THE UNIVERSITY OF ASTON IN BIRMINGHAM

ELECTRICAL ENGINEERING DEPARTMENT.

NEW STRATEGIES FCR CONTROL OF THE ELECTRICAL MAXIMUM DEMAND

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Thesis
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Ph.D. Thesis. January, 1971.

SUMMARY.

The object of this investigation is to develop new strategies that minimise the maximum demand on the electric power system, and maximise plant utilisation, for ensembles of electroheat processes.

Two classes of ensembles of similar on-off electroheat processes are thoroughly examined. The first ensemble is of magnitude ten or less, as when the processes are installed in a factory. The second is of large magnitude, as in domestic space and water heating.

New strategies and important design criteria are presented for demand control of loads that are switched (a) deterministically, and (b) by thermostat. In the latter case, control decisions are based upon prediction of load by monitoring of past energy consumption and of process temperature states.

New models are established to determine the statistical properties of the demand due to ensembles of similar two-position space heating processes. Results are applied to determine the penalty incurred, as a probability function, due to the demand exceeding a set level. These results are of particular value in assessing the economic viability of proposed space heating installations.

The models are verified by digital simulation using pseudo-random numbers, and by sampling from known probability distributions; computer programs are presented.

Switching characteristics are investigated experimentally for a number of similar thermostats, operating in turn within an environmental test chamber.

Two criteria of demand optimisation are thoroughly analysed:

1. The maintenance of a "minimum-comfort" level for space heating.

2. Minimisation of the cost per unit product for a specific industrial electroheat process. A case is presented for control of the maximum demand by digital computer.

A proportional plus integral controller is designed for continuous demand control of an electrothermal process. The system performance is simulated on an analogue computer.

ACKNOWLEDGEMENTS.

I wish to express my appreciation of the encouragement and support of this work by the late Professor W.K. Roots.

My thanks are due to my supervisor, Dr. G.K. Steel, for many useful discussions and for his guidance at all stages of the project.

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LIST OF SYMBOLS.

b	primary feedback.
b_2	secondary feedback.
e	actuating signal.
m	manipulated variable (0 or + 1).
$m\ (t)$	manipulated variable as a function of time.
$\bar{m}$	mean value of m under d.e.c. = $(1/t_q) \int_0^{t_q} m\ (t)\ dt$.
q	differential of discontinuous control element characteristic.
r	reference input.
s	Laplace complex variable.
t	general symbolfor time.
t_o	on-time = portion of t_q for which $m = + 1$.
t_p	off-time = portion of t_q for which $m = 0$.
t_q	d.e.c. period = $t_o + t_p$.
$\check{t}_q$	minimum t_q under d.e.c. conditions.
u	disturbance input.
A	transfer characteristic of reference input element.
B	secondary feedback loop compensated to give zero deadspace.
C	equivalent thermal capacitance of the controlled space.
C_1	equivalent thermal capacitance of heating apparatus.
F	thermal attenuation in an electroheat process.
G	thermal gain of heating apparatus in an electroheat process.
H_1	primary feedback element transfer function.
H_2	secondary feedback element transfer function.
I	integrating interval for demand evaluation.
L	transitdelay in an electroheat process.
R	general symbol for equivalent thermal resistance.
T	time constant of electroheat process.
$T(m)$	mode-dependent time constant of electroheat process.
T_A	active time constant of electroheat process when heating.
T_p	passive time constant of electroheat process when cooling.
θ	temperature of the process.
θ_a	heater output temperature.
$\theta\ (t)$	θ as a function of time.
θ_r	input command.
θ_u	general symbol for a disturbance.
d.e.c.	dynamic equilibrium cycling.
l.p.i.	location of particular interest.

Symbols not listed here are defined as they occur in the text.

CHAPTER 1.

INTRODUCTION.

As the use of electricity for industrial and domestic purposes continues to expand, the efficient utilisation of electrical energy becomes of increasing importance both to the producer and the consumer.

Current research into this problem includes the long-term planning of future energy requirements and the alleviation of the immediate demand upon the electricity supply. Heating load is anticipated from past records and by updated weather forecasting, and industrial consumers are encouraged to restrict their individual maximum demands. As a result, continuity of supply may be maintained except in abnormally cold weather conditions.

Minimisation of the maximum demand due to process ensembles of finite magnitude is of importance to the industrial consumer, who may arrange his load to take maximum advantage of existing tariff rates. Area Electricity Boards will be interested in the application of a probability cost function to large process ensembles, whereby the economic consequences of uncontrolled load may be appraised. The evaluation in monetary terms of the penalty incurred when exceeding a given demand can then form a basis for assessing the viability of a proposed installation.

The economics of demand control for any specific industrial project will need to be determined from operating experience, as the KWh per unit product are generally not independent of the production level. The application of electrical demand control is so widespread that it has been decided to concentrate in this thesis mainly upon on/off electroheat processes. However, mixed loads are also considered.

Briefly, the objectives of the investigations described in the following chapters are:

1. To develop an analytic basis for determining patterns of load behaviour for ensembles of two-position processes.
2. To develop strategies for control of the demand due to these ensembles.

3. To investigate a "minimum-comfort" criterionof optimisation for discontinuous temperature-control systems, and its effect upon the electrical demand.

4. To analyse the economic considerations arising as a result of the implementation of demand control.

The course of the investigations presented in this report is displayed in the flowchart Fig. 1.1.

INTRODUCTION.

EXISTING DEMAND CONTROL STRATEGIES.

IMPROVED STRATEGIES & CONTROL CRITERIA.

Small Ensembles of On-Off Processes (Incidence Deterministic.)

Large Ensembles of On-Off Processes (Incidence Normally Distributed).

Large Ensembles of On-Off Processes (Incidence Uniformly Distributed.)

SIMULATION STUDIES OF DEMAND DUE TO ON-OFF PROCESSES.

OPERATIONAL CRITERIA

Minimum-Comfort Level (Temperature and Demand Control for Temperature-Sensitive Loads).

Economic

Minimum Cost Per Unit Product.

Maximisation of Production.

CONCLUSIONS & APPENDICES.

FIG. 1.1 FLOWCHART OF THE DEVELOPMENT OF THE RESEARCH WORK.

CHAPTER 2.

TARIFFS AND STRATEGIES FOR THE DEMAND CONTROL OF PROCESS ENSEMBLES WITH ON-OFF LOADS.

2.1 INTRODUCTION.

The purpose of this chapter is to introduce the problems involved in the demand control by computer of process ensembles with on-off loads; also methods proposed by the author for dealing with these problems.

The computer will carry out a policy whereby the maximum demand on the operating system is minimised and the utilisation of plant equipment is maximised. These two conditions are in general compatible, with the particular exception where the maximum integrated demand is reduced by increasing the waiting time between successive cycles of the load.

An outline of tariff structures formulated by the Central Electricity Generating Board and Area Boards is given, together with a review of attempts which have been made to minimise load penalties in the consumer industry.

For the purpose of clarity, the term "indicated maximum demand" will be used to replace the term "maximum demand" quoted in current industrial electricity tariffs.

This is obtained as follows:

The instantaneous demand is integrated over a nominated period t_i, (30 minutes in the United Kingdom), and over consecutive periods throughout the Electricity Authority's account month; for each period the KVA-hours are metered to give a KVA demand of KVA-hours x $\frac{60}{t_i}$, the greatest value of which is taken as the "indicated maximum demand" for that month. However, alternative terms are available from different Area Boards- for example, in a tariff published by the Midlands Electricity Board, the maximum demand for the account month is taken as the greatest demand value within that month or the preceding eleven months, whichever is the greater.

2.2 ELECTRICITY SUPPLY TARIFFS.

The development of the electricity supply tariff structure in the United Kingdom since nationalisation of the industry has been reviewed by Sayers[1], so that it is now proposed only to survey the more recent developments.

2.2.1 Bulk Supply Tariff.

Electricity is purchased from the Central Electricity Generating Board by Area Boards (for resale to customers) under the terms of a national Bulk Supply Tariff, in which there is a demand charge of the order of £10 per KW of defined national "average peak demand."

Prior to the present structure, the national chargeable demand was defined as the average of the highest demands (summated for all Area Boards) set up in any one half-hour before 31st December and in any one half-hour after 31st December during the C.E.G.B. supply year terminating on the 31st March.

Under the revised structure[2], the highest demands before and after 31st December are again recorded to obtain an average peak demand, except that the times of recording are confined to "potential peak warning periods" in December and January. Readings are operative for charging purposes only if they occur in "potential peak periods" the aggregate of which does not exceed 50 hours a year.

Demand charges payable by an Area Board consist of a "peaking capacity" charge per KW which reflects the cost of plant to meet short-term peaks, together with a "basic capacity" charge per KW, which is based upon plant cost to meet the winter load plateau (defined as 90% of the national average peak demand).

Running rates are adjusted for day and night periods; also for peak periods during December and January.

2.2.2 Industrial Maximum Demand Tariffs.

Tariffs set up by Area Electricity Boards have been formed to reflect in practice the various cost-forming aspects of the pattern of electricity consumption.[51, 52]

Although considerable developments in electric space heating have taken place in the commercial and industrial fields, this demand represents a relatively insignificant part of the industrial load, in which the main use of electricity is for motive power or for electro-chemical or electro-thermal processes. On the other hand, it is the temperature- sensitive loads, of which space heating is foremost, that decide the day of peak demand.

Maximum demand charges are normally formed as a high-priced primary block followed by a number of lower-priced blocks, each proportional to the maximum demand. A reduction in the tariff may be offered if the customer is willing for part of his plant to be treated as a sheddable load at short notice.

The second part of the tariff is a flat rate charge for the energy consumed after payment of initial high-priced blocks of kilowatt-hours per KVA of maximum demand made in the month. The flat rate may be reduced for energy consumed between 11 p.m. and 7 a.m. if the consumer makes a capital contribution to the extra metering equipment required. The payment made per KWh supplied in each month may be subject to a stipulated variation according to change in the cost of fuel used for the purpose of electricity supply in bulk by the C.E.G.B.

Alternative Maximum Demand Tariffs for industrial supplies are available to the customer according to his load requirements; individual terms may be negotiated for supplies which have special load characteristics, e.g. when the supply is required exceptionally during the night compared with the day, or during the summer compared with the winter.

2.3 DIVERSIFICATION OF CONSUMER LOAD.

Owing to the diversity of consumer load requirements, it becomes uneconomic to supply generating or distribution plant capacity of magnitude approaching the total load connected to the electricity supply system.

Broadly speaking, electrical energy has to be produced as demanded, and cannot be stored in significant amounts. Although peak loading during the day can be alleviated by the use of night storage heaters, no facility exists for transfer of seasonal load.

Responsibility for the peak load problem during the winter months lies largely with unrestricted electric space heating, and various tariff suggestions have been made[3] for discouraging the further development of this load. Tariffs aimed at shifting domestic consumption to off-peak periods may effect considerable savings to Electricity Boards which more than outweigh the extra metering charges, and may also represent saving to the consumer.

High-capacity storage radiators have been developed for an eight-hour night charging period for domestic use, since day-time charging is no longer available for new customers. A White Meter tariff is offered by the Area Electricity Boards, in which the consumer's complete domestic supply is switched over to a cheap night rate during the restricted hours.

For industrial consumers whose production load is necessarily confined to the day time, electricity charges may be reduced by adopting control strategies that minimise the maximum demand.

As the size and complexity of process increases, demand control by computer becomes desirable.

On-line computer controls for optimising process products in the face of parameter variations are fairly widespread in industry [5, 6, 7]. However, the installation of a computer specifically to control the maximum demand must be justified by the savings thereby achievedin electricity and overall production costs.

At the Steel, Peech and Tozer steel-making factory a digital computer is used predominantly for this purpose.

The principles of this particular method of control, together with a number of maximum demand schemes currently in operation, are described briefly in section 2.4.

2.4 STRATEGIES FOR LIMITATION OF MAXIMUM DEMAND.

The choice of the maximum indicated demand level to which the load is restricted will be a major factor in the economics of any demand control scheme.

Once the optimum target demand is determined for a given load, any subsequent increase in production schedules will necessitate a re-assessment of the situation.

Methods of containing the total load within the nominated value include:

(1) Selective shedding of load, which, although serious where production is affected, in other cases may cause no more than temporary discomfort, e.g. in certain space heating applications.

(2) Increase of waiting period before re-cycling the process. Although this measure effects a reduction in indicated maximum demand, a limiting feature is the resulting inefficiency of plant usage.

(3) Staggering of incidence of the separate loads in a systematic manner. For a finite number of identical processes, the switching incidence of individual loads may be controlled closely by a master timer.

2.4.1 Predictive Computer Control at Steel, Peech and Tozer.

At Steel, Peech and Tozer, Rotherham, six 120-ton arc furnaces, each of rating 40 MVA, are used for steel-making and are currently controlled by a digital computer. The main task of the computer is to restrict the maximum demand to an economic level without incurring serious production loss.

Engineering details of the application [8, 9, 10, 11] will be confined to those which are necessary to outline the computer strategy.

2.4.1.1 Melting and Refining Program and Computer Control of Power Input.

The total charge of 120 tons of scrap is loaded into the furnace from two large baskets. The melting and refining program may be described by Fig. 2.1, which represents a typical power input program. The contents of the first basket are almost completely melted before the power is cut, and the second basket is then charged, after which power is restored and the melting process completed. There then follow two refining periods, before tapping of the charge and fettling of the furnace walls complete the cycle.

Automatic control of the power input level was achieved initially by a programmed controller, which effected given transformer tappings for pre-set times before on-load changing to new taps. The input was specified in terms of energy levels by counting pulses from the furnace integrating KWh meters.

This method of power control was superseded by the use of on-line digital computer, thus providing flexibility as a result of continuous monitoring.

2.4.1.2 Computer Control of Maximum Demand.

The maximum demand target level was obtained by plotting production loss curves and cost curves as functions of maximum demand for a model simulating the plant. The actual set value of maximum demand was a function of the total production level and, in fact, was assumed, for a given percentage loss of production, to be approximately proportional to the level of production.

The computer control strategy adopted is illustrated in Fig. 2.2.

If the maximum demand level P MW is assumed constant throughout the half-hour monitoring period, the energy consumed during this period is $T_p = P \times \frac{1}{2}$ MWh, represented by the straight line OT_p. Thus $P = 2T_p$. The line OXY represents the locus of actual energy consumed through the period.

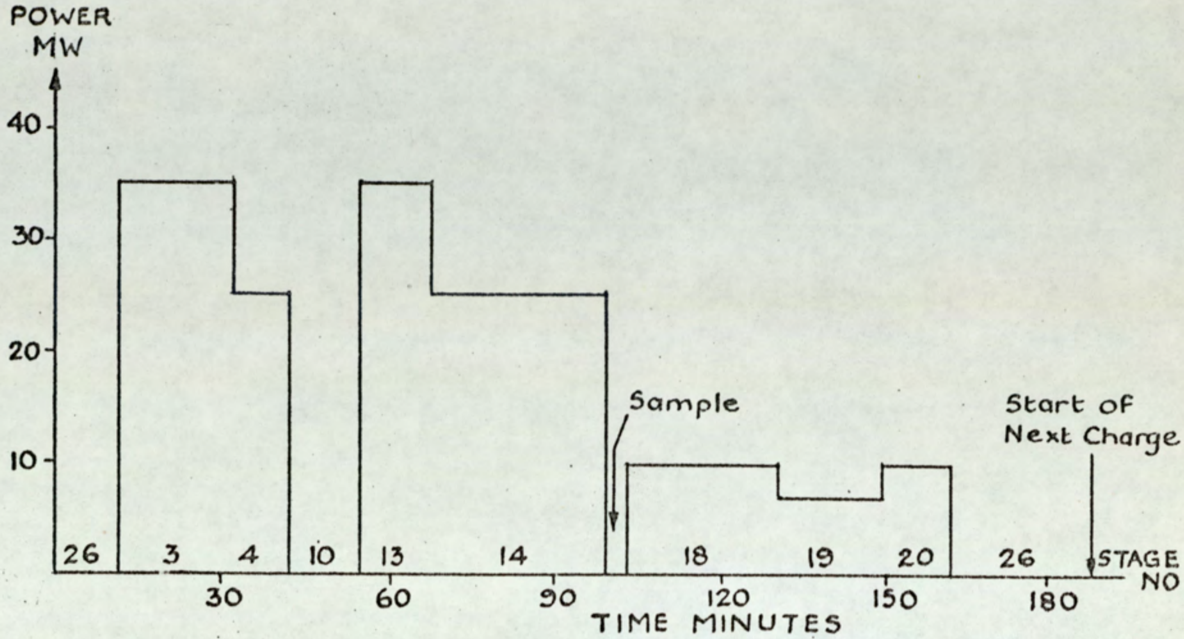
Predictive control may be established in either of two ways, the energy being monitored at 5-second intervals:

(1) At the monitoring point X, AA represents the tangent to curve OXY, and XT_p is the straight line joining X and the target point T_p. The condition $XX' > 0$ shows that the proportion of the quota T_p up to time OX" has been exceeded by the amount XX'.

The difference between the slopes of AA and XT_p gives the power shedding or permissible power increase in MW required to achieve the quota T_p. Increase of the energy consumption above T_p requires instant operation of the overload trip-out.

(2) At each monitoring point the energy E_C consumed from time zero to t = OX" is measured; in the remaining time 30 -tan energy amount $E_Q = T' - E_C$ is available before the quota is exceeded, where $T' = T_p - S$, the term S being a safety margin.

At the monitoring point the computer estimates that an energy E_E will be consumed by the furnace shop in the remaining time 30 - t. If $E_Q - E_E < 0$, corrective action must be taken



STAGE NoS.

26	Tapping and Charging 1st Basket
3	Melting 1st Basket
4	" " "
10	Charging 2nd Basket
13	Melting 2nd Basket
14	" " "
18	First Refine
19	Oxygen Lancing
20	Second Refine
26	Tapping and Fettling

FIG. 2.1 TYPICAL POWER INPUT PROGRAM
FOR A CAST OF STEEL (120-TON CHARGE)

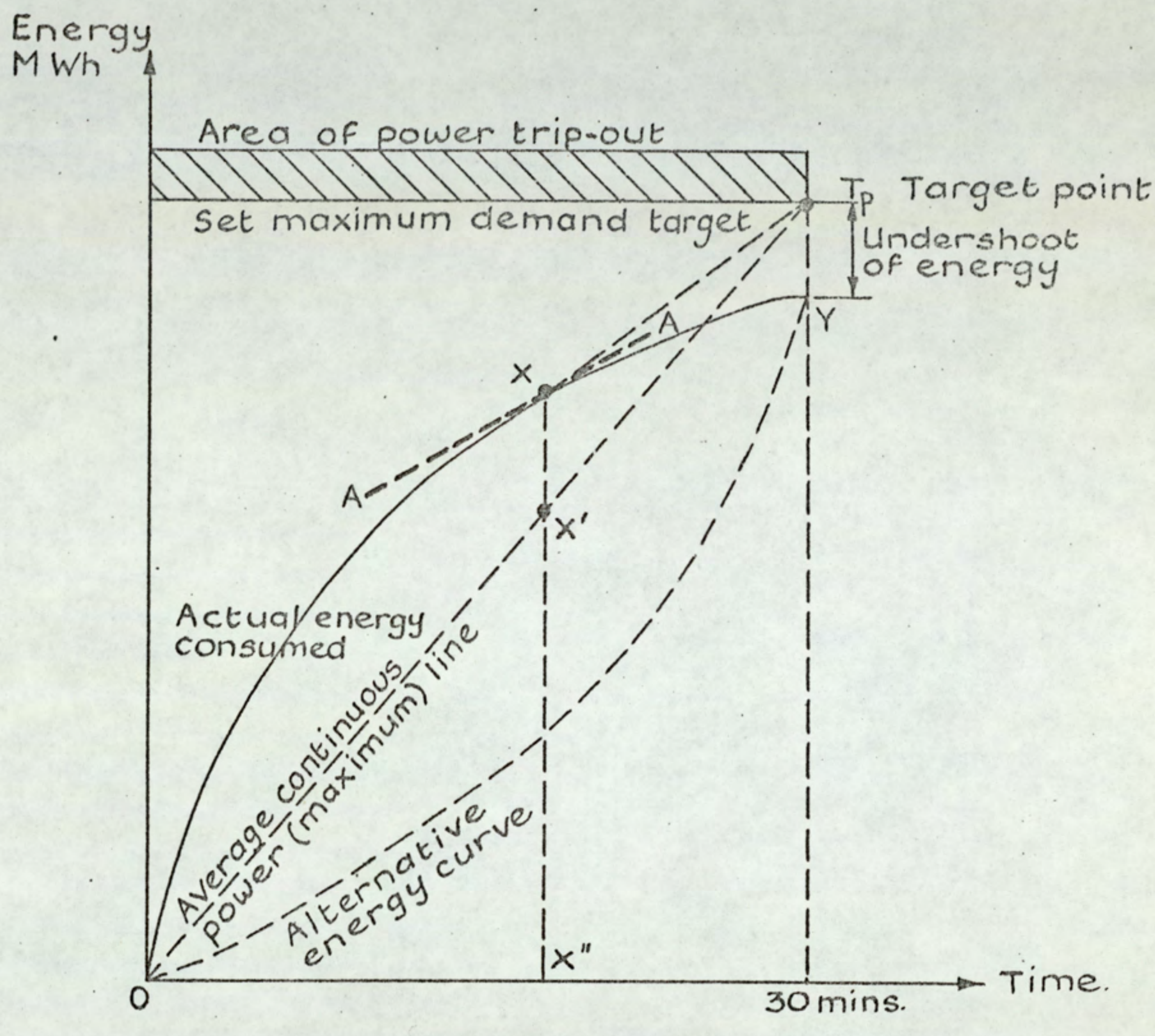


FIG. 2.2 TYPICAL ENERGY/TIME PLOT.

so that the condition $E_Q - E_E \geqslant 0$ is obtained by the time the monitoring period has been completed.

2.4.1.3 Principles of Ordered Load Cutting and Restoration.

Cuts were achieved on individual furnaces by reducing the supply transformer tapping to a point where the heat input from the arc balanced the heat losses. The first furnace to be cut was that one in the earliest stages of melting; if energy restoration were required, this was achieved in the reverse order to that for load cutting.

A cut became necessary when $E_Q - E_E < 0$, where E_Q was the energy available from the quota T_p during the remainder of the half-hour. However, the relation $E_Q - E_E < 0$ or $T_p - S - E_C - E_E < 0$ was modified to

$$T_p - S - E_C - E_E + f\,(30 - t) < 0\,. \qquad (2.1)$$

The function f (30 - t), which was taken as directly proportional to the remaining time 30-t, was introduced to restrict control action during the early part of the half-hour.

During the time of a cut a restoration of power was considered if

$$T_p - S - E_C - E_E > 0\,. \qquad (2.2)$$

The magnitude and time of the cut or restoration was computed so that the estimated energy E_E in the remaining time 30-t was in the zone defined by the inequalities (2.1) and (2.2).

Load was inhibited from coming on within the last three minutes of the monitored period, when maximum demand limitation could not have been implemented in time to maintain the energy consumption at or below T_p - S.

2.4.2 Maximum Demand Control Equipment.

2.4.2.1 At Stewarts and Lloyds.[13]

At the Bromford works of Stewarts and Lloyds, Birmingham, England, equipment has been installed by Thorn Automation automatically to control two electrical furnaces so that a nominated maximum-demand level is not exceeded.

The total load is divided into nine zones, each representing about 200 KW, which may be shed (or restored) sequentially to maintain the desired control.

The principle of control is similar to that employed at Steel, Peech and Tozer: the total consumption up to time t after the commencement of the tariff period is monitored by pulses at 5-KWh intervals, so that the energy available in the remaining time 30-t may be evaluated by a counter by comparison with the pre-set maximum-demand value.

A further counter, which is fed from a crystal oscillator and clock counter, provides a "periods-remaining" output every 90 seconds; a divider then divides the "remaining-energy" output by the "periods-remaining" output to give a "permissible" load. At the end of each 90-second sub-period the current "90-sec usage rate" is compared with the permissible load: the equipment is designed to initiate any load change in order to maintain the indicated demand $\left(\frac{\text{energy consumed over the tariff period}}{\text{tariff period}}\right)$ at or below the pre-set value.

An interesting feature of this system is that the Midland Electricity Board not only supply the 5-KWh summing pulses, but also the 30-minute signal at the end of each monitored demand period. This signal resets the equipment so that the control of the furnace load is maintained over the exact accounting period used by the Board.

Substantial savings in maximum-demand costs are claimed for the system as a result of installing this control equipment.

2.4.2.2 <u>At Alcan Industries.</u>[14]

At the Rogerstone Works of Alcan Industries Limited, maximum-demand control equipment has been installed by Thorn Automation with the object of minimising maximum-demand penalties by maximum utilisation of the available energy.

The principle of operation is as for the installation described in 2.4.2.1 , and the load may be reduced either manually or automatically, or increased under manual control. Digital displays are given of the maximum "permissible" load, "periods remaining", and the magnitude and direction of the load change required to achieve the desired maximum-demand control.

2.4.3 Staggering of Load by Ripple Control.

"Ripple control" [4, 12] is a method of control of electrical equipment by means of switching devices which respond to audio-frequency signals superimposed upon the 50 Hz supply, the conventional time switches being replaced by Ripple Control receivers.

Investigations into Ripple control have been carried out at the Midlands Board Feckenham 275/66KV grid supply point, from which considerable industrial and domestic load is drawn. The arrangement of injection circuits at the Feckenham sub-station is shown in Fig. 2.3.

The injection equipment consists of synchronous motor-generator sets giving a signal frequency of 300 Hz. This signal is made up of an initiating pulse, which starts the synchronous motors in all the receivers, succeeded by coded follow-up pulses, selected combinations being used to actuate particular receivers as required. The receivers contain a resonant circuit tuned to 300 Hz, a synchronous motor-driven cam selector, and separately controlled load switches for independent control of off-peak space heating and water heating.

Ripple control is claimed to be economically feasible [4] for the above two types of load, in which the energy is not necessarily used immediately upon production.

The advantage to Area Boards of replacing time switches by Ripple control receivers, responsive to different signals, is that a reduction in the maximum demand can be achieved by staggering of the storage heater load in cold spells.

2.5 APPLICATION OF DEMAND CONTROL TO ON-OFF LOADS.

A two-level generalised cyclic pattern for each process may be considered, for example, as in Fig. 2.4.

It is not desirable to achieve demand control by increasing the waiting time t_w significantly above the minimum value t_{wmin}, when the requirements of minimum maximum demand and maximum utilisation of generating plant will conflict. When the load is periodic, as for discontinuously-controlled electroheat systems,

$t_o = t_{o1} = t_{o2}$ and $t_{wmin} = t_p$, where $t_o + t_p = t_q$.

In Chapter 3 a control strategy of programmed switching

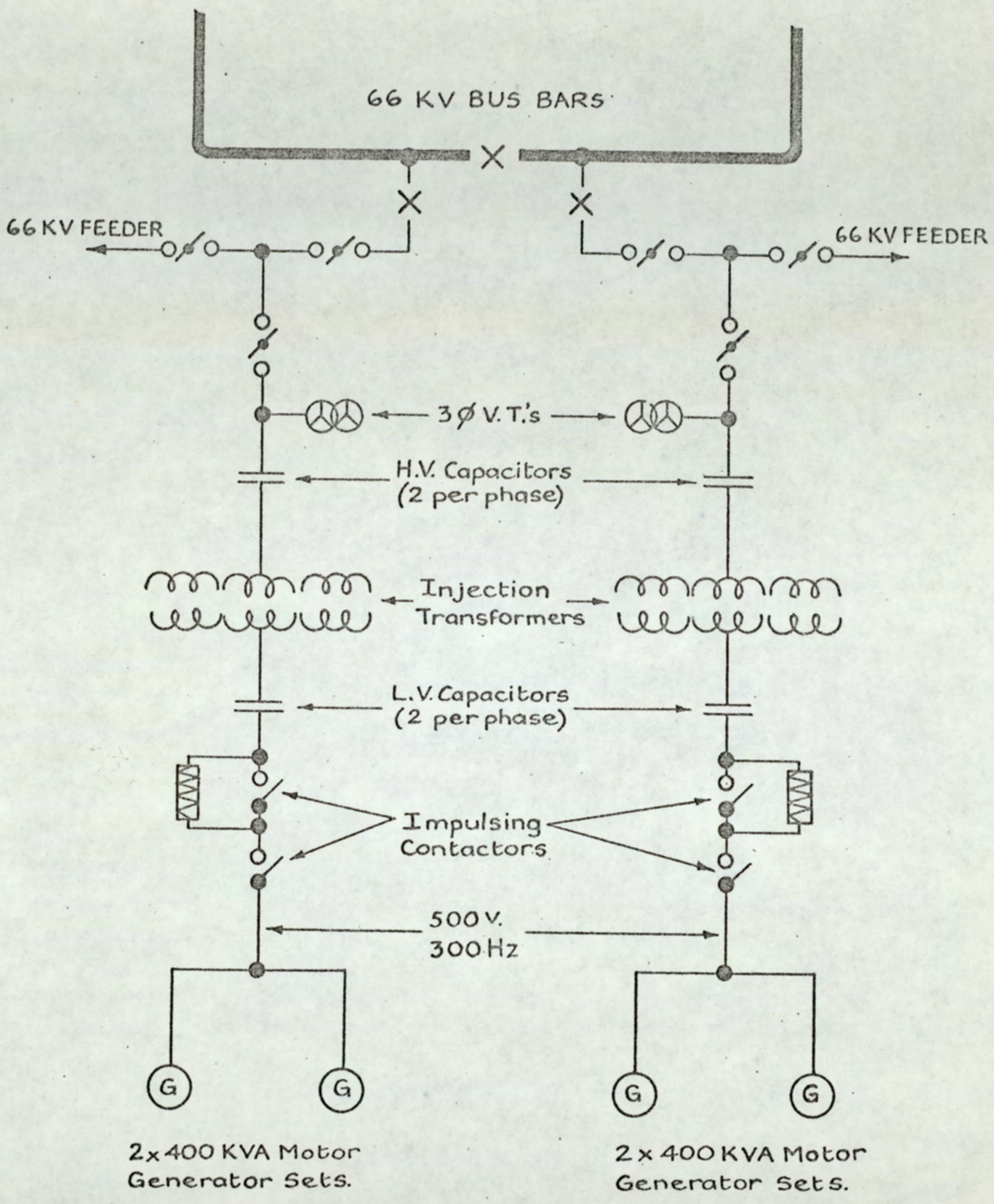
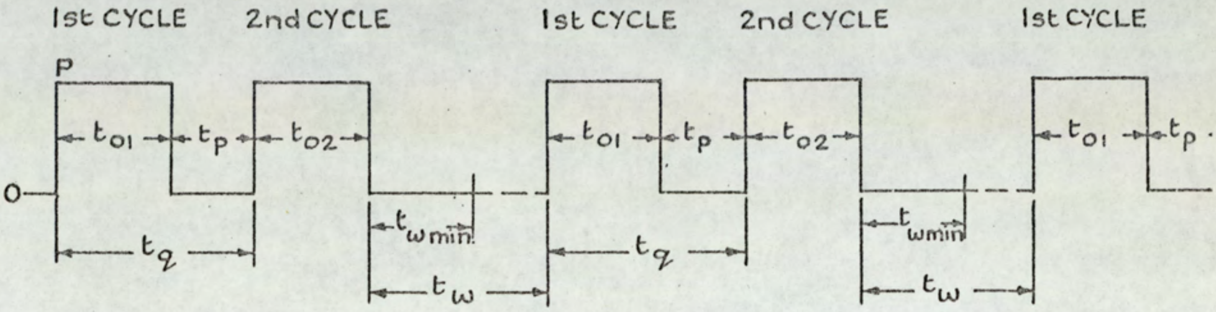


FIG. 2.3 PROPOSED INJECTION CIRCUITS FOR RIPPLE CONTROL.



t_q = cycle time

t_o = ON time per cycle (t_{01} for first of a pair of cycles; t_{02} " second " " " " ")

t_p = OFF time per cycle

$t_{wmin.}$ = minimum waiting time after every second cycle

t_w = actual " " " " " "

0,1 = values of switching index, corresponding to zero power and full power P, respectively.

FIG. 2.4 GENERALISED CYCLE PATTERN PER PROCESS.

is developed for minimising the maximum demand due to a small number of similar two-position processes.

Chapter 4 deals with the case of demand control for a large number of on-off processes where the incidence of loads is assumed normally distributed.

A strategy for control of the demand when the incidence of loads is random is presented in Chapter 7.

2.6 SUMMARY.

In this survey chapter electricity tariff structures relating to maximum demand are briefly reviewed, together with a number of demand control schemes currently in operation.

An introduction is given to the problems encountered in minimising the maximum demand in the most economical manner, which for a varying load pattern are indeed formidable.

If load shedding can be confined to one or two large loads, the task of demand control within each integrating half-hour period will be greatly simplified. Moreover, the accuracy of predicting future energy consumption may be improved significantly by curve fitting and extrapolation of the immediate past energy, leading to an improved quality of control.

CHAPTER 3.

DEMAND CONTROL OF TWO-POSITION PROCESS ENSEMBLES OF FINITE MAGNITUDE.

3.1 INTRODUCTION.

In this chapter, demand control is considered for a number of similar two-position processes of ten or less. A strategy is established for minimising the maximum demand by staggering of the loads in a systematic manner.

Implementation of the policy depends upon closely-defined switching of each process. No load prediction is required as the incidence of loads is controlled by program. However, a back-up policy is described whereby the plant may be operated at reduced power if, during the integrating half-hour, monitoring of demand indicates that the target level will otherwise be exceeded.

Since the switching is deterministic, this strategy cannot be applied to thermostatically-controlled processes.

3.2 PATTERNS CONSIDERED AND CONTRAINTS IMPOSED.

The specifications covered within this analysis are:

1. The switching index m will be restricted to the values 0 and 1, i.e. zero or full power P.
2. The load pattern for each process is a repetitive rectangular wave with a duty cycle defined by:
   t_o = on time per cycle.
   t_p = off time per cycle.
   t_q = cycle time = $t_o + t_p$,
   and average power $\bar{m} = t_o / t_q$.
3. The incidence of switching is deterministic.
4. The number of processes Q is given by $2 \leqslant Q \leqslant 10$.
5. All processes are identical in terms of m and $\bar{m}$, and the power is the same for each process.
6. The phasing of the loads is to be arranged so that the indicated maximum demand is minimised, and the load factor is maximised.

3.3 ASSESSMENT OF LOAD DUE TO ENSEMBLE OF SIMILAR ON-OFF PROCESSES.

A square wave pattern of amplitude E and period 2T, and switched on at t = 0 may be defined as:

$$\left.\begin{aligned} y &= E,\ rT < t < (r+1)\ T \\ &= -E,\ (r+1)\ T < t < (r+2)\ T \end{aligned}\right\} r = 0,2,4 - - - \quad (3.1)$$

or

$$y = E \left\{ H\ (t) - 2H\ (t - T) + 2H\ (t - 2T) - --- \right\} \quad (3.2)$$

where the Heaviside unit step function

$$\left.\begin{aligned} H\ (t) &= 0,\ t < 0 \\ &= 1,\ t > 0 \end{aligned}\right\}. \quad (3.3)$$

The Laplace transform may be expressed as:

$$\mathcal{L}\,(T,\ s) = \frac{E}{s} \left\{ 1 - 2e^{-sT} + 2e^{-2sT} - ----- \right\}$$

$$= \frac{E}{s} \quad \tan h\ \tfrac{1}{2}\ ST. \quad (3.4)$$

For Q loads the overall transform is:

$$\mathcal{L}\,(S) = \sum_{K=1}^{Q} \mathcal{L}\,(T_K, S), \quad (3.5)$$

where the transform $\mathcal{L}\,(T_K, S)$ for the Kth process which is switched on at a time $t = a_k \neq 0$ is obtained by the Heaviside's shifting theorem.

To the author's knowledge, there is no known method available for obtaining the time function which has a minimum value if the overall transform is known.

If we confine our attention to the time domain, we may sum individual load values at any time t on a digital computer using the relations:

<u>1st load.</u>

$$t/t_q = \text{an integer} + \text{a fraction } F_1$$
$$P_1 = 1,\ 0 \leqslant F_1 \leqslant \bar{m}$$
$$P_1 = 0,\ \bar{m} < F_1 \leqslant 1$$

<u>nth load.</u>

$$(t - (n-1)\,\alpha\ )/t_q = \text{an integer} + \text{a fraction } F_n$$
$$P_n = 1,\ 0 \leqslant F_n \leqslant \bar{m}$$
$$P_n = 0,\ \bar{m} < F_n \leqslant 1$$

where successive processes are staggered in time by a constant amount α, and the nth load lags the first by $(n - 1)\,\alpha$.

However, the most promising approach is to study the change of load level at each discontinuity and hence build up a general formula for the overall load pattern.

3.4 PHASING OF LOADS TO GIVE OPTIMAL OVERALL LOAD PATTERN.

The optimal load pattern is defined as that which minimises the maximum demand and maximises the load factor.

If the constant stagger is α (see Fig. 3.1), it is desired to find the optimum value α_o and the resulting minimum value of maximum demand; also over what range of α's this minimum value may be maintained.

We will consider first the simplest case $\overline{m} = \frac{1}{2}$ for Q either odd or even, followed by $\overline{m} = \frac{2}{3}$ for $\overline{m}\,Q$ both integral and non-integral.

A general analysis will be developed for $\overline{m}\,Q$ both integral and non-integral in the duty cycle range $0 \leqslant \overline{m} \leqslant 1$.

In this section the term "demand" refers to the instantaneous KW level, as distinct from the "indicated demand" already defined.

3.4.1 Case of $\overline{m} = \frac{1}{2}$, i.e. $t_o = \frac{1}{2}\,t_q$

The switching index is:

$$m = 1, \quad 0 \leqslant t \leqslant t_o$$

$$m = 0, \quad t_o < t < t_q,$$

and the duty cycle $\overline{m} = \frac{t_o}{t_q}$ is identical for all processes.

If the first process is switched on at $t = 0$, the nth process may be switched on at a time lagging the first process by $(n - 1)\,\alpha$. For Q = 2, the maximum demand (MD) is minimised at the constant value of P, once the overall load pattern is established, by making $\alpha = \frac{1}{2}\,t_q = \frac{1}{2}$, where t_q is normalised to have unity value. The criterion of maximum plant utilisation is also satisfied, since the load factor is unity.

Consideration of the overall pattern for Q processes leads to the following results:

1. The mean value of the total power, taken over a complete load cycle, is constant and independent of the phasing of individual loads.

2. An optimum phasing for minimum MD and maximum utilisation of generating plant is obtained by a uniform stagger α, where:

$$\alpha = \alpha_o = \frac{t_q}{Q} = \frac{1}{Q}.$$

3. The average number of processses on at any time is $\bar{m}Q$. For this phasing and $\bar{m}Q$ integral (Q even) the actual number will be the integer $\bar{m}Q$; for $\bar{m}Q$ non-integral the actual number will be either $[\bar{m}Q]$ or $[\bar{m}Q] + 1$, where $[\bar{m}Q]$ is the nearest integer $\leqslant \bar{m}Q$.

4. The <u>optimal pattern</u> is:

   <u>For Q even,</u> $\alpha_o = \frac{1}{Q}$. The overall KW are constant at $\frac{Q}{2}P$, and the load factor is unity.

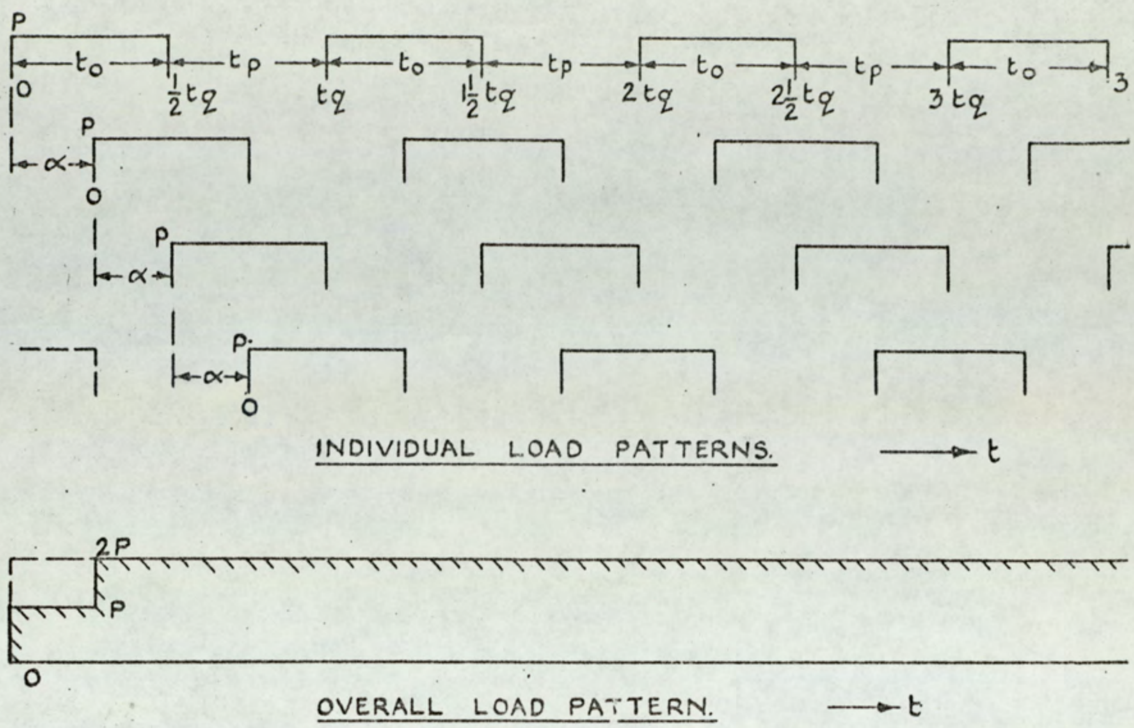
   <u>For Q odd,</u> $\alpha_o = \frac{1}{Q}$. The overall KW follow a square wave pattern, switching between the two levels $\frac{(Q+1)}{2}P$, and $\frac{(Q-1)}{2}P$, and the pulse width is $\frac{1}{2Q}$. The load factor is maximised at a value of $\frac{Q}{Q+1}$.

   The patterns for Q = 3 and Q = 4 are drawn in Fig. 3.1.

5. <u>For Q even,</u> $\alpha_o = \frac{1}{Q}$ is the only value which produces the minimum MD of $\frac{Q}{2}P$.

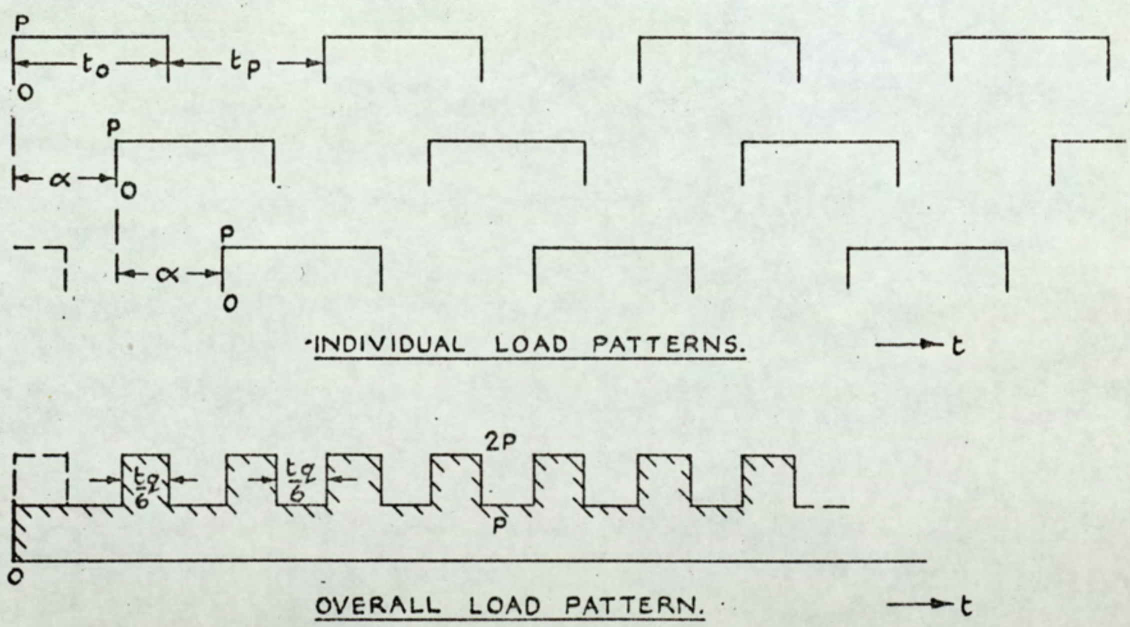
   <u>For Q odd,</u> however, there will be a range of α's which will give the same minimum MD of $\frac{(Q+1)}{2}P$ and the same load factor. For example, Fig. 3.2 shows the load patterns for Q = 3 and $\alpha = \frac{1}{4}, \frac{1}{2}, \frac{5}{8}$. The maximum demand and the load factor are the same as for $\alpha_o = \frac{1}{Q} = \frac{1}{3}$, but different rates of switching between the maximum and minimum levels are obtained. The complete range of α's for two-level switching with Q odd is defined by the condition that, at any instant, the number of individual load patterns overlapping must not exceed $\frac{Q+1}{2}$ and not be less than $\frac{Q-1}{2}$; also the staggering between succeeding loads must be uniform.

   For the simplest case $\bar{m} = \frac{1}{2}$ the range of α's for Q odd may be established by inspection of load patterns. Results are shown in Table 3.1.



Q=4 Symmetrical displacements between successive loads $\alpha = \frac{t_q}{4}$

$\bar{m} = {t_o}/{t_q} = \frac{1}{2}$ After initial transient, overall load constant at 2P.



Q=3 Symmetrical displacements between successive loads $\alpha = \frac{t_q}{3}$

$\bar{m} = {t_o}/{t_q} = \frac{1}{2}$ After initial transient, overall load switches between P and 2P.

FIG.3.1 LOAD PATTERNS FOR Q=3 AND Q=4. ($\bar{m} = \frac{1}{2}$).

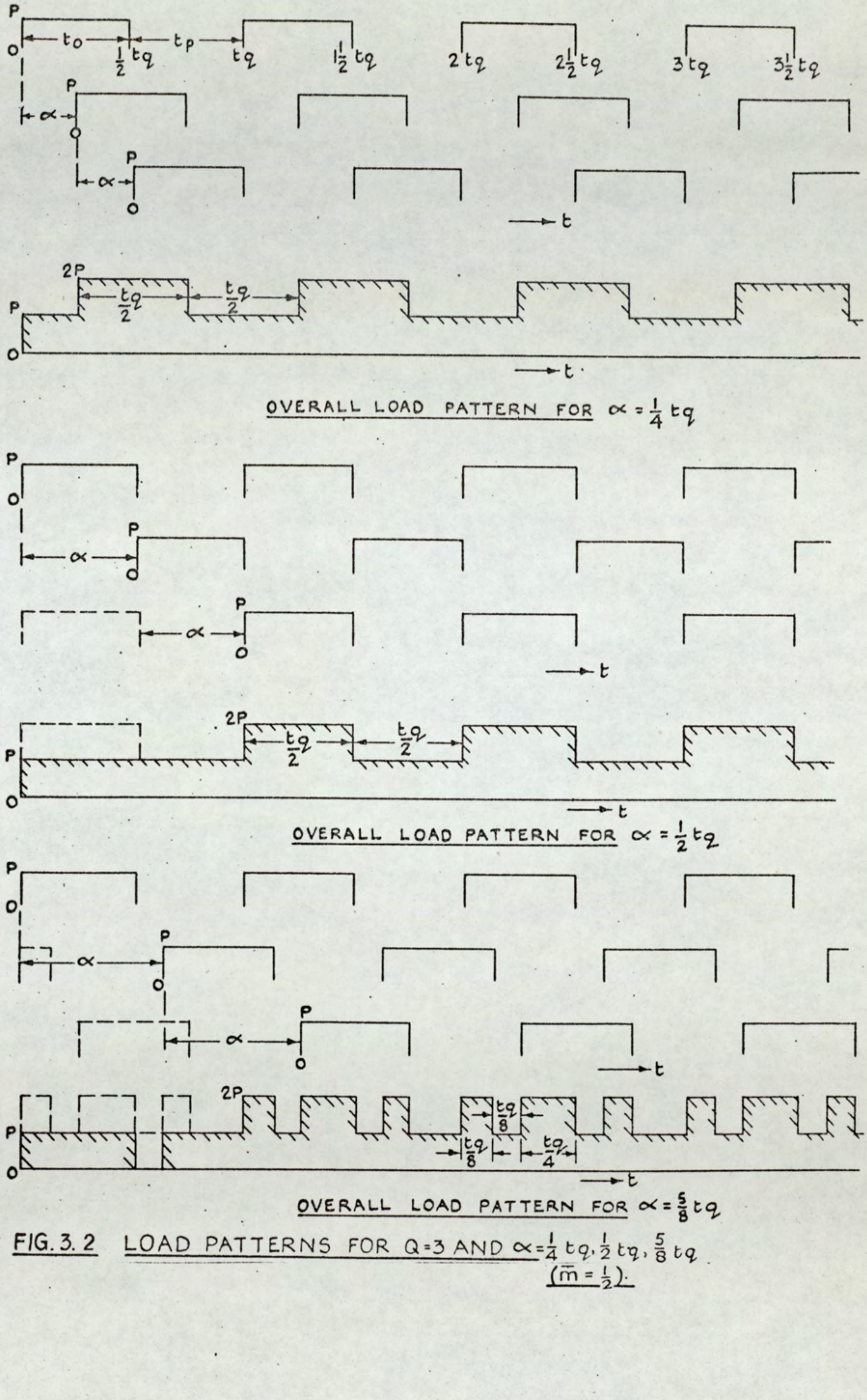


FIG. 3.2 LOAD PATTERNS FOR Q=3 AND $\alpha = \frac{1}{4}$ tq, $\frac{1}{2}$ tq, $\frac{5}{8}$ tq
($\bar{m} = \frac{1}{2}$).

Q	Minimum Stagger α min	Maximum Stagger α max
3	$\frac{1}{4}$	$\frac{3}{4}$
5	$\frac{1}{6}$	$\frac{1}{4}$
7	$\frac{1}{8}$	$\frac{1}{6}$
9	$\frac{1}{10}$	$\frac{1}{8}$

TABLE 3.1

6. There are successive bands α max to α min which will give two-level switching for $\bar{m}$ Q non-integral. The first band for Q = 3 is from $\alpha = \frac{1}{4}$ to $\frac{3}{4}$, the second band from $\alpha = 1\frac{1}{4}$ to $1\frac{3}{4}$, and so on. For Q = 5 the first band is from $\alpha = \frac{1}{6}$ to $\frac{1}{4}$, the second from $\frac{3}{8}$ to $\frac{5}{8}$, and so on.

However, the first band only need be considered in each case as of practical interest.

3.4.2 CASE OF $\bar{m} = \frac{2}{3}$, i.e. $t_o = \frac{2}{3}\ t_q$.

m and $\bar{m}$ are as defined in section 3.4.1, and are identical for all processes.

The cases to be considered here are:

$\bar{m}$ Q integral (given by Q = 3, 6, 9) and

$\bar{m}$ Q non-integral (Q = 2, 4, 5, 7, 8, 10).

t_q will again be normalised to have unity value.

The following results will apply:

1. The optimal pattern is:

For $\bar{m}$ Q integral, $\alpha_o = \frac{1}{Q}$; the overall KW are constant at $\bar{m}$ QP.

For $\bar{m}$ Q non-integral, $\alpha_o = \frac{1}{Q}$; the overall KW will be a rectangular wave pattern switching between the two levels $([\bar{m}Q] + 1)\ P = P_1$ and $[\bar{m}\ Q]\ P = P_2$. The value $\bar{m}$ Q may be expressed as an integer plus a fraction K_1 / K_2. If t_{m1} is the time of the overall pattern at level P_1 and t_{m2} is the time at level P_2, then $P_1 - P_2 = P$ and $t_{m1}/t_{m2} = \dfrac{K_1/K_2}{1 - K_1/K_2} = \dfrac{K_1}{K_2 - K_1}$.

Also $t_{m1} + t_{m2} = \dfrac{t_q}{Q}$, so that

$$t_{m1} = \frac{K_1}{K_2}\,\frac{t_q}{Q}, \quad t_{m2} = \frac{(K_2 - K_1)}{K_2}\,\frac{t_q}{Q}.$$

The patterns for $\alpha = \dfrac{t_q}{Q}$, $\bar{m} = \dfrac{2}{3}$ are drawn for Q = 3 and 4 in Fig. 3.3. The pulse widths for $\bar{m}$ Q non-integral in the range Q = 2 to 10 are shown in Table 3.2.

Q	$\dfrac{K_1}{K_2}$	$\dfrac{t_{m1}}{t_q}$	$\dfrac{t_{m2}}{t_q}$	$\dfrac{t_{m1} + t_{m2}}{t_q} = \dfrac{1}{Q}$
2	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$
4	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{4}$
5	$\frac{1}{3}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$
7	$\frac{2}{3}$	$\frac{2}{21}$	$\frac{1}{21}$	$\frac{1}{7}$
8	$\frac{1}{3}$	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{1}{8}$
10	$\frac{2}{3}$	$\frac{1}{15}$	$\frac{1}{30}$	$\frac{1}{10}$

TABLE 3.2

2. <u>For $\bar{m}$ Q integral,</u> the total demand is minimised at a constant value $\bar{m}$QP and the load factor is maximised.

<u>For $\bar{m}$ Q non-integral</u>, the variation about the mean demand of $\bar{m}$ QP is $P_1 - \bar{m}$ QP and $-(\bar{m}\text{ QP} - P_2)$, and the load factor is again maximised.

3. <u>For $\bar{m}$Q integral,</u> $\alpha_o = \dfrac{1}{Q}$ is the only value which produces the minimum MD of $\bar{m}$ QP.

<u>For $\bar{m}$ Q non-integral</u>, there will again be a range of α's which will give the same minimum MD of P_1, and also successive bands of stagger α max to α min over which switching is produced

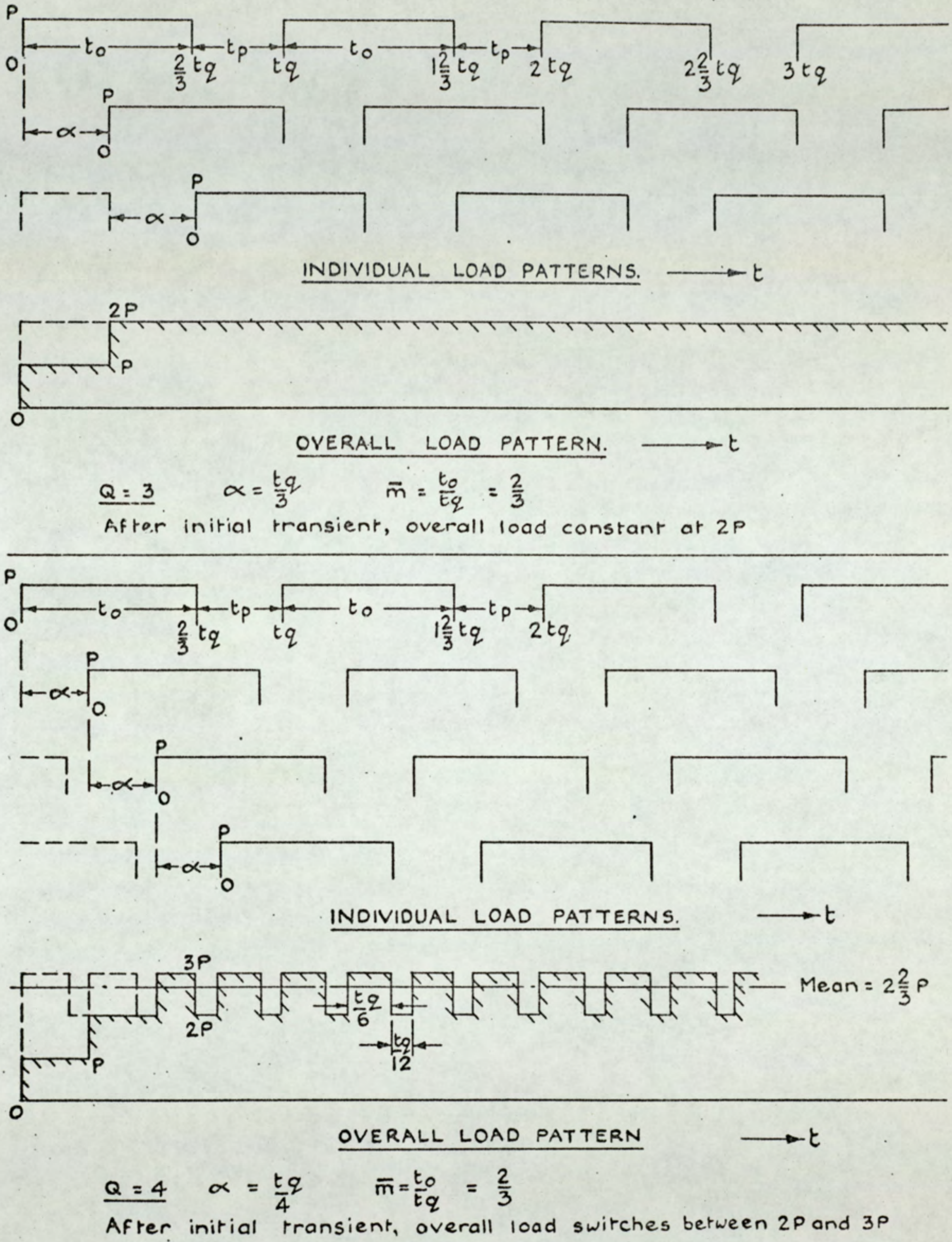


FIG. 3.3 LOAD PATTERNS FOR Q = 3 AND Q = 4. ($\bar{m} = \frac{2}{3}$).

between the two levels P_1 and P_2. The range of α 's to give two-level switching for $\bar{m}$ Q non-integral will be derived in the general analysis for $0 \leqslant \bar{m} \leqslant 1$ given in Section 3.4.3.

In Fig. 3.4 are shown the overall patterns for Q = 4, $\bar{m} = \frac{2}{3}$ with staggers $\alpha_{min} = \frac{2}{9}$ and $\alpha_{max} = \frac{1}{3}$.

As expected, the overall load pattern becomes asymmetrical as the value α_o is either decreased to α_{min} or increased to α_{max}, though in each case the average power per cycle remains at $\bar{m}$ QP.

3.4.3 GENERAL ANALYSIS FOR α_{min} AND α_{max} WHERE $0 \leqslant \bar{m} \leqslant 1$

The general analysis for $0 \leqslant \bar{m} \leqslant 1$ will be carried out for the more difficult case of $\bar{m}$ Q non-integral, and the theory extended to the case of $\bar{m}$ Q integral.

We will again consider the optimum stagger α_o, the effect of this choice of α upon the maximum demand, and the limiting values α_{max} and α_{min} beyond which two-level switching is no longer maintained.

3.4.3.1 $\bar{m}$ Q NON-INTEGRAL.

Analysis for α_{min}

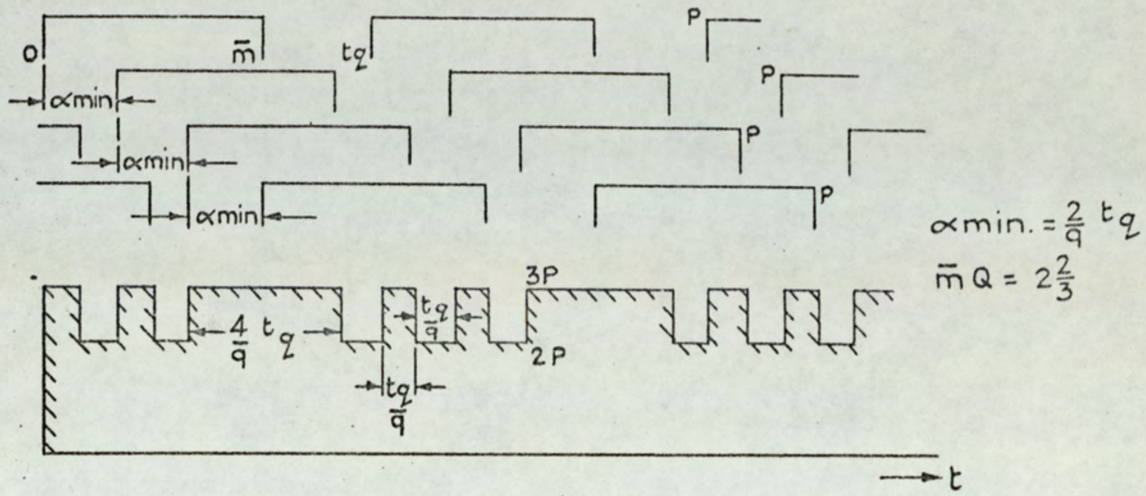
On average $\bar{m}$ Q processes will be on at any time, i.e. : there are either $[\bar{m}Q]$ or $[\bar{m}Q] + 1$ on at any time, where $[\bar{m}Q]$ is the next integer $\leqslant \bar{m}$ Q.

The two-level switching pattern given by $\alpha_o = \frac{t_q}{Q}$ can be spoiled in two ways:

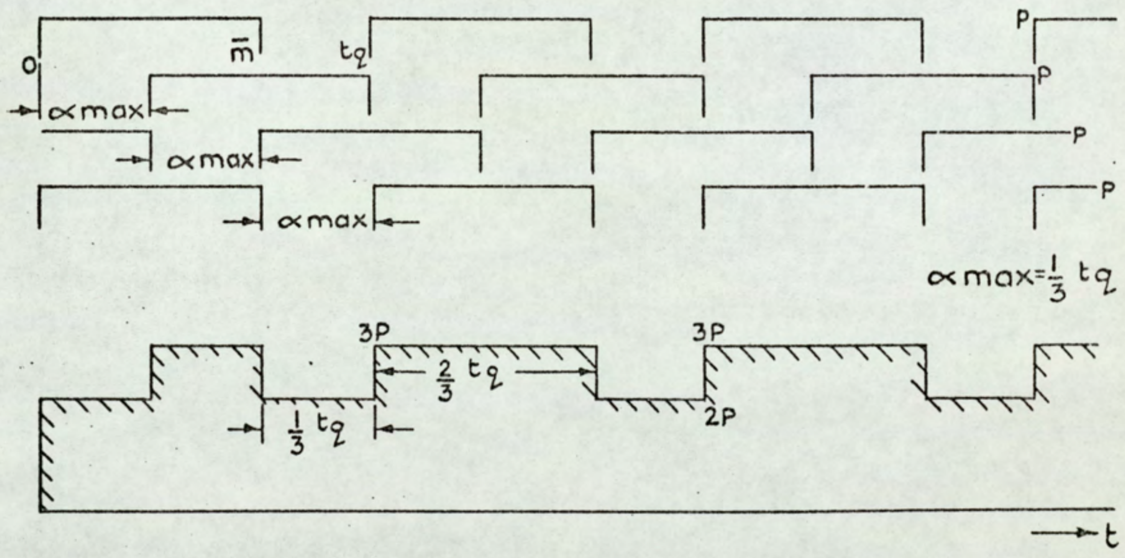
i) By the fewest number of processes on at once being reduced from $[\bar{m}Q]$ to $[\bar{m}Q] - 1$.

ii) By the largest number of processes on at once being increased from $[\bar{m}Q] + 1$ to $[\bar{m}Q] + 2$.

In Fig. 3.5 the pattern represents Q processes switched on at intervals $\alpha_o = \frac{1}{Q}$ (the value t_q being normalised to unity value).
Let the processes be numbered 0, 1, 2, ...Q - 1.
Case (i) occurs at point A, i.e. : t = 0 or t = 1, when one of the processes (process r_o) gets switched off at t = 1 as α is reduced below α_o. As $\bar{m}$ Q $>$ 1 except in the most trivial case, $\alpha_o < \bar{m}$ and process number Q - 1 is on at t = 0. There is



INDIVIDUAL AND OVERALL LOAD PATTERNS FOR STAGGER α min.



INDIVIDUAL AND OVERALL LOAD PATTERNS FOR STAGGER α max.

FIG. 3.4 LOAD PATTERNS FOR Q=4. ($\bar{m} = \frac{2}{3}$)
WITH STAGGER α min. AND αmax.

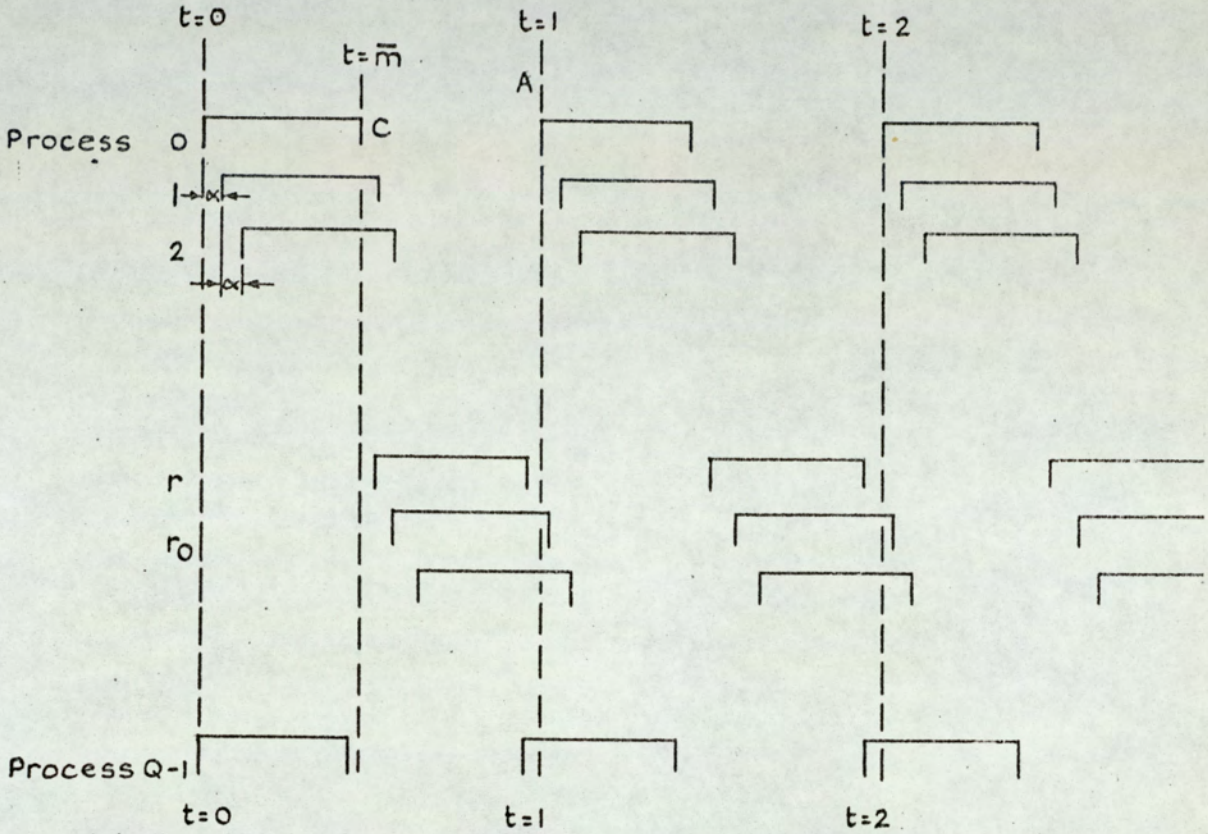


FIG. 3.5 GENERALISED LOAD PATTERNS FOR Q PROCESSES WITH STAGGER α.

no point in considering the case $\alpha(Q-1) > 1$, or $\alpha > \frac{1}{Q-1}$, since the pattern then becomes unsymmetrical in order of switching on (see Fig. 3.4). r represents the general number for any process, and r_o is the smallest process number such that $r_o \alpha_o + \bar{m} \geqslant 1$, i.e. $r_o \geqslant \frac{1-\bar{m}}{\alpha_o}$, or $r_o \geqslant Q - \bar{m}Q$.

Since $\bar{m}Q$ is not an integer, this is the same as $r_o \geqslant Q - [\bar{m}Q]$, and the smallest $r_o = Q - [\bar{m}Q]$.

Now let us reduce α from α_o to a smaller value such that process r_o is just being switched off at $t = 1$. This means reducing α to a value such that $r_o\alpha + \bar{m} = 1$, giving

$$\alpha = \alpha_{min} = \frac{1-\bar{m}}{r_o} = \frac{1-\bar{m}}{Q - [\bar{m}Q]} \quad \text{------------- Case (i)} \quad (3.6)$$

As α is reduced below α_o, case (ii) occurs at point C, when the process which is switched on just after $t = \bar{m}$ with value α_o becomes switched on at $t = \bar{m}$ by reducing α. This is process number r, where $r\alpha_o > \bar{m}$, i.e. $r > \frac{\bar{m}}{\alpha_o}$ or $\bar{m}Q$, so that $r = [\bar{m}Q] + 1$.

Therefore if this process is switched on at $t = \bar{m}$, the minimum value α_{min} is such that $r\alpha_{min} = \bar{m}$,

i.e.

$$\alpha_{min} = \frac{\bar{m}}{[\bar{m}Q] + 1} \quad \text{--------------- Case (ii)} \quad (3.7)$$

The minimum permissible value of α is therefore determined by which of cases (i) and (ii) occurs first if α is reduced below the value α_o.

Thus α_{min} must be the <u>greater</u> of $\frac{\bar{m}}{[\bar{m}Q] + 1}$ and $\frac{1-\bar{m}}{Q - [\bar{m}Q]}$.

If these alternatives are called $\alpha_{min\,1}$ and $\alpha_{min\,2}$,

then $\frac{\bar{m}}{[\bar{m}Q] + 1} > \frac{1-\bar{m}}{Q - [\bar{m}Q]}$ for $\alpha_{min\,1} > \alpha_{min\,2}$,

i.e. $\bar{m}Q - \bar{m}[\bar{m}Q] > [\bar{m}Q] + 1 - \bar{m}[\bar{m}Q] - \bar{m}$,

or: $\bar{m}Q - [\bar{m}Q] > 1 - \bar{m}$,

i.e. : $$F(\bar{m}Q) > 1 - \bar{m}, \quad (3.8)$$

where $F(\bar{m}Q)$ is the fractional part of $\bar{m}Q$.

As an example,

For $\bar{m} = \frac{2}{5}$, $Q = 3$, $F(\bar{m}Q) = \frac{1}{5}$, $1 - \bar{m} = \frac{3}{5}$, hence $\alpha_{min\ 2} > \alpha_{min\ 1}$.

For $\bar{m} = \frac{2}{5}$, $Q = 7$, $F(\bar{m}Q) = \frac{4}{5}$; $1 - \bar{m} = \frac{3}{5}$,

hence $\alpha_{min\ 1} > \alpha_{min\ 2}$.

When $\bar{m} = \frac{1}{4}$, $Q = 7$, $F(\bar{m}\,Q) = 1 - \bar{m}$, and $\alpha_{min\ 1} = \alpha_{min\ 2}$

$$= \frac{1}{8}.$$

For $\bar{m} = \frac{1}{2}$, all non-integral values of $\bar{m}\,Q$ will give

$\alpha_{min1} = \alpha_{min\ 2}$.

For cases where $\bar{m}$ is not a simple fraction, however, the easiest method of determining α_{min} is to evaluate $\alpha_{min\ 1}$ and $\alpha_{min\ 2}$ for the particular value of $\bar{m}$, and choose the larger of the two results.

ANALYSIS FOR α_{max}.

As α is increased from α_o, the two-level pattern can be spoiled in two ways:

(i) By the fewest number of processes on at once being reduced by 1.

(ii) By the largest number of processes on at once being increased by 1.

Consider again Fig. 3.5.

Case (i) occurs at point C, when the process $r - 1$ which is switched on just before $t = \bar{m}$ with value α_o becomes switched on at $t = \bar{m}$ by increasing α_o.

Thus $(r - 1)\,\alpha_o < \bar{m}$,
or $r - 1 < \bar{m}Q$, so that $r - 1 = \left[\bar{m}Q\right]$.

If this process is switched on at $t = \bar{m}$, the maximum value α_{max} is such that $(r - 1)\,\alpha_{max} = \bar{m}$,

i.e. $$\alpha_{max1} = \frac{\bar{m}}{\left[\bar{m}\,Q\right]} \quad \text{- - - - - - - - - - - Case (i)} \qquad (3.9)$$

Case (ii) occurs when process $r_o - 1$ gets switched off at $t = 1$,

i.e: $\alpha = \alpha_{max}$ where $(r_o - 1)\,\alpha_{max} + \bar{m} = 1$,

i.e: $$\alpha_{max\ 2} = \frac{1 - \bar{m}}{Q - \left[\bar{m}\,Q\right] - 1} \quad \text{- - - - - - - - - Case (ii)} \qquad (3.10)$$

Let the front edge of a load pattern be called F and the back edge B.

The two-level pattern changes when an edge F of one load catches up with an edge B of another load as α is increased from value α_o. This is condition (i) defined above.

It may also be spoiled due to an edge B of one load catching up with an edge F of another load. This is condition (ii) defined above.

A third possibility is due to an edge F of one load catching up with an edge F of another load (or to an edge B of one load catching up with an edge B of another load.) In this condition process Q - 1 will arrive in phase with process 0 at t = 1 when α is increased from α_o, i.e. : $(Q - 1) \ \alpha_{max} = 1$, or

$$\alpha_{max\ 3} = \frac{1}{Q-1} \quad \text{---------- Case (iii)} \qquad (3.11)$$

α_{max} should then be taken as the <u>least</u> of $\alpha_{max\ 1}$, $\alpha_{max\ 2}$, $\alpha_{max\ 3}$.

Now

$$\alpha_{max\ 1} > \alpha_{max\ 2} \quad \text{if } \bar{m}\,(Q - [\bar{m}\,Q] - 1) > (1 - \bar{m})[\bar{m}\,Q],$$

$$\text{i.e. if } \bar{m}\,Q - [\bar{m}\,Q] > \bar{m},$$

$$\text{i.e. if } F\,(\bar{m}\,Q) > \bar{m}, \qquad (3.12)$$

where $F\,(\bar{m}\,Q)$ is the fractional part of $\bar{m}\,Q$.

Also

$$\alpha_{max\ 1} > \alpha_{max\ 3} \quad \text{if } \bar{m}\,(Q - 1) > [\bar{m}\,Q],$$

$$\text{i.e.} \quad \text{if } \bar{m}\,Q - [\bar{m}\,Q] > \bar{m},$$

$$\text{i.e.} \quad \text{if } F\,(\bar{m}\,Q) > \bar{m}. \qquad (3.13)$$

Also

$$\alpha_{max\ 3} > \alpha_{max\ 2} \quad \text{if } Q - [\bar{m}\,Q] - 1 > (1 - \bar{m})\,(Q - 1),$$

$$\text{i.e. : if } Q - [\bar{m}\,Q] - 1 > Q - 1 - \bar{m}\,Q + \bar{m},$$

$$\text{i.e.} \quad \text{if } \bar{m}\,Q - [\bar{m}\,Q] > \bar{m},$$

$$\text{i.e.} \quad \text{if } F\,(\bar{m}\,Q) > \bar{m}. \qquad (3.14)$$

Therefore if $F\,(\bar{m}\,Q) > \bar{m}$, $\alpha_{max\ 1} > \alpha_{max\ 3} > \alpha_{max\ 2}$, and α_{max} should be taken as equal to $\alpha_{max\ 2}$.

If $F\,(\bar{m}\,Q) < \bar{m}$, $\alpha_{max\ 1} < \alpha_{max\ 3} < \alpha_{max\ 2}$, and α_{max} should be taken as equal to $\alpha_{max\ 1}$.

If $F(\bar{m}Q) = \bar{m}$, $\alpha_{max\,1} \doteq \alpha_{max\,2} = \alpha_{max\,3}$.

Thus $\alpha_{max\,3}$ is only equal to α_{max} when it is also equal to $\alpha_{max\,1}$ and $\alpha_{max\,2}$, so that α_{max} can be taken as the <u>lesser</u> of $\alpha_{max\,1}$ and $\alpha_{max\,2}$.

Therefore α_{max} is uniquely determined except in the case when the fractional part of $\bar{m}Q = \bar{m}$, and this case needs separate attention.

SPECIAL CASE FOR α_{max} WHEN $F(\bar{m}Q) = \bar{m}$

Examples are:

$\bar{m} = \frac{1}{4}$, $Q = 5$ or 9.

$\bar{m} = \frac{1}{2}$, $Q = 3, 5, 7, 9$.

For all cases except $Q = 3$, $\bar{m} = \frac{1}{2}$, the two-level pattern is spoiled when the last process $Q - 1$ is switched on at $t = 1$, where:

$$(Q - 1)\,\alpha_{max} = 1, \quad \text{or} \quad \alpha_{max} = \frac{1}{Q - 1}. \qquad (3.15)$$

In this position the spoiling of the two-level pattern is accomplished not at $t = 0$ or 1 but in between, and will require a minimum of two switching points in the range $0 < t < 1$.

For example, with $Q = 5$, $\bar{m} = \frac{1}{4}$, $\alpha_{max} = \frac{1}{4}$, and the two-level switching is spoiled at $t = \frac{1}{2}$ and $\frac{3}{4}$; for $\bar{m} = \frac{1}{2}$ it is spoiled at $t = \frac{1}{4}$ and $\frac{3}{4}$ (see Fig. 3.6).

The only exception to the rule is the particular case of $Q = 3$, $\bar{m} = \frac{1}{2}$, where the formula gives $\alpha_{max} = \frac{1}{2}$. In this case there is only one switching point for $\alpha_{max} = \frac{1}{2}$ (at $t = \frac{1}{2}$) in the range $0 < t < 1$, and the condition is not satisfied. Due to the small value of Q and the symmetry of load patterns given by $\bar{m} = \frac{1}{2}$, the two-level switching pattern is not disturbed until α_{max} is increased to a value of $\frac{3}{4}$.

SUMMARY OF ANALYSIS FOR $\bar{m}Q$ NON-INTEGRAL.

The results of the analysis for $\bar{m}Q$ non-integral may now be summarised as follows:

1. The optimum value of α is $\alpha_o = \frac{1}{Q}$.
2. Provided $[\bar{m}Q]$ is not less than 1 (the most trivial case):

   α_{min} should be taken as the <u>greater</u> value of $\dfrac{\bar{m}}{[\bar{m}Q] + 1}$

and $\dfrac{1 - \bar{m}}{Q - \left[\bar{m}\, Q\right]}$.

α_{max} should be taken as the lesser value of $\dfrac{\bar{m}}{\left[\bar{m}\, Q\right]}$

and $\dfrac{1 - \bar{m}}{Q - \left[\bar{m}\, Q\right] - 1}$.

3. For $F(\bar{m}\, Q) = \bar{m}$, $\alpha_{max} = \dfrac{1}{Q - 1}$ is identical to the above two values for α_{max}.

The only exception to the above relationships for α_{max} is for $Q = 3$, $\bar{m} = \frac{1}{2}$, when $\alpha_{max} = \frac{3}{4}$.

3.4.3.2 $\bar{m}$ Q INTEGRAL.

The results of the general analysis of section 3.4.3.1 may now be extended to the case of $\bar{m}$ Q integral.

If $\bar{m}$ Q is integral, $\alpha_{min\ 1} = \dfrac{1}{Q + \dfrac{1}{\bar{m}}}$ since

$\left[\bar{m}\, Q\right] = \bar{m}\, Q$.

Also $\alpha_{min\ 2} = \dfrac{1 - \bar{m}}{Q\ (1 - \bar{m})} = \dfrac{1}{Q}$.

Since $\bar{m}$ is positive, $\alpha_{min\ 2} > \alpha_{min\ 1}$ for all cases $\bar{m}$ Q integral. Taking the greater of the two values, $\alpha_{min} = \dfrac{1}{Q} = \alpha_o$, the optimum value as expected.

As far as α_{max} is concerned, a constant overall load is only maintained when, if α is increased, the trailing edge of process r reaches t = 1 just as the leading edge of process Q - 1 passes t = 1. This condition is satisfied by

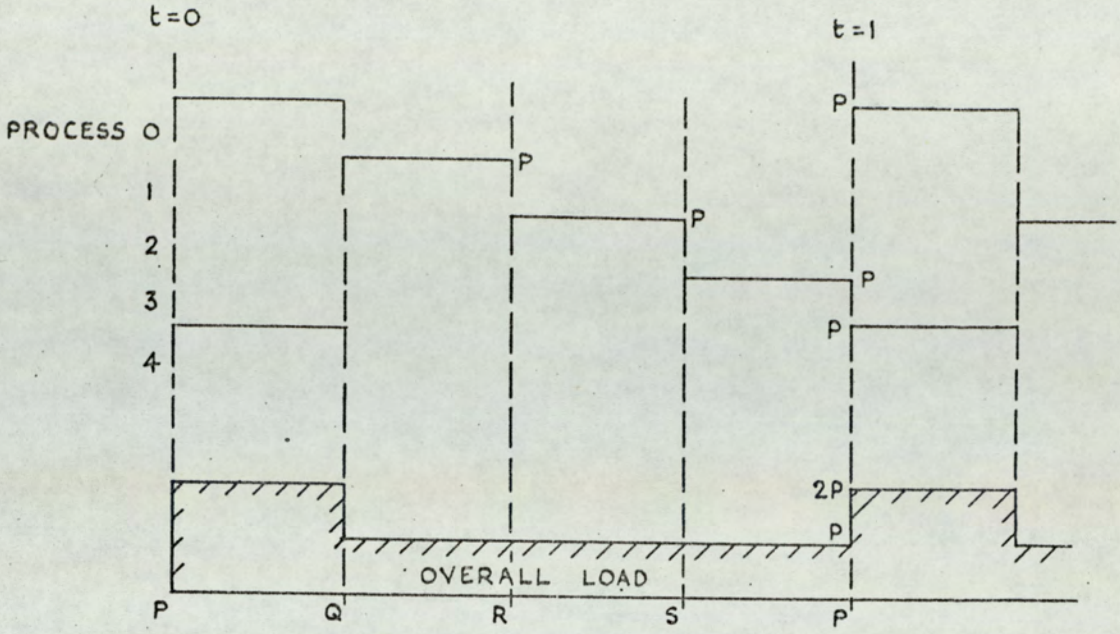
$1 - (\alpha r + \bar{m}) = 1 - (Q - 1)\alpha$, giving

$$\alpha = \frac{\bar{m}}{Q - 1 - (Q - \bar{m}\, Q) + 1} = \frac{1}{Q} = \alpha_o .$$

There is, in fact, no latitude in the choice of α for $\bar{m}$ **Q** integral, the only possible value for minimum constant total load being $\dfrac{1}{Q}$.

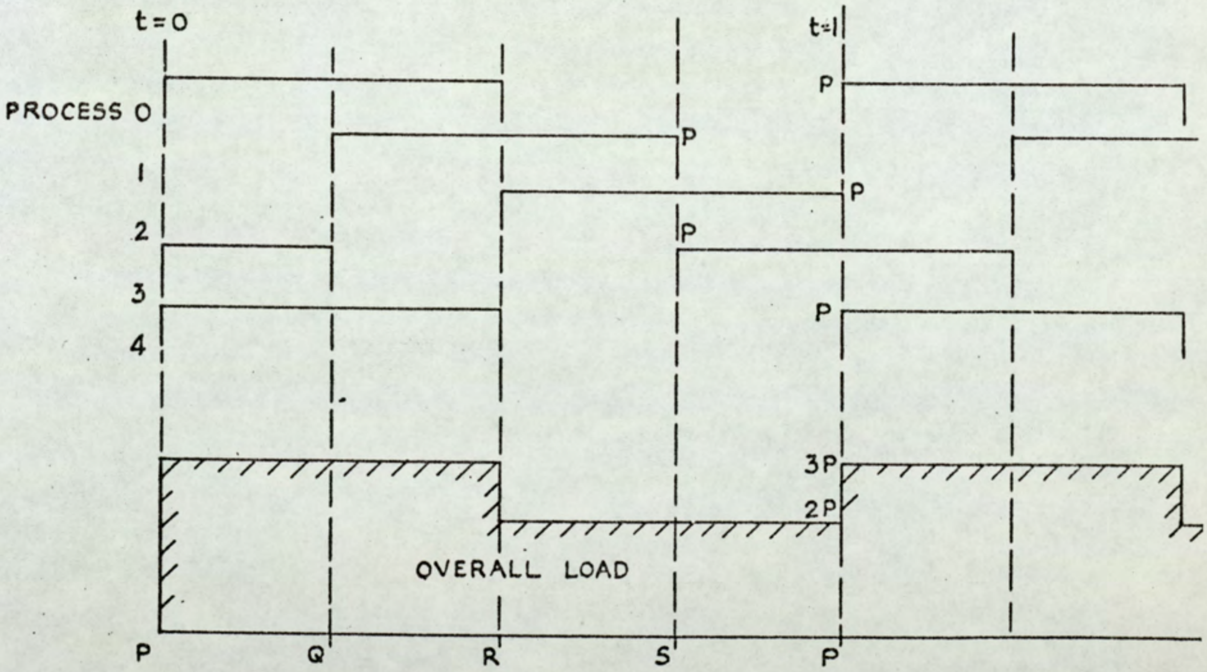
3.5 RESULTS OBTAINED FROM ANALYSIS OF 3.4.

We may now examine the effect of the value of Q upon the parameters α_{min} and α_{max}, and also determine the load factor



$\alpha \max = \frac{1}{Q-1} = \frac{1}{4}$. If α further increased, 2-level pattern disturbed at R, S.

<u>$Q = 5,\ \bar{m} = \frac{1}{4}$</u>



$\alpha \max = \frac{1}{Q-1} = \frac{1}{4}$. If α further increased, 2-level pattern disturbed at Q, S.

<u>$Q = 5,\ \bar{m} = \frac{1}{2}$</u>

FIG. 3.6 LOAD PATTERNS FOR $Q=5,\ \bar{m}=\frac{1}{4}$ AND $Q=5,\ \bar{m}=\frac{1}{2}$

$F(\bar{m}Q) = \bar{m}$.

as a function of Q.

We will consider first $\bar{m}\,Q$ non-integral and then $\bar{m}\,Q$ integral.

3.5.1 $\bar{m}\,Q$ NON-INTEGRAL.

Dependence of α_{min} and α_{max} upon Q.

Families of points may be plotted for α_{min} and α_{max} for $2 \leqslant Q \leqslant 10$ with fixed values of $\bar{m}$.

For $\bar{m} = \frac{1}{2}$, α_{min}, α_{max} may be found as follows:

Q	α_{min}			α_{max}			
	$\frac{\bar{m}}{[\bar{m}\,Q]+1}$	$\frac{1-\bar{m}}{Q-[\bar{m}\,Q]}$	value selected	$\frac{\bar{m}}{[\bar{m}\,Q]}$	$\frac{1-\bar{m}}{Q-[\bar{m}\,Q]-1}$	$\frac{1}{Q-1}$	value selected
3	1/4	1/4	1/4	1/2	1/2	1/2	3/4 *
5	1/6	1/6	1/6	1/4	1/4	1/4	1/4
7	1/8	1/8	1/8	1/6	1/6	1/6	1/6
9	1/10	1/10	1/10	1/8	1/8	1/8	1/8

* Exception to the rule.

TABLE 3.3

For $\bar{m} = \frac{2}{3}$, the results are:

Q	α_{min}			α_{max}			
	$\frac{\bar{m}}{[\bar{m}\,Q]+1}$	$\frac{1-\bar{m}}{Q-[\bar{m}\,Q]}$	value selected	$\frac{\bar{m}}{[\bar{m}\,Q]}$	$\frac{1-\bar{m}}{Q-[\bar{m}\,Q]-1}$	$\frac{1}{Q-1}$	value selected
2	1/3	1/3	1/3	2/3	∞	1	2/3
4	2/9	1/6	2/9	1/3	1/3	1/3	1/3
5	1/6	1/6	1/6	2/9	1/3	1/4	2/9
7	2/15	1/9	2/15	1/6	1/6	1/6	1/6
8	1/9	1/9	1/9	2/15	1/6	1/7	2/15
10	2/21	1/12	2/21	1/9	1/9	1/9	1/9

TABLE 3.4

These results are shown in Fig 3.7; adjacent points cannot, of course, be joined, since only integral values of Q are realisable. However, if a zone were to be imagined with boundaries defined by α_{max} and α_{min}, the range $\alpha_{max} - \alpha_{min}$ converges rapidly towards zero value as Q approaches 10. For $Q \gg 10$, therefore, a control policy for limitation of maximum demand with $\bar{m}Q$ integral would not differ from that for $\bar{m}Q$ non-integral.

CHOICE OF α.

The limits $\alpha_{max} - \alpha_o$ and $\alpha_o - \alpha_{min}$ for two-level switching to be maintained may be expressed in terms of α_o.

For example, for $\bar{m} = \frac{2}{3}$ the values of $(\alpha_{max} - \alpha_o)/\alpha_o$ and $(\alpha_o - \alpha_{min})/\alpha_o$ are given by:

33.3% and 11.1 % respectively for Q = 4 ,

16.7% and 6.67% respectively for Q = 7 ,

11.1% and 4.76% respectively for Q = 10 .

However an essential requirement of the switching policy is complete symmetry of load patterns, so that once α has been set anywhere within the range α_{max} to α_{min}, this value must be reproduced precisely for all succeeding processes. As a consequence, deterministic switching at a value of α set by the second process is essential.

The simplest switching policy to implement, therefore, will be to set α exactly to α_o for all processes after the first.

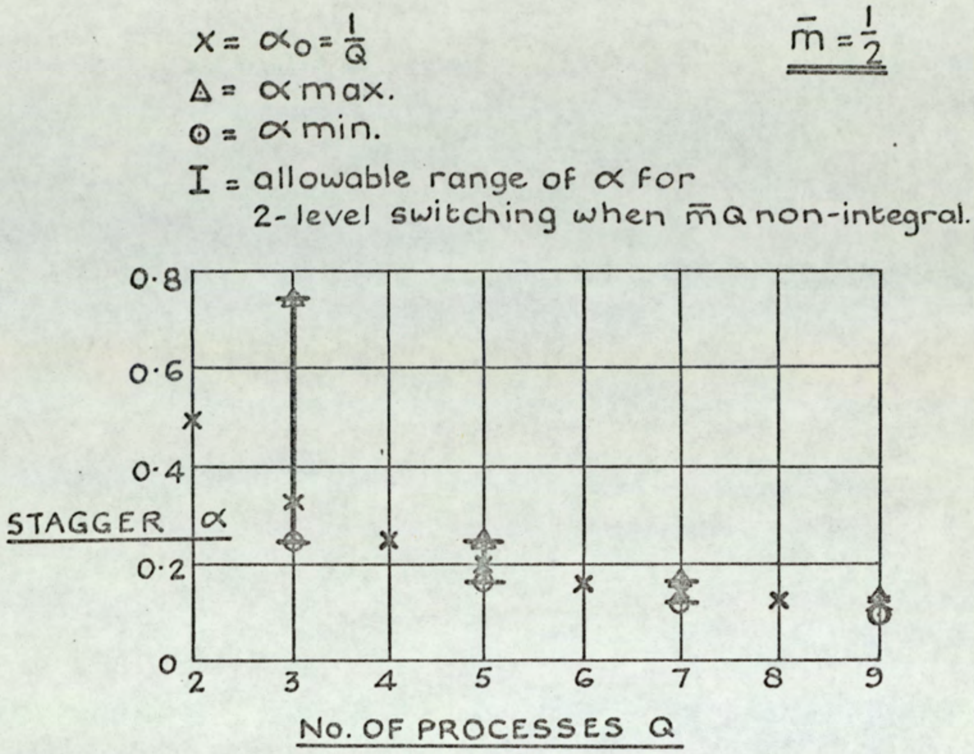
DEPENDENCE OF LOAD FACTOR UPON Q.

Provided two-level switching is maintained for $\bar{m}Q$ non-integral, the load factor is maximised in the range α_{max} to α_{min}. The load factor is defined by $\dfrac{\bar{m}Q}{[\bar{m}Q]+1}$, and is evaluated below for $\bar{m} = \frac{1}{2}$ and $\bar{m} = \frac{2}{3}$ for non-integral values of $\bar{m}Q$ in the range $2 \leqslant Q \leqslant 10$.

For $\bar{m} = \frac{1}{2}$ the load factor as a function of Q is:

Q	3	5	7	9
$\bar{m}Q$	$1\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$
$[\bar{m}Q]+1$	2	3	4	5
Load factor%	50	83.3	87.5	90

TABLE 3.5



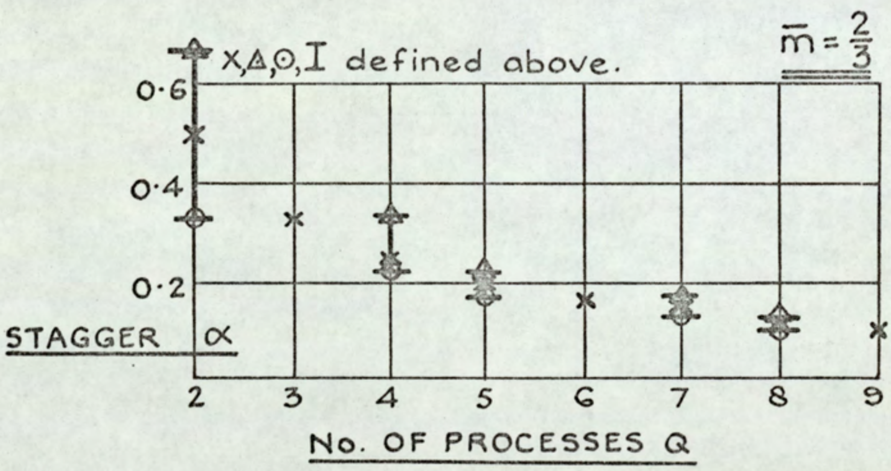


FIG. 3.7 ALLOWABLE RANGE OF α WHEN $\bar{m}Q$ NON INTEGRAL FOR $\bar{m} = \frac{1}{2}$ AND $\frac{2}{3}$.

For $\bar{m} = \frac{2}{3}$, the load factor is given by:

Q	2	4	5	7	8	10
$\bar{m}$ Q	$1\frac{1}{3}$	$2\frac{2}{3}$	$3\frac{1}{3}$	$4\frac{2}{3}$	$5\frac{1}{3}$	$6\frac{2}{3}$
$\left[\bar{m}\ Q\right] + 1$	2	3	4	5	6	7
Load factor %	66.7	88.9	83.3	93.3	88.9	95.2

TABLE 3.6

The load factor is plotted as a function of $\bar{m}$ and Q in Fig. 3.8.

As expected, the load factor increases with $\bar{m}$; for fixed $\bar{m}$ the trend of load factor is to increase with Q, individual values depending upon the fractional part of $\bar{m}$ Q.

3.5.2 $\bar{m}$ Q INTEGRAL.

Choice of α and Dependence of Load Factor upon Q.

As shown in Section 3.4.3.2, for $\bar{m}$ Q integral, the only choice for α which leads to a minimum constant load is α_o. This load has a value $\bar{m}$ QP and the load factor is 100%.

3.6 CONTROL STRATEGIES TO MINIMISE MAXIMUM DEMAND AND MAXIMISE PLANT UTILISATION.

3.6.1 Use of Computer to Implement Switching.

A small on-line computer may be used for implementation of the switching strategy, where α is set equal to α_o.

It must be emphasised that the control is essentially open-loop, in that the symmetry of the load pattern must be maintained by deterministic switching.

The function of the computer will be:

1. To provide the master switching pulses controlling the power supplied to each process.
2. To phase in processes at earlier times than dictated by the switching sequence without increasing the maximum load level.
3. To provide a back-up predictive control to ensure that a nominated "indicated demand" level is never exceeded by accident.

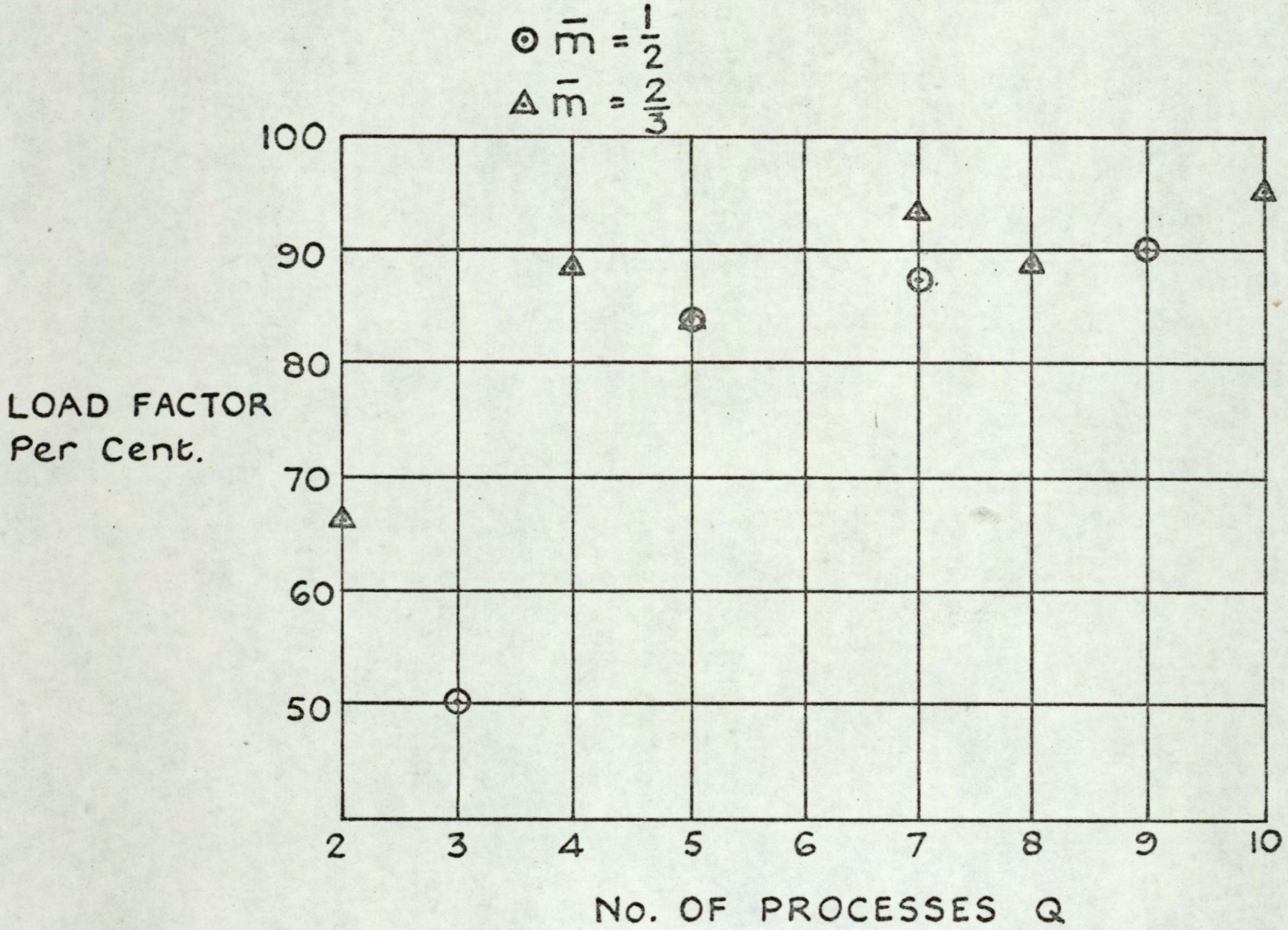


FIG. 3.8 LOAD FACTOR AS FUNCTION OF Q AND $\bar{m}$ FOR $\bar{m}Q$ NON-INTEGRAL.

(LOAD FACTOR = 100% FOR $\bar{m}Q$ INTEGRAL.)

3.6.2 Earlier Switching of Processes.

The sequence requires process Q - 1 to commence at $t = \left(\frac{Q-1}{Q}\right)t_q$, where there are Q processes numbered 0,1, 2,---r, ----Q -1.

In practice one may not wish to accept this delay, and power to the last process may be switched on at a time one cycle earlier without altering the switching symmetry.

A possible previous cycle of this process could commence theoretically at $t = -\left(1 - \frac{Q-1}{Q}\right)t_q$ or, in practice, be switched on at t = 0 and switched off at $t = \left(\bar{m} - \left(1 - \frac{Q-1}{Q}\right)\right)t_q$. The possibility also arises of switching on at t = 0 processes preceding the last, and accepting the first cycle to be only a portion of $\bar{m}$ before switch off, followed by the normal duty cycle. A process r which has to be re-introduced into service after repair or a power cut, must be switched on at the correct instant in order to maintain the switching symmetry when this extra load is placed on the supply.

If the application allows the first cycle of process r to be of reduced width, this cycle will be defined by:

$$t_q\left(\frac{r}{Q} - 1\right) \leqslant 0 \text{ - - - - - - - - - - - - - - - - - (i)} \qquad (3.16)$$

$$t_q\left(\frac{r}{Q} - 1 + \bar{m}\right) \geqslant 0 \text{ - - - - - - - - - - - - - - - - (ii).} \qquad (3.17)$$

Condition (i) defines the start of the first cycle (in practice at t = 0) and condition (ii) defines the finish. The second cycle will then commence at $t = r\,\frac{t_q}{Q}$, and so on.

3.6.3 Back-Up Predictive Control.

The principles of a computer back-up policy to ensure that the nominated level of indicated maximum demand is not exceeded are illustrated in Figs. 3.9 and 3.10. A curve of energy may be drawn to a base of time assuming a constant set demand of $\left\{\bar{m}\,Q\right\}P$, where $\left\{\bar{m}\,Q\right\}$ is the nearest integer $\geqslant \bar{m}\,Q$; this covers both cases of $\bar{m}\,Q$ integral and non-integral. The energy consumed up to time t with this set demand is $\left\{\bar{m}\,Q\right\}P \times t$, and the target energy consumption by the end of the monitored period will be $\left\{\bar{m}\,Q\right\}P \times t_i = T$.

The energy T_E consumed up to time t is sampled throughout t_i, (say at times $t = \alpha_o, 2\alpha_o, 3\alpha_o$ - - - - - - if $\alpha_o \leqslant t_i$).

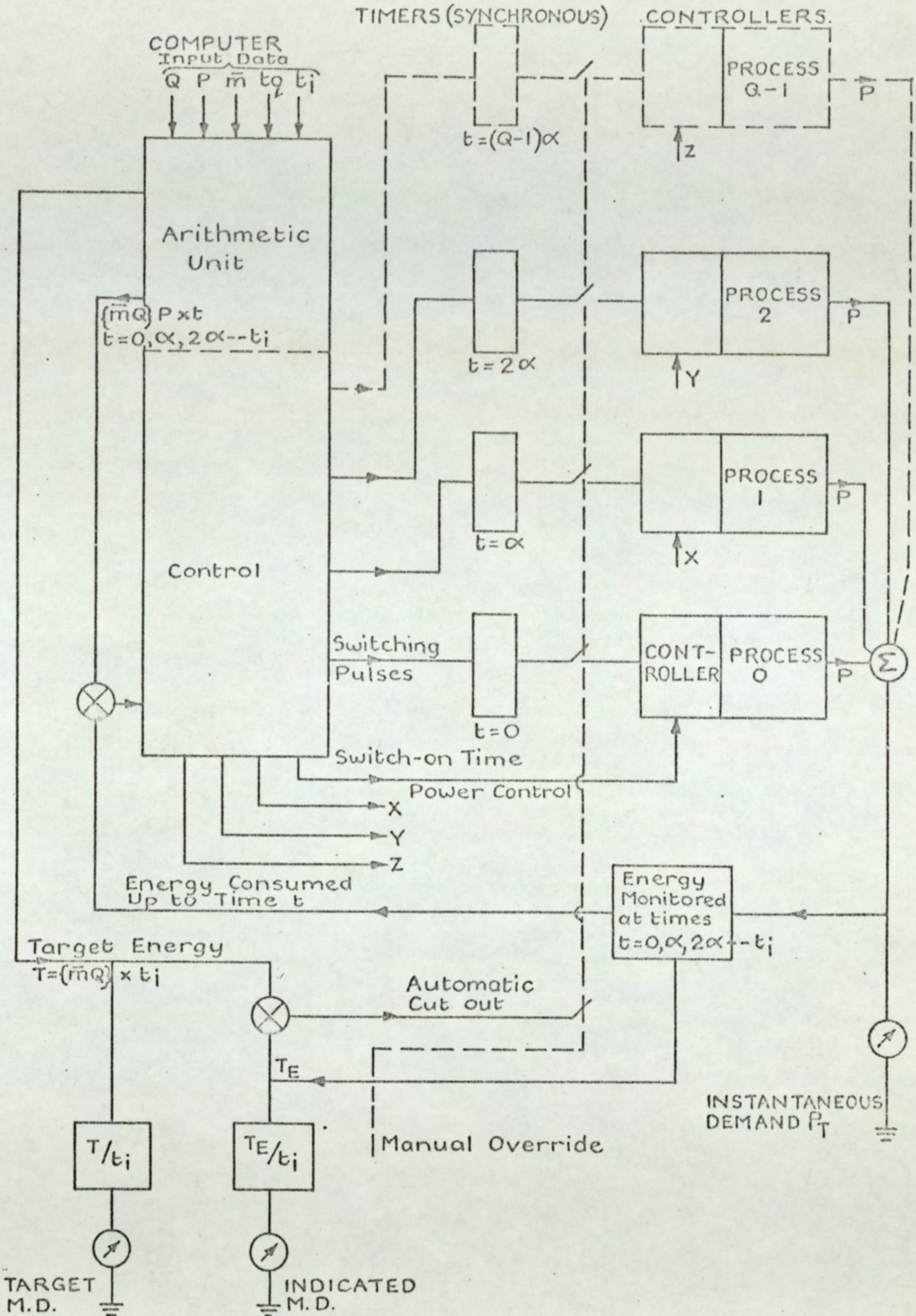
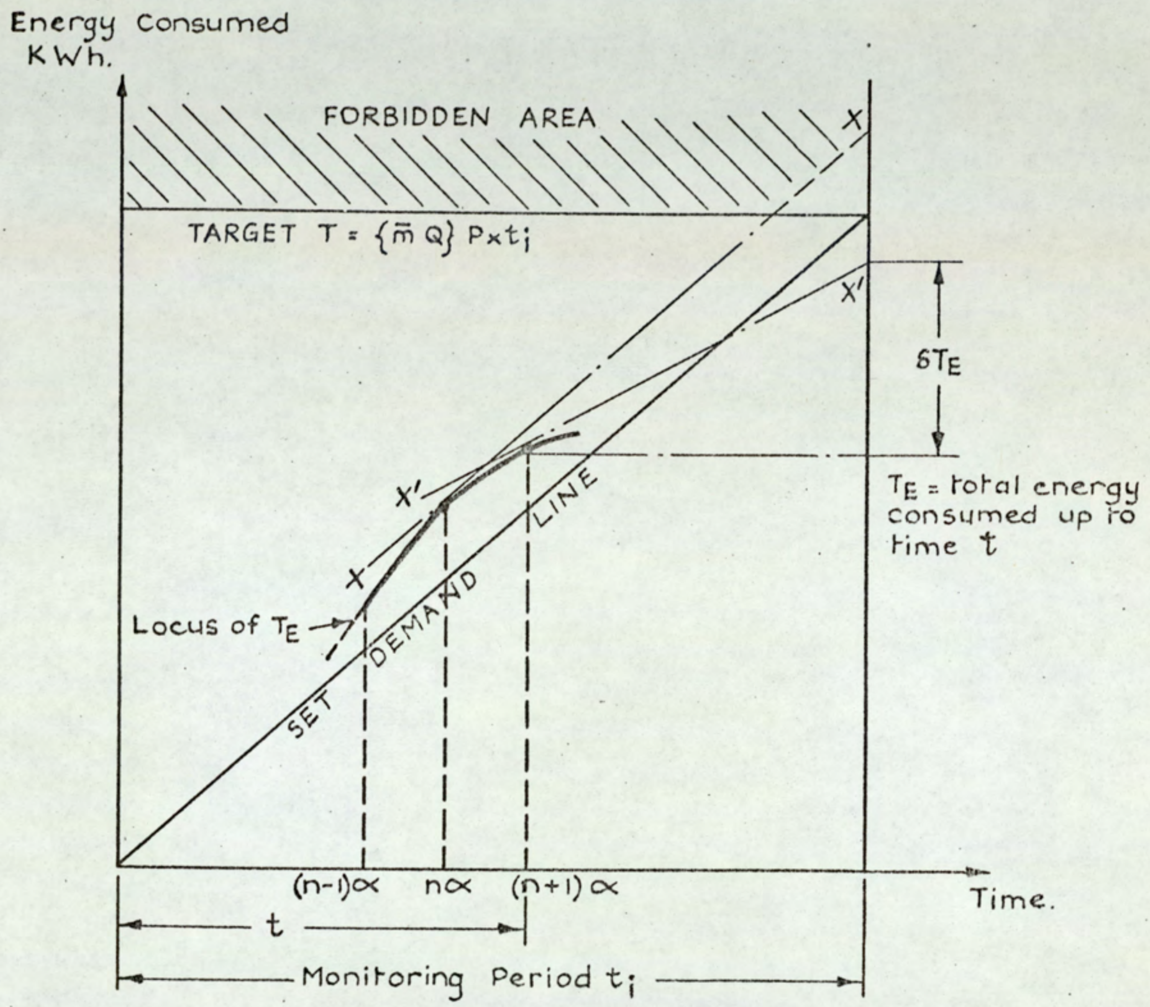


FIG. 3.9 SCHEMATIC FOR COMPUTER STRATEGY TO MINIMISE MAXIMUM DEMAND.



T_E SAMPLED AT TIMES 0, α, 2α ------ t_i

FIG. 3.10 COMPUTER STRATEGY IF $T_E > (\bar{m}\ Q)\ P \times t$.

At each sampling instant the energy is compared with that which would have been used if the demand were maintained constant at $\{\bar{m}\ Q\}\ P$.

Suppose in Fig. 3.10 that the predicted locus of T_E is given by the tangent XX at the sampling point previous to t. The locus must then be corrected so that the tangent X' X' at time t satisfies the condition $\delta T_E \leqslant T - T_E$, where $\delta T_E = (t_i - t) \times$ slope of T_E at time t.

Under normal operation the target energy will not be exceeded. If, however, load reduction becomes necessary, the relative phasing of the loads must not be changed. Limited nominated fluctuations in power level for individual processes may be tolerated, provided an adequate safety margin is included in setting the target energy value.

Visual indication of "target" and "indicated" demand give warning as the target energy is approached, and automatic cut out is effected if this value is exceeded.

3.6.4 Alternative Control Strategies.

Subject to the restriction that all process load patterns must be the same, the following alternative methods of control are possible:

1. Change of Q by integer values.
2. Change of $\bar{m}$ by the same amount for each process.
3. Voltage control so that P changes by the same amount for each process.

Suppose we have ten processes, each having the same $\bar{m}$ and t_q, and the stagger is $\alpha_o = t_q/10$. If the tenth process is removed as a result of demand control, the remaining nine may be switched to a cam system giving $\alpha_o = t_q/9$, so that the maximum demand is again minimised.

If it is required to re-energise the tenth process, we revert to the original stagger and phase this process into its original position in the switching sequence.

Alternatively, the position occupied by an existing load may be taken ; at the same time this process is temporarily shut down before being re-phased in the position which the tenth process would normally occupy. The electrical power system will recognise no change in the load pattern, and unnecessary delay may be avoided in re-commissioning the tenth process.

Thus the stagger α can always be maintained at its optimum value α_o corresponding to the number of processes in service, when the maximum demand will be continually minimised.

For an ensemble of two-position processes, the ratio energy per cycle/$t_q = \bar{m}\,QP$. Thus demand control can be effected by continuous control of $\bar{m}$ by the same amount for each process, which should prove easier than changing Q.

If equal voltage control to all processes is applied in conjunction with equal variation of $\bar{m}$, it will be possible both to supply the energy required and to maintain $\bar{m}\,Q$ integral, so that the power level is constant at $\bar{m}\,QP$ with $\alpha_o = t_q/Q$.

For $\bar{m}\,Q$ non-integral, the two-level load pattern with $\alpha_o = t_q/Q$ has a maximum value ($[\bar{m}\,Q] + 1$) P and a minimum $[\bar{m}\,Q]$ P. Thus the percentage ripple of load will be insignificant if $\bar{m}\,Q \gg 1$, when demand control by equal variation of $\bar{m}$ alone will be adequate.

3.6.5 Application of Control Strategies.

An application of deterministic switching is to bring a furnace to a required temperature by cyclic energising of a number of resistance heaters. The period and the phasing are so arranged that the maximum demand is minimised at a nominated value.

Greater versatility of load control is available than when the heating is effected by a smaller number of continuously rated elements, especially when the energy requirements vary from day to day.

Temperature control of individual processes within an ensemble cannot be applied owing to the symmetry requirements of the load pattern. However temperature control in an overall sense is possible, where the $\bar{m}$ of each process is adjusted by the same amount as the result of a closed-loop control.

The above strategies for maximum demand limitation may be applicable when an on-off process ensemble represents the controllable part of a larger factory load.

3.7 SUMMARY.

A generalised policy for computer control of maximum demand has been established where the maximum number of similar two-position processes to be controlled is of the order of ten. The load patterns arising from control of the demand have been thoroughly analysed.

The principle of load staggering to minimise the maximum demand is, of course, well known. However this new work shows that, if Q is the number of processes and t_q the cycle time:

1. For $\bar{m}Q$ integral.

The only choice for the stagger α between successive loads which leads to a minimisation of the maximum demand is given by $\alpha = \frac{t_q}{Q}$.

2. For $\bar{m}Q$ non-integral.

The above choice of α also results in a minimisation of the maximum demand. However, there is some latitude in the value of stagger which gives rise to this minimum maximum demand. Once chosen, the stagger is invariable and must be made equal for all processes succeeding the first.

The simplest switching policy to adopt for all value of $\bar{m}Q$, therefore, is to make α constant and equal to t_q/Q.

Where the process ensemble forms the controllable part of a larger factory load, scope exists for continuous minimisation of the maximum demand. As loads are shed due to the action of demand control, the stagger may be changed to the optimum value corresponding to the number of processes on load, at the same time maintaining a systematic pattern of incidence.

Strategies for control of the demand due to this class of load are given in section 3.6.4; an application is described in section 3.6.5.

These strategies will not be applicable where the switching sequence is random, as in situations where the load characteristics are thermostatically-controlled.

For large ensembles of processes the constraint of deterministic switching is not so severe, when the incidence of loads will be characterised by a distribution spread about a mean time.

CHAPTER 4.

DEMAND CONTROL IN LARGE ENSEMBLES OF TWO-POSITION PROCESSES.

4.1 INTRODUCTION.

A problem of current interest is when the number of processes becomes very large, such as off-peak water heating in utility supply areas.

The strategy adopted here is to sectionalise the total load into large blocks of power, which are staggered relative to each other in a systematic manner to minimise the demand. The switching incidence of the members comprising any block will have a distribution about the mean position, e.g. the spread in the operation of time switches.

This distribution has been assumed Normal.

The dependence of the load fluctuation upon the switching pattern will be evaluated for a range of switching spreads and for a range of duty cycles.

4.2 PATTERNS CONSIDERED AND CONSTRAINTS IMPOSED.

We will assume a large number Q_1 of similar two-position processes which are sectionalised into Q blocks, each containing, say, 100 processes. Thus $Q_1 = 100\,Q$.

The analysis of load demand will be subject to the following constraints:

1. The switching index m will be restricted to the values of 0 and 1, i.e. zero or full power P.

2. All processes have repetitive rectangular load patterns with the same values of cycle time t_q, the same $\bar{m} = t_o/t_q$, where $0 \leqslant \bar{m} \leqslant 1$, and the same power P.

The 100 members comprising a block will be assumed to be distributed normally about their mean position with a standard deviation equal to σ. The values of σ in respect of switching on and switching off will be taken as equal. However, if necessary, the overall load pattern may be determined in a similar manner for unequal values of σ and for other distributions.

4.3 ASSESSMENT OF LOAD PATTERN AS A FUNCTION OF TIME.

In order to minimise the indicated M.D. and maximise the load factor, the mean positions of successive blocks of power will be staggered by

$$\alpha = \frac{t_q}{Q}.$$

The 100 processes comprising a block will each be controlled by a time switch to operate nominally at the mean time specified for the block, but there will be a spread in the actual operating times.

Fig. 4.1 has been drawn for the case Q = 4, $\bar{m} = \frac{2}{3}$ to illustrate the switching sequence.

The probability of a process being switched on anywhere in the range $-\infty$ to $+\infty$ is

$$P(x) = \int_{-\infty}^{\infty} y(x)\, dx = 1, \qquad (4.1)$$

where x = time and y (x) is the probability density.

The probability of a process being switched on in the time band $t = x_2 - x_1$ is

$$P(x) = \int_{-\infty}^{x_2} f(x)\, dx - \int_{-\infty}^{x_1} f(x)\, dx, \qquad (4.2)$$

where for a normal distribution with a shift in the x - axis

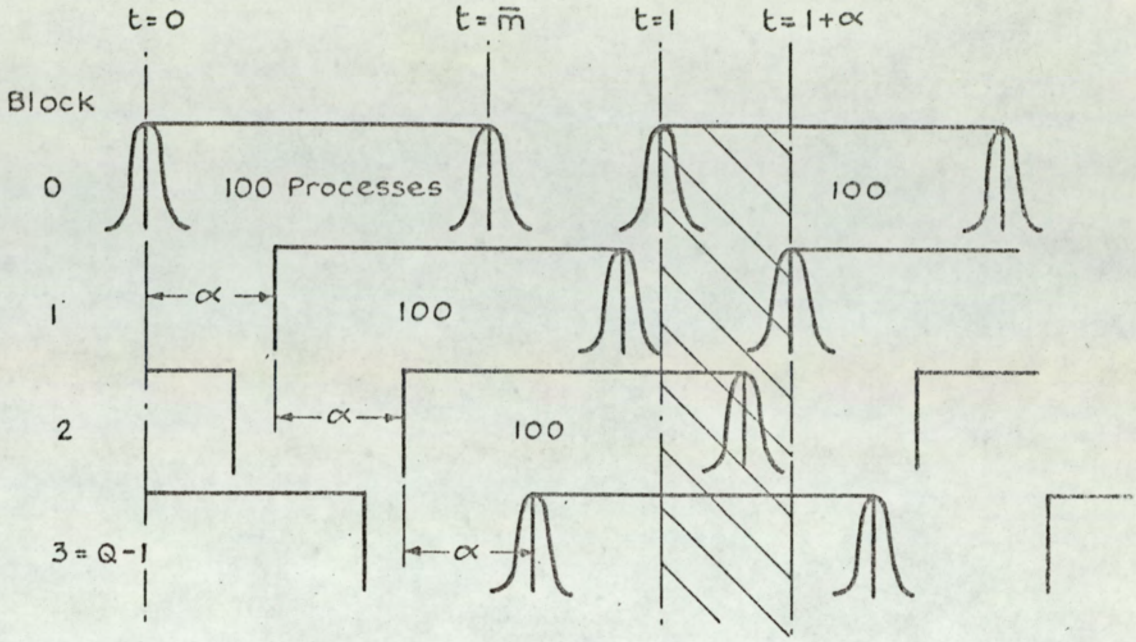
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-(x-\eta)^2 / (2\sigma^2)\right), \qquad (4.3)$$

in which σ^2 = variance and $x = \eta$ is the distribution midposition.

$$\text{Then } P\left\{x_1 \leqslant x \leqslant x_2\right\} = F(x_2) - F(x_1), \qquad (4.4)$$

$$\text{where } F(x) = \int_{-\infty}^{x} f(y)\, dy = \tfrac{1}{2} + \operatorname{erf} \frac{x-\eta}{\sigma} \qquad (4.5)$$

$$\text{and } \operatorname{erf} x = \frac{1}{\sqrt{2\pi}} \int_{0}^{x} \exp(-y^2/2)\, dy, \qquad (4.6)$$



Blocks being switched in band α are those with distributions falling within hatched zone.

Fig. 4.1(a) $\bar{m} = 2/3$ Blocks Q = 4 $\alpha = \frac{tq}{Q} = 1/4$
100 Processes per Block.

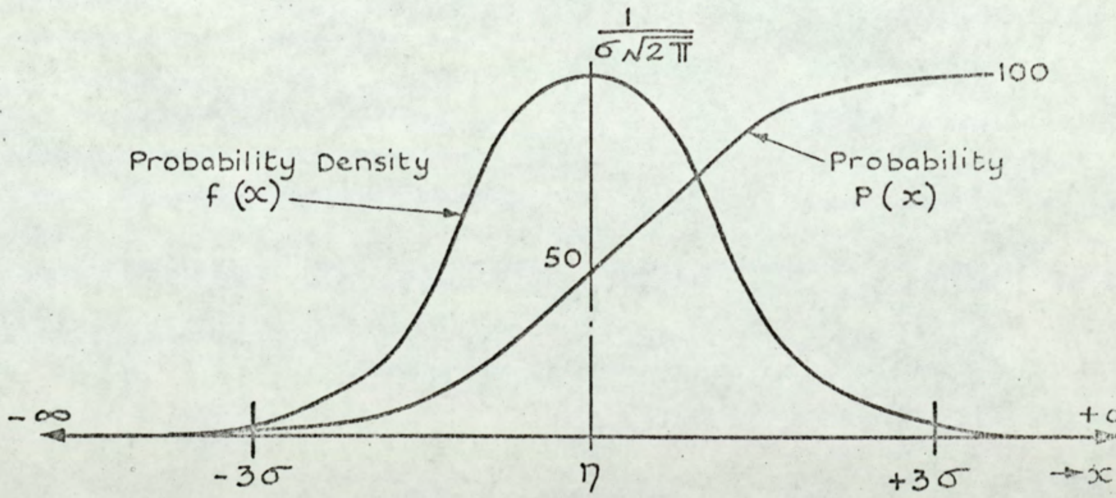


Fig. 4.1(b)

FIG. 4.1 SWITCHING SEQUENCE FOR Q = 4, $\bar{m} = \frac{2}{3}$ & NORMAL DISTRIBUTION.

$$\text{erf}(-x) = -\text{erf}(x), \tag{4.7}$$

$$\text{erf}\,\infty = \tfrac{1}{2}, \tag{4.8}$$

so that $$P\left\{x_1 \leqslant x \leqslant x_2\right\} = \text{erf}\left(\frac{x_2 - \eta}{\sigma}\right) - \text{erf}\left(\frac{x_1 - \eta}{\sigma}\right). \tag{4.9}$$

For example, if σ = 100 seconds and η = 300 seconds, the probability of a process being switched on between -1 minute and 0 minute of the mean is:

$$\text{erf}\,\frac{300 - 300}{100} - \text{erf}\,\frac{240 - 300}{100} = 0.2257\,.$$

Thus for 100 processes the probable number lying in this band is 22.57, or a total of 45 symmetrically positioned about the mean and within 1 minute of the mean.

Let us consider a time displacement from the mean position equal to three standard deviations. The area under the probability density curve outside the $\pm 3\sigma$ limits is less than 0.3%. Then the lower limit of integration in equation (4.2) may be changed without serious error from $-\infty$ to -3σ. If processes commence to be switched on after time -3σ, then the area under the probability density curve between -3σ and any time t_1 will give the probable number of processes switched on between these times. The number on at time t_1 may be taken without serious error as the corresponding ordinate of the probability curve (Fig. 4.2), values of which are taken from published tables [15, 16] and the amplitude scale multiplied by 100. If the time variable is stepped at regular intervals, the build-up of load between -3σ and t_1 may therefore be constructed.

4.3.1 Overall Load Pattern Assuming No Statistical Spreads.

The load pattern with no statistical spreads will be similar to that deduced in Chapter 3.

Thus if the stagger between successive blocks is $\alpha = \frac{1}{Q}$, where t_q is normalised to unit value: For $\bar{m}Q$ integral, the total load is constant at $\bar{m}Q$ (100P). For $\bar{m}Q$ non-integral, the total load has a two-level pattern, of maximum value $(\left[\bar{m}Q\right] + 1)$ 100P and minimum value $\left[\bar{m}Q\right]$ 100P.

4.3.2 Overall Load Pattern Assuming Normal Incidence of Loads per Block.

The Q blocks again have a stagger between midpositions of

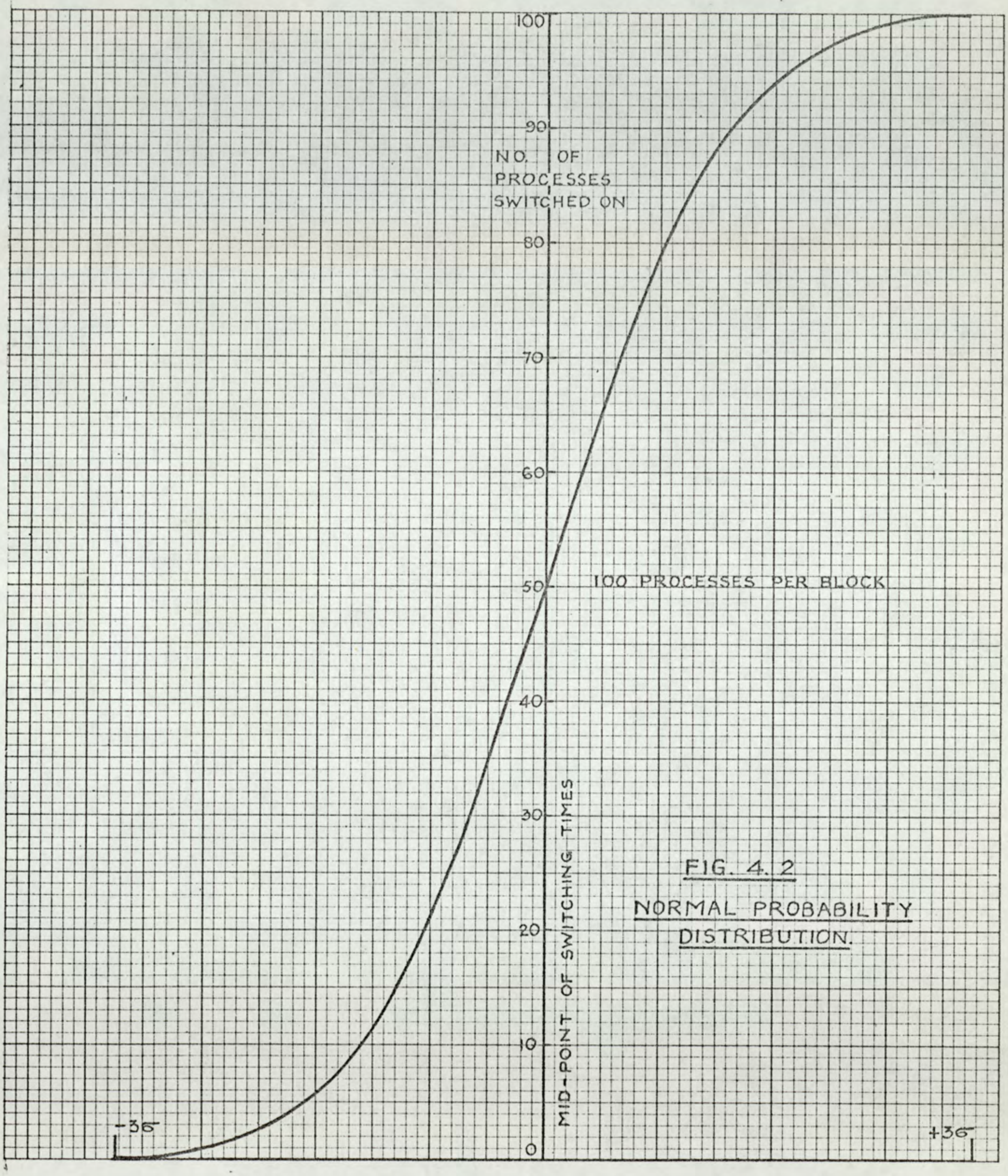


FIG. 4. 2
NORMAL PROBABILITY DISTRIBUTION.

$\alpha = \frac{1}{Q}$.

The incidence of the 100 loads per block is assumed normally distributed within a range 3σ on either side of the mean starting position. The same distribution is assumed for switching off as for switching on.

Fig. 4.1 (a) shows that from $t = 0$ to α (or 1 to $1 + \alpha$) the overall load pattern just spans this region, and is then repetitive at intervals of α. In this band some blocks will be completely on and some completely off, and for the remaining blocks the number of loads on will be given by the probability distribution curve. Where these distributions overlap they may be summed by the principle of superposition, the load pattern depending upon the ratio $\frac{3\sigma}{\alpha}$. The condition for a block not being entirely on or off is that the peak of the probability density "bell" is not more than 3σ displaced to the left of $t = 1$ or to the right of $t = 1 + \alpha$. For $\frac{3\sigma}{\alpha} < 1$ only a small number of bells fall in this category, but as $\frac{3\sigma}{\alpha} > 1$ more blocks of processes will be involved in switching.

4.4 MATHEMATICAL ANALYSIS OF LOAD PATTERN ASSUMING NORMAL INCIDENCE OF LOADS PER BLOCK.

Fig. 4.3 shows the general case in which the Q blocks are spaced at $\alpha = \frac{1}{Q}$.

Let the blocks be numbered 0, 1, 2 - - - r - - - R - - -- Q - 1.

The rth block comes on at time $r\alpha$ and goes off at $r\alpha + \bar{m}$.

The on and off times of the mean positions are:

	ON	OFF
Block 0	0	$\bar{m}$
Block 1	α	$\alpha + \bar{m}$
Block r	$r\alpha$	$r\alpha + \bar{m}$
Block Q - 1	$(Q - 1)\alpha$	$(Q - 1)\alpha + \bar{m}$

TABLE 4.1

It is required to find which switching distributions fall within the band α commencing at $t = 0$ or 1.

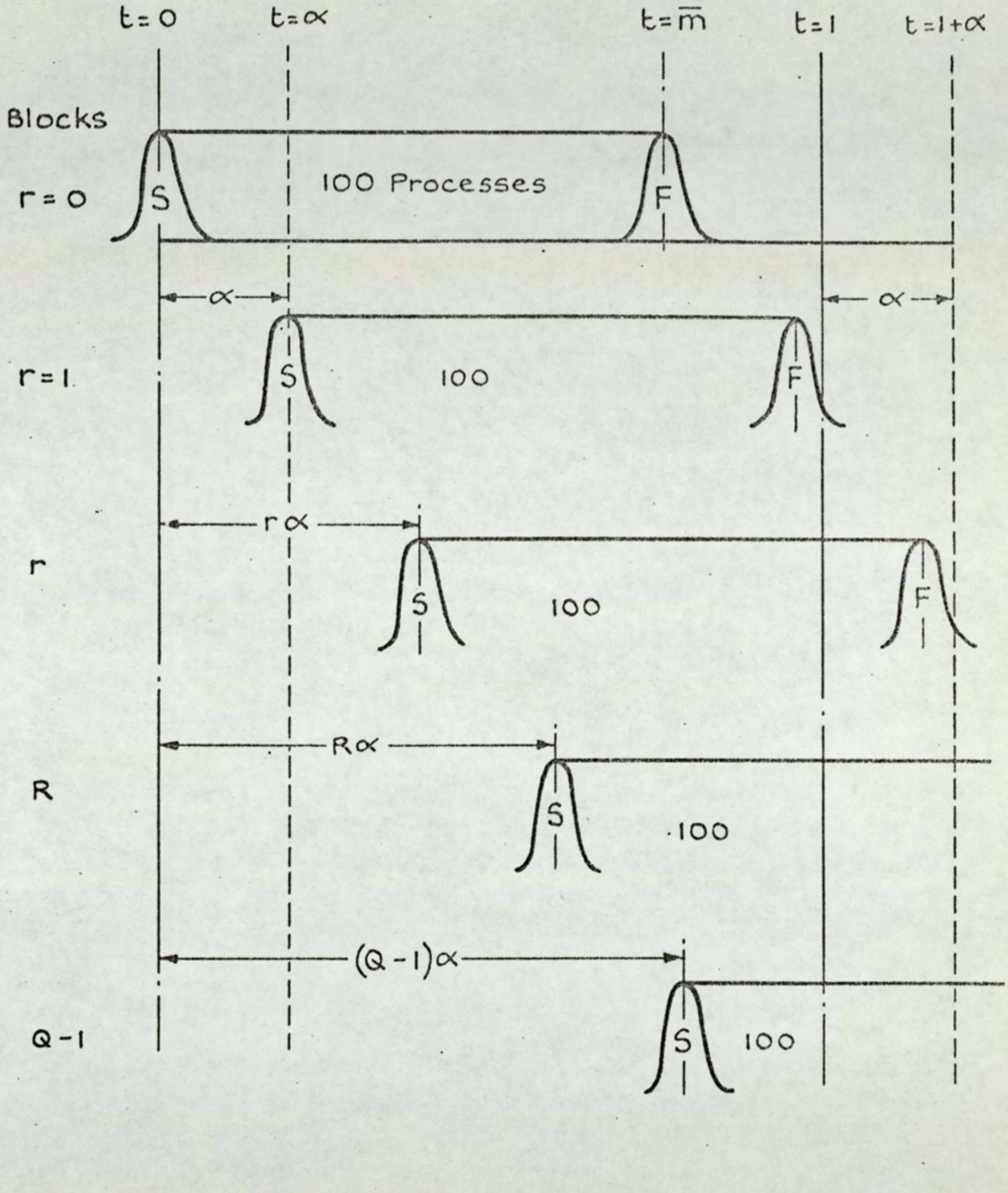


FIG. 4.3 GENERALISED SWITCHING SEQUENCE FOR Q BLOCKS.

We may assume that $\bar{m} + 3\sigma > 1$, i.e. $\bar{m} < 1 - 3\sigma$, so that block number 0 does not appear twice in the range $0 \leqslant t \leqslant \alpha$. If we consider first the <u>starts</u> S:-

By definition, block $r = 0$ and $r = 1$ are within the band α commencing at $t = 0$ and finishing at $t = \alpha$.

For other blocks the criterion of falling within this band is whether

$$r\alpha - 3\sigma < \alpha$$

i.e. $\underline{0 \leqslant r < 1 + p}$, where $p = \dfrac{3\sigma}{\alpha}$ - - - - - - - - -Case (i) (4.10)

The second criterion is derived at $t = 1$ at the beginning of the band α, and is whether

$$R\alpha + 3\sigma > 1,$$

i.e. $R > \dfrac{1 - 3\sigma}{\alpha}$, or $R > Q - p$.

But R must be $\leqslant Q - 1$,

so that

$\underline{Q - 1 \geqslant R > Q - p}$ - - - - - - - - - - - - -Case (ii) (4.11)

As an <u>example</u>, if $\dfrac{3\sigma}{\alpha} = 0.35$ (see Section 4.5),

$0 \leqslant r < 1 + 0.35$, i.e. $r = 0$ or 1.

Also $Q - 1 \geqslant R > Q - 0.35$, i.e. there are no R's.

If we now consider the <u>finishes</u> F:-

At $t = 1$ the criterion is whether

$$\bar{m} + r\alpha + 3\sigma > 1,$$

i.e. $r > \dfrac{1 - 3\sigma - \bar{m}}{\alpha}$,

or $\underline{r > \dfrac{1 - \bar{m}}{\alpha} - p}$ - - - - - - - - - - - - - - - - -Case(iii) (4.12)

At $t = 1 + \alpha$ the criterion is whether

$$\bar{m} + r\alpha - 3\sigma < 1 + \alpha,$$

i.e. $r < \dfrac{1 - \bar{m} + 3\sigma}{\alpha} + 1$,

or $\underline{r < \dfrac{1 - \bar{m}}{\alpha} + p + 1}$ - - - - - - - - - - - - - - --Case (iv) (4.13)

For <u>example</u>, if $\alpha = \dfrac{1}{100}$ and $\bar{m} = 0.7$, $p = 0.35$,

then $\left.\begin{array}{l} r > 30 - 0.35 \\ r < 31.35 \end{array}\right\}$ i.e. $r = 30$ and 31.

Suppose we take an example where

$$Q = 100, \quad \alpha = \frac{1}{100},$$

$$\text{and } \bar{m} = \frac{2}{3}, \quad \frac{3\sigma}{\alpha} = 1.5.$$

From Case (i),

$$0 \leqslant r < 2.5, \quad \text{i.e. } r = 0, 1, 2.$$

From Case (ii),

$$Q - 1 \geqslant R > Q - 1.5, \text{ i.e. } R = Q - 1 \text{ only, or } 99.$$

$\therefore$ only 4 of the 100 blocks start in the region t = 0 to α .

From Cases (iii) and (iv), the criterion of blocks finishing in the region 0 to α

is $\frac{1 - \bar{m}}{\alpha} - 1.5 < r < \frac{1 - \bar{m}}{\alpha} + 1.5 + 1$,

i.e. $33\frac{1}{3} - 1.5 < r < 33\frac{1}{3} + 2.5$,

i.e. $31.83 < r < 35.83$,

i.e. r = 32, 33, 34, 35.

$\therefore$ only 4 of the 100 blocks finish in the region 0 to α .

Clearly r = 3, 4, 5, 6 - - - - - - - 31 are not on at all in the period 0 - α ,

and r = 36, 37, 38 - - - - - - - - 98 are on fully in the period 0 - α .

Thus, of the 100 blocks, 63 are fully on and 29 are fully off, so that their total load may be easily summed.

Then we need only consider in detail the other 8 blocks.

Block r = 0 starts at t = 0, r = 1 at t = α ,

r = 2 at t = 2 α , r = 99 at t = - α .

Contribution from block r = 0 is f (t) .

Contribution from block r = 1 is f (t - α) .

Contribution from block r = 2 is f (t - 2 α) .

Contribution from block r = 99 is f (t + α),

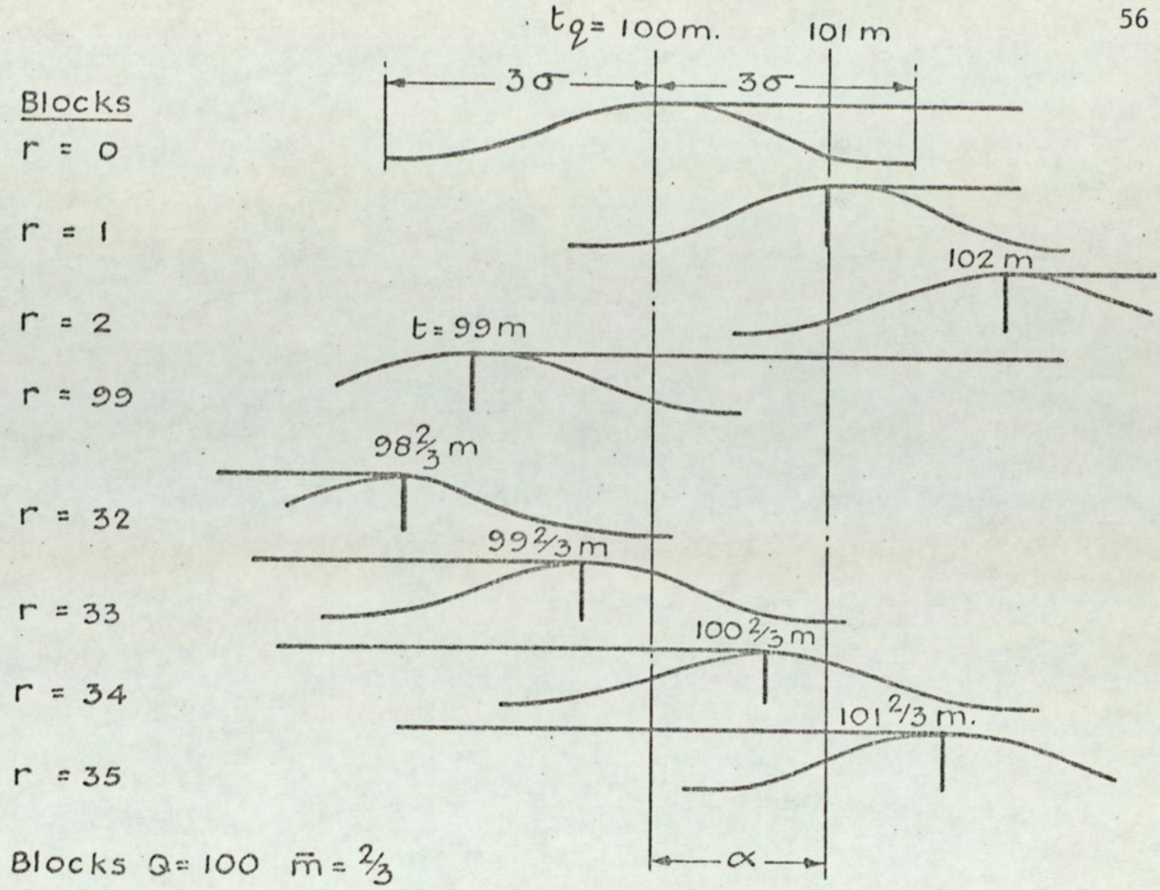
where f (t) is the amplitude of the probability curve (Fig. 4.2).

The contributions from the other 4 blocks may be found in a similar manner.

Fig. 4.4 (a) shows the positions of blocks r = 32, 33, 34, 35 relative to t = 1 for Q = 100, $\bar{m} = \frac{2}{3}$, $\frac{3\sigma}{\alpha} = 1.5$.

Contribution from block r = 32 is 100 - f (t + $1\frac{1}{3}$) .

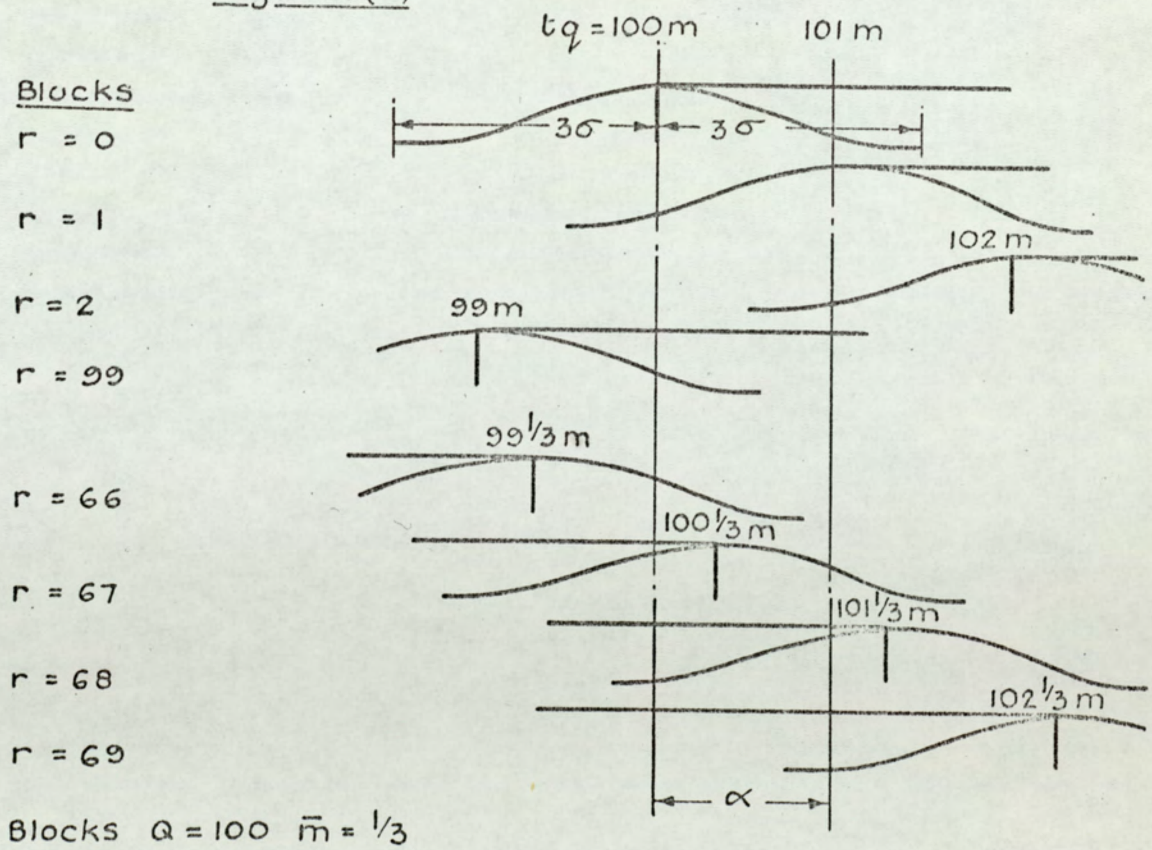
Contribution from block r = 33 is 100 - f (t + $\frac{1}{3}$) .



Blocks $Q = 100$ $\bar{m} = 2/3$

$\frac{3\sigma}{\alpha} = 1\cdot5$ $\alpha = \frac{t_q}{Q} = 1$ min.

Fig. 4.4 (a)



Blocks $Q = 100$ $\bar{m} = 1/3$

$\frac{3\sigma}{\alpha} = 1\cdot5$ $\alpha = 1$ min.

Fig. 4.4 (b)

FIG. 4.4 BLOCKS BEING SWITCHED.
(a) $Q = 100$, $\bar{m} = \frac{2}{3}$, $3\sigma/\alpha = 1\cdot5$. (b) $Q = 100$, $\bar{m} = \frac{1}{3}$, $3\sigma/\alpha = 1\cdot5$

Contribution from block r = 34 is $100 - f\ (t - \frac{2}{3})$.
Contribution from block r = 35 is $100 - f\ (t - 1\frac{2}{3})$,
since in the region t = 1 to $1 + \alpha$ these 4 blocks are in the stage of being switched off.

4.5 APPLICATION OF THEORY TO OBTAIN OVERALL LOAD PATTERN.

If t_q = 24 hours, say, then α = 14.4 minutes for Q = 100, so that $\frac{3\sigma}{\alpha}$ may reasonably be expected to have a value of less than unity (0.35 for 3σ = 5 minutes, for example). For t_q = 100 minutes, however, α = 1 minute, so that larger $\frac{3\sigma}{\alpha}$ ratios may merit consideration. It will be shown, however, that there is no point in considering values greater than 1.5, when the fluctuation in total load is virtually eliminated.

The stagger between the mid-switching points of successive blocks (50% of the processes within a block on) is $\alpha = \frac{t_q}{Q}$, and is taken as 1 minute for load summation purposes. By considering 3 -second intervals in the range 0 to α, the overall load pattern will be obtained for spreads given by:

$$\frac{3\sigma}{\alpha} = 0,\ \tfrac{1}{4},\ \tfrac{1}{2},\ 1 \text{ and } 1\tfrac{1}{2}.$$

The cases to be considered are: $\bar{m}$ Q integral and $\bar{m}$ Q non-integral.

4.5.1 Overall Load Pattern for $\bar{m}$ Q Integral.

Initially we will consider $\bar{m} = \frac{1}{4}$, Q = 100 (with 100 processes per block) and t_q = 100 minutes. Consider a spread $\frac{3\sigma}{\alpha} = p = \frac{1}{2}$.

By applying the criteria

$$0 \leqslant r < 1 + p, \tag{4.14}$$

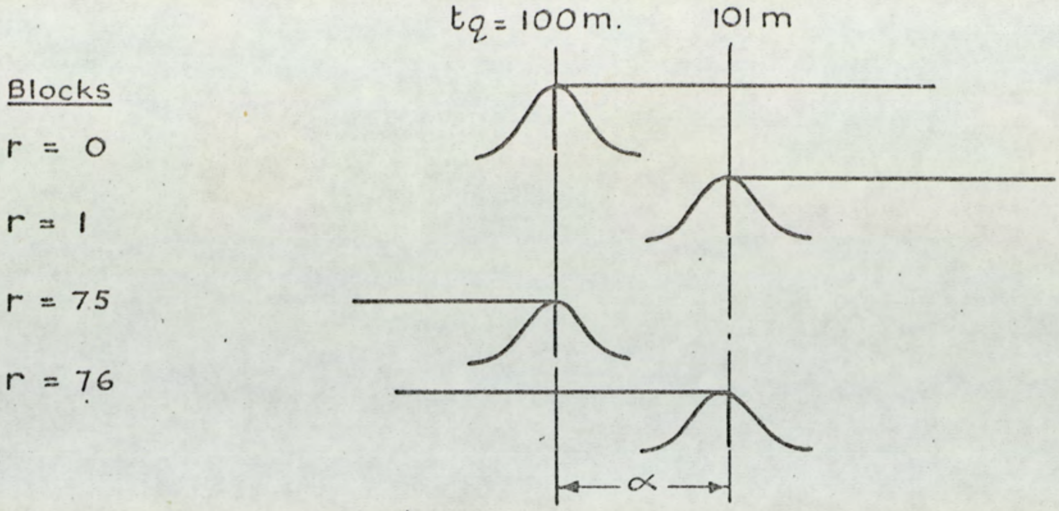
$$Q - 1 \geqslant R > Q - p, \tag{4.15}$$

$$\frac{1 - \bar{m}}{\alpha} - p < r < \frac{1 - \bar{m}}{\alpha} + p + 1, \tag{4.16}$$

we obtain in the period t = α to $1 + \alpha$:

Blocks r = 0, 1 are switching on and blocks 75, 76 are switching off; the relative spacings are shown in Fig. 4.5.
Blocks r = 2, 3, 4 - - - - - - - - 74 are not on at all (a total of 73).
Blocks r = 77, 78, 79 - - - - - -99 are fully on (a total of 23).

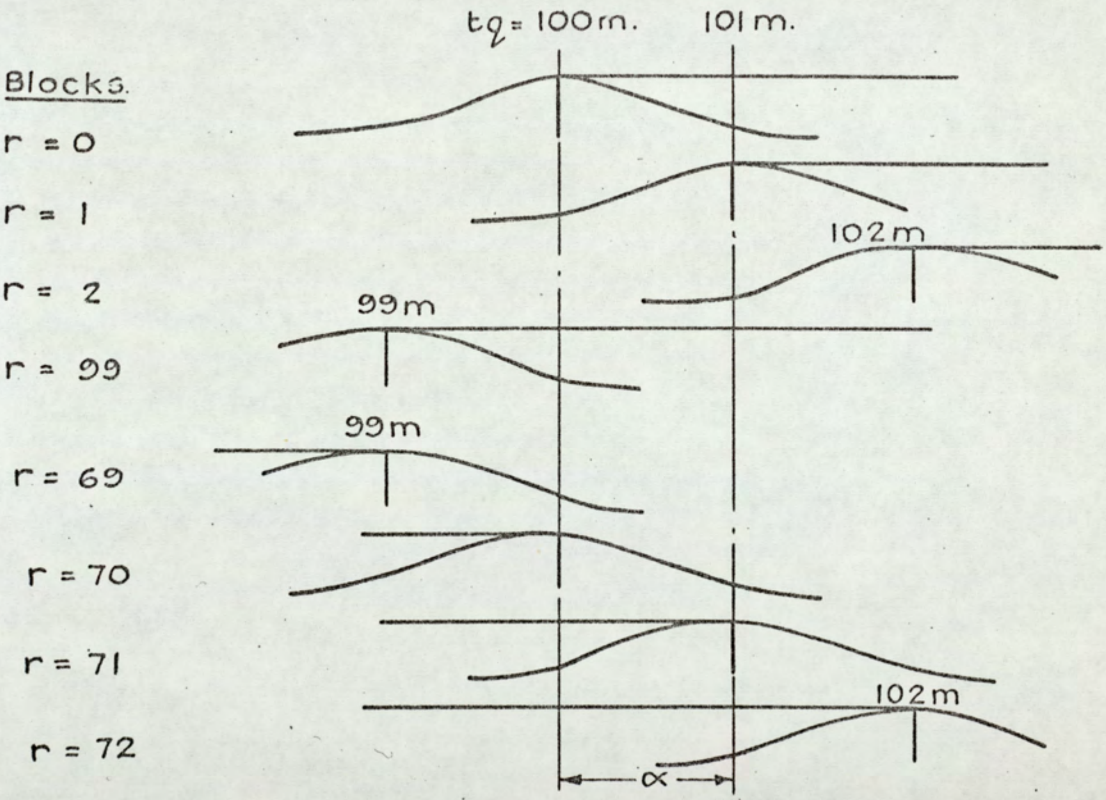
The number of processes in the act of switching in the period t = 1 to $1 + \alpha$ (or 0 to α) may now be summed from Fig. 4.2 as follows:



Blocks $Q = 100$ $\bar{m} = 1/4$

$\frac{3\sigma}{\alpha} = 0{\cdot}5$ $\alpha = 1$ min.

Fig. 4.5 (a.)



Blocks $Q = 100$ $\bar{m} = 0{\cdot}3$

$\frac{3\sigma}{\alpha} = 1{\cdot}5$ $\alpha = 1$ min.

Fig. 4.5 (b)

FIG. 4.5 BLOCKS BEING SWITCHED.
(a) $Q = 100$, $\bar{m} = \frac{1}{4}$, $3\sigma/\alpha = 0{\cdot}5$.
(b) $Q = 100$, $\bar{m} = 0{\cdot}3$, $3\sigma/\alpha = 1{\cdot}5$.

BLOCK r	NO. OF PROCESSES ON										
	t = 0s	3	6	9	12	15	18	21	24	27	30
0	50	62	72	81	89	93	96	98	99	100	100
1	0	0	0	0	0	0	0	0	0	0	0
75	50	38	28	19	11	7	4	2	1	0	0
76	100	100	100	100	100	100	100	100	100	100	100
	200	200	200	200	200	200	200	200	200	200	200

r	t = 33s	36	39	42	45	48	51	54	57	60	63
0	100	100	100	100	100	100	100	100	100	100	100
1	1	1	2	4	7	11	18	28	38	50	62
75	0	0	0	0	0	0	0	0	0	0	0
76	99	99	98	96	93	89	82	72	62	50	38
	200										

TABLE 4.2

The statistically-determined load is therefore constant throughout α. Adding the contributions due to the 23 blocks fully on, the total number of processes is constant at 200 + 23 x 100 = 2500, which is equal to 100 x $\bar{m}$ Q.

As the spread is increased, more blocks will be in the partially-switched state over the period α.

For example, for $\bar{m} = 0.3$, Q = 100, α = 1 minute and $\frac{3\sigma}{\alpha} = 1.5$, a total of 8 distributions need to be summed, giving a constant number of processes on equal to 400. There will be 26 blocks fully on, giving a total number of 400 + 26 x 100 = 3000, which is again equal to 100 x $\bar{m}$ Q. The relative spacing of the blocks in the state of switching in the interval t = 1 to 1 + α (100 minutes to 101 minutes for the example taken) is shown in Fig. 4.5.

Inspection of Fig. 4.5 shows that for every block switching on there is another block switching off with the same distribution, which gives rise to the constant total.

This symmetry of switching can only be obtained when:

The mid-switching points of the start of some blocks or the finish of some blocks coincide with either the point $t = 1$ or the point $t = 1 + \alpha$, or are displaced from $t = 1$ by integral multiples of α.

We may now examine under what situation these conditions obtain:

The start of any block r lags the commencement of block $r = 0$ by $r\alpha$ and must therefore precede $t = 1$ by $1 - r\alpha = (Q - r)\alpha$, where $Q - r$ is integral.

The finish of any block r lags the commencement of block $r = 0$ by $r\alpha + \bar{m}$. For a finish to coincide with $t = 1$, $r\alpha + \bar{m} = 1$, or $r = Q - \bar{m}Q$, which is only integral for $\bar{m}Q$ integral. The finishes of all other blocks will be displaced from that of r by integral multiples of α.

For constant t_q and the same symmetrical on and off switching distributions, the total load for $\bar{m}$ Q integral is constant irrespective of the spread, and equal to $100 \times \bar{m}Q$.

4.5.2 Overall Load Pattern for $\bar{m}$ Q Non-Integral.

For $\bar{m}$ Q non-integral the distribution of blocks switching on and off is asymmetrical with respect to the points $t = 1$ and $1 + \alpha$.

Furthermore, the blocks being switched on in the band α may be either odd or even, depending upon the value of $\bar{m}$ and upon the spread $\frac{3\sigma}{\alpha}$.

The loads have been summed for $\bar{m} = \frac{1}{8}, \frac{1}{3}, \frac{2}{3}, \frac{7}{8}$ for values of spread given by $\frac{3\sigma}{\alpha} = 0, \frac{1}{4}, \frac{1}{2}, 1$ and $1\frac{1}{2}$. The positions of the blocks being switched on relative to the period $t = 1$ to $1 + \alpha$ are shown in Fig. 4.4 for two of the above cases.

Detailed calculations are shown below for $\bar{m} = \frac{7}{8}$. The loads are summed throughout the period in Tables 4.3 to 4.6.

For $\frac{3\sigma}{\alpha} = 1\frac{1}{2}$, the fluctuation of total load about the mean value is too small to be observed with sufficient accuracy by readings taken from Fig. 4.2. For this value of spread, we may effect a numerical integration of the probability density function to obtain ordinates of the probability curve.

We will again assume α = 60 seconds, Q = 100, and number of processes per block = 100.

Detailed Calculations for $\bar{m} = \frac{7}{8}$

$\frac{3\sigma}{\alpha} = 0$

Blocks being switched in period t = 1 to 1 + α = 0, 13.

Blocks r = 14, 15,..................99 all on (86).

Blocks r = 1, 2, 3, ...12 all off (12).

Distribution of r = 0 is f (t), where t is in seconds, and 3 σ = 0.

Distribution of r = 13 is 100 - f (t - 30).

From t = 0 to 30 seconds, total load = 200 + 8600 = 8800.

From t = 30 to 60 seconds, total load = 100 + 8600 = 8700.

Load fluctuation = 100.

$\frac{3\sigma}{\alpha} = \frac{1}{4}$

Blocks Switched	Distribution.
r = 0	f (t) for 3 σ = 15 secs. (Fig. 4.2).
r = 1	f (t - 60)
r = 13	100 - f (t - 30)

r = 14, 15,............................99 all on (86).

r = 2, 3,..............................12 all off (11).

SUM OF LOADS.

Block r	t = 0s	3	6	9	12	15	18	21	24	27	30
0	50	72	89	96	99	100	100	100	100	100	100
1	0	0	0	0	0	0	0	0	0	0	0
13	100	100	100	100	100	100	99	96	89	72	50
	150	172	189	196	199	200	199	196	189	172	150
14 - 99	8600	→									
	8750	8772	8789	8796	8799	8800	8799	8796	8789	8772	8750

Block r	t = 33s	36	39	42	45	48	51	54	57	60
0	100	100	100	100	100	100	100	100	100	100
1	0	0	0	0	0	1	4	11	28	50
13	28	11	4	1	0	0	0	0	0	0
	128	111	104	101	100	101	104	111	128	150
14 - 99	8600 →									
	8728	8711	8704	8701	8700	8701	8704	8711	8728	8750

Load Fluctuation = 100 .

TABLE 4.3

$\frac{3\sigma}{\alpha} = \frac{1}{2}$

Blocks switched.	Distribution.
r = 0	f (t) (3 σ = 30 secs).
r = 1	f (t - 60)
r = 13	100 - f (t - 30)

r = 14, 15,....................99 all on (86).

r = 2, 3,......................12 all off (11).

SUM OF LOADS.

Block r	t = 0s	3	6	9	12	15	18	21	24	27	30
0	50	62	72	81	89	93	96	98	99	99	100
1	0	0	0	0	0	0	0	0	0	0	0
13	100	99	99	98	96	93	89	82	72	62	50
	150	161	171	179	185	186	185	180	171	161	150
14 - 99	8600 →										
	8750	8761	8771	8779	8785	8786	8785	8780	8771	8761	8750

Block r	t = 33s	36	39	42	45	48	51	54	57	60
0	100	100	100	100	100	100	100	100	100	100
1	1	1	2	4	7	11	18	28	38	50
13	38	28	19	11	7	4	2	1	1	0
	139	129	121	115	114	115	120	129	139	150
14 - 99	8600 →									
	8739	8729	8721	8715	8714	8715	8720	8729	8739	8750

Load Fluctuation = 72.

TABLE 4.4

$\frac{3\sigma}{\alpha} = 1$

Blocks switched.	Distribution.
r = 0	f (t) (3 σ = 60 secs.)
r = 1	f (t - 60)
r = 12	100 - f (t + 30)
r = 13	100 - f (t - 30)
r = 14	100 - f (t - 90)

r = 15, 16,....................99 all on (85).

r = 2, 3,.......................11 all off (10).

SUM OF LOADS.

Blocks	t = 0s	3	6	9	12	15	18	21	24	27	30
0 + 1 + 12	8750	8754	8756	8756	8757	8757	8757	8756	8756	8753	8750
+ 13 + 14 + 15 to 99	t = 33s	36	39	42	45	48	51	54	57	60	
(8500) Processes.	8745	8744	8743	8743	8742	8744	8744	8744	8747	8750	

Load Fluctuation = 15 .

TABLE 4.5

$\frac{3\sigma}{\alpha} = 1\frac{1}{2}$

Blocks Switched.	Distribution.
r = 0	f (t) (3 σ = 90 secs).
r = 1	f (t - 60)
r = 2	f (t - 120)
r = 99	f (t + 60)
r = 12	100 - f (t + 30)
r = 13	100 - f (t - 30)
r = 14	100 - f (t - 90)

r = 15, 16,..................................98 all on (84).

r = 3, 4,....................................11 all off (9).

Sum of Loads (Switched Loads Summed by Computer Program, See Section 4.5.2.1).

Blocks	t = 0s	3	6	9	12	15	18	21
0 + 1 + 2 + 99 + 12 + 13 + 14 + 15 to 98 (8400) Processes.	8750.0	8750.1	8750.2	8750.3	8750.4	8750.4	8750.4	8750.3
	t = 24s	27	30	33	36	39	42	45
	8750.2	8750.0	8749.8	8749.9	8749.8	8749.7	8749.6	8749.6
	t = 48s	51	54	57	60			
	8749.6	8749.7	8749.9	8750.0	8750.0			

Load Fluctuation = 0.8.

TABLE 4.6

Similar computations have been carried out for $\overline{m}$ = 1/8 (where the switching distribution is identical to that for $\overline{m}$ = 7/8), 2/3 and 1/3, with values of $\frac{3\sigma}{\alpha}$ = 0, ¼, ½, 1, 1½.

The overall load fluctuation over the period α is plotted for each case in Figs. 4.6 to 4.8.

For $\overline{m} = \frac{1}{8}$ and $\frac{7}{8}$ the fluctuation is symmetrical about the mean value owing to the symmetrical switching distributions relative to the period

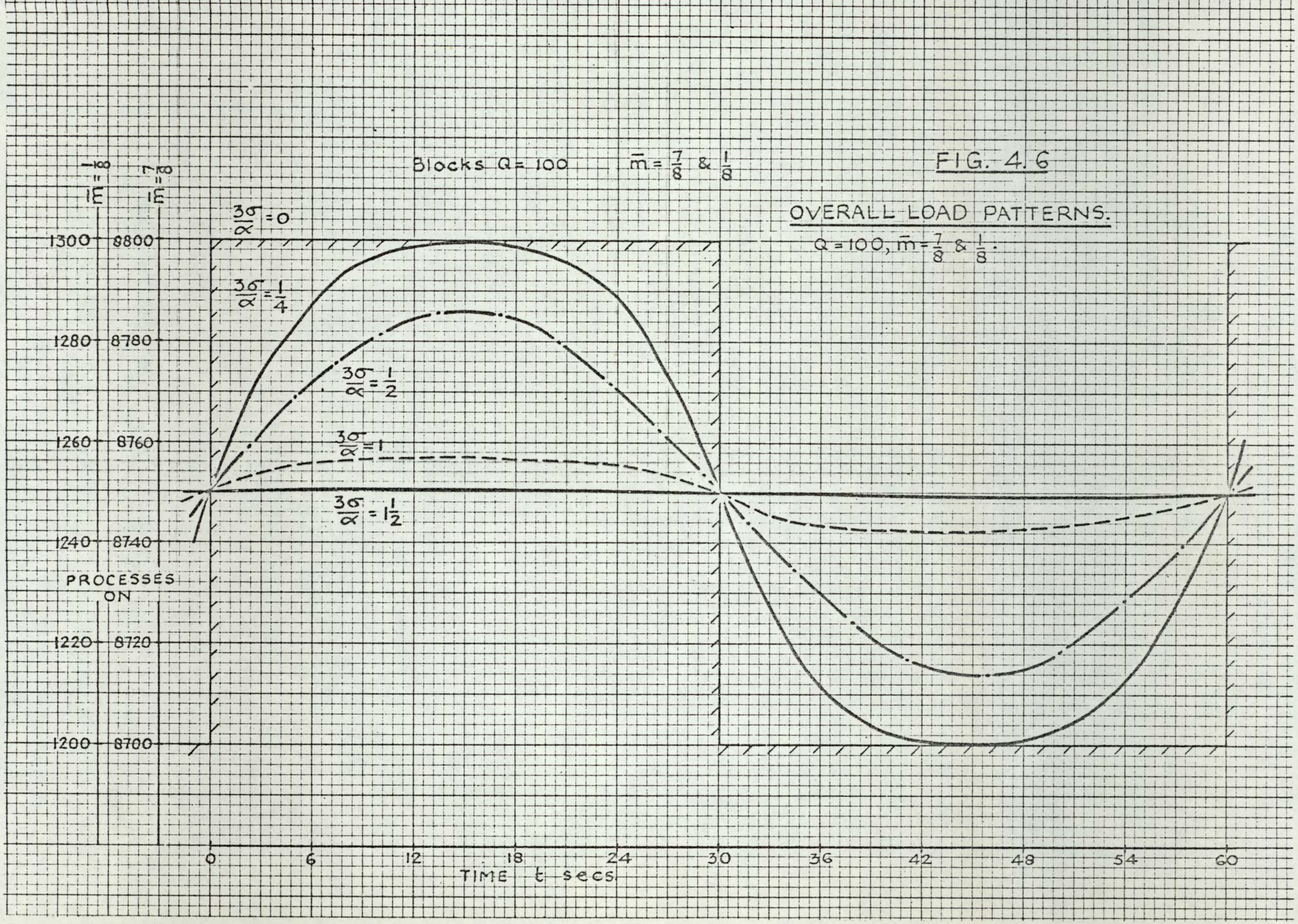


FIG. 4.6

OVERALL LOAD PATTERNS.

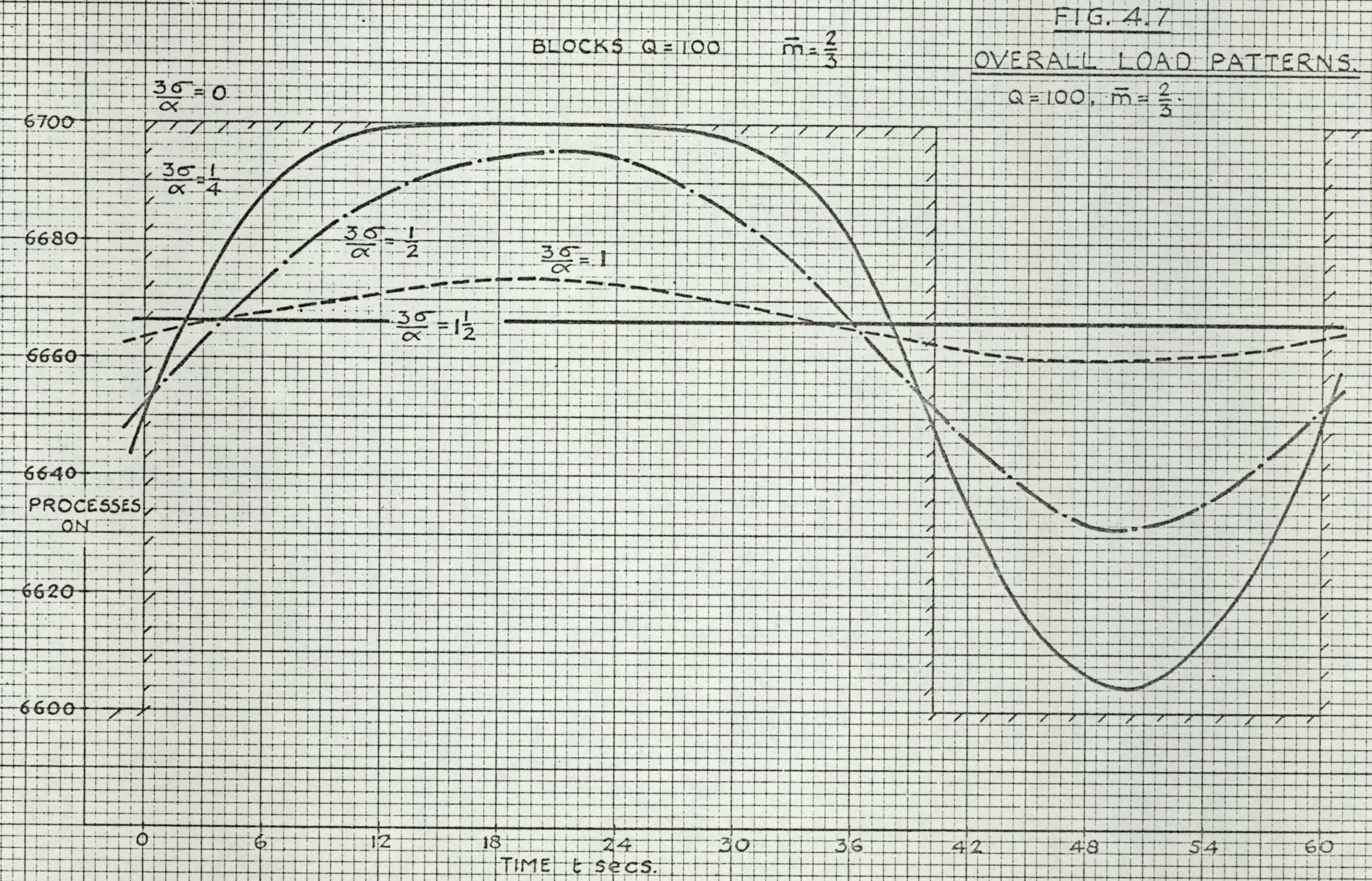


FIG. 4.7

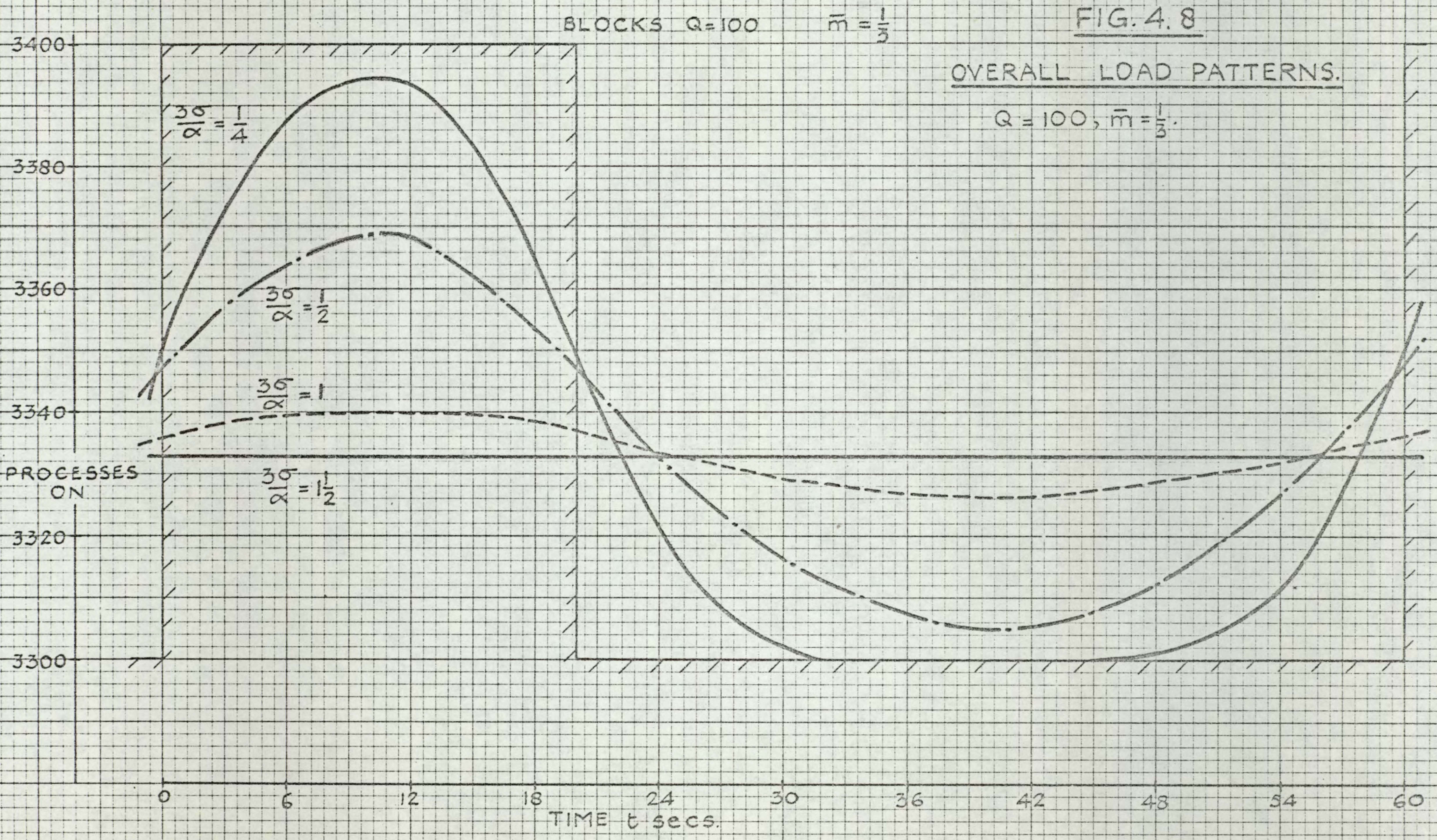
OVERALL LOAD PATTERNS.

$Q=100,\ \bar{m}=\frac{2}{3}$

FIG. 4.8

OVERALL LOAD PATTERNS.

$Q = 100, \bar{m} = \frac{1}{3}$.



1 to $1 + \alpha$. This does not apply to $\bar{m}$ = 1/3 and 2/3, however, (see Fig. 4.4), in which case the oscillation of load is asymmetric.

The load fluctuation as a percentage of 100 $\bar{m}$ Q is given in Table 4.7, and is plotted in Fig. 4.9 as a function of $\frac{3\sigma}{\alpha}$ for constant values of $\bar{m}$.

For $\bar{m}$ Q non-integral, the effect of an increased spread in the incidence of switching is to improve the load factor; this becomes pronounced for small values of $\bar{m}$.

$\frac{3\sigma}{\alpha}$	$\bar{m}$	7/8	2/3	1/3	1/8
	100 $\bar{m}$ Q	8750	6667	3333	1250
0	Load Fluctuation	100	100	100	100
	$\frac{\text{Fluctuation}}{100\,\bar{m}\,Q}$ %	1.143	1.5	3.0	8.0
1/4	Load Fluctuation	100	95	95	100
	$\frac{\text{Fluctuation}}{100\,\bar{m}\,Q}$ %	1.143	1.425	2.85	8.0
1/2	Load Fluctuation	72	63	63	72
	$\frac{\text{Fluctuation}}{100\,\bar{m}\,Q}$ %	0.823	0.945	1.89	5.76
1	Load Fluctuation	15	14	14	15
	$\frac{\text{Fluctuation}}{100\,\bar{m}\,Q}$ %	0.171	0.21	0.42	1.2
1½	Load Fluctuation	0.8	0.7	0.7	0.8
	$\frac{\text{Fluctuation}}{100\,\bar{m}\,Q}$ %	0.01	0.01	0.02	0.06

TABLE 4.7

4.5.2.1 Numerical Integration of Probability Density Fluctuation.

For $\bar{m}$ Q non-integral and $\frac{3\sigma}{\alpha} = 1.5$, the load fluctuation is too small to be read with accuracy from the probability curve of Fig. 4.2.

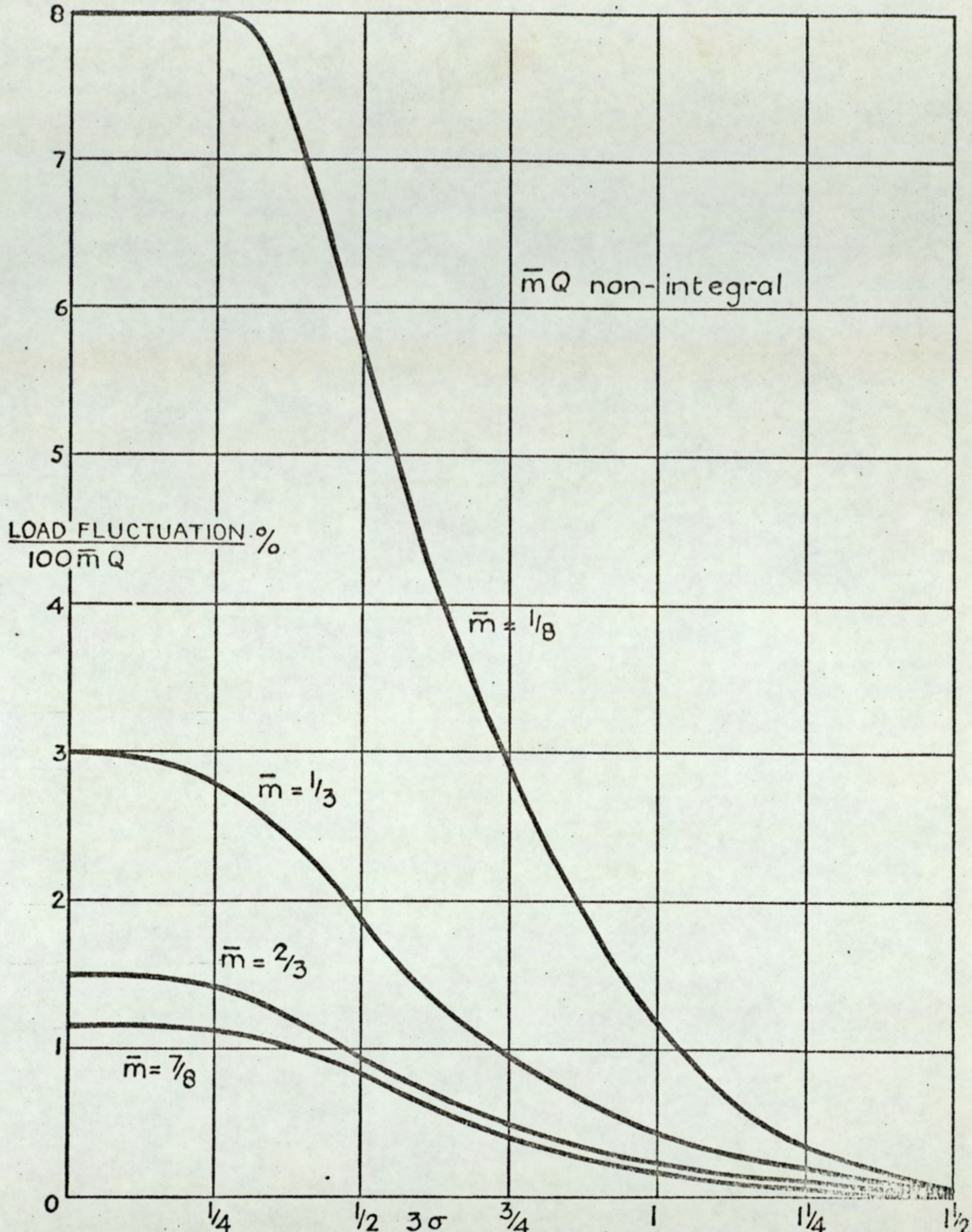


FIG. 4.9 PERCENTAGE LOAD FLUCTUATION AS A FUNCTION OF SWITCHING SPREAD.

For the sampled values of time over the 60-second period α, the distributions due to the processes being switched have been summed by digital computer for Q = 100 with 100 processes per block, and $\bar{m} = \frac{1}{8}, \frac{1}{3}, \frac{2}{3}, \frac{7}{8}$.

The probability values at the sampled points were obtained by numerical integration of the probability density function (Equation 4.3, section 4.3), between $t = -3\sigma$ (where the probability was assumed zero), and the point in question. Having obtained the probability ordinates for the first process being switched, the separate distributions were summed with the appropriate time displacements.

The computer program (NUMINT), flowchart and input data are shown in Appendix A.1.

4.6 CONTROL STRATEGY TO MINIMISE MAXIMUM DEMAND AND MAXIMISE PLANT UTILISATION.

For $\bar{m}$ Q integral, the total load is constant for a normal incidence of processes provided that one adheres to the switching policy described. For $\bar{m}$ Q non-integral, the worst case will be a two-level switching, with $\frac{3\sigma}{\alpha} = 0$, corresponding to the conditions shown in section 4.3.1.

Since the maximum demand charge is assessed on the greatest value of successive half-hourly integrated demands in a given accounting period, no advantage may be gained in attempting to position the overall load pattern relative to the monitoring intervals.

If it is desired to decrease the demand, power reduction may be applied equally to all processes. Alternatively, individual processes or complete blocks of power may be taken temporarily out of commission; upon re-introduction into service, the processes comprising any block need be switched on only nominally at the mean switching position allocated to the block.

A computer may be incorporated as an on-line control, whereby the integrated demand is inhibited from exceeding a nominated level.

The control strategy will be basically similar to that described in section 3.6.3. Control decisions are made at a number of points within the integrating period, when the energy is compared with that consumed due to a constant load equal to the set demand.

As before, the target energy is chosen to accommodate nominated variations of power.

4.7 SUMMARY.

In order to render the problem of demand control tractable to analysis, all loads are allocated the same $\bar{m}$ and the same cycle time t_q. This approximation will be realistic only when the processes are similar and are subject to a common environment.

This strategy may then be regarded as an extension of that described in Chapter 3 to the case where precise switching of individual processes is not possible.

Analysis of the load patterns arising out of demand control for this class of loads yields the following results:

Let the total number of similar on-off processes be split into Q equal blocks, and the mid-distribution points for each block be staggered sequentially by $\alpha = \frac{t_q}{Q}$, where t_q is the process cycle time. If the members comprising a block are distributed normally about their mean position and the distributions are the same in respect of switching on and switching off, then:

1. <u>For $\bar{m}Q$ integral.</u>

   The total load is constant irrespective of the distribution spread.

2. <u>For $\bar{m}Q$ non-integral.</u>

   The load fluctuation can be symmetric or asymmetric about the mean value. An increased spread in the distribution of the switching incidence results in an improvement of load factor, the effect becoming more pronounced for small values of $\bar{m}$.

Thus when the number of processes to be controlled is large, precise switching of individual loads is unnecessary provided that a measure of control is maintained over the average position of the separate blocks.

CHAPTER 5.

DEMAND DISTRIBUTION FOR THERMOSTATICALLY-CONTROLLED TWO-POSITION PROCESS ENSEMBLES.

5.1 INTRODUCTION.

An important class of processes contributing to the demand on the power system is that where the incidence of load is thermostatically-controlled, e.g. for space heating applications.

The duty cycle will be dependent upon the ambient conditions and external disturbances, so that a range of values of $\overline{m}$ must be considered.

A mathematical model is formulated which is representative of an ensemble of similar two-position processes operating under dynamic equilibrium cycling (d.e.c.) conditions.

A statistical analysis is performed to establish a confidence level that a given value of indicated demand will not be exceeded. This is used to predict the frequency of the load correction required to restrict the maximum demand penalty to a specified amount.

As a result of the analysis, a system design requirement is established for minimising the variance of the demand.

5.2 THERMOSTATICALLY-CONTROLLED THERMAL PROCESSES.

Recent developments in continuous and discontinuous control of electrothermal processes are described in the literature [17, 18, 19]. However, it is necessary briefly to introduce some of the more important performance characteristics, in that they affect the power and, in consequence, the electrical demand.

It has been shown[18] that many thermal processes may be represented by an electrical analogue consisting of at least one time constant and a transit delay. Fig. 5.1, for example, presents an analogue of one heater within a room subjected to two disturbances. G represents the gain of the heating apparatus in ^oC., and the manipulated variable m is a numeric representing the heater power dissipation (m = 1 with full rated dissipation and m = 0 with heater disconnected).

Control is achieved conventionally by sensing the temperature at the "location of particular interest" (l.p.i.), the thermostat closing the loop between the controlled variable θ and the manipulated variable m.

A block diagram representing the temperature control of an electrically-heated room by a thermostat with secondary-feedback deadspace compensation is shown in Fig. 5.2 (the parameters being defined in the "List of Symbols"). Fig. 5.3 gives a time-domain display of θ (t) and m(t) during the d.e.c. process.

In dynamic equilibrium cycling the controlled variable alternates between the two exponential trajectories

$$\theta(t)\Big|_{m=+1} = \theta_u + F.G \left\{ 1 - \exp\left(-\frac{t}{T_A}\right) \right\} \tag{5.1}$$

and

$$\theta(t)\Big|_{m=o} = \theta_u + F.G \left\{ \exp\left(-\frac{t}{T_P}\right) \right\}, \tag{5.2}$$

as it cycles between the limiting values θ_{max} and θ_{min}.

F is a numeric representing the thermal attenuation, the product F.G is the "runaway" temperature of the process when θ_u = o, and the mode-dependent thermal time contant T (m) has value T_A when heating and T_P when cooling.

Displays of the d.e.c. characteristics of thermostatically-controlled heating processes have commonly shown :[20, 21, 22]

(i) The variation of t_q as a function of the command θ_r for a steady-state disturbance θ_u.

(ii) The variation of t_q as a function of θ_u for a fixed command θ_r.

These two displays, which are mirror images of each other, are shown in Fig. 5.4. $\overline{m}$ is also displayed in each case as a function of the command. (The switching index m (t) is + 1 or 0, and $\overline{m} = \frac{1}{t_q}\int_o^{t_q} m(t)\,dt = \frac{t_o}{t_q}$).

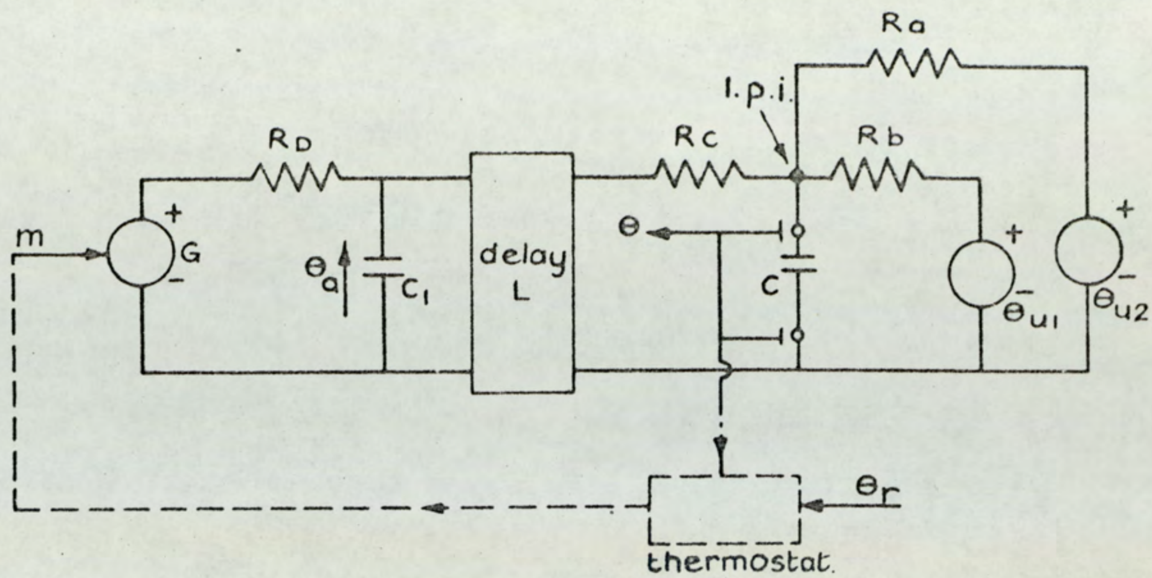
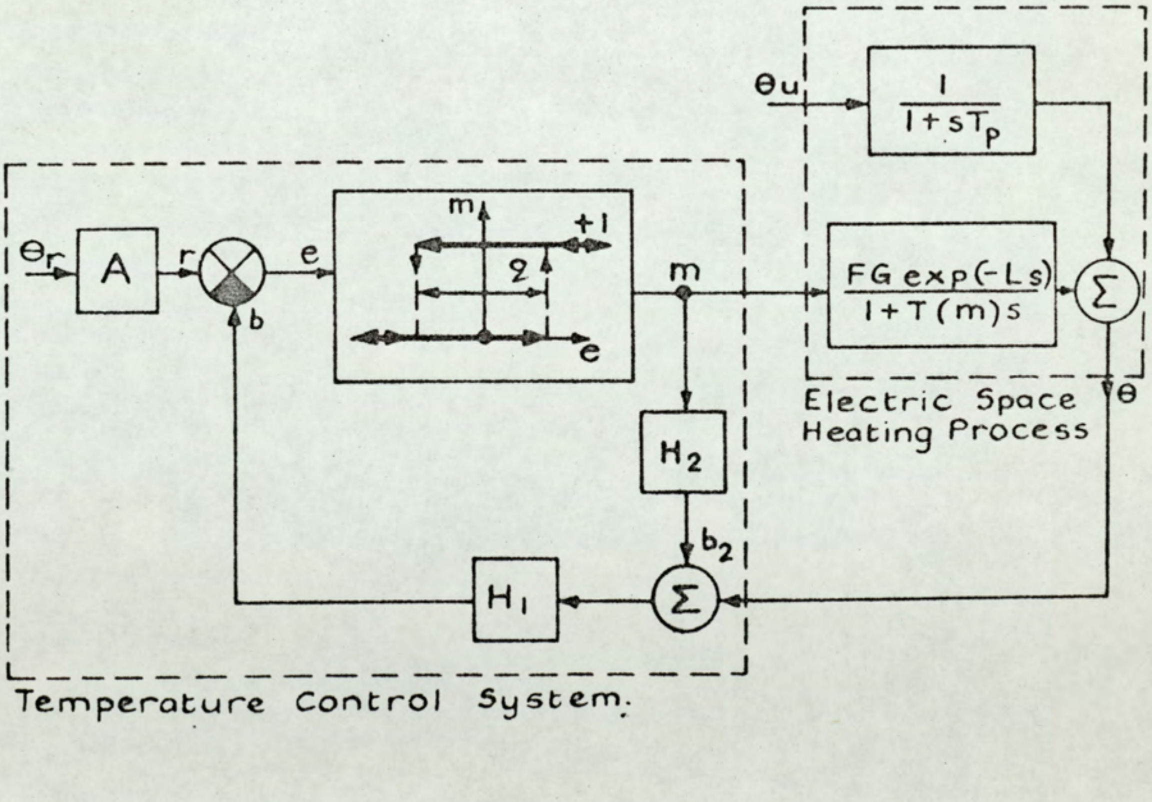


FIG.5.1 ELECTRICAL ANALOGUE - PROCESS CONSISTING OF HEATER WITHIN ROOM SUBJECTED TO TWO DISTURBANCES.



See List of Symbols for definition of terms.

FIG. 5.2 BLOCK DIAGRAM FOR THE THERMOSTATIC CONTROL OF ELECTRICALLY- HEATED ROOM.

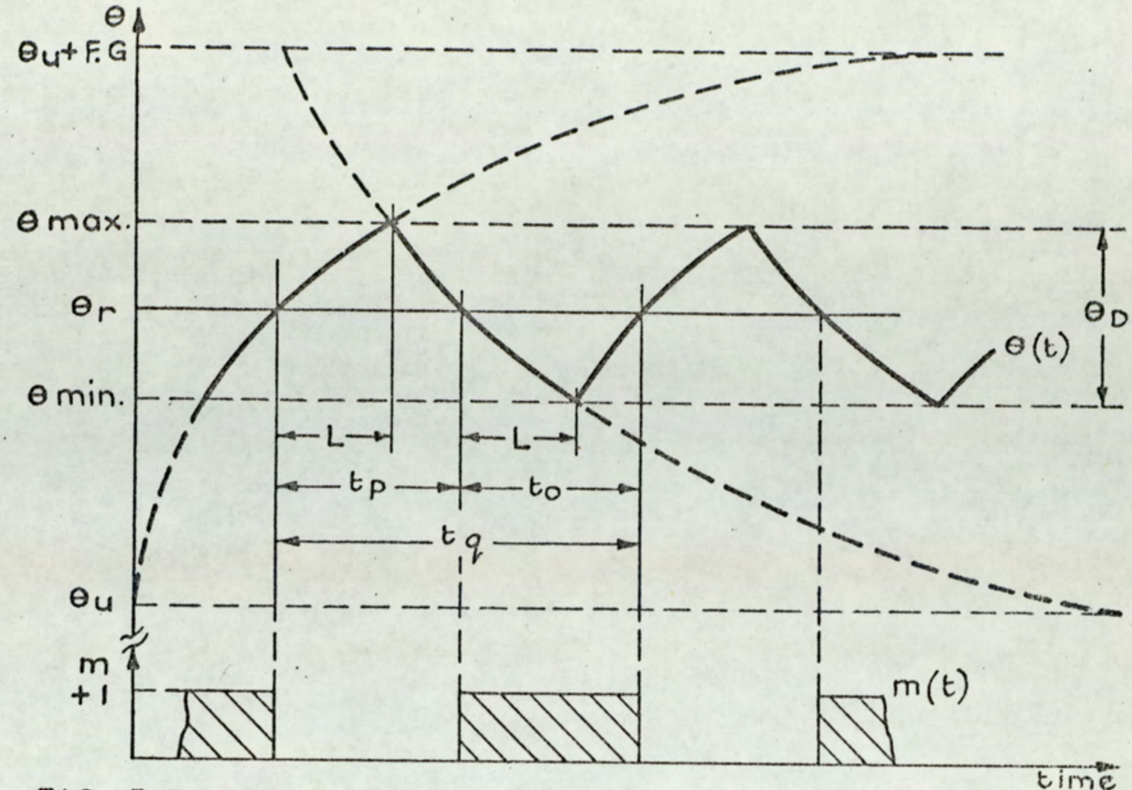


FIG. 5.3 TIME DOMAIN DISPLAY OF θ(t) AND m(t) DURING D.E.C.

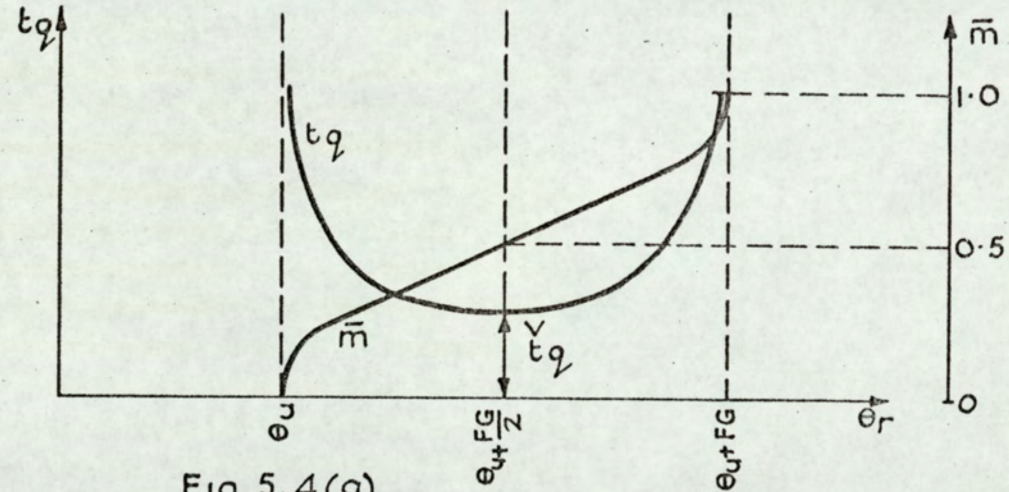


Fig. 5.4 (a)

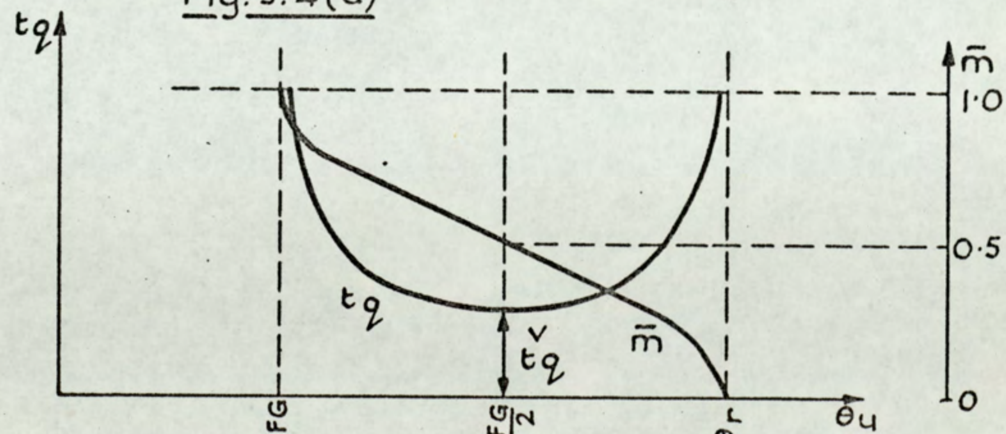


Fig. 5.4 (b)

FIG. 5.4 D.E.C. CHARACTERISTICS.

(a) STEADY STATE DISTURBANCE θ_u AND VARIABLE COMMAND θ_r

(b) FIXED COMMAND θ_r AND VARIABLE DISTURBANCE INPUT θ_u

It may be shown that, for mode-independent processes, the period t_q attains its minimum value $\check{t}_q$ when the on-and off-times are equal. In this condition the offset $\theta_r - \overline{\theta}$ is zero, where the d.e.c. mean temperature $\overline{\theta} = \frac{1}{t_q} \int_0^{t_q} \theta\ (t)\ dt$.

A convenient basis for comparing the d.e.c. characteristics of on-off electric space heating systems is to measure t_q at $\overline{m} = \frac{1}{2}$ and the amplitude of temperature oscillation $\theta_D = \theta_{max} - \theta_{min}$; also the offset which arises at other values of $\overline{m}$ (see Chapter 7).

5.3 STATISTICAL ANALYSIS OF LOAD.

In this section the load will be analysed for an ensemble of n similar two-position processes, each member being controlled by a separate thermostat.

After the cold-start, all the thermostats will settle down to dynamic equilibrium cycling. It will be assumed initially that all processes cycle with the same period t_q and a value of $\overline{m}$ equal to $\frac{1}{2}$. In practice, $\overline{m}$ and t_q will depend upon the room size and thermal capacity, the location and mechanical characteristics of the individual thermostat, and upon the external disturbance.

5.3.1 PROBABILITY OF MORE THAN A GIVEN NUMBER OF PROCESSES BEING ON AT ANY TIME.

The Binomial Probability Distribution P_x will give the chance of x processes out of an ensemble of size n being on at any time.

Thus

$$P_x = \binom{n}{x} p^x q^{n-x}, \tag{5.3}$$

where $x = 0, 1, - - - - - - n$

and $\binom{n}{x} = \frac{\lfloor n}{\lfloor n-x \lfloor x} = {}_nC_x$,

in which we have a sequence of n independent Bernouilli trials and compute the probability of getting x successes.

$P_{(success)} = p = \overline{m}.$

$P_{(failure)} = q$, where $p + q = 1$.

Thus for $p = q = \overline{m} = \frac{1}{2}$,

the chance of 1 process being on at any time is

$P_1 = {}_nC_1 \times (\frac{1}{2})^n = n \times 2^{-n}$, the chance of 2 processes being on at any time is

$$P_2 = {}_nC_2 \times (\tfrac{1}{2})^n = \frac{n\,(n-1)}{\lfloor 2} \times 2^{-n},$$

and the chance of n processes being on at any time is

$$P_n = 2^{-n}.$$

The probability that the number of processes on at any time exceeds a value x_1 is

$$P\left\{ x_1 \leqslant x \leqslant n \right\} = \sum_{x = x_1}^{n} \binom{n}{x} p^x q^{n-x}. \qquad (5.4)$$

For n small this expression may be evaluated directly without difficulty.

For n large, however, the term $\binom{n}{x}$ proves cumbersome. If np is of the order of 1, the Poisson theorem estimate may be used:

$$\binom{n}{x} p^x q^{n-x} \simeq e^{-np} \frac{(np)^x}{\lfloor x}$$

for x of the order of np.

For the cases to be considered, however, $npq \gg 1$, when the expression for $P\left\{ x_1 \leqslant x \leqslant n \right\}$ may be approximated by a normal distribution centred about a mean np with a variance npq, and

$$P\left\{ x_1 \leqslant x \leqslant n \right\} \simeq \operatorname{erf}\left(\frac{n - np}{\sqrt{npq}}\right) - \operatorname{erf}\left(\frac{x_1 - np}{\sqrt{npq}}\right). \qquad (5.5)$$

For $\overline{m} = \frac{1}{2}$ and n in the range 25 - 500, the probability $P\left\{ x_1 \leqslant x \leqslant n \right\}$ has been evaluated for $\frac{x_1}{n}$ = 55%, 60%, 65%, 70%. The results are plotted in Fig. 5.5 as a family of curves, each for constant $\frac{x_1}{n}$.

It should be noted that for n = 25 or less the normal approximation (5.5) loses validity, and the exact expression (5.4) must be evaluated.

Values of $\overline{m} = \frac{3}{4}$ and $\frac{1}{4}$ have also been considered. For $\overline{m} = \frac{3}{4}$, $\frac{x_1}{n}$ has been set at 80%, 85%, 90% and for $\overline{m} = \frac{1}{4}$, $\frac{x_1}{n}$ has been set at 30%, 35%, 40%. (Figs. 5.6 and 5.7).

It will be seen from Figs. 5.5 to 5.7 that, for a given $\overline{m}$, the ratio $\frac{x_1}{n}$ for a constant probability $P\left\{ x_1 \leqslant x \leqslant n \right\}$ becomes progressively smaller as n is increased. This is clearly illustrated in Fig. 5.8, where Fig. 5.5 is replotted as a family of curves, each

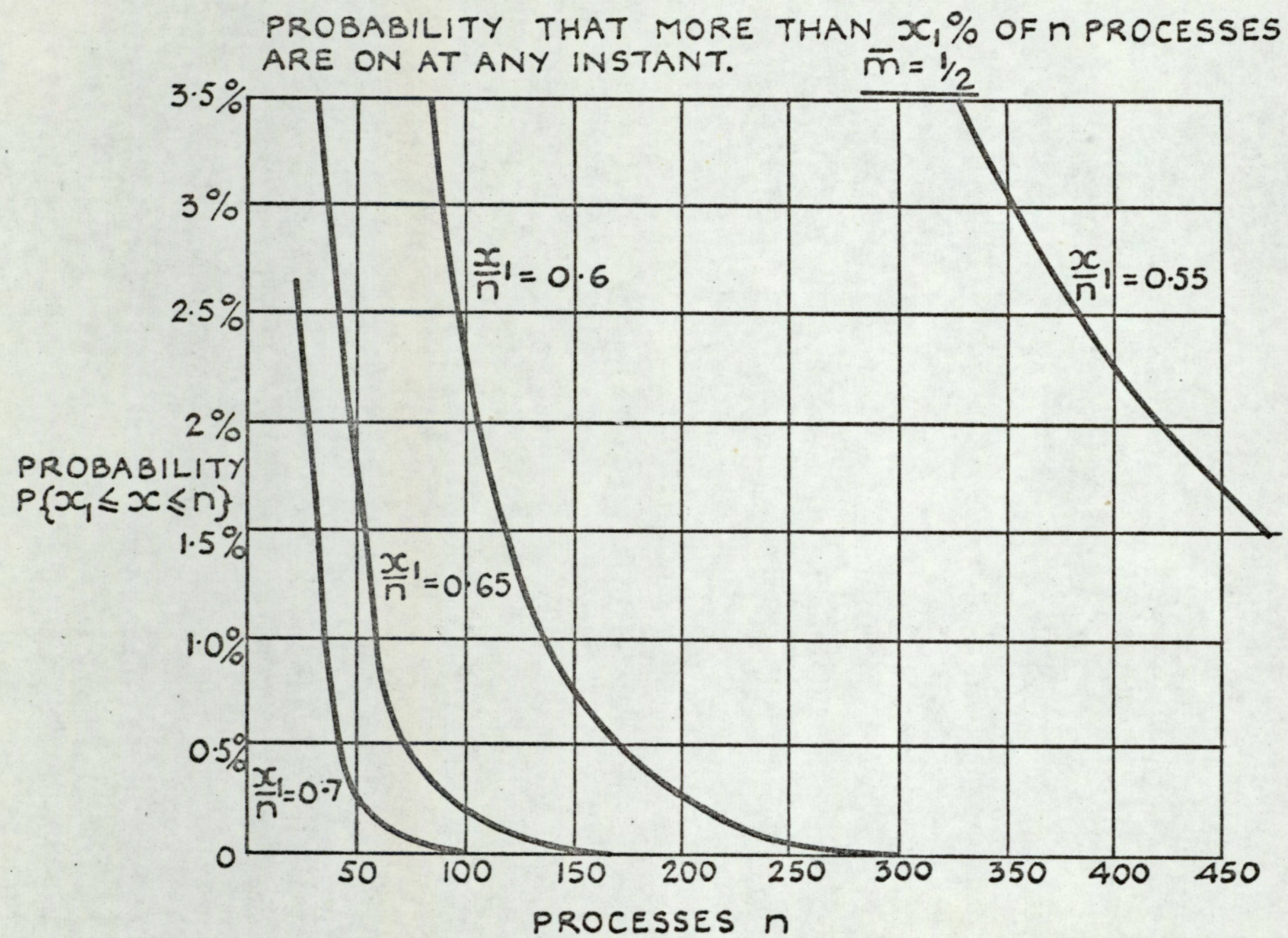


FIG.5.5 PROBABILITY THAT MORE THAN x_1% OF n PROCESSES ARE ON AT ANY INSTANT.

$(\bar{m} = \frac{1}{2})$.

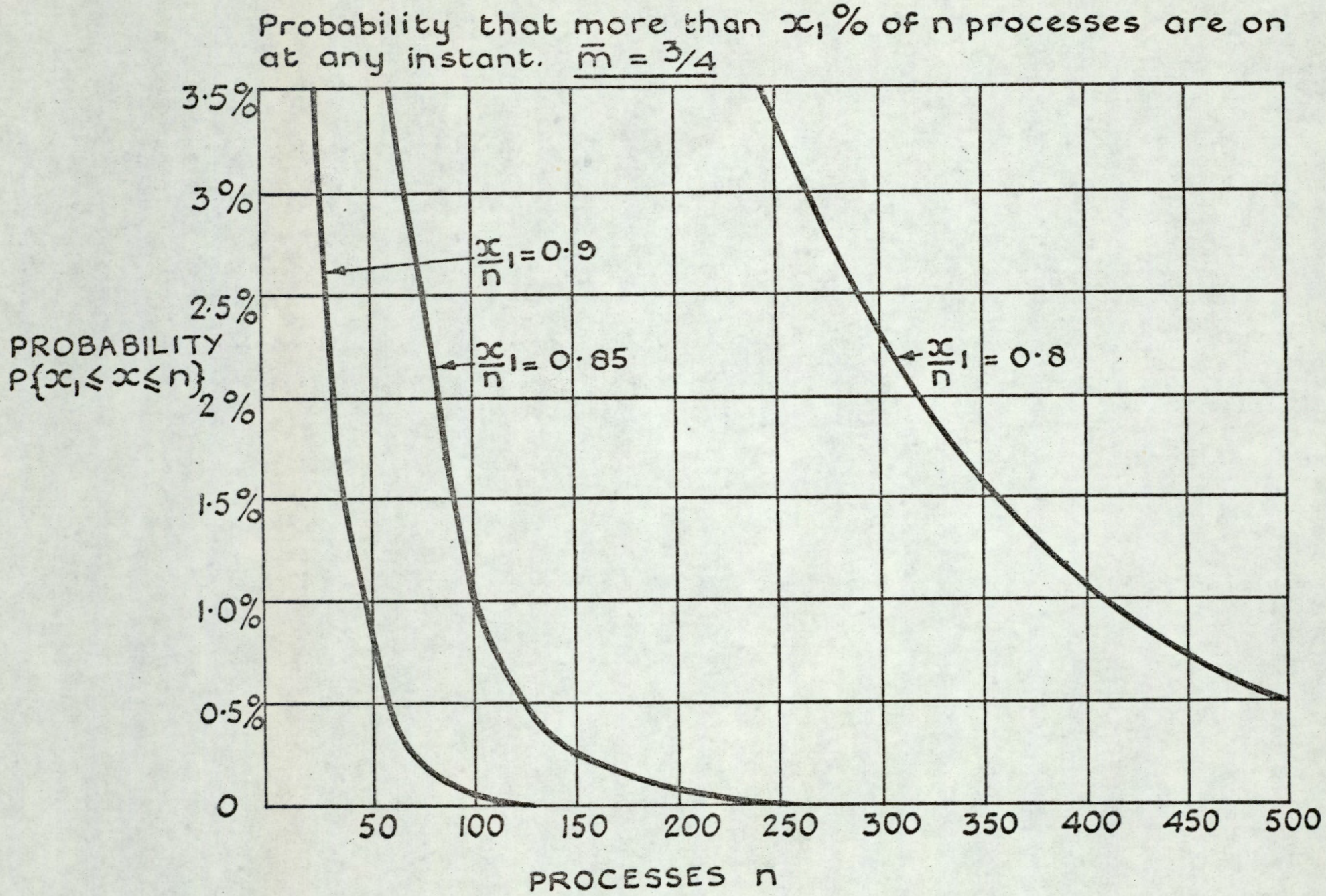


FIG. 5.6 PROBABILITY THAT MORE THAN x_1% OF n PROCESSES ARE ON AT ANY INSTANT. $(\bar{m} = \frac{3}{4})$.

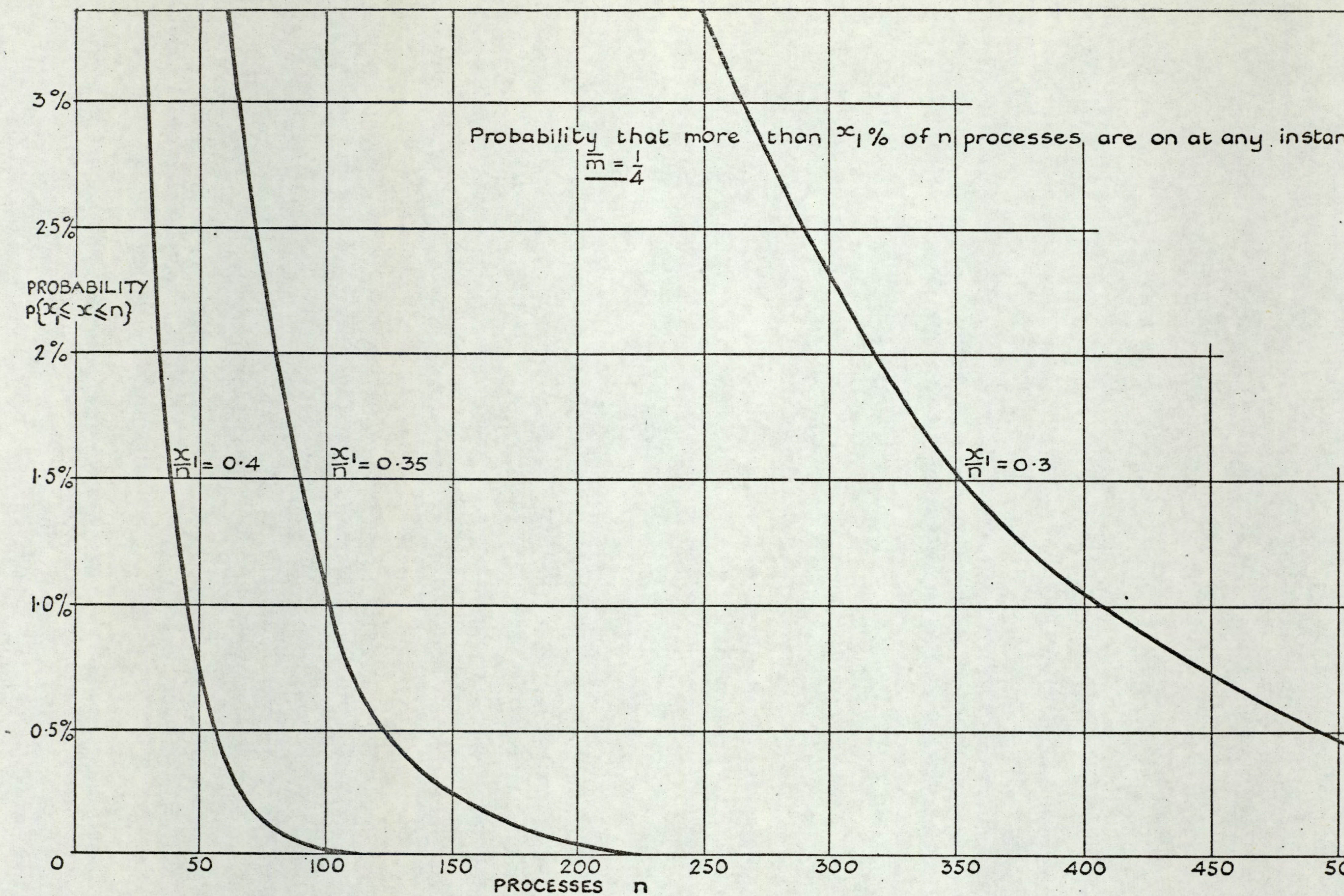


FIG. 5.7 PROBABILITY THAT MORE THAN x_1 % OF n PROCESSES ARE ON AT ANY INSTANT. ($\bar{m} = \frac{1}{4}$).

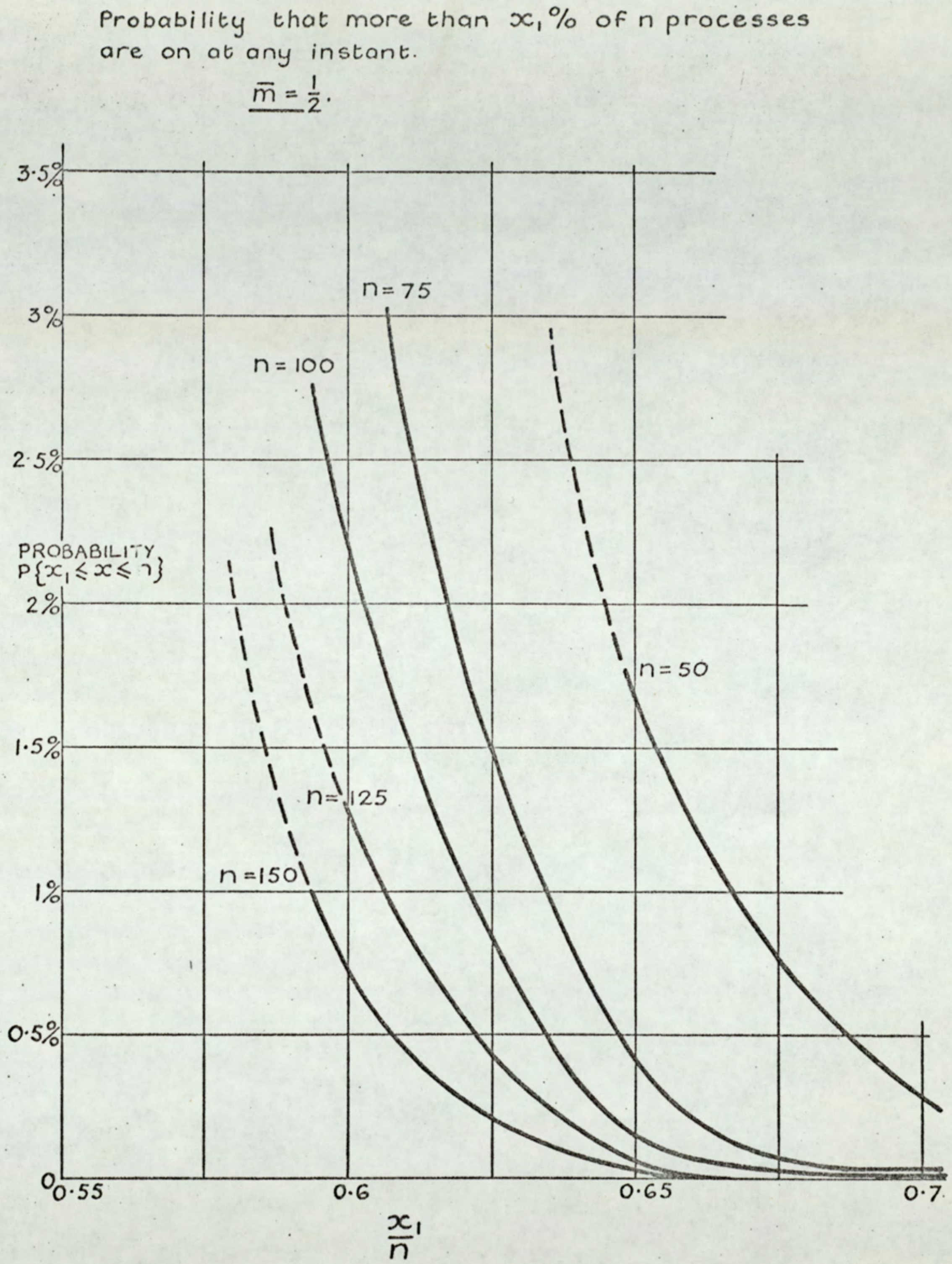


FIG.5.8 RE-PLOT OF FIG.5.5 FOR CONSTANT VALUES OF PARAMETER n.

of constant n.

Suppose that $P\left\{x_1 \leqslant x \leqslant n\right\} = 1\%$, say; then if this probability may be assumed the same at any instant, the condition $x > x_1$ would only occur for one half-hour in every 50 hours. However, the distribution of this half-hour is not known.

5.3.2 PROBABILITY OF MORE THAN A GIVEN NUMBER OF PROCESSES BEING ON CONTINUOUSLY FOR AN INTERVAL I.

For convenience of writing, the symbol t_q will be replaced by T for the remainder of this chapter and in chapter 6, and should not be confused with the symbol for time constant.

The overall load pattern for an ensemble of n equal-amplitude rectangular waveforms, each of period T, will also be cyclic with period T.

The probability that a process will be on continuously for time I will be zero for $I \geqslant \bar{m}\,T$, where $\bar{m} = \frac{t_o}{T}$.

To determine the probability that a process is on continuously for an interval $I \leqslant \bar{m}\,T$ we may consider its load pattern to be stationary, upon which is superimposed a window of width I (see Fig. 5.9). The limiting condition is that the right-hand edge of the window coincides with the point $t = \bar{m}\,T$, so that $\bar{m}\,T - I$ points are conducive to a continuous load. As there are a possible T points in the complete cycle, the probability $P\left[\text{process being on continuously for interval } I\right] = \frac{\bar{m}\,T - I}{T} = p'$. For n processes, the number x on during the interval I is Binomially distributed with probability

$$P\left[x_{on}\right] \doteq \binom{n}{x} (p')^x \, (1 - p')^{n-x}$$
$$= \binom{n}{x} \left(\frac{\bar{m}\,T - I}{T}\right)^x \left(\frac{I + T\,(1 - \bar{m})}{T}\right)^{n-x} \quad . \quad (5.6)$$

The probability that more than x_1 processes will be on continuously for a period I is

$$P\left\{x_1 \leqslant x \leqslant n\right\} = \sum_{x = x_1}^{n} \binom{n}{x} (p')^x \, (1 - p')^{n-x} \quad (5.7)$$

provided that $I \leqslant \bar{m}\,T$.

This can be solved for large n as in equation (5.5) provided $n\,p'\,(1 - p') \gg 1$.

Equation (5.7) is of the same form as (5.4), where p is replaced by p' and q by 1 - p'. The ordinates of Fig. 5.5 will therefore also give for $p' = \frac{1}{2}$ the probability that more than x_1 of

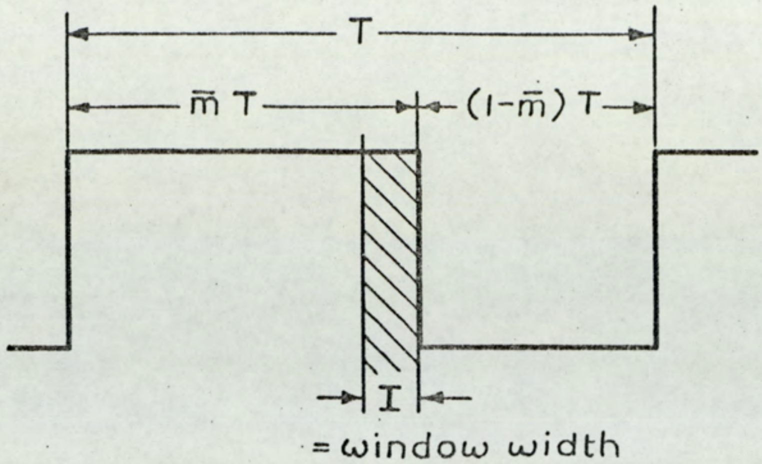
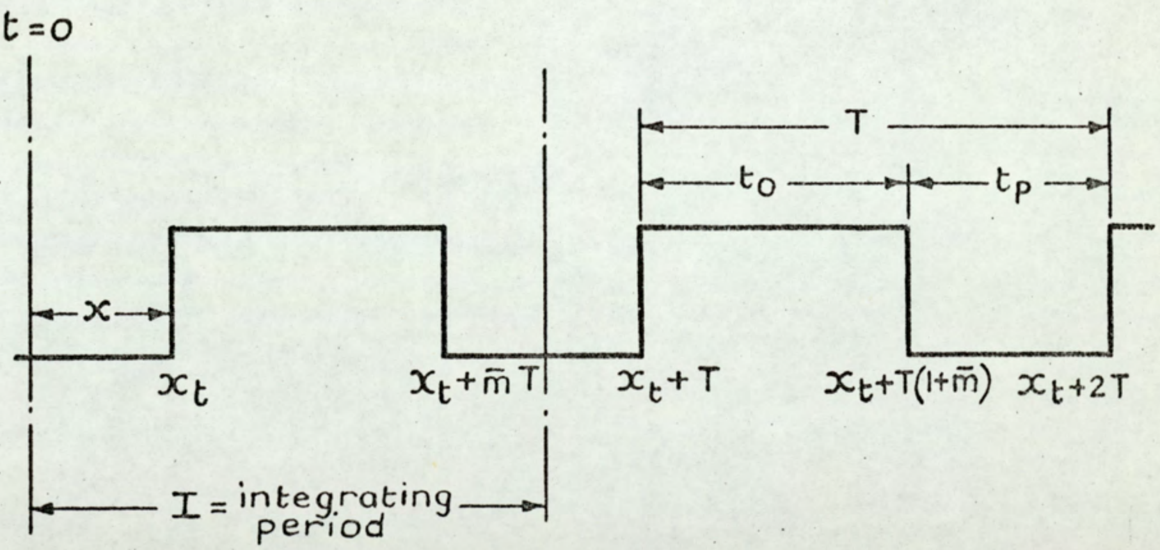


FIG. 5.9 WINDOW SUPERIMPOSED UPON LOAD PATTERN FOR ONE PROCESS.



$0 \leq x_t \leq T$, where T is period and $\bar{m} = \frac{t_o}{T}$.

FIG. 5.10 GENERALISED LOAD PATTERN FOR ONE PROCESS.

the total n processes will be on continuously for an interval I.

5.4 EVALUATION OF EXPECTED VALUE AND VARIANCE OF INDICATED DEMAND.

The expected value and the variance of the indicated demand D due to a single process will be determined; the analysis will then be extended to an ensemble of n independent processes, each having the same values of $\bar{m}$, T and P.

The power P may be normalised to have unity value.

If the integration is carried out from time t = 0 to I, then indicated demand

D = (area under power/time curve in interval 0 to I)/I.

A single process may be considered to start at time $t = x_t$, where x_t can have any value between 0 and T with equal probability (see Fig. 5.10). The process pattern may be expressed as:

$$f(t) = H(t - x_t) - H(t - (x_t + \bar{m} T)) + H(t - (x_t + T)) - H(t - (x_t + T(1 + \bar{m}))) + \text{ - - - } \quad (5.8)$$

where the Heaviside unit step function H (t) is defined in equation (3.3).

Fig. 5.11 (which is drawn for $x_t = 0$) shows a plot of D against t for $\bar{m} = \frac{1}{2}$ and $\frac{I}{T} = 1\frac{1}{2}$, where the integrating interval I commences at time t. This, of course, gives the same result as starting I from time zero and varying x_t.

Fig. 5.12 is drawn similarly for $\bar{m} = \frac{3}{4}$ and $\frac{1}{4}$, for the cases $\frac{I}{T} = \frac{1}{2}$ and 1.

The following analysis will deal, firstly, with a median value of $\bar{m} = \frac{1}{2}$, and then with the general case $0 \leqslant \bar{m} \leqslant 1$. The variance will be evaluated for a range of values of $\frac{I}{T}$ for the cases $\bar{m} = \frac{3}{4}$ and $\frac{1}{4}$, which for the purpose of analysis will be considered as reasonable limits on $\bar{m}$ for a well-designed heating system.

For convenience of writing, the variable x_t will be replaced by x for the remainder of this section.

The expected value of D will be denoted by E (D) and the variance by σ^2.

The order of calculations is as follows:

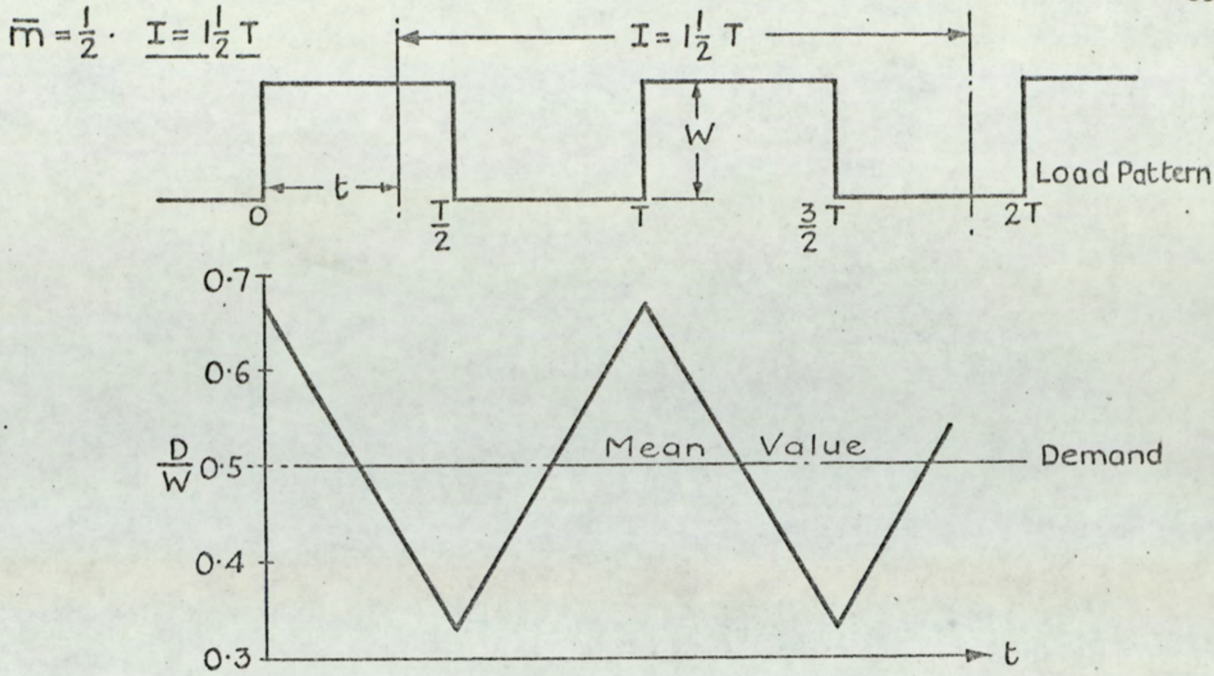


FIG. 5.11 PLOT OF INDICATED DEMAND AGAINST TIME.
($\bar{m} = \frac{1}{2}$).

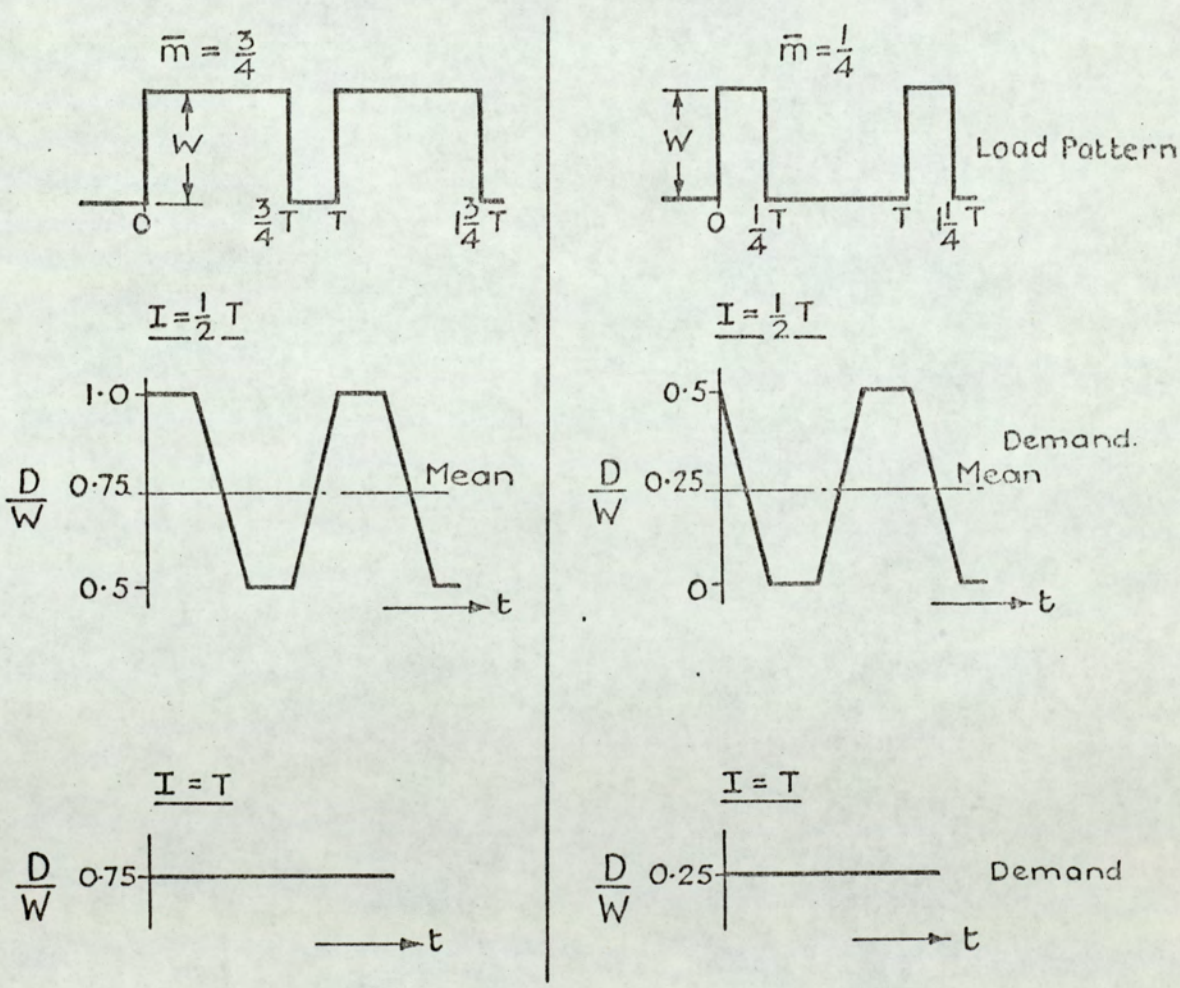


FIG. 5.12 PLOTS OF INDICATED DEMAND AGAINST TIME.
($\bar{m} = \frac{3}{4}$ and $\frac{1}{4}$).

1. Calculation of E (D) and σ^2 for $\bar{m} = \frac{1}{2}$ and $\frac{I}{T} \leqslant 1$.
2. Extension of results for $\bar{m} = \frac{1}{2}$ to cover the case $\frac{I}{T} \geqslant 1$.
3. Calculation of E (D) and σ^2 for $0 \leqslant \bar{m} \leqslant 1$ and $\frac{I}{T} \leqslant 1$.
4. Extension of results for $\bar{m}$ to cover the case $\frac{I}{T} \geqslant 1$.

5.4.1 EVALUATION OF E (D) AND VARIANCE σ^2 for $\bar{m} = \frac{1}{2}$.

Initially we will consider $\bar{m} = \frac{1}{2}$. Without loss of generality we can take the integrating interval I to be less than T, since for $I > T$ we can deduce the result from that for $I < T$. For example, if $I = 1.5\ T$ we can deduce the result from $I = 0.5\ T$ since we are only adding a complete period to I. If any one process is equally likely to start at any time x in the interval 0 to T, the probability of a value within the interval (x, x + dx) is $\frac{dx}{T}$ and the probability density $\frac{1}{T}$.

D is a function of x, and there are two cases to be considered, (i) when $I > \frac{T}{2}$ and (ii) when $I < \frac{T}{2}$.

Case (i). Suppose $I > \frac{T}{2}$, $\bar{m} = \frac{1}{2}$.

D as a function of x depends upon the position of x in relation to $\frac{T}{2}$, I, T (see Fig. 5.13).

If $0 \leqslant x \leqslant I - \frac{T}{2}$, then $D = \frac{T}{2I}$.

If $I - \frac{T}{2} \leqslant x \leqslant \frac{T}{2}$, then $D = \frac{I - x}{I}$.

If $\frac{T}{2} \leqslant x \leqslant I$, then $D = \dfrac{I - \frac{T}{2}}{I}$.

If $I \leqslant x \leqslant T$, then $D = \dfrac{x - \frac{T}{2}}{I}$.

(Since $I > \frac{T}{2}$ and $I < T$,

then $\dfrac{T}{2I} > \dfrac{I - \frac{T}{2}}{I}$).

D is plotted against x in Fig. 5.13 .

If the probability density of x is P (x) and that of D is P (D), then P (x) dx = P (D) dD (5.9)

and $P(D) = P(x) \cdot \dfrac{dx}{dD}$

$= \dfrac{1}{T} \cdot \dfrac{dx}{dD}$,

so that the probability distribution for D is

$$P(D)\,dD = \frac{dD}{T} \cdot \frac{dx}{dD} \qquad (5.10)$$

In order to obtain $\frac{dx}{dD}$, the curve of D against x is replotted in Fig. 5.13 with x as ordinate and D as abscissa.

Then as x goes from 0 to T, D goes from $\frac{T}{2I}$, down to $\frac{I - \frac{T}{2}}{I}$, and back again.

$\frac{dx}{dD}$ is either + I or - I, or $(I - \frac{T}{2})\ \delta(D)$ with the impulse function δ located at

$$D = \frac{T}{2I} \quad \text{or } 1 - \frac{T}{2I}.$$

Hence the probability distribution for D is

$$P(D)\,dD = \frac{dD}{T}\left[(I - \frac{T}{2})\,\delta(D - (1 - \frac{T}{2I})\,) + (I - \frac{T}{2})\ \delta(D - \frac{T}{2I}) + 2\,I \right].$$

A property of the impulse or delta function $\delta(D)$ is that

$$\int_{D_1}^{D_2} f(D)\,\delta(D - D_o)\,dD = f(D_o),$$
$$\text{if } D_1 < D_o < D_2 . \qquad (5.11)$$

(As a check that $\int P(D)\,dD = 1$:

$$\int P(D)\,dD = \frac{1}{T}\ (I - \frac{T}{2}) + \frac{1}{T}\ (I - \frac{T}{2}) + \frac{2I}{T}\ \left(\frac{T}{2I} - (1 - \frac{T}{2I})\right)$$

$$= \frac{2I}{T} - 1 + 1 - \frac{2I}{T} + 1 = +1\ ) .$$

The probability density distribution of D is shown in Fig. 5.13, i.e. P (D) against D.

It is now easy to find $E(D) = \overline{D}$ and $E(D - \overline{D})^2$.

Clearly from the symmetry of P (D),

$\overline{D}$ = the average of $1 - \frac{T}{2I}$ and $\frac{T}{2I}$,

$= \frac{1}{2}$.

Now variance $\sigma^2 = E(D - \overline{D})^2 = E(D^2) - \overline{D}^2$.

$$\therefore\ E(D^2) = \int P(D)\,D^2\,dD$$

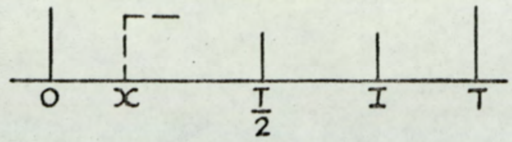
$$= \int \frac{D^2}{T}\ (I - \frac{T}{2})\ \delta\ (D - (1 - \frac{T}{2I}))\,dD + \int \frac{D^2\,dD}{T}\ (I - \frac{T}{2})\ (D - \frac{T}{2I})$$

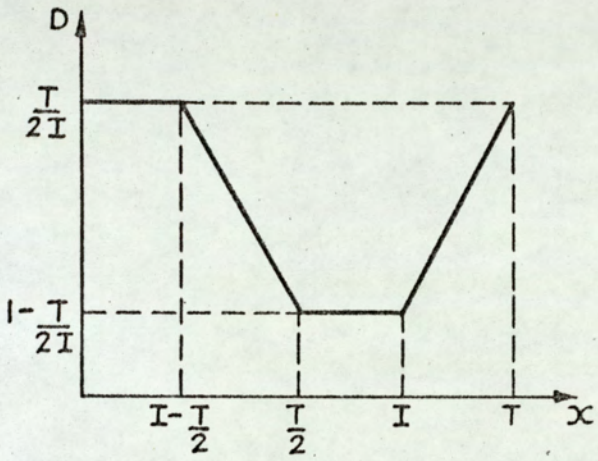
$$+ \int_{1 - \frac{T}{2I}}^{\frac{T}{2I}} \frac{2I}{T}\ D^2\,dD$$

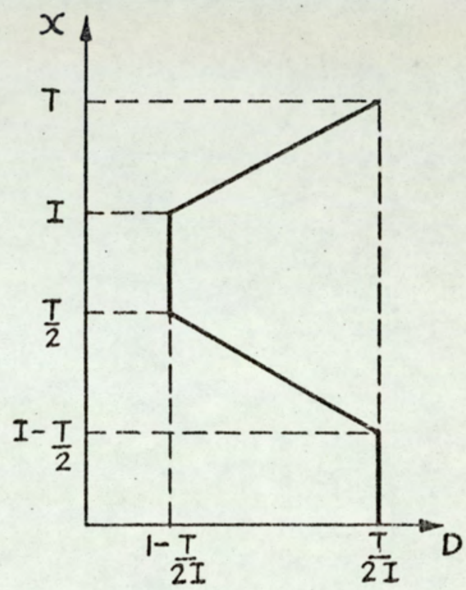
$$= \frac{(I - \frac{T}{2})}{T}\left(1 - \frac{T}{2I}\right)^2 + \frac{(I - \frac{T}{2})}{I}\left(\frac{T}{2I}\right)^2 + \frac{2I}{T}\left[\frac{D^3}{3}\right]_{1 - \frac{T}{2I}}^{\frac{T}{2I}} .$$

If we let $r = \frac{T}{2I}$ $(r < 1)$,

$\bar{m}=\frac{1}{2}$. Case (i). $I > \frac{T}{2}$.







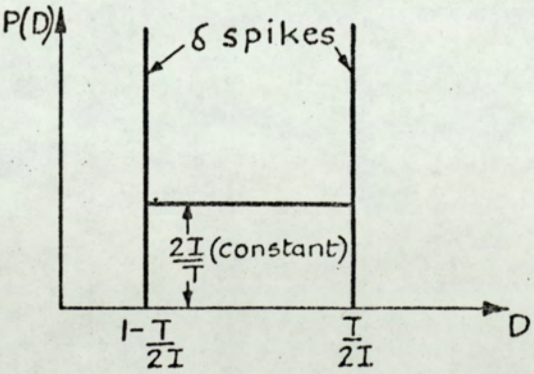


FIG. 5.13 PROBABILITY DENSITY DISTRIBUTION OF DEMAND

($\bar{m}=\frac{1}{2}$, $I > \frac{T}{2}$).

$$E(D^2) = \frac{I}{T}(1 - r)^3 + \frac{I}{T}(1 - r)\,r^2 + \frac{2I}{3T}\left(r^3 - (1 - r)^3\right)$$

$$= \frac{1}{6r}\left(1 - 3r + 6r^2 - 2r^3\right).$$

$$\therefore \quad \sigma^2 = \frac{1}{6r}\left(1 - 3r + 6r^2 - 2r^3\right) - \tfrac{1}{4}$$

$$= \underline{\frac{1}{12r}\,(2 - r)\,(2r - 1)^2} \quad . \qquad (5.12)$$

(As some check, when $I = T$, $r = \frac{1}{2}$ and $\sigma^2 = 0$).

<u>Case (ii)</u>.

Still for $\overline{m} = \frac{1}{2}$, $I < \frac{T}{2}$ is the final case. The treatment is similar to, but easier than, Case (i) (see Fig. 5.14).

If $0 \leqslant x \leqslant I$, then $D = \frac{I - x}{I}$.

If $I \leqslant x \leqslant \frac{T}{2}$, then $D = 0$.

If $\frac{T}{2} \leqslant x \leqslant I + \frac{T}{2}$, then $D = \frac{x - \frac{T}{2}}{I}$.

If $I + \frac{T}{2} \leqslant x \leqslant T$, then $D = 1$.

The curves of D against x and x against D are plotted in Fig. 5.14.

Again as in case (i), $\frac{dx}{dD}$ is either $+ I$, $- I$, or $(\frac{T}{2} - I)\,\delta(D)$.

Hence probability distribution for D is

$$P(D)\,dD = \frac{dD}{T}\left[\left(\frac{T}{2} - I\right)\delta(D) + \left(\frac{T}{2} - I\right)\delta(D - 1) + 2I\right].$$

The probability density distribution is shown in Fig. 5.14, and clearly $\underline{\overline{D} = \frac{1}{2}}$. (5.13)

As before,
$$E(D^2) = \int P(D)\,D^2\,dD$$
$$= \int \frac{D^2}{T}\left(\frac{T}{2} - I\right)\delta(D)\,dD + \int \frac{D^2}{T}\,dD\left(\frac{T}{2} - I\right)\delta(D - 1)$$
$$+ \int_0^1 \frac{2I}{T}\,D^2\,dD$$
$$= 0 + \left(\frac{T}{2} - I\right)\frac{I}{T} + \frac{2I}{T}\cdot\frac{1}{3}$$
$$= \tfrac{1}{2} - \frac{I}{3T} \quad .$$

$$\therefore \quad \sigma^2 = \tfrac{1}{2} - \frac{I}{3T} - \overline{D}^2$$

$$= \underline{\tfrac{1}{4} - \frac{1}{6r}}, \text{ where } r = \frac{T}{2I} \quad (r > 1). \qquad (5.14)$$

(As a check, when $I = 0$, $r = \infty$ and $\sigma^2 = \frac{1}{4}$.

This checks with the Binomial Distribution

$\bar{m} = \frac{1}{2}$. Case (ii). $I < \frac{T}{2}$.

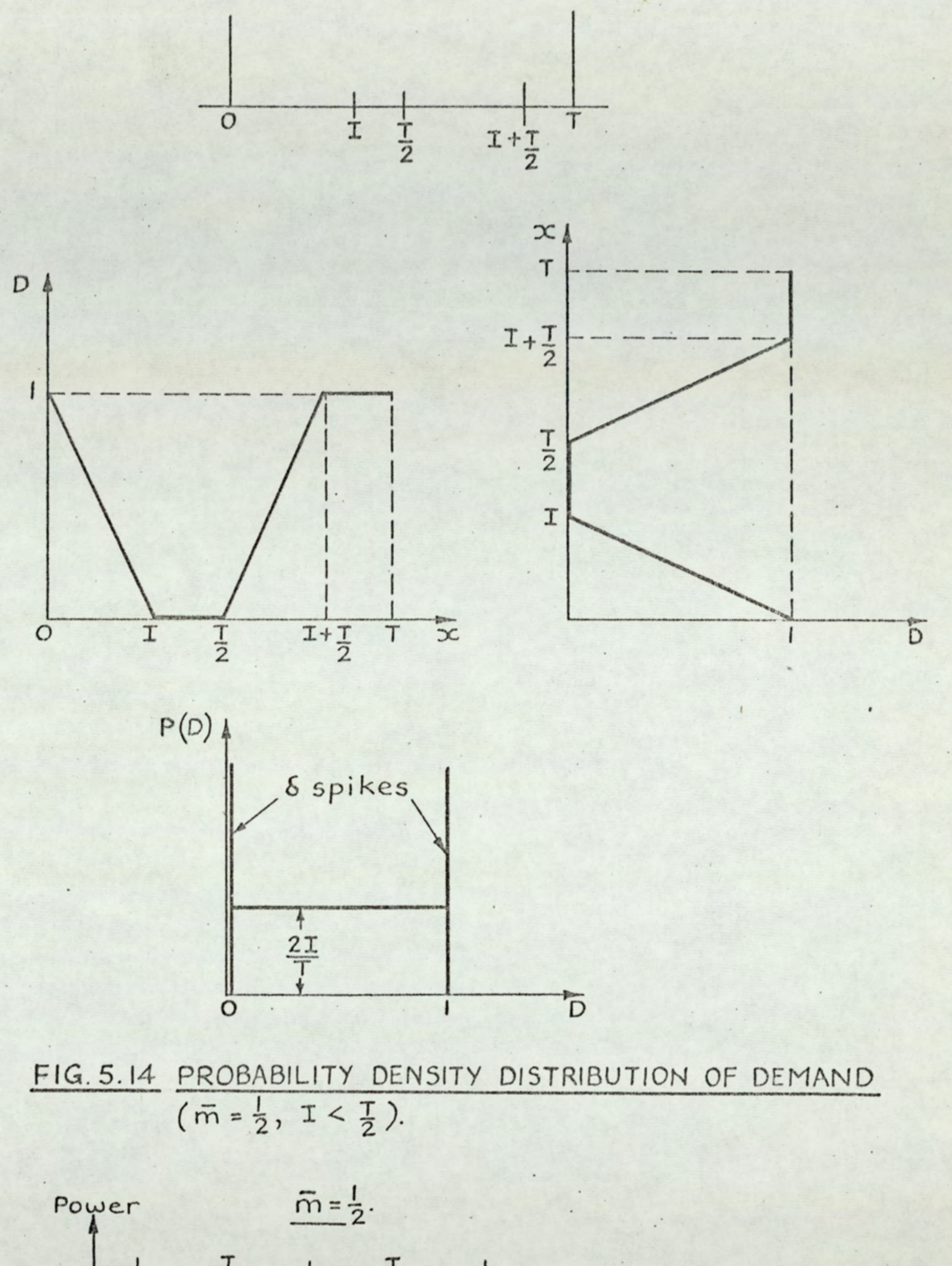


FIG. 5.14 PROBABILITY DENSITY DISTRIBUTION OF DEMAND ($\bar{m} = \frac{1}{2}$, $I < \frac{T}{2}$).

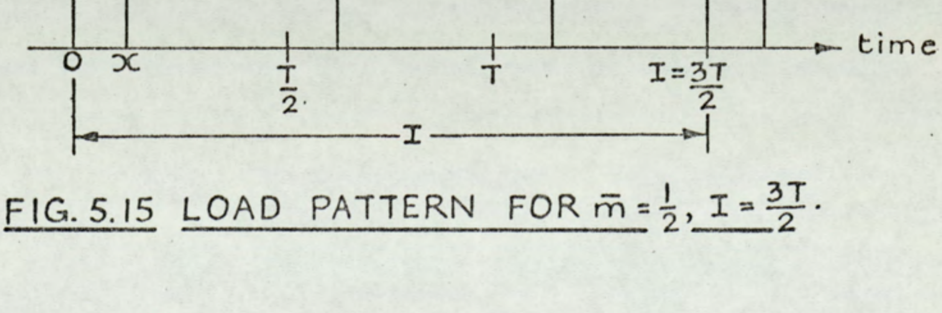


FIG. 5.15 LOAD PATTERN FOR $\bar{m} = \frac{1}{2}$, $I = \frac{3T}{2}$.

$\sigma = \sqrt{pq}$ with $p = q = \frac{1}{2}$;

when $I = \frac{T}{2}$, $r = 1$, and both equations (5.12) and (5.14) give $\sigma^2 = \frac{1}{12}$).

5.4.1.1 EXTENSION OF ANALYSIS FOR $\overline{m} = \frac{1}{2}$ and $\frac{I}{T} \geqslant 1$

We may now extend the results for $\frac{I}{T} < 1$ to the case when $\frac{I}{T} \geqslant 1$.

Let us examine the case where $I = \frac{3T}{2}$ (see Fig. 5.15).

This is similar to the case where $I = \frac{T}{2}$ except that the power is on for an extra time $\frac{T}{2}$.

If D is the demand when $I = \frac{T}{2}$,

then the demand when $I = \frac{3T}{2}$ is given by $\dfrac{D \times \frac{T}{2} + \frac{T}{2}}{\frac{3T}{2}} = \frac{D+1}{3}$, so that we must find the mean and variance of $\frac{D+1}{3}$.

The mean is $E\left(\frac{D}{3} + \frac{1}{3}\right) = \frac{1}{3} + \frac{\overline{D}}{3} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$.

The variance of $\frac{D}{3} + \frac{1}{3}$ is clearly the same as the variance of $\frac{D}{3}$, which is $\frac{1}{9}\sigma^2$.

Now when $I = \frac{T}{2}$, using either equation (5.12) or (5.14), $r = 1$ and $\sigma^2 = \frac{1}{12}$, so that variance in the case when $I = \frac{3T}{2}$ is $\frac{\sigma^2}{9} = \frac{1}{108}$.

In general,

if D_1 is the demand with an integrating interval I_1, where $0 \leqslant I_1 \leqslant T$, and D_2 is the demand with an integrating interval $I_1 + T$, then

$$D_2 = \frac{D_1 \times I_1 + \frac{1}{2}T}{I_1 + T}, \tag{5.15}$$

and variance of D_2 = variance of

$$\left(\frac{I_1}{I_1 + T}\right) D_1 = \left(\frac{I_1}{I_1 + T}\right)^2 \sigma^2_{I_1}, \tag{5.16}$$

where $\sigma^2_{I_1}$ is the variance of D_1.

All possible cases have now been dealt with for $\overline{m} = \frac{1}{2}$ for all values of $\frac{I}{T}$.

5.4.1.2 EVALUATION OF σ^2 for $\overline{m} = \frac{1}{2}$ and $0 \leqslant \frac{I}{T} \leqslant 1$.

We may now evaluate σ^2 for values of $\frac{I}{T}$ between 0 and 1 as follows:

$\bar{m} = \frac{1}{2} \quad \therefore \quad \bar{D} = \frac{1}{2}.$

Case (i). $I > \frac{T}{2}$. $r = \frac{T}{2I}$ (<1), $\sigma^2 = \frac{1}{12r}(2 - r)(2r - 1)^2$.

Case (ii). $I < \frac{T}{2}$. $r = \frac{T}{2I}$ (>1), $\sigma^2 = \frac{1}{4} - \frac{1}{6r}$.

$\frac{I}{T}$	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
r	$\frac{1}{2}$	$\frac{5}{9}$	$\frac{5}{8}$	$\frac{5}{7}$	$\frac{5}{6}$	1	$1\frac{1}{4}$	$1\frac{2}{3}$	$2\frac{1}{2}$	5	∞
σ^2	0	$\frac{13}{4860}$	$\frac{11}{960}$	$\frac{81}{2940}$	$\frac{7}{135}$	$\frac{1}{12}$	$\frac{7}{60}$	$\frac{3}{20}$	$\frac{11}{60}$	$\frac{13}{60}$	$\frac{1}{4}$
	0	0.00268	0.01145	0.0276	0.0519	0.0833	0.1167	0.15	0.1833	0.2165	0.25

$\underbrace{\hspace{6em}}_{\text{Case (i)}}\underbrace{\hspace{6em}}_{\text{Case (ii)}}$

TABLE 5.1

The variance is plotted in Fig 5.16 as a function of $\frac{I}{T}$. (Also shown are the results for $\bar{m} = \frac{1}{4}$ and $\frac{3}{4}$, which are evaluated in section 5.4.2.2).

5.4.1.3 EVALUATION OF E (D) AND VARIANCE FOR $\bar{m} = \frac{1}{2}$ and $\frac{I}{T} \geqslant 1$.

The values of E (D) and σ^2 for $\frac{I}{T} < 1$ may now be used to find the corresponding values for $\frac{I}{T} \geqslant 1$.

For example, for $\frac{I}{T} = 1.4$, equations (5.15) and (5.16) may be applied, together with the known result $\sigma^2_{I_1}$ for $\frac{I}{T} = 0.4$.

Thus demand $D_2 = \dfrac{D_1 \times 0.4T + 0.5T}{1.4T} = \frac{0.5}{1.4} + \frac{0.4}{1.4} D_1$.

Then mean $= E\left(\frac{0.5}{1.4} + \frac{0.4}{1.4} D_1\right) = \frac{0.5}{1.4} + \frac{0.4}{1.4} \bar{D}_1 = \frac{1}{2}$.

Variance = Variance of $\frac{0.4}{1.4} D_1$

$$= \left(\frac{0.4}{1.4}\right)^2 \sigma^2_{I_1} = \frac{4}{49} \times \frac{7}{60} = \frac{1}{105}.$$

Having obtained the variance of the demand for $\frac{I}{T} = 1.4$, the variance for $\frac{I}{T} = 2.4$ may be obtained in the same manner, and so on.

The mean of the demand for $\bar{m} = \frac{1}{2}$ is always $\frac{1}{2}$ whatever the value of $\frac{I}{T}$.

Taking the values of σ^2 from Table 5.1 for values of $\frac{I}{T}$

from 0 to 1, the variance has been calculated for values of $\frac{I}{T}$ up to 3.0.

The results are shown in Table 5.2, and the variance plotted in Fig. 5.17 as a function of $\frac{I}{T}$. (Also shown are the results for $\overline{m} = \frac{1}{4}$ and $\frac{3}{4}$, which are evaluated in Section 5.4.2.3).

$\frac{I}{T}$	1.1	1.2	1.3	1.4	1.5
Variance	$\frac{13}{7260}$	$\frac{11}{2160}$	$\frac{27}{3380}$	$\frac{1}{105}$	$\frac{1}{108}$
	0.00179	0.0051	0.008	0.00953	0.00927

$\frac{I}{T}$	1.6	1.7	1.8	1.9	2.0
Variance	$\frac{7}{960}$	$\frac{81}{17340}$	$\frac{11}{4860}$	$\frac{13}{21660}$	0
	0.00728	0.00466	0.00226	0.0006	0

$\frac{I}{T}$	2.1	2.2	2.3	2.4	2.5
Variance	$\frac{13}{2646}$	$\frac{1}{660}$	$\frac{27}{10580}$	$\frac{7}{2160}$	$\frac{1}{300}$
	0.000491	0.00152	0.00256	0.00325	0.00333

$\frac{I}{T}$	2.6	2.7	2.8	2.9	3.0
Variance	$\frac{7}{2535}$	$\frac{81}{43740}$	$\frac{11}{11760}$	$\frac{13}{50460}$	0
	0.00277	0.00186	0.000936	0.000257	0

Mean demand = ½ for all cases.

TABLE 5.2

5.4.2 Evaluation of E (D) and variance σ^2 for $0 \leqslant \overline{m} \leqslant 1$.

Having dealt with the particular case of $\overline{m} = \frac{1}{2}$, we may consider $0 \leqslant \overline{m} \leqslant 1$. (For those interested, the following detailed calculations are included to justify the results shown in Fig. 5.17).

FIG.5.16 VARIANCE OF DEMAND FOR $0 \leq \frac{I}{T} \leq 1.$

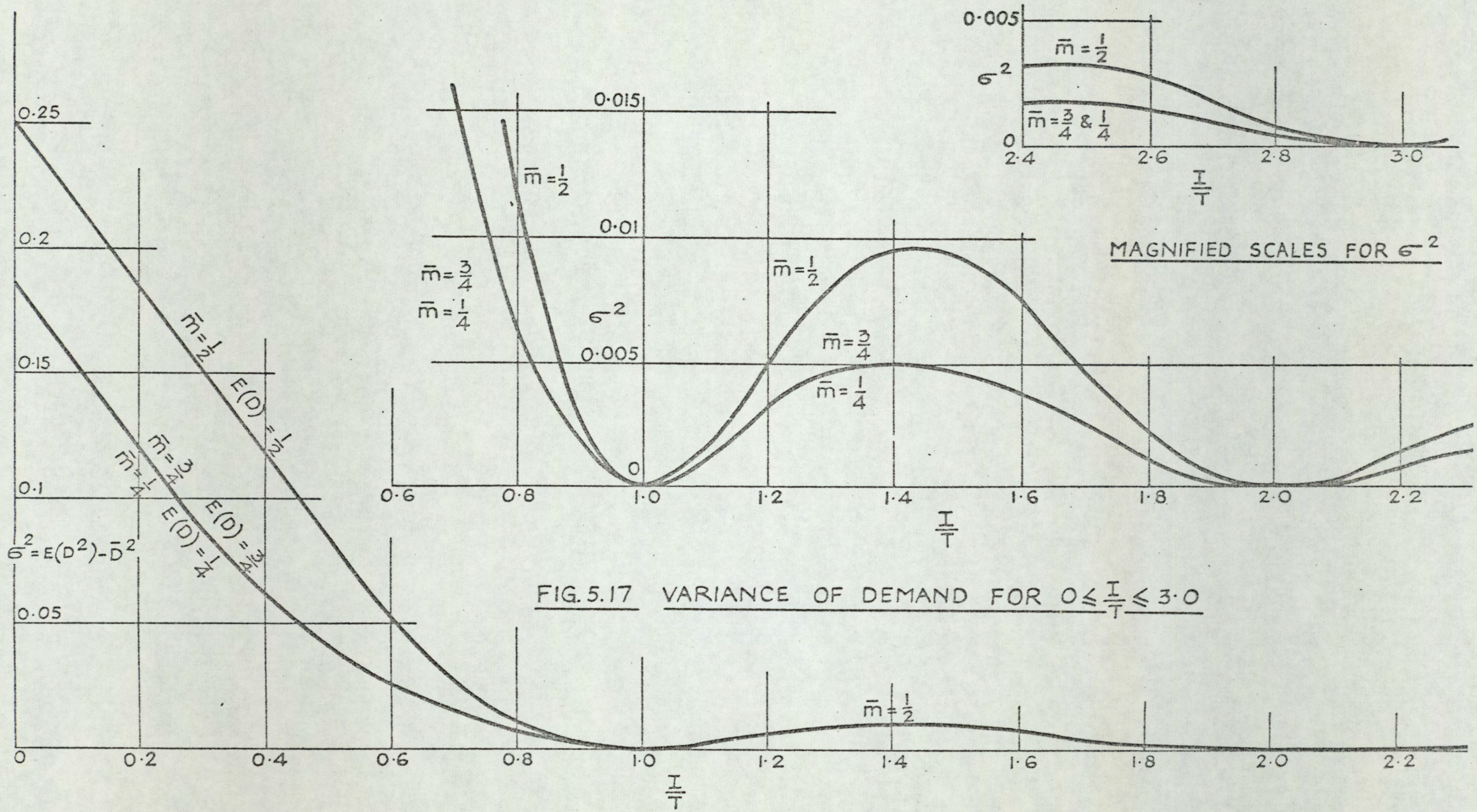


FIG. 5.17 VARIANCE OF DEMAND FOR $0 \leq \frac{I}{T} \leq 3 \cdot 0$

Again we can take the integrating interval I to be less than T, and the result for $I > T$ can be deduced from that for $I < T$.

D is a function of x, and we must consider both $\bar{m} > \frac{1}{2}$ and $\bar{m} < \frac{1}{2}$.

The treatment will be similar to that for $\bar{m} = \frac{1}{2}$, but there will now be a total of six cases to be considered:

$\underline{\bar{m} > \frac{1}{2}.}$

Case (i). $I > \bar{m}\,T$.

Case (ii). $I < (1 - \bar{m})\,T$.

Case (iii). $(1 - \bar{m})\,T < I < \bar{m}\,T$.

$\underline{\bar{m} < \frac{1}{2}.}$

Case (i). $I > (1 - \bar{m})\,T$.

Case (ii). $I < \bar{m}\,T$.

Case (iii). $\bar{m}\,T < I < (1 - \bar{m})\,T$.

$\underline{\bar{m} > \frac{1}{2}.}$ <u>Case (i).</u> $I > \bar{m}\,T$.

The position of x in relation to $\bar{m}\,T$, I, T is shown in Fig. 5.18.

If $0 \leqslant x \leqslant I - \bar{m}\,T$, then $D = \dfrac{\bar{m}\,T}{I}$.

If $I - \bar{m}\,T \leqslant x \leqslant (1 - \bar{m})\,T$, then $D = \dfrac{I - x}{I}$.

If $(1 - \bar{m})\,T \leqslant x \leqslant I$, then $D = \dfrac{I - (1 - \bar{m})\,T}{I} = 1 - (1 - \bar{m})\dfrac{T}{I}$.

If $I \leqslant x \leqslant T$, then $D = \dfrac{x - (1 - \bar{m})\,T}{I}$.

(Since $I < T$, then $\dfrac{\bar{m}\,T}{I} > \dfrac{I - (1 - \bar{m})\,T}{I}$).

The probability distribution for D is

$$P\,(D)\,dD = \frac{dD}{T}\left[\,(I - \bar{m}\,T)\;\delta\left(D - \frac{\bar{m}\,T}{I}\right) + (I - (1 - \bar{m})\,T)\,\delta\left(D - \left(1 - (1 - \bar{m})\frac{T}{I}\right)\right) + 2\,I\,\right].$$

Fig. 5.18 shows the probability density distribution, together with plots of D against x and x against D.

$$\text{Now } E\,(D) = \int P\,(D)\,D\;dD.$$

$$= \int \frac{D}{T}\;(I - \bar{m}\,T)\;\delta\left(D - \bar{m}\frac{T}{I}\right)dD$$

$$+ \int \frac{D}{T}\;(I - (1 - \bar{m})\,T)\;\delta\left(D - \left(1 - (1 - \bar{m})\frac{T}{I}\right)\right)dD$$

$$+ \int_{1 - (1 - \bar{m})\frac{T}{I}}^{\frac{\bar{m}\,T}{I}} 2\,\frac{I}{T}\;D\;dD$$

$\bar{m} > \frac{1}{2}$. Case (i). $I > \bar{m}T$.

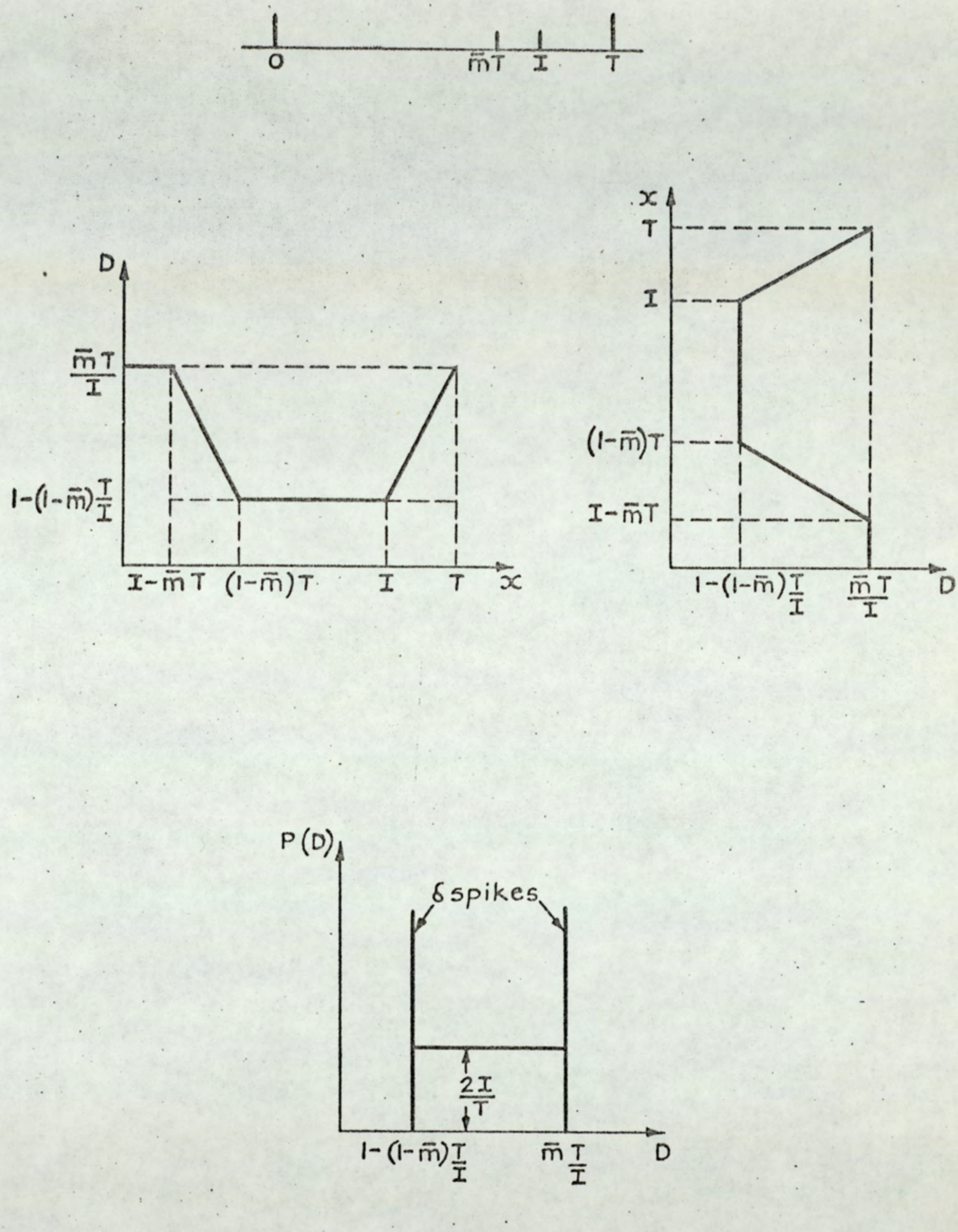


FIG. 5.18 PROBABILITY DENSITY DISTRIBUTION OF DEMAND.

($\bar{m} > \frac{1}{2}$, $I > \bar{m}T$).

$$= \frac{(I - \bar{m} T)}{T} \bar{m} \frac{T}{I} + \frac{(I - (1 - \bar{m}) T)}{T} (1 - (1 - \bar{m}) \frac{T}{I})$$

$$+ \frac{I}{T}\left[\left(\bar{m} \frac{T}{I}\right)^2 - (1 - (1 - \bar{m}) \frac{T}{I})^2\right].$$

If $r = \bar{m} \frac{T}{I}$ and $s = (1 - \bar{m}) \frac{T}{I}$,

then $E(D) = \frac{I}{T}(1 - r)r + \frac{I}{T}(1 - s)^2 + \frac{I}{T}(r^2 - (1 - s)^2)$

$$= \frac{I}{T}(r - r^2 + r^2) \quad = \underline{\bar{m}} \,. \tag{5.17}$$

Now $E(D^2) = \int P(D) D^2 \, dD$

$$= \int \frac{D^2}{T}(I - \bar{m} T)\, \delta(D - \bar{m} \frac{T}{I}) dD + \int \frac{D^2}{T}(I - (1 - \bar{m})T)$$

$$\delta(D - (1 - (1 - \bar{m}) \frac{T}{I})) dD$$

$$+ \int_{1 - (1 - \bar{m}) \frac{T}{I}}^{\frac{\bar{m} T}{I}} 2 \frac{I}{T} D^2 \, dD$$

$$= \frac{1}{T}\left[(I - \bar{m} T)\left(\bar{m} \frac{T}{I}\right)^2 + (I - (1 - \bar{m})T)(1 - (1 - \bar{m}) \frac{T}{I})^2\right.$$

$$\left. + \frac{2}{3} \frac{I}{T} \left[\left(\frac{\bar{m} T}{I}\right)^3 - (1 - (1 - \bar{m}) \frac{T}{I})^3\right]\right.$$

$$= \frac{\bar{m}}{3r}(3 r^2 - r^3 + (1 - s)^3).$$

$$\therefore \quad \sigma^2 = \frac{\bar{m}}{3r}(3r^2 - r^3 + (1 - s)^3) - \bar{m}^2$$

$$= \underline{\frac{\bar{m}}{3r}(3r^2 - r^3 + (1 - s)^3 - 3 \bar{m} r)} \tag{5.18}$$

(As a check, for $\bar{m} = \frac{1}{2}$, $\frac{I}{T} = 1$, $r = S = \frac{1}{2}$, and $\sigma^2 = 0$.

For $\bar{m} = \frac{1}{2}$, $\frac{I}{T} = \frac{1}{2}$, $r = S = 1$ and $\sigma^2 = \frac{1}{12}$.

For $\bar{m} = \frac{1}{2}$, $\frac{I}{T} = 0.8$, $r = S = \frac{5}{8}$ and $\sigma^2 = \frac{11}{960}$).

All results agree with those obtained from equation (5.12).

$\underline{\bar{m} > \frac{1}{2}.}$ Case (ii). $I < (1 - \bar{m}) T$.

The position of x in relation to $\bar{m}$ T, I, T is shown in Fig. 5.19.

If $0 \leqslant x \leqslant I$, then $D = \frac{I - x}{I}$.

If $I \leqslant x \leqslant (1 - \bar{m}) T$, then $D = 0$.

If $(1 - \bar{m}) T \leqslant x \leqslant I + (1 - \bar{m}) T$, then $D = \frac{x - (1 - \bar{m}) T}{I}$.

$\bar{m} > \frac{1}{2}$. Case (ii). $I < (1 - \bar{m})T$.

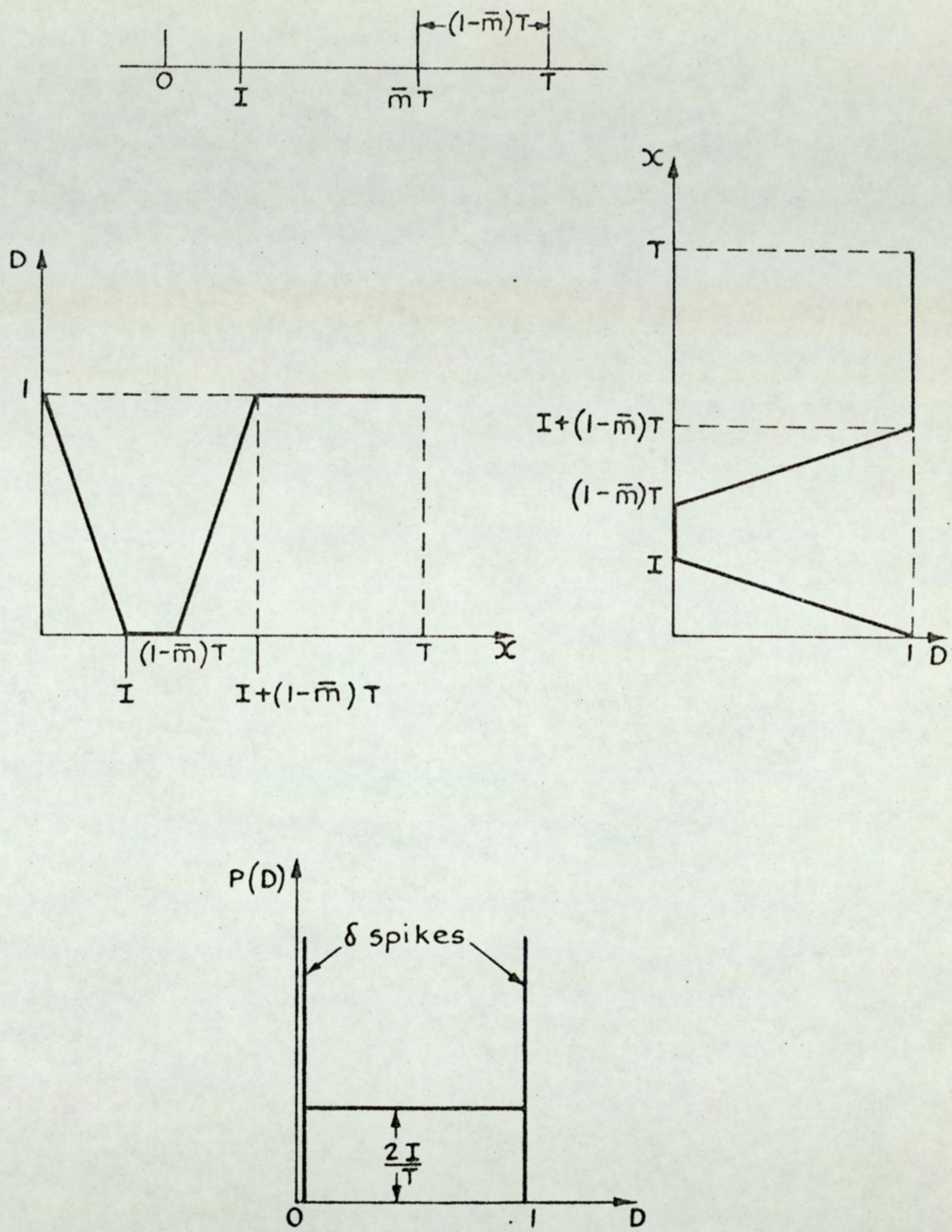


FIG. 5.19 PROBABILITY DENSITY DISTRIBUTION OF DEMAND.
($\bar{m} > \frac{1}{2}$, $I < (1-\bar{m})T$).

If $I + (1 - \bar{m}) T \leqslant x \leqslant T$, then $D = 1$.

The probability distribution for D is

$$P(D)\,dD = \frac{dD}{T}\left[((1 - \bar{m} T) - I)\,\delta(D - 0) + (T - (I + (1 - \bar{m}) T)\,\delta(D - 1) + 2I \right].$$

The probability density distribution and derivation are shown in Fig. 5.19.

$$E(D) = \int \frac{D}{T}\,((1 - \bar{m}) T - I))\,\delta(D)\,dD + \int \frac{D}{T}(T - (I + T - \bar{m} T)\,\delta(D - 1)\,dD + \int_0^1 2\frac{I}{T} D\,dD$$

$$= \frac{I}{T}\left[((1 - \bar{m}) T - I)\,0 + (\bar{m} T - I)\,1 + I \times 1^2\right]$$

$$= \underline{\bar{m}} \tag{5.19}$$

$$E(D^2) = \int \frac{D^2}{T}\,((1 - \bar{m} T) - I))\,\delta D\,dD + \int \frac{D^2}{T}\,(\bar{m} T - I)\,\delta(D - 1)\,dD + \int_0^1 2\frac{I}{T} D^2\,dD$$

$$= 0 + \frac{\bar{m} T - I}{T} + \frac{2}{3}\frac{I}{T} = \bar{m}\left(1 - \frac{1}{3r}\right)$$

$$\therefore \quad \sigma^2 = \bar{m}\left(1 - \frac{1}{3r}\right) - \bar{m}^2$$

$$= \underline{\bar{m}\left(1 - \bar{m} - \frac{1}{3r}\right)} \tag{5.20}$$

(As a check, for $\bar{m} = \frac{1}{2}$, $\frac{I}{T} = 0$, $r = \infty$ and $\sigma^2 = \frac{1}{4}$.

For $\bar{m} = \frac{1}{2}$, $\frac{I}{T} = \frac{1}{2}$, $r = 1$ and $\sigma^2 = \frac{1}{12}$).

$\underline{\bar{m} > \frac{1}{2}.}$ <u>Case (iii).</u> $(1 - \bar{m}) T < I < \bar{m} T$.

The position of x in relation to m T, I, T is shown in Fig.5.20.

If $0 \leqslant x \leqslant (1 - \bar{m}) T$, then $D = \frac{I - x}{I}$.

If $(1 - \bar{m}) T \leqslant x \leqslant I$, then $D = \frac{I - (1 - \bar{m}) T}{I}$.

If $I \leqslant x \leqslant I + (1 - \bar{m}) T$, then $D = \frac{x - (1 - \bar{m}) T}{I}$.

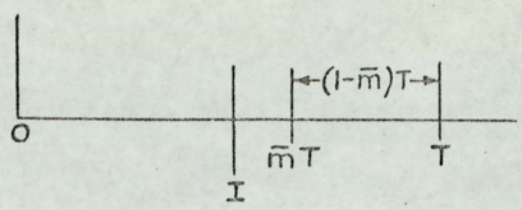
If $I + (1 - \bar{m}) T \leqslant x \leqslant T$, then $D = 1$.

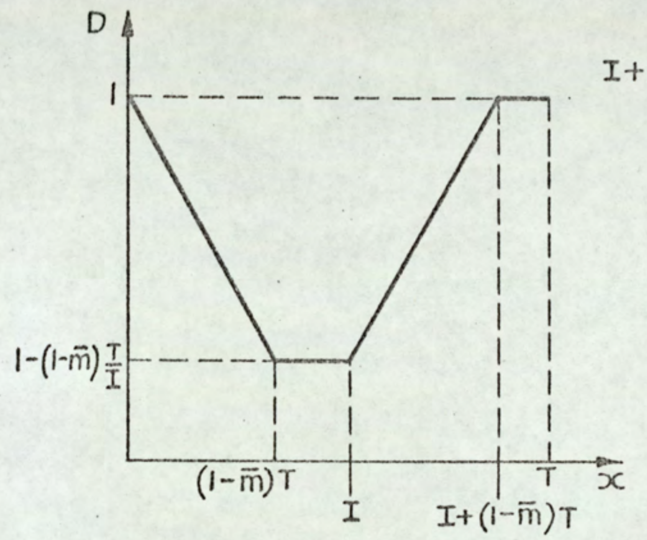
The probability distribution for D is

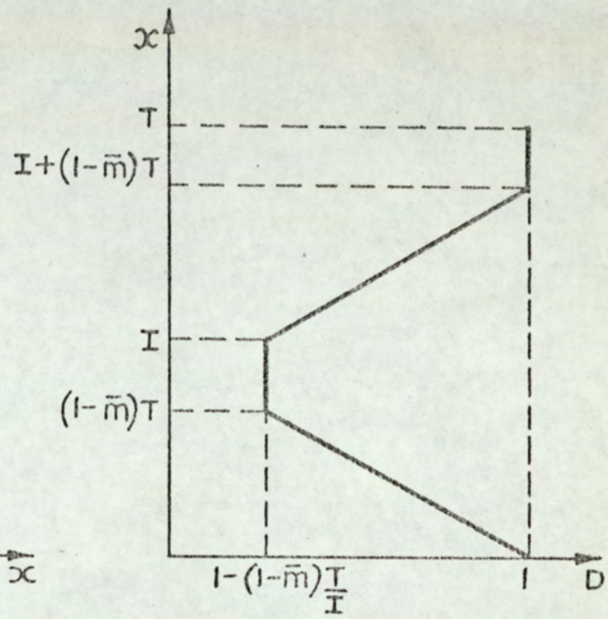
$$P(D)\,dD = \frac{dD}{T}\left[(I - (1 - \bar{m}) T\ \delta\left(D - \frac{(I - (1 - \bar{m}) T}{I}\right) + (T - (I + (1 - \bar{m}) T)\,\delta(D - 1) + 2I \right].$$

The probability density distribution and derivation are shown in Fig. 5.20.

$\underline{\bar{m} > \frac{1}{2}}$. $\underline{\text{Case (iii)} \ (1-\bar{m})T < I < \bar{m}T}$.







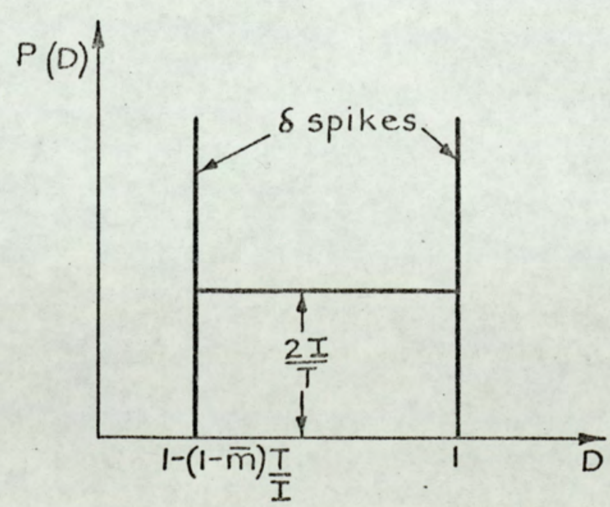


FIG. 5.20 PROBABILITY DENSITY DISTRIBUTION OF DEMAND.

$(\bar{m} > \frac{1}{2},\ (1-\bar{m})T < I < \bar{m}T)$

$$E(D) = \int \frac{D}{T}\,(I - (1 - \bar{m})\,T)\;\delta\left(D - \frac{(I - (1 - \bar{m})\,T)}{I}\right) dD$$

$$+ \int \frac{D}{T}\,(T - (I + (1 - \bar{m})\,T\,)\;\delta\,(D - 1)\;dD$$

$$+ \int_{\frac{I - (1 - \bar{m})\,T}{I}}^{1} 2\,\frac{I}{T}\;D\;dD$$

$$= \frac{I}{T}\left[(I - (1 - \bar{m})\,T)\frac{(I - (1 - \bar{m})\,T)}{I} + T - (I + (1 - \bar{m})\,T)\right.$$

$$\left. + I\left\{1 - \left(\frac{I - (1 - \bar{m})\,T}{I}\right)^2\right\}\right]$$

$$= \frac{I}{T}\left[(1 - S)^2 + \frac{T}{I} - (1 + S) + 1 - (1 - S)^2\right]$$

$$= \frac{I}{T}\;\left(\frac{T}{I} - S\right) \qquad\qquad = \underline{\bar{m}} \qquad\qquad (5.21)$$

$$E(D^2) = \int \frac{D^2}{T}\;(\,(I - (1 - \bar{m})\,T)\;\delta\left(D - \frac{(I - (1 - \bar{m})\,T\,)}{I}\right) dD$$

$$+ \int \frac{D^2}{T}\,(T - (I + (1 - \bar{m})\,T\,))\;\delta\;(D - 1)\;dD$$

$$+ \int_{\frac{I - (1 - \bar{m})\,T}{I}}^{1} 2\,\frac{I}{T}\;D^2\;dD$$

$$= \frac{I}{T}\left[(\,I - (1 - \bar{m})\,T)\left(\frac{I - (1 - \bar{m})\,T}{I}\right)^2 + T - (I + (1 - \bar{m})\,T)\right.$$

$$\left. + \frac{2}{3}\;I\left(1^3 - \left(\frac{I - (1 - \bar{m})\,T}{I}\right)^3\,\right)\right]$$

$$= 1 - \frac{\bar{m}}{3r}\;\left(1 + 3S - (1 - S)^3\,\right)$$

$$\therefore\quad \sigma^2 = 1 - \frac{\bar{m}}{3r}\;(1 + 3S - (1 - S)^3) - \bar{m}^2$$

$$= \underline{\quad 1 - \frac{\bar{m}}{3r}\;(1 + 3S + 3\bar{m}r - (1 - S)^3\,)\quad} \qquad\qquad (5.22)$$

(As a check, for $\bar{m} = \frac{1}{2}$, $\frac{I}{T} = \frac{1}{2}$, $r = S = 1$ and $\sigma^2 = \frac{1}{12}$).

This completes all cases for $\bar{m} > \frac{1}{2}$.

$\underline{\bar{m} < \frac{1}{2}.}$ Case (i). $I > (1 - \bar{m})\,T$.

The position of x in relation to $\bar{m}T$, I, T is shown in Fig. 5.21.

If $0 \leqslant x \leqslant I - \bar{m}\,T$, then $D = \bar{m}\,\frac{T}{I}$.

If $I - \bar{m}\,T \leqslant x \leqslant (1 - \bar{m})\,T$,

then $D = \frac{I - x}{I}$.

If $(1 - \bar{m})\,T \leqslant x \leqslant I$, then $D = \frac{I - (1 - \bar{m})\,T}{I}$.

If $I \leqslant x \leqslant T$,

then $D = \frac{x - (1 - \bar{m})\,T}{I}$.

This distribution is the same as for $\bar{m} > \frac{1}{2}$, case (i).

$$\therefore \quad E(D) = \bar{m} \tag{5.23}$$

$$\text{and} \quad \sigma^2 = \frac{\bar{m}}{3r}\left(3r^2 - r^3 + (1 - S)^3 - 3\bar{m}r\right). \tag{5.24}$$

$\bar{m} < \frac{1}{2}$. Case (ii). $I < \bar{m}\,T$.

The position of x in relation to $\bar{m}\,T$, I, T is shown in Fig. 5.21.

If $0 \leqslant x \leqslant I$, then $D = \frac{I - x}{I}$.

If $I \leqslant x \leqslant T - \bar{m}\,T$, then $D = 0$.

If $T - \bar{m}\,T \leqslant x \leqslant I + (1 - \bar{m})\,T$,

then $D = \frac{x - (1 - \bar{m})\,T}{I}$.

If $I + (1 - \bar{m})\,T \leqslant x \leqslant T$, then $D = 1$.

This distribution is the same as for $\bar{m} > \frac{1}{2}$, case (ii).

$$\therefore \quad E(D) = \bar{m} \tag{5.25}$$

$$\text{and} \quad \sigma^2 = \bar{m}\left(1 - \bar{m} - \frac{1}{3r}\right). \tag{5.26}$$

$\bar{m} < \frac{1}{2}$. Case (iii). $\bar{m}\,T < I < (1 - \bar{m})\,T$.

The position of x in relation to $\bar{m}\,T$, I, T is shown in Fig. 5.21.

If $0 \leqslant x \leqslant I - \bar{m}\,T$, then $D = \bar{m}\frac{T}{I}$.

If $I - \bar{m}\,T \leqslant x \leqslant I$, then $D = \frac{I - x}{I}$.

If $I \leqslant x \leqslant (1 - \bar{m})\,T$, then $D = 0$.

If $(1 - \bar{m})T \leqslant x \leqslant T$,

then $D = \frac{x - (1 - \bar{m})\,T}{I}$.

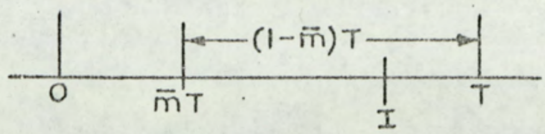
The probability distribution for D is

$$P(D)\,dD = \frac{dD}{T}\left[\left((1 - \bar{m})\,T - I\right)\delta(D - 0) + (I - \bar{m}\,T)\,\delta\left(D - \bar{m}\frac{T}{I}\right) + 2I\right].$$

The probability density distribution and derivation are shown in Fig. 5.21.

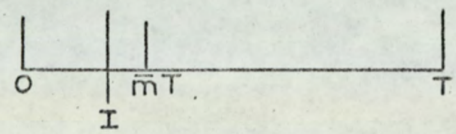
$$E(D) = \int \frac{D}{T}\left(\left((1 - \bar{m})\,T - I\right)\right)\delta D\,dD + \int \frac{D}{T}(I - \bar{m}\,T)\,\delta\left(D - \frac{\bar{m}\,T}{I}\right)dD + \int_0^{\bar{m}\frac{T}{I}} 2\frac{I}{T} D\,dD$$

$\bar{m} < \frac{1}{2}$. Case (i). $I > (1-\bar{m})T$.



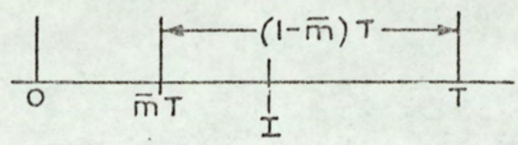
Probability Density Distribution the same as in FIG. 5.18.

$\bar{m} < \frac{1}{2}$. Case (ii). $I < \bar{m}T$.



Probability Density Distribution the same as in FIG. 5.19.

$\bar{m} < \frac{1}{2}$. Case (iii). $\bar{m}T < I < (1-\bar{m})T$.



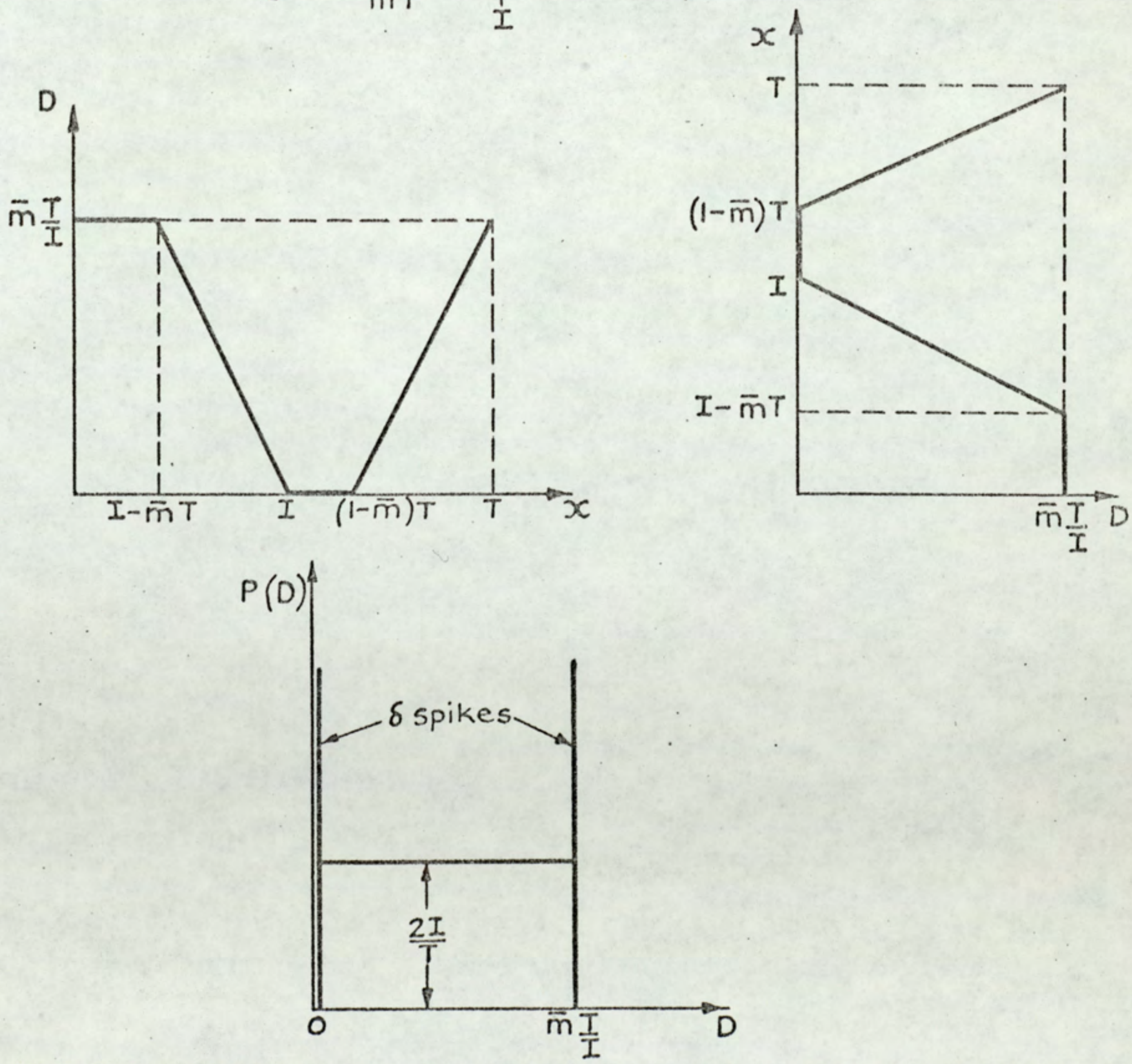


FIG. 5.21 PROBABILITY DENSITY DISTRIBUTIONS OF DEMAND.
($\bar{m} < \frac{1}{2}$, $I > (1-\bar{m})T$, $I < \bar{m}T$, $\bar{m}T < I < (1-\bar{m})T$).

$$= o + \frac{(I - \bar{m}T)}{T}\left(\bar{m}\frac{T}{I}\right) + \frac{I}{T}\left(\frac{\bar{m}T}{I}\right)^2$$

$$= \frac{I}{T}\left((1 - r)\,r + r^2\right) \qquad = \underline{\bar{m}} \tag{5.27}$$

$$E(D^2) = \int \frac{D^2}{T}\left(\left(1 - \bar{m}\right)T - I\right)\,\delta D\; dD$$

$$+ \int \frac{D^2}{T}\,(I - \bar{m}T)\,\delta\left(D - \bar{m}\frac{T}{I}\right) dD + \int_0^{\frac{\bar{m}T}{I}} \frac{2I}{T}\,D^2\,dD$$

$$= 0 + \frac{(I - \bar{m}T)}{T}\left(\frac{\bar{m}T}{I}\right)^2 + \frac{2}{3}\,\frac{I}{T}\left(\frac{\bar{m}T}{I}\right)^3$$

$$= \frac{I}{T}\left(r^2 - \frac{1}{3}r^3\right)$$

$$\therefore \quad \sigma^2 = \frac{I}{T}\left(r^2 - \frac{1}{3}r^3\right) - \bar{m}^2$$

$$= \underline{\frac{\bar{m}}{3r}\left(3r^2 - r^3 - 3\bar{m}r\right).} \tag{5.28}$$

(As a check, for $\bar{m} = \frac{1}{2}$, $I = \frac{1}{2}T$, $r = S = 1$ and $\sigma^2 = \frac{1}{12}$).

This completes all cases for $\bar{m} < \frac{1}{2}$.

5.4.2.1 EXTENSION OF ANALYSIS FOR $0 \leqslant \bar{m} \leqslant 1$ and $\frac{I}{T} \geqslant 1$.

We may now extend the results for $\frac{I}{T} < 1$ to the case where $\frac{I}{T} \geqslant 1$.

If the demand is D for the case $I = \bar{m}T$, then for the case $I = (1 + \bar{m})T$ the demand is given by

$$\frac{D \times \bar{m}T + \bar{m}T}{(1 + \bar{m})T} = D \times \frac{\bar{m}T}{(1 + \bar{m})T} + \frac{\bar{m}T}{(1 + \bar{m})T}.$$

Then Mean $= \dfrac{\bar{m}T}{(1 + \bar{m})T} + \dfrac{\bar{m}T}{(1 + \bar{m})T} \times \bar{D}$

$$= \underline{\bar{m}.} \tag{5.29}$$

The variance of $D \times \dfrac{\bar{m}T}{(1 + \bar{m})T} + \dfrac{\bar{m}T}{(1 + \bar{m})T}$

is the same as the variance of $D \times \dfrac{\bar{m}T}{(1 + \bar{m})T}$

$$= \underline{\left(\frac{\bar{m}}{1 + \bar{m}}\right)^2 \sigma^2.} \tag{5.30}$$

For <u>example</u>, with $\bar{m} = \frac{1}{2}$, $I = \frac{T}{2}$, $\sigma^2 = \frac{1}{12}$.

$\therefore$ variance when $I = \frac{3T}{2}$ is $\left(\frac{\frac{1}{2}}{\frac{3}{2}}\right)^2 \sigma^2 = \frac{\sigma^2}{9} = \frac{1}{108}$.

In general,

if D_1 is the demand with an integrating interval I_1, where $0 \leqslant I_1 \leqslant T$, and D_2 is the demand with an integrating interval $I_1 + T$,

$$\text{then } D_2 = \frac{D_1 \times I_1 + \bar{m}\,T}{I_1 + T} \tag{5.31}$$

$$\text{and variance of } D_2 = \text{variance of } \left(\frac{I_1}{I_1 + T}\right) D_1 = \left(\frac{I_1}{I_1 + T}\right)^2 \sigma_{I_1}^2, \tag{5.32}$$

where $\sigma_{I_1}^2$ is the variance of D_1.

The variance may thus be evaluated for any value of $\frac{I}{T} > 1$ from the value $\sigma_{I_1}^2$ for $\frac{I}{T} \leqslant 1$.

5.4.2.2 EVALUATION OF σ^2 for $\bar{m} = \frac{3}{4}$ and $\frac{1}{4}$, where $0 \leqslant \frac{I}{T} \leqslant 1$

We may now evaluate σ^2 for the particular values $\bar{m} = \frac{3}{4}$ and $\frac{1}{4}$.

$\bar{m} = \frac{3}{4}$. $\bar{D} = \frac{3}{4}$.

We apply the results for the three cases when $\bar{m} > \frac{1}{2}$.

Case (i). $I > \bar{m}\,T$.

$$r = \bar{m}\frac{T}{I}, \quad S = (1 - \bar{m})\frac{T}{I},$$

$$\sigma^2 = \frac{\bar{m}}{3r}\left(3r^2 - r^3 + (1 - S)^3 - 3\,\bar{m}r\right).$$

Case (ii). $I < (1 - \bar{m})\,T$. $\quad \sigma^2 = \bar{m}\left(1 - \bar{m} - \frac{1}{3r}\right).$

Case (iii). $(1 - \bar{m})\,T < I < \bar{m}\,T$. $\quad \sigma^2 = 1 - \frac{\bar{m}}{3r}\left(1 + 3S + 3\bar{m}r - (1 - S)^3\right).$

Applying the above relations for discrete values of $\frac{I}{T}$, we have:

$\frac{I}{T}$	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
r	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{15}{16}$	$\frac{15}{14}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{15}{8}$	$\frac{15}{6}$	$\frac{15}{4}$	$\frac{15}{2}$	∞
S	$\frac{1}{4}$	$\frac{5}{18}$	$\frac{5}{16}$	$\frac{5}{14}$	$\frac{5}{12}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{6}$	$\frac{5}{4}$	$\frac{5}{2}$	∞
σ^2	0	0.002	0.007	0.015	0.0272	0.0415	0.06	0.088	0.121	0.154	0.1875

Case (i) Case (iii) Case (ii)

TABLE 5.3

Cases (i) and (iii) cross-check at $\frac{I}{T} = 0.75$ ($\sigma^2 = \frac{5}{432}$)

and cases (ii) and (iii) cross-check at $\frac{I}{T} = 0.25$ ($\sigma^2 = \frac{5}{48}$).

The variance is plotted in Fig. 5.16 as a function of $\frac{I}{T}$.

$\bar{m} = \frac{1}{4}$. $\bar{D} = \frac{1}{4}$.

We apply the results for the three cases when $\bar{m} < \frac{1}{2}$.

Case (i). $I > (1 - \bar{m})\,T$. $r = \bar{m}\,\frac{T}{I}$, $S = (1 - \bar{m})\,\frac{T}{I}$;

$$\sigma^2 = \frac{\bar{m}}{3r}\,(3r^2 - r^3 + (1 - S)^3 - 3\,\bar{m}r).$$

Case (ii). $I < \bar{m}\,T$. $\sigma^2 = \bar{m}\,(1 - \bar{m} - \frac{1}{3r})$.

Case (iii). $\bar{m}\,T < I < (1 - \bar{m})\,T$. $\sigma^2 = \frac{\bar{m}}{3r}\,(3r^2 - r^3 - 3\,\bar{m}r)$.

Applying the above relations for discrete values of $\frac{I}{T}$ we have:

$\frac{I}{T}$	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
r	$\frac{1}{4}$	$\frac{5}{18}$	$\frac{5}{16}$	$\frac{5}{14}$	$\frac{5}{12}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{6}$	$\frac{5}{4}$	$\frac{5}{2}$	∞
S	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{15}{16}$	$\frac{15}{14}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{15}{8}$	$\frac{15}{6}$	$\frac{15}{4}$	$\frac{15}{2}$	∞
σ^2	0	0.002	0.007	0.015	0.0272	0.0415	0.06	0.088	0.121	0.154	0.1875
	Case (i)			Case (iii)				Case (ii)			

TABLE 5.4

Cases (i) and (iii) cross-check at $\frac{I}{T} = 0.75$ ($\sigma^2 = \frac{5}{432}$)

and cases (ii) and (iii) cross check at $\frac{I}{T} = 0.25$ ($\sigma^2 = \frac{5}{48}$).

The above results show that for a given value of $\frac{I}{T}$ the variance σ^2 of the demand for $\bar{m} = \frac{1}{4}$ has the same value as that for $\bar{m} = \frac{3}{4}$.

Fig. 5.12 has been drawn showing the demand as a function of x for $\bar{m} = \frac{3}{4}$ and $\frac{1}{4}$ respectively. This shows that the demand variation about the mean value for $\bar{m} = \frac{3}{4}$ is of the same shape and has the same magnitude as that for $\bar{m} = \frac{1}{4}$, but is displaced in time.

In general, for a given value of $\frac{I}{T}$ the values of σ^2 are the same for $\bar{m}$ and $1 - \bar{m}$, reaching a maximum at $\bar{m} = \frac{1}{2}$.

A rigorous proof of this statement necessitates differentiating with respect to $\bar{m}$ the expressions for σ^2, where $0 \leqslant \bar{m} \leqslant 1$, and equating the derivatives to zero.

We may consider equations (5.20) and (5.26), each of which are:

$$\sigma^2 = \bar{m} - \bar{m}^2 - \frac{1}{3K} \quad \text{(where } K = \frac{T}{I}\text{)},$$

subject to the conditions

$I \leqslant (1 - \bar{m})\, T$, or $K \geqslant \dfrac{1}{1 - \bar{m}}$ for $\bar{m} \geqslant \frac{1}{2}$,

and $I \leqslant \bar{m}\, T$, or $K \geqslant \dfrac{1}{\bar{m}}$ for $\bar{m} \leqslant \frac{1}{2}$.

Thus if K is taken as equal to 10, a range of $\bar{m}$ from 0.1 to 0.9 may be considered using the one equation. For K = 5 the relevant range of $\bar{m}$ is from 0.2 to 0.8. The equation remains valid for values of K not less than 2.

In Fig. 5.22 the variance is plotted as a function of $\bar{m}$ for values of K equal to 10, 5, 3 and 2. Each curve exhibits a single maximum at $\bar{m} = \frac{1}{2}$, and values of σ^2 for $\bar{m}$ are equal to those for $1 - \bar{m}$.

σ^2 may be plotted against $\bar{m}$ for fixed values of K of interest, in contrast to Fig. 5.17, where σ^2 is displayed as a function of $\frac{1}{K}$ for constant $\bar{m}$. Care must be taken to apply each individual equation for σ^2 only over the region of $\bar{m}$ for which it remains valid.

For example, for K = 5, equations (5.20) and (5.26) are relevant for $\frac{1}{5} \leqslant \bar{m} \leqslant \frac{4}{5}$, when $\frac{d}{dm}(\sigma^2) = 0$ gives $\bar{m} = \frac{1}{2}$ as the maximum variance.

(5.22) is relevant for $\frac{4}{5} \leqslant \bar{m} \leqslant 1$.

(5.28) is relevant for $0 \leqslant \bar{m} \leqslant \frac{1}{5}$.

(5.18) is not relevant, since for $\bar{m} \geqslant \frac{1}{2}$ the condition $\bar{m} \leqslant \frac{1}{K}$ is not realised with $K > 2$.

(5.24) is not relevant, since for $\bar{m} \leqslant \frac{1}{2}$ the condition $1 - \bar{m} \leqslant \frac{1}{K}$ is not realised with $K > 2$.

The equality of the variance values for any two values of $\bar{m}$ displaced equally above and below $\bar{m} = \frac{1}{2}$ may be verified from equations (5.18), (5.20), (5.22), (5.24), (5.26), (5,28).

Thus if in equations (5.18), (5.20), (5.22), we replace $\bar{m}$ by $1 - \bar{m}$, r by S, and S by r, we obtain equations (5.24), (5.26), (5.28) respectively, proving symmetry about $\bar{m} = \frac{1}{2}$ for a given value of K.

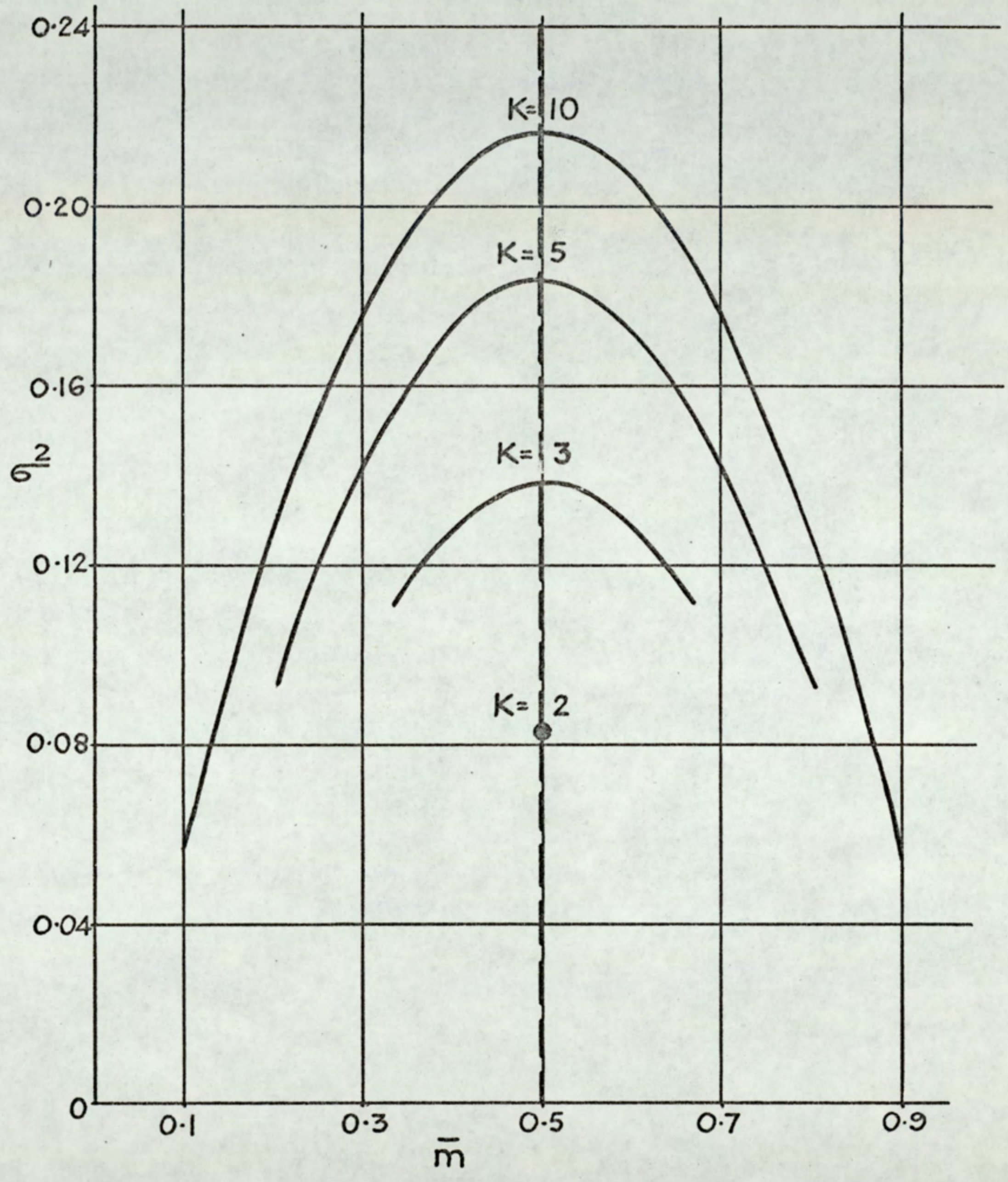


FIG. 5.22 DEMAND VARIANCE AS A FUNCTION OF $\bar{m}$ FOR CONSTANT $K = T/I$.

5.4.2.3 <u>EVALUATION OF E (D) AND VARIANCE FOR $0 \leqslant \bar{m} \leqslant 1$ AND $\frac{I}{T} \geqslant 1$.</u>

The values of E (D) and σ^2 for $\frac{I}{T} < 1$ may now be used to find the corresponding values for $\frac{I}{T} \geqslant 1$. Thus we apply equations (5.31) and (5.32), together with the known value $\sigma_{I_1}^2$ for $\frac{I}{T} \leqslant 1$ (Tables 5.3 and 5.4).

For example, for I = 1.4 and $\bar{m} = \frac{3}{4}$,

$$D_2 = \frac{D_1 \times 0.4\ T + 0.75\ T}{1.4\ T}$$

and mean $= \frac{0.75}{1.4} + \frac{0.4}{1.4} \times \bar{m}$ (Since $\bar{D}_1 = \bar{m} = 0.75$)

$= \bar{m}$.

Also variance $= \left(\frac{0.4}{1.4}\right)^2 \times \sigma^2_{I_1} = \frac{4}{49} \times 0.06 = 0.00489$.

Using this result, the variance of the demand for $\frac{I}{T} = 2.4$ may then be obtained in a similar manner.

Taking values of $\sigma^2_{I_1}$ from Table 5.3, the variance has been calculated for $\bar{m} = \frac{3}{4}$ for values of $\frac{I}{T}$ up to 3.0. The results for $\bar{m} = \frac{1}{4}$ will be identical to those for $\bar{m} = \frac{3}{4}$.

The results are shown in Table 5.5 and the variance plotted in Fig. 5.17 as a function of $\frac{I}{T}$.

$\frac{I}{T}$	1.1	1.2	1.3	1.4	1.5
Variance	0.00128	0.00335	0.00471	0.00489	0.00462

$\frac{I}{T}$	1.6	1.7	1.8	1.9	2.0
Variance	0.00382	0.00254	0.00138	0.000446	0

$\frac{I}{T}$	2.1	2.2	2.3	2.4	2.5
Variance	0.00035	0.000996	0.0015	0.00167	0.00166

$\frac{I}{T}$	2.6	2.7	2.8	2.9	3.0
Variance	0.00146	0.001	0.0005	0.00019	0

<u>TABLE 5.5</u>

For $\bar{m} = \frac{3}{4}$, mean $= \frac{3}{4}$.
For $\bar{m} = \frac{1}{4}$, mean $= \frac{1}{4}$.

5.4.3 USE OF MOMENT GENERATING FUNCTION.[23]

Having obtained for one process the probability density $P(D)$, the mean of D and the variance may be obtained alternatively to the method used by finding the moment generating function

$$E(e^{\Theta D}) = \int P(D)\, e^{\Theta D}\, dD = E\left(1 + \Theta D + \frac{\Theta^2}{\lfloor 2} D^2 + \text{- - - - - - - - - - -}\right)$$
$$= 1 + \Theta\, E(D) + \frac{\Theta^2}{\lfloor 2} E(D^2) + \text{- - - - -}, \tag{5.33}$$

where Θ is a dummy variable.

If the moment generating function for one process is

$$M = 1 + \lambda_1 \Theta + \lambda_2 \frac{\Theta^2}{\lfloor 2} + \text{- - - - - - - -}, \tag{5.34}$$

the mean of D will be given by λ_1 and the variance by $\lambda_2 - \lambda_1^2$.

For n processes, the moment generating function M may be raised to the nth power and expanded in terms of Θ, the new coefficients of Θ and $\frac{\Theta^2}{\lfloor 2}$ giving $E(D)$ and $E(D^2)$ respectively.

Taking, as the simplest example, the probability density distribution of Fig. 5.14 for $\bar{m} = \frac{1}{2}$ and the case where $I < \frac{T}{2}$:

$P(D)$ consists of spikes $a\,\delta(D)$ and $a\,\delta(D - 1)$ at $D = 0$ and 1 respectively, joined by a horizontal cross-bar of height b, where $a = \frac{1}{T}\left(\frac{T}{2} - I\right)$ and $b = \frac{2I}{T}$.

Since the probability, or total area under the probability density curve, is 1, then $2a + b = 1$.

For one process the m.g.f. is $M = \int_0^1 P(D)\, e^{\Theta D}\, dD$ (5.35)

$$= \int a\,\delta(D) e^{\Theta D} dD + \int a\,\delta(D - 1)\, e^{\Theta D}\, dD + \int_0^1 b\, e^{\Theta D}\, dD$$
$$= a\,(1 + e^{\Theta}) + \frac{b}{\Theta}(e^{\Theta} - 1);$$

then $M = a\left(2 + \Theta + \frac{\Theta^2}{\lfloor 2} + \text{- - - - -}\right) + b\left(1 + \frac{\Theta}{\lfloor 2} + \frac{\Theta^2}{\lfloor 3} + \text{- - - -}\right)$.

Then mean = coeff. of $\Theta = a + \frac{b}{2} = \underline{\tfrac{1}{2}}$.

$E(D^2)$ = coeff. of $\frac{\Theta^2}{\lfloor 2} = a + \frac{b}{3}$.

Then $\sigma^2 = E(D^2) - \bar{D}^2 = a + \frac{b}{3} - \frac{1}{4}$

$$= \frac{1 - b}{2} + \frac{b}{3} - \frac{1}{4} = \frac{1}{4} - \frac{b}{6}$$

$$= \underline{\frac{1}{4} - \frac{1}{6r}}, \text{ where } r = \frac{T}{2I},$$

agreeing with equation (5.14).

The characteristic function $E\,(e^{j\theta D})$ for one process is the same as the moment generating function with θ replaced by $j\theta$. The characteristic function is determined by the probability density of the distribution; conversely, the distribution is determined uniquely by its characteristic function, the relationships existing as Fourier transform pairs.[15,23]

To obtain the demand distribution for n independent processes we need to find $\left[M\,(j\theta)\right]^n$, and then the probability distribution of the demand y from the Fourier transform $\int_{-\infty}^{\infty}\left[M\,(j\theta)\right]^n e^{-j\theta y}\,d\theta$, which appears intractable analytically.

5.5 DISTRIBUTION OF DEMAND.

The distribution function of the demand due to one process, equal to $\int_{-\infty}^{D} P\,(D)dD$, will be obtained by integrating the probability density, and will depend upon the ratio $\frac{I}{T}$.

Fig. 5.23 shows the probability distribution for the case $\overline{m} = \frac{1}{2}$ and $I > \frac{T}{2}$. For $\frac{I}{T} = \frac{1}{2}$, the probability curve becomes a ramp with zero value at $D = 0$ and unity value at $D = 1$, and of mean value $\overline{D} = \frac{1}{2}$. For $\frac{I}{T} = 1$, however, the distribution is located entirely at the midpoint $D = \frac{1}{2}$ with zero variance, i.e. the most favourable distribution.

The distribution function may be compared with the error function for the worst case $\overline{m} = \frac{1}{2}$. We see that for $\frac{I}{T} = \frac{1}{2}$, $\sigma = \frac{1}{\sqrt{12}} = 0.29$, and the area under the probability density curve between $D = 0$ and $\overline{D} + \sigma = 0.5 + 0.5 \times \frac{0.29}{0.5} = 0.79$. Then the probability of the demand being not more than 0.29 above the mean is 0.79, compared with $0.5 + 0.341 = 0.841$ for the error function.

Thus for $\overline{m} = \frac{1}{2}$, the probability of the demand being more than σ above the mean is optimistic by only 6% for $\frac{I}{T} = \frac{1}{2}$ if a normal distribution is assumed. As the ratio $\frac{I}{T}$ is increased in the range $\frac{1}{2} < \frac{I}{T} < 1$, the assumption of a normal distribution becomes conservative. This is clearly seen from the probability distribution curve shown in Fig. 5.23.

If we have n similar independent processes, and the demand

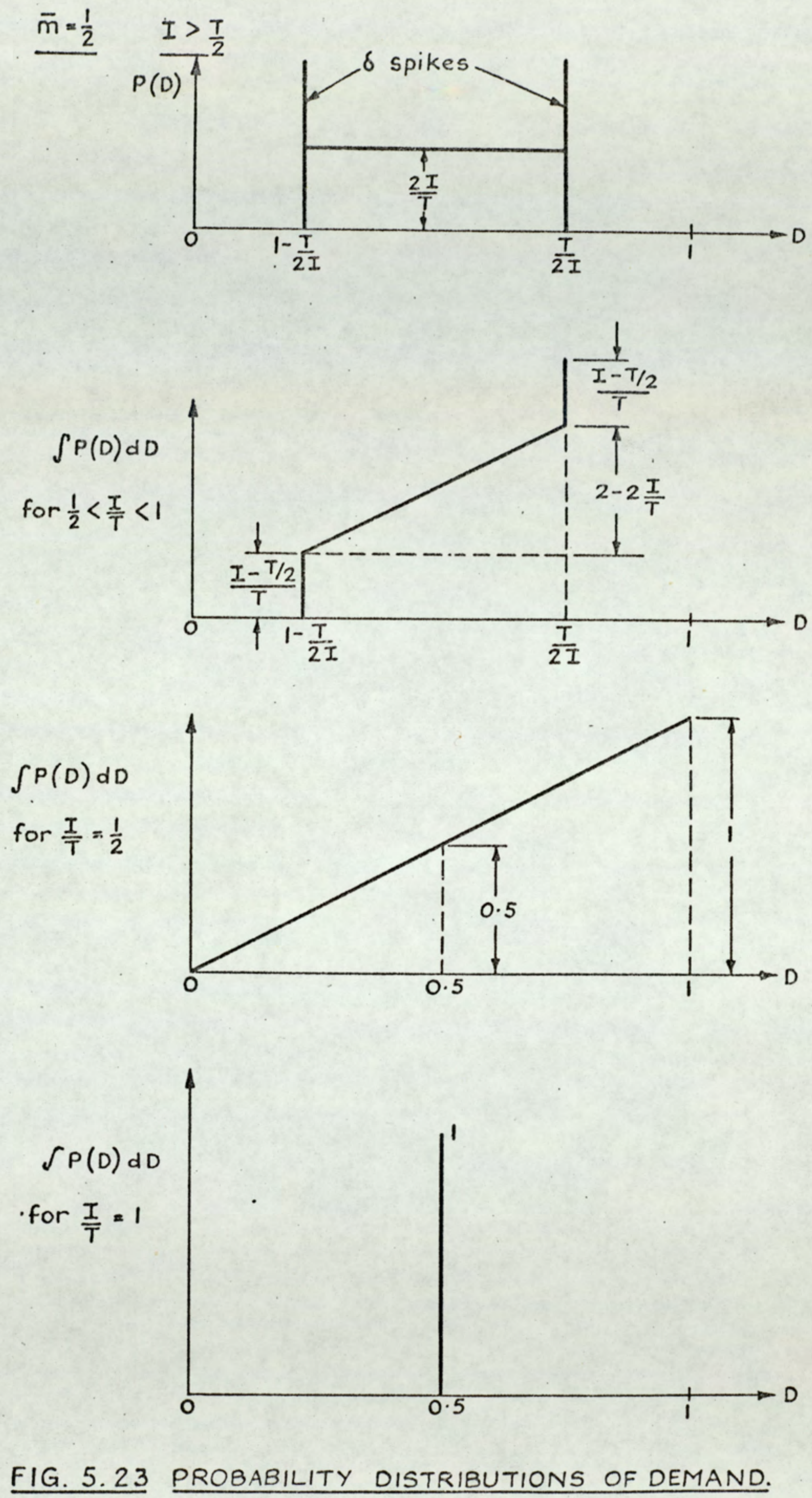


FIG. 5.23 PROBABILITY DISTRIBUTIONS OF DEMAND.
($\bar{m} = \frac{1}{2}$, $I > \frac{T}{2}$, $\frac{1}{2} < \frac{I}{T} < 1$, $\frac{I}{T} = \frac{1}{2}$ & 1).

due to each process has a mean value $\bar{m}$ and a variance σ^2, then:

the mean of the sum of the demands $= n\bar{m}$ (5.36)

and the variance of the sum of the demands $= n\sigma^2$. (5.37)

The mean of the distribution of means due to the several processes is $\frac{n\bar{m}}{n} = \bar{m}$, (5.38)

and the variance of the mean value of the separate values is

$$\frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}. \tag{5.39}$$

By the Central Limit theorem, as n increases towards infinity, the probability density distribution of the demands approaches normal. Thus the mean of the demands due to the n processes is normally distributed about the common mean $\bar{m}$, with a variance $\frac{\sigma^2}{n}$ which approaches zero as n approaches infinity.

Although for n finite we have not specified the probability distribution of the total demand, we may establish values for the mean and variance from equations (5.36) and 5.37). In order to establish a confidence that a given demand will not be exceeded, the best approach is to assume a normal distribution with these parameters.

5.5.1 MAXIMUM DEMAND PENALTY FOR AN ENSEMBLE OF n SIMILAR TWO-POSITION PROCESSES.

By means of the standard normal distribution we may determine a suitable set level of maximum demand:

$$P\left\{u_1 \leqslant u \leqslant u_2\right\} = \frac{1}{\sqrt{2\pi}}\int_{u_1}^{u_2} \exp\left(-\frac{u^2}{2}\right) du . \tag{5.40}$$

This gives the probability that u, chosen at random, has a value between u_1 and u_2,

$$\text{where } u = \frac{\text{observation } x - \text{mean value}}{\text{standard deviation}} = \frac{x - n\bar{m}}{\sqrt{n\sigma^2}} . \tag{5.41}$$

Values of u are tabulated[24].

For u = 1.0, 68% of the values of demand due to n processes lie in the range $n\bar{m} \pm 1.0\sqrt{n\sigma^2}$.

For u = 1.96, 95% of the distribution of the demand lies in the range

$n\bar{m} \pm 1.96\sqrt{n\sigma^2}$.

For u = 3.09, 99.8% of the distribution lies in the range $n\bar{m} \pm 3.09$

$\sqrt{n\sigma^2}$.

Example.

We will determine the maximum demand penalty due to an ensemble of twenty similar 50 KW on-off industrial loads at unity power factor.

The demand variation may be calculated for $\bar{m} = \frac{1}{2}$ and $\frac{3}{4}$ for the case $I/T = \frac{1}{2}$.

For $\bar{m} = \frac{1}{2}$, $\sigma^2 = 0.0833$ for one process, when the ensemble demand for the limits corresponding to u lies in the range $(10 \pm u \sqrt{20 \times 0.0833})$ 50 KW.

For $\bar{m} = \frac{3}{4}$, $\sigma^2 = 0.0415$ for one process, when the ensemble demand lies in the range $(15 \pm u\sqrt{20 \times 0.0415})$ 50 KW.

The upper limit of demand for given probabilities is plotted for each case in Fig. 5.24.

Also shown are the corresponding monthly maximum demand charges, which have been evaluated for the Midlands Electricity Board tariff No. 7 (see section 8.2). As the demand charge per KVA does not change over the range of demands considered, both the demand and cost curves will have normal distributions.

Suppose that $T = 2I = 60$ minutes and $\bar{m} = \frac{1}{2}$. For $u = 1.96$, corresponding to the 95% limits, the demand will exceed 626 KW in $\frac{1}{2} \times 5\%$ of 1440, or only 36 of the 1440 integrating periods in the account month.

The problem then arises of determining the economic demand level at which control action should be taken. The monthly demand penalty at a maximum demand of 626 KW is £520. This may be reduced to £500 by restricting the maximum demand to, say, 600 KW, but at the expense of lost production due to the increased control action. As shown in section 8.3.1, the demand penalty is doubly felt, since the tariff is so arranged that energy charges bear a portion of the charge due to the maximum demand.

The assessment of the production worth is a matter for the individual project. The economic maximum demand setting, as constrained by production requirements, is determined for a specific application in the first case study of section 8.5.

5.6 PRACTICAL APPLICATION OF RESULTS.

It has been shown that the maximum variance of the demand

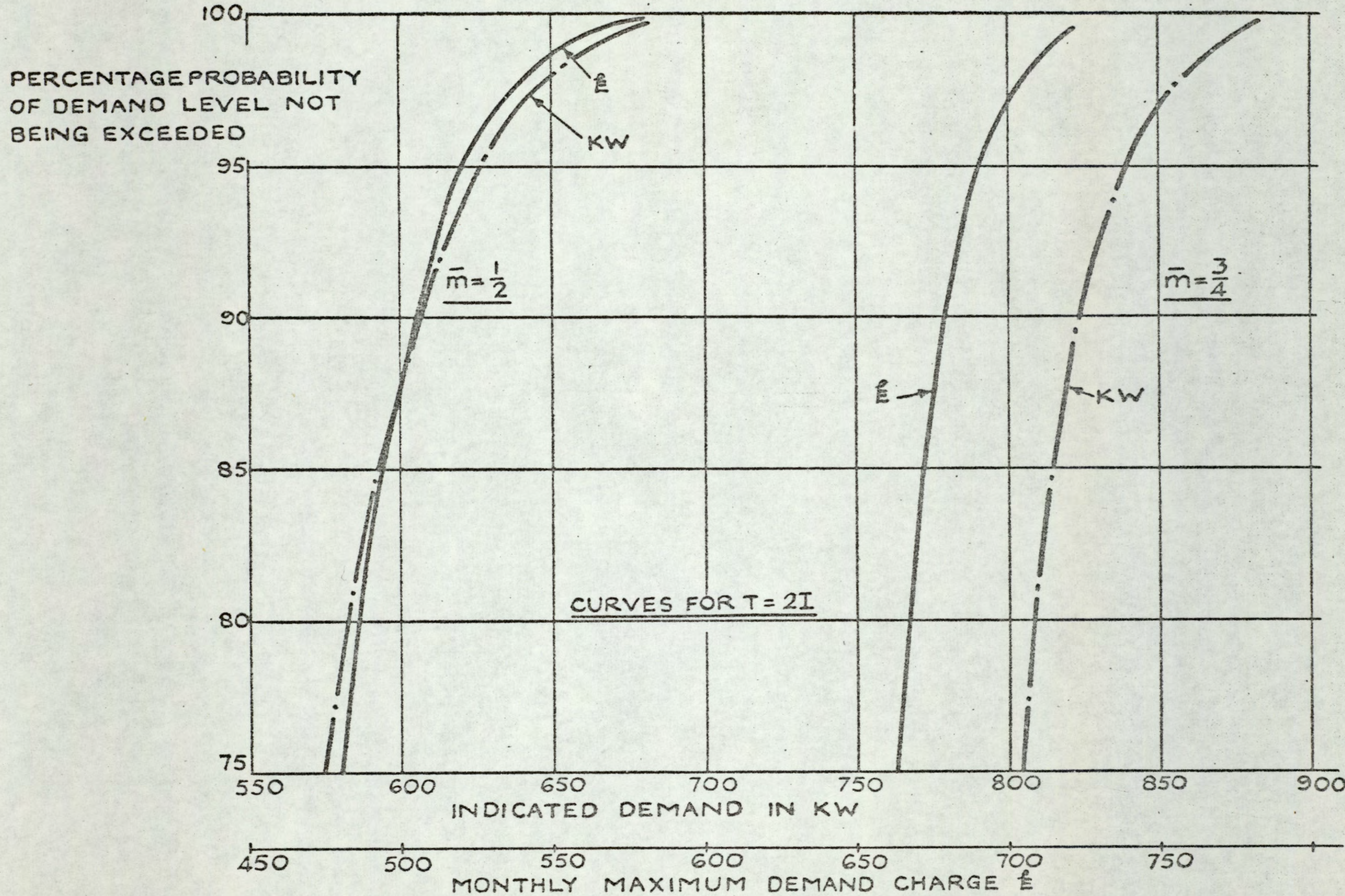


FIG. 5.24 MAXIMUM DEMAND PENALTY FOR ENSEMBLE OF TWENTY 50KW ON-OFF LOADS AT UNITY POWER FACTOR.

for a given ratio $\frac{I}{T}$ is obtained at $\bar{m} = \frac{1}{2}$; also that for minimisation of the demand variance, $\frac{I}{T}$ should have an integer value. In practice, this ratio should be as high as possible, and preferably not less than unity.

Roots, Woods and Wells[20] have shown that for electric space heating processes switched by thermostat, $\bar{m}$ and T do not vary independently. T is a minimum at $\bar{m} = \frac{1}{2}$, and will increase as $\bar{m}$ departs from this median value in either direction. If the system demand variance is to be minimised, therefore, T should ideally be less than I for the maximum value of $\bar{m}$ to be expected. Design methods for reducing T are discussed and analysed in Chapter 7.

5.7 SUMMARY.

If the incidence of processes is thermostatically-controlled a staggered start of blocks will be necessary, so that the maximum demand is determined by the steady-state load, and not the transient following a cold start.

In this chapter an analysis is presented of the statistical properties of the demand due to an ensemble of similar thermostatically-controlled processes operating under dynamic equilibrium cycling conditions.

If the demand variance is reduced, a closer control of the maximum demand can be achieved.

New work establishes the following important design requirement for minimising the variance of the demand due to the above load: The ratio integrating interval/process cycle time should be an integer. Otherwise, the value of this ratio should be as high as possible, and preferably not less than unity.

As the integrating interval has a fixed value, this shows that a rapid on/off sequence of load is desirable for reducing the demand variance, i.e. the period of the load should be minimised. Methods for achieving this object will be analysed in Chapter 7. It will be seen that the reduction in period also satisfies a "minimum-comfort" criterion of optimisation for discontinuously-controlled space heating systems in which the fluctuations of temperature are to be minimised.

The probability distribution of the demand due to an ensemble of similar thermostatically-controlled loads is shown to be a function of $\bar{m}$ and of the ratio integrating interval/load period. Analysis demonstrates that a confidence in not exceeding a given demand will be obtained conservatively by assuming a Normal distribution with the same values of mean and variance as calculated for the actual distribution.

The worth of the results of section 5.5 lies in the planning and costing of future projects requiring on-off loads of the type described. Knowing the likely demand costs and the energy charges, an economic assessment of the viability of each proposed installation may then by made, since each component of the tariff is referred to the maximum KVA demand. In determining the maximum demand penalty for a practical situation, curves such as those presented in Fig. 5.24 will be invaluable to the supply Authority.

An application where minimum demand variance can be obtained is in pulse width modulation control of electric space heating systems. The ratio integrating period/process cycle time may be set by the designer to have a fixed integer value; control of the temperature is then achieved by continuous adjustment of the parameter $\bar{m}$.

The object of this chapter has been to explore the extent to which analytical techniques may be applied to describe the probability distribution of the demand due to thermostatically-controlled loads. In order to render the problem tractable to analysis, therefore, all processes were assumed to have the same $\bar{m}$ and the same value of cycle time. These restrictions are overcome by simulation techniques in Chapter 6.

CHAPTER 6.

DIGITAL SIMULATIONS ON MODELS OF DEMAND DUE TO AN ENSEMBLE OF THERMOSTATICALLY-CONTROLLED TWO-POSITION PROCESSES.

6.1 INTRODUCTION.

The demand model established in chapter 5 is verified by the use of pseudo-random number sequences drawn from published tables. A PDP-9 digital computer is employed for generating further random variates, and also for carrying out the simulation computations.

By sampling from known theoretical distributions, the model is extended to take account of spreads in the parameters $\bar{m}$ and T. A simulation is then performed which yields realistic information of the statistical properties of the demand.

Experimental work is carried out to estimate the spread in performance parameters for a random sample of thermostats manufactured to the same specification.

6.2 PRINCIPLES OF SIMULATION BY GENERATION OF RANDOM VARIATES.

A requirement often arises for generating a reproducible "random number" sequence for check calculations. Techniques have been developed for the generation of pseudo-random numbers in a deterministic way for use in conjunction with a digital computer[25, 26]. The randomness of such sequences may be detected by subjection to statistical tests [27, 28].

Comprehensive tables of random digits, which may be used in collections as approximations to uniform random variates, are available [29, 30, 31]; also computer programs have been written for the generation of pseudo-random numbers [32, 33].

It is most desirable to sample random variates from empirical probability distributions for the system under consideration. However, in the absence of representative experimental data, it is necessary to sample from a theoretical probability distribution which adequately simulates the behaviour of the process. The accuracy of subsequent numerical calculations will depend upon the sample size.

6.3 MEAN AND VARIANCE OF DEMAND OF ENSEMBLE IN TERMS OF VALUES DUE TO A SINGLE PROCESS.

The "instantaneous demand" x for a cyclic load pattern, when averaged over an interval $I = t_1 + t_2$ (see Fig. 6.1), will have a mean given by:

$$\bar{x} = \frac{x_1 t_1 + x_2 t_2}{t_1 + t_2} \tag{6.1}$$

and a variance given by:

$$\begin{aligned} \text{Var } x &= (\text{average value of } x^2) - \bar{x}^2 \\ &= \frac{x_1^2 t_1 + x_2^2 t_2}{t_1 + t_2} - \bar{x}^2. \end{aligned} \tag{6.2}$$

The results of the analysis of section 5.4 to determine the corresponding parameters for the "indicated demand" may now be checked by numerical simulations using pseudo-random numbers.

6.3.1 MEAN AND VARIANCE OF INDICATED DEMAND DUE TO A SINGLE 2 - POSITION PROCESS.

We may read pseudo-random numbers into the computer line by line from published tables (or generate the sequence on the computer itself), to represent 1,000 events, say, of a single process with incidence X. The integrating interval I is measured commencing at time zero, and X is uniformly distributed in the range 0 to T (see Fig. 6.2).

If the period T is normalised to unity value, then X lies in the

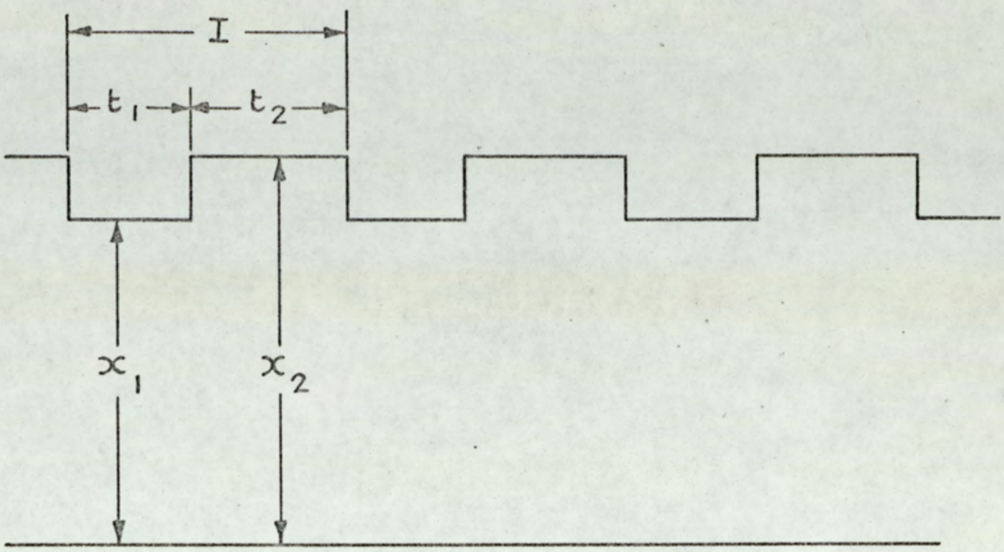


FIG. 6.1 INSTANTANEOUS DEMAND.

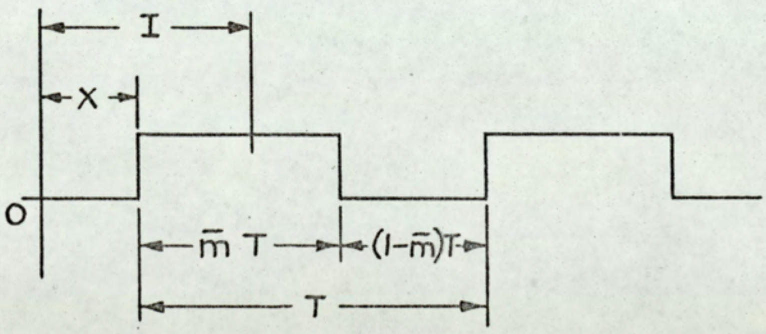


FIG. 6.2 DETERMINATION OF INDICATED DEMAND.

range $0 \leqslant X \leqslant 1$, so that an individual process coming on at X will go off at $X + \bar{m}$. The "indicated demand" D due to an individual process depends upon $\bar{m}$ and upon whether X is less than, equal to, or greater than I/T, where $I/T \leqslant 1$ (see Appendix A.2).

In the following sections the term "demand" will be understood to mean "indicated demand".

For 1,000 events of 1 process,

mean value of $D = \bar{D}$

$$= \frac{1}{1000} \sum_{n=1}^{1000} D_n = \frac{1}{1000I} \sum_{n=1}^{1000} A_n, \tag{6.3}$$

where A_n is the area under the load pattern in the interval 0 to I for the nth event.

Also, expected value of $D^2 = E(D^2)$

$$= \frac{1}{1000} \sum_{n=1}^{1000} D_n^2 = \frac{1}{1000I^2} \sum_{n=1}^{1000} A_n^2, \tag{6.4}$$

and variance of demand = $\sigma_D^2 = E(D^2) - \bar{D}^2$. (6.5)

6.3.2 MEAN AND VARIANCE OF DEMAND FOR AN ENSEMBLE OF N INDEPENDENT SIMILAR 2-POSITION PROCESSES.

If an installation consists of an ensemble of N similar processes with random incidence, a table of 1,000 random numbers will provide $\frac{1000}{N}$ groupings of the N processes.

Let the demand for the ensemble be W.

We will consider a value N = 20 for illustration. The question of the sample size required for given confidence levels will be subsequently investigated.

For 50 groupings of 20 processes,

mean value of demand $W = \bar{W}$

$$= \frac{1}{50I}\left(\sum_{n=1}^{20} A_n + \sum_{n=21}^{40} A_n + - - - - - - - - - + \sum_{n=981}^{1000} A_n\right)$$

$$= 20 \times (\bar{D} \text{ for 1000 events of 1 process}). \tag{6.6}$$

In general, for an ensemble of N similar independent processes, mean value of demand = $N \times \bar{D}$ for 1 process.

$$\text{Now } \underline{\text{Var } W} = E\left[(W - \bar{W})^2\right] \tag{6.7}$$

$$= E\left[\left[(D_1 + D_2 + - - - - - - + D_{20}) - (\bar{D}_1 + \bar{D}_2 + - - + \bar{D}_{20})\right]^2\right]$$

$$= E\left[\left[(D_1 - \bar{D}_1) + (D_2 - \bar{D}_2) + - - - - - - + (D_{20} - \bar{D}_{20})\right]^2\right]$$

$$= E\left[\left[\sum_{i=1}^{k} (D_i - \overline{D}_i)\right]^2\right] \quad \text{where } k = 20$$

$$= E\left[\sum_{i=1}^{k} (D_i - \overline{D}_i)^2 + \sum_{\substack{i=1 \\ i \neq j}}^{k} \sum_{j=1}^{k} (D_i - \overline{D}_i)(D_j - \overline{D}_j)\right]$$

$$= \sum_{i=1}^{k} E(D_i - \overline{D}_i)^2 + \sum_{\substack{i=1 \\ i \neq j}}^{k} \sum_{j=1}^{k} E(D_i - \overline{D}_i) E(D_j - \overline{D}_j)$$

(since if x_i, x_j are independent variates, $E(x_i x_j) = E(x_i) E(x_j)$)

$$= \sum_{i=1}^{k} \text{Var } D_i + 0, \text{ since } E(D_i - \overline{D}_i) = E(D_i) - E(\overline{D}_i)$$

$$= \mu - \mu = 0, \text{ where } E(D_i) = \mu.$$

Then if Var D = $\sigma_D^2 = \mu_2$,

Var W for an ensemble of 20 similar independent processes = $\mu_{2W} = 20\,\sigma_D^2 = 20\mu_2$, (6.8)

where μ_2 is the 2nd moment of the population about the mean for a single process, and μ_{2W} is the corresponding figure for the ensemble.

In general, for an ensemble of N similar independent processes, variance of demand = Nx variance of demand for 1 process.

The above relations connecting $\overline{W}$ and $\overline{D}$, and Var W and Var D respectively, were checked (see computer programs 1 and 2, Appendix A.2, and section 6.5).

Taking a sample of 1,000 random events[31], $\overline{W}$ was computed from the mean of 50 groupings, each of 20 independent processes. Var W was obtained from the relationship Var W = $E\left[W^2\right] - \overline{W}^2$.

A further check was made for 500 groupings by generating 10,000 pseudo-random numbers on the digital computer.

6.4 CONFIDENCE LIMITS AS A FUNCTION OF SAMPLE SIZE.

We must first determine a suitable sample size to obtain a high level of confidence in the demand computations.

Confidence limits will be expressed for a uniform variate x in terms of the sample size drawn from the population. They will then be adapted to apply to calculations of the parameters characterising the demand.

6.4.1 EFFECT OF SAMPLE SIZE OF VARIATE x UPON CONFIDENCE LIMITS.

The standard error of the mean and the variance of a random sample drawn from the population may be investigated for different sample sizes of uncorrelated sets of random numbers.

If a variate x is uniformly distributed in the range 0 to 1, its probability density function is a rectangular pulse given by:

$$f(x) = \begin{cases} 1 \text{ for } 0 \leqslant x \leqslant 1, \\ 0 \text{ elsewhere.} \end{cases}$$

The mean of this distribution is

$$E(x) = \int_0^1 f(x)\, x\, dx = \tfrac{1}{2},$$

and the variance is

$$E(x^2) - \left[E(x)\right]^2 = \int_0^1 f(x)\, x^2\, dx - (\tfrac{1}{2})^2 = \frac{1}{12}.$$

Consider a sample of size n, i.e. $x_1, x_2, x_3 - - - - - - - x_n$.

Then mean of the sample $= \bar{x}_s = \dfrac{1}{n} \sum_{i=1}^{n} x_i.$ (6.9)

Variance of the sample $= m_2 = \dfrac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x}_s)^2.$ (6.1C)

6.4.1.1 STANDARD ERROR OF MEAN OF THE SAMPLE.

By the Central Limit theorem, the distribution of $\bar{x}_s$ may be assumed to approach normal for large n, with mean μ and standard error[34] (i.e. square root of the sampling variance) equal to $\dfrac{\sigma}{\sqrt{n}} = \dfrac{1}{\sqrt{12n}}$, where μ and σ^2 are the mean and variance of the population from which the random sample of n values is drawn, equal to ½ and $\frac{1}{12}$ respectively.

Then the 95% fiducial or confidence limits for the mean of the population corresponding to the sample of size n are given by:[23]

$$\bar{x}_s - 1.96\, \sigma/\sqrt{n} < \mu < \bar{x}_s + 1.96\, \sigma/\sqrt{n},$$

or approximately 95% of the observed means will fall within ½ ± 2 x standard error

$$= \tfrac{1}{2} \pm \frac{2}{\sqrt{12n}}. \quad (6.11)$$

For a sample size n = 50 this approximates to ½ $(1 \pm \frac{1}{6})$, and for n = 500, to ½ $(1 \pm \frac{1}{19})$.

6.4.1.2 STANDARD ERROR OF VARIANCE OF THE SAMPLE.

The variance of the sample variance m_2 can be shown[34] to be:

$$\text{Var}(m_2) = \frac{(n-1)^2}{n^3}(\mu_4 - \mu_2^2) + \frac{2(n-1)}{n^3}\mu_2^2 \quad (6.12)$$

$$= \frac{(\mu_4 - \mu_2^2)}{n} \quad \text{to order } \frac{1}{n}, \quad (6.13)$$

where μ_r is the rth moment of the population about the mean, i.e. $\mu_r = \int_0^1 (x - \frac{1}{2})^r \, f(x) \, dx$, where $f(x) = 1$.

$$\therefore \quad \mu_4 = \int_0^1 (x - \tfrac{1}{2})^4 \, dx = \frac{1}{80}.$$

Now μ_2 = population variance = $\int_0^1 (x - \frac{1}{2})^2 \, dx = \frac{1}{12} = \sigma^2$.

$$\therefore \quad \text{Var}(m_2) \simeq \frac{\mu_4 - \mu_2^2}{n}$$

$$= \frac{\frac{1}{80} - \frac{1}{144}}{n} = \frac{1}{180n} .$$

$$\therefore \quad \text{Standard error}(m_2) = \frac{1}{\sqrt{180\,n}} . \qquad (6.14)$$

Assuming that the distribution of sample variances m_2 can be approximated by the Normal distribution for large n, approximately 95% of the observed values of the variance fall within

$$\frac{1}{12} \pm \frac{2}{\sqrt{180\,n}} . \qquad (6.15)$$

For n = 50, this approximates to $\frac{1}{12}(1 \pm \frac{1}{4})$, and for n = 500, to $\frac{1}{12}(1 \pm \frac{2}{25})$.

6.4.2 APPLICATION OF RESULTS OF SECTION 6.4.1 TO DEMAND D.

The results of section 6.4.1, which apply to random samples of the variate x, may now be adapted to determine the error in the values of demand and variance for one process as calculated for a given sample size n.

6.4.2.1 STANDARD ERROR OF MEAN VALUE OF DEMAND D FOR ONE PROCESS AS DETERMINED FOR A SAMPLE OF n VALUES OF x.

As shown in chapter 5, the mean value of the population D for a uniform distribution of x is equal to $\bar{m}$.

Further, the variance of the population D, which we may call σ_D^2, depends upon $\bar{m}$ and upon the ratio $\frac{I}{T}$. Then if the distribution of the values of D as determined from samples of size n may be assumed to approach normal for large n,

approximately 95% of the calculated means of D will fall within

$$\bar{m} \pm 2 \frac{\sigma_D}{\sqrt{n}} . \qquad (6.16)$$

As an example,

for $\bar{m} = \frac{1}{2}$ and $T \geqslant I \geqslant \frac{T}{2}$,

$$\sigma_D^2 = \frac{1}{12r} \quad (2 - r)\ (2r - 1)^2, \text{ where } \frac{I}{T} = \frac{1}{2r} ,$$

so that approximately 95% of the calculated means of D will fall within

$$\tfrac{1}{2} \quad \pm \ 2 \sqrt{\frac{1}{12rn} \ (2 - r)\ (2\ r - 1)^2} \ , \tag{6.17}$$

which may be evaluated for the value of r concerned.

6.4.2.2 STANDARD ERROR OF VARIANCE OF DEMAND D FOR ONE PROCESS AS DETERMINED FOR A SAMPLE OF n VALUES OF x.

Assuming that the distribution of the calculated sample variances of D can be approximated by the Normal distribution for large n, approximately 95% of these values will fall within

$$\sigma_D^2 \ \pm \ 2 \sqrt{\frac{\mu_4 - \mu_2^2}{n}} \ , \tag{6.18}$$

where $\mu_2 = \sigma_D^2$ and $\mu_4 = \int P\ (D)\ (D - \bar{m})^4\ dD$,

where P (D) is the probability density of D.

μ_4 may be integrated in parts as in chapter 5, where σ_D^2 and P (D) have been derived for all values of $\bar{m}$ in the range $0 \leqslant \bar{m} \leqslant 1$ and for all cases of $\frac{I}{T}$.

For the special case $\bar{m} = \frac{1}{2}$ and $\frac{I}{T} = \frac{1}{2}$, i.e. r = 1, equation (6.17) is identical to equation (6.11). Also P(D) is constant at $\frac{2I}{T} = 1$ (see Chapter 5, section 5.4.1), and equation (6.18) is identical to equation (6.15).

For all other cases, however, P (D) is not contant, and it is necessary to evaluate equations (6.16) and (6.18) for the values of $\bar{m}$ and $\frac{I}{T}$ of interest.

Taking the example quoted above where $\bar{m} = \frac{1}{2}$, but r given by $\frac{1}{2} \leqslant r \leqslant 1$,

$$\sigma_D^2 = \mu_2 = \frac{1}{12r}\ (2 - r)\ (2r - 1)^2 \text{ in the range } I \geqslant \frac{T}{2},$$

$$\text{and } \mu_4 = \int (I - \frac{T}{2})\ \delta\,(D - (1 - \frac{T}{2I})\ )\ \frac{(D - \bar{m})^4}{T}\ dD$$

$$+ \int (I - \frac{T}{2})\ \delta\,(D - \frac{T}{2I})\ \frac{(D - \bar{m})^4}{T}\ dD$$

$$+ \int_{1 - \frac{T}{2I}}^{\frac{T}{2I}} \frac{2I}{T}\ (D - \bar{m})^4\ dD$$

$$= \frac{(I - \frac{T}{2})}{T}\left[(1 - \frac{T}{2I})^4 - 4\bar{m}(1 - \frac{T}{2I})^3 + 6\bar{m}^2(1 - \frac{T}{2I})^2 - 4\bar{m}^3(1 - \frac{T}{2I}) + \bar{m}^4 \right]$$

$$+ \frac{(I - \frac{T}{2})}{T}\left[\left(\frac{T}{2I}\right)^4 - 4\bar{m}\left(\frac{T}{2I}\right)^3 + 6\bar{m}^2\left(\frac{T}{2I}\right)^2 - 4\bar{m}^3\left(\frac{T}{2I}\right) + \bar{m}^4 \right]$$

$$+ \frac{2I}{T}\left[\frac{D^5}{5} - \bar{m}D^4 + 2\bar{m}^2 D^3 - 2\bar{m}^3 D^2 + \bar{m}^4 D \right]_{1 - \frac{T}{2I}}^{\frac{T}{2I}},$$

from which equation (6.18) may be evaluated.

Taking Three Examples:

If $\frac{I}{T} = \frac{1}{2}$, r = 1. Then $\mu_4 = 0 + 0 + (\frac{1}{5} - \frac{1}{2} + \frac{1}{2} - \frac{1}{4} + \frac{1}{16}) = \frac{1}{80}$, as before, so that the 95% confidence limits for the calculated mean and variance of the demand corresponding to the sample size n are given by equations (6.11) and (6.15) respectively.

If $\frac{I}{T} = 1$, $r = \frac{1}{2}$ and $\sigma_D^2 = \mu_4 = 0$.

If $\frac{I}{T} = \frac{3}{4}$, $r = \frac{2}{3}$ and $\sigma_D^2 = \frac{1}{54}$, $\mu_4 = 4.75 \times 10^{-4}$.

Then from equation (6.17), 95% confidence limits for mean of demand corresponding to the sample size $= \frac{1}{2} \pm \frac{2}{\sqrt{54n}}$ ($\simeq \frac{1}{2}(1 \pm \frac{1}{41})$ for n = 500),

and from equation (6.18), 95% confidence limits for variance of demand corresponding to the sample size $= \frac{1}{54} \pm \frac{2}{\sqrt{7576n}}$ ($\simeq \frac{1}{54}(1 \pm \frac{1}{18})$ for n = 500).

For other cases of $\bar{m}$ and $\frac{I}{T}$, similar confidence limits may be evaluated using the appropriate value for P (D).

For the examples considered we may conclude that, for a single process, a sample size of not less than 500 will give sufficient accuracy for estimation of the population mean and variance.

We must now determine a suitable sample size where the demand is due to an ensemble of N independent processes. We will call this demand W.

6.4.2.3 STANDARD ERROR OF MEAN VALUE OF DEMAND W FOR AN ENSEMBLE OF N INDEPENDENT PROCESSES, AS DETERMINED BY n_W GROUPINGS OF THE ENSEMBLE.

It is assumed that the ensemble is made up of N uniformly distributed independent processes, so that mean value of demand

$W = N\,\overline{m}$,

where $\overline{m}$ is the mean value of the demand for one process. Also

variance of demand $W = N\,\sigma_D^{\;2}$,

where $\sigma_D^{\;2}$ is the variance of the demand for one process. Then approximately 95% of the calculated means of W will fall within

$$N\,\overline{m} \pm \frac{2}{\sqrt{n_W}}\sqrt{N}\;\sigma_D. \tag{6.19}$$

As an example, for $\frac{I}{T} = \frac{1}{2}$, N = 20 and n_W = 50 (corresponding to 1,000 values of the random variate x),

$$\sigma_D = \frac{1}{\sqrt{12}},$$

and the 95% fiducial limits corresponding to the sample n_W are

$$10 \pm \frac{2}{\sqrt{n_W}}\sqrt{\frac{10}{6}} \simeq 10\,(1 \pm \frac{1}{27}).$$

6.4.2.4 STANDARD ERROR OF VARIANCE OF DEMAND W FOR AN ENSEMBLE OF N INDEPENDENT PROCESSES AS DETERMINED BY n_W GROUPINGS OF THE ENSEMBLE.

Approximately 95% of the calculated values of the variance of W will fall within

$$N\,\sigma_D^{\;2} \pm \frac{2}{\sqrt{n_W}}\sqrt{\mu_{4W} - (\mu_{2W})^2}, \tag{6.20}$$

where $\mu_{2W} = N\,\mu_2 = N\,\sigma_D^{\;2}$,

and
$$\mu_{4W} = E\left[(W - \overline{W})^4\right]$$
$$= E\left[\left[\sum_{i=1}^{20}(D_i - \overline{D}_i)\right]^4\right].$$

$$\mu_{4W} = E\sum_{i=1}^{20}(D_i - \overline{D}_i)^4 + 4E\sum_{\substack{i=1\\ i \neq j}}^{20}\sum_{j=1}^{20}(D_i - \overline{D}_i)(D_j - \overline{D}_j)^3$$

$$+\,3E\sum_{\substack{i=1\\ i \neq j}}^{20}\sum_{j=1}^{20}(D_i - \overline{D}_i)^2(D_j - \overline{D}_j)^2$$

$$+\,a\,E\sum_{\substack{1=1\\ i \neq j \neq k}}^{20}\sum_{j=1}^{20}\sum_{k=1}^{20}(D_i - \overline{D}_i)^2(D_j - \overline{D}_j)(D_k - \overline{D}_k)$$

$$+\,b\,E\sum_{\substack{i=1\\ i \neq j \neq k \neq 1}}^{20}\sum_{j=1}^{20}\sum_{k=1}^{20}\sum_{1=1}^{20}(D_i - \overline{D}_i)(D_j - \overline{D}_j)(D_k - \overline{D}_k)(D_1 - \overline{D}_1)$$

The first term is $E\sum_{i=1}^{20}(D_i - \overline{D}_i)^4 = 20\,\mu_4$.

The second term may be expressed as $4\sum_{\substack{i=1\\ i \neq j}}^{20}\sum_{j=1}^{20} E\,(D_i - \overline{D}_i)E(D_j - \overline{D}_j)^3$

$= 0$, since $E(D_i - \bar{D}_i) = 0$.

The third term may be expressed as $3 \sum_{i=1}^{20} \sum_{\substack{j=1 \\ i \neq j}}^{20} E(D_i - \bar{D}_i)^2 E(D_j - \bar{D}_j)^2$

$= 3 \times 20 \times 19\, \sigma_D^4 = 1140\ \sigma_D^4$, since $E(D_i - \bar{D}_i)^2 = E(D_j - \bar{D}_j)^2 = \sigma_D^2$.

In both the fourth and fifth terms, the expected value of the product may be replaced by the product of the individual values such as $E(D_j - \bar{D}_j) = 0$, and is therefore equal to zero.

Thus $\mu_{4W} = 20\mu_4 + 1140\ \sigma_D^4$.

For example, for $\frac{I}{T} = \frac{1}{2}$, $\bar{m} = \frac{1}{2}$ ($P(D) = 1$ in this case), approximately 95% of the calculated variances of W for an ensemble of size N will fall within the range

$$N \times \frac{1}{12} \pm \frac{2}{\sqrt{n_w}} \sqrt{\frac{N}{80} + \frac{3N(N-1)}{144} - \left(\frac{N}{12}\right)^2}$$

$$= \frac{N}{12} \pm \frac{2}{\sqrt{n_w}} \sqrt{\frac{N^2}{72} - \frac{N}{120}}$$

$$\simeq \frac{N}{12} \pm \frac{2N}{\sqrt{n_w}} \sqrt{\frac{1}{72}} \quad \text{for } N \not< 10. \qquad (6.21)$$

For $\frac{I}{T} = \frac{1}{2}$, $N = 20$, the 95% confidence limits for the variance of W corresponding to $n_w = 500$ are $\frac{20}{12}(1 \pm 0.126)$, and $\frac{20}{12}(1 \pm 0.04)$ corresponding to $n_w = 5{,}000$.

6.5 CHECK PROGRAMS ON RESULTS OF CHAPTER 5.

The following FORTRAN check programs were carried out on the digital computer, as described in section 6.3.2, to obtain the mean and variance of the demand D for a single process; also of the demand W for an ensemble of 20 processes.

1,000 events of 1 process.

Demand for $\bar{m} = \frac{1}{2}$ with $\frac{I}{T} = 0.25, 0.5, 0.75$.
Demand for $\bar{m} = \frac{3}{4}$ with $\frac{I}{T} = 0.5$.
Demand for $\bar{m} = \frac{1}{4}$ with $\frac{I}{T} = 0.5$.

50 and 500 groupings of 20 similar independent processes.

Demand for $\bar{m} = \frac{1}{2}$ with $\frac{I}{T} = 0.5$. Incidence of processes uniformly distributed.

Detailed results, including pseudo-random number generating program and sample computation programs, are given in Appendix A.2.

Spot checks for groupings of 500 gave numerical results within 3% of the theoretical values, thus verifying the analysis of the model established in chapter 5.

6.6 EXTENSION OF MODEL TO TAKE ACCOUNT OF SPREAD OF PARAMETER VALUES AMONG PROCESS MEMBERS.

6.6.1 Philosophy of Demand Calculations.

The model for the ensemble demand in the d.e.c. condition may be improved by simulating $\bar{m}$, T and x by three independent variables, each of which has a distribution spread over a given range.

It has been shown that $\bar{m}$ and T do not vary independently for individual thermostatically-controlled space heating processes, so that for an ensemble we would expect correlation between the average values of these two parameters. If $\bar{m}$ and T are represented by random variables r_m, r_T respectively, we would then need to know their joint distribution function, since $E\ (r_m\ r_T) \neq E\ (r_m)\ E\ (r_T)$.

In practice, however, $\bar{m}$ and T are determined by the combined effect of diverse factors (room size and thermal capacity, disturbance, thermostat design, location, command setting, etc.). For an <u>ensemble</u> of processes, therefore, we are justified in representing $\bar{m}$ and T by uncorrelated independent variables.

A Monte Carlo approach will be adopted for establishing a model which takes into account these parameter variations.

In the absence of comprehensive data concerning the distribution of the parameters $\bar{m}$ and T, one must regard the philosophy of summing the loads by the use of known theoretical distributions as the best available approach to the problem.

Experimental work has been carried out for a small batch of similar thermostats in order to give some indication of the spreads to be expected in practice. (See section 6.7 and Appendix B).

Considerable spread in parameter values is apparent even when the thermostats function under similar environmental conditions.

In view of the diversity of operational factors affecting $\bar{m}$ and T, it appears logical to simulate their spread by a uniform distribution.

6.6.2 Effect of Transient u upon Steady-State Load Pattern.

Quite apart from the "steady-state" parameter values for a given value of u, the effect of an environmental transient will be to produce a shift in the respective distributions.

Conservative calculations of indicated demand may be based upon the "average-worst" value of u.

The "average-worst" temperature is defined as the average minimum daily temperature, taken over a number of years for a particular location, during the months of December, January and February (see chapter 7).

The transient effect may be simulated by performing a number of computations for small bands of u throughout the entire range of disturbances. The rate of change $\frac{du}{dt}$ will be sufficiently small for u to be regarded as constant for any individual computation. The requirement here is to assign realistic figures to the spreads of $\bar{m}$ and T for different values of u, which must be obtained as a result of operating experience.

6.6.3 Computation of Mean and Variance of Demand W for Ensemble of Similar On-Off Processes, assuming Uniform Distributions for x, $\bar{m}$ and T.

For computation purposes, the spreads of T and $\bar{m}$ for an "average-worst" value of u will be taken as:

$$15 \text{ minutes} \leqslant T \leqslant 45 \text{ minutes}, \tag{6.22}$$
$$\tfrac{1}{2} \leqslant \bar{m} \leqslant \tfrac{3}{4}, \tag{6.23}$$

each parameter being uniformly distributed over its range.

The principle of computation will be the same for extended ranges of T and $\bar{m}$, the demand being deduced as shown in chapter 5.

An ensemble of 20 processes is considered; the analysis is applicable to any ensemble size, however, provided the sample size is sufficiently large for the results obtained therefrom to be meaningful.

Assuming an incidence x of loads which is uniformly distributed in the range $0 \leqslant x \leqslant T$, the mean and variance of the indicated demand due to the ensemble will be computed.

T, $\bar{m}$ and x will each be generated as independent random variates; one grouping will require 20 values of each variate. A sample size of 500 groupings of the ensemble will be taken, requiring a total of

30,000 pseudo-random numbers.

These will be generated on the digital computer by an algorithm described by Naylor, Balintfy, Burdick and Chu.[32]

Procedure for the demand calculations is as follows:

The condition 15 mins. $\leqslant T \leqslant$ 45 mins. corresponds to $\frac{2}{3} \leqslant \frac{I}{T} \leqslant 2$, which for computational purposes will be split into the regions $\frac{2}{3} \leqslant \frac{I}{T} \leqslant 1$ and $1 \leqslant \frac{I}{T} \leqslant 2$ respectively, where I has the fixed value of 30 minutes.

For $\frac{1}{2} \leqslant \bar{m} \leqslant \frac{3}{4}$, the conditions to be considered for $\frac{2}{3} \leqslant \frac{I}{T} \leqslant 1$ are:

(i) $I \geqslant \bar{m}\, T$.

(ii) $(1 - \bar{m})T \leqslant I \leqslant \bar{m}\, T$.

For the purpose of obtaining the demand D due to one process for the condition $1 < \frac{I}{T} \leqslant 2$, the lower limit of $\frac{I}{T}$ is extended down to value zero, when the condition (iii) $I \leqslant (1 - \bar{m})T$ must also be considered.

<u>Region $\frac{2}{3} \leqslant \frac{I}{T} \leqslant 1$.</u>

<u>Case (i)</u>. $I \geqslant \bar{m}\, T$.

If $0 \leqslant x \leqslant I - \bar{m}T$, then demand $D = \frac{\bar{m}T}{I}$.

If $I - \bar{m}\, T \leqslant x \leqslant (1 - \bar{m})\, T$, $D = \frac{I - x}{I}$.

If $(1 - \bar{m})\, T \leqslant x \leqslant I$, $D = 1 - (1 - \bar{m})\, \frac{T}{I}$.

If $I \leqslant x \leqslant T$, $D = \frac{x - (1 - \bar{m})\, T}{I}$.

<u>Case (ii)</u>. $(1 - \bar{m})\, T \leqslant I \leqslant \bar{m}T$.

If $0 \leqslant x \leqslant (1 - \bar{m})\, T$, $D = \frac{I - x}{I}$.

If $(1 - \bar{m})T \leqslant x \leqslant I$, $D = \frac{I - (1 - \bar{m})\, T}{I}$.

If $I \leqslant x \leqslant I + (1 - \bar{m})\, T$, $D = \frac{x - (1 - \bar{m})\, T}{I}$.

If $I + (1 - \bar{m})\, T \leqslant x \leqslant T$, $D = 1$.

<u>Region $1 \leqslant \frac{I}{T} \leqslant 2$.</u>

The demand D for all values of $\frac{I}{T}$ in this region may be computed from (i), (ii) and the extra condition (iii) for $\frac{I}{T} \leqslant 1$.

<u>Case (iii)</u> for $\frac{I}{T} \leqslant 1$.

$I \leqslant (1 - \bar{m})\, T$.

If $0 \leqslant x \leqslant I$, $D = \frac{I - x}{I}$.

If $I \leqslant x \leqslant (1 - \bar{m})T$, $D = 0$.

If $(1 - \bar{m})T \leqslant x \leqslant I + (1 - \bar{m})T$, $D = \dfrac{x - (1 - \bar{m})T}{I}$.

If $I + (1 - \bar{m})T \leqslant x \leqslant T$, $D = 1$.

For $\frac{I}{T} > 1$ we first compute the demand D' due to one process for an integrating interval $I' = I - T$, using the appropriate case (i), (ii) or (iii) above.

Then demand D (for $1 \leqslant \frac{I}{T} \leqslant 2$) $= \dfrac{D'.I' + \bar{m}T}{I}$. (6.24)

(If the spread for T had included cases where $\frac{I}{T} > 2$, the analysis could have been extended, as shown in chapter 5, to obtain the demand).

Having covered all the possible cases which may occur for one process, we now consider the demand W due to an ensemble of 20 processes.

Now demand W_K for the Kth grouping of the ensemble

$= \sum_{n = 20K - 19}^{20K} D_n$, where the demand D_n due to one process is calculated from the sequentially generated random number n.

Thus $W_1 = \sum_{n=1}^{20} D_n$, $W_2 = \sum_{n=21}^{40} D_n$, etc..

For G groupings of the ensemble,

expected value $E(W) = \frac{1}{G} \sum_{K=1}^{G} W_K = \frac{1}{G} \sum_{K=1}^{G} \sum_{n=20K-19}^{20K} D_n$. (6.25)

Also $E(W^2) = \frac{1}{G} \sum_{K=1}^{G} \left(\sum_{n=20K-19}^{20K} D_n \right)^2$ (6.26)

and $\text{Var } W = E(W^2) - (E(W))^2$. (6.27)

The pseudo-random number algorithm generates a sequence of 2-digit numbers between 0 and 1 with uniform distribution. If these are taken in sequence as n_1, n_2, n_3 to represent T, $\bar{m}$ and x respectively, we may scale the numbers to represent the assumed ranges:

Period $T = 15 + n_1 \times 30$ mins. = A mins. (6.28)

$\bar{m} = \dfrac{n_2}{4} + \frac{1}{2}$ = B mins. (6.29)

so that

On-time $= \bar{m}T$ = A x B mins. (6.30)

Incidence $x = n_3 \times T$. (6.31)

The computer program DEMAND for determining the mean and variance of the demand for the ensemble of 20 similar independent processes is given in Appendix A.3. Also shown is the subroutine RANDOM for generating the pseudo-random number sequence, which requires initiating by an odd integer.

With a starting integer of 1, the results obtained were:

Mean value of demand = 13.23.

Variance of demand = 0.479.

Repetition of the program, with starting integers for the subprogram of 5 and 7 respectively, in each case gave values for the demand mean and variance within 3% of the above corresponding figures.

6.6.4 ALTERNATIVE DISTRIBUTIONS FOR $\bar{m}$ AND T.

Although we have assumed $\bar{m}$ and T to be uniformly distributed, similar computations may be effected if their spreads are described by other distributions. Specific techniques are available for generating random variates from some of the better-known probability distributions[26, 32, 33] by the use of a digital computer.

By the property of the Central Limit theorem, we may justifiably compare the foregoing results with those obtained by assuming $\bar{m}$ and T to be normally distributed.

6.6.5 COMPUTATION OF MEAN AND VARIANCE FOR ENSEMBLE DEMAND ASSUMING A NORMAL DISTRIBUTION FOR $\bar{m}$ AND T AND A UNIFORM DISTRIBUTION FOR x.

The probability density and cumulative distribution functions for the uniform and normal distributions are shown in Fig. 6.3.

Normally distributed random variates may be generated on a digital computer by taking the sum of K uniformly distributed random variates $r_1, r_2, ---r_i, ---r_K$, where $0 \leqslant r_i \leqslant 1$.

A single value of a normally distributed variate x_n is given by[32]:

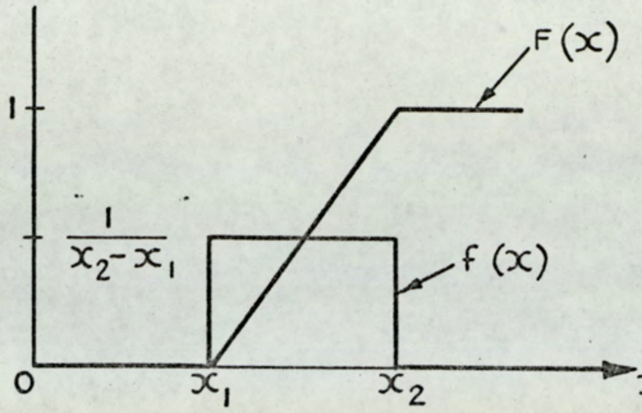
$$x_n = \sigma_x \left(\frac{12}{K}\right)^{\frac{1}{2}} \left(\sum_{i=1}^{K} r_i - \frac{K}{2} \right) + \mu_x, \tag{6.32}$$

where μ_x and σ_x^2 are the mean and variance respectively of the desired distribution, and K uniform variates are summed over the interval 0 to 1.

Taking K = 12 reduces the above expression to:

$$x_n = \sigma_x \left(\sum_{i=1}^{12} r_i - 6\right) + \mu_x. \tag{6.33}$$

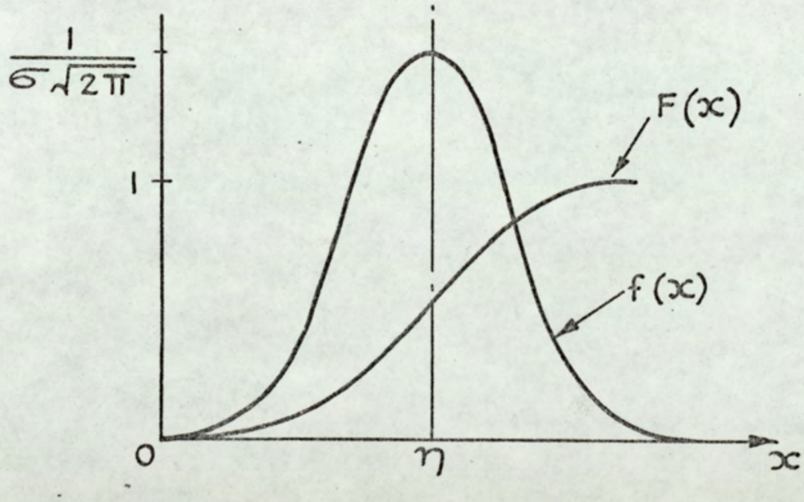
Now maximum value of $x_n = \sigma_x (12 - 6) + \mu_x = x_{n\,max}$.



$$f(x) = \text{density function} = \begin{cases} \dfrac{1}{x_2 - x_1} & \text{for } x_1 \leqslant x \leqslant x_2 \\ 0 & \text{elsewhere.} \end{cases}$$

$F(x)$ = distribution function

$$f(x) = \frac{d\,F(x)}{dx}$$



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \quad \exp\left(-(x-\eta)^2/2\sigma^2\right)$$

$$F(x) = \frac{1}{2} + \text{erf} \quad \frac{x-\eta}{\sigma}$$

FIG. 6.3 PROBABILITY DENSITY AND CUMULATIVE DISTRIBUTION FUNCTIONS.

and minimum value of $x_n = \sigma_x(0 - 6) + \mu_x = x_{n\ min}$.

If the normal variate distribution is truncated at 0 and 1 to give the same range as for the uniform variate, then $\mu_x = \frac{1}{2}$, and the only value of σ_x compatible with this requirement is given by:

$$x_{n\ max} = 6\ \sigma_x + \tfrac{1}{2} = 1 \tag{6.34}$$

or $$x_{n\ min} = -6\ \sigma_x + \tfrac{1}{2} = 0, \tag{6.35}$$

whence $\sigma_x = \frac{1}{12}$.

The resulting distribution is somewhat peaky (the maximum value of the normal density function being $\frac{1}{\sigma_x\sqrt{2\pi}} \doteq 4.75$, compared with unity for the uniform density function). It will therefore prove useful as a standard of comparison for results obtained using other distributions.

The normal distribution thus generated truncates at $\pm\ 6\ \sigma_x = \frac{1}{2}$; over 99% of the generated normal variates lie within the $\pm\ 3\ \sigma_x$ limits, beyond which the computation becomes unreliable. A choice of a less peaky distribution with truncation at 0 and 1 leads to a loss of the tails.

Choice of values for K much in excess of 12 will lead to excessive computation time in generating normal variates.

The computer program DNORM for determining $\overline{W}$ and Var W is shown in Appendix A.4.

A uniform distribution has been assumed for the incidence X of the individual processes comprising the ensemble, where $0 \leqslant X \leqslant T$. The parameters T and $\overline{m}$ have been taken as normally distributed over the ranges 15 mins. $\leqslant$ T $\leqslant$ 45 mins. and $\frac{1}{2} \leqslant \overline{m} \leqslant \frac{3}{4}$, and centred about T = 30 mins. and $\overline{m} = \frac{5}{8}$ respectively. Normal random variates between 0 and 1 have been generated for $\sigma_x = \frac{1}{12}$ by the use of subroutines RANDOM and NORMAL; the resulting variates have then been scaled as before to fit the required distributions.

500 groupings of the demand W for the ensemble of 20 processes have again been considered, requiring a total generation of 10,000 (1 + 2 x 12) = 250,000 pseudo-random numbers.

The results obtained were:

With starting value <u>N = 1</u> for initiating random variable generation,

<u>Mean value</u> of indicated demand for ensemble of 20 processes = 12.99.

Variance of indicated demand for ensemble of 20 processes = 0.505.

With starting value $N = 5$ to obtain a new set of random variables, the results were $\overline{W} = 12.96$ and Var $W = 0.524$.

For $\overline{m}$, T and X all uniformly distributed the results were:

$\overline{W} = 13.23$ and Var $W = 0.479$.

For purposes of comparison, we will take $\overline{W} = 12.99$ and Var $W = 0.505$ as the reference results.

Then for $\overline{m}$, T and X all uniformly distributed:

error in $\overline{W} = 1.8\%$,

and error in Var $W = 5.2\%$.

6.7 EXPERIMENTAL DETERMINATION OF PARAMETER SPREADS FOR LINE-VOLTAGE WALL-MOUNTED ROOM THERMOSTATS.

Details of tests for assessing thermostat performance when operating within standardised environmental chambers, are given for the United States in N.E.M A. and A.S.H.R.A.E. publications [35, 36], and in papers by Roots[37] and by Roots, Woods, Wells and Tull[38]. Comparable tests in the United Kingdom are detailed in a British Standard Specification [39]; field trials on sample thermostats produced by leading British manufacturers have been conducted by Area Electricity Boards.

It was decided to determine the spread in $\overline{m}$ and T, under standardised test conditions, for a random sample of twelve thermostats manufactured to the same specification. Although the sample size could have been increased by an analogue simulation, it was considered more representative of actual working conditions to test the thermostats themselves.

A wooden test chamber some 3½ ft. x 3 ft. x 2 ft. was thermally lagged with polystyrene and placed within a room maintained at a nominally constant temperature. This arrangement was preferred to creating an artificial environment by encasing the cabinet in a large water jacket.

The thermostat under test was mounted vertically in the chamber, and controlled the internal temperature by switching a small resistance heater.

Details of the experiments and the results achieved are given in Appendix B.

6.7.1 DISCUSSION OF TEST RESULTS.

It can be argued firstly, that the test conditions do not simulate fully the operational environment, and secondly, that the sample size is too small for the results to be meaningful.

If a diversity of loads subjected to a range of environmental temperatures is to be considered, demand estimates must, of necessity, be based upon past records. However for specific installations where similar two-position processes with a common environment are controlled to the same command temperature, the results will be of the right order for demand calculation purposes. It will be appreciated that, by its nature, a room thermostat is a difficult type of product from which to collect test results on a large batch basis. The reason for this is that equipment typically used by the manufacturers, e.g. a N.E.M.A. cabinet[36], does not have a high through-put rate of thermostats on test. Consequently, statistically accurate results are hard to obtain over a time scale extending into weeks. However, it is to be expected that the spread of parameter values obtained for the thermostats tested is typical of the performance of this design under standardised test conditions. No significance can be attributed, of course, to standard deviation values obtained for this small sample.

6.8 SUMMARY.

The validity of the analysis of the basic demand model developed in Chapter 5 for an ensemble of similar thermostatically-controlled processes has been verified by numerical simulation checks. Care has been taken to derive and to utilise an adequate sample size so that the results obtained for the demand mean and variance are representative of the population.

Extending the problem to the case where $\bar{m}$ and T can have a range of values, it is necessary practically to obtain the spreads in these two parameters for a given installation. It is shown in detail how the demand mean and variance may then be computed.

It appears logical to base the readings for $\bar{m}$ and T upon an "average-worst" winter daily temperature for the particular location, as defined in section 6.6.2. The choice of the set demand at which control action should be taken is an economic matter, as investigated in Chapters 7 and 8.

Justification is given for simulating the incidence, duty ratio and period each by independent uniform variates. As a standard of reference, comparison of the demand computations is made with a simulation where $\bar{m}$ and T are represented as normal variates over the same corresponding ranges as for a uniform distribution.

The results are as follows:

The mean value of indicated demand where $\bar{m}$ and T are uniformly distributed, agrees closely with the mean when these two parameters are normally distributed.

The standard deviation for the variates corresponding to normal distributions of $\bar{m}$ and T has been taken as $\frac{1}{12}$, compared with $\frac{1}{\sqrt{12}}$ where $\bar{m}$ and T are both uniformly distributed. However, the standard deviations of the indicated demands for these two cases differ by not more than about 5 per cent.

Thus, for simulation purposes, the choice of the Normal distribution assumed for $\bar{m}$ and T is justified for this ensemble size.

The appeal of the work in this chapter is that account has been taken of the distributions of $\bar{m}$ and T in determining the statistical properties of the demand. The restrictions of the analysis of Chapter 5 are therefore overcome, resulting in a simulation which is representative of a practical situation.

CHAPTER 7.

"MINIMUM-COMFORT" CRITERION OF OPTIMISATION AND DEMAND CONTROL FOR THERMOSTATICALLY-CONTROLLED TWO-POSITION PROCESSES.

7.1 INTRODUCTION.

An obvious criterion of optimisation when applying demand control to two-position thermostatically-controlled processes is one of minimum operating cost. For process quality control applications, however, an over-riding requirement may be the maintenance of the temperature excursions within prescribed limits for a range of disturbance inputs.

Although the chief merit of thermostats is their low cost, the operation of discontinously-controlled processes results in temperature fluctuations.

These are due to:

1. Hysteresis of the thermostat.

   The effect of hysteresis can be adequately compensated by the incorporation of secondary feedback [45].

2. Transit delay of process.

   Oscillations of temperature are caused primarily by the transit delay which is inherent in the space heating process.

Reduction of offset errors and of the amplitude and period of temperature oscillations may be effected by:

(i) Use of multi-position controllers[47].

(ii) Indirect control by means of a model with small transit delay[48].

(iii) Pulse width modulation [48 †].

(iv) Derivative discontinuous control [49].

Offset errors may be reduced by anticipatory switching of the power to the heaters by temperature-dependent thermistors, which automatically adjust t_o and t_p as the ambient temperature changes [65].

For domestic space heating, radical departure of the temperature θ at the l.p.i. from the command value θ_r implies discomfort. A minimum-comfort criterion can be that θ should not fall below a designated minimum θ_n and not exceed a maximum θ_m. The d.e.c. mean value $\overline{\theta}$ will be greater than θ_n, which implies energy wastage proportional to $\overline{\theta} - \theta_n$. Thus a minimum value of $\theta_D = \theta_m - \theta_n$ is required, corresponding to a minimisation of the period t_q [19, 40, 41].

In this investigation a design requirement will be that the temperature θ at the l.p.i. is restricted to the range $70^{\circ}F. \leqslant \theta \leqslant 80^{\circ}F.$ The range of disturbances u over which this criterion must be satisfied has been taken as $25^{\circ}F. \leqslant u \leqslant 50^{\circ}F.$, based upon the "average-worst" winter daily temperature (see section 6.6.2), as charted for selected areas of the United Kingdom [42]. The energy requirements to meet this specification will depend, of course, upon the particular application, (size of room, heat loss coefficients, etc.). The amplitude of temperature oscillation for a discontinously-controlled process increases as the ratio transit delay/process time constant is increased (equation 7.5). The design will be based upon a ratio of 0.1, which may be regarded as typical of space heating applications. [48]

It will be found that conventional thermostatic control is barely adequate to satisfy the above specification. The amplitude of temperature oscillation is therefore suitably reduced by a design incorporating derivative feedback. The effect of this modification upon the indicated demand is investigated.

For space heating processes where the need to maintain close temperature limits is not critical, demand control may be applied advantageously.

A strategy for control by computer of the demand due to two-position space heating processes is presented.

† Practical verification is provided in a project supervised by the author:

Obidi, U.E.:'P.W.M. Control of Electric Space Heating', <u>M.Sc. Project, Aston University</u>, October 1970.

7.2 JUSTIFICATION OF DESIGN SPECIFICATIONS.

Table 7.1 gives an extract from tables of average temperature for Birmingham/Edgbaston, published by the Meteorological Office[42] as a result of observations over a period of thirty years.

Analysis of Meteorological Office records[46] gives the number of days when the maximum temperature is less than a stated value, i.e. below that value all day. This information (excluding the night-time), is presented in Table 7.2 for maximum temperatures of interest in the months of January, February and December, as recorded at the Kew Observatory in the period 1955 - 69.

If these observations are taken as typical for Southern and Midland regions of the United Kingdom, a space heating design may reasonably be based upon the winter daily temperature variation specified in section 7.1.

Short-term drops below 25° F. will not unduly affect space heating ensembles of large thermal capacity unless close control of process temperature is required. Temporary raising of the thermostat command temperature may lead to an increase in the maximum demand, when the provision of non-electric standby heating equipment may be justified economically.

Period 1921 - 1950	Average of Daily (in 24 hours) Max.	Min.	Average of Highest each month.	Average of lowest each month.	Absolute extremes during entire period Max	Min.
	degrees F.					
January	42	35	53	25	59	11
February	43	35	53	25	59	13
March	49	37	61	27	70	19
April	53	40	66	31	75	28
May	60	45	73	35	85	30
June	66	50	79	42	87	37
July	69	54	81	47	92	44
August	68	53	79	46	91	42
September	63	50	74	41	83	37
October	55	45	66	34	79	28
November	47	39	57	30	57	25
December	43	37	53	26	58	20
Year	55	43	85*	21**	92	11
No. of years	30	30	30	30	30	30

* Average of highest each year. ** Average of lowest each year.

TABLE 7.1

Max. Daily Temperature.	No. of days in period 1955 - 69 when temperature is below the maximum stated in Column 1 all day (excluding night time).		
	January	February	December
35^o F.	35	22	31
30^o F.	7	9	7
25^o F.	-	1	-

TABLE 7.2

7.3 PERFORMANCE INDICES FOR CLOSED-LOOP DISCONTINUOUSLY-CONTROLLED ELECTRIC SPACE-HEATING PROCESSES.

Mathematical models for a wide range of processes involving electroheat, have been expressed by a number of investigators [43, 44] in terms of one significant time delay and one or two major time constants.

These models take the form:

$$G_1(s) = FG\,\exp(-sL)/(1 + sT_1(m)) \tag{7.1}$$

or

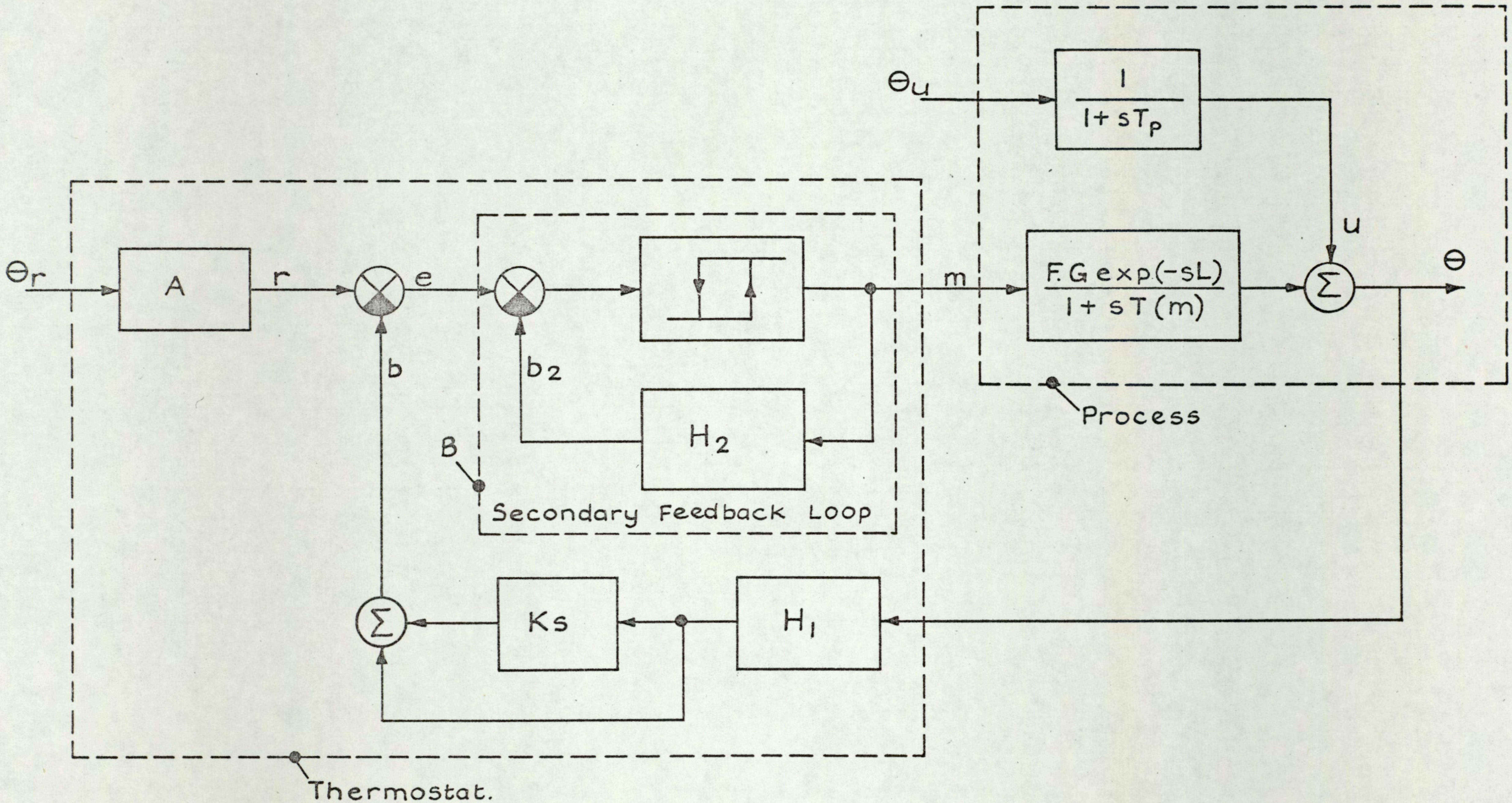
$$G_2(s) = FG\,\exp(-sL)/(\,(1 + sT_1(m))\,(1 + sT_2(m))\,), \tag{7.2}$$

where L is the transit delay which arises as a result of the distributed nature of the process. The parameters F, G and T(m) are defined in section 5.2.

Fig. 7.1 gives a block diagram representation of a discontinuously-controlled electric space heating process with a major mode-dependent time constant T(m) [38], where the time constant of the heating elements is negligible in comparison with that of the process. The secondary feedback loop may be optimally designed [45] so that the feedback b_2 compensates for the deadspace of the discontinuous control element. A derivative component of output has been added to the primary feedback in order to effect a reduction in the amplitude of temperature oscillations.

7.3.1 ON-OFF CONTROL.

It will be assumed that T(m) is mode independent and equal to T; also that the process performance is not significantly affected by the rate of change of the disturbance input.



<u>FIG. 7.1</u> <u>CLOSED-LOOP DISCONTINUOUS CONTROL OF ELECTRIC SPACE-HEATING PROCESSES.</u>

Fig. 7.2 gives the temperature/time response to a step input of power for the disturbance condition u = 0. When u has a finite value, the heating and cooling trajectories are asymptotic to $\theta = u + FG$ and $\theta = u$ respectively, as in Fig. 7.3.

On-off control of electroheat processes has been analysed elsewhere [19†], so that the results will be briefly quoted.

Maximum value of process temperature θ (t) is

$$\theta_m = \theta_r \exp(-L/T) + (u + FG)(1 - \exp(-L/T)). \tag{7.3}$$

Minimum value of θ is

$$\theta_n = \theta_r \exp(-L/T) + u(1 - \exp(-L/T)). \tag{7.4}$$

The amplitude of oscillation is

$$\theta_D = \theta_m - \theta_n = FG(1 - \exp(-L/T)). \tag{7.5}$$

The command value of θ is

$$\theta_r = \theta_n \exp(-t_1/T) + (u + FG)(1 - \exp(-t_1/T)) \tag{7.6}$$

$$= \theta_m \exp(-t_2/T) + u(1 - \exp(-t_2/T)). \tag{7.7}$$

The on-time is

$$t_o = L + t_1$$

$$= L + T \ln \left[(u + FG - \theta_n) / (u + FG - \theta_r) \right]. \tag{7.8}$$

The off-time is

$$t_p = L + t_2$$

$$= L + T\ln \left[(\theta m - u) / (\theta r - u) \right]. \tag{7.9}$$

The period is

$$t_q = 2L + T\ln \left[\left(\frac{\theta m - u}{\theta_r - u} \right) \left(\frac{u + FG - \theta n}{u + FG - \theta r} \right) \right]. \tag{7.10}$$

The d.e.c. mean value of θ (t) is

$$\overline{\theta} = u + \overline{m}\, FG, \text{ where } \overline{m} = t_o/t_q. \tag{7.11}$$

The offset error is

$$y = \theta_r - \overline{\theta}. \tag{7.12}$$

Zero offset will be attained at $\overline{m} = \frac{1}{2}$, when t_q has its minimum value $\check{t}_q$. This will occur at a command temperature $\theta_r = u + \frac{FG}{2}$.

7.4 DESIGN REQUIREMENTS TO SATISFY MINIMUM-COMFORT CRITERION.

7.4.1 On-Off Control.

The command θ_r may be fixed at the median value of 75° F. to

† The general case, where the plant has the property of mode dependence and the controller is hysteretic, has been analysed by Gönenç[48].

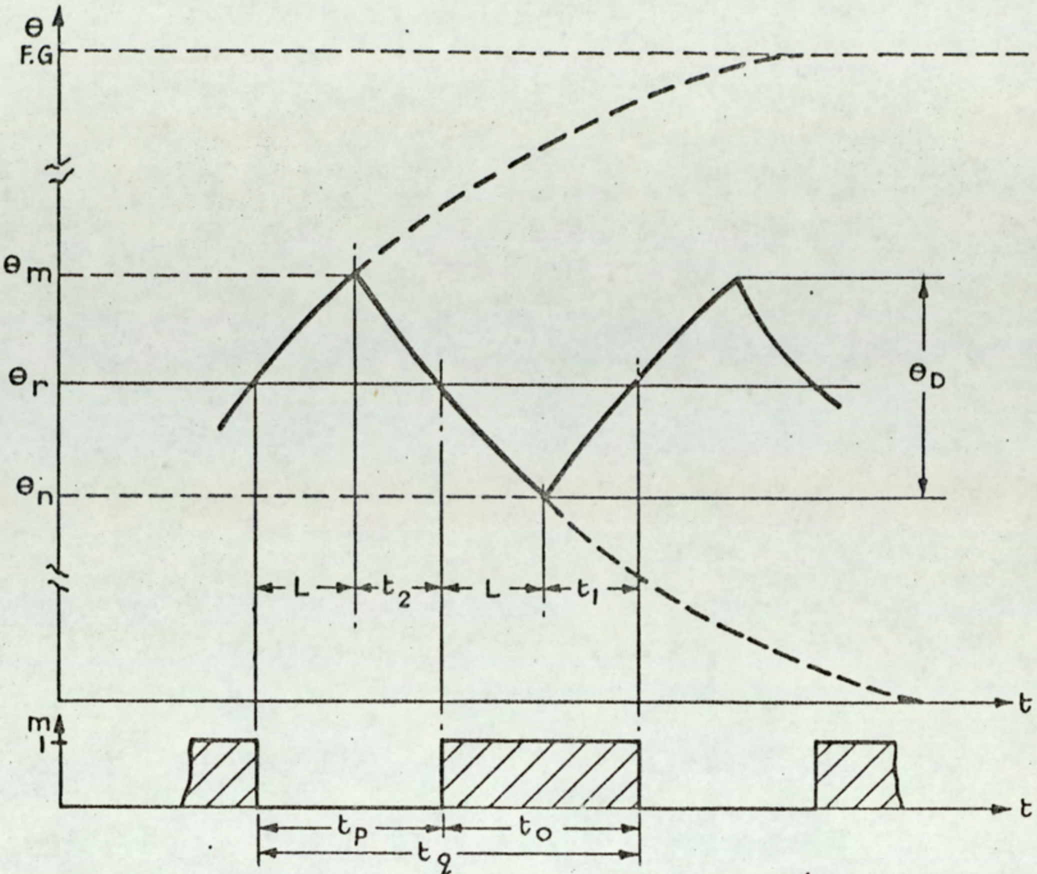


FIG. 7.2 RESPONSE TO STEP OF POWER (U = O).

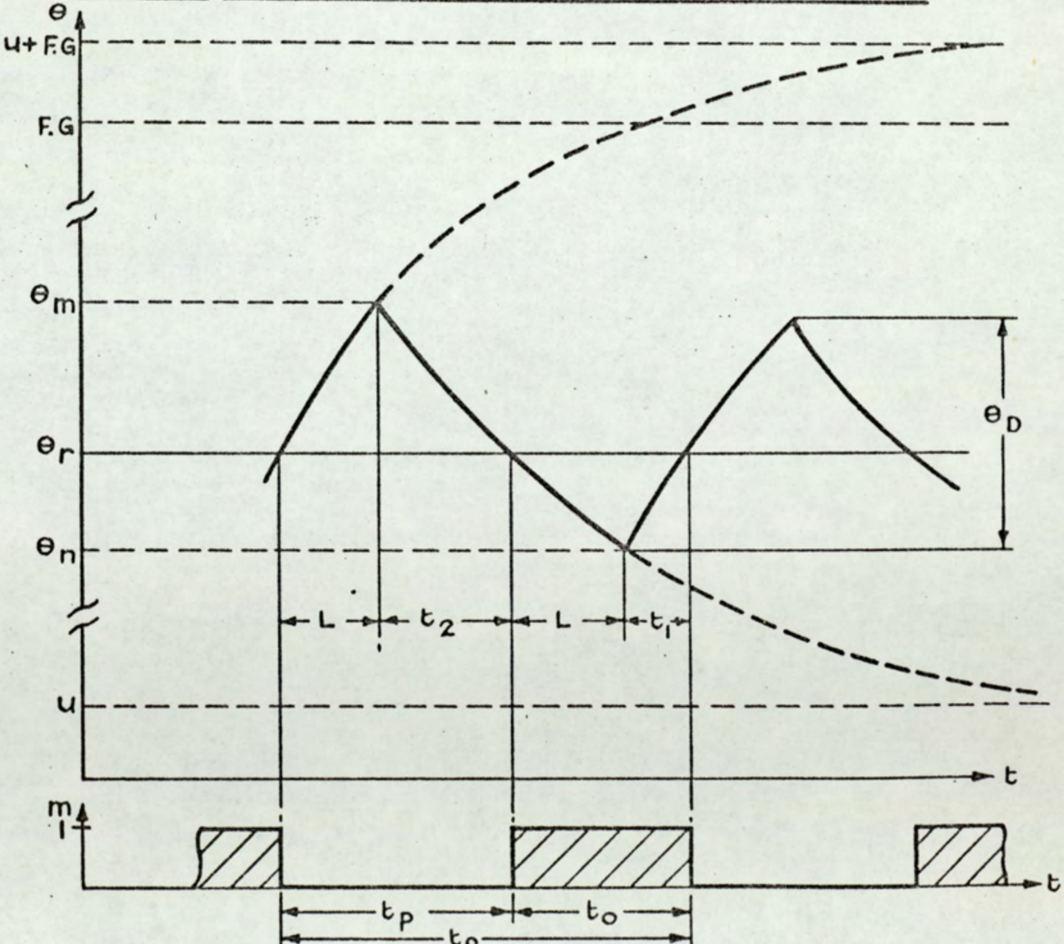


FIG. 7.3 RESPONSE TO STEP OF POWER (U POSITIVE).

give θ in the range 70° F. $\leqslant \theta \leqslant 80^\circ$ F.

The design will be based upon an L/T ratio of 0.1. If we make $FG = 2\,\theta_r = 150^\circ$ F., we see from equation (7. 5) that $\theta_D = 14.3^\circ$ F. Then $\overline{\theta} = \theta_r = 75^\circ$ F. only for a disturbance value $u = 0^\circ$ F.

It is required, however, to obtain a value $\overline{m} = \frac{1}{2}$ for a disturbance in the middle of the range specified for u. Also θ_D must satisfy the specification; this may be accomplished by reduction of G.

We will let $u_1 = 50^\circ$ F., $u_2 = 25^\circ$ F., and the median $u_3 = 37\frac{1}{2}^\circ$ F., at which value it is desired to make the offset zero.

Then $\theta_r = u_3 + \frac{FG}{2} = 75^\circ$ F., whence $FG = 75^\circ$ F.

Then $\left.\dfrac{\theta_r}{u + FG} = \dfrac{75}{125} = \dfrac{3}{5}\right|_{u = 50}$

$\left.\dfrac{\theta_r}{u + FG} = \dfrac{75}{100} = \dfrac{3}{4}\right|_{u = 25.}$

The design condition, therefore, is that $\frac{3}{5} < \dfrac{\theta_r}{u + FG} < \frac{3}{4}$.

The thermal constant F may be taken as unity without loss of generality. For ease of further calculation, exp (-L/T) is taken as 0.9 (when L/T = 0.1053). Applying equations (7.3) to (7.12) gives:

At $u_1 = 50^\circ$ F.

$\theta_m = 80.0^\circ$ F., $\theta_n = 72.5^\circ$ F., $\theta_D = 7.5^\circ$ F.

$\frac{t_o}{T} = 0.154$, $\frac{t_p}{T} = 0.288$, $\frac{t_q}{T} = 0.442$.

$\overline{m} = 0.349$, $\overline{\theta} = 76.2^\circ$ F. Offset $= -1.2^\circ$ F.

At $u_2 = 25^\circ$ F.

$\theta_m = 77.5^\circ$ F., $\theta_n = 70^\circ$ F., $\theta_D = 7.5^\circ$ F.

$\frac{t_o}{T} = 0.288$, $\frac{t_p}{T} = 0.154$, $\frac{t_q}{T} = 0.442$.

$\overline{m} = 0.651$, $\overline{\theta} = 73.8^\circ$ F. Offset $= +\,1.2^\circ$ F.

At $u_3 = 37\frac{1}{2}^\circ$ F.

$\theta_m = 78.75^\circ$ F., $\theta_n = 71.25^\circ$ F., $\theta_D = 7.5^\circ$ F.

$\frac{t_o}{T} = 0.201$, $\frac{t_p}{T} = 0.201$, $\frac{t_q}{T} = 0.402$.

$\overline{m} = 0.5$, $\overline{\theta} = 75^\circ$ F. Offset = 0.

The specification for θ_D is therefore satisfied. An undesirable feature of reducing G, however, is the increase in start-up time from the "cold" condition.

A design will now be effected using derivative discontinuous control, and the effect upon the indicated demand will be considered.

7.4.2 DERIVATIVE DISCONTINUOUS CONTROL.

In the control diagram Fig. 7.4, the derivative feedback is KS, where K is the derivative feedback coefficient. The secondary feedback loop is optimally designed to compensate for the discontinuous control element deadspace, i.e.

$$m(t) = 0 \Big|_{e < 0} \qquad = 1 \Big|_{e > 0} .$$

The reference and feedback transfer coefficients are taken as unity, and the gain figure G includes the thermal attenuation F. The disturbance is assumed steady or slowly varying.

An analysis due to Roots and Shridhar[49] gives the following relationships:

$$\text{Let } K/T = \alpha$$
$$L/T = \beta$$
$$\frac{\theta_r}{G}\Big|_{u=0} = \rho$$
$$\sigma = \exp(-\beta)$$
$$\beta = \ln(1/\sigma)$$
$$\gamma = 1 - \sigma .$$

1). If $\dfrac{\theta_r}{G} = \frac{1}{2}$ and $\alpha = \dfrac{K}{T} = \dfrac{1 - \exp(-\beta)}{2} = \dfrac{\gamma}{2}$,

and if $u = 0$, i.e. $\rho = \frac{1}{2}$,

then $t_q = 2L$ and $\theta_D = \theta_m - \theta_n = G \dfrac{\gamma}{1+\sigma}$, which tends to $\dfrac{G\gamma}{2}$ as $\sigma \rightarrow 1$, or $\dfrac{L}{T} \rightarrow 0$.

Comparing these results with equations (7.5) and (7.10) for conventional on-off control, we see that the effect of adding a derivative term to the primary feedback is to reduce both θ_D and t_q.

2). K cannot be increased arbitrarily without introducing instability. θ is no longer responsive to the command ρ for derivative feedback $\alpha > 1$, and for $\alpha > \dfrac{\gamma}{2}$ control of the process is lost for a range of commands symmetrically distributed about $\rho = \frac{1}{2}$.

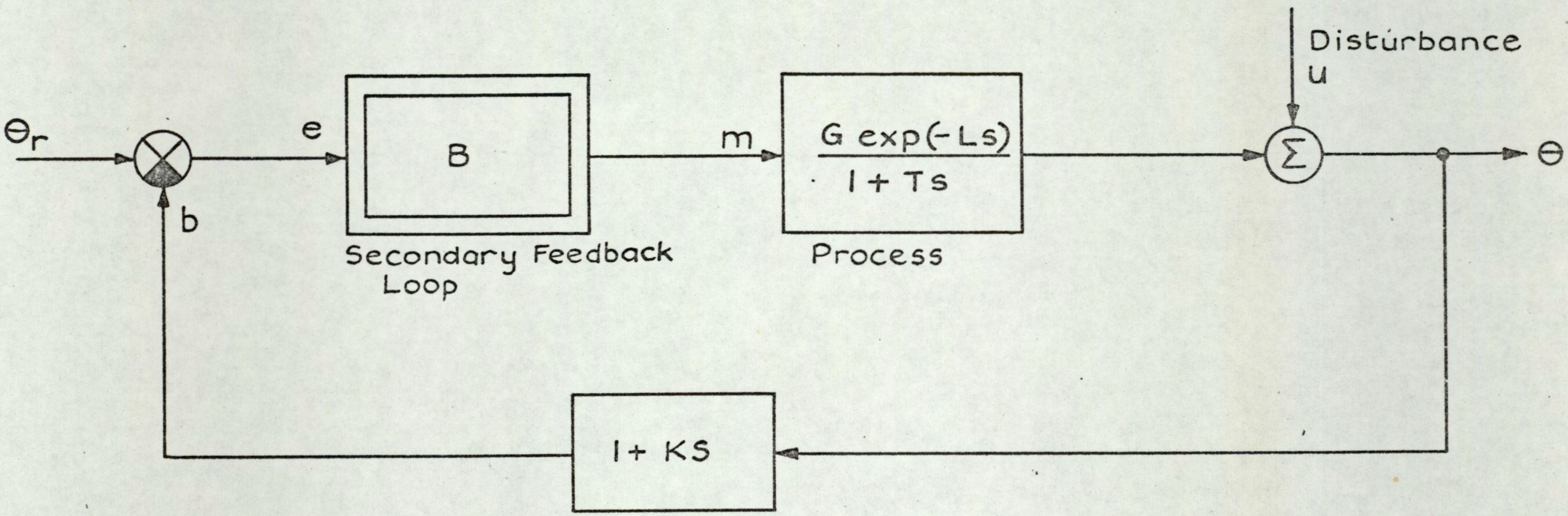


FIG. 7.4 BLOCK DIAGRAM FOR DERIVATIVE DISCONTINUOUS CONTROL.

3). For any command ρ, the droop and period are minimised if α is chosen so that

$$\alpha = \rho\gamma\Big|_{\rho \leqslant \frac{1}{2}} \quad \text{or } \alpha = \gamma(1-\rho)\Big|_{\rho \geqslant \frac{1}{2}} \; .$$

Roots and Shridhar considered the case where the disturbance u was constant or controlled within close limits, as for process control within a factory.

However the results may be extended to the situation where u can have a range of values, as in space heating.

Taking the parameters of the previous analysis:
$u_1 = 50^\circ$ F., $u_2 = 25^\circ$ F., $u_3 = 37\frac{1}{2}^\circ$ F., $\theta_r = 75^\circ$ F., $G = 75^\circ$ F..
For controllability, $u < \theta_r < u + G$.
Now $\theta_r = u_1 + \rho_1 G$, whence $\rho_1 = \dfrac{\theta_r - u_1}{G} = \dfrac{1}{3}$.

$\theta_r = u_2 + \rho_2 G$, whence $\rho_2 = \dfrac{\theta_r - u_2}{G} = \dfrac{2}{3}$.

Then $\alpha_1 = \rho_1\gamma\Big|_{\rho_1 = \frac{1}{3}} = \frac{1}{3} \times 0.1 = 0.0333.$

$\alpha_2 = \gamma(1-\rho_2)\Big|_{\rho_2 = \frac{2}{3}} = \frac{1}{3} \times 0.1 = 0.0333.$

The process is controllable since $\alpha = \alpha_1 = \alpha_2 < \frac{\gamma}{2}$.

The relevant parameters may now be evaluated.

<u>At $u_1 = 50^\circ$ F.</u>

$\alpha = 0.0333$, $\rho_1 = \frac{1}{3}$, $\gamma = 0.1$, $\sigma = 0.9$,

so that $\frac{\alpha}{\gamma} \leqslant \rho_1 \leqslant (1 - \frac{\alpha}{\gamma})$,

when the following equations apply:

$$\frac{\theta_m - u_1}{G} = \frac{\rho_1\sigma}{1-\alpha} + (1 - \frac{\sigma}{1-\alpha}) \tag{7.13}$$

$$\frac{\theta_n - u_1}{G} = \frac{\rho_1\sigma}{1-\alpha} \tag{7.14}$$

$$\frac{t_o}{T} = \ln\left[\frac{1-\alpha-\rho_1\sigma}{\sigma(1-\rho_1)}\right] \tag{7.15}$$

$$\frac{t_p}{T} = \ln\left[\frac{\rho_1\sigma + \gamma - \alpha}{\rho_1\sigma}\right], \tag{7.16}$$

whence $\theta_m = 78.5^\circ$ F., $\theta_n = 73.3^\circ$ F., $\theta_D = 5.2^\circ$ F.

$\frac{t_o}{T} = 0.1053$, $\frac{t_p}{T} = 0.2005$, $\frac{t_q}{T} = 0.3058$.

$\overline{m} = \frac{t_o}{t_q} = 0.345$, $\overline{\theta} = u_1 + \overline{m}\,G = 75.9^\circ$ F.

Offset = $-\ 0.9^\circ$ F.

At $u_2 = 25^\circ$ F.

$\alpha = 0.0333$, $\rho_2 = \frac{2}{3}$, $\gamma = 0.1$, $\sigma = 0.9$,

so that $(1 - \frac{\alpha}{\gamma}) \leqslant \rho_2 \leqslant 1$,

when the following equations apply:

$$\frac{\theta_m - u_2}{G} = \frac{\rho_2 \sigma}{1 - \alpha} + (1 - \frac{\sigma}{1 - \alpha}) \tag{7.17}$$

$$\frac{\theta_n - u_2}{G} = \frac{\rho_2 \sigma^2}{1 - \alpha} + \sigma (1 - \frac{\sigma}{1 - \alpha}) \tag{7.18}$$

$$\frac{t_o}{T} = \ln \left[\frac{\sigma^2 (1 - \rho_2) + \gamma (1 - \alpha)}{\sigma (1 - \rho_2)} \right] \tag{7.19}$$

$$\frac{t_p}{T} = \ln \left[\frac{1}{\sigma} \right] , \tag{7.20}$$

whence $\theta_m = 76.8^\circ$ F., $\theta_n = 71.6^\circ$ F., $\theta_D = 5.2^\circ$ F.

$\frac{t_o}{T} = 0.2005$, $\frac{t_p}{T} = 0.1053$, $\frac{t_q}{T} = 0.3058$.

$\overline{m} = 0.655$, $\overline{\theta} = u_2 + \overline{m}\,G = 74.1^\circ$ F.

Offset = $+\ 0.9^\circ$ F.

At $u_3 = 37\frac{1}{2}^\circ$ F.

$$\theta_r = u_3 + \rho_3 G, \text{ whence } \rho_3 = \frac{\theta_r - u_3}{G} = \tfrac{1}{2} .$$

In order to maintain controllability at the extremes u_1 and u_2, the derivative feedback has been restricted to

$$\alpha = \frac{K}{T} = 0.0333 .$$

Then $\alpha = 0.0333$, $\rho_3 = \frac{1}{2}$, $\gamma = 0.1$, $\sigma = 0.9$,

so that $\frac{\alpha}{\gamma} < \rho_3 < (1 - \frac{\alpha}{\gamma})$,

and equations (7.13) to (7.16) will apply,

whence $\theta m = 77.6^\circ$ F., $\theta n = 72.4^\circ$ F., $\theta_D = 5.2^\circ$ F.

$$\frac{t_o}{T} = 0.138, \frac{t_p}{T} = 0.138, \quad \frac{t_q}{T} = 0.276 .$$

$\overline{m} = 0.5$, $\overline{\theta} = u_3 + \overline{m}G = 75^\circ$ F., giving zero offset. Thus over the

complete range of u, maximum value of $\theta = 78.5^{\circ}$ F.
and minimum value of $\theta = 71.6^{\circ}$ F.

The design with restricted derivative feedback therefore satisfies the requirement 70° F. $\leqslant \theta \leqslant 80^{\circ}$ F. for 25° F. $\leqslant u \leqslant 50^{\circ}$ F.

The value $\theta_D = 5.2^{\circ}$ F. for constant u may be compared with $\theta_D = 7.5^{\circ}$ F. for $\alpha = 0$ (zero derivative component):

$$\frac{\theta_D \Big|_{\alpha = 0.0333}}{\theta_D \Big|_{\alpha = 0}} = 69.3\% .$$

Also the normalised period $\frac{t_q}{T}$ at $u = 37\frac{1}{2}^{\circ}$ F. has been reduced from

$$\frac{t_q}{T}\Big|_{\alpha = 0} = 0.402$$

to $\frac{t_q}{T}\Big|_{\alpha = 0.0333} = 0.276$, or $\frac{t_{q/T}\Big|_{\alpha = 0.0333}}{t_{q/T}\Big|_{\alpha = 0}} = 68.7\%.$

The design represents the best approach over the range 25°F. $\leqslant u \leqslant 50^{\circ}$F. to that for constant $u = u_o$,
when $\rho = \frac{1}{2}$, $\alpha = \rho\gamma = \frac{\gamma}{2}$, $\theta_D = G \frac{\gamma}{1 + \sigma} = 3.95^{\circ}$ F.,
and the smallest possible normalised period is $\frac{\check{t}_q}{T} = \frac{2L}{T} = 0.211$.

We may now consider the energy requirements to achieve this reduction in θ_D and t_q, and also the effect upon the electrical demand.

7.5 ENERGY REQUIREMENTS FOR DERIVATIVE DISCONTINUOUS CONTROL AND EFFECT UPON DEMAND.

7.5.1 ENERGY REQUIREMENTS.

As shown in section 7.1, a reduction in t_q will enable the command θ_r to be set at a level closer to the minimum specified temperature θ_n, and thus reduce energy wastage.

We will compare the results of the analyses of sections 7.4.1 and 7.4.2 for the same values of θ_r and G, i.e. with and without derivative feedback.

Now offset $y = \theta_r - \bar{\theta} = u + \rho G - (u + \bar{m} G)$,

so that $\bar{m} = \rho - y/G$, where $\rho = (\theta_r - u)/G$.

At $u = 37\frac{1}{2}^{\circ}$ F.

$\bar{m} = \rho = \frac{1}{2}$ for both cases, since droop = 0.

At $u = 50^{\circ}$ F.

$$\rho = \frac{1}{3}.$$

With derivative feedback, $\bar{\theta} = 75.9^{\circ}$ F., $y = -0.9^{\circ}$ F.,

whence $\bar{m} = \frac{1}{3} - \left(-\frac{0.9}{G}\right) = 0.345.$

Without derivative feedback, $\bar{\theta} = 76.2^{\circ}$F., $y = -1.2^{\circ}$ F.,

whence $\bar{m} = \frac{1}{3} + \frac{1.2}{75} = 0.349.$

At $u = 25^{\circ}$ F.

$$\rho = \frac{2}{3}.$$

With derivative feedback, $\bar{\theta} = 74.1^{\circ}$ F., $y = 0.9^{\circ}$ F.,

whence $\bar{m} = \frac{2}{3} - \frac{0.9}{75} = 0.655.$

Without derivative feedback, $\bar{\theta} = 73.8^{\circ}$F., $y = 1.2^{\circ}$F.,

whence $\bar{m} = \frac{2}{3} - \frac{1.2}{75} = 0.651.$

Thus for 25° F. $\leqslant u < 37\frac{1}{2}^{\circ}$ F.,

$$\bar{m}\Big|_{\alpha = \frac{K}{T}} > \bar{m}\Big|_{\alpha = 0} .$$

The reduced offset with derivative feedback is achieved, of course, only at the expense of increased energy.

For $37\frac{1}{2}^{\circ}$ F. $< u \leqslant 50^{\circ}$F., however,

$$\bar{m}\Big|_{\alpha = \frac{K}{T}} < \bar{m}\Big|_{\alpha = 0},$$

so that the energy consumed over the range 25° F. $\leqslant u \leqslant 50^{\circ}$F. is the same with or without derivative feedback.

7.5.2 REDUCTION OF PERIOD AND EFFECT UPON DEMAND.

The reduction in t_q due to derivative feedback is as follows:

Without derivative feedback,

$$\frac{t_q}{T}\Big|_{u = 37\frac{1}{2}^{\circ}\text{F.}} = 0.402 \text{ and } \frac{t_q}{T}\Big|_{\substack{u = 50^{\circ}\text{F.} \\ u = 25^{\circ}\text{F.}}} = 0.442.$$

With derivative feedback,

$$\frac{t_q}{T}\Big|_{u = 37\frac{1}{2}^{\circ}\text{ F.}} = 0.276 \text{ and } \frac{t_q}{T}\Big|_{\substack{u = 50^{\circ}\text{ F} \\ u = 25^{\circ}\text{ F.}}} = 0.3058.$$

This reduction in t_q brings about the following advantages:

(1) Since $\bar{m}$ changes only slightly from the value when $\alpha = 0$, the utilisation of generating plant is improved.

(2) Under certain conditions the "indicated maximum demand" may be reduced.

Let the account period be divided into integrating intervals I.

Then the maximum reading of the "indicated demand" due to a single process has a finite probability of being equal to the power P if $t_o \geqslant I$.

It is easily seen that a reduction in $\bar{m}\, t_q = t_o$ from $t_o \geqslant I$ to $t_o < I$ brings about a decreased maximum indicated demand.

As shown in chapter 5, for an ensemble of similar thermostatically-controlled processes a reduction in t_q leads to a smaller demand variance, resulting in greater confidence that a given indicated demand will not be exceeded.

7.6 DEMAND CONTROL FOR AN ENSEMBLE OF THERMOSTATICALLY-CONTROLLED SPACE HEATING PROCESSES.

7.6.1 Relation between weather and electrical demand.

Attempts have been made in recent years to establish a correlation between the weather and the demand due to the heating load. This research has been based mainly upon forecasting and the analysis of past load data. Upon the basic cyclic demand curve for the year is superimposed a predictable day-to-day variation, together with an irregular variation due to various meteorological factors.

Advance warning of expected increase in the heating load is essential for the generating stations. The reliability of short-term prediction will be dependent upon the accuracy of day-to-day forecasting and upon the speed of processing meteorological observations.

A summary of current techniques for predicting this demand is given by Matthewman and Nicholson[50].

7.6.2 Short-term load control.

The start-up time t_W to achieve d.e.c. from a cold-start is, from equation (7.8),

$$t_W = L + T \ln\left(\frac{1}{1 - (\theta_r - u)/(FG)}\right) . \tag{7.21}$$

For the switch-on transient after a load cut, the time t_W' taken to re-establish d.e.c. depends upon the minimum value θ_n' to which θ falls, and may be determined from (7.8), where θ_n is replaced by θ_n'.

t_W is readily calculable, so that individual processes may be staggered such that the maximum demand is determined by the d.e.c. condition and not the start-up. For switch-on transients after load cutting, however, t_W' will differ for each process according to the lowest temperature reached while power is removed.

Suppose in Fig. 7.5 that, due to demand control, power to a process is cut off at point P and subsequently restored at Q. Owing to the transit lag, the temperature rise upon switching on will be delayed to point R. The energy required to bring the temperature back into specification will result in the on-time t_o' being considerably longer than the d.e.c. on-time t_o. There is then an immediate danger of the set demand level being exceeded, causing yet a further power cut. The control of an individual process should therefore be considered not in respect of its own demand, but in the context of minimising the maximum demand due to the ensemble of which the process is a member.

Suppose we have an ensemble of 1,000 similar processes subjected to on-off heat cycling, and requiring equal power P per process. For load shedding purposes the ensemble may be subdivided into 10 groups, say, each of 100 processes.

In the integrating interval I the mean energy consumed under d.e.c. conditions is $1000\ \overline{m}P \times I$, where $\overline{m} = t_o/t_q$, and the mean demand for the ensemble = $1000\ \overline{m}P$.

Suppose that, as a result of a load cut, K groups remain cycling while 10-K groups are switched off.

Let y of the 10-K groups be returned subsequently to load. Assuming the worst condition where none of these groups attain d.e.c. within the next interval I, and working upon mean values, energy consumed in this interval = $(100\ K\ \overline{m}P + 100\ yP)\ I$.

$\therefore$ for the d.e.c. demand not to be exceeded on returning y groups to load,

$$1000\ \overline{m}P \times I \geqslant (100K\ \overline{m}P + 100\ yP)\ I,$$

$$\text{or } \overline{m}\ (10 - K) \geqslant y.$$

If $\overline{m}$ is taken typically as 0.5, we have:

When $K = 7$, $y \leqslant 1.5$.

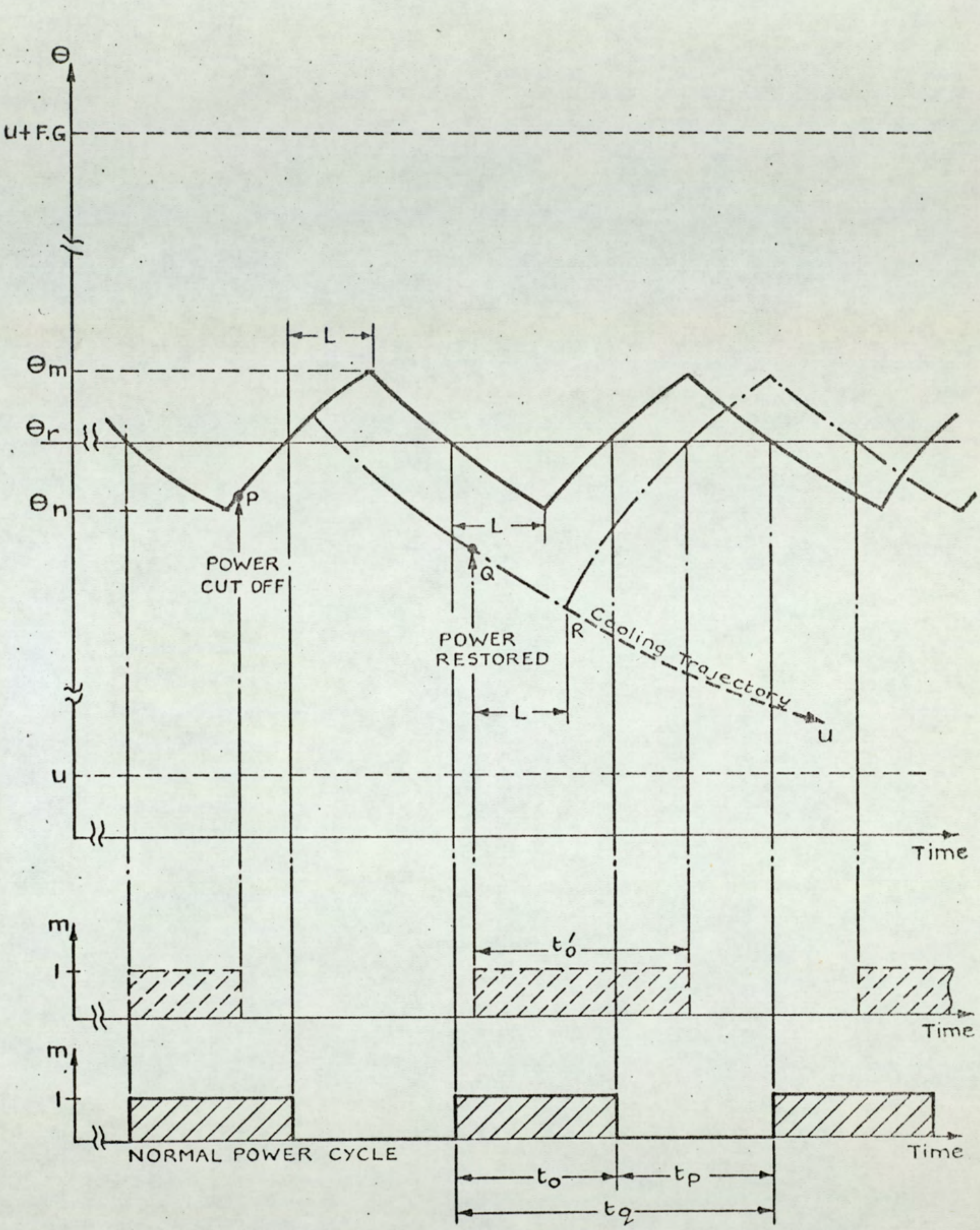


FIG. 7.5 HEATING AND COOLING TRAJECTORIES FOR THERMOSTATICALLY-CONTROLLED SPACE HEATING PROCESS.

When $K = 8$, $y \leqslant 1$.

When $K = 9$, $y \leqslant 0.5$.

The restoration rate of the remaining $10 - K - y$ groups will then be determined by the times taken for members of the y groups to re-achieve the d.e.c. condition.

In practice, the situation will be better than that analysed above, as some processes restored to load will attain d.e.c. while others are still operating at $\bar{m} = 1$. The optimum restoration rate may therefore be determined from operating experience, combined with a prediction of the future demand.

7.6.3 Demand Prediction as Processes are Returned to Load.

Demand control within each integrating half-hour may be effected as a result of two alternative strategies:

1. The KWh target is T_T = indicated demand in KW x $\frac{30}{60}$. The energy is monitored at time t after the commencement of the half-hour. If the demand were constant at the defined "indicated" value, KWh consumed up to time $t = T_T \times \frac{t}{30}$. Suppose that the energy is sampled at successive intervals α. Then for this demand, KWh consumed up to a time $t + n\alpha$ = $T_T \dfrac{(t + n\alpha)}{30}$,

   where $n = 0, 1, 2, - - - - - - - \dfrac{30 - t}{\alpha}$.

   Load cutting may be implemented if the actual KWh consumed up to time $t + n\alpha$

$$> T_T \frac{(t + n\alpha)}{30} \quad (7.22)$$

2. If the actual energy consumed up to time t is KWh_t, load cutting may be implemented if $T_T < KWh_t + (30 - t) \times$ rate of change of energy at time t, (7.23)

   giving a measure of prediction of future energy consumption.

   Suppose that, as a result of demand control, one or more selected processes are removed from load at a particular sampling instant t.

   It may be possible to restore power at the next sampling instant $t + \alpha$, at a rate determined by the following considerations:

   (a) Owing to the stochastic nature of the demand due to the loads still cycling, their contribution to the energy must be monitored at each sampling point.

At time $t + \alpha$ we extrapolate recent observed data to predict this contribution up to a subsequent sampling instant $t + \alpha(n + 2)$.

We will call this predicted energy KWh_p.

(b) Those processes which are returned to load will have a value of $\bar{m}$ equal to unity until their temperatures reach the θ_r level. The times to achieve d.e.c. may be obtained from the known heating trajectories.

The condition determining the rate of return of processes to load immediately subsequent to a measurement at time $t + \alpha$ is that, at a subsequent sampling instant $t + \alpha(n + 2)$:

the energy

(KWh_p + the KWh due to the processes returned to load)

$$\not> T_T \quad \frac{t + \alpha\,(n + 2)}{30} \tag{7.24}$$

for the first strategy described, or

$$KWh_{t + \alpha} + \alpha\,(n + 1)\left(\frac{T_T - KWh_{t + \alpha}}{30 - (t + \alpha)}\right) \tag{7.25}$$

for the second strategy described.

n will normally have zero value, since extrapolation to more than one sampling interval ahead may lead to a considerable prediction error.

In order to predict the maximum rate of return of load, it is necessary to know the temperature reached at a given time subsequent to load cutting or restoration. Thus for each process we must obtain the heating and cooling trajectories shown in Fig. 7.5.

The required information may be stored in a computer, and will be acquired experimentally for a specific plant as standard trajectories for constant values of u, stepped at, say, 5° F. increments. Any change in environment temperature will be sufficiently slow for u to be considered constant during the integrating period of interest. Knowing the instant when a process is cut relative to the d.e.c.switching instants, the temperature at the time of the cut and subsequently may be obtained by reference to the nearest standard trajectory. Hence we may obtain the time to re-achieve d.e.c. after load restoration.

7.6.3.1 Choice of Sampling Frequency.

The validity of extrapolation beyond the range of observations is somewhat open to question, since we assume the curve-fitting polynomial to be unaltered in the immediate future.

The accuracy of the energy prediction will depend upon the choice of sampling frequency, which will be related to the rate of load perturbation within each integrating half-hour. For fluctuating loads, infrequent sampling can result in incorrect decisions being made in respect of load restoration, as other than very recent observations will be too stale for use in extrapolation. On the other hand, a high sampling rate will be unnecessary for a slowly-changing load.

For several processes being switched, the computations of temperature states will become exceedingly complex. Control will become more practical if switching is confined to one large load whose effect upon the demand pattern is known.

EXAMPLE.

Suppose we have an ensemble of similar on-off electric space heaters, where the target energy for each integrating half-hour has been set at 100 KWh. Demand control is effected by switching of a single 10 KW heater.

Suppose that the energy consumption is monitored every 1 minute, and that, as a result of demand control, the 10 KW load is switched off at t = 19 mins.

At the sampling instant t = 20 mins., we must consider whether load may be restored immediately by predicting the energy consumption at t = 21 mins.

A polynomial is fitted to observations of the energy due to the cycling load at, and recent to, t = 20 mins. This curve is extrapolated 1 minute ahead by means of the "Gregory-Newton backwards' formula"[56, 57]:

$$f(a + \theta h) = Y_o + \theta \Delta Y_{-1} + \binom{\theta + 1}{2} \Delta^2 Y_{-2} + \text{-----} \text{---} + \binom{\theta + n - 1}{n} \Delta^n Y_{-n} \quad (7.26)$$

where we know the values Y_{-n} - - - - $-Y_{-2}$, Y_{-1}, Y_o of a function at regular intervals h of its argument, say a - nh, - - - - a - 2h, a-h, a,

and where $\binom{\theta}{n} = \dfrac{\theta\ (\theta - 1) - - - - - (\theta - n + 1).}{\underline{|n}}$

The polynomial f $(a + \theta h)$ of degree n in θ which passes through the n + 1 points $a - nh, - - - - a - 2h, a - h$, a and through a point $a + \theta h$, is obtained by constructing a difference table, where the forward differences are defined by:

$$\Delta Y_n = Y_{n+1} - Y_n, \tag{7.27}$$

$$\Delta^2 Y_n = \Delta Y_{n+1} - \Delta Y_n, \tag{7.28}$$

$$\Delta^3 Y_n = \Delta^2 Y_{n+1} - \Delta^2 Y_n, \tag{7.29}$$

etc.

Alternatively, differences may be measured backwards, commencing with the most recent information, where the backward difference operator ∇ is defined by

$$\nabla Y_n = Y_n - Y_{n-1}. \tag{7.30}$$

The degree of the collocation polynomial can then be determined by computing terms until they no longer appear significant.

This method of curve fitting is preferred to the Lagrange formula [57] or to the Method of Least Squares [24], which, although not requiring prior computation of the difference table, have the disadvantage that the degree of the polynomial must be chosen at the outset.

Suppose the monitored energy due to the continuously cycling loads is 47.15, 50.72, 54.39, 58.15, 62.0, 65.95 KWh at the times t = 15, 16, 17, 18, 19, 20 minutes respectively. The corresponding total monitored KWh (including the sheddable load) at these times are 48.40, 52.05, 55.80, 59.64, 63.59, up to t = 19 mins.

For a constant demand equal to the defined "indicated" value, the proportional values of the target energy are 50, 53.33, 56.67, 60, 63.33 KWh at t = 15, 16, 17, 18, 19 mins. respectively. If load cutting is effected at any sampling point when this proportion is exceeded, the 10 KW heater will be switched off at t = 19 mins.

At the next sampling instant t = 20 mins. we monitor the energy due to the cycling load as 65.95 KWh; we then predict this energy at t = 21 mins. by constructing a difference table for the values $f(a - 5h)$, $f(a - 4h)$, - - - - - - - $f(a)$, $f(a + \theta h)$, where $a = 20$ mins, $h = 1$ min., $\theta = 1$ (table 7.3).

Argument x mins.	KWh of cycling load = Y	DIFFERENCES ΔY	$\Delta^2 Y$	$\Delta^3 Y$	$\Delta^4 Y$	$\Delta^5 Y$
15	47.15					
		3.57				
16	50.72		0.10			
		3.67		-0.01		
17	54.39		0.09		0.01	
		3.76		0		0
18	58.15		0.09		0.01	
		3.85		0.01		0
19 (=a - h)	62.00		0.10		0.01	
		3.95		0.02		
20 (= a)	65.95		0.12			
		4.07				
a + θh	Predicted KWh f (a + θh) 70.02 At θ = 1					

TABLE 7.3

In the example chosen, the 5th difference column becomes zero for the given sampled values of cycling load. The polynomial representing the KWh due to the cycling load for a given argument $x \geqslant 15$ mins. is:

$$\begin{aligned} KWh_K &= 47.15(1 + \Delta)^K \\ &= 47.15\left(1 + K\Delta + \frac{K(K-1)}{\underline{|2}}\Delta^2 + \frac{K(K-1)(K-2)}{\underline{|3}}\Delta^3\right. \\ &\quad \left. + \frac{K(K-1)(K-2)(K-3)}{\underline{|4}}\Delta^4\right), \end{aligned} \tag{7.31}$$

where $K = x - 15$,

$\Delta = 3.57$, $\Delta^2 = 0.10$, $\Delta^3 = -0.01$, $\Delta^4 = 0.01$.

Alternatively, working back from the 5th difference column to the predicted point $f(a + \theta h)$, we obtain the consumption due to the cycling load as 70.02 KWh at $t = 21$ min., which exceeds the allowed target proportion 70.0 KWh even with the10 KW load removed.

This shows that if the target energy is not to be increased, and load shedding is confined to one process, we must increase the value of the sheddable load <u>for this particular energy/time pattern.</u>

We will take an <u>example</u> where the sheddable on-off load is 20 KW. The monitored energy due to the continuously cycling loads at $t = 15, 16, 17, 18, 19, 20$ mins. is 47.40, 50.65, 53.90, 57.10, 60.20, 63.25 KWh respectively, and the total monitored energy at these times is 49.90, 53.23, 56.56, 59.84, 63.53 KWh up to $t = 19$ mins. At this time the allotted proportion of target energy is exceeded, and the single load is again shed.

The difference table 7.4 may be constructed from observations recent to, and including, $t = 20$ mins. for the energy due to the cycling loads.

ARGUMENT x mins.	KWh of cycling load = Y	DIFFERENCES				
		ΔY	$\Delta^2 Y$	$\Delta^3 Y$	$\Delta^4 Y$	$\Delta^5 Y$
15	47.40					
		3.25				
16	50.65		0			
		3.25		-0.05		
17	53.90		-0.05		0	
		3.20		-0.05		0.10
18	57.10		-0.10		0.10	
		3.10		+0.05		
19	60.20		-0.05			
		3.05				
20	63.25					

<u>TABLE 7.4</u>

The 5th difference column content is sufficiently small for the KWh/time curve to be represented approximately by a quartic equation.

Applying equation (7.26) to find $f(a + \theta h)$ where $\theta = 1$, we obtain:

Predicted KWh of cycling load at $t = 21$ mins.

$$= 63.25 + 3.05 - 0.05 + 0.05 + 0.10$$

= 66.40 KWh if we neglect the error due to the 5th difference.

If required, values of predicted energy intermediate between $t = 20$ mins. and 21 mins. may be obtained by applying fractional values of θ.

From the stored temperature/time data for the appropriate value of u, we know the temperature state of the 20 KW load when it is switched off at $t = 19$ mins; also its subsequent value at $t = 20$ mins.

If this load were switched on again at $t = 20$ mins., it would operate at $\bar{m} = 1$ until the associated process temperature reached θ_r in, say, 30 seconds, and would then continue to cycle at its d.e.c. value of $\bar{m}$, say 0.5.

Then total predicted consumption up to $t = 21$ mins.

$= 66.40 + (20 \times \frac{1}{2} + 20 \times \frac{1}{2} \times \frac{1}{2})/60$

= 66.65 KWh, which is below the proportional target value of 70 KWh. The surplus of energy is due, of course, to the decreasing slope of the energy/time characteristic of the cycling load for $t > 17$ mins.

The computation shows that the 20 KW load may be safely switched on at $t = 20$ mins., when the complete ensemble is allowed to cycle until demand control is next effected. Any prediction error is corrected at each sampling point by updating the observed data upon which the prediction is based.

If we accept the premise that extrapolation beyond the range of observed data is justifiable, the question arises as to what degree of prediction accuracy is required.

In Table 7.4, the error in predicting the energy due to the cycling load at $t = 21$ mins., caused by the 5th difference column having a finite value, is $0.10/(63.25 + 3.05 - 0.05 + 0.1) = 0.15\%$.

It must be shown practically that if this is taken as the maximum allowable percentage error, an acceptable control of demand is obtained. The computer strategy will then be to continue to evaluate the difference table until this desired accuracy has been achieved. Difference columns may be terminated at the third for this example, though for the higher order polynomials describing fluctuating loads, more observations, and hence more columns, will be required.

Possible errors in successive columns eventually commence to build up due to rounding-off errors in the original readings, so that calculations should be made with at least two more figures than are required in the result.

Optimisation of the sheddable load value to minimise demand variations may result in this load being switched at an inconveniently high rate. It will then be more practical to sacrifice some quality of control by allowing a greater prediction error, and provide an adequate safety margin in the target energy value.

If the energy at any instant subsequent to time $t = 0$ is characterised by its mean and variance, then the predicted energy obtained from the difference table will be likewise specified.

It can be shown[24] that if $Z = x - y$ where x and y are independent variables, i.e. cov. $(x, y) = 0$, then

$$\text{mean } Z = \text{mean } x - \text{mean } y \qquad (7.32)$$

and

$$\text{Var }(Z) = \text{Var}(x) + \text{Var}(y). \qquad (7.33)$$

Thus if the elements in the difference table are specified statistically, there will be a cumulative build up of variances when evaluating the predicted energy.

For a given project, however, the sampling interval may best be chosen by actual observations of the magnitude and gradient of the energy perturbations.

7.7 SUMMARY.

The transit lag inherent in the electric space heating process gives rise to undesirable oscillations in the temperature when the heaters are switched by thermostat. By means of derivative feedback a design has been achieved where the amplitude of temperature oscillations has been reduced to a value compatible with the specification given in section 7.1.

The use of derivative control gives rise to the further advantages:

1. Owing to the reduced value of θ_D, the command temperature θ_r may be set at a value closer to the minimum specified temperature θ_n, thus reducing the energy requirements.
2. A reduction is achieved in the offset $\theta_r - \overline{\theta}$ occurring in discontinuously-controlled space heating systems due to variations in the ambient temperature.

3. A significant reduction in the period t_q is obtained. Thus not only is the "minimum-comfort" criterion of optimisation satisfied, but for an ensemble of similar thermostatically-controlled space heaters this reduction will lead to a smaller demand variance (see Chapter 5).

Demand control may be applied advantageously to an ensemble of temperature-sensitive loads provided the heating energy requirements are met at the same time, i.e. control action will be limited by the allowable drop in temperature. The quality of control that can be obtained is a function of the energy/time characteristic, and depends upon the accuracy of load prediction.

Prediction accuracy will increase with the size of ensemble, but is limited by the inherent load fluctuations due to the switching action of individual process members.

The scheme described in section 7.6.3 may be readily applied where a switching strategy can be imposed upon a single sheddable load within a process ensemble in order to achieve control of the overall demand. At each monitoring point a control decision can then be made by comparing actual energy and proportional target consumption without the necessity of computing the power to be shed. The more complex problem when several loads are shed together requires further consideration.

Demand control of thermostatically-controlled loads may be applied to the situation where blocks of flats are provided with underfloor heating or "Electricaire" units.

Control will be provided by a centralised digital computer upon receipt of telemetered signals of the demand due to each block.

Owing to the high thermal inertia of the heated space, interruption of the supply to each block for a few minutes within every integrating half-hour period will cause little noticeable fall in temperature: load shedding may be applied sequentially to ensure infrequent cessation of power to any one block. In this way, significant demand reduction may be obtained at peak times without discomfort to residents.

When estimating the cooling effected during a load cut, it will be inconvenient to monitor the temperatures of individual premises within the block being switched.

The best compromise will be to assume an equivalent cooling trajectory which commences at an initial temperature value of 70° F. at the instant of switch-off, and which corresponds to the average load recorded for the block in the previous sampling interval. As may be seen from the worked example in section 7.6.3, this assumption will give insignificant error in the total predicted energy at the next sampling instant.

Optimisation of the load to be shed may be obtained by monitoring the space heating demand pattern at the incoming feeder to each block, and by observing the effects upon the overall demand of removal and subsequent restoration of the associated load. The further implications of this technique could form the basis of subsequent research.

CHAPTER 8.

ECONOMIC ASPECTS OF DEMAND LIMITATION.

8.1. INTRODUCTION

In this chapter an appraisal is made of production costs for electroheat processes, as affected by the KVA maximum demand, energy consumption, power factor and load factor.

Calculations are based upon Maximum Demand tariffs published by the Midlands Electricity Area Board.

The economic advantages of maximum demand limitation are analysed in two case studies:

1). For the melting cycle in a foundry producing cast iron, where the savings thereby effected are compared with the cost of lost production.

2). For a factory producing a variety of metal products involving the use of electroheat, where the need to maximise production restricts the implementation of demand control.

8.2 MAXIMUM DEMAND TARIFFS FOR INDUSTRIAL SUPPLIES.

The following economic calculations will be based upon industrial tariffs published by the Midlands Electricity Board (M.E.B.), which became effective for electricity used after 30th September, 1967. These are the two-part tariffs Nos. 7 and 7 (a).

8.2.1 MAXIMUM DEMAND TARIFFS Nos. 7 and 7 (a).

The following information is relevant to the subsequent calculations:

Tariff No. 7.

Maximum Demand (M.D). Charge each month:

Each KVA of the first 200 KVA of M.D. = C_1 = 17s. 2d.
Each KVA of the next 300 KVA of M.D. = C_2 = 16s. 7d.
Each KVA of the next 500 KVA of M.D. = C_3 = 16s. 0d.
Each KVA of the next 4000 KVA of M.D. = C_4 = 15s. 5d.
Each additional KVA of M.D. = C_5 = 14s.10d.

For demands exceeding 20 KVA,"maximum demand" means twice the greatest number of kilovolt-ampere hours during any thirty consecutive minutes determined by the M.E.B. during the account month.

Kilowatt-hour (Unit) Charge each month:

Each of the first 180 KWh per KVA of M.D. = D_1 = 1.14d.
Each of the next 180 KWh per KVA of M.D. = D_2 = 0.95d.
Each additional KWh supplied = D_3 = 0.82d.

A rebate of 0.13d. is allowed in respect of each KWh consumed between 23.00 hours and 07.00 hours, subject to the consumer making a capital contribution to cover the cost of additional metering equipment required.

An adjustment charge is payable to cover variations in fuel costs; a supplementary charge is made for metering at 650 volts or less.

Tariff No. 7 (a).

Each KVA of the first 200 KVA of M.D. = 19s. 2d.
Each KVA of the next 300 KVA of M.D. = 18s. 7d.
Each KVA of the next 500 KVA of M.D. = 18s. 0d.
Each KVA of the next 4000 KVA of M.D. = 17s. 5d.
Each additional KVA of M.D. = 16s. 9d.

An important feature of this tariff is that"maximum demand" means twice the greatest number of kilovolt-ampere hours supplied during any thirty consecutive minutes during the account month or in any of the preceding eleven months whichever is the greater.

Kilowatt-hour (Unit) Charge each month:

Each of the first 180 KWh per KVA of M.D. made in the month = 0.99d.
Each of the next 180 KWh per KVA of M.D. made in the month = 0.82d.
Each additional KWh supplied = 0.70d.

The Off-peak rebate rate and conditions are the same as for Tariff No. 7. A fuel costs adjustment charge is payable; metering is at over 650 volts.

It should be pointed out, however, that Area Board tariffs are now tending to reflect the Bulk Supply Tariff structure by charges which relate the demand costs to specified seasonal peak periods[58].

8.3 COST PER UNIT OF PURCHASED INDUSTRIAL POWER AS AFFECTED BY LOAD FACTOR AND DEMAND, ENERGY CONSUMED AND POWER FACTOR.

8.3.1 Load Factor, Demand and Energy Consumed.

We may evaluate the cost per KWh of energy purchased for the rates given by tariff No. 7.

Now load factor $= \dfrac{\text{KWh supplied per month}}{\text{M.D. in KW} \times 720}$.

Let M.D. in KVA = M

Power factor = $\cos \emptyset$

Load factor = F

KWh supplied per month = K.

Then $K = M \cos \emptyset \times 720\ F.$ (8.1)

Maximum demand Charge.

The maximum demand charges C_1 to C_5 above are first converted to pence to give equivalent charges C_6 to C_{10} respectively.

Then $C_6 = 206$, $C_7 = 199$, $C_8 = 192$,
$C_9 = 185$, $C_{10} = 178$, each in pence.

For $0 \leqslant M \leqslant 200$,

maximum demand charge = $C_6\ M$ pence. (8.2)

For $200 \leqslant M \leqslant 500$,

maximum demand charge

$= C_6 \times 200 + C_7 (M - 200)$ pence. (8.3)

For $500 \leqslant M \leqslant 1000$,

maximum demand charge

$= C_6 \times 200 + C_7 \times 300 + C_8 (M - 500)$ pence. (8.4)

For $1000 \leqslant M \leqslant 5000$,

maximum demand charge

$= C_6 \times 200 + C_7 \times 300 + C_8 \times 500$
$+ C_9 (M - 1000)$ pence. (8.5)

For $5000 \leqslant M$,

maximum demand charge

$$= C_6 \times 200 + C_7 \times 300 + C_8 \times 500 + C_9 \times 4000 + C_{10} (M - 5000) \text{ pence.} \quad (8.6)$$

Equations (8.2) to (8.6) represent, of course, linear relationships between the maximum demand charge and M.

Kilowatt-hour charge.

Let $D_1 = 1.14$ pence, $D_2 = 0.95$ pence, $D_3 = 0.82$ pence.

For $0 \leqslant \frac{K}{M} \leqslant 180$,

$$\text{KWh charge} = D_1 M \times \frac{K}{M} \text{ pence.} \quad (8.7)$$

For $180 \leqslant \frac{K}{M} \leqslant 360$,

KWh charge

$$= D_1 \times 180\,M + D_2 M \left(\frac{K}{M} - 180\right) \text{pence.} \quad (8.8)$$

For $360 \leqslant \frac{K}{M}$,

KWh charge

$$= D_1 \times 180M + D_2 \times 180M + D_3\, M\left(\frac{K}{M} - 360\right) \text{ pence.} \quad (8.9)$$

We may take a load which has a maximum demand level of 1,000 KVA at 0.9 power factor and a load factor of 80%.

From (8.1), KWh supplied per month = 1000 x 518.4.

Then from (8.4), M.D. charge per KWh supplied = 0.38d., and from (8.9), kilowatt-hour (unit) charge per KWh supplied = 0.976d.

Thus total charge of purchased power

= 1.356 pence per KWh of energy supplied.

If a load has a maximum demand of 10,000 KVA at 0.9 power factor and a load factor of 80%,

KWh supplied per month = 10,000 x 518.4.

From (8.6) and (8.9) we may again evaluate the maximum demand and kilowatt-hour charges per KWh supplied.

Then total charge of purchased power

= 0.353 + 0.976, or 1.329 pence per KWh of energy supplied.

For a power factor of 0.9, the charge per KWh of purchased electricity has been calculated for values of KVA maximum demand ranging from 100 to 10,000 with load factors of 20 - 100%.

The total charges per month are shown in Fig. 8.1 as a function of KVA maximum demand, load factor, and energy supplied. The proportions of the total charges directly due to the maximum demand are shown in Fig. 8.2. No account has been taken of the rebate allowed for energy consumed during the off-peak period.

These curves show clearly how the pence per KWh may be reduced by increase of the energy consumption and by improvement of load factor (or reduction of maximum demand).

For example, if the energy required per month is 3×10^6 KWh, the load may have a maximum demand of 7500 KVA with load factor 62% and p.f. 0.9, when the cost of electricity is 1.49 pence per KWh; improvement of the load factor to 80% with a maximum demand of 6000 KVA reduces the electricity cost to 1.34 pence per KWh. Thus a monthly saving of £1875 is effected.

The effect of load factor upon the total costs is also clearly illustrated in Fig. 8.3. This is drawn for a load with an M.D. of 5000 KVA, and shows how the total charge for electricity supplied is apportioned between maximum demand and energy charges.

As the energy is increased, reduced electricity charges per KWh become payable owing to the graduated scale of energy charges shown in equations (8.7) to (8.9).

It will be appreciated that the tariffs Nos. 7 and 7 (a) are so framed that the running costs bear part of the charges due to maximum demand. In other words, a high maximum demand incurs a double penalty.

8.3.2 Power Factor.

We may now consider the effect of power factor upon the unit charge.

It is well known that, for highly inductive loads, power factor correction is not only economically desirable, but is insisted upon by the supply authorities, owing to the otherwise high loading upon the feeders and supply equipment.

Maximum demand tariffs based upon maximum kilowatt demand normally include a penalty charge for average power factors below a stipulated value, say 0.9. For tariffs which are based upon maximum KVA demand, a poor power factor is inherently penalised, since the KVA demand is inversely proportional to the power factor for a given KW rating.

Suppose for a given power that the power factor is improved from $\cos \phi_1$, for an uncompensated system, to $\cos \phi_2$ as a result of p.f.

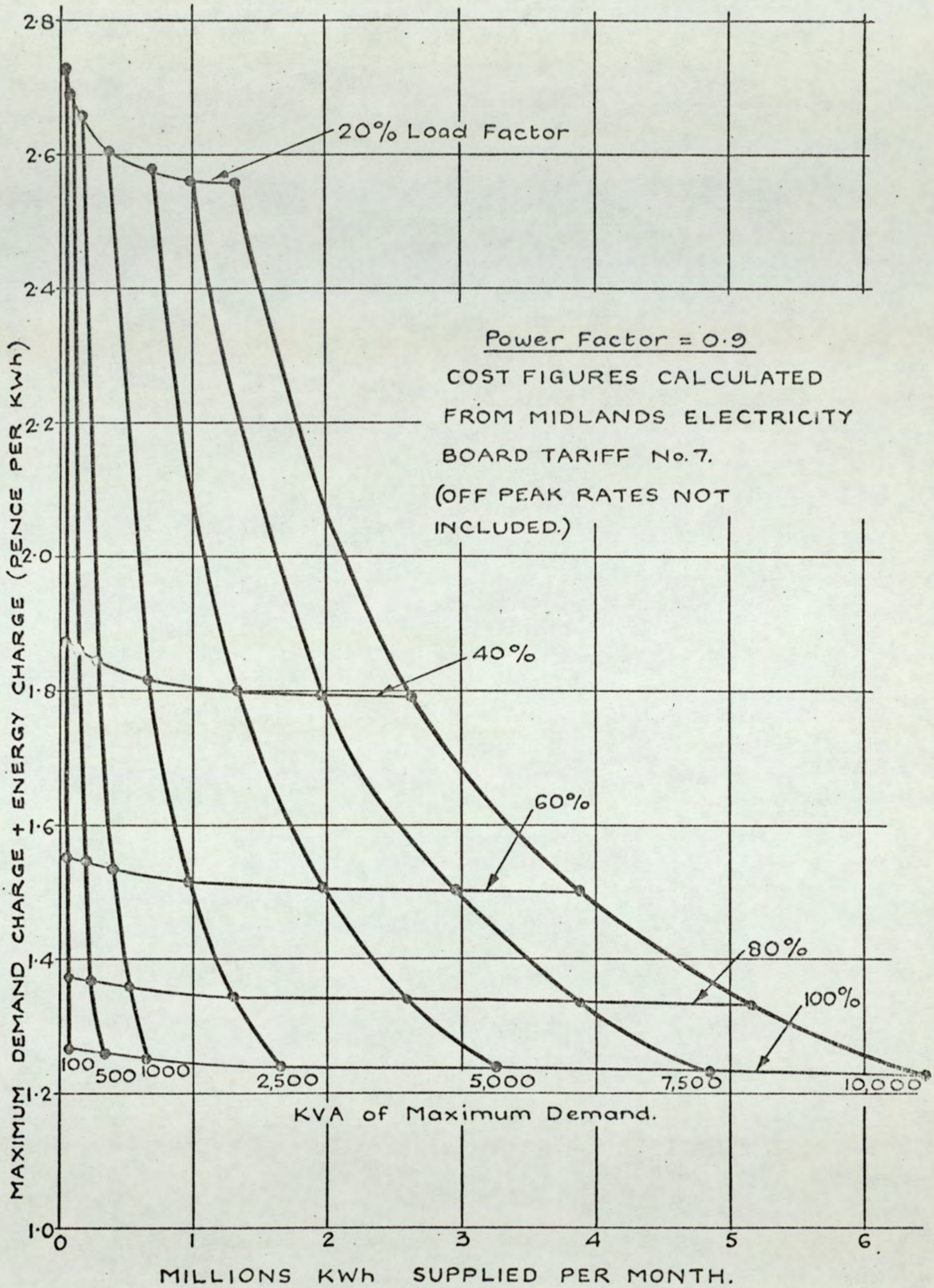


FIG. 8.1 TOTAL CHARGE OF PURCHASED ELECTRICITY PER K.W.h. DELIVERED.

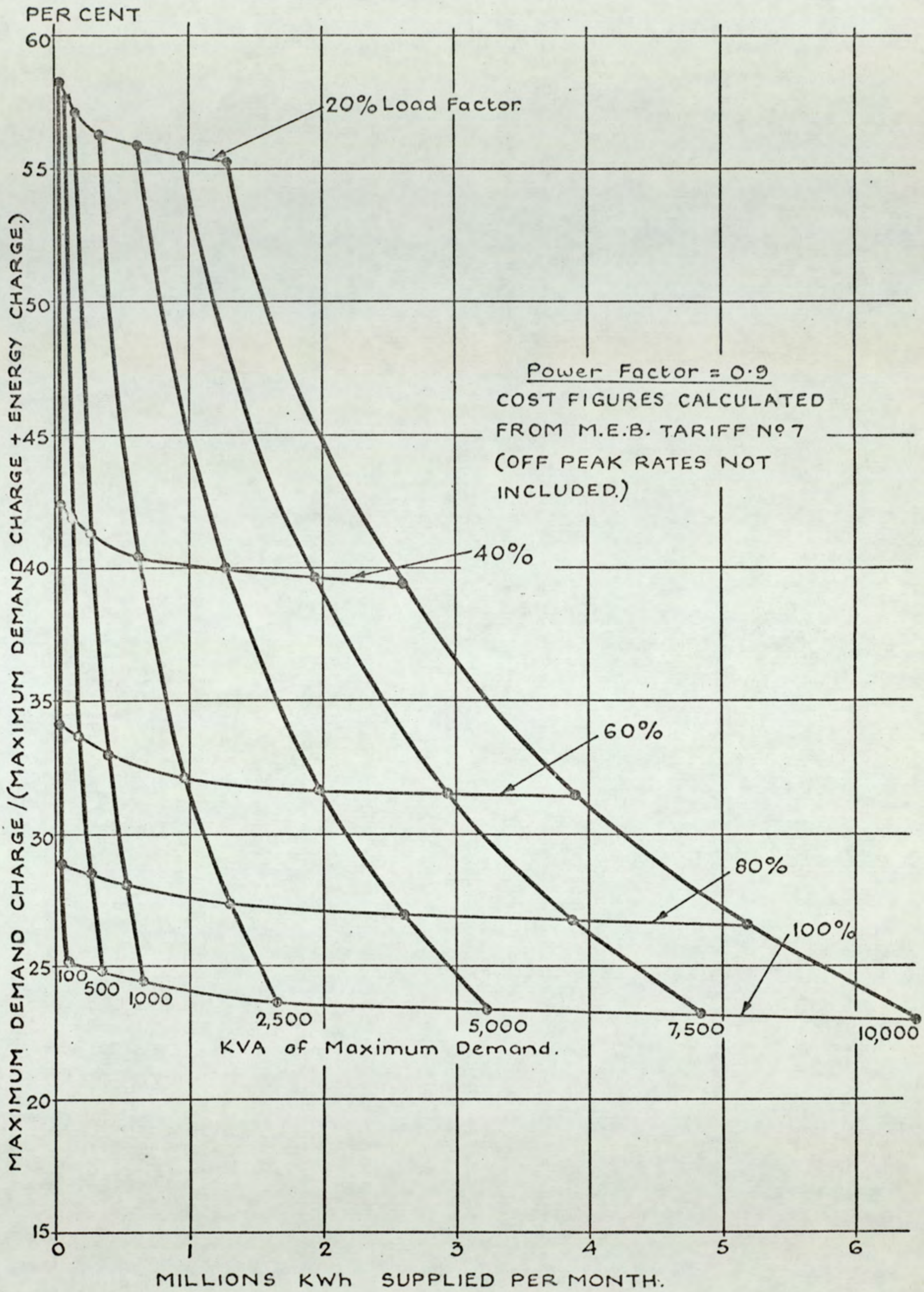


FIG. 8.2 PROPORTION OF ELECTRICITY CHARGES DUE TO MAXIMUM DEMAND.

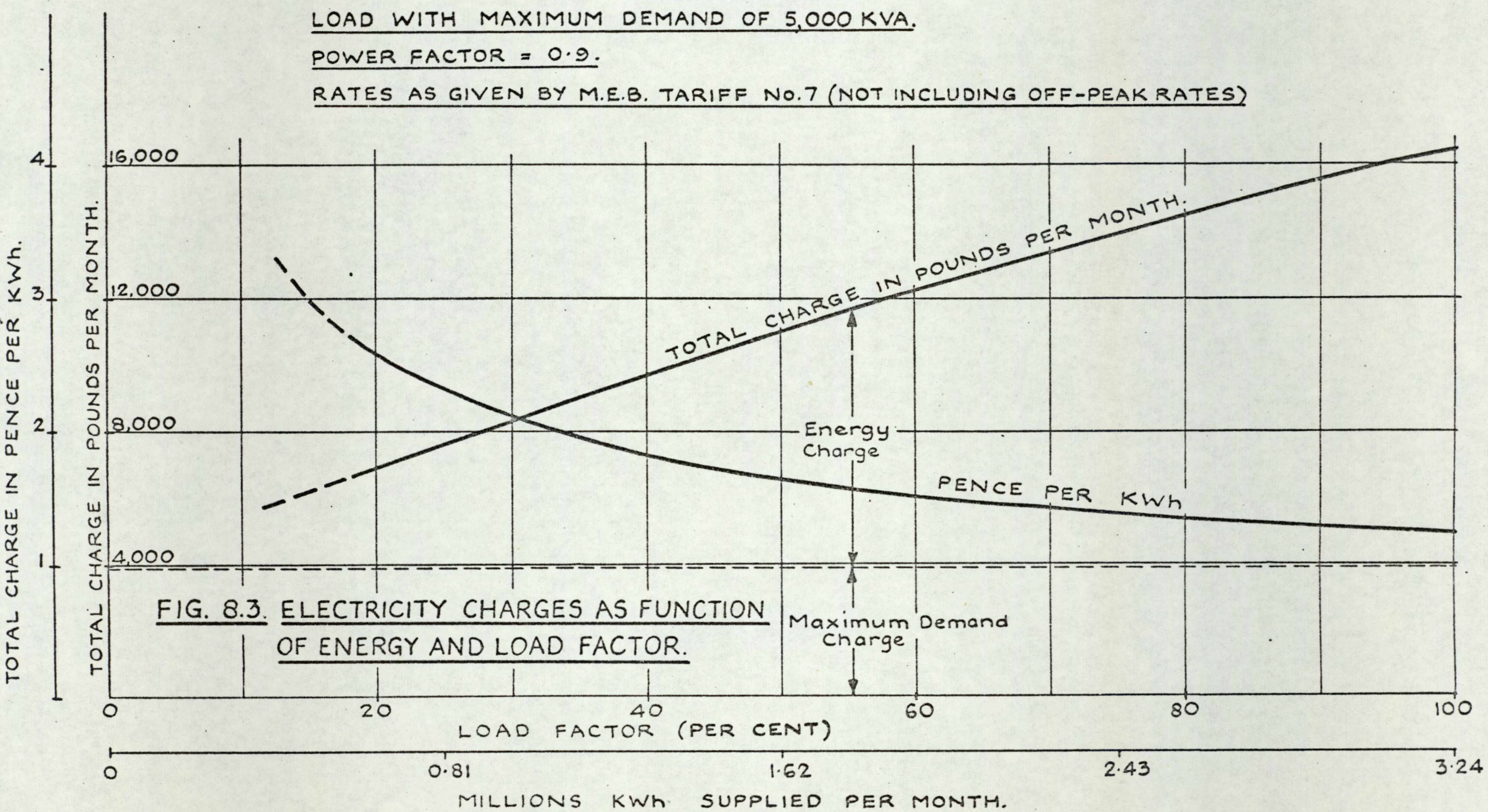


FIG. 8.3. ELECTRICITY CHARGES AS FUNCTION OF ENERGY AND LOAD FACTOR.

correction.

It is shown in standard texts that, for tariffs based upon maximum KVA demand plus a flat rate per KWh, the M.D. charges are minimised when

$$\text{Cos } \phi_2 = \sqrt{1 - (Bp/(100A))^2}, \qquad (8.10)$$

where A is the yearly charge per KVA of maximum demand, B the cost per KVAR of p.f. improvement capacitors, and p the percentage annual interest and depreciation. If we take A and B to be £10 per year per KVA of M.D., and £10 per KVAR of phase-advance equipment respectively, and p = 20 per cent per annum, we obtain the result $\phi_2 = \text{Cos}^{-1}\ 0.98$, which is independent of the original power factor. Since the leading KVAR increases as $\tan \phi_1 - \tan \phi_2$, further power factor improvement leads to the situation where the cost of the phase-advance equipment outweighs the savings in maximum demand charges.

These basic results are modified somewhat for industrial tariffs where the standing charge per KVA of maximum demand has a sliding scale; also any reduction achieved in maximum demand is reflected as a saving in the energy charge, which is therefore not a flat rate per KWh.

Suppose a consumer's load has a maximum demand of M_1 KVA at p.f. $\text{Cos } \phi_1$, and the load factor is F. Compensation by capacitors improves the power factor to $\text{Cos } \phi_2$, i.e. M_1 is reduced to a value $M_2 = M_1 \text{ Cos } \phi_1 / \text{Cos } \phi_2$.

Monthly savings in maximum demand and energy charges may be obtained from equations (8.2) to (8.9). We will consider the economy effected by power factor compensation in four cases.

Case (i).

$M_1 = 1000$, $\text{Cos } \phi_1 = 0.6$, $F = 80\%$.

The power factor is improved to $\text{Cos } \phi_2 = 0.9$, whence $M_2 = 666.67$.

Maximum demand charge per month.

Equation (8.4) applies both before and after compensation.

$$\text{Then saving effected} = \frac{C_8}{240}\left[(M_1 - 500) - (M_2 - 500)\right]$$

$$= £266.67.$$

Energy charge per month.

From (8.1), $K = M_1 \text{ Cos } \phi_1 \times 720F = 345{,}600$ KWh per month.

Before compensation,

$180 < \frac{K}{M} < 360$, when (8.8) applies, where $M = M_1$.

After compensation,

$360 < \frac{K}{M_2}$, when (8.9) applies, where $M = M_2$, whence saving effected $= £104.73$.

Then total saving per month $\simeq$ £371, and we may calculate the time required to recoup the capital cost of the power-factor correction equipment.

Compensating KVAR to be supplied

$= M_1 \sin \emptyset_1 - M_2 \sin \emptyset_2 = 510.$

If this capacitor cost if £x per KVAR, and is recouped in N months as a result of the saving effected, N = 510x/371.

For x = £10 per KVAR, for example, N = <u>13¾ months.</u>

<u>Case (ii).</u>

Suppose now that the consumer has a maximum demand M_1 of 1,000 KVA at $\cos \emptyset_1 = 0.6$, but that the load factor F is 60%. The power factor is again improved to $\cos \emptyset_2 = 0.9$. It is found that:

Monthly saving in M.D. charge = £266.67 as before,

but monthly saving in energy charge = £57.9,

whence N = <u>15¾ months</u> if x is as above.

<u>Case (iii).</u>

This case is the same as case (i) except that the power factor is improved to $\cos \emptyset_2 = 0.95$ instead of 0.9.

The monthly saving in M.D. charge = £294.7.

The monthly saving in energy charge = £116.5.

The required compensating KVAR, however, has now increased from 510 to 600, whence N = <u>14¾ months.</u>

<u>Case (iv).</u>

This case is the same as case (ii) except that the power factor is improved to 0.95 instead of 0.9.

The monthly saving in M.D. charge = £294.7.

The monthly saving in energy charge = £86.4,

whence N = <u>15¾ months.</u>

We may conclude that power factor compensation to a value of at least 0.95 remains economically attractive in spite of the extra compensating KVA required, as worthwhile savings may be achieved once the cost of this equipment is recouped.

The economics of buying leading KVAR has been considered in the light of power factor improvement for a fixed load. When demand control is applied, however, we have the situation of a fixed capacitor in parallel with a variable inductive load, and the power factor will change as load is shed.

However, the sheddable load normally represents only a small proportion of the total, so that the electricity cost calculations will not be substantially modified.

8.3.3 Relative Merits of Tariffs Nos. 7 and 7 (a).

Electricity charges for tariff No. 7 (a) have been evaluated in the same manner as for tariff No. 7. The results are shown in Figs. 8.4 and 8.5, which correspond to Figs. 8.1 and 8.2 respectively.

Comparison of corresponding curves shows that, for load factors of 40% and above, the more economical of the two tariffs is No. 7 (a). Tariff 7 (a) will therefore be suitable for demands of high and relatively constant load factor, e.g. water pumping stations.

An undesirable feature of this tariff is the lack of flexibility over short periods. Thus an isolated high demand, once registered in any monitoring half-hour, brings about a high maximum demand penalty for a complete year.

For tariff 7, however, this penalty applies to each month taken separately. Where the demand is mainly seasonal, e.g. due to space heating, economies may be effected by judicious adjustment of the set demand at which load is shed. Production involving the use of electroheat may also be maximised over short periods, when the profit resulting from early completion of urgent orders may outweigh the high demand charges thus incurred.

8.4 IMPLEMENTATION AND ADVANTAGES OF DEMAND LIMITATION.

Limitation of the demand over each half-hour to a nominated "indicated" value may be termed tactical control of load.

In the long term, strategic control is required, whereby the load factor is maximised over an extended period of time.

The principles of tactical control of load are described in chapter 2. The Duomax and Trivector meters have been developed for use in conjunction with such schemes [53, 59].

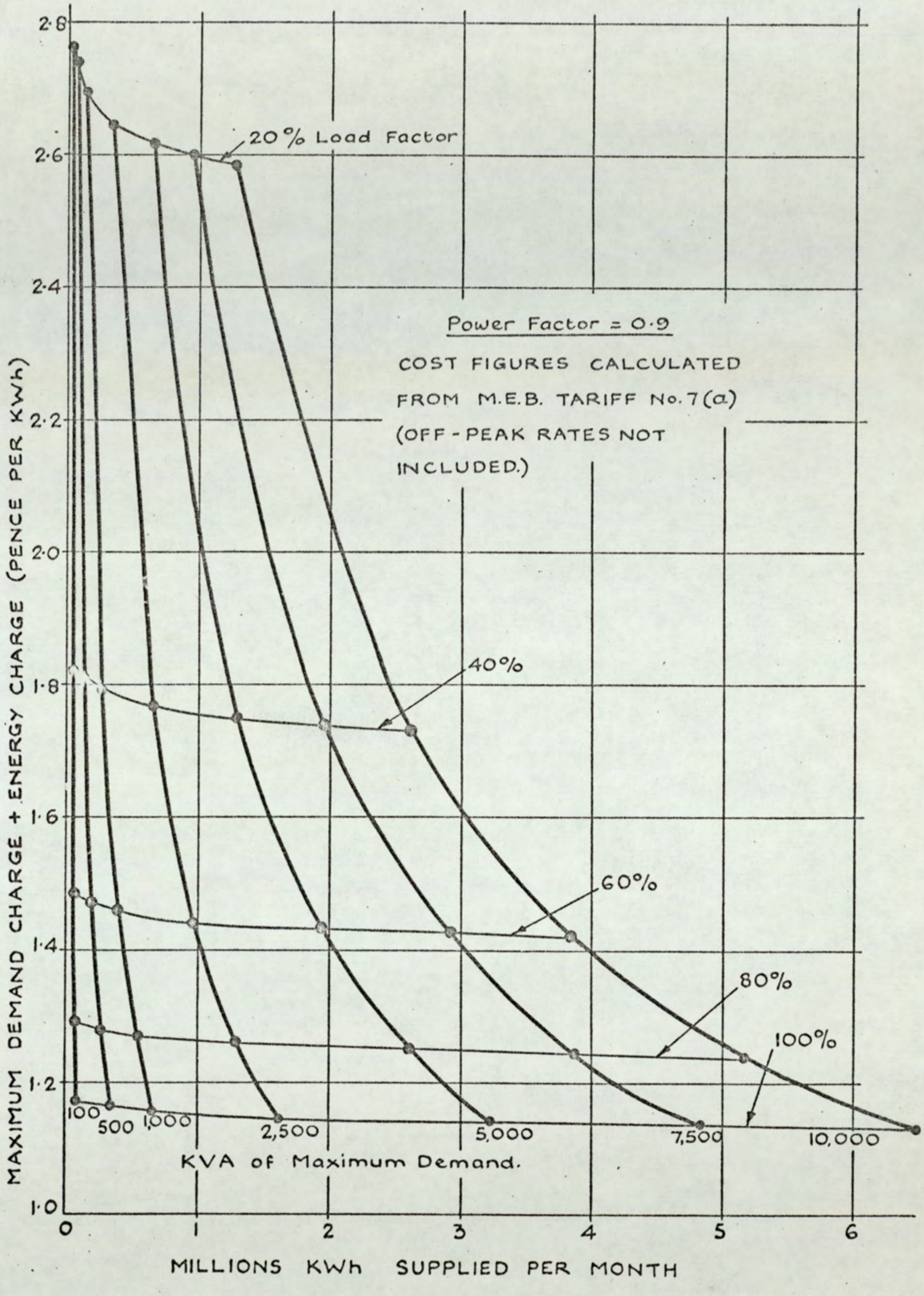


FIG. 8.4 TOTAL CHARGE OF PURCHASED ELECTRICITY PER KWh DELIVERED.

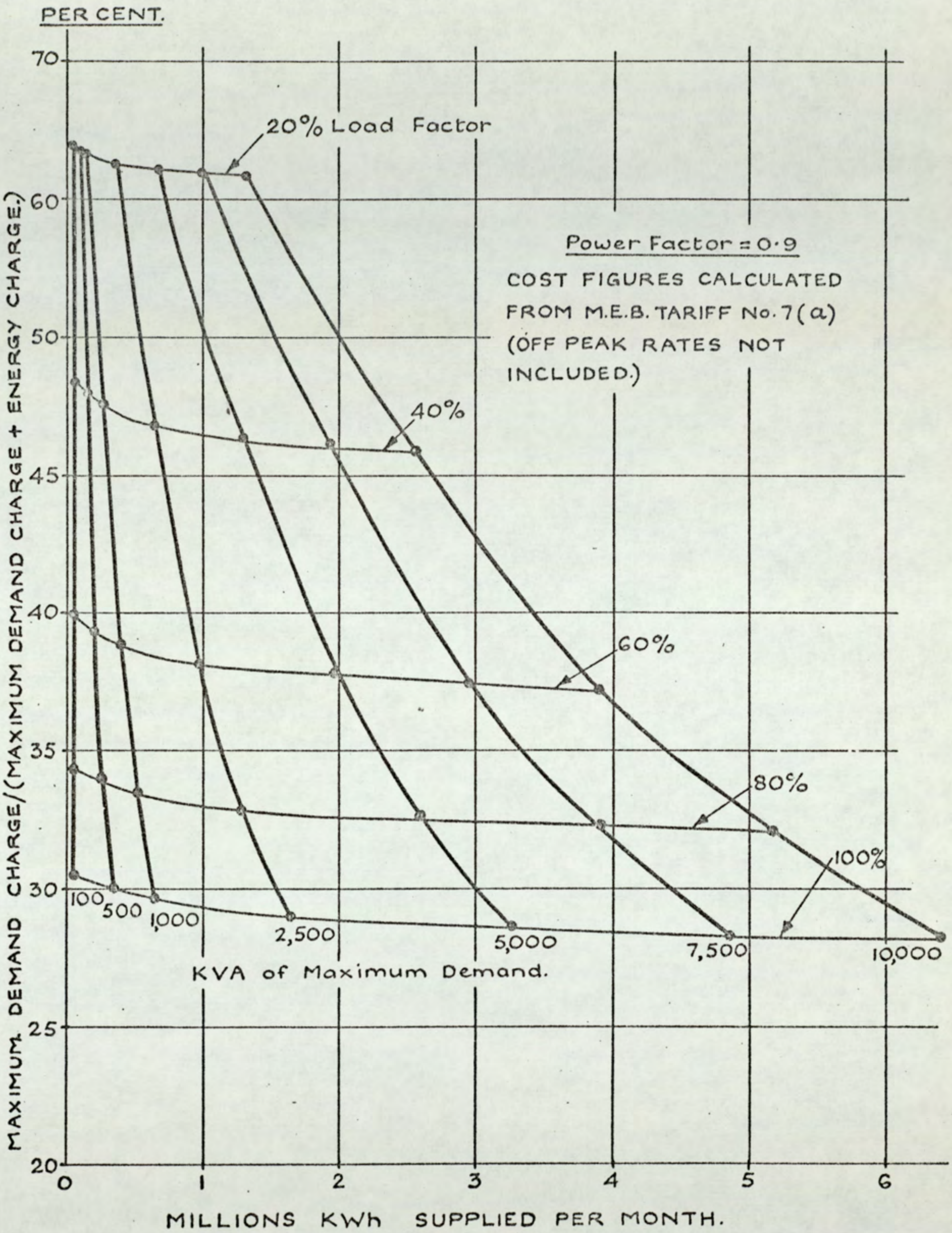


FIG. 8.5 PROPORTION OF ELECTRICITY CHARGES DUE TO MAXIMUM DEMAND.

Industrial applications for automatic control of load are described by Severin[54], and include power interruption of electric arc furnaces, on-load tap changing of furnace transformers, and step control of discrete loads.

For new installations the incidence of load may be planned to avoid excessive demand peaks.

For existing plants, however, major reorganisation of production to achieve significant load factor improvement may often prove too costly to implement.

8.4.1 Advantages of Demand Limitation.

As far as the industrial consumer is concerned, a criterion of optimisation is the cost per unit of production, say the cost per ton of metal produced.

If the load factor is poor, significant M.D. reduction may be achieved by peak lopping, with only minor reduction in the energy available for the process. Implementation of this method of control must, of course, be subject to the constraint that the level of production is not thereby seriously reduced.

Let us suppose that the production cost per ton is £P and that a maximum demand reduction of, say, 10 per cent, has a p_1 per cent effect on production costs. The value of p_1 will depend both upon the loss of energy due to load shedding and upon the proportion of the total load which is not related to production.

Let the electricity costs be p_2 per cent of the total production costs, and the monthly saving in electricity charges due to a 10 per cent maximum demand reduction be p_3 per cent.

Then saving in production cost as a result of reduced electrical charges = $p_3\ \% \times p_2\ \%$ of P.N, where N tons per month are produced before maximum demand reduction. (8.11)

Also cost of production loss per month = $p_1\%$ of P.N. Thus net saving per month due to maximum demand reduction

$$= P.N(p_2\% \times p_3\% - p_1\%), \qquad (8.12)$$

which is positive for $p_2\ p_3 > 100\ p_1$.

Progressive reduction of the maximum demand will eventually lead to the situation where the cost of production loss exceeds the saving achieved in electricity charges.

Realistic values can only be assigned to the above parameters for a specific process, as they will vary considerably according to the type of product, electrical load pattern and tariff rate.

We will now obtain the M.D. reduction for a specific project, the application of which results in minimum overall cost per unit product.

8.5 A SPECIFIC CASE STUDY - A MELTING CYCLE IN AN IRON FOUNDRY.

The case to be studied concerns an existing foundry which operates cupola equipment for producing Grade 17 iron to B.S.S. 1452. After machining the cast material, it has proved economical to install a coreless induction furnace, of nominal rating 800 KW, to melt the resulting borings. Under normal practice, the maximum indicated demand for the foundry does not exceed 2,000 KW.

Records of the indicated demand in KW are available for all the 1440 half-hourly integrating periods of the account month at a time of high production.

Demand recordings for seven days and nights are shown in Figs. 8.6, 8.7 and 8.8, and include those for the day when the month's maximum demand occurs. The energy consumption and load pattern vary from day to day owing to factors such as maintenance, week-end interruption of work, holidays, etc..

Significant demand reduction by load redistribution cannot be achieved without major expense. At a time of full employment, transfer of load to night-time and to week-ends will necessitate the introduction of costly bonus schemes.

8.5.1 Saving in Electricity Charges due to Reduction of Maximum Demand.

Total energy consumed in the account month = 562,720 KWh.

The maximum recorded demand is 2000 KW, so that

$$\text{load factor} = \frac{562,720}{2000 \times 720} = 0.391.$$

We will consider lopping of the peak demand by values increasing from zero to 20 per cent at 2 per cent increments. Owing to the high corrected power factor, the KVA may be taken as equal to the KW with an error of less than 2 per cent.

The demand peaks of interest and their summed durations over

MONDAY 8.7.68.

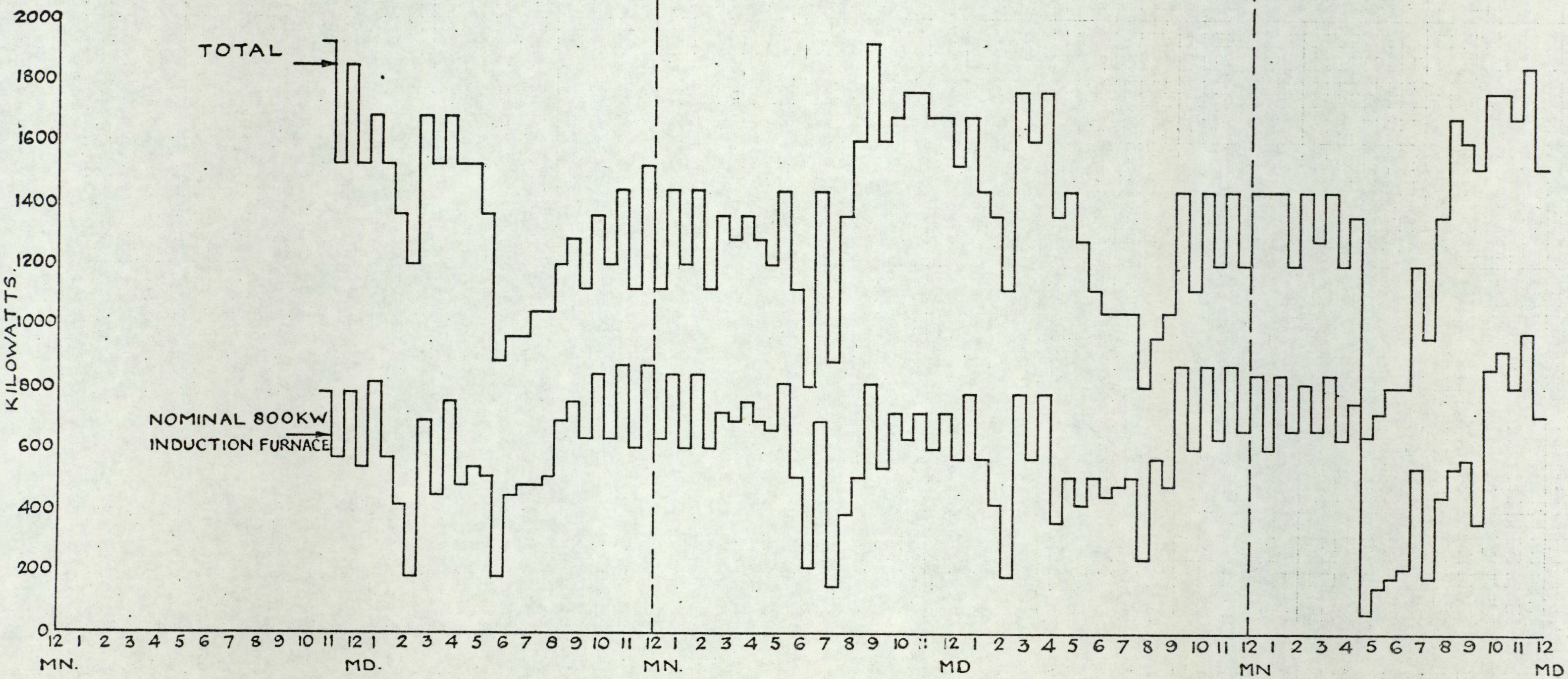
TUESDAY 9.7.68.

Total 32,360 KWh.
Furnace 14,355 KWh.

WEDNESDAY 10.7.68.

Total 30,840 KWh.
Furnace 14,160 KWh.

FIG. 8.6 HISTOGRAMS OF INDICATED DEMAND OVER ACCOUNT MONTH.



2.

WEDNESDAY 10·7·68.

THURSDAY 11·7·68.

Total 33,880 KWh.
Furnace 15,525 KWh.

FRIDAY 12·7·68.

Total 30,280 KWh.
Furnace 11,985 KWh.

FIG. 8.7 HISTOGRAMS OF INDICATED DEMAND.


3.
SATURDAY 13·7·68.
Total 23,520 KWh.
Furnace 8,040 KWh.
SUNDAY 14·7·68.
Total 17,160 KWh.
Furnace 5,895 KWh.
MONDAY 15·7·68.
Total 27,560 KWh.
Furnace 10,320 KWh.
FIG. 8.8
HISTOGRAMS OF INDICATED DEMAND.
KILOWATTS
0
200
400
600
800
1000
1200
1400
1600
1800
2000
12 1 2 3 4 5 6 7 8 9 10 11 12 1 2 3 4 5 6 7 8 9 10 11 12 1 2 3 4 5 6 7 8 9 10 11 12 1 2 3 4 5 6 7 8 9 10 11 12 1 2 3 4 5 6 7 8 9 10 11 12
MN.
MD.
MN.
MD.
MN.
MD.
CONTINUED

the account month are given in Table 8.1.

INDICATED DEMAND KVA	DURATION Hours	INDICATED DEMAND KVA	DURATION Hours
2000	0.5	1760	10.5
1960	-	1720	-
1920	0.5	1680	14.5
1880	-	1640	-
1840	5.0	1600	17.5
1800	-		

TABLE 8.1

Electricity is supplied by the M.E.B., the charges being given by tariff No. 7.

On no day does the indicated demand in the off-peak hours exceed 1600 KVA.

25.4% of the total energy delivered per month is consumed in the hours 11 p.m. - 7 a.m.; a rebate of 0.13 pence is due in respect of each KWh consumed in this period.

The cost of electricity per month before load cutting is £4025 (made up of £1590 M.D. and £2435 energy charges), or 1.717 pence per KWh.

The M.D. and energy savings per month effected by M.D. reduction have been evaluated as follows:

M.D. Reduction Per cent	Monthly Saving in M.D. charges £	Monthly Saving in Energy Charges £	Total Monthly Saving in Charges £	Pence per KWh	Energy Reduction per month KWh
0	0	0	0	1.717	0
2	30.83	5.78	36.61	1.701	20
4	61.66	11.55	73.21	1.686	40
6	92.49	17.42	109.91	1.67	80
8	123.33	23.28	146.61	1.654	120
10	154.17	29.92	184.09	1.639	360
12	185.0	36.58	221.58	1.624	600
14	215.83	44.89	260.72	1.609	1260
16	246.66	53.20	299.86	1.594	1920
18	277.50	63.81	341.31	1.58	3160
20	308.33	74.42	382.75	1.566	4400

TABLE 8.2

The savings achieved in M.D. and energy charges as a result of load limitation, together with the pence per KWh, are plotted as a function of M.D. reduction in Fig. 8.9.

Savings in M.D. charges are obviously directly proportional to the maximum demand reduction, since the range of maximum demand considered does not involve movement to a new charge rate (see equation 8.5).

Savings in energy charges are a function both of the demand pattern and the maximum demand reduction. However the reduction in M.D. charges represents a sufficiently high proportion of the total costs saved to linearise the overall characteristic.

8.5.2 Effect of Demand Reduction upon Production Costs.

For a foundry producing a variety of products, each of which requires heat and metallurgical processing, operating experience may present the only way of determining the effect of maximum demand reduction upon production costs.

However, for steel melting or heat treatment, experience has shown that the KWh per ton are substantially constant. It is therefore reasonable to assume that the percentage saving in energy due to load cutting produces an approximately equal percentage loss of production, provided the resulting heat loss over the increased holding period does not become appreciable.

This assumption is justified by experience at the Templeborough electric melting shop of the United Steel Companies Limited, where the maximum demand due to six arc furnaces for producing steel is controlled by computer (section 2.4.1).

The production curves of Fig. 8.10 are replotted in Fig. 8.11 as percentages of the values obtained before load cutting. The curves of percentage production lost and percentage energy saved diverge appreciably only when the M.D. reduction exceeds about 20 per cent.

This divergence may be explained as follows:

The heat supplied to the furnaces may be divided into:

(a) That required to produce a change of state, including latent heat plus any endothermic reaction.

(b) That required to maintain the melt temperature by replacing the radiation losses over a period of time.

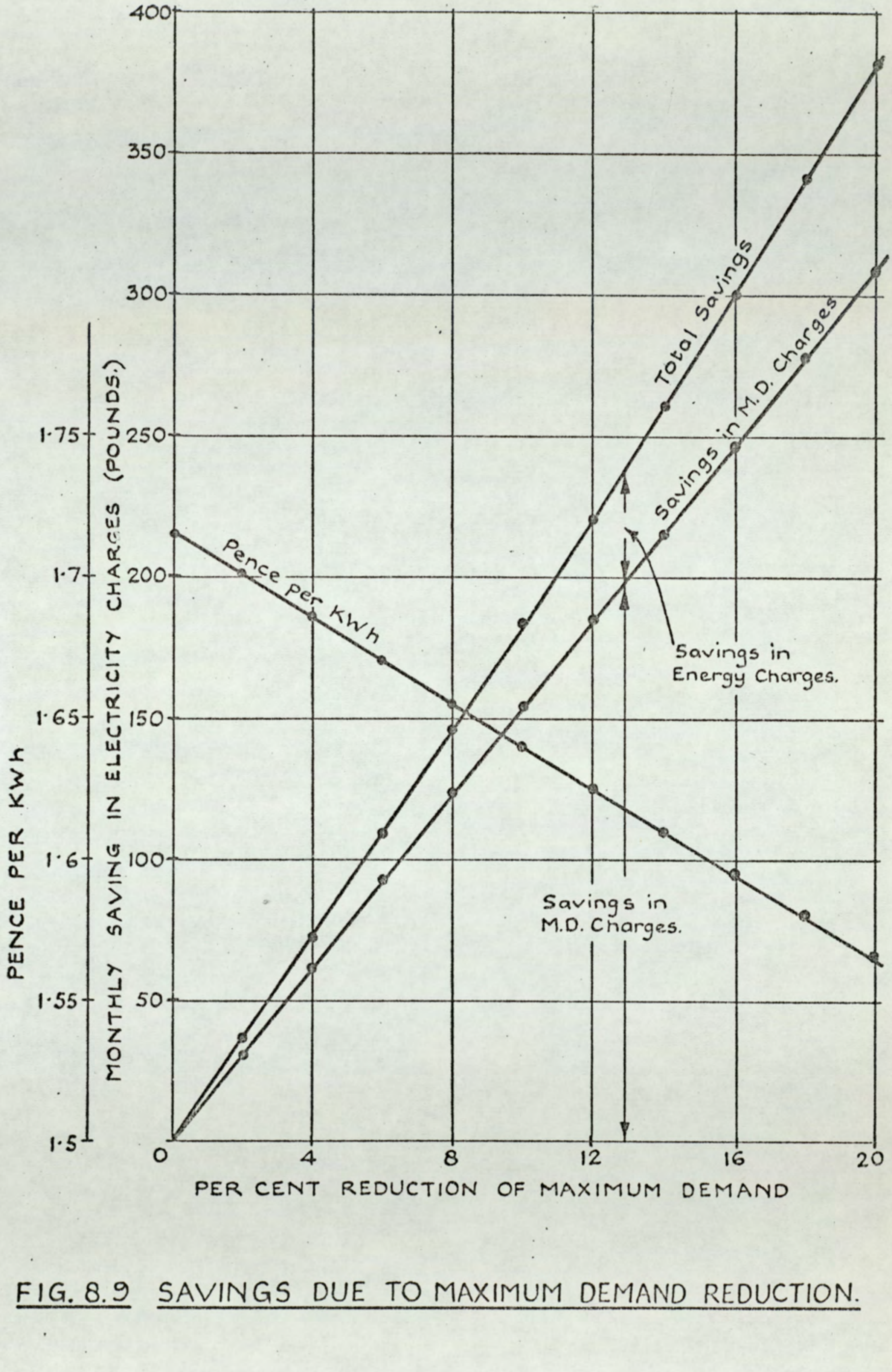


FIG. 8.9 SAVINGS DUE TO MAXIMUM DEMAND REDUCTION.

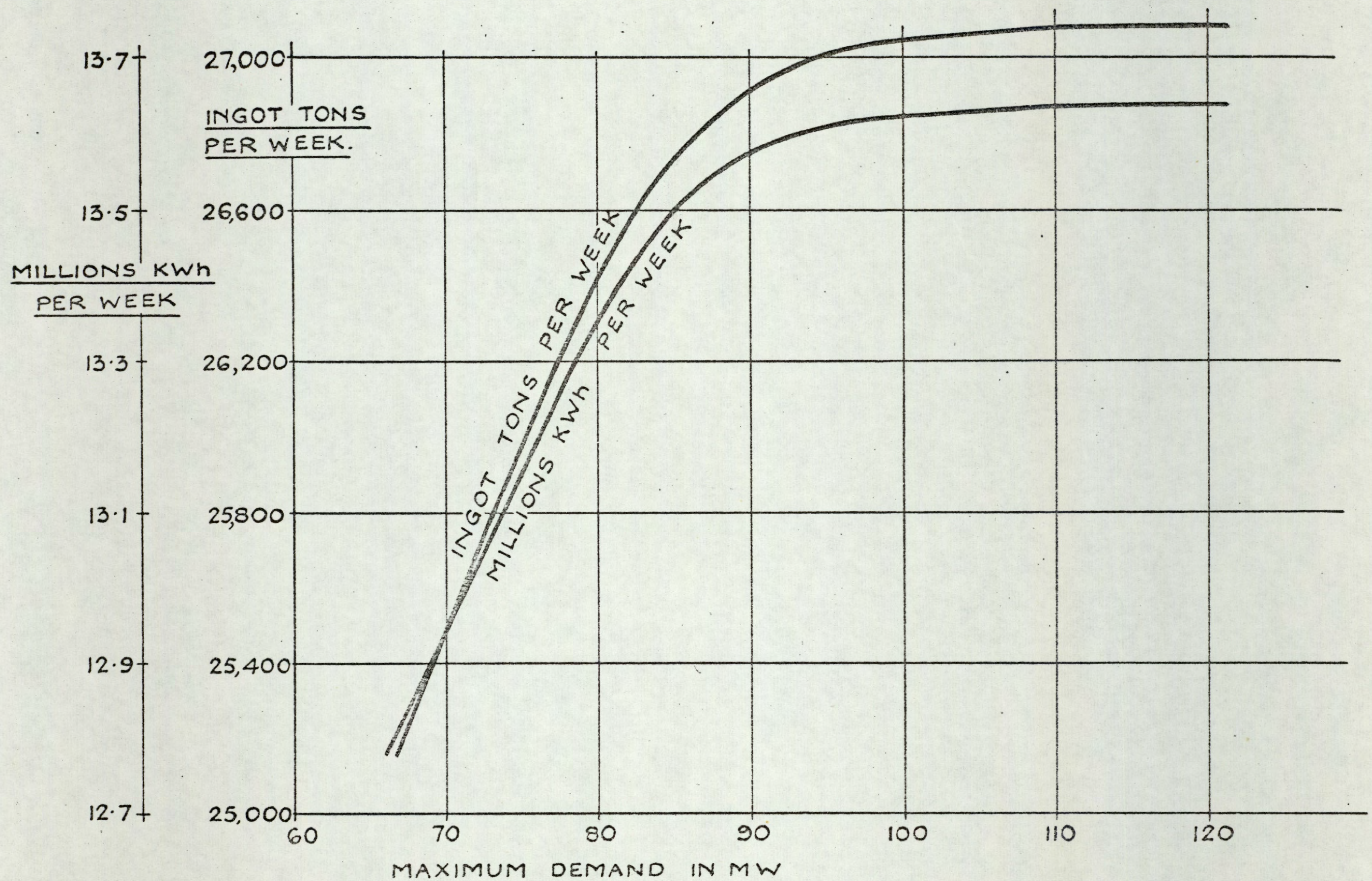


FIG. 8.10 EFFECT OF MAXIMUM DEMAND REDUCTION UPON PRODUCTION & ENERGY USAGE.

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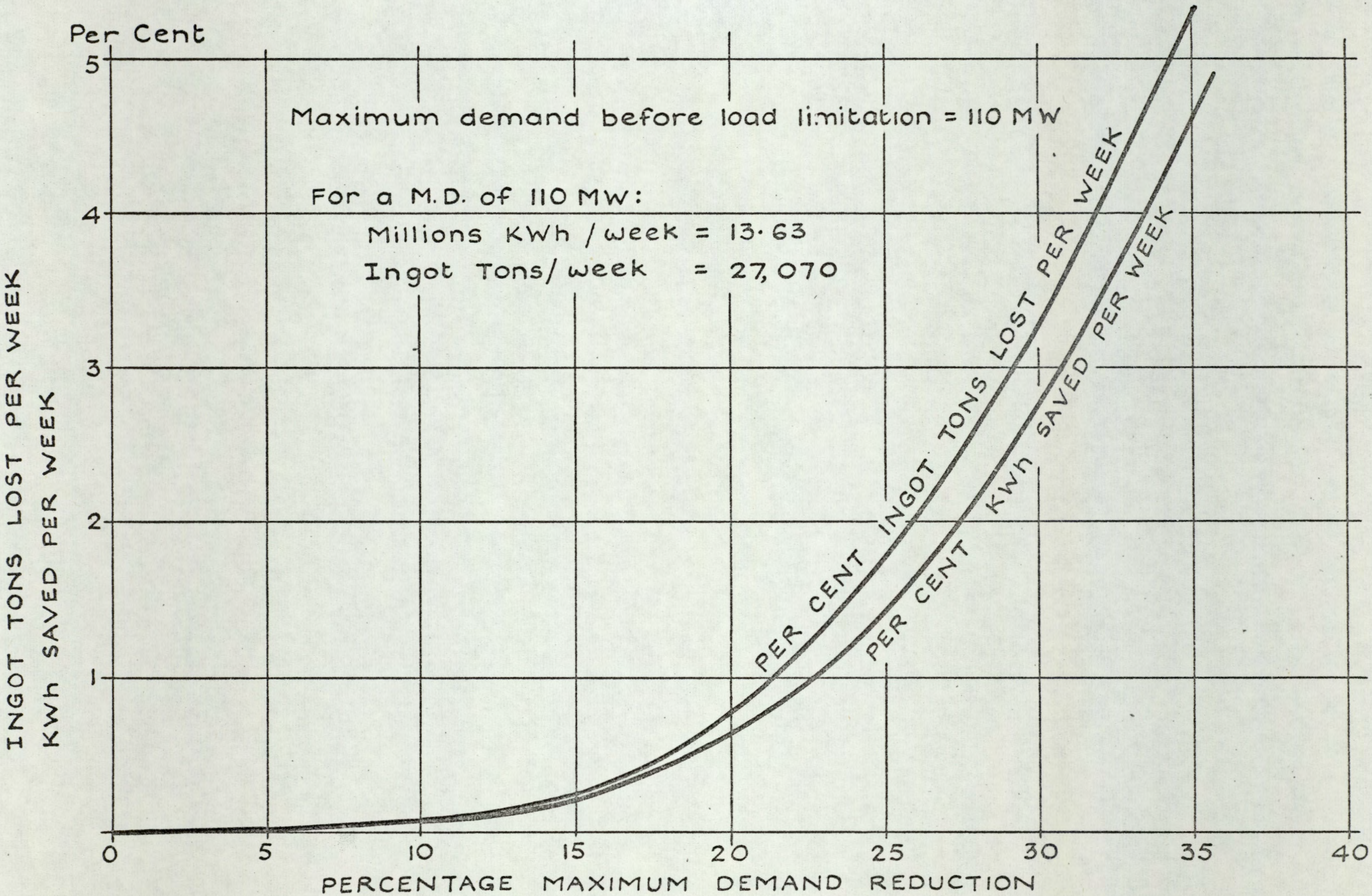


FIG. 8.11 EFFECT OF MAXIMUM DEMAND REDUCTION UPON PRODUCTION & ENERGY USAGE.

M.D. REDUCTION PER CENT	MONTHLY M.D. CHARGES £	MONTHLY ENERGY CHARGES £	TOTAL MONTHLY CHARGES £	TOTAL MONTHLY SAVING IN CHARGES £	PENCE PER KWh	REDUCTION IN ENERGY PER MONTH	
						KWh	Per cent
0	740.42	941.08	1681.50	0	1.880	0	0
2	726.02	938.37	1664.39	17.11	1.862	36	0.017
4	711.62	935.43	1647.05	34.45	1.844	141	0.066
6	697.22	931.99	1629.21	52.29	1.825	384	0.179
8	682.82	927.75	1610.57	70.93	1.808	807	0.376
10	668.42	923.26	1591.68	89.82	1.791	1320	0.615
12	654.02	917.48	1571.50	110.00	1.776	2221	1.035
14	639.62	911.63	1551.25	130.25	1.760	3024	1.409
16	625.22	904.80	1530.02	151.48	1.745	4138	1.929
18	610.82	897.59	1508.41	173.09	1.730	5350	2.494
20	596.42	890.07	1486.49	195.01	1.716	6640	3.094

TABLE 8.4

For a given charge weight and composition, the energy represented by (a) will be virtually constant. However, if the effect of load reduction is to spread the production cycle over an extended period, the requirement to maintain the temperature for an increased time will eventually result in a loss of production not compensated by the energy saved.

In this case study, details are not available of the operating costs for the whole plant. However, information is published[55] of the operating costs for a foundry using an induction melting furnace of identical type and rating to that described, and producing the same product. We will therefore examine a hypothetical case where the induction furnace represents the main factory load, and has sufficient rating to be available for demand reduction.

8.5.3 Reduction of Maximum Demand due to Induction Furnace.

From the demand histograms for the furnace alone:

Total energy consumed in the account month = 214,600 KWh.
Maximum recorded demand in the 1440 integrating periods = 900 KVA, giving a load factor of 33.8 per cent, based upon a 720 - hour month.

The corrected power factor is 0.98, so that KW $\simeq$ KVA.

Reductions in maximum demand up to 20 per cent of 900 KVA will be considered, when saving will be achieved both in day-time and off-peak rates.

The relevant demand peaks and their durations are as follows:

INDICATED DEMAND KVA	DURATION	
	On-Peak Hours	Restricted Hours 11 p.m. to 7 a.m.
900	2	-
870	8	3.5
840	8	7
810	13.5	2.5
780	16	1
770	0.5	-
750	6	2
740	1.5	-

TABLE 8.3

Of the total energy delivered per month, 67,620 KWh, or 31.5 per cent, are consumed in the restricted hours. The cost of electricity per month before load cutting is:

Monthly M.D. charge.

200 KVA at 17s. 2d.	=	£171. 13. 4.
300 KVA at 16s. 7d.	=	£248. 15. 0.
400 KVA at 16s. 0d.	=	£320. 0. 0.
		£740. 8. 4.

Monthly KWh charge.

180 x 900 x 1.14d.	=	£769. 10. 0.
(214,600 - 162,000) x 0.95d.	=	£208. 4. 2.
		£977. 14. 2.
Less 67,620 x 0.13d. (off-peak rebate)	=	36. 12. 7.
		£941. 1. 7.

Then charge/KWh $= \frac{£1681.\ 9.\ 11}{214,600} = 1.88$ pence.

For a 4 per cent reduction of maximum demand, 120 KWh are saved at the day-time rate and 21 KWh at the off-peak rate.

Then monthly M.D. charge = £711. 12. 4 ,

and monthly KWh charge

$= 180 \times 864 \times 1.14 + (214,459 - 155,520) \times 0.95$

$- 67,599 \times 0.13$ pence.

= £935. 8. 8.

Then charge/KWh $= \frac{£1647.\ 1.\ 0}{214,459} = 1.844$ pence.

The monthly savings in electricity charges for peak lopping up to 20 per cent of 900 KVA have been evaluated for the above furnace load pattern.

The results are given in Table 8.4 and are plotted in Fig. 8.12, showing the effect of maximum demand reduction upon:

(a) The monthly saving in electricity charges. (£).

(b) The monthly saving in electricity usage (KWh).

(c) Cost of electricity (pence per KWh).

8.5.4 Production Costs.

The electrical load consists of a line frequency coreless induction furnace for iron melting, of nominal electrical rating 800 KW and capacity 5 tons, with a gross melt rate of 1.22 tons per hour. Feedstock to the furnace consists entirely of borings recovered from

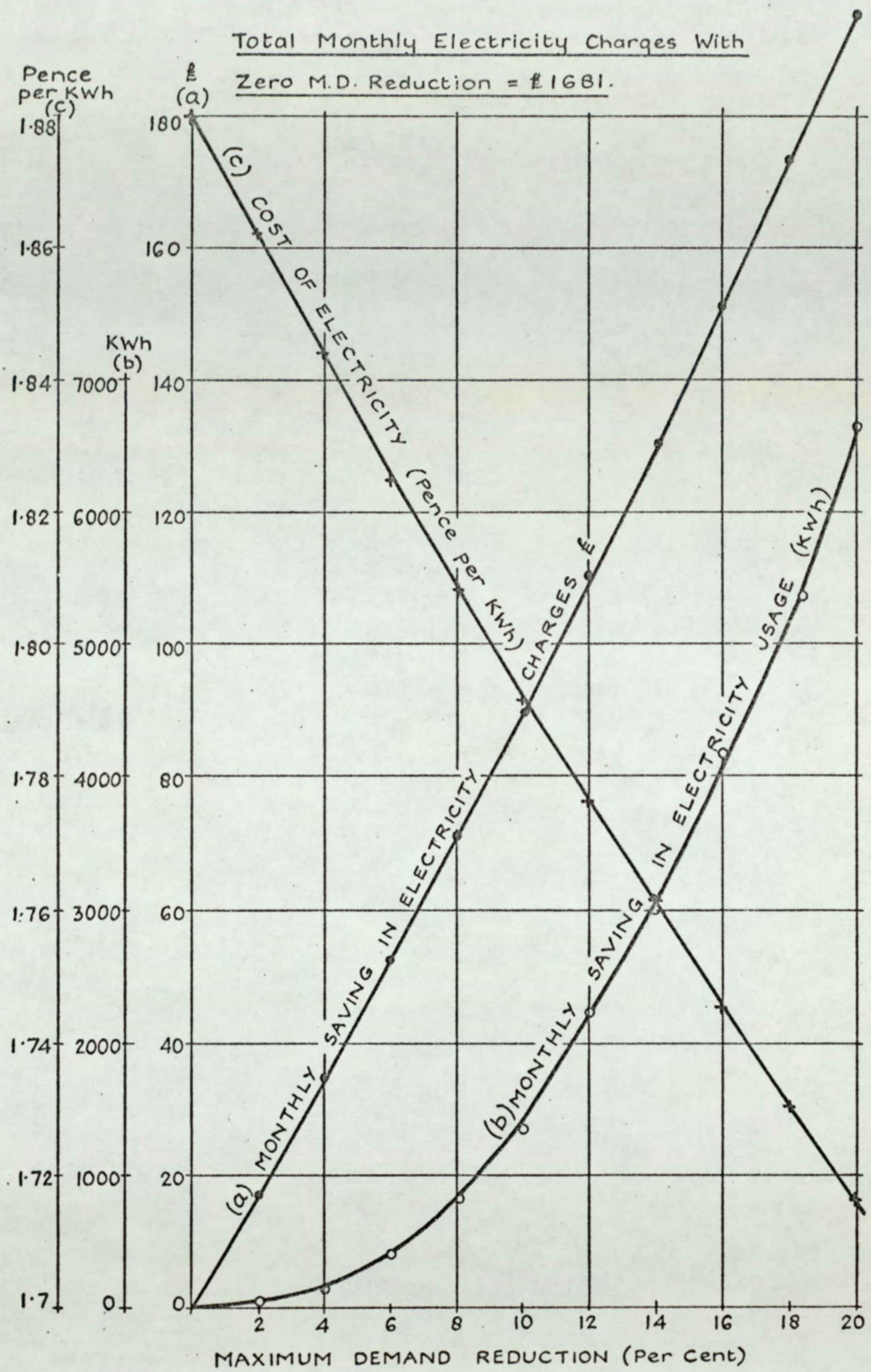


FIG. 8.12 EFFECT OF M.D. REDUCTION UPON ELECTRICITY COSTS & USAGE.

machined castings and costing £6 per ton.

Raw materials costs and cost of purchased energy are at prices appertaining in May 1968.

An analysis of operating costs (but excluding overheads not related to production) is as follows:

Capital cost of furnace, including installation = £40,000.

Output for month considered = 275 tons.

Maximum demand charge for month.	£740
Energy charges for month.	941
Fuel adjustment charge (assumed zero for this month).	-
Labour (4 men each at £1000 p.a).	333
Refractory.	100
Depreciation (assumed life of 10 years)	333
Maintenance and spares.	50
	£ 2947
Then melting cost per ton =	£9.08
Adding the cost of the charge:	
Borings at £6 per ton.	6.0
Melting loss (5%).	0.3
Metallurgical additives.	1.0
Therefore the overall metal cost per ton =	£16.38

TABLE 8.5

For each value of M.D. reduction we may repeat the analysis of Table 8.5, using the results presented in Table 8.4. The percentage loss of production due to the reduction of peak demand is taken to equal the percentage decrease in energy. The production-related overheads may be assumed constant for the change in production level involved.

Table 8.6 shows the calculated melting and total metal costs for each value of maximum demand.

For any production plant certain overheads must be carried which are, broadly speaking, independent of production, e.g. administration, lighting and heating of offices; these can vary considerably, of course, according to the premises, management efficiency, and diversity of product. Ascribing a range of fixed costs to these overheads, the overall operating costs have been evaluated in Table 8.7.

Maximum Demand Before Peak Lopping = 900 KVA. TABLE 8.6

M.D. Reduction per cent	Total Monthly Charges (M.D. + Energy) £	Reduction of Output per cent	Monthly Output Tons	Melting Cost per month £	Feedstock Cost (at £7.3 per ton) £	Total Metal Cost per month £
0	1681	0	275	2497	2008	4505
2	1664	0.017	274.95	2480	2007	4487
4	1647	0.066	274.82	2463	2006	4469
6	1629	0.179	274.51	2445	2004	4449
8	1611	0.376	273.97	2427	2000	4427
10	1592	0.615	273.31	2408	1995	4403
12	1572	1.035	272.15	2388	1987	4375
14	1551	1.409	271.13	2367	1979	4346
16	1530	1.929	269.69	2346	1969	4315
18	1508	2.494	268.14	2324	1957	4281
20	1486	3.094	266.49	2302	1945	4247

M.D. Reduction Per cent	Monthly Output Tons	Total Metal Cost £	OVERALL OPERATING COSTS							
			Overall Cost/month £	£/Ton	Overall Cost/month £	£/Ton	Overall Cost/month £	£/Ton	Overall Cost/ Month £	£/Ton
0	275	4505	4505	16.38	6505	23.65	7005	25.48	9505	34.56
2	274.95	4487	4487	16.32	6487	23.60	6987	25.41	9487	34.50
4	274.82	4469	4469	16.26	6469	23.54	6969	25.36	9469	34.45
6	274.51	4449	4449	16.21	6449	23.49	6949	25.31	9449	34.42
8	273.97	4427	4427	16.15	6427	23.45	6927	25.27	9427	34.40
10	273.31	4403	4403	16.11	6403	23.43	6903	25.24	9403	34.40
12	272.15	4375	4375	16.07	6375	23.42	6875	25.24	9375	34.43
14	271.13	4346	4346	16.03	6346	23.41	6846	25.25	9346	34.46
16	269.69	4315	4315	16.00	6315	23.41	6815	25.26	9315	34.53
18	268.14	4281	4281	15.97	6281	23.42	6781	25.29	9281	34.61
20	266.49	4247	4247	15.94	6247	23.44	6747	25.32	9247	34.69
			A		B		C		D	

A - Overall operating cost with overheads not related to production equal to zero.

B - Overall operating cost with overheads not related to production equal to £2000.

C - Overall operating cost with overheads not related to production equal to £2500.

D - Overall operating cost with overheads not related to production equal to £5000.

TABLE 8.7

The calculation may be expressed in algebraic form:-

Let M_1 = initial maximum demand, and charge/month = M_{1c}

E_1 = initial energy, and charge/month = E_{1c}

C_p = production-related overheads/month.

C_N = overheads per month not related to production.

P_1 = monthly production in tons.

K = cost of metal and additives/ton.

Then total metal cost per ton $= \dfrac{M_{1c} + E_{1c} + C_p}{P_1} + K,$ (8.13)

and overall cost per ton $= \dfrac{M_{1c} + E_{1c} + C_p + KP_1 + C_N}{P_1}.$ (8.14)

Let the values of maximum demand and energy subsequent to load cutting be M_2 and E_2 respectively, and their corresponding monthly charges M_{2c} and E_{2c}.

Then new overall cost per ton

$$= \frac{M_{2c} + E_{2c} + C_p + KP_2 + C_N}{P_2}, \qquad (8.15)$$

where P_2 is the corresponding monthly production in tons.

We assume that $\dfrac{E_1 - E_2}{E_1} = \dfrac{P_1 - P_2}{P_1},$ (8.16)

so that $P_2 = P_1 \left(1 - \dfrac{E_1 - E_2}{E_1}\right).$ (8.17)

If $E_1 - E_2 = \Delta E,$ (8.18)

then $\Delta E = f(\Delta M),$ (8.19)

where $\Delta M = M_1 - M_2.$ (8.20)

The costs M_{2c}, E_{2c} may be expressed in terms of M_{1c}, E_{1c}, respectively by applying the appropriate equations (8.2) - (8.9).

However the relation (8.19) may be determined only by observations of the demand pattern for the specific project. Minimisation of the overall cost per ton then requires numerical evaluation.

The information derived in Table 8.7 is presented graphically as follows:

1. The effect of maximum demand reduction upon:
   (i) Operating cost per ton (Fig. 8.13).
   (ii) Production loss per month (Fig. 8.14).
   (iii) Operating cost per month (Fig. 8.14).

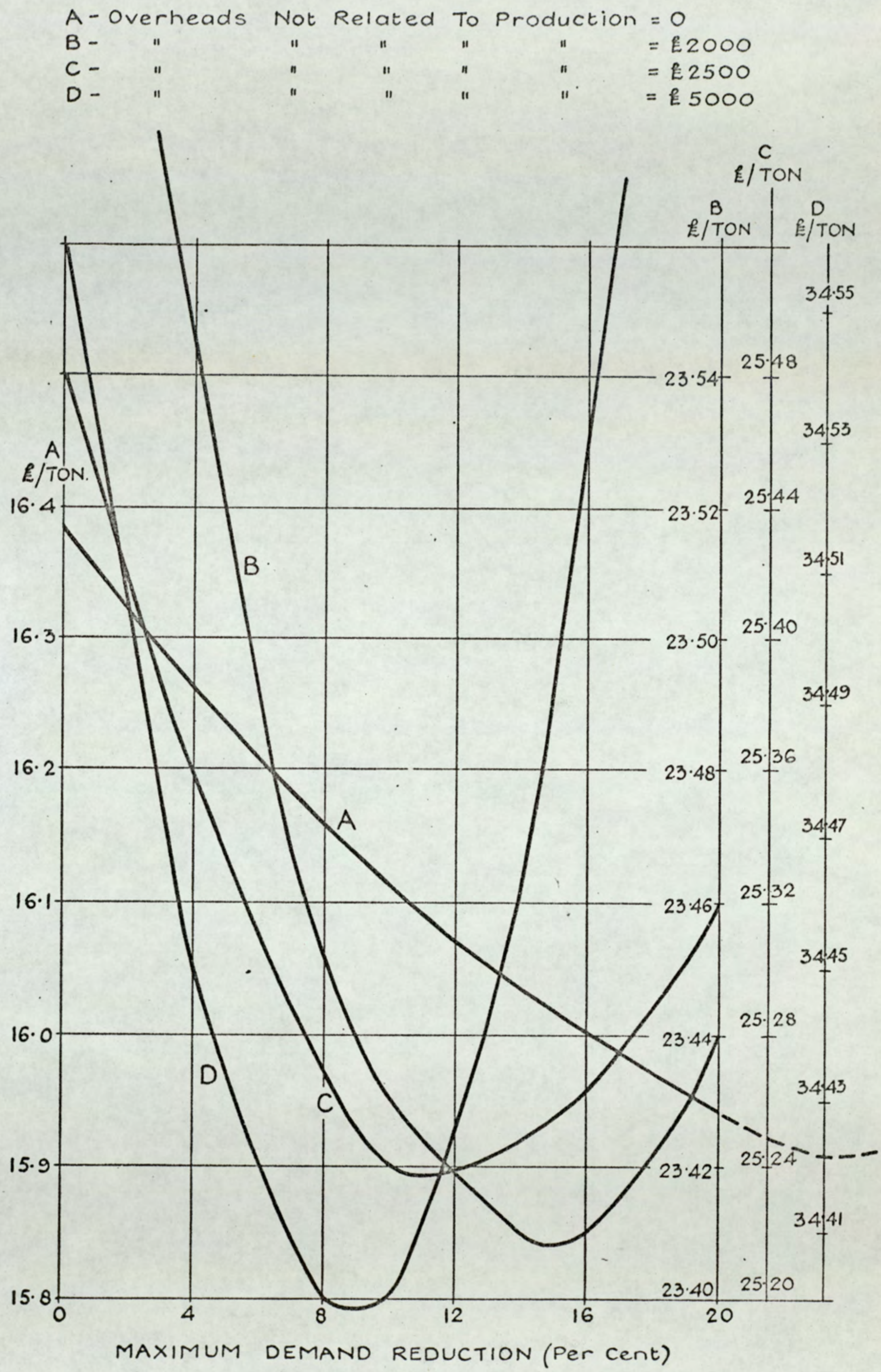


FIG. 8.13 OPERATING COST PER TON.

(b) Overheads not related to production = £2000
(c) " " " " " = £5000

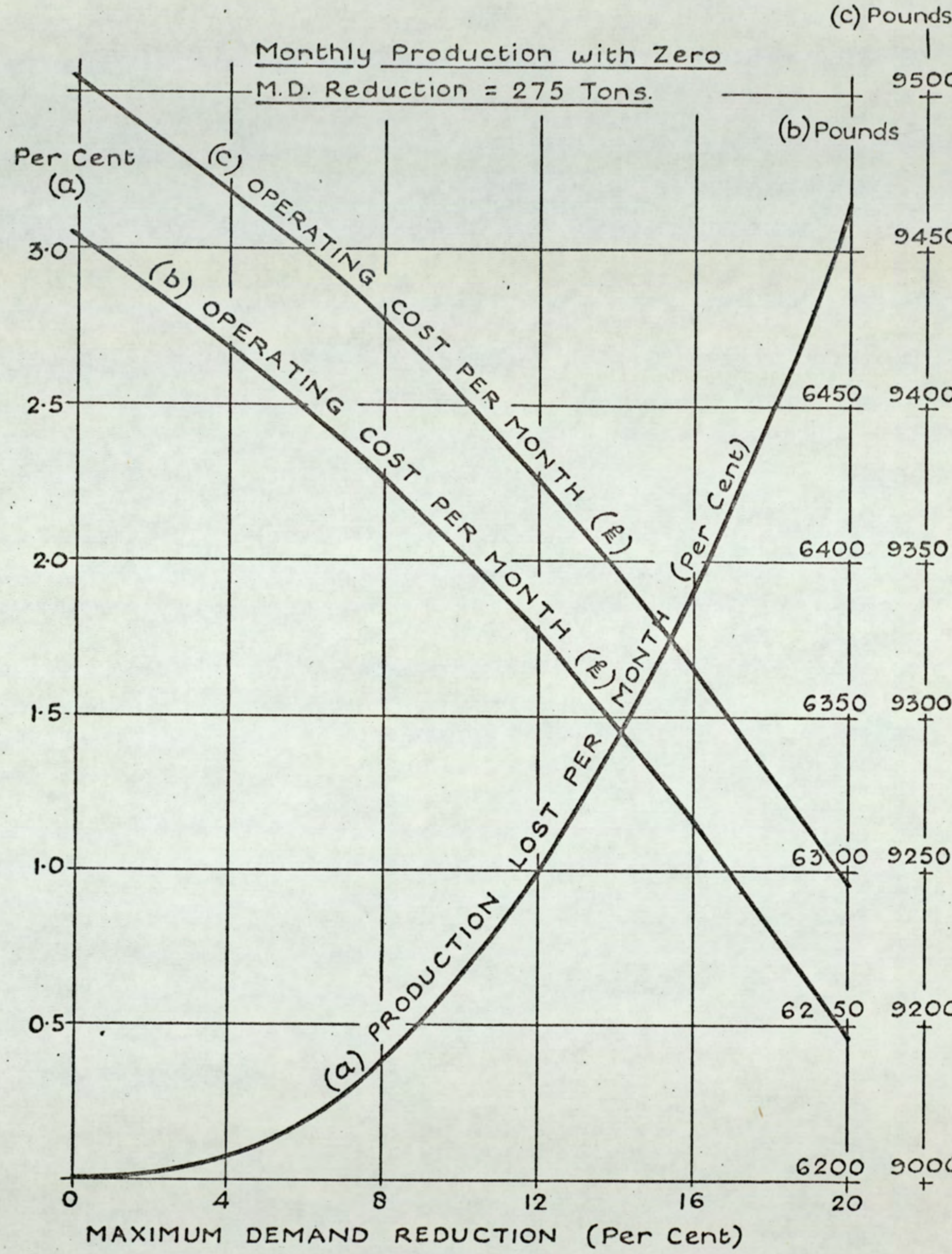


FIG. 8.14 EFFECT OF M.D. REDUCTION UPON PRODUCTION & OPERATING COSTS.

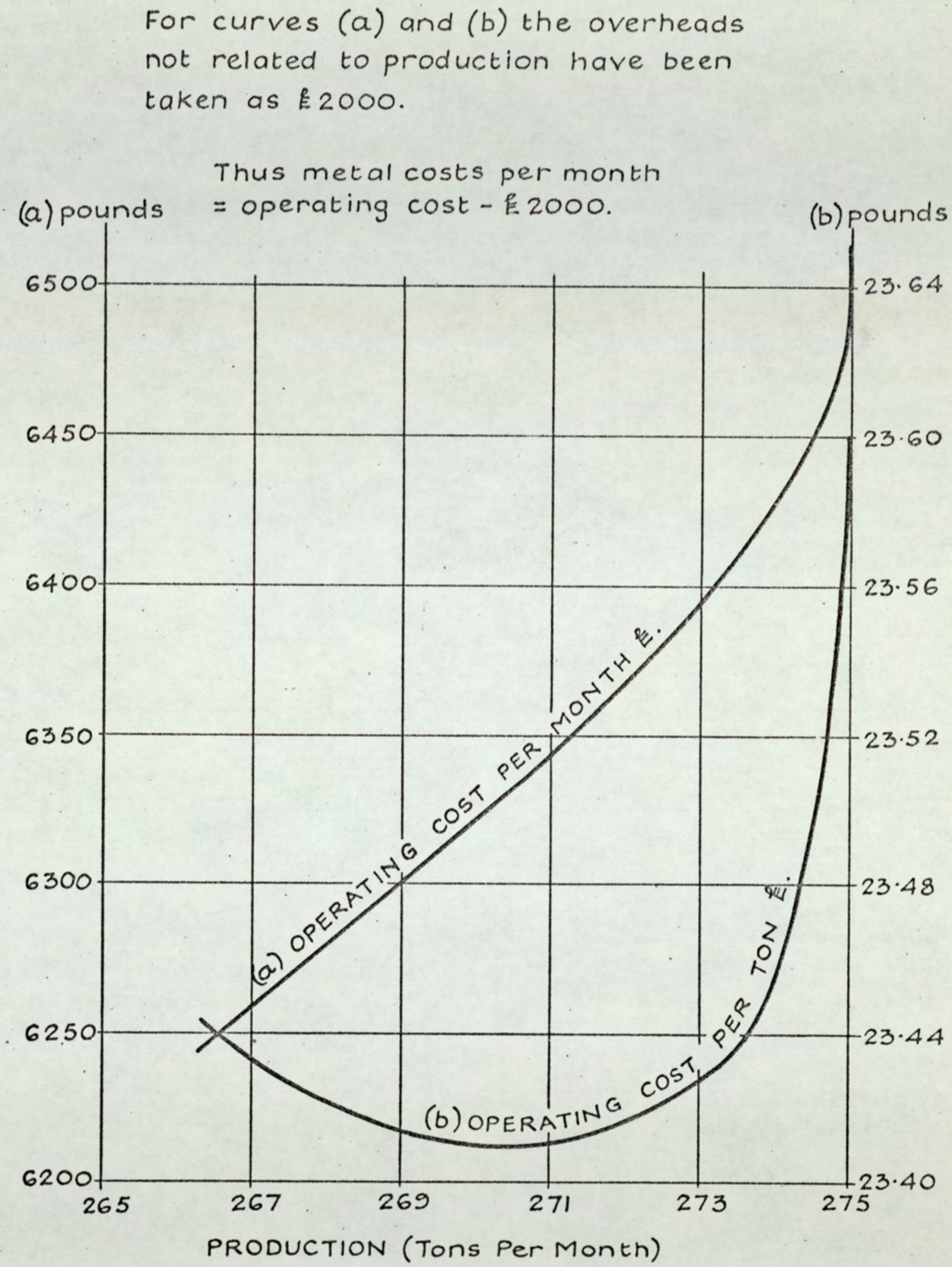


FIG. 8.15 OPERATING COSTS v PRODUCTION.

Also presented are:

2 . Operating cost as a function of monthly production in tons (Fig.8.15).

3 . Operating cost per ton as a function of monthly production (Fig.8.15).

8.5.5 Discussion of Results.

In considering the characteristics of Fig. 8.13 we may refer to equation (8.14).

For the variation of production considered, C_p and C_N may be taken as substantially constant for a given account month, although C_N will have seasonal variations, e.g. office heating load.

Rearranging (8.14) in general terms,

overall cost per ton

$$= \frac{M_c + E_c}{P} + \frac{C_p}{P} + \frac{C_N}{P} + K \quad . \qquad (8.21)$$

Over the M.D. range considered, the magnitude of the first term of (8.21) decreases with progressive reduction of the maximum demand; however, the contributions of the second and third terms to the cost per ton become of increasing significance as production falls, so that a minimum appears in the overall characteristic. For C_N = £2000, £2500, £5000, the values of maximum demand reduction which give minimum operating cost per ton are 15, 11 and 9 per cent respectively.

An outstanding feature of Fig. 8.13 is the importance of reducing overheads which are not related to production. A twofold advantage is thereby obtained; firstly, the overall operating cost is reduced, and secondly, economic production is obtained with a reduced maximum demand, leading to smaller electricity charges.

A constraint upon the implementation of maximum demand reduction will be the resulting fall in production rate.

We see from Fig. 8.15 that, for C_N = £2,000, minimum operating cost is achieved at a production rate of 270.3 tons per month, a loss of 1.71 per cent compared with the production with no demand restriction. An optimum maximum demand reduction of 15 per cent will therefore be based upon a monthly production schedule of not less than 270 tons.

If, however, production must not be restricted by more than 0.5 per cent, or to a minimum of 273.6 tons per month, the reduction of maximum demand is limited to 9 per cent (Fig. 8.14). The resulting operating cost is £23.44 per month, compared with the figure of £23.41

if the maximum demand were reduced by 15 per cent.

The load factor is insufficiently high to warrant a tariff charge from No. 7 to 7 (a). The results apply only for this level of production and for the electrical demands recorded. They will be modified, of course, by subsequent changes of raw material prices, wages and tariff rates.

Owing to loss of production due to demand control, the economic effect of a penalty charge for unfulfilled orders may have to be considered.

In planning the monthly production, it may be found that operation at a maximum demand corresponding to the minimum of a curve shown in Fig. 8.13 gives a satisfactory production rate. This economic maximum demand level will have a stochastic variation, since the demand for each month is predicted from the previous month's recorded pattern. Operation to the left of the minimum of the characteristic ensures overproduction, but at the expense of increased cost per ton. Operation to the right of the minimum also results in production less efficient than optimum; moreover, the reduction of overall costs thus achieved may be insufficient to compensate for the incompleted quota penalty.

Quite apart from such considerations, objective management decisions will have to be taken on production schedules in the light of fluctuating market requirements.

8.6 A SECOND CASE STUDY - APPLICATION OF DEMAND CONTROL IN A FACTORY PRODUCING A DIVERSITY OF METAL PRODUCTS.

This study concerns an existing factory operating a number of furnaces in cascade to produce a variety of iron castings of varying controlled composition.

The feedstock is melted in a coke-fired cupola which, by its nature, is essentially a melting furnace producing a continuous flow of metal. In order immediately to meet the intermittent flow required by the casting production line, the metal is poured into a 30-ton holding furnace, which maintains a full reserve of molten metal. In the succeeding chain are included an induction furnace for raising the temperature of the melt, and a group of resistance-heated furnaces, of total rating 1,000 KVA, for diverse annealing of batch products.

8.6.1 Strategy Adopted for Minimising Electrical Demand.

Owing to a full order book, priority was given to maximising production, which was therefore not obtained at minimum cost per ton. The aim of this policy was to capture a rising market and to foster producer-customer relations.

A Duomax maximum demand control system was installed to give warning when load reduction was necessary.

The principle of implementation was that load control would be effected by the operator, but only at times when, in so doing, no blockage or delay in the overall production flow would be caused. Reduction of electrical charges due to load restriction was thus regarded as a bonus to be obtained whenever production allowed.

At a time of full employment, production had to be confined mainly to the day, in spite of incentive bonuses for night-time work. Load restriction could therefore be applied freely in the early part of the morning, but became more difficult to implement as the full electrical demand was reached by mid-morning.

Setting of the maximum demand level was obtained as a result of extensive operating experience of the plant. Too low a level would have resulted in operating staff ignoring frequent warnings of impending completion of the half-hourly energy quota, whilst too high a level would have been largely ineffective in reducing electricity costs.

Tariff 7 (a) was considered unsuitable for this application owing to the production variability and relatively low load factor. Thus the maximum demand control level was reset each month to take best advantage of the rates given by tariff 7.

In Figs. 8.16 and 8.17, histograms are shown of the half-hourly integrated demands for four days of production. The greatest demand always occurs during the day, at either mid-morning or mid-afternoon. Production is low at weekends.

The monthly maximum demand settings over a 2-year period are shown in Table 8.8.

As expected, the cost per KWh increases substantially for months of poor load factor. The table clearly shows how the maximum demand settings must be raised in the winter months as a result of the increased load due to unrestricted office heating and lighting.

8.6.2 Scope for Reduction of Operating Costs .

In this application, the worth of production for an individual

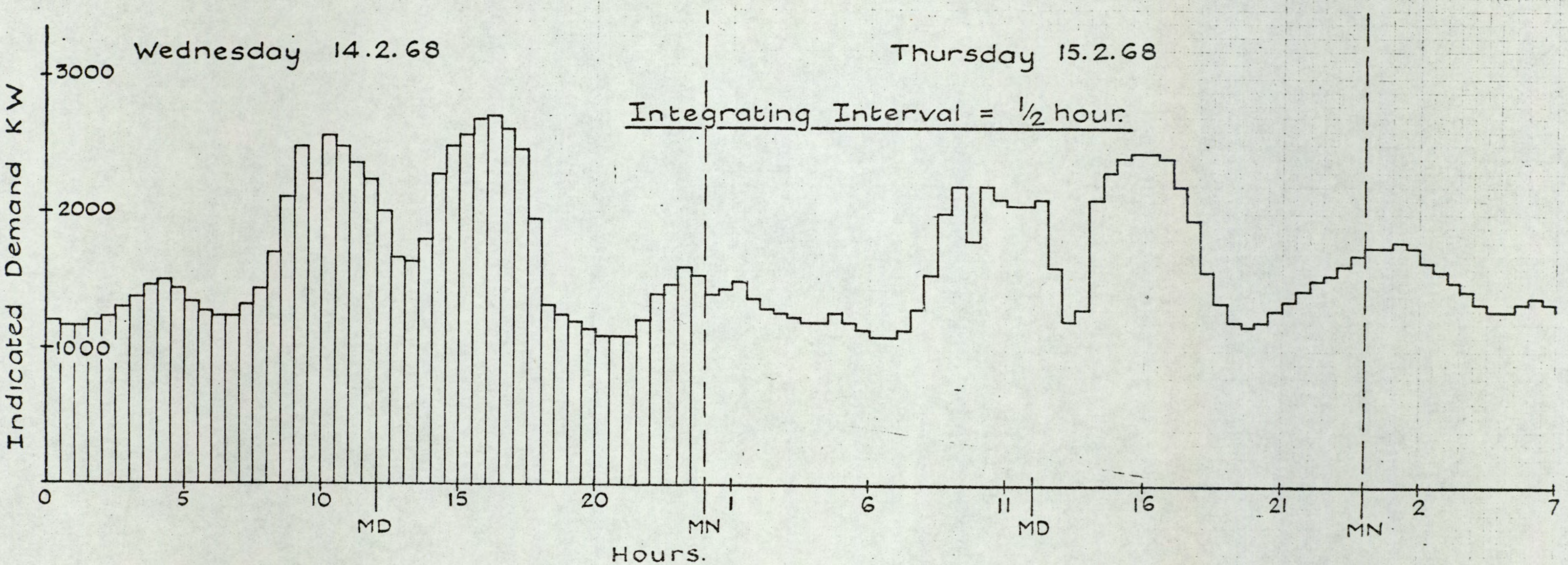


FIG. 8.16 VARIATION OF INDICATED DEMAND.

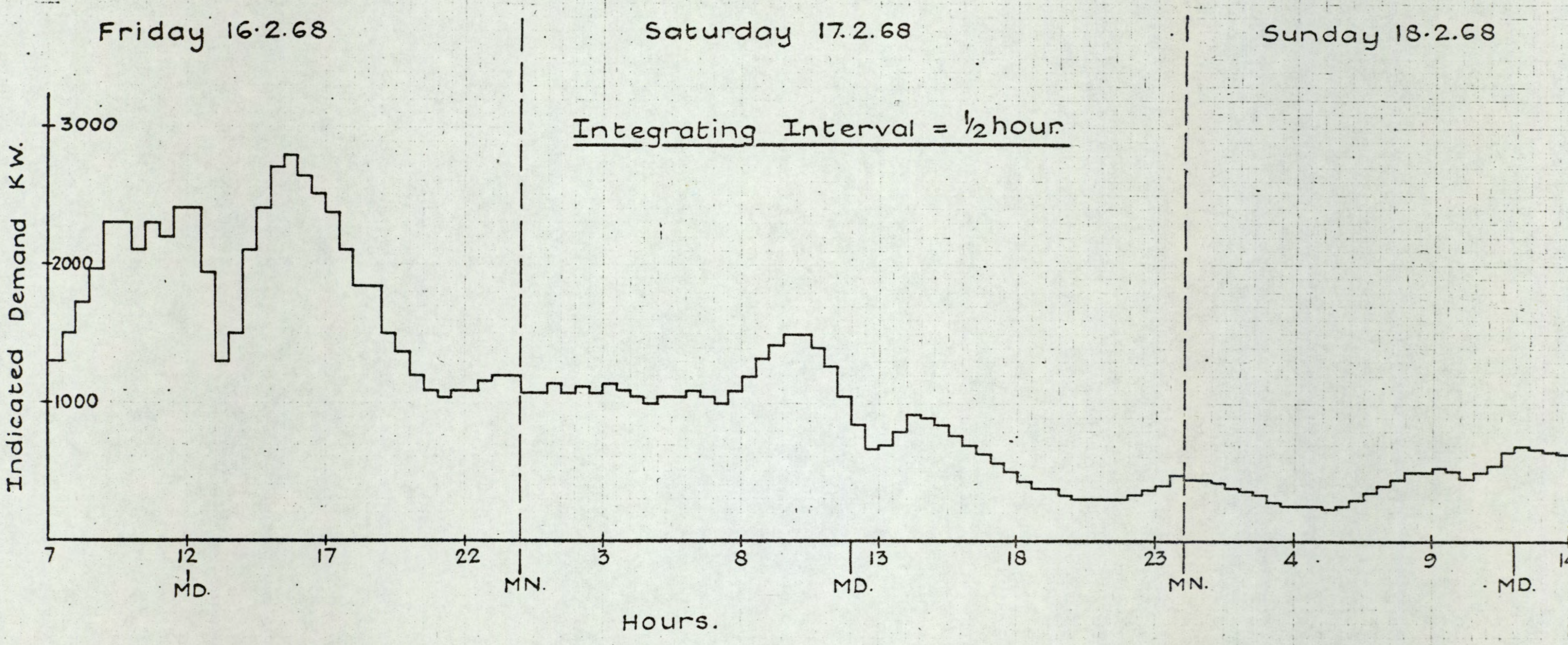


FIG. 8.17 VARIATION OF INDICATED DEMAND.

ELECTRICITY COSTS TO MIDLANDS ELECTRICITY BOARD.

Year & Month.	KVA Max. Demand.	KVA Setting on M.D. Control Equipment.	Units KWh.	Load Factor Per Cent.	Total Bill £	Cost per Unit. Pence.
1967						
October	2900	2880	942,300	44.51	6612	1.68
November	2950	2880	866,000	40.21	6300	1.74
December	3200	3000	872,900	37.37	6557	1.82
1968						
January	3350	3180	1,108,300	45.32	7367	1.59
February	3850	3120	1,017,200	36.55	7687	1.81
March	3190	3120	1,049,100	45.05	7201	1.65
April	3200	3000	769,300	32.93	6098	1.90
May	2950	2820	892,900	35.72	6395	1.71
June	2675	2640	691,700	35.42	5310	1.84
July	2900	2760	677,300	31.99	5445	1.93
August	2750	2880	693,600	34.55	5410	1.87
September	3350	3180	847,500	34.65	6569	1.86
October	3250	3000	1,104,600	46.55	7554	1.64
November	3300	3000	1,106,900	45.94	7576	1.64
December	3450	3300	1,010,000	40.10	7349	1.74
1969						
January	3400	3300	1,098,000	44.24	7668	1.67
February	3300	3180	1,047,200	43.47	7287	1.67
March	3250	3180	1,090,700	45.97	7427	1.63
April	3050	3000	916,300	41.15	6584	1.72
May	3100	2900	928,000	41.01	6642	1.72
June	3000	2850	943,200	43.07	6650	1.69
July	2800	2750	886,300	43.36	6237	1.69
August	2900	2900	662,000	31.27	5386	1.95

TABLE 8.8

furnace must be assessed in relation to the complete chain, since, for optimum working, no stage of the process must be held up for lack of metal.

Although manufacturing schedules and metallurgical requirements vary from week to week, it should eventually be possible, from experience gained, to predict the demand due to various combinations of production routines. Should the time come when priority is not given to maximising production, the purchase of a digital computer to determine and implement the economic maximum demand reduction may be warranted by the savings thereby achieved.

In this application, alternative fuels to electricity may be considered for heat treatment in the day-time. The high energy required for the melting cycle may then be supplied by electricity in the off-peak hours at reduced rates.

8.7 SUMMARY.

Figures 8.1 to 8.5 portray in a lucid manner the charges due to the M.E.B. maximum-demand tariffs 7 and 7 (a), and the economic effects of varying the parameters upon which they depend.

Section 8.3.2 has been included to illustrate the effect of the sliding tariff scales upon the savings achieved in maximum demand and energy charges due to power factor improvement. The results are expressed in terms of the pay-back period which is commonly used in calculating the economics of power factor correction.

The case studies of sections 8.5 and 8.6 are of importance, in that they deal with practical situations where the energy and power requirements fluctuate with time. They are, of course, only relevant for the particular tariff quoted, though the principle of optimising the maximum demand still applies for other two-part tariffs where each part is referred to the maximum KVA demand. If the control of demand is to be implemented by computer, the savings thereby effected must be sufficient to cover the capital cost and running expenses of the computer itself.

An analysis of the economic viability of maximum demand reduction depends largely upon a knowledge of the relation between the energy saved and the loss of production. For an existing process the appropriate cost figures may be obtained from operating experience. For a new project, however, postulation of the effect of maximum demand reduction upon manufacturing and operating costs will necessitate a complete understanding of the process. Implementation of the economic maximum demand may be expected to produce significant savings when electricity charges form a substantial proportion of the total costs, e.g. in electric melting.

CHAPTER 9.

GENERAL CONCLUSIONS.

The general conclusions of this work may be summarised as follows:

1). The maximum demand due to a finite number of similar on-off processes may be minimised by staggering of the load incidence in a deterministic manner, i.e. by programmed switching. New work in Chapter 3 shows that, under certain conditions, there is an allowable tolerance in the choice of the stagger between successive processes without increasing the minimum maximum demand. However, this stagger, once selected, must be maintained constant and equal for all processes.

The demand patterns arising from control of the load incidence are thoroughly analysed. Methods are given for controlling the demand without altering the symmetry of the switching.

Where a large number of processes is concerned, a statistical approach to demand control becomes necessary, although the principle of load staggering still applies.

In Chapter 4 a demand control strategy is established where the total number of processes is split into several equal blocks. which are staggered sequentially. An important feature of this policy is that the incidence of individual processes comprising a block need not be precisely controlled, but may be characterised by a distribution about a mean position.

2). In Chapters 5 and 6 new models have been established and proven for determining the mean and variance of the demand due to an ensemble of similar on-off processes whose incidence is random.

Applying the results to an installation of similar discontinuously-controlled space heating processes, the following important design requirement is established:

For minimisation of the variance of the demand, the ratio integrating interval/periodic time should have an integer value. Otherwise, the ratio should be as high as possible, and preferably not less than unity.

The principle of minimising individual load periods in order to reduce the demand variance still applies when there is a spread of cycle times. The reduction of load period also reduces the amplitude of temperature oscillations, thus better satisfying the "minimum-comfort" specification described in Chapter 7.

It has been shown that the distribution of the demand may be represented conservatively by a Normal distribution. This result has been used to determine the probability of exceeding a given demand, and hence the maximum demand penalty incurred.

Valuable information is thus provided for assessing the economic viability of proposed electric space heating projects.

These analyses represent a new and original approach to the determination and control of demand due to on-off processes.

3). In Chapter 7 a new strategy has been established for control of the demand due to thermostatically-controlled loads. This is based upon accurate prediction of future energy consumption within the integrating half-hour as a result of:

(i) Extrapolation of recent observed data by curve fitting of the energy locus at a number of sampled points.

(ii) Monitoring of temperature states of the cycling loads.

The appeal of this scheme lies in situations where the demand due to an ensemble of such processes is adequately controlled by exercising a switching strategy upon a single load. Control decisions may then be based upon a comparison of actual consumption and proportional target energy without the necessity for deriving the power to be shed from slopes of the energy locus.

An application for this strategy is in the control of the demand due to electric space heating of blocks of flats, and is described in Chapter 7. The quality of control achieved will be dependent upon the prediction accuracy, which is a function of the magnitude and rate of change of the energy consumption.

This application of demand control to thermostatically-controlled loads represents a considerable advance beyond existing work in this area.

In Chapter 7 an on-off space heating system has been designed where the temperature fluctuation is minimised over a prescribed range of environmental temperatures by means of derivative feedback.

This has the added advantages over conventional thermostatic control of reducing the offset and of minimising the energy requirements, since the command temperature may be lowered for a specified minimum process temperature.

4). In Chapter 8 the cost figures of current maximum demand tariffs, and the economic consequences of varying the parameters upon which they depend, have been presented in a lucid manner not readily apparent from the tariff format.

Although calculations are based upon a particular Area Board tariff, the principles still apply for two-part tariffs where each part is referred to the KVA maximum demand.

5). In Chapters 7 and 8 "optimisation" of electrothermal processes has been considered from the following viewpoints:

(i) A "minimum-comfort" level for discontinuously-controlled space heating processes.

(ii) A minimum overall cost per ton for an ironfoundry producing metal castings.

(iii) Maximising production of diverse products where the use of electroheat is involved.

It has been shown that the requirements of (ii) and (iii) are compatible only to a limited extent.

These analyses provide a significant contribution to work upon the determination of the economic maximum demand for electroheat processes. No generalised analysis can be given, of course, owing to the wide variations of efficiency and costs in the manufacture of diverse products involving the use of electroheat.

As shown in Chapter 8, the economic maximum reduction depends largely upon the relation between energy saved and the resulting lost production. For new projects, postulation of this relationship and of the cost factors involved will therefore require a complete understanding of all aspects of the process.

6). Areas for future research lie in the field of strategic control, particularly in load prediction techniques based upon past weather records and short-term forecasting.

It is suggested that control of the maximum demand in the future will be carried out continuously by digital computer, when substantial economic advantages will accrue to both the user and the supply Authority. Prediction will be based upon previous recorded demand patterns, and the computer will evaluate the economic maximum demand upon receipt of updated information of electricity and production costs.

The savings in charges as a result of computer control must be sufficient to justify the capital cost of the computer itself, though running costs may be diversified by time sharing with other duties.

APPENDIX A.

SAMPLES OF COMPUTER PROGRAMS.

Computations were performed on a PDP-9 digital computer using the FORTRAN IV language.

A.1 NUMERICAL INTEGRATION. (See section 4.5.2.1).

Numerical integration of the probability density function, equation (4.3), is achieved by program NUMINT for the condition $\frac{3\sigma}{\alpha} = 1\frac{1}{2}$ and $\bar{m} = \frac{7}{8}, \frac{1}{8}, \frac{2}{3}, \frac{1}{3}$. The probability distributions are summed for the processes being switched in the band of width α.

Integration is performed by Simpson's rule between -3σ and the point in question, which is stepped at intervals of 3 seconds between -3σ and $+3\sigma$.

The input data, flowchart and program are shown below. The results are incorporated within section 4.5.2.

Program identifiers are as follows:

Y = amplitude of probability density function.

AMP = area under probability density curve between -3σ and sampled point.

Z = time displacement of displaced probability density function f (t - Z)

SIGMA = standard deviation = 30 seconds.

TOT 78 = sum of switched loads in band α for $\bar{m} = \frac{7}{8}, \frac{1}{8}$.

TOT 23 = sum of switched loads in band α for $\bar{m} = \frac{2}{3}$.

TOT 13 = sum of switched loads in band α for $\bar{m} = \frac{1}{3}$.

The following computations are performed in the blocks A, B, C, D shown in the flowchart of Fig. A.2.

BLOCK A.

$Y = YA \times YB$,

where

$$YA = \frac{100}{\sigma\sqrt{2\pi}},$$

$$YB = \exp\left(-0.5\,(R/\sigma)^2\right)$$

where $R = |R|$ if $R \neq 0$,

or $YB = 1$ if $R = 0$.

BLOCK B.

Using Simpson's rule,

$$\text{area AMP} = \frac{C}{3 \times 24}\left(Y_1 + 4\sum \text{even ordinates} + 2\sum \text{odd ordinates} + Y_{25}\right).$$

BLOCK C.

Area printed out every 5th time round J loop.

BLOCK D.

$$TOT\ 78 = \left(\sum AMP(J)\ J = 1, 4\right) + \left(300 - \left(\sum AMP\ (J)\ J = 5, 7\right)\right)$$

$$TOT\ 23 = \left(\sum AMP(J)\ J = 1, 4\right) + \left(400 - \left(\sum AMP\ (J)\ J = 8,11\right)\right)$$

$$TOT\ 13 = \left(\sum AMP(J)\ J = 1, 4\right) + \left(400 - \left(\sum AMP\ (J)\ J = 12, 15\right)\right)$$

Z(1)	0.0		
Z(2)	-60.0		
Z(3)	-120.0		
Z(4)	60.0		
Z(5)	30.0		
Z(6)	-30.0		
Z(7)	-90.0		
Z(8)	80.0	$\bar{m} = \frac{7}{8}, \frac{1}{8}$	- Z (1) to Z (7)
Z(9)	20.0		
Z(10)	-40.0	$\bar{m} = \frac{2}{3}$	- Z (1) to Z (4) & Z (8) to Z (11).
Z(11)	-100.0		
Z(12)	40.0		
Z(13)	-20.0	$\bar{m} = \frac{1}{3}$	- Z (1) to Z (4) & Z (12) to Z (15).
Z(14)	-80.0		
Z(15)	-140.0		

FIG. A.1 INPUT DATA FOR PROGRAM " NUMINT".

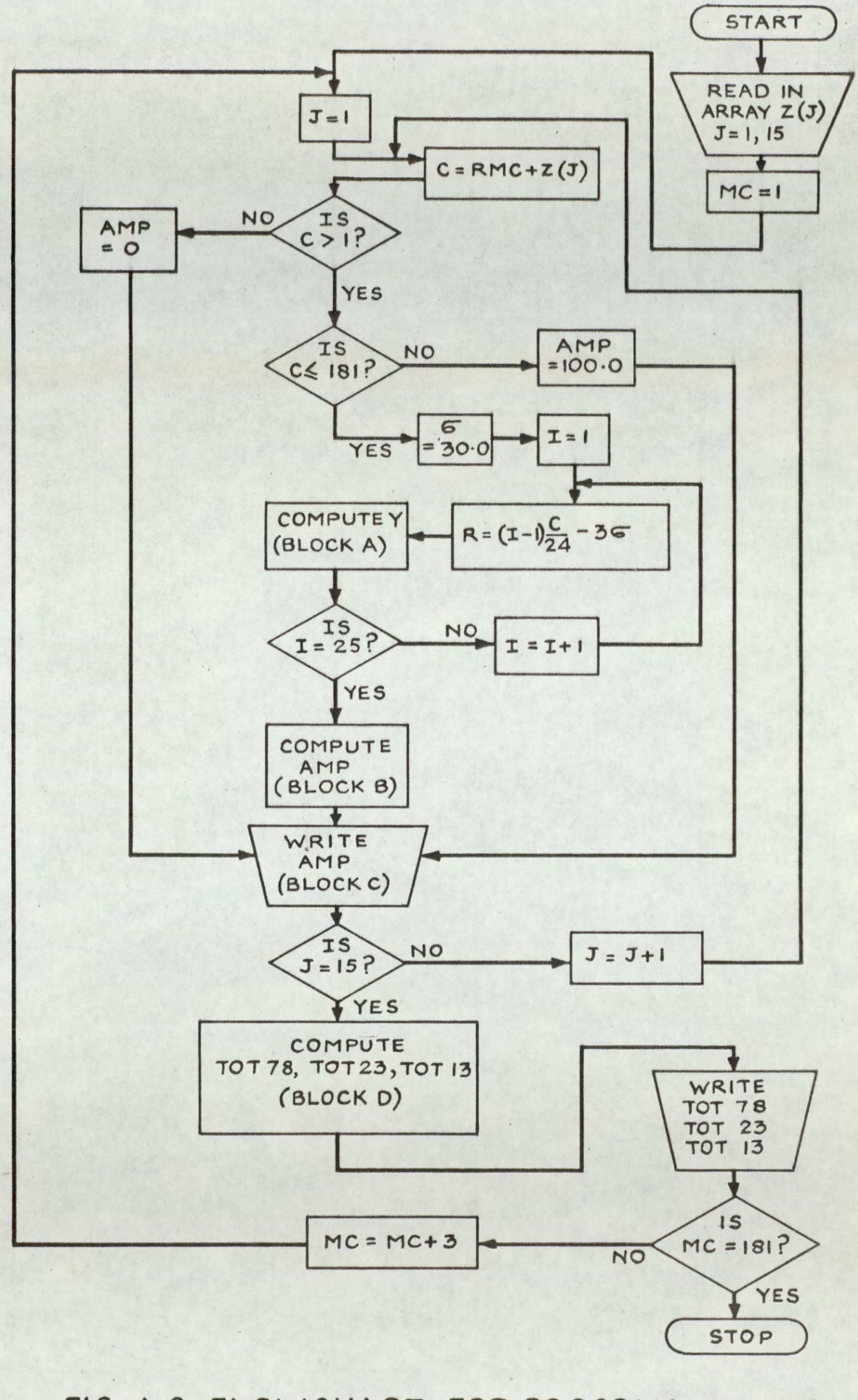


FIG. A.2 FLOWCHART FOR PROGRAM NUMINT.

```
      DIMENSION Y(200),AMP(15),Z(15)
   77 READ (3,99)(Z(J),J=1,15)
   99 FORMAT(6X,F8.3)
      DO 191 MC=1,181,3
      IK=5
      JK=1
      DO 90 J=1,15
      RMC=MC
   72 C=RMC+Z(J)
      IF(C-1.)22,22,25
22    AMP(J)=0.0
      GO TO 19
   25 IF(C-181.0)31,31,32
   32 AMP(J)=100.0
      GO TO 19
   31 SIGMA=30.0
      DO 12 I=1,25
      T=I-1
      YA=100.0/SIGMA/SQRT(2.0*3.14159)
      R=T*C/24.0-3.0*SIGMA
      IF(R.LT.0.0)R=-R
      IF(R.EQ.0.0)YB=1.0
      IF(R.NE.0.0)YB=EXP(-0.5*(R/SIGMA)**2.0)
   12 Y(I)=YA*YB
      EVEN=0.0
      ODD=0.0
      DO 15 JA=2,24,2
   15 EVEN=EVEN+Y(JA)
      DO 16 JA=3,23,2
   16 ODD=ODD+Y(JA)
      AMP(J)=C*(Y(1)+4.0*EVEN+2.0*ODD+Y(25))/72.0
   19 K=J-IK
      IF(K)90,100,90
  100 WRITE (2,26) (AMP(II),II=JK,J)
      IK=IK+5
      JK=JK+5
   26 FORMAT(1X,5F9.3)
   90 CONTINUE
      SUMS=0.0
      DO 51 J=1,4
   51 SUMS=SUMS+AMP(J)
      SUM78=0.0
      DO 50 J=5,7
   50 SUM78=SUM78-AMP(J)
      TOT78=SUM78+300.0+SUMS
      SUM23=0.0
      DO 52 J=8,11
   52 SUM23=SUM23-AMP(J)
      TOT23=SUM23+400.0+SUMS
      SUM13=0.0
      DO 53J=12,15
   53 SUM13=SUM13-AMP(J)
      TOT13=SUM13+400.0+SUMS
      WRITE (2,44) TOT78,TOT23,TOT13
   44 FORMAT (1X,3F8.3)
  191 CONTINUE
      STOP
      END
```

FIG. A.3 PROGRAM "NUMINT".

A.2 SIMULATION OF DEMAND CALCULATIONS USING PSEUDO-RANDOM NUMBERS

(See sections 6.3.1, 6.3.2, and 6.5).

Demand check programs were carried out as described in section 6.5, using a published pseudo-random number table [31], a sample of which is given in Table A.1.

Let N be a two-digit random number drawn sequentially from the table.

If a process commences at time X with equal probability anywhere in the range $0 \leqslant X \leqslant T$, where the period T is normalised to unity value, then $X = \frac{N}{100}$.

The indicated demand D will be a function of X, $\bar{m}$ and I, where $\frac{I}{T} \leqslant 1$.

For example, if the power per process is normalised to unity value:

For $\bar{m} = \frac{1}{2}$ and $I = 0.75$.

If $X < 0.75$:

Then if $X \leqslant 0.25$, $D = \frac{2}{3}$;

if $0.25 < X < 0.5$, $D = \frac{0.75 - X}{0.75}$;

if $0.5 \leqslant X$, $D = \frac{1}{3}$.

If $X = 0.75$:

Then $D = \frac{1}{3}$.

If $X > 0.75$:

Then $D = \frac{X - 0.5}{0.75}$.

Programs 1, 2 and 4 and sample flowcharts are shown below.

Results were as follows:

Program 1. $\bar{m} = \frac{1}{2}$, $\frac{I}{T} = 0.5$. (1,000 events of 1 process).

The theoretical results are:

$\bar{D} = 0.5$, Var D = 0.0833.

The computation gave:

$\bar{D} = 0.4965$ (0.7% error) and Var D = 0.0859 (3.2% error).

Program 2. $\bar{m} = \frac{1}{2}$, $\frac{I}{T} = 0.5$ (50 groupings of 20 independent processes using 1,000 random numbers).

The theoretical results are:

$\bar{W} = 20 \times 0.5 = 10$,

Var W = 20 x 0.0833 = 1.667.

The computation gave:

$\overline{W} = 9.93$ (0.7 % error) and Var W = 1.575 (5.5% error).

A repetition of the computation for a further independent set of 1,000 random numbers gave $\overline{W} = 9.971$ (0.3% error) and Var W = 1.717 (3.0% error). The sample size n_W was increased to 500 by generating 10,000 pseudo-random numbers on the computer; computations for two independent sets of random numbers each gave $\overline{W}$ and Var W within 3% of their theoretical values.

<u>Program 3.</u> $\overline{m} = \frac{1}{2}$, $\frac{I}{T} = 0.25$ (1000 events of 1 process).

The theoretical results are:

$$\overline{D} = \overline{m} = 0.5.$$

$$\text{Var } D = \tfrac{1}{4} - \frac{1}{6r}, \text{ where } r = \frac{T}{2I} = 2,$$

whence Var D = 0.1667.

The computation gave:

$\overline{D} = 0.4799$ (4.0% error) and Var D = 0.1692 (1.5% error).

<u>Program 4.</u> $\overline{m} = \frac{3}{4}$, $\frac{I}{T} = 0.5$ (1000 events of 1 process).

The theoretical results are:

$$\overline{D} = 0.75.$$

$$\text{Var } D = 1 - \frac{\overline{m}}{3r} \left(1 + 3s + 3\,\overline{m}r - (1 - S)^3\right)$$

$$\text{where } r = \frac{\overline{m}T}{I} = 1.5, \quad s = (1 - \overline{m})\frac{T}{I} = 0.5,$$

whence Var D = 0.0415.

The computation gave:

$\overline{D} = 0.7399$ (1.3% error) and Var D = 0.0423 (1.9% error).

<u>Program 5.</u> $\overline{m} = \frac{1}{4}$, $\frac{I}{T} = 0.5$ (1000 events of 1 process).

The theoretical results are:

$$\overline{D} = 0.25,$$

$$\text{Var } D = \frac{\overline{m}}{3r}\left(3r^2 - r^3 - 3\,\overline{m}r\right), \text{ where } r = \frac{\overline{m}T}{I} = \tfrac{1}{2},$$

whence Var D = 0.0415.

The computation gave:

$\overline{D} = 0.2479$ (0.8% error) and Var D = 0.0419 (1.0% error).

<u>Program 6.</u> $\overline{m} = \frac{1}{2}$, $\frac{I}{T} = 0.75$ (1000 events of 1 process).

The theoretical results are:

$$\overline{D} = 0.5,$$

$$\text{Var } D = \frac{1}{12r}(2 - r)(2r - 1)^2, \text{ where } r = \frac{T}{2I} = \frac{2}{3},$$

whence Var D = 0.0185.

The computation gave:

$\overline{D}$ = 0.4915 (1.7% error) and Var D = 0.0193 (4.1% error).

A.2.1 GENERATION OF PSEUDO-RANDOM NUMBERS ON THE DIGITAL COMPUTER.

Pseudo-random numbers are generated by the multiplicative congruential method [32] according to the rule $N_{i+1} \equiv aN_i \pmod{m}$, where N_i, a and m are all non-negative integers.

The binary computer has an 18- bit word length, and m is chosen as $2^b = 2^{17}$.

The procedure is as follows:

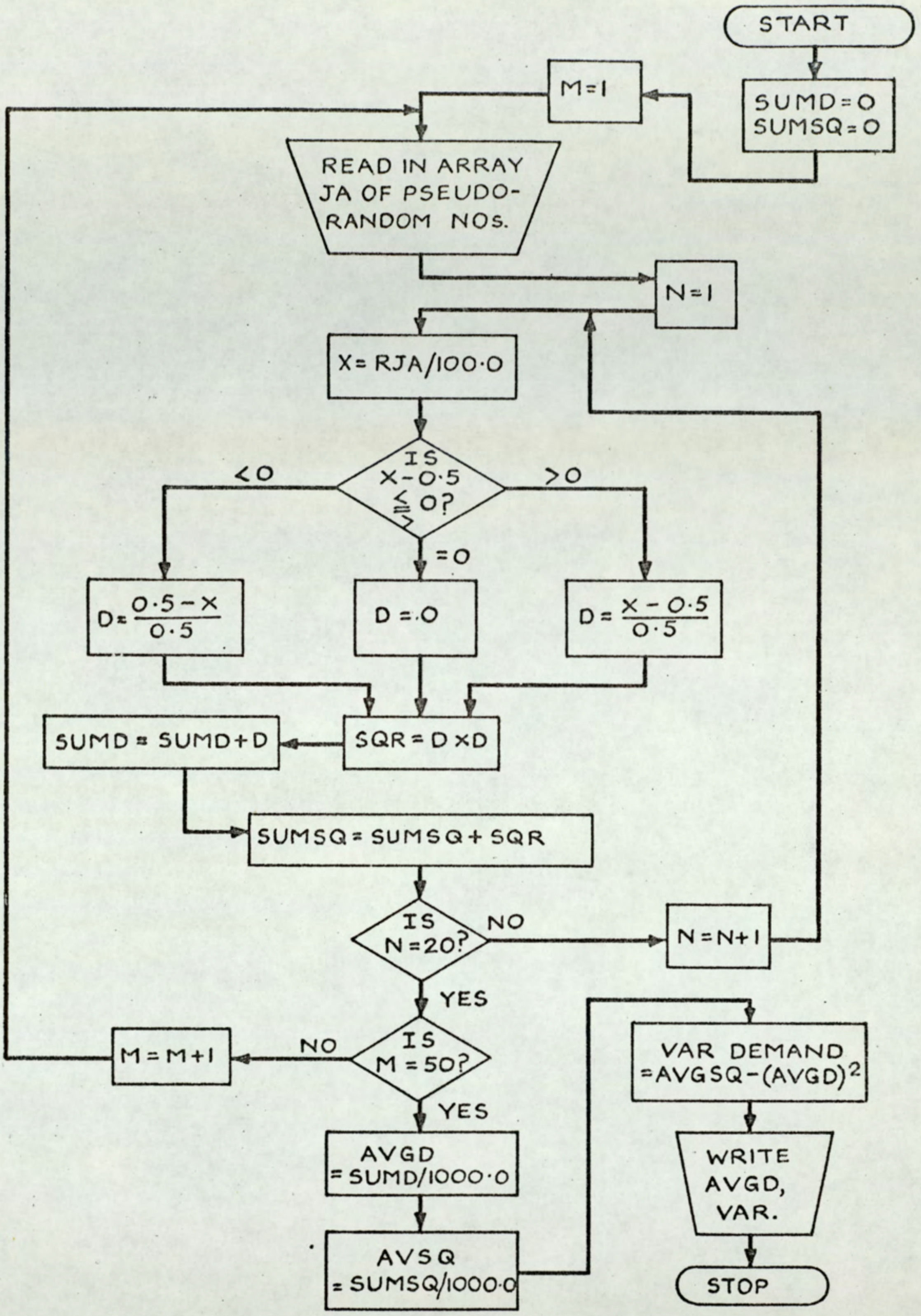
1. The first entry N_o from the main program to the subroutine to start the sequence is required to be an odd integer.
2. Choose an integer $a = 8t \pm 3$ close to $2^{b/2}$, where t is any positive integer.
   $b = 17$, whence $a = 2^8 + 3 = 259$.
3. Compute a N_o using fixed point arithmetic. Overflow of the store in FORTRAN results in retention of the low-order bits N_1 and discarding of high-order bits.
4. Calculate $R = N_1/2^b = N_1/131{,}072$ to obtain a uniformly distributed variate on the unit interval.
5. Each subsequent entry from the main program is required to be the result of the previous exit from the routine.
   The multiplicative procedure will produce 2^{b-2} = 32,768 random numbers before repeating. Program 7 gives the subroutine and a suitable test program to generate 1,000 two-digit numbers uniformly distributed between 0 and 1; each digit is an independent sample from a population in which the digits 0 to 9 are equally likely.

TABLE 8

16 16	57 04	81 71	17 46	53 29	73 46	42 73	77 63	62 58	60 59
98 63	89 52	77 23	61 08	63 90	80 38	42 71	85 70	04 81	05 50
01 03	09 35	02 54	51 96	92 75	58 29	24 23	25 19	89 97	91 29
29 07	16 34	49 22	52 96	89 34	17 11	06 91	24 38	55 06	83 59
72 61	80 54	70 99	24 64	11 38	83 65	27 23	40 37	84 58	48 53
71 11	41 82	79 37	00 45	98 54	52 89	26 34	40 13	60 38	08 86
61 05	66 18	76 82	11 18	61 90	90 63	78 57	32 06	39 95	75 94
81 89	42 34	00 49	97 53	33 16	26 91	57 58	42 48	51 05	48 27
10 24	90 84	22 16	26 96	54 11	01 96	58 81	37 97	80 98	72 81
14 28	33 43	01 32	58 39	19 54	56 57	23 58	24 87	77 36	20 97
35 41	17 89	87 04	28 32	13 45	59 03	91 08	69 24	84 44	42 83
07 89	36 87	98 73	77 64	75 19	05 61	11 64	31 75	49 38	96 60
27 59	15 58	19 68	95 47	25 69	11 90	26 19	07 40	83 59	90 95
95 98	45 52	27 35	86 81	16 29	37 60	39 35	05 24	49 00	29 07
12 95	72 72	81 84	36 58	05 10	70 50	31 04	12 67	74 01	72 90
35 23	06 68	52 50	39 55	92 28	28 89	64 87	80 00	84 53	97 97
86 33	95 73	80 92	26 49	54 50	41 21	06 62	73 91	35 05	21 37
02 82	96 23	16 46	15 51	60 31	55 27	84 14	71 58	94 71	48 35
44 46	34 96	32 68	48 22	40 17	43 25	33 31	26 26	59 34	99 00
08 77	07 19	94 46	17 51	03 73	99 89	28 44	16 87	56 16	56 09
61 59	37 08	08 46	56 76	29 48	33 87	70 79	03 80	96 81	79 68
67 70	18 01	67 19	29 49	58 67	08 56	27 24	20 70	46 31	04 32
23 09	08 79	18 78	00 32	86 74	78 55	55 72	58 54	76 07	53 73
89 40	26 39	74 58	59 55	87 11	74 06	49 46	31 94	86 66	66 97
84 95	66 42	90 74	13 71	00 71	24 41	67 62	38 92	39 26	30 29
52 14	49 02	19 31	28 15	51 01	19 09	97 94	52 43	22 21	17 66
89 56	31 41	37 87	28 16	62 48	01 84	46 06	04 39	94 10	76 21
65 94	05 93	06 68	34 72	73 17	65 34	00 65	75 78	23 97	13 04
13 08	15 75	02 83	48 26	53 77	62 96	56 52	28 26	12 15	75 53
03 18	33 57	16 71	60 27	15 18	39 32	37 01	05 86	25 14	35 41
10 04	00 95	85 04	32 80	19 01	85 03	29 29	80 04	21 52	14 76
23 94	97 28	60 43	42 25	26 48	48 13	34 68	39 22	74 85	03 25
35 63	42 90	90 74	33 17	58 77	83 36	76 22	00 89	61 55	13 17
42 86	03 36	45 33	60 77	72 92	10 76	22 55	11 00	37 60	47 73
67 26	92 87	09 96	85 37	82 61	39 01	70 05	12 66	17 39	99 34
91 93	88 56	35 76	97 35	19 37	14 66	07 57	24 41	06 90	07 72
37 14	73 35	32 01	07 94	78 28	90 33	71 56	63 77	89 24	24 28
07 46	50 58	08 73	42 97	20 42	64 68	48 35	04 38	28 28	36 94
92 18	09 46	94 99	17 41	28 60	67 94	26 54	63 70	84 73	76 61
00 49	98 43	39 67	68 40	41 31	92 28	49 57	15 55	11 81	41 89
08 59	41 41	33 59	43 28	14 51	02 71	24 45	41 57	22 11	79 79
67 05	19 54	32 33	34 68	27 93	39 35	62 51	35 55	40 99	46 19
24 99	48 06	96 41	21 25	29 03	57 71	96 49	94 74	98 90	21 52
65 86	27 46	70 93	27 39	64 37	01 63	21 03	43 78	18 74	77 07
52 70	03 20	84 96	14 37	51 05	63 99	81 02	84 56	17 78	48 45
32 88	29 93	58 21	71 05	68 58	79 08	86 37	98 76	70 45	66 23
54 16	39 40	98 57	02 05	65 15	73 23	51 51	75 06	38 13	51 68
95 22	18 59	54 57	44 22	72 35	81 24	14 94	24 04	42 26	92 14
93 10	27 94	90 45	39 33	50 26	88 46	90 57	40 47	71 63	62 59
19 20	85 20	15 67	78 03	32 23	50 59	24 83	64 99	18 00	78 50

Each digit is an independent sample from a population in which the digits 0 to 9 are equally likely, that is each has a probability of $\frac{1}{10}$.

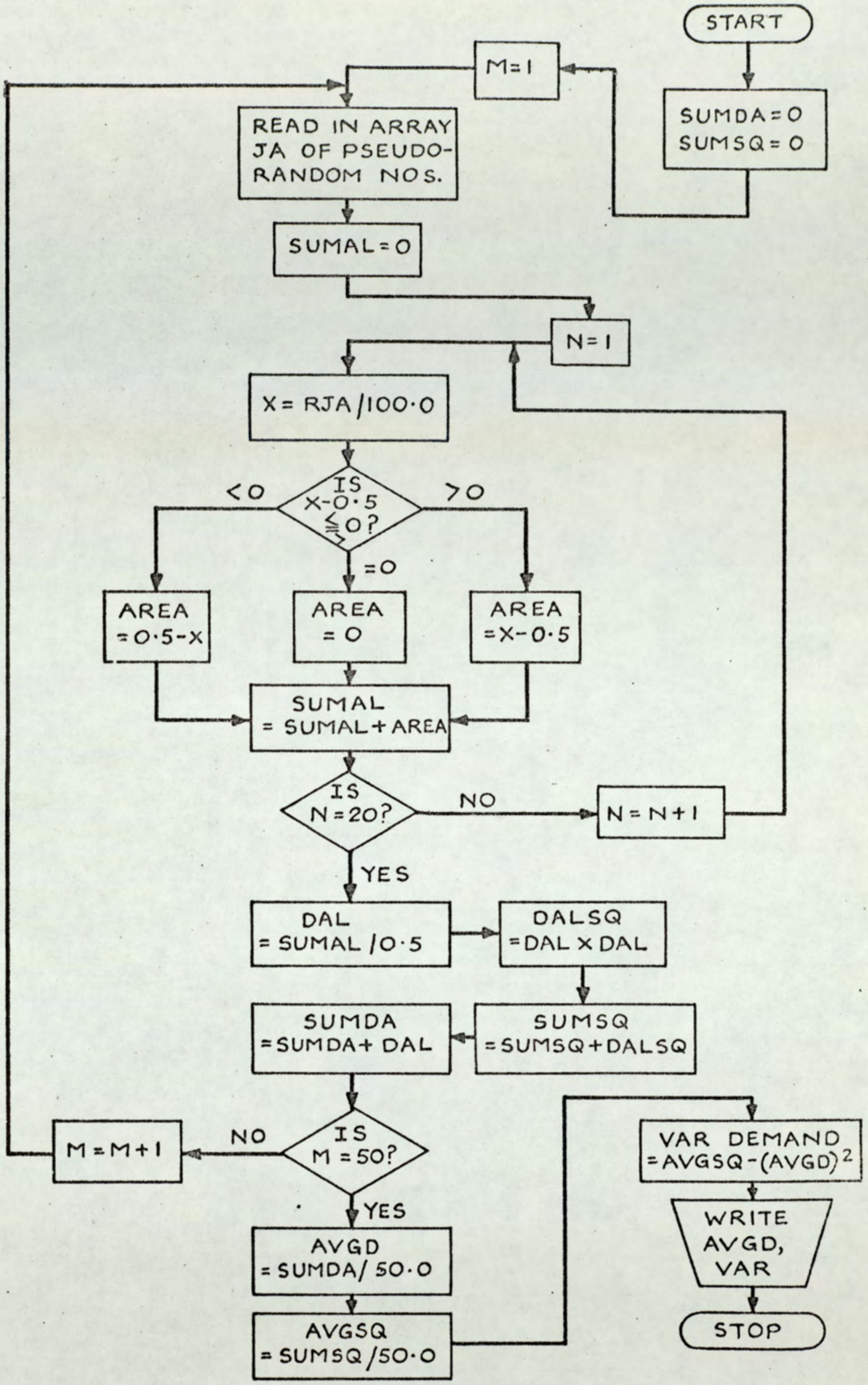
TABLE A.1.



Mean value of demand $D = \frac{1}{1000} \sum_{M=1}^{50} \left(\sum_{N=1}^{20} D_n \right) = \frac{SUMD}{1000}$

Expected value of $D = \frac{1}{1000} \sum_{M=1}^{50} \left(\sum_{N=1}^{20} D_n^2 \right) = \frac{SUMSQ}{1000}$

FIG.A.4 FLOWCHART FOR PROGRAM I.



Mean value of demand $W = \frac{1}{50}\sum_{M=1}^{50}\left(\sum_{N=1}^{20}\frac{A_n}{0\cdot5}\right) = \frac{SUMDA}{50}$

Expected value of $W^2 = \frac{1}{50}\sum_{M=1}^{50}\left[\left(\sum_{N=1}^{20}\frac{A_n}{0\cdot5}\right)^2\right] = \frac{SUMSQ}{50}$

FIG. A.5 FLOWCHART FOR PROGRAM 2.

```
      DIMENSION JA(50)
      SUMD=0.0
      SUMSQ=0.0
      DO 90 M=1,50
      READ(3,1)(JA(N),N=1,20)
    1 FORMAT(20I3)
      DO 88 N=1,20
      RJA=JA(N)
      X=RJA/100.0
   25 IF(X-0.5)30,32,34
   30 D=(0.5-X)/0.5
      GO TO 36
   32 D=0.0
      GO TO 36
   34 D=(X-0.5)/0.5
   36 SQR=D*D
      SUMD=SUMD+D
      SUMSQ=SUMSQ+SQR
   88 CONTINUE
   90 CONTINUE
      AVGD=SUMD/1000.0
      AVGSQ=SUMSQ/1000.0
      DB=AVGD
      VAR=AVGSQ-DB*DB
      WRITE(2,44)
      WRITE(2,46) DB,VAR
   44 FORMAT(3X,5HI=0.5,3X,8HMBAR=0.5//)
   46 FORMAT(5X,2F9.5)
      STOP
      END
```

PROGRAM I

DEMAND D FOR 1 PROCESS.

($\bar{m} = \frac{1}{2}$, I/T = 0.5).

FIG. A.6 (a)

```
      DIMENSION JA(50)
      SUMDA=0.0
      SUMSQ=0.0
      DO 100 M=1,50
100   READ(3,1)IA
      DO 90 M=51,100
       READ(3,1)(JA(N),N=1,20)
 1    FORMAT(20I3)
      SUMAL=0.0
      DO 70 N=1,20
      RJA=JA(N)
      X=RJA/100.0
25    IF(X-0.5)30,32,34
30    AREA=0.5-X
      GO TO 36
32    AREA=0.0
      GO TO 36
34    AREA=X-0.5
36    SUMAL=SUMAL+AREA
70    CONTINUE
      DAL=SUMAL/0.5
      DALSQ=DAL*DAL
      SUMSQ=SUMSQ+DALSQ
      SUMDA=SUMDA+DAL
90    CONTINUE
      AVGD=SUMDA/50.0
      AVGSQ=SUMSQ/50.0
      DB=AVGD
      VAR=AVGSQ-DB*DB
      WRITE(2,44)
      WRITE(2,46) DB,VAR
44    FORMAT(3X,5HI=0.5,8HMBAR=0.5,22H50 EVENTS 20 PROCESSES//)
46    FORMAT(5X,2F9.5)
      STOP
      END
```

PROGRAM 2.

DEMAND W FOR ENSEMBLE OF 20 PROCESSES.

($\bar{m} = \frac{1}{2}$, I/T = 0.5).

FIG. A.6 (b)

PROGRAM 4

DEMAND D FOR 1 PROCESS.

($\bar{m}$ = ¾, I/T = 0.5.

```
      DIMENSION JA(50)
      SUMD=0.0
      SUMSQ=0.0
      DO 90 M=1,50
      READ(3,1) (JA(N),N=1,20)
    1 FORMAT(20I3)
      DO 88 N=1,20
      RJA=JA(N)
      X=RJA/100.0
   25 IF(X-0.25)30,32,34
   30 D=(0.5-X)/0.5
      GO TO 36
   32 D=0.5
      GO TO 36
   34 IF(X-0.5)38,38,42
   38 D=0.5
      GO TO 36
   42 IF(X-0.75)51,53,53
   51 D=(X-0.25)/0.5
      GO TO 36
   53 D=1.0
   36 SQR=D*D
      SUMD=SUMD+D
      SUMSQ=SUMSQ+SQR
   88 CONTINUE
   90 CONTINUE
      AVGD=SUMD/1000.0
      AVGSQ=SUMSQ/1000.0
      DB=AVGD
      VAR=AVGSQ-DB*DB
      WRITE(2,44)
      WRITE(2,46)DB,VAR
   44 FORMAT(3X,5HI=0.5,3X,9HMBAR=0.75//)
   46 FORMAT(5X,2F9.5)
      STOP
      END
```

FIG. A.7. DEMAND PROGRAM 4.

PROGRAM 7

```
      DIMENSION A(10)
      N=1
      DO 1 K=1,100
      DO 2 J=1,10
      CALL RANDOM(N,R)
      A(J)=R
    2 CONTINUE
      WRITE(2,100)A
    1 CONTINUE
  100 FORMAT(10F5.2)
      STOP
      END

      SUBROUTINE RANDOM(N,R)
      N=259*N
      R=100.0*FLOAT(N)/131072.0+0.5
      R=AINT(R)/100.0
      RETURN
      END
```

FIG. A.8 TEST PROGRAM FOR GENERATING RANDOM NUMBERS ON THE DIGITAL COMPUTER.

A.3 PROGRAM DEMAND.

This program is used to determine the mean and variance of the demand due to an ensemble of 20 similar independent 2-position processes.

The distributions of T, $\bar{m}$ and x are given by:

T uniformly distributed over the range 15 mins $\leqslant T \leqslant$ 45 mins,

$\bar{m}$ uniformly distributed over the range $\frac{1}{2} \leqslant \bar{m} \leqslant \frac{3}{4}$,

x uniformly distributed over the range $0 \leqslant x \leqslant T$.

500 groupings of the ensemble are considered.

Pseudo-random numbers are generated using the subroutine RANDOM given in program 7.

The development of the program, and the results obtained, are given in section 6.6.3.

The flowchart and listing are given in Figs. A.9 and A.10 (program 8) respectively.

Program identifiers are as follows:

AI = integrating interval I = 30 minutes.

T = period

EMB = $\bar{m}$

CEMB = $(1 - \bar{m})$ T

TION = $\bar{m}$T

CIM = $(1 - \bar{m})T + I$

DIM = $I - \bar{m}\,T$

X = incidence.

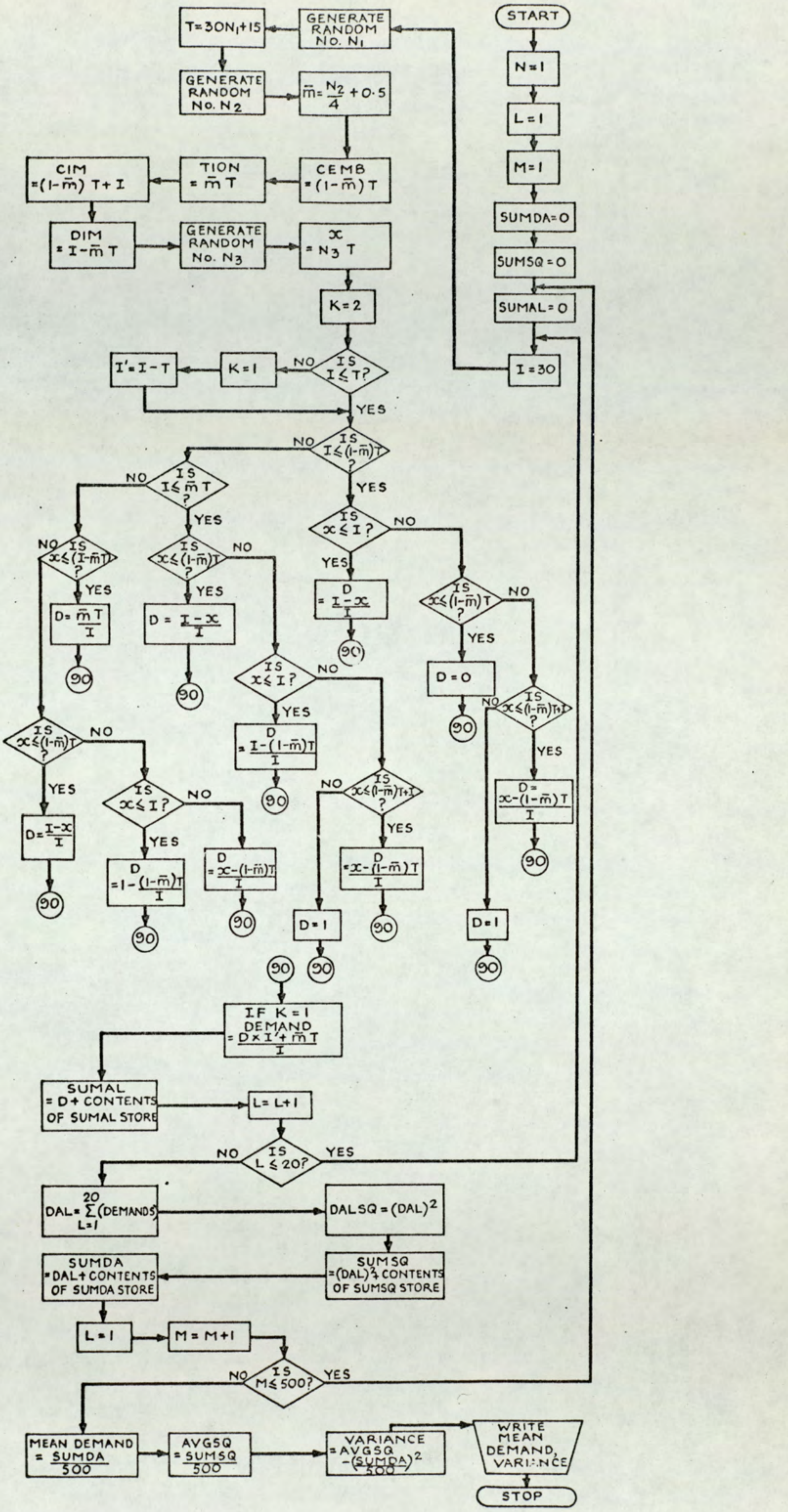


FIG. A.9 FLOWCHART FOR PROGRAM 8.

```
      N=1
      L=1
      M=1
      SUMDA=0.0
      SUMSQ=0.0
    3 SUMAL=0.0
    7 AI=30.0
      CALL RANDOM(N,T)
      T=30.0*T+15.0
      CALL RANDOM(N,EMB)
      EMB=EMB/4.0+0.5
      CEMB=(1.0-EMB)*T
      TION=T*EMB
      CIM=CEMB+AI
      DIM=AI-TION
      CALL RANDOM(N,X)
      X=X*T
      K=2
        IF(AI-T)20,20,91
   20 IF(AI-CEMB)21,21,25
   21 IF(X-AI)27,27,31
   27 D=(AI-X)/AI
      GO TO 90
   31 IF(X-CEMB)33,33,37
   33 D=0.0
      GO TO 90
   37 IF(X-CIM)39,39,45
   39 D=(X-CEMB)/AI
      GO TO 90
   45 D=1.0
      GO TO 90
   25 IF(AI-TION)51,51,53
   51 IF(X-CEMB)55,55,57
   55 D=(AI-X)/AI
      GO TO 90
   57 IF(X-AI)59,59,61
   59 D=(AI-CEMB)/AI
      GO TO 90
   61 IF(X-CIM)63,63,67
   63 D=(X-CEMB)/AI
      GO TO 90
   67 D=1.0
      GO TO 90
   53 IF(X-DIM)71,71,73
   71 D=TION/AI
      GO TO 90
   73 IF(X-CEMB)75,75,77
   75 D=(AI-X)/AI
      GO TO 90
   77 IF(X-AI)79,79,83
   79 D=1.0-CEMB/AI
      GO TO 90
```

PROGRAM 8

DEMAND W FOR ENSEMBLE

OF 20 PROCESSES.

(T, $\bar{m}$, X all uniformly distributed).

FIG. A.10 PROGRAM "DEMAND"

PROGRAM 8 (CONT.)

```
83   D=(X-CEMB)/AI
     GO TO 90
91   K=1
     AAT=AI
     AI=AI-T
     GO TO 20
90   IF(K.EQ.1) D=(D*AI+TION)/AAT
        SUMAL=SUMAL+D
     L=L+1
     IF(L-20)7,7,8
 8   DAL=SUMAL
     DALSQ=DAL*DAL
     SUMSQ=SUMSQ+DALSQ
     SUMDA=SUMDA+DAL
     L=1
      M=M+1
     IF(M-500)3,3,10
10   AVGD=SUMDA/500.0
     AVGSQ=SUMSQ/500.0
     DB=AVGD
     VAR=AVGSQ-DB*DB
     WRITE(2,44)
     WRITE(2,46)DB,VAR
44   FORMAT(3X,23H500 EVENTS 20 PROCESSES//)
46   FORMAT(5X,2F9.5)
     STOP
     END

     SUBROUTINE RANDOM(N,R)
     N=259*N
     R=100.0*FLOAT(N)/131072.0+0.5
     R=AINT(R)/100.0
     RETURN
     END
```

A.4 PROGRAM DNORM.

In this program the distributions of T, $\bar{m}$ and x are:

T normally distributed over same range as in section A.3,

$\bar{m}$ normally distributed over same range as in section A.3,

x uniformly distributed over same range as in section A.3.

500 groupings of the ensemble of 20 processes are again considered.

Program DNORM, which is used to compute the mean and variance of the ensemble demand, is given in Fig. A.11 (program 9). Also included are subroutine RANDOM for generation of uniform variates, and subroutine NORMAL for generation of normal variates.

The development of subroutine NORMAL, and the results obtained for the computation, are given in section 6.6.5.

Apart from the generation of the variates, the program flow will be as for program 8.

PROGRAM 9

DEMAND W FOR ENSEMBLE
OF 20 PROCESSES

(T, $\bar{m}$ normally
distributed.
X uniformly distributed)

```
      EMU=0.5
      SIG=1.0/12.0
      STDD=SIG
      N=1
      L=1
      M=1
      SUMDA=0.0
      SUMSQ=0.0
  3   SUMAL=0.0
  7   AI=30.0
      CALL NORMAL(EMU,STDD,N,R,T)
        T=30.0*T+15.0
      CALL NORMAL(EMU,STDD,N,R,EMB)
            EMB=EMB/4.0+0.5
      CEMB=(1.0-EMB)*T
      TION=T*EMB
      CIM=CEMB+AI
      DIM=AI-TION
      CALL RANDOM(N,X)
      X=X*T
      K=2
        IF(AI-T)20,20,91
 20   IF(AI-CEMB)21,21,25
 21   IF(X-AI)27,27,31
 27   D=(AI-X)/AI
      GO TO 90
 31   IF(X-CEMB)33,33,37
 33   D=0.0
      GO TO 90

      SUCCEEDING STATEMENTS
      EXACTLY AS FOR MAIN
      PROGRAM OF PROGRAM 8
      SUBROUTINE NORMAL(EMU,STDD,N,R,Y)
      JJ=1
      SUM=0.0
 13   N=259*N
      R=100.0*FLOAT(N)/131072.0+0.5
      R=AINT(R)/100.0
      SUM=SUM+R
      JJ=JJ+1
      IF(JJ-12)13,13,14
 14   Y=STDD*(SUM-6.0)+EMU
      RETURN
      END

      SUBROUTINE RANDOM(N,R)
      N=259*N
      R=100.0*FLOAT(N)/131072.0+0.5
      R=AINT(R)/100.0
      RETURN
      END
```

FIG. A.11 PROGRAM "D NORM"

APPENDIX B.

EXPERIMENTAL DETERMINATION OF PARAMETER SPREADS FOR LINE-VOLTAGE WALL-MOUNTED ROOM THERMOSTATS.

B.1 Test Cabinet and Thermostat.

Photographs of the test cabinet described in section 6.7 are given in Figs. B.1 (a) and B.1 (b).

By insertion of heating elements within the cabinet, the performance of the thermostat under test can be assessed at prescribed temperature rise rates. A convected air flow may be induced over the thermostat by an extractor fan mounted in the side of the box.

The thermostat is designed for operation at line voltage with a 14-amp . single-pole snap-action switch, bimetallic scroll and anticipator resistance. As shown in Fig. B.2, the thermostat is contained in a ventilated housing suitable for wall mounting.

B.2 Possible Tests for Assessment of Thermostat Performance.

Three alternative tests may be performed which are reasonably representative of operating conditions:

1. A small heater may be placed within the test box to be switched by

the thermostat at the command temperature. The thermostat secondary feedback is relatively inactive owing to the small current load.

2. The thermostat may be used to create its own environmental temperature by switching of full load, causing significant I^2R and contact heating within the thermostat case. A space heater which the thermostat switches is placed remotely from the test cabinet, and acts purely as a current load.

This method provides an opportunity of studying the thermostat performance due to the secondary feedback alone. Owing to the comparatively small heating effected, cyclic switching will only be possible for a small rate of heat loss from the cabinet.

3. The best simulation of practical conditions is given by a combination of the two previous tests. The local heating effect may be obtained by allowing the thermostat to switch about 90% full load (a 3KW space heater placed remotely from the test box). In parallel with this load, but placed inside the test cabinet, may be placed a 30W heater element which is also switched by the thermostat.

B.3 Test Details.

The connection diagram for test 3 is shown in Fig. B.3.

The temperature of the room containing the test cabinet was independently controlled to a value of 70° F. to give a standardised environment; this necessitated setting the command level for the thermostat within the cabinet to 78°F.

In test 2 it was found that, for an ambient temperature lower than 70° F., the rate of heat loss from the cabinet was too great to enable the thermostat to switch. Heating in this test was determined mainly by the losses of the connecting cable.

Twelve thermostats, produced by one manufacturer to the same specification, were subjected in turn to tests 2 and 3. The thermostat on test was mounted vertically in the cabinet about 1½ feet from the 30W heater; this rating was chosen to give on- and off-times of the same order.

The command temperature of 78° F. was standardised for all thermostats independently of individual dial settings.

It was found that the period of the d.e.c. cycle was not significantly affected by the orientation of the thermostat.

Visual indication of switching was provided, and the on-off load pattern was registered on a Miniscript recorder. The temperature adjacent to the thermostat housing was monitored by calibrated series-connected NiCr - NiAl thermocouples with the cold junctions in icy water. Oscillations of temperature were recorded on a Honeywell chart recorder with suppressed zero.

Typical recordings of load pattern and temperature are given in Figs. B.4 and B.5 respectively.

B.4 Test Results.

Heating and cooling trajectories showed the active and passive time constants for the cabinet to be 50 mins. and 65 mins. respectively, and the transit lag $3\frac{1}{2}$ mins. (see Appendix C.2).

Typical test results were as follows:

Test 2.

Thermostat mounted vertically in centre of test box and passing full load current of 13 amp. Operation of thermostat due to effect of local heating.

Temp. of room = 70^{o} F.

Thermostat command temp. = 78^{o} F.

Natural air convection within test cabinet.

D.e.c. values for T and $\overline{m}$ were as follows:

THERMOSTAT NO.	PERIOD T mins.	$\overline{m}$
1	12	0.55
2	10	0.5
3	11	0.55
4	18	0.69
5	13	0.69
6	11	0.58
7	18	0.65
8	15	0.6
9	16	0.62
10	7	0.57
11	15	0.6
12	10	0.5

TABLE B.1

The period is in the range 7 mins. $\leqslant T \leqslant$ 18 mins., and $\bar{m}$ is in the range $0.5 \leqslant \bar{m} \leqslant 0.69$. The mean values for T and $\bar{m}$ are 13 mins. and 0.59 respectively.

Histograms of the parameter distributions are given in Fig. B.6. The sample is too small to obtain meaningful figures for the standard deviations of the parameter values.

Test 3.

Sensing of temperature due to combined effect of:

(i) Local heating caused by 12.5 amp. current load,

(ii) Heating due to 30 W element within cabinet.

Temp. of room = 70^{o} F.

Thermostat command temperature = 78^{o} F.

Natural air convection within test box.

D.e.c. values for T and $\bar{m}$ were as follows:

Thermostat No.	Period T mins.	$\bar{m}$
1	25	0.2
2	20	0.2
3	18	0.22
4	44	0.09
5	50	0.2
6	18	0.25
7	38	0.18
8	30	0.3
9	29	0.33
10	6	0.33
11	17	0.3
12	20	0.18

TABLE B.2

The period is in the range 6 mins. $\leqslant T \leqslant$ 50 min., and $\bar{m}$ is in the range $0.09 \leqslant \bar{m} \leqslant 0.33$. The mean values for T and $\bar{m}$ are 26 mins. and 0.23 respectively.

Histograms of the parameter distributions are given in Fig. B.7. The results clearly show the decrease in $\bar{m}$ and the increase in T due to energising the internal heater.

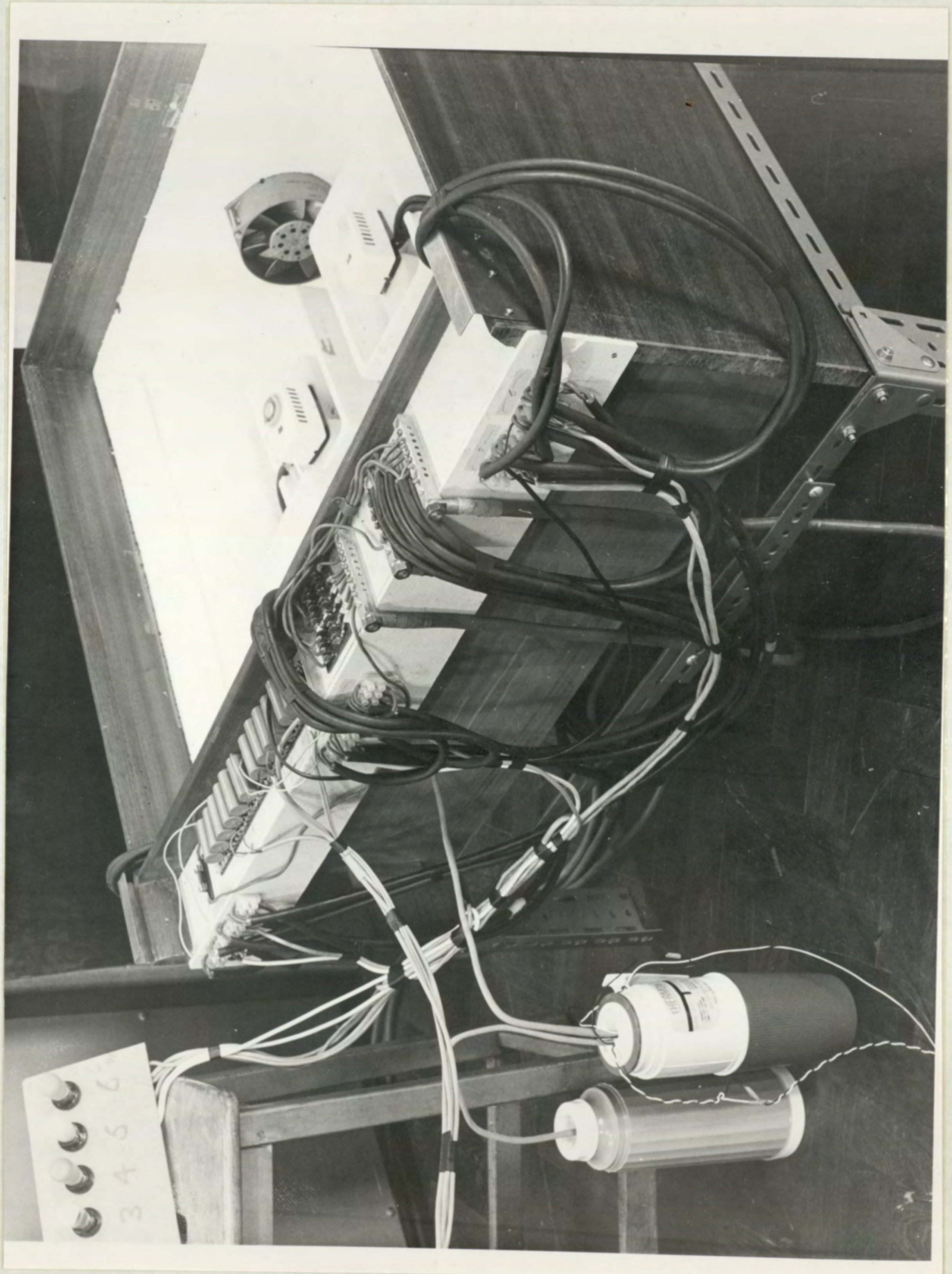


FIG. B.1 (a).

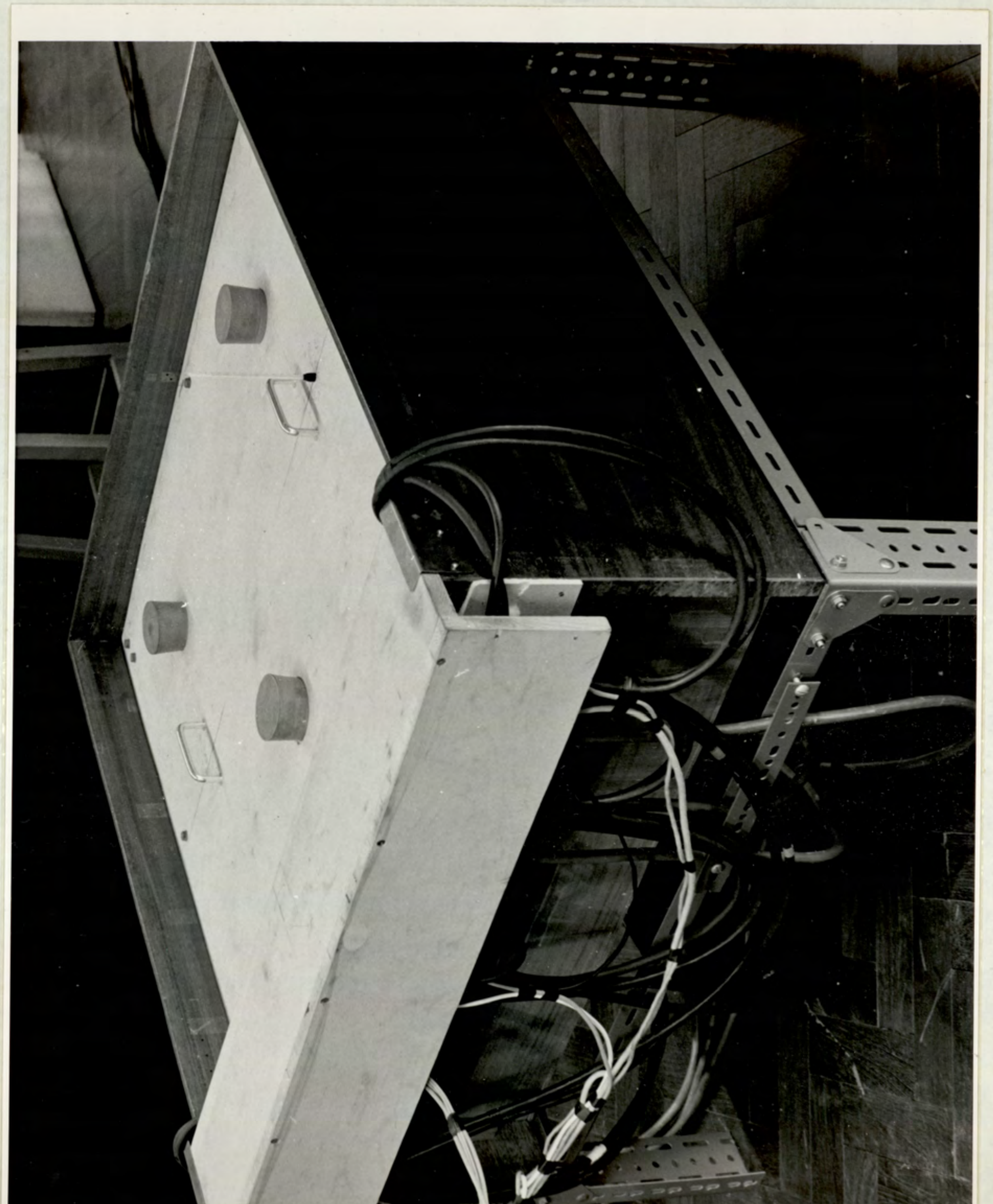


FIG. B. 1 (b).

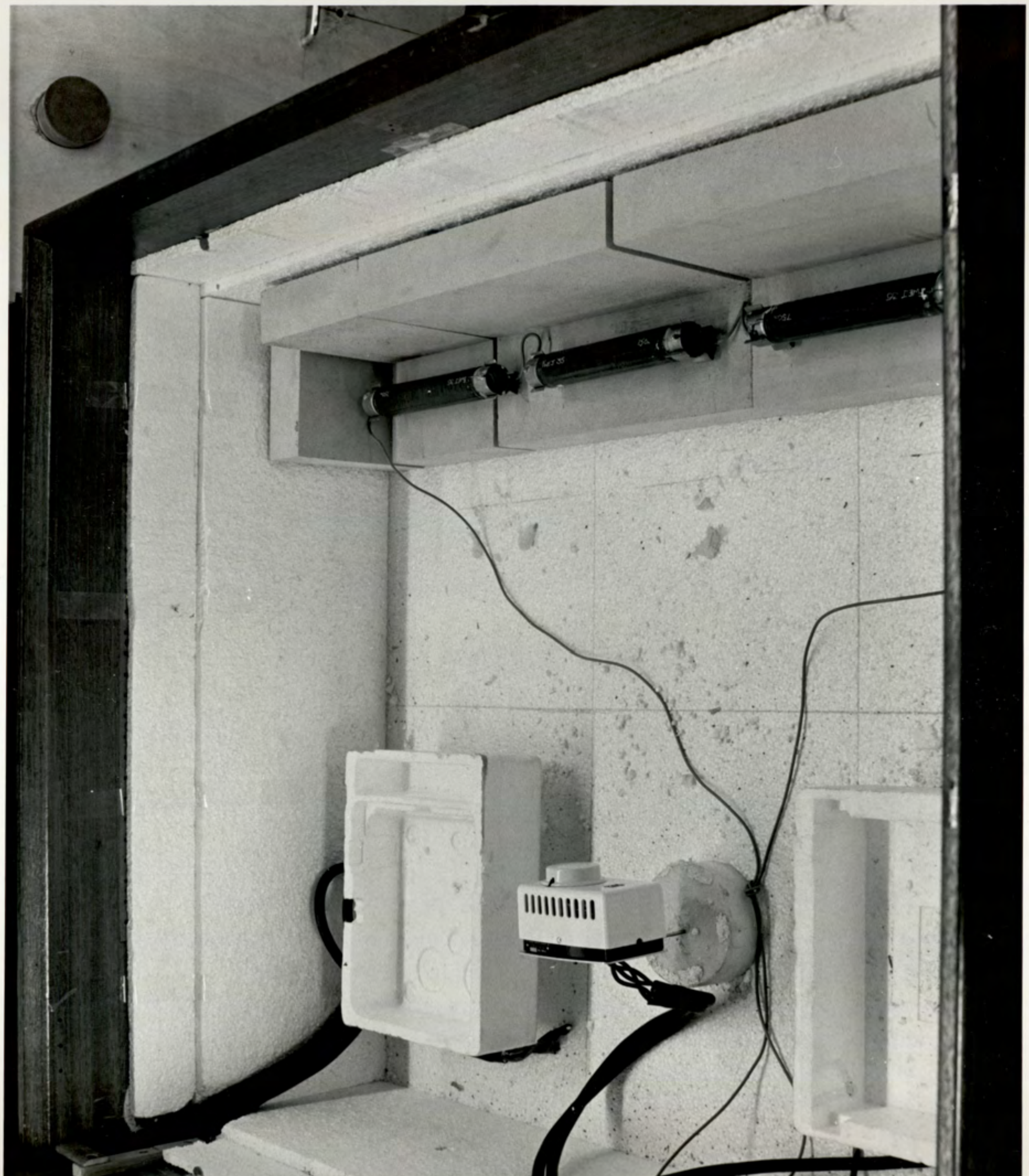


FIG. B.2.

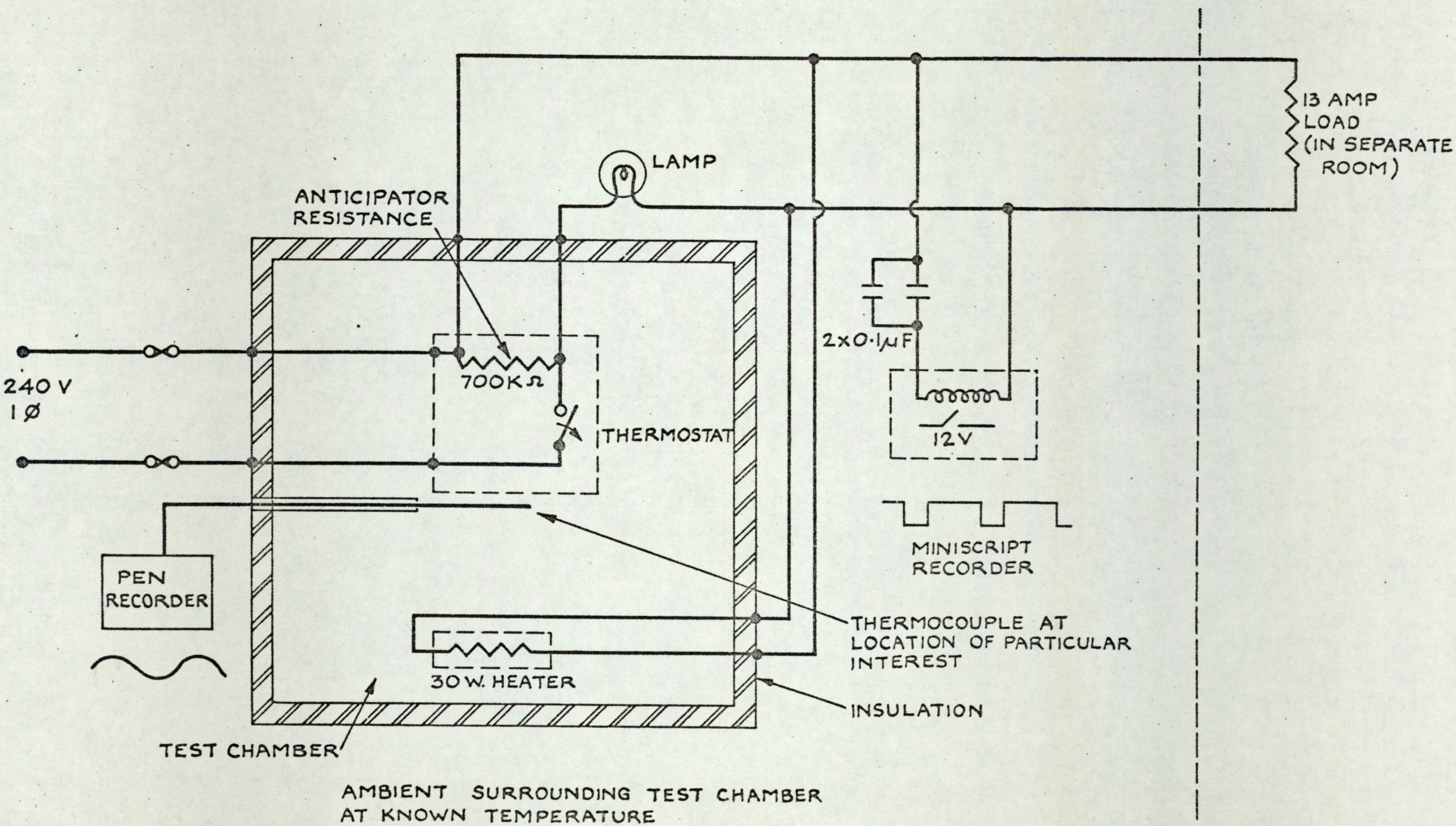


FIG. B.3 CONNECTIONS FOR TEST ON THERMOSTATS.

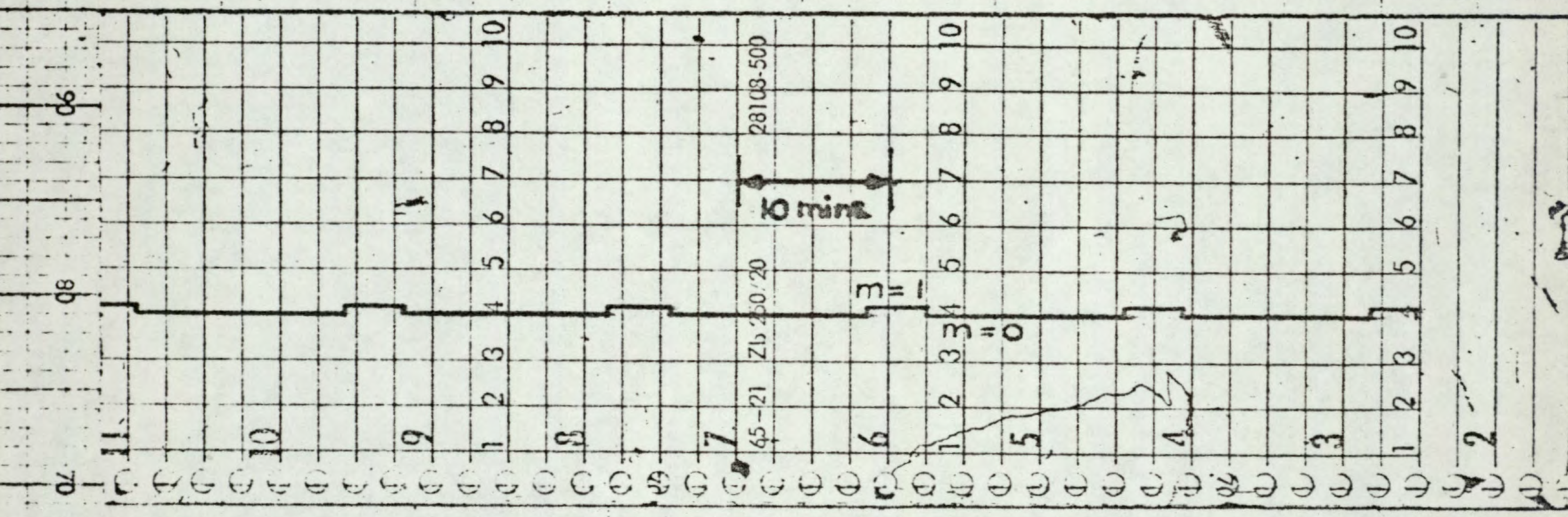


FIG. B 4 LOAD PATTERN.

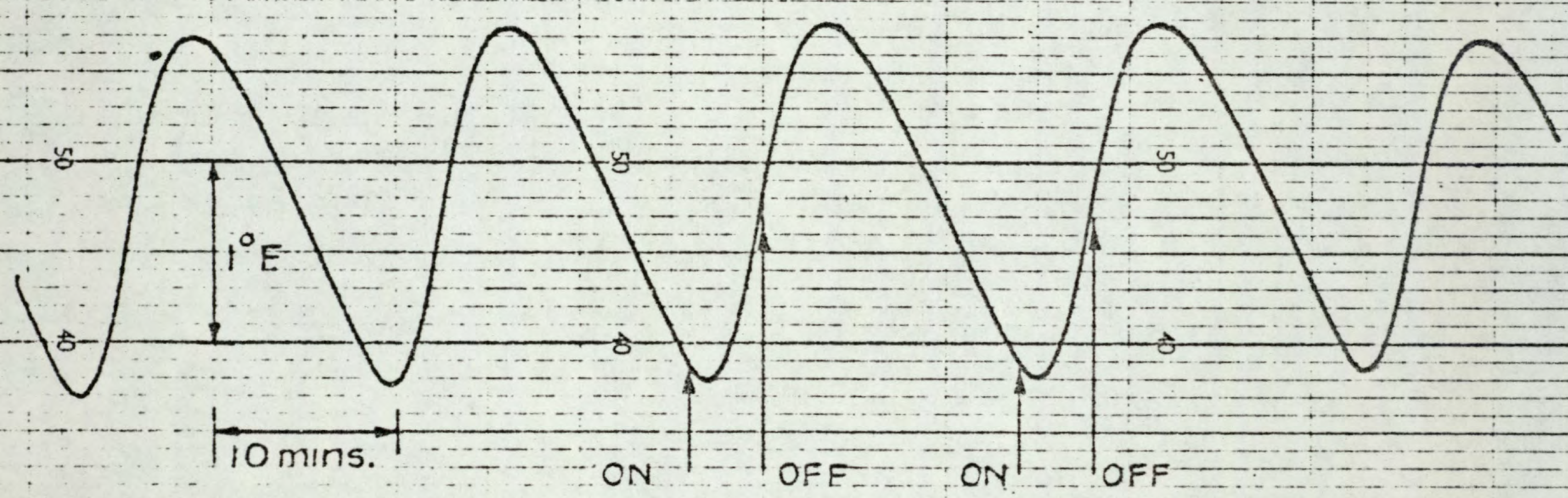


FIG B.5 TEMPERATURE OSCILLATIONS

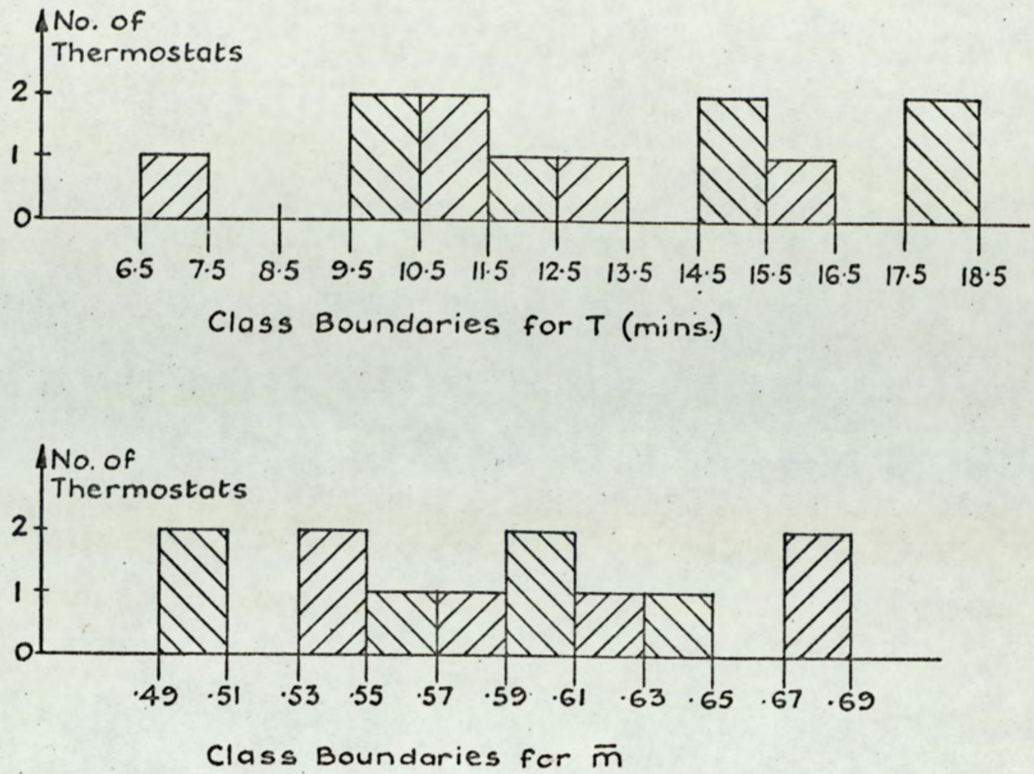


FIG. B.6 SPREADS FOR T AND $\bar{m}$.

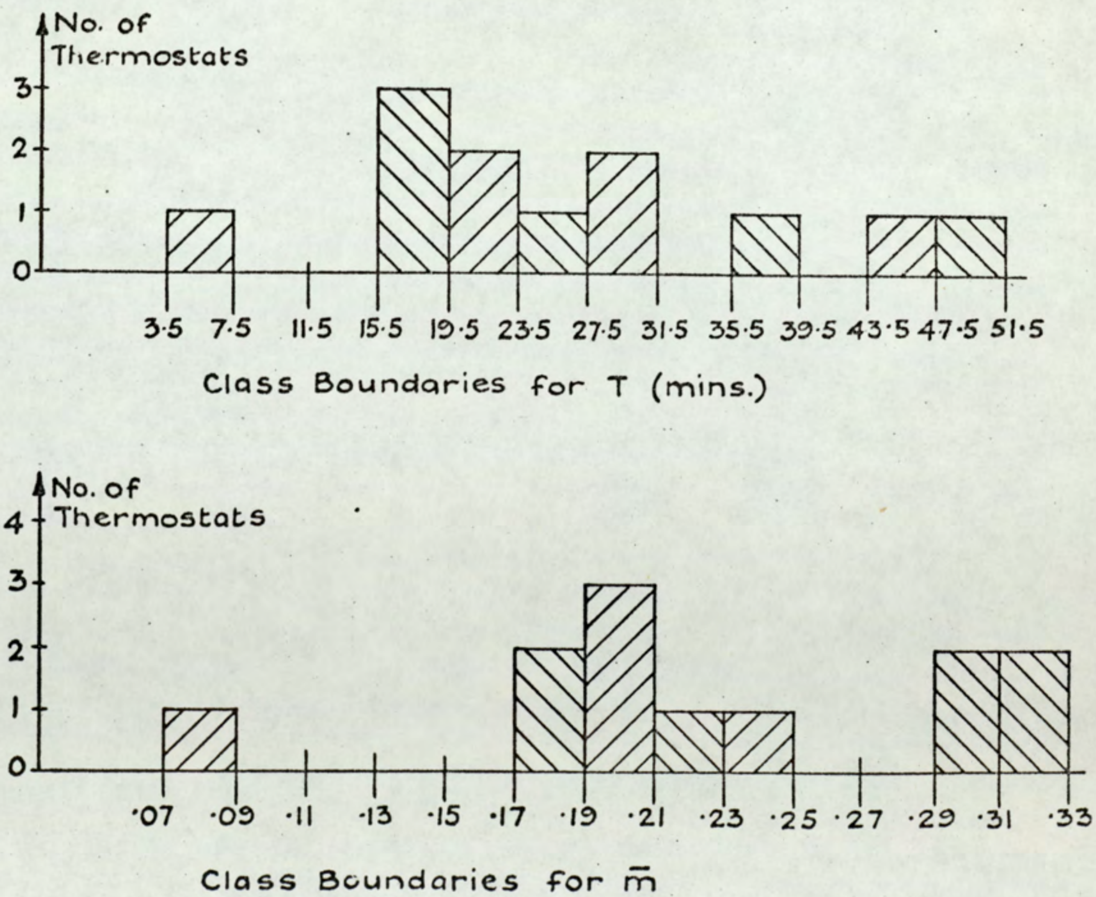


FIG. B.7 SPREADS FOR T AND $\bar{m}$.

APPENDIX C.

A PROPORTIONAL PLUS INTEGRAL CONTROLLER FOR CONTINUOUS CONTROL OF DEMAND.

C.1 Application of Proportional Plus Integral Control.

A need arises for continuous control of electroheat processes which are subject to sustained disturbances, e.g. variations of load and power supply, or of ambient temperature. These disturbances will occur simultaneously in a random manner, and the action of the controller will be continuously to adjust the manipulated variable towards the steady-state condition where the offset is zero.

Thus the power may be continuously corrected to that of constant demand.

The disadvantages of proportional plus integral control for a system with transit lag are:

1. The delay between the transient occurrence and the correcting action of the controller.
2. The long settling time.

Thus large load fluctuations immediately before the end of the integration period must be inhibited so that the target energy is not exceeded.

C.2 Use of Ziegler-Nichols Model.

Electroheat processes are modelled by irrational transfer functions of the form of equations (7.1) and (7.2). In the Ziegler-Nichols model[60,61] the analysis of higher-order control systems is approximated by one of first order with transit delay. The transfer function is

$$F.G. \exp(-sL)/(1 + sT),$$

where the parameters L and T are determined as shown in Fig. C.1. $h(t)$ is the step response of the actual process, upon which is drawn the tangent QR at the inflection point P. The time constant of the model is given by the inverse slope of QR, and the equivalent delay by the intersection of QR with the time axis.

Closed-loop continuous load control of an electrothermal process may be provided as shown in Fig. C.2. If the set point is θ_r, the effect of a disturbance θ_D is to produce an offset, which is brought to zero in a time dependent upon the process parameters and the controller design.

C.3 Controller Design and Transient Response.

The transient response may be determined by the root-locus method showing the locus of the closed-loop poles as the gain is altered.

The constants F, G may be omitted without loss of generality. If the controller proportional and integration constants are K_1 and $1/K_2$ respectively, then the open-loop gain $\emptyset(s)$ will be:

$$\emptyset(s) = \left(K_1 + \frac{1}{K_2 s}\right) \frac{e^{-sL}}{1 + sT}$$

$$= \frac{(sK_1K_2 + 1)\ e^{-sL}}{K_2 s\ (1 + sT)} \qquad \text{(C.1)}$$

A design procedure for the controller must be a compromise between a high degree of stability and the time taken to reduce any offset to an acceptable level. Thus whilst a fast integral action due to reducing K_2 produces an oscillatory response, a large integral time constant will lead to sluggish corrective action. If we take a second-order system as a standard of reference, the step response for a damping factor of about 0.7 exhibits a single overshoot of some 5 per cent.

We will consider first the open-loop transfer function in the absence of the delay term e^{-sL}.

Open-loop poles will be located on the s-plane at the origin and

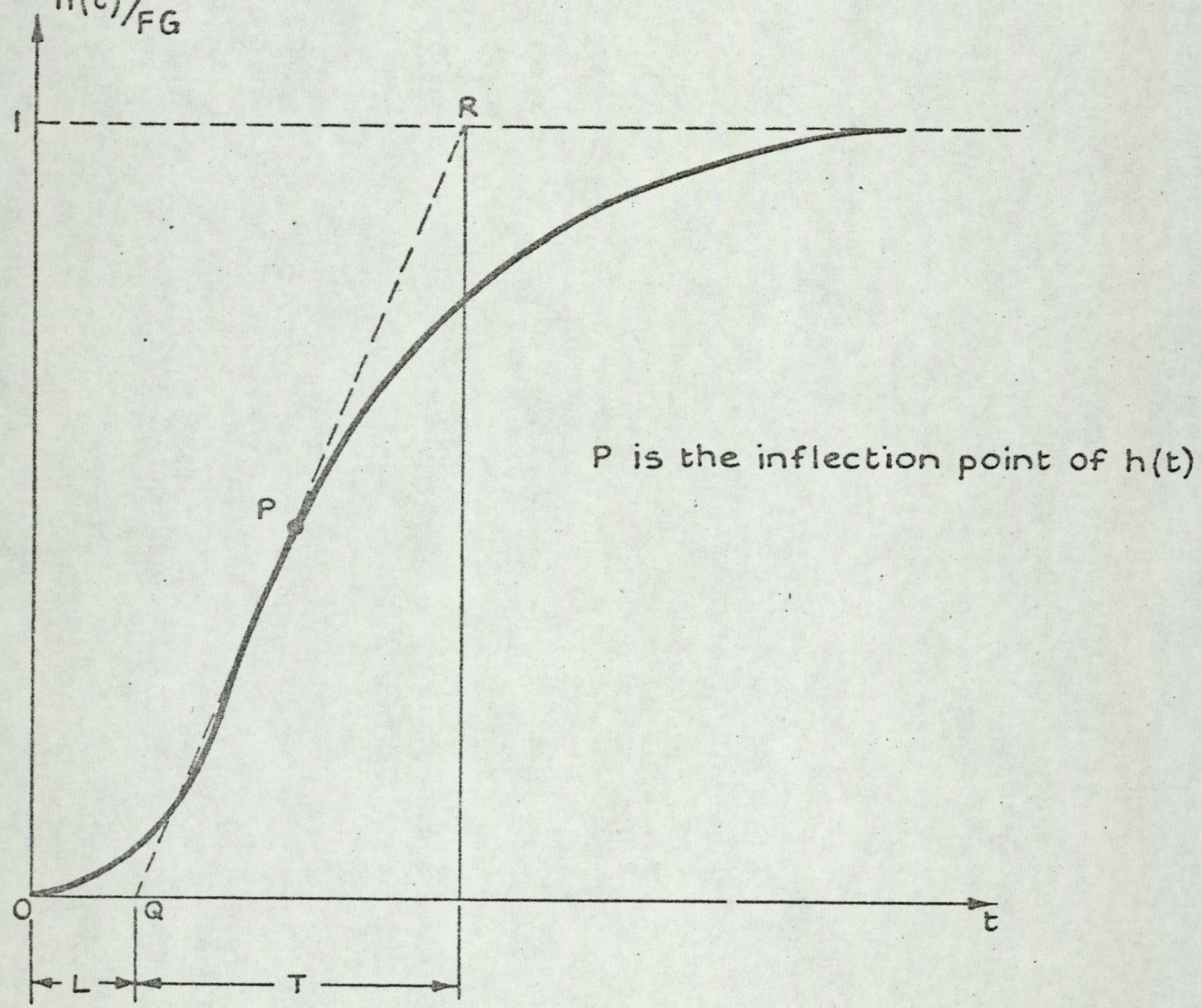


FIG.C.1 DETERMINATION OF EQUIVALENT L AND T FROM THE STEP RESPONSE OF THE PROCESS.

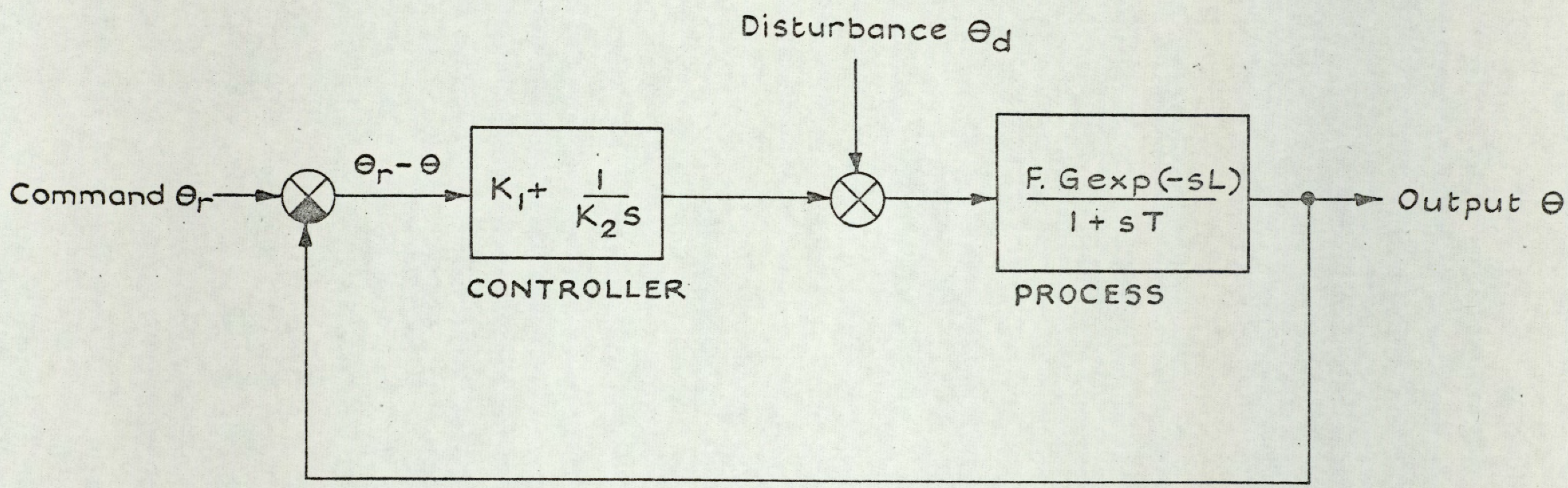


FIG. C.2 CLOSED-LOOP CONTINUOUS CONTROL USING PROPORTIONAL PLUS INTEGRAL CONTROLLER.

at $s_A = -\frac{1}{T}$, and there will be a zero at $s_B = -\frac{1}{K_1K_2}$. If s_B lies to the right of s_A the system will be non-oscillatory, but the condition where s_B lies to the left of s_A corresponds to conjugate closed-loop poles and an oscillatory response (see Fig. C.3). A limiting condition for no oscillations is when the pole s_B is moved away from the origin to cancel out the pole s_A at $s = -\frac{1}{T}$.

A suitable design procedure to produce the quickest stable response, therefore, is to adjust the controller constants K_1 and K_2 so that $s_B = -\frac{1}{K_1K_2}$ lies in the vicinity of $s_A = -\frac{1}{T}$. For this condition $K_1 K_2 \simeq T$, or $\phi(s) \simeq \frac{1}{K_2 s}$, leaving an open-loop pole at the origin, and producing integrating action.

We may now consider the effect of the transit delay. Graphical methods for determining the root loci for a system with dead time have been developed by superposition of constant gain and constant phase loci[47,62].

The transient response will be determined by the nature of the closed-loop poles, which are the roots of the characteristic equation

$$1 + \phi(s) = 0, \tag{C.2}$$

where the open-loop gain is

$$\phi(s) = \frac{1}{K_2 s} e^{-sL} = \frac{k}{s} e^{-sL}. \tag{C.3}$$

For each constant gain contour there will be an infinite number of roots p_1, p_1^*, p_2, p_2^* - - - - - - - - - corresponding to phase loci $180^\circ + N.360^\circ$ (N = 0, 1, 2 - - - -),

where

$$p_1 = \sigma_1 + j\omega_1,\ p_1^* = \sigma_1 - j\omega_1,$$

$$p_2 = \sigma_2 + j\omega_2,\ p_2^* = \sigma_2 - j\omega_2, \text{ etc.,}$$

and for stability all roots will have negative real parts.

The closed-loop transfer function is

$$\frac{\phi(s)}{1 + \phi(s)} = \frac{ke^{-sL}}{s + ke^{-sL}}, \tag{C.4}$$

giving a step response

$$f(s) = \frac{1}{s}\left(\frac{ke^{-sL}}{s + ke^{-sL}}\right). \tag{C.5}$$

By the Final Value theorem, the steady-state value of the transit response as $t \to \infty$

is $\lim_{s \to 0} s \left[\frac{1}{s}\left(\frac{ke^{-sL}}{s + ke^{-sL}}\right)\right] = 1.$

$$\Phi(s) = \frac{sK_1K_2+1}{K_2 s(1+sT)}$$

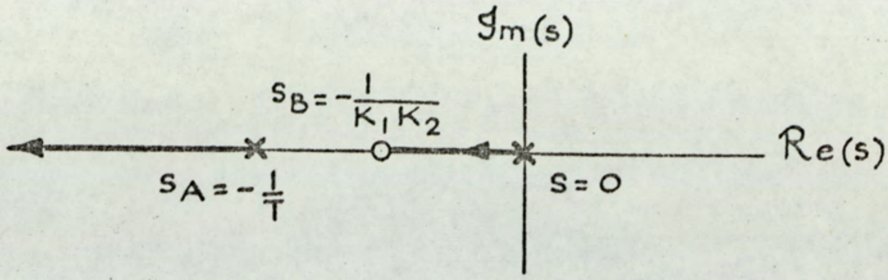


Fig. C.3 (a) $\frac{1}{T} > \frac{1}{K_1K_2}$ Non-oscillatory response.

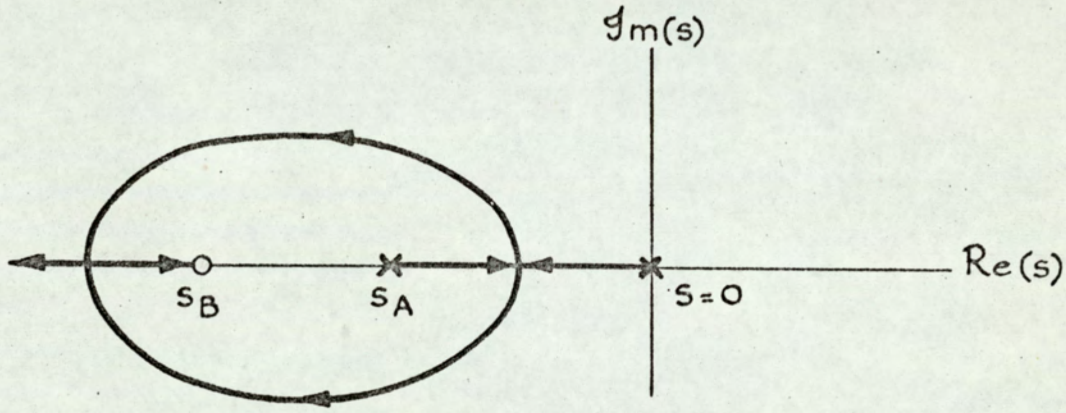


Fig. C.3 (b) $\frac{1}{T} < \frac{1}{K_1K_2}$. Oscillatory response.

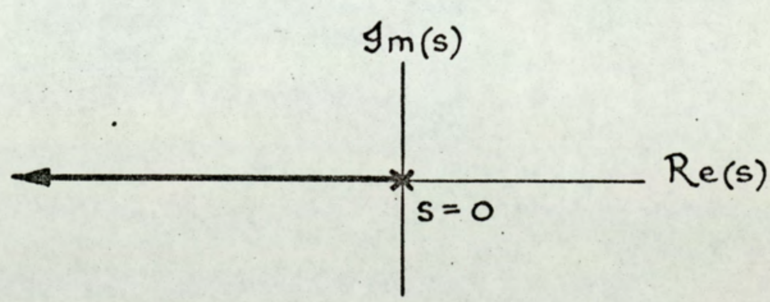


Fig. C.3 (c) $\frac{1}{T} = \frac{1}{K_1K_2}$, when $\Phi(s) = \frac{1}{K_2 s}$

FIG. C.3 ROOT LOCUS PLOTS.

Shridhar [47] develops a graphical method for determining the gain and the location of the closed-loop poles where the step response of the controlled process is required to have an overshoot of about 5 per cent.

Account is taken only of the roots on the branches of root loci corresponding to phase loci of 180° and 540°, since the roots on the other branches may be ignored owing to their large negative real parts.

C.4 Analogue Simulation of the Transient Response.

Graphical methods for location of the predominant closed-loop poles can prove time-consuming for systems with dead time, even when using the Ziegler-Nichols approximation.

In practice, however, the system step response may be studied exhaustively and in a relatively quick manner by means of an analogue simulation. Thus K_1 and K_2 may be set experimentally to achieve a desired response.

Simulation of the control system represented in Fig. C.2 was realised on an accelerated time scale using an E.A.L.48 hybrid computer. The input was triggered repetitively to facilitate observation by oscilloscope of the system performance. The time scale could be made slower by a factor of 1,000 to enable the plotting of the transient response by an X - Y recorder.

Generation of Time Delay.

Time delay may be simulated upon an analogue computer by means of Padé approximations [63, 64]; alternatively by use of a tape recorder with spatial separation of record and playback heads.

The method adopted here makes use of the relation:

$$\lim_{n \to \infty} \left(1 + \frac{sL}{n}\right)^n = e^{sL}, \qquad \text{(C.6)}$$

or $e^{-sL} \simeq \dfrac{1}{\left(1 + s\frac{L}{n}\right)^n}$, with increasing accuracy as n becomes larger. The transit delay may therefore be simulated by cascading n first-order systems each with time constant $\frac{L}{n}$. An even number of stages is desirable to avoid an overall sign change.

Six stages were found to produce a relatively noise-free delayed signal of sufficient accuracy.

As shown in Fig. C.4, twelve operational amplifiers were interconnected via the computer patchboard to simulate the control system

FIG. C.4 ANALOGUE SIMULATION OF TRANSIENT RESPONSE.

represented in Fig. C.2. In order to observe the response for a wide range of L/T ratios, the value of T was standardised as 1 sec. and L chosen as either 60 millisec. (L/T = 1/16.7) or 600 millisec. (L/T = 1/1.67).

The reference Θ_r was set to zero, and the magnitude of Θ_d was adjusted so that no amplifier was driven beyond its linear region of operation upon application of the step. The step response was optimised for potentiometer settings of K_1 and K_2 in turn to give about 5 per cent overshoot.

The optimal settings were found to be:
$K_1 = 2.75$, $1/K_2 = 2.9$ sec.$^{-1}$, so that $K_1 K_2 = 0.95$ sec., compared with T = 1 sec.

The following pen recordings of the transient response are shown in Figs. C.5 to C.7:

1). With zero integral correction to give proportional control alone.

2). With maximum integral correction, resulting in instability.

3). With K_1 and K_2 optimised as shown above.

In Fig. C.7 we see that, provided linearity is maintained, the time between the application of Θ_d and the return of Θ to the value Θ_r is independent of the step magnitude, and is of the order of 3T. However, the "settling time" becomes amplitude dependent when defined as the time, subsequent to a disturbance, for Θ to be reduced to a value within, say, 10 per cent of Θ_r.

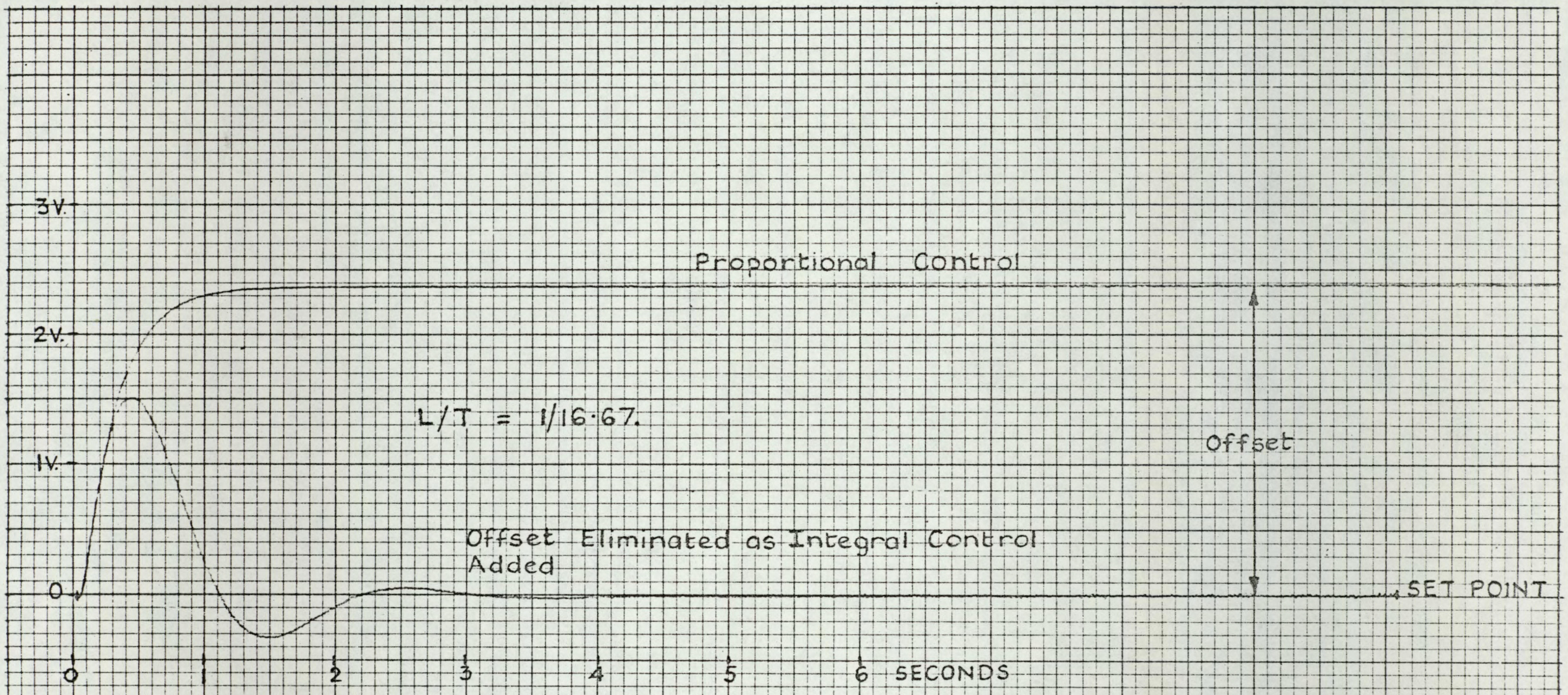


FIG. C.5 OFFSET PRODUCED BY DISTURBANCE WITH PROPORTIONAL CONTROL ALONE.

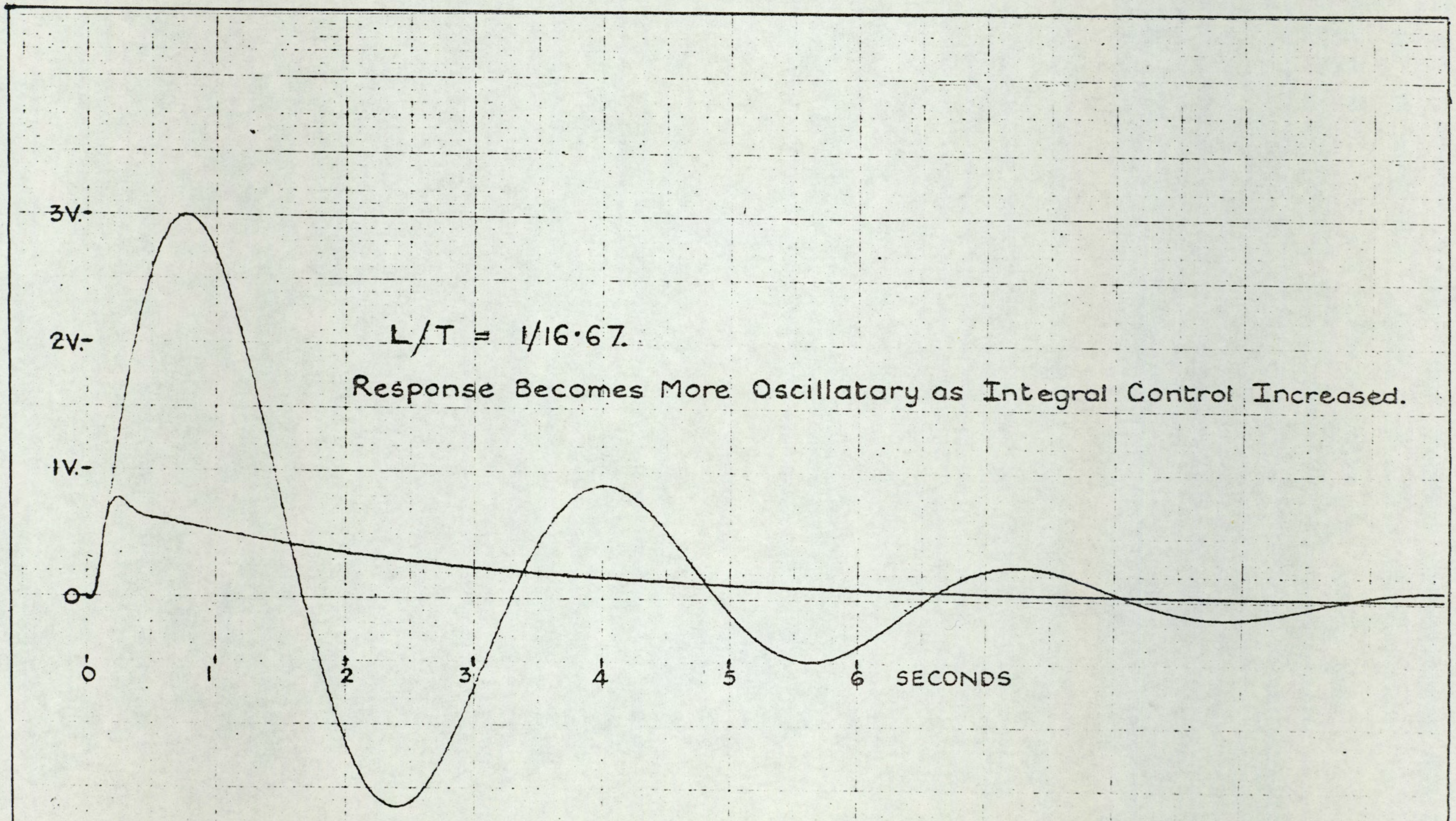


FIG. C.6 RESPONSE TO DISTURBANCE WITH PROPORTIONAL PLUS INTEGRAL CONTROL.

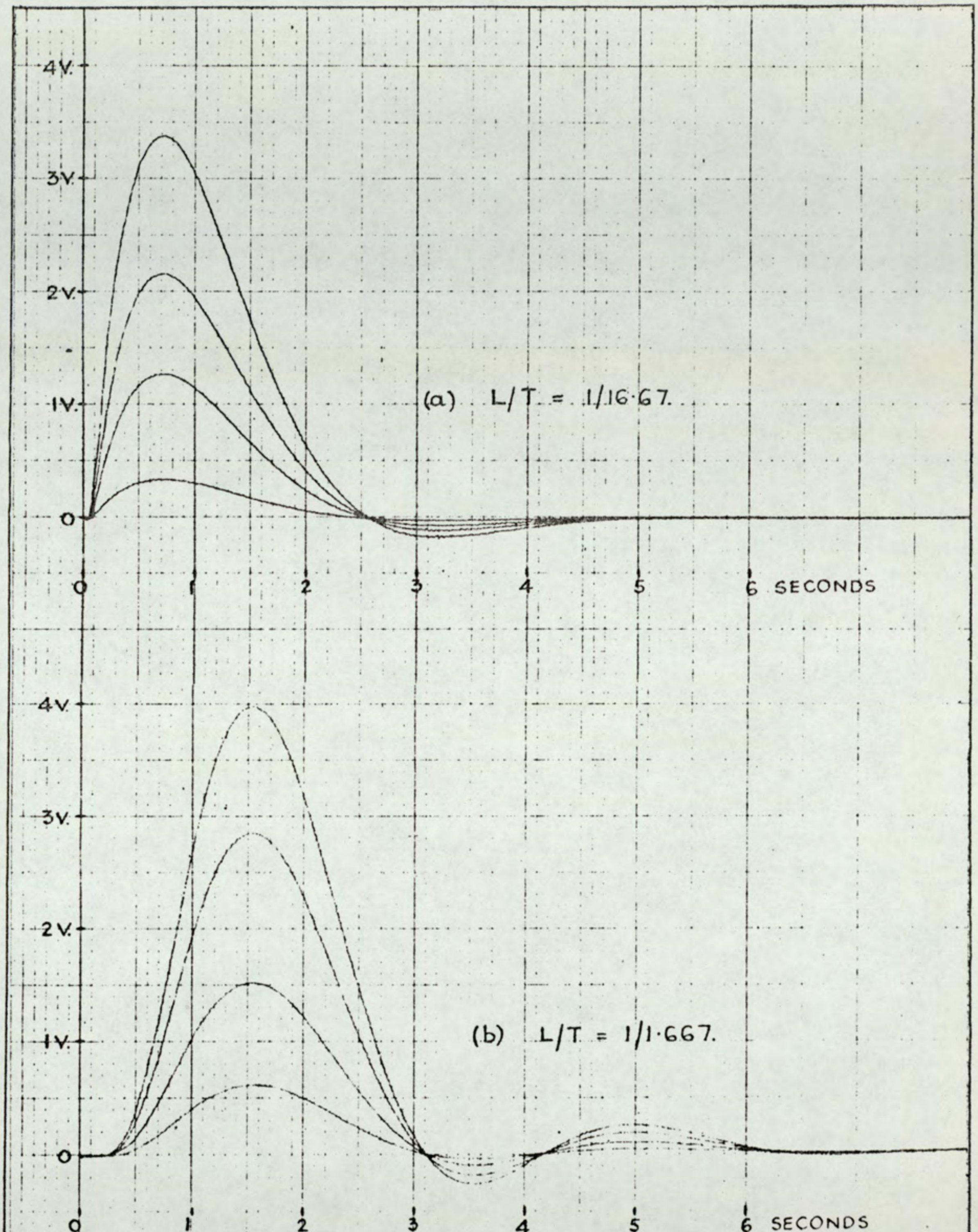


FIG. C.7 OPTIMISED TRANSIENT RESPONSE WITH PROPORTIONAL PLUS INTEGRAL CONTROL.

PUBLICATION OF RESULTS.

The results of chapter 3 have been accepted for publication: "Demand Control of Two-position Process Ensembles of Finite Magnitude", I.E.E.E. Trans. Industry and General Applications, Paper 69 TP 157 - IGA, Jan./Feb.1971.

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