PLASTIC DESIGN OF MULTI - STOREY SWAY FRAMES

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SYNOPSIS

Ultimate load methods have become increasingly popular in recent years, and are now widely used in preference to the traditional allowable stress approach. In particular, the simple plastic theory has been applied to the design of a large number of steel structures. Its use has, however, been restricted to frameworks with comparatively few storeys, since, in tall buildings, the presence of high axial loads in the columns invalidates one of the basic assumptions of the theory. In addition, simple plastic design is prohibited for tall frameworks which are required to resist wind loading without the aid of bracing, since the large sidesway deflections which are produced again violate the basic assumption that changes of geometry are small and may therefore be neglected.

This thesis describes the development of a rational form of ultimate load design for multi-storey sway frames. It is shown that the simple plastic equations only require slight modification in order to take account of the instability effects. Furthermore, iterative use of these modified equations produces a design which is economical, whilst satisfying an exacting set of design criteria.

A variety of design aids have been developed in order to reduce the design time and the quantity of arithmetical work involved, with the result that frameworks of any size may be designed without the aid of an electronic computer.

To assess the validity of the proposed design equations, a range of frameworks designed by the method has been analysed using an accurate computer program, which traces the complete load-deformation behaviour up to collapse. The economy of the design method has been examined by comparison with several frameworks designed by other methods. Due to approximations in the design approach, it is not possible to produce the minimum-weight solution, but a safe, extremely economical framework is obtained.

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NOTATION

A	beam magnification factor; area of an I-section;
A_i	initially assumed value for the magnification factor;
A_c	column magnification factor;
A'_{c2}	effective magnification factor for the internal columns of the lowest storey;
$A'_{c2}(sp)$	value of A'_{c2} under simple plastic conditions (i.e., for zero axial load);
a^{α}_{12}	real distribution factor in the elastic range for member 12, under horizontal load αH;
a^{Δ}_{12}	real distribution factor in the plastic range for member 12, under horizontal load $(\lambda_2-\alpha)H$;
a^{1}_{12}	real distribution factor for member 12 at working load (combined loading);
a^{v}_{12}	real distribution factor for member 12 of a single-bay frame under vertical load alone;
$a^{\Delta'}_{12}$	real distribution factor in the plastic range for member 12 - leeward column of a single-bay frame;
a_1, a_2	constants;
a_b, a_d	combined areas of the bending stress-blocks, and area of the direct compression stress-block of an I-section;
B	required fully plastic moment for a beam; flange width of an I-section;
B_1, B_2	required fully plastic moments for the beams in Storeys 1 and 2 (counting from the top);
b	reference number of a section selected for a beam;
C_E	required plastic moment for an external column;
C_I	required plastic moment for an internal column;
c_e	reference number of a section selected for an external column;
c_i	reference number of a section selected for an internal column;
$c, d, e,$	reduction factors for calculating the plastic modulus of an I-section in the presence of axial load;
D	overall depth of an I-section;
$D, E, F, G,$	factors for checking the "working-load elasticity condition" for a beam in the lowest storey;

E Young's Modulus of Elasticity;

f_y yield stress;

$f(B)$ non-dimensionalized function of B $\left(= B \Big/ \lambda_2 \frac{WL}{4}\right)$;

$F.E.M._2$ total fixed-end moment or initial sway moment at joint 2;

grad gradient of a straight line;

H shear force per bay in any storey;

$\Sigma(H)$ total shear force in any storey;

h storey height;

h_b, h_d combined depth of the bending stress-blocks, and depth of the direct compression block of an I-section;

$(mHh)_{av}$ average value of (mHh) for two consecutive storeys;

I second moment of area of a section;

I_b, I_c second moment of area of a beam section and a column section respectively;

I_i, I_e second moment of area of an internal column section and an external column section, respectively;

$\Sigma(I)$ sum of the second moments of area of all the columns in a storey;

K_b $\frac{EI_b}{L}$ for a beam;

K_c $\frac{EI_c}{h}$ for a column;

$K_{b(j)}$ $\Sigma\left(\frac{EI_b}{L}\right)$ for all the beams in Storey (j);

$K_{c(j)}$ $\Sigma\left(\frac{EI_c}{h}\right)$ for all the columns in Storey (j);

$\bar{K}$ ratio of the flexural rigidities of the two columns at a joint $= \frac{K_{c1}}{K_{c2}}$, where suffixes 1 and 2 refer to the columns above and below the joint, respectively;

K_2 ratio of the flexural rigidities of the lower column and the beam at a joint $= \frac{K_{c2}}{K_{b2}}$;

L length of any beam;

m Merchant's[42] magnification factor, allowing for the eccentricity of axial load in a member

$$= \frac{2s(1+c)}{2s(1+c) - \pi^2\rho}$$

m^1	value of m at working load;
M_A	sum of the first moments of area of the bending stress-blocks of an I-section about the neutral axis;
M_{12}	bending moment at end 1 of member 12;
M_W	bending moment due to wind loading alone;
$\Sigma M_{(j)}$	sum of the bending moments at joint (j);
Mp_b	fully plastic moment of a selected beam section;
NA	neutral axis;
n	reference number for a general storey;
	stability function[5] $= s\left[1 - \frac{m(1+c)}{2}\right]$;
	ratio of axial stress to yield stress $= \frac{P}{Af_y}$;
n'	"change-over" value[56] of $n = \frac{P}{Af_y}$;
o	stability function[5] $= s\left[-c + \frac{m(1+c)}{2}\right]$;
P	axial load in a member;
P_I, P_E	axial load in an internal column and an external column, respectively.
P_E^V, P_E^H,	components of P_E due to vertical loading and wind loading, respectively;
P_e	Euler load $= \frac{\pi^2 EI_c}{h^2}$;
p	wind load intensity (force/unit area);
p_1, p_2	ratios $\frac{(mHh)av}{m_1H_1h_1}$ and $\frac{(mHh)av}{m_2H_2h_2}$ respectively, where suffixes 1 and 2 refer to the storeys above and below the beam;
q	total number of storeys in a frame;
r	total number of bays in a frame;
R	ratio of joint rotations at the ends of member 23 $= \frac{\theta_3}{\theta_2}$;
S	spacing of frames;
S_{12}^r	real rotational stiffness at end 1 of member 12;
ΣS_2^r	sum of the real stiffnesses of all members meeting at joint 2;

s	rotational stiffness factor for a member with the far end fixed;
t	ratio of shear forces induced in the external columns of the top two storeys of a frame - vertical load alone; web thickness of an I-section;
T	flange thickness of an I-section;
V	combined stablility function for the two columns meeting at a joint = $(n_1-o_1)\overline{K} + (n_2-o_2)$;
V'	value of V at a joint in the lowest storey = $(n_1-o_1)\overline{K}+n_2$;
V^1, V'^1	values of V and V' at working load;
w	unit floor loading;
W	total load on any beam;
W_1, W_2	loads on beams in Storeys 1 and 2;
wr	wind ratio = $\frac{A(mHh)av}{WL}$;
wr_i	initial value of wr = $\frac{(Hh)av}{WL}$;
wr'	effective wind ratio in the lowest storey;
x	ratio of the sum of the flexural rigidities of the columns to that of the beam at any storey in a single-bay frame = $\frac{K_{c1} + K_{c2}}{K_{b2}}$;
x'	modified version of x for the lowest storey of a single-bay frame = $\frac{3K_{c1} + 2K_{c2}}{3K_{b2}}$;
X	shear force in an external column due to vertical load alone;
Y_1, Y_2	assumed axial forces in the beams of the leeward external bays of Storeys 1 and 2;
Zp_b	full plastic modulus of a beam section;
Zp_c	full plastic modulus of a column section;
Zp_c'	reduced plastic modulus of a column section (in the presence of axial load);
zp_b	required plastic modulus for a beam;
zp_{ci}, zp_{ce}	required plastic moduli for an internal and an external column, respectively;
α	the load factor for wind loading at which the first plastic hinge forms in a beam;

γ	function used in the design of the lowest storey of a single-bay frame = $\frac{2K_2V'}{2K_2V' + 3}$;
δ, Δ	sway deflection of a single storey or a complete frame;
Δf	sway deflection due to column flexibility;
Δr	sway deflection due to rotation of the joints;
η	ratio of the actual plastic moment of a selected beam section to the plastic moment required for the beam = $\frac{Mp_b}{B} = \frac{Zp_b}{zp_b}$;
ρ	Euler ratio = $\frac{P}{P_e}$;
ρ^1	value of ρ at working load;
$\rho_{(j)}$	average ρ value for all the columns of Storey (j);
θ	free rotation of a joint after plastic collapse;
$\theta_{(j)}$	true rotation of a joint(j);
$\phi_{(j)}$	deflection index for Storey (j) = $\frac{\Delta_{(j)}}{h_{(j)}}$;
λ	general value of load factor;
λ_1	design load factor for vertical load alone;
λ_2	design load factor for combined loading;
λ_C	elastic critical load;
λ_F	failure load;
λ_P	rigid plastic collapse load;

CHAPTER 1

INTRODUCTION

The work presented in this thesis consists of the development of a method of structural design suitable for multi-storey plane frameworks, originally proposed by S.N.Gandhi.[1] The limitations and advantages of this method are critically assessed in the thesis by accurate analysis of a series of design examples and by comparison with other design methods.

It is shown that the method may be easily applied to produce a structure which satisfies a stringent set of design criteria. In addition, it is demonstrated that the structure is capable of carrying its applied loads in a rational and efficient manner, with an even distribution of strength throughout the framework.

The proposed method produces a design which is both economical and aesthetically pleasing in its structural efficiency.

1.1. CRITICISM OF ELASTIC DESIGN METHODS

One of the principal reasons for the development of the plastic theory was the realisation that most structures designed by the elastic method are necessarily uneconomical. The basis of elastic design is that the stresses developed in any part of a structure under normal working loads shall not exceed certain permissable values, these being given in the British Standard Specification No. 449.[2]

The irrational nature of this method may be clearly demonstrated[3] by comparing the relative strengths of a simply supported beam and an encastré beam when both are designed elastically.

B.S.S. No. 449 allows a maximum stress at working load of 10.5 tons per sq. in. for a material with a guaranteed yield stress of

16 tons per sq. in. It follows, therefore, that the loads could be increased by a factor of $\frac{16}{10.5}$, or 1.524, before yielding would occur in the extreme fibres at the most critical cross-section in the structure. This is true for any structure designed in this way, assuming linear elastic behaviour. Elastic design therefore provides a safety factor of 1.524 against the onset of yielding.

However, the structure will support additional load after yielding first occurs. This is due to the fact that the plastic modulus of a cross-section is greater than the corresponding elastic modulus. The ratio of the two is called the shape factor and for an I-section is approximately equal to 1.15. Accordingly this implies that the fully plastic moment of the section is 1.15 times as great as the moment at which yielding first occurs. Therefore, for any elastically designed structure the safety factor against the occurrence of the first plastic hinge is 1.15 x 1.524, or 1.75 times the working loads.

Thus, in the case of a simply supported beam carrying a uniformly distributed load, where only one plastic hinge is required to cause collapse, the load factor (i.e. the ratio of collapse load to working load) has the same value as the safety factor against a plastic hinge, both being equal to 1.75.

However, in the case of an encastré beam more than one plastic hinge is required to produce a collapse mechanism. As for the simply supported beam the first plastic hinges occur at 1.75 times the working load at the ends of the beam. In this case, however, collapse does not occur until a third plastic hinge forms at the centre of the beam at 2.34 times the working load.

So, both beams have been provided with the same safety factor against the formation of a plastic hinge, but in one case the load factor is 1.75 and in the other it is 2.34. The real strengths of the two beams are therefore different.

Any framed structure which consists of a number of beams and columns with different end conditions and applied loads will exhibit similar behaviour. The load factor against collapse of every member of the frame is likely to be different. This is clearly undesirable since it implies that the majority of the members in the frame are stronger than they are required to be.

To be more precise, this means that any member designed elastically which is not pinned at both ends is overdesigned. This will be true of every single member in a rigid jointed frame.

A far more rational and considerably more economical approach to the design of a framework is to select the members in such a way that the complete framework has some definite load factor against collapse. In addition, it would be desirable for every single member to have a similar strength, although this is seldom possible. This is the basis of plastic design, of which the method proposed in the subsequent chapters is a modified form.

Plastic design methods have the additional advantage that the analysis techniques involved are generally considerably less demanding than those required for an accurate elastic design, due to the fact that the framework under consideration has far fewer redundancies than the corresponding fully elastic structure. It can also be shown that the residual stresses developed by rolling and welding the steel sections, and relative settlement of the supports of a structure have no effect on the simple plastic collapse load, whereas they should be considered when using the allowable stress approach of elastic design.

1.2. LIMITATIONS OF SIMPLE PLASTIC DESIGN

Two of the basic assumptions of the simple plastic theory are as follows:-

(a) The equilibrium equations are based on the undeformed structure; i.e. deflections must be small.

(b) Local instability of any individual member or overall frame instability does not occur.

Both these assumptions are quite valid when dealing with comparatively small structures. However, for multi-storey frameworks neither of them may be assumed to be true, particularly if the members of the frame carry large axial forces or if the frame itself is subjected to heavy applied wind loading.

The necessity to recognise the possibility of early collapse due to instability effects has been emphasized by several people, a review of this work being contained in Chapter 2. In particular, R.H.Wood[4] clearly demonstrated that there is a serious deterioration in the elastic critical load of a structure due to the formation of plastic hinges, and that failure may occur due to the partially plastic frame becoming unstable long before a complete plastic collapse mechanism is attained.

The procedures incorporated in the proposed design method to allow for the above limitations are briefly described in the following section.

1.3. PROPOSED DESIGN METHOD

For frames in which the instability effects are negligible the simple plastic theory produces a safe and economical design. However, as already stated,for frames with more than two or three storeys, or for those acted on by heavy wind loading the beams and columns selected by this method cease to be adequate and failure would occur below the specified load factor for a complete mechanism.

These selected sections do however provide a lower bound to those required for a safe design. The degree by which these sections have to be increased depends on a variety of factors, but it is shown in Chapter 3 that all these effects may be incorporated into one function, and this has been termed the "magnification factor". This factor is basically a function of the relative stiffnesses of the selected sections, which are in turn functions of the section properties and the stability functions derived by Livesley and Chandler.[5] It is also shown that if the value of this magnification factor was known initially it would only be necessary to multiply the wind loading by this value and apply the original simple plastic design equations directly to the frame under the modified wind loading to give a design which would automatically allow for the instability effects.

However, the ultimate value of the magnification factor is not known initially and the proposed design method is directed towards calculating its value using an iterative procedure, thereby converging to the required set of sections rather than by obtaining them directly. Throughout the procedure the equations used to select sections are similar in form to the simple plastic design equations, the only difference being the introduction of the magnification factor. The determination of the value of this factor during each iteration occupies the bulk of the computational work.

No attempt has been made in this introduction to give anything other than the basic philosophy of the design method and its relationship with traditional elastic and simple plastic methods. A detailed derivation of the design equations appears in Chapters 3,4 and 5, together with a statement of the basic design criteria and the assumptions involved.

The following chapter consists of an historical review of associated work in the fields of plastic design and instability of structures.

CHAPTER 2

HISTORICAL REVIEW OF RELATED WORK

2.1. INTRODUCTION

Prior to the development of the plastic theory in a form which was suitable for design purposes, all multi-storey frameworks were designed using modifications of the traditional elastic methods. Directly and indirectly, this had the following effects:-

(a) As indicated in Chapter 1, the frames were generally safe, but they were uneconomical and inefficient in their structural behaviour.

(b) The understanding of instability theory was delayed, since any tendency towards reduction in stiffness in complete structures due to axial load effects was disguised by the nature of the design methods. For instance :-

(i) The majority of frameworks were restrained against sway and were therefore less susceptible to instability failure.

(ii) In general the design methods neglected the stiffening effects of cladding, and this, together with the additional lack of economy mentioned in (a), ensured that the elastic critical load of the structure was sufficiently high for acceptable designs to be produced.

In 1929 the Steel Structures Research Committee was formed in order to investigate the possibility of applying modern theories of structures to produce a more economical and rational elastic design method. Their final report published in 1936(6) proposed a design procedure which utilized the additional stiffness contributed to the ends of a member by those members connected to it. This was in contrast with the previous methods, from which a deliberately safe design had been obtained by underestimating the stiffnesses of the connections, for example by designing the beams in a framework as pin-ended. The work of this committee, which included the first full-scale tests on steel structures, has been described in detail by Baker[7] who stresses that despite the fact that the original aims were largely satisfied, the proposed design method could not have been particularly acceptable to industry since it was not generally adopted. The procedure was complex for design office purposes and this was the principal reason for its lack of popularity.

Subsequently, several attempts were made to simplify the S.S.R.C. proposals by Baker and Williams,[8] Horne,[9] and Wood.[10,11] The latter produced a non-mathematical method for assessing the critical moments in each member of a frame, and this resulted in a design in which the beams were economically selected, the columns having an additional safety factor.

Despite these modifications the basic lack of economy, the complexity and the irrational approach of elastic design still remained, and it was natural for the attention of research workers to turn towards the more logical philosophy of plastic design. In turn, this was to demand a definite understanding of instability theory. The remainder of this chapter is devoted to a review of these fields of research.

2.2. DEVELOPMENT OF THE SIMPLE PLASTIC THEORY

The principal reason for the introduction of mild steel in preference to other structural materials such as cast iron was its marked ductility. It had been appreciated that stress concentrations in the regions of rivet and bolt holes and at sudden changes of cross-section could be easily absorbed with a material such as this without the occurrence of local failure due to brittle fracture. Nevertheless, this ductility, which is associated with behaviour outside the elastic range, was completely disregarded in the conventional elastic design of the major structural components.

Although the plastic behaviour of steel had been observed much earlier, it was not until 1914 in Hungary that the first published work appears to have been produced describing what has now become to be known as a plastic collapse mechanism. In this paper[12], Kazinczy noted that if the load on an encastré beam was steadily increased, "failure" was found to occur when three independent cross-sections of the beam had yielded completely. Similar observations were made by Kist[13] in Holland in 1917, but Kazinczy, being the first to appreciate the fundamental concept of a plastic hinge, effectively laid the foundations for all the subsequent work on the plastic theory.

Following the publication of these two papers, a considerable quantity of research was performed in Germany, notably under the supervision of Gruning[14], Maier-Leibnitz[15], Bleich[16] and Girkmann[17]. The tests performed by Maier-Leibnitz were particularly valuable, and he demonstrated collapse mechanisms for a variety of single-span and continuous beams. In addition, he was the first to show that the collapse load of a continuous beam is unaffected by settlement of the supports. This one fact has remained a powerful argument in favour of ultimate load methods of analysis. It is well known by engineers that considerable difficulties are introduced in elastic analysis by differential movement of supports.

Maier-Leibnitz was also largely responsible for the stimulation of interest in plastic methods in Britain. His paper in Berlin in 1936[18] summarized much of the relevant work in the previous ten years in Europe, and following this he was invited to a meeting with members of the Steel Structures Research Committee, who, as mentioned before, had become increasingly critical of elastic design methods whilst preparing their final report. This meeting led directly to the instigation of a research programme at the University of Bristol by Baker and Roderick[19,20] who attempted to verify and extend Maier-Leibnitz's results. Unfortunately, this programme had to be curtailed due to the outbreak of the Second World War, but by this time a series of eight portal frames had been successfully tested in the plastic range and the early work had been largely substantiated.

Interest had also been aroused in America and in 1940 van den Brock[21] restated the basic ideas of the plastic theory under the new name of "Limit Design". The first comprehensive definition of the general principles of simple plastic behaviour, later to be known as the kinematical, statical and uniqueness theorems, was given by Horne[22] in 1950, this paper preceding a similar publication in America by Greenberg and Prager.[23] Subsequently the most important developments came from firstly Neal and Symonds,[24,25,26] who extended the basic philosophy of simple plastic theory in producing several rapid methods of analysis, in particular the method of "combination of elementary mechanisms", and secondly from Horne,[27] who devised the "plastic moment distribution" technique.

In 1956 all these suggestions were correlated by the publication of the second volume of "The Steel Skeleton",[28] which deals exclusively with the plastic behaviour of steel structures. This book not only states all the assumptions on which the simple plastic theory is based but also shows in great detail how it may be used for both design and analysis. In addition, it gives a clear warning of the limitations of the theory, with particular reference to the instability problem, and at the time of publication preceded much important research directed towards developing an understanding of this phenomenon. This work is the subject of the following section.

2.3. THE INSTABILITY PROBLEM

It has been suggested in 2.1. that prior to the development of the plastic theory the possible incidence of instability in real frameworks was often disguised by the fact that the majority of structures had a large reserve of strength. This is not to say that the dangers of buckling were not appreciated, for several recommendations already existed in the codes of practice to counteract failure of individual members. However, there was generally no great necessity to consider overall frame instability, and most of the work designed to obviate this type of failure has been carried out during the past fifteen years.

It is considered suitable therefore to discuss the history of this subject in two sections, which are necessarily related, but which both chronologically and in content have tended to constitute two distinct fields of research.

2.3.(a) MEMBER INSTABILITY

It is well known that the elastic failure of a slender pin-ended strut under increasing axial load was analysed initially by the German mathematician Euler in 1759. This analysis did in fact provide the basis for all subsequent development in the field of member instability.

In 1886 Euler's theory for a perfect mathematical model was extended to suit the more practical case of an imperfect strut. It was realised that in all structures there exists a variety of imperfections due to errors in rolling, variation in material properties, lack of straightness and eccentricity of loading. Ayrton and Perry[(29)] suggested that the combined effect of all such imperfections could be represented as either an equivalent eccentricity or as an equivalent curvature, and using such an idealisation they derived a formula for estimating the applied load that is required to produce the onset of yielding in the most highly stressed section of a strut. Moncrieff[30] developed a modified version of this "Perry formula" which, although inconvenient to use, was popular in British practice for many years. The main difficulty with the application of this method was the individual assessment of how best to represent the imperfections in the strut. In 1925 Robertson[31] suggested a parameter based on a conservative view of the results of his and other people's tests on struts, and the Steel Structures Research Committee, of which Robertson was a member, later incorporated this function in an alternative form of Perry's original equation. The Committee recommended the use of this "Perry-Robertson formula" in its preliminary report in 1931. In addition, the formula was applied to calculate a range of safe working stresses for elastic design purposes and these were reproduced in B.S.S.No.449 in 1932. The selection of the appropriate design stress did however depend on the engineer's assessment of the effective length of the stanchion under consideration, and initially no guidance was given for estimating this parameter. Several explanatory clauses have since appeared in the code of practice to enable the designer to assess the degree of fixity at the ends of a column length. However, there is still considerable room for error, much of the assessment being based on past experience, and it is certain that in some cases large inaccuracies have been introduced by strict observance of the recommended procedure in B.S.S.No.449.

More recently, following the introduction of plastic methods, attention has turned towards the elasto-plastic behaviour of struts and the stability characteristics of beam to column connections. In Britain, the first qualitative failure tests on stanchions were performed by Baker and Roderick, initially at the University of Bristol and later, after the end of World War 2, at the University of Cambridge. Their most significant observations were firstly that the ultimate load of a continuous stanchion is considerably greater than the load which is required to cause the onset of yielding at the most highly stressed section, and secondly that in general the loads applied to the beams connected to the ends of the stanchion have relatively little effect on the axial force required to cause failure. This second result is now fairly well known. It is the beam stiffness at the connection and not the initial moment at the end of the beam that largely controls the collapse load of the column.

These early tests were performed on both rectangular and I-section columns bending in both single and double curvature in one plane about the minor axis, the results being published in three papers between 1942 and 1948(32,33,34).

Following this work an attempt was made by Baker, Roderick and Horne(35) to produce a method of analysis for estimating the collapse load of a stanchion bent about the minor-axis. Although this was found to explain adequately the behaviour of the columns in the tests, the theory was complicated and was therefore difficult to incorporate in any simple design method. Similar analytical procedures were later developed by Horne(36) to deal with the more practical cases of stanchions bent about the major axis and for simultaneous bending about both axes. Initially these methods were also complex and not in common use. Moreover, as will be seen in the following section, the majority of current design methods do in fact attempt to restrict plasticity to the beams rather than the columns. It has generally been found necessary to use either this "weak-beam, strong-column" approach, or occasionally the "strong-beam, weak-column" approach in which plastic hinges are restricted to the columns, the beams remaining elastic. Design methods which rely on an indiscriminate plastic hinge pattern in both beams and columns have often been shown to result in a structure which is very susceptible to overall instability.

Horne has recently simplified his analysis methods considerably and has also produced a method of elastic analysis which guards against lateral torsional buckling for a column undergoing major and minor axis bending together with axial load.[37,38]

The second type of member instability, that of lateral buckling of the beams due to the development of high compressive stresses in bending, is unlikely to occur in a regular multi-storey framework where generally adequate restraint is provided by composite action with the floor slabs. However, when dealing with certain other types of structure, for example a pitched-roof portal frame, this type of failure may be more significant. A comprehensive treatment of this subject is given in "The Steel Skeleton", Vol.2[28].

2.3.(b). FRAME INSTABILITY

Overall frame instability is a far more complex problem than that of individual member instability since it becomes necessary to consider the interaction of every single member in the structure. Nevertheless, the behaviour of a single member, such as the pin-ended strut discussed in 2.3.(a), indicates many important principles which apply to a framework as a whole.

The Euler load is in fact the first elastic critical load of a pin-ended strut, and this is accompanied by a series of elastic critical loads, each of which corresponds to a different mode of deformation. In a similar way, a complete framework has a large number of elastic critical loads, although the modes of deformation associated with these are of course far more varied than those of a single strut. Comparisons such as this have been used by several research workers to assist in qualitatively explaining the behaviour of rigid-jointed frameworks, and it is this concept of the elastic critical load which provides the basis of all stability analyses for both the elastic and post-elastic ranges.

Stability functions, designed to allow automatically for the effects of axial load on the stiffness of a structure, have been developed in several forms since they were first suggested by Berry(39) in 1916. In 1935, James(40) demonstrated how the moment distribution procedure could be modified to allow for stiffness changes due to axial load, his work later being extended by Lúndquist(41) who attempted to use stability functions in order to determine the elastic critical loads of frameworks. However, it is only comparatively recently that it has been generally realised that axial load effects may be responsible for reducing the entire stiffness of a perfectly elastic frame to zero, and Lundquist's work was never fully appreciated until more sophisticated methods of analysis were produced several years later. Even then, the elastic instability of frameworks was considered by most engineers to be relatively unimportant in practical terms. This apparent apathy is understandable since the elastic critical load of a structure is generally several times greater than the normal working loads, so that there is little chance of overall instability provided that the framework remains elastic.

More recently, however, Merchant(42) and Livesley and Chandler(5) have developed a wide range of readily applicable stability functions, and there now exist several well established methods for calculating the critical loads of rigidly-jointed frames. These methods have become essential tools for dealing with the more complicated subject of elasto-plastic frame instability.

The first detailed account of the complete frame instability problem was given by Merchant(43) in 1954. As an approximate method for the evaluation of the elasto-plastic failure load he suggested a relationship based on the Rankine formula as applied to struts.

The "Merchant-Rankine formula" is as follows:-

$$\frac{1}{\lambda_F} = \frac{1}{\lambda_C} + \frac{1}{\lambda_P}$$

where λ_F represents the actual failure load of the frame, λ_C the elastic critical load and λ_P the rigid plastic failure load.

One of the main criticisms of this formula must be that despite the fact that an approximate solution is obtained, calculation of the elastic critical load is still required, which is in itself a complex procedure for anything other than a simple framework. Nevertheless, for small frames, for which the computation is not too laborious, the Merchant-Rankine formula generally provides a lower bound solution to the actual failure load, and as such may be a useful approximation.

The real significance of the elastic critical load of a frame was first demonstrated conclusively by Wood(4) in 1958. He showed that despite the fact that the critical load may initially appear to be deceptively high, a serious deterioration in the overall stiffness of a frame may occur whenever a fully plastic hinge develops in a member. Using the hypothesis that a plastic hinge contributes no more stiffness to a structure than a real hinge, provided that rotation continues in the same direction as before, Wood suggested that a new reduced critical load may be obtained for the modified structure by effectively substituting a real hinge for the plastic hinge. Furthermore, as successive plastic hinges form this "deteriorated critical load" becomes so low that failure may occur due to elasto-plastic instability well before the attainment of the rigid plastic collapse load factor, and with far fewer plastic hinges than are required for a complete collapse mechanism. It was also proposed that the deteriorating critical load may be considered to be a continuously descending function and that failure occurs when the value of this function coincides with the increasing applied load. The elastic critical load of the frame is in fact the initial value of this decreasing function, and if the ratio of this load to the plastic collapse load is high, then the true failure load will tend to be fairly close to the plastic collapse load since the point of coincidence of the two curves will be delayed. This last observation by Wood is also indicated by the Merchant-Rankine formula, from which it may be seen that as λ_C increases, λ_F tends to λ_P.

Whilst generally supporting Merchant's equation, Wood did in fact make one important criticism. He pointed out that for frames with a low elastic critical load, such as those bending about the minor-axis, the formula tends to underestimate the failure load considerably.

In the discussion on Wood's paper(44) Merchant commented on the revolutionary nature of the material presented in it . Having been particularly involved himself in the development of this branch of structural analysis, Merchant was possibly in a better position than most to grasp the full significance of Wood's suggestions, which for the first time gave a clear account of the relative roles played by plasticity and instability in determining the ultimate load behaviour of tall frameworks.

Recently, several sophisticated analysis methods have been developed in order to trace the load-deformation history of large structures up to collapse. A variety of design methods have been produced, some relying on iterative use of analytical procedures, others being of a more direct nature. A review of several of these methods is given in the following section.

2.4. RECENT DESIGN AND ANALYSIS METHODS

In 1957 a joint committee of The Institution of Structural Engineers and The Institute of Welding was formed in order to produce a simple and economical design method for fully rigid multi-storey steel frames. It was decided initially to consider no-sway frames, and the final recommendations of the committee for these were given in a report in 1964[45]. Briefly, it was suggested that the major-axis beams in a framework should be designed by the simple plastic theory, thus giving maximum economy, and that the columns should remain elastic in order to guard against local buckling and overall frame instability. The critical moments in any particular member were to be obtained by analysing only a small portion of the framework, consisting of the member being designed plus all other members connected to it, the remote ends of these members being regarded as encastré.

This committee has recently begun to consider the design of unbraced frames, which is necessarily a far more complex procedure. The provision of adequate bracing in a framework ensures that sidesway is negligible, and the analysis methods, which are an integral part of any design process, are simplified accordingly. In addition, one of the less obvious but most important subsidiary effects of bracing is that it provides the frame with a comparatively high first elastic critical load since it eliminates all those modes of deformation which are generally decisive, namely those associated with sidesway. Accordingly, as implied previously in 2.3.(b) there is generally little danger of overall buckling about the major axis of a braced frame, and the mode of failure will almost invariably be a simple beam mechanism provided that adequate provision has been made against local column instability. In addition, this indicates that there is no necessity to ensure that the columns in such a frame are designed elastically, as, for instance, in the method described above. It would appear that greater economy may be obtained with no loss of safety by an elasto-plastic column design, for example using the method suggested by Horne[37].

A sway frame does however require more careful treatment. Not only are the basic analytical procedures more complex due to the presence of sidesway, but also the overall stiffness of the frame is always considerably lower than that of the corresponding braced frame, and this is particularly so in the post-elastic range.

In 1960 Heyman[46] proposed an approach to the design of regular multi-storey sway frames. In this method the structure is idealised initially by assuming points of contraflexure to exist at the midheights of the columns thus enabling one storey to be isolated from the adjacent ones by sectioning through these points. A pattern of hinges in both the beams and columns is then assumed in order to reduce the storey to a mechanism, and the required beam size may then be obtained by any of the methods of simple plastic analysis. The column size is selected in such a way that it will remain elastic under the factored loads.

Heyman suggested that the stiffening effect of cladding would generally be sufficient to eliminate the stability problem, although in the discussion on his paper[47] some doubt was expressed concerning the effectiveness of this safeguard. In fact, the method he proposed generally produces a conservative design, and this lack of economy contributes towares the safety of the frame. The method may however be criticised for its lack of direct consideration of instability and in the discussion[47] it was Calladine who suggested a possible procedure for developing a rational design allowing for the instability effects. This was investigated further by Holmes and Gandhi[1,48] and an experimental research programme designed to substantiate their work was later conducted by Clough[49].

The design methods that have been discussed so far all rely to some extent on the individual judgement of the engineer, and are based on a variety of assumptions and approximations. An alternative approach is that of automatic design by computer. The most sophisticated method available is that suggested recently by Majid and Anderson[50]. This involves iterative use of a non-linear elasto-plastic analysis procedure which is based on the matrix displacement method. The effects of axial load on stiffness are allowed for by the introduction of stability functions, while the loss of stiffness due to the formation of plastic hinges is automatically recorded by successive alteration to the overall stiffness matrix as each hinge forms. The design method depends on first of all analysing the frame with an initially assumed set of sections and then altering those sections which are either inadequate or oversafe. The frame is then re-analysed and the procedure is repeated until the method converges to a unique set of sections which economically satisfy the design criteria. Although the method does not pretend to provide the minimum-weight solution it undoubtably produces a very economical design and may be applied to large frameworks of extremely irregular shape. The elasto-plastic analysis procedure was first produced by Jennings and Majid(51) and has subsequently been developed by Majid and Anderson(52).

A computer program has also been written for the method proposed by Holmes and Gandhi(53), although it must be stressed that this is purely intended to reduce the quantity of repetitive arithmetical work. This design method is not dependent on the availability of a computer, unlike the true "computer method" mentioned above. As will be seen later in this thesis, the Holmes and Gandhi method may be suitably adapted for rapid design by hand, and as such enjoys a measure of flexibility which is lacking in certain other methods.

CHAPTER 3

GENERAL THEORY

3.1. INTRODUCTION

In this chapter the general theory is developed for the design of a typical internal bay in the intermediate storeys of a regular multi-bay, multi-storey sway frame. Plastic thory is used as the basis of this design method, which is an extension of a method originally proposed by Gandhi[1]. In the early part of the chapter the basic design criteria and the assumptions involved in using the design method are given. These are followed by the development of a set of simple plastic design equations, which are later modified to compensate for instability effects.

In subsequent chapters the general theory is extended to cater for the design of the boundary regions in the framework, and is also modified to deal with single-bay frames.

3.2. DESIGN CRITERIA

The design criteria that have been adopted in the proposed method are as follows:-

(1) The framework shall be capable of withstanding dead load plus vertical superload at some load factor, λ_1.

(2) The framework shall be capable of withstanding a combination of dead load plus superload plus wind load at some load factor, λ_2.

(3) The framework shall be fully elastic at working load.

(4) Plastic hinges shall not form in the columns below the design load factor under either system

of loading.

Conditions (1) and (2) are the basic design criteria in that they ensure that the framework has the required strength. The values of load factor which have been adopted in this thesis are those which are generally accepted in this country, namely 1.75 for λ_1, and 1.40 for λ_2. The reduced value for λ_2 corresponds to the 25% increase in allowable stresses in the presence of wind loading, as permitted in B.S.S.No.449[2]. These load factors are not however obligatory, and the designer is free to use his own judgement in selecting appropriate values.

Condition (3) is an additional design criterion that has been imposed in order to help control the sway deflections in the frame at working load. As such, it is not directly related to the strength requirements of the structure. Whilst it is appreciated that even if a frame is fully elastic, excessive sway deflections may occur, the deflection problem is aggravated considerably by the formation of early plastic hinges, and calculation of the corresponding elasto-plastic deflections involves more complicated analytical procedures. The approach which has been adopted is, therefore, to ensure first of all that the frame is fully elastic at working load, and then to base the deflection calculations on this assumption. The elastic analysis for deflections is given in Chapter 9.

Condition (4) is applied in order to control the pattern of hinge formation in the framework. This again is not strictly necessary for strength purposes, but ensures that the "weak-beam, strong-column" design, mentioned in 2.3., is achieved. This tends to reduce the likelihood of early failure due to indiscriminate plastic hinge formation. In addition, the analysis is simplified by restricting the formation of plastic hinges to the beams.

3.3. BASIC ASSUMPTIONS

The success of any design method is based initially on the assumptions that are inherent in its use. These generally fall into two catagories. In all methods it is necessary to idealise the actual behaviour of the material, in this case, steel. Also, if, as in this thesis, a design method is to be produced which does not require the use of an electronic computer and which attempts to deal with large frameworks, it is necessary to make certain approximations about the behaviour of the structure. The assumptions that have been adopted in this approach are given in the two sub-sections below.

3.3.(a) IDEALISATION OF THE MATERIAL

The following assumptions have been made concerning the plastic behaviour of steel:-

(1) The idealised stress-strain relationship shown in Figure 1 represents the behaviour of every fibre in the cross-section of a steel member subject to bending, in both the tensile and compressive zones. The assumed yield stress, fy, is equal to the lower yield stress of the material.

(2) As indicated by Figure 1, strain hardening effects are ignored.

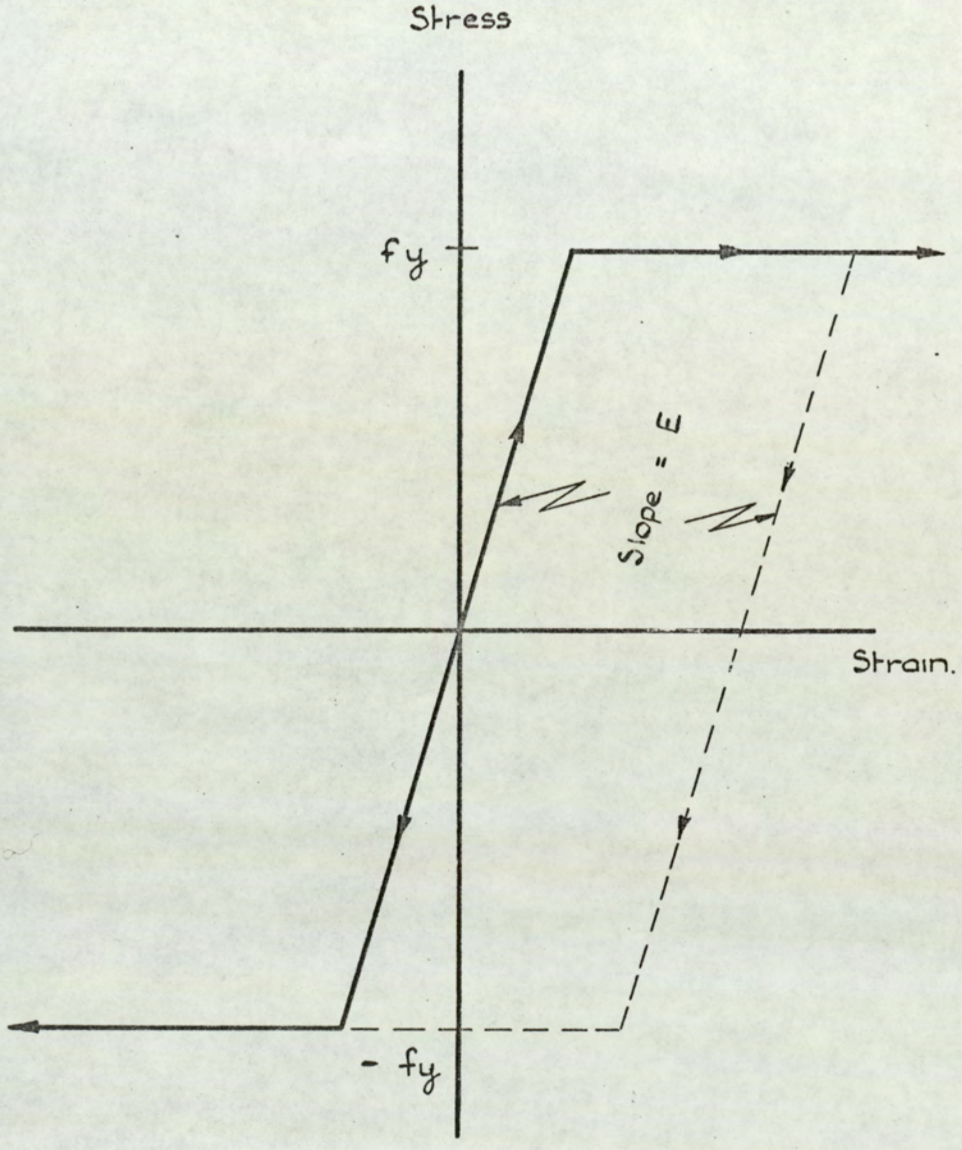
(3) Plane transverse sections in any member remain plane and normal to the longitudinal axis after bending, the effect of shear being neglected.

(4) Fully plastic hinges occur at discrete cross-sections, the longitudinal spread of plasticity being ignored.

(5) The material is homogeneous and isotropic in both the elastic and plastic ranges.

3.3.(b) IDEALISATION OF THE STRUCTURE

The theory will be developed for a general rigid-jointed sway



Idealised stress - strain relationship

FIGURE 1

frame having r bays, each of span L, and q storeys, each of height h, as shown in Figure 2. The assumptions concerning the structure and its loading are as follows:-

(1) The number of bays and the number of storeys are large, so that the effect on a typical internal bay of the unsymmetrical loading conditions on the end bays is negligible.

(2) The span loading is uniformly distributed and of magnitude W per bay at working load.

(3) No bending moments, and therefore no shear forces, are induced in the internal columns by the vertical loading, as implied by (1).

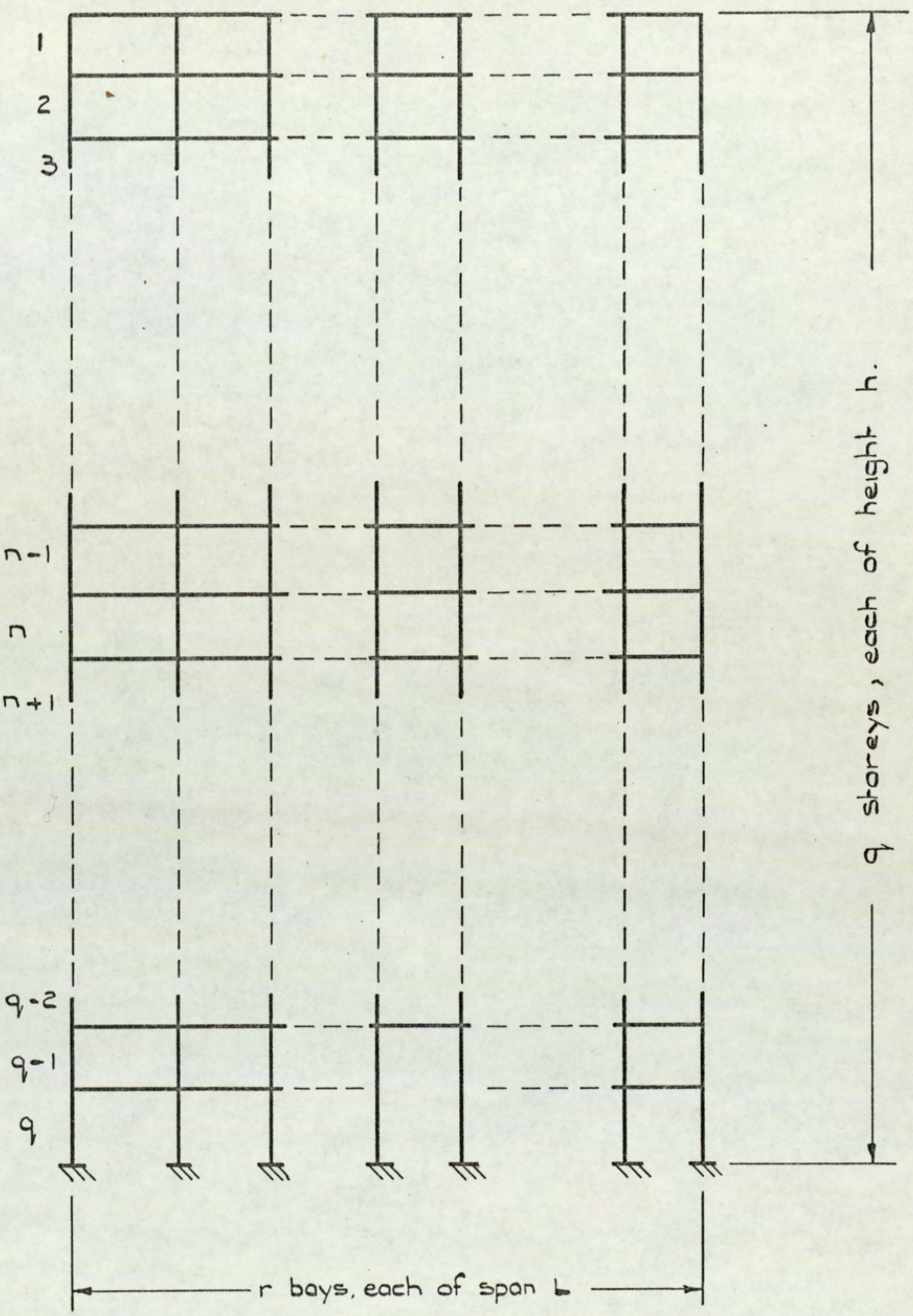
(4) An equal shear force, H, is induced in each internal column in a storey by the application of wind loading. H is defined as the sum of all the applied wind forces above the storey in question, divided by the number of bays.

(5) Due to wind loading, the columns bend in double curvature in such a way that points of contraflexure exist at their mid-heights.

(6) The axial forces developed in the beams are small and may be neglected.

(7) The axial force in an internal column is equal to the force in the column above, plus half the loads on the adjoining beams.

It must be stressed that these assumptions apply to the behaviour of the internal regions of the framework. The derivation of design equations for the boundary regions, which will be described in Chapter 4, requires that certain of these hypotheses be modified. For example, when dealing with the lowest storey of the framework, it is no longer satisfactory to assume that points of contraflexure exist at mid-heights of the columns, as stated in (5). Similarly, the assumptions (3), (4) and (7) do not apply to the design of the external



The general framework

FIGURE 2.

columns.

At this stage, using the basic assumptions, it is possible to isolate each beam in the frame by sectioning through the assumed points of contraflexure in the columns. The resulting "storey subassemblage" is used as the basic structure in the derivation of the general design theory throughout this chapter. The way in which this general theory may be applied to the design of a complete framework will be described in Chapter 6.

The subassemblage for the internal bays of the typical intermediate storey (e.g. storey n in Figure 2) is shown in Figure 3(a) under the action of vertical load alone, and in Figure3(b) under wind load alone.

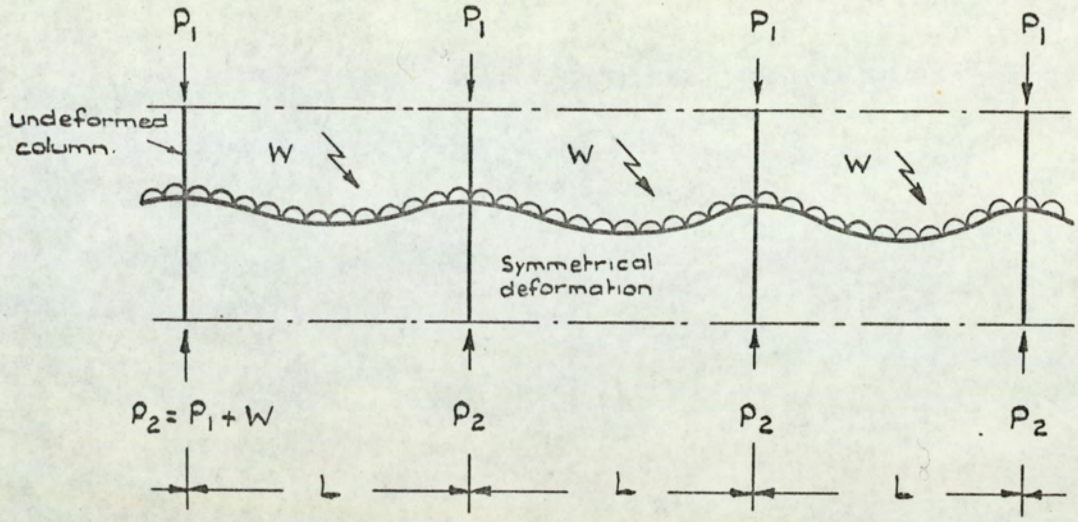
3.4. DERIVATION OF THE SIMPLE PLASTIC DESIGN EQUATIONS

In this section, the simple plastic theory is applied in order to produce a set of equations for the design of the general storey subassemblage. These equations supply the values of plastic moment that are required for the beams and columns of the subassemblage so that it may be capable of satisfying the design criteria given in 3.2. Since there are two types of loading free to act on the framework, it is necessary to derive suitable design equations for each loading case and to select the required sections by using the most critical of these equations. Instability effects are ignored at this stage.

3.4.(a) FAILURE UNDER VERTICAL LOAD

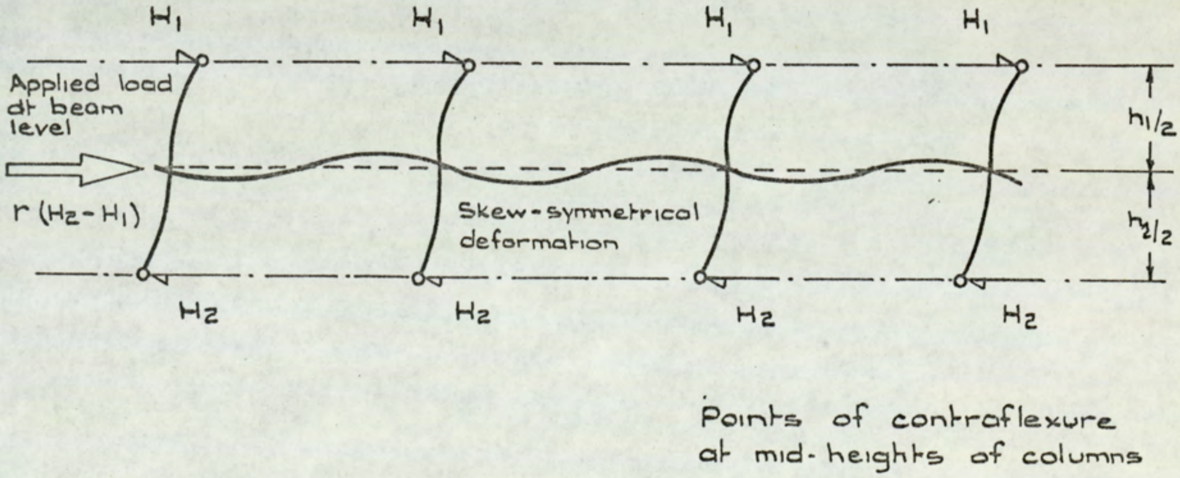
Under purely vertical loading the structure deflects as shown in Figure 3(a). The beam bends symmetrically with zero rotation at the ends, and no bending moments are induced in the columns.

The only possible mode of failure under this type of loading involves the independent collapse of each beam, as shown in Figure 4. B is defined as the fully plastic moment required for this simple



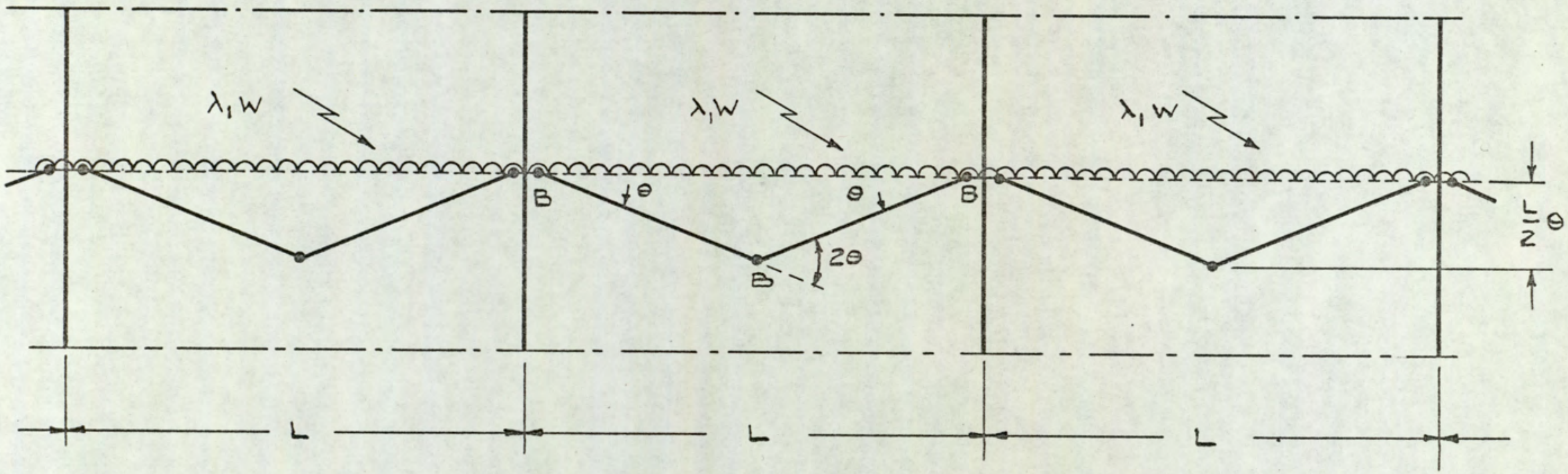
Subassemblage under vertical load alone; $\lambda = 1 \cdot 0$

FIGURE 3 (a)



Subassemblage under wind load alone; $\lambda = 1 \cdot 0$;

FIGURE 3 (b).



Collapse mechanism under vertical load alone : $\lambda = \lambda_1$.

FIGURE 4.

beam mechanism to occur at load factor λ_1, and may be obtained by equating the work done by the loads to the work absorbed by the plastic hinges. Thus, referring to Figure 4, for any beam:-

$$B(2\theta+\theta+\theta) = \frac{\lambda_1 W}{2}\cdot\frac{L}{2}\cdot\theta$$

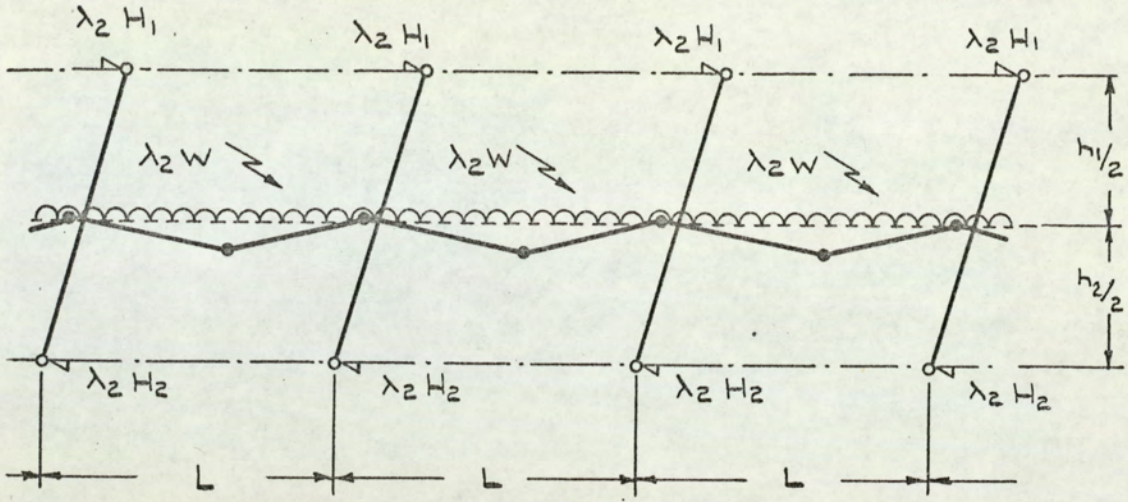
Therefore,

$$B = \lambda_1 \frac{WL}{16} \tag{1}$$

Theoretically, since the internal column carries only axial load, its required value of plastic moment, C_I, may be taken as zero.

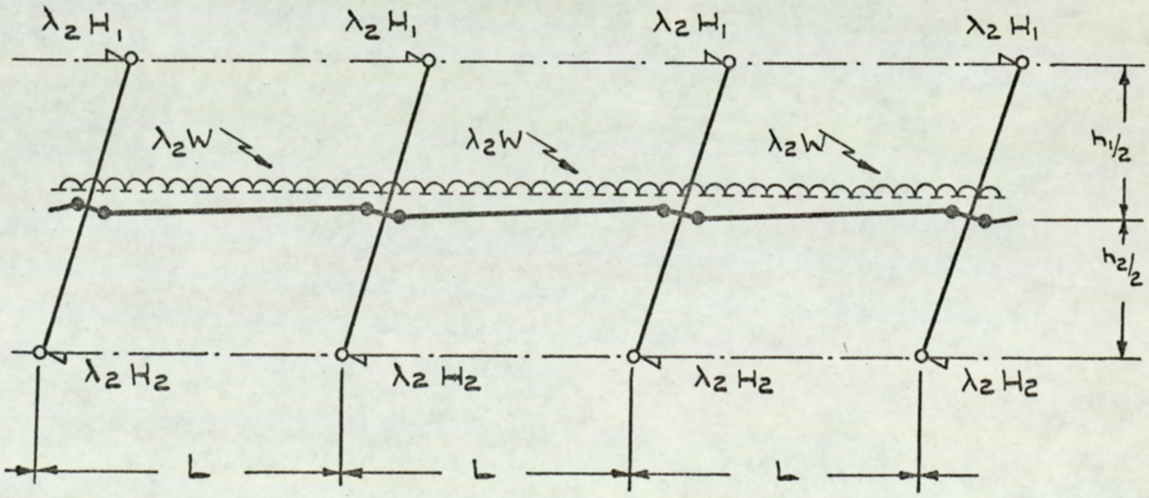
3.4.(b) FAILURE UNDER COMBINED LOAD

Under a combination of vertical and horizontal loading the subassemblage may be reduced to a mechanism in one of two ways, both of which are accompanied by sway deformation. Only two plastic hinges are required in each beam to create these mechanisms. In both cases, a hinge is bound to form at the leeward end of the beam, since at this location the bending moments induced by the vertical and horizontal loadings act in the same sense, and are therefore cumulative. The position of the second plastic hinge that is required to cause failure depends on the relative magnitudes of the moments induced by the two types of loading. In general, this second hinge forms in the span of the beam, producing the "combined mechanism" shown in Figure 5(a). However, in cases of very heavy wind loading, the second hinge may form at the windward end of the beam, producing the "sway mechanism" shown in Figure 5(b). Basically, this sway mechanism is the particular case of the combined mechanism in which the span hinge has "shifted" from the centre of the beam to its windward end. However, in this design method, it is assumed that these are distinct modes of failure, and that in the combined mechanism the span hinge forms exactly in the centre of the beam. The validity of these assumptions will be discussed in



Combined mechanism; $\lambda = \lambda_2$

FIGURE 5 (a).



Sway mechanism ; $\lambda = \lambda_2$

FIGURE 5 (b)

detail later in this chapter.

For both types of mechanism, the required values of fully plastic moment may be obtained by applying the work equation to an isolated bay consisting of one beam and one column, as shown in Figures 6(a) and 6(b). This follows from the initial assumptions concerning the distribution of shear forces in the columns. The work equation for the complete storey assemblage would be an exact multiple of that for each single bay.

Consider first the combined mechanism in Figure 6(a). If the column sways through a small angle θ, each plastic hinge rotates by 2θ, and work is done by both vertical and horizontal loads. The corresponding work equation is:-

$$4B\theta = \frac{\lambda_2 W}{2}.\frac{L}{2}.\theta + \lambda_2 H_1.\frac{h_1}{2}.\theta + \lambda_2 H_2.\frac{h_2}{2}.\theta$$

Therefore,

$$B = \lambda_2 \frac{WL}{16} + \lambda_2 \frac{1}{4}\left(\frac{H_1 h_1 + H_2 h_2}{2}\right)$$

or,

$$B = \lambda_2 \frac{WL}{16} + \lambda_2 \frac{(Hh)av}{4}$$

where,

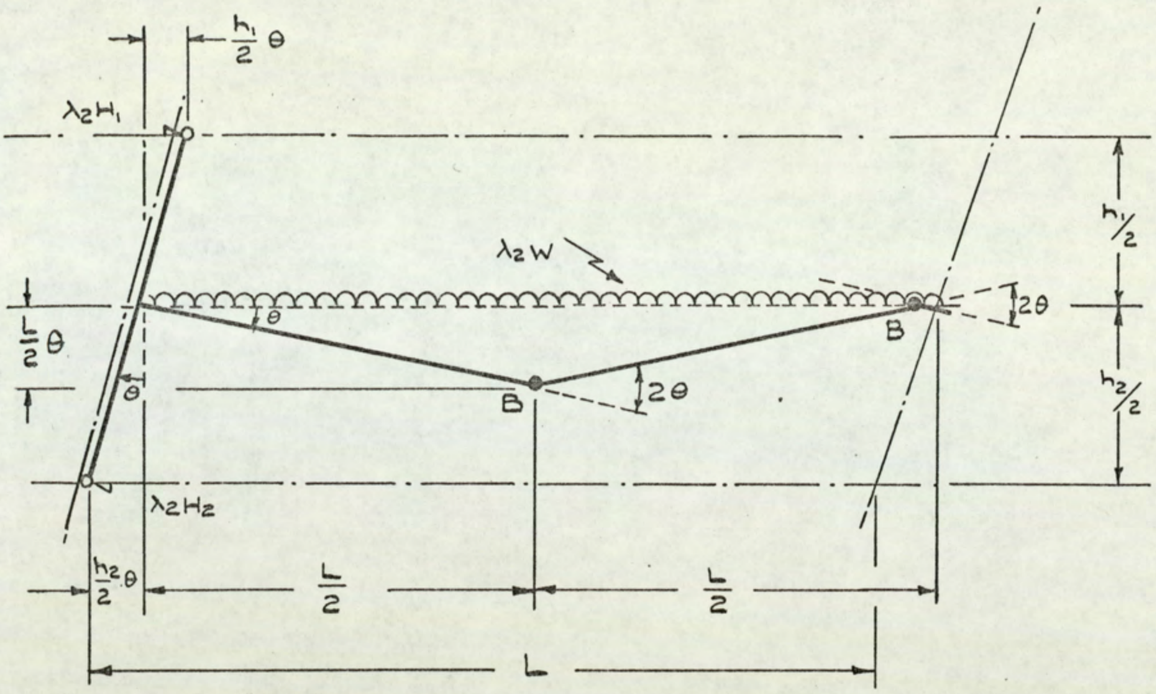
$$(Hh)av = \left(\frac{H_1 h_1 + H_2 h_2}{2}\right)$$

In an alternative form,

$$B = \lambda_2 \frac{WL}{4} \left[\frac{1}{4} + \frac{(Hh)av}{WL}\right] \qquad (2)$$

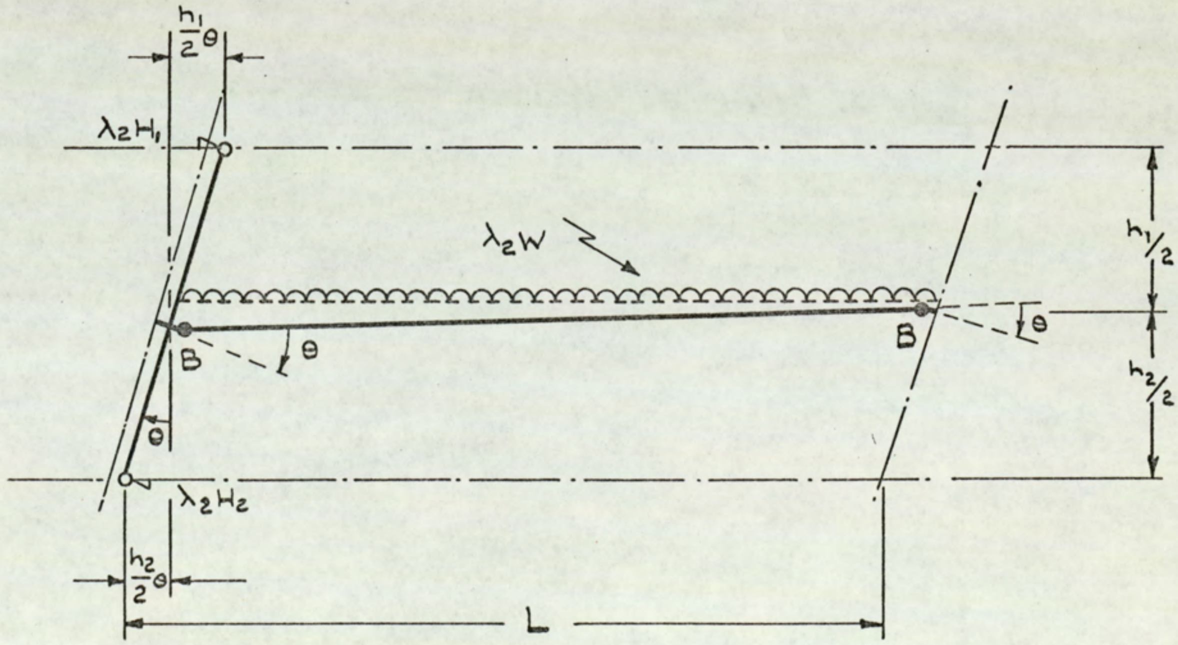
The quantity $\frac{(Hh)av}{WL}$ is hereafter referred to as the "wind ratio", representing as it does the relative intensities of wind loading and vertical loading.

Consider now the sway mechanism of Figure 6(b). In this case, if the column sways through an angle θ, the plastic hinges both rotate by θ, and the vertical loading does no work. The resulting work equation is:-



Combined mechanism - isolated bay

FIGURE 6 (a)



Sway mechanism - isolated bay

FIGURE 6 (b).

$$2B\theta = \lambda_2 H_1 . \frac{h_1}{2} . \theta + \lambda_2 H_2 . \frac{h_2}{2} . \theta$$

Therefore, in this case, the fully plastic moment of the beam is,

$$B = \lambda_2 \frac{(Hh)av}{2} \qquad (3)$$

For either the combined or sway mechanisms the maximum column moments occur at the level of the beam, and are simply equal to the shear in the column multiplied by the distance from the joint to the assumed point of contraflexure. Therefore, for both types of mechanism a plastic hinge will not form in the column if:-

$$C_I > \lambda_2 \frac{Hh}{2} \qquad (4)$$

where C_I represents the reduced plastic moment of the column in the presence of axial load.

3.4.(c) SELECTION OF THE CRITICAL EQUATION:

Equation (1) gives the value of fully plastic moment which is required for the beams of the subassemblage under purely vertical loading. If a section with any smaller value of B is supplied to the structure, collapse will occur before the design load factor, λ_1, is attained. This would result in an immediate violation of the first of the design criteria given in 3.2. The value of B which is given by equation (1) is therefore the minimum allowable fully plastic moment for this loading case.

Equations (2) and (3) give the values of fully plastic moment for the beam which ensures that the two mechanisms which may form under combined loading just occur at load factor λ_2. Therefore, in order to satisfy the second design criterion, it is necessary to supply the subassemblage with a beam that has a value of B equal to, or greater than, the larger of the two values obtained from equations (2) and(3).

It follows, therefore, that the two basic design criteria, corresponding to the different loading conditions, may only be mutually fulfilled by selecting a beam with a plastic moment which is at least as great as the largest of the three values given by equations (1),(2) and (3). It is the wind ratio $\left(\text{i.e. } \frac{(Hh)av}{WL}\right)$ that determines which of these equations leads to the correct beam size.

The basic equations may be rewritten in a more convenient form for comparison purposes. Since $\lambda_1 = \frac{5}{4}.\lambda_2$, equation (1) becomes:-

$$B = \frac{5}{4}.\lambda_2.\frac{WL}{16}$$

or, alternatively,

$$B = \lambda_2 \frac{WL}{4}.\frac{5}{16}$$

Equation (2) is left in its original form; i.e.,

$$B = \lambda_2 \frac{WL}{4} \left[\frac{1}{4} + \frac{(Hh)av}{WL}\right]$$

Equation (3) is rewritten as:-

$$B = \lambda_2 \frac{WL}{4}.2 \frac{(Hh)av}{WL}$$

These three equations may be non-dimensionalised by dividing by the common factor, $\lambda_2 \frac{WL}{4}$. The following notation is introduced for simplicity:-

$$f(B) = \frac{B}{\lambda_2 \frac{WL}{4}}$$

and,

$$wr = \frac{(Hh)av}{WL}$$

The basic equations therefore become:-

$$f(B) = \frac{5}{16} \qquad (1')$$

corresponding to the simple beam mechanism;

$$f(B) = \frac{1}{4} + wr \qquad (2')$$

corresponding to the combined mechanism;

$$f(B) = 2wr \qquad (3')$$

corresponding to the sway mechanism.

These three linear relationships are shown together in Figure 7, from which it can be seen that the envelope of the three lines gives the critical value of f(B) for any wind ratio. Initially, for low values of the wind ratio, this envelope is represented by equation (1'). Therefore, for a storey under light wind loading, equation (1) gives the required fully plastic moment for the beam. As the wind ratio increases, equation (2) becomes dominant, and eventually, for conditions of very heavy wind loading, the critical value is to obtained using equation (3).

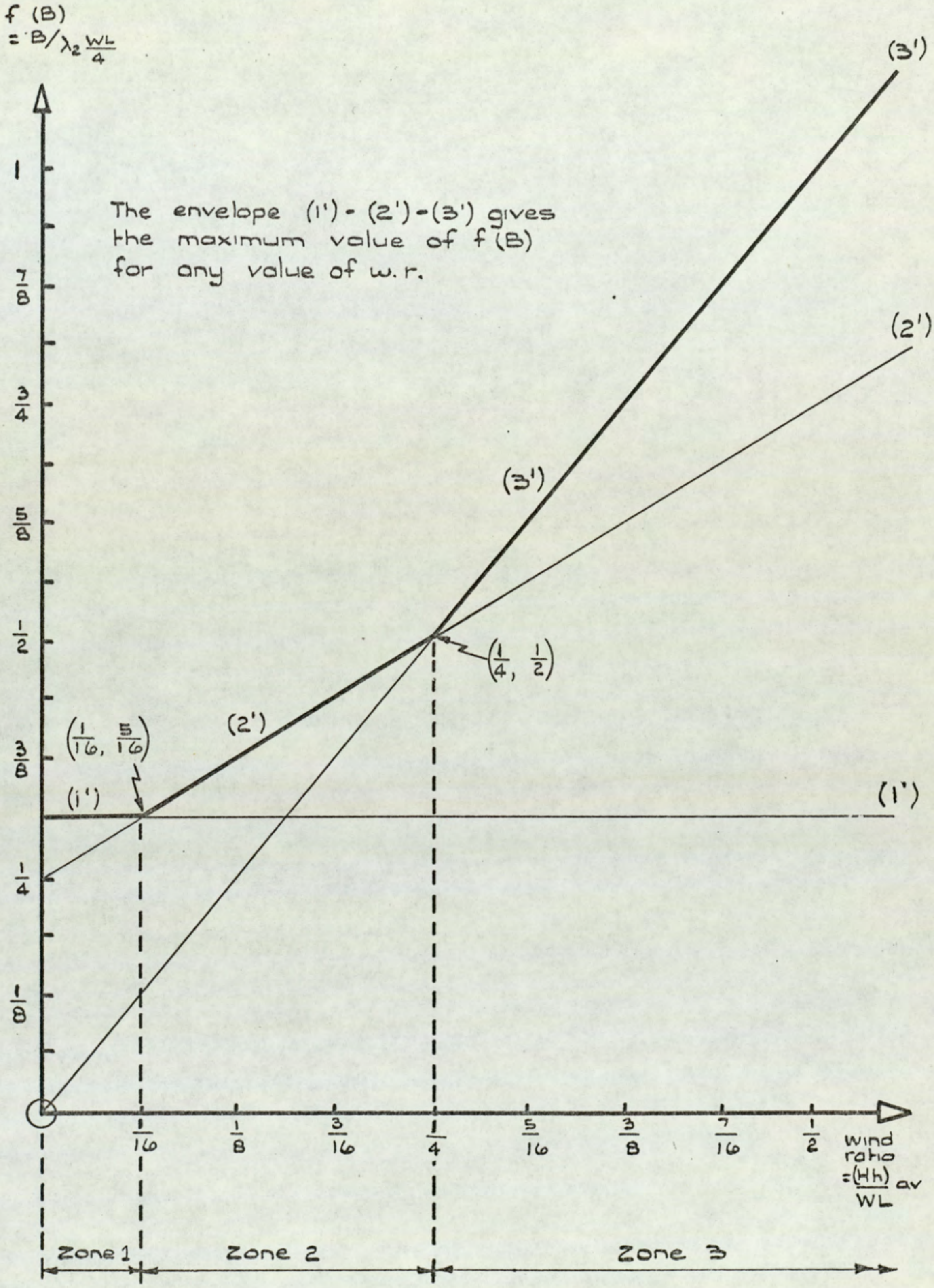
Figure 7 also shows that the range of application of each equation is bounded by distinct values of the wind ratio, as follows:-

$$0 \leqslant \frac{(Hh)av}{WL} \leqslant \frac{1}{16}$$

In this range, the first design criterion is critical, and equation (1) must be used to obtain B. The subassemblage fails by a simple beam mechanism under vertical load alone at λ_1, but has a reserve of strength under combined load, collapsing in this case at a load factor greater than λ_2. Storeys which have a wind ratio falling within the above limits will subsequently be described as being in Zone 1.

$$\frac{1}{16} \leqslant \frac{(Hh)av}{WL} \leqslant \frac{1}{4}$$

In this case, the second design criterion is critical, and equation (2) gives the required value of B. That is to say, under a combination of vertical and horizontal loading the subassemblage fails by the



f (B)
= B/λ₂ WL/4
1
7/8
3/4
5/8
1/2
3/8
1/4
1/8
The envelope (1')-(2')-(3') gives
the maximum value of f (B)
for any value of w.r.
(3')
(2')
(1')
(1/16, 5/16)
(1/4, 1/2)
1/16
1/8
3/16
1/4
5/16
3/8
7/16
1/2
wind ratio =(Hh)av/WL
Zone 1
Zone 2
Zone 3

FIGURE 7

combined mechanism at λ_2, the load factor against a sway mechanism being greater than λ_2. In addition, under vertical load alone, the selected beam is stronger than required, and a beam mechanism does not form until a load factor greater than λ_1. This is referred to as Zone 2.

$$\frac{1}{4} \leqslant \frac{(Hh)av}{WL}$$

For heavy wind loading, the second design criterion is again critical, but B must be obtained by using equation (3). This implies that under combined loading, failure occurs due to the sway mechanism at load factor λ_2, the beam having an additional safety factor against the combined mechanism. Under vertical load alone at λ_1 there is a large reserve of strength against a beam mechanism. This is referred to as Zone 3.

In addition, for designs in all three zones, the column size is selected using equation (4). This equation, which is based on the shear force in the column, is independent of the wind ratio. Under vertical load alone, the column carries only axial forces, and so equation (4) always dictates the required plastic moment, and automatically satisfies the final design criterion given in 3.2.

The third design criterion, which requires that the frame shall be fully elastic at working load, is discussed later in this chapter.

The simple plastic design equations are summarized below, and in the following sub-section their validity is examined.

SUMMARY

ZONE 1

$$0 \leqslant \frac{(Hh)av}{WL} \leqslant \frac{1}{16}$$

$$B = \lambda_1 \frac{WL}{16} \qquad (1)$$

ZONE 2

$$\frac{1}{16} \leqslant \frac{(Hh)av}{WL} \leqslant \frac{1}{4}$$

$$B = \lambda_2 \frac{WL}{4} \left[\frac{1}{4} + \frac{(Hh)av}{WL}\right] \qquad (2)$$

ZONE 3

$$\frac{1}{4} \leqslant \frac{(Hh)av}{WL}$$

$$B = \lambda_2 \frac{(Hh)av}{2} \qquad (3)$$

In all zones,

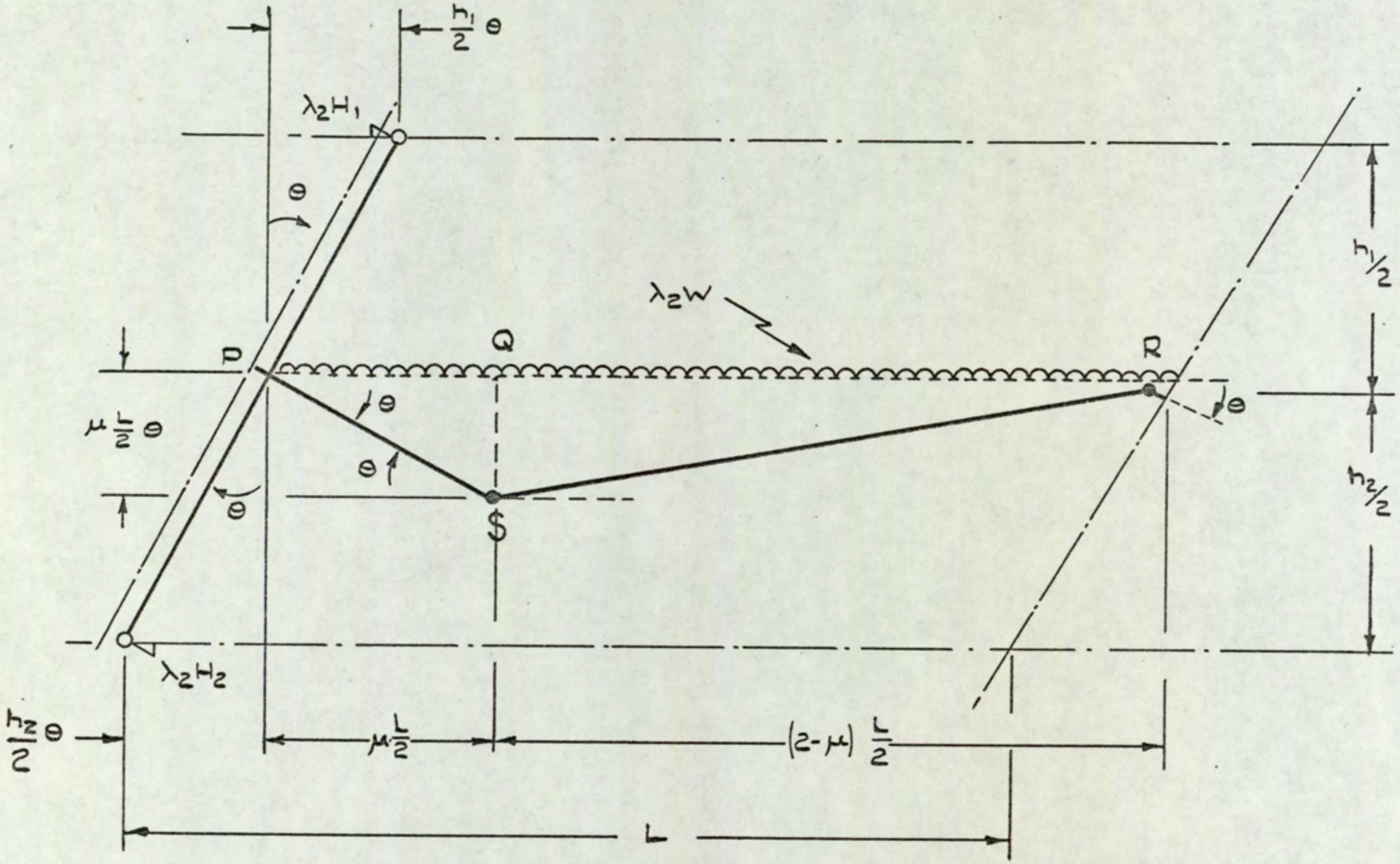
$$C_I > \lambda_2 \frac{Hh}{2} \qquad (4)$$

3.4.(d) CRITICISM OF THE EQUATIONS

In 3.4.(b), it was assumed that in the combined mechanism, shown in Figure 6(a), the span hinge always forms at the centre of the beam, irrespective of the intensity of wind loading. This, as suggested before, is not strictly true. In theory, the span hinge only forms in this position if the wind ratio is zero. As the wind ratio increases, the hinge displaces towards the windward end of the beam, always forming at the position of maximum bending moment in the span. At some value of the wind ratio, this hinge does in fact form at the end of the beam, producing the sway mechanism of Figure 6(b), which continues to occur for all higher intensities of wind loading.

Equation (2) is therefore an approximate expression for obtaining the required fully plastic moment of the beam for the combined mechanism. In order to assess the validity of this equation, and to estimate the errors involved in using all three equations within the ranges of wind ratio stated, it is necessary to consider the true behaviour of the subassemblage under combined load. The accurate combined mechanism, in which the span hinge is not assumed to form at the centre of the beam, is shown in Figure 8.

As before, the column sways through an angle θ. Consider triangles PQS and RQS.



Accurate combined mechanism

FIGURE 8

If θ is small, then

$$QS = PQ.\theta = \mu.\frac{L}{2}.\theta$$

Therefore,

$$Q\hat{R}S = \frac{QS}{QR} = \mu.\frac{L}{2}.\theta.\ \frac{2}{(2-\mu)L}$$

i.e.

$$Q\hat{R}S = \frac{\mu}{(2-\mu)}.\theta$$

Therefore, the rotation of each plastic hinge is

$$\theta + Q\hat{R}S = \left[1 + \frac{\mu}{(2-\mu)}\right]\theta = \frac{2}{(2-\mu)}.\theta$$

Referring to Figure 8, the work equation is,

$$\frac{4}{(2-\mu)}.B\theta = \frac{\lambda_2 W}{2}.\frac{\mu L}{2}.\theta + \lambda_2\ \frac{H_1 h_1}{2}.\theta + \lambda_2\ \frac{H_2 h_2}{2}.\theta$$

Therefore,

$$B = \frac{(2-\mu)}{4}\left[\mu.\lambda_2\ \frac{WL}{4} + \lambda_2 (Hh)av\right]$$

or,

$$B = \lambda_2\ \frac{WL}{4}.(2-\mu)\left[\mu.\frac{1}{4} + \frac{(Hh)av}{WL}\right] \qquad (5)$$

This may be rewritten in the non-dimensionalised form of equations (1′),(2′) and (3′); i.e.,

$$f(B) = (2-\mu)\left[\mu.\frac{1}{4} + wr\right]$$

For any particular value of the wind ratio, an infinite number of solutions of this equation are available, each depending on a different value of μ. These solutions represent separate combined mechanisms, the position of the span hinge being different in each case. In accordance with the kinematical theorem, the true solution is obtained by using the particular value of μ which gives the maximum value of B.

Expanding the above equation:-

$$f(B) = -\frac{\mu^2}{4} + \mu\left(\frac{1}{2} - wr\right) + 2wr$$

For a given value of wr, f(B) is a maximum when $\frac{df(B)}{d\mu} = 0$.

But,

$$\frac{df(B)}{d\mu} = -\frac{\mu}{2} + \frac{1}{2} - wr$$

Therefore, B is a maximum when,

$$\mu = 1 - 2wr$$

Substituting for μ,

$$f(B) = [2-(1-wr)]\left[\frac{(1-2wr)}{4} + wr\right]$$

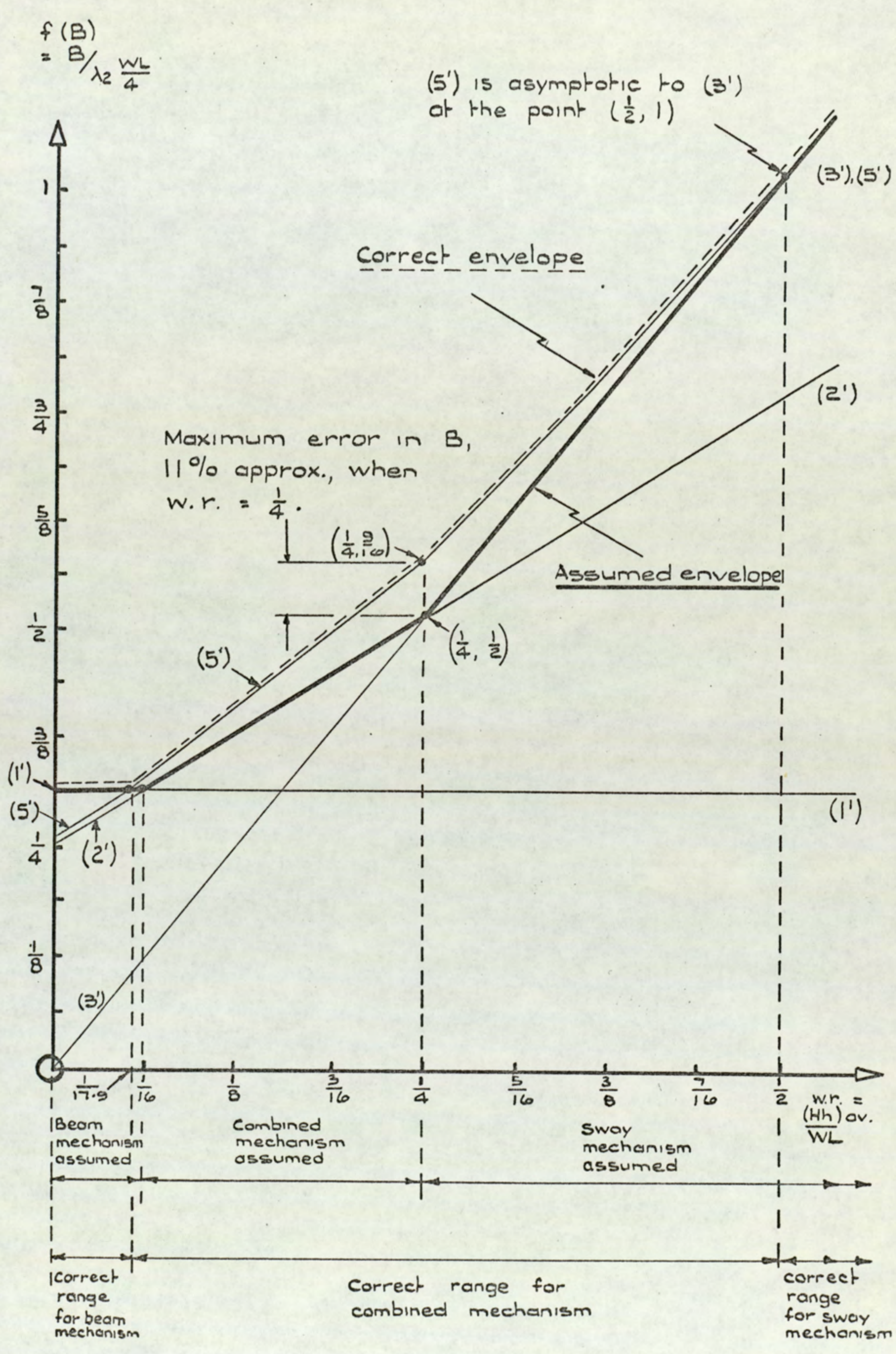
This reduces to:-

$$f(B) = \frac{1}{4} + wr + wr^2 \qquad (5')$$

This equation enables the true value of B to be calculated for any wind ratio leading to the formation of a combined mechanism. Equation (2′), the previously derived approximate expression, is reproduced below for comparison:-

$$f(B) = \frac{1}{4} + wr \qquad (2')$$

It may be seen that the approximate value of f(B), obtained from (2′), is always less than the exact value, given by (5′), by an amount equal to the square of the wind ratio. Figure 9 indicates the order of magnitude of this discrepancy. In the figure, equation (5′) is superimposed on the assumed design envelope, originally given in Figure 7. The following observations may be made concerning the degree of error involved in using the basic design equations:-



f (B)
= B/λ2 WL/4
(5') is asymptotic to (3')
at the point (1/2, 1)
(3'),(5')
Correct envelope
(2')
Maximum error in B,
11% approx., when
w. r. = 1/4.
(1/4, 9/16)
Assumed envelope
(1/4, 1/2)
(5')
(1')
(5')
(2')
(1')
(3')
1/17·9
1/16
1/8
3/16
1/4
5/16
3/8
7/16
1/2
w.r. = (Hh) av./WL
1
7/8
3/4
5/8
1/2
3/8
1/4
1/8
Beam mechanism assumed
Combined mechanism assumed
Sway mechanism assumed
Correct range for beam mechanism
Correct range for combined mechanism
Correct range for sway mechanism

FIGURE 9.

(i) The range of application given for Zone 1, $0 \leqslant wr \leqslant \frac{1}{16}$, is slightly excessive. The beam mechanism will in fact be the critical mode of failure within the limits $0 \leqslant wr \leqslant \frac{1}{17.9}$. However, the error involved in applying equation (1) within the limits for Zone 1 is very small (less than 1% of B).

(ii) Within the range $\frac{1}{17.9} \leqslant wr \leqslant \frac{1}{2}$, theoretically the combined mechanism is the mode of failure, and equation (5′) gives the exact required value for B. However, the error introduced by using the approximate design equations (2) and (3) is not large.

For $\frac{1}{16} \leqslant wr \leqslant \frac{1}{4}$, which is specified as Zone 2, the correct mechanism is predicted, and the value of B obtained using equation (2) is between 1% and 11% below the correct value. The maximum error occurs when $wr = \frac{1}{4}$.

In the range $\frac{1}{4} \leqslant wr \leqslant \frac{1}{2}$, which is specified as part of Zone 3, the predicted sway mechanism does not occur, but nevertheless, as for equation (2), the application of equation (3) does not underestimate B by more than 11%. This error reduces as the wind ratio increases, and it is eliminated when $wr = \frac{1}{2}$.

(iii) For heavy wind loading, when $wr \geqslant \frac{1}{2}$, equation (3) gives the correct value of B, and Zone 3 accurately describes the failure mechanism.

Therefore, it may be seen that the basic design equations, (1), (2) and (3), applied within the ranges of wind ratio specified for the three zones, predict values of fully plastic moment which are close to those theoretically required. The aim of this design method is eventually to produce adequate sections to satisfy the original design criteria. It is believed that the basic equations are sufficiently accurate for that aim to be fulfilled, any discrepancies being counteracted later by additional conservative elements in the final design procedure. In Chapter 7, which is devoted to the analysis of a series of design examples, it is shown that this assumption is justified.

3.5. MODIFICATIONS TO THE SIMPLE PLASTIC DESIGN PROCEDURE

The simple plastic design equations for a typical intermediate storey are summarized in 3.4.(c). These are adequate for the design of a framework in which the instability effects are small. However, in general, a multi-storey unbraced frame is subject to considerable sway deflection, and the presence of high axial forces, particularly in the comparatively slender upper columns, also reduces the stiffness of the structure. The basic equations are liable to underestimate the sections required for such a framework, and they must therefore be modified to allow for instability.

Consider first the general column shown in Figures 10(a) and 10(b). This is in equilibrium under the action of moments M_1 and M_2, vertical force P, and shear force H. If sidesway is ignored, as in Figure 10(a), the equilibrium equation is as follows:-

$$M_1 + M_2 + Hh = 0$$

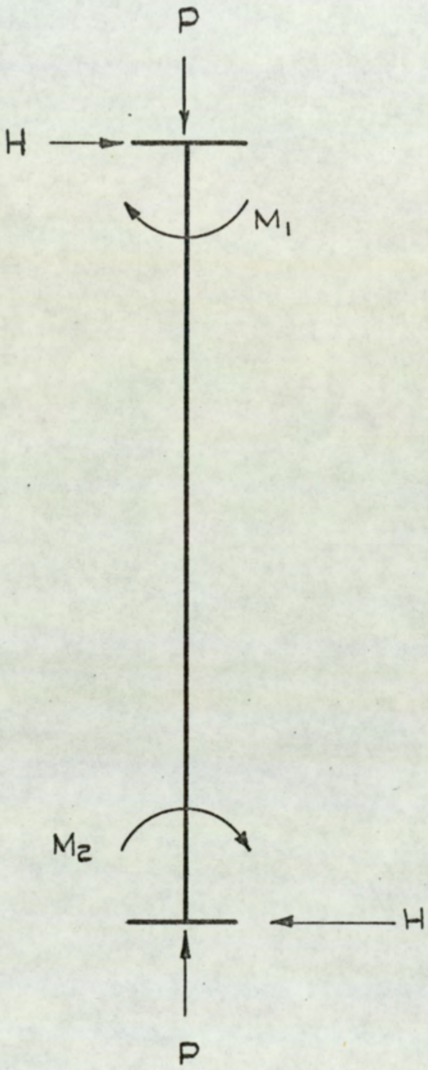
However, in practice, sidesway always occurs as shown in Figure 10(b). If the horizontal displacement, without joint rotation, is denoted by δ, then the equilibrium equation for this case is:-

$$M_1 + M_2 + Hh + P\delta = 0$$

Merchant[42] has shown that this additional moment, which occurs due to the eccentricity of the axial force in the column, may be allowed for by rewriting the equilibrium equation in the following way:-

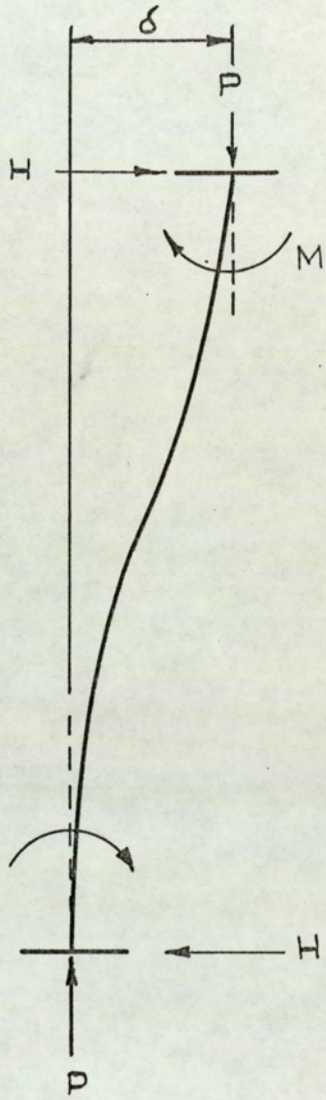
$$M_1 + M_2 + mHh = 0$$

The variable, m is a function of the ratio of axial load in the column to its Euler load $\left(\rho = \frac{P}{P_e}\right)$. Livesley and Chandler[5] have tabulated the complete range of values of m.



No sway.

FIGURE 10 (a)



Sway, without joint rotation.

FIGURE 10 (b)

In addition to the "Pδ effect", a variety of other factors require consideration when modifying the simple plastic design approach. These are:-

(i) The axial forces cause a reduction in column stiffness.

(ii) Additional sidesway deflections, accompanied by additional bending moments, are introduced by joint rotations.

(iii) Whenever a plastic hinge forms in a beam, its effective stiffness is reduced.

In the following sections it will be shown that the combined effect of these factors is equivalent to multiplying the design wind loading by a quantity A for the beams, and A_c for the columns. A and A_c are subsequently referred to as the magnification factors, their values depending on which zone is under consideration.

The modified design equations, which are assumed to allow automatically for the instability effects, may therefore be summarized as follows:-

<u>ZONE 1</u>

$$0 \leq \frac{A(mHh)av}{WL} \leq \frac{1}{16}$$

$$B = \lambda_1 \frac{WL}{16} \tag{6}$$

<u>ZONE 2</u>

$$\frac{1}{16} \leq \frac{A(mHh)av}{WL} \leq \frac{1}{4}$$

$$B = \lambda_2 \frac{WL}{4}\left[\frac{1}{4} + \frac{A(mHh)av}{WL}\right] \tag{7}$$

<u>ZONE 3</u>

$$\frac{1}{4} \leq \frac{A(mHh)av}{WL}$$

$$B = \lambda_2 \frac{A(mHh)av}{2} \tag{8}$$

In all zones,

$$C_I > \lambda_2 \frac{A_c mHh}{2} \tag{9}$$

Equations (6),(7) and (8) correspond to equations (1),(2) and (3) in 3.4.(c). Equation (9) is equivalent to equation (4), and although it is basically identical for each zone, the expression for A_c will be shown to vary, depending on which zone is under consideration.

3.6. THE LOADING SEQUENCE

In deriving the initial design equations, no mention was made of the order in which the loads are applied to the structure. For combined loading, the equations have been obtained by applying the principle of virtual work to assumed mechanisms, with both the vertical and horizontal loads at the ultimate load factor, λ_2 . The manner in which each load approaches λ_2 is unimportant.

However, although the simple plastic design equations are independent of the loading sequence, the final design equations include the magnification factors, A and A_c, which may only be obtained by observation of the behaviour of the subassemblage under increasing load. The values of A and A_c which are derived in this way vary for different loading cases.

The most likely sequence of loading on a building frame is that the wind loading increases from zero to its maximum intensity irrespective of the vertical loading. A system of proportionate loading, in which the change in horizontal loading is directly proportional to the change in vertical loading, is obviously unrealistic. The vertical dead load of the structure, which is generally the greater part of the total vertical load, remains unaltered throughout the life of the framework.

In addition, proportionate loading seldom produces the most critical set of deformations and moments in the frame, particularly in the case where large axial forces in the columns are instrumental in seriously reducing the overall stiffness. At any value of wind load, the stiffness of a frame analysed under proportionate loading is bound to be greater than its stiffness under this wind load plus the total vertical load, since the axial forces in the columns are less. Accordingly, the sway deformations will be smaller, and it can be seen that at any stage in the proportionate loading analysis, the frame is acted upon by a less critical combination of loads.

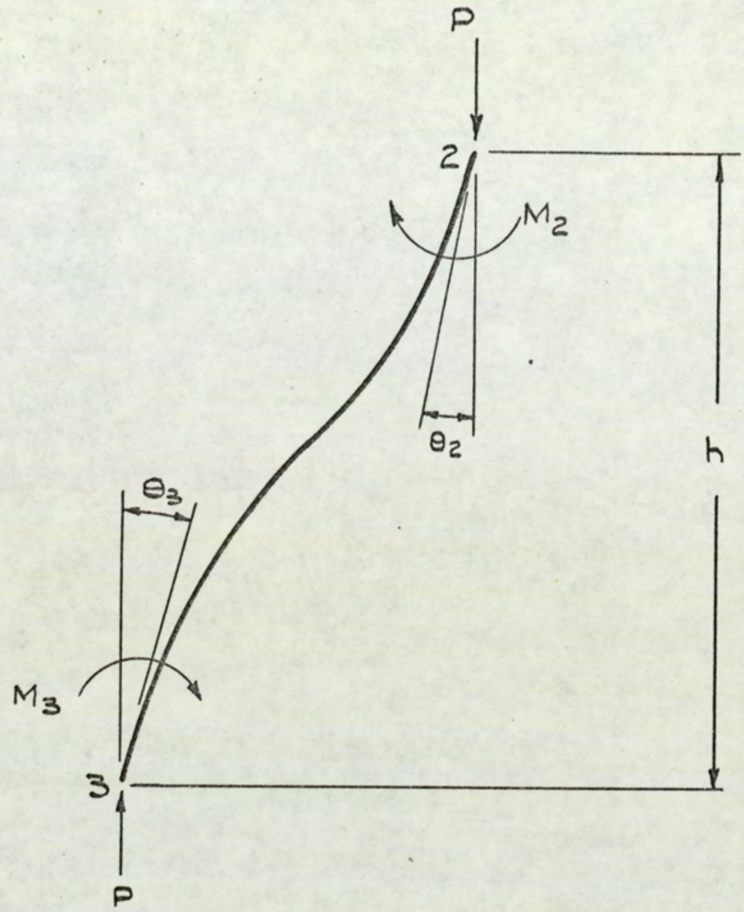
In accordance with the above argument, it is proposed that the magnification factors should be derived by first considering the full vertical loading acting on the frame before any wind load is applied.

3.7. DIRECT MOMENT DISTRIBUTION

The traditional moment distribution procedure uses "stiffnesses" which are related to arbitrarily assumed conditions at the far end of a member. For example, the stiffness at one end of a member with the far end fixed is generally denoted by $\frac{4EI}{L}$. If the far end is pinned, the effective stiffness becomes $\frac{3EI}{L}$. The initial fixed-end moments at the joints are distributed in the ratios of these stiffnesses in such a way that the conditions of joint equilibrium are satisfied. The introduction of the distributed moments induces additional moments at the far ends of the members, and again these moments must be distributed in order to satisfy joint equilibrium. Successive iteration of this procedure eventually leads to a set of bending moments which satisfies both the principles of equilibrium and compatibility.

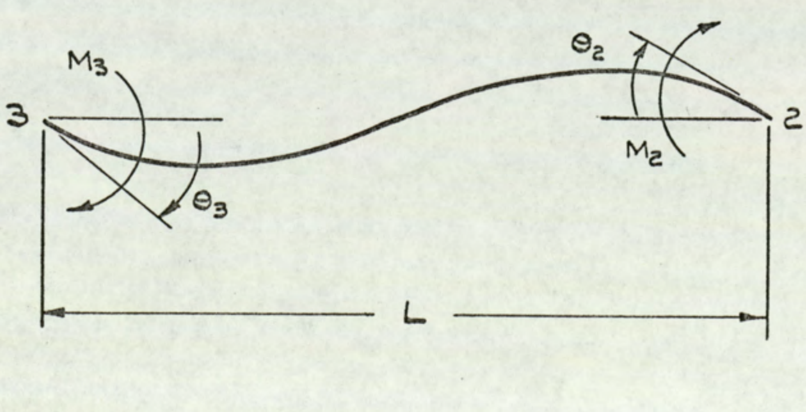
In the method of direct moment distribution, as described by Holmes and Gandhi(54), the stiffnesses are related to the real rotations which occur at the ends of a member when it forms part of a structure. In this case it is only necessary to write down the fixed-end moments and then balance at the joints, using "real distribution factors", which are calculated from "real stiffnesses". No carry over of moment from one end of a member to the other is required, and thus the solution is obtained directly without iteration.

Figure 11(a) shows a column subjected to both axial load and



Typical column member - sway with zero shear.

FIGURE 11 (a)



Typical beam member - no sway and zero axial force.

FIGURE 11 (b)

bending moments <u>without shear</u>. The real stiffnesses, S^r, at the ends of the member for this no-shear case are as follows:-

At end 2,

$$S^r_{23} = \frac{M_2}{\theta_2} = (n-R_3 o)\,\frac{EI}{h} \tag{10}$$

At end 3,

$$S^r_{32} = \frac{M_3}{\theta_3} = (n-{}^{o}/_{R_3})\,\frac{EI}{h} \tag{11}$$

where R_3 is equal to the ratio of the end rotations, $\frac{\theta_3}{\theta_2}$, and n and o are stability functions at Euler ratio ρ, as tabulated by Livesley and Chandler[5].

Figure 11(b) shows a fully-elastic beam bending in double curvature, in which the axial load and the lateral displacement of the ends are assumed to be negligible. For this member the real stiffnesses are:-

$$S^r_{23} = (4+2R_3)\frac{EI}{L} \tag{12}$$

and,

$$S^r_{32} = (4+{}^{2}/_{R_3})\,\frac{EI}{L} \tag{13}$$

The real stiffnesses of all the members of a multi-storey frame may be obtained by substituting suitable values of R in equations (10) to (13).

It has already been assumed in 3.3(b) that the columns bend in double curvature under the action of wind loading, with points of contraflexure existing at their mid-heights. This implies that the end rotations of any column must be equal, so that R = 1. Therefore, the approximate expressions for the no-shear stiffnesses of any column are given by:-

$$S^r_{23} = S^r_{32} = (n-o)\,\frac{EI}{h} \tag{14}$$

It was also stated in 3.3(b) that, for the purpose of developing the general theory for a typical internal bay, the total number of bays, r, is considered to be large. Therefore, since the same sections are used for each internal column, each bay behaves in an identical manner, and every beam will bend in exact double curvature under the action of horizontal loading, as shown previously in Figure 3(b). The end rotations of the beam will be equal, so that R is again equal to unity. Therefore, substituting in equations (12) and (13), the approximate values for the real stiffnesses of the beams are:-

$$S^r_{23} = S^r_{32} = 6\,\frac{EI}{L} \tag{15}$$

The use of these approximate values of real stiffness results in a considerable saving in design time, without introducing any large degree of error. The precise way in which they are applied to develop expressions for the magnification factors, using the direct moment distribution technique, is shown in the following section.

3.8. DERIVATION OF THE MAGNIFICATION FACTORS

The magnification factors, A and A_c, are derived for each zone in turn.

3.8.(a) ZONE 1

$$0 \leqslant \frac{A(mHh)av}{WL} \leqslant \frac{1}{16}$$

In this zone the beam and column sizes are selected using equations (6) and (9) respectively. Although the condition of vertical load alone is critical in selecting the beam size, the combined loading condition dictates the column size. The beam is independent of the magnitude of wind loading and of the magnification factor, which must be obtained therefore by considering the combined loading case. This is identical to the treatment for Zone 2.

3.8.(b) ZONE 2

$$\frac{1}{16} \leq \frac{A(mHh)av}{WL} \leq \frac{1}{4}$$

The loading sequence recommended in 3.6. is adopted in deriving the values of A and A_c. Initially the factored vertical load is applied to the subassemblage, and this is followed by the horizontal loading. In this zone, under the vertical load alone, the beam may remain elastic or may develop plastic hinges, and it is necessary to differentiate between these two types of behaviour.

Consider the beam in Figure 12(a) under the action of vertical load at load factor λ_2. Due to symmetry, the joints do not rotate, and the bending moment diagram is as shown in Figure 12(b). The required fully plastic moment of a beam falling in this zone is given by equation (7); i.e.

$$B = \lambda_2 \frac{WL}{4} \left[\frac{1}{4} + \frac{A(mHh)av}{WL}\right]$$

Therefore, under this load, plastic hinges will have formed at the ends of the beam if,

$$\lambda_2 \frac{WL}{12} \geq \lambda_2 \frac{WL}{4} \left[\frac{1}{4} + \frac{A(mHh)av}{WL}\right]$$

which reduces to the condition:-

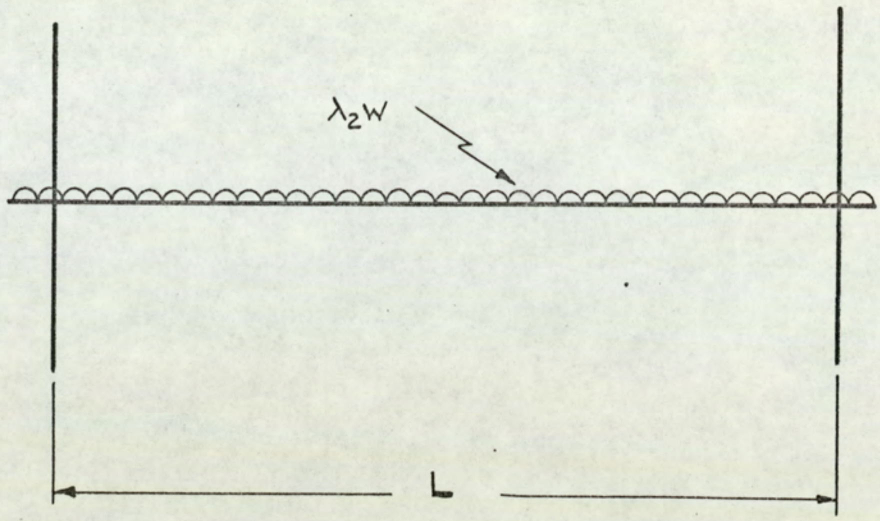
$$\frac{A(mHh)av}{WL} \leq \frac{1}{12}$$

This zone may therefore be sub-divided into two separate zones, which are considered in turn below.

3.8.(b)(i) ZONE 2(i)

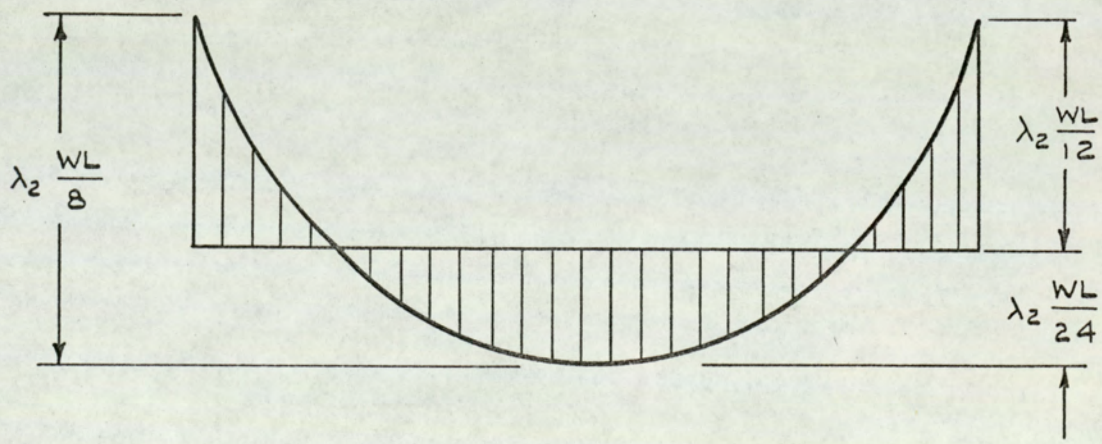
$$\frac{1}{16} \leq \frac{A(mHh)av}{WL} \leq \frac{1}{12}$$

Plastic hinges form at the ends of each beam before the full vertical loading has been applied. At load factor λ_2, the frame is as



Fully - elastic beam - vertical load alone; $\lambda = \lambda_2$

FIGURE 12 (a).



Bending moment diagram.

FIGURE 12 (b)

shown in Figure 13(a), and the corresponding bending moment diagram is given in Figure 13(b).

On application of the wind loading, the plastic hinge at the leeward end of the beam continues to rotate as before. However, the hinge at the windward end tends to rotate in the opposite direction, and therefore immediately disappears. Accordingly, the frame for analysis under horizontal loading is as shown in Figure 14(a). The internal column stiffness in each bay is identical, and the presence of a plastic hinge at the end of each beam effectively isolates one bay from the next, as indicated by Figure 14(b). Assuming the number of bays to be large, this part of the frame satisfies the "principle of multiples", as described by Lightfoot(55), and the method of direct moment distribution may be applied to each single bay. In the typical bay in Figure 15(a), points of contraflexure are assumed to exist at the mid-heights of the columns, and K_{c1}, K_{c2} and K_{b2} represent the flexural rigidities of the members.

The real stiffnesses of the columns at joint 2 may be obtained from equation (14). Thus,

$$S_{21}^{r} = (n_1 - o_1)K_{c1}$$

and,

$$S_{23}^{r} = (n_2 - o_2)K_{c2}$$

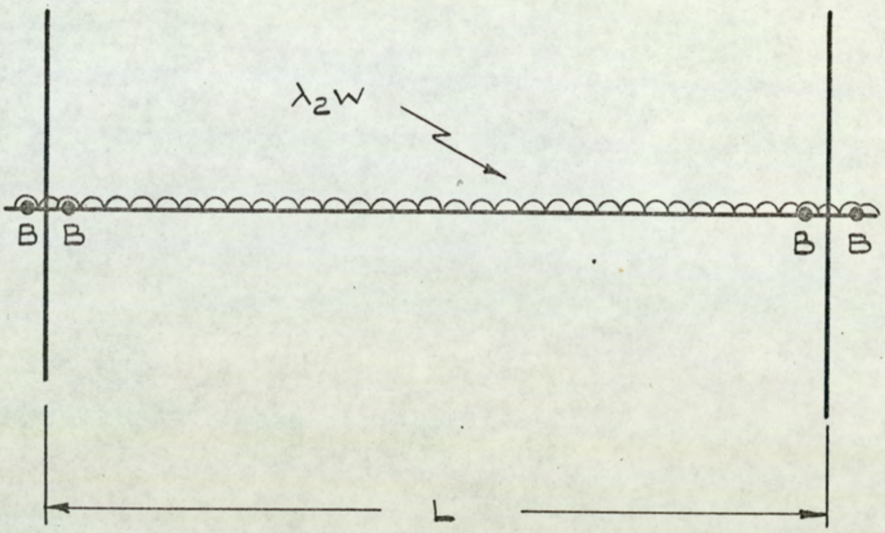
Also, since the plastic hinge at 2′ may be considered to behave in an identical manner to a real hinge, the real stiffness of the beam at joint 2 is given by:-

$$S_{22}^{r}{}' = 3K_{b2}$$

Therefore, the total stiffness at joint 2 is:-

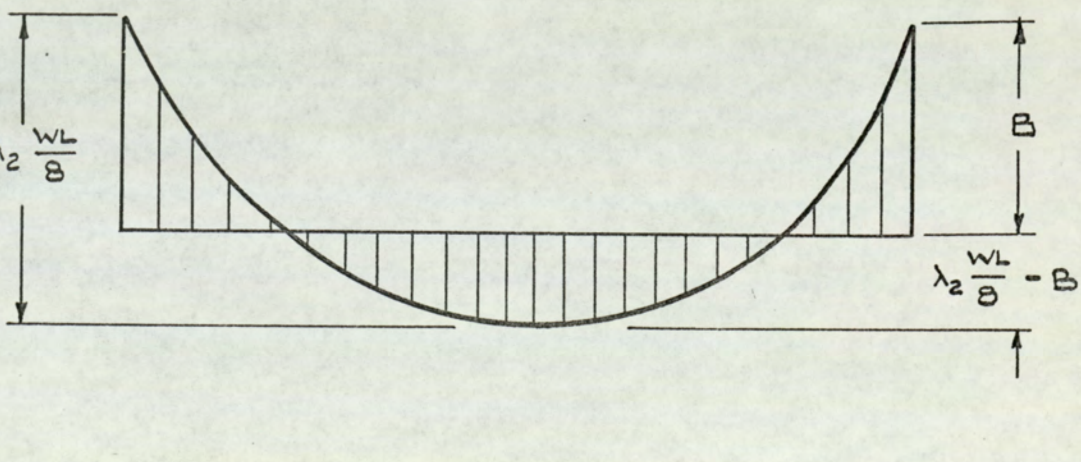
$$\Sigma S_{2}^{r} = (n_1 - o_1)K_{c1} + (n_2 - o_2)K_{c2} + 3K_{b2}$$

Let a^{Δ} represent the real distribution factors at 2 in the presence of



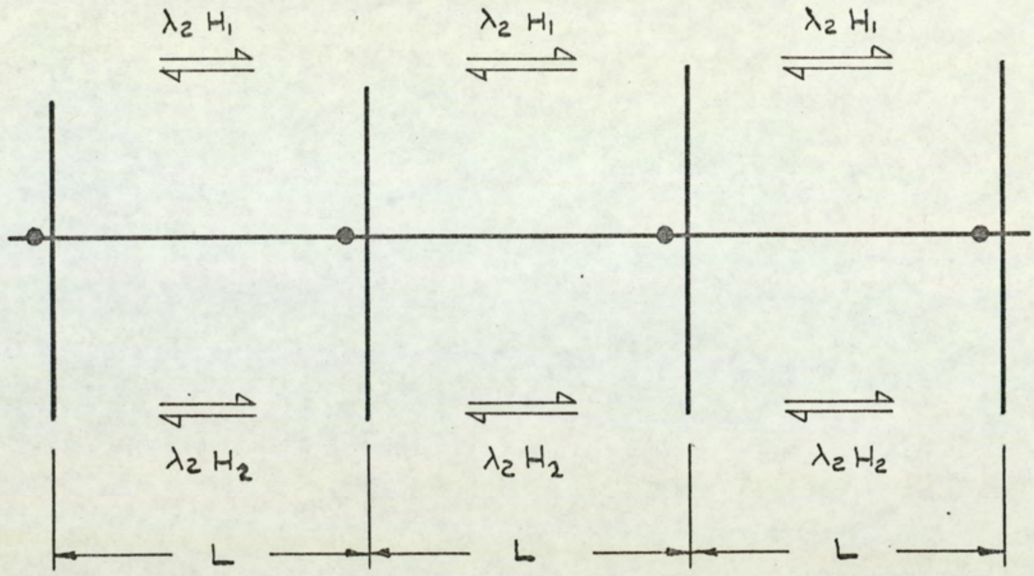
Partially plastic beam - vertical load alone ; $\lambda = \lambda_2$

FIGURE 13 (a)



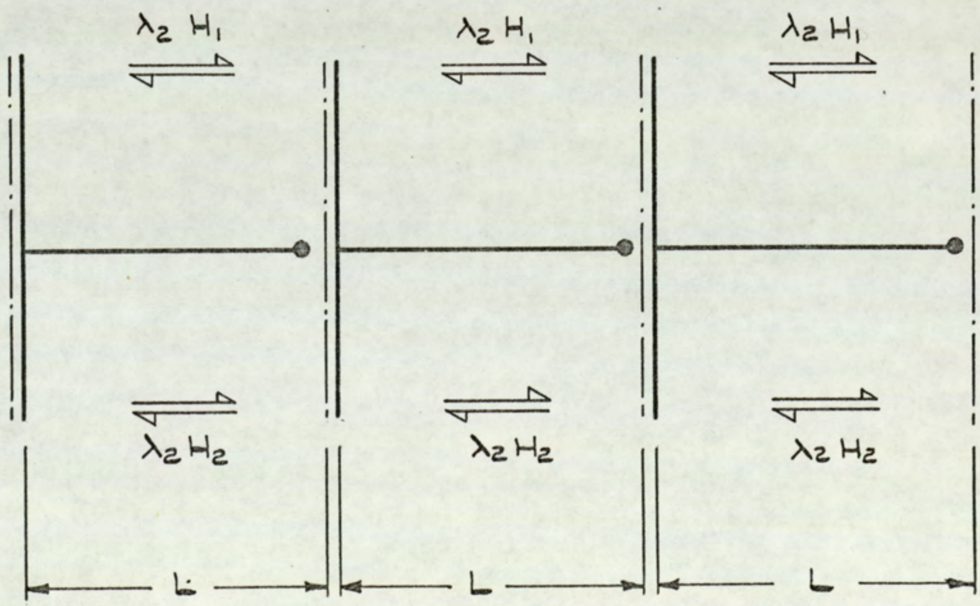
Bending moment diagram

FIGURE 13 (b).



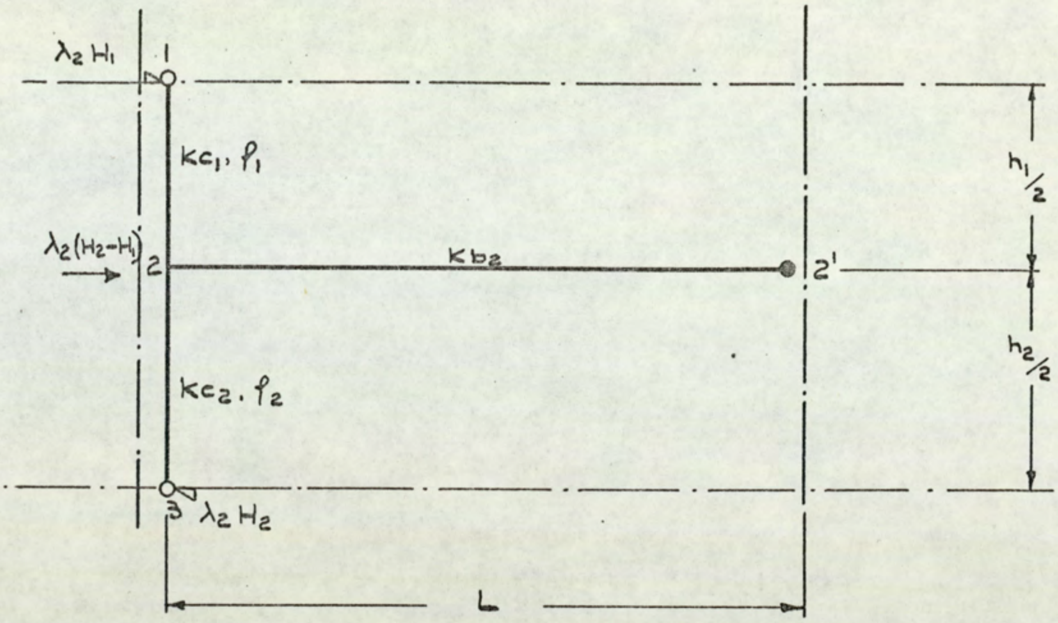
Frame for analysis under wind loading - Zone 2 (1)

FIGURE 14 (a)



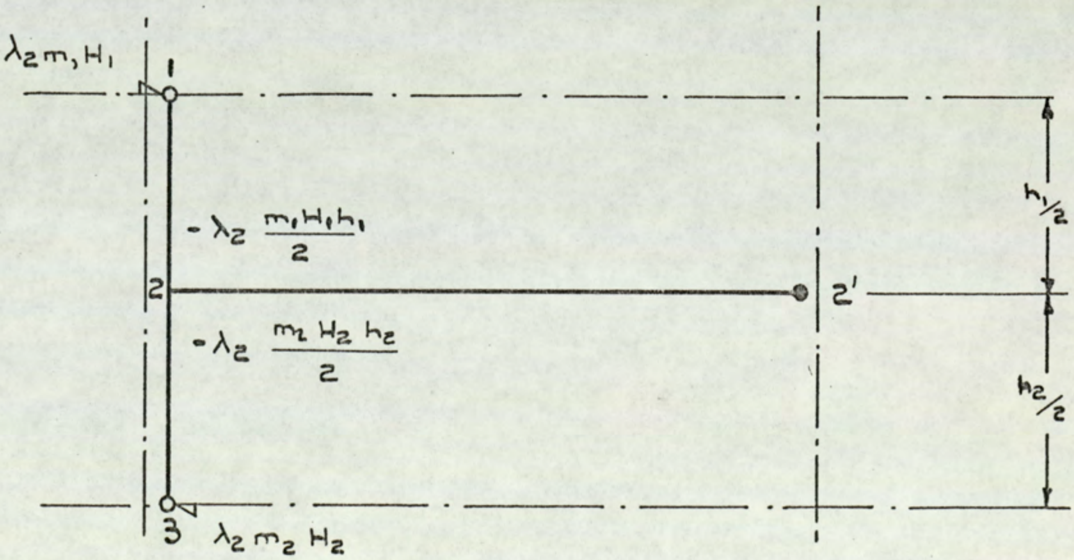
Isolation of each bay

FIGURE 14 (b)



Isolated bay - joint numbers, $\frac{"EI"}{l}$ values and Euler ratios.

FIGURE 15 (a).



Initial sway moments, allowing for the Pδ effect - Zone 2(i)

FIGURE. 15 (b).

a plastic hinge at 2'. Then:-

$$a^{\Delta}_{21} = \frac{S^r_{21}}{\Sigma S^r_2} = \frac{(n_1-o_1)K_{c1}}{\Sigma S^r_2}$$

$$a^{\Delta}_{23} = \frac{S^r_{23}}{\Sigma S^r_2} = \frac{(n_2-o_2)K_{c2}}{\Sigma S^r_2}$$

$$a^{\Delta}_{22'} = \frac{S^r_{22'}}{\Sigma S^r_2} = \frac{3K_{b2}}{\Sigma S^r_2}$$

These may be rewritten in the following form:-

$$a^{\Delta}_{21} = \frac{(n_1-o_1)\overline{K}K_2}{K_2V + 3} \qquad (16)$$

$$a^{\Delta}_{23} = \frac{(n_2-o_2)K_2}{K_2V + 3} \qquad (17)$$

$$a^{\Delta}_{22'} = \frac{3}{K_2V + 3} \qquad (18)$$

where,

$$K_2 = \frac{K_{c2}}{K_{b2}}, \quad \overline{K} = \frac{K_{c1}}{K_{c2}},$$

and,

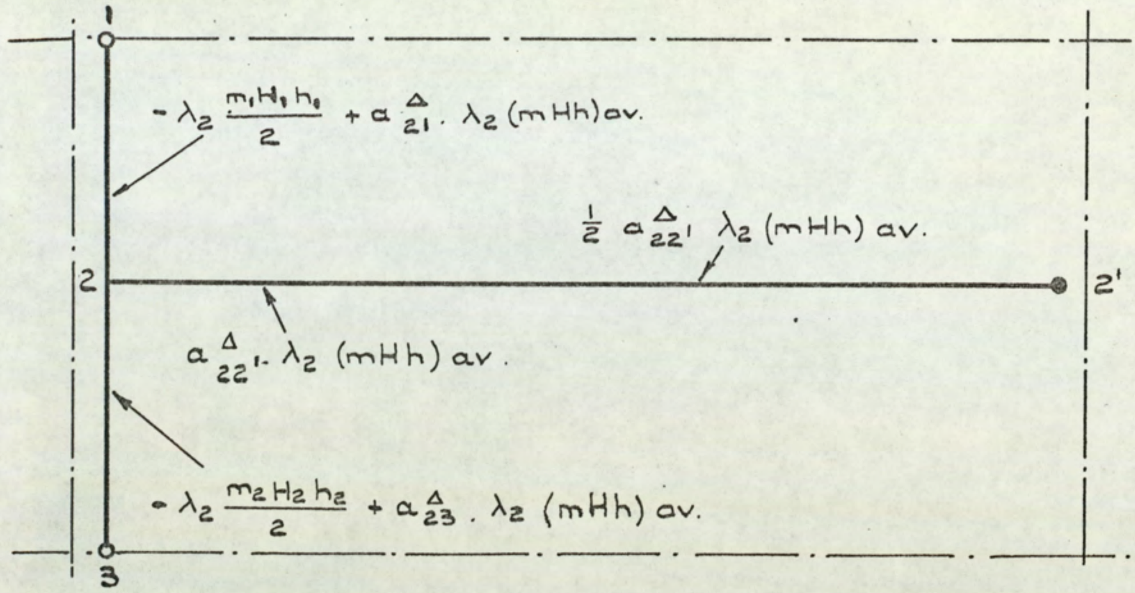
$$V = \overline{K}(n_1-o_1) + (n_2-o_2)$$

The initial sway moments due to wind loading, increased to allow for the $P\delta$ effect, are shown in Figure 15(b). The total out of balance moment at joint 2 is:-

$$F.E.M._2 = -\lambda_2\frac{m_1H_1h_1}{2} - \lambda_2\frac{m_2H_2h_2}{2} = -\lambda_2(mHh)av$$

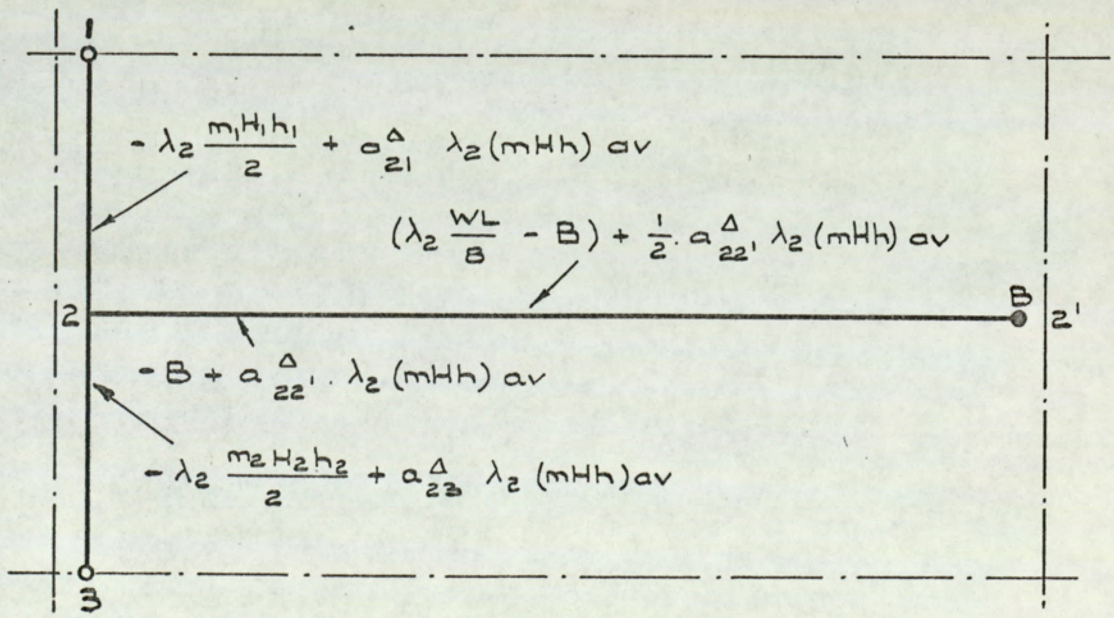
Therefore, the total moment to be distributed at joint 2 in order to satisfy joint equilibrium is $+\lambda_2(mHh)av$. The final moments at the joint due to wind loading alone are shown in Figure 16(a), whilst the moments due to combined vertical and horizontal loading at load factor λ_2 are given in Figure 16(b).

In this zone, failure occurs due to the formation of a combined mechanism, and the span hinge is assumed to form exactly in the centre



Bending moments due to wind load alone - Zone 2(i).

FIGURE 16 (a).



Bending moments at ultimate combined load - Zone 2(i)

FIGURE 16 (b).

of the beam. The bending moment at this location must therefore be equal to the fully plastic moment of the beam. Thus, referring to Figure 16(b):-

$$\left(\lambda_2 \frac{WL}{8} - B\right) + \frac{1}{2}.\frac{a^{\Delta}_{22}{}'}{2}.\lambda_2(mHh)av = B$$

Therefore,

$$B = \lambda_2 \frac{WL}{16} + a^{\Delta}_{22}{}'.\lambda_2 \frac{(mHh)av}{4}$$

$$= \lambda_2 \frac{WL}{4} \left[\frac{1}{4} + a^{\Delta}_{22}{}'. \frac{(mHh)av}{WL}\right]$$

However, in this zone,

$$B = \lambda_2 \frac{WL}{4} \left[\frac{1}{4} + A.\frac{(mHh)av}{WL}\right]$$

Therefore,

$$A = a^{\Delta}_{22}{}'$$

Referring again to Figure 16(b), the moment in the lower column is:-

$$M_{23} = - \lambda_2 \frac{m_2H_2h_2}{2} + a^{\Delta}_{23}.\lambda_2(mHh)av$$

$$= - \lambda_2 \frac{m_2H_2h_2}{2} \left[1-2a^{\Delta}_{23}. \frac{(mHh)av}{m_2H_2h_2}\right]$$

$$= - \lambda_2.(1-2a^{\Delta}_{23}.p_2).\frac{m_2H_2h_2}{2}$$

where,

$$p_2 = \frac{(mHh)av}{m_2H_2h_2}$$

However,

$$C_I > |M_{23}| = \lambda_2.A_c.\frac{m_2H_2h_2}{2}$$

Therefore,

$$A_c = 1 - 2a^{\Delta}_{23}.p_2$$

<u>3.8(b)(ii)ZONE 2(ii)</u>

$$\frac{1}{12} \leq \frac{A(mHh)av}{WL} \leq \frac{1}{4}$$

In this zone, the beams remain completely elastic under the factored vertical load alone, and the bending moment diagram is as shown previously in Figure 12(b). On application of the horizontal loading, the moment at the windward end reduces, and that at the leeward end increases. The beam deflects skew-symmetrically, with the central bending moment remaining unaltered. The real stiffness of each beam connected to a joint is therefore given by equation (15); i.e.

$$S^r_{22}{}' = 6K_{b2}$$

Since there are two beams connected to each joint, the total joint stiffness in this case is given by:-

$$\Sigma S^r_2 = (n_1 - o_1)K_{c1} + (n_2 - o_2)K_{c2} + 12K_{b2}$$

Let a^{α} denote the real distribution factors for the case when the beam is fully elastic. With the same notation as before,

$$a^{\alpha}_{21} = \frac{S^r_{21}}{\Sigma S^r_2} = \frac{(n_1 - o_1)\overline{K}K_2}{K_2V + 12} \qquad (19)$$

$$a^{\alpha}_{23} = \frac{S^r_{23}}{\Sigma S^r_2} = \frac{(n_2 - o_2)K_2}{K_2V + 12} \qquad (20)$$

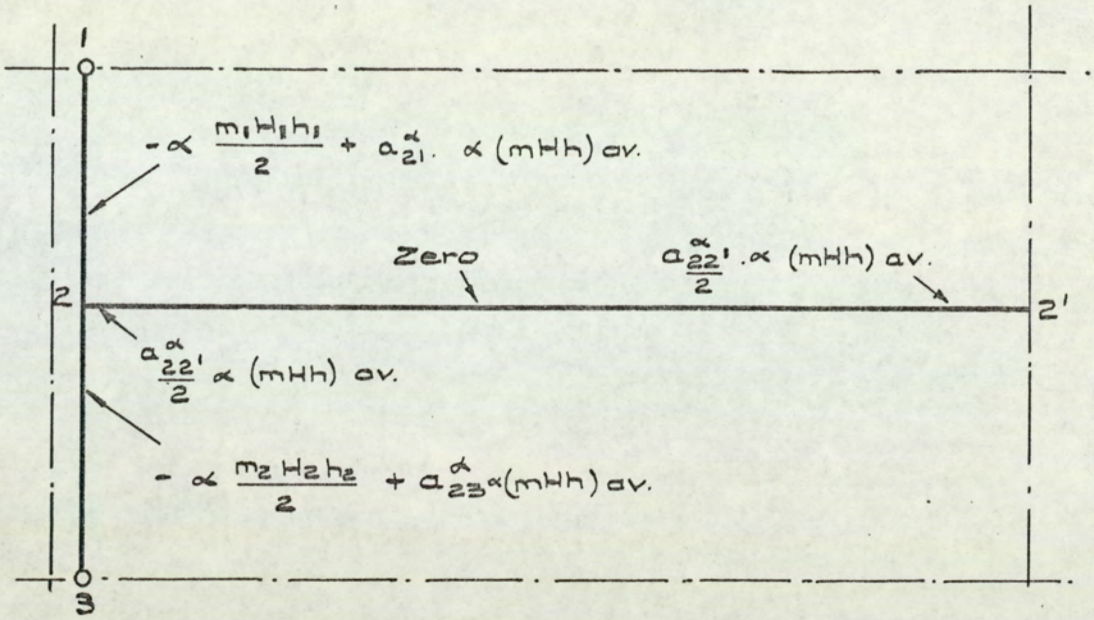
$$a^{\alpha}_{22}{}' = \frac{S^r_{22}{}'}{\Sigma S^r_2} = \frac{12}{K_2V + 12} \qquad (21)$$

It may be seen that the real distribution factor for each beam is equal to $\frac{a^{\alpha}_{22}{}'}{2}$.

As the wind load increases, eventually a plastic hinge forms at the leeward end of the beam, at some load factor, α say, where α is less than λ_2. For this intensity of wind loading, the initial out of balance moment at joint 2 is given by:-

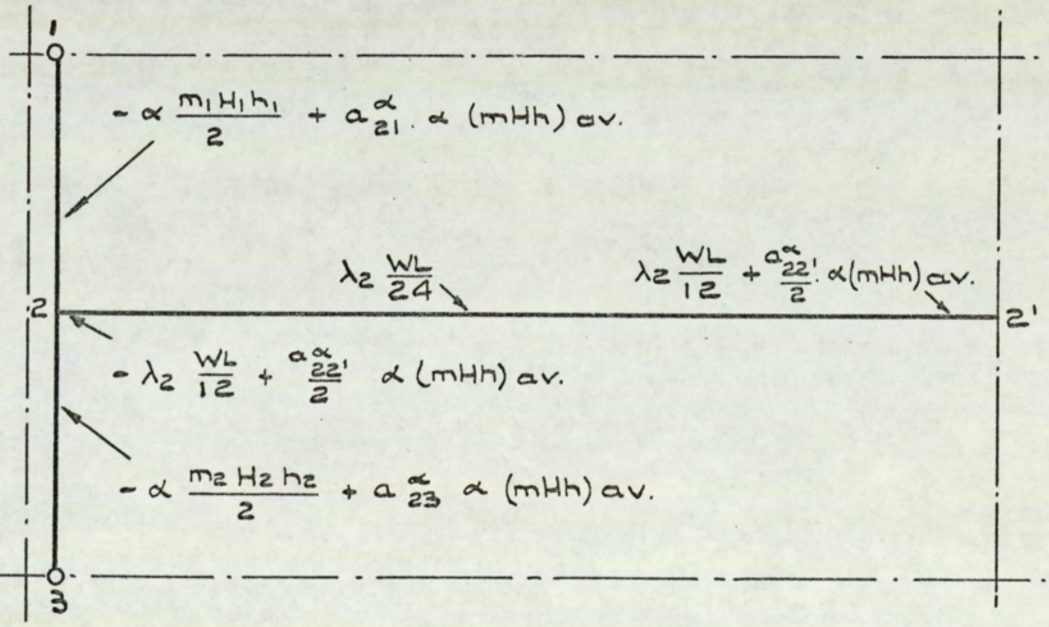
$$F.E.M._2 = -\alpha(mHh)av$$

The bending moments obtained after distribution of the balancing moment are given in Figure 17(a).



Bending moments due to wind load at load factor α - Zone 2(ii)

<u>FIGURE 17 (a).</u>



Bending moments due to vertical load at λ_2 plus wind load at α — Zone 2 (ii)

<u>FIGURE 17 (b).</u>

Figure 17(b) shows the bending moments due to the combination of vertical load at load factor λ_2 plus wind load at load factor α. For a plastic hinge to occur at $2'$ under these loads:-

$$\lambda_2 \frac{WL}{12} + \frac{a_{22}^{\alpha}{}'}{2} . \alpha(mHh)av = B \qquad (22)$$

However, as before, for Zone 2,

$$B = \lambda_2 \frac{WL}{4} \left[\frac{1}{4} + \frac{A(mHh)av}{WL}\right]$$

Therefore, eliminating B in equation (22),

$$\alpha \left[\frac{a_{22}^{\alpha}{}'}{2}(mHh)av\right] = - \lambda_2 \frac{WL}{48} + \lambda_2 \frac{A(mHh)av}{4}$$

or, in a more convenient form, α is given by:-

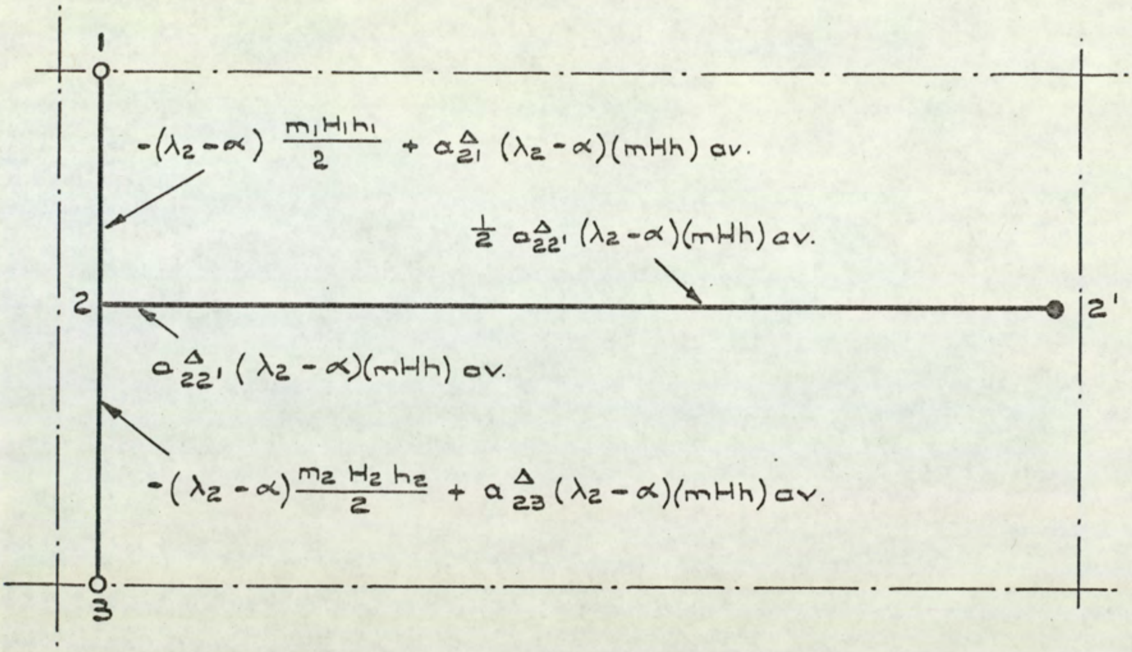
$$\frac{\alpha}{\lambda_2} = \frac{A}{a_{22}^{\alpha}{}'}\left[\frac{1}{2} - \frac{WL}{24A(mHh)av}\right]$$

At load factor α, a plastic hinge forms at $2'$, and the stiffness of the beam at 2 is reduced to $3K_{b2}$. Therefore, for subsequent wind loading, in the range α to λ_2, the real distribution factors are identical to those used in Zone 2(i), namely a_{21}^{Δ}, a_{23}^{Δ} and $a_{22}^{\Delta}{}'$. Due to the additional horizontal loading, the initial sway moment at the joint is given by:-

$$F.E.M._2 = - (\lambda_2 - \alpha)(mHh)av$$

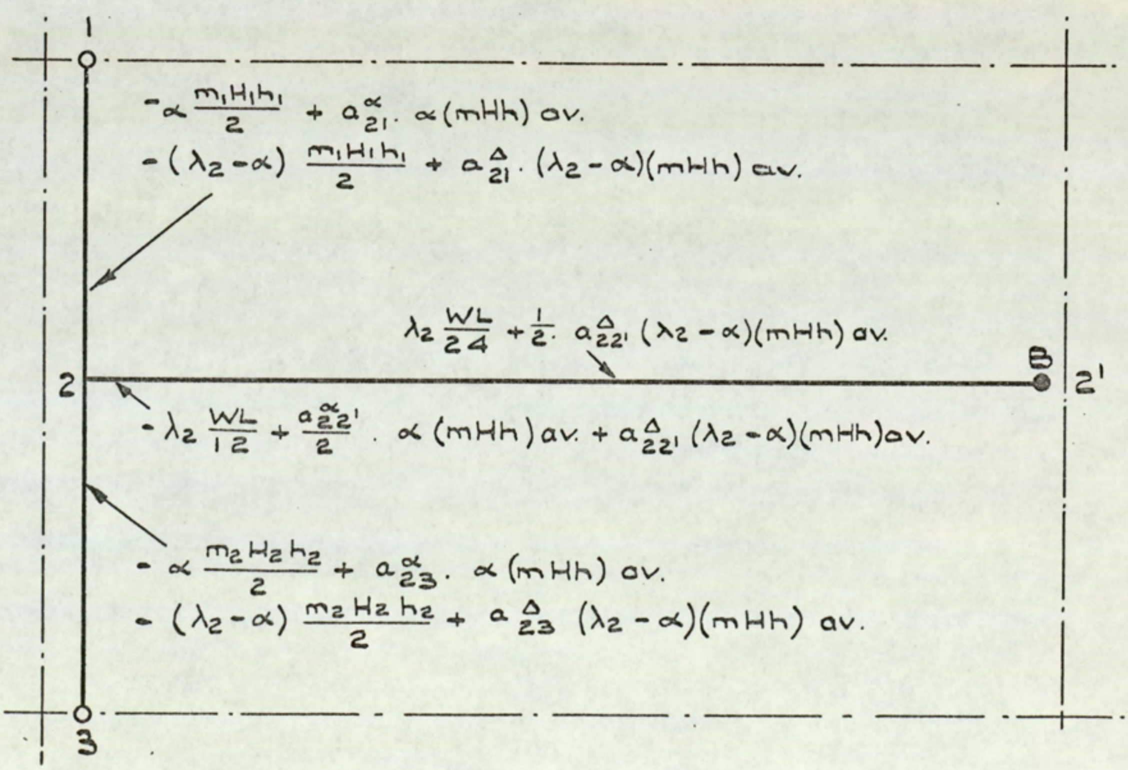
The bending moments produced at the critical locations due to this loading are given in Figure 18(a), which is similar to Figure 16(a), all the moments being factored by the ratio $\frac{(\lambda_2 - \alpha)}{\lambda_2}$.

Figure 18(b) is obtained by superimposing Figures 17(b) and 18(a), and shows the resulting moments under combined load at λ_2. Again, in this zone, failure occurs when a plastic hinge forms at the centre of the



Bending moments due to wind load in the range α to λ_2 — Zone 2 (ii)

FIGURE 18 (a)



Bending moment at ultimate combined load - Zone 2 (ii)

FIGURE 18 (b).

span. Therefore:-

$$\lambda_2 \frac{WL}{24} + \frac{1}{2}.a_{22}'(\lambda_2-\alpha)(mHh)av = B$$

This may be rewritten as:-

$$B = \lambda_2 \frac{WL}{8} + \frac{a_{22}^{\Delta}{}'}{2}(\lambda_2-\alpha)(mHh)av - \lambda_2 \frac{WL}{12}$$

But, from equation (22),

$$\lambda_2 \frac{WL}{12} = B - \frac{a_{22}^{\alpha}{}'}{2}.\alpha(mHh)av$$

Eliminating $\lambda_2 \frac{WL}{12}$ leads to:-

$$B = \lambda_2 \frac{WL}{8} + \frac{1}{2}\left[a_{22}^{\alpha}{}'.\alpha + a_{22}^{\Delta}{}'.(\lambda_2-\alpha)\right](mHh)av - B$$

Therefore,

$$B = \lambda_2 \frac{WL}{16} + \left[a_{22}^{\alpha}{}'.\alpha + a_{22}^{\Delta}{}'.(\lambda_2-\alpha)\right]\frac{(mHh)av}{4}$$

Alternatively,

$$B = \lambda_2 \frac{WL}{4}\left\{\frac{1}{4} + \left[\frac{\alpha}{\lambda_2}.a_{22}^{\alpha}{}' + \left(1-\frac{\alpha}{\lambda_2}\right).a_{22}^{\Delta}{}'\right]\frac{(mHh)av}{WL}\right\}$$

But,

$$B = \lambda_2 \frac{WL}{4}\left[\frac{1}{4} + A.\frac{(mHh)av}{WL}\right]$$

Therefore, for this zone the beam magnification factor is given by,

$$A = \frac{\alpha}{\lambda_2}.a_{22}^{\alpha}{}' + \left(1-\frac{\alpha}{\lambda_2}\right).a_{22}^{\Delta}{}'$$

Similarly, the resulting moment in the lower column may be written as:-

$$M_{23} = -\frac{m_2H_2h_2}{2}\left[\alpha(1-2a_{23}^{\alpha}.p_2) + (\lambda_2-\alpha)(1-2a_{23}^{\Delta}.p_2)\right]$$

$$= -\lambda_2.\left[\frac{\alpha}{\lambda_2}.(1-2a_{23}^{\alpha}.p_2) + (1-\frac{\alpha}{\lambda_2}).(1-2a_{23}^{\Delta}.p_2)\right].\frac{m_2H_2h_2}{2}$$

As before,

$$C_I > |M_{23}| = \lambda_2.A_c.\frac{m_2H_2h_2}{2}$$

Therefore, the column magnification factor is given by:-

$$A_c = \frac{\alpha}{\lambda_2}.(1-2a_{23}^{\alpha}.p_2) + (1 - \frac{\alpha}{\lambda_2}).(1-2a_{23}^{\Delta}.p_2)$$

3.8(c) ZONE 3

$$\frac{1}{4} = \frac{A(mHh)av}{WL}$$

In Zone 3, the beam is fully elastic under vertical load alone. The expressions for A and A_c are identical to those derived for Zone 2(ii), but the value of α, the load factor for horizontal loading at which the first plastic hinge forms, is different.

The bending moments under full vertical load plus wind load at load factor α are identical to those shown in Figure 17(b), and the condition for the first hinge to form in the beam is again given by equation (22); i.e.,

$$\lambda_2 \frac{WL}{12} + \frac{a_{22}^{\alpha}{}'}{2}.\alpha(mHh)av = B$$

In this case, however,

$$B = \lambda_2 \frac{A(mHh)av}{2}$$

Therefore,

$$\alpha.\left[\frac{a_{22}^{\alpha}{}'}{2}(mHh)av\right] = -\lambda_2 \frac{WL}{12} + \lambda_2 \frac{A(mHh)av}{2}$$

which reduces to:-

$$\frac{\alpha}{\lambda_2} = \frac{A}{a_{22}^{\alpha}{}'}\left[1 - \frac{WL}{6A(mHh)av}\right]$$

3.9. SUMMARY OF THE MAGNIFICATION FACTORS

A summary of the basic design equations has been given in 3.5. It can be seen that these are easily solved if values of the magnification factors, A and A_c, are known. These factors are summarized overleaf.

ZONE 1 $$0 \leqslant \frac{A(mHh)av}{WL} \leqslant \frac{1}{16}$$

$$A = a_{22}^{\Delta}{}' \tag{23}$$

$$A_c = 1 - 2a_{23}^{\Delta}.p_2 \tag{24}$$

ZONE 2(i) $$\frac{1}{16} \leqslant \frac{A(mHh)av}{WL} \leqslant \frac{1}{12}$$

The expressions for A and A_c are identical to those for Zone 1; i.e. equations (23) and (24).

ZONE 2(ii) $$\frac{1}{12} \leqslant \frac{A(mHh)av}{WL} \leqslant \frac{1}{4}$$

$$A = \frac{\alpha}{\lambda_2}.a_{22}^{\alpha}{}' + (1 - \frac{\alpha}{\lambda_2}).a_{22}^{\Delta}{}' \tag{25}$$

$$A_c = \frac{\alpha}{\lambda_2}.(1-2a_{23}^{\alpha}.p_2) + (1 - \frac{\alpha}{\lambda_2}).(1-2a_{23}^{\Delta}.p_2) \tag{26}$$

$$\frac{\alpha}{\lambda_2} = \frac{A}{a_{22}^{\alpha}{}'}\left[\frac{1}{2} - \frac{WL}{24A(mHh)av}\right] \tag{27}$$

ZONE 3 $$\frac{1}{4} \leqslant \frac{A(mHh)av}{WL}$$

The expressions for A and A_c are identical to those for Zone 2(ii); i.e. equations (25) and (26).

$$\frac{\alpha}{\lambda_2} = \frac{A}{a_{22}^{\alpha}{}'}\left[1 - \frac{WL}{6A(mHh)av}\right] \tag{28}$$

In all three zones:-

$$p_2 = \frac{(mHh)av}{m_2H_2h_2}$$

$$a_{22}^{\Delta}{}' = \frac{3}{K_2V + 3}; \quad a_{23}^{\Delta} = \frac{(n_2-o_2)K_2}{K_2V + 3};$$

$$a_{22}^{\alpha}{}' = \frac{12}{K_2V + 12}; \quad a_{23}^{\alpha} = \frac{(n_2-o_2)K_2}{K_2V + 12};$$

Also,

$$V = (n_1-o_1)\overline{K} + (n_2-o_2)$$

and,

$$\overline{K} = \frac{K_{c1}}{K_{c2}}; \quad K_2 = \frac{K_{c2}}{K_{b2}};$$

Under simple plastic conditions (i.e. with zero axial forces in the columns), A and A_c are both equal to unity in all zones. This is so since, when $\rho = 0$, $(n-o) = 0$ and therefore $V = 0$. Thus, $\overset{\Delta}{a}_{22}{}' = \overset{\alpha}{a}_{22}{}' = 1$, and $\overset{\Delta}{a}_{23} = \overset{\alpha}{a}_{23} = 0$, and all the magnification factors become unity.

As ρ increases, A and A_c increase in all zones since (n-o), and therefore V, become negative. This leads to values of $\overset{\Delta}{a}_{22}{}'$ and $\overset{\alpha}{a}_{22}{}'$ greater than unity, and to negative values of $\overset{\Delta}{a}_{23}$ and $\overset{\alpha}{a}_{23}$.

In addition, it may be seen that the ratio of beam stiffness to column stiffness, K_2, controls the values of the magnification factors to a large extent. This important observation will be discussed in Chapter 6, which deals with the design procedure and the methods that have been adopted to ensure both rapid convergence and control of instability.

3.10. ELASTICITY OF THE BEAM AT WORKING LOAD

The equations that have been developed so far ensure that the design criteria (1), (2) and (4), given in 3.2., will be satisfied. The third design criterion demands that the framework shall be fully elastic at working load.

Consider first the calculation of the maximum bending moment occurring in the beam at working load.

Under vertical load alone, the bending moment diagram is as shown previously in Figure 12(b). The maximum moments exist at the ends of the beam, and are of magnitude $\frac{WL}{12}$.

In application of the wind loading, the moment at the leeward end increases and is always greater than that at the windward end. The leeward end is therefore the critical section in the beam.

At load factor unity for both vertical and horizontal loading, the member stiffnesses are as shown in Figure 19, corresponding to Euler ratios of ρ_1^1 and ρ_2^1 in the columns. The total stiffness of any joint is given by:-

$$\Sigma S^r = (n_1 - o_1)^1 K_{c1} + (n_2 - o_2)^1 K_{c2} + 12K_{b2}$$

The out of balance moment due to wind loading is given by:-

$$F.E.M.^1 = -(m^1 Hh)av$$

The real distribution factor for the two beams meeting at the joint is similar in form to $a_{22}^{\alpha}{}'$, derived previously; i.e.,

$$a_{22}^1{}' = \frac{12K_{b2}}{\Sigma S^r} = \frac{12}{K_2 V^1 + 12} \qquad (29)$$

where,

$$V^1 = (n_1 - o_1)^1 \overline{K} + (n_2 - o_2)^1$$

With this notation, the total moment at working load at the leeward end of the beam is:-

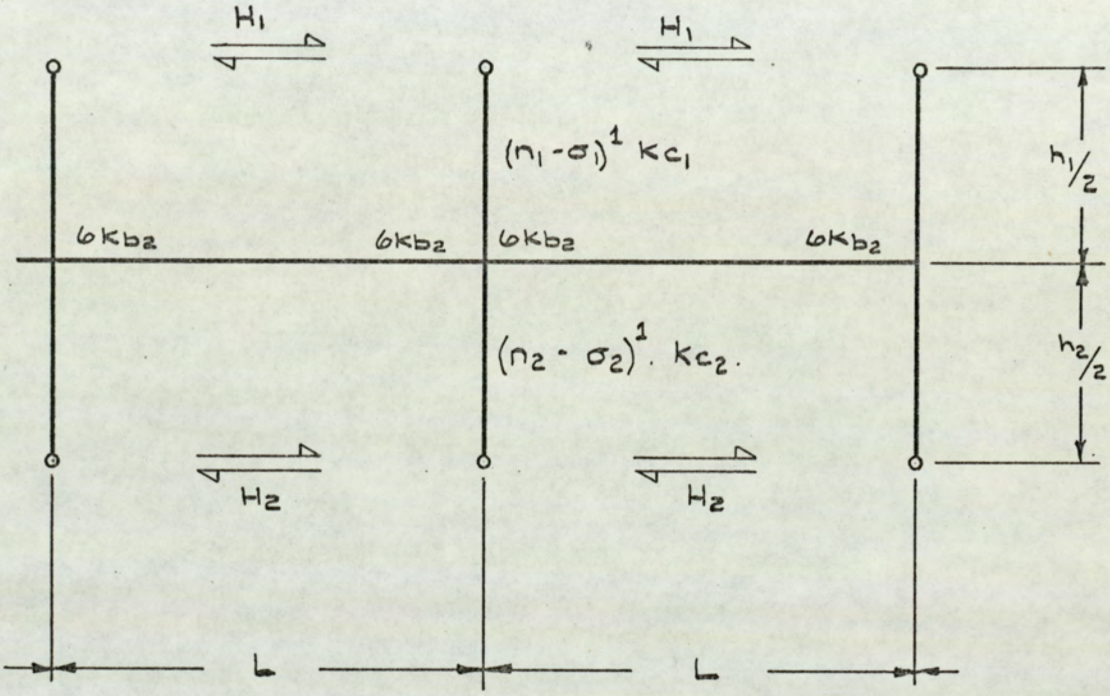
$$M_{max} = \frac{WL}{12} + \frac{a_{22}^1{}'}{2}.(m^1 Hh)av$$

Therefore, irrespective of the zone, no plastic hinge will form in the beam below working load provided that:-

$$\frac{WL}{12} + \frac{a_{22}^1{}'}{2}.(m^1 Hh)av \leq Mp_b \qquad (30)$$

where Mp_b is the fully plastic moment of the selected beam.

In Chapter 6, it is shown that it is possible to derive a general equation for each zone from this basic equation in terms of the quantities A and $\frac{A(mHh)av}{WL}$, which correspond to load factor λ_2, and which are necessarily calculated during the design of each storey. Furthermore, if these equations are represented graphically, the "working load



Member stiffnesses at combined working load

FIGURE 19.

elasticity condition" may be checked immediately, without calculation, by using the known values of the two variables.

In the following chapter, suitable design equations are developed for the boundary regions of the framework, namely the top and bottom storeys, and the external columns.

CHAPTER 4

THE BOUNDARY REGIONS

4.1. INTRODUCTION

In the previous chapter, equations have been derived for the design of the beams and the internal columns of a typical intermediate storey in a framework. However, these basic equations are not suitable for the members which constitute the boundaries of the frame, since, in these areas, the structural behaviour may be considerably different from that of the internal regions.

It is necessary, therefore, to consider each of the limiting areas of the framework separately, to make suitable assumptions about the behaviour of that area, and to produce an appropriate set of design equations. This is the subject of the current chapter, which deals in turn with the design of the top storey, the lowest storey, and the external columns of the frame.

4.2. THE TOP STOREY

Under vertical load alone, the top storey beam, like those for the other storeys, may only fail by a simple beam mechanism. If the required fully plastic moment of the beam is denoted by B_1, and the vertical working load by W_1, then, as before:-

$$B_1 = \lambda_1 \frac{W_1 L}{16} \tag{31}$$

Under a combination of vertical and horizontal loading at λ_2, there is a possibility that failure may occur by either a combined mechanism or a sway mechanism, provided that the columns remain elastic. In deriving design equations for this storey, however, the general subassemblage, described in Chapter 3, is no longer valid. It is proposed, therefore, to consider the top two storeys of the framework as one subassemblage, with points of contraflexure being assumed to exist at the centres of the columns in the lower storey. Furthermore, it is assumed that the lower storey has already been designed, and that it has been supplied with a beam of fully plastic moment, B_2, using equations (1) to (3) in Chapter 3.

Under combined load, a possible failure mechanism for this two-storey subassemblage is as shown in Figure 20. In this case, both beams collapse due to the combined mechanism. This is in fact the only mode of failure for the complete subassemblage if plastic hinges corresponding to the combined mechanism form in the lower beam, since the wind loading on the upper storey is considerably less than that on the lower storey. It follows, that if hinges corresponding to the sway mechanism were to form in the lower beam, then either the combined or the sway mechanism would be possible for the upper beam. However, these two cases do not require consideration, as will be shown subsequently.

Consider, therefore, the overall combined mechanism in Figure 20. As for the intermediate storeys, the work equation may be applied to an isolated bay of the subassemblage, and this is shown in Figure 21, where the column is considered to rotate through an angle θ about the joint with the lower beam. The work equation is:-

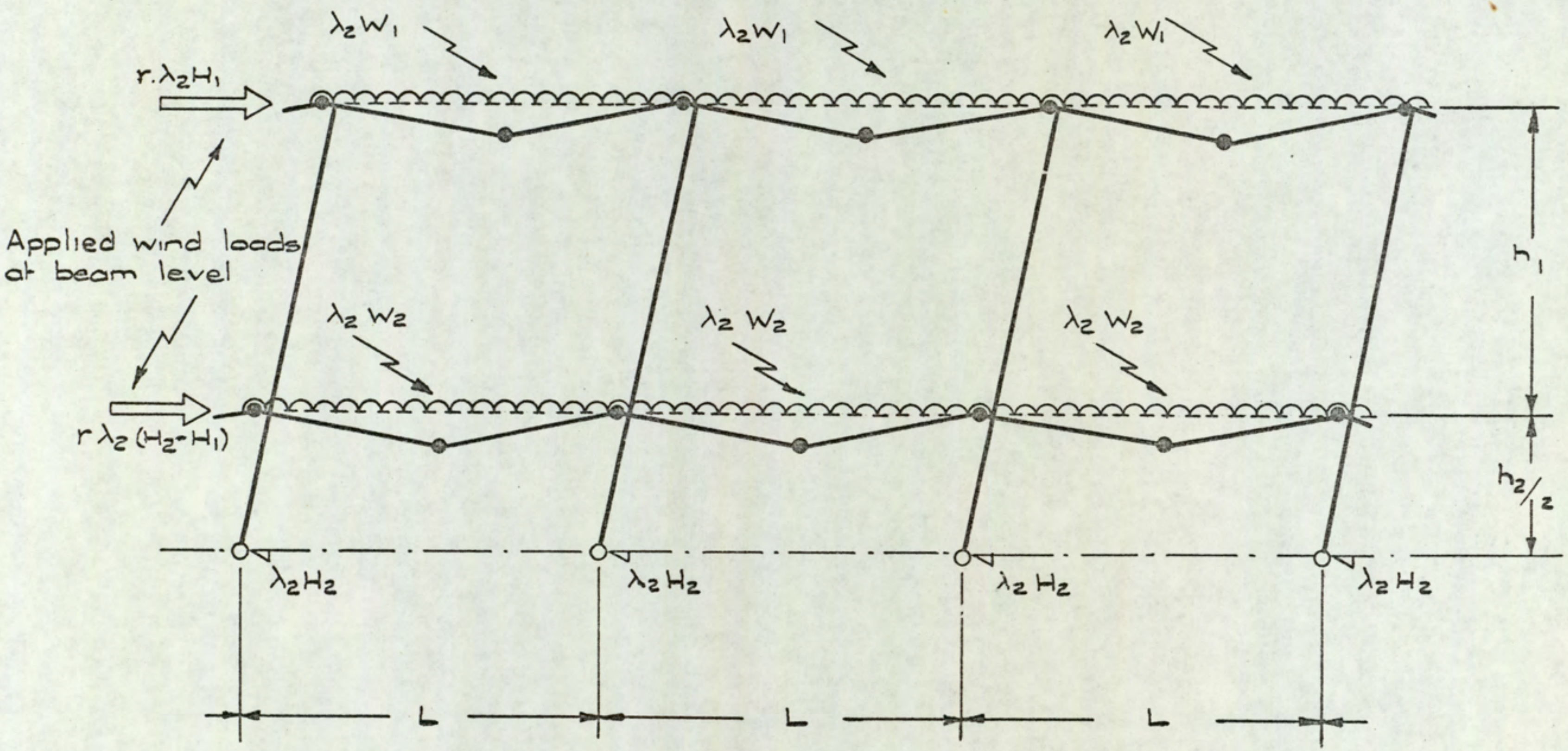
$$4B_1\theta + 4B_2\theta = \frac{\lambda_2 W_1}{2}.\frac{L}{2}.\theta + \frac{\lambda_2 W_2}{2}.\frac{L}{2}.\theta + \lambda_2 H_1.h_1.\theta + \lambda_2 H_2.\frac{h_2}{2}.\theta$$

Eliminating θ, and regrouping:-

$$4B_1 + 4B_2 = \lambda_2 \frac{W_1 L}{4} + \lambda_2 \frac{H_1 h_1}{2} + \lambda_2 \frac{W_2 L}{4} + \lambda_2 (Hh)av$$

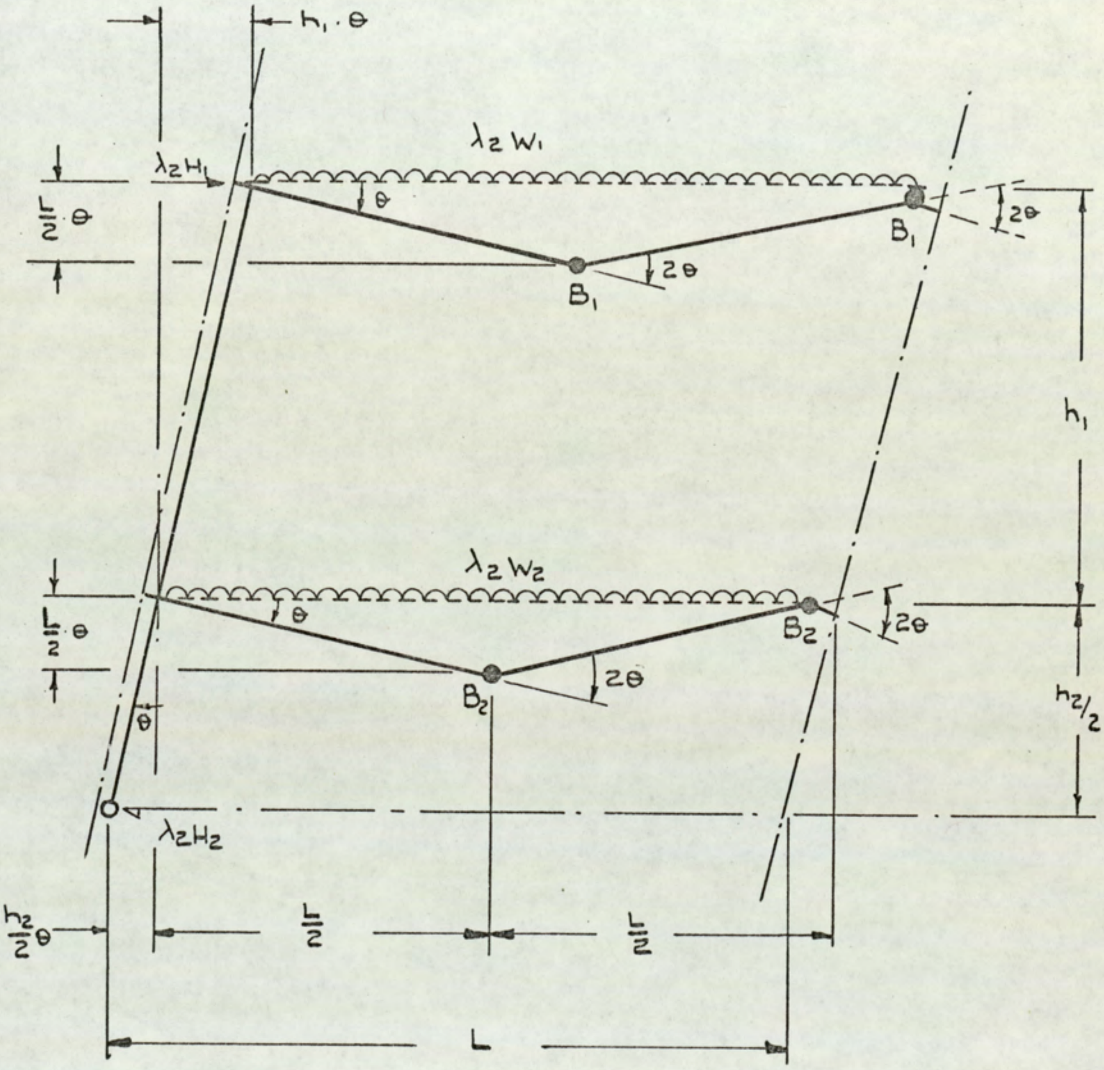
But, from equation (2) in Chapter 3, the lower beam will have been selected with:-

$$B_2 = \lambda_2 \frac{W_2 L}{16} + \lambda_2 \frac{(Hh)av}{4}$$



Top two storeys - combined mechanism at λ_2

FIGURE 20.



Top two storeys - Combined mechanism - isolated bay

FIGURE 21

Therefore, eliminating B_2:-

$$4B_1 = \lambda_2 \frac{W_1L}{4} + \lambda_2 \frac{H_1h_1}{2}$$

Therefore,

$$B_1 = \lambda_2 \frac{W_1L}{16} + \lambda_2 \frac{H_1h_1}{8} \qquad (32)$$

So, in order to satisfy the basic design criteria, neglecting any other types of mechanism at this stage, the top storey beam must be supplied with a value of fully plastic moment at least as great as the larger of the two values given by equations (31) and (32). Equation (31) will give the required value if:-

$$\lambda_1 \frac{W_1L}{16} \geq \lambda_2 \frac{W_1L}{16} + \lambda_2 \frac{H_1h_1}{8}$$

This condition may be rewritten as:-

$$\frac{5}{64} \lambda_2 W_1L - \lambda_2 \frac{W_1L}{16} \geq \lambda_2 \frac{H_1h_1}{8}$$

which reduces to:-

$$\frac{H_1h_1}{W_1L} \leq \frac{1}{8} \qquad (33)$$

Now, in general, the storey heights in a framework are approximately equal; i.e. $h_1 \simeq h_2$. Also, when the wind loading is uniformly distributed, $H_2 \simeq 3H_1$. Therefore,

$$(Hh)av = \frac{H_1h_1 + H_2h_2}{2} \simeq \frac{(H_1 + 3H_1)h_1}{2} = 2H_1h_1$$

i.e.

$$H_1h_1 \simeq \frac{(Hh)av}{2}$$

Equation (33) may therefore be rewritten in an approximate form as:-

$$\frac{(Hh)av}{W_1L} \leq \frac{1}{4} \qquad (34)$$

In addition, the vertical load on the top storey of a frame is generally less than or equal to the load on the intermediate storeys. If, for instance, $W_1 = W_2$, then equation (34), which is the condition for the simple beam mechanism to be the critical mode of failure, becomes:-

$$\frac{(Hh)av}{W_2 L} \leq \frac{1}{4}$$

If, however, $W_1 = \frac{1}{2}W_2$, then,

$$\frac{(Hh)av}{W_2 L} \leq \frac{1}{8}$$

It may be seen that the quantity $\frac{(Hh)av}{W_2 L}$ is equal to the wind ratio for the lower storey. In general, as will be demonstrated by the design examples in Chapters 7 and 8, the design of the second storey lies in Zone 1, where $0 \leq$ wind ratio $\leq \frac{1}{16}$. In only one framework, which is designed for very heavy wind loading, does this storey fall in Zone 2, and in this case the wind ratio is equal to $\frac{1}{15}$. It is therefore extremely unlikely that any frame will have a wind ratio for the second storey in the region of $\frac{1}{8}$, and so it may be assumed that the beam mechanism is always the dominant mode of failure for the top storey.

As suggested previously, there is no necessity to consider the other possible mechanisms for the two-storey subassemblage, since these may only occur at even higher values of the wind ratio than $\frac{1}{8}$.

Therefore, in all cases, the beams in the top storey may be selected by equation (31). It is, however, difficult to assess the exact plastic moment required for the internal columns, since the positions of the points of contraflexure are unknown. It is suggested that this value shall be assumed to be equal to the required plastic moment for the columns in the second storey. This does not involve any great loss of economy, since the columns in this region of the frame are comparatively slender. In addition, the selection of a slightly conservative column provides an additional safeguard against instability.

The design equations for the top storey are summarized below.

$$B = \lambda_1 \frac{W_1 L}{16}$$

where W_1 is the load on the top storey beam.

C_I > required plastic moment for the column in the storey below.

4.3. THE LOWEST STOREY

Before deriving a set of design equations for the internal bays of the lowest storey, it is necessary to consider the way in which the internal shear forces are distributed in the columns.

4.3(a). ESTIMATE OF INTERNAL COLUMN SHEAR FORCES

One of the assumptions given in 3.3(b), for the derivation of the general theory, is that the shear force, H, induced in each internal column of an intermediate storey, is equal to the total wind loading above that storey, divided by the number of bays. If there are r bays in the storey, then the total shear in the storey is equal to rH. However, the number of internal columns is equal to (r-1), so that the total shear in these members is (r-1)H. It follows, therefore, that each external column must carry a shear force of $\frac{H}{2}$ due to wind loading.

This assumed distribution of shear forces is satisfactory for the design of the intermediate storeys, where, in the resulting structure, the stiffness of an external column is found to be approximately half that of an internal column.

However, as will be shown subsequently, a modified design approach has been adopted for the lowest storey, and this results in an internal column which is considerably greater than that in any of the storeys above. In contrast, the external column in the bottom storey is generally similar in size to that in the previous storey. This results in a ratio of external to internal column stiffness in the lowest storey which is less than a half, and a similar ratio of shear forces in these members is to be expected. Thus, the design shear forces for the internal columns would be underestimated if they were obtained by simply dividing the total shear by the number of bays, and this error would be particularly apparent when dealing with a framework with comparatively few bays.

Figure 22 shows the assumed subassemblage for the interior bays of the bottom storey, together with the applied working loads. It may be seen that points of contraflexure are still considered to exist at the mid-heights of the columns in the upper storey, and each column carries a shear force of H_1, the total shear being rH_1. However, in the lowest storey, where the columns are assumed to be fully fixed at the base, the total shear force is given by:-

$$(r-1)H_2 + 2H_2'$$

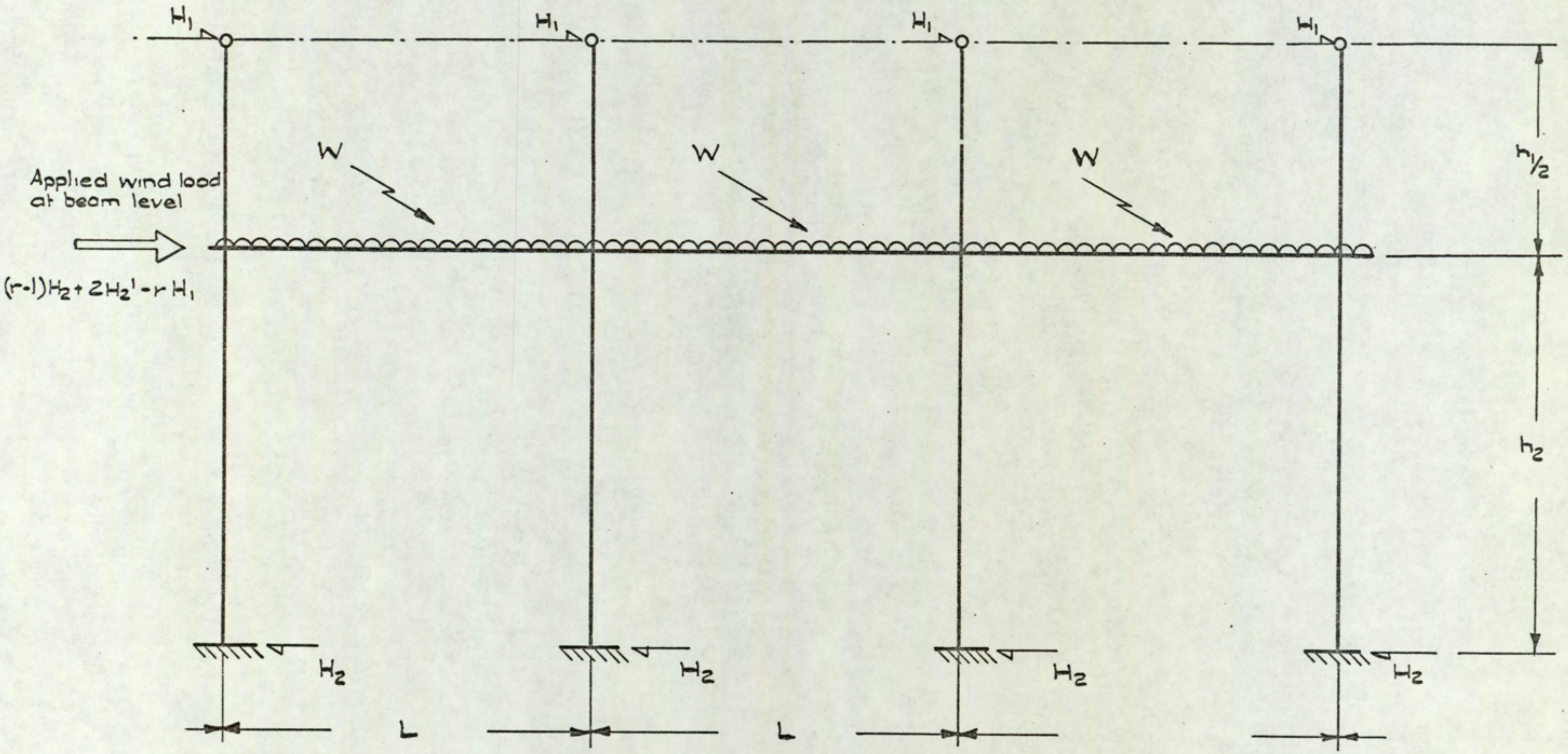
where H_2 is the shear on each internal column, and H_2' the shear on each external column. H_2' is not assumed to be equal to $\frac{H_2}{2}$.

The wind loading on this storey is still assumed to act at the level of the beam, but will of course be greater than that applied to the upper storeys, since this force is the summation of a distributed wind loading acting over half the height of the upper column plus the total height of the lower column. This is a conservative assumption, since the true centroid of this force system is at a level of $\frac{1}{2}\left(\frac{h_1}{2} + h_2\right)$ above the ground.

The following empirical formula is suggested for calculating the magnitude of the internal column shear forces:-

$$H_2 = \Sigma(H)_2 . \frac{1.05(3+17wr')}{[2+(r-1)(3+17wr')]} \qquad (35)$$

where $\Sigma(H)_2$ is the total shear in the bottom storey (i.e. the sum of the applied wind forces above ground level), and where wr' is the wind ratio that would be obtained if the bottom storey shears were calculated by dividing the total shear by the number of bays $\left(\text{i.e. } wr' = \frac{[\Sigma(H)h]av}{rWL}\right)$



Lowest storey - assumed subassemblage at working load.

FIGURE 22

The development of this empirical formula is given in a later chapter, where it will be shown that it derives from the results of a series of accurate computer analyses. This chapter is more concerned with the derivation of a set of design equations, the initial design loads having been assumed.

4.3.(b) FAILURE OF THE LOWEST STOREY

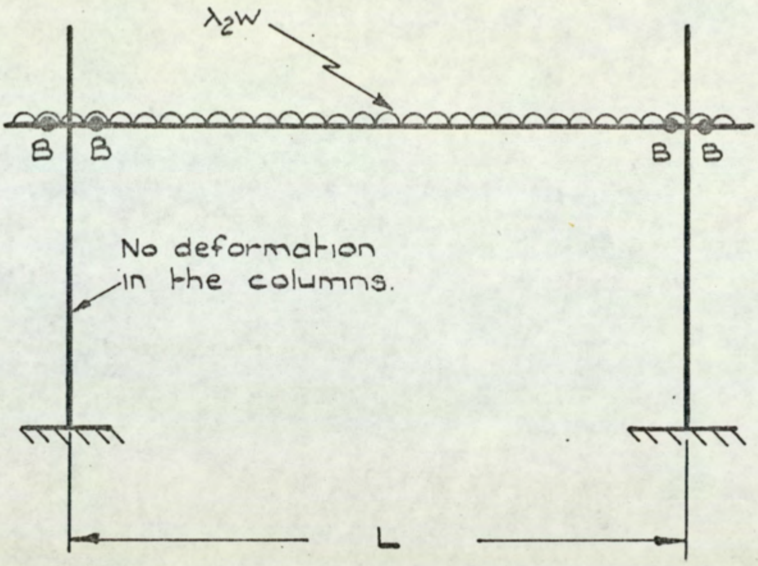
The design of the lowest storey is based on a slightly different approach to that used for the other storeys. One of the basic design criteria is that all the columns in the framework shall be fully elastic at the design load factor. Therefore, since the column feet are fully fixed in this storey and sway is largely eliminated, the only true mechanism possible is the simple beam mechanism, under vertical load alone. Thus, irrespective of the wind ratio, for the bottom storey,

$$B = \lambda_1 \frac{WL}{16}$$

The internal column size must, however, be assessed from consideration of the bending moments produced in the combined loading case. As for the upper storeys, the vertical load is considered to be applied first, followed by the wind loading.

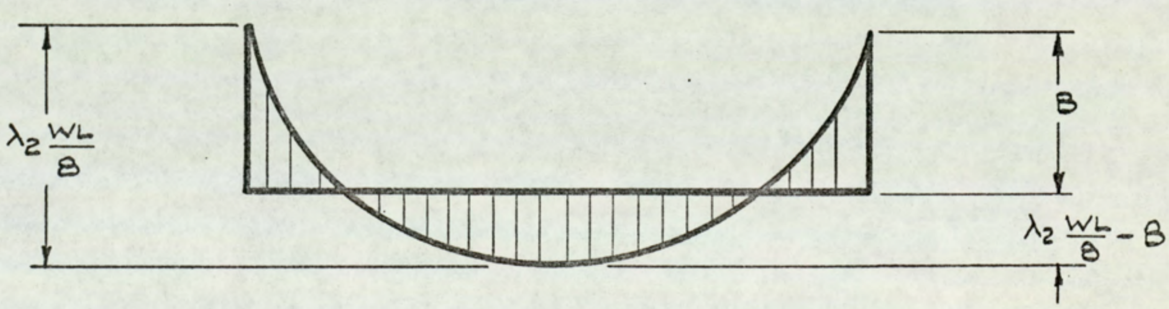
Under vertical load alone, no bending moments are produced in the internal columns. Each beam behaves as if encastré, with a plastic hinge forming at each end at a load factor below λ_2. At load factor λ_2, the frame is as shown in Figure 23(a), the corresponding bending moments being given in Figure 23(b).

On application of the horizontal loading, the windward hinge immediately disappears, and the isolated bay for analysis is shown in Figure 24(a), where the flexural rigidities of the members are K_{c1}, K_{c2} and K_{b2}, and the Euler ratios of the columns are ρ_1 and ρ_2. The real stiffnesses of the columns may be obtained from equations (10) and (11)



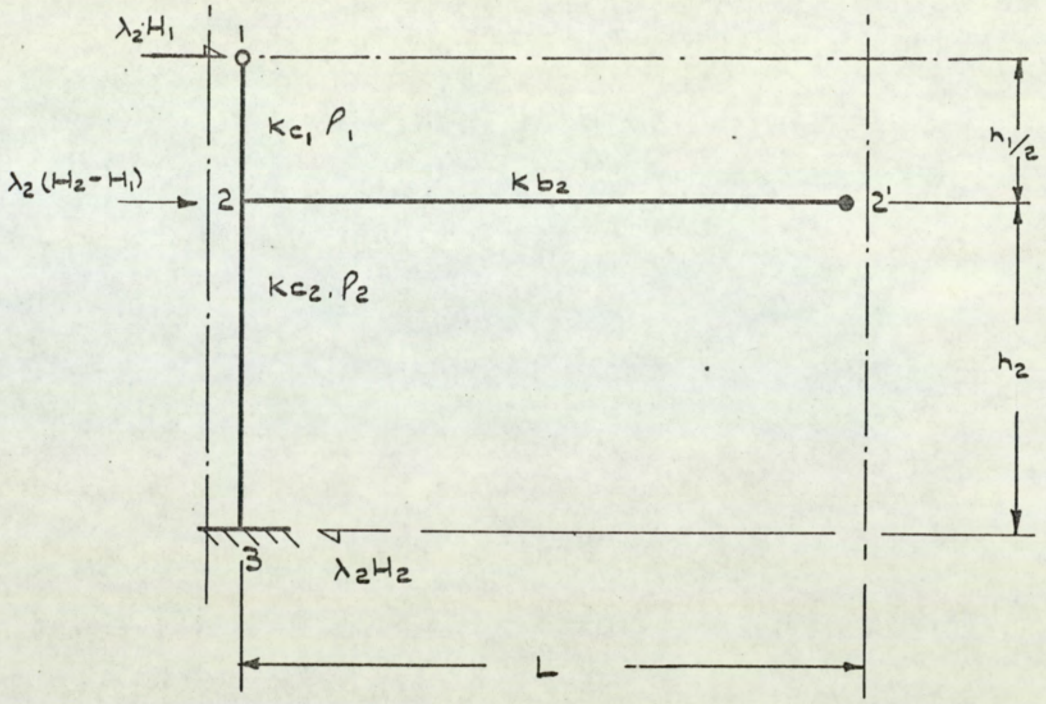
Lowest storey - vertical load alone at λ_2

FIGURE 23 (a)



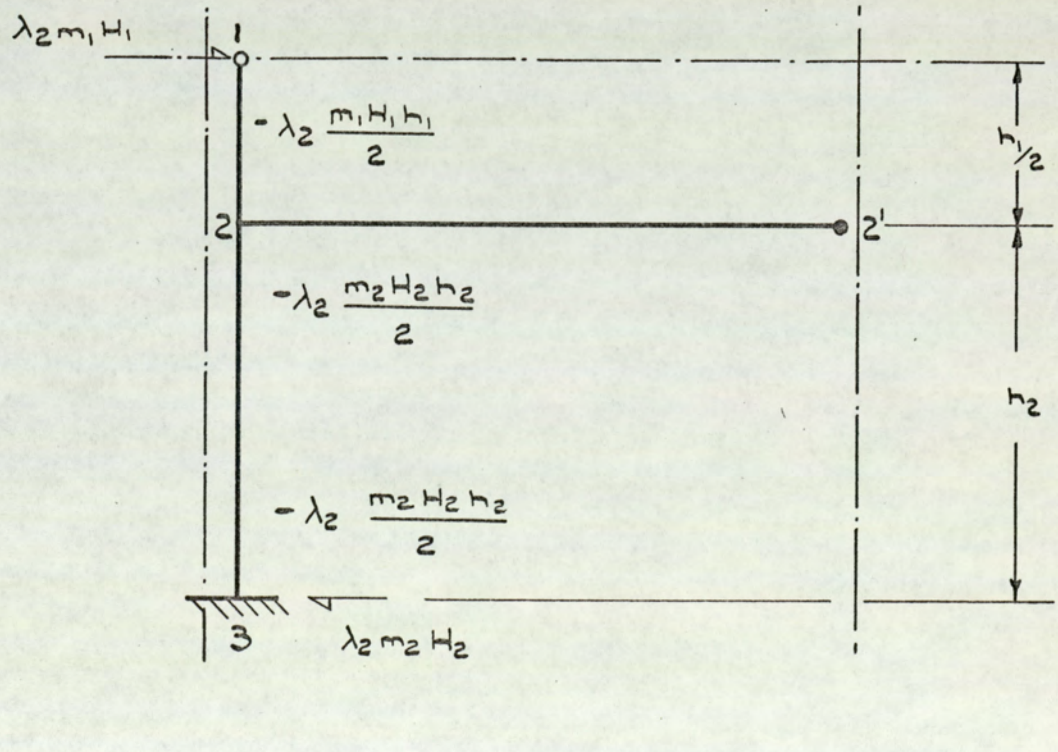
Bending moment diagram

FIGURE 23 (b).



Isolated bay - joint numbers, $\frac{"EI"}{\ell}$ values and Euler ratios

<u>FIGURE 24 (a)</u>



Initial sway moments, allowing for the Pδ effect

<u>FIGURE 24 (b)</u>

in Chapter 3.

For the upper column, R = 1, so that,

$$S^r_{21} = (n_1 - o_1)K_{c1}$$

For the lower column, since end 3 is fully fixed, R = 0, so that from equation (10),

$$S^r_{23} = n_2 K_{c2}$$

and from equation (11),

$$S^r_{32} = \infty$$

In addition, there is a carry-over of $\left(-\frac{o_2}{n_2}\right)$ times the moment distributed at section 23 to the base of the column. Since there is a plastic hinge at 2', the real stiffness of the beam at joint 2 is given by:-

$$S^r_{22}{}' = 3K_{b2}$$

The total stiffness at joint 2 is therefore:-

$$\Sigma S^r_2 = (n_1 - o_1)K_{c1} + n_2 K_{c2} + 3K_{b2}$$

and the distribution factors are:-

$$a^{\Delta}_{21} = \frac{S^r_{21}}{\Sigma S^r_2} = \frac{(n_1 - o_1)K_{c1}}{\Sigma S^r_2}$$

$$a^{\Delta}_{23} = \frac{S^r_{23}}{\Sigma S^r_2} = \frac{n_2 K_{c2}}{\Sigma S^r_2}$$

$$a^{\Delta}_{22}{}' = \frac{S^r_{22}{}'}{\Sigma S^r_2} = \frac{3K_{b2}}{\Sigma S^r_2}$$

In an alternative form:-

$$a^{\Delta}_{21} = \frac{(n_1 - o_1)\overline{K}K_2}{K_2 V' + 3}$$

$$\overset{\Delta}{a}_{23} = \frac{n_2 K_2}{K_2 V' + 3}$$

$$\overset{\Delta}{a}_{22}{}' = \frac{3}{K_2 V' + 3}$$

where,

$$V' = \overline{K}(n_1 - o_1) + n_2$$

and,

$$\overline{K} = \frac{K_{c1}}{K_{c2}}; \quad K_2 = \frac{K_{c2}}{K_{b2}};$$

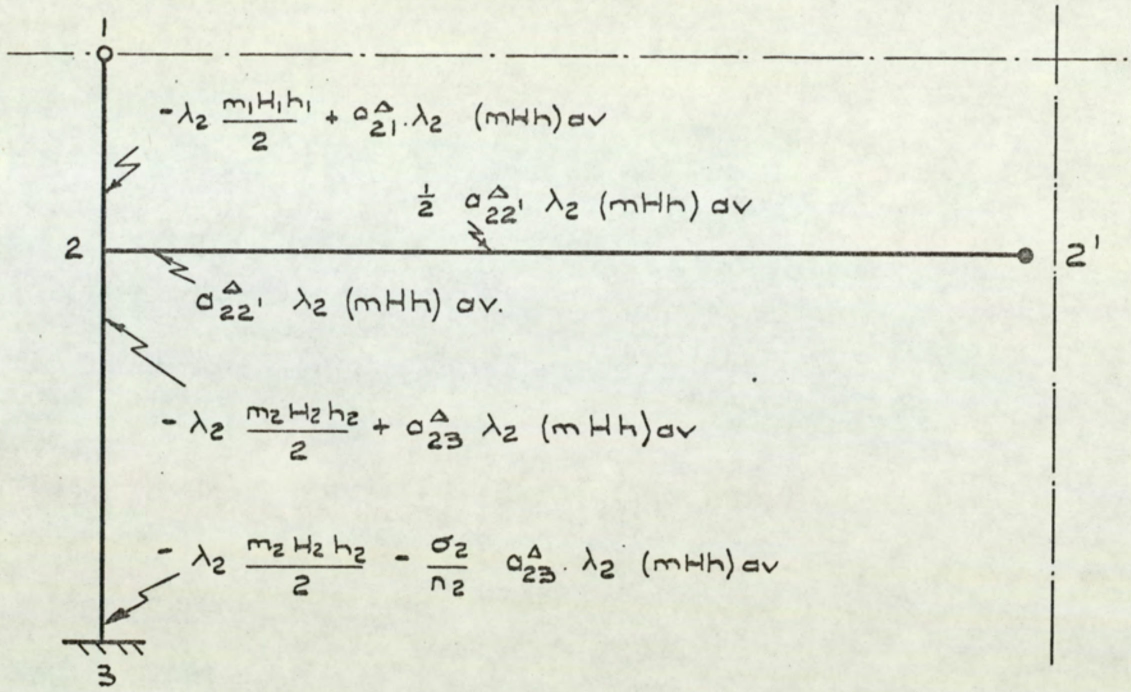
The initial sway moments due to wind loading at λ_2, magnified to allow for the Pδ effect, are shown in Figure 24(b). The total out of balance moment at joint 2 is therefore:-

$$F.E.M._2 = - \lambda_2 (mHh)av$$

Figure 25(a) shows the moments, due to wind loading alone, after distribution. The resulting moments under combined loading at load factor λ_2 are given in Figure 25(b). It may be seen that on the lower column both the initial fixing moments are negative.

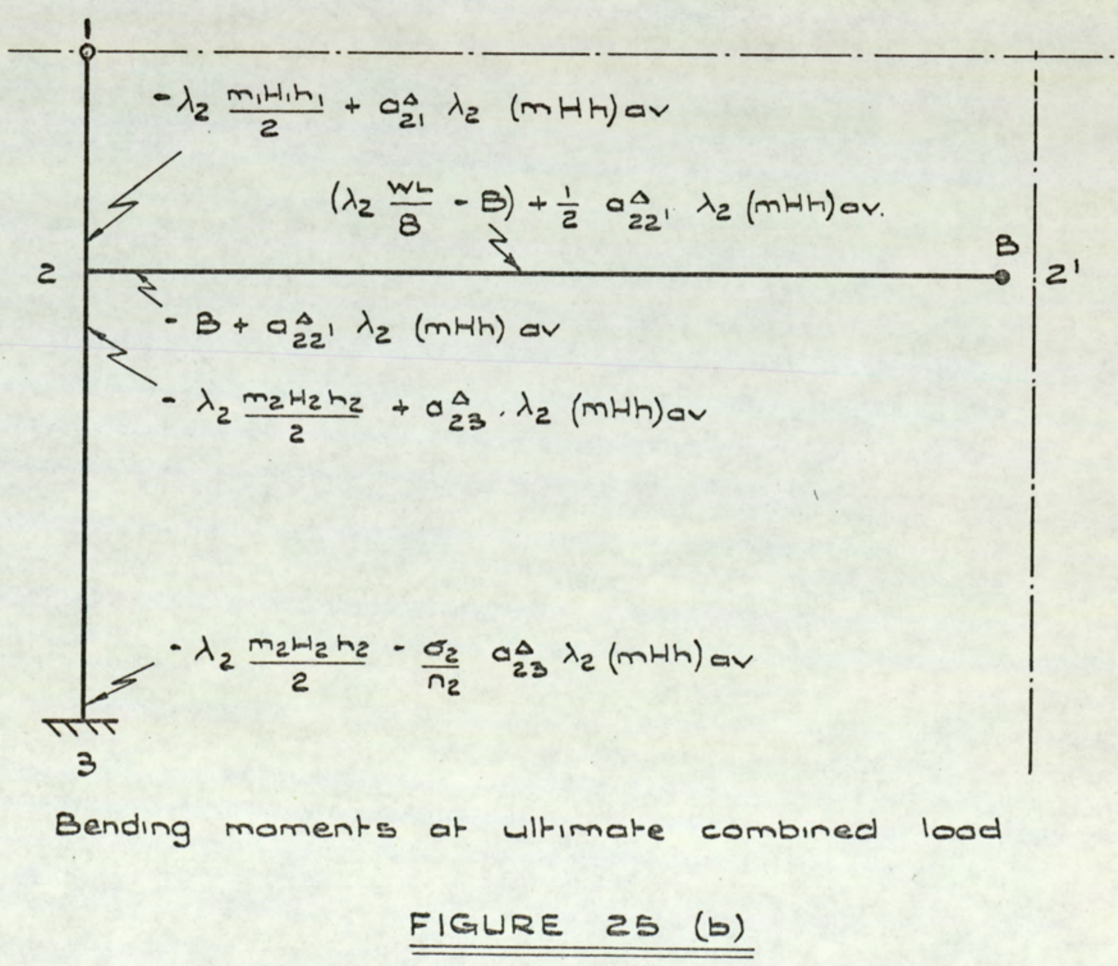
When $\rho = 0$, $V' = 1$, since $n = 1$ and $(n-o) = 0$. As ρ increases, n reduces and (n-o) becomes negative, so that V' reduces. However, n' is still the dominant term, so that V' is always positive. Therefore, $\overset{\Delta}{a}_{21}$ is always negative, and $\overset{\Delta}{a}_{23}$ is always positive.

Thus, the moment distributed at 21 is in the same sense as the initial fixing moment at this location. On the lower column, the moment distributed at 23 is in the opposite sense to the initial moment, whereas, since $\frac{o}{n}$ is always positive, that carried over to joint 3 increases the moment at this point, Thus, the final bending moment at 3 is always greater in magnitude than that at 2, and this is therefore the potential position for a plastic hinge in the lower column.



Bending moments due to wind load alone

FIGURE 25 (a)



Bending moments at ultimate combined load

FIGURE 25 (b)

On the upper column, the moment at the joint is given by:-

$$M_{21} = -\lambda_2 \frac{m_1H_1h_1}{2} + \overset{\Delta}{a}_{21}.\lambda_2(mHh)av$$

$$= -\lambda_2 \frac{m_1H_1h_1}{2} [1-2\overset{\Delta}{a}_{21}.p_1]$$

where,

$$p_1 = \frac{(mHh)av}{m_1H_1h_1}$$

Therefore, a plastic hinge will not form in the upper column below λ_2, if:-

$$C_{I_1} > |M_{21}| = \lambda_2 \frac{A_{c1}m_1H_1h_1}{2}$$

where,

$$A_{c1} = 1-2\overset{\Delta}{a}_{21}.p_1$$

The moment at joint 3 is given by:-

$$M_{32} = -\lambda_2 \frac{m_2H_2h_2}{2} - \frac{o_2}{n_2}.\overset{\Delta}{a}_{23}.\lambda_2(mHh)av$$

$$= -\lambda_2 \frac{m_2H_2h_2}{2} \left[1+2\frac{o_2}{n_2}.\overset{\Delta}{a}_{23}.p_2\right]$$

where,

$$p_2 = \frac{(mHh)av}{m_2H_2h_2}$$

Therefore, a plastic hinge will not form in the lower column below λ_2, if:-

$$C_{I_2} > |M_{32}| = \lambda_2 \frac{A'_{c2}m_2H_2h_2}{2}$$

where,

$$A'_{c2} = 1 + 2\frac{o_2}{n_2}.\overset{\Delta}{a}_{23}.p_2$$

It is not necessary, in this case, to derive a magnification factor for the beam, since its size is determined initially from the simple beam mechanism. In practice, under horizontal loading, there is a possibility (although remote) of the hinge pattern corresponding to the combined mechanism occurring in the beam. However, the safety of the frame would not be seriously affected by the formation of a second hinge in the span, due to the rigidity of the lowest storey column.

The magnification factor for the upper storey, A_{c1}, is similar in form to that for the other intermediate storeys, and has the same significance. It is equal to unity under simple plastic conditions, and increases with the axial force in the column. As will be shown in Chapter 6, it is necessary to estimate the required plastic moment for this column during the design of the bottom storey, using the equation above. This value should then be checked against that which has previously been obtained when using the eqations for the intermediate storeys, and the larger of the two values must be selected.

The function A'_{c2}, referring to the moment at the base of the bottom storey column, is not a true magnification factor. Under simple plastic conditions, it is not equal to unity, and takes the value,

$$1 + 2\overset{\Delta}{a}_{23}.p_2 = 1 + 2\,\frac{K_2}{K_2 + 3}.p_2 = A'_{c2}(sp), \text{ say.}$$

Since K_2 tends to be large in this storey, and since $p_2 \simeq 1$, $A'_{c2}(sp) \simeq 3$. The true magnification factor for this storey (i.e. the degree by which the moment at the base of the column is magnified due to instability effects) is in fact given by the ratio $\frac{A'_{c2}}{A'_{c2}(sp)}$. This is generally very close to unity, indicating that the instability effects are small in this region of the frame. Despite the fact that very large axial forces tend to exist in the columns of the lowest storey, it is not generally realised that the high flexural rigidity of these columns, and the fixity at the foundations, tend to largely eliminate the likelihood of instability failure.

A summary of the design equations for the beams and columns of the lowest storey is given in the following sub-section, and this is followed by consideration of the condition for elasticity of the beam at working load.

4.3(c) SUMMARY

In the bottom storey, the beam is selected using:-

$$B = \lambda_1 \frac{WL}{16} \qquad (36)$$

The columns above and below the beam, respectively, are selected from:-

$$C_{I_1} > \lambda_2 \frac{A_{c1} m_1 H_1 h_1}{2} \qquad (37)$$

where,

$$A_{c1} = 1 - 2\overset{\Delta}{a}_{21} . p_1 \qquad (38)$$

and,

$$C_{I_2} > \lambda_2 \frac{A'_{c2} m_2 H_2 h_2}{2} \qquad (39)$$

where,

$$A'_{c2} = 1 + 2 \frac{o_2}{n_2} . \overset{\Delta}{a}_{23} . p_2 \qquad (40)$$

Also, in the equations above,

$$p_1 = \frac{(mHh)av}{m_1 H_1 h_1}; \quad p_2 = \frac{(mHh)av}{m_2 H_2 h_2};$$

$$\overset{\Delta}{a}_{21} = \frac{(n_1 - o_1)\overline{K}K_2}{K_2 V' + 3}; \quad \overset{\Delta}{a}_{23} = \frac{n_2 K_2}{K_2 V' + 3}$$

where,

$$V' = (n_1 - o_1)\overline{K} + n_2$$

and,

$$\overline{K} = \frac{K_{c1}}{K_{c2}}; \quad K_2 = \frac{K_{c2}}{K_{b2}};$$

4.3.(d) ELASTICITY OF THE BEAM AT WORKING LOAD

The basic condition for the beams of the intermediate storeys to remain elastic at working load has been derived previously in 3.10., and an identical expression is obtained for the lowest storey; i.e.,

$$\frac{WL}{12} + \frac{a^1_{22}{}'}{2}(m^1 Hh)av \leq Mp_b \qquad (41)$$

However, in this case, the real stiffnesses of the columns are different, and the real distribution factor for the beam becomes:-

$$a_{22}^{1\ \prime} = \frac{12}{K_2 V'^1 + 12}$$

where,

$$V'^1 = (n_1 - o_1)^1\overline{K} + n_2^1$$

It has been found that this condition is often critical in the bottom storey, and the beam has to be increased to satisfy all the design criteria. Theoretically, it is possible to redesign the columns with the new increased beam size, with the intention of obtaining a more economical design. However, in general, identical columns are obtained. This is due to the fact that although there may be a high percentage increase in the second moment of area of the beam, it is still small compared with the size of the column. In addition, due to the large difference in the properties of successive Universal Columns, normally a considerable alteration in the required size of the column is necessary in order to dictate a new section.

It is suggested, therefore, that the column sizes be selected using the basic equations, (37) to (40), and that at the end of the design, the beam, which has been selected using equation (36), should simply be increased, if required, in order to satisfy equation (41). No further alteration to the section sizes is necessary. Equation (41) is developed into a more readily applicable form in Chapter 6.

4.4. THE EXTERNAL COLUMNS

The behaviour of the external columns differs from that of the internal columns, since the former are subjected to unsymmetrical loading conditions. Before deriving design equations for these members, it is necessary to make several additional assumptions concerning the distribution of the internal forces in the framework, with particular reference to the most critically loaded external column.

4.4(a) DISTRIBUTION OF FORCES

Under combined vertical and horizontal loading, the shear forces developed in the external columns of a framework are not purely a function of a horizontal loading, unlike those induced in the internal columns. Due to vertical load alone, the external columns deform, and shear forces are introduced. The following assumptions have been made:-

(1) The external columns bend in double curvature due to both horizontal and vertical loading, with points of contraflexure occurring at their mid-heights.

(2) As previously stated, in the intermediate storeys, due to horizontal load alone, the external columns carry shear forces equal to half the shear forces in the internal columns.

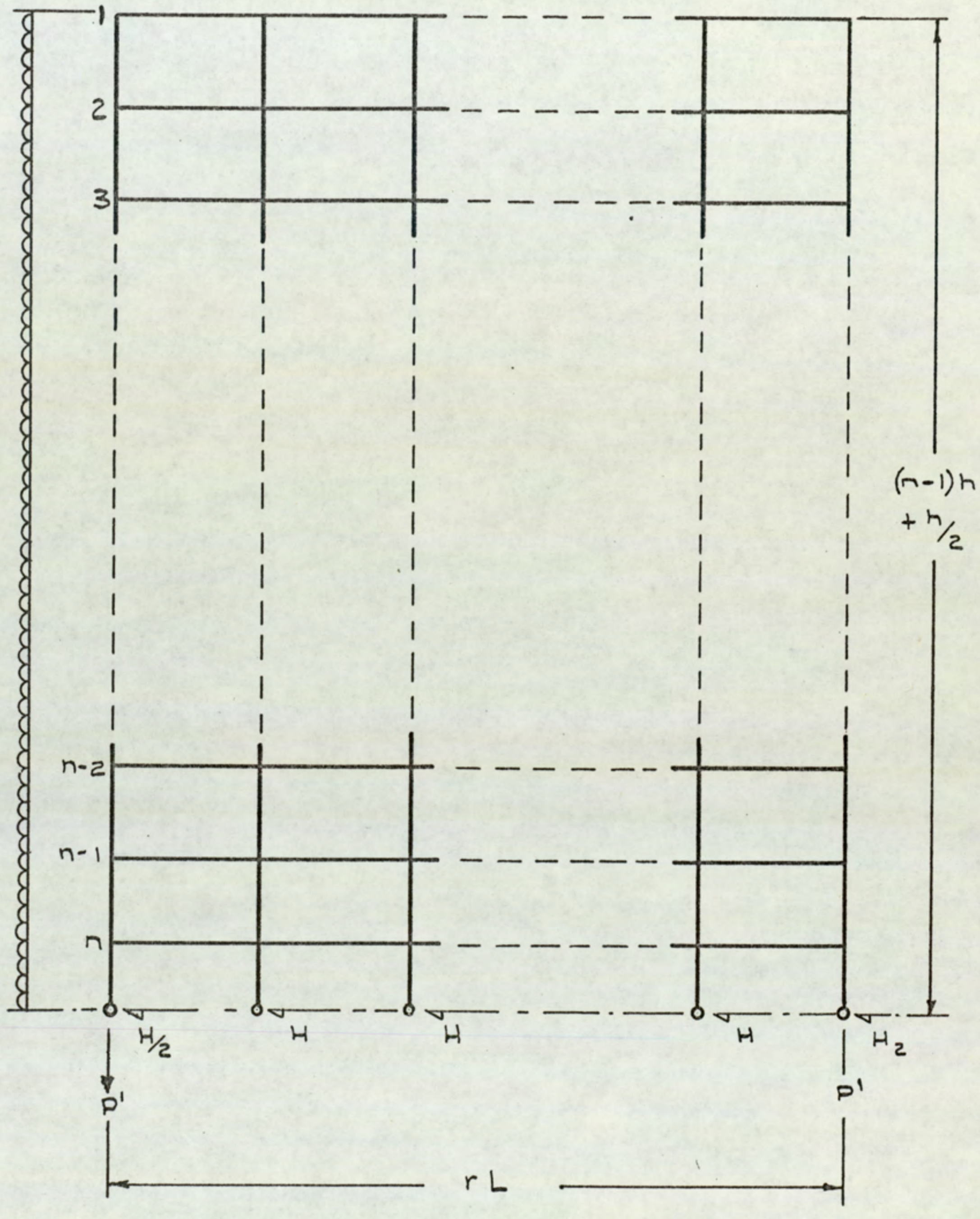
(3) The axial force in an external column, due to vertical load alone, is equal to the force in the column above plus half the load on the beam.

(4) Due to wind loading, equal axial forces are introduced in each external column in a storey, the force in the windward column being tensile, and that in the leeward column being compressive.

(5) At ultimate load, plastic hinges exist at the leeward ends of all the beams.

Assumption (4) follows automatically from the original assumption in 3.3(b) that there are no axial forces in the internal columns under wind loading. This is illustrated in Figure 26, where a section line has been drawn through the points of contraflexure in all the columns of a storey. In order to maintain vertical equilibrium, the external column forces must be equal in magnitude. Also, the couple formed by these two forces, the lever arm of which is equal to the total width of the frame, must balance the moment of all the wind forces above the points of contraflexure.

It follows, therefore, that under vertical and horizontal loading



Distribution of axial and shear forces at the points of contraflexure of the general storey n, due to wind loading alone.

FIGURE 26

together, the leeward external column contains the greater axial force, and as such is the critical column for design purposes. Both external columns must nevertheless be provided with the same section, since the wind loading is free to act from either side of the frame.

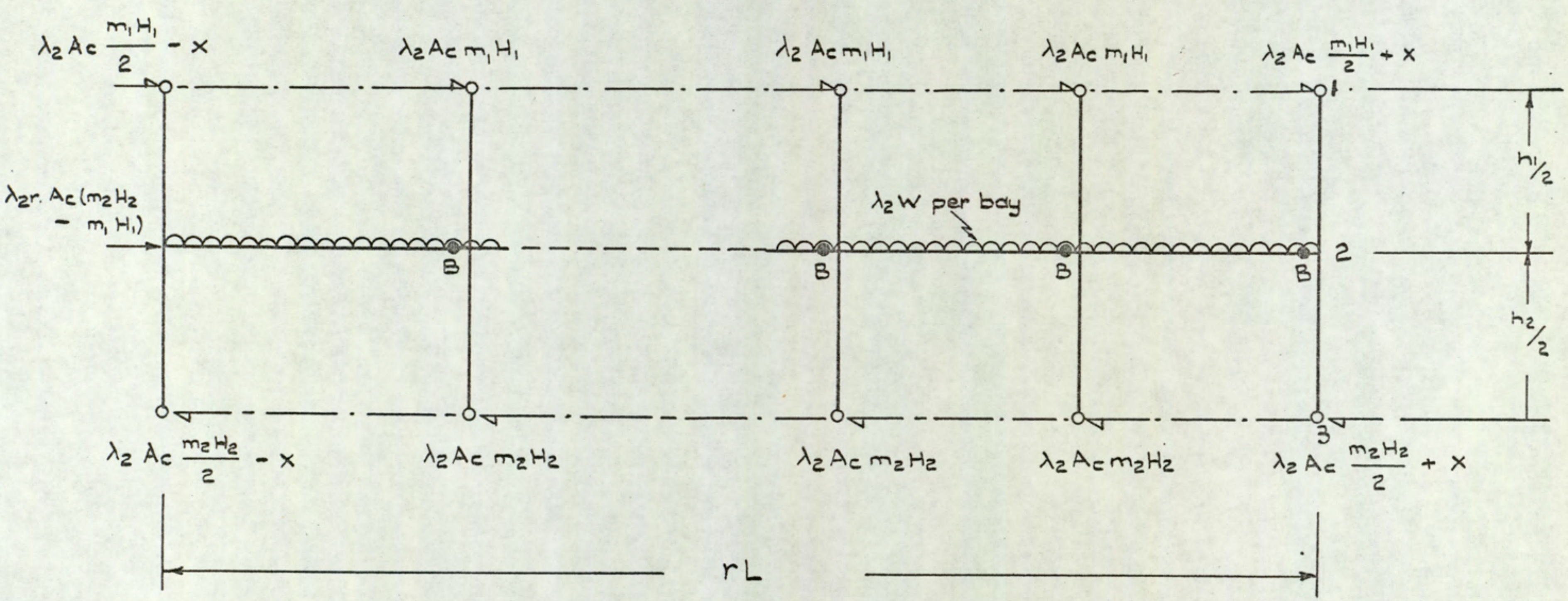
The derivation of the design equations for this critical external column is considered in the following three sub-sections, which deal in turn with the intermediate storeys, the upper storeys, and finally with the lower storeys.

4.4.(b) THE INTERMEDIATE STOREYS

Consider Figure 27, which shows the complete subassemblage for both the internal and external bays of an intermediate storey, at ultimate load. In accordance with the assumptions in 4.4(a), plastic hinges exist at the leeward ends of the beams, and equal shear forces are induced as shown in the internal columns.

In order to simplify the development of the design equations, it is also assumed that the basic shear forces due to wind loading in the external columns $\left(\text{i.e. } \frac{H_1}{2} \text{ and } \frac{H_2}{2}\right)$ are magnified by the same factors as the shear forces in the internal columns (i.e., by $A_c m_1$ and $A_c m_2$). Thus, the bending moment in any column at the level of the beam, magnified to allow for the instability effects, may be obtained automatically by multiplying the shear force in the column by the distance to its point of contraflexure.

Additional shear forces, denoted by X, are assumed to exist in each external column, both above and below the beam, due to the action of the vertical loading at load factor λ_2. In the leeward columns, these act in the same sense as the shear forces due to wind loading. However, in the windward column, the two components of shear act in opposite directions. The assumption that these additional shear forces are equal both above and below the beam has been found to be reasonable for the general case, where the beam loads, and the column heights, are



Intermediate storey subassemblage - effective shear forces at ultimate load

FIGURE 27

approximately the same in each storey.

Consider the equilibrium of the critical leeward column in Figure 27. For equilibrium of bending moments at joint 2:-

$$B = \left(\lambda_2 \frac{A_c m_1 H_1}{2} + X\right)\frac{h_1}{2} + \left(\lambda_2 \frac{A_c m_2 H_2}{2} + X\right)\frac{h_2}{2}$$

Solving for X:-

$$X = \frac{\left[B - \lambda_2 \frac{A_c (mHh)av}{2}\right]}{\frac{(h_1 + h_2)}{2}}$$

However, the moment on the lower column is given by:-

$$M_{23} = - \left[\lambda_2 \frac{A_c m_2 H_2}{2} + X\right] \frac{h_2}{2}$$

Substituting for X:-

$$M_{23} = - \lambda_2 \frac{A_c m_2 H_2 h_2}{4} - \frac{h_2}{h_1 + h_2}.B + \frac{h_2}{h_1 + h_2}.\lambda_2 \frac{A_c (mHh)av}{2}$$

$$= - \frac{h_2}{h_1 + h_2}.B - \lambda_2 \frac{A_c m_2 H_2 h_2}{4}\left(1 - \frac{h_2}{h_1 + h_2}.2.\frac{(mHh)av}{m_2 H_2 h_2}\right)$$

$$= - \frac{h_2}{h_1 + h_2}.B - C_I.\frac{1}{2}\left(1 - \frac{h_2}{h_1 + h_2}.2p_2\right)$$

where C_I is the required plastic moment for the internal column in the lower storey, as obtained from equation (4) in Chapter 3. For the lower external column to remain elastic, its plastic moment, C_E, must be given by:-

$$C_E > |M_{23}| = \frac{h_2}{h_1 + h_2}.B + C_I.\frac{1}{2}\left(1 - \frac{h_2}{h_1 + h_2}.2p_2\right) \qquad (42)$$

In the particular case when $h_1 = h_2$,

$$C_E > \frac{B}{2} + C_I.\frac{1}{2}(1 - p_2) \qquad (43)$$

Also, for the upper column in the subassemblage, the following value of plastic moment is suggested:-

$$C_E > \frac{B}{2} \qquad (44)$$

Equations (42) and (43) together supply the two columns at the joint with a total moment capacity which is greater than the fully plastic moment of the beam. This slightly conservative condition is considered to be necessary. It must be remembered that all the design equations developed in this thesis are, to some degree, approximations. In this particular case, experience has shown that a small reserve of strength is advisable in order to ensure that a plastic hinge does not form in either column below the design load factor.

It may also be seen that, in applying equations (42) and (43) at every joint down the external column, two values of plastic moment are suggested for each column. Equation (42) gives the required plastic moment at the top of the column, whilst equation (43), when applied to the next storey down, supplies the value for the base of the column. For every column, therefore, the larger of these two values must be selected.

4.4(c) THE UPPER STOREYS

In this sub-section, the "upper storeys" are considered to consist of the top storey plus the second storey. These will be considered in turn.

4.4(c)(i) THE TOP STOREY

The design of the external column in this storey is straightforward. At the top of the leeward column, only two members are connected, and so, at any stage in the loading, the bending moments in both these members must be equal. The maximum possible moment in the beam is equal to its fully plastic moment, and a plastic hinge will form in the beam rather than in the column provided that:-

$$C_E > B$$

In fact, it may be argued that it is not necessary to restrict the plastic hinge to the beam at this location. The effect on the overall frame stiffness of the hinge forming in either the beam or the column is identical. However, in order to guard against local column buckling, and to be consistent with the basic design criteria, it is advisable to satisfy the condition above.

4.4(c)(ii) THE SECOND STOREY

The equations derived in 4.4(a), for the design of the external columns in the intermediate storeys, have been found to be unsatisfactory when applied to the second storey, particularly in the case when there is a large difference between the beam loads in the top two storeys.

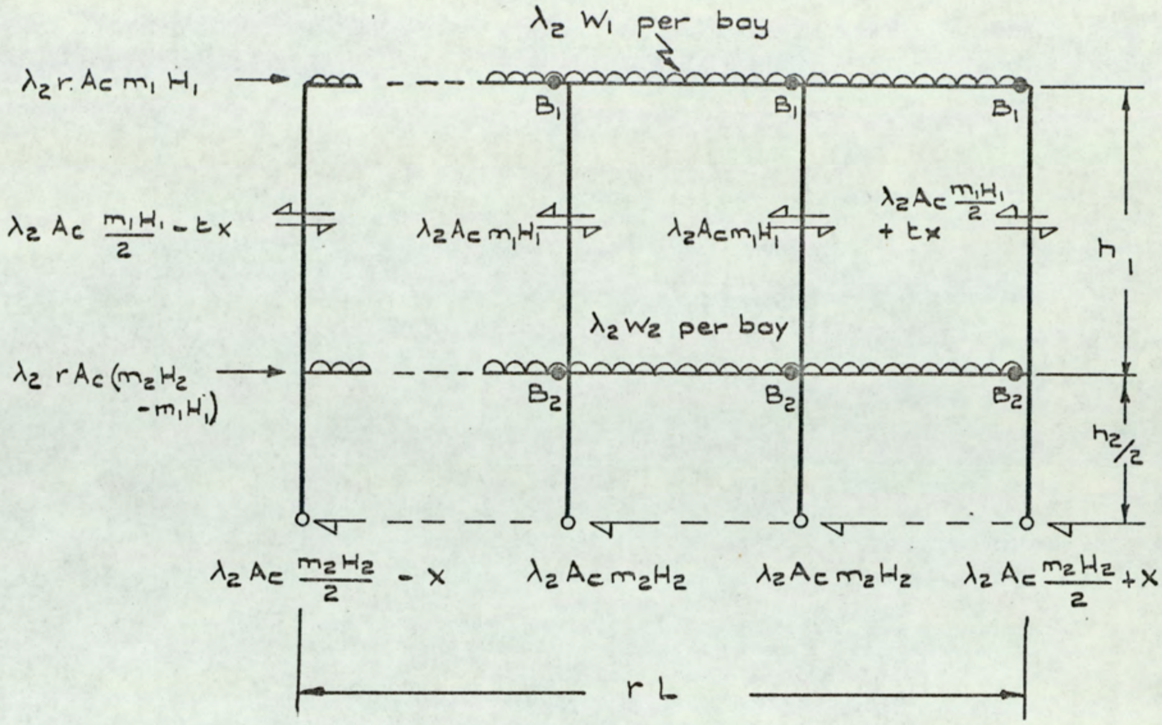
Therefore, a slightly modified approach is adopted in developing the equations for this storey, using the two-storey subassemblage shown in Figure 28(a). The shear forces due to vertical load alone in the two storeys are not assumed to be equal in this case, that in the lower external column being denoted by X, and that in the upper column by tX, where t is constant. Figure 28(a) shows the resultant shear forces under combined load, the components due to wind loading having been magnified in the same way as for the intermediate storeys of Figure 27.

Figure 28(b) shows the leeward column, isolated from the remainder of the subassemblage. In deriving this particular part of the theory, it is assumed that axial forces Y_1 and Y_2 exist in the beams, as shown. For horizontal equilibrium at joint 1:-

$$Y_1 = \lambda_2 \frac{A_C m_1 H_1}{2} + tX$$

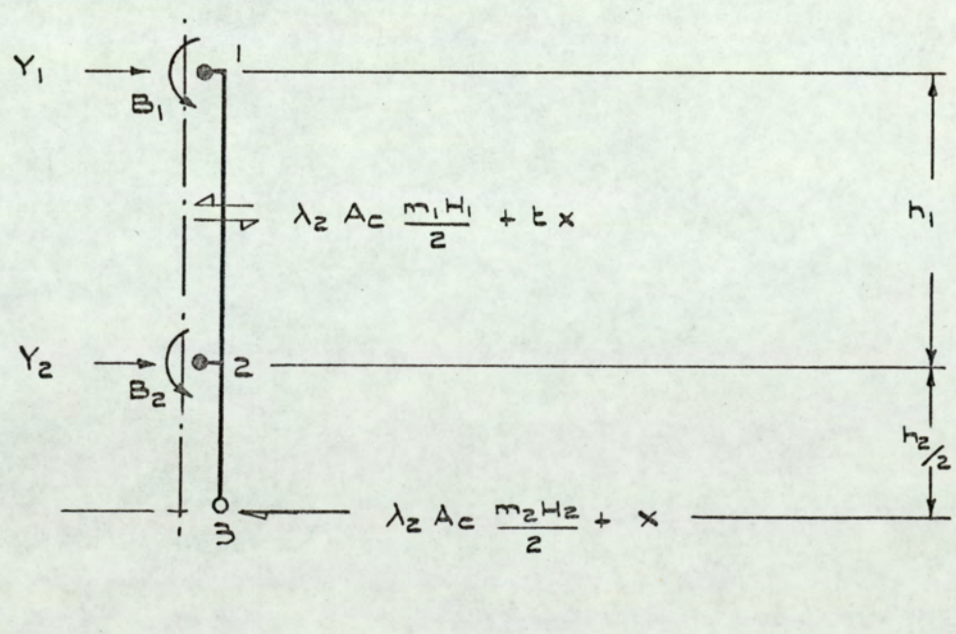
For horizontal equilibrium at joint 2:-

$$Y_2 = \lambda_2 \frac{A_C}{2} (m_2 H_2 - m_1 H_1) + X (1-t)$$



Subassemblage for the upper storeys — effective shear forces at ultimate load

FIGURE 28 (a)



Leeward external column - upper storeys

FIGURE 28 (b)

Taking moments about joint 3 (the point of contraflexure):-

$$Y_1.\left(h_1 + \frac{h_2}{2}\right) + Y_2.\frac{h_2}{2} - B_1 - B_2 = 0$$

Eliminating Y_1 and Y_2:-

$$\left[\lambda_2 \frac{A_c m_1 H_1}{2} + tX\right]\left(h_1 + \frac{h_2}{2}\right) + \left[\lambda_2 \frac{A_c}{2}(m_2H_2 - m_1H_1) + X(1-t)\right]\frac{h_2}{2}$$

$$= B_1 + B_2$$

Solving for X leads to:-

$$X = \frac{(B_1 + B_2) - \lambda_2 \frac{A_c}{2}\left[(mHh)av + \frac{m_1H_1h_1}{2}\right]}{\left(th_1 + \frac{h_2}{2}\right)}$$

The moment on the lower column is given by -

$$M_{23} = -\left(\lambda_2 \frac{A_c m_2 H_2}{2} + X\right)\frac{h_2}{2}$$

Substituting for X, and re-arranging, the following expression is obtained:-

$$M_{23} = -\frac{h_2}{(2th_1+h_2)}(B_1+B_2) - \lambda_2 \frac{A_c m_2 H_2 h_2}{4}\left\{1 - \frac{h_2}{(2th_1+h_2)}\left[2p_2 + \frac{m_1H_1h_1}{m_2H_2h_2}\right]\right\}$$

$$= -\frac{h_2}{(2th_1+h_2)}(B_1+B_2) - C_I . \frac{1}{2}\left[1 - \frac{h_2}{(2th_1+h_2)}(4p_2-1)\right]$$

where C_I is the required plastic moment for the internal column in the second storey, and since,

$$\frac{m_1H_1h_1}{m_2H_2h_2} = 2p_2 - 1$$

Therefore, if there is to be no plastic hinge in the external column,

$$C_E > |M_{23}| = \frac{h_2}{(2th_1+h_2)}(B_1+B_2) + C_I.\frac{1}{2}\left[1 - \frac{h_2}{(2th_1+h_2)}(4p_2-1)\right] \qquad (45)$$

The factor t, the ratio of the shears due to vertical load in the top two storeys, has been found to depend primarily on the ratio of the beam loads in these storeys (i.e. $\frac{W_1}{W_2}$). The following empirical values of t, obtained by inspection from the accurate analysis of several frames, are suggested:-

$$\text{When } W_1 = W_2, \; t \simeq 1;$$

$$\text{When } W_1 = \tfrac{1}{2}W_2, \; t \simeq \tfrac{3}{4};$$

For ratios of $\frac{W_1}{W_2}$ intermediate between $\frac{1}{2}$ and 1, t may be obtained from the following approximate formula, which tends to lead to a slightly conservative value for C_E:-

$$t = \frac{1}{2}\left(\frac{W_1}{W_2} + 1\right)$$

In the particular case when $h_1 = h_2$ (i.e. when the storey heights are equal), the general formula for C_E may be simplified considerably. For example:-

If $W_1 = W_2$,

$$C_E > \frac{1}{3}(B_1+B_2) + C_I.\frac{1}{2}\left[1 - \frac{1}{3}(4p_2-1)\right] \qquad (46)$$

If $W_1 = \frac{1}{2}W_2$,

$$C_E > \frac{2}{5}(B_1+B_2) + C_I.\frac{1}{2}\left[1 - \frac{2}{5}(4p_2-1)\right] \qquad (47)$$

Also, as in the intermediate storeys, the plastic moment of the upper storey column, at the lower joint, is given by:-

$$C_E > \frac{B_2}{2}$$

and this value should be checked against the value previously obtained for this column(i.e. B_1).

4.4(d) THE LOWER STOREYS

In the bottom storey, as in any other, the external column must be at least as large as that already selected for the storey above, in order to prevent "reverse column taper". This provisional section is generally found to be adequate, for the bending moments in the bottom storey column are normally smaller than those in the column above. Despite the fact that the selected column has a lower reduced plastic moment due to the presence of higher axial force in the bottom storey, there is sufficient reserve of strength.

The only exceptions to this are likely to occur in frameworks with relatively few storeys, and in which the wind ratio in the bottom storey is low. In these cases, the two column moments at the level of the bottom storey beam become similar in magnitude, and, in addition, both the moments in the lower column tend to be of the same order.

This argument is summarized below, with the aid of the notation in Figure 29.

In general:-

$M_{32} >>>> M_{23}$; $M_{21} > M_{32}$;

Column 23 has sufficient reserve of strength if given the same section as 21.

In cases of very low wind ratio:-

$M_{23} \rightarrow M_{32}$, and $M_{23} \rightarrow M_{21}$

Since $M_{21} + M_{23} = B$, then,

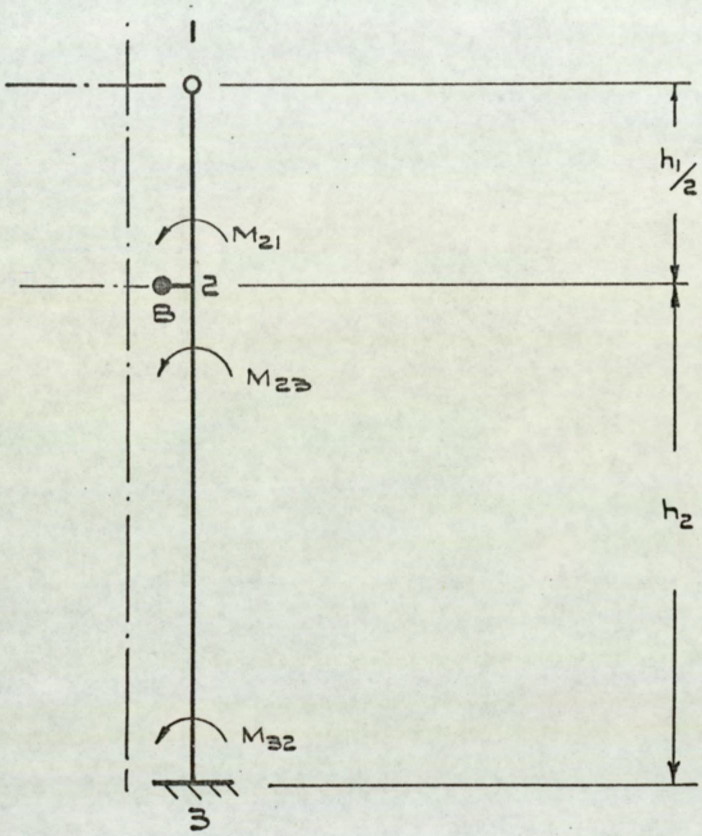
$M_{21} \rightarrow M_{23} \rightarrow M_{32} \rightarrow \frac{B}{2}$;

i.e., all moments tend to $\frac{B}{2}$.

Therefore, in this storey,

$C_E > \frac{B}{2}$

although, generally, the upper column is dominant.



Leeward external column - lower storeys - notation

FIGURE 29

4.4(e) SUMMARY OF EXTERNAL COLUMN DESIGN

STOREY No. 1:-

$$C_E > B$$

STOREY No. 2:-

In general,

$$C_E > \frac{h_2}{(2th_1+h_2)} (B_1+B_2) + C_I.\frac{1}{2}\left[1 - \frac{h_2}{(2th_1+h_2)} (4p_2-1)\right]$$

If $h_1 = h_2$, $W_1 = W_2$,

$$C_E > \frac{1}{3} (B_1+B_2) + C_I.\frac{1}{2}\left[1 - \frac{1}{3}(4p_2-1)\right]$$

If $h_1 = h_2$, $W_1 = \frac{1}{2}W_2$,

$$C_E > \frac{2}{5}(B_1+B_2) + C_I.\frac{1}{2}\left[1 - \frac{2}{5}(4p_2-1)\right]$$

INTERMEDIATE STOREYS:-

In general,

$$C_E > \frac{h_2}{h_1+h_2} .B + C_I.\frac{1}{2}\left(1 - \frac{h_2}{h_1+h_2}.2p_2\right)$$

If $h_1 = h_2$,

$$C_E > \frac{B}{2} + C_I.\frac{1}{2}(1-p_2)$$

BOTTOM STOREY:-

$$C_E > \frac{B}{2}$$

The relationships given above refer to the lower column at a joint. In addition, at any joint, the plastic moment of the upper column is given by:-

$$C_E > \frac{B}{2}$$

and this must be checked against the value obtained for this column at the joint above, the larger of the two values being selected. Furthermore, "reverse column taper" must be prevented.

The derivation of the design equations for multi-bay frames is now complete. The information contained in these two chapters is correlated in Chapters 6 and 7, where the design procedure is explained and a detailed design example is given. In the following chapter, the theory is modified to cater for the design of single-bay frames.

CHAPTER 5

SINGLE-BAY FRAMES

5.1. INTRODUCTION

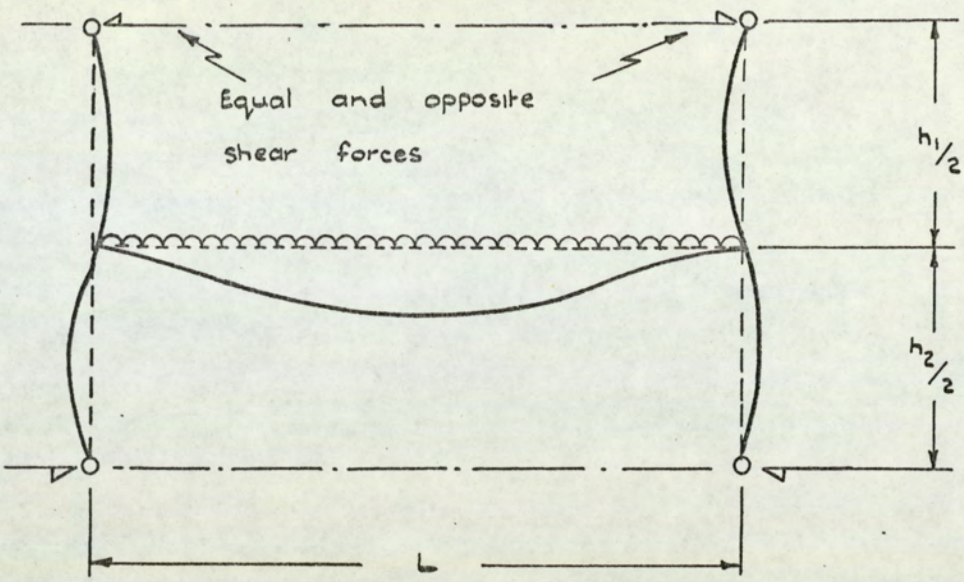
In the two preceding chapters, equations have been derived for the design of multi-bay frames, and these will now be modified, where appropriate, in order to cater for the particular case of a single-bay frame. The approach is similar to before, the general theory for a typical intermediate storey being developed initially, followed by consideration of the upper and lower storeys of the framework. The complete derivation of the design equations is not given in the cases where simple substitution may be made to equations developed previously.

5.2. THE INTERMEDIATE STOREYS

As for the multi-bay frames, each intermediate storey may be isolated from the others by assuming points of contraflexure to exist at the mid-heights of the columns. The basic subassemblage is shown in Figures 30(a) and 30(b) under the action of vertical load and wind load respectively.

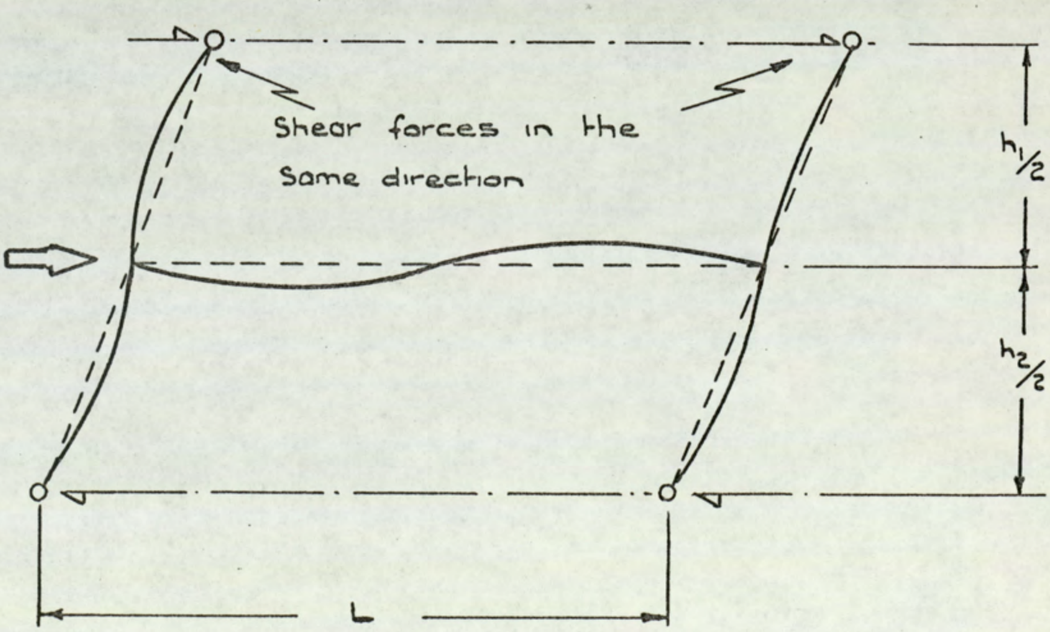
Under vertical load alone, the beam bends symmetrically, but in contrast with the multi-bay frame, the joints at the ends of the beam rotate, and bending moments and shear forces are induced in the columns. The shear forces at the points of contraflexure in each column of a storey are equal in magnitude, but act in opposite directions, as shown.

Under wind load alone, all the members deform skew-symmetrically, and the shear forces induced in each column of a storey act in the same direction.



Single - bay subassemblage - deformation
under vertical load alone

FIGURE 30 (a)



Single - bay subassemblage - deformation
under horizontal load alone

FIGURE 30 (b)

5.2(a) BEAM DESIGN

Provided that the columns remain elastic, under simple plastic conditions the subassemblage may fail by one of the three basic mechanisms described for the beams of the multi-bay frames. These are reproduced in Figures 31(a),(b) and (c), where all the loads which do not enter into the virtual work equations are omitted, and where each mechanism is based on the undeformed geometry of the subassemblage, in accordance with the simple plastic theory.

Thus, in Figure 31(a), the elastic rotation at the ends of the beam may be neglected. Also, in Figures 31(b) and (c), the shear forces due to vertical load alone are not included, since, considered together, they do no work as the structure sways. In developing a work equation for either the combined or the sway mechanism, it is not necessary to state the exact proportions in which the shear in a storey is distributed to the two columns. The sum of the work done by the individual column shears is simply equal to the work done by the total shear.

It follows, therefore, that in each case, the expression for the fully plastic moment of the beam is identical to that already obtained in Chapter 3, and, as before, the design may be considered to fall into one of three zones, depending on the wind ratio. Introducing the magnification factors, the beam design may therefore be summarized as follows:-

ZONE 1

$$0 \leq \frac{A(mHh)av}{WL} \leq \frac{1}{16}$$

$$B = \lambda_1 \frac{WL}{16}$$

ZONE 2

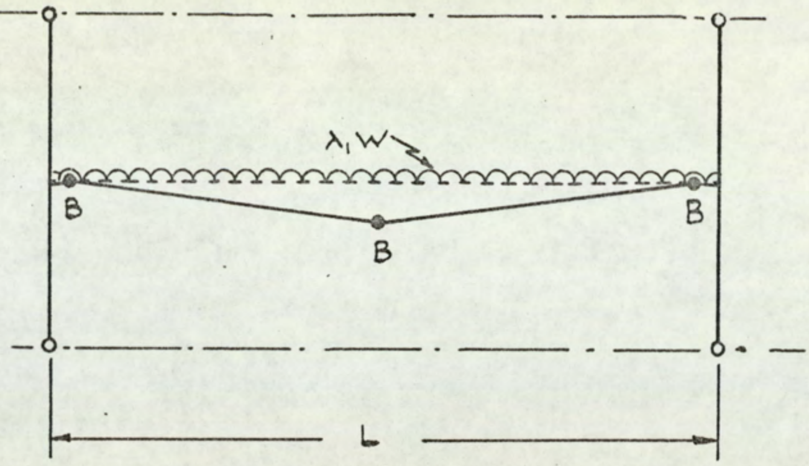
$$\frac{1}{16} \leq \frac{A(mHh)av}{WL} \leq \frac{1}{4}$$

$$B = \lambda_2 \frac{WL}{4}\left[\frac{1}{4} + \frac{A(mHh)av}{WL}\right]$$

ZONE 3

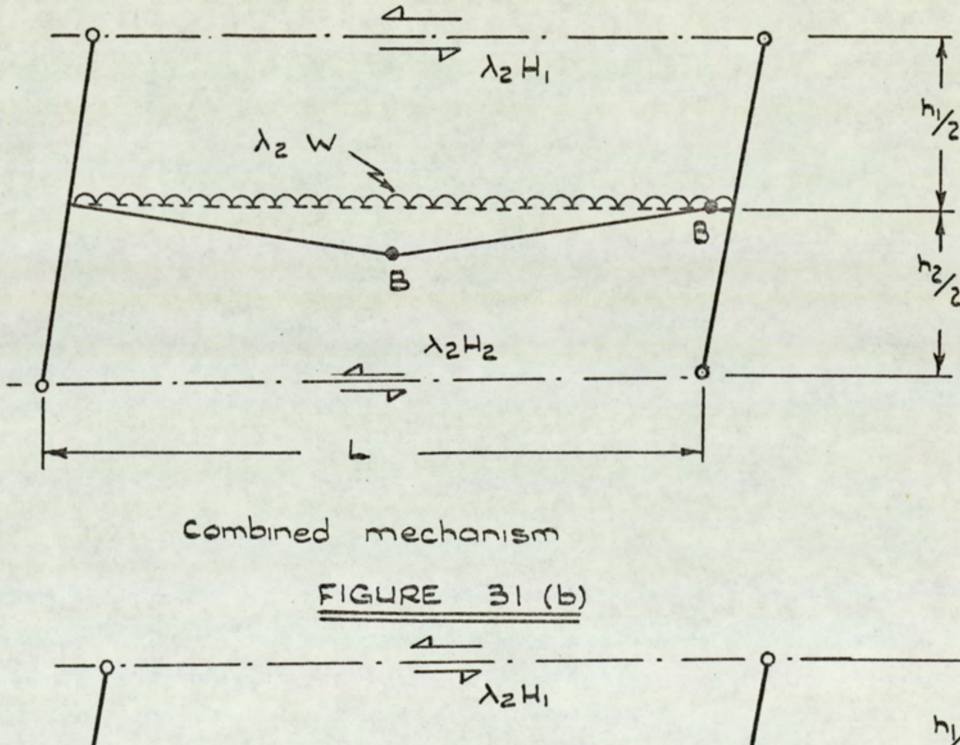
$$\frac{1}{4} \leq \frac{A(mHh)av}{WL}$$

$$B = \lambda_2 \frac{A(mHh)av}{2}$$



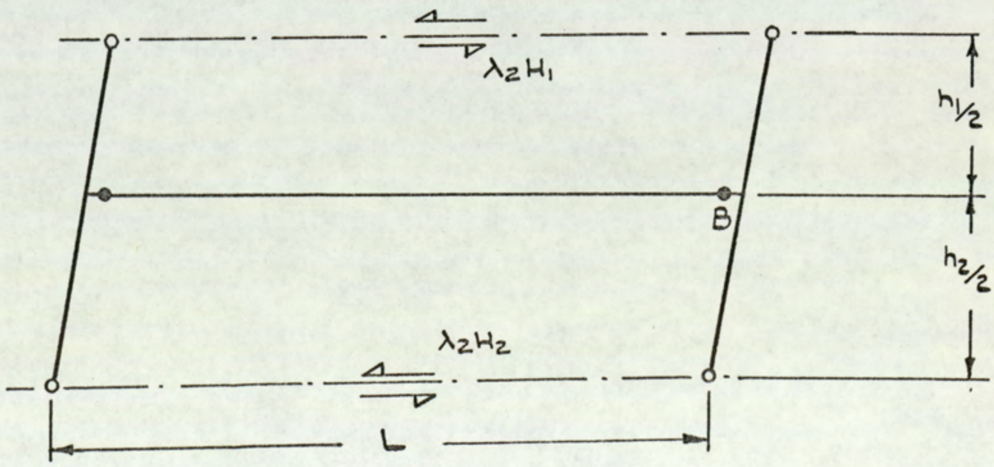
Beam mechanism

FIGURE 31 (a)



Combined mechanism

FIGURE 31 (b)



Sway mechanism

FIGURE 31 (c)

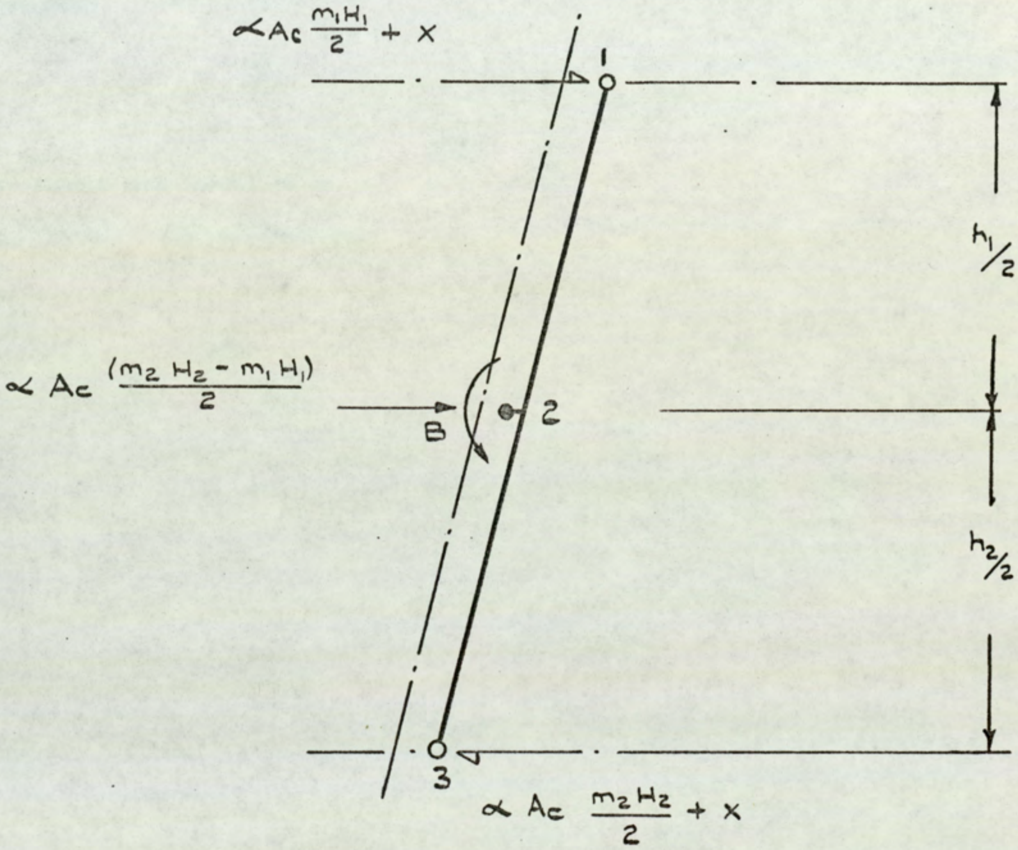
5.2(b) COLUMN DESIGN

The columns of a single-bay frame behave in a similar manner to the external columns of a multi-bay frame. Shear forces are induced by both the vertical and horizontal components of loading, and as shown in Figures 30(a) and 30(b), in the leeward column these shears act in the same direction, and are therefore cumulative. The combined shear forces in the windward column are always less than those in the leeward column, which is therefore the critical member for design.

In deriving the design equations for this critical column, it is assumed that at some stage in the loading, before the ultimate load is attained, a plastic hinge forms at the leeward end of the beam. Prior to the formation of this hinge, while the beam is still elastic, the stiffnesses of each joint at the ends of the beam are approximately equal, so that the total shear force in the storey due to wind loading is divided equally between the two columns. However, after the hinge has formed, the stiffness of the joint at the leeward end may be considered to be negligible compared to that at the windward end, and it is assumed that under additional wind loading, no more shear is distributed to the leeward column.

The shear forces in the leeward column at ultimate load, magnified to allow for instability, may therefore be represented statically as shown in Figure 32. X is the component of shear due to vertical load alone at λ_2, and, as before, this is assumed to be equal in the storeys above and below the beam. $\acute{\alpha}$ is the load factor for wind loading at which the first beam hinge forms, and A_c is the column magnification factor corresponding to α.

This distribution of shear forces is similar to that assumed for the leeward external column of a multi-bay frame, which is part of the sub-assemblage given in Figure 27 of Chapter 4. Comparing Figure 27 and 32, it may be seen that the only difference in the shear forces occurs in



Equilibrium of leeward column - ultimate combined load

FIGURE 32

the load factors applied to the wind components. It follows that similar expressions for the required plastic moment of the column may be obtained in each case. Referring to Section 4.4(b), it may be seen that the value of C_E for the single-bay frame may be obtained simply by substituting the quantity $\alpha \frac{A_c m_2 H_2 h_2}{2}$ for C_I in equations (42) and (43). Thus, from equation (42), in the general case when the storey heights vary:-

$$C_E > \frac{h_2}{h_1+h_2}.B + \alpha \frac{A_c m_2 H_2 h_2}{4}\left[1 - \frac{h_2}{h_1+h_2}.2p_2\right] \qquad (48)$$

In the particular case when $h_1 = h_2$, from equation (43):-

$$C_E > \frac{B}{2} + \alpha \frac{A_c m_2 H_2 h_2}{4}(1-p_2) \qquad (49)$$

In Section 4.4(c)(ii), corresponding expressions were derived for the external columns in the second storey from the top of a multi-bay framework, and the same substitution for C_I may be used to modify these. Therefore, from equation (45), in the second storey of a single bay frame:-

$$C_E > \frac{h_2}{(2th_1+h_2)}(B_1+B_2) + \alpha \frac{A_c m_2 H_2 h_2}{4}\left[1 - \frac{h_2}{(2th_1+h_2)}.(4p_2-1)\right] \qquad (50)$$

With the same values for t as before, in the particular case when $h_1 = h_2$, equations (46) and (47) lead to the following expressions:-

If $W_1 = W_2$,

$$C_E > \frac{1}{3}(B_1+B_2) + \alpha \frac{A_c m_2 H_2 h_2}{4}\left[1 - \frac{1}{3}(4p_2-1)\right] \qquad (51)$$

If $W_1 = \frac{1}{2}W_2$,

$$C_E > \frac{2}{5}(B_1+B_2) + \alpha \frac{A_c m_2 H_2 h_2}{4}\left[1 - \frac{2}{5}(4p_2-1)\right] \qquad (52)$$

5.2(c) DERIVATION OF THE MAGNIFICATION FACTORS

5.2(c)(i) ZONE 1

$$0 \leqslant \frac{A(mHh)av}{WL} \leqslant \frac{1}{16}$$

In this zone, under the vertical load alone, at load factor λ_1, a simple beam mechanism forms. However, the magnification factors are derived by consideration of the combined loading case.

Consider first the application of the vertical load component at load factor λ_2. The subassemblage, assumed to be fully elastic at this stage, is shown in Figure 33(a), where K_{c1}, K_{c2} and K_{b2} represent the flexural rigidities of the members, and where ρ_1 and ρ_2 are the Euler ratios.

Under this loading, the beam bends symmetrically, so that its real stiffness at either end may be taken as $2K_{b2}$. The columns bend skew-symmetrically, and since there is no side-sway, their stiffness may be taken as $6K_c$. In practice, due to the presence of axial load in the columns, their real stiffnesses are slightly less than $6K_c$, but this small reduction may be neglected for the normal range of Euler ratios. So, for vertical load alone, the total stiffness of either joint is given approximately by:-

$$\Sigma \overset{r}{s} = 6(K_{c1}+K_{c2}) + 2K_{b2}$$

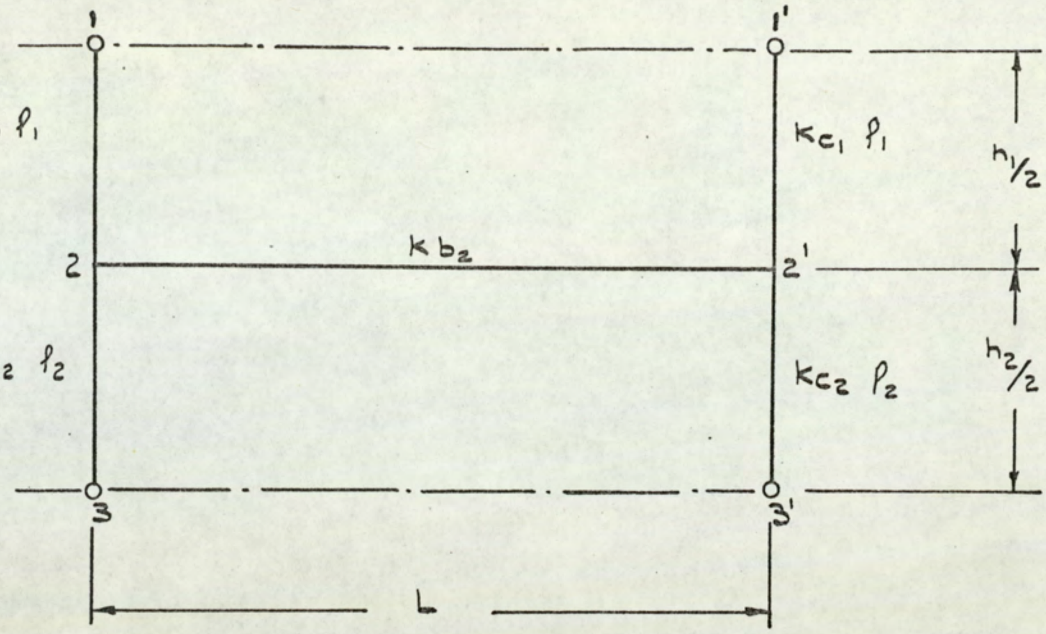
$$= 2K_{b2}\left[3\,\frac{K_{c1}+K_{c2}}{K_{b2}} + 1\right]$$

$$= 2K_{b2}\,(3x + 1)$$

where,

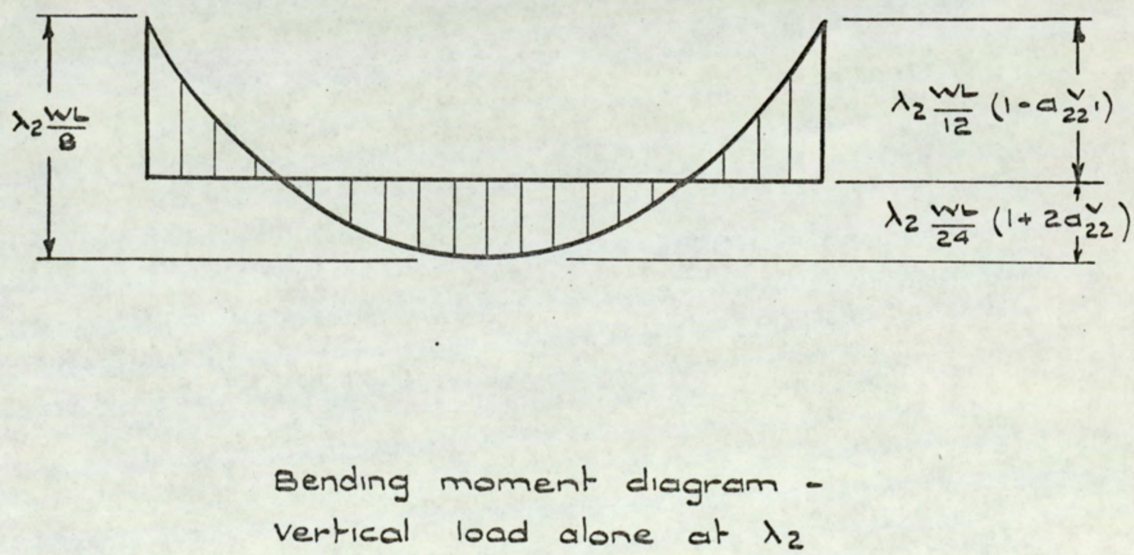
$$x = \frac{K_{c1}+K_{c2}}{K_{b2}} = K_2(\overline{K}+1)$$

The real distribution factor for either end of the beam is:-



Single-bay subassemblage - joint numbers, $\frac{"EI"}{L}$ values and Euler ratios

FIGURE 33 (a)



Bending moment diagram - vertical load alone at λ_2

FIGURE 33 (b)

$$a_{22}^{v\prime} = \frac{2K_{b2}}{\Sigma s^r} = \frac{1}{3x+1}$$

At load factor λ_2, the fixed-end moments are equal to $\pm \lambda_2 \frac{WL}{12}$, and the resulting bending moments in the beam are given in Figure 33(b). The maximum moments occur at the ends of the beam, and are of magnitude $\lambda_2 \frac{WL}{12}(1-a_{22}^{v\prime})$.

In this zone;

$$B = \lambda_1 \frac{WL}{16} = \frac{5}{64} \lambda_2 WL$$

Therefore, no plastic hinge will form in the beam under vertical load alone if:-

$$\lambda_2 \frac{WL}{12} (1-a_{22}^{v\prime}) < \frac{5}{64} \lambda_2 WL$$

Eliminating $\lambda_2 WL$, and substituting for $a_{22}^{v\prime}$, the condition becomes:-

$$\frac{1}{12}\left[1 - \frac{1}{3x+1}\right] < \frac{5}{64}$$

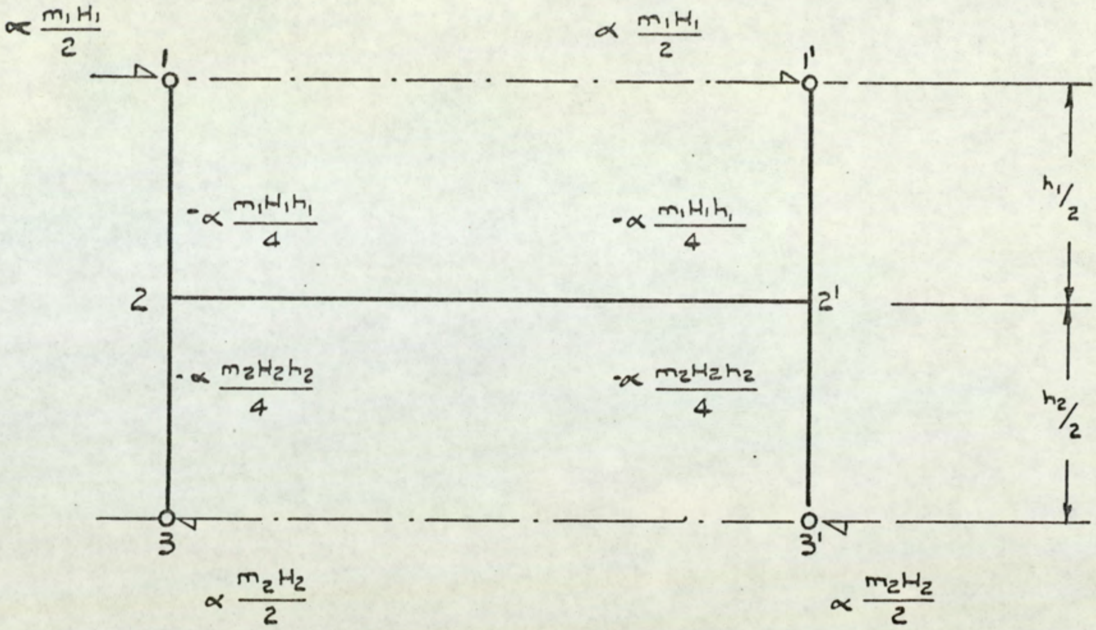
i.e:-

$$x < 5$$

or, alternatively,

$$K_{c1} + K_{c2} < 5K_{b2}$$

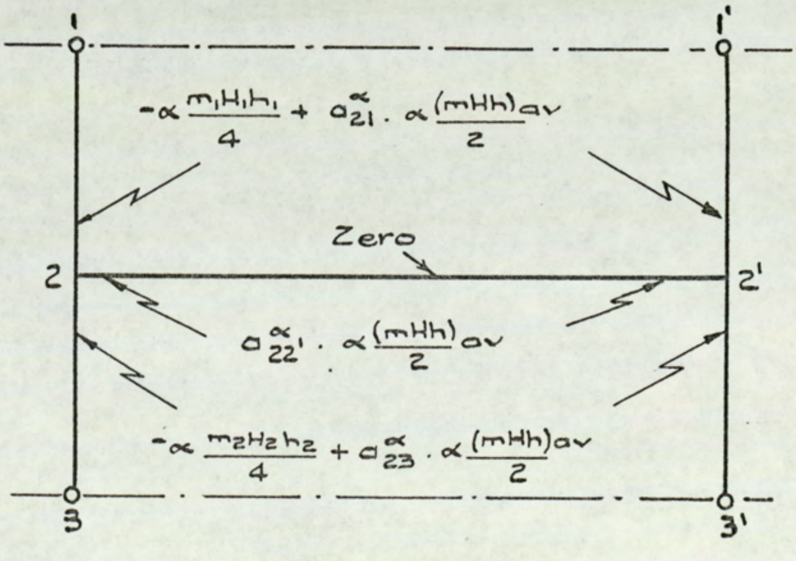
It is extremely unlikely that this condition will ever be violated, and even if it is, the effect on the overall design will be very small. Therefore, it is safe to assume that plastic hinges never form in the beam under vertical load alone, at load factor λ_2, so that the subassemblage may be considered to be fully elastic on application of the horizontal loading. While the beam remains elastic, the joint stiffnesses at each end are equal, so that half of the shear is applied to each column. Figure 34(a) shows the initial sway moments, magnified to allow for the Pδ effect, due to wind loading at load factor α. The total out of balance



Initial sway moments —

wind load alone at load factor α

FIGURE 34 (a).



Final moments -

wind load alone at load factor α.

FIGURE 34 (b)

moment at each end of the beam is given by:-

$$F.E.M. = - \alpha \frac{(mHh)av}{2}$$

The real stiffnesses of the members are identical on either side of the frame under this loading; i.e.,

$$S^r_{21} = (n_1-o_1)\ K_{c1}$$

$$S^r_{23} = (n_2-o_2)\ K_{c2}$$

$$S^r_{22}{}' = 6K_{b2}$$

The joint stiffnesses are therefore equal, and given by:-

$$\Sigma S^r = (n_1-o_1)\ K_{c1} + (n_2-o_2)\ K_{c2} + 6K_{b2}$$

The real distribution factors become:-

$$a^\alpha_{21} = \frac{(n_1-o_1)\overline{K}K_2}{K_2V + 6}$$

$$a^\alpha_{23} = \frac{(n_2-o_2)K_2}{K_2V + 6}$$

$$a^\alpha_{22}{}' = \frac{6}{K_2V + 6}$$

where,

$$V = (n_1-o_1)\overline{K} + (n_2-o_2)$$

The resulting bending moments due to wind loading alone are given in Figure 34(b). For a combination of vertical and horizontal loading, the bending moments in the beam may be obtained by superimposing Figures 33(b) and 34(b). The critical beam moment occurs at joint 2′, and a plastic hinge will form at this location at some value of load factor, $\overline{\alpha}$ say, given by:-

$$\lambda_2\frac{WL}{12}(1-a^v_{22}{}') + a^\alpha_{22}{}'.\overline{\alpha}\ \frac{(mHh)av}{2} = B = \frac{5}{64}\ \lambda_2 WL \qquad (53)$$

Therefore,

$$\bar{\alpha}.a_{22}^{\alpha}{}'.(mHh)av = \lambda_2 WL \left[\frac{5}{32} - \frac{1}{6}\,(1-a_{22}^{v}{}')\right]$$

$$= \lambda_2 \frac{WL}{32} \left[5 - \frac{16}{3}\left(1 - \frac{1}{3x+1}\right)\right]$$

$$= \lambda_2 \frac{WL}{32} \left[5 - \frac{16x}{3x+1}\right]$$

i.e.,

$$\bar{\alpha} = \lambda_2 \frac{WL}{32a_{22}^{\alpha}{}'(mHh)av} \left[\frac{5-x}{3x+1}\right]$$

$\bar{\alpha}$ may be either greater than or less than λ_2. If, for instance, $\bar{\alpha} > \lambda_2$, then,

$$\lambda_2 \frac{WL}{32a_{22}^{\alpha}{}'(mHh)av} < \frac{1}{32} \left[\frac{5-x}{3x+1}\right]$$

i.e.

$$\frac{a_{22}^{\alpha}{}'(mHh)av}{WL} < \frac{1}{32} \left[\frac{5-x}{3x+1}\right]$$

In this case, the beam, which is designed basically to withstand vertical load alone at λ_1, is in fact still fully elastic under combined loading at λ_2. At this load factor, the bending moments at the critical locations, due to wind loading alone, are as shown in Figure 35, which indicates that the beam distribution factor, $a_{22}^{\alpha}{}'$, is in fact the magnification factor. This is referred to as Zone 1(i).

ZONE 1(i)

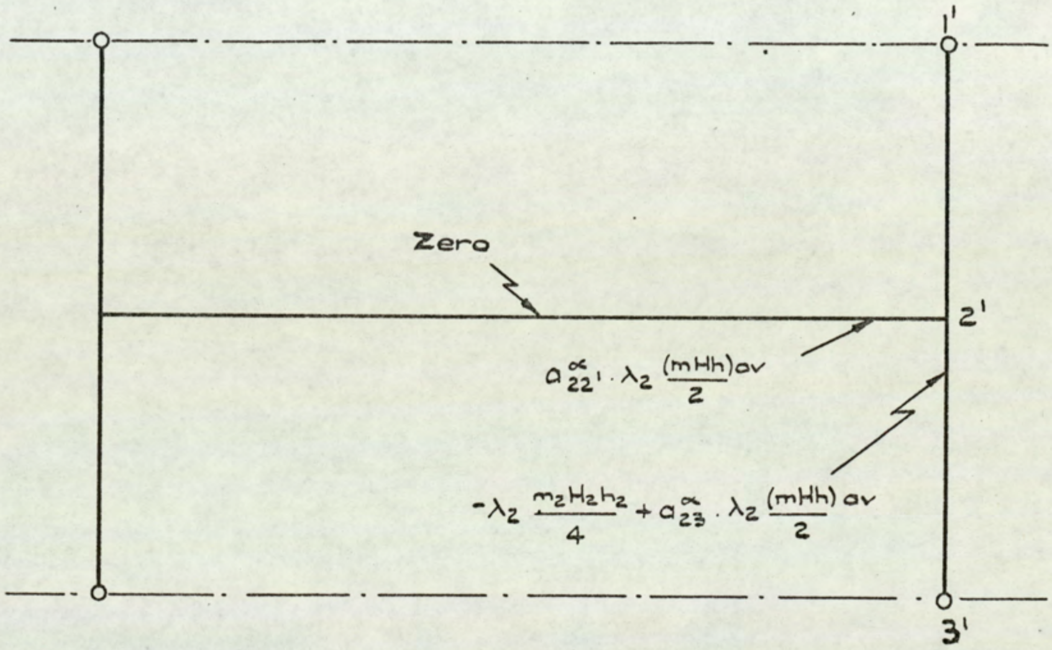
$$0 \leqslant \frac{A(mHh)av}{WL} \leqslant \frac{1}{32} \left[\frac{5-x}{3x+1}\right]$$

$$A = a_{22}^{\alpha}{}'$$

Referring to Figure 35, the component of the total bending moment in the lower column which is due to wind loading is given by:-

$$M_W = -\lambda_2 \frac{m_2H_2h_2}{4} + a_{23}^{\alpha}.\lambda_2 \frac{(mHh)av}{2}$$

$$= -\lambda_2 \frac{m_2H_2h_2}{4}\,(1-2a_{23}^{\alpha}.p_2)$$



Resultant bending moments due to wind loading -

Zone 1 (i)

FIGURE 35

Under simple plastic conditions,

$$a_{23}^{\alpha} = 0$$

so that

$$M_W = -\lambda_2 \frac{H_2h_2}{4}$$

Therefore, the factor $(1-2a_{23}^{\alpha}.p_2)$ represents the degree by which the basic moment in the lower column is magnified by instability effects. i.e.,

$$A_c = 1 - 2a_{23}^{\alpha}.p_2$$

ZONE 1(ii) $\quad \frac{1}{32}\left[\frac{5-x}{3x+1}\right] \leq \frac{A(mHh)av}{WL} \leq \frac{1}{16}$

In this zone, a plastic hinge develops at the leeward end of the beam before the full wind load is applied, at a load factor α, where:-

$$\frac{\alpha}{\lambda_2} = \frac{WL}{32a_{22}^{\alpha}/(mHh)av}\left[\frac{5-x}{3x+1}\right]$$

The behaviour of the frame is identical to that for Zones 2 and 3, and the magnification factors are of the same form.

5.2(c)(ii) ZONE 2

$$\frac{1}{16} \leq \frac{A(mHh)av}{WL} \leq \frac{1}{4}$$

In this zone, as for Zone 1(ii), the beam is fully elastic after the full vertical load has been applied, and remains so under wind loading until some load factor α. The bending moments due to the vertical and horizontal loading respectively are as given previously in Figures 33(b) and 34(b). In this zone, however,

$$B = \lambda_2 \frac{WL}{4}\left[\frac{1}{4} + \frac{A(mHh)av}{WL}\right]$$

Therefore, for a plastic hinge to form at 2$'$ at load factor α:-

$$\lambda_2 \frac{WL}{12}\left(1-a_{22}^{v}{}'\right) + a_{22}^{\alpha}{}'.\alpha\,\frac{(mHh)av}{2} = B = \lambda_2\frac{WL}{16} + \lambda_2\,\frac{A(mHh)av}{4} \qquad (54)$$

Solving for α:-

$$\alpha = \frac{2}{a_{22}^{\alpha}{}'.(mHh)av}\left\{\lambda_2\frac{WL}{4}\left[\frac{1}{4} - \frac{(1-a_{22}^{v}{}')}{3}\right] + \lambda_2\,\frac{A(mHh)av}{4}\right\}$$

$$= \frac{\lambda_2}{a_{22}^{\alpha}{}'}\left\{\frac{WL}{2(mHh)av}\left[\frac{1}{4} - \frac{1}{3}\left(1 - \frac{1}{3x+1}\right)\right] + \frac{A}{2}\right\}$$

In an alternative form:-

$$\frac{\alpha}{\lambda_2} = \frac{A}{a_{22}^{\alpha}{}'}\left\{\frac{1}{2} + \frac{WL}{2A(mHh)av}\left[\frac{1}{4} - \frac{x}{3x+1}\right]\right\}$$

$$= \frac{A}{a_{22}^{\alpha}{}'}\left\{\frac{1}{2} + \frac{WL}{8A(mHh)av}\left[\frac{1-x}{3x+1}\right]\right\}$$

On application of further wind loading, in the range α to λ_2, all the shear may be considered to act on the windward column, the moments in the leeward column remaining unaltered. The initial sway moments for this additional wind load are shown in Figure 36(a), from which it may be seen that the total out of balance moment at joint 2 is given by:-

$$F.E.M._2 = -(\lambda_2-\alpha)(mHh)av$$

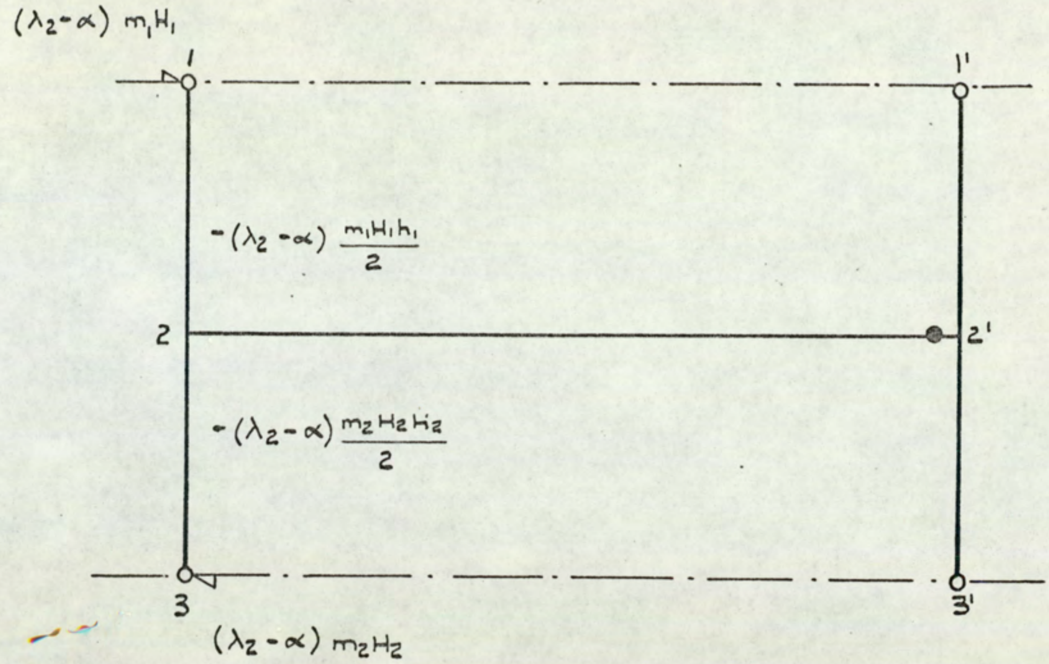
Due to the formation of the plastic hinge at 2$'$, the beam stiffness at 2 is reduced to $3K_{b2}$, and the real distribution factors become:-

$$a_{21}^{\Delta} = \frac{(n_1-o_1)\overline{K}K_2}{K_2V + 3}$$

$$a_{23}^{\Delta} = \frac{(n_2-o_2)K_2}{K_2V + 3}$$

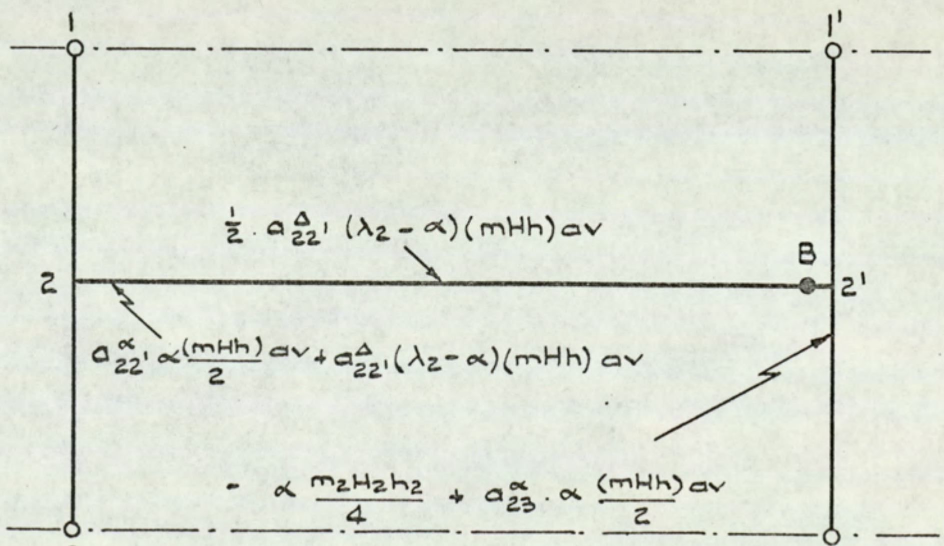
$$a_{22}^{\Delta}{}' = \frac{3}{K_2V + 3}$$

These are identical to those obtained previously for the multi-bay frames.



Zones 1 (ii), 2 and 3 - initial sway moments due to wind load in the range α to λ_2

FIGURE 36 (a)



Zones 1 (ii), 2 and 3 - final moments due to wind load at λ_2

FIGURE 36 (b).

The final moments at the critical location due to the full wind loading are shown in Figure 36(b). In this zone, collapse occurs when a second plastic hinge forms at the centre of the beam. Therefore, combining the moments from Figures 33(b) and 36(b):-

$$\lambda_2 \frac{WL}{24} (1+2a_{22}^{v}{}') + \frac{1}{2}.a_{22}^{\Delta}{}'(\lambda_2-\alpha)(mHh)av = B \qquad (55)$$

However, from equation (54),

$$\lambda_2 \frac{WL}{12} (1-a_{22}^{v}{}') = B - a_{22}^{\alpha}{}'.\alpha \frac{(mHh)av}{2}$$

Therefore,

$$\lambda_2 \frac{WL}{12}.a_{22}^{v}{}' = \lambda_2 \frac{WL}{12} - B + a_{22}^{\alpha}{}'.\alpha \frac{(mHh)av}{2}$$

Eliminating $a_{22}^{v}{}'$ in equation (55):-

$$\lambda_2 \frac{WL}{24} + \lambda_2 \frac{WL}{12} - B + a_{22}^{\alpha}{}'.\alpha \frac{(mHh)av}{2} + \frac{1}{2}.a_{22}^{\Delta}{}'(\lambda_2-\alpha)(mHh)av = B$$

Therefore,

$$2B = \lambda_2 \frac{WL}{8} + \frac{(mHh)av}{2}\left[\alpha.a_{22}^{\alpha}{}' + (\lambda_2-\alpha)a_{22}^{\Delta}{}'\right]$$

i.e.,

$$B = \lambda_2 \frac{WL}{4}\left\{\frac{1}{4} + \left[\frac{\alpha}{\lambda_2}.a_{22}^{\alpha}{}' + \left(1-\frac{\alpha}{\lambda_2}\right).a_{22}^{\Delta}{}'\right].\frac{(mHh)av}{WL}\right\}$$

However,

$$B = \lambda_2 \frac{WL}{4}\left[\frac{1}{4} + A.\ \frac{(mHh)av}{WL}\right]$$

Therefore,

$$A = \frac{\alpha}{\lambda_2}.a_{22}^{\alpha}{}' + \left(1 - \frac{\alpha}{\lambda_2}\right).\ a_{22}^{\Delta}{}'$$

The moment due to wind loading alone on the leeward column is again given by:-

$$M_W = - \alpha \frac{m_2H_2h_2}{4} + a_{23}^{\alpha}.\alpha \frac{(mHh)av}{2}$$

$$= - \alpha\frac{m_2H_2h_2}{4} (1-2a_{23}^{\alpha}.p_2)$$

i.e., as in zone 1,

$$A_c = 1-2a_{23}^{\alpha}.p_2$$

5.2(c)(iii) ZONE 3

$$\frac{1}{4} \leq \frac{A(mHh)av}{WL}$$

In this zone, the expressions for A and A_c are identical to those for Zone 2. However, the value of α is different. The maximum bending moment in the beam is as given in equation (54), but, in this case,

$$B = \lambda_2 \frac{A(mHh)av}{2}$$

Therefore, α is obtained from:-

$$\lambda_2\frac{WL}{12} (1-a_{22}^{v}{}') + a_{22}^{\alpha}{}'.\alpha \frac{(mHh)av}{2} = B = \lambda_2 \frac{A(mHh)av}{2} \qquad (56)$$

This relationship eventually reduces to :-

$$\frac{\alpha}{\lambda_2} = \frac{A}{a_{22}^{\alpha}{}'} \left\{1 - \frac{WL}{2A(mHh)av} \left[\frac{x}{3x+1}\right]\right\}$$

The design equations for all the zones are summarized in the following sub-section.

5.2(d) SUMMARY OF DESIGN EQUATIONS

ZONE 1(i) $\quad 0 \leq \frac{A(mHh)av}{WL} \leq \frac{1}{32} \left[\frac{5-x}{3x+1}\right]$

$$B = \lambda_1 \frac{WL}{16}$$

$$A = a_{22}^{\alpha}{}'$$

For column design,

$$\alpha = \lambda_2$$

ZONE 1(ii) $\frac{1}{32}\left[\frac{5-x}{3x+1}\right] \leqslant \frac{A(mHh)av}{WL} \leqslant \frac{1}{16}$

$$B = \lambda_1 \frac{WL}{16}$$

$$A = \frac{\alpha}{\lambda_2}.a_{22}^{\alpha}{}' + \left(1 - \frac{\alpha}{\lambda_2}\right).\ a_{22}^{\Delta}{}'$$

$$\frac{\alpha}{\lambda_2} = \frac{WL}{32a_{22}^{\alpha}{}'(mHh)av}\ \left[\frac{5-x}{3x+1}\right]$$

ZONE 2 $\frac{1}{16} \leqslant \frac{A(mHh)av}{WL} \leqslant \frac{1}{4}$

$$B = \lambda_2\ \frac{WL}{4}\ \left[\frac{1}{4} + \frac{A(mHh)av}{WL}\right]$$

$$A = \frac{\alpha}{\lambda_2}.\ a_{22}^{\alpha}{}' + \left(1 - \frac{\alpha}{\lambda_2}\right).\ a_{22}^{\Delta}{}'$$

$$\frac{\alpha}{\lambda_2} = \frac{A}{a_{22}^{\alpha}{}'}\left\{\frac{1}{2} + \frac{WL}{8A(mHh)av}\ \left[\frac{1-x}{3x+1}\right]\right.$$

ZONE 3 $\frac{1}{4} \leqslant \frac{A(mHh)av}{WL}$

$$B = \lambda_2\ \frac{A(mHh)av}{2}$$

$$A = \frac{\alpha}{\lambda_2}.a_{22}^{\alpha}{}' + \left(1 - \frac{\alpha}{\lambda_2}\right).a_{22}^{\Delta}{}'$$

$$\frac{\alpha}{\lambda_2} = \frac{A}{a_{22}^{\alpha}{}'}\left\{1 - \frac{WL}{2A(mHh)av}\ \left[\frac{x}{3x+1}\right]\right\}$$

In all zones,

$$A_c = 1 - 2a_{23}^{\Delta}.p_2$$

The column size is to be selected from the following equations.

In the second storey from the top, in general,

$$C_E > \frac{h_2}{(2th_1+h_2)}(B_1+B_2) + \alpha\frac{A_c m_2 H_2 h_2}{4}\left[1 - \frac{h_2}{(2th_1+h_2)}.(4p_2-1)\right]$$

For the particular case when $h_1 = h_2$:-

if $W_1 = W_2$, then

$$C_E > \frac{1}{3}(B_1 + B_2) + \alpha\frac{A_c m_2 H_2 h_2}{4}\left[1 - \frac{1}{3}(4p_2 - 1)\right]$$

if $W_1 = \frac{1}{2}W_2$, then,

$$C_E > \frac{2}{5}(B_1+ B_2) + \alpha\frac{A_c m_2 H_2 h_2}{4}\left[1 - \frac{2}{5}(4p_2-1)\right]$$

In all other intermediate storeys, in general,

$$C_E > \frac{h_2}{h_1+h_2}.B + \alpha\frac{A_c m_2 H_2 h_2}{4}\left[1 - \frac{h_2}{h_1 + h_2}.2p_2\right]$$

For the particular case when $h_1 = h_2$,

$$C_E > \frac{B}{2} + \alpha\frac{A_c m_2 H_2 h_2}{4}(1-p_2)$$

The equations above all apply to the lower column in the design of any storey. In addition, at any joint, the following condition for the required plastic moment of the upper column must be checked:-

$$C_E > \frac{B}{2}$$

The distribution factors are defined as follows:-

$$a^{\alpha}_{22}{}' = \frac{6}{K_2V + 6}; \quad a^{\alpha}_{23} = \frac{(n_2-o_2)K_2}{K_2V + 6};$$

$$a^{\Delta}_{22}{}' = \frac{3}{K_2V + 6};$$

As before,

$$V = (n_1-o_1)\overline{K} + (n_2-o_2)$$

and,

$$\overline{K} = \frac{K_{c1}}{K_{c2}}; \quad K_2 = \frac{K_{c2}}{K_{b2}};$$

Also,

$$x = \frac{K_{c1} + K_{c2}}{K_{b2}}$$

and,

$$p_2 = \frac{(mHh)av}{m_2H_2h_2}$$

5.2(e) ELASTICITY OF THE BEAM AT WORKING LOAD

The condition for the beams of a multi-bay frame to remain elastic at working load has been derived previously in 3.10. In the case of a single-bay frame, a similar expression is obtained for the critical moment at the leeward end, and the condition becomes,

$$\frac{WL}{12}(1-a^{v}_{22}{}') + a^{1}_{22}{}' \frac{(m^1Hh)av}{2} \leqslant Mp_b \qquad (57)$$

where, in this case,

$$a^1_{22}{}' = \frac{6}{K_2V^1 + 6}$$

and, as before,

$$V^1 = (n_1-o_1)^1\overline{K} + (n_2-o_2)^1$$

5.3. THE TOP STOREY

In section 4.2., it has been shown that, for multi-bay frames, the simple beam mechanism is the critical mode of failure for the top storey, provided that:-

If $W_1 = W_2$:-

$$\frac{(Hh)av}{W_2L} \leqslant \frac{1}{4}$$

and if $W_1 = \frac{1}{2}W_2$:-

$$\frac{(Hh)av}{W_2L} \leq \frac{1}{8}$$

where,

W_1 = the load on the top storey beam.

W_2 = the load on the second storey beam.

$\frac{(Hh)av}{W_2L}$ = the wind ratio for the second storey.

Identical conditions are obtained when considering single-bay frames. However, the wind ratio in any particular storey of a single-bay frame is likely to be greater than that in the corresponding multi-bay frame. Nevertheless, it is still unlikely that these conditions will ever be violated. In the design examples of Chapter 7 and 8, the maximum value of wind ratio obtained for the second storey of a single-bay frame was $\frac{1}{9}$, and this occurred for a frame under extremely heavy wind loading, and with $W_1 = W_2$.

Thus, as before, it may be assumed that in any frame the size of the top storey beam is controlled by the vertical loading case. Also, in order to restrict the formation of plastic hinges to the beam, the reduced plastic moment of the column must always be greater than the fully plastic moment of the beam.

The design equations for the top storey may therefore be summarized as follows:-

$$B = \lambda_1 \frac{W_1L}{16}$$

$$C_E > B$$

5.4. THE LOWEST STOREY

5.4(a) BEAM DESIGN

In this region, as in the multi-bay frames, due to the fixity of the column feet and the fact that no plastic hinges are allowed in the columns below the design load factor, the only true mechanism possible is the simple beam mechanism. Therefore, irrespective of the intensity of wind loading,

$$B = \lambda_1 \frac{WL}{16}$$

5.4(b) COLUMN DESIGN

The column size is assessed from consideration of the combined loading case. Points of contraflexure are assumed to exist at the mid-heights of the upper column, and the resulting subassemblage is shown in Figure 37(a).

Under vertical load alone, the subassemblage deforms symmetrically. As stated in 5.2(c)(i), since there is no sidesway, the effect of axial loads on the real stiffnesses of the columns is very small, and may be considered to be negligible. Therefore, due to symmetry,

$$S^r_{22}{}' = 2K_{b2}$$

Due to skew-symmetry in the upper column,

$$S^r_{21} = 6K_{c1}$$

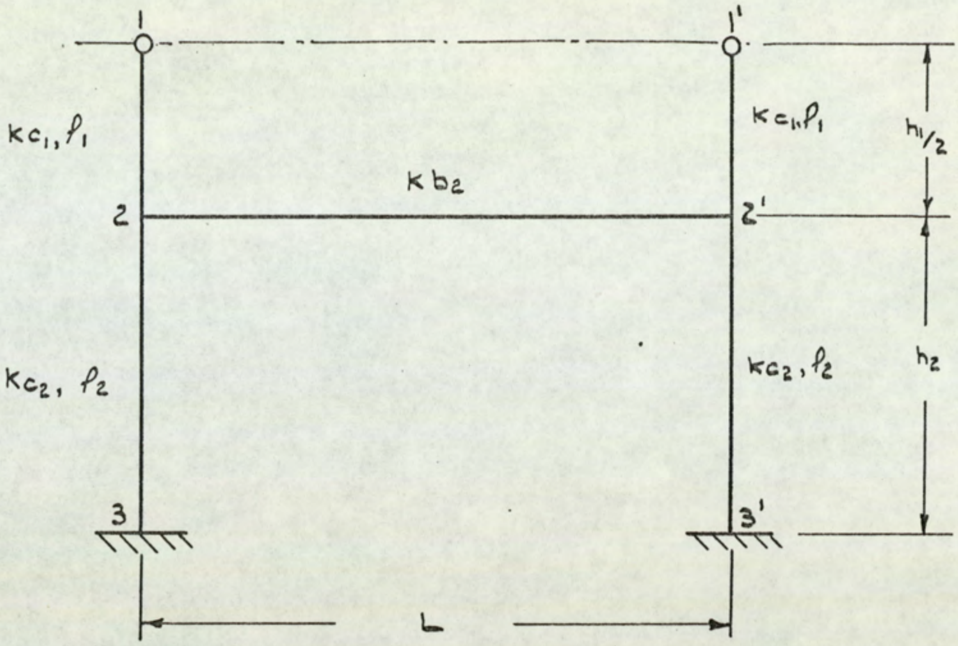
Due to the fixity at the base of the lower column,

$$S^r_{23} = 4K_{c2}$$

At either end of the beam:-

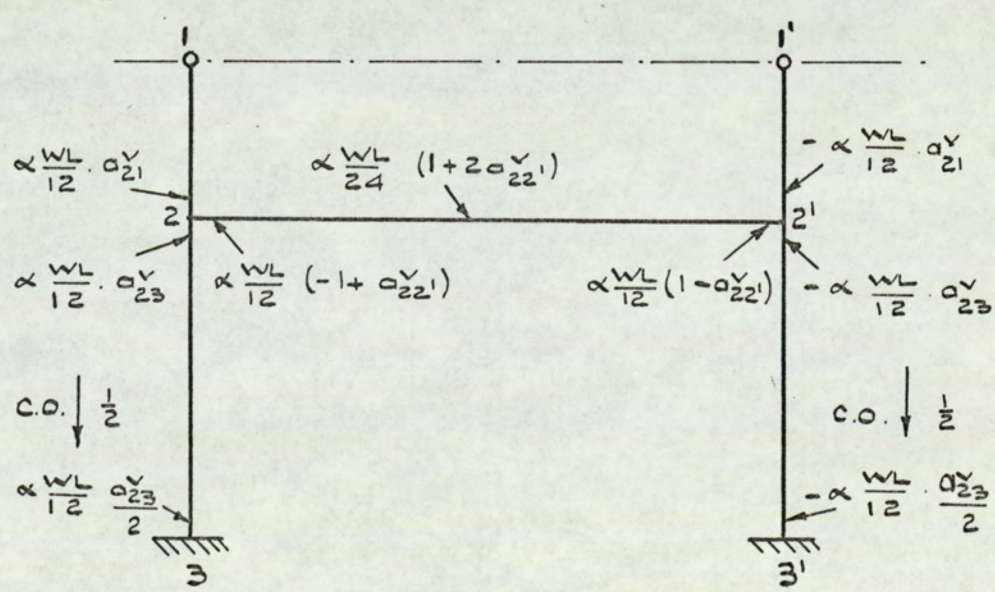
$$\Sigma S^r = 6\ K_{c1} + 4K_{c2} + 2K_{b2}$$

$$= 2K_{b2}\left[\frac{3K_{c1} + 2K_{c2}}{K_{b2}} + 1\right]$$



Subassemblage for the lowest storey - single - bay frames.

FIGURE 37 (a)



Bending moments at load factor α – vertical load alone

FIGURE 37 (b)

$$= 2K_{b2}\,(3x' + 1)$$

where,

$$x' = \frac{3K_{c1} + 2K_{c2}}{3K_{b2}} = K_2(\overline{K} + \frac{2}{3})$$

The real distribution factors are therefore:-

$$a^{v}_{21} = \frac{6K_{c1}}{2K_{b2}(3x'+1)} = \frac{3\overline{K}K_2}{3x'+1}$$

$$a^{v}_{23} = \frac{4K_{c2}}{2K_{b2}(3x'+1)} = \frac{2K_2}{3x'+1}$$

$$a^{v}_{22}{}' = \frac{2K_{b2}}{2K_{b2}(3x'+1)} = \frac{1}{3x'+1}$$

Under vertical load alone, at some load factor, α, the fixed-end moments are $\pm\ \alpha\,\frac{WL}{12}$. The final bending moments, after distribution, are as shown in Figure 37(b). The maximum beam moments occur at the joints, and a plastic hinge will form at either end when:-

$$\alpha\,\frac{WL}{12}\,(1-a^{v}_{22}{}') = B = \lambda_1\,\frac{WL}{16} = \frac{5}{64}\,\lambda_2 WL$$

i.e., a plastic hinge forms when:-

$$\alpha = \lambda_2\,\frac{15}{16(1-a^{v}_{22}{}')} = \lambda_2 .\ \frac{15}{16} .\ \frac{3x'+1}{3x'}$$

In the bottom storey, due to the selection of the minimum beam size initially, the beam is comparatively slender in relation to the columns, and it has been found that generally, $3 < x' < 15$. Substituting these limiting values in the expression above, the following limits are obtained for α:-

$$\lambda_2 .\ \frac{15}{16} .\ \frac{10}{9} > \alpha > \lambda_2 .\ \frac{15}{16} .\ \frac{46}{45}$$

i.e.,

$$1.04\lambda_2 > \alpha > 0.96\lambda_2$$

This implies that α is always very close to λ_2. Therefore, it is reasonably accurate to assume that the first plastic hinges form at the ends of the beam just as the full vertical load is applied. Little error is involved in making this assumption, and the analysis is simplified considerably.

So, at the end of the vertical loading, the bending moments are as shown in Figure 38(a). When the horizontal load is applied, the plastic hinge at the windward end immediately disappears, and the frame for analysis is as shown in Figure 38(b). The real stiffnesses are as follows:-

Due to the hinge at $2'$,

$$S^r_{22'} = 3K_{b2}$$

Also,

$$S^r_{21} = S^r_{21'} = (n_1 - o_1)K_{c1}$$

and,

$$S^r_{23} = S^r_{23}{}' = n_2K_{c2}$$

The total joint stiffness at 2 is given by:-

$$S^r_2 = (n_1-o_1)K_{c1} + n_2K_{c2} + 3K_{b2}$$

$$= K_{c2}\ [(n_1-o_1)\overline{K} + n_2] + 3K_{b2}$$

$$= \left(K_2[(n_1-o_1)\overline{K} + n_2] + 3\right)K_{b2}$$

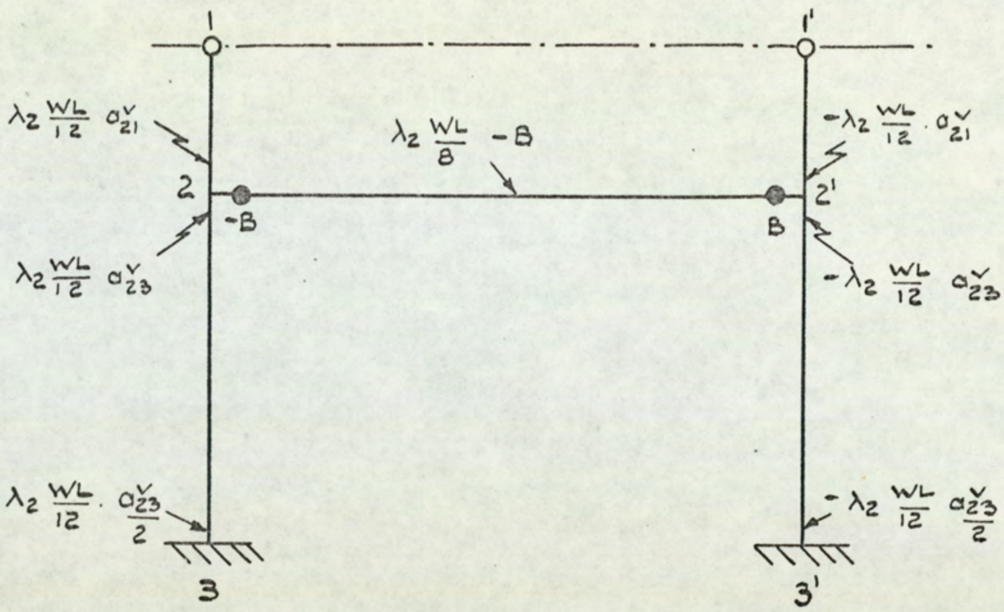
$$= (K_2V' + 3)K_{b2}$$

where,

$$V' = (n_1-o_1)\overline{K} + n_2$$

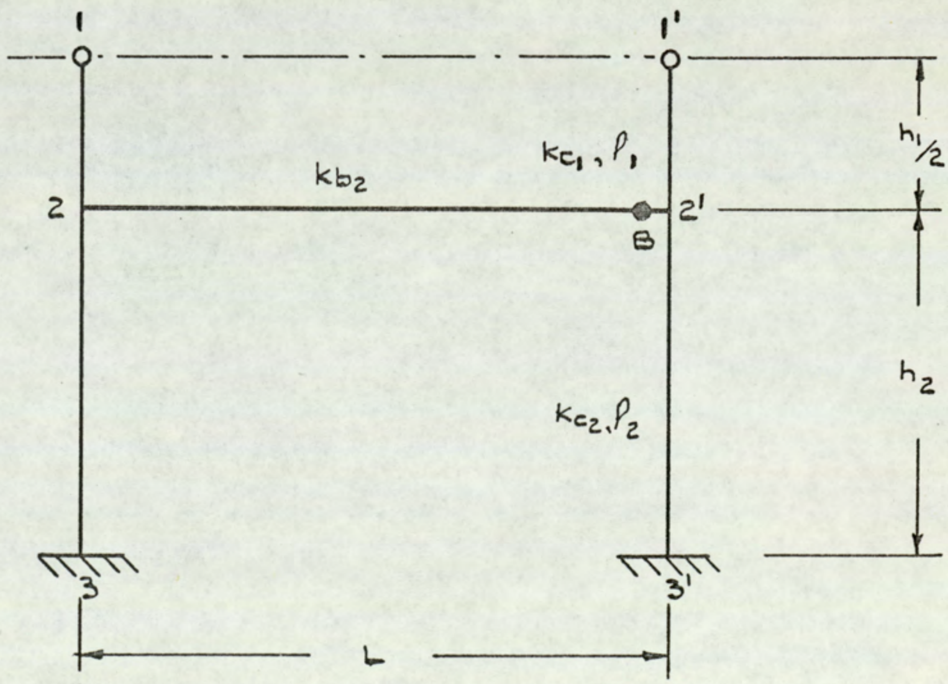
The total joint stiffness at $2'$ is given by:-

$$S^r_{2'} = (n_1-o_1)K_{c1} + n_2K_{c2}$$



Bending moments due to vertical load alone at λ_2

FIGURE 38 (a)



Frame for analysis under horizontal loading

FIGURE 38 (b)

$$= K_{c2}[(n_1 - o_1)\overline{K} + n_2]$$

$$= K_2 V' . K_{b2}$$

Therefore, the total stiffness at $2'$ is less than that at 2, and the leeward column carries less shear than the windward column. The leeward column is still the critical member for design, since the moments due to the horizontal and vertical loadings are cumulative.

Let the initial shear in the leeward column due to wind load alone at λ_2 be $\gamma.\lambda_2 \frac{H}{2}$, where γ is a constant. The shear on the windward column is therefore equal to $\lambda_2 H - \gamma.\lambda_2 \frac{H}{2} = \lambda_2 \frac{H}{2}(2 - \gamma)$. The ratio of these two shears must be equal to the ratio of the individual joint stiffnesses: i.e.,

$$\frac{\gamma.\lambda_2 \frac{H}{2}}{(2-\gamma)\lambda_2 \frac{H}{2}} = \frac{K_2 V'.K_{b2}}{(K_2 V' + 3)K_{b2}}$$

Solving this equation for γ,

$$\gamma = \frac{2K_2 V'}{2K_2 V' + 3}$$

The initial sway moments on the leeward column for this assumed distribution of shear forces, magnified to allow for the Pδ effect, are shown in Figure 39(a). The real distribution factors for the columns at $2'$ are as follows:-

$$a_{21}^{\Delta'} = \frac{(n_1 - o_1)K_{c1}}{(n_1 - o_1)K_{c1} + n_2 K_{c2}} = \frac{(n_1 - o_1)\overline{K}}{V'}$$

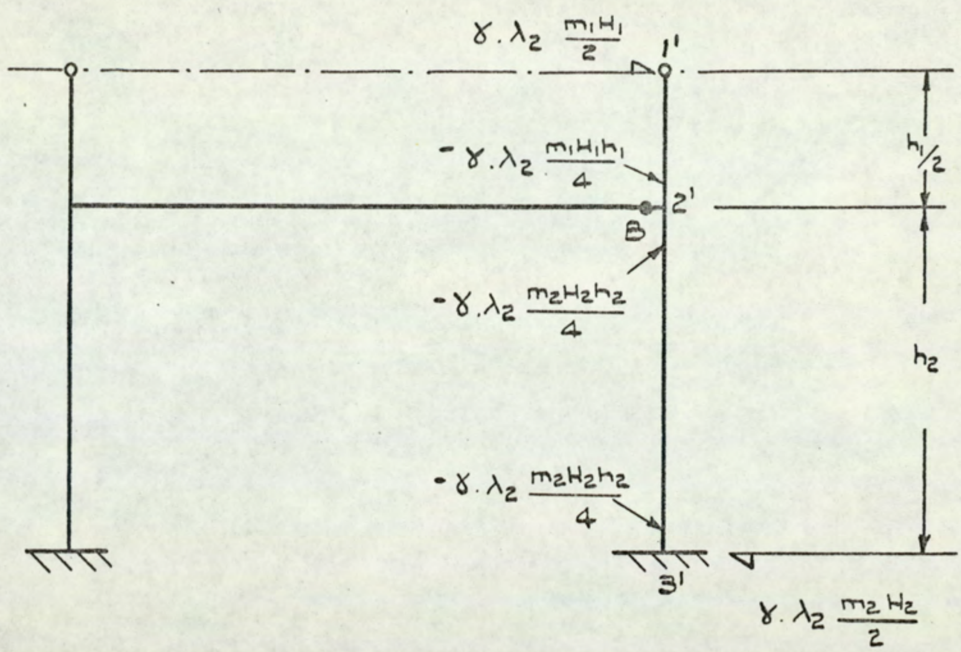
and,

$$a_{23}^{\Delta'} = \frac{n_2 K_{c2}}{(n_1 - o_1)K_{c1} + n_2 K_{c1}} = \frac{n_2}{V'}$$

Also since the base of the lower column is fully fixed, there is a carry-over of $-\frac{o_2}{n_2}$ times the moment distributed at the top of the column. The total out of balance moment at $2'$ is given by:-

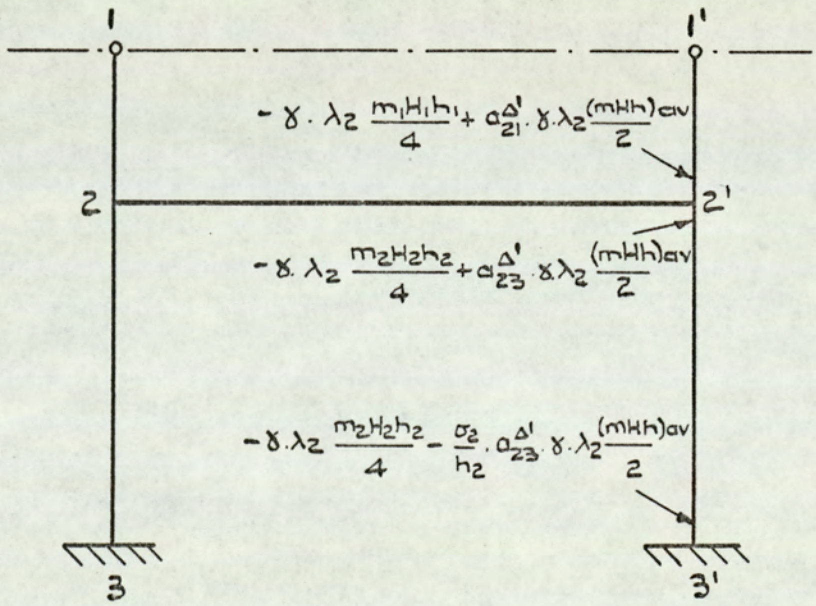
$$F.E.M_{2'} = -\gamma.\lambda_2 \frac{(mHh)av}{2}$$

The final moments due to wind loading are shown in Figure 39(b). Consider



Initial sway moments on the leeward column.

FIGURE 39. (a)



$-\gamma.\lambda_2 \frac{m_1H_1h_1}{4} + a^{\Delta'}_{21}.\gamma.\lambda_2 \frac{(mHh)_{av}}{2}$

$-\gamma.\lambda_2 \frac{m_2H_2h_2}{4} + a^{\Delta'}_{23}.\gamma.\lambda_2 \frac{(mHh)_{av}}{2}$

$-\gamma.\lambda_2 \frac{m_2H_2h_2}{4} - \frac{\sigma_2}{h_2}.a^{\Delta'}_{23}.\gamma.\lambda_2 \frac{(mHh)_{av}}{2}$

Final moments in the leeward column due to wind load at λ_2

FIGURE 39 (b)

the upper column first. Due to the combined vertical and horizontal loading,

$$M_{21} = -\lambda_2 \frac{WL}{12}.a_{21}^{v} - \gamma.\lambda_2 \frac{m_1H_1h_1}{4} + a_{21}^{\Delta'}.\gamma.\lambda_2 \frac{(mHh)av}{2}$$

$$= -\lambda_2 \frac{WL}{12}.a_{21}^{v} - \gamma.\lambda_2 \frac{m_1H_1h_1}{4}(1-2a_{21}^{\Delta'}.p_1)$$

where,

$$p_1 = \frac{(mHh)av}{m_1H_1h_1}$$

Therefore, if no plastic hinge is to occur at this location below λ_2, then,

$$C_{E_1} > |M_{21}| = \lambda_2 \frac{WL}{12}.a_{21}^{v} + \gamma.\lambda_2 \frac{A_{c1}m_1H_1h_1}{4}$$

where,

$$A_{c1} = 1 - 2a_{21}^{\Delta'}.p_1$$

At the base of the lower column, the moment is:-

$$M_{32} = -\lambda_2 \frac{WL}{12}.\frac{a_{23}^{v}}{2} - \gamma.\lambda_2 \frac{m_2H_2h_2}{4} - \frac{o_2}{n_2}.a_{23}^{\Delta'}.\gamma.\lambda_2 \frac{(mHh)av}{2}$$

$$= -\lambda_2 \frac{WL}{24}.a_{23}^{v} - \gamma.\lambda_2 \frac{m_2H_2h_2}{4}\left[1 + 2\frac{o_2}{n_2}.a_{23}^{\Delta'}.p_2\right]$$

where,

$$p_2 = \frac{(mHh)av}{m_2H_2h_2}$$

Therefore, the lower column must be selected from:-

$$C_{E_2} > |M_{32}| = \lambda_2 \frac{WL}{24}.a_{23}^{v} + \gamma.\lambda_2 \frac{A_{c2}'\,m_2H_2h_2}{4}$$

where,

$$A_{c2}' = 1 + 2\frac{o_2}{n_2}.a_{23}^{\Delta'}.p_2$$

5.4(c) SUMMARY

In the bottom storey, the beam is selected using:-

$$B = \lambda_1 \frac{WL}{16} \qquad (58)$$

The columns above and below the beam respectively are selected from:-

$$C_{E_1} > \lambda_2 \frac{WL}{12}.a^{v}_{21} + \gamma.\lambda_2 \frac{A_{c1}m_1H_1h_1}{4} \tag{59}$$

where,

$$A_{c1} = 1 - 2a^{\Delta'}_{21}.p_1 \tag{60}$$

and,

$$C_{E_2} > \lambda_2 \frac{WL}{24}.a^{v}_{23} + \gamma.\lambda_2 \frac{A'_{c2}m_2H_2h_2}{4} \tag{61}$$

where,

$$A'_{c2} = 1 + 2\frac{o_2}{n_2}.a^{\Delta'}_{23}.p_2 \tag{62}$$

In the equation above,

$$a^{v}_{21} = \frac{3\overline{K}K_2}{3x' + 1}; \quad a^{v}_{23} = \frac{2K_2}{3x' + 1}$$

where,

$$x' = K_2(\overline{K} + \frac{2}{3})$$

and, as before,

$$\overline{K} = \frac{K_{c1}}{K_{c2}}; \quad K_2 = \frac{K_{c2}}{K_{b2}}$$

Also,

$$p_1 = \frac{(mHh)av}{m_1H_1h_1}; \quad p_2 = \frac{(mHh)av}{m_2H_2h_2}$$

$$a^{\Delta'}_{21} = \frac{(n_1-o_1)\overline{K}}{V'}; \quad a^{\Delta'}_{23} = \frac{n_2}{V'};$$

$$\gamma = \frac{2K_2V'}{2K_2V' + 3};$$

where,

$$V' = (n_1-o_1)\overline{K} + n_2$$

5.4(d) ELASTICITY OF THE BEAM AT WORKING LOAD

The basic condition for the beams of the intermediate storeys to remain elastic at working load has been derived in 5.2(e). An identical expression is obtained for the lowest storey; i.e.,

$$\frac{WL}{12}(1-a^{v}_{22'}) + a^{1}_{22'}\frac{(m^1Hh)av}{2} \leq Mp_b \tag{63}$$

However, in this case, the real distribution factors are different:-

$$a_{22}^{v}{}' = \frac{1}{3x' + 1}$$

where,

$$x' = K_2(\overline{K} + \frac{2}{3});$$

$$a_{22}^{1}{}' = \frac{1}{K_2 V'^1 + 6}$$

where,

$$V'^1 = (n_1 - o_1)^1 \overline{K} + n_2{}^1$$

As for the lowest storey of the multi-bay frames, this condition is often found to be violated after the basic design has been completed, and it is necessary to increase the beam. However, as stated previously, it is not generally beneficial to repeat the design with this new beam size.

The following chapter deals specifically with the design procedure. It is shown how the equations developed for the different regions of both multi-bay and single-bay frames may be applied efficiently for the design of a complete framework. In particular, emphasis is placed on methods of reducing the design time to a minimum.

CHAPTER 6

DESIGN PROCEDURE

6.1. INTRODUCTION

In the previous three chapters, equations have been derived for a variety of subassemblages, each of which is assumed to represent a particular region of either a multi-bay or a single-bay framework. The current chapter attempts to co-ordinate this information, and shows how a complete design is obtained by applying each of these sets of equations in turn. Particular emphasis is placed on the efficiency of the iterative procedures involved, and, where appropriate, suggestions are made which enable both the design time and the volume of computational work to be reduced considerably.

In the following section, the design of multi-bay frames is considered, and this is later extended to deal with single-bay frames.

6.2. DESIGN OF MULTI-BAY FRAMES

The design procedure for any multi-bay frame may be summarized as follows:-

(a) Calculation of the initial design loads.

(b) Design of each intermediate storey in turn, starting at the second storey from the top, and using the equations developed in Chapter 3 with the following schedule:-

(i) Predict initial values of the magnification factors, A and A_c, using the basic wind ratio, $\frac{(Hh)av}{WL}$.

(ii) Calculate the modified wind ratio, $A\frac{(mHh)av}{WL}$, and select the correct zone for design.

Determine the required plastic moments for the beams and the internal columns.

Select appropriate Universal Beam and Universal Column sections.

(iii) Calculate the Euler ratios, stability functions, relative flexural rigidities and appropriate real distribution factors for the zone under consideration.

Determine p_2, and also $\frac{\alpha}{\lambda_2}$ if required.

Calculate new values of A and A_c.

Repeat step (ii) - if the sections obtained are identical to those selected initially, the design of the storey is complete - if not, repeat steps (iii) and (ii) until convergence is obtained.

(iv) Check the "working load elasticity condition".

(c) Design of the top storey, using the equations developed in Chapter 4.

(d) Design of the lowest storey, using the equations developed in Chapter 4. The procedure is similar to that for the intermediate storeys, except that the beam size is fixed, and there is no beam magnification factor to be calculated. However, two separate column magnification factors are required.

(e) Design of the external column, using the equations developed in Chapter 4.

The following sub-section, 6.2(a) to 6.2(e), refer to each of these topics more fully, and describe several of the design aids which have been developed. An additional sub-section, 6.2(f), contains details of the design of a typical framework, and is intended to clarify any outstanding points.

6.2(a) CALCULATION OF THE DESIGN LOADS

In general, the basic frame geometry is given, together with the assumed dead load of the structure, and an estimate of the vertical and horizontal live loads which may be expected to be applied. The total load on each beam, and the shear force and axial force in each column must be calculated from the initial data.

6.2(a)(i) BEAM LOADS

Figure 40 represents a section of a typical floor in a multi-bay framework. The combined dead load plus live load per unit area is denoted by w, the length of each primary beam by L, and the frame spacing (i.e. the lengths of each secondary beam) by S.

The primary beam is assumed to carry that portion of the loading which acts on the adjacent trapezoidal floor areas. Therefore, if the total load on the beam is W, then:-

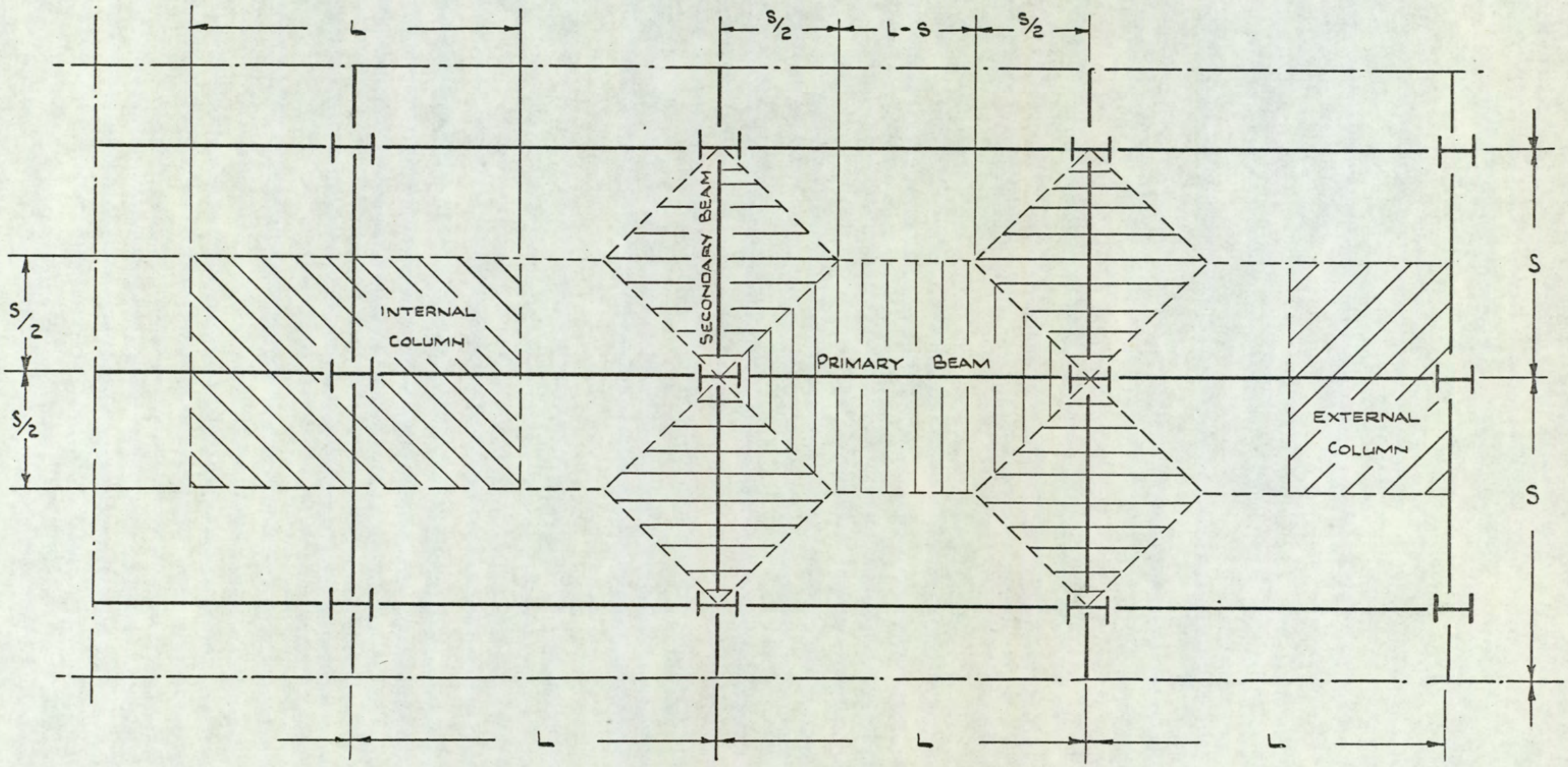
$$W = w \times 2\left(\frac{1}{2}.\frac{S}{2}.[L + (L-S)]\right)$$

i.e.,

$$W = \frac{w.S}{2}\,(2L-S) \qquad (64)$$

It is recommended that equation (64) should normally be used to calculate W, the remainder of the floor load being carried by the secondary beams. However, in certain of the design examples in Chapters 7 and 8, W is calculated by assuming that the beam carries half the loading on each floor panel. These particular examples are used to compare the economy of the design method with several other methods, and in order to do this, it is necessary to simulate the loading conditions used in these other designs. In general, however, the distribution of loading given in Figure 40 is considered to be more realistic.

In addition, in developing the design theory in Chapter 3, the total beam load is assumed to be uniformly distributed. Thus, when applying the design equations to a real framework, W, as calculated by equation (64), must also be assumed to act in this manner.



Dead load + live load = w/unit area.

Distribution of vertical loading to each member.

FIGURE 40

6.2(a)(ii) SHEAR FORCES

Consider Figure 41. This shows a section of the framework under two equivalent sets of wind loading. The actual loading is uniformly distributed, and is assumed to be of magnitude p per unit area. This distributed loading may be idealised as a series of point loads acting at the beam levels, each load being equal to the wind pressure, multiplied by the frame spacing, multiplied by half the sum of the heights of the columns above and below the beam. Therefore, the load applied at the beam in the general storey, n, is equal to $p.S\left(\frac{h_{n-1} + h_n}{2}\right)$.

As stated previously in Chapter 3, the shear force in any internal column of a storey is assumed to be equal to the sum of the wind loads above the points of contraflexure in the columns, divided by the number of bays. Thus, referring to Figure 41, considering either system of loads,

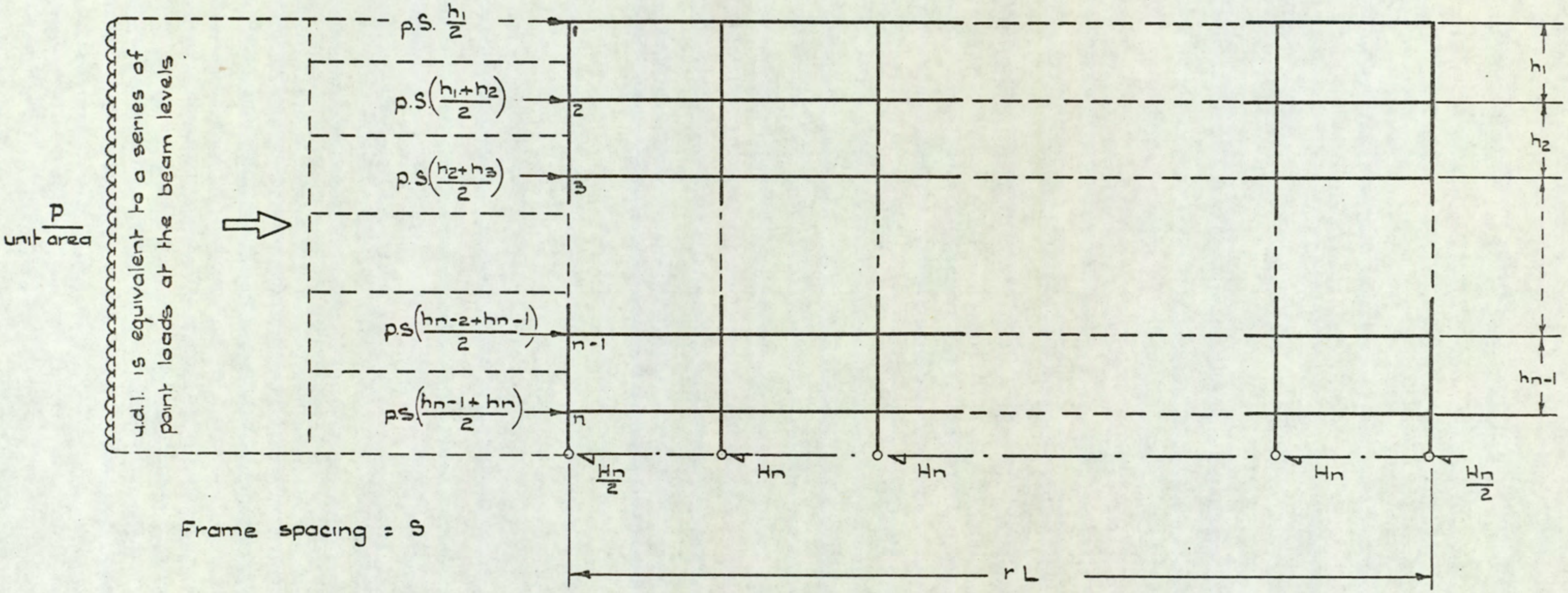
$$H_n = \frac{1}{r}.p.S.(h_1 + h_2 + - - - - - - + h_{n-1} + \frac{h_n}{2})$$

i.e.,

$$H_n = \frac{p.S}{r}.\left[\left(\sum_1^n h\right) - \frac{h_n}{2}\right] \qquad (65)$$

This is the general expression for calculating the shear force in each internal column of the top and intermediate storeys. It must not, however, be used for the lowest storey columns.

In the initial stages of this research project, the series of frames described in Chapter 7 was designed by assuming that the shear in the lowest storey was as given by equation (65). These frames were subsequently analysed, under combined loading, and in several of the analyses, plastic hinges were found to occur in the lowest storey internal columns below the design load factor λ_2. Furthermore, the analyses indicated that the shear forces at working load in these members were considerably greater than those assumed in the design. In order to compensate for this error, a modified formula for these shears was developed from inspection of the



Idealization of wind loading and shear forces.

FIGURE 41.

analysis results. This has been given previously by equation (35) in 4.3(a), and was derived using the following notation:-

H_1, H_2 = shears in the upper and lower columns of the subassemblage (which is shown in Figure 22).

$\Sigma(H)_1, \Sigma(H)_2$ = the corresponding total shears in each storey.

For the lower column only,

I_i, I_e = second moments of area of the internal and external columns, respectively.

$\Sigma(I)$ = the sum of the second moments of area of all the columns.

Therefore, for r bays,

$$\Sigma(I) = 2I_e + (r-1)\, I_i = I_e\left[2 + (r-1)\frac{I_i}{I_e}\right]$$

For each of the eight frameworks, the ratio $\frac{H_2}{\Sigma(H)_2}$ was obtained from the results of the analyses, and the ratio $\frac{I_i}{\Sigma(I)}$ was calculated from the original designs. Figure 42(a) shows these results graphically, and it may be seen that the two ratios are linearly related. The equation of this straight line is:-

$$\frac{H_2}{\Sigma(H)_2} = 1.05 \times \frac{I_i}{\Sigma(I)}$$

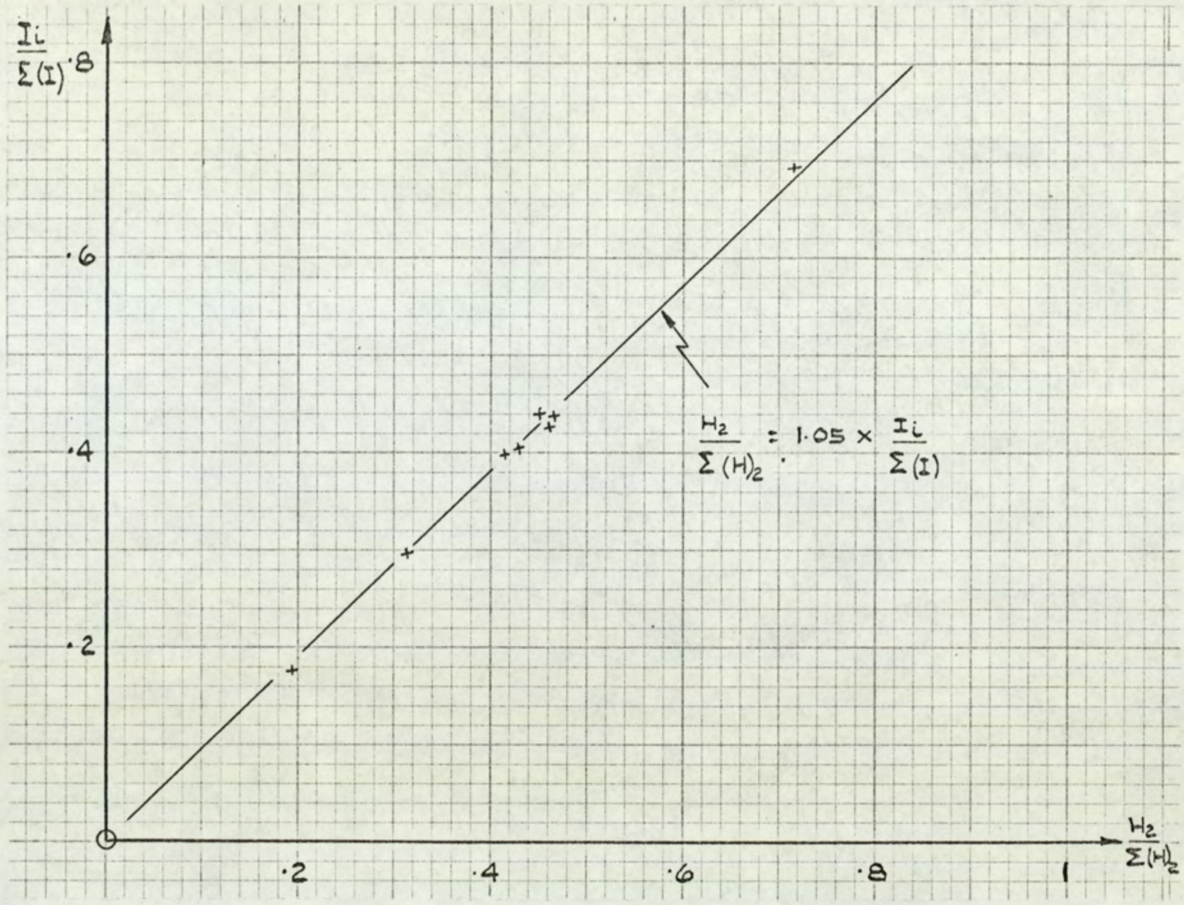
Substituting for $\Sigma(I)$:-

$$H_2 = \Sigma(H)_2 . \frac{1.05\,\frac{I_i}{I_e}}{\left[2 + (r-1)\frac{I_i}{I_e}\right]}$$

However, $\frac{I_i}{I_e}$ is unknown at the start of the design procedure, so in order to obtain an expression for estimating H_2, a further substitution is required. Now, the relative intensities of horizontal and vertical loading may be represented by an "average" wind ratio per bay;i.e.,

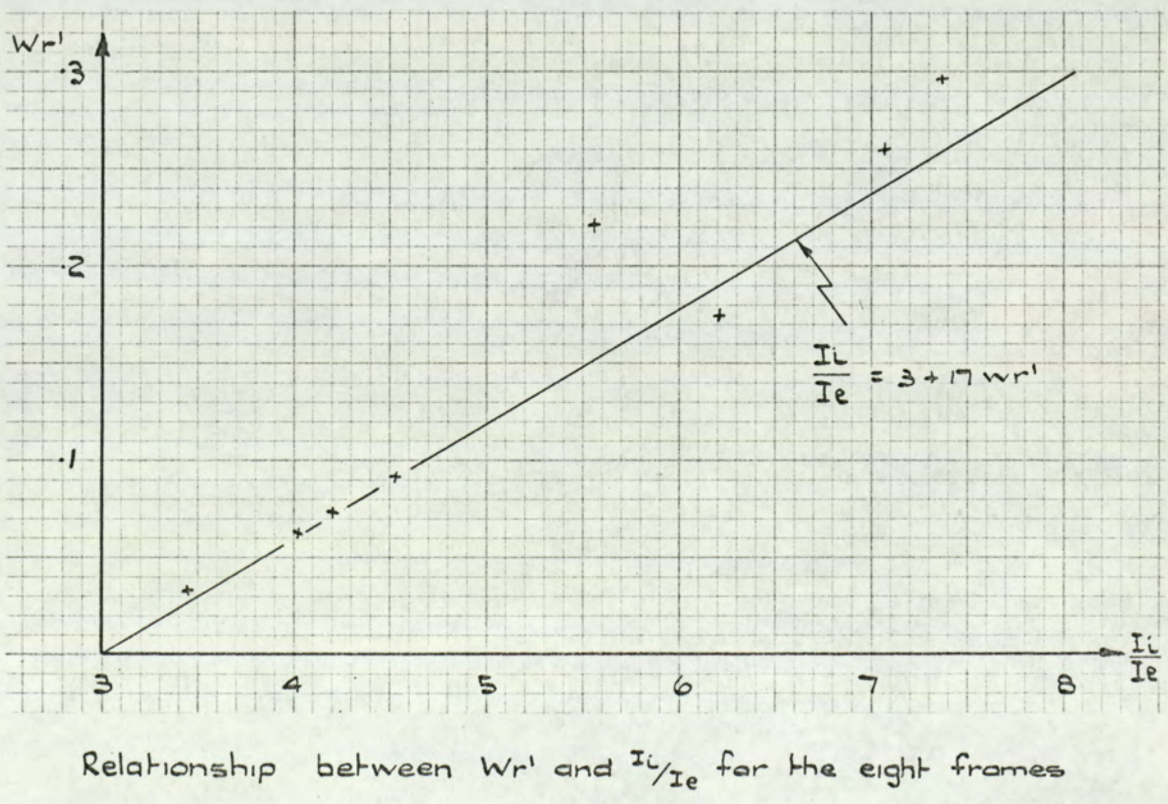
$$wr' = \frac{1}{r}.\frac{1}{WL}\left[\frac{\Sigma(H)_1 h_1 + \Sigma(H)_2 h_2}{2}\right] = \frac{[\Sigma(H)h]av}{rWL}$$

This quantity depends on the initial loading conditions and the



Relationship between the ratios $^{I_i}/_{\Sigma(I)}$ and $^{H_2}/_{\Sigma(H)_2}$ from the analyses of the preliminary designs of eight frames.

FIGURE 42 (a)



Relationship between Wr' and $^{I_i}/_{I_e}$ for the eight frames

FIGURE 42 (b)

frame geometry. Thus, if a relationship between $\frac{I_i}{I_e}$ and wr' may be found, H_2 may be calculated immediately. Such a relationship is derived from Figure 42(b). Each point on the graph corresponds to the values of $\frac{I_i}{I_e}$ and wr' obtained in the design of a particular frame. The straight line shown is assumed to represent all these points, and its equation is:-

$$\frac{I_i}{I_e} = 3 + 17wr'$$

Therefore, eliminating $\frac{I_i}{I_e}$,

$$H_2 = \Sigma(H)_2 . \frac{1.05(3+17wr')}{[2+(r-1)(3+17wr')]}$$

and this is the required expression, previously given by equation (35).

Despite the fact that the straight line shown in Figure 42(b) does not pass very close to the points for four of the frames, the resulting expression for H_2 gives a very accurate estimate of the lowest storey shear in these frames. For all the frames, this value of H_2 was found to be within four per cent of that obtained from the computer analysis. Furthermore, as will be shown in Chapters 7 and 8, the use of this equation has been found to lead to a safe and economical design in every case.

6.2(a)(iii) COLUMN LOADS

The final design assumption in 3.3(b) states that the axial force in each internal column is equal to the force in the column above, plus half the loads on the adjoining beams. Referring again to Figure 40, there are four beams framing into each internal column. It may be seen that half of the loads on these four beams is equivalent to the load acting on the rectangular area surrounding the internal column shown on the left of the figure. This area is simply equal to the beam length multiplied by the frame spacing. Therefore, if the axial load in each internal column of the general storey, n, is denoted by P_{In}, then,

$$P_{In} = P_{In-1} + wLS \tag{66}$$

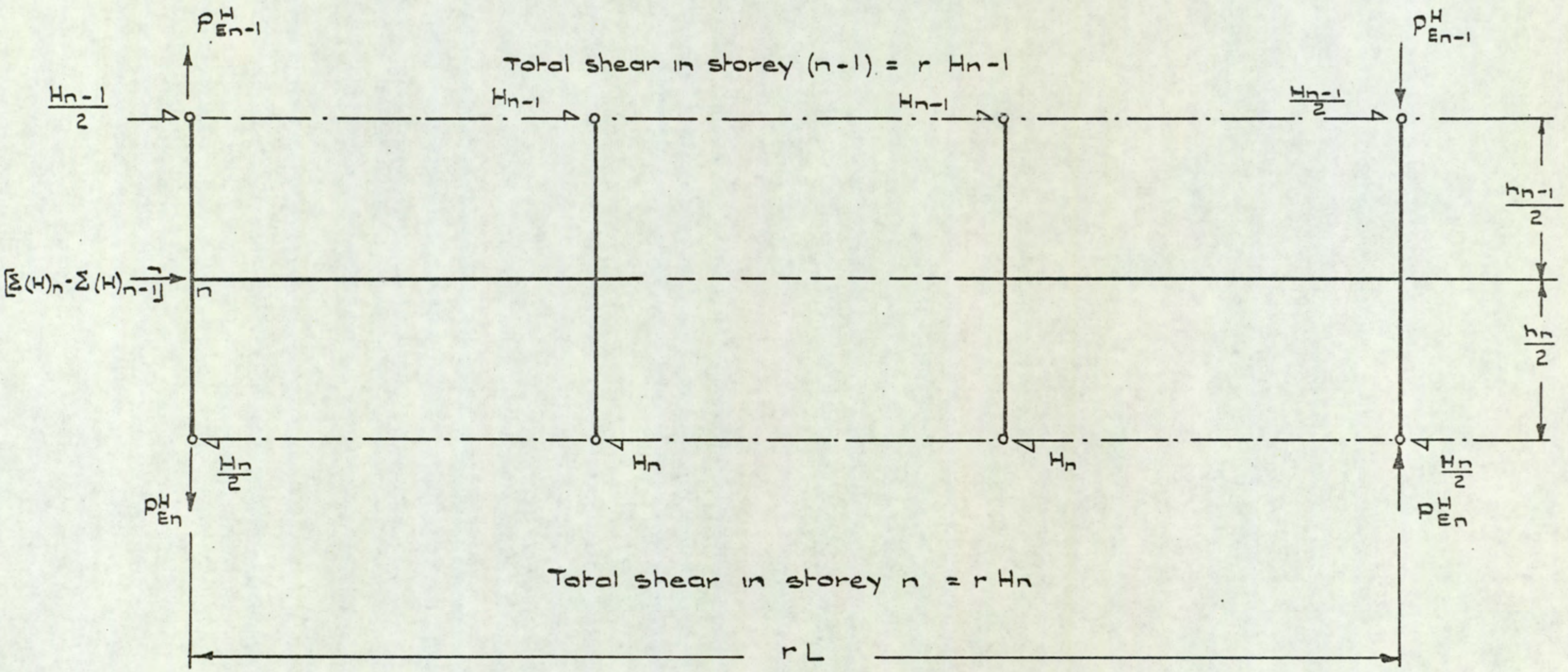
In the British Standard Code of Practice, CP3, it is suggested that the axial load to be used for the design of any column may be obtained by assuming a certain percentage reduction in the live load, depending on the number of floors carried by that column. It is the Author's opinion that this reduction should not be applied when designing by ultimate load methods. Since the framework is designed to have a definite load factor against collapse, it would seem to be an unnecessary and possibly dangerous practice to reduce the design loads for the columns on the strength of a statistical probability. A far more acceptable philosophy is to base the design on the most critical loading conditions. If these are considered to be excessive, the design load factor may be reduced, but the knowledge that a definite load factor will be attained must surely remain the fundamental advantage of ultimate load methods.

It has been necessary, however, in certain of the design examples, to include this "live-load reduction" in order to reproduce the design loads adopted by other research workers. In such cases, it will be stated if this has been done.

Consider now the axial loads developed in the external columns. In Chapter 4, it was stated that each external column carries an axial load equal to that in the column above, plus half the beam loads, plus a component due to wind loading. Referring again to Figure 40, the axial load due to vertical loading alone in the external column of storey n, P_{En}^{V}, say, is given by:-

$$P_{En}^{V} = P_{En-1}^{V} + \frac{wLS}{2}$$

The axial load component due to wind loading, P_{En}^{H}, may be obtained with the aid of Figure 43, which shows the shear forces and axial loads



Force distibution for calculation of the axial load component in the external column due to wind loading.

FIGURE 43.

induced in the complete storey subassemblage. The applied wind load at the beam level may be expressed in the form $[\Sigma(H)_n - \Sigma(H)_{n-1}]$ since, for horizontal equilibrium, this force must represent the difference between the total shear forces in the storeys above and below the beam. For equilibrium of moments about the points of contraflexure in the lower windward column:-

$$P_{En}^{H}.rL = P_{En-1}^{H}.rL + r.H_{n-1}\left(\frac{h_{n-1} + h_n}{2}\right) + [\Sigma(H)_n - \Sigma(H)_{n-1}]\frac{h_n}{2}$$

But, in either storey,

$$\Sigma(H) = r.H$$

Using this substitution, and rearranging:-

$$P_{En}^{H} = P_{En-1}^{H} + \frac{(Hh)av}{L}$$

Now, the total axial load in the external column is,

$$P_{En} = P_{En}^{V} + P_{En}^{H}$$

$$= P_{En-1}^{V} + \frac{wLS}{2} + P_{En-1}^{H} + \frac{(Hh)av}{L}$$

But,

$$P_{En-1}^{V} + P_{En-1}^{H} = P_{En-1}$$

Therefore,

$$P_{En} = P_{En-1} + \frac{wLS}{2} + \frac{(Hh)av}{L} \qquad (67)$$

This is the general expression for the external column loads in the top and intermediate storeys. It may also be used for the lowest storey in the slightly modified form:-

$$P_{En} = P_{En-1} + \frac{wLS}{2} + \frac{[\Sigma(H)h]av}{r.L}$$

As for the internal column loads, the "live-load reduction" may be applied if required, but again, this is not recommended.

6.2(b) THE INTERMEDIATE STOREYS

The design equations for the intermediate storeys have been derived in Chapter 3. In particular, the magnification factors are summarized by equations (23) to (28) in Section 3.9. The significance of these equations is now examined, and following this, an empirical method is developed for predicting initial values of A and A_c which ensure rapid convergence of the iterative design procedure.

6.2(b)(i) PREDICTION OF THE MAGNIFICATION FACTORS

The magnification factors for all the zones, for both beams and columns, are basically functions of the same variables, and themselves vary in a similar manner. Therefore, it is only necessary to consider one of them in order to develop an understanding of their physical significance. Consider, for example, equation (25), which represents the beam magnification factor for Zone 2(ii):-

$$A = \frac{\alpha}{\lambda_2}.\overset{\alpha}{a}_{22}{}' + \left(1 - \frac{\alpha}{\lambda_2}\right).\overset{\Delta}{a}_{22}{}'$$

where,

$$\frac{\alpha}{\lambda_2} = \frac{A}{\overset{\alpha}{a}_{22}{}'}\left[\frac{1}{2} - \frac{WL}{24A(mHh)av}\right];$$

$$\overset{\alpha}{a}_{22}{}' = \frac{12}{K_2V + 12}; \quad \overset{\Delta}{a}_{22}{}' = \frac{3}{K_2V + 3};$$

$$V = (n_1 - o_1)\overline{K} + (n_2 - o_2)$$

For the purpose of this discussion, it is convenient to derive an alternative expression for A.

Let $\quad \frac{A(mHh)av}{WL} = wr$

Therefore,

$$\frac{\alpha}{\lambda_2} = \frac{A}{2a_{22}^{\alpha\prime}}\left[1 - \frac{1}{12wr}\right]$$

Also,

$$A = \frac{\alpha}{\lambda_2}\left[a_{22}^{\alpha\prime} - a_{22}^{\Delta\prime}\right] + a_{22}^{\Delta\prime}$$

Eliminating $\frac{\alpha}{\lambda_2}$:-

$$A = \frac{A}{2a_{22}^{\alpha\prime}}\left[1 - \frac{1}{12wr}\right](a_{22}^{\alpha\prime} - a_{22}^{\Delta\prime}) + a_{22}^{\Delta\prime}$$

which reduces to :-

$$A = \frac{2a_{22}^{\alpha\prime}.a_{22}^{\Delta\prime}}{2a_{22}^{\alpha\prime} - \left[1 - \frac{1}{12wr}\right](a_{22}^{\alpha\prime} - a_{22}^{\Delta\prime})}$$

Substituting for $a_{22}^{\alpha\prime}$ and $a_{22}^{\Delta\prime}$, the following general relationship between A, K_2V and wr is obtained:-

$$A = \frac{24}{8(K_2V+3) - 3K_2V\left[1 - \frac{1}{12wr}\right]} \qquad (68)$$

However, in Zone 2(ii),

$$\frac{1}{12} \leqslant wr \leqslant \frac{1}{4}$$

These limiting values may be substituted in equation (68).

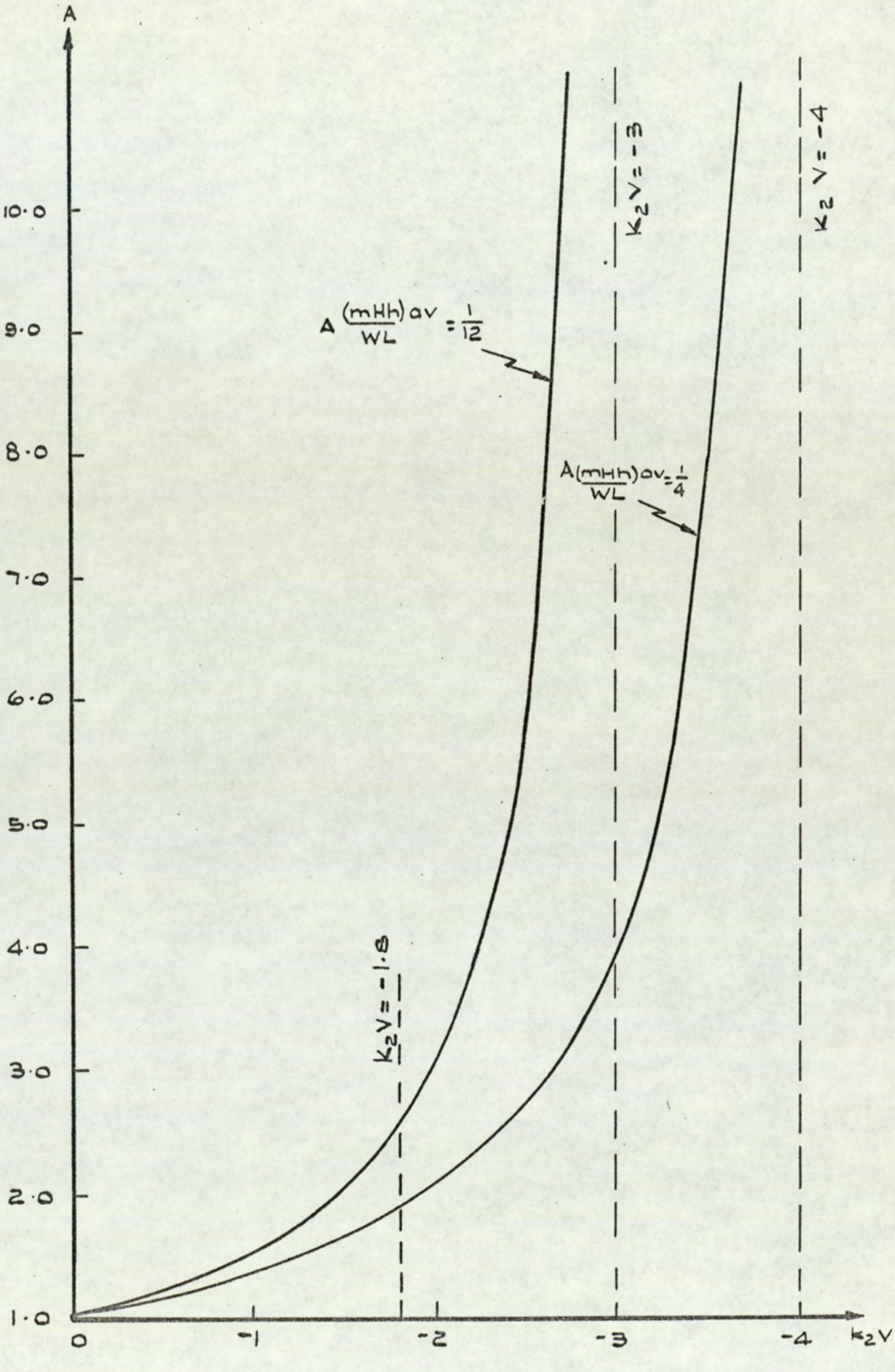
When $wr = \frac{1}{12}$,

$$A = \frac{3}{K_2V + 3} \qquad (69)$$

When $wr = \frac{1}{4}$,

$$A = \frac{4}{K_2V + 4} \qquad (70)$$

Equations (69) and (70) are represented graphically in Figure 44, and it may be seen that they are the limiting curves of the general set for Zone 2(ii) given by equation(68). In addition, equation (69) is in fact identical to the expression for A in Zones 1 and 2(i), and equation



Variation of A with k_2V – Zone 2 (ii)

FIGURE 44

(70) also represents the upper bound curve for Zone 3. As stated before, these curves are therefore typical of those for all three zones.

It will be remembered that A is defined originally as the degree by which the bending moments are magnified due to the instability effects. This is clearly indicated by Figure 44; i.e.,

(1) $A = 1$ when $K_2V = 0$

Now, $K_2V = 0$ when $\rho = 0$; i.e., when there are no axial loads in the columns, A is equal to unity. This is the "simple plastic" condition.

(2) A increases as K_2V reduces.

Now, $K_2V < 0$ when $\rho > 0$, so that as the axial loads increase, A increases. The bending moments are therefore greater than those assumed in the simple plastic design. Any frame which is designed assuming $A = 1$, whilst carrying these axial loads, is unlikely to attain the required load factor.

(3) $A \rightarrow \infty$ as K_2V tends to some limiting value.

Under very heavy axial loads, A becomes very large, indicating that the instability effects are predominant and that the simple plastic design equations are hopelessly inadequate. If such a frame is designed assuming $A = 1$, failure will occur at a very low load factor, with far fewer plastic hinges than are required for a mechanism. In fact, in the limiting case, when $A = \infty$, failure will occur due to elastic instability. This infinite value of A occurs when the total joint stiffness is zero.

Theoretically, any point on the curve below the instability point represents a stable condition, and provided that the correct value of A is taken into account, a satisfactory design could be obtained. However, it may be seen that due to the non-linearity of the curve, the magnification factor begins to increase very rapidly long before the limiting value of K_2V.

The proposed design method contains many approximations, and although all of these are considered to be justifiable, it is clearly inadvisable to permit high values of the magnification factors. Any such value indicates that the frame is very close to the point of instability, and any small error in the calculations would be likely to produce an unsafe design.

In order to control the instability effects, for design purposes, a limit of - 1.80 has been imposed on the value of K_2V. This limiting ordinate is shown in Figure 44, and it may be seen that reasonably high values of A are still possible. The maximum values of the distribution factors corresponding to this limit are 2.5 for $a_{22}^{\Delta}{}'$ and 1.176 for $a_{22}^{\alpha}{}'$, and, in any zone, these lead to a maximum value of 2.5 for A.

If, during the design of a particular storey, K_2V is found to be less than - 1.80, its value must be increased by suitable alteration to the selected sections. This is considered below:-

Expanding K_2V,

$$K_2V = \frac{K_{c2}}{K_{b2}}\left[(n_1-o_1)\overline{K} + (n_2-o_2)\right]$$

$$= \frac{I_{c2}}{I_{b2}}\cdot\frac{L}{h_2}\left[(n_1-o_1)\,\frac{I_{c1}}{I_{c2}}\cdot\frac{h_2}{h_1} + (n_2-o_2)\right]$$

Now, by inspection of the stability functions, (n-o) is approximately proportional to ρ, the Euler ratio. Also, ρ is inversely proportional to the second moment of area of the column. Therefore, approximately,

$$(n_1-o_1) \propto \rho_1 \propto \frac{1}{I_{c1}}$$

and,

$$(n_2-o_2) \propto \rho_2 \propto \frac{1}{I_{c2}}$$

Alternatively,

$$(n_1 - o_1) = \frac{a_1}{I_{c1}}$$

and,

$$(n_2 - o_2) = \frac{a_2}{I_{c2}}$$

where a_1 and a_2 are constants. Therefore, K_2V may be written as follows:-

$$K_2V = \frac{I_{c2}}{I_{b2}} \cdot \frac{L}{h_2} \left[\frac{a_1}{I_{c1}} \cdot \frac{I_{c1}}{I_{c2}} \cdot \frac{h_2}{h_1} + \frac{a_2}{I_{c2}} \right]$$

$$= \frac{1}{I_{b2}} \cdot \frac{L}{h_2} \left[a_1 \cdot \frac{h_2}{h_1} + a_2 \right]$$

All the terms apart from I_{b2} are constants, so that:-

$$K_2V \propto \frac{1}{I_{b2}}$$

Thus, any alteration in the column size has a negligible effect on the magnitude of K_2V, which is basically a function of I_{b2}. Therefore, in order to increase K_2V above the limiting value of - 1.80, (i.e. to reduce its modulus), it is simply necessary to select a new beam with a larger second moment of area. It is the beam size, and not the column size, that controls the instability effects.

The imposition of a limit on the magnification factors has the additional beneficial effects of reducing the sidesway deflections and ensuring more rapid convergence of the iterative procedure.

However, as mentioned previously, the most efficient way of encouraging convergence in the design of any storey is to make an initial estimate of the values likely to be obtained for the magnification factors. This is the first step in the design schedule given for the intermediate storeys at the beginning of 6.2, and is the basic subject of this section. Having examined the significance of the magnification factors, the development of the method of predicting their values may now be considered. As will be seen subsequently, this method is entirely empirical, and is based on the results obtained from the design of the wide range of frameworks given in Chapter 7. For any storey, the following notation is used:-

Final wind ratio = $\frac{A(mHh)av}{WL}$ = wr.

Final magnification factor = A

Initial wind ratio = $\frac{(Hh)av}{WL}$ = wr_i

Initial magnification factor = A_i

i.e., wr and A are the values obtained after the design of the storey has converged, wr_i is the basic wind ratio, which may be calculated from the initial design loads, and A_i is the initially assumed value for A which will ensure rapid convergence.

Now, in the design of any storey, m, the Merchant magnification factor, is approximately equal in the columns above and below the beam. Therefore,

$$wr = \frac{A(mHh)av}{WL} \simeq A.m. \frac{(Hh)av}{WL}$$

i.e.

$$wr \simeq A.m.wr_i \qquad (71)$$

It will be shown subsequently that the design of any storey is started by multiplying the initial wind ratio by the initial magnification factor, thus giving an initial modified wind ratio, which leads to the selection of the correct zone. In this case, convergence is bound to be rapid if this initial modified value of wind ratio is close to the value which is ultimately obtained. Therefore, in general, the following relationship is required for rapid convergence:-

$$A_i.wr_i \simeq wr \qquad (72)$$

Thus, eliminating wr from equations (71) and (72),

$$A_i \simeq m.A \qquad (73)$$

Since m is always greater than unity, A_i must therefore be greater than A; i.e., rapid convergence is obtained by initially overestimating the final value of A.

Consider first the design of the second storey from the top (denoted by Storey 2). In all the frames described in Chapter 7 which are designed with Universal Beams and Universal Columns, the design of Storey 2 lies in Zone 1, and the final magnification factor lies within the limits $1.17 < A < 1.24$, the lower limit applying to the frame carrying the heaviest wind loading. This is indicated in Figure 45, where each of the points denoted by "2" represents the final values of A and wr obtained in the design of Storey 2 in each of the frames. The remaining points, denoted by "3", represent the results of the designs for Storey 3, and these will be referred to subsequently.

As will be shown in Chapter 7, the six frames given in Figure 45 constitute a representative sample, since they have been deliberately selected in order to test a wide range of design parameters. Therefore, considering the distribution of points "2", it may be assumed that a relationship exists between A and wr in Storey 2 of any frame, given approximately by the equation of the straightline shown in the figure; i.e.,

$$A = 1.27 - 2.0\ wr$$

However, in order to predict an initial value for A, a relationship between A_i and wr_i is required. Substituting for wr from equation (72) and for A from equation (73):-

$$\frac{A_i}{m} = 1.27 - 2.0\ A_i wr_i$$

Therefore,

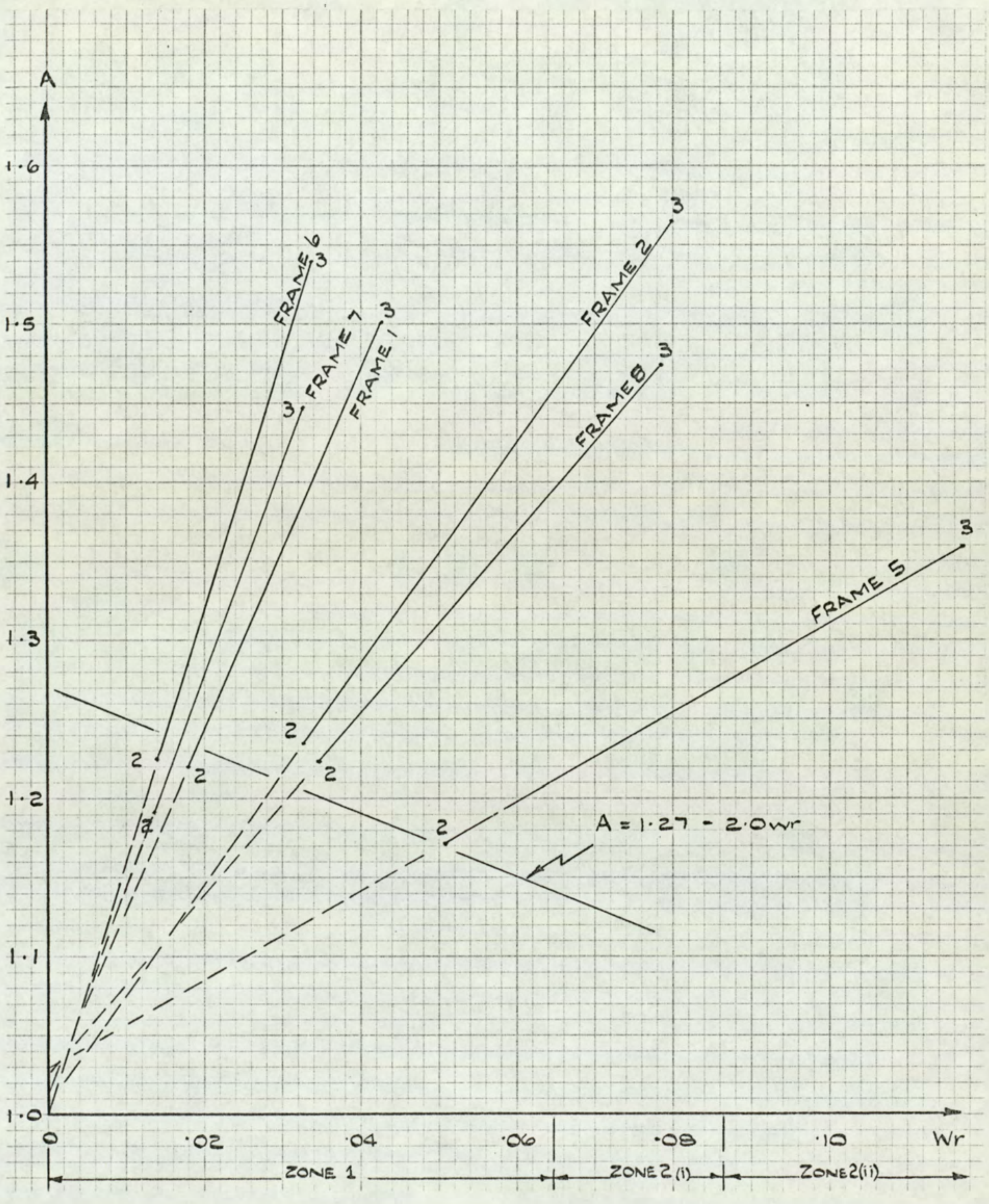
$$A_i(1 + 2.0m\ wr_i) = 1.27m$$

i.e.,

$$A_i = \frac{1.27m}{1 + 2.0m\ wr_i}$$

Now, in general, in Storey 2, $m \simeq 1.1$, so that:-

$$A_{i2} = \frac{1.4}{1 + 2.2\ wr_{i2}} \qquad (74)$$



Variation of A with wr for Storeys 2 and 3
in each of the six frames designed with U.B's and U.C's

FIGURE 45

For every frame which has been designed, this expression has been found to give an initial estimate of A which yields beam and column sections in Storey 2 which are identical to those finally required. Thus, in every case, the design converges immediately, with only one iteration being necessary.

Figure 45 may also be used to develop a formula for A_{i3}, the initial magnification factor for Storey 3. For each frame, it may be seen that the points 2 and 3 lie approximately on a straight line passing through the point (0,1). Assuming this to be true for any general frame, and describing the points 2 and 3 for this frame by the co-ordinates (wr_2, A_2) and (wr_3, A_3) respectively, it follows that the general equation of the straight line is:-

$$A_3 = 1 + (A_2 - 1)\,\frac{wr_3}{wr_2}$$

Now, from equation (72),

$$wr_3 = A_{i3}.wr_{i3}$$

Also, since $m \simeq 1.05$ in Storey 3, from equation (73),

$$A_3 = \frac{A_{i3}}{1.05} = 0.95\ A_{i3}$$

Therefore,

$$0.95\ A_{i3} = 1 + \frac{(A_2 - 1)}{wr_2}.A_{i3}.wr_{i3}$$

i.e.,

$$A_{i3}\left[0.95 - \frac{(A_2-1)}{wr_2}.wr_{i3}\right] = 1$$

Therefore,

$$A_{i3} = \frac{1}{\left[0.95 - \frac{(A_2-1)}{wr_2}.wr_{i3}\right]} \qquad (75)$$

So, for Storey 3, the initial value of A may be calculated from the final values, A_2 and wr_2, obtained in the design of Storey 2, and the basic wind ratio, wr_{i3}. Again, using this value of A_i, it has been found that convergence occurs immediately in every frame.

Equations (74) and (75) are functions of the initial values of magnification factor and wind ratio, A_i and wr_i. However, in the remaining intermediate storeys, no such simple relationships exist, and it has been found necessary to adopt a graphical approach, relating the functions A and wr_i. Having obtained A for any storey, A_i may then be estimated using equation (73), with m = 1.05. The development of this method is given below.

The final value of A in any storey is dependent on the zone in which that storey eventually lies, which in turn depends on the final wind ratio, wr; for example, in Zone 1, $0 \leq wr \leq \frac{1}{16}$, etc. The demarcation between one zone and the next may also be defined in terms of the initial wind ratio, wr_i, as follows:-

From equation (71), if m = 1.05,

$$wr = 1.05 \; A \; wr_i$$

Thus, for example, substituting in the bounds for Zone 1,

$$0 \leq 1.05 \; A \; wr_i \leq \frac{1}{16}$$

Dividing by 1.05 A, Zone 1 may be described by:-

$$0 \leq wr_i \leq \frac{1}{16.8A}$$

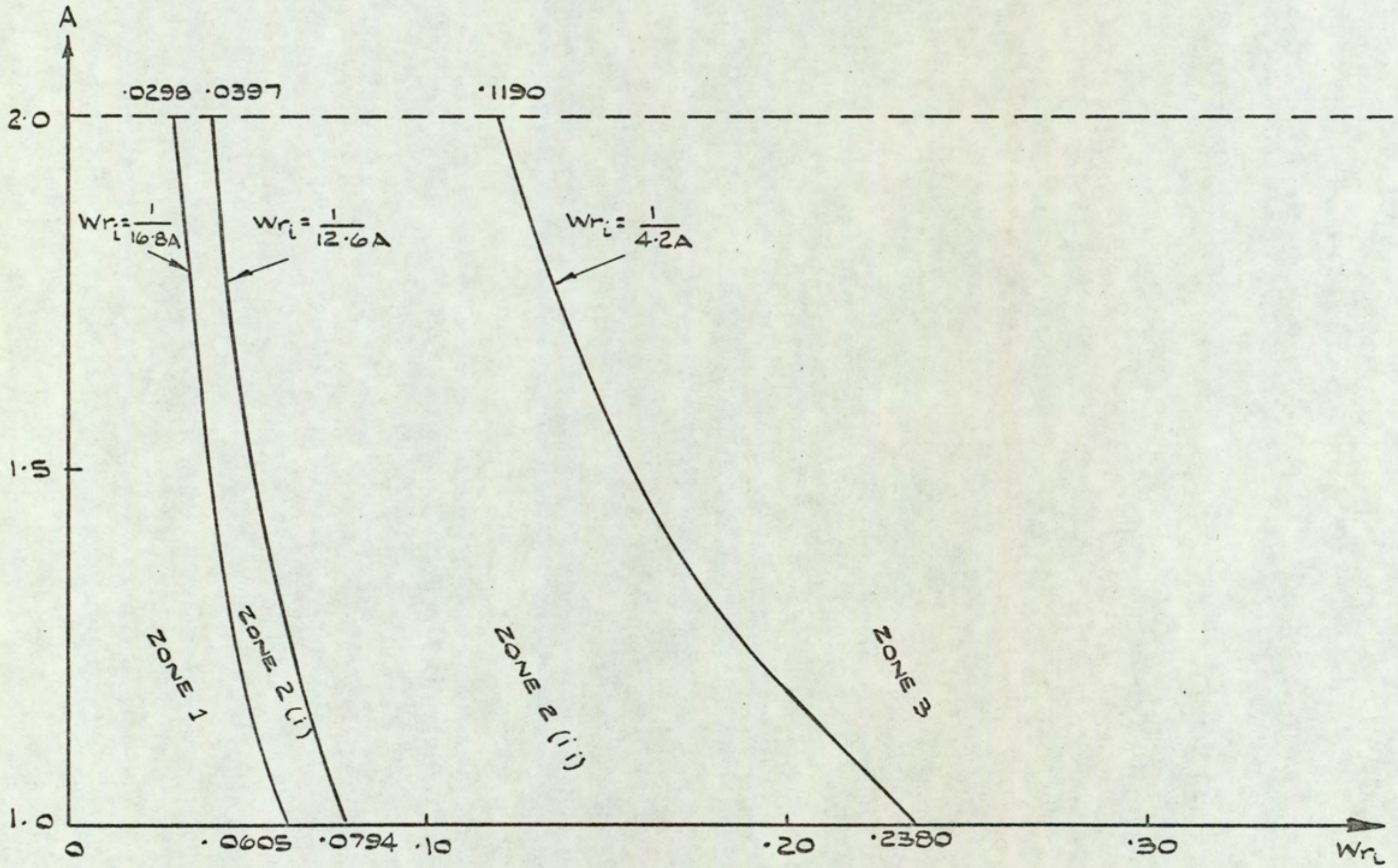
In a similar way, the limits on wr_i in the other zones are as follows:-

Zone 2(i): $\frac{1}{16.8A} \leq wr_i \leq \frac{1}{12.6A}$

Zone 2(ii): $\frac{1}{12.6A} \leq wr_i \leq \frac{1}{4.2A}$

Zone 3: $\frac{1}{4.2A} \leq wr_i$

These modified zone boundaries are shown in Figure 46, within the limits 1 < A < 2. Although derived by an approximate method, these lines



Zone boundaries, referred to the final magnification factor, A, and the initial wind ratio, $Wr_i = \frac{(Hh)_{av}}{WL}$

FIGURE 46.

have been found to represent the demarcation between the zones very accurately. In every storey of each of the frames described in Chapter 7, the point (wr_i,A) does in fact indicate the same zone as that given in the design by the final value of wr, despite the fact that several storeys are close to the border between two zones.

Further inspection of the variation of A with wr_i in successive storeys of each of the frames indicates certain definite trends, and these are described below with the aid of Figure 47. In the figure, lines A,B,C and D refer to four fictitious frames, under different intensities of wind loading, and these are taken to be representative of the range of real frameworks given in Chapter 7. The general observations are as follows:-

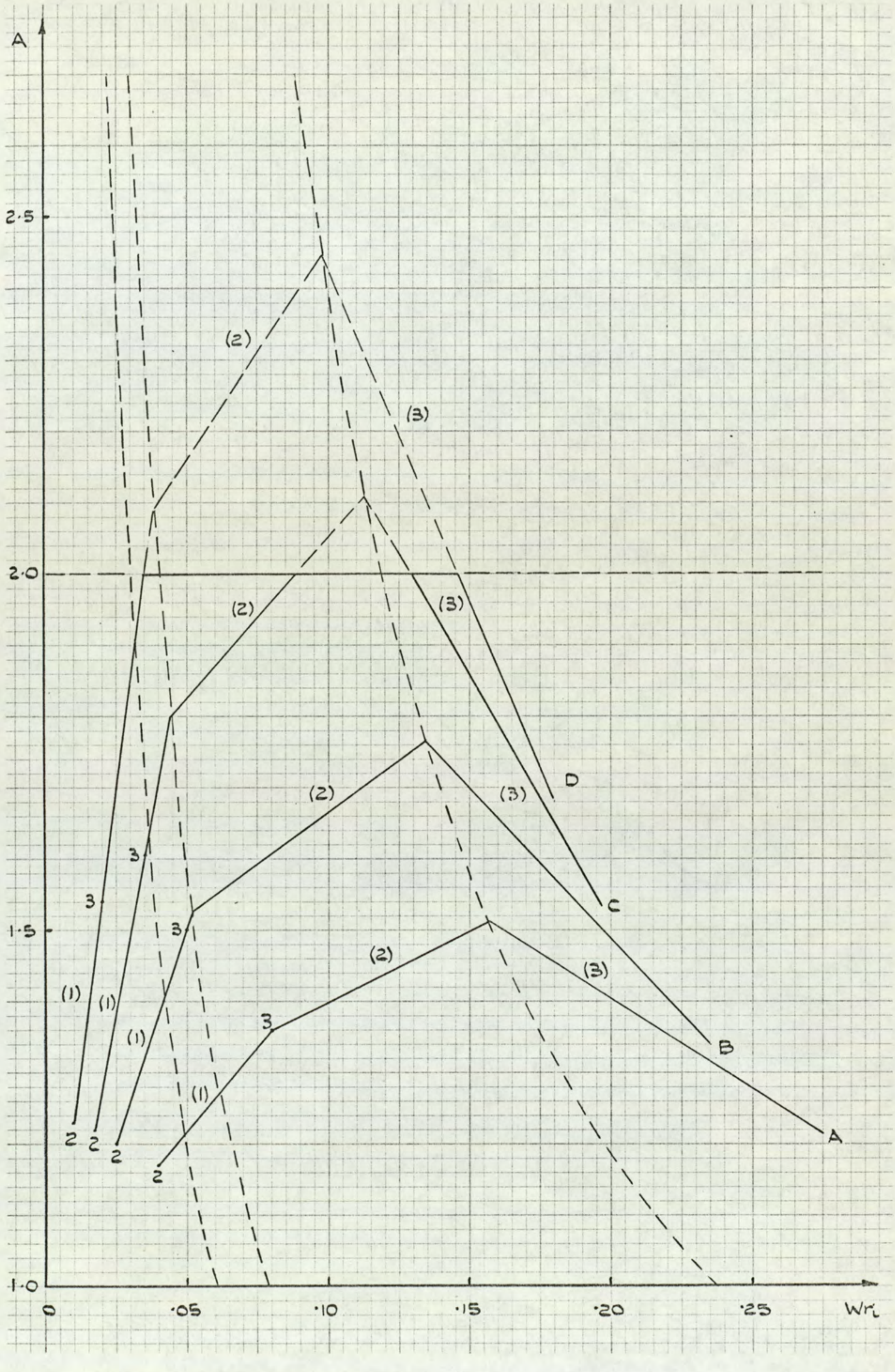
(a) The magnification factor in a particular frame under heavy wind loading (e.g. Frame A in Figure 47) ia always less than that in the corresponding storey of a frame under light wind loading (e.g. Frame D).

(b) As the initial wind ratio in successive storeys increases, A tends to increase in Zones 1 and 2, but reduces in Zone 3.

(c) The variation of A with wr_i may be considered to consist basically of three straight lines, denoted by (1),(2) and (3) for each frame in Figure 47. These are described below:-

   (1) This portion connects the points referring to Storeys 2 and 3, as shown. If, as in B, C and D, Storey 3 lies in Zones 1 or 2(i), the line continues to the boundary between Zones 2(i) and 2(ii).

   (2) The second portion connects the point on the boundary between Zones 2(i) and 2(ii), or, as for Frame A, the point referring to Storey 3, to a point on the boundary of Zone 3. It has been found that there is an approximate relationship between the gradients of lines (1) and (2), as follows:-



Variation of A with wri for four imaginary frames

FIGURE 47

$$\text{grad }(2) \simeq 0.15 \text{ grad }(1) + 1.20 \qquad (76)$$

(3) The final portion of the curve lies in Zone 3, and has a negative gradient, given approximately by:-

$$\text{grad }(3) \simeq -1.70 \text{ grad }(2) + 0.80 \qquad (77)$$

(d) Due to the limitation of - 1.80 on K_2V, as described earlier in this section, A is seldom found to be greater than 2.0. For frames under comparatively light wind loading (e.g. Frames C and D), this upper limit curtails lines (1), (2) and (3), as shown.

It may be seen that each of these curves depends exclusively on the values of A and wr_i for Storeys 2 and 3. Thus, for any frame, the procedure for estimating the value of A_i in any storey is as follows:-

(i) Design Storeys 2 and 3 in turn, using the values of A_i given be equations (74) and (75).

(ii) Plot the points (wr_i,A) for these two storeys. Join these points and, if Storey 3 lies in Zones 1 or 2(i), continue to the boundary of Zones 2(i) and 2(ii). Calculate the gradient of this line, grad (1).

(iii) Construct a line from the last point to the boundary of Zones 2(ii) and 3, with gradient, grad (2), given by equation (76).

(iv) Construct a line in Zone 3 from the last point, with gradient, grad (3), given by equation (77).

(v) For any subsequent storey, knowing wr_i from the initial design loads, A may be estimated from the graph, with an upper limit of A = 2.0. The initial magnification factor to be used in the design is given by A_i = 1.05A.

It should also be noted that, in those frames with comparatively few storeys, or with light wind loading, it is unlikely that any part of the design will fall in Zone 3. Occasionally, the complete frame design may in fact fall in Zones 1 and 2(i), in which case, portions

(2) and (3) of the curve are not required. Therefore, in order to eliminate unnecessary calculation, after portion (1) has been constructed the maximum initial wind ratio (which occurs in the penultimate storey: i.e. the last "intermediate" storey), should be inspected to see if it is necessary to construct portion (2). The same procedure should be employed after portion (2) has been constructed.

For example, in Frame C of Figure 47, portion (3) is not required if the maximum value of wr_i in any storey is less than 0.1190. Similarly, in Frame D, neither portions (2) or (3) are required if the maximum wr_i is less than 0.1190, the upper limit, A = 2.0, obviously being dominant in most storeys. In this particular frame, since $wr_{i2} \simeq .01$, and since wr_i increases linearly as the number of storeys increases, the frame would require at least twelve storeys $\left(\text{i.e. } \frac{.1190}{.01}\right)$ before portion 3 would be required.

Using this method of predicting A, the saving in design time is considerable. If no initial estimate is made, the design of any storey has to be started using the simple plastic design equation, for which all the magnification factors are unity. If the instability effects are not predominant, convergence is obtained quite rapidly using these initial values. However, the simple plastic sections are often quite inadequate, and lead to extremely high values for A and A_c. If there is no intervention, these magnification factors produce massive sections which in turn give values of A and A_c very close to unity, and this oscillation continues until convergence is slowly obtained.

In contrast, as will be shown in the design example in 6.2(f), using a sensible starting value for A, convergence is extremely rapid.

6.2(b)(ii) SELECTION OF SECTIONS

The second part of the design procedure for the intermediate storeys is concerned with the selection of suitable sections for the beams and internal columns. In general, the sections used in the design examples in this thesis are the standard Universal Beams and Universal Columns. However, any type of section may be used provided that it has a shape factor approximately equal to that of an I-section.

Having estimated the initial value of the magnification factor, A_i, for the storey under consideration, the modified wind ratio, wr, is calculated using equation (72); i.e.

$$wr = A_i . wr_i$$

The correct zone for design is then chosen, and, assuming that $A = A_c = A_i$, the appropriate values of fully plastic moment required for the beams and internal columns of the storey, B and C_I, are calculated using equations (6) to (9) in Chapter 3. The corresponding required plastic moduli, zp_b and zp_{ci} are obtained by dividing by the yield stress. Consider first the selection of the most suitable Universal Beam.

There are seventy-four Universal Beams given in the standard tables(56). However, the most economical design may be obtained by using a limited range of thirty-one of these. Each of the remaining forty-three sections has a plastic modulus which is less than that of one of the other thirty-one, whereas its weight per unit length is at least as great. Thus, none of these forty-three need to be considered, since, when selecting a beam, there is always another section which is stronger, yet no heavier. For reference purposes, the limited range of Universal Beams is given in Table 1, from which it may be seen that the section weights increase as the plastic modulus values increase. For simplicity, the section reference numbers, given on the left of the table, have been adopted in the design examples.

In deriving the design equations, it has been assumed that the axial

REFERENCE NUMBER	FULLY PLASTIC MODULUS Zp_b (in.^{3})	2nd. MOMENT OF AREA I_b (in.^{4})	CROSS-SECTIONAL AREA A (in.^{2})	STANDARD SPECIFICATION
1	16.0	68.8	4.40	10x4x15
2	20.6	105.3	4.86	12x4x16.5
3	24.8	130.1	5.62	12x4x19
4	32.8	196.2	6.47	14x5x22
5	32.9	171.6	7.35	12x5x25
6	43.9	298.1	7.64	16x5.5x26
7	47.1	289.6	8.81	14x6.75x30
8	54.1	374.9	9.12	16x5.5x31
9	54.5	339.2	10.00	14x6.75x34
10	58.5	385.5	10.29	15x6x35
11	63.8	446.3	10.59	16x7x36
12	72.7	515.5	11.77	16x7x40
13	89.6	704.8	13.23	18x7.5x45
14	100.9	800.6	14.71	18x7.5x50
15	125.2	1137.9	16.17	21x8.25x55
16	144.1	1326.8	18.23	21x8.25x62
17	175.6	1815.1	20.00	24x9x68
18	200.3	2096.4	22.37	24x9x76
19	243.3	2827.7	24.71	27x10x84
20	277.8	3266.8	27.65	27x10x94
21	315.1	4049.1	29.11	30x10.5x99
22	342.7	4080.5	33.53	27x10x114
23	377.5	4919.1	34.13	30x10.5x116
24	414.5	5896.0	34.69	33x11.5x118
25	465.9	6699.0	38.26	33x11.5x130
26	509.2	7801.3	39.69	36x12x135
27	580.0	9012.1	44.16	36x12x150
28	667.0	10470	49.98	36x12x170
29	766.8	12103	57.11	36x12x194
30	942.5	14988	67.73	36x16.5x230
31	1076	17234	76.56	36x16.5x260

Section properties of 31 Universal Beams.

TABLE 1

loads in the beams are small, and that they may therefore be neglected. Thus, in selecting the most suitable sections for these members, the moment capacity of each Universal Beam may be considered to be equal to its fully plastic moment. The procedure is therefore straightforward, the correct section, denoted by b, being the one of the smallest weight for which:-

$$Zp_b > zp_b$$

where Zp_b is the actual plastic modulus of the section, and zp_b is the required plastic modulus for the beam.

The standard range of Universal Columns consists of thirty-two sections, although, when using mild steel, only twenty-eight of these are suitable for plastic design purposes[37]. In the remaining four sections, the ratio of flange width to flange thickness is excessive, and the sections are liable to premature failure due to flange buckling before the development of a fully plastic hinge. These four sections are given below:-

$14 \times 14\frac{1}{2} \times 87$ U.C.

$12 \times 12 \times 65$ U.C.

$8 \times 8 \times 31$ U.C.

$6 \times 6 \times 15.7$ U.C.

It should also be noted that several other Universal Columns, and some Universal Beams, are unsuitable for plastic design when using high yield stress steel. These are given in the standard tables[56].

The properties of the twenty-eight Universal Columns, together with the reference numbers adopted in this thesis, are given in Table 2. The sections are arranged as before in order of increasing plastic modulus (Zp_c).

REFERENCE NUMBER	FULLY PLASTIC MODULUS Z_{p_c} (in.^{3})	2nd. MOMENT OF AREA I_c (in.^{4})	CROSS SECTIONAL AREA A (in.^{2})	STANDARD SPECIFICATION
32	15.1	41.9	5.93	6x6x20
33	18.9	53.3	7.35	6x6x25
34	34.7	126.5	10.30	8x8x35
35	39.8	146.3	11.76	8x8x40
36	49.0	183.7	14.11	8x8x48
37	59.7	227.3	17.06	8x8x58
38	60.3	272.9	14.40	10x10x49
39	75.0	343.7	17.66	10x10x60
40	90.6	420.7	21.18	10x10x72
41	114.4	542.4	26.19	10x10x89
42	119.2	663.1	23.22	12x12x79
43	140.3	788.9	27.06	12x12x92
44	147.5	718.7	32.92	10x10x112
45	163.5	930.7	31.19	12x12x106
46	180.9	1165.8	30.26	14x14.5x103
47	209.7	1221.3	39.11	12x12x133
48	211.0	1373.1	34.99	14x14.5x119
49	242.7	1593.0	39.98	14x14.5x136
50	259.1	1541.9	47.38	12x12x161
51	286.1	1900.1	46.47	14x16x158
52	311.3	1892.6	55.86	12x12x190
53	355.0	2402.4	56.73	14x16x193
54	426.8	2942.4	67.06	14x16x228
55	502.2	3526.0	77.63	14x16x264
56	591.9	4141.7	94.12	Column Core
57	610.8	4399.4	92.30	14x16x314
58	737.1	5454.2	108.78	14x16x370
59	869.4	6610.3	125.25	14x16x426

Section properties of 28 Universal Columns.

TABLE 2

In selecting the appropriate Universal Column, due allowance must be made for the reduction in plastic modulus due to the presence of axial load. For any section, the reduced plastic modulus, Zp_c' may be calculated using the formulae given in the standard tables(56); i.e.,

$$Zp_c' = Zp_c - cn^2 \tag{78}$$

for all values of n less than the "change-over" value given in the tables, and,

$$Zp_c' = d(1-n)(e+n) \tag{79}$$

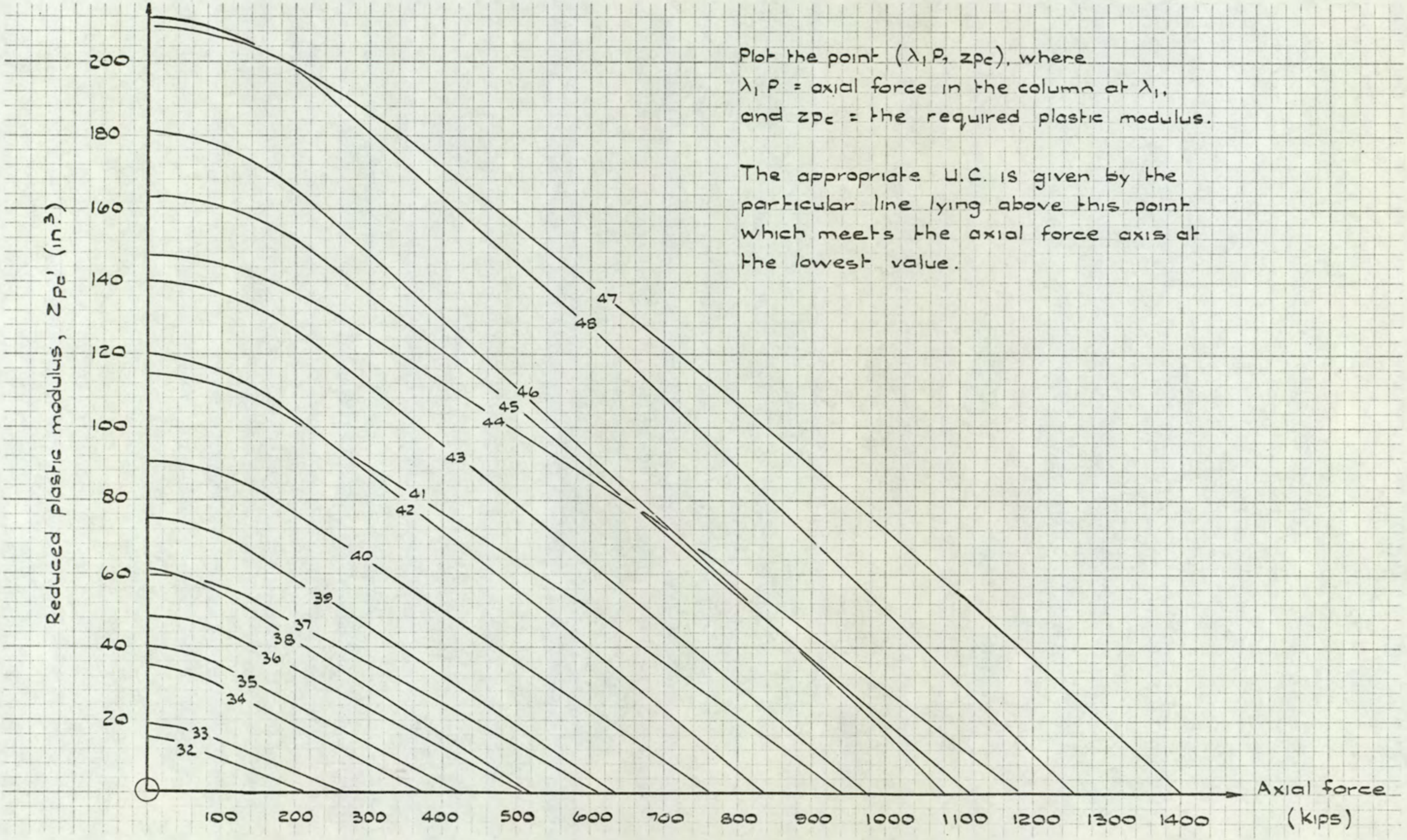
for all greater values of n, where,

$$n = \frac{\text{mean axial stress}}{\text{yield stress}} = \frac{\text{axial force}}{\text{area of section} \times \text{yield stress}} \tag{80}$$

and where c,d and e are constants for each section.

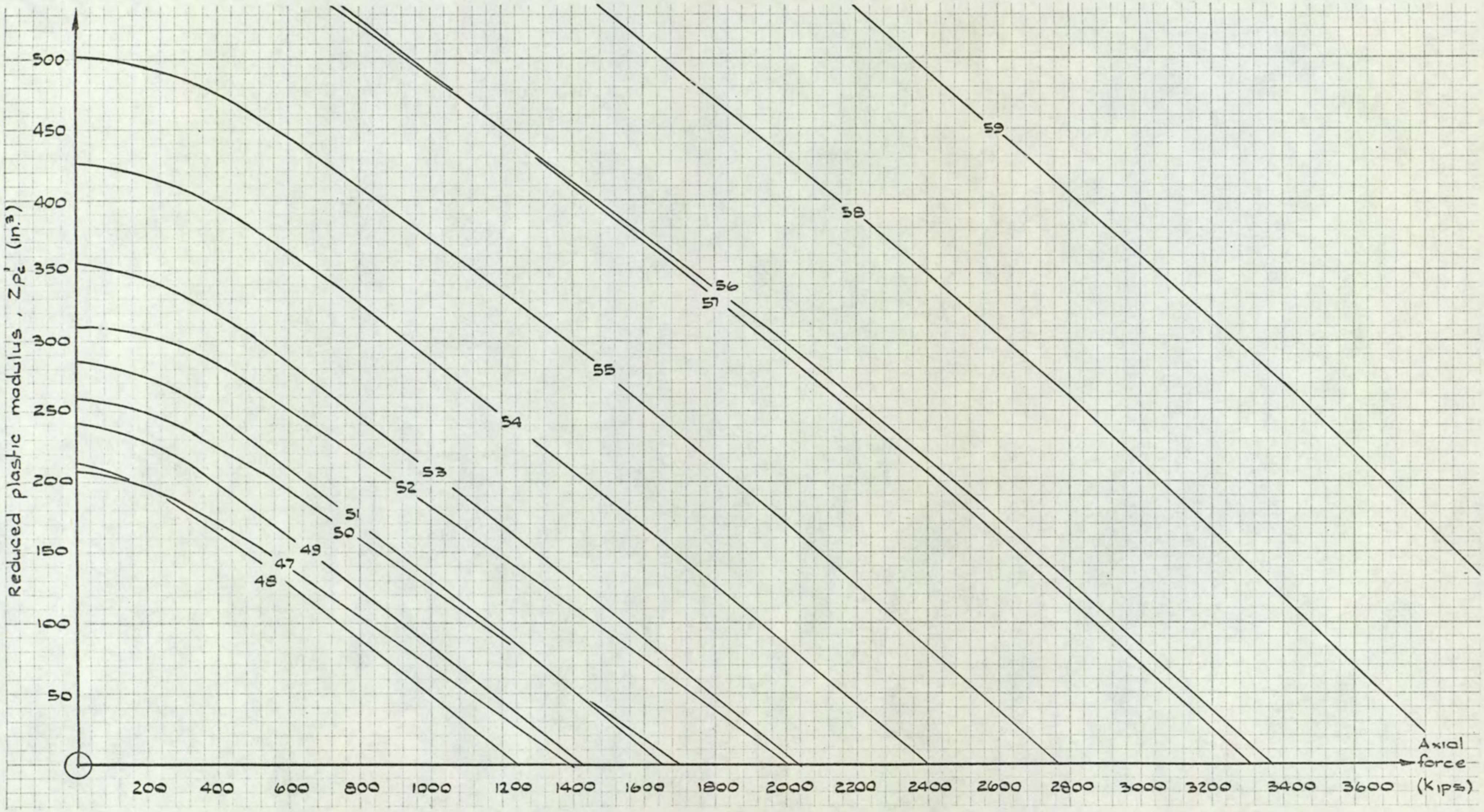
For a particular value of yield stress, for any specific column, it may be seen that n is purely a function of the axial force in the column, whereas Zp_c' is purely a function of n. Thus a direct relationship exists between the axial force and the reduced plastic modulus.

This relationship is shown for each of the twenty-eight Universal Columns in Figures 48(a) and 48(b). These curves have been constructed for mild steel, assuming a yield stress of 35.84 kips per sq. in. (i.e., 16 tons per sq. in.), and the reference numbers are those given in Table 2. For any section, the plastic modulus is seen to reduce from its full value under zero axial load (Zp_c in Table 2) to zero at a load equal to the area of the section multiplied by the yield stress. At this maximum load, the cross-section is incapable of resisting any bending moment. Also, since this maximum load is directly proportional to the area of the section, it is also directly proportional to its weight per unit length. For example, section 38 is lighter than section 37, despite the fact that its fully plastic modulus is greater. Therefore, these curves



Variation in plastic modulus with axial load for Universal Columns 32 to 48; $f_y = 35 \cdot 84^{k}/\text{sq.in.}$

FIGURE 48 (a)



Variation in plastic modulus with axial load for Universal Columns 47 to 59; f_y = 35·84 k/sq. in.

FIGURE 48 (b)

not only indicate the relative strengths of the sections, but also their relative weights.

In the design of any intermediate storey, Universal Column sections must be selected for the columns above and below the beam. Consider the lower column first. The required plastic modulus of this member, zp_{ci}, is determined from C_I in equation (9), which has been derived for combined loading at load factor λ_2. However, the maximum axial force in the column occurs under vertical loading alone at load factor λ_1, and is equal to $\lambda_1 P_2$, where P_2 is the axial force at working load. Therefore, any Universal Column which has a reduced plastic modulus greater than zp_{ci}, under axial force $\lambda_1 P_2$, will remain elastic at the design load factor for either system of loading. The appropriate Universal Column is readily obtained from Figures 48(a) and 48(b). If the point with co-ordinates $(\lambda_1 P_2,\ zp_{ci})$ is plotted, it may be seen that all the Universal Columns which are represented by curves which lie above this point are adequate. The most economical section is the one with the lowest weight, represented, as stated previously, by the curve which meets the axial load axis at the lowest value. In the design example in 6.2(f), the selected section for the lower column is denoted by c_{i2}.

To obtain the section required for the upper column, the point $(\lambda_1 P_1, zp_{ci})$ is plotted, where P_1 represents the axial force in this section at working load. The selected section is denoted by c_{i1}.

The use of these simple design charts leads to a considerable saving in the design time. Traditionally, equations (78) to (80) must be used to select the columns, and the only way of doing this is by a "trial and error" approach. A particular Universal Column is chosen, n is calculated from equation (80), and using either equation (78) or (79), Zp_c' is compared with zp_{ci} to see if the column is adequate. If not, another column is chosen, and the calculation repeated. Even when a column with sufficient strength has been found, it is not necessarily the most economical, and generally the operation must be repeated for one or two more sections to check if this is so. Thus, in selecting the appropriate section for any member, even with experience, at least three trials are necessary. Accordingly, in the design of any intermediate storey, where sections have to be obtained for both upper and lower columns separately, generally at least five operations on equations (78) to (80) are required, and these must be repeated at the end of each iteration in the design of that storey. Sections must also be selected in a similar way for the external columns. Therefore, in designing for example a ten storey frame, the total number of times that equations (78) to (80) would have to be applied is certainly in excess of one hundred. Despite the fact that these equations are comparatively simple in form, such repetitive calculation is not only very time consuming but also extremely tedious, and is completely eliminated by the use of Figures 48(a) and 48(b).

6.2(b)(iii) CALCULATION OF THE MAGNIFICATION FACTORS

Having selected preliminary sections for the beams and internal columns of the intermediate storey, the next step in the design procedure is to determine the values of the magnification factors corresponding to these sections. Equations (23) to (28) in Chapter 3 are used for this purpose, and the relevant stability functions may be selected from Table 3.

In the design example in 6.2(f), specific calculations are given for the magnification factors in each zone, and these amply illustrate the order of calculation of the different parameters. The resulting values of A and A_c for any storey are always found to be similar, the reason for this being examined below.

Consider, for example, the values of A and A_c in Zone 1, as given by equations (23) and (24); i.e.,

ρ	m	n - o	n	$\frac{o}{n}$	s(1+c)
0	1.0000	0	1.0000	1.000	6.0000
0.01	1.0083	-0.0497	0.9669	1.051	5.9901
0.02	1.0168	-0.1004	0.9333	1.107	5.9802
0.03	1.0254	-0.1518	0.8993	1.169	5.9703
0.04	1.0343	-0.2042	0.8648	1.236	5.9604
0.05	1.0433	-0.2574	0.8298	1.310	5.9504
0.06	1.0525	-0.3113	0.7943	1.392	5.9405
0.07	1.0618	-0.3668	0.7584	1.484	5.9306
0.08	1.0714	-0.4230	0.7218	1.586	5.9206
0.09	1.0812	-0.4802	0.6848	1.701	5.9106
0.10	1.0913	-0.5385	0.6471	1.832	5.9006
0.11	1.1015	-0.5979	0.6089	1.982	5.8905
0.12	1.1120	-0.6584	0.5701	2.155	5.8805
0.13	1.1227	-0.7202	0.5306	2.357	5.8704
0.14	1.1336	-0.7832	0.4905	2.597	5.8604
0.15	1.1449	-0.8474	0.4498	2.884	5.8503
0.16	1.1563	-0.9130	0.4083	3.236	5.8403
0.17	1.1681	-0.9800	0.3661	3.677	5.8301
0.18	1.1801	-1.0482	0.3233	4.242	5.8200
0.19	1.1924	-1.1180	0.2796	4.998	5.8099
0.20	1.2051	-1.1894	0.2351	6.059	5.7998
0.21	1.2180	-1.2622	0.1899	7.647	5.7896
0.22	1.2313	-1.3367	0.1438	10.295	5.7794
0.23	1.2449	-1.4130	0.0968	15.597	5.7692
0.24	1.2589	-1.4909	0.0489	31.489	5.7590
0.25	1.2732	-1.5708	0	∞	5.7487

Stability functions for the range $0 \leqslant \rho \leqslant 0.25$

<u>TABLE 3</u>

$$A = a_{22}^{\Delta}{}' = \frac{3}{K_2V + 3}$$

and,

$$A_c = 1 - 2a_{23}^{\Delta}.p_2$$

Expanding A_c:-

$$A_c = 1 - 2.\frac{(n_2-o_2)K_2}{K_2V + 3}.p_2$$

$$= \frac{K_2V + 3 - 2(n_2-o_2)K_2p_2}{K_2V + 3}$$

$$= \frac{3}{K_2V + 3} + \frac{K_2[V - 2(n_2-o_2)p_2]}{K_2V + 3}$$

Now,

$$V = (n_1-o_1)\overline{K} + (n_2-o_2)$$

Therefore,

$$A_c = \frac{3}{K_2V + 3} + \frac{K_2[(n_1-o_1)\overline{K} - (n_2-o_2)(2p_2-1)]}{K_2V + 3}$$

For most intermediate storeys, $\overline{K}$ and p_2 are generally close to unity, and $(n_1-o_1) \simeq (n_2-o_2)$. Thus, it may be seen that the second term in A_c is generally very small, and it follows that :-

$$A_c \simeq \frac{3}{K_2V + 3}$$

i.e.,

$$A_c \simeq A$$

This is also true in Zones 2 and 3. In any frame, the greatest deviation from this result is likely to occur in Storey 2, where $p_2 \simeq \frac{2}{3}$. However, the second term in A_c is still small in comparison to the first, and, in all the frames that have been designed, A and A_c have never been found to differ by more than 3%. Generally, the difference is far less than this.

This fact serves as a useful check on the validity of the values obtained for the magnification factors. If a substantial difference is found between A and A_c, it is certain that an arithmetical mistake has been made, and the calculation must be repeated.

The design procedure continues by recalculating the wind ratio using the current value of A, and selecting a new set of sections for the beams and columns. If these are different to the initial set, the magnification factors are recalculated and a further set of sections is obtained. The process is continued until a unique set is obtained in two successive iterations.

As stated previously, provided that a rational estimate is made for the magnification factors, the design generally converges very rapidly. However, occasionally it is found that the design tends to oscillate between two distinct section sizes for either the beams or the columns. In such cases, new trial values of the magnification factors, intermediate between those obtained for the different sets of sections, may be assumed in order to encourage convergence. If this is unsuccessful, then the larger of the two sections must be selected.

6.2(b)(iv) "THE WORKING-LOAD ELASTICITY CONDITION"

The final stage in the design procedure for each intermediate storey is to check that the beam remains elastic at working load. The basic condition has been derived in Section 3.10, and is represented by

$$\frac{WL}{12} + \frac{a_{22}^{1\,\prime}}{2}(m^1 Hh)av \leq Mp_b$$

This relationship is now examined in more detail. Consider first the possible range of values for the distribution factor, which is given by:-

$$a_{22}^{1\,\prime} = \frac{12}{K_2 V^1 + 12}$$

where,

$$V^1 = (n_1 - o_1)^1 \overline{K} + (n_2 - o_2)^1$$

In 6.2(b)(i), it has been shown that it is necessary to restrict the value of K_2V(which corresponds to Euler ratios at load factor λ_2) in the design of any storey in order to control the instability effects, and a limit of - 1.80 has been chosen. There must therefore be a corresponding limit on K_2V^1, and this may easily be calculated provided that a relationship may be found between V and V^1.

Now, since the Euler ratio in any column is directly proportional to the axial load in that column:-

$$\rho = \lambda_2 \rho^1$$

Also, from inspection of the stability functions, approximately:-

$$(n - o) \propto \rho$$

Therefore,

$$(n_1 - o_1) \simeq \lambda_2 (n_1 - o_1)^1$$

and

$$(n_2 - o_2) \simeq \lambda_2 (n_2 - o_2)^1$$

i.e.,

$$V \simeq \lambda_2 V^1 = 1.4V^1$$

Thus, if the limiting value of K_2V is - 1.80, the corresponding limit on K_2V^1 is approximately equal to $\frac{-1.80}{1.4} = -1.284$. This leads to a maximum value of $a_{22}^{1\prime}$ given by:-

$$a_{22}^{1\prime} = \frac{12}{10.716} = 1.120$$

Now, it may be seen that the working load elasticity condition becomes more difficult to satisfy as $a_{22}^{1\prime}$ increases. Therefore, the assumption that $a_{22}^{1\prime}$ is always equal to its maximum value leads to an approximate conservative condition, as follows:-

$$\frac{WL}{12} + 0.56\ (m^1Hh)av \leq Mp_b$$

The condition may be modified further by assuming $m^1 \simeq m$, which is again slightly conservative. In addition, let η be the ratio of the actual plastic moment of the selected Universal Beam to the required plastic moment for the beam; i.e.,

$$\eta = \frac{Mp_b}{B} \geq 1$$

Therefore, in any zone, the working load elasticity condition becomes:-

$$\frac{WL}{12} + 0.56(mHh)av \leq \eta.B \qquad (81)$$

Each zone may be considered in turn, with the approximate value of B substituted in equation (81).

ZONE 1 $\qquad B = \lambda_1 \frac{WL}{16} = \frac{5}{64}\lambda_2\ WL$

Therefore,

$$\frac{WL}{12} + 0.56(mHh)av \leq \eta.\frac{5}{64}\lambda_2\ WL$$

Dividing be 0.56WL, and regrouping:-

$$\frac{(mHh)av}{WL} \leq \frac{1}{0.56}\left[\frac{5}{64}.\lambda_2.\eta - \frac{1}{12}\right] = \frac{1}{0.56}\left[\frac{15\lambda_2\eta-16}{192}\right]$$

However, denoting the final wind ratio by wr,

$$\frac{(mHh)av}{WL} = \frac{wr}{A}$$

Therefore, the condition becomes:-

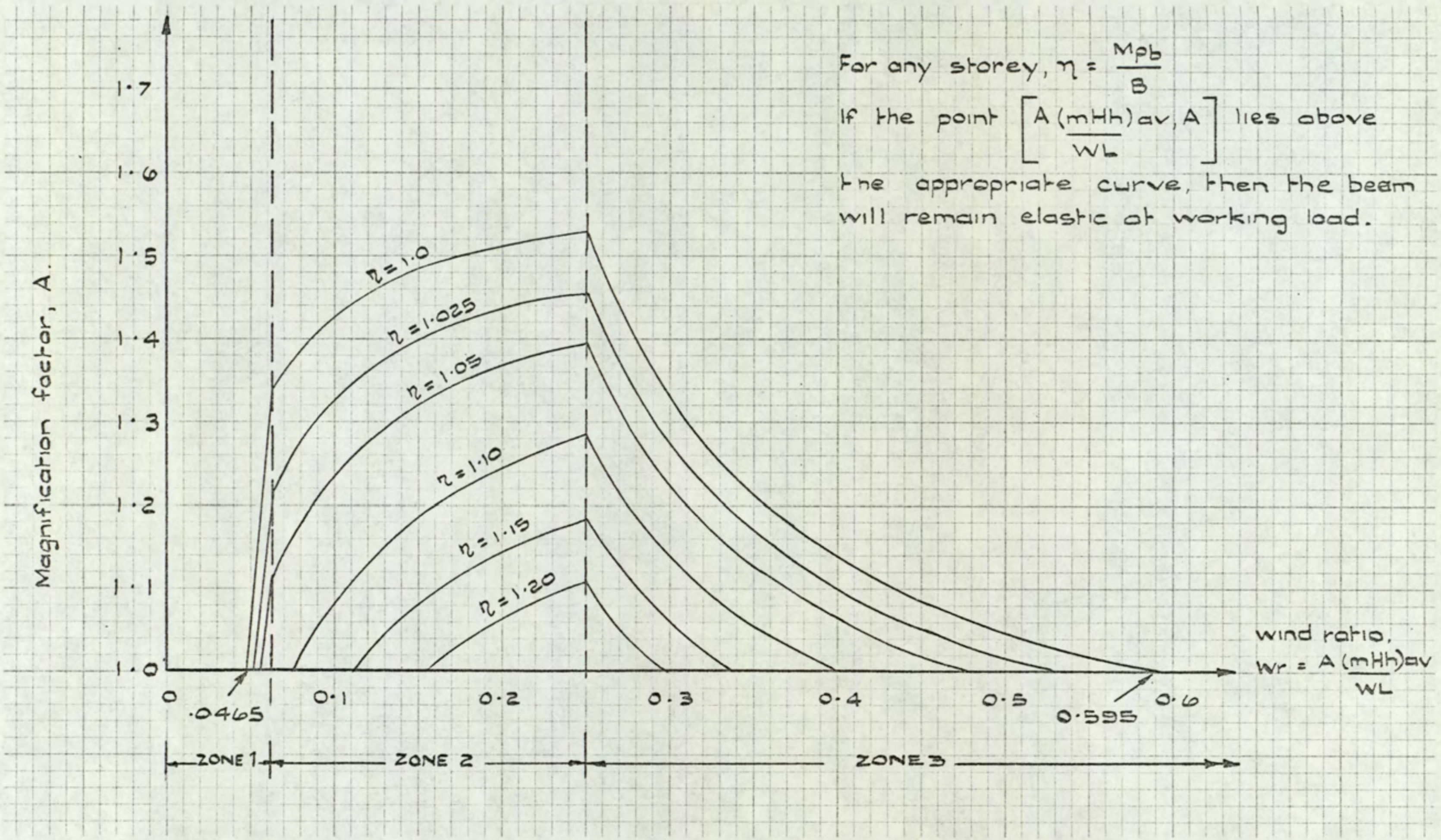
$$\frac{wr}{A} \leq \frac{15\lambda_2\eta-16}{107.5}$$

Substituting $\lambda_2 = 1.4$,

$$\frac{wr}{A} \leq \frac{21\eta-16}{107.5}$$

Alternatively,

$$A \geq \frac{107.5wr}{21\eta-16} \qquad (82)$$



"Working - load elastistity condition" - Multi- bay frames

FIGURE 49

ZONE 2 $\quad B = \lambda_2 \frac{WL}{16} + \lambda_2 \frac{A(mHh)av}{4}$

Substituting B in expression (81), and rearranging as before, the following condition is obtained for Zone 2:-

$$A \geqslant \frac{134.4\ wr}{84\eta.wr + (21\eta - 20)} \qquad (83)$$

ZONE 3 $\quad B = \lambda_2 \frac{A(mHh)av}{2}$

In this case, the condition is:-

$$A \geqslant \frac{6.72\ wr}{8.4\ \eta wr - 1} \qquad (84)$$

The limiting cases of these three conditions are represented graphically in Figure 49, for values of η varying between 1.0 and 1.2. Thus, to check the elasticity of any beam at working load, it is simply necessary to calculate η and select the appropriate curve. Then, if the point (wr,A) lies above that curve, the beam will remain elastic at working load.

Although based on several approximations, these curves appear to give an accurate representation of the original working load elasticity condition. In fact, it has not been found necessary to increase the beams in any of the frames described in this thesis, although, in one frame, which was designed with a fictitious range of sections (which will be discussed later), two storeys were found to lie very close to the limiting curve. In this particular frame, the accurate computer analysis predicted plastic hinges in these two storeys at load factors fractionally below the working load (0.991 and 0.996). In every other case the analysis indicated that the beams remained elastic at working load, as predicted by the curves in Figure 49.

6.2(c) THE TOP STOREY

In section 2 of Chapter 4 it has been shown that the critical mode of failure in the top storey is always the simple beam mechanism under vertical load alone at load factor λ_1. Even under extremely heavy wind loading, in this region of the frame there is no likelihood of a combined or sway mechanism at load factor λ_2.

However, as for the intermediate storeys, the maximum column bending moments occur in the combined loading case. The shear force in each column of the top storey is considerably less than that in the storey below (Storey 2), and a similar ratio is bound to exist between the bending moments. Thus, as suggested previously, the value of plastic moment required for the column in Storey 2 is also adequate for the top storey. The Universal Column required for the top storey must therefore be identical to that denoted by c_{i1} in the design of Storey 2, since this particular section is selected using the required plastic modulus in Storey 2, together with the axial load corresponding to the top storey.

The design of the top storey is therefore straightforward, and does not involve any iteration.

6.2(d) THE LOWEST STOREY

The design equations for the lowest storey have been derived in Chapter 4, and they are summarized in 4.3(c). The design procedure follows a similar schedule to that for the intermediate storeys, although, of course, different design equations are involved.

It will be remembered that the design of this storey is always based on failure by the simple beam mechanism. Therefore, the beam size remains unaltered, and no beam magnification factor is required. There are, however, two column magnification factors to be calculated. These are given by equations (38) and (40) in 4.3(c); i.e.,

$$A_{c1} = 1-2a_{21}^{\Delta}.p_1$$

and,

$$A'_{c2} = 1 + 2\,\frac{o_2}{n_2}.\overset{\Delta}{a}_{23}.p_2$$

The significance of the second of these factors, A'_{c2}, has already been discussed in 4.3(b), where it was shown that this is not a true magnification factor. Under simple plastic conditions, its value is considerably greater than unity. Furthermore, this value does not increase substantially as the axial loads increase, indicating that the instability effects in the lowest storey are small. For this reason, and also because the beam remains the same in each iteration, the design invariably converges to the correct solution very quickly. Therefore, unlike in the design of the intermediate storeys, it is not strictly necessary to predict particularly accurate initial values for A'_{c1} and A'_{c2} in order to encourage convergence.

In all but one of the frameworks described in this thesis, the design has been found to converge immediately by using the approximate values, $A_{c1} = 1.4$ and $A'_{c2} = 2.7$. In the remaining frame, only two iterations were required, despite the fact that the final value of A'_{c2} was as low as 1.84. Therefore, in any frame, these initial values for the magnification factors may be assumed to produce a rapid design.

As for the intermediate storeys, the design of the lowest storey proceeds by calculating the value of plastic moment for each member, using the appropriate equations in 4.3(c). Column sections are selected as before using Figures 48(a) and 48(b). The Euler ratios, stability functions and distribution factors are then determined, and new values are calculated for A_{c1} and A'_{c2}. As before, if the resulting sections are different to those selected initially, the procedure is repeated until convergence is obtained.

The working-load elasticity condition must now be checked. In this case:-

$$\frac{WL}{12} + \frac{a_{22}^{1}{}'}{2}(m^1 Hh)av \leq Mp_b$$

where,

$$a_{22}^{1}{}' = \frac{12}{K_2 V'^1 + 12}$$

and,

$$V'^1 = (n_1 - o_1)^1 \overline{K} + n_2^{\,1}$$

In deriving the approximate form of the working-load elasticity condition for the intermediate storeys, in 6.2(b)(iv), a simple relationship was obtained between the quantities V and V^1. However, no such relationship exists between V^1 and V'^1, since each of these depends on two different stability functions, (n-o) and n.

Therefore, in this storey, it is necessary to check the condition arithmetically rather than by using an approximate graphical approach. For this purpose, it has been found convenient to derive an alternative form of the basic condition in terms of the section properties, I_b and Zp_b, which, for any Universal Beam, may be read directly from Table 1. Expanding $a_{22}^{1}{}'$, the condition becomes:-

$$\frac{WL}{12} + \frac{1}{2}.\frac{12}{K_2 V'^1 + 12}.(m^1 Hh)av \leq Mp_b$$

Dividing throughout by the yield stress, f_y, and expanding K_2:-

$$\frac{WL}{12f_y} + \frac{6(m^1 Hh)av}{f_y\left[\frac{1}{I_b}.I_{c2}.\frac{L}{h_2}.V'^1 + 12\right]} \leq Zp_b$$

Alternatively,

$$E + \frac{F}{\left[\frac{G}{I_b} + 12\right]} \leq Zp_b \qquad (85)$$

where,

$$E = \frac{WL}{12f_y}; \quad F = \frac{6(m^1 Hh)av}{f_y}; \quad G = I_{c2}.\frac{L}{h_2}.V'^1$$

The quantities E, F and G are independent of the properties of the selected Universal Beam. Therefore, having determined these functions, the adequacy of the beam may be checked quite simply by substituting I_b, calculating the left-hand side of the condition, and comparing this value with Zp_b. If the condition is violated, another beam is selected with a greater value of I_b, and the calculation is repeated, with no alteration being required to E, F and G.

This method of checking the elasticity of the beam at working load is more time-consuming than that developed for the intermediate storeys. However, it is more accurate, and in this respect it is preferable to a graphical approach. In the frames that have been designed and then analysed, it has been found that the lowest storey beam is generally the member in which the first plastic hinge forms. It is therefore advisable to use an accurate method of checking this critical section, and, in every frame, the computer analysis has confirmed the validity of the proposed condition.

6.2(e) THE EXTERNAL COLUMN

Having selected sections for every beam and internal column in the framework, only the external columns remain to be designed. The various equations for determining C_E are summarized in 4.4(e). It may be seen that each equation is a function of B, C_I and p_2. These are all known quantities, so that the design of the external column is straightforward, and no iteration is involved.

In calculating C_E, it must be remembered that B and C_I refer to the required values of plastic moment for the beam and internal column. C_I

appears in the equations for C_E due to direct substitution during the derivation of these equations. B, however, appears due to the initial assumption that a plastic hinge exists at the leeward end of each beam. It may be argued that the value Mp_b, the actual plastic moment of the beam, should be used in place of B. However, since B represents the basic strength requirement of the beam, it is considered that this should be the value with which the strength requirement of the column is assessed. The fact that the actual beam has a higher plastic moment should only increase the overall strength of the framework. In addition, in selecting the column sections, the two columns meeting at a joint are together likely to have a reserve of strength at least as great as that assigned to the beam. Nevertheless, it is fully appreciated that by selecting the sections in this way, there is a slight possibility that a plastic hinge may form in a column below the design load factor. This has not, however, been found to occur in any framework designed with Universal Beams and Universal Columns.

A typical calculation for the external column is included in the design example which follows.

6.2(f) DESIGN EXAMPLE

This section describes the design of specific regions of the ten storey, four bay framework shown in Figure 50. In order to illustrate as many points as possible in the design procedure, to demonstrate the use of the design aids, and to provide a useful reference to the most efficient order of calculation of the numerous design parameters, a fairly comprehensive treatment is given for each of these regions. However, the calculation for four of the intermediate storeys are omitted, since they add little to the discussion, and, whenever possible, repetitive arithmetical work is abridged.

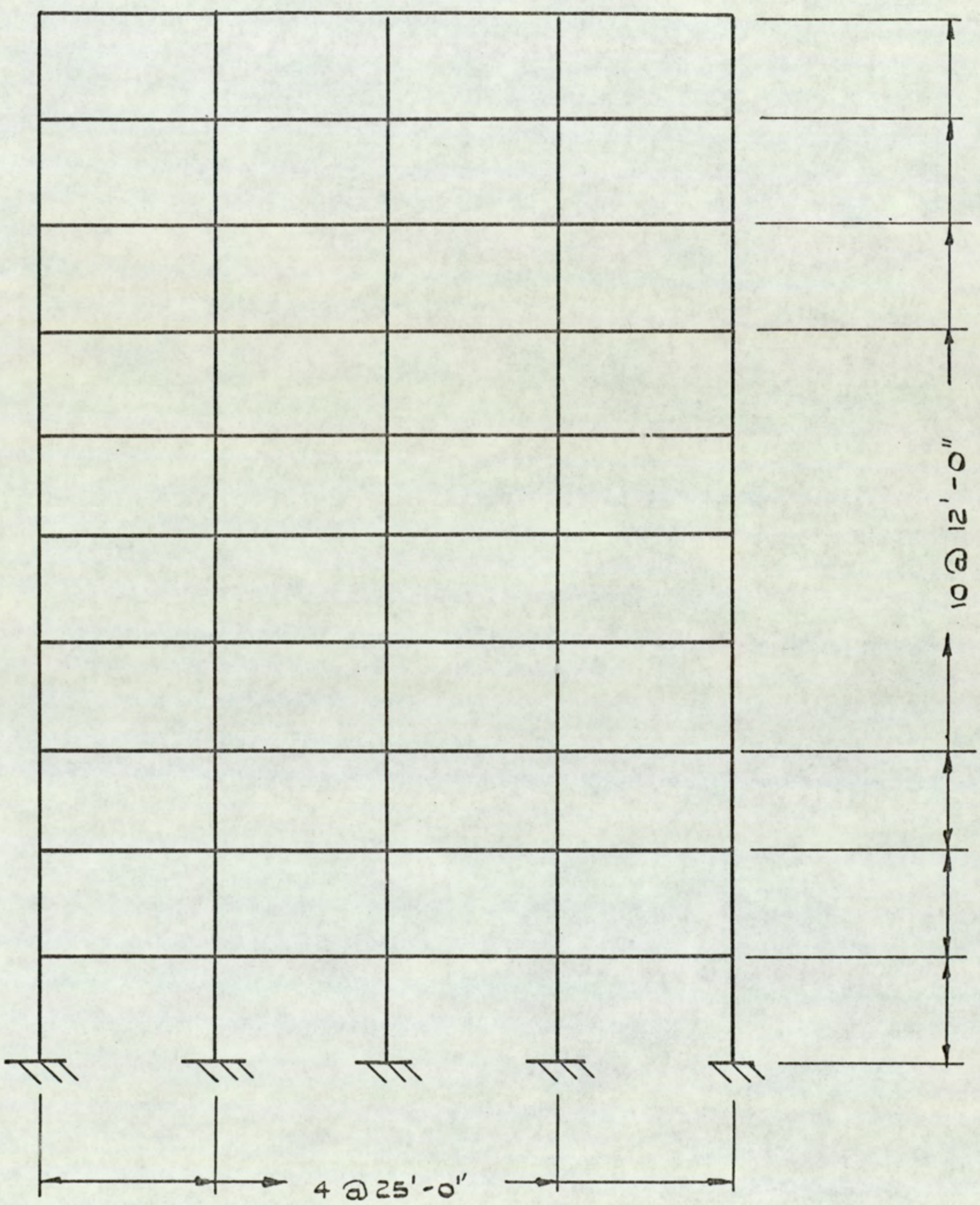
This particular frame has been selected as the design example, since

Dead + live load on top storey = ·05 k./sq. ft.

Dead + live load on other storeys = ·10 k/sq ft.

Wind load = ·025 k./sq. ft

Frame spacing : 20'-0"



10 Storey, 4 bay frame.

FIGURE 50

it contains storeys lying in all of the four zones, 1, 2(i), 2(ii) and 3. The frame specification is given below:-

SPECIFICATION

Number of storeys, $q = 10$; Number of bays, $r = 4$;

Beams lengths, $L = 25$ft.; Column heights, $h = 12$ft.,in all storeys;

Frame spacing, $S = 20$ft.;

In the top storey, $w_1 = .05$k. per sq. ft..

In all other storeys, $w = .10$k. per sq. ft.

Wind loading, $p = .025$k. per sq. ft.

Design load factors: $\lambda_1 = 1.75$; $\lambda_2 = 1.4$;

Material: Mild steel; $f_y = 35.84$k. per sq. in.; $E = 30000$k. per sq.in.

THE DESIGN LOADS

(i) Beam loads [equation (64)]:-

In the top storey, $W_1 = \frac{.05 \times 20}{2}(2 \times 25 - 20) = 15$k.

In all other storeys, $W = 30$k.

Also, $WL = 30 \times 25 = 750$k.ft.

(ii) Shear forces [equation (65)]:-

All storey heights are equal. Therefore, equation (65) reduces to:-

$$H_n = \frac{p.S.h}{r}\left[n - \frac{1}{2}\right]$$

Therefore, in all intermediate storeys:-

$$H_n = 1.5\left[n - \frac{1}{2}\right]$$

(iii)Internal column loads [equation (66)]:-

In the top storey, $P_I = 25$k

In all other storeys, $P_{In} = P_{In-1} + 50$

(iv) External column loads [equation (67)]:-

Equation (67) reduces to:-

$$P_{En} = P_{En-1} + \frac{wLS}{2} + \frac{h}{2L}\left(H_{n-1} + H_n\right)$$

n	Shear per bay; $H_n = 1.5\,[n - \frac{1}{2}]$	Internal column load; $P_{I_1} = 25$ $P_{In} = P_{In-1} + 50$	$0.24(H_{n-1} + H_n)$ = (x), say.	External column load; $P_{E_1} = 12.5 + (x)$ $P_{En} = P_{En-1} + 25 + (x)$	Initial wind ratio; $wr_i = .008(H_{n-1} + H_n)$
1	0.75	25	0.2	12.7	-
2	2.25	75	0.7	38.4	.024
3	3.75	125	1.4	64.8	.048
4	5.25	175	2.2	92.0	.072
5	6.75	225	2.9	119.9	.096
6	8.25	275	3.6	148.5	.120
7	9.75	325	4.3	177.8	.144
8	11.25	375	5.0	207.8	.168
9	12.75	425	5.8	238.6	.192
10	$\Sigma(H) = 60$	475	6.5	270.2	-

Calcuation of initial design load - 10 storey, 4 bay frame - all loads in kips.

<u>TABLE 4</u>

In the top storey, n = 1, so that:-

$$P_E = 12.5 + 0.24\ H$$

In all other storeys,

$$P_{En} = P_{En-1} + 25 + 0.24\left(H_{n-1} + H_n\right)$$

(v) Initial wind ratios:-

Since h is constant, in all intermediate storeys:-

$$wr_i = \frac{(Hh)av}{WL} = \frac{h}{2WL}\left(H_{n-1} + H_n\right)$$

i.e., $wr_i = .008\left(H_{n-1} + H_n\right)$

The calculation of these initial loads is shown in Table 4. The only modification occurs in the calculation of the external column load in the lowest storey, for which the wind load component is given by:-

$$(X) = 0.24\left(H_9 + \frac{\Sigma(H)_{10}}{r}\right)$$

DESIGN FORMULAE

Many of the design formulae may be simplified considerably for any particular frame by substituting the known values of h, L, W, λ_1 and λ_2. In this case, the following simplified formulae are obtained:-

$$wr = .008\ A\Sigma(mH)$$

where, for any storey,

$$\Sigma(mH) = (m_1H_1 + m_2H_2)\ \text{kips}$$

In all zones,

$$\overline{K} = \frac{I_{c1}}{I_{c2}};\ K_2 = 2.083\ \frac{I_{c2}}{I_{b2}};\ p_2 = \frac{\Sigma(mH)}{m_2H_2}$$

$$C_I = 100.8\ A_c m_2 H_2;\ \rho = .0980\ \frac{P}{I}$$

C_I is measured in the units kip.inches.

In Zone 1,

$$B = \lambda_1\ \frac{WL}{16} = 984\ \text{k.ins}$$

$$zp_b = \frac{984}{35.84} = 27.5 \text{ in}^3$$

Therefore, for all storeys lying in Zone 1, from Table 1,

$$b = 4$$

In Zone 2,

$$B = \lambda_2 \frac{WL}{4} \left[\frac{1}{4} + \frac{A(mHh)av}{WL}\right] = 3150\ (0.25 + wr)$$

$$\frac{\alpha}{\lambda_2} = \frac{A}{a_{22}^{\alpha}/} \left[\frac{1}{2} - \frac{WL}{24A(mHh)av}\right] = \frac{A}{a_{22}^{\alpha}/} \left[0.5 - \frac{5.21}{A\Sigma(mH)}\right]$$

In Zone 3,

$$B = \lambda_2 \frac{A(mHh)av}{2} = 50.4\ A\Sigma(mH)$$

$$\frac{\alpha}{\lambda_2} = \frac{A}{a_{22}^{\alpha}/} \left[1 - \frac{WL}{6A(mHh)av}\right] = \frac{A}{a_{22}^{\alpha}/} \left[1 - \frac{20.84}{A\Sigma(mH)}\right]$$

The function $\Sigma(mH)$ is included as shown, since it is always determined during the calculation of p_2, and may then be substituted directly.

DESIGN OF THE INTERMEDIATE STOREYS

Design calculations are given in full for Storeys 2,3,6 and 7, each of which lies in a different zone. The design of the remaining intermediate storeys is summarized. The method of estimating the values of A_i is also included. In addition, the design of Storey 7 demonstrates the case when the beam has to be increased to control the instability effects.

Storey 2:

$$H_1 = 0.75;\quad H_2 = 2.25;\quad P_1 = 25;\quad P_2 = 75$$

$$wr_i = .024$$

From equation (74):-

$$A_i = \frac{1.4}{1 + 2.2 \times .024} = 1.33$$

Assume $A = A_c = A_i$; $m_1 = m_2 = 1.0$

The initial modified wind ratio is:-

$$wr = A_i . wr_i = .0319 \simeq \frac{1}{31} \rightarrow \text{Zone 1.}$$

Therefore, it is predicted that the design lies in Zone 1, for which the beam size is already known [b = 4]. The column sections must be selected with the aid of Figures 48(a) and 48(b).

$$C_I = 100.8 \times 1.33 \times 2.25 = 303.$$

$$zp_{ci} = 8.4$$

	$\lambda_1 P_1 = 44$	$\lambda_1 P_2 = 131$
$b = 4$	$c_{i1} = 32$	$c_{i2} = 33$
$I_{b2} = 196$	$I_{c1} = 42$	$I_{c2} = 53$

The Euler ratios are now calculated, and the stability functions are obtained from Table 3, using linear interpolation.

$$\rho_1 = .0980 \times \frac{25}{42} = .0582$$

$$\rho_2 = .0980 \times \frac{75}{53} = .1385$$

$(n_1 - o_1) = - 0.302$; $(n_2 - o_2) = - 0.774$; $m_1 = 1.051$; $m_2 = 1.132$;

The functions $\overline{K}$, V, K_2, K_2V, (K_2V+3) are calculated in turn.

$$\overline{K} = \frac{42}{53} = 0.792; \quad V = - 0.302 \times 0.792 - 0.774 = - 1.013;$$

$$K_2 = 2.083 \times \frac{53}{196} = 0.562; \quad K_2V = -0.569; \quad K_2V + 3 = 2.431;$$

In Zone 1, the distribution factors $\overset{\Delta}{a}_{22}{}'$ and $\overset{\Delta}{a}_{23}$ are required.

$$\overset{\Delta}{a}_{22}{}' = \frac{3}{2.431} = 1.235; \quad \overset{\Delta}{a}_{23} = \frac{- 0.792 \times 0.562}{2.431} = - 0.183;$$

Also,

$$p_2 = \frac{1.051 \times 0.75 + 1.132 \times 2.25}{2 \times 1.132 \times 2.25} = \frac{0.789 + 2.546}{5.092} = \frac{3.335}{5.092} = 0.655;$$

The magnification factors are obtained from equations (23) and (24).

$A = 1.235$;

$A_c = 1 - 2 \times (-0.183) \times .655 = 1.240$;

Recalculating the wind ratio (abstracting $\Sigma(mH)$ from p_2):-

$$wr = .008 \times 1.235 \times 3.335 = .0329 \simeq \frac{1}{30} \rightarrow \text{Zone 1.}$$

$$C_I = 100.8 \times 1.240 \times 2.546 = 321$$

$$zp_{ci} = 9.0$$

$b = 4$ $\qquad$ $c_{i1} = 32$ $\qquad$ $c_{i2} = 33$

Thus, the sections are identical to those selected initially, and the design has converged. Referring to Figure 49, it may be seen that the working-load elasticity condition is bound to be satisfied, since $wr < .0465$. Therefore, the value of η need not be calculated. The sections selected for Storey 2 are therefore:-

<u>Beam</u>: 4 $\qquad$ <u>Internal Column</u>: 33

<u>Storey 3</u>:

$$H_1 = 2.25; \quad H_2 = 3.75; \quad P_1 = 75; \quad P_2 = 125;$$

$$wr_i = .048$$

From the design of Storey 2, $A_2 = 1.235$; $wr_2 = .0329$;

Therefore, in this storey, from equation (75):-

$$A_i = \frac{1}{\left[0.95 - \frac{0.235}{.0329} \times .048\right]} = \frac{1}{0.61} = 1.64$$

The design proceeds as before, and in this case, no explanation is given:-

$$wr = 1.64 \times .048 = .0784 \simeq \frac{1}{13} \rightarrow \text{Zone 2(i)}$$

$$B = 3150 \times (.25 + .0784) = 1035;$$

$$C_I = 100.8 \times 1.64 \times 3.75 = 620;$$

$zp_b = 28.9$ $\qquad$ $zp_{ci} = 17.3$

$\lambda_1 P_1 = 131$ $\qquad$ $\lambda_1 P_2 = 219$

$b = 4$ $\qquad$ $c_{i1} = 34$ $\qquad$ $c_{i2} = 35$

$I_{b2} = 196$ $\qquad$ $I_{c1} = 127$ $\qquad$ $I_{c2} = 146$

$$\rho_1 = .0980 \times \frac{75}{127} = .0579$$

$$\rho_2 = .0980 \times \frac{125}{146} = .0839$$

$(n_1 - o_1) = -\ 0.300;\quad (n_2 - o_2) = -\ 0.445;\quad m_1 = 1.051;\quad m_2 = 1.075;$

$\overline{K} = \frac{127}{146} = 0.870;\quad V = -\ 0.300 \times 0.870 - 0.445 = -\ 0.706;$

$K_2 = 2.083 \times \frac{146}{196} = 1.530;\quad K_2V = -\ 1.080;\quad K_2V + 3 = 1.920;$

$a^{\Delta}_{22}{}' = \frac{3}{1.920} = 1.565;\quad a^{\Delta}_{23} = \frac{-0.445 \times 1.530}{1.920} = -0.355;$

$p_2 = \frac{1.051 \times 2.25 + 1.075 \times 3.75}{2 \times 1.075 \times 3.75} = \frac{2.36 + 4.02}{8.04} = \frac{6.38}{8.04} = 0.791;$

$A = 1.565;$

$A_c = 1 + 2 \times 0.355 \times 0.791 = 1.562;$

$$wr = .008 \times 1.565 \times 6.38 = .0799 \simeq \frac{1}{12.5} \rightarrow \text{Zone } 2(i).$$

$$B = 3150 \times (.25 + .0799) = 1040;$$

$$C_I = 100.8 \times 1.562 \times 4.02 = 633;$$

$zp_b = 29.0$ $\qquad$ $zp_{ci} = 17.7$

$b = 4$ $\qquad$ $c_{i1} = 34$ $\qquad$ $c_{i2} = 35$

The sections are identical to the previous set, so that the design has converged. Again, in this storey, there is no necessity to calculate η, since, referring to Figure 49, the point (wr,A) lies above the most critical curve, $\eta = 1.0$. The selected sections for Storey 3 are therefore:-

<u>Beam</u>: 4 $\qquad$ <u>Internal Column</u>: 35

Estimate of A_i for the remaining storeys

Having designed Storeys 2 and 3, the curves for predicting the magnification factors in the remaining intermediate storeys may be constructed in the manner described in 6.2(b)(i). The two points corresponding to Storey 2 and Storey 3, (.024, 1.235) and (.048,1.565) are plotted as shown in Figure 51. The line through these points is continued to the boundary of Zones 2(i) and 2(ii), and its gradient, grad (1), is found to be equal to 13.5. Therefore, from equation (76),

$$\text{grad } (2) = 0.15 \times 13.5 + 1.20 = 3.23;$$

From equation (77),

$$\text{grad } (3) = -1.7 \times 3.23 + 0.80 = -4.70;$$

The two lines corresponding to Zone 2(ii) and 3 are constructed as shown. It may be seen that the upper limit, $A = 2$, is not required. The points 4 to 9 represent Storeys 4 to 9 respectively and are obtained by constructing the ordinates corresponding to the appropriate values of wr_i (these values being given in Table 4.) Thus, for any storey, A_i is obtained by reading the value of A from the graph, and multiplying by 1.05.

Storey 4: The design of this storey lies in Zone 2(ii), and the results are summarized below.

$$A_i = 1.67 \rightarrow \text{Zone } 2(\text{ii})$$

Initial sections:-

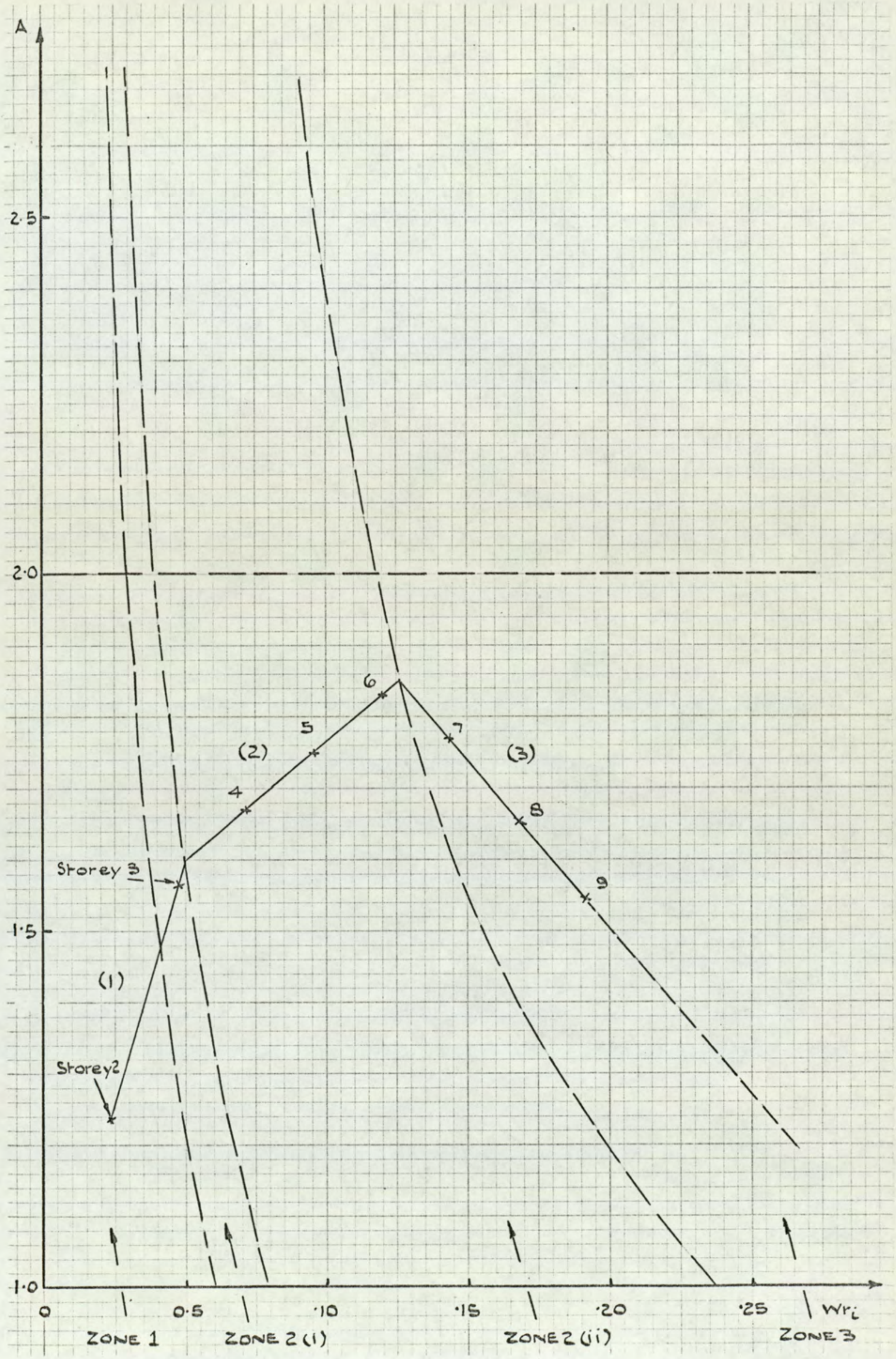
$b = 6$ $\qquad c_{i1} = 36$ $\qquad c_{i2} = 38$

Magnification factors:-

$A = 1.433$; $\quad A_c = 1.447$; $\rightarrow$ Zone 2(ii)

Resulting sections:-

$b = 4$ $\qquad c_{i1} = 36$ $\qquad c_{i2} = 38$



Prediction of the magnification factors for the 10 storey, 4 bay frame

FIGURE 51

The column design has converged after one iteration. However, the beam section is different, so that a further iteration is required. This leads to higher values for A and A_c, and the beam section again becomes b = 6. The design tends to oscillate between these two sections, and it is impossible to obtain convergence. The larger set of sections is therefore selected; i.e., in Storey 4:-

<u>Beam</u>: 6   <u>Internal Column</u>: 38

<u>Storey 5</u>:- This storey also lies in Zone 2(ii). The design is summarized below.

$$A_i = 1.83 \rightarrow \text{Zone } 2(\text{ii})$$

Initial sections:-

$b = 6$ $c_{i1} = 37$ $c_{i2} = 40$

Magnification factors:-

$A = 1.567$; $A_c = 1.560 \rightarrow$ Zone 2(ii)

Resulting sections:-

$b = 6$ $c_{i1} = 37$ $c_{i2} = 39$

The beam has converged, but the size of the lower column is different. A second iteration leads to:-

$A = 1.668$; $A_c = 1.658 \rightarrow$ Zone 2(ii)

$b = 6$ $c_{i1} = 37$ $c_{i2} = 39$

The design has therefore converged in only two iterations, and the final sections are:-

<u>Beam</u>: 6   <u>Internal Column</u>: 39

<u>Storey 6</u>:- The procedure for any Storey lying in Zone 2(ii) is identical to that given below for Storey 6.

$$H_1 = 6.75;\ H_2 = 8.25;\ P_1 = 225;\ P_2 = 275;$$

$$wr_i = .120$$

From Figure 51, $A \simeq 1.83$

Therefore,

$$A_i = 1.05 \times 1.83 = 1.92 \rightarrow wr = 1.92 \times .120 = .230 \rightarrow \text{Zone } 2(\text{ii})$$

$$B = 3150 \times (.25 + .230) = 1512,$$

$$C_I = 100.8 \times 1.92 \times 8.25 = 1600;$$

$zp_b = 42.1$ $\quad$ $zp_{ci} = 44.6$

$\lambda_1 P_1 = 394$ $\quad$ $\lambda_1 P_2 = 481$

$b = 6$ $\quad$ $c_{i1} = 40$ $\quad$ $c_{i2} = 42$

$I_b = 298$ $\quad$ $I_{c1} = 421$ $\quad$ $I_{c2} = 663$

$$\rho_1 = .0980 \times \frac{225}{421} = .0523;$$

$$\rho_2 = .0980 \times \frac{275}{663} = .0406;$$

$$(n_1 - o_i) = -0.270;\ (n_2 - o_2) = -0.207;\ m_1 = 1.045;\ m_2 = 1.035;$$

$$\overline{K} = \frac{421}{663} = 0.635;\ V = -0.270 \times 0.635 - 0.207 = -0.379;$$

$$K_2 = 2.083 \times \frac{663}{298} = 4.64;\ K_2V = -1.755;\ K_2V + 3 = 1.245;\ K_2V + 12 = 10.245;$$

In Zone 2(ii), four distribution factors are required:-

$$a^{\Delta}_{22}{}' = \frac{3}{1.245} = 2.405; \quad a^{\Delta}_{23} = \frac{-0.207 \times 4.64}{1.245} = -0.771;$$

$$a^{\alpha}_{22}{}' = \frac{12}{10.425} = 1.170; \quad a^{\alpha}_{23} = \frac{-0.207 \times 4.64}{10.245} = -0.0939;$$

$$p_2 = \frac{1.045 \times 6.75 + 1.035 \times 8.25}{2 \times 1.035 \times 8.25} = \frac{7.06 + 8.52}{17.04} = \frac{15.58}{17.04} = 0.912;$$

$$\frac{\alpha}{\lambda_2} = \frac{1.92}{1.170}\left[0.5 - \frac{5.21}{1.92 \times 15.58}\right] = 0.533;\ \left(1 - \frac{\alpha}{\lambda_2}\right) = 0.467;$$

Therefore, from equations (25) and (26),

$$A = 0.533 \times 1.170 + 0.467 \times 2.405 = 0.624 + 1.123 = 1.747;$$

$$A_c = 0.533\ [1 + 2 \times 0.0939 \times 0.912] + 0.467\ [1 + 2 \times 0.771 \times 0.912]$$

$$= 0.533 \times 1.171 + 0.467 \times 2.405 = 0.624 + 1.123 = 1.747;$$

Recalculating the wind ratio:-

$$wr = .008 \times 1.747 \times 15.58 = .218 \rightarrow \text{Zone } 2(\text{ii})$$

$$B = 3150 \times (.25 + 218) = 1475;$$

$$C_I = 100.8 \times 1.747 \times 8.52 = 1500;$$

$zp_b = 41.1$ $zp_{ci} = 41.9$

$b = 6$ $c_{i1} = 40$ $c_{i2} = 42$

i.e., the new sections are identical to those selected initially, so that the design has converged in only one iteration. The final design lies close to the boundary of Zone 2(ii), as indicated initially in Figure 51. Therefore, the final sections for Storey 6 are:-

Beam: 6 Internal Column: 42

Storey 7: The design of this storey is typical of those lying in Zone 3, and is therefore given in some detail. However, the equations for calculating A and A_c are identical to those for Zone 2(ii), except that $\frac{\alpha}{\lambda_2}$ is different. Certain sections of the calculations are abridged accordingly.

$$H_1 = 8.25; \; H_2 = 9.75; \; P_1 = 275; \; P_2 = 325;$$

$$wr_i = .144$$

From Figure 51, $A \simeq 1.77$;

Therefore,

$A_i = 1.05 \times 1.77 = 1.86 \rightarrow wr = 1.86 \times .144 = .268 \rightarrow$ Zone 3;

$B = 50.4 \times 1.86 \times (8.25 + 9.75) = 1690;$

$C_I = 100.8 \times 1.86 \times 9.75 = 1830;$

$zp_b = 47.0$ $zp_{ci} = 51.0$

$\lambda_1 P_1 = 481$ $\lambda_1 P_2 = 568$

$b = 7$ $c_{i1} = 42$ $c_{i2} = 43$

$I_{b2} = 290$ $I_{c1} = 663$ $I_{c2} = 789$

The column $c_{i2} = 41$ is in fact strong enough [Figure 48(a)]. However, in Storey 6, section 42 has been selected, and the dimensions of this section are greater than those of section 41. The latter section therefore cannot be used in this storey, since otherwise, "reverse column taper" would occur. The section $c_{i2} = 43$ is selected instead. The calculation proceeds as before:-

$\rho_1 = .0406$

$\rho_2 = .0404$

$(n_1-o_1) = -0.207$; $(n_2-o_2) = -0.206$; $m_1 = 1.035$; $m_2 = 1.035$;

$\overline{K} = 0.841$; $V = -.370$; $K_2 = 5.65$; $K_2V = -2.09$;

Thus, $K_2V < -1.80$, and the beam must be increased. For this value of V (which is purely a function of the column sizes), the limiting value of K_2 is $\frac{-1.80}{-0.370} = 4.86$;

Now,

$$K_2 = 2.083 \times \frac{I_{c2}}{I_{b2}} = \frac{1640}{I_{b2}};$$

Thus, for any beam to satisfy the limit on K_2V,

$$\frac{1640}{I_{b2}} < 4.86$$

i.e.,

$$I_{b2} > \frac{1640}{4.86} = 338;$$

The beam of least weight which satisfies this condition is b = 9.

The calculation continues with this increased section:-

$b = 9$

$I_b = 339$

$K_2 = 2.083 \times \frac{789}{339} = 4.84$; $K_2V = -1.790$; $K_2V + 3 = 1.210$; $K_2V + 12 = 10.210$;

The expressions for the distribution factors are identical to those for Zone 2(ii):-

$a^{\Delta}_{22}{}' = 2.480$; $a^{\Delta}_{23} = -0.826$; $a^{\alpha}_{22}{}' = 1.174$; $a^{\alpha}_{23} = -0.0976$;

$$p_2 = \frac{1.035 \times 8.25 + 1.025 \times 9.75}{2 \times 1.025 \times 9.75} = \frac{8.52 + 10.07}{20.14} = \frac{18.59}{20.14} = 0.921;$$

The expression for $\frac{\alpha}{\lambda_2}$ is different from that in Zone 2(ii):-

$$\frac{\alpha}{\lambda_2} = \frac{1.86}{1.174}\left[1 - \frac{20.84}{1.86 \times 18.59}\right] = 0.630; \left(1 - \frac{\alpha}{\lambda_2}\right) = 0.370;$$

$A = 0.630 \times 1.174 + 0.370 \times 2.480 = 0.739 + 0.918 = 1.657;$

$$A_c = 0.630\,[1 + 2 \times 0.0976 \times 0.921] + 0.370\,[1 + 2 \times 0.826 \times 0.921]$$
$$= 0.630 \times 1.180 + 0.370 \times 2.521 = 0.742 + 0.933 = 1.675;$$

The remainder of the design of this storey is summarized.

These values of A and A_c lead to the following sections:-

$b = 6$ $c_{i1} = 42$ $c_{i2} = 43$

i.e., the columns are as before, whereas a smaller beam than $b = 9$ is obtained. Thus, in the sectond iteration, it is again found that $K_2V < -1.80$, and the beam must once more be increased to $b = 9$. Therefore, the same distribution factors are obtained, but, in this case:-

$$\frac{\alpha}{\lambda_2} = \frac{1.657}{1.174}\left[1 - \frac{20.84}{1.657 \times 18.59}\right] = 0.458;$$

The corresponding magnification factors are:-

$A = 1.879$; $A_c = 1.903$;

The new sections are:-

$b = 8$ $c_{i1} = 42$ $c_{i2} = 43$

Again, the column sizes are identical, and the beam is less than $b = 9$, and will therefore have to be increased as before. Thus, the design has converged in two iterations, and the effectiveness of the limit on K_2V in encouraging convergence is demonstrated. The final sections for Storey 7 are therefore:-

<u>Beam</u>: 9 <u>Internal Column</u>: 43

<u>Storey 8</u>: This storey also lies in Zone 3, and the results are summarized below:-

$A_i = 1.74 \rightarrow$ Zone 3

Initial sections:-

$b = 8$ $c_{i1} = 43$ $c_{i2} = 46$

The beam is subsequently increased to $b = 11$, since $K_2V < -1.80$.

The magnification factors are:-

$A = 1.487$; $A_c = 1.487$;

The new sections are:-

$b = 7$ $c_{i1} = 43$ $c_{i2} = 43$

Again, it is found that the beam must be increased to $b = 11$.

At the end of the second iteration:-

$A = 1.718; \qquad A_c = 1.720;$

and,

$b = 8 \qquad c_{i1} = 43 \qquad c_{i2} = 46$

i.e., after two iterations the initial sections are again obtained, and, if continued, the design oscillates between two different sets. It is found that convergence may be obtained by restarting the procedure with the average values of the two sets of magnification factors; i.e.,

$A = 1.603; \qquad A_c = 1.604;$

These give the following sections:-

$b = 8 \qquad c_{i1} = 43 \qquad c_{i2} = 43,$

and, after the beam is increased to $b = 11$,

$A = 1.608 \qquad A_c = 1.608,$

for which the same sections are obtained. Therefore, the final sections for Storey 8 are:-

Beam: 11 Internal Column: 43

Storey 9: This storey also lies in Zone 3, and the design converges in two iterations, as follows:-

$A_i = 1.63 \rightarrow$ Zone 3

Initial sections:

$b = 10 \qquad c_{i1} = 46 \qquad c_{i2} = 46$

The beam is subsequently increased to $b = 12$.

Magnification factors:

$A = 1.469; \qquad A_c = 1.466;$

Resulting sections:

$b = 8 \qquad c_{i1} = 46 \qquad c_{i2} = 46$

The beam is again increased to $b = 12$, and the final magnification factors are:

$A = 1.608; \qquad A_c = 1.610;$

Therefore, the final sections for Storey 9 are:-

Beam: 12 Internal Column: 46

THE BOUNDARY REGIONS

Storey 1 (the top storey):

$$W_1 = 15k.$$

$$L = 25ft.$$

Therefore,

$$B = \lambda_1 \frac{W_1 L}{16} = 1.75 \times \frac{15 \times 25 \times 12}{16} = 492k.ins.$$

$zp_b = 13.7$

$b = 1$

Also, from the design of Storey 2,

c_{i2} = (c_{i1} in Storey 2) = 32

Therefore, the final sections for the top storey are:-

Beam: 1 Internal Column: 32

Storey 10 (the lowest storey)

$r = 4$; $WL = 750$; $h_1 = h_2 = 12$, $\Sigma(H)_1 = 51$; $\Sigma(H_2) = 60$.

Therefore,

$$wr' = \frac{[51 + 60] \times 12}{4 \times 750 \times 2} = .222;$$

$17wr' = 3.77$

$(3 + 17wr') = 6.77$

Thus, from equation (35),

$$H_2 = 60 \times \frac{1.05 \times 6.77}{[2 + 3 \times 6.77]} = 19.1$$

The basic loads are therefore:-

$H_1 = 12.75$; $H_2 = 19.1$; $P_1 = 425$; $P_2 = 475$;

Also, assume:-

$$A_{c1} = 1.4; \quad A'_{c2} = 2.7$$

The beam is assumed to fail by the simple beam mechanism; i.e., b = 4.

The design proceeds as follows:-

$$C_{I_1} = 100.8 \times 1.4 \times 12.75 = 1802;$$

$$C_{I_2} = 100.8 \times 2.7 \times 19.1 = 5200;$$

	$zp_{ci1} = 50.3$	$zp_{ci2} = 145$
	$\lambda_1 P_1 = 743$	$\lambda_1 P_2 = 831$
$b = 4$	$c_{i1} = 46$	$c_{i2} = 51$
$I_b = 196$	$I_{c1} = 1166$	$I_{c2} = 1901$

$$\rho_1 = .0980 \times \frac{425}{1166} = .0356;$$
$$\rho_2 = .0980 \times \frac{475}{1901} = .0244;$$

$(n_1 - o_1) = -0.180$; $n_2 = 0.918$; $\frac{o_2}{n_2} = 1.134$; $m_1 = 1.030$; $m_2 = 1.021$;

$\overline{K} = \frac{1166}{1901} = 0.614$; $V' = -0.180 \times 0.614 + 0.918 = -0.111 + 0.918 = 0.807$;

$K_2 = 2.083 \times \frac{1901}{196} = 20.15$; $K_2V' = 16.28$; $K_2V' + 3 = 19.28$;

The column distribution factors are as follows:-

$$a^{\Delta}_{21} = \frac{-0.111 \times 20.15}{19.28} = -0.116; \quad a^{\Delta}_{23} = \frac{0.918 \times 20.15}{19.28} = 0.961;$$

$$p_1 = \frac{1.030 \times 12.75 + 1.021 \times 19.1}{2 \times 1.030 \times 12.75} = \frac{13.16 + 19.50}{26.32} = \frac{32.66}{26.32} = 1.243;$$

$$p_2 = \frac{32.66}{2 \times 1.021 \times 19.1} = \frac{32.66}{39.00} = 0.839;$$

Therefore, from equations (38) and (40),

$$A_{c1} = 1 + 2 \times 0.116 \times 1.243 = 1.289;$$

$$A'_{c2} = 1 + 2 \times 1.134 \times 0.961 \times 0.839 = 2.823;$$

The new sections are obtained from:-

$$C_{I_1} = 100.8 \times 1.289 \times 13.16 = 1712;$$

$$C_{I_2} = 100.8 \times 2.823 \times 19.50 = 5560;$$

$zp_{ci} = 47.7$	$zp_{ci2} = 155$

$b = 4$ $\qquad$ $c_{i1} = 46$ $\qquad$ $c_{i2} = 51$

These are identical to the initial sections, so that the design has converged immediately. The working load elasticity condition, given by equation (85), must now be checked. The quantities E,F and G are calculated initially.

$$\rho_1^1 = \frac{.0356}{1.4} = .0255;$$

$$\rho_2^1 = \frac{.0244}{1.4} = .0174;$$

$(n_1\text{-}o_1)^1 = -0.129;\ n_2^{\ 1} = 0.942;\ m_1^{\ 1} = 1.022;\ m_2^{\ 1} = 1.015;$

Since $\overline{K} = 0.614$, $V'^1 = -0.079 + 0.942 = 0.863;$

$$E = \frac{WL}{12f_y} = \frac{750 \times 12}{12 \times 35.84} = 20.9;$$

$$F = \frac{6(m^1 Hh)av}{f_y} = \frac{6 \times 12 \times 12}{35.84 \times 2}\left[1.022 \times 12.75 + 1.015 \times 19.1\right] = 390;$$

$$G = I_{c2}.\ \frac{L}{h_2}.\ V'^1 = 1901 \times 2.083 \times 0.863 = 3410;$$

Therefore, the condition reduces to:-

$$20.9 + \frac{390}{\left[\frac{3410}{I_b} + 12\right]} \leq Zp_b$$

This condition is checked in Table 5, below, where it is seen that sections 4 and 5 are inadequate, and that eventually, section 6 is selected.

b	I_b	$\frac{3410}{I_b}$	$\frac{3410}{I_b} + 12$	$\frac{390}{\frac{3410}{I_b} + 12}$ = (A),say.	Left hand side = 20.9 + (A)	Right hand side = Zp_b	Comment
4	196	17.4	29.4	13.3	34.2	32.8	LHS>RHS
5	172	19.8	31.8	12.3	33.2	32.9	LHS>RHS
6	298	11.4	23.4	16.7	37.6	43.9	LHS<RHS

"Working-load elasticity" - lowest storey - 10st., 4 bay frame.

TABLE 5

i.e. The final sections for Storey 10 are:-

Beam: 6 $\qquad$ Internal Column: 51

n	B	C_I	p_2	$\frac{1}{2}\left[1-\frac{2}{5}(4p_2-1)\right]$ in Storey 2. $\frac{1}{2}(1-p_2)$ in other storeys (a)	B, or $\frac{2}{5}(B_1+B_2)$, or $\frac{B}{2}$ where appropriate (b)	$C_I \times (a)$ (c)	$C_E=(b)+(c)$ / $C_E = \frac{B}{2}$ from storey below.	C_E (max)	zp_{ce}	$\lambda_1 P_E$	Basic U.C. (for strength)	Final U.C. (for no reverse column taper)
1	492	-	-	-	492	-	492 / 492	492	13.8	22	32	32
2	984	321	.655	.176	590	57	650 / 520	650	18.1	67	34	34
3	1040	633	.791	.105	520	67	590 / 605	605	16.9	113	34	34
4	1210	900	.856	.072	605	65	670 / 665	670	18.7	161	34	34
5	1330	1210	.893	.054	665	64	730 / 740	740	20.6	209	35	35
6	1475	1500	.912	.044	740	66	810 / 880	880	24.6	260	36	36
7	1760	1940	.921	.040	880	78	960 / 890	960	26.9	311	38	37
8	1780	1940	.934	.033	890	64	960 / 1005	1005	28.1	363	37	37
9	2010	2150	.941	.030	1005	64	1070 / 492	1070	29.8	417	39	39
10	984	-	-	-	492	-	492 / -	492	13.8	473	37	39

External column design for the 10 storey, 4 bay frame.

TABLE 6

External column:

In this particular frame, all the storey heights are equal. The design equations for the external columns are as follows:-

In the top storey:-

$$C_E > B$$

In Storey 2, since $W_1 = \frac{1}{2}W_2$,

$$C_E > \frac{2}{5}(B_1+B_2) + C_I.\frac{1}{2}\left[1 - \frac{2}{5}(4p_2-1)\right]$$

In the remaining intermediate storeys,

$$C_E > \frac{B}{2} + C_I.\frac{1}{2}(1-p_2).$$

In the lowest storey,

$$C_E > \frac{B}{2}$$

In addition, at any joint, for the upper column,

$$C_E > \frac{B}{2}$$

The design calculations for all the storeys are given in Table 6. The final column shows the sections which are selected in order to prevent "reverse column taper".

6.3 DESIGN OF SINGLE-BAY FRAMES

The design procedure for the single-bay frame is very similar to that given for the multi-bay frame at the beginning of 6.2., although, of course, there are no internal columns to consider. Due to the similarity between the two methods, it is only necessary to summarize the procedure for single-bay frames, and to expand the main points of difference when they arise.

The initial design loads are calculated using the equations already derived in 6.2(a). Equation (64) gives the beam loads, equation (65) the shear forces, and equation (67) the column loads.

The design of the intermediate storeys proceeds as before, with the initial values being predicted for the magnification factors. In this case, however, the values of A and A_c are always small in comparison with those obtained for the multi-bay frame. Initial values of A_i = 1.25 and A_{ci} = 1.20 have been found to ensure that the method converges immediately in nearly every storey of every frame. In fact, if the simple plastic values, $A = A_c = 1$, are used, convergence also occurs rapidly in many frames, but it is generally advisable to over-estimate, rather than underestimate tha magnification factors. The reason for the low magnification factors is that the columns have to be selected to resist a substantial moment due to beam loading, and this results in comparatively low Euler ratios. In addition, as implied above, A_c is generally less than A.

Referring to equations (48) to (57) in 5.2(b), it will be seen that initial values are also required for the quantities p_2 and $\frac{\alpha}{\lambda_2}$ in order to calculate the required plastic moment for the column. p_2 may be obtained quite simply from the initial loading, assuming that $m_1 = m_2 = 1$. Also, $\frac{\alpha}{\lambda_2}$ occurs in the second part of the expression for C_E, and this second part is always small in comparison to the total. Therefore, there is no necessity to predict $\frac{\alpha}{\lambda_2}$ very accurately, and a value of 0.5 is suggested.

Having calculated the required plastic moments, the sections are selected as before, and the magnification factors are calculated using the equations given in 5.2(d). The process is repeated until convergence is obtained.

The working load elasticity condition must now be checked, and this is given by equation (58) and 5.2(e); i.e.,

$$\frac{WL}{12}\left(1 - a^{v}_{22}{}'\right) + a^{1}_{22}{}' \, \frac{(m^{1}Hh)av}{2} \leqslant Mp_b$$

where,

$$a^{1}_{22}{}' = \frac{6}{K_2V^{1} + 6}; \; a^{v}_{22}{}' = \frac{1}{3x + 1};$$

$V^1 = (n_1 - o_1)^1\overline{K} + (n_2 - o_2)^1$; $x = \frac{K_{c1} + K_{c2}}{K_{b2}} = K_2(\overline{K} + 1)$;

An approximate form of this condition may be derived in a similar way to that given in 6.2(b)(iv) for the intermediate storeys of the multi-bay frame. Consider first the values of the factors $a_{22}^{v}{}'$ and $a_{22}^{1}{}'$.

The working load elasticity condition is most likely to be violated in frames designed for heavy wind loading. In these frames, the beam stiffnesses are comparatively large, and it may be assumed that $x \simeq 1$. The corresponding value of $a_{22}^{v}{}'$ is $\frac{1}{4}$.

In addition, as the wind ratio increases, the beam distribution factor for combined load at load factor λ_2, $a_{22}^{\alpha}{}'$, increases. In all the frames which have been designed, the maximum value of $a_{22}^{\alpha}{}'$ in any storey is 1.08. Therefore, a limiting value of K_2V is obtained from:-

$$a_{22}^{\alpha}{}' = \frac{6}{K_2V + 6} = 1.08$$

i.e.,

$$K_2V = -0.444;$$

As for the multi-bay frame, however,

$$K_2V \simeq \lambda_2 K_2 V^1$$

Therefore, the limiting value of K_2V^1 is approximately:-

$$K_2V^1 = \frac{-0.444}{1.4} = -0.316$$

The corresponding maximum value for the distribution factor at working load is therefore:-

$$a_{22}^{1}{}' = \frac{6}{5.684} = 1.05$$

Also, as before, the basic condition may be simplified by assuming that $m^1 \simeq m$, and by substituting $Mp_b = \eta.B$; i.e.,

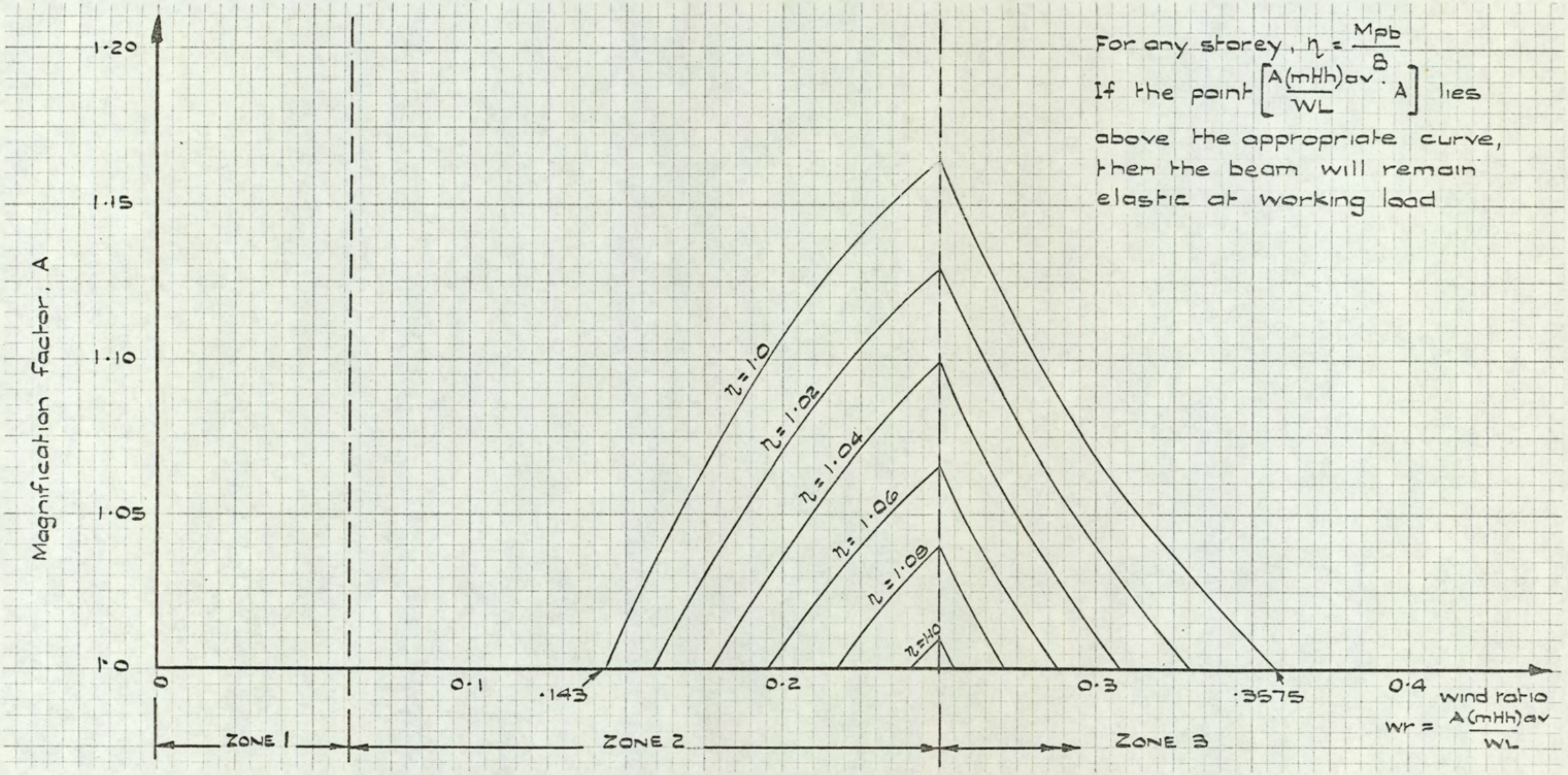
$$\frac{WL}{12}\cdot\frac{3}{4} + 1.05\frac{(mHh)av}{2} \leqslant \eta.B$$

Rearranging,

$$1.05\,\frac{(mHh)av}{WL} \leqslant 2\eta.\frac{B}{WL} - \frac{1}{8}$$

i.e.,

$$1.05\,\frac{wr}{A} \leqslant 2\eta.\frac{B}{WL} - \frac{1}{8} \qquad (86)$$



"Working - load elasticity condition" - Single - bay frames.

FIGURE 52.

If the expressions for B for each zone are substituted in equation (86) the following approximate conditions are obtained:

ZONE 1: $$A \geq \frac{33.6\ wr}{7\eta - 4} \qquad (87)$$

ZONE 2: $$A \geq \frac{8.4\ \eta}{5.6\ \eta.wr + (1.4\ \eta-1)} \qquad (88)$$

ZONE 3: $$A \geq \frac{8.4\ \eta}{11.2\ \eta.wr-1} \qquad (89)$$

The limiting values of these relationships are expressed graphically in Figure 52, for the range $1.0 \leq \eta \leq 1.10$. These curves have been found to give satisfactory results in every frame, with the point (wr, A) lying close to the relevant curve in those cases where the analysis indicates that the beam is just adequate.

Having designed all the intermediate storeys, the top storey beam is designed in the same way as that in the multi-bay frame, the simple beam mechanism being the assumed mode of failure. The top storey column is supplied with a reduced plastic moment greater than the required fully plastic moment of the beam.

The design equations for the lowest storeys are summarized in 5.4(c). In this case, it may be seen that initial values are required for the functions A_{c1}, A'_{c2}, $\overset{v}{a}_{21}$, $\overset{v}{a}_{23}$ and γ. Again, the design of this storey tends to converge rapidly, and no great accuracy is required in making the initial estimates. The following values have been found to give satisfactory results:-

$$A_{c1} = 1.3;\ A'_{c2} = 3.2;$$

$$\overset{v}{a}_{21} = 0.4;\ \overset{v}{a}_{23} = 0.6;$$

$$\gamma = 0.75;$$

The procedure is similar to that for the lowest storey of the multi-bay frame, and convergence generally occurs with no more than two iterations.

The working load elasticity condition for this storey is given in 5.4(d), the basic expression being identical to that for the intermediate storeys; i.e.,

$$\frac{WL}{12}(1 - a_{22}^{v\,\prime}) + a_{22}^{1\,\prime}\,\frac{(m^1Hh)av}{2} \leq Mp_b$$

In the lowest storey, however,

$$a_{22}^{1\,\prime} = \frac{6}{K_2V^{\prime 1} + 6};\quad a_{22}^{v\,\prime} = \frac{1}{3x + 1};$$

where,

$$V^{\prime 1} = (n_1 - o_1)^1\overline{K} + n_2^1;\quad x^{\prime} = \frac{K_{c1} + \frac{2}{3}K_{c2}}{K_{b2}} = K_2(\overline{K} + \tfrac{2}{3});$$

As for the lowest storeys of the multi-bay frame, an alternative form of the basic condition may be derived in terms of the section properties I_b and Zp_b. Expanding $a_{22}^{1\,\prime}$ and $a_{22}^{v\,\prime}$, the condition becomes:-

$$\frac{WL}{12}\cdot\frac{3x^{\prime}}{3x^{\prime}+1} + \frac{6}{K_2V^{\prime 1} + 6}\,\frac{(m^1Hh)av}{2} \leq Mp_b$$

Substituting for $x^{\prime}$, and dividing by f_y,

$$\frac{WL}{12f_y}\cdot\frac{1}{\left[1 + \frac{1}{K_2(3\overline{K}+2)}\right]} + \frac{3(m^1Hh)av}{f_y(K_2V^{\prime 1}+6)} \leq Zp_b$$

Expanding K_2:-

$$\frac{WL}{12f_y}\cdot\frac{1}{\left[1 + I_b\cdot\frac{h_2}{I_{c2}\cdot L(3\overline{K}+2)}\right]} + \frac{3(m^1Hh)av}{f_y\left[\frac{1}{I_b}\cdot I_{c2}\cdot\frac{L}{h_2}\cdot V^{\prime 1} + 6\right]} \leq Zp_b$$

Alternatively,

$$\frac{E}{1 + D.I_b} + \frac{F}{\left[\frac{G}{I_b} + 6\right]} \leq Zp_b \qquad (90)$$

where,

$$D = \frac{h_2}{I_{c2}.L.(3\overline{K}+2)};\quad E = \frac{WL}{12f_y};\quad F = \frac{3(m^1Hh)av}{f_y};\quad G = I_{c2}\cdot\frac{L}{h_2}\cdot V^{\prime 1};$$

As in the multi-bay frame, due to the selection of the minimum beam in the lowest storey, this condition is often violated, in which case a larger beam must be selected.

The exhaustive treatment that has been given to this discussion on the design procedure is believed to be justified, since the design aids which have been developed have been found to reduce the design time dramatically. For example, using this procedure, a thirty storey, five bay frame, which will be described in Chapter 8, has been designed in less than fifteen hours. The Author doubts if there is any other "hand-method" available which will produce as economical a design in so short a time.

In the following chapter, the accuracy of the design equations is assessed by analysing a wide range of frameworks, and it is shown that a safe and efficient structure is always obtained.

CHAPTER 7

VERIFICATION OF THE DESIGN METHOD

7.1. INTRODUCTION

It is essential that the validity of any approximate design method should be examined as extensively as possible. Verification may be obtained by physical testing of full scale or model structures, by comparison with other design methods, or by the use of accurate methods of structural analysis.

The first of these alternatives, that of direct experimentation, is extremely useful when dealing with comparatively small structures, but tends to be both expensive and laborious even when applied, for example, to the model of a multi-storey framework. Furthermore, unless excellent experimental techniques are adopted, with the numerous subsidiary effects of introducing a "scale factor" being taken into account, the results obtained may often be inconclusive, and may in fact tend to obscure certain deficiencies in the design approach.

The second alternative, that of comparison of design examples is useful when estimating the relative economy of different design approaches. Also, the adequacy of any design may be assessed qualitatively in this way. However, little information is obtained concerning the validity of the design equations.

In contrast, if the complete load-deformation history of a framework is obtained by using an established method of structural analysis, the accuracy of the method with which that framework is designed may be readily assessed. This is the procedure which has been adopted in this research project, and which is described in the current chapter.

In all, eleven multi-storey frames have been designed using the proposed method, and these have subsequently been analysed using the computer program developed by Majid and Anderson[52]. A description of this elasto-plastic analysis program has been given previously in Section 2.4.

Furthermore, each frame has been designed using a fictitious range of beam and column sections. These are similar in form to the standard Universal Beams and Universal Columns, and allow the designer to select the exact value of required plastic moment for any section. Thus, the additional reserve of strength due to the "lack of availability of sections" is eliminated, enabling precise information to be obtained about the accuracy of the design equations. The derivation of these fictitious beams and columns is given in the following section. Later in the chapter, the test frames are described, and details are given of their designs and analyses using both the fictitious sections and the Universal Beams and Universal Columns which are currently manufactured.

7.2. THE FICTITIOUS SECTIONS

Figure 53(a) shows the general dimensions of an I-section, which, for simplicity, is considered to consist of purely rectangular areas. The section is symmetrical about the major axis, xx, and the following standard notation is adopted:-

D = the overall depth of the section.

B = the width of the section.

T = the thickness of the flange.

t = the thickness of the web.

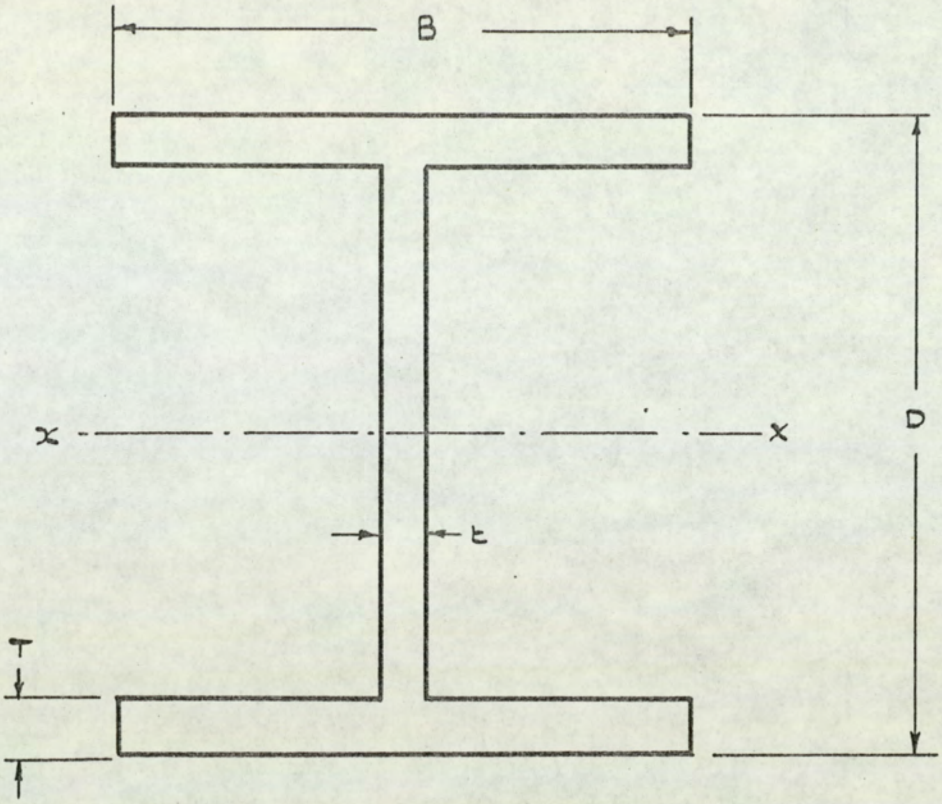
The section properties may be calculated in terms of these variables. The area of the cross-section is given by:-

$$A = 2BT + t(D-2T) \tag{91}$$

The second moment of area about the major axis is given by:-

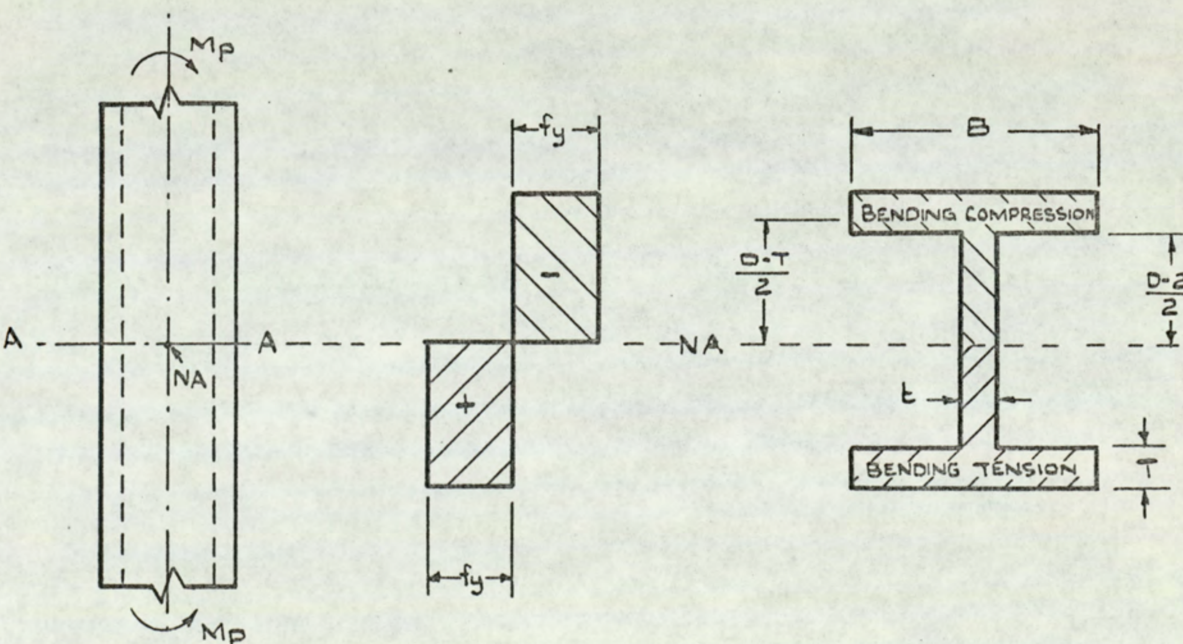
$$I = 2.\frac{BT^3}{12} + 2.BT\left[\frac{(D-T)}{2}\right]^2 + \frac{t(D-2T)^3}{12}$$

i.e.,



Dimensions of a general I-section

FIGURE 53 (a)



I-section under pure bending

Stress distribution at failure — Section A A

Section A A — "Stress - blocks" due to bending

FIGURE 53 (b).

$$I = \frac{BT^3}{12} + \frac{BT(D-T)^2}{2} + \frac{t(D-2T)^3}{12} \qquad (92)$$

Figure 53(b) shows the I-section fully stressed under pure bending. The neutral axis, denoted by NA, corresponds to the major axis, xx in Figure 53(a). The fully plastic modulus is equal to the sum of the first moments of area of the tensile and compressive "stress-blocks" about the major axis, M_A say. Thus, considering the flange and web areas separately,

$$Zp = M_A = 2\left[BT.\frac{(D-T)}{2} + t.\frac{(D-2T)^2}{8}\right]$$

i.e.,

$$Zp = BT(D-T) + \frac{t(D-2T)^2}{4} \qquad (93)$$

Under a combination of bending and axial load, the neutral axis lies somewhere between the major axis and the outer edge of the tension flange. If the axial force is small, the stress distribution at failure is as shown in Figure 54(a), with the neutral axis lying in the web. This stress distribution may be considered to consist of two components, one due to axial force, and the other due to bending, and these are given in Figure 54(b), together with the corresponding stress-blocks.

Let a_d and h_d be the area and depth respectively of the direct compression block, and let a_b and h_b be the combined area and the combined depth of the bending tension and bending compression blocks. If M_{a_d} is the sum of the first moments of area of each half of the direct compression block, then,

$$M_{a_d} = 2.\frac{a_d}{2}.\frac{h_d}{4}$$

But, since the whole of the direct compression block lies in the web:-

$$h_d = \frac{a_d}{t}$$

Therefore,

$$M_{a_d} = \frac{a_d^2}{4t}$$

However, considering the direct stress component,

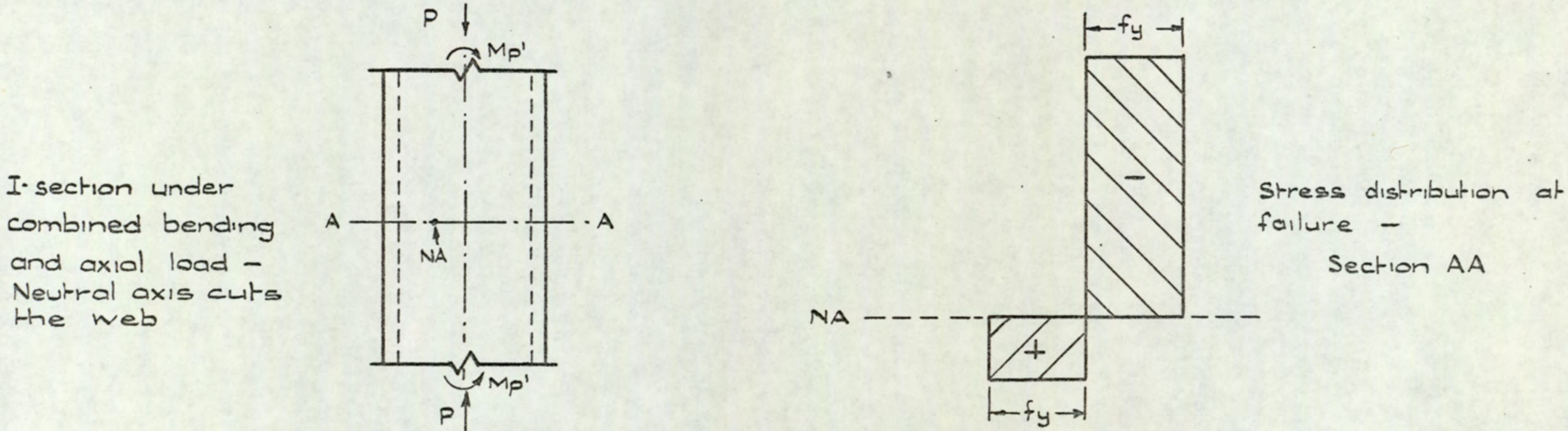
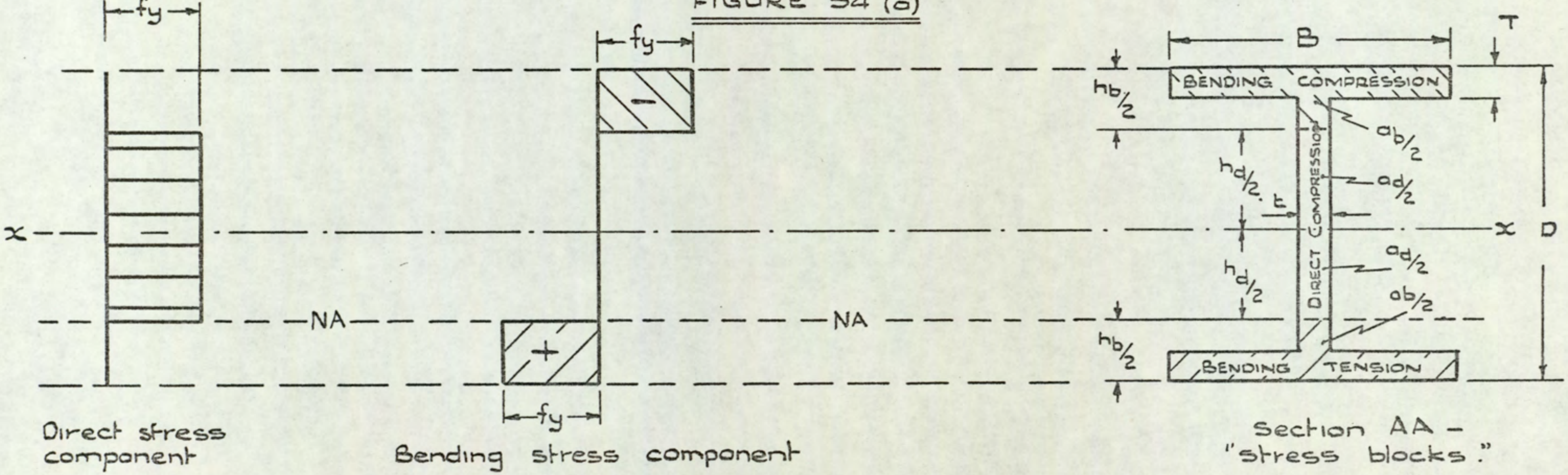


FIGURE 54 (a)



Equivalent stress distributions. FIGURE 54 (b)

$$a_d = \frac{P}{f_y}$$

Also, by definition, from equation (80) in Chapter 6,

$$n = \frac{P}{A.f_y}$$

Therefore,

$$a_d = n.A$$

so that,

$$M_{a_d} = \frac{n^2A^2}{4t}$$

Now, the reduced plastic modulus, $Z\acute{p}$ is equal to the sum of the first moments of area of the bending compression and bending tension blocks about the major axis, M_{a_b} say. However,

$$M_A = M_{a_b} + M_{a_d}$$

and,

$$M_A = Zp$$

Therefore,

$$Z\acute{p} = M_{a_b} = M_A - M_{a_d} = Zp - M_{a_d}$$

i.e.,

$$Z\acute{p} = Zp - \frac{n^2A^2}{4t}$$

In the same form as equation (78) of Chapter 6, the reduced plastic modulus is given by:-

$$Z\acute{p} = Zp - cn^2 \qquad (94)$$

where,

$$c = \frac{A^2}{4t}$$

This equation only applies for the case when the neutral axis cuts the web. Referring to Figure 54(b), it may be seen that the neutral axis coincides with the edge of the tension flange when:-

$$\frac{h_d}{2} = \frac{D-2T}{2}$$

Substituting for h_d, the condition is:-

$$\frac{a_d}{t} = D - 2T$$

Substituting for a_d,

$$nA = t(D-2T)$$

i.e., if the critical value of n is denoted by n', then,

$$n' = \frac{t(D-2T)}{A}$$

If $n < n'$, then equation (94) may be used to calculate Zp'. If $n > n'$, then the neutral axis lies in the tension flange, as shown in Figure 55(a). The equivalent stress distribution and corresponding stress-blocks are given in Figure 55(b). The expression for Zp' is derived as follows. As before:-

$$a_d = nA$$

Therefore,

$$a_b = A - a_d = A(1-n)$$

In this case, from Figure 55(b),

$$a_d = t(D - 2T) + B\,[h_d - (D - 2T)] = nA$$

Therefore,

$$h_d = \frac{1}{B}\left[B(D - 2T) - t(D - 2T) + nA\right]$$

Also,

$$h_b = D - h_d$$

Now,

$$M_{a_b} = 2.\frac{a_b}{2}\left[\frac{D}{2} - \frac{1}{2}.\frac{h_b}{2}\right] = \frac{a_b}{4}\,(2D - h_b)$$

Since $Zp' = M_{a_b}$, substitutung for a_b and h_b from above,

$$Zp' = \frac{A(1-n)}{4}\,(D + h_d)$$

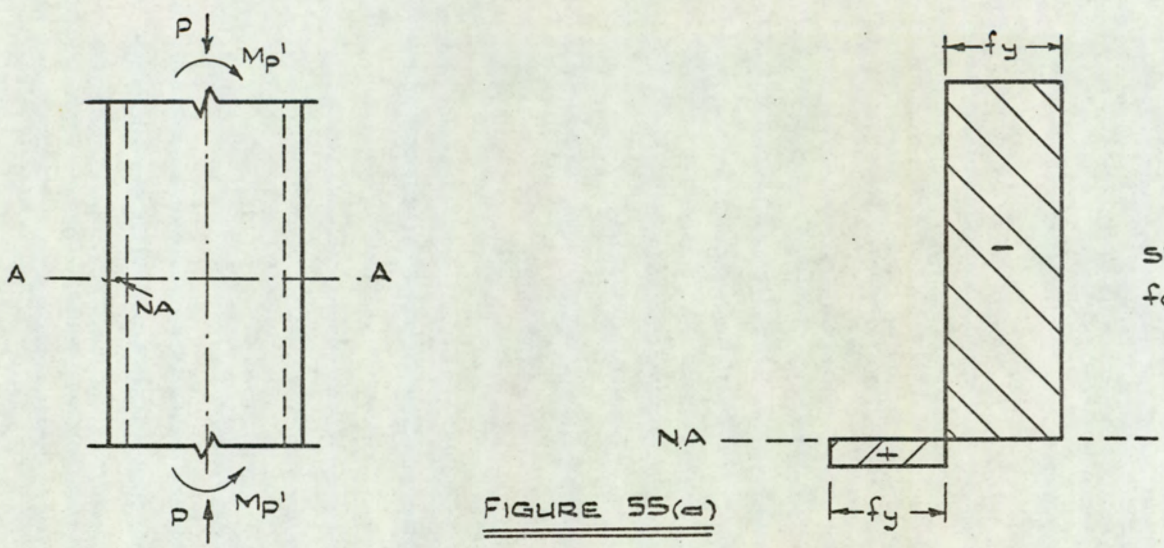
Substituting for h_d,

$$Zp' = \frac{A(1-n)}{4}\left\{D + \frac{1}{B}\left[B(D - 2T) - t(D - 2T) + nA\right]\right\}$$

$$= \frac{A}{4B}.(1-n)\left\{BD + B(D - 2T) - t(D - 2T) + nA\right\}$$

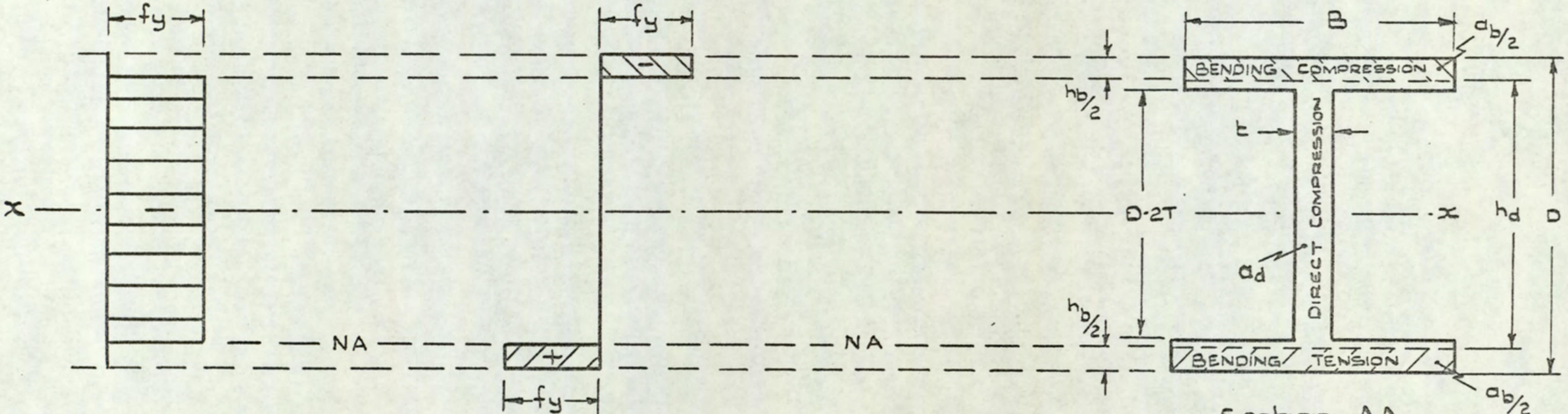
Alternatively,

I-section under combined bending and axial force - Neutral axis lies in the flange.



Stress distribution at failure - Section AA

FIGURE 55(a)



Direct stress component

Bending stress component

Equivalent stress distributions

Section AA - "Stress blocks"

FIGURE 55 (b)

$$Zp' = \frac{A^2}{4B}(1-n)\left\{\frac{1}{A}\left[2B(D - T) - t(D - 2T)\right] + n\right\}$$

In the same form as equation (79) of Chapter 6,

$$Zp' = d(1-n)(e+n) \tag{95}$$

where,

$$d = \frac{A^2}{4B}; \quad e = \frac{1}{A}\left[2B(D - T) - t(D - 2T)\right];$$

Equations (91) to (95) do not apply exactly to the standard range of Universal Beams and Universal Columns, since they have been derived assuming that the cross-section consists entirely of rectangular areas. The Universal Beams are manufactured with a flange taper of 5°, and with a fillet between web and flange. Nevertheless, if the properties of any Universal Beam are calculated using these equations, the values are always accurate to within one per cent of the true values. When applied to the Universal Columns, the difference is negligible, since these members are manufactured with parallel flanges, and therefore only the effect of the fillet is ignored.

Consider first the development of a range of fictitious beam sections. From the standard tables(56), it has been found that all the Universal Beams are dimensionally similar, and that the following relationships between the basic dimensions, D, B, T and t, are approximately true for the majority of sections:-

$$D = 2.5B$$

$$T = 1.5t$$

$$B = 13T$$

Alternatively, B, T and t may be expressed in terms of the single variable, D; i.e.,

$$B = 0.4D$$

$$T = \frac{B}{13} = 0.0308D$$

$$t = \frac{T}{1.5} = 0.0205D$$

If these values are substituted in equations (91), (92) and (93), the following expressions are obtained:-

$$A = 0.04387\ D^2 \qquad (96)$$

$$I = 0.007197\ D^4 \qquad (97)$$

$$Zp = 0.01645\ D^3 \qquad (98)$$

Equations (96), (97) and (98) therefore represent the values of A, I and Zp of an infinite range of fictitious beams, all dimensionally identical, and similar in form to the majority of the Standard Universal Beams. For any value of D, an appropriate set of section properties may be obtained.

Consider, for example, the fictitious section denoted by D = 15in. This is found to be similar to the 15 × 6 × 35 U.B.(section 10 in Table 1), as shown below. The properties of the real section are given in brackets:-

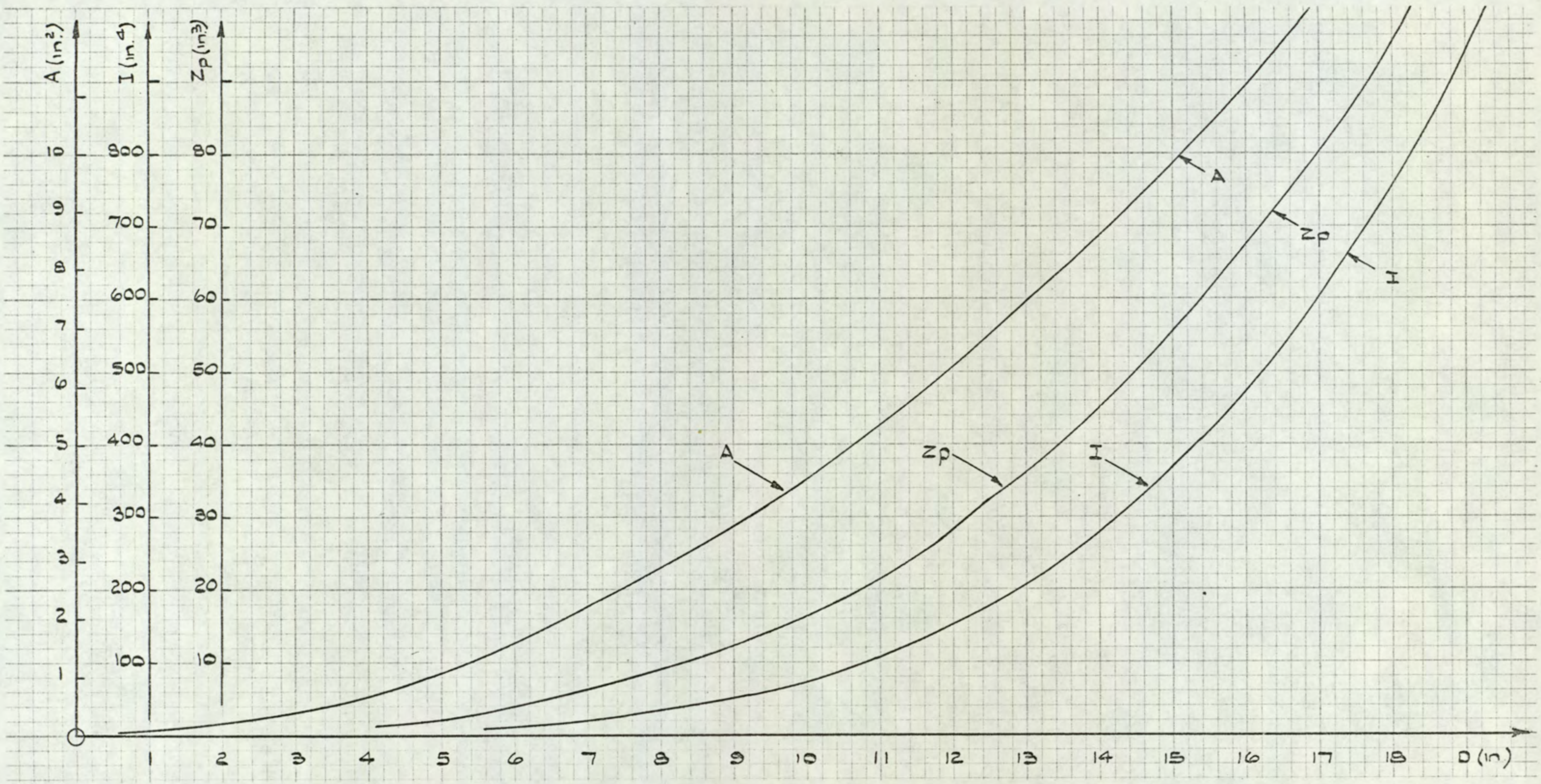
D = 15in.(15.00); B = 6in.(6.000); T = 0.5in.(0.490); t = 0.333in.(0.306);

A = 9.86in.^{2} (10.29); I = 364in.^{4} (385.5); Zp = 55.6in.^{3} (58.5);

Equations (94) and (95), which give the values of reduced plastic modulus, are not required for the beam sections since the axial force in these members is assumed to be negligible.

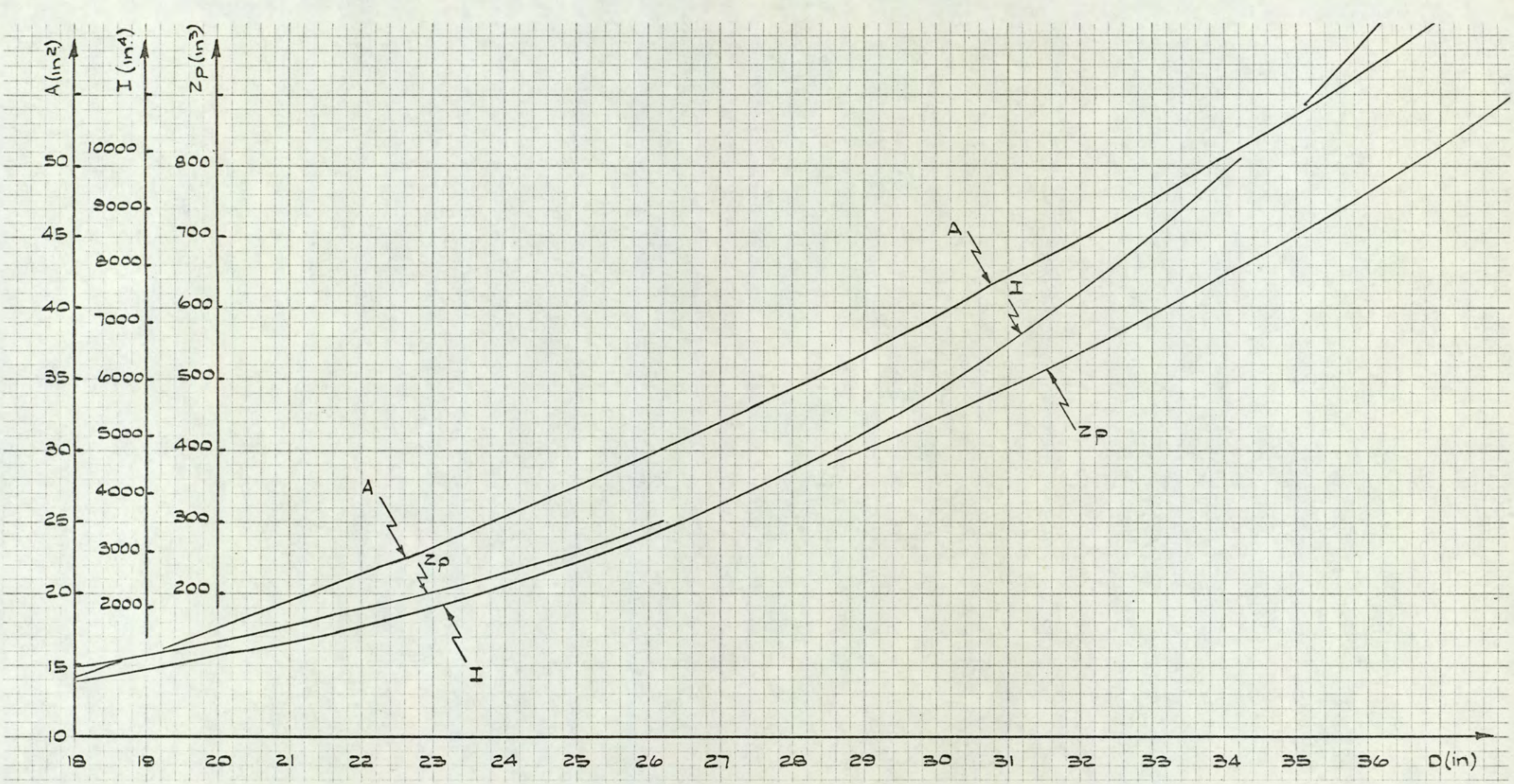
The variation of A, I and Zp with D for the fictitious beams is given in Figures 56(a) and 56(b). During the design of any storey, the beam is selected so that its value of Zp is exactly equal to the required value, zp_b. For example, if it was found that zp_b = 50in.^{3}, then, from Figure 56(a), the selected beam would have a depth given by D $\simeq$ 14.5in. For this value of D, from the same figure, A = 9.2in.^{2}, and I = 320in.^{4}

The fictitious column sections have been derived using the



Section properties, A, I and Zp for the fictitious beams : $0 < D < 18''$

FIGURE 56 (a)



Section properties, A. I and Zp, for the fictitious beams ; 18" < D > 36"

FIGURE 56 (b)

relationships:-

$$D = B$$

$$T = 1.6t$$

$$B = 12T$$

Again these have been assumed from inspection of the standard range of Universal Columns. Expressing B, T and t in terms of D:-

$$B = D$$

$$T = \frac{B}{12} = 0.0833D$$

$$t = \frac{T}{1.6} = 0.0521D$$

Equations (91), (92) and (93) become:-

$$A = 0.2100\ D^2 \qquad (99)$$

$$I = 0.0376\ D^4 \qquad (100)$$

$$Zp = 0.0855\ D^3 \qquad (101)$$

Also, for calculating the reduced plastic modulus, Zp', from equations (94) and (95), the factors c,d,e and n' become:-

$c = .2118\ D^3$; $d = .011025\ D^3$; $e = 8.524$; $n' = 0.207$;

Therefore, if $n < 0.207$, from equation (94),

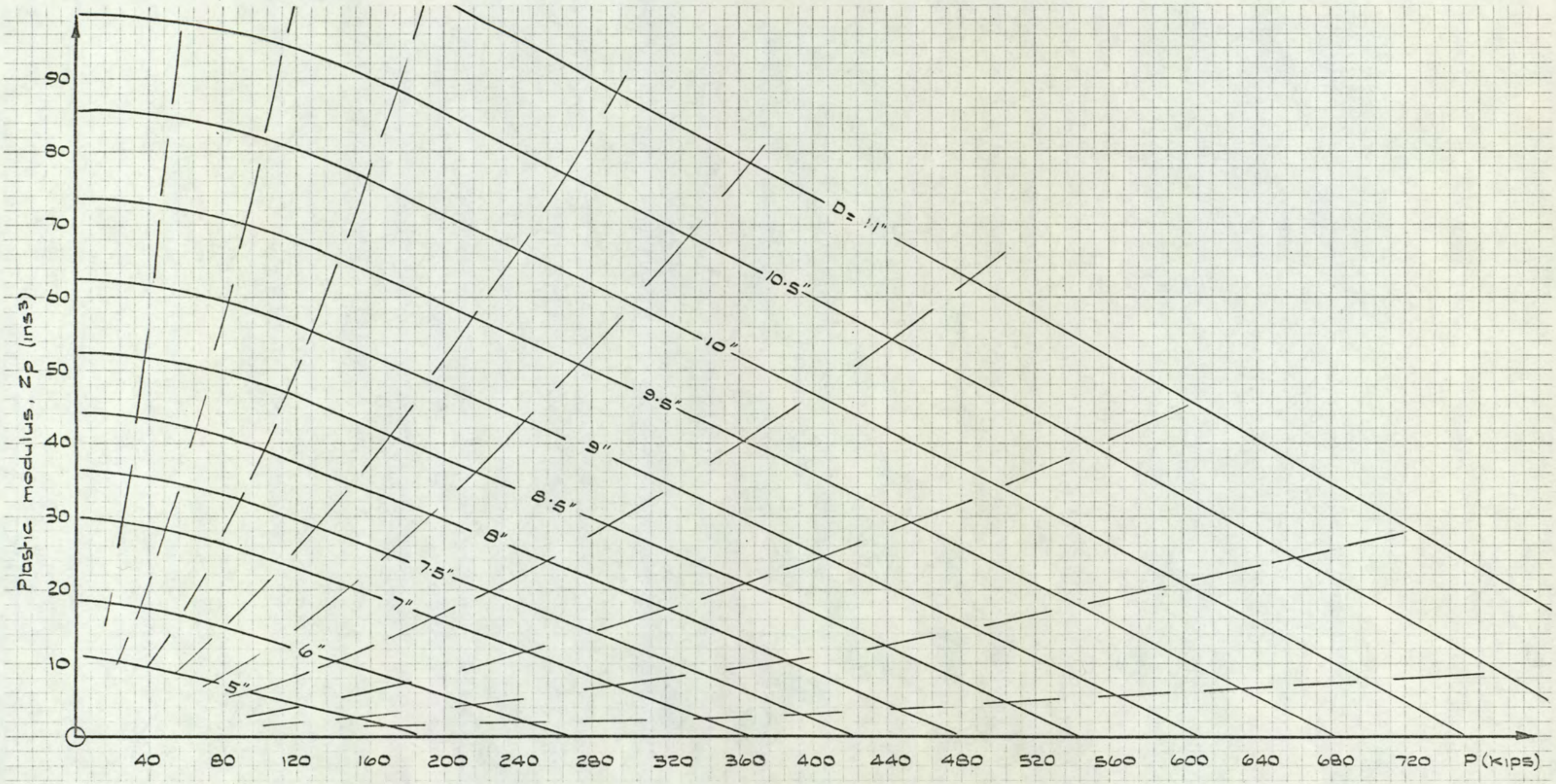
$$Zp' = Zp - .2118\ D^3n^2 \qquad (102)$$

If, however, $n > 0.207$, from equation (95),

$$Zp' = .011025\ D^3(1-n)(8.524 + n) \qquad (103)$$

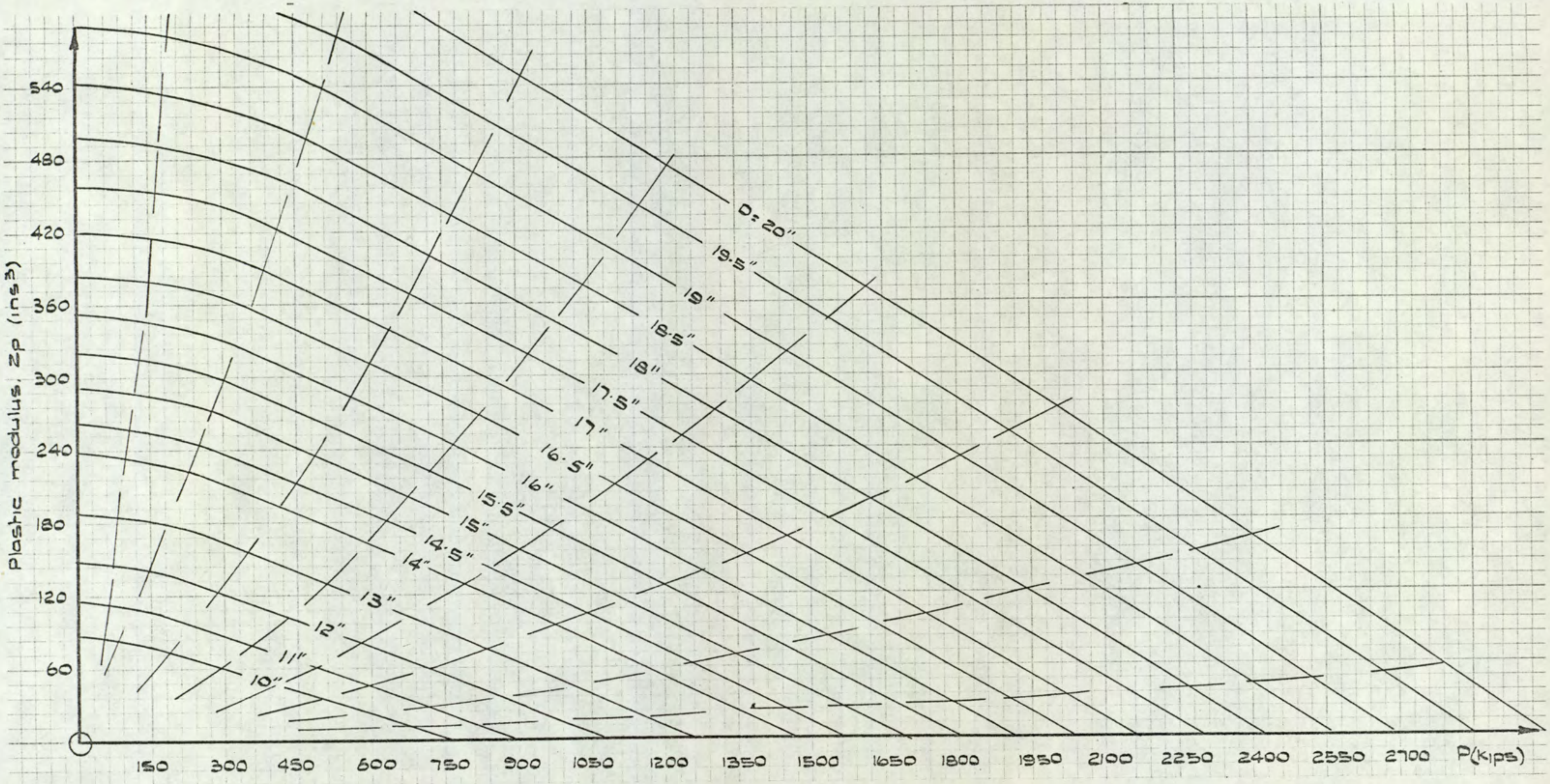
Equations (99) to (103) represent the section properties of an infinite range of fictitious columns, which as for the beams, are all dimensionally identical, and which are similar to most of the Universal Columns.

In order to select the appropriate column, the procedure is similar to that described for the Universal Columns in Section 6.2(b)(ii). Figures 57(a) and 57(b) have been constructed from equations (102) and (103), for a yield stress of 35.84 kips per sq. in., and show the variation in reduced plastic modulus with axial load for different values of D. These



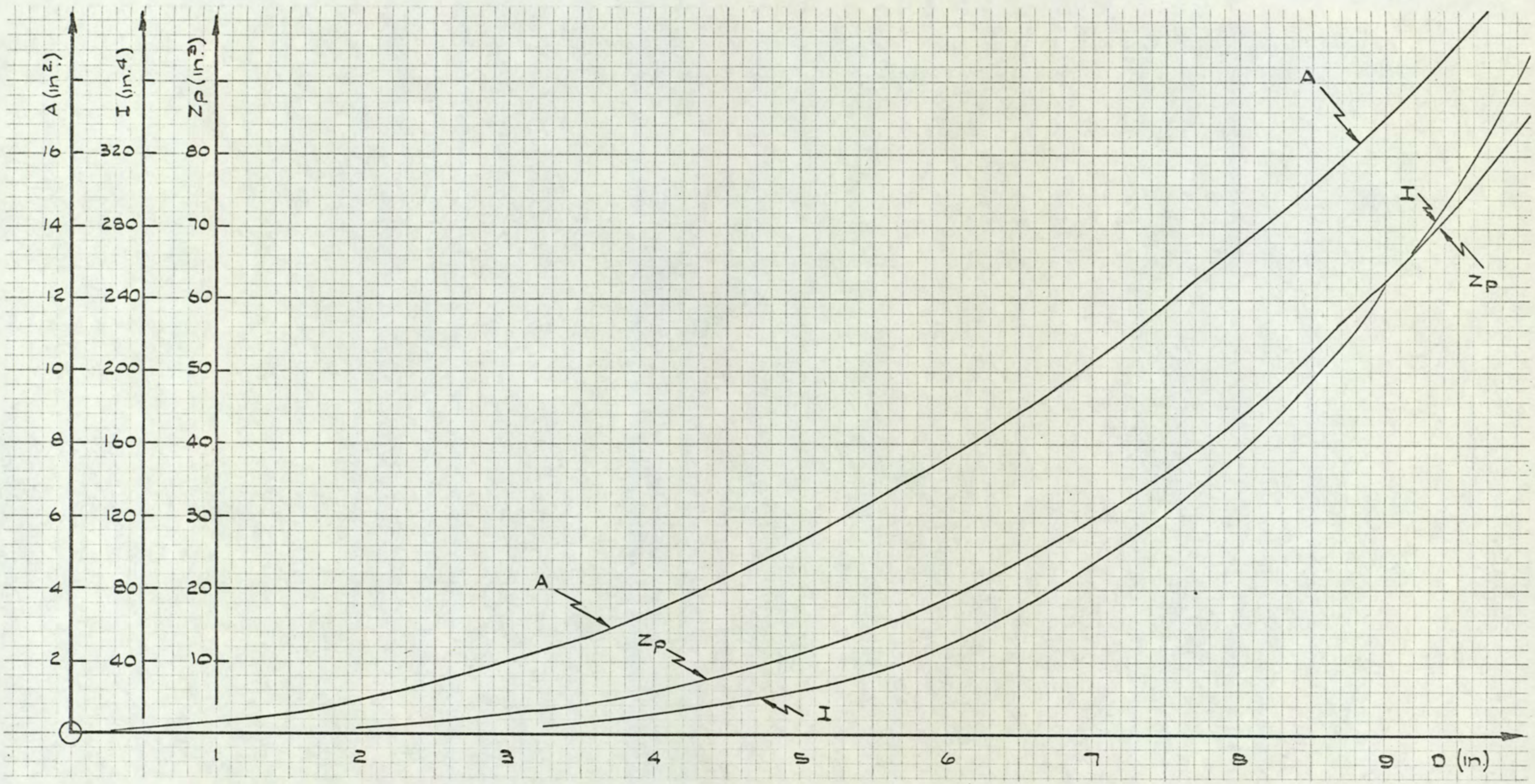
Variation in plastic modulus with axial load for the fictitious columns ; 5" < D < 11"; f_y = 35·84 K/sq.in

FIGURE 57 (a).



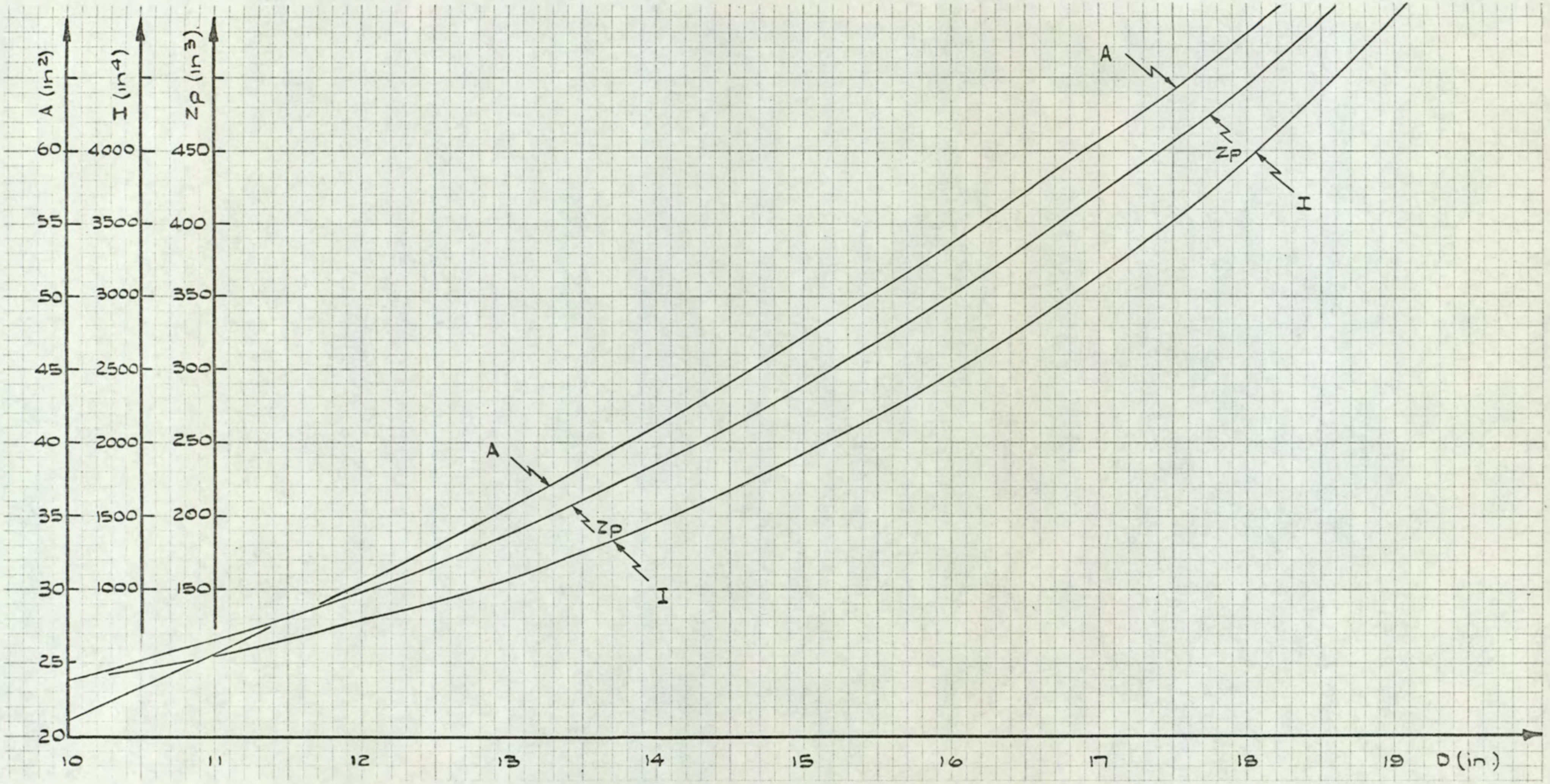
Variation in plastic modulus with axial load for the fictitious columns; 10" < D < 20"; f_y = 35.84 k/sq.in.

FIGURE 57 (b)



Section properties, A, I and Zp, for the fictitious columns; 0 < D < 10"

FIGURE 58 (a)



Section properties, A, I and Zp, for the fictitious columns; 10" < D < 18"

FIGURE 58 (b)

curves correspond to those given for the Universal Columns in Figures 48(a) and 48(b). Thus, for any column, the point ($zp_c, \lambda_1 P$) is plotted on the graph. The depth of the required section may be estimated quite accurately by linear interpolation between the values corresponding to the lines above and below the point. Knowing D, the section properties A, I and Zp are then obtained from Figure 58(a) and 58(b), which represent equations (99), (100) and (101).

For example, suppose that during the design of a particular storey, it was found that $zp_{ci} = 40 in^3$, and $\lambda_1 P_2 = 200$ kips. Then, from Figure 57(a), it may be seen that the lower column in this storey would require a depth in the range 8.5in. < D < 9in. By interpolation, in the direction of the "broken" lines which cut the reduced plastic modulus curves, it is found that D = 8.61in. Then, from Figure 58(a), for this value of D, A = $15.6 in^2$, I = $207 in^4$, and Zp = $55 in^3$

Thus, using the fictitious sections, a complete set of beams and columns may be selected for any framework, and each of these has the precise strength given by the design equations. If such a frame is analysed, then any violations of the design criteria, or any excessive reserve of strength, may be attributed directly to deficiencies in these equations.

In the following section, a description is given of the range of frames which has been designed with both these fictitious sections and with the Universal Beams and Universal Columns.

7.3. THE TEST FRAMES

In order to test the validity of the design equations, eight multi-bay and three single-bay frameworks were designed. These frames were deliberately devised to show the effect of the variation of each individual design parameter on the accuracy of the design method, and the design loads and geometry of every frame are summarized in Table 7. The characteristics of each frame will now be discussed briefly, and any deviations from the

	Frame	Number of bays, r	Number of storeys, q	Beam lengths, L (ft.)	Column heights, h (ft.)	Frame spacing, S (ft.)	Dead load + live load, w (k/sq.ft.) Top storey / Other storeys	Wind loading, p (k/sq.ft.)	Frame description
Multi-bay	1	2	8	26	11	18	.130 / .130	.0125	8 storey, 2 bay; *
	2	4	10	25	12	20	.050 / .100	.025	10 storey, 4 bay;
	3	3	8	25	16	20	.0625 / .125	.020	8 storey, 3 bay, tall columns;
	4	3	8	35	12	20	.0625 / .125	.020	8 storey, 3 bay, long beams;
	5	3	8	25	12	20	.0625 / .125	.040	8 storey 3 bay, heavy wind load;
	6	3	8	25	12	20	.125 / .250	.020	8 storey, 3 bay, heavy beam load;
	7	6	4	25	12	20	.0625 / .125	.020	4 storey, 6 bay, instability effects small;
	8	3	8	25	*	20	.0625 / .125	.020	8 storey, 3 bay, variable column heights;
Single-bay	9	1	4	30	12	20	.100 / .100	.030	4 storey, single bay; *
	10	1	6	30	12	20	.200 / .200	.020	6 storey, single bay, heavy beam load;
	11	1	6	30	12	20	.100 / .100	.040	6 storey, single bay, heavy wind load.

* See text.

Design parameters for eight multi-bay and three single-bay test frames.

TABLE 7

normal design procedure will be described.

Frame 1:-

This frame is similar to an eight storey, two bay frame with unequal bays, which has been designed previously by several other research workers[49,57,58], and which will be described in the following chapter. In this case, however, the bays were both assumed to be equal. Also, in deriving the initial design loads, the "live-load reduction" was included for the design of the columns, since the example was originally to be used for comparison purposes. The following floor loads were assumed:-

dead load = .080 k.per sq. ft.

live load = .050 k. per sq. ft.

Apart from the calculation of the column loads, the standard design procedure was used.

Frame 2:-

This is the ten storey, four bay frame, described previously in Figure 50, which was used in Chapter 6 to illustrate the design procedure. It was chosen in order to verify the design method for a comparatively large framework containing several bays. Although, of course, much larger frames do exist, they are difficult to analyse with an accurate computer program, and this particular frame was the largest of those which were designed and subsequently analysed.

Frames 3,4,5 and 6:-

These four frames all contain eight storeys and three bays, and in each case, all but one of the design parameters are identical. Frames 3 and 4 were chosen to represent typical frames with tall columns and long beams respectively. Frames 5 and 6 were devised to demonstrate two extreme loading conditions, those of heavy wind loading and heavy beam loading. The value of p = .040 k. per sq. ft. for Frame 5 is unlikely to be exceeded in any normal design. In fact, this magnitude of wind loading is greater than the maximum value recommended in CP3, Chapter V, (1952)[59]. Similarly, the value of w = .250 k. per sq. ft. for Frame 6 represents a very heavy intensity of floor loading.

All these four frames were designed using the standard procedure in Chapter 6.

Frame 7:-

The four storey, six bay frame was selected in order to test the accuracy of the method when applied to a frame in which the design is governed by the vertical loading, and in which the instability effects are small.

Frame 8:-

This eight storey, three bay frame contains storeys of different height, and was devised to test the generality of the design equations. Starting at the top storey, the column heights are as follows:- 12,15, 12,12,9,12,12,15ft.

Frame 9:-

The four storey, single bay frame was originally designed by Heyman[46], and will subsequently be used for comparison purposes in Chapter 8. The loadings given in Table 7 are those adopted by Heyman, and in order to simulate his design, the beam loads were calculated assuming that each beam carries half the loads on the adjoining floor panels. Thus, in every storey, the total uniformly distributed load on each beam is equal to 60 kips. Also, the applied wind loads at each beam level were assumed to be equal to 7.2 kips, including the lowest storey, with 3.6 kips applied at the top storey. Otherwise, the standard design procedure was used.

Frames 10 and 11:-

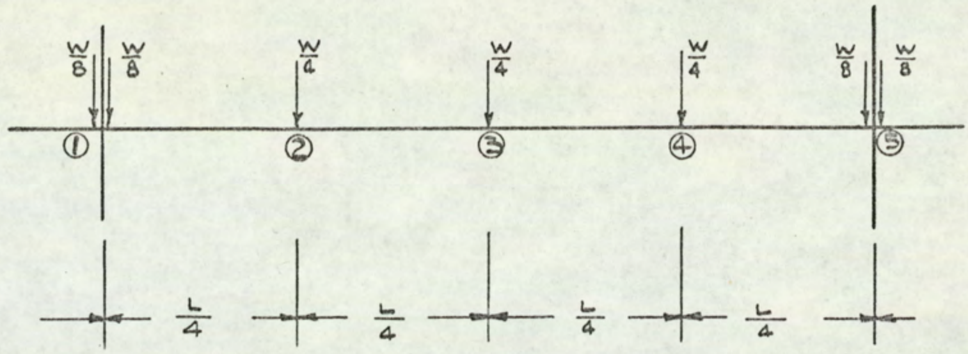
The geometry of both these six storey, single bay frames is identical. Frame 10 represents the case of heavy beam loading, and Frame 11 the case of heavy wind loading. It may be seen that the basic wind ratio in any storey of Frame 11 is four times that in the corresponding storey of Frame 10. [The same is true of Frames 6 and 5 respectively of the multi-bay range].

The following two sections contain the results obtained from the designs and analyses of these eleven frames, for both the fictitious sections and the standard Universal Beams and Universal Columns. This information is summarized at the end of the chapter.

7.4. DESIGN AND ANALYSIS USING THE FICTITIOUS SECTIONS

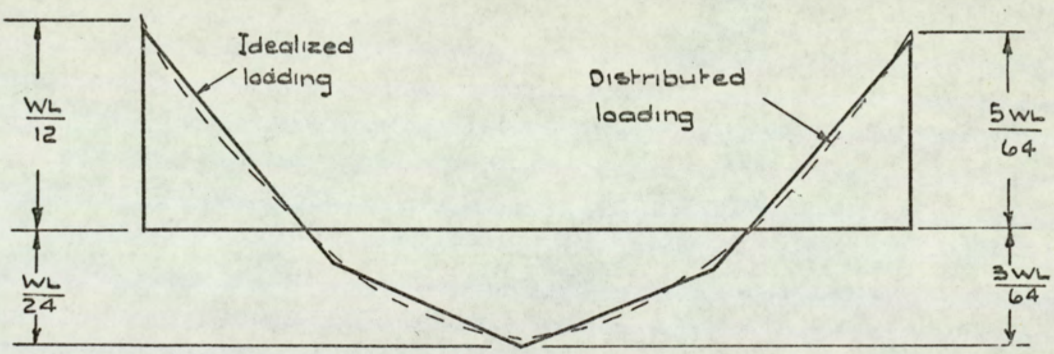
In order to analyse the frameworks, it was necessary to idealize the uniformly distributed beam loading as a series of point loads, since the computer program demands that loads may only be applied at distinct joints. The loading pattern which was assumed, and the corresponding "artificial" joints are shown in Figure 59(a). From Figures 59(b) and 59(c), it may be seen that there is little difference between the true and assumed bending moment diagrams, either at working load, when the beam is fully elastic, or at ultimate load.

Under combined load, with the wind acting from the left, say, there is a possibility that the span hinge will form at joint 2, rather than at joints 1 or 3 as predicted in the design. This has occurred in the analyses of several frames, and although this hinge pattern is not assumed in the design method, it is in fact a valid mode of failure when a uniformly distributed load acts on the beam. It will be remembered from Section 3.4(d), in which the basic design equations have been criticised, that in practice the span hinge "shifts" from the centre of the beam to the windward



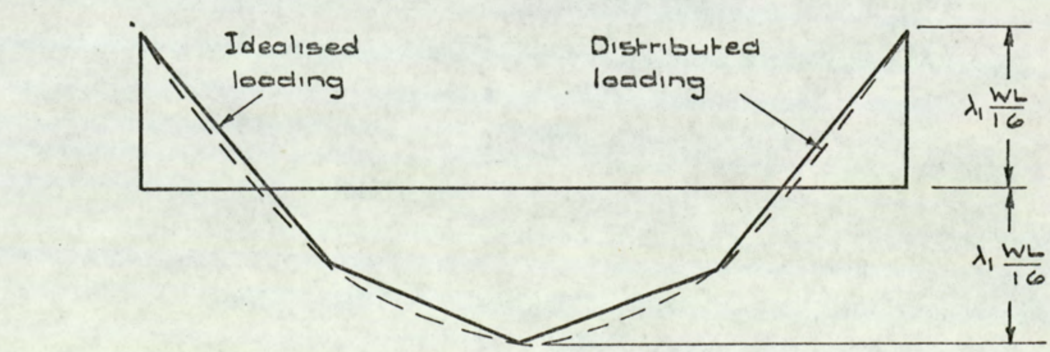
Idealization of the uniformly distributed beam loads;
(for analysis purposes)

FIGURE 59 (a)



Bending moment diagrams at working load

FIGURE 59 (b)



Bending moment diagrams at collapse - vertical load alone at λ_1

FIGURE 59 (c).

end as the wind ratio increases. The idealized loads allow the formation of this hinge in an intermediate position, and as such represent a more realistic distribution than the more popular system in which a single load of $\frac{W}{2}$ is applied at the centre of the beam, together with loads of $\frac{W}{4}$ at each end.

A slightly different system of loads to that given in Figure 59(a) was used in the analysis of Frame 1. As for the other frames, loads of $\frac{W}{4}$ were applied at the "quarter-points" on the beams, but the loads at the top of the columns were adjusted in order to allow for the fact that these members were designed assuming the "live-load reduction". This was simply done by reducing the applied load at each of these locations by the same amount as that assumed in the design. Without this modification, the analysis would not have been a realistic check on the adequacy of the design.

The results obtained for the multi-bay and the single-bay frames will be considered separately.

7.4(a) MULTI-BAY FRAMES

The fictitious sections obtained during the design of each multi-bay frame are given in Table 8, where each section is defined by its overall depth, D. If required, the corresponding section properties may be derived from Figures 56(a) and 56(b) for the beams, and from Figures 58(a) and 58(b) for the columns. The results of the analyses of these frames are discussed below.

Combined load analyses (Design load factor, $\lambda_2 = 1.4$)

Figures 60(a) and 60(b) show the order of hinge formation obtained from the combined load analyses of Frames 5 and 6 respectively. These hinge patterns are typical of those for any multi-bay frame designed by the proposed method.

Frame 5 was designed to carry very heavy wind loading, and accordingly

Frame	Member	Storey number (from the top)									
		1	2	3	4	5	6	7	8	9	10
1	b	13.20	13.20	13.20	13.60	14.00	14.45	15.10	13.54		
	c_i	6.30	6.30	7.72	8.83	9.66	10.49	11.14	13.15		
	c_e	7.69	7.69	7.69	7.69	7.98	8.36	8.89	8.89		
2	b	9.40	11.90	12.22	12.88	13.48	14.20	14.88	15.42	15.95	12.84
	c_i	5.88	5.88	7.51	8.72	9.73	10.52	11.17	11.82	12.37	14.84
	c_e	5.50	6.27	6.50	7.07	7.56	8.15	8.69	9.16	9.59	9.59
3	b	10.20	12.80	13.76	14.62	15.60	16.42	17.29	14.87		
	c_i	7.17	7.17	8.97	10.32	11.26	12.07	12.88	16.48		
	c_e	5.95	6.95	7.38	8.05	8.78	9.40	10.00	10.00		
4	b	13.43	16.98	16.98	16.98	17.26	17.78	18.56	16.98		
	c_i	6.65	6.65	8.47	10.04	11.44	12.67	13.50	15.31		
	c_e	7.84	8.61	8.67	9.13	9.66	10.27	10.94	10.94		
5	b	10.20	12.80	13.61	14.42	15.26	16.21	16.94	14.48		
	c_i	7.02	7.02	8.78	10.09	11.16	11.99	12.75	16.61		
	c_e	5.95	6.90	7.29	7.99	8.64	9.37	9.96	9.96		
6	b	12.80	16.12	16.12	16.20	16.88	17.83	18.64	16.12		
	c_i	7.50	7.50	9.60	11.48	13.01	14.15	15.24	17.05		
	c_e	7.55	8.78	8.89	9.54	10.40	11.21	13.02	12.02		
7	b	10.20	12.80	12.80	12.80						
	c_i	5.67	5.67	7.33	8.92						
	c_e	5.95	6.66	6.77	7.10						
8	b	10.20	12.80	13.30	13.72	14.08	14.34	15.70	14.00		
	c_i	6.75	6.75	8.15	9.39	9.90	11.33	12.07	15.00		
	c_e	5.95	7.02	7.02	7.61	7.89	8.77	9.38	9.38		

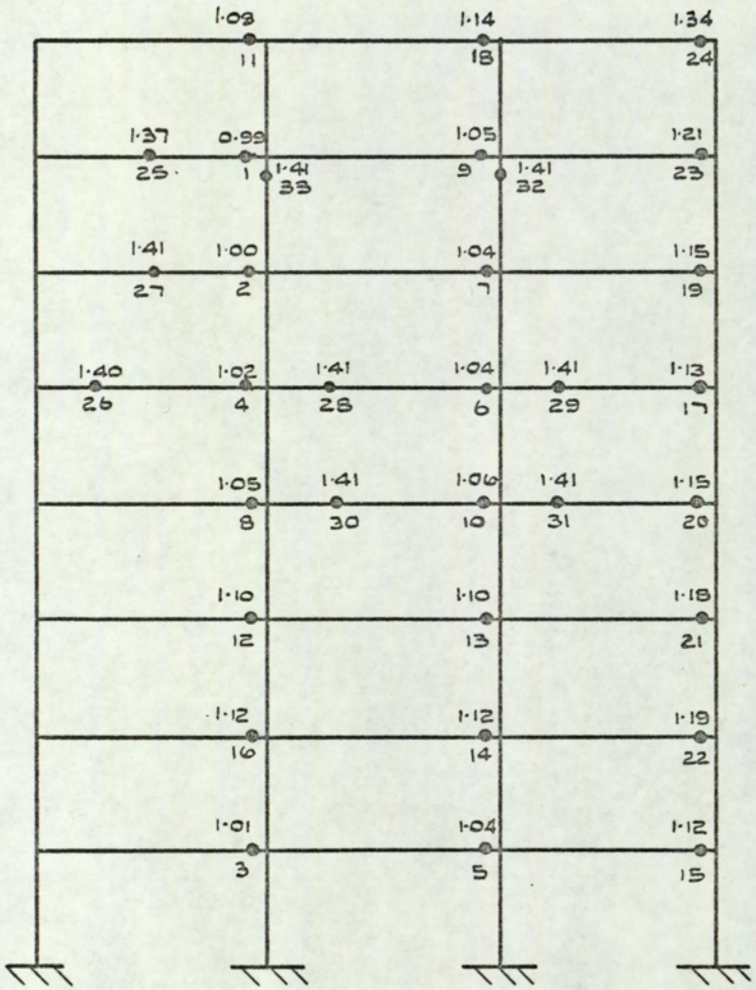
Properties of fictitious beams are given in Figures 56(a) and 56(b).

" " columns " " " 58(a) and 58(b).

Fictitious sections selected for Frames 1 to 8 (Multi-bay);

each section is defined by its overall depth, D ins.

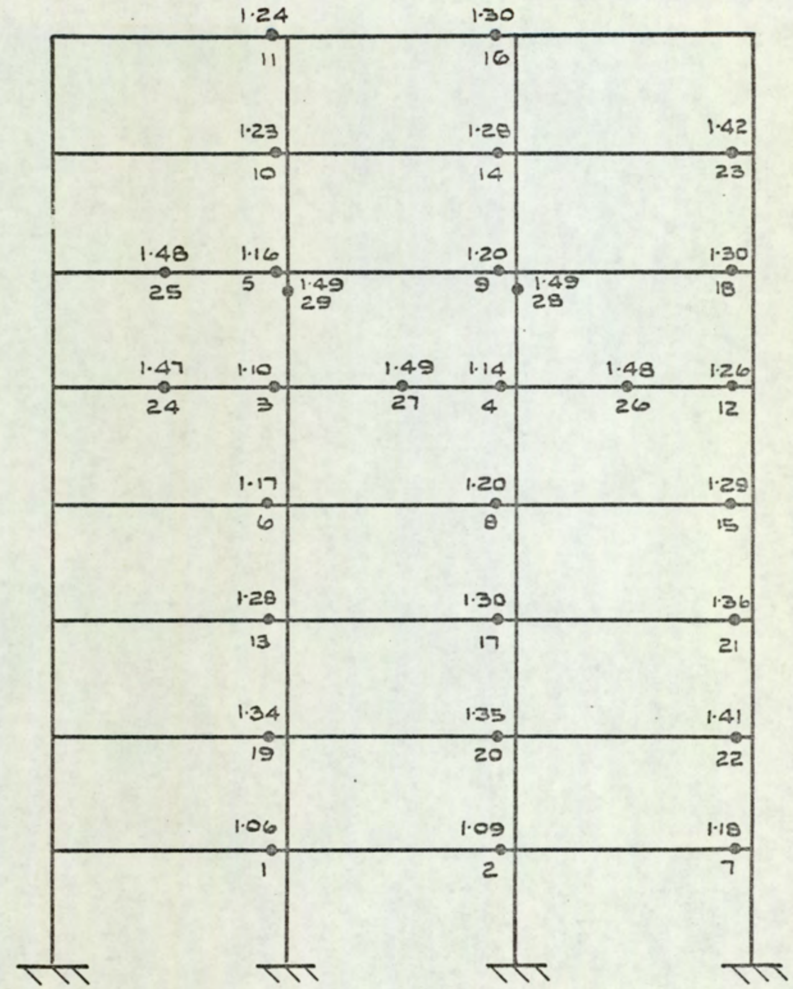
TABLE 8



$\lambda_F = 1{\cdot}41$

Frame 5 - Fictitious sections - Combined load analysis.

FIGURE 60 (a)



$\lambda_F = 1{\cdot}49.$

Frame 6 Fictitious sections - Combined load analysis.

FIGURE 60 (b)

the majority of its storeys were found to lie in Zones 2 and 3, so that the combined loading case generally dictated the required section sizes. This is reflected in Figure 60(a), where it may be seen that failure occurs shortly after the design load factor, $\lambda_2 = 1.4$, is attained.

In contrast, the design of Frame 6 was dominated by the vertical loading case, and several storeys were found to lie in Zone 1. This is also indicated by comparison between Figures 60(a) and 60(b). Frame 6 fails at a higher load factor than Frame 5 under combined loading, and plasticity develops in every beam much later.

The results of all the analyses of the multi-bay frames are summarized in Table 9. Considering just the combined load analyses, the following general observations may be made:-

(1) Every frame fails within the range $1.412 < \lambda_F < 1.492$. Thus, the first design criterion, $\lambda_F \geq 1.4$, is always satisfied, and none of the frames has an excessive reserve of strength.

(2) In seven of the frames, there are no plastic hinges in the beams below working load, so that the final design criterion is satisfied. Also, in each of these frames, the first hinge is found to occur in the lowest storey beam. During the design of Frames 1, 2, 3, and 8, this beam was increased in order to satisfy the "working load elasticity condition", and, referring to the results for these particular frames in Table 9, it may be seen that the condition gives an extremely accurate solution, the first hinges forming in the range $1.009 < \lambda < 1.033$.
In Frame 5, however, plastic hinges occur just below the working load in Storeys 2 and 3, at load factors 0.991 and 0.996 respectively. As stated previously in Section 6.2(b)(iv), during the design of this frame, the "working load elasticity condition" for the intermediate storeys indicated that these two beams were just satisfactory, whereas the analysis indicates that they

Frame number	Combined load analysis; $\lambda_2 = 1.40$;							Vertical load analysis; $\lambda_1 = 1.75$;		
	λ_F	No. of beam hinges.	No. of column hinges.	First beam hinge forms at:-	First column hinge forms at:-	No. of beam hinges below working load.	No. of column hinges before $\lambda = 1.40$.	λ_F	Total number of hinges.	First beam hinge forms at:-
1	1.475	19	2	1.011	1.465	0	0	1.733	26	1.233
2	1.440	44	4	1.033	1.438	0	0	1.734	28	1.252
3	1.423	29	0	1.009	-	0	0	1.731	16	1.218
4	1.469	27	8	1.049	1.450	0	0	1.723	43	1.297
5	1.412	31	2	0.991	1.412	2	0	1.731	16	1.232
6	1.488	27	2	1.055	1.488	0	0	1.736	36	1.289
7	1.492	26	7	1.050	1.396	0	1	1.730	48	1.247
8	1.463	29	0	1.022	-	0	0	1.730	22	1.240

Analysis results for Frames 1 to 8, designed with fictitious sections.

[Collapse load factor = λ_F]

TABLE 9

should have been increased. However, the error is very small, and only a marginal increase is required. Furthermore, the extreme loading conditions on this frame are unlikely to be encountered in practice.

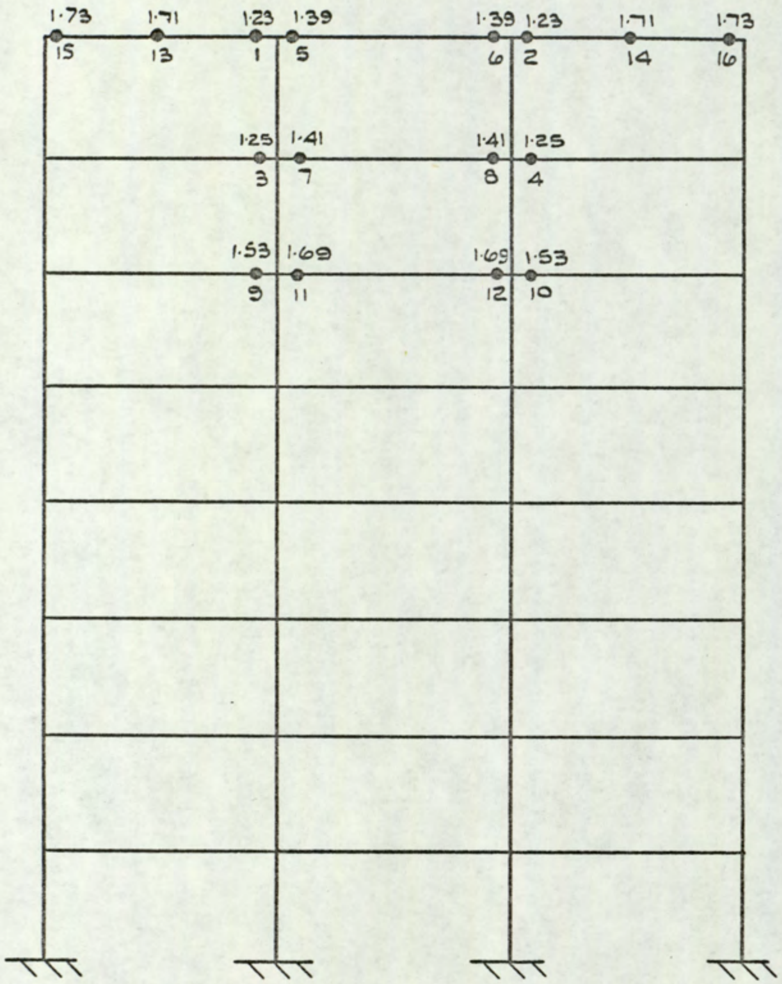
(3) The third design criterion, which demands that there shall be no plastic hinges in the columns below the design load factor, is satisfied in all but one of the frames. In the remaining case, Frame 7 (the four storey, six bay frame), a hinge occurs at λ = 1.396 at the base of the leeward external column of Storey 3. Again, the deficiency is very small, and, as for Frame 5, this violation of one of the subsidiary design criteria has little effect on the overall strength of the frame. This particular frame does in fact have a higher collapse load factor than any other, and this is as expected, since the complete design was found to lie in Zone 1.

Vertical load analyses (design load factor, λ_1 = 1.75)

The hinge patterns from the vertical load analyses of Frames 5 and 6 are given in Figures 61(a) and 61(b) respectively, and these may again be seen to reflect the manner in which each of these frames was designed.

Since the combined loading case dominated the design of most of the storeys in Frame 5, these storeys have a large reserve of strength under vertical load alone. In contrast, Frame 6 exhibits an extensive pattern of plastic hinges under this loading. Both frames nevertheless fail at approximately the same load factor, since they both contain at least one beam for which the vertical loading case was the critical design condition. Failure may occur in any frame due to the localized collapse of a single member, even if every other member is still fully elastic.

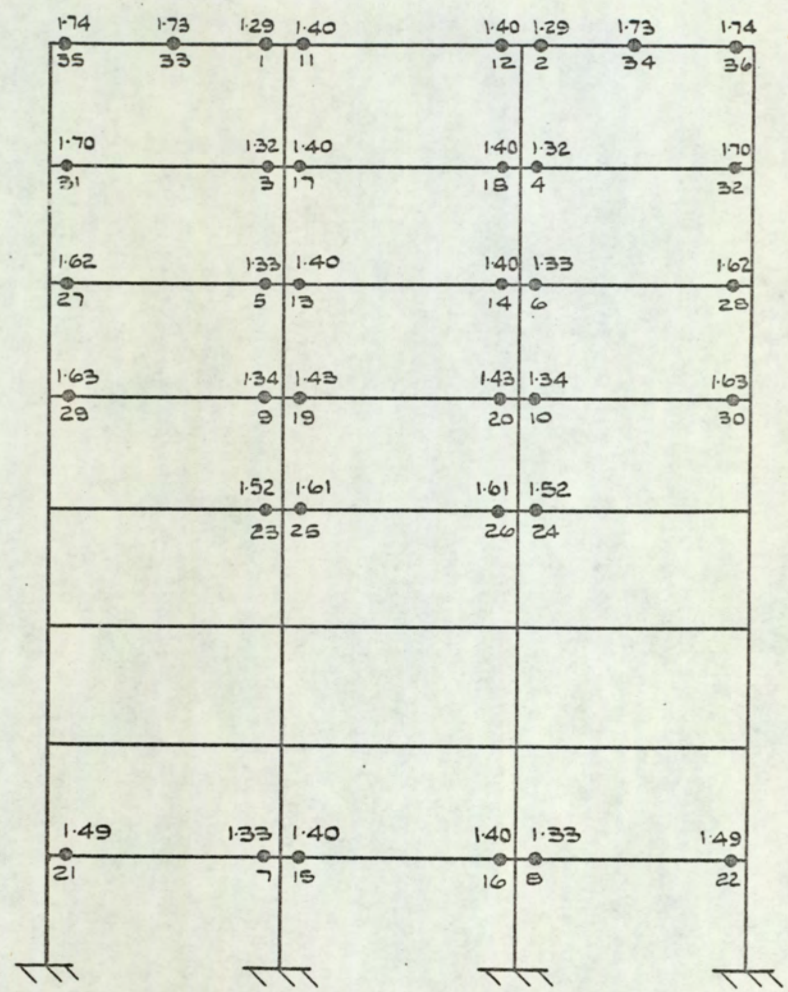
Referring again to Table 9, the following general observations may be made concerning the vertical load analyses of all the frames:-



$\lambda_F = 1{\cdot}73$

Frame 5 - Fictitious sections - Vertical load analysis

FIGURE 61 (a)



$\lambda_F = 1{\cdot}74$

Frame 6 - Fictitious sections - Vertical load analysis.

FIGURE 61 (b).

(1) Under vertical load alone, all eight frames collapse due to the formation of a simple beam mechanism in the external bays of the top storey, at a load factor in the range 1.723 < λ_F < 1.736. Thus, in theory, every frame fails to satisfy the second design criterion, $\lambda_F \geq 1.75$, despite the fact that the mode of failure is identical to that assumed for the design of the top storey. The reason for this is that the design equation for the top storey beam, $B_1 = \lambda_1 \frac{W_1 L}{16}$, is based on the "simple plastic" assumption that the structure remains completely undeformed up to collapse. In this particular case, the external columns in the top storey are comparatively flexible. Quite large deformations occur at the top of these columns long before the collapse load factor is reached, and these lead to slightly premature failure.

(2) In all the frames, the first plastic hinge forms in the range 1.218 < λ < 1.297, and no hinges occur in the columns under vertical load alone. Thus, the third and fourth design criteria are always satisfied for this loading case.

7.4(b) SINGLE-BAY FRAMES

The fictitious sections obtained for the three single-bay frames are given in Table 10, and the analysis results for these designs are summarized in Table 11.

Combined load analyses:-

The hinge patterns obtained for Frames 10 and 11 are shown in Figures 62(a) and 62(b). These are typical of those obtained for any single-bay frame, and again reflect the manner in which the frames were designed. Frame 11, which was designed for heavy wind loading, fails at a lower load factor than Frame 10, which was designed for heavy beam loading.

From Table 11, it may also be seen that the first design criterion,

Frame	Member	Storey number (from the top).					
		1	2	3	4	5	6
9	b	15.86	15.86	16.68	16.80		
	c_e	9.30	9.30	9.30	10.92		
10	b	17.48	17.48	17.48	18.10	18.66	18.00
	c_e	10.25	10.25	10.25	10.49	11.55	13.60
11	b	13.84	14.55	16.00	17.85	19.50	16.63
	c_e	8.12	8.12	8.86	9.91	10.99	14.95

Fictitious sections selected for Frames 9 to 11(Single-bay);

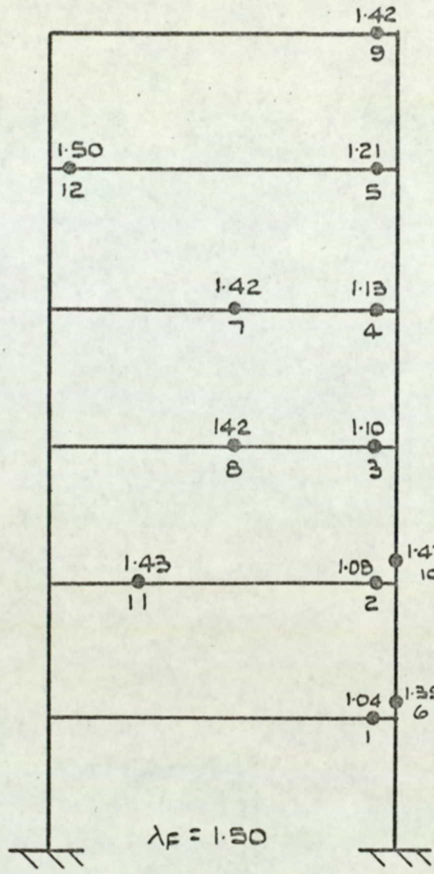
TABLE 10

Frame number	Combined load analysis; λ_2 = 1.40;							Vertical load analysis; λ_1 = 1.75;		
	λ_F	No. of beam hinges.	No. of column hinges.	First beam hinge forms at:-	First column hinge forms at:-	No. of beam hinges below working load.	No. of column hinges before λ = 1.40.	λ_F	Total number of hinges.	First beam hinge forms at:-
9	1.493	7	1	1.046	1.493	0	0	1.731	5	1.510
10	1.499	10	2	1.044	1.386	0	1	1.733	9	1.509
11	1.396	10	3	1.015	1.389	0	3	1.730	3	1.701

Analysis results for Frames 9 to 11, designed with fictitious sections.

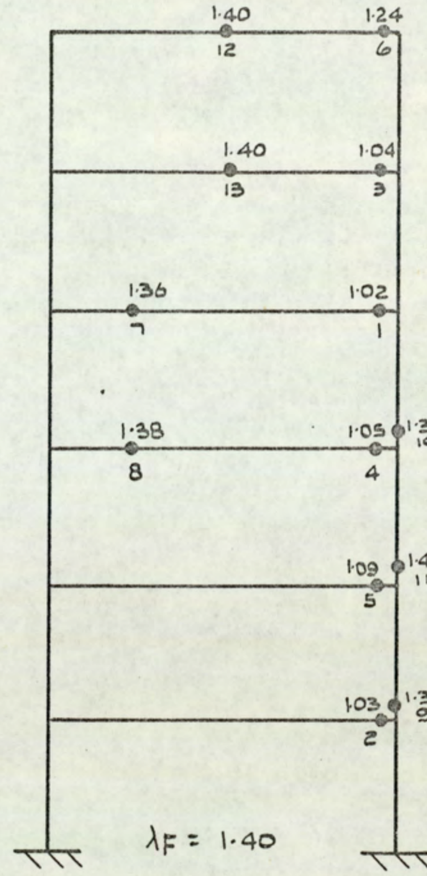
[Collapse load factor = λ_F]

TABLE 11



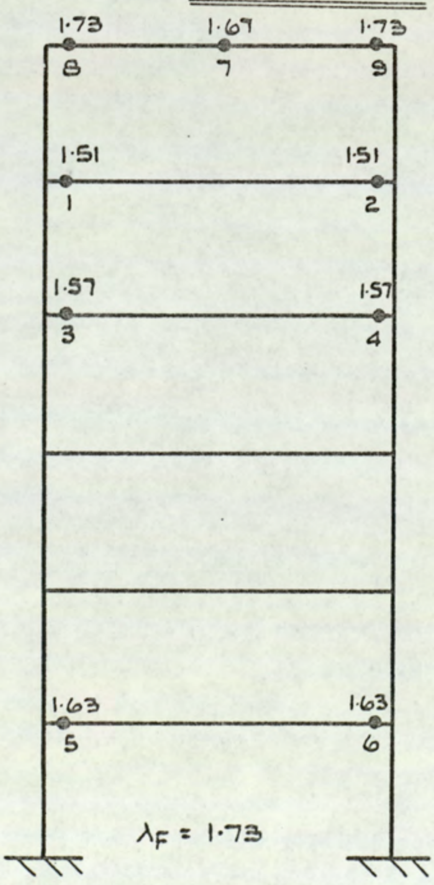
Frame 10 - Fictitious sections - Combined load.

FIGURE 62 (a)



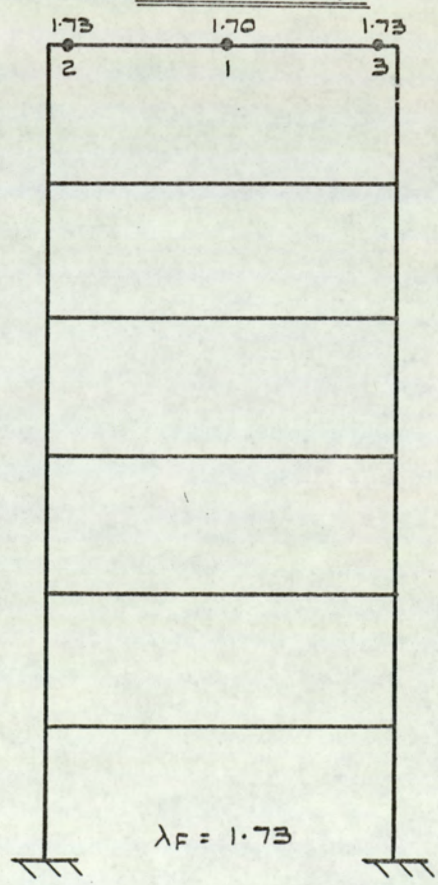
Frame 11 - Fictitious sections - Combined load

FIGURE 62 (b)



Frame 10 - Fictitious sections - Vertical. load

FIGURE 63 (a)



Frame 11 - Fictitious sections - Vertical load

FIGURE 63 (b).

$\lambda_F \geq 1.4$, is satisfied for Frames 9 and 10, but is violated slightly in Frame 11, collapse occurring in this case at $\lambda = 1.396$. Also, in all three frames, there are no early beam hinges, so that the third design criterion is satisfied. As in the multi-bay frames, the lowest storey beam in each of these single-bay frames was increased in order to satisfy the "working load elasticity condition", and the accuracy of this condition is again confirmed.

In Frames 10 and 11, plastic hinges occur in the column shortly before the design load factor is reached, so that the final design criterion is violated. However, as for the multi-bay frames, the error is very small, the earliest of these hinges occurring at $\lambda = 1.386$ (within one per cent of λ_2).

Vertical load analyses:-

Figures 63(a) and 63(b) give the hinge formation for Frames 10 and 11 under vertical load alone. As expected, Frame 10 develops more plasticity than Frame 11, since its design was controlled by the vertical loading case. As before, each frame fails slightly before the design load factor, $\lambda_1 = 1.75$, with the formation of a simple beam mechanism in the top storey, and again this is due to the hinge flexibility of the top storey columns. The values of λ_F for each frame are given in Table 11. The remaining design criteria for this loading condition are satisfied in each case.

7.5. DESIGN AND ANALYSIS USING UNIVERSAL BEAMS AND UNIVERSAL COLUMNS

As in the previous section, the results obtained for the multi-bay and the single-bay frames will be considered separately.

7.5(a) MULTI-BAY FRAMES

Referring back to Table 8, which gives the fictitious sections for each multi-bay frame, and to Table 9, which gives the analysis results, it may be seen that both the designs and the analyses of Frames 3 and 5 are

Frame	Member	Storey number (from the top).									
		1	2	3	4	5	6	7	8	9	10
1	b	6	6	6	6	6	8	8	6		
	c_i	34	34	35	37	40	42	43	48		
	c_e	35	35	35	35	36	36	37	37		
2	b	1	4	4	6	6	6	9	11	12	6
	c_i	32	33	35	38	39	42	43	43	46	51
	c_e	32	34	34	34	35	36	37	37	39	39
5	b	2	6	6	8	10	12	13	8		
	c_i	34	34	38	40	41	46	48	54		
	c_e	33	34	34	36	38	39	40	40		
6	b	6	12	12	12	13	14	15	12		
	c_i	35	35	40	43	48	51	53	54		
	c_e	35	38	39	40	42	43	46	46		
7	b	2	6	6	6						
	c_i	33	33	34	37						
	c_e	33	34	34	34						
8	b	2	6	6	6	6	8	11	7		
	c_i	34	34	36	39	40	41	46	51		
	c_e	33	34	34	35	36	37	39	39		

The properties of the Universal Beams are given if Table 1, and those of the Universal Columns are given in Table 2,

Real sections selected for Frames 1, 2, 5, 6, 7 and 8.

TABLE 12

Frame number	Combined load analysis; $\lambda_2 = 1.40$;							Vertical load analysis; $\lambda_1 = 1.75$;		
	λ_F	No. of beam hinges.	No. of column hinges.	First beam hinge forms at:-	First column hinge forms at:-	No. of beam hinges below working load.	No. of column hinges before $\lambda = 1.40$.	λ_F	Total number of hinges.	First beam hinge forms at:-
1	1.553	18	4	1.061	1.502	0	0	1.948	30	1.409
2	1.457	39	2	1.151	1.457	0	0	1.971	26	1.451
5	1.521	29	6	1.055	1.419	0	0	2.016	18	1.453
6	1.596	27	5	1.136	1.500	0	0	1.851	35	1.403
7	1.740	24	10	1.326	1.668	0	0	2.015	44	1.483
8	1.542	25	6	1.075	1.475	0	0	2.015	30	1.463

Analysis results for Frames 1,2,5,6,7 and 8, designed with real sections.

[Collapse load factor = λ_F]

TABLE 13

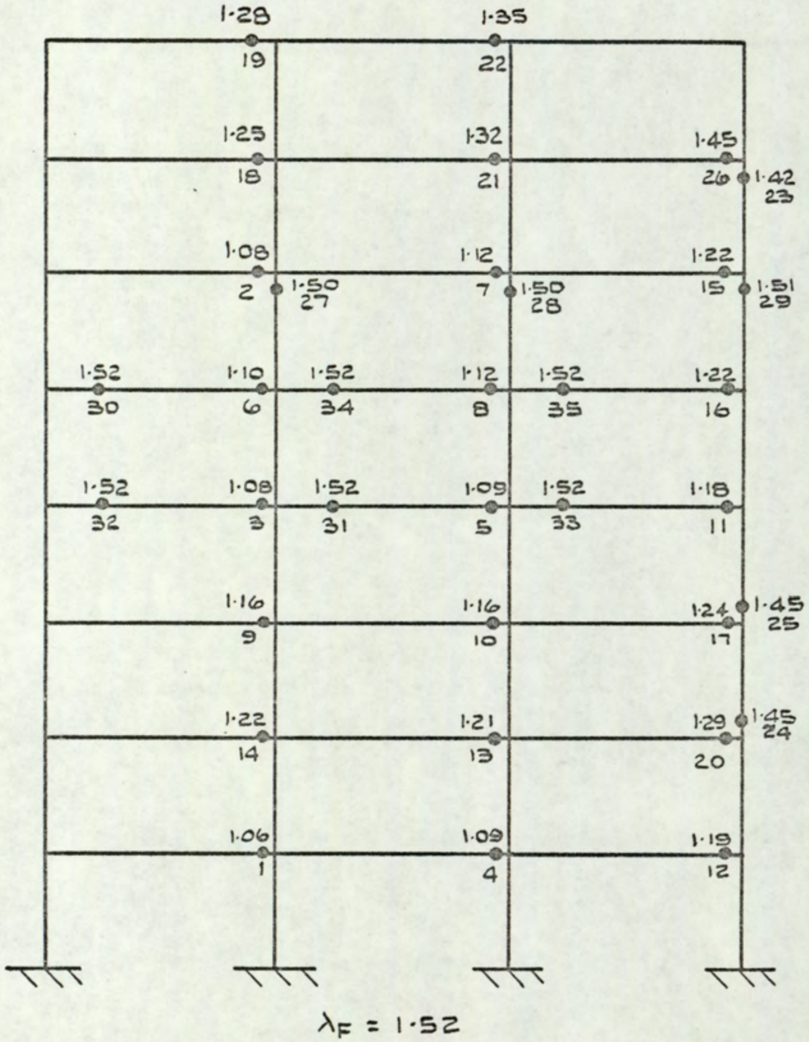
very similar. The reason for this is that both frames have been designed for high values of wind ratio, Frame 3 containing tall columns, and Frame 5 being required to withstand heavy wind loading. The results obtained for either frame add little to the information already supplied by the results for the other. Therefore, Frame 3 has not been redesigned using the Universal Beams and Universal Columns. In the same way, Frame 4 has not been redesigned, since it is similar to Frame 6.

The "real" sections obtained for the remaining six multi-bay frames are given in Table 12. The reference numbers are those adopted in Tables 1 and 2 of Chapter 6. The analysis results are given in Table 13, and are discussed below.

Combined load analysis:-

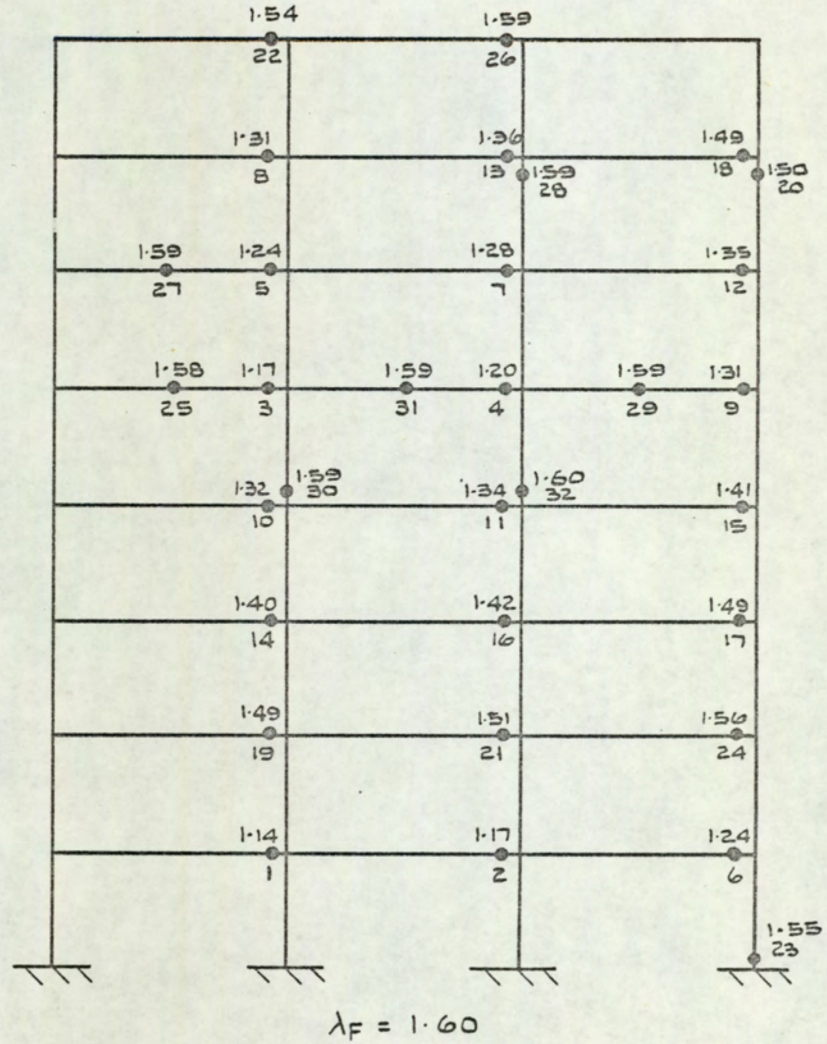
Figures 64(a) and 64(b) show the hinge pattern for Frames 5 and 6, and these may be compared directly with those given previously in Figures 60(a) and 60(b), which referred to the design with fictitious sections. It may be seen that each frame fails in a very similar manner when designed with the two different types of section. The use of the real sections increases the collapse load factor in each case. As before, comparing Figures 64(a) and 64(b), Frame 6, which was designed for heavy beam loading, has an additional reserve of strength when analysed under combined load, and plasticity develops in the beams much later than in Frame 5.

The results given in Table 13 for the combined load analyses indicate that all the design criteria are satisfied when the frames are designed using the real sections. Frame 7 has a particularly high value of λ_F, since its design was governed entirely by the vertical loading case. Also, the reason for the comparatively low collapse load in Frame 2 is that failure occurs in this case due to the formation of a "joint mechanism" at the junction of the beam and the leeward external column in Storey 9. Plastic hinges form simultaneously in all three members at this location, with the



Frame 5 - Real sections - Combined load analysis.

FIGURE 64 (a)



Frame 6 - Real sections - Combined load analysis

FIGURE 64 (b).

result that the stiffness of the joint, and therefore that of the whole frame, suddenly becomes zero. However, although unexpected, this is a perfectly legitimate mode of collapse, since the hinges in the columns do not form below the design load factor.

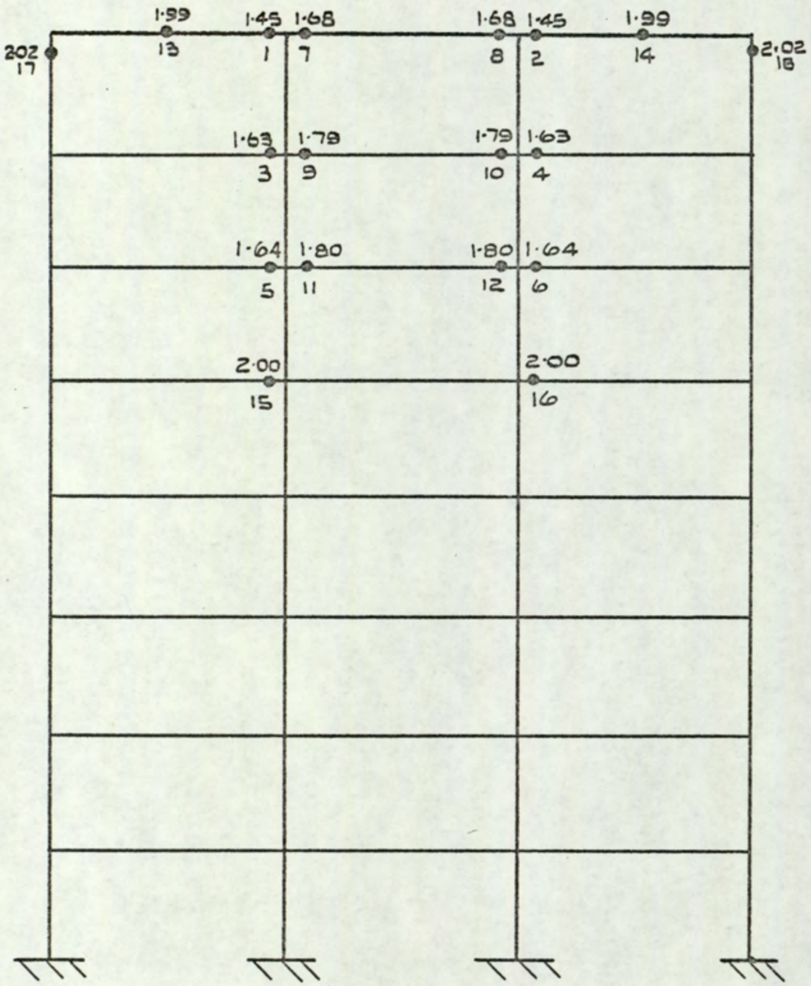
Vertical load analyses:-

The hinge patterns obtained due to vertical loading on Frames 5 and 6 are given in Figures 65(a) and 65(b). Comparing these with Figures 61(a) and 61(b), it may again be seen that the frames fail in a similar manner whether designed with the real or fictitious sections.

As expected, Frame 5 has a considerable reserve of strength under this loading, since only the top two beams were designed to fail by a simple beam mechanism. Both these beams were supplied with a fully plastic moment considerably greater than that which was required. This indicates that the "availability of sections" can have a large effect on the collapse load factor of any frame. For example, Frame 7 collapses under vertical load alone at $\lambda_F = 2.015$, despite the fact that every beam of this frame was designed to fail by a simple beam mechanism.

Also, in Frame 5, the "beam mechanism" in the top storey eventually occurs when a plastic hinge forms in the column, as shown in Figure 65(a). This is again due to the fact that the beam is considerably stronger than required. The column was selected to have a reduced plastic moment greater than the fully plastic moment of the beam at $\lambda_1 = 1.75$, but, at higher load factors, this reduced plastic moment becomes smaller than that of the beam, and the column becomes the potential position for a plastic hinge.

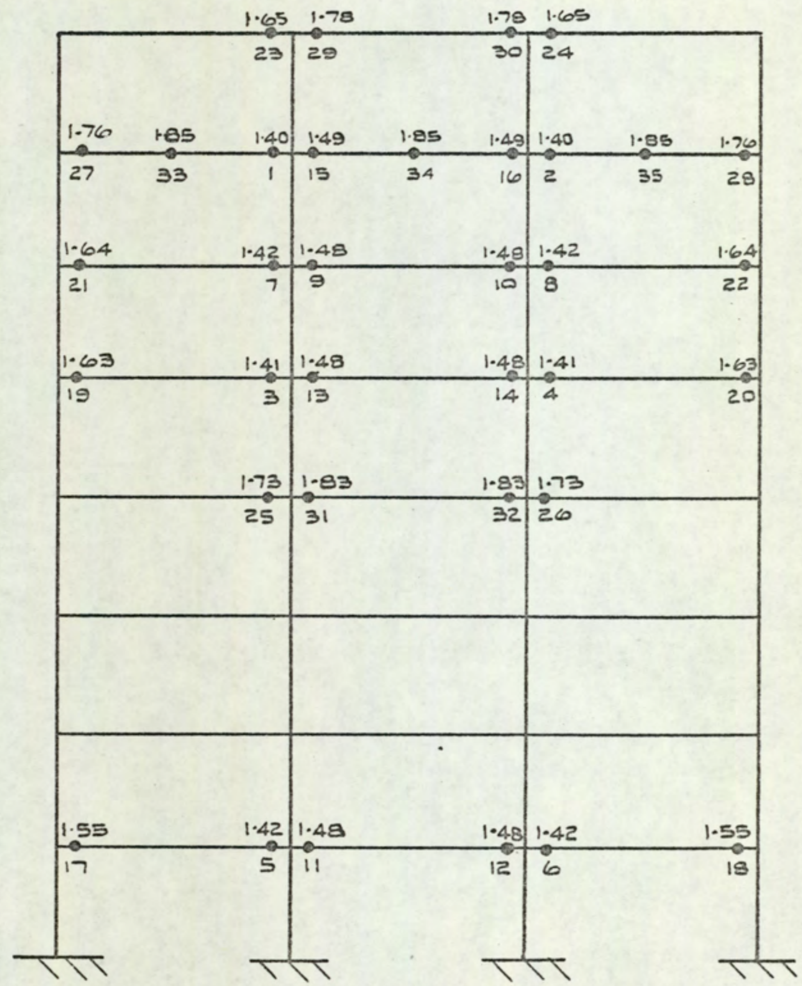
Nevertheless, no hinges form in the columns below the design load factor, and, as may be seen from Table 13, every frame has the required strength.



$\lambda_F = 2 \cdot 02$

Frame 5 - Real sections - Vertical load analysis

FIGURE 65 (a)



$\lambda_F = 1 \cdot 85$

Frame 6 - Real sections - Vertical load analysis.

FIGURE 65 (b).

7.5(b) SINGLE-BAY FRAMES

The Universal Beams and Universal Columns obtained for Frames 9, 10 and 11 are given in Table 14. The analysis results are summarized in Table 15, and are discussed below.

Combined load analyses

Figures 66(a) and 66(b) show the combined load analyses of Frames 10 and 11. These may be compared with Figures 62(a) and 62(b), which correspond to the designs for these frames using the fictitious sections.

Table 15 indicates that all three frames fail at a value of λ_2 above the design load factor. Also, there are no hinges in the beams below working load, and the early column hinges, which formed in the fictitious sections, do not appear in these designs. Thus, all the design criteria are satisfied.

Vertical load analyses:-

The hinge patterns for Frames 10 and 11 under vertical load alone are given in Figures 67(a) and 67(b), which correspond to Figures 63(a) and 63(b). Frame 11 again fails by the simple beam mechanism in the top storey, the remaining storeys being fully elastic at collapse. In contrast, five of the beams in Frame 10 contain plastic hinges, the design of this frame being largely controlled by this loading case.

As shown in Table 15, each frame attains the design load factor, λ_1, and all the design criteria are satisfied.

7.6 CONCLUSIONS

By using the accurate elasto-plastic analysis program to test a wide range of frameworks designed using the fictitious sections, an extremely critical appraisal of the validity of the design equations has been made. In general, it is seen that these equations predict the required sections very accurately, although slight violations of the design criteria do

Frame	Member	Storey number(from the top).					
		1	2	3	4	5	6
9	b	12	12	13	13		
	c_e	39	39	39	42		
10	b	13	13	13	14	15	14
	c_e	42	42	42	42	46	49
11	b	7	8	12	14	15	13
	c_e	36	36	37	40	42	51

Real sections selected for Frames 9 to 11.

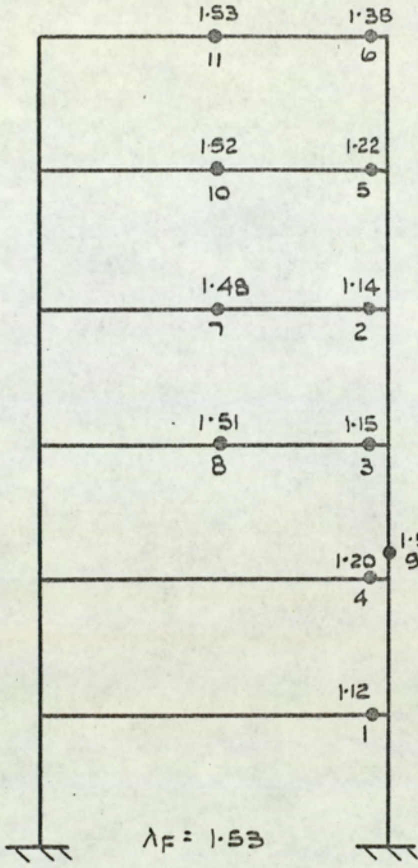
TABLE 14

Frame number	Combined load analysis; λ_2 = 1.40;							Vertical load analysis; λ_1 = 1.75;		
	λ_F	No. of beam hinges.	No. of column hinges.	First beam hinge forms at:-	First column hinge forms at:-	No. of beam hinges below working load.	No. of column hinges before λ = 1.40.	λ_F	Total number of hinges.	First beam hinge forms at:-
9	1.678	7	2	1.205	1.635	0	0	1.897	5	1.654
10	1.526	10	1	1.123	1.522	0	0	1.781	11	1.496
11	1.496	11	2	1.102	1.414	0	0	1.842	3	1.802

Analysis results for Frames 9 to 11, designed with real sections.

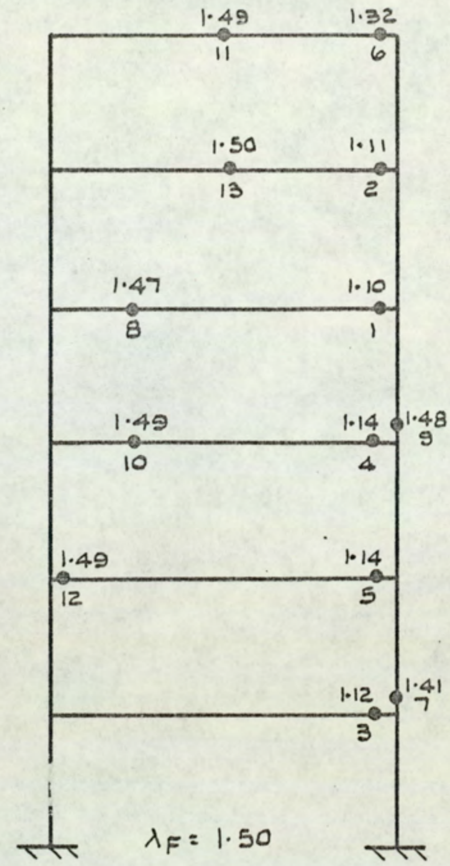
[Collapse load factor = λ_F]

TABLE 15



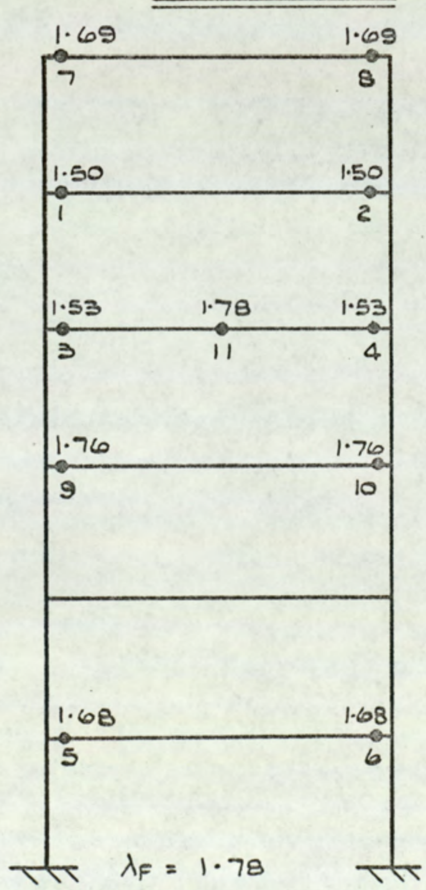
Frame 10 - Real sections - Combined load

FIGURE 66 (a)



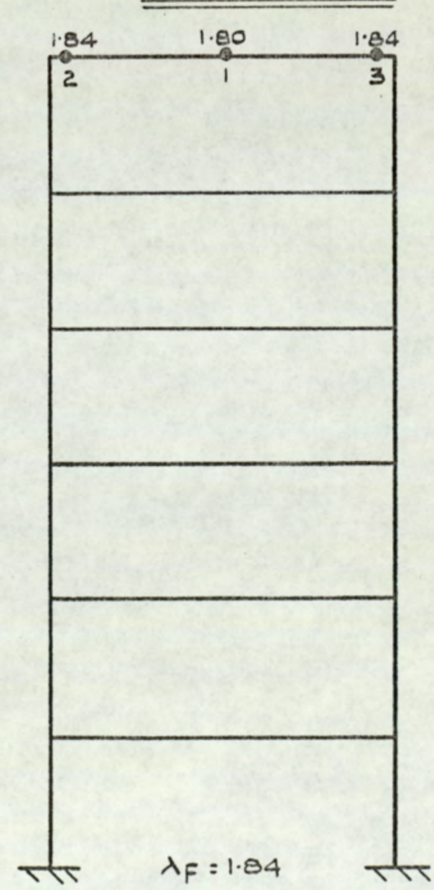
Frame 11 - Real sections - Combined load

FIGURE 66 (b)



Frame 10 - Real sections - Vertical load.

FIGURE 67 (a)



Frame 11 - Real sections - Vertical load

FIGURE 67 (b)

occur. However, this is to be expected, since the equations have been developed using a variety of approximations, whereas extremely sensitive techniques have been used in order to assess the degree of error inherent in their use.

In practice, when designed with the standard Universal Beams and Universal Columns, it has been shown that every frame is supplied with an additional reserve of strength, and that all the design criteria are satisfied. In general, it may be assumed that the collapse load factor of the majority of frames designed by the proposed method will lie approximately in the range $1.5 < \lambda_F < 1.6$ for combined loading, and $1.8 < \lambda_F < 2.0$ for vertical loading. Considering the approximate nature of the method, this is believed to be a very satisfactory result, and it is not considered to be necessary to modify the design equations further in order to compensate for the slight deficiencies which arise in theory.

In the following chapter, the economy of the proposed method is examined by comparison with certain frameworks which have been designed by other methods. It is shown that a competitive design is always obtained.

CHAPTER 8

DESIGN ECONOMY

8.1. INTRODUCTION

In the previous chapter, it has been shown that the proposed design method may be expected to produce an adequate set of beam and column sections for any regular framework. The resulting structure is not only safe, but is also extremely efficient, for it satisfies all the original design criteria, without having an excessive reserve of strength.

However, although the exhibition of "structural efficiency" is attractive, the popularity of any design method depends primarily on its ability to produce an economical design. This is the subject of the current chapter.

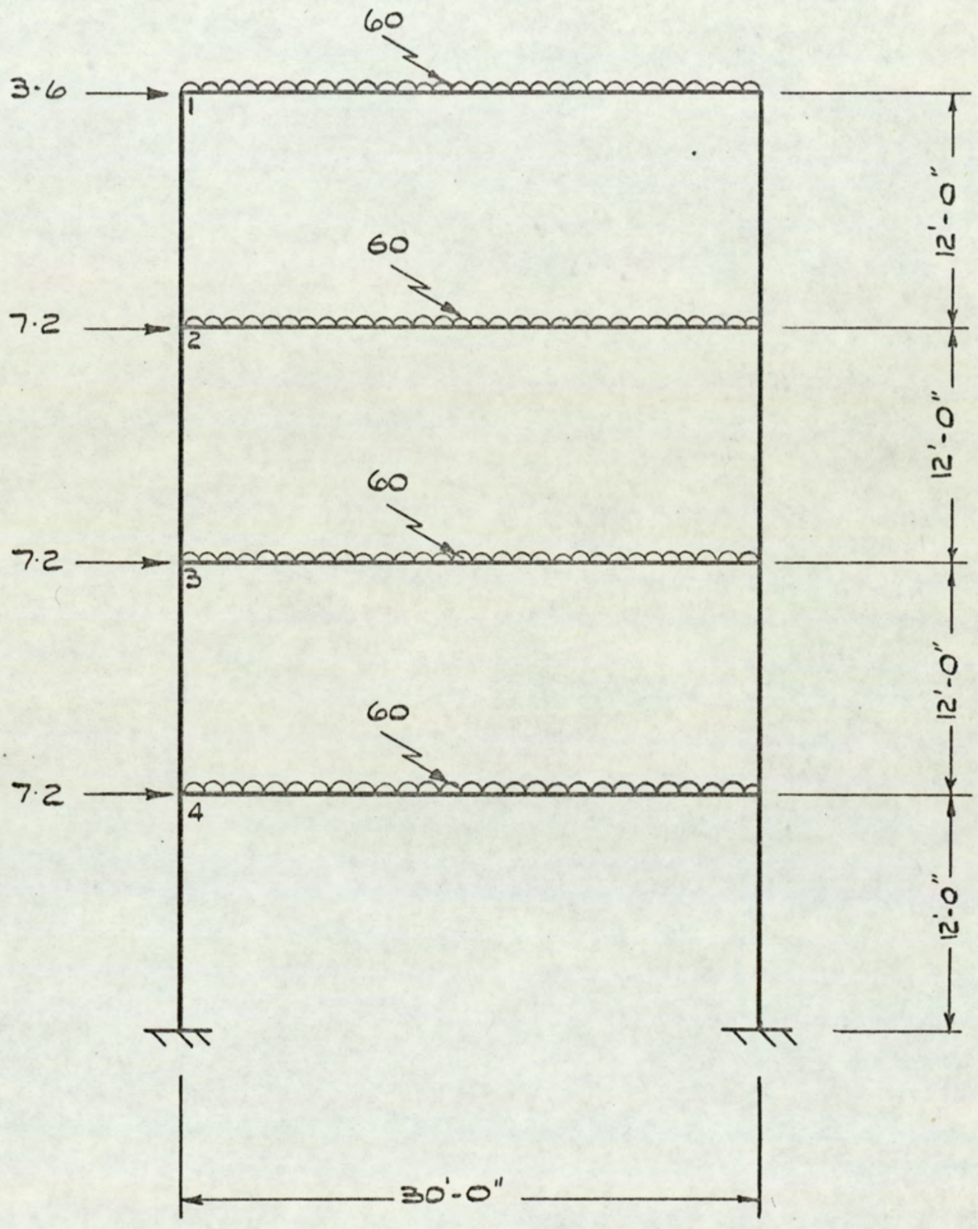
Four multi-storey frames have been designed by the proposed method, and in each case the sections obtained have been compared with those predicted by alternative methods. The design examples consist of one single-bay frame, a completely regular multi-bay frame, and two slightly irregular frames. Each of these is considered in turn in the following four sub-sections.

8.2. FOUR-STOREY, SINGLE-BAY FRAME

The first design example is the four-storey, single-bay frame which was used as one of the test frames in Chapter 7 (previously referred to as Frame 9). This frame was originally designed by Heyman[46], using the loads shown in Figure 68. Table 16 gives the sections selected for the frame using the proposed design method, the automatic elasto-plastic method devised by Majid and Anderson[50], and Heyman's method. The following observations may be made from the table:-

(1) The proposed method yields slightly larger beams in Storeys 3 and 4 than Majid and Anderson's method. Otherwise, the two designs

All loads in kips.



Four - storey, single - bay frame.

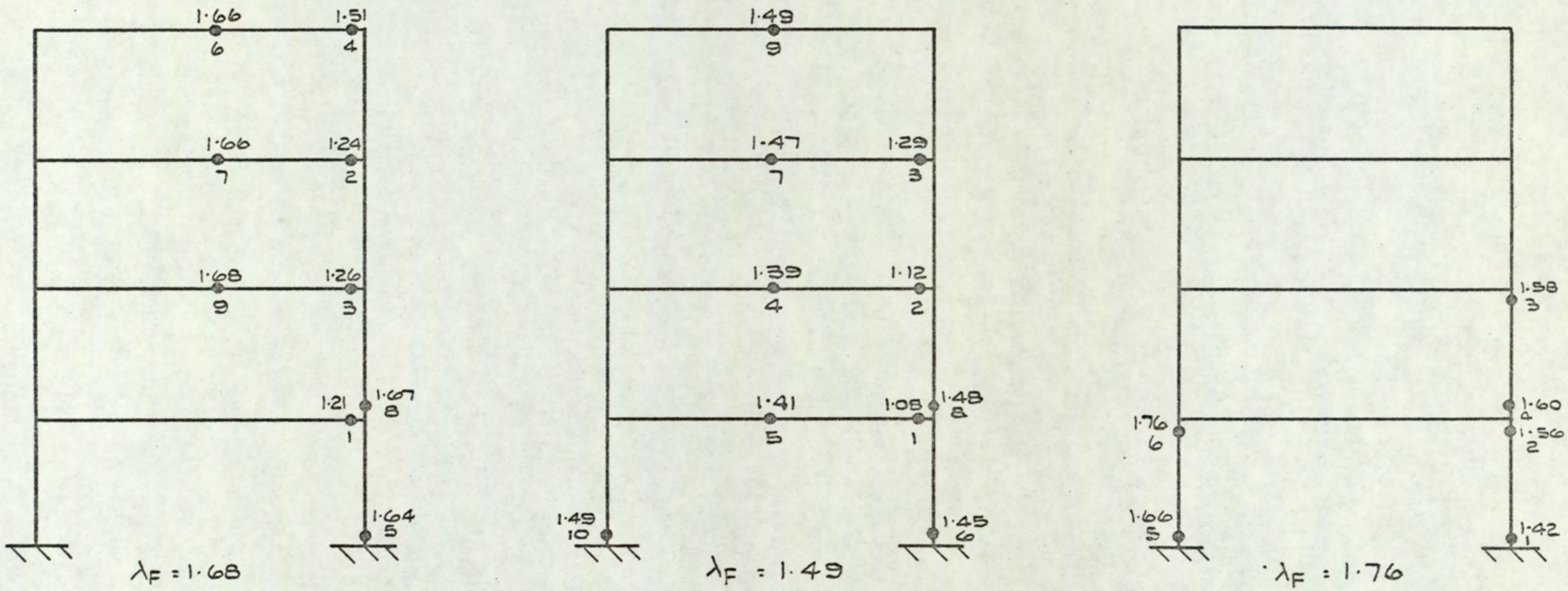
FIGURE 68

	Storey	Proposed design method		Majid and Anderson's method		Heyman's method	
		Section	Zp (in^3)	Section	Zp (in^3)	Section	Zp (in^3)
Beams	1	16 × 7 × 40	72.7	16 × 7 × 40	72.7	18 × 7.5 × 55	111.7
	2	16 × 7 × 40	72.7	16 × 7 × 40	72.7	18 × 7.5 × 55	111.7
	3	18 × 7.5 × 45	89.6	16 × 7 × 40	72.7	18 × 7.5 × 60	122.8
	4	18 × 7.5 × 45	89.6	16 × 7 × 40	72.7	18 × 7.5 × 60	122.8
Columns	1	10 × 10 × 60	75.0	10 × 10 × 60	75.0	10 × 10 × 60	75.0
	2	10 × 10 × 60	75.0	10 × 10 × 60	75.0	10 × 10 × 60	75.0
	3	10 × 10 × 60	75.0	10 × 10 × 60	75.0	10 × 10 × 60	75.0
	4	12 × 12 × 79	119.2	12 × 12 × 79	119.2	10 × 10 × 60	75.0
	Frame weight	5.07 Tons		4.91 Tons		5.64 Tons	

All sections are Universal Beams or Universal Columns

Four-storey, single-bay frame; selected sections.

TABLE 16



Proposed method
(a)

Majid and Anderson's method
(b)

Heyman's method
(c)

Four - storey, single - bay frame - combined load analyses for different designs

FIGURE 69.

are identical, and the difference in the weights of the two frames is less than 4%.

(2) Heyman's frame is approximately 12% heavier than that obtained by the proposed method, the beam sections being considerably larger in each storey.

Figure 69 shows the order of hinge formation for each of the designs under combined loading. It may be seen that every beam in Heyman's frame [Figure (c)] remains elastic up to the collapse load factor, $\lambda_F = 1.76$, which is considerably greater than the design load factor, $\lambda_2 = 1.4$. Although the collapse load factor for the Author's design [Figure (a)], $\lambda_F = 1.68$, is also quite high, there is a distribution of plasticity throughout the frame which compares very favourably with that shown for Majid and Anderson's frame [Figure (b)].

The vertical load analyses for these frames are not shown, since they add little to the discussion. Under this loading, the frame designed by the proposed method fails at $\lambda_F = 1.90$, due to the formation of a simple beam mechanism in the top storey. Majid and Anderson's frame fails in the same way, at $\lambda_F = 1.80$.

8.3. THIRTY-STOREY, FIVE-BAY FRAME

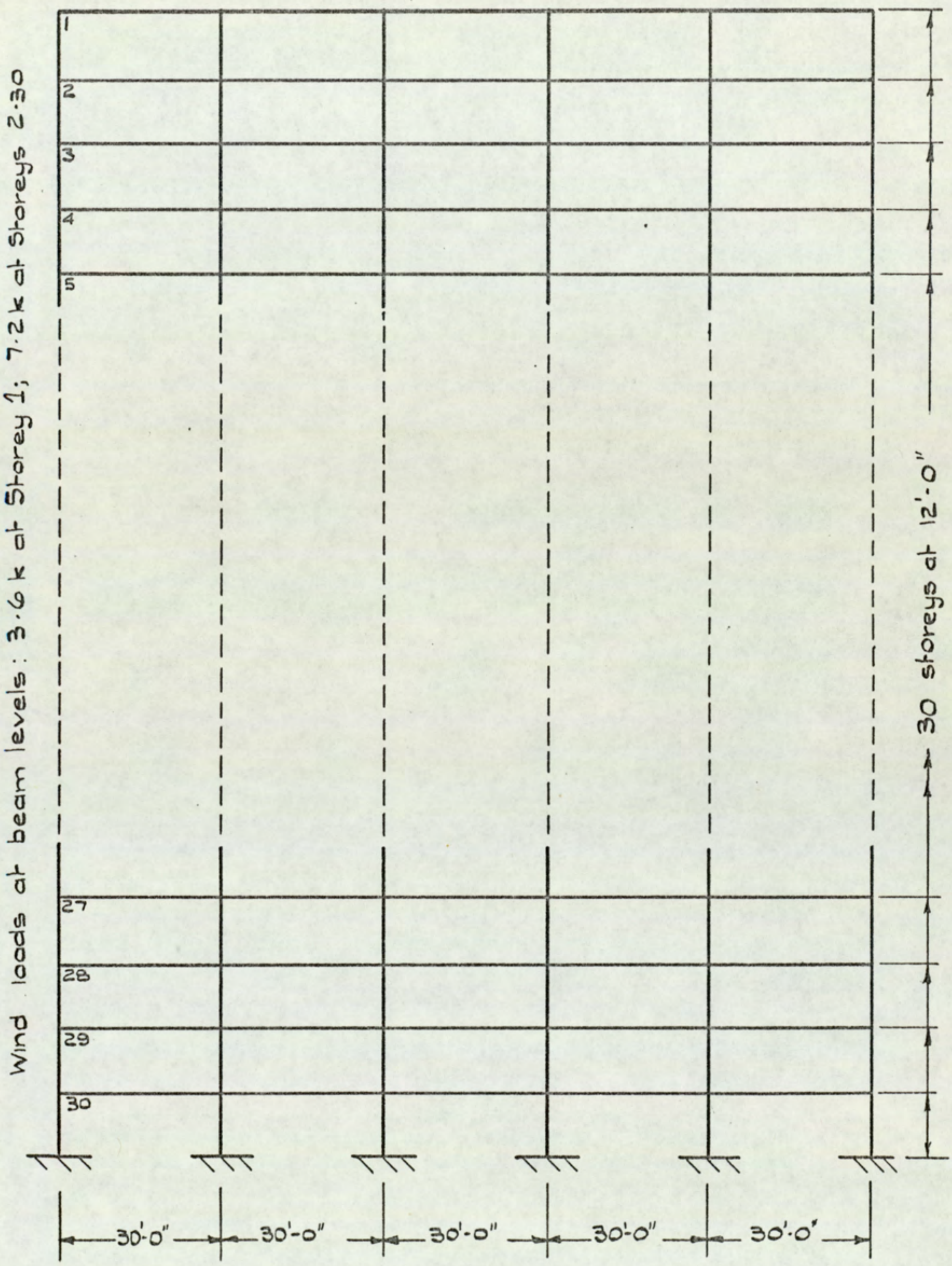
This large framework was also originally designed by Heyman(46), and the frame geometry and design loads are shown in Figure 70. As for the previous design example, these loads were devised from the following load intensities:-

Dead and live load = 0.100 kips per sq. ft.

Wind load = 0.030 kips per sq. ft.

In performing his design, Heyman calculated the column loads using the "live-load reduction" permitted in CP3(59). However, this reduction was in fact applied to the total dead load plus live load, and although this is an incorrect procedure, the same loads were assumed in the Author's

U. d. L. per bay on every floor = 60 K.



Thirty - storey, five - bay frame.

FIGURE 70

Storey	Beams (Universal Beams)		Internal columns (Universal Columns)		External columns (Universal Columns)	
	X	Y	X	Y	X	Y
1	16×7×40	16×7×45	8×8×35	10×10×60	10×10×60	10×10×60
2	"	"	"	"	"	"
3	"	"	8×8×48	"	"	"
4	"	"	"	"	"	"
5	"	"	10×10×60	"	"	"
6	"	18×7.5×50	"	"	"	"
7	"	"	12×12×79	12×12×79	"	"
8	18×7.5×45	"	"	"	"	"
9	"	"	14×14.5×103	12×12×92	10×10×72	"
10	"	"	"	"	"	"
11	"	"	14×14.5×119	12×12×106	"	"
12	18×7.5×50	"	"	14×14.5×119	"	"
13	"	"	14×14.5×136	"	12×12×79	12×12×79
14	"	"	"	14×14.5×136	"	"
15	21×8.25×55	"	14×16×158	"	12×12×92	"
16	"	"	"	14×16×158	"	"
17	"	18×7.5×55	"	"	14×14.5×103	"
18	"	"	"	14×16×202	"	12×12×92
19	"	"	14×16×193	"	14×14.5×119	"
20	"	"	"	"	"	"
21	21×8.25×62	18×7.5×60	"	"	"	12×12×106
22	"	"	"	"	"	"
23	"	"	14×16×228	"	14×14.5×136	"
24	"	"	"	14×16×264	"	14×14.5×119
25	24×9×68	"	"	"	14×16×158	"
26	"	21×8.25×62	"	"	"	"
27	"	"	14×16×264	"	"	"
28	"	"	"	"	"	14×14.5×136
29	"	"	14×16×370	"	14×16×193	"
30	18×7.5×45	"	"	"	"	"
Weight	105.2 Tons	107.7 Tons	101.7 Tons	100.4 Tons	33.0 Tons	27.8 Tons

X = Author's design; Y = Heyman's design;

Thirty-storey, five-bay frame; selected sections.

TABLE 17

design in order to provide a fair comparison between the two methods. Also, it was assumed that the columns were to be continuous over at least two storeys. The sections obtained by the two methods are given in Table 17.

The total weight of Heyman's frame is 235.9 Tons, as opposed to 239.9 Tons for the frame designed by the proposed method. These two weights are extremely close, although referring to Table 17, there is considerable variation between the sections selected by the two methods. Since the proposed method has been amply verified in the previous chapter, it may be seen that Heyman's method tends to overestimate the section sizes in the upper storeys, but underestimates them in the lower regions of the frame.

Unfortunately, it has not been possible to analyse these designs, since the size of the frame prohibits the use of the computer program.

8.4. EIGHT-STOREY, TWO-BAY FRAME

The geometry and design loads for this frame are shown in Figure 71. In order to allow for the variable bay widths, a slightly modified procedure was used to obtain an economical set of set of sections:-

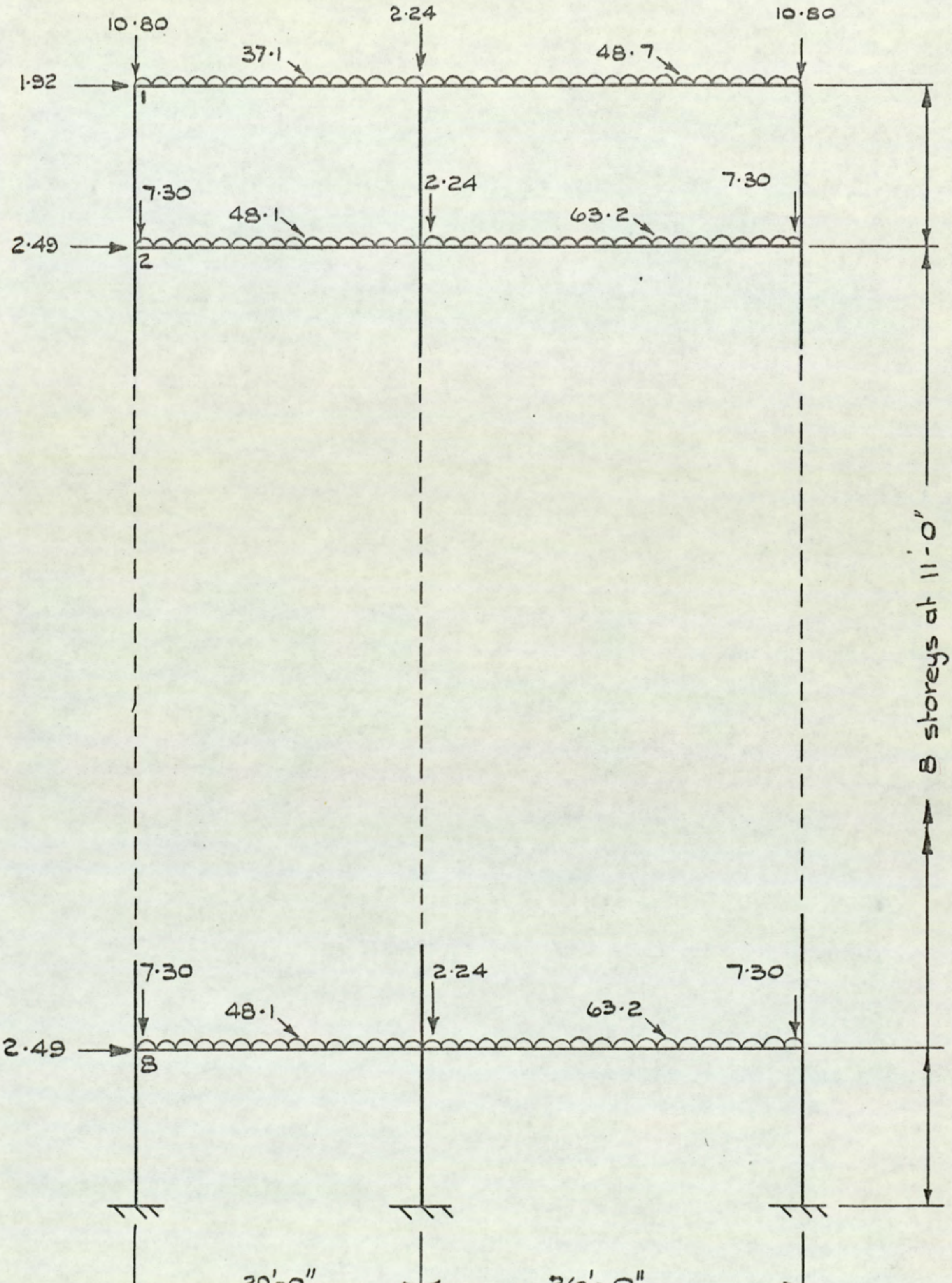
8.4(a) MODIFIED DESIGN PROCEDURE

(a) Calculation of the initial design loads:-

(1) The shear force in the internal column of each storey was assumed to be equal to half the total storey shear, in the normal way.

(2) The axial force in each internal column was assumed to be equal to the force in the column above, plus half the loads on the adjoining beams, plus any additional axial force at the joint.

(3) The axial force in the right-hand external column was assumed to be equal to the load in the column above, plus half the load on the 26ft. beam, plus any axial load at the joint, plus a component due to wind loading. This wind loading component was derived with the wind acting from the left, as shown in Figure 71, and was based on the total storey shear and the total span of the frame.



All loads in kips

Eight - storey, two - bay frame, with unequal bays

FIGURE 71.

(4) The axial force in the left-hand external column was based on the load applied to the 20ft. beam. In this case, it was assumed that the wind was acting from the right, the wind load component having the same value as that calculated in (3).

(b) Design based on the larger of the two bays:-

Using the forces obtained in (1), (2) and (3) above, the frame was designed by the standard procedure, assuming both bays to be 26ft. wide, and with the wind loading acting from the left.

(c) Design based on the smaller of the two bays:-

Using the forces obtained in (1), (2) and (4), the frame was designed assuming both bays to be 20ft. wide, and with the wind loading acting from the right.

(d) Selection of the sections:-

(1) The sections selected in (b) for the beams and external columns were adopted for the 26ft. beams and the right-hand external columns of the real frame shown in Figure 71.

(2) The sections selected in (c) for the beams and external columns were adopted for the 20ft. beams and the left-hand external columns of the real frame.

(3) In any storey, the larger of the two internal columns given by (b) and (c) was selected for the real frame.

8.4(b) DESIGN AND ANALYSIS RESULTS

The design loads shown in Figure 71 were taken from the B.C.S.A. Publication No. 16[57], which uses this frame to demonstrate the elastic design procedure, in accordance with B.S.S.No. 449[2]. In the B.C.S.A. design it was assumed that the columns were continuous over four storeys, and so the same procedure was adopted when redesigning by the proposed method. The frame has also been designed by the elasto-plastic method of Majid and Anderson[60], and the results of this design are given in Table 18, together with the sections obtained by the proposed method.

Storey	Left-hand beams		Right-hand beams		Left-hand external columns		Internal columns		Right-hand external columns	
	X	Y	X	Y	X	Y	X	Y	X	Y
1	14×5×22	14×5×22	14×6.75×30	16×5.5×31	8×8×40	8×8×35	10×10×49	10×10×49	10×10×49	8×8×48
2	16×5.5×26	16×5.5×26	16×7×36	16×7×36	"	"	"	"	"	"
3	"	"	"	"	"	"	"	"	"	"
4	"	"	"	"	"	"	"	"	"	"
5	"	"	"	"	8×8×58	10×10×60	14×14.5×119	12×12×92	12×12×79	10×10×72
6	"	"	"	"	"	"	"	"	"	"
7	16×5.5×31	"	16×7×40	"	"	"	"	"	"	"
8	16×5.5×26	"	16×7×36	"	"	"	"	"	"	"
Weight	1.87 Tons	1.82 Tons	3.32 Tons	3.27 Tons	1.92 Tons	1.87 Tons	3.30 Tons	2.72 Tons	2.51 Tons	2.35 Tons

All sections are Universal Beams or Universal Columns.

X = Author's design; Y = Majid and Anderson's design;

Eight-storey, two-bay frame; selected sections.

TABLE 18

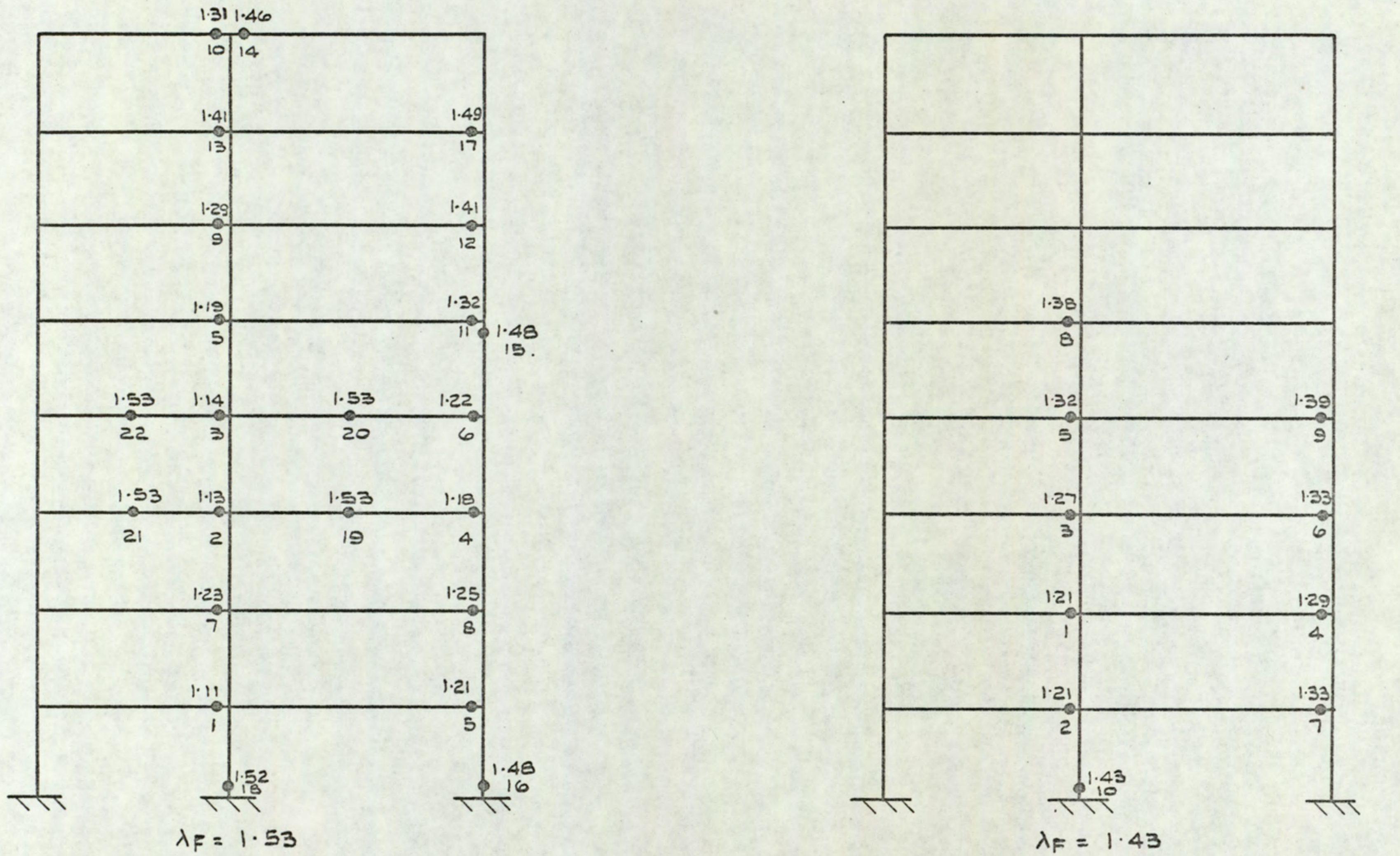
It may be seen that the two designs are very similar, the main difference in weight occurring in the internal columns of the lower four storeys. The total weight of the Author's frame is 12.92 Tons. Majid and Anderson's frame which weighs 12.03 Tons, is approximately 7% lighter. In contrast, the elastic B.C.S.A. design yields a weight of 15.12 Tons, approximately 17% greater than that obtained by the proposed method, despite the fact that this elastic method allows for the "live-load reduction" in the design of the columns. Thus, the proposed method leads to a frame which is not as economical as that designed by the accurate computer method, as one would expect, but which is considerably lighter than the frame designed by the traditional elastic approach.

Although the main purpose of this chapter is to assess the economy of the design method by direct comparison of design examples, it was considered to be adviseable to analyse this particular frame in detail, since a modified design approach was used to allow for the unequal bay widths. The analysis results, together with those for Majid and Anderson's design, are given in Figures 72, 73 and 74, and these will be considered in turn.

Figure 72 shows the analysis of the two different designs for the case of combined loading with the wind acting from the left. It may be seen that the Author's frame [Figure (a)] satisfies all the design criteria for this loading case. Failure occurs at $\lambda_F = 1.53$, with an extensive distribution of plasticity throughout the frame.

Figure 73 shows the corresponding analyses with the wind acting from the right. Again, the hinge formation in the frame designed by the proposed method is very similar to before, failure occurring in this case at a slightly lower load factor. All the design criteria are satisfied. Majid and Anderson's frame fails at $\lambda_F = 1.40$, with the formation of a hinge at the base of the internal column.

The analyses under vertical load are shown in Figure 74. In the Author's frame, a simple beam mechanism occurs in the top storey at λ_F =

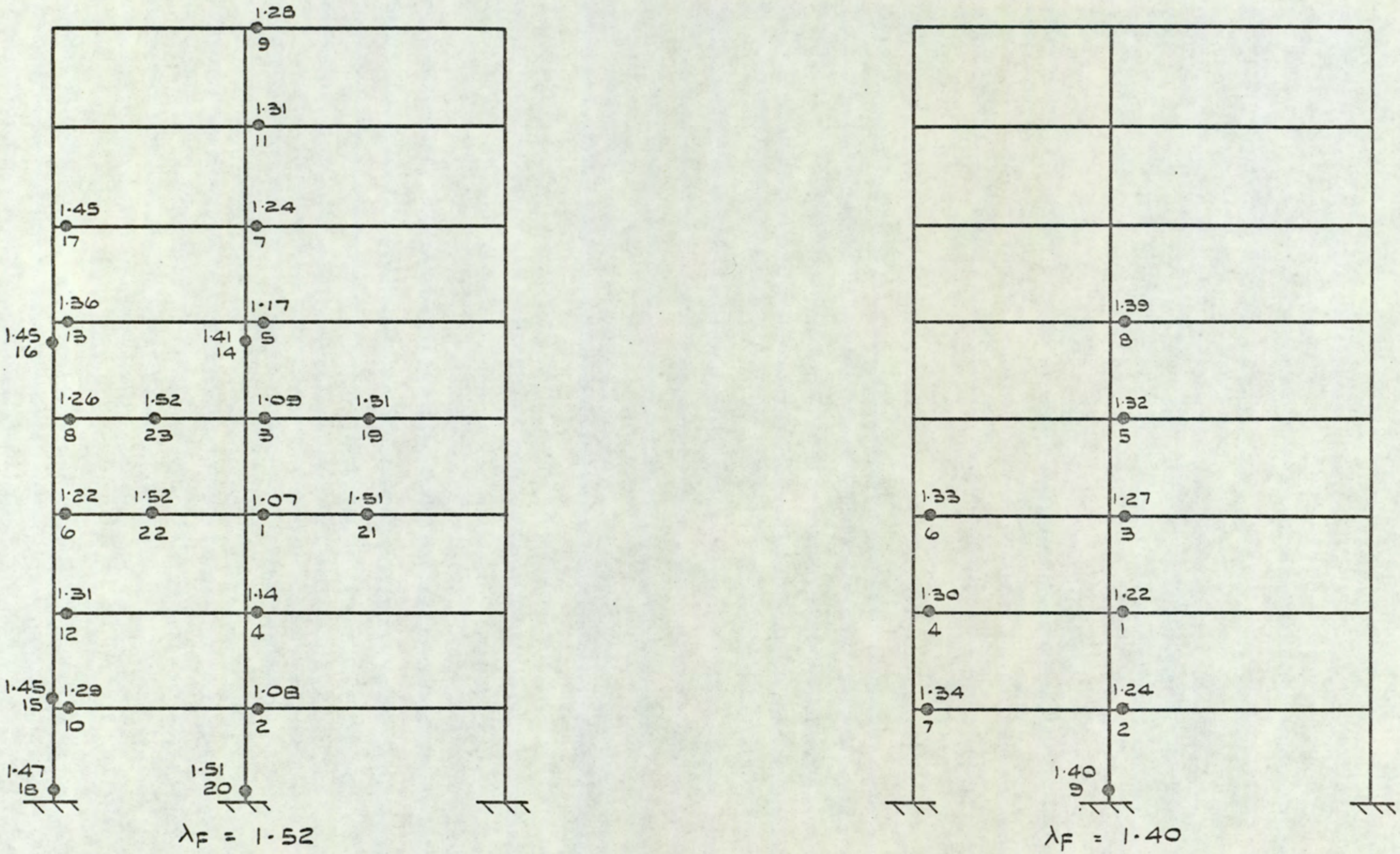


(a) Proposed method.

(b) Majid and Anderson's method

Combined load analyses – wind from left.

FIGURE 72

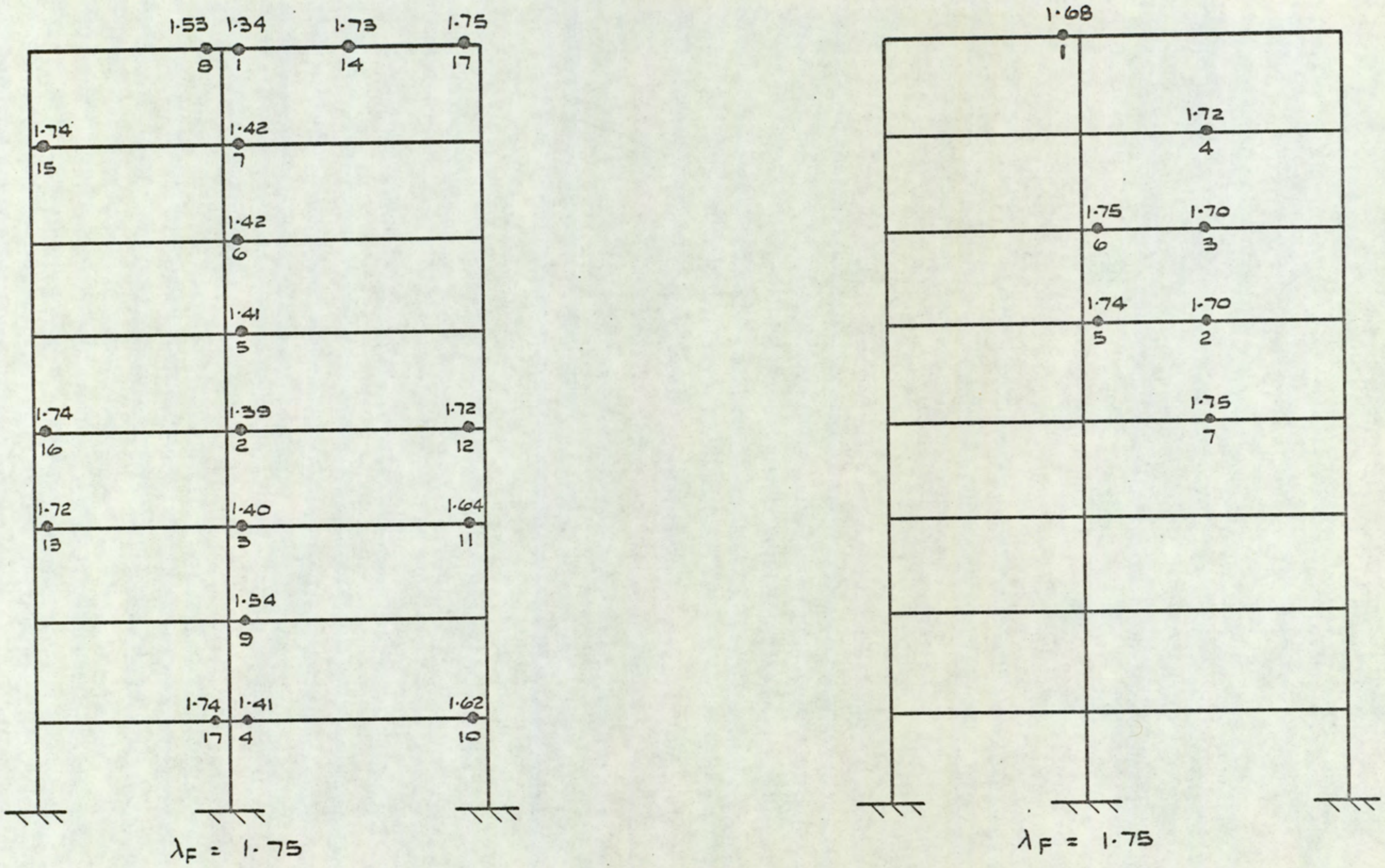


(a) Proposed method.

(b) Majid and Anderson's method

Combined load analyses - wind from right

FIGURE 73



(a) Proposed method

(b) Majid and Anderson's method

Vertical load analyses

FIGURE 74

1.75, and all the design criteria are satisfied. Majid and Anderson's frame also fails at $\lambda_F = 1.75$, but without the formation of a simple mechanism. The axial load effects in the comparatively light column sections of this frame encourage instability.

8.5 TEN-STOREY, THREE-BAY FRAME

The final example is a ten-storey, three-bay frame with unequal bay widths, which has previously been designed as part of the research programme at the University of Lehigh(61). The original frame specification is shown in Figure 75.

In order to be able to make a valid comparison with the Lehigh design, and also to be able to analyse the resulting structures using the computer program, it was necessary to make considerable alterations to the design loads. This is the subject of the following sub-section.

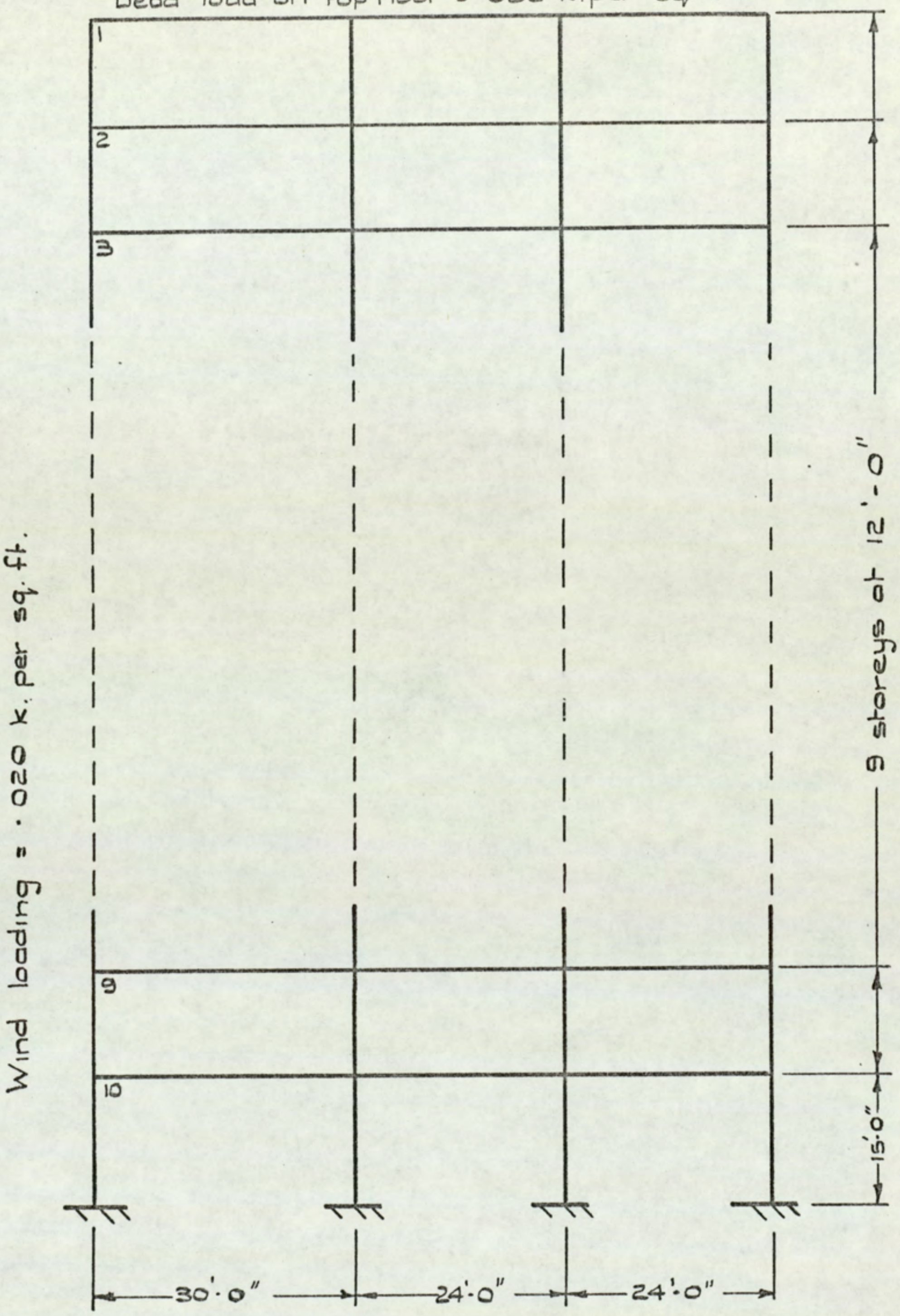
8.5(a) DEVELOPMENT OF AN EQUIVALENT FRAMEWORK

Figure 76(a) shows the beam loads and column axial forces which were obtained in the Lehigh design. These loads were derived from the initial frame specification in Figure 75, using the allowable "live-load reductions" permitted for the design of both beams and columns in the American code of practice, ASA A58.1(62).

Due to these live-load reductions, it may be seen that the assumed column loads in Figure 76(a) are no longer in equilibrium with the assumed beam loads. For example, in the top storey, the sum of the column loads is 180.5 kips, whereas the total beam load is only 168 kips. Thus, these assumed design loads could only be obtained simultaneously in an analysis if additional vertical loads were applied at the joints. Such an equivalent system is shown in Figure 76(b).

In deriving this equivalent frame, the spans have all been reduced in order to simulate the design conditions further, since, in the Lehigh

live load on top floor = ·030 k. per sq. ft.
Dead load on top floor = ·060 k. per sq. ft



On all floors except the top [Storeys 2 to 10]:-

Dead load = ·080 k. per sq. ft.

Live load = ·080 k. per sq. ft.

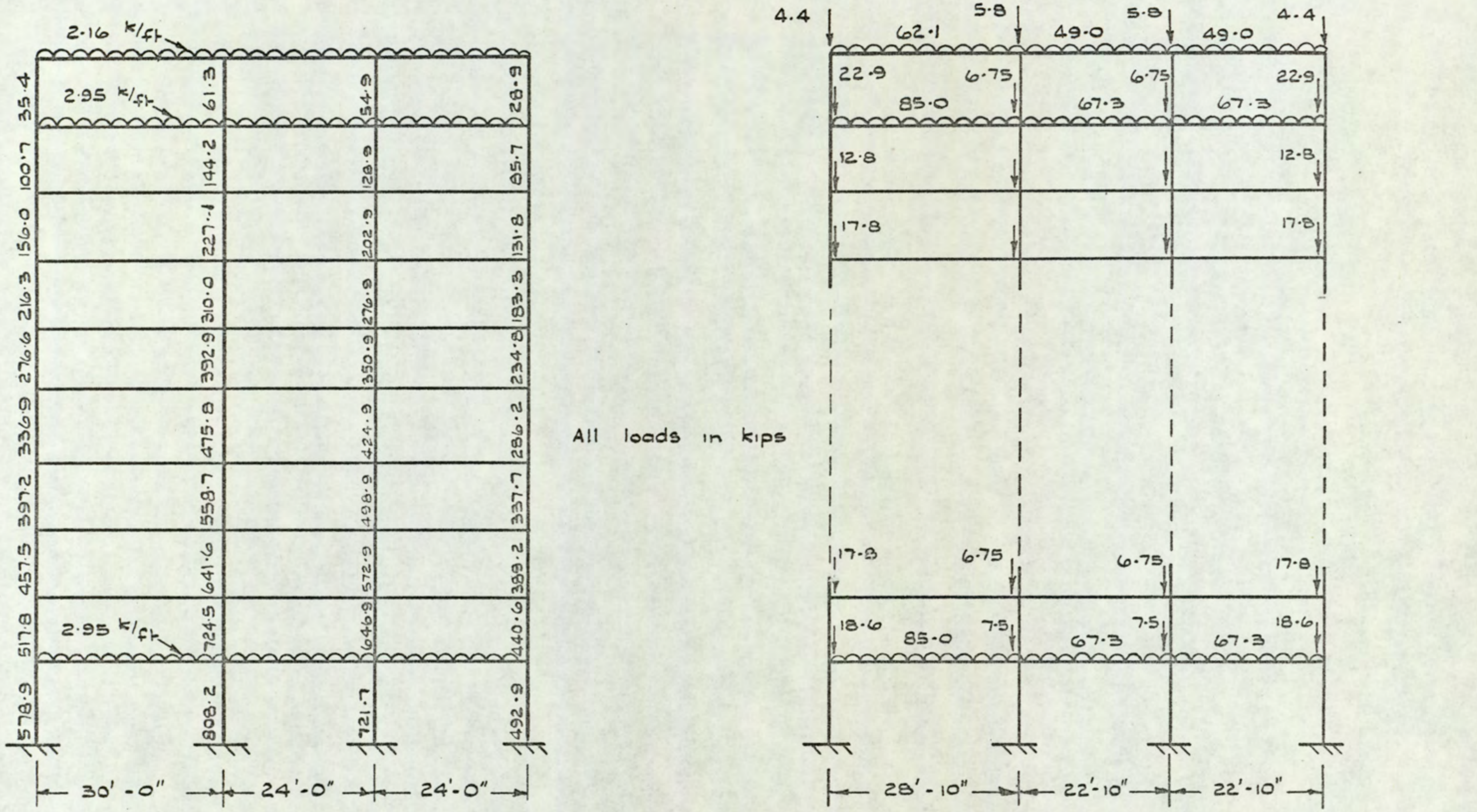
Frame spacing = 24'-0"

Dead weight of external walls = ·045 k. per sq. ft.

Self weight of columns = ·025 k. per ft.

Ten-storey, three-bay frame; original specification.

FIGURE 75.



(a) Beam and column working loads.

(b) Equivalent loading system, which yields the same Column loads as those given in (a); spans are the maximum beam lengths in which plastic hinges may form.

Ten - storey, three - bay frame; equivalent loading systems for the Lehigh design.

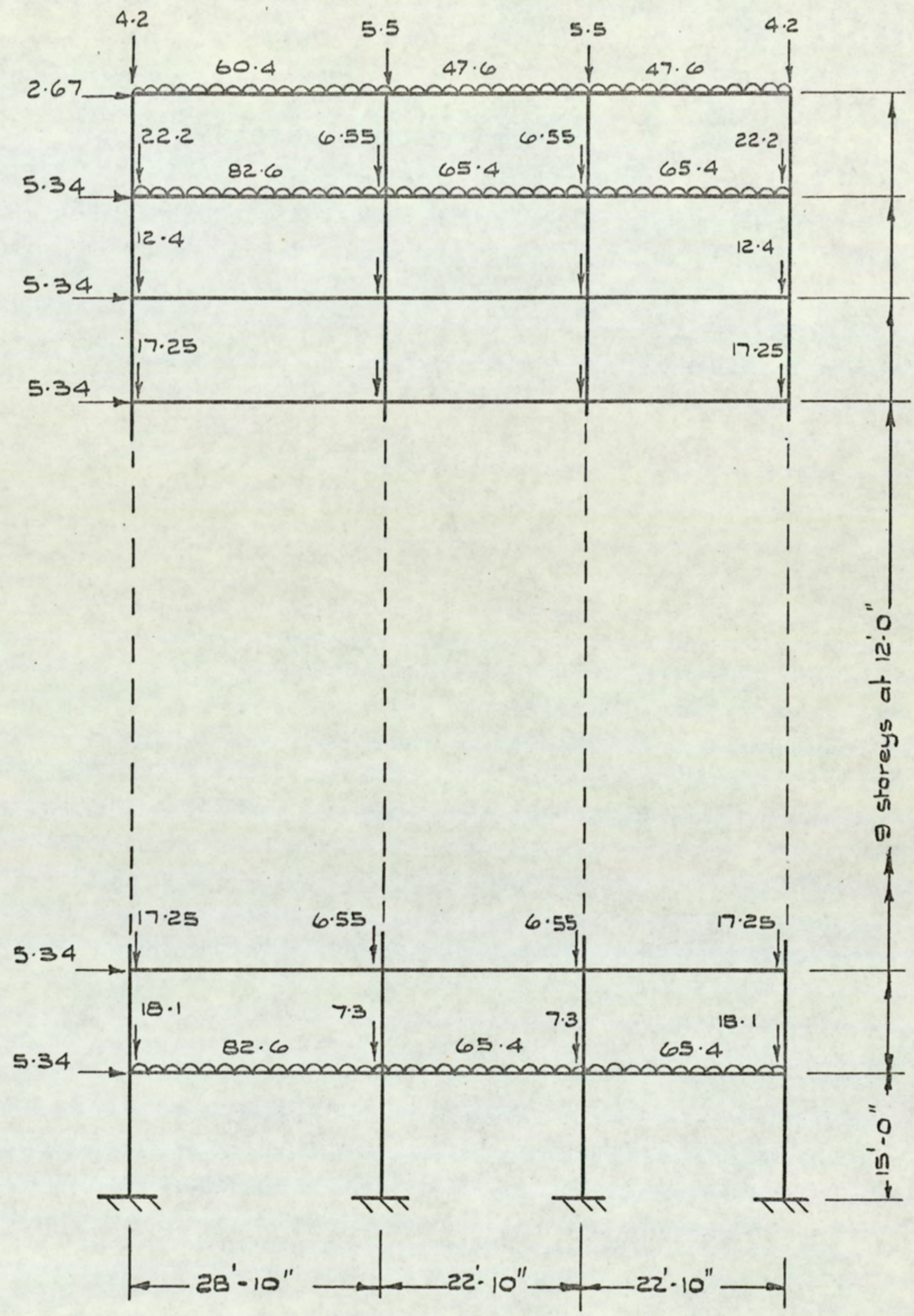
FIGURE 76.

design, it was assumed that plastic hinges could only form in the beams between the column faces, and that every column was 14 in. deep. Each beam load shown in Figure 76(b) was calculated by multiplying the reduced span by the uniformly distributed load given in Figure 76(a). Appropriate vertical loads were then added at the joints so that the column loads obtained from Figure 76(b) were identical to those originally assumed in Figure 76(a). The equivalent frame not only gives identical working loads to those assumed in the Lehigh design, but may also be analysed to give a valid check on the accuracy of this design.

However, in order to redesign the frame using the proposed method, it was necessary to make a further modification to the equivalent frame in Figure 76(b), since the Lehigh design was based on the load factors $\lambda_1 = 1.70$ and $\lambda_2 = 1.30$. The two methods obviously had to be compared using frameworks designed for the same ultimate loads rather than the same working loads. Therefore, a new set of working loads was required for the proposed method, such that when factored by $\lambda_1 = 1.75$ and $\lambda_2 = 1.40$, they would give the same ultimate loads as before.

The modified working loads which were adopted for the proposed method are shown in Figure 77. The vertical loads were obtained by multiplying the corresponding loads in Figure 76(b) by the factor $\frac{1.70}{1.75}$. The horizontal loads were obtained by multiplying those used in the Lehigh design by $\frac{1.30}{1.40}$.

These two equivalent loading systems for the two designs do not correspond exactly, although the error is very small. Under vertical load alone, identical ultimate loads are obtained from either system using the appropriate value of λ_1. However, under combined load at λ_2, only the ultimate wind loads are identical, the vertical load component assumed for the proposed method being about 5% greater than that for the Lehigh method. It is not possible to obtain identical sets of ultimate loads for both loading cases, since the ratio $\frac{\lambda_1}{\lambda_2}$ is different in the two designs. However, for comparison purposes, the system shown in Figure 77 is



All loads in kips

Ten-storey, three-bay frame; loadings and beam lengths to be used for comparison with the Lehigh design.

FIGURE 77

conservative, in that the proposed design is based on a slightly more severe set of loads.

8.5(b) MODIFIED DESIGN PROCEDURE

As in the previous example described in 8.4, a modified design procedure was adopted in order to allow for the unequal bay widths. This is summarized below:-

(1) The initial design loads were calculated from Figure 77. Each internal column was assumed to carry one-third of the total storey shear. The axial loads were calculated as before, each column carrying the load in the column above, plus half the loads on the adjoining beams, plus any additional axial force at the joint. Equal components due to wind loading were added to the forces in the two external columns.

(2) The frame was designed using the standard procedure, with the wind loading acting from the right, assuming each bay to be 28ft. 10in. wide. The column sections were selected using the axial forces obtained in (1) for the left-hand internal and external columns.

(3) The frame was redesigned with the wind loading acting from the left, assuming each bay to be 22ft. 10in. wide, and using the axial forces obtained in (1) for the right-hand internal and external columns.

(4) The sections obtained in (2) were adopted for the longer beams and the left-hand internal and external columns.

(5) The sections obtained in (3) were adopted for the shorter beams and the right-hand internal and external columns.

8.5(c) DESIGN AND ANALYSIS RESULTS

The Lehigh design was performed using the American range of WF sections, which are similar in form to the standard range of Universal Beams and Universal Columns. These wide flange sections are considerably more economical than the British sections, since they are far more numerous, and accordingly less "widely spaced". Therefore, they were also used in the Author's design in order to give a closer comparison with the Lehigh method. Also, in both designs, the columns were considered to be continuous over two storeys. The sections obtained using the two different approaches are given in Table 19.

The two designs may be seen to give similar results, the proposed method yielding lighter beams but heavier columns. The total weight of the Lehigh frame is 33.2 Tons, which is approximately 5% less than that of the Author's frame, which was designed for a slightly more critical set of loads, and which weighs 34.9 Tons. At first sight, therefore, it would appear that the Lehigh design is slightly more economical. However, as will be shown subsequently, this design is inadequate, failure occurring under vertical load alone below the design load factor.

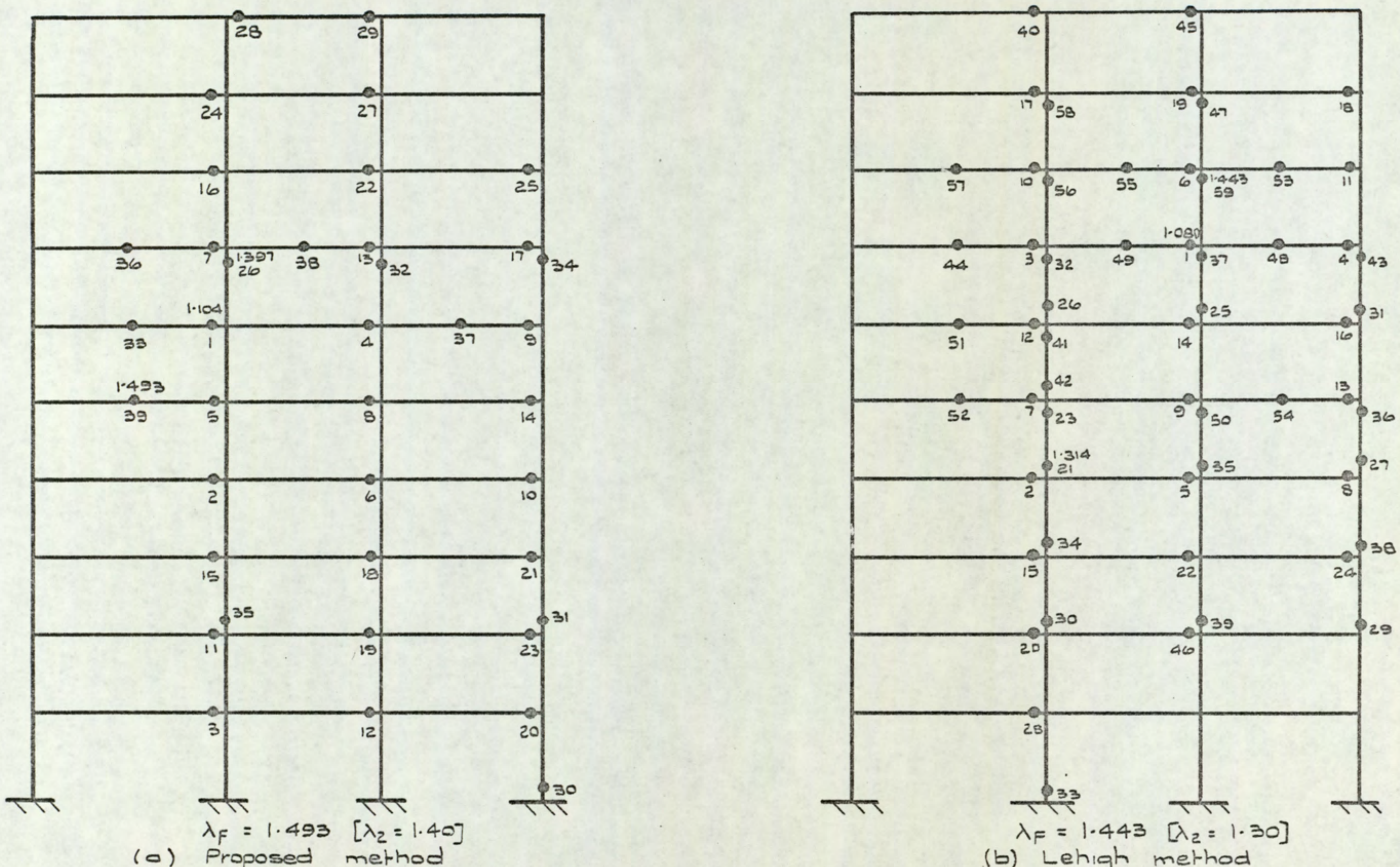
The analysis results for the two different designs are given in Figures 78, 79 and 80. Under combined loading, with the wind acting from the left, the frame designed by the proposed method fails at $\lambda_F = 1.493$, as shown in Figure 78(a). The first beam hinge forms above the working load, at $\lambda = 1.104$. The first column hinge forms at $\lambda = 1.397$, which is fractionally below the design load factor, $\lambda_2 = 1.40$, so that the third design criterion is not quite satisfied. This early column hinge occurs due to the fact that the design procedure neglected the bending moments produced in the internal columns by the vertical loading. However, the error is very small, and the overall strength of the framework is not seriously affected. Figure 78(b) shows the corresponding analysis for the Lehigh design. In this case, failure occurs at $\lambda_F = 1.443$, which is above the design load factor, $\lambda_2 = 1.30$. Comparing Figures 78(a) and 78(b), it may be seen that the hinge patterns obtained for the two designs are quite different, a large number of plastic hinges forming in the columns of the Lehigh frame before collapse occurs.

Storey	30ft. Beams		24ft. Beams		Left-hand external column		Left-hand internal column		Right-hand internal column		Right-hand external column.	
	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y
1	16WF40	16WF40	16 B 26	16 B 26	14WF43	14WF43	10WF39	12WF40	8WF28	8WF24	10WF39	12WF40
2	18WF45	18WF45	16WF36	14WF34	"	"	"	"	"	"	"	"
3	"	"	"	"	14WF61	14WF53	10WF66	14WF61	12WF58	14WF48	14WF48	14WF43
4	"	"	"	"	"	"	"	"	"	"	"	"
5	"	18WF50	"	16WF40	14WF84	14WF74	12WF106	14WF84	12WF99	14WF84	14WF68	14WF61
6	18WF50	"	16WF40	"	"	"	"	"	"	"	"	"
7	"	"	"	"	14WF111	14WF111	14WF127	14WF119	14WF127	14WF119	14WF84	14WF84
8	21WF55	18WF55	18WF45	18WF55	"	"	"	"	"	"	"	"
9	"	21WF55	"	21WF55	14WF127	14WF136	14WF219	14WF142	14WF219	14WF142	14WF111	14WF136
10	18WF50	"	16WF40	"	"	"	"	"	"	"	"	"
Weight	6.20 T	6.32 T	7.80 T	8.46 T	4.76 T	4.68 T	6.28 T	4.98 T	5.99 T	4.67 T	3.91 T	4.10 T

X = Author's design; Y = Lehigh design;

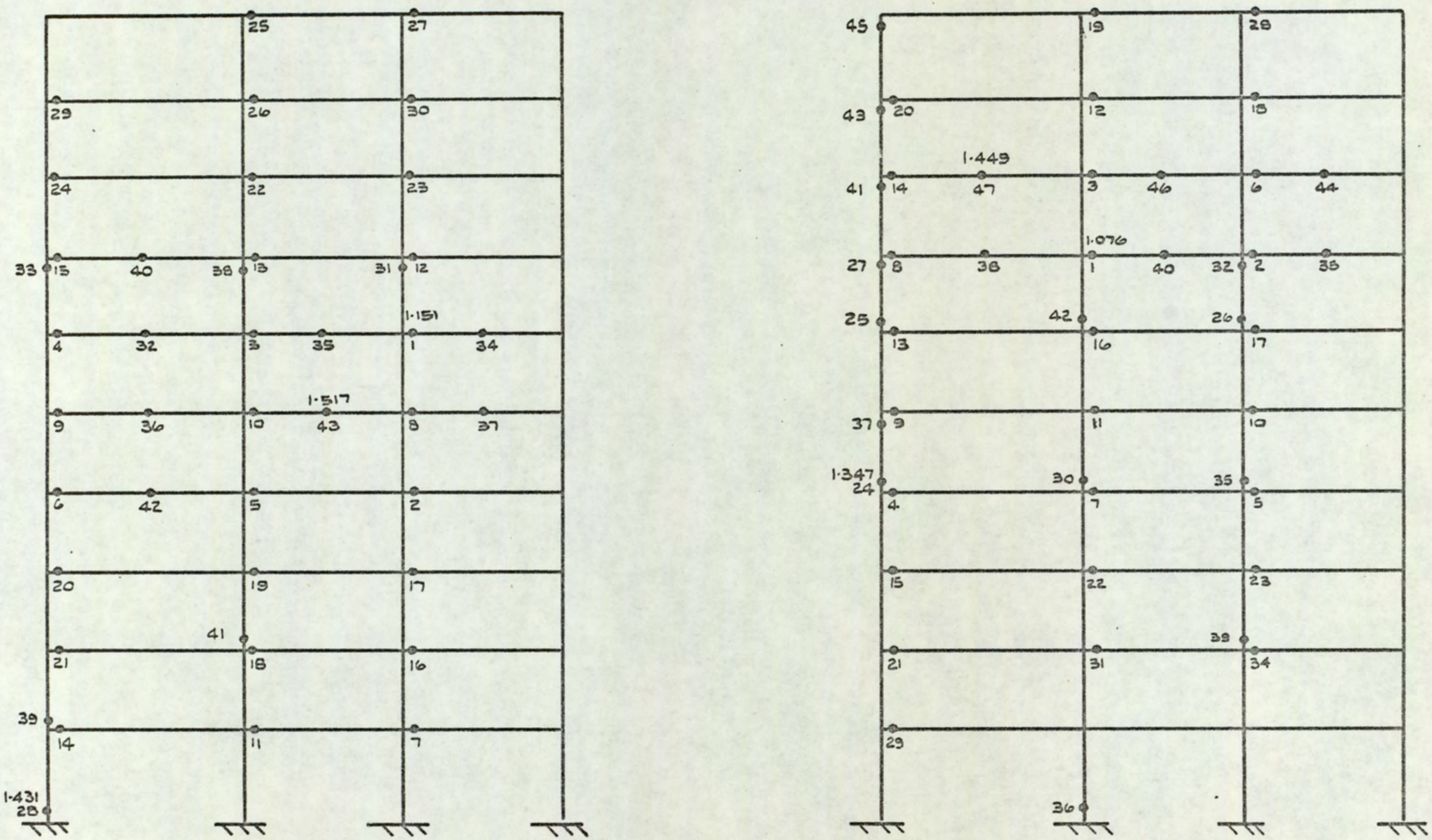
Ten-storey, three-bay frame; selected sections.

TABLE 19



Combined load analysis - wind from left.

FIGURE 78.

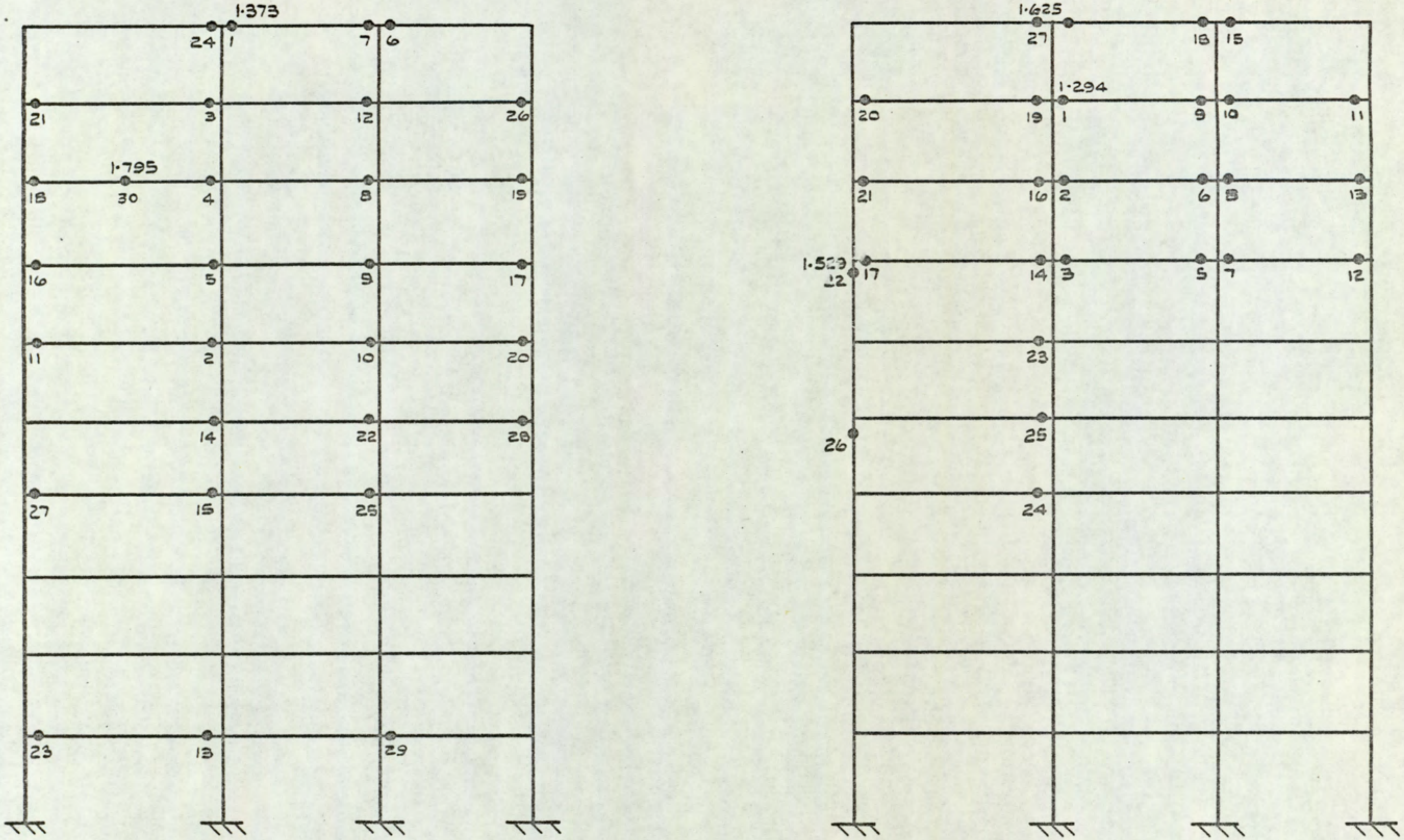


$\lambda_F = 1{\cdot}517$ [$\lambda_2 = 1{\cdot}40$]
(a) Proposed method

$\lambda_F = 1{\cdot}449$ [$\lambda_2 = 1{\cdot}30$]
(b) Lehigh method.

Combined load analyses - wind from right

FIGURE 79



$\lambda_F = 1{\cdot}795$ [$\lambda_1 = 1{\cdot}75$]

(a) Proposed method.

$\lambda_F = 1{\cdot}625$ [$\lambda_1 = 1{\cdot}70$]

(b) Lehigh method

Vertical load analyses.

FIGURE 80

Figure 79 shows the analysis results for combined loading with the wind acting from the right. Under this loading system, the Author's frame satisfies all the design criteria, failure occurring at λ_F = 1.517. The Lehigh frame is also safe, failure occurring at λ_F = 1.449 with fewer hinges than were required for collapse under the previous loading system.

Figure 80(a) gives the vertical load analysis for the Author's frame, and indicates that the design is adequate. The collapse load factor is given by λ_F = 1.795, which is greater than the design load factor, λ_1 = 1.75. Despite the fact that sway deflections are induced due to the unsymmetry of the structure, failure still occurs due to the formation of a simple beam mechanism.

In contrast, the Lehigh frame fails at λ_F = 1.625, considerably below the design load factor, λ_1 = 1.70, as shown in Figure 80(b). The previous analyses under combined loading indicate that the columns of this frame are comparatively weak, and this is confirmed by the vertical loading analysis. Failure occurs due to overall frame instability, and this results directly from the selection of a set of column sections which are considerably smaller than those predicted by the proposed method.

8.6. SUMMARY

The following conclusions may be made from the results presented in this chapter:-

(1) The proposed design method leads to a far more economical framework than the traditional elastic approach, as one would expect.

(2) For frames in which the instability effects are not predominant, the proposed method produces a considerably lighter framework than Heyman's ultimate load approach. In large frameworks, Heyman's method is liable to underestimate the required section sizes, since no specific allowance is made for the possibility of

overall frame instability. [Heyman assumed that by ensuring that the frame was fully elastic at working load, there would be an adequate reserve of strength, and that the danger of instability would be further reduced by the stiffening effect of the cladding.] In such cases, the proposed method may be used to give a safe and economical design.

(3) Majid and Anderson's computer method is always likely to yield a lighter structure, since, by iterative use of an accurate analysis procedure, the design may be continued until all the design criteria are just satisfied. In contrast, the proposed method depends on a variety of approximations, and an accurate analysis of the complete frame is not inherent in its use. Nevertheless, it has been shown that the two approaches lead to similar sets of sections, and in general, it is unlikely that the weights obtained by the two methods for any regular framework will differ by more than 10%. This difference is suprisingly low, considering that Majid and Anderson's method is extremely economical.

(4) Any frame designed by the Lehigh method must be regarded with suspicion, since there is a likelihood that it will fail under vertical load alone before the design load factor is attained. However, by selecting slightly heavier columns, the proposed method yields a safe design, with a frame weight which is only slightly greater.

(5) The proposed method may also be applied to frames in which the bay widths vary, using modifications similar to those described for the final two design examples. However, since little work has beem done on this subject, no specific design procedure has been advocated for a general unsymmetrical framework, this being left to the discretion of the designer. The development of such a general method would of course be a logical extension of the work presented in this thesis.

In conclusion, it is believed that the proposed method is the only "hand" design procedure available which will consistently produce such an economical design without any serious violation of the basic design criteria. A lighter frame may only be designed with confidence if an accurate elasto-plastic analysis is used to check the adequacy of the structure.

However, the selection of an economical set of sections implies that considerable sway deflections may occur in the framework due to wind loading. The calculation of these deflections is the subject of the following chapter.

CHAPTER 9

DEFLECTIONS

9.1. INTRODUCTION

In the previous two chapters, it has been shown that the proposed design method produces an economical structure which satisfies all the original design criteria. However, this method has been evolved from consideration of the behaviour of the framework at ultimate load, and although steps have been taken to ensure that the frame is completely elastic at working load, there is a possibility that excessive working load deflections may occur.

Large deformations give rise to many minor problems in the maintenance of tall buildings. Plaster ceilings may be damaged by vertical deflections of the beams, and window frames and partitions tend to distort due to shear deformation. Furthermore, particularly in unbraced frames, considerable discomfort may be caused to the occupants of a building by excessive sway deflections.

In the following section, a method is derived for calculating the sway deflections at working load, and this is subsequently checked by comparison with the accurate computer analysis. Later in the chapter, the current opinion on the selection of suitable deflection limits is reviewed, and the deflections obtained for the frames which have been designed by the proposed method are examined in the light of this discussion.

9.2. DEFLECTION CALCULATIONS

The development of an equation for estimating the sway deflections may be based entirely on elastic methods of analysis, since it has been shown that no plastic hinges form below the working load in any frame designed by the proposed method. An approximate, but reasonably accurate deflection formula may be obtained using the "equivalent column" technique, as described by Lightfoot[55].

The equivalent column for a typical frame is shown in Figure 81, and it is derived by summing the flexural rigidities of the beams and columns in every storey of the real frame. The Euler ratio for each storey is assumed to equal the average of the Euler ratios for all the columns of that storey, and the shear forces shown in the figure are the total storey shears at working load.

The generalized slope-deflection equations for Storey (j) are given by:-

$$M_{(j,j+1)} = K_{c(j)}\left\{s_{(j)}\theta_{(j)} + (sc)_{(j)}\theta_{(j+1)} - [s(1+c)]_{(j)}\phi_{(j)}\right\} \quad (104)$$

and,

$$M_{(j+1,j)} = K_{c(j)}\left\{(sc)_{(j)}\theta_{(j)} + s_{(j)}\theta_{(j+1)} - [s(1+c)]_{(j)}\phi_{(j)}\right\} \quad (105)$$

where $\phi_{(j)}$ is the deflection index [i.e. the relative horizontal displacement at the ends of the column, $\Delta_{(j)}$, divided by the height, $h_{(j)}$] and s and c are the standard stability functions[5]. Adding equations (104) and (105):-

$$M_{(j,j+1)} + M_{(j+1,j)} = [s(1+c)]_{(j)}K_{c(j)}[\theta_{(j)} + \theta_{(j+1)} - 2\phi_{(j)}]$$

Therefore,

$$\phi_{(j)} = \frac{\theta_{(j)} + \theta_{(j+1)}}{2} - \frac{[M_{(j,j+1)} + M_{(j+1,j)}]}{2[s(1+c)]_{(j)}K_{c(j)}}$$

Now,

$M_{(j,j+1)} + M_{(j+1,j)}$ = the sum of the column moments in Storey (j)

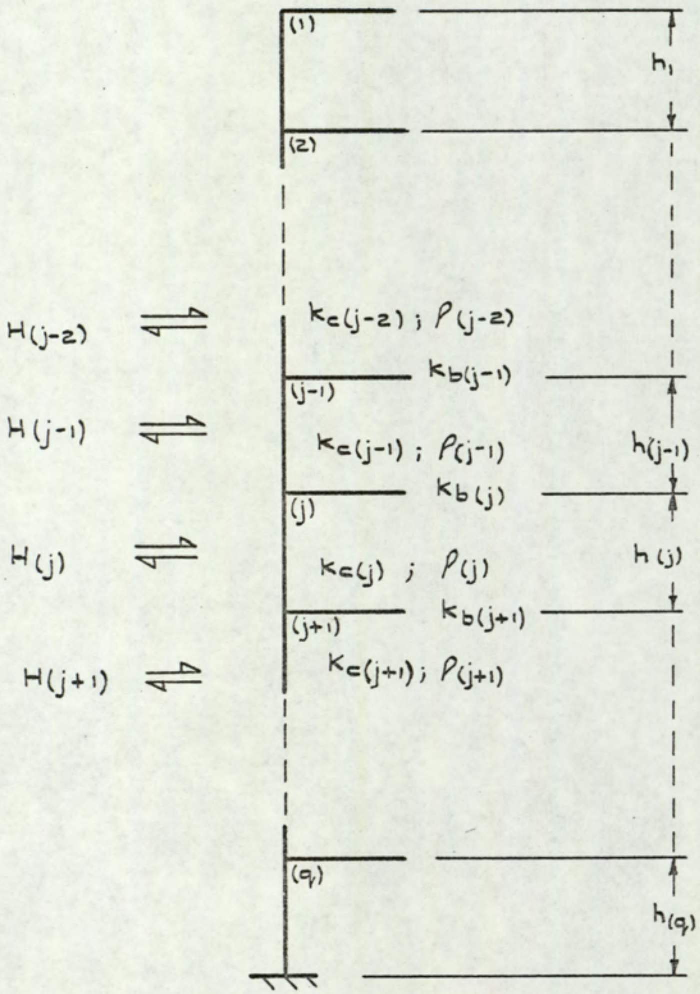
= the initial sway moment for Storey (j).

i.e., allowing for the Pδ effect,

$$M_{(j,j+1)} + M_{(j+1,j)} = -\, m_{(j)}H_{(j)}h_{(j)}$$

Therefore,

$$\phi_{(j)} = \frac{\theta_{(j)} + \theta_{(j+1)}}{2} + \frac{m_{(j)}H_{(j)}h_{(j)}}{2[s(1+c)]_{(j)}K_{c(j)}}$$



$H_{(j)}$ = the total shear force in Storey (j)

$K_{b(j)} = \Sigma\left(\frac{E I_b}{L}\right)$ for all beams in storey (j)

$K_{c(j)} = \Sigma\left(\frac{E I_c}{h}\right)$ for all columns in storey (j)

$\rho_{(j)}$ = average Euler ratio for all columns in storey (j)

Equivalent column for calculation of sway deflections at working load.

FIGURE 81

Also, let $\Sigma M_{(j)}$ be the sum of the initial sway moments at joint (j). Then, by definition,

$$\theta_{(j)} = \frac{\Sigma M_{(j)}}{\Sigma S^r_{(j)}}$$

and,

$$\theta_{(j+1)} = \frac{\Sigma M_{(j+1)}}{\Sigma S^r_{(j+1)}}$$

However,

$$\Sigma M_{(j)} = \frac{m_{(j-1)}H_{(j-1)}h_{(j-1)}}{2} + \frac{m_{(j)}H_{(j)}h_{(j)}}{2}$$

$$= (mHh)av_{(j)}$$

Similarly,

$$\Sigma M_{(j+1)} = (mHh)av_{(j+1)}$$

Thus, the deflection in any storey may be obtained from the following expression:-

$$\phi_{(j)} = \frac{\Delta_{(j)}}{h_{(j)}} = \frac{1}{2}\left[\frac{(mHh)av_{(j)}}{\Sigma S^r_{(j)}} + \frac{(mHh)av_{(j+1)}}{\Sigma S^r_{(j+1)}}\right] + \frac{m_{(j)}H_{(j)}h_{(j)}}{2[s(1+c)]_{(j)}K_{c(j)}} \qquad (106)$$

where,

$$\Sigma S^r_{(j)} = (n\text{-}o)_{(j-1)}K_{c(j-1)} + (n\text{-}o)_{(j)}K_{c(j)} + 12K_{b(j)} \qquad (107)$$

and,

$$\Sigma S^r_{(j+1)} = (n\text{-}o)_{(j)}K_{c(j)} + (n\text{-}o)_{(j+1)}K_{c(j+1)} + 12K_{b(j+1)} \qquad (108)$$

The total deflection of any frame may be found by applying equation (106) to each storey in turn, the solution being obtained quite simply provided that the section properties and loading conditions are known. The variation in the stability functions, s(1+c) and (n-o), as the Euler ratio increases, has been given previously in Table 3. Equation (106) is in fact the summation of the two types of deformation which contribute to the total sidesway deflection of a storey, the first part representing the deflection due to the rotation of the joints, and the second part

Frame	Approximate solution [equation (106)]			Accurate solution;	Percentage	ϕ (from the
	Δr (inches)	Δf (inches)	$\Delta = \Delta r + \Delta f$ (inches)	Δ (inches)	error	accurate solution)
1	2.09	0.88	2.97	3.04	-2.3%	0.00288
2	4.46	1.82	6.28	6.39	-1.7%	0.00443
3	5.20	2.86	8.06	7.94	+1.5%	0.00517
4	1.71	0.73	2.44	2.57	-5.0%	0.00223
5	4.24	1.85	6.09	5.76	+5.7%	0.00499
6	1.33	0.53	1.86	1.94	-6.2%	0.00169
7	0.50	0.53	1.03	1.12	-8.0%	0.00194
8	3.16	1.47	4.63	4.60	+0.7%	0.00388
9	1.56	0.68	2.24	2.24	-	0.00388
10	1.94	0.63	2.57	2.60	-1.2%	0.00300
11	4.20	1.66	5.86	5.86	-	0.00676

Δr = deflection due to rotation of the joints.

Δf = deflection due to flexibility of the columns.

ϕ = deflection index = $\frac{\Delta}{h}$.

Total sidesway deflections at working load; Frames 1 to 11.

(designed with fictitious sections)

TABLE 20

representing the deflection due to the flexibility of the columns.

The validity of this approximate formula has been examined by calculating the working load deflections for each of the eleven test frames described in Chapter 7, and by comparing these with the deflections obtained using the accurate computer analysis. The results are given in Table 20, from which it may be seen that there is little difference between the two sets of solutions. It may be assumed that any deflection computed from equation (106) will always be within 10% of the true value, and this is considered to be a satisfactory degree of accuracy.

9.3. LIMITATION OF DEFLECTIONS

As stated previously, the proposed design method is not in itself sufficient to safeguard against the numerous effects of excessive deflection, despite the fact that the resulting framework remains elastic at working load. The degree of deformation which may be tolerated must depend to a large extent on the functional requirements of the structure, and even a frame designed by the traditional elastic approach may be deemed to be unsatisfactory in certain circumstances. For this reason, there has been a general reticence on the part of the standard codes of practice to insist upon a universally applicable set of deflection limits, although certain recommendations do of course exist.

The vertical deflection of a beam due to the action of live load is limited by B.S.S. No. 449[2] to $\frac{1}{360}$ of the span. It is extremely unlikely that this limit will ever be exceeded in frames with fully-rigid joints, and it has certainly never been found to be violated in any of the frames designed by the proposed method.

As yet, no regulations exist in this country concerning the overall sway deflections of a tall structure, although the horizontal displacement of stanchions in single-storey buildings is limited by B.S.S. No. 449[2] to $\frac{1}{325}$ of the height. The code however does state that this limit, and also that governing the vertical deflection of beams, may be exceeded in cases where greater deflection would not impair the efficiency of the structure or lead to local damage.

The only comprehensive discussion on deflections in tall structures appears in the Report of the Sub-Committee on Wind Bracing of the A.S.C.E.[63] published in 1940, where a limiting deflection index of $\phi = 0.002$ was recommended. The same value has been suggested by other sources[64,65] in more recent times. Nevertheless, Grinter[66] has pointed out that many buildings in the U.S.A. have been found to behave satisfactorily when designed using a value of ϕ considerably greater than 0.002, and reference may also be found[65] to frames designed for a deflection index in excess of 0.005.

The justification for allowing such high values of sidesway for the bare frame is that the deflection calculations completely ignore the stiffening effects of the external cladding and the partition walls. Although some forms of lightweight cladding possibly have little effect on the sidesway, it is certain that deflection calculations based on the bare frame will always be conservative. Heavy cladding is likely to reduce the sway deflections by a substantial amount, as demonstrated, for example, by tests on models of the Empire State Building[65], which suggested that the masonry walls increased the rigidity of the structure about 350% above the rigidity of the steel frame.

Thus, while no specific allowance is made for the increase in stiffness due to the effect of cladding, it would appear to be common practice to exercise a considerable degree of tolerance if the bare frame deflection is found to exceed $\phi = 0.002$. The decision as to what constitutes an "acceptable deflection" is generally left to the discretion of the designer.

Referring back to Table 20, in which the overall deflections of the eleven test frames are summarized, it may be seen that the values of ϕ vary between 0.00169 for Frame 6 and 0.00676 for Frame 11. The sidesway of several of these frames is probably excessive, but it must be remembered that the designs were performed for extreme loading conditions, using the fictitious range of sections. The design examples given in Chapter 8 are however more realistic, and the deflections computed for these frames will now be examined in more detail.

(a) Four-storey, single-bay frame: [Section 8.2.]

This is the first design example in Chapter 8. The deflections obtained for each storey are summarized in Table 21:-

Storey	Approximate solution [equation(106)]				Accurate value of ϕ
	Δr (inches)	Δf (inches)	$\Delta = \Delta r + \Delta f$ (inches)	$\phi = \frac{\Delta}{h}$	
1	0.187	0.044	0.231	0.00161	0.00186
2	0.371	0.132	0.503	0.00350	0.00349
3	0.459	0.221	0.680	0.00473	0.00475
4	0.237	0.158	0.395	0.00275	0.00279
TOTALS:-	1.254	0.555	1.809	0.00314	0.00321

Four-storey, single-bay frame; deflections at working load.

TABLE 21

The overall deflection index for the bare frame, ϕ = 0.00321, would not appear to be excessive. Heyman suggested that a value of two inches would be acceptable as the total sidesway of a 48ft. building, and this corresponds to a value of ϕ equal to 0.00348, greater than that obtained for the design by the proposed method.

The deflection index for Storey 3, ϕ = 0.00475, is considerably higher than those calculated for the other storeys, and it could be argued that this particular value is excessive. A high value of relative deflection for any one storey is the primary cause of local damage, and any limiting deflection index should logically be applied to every storey in a frame. The possible procedures which are available to the designer when a permissable deflection limit is exceeded will be discussed later in this chapter.

(b) Thirty-storey, five-bay frame: [Section 8.3.]

The deflections obtained for this frame using the approximate expressions given by equation (106) are summarized in Table 22. The overall deflection index is ϕ = 0.00364, and again this is considerably greater that the generally recommended value of ϕ = 0.002. However, as before, this bare frame deflection is unlikely to lead to any undesirable effects, since only a small proportion of it will be realised in practice. The maximum deflection index for any storey is ϕ = 0.00470 is Storey 23.

The overall deflection index for Heyman's design was found to be 0.0045 approximately, representing a sway of 19.5 inches at the level of the top storey beam (as opposed to 15.7 inches for the Author's design).

(c) Eight-storey, two-bay frame: [Section 8.4.]

This framework has two unequal bays, so that the sway deflections cannot be obtained simply by using equation (106). However, the frame has been analysed using the computer program, and the results are given in Table 23 for the two cases when the wind loading acts from opposite sides of the frame. The maximum sidesway occurs with the wind loading acting from the right, and the corresponding overall deflection index for the bare frame is ϕ = 0.00202, which is quite satisfactory. Storey 4 exhibits the greatest relative deflection, with ϕ = 0.00258.

(d) Ten-storey, three-bay frame: [Section 8.5.]

The final design example in Chpater 8 also contains unequal bays, and again the deflections have been obtained using the computer program. The

Storey	Δr (inches)	Δf (inches)	Δ = Δr + Δf (inches)	ϕ
1	0.039	0.026	0.065	0.00045
2	0.095	0.078	0.173	0.00120
3	0.160	0.110	0.270	0.00188
4	0.222	0.155	0.377	0.00261
5	0.284	0.134	0.418	0.00290
6	0.350	0.166	0.516	0.00359
7	0.356	0.120	0.476	0.00331
8	0.351	0.139	0.490	0.00340
9	0.405	0.095	0.500	0.00347
10	0.460	0.106	0.566	0.00393
11	0.480	0.103	0.583	0.00405
12	0.495	0.112	0.607	0.00421
13	0.538	0.100	0.638	0.00444
14	0.486	0.108	0.594	0.00413
15	0.430	0.098	0.528	0.00366
16	0.459	0.104	0.563	0.00390
17	0.488	0.102	0.590	0.00410
18	0.522	0.108	0.630	0.00437
19	0.556	0.092	0.648	0.00450
20	0.542	0.097	0.639	0.00444
21	0.526	0.102	0.628	0.00436
22	0.556	0.107	0.663	0.00460
23	0.584	0.093	0.677	0.00470
24	0.521	0.097	0.618	0.00429
25	0.453	0.097	0.550	0.00381
26	0.474	0.101	0.575	0.00399
27	0.496	0.091	0.587	0.00407
28	0.519	0.094	0.613	0.00420
29	0.535	0.066	0.601	0.00417
30	0.270	0.067	0.337	0.00234
TOTALS:-	12.652	3.068	15.720	0.00364

Thirty-storey, five-bay frame; deflections at working load.

<u>TABLE 22</u>

Storey	Wind from left		Wind from right	
	Δ (inches)	ϕ	Δ (inches)	ϕ
1	0.029	0.00022	0.147	0.00111
2	0.091	0.00069	0.189	0.00143
3	0.170	0.00129	0.271	0.00205
4	0.257	0.00195	0.341	0.00258
5	0.262	0.00198	0.321	0.00244
6	0.286	0.00216	0.339	0.00256
7	0.291	0.00221	0.335	0.00254
8	0.176	0.00133	0.198	0.00150
TOTALS:-	1.562	0.00148	2.141	0.00202

Eight-storey, two-bay frame; deflections at working load.

TABLE 23

Storey	Wind from left		Wind from right	
	Δ (inches)	ϕ	Δ (inches)	ϕ
1	0.200	0.00139	0.036	0.00025
2	0.275	0.00191	0.128	0.00089
3	0.293	0.00204	0.220	0.00153
4	0.364	0.00253	0.307	0.00213
5	0.375	0.00260	0.350	0.00243
6	0.410	0.00285	0.395	0.00274
7	0.399	0.00276	0.387	0.00269
8	0.391	0.00272	0.387	0.00269
9	0.364	0.00253	0.366	0.00254
10	0.204	0.00113	0.207	0.00115
TOTALS:-	3.275	0.00221	2.783	0.00188

Ten-storey, three-bay frame; deflections at working load.

TABLE 24

results are given in Table 24. In this case, the sidesway is more pronounced when the wind loading acts from the left, and the overall deflection index is $\phi = 0.00221$. The maximum individual storey deflection occurs in Storey 6, for which $\phi = 0.00285$. Neither of these values is considered to be excessive.

9.4. PROCEDURE FOR REDUCING DEFLECTIONS

In accordance with the discussion in the previous section, no specific limit on the deflection index has been recommended in this chapter. Nevertheless, it has been suggested that there are many circumstances in which the designer may wish to restrict sidesway deformation, and there are several ways in which this may be done.

Clough[49] has suggested that acceptable deflections may be obtained by choosing values for the load factors which ensure that the frame remains elastic at working load. However, the deflection calculations that have been given for the four design examples indicate that considerable sidesway may occur even in a fully-elastic framework. Furthermore, the selection of higher load factors may well lead to a considerable loss of economy, since every member in the frame is liable to be increased, whereas excessive deflections may have only been found to occur in specific regions.

This fact is amply demonstrated by observation of the deflections in Table 21 for the four-storey, single-bay framework. The deflection index for Storey 3 is considerably greater than those for the remaining three storeys, and the designer may only be required to reduce this particular value. The obvious way to do this is to increase the members in the immediate vicinity of Storey 3, rather than by increasing every single member in the frame. Despite the fact that the deflections are quite high, it was shown in Chapter 8 that this frame has a considerable reserve of strength, failure occurring at $\lambda_F = 1.68$ under combined loading, and $\lambda_F = 1.90$ under vertical load alone. Thus, the selection of a higher design load factor would obviously be unduly conservative.

Further inspection of the results in the previous section indicates that the deflection due to rotation of the joints in any storey, Δr, is generally far greater than that due to the flexibility of the columns, Δf. Since Δr is a function of the total joint stiffness, which is controlled to a large extent by the stiffness of the beam, it may be seen that the most effective way of reducing deflection is to select a beam with a higher second moment of area. Admittedly, this will also supply the frame with an additional safety factor, but provided that the design load factor is attained, the designer should not be too concerned with the load factor at which failure eventually occurs. Once the basic design criteria have been satisfied, the decision to increase any section size must be considered in economic terms.

Thus, it is suggested that if any deflections are believed to be excessive, the appropriate beam sizes should be increased and the deflections recalculated using equation (106). Referring back to equations (106), (107) and (108), it may be seen that little computational work is involved, since the only terms affected by any alteration in the beams are the overall joint stiffnesses, $\Sigma S^r_{(j)}$ and $\Sigma S^r_{(j+1)}$. The required beam size may therefore be obtained quite rapidly.

9.5. SUMMARY

The subject of deflections has not been considered in any great detail in this chapter, since it does not constitute a major part of this research project. However, it has been shown that the sidesway of any fully elastic regular framework may be obtained quite accurately by reducing the structure to an equivalent column. Furthermore, a large number of results have been given to illustrate the order of magnitude of this sidesway in frames designed by the proposed method. Although it is considered that these deflections will generally be acceptable, a procedure has been described for reducing the sidesway should the circumstances demand that a specific limit be applied to the deflection index.

In the following chapter, the advantages and limitations of the proposed design method are examined, and the thesis is concluded by a brief discussion on the way in which the range of applicability of the method could be extended through additional research.

CHAPTER 10

CONCLUSIONS

10.1 INTRODUCTION

Throughout this research project, considerable attention has been paid towards assessing the validity and economy of the proposed design method, which has been found to possess distinct advantages over many other methods. These advantages are the subject of the following section. Subsequently, the limitations of the method are examined, and finally suggestions are made for additional research which would help to develop the full potential of this design approach.

10.2. ADVANTAGES OF THE DESIGN METHOD

By using the fictitious range of sections and the elasto-plastic computer analysis, it has been shown in Chapter 7 that the design equations accurately represent the strength requirements of any member in a regular multi-storey frame. The eleven test frames were all found to behave in a satisfactory manner under both sets of design loads, and although slight violations of the design criteria occurred, it was subsequently shown that these criteria were satisfied in every case when the frames were redesigned using the standard Universal Beams and Universal Columns. Furthermore, none of the frames had an excessive reserve of strength, and there was found to be an extensive distribution of plasticity throughout each frame at the design load factor. The hinge patterns which were obtained from the computer analysis were shown to reflect the manner in which each frame was designed.

Since these eleven frames covered a wide range of design parameters, it may be concluded that any frame designed for parameters lying within this range is likely to exhibit the same degree of safety and "structural efficiency", and this is a powerful argument in favour of the proposed method. Since the method does not depend on the use of a computer analysis, it is essential that the designer should be able to express confidence in the ability of the resulting structure to carry its applied loads in a safe and efficient manner.

This "predictability" has been shown to be lacking in certain other methods. For example, Figure 69(c) in Chpater 8 demonstrates that the four-storey, single-bay frame designed by Heyman(46) fails without the formation of a single plastic hinge in the beams, whereas Heyman's design method is intended to be a "weak-beam, strong-column" approach. Although this frame has a large reserve of strength, it is disconcerting to find that failure occurs in a somewhat unexpected manner, and it is reasonable to assume that this lack of predictability could lead to premature collapse in a different framework. It has also been shown that the ten-storey, three-bay frame designed by the Lehigh method(61) collapses below the design load factor under vertical load alone, and so any subsequent designs by this method must be treated with suspicion.

The subject of design economy has been examined in Chapter 8, and again, it was shown that the proposed method compares favourably with the alternative methods. The accurate computer method devised by Majid and Anderson(50) was shown to be the only design approach which consistently produces a more economical structure than the proposed method, without any loss of safety. The fact that the proposed method leads to a framework with a similar weight to that designed by Majid and Anderson's method again recommends it for use as a standard procedure, for it indicates that little economy has been sacrificed in producing a much simpler and considerably less expensive design approach. The method also enjoys the flexibility of being independent of the availability of an electronic computer, and furthermore, its ease of application is independent of the size of the frame.

The design procedure was explained in detail in Chapter 6. In particular, the design example in Section 6.2(f) clearly illustrates another useful feature of the proposed method, namely that it is an automatic iterative process, and as such is to some extent self-checking. Any error in the selection of sections or in the arithmetical calculations is unlikely to remain unnoticed.

10.3 LIMITATIONS OF THE DESIGN METHOD

Although it has been shown that the proposed method may be used to obtain a satisfactory design for a wide range of frameworks, it is appreciated that there is a slight possibility of early failure with, for example, very extreme wind loading or extraordinarily tall columns. However, this is not considered to be a serious limitation on the use of the method, since such exceptional framewo ks would normally receive special attention when designed by any alternative approach. In such cases, it is recommended that the safety of the structure should be assessed using an accurate elasto-plastic computer analysis.

Similarly, it has been shown in Chapter 9 that comparatively high sway deflections may occur in frames designed for heavy wind loading, and on certain occasions the designer may consider these to be excessive. Again, this does not necessarily imply that the range of applicability of the proposed method is any smaller than that of the other methods which have been discussed, since even an elastic design of any such frame would also be likely to deflect excessively. The only way in which a satisfactory design could be obtained in these circumstances would be to introduce some form of bracing, and the argument reduces to one between the general philosophies of the design of sway and no-sway frames. Thus, as long as justification exists for the design of unbraced frames, the proposed method is no less valid than any other related method, concerning the subject of permissable sway deflections.

One further criticism of the design method at its current stage of development is that it may only be applied to regular frameworks. However, it has been shown in Chapter 8 that certain forms of irregularity may be allowed for quite simply by appropriate modifications to the design approach, and it is believed that the design theory may be extended to cater for any frame with variable bay-widths. It may also be possible to deal with certain other types of unsymmetry. This subject is discussed more fully in the following section.

10.4. SUGGESTIONS FOR FUTURE RESEARCH

There are many ways in which the proposed method may be extended in order to produce a more generalized or more economical design procedure. Several suggestions are made in the following sub-sections.

10.4(a) DESIGN OF UNSYMMETRICAL FRAMES

Unsymmetrical frame behaviour may occur due to either unsymmetrical loading or unsymmetry of the geometrical shape. The first of these causes has been investigated by Gandhi[1], who derived expressions for the bending moments induced in the internal columns when they are deformed in single-curvature due to the action of "pattern live-loading". It was found that the sections selected by the basic design method, which ignores these additional column moments, were generally satisfactory, although occasionally small increases were required in the comparatively slender upper storey columns. In the lower storeys, the substantial reduction in the column axial loads due to the removal of the live loads on certain beams adequately compensated for any additional bending moments. These results would form a useful basis for any future research on the subject of unsymmetrical loading conditions.

The problems associated with unsymmetrical frame geometry are however far more numerous, and it is not suggested that the proposed method could ever be applied to completely irregular structures. Nevertheless, as mentioned in the previous section, and as described in Chapter 8, successful designs for two slightly irregular frameworks have already been achieved by making only small modifications to the basic design procedure. The approach adopted in these cases was based to a large extent on experience, and is not necessarily recommended for any frame with unequal bays. However, the results of these two design examples imply that certain general rules do exist to cater for this type of unsymmetry, and it is suggested that additional work on this subject would certainly be fruitful. The proposed method already allows for unequal storey heights, and this further modification would make it applicable to a very wide range of practical frameworks.

Also, the proposed method may possibly be extended to cater for the design of frames in which the number of bays suddenly alters at a particular level. Many frameworks are constructed in this manner, with, for example, the lower few storeys containing many more bays than the main tower-block. In such cases, the standard procedure could be applied to each storey of the tower-block in turn (for it will be remembered that the design proceeds from the top storey downwards, and at any level the sections obtained are independent of the geometry of that part of the frame which lies below this level). Provided that means could be found of effectively representing the structural action of the tower-block on the lower storeys, the design for the remainder of the frame could be continued as normal. Again, such modification would extend the range of applicability of the design method considerably.

10.4(b) COMPOSITE DESIGN

There are many advantages to be obtained by considering a structure as a composite system rather than as a series of individual components, each having only one primary function. Whenever any continuity exists between two components, there is bound to be a certain degree of structural interaction, and the basic philosophy of composite design is that this interaction should be both recognised and allowed for during the design process. There is no reason why a similar philosophy should not be applied in order to extend the proposed design method.

The most obvious way to produce a more economical design is to allow for composite action between the concrete floor slabs and the steel beams. However, the problems involved in modifying a plastic design method to allow for this composite action are considerable. The composite beam does not have uniform properties, and the value of fully plastic moment to be assumed in design varies according to whether the concrete slab is in tension or compression. Similarly, the second moment of area is variable, and the derivation of suitable magnification factors to allow for the instability effects would necessarily be a complicated process. Nevertheless, it is believed that this would be justified, since considerable economy would be achieved, and the continuity between the beams and slabs would undoubtably reduce the sidesway deflections enormously.

Also, should it be required to use reinforced concrete casing for the columns of a frame in order to satisfy the fire-proofing requirements[2], then it would be possible to modify the proposed design method in order to take full advantage of the extra strength available. In the proposed method, it has been shown that the column ends are not required to undergo any rotation due to the formation of a fully plastic hinge. Therefore, the concrete casing could be assumed to carry a proportion of the total axial load in the member, thus relieving the steel section of a certain amount of direct stress. In the lower regions of the frame, where selection of a suitable column is largely controlled by the axial load in the member, considerably lighter Universal Columns would be found to be adequate.

The suggestions which have been made in this section involve quite radical changes in the design philosophy, and are likely to lead to substantial saving in the weight of the frame. There are of course many other refinements which could be made to the work presented in this thesis, but it must be remembered that the proposed method is based on a large number of assumptions, and despite the fact that these lead to a surprisingly accurate design, it does not claim to be anything other than an approximate approach. The success of any derived version of the method must still depend of the validity of the initial hypotheses, many of which were devised in order to simplify the design procedure. It would therefore be quite illogical to attempt to introduce a large number of sophisticated analysis techniques in order to obtain only a small increase in economy, and this should be borne in mind when planning any future research programme. The Author believes that, if possible, the basic simplicity of the design method should be preserved.

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