The University of Aston in Birmingham

# Improvements and Extensions to Recent Electroheat Control Contributions

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Ph. D Thesis

September 1969

ACKNOWLEDGEMENTS

I gratefully acknowledge the guidance of Professor W.K. Roots at every stage of this project. I wish to thank Professor J. E. Flood who kindly consented to be my adviser after the departure of Professor W.K. Roots.

My thanks are also due to Mr. J. L. Murgatroyd for the many useful discussions and his guidance after the departure of Professor W.K. Roots.

SUMMARY

The increasing use of electroheat in industrial processes necessitates a thorough investigation into the current state of control techniques for electric process heating.

A new unified approach to the assessment of stability of continuously and discontinuously controlled electroheat processes has been derived. It utilises a newly defined critical frequency locus in conjunction with the describing function.

An exponential describing function is proposed for determining the conditions for a critically damped response to step-inputs in discontinuously controlled electroheat processes. The results obtained closely correlate with actual test results.

The improvements obtainable by incorporating derivative feedback in discontinuously controlled processes has been analytically investigated. Facile displays have been developed that determine the optimum derivative content in the feedback loop.

The computer control of electroheat processes has been thoroughly examined. A design of model-referenced integrating controller has been established from first principles. It eliminates offset errors due to disturbance inputs.

A time-optimal control strategy has been established. It combines the advantages of discontinuous control with the advantages of the new model-referenced integrating control. It results in a performance that is greatly superior to that obtained with conventional control techniques for these processes.

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## LIST OF SYMBOLS

| | |
|---|---|
| $e$ | actuating signal in a controlled process |
| $e(t)$ | actuating signal as a function of time |
| $e^*(t)$ | actuating signal in a sampled-data system |
| $i$ | symbol for integer |
| $j$ | complex number = $\sqrt{(-1)}$ |
| $k$ | general symbol for integer |
| $m$ | manipulated variable (output of the control element) |
| $m(t)$ | manipulated variable as a function of time |
| $m^*(t)$ | manipulated variable in a sampled-data system |
| $s$ | Laplace complex variable |
| $t$ | general symbol for time |
| $u$ | complex variable obtained from Laplace complex variable s by a transformation |
| $A$ | transfer characteristics of reference input element |
| $D_c(Z)$ | Z-transform of the closed-loop digital compensator |
| $D_s(Z)$ | Z-transform of open-loop compensator |
| $E(Z)$ | Z-transform of the sampled actuating signal |
| $G$ | gain of the heating apparatus in an electro-heat process |
| $G_1(Z)$ | Z-transform of electroheat process and zero order hold |
| $G_1(Z,m)$ | modified Z-transform of electroheat process and zero order hold |
| $G_i(.)$ | general symbol for a transfer function |
| $H$ | transfer characteristics of feedback unit in a controlled process |
| $K$ | sensitivity of control element |
| $L$ | transit delay in an electroheat process |
| $M$ | parameter of multiposition discontinuous control element = $(Q-1)/2$ |

| | |
|---|---|
| $N$ | describing function of nonlinear control element |
| $N_m$ | maximum value of the describing function |
| $Q$ | number of possible outputs of a multi-position discontinuous control element |
| $T$, $T_1$, $T_2$ | time constants of electroheat processes |
| $T_s$ | sampling interval in a sampled-data system |
| $Z$ | complex variable in Z-transform |
| $\eta$ | gain of controlled process in the u-plane |
| $\theta$ | temperature of the process |
| $\theta_r$ | input command or the desired temperature |
| $\theta_u$ | disturbance input in a controlled process |
| $\mu$ | gain of controlled process |
| $\omega$ | general symbol for frequency |
| $\Delta e$, $\Delta e_1$, $\Delta e_2$ | calibration increments of multiposition discontinuous control elements |
| $\Delta m$, $\Delta m_1$, $\Delta m_2$ | increments of control effort of multiposition discontinuous control elements |

Only those symbols that are consistently used are listed here. Other symbols are defined as they occur in the various chapters.

# Chapter 1

# INTRODUCTION

# 1 INTRODUCTION

The increasing use of electroheat in industrial processes prompted the Electrical Engineering Department of the University of Aston in Birmingham, to examine in depth the current control technique for such processes and develop new strategies for vastly improved performance.

With a view to developing new control techniques that could be applied to a class of electroheat processes, it was decided to restrict the investigation to those processes that had the following common characteristics:

1) The processes possessed distributed parameters.
2) The processes were essentially of the single variable type i.e., temperature was the most significant parameter of the process.

Typical examples of such processes are the electrically heated fluidized beds, boilers without superheaters, induction furnaces, plasma torches, vats, ovens, kilns, etc. The mathematical model for such processes, established by several workers[1 2 3 4] were found to comprise one significant transit delay and one or two major time constants. Expressed mathematically, these models take the following forms:

$$G_1(s) = G \exp(-sL)/(1 + sT) \qquad (1.1)$$

$$G_2(s) = G \exp(-sL)/(1 + sT_1)(1 + sT_2) \qquad (1.2)$$

where G is the gain of the heating apparatus.

$T$, $T_1$, $T_2$ denote the major time constants.

L is a significant transit delay of the process.

In general, the transit delay L arises due to the distributed nature of the process, while the time constants are due to the thermal resistance and the thermal capacitance of the process.

A typical response of such processes to step temperature commands is displayed in Figure 1.1. In practice such pure delays are not normally encountered, but, the temperature rise for the period L is so small compared to the magnitude of the step command that it may be ignored.

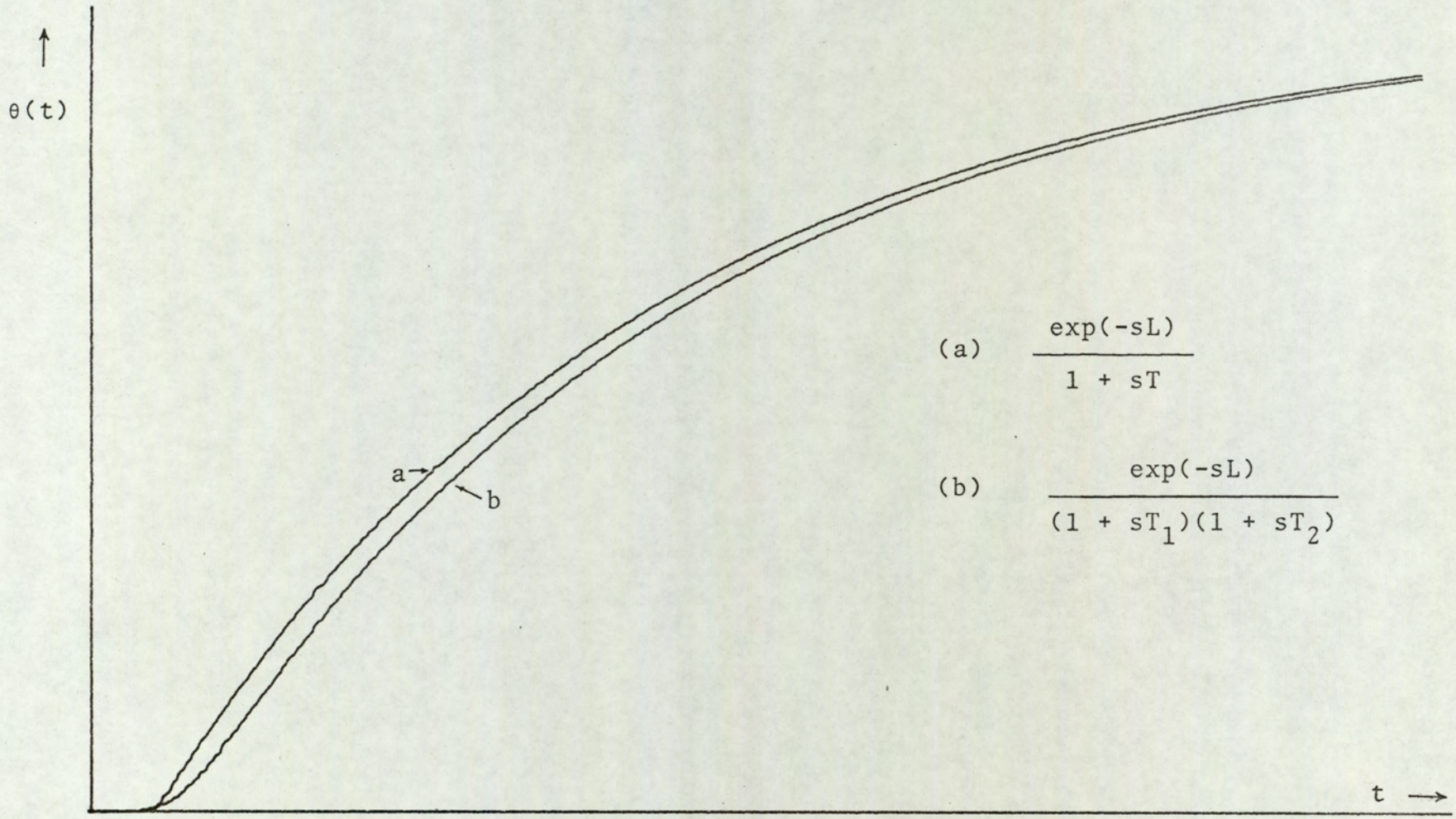


Fig. 1.1 Typical step response of electroheat processes and their mathematical models.

In the course of the investigation into the control of such processes, it was found that, until the recent past, the design of controllers was based on empiricism, rather than an analytical understanding of the behaviour of closed-loop controlled processes. In fact the optimum control settings derived empirically by Ziegler and Nichols[5] in 1942 are still being used widely in process control. However in the past few years, considerable work has been done in developing analytical techniques for the design of temperature controllers, especially by the International Union for Electroheat, BISRA, and their allied organisations.

Briefly, the objective of the investigations, described in detail in the ensuing chapters of this report were:

1) Development of an unified stability criteria for both continuously and discontinuously controlled processes.
2) Determination of critically damped response(non-oscillatory response) criteria for discontinuously controlled processes.
3) New strategies for better performance characteristics when continuous or discontinuous controllers are used.
4) Development of time-optimal controllers that utilise the digital computer for both process simulation and on-line control. The objective here is to minimise the storage requirements on the computer so as to facilitate time-shared control.

The flow chart displayed in Figure 1.2 traces the course of the investigations and the presentation of this report.

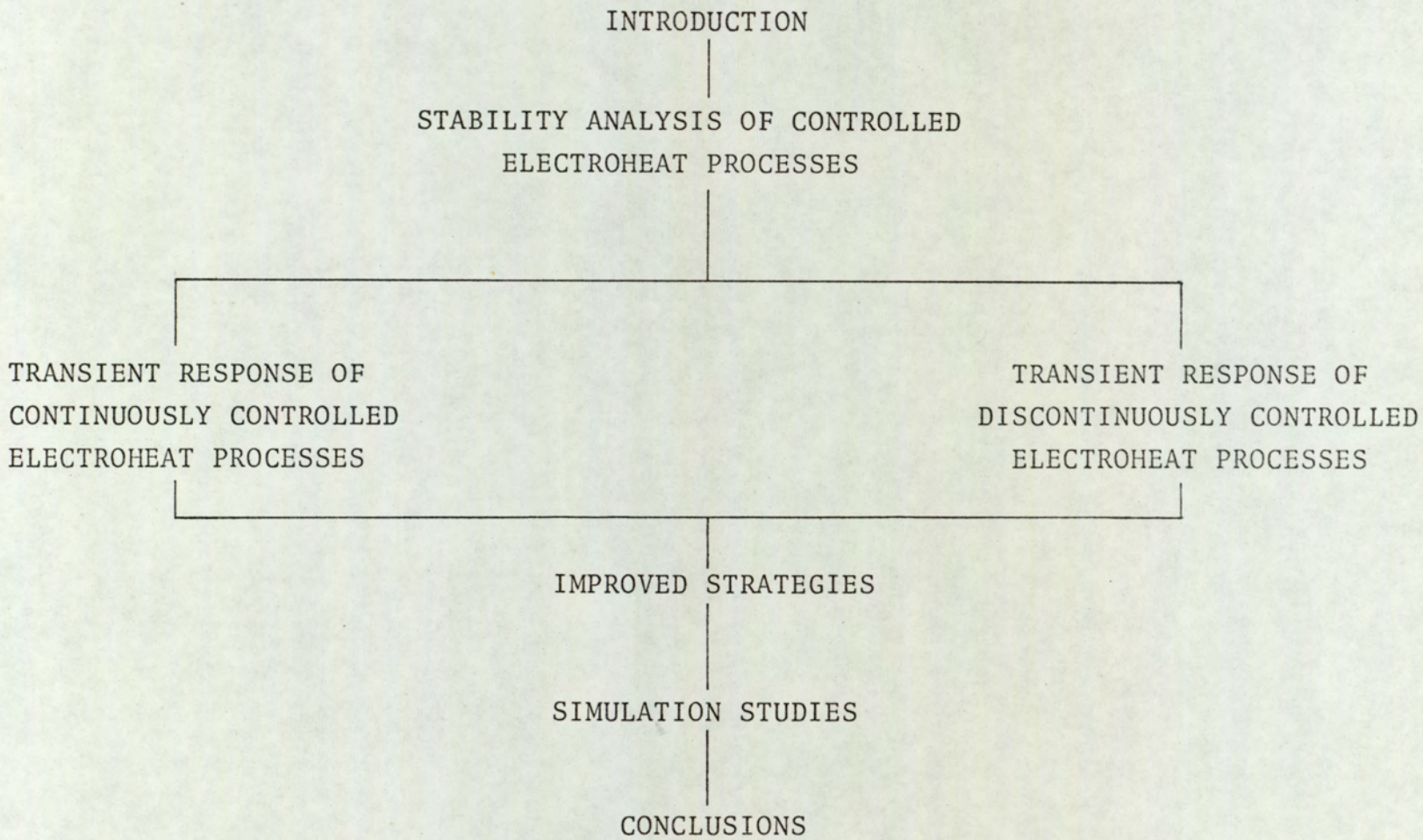


Fig. 1.2 Flow chart of the development of the research work and the presentation of this report.

# Chapter 2

## STABILITY CRITERIA FOR ELECTROHEAT PROCESSES

# STABILITY CRITERIA FOR ELECTROHEAT PROCESSES

## 2.1 INTRODUCTION

The determination of stability of controlled electroheat processes by classical methods viz. Bode plots, Nyquist's criterion, root-loci etc. is very cumbersome for processes with distributed parameters. In this chapter a critical frequency locus for the determination of stability of controlled electroheat processes, is proposed; this locus determines the critical stability condition directly in terms of the process parameters. It is further shown that the use of the describing function technique leads to a unified stability criteria for both continuously and discontinuously controlled processes.

It is also shown that the use of the conventional describing function in the analysis of discontinuous controllers leads to erroneous results when processes are modelled by equation (1.1). The RMS describing function, initially proposed by Gibson[6], is shown to be more accurate for discontinuous controllers.

Experimental investigations have established the accuracy of the describing function technique in the analysis of stability of controlled electroheat processes. These are described in chapter 5.

## 2.2 CHARACTERISTICS OF NONLINEAR CONTROL ELEMENTS

The characteristics of the commonly encountered nonlinear control elements in electroheat processes are displayed in Figure 2.1. These are the on-off, three-position, five-position, multiposition and proportional control elements; and their characteristics are displayed in Figures 2.1 (a), (b), (c), (d), and (e) respectively. The control elements described have odd-symmetrical characteristics; but asymmetrical control elements are often used in the control

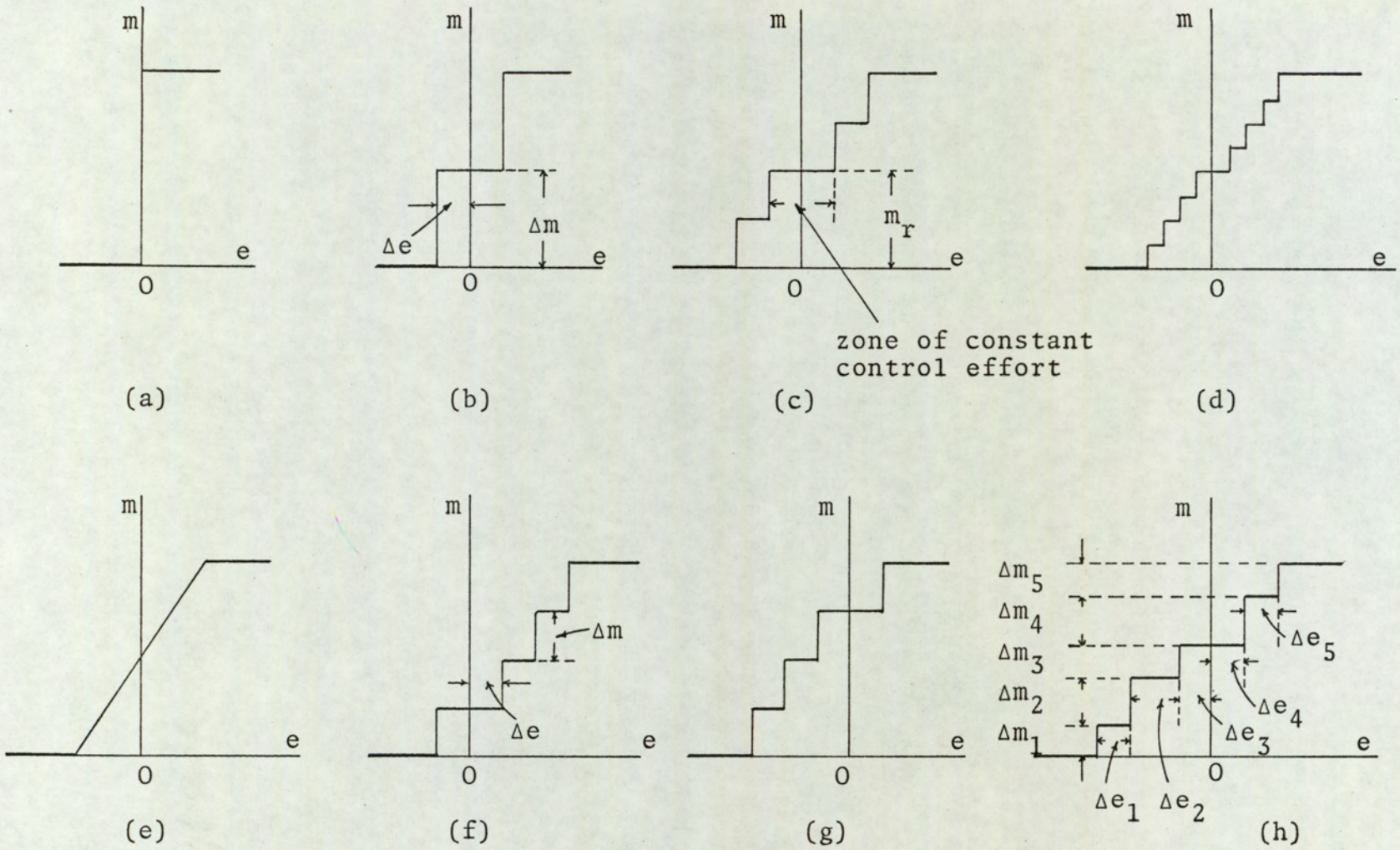


Fig. 2.1 Characteristics of commonly encountered discontinuous control elements.

of electroheat processes and typical characteristics have the forms shown in Figures 2.1 (f) and (g).

The range of the actuating signal over which the output of the control element is constant, is referred to as the calibration increment; while the magnitude of the change in the output, as the actuating signal varies from one range to another, is referred to as the increment of control effort. In Figures 2.1 (a)-(g) the calibration increment and the increment of control effort are denoted by $\Delta e$ and $\Delta m$ respectively. The characteristics of the control element are described in terms of the distribution of the calibration increments and the increments of control effort. A general multiposition discontinuous control element characteristic is displayed in Figure 2.1(h). The maximum and the minimum values of the output are fixed at 0 and 1.

The control element is said to be <u>linearly distributed</u> if (see Figure 2.1 (h) )

$$\Delta e_1 = \Delta e_2 = \cdots\cdots\cdots\cdots = \Delta e_n = \Delta e$$

and

$$\Delta m_1 = \Delta m_2 = \cdots\cdots\cdots\cdots \quad \Delta m_n = \Delta m$$

One may then define a <u>slope</u> for the control element as

$$K = (\Delta m/\Delta e)$$

For the linear case, if the control element has Q positions then

$$\Delta m = 1/(Q-1)$$

Therefore

$$K = 1/(Q-1)\Delta e$$

The control element can therefore be described in terms of the

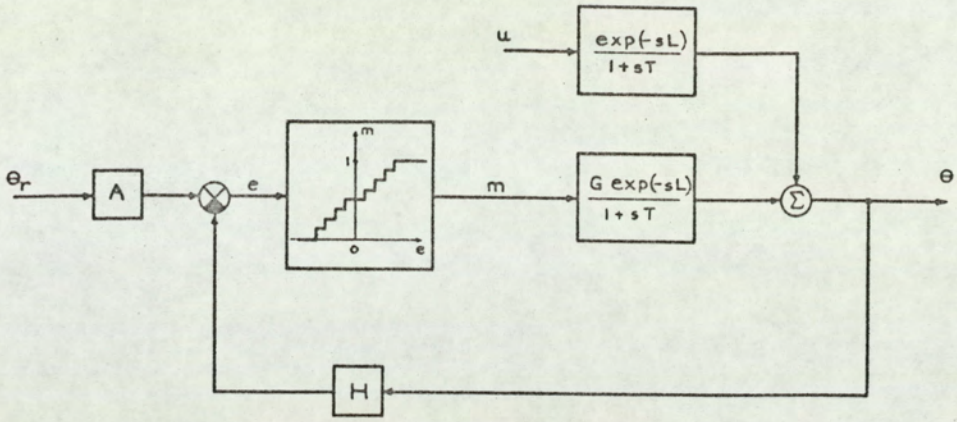


Fig. 2.2 Block diagram of a typical control system for an electroheat process.

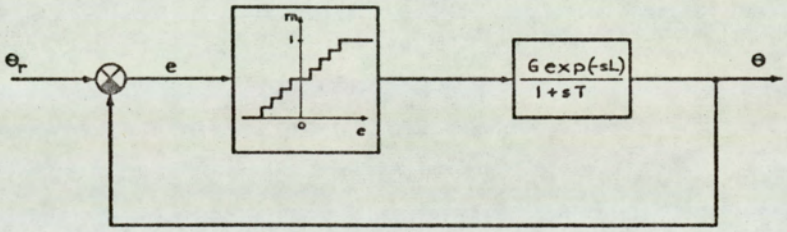


Fig. 2.3 Simplified block diagram of the control system of Fig. 2.2.

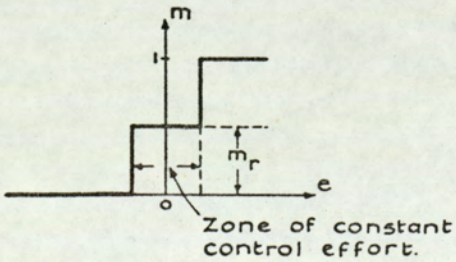


Fig. 2.4 The zone of constant control effort in a discontinuous control element.

number of positions Q and the slope K.

The analysis of stability will be presented for the linearly distributed control element, since, these are the commonly used control elements in electroheat process control.

The block diagram of a typical temperature control system for electroheat processes is shown in Figure 2.2. A and H represent the transfer characteristics of the reference input element and the feedback unit respectively. These are usually constants and, without loss of generality, can be assumed to be unity:

$$A = 1$$
$$H = 1$$

u is the disturbance input to the system. For the analysis of stability, only the disturbance-free system need be considered. Thus

$$u = 0$$
$$\dot{u} = 0$$

Under these conditions, the block diagram of Figure 2.2 simplifies to Figure 2.3.

Before the criteria for stability are developed, the following observations should be made as to the nature of the input command :

1) Discontinuous controllers of the type shown in Figure 2.1 are usually used only for constant input commands, i.e. $D\theta_r = 0$
2) A necessary condition for no oscillations in the system is that the control element possesses a finite zone of constant control effort about the e = 0 axis of the control element characteristic. This is shown in Figure 2.4 for a multiposition controller.

3) A further condition[7] for no oscillations and no offset (or droop) errors is that the control effort around the e = 0 axis be equal to the normalised input command, i.e.

$$m_r = \theta_r/G$$

where $G$ is the gain of the heating apparatus and $\theta_r$ is the input command.

For a linearly distributed control element this condition reduces to

$$\theta_r/G = n.\Delta m = n/(Q-1)$$

where $n$ is an integer less than $(Q-1)$. $\Delta m$ is the increment of control effort.

Summarising, stability analysis will be presented for the following conditions:

a) $A = 1$, $H = 1$
b) $u = 0$, $Du = 0$, $D\theta_r = 0$
c) There is a zone of constant control effort about the e = 0 axis.
d) The normalised input command $\theta_r/G = n.\Delta m$.

## 2.3 THE CONVENTIONAL DESCRIBING FUNCTION FOR DISCONTINUOUS CONTROL ELEMENTS

The describing function of a nonlinear control element is defined as the ratio of the fundamental component of the output to the input amplitude, when the input signal is sinusoidal.

For the odd-symmetrical control element shown in Figure 2.5 the direct component and the fundamental component of the output is found to be[8]

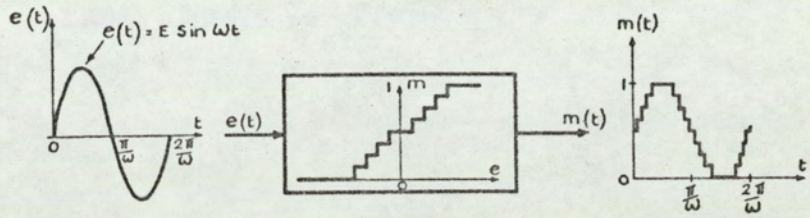


Fig. 2.5 Output of a discontinuous control element to a sinusoidal input signal.

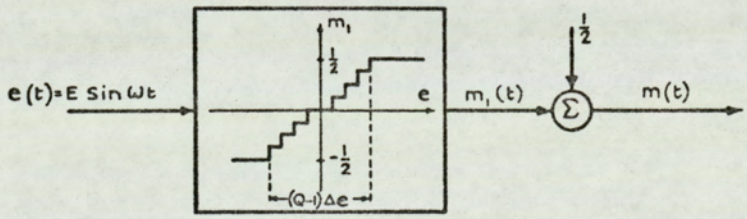


Fig. 2.6 Alternate representation of the control element of Fig. 2.5.

$$m(t) = \frac{1}{2} + \frac{4}{(Q-1)\pi}\left\{\sum_{i=1}^{k}\left[1 - \left(\frac{i.\Delta e}{E}\right)^2\right]^{\frac{1}{2}}\right\}\sin(\omega t) \qquad (2.1)$$

$$k = 1,2,3,\cdots\cdots\cdots,\ (Q-1)/2$$

where $E$ is the peak value of the input signal and $E > k.\Delta e$

Since the describing function relates only the fundamental component of the output to the input amplitude, an alternate representation of the control element is required, to take into consideration, the direct component in the output. The form of equation (2.1) suggests that the control element of Figure 2.5 can be split into its direct and oscillatory parts as shown in Figure 2.6.

The describing function of the oscillatory part is then

$$m_1/E = \frac{4}{\pi(Q-1)E}\left\{\sum_{i=1}^{k}\left[1 - \left(\frac{i.\Delta e}{E}\right)^2\right]^{\frac{1}{2}}\right\} \qquad (2.2)$$

Dividing $m_1/E$ by $K$ one obtains

$$N = m_1/KE = \frac{4}{\pi(Q-1)KE}\left\{\sum_{i=1}^{k}\left[1 - \left(\frac{i.\Delta e}{E}\right)^2\right]^{\frac{1}{2}}\right\} \qquad (2.3)$$

$N$ is displayed as a function of $E/\Delta e$ in Figure 2.7. This curve can be used to determine $N$ for any Q-position odd-symmetrical control element up to $Q = 33$. $m_1/E$ is obtained from by the relation

$$m_1/E = KN$$

The plot of Figure 2.7 shows that there are $(Q-1)/2$ maxima and these occur at

$$(E/\Delta e)_n = n + \sqrt{2} - 1 \qquad (2.4)$$

where $n = 1,2,\cdots\cdots\cdots,\ (Q-1)/2$

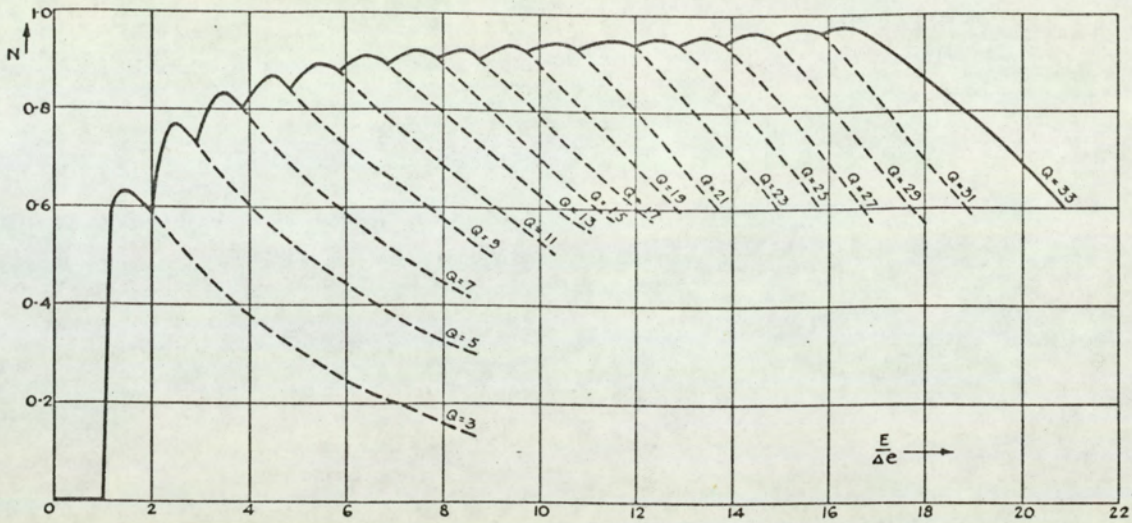


Fig. 2.7 Describing function of a multiposition discontinuous control element. The output of the control element can assume Q values.

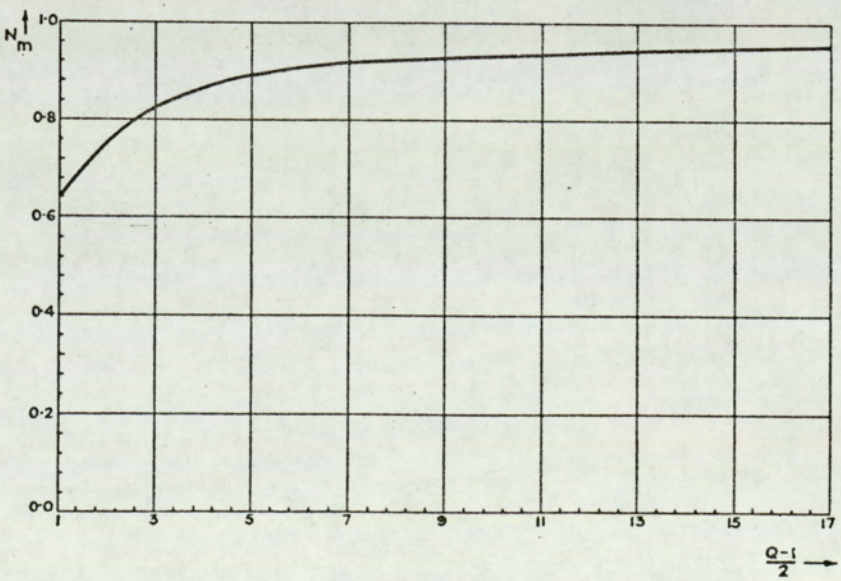


Fig. 2.8 The maximum value of the describing function $N_m$ of discontinuous control elements as a function of Q (Q is an odd integer).

$(E/\Delta e)_n$ is the location of the n th maxima

It is also found that the value of N at the values of $(E/\Delta e)_n$ given by equation (2.4) increases with n. The global maximum occured at

$$(E/\Delta e)\Big|_{n = (Q - 1)/2} = (Q + 2\sqrt{2} - 3)/2 \qquad (2.5)$$

The corresponding value of N at this value of $(E/\Delta e)$ is found to be

$$N_m = N_{E/\Delta e = (Q + 2\sqrt{2} - 3)/2} =$$

$$\frac{8}{\pi M_1} \left\{ \sum_{i=1}^{k} \left[ 1 - \left( \frac{2i}{M_1} \right)^2 \right]^{\frac{1}{2}} \right\} \qquad (2.6)$$

where $M_1 = Q + 2\sqrt{2} - 3$

Figure 2.8 displays $N_m$ as a function of $(Q - 1)/2$. Figures 2.7 and 2.8 enable the determination of the describing function for any linearly distributed odd-symmetrical control element. The use of these curves in the analysis of stability of electroheat processes will be explained in sections 2.5 and 2.6.

## 2.4 THE RMS DESCRIBING FUNCTION OF NONLINEAR CONTROL ELEMENTS

In using the conventional describing function for the analysis of stability, it is assumed, often with justification, that the higher harmonics in the output of the nonlinear control element do not contribute significantly to the output of the process. This assumption is valid, only if the higher harmonics are sharply attenuated by the process.

In the case of processes modelled by equation (1.1) and controlled by a Q-position discontinuous control elememt this assumption is not valid. A typical temperature waveform of a process controlled by a three-position controller is shown

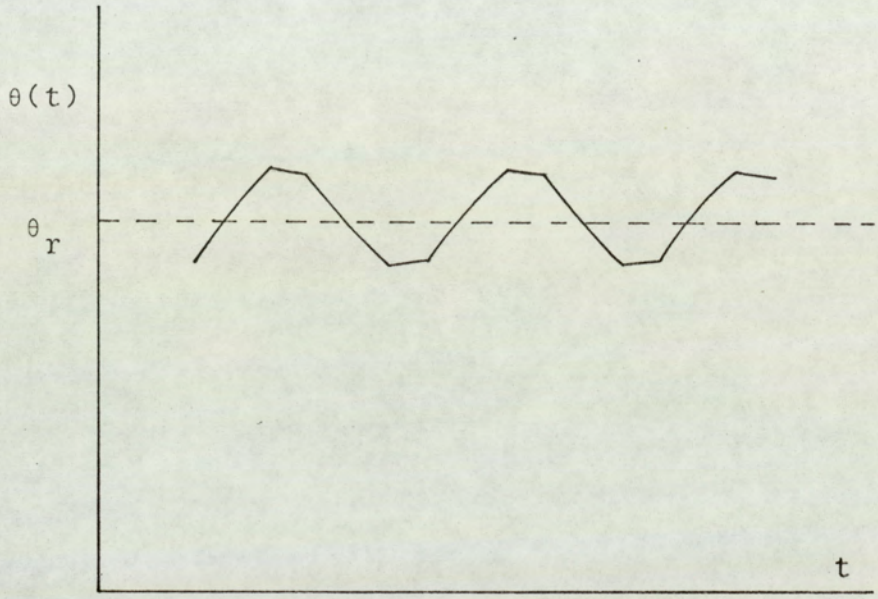


Fig. 2.9 Steady state oscillation in a 3-position controlled process.

in Figure 2.9. The amplitude of the third harmonic is found from Fourier analysis to be nearly a third of the fundamental component. The conventional describing function, which ignores the higher harmonics, could therefore lead to erroneous results. The accuracy of the describing function analysis is improved, if the third harmonic in the output of the nonlinear control element is also considered; however except in the case of the simplest nonlinear elements this analysis becomes very complicated.[9]

The RMS describing function which considers the RMS value of the output accounts for the effects of the higher harmonics in the output signal. This form of quasi-linearization of the nonlinear characteristics of the control element is justified in terms of energy equivalence[6].

For the odd-symmetrical control element displayed in Figure 2.5 the RMS value of the output can be shown to be

$$m_{rms} = \frac{1}{2} + \left\{ \frac{1}{\pi} \left[ \frac{k^2\pi}{4M^2} - \sum_{j=1}^{k} \frac{2j-1}{2M^2} \sin^{-1}\left(\frac{j.\Delta e}{E}\right) \right]^{\frac{1}{2}} \right\} \qquad (2.7)$$

where $k = 1,2,\cdots\cdots\cdots M$

and $M = (Q - 1)/2$

The RMS value of the input sinusoidal signal is $E/\sqrt{2}$. The RMS describing function of a nonlinear control element is defined as the ratio of the RMS values of the output to the input, when the output has no direct component. However as in the previous section, the control element can be split into its direct and oscillatory parts as shown in Figure 2.6. The describing function of the oscillatory part is then

$$m_1/E = \left\{ \frac{2}{\pi E^2} \left[ \frac{k^2\pi}{4M^2} - \sum_{j=1}^{k} \frac{2j-1}{2M^2} \sin^{-1}\left(\frac{j.\Delta e}{E}\right) \right] \right\}^{\frac{1}{2}} \qquad (2.8)$$

Normalising the above with respect to the slope of the control element, one obtains

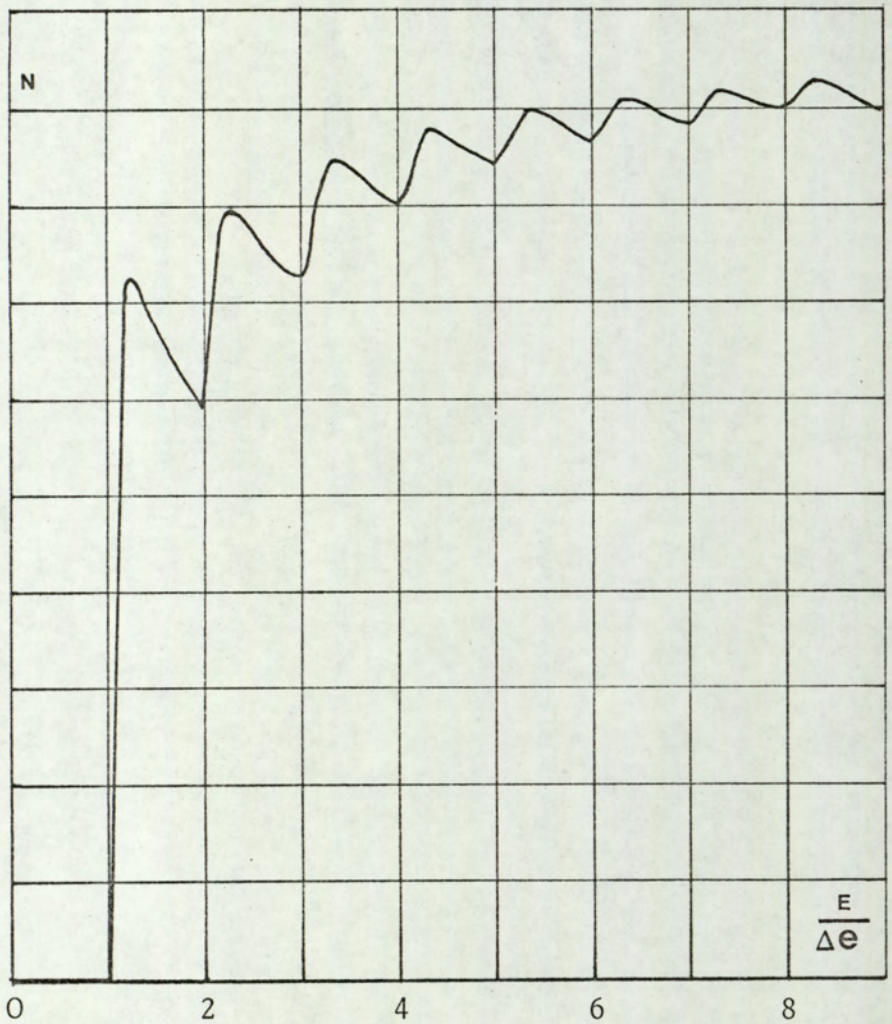


Fig. 2.10 RMS describing function of multi-position control elements.

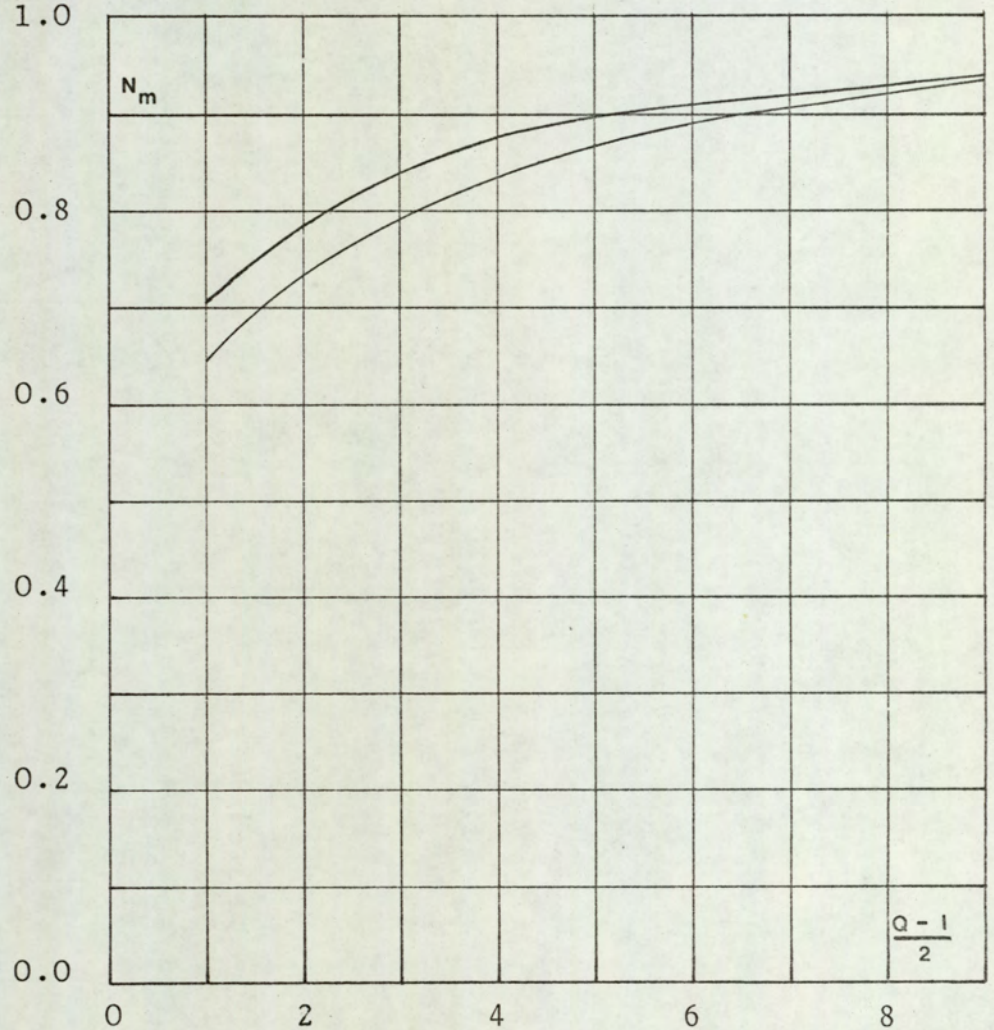


Fig. 2.11 Global maxima of RMS and conventional describing functions.

$$N = (m_1/E).2M.\Delta e = \left\{ \frac{8}{\pi y^2} \left[ \frac{k^2\pi}{4M^2} - \sum_{j=1}^{k} \frac{2j-1}{2M^2} \sin^{-1}\left(\frac{j.\Delta e}{E}\right) \right]^{\frac{1}{2}} \right\} \quad (2.8)$$

where $y = E/M.\Delta e$

$k$ is the integer part of $E/\Delta e$

and $k < M$

The RMS describing function is similar in many respects to the conventional describing function. A plot of N vs $E/\Delta e$ reveals M maxima, the value of N at these maxima, increasing with $E/\Delta e$. As before the global maxima occurs when $k = M$. The RMS describing function is displayed in Figure 2.10. Figure 2.11 displays the global maximum of the describing function as a function of $(Q - 1)/2$ for both the RMS and the conventional describing functions. It is observed that the RMS describing function yields higher values for $N_m$ than the conventional describing function. As Q is increased, however this difference is reduced.

COROLLARY

The proportional control element described in section 2.1 may be considered as an infinite-position control element with the slope given by

$$K = \lim_{\substack{Q \to \infty \\ \Delta e \to 0}} 1/(Q - 1)\Delta e$$

The RMS describing function is displayed in Figure 2.12. It is observed that the global maximum value of the describing function $N_m$ is 1.

## 2.5 STABILITY CRITERIA

The block diagram of Figure 2.3 can be redrawn as shown in Figure 2.13 with the control element split into its direct and oscillatory parts. KN is the describing function of the

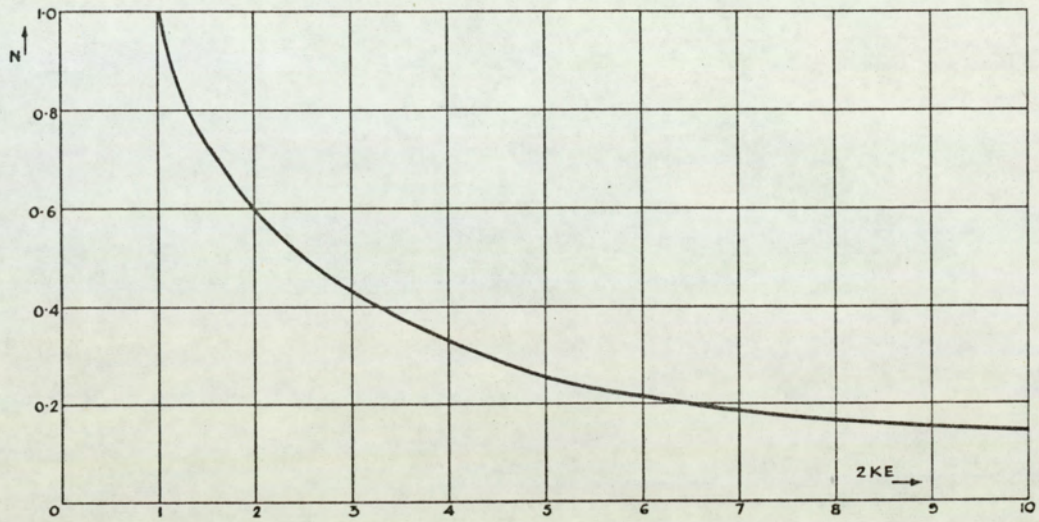


Fig. 2.12 RMS describing function of a proportional control element. Note that the maximum value of the describing function is 1.

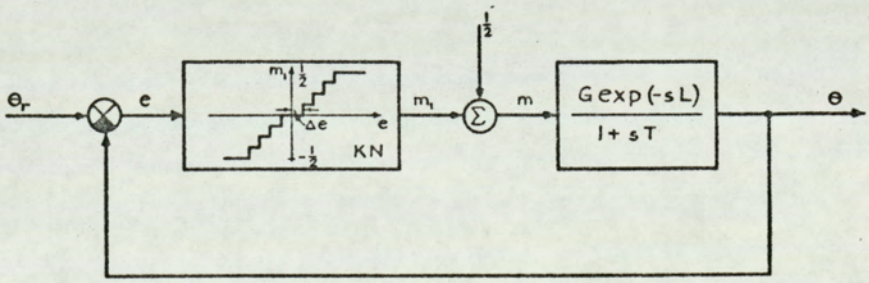


Fig. 2.13 Block diagram of the control system of Fig. 2.3 with the control element split into its direct and oscillatory components.

oscillatory component of the control element; while the direct component is shown as a separate input into the process. Since the control element considered is odd-symmetrical, condition 2 formulated in section 2.2 requires that

$$\theta_r/G = 1/2$$

The stability analysis, when asymmetrical control elements are used, will be presented in section 2.6.

The transfer function of the controlled electroheat process, shown in Figure 2.13 is

$$\frac{\theta}{\theta_r}(j\omega) = \frac{(1 + KNG).G_p(j\omega)}{1 + KNG.\ G_p(j\omega)} \tag{2.10}$$

where $G_p(j\omega)$ is the transfer function of the process and has the form given by equation (1.1) or (1.2).

Defining $\mu$ as $\mu = KG$, equation (2.10) can be written as

$$\frac{\theta}{\theta_r}(j\omega) = \frac{(1 + N\mu).G_p(j\omega)}{1 + N\mu.G_p(j\omega)} \tag{2.11}$$

The stability of the control system is determined by the location of the zeros of the characteristic equation i.e. the zeros of

$$1 + N\mu.G_p(j\omega) = 0 \tag{2.12}$$

This requires

$$|N\mu.G_p(j\omega)| = 1$$

and

$$\arg\ (G_p(j\omega)) = \pi \tag{2.13}$$

Equations (2.13) which can be solved graphically or analytically determine the stability boundaries for the controlled process.

For processes that can be modelled by equation (1.1) where

$$G_p(j\omega) = \exp(-j\omega L)/(1 + j\omega T)$$

equations (2.13) take the form

$$N\mu = (1 + \omega^2 T^2)^{\frac{1}{2}} \tag{2.14}$$

$$\omega L + \tan^{-1}(\omega T) = \pi \tag{2.15}$$

Eliminating $\omega T$ from the above equations, one obtains

$$L/T = \frac{\pi - \tan^{-1}(N^2\mu^2 - 1)^{\frac{1}{2}}}{(N^2\mu^2 - 1)^{\frac{1}{2}}} \tag{2.16}$$

Equation (2.16) determines the stability boundaries for processes modelled by equation (1.1). Figures 2.14 (a),(b), and (c) display $L/T$ as a function of $N_m.\mu$. Since $N_m$ is known from Figure 2.11, $\mu$ and therefore the slope of the control element is easily determined.

It is worth noting that equation (2.16) may also be used for the determination of stability of processes controlled by proportional control elements. In this case, the maximum value of N is 1; hence equation (2.16) reduces to

$$L/T = \frac{\pi - \tan^{-1}(\mu^2 - 1)^{\frac{1}{2}}}{(\mu^2 - 1)^{\frac{1}{2}}} \tag{2.16a}$$

The above result corresponds with the stability criterion developed by Roots and Wu[10].

### 2.5.1 EXAMPLE

An example will be worked out that illustrates the ease with which stability conditions can be evaluated. An electroheat process with a time constant of 1 hour, which can develop a maximum temperature of 1000°C, is to be

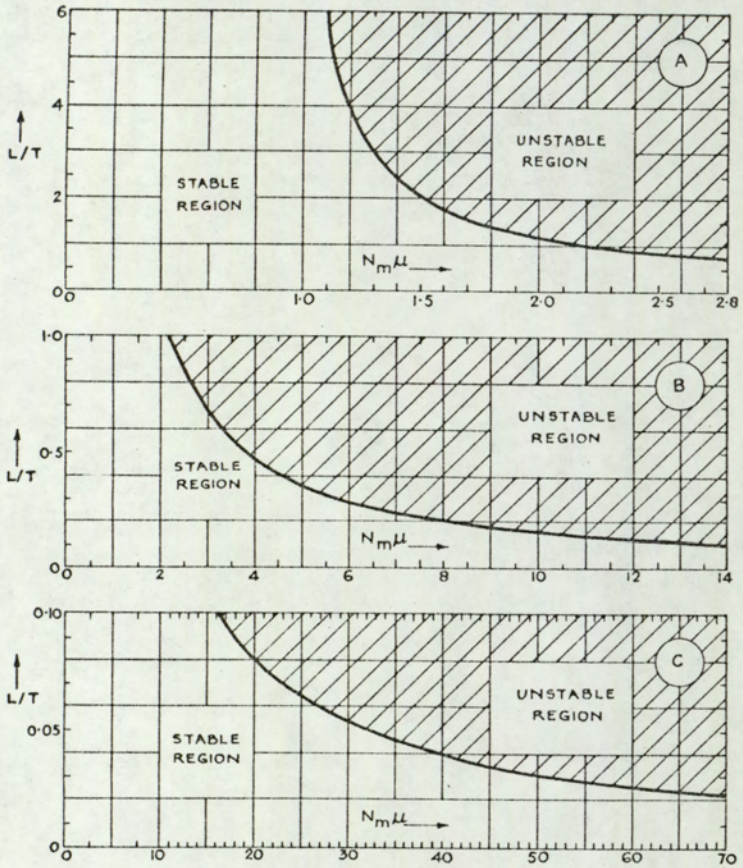


Fig. 2.14 Stability boundaries of a first order transit delay process.

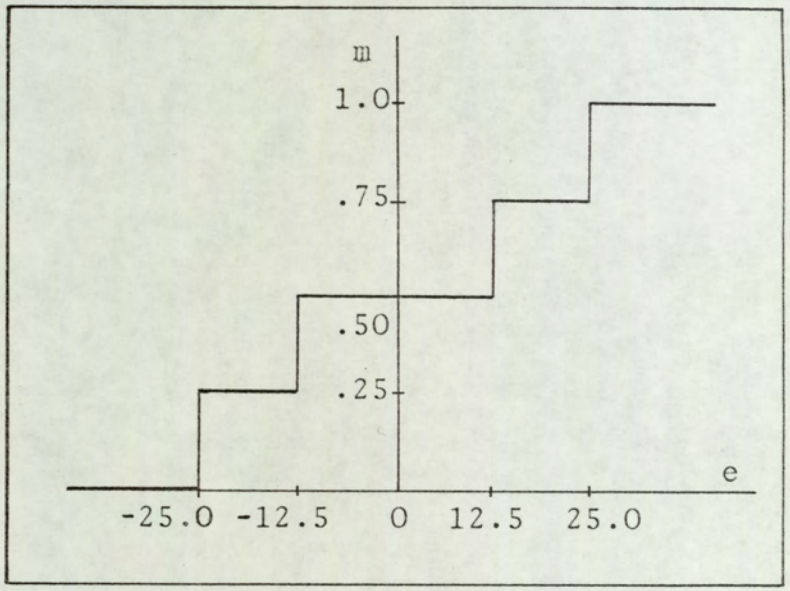


Fig. 2.15 Critical control element characteristic for the example of section 2.5.1.

controlled by a five-position controller that enables the following output values to be realised: 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1. It is desired to control the temperature at 500°C. The transit delay due to the distributed nature of the thermal system is 6 minutes. Determine the calibration settings for the controller so that the process operation is stable. The process is modelled by equation (1.1).

For this example, $L/T = 0.1$, $G = 1000^{\circ}C$, $\theta_r/G = \frac{1}{2}$

and $$Q = 5, \quad (Q - 1)/2 = 2$$

From Figure 2.11, $N_m$ (RMS) for a five-position controller is found to be 0.8021. From Figure 2.14 (c), $N_m\mu$ for $L/T = 0.1$ is found to be 16.

Therefore $$\mu = 16/0.8021$$

$$= 19.948$$

For a linearly distributed control element, the calibration increment is found to be

$$\Delta e = G/\mu(Q - 1) = 1000/(19.948).(4)$$

$$= 12.531^{\circ}C$$

Any calibration increment less than the above value will yield a stable control of the process. The critical control element characteristic is shown in Figure 2.15.

<u>For electroheat processes modelled by equation (1.2)</u>, the analytical procedure outlined above, may be used; but the graphical procedure, discussed in the subsequent development is shown to be more straightforward and simpler. This method is a modification of the Bode diagrams, specially adapted to the evaluation of stability boundaries for electroheat processes.

In this method, equations (2.13) are written in terms of the

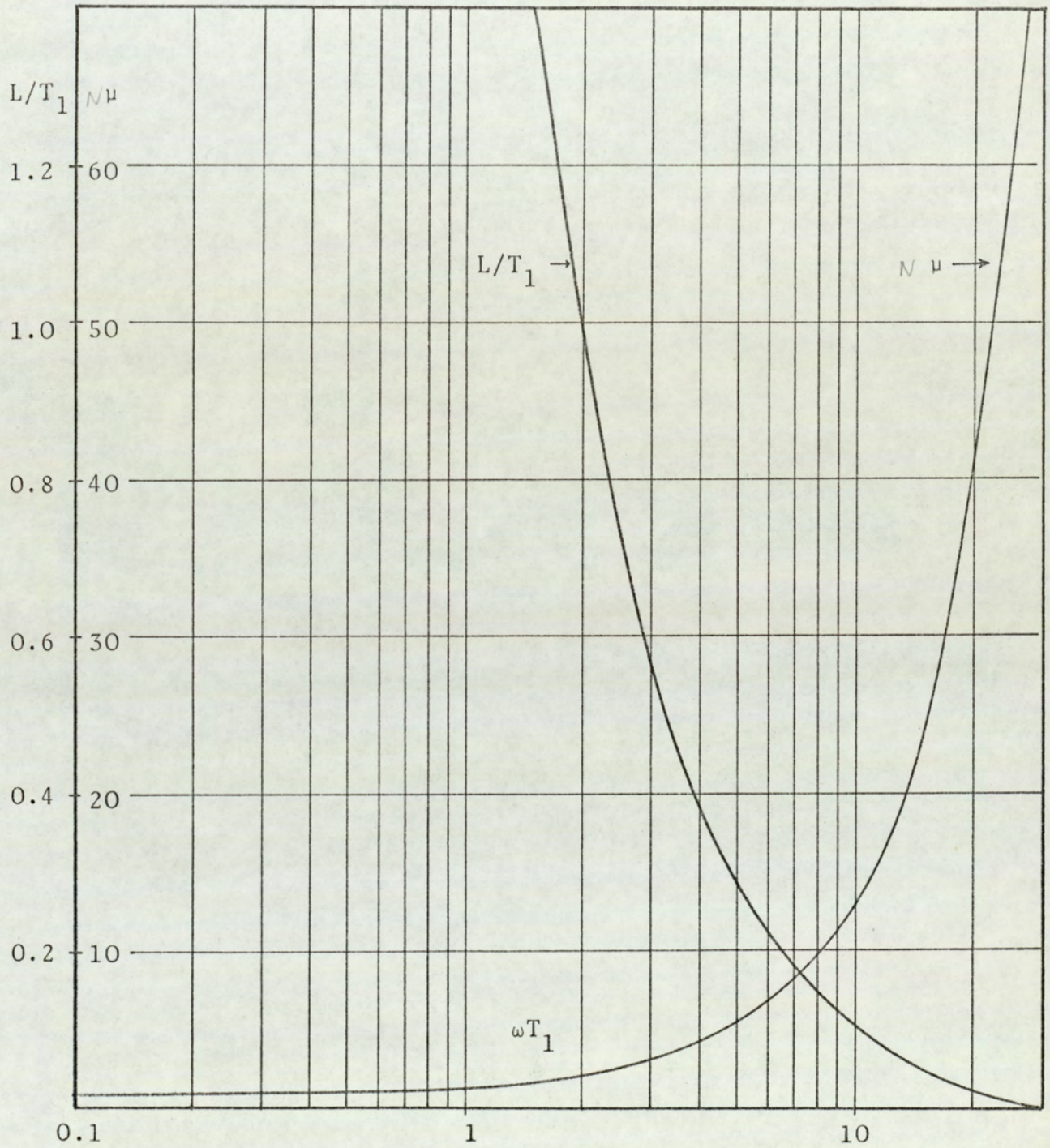


Fig. 2.16 Critical frequency loci for second order electro-heat process with transit delay.

process parameters. In the case of processes modelled by equation (1.1), equations (2.13) take the following form:

$$N\mu = \{(1 + \omega^2 T_1{}^2)(1 + \omega^2 T_2{}^2)\}^{\frac{1}{2}}$$

$$L/T_1 = \frac{\pi - \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)}{\omega T_1} \qquad (2.18)$$

Equation (2.18) represents two functions of $\omega T_1$ and these are plotted in Figure 2.16 for $T_2/T_1 = 0.1$. The evaluation of stability, now reduces to the determination of $N\mu$ for a given $L/T_1$ or vice-versa. For a given $L/T_1$, the critical frequency is determined from Figure 2.16; the corresponding value of $N\mu$ at this critical frequency, is the maximum gain allowed for the process.

For example, if $L/T_1 = \frac{1}{8}$, and $T_2/T_1 = 0.1$, the critical frequency $(\omega_c . T_1)$ is found to be 8.1. The value of $N\mu$ at this frequency is 10.5. Therefore for stability

$$N\mu < 10.5$$

or

$$\mu < \text{Min}(10.5/N) = 10.5/N_m$$

$N_m$ is obtained from Figure 2.11.

The above procedure illustrates the ease with which stability conditions are determined. This method obviates the need for Bode diagrams, Nyquist plots and complex analytical methods. Using the above procedure, the stability boundaries for processes modelled by equation (1.2), were determined for the following typical process parameters:

$$T_2/T_1 = 0.1, 0.5 \text{ and } 1.0$$

These are displayed on a logarithmic scale in Figure 2.17.

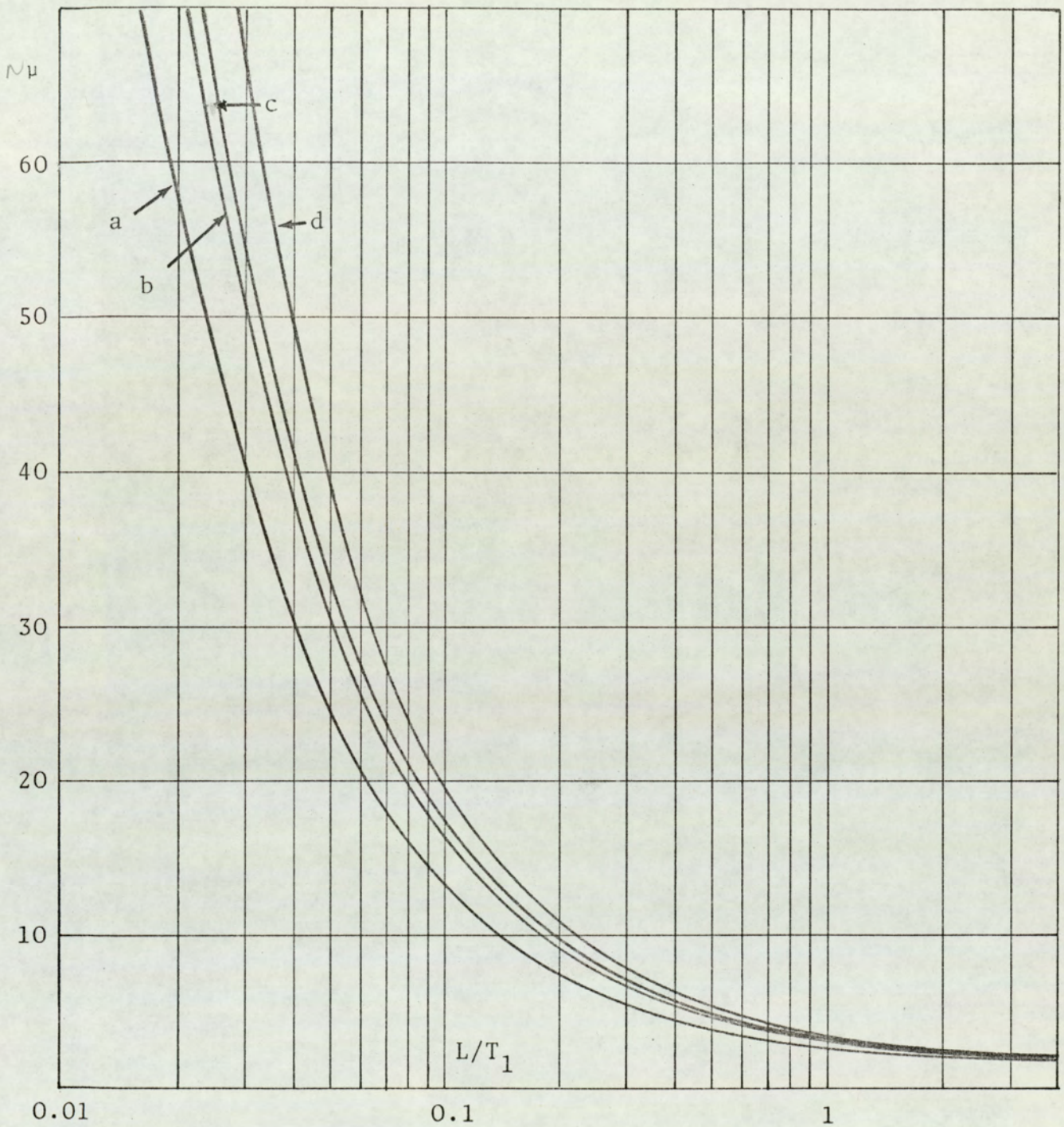


Fig. 2.17 Stability boundaries for second order electroheat processes with transit delay.

a) $T_2/T_1 = 0.1$ b) $T_2/T_1 = 0.5$ c) $T_2/T_1 = 0.0$
d) $T_2/T_1 = 1.0$ .

## 2.6 STABILITY OF PROCESSES WITH ASYMMETRICALLY DISTRIBUTED CONTROL ELEMENTS

In section 2.5, the stability criteria for processes, where the normalised input command $\theta_r/G = \frac{1}{2}$, were derived. The analysis of stability, when asymmetrical control elements are used, is complicated by the fact that in the event of instability, the input to the control element is comprised of a direct component (offset or droop) in addition to the oscillatory signal (assumed sinusoidal). A dual-input describing function has been proposed by Douce et al[11], for the analysis of stability in such cases. The procedure, however, is very cumbersome and not easy to apply.

Fortunately, electroheat processes designed for input commands $\theta_r/G = \frac{1}{2}$, were also stable when $\theta_r/G \neq 0.5$. The critical condition for stability in such systems was found to be determined only by the symmetrical region of the control element. This is illustrated in Figure 2.18 (a) for a five-position control element with the normalised input command $\theta_r/G = \frac{1}{4}$. The output of the control element assumed any of the following values: 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1. The symmetrical region in this case is A-B-C-D-E; and the slope of the control element K for a stable operation of the process is the same as the slope of a three-position control element with $\theta_r/G = \frac{1}{2}$. Similarly, if $\theta_r/G = \frac{3}{4}$ the five-position control element has the characteristics shown in Figure 2.18(b). The symmetrical region in this case is P-Q-R-S-T and the slope K for stable operation is the same as the slope of a three-position control element with $\theta_r/G = \frac{1}{2}$.

Thus the stability of electroheat processes controlled in this manner, can be easily determined in terms of the stability of processes controlled by odd-symmetrical control elements. The following example will illustrate the application of this technique.

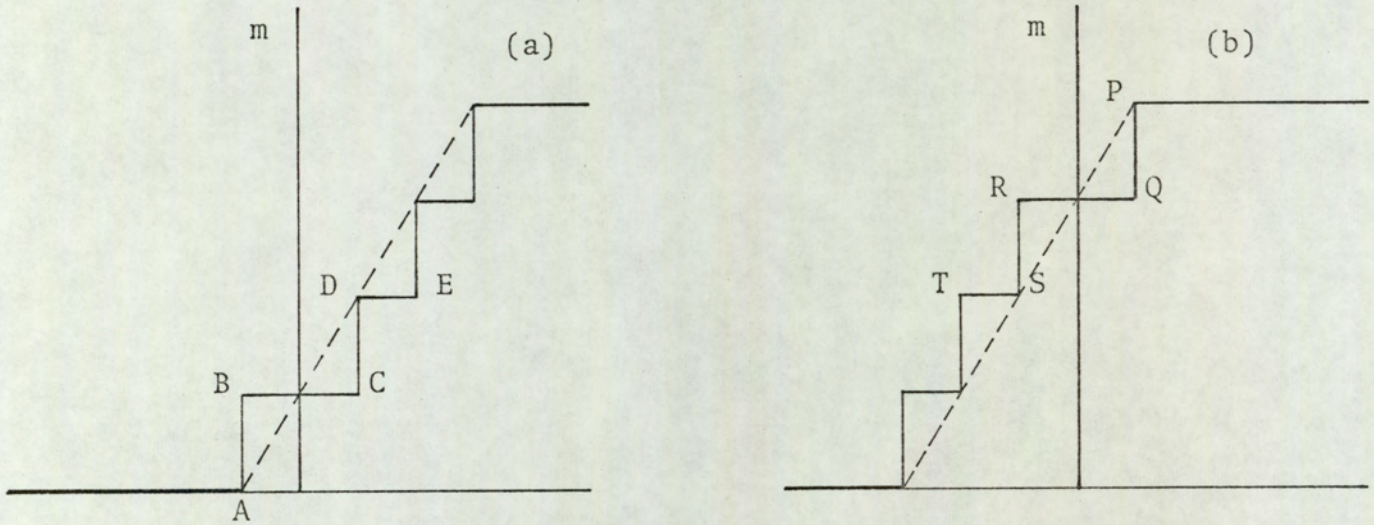


Fig. 2.18 Symmetrical regions of asymmetrical control elements.

An electric furnace has the following operating conditions:

1) The process has the following transfer function:

$$G_p(s) = \frac{G \exp(-sL)}{1 + sT}$$

where $G = 1000^\circ C$
$L/T = 0.1$

Determine the stable operating conditions for the following input commands:

$$\theta_r = 250^\circ C,\ 500^\circ C \text{ and } 750^\circ C$$

2) A five-position control element whose output can assume the following values: 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1. is to be used for the control of temperature.

<u>SOLUTION</u>:

The calibration increment for $\theta_r = 500^\circ C$ has been found from the example(section 2.5.1) to be

$$\Delta e \Big|_{\theta_r = 500^\circ C} = 12.531^\circ C$$

The input commands $\theta_r = 250^\circ C$ and $750^\circ C$ correspond to $\theta_r/G$ ratios of $\frac{1}{4}$ and $\frac{3}{4}$ respectively. As outlined earlier the stability for such commands, depends only on the symmetrical region of the control element; for a five-position control element, the calibration increment $\Delta e$ is the same as for a three-position control element with the following operating conditions:

$$\theta_r/G = \tfrac{1}{2}$$

$$G = 500^\circ C$$

From Figure 2.11, $N_m$ for a three position control element is found to be 0.7242. From figure 2.14, $N_m\mu$ for L/T = 0.1 is

found to be 16. Therefore

$$\mu = 16/0.7242 = 22.2$$

$$\Delta e = 500/(22.2)(2) = 11.2^{\circ}C$$

The complete solution is then

$$\Delta e \Big|_{\theta_r = 500^{\circ}C} = 12.53^{\circ}C$$

$$\Delta e \Big|_{\theta_r = \{^{250^{\circ}C}_{750^{\circ}C}} = 11.2^{\circ}C$$

A control system designed for $\theta_r/G = \frac{1}{2}$, is also stable for other input commands, provided, the conditions formulated in section2.2 are fulfilled. An asymmetrical control element permits a higher critical gain (smaller calibration increment) for the same process.The critical gain, however is not altered, if a proportional control element is used as the control element, since the stability boudaries are independent of the input command.

## 2.7 DISCUSSION

The stability boundaries for electroheat processes modelled by equations (1.1) and (1.2) were determined in sections 2.5 and 2.6; facile displays that enabled stability evaluation in terms of normalised parameters of the process were presented.

From the displays presented in the earlier sections, it is observed that the normalised transit delay ($L/T$ or $L/T_1$) is the most significant parameter of the process in the analysis of stability. While a small delay permits high gain, a large delay causes the process to go unstable even when the gain of the controller is small; this is particularly important for processes in which the delay is caused by the location of the temperature transducer. For such processes the stability analysis yields valuable information on the

maximum permissible delay in the process.

In the case of processes that can be represented by equation (1.2), the stability is also affected by the relative magnitude of the two time constants $T_1$ and $T_2$. Frequently, the smaller time constant is due to the inertia of the heating apparatus; it is therefore assumed often with justification, that the smaller time constant $T_2$ is negligibly small. This assumption is valid, only, if the smaller time constant is at least twenty times smaller than the major time constant i.e. $T_2/T_1 < 0.05$. Comparison of the stability boundaries of the processes modelled by equation (1.2) reveals the following:

1) For $0 < (T_2/T_1) < 0.05$, the stability of the processes is essentially determined by the major time constant $T_1$.
2) When $T_2/T_1 \simeq 0.1$, the stability is decreased.
3) When $T_2/T_1 \simeq 0.5$, the stability is improved and is found to be very nearly the same as that of a first order process.
4) When $T_2/T_1 \simeq 1.0$, the stability of the process is further improved.
5) The frequency at which instability occurs, is however different for the four cases mentioned above.

The effect of the transit delay and the relative magnitude of the two time constants, is easily observed in Figure 2.17. Test results based on a digital simulation of electroheat processes is discussed in chapter 5.

SUMMARY

A unified approach to the determination of stability of electroheat processes controlled continuously or discontinuously, has been presented. Facile displays that determine the stability boundaries in terms of the normalised parameters of the process are intended for the professional engineer

engaged in the design of control systems for electroheat processes.

Chapter 3

# TRANSIENT RESPONSE OF ELECTROHEAT PROCESSES

# TRANSIENT RESPONSE OF ELECTROHEAT PROCESSES

## 3.1. INTRODUCTION

The analysis of stability presented in Chapter 2, does not yield any information on the transient behaviour of closed-loop controlled electroheat processes. The prediction of transient behaviour of such processes especially when they are controlled discontinuously, is very difficult; this is due to the lack of a precise solution of the differential equations governing such processes. The difficulty is further enhanced by the transit delay inherent in electroheat processes. In this chapter new generalised graphical techniques are presented that enable the design of closed-loop control systems to given specifications.

A new exponential describing function, which has not been proposed before, is described; this describing function derives conditions for non-oscillatory response, when electroheat processes are controlled discontinuously. Exact solutions, where possible, are derived for non-oscillatory response to step inputs, when discontinuous controllers are used.

## 3.2 TRANSIENT RESPONSE OF CONTINUOUSLY CONTROLLED PROCESSES

### 3.2.1. Proportional control

The prediction of transient response of any controlled system is based on the knowledge of the distribution of closed loop poles and zeros. The distribution of the poles and zeros depends on the open loop gain and the open loop poles and zeros.

If the open loop system has a transfer function G(s) and the open loop gain is $\mu$ , then the closed-loop transfer function of a unity feedback system is

$$\frac{\theta}{\theta_r}(s) = \mu G(s)/\{1 + \mu G(s)\}$$

The closed loop zeros are therefore the same as the open-loop zeros; whilst the closed loop poles are the roots of the characteristic equation

$$1 + \mu G(s) = 0 \tag{3.1a}$$

or the values of s that satisfy

$$G(s) = -1/\mu \tag{3.1b}$$

The following relations exist between the nature of closed loop poles and the transient response:

1) If all the roots of the characteristic equation are real and negative, then the transient response to any of the standard inputs (viz. step, ramp, impulse etc.) is non-oscillatory.

2) If any of the roots are complex, having negative real parts, the response is damped oscillatory.

3) If any of the roots are imaginary, then the transient response is undamped oscillatory.

4) If one or more of the roots have positive real parts, then the system is unstable.

Thus it is essential to know the nature of the roots of equation (3.1), for predicting transient response.

For a process modelled by (1.1) the characteristic equation has been shown to be

$$1 + sT + \mu\exp(-sL) = 0 \tag{3.2a}$$

or

$$\exp(-sL)/(1 + sT) = -1/\mu \tag{3.2b}$$

The solution of equations (3.2) may be obtained graphically by the root locus technique[13]. The technique may however be modified so that a general technique for the design of control systems is available.

A generalised root technique, applicable to processes with any L and T is achieved by the transformation of the Laplace complex variable s as follows:

The transfer function of the process is given by:

$$G_1(s) = \mu\exp(-sL)/(1 + sT)$$

This may be written as

$$G_1(s) = \mu\exp(L/T).\exp\{-(sL + L/T)\}/(1 + sT)$$

$$= \mu\exp(L/T).\exp\{-(sL + L/T)\}/(sL + L/T)(T/L)$$

If one defines $u = sL + L/T$

and $\eta = (\mu L/T)\exp(L/T)$

the transfer function of the process may be written as:

$$G_1(u) = \eta\exp(-u)/u$$

With the above transformation, equation (3.2b) takes the form:

$$\exp(-u)/u = -1/\eta \tag{3.3}$$

Since equation (3.3) is independent of L and T, the solution of this equation is applicable to electroheat processes with any combination of L and T.

The complete solution of equation (3.3) is obtained by plotting the root locus as $\eta$ is varied and superimposing on these the constant gain ($\eta$ ) contours.

The root loci is plotted as follows [14]:

i) Construct the phase-loci for exp(-u) and (1/u). These are displayed in Figures 3.1 (a) and (b).

ii) Superimpose the phase locus of exp(-u) on the phase-locus of (1/u) as shown in Figure 3.2.

iii) Join all the points where the sum of the two phase angles is $\pm\pi$ ($180^{\circ}$), by a smooth curve. This curve is then a branch of the root locus for a phase angle of $\pm\pi$. Similar branches may then be constructed for other phase angles by joining those points where the total phase is $\pm(2i - 1)\pi$, $i = 1,2,\cdots\cdots\cdots\cdots$. Figure 3.2 displays the branches for phase angles of $\pi$ and $3\pi$. It will be shown later that it is sufficient to consider these two branches in the design of controllers for electroheat processes.

iv) From (iii), it is clear that the root-loci for processes with transit delay, will have infinite branches.

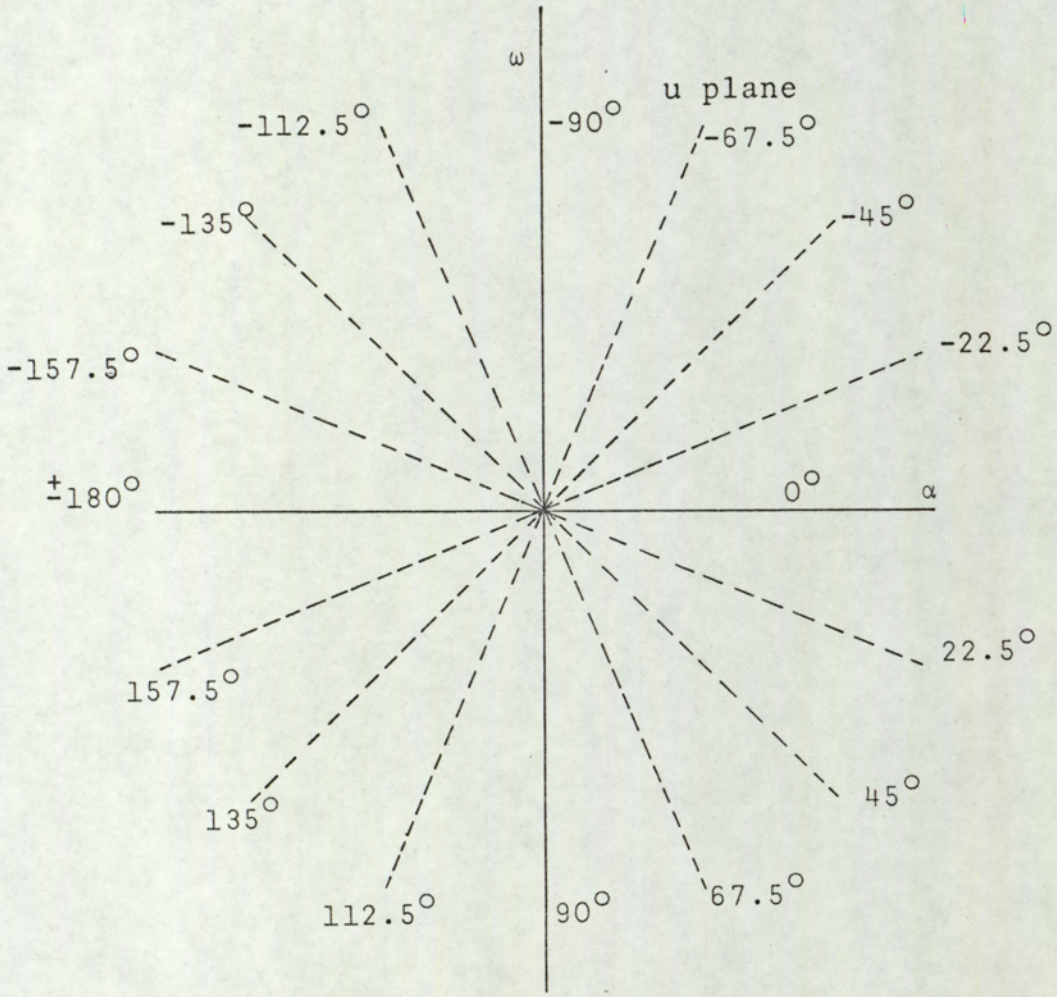


Fig. 3.1 (a) Phase-loci of (1/u).

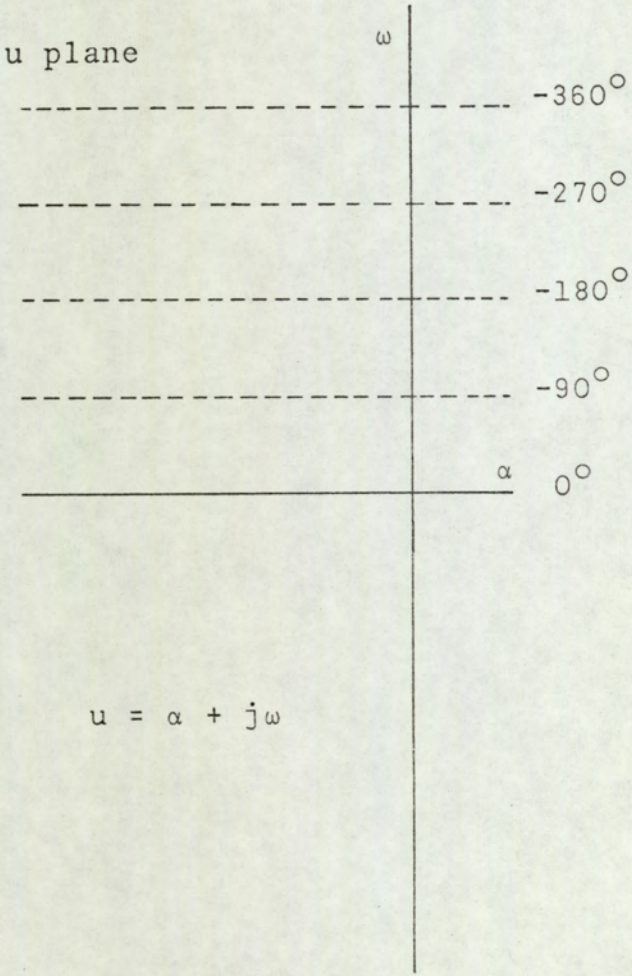


Fig. 3.1 (b) Phase-loci of exp(-u)

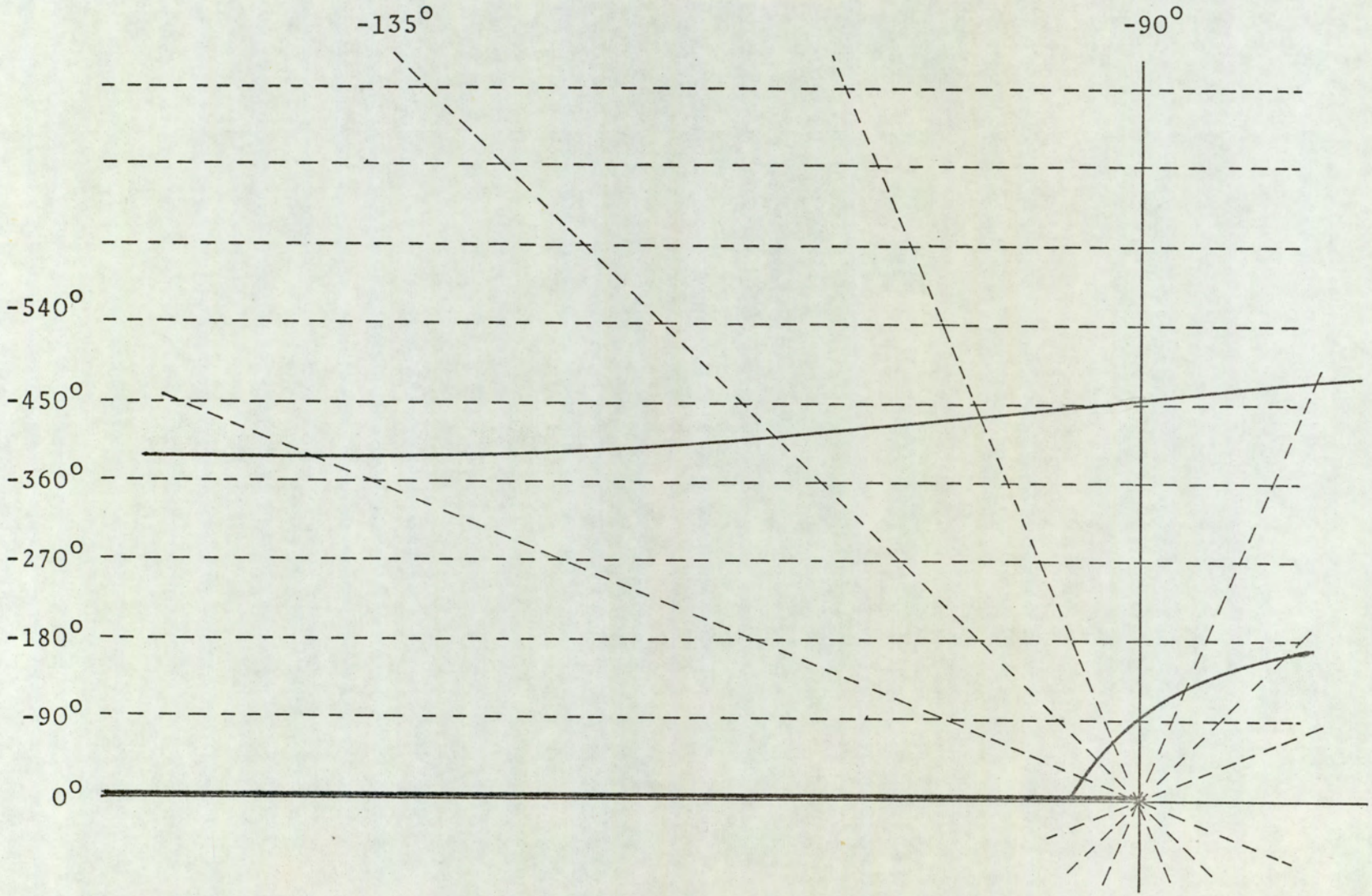


Fig. 3.2 Root-loci plots for a first order process with transit delay.

Since the root-loci are symmetrical about the real axis (all complex roots occur in conjugate pairs), only one half of the loci need be shown.

The gain $\eta$ corresponding to any root ( a point on the root loci) is determined by the relation

$$\eta = |1/G_1(u)|$$

$$= |u.\exp(u)|$$

If $u = \alpha + j\omega$ is a root, then $\eta$ is given by

$$\eta = \exp(\alpha)\{\alpha^2 + \omega^2\}^{\frac{1}{2}} \qquad (3.4)$$

While there is only one value of $\eta$ corresponding to a given root (ie: a point on the root loci) the converse is not true. Thus for a given $\eta$ there are infinite roots one on each branch of the root loci. In order to predict the transient response characteristics of a process it is necessary to have a knowledge of the distribution of the roots for a given gain $\eta$ . This is facilitated by constructing constant gain ($\eta$ ) contours and superimposing these on the root loci plots of Figure 3.2. The equation governing the constant gain contour is derived from equation (3.4), where

$$\eta = \exp(\alpha)\{\alpha^2 + \omega^2\}^{\frac{1}{2}}$$

Expressing $\omega$ in terms of $\eta$ and $\alpha$ one obtains

$$\omega^2 = \eta^2\exp(-2\alpha) - \alpha^2$$

Then for a given $\eta$, $\omega$ is a function of $\alpha$ and this is plotted on the u-plane. By constructing families of such curves for various values of $\eta$ the constant gain contours are obtained. These are shown superimposed on the root-loci plots in Figure 3.3. The intersections of the constant gain contour with the branches of the root loci determine the various roots corresponding to a given gain $\eta$ . By the use of the inverse

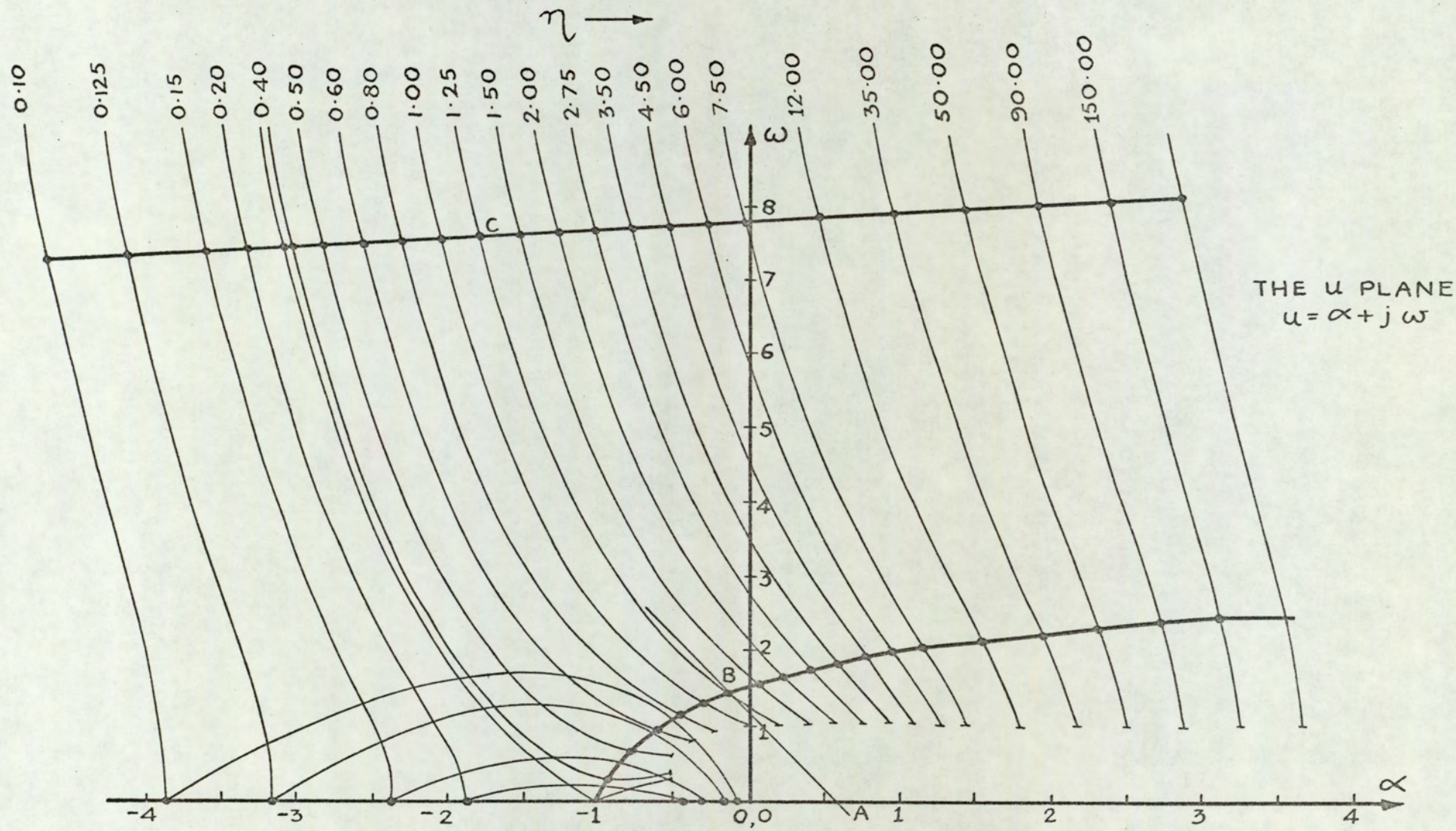


Fig. 3.3 Constant gain ($\eta$) contours superimposed on the root-loci plots of Fig. 3.2.

transformation.

$$s = (u - L/T)/L$$

and

$$\mu = \eta \exp(-L/T)/(L/T)$$

The closed loop poles of the controlled electroheat process in the s-domain, corresponding to a given $\mu$ are easily determined.

It is seen that by the use of the generalised root loci and the constant gain contours* the complete solution of a controlled electroheat problem can be obtained. Also the transformation of the complex variable s to u enables the same curves to be used for any process that can be modelled by equation ( 1.1).

The problem of controller design is now reduced to the choice of gain $\mu$ (or $\eta$ ) that yields the desired response. From Figure 3.3 it is observed that, to a given choice of $\eta$ (and therefore $\mu$ ) there are at the most two real roots, while all the other roots are complex. One may then conclude that for processes that have transit delay, a <u>nonoscillatory response cannot be obtained</u>. This directly contradicts the findings of Roots and Wu [15]. This contradiction is due to the fact that previous work[15] considered only one branch of the root-loci, namely the branch corresponding to a phase-lag of $180^{o}$( $\pi$ ); it has clearly been shown that root-loci for the processes considered here have infinite branches and that real roots are found on only the branch corresponding to a phase of $180^{o}$.

In selecting the gain $\mu$ (or $\eta$ ) for the process it is sufficient to take into account the roots on the two branches of root loci corresponding to a phase lag of $180^{o}$ ( $\pi$ ) and $540^{o}$ (3 $\pi$). The roots on the other branches have such large negative real parts that they may be safely ignored.

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* The author acknowledges the help of Prof. L.D.Meeker in the formulation and plotting of constant gain contours.

The procedure for selecting the gain $\mu$ for a desired response subject to the restrictions mentioned above, is:

i) Since in most electroheat processes, it is required to restrict overshoot to less than 5% the required damping factor $\zeta$ is 0.7 [13]. This corresponds to predominant poles located in the s-plane at an angle of $45^{o}$.

ii) In the u-plane, locate the point A (Figure 3.3) on the positive real axis such that OA = L/T. At A construct an angle of $45^{o}$ intersecting the lower branches of root-loci at B as shown in Figure 3.3.

iii) Determine the gain $\eta$ at the point B. By using the constant gain ($\eta$) contours locate the root on the higher branch (point C in Figure 3.3).

iv) If the roots at B and C are denoted by $u_1$ and $u_2$ then the corresponding poles on the s-plane are obtained from the relation

$$p_i = (u_i - L/T)/L, \quad i = 1,2$$

v) Since the root-loci are symmetrical about the real axis ther will be additional poles $p_1^*$ and $p_2^*$ being the complex conjugates of $p_1$ and $p_2$ respectively.

vi) The required gain $\mu$ is obtained from $\eta$ by the relation

$$\mu = \eta \exp(-L/T)/(L/T)$$

vii) The closed-loop transfer function is then

$$\frac{\mu \exp(-sL)}{1 + sT + \mu \exp(-sL)} \simeq \frac{\mu p_1 p_2 p_1^* p_2^* \exp(-sL)}{(1+\mu)(s-p_1)(s-p_1^*)(s-p_2)(s-p_2^*)} \qquad (35)$$

where $p_1$, $p_2$, $p_1^*$, $p_2^*$ are obtained from (iv) and (v).

viii) It is only required to shift the point A for any other value of L/T. The same procedure may then be repeated.

It may be easily verified from any standard text on control systems that the step response of a controlled process whose transfer function is given by equation (3.5) has an overshoot of about 5-6% provided $\mu$ is chosen according to the method outlined above.

### 3.2.2. Proportional-integral control

If an electroheat process is subject to sustained disturbances (viz. load variations, power supply variations, ambient temperature changes etc) a proportional controller alone does not yield an acceptable performance; this requires an additional integral control, whereby the manipulated variable is continuously adjusted until in the steady state the error is eliminated.

A method will be outlined whereby the same set of curves developed in the previous section for proportional control, is used to choose the controller parameters for the desired response.

The transfer function of the electroheat process in cascade with a proportional-integral controller has the form

$$G_c(s).G_p(s) = \frac{\{K + (1/T_R s)\}G\exp(-sL)}{1 + sT}$$

where
- $K$ = sensitivity of proportional control
- $T_R$ = Reset time of integral control
- $G$ = Gain of heating apparatus
- $G_c(s) = K + (1/T_R s)$ = controller transfer function
- $G_p(s) = G\exp(-sL)/(1+sT)$ = process transfer function
- $L$, $T$ = process parameters

The transfer function after simplification may be written as

$$G_c(s).G_p(s) = \frac{\{KT_R s + 1\}G.\exp(-sL)}{T_R s\ (1 + sT)} \qquad (3.6)$$

If $T_R$ in equation (3.6) is made large such that $KT_R >> 1$ then

$$G_c(s).G_p(s) \simeq KG \exp(-sL)/(1 + sT)$$
(proportional control)

If $T_R$ is chosen small such that $KT_R << 1$ then

$$G_c(s).G_p(s) \simeq G \exp(-sL)/T_R s(1 + sT)$$
(integral control)

It is seen that for large $T_R$ the controller action is similar to proportional control, and for small $T_R$ an integral control is realised. With small $T_R$, however, the gain of the system is increased to a large value and consequently results in a poor performance. A large $T_R$ on the other hand yields a response similar to proportional control; consequently the response to disturbance inputs is very sluggish.

Experimental investigations[5 12] and theoretical considerations (outlined above) reveal that the benefits of integral control are realised if $T_R$ is chosen such that

$$KT_R = T \quad \text{or} \quad K = T/T_R$$

Equation (3.6) with $T_R$ as above reduces to

$$G_c(s).G_p(s) = G \exp(-sL)/T_R s \tag{3.7}$$

If one defines

$$u = sL$$
$$\text{and} \quad \eta = L.G/T_R$$

equation (3.7) may be written as

$$G_c(u).G_p(u) = \eta \exp(-u)/u$$

The characteristic equation then takes the form

$$\exp(-u)/u = -1/\eta$$

This is the same as equation (3.3). Hence the generalised root-loci technique developed in section 3.2.1 may be utilised to determine $\eta$ for the desired response. The only difference in the procedure for the choice of $\eta$ is that the angle of $45^{\circ}$ is constructed at the origin and the intersection of this with the branch of root-loci corresponding to a phase lag of $\pi$, yields the predominant closed loop poles. The corresponding value of $\eta$ and therefore K, $T_R$ is then easily obtained from the relation

$$\eta = |1/G_c(u)\ G_p(u)|$$

Thus the generalised root-loci enable the design of both proportional and proportional-integral controllers for electroheat processes, by the use of the same set of root-loci and constant gain contours. It is in the author's opinion, a simple, yet analytically sound approach to the design of continuous controllers for electroheat processes.

## 3.3 TRANSIENT RESPONSE OF DISCONTINUOUSLY CONTROLLED ELECTROHEAT PROCESSES

The transient response of electroheat processes when discontinuously controlled presents a unique problem. An exact solution will be outlined for On-Off and three-position controllers when used for processes modelled by (1.1). An exponential describing function for the analysis of transient response characteristics for multi-position discontinuously controlled processes is also presented.

### 3.3.1 EXACT SOLUTION OF TRANSIENT RESPONSE CHARACTERISTICS

#### a) ON-OFF CONTROL

The On-Off control of electroheat processes has been exhaustively treated in several articles[16] and the results will be briefly quoted here.

The response of On-Off controlled processes to step temperature commands is always oscillatory, the period and the offset error being dependant on the magnitude of the input command. The amplitude of oscillation, the period of oscillation, the type of

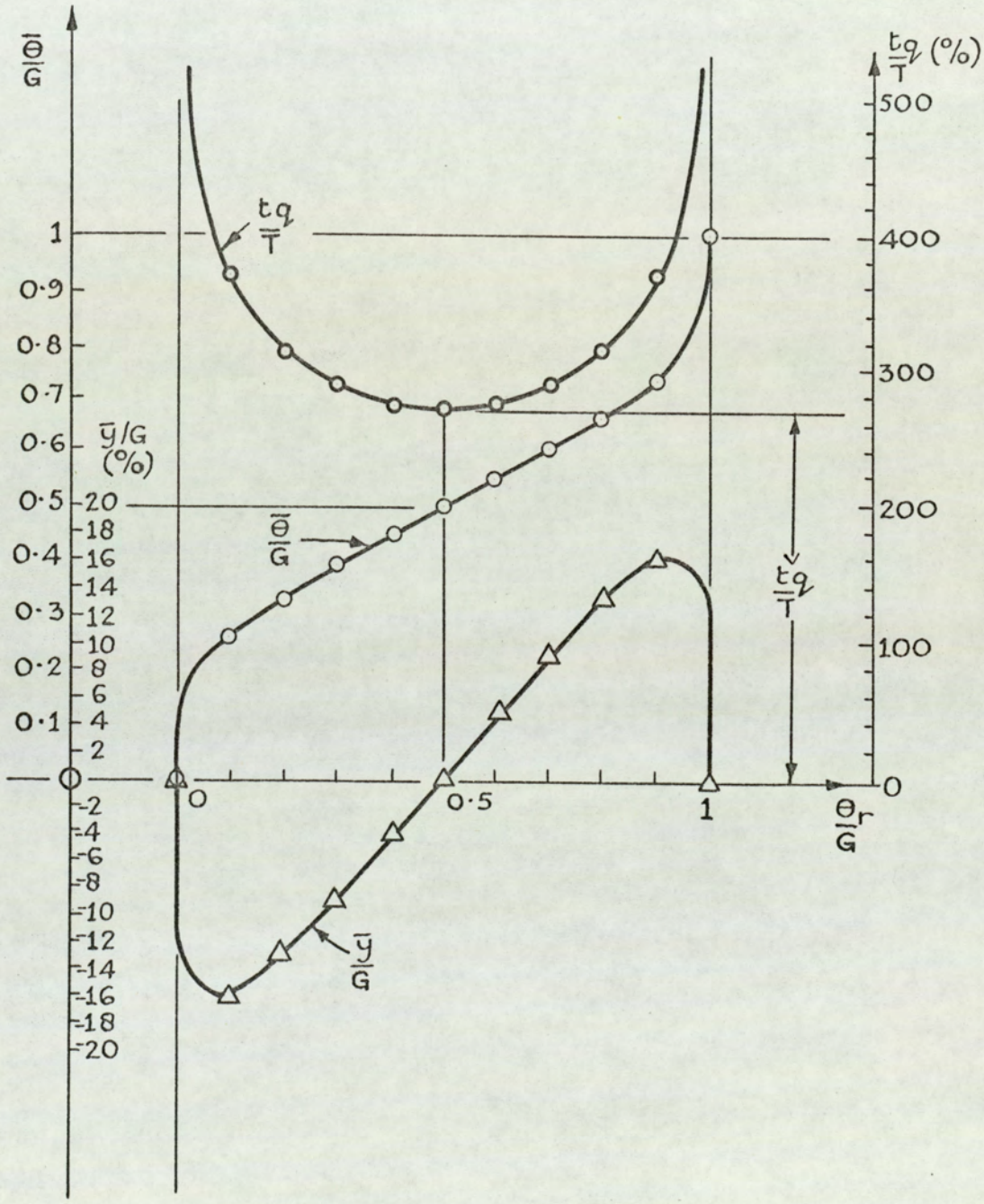


Fig. 3.5 Effect of input command on the parameters of an on-off controlled electroheat process.

oscillation and the offset error are given by

Amplitude $\theta_d = G\{1 - \exp(-L/T)\}$

Period $t_q = T.\ln \dfrac{G - \theta_n}{G - \theta_n} \dfrac{\theta_m}{\theta_n}$

where $\theta_m = \theta_r.\exp(-L/T) + G\{1 - \exp(-L/T)\}$ (maximum value of $\theta$)

$\theta_n = \theta_r.\exp(-L/T)$ (minimum value of $\theta$)

where

$\theta(t)$ is the temperature of the process
$L$ is the transit delay
$T$ is the time constant

Offset error $\theta_r - \bar{\theta}$

where $\bar{\theta}$ is the average value of $\theta(t)$

Average value $\bar{\theta} = \dfrac{1}{t_q} \displaystyle\int_0^{t_q} \theta(t)\, dt$

Figure 3.5 displays the variation of $\theta_d$, $t_q$, $\bar{y}$ in terms of the normalised parameters.

Figure 3.6 depicts the response of the process to a step-input.

From the displays of Figure 3.5 and 3.6 the following conclusions may be reached:

1) The response is always oscillatory.
2) The amplitude of oscillation increases as L/T is increased.
3) The period of oscillation is a function of the input command $\theta_r/G$ and is a minimum when $\theta_r/G = 0.5$.

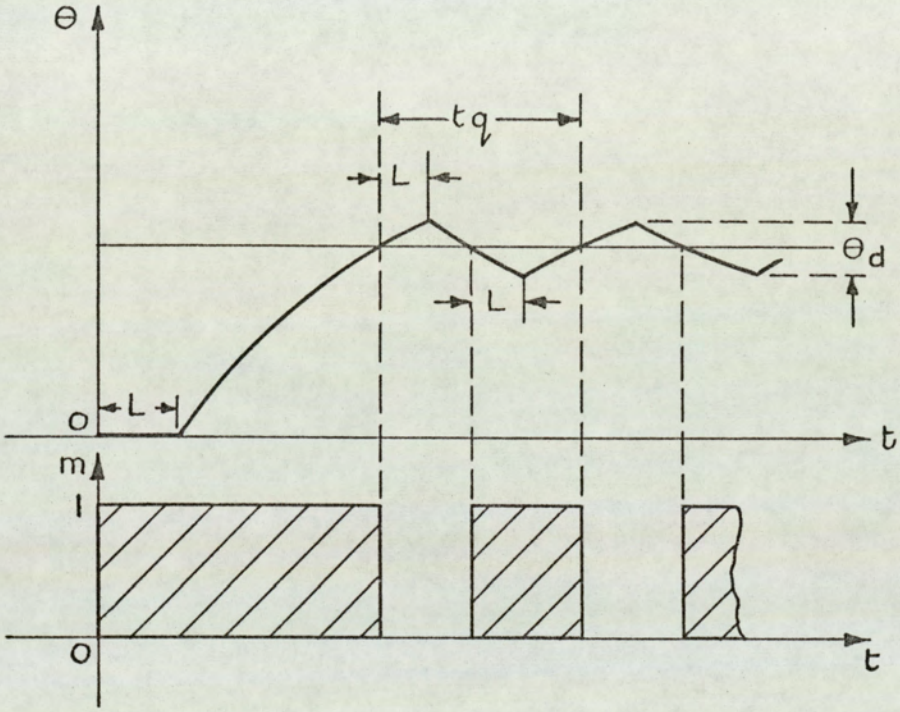


Fig. 3.6 Step response of an on-off controlled electroheat process.

4) The offset error is zero only if $\theta_r/G = 0.5$. It is negative if $\theta_r/G < 0.5$ and positive if $\theta_r/G > 0.5$

b) <u>3-position control</u>

In the case of a 3-position control, the output of the control element assumes three values viz. $m = 0, m_1, 1$. As mentioned in Chapter 2 (Section 2.2) it is required that

$$m_1 = \theta_r/G\big|_{\theta_r = \text{constant}}$$

The analysis is presented in the time-domain . The differential equation of the process is

$$T.D\theta(t) + \theta(t) = G.m(t - L)$$

The following characteristic will be assumed for the 3-position control element (see Figure 3.7a).

Assuming zero initial conditions ie:

$$\theta(0) = 0$$

$$\theta_r(t) = \theta_r, \quad t > 0$$

the initial trajectory of $\theta(t)$ is

$$\theta(t) = 0, \quad 0 < t < L \tag{3.8}$$

$$\theta(t) = G\left[1 - \exp\{-(t - L)/T\}\right], \quad t > L$$

and $m(t)$ is given by

$$m(t) = 1 \text{ for } t > 0 \text{ and } \theta(t) < \theta_r - \Delta e_1$$

The first switching ($m(t) = 1 \rightarrow m(t) = \theta_r/G$) will occur when $\theta(t) = \theta_r - \Delta e_1$. If the switching instant is denoted by $t_1$

$$\theta(t_1) = \theta_r - \Delta e_1 = G\{1 - \exp(-(t_1 - L)/T)\} \tag{3.9}$$

and

$$m(t_1) = \theta_r/G$$

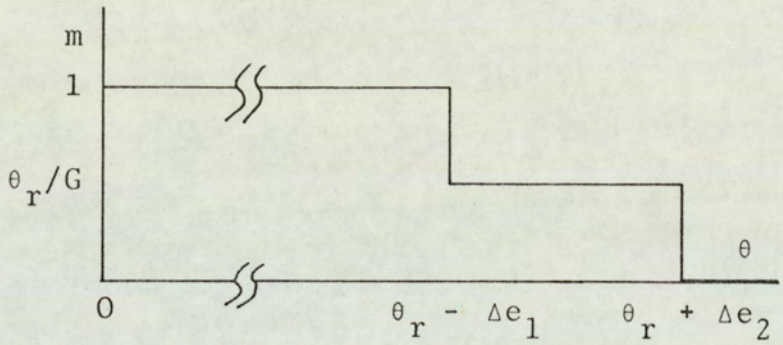


Fig. 3.7a Three-position control element characteristic as a function of output temperature of process.

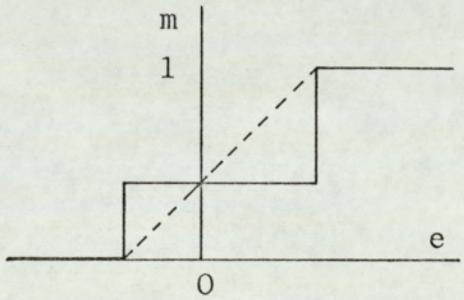


Fig. 3.7b Three-position control element characteristic for a nonoscillatory temperature response.

But $\theta(t)$ will continue along the trajectory given by equation (8) for the duration of the transit delay L, after switching. Thus

$$\theta(t_1 + L) = G\{1 - \exp(-t_1/T)\} \tag{3.10}$$

The conditions for a desired transient response may now be determined

If it is required that there be no overshoot then the condition is that

$$\theta(t_1 + L) = G\{1 - \exp(-t_1/T)\} \leq \theta_r \tag{3.11}$$

From equation (9) it may also be found that

$$\theta(t_1 + L) = (\theta_r - \Delta e_1)\exp(-L/T) + G\{1 - \exp(-L/T)\}$$

Condition (11) therefore reduces to

$$(\theta_r - \Delta e_1)\exp(-L/T) + G\{1 - \exp(-L/T)\} < \theta_r$$

Simplifying one obtains

$$\Delta e_1 > (G - \theta_r)\{\exp(L/T) - 1\} \tag{3.12}$$

The transient response of the process when $\Delta e_1$ is chosen according to the condition (3.12) is shown in Figure 3.7c

If the initial conditions were

$$\theta_o = \theta(0) > \theta_r + \Delta e_2$$

$$\theta_r(t) = \theta_r, \quad t > 0$$

then the $\theta(t)$ trajectory is given by

$$\theta(t) = \theta(0)\exp(-t/T), \quad m(t) = 0 \tag{3.13}$$

If the first switching occurs at $t = t_2$ then

$$\theta(t_2) = \theta_r + \Delta e_2 = \theta_o \exp(-t_2/T)$$

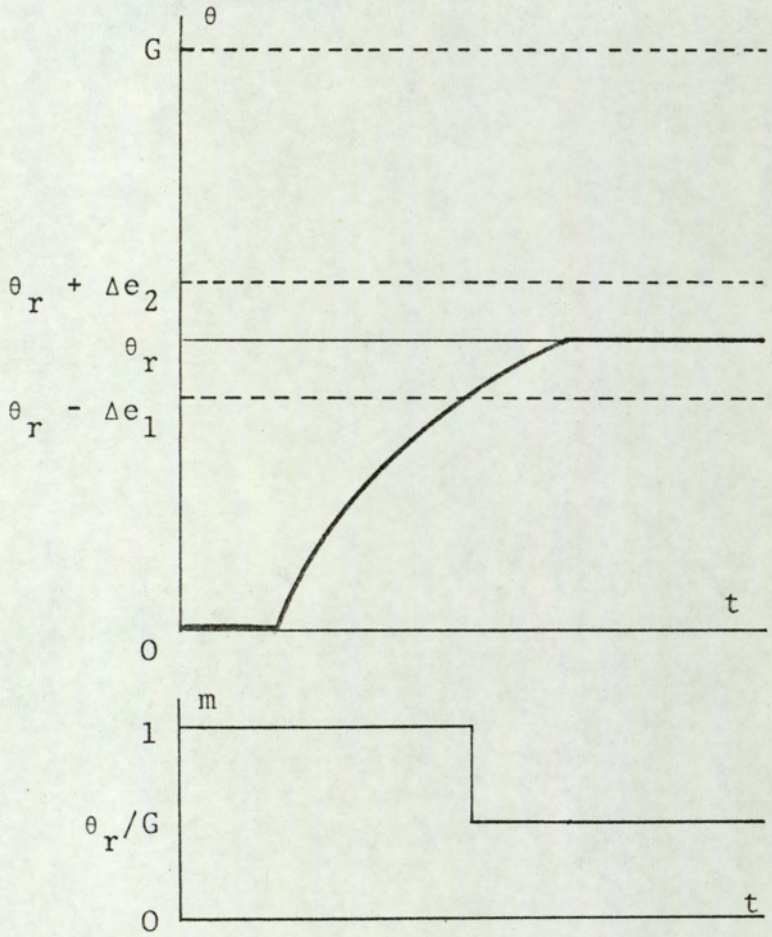


Fig. 3.7c Nonoscillatory response of a process with zero initial conditions.

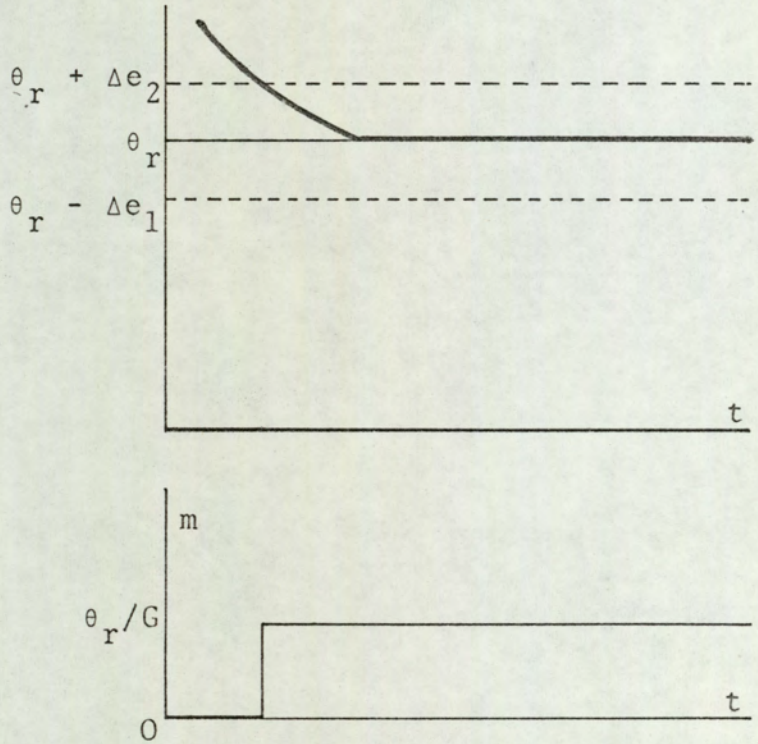


Fig. 3.7d Nonoscillatory response of a process with nonzero initial conditions.

and

$$m(t_2) = \theta_r/G$$

But $\theta(t)$ continues to follow the trajectory of equation (3.13) for the duration of the transit delay (L). Then

$$\theta(t_2 + L) = \theta_o \exp(-(t_2 + L)/T)$$

$$= (\theta_r + \Delta e_2)\exp(-L/T) \qquad (3.14)$$

For no overshoot, it is required that

$$(\theta_r + \Delta e_2)\exp(-L/T) > \theta_r$$

ie. $$\Delta e_2 > \theta_r\{\exp(+L/T) - 1\} \qquad (3.15)$$

Figure 3.7d displays the transient response to a step input when $\Delta e_2$ is chosen according to (3.15).

Thus the criteria for nonoscillatory response depends on the initial conditions. For a three-position controller, the best results are obtained with an asymmetrical distribution of the calibration increment about the e = 0 axis as shown in Figure 3.7b.

The choice of $\Delta e_1$ and $\Delta e_2$ is governed by the inequalities (3.12) and (3.15). It is worth pointing out that the sum of the two increments $\Delta e_1$ and $\Delta e_2$ is a constant when the controller is designed for critical damping. Thus

$$\Delta e_1 + \Delta e_2 = G\{\exp(+L/T) - 1\} \qquad (3.16a)$$

Under these conditions, one can define the slope of the control element as

$$K = 1/(\Delta e_1 + \Delta e_2)$$

or $$\mu = 1/(\exp(L/T) - 1) \qquad (3.16b)$$

With the choice of $\Delta e_1, \Delta e_2$ as noted above critically damped response is obtained under any given initial conditions.

### 3.3.2. Exponential Describing Function

An exponential describing function for determining the transient response of electroheat processes controlled by multiposition discontinuous elements is presented here. The results derived by using the describing function closely check with actual test results. This method, however, determines only the conditions for non-oscillatory response. Since in most electroheat processes, large overshoots (greater than 3-4%) are not acceptable, the analysis presented here will be of significance in the design of multiposition discontinuous controllers.

It may be recalled, that, in the analysis of stability, it was assumed that the input to the control element was a sinusoidal signal. The output was then related to the input by a sinusoidal describing function. It was therefore decided to investigate the relation of the output of the control element to the input when the input is an exponential function (as would be the case in the event of a nonoscillatory response).

Exponential describing functions have been used before in the transient analysis of nonlinear control systems [17]. But the results derived were very inaccurate when applied to processes with transit delay. The method described below was found to yield reasonably accurate results.

The analysis will be presented for the odd-symmetrical control elements shown in Figure 2.1. It will be assumed that the conditions formulated in Chapter 2 (Section 2.2) are fulfilled. Then the exponential input to the control element will be of the form

$$e(t) = G.\exp(-t/T_1)/2$$

where G is the gain of the heating apparatus
$T_1$ an arbitrary time constant.

The output of the control element will be as shown in Figure 3.8.

The output is seen to comprise a d.c. component and a transient component. The d.c. component whose magnitude is 1/2 indicates the power required to maintain the temperature at the value G/2; while the transient component may be considered as the excess power that raises the temperature from 0 to G/2.

The shaded area shown in Figure 3.8 then represents the heat energy consumed in the transient state. This area can be easily shown to be

$$N(M) = \frac{T_1}{2}\left[\ln\left(\frac{G}{2M.\Delta e}\right) + \sum_{j=1}^{M-1}\left(1 - \frac{j}{M}\right).\ln\left\{1 + \frac{1}{M - j}\right\}\right] \qquad M > 1 \tag{3.19}$$

$$= \frac{T_1}{2}.\ \ln\left(\frac{G}{2\Delta e}\right), \qquad M = 1$$

where $\Delta e$ is the calibration increment of the control element.

$M = \frac{Q - 1}{2}$ , Q-the number of possible values of M.

If K(M) is the slope of the control element = $\frac{1}{(Q-1).\Delta e}$

and $\mu(M) = K(M).G$

then

$$N(M) = \frac{T_1}{2}\{\ln\mu(M) + A(M)\} \tag{3.20}$$

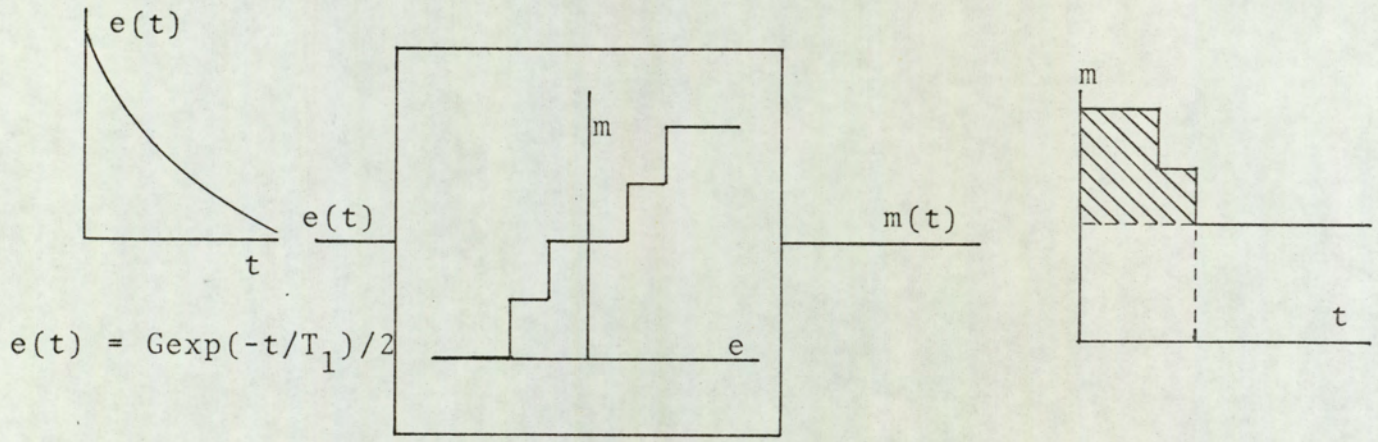


Fig. 3.8 Output of a multiposition discontinuous control element to an exponential input signal.

$$A(M) = 0 \ , \quad M = 1$$

$$A(M) = \sum_{j=1}^{M-1} \left[1 - (j/M)\right] \ln\left\{1 + \frac{1}{M - j}\right\} \qquad M > 1$$

$$A(M) = 1 \ , \quad M \to \infty$$

A plot of A(M) as a function of M is displayed in Figure 3.9.

The values of N derived above which represents the energy in the transient state should be equal, irrespective of the form of control (assuming a critically damped response). Thus

$$N(1) = N(2) = \cdots\cdots = N(M) = \cdots = N(\infty) \qquad (3.21)$$

An exact solution has been found for the transient response analysis of processes controlled by a 3-position controller. Then using equation (3.21) and the value of the gain $\mu$ obtained from the exact analysis for a 3-position controller (M = 1), the conditions for a nonoscillatory response to step input commands may be derived for multiposition discontinuous controllers (M > 2).

For a 3-position controller it was shown that (equation(3.16b))

$$\mu(1) = 1/(\exp(L/T) - 1)$$

From equation (3.21)

$$N(1) = N(M) \ , \quad M = 2,3, \cdots\cdots$$

ie: $$T_1.\ln\{\mu(1)\}/2 = T_1.\ \ln\{\mu(M) + A(M)\}$$

Simplifying

$$\mu(M) = \mu(1).\exp\{-A(M)\} \qquad (3.22)$$

where M = (Q - 1)/2 and Q is the number of values m assumes.

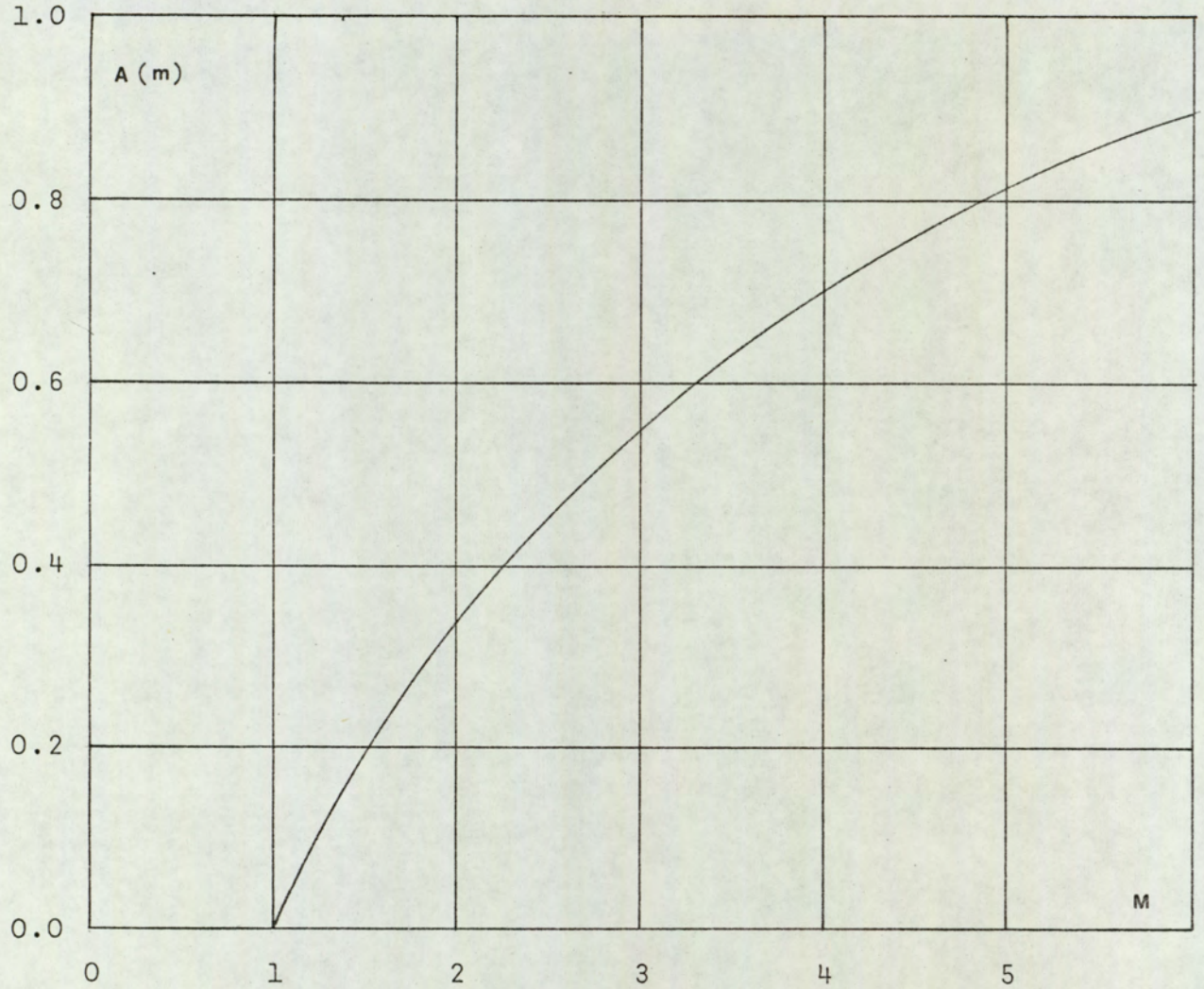


Fig. 3.9 Exponential describing function of multiposition discontinuous control elements.

Equation (3.22) enables the evaluation of $\mu(M)$ for a critically damped response when multiposition discontinuous controllers are used. $A(M)$ is obtained from Figure 3.9. Figure 3.10 displays $\mu(M)$ as a function of M for a process modelled by (1.1) where $L/T = 0.1, 0.25, 1$. The exact values derived from simulation studies is also displayed for comparison It is seen that for $L/T = 0.25$ the exponential describing function yields accurate results. The error is seen to increase with L/T. And the error is about 14% when $L/T = 1$. But such high values of L/T are not encountered in electroheat processes. In any case the models used for such processes are themselves only approximate; therefore values obtained within 10% accuracy are quite acceptable.

When the control element has an assymmetrical distribution the describing function postulated above may still be used; however a general describing function is no longer realisable, since the common features of symmetrical control elements do not exist in an asymmetrical control element. The procedure for determining the gain for a critically damped response will be illustrated for a 5-position control element.

When the input to the control element is an exponential function, the output is as shown in Figure 3.11 for $\theta_r/G = \frac{1}{4}$.

The transient output indicated by the shaded area in Figure 3.11 is given by

$$N(2) = \frac{3T_1}{4} \ln \frac{G}{12E} + \frac{T_1}{2} \ln \frac{3}{2} + \frac{T_1}{4} \ln 2$$

$$= \frac{3T_1}{4} \ln \frac{G}{12E} + \frac{T_1}{4} \ln \frac{9}{2} \qquad (3.23)$$

For a 3-position controller where exact solutions have been found, it can be shown that

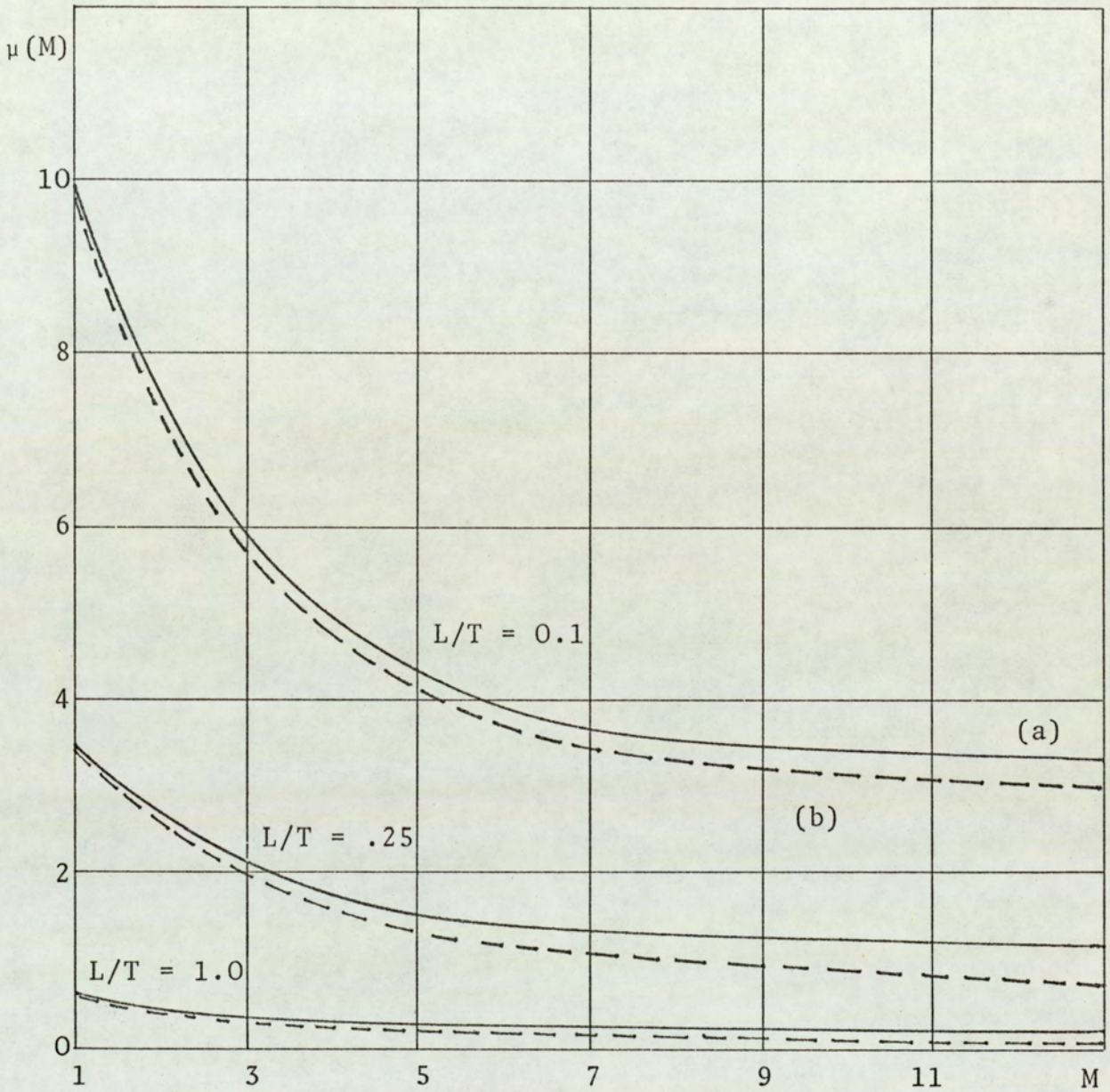


Fig. 3.10 a Boundaries for nonoscillatory response in a discontinuously controlled electroheat process as determined by (a) exponential describing function and (b) simulation on the digital computer. See Fig. 3.10 b for the determination of gain for nonoscillatory response.

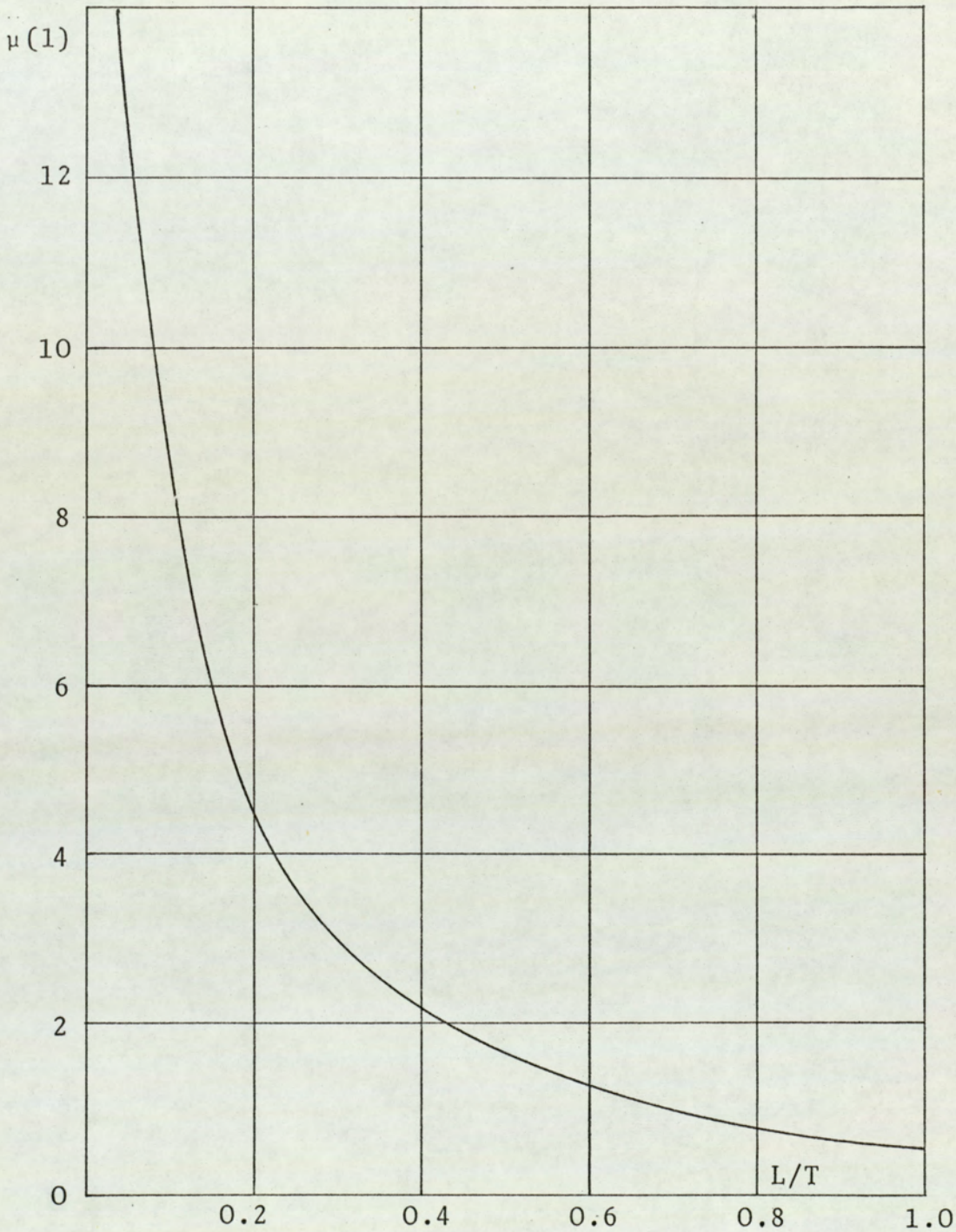


Fig. 3.10b Facile display for determining the gain of multi-position discontinuous controller for nonoscillatory process response. $\mu(M)$ is obtained from $\mu(1)$ by the relation $\mu(M) = \mu(1).\exp\{-A(M)\}$ and $A(M)$ is obtained from Fig.3.9.

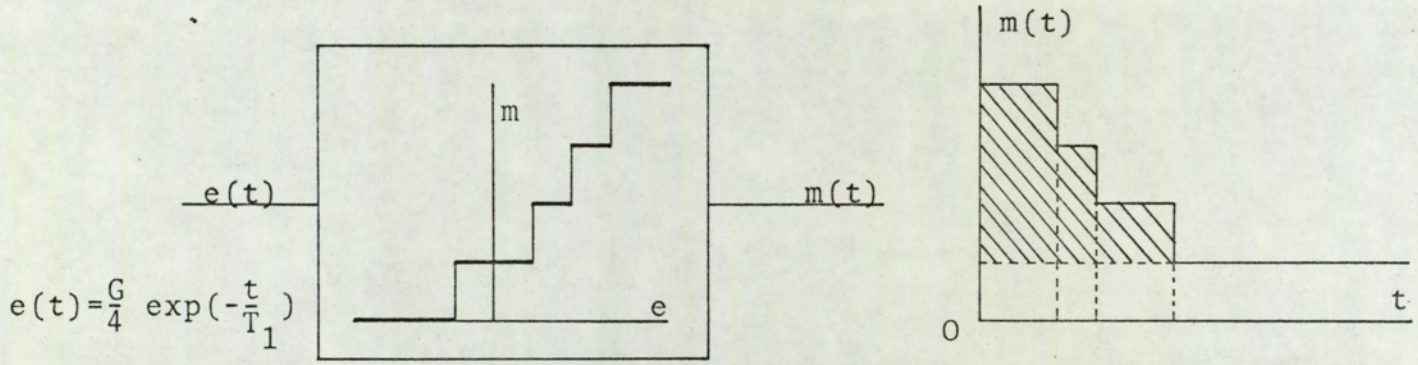


Fig. 3.11 Output of an asymmetrical control element to an exponential input signal.

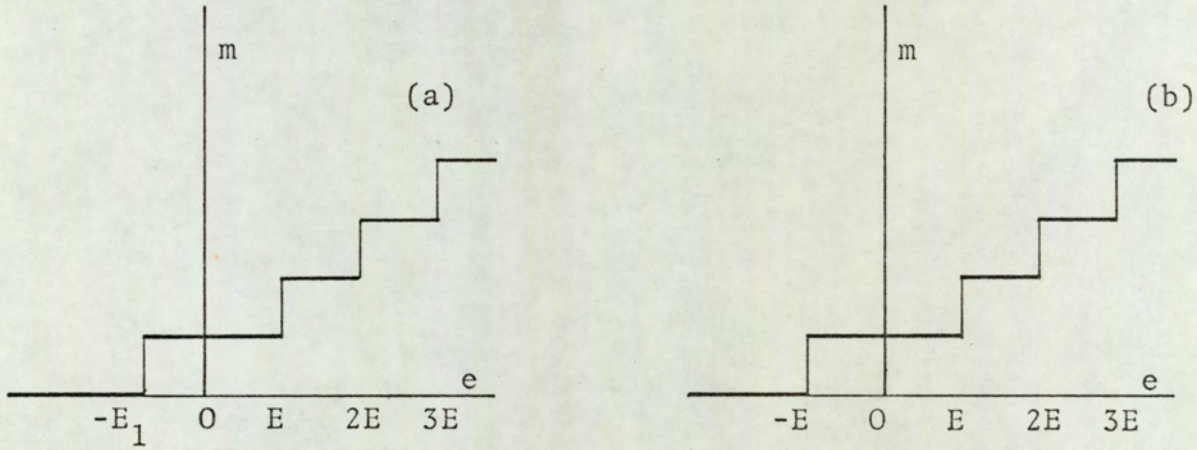


Fig. 3.12 Asymmetrical control elements for nonoscillatory response.
(a) Nonlinear control element (b) Linear control element.

$$N(1) = (3T_1/4).\ln(G/4\Delta e) \tag{3.24}$$

It is required that N(1) = N(2) as before. Equating (3.23) and (3.24) and simplifying

$$\frac{3}{4}\ln\frac{G}{12E} + \frac{1}{4}\ln\frac{9}{2} = \frac{3}{4}\ln\frac{G}{4\Delta e_1}$$

Therefore $$E = \frac{\Delta e_1}{3}\exp(1/2) = 0.55\Delta e_1$$

$\Delta e_1$ is found from equation (3.12) to be

$$\Delta e_1 = \frac{3G}{4}\{\exp(L/T) - 1\}$$

Therefore

$$E = 0.41\{G\exp(L/T) - 1\} \tag{3.25}$$

This is the value of the calibration increment when the system starts from zero initial conditions.

When the initial conditions for the process are

$$\theta(O) > \theta_r + E_1$$

Then the transient response is the same as that of a 3-position controller with m = O, ½, 1 and $\theta_r/G' = \frac{1}{2}$ and $G' = G/2$

Then the calibration increment $E_1$ may be found from (3.15) and is given by

$$E_1 = G\{\exp(L/T) - 1\}/4 \tag{3.26}$$

$E_1$ is thus seen to be less than E as given by equation (3.25) A non-linear control element would therefore give a critically damped response under any given initial conditions. If it is required to use a linearly distributed control element then the calibration increment is chosen according to equation (3.25) Figure 3.12 displays the linear and non-linear control element for a 5-position controller.

The use of exponential describing function thus enables the design of discontinuous controllers for electroheat processes. The procedure is quite simple, yet the results are reasonably accurate.

The analysis presented here has so far been restricted to processes modelled by (1.1). The extension to second order processes (1.2) is quite straight forward, provided the ratio of the two time constants $(T_2/T_1)$ is less than 0.1. Inaccuracy results if $(T_2/T_1)$ is greater than 0.1, since the input to the control element is no longer an exponential function. However rough estimates of the gain required may be obtained if the second-order process is simplified to first order process by the Zeigler-Nichols approximation [18].

## 3.4 SUMMARY

1) A new generalised root-locus technique has been presented that enables the design of continuous control systems to desired specifications.

2) Exact solutions have been derived for the transient response of electroheat processes when controlled by On-Off (two-step) 3-position controllers.

3) A new exponential describing function has been formulated that determines the criteria for critically damped response of multiposition discontinuous controllers. This is hoped to be a fruitful area for further research.

4) The unified stability analysis presented in Chapter 2 and the transient response characteristic presented in this Chapter have established powerful techniques for the design of controlled electroheat processes.

Chapter 4

# NEW CONTROL STRATEGIES FOR ELECTROHEAT PROCESSES

# NEW CONTROL STRATEGIES FOR ELECTROHEAT PROCESSES

## 4.1 INTRODUCTION

The design of proportional controllers for electroheat processes was discussed in Chapter 3. As is well known, the proportional controllers are incapable of compensating for sustained load disturbances; consequently, offset errors occur, often impairing the quality of the final product. For electroheat processes that are subject to sustained disturbances, the use of proportional- integrating controllers is recommended. The design of such controllers has been well covered in literature [5 19 20 21]. In this chapter new control strategies that lead to improved process performance are presented. In particular the following control schemes are investigated in detail:

i) The effect of derivative feedback in discontinuously controlled processes.

ii) The development of a nonsaturating model-referenced integrating controller that utilises the digital computer for both process simulation and on-line control.

iii) Development of a time optimal controller that incorporates a 3-position discontinuous controller with derivative feedback in cascade with a model-referenced integrating controller.

The analysis is mainly restricted to processes modelled by (1.1) . The extension of the analyses to processes modelled by (1.2) is indicated.

## 4.2 EFFECT OF DERIVATIVE FEEDBACK IN DISCONTINUOUSLY CONTROLLED ELECTROHEAT PROCESSES

The improvement in process performance that results, when derivative content is incorporated in the feedback loop of discon-

tinuously controlled processes is analytically examined. The analysis is presented for two different kinds of discontinuous controllers that are commonly used in electroheat control. These are the on-off (two-position) controller and the 3-position controller.

### 4.2.1. On-off controllers with derivative feedback

#### (a) FIRST ORDER PROCESS

The improvement in process performance as a result of derivative feedback in on-off controllers is well known and has been reported by several workers [22 23]. However the choice of the derivative content had always been empirical; this often resulted in on-off controllers designed with arbitrarily chosen derivative content causing droop errors and even uncontrollability (instability)[24]. An investigation was conducted into the effects of derivative feedback with the purpose of establishing a sound basis for the optimum derivative content in on-off controllers. The findings of this investigation have been published and is attached as Appendix 4A [25].

#### (b) SECOND ORDER PROCESS

For second order process (modelled by 1.2) an analytical solution is obtained by approximating the step-response as outlined by Zeigler-Nichols [26].

The step-response of a second order process is displayed in Figure 4.1a. This response may be approximated to a first order response, by incorporating a time-lag and an equivalent time constant as shown in Figure 4.1 b.

The procedure for this approximation is as follows:

1) Locate the point of inflection in the step response of the second-order system (point P in Figure 4.1 a). This point may also be determined analytically by finding the value of $\theta$, where the slope has a maximum value.

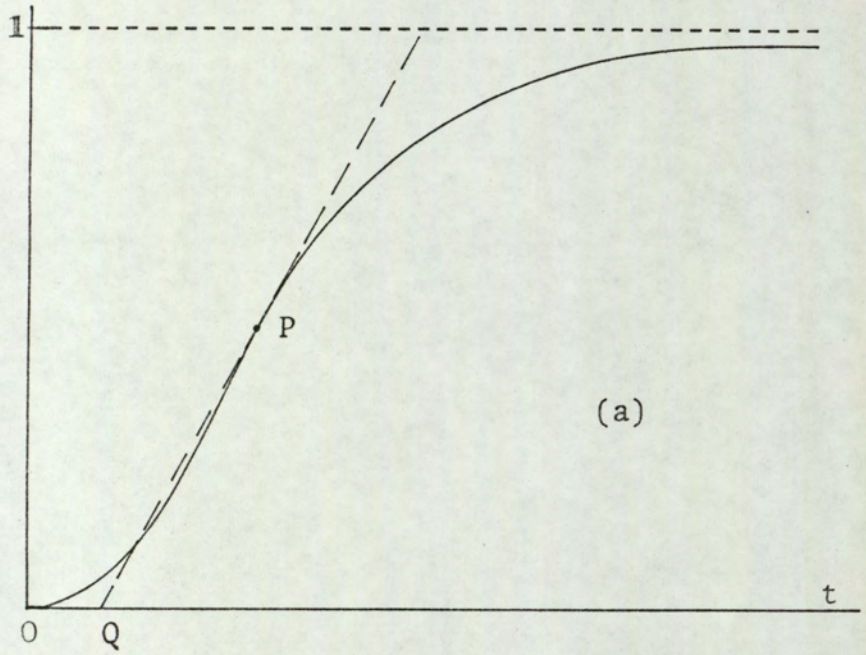


Fig. 4.1a Step response of a second order process.

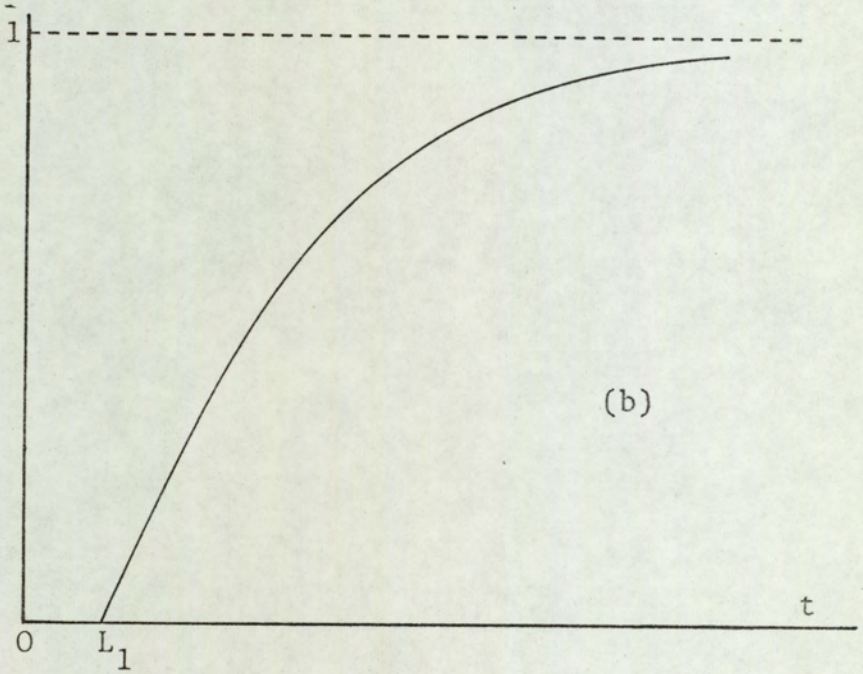


Fig. 4.1b Equivalent first order, transit delay model of second order process.

2) Determine the slope K of the step-response at the point P. Locate the intersection of this slope with the time axis (point Q in Figure 4.1a )

3) Then the first-order approximation of this process is given by:

$$\theta_a(s) = \exp(-sL_1)/(1 + sT)$$

where $L_1$ is the equivalent delay (OQ in Figure 4.1a )

$T$ is the equivalent time constant $= \frac{1}{K}$

With the above approximation, the second-order process with delay (1.2) may be written as

$$G_a(s) = \exp\{-s(L + L_1)\}/(1 + sT)$$

The analysis of derivative feedback may then be carried out, using the method outlined above.

### 4.2.2. 3-position controller with derivative feedback

The conditions for a critically damped response were derived for a 3-position controller, in Chapter 3 (Section 3.3.1.). It is shown in this section the improvement in process response when derivative feedback is incorporated in the controller. The optimum derivative content for the best performance is shown to result in an asymmetrically distributed control element. The analysis is presented in the time-domain. The block diagram of the control system is shown in Figure 4.2.

The differential equation of the controlled process is given by

$$TD\theta(t) + \theta(t) = Gm(t - L)$$

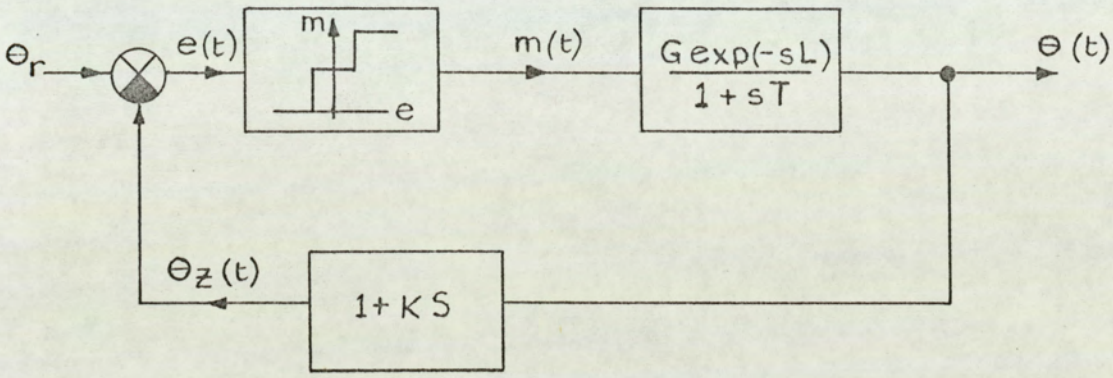


Fig. 4.2a Block diagram of a three-position control system incorporating derivative feedback.

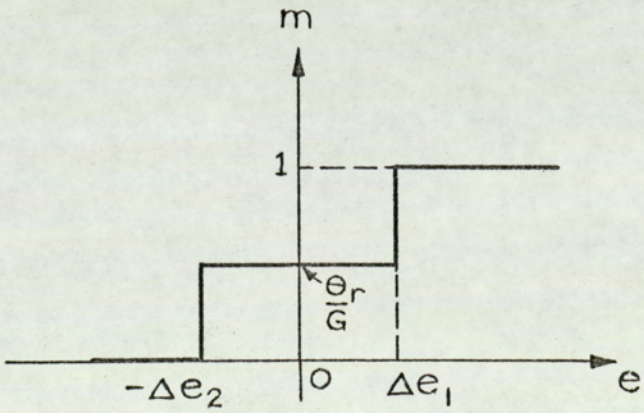


Fig. 4.2b Three-position control element characteristic for a nonoscillatory response.

If the system starts from the ambient temperature, then the temperature of the process is given by

$$\theta(t) = 0, \qquad 0 < t < L$$

$$\theta(t) = G(1 - \exp(L - t)/T), \qquad t > L$$

The rate of change of temperature is given by

$$D\theta(t) = 0, \qquad 0 < t < L$$

$$D\theta(t) = (G/T)\exp(-(t - L)/T), \quad t > L$$

The feedback signal is given by

$$\theta_f(t) = \theta(t) + KD\theta(t)$$

$$= G\{1 - \exp(-(t - L)/T)(1 - K/T\} \qquad (4.1)$$

where K is the derivative gain.

A switching action ($m = 1 \rightarrow m = \theta_r/G$) occurs at $t = t_1$, when

$$\theta_f(t) = \theta_r - \Delta e_1$$

Therefore

$$\theta_f(t_1) = G\{1 - \exp(-(t - L)/T)(1 - K/T)\}$$

$$= \theta_r - \Delta e_1 \qquad (4.2)$$

Due to the transit delay 1, $\theta_f(t)$ follows the m = 1 trajectory for the duration of the delay.

Then

$$\theta_f(t_1 + L) = G\{1 - \exp(-(t - L)/T)\ 1 - K/T\} \qquad (4.3)$$

Substituting the value of $\exp(-t_1/T)$ obtained from equation (4.2) in equation (4.3), one obtains

$$\theta_f(t_1 + L) = G - (G - \theta_r + \Delta e_1)\exp(-L/T) \qquad (4.4)$$

The corresponding value of $\theta(t)$ at $t = t_1 + L$ can be shown to be

$$\theta(t_1 + L) = G(1 - \exp(-t_1/T))$$

$$= \frac{G(1 - \exp(-L/T) - K/T) + (\theta_r - \Delta e_1)\exp(-L/T)}{(1 - K/T)} \quad (4.5)$$

For a critically damped response, it is required that

$$\theta_f(t_1 + L) < \theta_r + \Delta e_2 \quad (4.6a)$$

and

$$\theta(t_1 + L) = \theta_r \quad (4.6b)$$

Equation (4.6b) when solved for $\Delta e_1$, gives

$$\Delta e_1 = (G - \theta_r)(1 - \exp(-L/T) - K/T)\exp(L/T) \quad (4.7)$$

Substituting the value of $\Delta e_1$ in equation(4.4) and simplifying equation (4.6b), one obtains

$$\Delta e_2 > (K/T)(G - \theta_r) \quad (4.8)$$

Conditions (4.7) and (4.8) establish the criteria for a critically damped response (nonoscillatory), when the process starts from ambient temperature.

When the process initial conditions are different, ie. $\theta(0) = \theta_1 > \theta_r + \Delta e_2$, then a similar analysis yields the following conditions:

$$\Delta e_2 = \theta_r(1 - \exp(-L/T) - K/T)\exp(L/T) \quad (4.9)$$

$$\Delta e_1 > K.\theta_r/T \quad (4.10)$$

Substituting the value of $\Delta e_1$ and $\Delta e_2$ from equations (4.7) and (4.9) in the conditions (4.8) and (4.10) and solving for K, it is observed that

$$\frac{K}{T} < \frac{(G - \theta_r)(1 - \exp(-L/T))}{G - \theta_r(1 - \exp(-L/T)}$$

and

$$\frac{K}{T} < \frac{\theta_r(1 - \exp(-L/T))}{\theta_r + (G - \theta_r)\exp(-L/T)} \tag{4.11}$$

Conditions (4.7) - (4.11) constitute the complete solution of a three-position controlled process for a nonoscillatory response to step temperature commands.

The results may now be summarised. The calibration increments $\Delta e_1$, $\Delta e_2$ and the optimum derivative content K are given by

$$\left.\begin{aligned}
\Delta e_1 &= (G - \theta_r)(1 - \exp(-L/T) - K/T)\exp(L/T) \\
\Delta e_2 &= \theta_r(1 - \exp(-L/T) - K/T)\exp(L/T) \\
\frac{K}{T} &< \left.\frac{(G - \theta_r)(1 - \exp(-L/T))}{G - \theta_r(1 - \exp(-L/T))}\right|_{\theta_r > G/2} \\
\frac{K}{T} &< \left.\frac{1 - \exp(-L/T)}{1 + \exp(-L/T)}\right|_{\theta_r = G/2} \\
\frac{K}{T} &< \left.\frac{\theta_r(1 - \exp(-L/T))}{\theta_r + (G - \theta_r)\exp(-L/T)}\right|_{\theta_r < G/2}
\end{aligned}\right\} \tag{4.12}$$

The above equations indicate that the calibration increments and the optimum derivative content are dependent on the input command. Consequently an asymmetrically distributed control element is required to be used to achieve a nonoscillatory response for step temperature commands.

The effect of derivative feedback on the process response is best understood, by determining the calibration increments of the control element with and without derivative feedback, for a nonoscillatory step response. This is illustrared in Figure 4.3.

The principal advantage of incorporating derivative signal in the feedback loop is that, the dead-zone (or the zone of constant control effort) is now smaller. In any three position controller with a dead-zone, the offset errors due to disturbances are not eliminated, since there is no control action within the dead-zone. Hence a better performance is obtained, if the width of the dead-zone is minimised. This is achieved by incorporating derivative feedback. The improvement that results is futher illustrated by the following example.

EXAMPLE

Given that the process is modelled by equation (1.1) with the following data: $G = 100^{o}C$, $L/T = 1/8$, $\theta_r = 50^{o}C$

Determine the calibration increments for a three-position controller with and without derivative feedback for a nonoscillatory response to a step input command.

For the controller without derivative feedback, the calibration increments are given by equations (3.12) and (3.15). They are found to be:

$$\Delta e_1\Big|_{K=0} = (G - \theta_r)(\exp(L/T) - 1)$$

$$= 7.1^{o}C$$

$$\Delta e_2\Big|_{K=0} = \theta_r(\exp(L/T) - 1)$$

$$= 7.1^{o}C$$

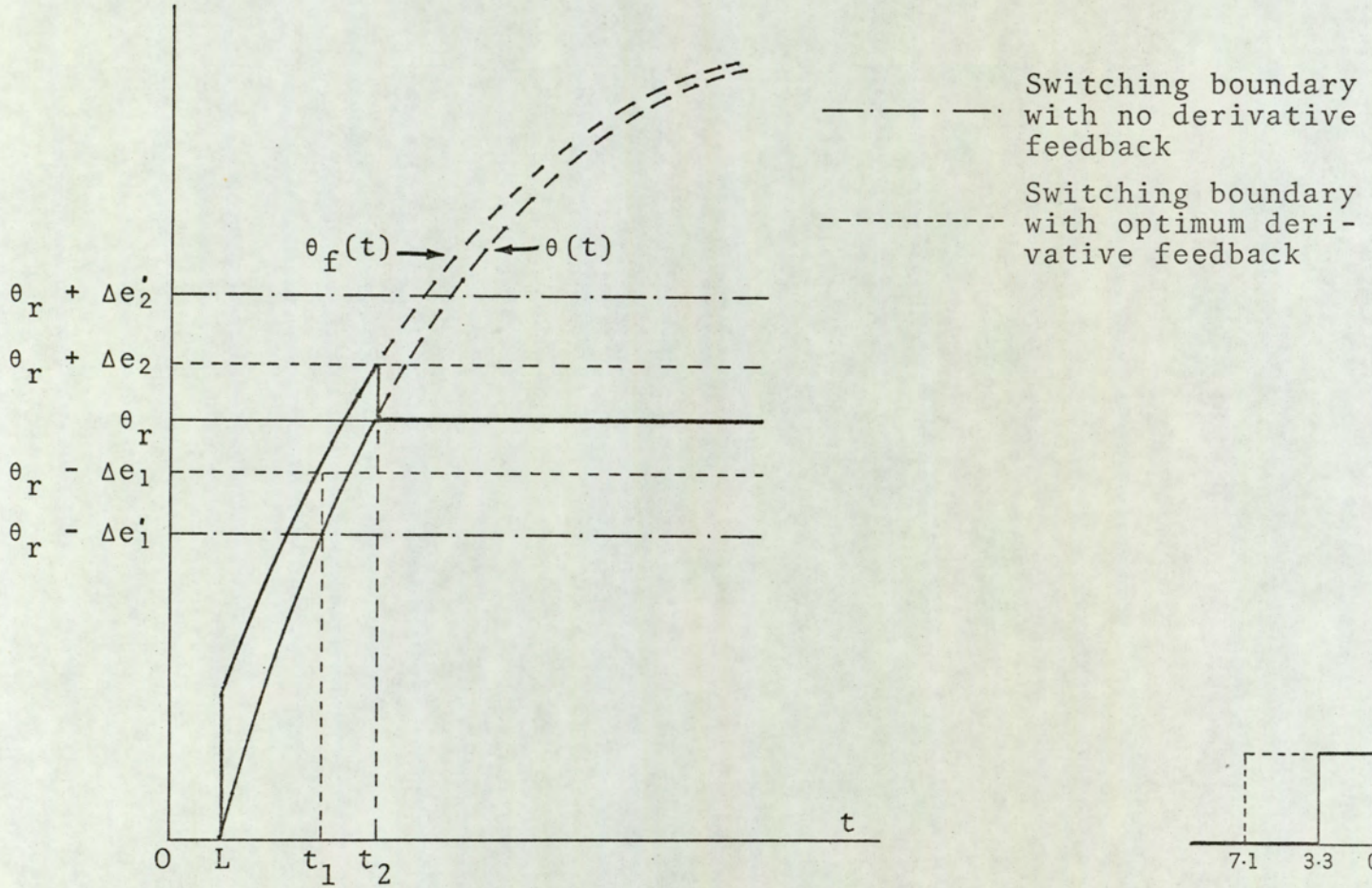


Fig. 4.3 Switching boundaries for a 3-position controlled process with and without derivative feedback. $t_1$ is the switching instant; $t_2$ is the settling time.

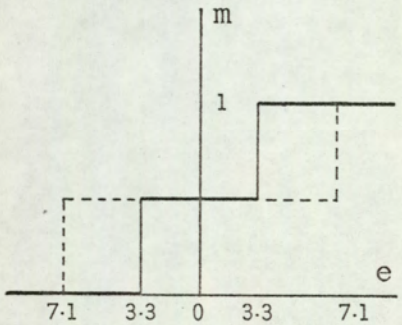


Fig. 4.4 Controller characteristics for the example.

If derivative feedback is incorporated according to equations (4.12), then

$$\Delta e_1 \Big|_{K \neq 0} = 3.33^{\circ}C$$

$$\Delta e_2 \Big|_{K \neq 0} = 3.33^{\circ}C$$

The width of this dead-zone without derivative feedback is given by

$$\Delta e \Big|_{K = 0} = (\Delta e_1 + \Delta e_2) \Big|_{K = 0} = 14.2^{\circ}C$$

If optimum derivative feedback is incorporated then

$$\Delta e \Big|_{K \neq 0} = (\Delta e_1 + \Delta e_2) \Big|_{K \neq 0} = 6.66^{\circ}C$$

The controller characteristics for critically damped response with and without derivative feedback is displayed in Figure 4.4.

The above example clearly shows that with optimum derivative feedback, the width of the dead-zone is more than halved; consequently a faster response to disturbance input is obtained. For the example considered above, the process does not respond to disturbance input less than $7.1^{\circ}C$ in magnitude, if the controller has no derivative feedback; while if optimum derivative feedback is incorporated into the controller, this range of insensitivity is reduced to $3.33^{\circ}C$. More accurate control is thereby achieved.

Since the calibration increments are now a function of the derivative content K and the optimum derivative content varies with the input command, 3-position controllers in general have a nonlinear distribution.

The optimum derivative content for processes modelled by (1.2) is derived by the use of the Zeigler-Nichols approximation as explained in Section 4.2.1. (b).

## 4.3. MODEL REFERENCED INTEGRATING CONTROLLER

The use of process models in the design of control systems is well known [27] [28]; their application in the design of adaptive control systems has been the subject of extensive investigations [27]. In this section, the use of the model as a control element will be indicated. The forerunner to this investigation is the Smith's Linear predictor [28] for processes with transit delay or transportation lag. The Smith's Linear predictor was designed only to overcome the destabilising effect of the transit delay. The scheme proposed in this section utilises the process model itself as an integrating control element.

If a process is subject to disturbances, an integrator is required to eliminate offset errors that might arise. The proportional controller and the discontiuous controller discussed earlier are not satisfactory since these do not adequately compensate for disturbances. In the case of proportional control, a step disturbance causes an offset error, whose magnitude is a function of the gain of the process; while if the process is discontinuously controlled, in addition to an offset error, there are also oscillations.

The model-referenced integrator will be presented as the logical outcome of the design of nonsaturating digital controller for a finite time deadheat response to step inputs. It will further be shown that by conbining the advantages of discontinuous control with those of integrating control, a time-optimal controller for electroheat process is realised. The analysis is mainly restricted to processes modelled by (1.1) since these represent a class of common thermal processes. The extension of the results to processes modelled by (1.2) by the procedure indicated in section 4.2.1 (b) is straightforward.

### 4.3.1. Digital controller for critically damped response

In this section, a simple time-domain technique of synthesizing digital controllers for deadbeat response to step input commands will be presented. The strategy developed here takes into consideration the plant saturation; further the method outlined facilitates the use of computers as on-line controllers for electroheat processes.

The open-loop digital control of electroheat process will be considered first; the required closed-loop control for the same response characteristics is then easily determined.

Figure 4.5 shows the closed loop digitally controlled electroheat process. The closed loop controller is denoted by its impulse response $D_c$ (Z) Figure 4.5 b shows the equivalent open loop control of the same process. The open loop controller is denoted by $D_s$ (Z).

The first consideration in the design of digital controller is the choice of the sampling period. Strictly, the choice of the sampling period is governed by Shannon's theorem,[29] which states that the sampling frequency $\frac{1}{T_s}$ should be greater than twice the highest frequency of the input signal. In practice, especially in electroheat process, there are no band-limited input signals; hence a more meaningful criteria for the choice of sampling period, is required.

Since the electroheat process has a low pass filter characteristic, it is sufficient to consider such frequencies where attenuation by the process is less than a given value. Thus the amplitude of the process modelled by (1.1) is given by

$$|G(j\omega)| = 1/\sqrt{(1 + \omega^2 T^2)}$$

For high frequency ($\omega T >> 1$) this may be approximated as

$$|(G(j\omega))| \simeq 1/\omega T$$

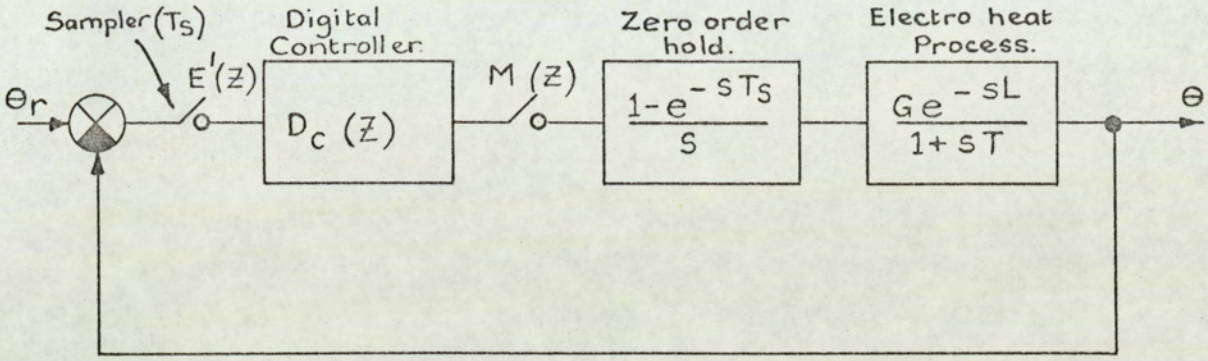


Fig. 4.5a Electroheat process controlled by a digital controller.

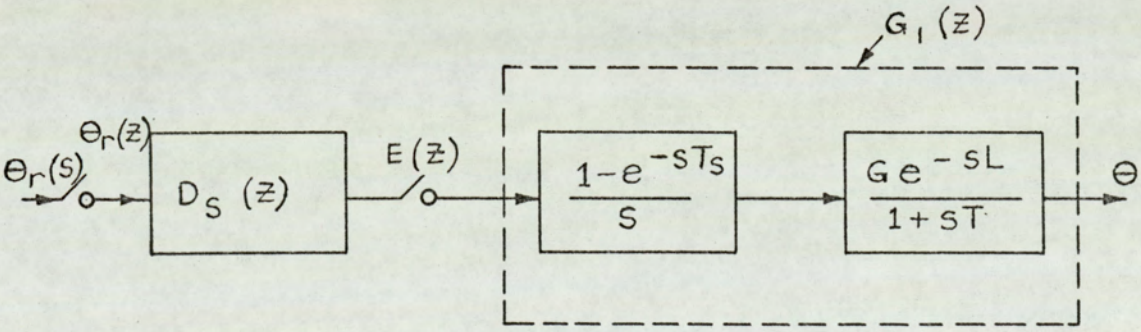


Fig. 4.5b Equivalent open loop digital control of an electroheat process.

If the highest frequency of interest is considered as that which gives a value of $|G(j\omega)| = 0.01$ then the value of this frequency is given by

$$1/\omega T = 0.01$$

Therefore $\omega = 100/T$

Now the application of Shannon's theorem leads to a sampling frequency $\omega_s$ of twice this value, ie:

$$\omega_s = 2\omega = (2)(100)/T$$

The sampling period is therefore

$$T_s = 2\pi/\omega_s = \pi T/100$$

$$T_s/T = \pi/100 = 0.03 \qquad (4.13)$$

The sampling period chosen according to the above method yields a reasonably acceptable performance [30]. It will be assumed in the following analysis that $T_s$ is chosen according to the criterion outlined above.

Considering the block diagram of Figure 4.5 (b), let

$$\theta_r(Z) = \theta_r/(1 - Z^{-1}) \quad \text{(a step function of magnitude } \theta_r\text{)}$$

The output of the open-loop controller $D_s$ (Z) is then

$$E(Z) = \theta_r(Z).D_s(Z)$$

The modified Z -transform of the process and the hold can be shown to be

$$G_1(Z,m) = \frac{GZ^{-(k+1)}\{1 - a(m)\} + GZ^{-(k+2)}\{a(m) - a(1)\}}{1 - a(1).Z^{-1}} \quad (4.14)$$

where $a(m) = \exp(-mT_s/T)$

$a(1) = \exp(-T_s/T)$

$L = (k + \delta)T_s \quad 0<\delta< 1$

and m is a Z-transform parameter such that $0<m<1$

The ordinary Z-transform is obtained from the relation

$$ZG_1(Z,m)\Big|_{m\to 0} = G_1(Z)$$

The use of modified Z-transform enables the output to be determined between the sampling instants. The output is then given by

$$\theta(Z,m) = E(Z).G_1(Z,m) = \theta_r(Z).D_s(Z).G_1(Z,m) \quad (4.15)$$

The output at each sampling instant and between the sampling instants is easily determined by a series expansion of $\theta(Z,m)$

$$\theta(Z,m) = GZ^{-(k+1)}\{e_o + e_1Z^{-1} + e_2Z^{-2} + \cdots\cdots\cdots\}\{1 - a(m) +$$

$$a(m)\{1 - a(1)\}Z^{-1} + a(1)a(m)\{1 - a(1)\}Z^{-2} + \cdots\} \quad (4.16)$$

where $e_o$, $e_1$ are the coefficients of the various powers of $Z^{-1}$ in the series expansion of E(Z). The second series on the right is the series expansion of $G_1(Z,m)$. The product of the two series of (4.16) is given by

$$\theta(Z,m) = GZ^{-(k+1)}\Big[e_o b_o + (e_o b_1 + e_1 b_o)Z^{-1} + (e_o b_2 + e_1 b_1 + e_2 b_o)Z^{-2} + \cdots\cdots\cdots\Big] \quad (4.17)$$

where $b_o = 1 - a(m)$

$b_1 = a(m)\{1 - a(1)\}$

$b_2 = a(m)a(1)\{1 - a(1)\}$ etc.

The output (ie: the temperature of the process) between the sampling instants $nT_s$ and $(n+1)T_s$ is given by the coefficient of $Z^{-n}$ in (4.17). The output at the sampling instant is obtained with m = 0 and the output between the sampling instants is obtained by varying m between 0 and 1.

From (4.17) it is observed that the temperature is zero for t=0 to $t=kT_s$. The output between $(k+1)T_s$ and $(k+2)T_s$ is given by

$$\theta(t) = Ge_o b_o = Ge_o\{1 - a(m)\}, \quad (k+1)T_s < t < (k+2)T_s$$

$$\theta(t) = Ge_o\{1 - a(m)\}\Big|_{m=0} = 0, \quad t = (k+2)T_s \quad (4.18)$$

$$\theta(t) = Ge_o\{1 - a(m)\}\Big|_{m=1} = Ge_o\{1 - a(1)\}, \quad t = (k+2)T_s$$

It is required to choose $e_o$, $e_1$,....so that the output temperature is raised to the value of the command $\theta_r$ in the fastest time without overshoot. The constraints on the digital controller are such that

$$0 < e_i < 1, \quad i = 1, 2, \cdots\cdots\cdots\cdots$$

The constraint on the input command is

$$0<\theta_r<G$$

Since the value of the command cannot exceed the maximum temperature G, which the process can attain (with full power). For similar reasons, it is required that $0<e_i<1$ since $e_i$ which represents the input power cannot exceed the power rating of the heating apparatus; also since there is no forced cooling $e_i$ should be greater than zero.

The design of controllers for plants subject to saturation has been studied by several authors [32 33]. The design is based on the assumption that the maximum value of the input command to the system is known. If, for example, this maximum value is $\theta_m$ and the settling time for deadheat response is $nT_s$ then the process will respond to any step command $\theta_r<\theta_m$ with the same settling time. Figure 4.6. displays the step response of such a control for various input commands.

If the input command exceeds $\theta_m$ then saturation occurs, and the system response then exhibits overshoot and the settling time is no longer finite, also the form of the controller $D_s(Z)$ required for such processes is complex and requires increased storage location if a digital computer is used.

In the scheme suggested below, the digital controller is designed for the maximum input command, ie: $\theta_r = G$. When this controller is used in conjunction with a 3-position controller incorporating derivative feedback a much faster response results.

The design of the digital controller will now be illustrated for a step input command.

$$\theta_r(Z) = G/(1 - Z^{-1})$$

the magnitude of the step command being G.

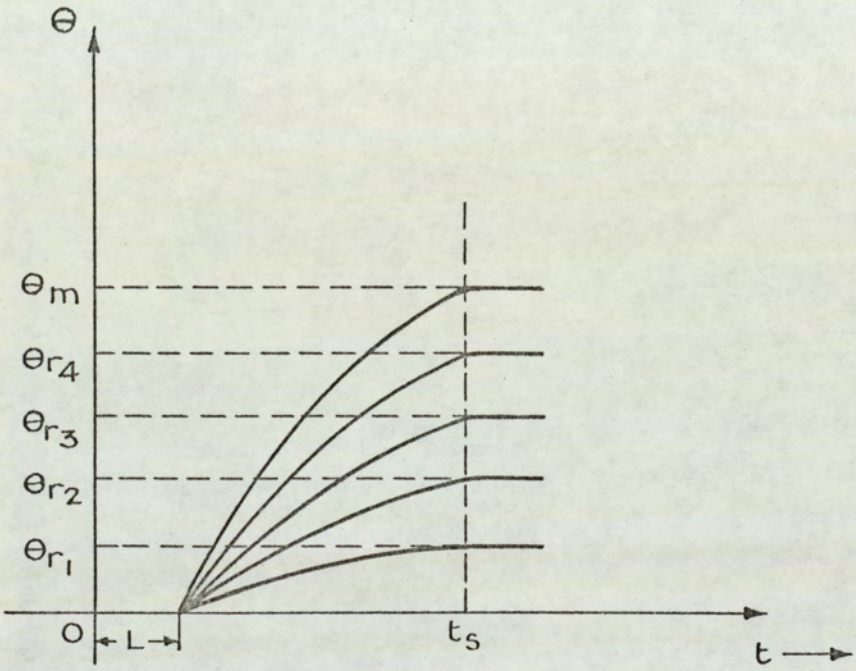


Fig. 4.6 Step response of digitally controlled electroheat process designed for constant settling time.

From equation (4.18) it is seen that the first nonzero output occurs for $t > (K+2)T_s$.

The output at the sampling instant $t = (K+2)T_s$ is

$$\theta(t) = G.e_o\{1 - a(1)\}, \quad t = (k + 2)T_s$$

Since the maximum value of $e_o = 1$ the output is then

$$\theta(t) = G\{1 - a(1)\}, \qquad t = (k + 2)T_s$$

The output between $(K+2)T_s$ and $(K+3)T_s$ is given by

$$\theta(t) = G(e_o b_1 + e_1 b_o)$$

$$= G(b_1 + e_1 b_o) \quad \text{since} \quad e_o = 1$$

$$= G\left[a(m)\{1 - a(1)\} + e_1\{1 - a(m)\}\right]$$

$$(k + 2)T_s < t < (k + 3)T_s$$

At $t = (k + 3)T_s$

$$\theta(t) = G\left[a(1)\{1 - a(1)\} + e_1\{1 - a(1)\}\right]$$

$$= G\{1 - a(1)\}\{a(1) + e_1\}$$

With $e_1 = 1$, $\theta(t) = G\{1 - a(1)\}\{1 + a(1)\}$

$$= G\{1 - a(1)^2\} < G$$

It can be similarly shown that with $e_2 = e_3 = \ldots\ldots = 1$ the successive outputs are given by

$$\theta(t) = G\{1 - a(1)^n\}, \quad t = (k + n)T_s \tag{4.19}$$

$$n = 0, 1, 2, \ldots\ldots$$

Thus the process attains the temperature G only at $t \to \infty$. Also the values of $e_o$, $e_1 \ldots\ldots e_{n1} \ldots$ are equal, ie: $e_i = 1$.

This gives

$$E(Z) = e_o + e_1 Z^{-1} + e_2 Z^{-2} + \cdots\cdots$$

$$= 1 + Z^{-1} + Z^{-2} + \cdots\cdots\cdots\cdots \qquad (4.20)$$

$$= 1/(1 - Z^{-1})$$

Since

$$E(Z) = \theta_r(Z).D_s(Z) = G\ D_s(Z)/(1 - Z^{-1})$$

Therefore

$$D_s(Z) = 1/G \qquad (4.21)$$

With the open loop digital controller, the process responds to any input command $\theta_r < G$ without saturation effects. This is easily observed by evaluating E(Z) for an input command $\theta_r(Z)$ given by

$$\theta_r(Z) = \theta_r/(1 - Z^{-1}), \qquad \theta_r < G$$

$$E(Z) = \theta_r(Z)\ D_s(Z) = \theta_r/G(1 - Z^{-1})$$

Thus the output of E(Z) which indicates the power input to the electroheat process is equal to the power that raises the process to the temperature $\theta_r$, and maintains it at $\theta_r$. The step response of such a controlled process for various magnitudes of input command is displayed in Figure 4.7. In the time domain the output temperature at any instant is given by

$$\theta(t) = \theta_r\{1 - \exp(-t/T)\}, \quad 0 < t < \infty$$

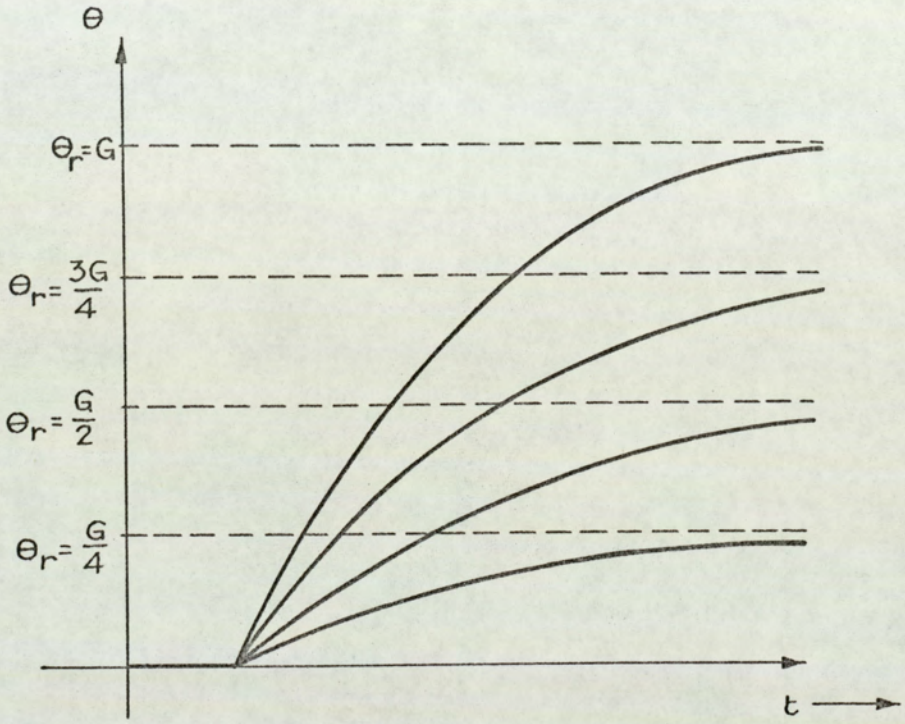


Fig. 4.7 Step response of electroheat process controlled by a nonsaturating digital controller.

Admittedly this response (Figure 4.7) is very slow compared to the responses obtainable (Figure 4.6) by other methods[32 33]. The advantage of the scheme proposed above is that the process never saturates. Also the form of $D_s(Z)$ given by (4.21) is very simple to implement if only open loop control is desired; further the closed loop controller $D_c(Z)$ (Figure 4.5a) that yields the same response is also much simpler to implement.

The closed loop controller $D_c(Z)$ that yields the same step response as the open loop controller $D_s(Z)$ is derived as follows:

For open loop control:

$$\theta(Z,m) = \theta_r(Z)\ D_s(Z)\ G_1(Z,m)$$

For the closed loop control:

$$\theta(Z,m) = E'(Z)\ D_c(Z)\ G_1(Z,m)$$

For the two responses to be identical:

$$\theta_r(Z)\ D_s(Z) = E'(Z)\ D_c(Z)$$

From Figure 4.5a $E'(Z)$ can be seen to be:

$$E'(Z) = \theta_r(Z)/\{1 + D_c(Z)\ G_1(Z)\}$$

$$D_s(Z) = D_c(Z)/\{1 + D_c(Z)\ G_1(Z)\}$$

Simplifying:

$$D_c(Z) = D_s(Z)/\ 1 - D_s(Z)\ G_1(Z) \qquad (4.22)$$

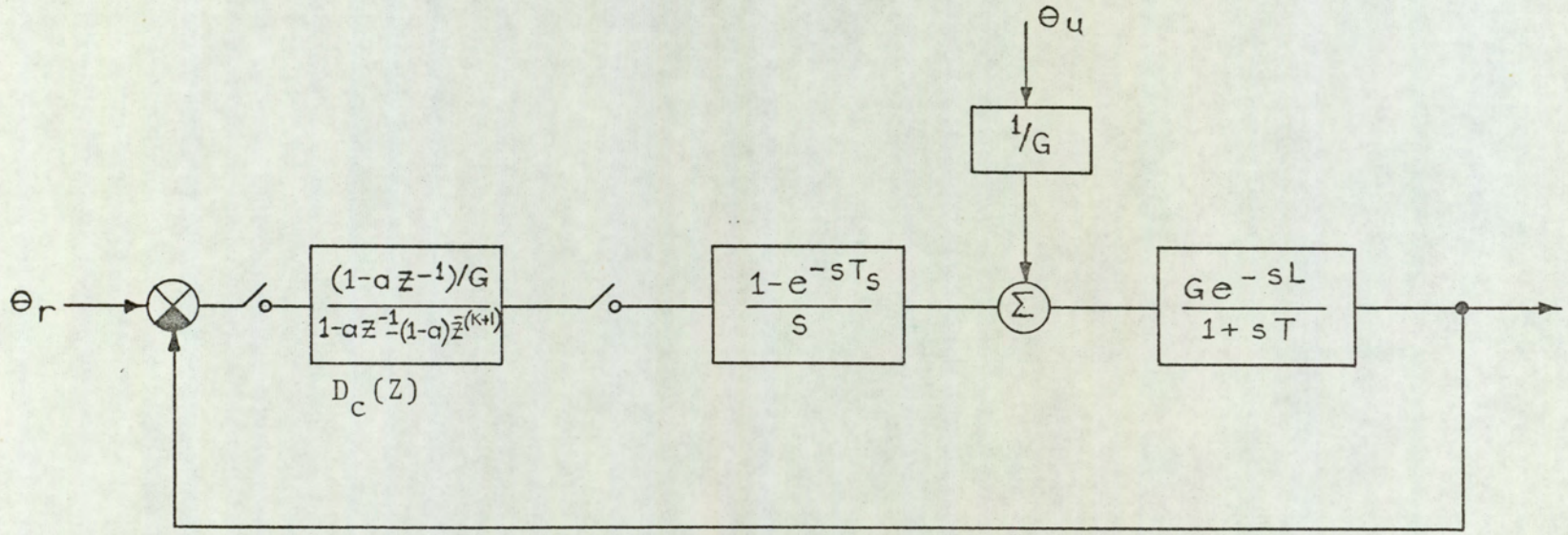


Fig. 4.8 Closed loop nonsaturating digital control of electroheat process.

Substituting the value of $G_1(Z)$ from equation (4.14) with $m = 1$, the closed loop compensator has the form

$$D_c(Z) = (1 - aZ^{-1})/G(1 - aZ^{-1} - (1 - a)Z^{-(k + 1)}) \qquad (4.23)$$

This form of $D_c(Z)$ is a special integrator and may be verified by substituting $Z = 1$ in equation (4.23). The denominator becomes zero with $Z = 1$, thus revealing a pole at $Z = 1$. Equation 4.23 may also be written as:

$$D_c(Z) = \frac{1/G}{1 - (1 - a)Z^{-(k + 1)}/(1 - aZ^{-1})}$$

$$= \frac{1/G}{1 - G_1(Z)/G} = \frac{1}{G - G_1(Z)} \qquad (4.24)$$

Thus $D_c(Z)$ is seen to incorporate the plant characteristics $G_1(Z)$. This form suggests that the flow diagram for realising $D_c(Z)$ may be as shown in Figure 4.9. The path identified by $\{G_1(Z)\}_m$ denotes a digital model of the process. Thus $D_c(Z)$ given by equation (4.23) is termed <u>a model-referenced integrator</u>[34]. The simulation of the model on the digital computer is shown in Figure 4.10. The closed loop transfer function of the controlled process for an input command $\theta_r$ in the absence of disturbance inputs is given by (see Figure 4.8)

$$\theta/\theta_r(Z,m) = G_1(Z,m)\, D_c(Z)/\{1 + G_1(Z)\, D_c(Z)\}$$

Substituting the expressions for $G_1(Z,m)$, $G_1(Z)$ and $D_c(Z)$ from equations (4.14) and (4.23) and simplifying one obtains

$$\theta/\theta_r(Z,m) = G_1(Z,m)/G \qquad (4.25)$$

Equation (4.25) with the substitution of the expression for $G_1(Z,m)$ becomes

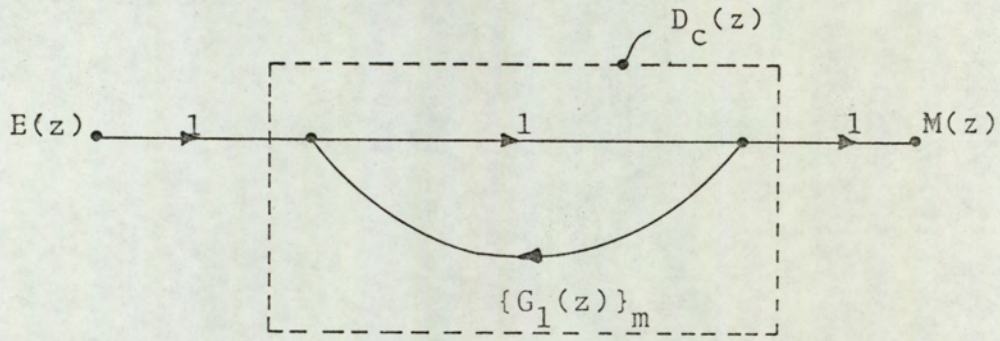


Fig. 4.9 Flow diagram of model-referenced integrator.

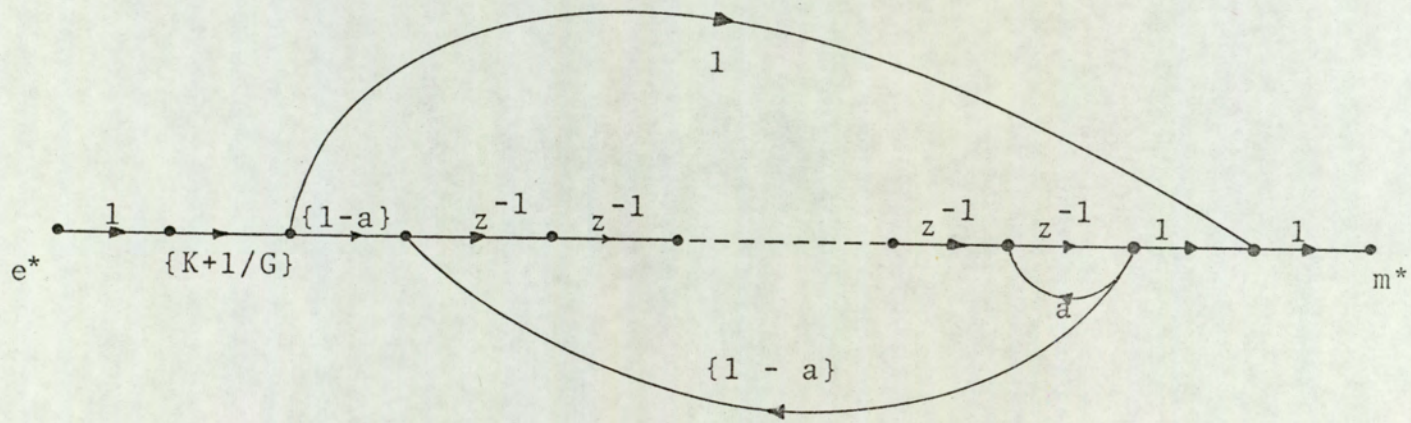


Fig. 4.10 Flow diagram for simulation of model-referenced integrator on the digital computer.

$$\frac{\theta}{\theta_r}(Z,m) = \frac{Z^{-(k+1)}(1 - a(m)) + Z^{-(k+2)}(a(m) - a(1))}{1 - a(1)Z^{-1}}$$

It is seen that with $D_c(Z)$ as given by equation (4.23), the closed loop transfer function has the same form as the transfer function of the process, except for the constant term.

Similarly, it may be easily shown that the transfer function of the output for a disturbance input $\theta_u$ in the absence of input command $\theta_r$ (ie. $\theta_r = 0$) is

$$\theta(Z,m) = \theta_u G(Z,m)/\{1 + D_c(Z)\, G_1(Z)\}\Big|_{\theta_r = 0}$$

$$\frac{\theta_u G(Z,m)\{1 - aZ^{-1} - (1 - a)Z^{-(k+1)}\}}{G(1 - aZ^{-1})} \qquad (4.26)$$

where

$$\theta_u G(Z,m) = Z_m\left[\theta_u(s)\, G(s)\right]$$

$$= Z_m\left[\theta_u(s).G\exp(-sL)/(1 + sT)\right]$$

Once $\theta_u(s)$ is specified (ie. a step or ramp function), the modified Z-transform of the above function may be obtained from any standard table of Z-transforms[30]. It should be noted that in general

$$\theta_u G(Z,m) \neq Z_m\left[\theta_u(s)\right].\, Z_m\left[G(s)\right]$$

The compensating action of the above control scheme will now be shown by examining the steady state value of the output, when the disturbance input is a step function.

For a step disturbance ($\theta_u(s) = \theta_u/s$) an application of the final value theorem yields the steady state output. Thus

$$\lim_{t \to \infty} \theta(t) = \lim_{z \to 1} \left\{ \frac{(1 - Z^{-1}).\theta_u G(Z)\{1 - aZ^{-1} - (1-a)Z^{-(k+1)}\}}{G(1 - aZ^{-1})} \right\}$$

$$\text{where} \quad \theta_u G(Z) = \frac{\theta_u Z^{-(k+1)}(1 - a)}{(1 - Z^{-1})(1 - aZ^{-1})}$$

Therefore

$$\lim_{t \to \infty} \theta(t) = \lim_{Z \to 1} \left[ \frac{\theta_u\{1 - aZ^{-1} - (1-a)Z^{-(k+1)}\}(1-a)}{G(1 - aZ^{-1})^2} \right] = 0$$

The control scheme of Figure 4.8 with $D_c(Z)$ as given by equation (4.23) gives an open loop control for input command $\theta_r$ and as an integrator for disturbance inputs $\theta_u$.

The controller scheme may also be extended to continuously controlled electroheat processes. In this case the digital controller, the zero-order hold and the sampler are replaced by a continuous (or analogue) controller having the form:

$$G_c(s) = 1/[G - \{G(s)\}_m] \tag{4.27}$$

where $G(s) = G.\exp(-sL)/(1 + sT)$

The control system is then as shown in Figure 4.11 $|G(s)|_m$ denotes the model of the electroheat process.

Electroheat processes involving large power are difficult to simulate by analogue methods; hence the use of digital computer is required, in which case the control scheme of 4.5(a) is adopted. Since only a few storage locations are involved in the implementation of such a control scheme, time-shared computer control is facilitated. The control scheme of Figure 4.5(a) is displayed as block diagram for computer control in Figure 4.8 . Experimental investigations (reported in Chapter 5) reveal, that the simulation of the model even when inexact leads to imporved performance.

## 4.4 OPTIMAL CONTROL OF ELECTROHEAT PROCESSES

One of the main disadvantages of the control scheme discussed in the previous section is the slow response as evidenced in Figure 4.7. It was shown in Section 4.2.2. that a 3-position controller with derivative feedback yields a finite time deadheat response if the calibration increments and the derivative content are optimally chosen (equations 4.12) It can be easily shown that the step response of a 3-position controller is a time-optimal response.

Since the plant is subject to saturation, the maximum value of the input command is G. Thus

$$\theta_r < G$$

In other words the optimal trajectory for an input command is given by

$$\theta_{opt}(t) = G\{1 - \exp(-t/T)\} \qquad (4.28)$$

If for any given input command $\theta_r$, the process response is made to follow the optimal trajectory of equation (4.28) until the process temperature $\theta_r$ is attained, then optimal control is

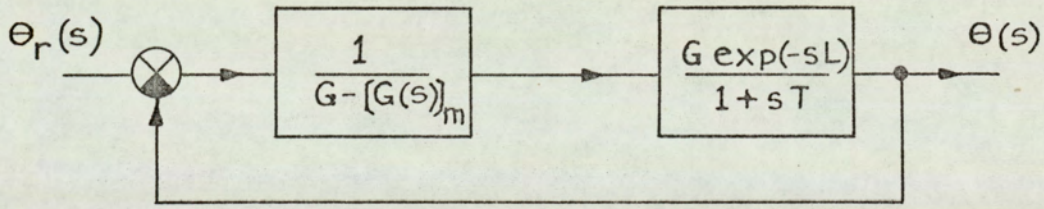


Fig. 4.11 Control scheme for continuous nonsaturating control of electroheat processes.

realised. The desired response for various input command is displayed in Figure 4.12. This response is superior to those obtainable by the methods proposed by several authors [31], where the settling time was fixed for any input command. Further the process of saturation caused further complications.

The response of Figure 4.12 is easily realised by a 3-position controller with derivative feedback. But an electroheat process subject to disturbance inputs requires an integrator for elimination of offset errors. This compensation may be achieved by incorporating the model referenced controller proposed earlier (Section 4.4.2.) Thus combining the 3-position controller,time-optimal controller, capable of counteracting the effects of disturbances is realised.

The control scheme is displayed in Figure 4.13. This is a multiple loop control system incorporating a continuous sampled data controller and a discontinuous controller with derivative feedback. If an analogue model of the process can be simulated, then the digital controller and the zero-order hold may be replaced by the controller $G_c(s)$ (equation 4.27). It is also seen from Figure 4.13 that the input command to the model-referenced controller is the output of the 3-position controller. This control-scheme may then be termed as the <u>input-modulated optimal control</u>. It is assumed that the 3-position controller characteristics satisfy equations (4.12).

The following relations exist for this control system:

a) Disturbance input $\theta_u = 0$

From equation (4.25) this is observed to be

$$\frac{\theta}{r}(Z,m) = G_1(Z,m)/G$$

Since r is the output of the 3-position controller,

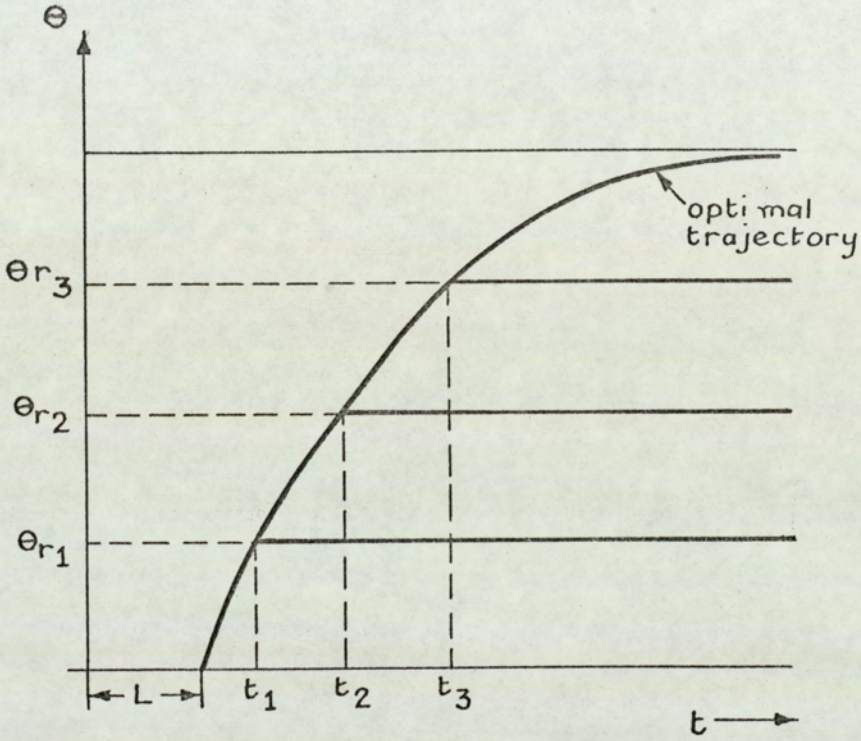


Fig. 4.12 Desired step response of electroheat process for various input commands.

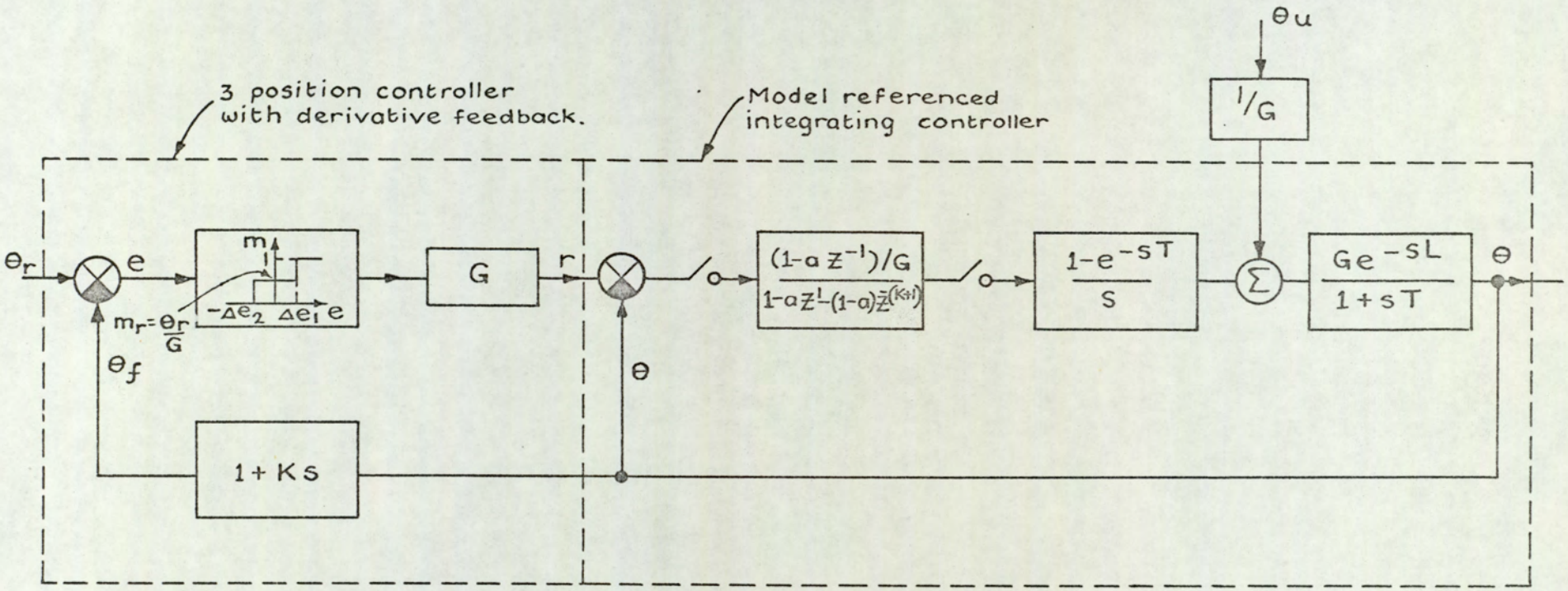


Fig. 4.13 Cascade combination of 3-position controller and the model-referenced integrating controller that leads to the optimal control of electroheat process.

$$r(t) = G\Big|_{e(t) > \Delta e_1}$$

$$r(t) = \theta_r\Big|_{-\Delta e_2 < e(t) < \Delta e_1}$$

$$r(t) = 0\Big|_{e(t) < -\Delta e_2}$$

In view of the above, it may be concluded that in the absence of disturbance inputs, the step-response of the controlled process is the same as that of a 3-position controlled process with derivative feedback.

b) (i) Disturbance input $\theta_u$ 0

(ii) $-\Delta e_2 < \theta_u < \Delta e_1$

(iii) $\theta(t) = \theta_r$, $t = 0$

It will be assumed that a disturbance input $\theta_u$ is applied to the process, at time $t > 0$, when the system is in the steady-state, ie: $\theta(t) = \theta_r$. In view of (iii) the three position controller is inactive, the magnitude of $\theta_u$ is within the limits of the deadzone.

The output temperature is then governed by equation (4.26) with the qualification that $\theta_r \neq 0$. Then

$$\theta(Z,m) = \frac{\theta_r}{1-Z^{-1}} + \frac{\theta_u G(Z,m)\{1 - aZ^{-1} - (1-a)Z^{-(k+1)}\}}{G(1 - aZ^{-1})} \qquad (4.29)$$

The application of the final value theorem to $\theta(Z,m)$ it may be shown that the steady state value of the output temperature is

$$\lim_{t \to \infty} \theta(t) = \lim_{Z \to 1} (1 - Z^{-1})\, \theta(Z,m) = \theta_r$$

Thus for disturbances whose magnitudes are within the limits of the deadzone of the 3-position controller, the compensation is performed by the integrator alone.

c) (i) $\theta_u \neq 0$

(ii) $\theta(t) = \theta_r$, $t = 0$

(iii) $\theta_r > \theta_u > \Delta e_1$, or $\theta_r - G < \theta(u) < -\Delta e_2$

Since the magnitude of the disturbance $_u$ is now greater than the width of the deadzone, the three position controller is also activated in addition to the compensating action of the integrator. The 3-position controller brings the process temperature within the deadzone in the fastest possible manner, while the action of the integrator is more predominant when the error is within the deadzone.

d) (i) $\theta(u) \neq 0$

(ii) $\theta(t) = \theta_r$, $t = 0$

(iii) $\theta_u > \theta_r$, or $\theta_u < \theta_r - G < 0$

For disturbances of such magnitudes, the controller is not adequate to compensate for the offset errors since, for $\theta_u > \theta_r$, forced cooling is necessary to maintain the temperature at $\theta_r$; while for $\theta_u < \theta_r - G < 0$, the power input to the heating apparatus is required to be greater than the rated power.

The above observation is in general true for all electroheat processes; such disturbances however are not normally encountered.

The experimental investigations the actual step response to disturbance inputs are discussed in Chapter 5.

In passing, the following observations should be noted:

1) The integrating controller is applicable to any electroheat process; the only requirement is that a digital simulation of the process be possible. Thus processes modelled by (1.2) and other more complex processes are amenable to control in an optimal manner.

2) Strictly deadheat response is assured, only, if the digital controller (equation 4.23) is replaced by its analogue equivalent (equation 4.27). However, if a sufficiently small sampling period is chosen, an almost deadheat response, sufficient for practical applications, is realised.

3) The design of optimal controllers for processes modelled by (1.2) is facilitated by equation (4.24) and the Zeigler-Nichols approximation discussed in Section 4.2.1 (b).

## 4.5 SUMMARY

The improvements that result with derivative feedback in discontinuously controlled processes has been discussed and exact results that enable the design of control elements discussed.

A model referenced integrating controller suitable for plants subject to saturation has been proposed.

The realisation of such controllers on the digital computer has been indicated.

An optimal controller that combines the advantages of a 3-position controller incorporating derivative feedback, with those of the model referenced integrating controller, has been shown to result in an improved performance of the process. The performance is better than optimal controllers designed according to the conventional methods.

# APPENDIX 4A

# DISCONTINUOUS "PD" CONTROL OF THERMAL PROCESSES

*by*

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Reprinted from
***TRANSACTIONS OF THE INSTITUTE OF MEASUREMENT AND CONTROL***
**Vol. 2, No. 5, May 1969. pp. T57 - T64**

# iscontinuous 'PD' Control of Thermal Processes

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*hermal processes (boilers, furnaces, torches etc.) that are often ontinuously controlled (to reduce the cost of control) a great, of confusion has existed as to the effect of derivative signals in feedback loop. This has resulted in empirical design rules for the vative feedback settings of their controllers. This paper presents nalysis of the effect of derivative feedback upon the perform- indices of such processes. Methods of evaluating these indices, various combinations of command and derivative feedback, are n. The boundaries are clearly defined for the process instability results from excessive derivative feedback. It is shown that well-known cycling characteristic display is inadequate for eying to the practising engineer the effect of derivative feed- k. Two new displays have been derived that clearly show the t of command and derivative feedback upon the process per- nance. Finally, the optimum derivative feedback conditions are n for the typical process. The presentation is intended for the essional engineer concerned with the control of industrial esses.*

## NTRODUCTION

he results reported here arise from an investigation into the trol of processes involving an electrothermal energy con- ion (Induction furnaces, Arc furnaces, Plasma torches, etc.), these results can be applied to the many thermal processes ilers, Ovens, Vats etc.) that previous work[1,2] has shown to be esentable by the cascade connection of a transit delay ($L$) a time constant ($T$). The transit delay arises because of the ributed nature of many thermal processes, and can be rded as a cascade of an infinite number of time parameters:

$$\exp(-sL) = \lim_{n \to \infty} [1/(1 + sL/n)]^n$$

ny of these processes are discontinuously controlled because represents an economical form of process control[3,4] and the mon form of this control is 'two-position' or 'bang-bang' in ch the thermal element is either inoperative or operating at imum capacity. The mode dependent nature of the time stant of some thermal processes will not be considered as the lts reported here can be extended to cover such cases in the ner reported elsewhere.[5,6]

he effect of a derivative signal on the control of such processes not analytically understood. Consequently many control ineers tend to avoid incorporating derivative feedback when trolling these processes. Alternatively, when derivative feed- k is used, the manner in which it is incorporated into the cess-controller is often based upon empiricism. This fre- ntly results in permanent offset errors that displace the trolled temperature from the command (or set point) and, infrequently, also cause instability. This paper presents an lytic basis for the use of derivative feedback in all discon- ously-controlled thermal processes.

f. Roots was, and Mr. Shridhar is, with the Electrical Engineering Department, ersity of Aston in Birmingham. Prof. Roots is Head of Electrical Engineering, ersity of Windsor, Ontario, Canada.

**NOMENCLATURE**

| | |
|---|---|
| $b$ | = primary feedback |
| $e$ | = actuating signal |
| $m$ | = manipulated variable |
| $s$ | = Laplace complex variable |
| sgn ($e$) | = $e/\lvert e\rvert = +1$ or $-1$ |
| $t$ | = general symbol for time |
| $t_o$ | = ON time (portion of $t_q$ for which $m = +1$) |
| $t_p$ | = OFF time (portion of $t_q$ for which $m = 0$) |
| $t_q$ | = period $= t_o + t_p$ |
| $u$ | = disturbance input |
| $y$ | = $\theta_r - \bar{\theta}$ = offset error |
| $B$ | = two position controller characteristic with zero deadspace (achieved by a secondary feedback loop if necessary) |
| $D$ | = $d/dt$ |
| $G$ | = gain of a thermal process. Incorporates any thermal attenuation within the process. |
| $K$ | = derivative feedback coefficient |
| $L$ | = transit delay |
| $T$ | = process time constant |
| $\alpha$ | = $K/T$ |
| $\beta$ | = $L/T$ |
| $\gamma$ | = $1 - \exp(-\beta) = 1 - \exp(-L/T)$ (= index of process time parameters) |
| $\theta$ | = controlled variable |
| $\bar{\theta}$ | = mean value of $\theta$ |
| $\theta_d$ | = $\theta_m - \theta_n$ (= cycling amplitude) |
| $\theta_m$ | = maxima of $\theta$ |
| $\theta_n$ | = minima of $\theta$ |
| $\theta_r$ | = command |
| $\lambda$ | = $[(u + G) - \theta_r]/[\theta_r - u]$ |
| $\rho$ | = $\theta_r/G \mid_{u=0}$ |
| $\sigma$ | = $\exp(-L/T) = \exp(-\beta) = 1 - \gamma$ |

## 2. CONTROLLED THERMAL PROCESSES

Figure 1 shows the general block diagram for a controlled process with a derivative content ($K$) in the feedback, all symbols being designated in the previous nomenclature section. Figure 1

TABLE 2

**vation of the performance indices in terms of the operating conditions when the derivative feedback is negligible ($\alpha = 0$).**

| *Performance Index* | *Derivation* |
|---|---|
| Period ($t_q$) | $\frac{t_q}{T} = ln\left[\frac{(1-\rho\sigma)\,\gamma\,(1+\rho)}{\sigma^2\rho\,(1-\rho)}\right]$ |
| Amplitude ($\theta_d$) | $\frac{\theta_d}{G} = \gamma$ |
| Error ($y$) | $\frac{y}{G} = \rho - \frac{ln\left[\frac{1-\rho\sigma}{\sigma\,(1-\rho)}\right]}{[t_q/T]}$ |

symbols $\alpha$, $\beta$, $\rho$ will be used to identify the normalised ating conditions of (4), (5), (6). Earlier electroheat litera-[1,7,9] also used the symbols $\gamma$, $\sigma$, $\lambda$ which are identified in endix B; which also summarises the interrelationships een these earlier symbols and the simplified nomenclature is paper.

o understand controlled thermal process behaviour it is ortant to be able to evaluate the performance indices (1), (3) in terms of the operating conditions. This is important hree reasons:

) To be able to design new processes and to predict their performances in the conceptual stage.

) To design systems for the control of such processes, or to specify controlled-performance requirements that are practicable.

) To design algorithms for the computer control of such processes.

hen the derivative feedback content is negligible ($\alpha = 0$) e performances indices can be expressed in terms of the ating conditions by the equations displayed in Table 2. y professional engineers know, from practical experience, these performance indices will change as the derivative back becomes significant ($\alpha > 0$). To fully understand vative-discontinuous control, and to gain the conceptual hts necessary for the effective use of computers in controlling processes, one must know how these equations (Table 2) affected by the presence of derivative feedback and how to ent this knowledge in practical terms. This is accomplished e next two sections.

## ESTRICTED DERIVATIVE FEEDBACK

ble 2 considered controlled processes that had a negligible vative content in the feedback. It is known that when the

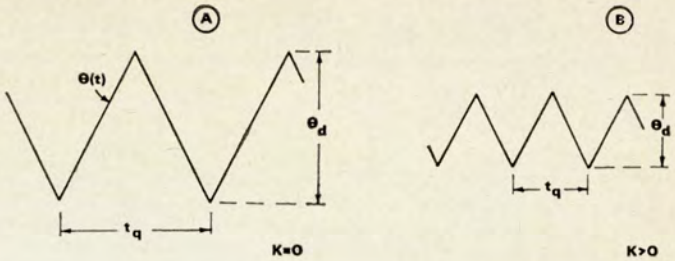



Figure 3. The reduction of the amplitude ($\theta_d$) and period ($t_q$) due a derivative content ($K$) in the primary feedback loop. (A) $K = 0$ (B) $K > 0$.

derivative content is increased, one effect is to reduce $t_q$ and $\theta_d$ (as shown in Figure 3) which improves the process performance. It is equally known that if this derivative content is increased arbitrarily, the process can become unstable (ie. become uncontrollable). In practice, a controlled process with derivative content has *two modes of behaviour*. The first is when the derivative content is small, and this will be called *restricted derivative feedback*. The second mode of behaviour is when the derivative content is further increased, and this will be called *increased derivative feedback*. What is not generally realised is that the boundary condition between these two behavioural modes is the quantity $\gamma$ (see Appendix B) and that they can be defined as follows in terms of $\alpha$ (ie. in terms of $K$):

*Restricted Derivative Feedback:*

$$0 < \alpha \leqslant \gamma/2 \tag{7}$$

*Increased Derivative Feedback:*

$$\gamma/2 < \alpha \leqslant 1 \tag{8}$$

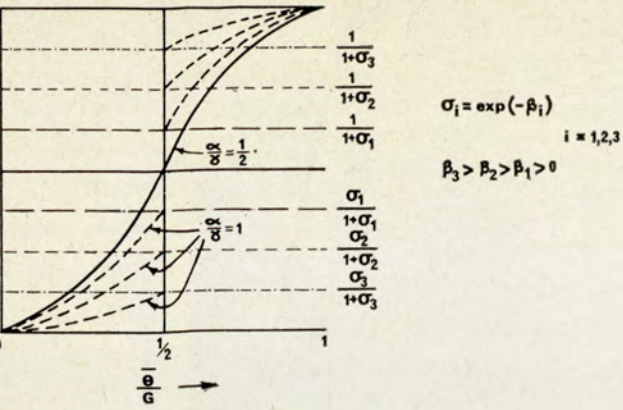
The important practical implication is that in a real process the parameters $L$ and $T$ (and hence $\gamma$) define the boundary condition between these two modes of behaviour. The spectrum of all commands possible for a process (ie. $0 < \rho < 1$) divides into three ranges, each of which denotes a *particular performance mode* of the process. In other words as the command changes from one range to another, the performance of the process changes from one behavioural mode to another (distinctly different) behavioural mode. In the case of *restricted* derivative feedback (7) the process is controllable (ie. stable) in each of these three command ranges. In the next section (Section 5) it will be shown that with *increased* derivative feedback (8) these ranges have different boundaries, also in one of these ranges (the middle range) the process can become uncontrollable (ie. unstable).

For a process with restricted derivative feedback (7) these

TABLE 3

**Derivation of normalised performance indices in a process with restricted derivative feedback.**

| *Line* | *Command* | $\theta_d/G$ | $t_q/T$ |
|---|---|---|---|
| 1 | $o < \rho < \alpha/\gamma$ | $\gamma\left[1 - \frac{\sigma\rho}{(1-\alpha)}\right]$ | $\beta + ln\left[\frac{\sigma^2\rho + \gamma(1-\alpha)}{\sigma\rho}\right]$ |
| 2 | $\alpha/\gamma < \rho < (1 - \alpha/\gamma)$ | $\left[1 - \frac{\sigma}{(1-\alpha)}\right]$ | $ln\left[\frac{(1-\alpha-\rho\sigma)(\gamma-\alpha-\rho\sigma)}{\sigma^2\rho\,(1-\rho)}\right]$ |
| 3 | $(1 - \alpha/\gamma) < \rho < 1$ | $\gamma\left\{\frac{\sigma\rho}{(1-\alpha)} + \left[1 - \frac{\sigma}{(1-\alpha)}\right]\right\}$ | $\beta + ln\left[\frac{\sigma^2(1-\rho) + \gamma(1-\alpha)}{\sigma(1-\rho)}\right]$ |




e 5. The relationship between $\theta_r$ and $\bar{\theta}$. Note that when $\alpha/\gamma > 1/2$ is a discontinuity in this relationship indicating a region in n control of the process is lost.

*r Command Range*

$$0 < \rho < 1/2 \tag{10}$$

*r Command Range*

$$1/2 < \rho < 1 \tag{11}$$

means that the (middle) region of uncontrollability is ced to a line (at $\rho = 1/2$). The performance indices in the r (10) and upper (11) command ranges are displayed in e 5, and it will be noted that they are all *command dependent* nfluenced by the command). Appendix A summarises the ation of Table 5.

th the criterion for selecting an upper limit for the derivative back, and the boundaries of the (middle) command range of ntrollability in Table 4, can be conveniently summarised by ew display of Figure 5. This shows the normalised relation- between the command $\theta_r$ and the mean controlled variable $\bar{\theta}$ number of processes; the difference between $\theta_r$ and $\bar{\theta}$ the error $y (= \theta_r - \bar{\theta})$. It will be noted that when $\alpha/\gamma = 1/2$ is no discontinuity in the relationship between $\theta_r$ and $\bar{\theta}$ , although this relationship may be nonlinear (ie. an error $y$ s), there is no middle region of command in which $\theta$ is pendent of $\theta_r$. However when $\alpha/\gamma > 1/2$ discontinuity occurs is $\theta_r : \bar{\theta}$ relationship. The limits of this discontinuity are the daries of the middle command region in which $\theta$ is inde- ent of $\theta_r$ (ie. the process is uncontrollable). It will also be d that these boundaries of the uncontrollability region vary $\sigma$ (and hence with $\beta$). As $\beta$ increases so the boundaries of iddle (uncontrollable) command region increase.

## EW PERFORMANCE DISPLAYS

the past[10] the so-called 'cycling characteristic' diagram was to summarise the complex mathematical relationships ved in the performance of the process. The most useful ay was the parabolic relationship between the period $t_q$ and ommand $\theta_r$ (or, in normalised form, between $t_q/T$ and $\rho$). n the feedback contains derivative signals such displays are nger a useful summary. For instance Figure 6 displays the

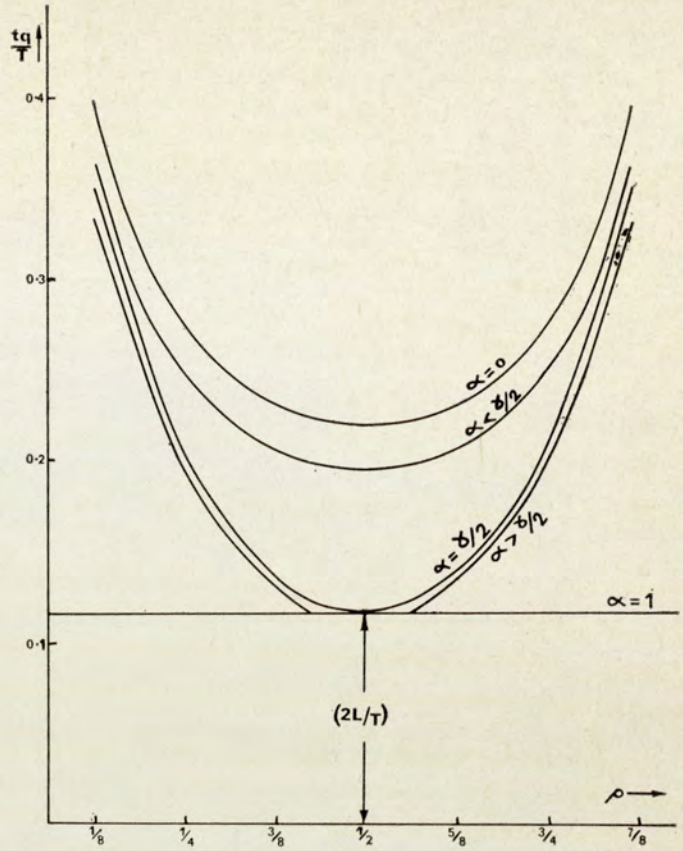



Figure 6. Conventional display of $t_q/T$ as a function of $\rho$ for the derivative feedback range $0 < \alpha \leqslant \gamma/2$.

relationship between $t_q/T$ and $\rho$ for a controlled process with varying amounts of derivative feedback. With no derivative feedback ($\alpha = 0$) the display follows the familiar form[10] with a minimum $\check{t}_q/T$ at $\rho = 1/2$. As the derivative content increases (ie. $\alpha$ increases) so this minimum reduces until at $\alpha = \gamma/2$ the limiting condition ($\check{t}_q/T = 2\beta$) is reached at $\rho = 1/2$. Thereafter, as $\alpha$ is still further increased ($\alpha > \gamma/2$), the deadband becomes apparent in which $t_q/T$ is independent of the command (ie. is fixed at $2\beta$) and the process is uncontrollable. However this is not a satisfactory display as the important relationships of the previous sections are not readily discernible.

Similarly the classical display of the mean manipulated variable ($\bar{m}$) as a function or command ($\rho$) is not satisfactory, as it is, difficult to clearly identify the effect of varying $\alpha$. A slight improvement is obtained if the normalized error ($y/G$) is displayed as a function of $\rho$, and this gives $\bar{m}$ information because:

$$\left.\begin{aligned} y &= \theta_r - \bar{\theta} \\ y/G &= \rho - \bar{m} \end{aligned}\right\} \tag{12}$$

There will be a change in the polarity of $y$ according as to whether $\rho$ is greater than, or less than, one half. Consequently, a more effective display is obtained if $|y|$ is plotted as in

TABLE 5

**Derivation of normalised performance indices in a process with derivative feedback at the recommended upper limited $\alpha = \gamma/2$**

| *Line* | *Command* | $\theta_d/G$ | $t_q/T$ |
|---|---|---|---|
| 1 | $0 < \rho < \frac{1}{2}$ | $\gamma\left[\frac{1+\sigma-2\rho\sigma}{1+\sigma}\right]$ | $ln\left[\frac{2\sigma^2\rho+\gamma(1+\sigma)}{\rho\sigma}\right]$ |
| 2 | $\frac{1}{2} < \rho < 1$ | $\gamma\left[\frac{1+2\rho\sigma}{1+\sigma}\right]$ | $ln\left[1+\frac{\gamma(1+\sigma)}{2\sigma^2(1-\rho)}\right]$ |

## ONCLUSIONS

An analytic basis has been presented from which the rmance characteristics of discontinuously-controlled ther- processes can be calculated for varying amounts of derivative ack.

The performance, of the process so controlled, is deter- d by the important index $\gamma$:

$$\gamma = 1 - \exp(-L/T)$$

The performance indices vary significantly according as to ner the normalised derivative feedback $\alpha(= K/T)$ lies below ove $\alpha/2$. It is, therefore, convenient to consider two ranges rivative feedback:

$0 < \alpha \leqslant \gamma/2$ (restricted derivative feedback)

$\gamma/2 < \alpha \leqslant 1$ (increased derivative feedback).

eater derivative feedback (ie. $\alpha > 1$) has no meaning as ol of the process is lost. For increased derivative feedback $< \alpha < 1$) there will *also* be a range of commands, sym- cally distributed about $\rho = 1/2$ in which control of the ss is lost. As $\alpha \to 1$ this range extends to encompass all ble commands ($0 < \rho < 1$).

If only restricted derivative feedback is used ($0 < \alpha < \gamma/2$) erformance indices will be command dependent, and the ss will be controllable.

For many processes the optimum value of derivative feed- will be determined by:

$$\alpha = \rho\gamma \Big|_{\rho \leqslant 1/2} \quad \text{or} \quad \alpha = \gamma(1-\rho)\Big|_{\rho \geqslant 1/2}$$

se conditions are observed the normalised error $|y|/G$ will a minimum, and the maximum practical benefit will be ned in the reduction of the normalised period $t_q/T$.

If the gain $G$ can be so selected that the process normally ites with commands in the region of:

$$\rho \simeq 1/2$$

est possible process performance will be obtained as the $|y|/G$ will be zero; and if:

$$\alpha = \gamma\rho \quad \text{or} \quad \gamma(1-\rho)$$

nallest possible period will result:

$$t_q/T = 2\beta\ (= 2\,L/T)$$

Two new displays have been developed that portray the effect of derivative feedback upon the period (Figure 8) and the error (Figure 9). These simplify the prediction of the process performance that can be obtained for various combinations of command and derivative signal.


## 7. ACKNOWLEDGEMENTS

This research was supported by the National Research Council, of Canada, Grant A7035.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# APPENDIX A

e equations of Tables 2, 3, 4 and 5, are derived on the nption that $\theta(t)$ is cycling in dynamic equilibrium between axima $\theta_m$ and minima $\theta_n$

Let $\theta(t) = \theta_m$ at $t = 0$

Then $\theta(t) = \theta_m \exp(-t/T)$; $t \geqslant 0$

$b(t) = \theta(t) + KD\theta(t) = \theta_m[\exp(-t/T)](1-\alpha)$

$b(0) = \theta_m(1-\alpha)$

**(i)** If $\theta_m(1-\alpha) \leqslant \theta_r$ (A1)

a switching operation at $t = 0$ results in $m = 0 \to m = +1$. eaches $\theta_n$ a time $L$ after the switching instant.

efore:

$$\theta_n = \theta_m \exp(-L/T) \tag{A2}$$

**(ii)** If $\theta_m(1-\alpha) > \theta_r$ (A3)

at $t = t_1$; $b(t_1) = \theta_m(1-\alpha)\exp(-t_1/T) = \theta_r$

and a switching operation at $t = t_1$ results in $m = 0 \to m = +1$.

$\theta(t)$ reaches $\theta_n$ at $t = t_1 + L$:

$$\theta_n = \theta_m \exp[-(t_1+L)/T] = \theta_r \exp(-L/T)/(1-\alpha) \tag{A4}$$

If: $\theta(t) = \theta_n$ at $t = 0$

then $\theta(t) = \theta_n \exp(-t/T) + G[1 - \exp(-t/T)]$; $t \geqslant 0$
$b(t) = \theta_n(1-\alpha)\exp(-t/T) +$

$G[1 - \exp(-t/T)] + \alpha G \exp(-t/T)$

$b(0) = \theta_n(1-\alpha) + \alpha G$

**Case (iii)** If $\theta_n(1-\alpha) + \alpha G \geqslant \theta_r$ (A5)

then a switching operation at $t = 0$ results in $m = +1 \to m = 0$. $\theta(t)$ reaches $\theta_m$ a time $L$ after the switching instant.

Therefore:

$$\theta_m = \theta_n \exp(-L/T) + G[1 - \exp(-L/T)] \tag{A6}$$

Chapter 5

# SIMULATION OF ELECTROHEAT PROCESSES ON THE DIGITAL AND ANALOGUE COMPUTER

# SIMULATION OF ELECTROHEAT PROCESSES ON THE DIGITAL AND ANALOGUE COMPUTER

## 5.1. INTRODUCTION

An experimental verification of the analytical derivations and the new control strategies developed in earlier chapters is presented in this chapter. The process and the associated control systems are simulated on the digital or the analogue computer. Useful computer programmes for the study of electroheat processes controlled continuously or discontinuously are appended.

## 5.2. STABILITY AND TRANSIENT RESPONSE ANALYSIS BY DIGITAL COMPUTER SIMULATION

The digital computer simulation of electroheat processes, and the associated control systems, essentially involves sampling; hence the response one observes in a continuous process cannot be simulated in a digital computer. However, by choosing a sufficiently small sampling period, this difficulty is overcome. The basis for digital computer simulation of electroheat processes is the differential equation governing the process behaviour and the corresponding difference equation.

The differential equation of a controlled electroheat process modelled by (1.1) is

$$T.D\theta(t) + \theta(t) = G.f(\theta_r - \theta(t - L)) \qquad (5.1)$$

where $\theta(t)$ is temperature of the process
$T$ is the time constant
$L$ is the transit delay
$\theta_r$ is the input command
$f$ is the control law
$G$ is the gain of the process

For a constant input command $\theta_r$ equation (5.1) may be written as

$$T.D\theta(t) + \theta(t) = G.m(t - L) \tag{5.2}$$

where m is the manipulated variable.

and $m(t-L) = f(\theta_r - \theta(t-L))$

The system of equation (5.2) when simulated on the digital computer has the form shown in Figure 5.1. Thus digital simulation makes it necessary to incorporate a sampler and a zero-order hold. The input to the zero-order hold is a series of impulses spaced $T_s$ (sampling period) apart and the output is a series of pulses (this is also the input to the process) This is illustrated in Figure 5.2.

Since only digital simulation (as distinct from digital control) is considered here it is assumed, without loss of generality, that the transit delay is an integral multiple of the sampling interval. Thus:

$$L = K\,T_s$$

where K is an integer.

The output between any two sampling instants $nT_s$ and $(n + 1)T_s$ is given by

$$\theta(t) = \theta(nT_s)\exp(-t/T) + m(t - KT_s)G\{1 - \exp(-t/T)\} \tag{5.3}$$

$$nT_s < t < (n + 1)T_s \Big|_{n \geq K}$$

$$\theta(t) = 0, \qquad 0 < t < KT_s$$

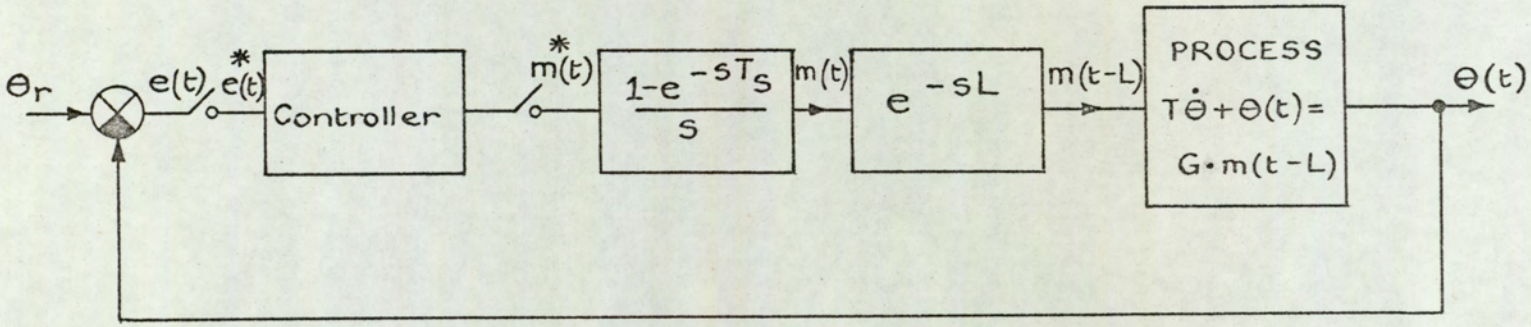


Fig. 5.1 Simulation of an electroheat process control system on the digital computer.

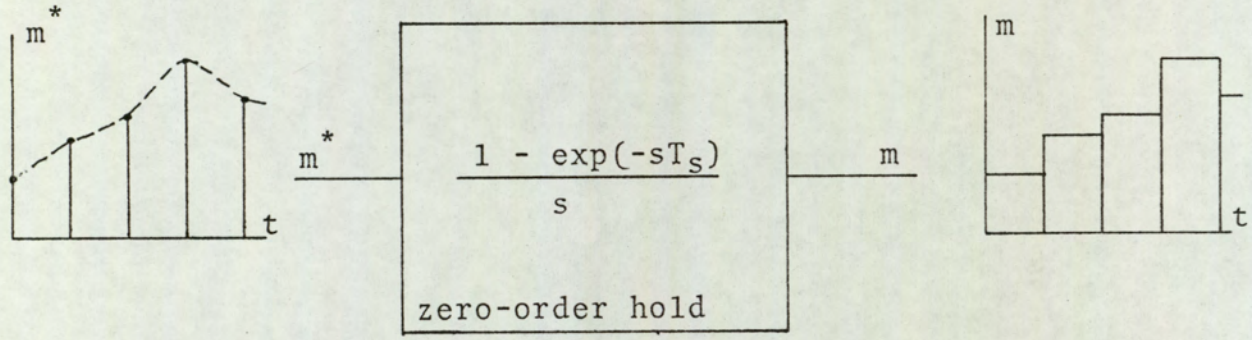


Fig. 5.2 Output of a zero-order hold in a sampled-data system.

Equation (5.3) is the basis for digital computer simulation of controlled electroheat processes. The input to the process, the manipulated variable m(t-L) is the output of the controller delayed by L seconds. By defining the appropriate control law, the step-response of electroheat processes when closed-loop controlled is obtained. The controller may be of any type, ie: continuous or multiposition discontinuous.

The computer program, written in Fortran IV, for electroheat processes modelled by (1.1) is included as Appendix 5A. This program consists of the following:

1) The main program defines the initial conditions and determines $\theta(t)$ according to equation (5.3).

2) Sub-routines for defining controller characteristics.
   a) Proportional control.
   b) Multiposition discontinuous control.

3) Sub-routines for the form of output, ie: graphic plots, or tabulated results.

With this program, criteria for critically damped response and stability were obtained. These are displayed in Figure 5.3 and Figure 5.4 respectively, which also display the criteria derived analytically in Chapters 2 and 3.

The following observations are made on the accuracy of the describing function technique in the analysis of stability; and transient response.

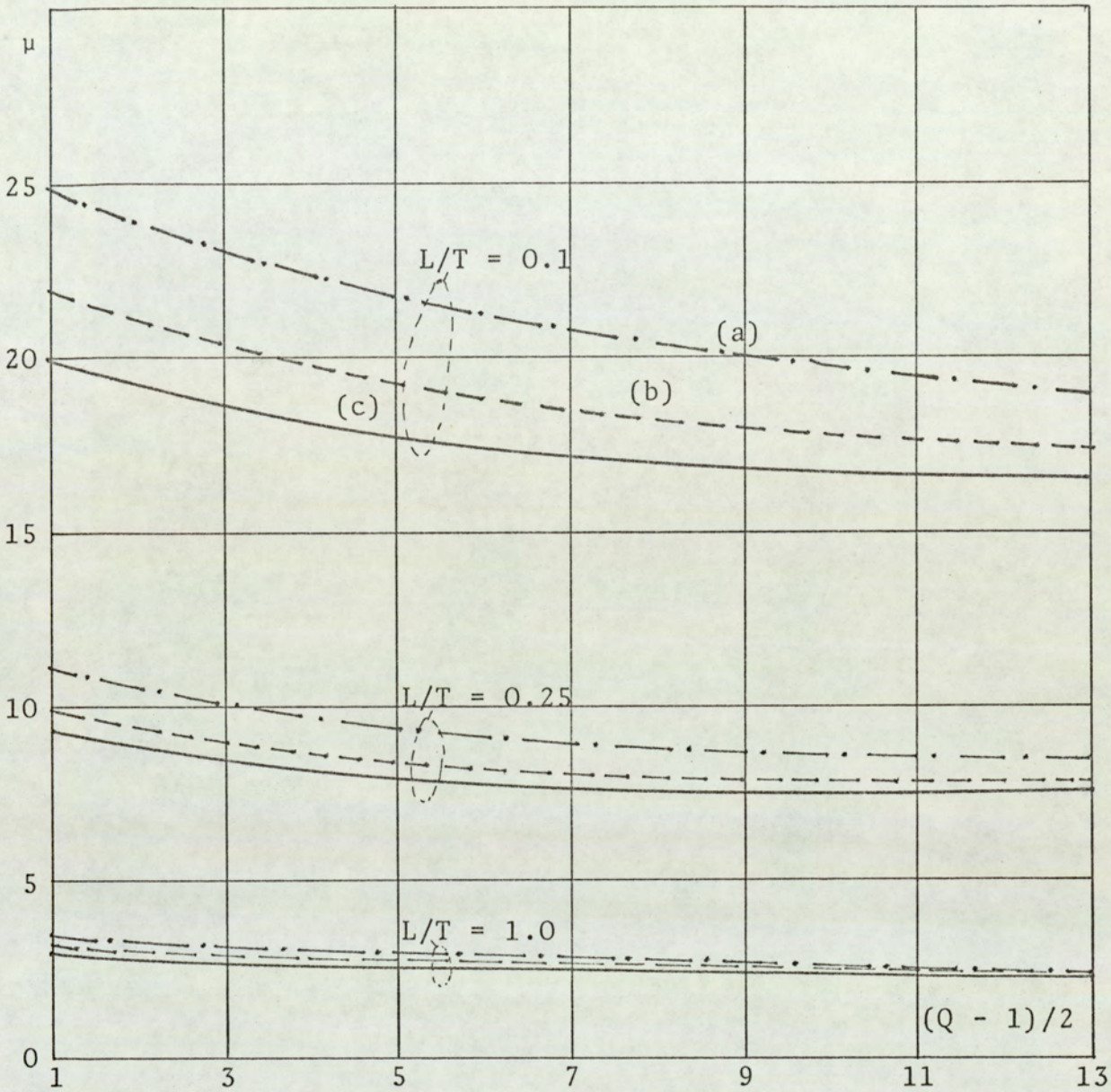


Fig. 5.3 Stability boundaries of discontinuously controlled processes, as determined by (a) conventional describing function (b) RMS describing function and (c) digital computer simulation of the controlled process.

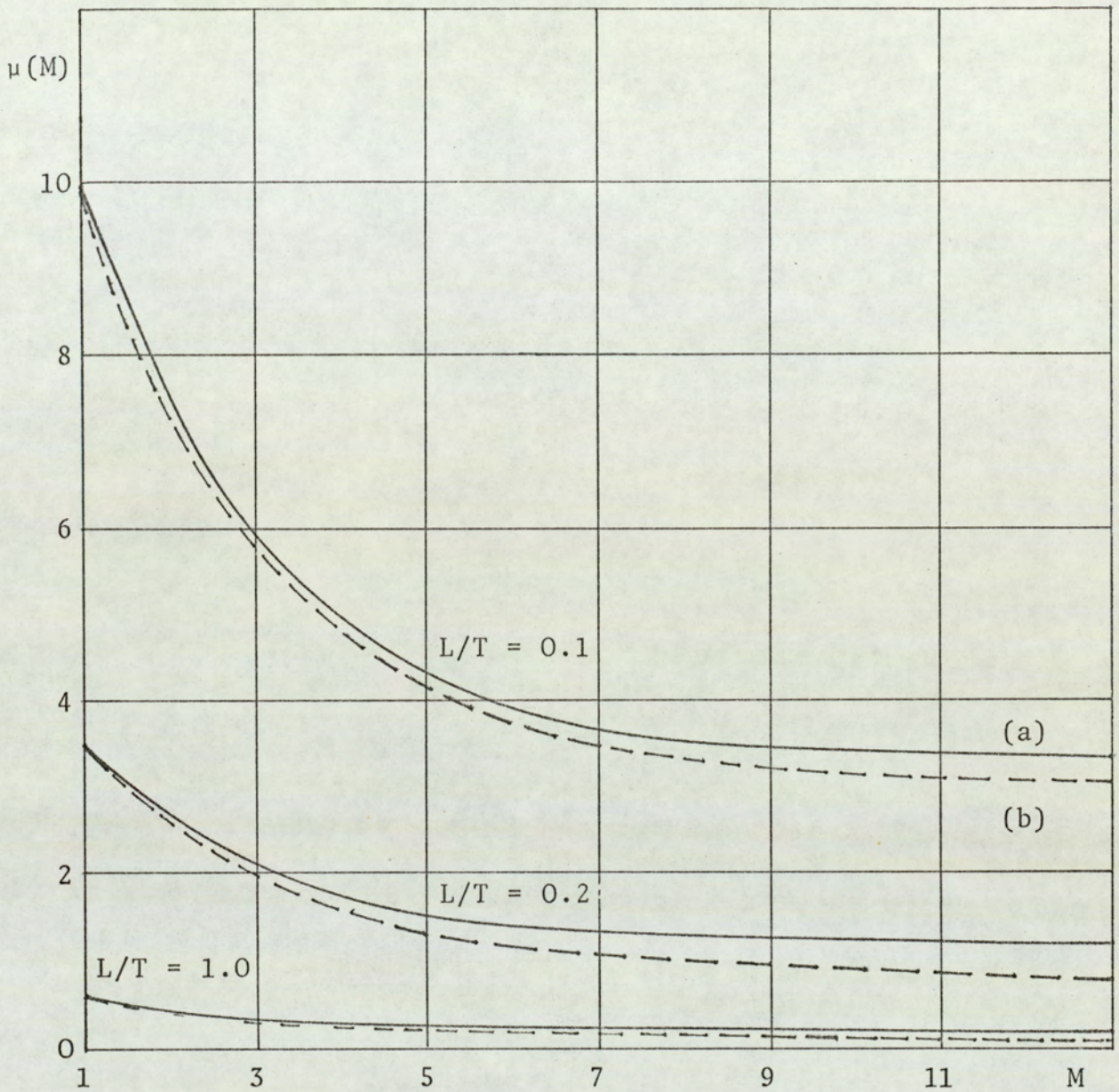


Fig. 5.4 Boundaries for nonoscillatory response in a discontinuously controlled electroheat process as determined by (a) exponential describing function and (b) simulation on the digital computer.

1) Both RMS describing function and the conventional describing function yield higher values of critical gain than the actual value.

2) The RMS describing function yields more accurate results for multiposition discontinuously controlled processes.

3) The exponential describing function facilitates determination of calibration increments for multiposition discontinuously controlled processes, to obtain a non-oscillatory response. The results obtained from theoretical considerations, which involved simplification and approximation, are quite close to the actual values obtained from simulation.

The stability and transient response analysis by the sinusoidal describing function and the newly defined exponential describing function respectively, described in Chapters 2 and 3, constitute a powerful technique for design of controlled electroheat processes. The digital computer simulation of controlled electroheat processes enables a verification of the theoretical techniques and the determination of more accurate settings. The computer programs presented in the appendix also enable the determination of the stability and transient response characteristics of sampled-date controlled electroheat processes.

## 5.3 DISCONTINUOUS 'PD' CONTROL OF ELECTROHEAT PROCESSES

### 5.3.1. Derivative feedback in on-off control

The improvement in process performance when derivative feedback is incorporated in discontinuous controllers (on-off and three position) was theoretically established in Chapter 4. The results were verified by a simulation of the process and the control system on the analogue

computer. For purposes of simulation it was assumed that the transit delay was due to a cascade of eight equal time constants, ie:

$$\exp(-sL) \simeq 1/(1 + sL/8)^8$$

Figure 5.5 displays the effect of derivative feedback on the period and amplitude of oscillations for the symmetrical case ($\theta_r/G = \frac{1}{2}$) in an on-off controlled process. Both the reduction in amplitude and period of oscillations are clearly observed.

It was further pointed out that excessive derivative feedback can lead to increased droop errors and even uncontrollability. This is illustrated in Figure 5.6 for $\theta_r/G = 0.25$. As the derivative content is arbitrarily increased the output temperature tends to oscillate about the mean value $\bar{\theta}$ (where $\theta_r/G = \frac{1}{2}$)

In the simulation of electroheat processes on the analogue computer the following scaling was adopted:

$$L = 0.5 \text{ secs.}$$
$$T = 48 \text{ secs.}$$
$$G = 10 \text{ volts.}$$
$$\theta_r = 2.5 \text{ v.}$$

The value of the derivative gain K that minimised the droop error ($\bar{y}$) was found to be (from experiments)

$$K = .127$$

From theoretical considerations, the gain K for minimum droop error is found to be

$$K = T.\theta_r/G\ (1-\exp(-L/T)\ ) = .125$$

The agreement is quite close, if one considers that the mathematical model used for theoretical studies was

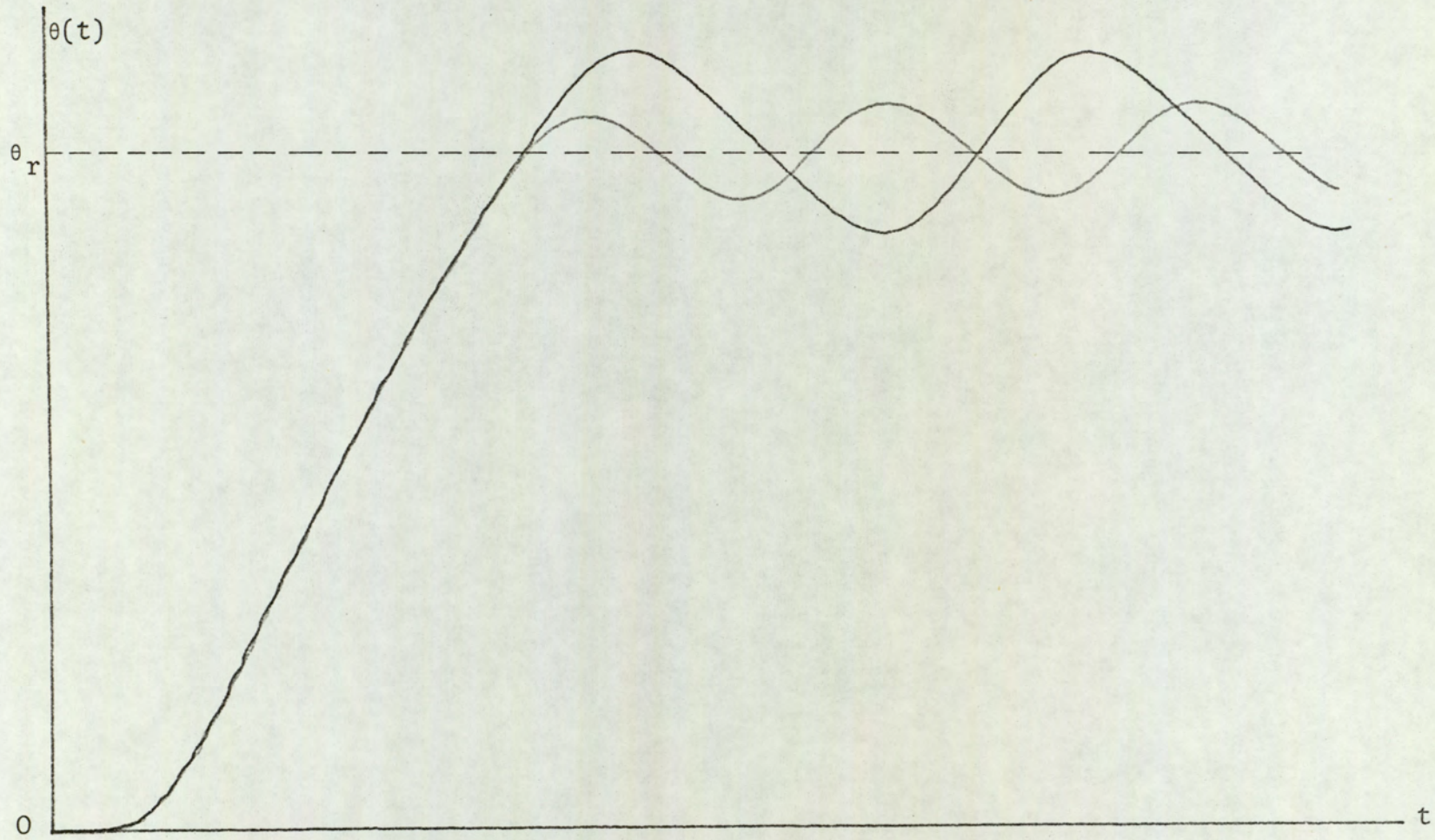


Fig. 5.5 Effect of derivative feedback on the response of an on-off controlled electroheat process. Note the reduction in the amplitude and period of oscillation due to derivative feedback (response indicated in blue).

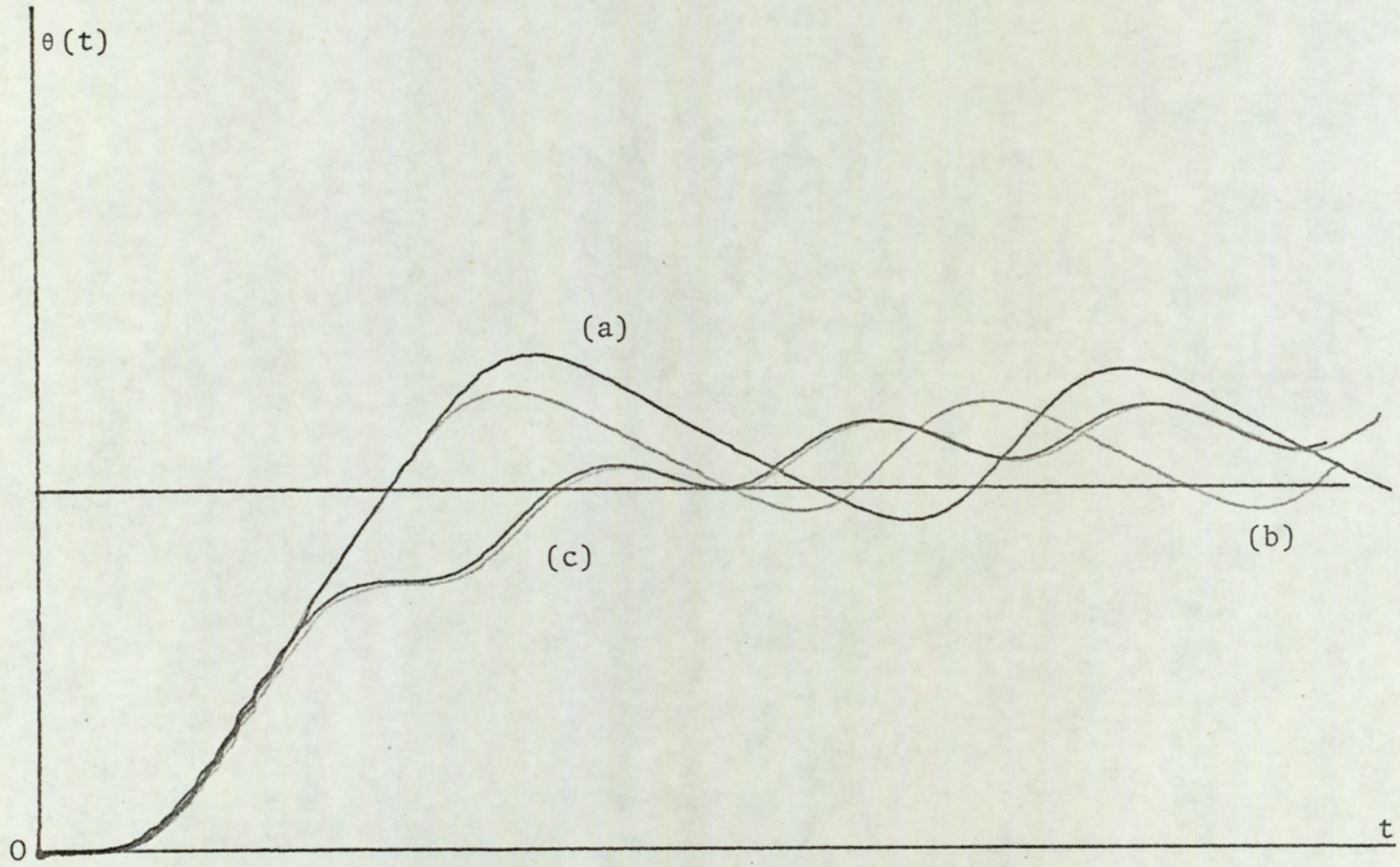


Fig. 5.6 Offset errors due to excessive derivative feedback. (a) no derivative feedback (b) restricted derivative feedback (c) excessive derivative feedback.

different from the model simulated on the analogue computer.

### 5.3.2. Three-position control with derivative feedback

The main improvement in process performance that results when derivative control is incorporated in three-position controllers is the faster response to load disturbances since narrower calibration increments are permitted.

Figure 5.7 displays the step response of an electroheat process controlled in the following modes:

1) Open-loop control.
2) Three-position closed-loop control without derivative feedback.
3) Three-position closed-loop control with derivative feedback.

Clearly, the best response is obtained when a three-position controller with derivative feedback is used. The response is also time-optimal since the process utilises the maximum available power until the desired temperature is attained. A close correlation between the theoretical results and the experimental results was observed when the process was simulated on the analogue computer. The simulated process had the same characteristics as in Section 5.3.1. The optimum derivative content K was found to be ( for $\theta_r/G = 0.5$)

$$K_{opt} = 0.26$$

The theoretically derived value of $K_{opt}$ is found to be (from equation 3)

$$K_{opt} = T \frac{1 - \exp(-L/T)}{1 + \exp(-L/T)} = 0.27$$

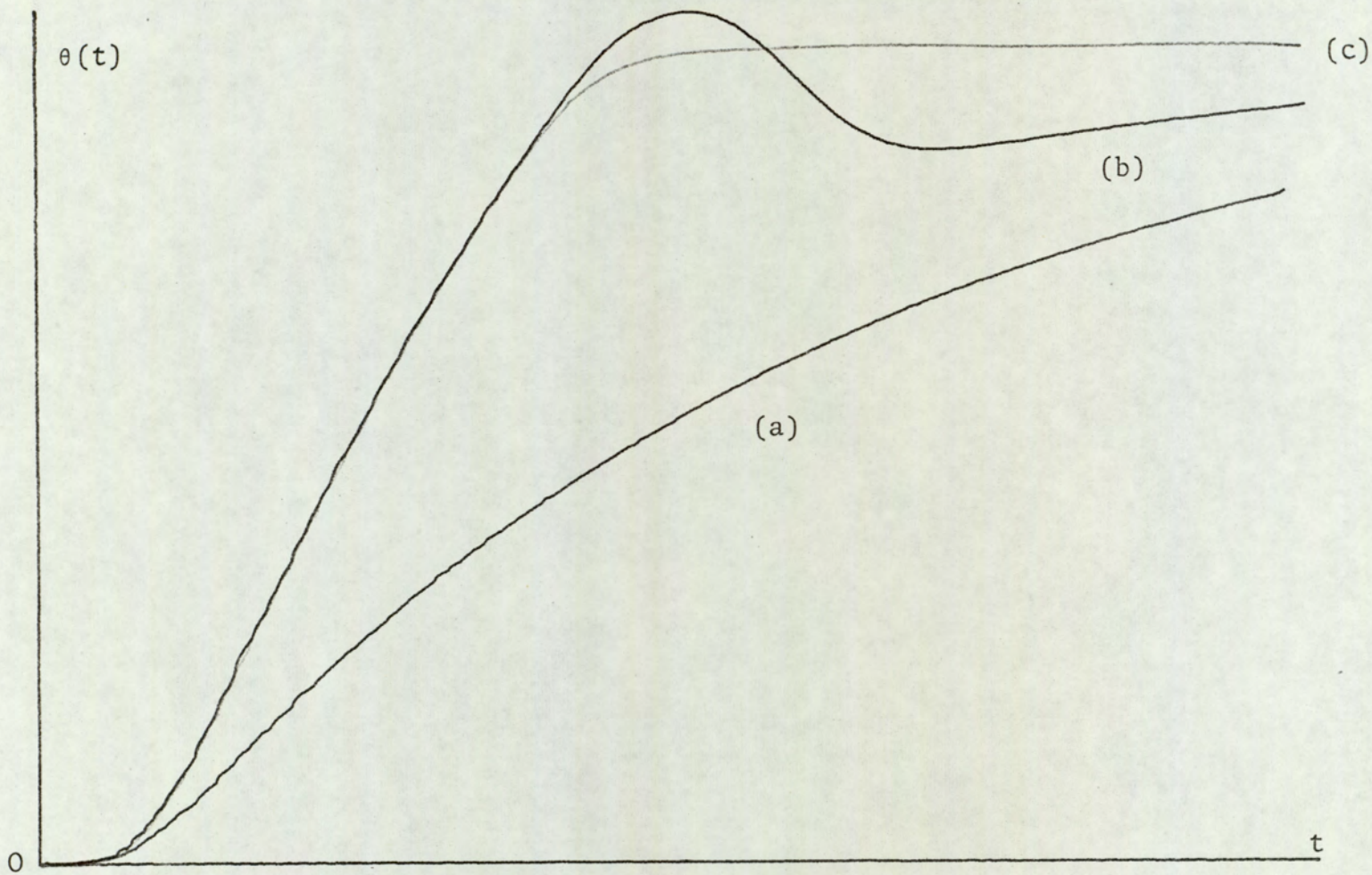


Fig. 5.7 Step response of a three-position controlled electroheat process. (a) open loop control (b) three-position control without derivative feedback and (c) three-position control with optimum derivative feedback.

The close agreement between the analytical and experimental results was verified for various values of the parameters. The theoretically derived results were found to hold even if the simulated process was inexact.

## 5.4 MODEL-REFERENCED OPTIMAL CONTROLLER FOR ELECTROHEAT PROCESSES

The model-referenced optimal controller developed in Chapter 4, section 4.5, was simulated on the analogue computer. The block diagram of the control system is shown in Figure 4.13 . The response to a constant input command $\theta_r$ was observed for the following configurations of the control system:

i) model-referenced controller without the cascaded three-position controller.
ii) model-referenced controller with three-position controller in cascade.
iii) model-referenced controller in cascade with three-position controller, but with an inexact model.

The responses are displayed in Figure 5.8 for the following process characteristics:

$G = 10v$
$\theta_r = 5\ v$
$L = 0.5$ sec.
$T = 4$ secs.

For the inexact model L and T had the following values:

$L_{model} = 0.3$ sec.
$T_{model} = 5.0$ secs.

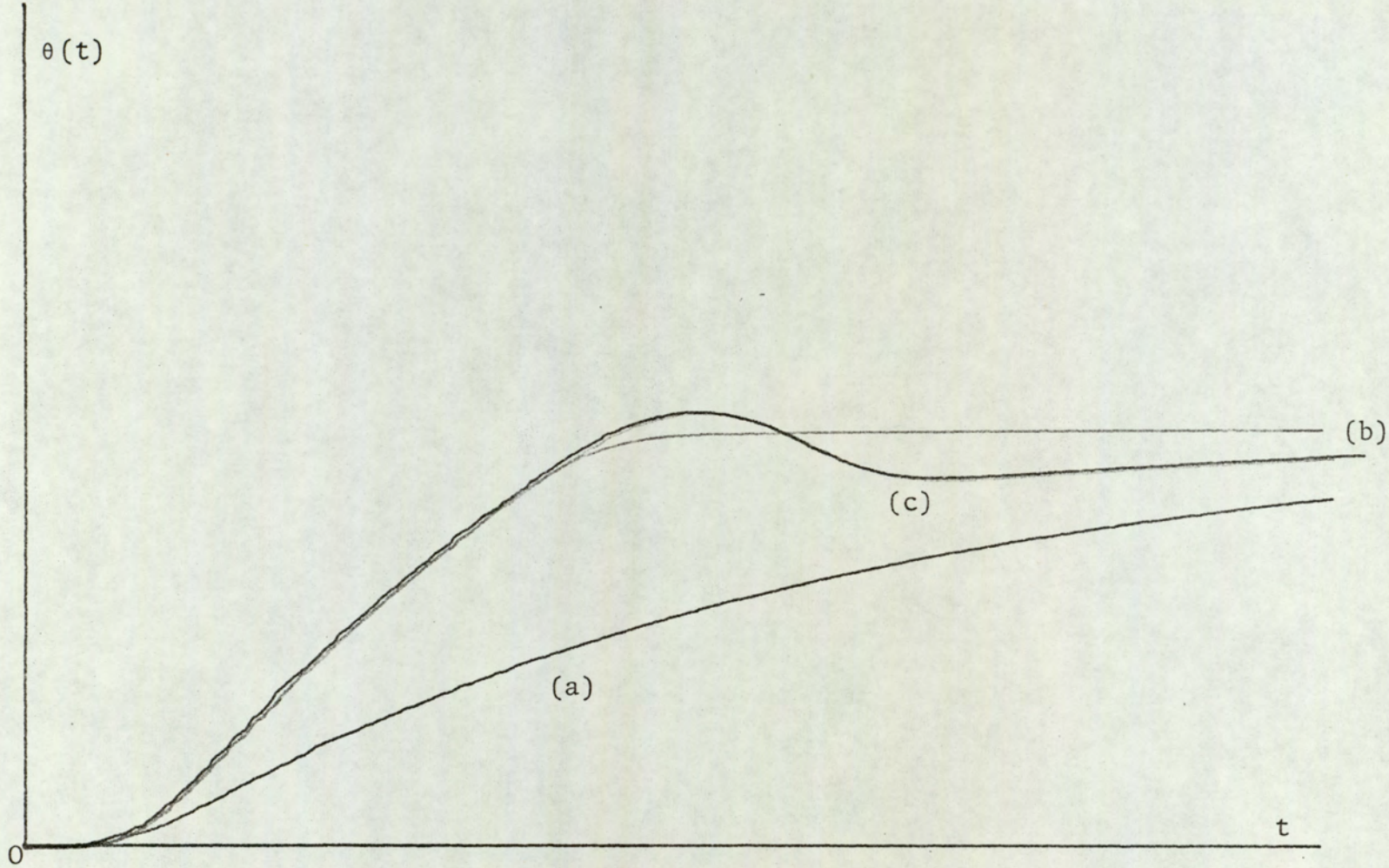


Fig. 5.8 Optimal control of electroheat process. (a) model-referenced control (b) model-referenced control in cascade with 3-position control (optimal control) and (c) same as (b), but with inexact model.

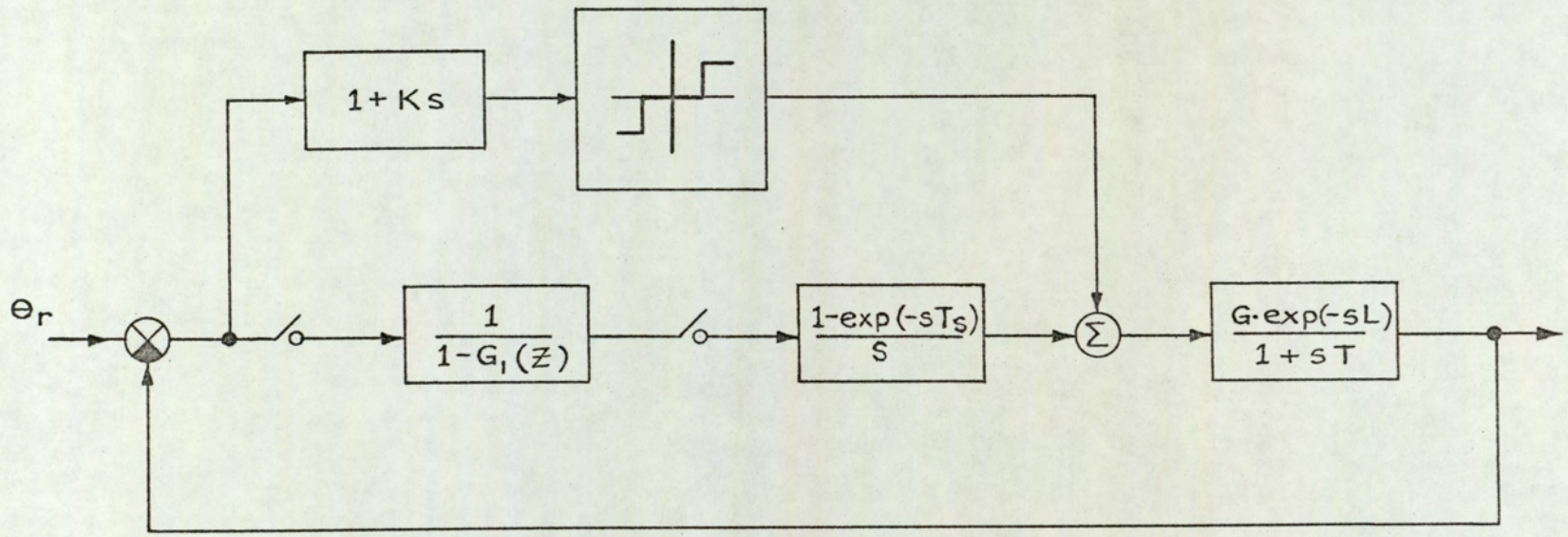


Fig. 5.9 Control scheme for fail-safe operation of electroheat process.

A study of the response under the aforementioned conditions revealed the following advantages:

i) The system was highly stable.

ii) The process attained the desired value $_r$ even when the three-position controller failed.

iii) The response of the process when the model was inexact was still acceptable and there was only a small overshoot.

iv) It was further observed that fail safe (or a stand by) control system is attainable by a parallel combination of the three-position controller and the model-referenced controller, as shown in Figure 5.9. When either of the controllers failed or was disconnected the other controller was capable of restoring the output variable to the desired value.When both were operative, a time-optimal response was obtained. The control scheme of Figure 5.9 was not merely a fail-safe or a standby system, but was also a time-optimal control system under normal operating conditions.

It was also observed that the response to a given input command $\theta_r$ followed an optimal trajectory given by (see Figure 5.10):

$$\theta_{opt}(t) = G\left\{1 - \exp\{-(t - L)/T\}\right\} \quad t > L$$

When the desired value $\theta_r$ was attained by the process, the switching action of the three-position controller caused the output of the process to stabilise at the value $\theta_r$. The response of the process to various input commands are displayed in Figure 5.10.

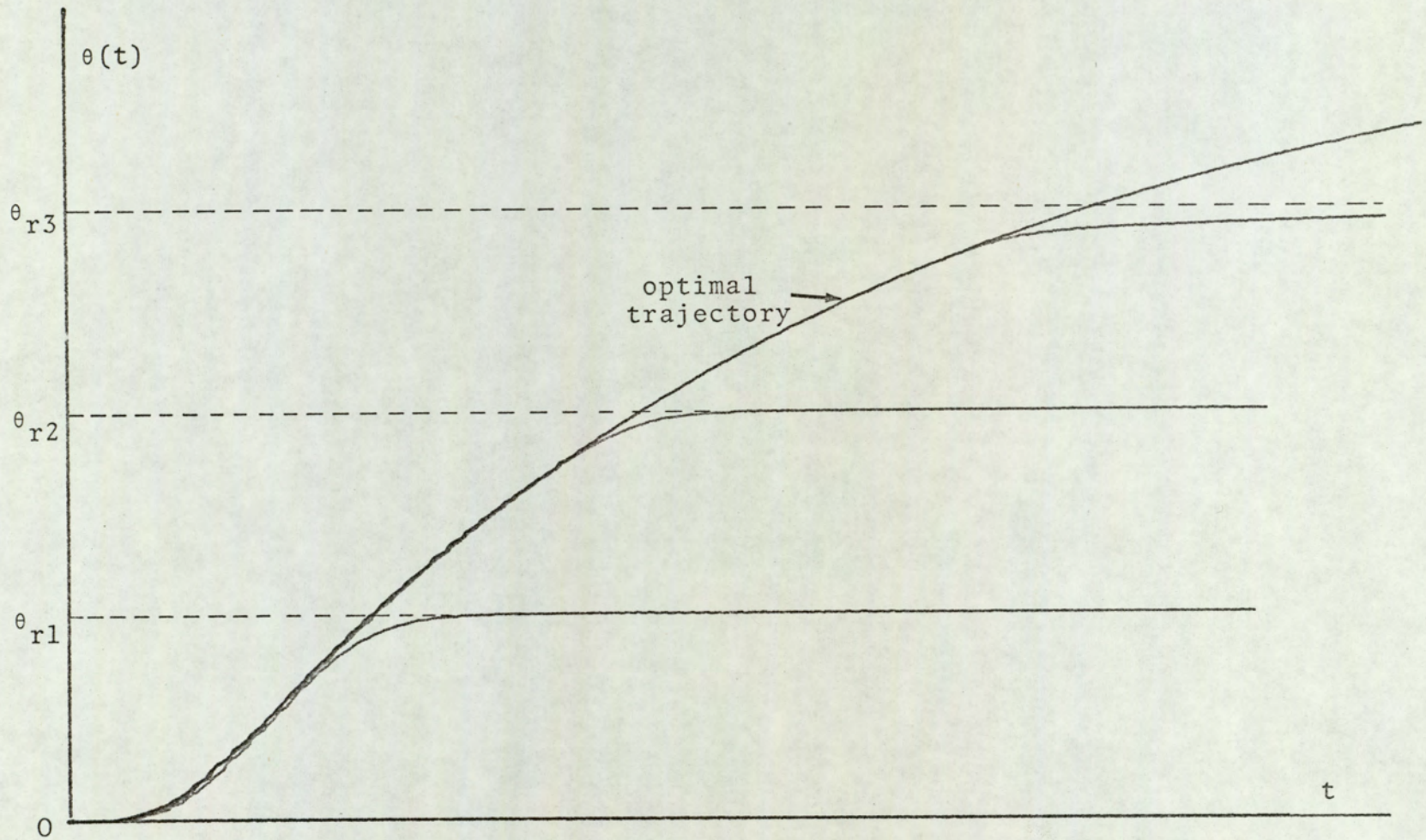


Fig. 5.10 Step response of optimally controlled electroheat process to various input commands.

The response of the model-referenced control scheme was also compared with the response of a PI controlled process with optimal control settings recommended by Zeigler and Nichols[5 18], Harriott[21] and Jackson [20] . The PI controller settings for a process whose transfer function was of the form (1.1) were:

Proportional gain $K_p = \frac{T}{L.G}$

Reset time $T_r = \frac{L.G}{0.3}$

where L is the transit delay of the process and T is the time constant of the process.

and the controller transfer function had the form

$$G_c(s) = (K_p + \frac{1}{T_R.s})$$

The response of the conventional PI controlled process and the model-referenced controlled process are shown in Figure 5.11. The response of the model-referenced control scheme is clearly better than the conventional PI control scheme.

## 5.5 SUMMARY

1) Experimental studies on the analogue computer and the digital computer have established the accuracy of the describing function technique in the stability analysis of multiposition discontinuously controlled electroheat processes.

2) The newly defined exponential describing function has been found to yield reasonably accurate results for establishing the critically damped response criteria.

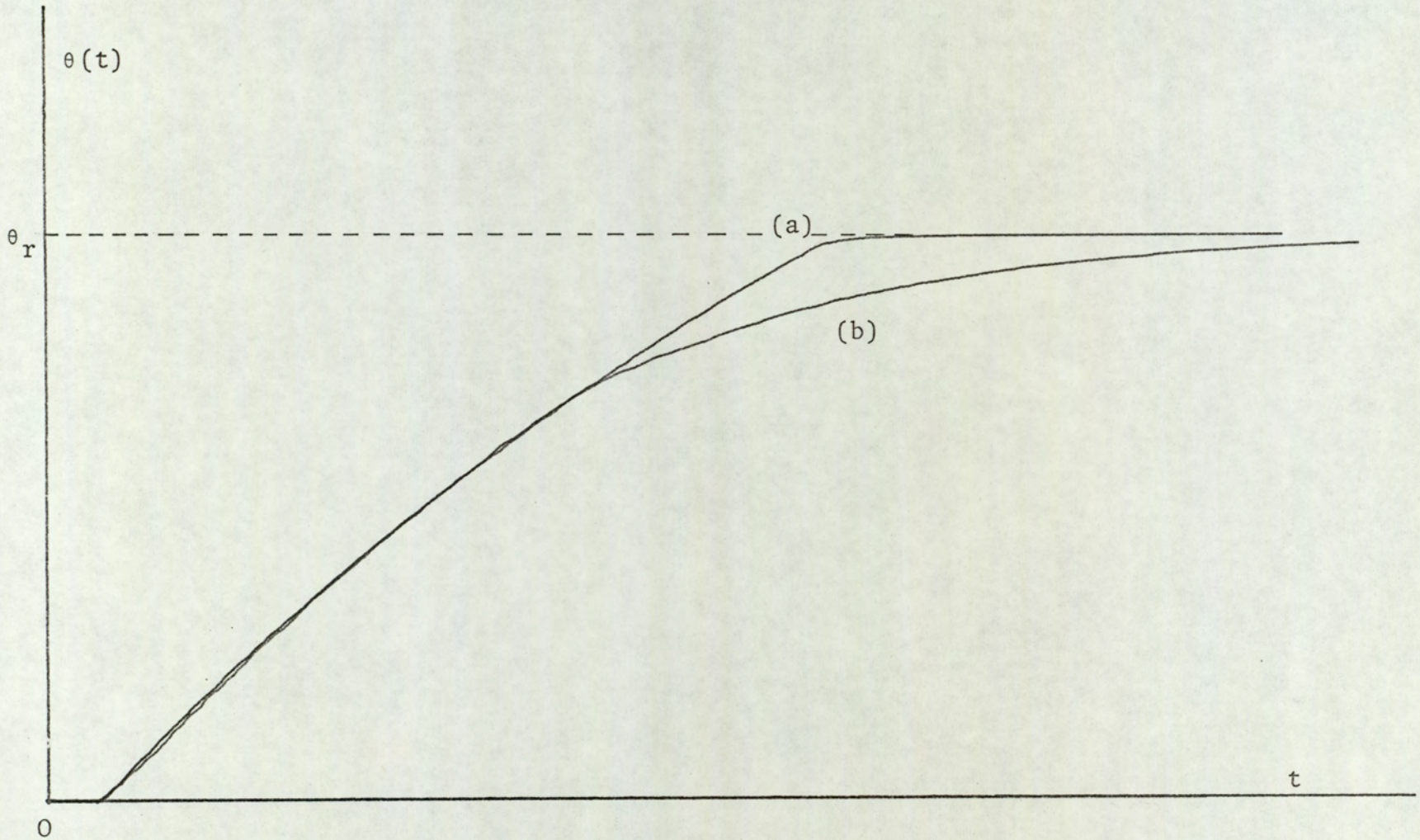


Fig. 5.11 Comparison of Conventional control with the optimal control strategy for electroheat process. (a) optimal control (b) conventional PI control.

3) The feasibility of incorporating derivative feedback for discontinuous controllers has been demonstrated on the analogue computer.

4) The model-referenced control scheme has been shown to be superior to the current control strategies in the field of electroheat processes.

5) The combination of model-referenced controller and the three-position discontinuous controller has been shown to yield a time-optimal control strategy (when operating together) or a standby control system (when operating individually).

## APPENDIX 5A

1) The main program simulates electroheat processes modelled by equation (1.1) and evaluates the temperature according to control law.

```
C  SIMULATION OF FIRST ORDER PROCESS WITH DELAY. THIS
C  PROGRAM SHOULD BE USED WITH TWO SUBROUTINES. THE  FIRST
C  SUBROUTINE DEFINES THE CONTROL LAW; THE SECOND DEFINES
C  THE FORM OF OUTPUT DESIRED,VIZ. GRAPHIC OR TABULAR. THE
C  PROCESS TRANSFER FUNCTION HAS THE FORM
C    G(S) = EXP(-SL)/(1 + ST).  THE INPUT TO THE CONTROL
C  ELEMENT IS A STEP FUCTION OF MAGNITUDE R.
C  THE OUTPUT TEMPERATURE WILL BE DENOTED BY THETA.
C  G IS THE GAIN OF THE HEATING APPARATUS. L IS THE
C  TRANSIT DELAY, T IS THE TIME CONSTANT. M IS THE MANI-
C  PULATED VARIABLE INDICATING THE INPUT HEAT POWER TO
C  THE PROCESS. K, MYU, E, REFER TO THE CONTROLLER PARA-
C  METERS. CON REFERS TO THE FORM OF CONTROL; IF CON
C  EQUALS 1 THEN DISCONTINUOUS CONTROL; IF CON EQUALS 0
C  WE HAVE CONTINUOUS PROPORTIONAL CONTROL. K REPRESENTS
C  THE DISCONTINUOUS CONTROL CHARACTERISTIC; E REPRE-
C  SENTS THE CALIBRATION INCREMENT; MYU INDICATES THE
C  GAIN OR SENSITIVITY OF THE CONTROLLER.
C  THE PROGRAM BEGINS.
C  DECLARE THE VARIABLES VIZ. REAL OR INTEGER.
C     REAL T, THETA,M,MYU,CON
      INTEGER L,K
      DIMENSION THETA(150),M(150)
C  READ IN THE VALUES OF L,T,THETA(1),G,MYU,K,CON,R
      READ(1,1)L,K,T,G,R,MYU,THETA(1),CON
    1 FORMAT(2I3,5F6.3,F3.1)
      N1=1
      Z=L
      ELBYT=Z/T
      IF(CON.EQ.0.0)GOTO7
      IF(CON.EQ.1.0)GOTO8
    7 WRITE(2,19)ELBYT,G,MYU,R
   19 FORMAT(29H PROPORTIONAL CONTROL//6H L/T =,F6.3,5X,3H
     1G =,F6.3//16H MYU =,F6.3,5X,3HR =,F6.3//)
      GOTO9
    8 WRITE(2,3)ELBYT,G,MYU,R,K
    3 FORMAT(29H DISCONTINUOUS CONTROL//6H L/T =,F6.3,5X,3H
     2G =,F6.3//6H MYU =,F6.3,5X,3HK =,I3//)
C  DEFINE INITIAL CONDITIONS ON M
    9 DO2J=1,L
    2 M(J)=0.0
C  CHOOSE THE APPROPRIATE CONTROL LAW FOR M(L+1)
   15 A=M(L+1)
      D=THETA(1)
      CALL CONLAW(A,D,R,G,MYU,K)
      M(L+1)=A
      CALL OUTPUT(D,R,N1)
      I=L+2
C  EVALUATE THETA FOR TIME (L+2)
      DO6J=2,I
      THETA(J)=THETA(J-1)*EXP(-1./T)+M(J-1)*(1.-EXP(-1./T))
C  PRINT OUTPUT RESULT
      D=THETA(J)
      CALL OUTPUT(D,R,N1)
    6 CONTINUE
C  EVALUATE M(L+2)
      A=M(L+2)
      D=THETA(2)
      CALL CONLAW(A,D,R,G,MYU,K)
      M(L+2)=A
C  TO SAVE STORAGE, REPEAT CYCLE I.E. C(L+3)=C(1)
      THETA(1)=THETA(L+2)*EXP(-1./T)+M(L+2)*(1.-EXP(-1./T))
      J=1
   10 A=M(J)
      D=THETA(J+2)
C  EVALUATE M(J) FOR J=1 TO J=L
      CALL CONLAW(A,D,R,G,MYU,K)
      M(J)=A
      J=J+1
       IF(J.GT.L)GOTO15
      GOTO10
      END
```

2) Subroutine for proportional control of electroheat process.

```
C  CONTROL LAW FOR PROPORTIONAL CONTROLLER WITH SATURATION.
C  SUBROUTINE FOR DEFINING THE PROPORTIONAL CONTROLLER WITH
C  SATURATION. THE VALUE OF M AT ZERO ERROR IS THE NORMALISED
C  INPUT COMMAND.
      SUBROUTINE CONLAW(A,D,R,G,MYU,K)
      REAL MYU
C  DEFINE CONTROL LAW
      A=R*(1.+MYU)-MYU*D
      IF(A.GT.G)A=G
      IF(A.LT.0.0)A=0.0
      RETURN
      END
```

3) Subroutine for multiposition discontinuous control of electroheat process.

```
C  SUBROUTINE FOR A CONTROL LAW THAT CORRESPONDS TO THE
C  MULTIPOSITION DISCONTINUOUS CONTROL ELEMENT CHARAC-
C  TERISTIC. THE DEAD ZONE AROUND THE E = 0 AXIS HAS A
C  WIDTH 2E; WHILE THE OTHER ZONES OF CONSTANT CONTROL
C  EFFORTHAVE WIDTH E . THE DISTRIBUTION OF THE CALIB-
C  RATION INCREMENTS IS LINEAR.
      SUBROUTINE CONLAW(A,D,R,G,MYU,K)
      REAL MYU
C  DEFINE CONTROL LAW
      B1=K
      E=G/(2.*B1*MYU)
      IF(D.LT.R-B1*E)A=G
      IF(D.GT.R+B1*E)A=0.0
      DO71J=1,K
      Y1=J
      Y2=J-1
      IF(D.LT.R-Y2*E.AND.D.GT.R-Y1*E)A=R+G*((Y2/(2.*B1)))
      IF(D.GT.R+Y2*E.AND.D.LT.R+Y1*E)A=R-G*((Y2/(2.*B1)))
      IF(D.LT.0.0)D=0.0
   71 CONTINUE
      RETURN
      END
```

4) Subroutine for plotting output temperature of controlled electroheat process against time.

```
C  SUBROUTINE FOR GRAPHIC PLOT
      SUBROUTINE OUTPUT(D,R,N1)
      REAL LINE
      DIMENSION LINE(61)
      DATA BLANK,DOT,PLUS,STAR,HORIZ/1H ,1H.,1H+,1H*,1H-/
      IF(N1.GT.1)GOTO2
      DO101J=1,61
  101 LINE(J)=DOT
      DO102J=1,61,5
  102 LINE(J)=PLUS
      WRITE(2,103)LINE
  103 FORMAT(1H ,61A1)
      DO104J=1,61
  104 LINE(J)=BLANK
      LINE(1)=DOT
    2 N=D+1.5
      IF(N.LT.1)N=1
      DO106J=1,61,5
  106 LINE(J)=PLUS
      N=R+1.0
      LINE(N)=DOT
      LINE(N)=STAR
      WRITE(2,107)LINE,D
  107 FORMAT(1H ,61A1,F8.3)
      LINE(N)=BLANK
      N1=2
      RETURN
      END
```

5) Subroutine for tabulating output temperature of controlled electroheat process.

```
C  SUBROUTINE FOR TABULATING TWO PARAMETERS OF AN
C  ELECTROHEAT PROCESS CONTROL SIMULATION PROBLEM
      SUBROUTINE OUTPUT(D,R,N1)
      WRITE(2,1)D,R
    1 FORMAT(2F12.5)
      RETURN
      END
```

# APPENDIX 5A

Chapter 6

# GENERAL CONCLUSIONS

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# GENERAL CONCLUSIONS.

Individual conclusions have been summarised at the end of each chapter. Here the general conclusions are listed together with their implications on future developments in the control of electroheat processes for industrial applications. Also included are details of where these results have been published.

1) The control of an important class of electroheat processes has been examined in depth. These are processes with distributed parameters and are common in industry. Typical examples are induction furnaces, electroheated fluidised beds, plasma torches, boilers, vats, kilns, etc.

2) As a result of the application of the established describing function technique to this problem, a new unified approach to the determination of stability in controlled electroheat processes has been established. The most important feature of this contribution is that the same approach covers both continuous and discontinuous electroheat. It therefore represents a considerable advance on Roots and Wu [10] which was restricted to continuous control.

3) A new exponential describing function has been developed and tested. This greatly simplifies determination of the criteria for nonoscillatory response for discontinuous controllers of all classes (ie: two-position, multi-position, quasi-continuous, etc) and also the proportional controller with saturation (which is the limiting case of quasi-continuous control). This combination is in the author's opinion even more important than the new stability determination of item 2, as it readily determines the controller characteristics

for desired transient response. Because it covers all classes of discontinuous and continuous controllers it is a significant advance over the second Roots and Wu contribution [15] as they only considered the continuous case.

4) New strategies have been developed and proven for improving the performance of discontinuously controlled electroheat processes. The salient contributions are:

   i) The first analytic derivation of the optimal controller parameters for derivative discontinuous control of electroheat.

   ii) Item (i) was extended to cover three-position control and criteria were established for the minimization of deadzone in such controllers.

   The above represents a considerable advance beyond the contribution of Roots and Walker [22].

5) New strategies have been developed and proven for improving the performance of continuously controlled electroheat. The salient contributions are:

   i) A generalised root-locus technique for the design of continuous controllers of given performance specifications when controlling electroheat processes.

   ii) Developing and proving the concept of a model-referenced controller. This has limited computer storage requirements and thus facilitates time-shared control with low-priced digital computers.

iii) A fascinating feature of (ii) is that control of a standard acceptable in many industrial operations is obtained even when the model is inaccurate.

The above represents a significant improvement over the approach of Chu [14] and Harriott [21] whose work mainly form the basis of industrial electroheat control.

6) The new concept of controlling electroheat by the combination of a model-referenced computer controller and a three-position discontinuous controller was developed. The significant contributions are:

i) The time-optimal control of the process is easily achieved.

ii) The two types of controllers can be in cascade or parallel. If in cascade the combination never saturates. If in parallel it gives fail-safe or standby control.

iii) This combination gives better command response and disturbance compensation than any electroheat controller currently in use.

If one takes Roots [35] or Harriott [21] as the latest state of the art of electroheat control, this new concept is a considerable advance over either of them.

## PUBLICATION OF THESE RESULTS

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B) "Discontinuous 'PD' control of thermal processes". Trans. Inst. Measurement and Control, Vol.2, No.5, May 1969, pp.T57-T64.

C) "Derivative discontinuous electroheat control". IEEE Trans. Industry and General Applications, accepted.

D) "Temperature control strategies for industrial processes". IEEE Trans. Industry and General Applications, accepted.

E) "Stability criteria for discontinuous electroheat processes". Accepted for IEEE Industry and General Applications Annual Meeting, Detroit, October 1969, and for IEEE Trans. Industry and General Applications.

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