles by a vertical surface are shown for both conditions in (fig B-2).





Velocity profiles in free and forced convection

The chief difference is that in the forced convection case the body of the liquid is in motion, whilst in free convection there is a maximum velocity region adjacent to the heated vertical surface. The Grashof number plays the same role in free convection that Reynolds number plays in forced convection. The higher Grashof numbers indicate turbulent flow. Theoretical solutions can be derived for some simple laminar flow problems. For the flat plate problem of height l

$$\overline{\mathrm{Nu}}_{1} = C \left(\mathrm{Gr}_{1} \mathrm{Pr}\right)^{\frac{1}{4}}$$
 (B-8)

where $\overline{Nu_1}$ is the average Nusselt number, and the values of the physical constants are taken at the mean film temperature, which in this case is the mean of the wall temperature and the bulk fluid temperature. C is given approximately 0.52 and varies slightly with the Prandtl number. For Grashof numbers > 10¹⁰ when the flow might be expected to be turbulent

$$Nu_1 = 0.13(Gr_1 Pr)^{\frac{1}{3}}$$
 (B-9)

for a flat plate.

However, the above results do not strictly apply to flow within a vessel. Firstly, because the vessel is essentially cylindrical in nature and 23 gives

$$Nu_{1} = 0.01(d/1)^{3}(Gr_{1} Pr)$$
 (B-10)

for an open cylindrical condition. Secondly, because the fluid is in a circulatory system. A possible flow pattern is shown below which differs from the conditions for B-10 in that the velocity varies vertically. This is



Fig. B-3

Fluid flow pattern in a vessel heater.

borne out by the results of experiment 5-2. Because of the complicated nature of the flow pattern there is some confusion as to which formula is applicable. Moreover, a common result of these formulae is that the heat transfer coefficient is power density dependant. This is a further complication when trying to decide a suitable heating power density for an induction vessel heater.

B-2 Influence of the thermal sub-layer

Generally an improvement in the surface heat transfer coefficient within the vessel results in a reduction of the first cost of an induction heater. The chief barrier to heat flow is the thermal sub-layer (fig B-1). Some idea of the thickness of the thermal sub-layer is given in the table below

	Thermal diffusivity	Kinematic viscosity	Thermal sub-layer		
	к	v	δ		
	$10^{-6} \text{ m}^2/\text{s}$	$10^{-6} \text{ m}^2/\text{s}$	mm		
Water	0-14	1.0	0.7		
Oil (Mobile velocite 6)	0.08	19	4-6		
Glycerol	0.05	650	53		
	Stream velocity of 0.06 m/s				

Table B-1

Table B-1 has been calculated from the formula 19

$$6 = 8.9 \kappa^3 v^3 / v^1$$
 (B-11)

where v' is the fluctuation velocity in the turbulent region of the fluid and is taken as 10% of the stream velocity. Clearly any mechanism which can reduce this barrier must improve the heat transfer coefficient. Moreover, the thermal sub-layer becomes increasingly important with viscous liquids and its reduction becomes a necessity. Equation B-11 shows that increasing the fluid velocity reduces the resistance to heat transfer. It is usual to provide stirrers when heating paint resins or similar fluids. The thermal sub-layer can be reduced by the addition of roughness elements, or by the use of a mechanism to mechanically break down the layer, such as scraped film heat exchangers 3 . Other methods of improvement rely on increasing the vibration energy of the heat transfer surface. This method functions by increasing the turbulence near the surface.

B-3 Effect of vibration on the heat transfer coefficient

Experimental studies have been undertaken by various authors into the effects of vibration on heat transfer. Recently Wong and Chon 62 investigated the effects of ultrasonic vibration on heat transfer. They used frequencies in the range from 20.6kHz to 306kHz to increase the heat transfer coefficient from a wire suspended in water or methanol. They applied vibration to the liquid and not directly to the wire. However, their results show a nearly double heat transfer coefficient in the natural convection region and up an 800% increase in the nucleate boiling region. Other researchers 2 , 21 have reported increases in heat transfer rate with the direct application of ultrasonic vibration to the heat transfer surface. Edwards, Nellist and Wilkinson 24 applied a pulsed vibration to fluid in a heated pipe line. The vibration in these experiments were of the order of 0.5Hz and under certain conditions an improvement in heat transfer was noted.

B-4 Possible thermal side-effects of induction heating

Induction heating contains a proportion of vibration loss. Although the vibration energy is small compared with the eddy-current loss and the hysteresis loss, even a relatively small component of vibration power would be of the same order as that used by other authors to enhance heat transfer⁶² The major component of vibration in mains frequency induction heating is 100 Hz which is intermediate between the ultrasonic frequency heat transfer work and the low frequency heat transfer experiments. It seems highly likely that this vibration energy may have a beneficial effect on the heat transfer coefficient. There appears to be no experimental work directly relevant to this effect at 100 Hz. An investigation into the level of vibration in induction vessel heating and its influence on heat transfer rates promises sound engineering benefits.

AFPENDIX C GRAPHS FROM CHAPTER 5

NOTE - The graph numbering system is as follows:-The first number applies to the section of Chapter 5, and the second group of numbers is the number of the graph in that section.


Graph 2-1





Graph 2-3

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Vessel Power & Supply VAr v Supply Frequency (at Supply Current=9A)

Graph 3-1





156mm 166 mm od

Graph 3-2



Graph 3-3

Heater Performancev Frequency & Scale







- 211 -





× Magnetising searchccoil Volts Work 24cm long + Leakage 14cm diam.

Search coil voltage v Vessel power

Graph 5-1



Vessel 570mm 60mm od. Coil .308 turns

Phase angle

Leakage flux to Magnetising flux

Coil supply voltage

V

Graph 6-1



<u>Vessel length</u> <u>0 2 40 mm</u> <u>* 3 20 mm</u> <u>0 4 80 mm</u> <u>+ 1.8 m</u>

Rower absorbed by the Vessel v Leakage Current



Graph 7-2

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Vessel dia	140 mm	Vessel length D	240 min
		×	320 mm
		0	4 80 mm
		+	1.8 m

Magnetising permeance v Vesselpower



Vessel diameter 140 mm								
I Vessel length 240 mm								
×	m	-	320 111					
0		n	480mm					
+	6	v	1.8m					

<u>Coil 308 turns 115 mm long</u> <u>180 mm jd, 220 mm od</u>.

Leakage Permeance v Vessel Power



Vessel diameter 140 mm

Measured Magnetising permeance

v Vessel length



Leakage Permeance v Vessel length (taken from graph 8-4)

Graph7.6



Supply Conditions

Voltage	Current	Vessel Power
V	A	W
+ 106 • 143 × 178 • 214	4.07 5.72 7.70 9.50	119 250 510 681

Typical Mid-plane External Search

Coil Voltages v Position

Graph 8-1



Vessel diameter 140 mm

Vessel Length	Graph Slope	
240 mm 320 mm 480 mm L.8 m	-1.44 -1.24 -1.12 -1.07	0×0+

(n.b. V_S Normalised w.r.t. coil surface voltage.) <u>Normalised Logarithmic Plots of External Search</u> <u>Coil Voltages v Relative Distance from the Coil</u> <u>Graph 8-2</u>



Graph 8-3



<u>Vessel Power v SupplyVoltage and Current</u> <u>Graph 9-1</u>

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Power Densities v Mean Gap

Graph 9-2





Graph 9-3

Transient Surface Temperature, B





E

G

<u>Transient Temperature</u> <u>Tests</u> <u>Surface Temperature</u> <u>changes at switch off</u>

Graph 10-2





<u>Graph 10-3</u>



Coil 66 turns, 0·6 m long Vessel 1·2 m long,0·5 m dia

<u>Normalised Surface E Distribution</u> <u>Graph 11-1</u>









<u>Graph 11- 2</u> Normalised Power Density Distribution Curves

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APPENDIX D DESIGN METHOD

APPENDIX SUMMARY

This Appendix contains the thesis design method derived in Chapter 6, together with the necessary design tables and graphs.

D. Design procedure for large cylindrical vessels.

 Choose the coil length on the basis of power density and the coil vessel gap to give a reasonable heat loss with the design vessel temperatures and thermal insulation.

2. Look up W_n , ϕ_n , S, O_f , G and Y.

- 3. Calculate the coil output power density.
- 4. Calculate ϕ_1 and ϕ_m from equations (6-46) and (6-45). Hence, calculate total flux and ϕ_1/ϕ_m to give ϕ_1 -E from table D-1A.
- 5. Calculate I_m/I_1 from (6-48) and obtain $\phi_1 I_t$ from D.1B.
- 6. Knowing the electrical efficiency and the internal powerfactor angle E-I_t, look up the factor K in graph D.2, and the overall powerfactor angle in table D-4.
- 7. Calculate the number of turns from (6-52).
- 8. Calculate the total current from the powerfactor and voltage. Then calculate the coil resistance from the coil loss and total current. Hence choose the coil conductor.
- Calculate l_c if this is not equal to that assumed, return to step 2 and repeat.

Angle $I_t - \phi_1$ for I_m / I_1

				-		
		0.00	0.01	0.02	0.03	0.04
-	0.00	0.00	0.46	0.91	1.36	1.80
	0.05	2.23	2.66	3.09	3.51	3.92
	0.10	4.33	4.74	5.14	5.54	5.93
1	0.15	6.31	6.69	7.07	7.44	7.81
	0.20	8.17	8.53	8.88	9.23	9.57
	0.25	9.91	10.25	10.58	10.91	11.23
	0.30	11.55	11.87	12.18	12.49	12.79
	0.35	13.10	13.39	13.69	13.98	14.26
	0.40	14.54	14.82	15.10	15.37	15.64
	0.45	15.91	16.17	16.43	16.69	16.94
	0.50	17.19	17.44	17.68	17.93	18.17
	0.55	18.4.0	18.64	18.87	19.10	19.32
	0.60	19.54	19.77	19.98	20.20	20.41
3	0.65	20.62	20.83	21.04	21.24	21.44
	0.70	21.64	21.84	22.04	22.25	22.42
	0.75	22.61	22.19	22.90	23.16	23.34
	03.0	23.52	23.69	23.01	24.04	24.21
	0.05	24.30	24.55	24. /1	24.00	22:04
	0.90	25.20	25.00	23.52	20.01	29.09
	1.00	25.90	26.15	27.00	20.42	20.01
	1.00	20.12	20.00	21.00	21012	21020

Angle $\phi_1 - E$ for ϕ_1/ϕ_m

I		0.00	0.01	0.02	0.03	0.01+
	0.00	36.56	37.02	37.47	37.92	38.36
	0.05	38.79	39.23	39.65	4.0.07	40.49
	0.10	40.90	41.30	41.70	4.2.10	42.49
1	0.15	42.87	43.25	43.63	44.00	44.37
	0.20	44.73	45.09	45.44	45.79	46.13
	0.25	46.47	46.81	47.14	47.47	47.79
	0,30	4.8.11	48.43	40.74	49.05	4.9.36
	0.35	49.65	49.95	50.25	50.53	50.82
	0.10	51.11	51.39	51.66	51.95	52.20
	0.45	52,41	52.15	52.99	53.25	53.50
	0.50	53.10	54.00	54.27	24.49	55 88
	0.55	56 11	56 33	56 55	56 76	56 97
1	0.65	57 19	57.39	57.60	57.80	58.00
	0.70	58 20	58.10	58.60	58.79	58.98
	0.75	59.17	59.35	59.54	59.72	59.90
	0.80	60.08	60.26	60.1.3	60.60	60.77
	0.85	60.94	61.11	61.28	61.44	61.60
	0.90	61.76	61.92	62.08	62.23	62.39
	0.95	62.54	62.69	62.84	62.99	63.14
	1.00	63.28	63.42	63.57	63.71	63.85

Tables D-1-A and D-1-B

for mild steel where B= bH0.1

POWER DISTRIBUTION FUNCTION

S(g/1c,1w/1c)

Gap Coil length		Vessel length Coil length						
	1	2	3	4	5	10		
0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45	0.8934 0.8305 0.7932 0.7649 0.7516 0.7380 0.7334 0.7265 0.7317	0.9457 0.9232 0.9088 0.9052 0.9127 0.9233 0.9297 0.9395 0.9471	0.9517 0.9325 0.9323 0.9428 0.9572 0.9734 0.9934 1.0157 1.0344	0.9504 0.9437 0.9451 0.9583 0.9780 0.9957 1.0223 1.0471 1.0711	0.9498 0.9373 0.9433 0.9590 0.9824 1.0127 1.0335 1.0617 1.0916	0.9490 0.9442 0.9487 0.9663 0.9924 1.0212 1.0548 1.0872 1.1263		

(from equations 6-18 and 6-20)

Table D-2

LEAKAGE FLUX FUNCTION

G	g
	10

				A	
Gap Coil Length	0.00 0.05	0.01	0.02 0.07	0.03 0.08	0.04
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45	0.4336 0.4704 0.5195 0.5822 0.6583 0.7467 0.8458 0.9544 1.0714	0.4105 0.4402 0.4792 0.5309 0.5964 0.6751 0.7657 0.8668 0.9772 1.0957	0.4159 0.4471 0.4884 0.5429 0.6111 0.6923 0.7851 0.8882 1.0003 1.1203	0.4216 0.4544 0.4982 0.5555 0.6263 0.7100 0.8049 0.9099 1.0237 1.1451	0.4274 0.4622 0.5086 0.5686 0.6421 0.7281 0.8252 0.9320 1.0474 1.1702

(from equations 6-24 and 6-25)

Table D-3

Arctan[Efficiency_TanA]

A	Efficiency							
	60	65	70	75	80	85	90	95
30	19.11	20.57	22.01	23.41	24.79	26.14	27.46	28.75
31	19.83	21.33	22.81	24.26	25.67	27.06	28.40	29.72
32	20.55	22.11	23.63	25.11	26.56	27.98	29.35	30.70
33	21.29	22.89	24.45	25.97	27.45	28.90	30.31.	31.67
34	22.03	23.68	25.28	26.83	28.35	29.83	31.26	32.65
35	22.79	24.47	26.11	27.71	29.26	30.76	32.22	33.63
36	23.55	25.28	26.96	28.59	30.17	31.70	33.18	34.62
37	24.33	26.10	27.81	29.47	31.08	32.64	34.15	35.60
38	25.12	26.92	28.68	30.37	32.01	33.59	35.11	36.58
39	25.91	27.76	29.55	31.27	32.94	34.54	36.09	37.57
40 41 42 43 44 45 46 47 48 49	26.72 27.55 28.38 29.23 30.09 30.96 31.85 32.76 33.68 34.62	28.68 29.47 30.34 31.22 32.12 33.02 33.95 34.88 35.83 36.79	30.43 31.32 32.22 33.14 34.06 34.99 35.94 36.90 37.86 38.84	32.18 33.10 34.03 34.97 35.92 36.87 37.84 38.81 39.79 40.79	33.87 34.82 35.77 36.72 37.69 38.66 39.64 40.63 41.62 42.62	35.50 36.46 37.43 38.40 39.38 40.37 41.36 42.35 43.35 44.36	37.06 38.04 39.02 40.01 41.00 41.99 42.98 43.98 44.99 46.00	38.56 39.55 40.54 41.54 42.53 43.53 44.53 44.53 45.53 46.54 47.54
50 51 52 53 55 55 55 57 58 59	35.57 36.54 37.52 38.53 39.55 40.59 41.66 42.74 43.84 44.96	37.76 38.75 39.76 40.78 41.82 42.87 43.94 45.03 46.13 47.25	39.84 40.84 41.86 42.89 43.94 44.99 46.06 47.15 48.25 49.36	41.79 42.81 43.83 44.87 45.91 46.97 48.03 49.11 50.20 51.30	43.63 44.65 45.68 46.71 47.76 48.81 49.87 50.93 52.01 53.09	45.37 46.39 47.41 48.44 49.48 50.52 51.57 52.62 53.68 54.74	47.01 48.02 49.04 50.06 51.09 52.12 53.15 54.19 55.23 56.27	48.55 49.56 50.57 51.58 52.59 53.61 54.63 55.64 55.64 56.67 57.69
60	46.10	48.39	50.49	52.41	54.18	55.82	57.32	58.71
61	47.27	49.54	51.63	53.53	55.28	56.89	58.37	59.74
62	48.45	50.72	52.78	54.67	56.39	57.97	59.43	60.77
63	49.66	51.91	53.95	55.81	57.51	59.06	60.49	60.79
64	50.89	53.12	55.13	56.96	58.63	60.15	61.55	62.82
65	52.15	54.35	56.33	58.13	59.76	61.25	62.61	63.86
66	53.42	55.59	57.54	59.31	60.90	62.36	63.68	64.89
67	54.72	56.85	58.77	60.49	62.05	63.46	64.75	65.93
68	56.05	58.14	60.01	61.69	63.21	64.58	65.82	66.96
9	57.39	59.44	61.26	62.90	64.37	65.70	66.90	68.00
70	58.76	60.75	62.53	64.11	65.54	66.82	67.98	69.04
71	60.15	62.09	63.81	65.34	66.71	67.95	69.06	70.08
72	61.56	63.44	65.10	66.58	67.90	69.08	70.15	71.12
73	63.00	64.81	66.41	67.82	69.09	70.22	71.24	72.16
74	64.46	66.20	67.73	69.08	70.28	71.36	72.33	73.21
75	65.94	67.60	69.05	70.34	71.48	72.50	73.42	74.25
76	67.44	69.02	70.40	71.61	72.69	73.65	74.52	75.30
77	68.96	70.45	71.75	72.89	73.90	74.81	75.61	76.34
78	70.49	71.89	73.11	74.18	75.12	75.96	76.71	77.39
79	72.05	73.35	74.48	75.47	76.34	77.12	77.81	78.44
80	73.62	74.82	75.86	76.77	77.57	78.28	78.92	79.49
81	75.21	76.31	77.25	78.08	78.80	79.45	80.02	80.54
82	76.82	77.80	78.65	79.39	80.04	80.61	81.13	81.59
83	78.44	79.30	80.05	80.70	81.28	81.78	82.23	82.64
84	80.07	80.82	81.46	82.02	82.52	82.95	83.34	83.69
85	81.70	82.34	82.88	83.35	83.76	84.12	84.45	84.74
86	83.35	83.86	84.30	84.67	85.01	85.30	85.56	85.79
87	85.01	85.39	85.72	86.00	86.25	86.47	86.67	86.84
88	86.67	86.93	87.15	87.34	87.50	87.65	87.78	87.90
89	88.33	88.46	88.57	88.67	88.75	88.82	88.89	88.95

Table D-4.





Graph D-1







Vessel Length Correction Factor for Leakage Permeance Calculations

Graph D-3
APPENDIX E THE DREYFUS LOSS THEORIES FOR SATURATED STEEL APPENDIX SUMMARY

The work of Dreyfus concerning eddy-current loss in cold steel is restated in M.K.Sunits. It is shown that the semiinfinite flat plate solution may be normalised; this theory is applied in Chapter 6.

APPENDIX CONTENTS

- E-1 The Dreyfus loss theories for saturated steel
- E-2 Eddy-current loss in a semi-infinite flat plate
- E-3 Extension of the semi-infinite flat plate solution
- E-4 Eddy-current loss in a solid cylinder

E-1 Dreyfus loss theories for saturated steel

Extensive use is made in this thesis of the theories of Dreyfus and Nejman for the eddy-current loss in saturated steel. The foundations and implications of these theories are discussed in section 4-2-7. However, the Dreyfus theories are given in a mixed form of c.g.s units. whilst the work of this thesis is in SI units Hence it is felt that would be instructive to rework the solutions in the m.k.s system using the equation $B = bH^m$ for the initial magnetisation curve.

E-2 Eddy-current loss in a semi-infinite flat plate.

The solution is only concerned with the variation of field quantities with depth, and is one dimensional. It starts from Maxwell's equations.

 $CurlH = J \tag{E-1}$

and

$$\operatorname{Curl} \mathbf{E} = -\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}\mathbf{t}}$$

The resistivity o of the steel is assumed constant and the flux density is assumed to vary sinusoidally. Equations 1 and 2 may be written in one dimensional form as -

(E-2)

$$\frac{dH}{dx} = J \qquad \qquad A/m^2 (E-3)$$

and

$$\frac{dJ}{dx} = \frac{j w B}{\sqrt{2}} \qquad A/m^3 (E-4)$$

Combining these equations and normalising in terms of surface values gives:

$$\frac{d (H/H_s)}{d(x/h)} = \frac{j(\frac{h}{\delta})^2 (\frac{\mu}{\mu_s}) (\frac{H}{H_s})}{s}$$
(E-5)

where

8

$$m = \sqrt{\frac{2\rho}{W\mu_s}}$$
 m (E-6)

and

$$h = \frac{\delta_{s}}{(1-m)} \sqrt{(3+m)} \sqrt{\frac{(1+m)}{2}} m$$
 (E-7)

Equation 5 is solved by the relationships -

$$\frac{H}{H_{s}} = \left(1 - \frac{x}{h}\right)^{\left(1 + j\sqrt{(1+m)/2}\right)\left(2/(1-m)\right)}$$
(E-8)

and

$$\frac{u}{s} = \left(1 - \frac{x}{h}\right)^{-2}$$
(E-9)

The power flow per unit area is calculated from the Poynting vector as:

$$W = \frac{H_{s}^{2}}{h} \frac{2}{(1-m)} (1 + j\sqrt{(1+m)/2}) \qquad W/m^{2} (E-10)$$

Substituting into equation 10 from equations 6 and 7 and simplifying gives:

$$W = H_{s} \frac{(3+m)/2}{\sqrt{\rho Wb} \left(\frac{2}{3+m}\right) \sqrt{\frac{2}{1+m}} (1+j\sqrt{(1+m)/2})} W/m^{2} (E-10)$$

Hence the power factor is independent of the level of excitation providing that the steel is saturated and that the index is constant.

$$pf = \sqrt{\frac{2}{3+m}}$$
(E-11)

When m= 0.1 the power-factor is 0.81

Dreyfus explains that as a consequence of equation 9 there is no electrical activity below the depth h. At h the permeability becomes infinite and this effectively traps the flux. He goes on to define an effective depth of power penetration: which is the depth of steel required to dissapate power at the surface loss rate to give the same total loss. This is shown to be :

$$h_{eff} = \frac{h(m-1)}{m+3} \qquad m \quad (E-12)$$

Combining this with equation 7

$$h_{eff} = \delta \sqrt{\frac{1}{3+m} \sqrt{\frac{1+m}{2}}}$$
 m (E-13)

Now since

$$1 \le m \le 0$$
 then $\delta/2 \le h_{off} \le \delta/2.3$

the power penetration depth is very close to its linear equivalent.

E-3 Extension of the semi-infinite flat plate solution.

The previous solution may be usefully extended for induction heating purposes, where the circuit is voltage is voltage driven. Firstly by calculating the magnetising flux and secondly by normalising the loss equations so that the difficult expressions need only be calculated once.

The value of electric field strength at the surface E_s may be derived from Ohm's law and equations 3 and 8

$$E_{s} = H_{s}\left(\frac{\rho}{h}\right)\left(\frac{2}{1-m}\right)\sqrt{\frac{1+m}{2}}\left(\sqrt{\frac{1+m}{2}}+j\right) \qquad V/m \quad (E-14)$$

The flux per unit length of periphery may be calculated from this equation and may be simplified to -

$$|\phi| = H_{s} \frac{(1+m)/2}{b_{w}} = \frac{4}{\sqrt{2(1+m)}}$$
 Wb/m (E-15)

Turning to the problem of normalisation, it is explained in section 6-4-2 that a unique point is required on the B-H curve. The permeability of steel rises to a maximum value and then decays with saturation the value of H at this knee point may be used as a normalising quantity. Whilst the normalising value of B is defined by the equation $B = bH^m$ where the values of b and m are chosen to give the best fit in the saturated region of the B-H curve. The other modification to the theory is brought about by normalising. The calculaions of loss and, flux are complicated, but may be readily scaled from a known loss condition. It is convenient to define a hypothetical knee point on the B-H curve. Where H_n is the value of H at the knee point on the true B-H curve but where B_n is the value given by the $B = bH^m$ law fitted to the saturated region of the magnetisation curve.



The process of normalisation yields the following :

$$\frac{W}{W_n} = \left(\frac{H}{H_n}\right)^{(3+m)/2}$$
(E-16)

where

$$W_n = H_n^{(3+m)/2} \sqrt{\text{owb}} \sqrt{\frac{2}{3+m}} \sqrt{\frac{2}{1+m}} W$$
 (E-17)

and also

$$\frac{N}{N_n} = \left(\frac{\phi}{\phi_n}\right)^{\frac{3+m}{1+m}}$$
(E-18)

where

$$\phi_n = {}^{b} H_n^{(1+m)/2} \sqrt{\frac{2\rho}{wb}} \sqrt[4]{2(1+m)}$$
 Wb/m (E=19)

and finally

$$\frac{W}{W_n} = \left(\frac{H}{H_n}\right)^{3+m}$$
(E-20)

with

$$h_n = H_n^{(1-m)/2} \sqrt{\frac{2\rho}{wb}} \sqrt{3+m} \sqrt{\frac{1+m}{2}} m$$
 (E-21)

Equations 16, 18 and 20are the practicle forms of the theory

E-4 Eddy-current loss in a solid cylinder

The solution closely resembles that of the previous problem. Dreyfus starts from Maxwells equations in cylindrical co-ordinates of the form

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dH}{dr}\right) = \frac{2j}{\delta}\frac{\mu}{\mu_{s}}H \qquad At/m^{3} (E-22)$$

This equation is solved using the relationship

$$\frac{H}{H}_{s} = \left[\frac{\frac{r}{r_{b}} - 1}{\frac{r}{s} - 1}\right]^{(x + jy)}$$
(E-23)

Where

r_h = radius to a depth h from the surface r_s = radius to the surface x,y = real constants

This is shown to be an approximate solution and equation 23 is substituted to give values for x and y

$$x = \frac{2}{1 - m}$$
 (E-24)

and

$$y = \frac{2}{1 - m} \sqrt{\frac{1 + m}{2} + \frac{1 - m}{2} \cdot \frac{r - r_h}{r}}$$
(E-25)

Thus equation 25 is not strictly constant and independant of radius

- 247 -

but as a mean value

$$\frac{\mathbf{r} - \mathbf{r}_{\rm h}}{\mathbf{r}} = \frac{\mathbf{n}}{2\mathbf{r}_{\rm m}} \tag{E-26}$$

Where

$$r_{\rm m} = (r_{\rm s} + r_{\rm h})/2$$
 m (E-27)

and again by matching constants

$$\frac{h}{\delta_{s}} = \frac{\frac{1}{1-m}}{1-\frac{\delta_{s}}{r_{m}}} \cdot \frac{\frac{5+3m}{2}}{16+16m} \cdot \sqrt{\frac{1}{3+m}} \cdot \frac{4}{2}$$
(E-28)

At this point it is interesting to compare equation 28 with equation 7 from the flat slab solution. For a steel where m = 0.1

$$\frac{h(flat)}{h(cylindrical)} = 1 - 0.1473(\delta_s/r_m)$$
(E-29)

The curvature does not greatly affect the thickness of steel required for a given heating condition.

The poynting vector is used to show that the power flow is $W = H_s^2 \rho (x + jy)$ W/m^2 (E-30)

Which gives rise to a powerfactor of

$$pf = \sqrt{\frac{2}{3 + m + (1 - m)h}}$$
(E-31)

and for a steel where m=0.1 this is approximately

$$pf = 0.803 - 0.218(\delta_{s}/r_{m})$$
(E-32)

Whilst the power input per unit surface area becomes

$$W = \frac{H_{s}^{2}}{\delta_{s}} \frac{W/m^{2}}{(0.758 + 0.064 + (\delta_{s}/r_{m}))}$$
 W/m² (E-33)

The cylindrical problem unlike the slab does not give simple functions of H and δ_s , and does not predict a constant power factor.

The reversion of these equations into dependance on power density poses a difficult problem. However they show that for a vessel whose radius is greater than ten times the skin depth the error in a slab solution is less than one per cent. APPENDIX F PLATES



Preliminary Test Apparatus

Plate 1.



Steel Loss Apparatus Plate 2.



Full Scale Apparatus

Plate 3.



Flux Shielding Apparatus

Plate 4.



Flux Guided Heater Plate 5. REFERENCES

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