HIGH FREQUENCY NOISE AND VIBRATION OF AUTOMOBILE TRANSMISSION SYSTEMS

by

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SUMMARY

Investigations are discussed which were aimed at establishing the characteristics of rear axle gear noise which was considered to be a typical example of a power transmission noise.

A series of preliminary investigations gave quantitative information on which the ensuing research could be based. It was hypothesised from this information that the amplification of the vibrations by the resonant response of the propeller shaft was the cause of the audible noise.

Investigation of the influence of the propeller shaft showed that its resonant behaviour could not be the cause of the audible noise and that it was sufficiently well isolated to be disregarded in this respect. Further appraisal of the results indicated the possibility of transmission of low level vibrations to the vehicle body.

A study of the rear suspension system showed that rear axle vibrations were transmitted to the body. It was concluded that the vibrations became audible noise due to the coupling of the acoustics of the passenger compartment with the vibrations of the enclosing panels. Redesign of the rear suspension system with particular regard to the dynamic properties of the rubber bushes is recommended as a course of action likely to lead to a reduction in the noise.

Theoretical studies into the prediction of the

natural frequencies and mode shapes of the power transmission system by numerical methods were pursued. The transfer matrix method of evaluation was found to be an excellent technique for this work and if larger computor facilities had been available extension of the analysis could have yielded useful results.

It is shown that random process techniques must be used to obtain reliable results from road test data, and information about the analog data analysis systems developed for the analysis is presented.

CONTENTS.

LIST OF FIGURES

NOTATION

B = Bandwidth (data or filter)

C = Co-spectral density function or capacitance

c = Speed of sound

E = Young's modulus

f = Temporal frequency

G = One sided power spectral density function

H = Flexibility matrix

h = Spacing of correlation points

I = Second moment of area

K = Time constant of RC integrators

l = Length

M = Mass matrix or bending moment

m = mass

N = Number of nodes

p = Probability density

Q = Quadrature-spectral density function

R = Correlation function of resistance

S = Stiffness matrix or road speed of vehicle

s = Spring stiffness

T = Transfer matrix or sample time

t = Time

V = Shear force

w = Deflection of beam

Z = State vector

δ = Dirac delta function

θ = Phase angle

ψ^2 = Mean square value

ϕ = Slope

σ = Mean square value about mean value

τ = Time delay

μ = Mass per unit length or mean value

ω = Circular frequency

CHAPTER 1 INTRODUCTION

1-1) Objects of the research

The primary object of the research discussed in this thesis was the investigation of rear axle gear noise which was audible in the passenger compartments of some medium sized automobiles. It was intended that the investigations should establish the characteristics of the audible noise and also establish the mode of transmission between the rear axle gears and the passenger compartment. Resulting from these investigations it was envisaged that modifications could be made to the vehicles which would either eliminate the audible noise or at least reduce it to a more tolerable level.

This noise problem was situated in the mid-audio frequency range of the noise spectrum (250-3000 Hz) a section of the spectrum which had received very little attention from previous researchers. Consequently due to a lack of suitable equipment and experimental procedures a secondary object of the research was the development of new, and the assessment of existing, data acquisition equipment and procedures for automobile testing.

1-2) Characteristics of the problem

At the commencement of the research the only information available about the audible rear axle gear noise in the model of vehicle chosen for the investigations was customer complaint statistics and the subjective assessments of the noise by the engineers concerned with the problem. These two sources provided the following information:-

i) The noise was assessed to be periodic at a frequency of 500 Hz.

ii) The noise was normally audible in a narrow speed range in the region of 50 mile/hr.

iii) The intensity of the noise was such as to become annoying after only a short period of time.

iv) Approximately 10% of the total production of the model suffered with audible rear axle noise.

v) A good proportion of this 10% could be cured by changing the rear axle gear set.

vi) There was no significantly measurable difference between a noisy gear set and a quiet gear set.

vii) All noise complaints and assessment were associated with steady cruising in top gear.

As can be seen the prior knowledge was rather limited. It was all based on subjective assessment which, though the ultimate test of any cure of this type of problem, is a notoriously bad method of measuring trends in noise levels.

The initial assumptions were made based on the above data:-

i) It was assumed that, since the incidence of occurrence of the audible noise was low (10%) and apparently negligible changes could cure the problem, the noise was only slightly above the threshold of perception in the general noise environment of the passenger compartment.

ii) It was assumed that, since the noise was only audible in a consistently reproducible small speed range, the noise or source vibration was amplified by a resonant component in the vehicle system.

1-3) Review of published work

a) Published work relating to rear axle noise.

Prior to this work three papers with some direct reference to rear axle noise had been published.

Farnham in his paper Control of Noise and Vibration in the Unibody (1) outlines how the change to unit construction had removed the inherent damping which existed in the chassis to body connections of earlier cars. He then explains that the major problem encountered by Chrysler when they changed to unit construction in 1960 was one of rear axle noise which in this case was amplified by the torsional resonance of the propeller shaft. The rest of the paper was a fairly general but extensive discussion of the research techniques and curative measures used in overcoming the noise and vibration generated by the road and engine.

The most significant fact mentioned in this paper was that the conventional methods of specifying rubber properties for mounts was inadequate in respect to high frequency isolation.

Stafeld in his paper Computor Analysis of Automotive

Drivelines (2) briefly discusses the use of a digital computor to evaluate the natural frequencies and modal shapes of vibration of conventional transmission systems. He emphasises that the whole system should be considered and not just the propeller shaft. The method employed was a transfer matrix method which was reasonably adaptable to design modifications and could include measured data.

A second paper by Farnham, Power Train Tuning for Quiet Cars (3) was a repeat of those sections of ref (1) which dealt with the problem of rear axle noise and no new information was presented.

The topic of rear axle whine is mentioned in the Handbook of Noise Control (4) but only to the extent of saying that it exists in most automobiles and occures in a speed range of 40 to 60 mile/hr. and at a frequency of approximately 500 Hz.

b) Other published work relating to noise and vibration in automobiles.

Gladwell (5) reviewed the majority of published work in this field and it was clear from this review that the major effort in automobile noise and vibration research had been concentrated on the low frequency engine and road generated phenomena and had little bearing on the work which was to be the subject of this thesis. The general conclusion of this review was that the American automobile manufacturers were more aware of the part played by research in the development of automobiles.

Only a few other papers had been published after Gladwell's review and they were concerned with the low frequency ride and road noise problems.

1-4) Description of the test vehicle

The vehicle chosen for the tests was a typical example of a current production vehicle which suffered with audible rear axle noise. Incedence of customer complaints of rear axle noise for this particular model was reported by the manufacturer to be 10% and the test vehicle was described as typical of this 10%.

The vehicle was a medium sized saloon car with the following general specification:-

Body	Unit Construction
Wheelbase	99 in
Track front	48.5 in
Track rear	50 in
Weight	2470 lbf
Engine:	
Type	Water-cooled in line o.h.v. petrol engine
Number of cylinders	4
Displacement	1622 cm^3
Gear box:-	
Type	manual
Ratios	1st 3.636:1, 2nd 2.214:1, 3rd 1.374:1, 4th 1.000:1

Rear axle gearratio 4.3:1
(10 pinnion and 43 crownwheel teeth)

1-4-1) Power Transmission System

For this thesis the term power transmission system will be restricted to refer to the propeller shaft and rear axle assemblies.

The propeller shaft was a hollow steel tube of 2.5 in o.d. fitted with Hardy-Spicer universal joints to give angular freedom and a splined drive from the gear box to give longitudinal freedom. Length of the shaft was 42.75 in centre to centre of the universal joints.

The final drive gears were a hypoid pinnion and crownwheel pair which drove the half-shafts through a standard set of differential gears. The main casing of the rear axle consisted of two steel pressings which formed the upper and lower halves of the axle and were welded together along two horizontal seams. An aluminium casting was used for the gear carrier which held the final drive gears and attachment to the main casing was by means of studs and nuts.

1-4-2) Rear Suspension System.

Rear suspension system was of the conventional semi-elliptic leaf spring form and was designed for a working load of 625 lbf per spring. Viscous damping was provided by double acting lever type shock absorbers (one per spring) and some degree of roll stiffening provided by an anti-roll bar fitted between the shock absorber levers. All the mounting and pivot points other than the mounting of the shock absorbers to the body contained some degree of rubber bushing. No information could be obtained for the dynamic characteristics of the rubber bushes.

1-5) Outline of work

First stage of the research was a preliminary investigation of a superficial nature in order to obtain a feel for the problems likely to be encountered in the main investigation, and to obtain some quantitative facts to form a base for the future work.

These preliminary investigations, which are discussed in full in chapter 2, included a response test on a laboratory rig of the transmission system and a road test of a production vehicle. Four experimental problems were found to exist:-

i) reliable measurement of the input force to the laboratory rig,

ii) lack of repeatable phase response of the laboratory rig,

iii) Lack of suitable analog data recording equipment for the road test and

iv) Lack of suitable data analysis systems for the road test recordings.

The first problem was overcome by constructing our own load cell and the second was found to be due to the inherent non-linearities of the transmission system. The latter two problems were not overcome at this stage in the work but the difficulties were noted and provided useful guide lines for the specification of future equipment.

Implications of the results of these initial investigations were that the audible rear axle gear noise was due to the resonant response of the propeller shaft in its second bending mode of vibration.

Chapter 3 describes and discusses the experiments performed to substantiate the conclusions drawn from the preliminary investigations. Response tests and road tests were again the main tool but they were used to a much greater degree.

In this case the vibration response tests of the transmission system were performed on the system mounted in a vehicle and for some of the tests an automatic response test instrumentation system was employed which though it placed some restriction on the forcing levels did enable results to be obtained very quickly. No major problems were encountered in these response tests since the instrumentation and procedures were similar to those developed for the preliminary test.

Serious difficulties did arise in the road tests performed at this stage in the research. These were associated with the data recording, reproduction and analysis equipment. The problem with the recording and reproduction equipment was the lack of a high quality instrumentation magnetic tape recorder suitable for portable use. Most of the problems with the data analysis equipment were connected with the construction of the automatic system required to handle the large amounts of data collected in the road tests. These problems were overcome with reasonable ease but one feature of the equipment used became the major limitation of the system. This limitation was the relatively poor frequency resolution of the constant percentage bandwidth analyser used as the central unit of the system. Due to the prime role of the data analysis system in this investigation, the discussion of the theory and practice of wave analysis and of the construction of the system used at this stage of the research has been separated into chapter 4.

Response tests confirmed that the most intense resonant responses of the transmissions system were those of the propeller shaft. However, the hypothesis that these propeller shaft responses were instrumental in creating the audible rear axle noise was not confirmed since the resonant frequencies were significantly different from the expected rear axle noise frequency.

Generally the confidence placed in the interpretation of the wave analysis results of the road test data was low due to the limitations of the equipment used and also due to a basic error in assuming the data to be complex periodic in character. Sufficient confidence could be placed in the results however, to show that the resonant response of the propeller shaft was definitely not the cause of the audible rear axle gear noise.

The only promising information to emerge from these tests was evidence from the road tests of a mechanical vibration at a frequency of 440Hz, at the rear spring rear axle interfaces which appeared to reach a maximum at a speed of 48 mile/hr. It was also quite clear that a revision of the assumptions about the character of the road test data was required with consequent changes to the analysis procedures and equipment.

In parallel with the physical investigations of the influence of the propeller shaft some theoretical investigations as discussed in chapter 5 were carried out. The object was to study the feasibility of predicting natural frequencies and the associated modal shape of vibration of automobile transmission systems by the use of numerical methods and a digital computor.

Three numerical methods for evaluating natural frequencies and modal shapes were considered from which the Transfer Matrix method was selected as the most suitable. Within the capacity of the computor available at the University it was found that answers could be obtained which had a reasonable accuracy and with a faster computor they would be obtained in a reasonable time.

Since it had been found that the road test data was predominantly random in character and not complex periodic as originally assumed, it was necessary to change from wave analysis techniques of analysis to the statistical describing function analysis used for random process analysis. The theory of the functions used in these researches and the practical methods of measuring them are presented in chapter 6.

At this stage of the work it had become obvious that the most likely cause of the audible rear axle noise was the mechanical transmission of a relatively low level vibration of the rear axle to the body of the vehicle via the rear suspension system. In order to investigate this hypothesis, further tests were performed which were divided into two sections, first an investigation of the part played by the suspension system in the transmission of the vibration and, second an investigation of the likely influence of the acoustic response of the passenger compartment.

The investigation of the mechanical transmission of the vibration by the rear suspension system (chapter 7) was approached with a different measurement philosophy both for the response tests and the road tests. For the response tests a total system response measurement concept was applied by measuring the acoustic response in the passenger compartment due to vibration applied to various points of the rear suspension system. In the case of the road tests, the major change of approach was the adoption of random process analysis techniques.

As was expected, the acoustic response of the passenger compartment to mechanical excitation of the rear suspension system was extremely complex containing numerous peaks even in the narrow band adopted for these tests. However, it was quite clear that the rear suspension system did play a major role in determining the levels of the measured response. In particular, a major response level was noted at 440Hz which was apparently due to transmission of vibration via the rear attachment of the rear springs.

Results of the analysis of the data obtained from further road tests showed a similar influence of the rear suspension system to that noted in the laboratory tests. Use of random process analysis techniques enabled the rear axle gear meshing frequency components of the noise and vibration to be extracted more positively than previously. These components showed conclusively that the audible rear axle gear noise was due to mechanical transmission of the vibration to the vehicle body from the rear axle by the rear suspension system.

Statistical modal density theory of sound transmission in large enclosures is presented in chapter 8 together with computed modal densities which show that this general theory is insufficient for small enclosures particularly if there is any likelihood of narrow band excitation.

Practical measurements of the acoustic response of the passenger compartment showed some evidence of a resonant response at the rear axle gear noise frequency, but of insufficient magnitude to justify a positive conclusion being drawn about its role in the amplification of the noise.

CHAPTER 2 PRELIMINARY INVESTIGATIONS

2-1) Summary

The results obtained from frequency response tests of a laboratory rig of the vehicle transmission system, which are presented in this chapter showed that the most likely cause of the amplification of the rear axle distrubance to a high level was the vibration of the propeller shaft in its second mode of bending. The road test results also presented in this chapter supported this conclusion and also gave indications of the problems likely to be encountered in the later analysis of multi channel road test data.

2-2) Objects of the investigations

There were three main objects of these preliminary investigations:-

i) to attempt to obtain some quantitative figures for frequency and intensity of the offending noise.

ii) to discover if any component of the transmission system had a lightly damped resonant response in the region of 500 Hz (noise frequency from subjective assessment).

iii) to assess the difficulties likely to be encountered in the data analysis of analog tape recordings of the noise and vibration measured during road tests.

2-3) Description of the laboratory rig

A rig was used for response tests in preference to the full vehicle since the components of interest were more accessible and also at the time these tests were performed no facilities were available for the laboratory testing of complete vehicles. It was expected that the response results obtained from the rig in the frequency range 100 to 2500 Hz would be a good representation of the response of a transmission system in its normal environment. This assumption was based on the coupling of the components of the rig to the vehicle being by low frequency systems, which it was considered, would act as isolators in the frequency range of interest.

The rig was constructed from the following major components;- engine and gear box assembly, propeller shaft, rear axle assembly, rear wheels and rear suspension springs and shock absorbers. A frame was built to support the engine by its normal rubber mounts in the normal configuration relative to the floor. The propeller shaft and rear axle were added to the engine and arranged with the rear wheels standing on the floor, so that the normal alignment of the components was maintained. The rear leaf springs were fitted to the axle and a loading frame was in turn attached to the springs using the normal mounting brackets. This loading frame also carried mounting points for the hydraulic shock absorbers.

The loading frame on the rear springs was loaded with steel blocks until the deflexion of the springs was the same as the deflection of the rear springs in an unladen vehicle.

In this loaded state it was found to be necessary to stabilize the loading frame with vertical stays at each corner fixed to the floor with anti-vibration mounts. The whole rig is shown in fig 2-1

2-4) Instrumentation of the laboratory rig

A block diagram of the instrumentation used to measure the response characteristics of the laboratory rig is shown in fig. 2-2.

Operation of the system was as follows. A sine wave of known frequency was obtained from a decade oscillator, and after amplification by a 1 kilowatt power amplifier used to drive an electro-dynamic exciter. The resultant forcing applied to the system was measured by a force transducer mounted in the coupling between the exciter and the centre of the rear axle. The response of the system to this forcing was measured with piezo-electric accelerometers mounted in the positions shown in fig. 2-3. The outputs of these accelerometers were fed via a selector switch to a valve voltmeter for amplitude measurement and together with the output of the force transducer to a phase meter for the measurement of the phase relative to the forcing.

A common calibration was achieved for the accelerometers by the adjustment of the gain of the cathode followers. This calibration was standardised prior to measurement by a preamplifier which gave an output of 1 volt r.m.s./peak g.

The only difficulty encountered in this instrumentation system was in obtaining a suitable load cell. Load cells commercially available at the time of the tests were neither small enough, nor of sufficiently low mass for use without modification of the system dynamics being incurred. To overcome this problem a load cell was designed and built in the laboratory.

The first cell built consisted of a steel cylinder with a piezo-resistive strain gauge, having a gauge factor of 120, bonded to the outside. This design was unsatisfactory due to the low signal to noise ratio and the high variation of sensitivity with temperature encountered. The second of these limitations was a particular embarrassment, since it was impossible to isolate the cell thermally from the exciter whose temperature varied over a wide range with variation in load.

A second design of load cell employed an inductive proximity probe. This type of proximity probe used a variable inductance to produce a frequency modulated signal in which the modulation was proportional to the change of gap between the inductance and a metal surface. The modulated frequency could then be demodulated to give a voltage proportional to the gap. Fig. 2-4 shows the final design of the load cell which consisted basically of a thin walled cylinder (A), and two solid cylinders (B&C). The load was transmitted axially by the tube and the resultant axial deflexion was measured by the inductive probe mounted in the end of cylinder (B).

To ensure that the linearity of the cell would not be affected by temperature all the parts were made from the same material. A further precaution taken during assembly, was to smear the threads of cylinders (B&C) with 'Araldite' to ensure that they would remain tight after assembly.

The cell had the following limitations, relatively delicate construction, low stiffness and lack of protection against bending. Both the first and second limitations were due to the thin walls required to give a reasonable deflexion for measurement by the proximity probe. The third limitation could have resulted in the cell being fractured, but did not effect the measurements since the sensing element was on the axis of symmetry where the mean gap remained sensibly constant. The low stiffness of the cell was the main limitation since in combination with the moving parts of the exciter, it formed a dynamic absorber at high frequencies. In use it was found that the cell gave a good clean signal, and that the useful frequency range was 0-2500 Hz

2-5) Response test techniques

Throughout the preliminary tests the exciting force was applied to the rear axle oil drain plug which was situated at the central point of the underside of the rear axle. Forcing level was maintained at 4.5 lbf r.m.s., and both amplitude and phase responses were measured, for each of the measuring stations, over the frequency range 70 to 2500Hz.

The following procedure was adopted to ensure that the forcing conditions remained as constant as possible while each set of measurements was being made

a) the desired frequency was selected on the decade oscillator and checked with a digital counter.

b) The phase meter was switched to the reference channel i.e. force signal, and the filter tuned to give a maximum reading on the meter

c) The force level was set using the phase meter reference level as a reference.

d) the amplitude and phase responses were measured at each position, the frequency and forcing level being checked between each pair of measurements.

The measurements were made at 5Hz intervals in the frequency range 70 to 2500 Hz.

2-6) Road tests

These tests were performed with the vehicle described in the introduction. This vehicle was reported to have an audible rear axle gear whine in the speed range 50 to 60 mile/hr.

Instrumentation for this test series consisted of an accelerometer mounted on the rear axle drain plug, and a microphone mounted on a short tube which fitted into the oil filler hole in the rear axle back plate. The signals from these transducers were recorded on two channel magnetic tape.

Recordings were made at constant speed at 5 mile/hr intervals in the speed range 25 to 60 mile/hr inclusive.

Analysis of the recorded signals was performed by cutting the original tape and forming continuous loops, one for each speed, for replay into a wave analyser. The analyser was equipped with an automatic frequency sweep and an analog output of the meter reading, enabling a pen recorder to be used to obtain amplitude v frequency curves. The analyser was not provided with any method of producing a frequency scale on the pen recording, and consequently the presence of an operator was required to mark the scale on the records.

2-7) Results

a) Resonance Test Results

The amplitude response curves were fairly well defined even though the frequency increments between measurements were rather large. This was not the case for the phase response curves since the system was lightly damped, and consequently the phase changes were extremely rapid with respect to frequency. Two other factors accounted for the poor phase measurements, the phase meter used would not measure the phase of low level signals resulting in gaps occuring in the curves, and small movements of the components of the rig caused quite large changes in the phase response. This latter condition was due to the non-linearities in the couplings etc.

The peaks in the response curves were not exhaustively identified in terms of resonating components, but those labled A, B & C in figs. 2-9 to 12 were identified as resulting from the resonance of the propeller shaft in the first, second and third bending modes of vibrations respectively. If the propeller shaft was considered to be a pinned-pinned beam then the natural frequencies of transverse vibration would have been given by:-

$$f_n = 2\pi^3 n^2 \sqrt{\frac{EI}{\mu l^4}}$$

Which gave $f_1 = 164$ Hz, $f_2 = 656$ Hz, and $f_3 = 1467$ Hz compared with $f_1 = 160$ Hz, $f_2 = 550$ Hz and $f_3 = 1250$ Hz measured from the rig. The discrepancies in the frequencies for the higher modes were most likely due to the influence of the flexibilities of the end fixings of the propeller shaft which were not pin joints to earth as assumed in the theory. This conclusion was supported by the occurance of peaks at the frequencies quoted above in the response curves for the two measuring positions immediately adjacent to the ends of the propeller shaft, i.e. on the gear box extention, and on the rear axle gear carrier, figs. 2-8 and 2-12.

In order to establish if there was a resonant response of a component or group of components which might have been instrumental in creating the audible noise, the responses in the frequency range 400 - 600 Hz were scrutinized. This frequency range was equal to the first order rear axle gear meshing frequency range corresponding to a road speed range of approximately 42 to 64 mile/hr. In this range the only major resonant response was the second bending mode of the propeller shaft at 550 Hz. A road speed of 57 mile/hr would have been required for the first order rear axle gear meshing frequency to be equal to this frequency. This speed did not compare well with the reported speed of 50 mile/hr. for the maximum noise, but it was considered that the differences between static and dynamic conditions and rig and actual environment, could possibley account for the discrepancy.

If this resonance was the cause of the amplification of the rear axle gear meshing forcing it appeared unlikely that the transmission of the disturbance to the passenger compartment was by mechanical means since the amplitudes of the responses at the rear spring-rear axle interfaces were very small at this frequency, figs. 2-5 and 6. It was indicated therefore, that the most likely mode of transmission was by airborn radiation from the propeller shaft.

b) Road Test Results

Figs. 2-13 and 2-14 show reduced copies of the wave analysis traces obtained from the magnetic tape recordings made during the road tests. Due to the linear form of recording used in the pen recorder, it was necessary to adjust the gain of the wave analyser at some stages in the analysis to obtain the required dynamic range , and this gave a discontinuous record.

All the analyses obtained from the microphone recordings fig. 2-13, showed a large amount of noise below 300 Hz, which was due to road noise and oil slop noises. There was also a considerable amount of noise above 3000 Hz generated by the ball and roller bearings. Neither of these regions of noise was considered to be of importance in respect to the rear axle gear noise problem due to their fairly even spectra, and lack of any marked changes with change of speed.

In the frequency range 400 to 600 Hz there was a peak in the noise analysis which reached a maximum at 55 mile/hr., and at a frequency of 525 Hz which coincided with gear meshing frequency at that speed. At the same speed peaks occurred in the noise analysis at frequencies of 1150 and 1575 Hz, and were most likely due to harmonics of the basic frequency.

From the wave analysis of the accelerometer signals fig. 2-14, it was found that there was a large proportion of low frequency components due to engine and road generated vibrations. Unlike the noise inside the rear axle the vibration at the accelerometer position did not show the dominant high frequency components due to the bearing noise. Components of the vibrations at first order rear axle gear meshing frequency were much more predominant than was the case for the noise but again showing a maximum amplitude at a frequency of 525 Hz (road speed 55 mile/hr.) The harmonics of the gear meshing frequency were not present in the vibration these apparently being a purely acoustic phenomina.

Lack of similarity between the noise and vibrations analyses showed that the microphone was reasonably insensitive to vibration and was consequently giving reliable results.

Comparison of the road and laboratory test results showed that the high level of noise and vibration detected in the region of 525 Hz correlates well with the resonance of the propeller shaft in its second bending mode of vibration. Other than this there was little agreement between the two sets of results.

2-8) Conclusions and decisions on further work

It was concluded that the most likely cause of the audible rear axle noise was the resonant response of the propeller shaft, in its second bending mode of vibration, to the first order rear axle gear meshing disturbance. Further, it was considered that if the above conclusions were true, then the most probable form of transmission of the disturbance to the passenger compartment was by direct airborn transmission from the propeller shaft to the floor of the vehicle and hence to the passenger compartment.

Based on the above hypothyses, the following guide lines were set out for further work:-

i) Further response tests should be performed on the laboratory rig, or preferably on a transmission system in its normal environment in a vehicle.

ii) More road tests should be carried out employing more comprehensive instrumentation, and multi-channel magnetic tape recording equipment.

iii) An investigation of methods of computing the natural frequencies and associated modes of vibration should be commenced.

iv) All the above studies should include some variation of a major parameter of the propeller shaft in order to fully assess the influence of the propeller shaft.

From the experience gained in analysing the data collected in the road tests, it was considered that the following points should form the basis of a specification for an analysis system:-

i) Original recordings should not need to be cut to form loops for analysis but should be copied onto secondary loops.

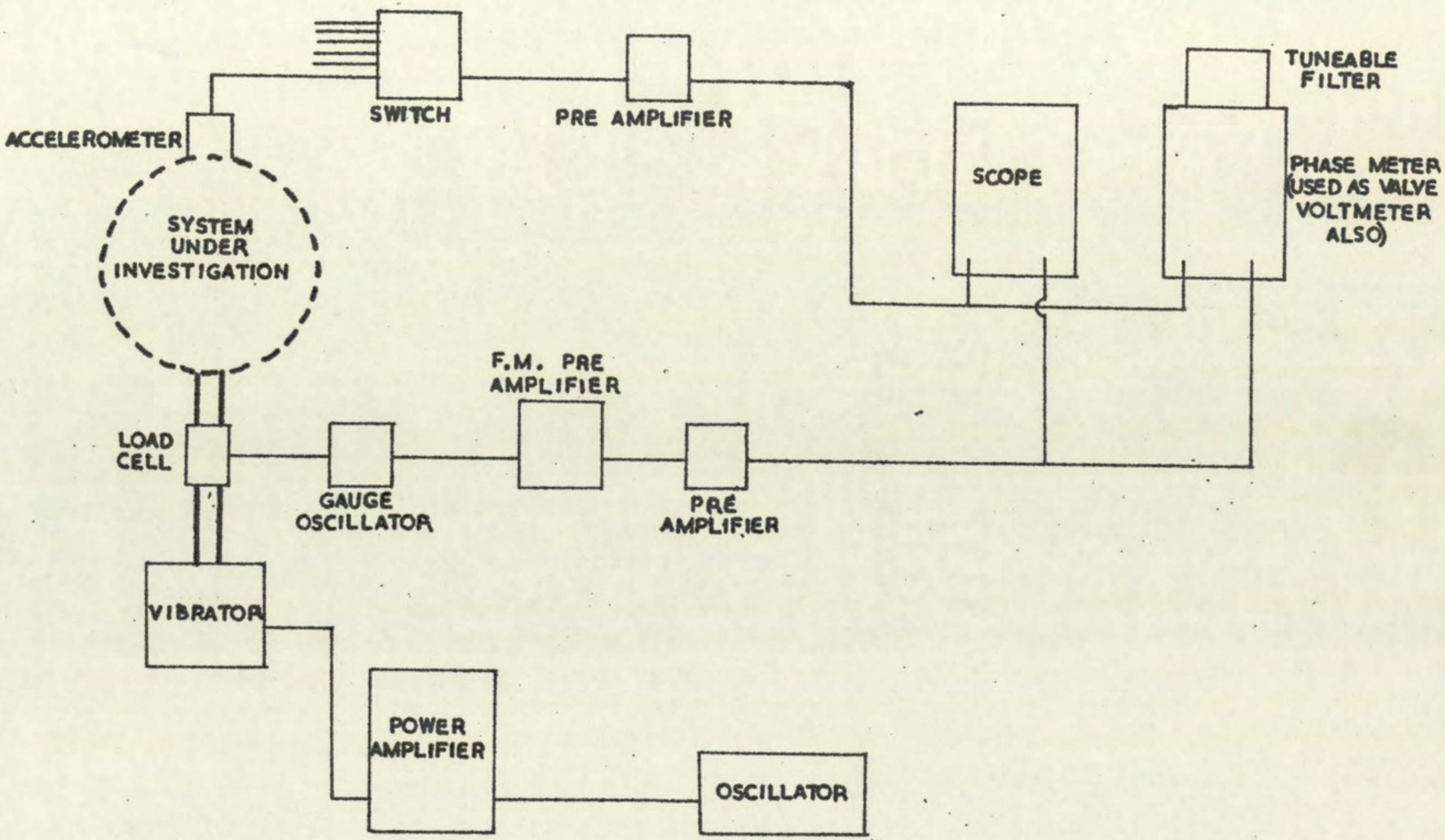
ii) Output of the system should be fully calibrated graphic form.

iii) If possible, provisions should be made for the analysis of multi-channel data either in parallel or sequentially.

iv) The system should require the absolute minimum of operator attention while running.

TRANSMISSION SYSTEM TEST RIG

FIG 2-1



BLOCK DIAGRAM OF INSTRUMENTATION FOR LABORATORY RIG TESTS

FIG 2-2

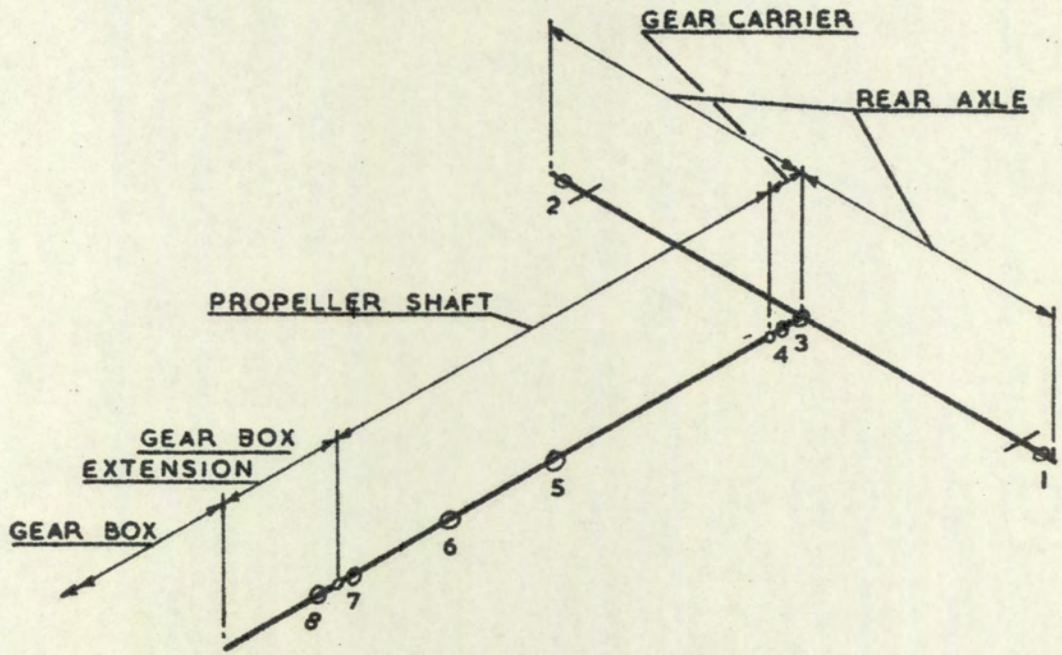
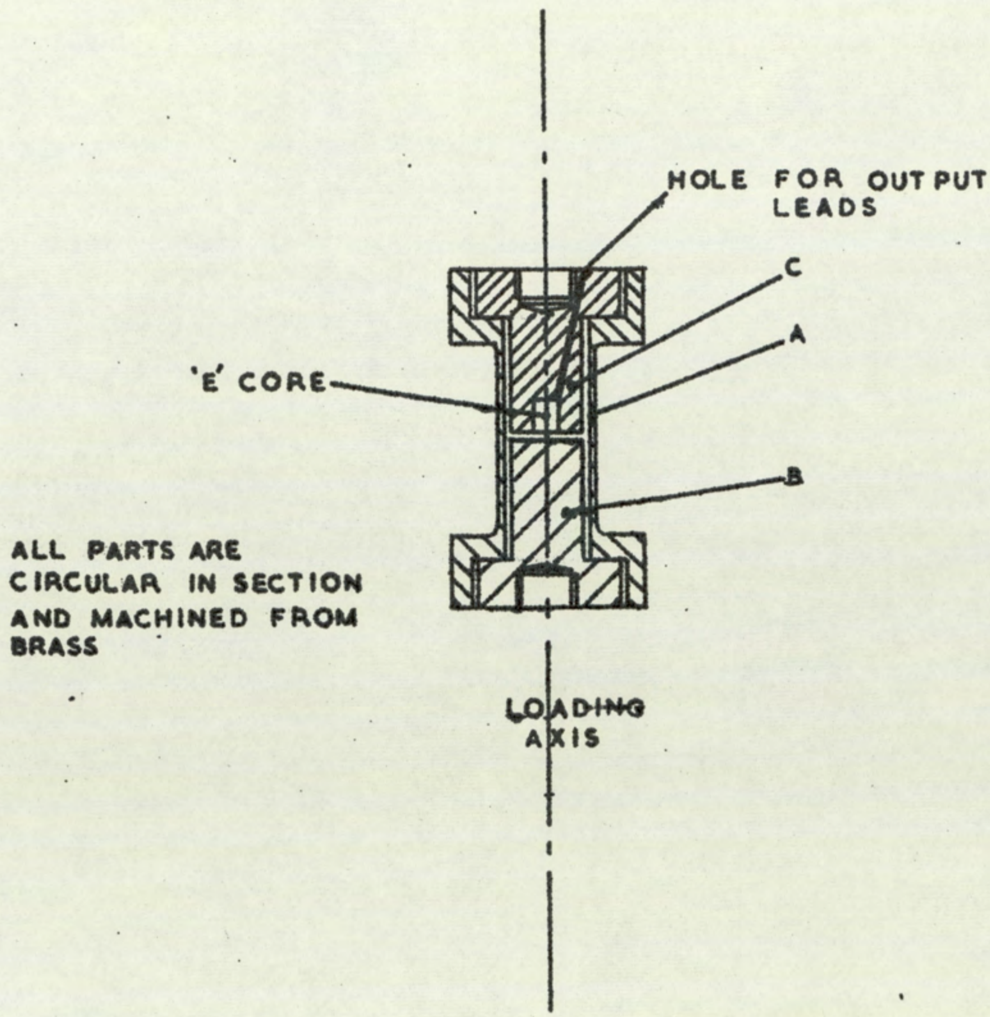


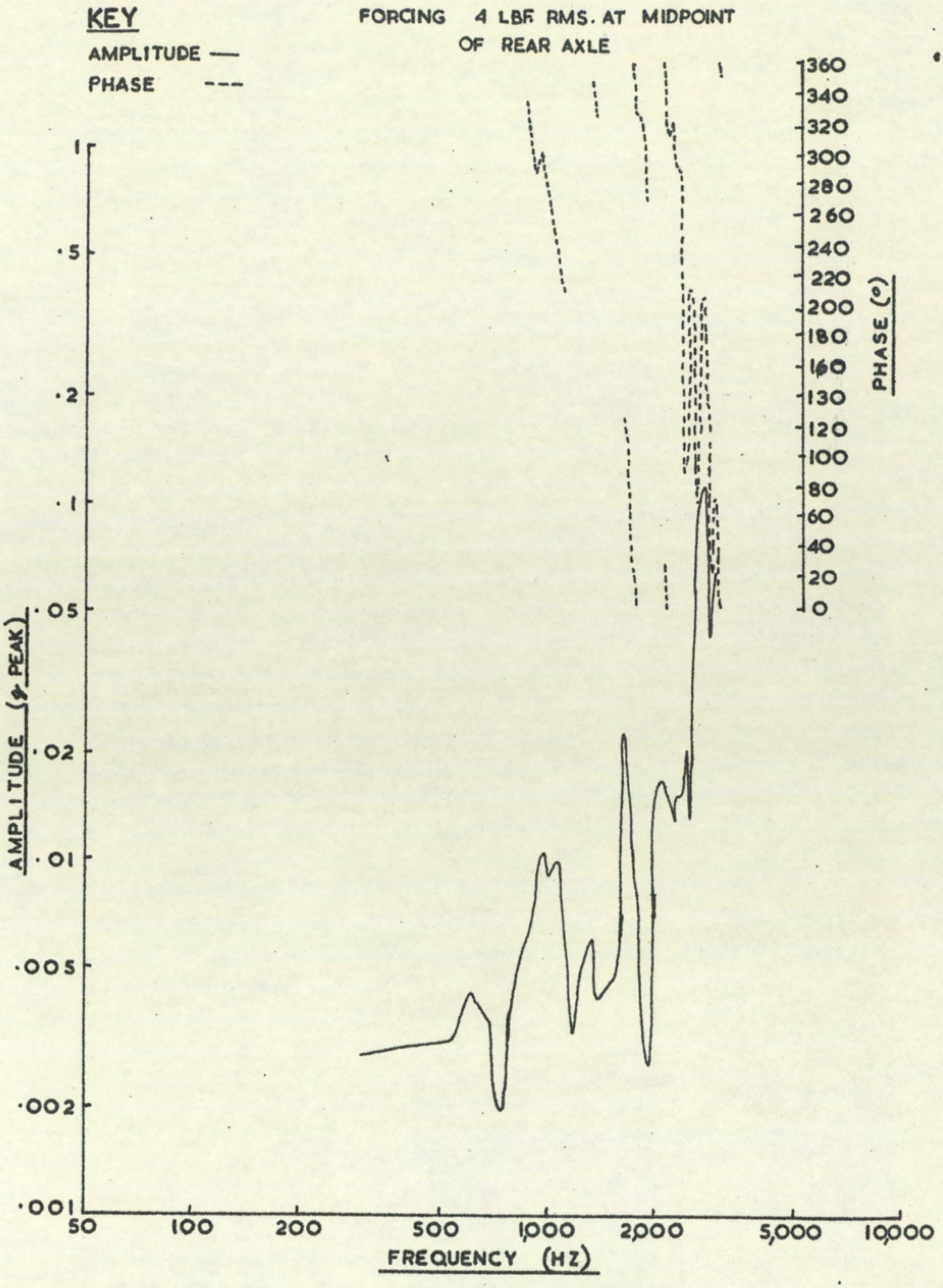
FIG 2-3

MEASUREMENT POSITIONS FOR RIG RESPONSE TESTS



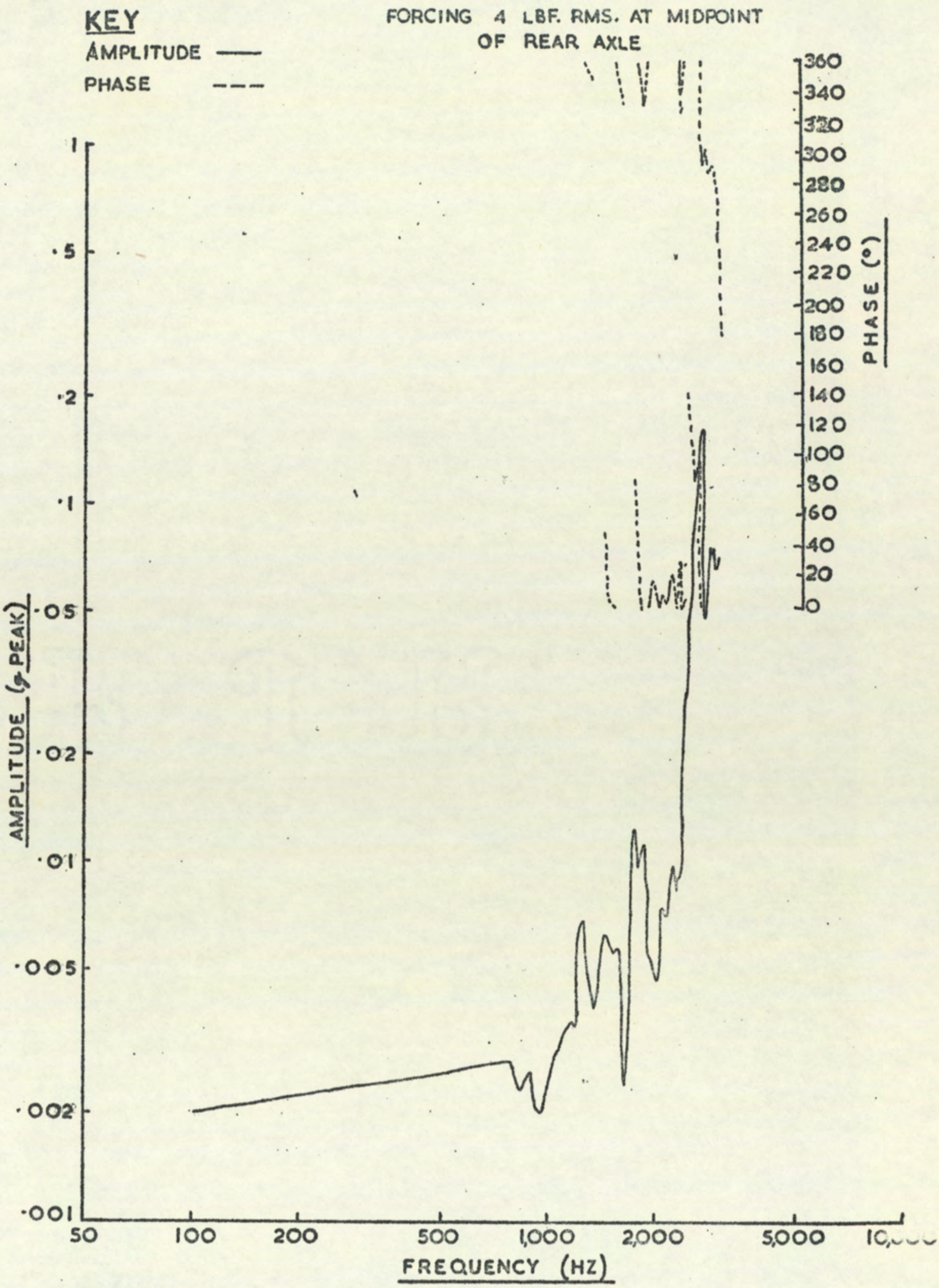
LOAD CELL FOR USE WITH SOUTHERN INSTRUMENTS FRQUENCY MODULATED DETECTION SYSTEM

FIG 2-4



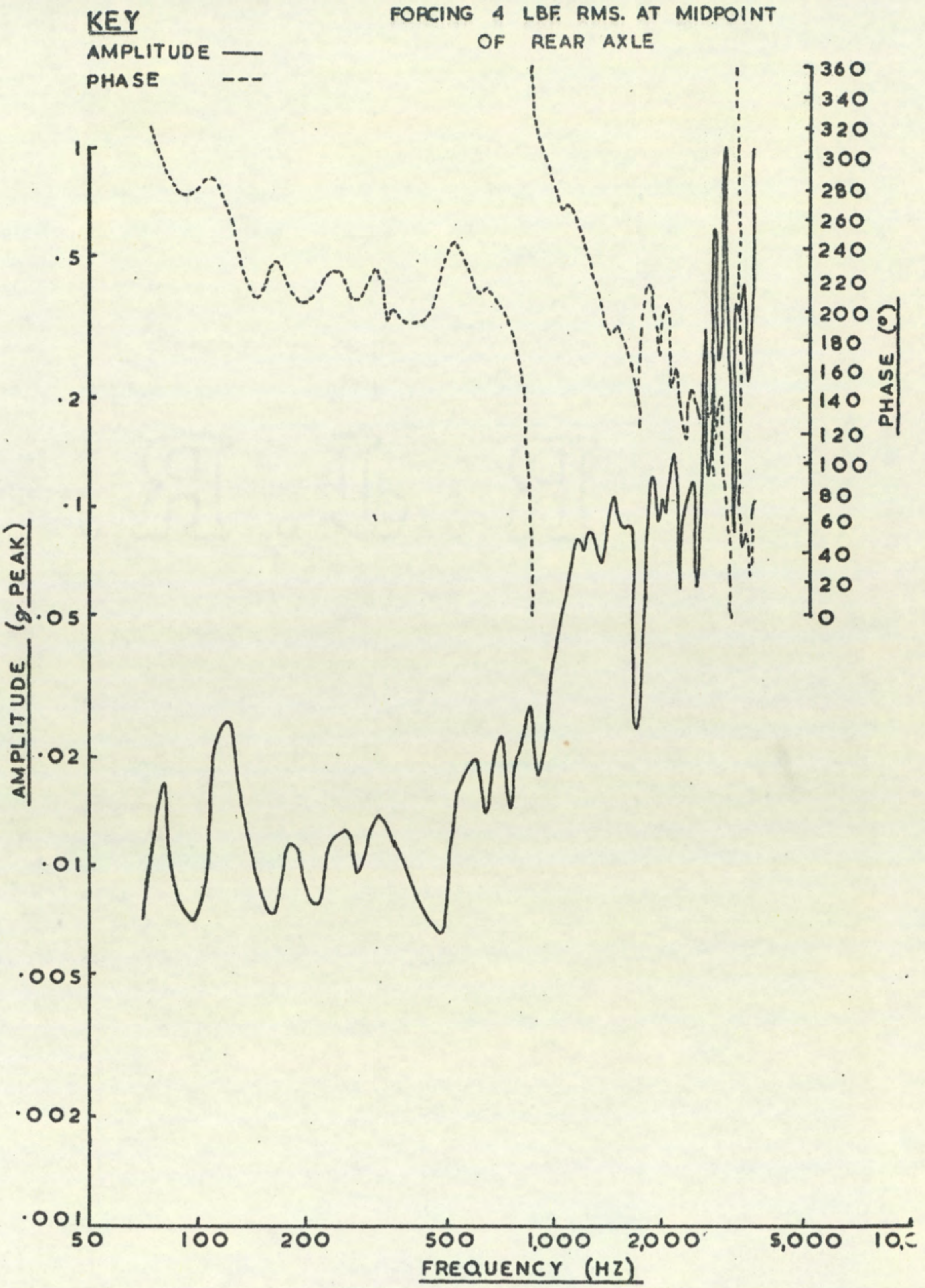
RESPONSE CURVE FOR POSITION No. 1.

FIG 2-5



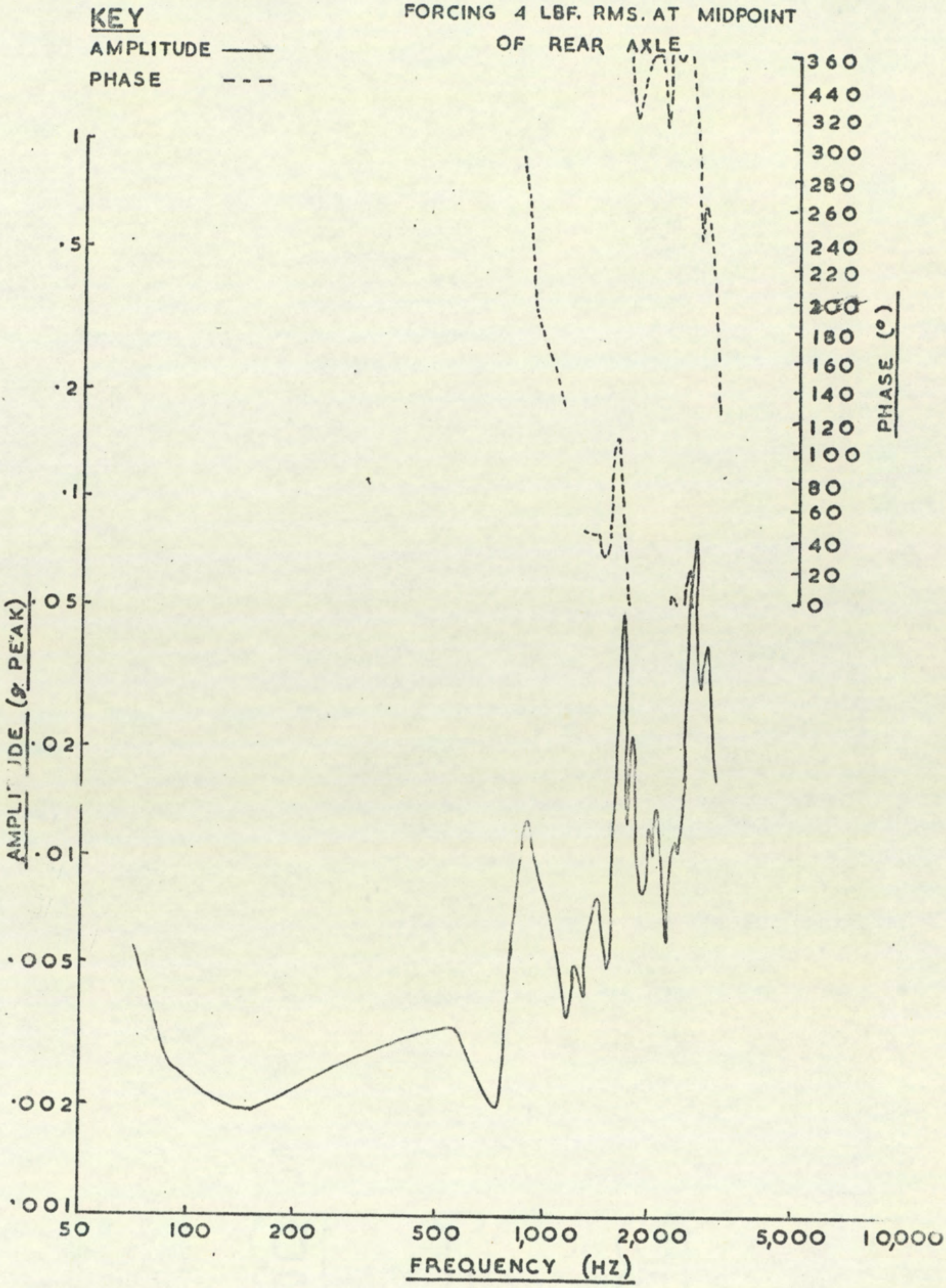
RESPONSE CURVE FOR POSITION No. 2.

FIG 2-6



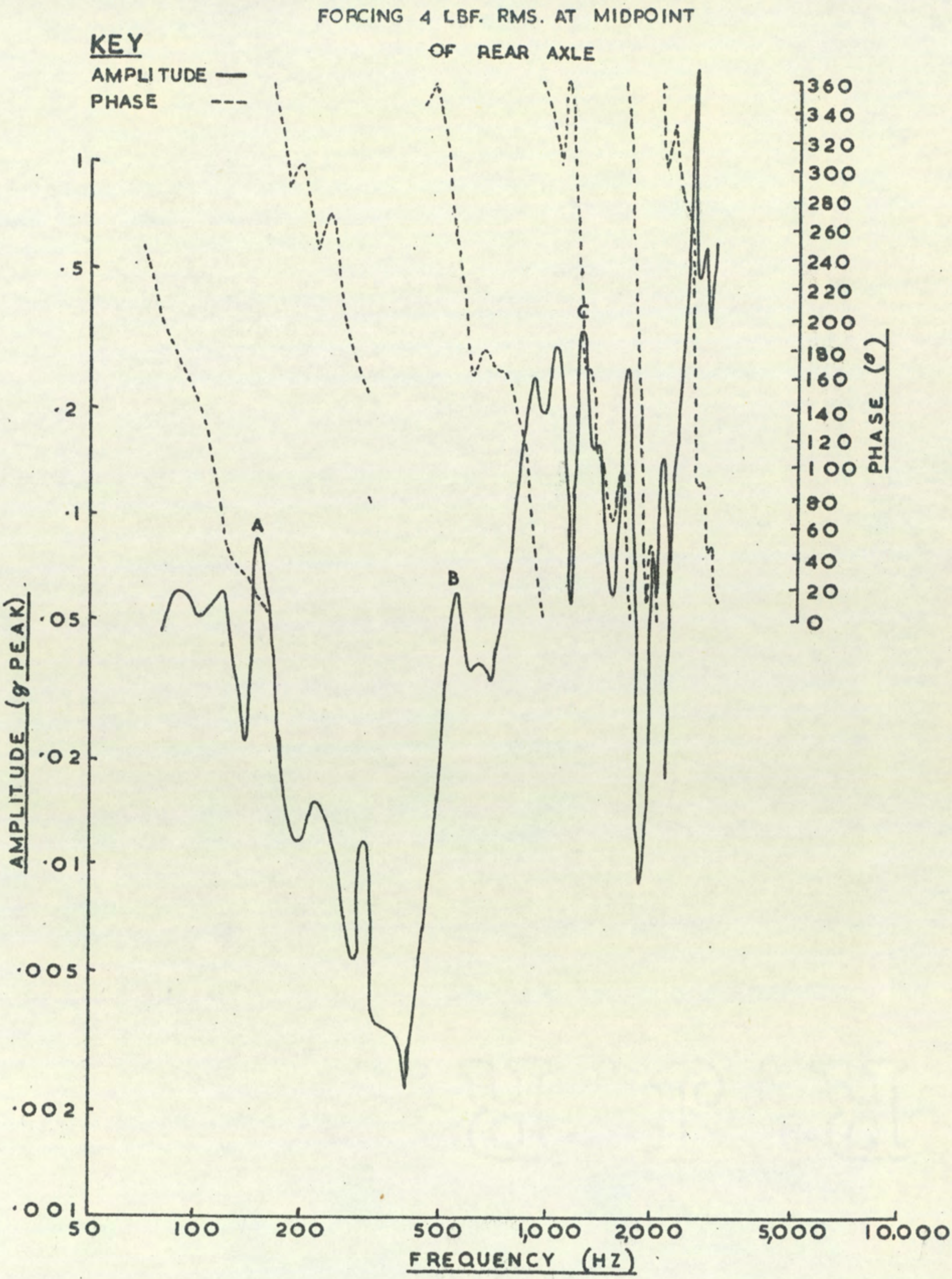
RESPONSE CURVE FOR POSITION No. 3.

FIG 2-7



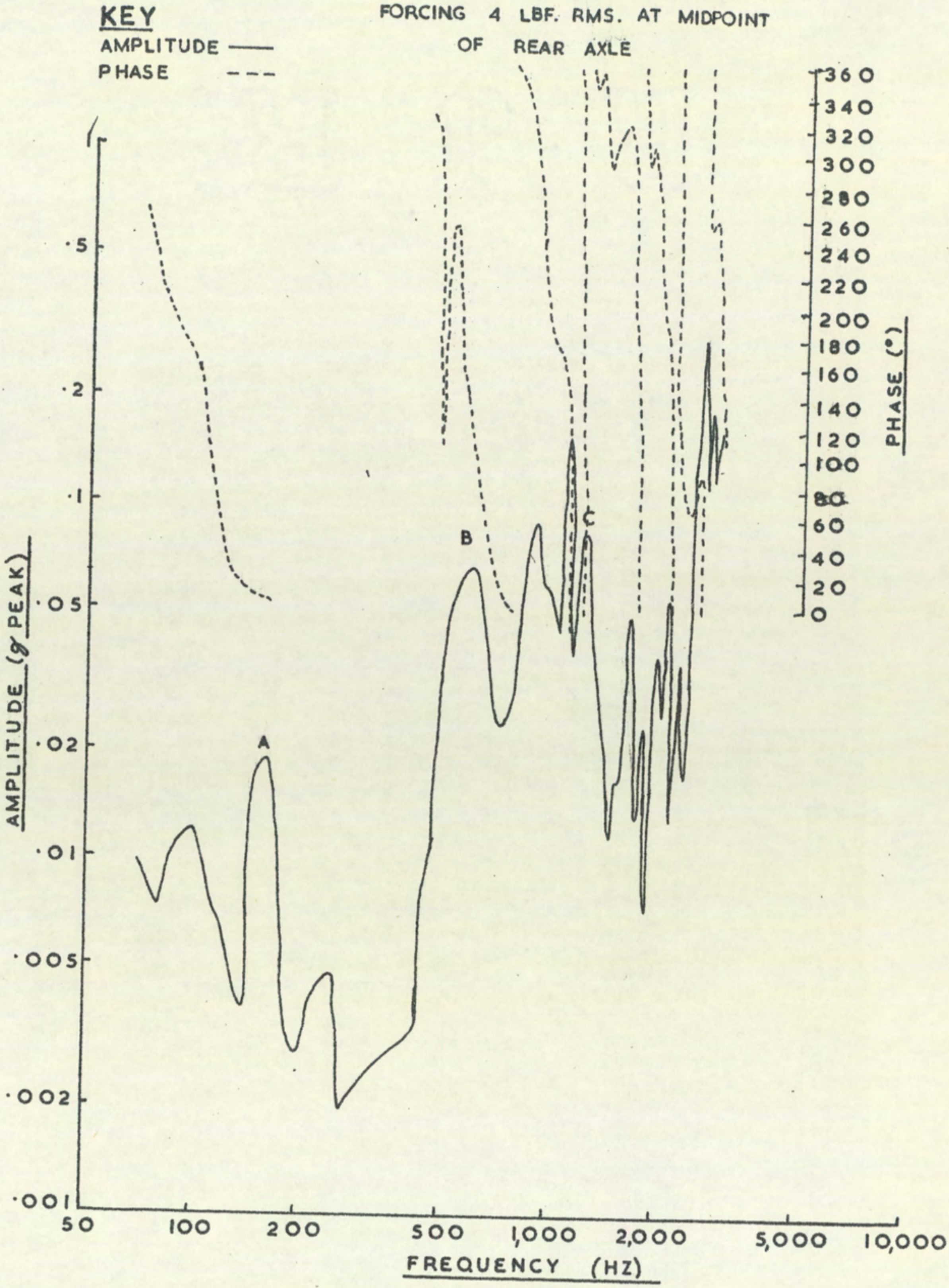
RESPONSE CURVE FOR POSITION No. 4.

FIG 2-8



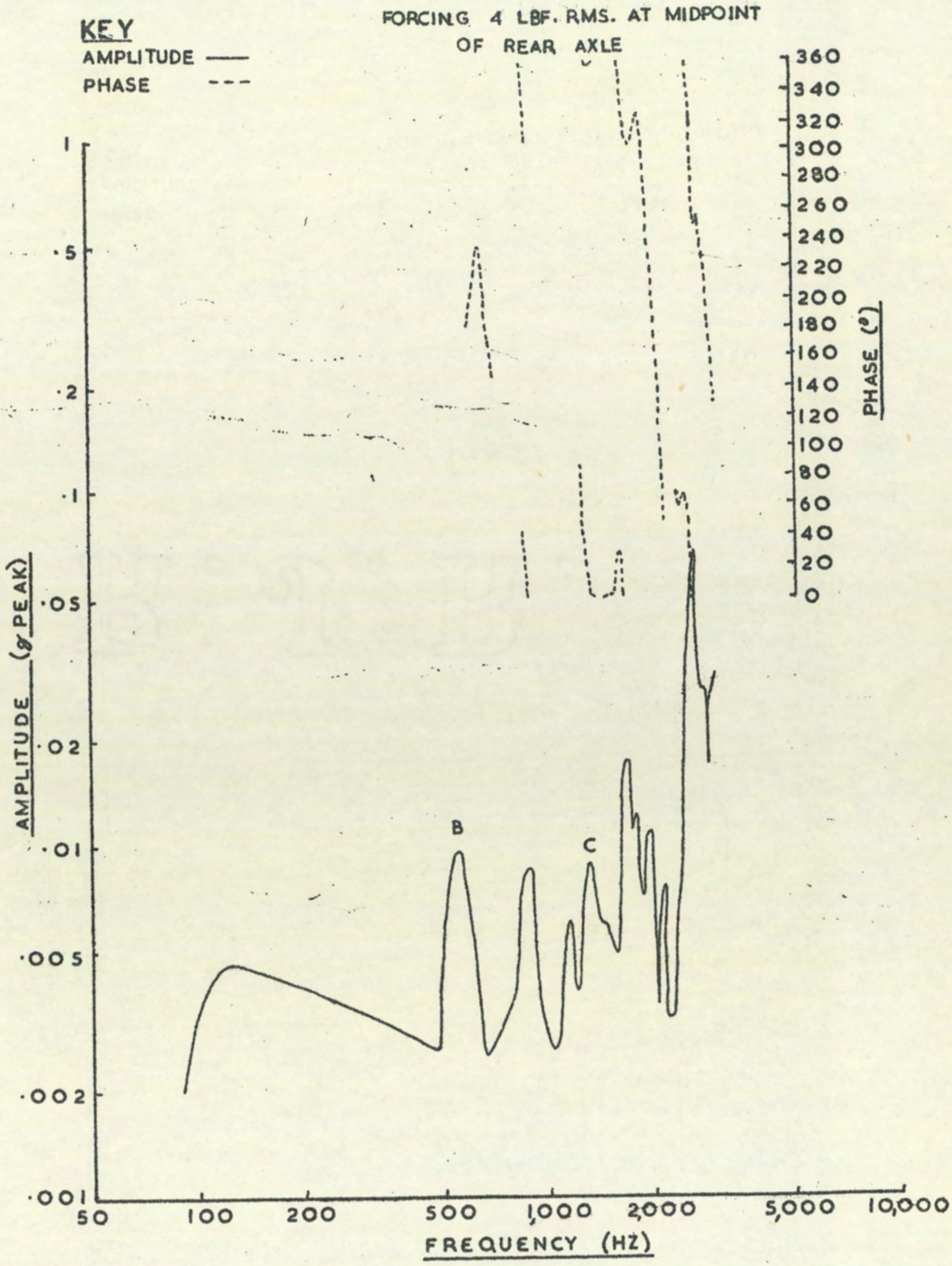
RESPONSE CURVE FOR POSITION No. 5.

FIG 2-9



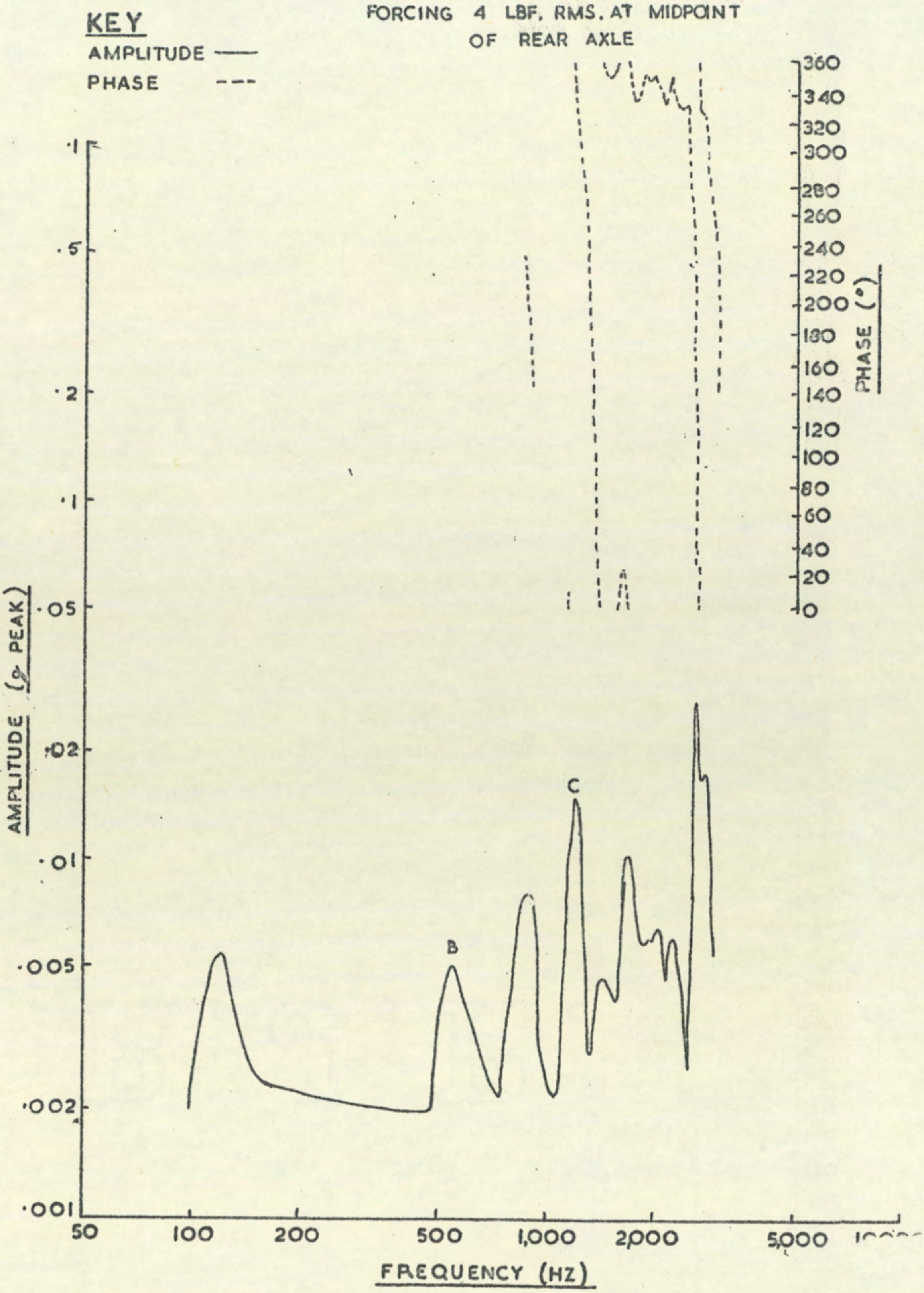
RESPONSE CURVE FOR POSITION No. 6.

FIG 2-10



RESPONSE CURVE FOR POSITION No. 7.

FIG 2-11



RESPONSE CURVE FOR POSITION No. 8.

FIG 2-12

FIG 2-13

ANALYSIS OF NOISE INSIDE REAR AXLE

FIG 2-14

ANALYSIS OF ACCELERATION OF CENTRE OF REAR AXLE

CHAPTER 3 . INFLUENCE OF THE PROPELLER SHAFT.

3-1) Summary

Tests are described which were designed to test the hypothesis, based on the preliminary investigations of Chapter 2, that:- the resonant response of the propeller shaft in its second bending mode of vibration was a prime cause of the audible rear axle noise in the passenger compartment. Both static resonance tests and road tests were performed, using five propeller shafts having different diameters.

It is shown that the results of the tests disproved the hypothesis, but they did indicate that an investigation of the possible mechanical transmission of a disturbance via the rear suspension system would be likely to yield more positive results. It is also shown that the assumptions made about the character of the road test data were incorrect, and that the analysis techniques employed were consequently not entirely satisfactory.

3-2) Details of experiments

i) Response tests of transmission systems.

All the response tests discussed in this chapter were performed on transmission systems fitted to a standard vehicle. The only parameter varied was the diameter of the propeller shaft which was chosen due to the relative ease with which it could be varied.

Initial response tests were on the transmission system fitted with a standard 2.5 in. diameter propeller shaft. The procedure employed for these response tests was basically the same as that used for the preliminary rig tests, detailed in Chapter 2 section 5. Modifications to the procedure were the omission of the phase measurement due to the difficulty of obtaining a consistent measurement, and the increase in the number of measurements taken, 1 Hz intervals from 70 to 2500 Hz. Instrumentation was the same as that used for the preliminary tests, Chapter 2 section 4, except that a greater number of measurement points were employed, as shown in fig. 3-1, and the change of the exciting point to the externally accessible end of the pinion shaft. This latter change was to obtain a more accurate representation of the excitation due to the meshing of the rear axle gears.

At this stage in the work an automatic response test instrumentation system was purchased, and this system, which is shown in block diagram form in fig. 3-2, was employed in the further tests of the transmission system with the non-standard propeller shafts fitted. This sytem consisted of a beat frequency oscillator supplying an electro-dynamic exciter via a 1 kilowatt power amplifier. The force output of the exciter was maintained constant by a feedback signal from the load cell to the oscillator ouput compressor circuit. A level recorder was used to record the outputs of the accelerometers, and was syncronized to the oscillator by virtue of the fact that the oscillator frequency sweep drive was provided by a mechanical coupling from the level recorder. Since only one channel was available on the level recorder, the outputs of the accelerometers were recorded in turn on successive frequency sweeps.

The main disadvantage of this automatic system was the lack of any means of filtering the accelerometer signals to ensure that only the fundamental response was being recorded. To overcome this, it was necessary to perform the tests at a low excitation level in order to reduce the harmonic excitation to a minimum.

For the tests of the transmission system with non-standard propeller shafts fitted, measurements were restricted to a reduced number of positions in order to reduce the time required for the tests. The positions at which measurements were taken were 1, 3, 9, 10, 12 and from an accelerometer mounted in the exciter coupling see fig 3-1.

Response tests were performed with this system on the transmission system fitted with propeller shafts having diameters of 2, 2.25, 2.5, 2.75 and 3 in.

An attempt was made to measure the acoustic response in the passenger compartment during these tests, but was unsuccessful due to the high background noise and lack of suitable filtering equipment.

ii) Road Tests

The road tests described in this section were carried out on the high speed circuit of the Motor Industry Research Association's proving ground. The vehicle used for these tests being the same vehicle used in the resonance tests described in the previous section.

Measurements were taken during the tests with the transducers listed below:-

i) Microphone in passenger compartment, suspended at ear level midway between and slightly behind the driver's and front passenger's heads.

ii) Microphone inserted through the propeller shaft tunnel at a point $\frac{1}{4}$ length of propeller shaft from the rear hookes joint.

iii) Microphone by rear axle gear carrier.

iv) Accelerometers at positions 1, 3, 9, 10 and 12 (see fig. 3-1)

The microphone in the passenger compartment was positioned as stated above, since this seemed to be a representative position at which to measure the noise audible to the occupants.

The microphone inserted through the propeller shaft tunnel was intended to measure the noise above the propeller shaft in order to test the hypothesis that if the propeller shaft was responsible for the amplification of the disturbance the noise would be transmitted to the passenger compartment by direct airborn noise. The positioning of this microphone was such that it was above an anti-node of the second mode of transverse vibration of the propeller shaft which was the mode of most interest in this part of the investigation.

The third microphone by the rear axle gear carrier was to detect if there was any direct airborn transmission from the gears.

Accelerometer positions 1 and 3 were chosen to give information about the behaviour of the propeller shaft, by inferance, since it was impossible to measure directly due to the rotation. The further positions 9, 10 and 12 were chosen to give information about the behaviour of the rear axle and to correspond to the positions used in the latter response tests.

Microphones were B and K condenser microphones, and were powered by battery packs. Accelerometers were Langham Thompson type XA2 piezo-crystal with battery cathode followers of our own design and construction.

Output signals from the eight transducers were recorded on eight channel magnetic tape by a battery powered tape recorder which employed a F. M. (frequency modulated) recording system. The F. M. recording system was chosen due to the inherent stability of the recorded signals. D.C. capability of the F.M. system was sacrificed in favour of an A.C. system with a bandwidth of 50 to 2500 Hz and an increased sensitivity suitable for the direct recording of the transducer signals. A further advantage of the high lower limiting frequency of the recording system was the filtering out of the low frequency, large amplitude components of the signals with a consequent gain in the sensitivity to the signals of interest.

A series of tests were performed with each of the five different propeller shafts fitted to the transmission system. Each series of tests consisted of constant speed runs at 5 mile/hr. intervals in the speed range 25 to 85 mile/hr. inclusive.

Wave analysis of the recordings was performed by copying the recordings onto multi-channel magnetic tape loops for replay into the automatic wave analysis equipment which is described in Chapter 4.

3-3) Discussion of results.

i) Response tests of transmission system with standard propeller shaft.

Response curves for all the measuring positions other than position 8 are shown in figs. 3-3 to 15. (No results were obtained for position 8 due to a circuit failure at an early stage in the tests).

The propeller shaft was confirmed to be the most active component by the much greater magnitude of the vibrations measured at positions 13 and 14 (figs. 3-14 and 15), compared to those measured at other positions.

The following natural frequencies were identified from the curves for the response of the propeller shaft in the vertical plane. (figs. 3-14 and 15).

1st	flexural	mode	of	propeller	shaft	150Hz
2nd	"	"	"	"	"	575Hz
3rd	"	"	"	"	"	1100Hz
4th	"	"	"	"	"	1800Hz

These frequencies agreed well with the values obtained from the preliminary response tests performed on the laboratory rig which gave 160, 550 and 1250 for the first three frequencies. The small differences which did occur were due to the change of environment of the transmission system and the fact that the components fitted to the vehicle were not the same as those used in the rig.

If the resonance of the propeller shaft in its second flexural mode was responsible for the audible rear axle gear meshing noise, then these results showed that the noise would be a maximum at a road speed of approximately 58 mile/hr. This did not agree with the subjectively assessed maximum noise which occurred at a speed of approximately 50 mile/hr. It was unlikely that the change in end conditions of the propeller shaft between the static and dynamic conditions would be of a sufficient magnitude to produce the required change in frequency of the second bending mode of vibration of the propeller shaft to make it resonate at 50 mile/hr., when forced by the rear axle gear meshing.

Response curves for the measuring positions on the rear axle showed a very complex response picture, and no particular resonances could be identified. There was a marked lack of symmetry in the response about the axis of the propeller shaft as is shown by fig. 3-16, which is a comparison of the vertical response at the two ends of the rear axle. There were two reasons for this lack of symmetry:- the complex input output mechanisms of the internal components of the rear axle, and the fact that the mass of the axle was not evenly distributed about the axis of the propeller shaft. The first of these factors would almost certainly be the major factor in producing the asymmetric response, since the mass was not excessively asymmetric, and would have its major effect in the lower frequency ranges. A further factor which might have influenced the symmetry of the response was the leaf springs which may have had slightly different frequency responses and damping characteristics.

ii) Response tests with non standard propeller shafts fitted to the transmission system.

The response curves obtained from these tests are shown in figs. 3-17 to 22. Amplitudes are plotted on a Db scale relative to 0 Db = 0.01 g peak and the curves are displaced upwards by 15 Db for each increase in propeller shaft diameter.

Influence of the propeller shafts on these responses was identified by the peaks labled A, B and C which were due to the resonant response of the propeller shafts in their first three bending modes. These frequencies together with the frequencies of the other major peaks in the response curves, are plotted against propeller shaft diameter in figs. 3-23 to 28.

It can be shown that if the propeller shaft is treated as a simple pinned-pinned beam, then the natural frequencies will be proportional to the change in diameter for small changes of diameter. This simple theory was not supported by the result obtained from the response tests. The main reason for the deviation from the simple theory was the influence on the propeller shaft of the flexabilities of the rear axle and gear box. These deviations would be particularly marked where natural frequencies of modes of the rear axle or gear box existed close to the propeller shaft frequencies.

The propeller shaft also influenced the response of the rear axle at frequencies other than the natural frequencies of the propeller shafts as can be seen by the effects on the two peaks in the response curves in the region of 125 Hz. The interaction of the various modes of the propeller shaft and the rear axle will be discussed further in chapter 5, which deals with the theoretical investigations of the transmission system.

A considerable shift in the second natural frequency of the propeller shaft was achieved by the variation of diameter. Total range of the frequency shift was 520 to 700 Hz and this was considered to be more than sufficient to be detectable in the noise in the passenger compartment during the road tests.

In the frequency range containing the second natural frequency of the propeller shafts the response at the spring axle interface was in all cases very small, fig. 3-18. This strengthened the hypothesis drawn from the preliminary survey that if the audible rear axle gear meshing noise was a function of the resonance of the propeller shaft in its second bending mode, then the transmission of the disturbance to the passenger compartment would be by direct airborn radiation from the propeller shaft. It was noted that in all the response curves for the spring axle interface, there was a small peak in the response at a frequency slightly below 500 Hz which was a frequency more consistant with the reported speed at which the maximum audible rear axle gear meshing noise was heard. It therefore seemed possible, that there was a mechanical transmission of the gear meshing frequency disturbance at 50 mile/hr. which was independent of the behaviour of the propeller shaft. It was hoped that the road test results would clarify which of these two possible transmission paths was the dominant one.

iii) Road test results.

Magnetic tape recordings obtained from the road tests were first analysed on the assumption that the data would be complex periodic with a small random content. A wave analysis system which is discussed fully in Chapter 4 was used to obtain the wave analyses of the recordings with the following settings on the B and K level recorder.

Dynamic range	50 Db
Lower limiting frequency	20 Hz
Range	50 Db
Writing speed	16 mm/s
Paper speed	0.3 mm/s

From Ref. 10 these settings were found to give an averaging time for the output of 1.6 s. Two typical output traces are shown in figs. 3-29 and 30.

The traces from this first analysis were found to be spikey and consequently difficult to interpret. Two factors contributed to these poor results, the low averaging time, and the long data sample. This combination would have been satisfactory if the data had been stationary periodic as assumed.

In an attempt to improve the confidence in the results the same magnetic tape recordings were re-analysed using the following settings of the level recorder:-

Dynamic range	50 Db
Lower limiting frequency	20 Hz
Range	10 Hz
Writing speed	2 mm/s
Paper Speed	0.3 mm/s

These gave an effective averaging time of 2.5 s and the data sample was also reduced to 2.5 s to be compatable with this averaging time. The traces obtained from this second analysis were much smoother as is shown in figs. 3-29 and 30. This improvement in the wave analysis traces showed that the initial assumption of stationary complex periodic signals was incorrect, and that the signals were in fact non-stationary, but could be treated as stationary over short time samples.

The first set of wave analysis traces were reduced to a usable form as follows. Four frequency ranges were chosen, 100 - 200, 200 - 400, 400 - 800 and 800 - 2500 Hz, and in each range the mean lower level (or approximate signal noise level) was noted together with the frequency and amplitude of any peaks in the trace.

The data obtained from this first method of reduction was plotted against road speed as is shown in the examples presented in fig. 3-31 to 34.

The most obvious characteristic of all these plots was the increase of amplitudes with increase in speed. This was expected since the power input to the vehicle in all forms was expected to increase with speed and it was reasonable to assume that the total power would be distributed in a similar manner, with respect to frequency at all speeds.

In general most of the low frequency peaks in the analyses were identifiable in terms of harmonics of engine rotational frequency or resonant frequencies of components. Repeatability of both frequency and amplitude measurements were quite good in the **two** lower frequency ranges showing that the signals most probably were complex periodic as originally expected. Repeatability was not so good in the higher frequency bands due to either the poor analysis techniques, or incorrect assumption about the form of the signals, or a combination of both these factors.

The analyses of the noise in the passenger compartment did not show conclusively that rear axle gear meshing noise was present. A peak at rear axle gear meshing frequency only occurring in three of the five sets wave analyses, and then it was not a very significant peak. No evidence of the propeller shaft influencing these rear axle gear meshing frequency components of the noise was found. These components may be observed fig. 3-34 where the frequency of the rear axle gears meshing is given in Hz by approximately 10 times the road speed in mile/hr.

A major factor which accounted for the lack of detection of the rear axle gear meshing frequency components of the noise in the passenger compartment, was the position of the microphone. The microphone was placed almost exactly in the centre of the compartment where it would have only detected activity in one eighth of the acoustic modes of the enclosure. The correct position for the microphone was in one of the corners of the compartment where all the modes of the enclosure have pressure anti-nodes.

Other factors which contributed to the failure to detect the rear axle gear meshing frequency components were the large speed increments used (5 mile/hr.), which could have resulted in a high Q resonance being straddled, and the rather broad bandwidth of the analyser resulting in the signal of interest being buried in noise.

As was stated to be the general case, the lower frequency peaks in the passenger compartment noise analyses, were due to harmonics of engine speed, and were unaffected in respect to frequency and amplitude by the changes of the propeller shaft.

The amplitues of peaks in the analyses of the noise in the passenger compartment in the upper frequency bands showed variations from test to test, of as much as 10 Db, but not in any particular pattern which would establish the influence of the propeller shaft. These variations would have been reasonable if the noises were due to external influences such as wind noise which would require identical test conditions for a reasonable repeatability. Alternatively, the noises could have been due to high Q resonances which would require the test speeds to be exactly equal for repeatability.

Noise above the propeller shaft was of a greater magnitude than the noise in the passenger compartment, and the trend of the sound pressure level was more linear with the increase of speed. A more detailed comparison of the noise with the noise in the passenger compartment showed that the attenuation of the floor of the vehicle was good, and that the noise transmitted through the floor formed only a small component of the total noise in the passenger compartment.

There was a well defined peak in the analyses of the noise above the propeller shaft at the frequencies associated with the vibration of the propeller shafts in their second mode of flexural vibration. These peaks existed at all speed, and did not show any marked increase in amplitude when the rear axle gear meshing frequency was equal to the natural frequencies. The frequencies obtained for the natural frequencies of the second mode of flexural vibration of the propeller shafts, were slightly lower than those obtained from the response tests due to the change of end conditions resulting from the rotation of the hookes joints.

The only component of the noise above the propeller shaft in the region of 500 Hz which showed any predominance was a peak in the analyses at 500 Hz at 70 mile/hr. but this was not related to rear axle gear meshing noise, since it was well below the gear meshing frequency at this speed.

Results obtained from the microphone near to the rear axle gear carrier were of a much greater level than those obtained from either of the other microphones. This was due to the exposed position of the microphone resulting in it recording mainly wind noise, which being random in nature, tended to swamp the noises of interest and hence nulify the measurements taken with this transducer.

Analysis of the accelerations measured at the measuring positions on the rear axle i.e. positions 1, 3 and 9, showed that the discreet frequency components were of a small magnitude throughout the frequency range considered. In general the lower frequency components were the largest in amplitude, but not to such a marked extent as in noise analyses.

In relation to the rear axle gear meshing noise problem, there was a component of the vibration at the meshing frequency which reached a maximum at 45 mile/hr. approximately. This component was detected in the signals from each of the three positions, but was particularly marked in the vibration of the rear axle rear spring interface (position 1 fig. 3-31). The frequency and speed at which the maximum amplitude occurred were independent of the change of propeller shaft.

Comparisons of the amplitudes of the gear meshing frequency component in the vibration of the rear axle at 40 and 50 mile/hr. indicated that the absolute maximum of this components amplitude occurred between 45 and 50 mile/hr. This fact, combined with the frequency of the vibration of 440 Hz approximately agreed much more closely with the subjective assessments of the rear axle noise than did the natural frequencies of the second mode of flexure of the propeller shafts. These results indicate that there was a high probability that the audible noise was due to the mechanical coupling of the rear axle to the body via the rear springs, and not due to direct airborn transmission from the propeller shaft.

None of the peaks which occurred in all three analyses of the vibrations of the rear axle showed any variation of amplitude between the measuring positions of a sufficient magnitude for identification of modal shapes. Lack of resolution of the analyser was the most likely cause of this shortcoming of the results, although if the vibrations were random in nature, the unsuitable analysis techniques could have caused this.

Since the vibration of the propeller shaft could not be measured directly due to its rotation, it was necessary to infer its behaviour from the measurements at positions 11 and 12. The vibration at these two points were of the order of 5 times greater in magnitude than those measured on the rear axle. This gave strong support to the hypothysis that the propeller shaft was the least damped component of the transmission system.

No evidence of coupling between the rear axle gear meshing forcing and the second mode of flexural vibration of the propeller shaft was found in the results obtained from position 11. This may have been due to the measuring position being close to a nodal point of the mode of vibration, but was more likely due to the good isolation of the propeller shaft from the pinion shaft by the hookes joint.

None of the vibration measurements showed any signs of local resonances which could have been amplifying factor in the generation of the audible noise, other than the response of the rear axle at 440 Hz.

The results obtained from this analysis and interpretation technique, showed poor repeatability, and not much confidence could be placed in them. It was, therefore, decided to re-analyse the original recordings with improved analysis parameters. This was indicated earlier in the discussion.

A different technique of reducing the initial wave analysis traces was employed in order to increase the confidence in the results, and also to speed up the reduction. Only the amplitues and frequency of peaks in the wave analysis were considered in this reduction, since it was realised at this stage that the base level of the traces was strongly influenced by the poor filter shape, and was not representative of the true data. The amplitudes and frequencies of the peaks were tabulated and copied on to puched tape for computer processing.

The first stage of the processing was to correct the amplitues according to the frequency response characteristics of the magnetic tape equipment, and to convert to g levels in the case of the acceleration results. The second part of the processing programme was used to plot frequency v speed on a log-log scale on the "on line" digital plotter. (An example of the frequency v road speed plots obtained is shown in fig. 3-36.)

From the frequency v road speed plots it was possible to decide which of the peaks in the wave analyses had occurred at constant frequencies irrespective of road speed and which had occurred at multiples of engine speed. This information was used to control a sorting programme which obtained from the ouput of the first programme lists of the amplitudes for the constant frequency peaks, and for the constant multiples of engine frequency peaks. A selection of the curves plotted from these lists are presented in figs. 3-37 to 44. In the following discussion, the constant frequency resultsare dealt with first, and then the multiples of engine speed results.

Noise in the passenger compartment exhibited six major frequencies at which peaks occurred in the wave analyses for a large number of conditions. These frequencies were:-

79 Hz

215 Hz

307 Hz

417 Hz

501 Hz

682 Hz

Of the amplitude v speed curves for these frequencies only those for the 79 Hz frequency showed a distinct maximum which occurred at 40 mile/hr. approximately. This frequency was the fundamental acoustic resonant frequency of the enclosure and the maximum response was due to the engine firing disturbance.

Since the maximum audible rear axle gear meshing noise was reported to occur at 50 mile/hr. it should have been detected as a maximum in the 501 Hz amplitude v speed curves. There was no evidence of such a condition existing in any of these curves, probably due to the poor positioning of the microphone as previously mentioned. None of the other curves showed any evidence of any response to rear axle gear meshing disturbance.

The major constant frequency components occurring in the analyses of the noise above the propeller shaft were:-

75 Hz

217 Hz

491 Hz

848 Hz

974 Hz

together with a series of constant frequency components which varied with propeller shaft diameter as follows:

P.S. DIA.	Freq.
2.00 in	500.8 Hz
2.25 in	549.6 Hz

P.S. DIA.	FREQ.
2.50 in	577.3 Hz
2.75 in	642.0 Hz
3.00 in	669.3 Hz

These latter frequencies gave a slightly different line for the variation of the second flexural resonant frequency of the propeller shafts to those obtained from the response tests and the first analysis of the road tests, see fig. 3-36.

None of the amplitude v speed curves for these constant frequency components of the noise above the propeller shaft showed any large increase in amplitude from their general trend as would be expected if a resonant condition existed. This lack of any resonant condition was also the case for the second node of vibration of the propeller shaft as was evidenced by the sound pressure level at the appropriate frequencies being nearly linear with speed and relatively unaffected by the variation of propeller shaft.

Sound pressure levels above the propeller shaft were considerably higher than those in the passenger compartment at equivalent frequencies, and the major constant frequency components of the two noises were not all the same. These two results showed that the acoustic isolation of the passenger compartment from the below floor noises was good. It appeared from this result that the airborn transmission of the rear axle gear noise direct from the transmission system was negligible.

As was stated in the discussion of the first analysis, the measurements taken with the microphone close to the rear axle gear carrier were negated by the excessive amount of wind noise.

Only two constant frequency components worthy of note occurred in the analyses of the vibrations of the rear spring-rear axle interface (position 1) 435 and 940 Hz. In two of the five cases, the curves of amplitude v speed showed a sharp increase at 45 mile/hr., i.e. when the rear axle gear meshing frequency was approximately equal to 435 Hz. As was stated earlier this was considered to be the most significant component of the vibration of the rear axle in relation to the audible rear axle gear meshing noise.

Three major constant frequencies were detected in the results for positions 3 and 9, they were 70 , 437 and 934 Hz. The 70 Hz components did not show any distinct maxima, but the 435 and 934 Hz components both showed maxima in the region of 45 to 50 mile/hr. Comparison of the levels obtained at the 437 and 934 Hz frequencies with those obtained at position 1 for the corresponding components showed good agreement with the results of the response tests at those frequencies.

The characteristics of the vibration at the nose of the rear axle gear carrier in terms of the constant frequency components were quite different from those of the rear axle proper, 215 Hz and the second flexural frequencies of the propeller shaft being the major components. The 215 Hz components were much greater in amplitude than the response of the rear axle at similar frequency (20 Db up) and there were maxima in these components at approximately 60 mile/hr.

There was some evidence of the rear axle gear meshing causing resonance of the propeller shafts in their second flexural modes, but in all cases these were occurring above the suspect speed of 50 mile/hr.

Only one major constant frequency component was detected in the vibration of the end of the gear box extension 626 Hz, and the amplitudes of these components did not show any maxima but only a steady increase with speed.

Predominant constant engine order components in the noise in the passenger compartment were first and second both at approximately 90 Db sound pressure level. Some evidence of tenth engine order rear axle rear meshing frequency components was found, but no maxima were found in the corresponding s.p.l. v speed curves.

In the noise over the propeller shaft, the major constant engine order components detected were first, second and fourth. Sound pressure level for these components was about 100 Db showing that the passenger compartment acoustic treatment was not so good at these lower frequencies. Changing the propeller shaft did not have any measurable effect on these results, and no evidence was found of the tenth engine order harmonic being present.

The vibration of the rear axle showed similar constant engine order components at all three measuring positions. Major orders detected were first, second and fourth with measurable components also occurring at 1.2, 1.4, 1.6, 2.5 and 3 times engine speed. These strange order components were presumably due to the rotation of the half shafts and the differential gears. Rear axle gear meshing frequency (tenth engine order) components were only detected in the vibration at position 9 which was the nearest to the gears. There was resonant peaking in these tenth order components in the region of 45 to 50 mile/hr., but only to the extent of two to three Db and independent of the size of propeller shaft fitted.

Constant engine order components of the vibration of the nose of the rear axle gear carrier were similar to the rear axle vibration components but 10 to 15 Db greater in amplitude. There was some influence of the propeller shafts on the amplitudes of the lower orders, but not on the tenth engine order which was not a well defined component.

As was expected, the highest vibration levels at constant engine order frequencies were measured at the end of the gear box extension. The first, second and fourth order components were the strongest due to the direct mechanical connection of this measuring point to the engine block.

In order to establish which frequencies and constant engine orders were predominant but independent of the change of propeller shaft, a further quick analysis of the data from the second wave analysis was performed. For each of the transducers the frequencies which lay within 10% of each other in the five tests with the different propeller shafts were averaged to give a mean frequency independent of the propeller shaft for each speed. The results of this data reduction are shown in figs. 3-45 to 52 on which the major frequencies and engine orders detected are shown.

For all the transducers the constant engine order components were predominating characteristics with a small number of constant frequency components. The strength of these constant engine order components was such as to make the interpretation of the constant frequency components in the lower frequency region almost impossible.

Noise in the passenger compartment revealed four constant frequency components by this reduction, i.e. 125, 215, 305 and 500 Hz. The 500 Hz component was only consistently detected at 65 mile/hr., though there was evidence of it being associated with the 11th order component at 50 mile/hr. There was no evidence of a rear axle gear meshing frequency component and only slight evidence of a 440 Hz constant frequency component which had been indicated to be the frequency at which the audible rear axle gear noise would be a maximum.

No constant frequency components were detected in the noise above the propeller shaft, or in the noise near the rear axle gear carrier. At neither of these positions did the noise have any rear axle gear meshing frequency components which added further strength to the hypothysis that the audible noise was mechanically transmitted to the body.

Strong evidence of the mechanical transmission of the rear axle gear meshing vibration to the body of the vehicle was given by the fairly well defined tenth engine order component and the 440 Hz constant frequency component in the vibration of the spring axle interface. These two components coincided at approximately 48 mile/hr. which could have accounted for the lack of a measurable component in the passenger compartment, since this speed lay between two of the test speeds. These results also showed some evidence of a constant frequency component at 950 Hz and a twentieth engine order component which indicated that the first harmonic of rear axle gear meshing frequency might also be important in the audible noise in the passenger compartment.

Further support for the mechanical transmission of the vibration was found in the well defined tenth engine order component of the vibrations at the other two positions on the rear axle. However, neither of these positions showed the 440 Hz constant frequency component in their vibrations, and it was considered that this component was due to the springs.

The components of the vibrations at measuring positions 11 and 12 were all low engine order components, and were unimportant in relation to the rear axle gear noise problem.

3-4) Conclusions

The tests described in this chapter showed conclusively that the hypothesis that the resonant response of the propeller shaft in its second bending mode of vibration was a prime cause of the audible rear axle noise was incorrect. The reasons for this conclusion were:

a) The frequency at which the second mode of bending vibration was excited was found to be well above the rear axle gear meshing frequency at the suspect road speed of 50 mile/hr.

b) The variation of propeller shaft diameter produced no measurable effect on the noise in the passenger compartment.

c) The secondary hypothesis that the noise would be transmitted by direct airborn transmission from the propeller shaft was not proved, and was further more shown to be unlikely.

There was strong evidence, both from the static response tests and from the road tests that the audible noise was in fact due to the mechanical transmission, of a disturbance centred at a frequency of 440 Hz, via the rear suspension system to the body of the vehicle. The road test results also showed that the maximum noise was likely to occur at a road speed of 48 mile/hr., and that it was likely to be associated with a high Q resonance.

The poor quality of the results obtained from the road tests showed that greater attention to test technique was required for the actual tests. The results also showed that the initial assumptions about the form of the data were wrong, and indicated that:-

a) the data was in fact, random, and not complex periodic.

b) the wave analysis techniques employed for the data reduction were not the most efficient techniques for analysing this form of data.

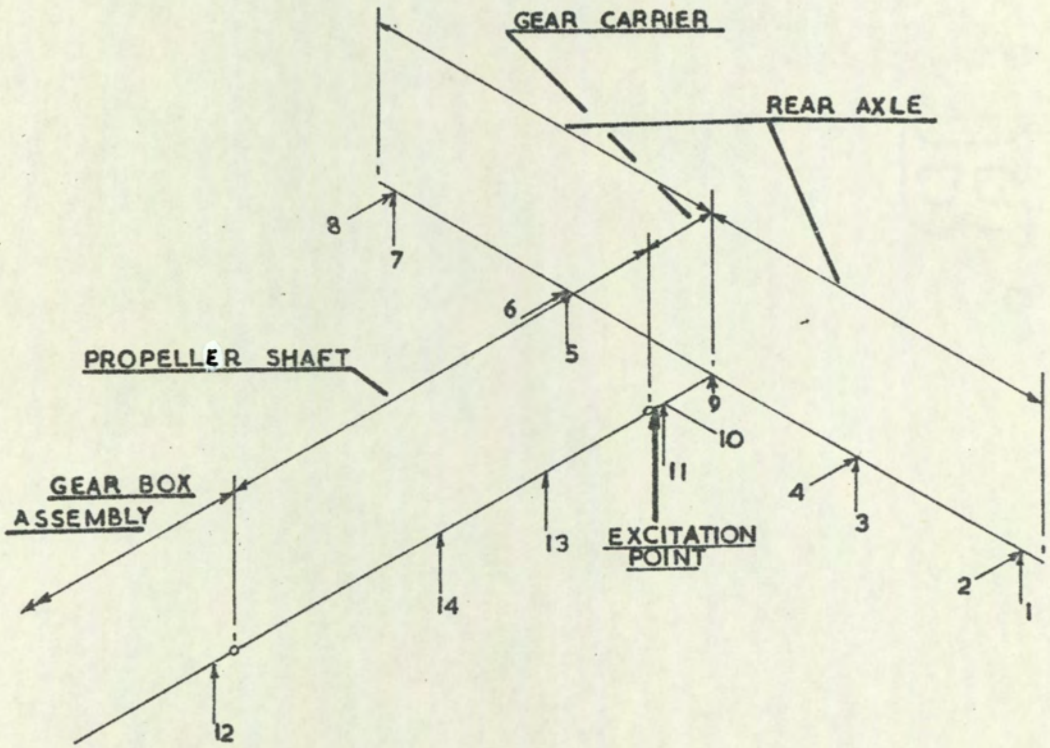
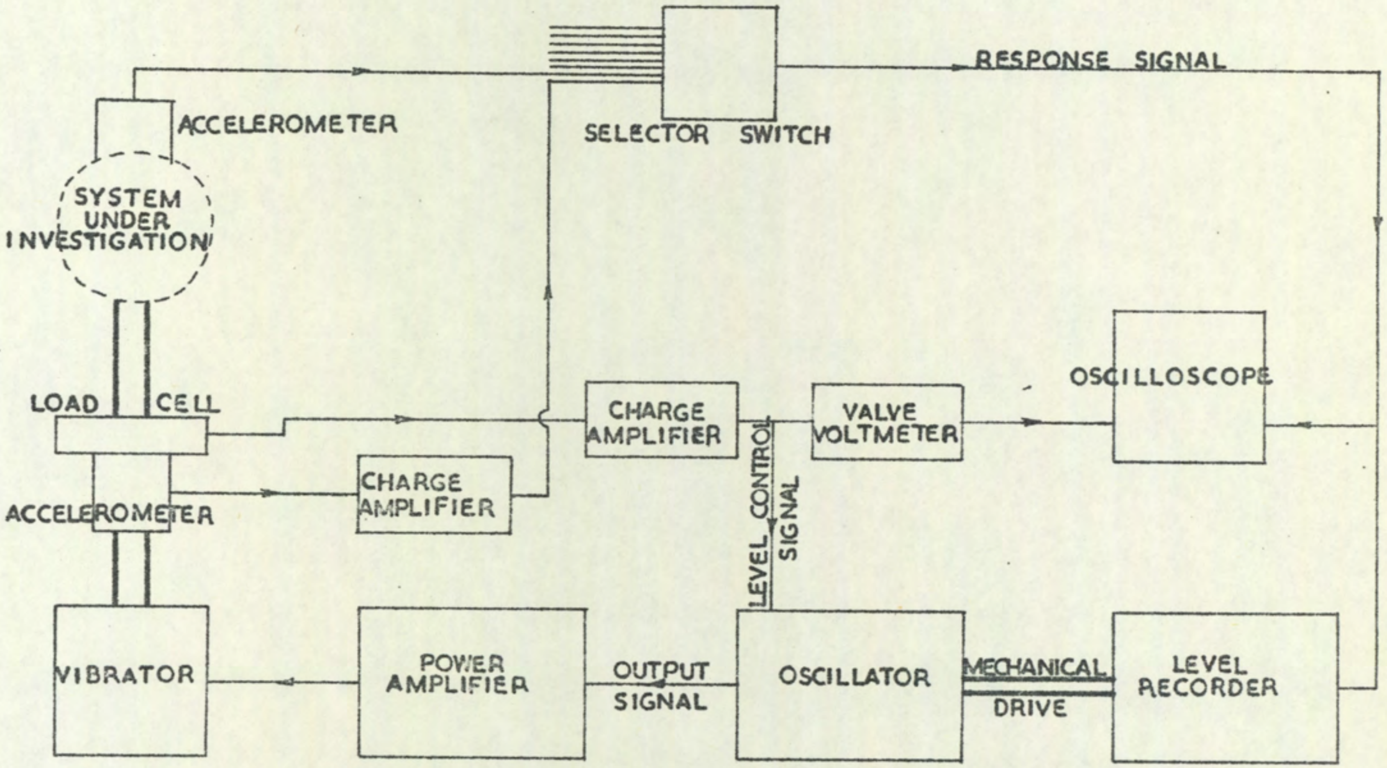


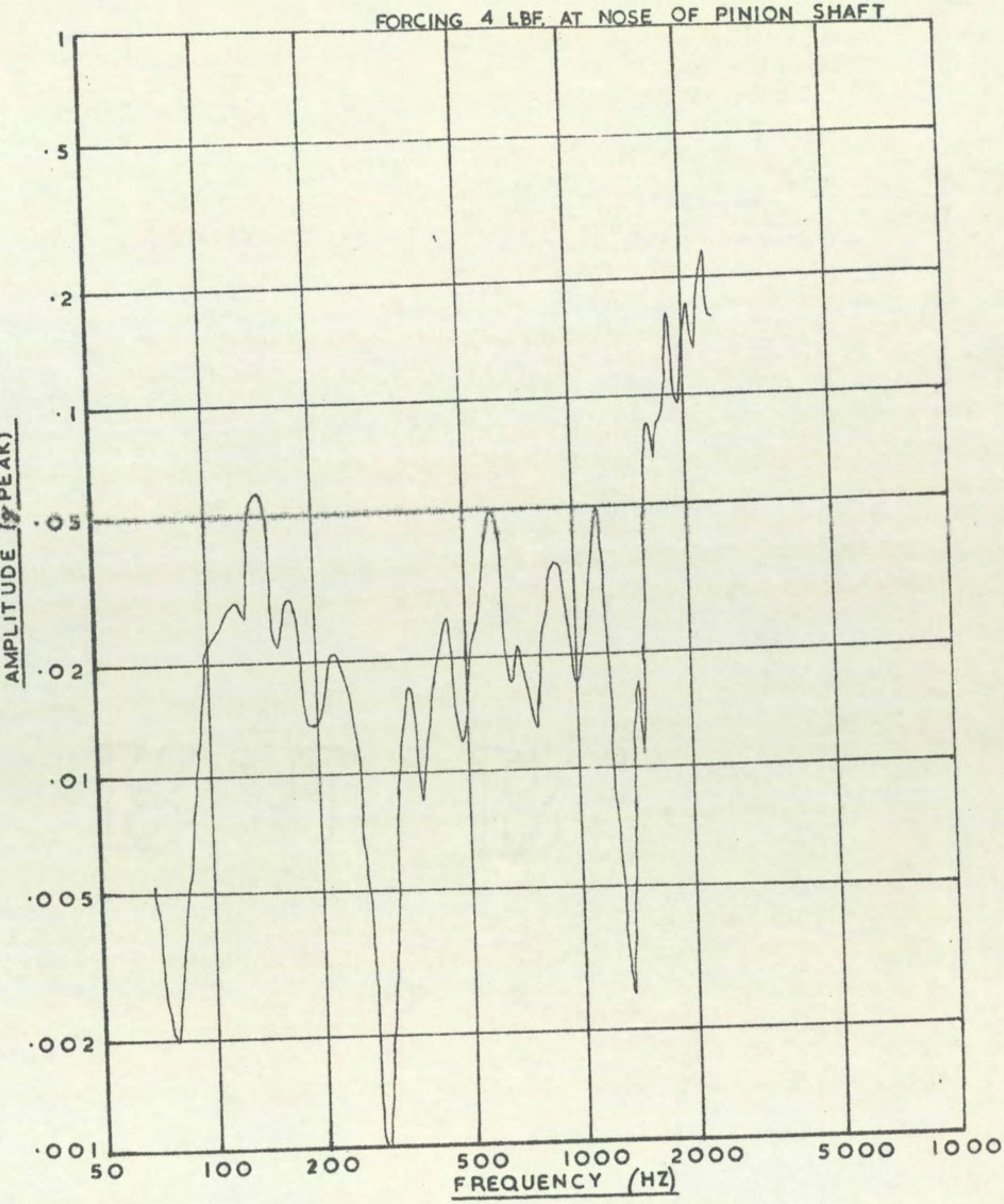
FIG 3-1.

MEASUREMENT POSITIONS FOR RESPONSE TESTS AND ROAD TESTS



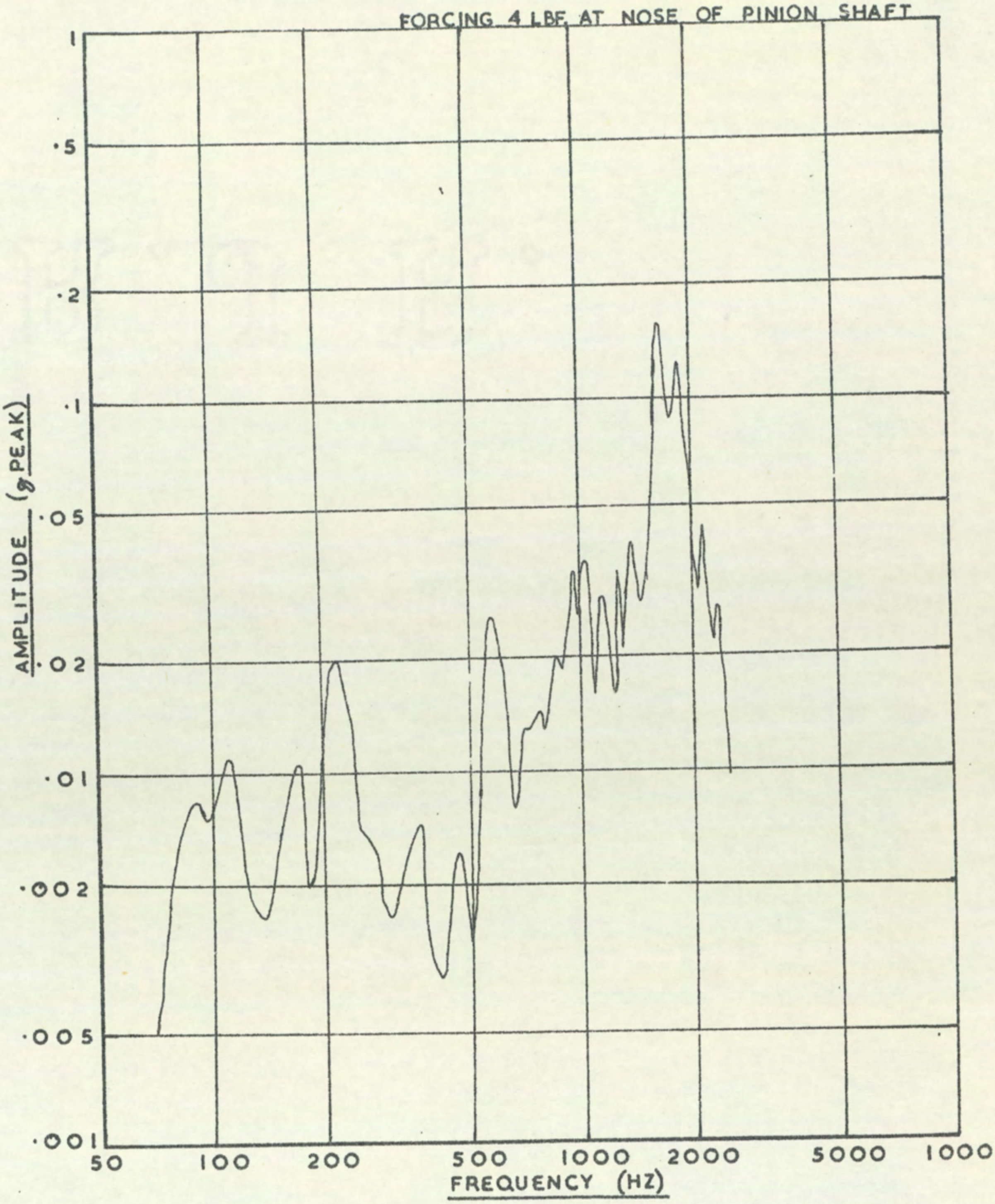
BLOCK DIAGRAM OF AUTOMATIC RESPONSE TEST INSTRUMENTATION

FIG 3-2



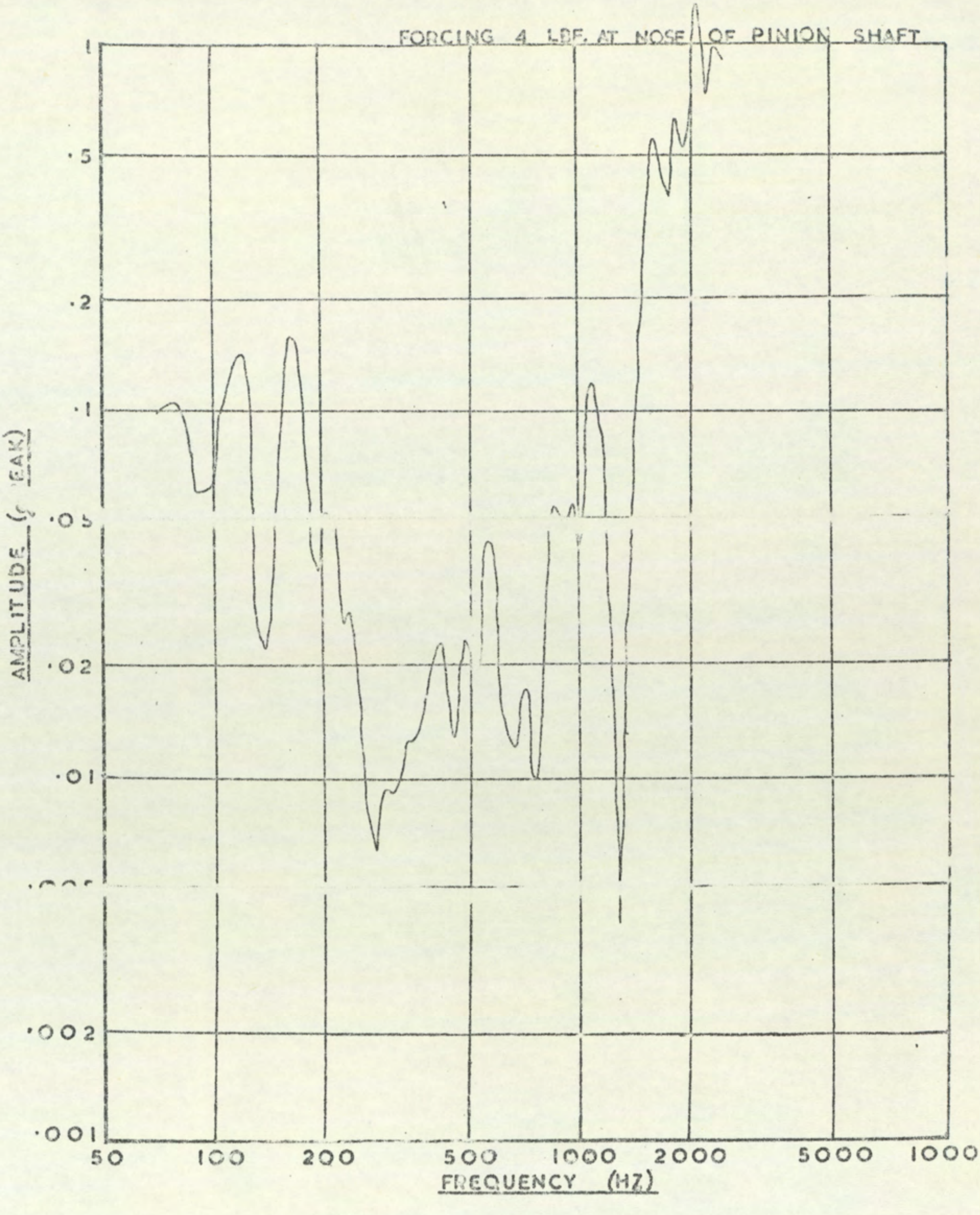
RESPONSE CURVE FOR POSITION No. 1.

FIG 3-3



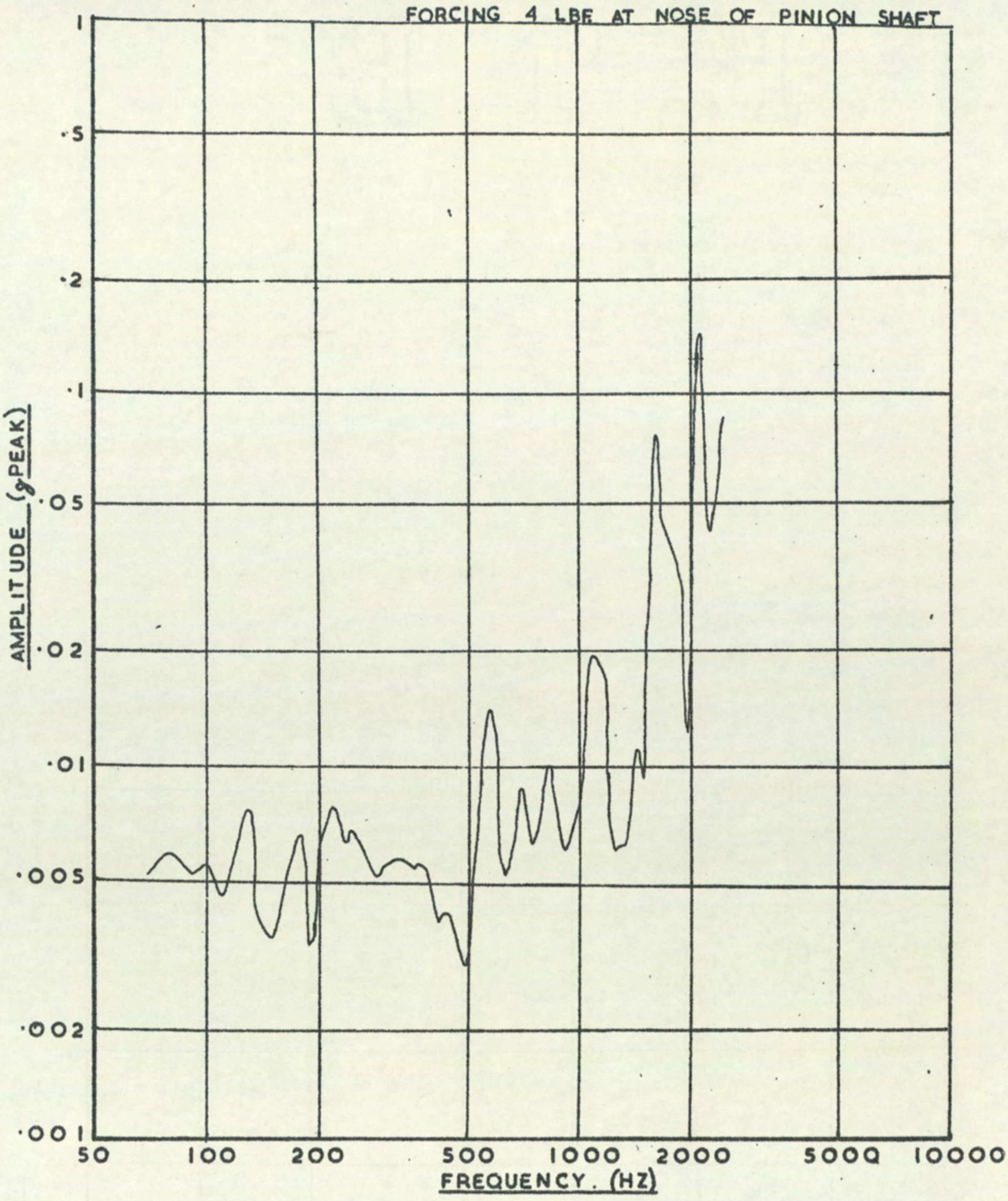
RESPONSE CURVE FOR POSITION No. 2.

FIG 3-4



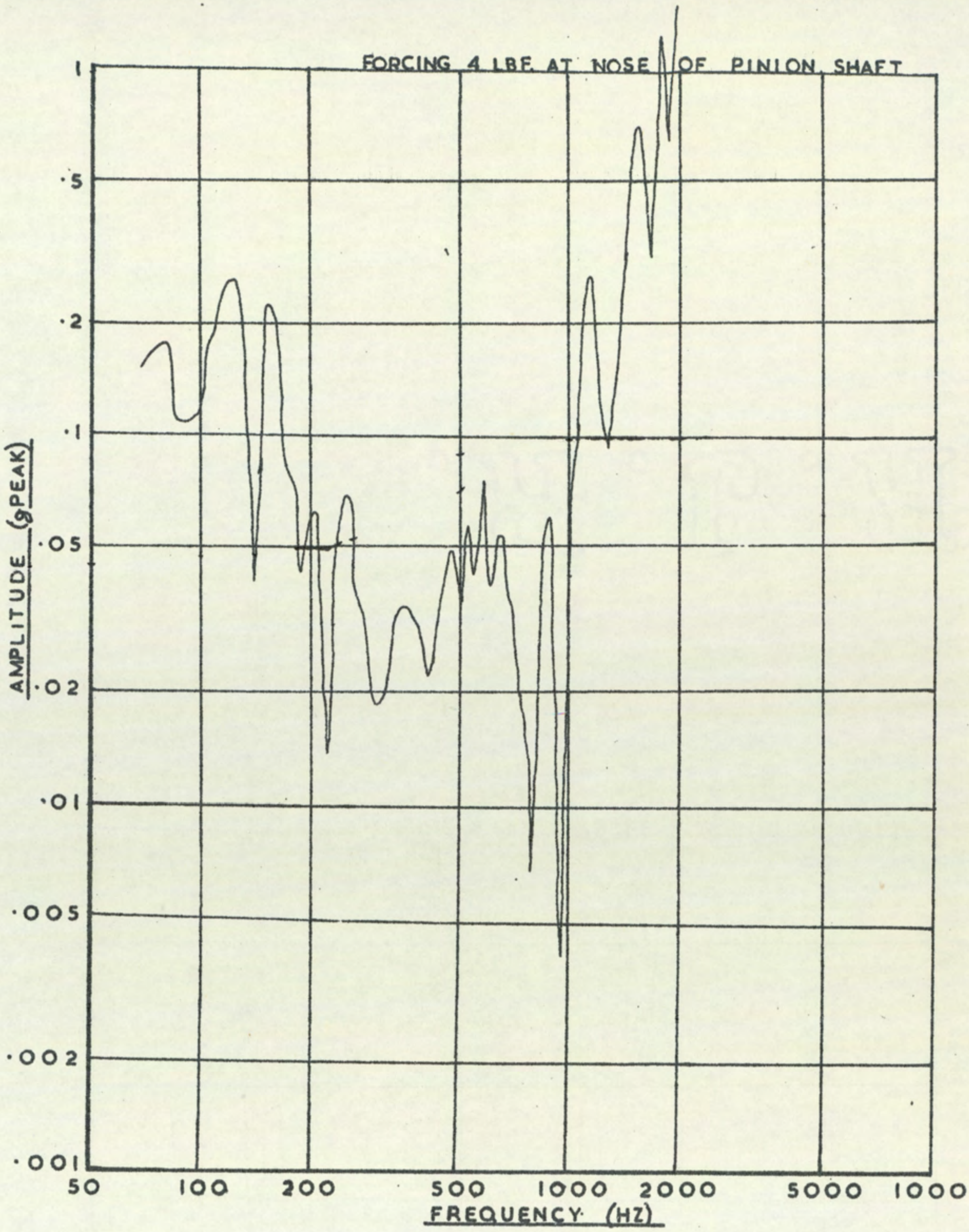
RESPONSE CURVE FOR POSITION No. 3.

FIG 3



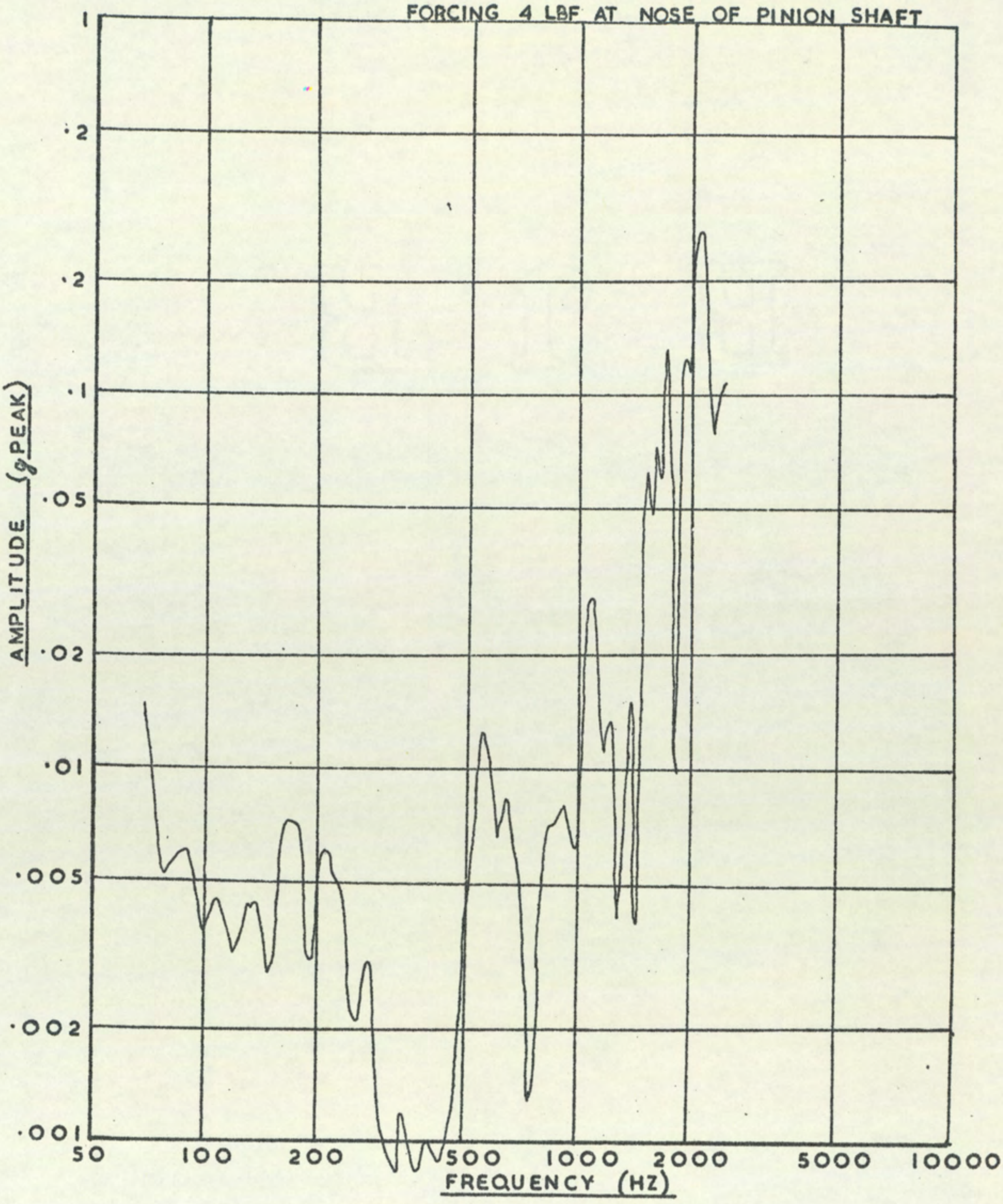
RESPONSE CURVE FOR POSITION No. 4.

FIG 3-6



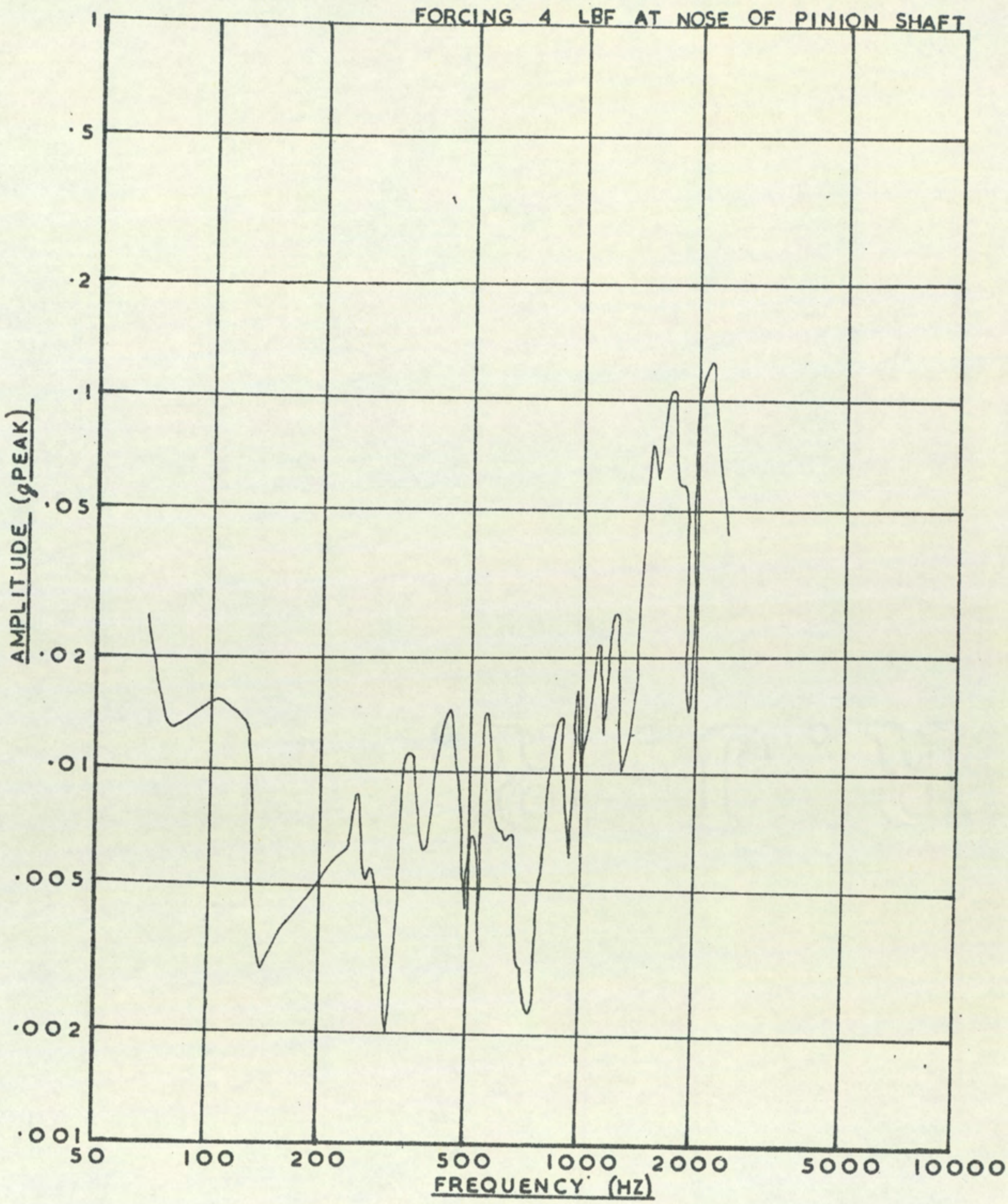
RESPONSE CURVE FOR POSITION No. 5

FIG 3-7



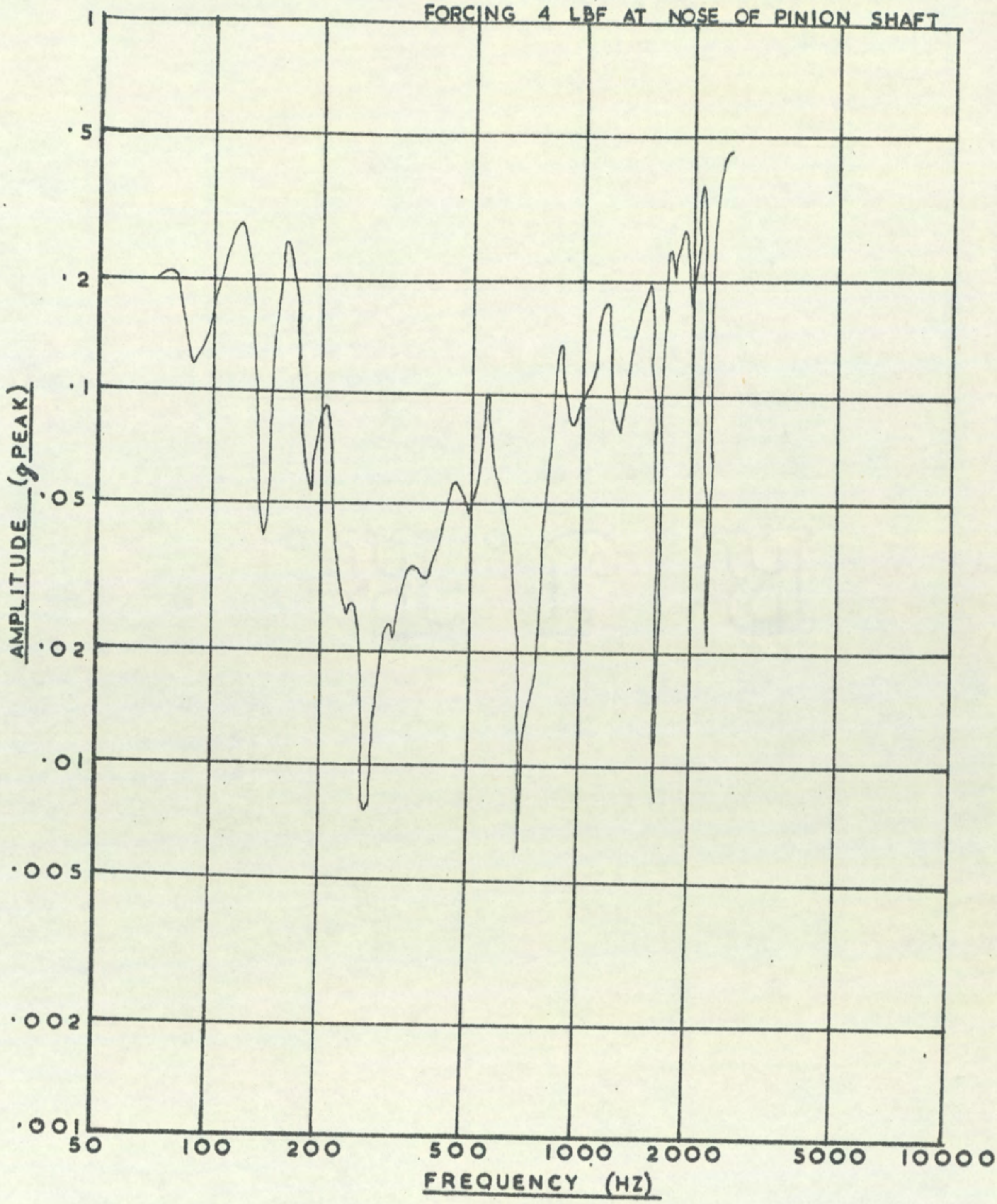
RESPONSE CURVE FOR POSITION No. 6.

FIG 3-8



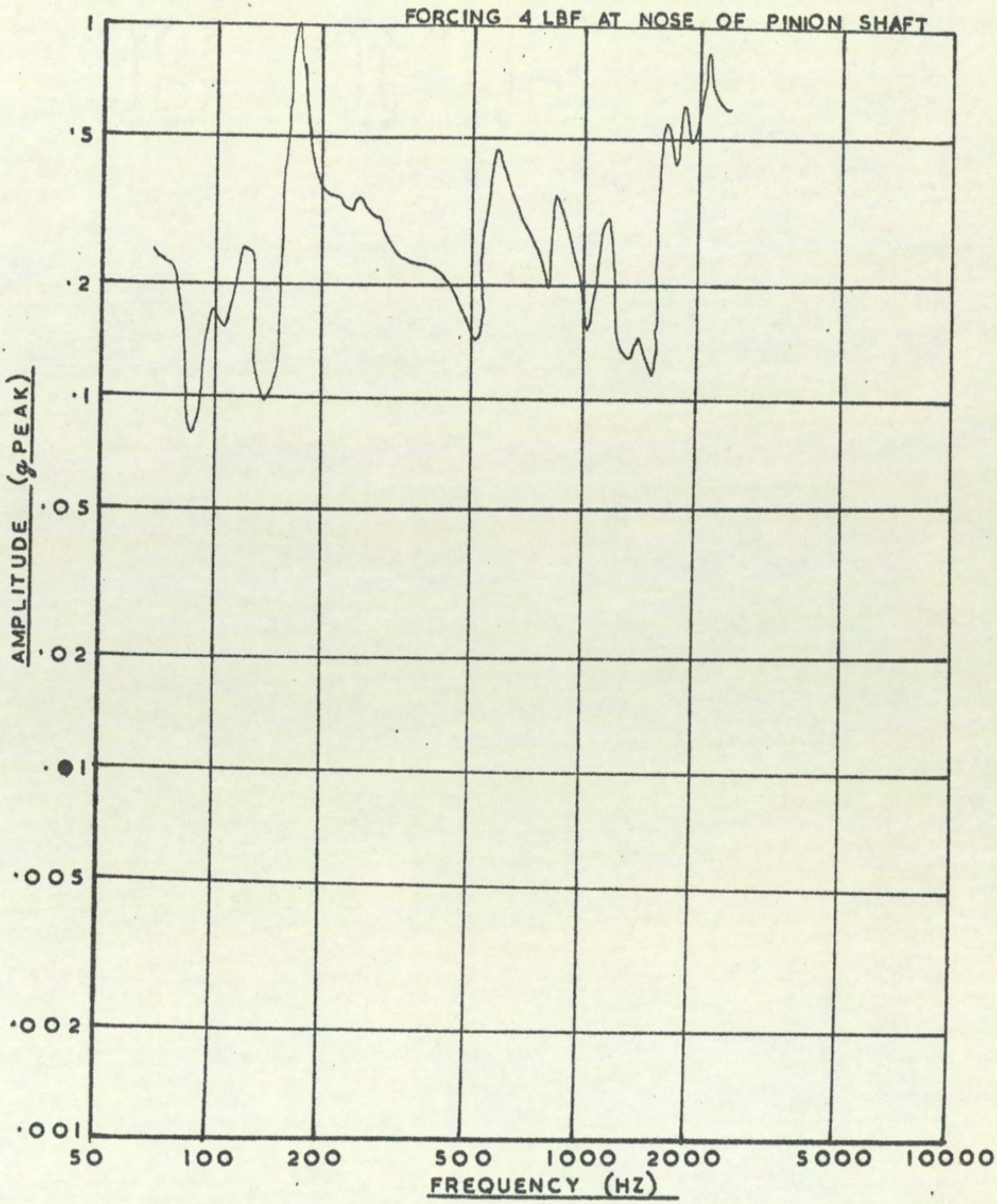
RESPONSE CURVE FOR POSITION No. 7.

FIG 3-9



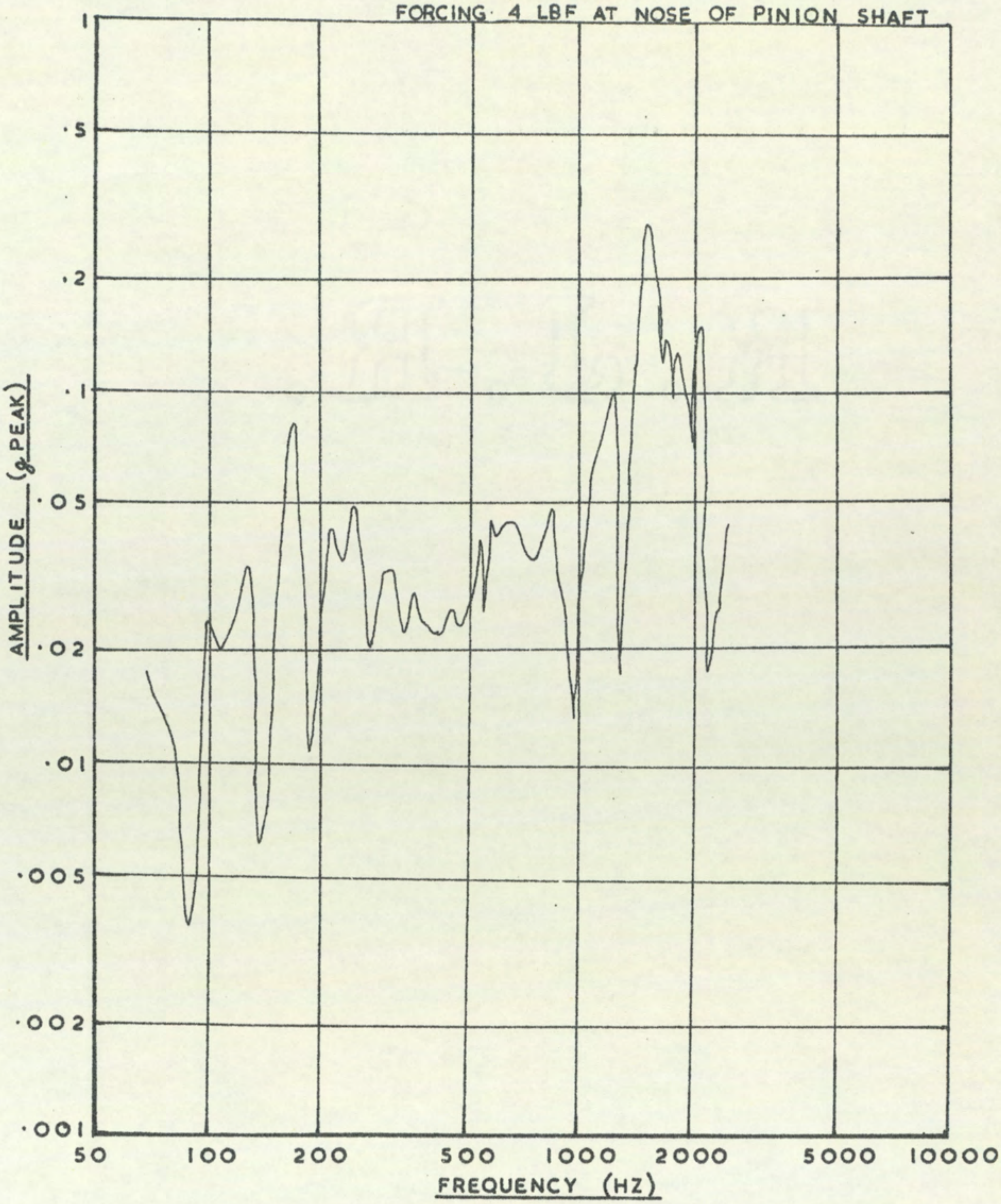
RESPONSE CURVE FOR POSITION No. 9.

FIG 3-10



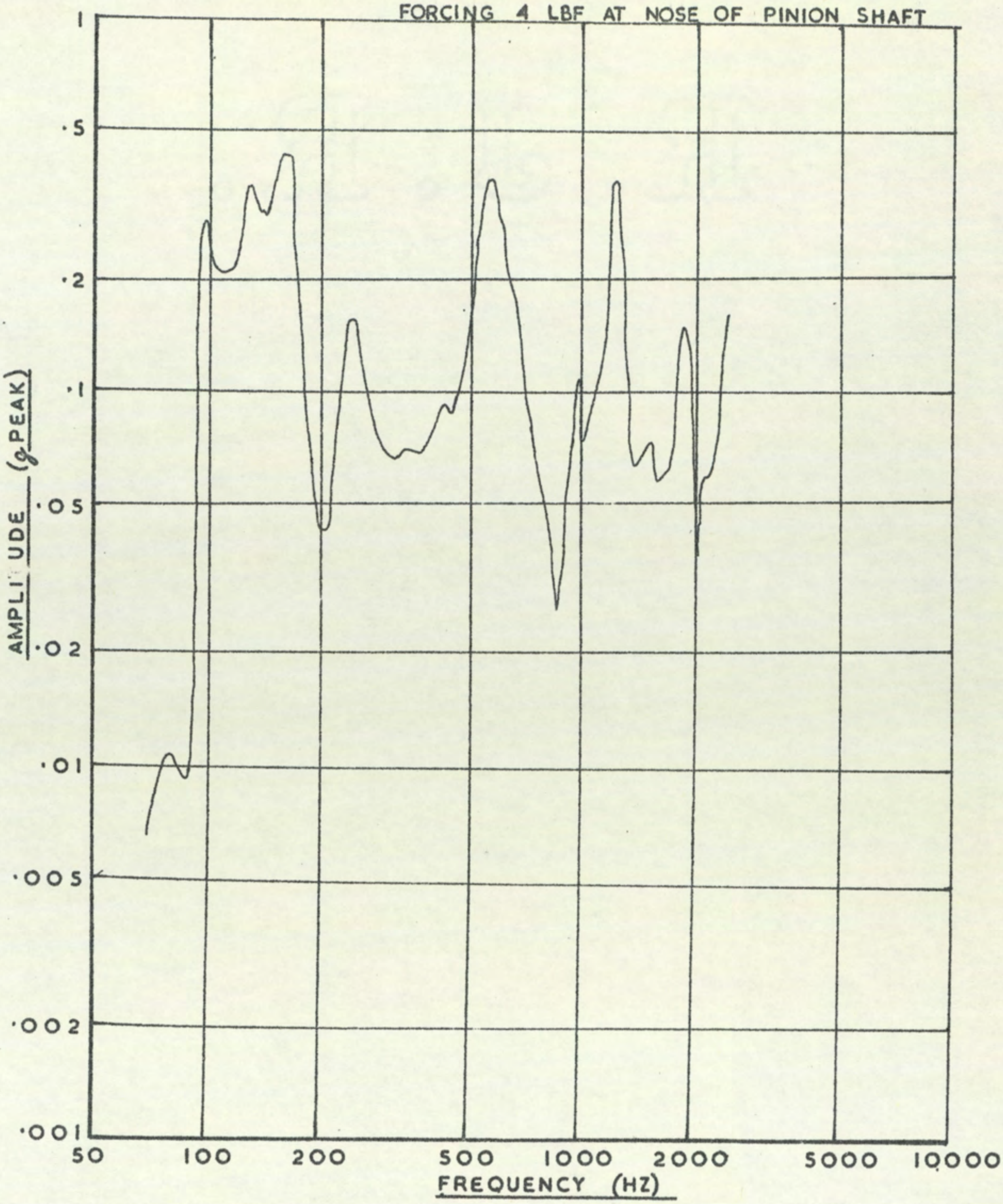
RESPONSE CURVE FOR POSITION No. 10.

FIG 3-11



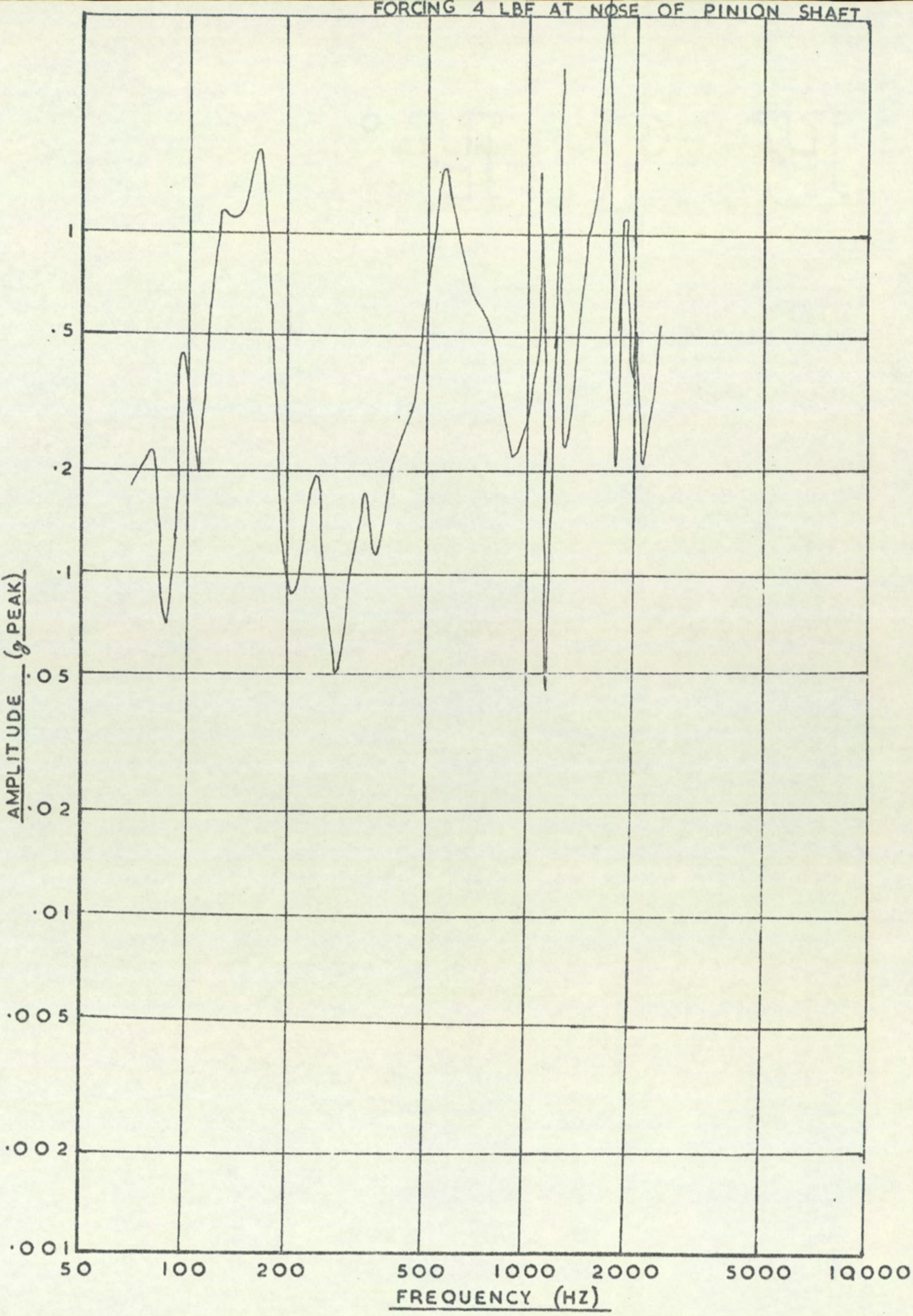
RESPONSE CURVE FOR POSITION No. 11.

FIG 3-12



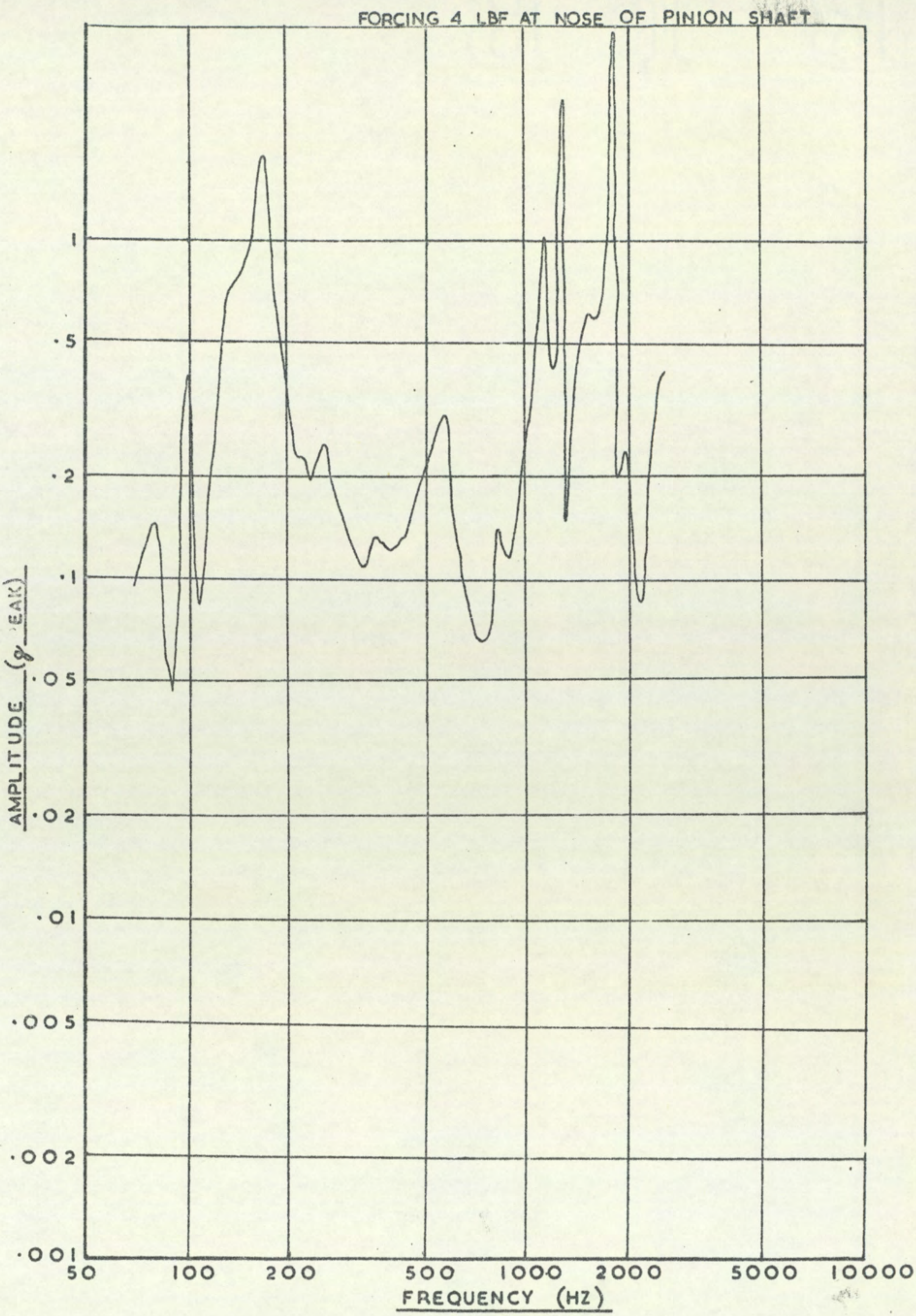
RESPONSE CURVE FOR POSITION No. 12.

FIG 3-13



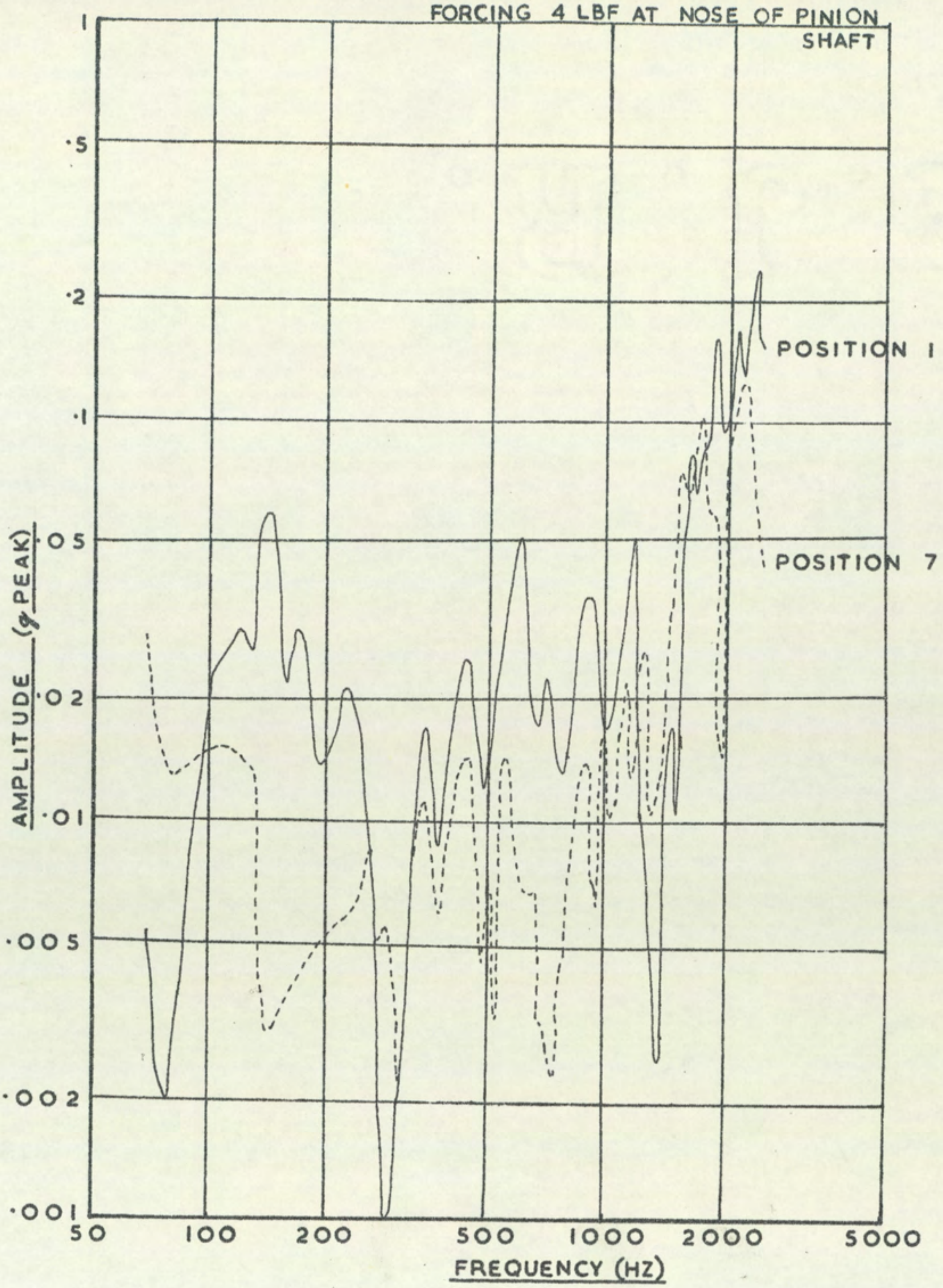
RESPONSE CURVE FOR POSITION No. 13.

FIG 3-14



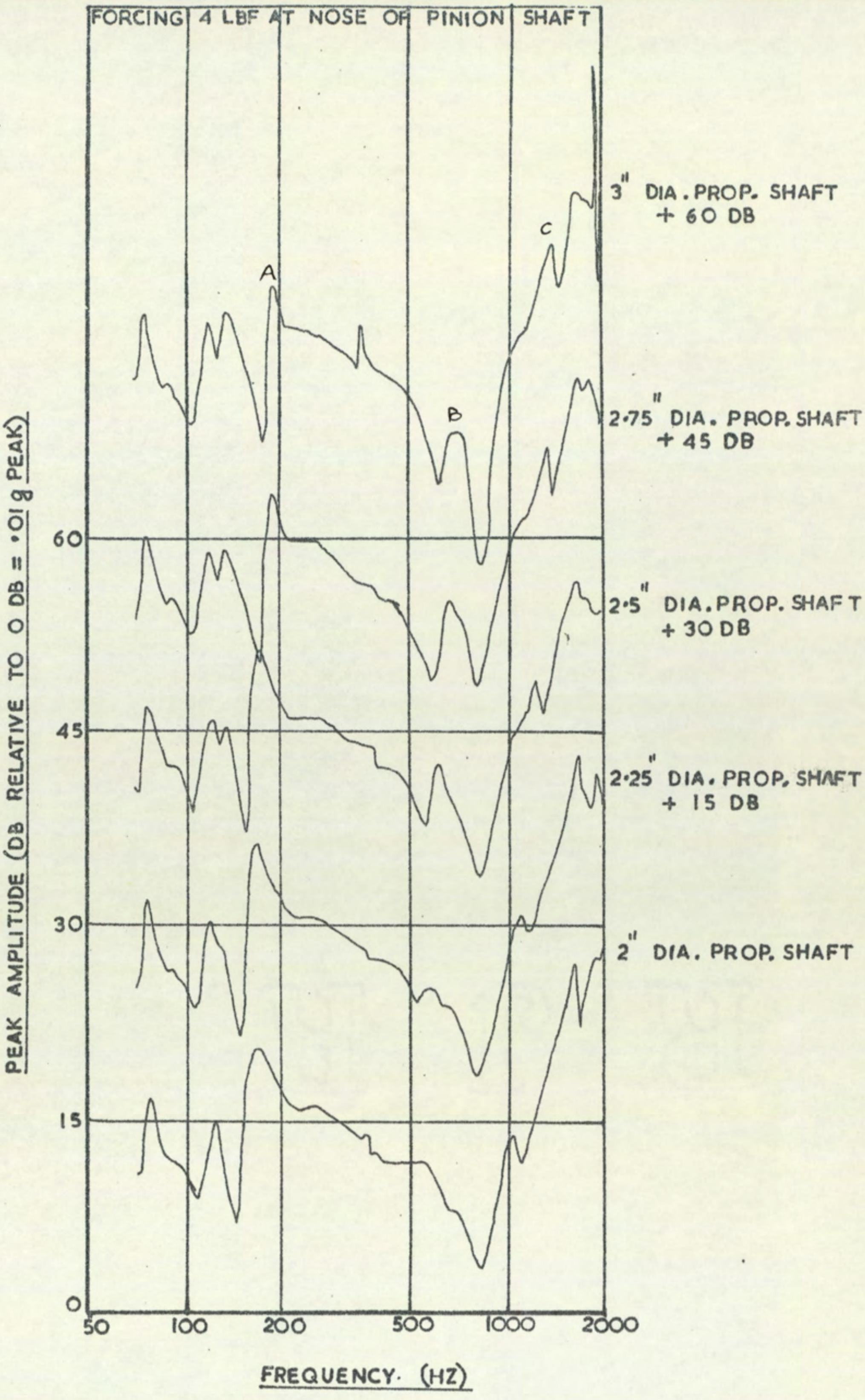
RESPONSE CURVE FOR POSITION No. 14.

FIG 3-15



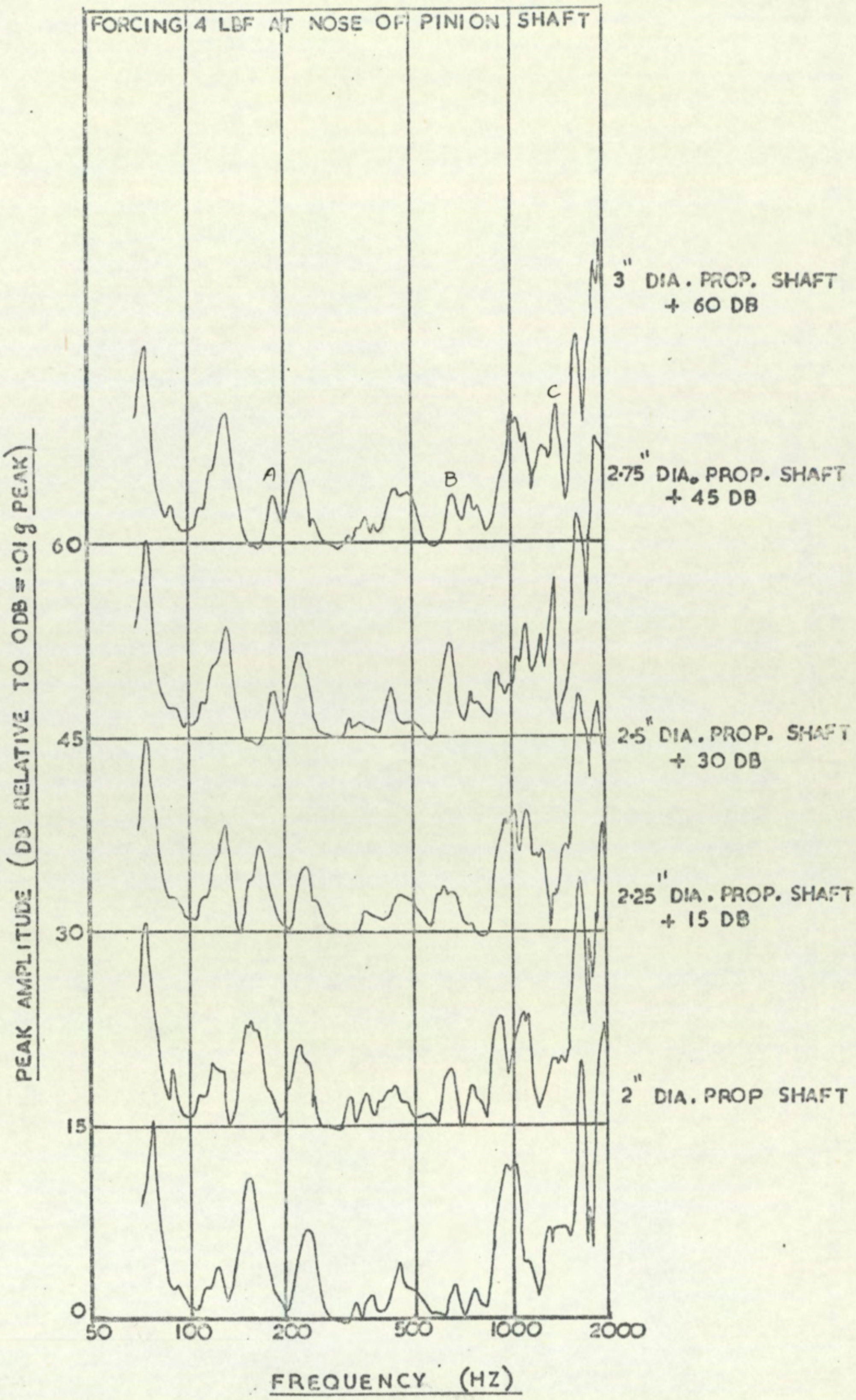
COMPARISON OF RESPONSE AT ENDS OF REAR AXLE

FIG 3-16



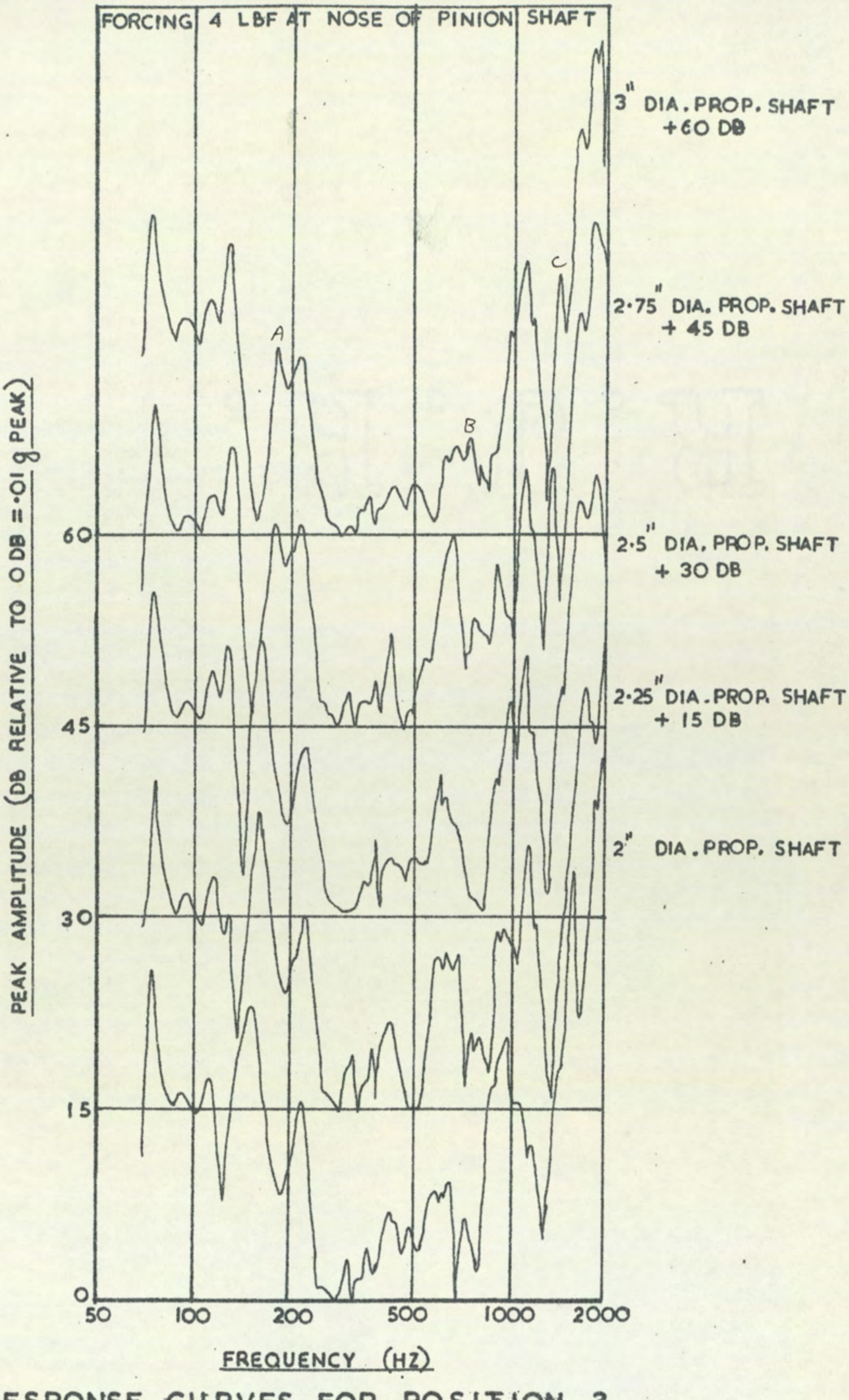
RESPONSE CURVES FOR FORCING INPUT POSITION
(VARIOUS DIAMETER PROPELLER SHAFTS)

FIG 3-17



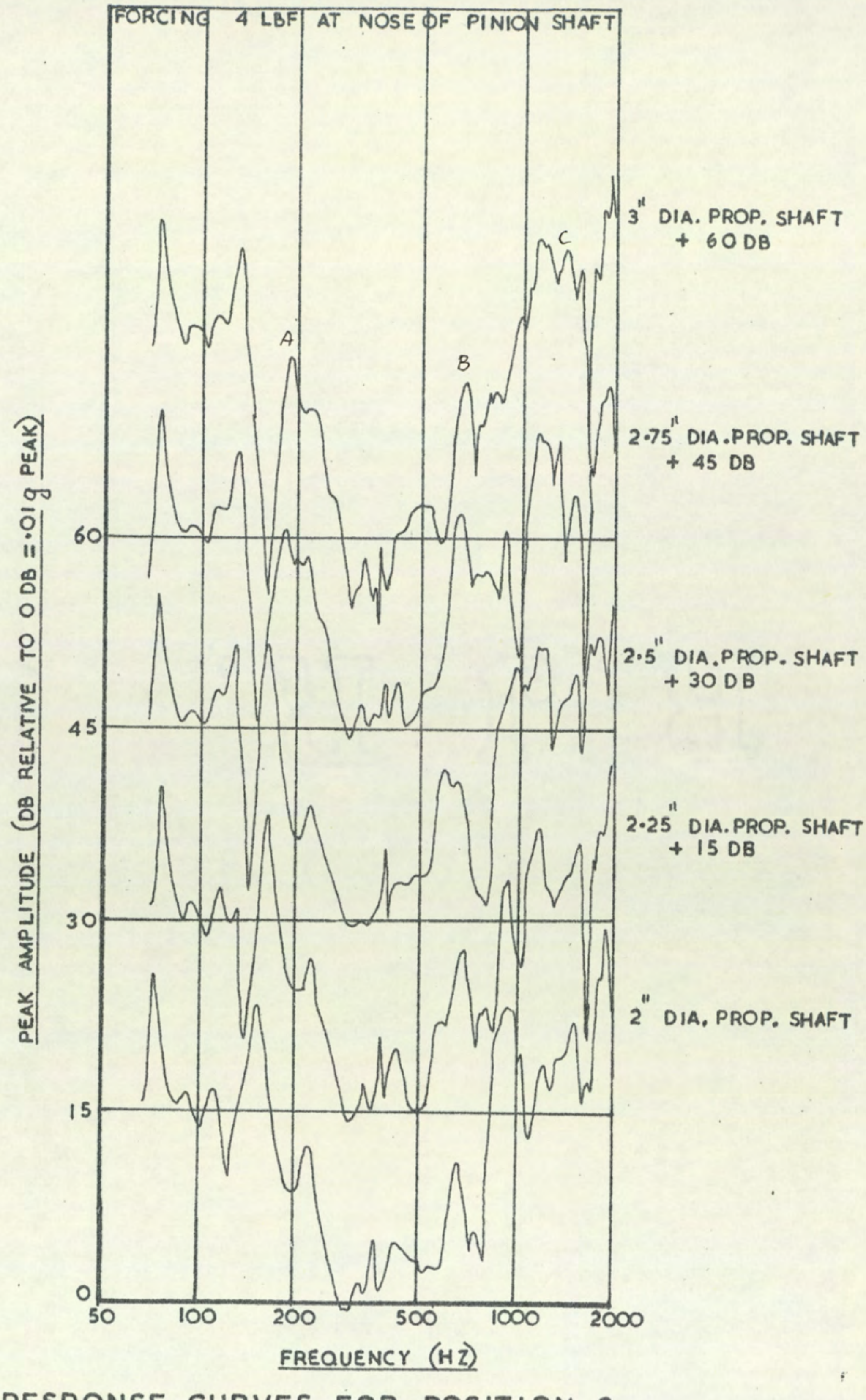
RESPONSE CURVES FOR POSITION I
(VARIOUS DIAMETER PROPELLER SHAFTS)

FIG 3-18



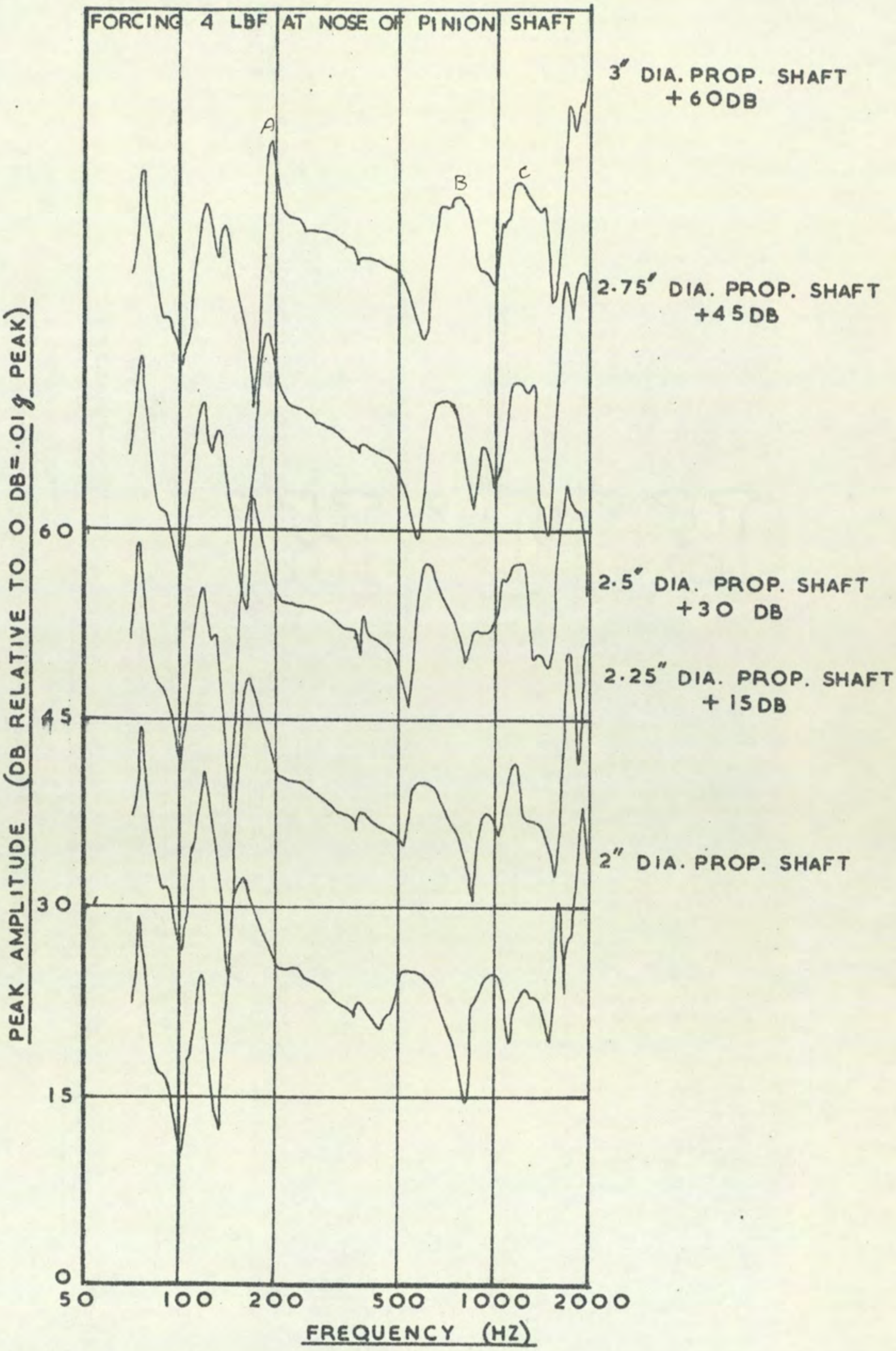
RESPONSE CURVES FOR POSITION 3
(VARIOUS DIAMETER PROPELLER SHAFTS)

FIG 3-19



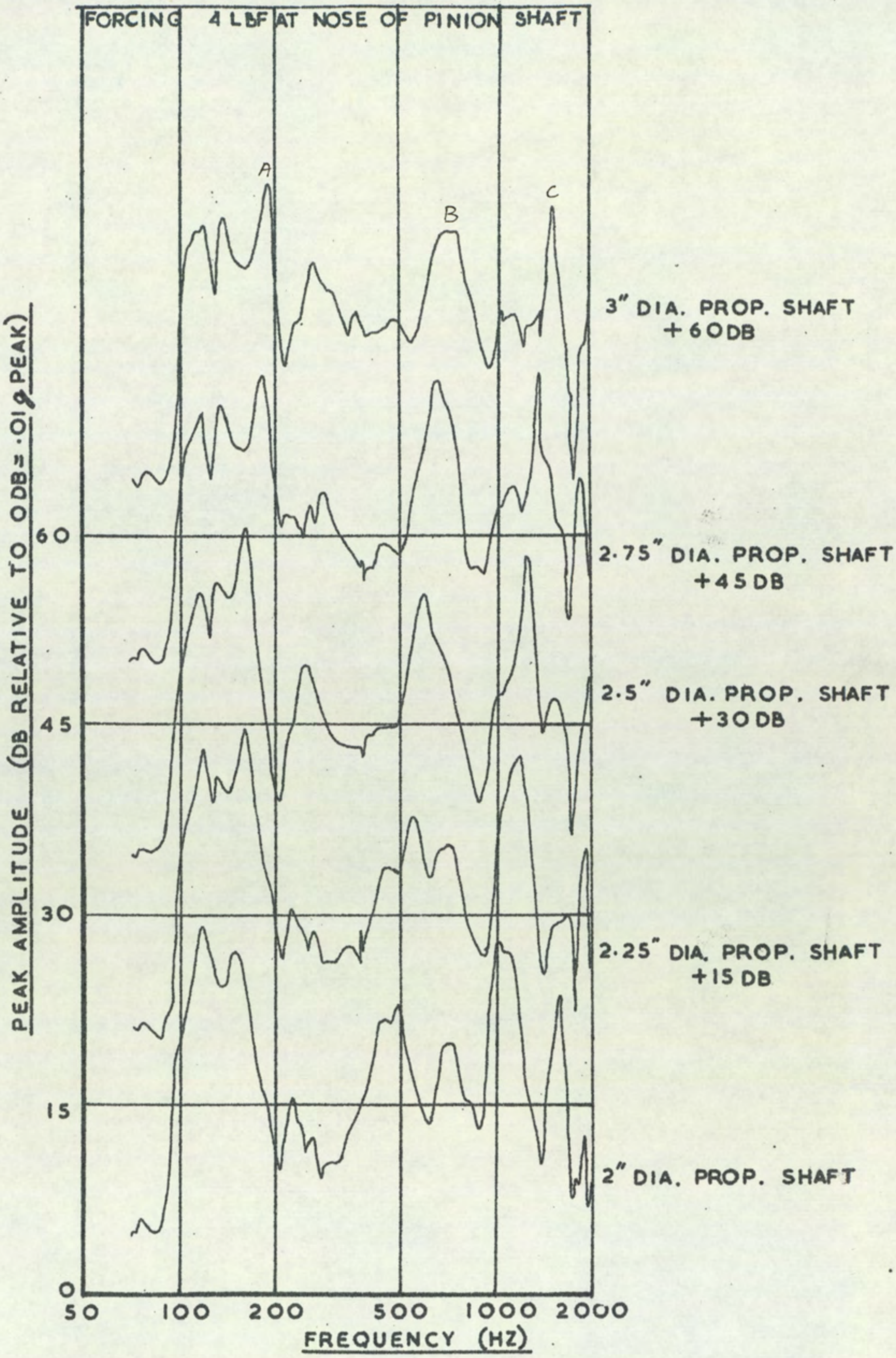
RESPONSE CURVES FOR POSITION 9
(VARIOUS DIAMETER PROPELLER SHAFTS)

FIG 3-20



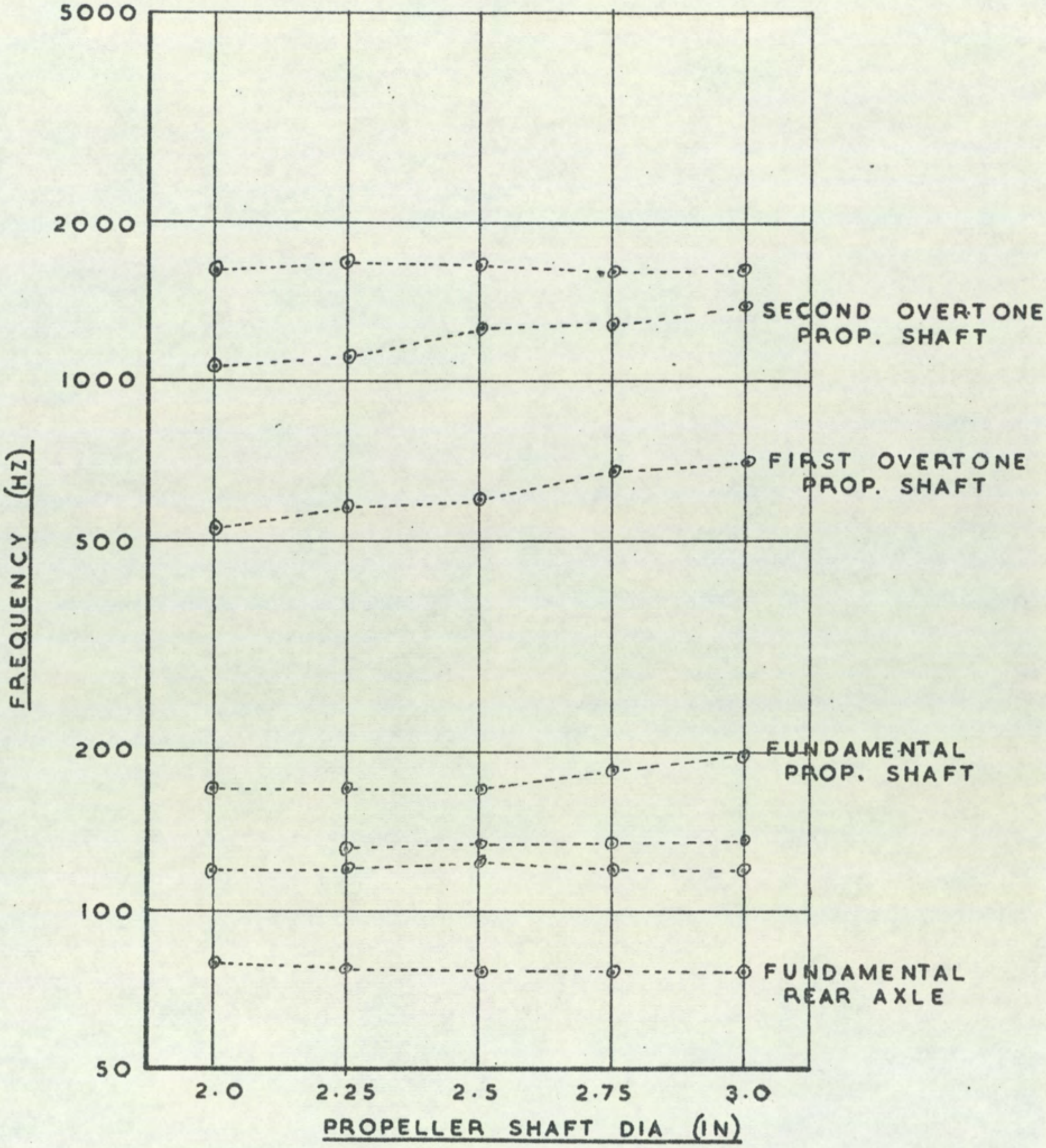
RESPONSE CURVES FOR POSITION 11
(VARIOUS DIAMETER PROPELLER SHAFTS)

FIG 3-21



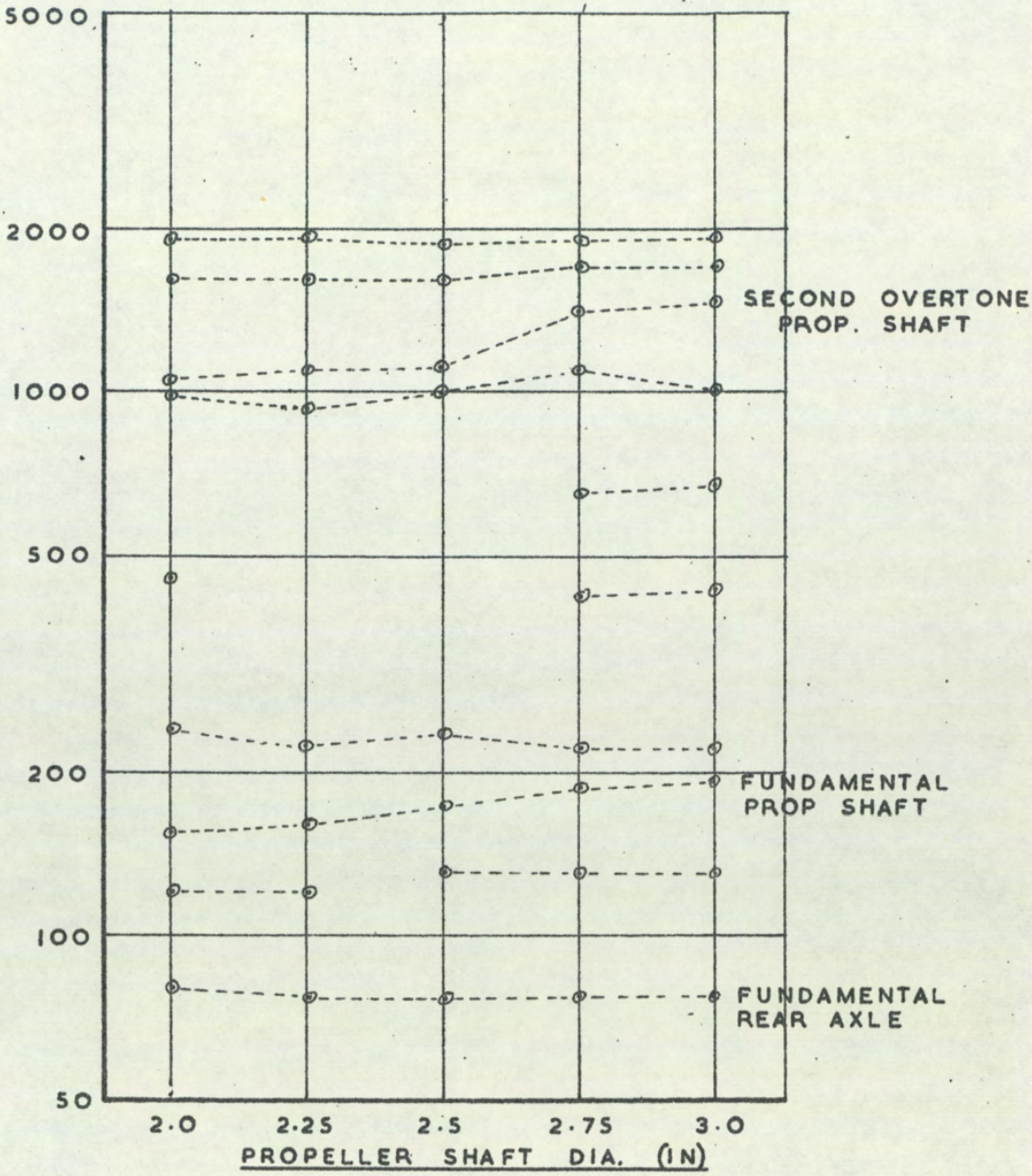
RESPONSE CURVES FOR POSITION 12
(VARIOUS DIAMETER PROPELLER SHAFTS)

FIG 3-22



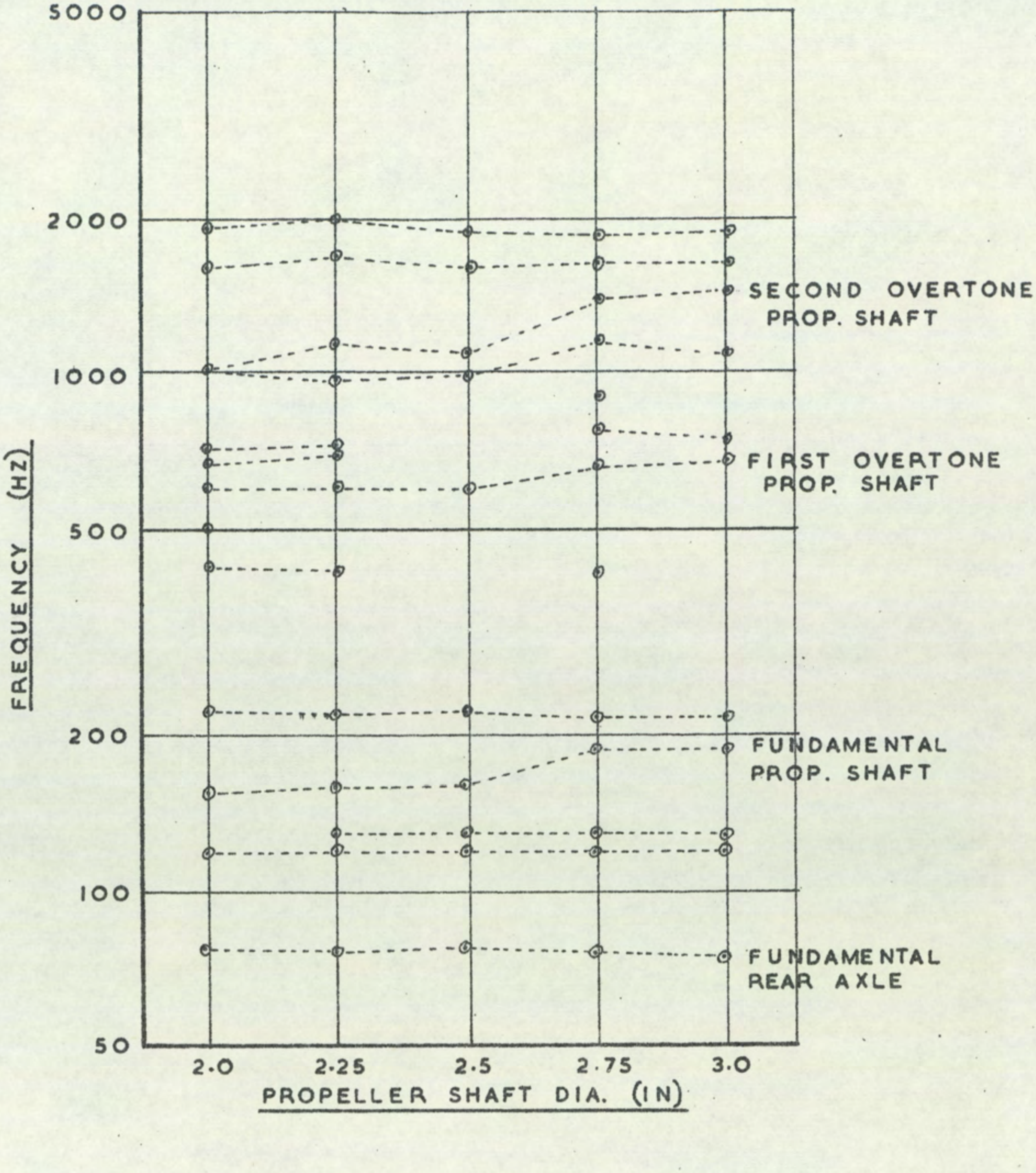
EFFECT OF DIAMETER OF PROPELLER SHAFT ON RESONANT FREQUENCIES (INPUT POINT)

FIG 3-23



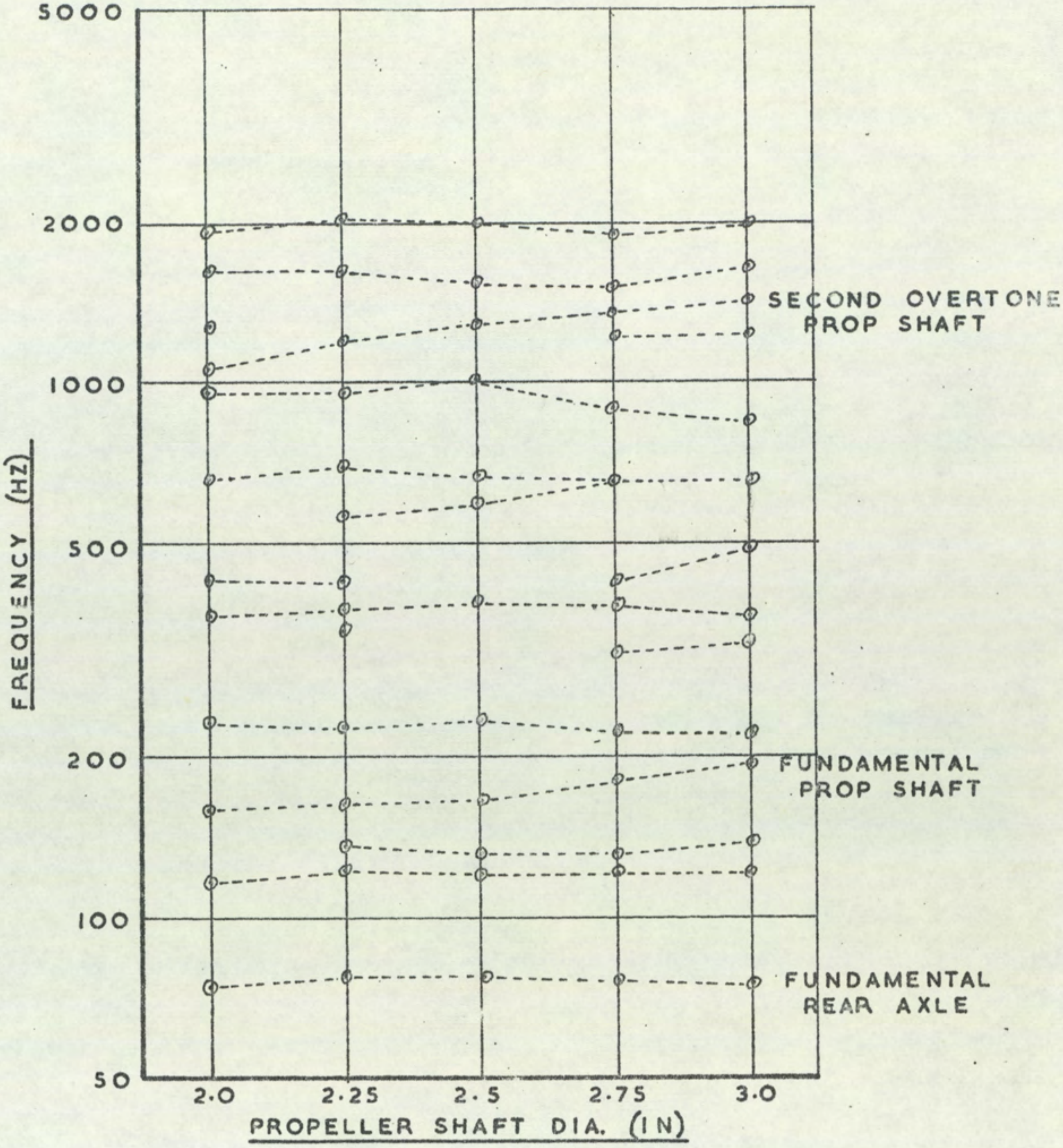
EFFECT OF DIAMETER OF PROPELLER SHAFT ON RESONANT FREQUENCIES (POSITION 1)

FIG 3-24



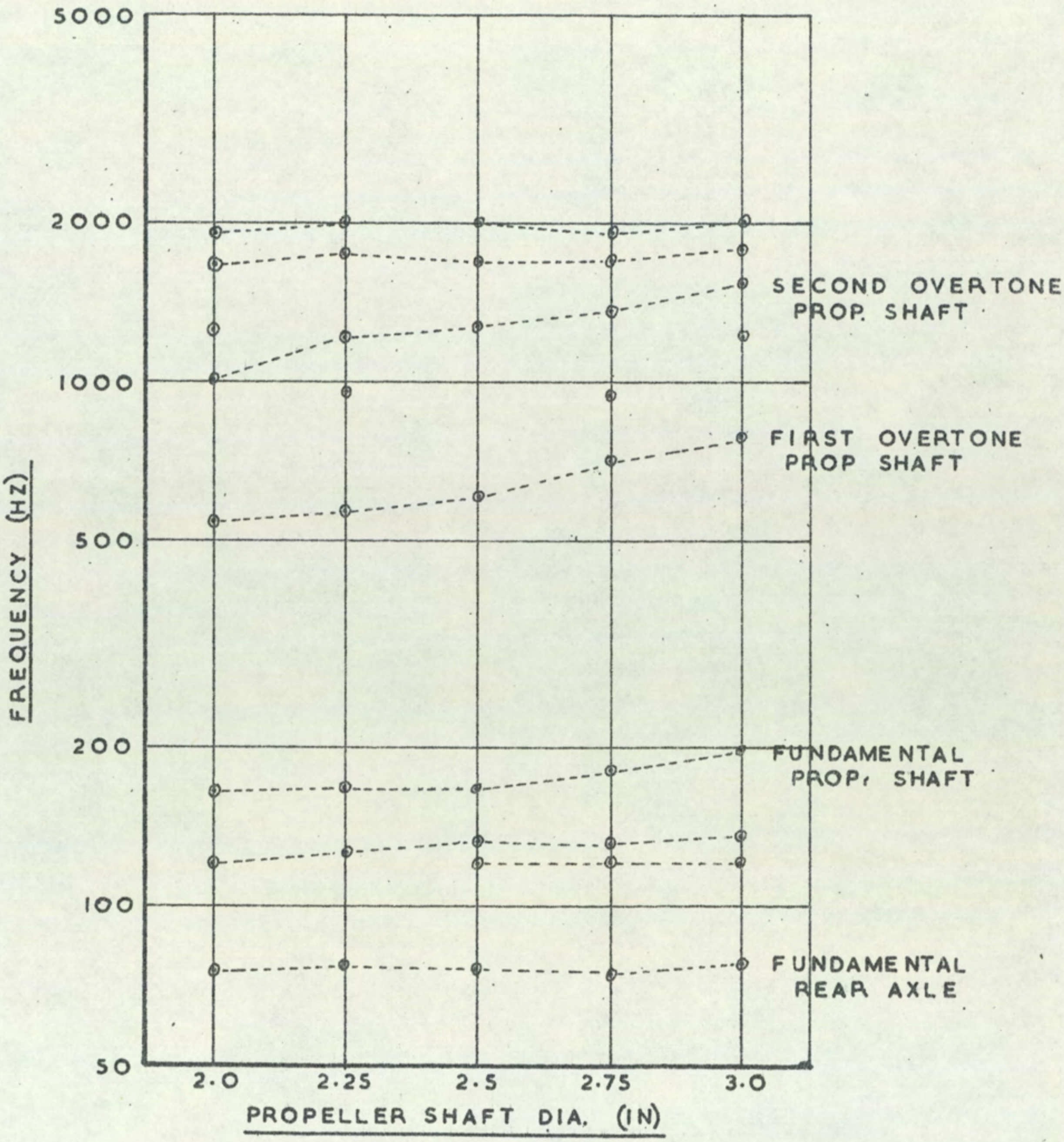
EFFECT OF DIAMETER OF PROPELLER SHAFT ON RESONANT FREQUENCIES (POSITION 3)

FIG 3-25



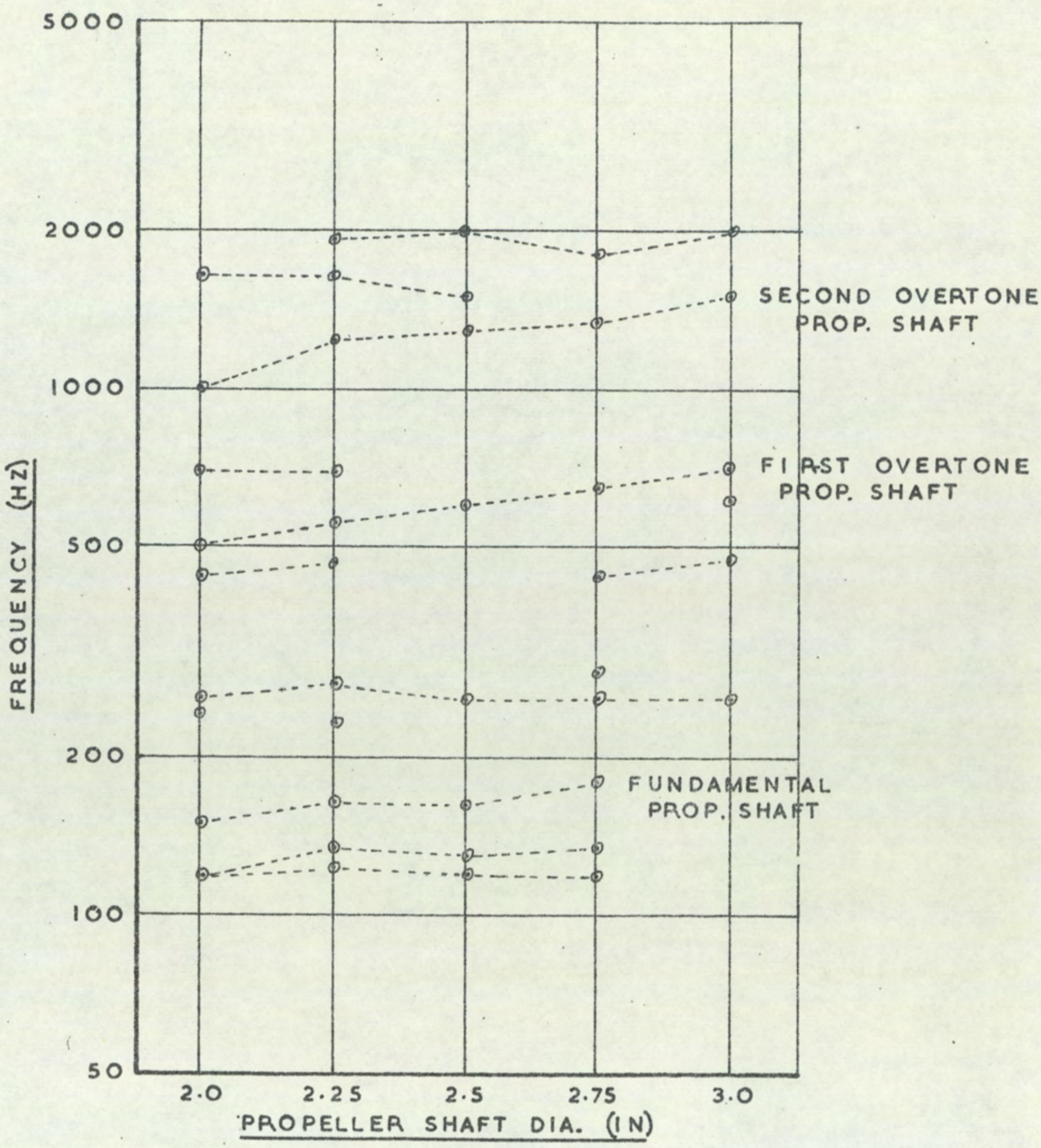
EFFECT OF DIAMETER OF PROPELLER SHAFT ON RESONANT FREQUENCIES (POSITION 9)

FIG 3-26



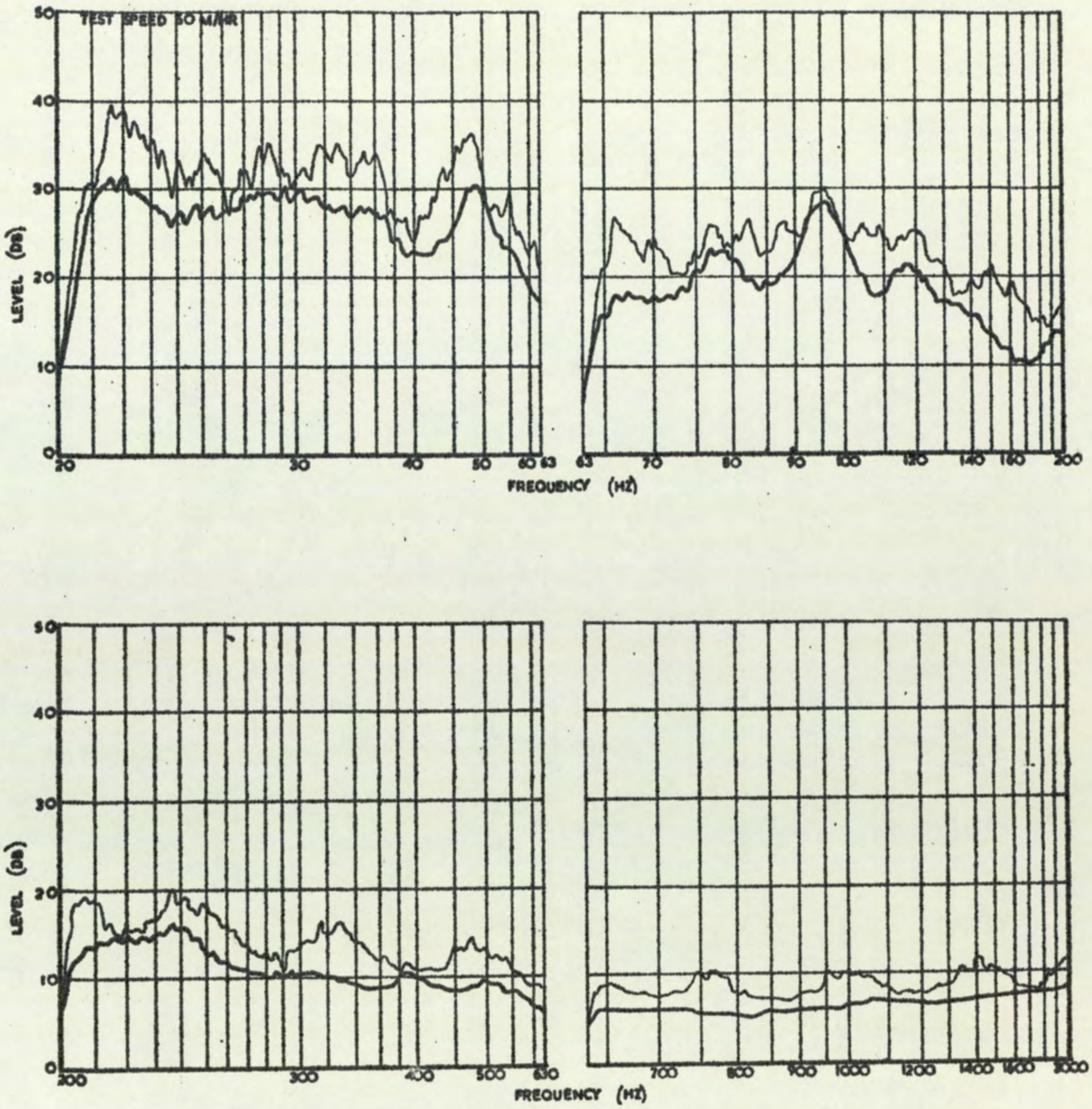
EFFECT OF DIAMETER OF PROPELLER SHAFT ON RESONANT FREQUENCIES (POSITION II)

FIG 3-27



EFFECT OF DIAMETER OF PROPELLER SHAFT ON RESONANT FREQUENCIES (POSITION 12)

FIG 3-28.



KEY

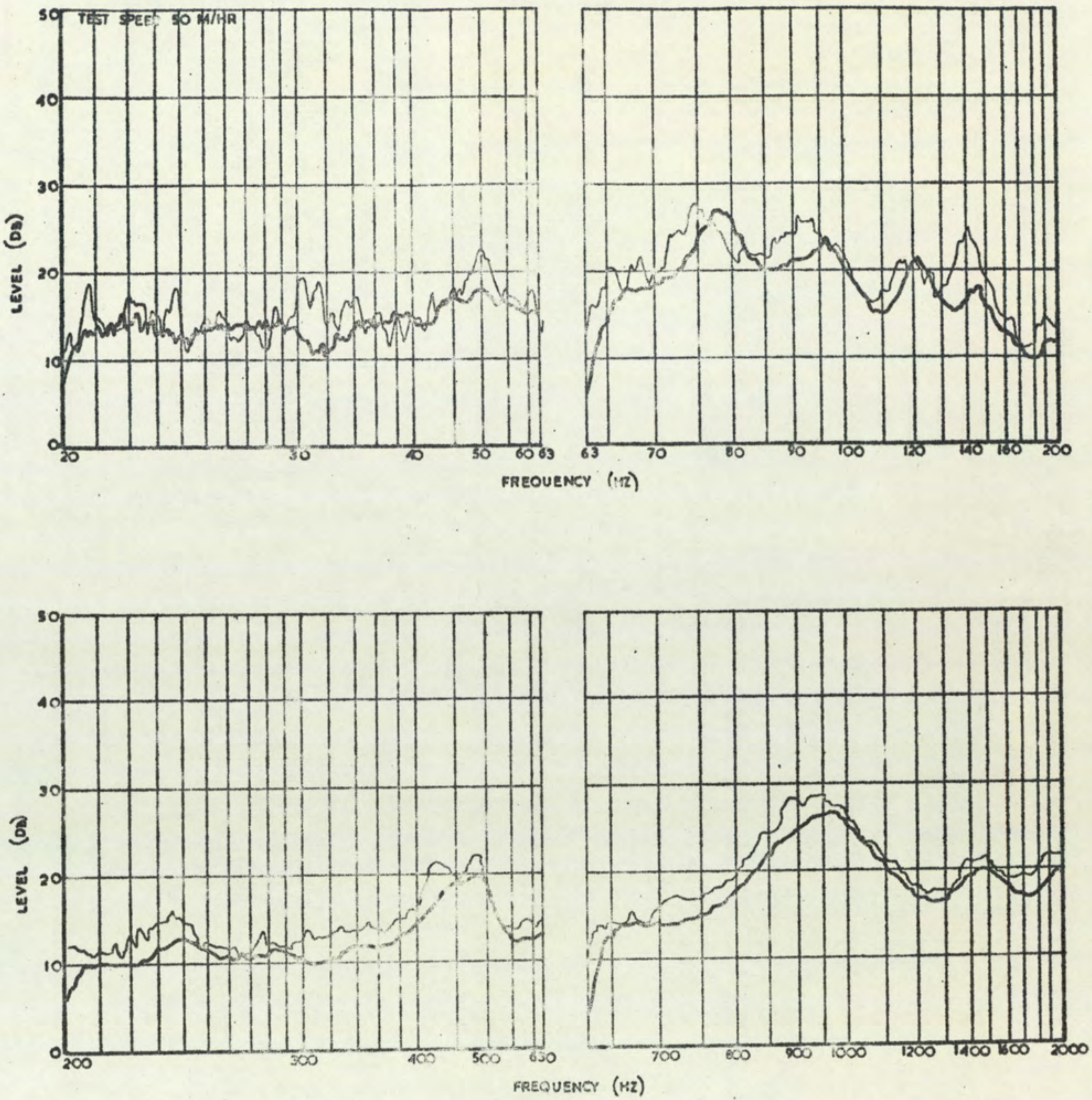
FIRST ANALYSIS —— 10 S DATA LOOP, 1·6 S AVERAGING TIME

SECOND ANALYSIS —— 3 S DATA LOOP, 2·5 S AVERAGING TIME

SOUND PRESSURE LEVEL = MEASURED LEVEL + 62·5 DB

EFFECT OF DATA SAMPLE LENGTH AND ANALYSER AVERAGING TIME ON WAVE ANALYSIS OF 'MICROPHONE IN CAB' RECORDINGS

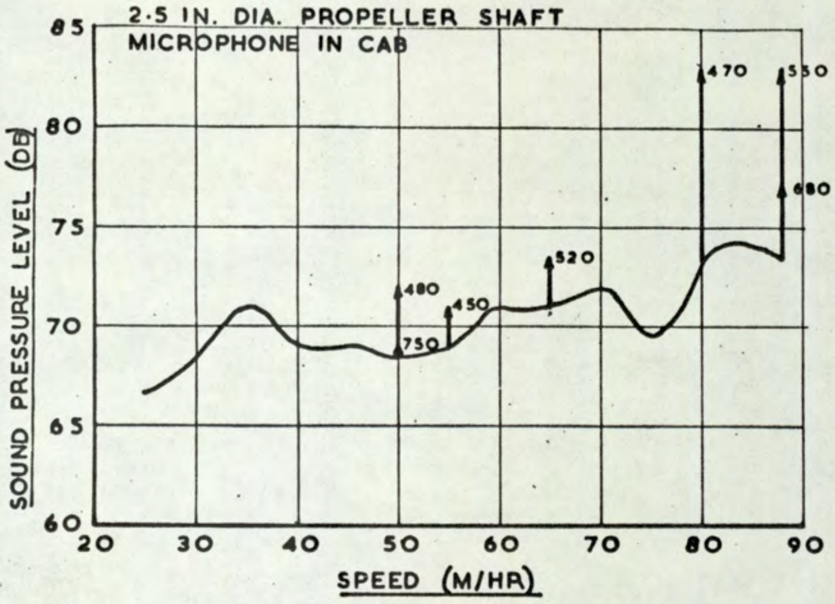
FIG 3-29

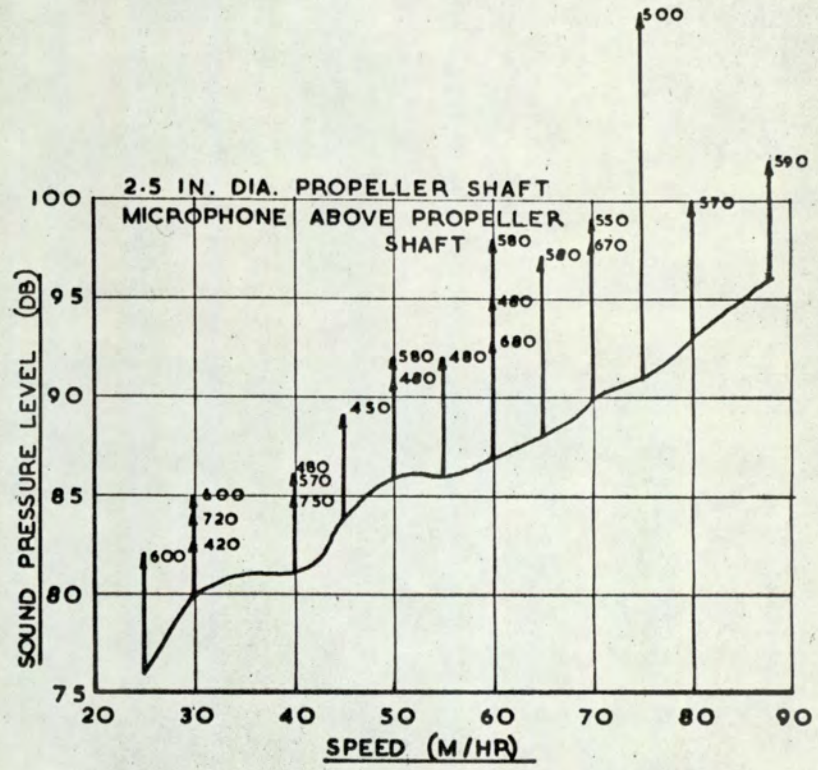


KEY
FIRST ANALYSIS ——— 10 S DATA LOOP, 1·6 S AVERAGING TIME
SECOND ANALYSIS ——— 3 S DATA LOOP, 2·5 S AVERAGING TIME
SOUND PRESSURE LEVEL = MEASURED LEVEL + 62·5 DB

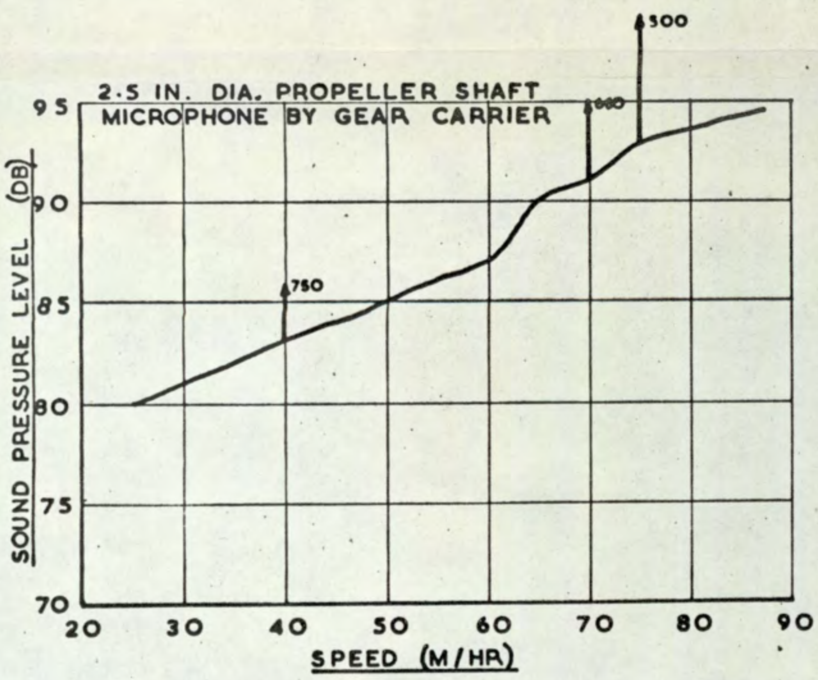
EFFECT OF DATA SAMPLE LENGTH AND ANALYSER AVERAGING TIME ON WAVE ANALYSIS OF 'ACCELEROMETER No 3' RECORDINGS

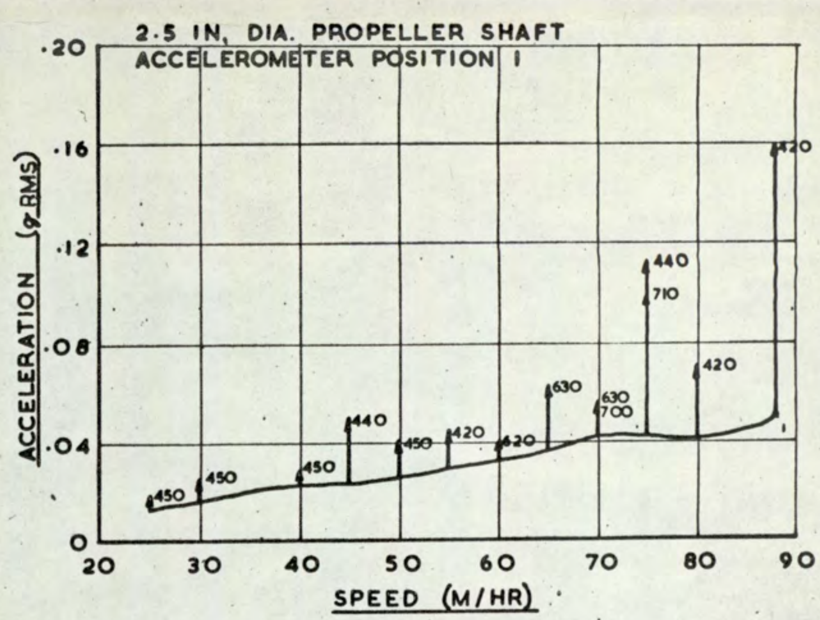
FIG 3-30





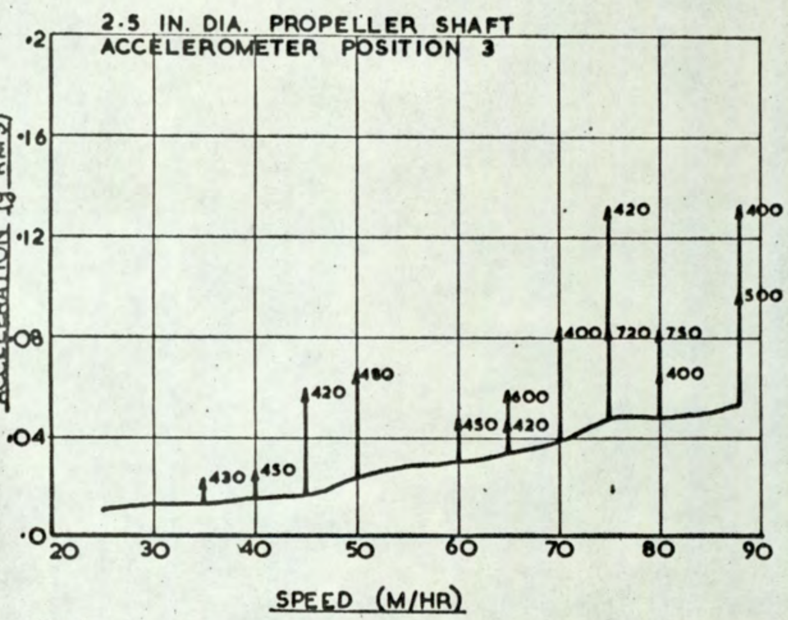
KEY
MEAN LOWER LEVEL OF WAVE ANALYSIS IN FREQUENCY BAND 400 – 800 HZ
AMPLITUDE AND FREQUENCY OF PEAKS IN WAVE ANALYSIS 450

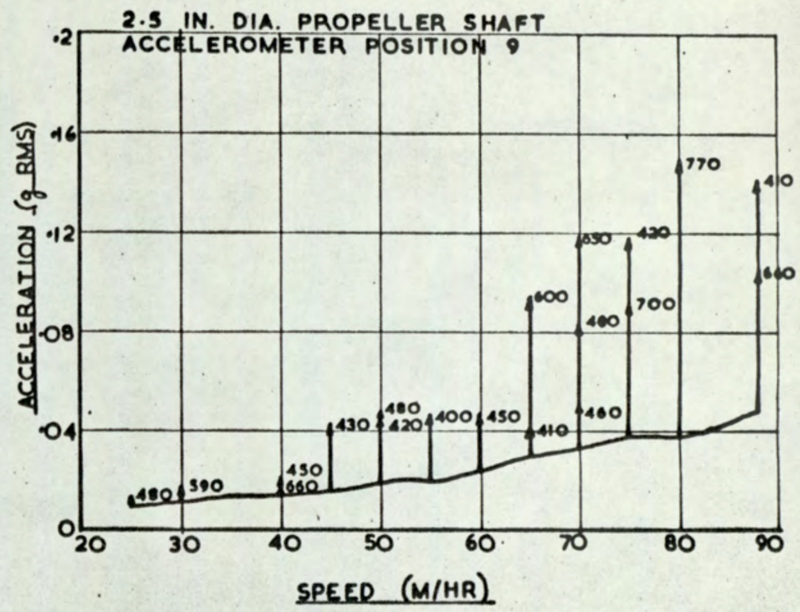




RESULTS OF WAVE ANALYSIS OF ROAD TEST RECORDINGS (400 – 800 HZ)
(STANDARD PROPELLER SHAFT)

FIG 3 – 31

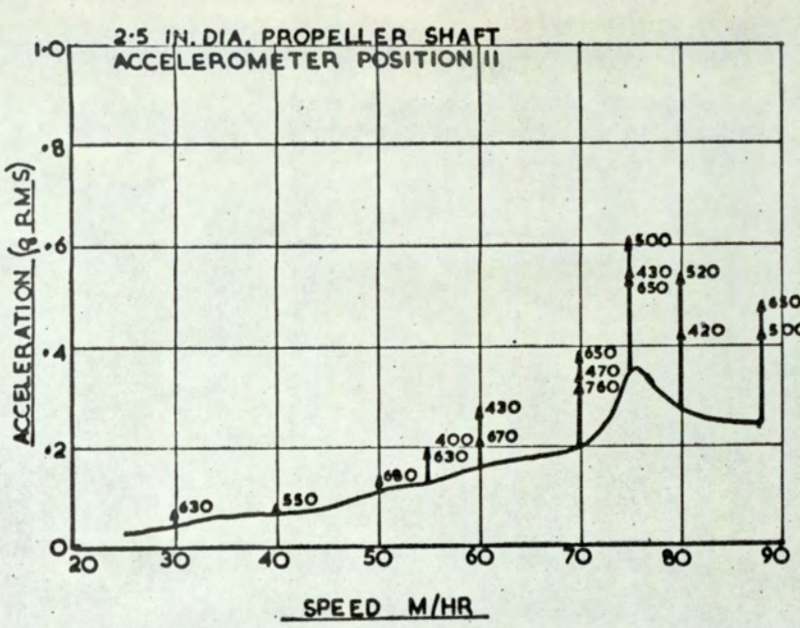


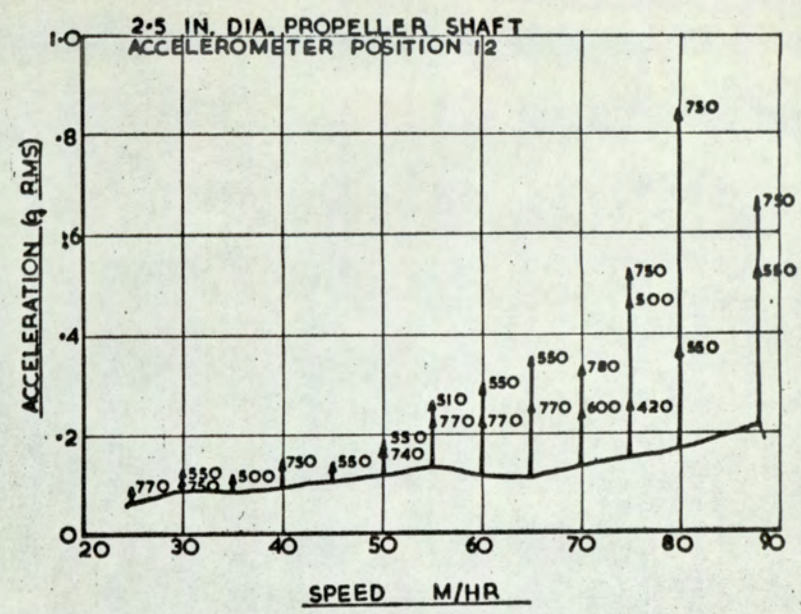


KEY

MEAN LOWER LEVEL OF WAVE ANALYSIS IN FREQUENCY BAND ———

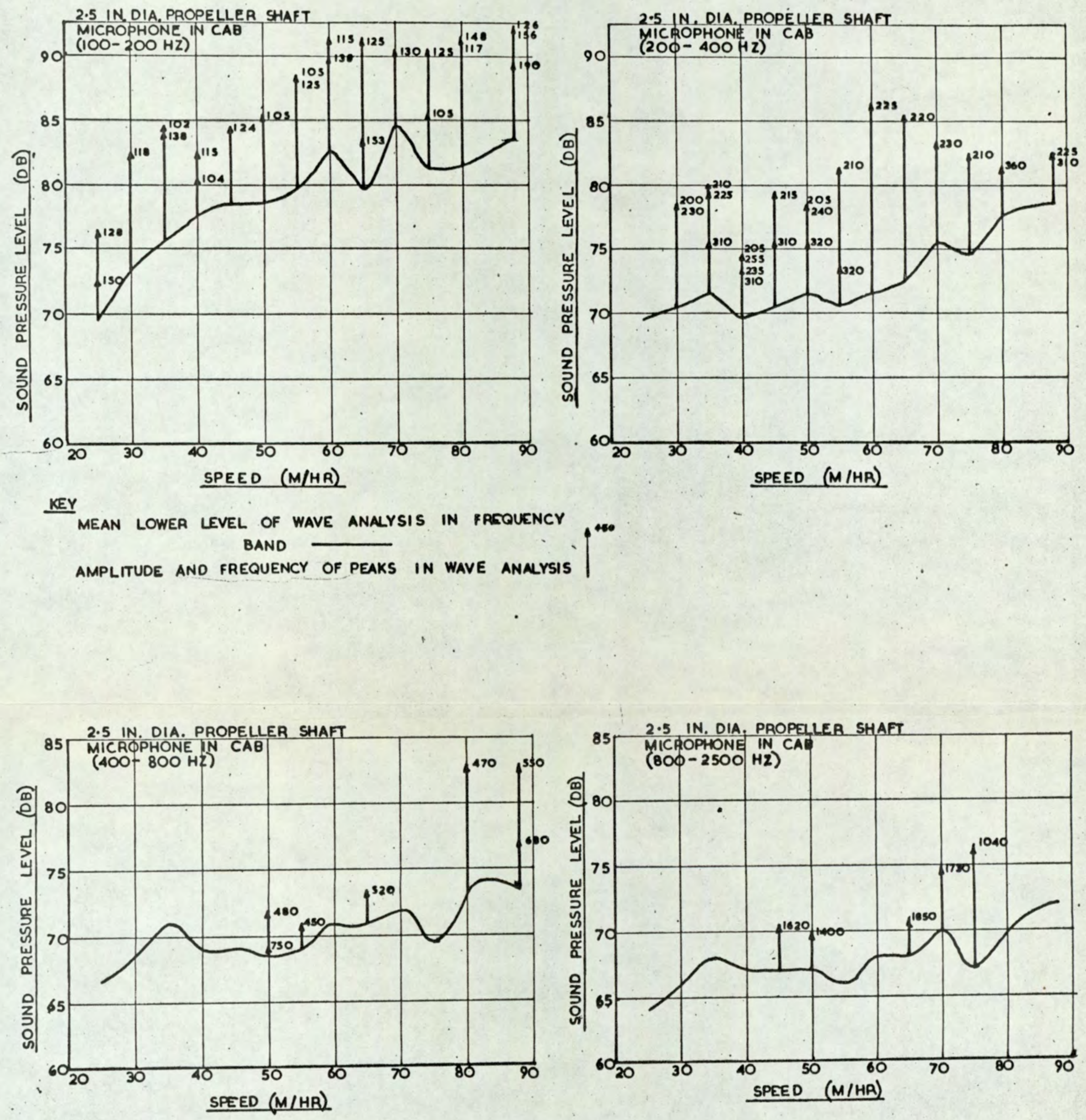
AMPLITUDE AND FREQUENCY OF PEAKS IN WAVE ANALYSIS 450





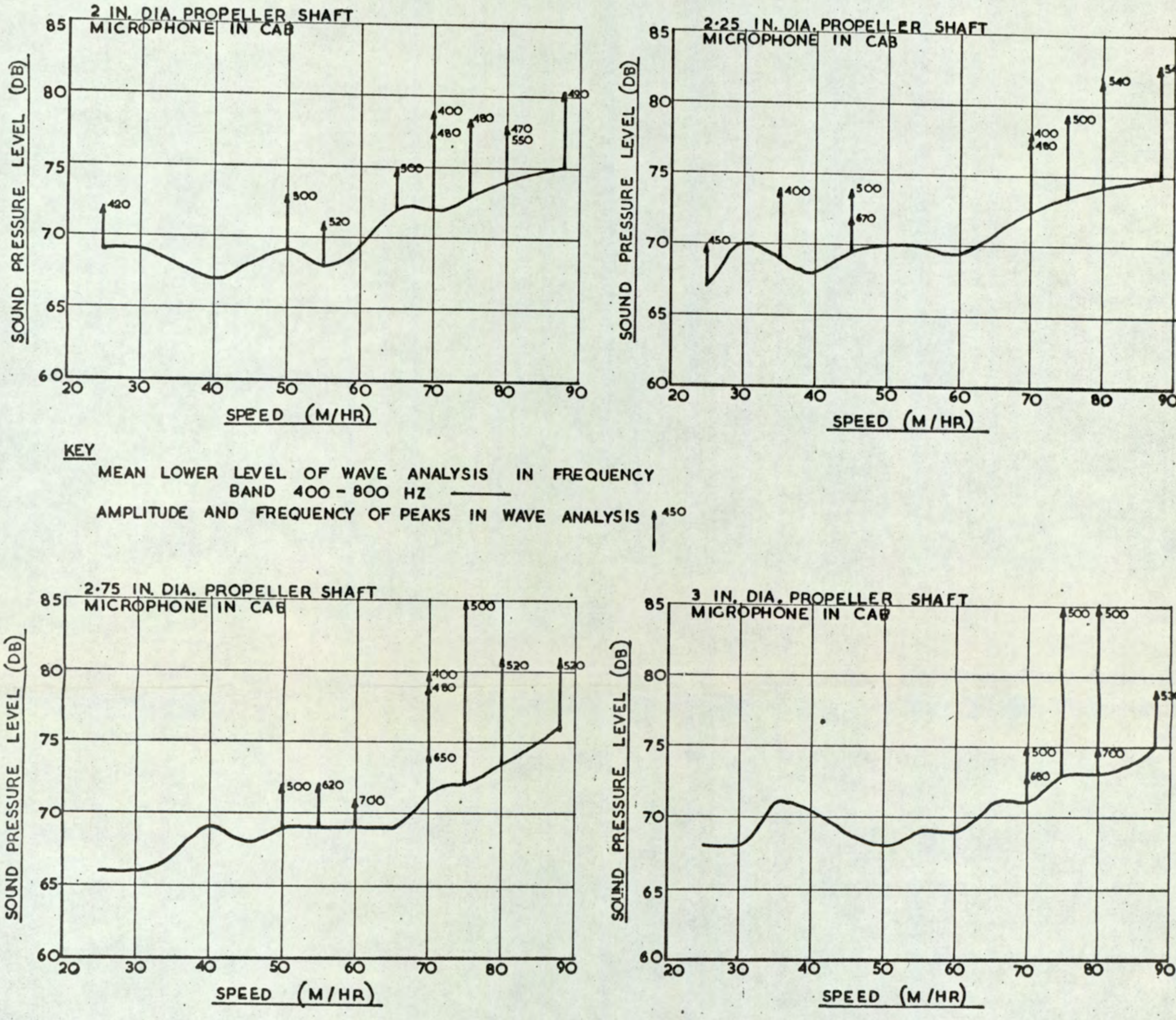
RESULTS OF WAVE ANALYSIS OF ROAD TEST RECORDINGS (400 - 800 HZ)
(STANDARD PROPELLER SHAFT)

FIG 3-32



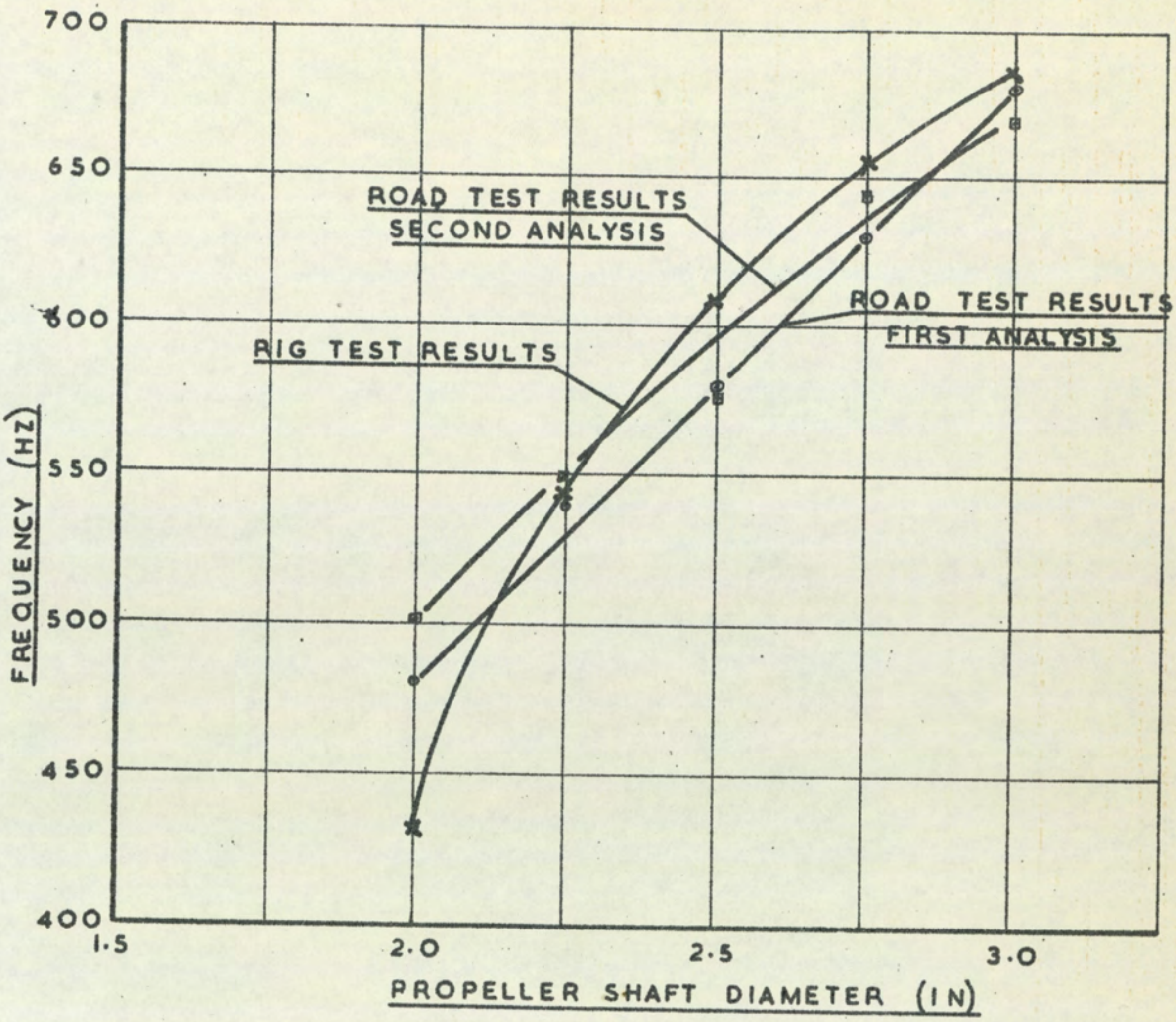
RESULTS OF WAVE ANALYSIS OF ROAD TEST RECORDINGS
STANDARD PROPELLER SHAFT

FIG 3-33



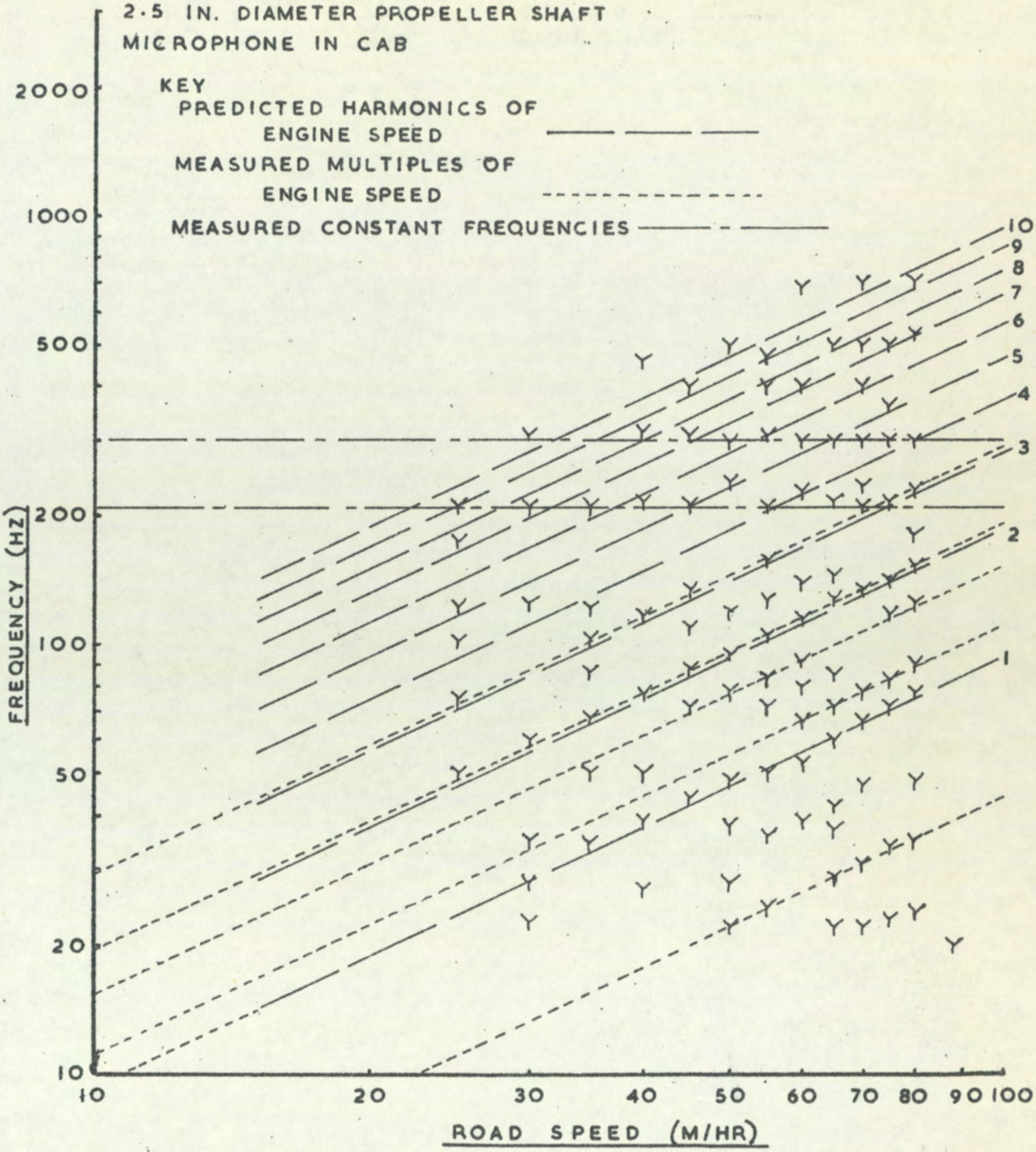
RESULTS OF WAVE ANALYSIS OF ROAD TEST RECORDINGS (400 - 800 HZ)
(NON STANDARD PROPELLER SHAFTS)

FIG 3-34



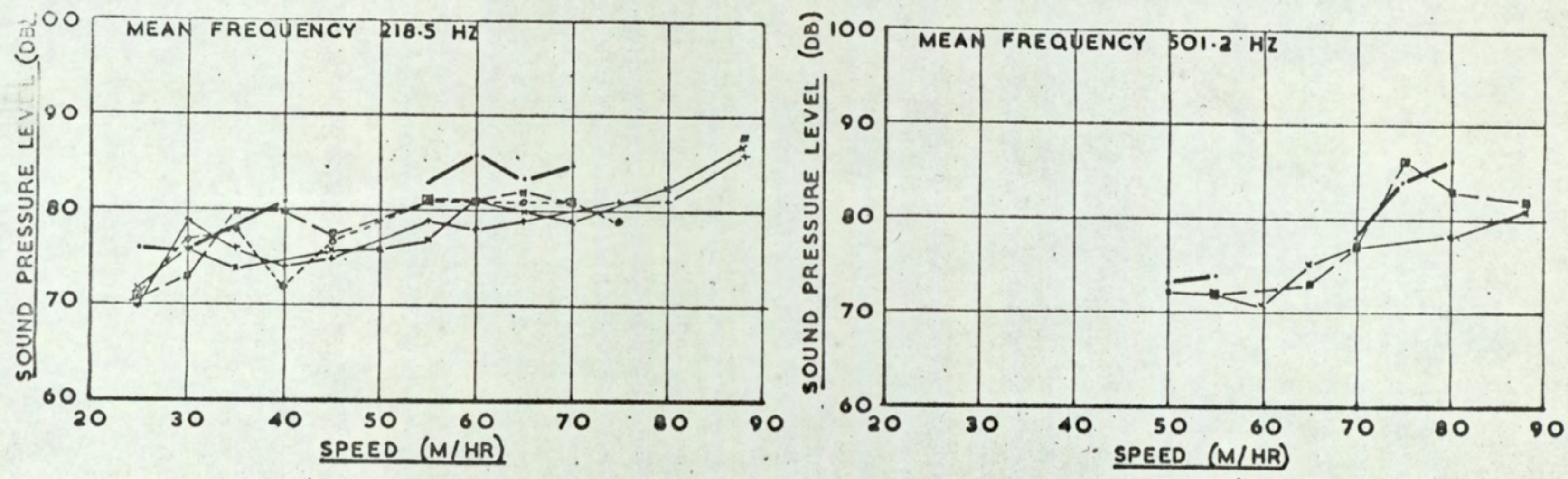
SECOND MODE OF TRANSVERSE VIBRATION OF PROPELLER SHAFT EFFECT OF DIAMETER ON NATURAL FREQUENCY

FIG 3-35



FREQUENCY V SPEED PLOT FOR MICROPHONE IN CAB

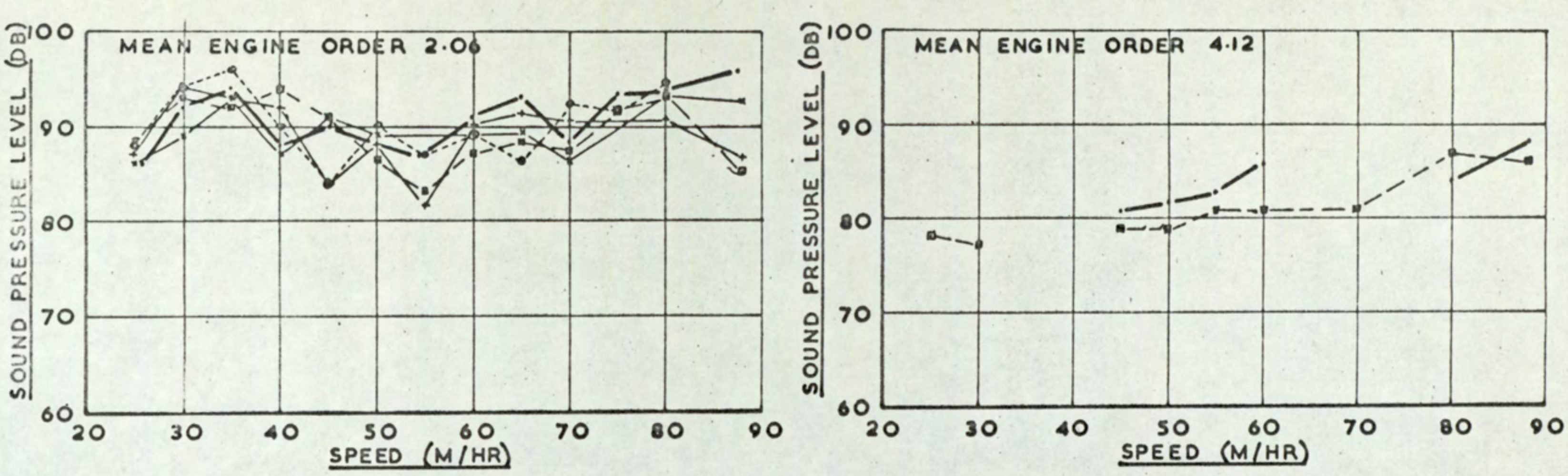
FIG 3-36



(A) CONSTANT FRQUENCY RESPONSES

KEY

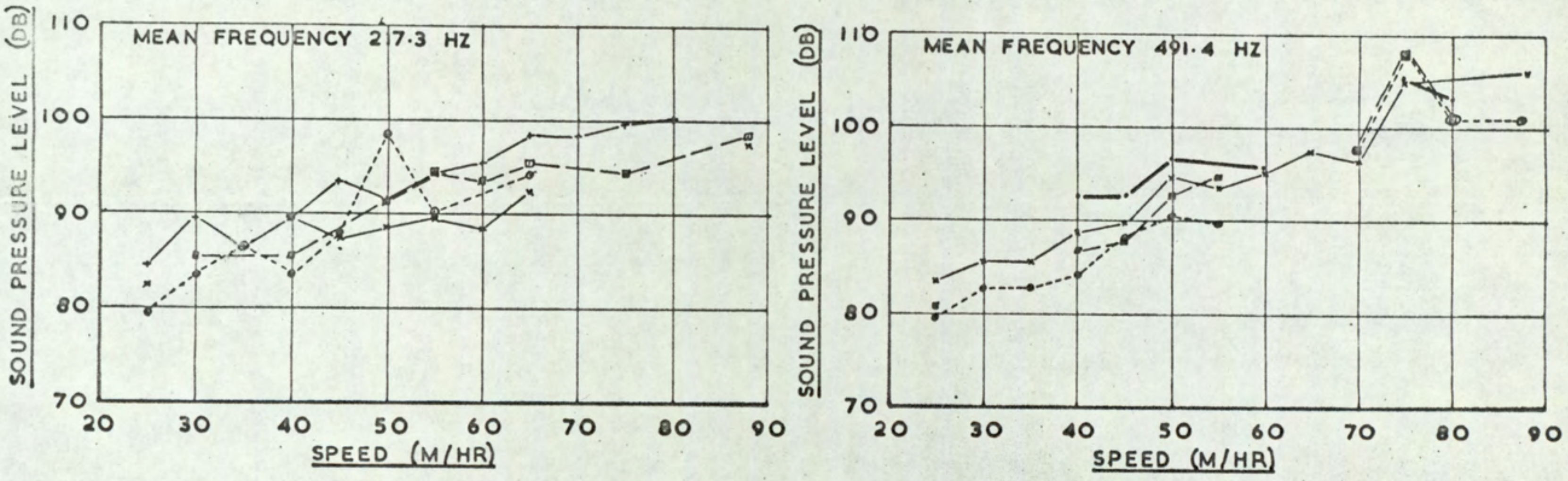
2.00 IN. DIA. PROP. SHAFT
2.25 " " " "
2.50 " " " "
2.75 " " " "
3.00 " " " "



(B) CONSTANT ENGINE ORDER RESPONSES

SPECIMEN RESULTS OF SECOND ANALYSIS OF ROAD TEST RECORDINGS
(NOISE IN PASSENGER COMPARTMENT)

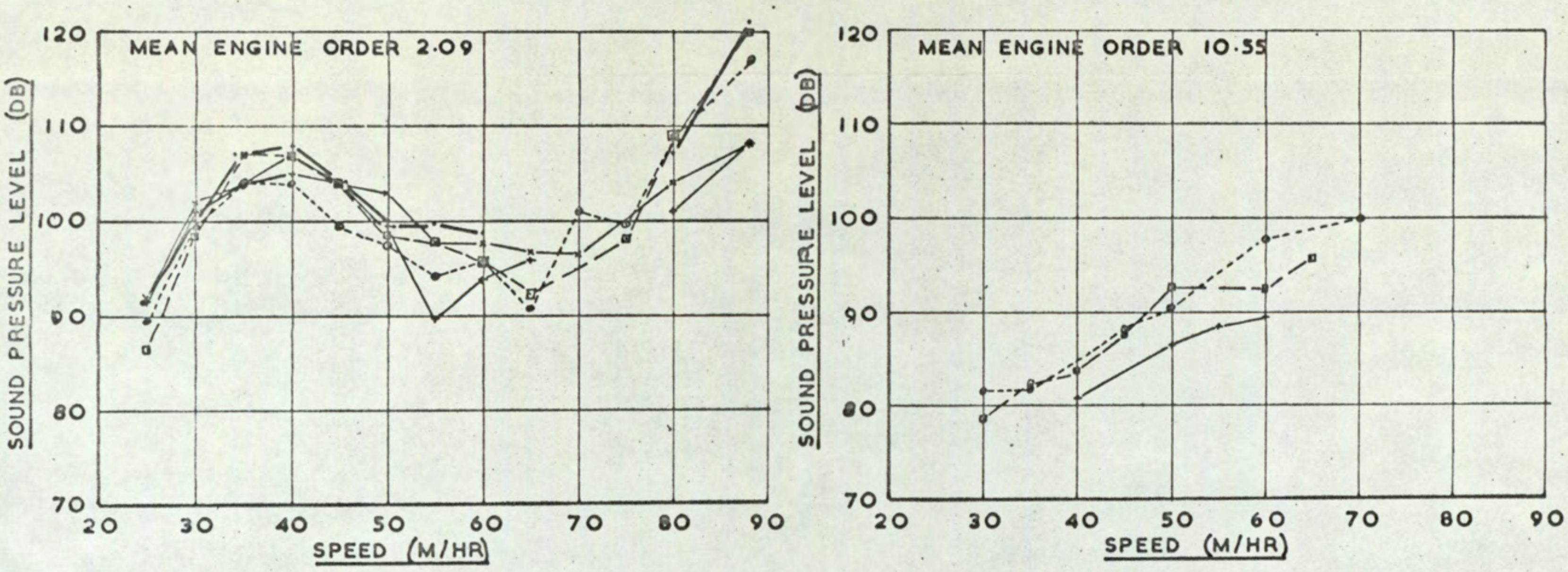
FIG 3-37



(A) CONSTANT FREQUENCY RESPONSES

KEY

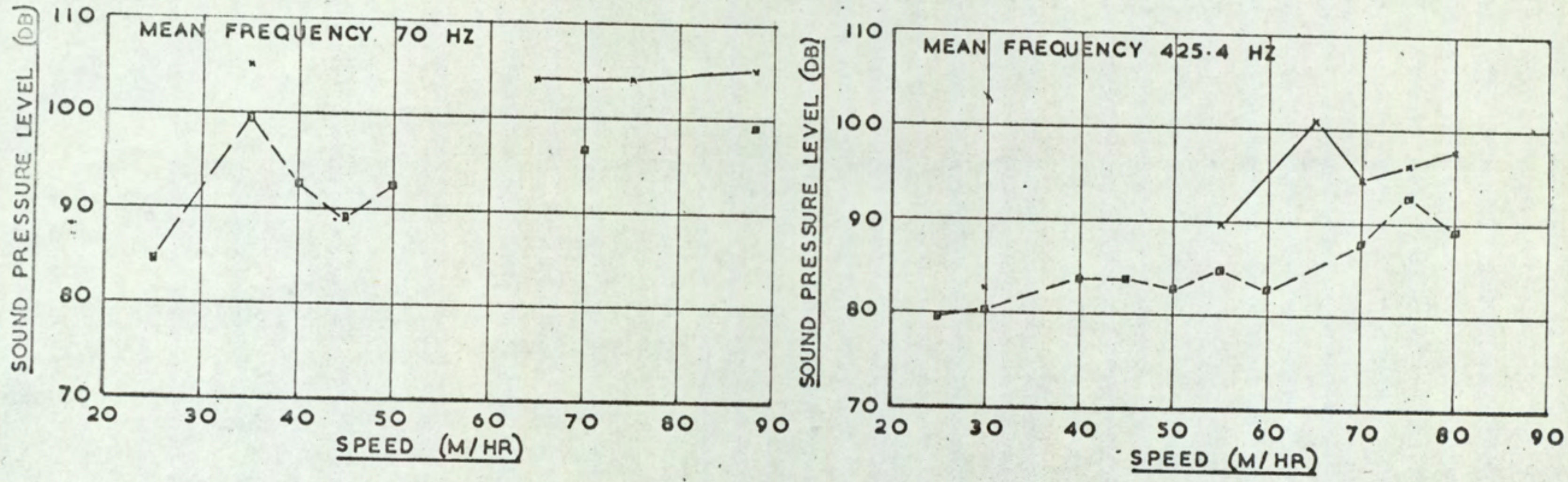
2·00 IN. DIA. PROP. SHAFT
2·25 .. ..
2·50 .. ..
2·75 .. ..
3·00 .. ..



(B) CONSTANT ENGINE ORDER RESPONCES

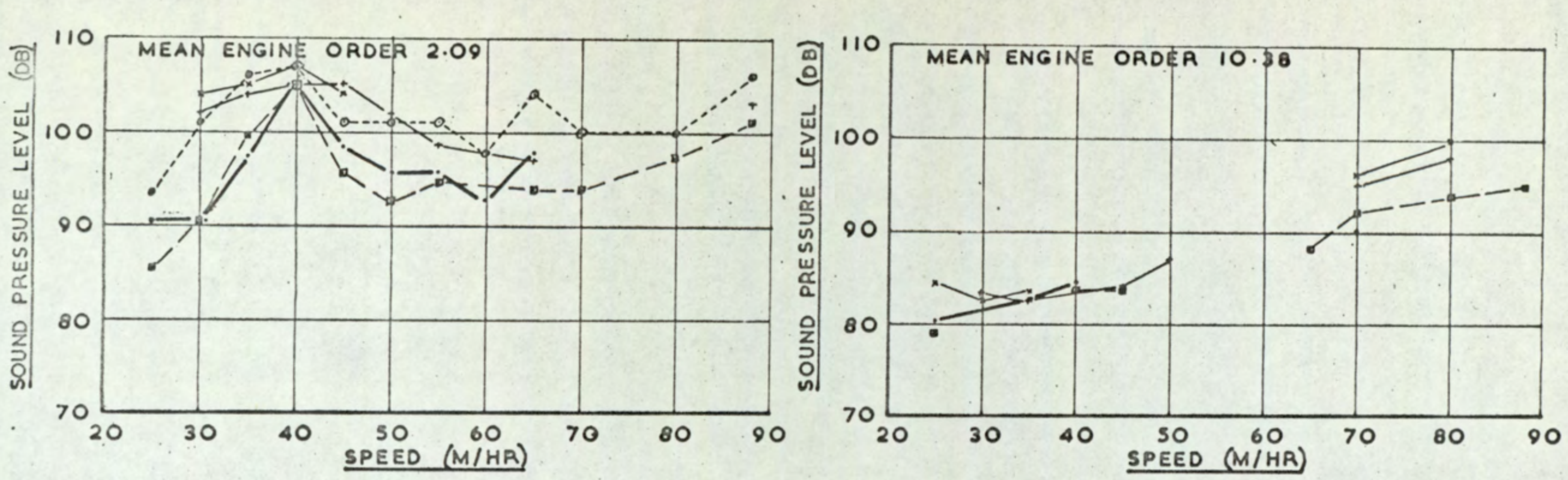
SPECIMEN RESULTS OF SECOND ANALYSIS OF ROAD TEST RECORDINGS (NOISE ABOVE PROPELLER SHAFT)

FIG 3-38



(A) CONSTANT FREQUENCY RESPONSES

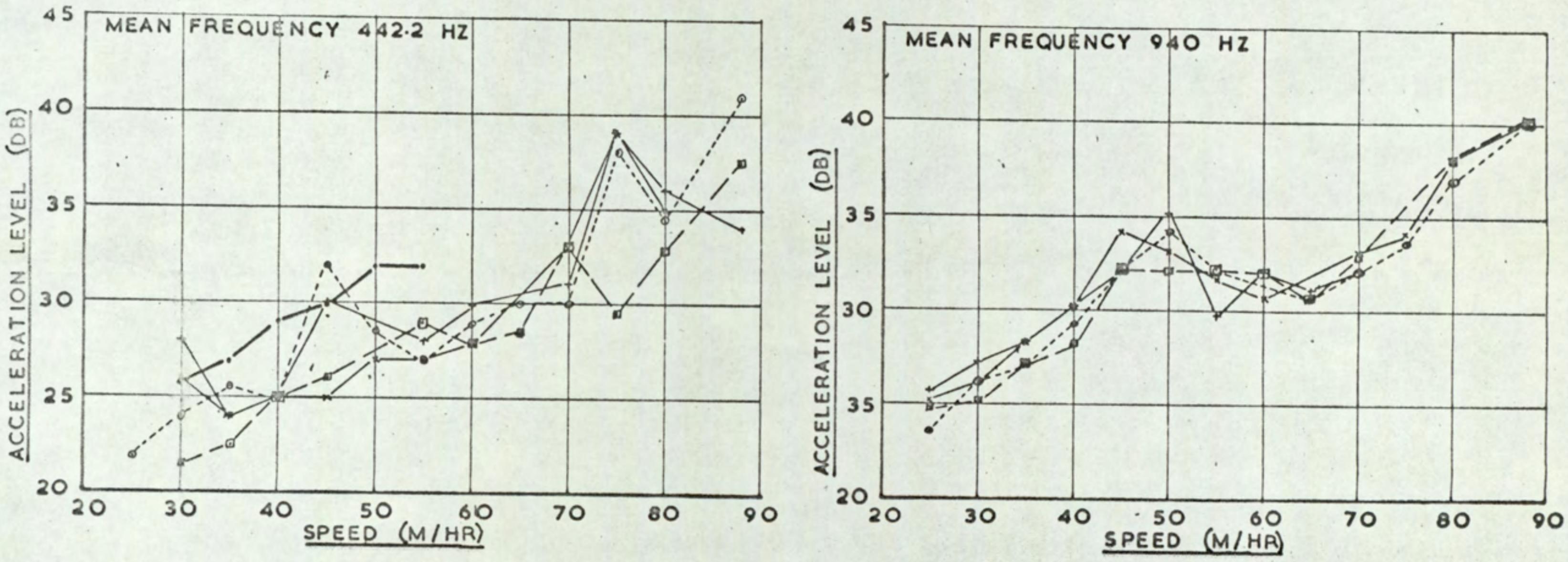
KEY
2·00 IN. DIA. PROP. SHAFT
2·25 .. ..
2·50 .. ..
2·75 .. ..
3·00 .. ..



(B) CONSTANT ENGINE ORDER RESPONSES

SPECIMEN RESULTS OF SECOND ANALYSIS OF ROAD TEST RECORDINGS
(NOISE BY REAR AXLE GEAR CARRIER)

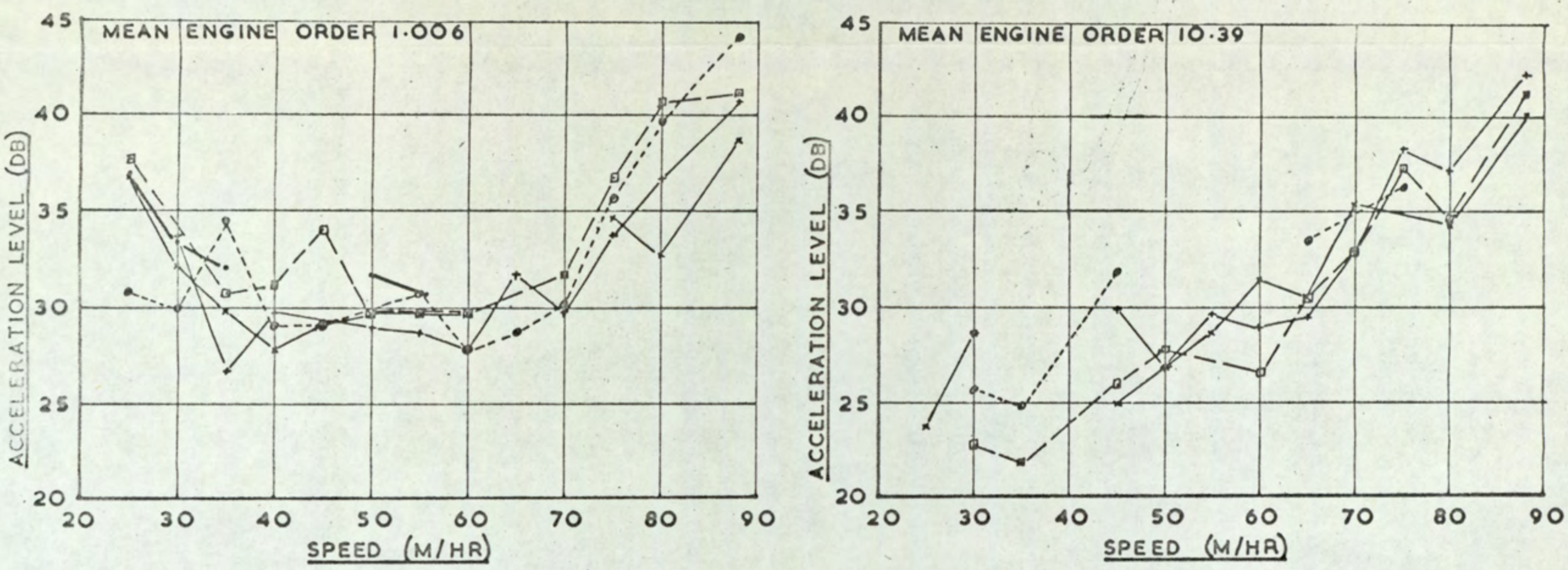
FIG 3-39



(A) CONSTANT FREQUENCY RESPONSES

KEY

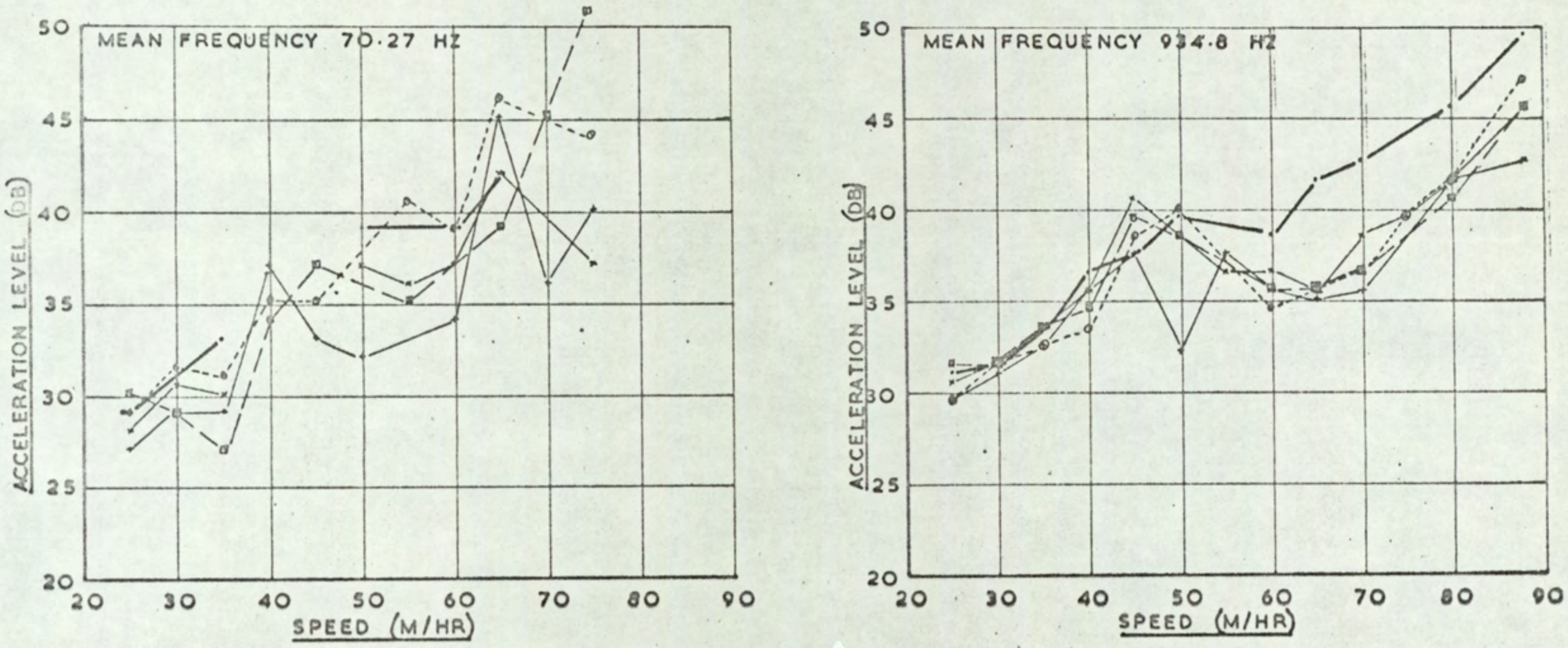
2·00 IN. DIA. PROP. SHAFT
2·25 .. ..
2·50 .. ..
2·75 .. ..
3·00 .. ..
0 DB = 0·001g



(B) CONSTANT ENGINE ORDER RESPONSES

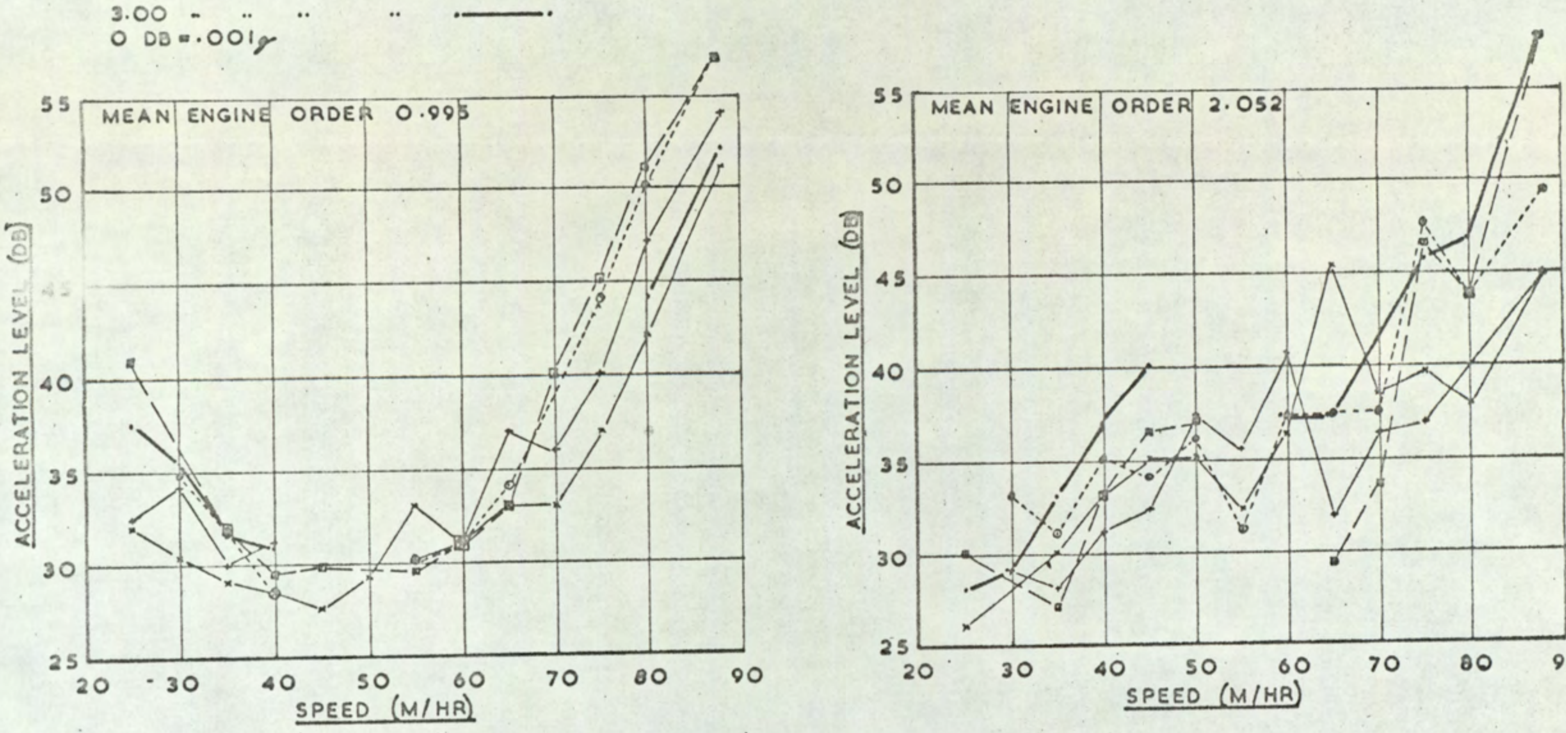
SPECIMEN RESULTS OF SECOND ANALYSIS OF ROAD TEST RECORDINGS
(VIBRATION AT POSITION 1)

FIG 3-40



(A) CONSTANT FREQUENCY RESPONSES

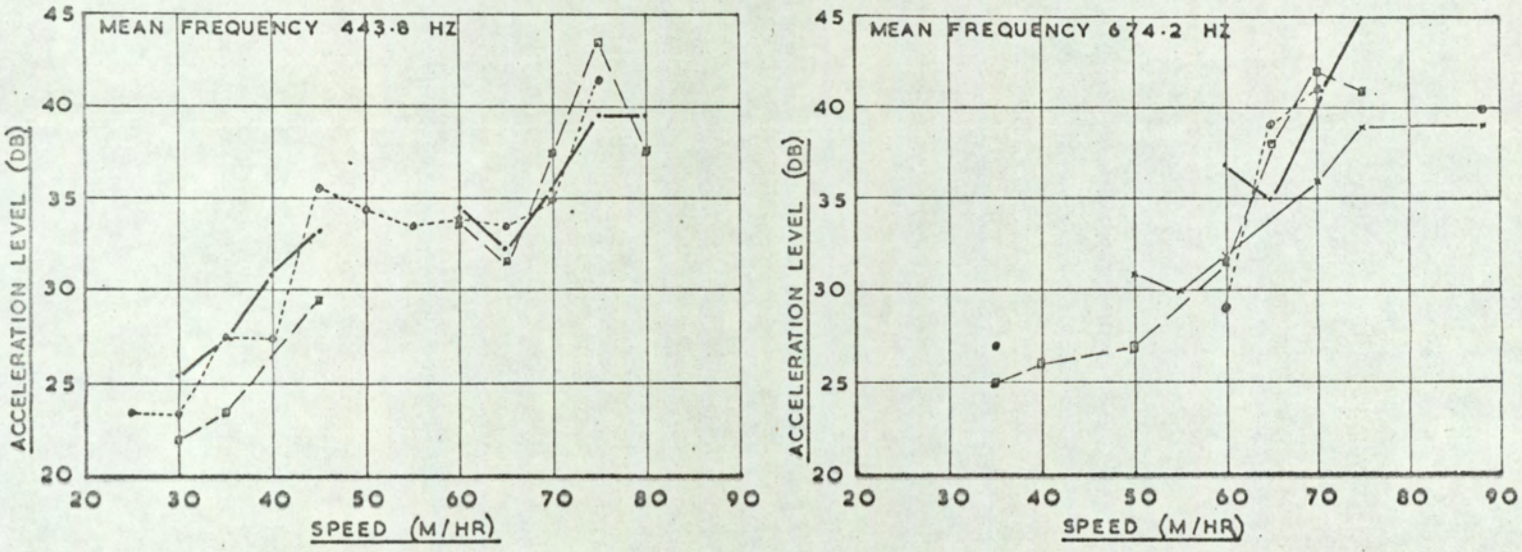
KEY
2·00 IN. DIA. PROP. SHAFT
2·25 " " " "
2·50 " " " "
2·75 " " " "
3·00 " " " "
0 DB = ·001g



(B) CONSTANT ENGINE ORDER RESPONSES

SPECIMEN RESULTS OF SECOND ANALYSIS OF ROAD TEST RECORDINGS
(VIBRATION AT POSITION 3)

FIG 3-41

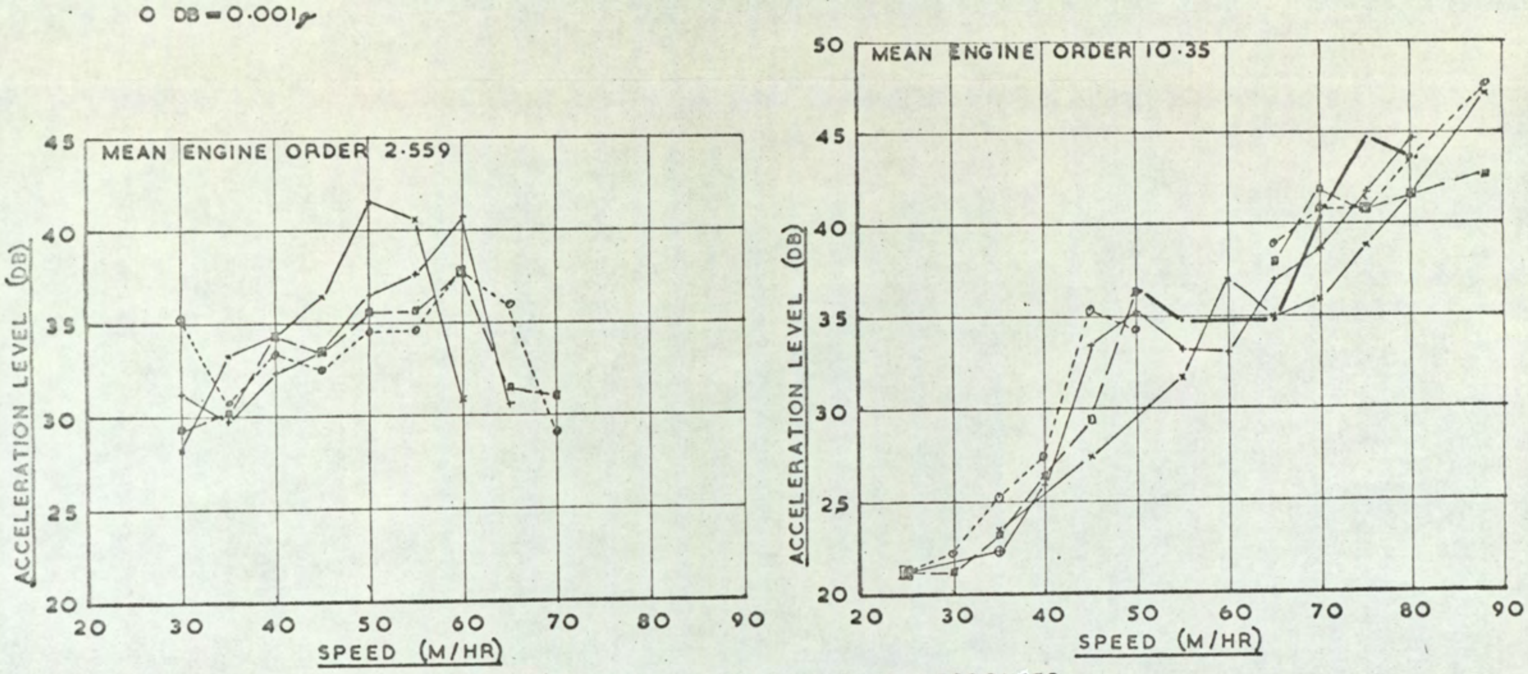


(A) CONSTANT FREQUENCY RESPONSES

KEY

2·00 IN. DIA. PROP. SHAFT
2·25 .. ..
2·50 .. ..
2·75 .. ..
3·00 .. ..

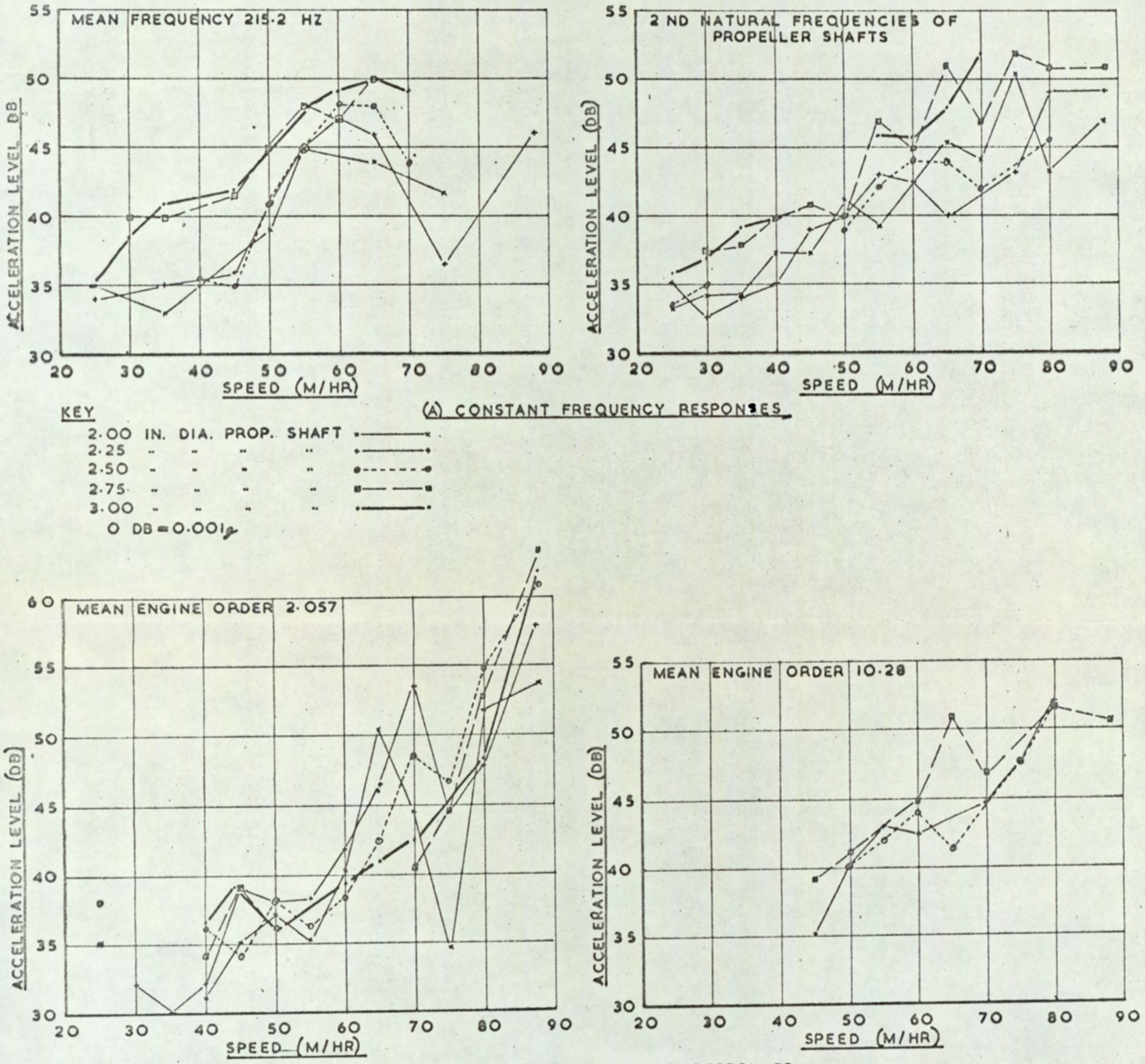
0 DB = 0·001g



(B) CONSTANT ENGINE ORDER RESPONSES

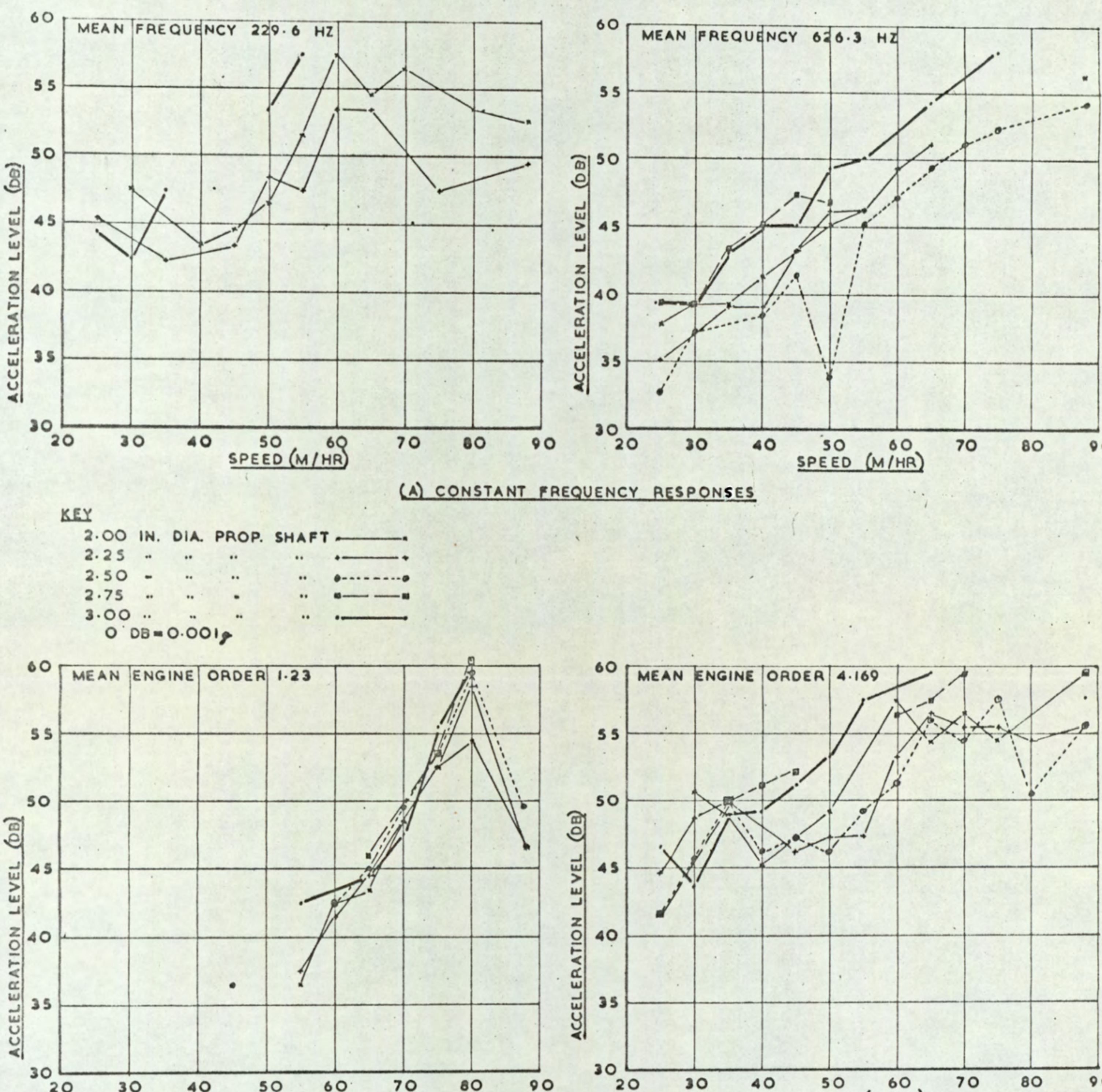
SPECIMEN RESULTS OF SECOND ANALYSIS OF ROAD TEST RECORDINGS
(VIBRATION AT POSITION 9)

FIG 3-42



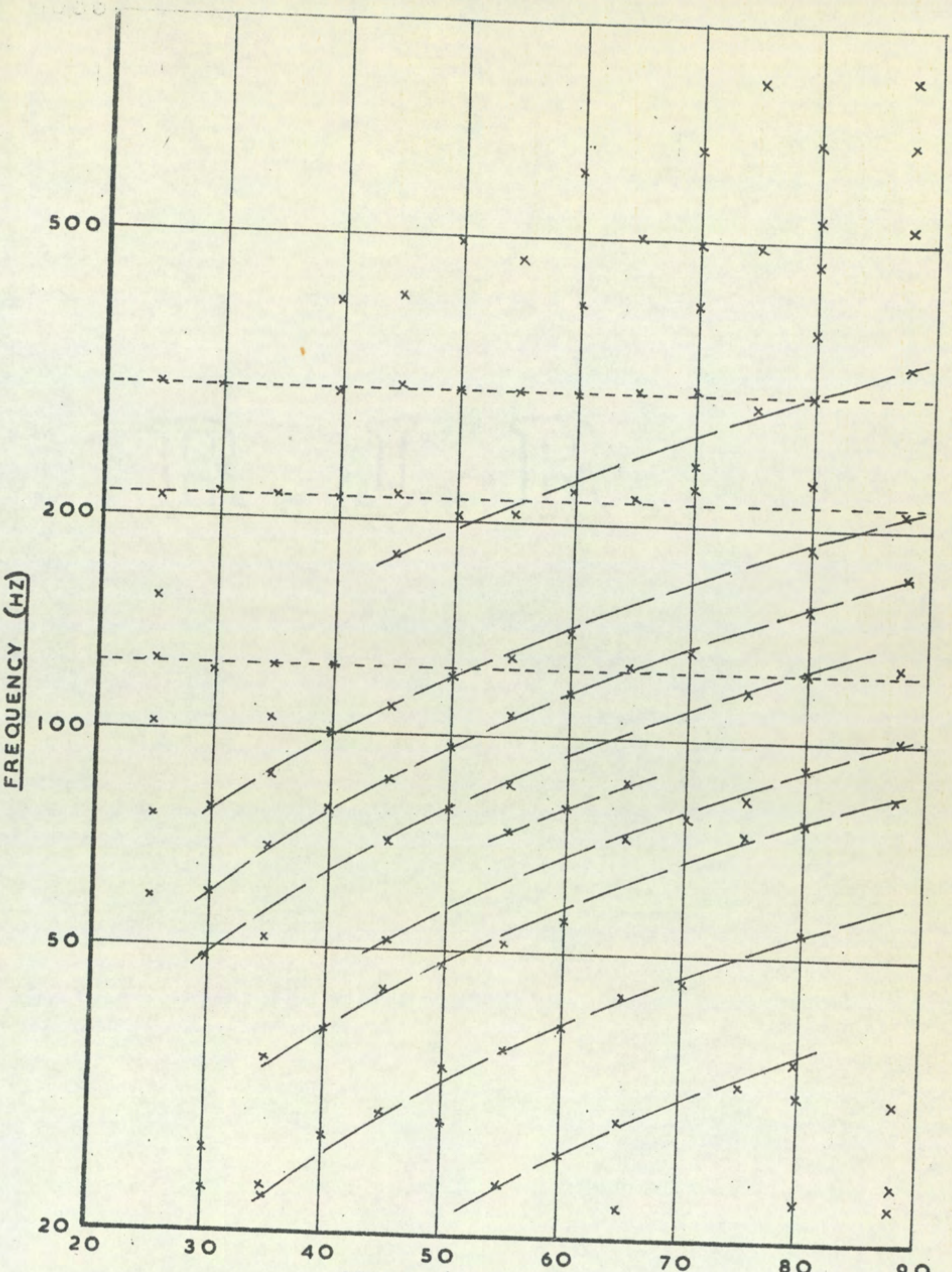
SPECIMEN RESULTS OF SECOND ANALYSIS OF ROAD TEST RECORDINGS
(VIBRATION AT POSITION 11)

FIG 3-43



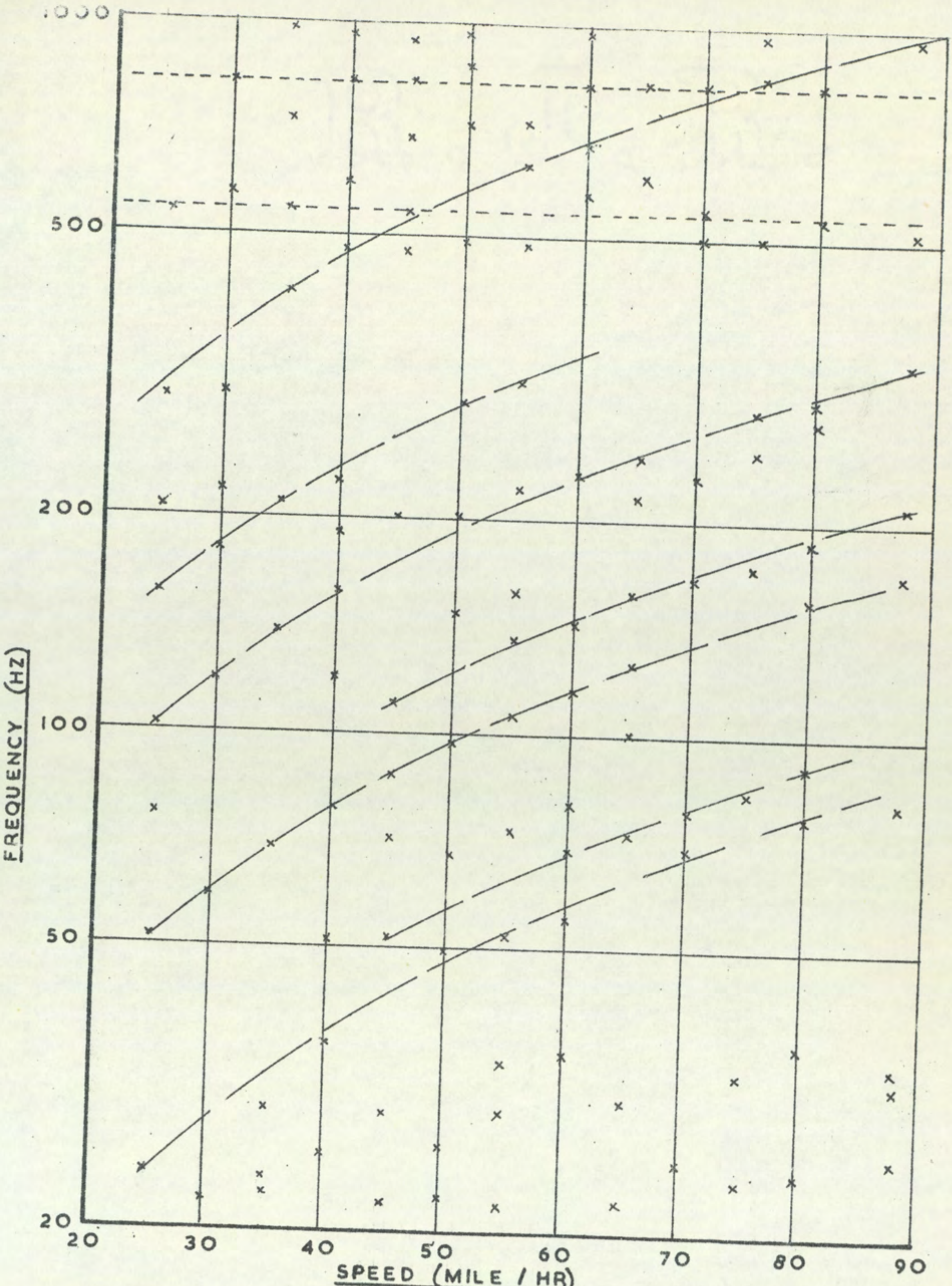
SPECIMEN RESULTS OF SECOND ANALYSIS OF ROAD TEST RECORDINGS
(VIBRATION AT POSITION 12)

FIG 3-44



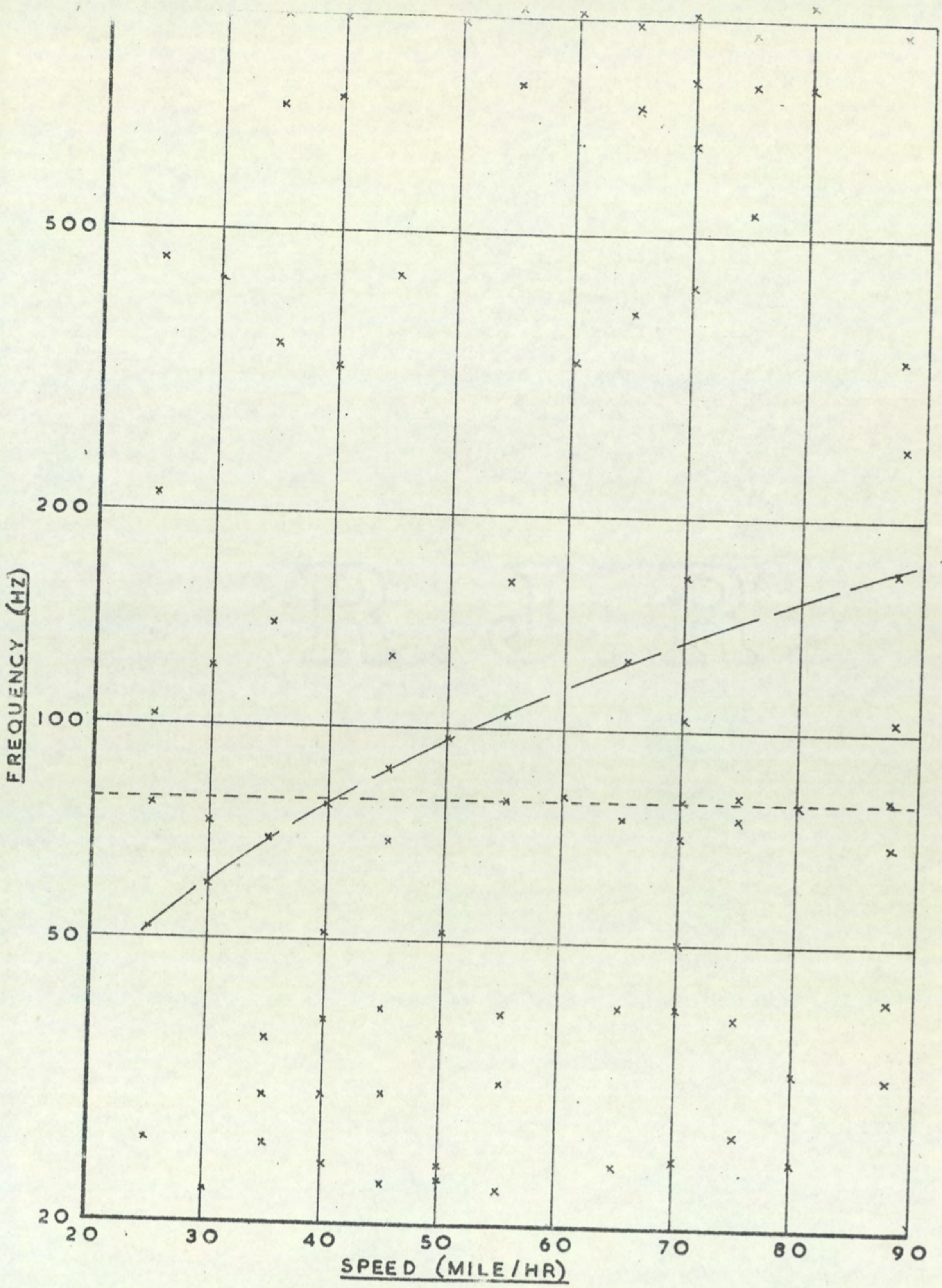
FREQUENCIES COMMON TO ALL PROPELLER SHAFTS
NOISE IN CAB

FIG 3-45



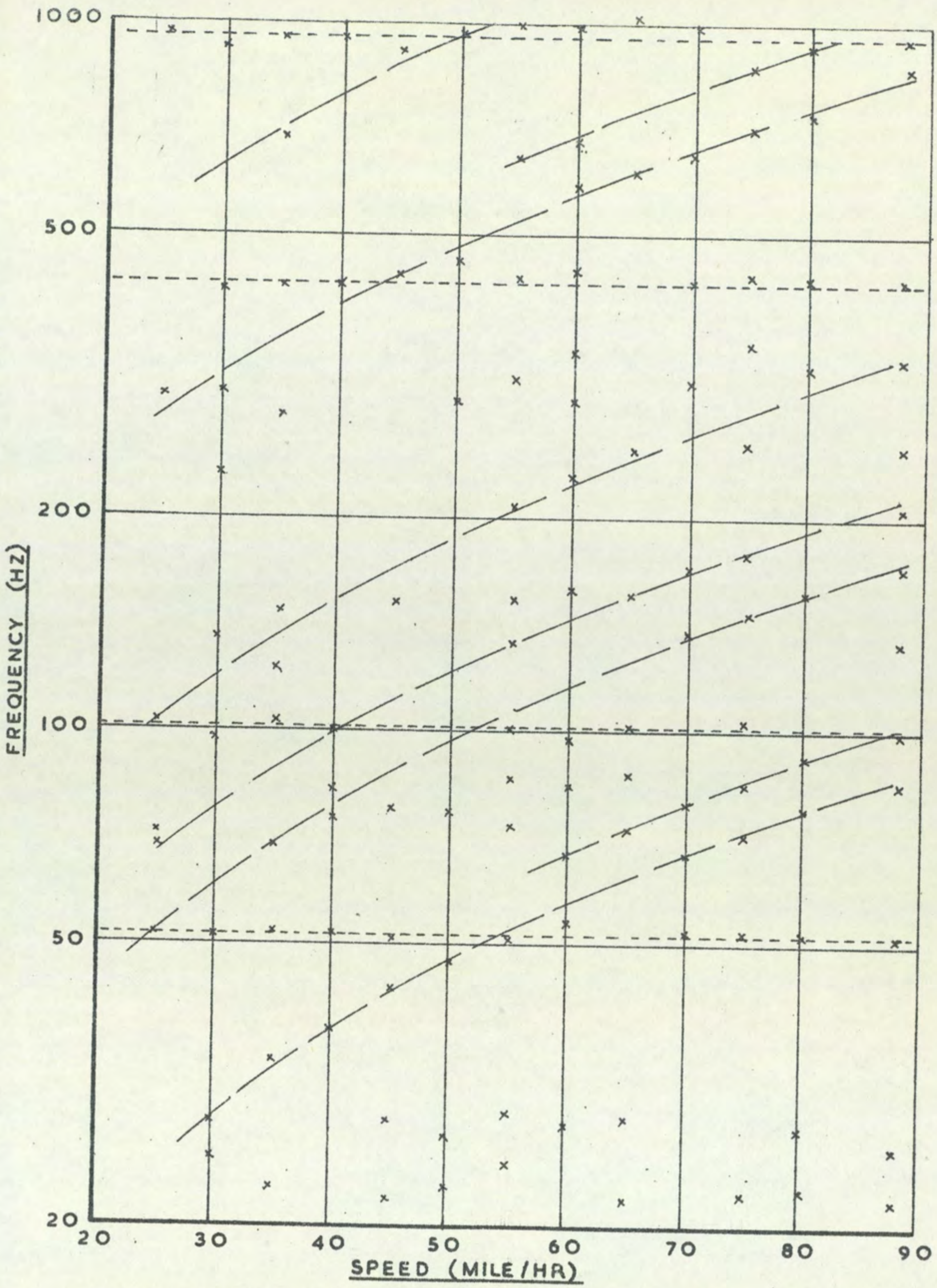
FREQUENCIES COMMON TO ALL PROPELLER SHAFTS
NOISE ABOVE PROPELLER SHAFT

FIG 3-46



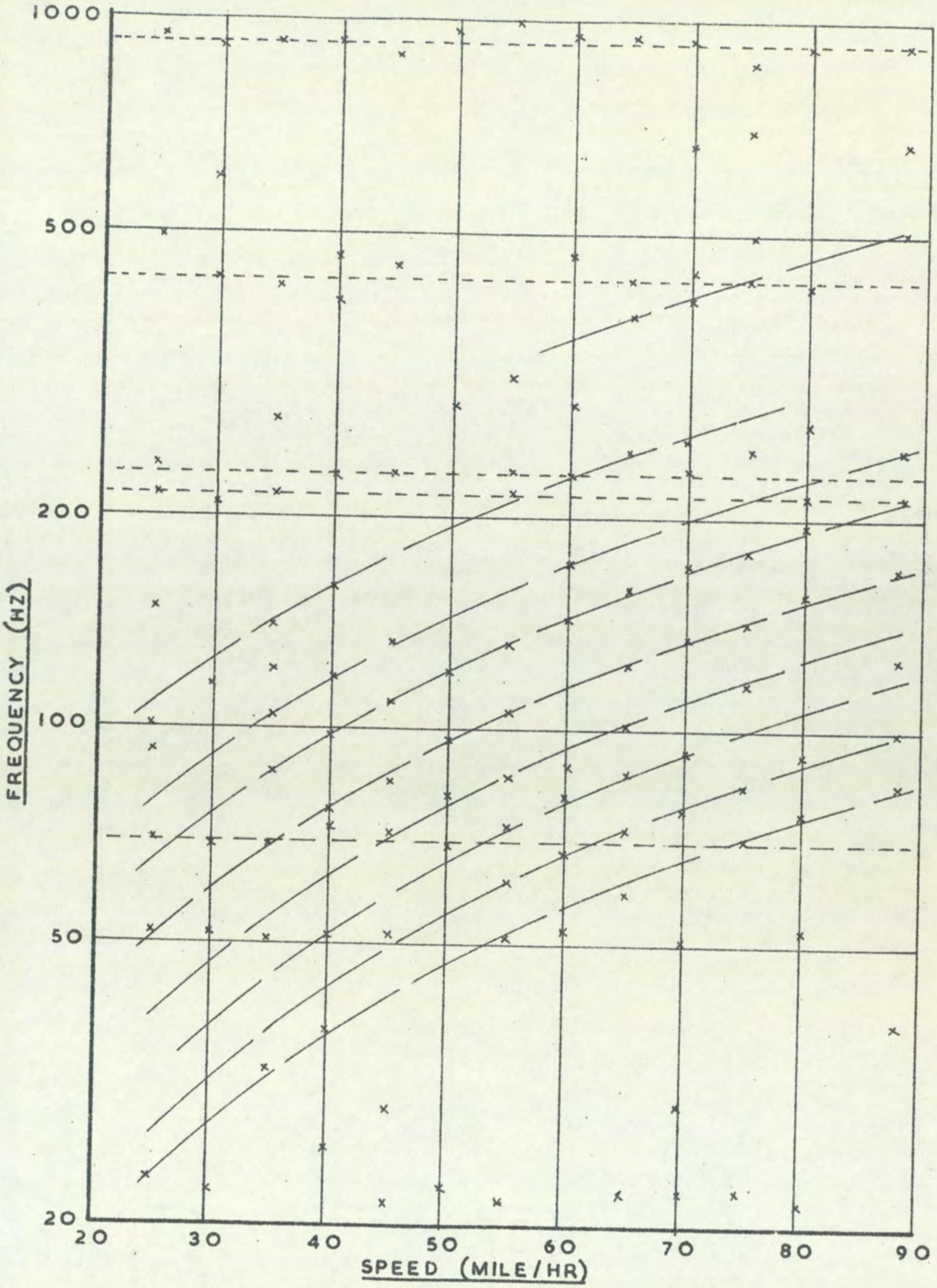
FREQUENCIES COMMON TO ALL PROPELLER SHAFTS
NOISE BY REAR AXLE GEAR CARRIER

FIG 3-47



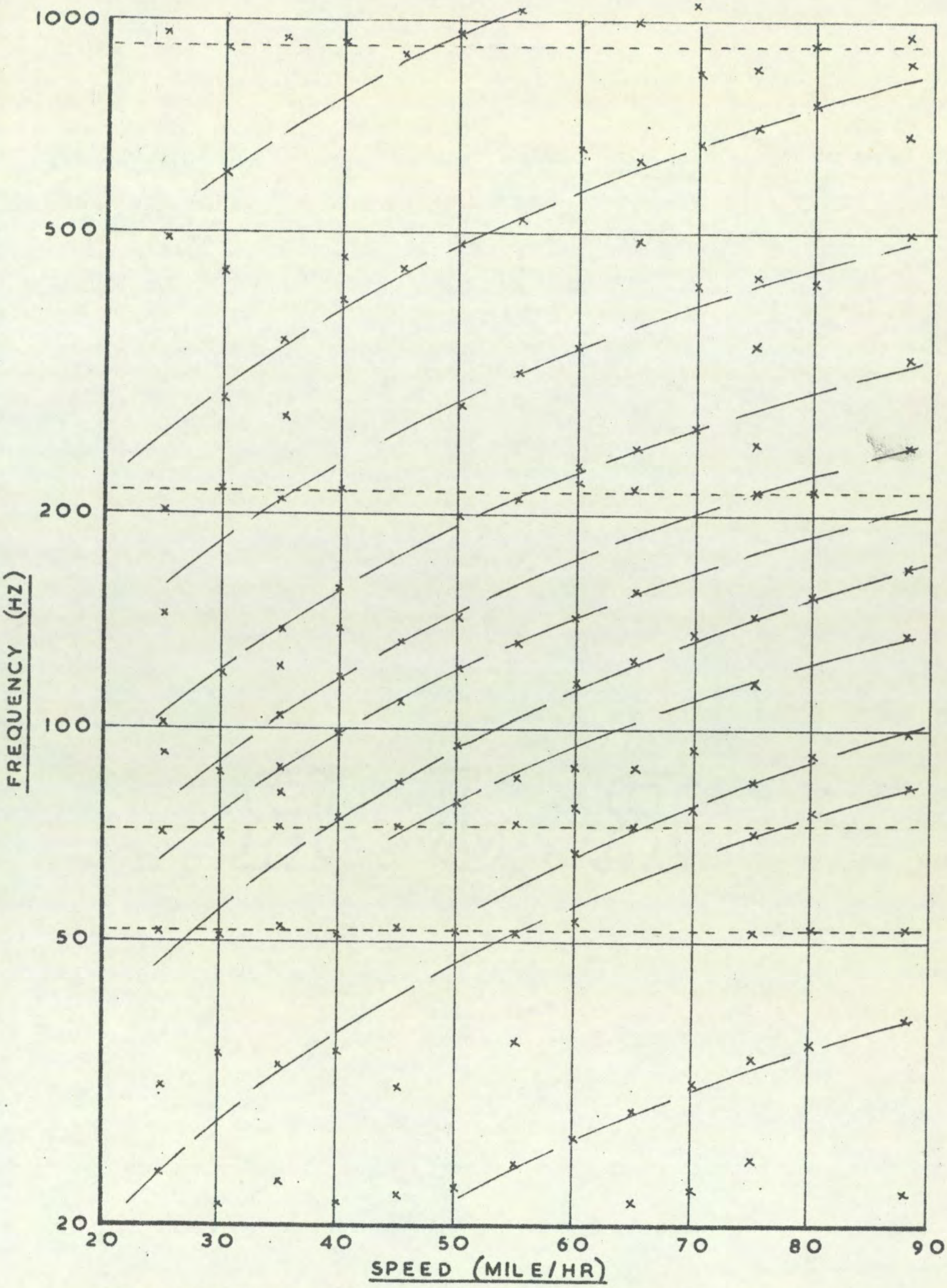
FREQUENCIES COMMON TO ALL PROPELLER SHAFTS' VIBRATION AT POSITION I

FIG 3-48



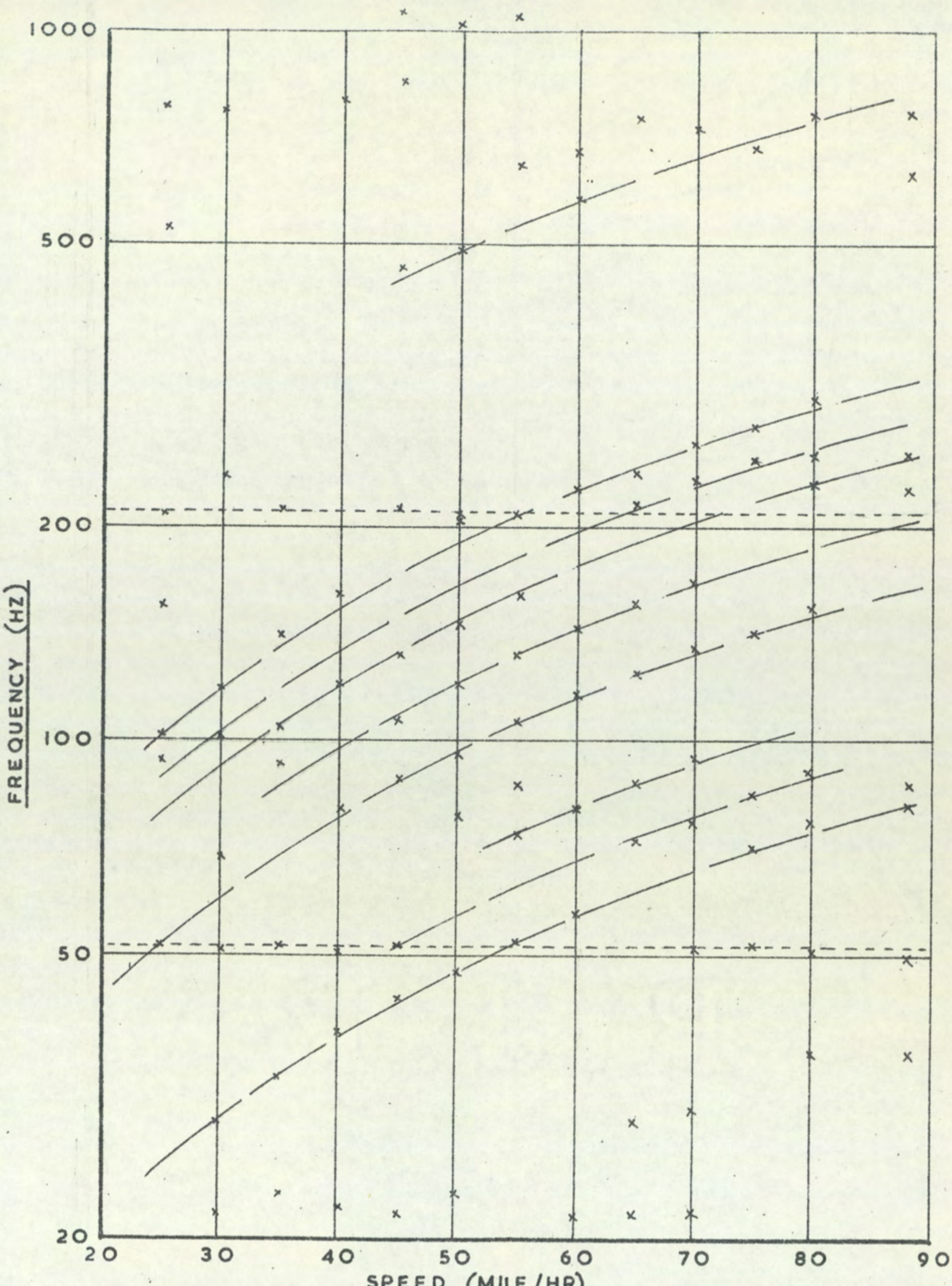
FREQUENCIES COMMON TO ALL PROPELLER SHAFTS
VIBRATION AT POSITION 3

FIG 3-49



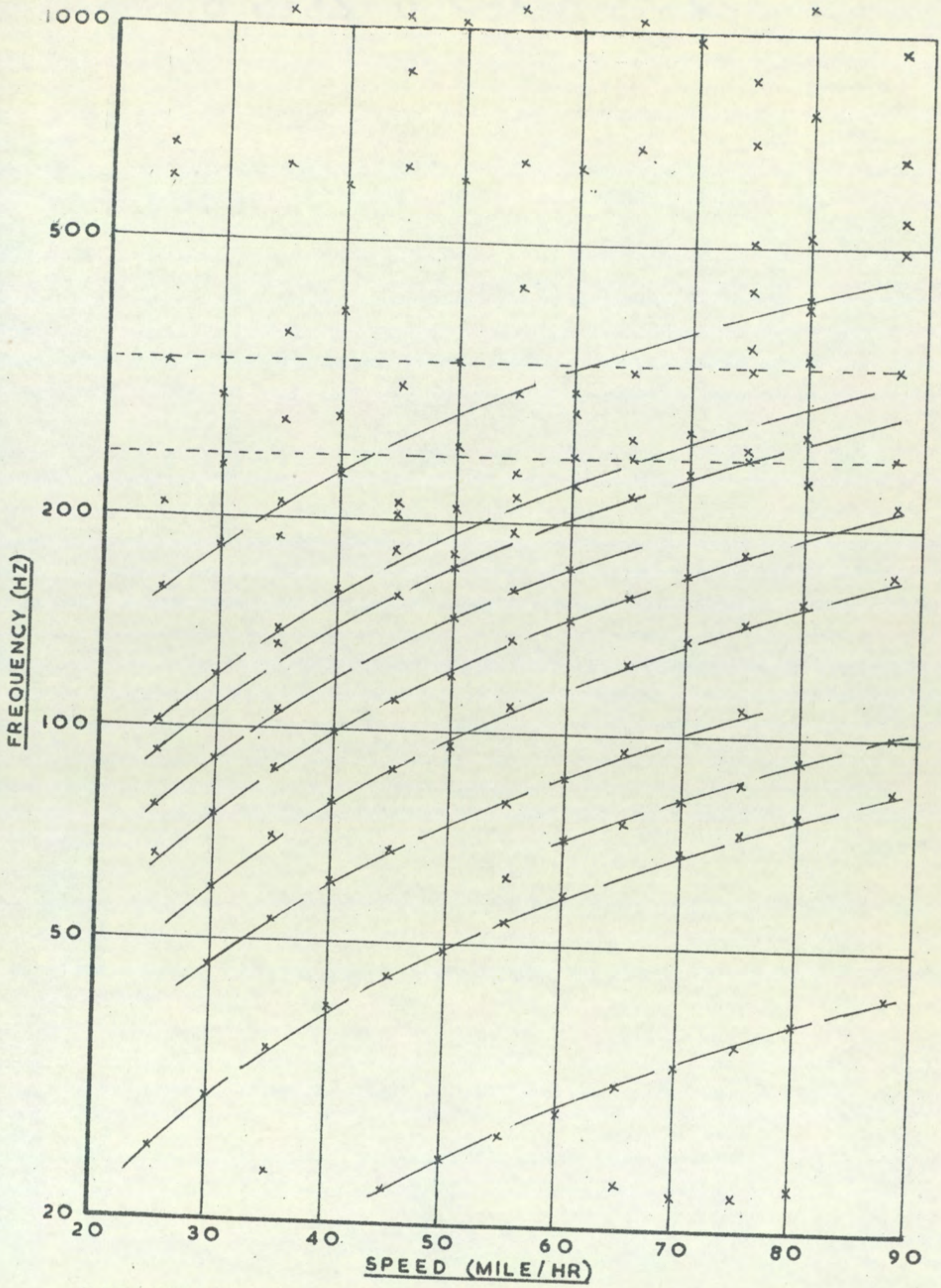
FREQUENCIES COMMON TO ALL PROPELLER SHAFTS
VIBRATION AT POSITION 9

FIG 3-50



FREQUENCIES COMMON TO ALL PROPELLER SHAFTS
VIBRATION AT POSITION 11

FIG 3-51



FREQUENCIES COMMON TO ALL PROPELLER SHAFTS
VIBRATION AT POSITION 12

FIG 3-52

CHAPTER 4 THEORY AND PRACTICE OF WAVE ANALYSIS

4-1) Summary

The theory of wave analysis is discussed and practical methods of performing analyses of electrical analog signals electronically are described. Equipment used for the wave analyses of road test results is described fully, and its shortcomings detailed.

4-2 Types of data to which wave analysis may be applied

In wave analysis we are considering two types of data, stationary periodic, and stationary complex periodic. Both of these types of data are determanistic, i.e. they can be described by suitable equations in the independent variable.

Stationary periodic data can be expressed in equation form by a fourier series as below:

$$Y(t) = K_1 \ (\sin 2\pi ft + \theta_1) + K_2 \sin (4\pi ft + \theta_2) + \text{------} \qquad 4\text{-}1a$$

or

$$Y(t) = \sum_{n=1}^{\infty} K_n \ \sin \ (2\pi fnt + \theta_n) \qquad 4\text{-}1b$$

Where K_n = the peak amplitude of the n^{th} harmonic

f = the fundamental frequency

θ_n = the phase of the n^{th} harmonic with respect to the fundamental.

Stationary complex signals are a mixture of sinusiods which are not necessarily related harmonically, but which have constant amplitudes. The mixtures may be products as well as sums of the sinusiods in any combination. This type of data is obviously not capable of such a neat mathematical description as the previous case.

4-3) Types of wave analysers

The only way of performing true fourier series analysis of data is to process it on a digital computor, but this requires expensive digitizing equipment for the conversion of analog signals to digital samples, and there is also the problem of establishing the fundamental period of the signal. In order to overcome these problems, the most usual way of analysing electrical analog signals is with an electronic analog wave analyser.

There are two basic types of analog wave analyser:-

i) contiguous band analyser

ii) swept filter analyser

The swept filter analyser can be further sub-divided into two classes, constant bandwidth, or constant percentage bandwidth. All the above analysers give an assessment of the amplitudes of the periodic components of the signal being processed, but no phase information. This lack of phase information is normally of little consequence in vibration analysis, and was no disadvantage in the investigations described in this thesis.

A block diagram of a contigous band wave analyser is shown in fig. 4-1. An analyser of this type consists of a set of filters (band pass) each centred at a different frequency, and a set of detection and averaging circuits, one set for each filter. The signal to be analysed is applied simultaneously to the inputs of all the filters, and after a short time lapse the outputs of the detection circuits are logged.

Quick look analysis can be performed very efficiently on a contigoous band analyser employing fairly wide band filters not necessarily all of the same bandwidth. However, for detailed analysis particularly of closely spaced components, the cost of the contigeous band analyser becomes prohibative. Due to the lower cost, the swept filter analyser is the most common form of analyser used for detailed analysis, and of the two classes of analyser, the constant percentage bandwidth analysers is the most frequently used.

As the name implies, the constant percentage bandwidth analyser is based on a filter which has a bandwidth proportional to the centre frequency to which the filter is tuned. These filters consist of tuned rejection circuits used in the feedback of an operational amplifier as shown in the block diagram in fig. 4-2.

The main disadvantage of constant percentage bandwidth analysers is the poor filter frequency response function shapes obtained due to the requirement that the components of the rejection circuits must be variable in order that the centre frequency of the filter can be varied. A further limitation is the low ultimate rejection of the filters resulting from the gain obtainable from the operational amplifiers.

The constant bandwidth analyser is not described in this chapter since it is discussed in detail in chapter 6.

4-4) <u>System used for the analysis of road test recordings</u>

The basic unit of the system was a Bruel and Kjoer Type 2107 narrow band analyser. This instrument was a constant percentage bandwidth analyser operating on the principle described above. Automatic sweep of the filter centre frequency was achieved by a mechanical drive from a Bruel and Kjoer Type 2305 level recorder which also provided the detection, averaging and pen recording facilities for the analysis.

Since the analyser was found to require approximately 12 min of signal to produce a complete analysis, it was necessary to employ a continuous loop magnetic recorder to obtain this length of record by recirculation of a short segment of the source recording. This system gave satisfactory results, but required rather frequent operator attention to change the recordings.

To overcome the problem of frequent operator intervention two features of the equipment were used:-

i) The analyser drive was arranged such that on the completion of one complete frequency sweep, it automatically commenced a further sweep.

ii) The continuous loop magnetic tape recorder used for storing the analog signals had 16 tracks.

These features were combined by the use of a uniselector which was arranged to select the 16 channels sequentially. The uniselector was triggered by the closing of a pair of contacts which were operated by a cam on the analyser. The cam on the analyser was set to close the contacts at the end of a complete frequency sweep, so enabling 16 channels of data to be analysed automatically. A block diagram of the complete system is shown in fig. 4-3, and a picture of the equipment in fig. 4-5.

4-5) Limitations and practical difficulties of wave analysis of road test recordings

The major limiting factor in the use of the equipment described above for the analysis of the magnetic tape recordings was the poor filter characteristics. As can be seen from fig. 4-4, the filter frequency response function had relatively wide skirts, and a poor ultimate rejection of only 56 Db. The first of these factors resulted in the running together of any closely spaced component of the signal being analysed with a resulting loss in resolution. The second factor also resulted in a loss of resolution due to the filter passing fairly large amounts of noise signal which when integrated over the full bandwidth of the analyser was sufficient to obscure the lower level components.

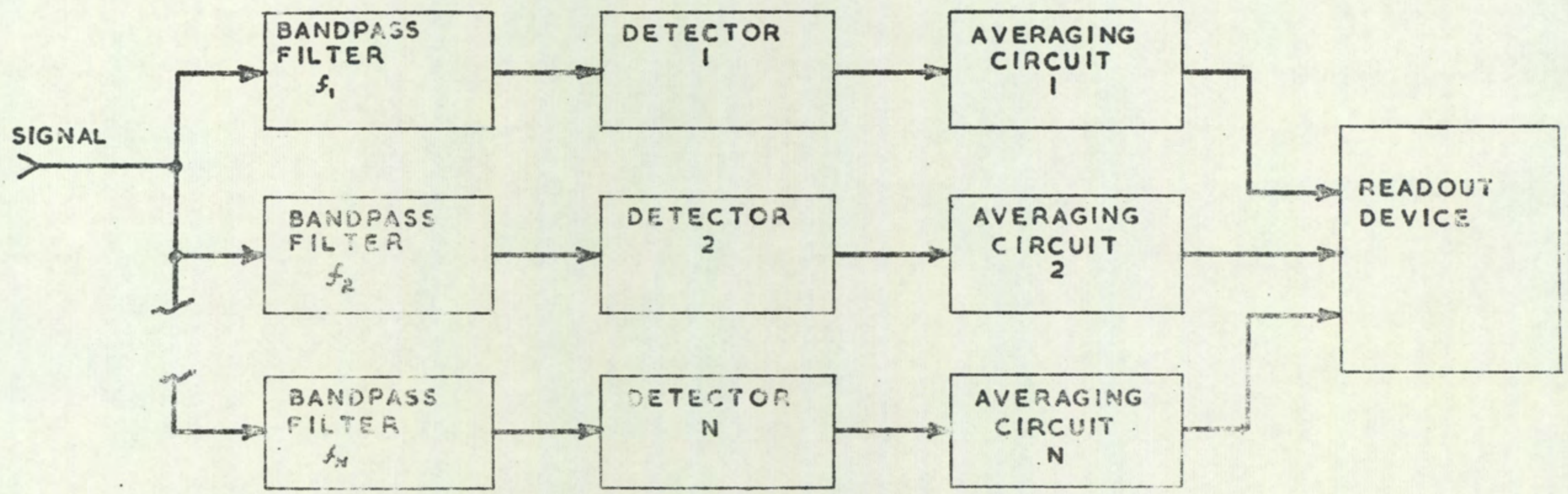
The main difficulties associated with the analysis of the particular signals obtained from the road tests have already been discussed in the previous chapter. However, one significant fact should be mentioned here in relation to the type of data being analysed. Reference to figs. 3-29 and 30 will show that the wave analyses showed an upward trend with frequency, particularly in the lower amplitude regions of the spectrum. This was due to the increase in bandwidth of the filter with centre frequency and was a strong indication that the data was broad band random, and not stationary complex periodic as originally assumed.

A major problem encountered on the operational side of the complete system arose as a result of the use of a loop recorder. The joint in the magnetic tape used to form the continuous loop had the effect of producing a large transient output from the recorder which caused the filter of the analyser to ring particularly at low frequencies. Increasing the averaging time to equal the loop repeatition time reduced the effect of these transients, but did not cure the problem completely.

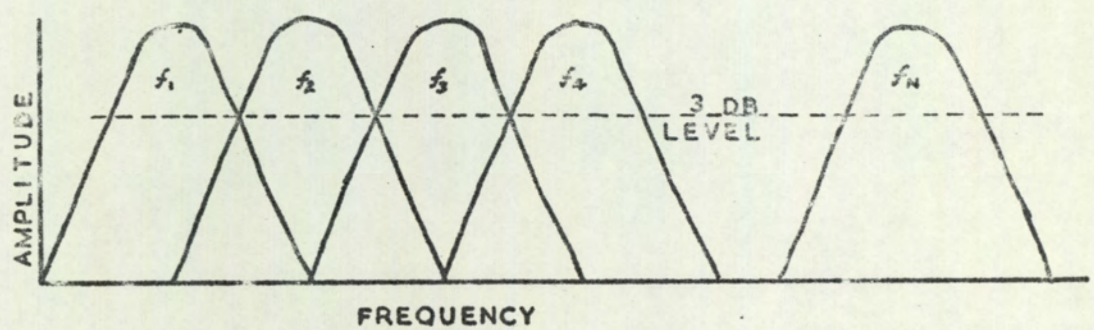
The best method of completely removing the transient output due to tape joints would be to remove the joint by employing truely continuous loops of tape, but none of the magnetic tape manufacturers would produce them due to the small demand for them. Since continuous loops were not available, it was necessary to attempt to reduce the output of the recorder to zero during the passage of the joint over the replay heads. This was achieved with some measure of success by the use of a photo-electric sensing device, and a reed relay arranged to earth the output signal of the tape recorder during the passage of the joint.

4-6) Conclusions relating to wave analysis

Though the initial assumption that the signals obtained from road tests of a vehicle would be stationary complex periodic was not unreasonable, the evidence of the results obtained led to the conclusion that they were in fact, random processes. This being the case, the type of equipment described in this chapter was not really suited to their analysis, and random process analysis equipment and techniques should have been employed.



a) BLOCK DIAGRAM



b) FILTER CONFIGURATION

FIG 4-1

CONTIGUOUS BAND ANALYSER

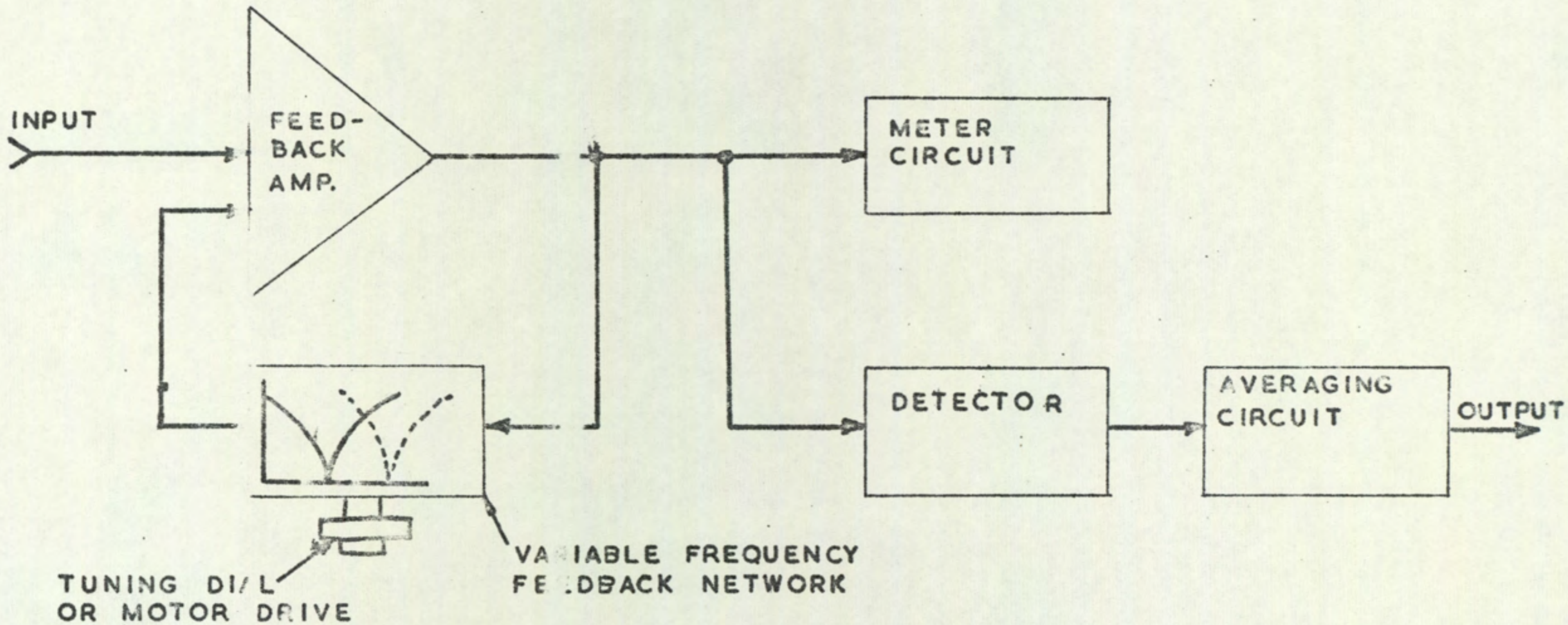


FIG 4-2

CONSTANT PERCENTAGE BANDWIDTH ANALYSER

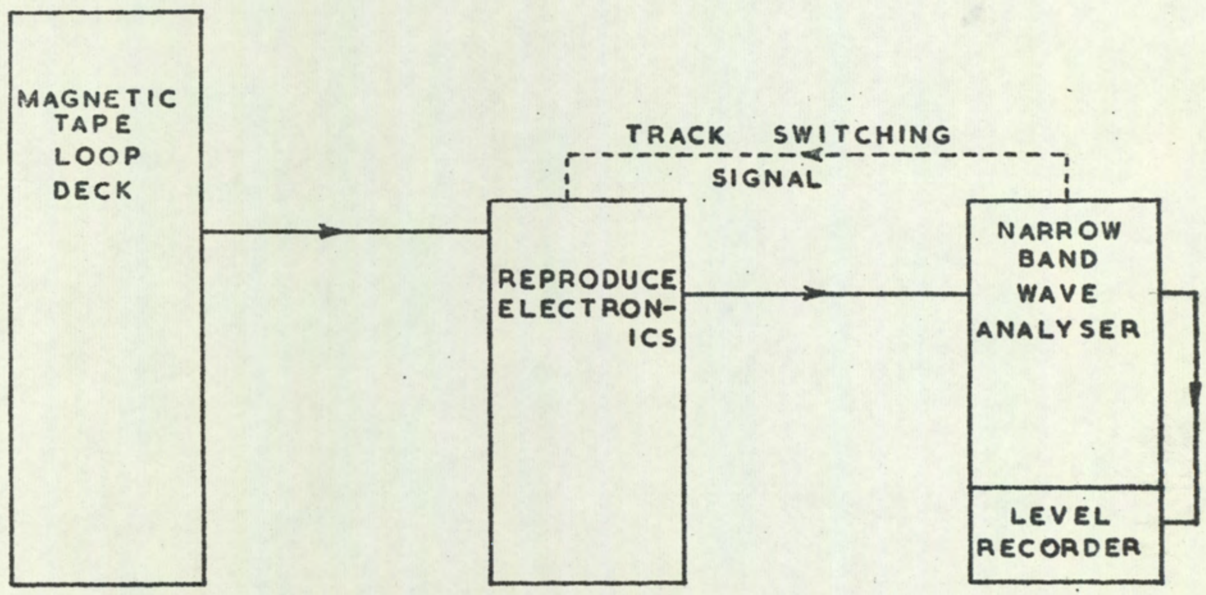
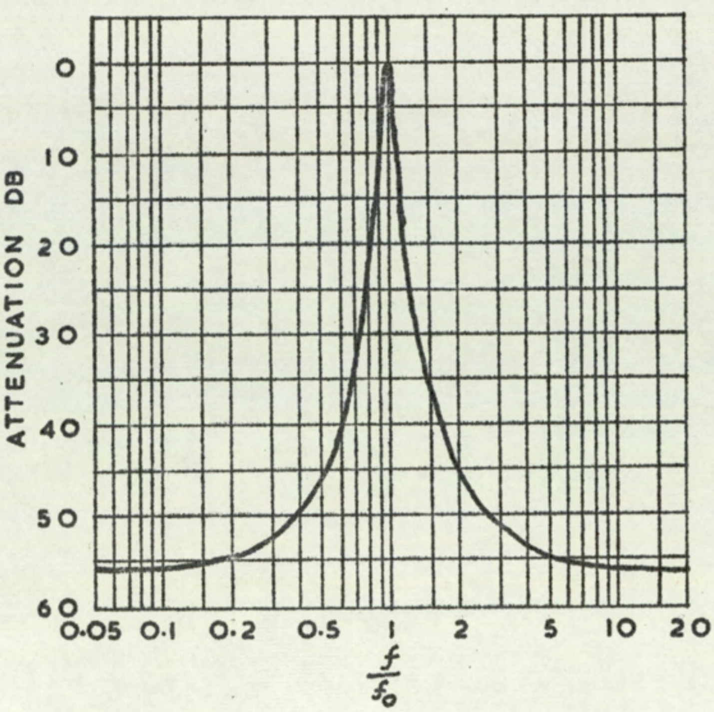


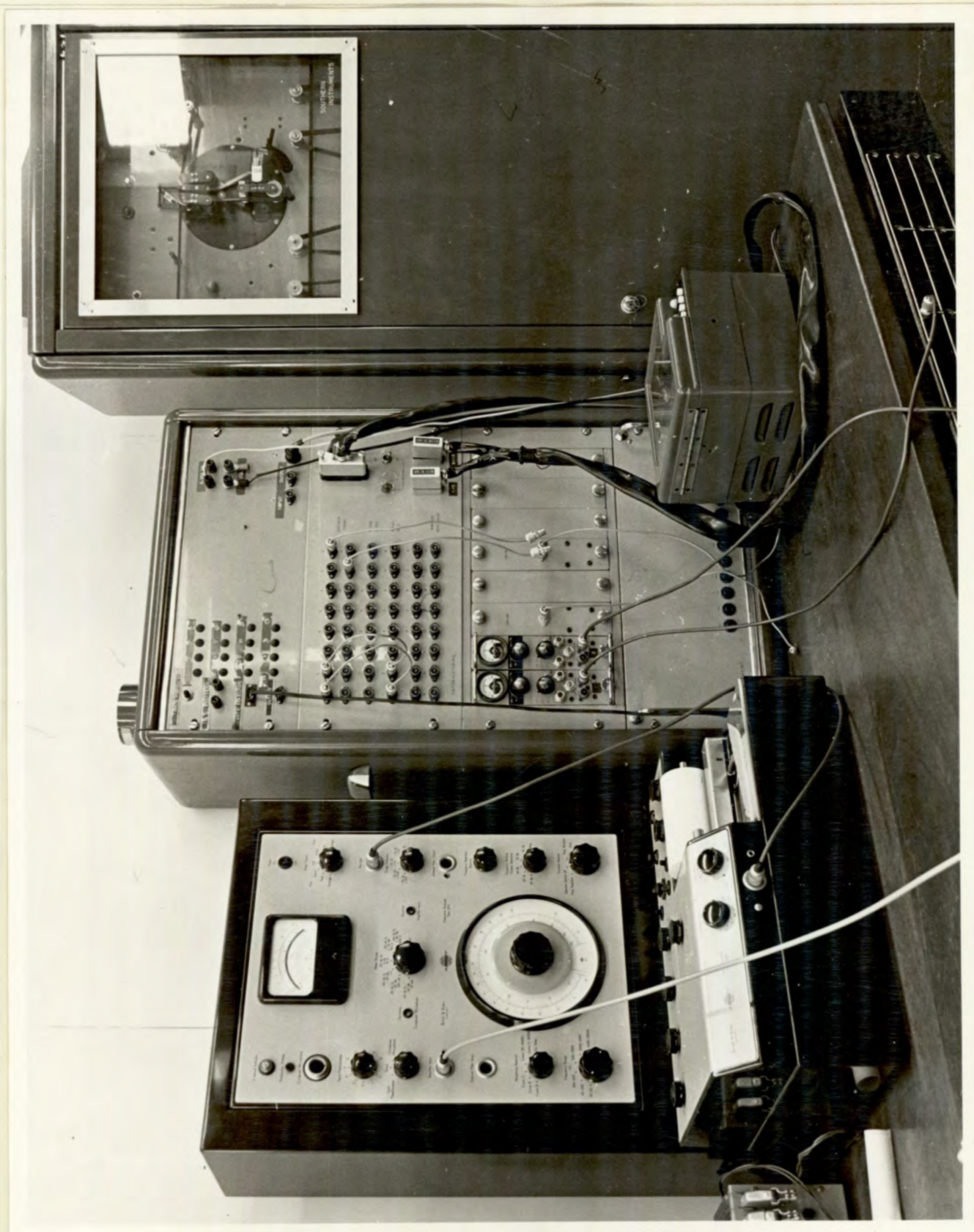
FIG 4-3

BLOCK DIAGRAM OF AUTOMATIC WAVE ANALYSIS EQUIPMENT



SELECTIVITY CURVE OF FILTER IN THE BRUEL & KJAER NARROW BAND ANALYSER TYPE 2107

FIG 4-4



WAVE ANALYSIS EQUIPMENT

FIG 4-5

CHAPTER 5 THEORETICAL INVESTIGATIONS

5-1) Summary

Work described in this chapter was an investigation of possible numerical methods for evaluating the natural frequencies and modal shapes of vibration of the transmission system. Three methods were considered, the Eigen value method, Transfer matrix method, and the Branch Mode Analysis method. Of these three, the Transfer matrix method was selected as the most suitable and was developed sufficiently to prove its value.

Results are presented which were calculated using the transfer matrix method, and a simplified mathematical model of the transmission system used in the experimental car. These results are discussed in respect to the interaction of the components of the transmission system, and in relation to the problems of computing.

5-2) The Eigen value method

This method is basically a restatement in terms of matrix algebra of the classical multi-degree of freedom analysis. In multi-degree of freedom analysis, the equations of motion are expressed as follows:

$$
\left.
\begin{aligned}
-m_1\ddot{x}_1 &= s_{11}x_1+s_{12}x_2+s_{13}x_3+ \;\ldots\ldots\; s_{1n}x_n \\
-m_2\ddot{x}_2 &= s_{21}x_1+s_{22}x_2+s_{23}x_3+ \;\ldots\ldots\; s_{2n}x_n \\
-m_3\ddot{x}_3 &= s_{31}x_1+s_{32}x_2+s_{33}x_3+ \;\ldots\ldots\; s_{3n}x_n \\
&\;\;\vdots \\
-m_n\ddot{x}_n &= s_{n1}x_1+s_{n2}x_2+s_{n3}x_3+ \;\ldots\ldots\; s_{nn}x_n
\end{aligned}
\right\} \quad 4\text{-}1
$$

These n equations can be replaced by the simple matrix equation:-

$$-M\ddot{X} = SX \qquad 4\text{-}2$$

Where M is a diagonal matrix of the mass terms

S is a nxn matrix of the spring terms

and X is a column matrix of displacements

If we assume a solution of the form $X = X \cos\omega t$ substituting for X we get

$$SX = \omega^2 MX \qquad 4\text{-}3$$

premultiplying both sides by M^{-1} we get

$$M^{-1}SX = \omega^2 X \qquad 4\text{-}4$$

or alternatively premultiplying by $H=S^{-1}$ gives

$$HMX = \omega^{-2}X \qquad 4\text{-}5$$

Equations 4-4 and 4-5 are alternative forms, and either may be solved for the Eigen values (natural frequencies) and the Eigen vectors (modal shapes). The choice of which equation to solve is indicated by which frequencies are required to the greatest accuracy. Equation 4-4 will give the lowest frequencies to a high degree of accuracy and equation 4-5 the highest frequencies. However, as in all numerical methods, the accuracy can only be as good as the input data will permit.

The methods of solving these equations is not discussed here since there are standard computer programmes for this function.

This method in its simple form set out above is obviously very good for analysing discreet systems where the springs and masses are clearly defined. The application to a continuous system such as a beam, however, is not so straight forward. To apply the method in such a case the continuous system is first approximated by a lumped mass system which in the case of a beam, is achieved by dividing the beam into convenient lengths and concentrating half the mass of each length at each end of the segment. The resulting model is a massless beam with a series of concentrated masses placed at each of the junctions of the selected lengths. Each length of massless beam is considered to retain its original bending resistance.

Evaluation of the mass matrix of a lumped mass model of the type described above is simple, since it consists of only the leading diagonal of the matrix. However, this is not the case for the stiffness matrix, since the displacements of the beam in terms of the forces at the mass point, are statically indeterminate. To overcome this problem the beam is divided into elements, which are small enough for their rotational inertia to be neglected and use is made of an enlarged stiffness matrix which includes the bending moments. It is shown in ref. 11 that this enlarged stiffness matrix can be reduced as a result of the zero rotational inertias to give the required stiffness matrix. This reduction of the enlarged stiffness matrix requires a considerable amount of matrix evaluation, and also the evaluation of the elements of the enlarged matrix can be quite tedious.

Inflexibility of this method when applied to continuous systems was its great disadvantage. This is particularly the case when it is desired to find the effect of minor design changes on the natural frequencies of a system, since the whole of the basic stiffness matrix needs to be recalculated.

5-3) Transfer matrix method

Pestel and Leckie have described the transfer matrix method in great detail in their book "Matrix Methods in Elasto-Mechanics", ref. 6, consequently only the application to beam like systems will be discussed.

The transfer matrix method of evaluating natural frequencies and mode shapes can be easily adapted to beam like systems since it is a lumped parameter process, and this is the most usual form of representation of beams.

A transfer matrix is the matrix expression of the coefficients of the equations which relate the parameters describing the state of a system at one point to those parameters which describe its state at a second point. Consider an element E_n of a system, then if the state of the left hand side of the element is described by the set of known parameters $x_1, x_2, \ldots\ldots\ldots x_m$ then the state at the right hand side is given by the equation:-

$$\underset{\text{R.H.S.}}{\begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_m \end{bmatrix}} = T_n \underset{\text{L.H.S.}}{\begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_m \end{bmatrix}} \qquad 4\text{-}6$$

Where the matrix T_n is the transfer matrix for the element E_n and the terms of T_n are evaluated from the physical properties of the element.

By the careful selection of the sign convention for the equations and describing paraments, it can be arranged that the R.H.S. state vector of element E_n is equal to the L.H.S. state vector of element E_{n+1}. Hence with a slight change in notation we get:-

$$\begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}_{R.F_n} = T_n \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_m \end{bmatrix}_{L.E_n} = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}_{L.E_{n+1}} \qquad 4\text{-}7$$

To simplify the writing the column matrices of the state paramenters are represented by the symbol Z and from 4-7 we get

$$T_{n+1}T_{n}Z_{L.E_{n}} = Z_{R.E_{n+1}} \qquad 4\text{-}8$$

and for a total system of k elements we get

$$Z_R = T_k T_{k-1} \cdots\cdots\cdots\cdots T_3 T_2 T_1 Z_L \qquad 4\text{-}9$$

In most cases half the elements of the two boundary state vectors will have known values, and it is therefore, possible to evaluate the matrix chain and solve for any variable terms in order to satisfy the known restraints. This technique will be explained fully in the following devdopment of the method for the solution of beam vibrations.

Considering the transverse vibration of a beam, i.e. vibration of the beam in a plain containing its longitudinal axis, there are four necessary and sufficient parameters in the state vector:-

deflection	w
slope	ø
bending moment	M
shear force	V

For reasons of symmetry of the transfer matricies the parameters are usually arranged in the order given above.

The co-ordinate system used is the right handed cartesian co-ordinate system, with the x axis coinciding with the longitudinal axis of the beam. If the beam is cut at any point then two faces will be exposed, and the face whose outward normal points in positive direction of the x axis is defined as the positive face. Positive displacements are in the positive direction of their respective co-ordinates, and forces are positive if when acting on the positive (negative) face their vectors are in the positive (negative) direction. These definitions of the sign conventions and co-ordinates ensures that the right hand state vector for one element of the beam will be equal to the left hand state vector of the next element.

To enable the transfer matrix method to be applied to a beam it is first necessary to represent the beam as a discreet mass model of the type previously described (5-2). In this case, the elements are considered to be of two distinct types, massless beam elements, and point mass elements. Each of these elements is treated separately as follows:

i) massless beam element

The forces and displacements for a massless beam element are shown in fig. 5-10. Equilibrium requires that the sum of the forces be zero, and that the sum of the moments about say point i-1 be zero. This gives two equilibrium equations:-

$$V^L_i - V^R_{i-1} = 0 \qquad 5\text{-}10$$

$$M^L_i - M^R_{i-1} - V^L_i l_i = 0 \qquad 5\text{-}11$$

two further equations are obtained for the end deflexion, and slope of the element treated as a cantilever of flexural stiffness EI subjected to a bending moment and a shear force at its free end (i). From simple beam theory these two equations are:-

$$w = \frac{Ml^2}{2EI} + \frac{Vl^3}{3EI} \qquad 5\text{-}12$$

$$\phi = \frac{Ml}{EI} - \frac{Vl^2}{2EI} \qquad 5\text{-}13$$

Applying these equations to the element and remembering that we already have a deflexion and a slope at point i-1 we get:

$$w_i^L = w_{I-1}^R - \phi_{i-1}^R l_i - M_i^L \frac{l^2}{2EI_i} + V_i^L \frac{l^3}{3EI_i} \qquad 5\text{-}14$$

$$\phi_i^L = \phi_{i-1}^R + M_i^L \frac{li}{(EI)_i} - V_i^L \frac{l}{2(EI)_i} \qquad 5\text{-}15$$

We note from equations 9 and 10 that

$$V_i^L = V_{i-1}^R \quad \text{and} \quad M_i^L = M_{i-1}^R + V_{i-1}^R li$$

Hence equations 5-14 and 15 can be rewritten such that all the state vector parameters at point i^L can be expressed in terms of those at point $(i-1)^R$

$$\left.\begin{aligned} -w_i^L &= -w_{i-1}^R + l_i\phi_{i-1}^R + \frac{l_i^2}{2(EI)_i} M_{i-1}^R + \frac{l^3}{6(EI)_i} V_{i-1}^R \\ \phi_i^L &= \phi_{i-1}^R + \frac{l_i}{(EI)_i} M_{i-1}^R + \frac{l^2}{2(EI)_i} V_{i-1}^R \\ M_i^L &= M_{i-1}^R + l_i V_{i-1}^R \\ V_i^L &= V_{i-1}^R \end{aligned}\right\} \quad 5\text{-}17$$

Choosing the first element of the state vector to be $-w_i$ has the advantage that all the elements of the matrix will be positive and the matrix will be cross-symmetric.

The resulting equation will be:-

$$\begin{bmatrix} -w \\ \phi \\ M \\ V \end{bmatrix}_i^L = \begin{bmatrix} 1 & \ell & \frac{\ell^2}{2EI} & \frac{\ell^3}{6EI} \\ 0 & 1 & \frac{\ell}{EI} & \frac{\ell^2}{2EI} \\ 0 & 0 & 1 & \ell \\ 0 & 0 & 0 & 1 \end{bmatrix}_i \begin{bmatrix} -w \\ \phi \\ M \\ V \end{bmatrix}_{i-1}^R \qquad 5\text{-}18$$

or

$$Z_i^L = F_i Z_{i-1}^R \qquad 5\text{-}19$$

ii) point mass

For the concentrated mass at point i the transfer matrix can be evaluated from the known fact that deflexion, slope and providing the rotational inertia of the mass is negligible bending moment are continuous giving

$$w_i^R = w_i^L, \quad \phi_i^R = \phi_i^L, \quad M_i^R = M_i^L \qquad 5\text{-}20$$

However, the mass introduces a discontinuity in the shear force due to the inertia force hence:

$$V_i^R = V_I^L - m_i \omega^2 w_i \qquad 5\text{-}21$$

From these the following matrix relation is obtained:

$$\begin{bmatrix} -w \\ \phi \\ M \\ V \end{bmatrix}_i^R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ m^2 & 0 & 0 & 1 \end{bmatrix}_i \begin{bmatrix} -w \\ \phi \\ M \\ V \end{bmatrix}_{i\cdot 1}^L \qquad 5\text{-}22$$

or

$$Z_i^R = P_i Z_i^L \qquad 5\text{-}23$$

A total system containing k mass points would then be represented by the product of a chain of matrices as below:

$$Z^R = P_k F_k P_{k-1} F_{k-1} \;\ldots\ldots\ldots\ldots\; P_2 F_2 P_1 F_1 P_0 Z^L \qquad 5\text{-}24$$

Where the suffixes of the field (massless beam elements) matrices are the same as their right hand terminating mass points when the evaluation is from left to right.

If the equation 5-24 is expressed as below

$$Z^R = UZ^L$$

then U is the transfer matrix for the total system, and is given by the product of the transfer matrices for the elements taken in order.

Taking as an example a pinned-pinned beam, we know that at both ends the displacement w and the bending moment M will both be zero giving the following expression

$$\begin{bmatrix} 0 \\ \phi \\ 0 \\ V \end{bmatrix}^R = \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \phi \\ 0 \\ V \end{bmatrix}^L \qquad 5\text{-}25$$

This gives the following expressions which must be satisfied if the right hand end conditions are to be satisfied:

$$0 = u_{12}\phi^{L} + u_{14}V^{L}$$
$$0 = u_{32}\phi^{L} + u_{34}V^{L} \qquad 5\text{-}26$$

The necessary condition for these two equations to be satisfied is that the determinant of the coefficients be zero, i.e.,

$$\begin{vmatrix} u_{12} & u_{14} \\ u_{32} & u_{34} \end{vmatrix} = 0 \qquad 5\text{-}27$$

For simple systems with only two or three mass points the end condition determinant could be evaluated analytically but for complex systems this is not possible, due to the high powers of omega involved. The technique applied to extensive systems is to evaluate the system matrix for a given value of omega, and then evaluate the determinant. This is repeated for small increments of omega until a change of sign in the value of the determinant is detected. The natural frequency can then be determined either by plotting the value of the determinant against omega, or by taking an interpolation for the zero value.

Intermediate conditions of the form of simple springs etc., can be introduced by means of simple point matrices. Where an intermediate condition is more complex, i.e. when a sub-system joins the main system, then the effective stiffness of the intermediate condition is calculated for each frequency and then added to the main system as a point matrix as before.

This method of evaluating natural frequencies has the great advantage of being very flexible in respect of changes in the components of the system, since often only one or two of the basic transfer matrices need to be changed. A second advantage is that transfer matrices have been evaluated for a large number of typical components of systems, and these have been catalogued for example see ref. 6. These two advantages are offset by the large number of calculations involved in finding a natural frequency due to the necessity of evaluating the matrix chain for a large number of frequencies.

5-4) Branch mode analysis

This method depends on dividing the system to be analysed into a number of sub-systems or branches, and evaluating the natural frequencies and modal shapes of each of these branches, treating the other branches in each case as rigid bodies. The frequencies and modes so obtained are then combined in a Rayleigh-Ritz analysis to give the natural frequencies and modal shapes of the total system. A full development of the method is given in ref. 7.

This method has the advantage that a change in one part of the system does not necessarily require the total computation to be repeated. However, it requires the use of an Eigen value method for the initial evaluation of the branch frequencies and modes, and these have already been shown to be rather inflexible to changes in design.

The method obviously has great advantages in the one off analysis of complicated systems, but is felt by the author to be rather cumbersome for the type of system considered in this thesis.

5-5) Mathematical model of the transmission system

For the numerical analysis of natural frequencies and mode shapes by the transfer matrix method, a very simple model was used consisting of the propeller shaft and rear axle only. The propeller shaft was considered to be pin jointed to earth at the forward end and was represented by 11 masses and 12 massless beams. The two end beams were considered to be rigid since they represented the yokes of the Hookes joints which were cast iron, and of much greater stiffness than the main tube of the propeller shaft.

Representation of the rear axle was not very good since only five masses and four massless beams were used, and also both the leaf springs and the tyres were treated as simple springs acting at the extreme ends of the axle. Even with this limited representation it was still possible to form some idea of the interaction of the propeller shaft and the rear axle in terms of natural frequencies and mode shapes.

A diagram of the model is shown in fig. 5-1. The values of the various masses and stiffnesses were evaluated for the system fitted with the five different diameter propeller shafts used in the tests described in Chapter 3, and are shown in table 1. Since these calculations were only intended as an appraisal of the method of evaluating the natural frequencies and mode shapes, the values of the masses and stiffnesses were not checked against measured values.

5-6) Computation procedure

The calculations were performed on an Elliott 803 digital computer. This was a small machine of medium speed which included an algol compiler in its basic software.

Use of the Algol language in programming was an advantage due to the facility to directly address multi-dimensional arrays. However, the programmes produced from this source language were rather slow in operation, and any more extensive computations with this method would either have to be reprogrammed in a more basic machine language, or preferably be run on a more powerful machine.

The basic data input to the programme was in the form of parameters of the system, and these were reduced to a minimum by arranging that the programme evaluated the maximum amount of data internally. As is shown in the flow diagram fig. 5-2, the main calculation was controlled by a further data input consisting of the frequency range of interest, and some logical control data. The programme could be re-entered at this second data input as often as required allowing the use of coarse and fine frequency scans on the same basic data.

Calculation procedure in the main body of the programme was as follows:

i) evaluate the total transfer matrix of the propeller shaft and hence the effective spring stiffness to be added to the centre mass point of the rear axle.

ii) Evaluate the transfer matrix for the rear axle with the effective stiffness of the propeller shaft added

iii) evaluate the end condition determinant and test to see if its sign had changed compared to that previously computed.

iv) if a sign change was detected then a linear interpolation was performed to obtain an approximate natural frequency, otherwise the frequency was incremented and the computation retuned to i).

v) the mode shape corresponding to the approximate natural frequency was evaluated if required, and control returned to the control data input point.

Continuous monitoring of the programme during running was achieved by printing out the values of frequency effective stiffness of the propeller shaft, and the value of the end condition determinant. The final output of the programme was the approximate natural frequencies followed by the mode shape when called for. The mode shape was in the form of displacements normalised to the maximum displacement in each mode.

5-7) Results

Typical nodeal shapes obtained for the transmission system with the standard 2.5 in. diameter propeller shaft fitted are shown in figs. 5-3 to 8. For comparison, the natural frequencies and nodal shape figures for the transmission system fitted with each of the five different propeller shafts are presented in tables 2 to 7.

As was expected due to the lightness of the propeller shafts they had very little influence on the rigid body modes which were dominated by the rear axle bouncing on the combined stiffness of the tyres and rear springs. The only rear axle mode influenced by the change in the propeller shaft was the first flexural mode, but this influence reduced as the natural frequency of the propeller shaft in its first flexural mode increased. The reason for this increase is clearly seen from the mode shape fig. 5-5 where it is seen that the propeller shaft was assuming a considerable dynamic deflexion.

The natural frequencies and mode shapes of the propeller shafts were only effected to a slight extent by the presence of the rear axle and then only the first flexural mode was influenced. The only evidence of this interaction of the rear axle on the propeller shaft natural frequencies was the changing of the ratio of second to first mode from the theoretical 4 to 3.94. The simple theory mentioned in Chapter 3 which predicted that the propeller shaft natural frequencies would be proportional to the diameter was confirmed by the values of frequencies obtained for both first and second modes.

5-8) Comparison with practical results

Calculated and measured results for the flexural frequencies of the rear axle did not agree. This was expected due to the following shortcomings of the representation of the rear axle:

i) The small number of masses used in the model. This would be easily overcome with a little more preparitory work.

ii) The poor positioning of the springs representing the rear leaf springs, and the assumption that they could be considered to be simple springs.

iii) The approximation to the stiffness of the tyres which were assumed to be simple springs of stiffness equal to their static stiffness.

These last two assumptions raise the problem of how to account for frequency dependent springs. In the first case, the leaf springs could reasonably be represented as further sub-systems to the main system, and be treated in the same way as the propeller shaft. However, the case of the tyres is not so simple since it is extremely difficult to derive a reasonable mathematical model of a tyre.

Results for the propeller shaft natural frequencies showed good agreement with the response test results for both the first and second modes. The agreement with the road test values was better than with the response test results presumably due to the relieving of the stiction in the Hookes joints, due to the rotation of the propeller shaft making them closer to true pin joints.

Overall the comparison of results was sufficiently good to show that, with improvements in the model and development of the programme to include the vibrations of the system in the horizontal plane, good predictions of the natural frequencies and nodal shapes could be obtained. Such a development required the use of a much more powerful computer than was available at the time this work was carried out, and consequently was not attempted.

PROPELLER SHAFT DIA (in)	2	2.25	2.5	2.75	3
ℓ_1 (in)	1.5	1.5	1.5	1.5	1.5
ℓ_2 (in)	4.1	4.1	4.1	4.1	4.1
ℓ_3 (in)	9	9	9	9	9
m_1, m_{11} (lb)	.2214	.2500	.2785	.3071	.3357
m_2 – $_{10}$ (lb)	.4429	.5000	.55717	.6143	.6715
m_{13}, m_{16} (lb)	34.3618	34.3618	34.3618	34.3618	34.3618
m_{14}, m_{15} (lb)	2.7237	2.7237	2.7237	2.7237	2.7237
m_{12} (lb)	30.3618	30.3618	30.3618	30.3618	30.3618
I p.s. (in^4)	.17866	.25707	.35561	.47658	.62228
I r.a. (in^4)	.74214	.74214	.74214	.74214	.74214
S (lbf/in)	1140	1140	1140	1140	1140

Table 1. Parameters for Numerical Calculations.

PROPELLER SHAFT DIA. (IN)		2	2.25	2.5	2.75	3
FREQUENCY (C.P.S)		98.03	99.01	[illegible].39	99.62	99.37
	POSITION	MODE SHAPE				
REAR AXLE	1	-0.227	-0.442	-0.[illegible]51	-0.508	-0.537
	2	0.209	0.429	0.457	0.495	0.522
	3	0.405	0.824	0.[illegible]6	0.943	0.998
	4	0.209	0.442	[illegible]37	0.488	0.522
	5	-0.227	-0.415	[illegible]51	-0.521	-0.537
PROPELLER SHAFT	1	0.097	0.101	[illegible]081	0.074	0.067
	2	0.356	0.372	[illegible]297	0.272	0.2[illegible]7
	3	0.589	0.625	[illegible]499	0.458	0.418
	4	0.781	0.576	[illegible]675	0.625	0.573
	5	0.918	0.756	[illegible]817	0.765	0.707
	6	0.993	0.889	[illegible]920	0.873	0.817
	7	1.000	0.97[illegible]	[illegible]981	0.948	0.900
	8	0.942	1.000	1.000	0.989	0.956
	9	0.827	0.981	0.981	1.000	0.988
	10	0.666	0.931	0.932	0.986	1.000
	11	0.477	0.851	0.863	0.956	0.999
	12	0.405	0.82[illegible]	0.836	0.943	0.998

TABLE 4 MODE SHAPE FIRST FLEXURAL MODE OF REAR AXLE.

PROPELLER SHAFT DIA. (IN)		2	2.25	2.5	2.75	3
FREQUENCY (C.P.S)		117.13	130.95	145.65	160.41	175.29
	POSITION	MODE SHAPE				
REAR AXLE	1	0.045	0.026	0.020	0.015	0.012
	2	-0.077	-0.061	-0.060	-0.057	-0.059
	3	-0.132	-0.099	-0.093	-0.088	-0.089
	4	-0.077	-0.061	-0.054	-0.055	-0.055
	5	0.046	0.027	0.032	0.019	0.020
PROPELLER SHAFT	1	0.109	0.109	0.109	0.108	0.109
	2	0.399	0.397	0.396	0.395	0.396
	3	0.654	0.656	0.649	0.648	0.649
	4	0.849	0.845	0.845	0.843	0.844
	5	0.968	0.966	0.965	0.965	0.964
	6	1.000	1.000	1.000	1.000	1.000
	7	0.941	0.945	0.946	0.946	0.946
	8	0.795	0.804	0.806	0.808	0.807
	9	0.574	0.590	0.593	0.596	0.594
	10	0.297	0.320	0.324	0.327	0.326
	11	-0.015	0.015	0.021	0.025	0.024
	12	-0.132	-0.099	-0.093	-0.088	-0.089

TABLE 5 MODE SHAPE FIRST FLEXURAL MODE OF PROPELLER SHAFT.

PROPELLER SHAFT DIA. (IN)		2	2.25	2.5	2.75	3
FREQUENCY (C.P.S)		454.23	518.42	573.18	631.76	689.58
	POSITION	MODE SHAPE				
REAR AXLE	1	-0.001	-0.005	-0.013	-0.001	-0.004
	2	0.024	0.073	0.193	0.039	0.093
	3	0.023	0.031	-0.004	0.030	0.034
	4	-0.001	-0.105	-0.480	0.003	-0.070
	5	-0.034	-0.219	-0.687	-0.029	-0.095
PROPELLER SHAFT	1	0.213	0.213	-0.213	0.212	0.213
	2	0.728	0.728	-0.728	0.723	0.728
	3	1.000	1.000	-0.999	1.000	1.000
	4	0.936	0.936	-0.934	0.939	0.935
	5	0.558	0.556	-0.553	0.560	0.556
	6	-0.008	-0.010	0.015	-0.006	-0.010
	7	-0.571	-0.5[illegible]3	0.578	-0.571	-0.573
	8	-0.941	-0.942	0.948	-0.943	-0.942
	9	-0.995	-0.993	1.000	-0.996	-0.992
	10	-0.714	-0.709	0.720	-0.712	-0.708
	11	-0.192	-0.184	0.206	-0.186	-0.182
	12	0.023	0.031	-0.003	0.030	0.034

TABLE 6 MODE SHAPE SECOND FLEXURAL MODE OF PROPELLER SHAFT.

PROPELLER SHAFT DIA (IN)		2	2.25	2.5	2.75	3
FREQUENCY (C.P.S.)		819.18	819.18	819.18	819.20	819.16
	POSITION	MODE SHAPE				
REAR AXLE	1	0.039	0.039	0.039	0.039	0.040
	2	-1.000	-1.000	-1.000	-1.000	-1.000
	3	0.000	0.000	0.000	0.000	0.000
	4	1.000	1.000	1.000	1.000	1.000
	5	-0.039	-0.039	-0.039	-0.040	-0.039
PROPELLER SHAFT	1	0.000	0.000	0.000	0.000	0.000
	2	0.000	0.000	0.000	0.000	0.000
	3	0.000	0.000	0.000	0.000	0.000
	4	0.000	0.000	0.000	0.000	0.000
	5	0.000	0.000	0.000	0.000	0.000
	6	0.000	0.000	0.000	0.000	0.000
	7	0.000	0.000	0.000	0.000	0.000
	8	0.000	0.000	0.000	0.000	0.000
	9	0.000	0.000	0.000	0.000	0.000
	10	0.000	0.000	0.000	0.000	0.000
	11	0.000	0.000	0.000	0.000	0.000
	12	0.000	0.000	0.000	0.000	0.000

TABLE 7 MODE SHAPE SECOND FLEXURAL MODE OF REAR AXLE.

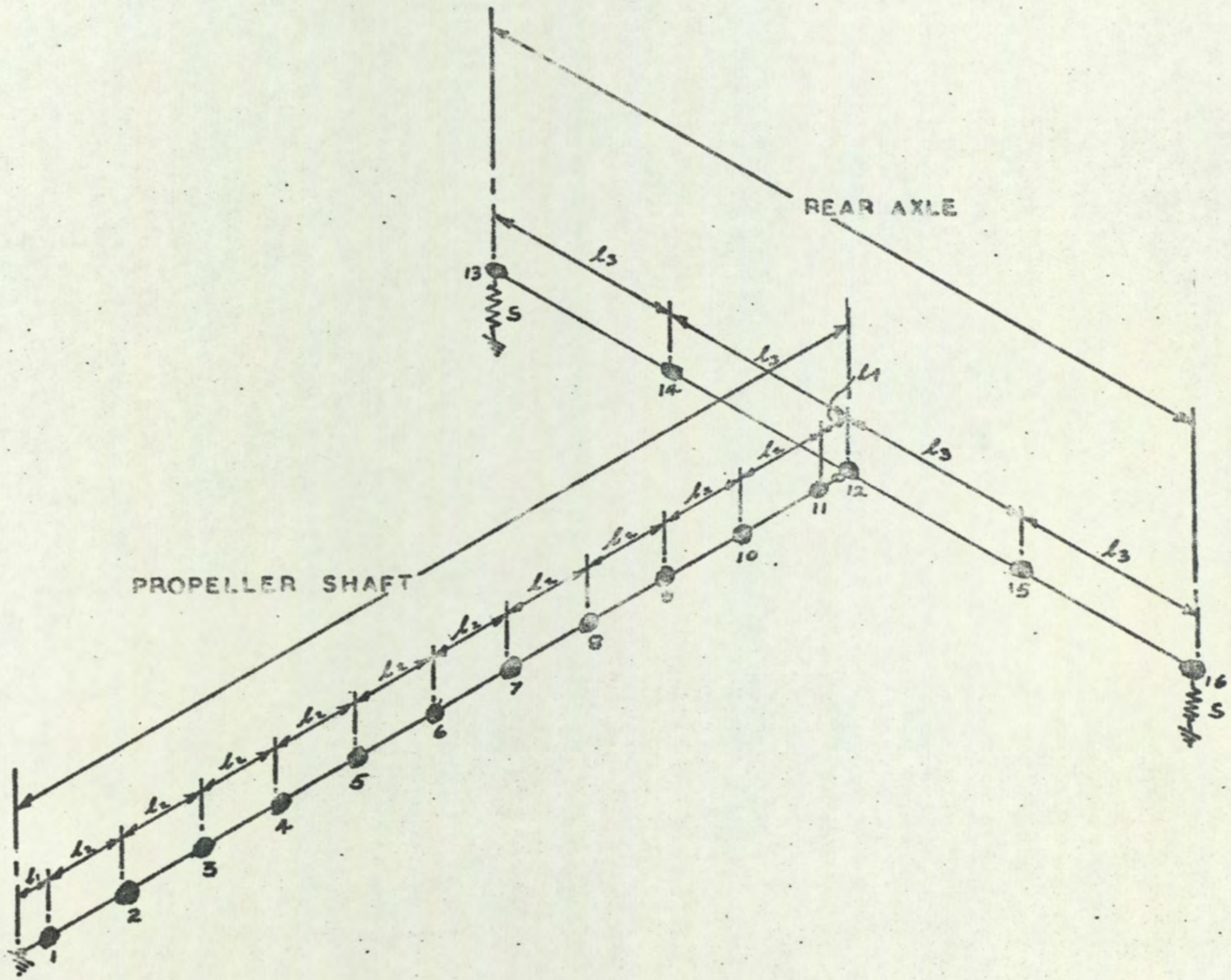
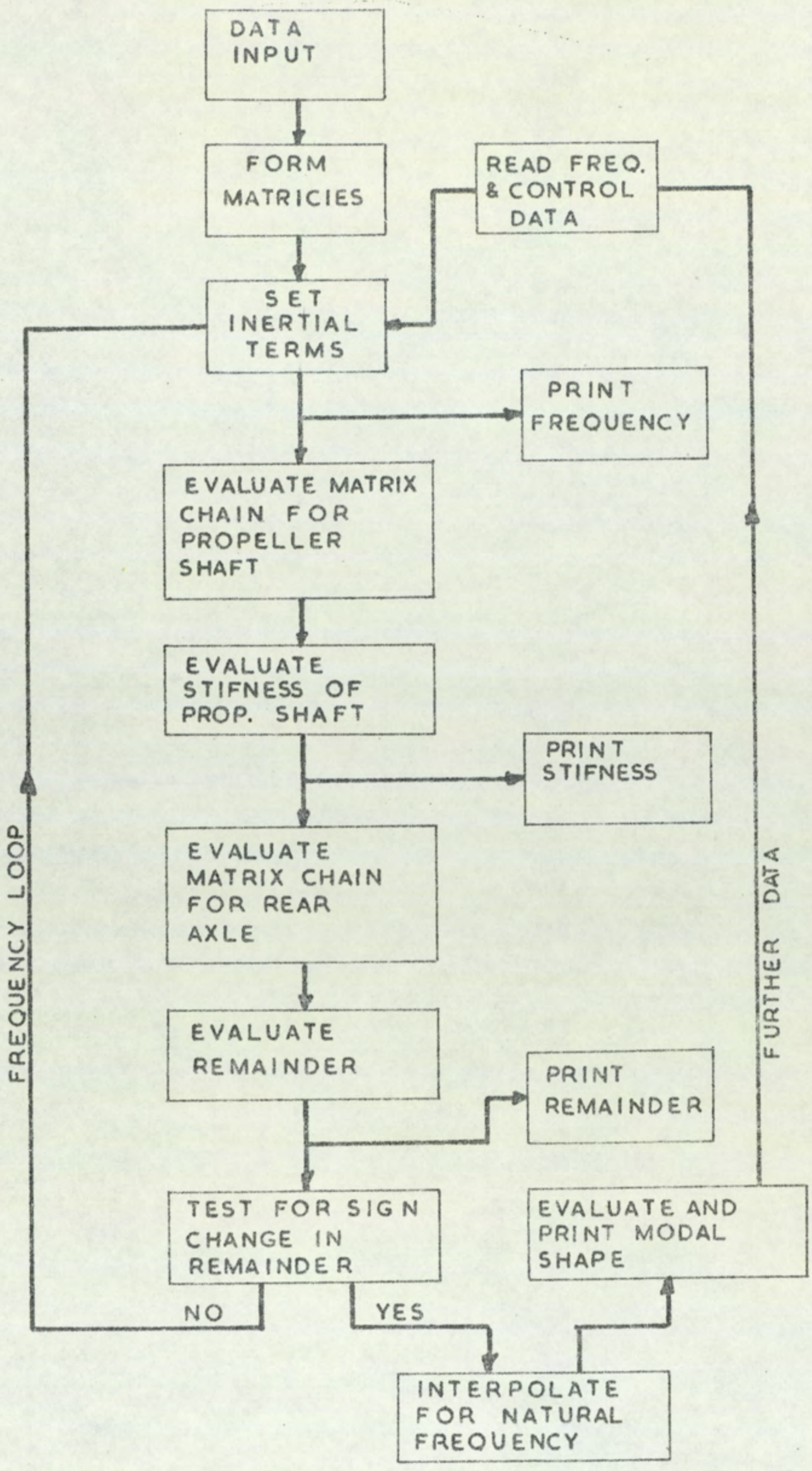


FIG 5-1

LUMPED PARAMETER MODEL FOR NUMERICAL ANALYSIS.



FLOW DIAGRAM FOR TRANSFER MATRIX COMPUTOR PROGRAMME

FIG 5-2

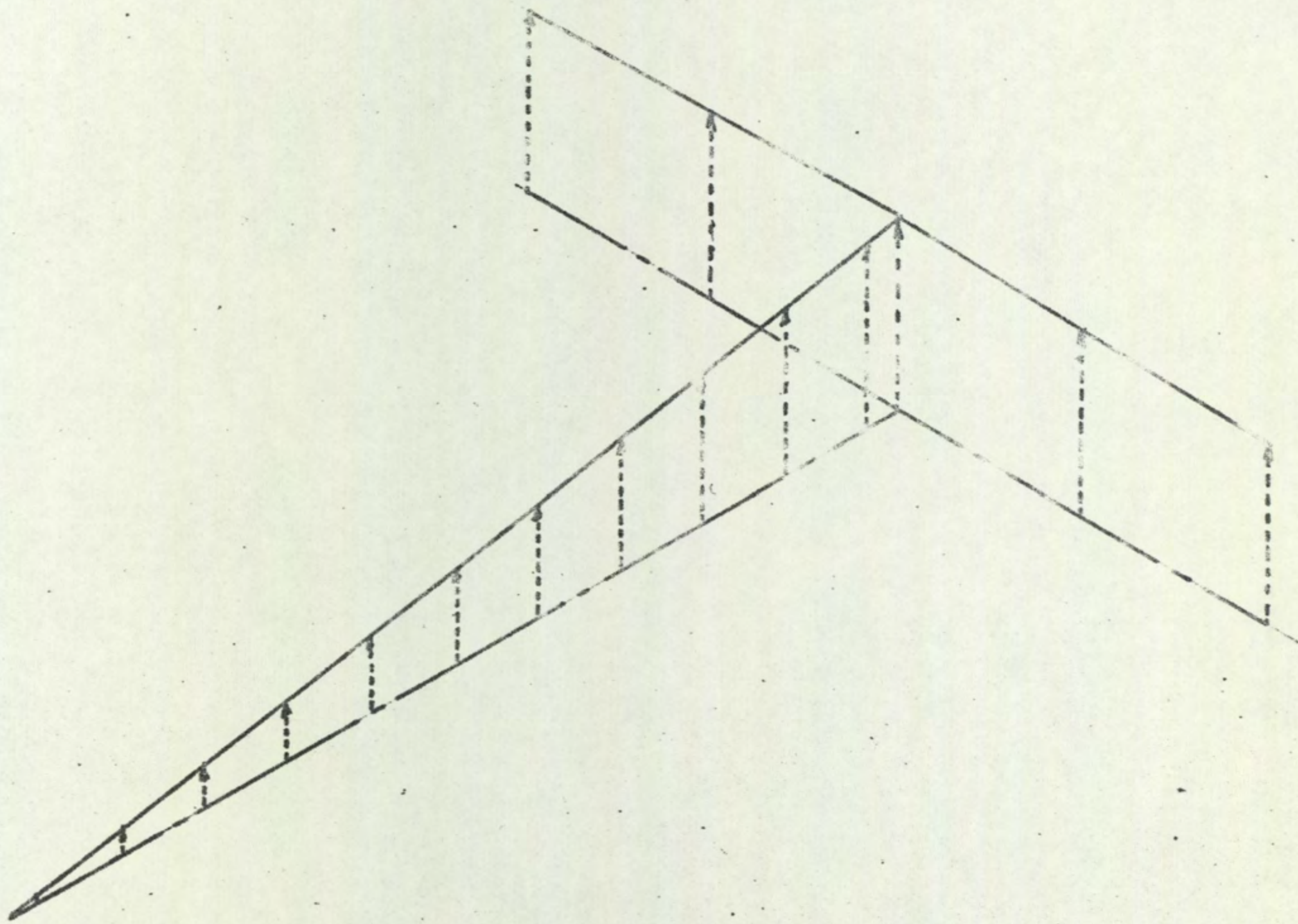
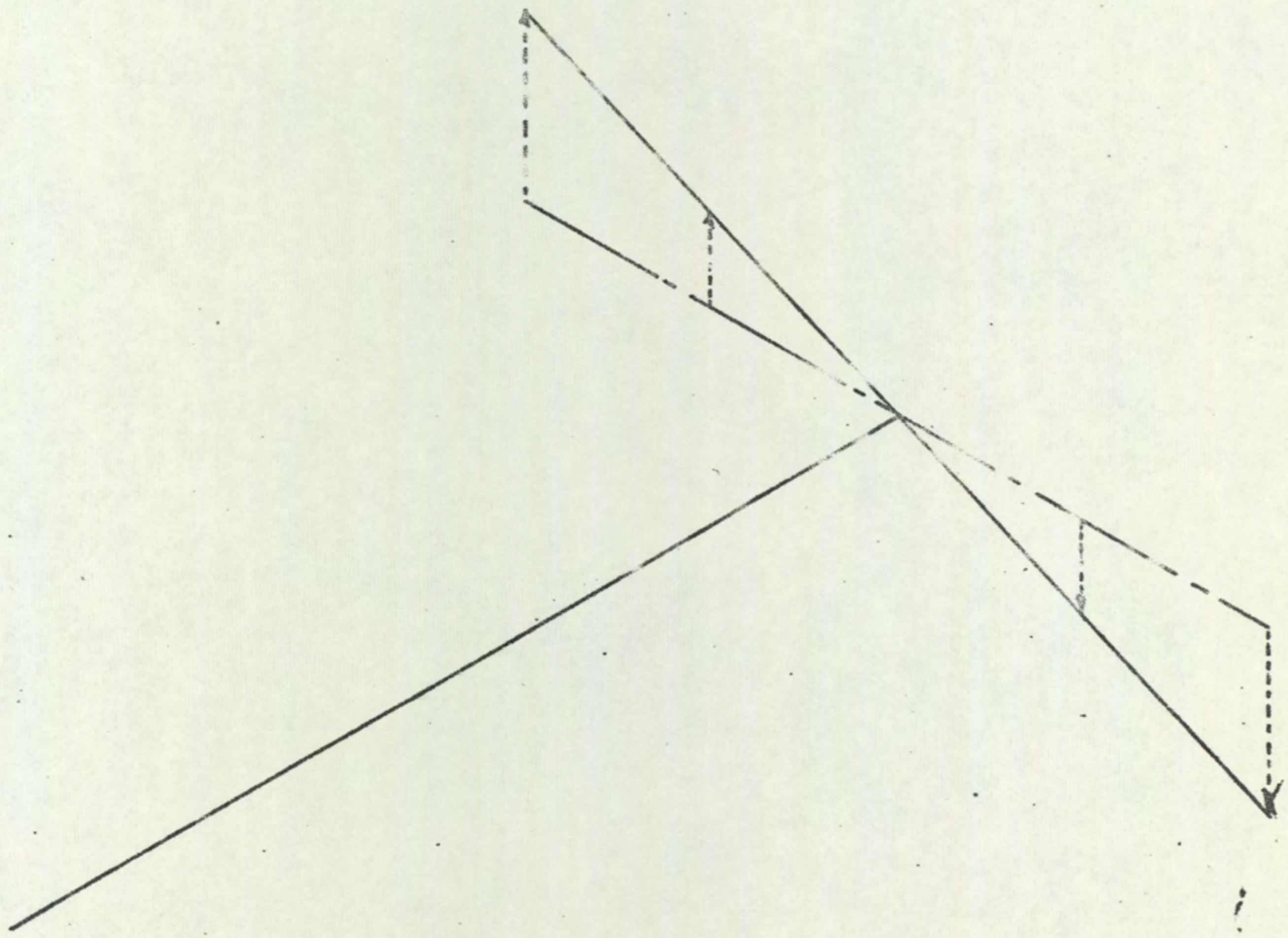


FIG 5.2

FIRST RIGID BODY MODE.



SECOND RIGID BODY MODE.

FIG 5-4

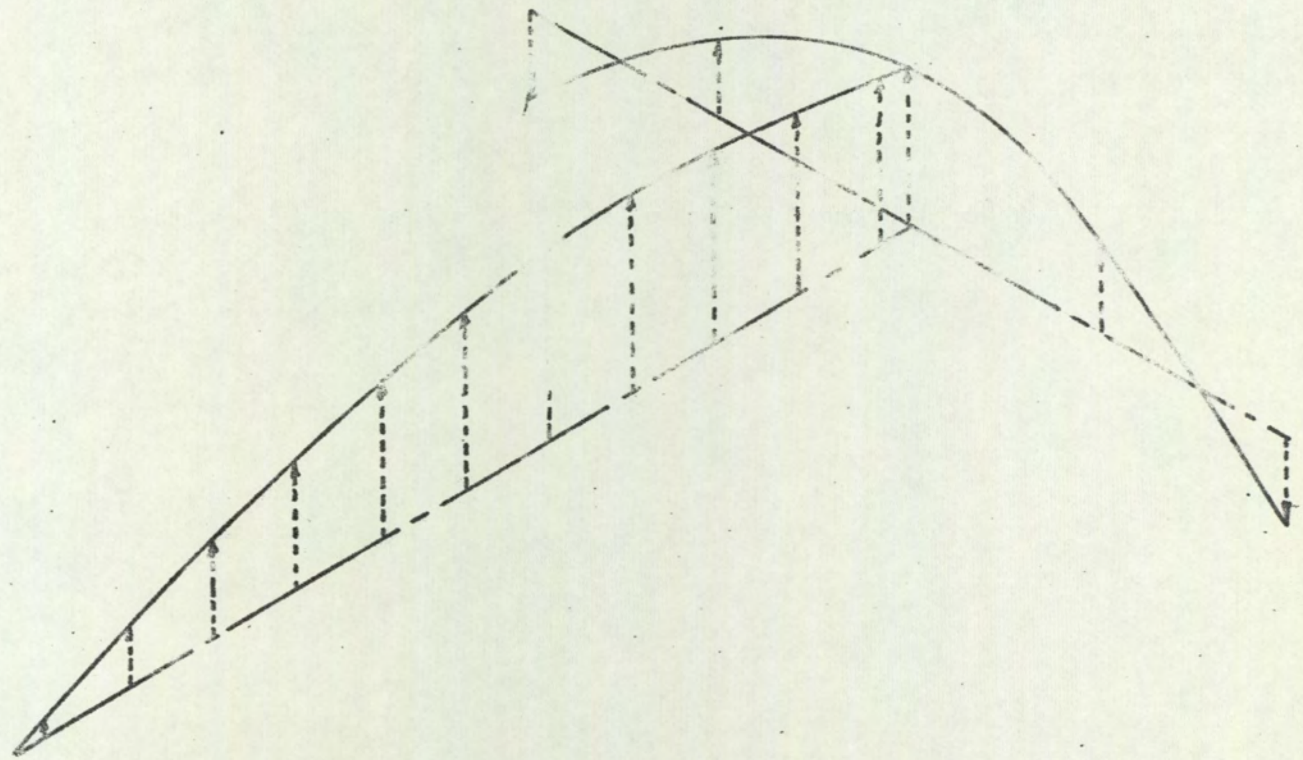
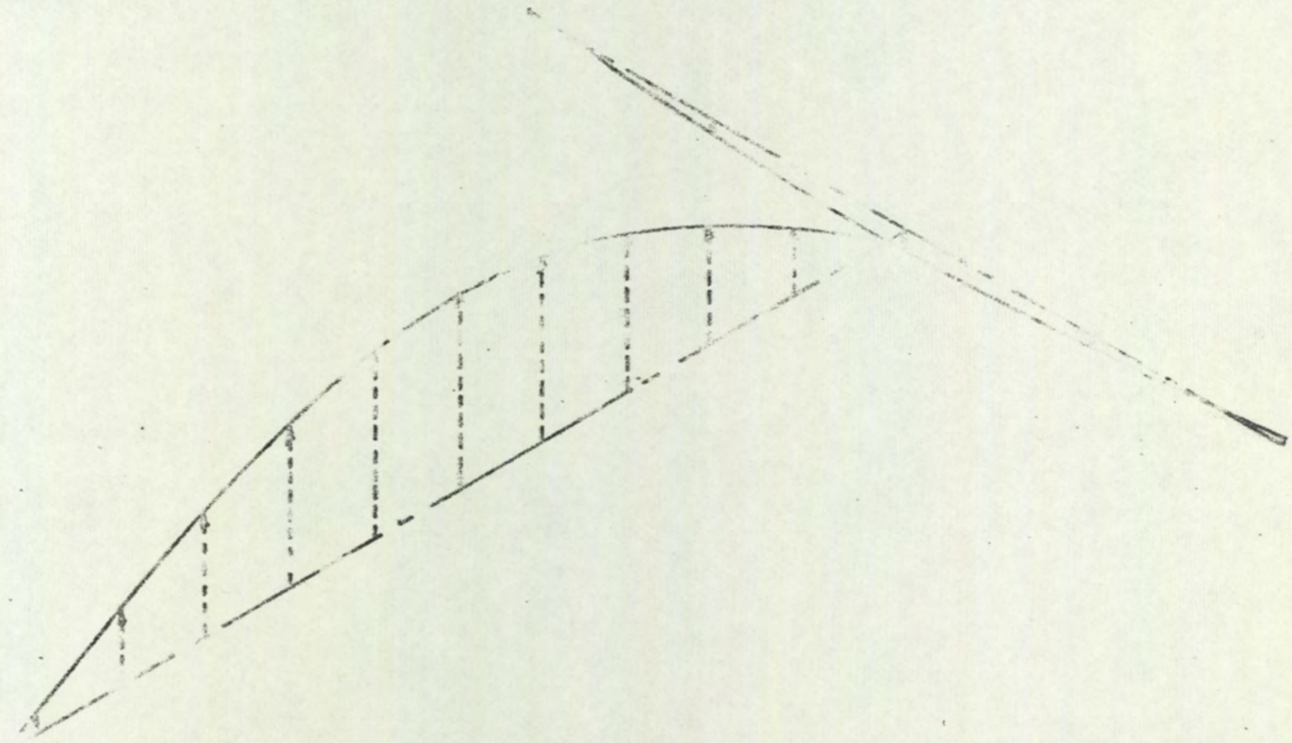


FIG 5-5

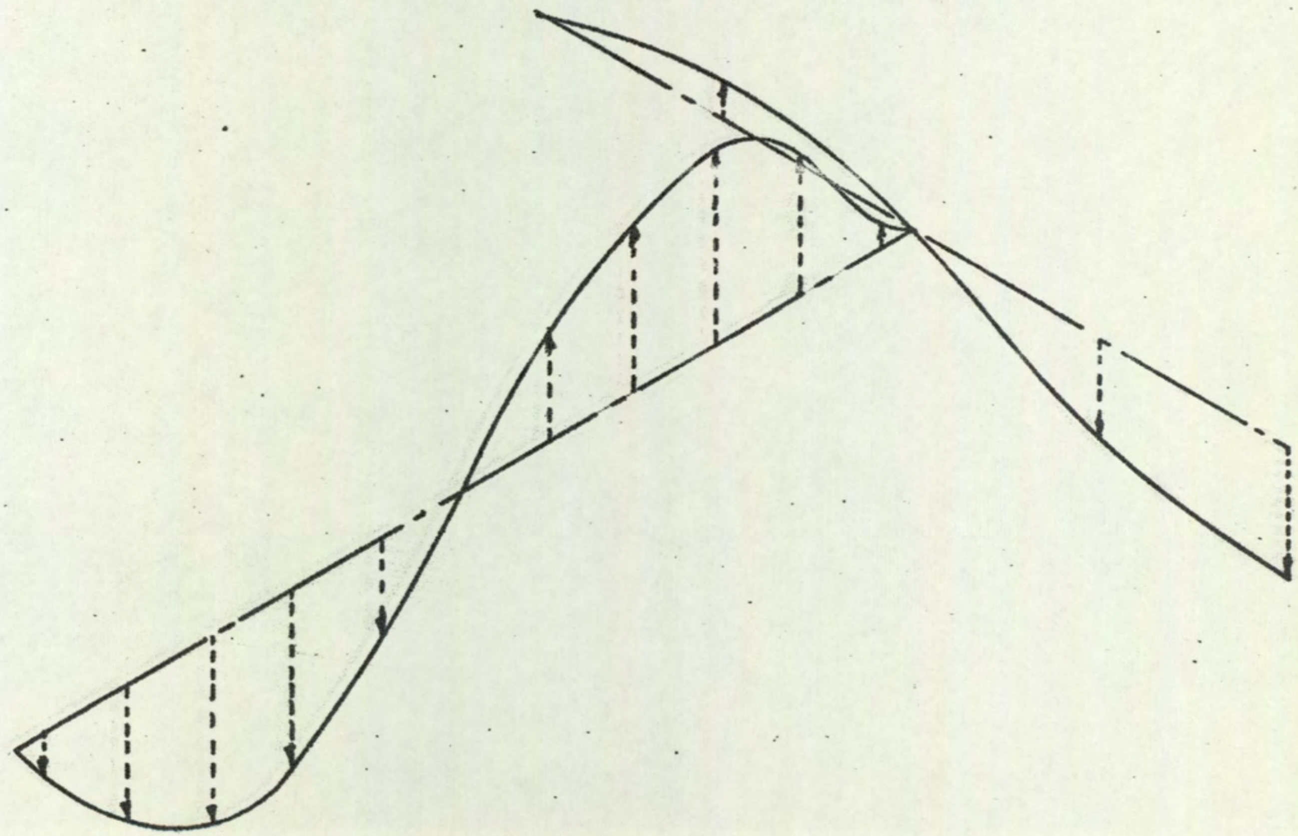
FIRST FLEXURAL MODE OF REAR AXLE

FIG 5-6



FIRST FLEXURAL MODE OF PROPELLER SHAFT

FIG 5-7



SECOND FLEXURAL MODE OF PROPELLER SHAFT.

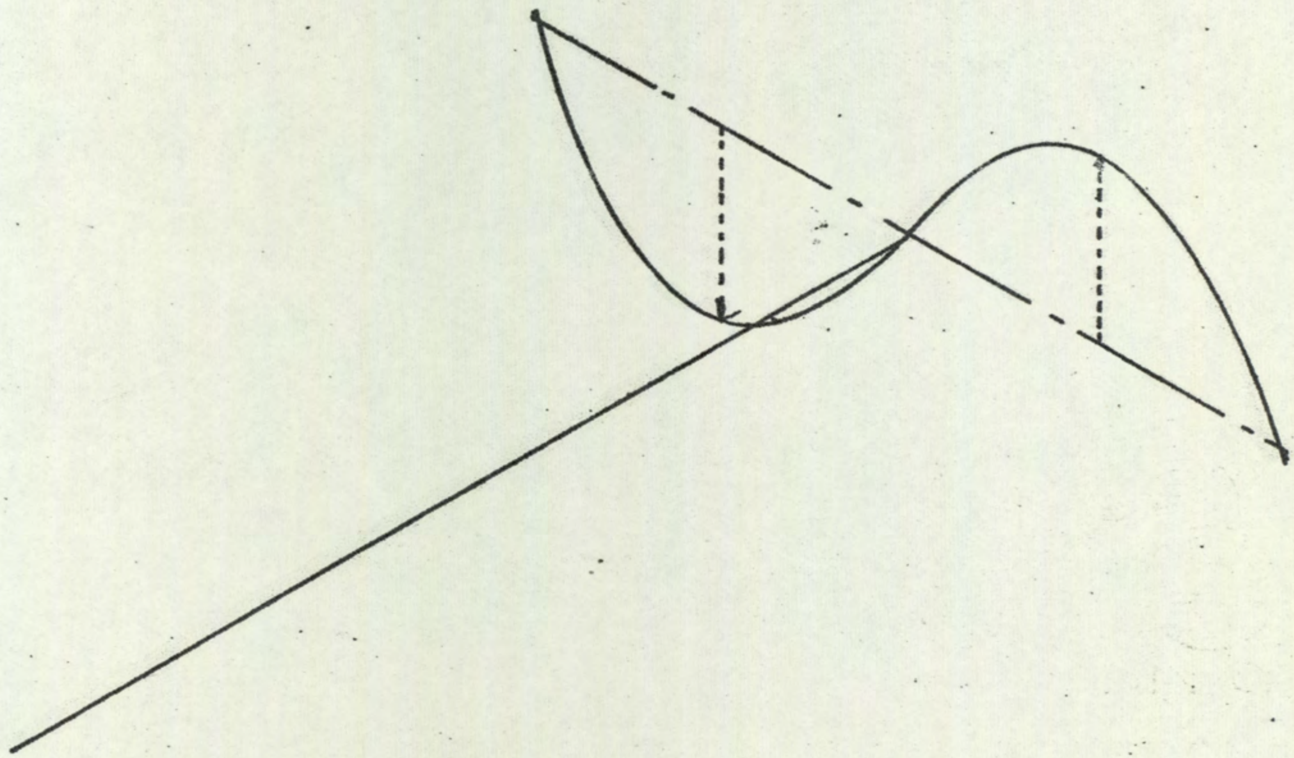
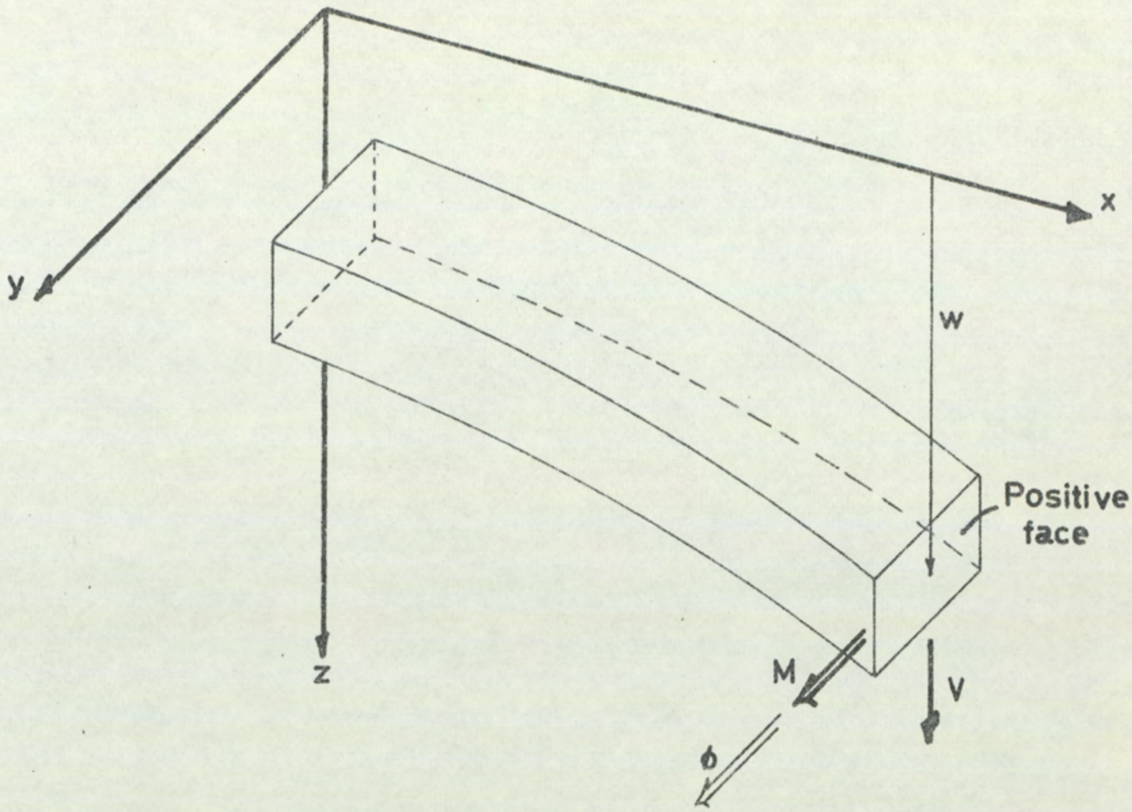


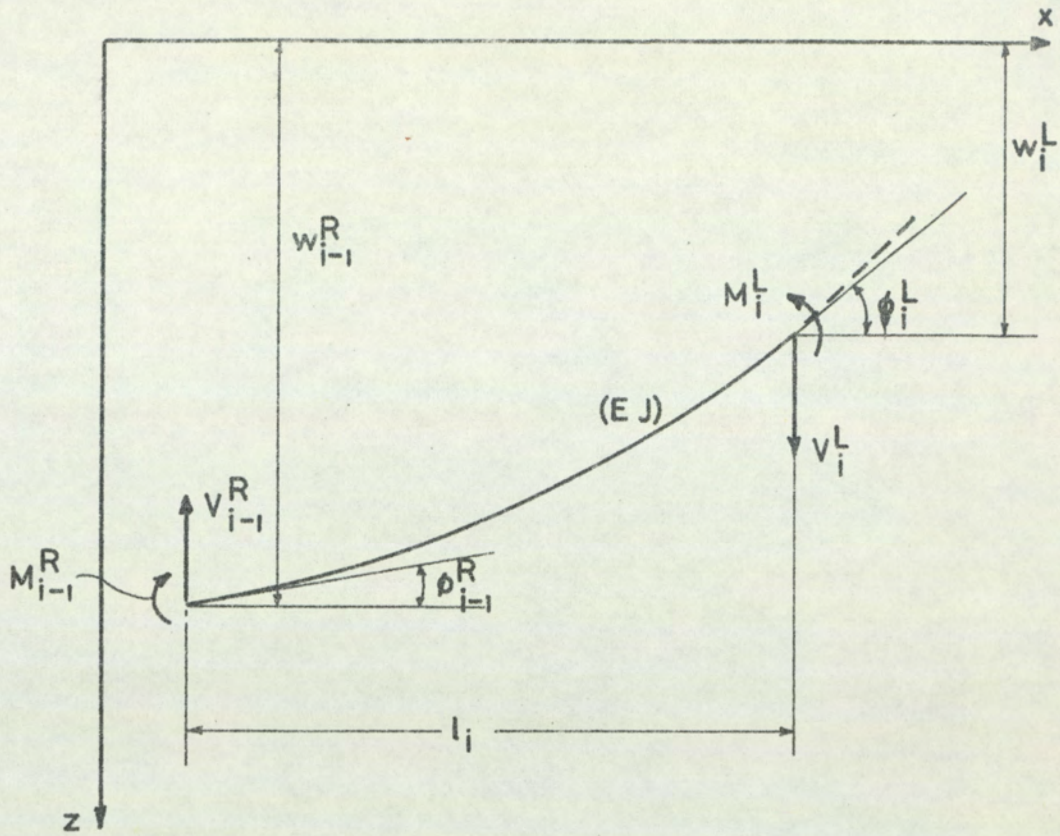
FIG 5-8

SECOND FLEXURAL MODE OF REAR AXLE.



SIGN CONVENTION FOR VERTICAL PLANE OF BEAM

FIG 5-9



FREE-BODY DIAGRAM FOR A MASSLESS BEAM

FIG 5-10

CHAPTER 6 BASIC THEORY AND PRACTICE OF RANDOM PROCESS ANALYSIS

6-1) Summary

A random process is defined together with the basic describing functions used in the analysis of random processes. Methods of evaluating the describing functions are discussed with particular emphasis on the analog equipment used to evaluate power spectral density functions and correlation functions.

6-2) Definition and classification of a random process

A random process in the context of this thesis is a time variant process which cannot be described by a mathematical function of time. Resulting from this definition each time history observation of the process will be unique, and the process should be shown as an ensemble of time histories as the example in fig. 6-1.

Random data is divided into two basic classifications:

i) None stationary data. This is data for which the describing functions vary with time.

ii) Stationary data. This is data for which the describing functions do not vary with time.

In these discussions we shall be considering the latter of these classifications, and this class of data can be divided into two further sub-classes:-

a) non-ergodic data, which is data for which the describing functions must be derived from averages across the ensemble.

b) Ergodic data, which is data for which ensemble averages can be replaced by simple time averages taken along any one of the member time histories of the ensemble.

6-3) Justification of the assumption of random data for road test recordings

Prior to detailing the theory of random process analysis the reasons for the assumption of random data being applied to the road test recordings will be set out.

Let us consider the factors likely to influence the rear axle gear noise as an example. The main factors affecting the meshing of the crown wheel and pinion are:-

i) The none uniform rotation of the propeller shaft due to the small variations of engine speed and the influence of the Hookes joints.

ii) The none uniform meshing force due to the variations in engine output torque, and the variation of driving force required at the rear wheels due to friction windage, gradients and inertia.

The resulting gear meshing process will be a narrow band random process centred on the nominal meshing frequency of the gears.

Similar arguements can be applied to many of the other noise and vibration source in an automobile, and as a result of the practical implications of the central limit theorm, the process resulting from the summation of these processes will be a normally distributed random process.

Further support for these assumptions was obtained from the analyses of the road test recordings obtained while investigating the influence of the propeller shaft. These recordings showed many of the characteristics of random processes.

6-4) The characteristic describing functions of random processes

Since a random process is not capable of exact mathematical description it is necessary to use statistical techniques to derive describing functions. The describing functions which are pertinent to the work discussed in this thesis are developed below.

a) Mean square values (mean values and variances)

The mean square value gives an overall measure of the intensity of the process. For stationary ergodic processes, which is the classification of data, we shall be dealing with in all these definitions, the mean square value is the time average of the instantaneous square of the process amplitude taken over an infinite time. In equation form the function is given by:-

$$\psi_x^2 = \left(\lim_{T \to \infty} \int_0^T x^2(t)\,dt \right) \frac{1}{T} \qquad 6\text{-}1$$

The more common root mean square (r.m.s.) value is the positive square root of this function.

For convenience it is often desirable to think of a process as the summation of a time invariant component and a dynamic component. The static component is given by the mean value or time average of the process, which in equation form is given by:-

$$\mu_x = \left(\lim_{T\to\infty} \int_0^T x(t)\,dt \right) \frac{1}{T} \qquad 6\text{-}2$$

and the describing function for the dynamic component is the variance which is the mean square value about the mean value given in equation form by:-

$$\sigma_x^2 = \left[\lim_{T\to\infty} \int_0^T (x(t) - \mu_x)^2 dt \right] \frac{1}{T} \qquad 6\text{-}3$$

$$= \psi_x^2 - \mu_x^2$$

The positive square root of the variance is the standard deviation and, in the analog analysis of electrical signals with RC coupled instruments, is often referred to as the root mean square value.

b) Autocorrelation functions

An autocorrelation function gives information about the process in the time domain. The function describes the dependence of the amplitude of the function at one time on the amplitude at another time.

Referring to the time history shown in fig. 6-3, an estimate of the autocorrelation function is obtained by taking a time average of the product of the amplitude at time t and the amplitude at time $t-\tau$ over the sample time T. This becomes an exact autocorrelation function as T tends to infinity or in equation form

$$R(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)\,x(t-\tau)\,dt \qquad 6\text{-}4$$

The quantity $R_x(\tau)$ is always a real valued even function having a maximum at $\tau = 0$, and may be positive or negative.

$$\left.\begin{aligned} R_x(\tau) &= R_x(-\tau) \\ R_x(0) &\geqslant R_x(\tau) \quad \text{for all } \tau \end{aligned}\right\} \qquad 6\text{-}5$$

Fig. 6-4 shows the autocorrelation functions corresponding to the time histories of fig. 6-2. The autocorrelation function of a sine wave is always a cosine wave in τ, as shown in fig. 6-4a, with the same period in τ as the original wave had in t. but the phase information relative to $t = 0$ is lost i.e. for

$$x(t) = X\sin\,(\omega t-\theta) \qquad R_x(\tau) = \frac{X^2}{2}\cos\omega\tau \qquad 6\text{-}6$$

This feature of the autocorrelation function of a sine wave supplies the main application for the use of the autocorrelation function in its own right. Since all periodic functions of time are the sum of a series of sine waves then the autocorrelation function of any periodic wave will always be periodic, since it will also be a sum of similar sine waves in τ but with zero phase in each case. This means that the autocorrelation function can be used to detect the presence of otherwise hidden periodic functions in random signals.

c) Power spectral density functions

The power spectral density of random data is, as the term spectral implies, an interpretation of the relevant process in the frequency domain. It describes the distribution of the process with respect to frequency in terms of the spectral density of its mean square value.

A mathematical interpretation of the power spectral density is obtained from the following arguement. The mean square value of a sample process in the frequency range f to f + δf is obtained by filtering the process with a band pass filter having sharp cut-off characteristics, and computing the average of the squared output from this filter. This average squared value will approach an exact mean square as the sample time approaches infinity, or

$$\psi^2(f,f+\delta f) = \lim_{T\to\infty} \frac{1}{T}\int_0^T x^2(t,f,\delta f)\,dt \qquad 6\text{-}7$$

where $x(t,f,\delta f)$ is that portion of $x(t)$ in the frequency range f to f+δf.

For small values of δf the power spectral density is defined and given by

$$G_x(f) = \lim_{\delta f\to 0}\frac{\psi^2(f,f+\delta f)}{\delta f} = \lim_{\delta f\to 0}\lim_{T\to\infty}\frac{1}{\delta f T}\int_0^T x^2(t,f,\delta f)\,dt \qquad 6\text{-}8$$

the quantity $G_x(f)$ is always a real-valued non-negative function.

For stationary data the power spectral density and the autocorrelation function are related by a fourier transform as follows:-

$$G_x(f) = 2\int_{-\infty}^{+\infty} R_x(\tau)\, e^{i2\pi f\tau}\, d\tau = 4\int_0^{+\infty} R_x(\tau) \cos 2\pi f\tau \, d\tau \qquad 6\text{-}9$$

It is on the basis of this relationship that the numerical evaluation of power spectral density is performed.

Power spectral density curves corresponding to the time histories shown in fig. 6-2 are shown in fig. 6-5. The main difficulty with power spectral density functions is the representation of a sine wave function. From the definition of power spectral density it can be seen that the theoretical power density for a sine wave is zero throughout the spectrum except at the frequency of the wave where it is infinite. This condition can be expressed mathematically as follows:

$$G_x(f) = \delta(f-f_o)\frac{X^2}{2} \qquad 6\text{-}10$$

where δ is the Dirac delta function and f_o is the frequency of the wave. This obviously, is not a physically realisable condition, and any measured power spectral density of a sine wave will exhibit a finite bandwidth and amplitude.

d) Cross-correlation functions

The time domain dependence of one process on a second process is given by the cross-correlation function. This function is given by the following expression:-

$$R_{xy}(\tau) = \lim \frac{1}{T}\int_0^T x(t)\, y(t+\tau)\, dt \qquad 6\text{-}11$$

This includes the autocorrelation function as a special case when $y(t) = x(t)$.

$R_{xy}(\tau)$ is always a real-valued function which may be either positive or negative, but unlike the autocorrelation function it does not necessarily have a maximum at $\tau = 0$, and is not an even function.

Fig. 6-6 shows a typical cross-correlation function of two processes which have a strong correlation at some time delay as is shown by the sharp peaks in the function.

There are three major application of cross-correlation functions:-

i) The measurement of time delays. If it is required to establish the time taken for a process to pass through a linear system then the cross-correlation between the input and the output processes will furnish the information directly. This is because the cross-correlation function will reach a maximum when the value of τ is equal to the delay introduced into the output process (relative to the input process) due to the passage of the process through the system. Any frequency sensitivity of the delay time introduced by the system, as occurs in plate vibrations for instance, will reduce the efficiency of this technique unless careful filtering is employed.

ii) The determination of transmission paths. This application arises directly out of the previous application since if a disturbance is transmitted between two points in a system by more than one path, then each path will have a different time delay.

The resultant output process will be the sum of a number of components each having a different delay and amplitude relative to the source process. The relative delays and intensities of the components of the output process can be measured by cross-correlating the two processes, when a correlogram with peaks at each of the appropriate delays will be obtained.

iii) Detection of known signals in noise. This is an extension of the use of the autocorrelation function for the detection of periodic signals in noise signals. If a signal is known before it is combined with a noise signal then the noisey signal can be cross-correlated with the known signal to reveal the required signal. The use of the cross-correlation function for periodic signals will give higher signal to noise ratio in the output wave form than the autocorrelation function but it also has the advantage that it is not restricted to periodic signals.

6-5) Factors affecting the measurement of power spectral density

The theoretical requirements for the analog measurement of power spectral density are an infinitely long sample process, an infinitely narrow bandwidth filter, and true square law detection circuit. The first of these requirements could be approximated by the use of long magnetic tape recordings, but is undesirable due to the difficulty of obtaining such a record, and the excessive analysis time which would result. Physical limitations reduce the second requirement to a finite narrow filter bandwidth and as will be shown later, this is also desirable from the point of view of analysis time. True square law detectors can be produced, but are often replaced with averaging circuits since these have a greater dynamic range and with correctly chosen filter bandwidths they will give comparable accuracy. The result of these various limitations is that certain compromises are necessary in order to obtain reasonable accuracy in the analyses.

a) Statistical accuracy. Since the analysis of the process will only give an estimate of the true power spectral density, it is desirable to reduce the expected error in the measurement to a minimum. The mean square error for the measurement of power spectral density can be shown to consist of two parts, a bias term which can be neglected providing the estimate is properly resolved, and a variance term. For an analyser with a filter bandwidth of B and a true averaging time of T the standard deviation of the measurement about the true value is given by;-

$$\sigma_{G(f)} = \frac{\hat{G}(f)}{BT} \qquad 6\text{-}12$$

Or in terms of the normalised standard error

$$\sigma = \frac{\hat{G}(f)}{G(f)} = \frac{1}{BT} \qquad 6\text{-}13$$

Where the $\hat{}$ is used to show the estimated value.

Equation 6-21 means that with a confidence level of 67% the measured value will lie within $\pm\sigma$ of the true value. The above two formulae only apply if $\sigma < 1$ if σ is greater than this then the actual sampling distribution must be used to obtain the deviation of the estimate.

Providing the output of the analyser filter has a Gaussian amplitude distribution then the estimate of the mean square of the filter output will be distributed about the true mean square with a Chi-square distribution with n degrees of freedom. In equation form this gives:-

$$\frac{G(f)}{\hat{G}(f)} = \frac{\chi^2_n}{n} \qquad 6\text{-}14$$

The values of χ^2 for various confidence limits are tabulated in books of statistical tables.

It has been shown (ref. 8), that the number of degrees of freedom n is given by:-

$$n = 2BT \qquad 6\text{-}15$$

b) Filter bandwidth. It was shown above that one factor effecting the selection of the filter bandwidth is the desired accuracy of the estimate of power spectral density. It was shown that in order to increase the accuracy of the measurement it was required to increase either the filter bandwidth or the averaging time. Since the averaging time is usually controlled by the length of sample process available, increased accuracy can only be achieved by increasing the filter bandwidth. However, one of the requirements of the statistical accuracy discussions was that for the bias error to be neglected the filter output should be properly resolved.

The resolution requirements for power spectral density analysis are best explained by example. Consider a signal which has a true power spectrum consisting of a narrow band of random process only 20 Hz wide. If this signal were analysed using a filter bandwidth of 50 Hz then the peak in the measured spectrum would appear to have a bandwidth of slightly greater than 50 Hz since the analyser will indicate the presence of the signal for all frequencies for which the filter bandwidth contains all or some of the signal. Secondly the peak would be displayed with an amplitude of 2/5 the correct amplitude due to the bandwidth division. Obviously as the filter bandwidth is reduced these errors are reduced, and it has been found (ref. 8) that the errors can be reduced to 10% if the filter bandwidth is made 1/4 of the bandwidth of the narrowest peak in the spectrum.

This requires some pre-knowledge of the spectrum, but this can often be obtained by quick look analysis, and prior knowledge of the type of system from which the signals were obtained. Resonant mechanical systems for instance, almost invariably have resonant bandwidths which are proportional to the frequency of excitation, and this has the advantage that the filter bandwidth can be increased as the analyser centre frequency is increased.

c) Detection. For absolute accuracy of detection a true square law device should be employed after the narrow band filter of an analog power spectral density analyser.

This type of circuit is difficult and expensive to construct, particularly for the wide dynamic range required.

Under certain conditions the true square law circuit can be replaced by an absolute value averaging circuit. This type of detection circuit is inexpensive and easy to produce, and has the wide dynamic range required for power spectral density analysis.

The main condition pertaining to the use of an absolute averaging circuit is that the filter output should have a Gaussian distribution. Burrow (ref. 9) has shown that providing the bandwidth of the filter is less than, or equal to the bandwidth of any narrow band random content in the signal being analysed then the above condition will be satisfied with negligable error. Hence providing the resolution requirements detailed above are met, then the use of an absolute average detection circuit is acceptable.

Calibration of an absolute average detection circuit depends on the known relation between the average value of a Gaussian distributed process and its r.m.s. value, and the use of a log convertor to give the mean square value simply as a scale factor. Obviously in this case the averaging of the output must take place prior to the log conversion since it is average mean square value that is required, and not the average of the log of the mean square value.

d) Averaging.time and methods. Almost invariably the maximum averaging time that can be used is determined by the available sample record length. However, in cases where either very long records are available or wide bandwidth filters are being used, the averaging time will be determined by the desired statistical accuracy according to equation 6-13, or by the desired number of degrees of freedom in the Chi-squared distribution of the estimate as given by equation 6-15.

There are two methods of obtaining the desired average with electrical analog circuits true averaging with an integrating operational amplifier or time weighed averaging with passive resistor capasitor networks.

The true averaging circuits are most commonly used with analysers which operate on a stepping frequency system, and require a large amount of logic to control the sequencing of zeroing, readout, etc.

Time weighed averaging is the most common method employed to obtain a time integral of an analog signal due to its simplicity as is shown in fig. 6-7. These circuits are often referred to as RC averaging circuits due to their construction. The time constant of this circuit is given by the product of the values of the two components of the network, i.e.,

$$K = RC \qquad 6\text{-}16$$

Where R will also include the output resistence of the preceding circuit. It is shown by Bendat and Persol (ref.8) that the effective averaging time of an RC circuit is twice its time constant, i.e.

$$T = 2K \qquad 6\text{-}17$$

Bendat and Piersol also show that the best setting of an RC averaging device is with

$$K = T_r \qquad 6\text{-}18$$

which will give an error of 4% relative to true averaging.

Fig. 6-13 shows the response of an RC averaging network to stationary periodic and stationary random signals. It can be seen from those curves that approximately 4K seconds are required for the output of the network to reach its full value after a change of input condition. This response characteristic determines the maximum operating speed of an analyser since with a stepping analyser, at least 4K seconds must be allowed prior to readout and with a sweep analyser, the sweep rate must not exceed $\frac{B}{4RC}$ which is only $\frac{1}{4}$ the rate achieved with true averaging.

The advantages of RC averaging over true averaging are the great simplification of control circuit, and the production of a continuous curve rather than a series of points.

6-6) Factors influencing correlation measurements

The generation of correlation functions by analog methods requires the delaying of one signal relative to a second signal, the multiplication of the resulting signals, and the time averaging of the product. Consequently the factors to be considered in the measurements are the statistical accuracy, the lag time for proper resolution, and the averaging time.

a) Statistical accuracy. In the case of cross-correlation of two signals of different bandwidths, it is impossible to evaluate the error in simple mathematical terms, since it is a function of the characteristics of both signals and the correlation being measured. For the case of two Gaussian distributed signals, both having the same bandwidth B, then the error is given by:-

$$e = \frac{\alpha(\hat{R}_{XY}(\tau))}{R_{xy}(\tau)} = \frac{1}{BT}\left(\frac{1+R_x(0)R_y(0)}{R_{xy}^2(\tau)}\right)^{\frac{1}{2}} \qquad 6\text{-}19$$

This equation includes the case of autocorrelation as a special case when $y(t) = x(t)$.

b) Resolution. There are two cases to be considered here, firstly, the number of points required in the total lag to correctly resolve the correlogram for data with a given maximum frequency content and secondly, the scan rate required for a continuously variable lag time correlation analyser employing RC averaging.

In the first case the spacing of the points on the scale is fairly simply established from the general rule that a minimum of two points per period are required to define data at any frequency. Thus the minimum resolution requirement is given by:-

$$h = \frac{1}{2f_m} \qquad 6\text{-}20$$

In the second case a continuous correlogram is produced which appears to be defined for all τ but is in fact a time weighed average of the previous values, and this tends to smooth out the sharp peaks of the correlogram. Consequently, the resolution will be a function of the scan rate of the analyser which is discussed later.

c) Averaging time. In the case of step by step analysis employing true integration the averaging time should obviously be equal to the sample time in order to obtain the maximum statistical accuracy. The time constant for the case of RC averaging should be selected to equal the record length as was the case for power spectral density analysis.

d) Scan rate. For the discreet τ value analyser, the scan rate is determined by the integration time, and the step length, i.e.

$$S.R. = \frac{h}{T} \tag{6-21}$$

For the continuous scan case the RC time constant and the effective resolution required will fix the scan rate and as in the case of power spectral density measurements four time constants should be allowed between the equivalent measurement points, i.e.

$$S.R. = \frac{h}{4T} \tag{6-22}$$

6-7) Practical methods of analog measurement of power spectral density

In the foregoing discussions it was stated that most power spectra derived from the vibrations of mechanical systems will tend to contain narrow band peaks which will have bandwidths proportional to the centre frequency of the peak. Based on this constant percentage bandwidth filters would appear to be the logical choise for use in power spectral density analysers. However, there are a number of limitations which make them undesirable from the practical point of view. The major limitation is the construction of a filter having a suitable shape factor[1] which should be 4 or less. Even if this problem could be overcome, there would still remain the difficulties of providing a variable RC time constant, and a variable sweep rate to make full advantage of the characteristics of the filter.

To simplify the construction and operation of power spectral density analysers, most commercial analysers employ constant bandwidth filters. There are two types of constant bandwidth filters both using the hetrodyning principle of operation where the signal to be analysed is hetrodyned with a high frequency signal in order to shift it to a high frequency where it is filtered by a crystal filter. The difference between the two is that the first hetrodynes the signal with a high frequency signal derived from a local variable high frequency oscillator as shown in fig. 6-9, whereas the second generates the hetrodyning signal from a

1 - The shape factor of a filter is defined as the ratio of the 60 Db bandwidth to the 3 Db bandwidth.

fixed frequency oscillator, and an audio frequency tuning signal, as shown in fig. 6-10.

The commercial analyser used to measure the power spectral density functions described in the next chapter used a filter of the second type. In the analyser the single filter of fig. 6-10 was replaced by a set of five filters which could be switched automatically as the analysis frequency increased. Each filter was equipped with its own calibration, sweep rate and RC time constant controls to ensure that the maximum utilisation was achieved. A block diagram of the basic system is shown in fig. 6-11.

This analyser with the availability of automatic filter changing with frequency enabled some of the advantages of the constant percentage bandwidth analyser to be gained with less complication. The full operating parameters for such an analyser are set out in ref 9, and since these are purely mechanical, they are not repeated here.

Two main problems were encountered in the use of the equipment. Due to the use of magnetic tape loop recording to obtain records of sufficient duration for the analysis, trouble was encountered from the splice in the tape creating transient inputs to the analyser. This was overcome by ensuring that the RC time constant was always equal to the loop repetition time. Use of magnetic tape recording supplied the answer to the second problem which was the excessive time required for each analysis, since if the tape speed was increased for play back then the time was reduced proportionally.

The complete equipment used for the analysis is shown in fig. 6-12.

6-8) The practical methods of analog measurement of correlation functions

The most common form of analog correlation analyser is the moving head magnetic tape or drum system. In this system time delay is achieved by recording the two signals simultaneously onto two tracks of multi-channel magnetic tape loop or drum, and then replaying from different heads one of which can be moved relative to the other, hence creating a variable delay time between the signals. This type of system is very good for low frequency signals, but for high frequency signals the high tape speeds and high degree of precision required in the mechanical components makes them excessively expensive.

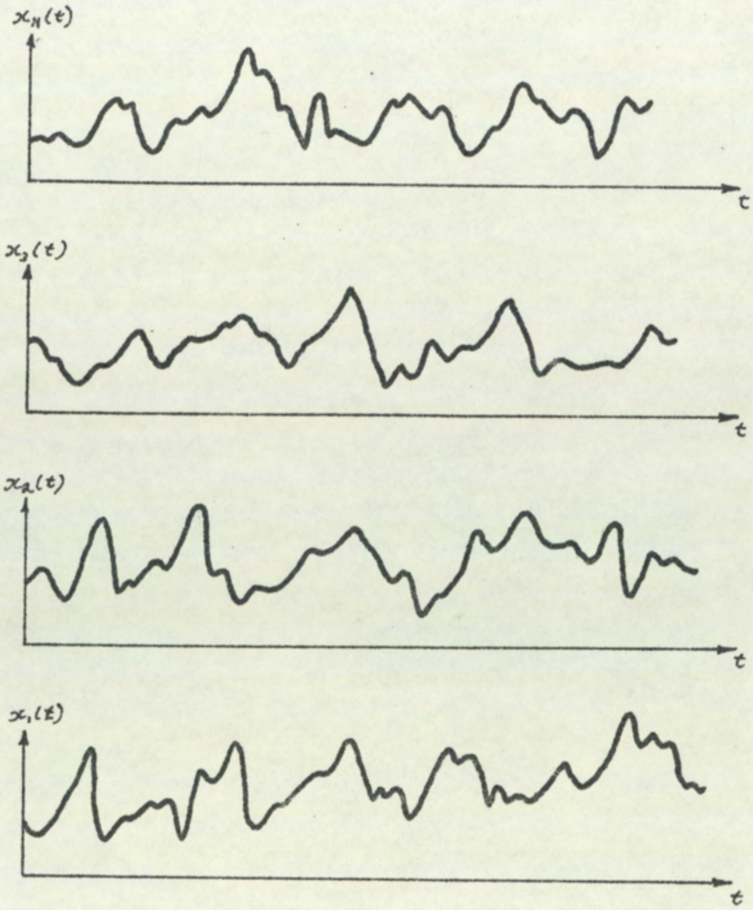
Ordinary delay lines can be used where only short delays are of interest, but they are not suitable for long delays due to the possibilities of non-linear phase frequency characteristics. The use of magnetic tape time transforms can extend the use of delay lines, providing the bandwidth of the signals is not too great.

The commercial analyser employed for correlation measurements of vibration data from road tests used a hybred analog pulse system. The process employed was to sample one signal at regular intervals and store the sample voltages in a sample hold circuit. After the required time delay, the second signal was sampled. Multiplication of the two samples was performed by using the first sample to create a pulse of constant amplitude but with a duration determined by the sample amplitude, and then varying the amplitude of the pulse by the second sample. A width and amplitude modulated pulse train was thus created, the mean value of which was proportional to the required correlation function. The mean value was obtained by RC averaging the pulse train and this enabled a continuous sweep of delay time to be used.

This system had the great advantage of simplicity of operation and also was relatively inexpensive. It had a very wide bandwidth 50 to 250,000 Hz, but the maximum time delay was limited to 17 ms.

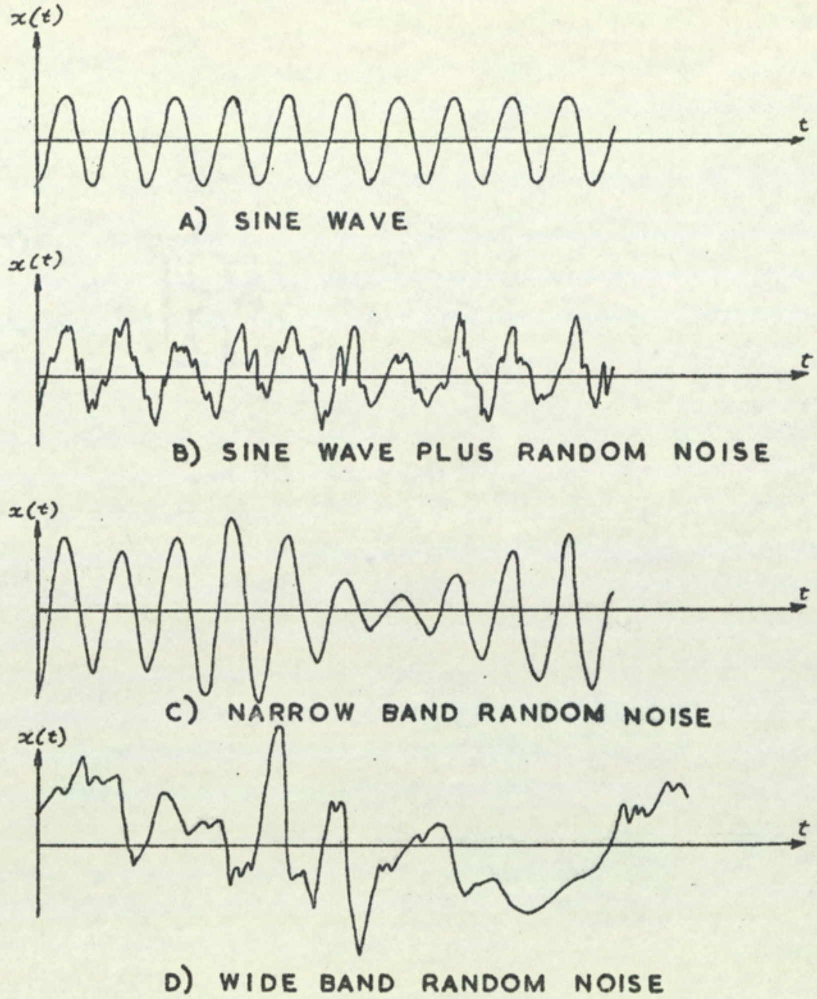
The total instrumentation systems for correlation analysis are shown in block diagram form in figs. 6-13 and 14.

Tape loop splice transients again gave trouble, particularly in the case of autocorrelation at small time delays, but careful pre-recording treatment to the signals to ensure the full utilisation of the FM deviation of the recorder, and the use of a limiting amplifier to clip the transients on output from the recorder reduced this to a minimum.



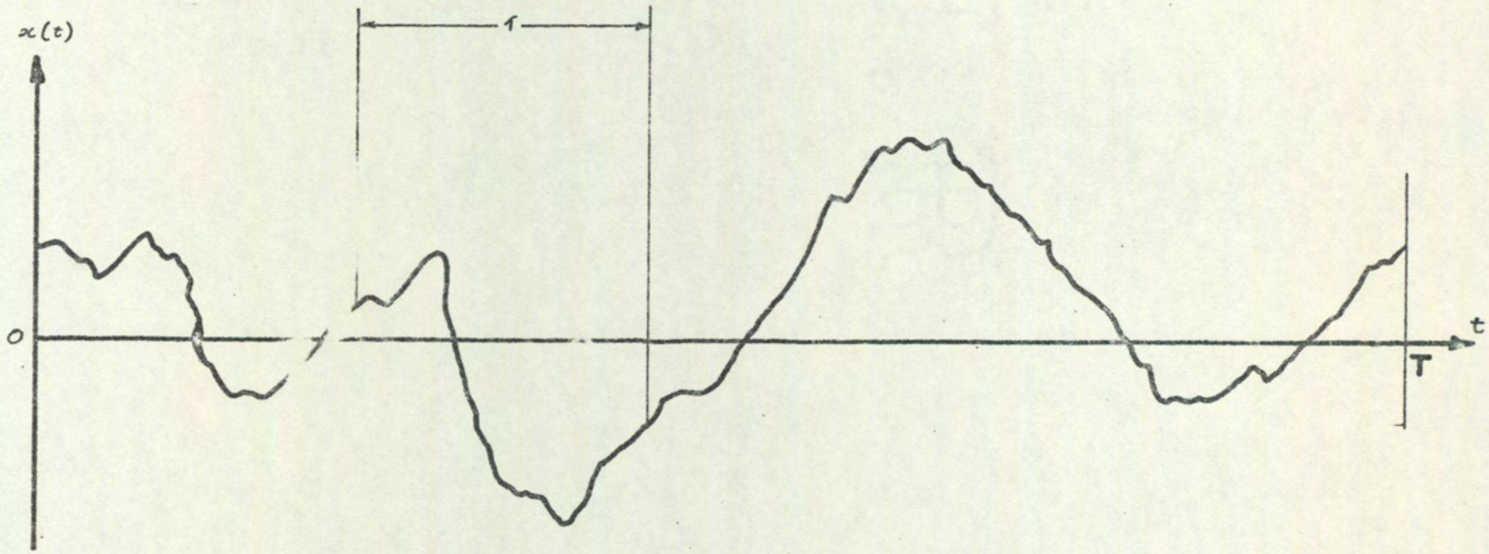
ENSEMBLE OF SAMPLE FUNCTIONS FORMING A RANDOM PROCESS

FIG 6-1



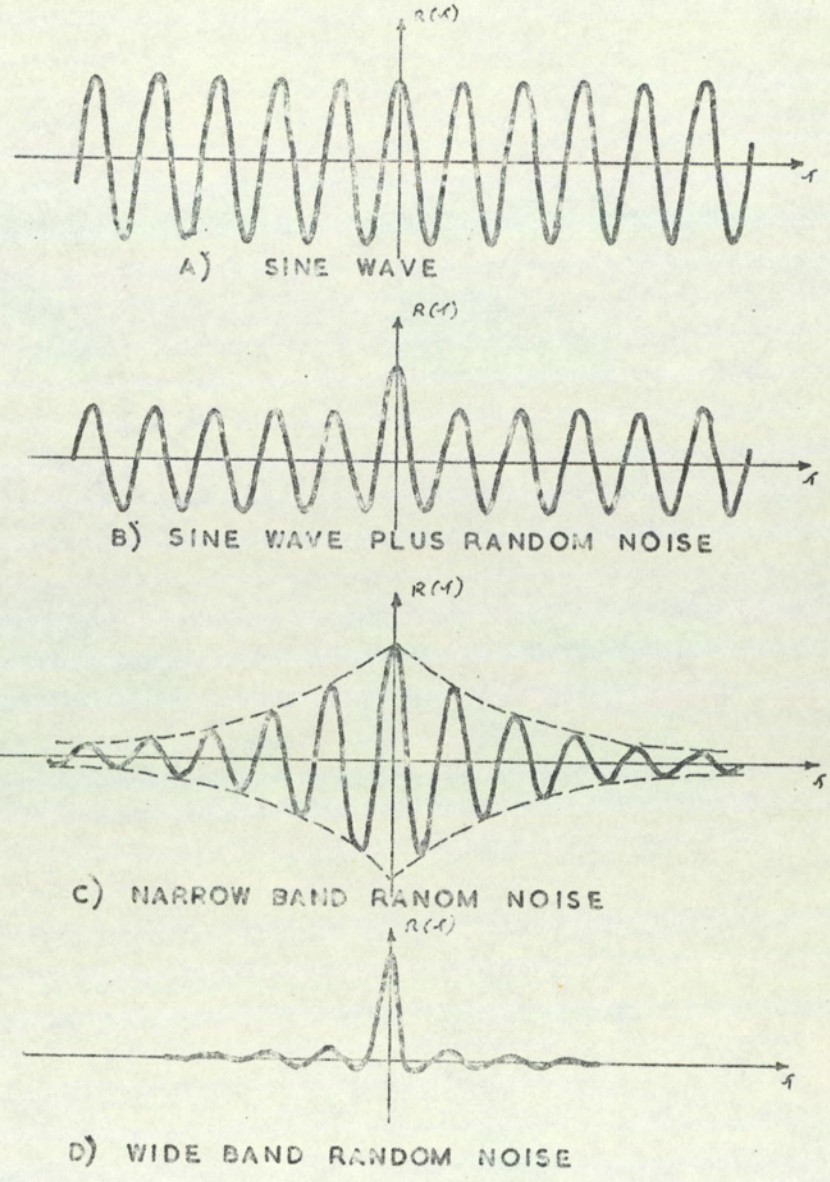
FOUR EXAMPLE TIME HISTORIES

FIG 6-2



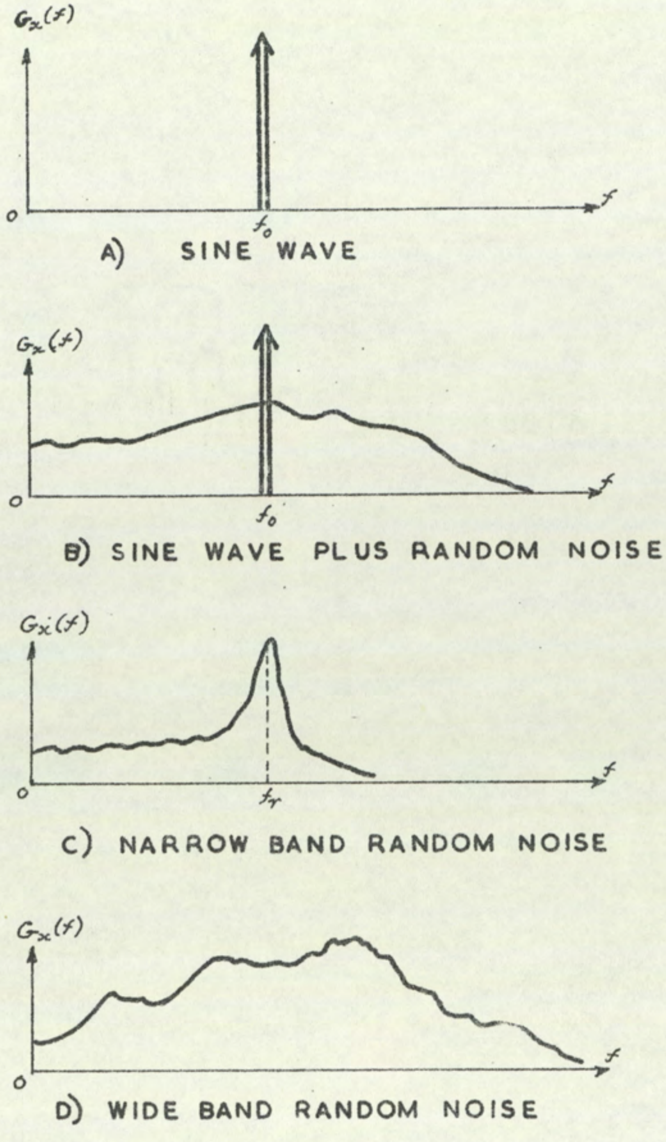
MEASUREMENT OF AUTOCORRELATION FUNCTION

FIG 6-3



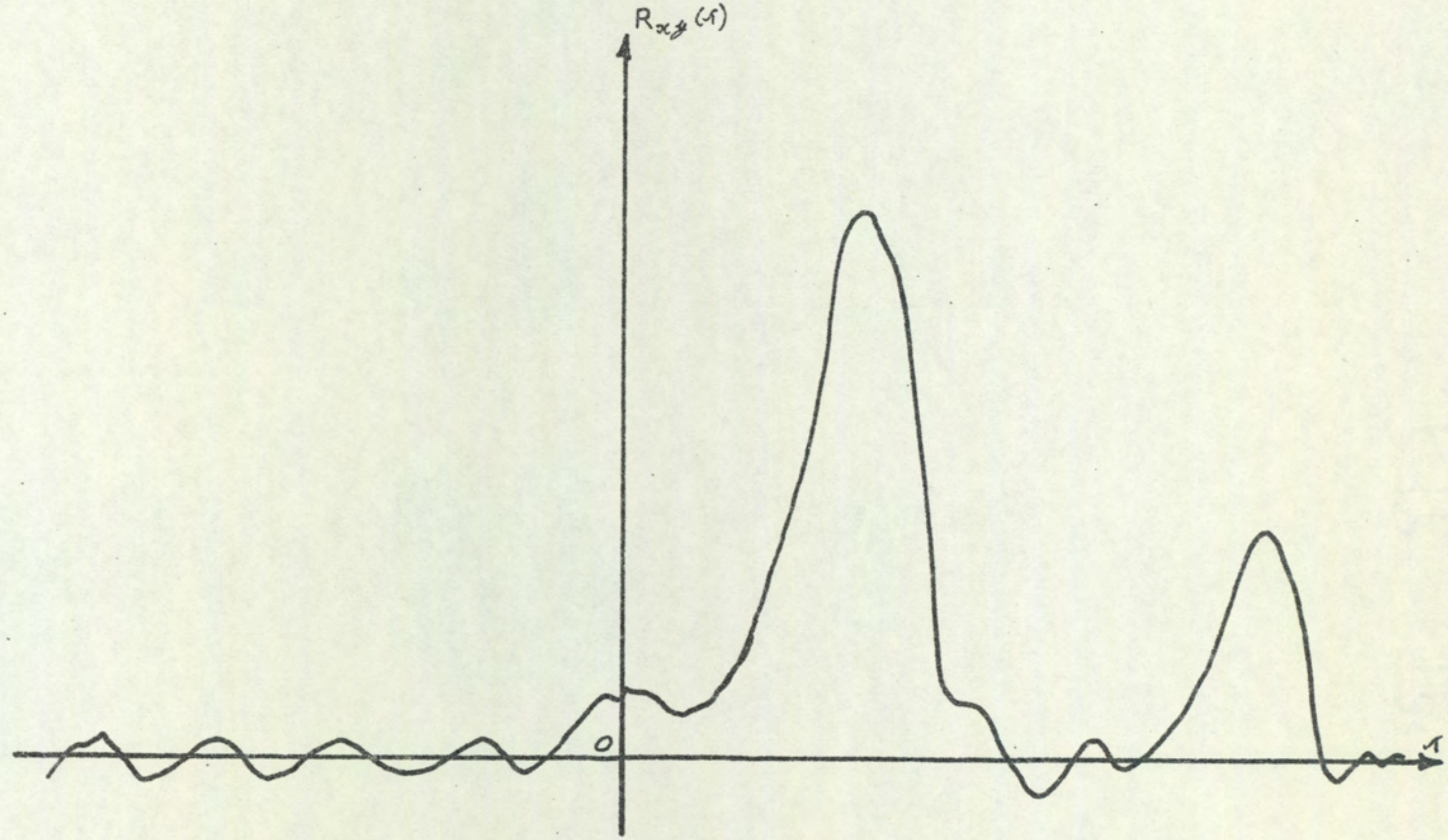
AUTOCORRELATION FUNCTIONS CORRESPONDING TO THE TIME HISTORIES SHOWN IN FIG 6-2

FIG 6-4



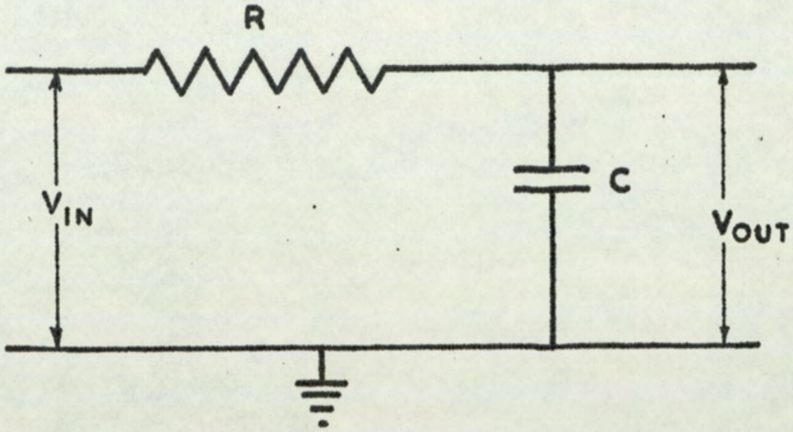
POWER SPECTRAL DENSITY FUNCTIONS CORRESPONDING TO THE TIME HISTORIES SHOWN IN FIG 6-2

FIG 6-5



A TYPICAL CROSS-CORRELATION PLOT (CROSS-CORRELOGRAM)

FIG 6-6



TIME CONSTANT $K = RC$

RC AVERAGING NETWORK

FIG 6-7

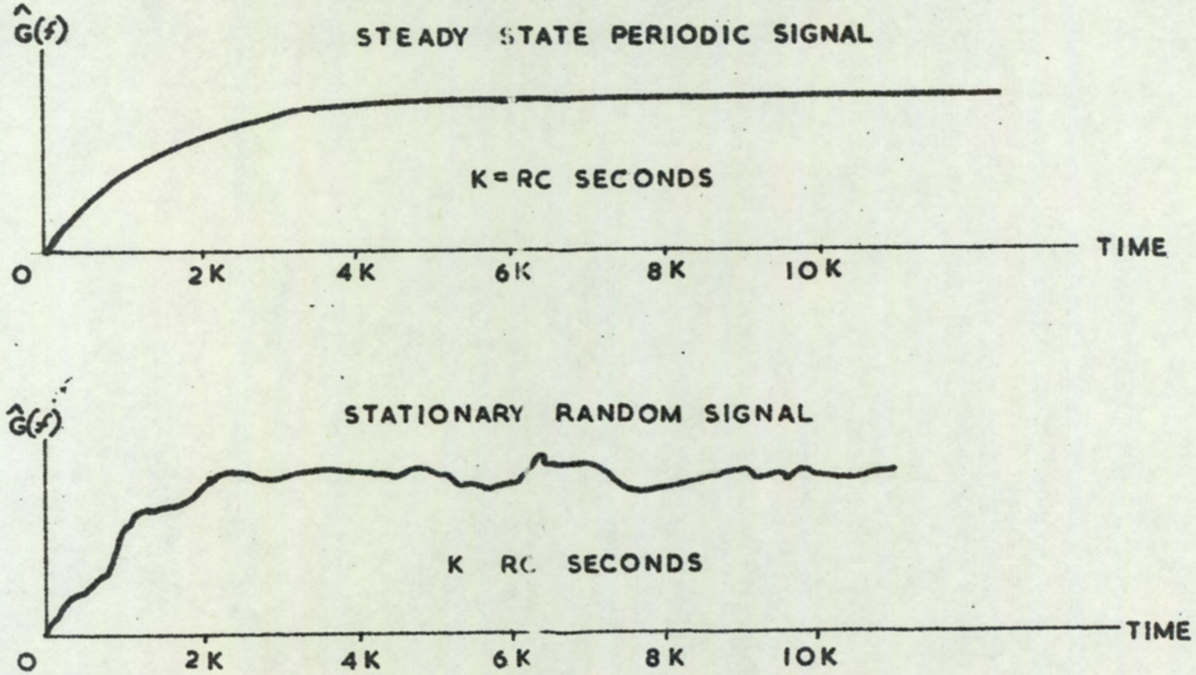
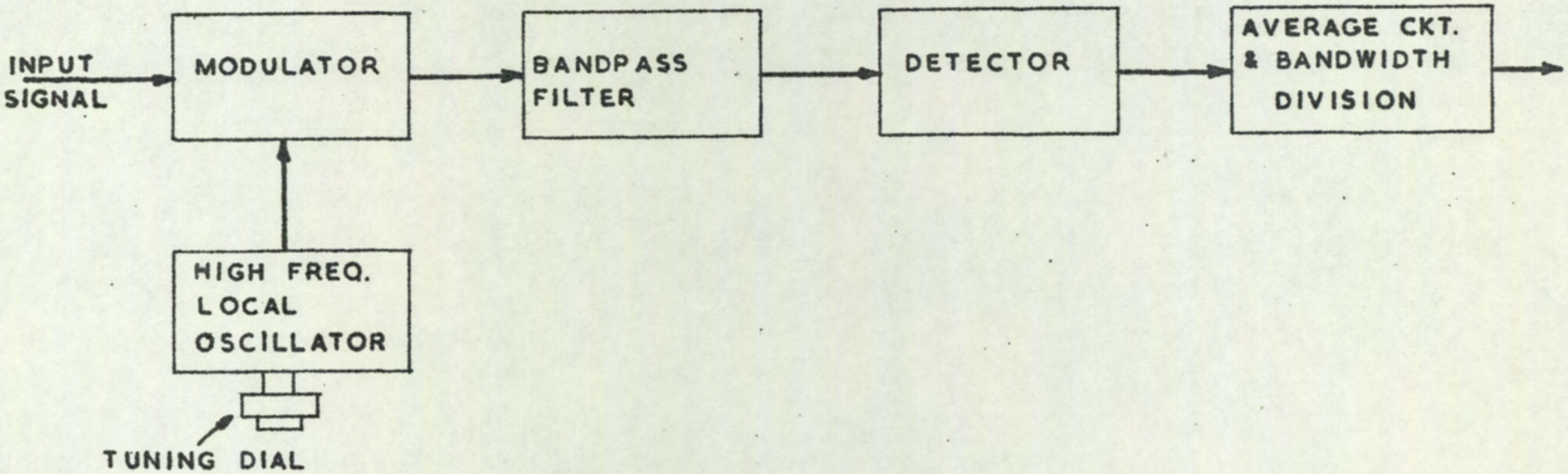


FIG 6-8

TIME HISTORIES FOR RC AVERAGED POWER SPECTRAL DENSITY



CONSTANT BANDWIDTH ANALYSER USING LOCAL OSCILLATOR

FIG 6-9

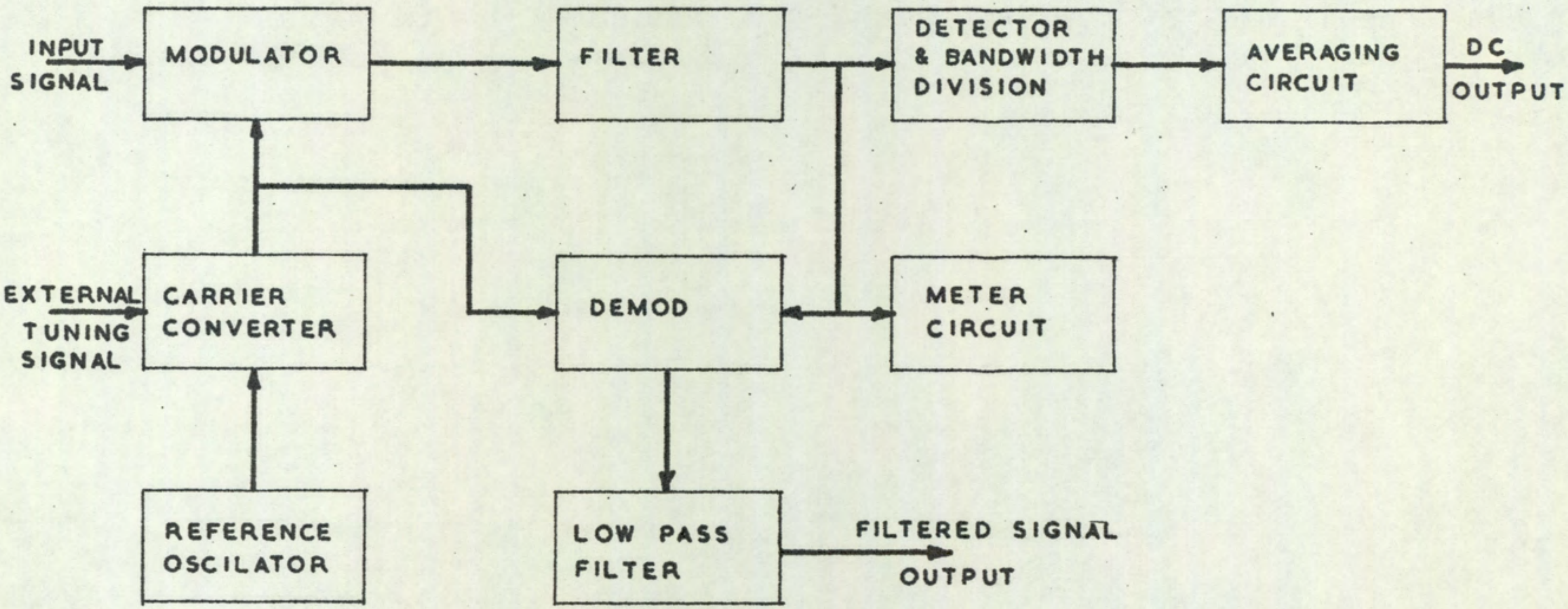


FIG 6-10

CONSTANT BANDWIDTH TRACKING FILTER

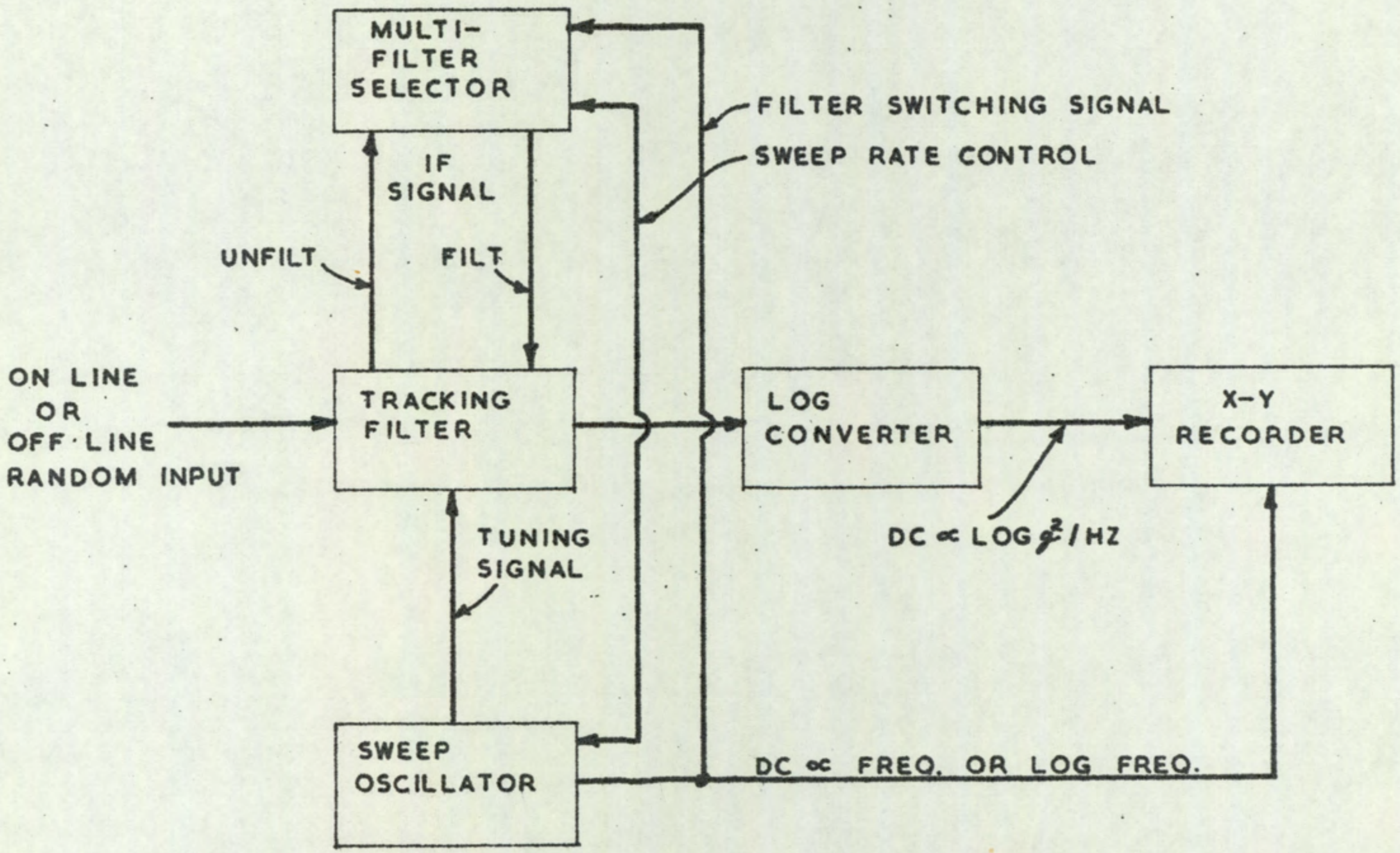
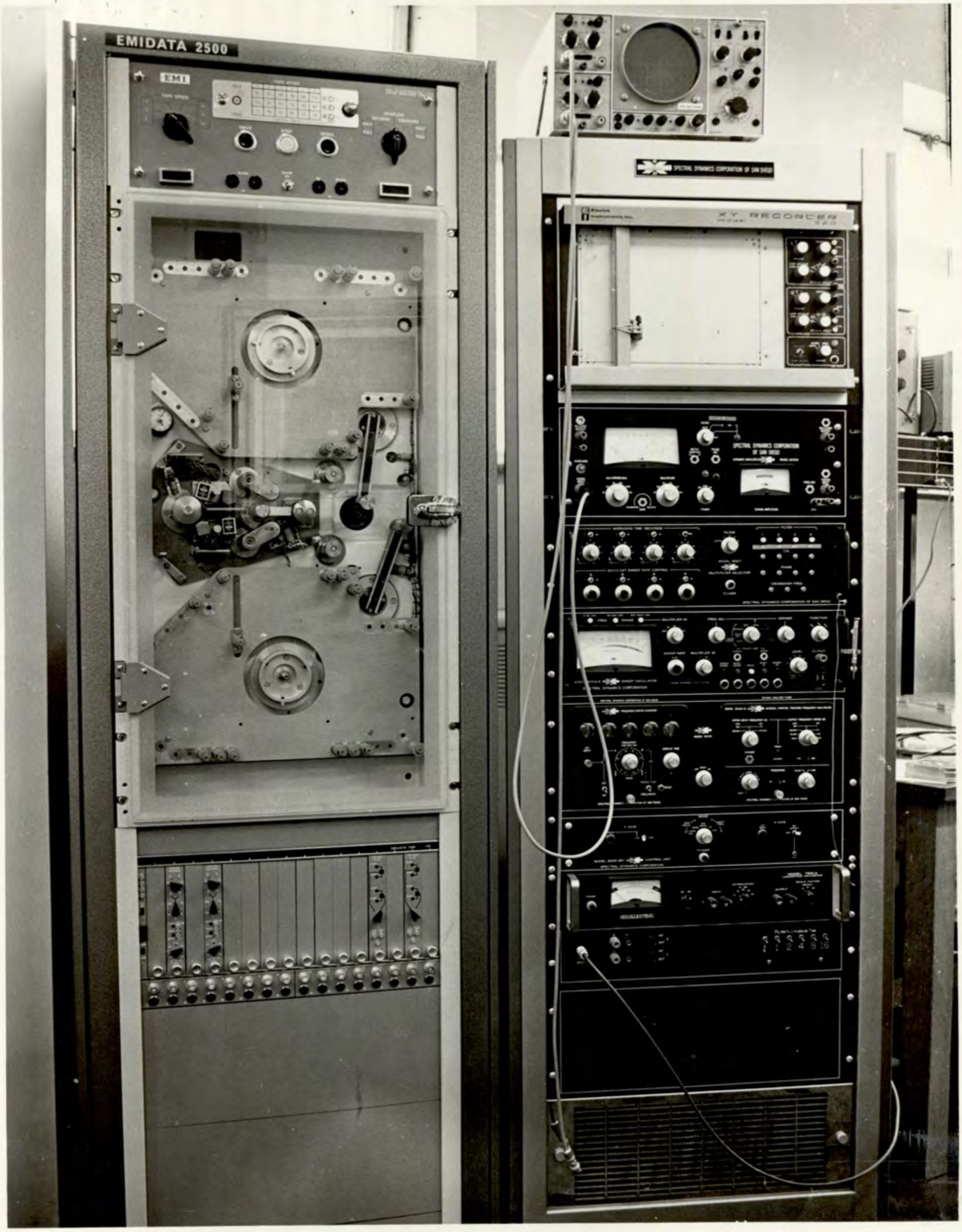


FIG 6-11

POWER SPECTRAL DENSITY ANALYSER



POWER SPECTRAL DENSITY ANALYSIS EQUIPMENT

FIG 6-12

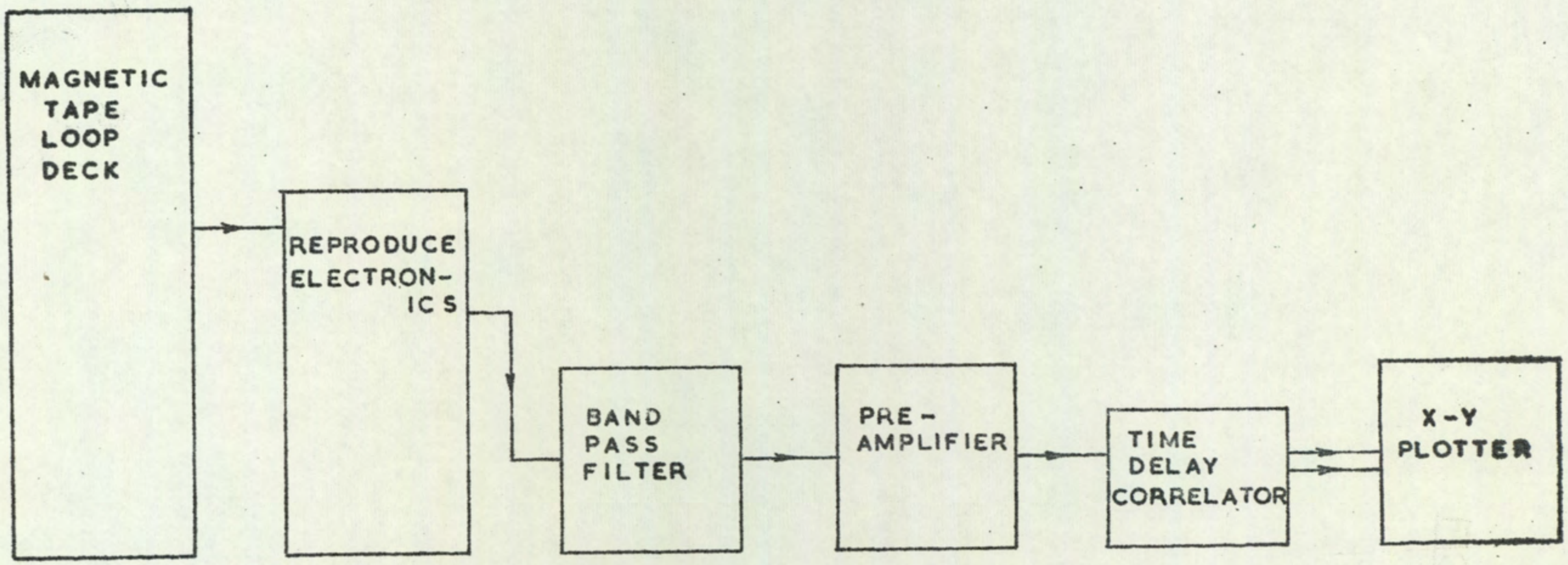
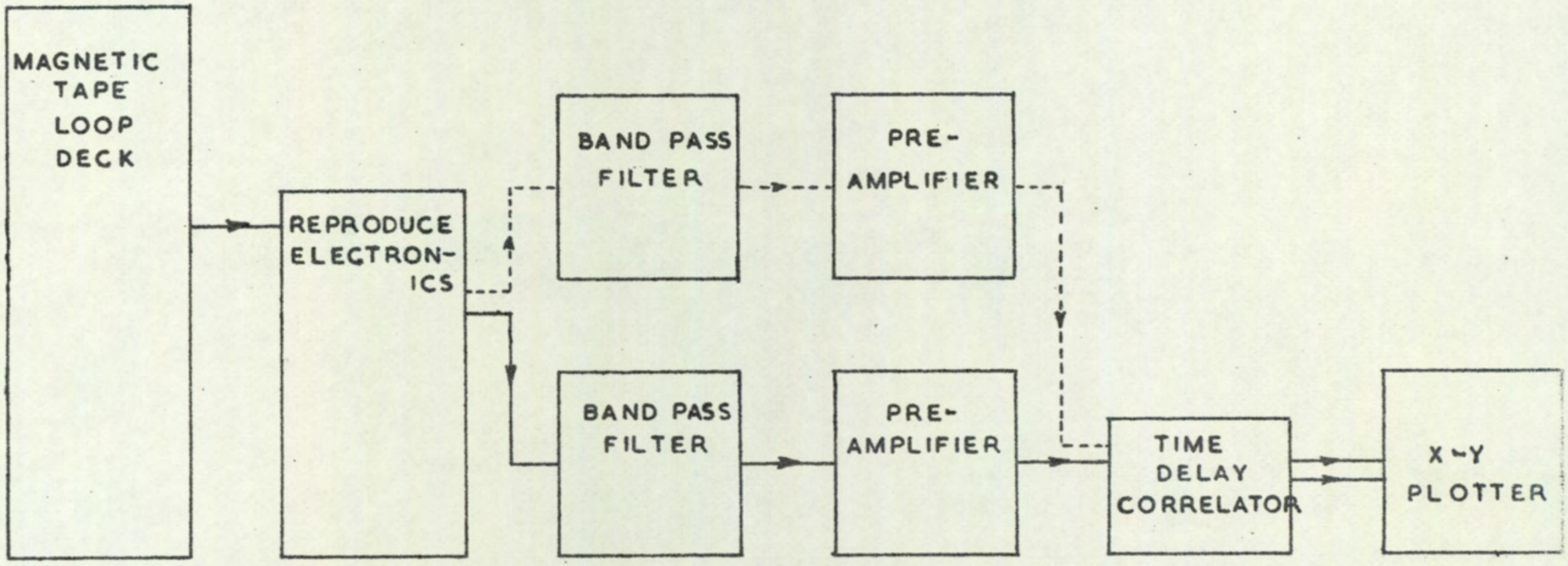


FIG 6-13

ANALOG DATA PROCESSING SYSTEM FOR AUTOCORRELATION



ANALOG DATA PROCESSING SYSTEM FOR CROSS-CORRELATION

FIG 6-14

CHAPTER 7 INFLUENCE OF THE REAR SUSPENSION SYSTEM.

7-1) Summary

Laboratory and road tests designed to establish the influence of the rear suspension in the mechanical transmission of the rear axle vibration are described, and the results obtained discussed. These tests employed the more advanced analysis equipment required for random process analysis, and improved techniques for controlling the road test conditions.

7-2) Description of laboratory test

The purpose of the laboratory tests was to establish if there was a critical path for the transmission of the rear axle gear meshing frequency disturbance to the vehicle body, and hence to the passenger compartment. Since the acoustic response in the passenger compartment was the prime factor of interest, this was the major response function measured, together with the acceleration response at the force input points.

In order that only the influence of the suspension system was studied, the rear axle and propeller shaft were removed, and the rear end of the vehicle suspended by nylon cords which had a sufficiently low spring rate in tension to act as isolators for the frequency range of the tests.

The instrumentation system used is shown in block diagram form in fig. 7-1, and was very similar to the automatic response test equipment detailed in Chapter 3. The acoustic response was measured in all cases at the near side rear passengers head position with a condenser microphone. Input force and input point acceleration were measured with an impedence head which consisted of series mounted load cell, and accelerometer, both of which were crystal transducers, operated in conjunction with charge amplifiers. Either the input force or the input acceleration could be used for the level control amplifier, and tests were carried out under both constant force input, and constant acceleration input conditions.

To discover if either of the three body mounting points was more important as an input point, the springs and dampers were removed completely, and forcing applied to each of the near side mounting points in turn. Further tests were performed with input via the damper only, spring only, and the damper and spring system complete.

For the tests with the springs, it was necessary to use two short pieces of round bar to replace the axle in order that the U bolts and locating and mounting plates could be used to clamp the spring in the normal manner. To obtain the correct loading of the springs two rubber blocks having a stiffness roughly equal to the stiffness of a tyre were used to stand the centre of each spring on. So that the excitation could be applied to the centre of the spring the rubber block used on the near side had a hole through the centre large enough to take the impedence head.

7-3) Description of road tests

These road tests were performed with two objectives in view.

a) to confirm that more reliable information could be obtained by the use of random process analysis and

b) to evaluate the influence if any, of the rear suspension system on the audible rear axle noise.

Two series of tests were performed:

The first series of tests employed 7 transducers in the following positions:

1. microphone in the passenger compartment in the near side top corner
2. accelerometer on nose of gear box extension
3. accelerometer on nose of rear axle gear carrier
4. accelerometer at centre of the rear axle
5. accelerometer at spring axle interface
6. accelerometer at shock absorber body mounting
7. accelerometer at forward spring body mounting.

All the accelerometers were orientated to measure in the vertical plane.

In addition to these transducers, a ten toothed wheel was fitted between the pinion and propeller shaft.

mounting flanges, and an inductive pickup was used in conjunction with this to provide a reference frequency at rear axle gear meshing frequency. This method of obtaining a reference frequency was used to enable cross-correlation detection of the rear axle gear meshing frequency components of the noise and vibrations to be performed.

Charge amplifiers were used with the accelerometers to obtain a higher gain and a higher signal to noise ratio than had been possible with the cathode followers used in the previous tests. This necessitated a rather large weight of accumulators in the boot of the vehicle, but it was considered that this would have negligible effect at the frequencies of interest.

Tests were performed at constant speed at 2 mile/hr. intervals in the speed range 40 to 60 mile/hr. inclusive. Recordings were taken for 30 seconds at each test speed.

For the second series of tests only six transducers were used as follows:-

1. microphone in passenger compartment in near side rear top corner.
2. accelerometer at centre of rear axle.
3. accelerometer at spring axle interface.
4. accelerometer at forward spring body mount.
5. accelerometer at shock absorber body mount.
6. accelerometer at rear spring body mount.

Transducers 3-6 were all on the off side rear suspension system and transducers 4-6 were on the body side of the mounts. All the accelerometers were aligned to measure vertical acceleration. As in the first series of tests described above, a reference frequency signal was obtained from the ten toothed wheel on the pinion shaft.

To improve the drivers control of the speed of the vehicle and hence the stationarity of the signals, the following speed indication system was devised. The reference frequency from the ten toothed wheel was applied to a frequency to voltage convertor, and the resulting voltage analog of speed indicated on a simple voltmeter. This voltmeter was placed above the dash board of the vehicle in front of the driver, and hence only slightly below his normal line of vision. By this system the control of speed obtained was greatly improved due to the increased response over the speedometer, and the fact that once the desired speed had been reached, the driver could view the meter by peripheral vision.

Signal conditioning and recording were as in first test series, but the test speeds were at given meter settings, and were such as to give approximately 20 Hz intervals of rear axle gear meshing frequency in the range 300 to 600 Hz inclusive.

Tests were performed with standard springs and with springs with enlarged rubber bushes at the forward end. Unfortunately, no time was available for these tests to be repeated with other variations of suspension bushes.

7-4) Results of acoustic response tests

The acoustic response in the passenger compartment due to excitation at the body mounting points of the rear suspension system are shown in fig. 7-2. It can be seen that these responses were very complex with a considerable number of resonant peaks in the frequency range 340 to 700 Hz. This complexity resulted from the coupling of the numerous acoustic modes of the passenger compartment with the equally numerous vibratory modes of the body shell. In respect to the possible transmission of rear axle gear meshing vibration at 440 Hz, all the curves showed a fairly high response in this frequency region, and in particular, the response due to excitation at the rear spring mount showed a distinct resonant peak at this frequency.

The effect of placing the shock absorber in the transmission path in the case of excitation at the shock absorber mount is shown in fig. 7-3. The overall influence of the shock absorber was not very marked, but in the region of 440 Hz there was a marked reduction in the level of response.

Responses with the rear springs fitted due to excitation at the spring axle interface, both with and without the shock absorbers fitted, are shown in fig. 7-5. The tests were performed with three sets of springs, a pair of used standard springs, a pair of unused standard springs, and a pair of used springs which had enlarged rubber bushes at the front ends.

As was the case for the responses due to excitation at the spring mounting point, the responses due to excitation via the springs was complex in nature. In the case of the used standard springs, which were taken as the standard condition, there was a marked resonant condition surrounding 440 Hz, both with and without the shock absorber fitted. With the unused standard spring fitted the response levels were of almost identical intensity, but there was some change in the characteristics in respect to the resonant frequencies. These changes were only to the extent that could be accounted for in manufacturing deviations, and wear of the used springs. The most marked change was obtained when the springs with the enlarged rubber bushes were fitted. These springs gave a reduction of the order of 10 Db relative to the response with the standard springs for the condition without the shock absorbers, and about 5 Db with the shock absorbers. The difference between the two cases being due to only the transmission via one of the three mounting points being directly influenced. The resonant condition at 440 Hz was only influenced to the same extent as the rest of the response, presumably due to this resonance resulting from transmission via the rear spring mount.

The resonant condition in the region of 440 Hz due to excitation via the complete suspension system was not in itself of sufficient intensity to be the sole amplifier of the rear axle disturbance, but if coupled with a resonant condition of the rear axle it would undoubtably be significant.

7-5) <u>Road test results</u>

i) Relation between rear axle gear meshing frequency and road speed.

The reference frequency generated by the toothed wheel on the end of the pinion shaft was used to obtain the actual gear meshing frequency of the rear axle gears for each of the test speeds used in the first series of tests. Since the speed tended to vary slightly during each test run, a number of measurements of the frequency were taken for each recording. Measurements were taken with a digital frequency meter over one second samples, and the results of these measurements are shown in fig. 7-5. The best straight line was drawn through the point, and it was found to have the following law:-

$$f = (8.8S_s + 29) \text{ Hz}$$

where f is the rear axle gear meshing frequency

and S_s is the road speed in mile/hr. measured by the speedometer.

The theoretical line also shown in fig. 7-5 was evaluated from the rolling radius of the rear wheels which was 12.5 in giving the relationship:-

$$f = 9.86S$$

where S is the true road speed in mile/hr.

The empirical relationship was used for all conversions from frequency to road speed and vice versa since all previous analyses had been relative to speedometer reading.

ii) First series of road tests.

These tests were treated as exploratory tests since they were the first tests to be analysed using random process analysis equipment.

Two forms of analysis were applied to the recorded data, power spectral density analysis, and cross-correlation analysis. Power spectral density analysis was performed with an analyser of the type described in detail in Chapter 6. Advantage was taken of the fact that the main interest was in components of the data in the region of the gear meshing frequency and this gave a frequency range of 350 to 600 Hz for the range of road speeds used in these tests. Speeding up of the data by a factor of 4 changed this bandwidth to 1.4 to 2.4 kHz which was still within the 3 kHz bandwidth of the continuous loop magnetic tape recorder which was in use at the time of these analyses. Use of this speeding up technique enabled the analyses to be performed in a reasonable time.

The actual bandwidth analysed for all tests was 200 to 800 Hz real time, i.e. 800 to 3200 Hz analysis time. A 47.5 Hz filter was used throughout this frequency band which gave a resolution of 2.4% at 500 Hz (real time) compared to the 6% of the analyser used for the previous road test analyses. The RC time constant of the analyser was set to 2.5 seconds giving the resultant power spectral density estimate 475 degrees of freedom in its Chi-squared distribution.

The cross-correlation analysis was carried out with a time delay correlator of the type described in Chapter 6. The purpose of the analysis was to extract any periodic component at the rear axle gear meshing frequency from the noise and vibration data. This was achieved by cross-correlating the noise and vibration signals with the reference signal. Due to the waveform generated by the toothed wheel and inductive pickup not being a pure sine wave it was necessary to pre-condition it prior to its use for cross-correlation as follows: the recorded signal was first applied to a limiting amplifier which produced a near squarewave of constant amplitude at the same frequency, and this squarewave was then filtered to give a good sine wave at the rear axle gear meshing frequency. It was also found desirable to precondition the noise and vibration signals to increase the gain of the correlation, by filtering with a bandpass filter. Both the reference and the transducer signals were adjusted to give 1 volt peak input to the correlator, careful note being kept of the gain applied to the transducer signal. A block diagram of the analysis system is shown in fig. 7-6.

An example of a power spectral density curve is shown in fig. 7-7. This curve was obtained from the recording of the noise in the passenger compartment at a speed of 52 mile/hr., and has a well defined peak at the rear axle gear meshing frequency of 490 Hz. From this single measurement there was no way of deciding whether this peak was due to a narrow band random component of the noise, or a periodic component of the noise. If the component of the noise which created the peak were not present, it would be reasonable to assume that the power spectral density would have followed the dotted line across the base of the peak in fig. 7-7. This line may be considered to give the inherent noise at the rear axle gear meshing frequency, and the power of this noise referred to by the subscript n, may be removed from the total power of the peak, subscript t, to give the power of the components creating the peak, subscript p, as follows:

$$G_p = 10\mathrm{Log}_{10}\left(\mathrm{Log}^{-1}\frac{G_t}{10} - \mathrm{Log}^{-1}\frac{G_n}{10}\right) \qquad 7\text{-}1$$

Since the amplitudes of any periodic component of the signals at the rear axle gear meshing frequency was obtained from the cross-correlation measurements a similar formula was used to find the narrow band random content of the peak. Before the power of the periodic signal component estimated from the cross-correlation analysis could be used for the correction of the power spectral density estimates, it was necessary to correct it for the bandwidth division, and the detector constant of the power spectral density analyser. This correction was found to be -16.23 Db for this case. Referring to the periodic component by the subscript g we obtain the power of the narrow band random component subscript r from the following:-

$$G_r = 10\text{Log}_{10}\left(\text{Log}^{-1}\frac{G_p}{10} - \text{Log}^{-1}\frac{G_g}{10}\right) \qquad 7\text{-}2$$

The relative powers of these components of the peaks at the rear axle gear meshing frequency for the noise in the passenger compartment are shown in fig. 7-8. The power shown for the periodic component is the corrected power to give a direct comparison with the other curves. The hypothesis that the rear axle gear meshing noise would be basically narrow band random in nature was strongly supported by these results, since the difference between the peak power and the narrow band random power was only of the order of 0.5 Db while the difference between the periodic and narrow band random component powers was -10 Db or greater. In terms of the relative noise powers in the passenger compartment the total power of the narrow band random component would be 17 to 20 Db greater than the periodic component.

Similar comparisons of the peaks in the power spectra at the rear axle gear meshing frequency and the cross-correlation results for the vibration measurements showed that the narrow band random content of the peaks was even more predominant than was the case for the noise in the passenger compartment. This may have been due to the refining effect of the high Q resonances of the acoustic modes of the passenger compartment.

The springs introduced an attenuation factor of approximately 10 into the vibration characteristics at the rear axle gear meshing frequency as can be seen from fig. 7-9 which shows the variation of total power spectra density with gear meshing frequency. From this figure it was also seen that the major resonance of the rear axle was in the region of 400 to 440 Hz, and the major transmission through the suspension system was also in this frequency range. However, the major response in the passenger compartment occurred at a higher frequency of 485 Hz indicating that the mechanics of the overall system beyond the suspension system may have played an important part in the amplification of the source disturbance to an audible level.

The power spectral density characteristics of the rear axle gear meshing frequency peaks with the inherent noise power removed, which may also be taken as the narrow band component power, are shown in fig. 7-10. These showed a similar pattern to the total power with the 420 to 440 Hz response in the vibration curves becoming even more predominant.

iii) Second series of road tests

Since the first series of road tests were only intended as an exploratory series, they were closely reviewed to see if any of the test techniques could be improved to increase the confidence in the results of this second series. The most obvious area for lack of confidence was in the control of the major test parameter the road speed, as was shown in scatter obtained in the rear axle gear meshing frequency measurements fig. 7-5. The use of the analog frequency meter near to the driver's line of vision considerably improved his ability to control the vehicle speed and resulted in an improvement in the resolution, and statistical accuracy of the power spectra obtained.

To obtain an overall picture of the power spectral density in the passenger compartment an oblique projection plot of the spectra against speed and frequency was drawn fig. 7-11. As can be seen in the frequency band over which the analysis was performed, the general characteristics of the noise spectra were of increase with speed and decrease with frequency. No particularly distinct peaks occurred in the spectra obtained from the lower speed tests, but in the lower frequency region an increasing number of sharply defined peaks did occur as the speed was increased. These low frequency peaks were due to the harmonic excitation from the engine firing disturbance, and were almost certainly periodic components, with random amplitudes due to the inconsistancies of the combustion processes.

In the region of 400 to 500 Hz there was a hump in the basically falling characteristic of the spectra. This hump could be due to two factors, either a broad resonant condition in the acoustics of the passenger compartment, or a high spectral density in one or more of the input disturbances which were creating the measured spectra. Of these two factors, the latter was the most likely since acoustic resonances tend to be high Q resonances, and would consequently produce much sharper peaks in the noise spectra.

The audible maximum of the rear axle gear meshing noise occurred at approximately 47 mile/hr. speedometer reading, and this gave a meshing frequency of 440 Hz. A combination of amplification of the narrow band random disturbance due to the rear axle gears meshing by the broad resonant condition which created the hump in the noise spectra, and then coupling with a high Q acoustic or pannel resonance was the most likely cause of this audible noise. This maximum of the noise at rear axle gear meshing frequency was also evident from the power spectral measurements and is shown in fig. 7-11.

A set of example spectra for the tests speed at which the audible rear axle gear meshing noise was subjectively assessed to be a maximum by the driver of the vehicle, and the author, are presented in figs. 7-12 to 17. The vibration spectra did not show the falling characteristic of noise spectra but they did exhibit the same broad resonant condition in the region of 400 to 500 Hz. This confirmed that the hump in the noise spectra was due to the characteristics of the source disturbances, and not acoustic resonances, and it would seem in fact, that the hump was primarily due to the transmission of the rear axle vibration to the passenger compartment.

Reference to the response curves for the rear axle presented in Chapter 3 will show that no broad low Q resonance was detected in this frequency region. However, since the hump in the vibration spectra occurred in the results for all the tests speeds to the same relative extent, it was almost certainly due to a resonant condition of the transmission system. This lead to the hypothesis that the response characteristics of the transmission system were considerably altered under dynamic conditions in respect to the static case.

A considerable number of factors would have influenced the response characteristics of the transmission system under dynamic conditions. The factors most likely to have created significant changes were the torque loading on the rotating parts, and the change in the loading of the ball and roller bearings.

Also shown in figs. 7-12 to 17 are the spectra obtained with the none standard springs fitted, i.e. the springs with the enlarged rubber bushes at the forward mounting points. The general influence of this modification to the springs was fairly small and reasonably independent of frequency.

The repeatability of the source vibration in respect to the rear axle gear noise was demonstrated by the small difference in the acceleration power spectra measured at the centre of the rear axle for the two series of tests, fig. 7-13. The slight lack of agreement between the two test measurements were due to the inherent statistical errors and the impossibility of achieving exactly ergodic test conditions for tests performed with a considerable lapse of time between them. A relatively high degree of repeatability was also achieved for the measurements taken at the spring axle interface. These high repeatabilities were expected since both these measuring positions were prior to the modification to the system in terms of the transmission of the disturbances.

Unfortunately, of the six sets of acceleration measurements taken at the body mounting points for the rear suspension, two sets were lost due to instrumentation faults. In the tests with the standard springs the records of the vibrations at the rear mount were lost due to a cable fault, and a similar fault resulted in the loss of recordings of the vibration at the forward mount in the tests with the none standard springs. The latter loss was the most serious since this was the point where the most significatn changes would have occurred due to the modification to the rubber bushes.

It was not possible to repeat these tests due to shortage of time at the end of the research programme.

Considerable attenuation of the accelerations took place in the transmission through the springs as can be seen by the comparison of the vibration spectra for the spring axle interface, 4ig. 7-14, with the spectra for the body mounting points, figs. 7-15 to 17. In terms of the power of the signals the attenuation amounted to a factor of the order of 10. As far as could be estimated from the acceleration power spectra measured at the damper body mounting, the only position for which spectra for the standard and non-standard springs, were available, the fitting of the enlarged rubber bushes did not increase this attenuation.

The relative amplitudes of the narrow band random components at rear axle gear meshing frequency in the acceleration spectra of the body mounting points indicated that the major transmission path for this disturbance was via the rear body mounts of the rear suspension system. It was obvious after the series of tests with the none-standard springs that a modification in the rear mounting bushes would have probably produced more marked changes. This course of action was not followed due to lack of time in which to obtain further modified springs.

Power spectral densities of the narrow band random peaks at rear axle gear meshing frequency were summarised to give the variation with frequency figs. 7-18 to 23.

Taking the vibration at the centre of the rear axle to be a good representation of the source disturbance fig. 7-19 shows that the repeatability of the rear axle gear meshing disturbance was quite good for the two series of tests. This was particularly true at 440 Hz which was the frequency at which the maximum audible noise was observed in the passenger compartment. A further peak in these curves at 570 Hz also showed a high degree of repeatability, and was a result of the resonant response of the propeller shaft in its second flexural mode of vibration. The good resolution of this resonant response of the propeller shaft showed the greater power of the power spectral density analysis over wave analysis for the reduction of the type of data encountered in this type of test.

Results obtained from the vibration of the spring axle interface fig. 7-20 showed a much lower degree of repeatability than the results obtained from the centre of the rear axle. This indicated that the major peak at 440 Hz in these curves was most likely due to a resonant component near to the rear axle gears or that the tyre-road contact generated vibration formed a major part of the vibration at this measurement position. In the case of the standard springs the 440 Hz peak in the curve was clearly the dominant one, and appeared to be a fairly narrow band process as far as could be judged from the limited number of points available. Changing the suspension springs had a marked effect on the other peaks making them more dominant while reducing the amplitude of the 440 Hz peak. Whether these changes were due to the change of the rubber bushes or simply due to the differences between the two spring sets was not clear, but they were most likely due to the latter.

From the results obtained from the vibration measurements on the body side of the springs, it was found that the springs had a fairly even attenuation factor of the order of 10 over the whole of the frequency range of the analyses. This resulted in the 440 Hz peak remaining the most dominant feature of the curves figs. 7-21 to 23. There was no evidence of the resonances of the springs adding further to the amplification of the rear axle gear meshing frequency vibration above that already obtained in the rear axle itself. As far as could be judged the changing of the forward rubber bushes did not have a marked effect on the transmissibility of the rear suspension system, but this could not be concluded with a high degree of certainty due to the loss of the recordings for the forward spring body mount when the non-standard springs were fitted.

The power density of the peak centred on the rear axle gear meshing frequency in the power spectra of the noise in the passenger compartment followed a very similar characteristic to the acceleration power spectra measured at the suspension body mounts. It was, therefore, confirmed that the rear axle gear meshing frequency components in the noise in the passenger compartment were almost solely due to the mechanical transmission of the source disturbance to the passenger compartment. The slight differences in the shapes of the noise and vibration curves were due to the acoustic resonances, and to the complex coupling of the pannel modes of vibration with the acoustic modes.

The power of the rear axle gear meshing frequency component in the noise in the passenger compartment was reduced at the critical frequency of 440 Hz but it was increased at other frequencies. However, the considerable changes in the spectra resulting from the change of springs did show that a re-design of the suspension system with particular emphasis on the isolation of the high frequency vibrations would have achieved a considerable improvement in the noise in the passenger compartment.

7-6) Conclusions on the influence of the rear suspension system

That a change in the rear suspension system, even of a minor nature, resulted in a considerable change in the acoustic response of the passenger compartment to a disturbance having its source at or near the centre of the rear axle has been shown by the results presented in this chapter. It was clearly indicated, therefore, that a re-design of the rear suspension system with particular regard to the transmissibility in the region of 300 to 600 Hz would reduce the rear axle gear meshing noise nuisance. This re-design could almost certainly be achieved without degrading the low frequency characteristics and hence the handling to any great extent. In fact all that may be required is a closer control of the properties of the rubber isolating bushes.

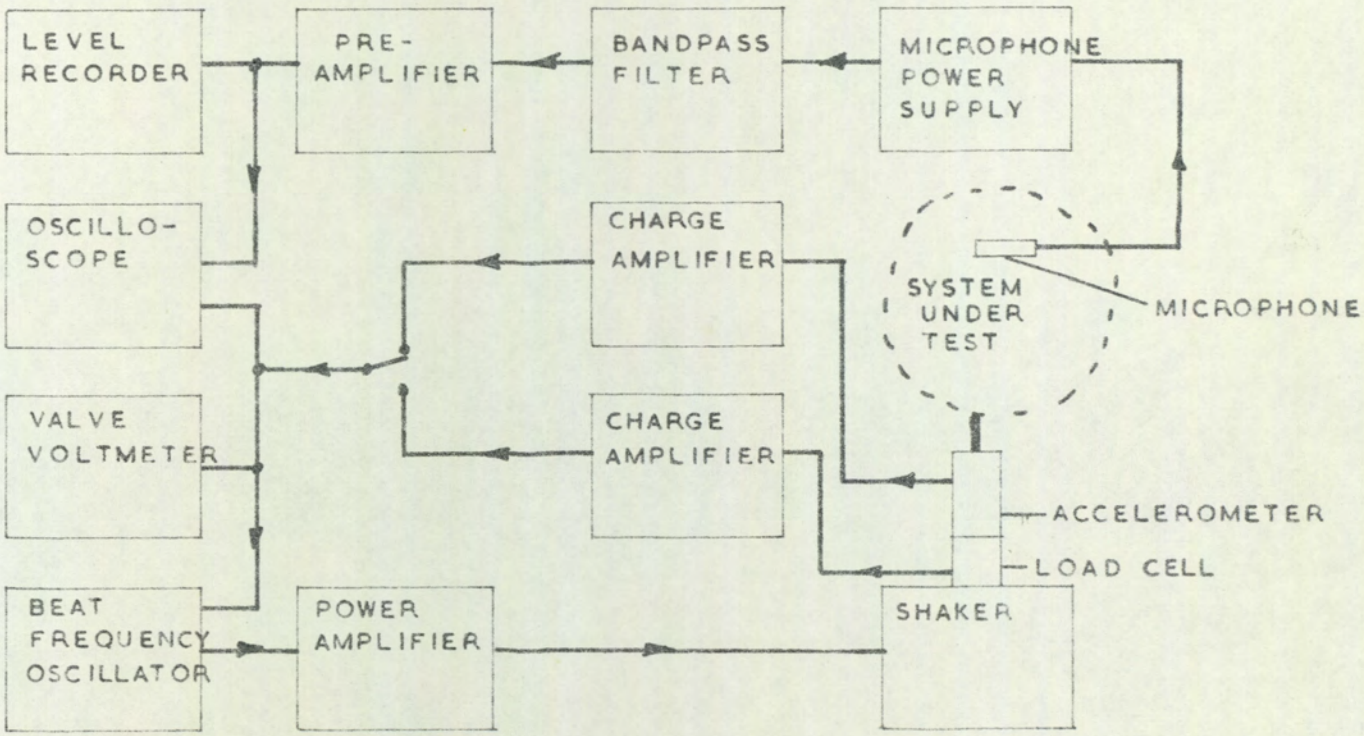
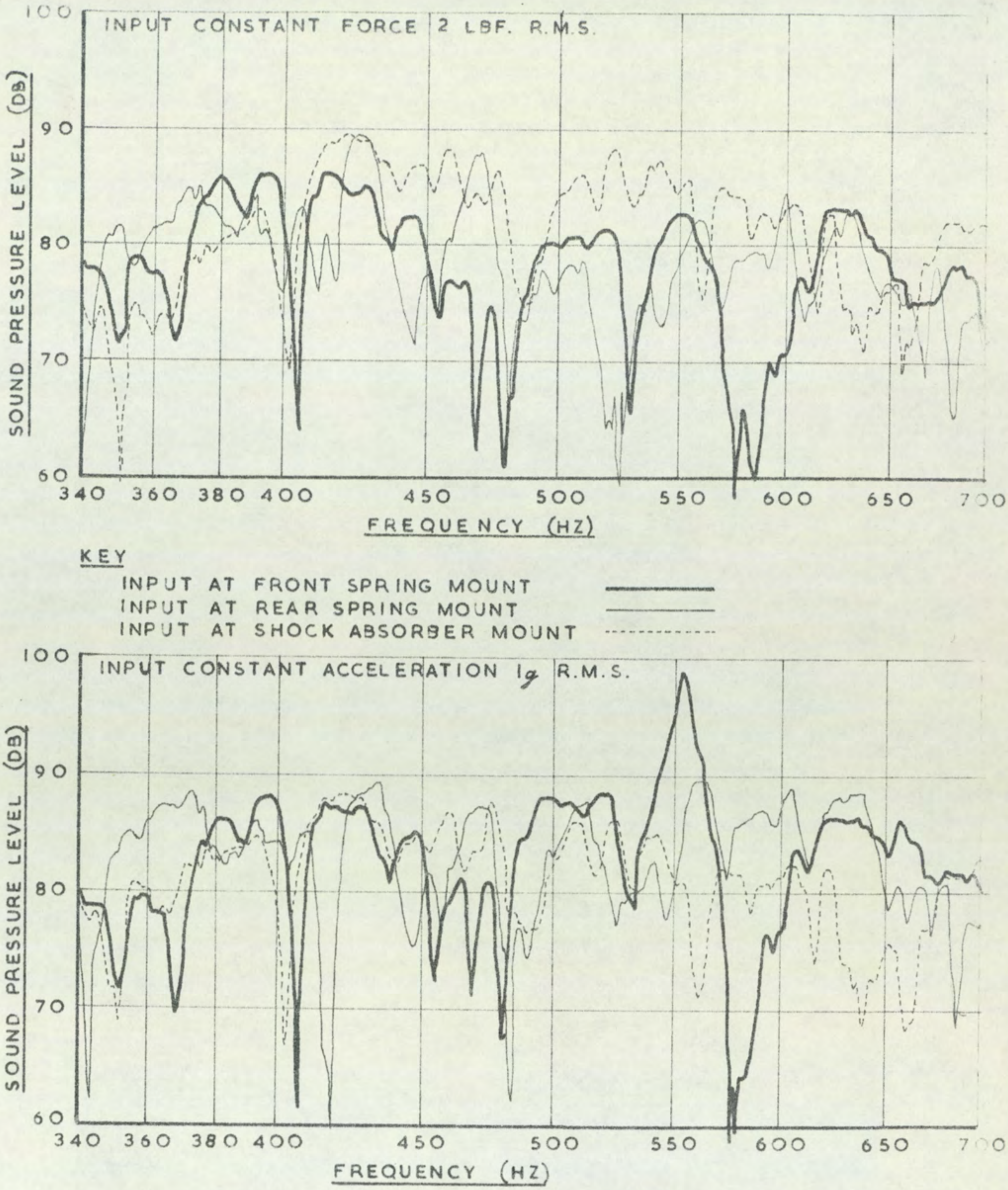


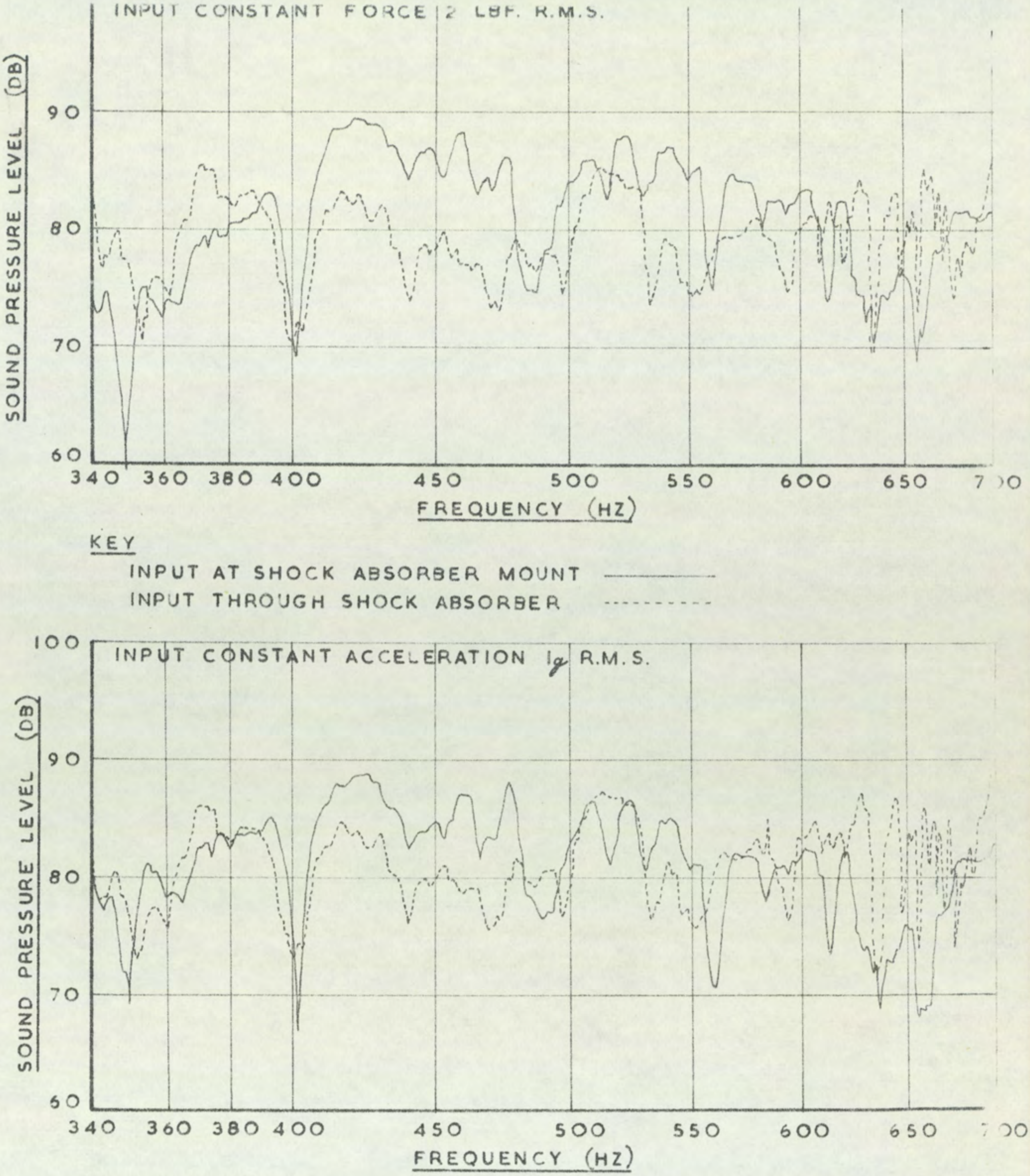
FIG 7-1

INSTRUMENTATION FOR ACOUSTIC RESPONSE TESTS



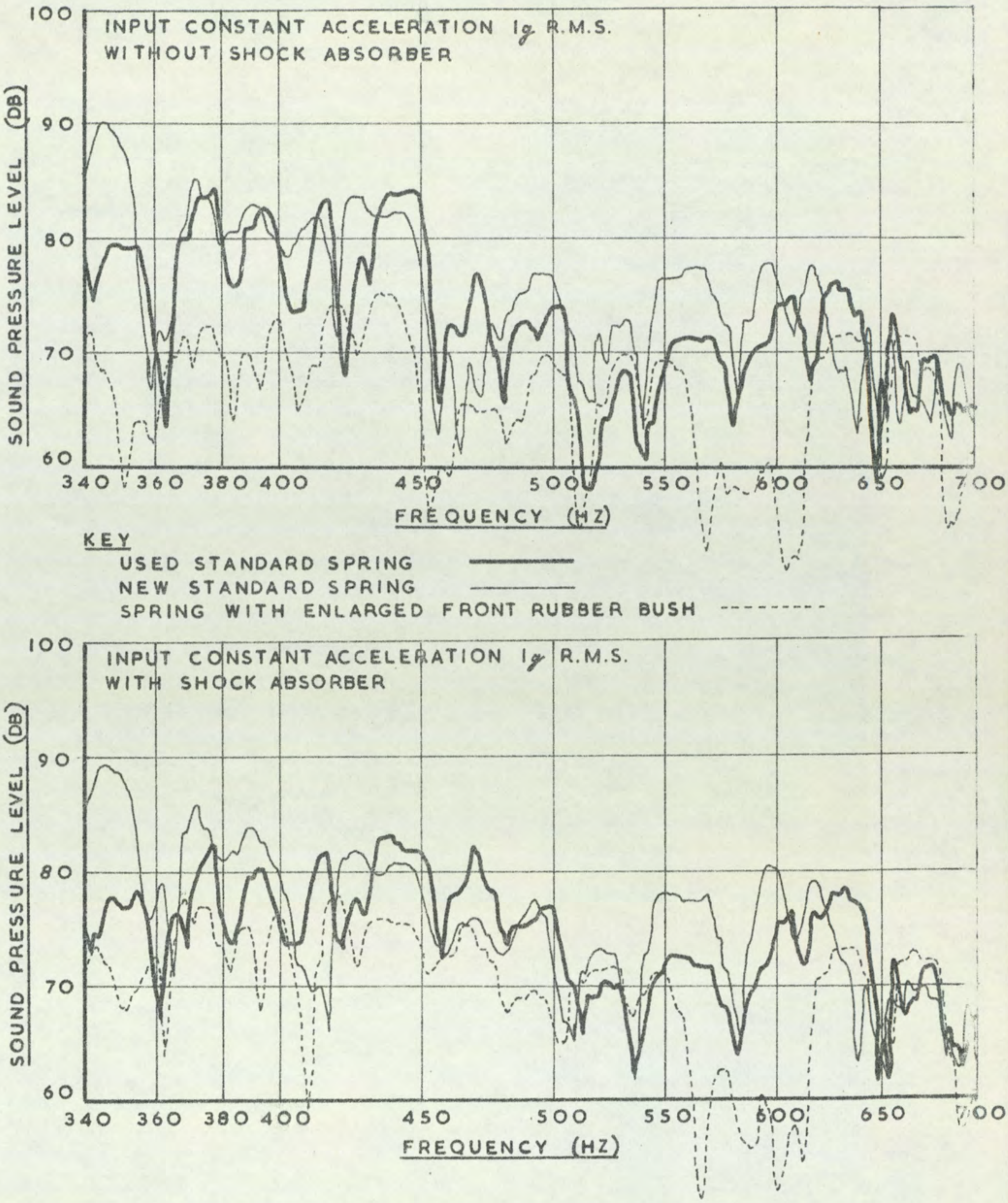
ACOUSTIC RESPONSE IN PASSENGER COMPARTMENT TO SINUSOIDAL EXCITATION AT REAR SUSPENSION MOUNTS

FIG 7-2



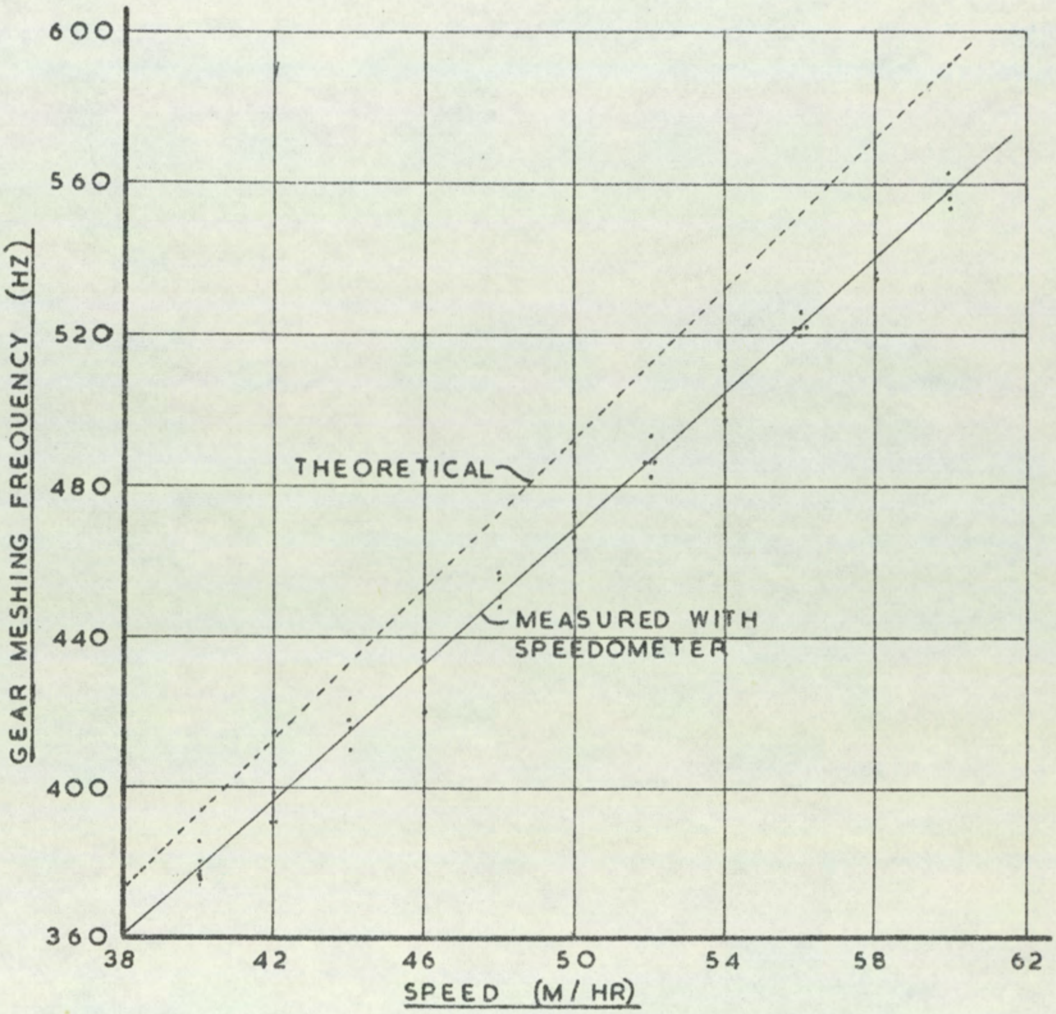
EFFECT OF SHOCK ABSORBER ON ACOUSTIC RESPONSE IN PASSENGER COMPARTMENT

FIG 7-3



ACOUSTIC RESPONSE TO SINUSOIDAL EXCITATION THROUGH SUSPENSION SYSTEM

FIG 7-4



RELATION OF GEAR MESHING FREQUENCY TO ROAD SPEED

FIG 7-5

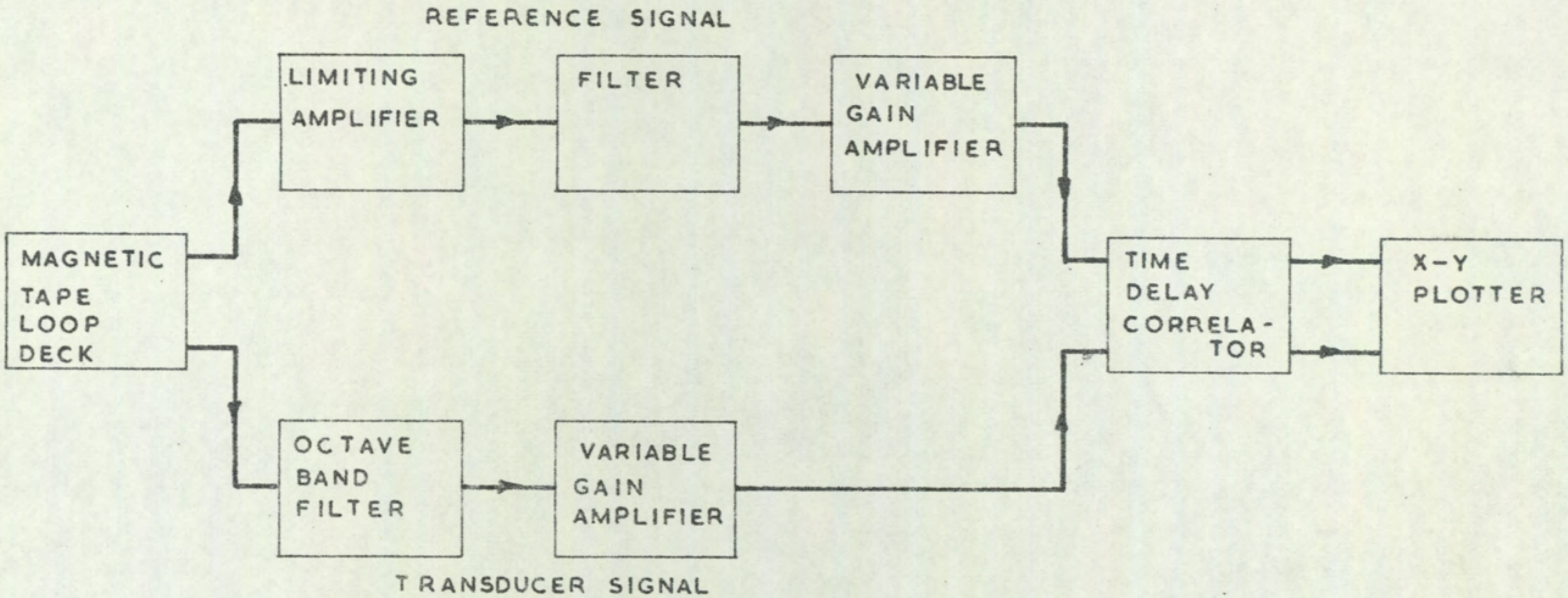
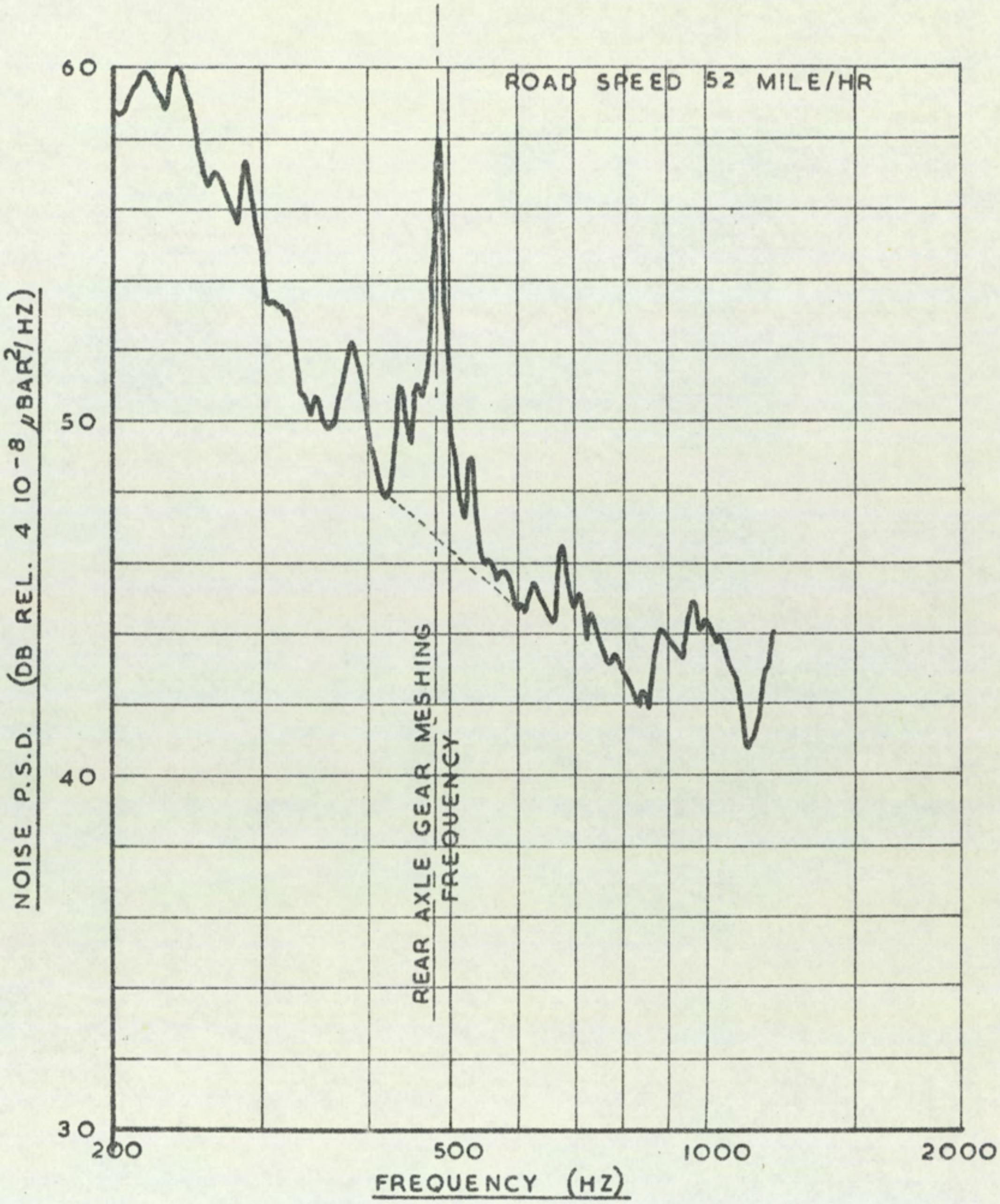


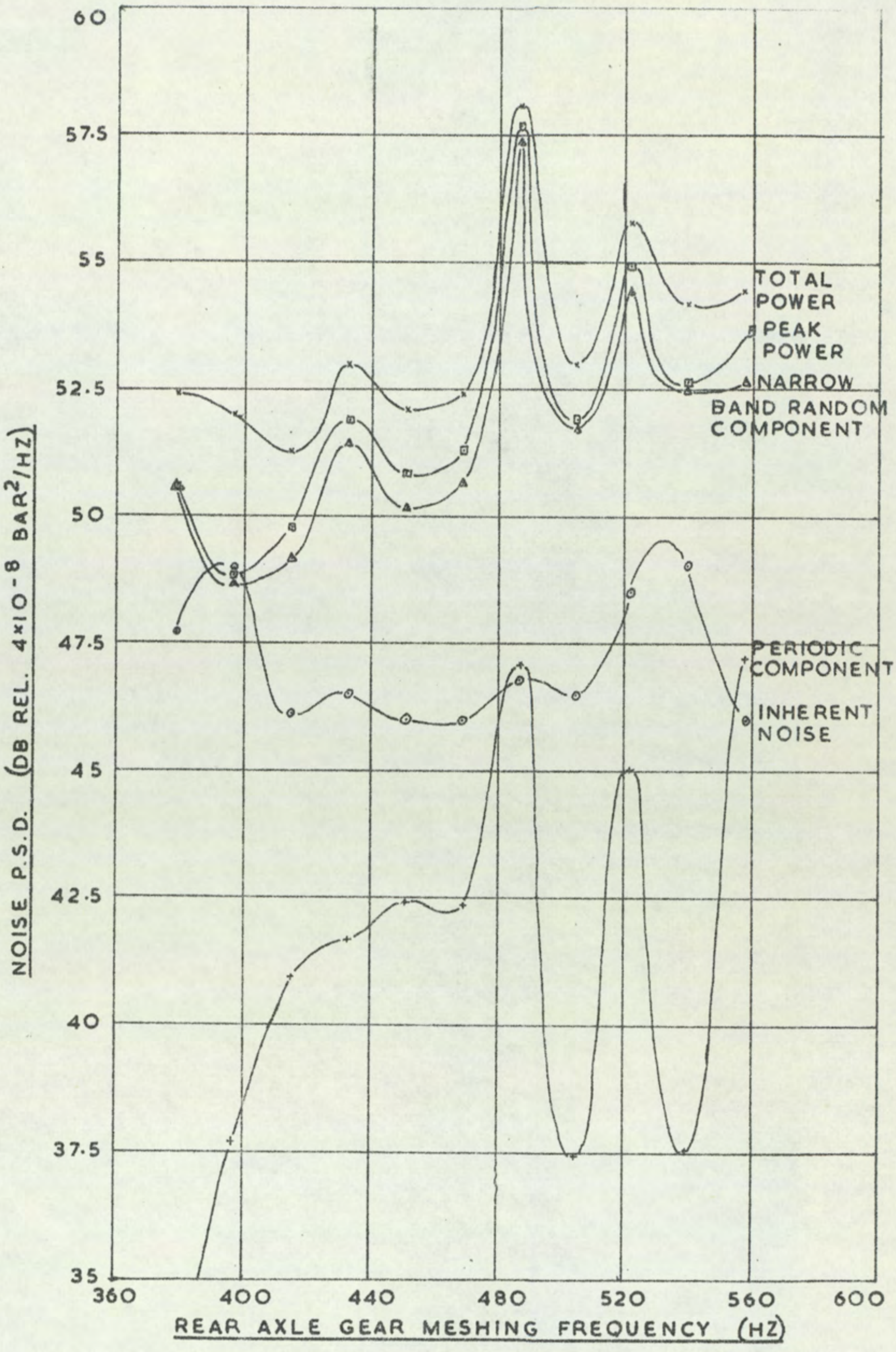
FIG 7-6

SIGNAL CONDITIONING FOR THE DETECTION OF PERIODIC COMPONENTS OF NOISY SIGNALS BY CROSS-CORRELATION TECHNIQUES



EXAMPLE OF POWER SPECTRAL DENSITY OF NOISE IN PASSENGER COMPARTMENT

FIG 7-7

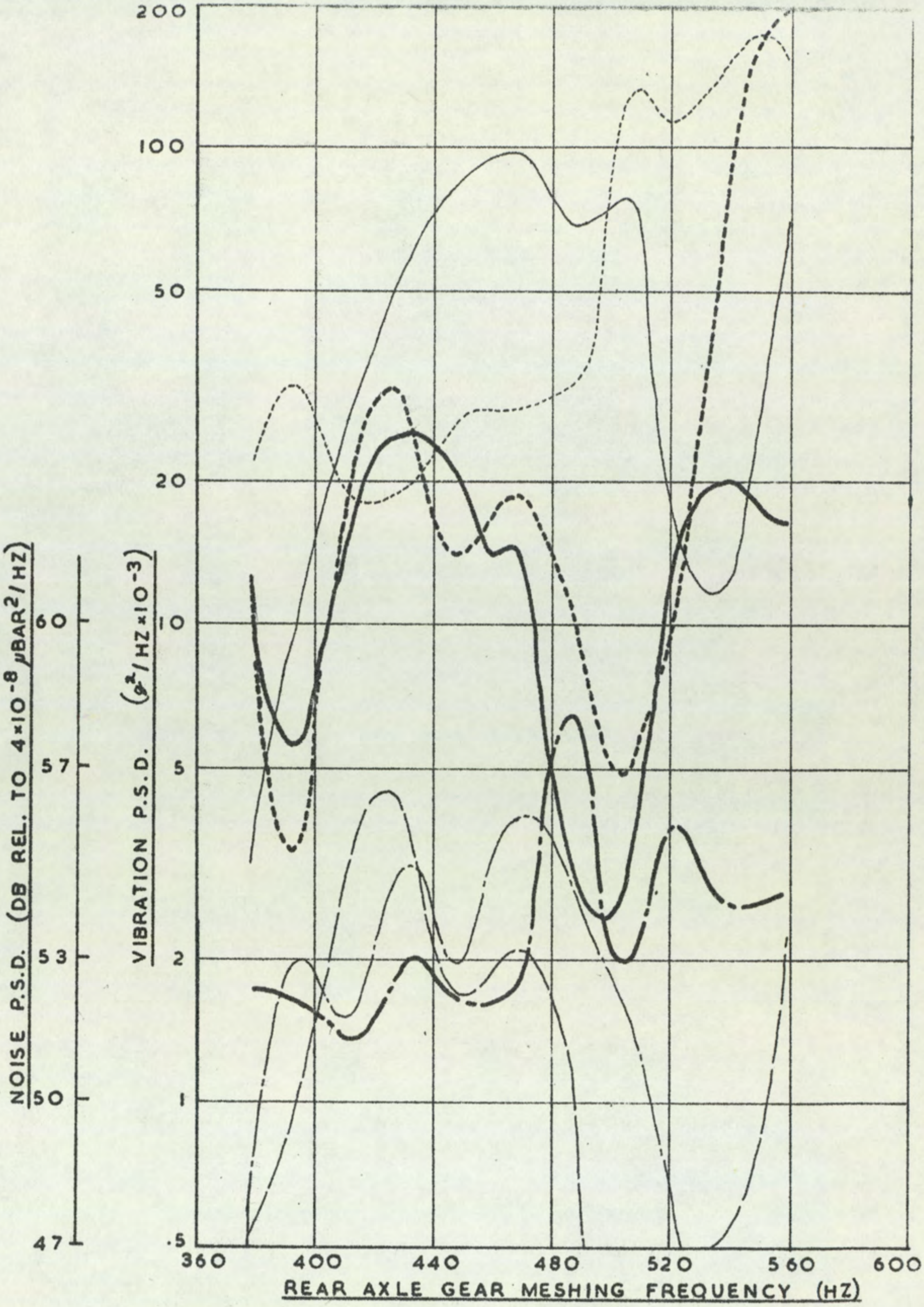


COMPONENTS OF REAR AXLE GEAR MESHING FREQUENCY PEAK IN P.S.D. OF NOISE IN PASSENGER COMPARTMENT

FIG 7-8

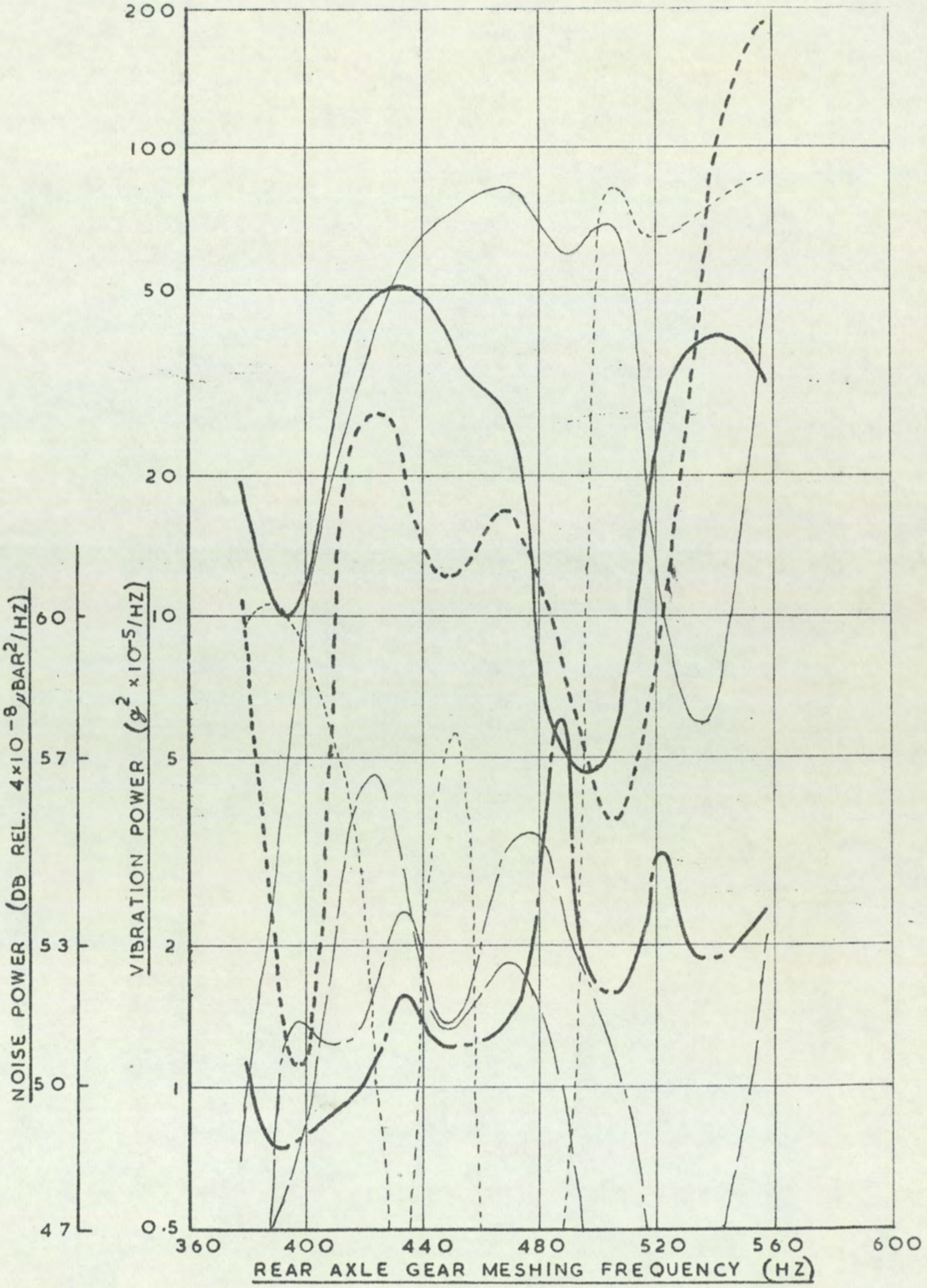
NOISE IN PASSENGER COMPARTMENT	— - ——— - —
VIBRATION AT REAR END OF GEAR BOX EXTENSION	------------
VIBRATION AT NOSE OF REAR AXLE GEAR CARRIER	————————
VIBRATION AT CENTRE OF REAR AXLE	————————
VIBRATION AT N.S. REAR SPRING-REAR AXLE INTERFACE	------------
VIBRATION AT N.S. REAR SHOCK ABSORBER BODY MOUNT	—— —— ——
VIBRATION AT N.S. REAR SPRING FORWARD BODY MOUNT	—·————

KEY FOR FIGS. 7-9&10



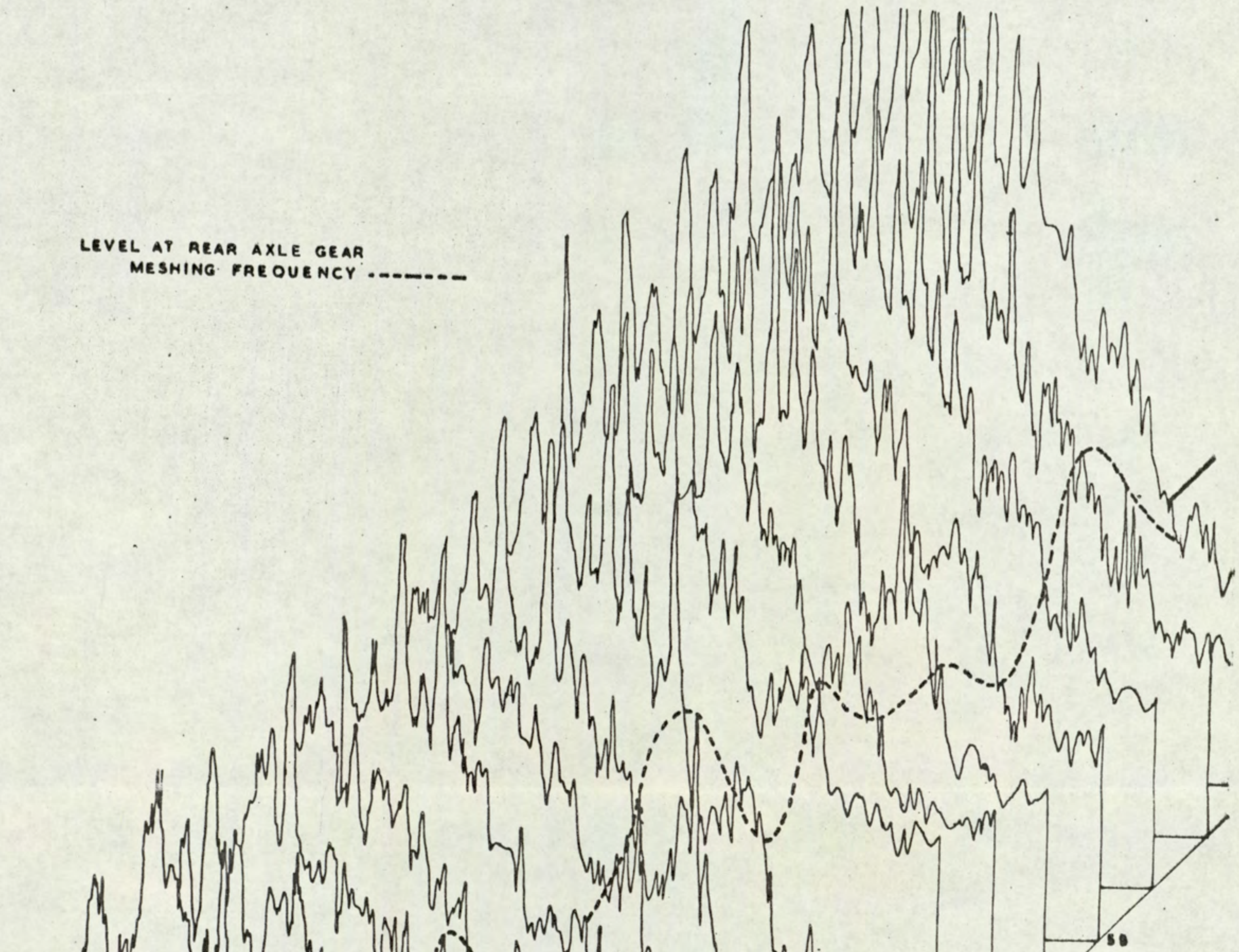
TOTAL POWER SPECTRAL DENSITIES AT REAR AXLE GEAR MESHING FREQUENCY

FIG 7-9



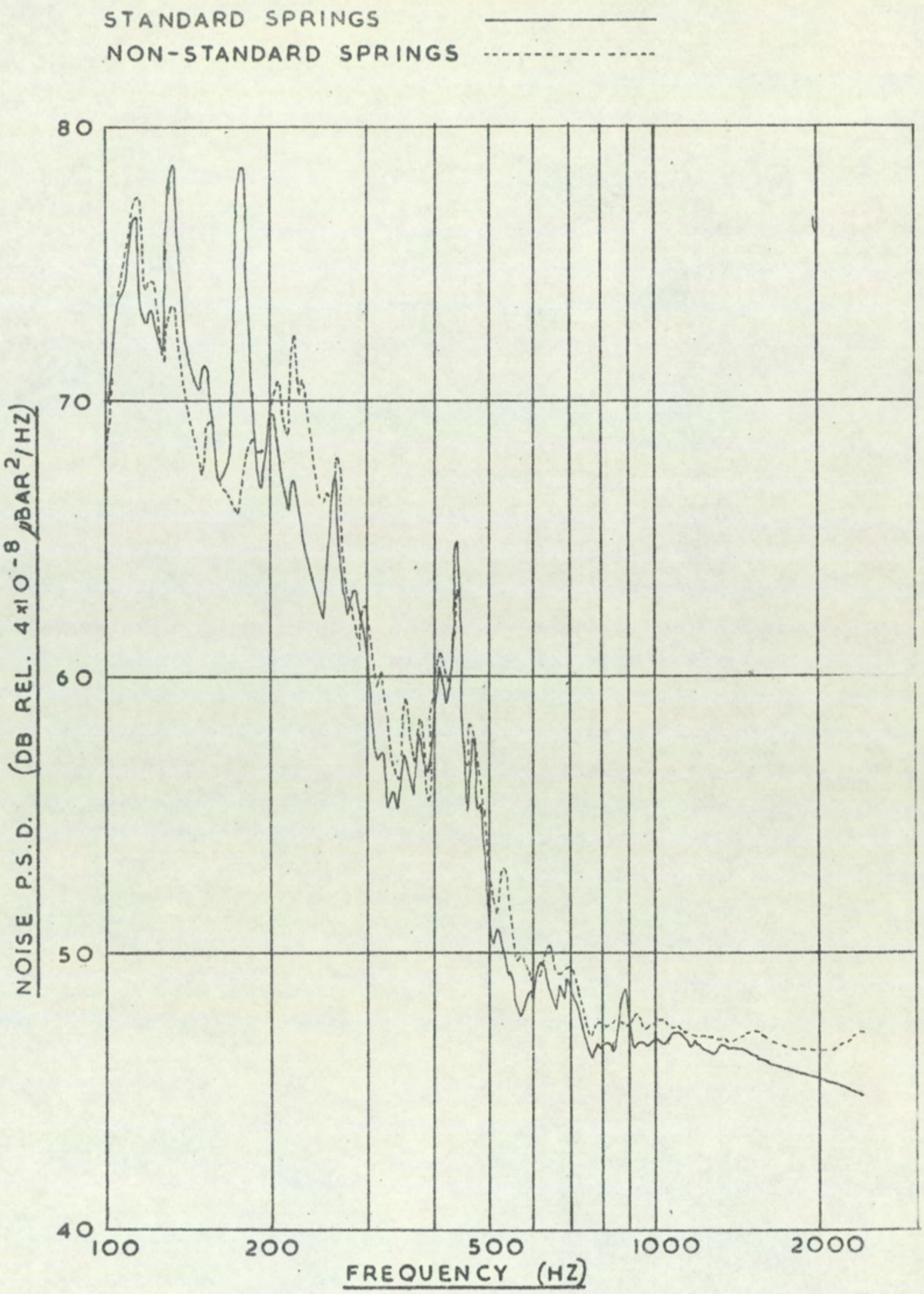
POWER OF GEAR MESHING FREQUENCY PEAK IN POWER SPECTRA

FIG 7-10



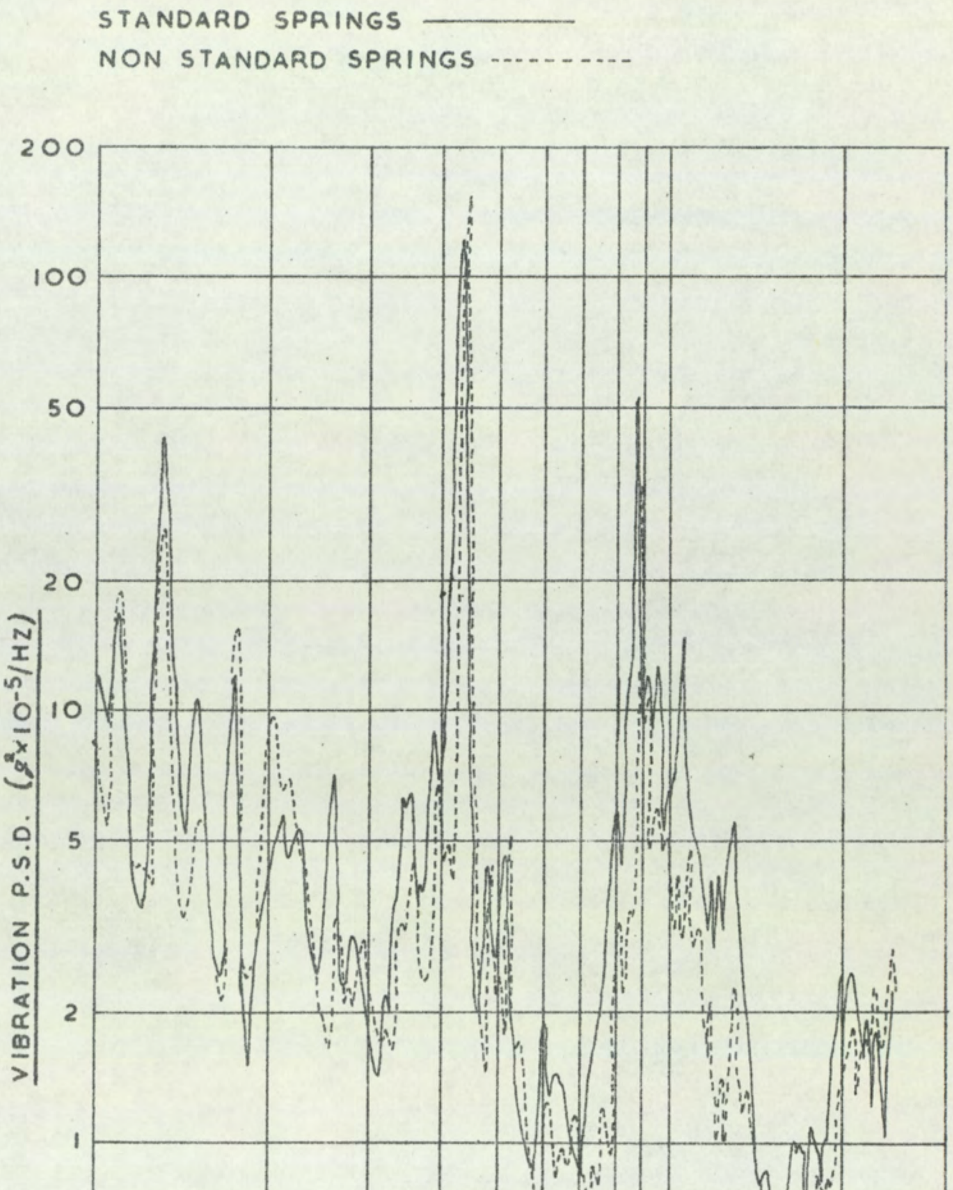
POWER SPECTRA OF NOISE IN PASSENGER COMPARTMENT FOR VARIOUS ROAD SPEEDS

FIG 7-11



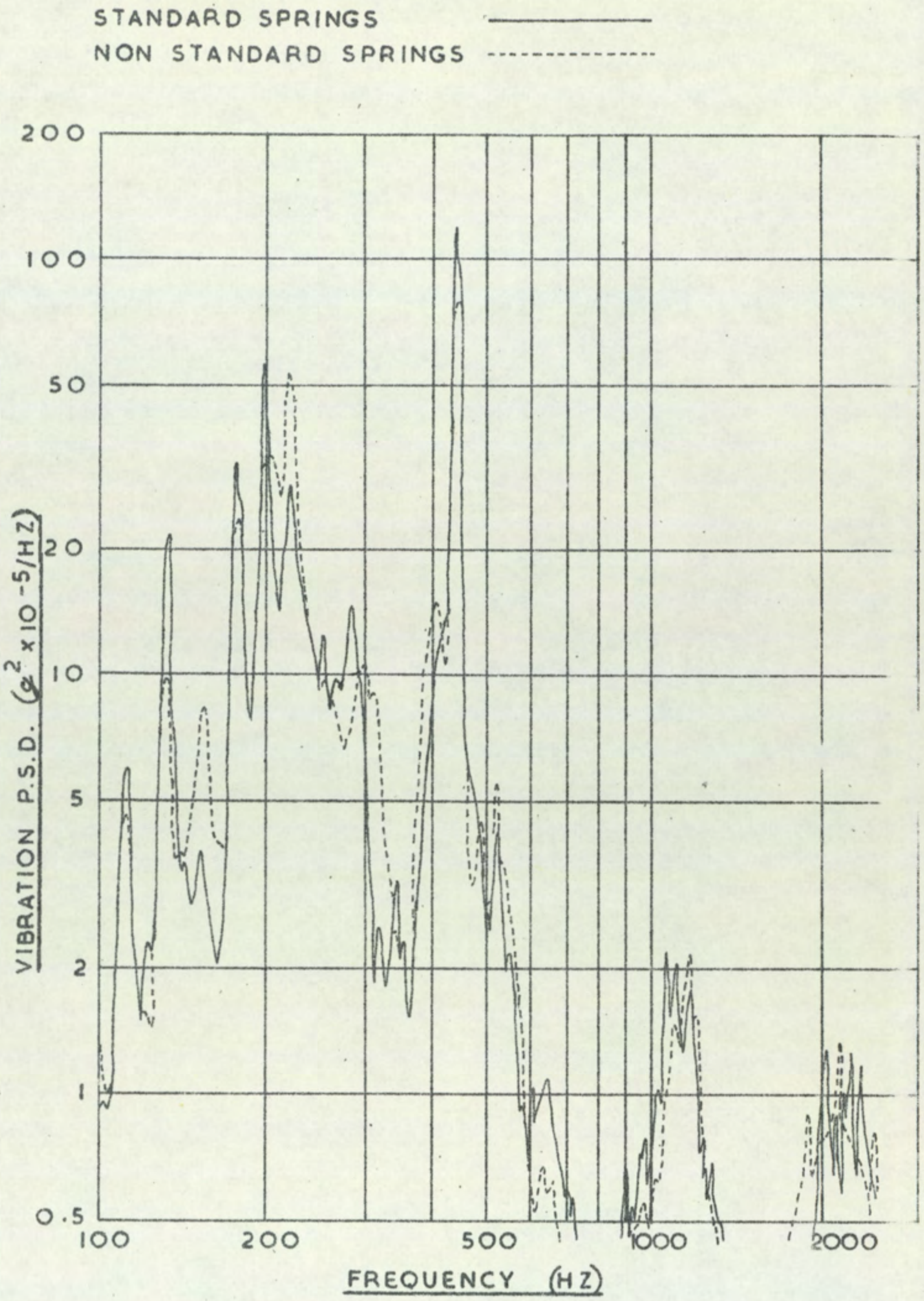
POWER SPECTRA OF NOISE IN PASSENGER COMPARTMENT AT A ROAD SPEED OF APPROXIMATELY 47 MILE/HR

FIG 7-12



POWER SPECTRA OF VIBRATION AT CENTRE OF REAR AXLE AT A ROAD SPEED OF APPROXIMATELY 47 MILE/HR

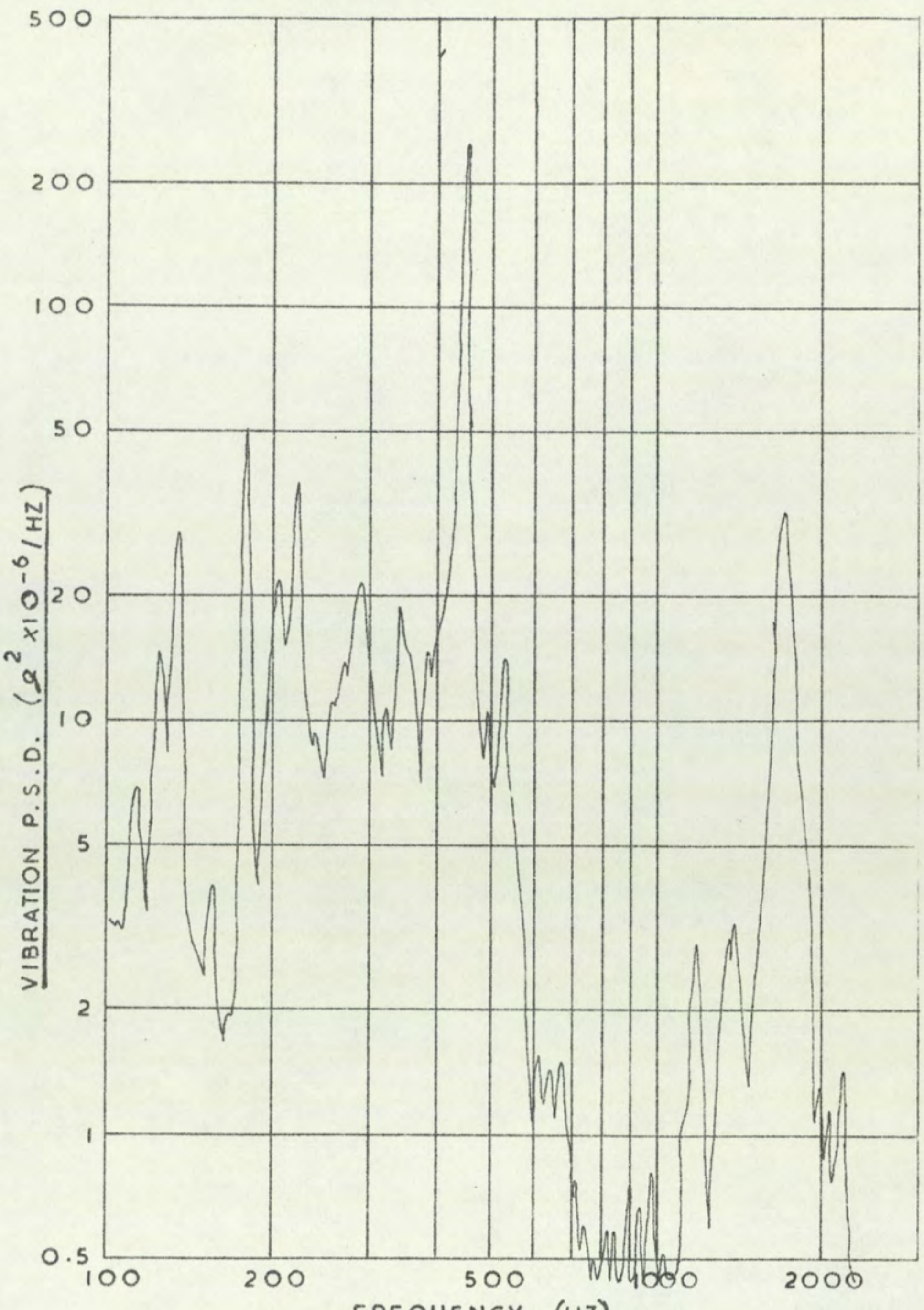
FIG 7-13



POWER SPECTRA OF VIBRATION AT SPRING-AXLE INTERFACE AT A ROAD SPEED OF APPROXIMATELY 47 MILE/HR

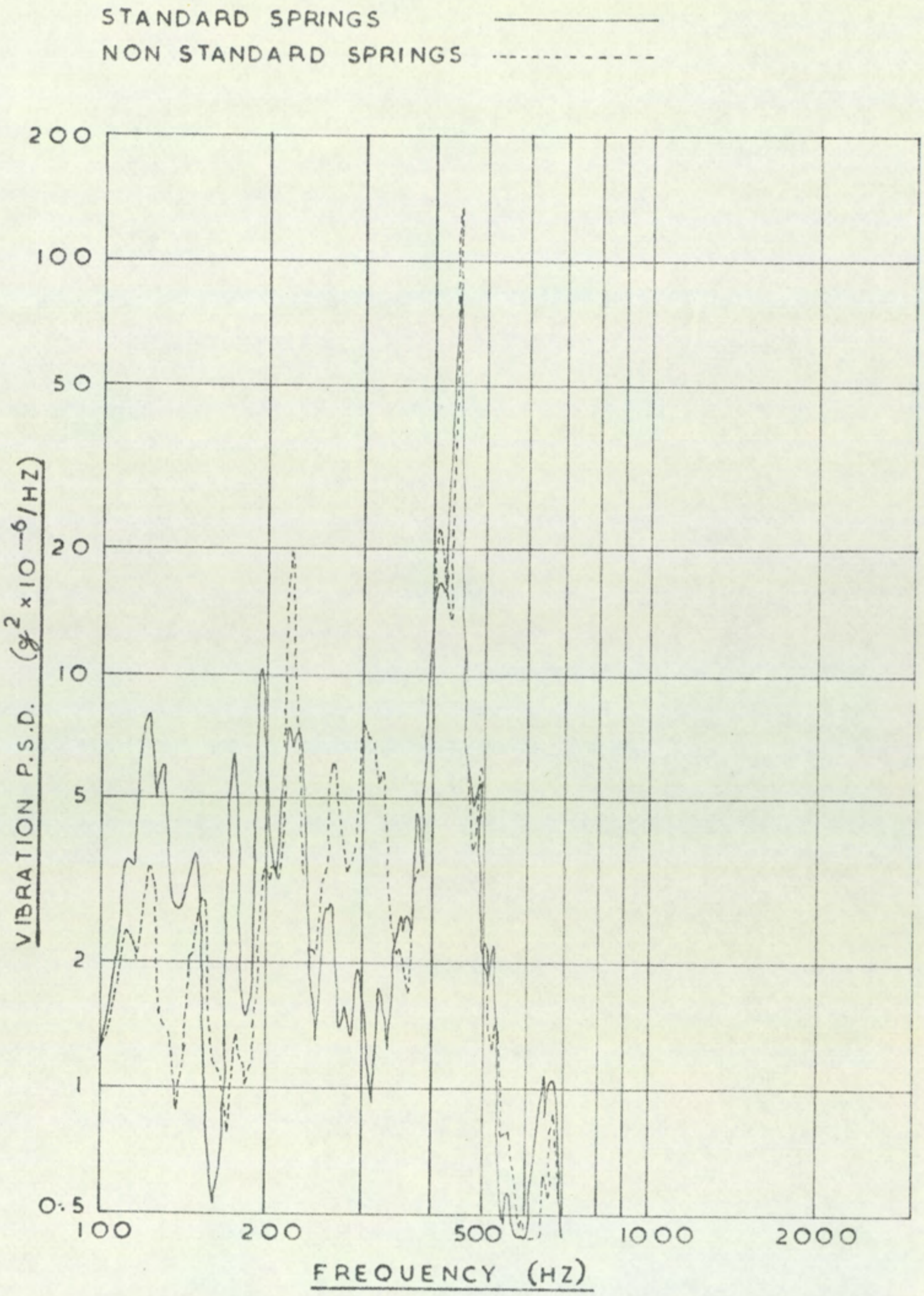
FIG 7-14

STANDARD SPRINGS ONLY



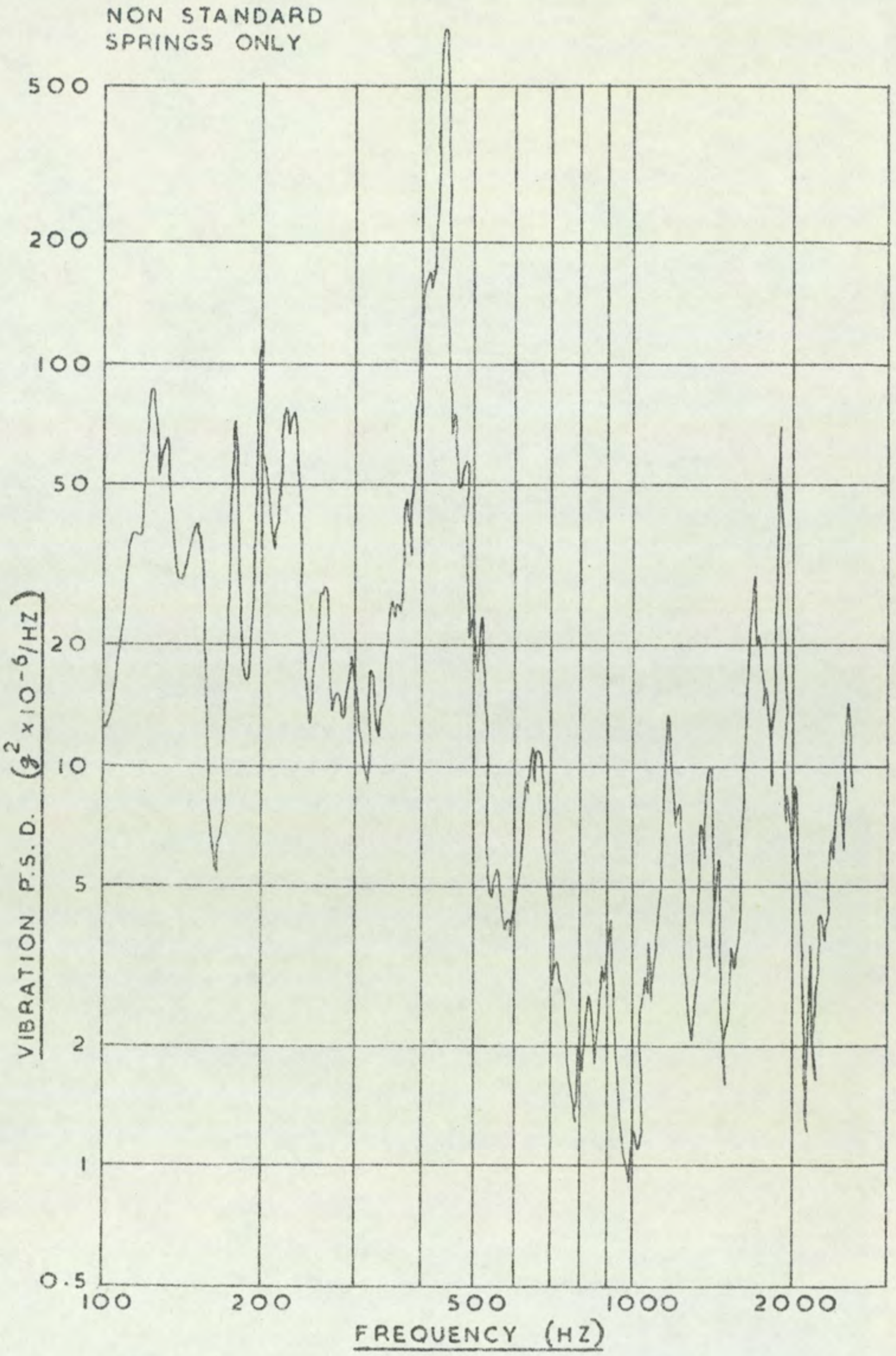
POWER SPECTRA OF VIBRATION AT FORWARD SPRING-BODY MOUNTING AT A ROAD SPEED OF APPROXIMATELY 47 MILE/H

FIG 7-15



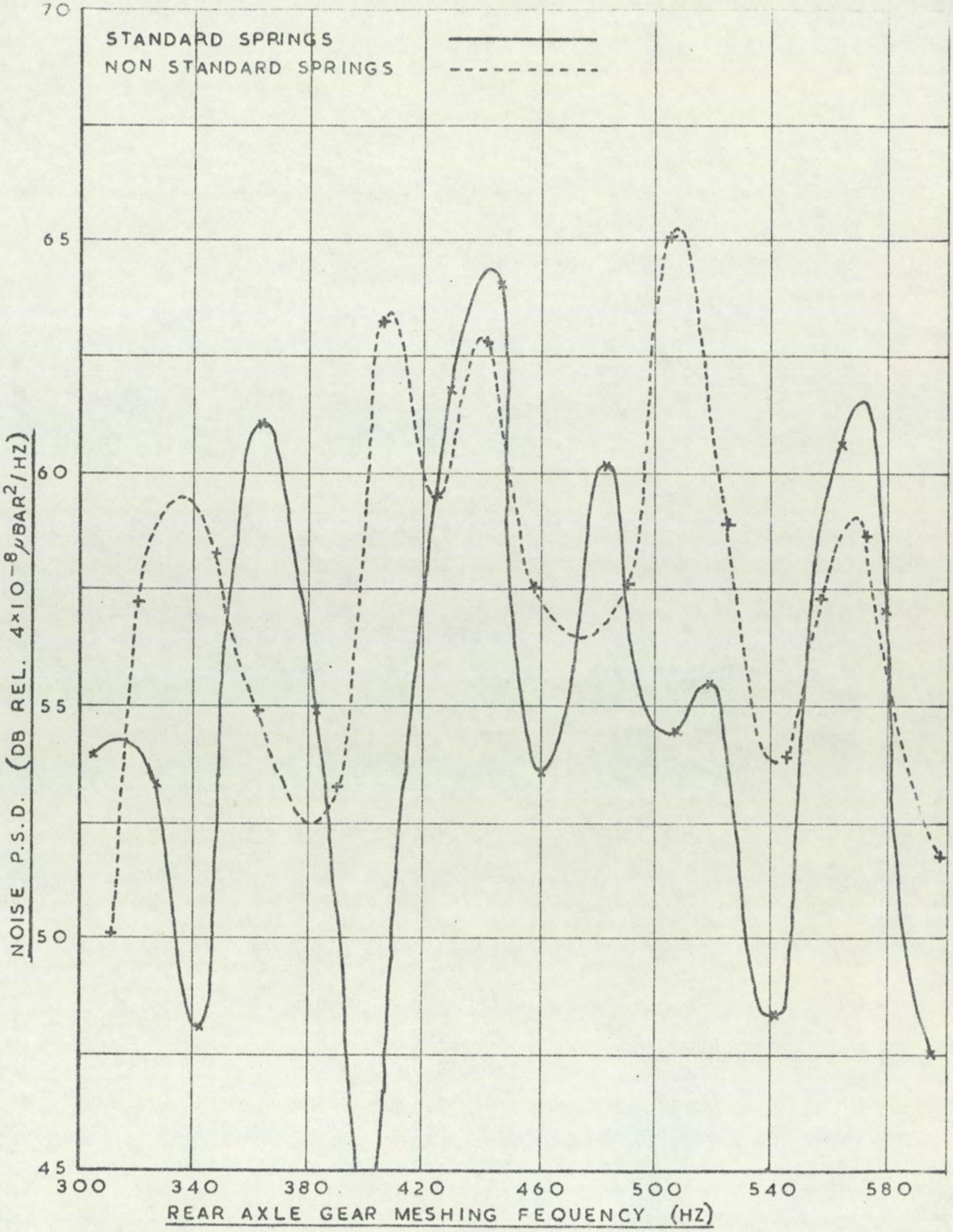
POWER SPECTRA OF VIBRATION AT SHOCK ABSORBER-BODY MOUNTING AT A ROAD SPEED OF APPROXIMATELY 47 MILE/ HR

FIG 7-16



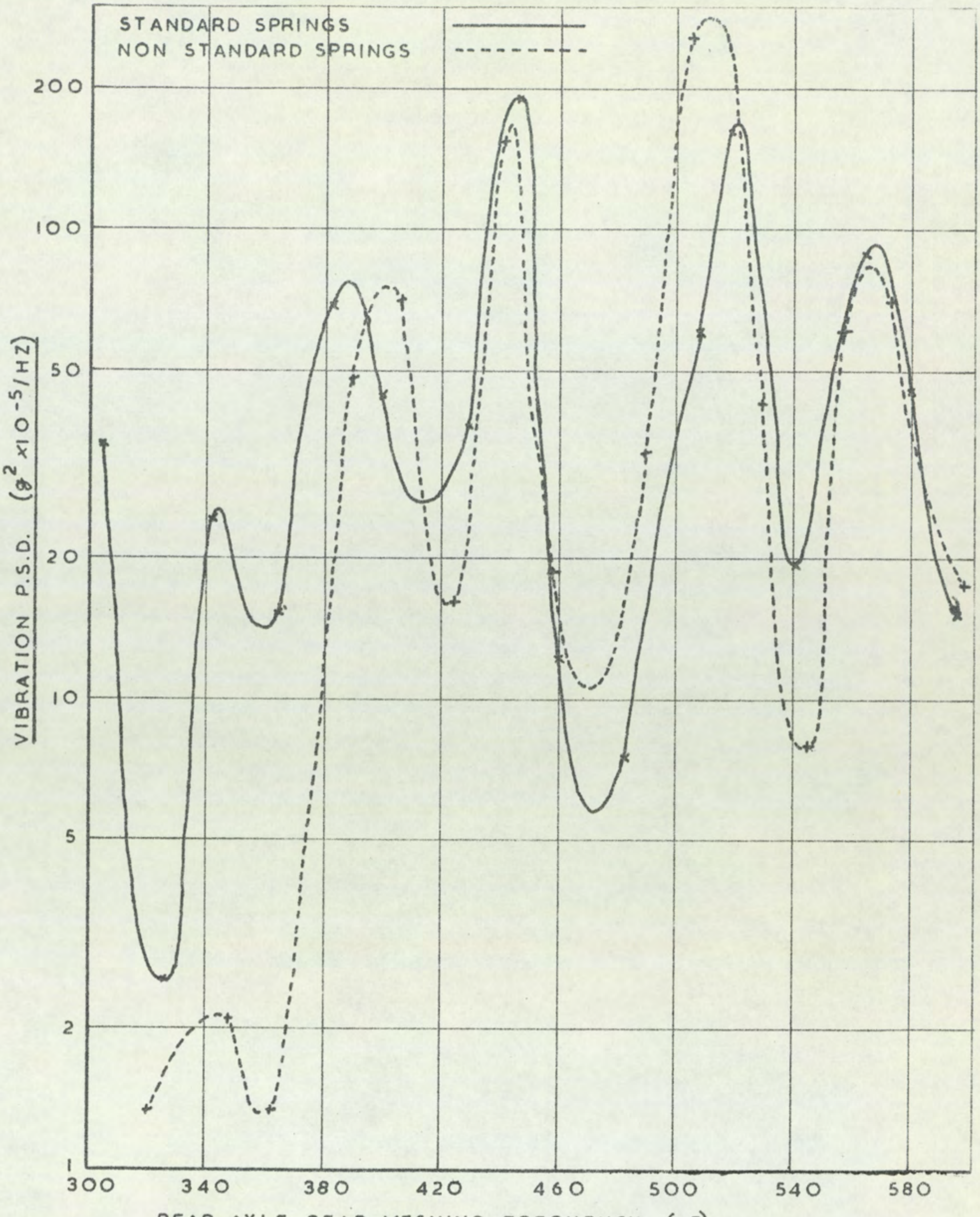
POWER SPECTRA OF VIBRATION AT REAR SPRING-BODY MOUNTING AT A ROAD SPEED OF APPROXIMATELY 47 MILE/HR

FIG 7-17



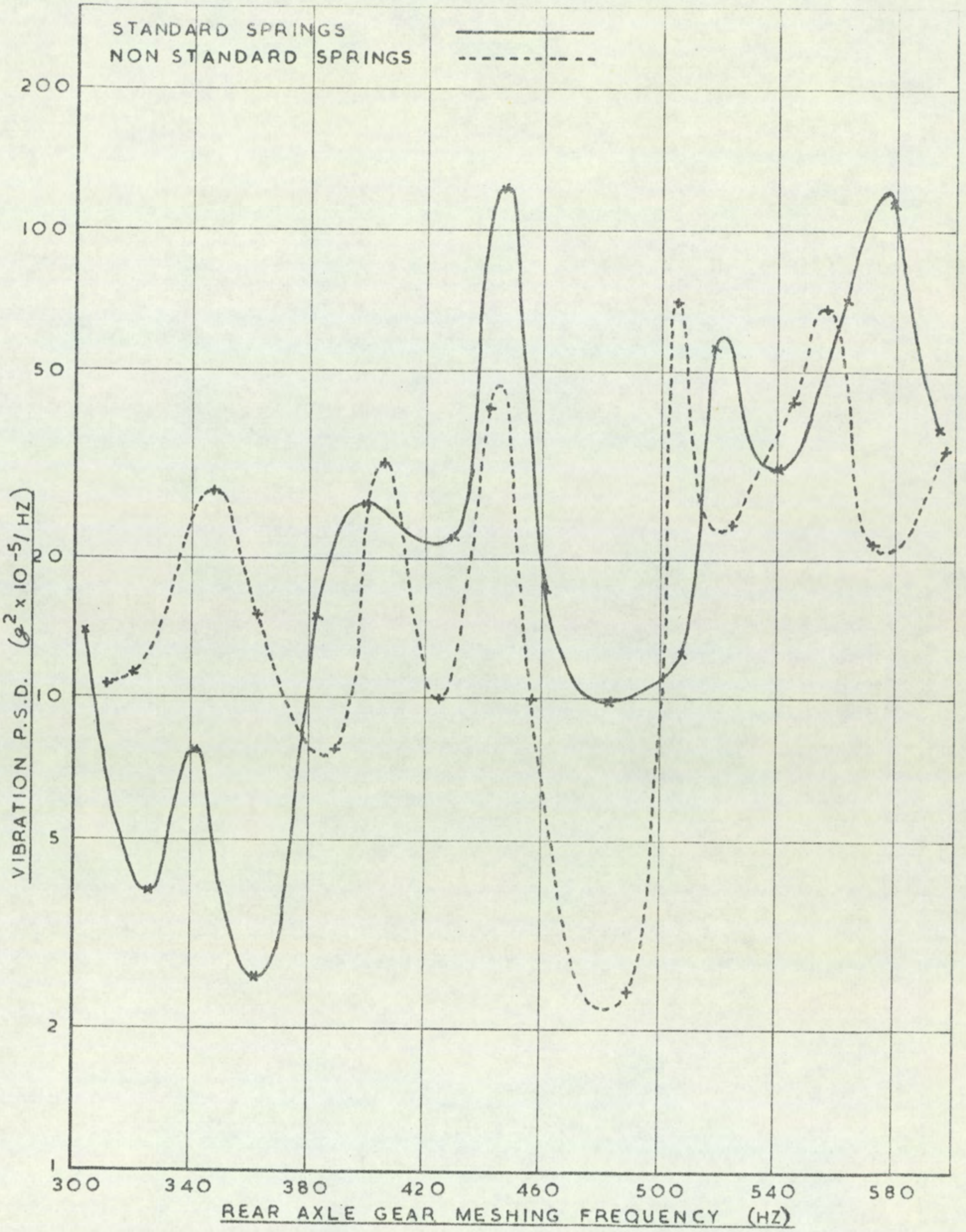
PEAK COMPONENT OF SPECTRA AT GEAR MESHING FREQUENCY OF NOISE IN PASSENGER COMPARTMENT

FIG 7-18



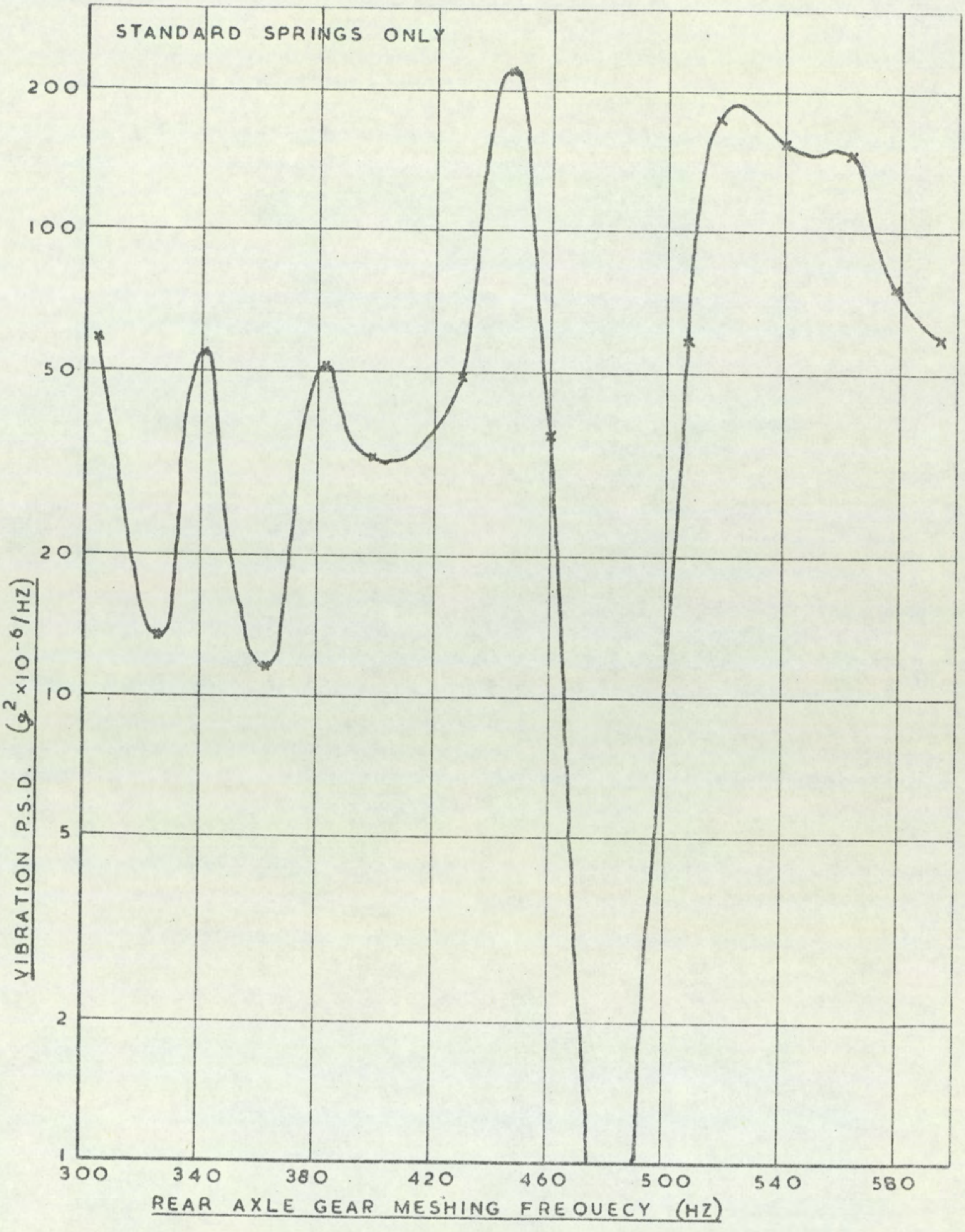
PEAK COMPONENT OF SPECTRA AT GEAR MESHING FREQUENCY OF VIBRATION AT CENTRE OF REAR AXLE

FIG 7-19



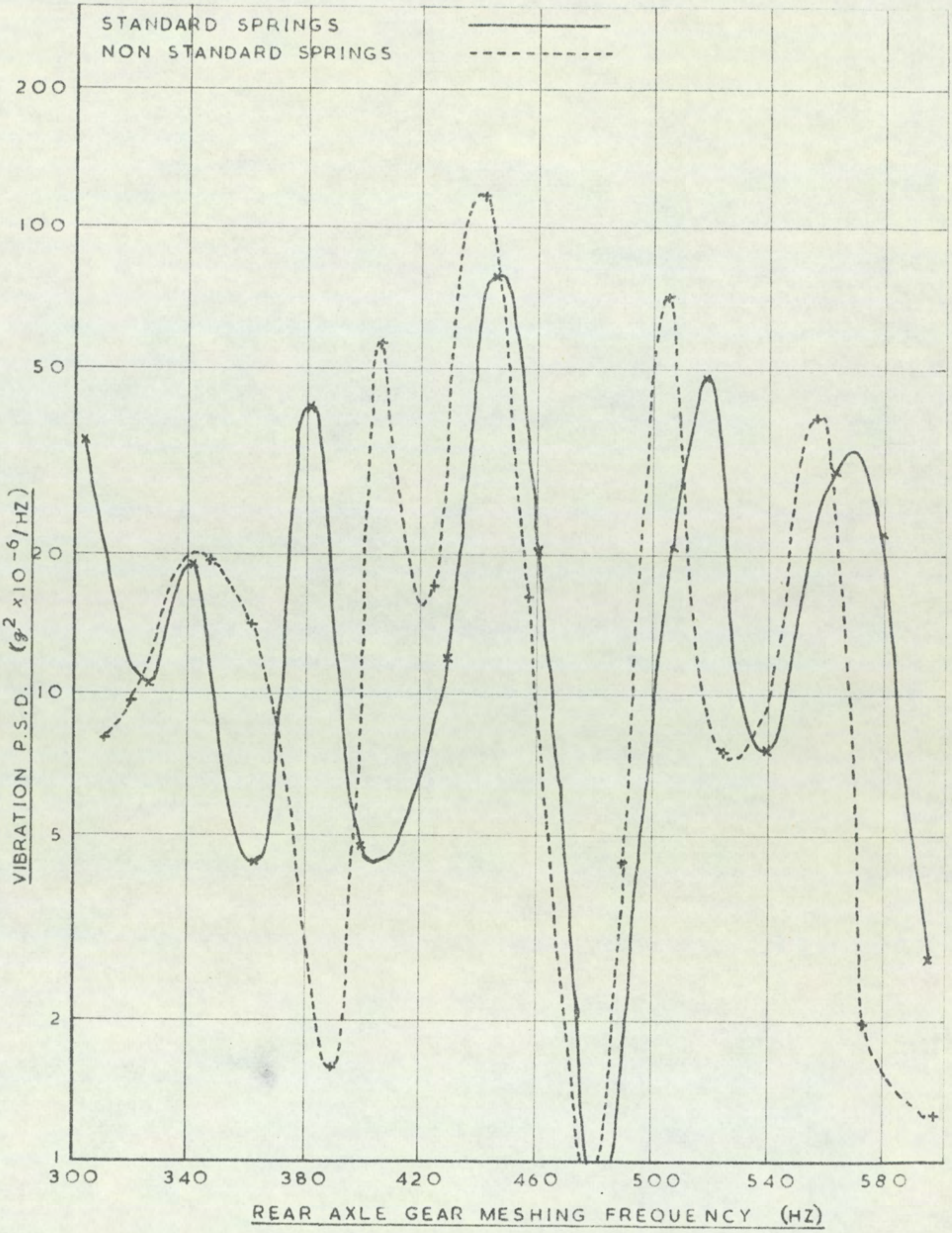
PEAK COMPONENT OF SPECTRA AT GEAR MESHING FREQUENCY OF VIBRATION AT SPRING-REAR AXLE INTERFACE

FIG 7-20



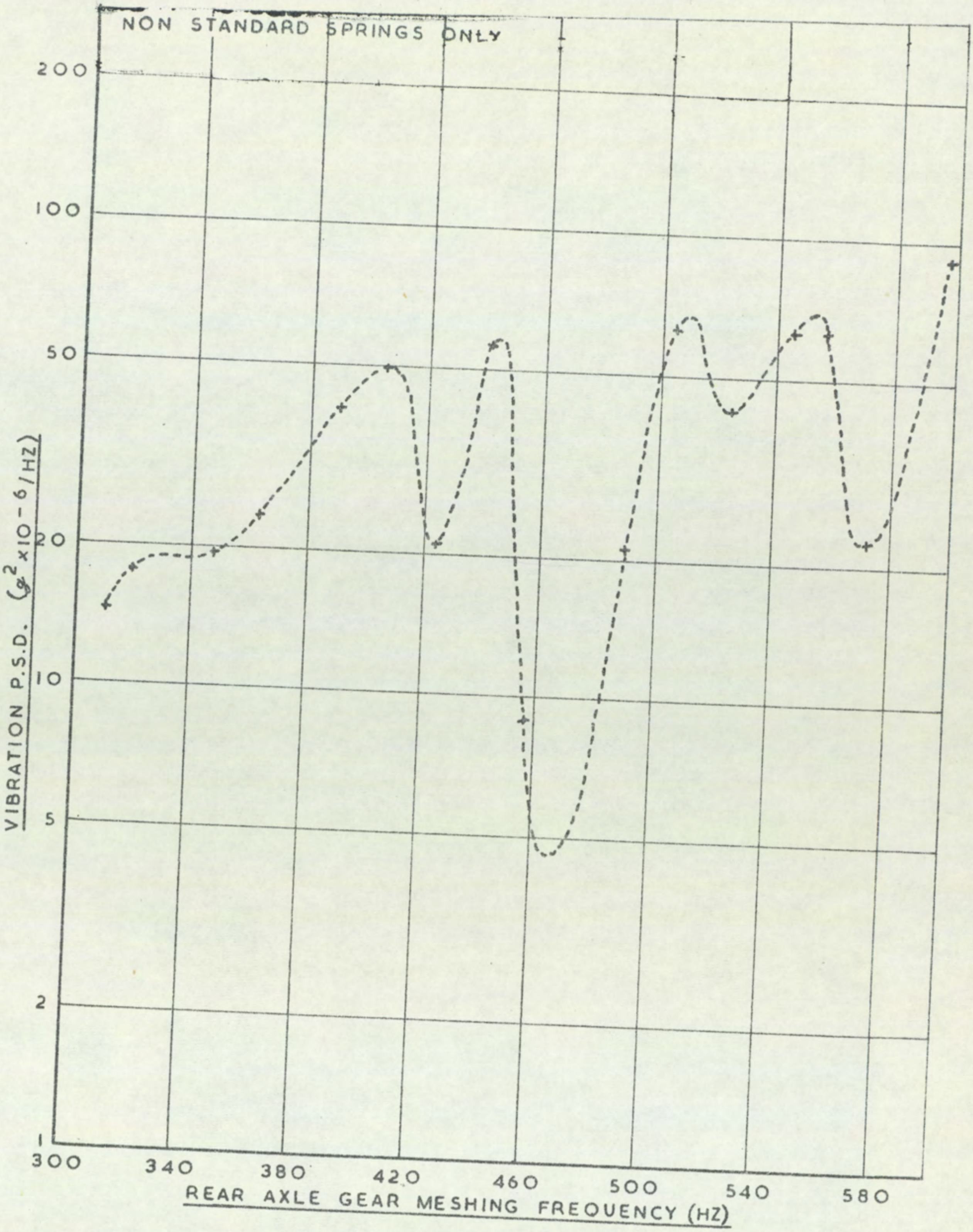
PEAK COMPONENT OF SPECTRA AT GEAR MESHING FREQUENCY OF VIBRATION AT FORWARD REAR SPRING-BODY MOUNTING

FIG 7-21



PEAK COMPONENT OF SPECTRA AT GEAR MESHING FREQUENCY OF VIBRATION AT SHOCK ABSORBER-BODY MOUNTING

FIG 7-22



PEAK COMPONENT OF SPECTRA AT GEAR MESHING FREQUENCY OF VIBRATION AT REAR SPRING-REAR BODY MOUNT

FIG 7-23

CHAPTER 8 ACOUSTICS OF THE PASSENGER COMPARTMENT

8-1) Summary

The factors influencing the acoustic transmission of noise in a closed room are discussed, and the statistical theory of modal density for a rectangular room is demonstrated. Computed results for a rectangular room are presented and compared to the statistical theory to demonstrate the short comings of the theory in relation to narrow band noise. Acoustic response test results are presented and discussed in relation to the deviations of the true modal density of the passenger compartment of the vehicle from the statistical estimate for a similar sized rectangular room.

8-2) Factors affecting the intensity of sound in a closed room

It is well known that the intensity of sound in free air varies as the inverse square of the distance of the point of observation from the source. In a closed room the inverse square law does not hold, and the condition can arise where the intensity of the sound at some distance from the source is greater than the intensity adjacent to the source. This condition arises because in a closed room the source noise excites one or more of the acoustic modes of the enclosure and the resultant intensity at other points in the room is the sum of the intensities of the modes excited.

The actual intensity of the sound at any given point in a closed room will depend on a number of factors namely:- the intensity of the source, the position of the source, the absorbancy of the walls, and most important, the number of modes excited by the source. The number of modes excited by a given source noise will depend on the frequency of the noise and the modal density of the room at that frequency. At low frequencies only one or two modes would be excited by a pure tone, but as the frequency is increased, the number of modes excited will increase according to a square law relationship.

For simplicity in the development of the statistical theory of modal density, the acoustics of a rectangular room will be considered. Let the three dimensions of the room be l_x, l_y and l_z. It can be shown that the natural frequencies of the room are given by:-

$$f = \frac{c}{2}\left\{\left(\frac{n_x}{l_x}\right)^2 + \left(\frac{n_y}{l_y}\right)^2 + \left(\frac{n_z}{l_z}\right)^2\right\}^{\frac{1}{2}} \qquad 8\text{-}1$$

where c = the speed of sound.

This expression can be considered as a vector having components f_x, f_y, and f_z given by $\frac{n_x}{l_x}$, $\frac{n_y}{l_y}$, and $\frac{n_z}{l_z}$ respectively.

The length of the vector gives the frequency of the standing wave and the direction of the vector is the same as the direction of the standing wave. Hence a normal mode of the enclosure can be considered as a point in frequency space as shown in fig. 8-1. These modes can be separated into three categories and seven classes:-

Axial waves (for which two ns are zero)

x-axial waves, parallel to the x-axis. ($n_y = n_z = 0$)

y-axial waves, parallel to the y-axis. ($n_x = n_z = 0$)

z-axial waves, parallel to the z-axis. ($n_x = n_y = 0$)

Tangential waves (for which one of the ns is zero.)

y, z-tangential, parallel to the y, z-plane ($n_x = 0$)

x, z-tangential, parallel to the x, z-plane ($n_y = 0$)

x, y-tangential, parallel to the x, y-plane ($n_z = 0$)

Oblique waves (for which no n is zero).

These three categories of waves are influenced by the walls to different extents, depending on their incodence on the walls. Waves travelling parallel to a wall are absorbed by that wall to a lesser extent than waves having an oblique incodence on the wall. Hence the axial waves will be the waves with the lowest damping since they are parallel to four of the six walls followed by the tangential, oblique to four walls and the oblique waves, oblique to all the walls.

For the prediction of the response of an enclosure it is desirable to evaluate the numbers of each type of mode below any given frequency, and the following method will give approximate values. Consider the frequency space diagram shown in fig. 8-1. The natural frequencies are given by the intersections of the x, y, z lattice shown. If the lattice points are considered to occupy a volume of dimensions $(c/2l_x)$, $(c/2l_y)$, $(c/2l_z)$ with the lattice point at the centre of the volume then the average number of frequencies below a given frequency is given by dividing the volume described by that frequency vector by the above volume. For x-axial waves the number of natural frequencies less than f is given by f divided by the frequency spacing in the f_x direction, $(2fl_x/c)$, (i.e. the number of volumes in a rod of cross-section $(c^2/4l_y l_z)$ and length f, and the total number of axial waves with frequency less than f is:

$$N_{ax} = \frac{fL}{2c} \qquad 8\text{-}2$$

where L is the sum of the lengths of the edges of the room

$$L = 4(l_x + l_y + l_z)$$

The number of y,z-tangential waves with frequency less than f is the number of volumes in a quarter of a disc of thickness $(c/2l_x)$ and of radius f less a correction to allow for the axial waves which have been counted previously. This correction is one half of the volume occupied by the y and z axial lattice point with frequency less than f, i.e.

$$(fc^2/8V)\ (l_y + l_z) \qquad 8\text{-}3$$

where V is the volume of air in the room i.e. $V = l_x l_y l_z$. Hence the number of y, z-tangential waves with a frequency less than f is:

$$N_{ta,yz} = \frac{f^2 l_y l_z}{c^2} - \frac{f}{c}(l_y + l_z) \qquad 8\text{-}4$$

and the total number of tangential waves with frequency less than f is:

$$N_{ta} = \frac{f^2 A}{2c^2} - \frac{fL}{2c} \qquad 8\text{-}5$$

where A is the total wall area $2(l_x l_y + l_y l_z + l_z l_x)$.

The number of oblique waves is given by the volume of one eigth of a sphere of radius f less the volumes already accounted for by the categories already counted divded by the mode volume, i.e.

$$N_{ob} = \frac{4 f^3 V}{3c^3} - \frac{f^2 A}{4c^2} + \frac{fL}{3c} \qquad 8\text{-}6$$

The total number of standing waves is given by the sum of categories, i.e.

$$N = \frac{4 f^3 V}{4c^3} + \frac{f^2 A}{2c^2} + \frac{fL}{8c} \qquad 8\text{-}7$$

Differentiating these formulas gives the number of waves having frequencies in the band df:-

$$\left.\begin{aligned} dN_{ax} &= (L/2c)\,df \\ dN_{ta} &= ((fA/c^2) - (L/2c))\,df \\ dN_{ob} &= ((4 f^2 V/c^3) - (fA/{}^2c^2) + (L/8c))\,df \\ dN &= ((4 f^2 V/c^3) + (fA/2c^2) + (L/8c))\,df \end{aligned}\right\} \qquad 8\text{-}8$$

In order to get some indication of the order of modal densities to be considered for the passenger compartment, of the test vehicle, it was treated as a rectangular room of dimensions $l_x = 6.5$ ft., $l_y = 4.25$ ft. and $l_z = 4.5$ ft. These dimensions gave a fundamental frequency of 85 Hz, and the bandwidth was chosen to be one tenth of the fundamental frequency, i.e. 8.5 Hz. The modal density curves obtained are shown in fig. 8-2 from which it can be seen that the oblique modes are the most numerous as was expected. A square law characteristic predominates in the total modes curve giving a rapid increase in modal density with increasing frequency. Hence with wide band white noise an increasing power density spectrum would be expected.

8-3) Limitations of the statistical theory

Modal densities given by the equations 8-8 are smoothed estimates, and would give reasonable results in the study of broad band excitation or high frequency excitation of the enclosure. However, in the case of rear axle gear meshing noise we are considering a narrow band phenomina at a frequency only approximately five times the fundamental frequency.

Comparisons of the statistical curves and the true modal density plots obtained from a computer evaluation using the same room dimensions are shown in figs. 8-3 to 5. The estimates for tangential and oblique waves were high and low respectively, but the estimate for the total modes was a good average.

Considering the total modes curves fig. 8-5, since this was the most important, it was obvious that though the estimate was a good average, the deviations from it were quite large. At the high frequency end where the bandwidth of the exciting processes tends to increase, a wider bandwidth could have been used in evaluating the modal densities which would have given a much better correlation of the curves. At the low frequency end of the curves no such improvement can be achieved, and it was obvious that the true modal density curve should be used in predictions of the response of the enclosure.

Reference to fig. 8-6 which shows the low frequency end of the modal density curve for the total modes to a much larger scale will show that in the region of rear axle gear meshing frequency at 47 mile/hr. there was a local increase in the modal density which could have been one of the contributing causes of the amplification of the gear meshing disturbance to an audible level. It must be remembered that the above statements relate to an idealised room, and not to the actual passenger compartment of the vehicle which would have been very difficult to analyse due to its complex shape.

The major limitation of the statistical theory of acoustic modal density are its poor estimation of the low frequency densities, and the evaluation of densities for complex shaped enclosures.

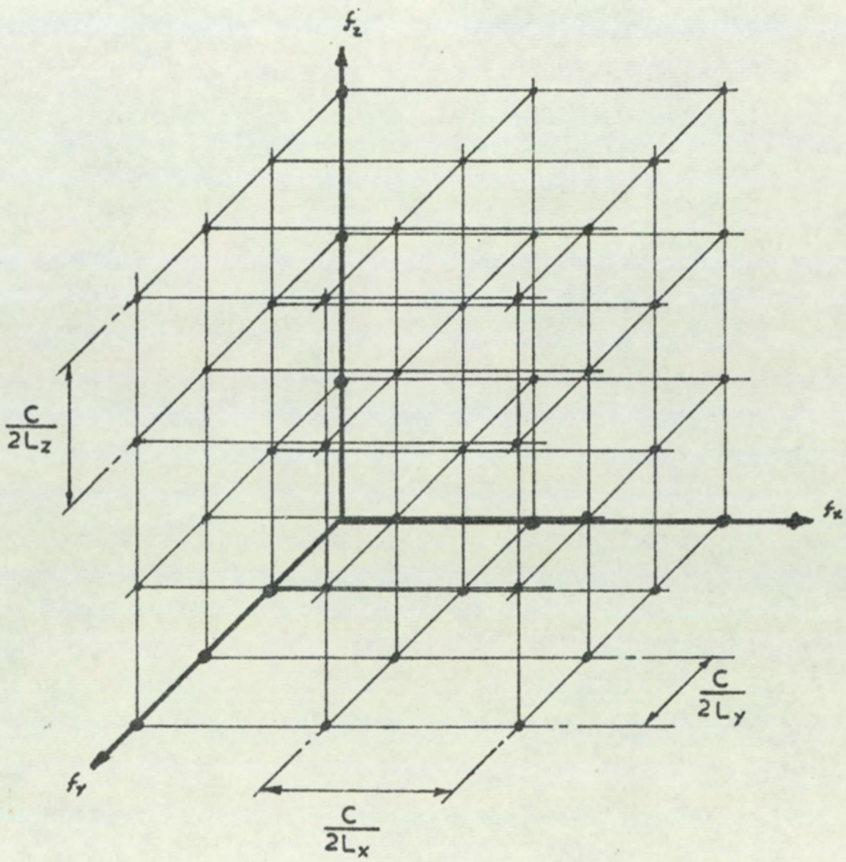
8-4) Actual response measurements of the acoustics of the passenger compartment.

Since it was possible that the acoustics of the passenger compartment might be important in the amplification of the rear axle gear noise, an attempt to measure the frequency response was made. Random excitation was used rather than periodic due to the difficulties of controlling the level of excitation from a loud speaker when periodic waves are used.

The procedure adopted was as follows:- A small loud speaker was placed in the passenger compartment of the test vehicle in the near side front lower corner, and the resultant noise measured with a condenser microphone. The speaker was with a wide band random signal, and the resulting microphone signals were recorded on magnetic tape for later power spectral density analysis. The measurements were taken at a random selection of points in the compartment to obtain an overall picture of the response of the compartment.

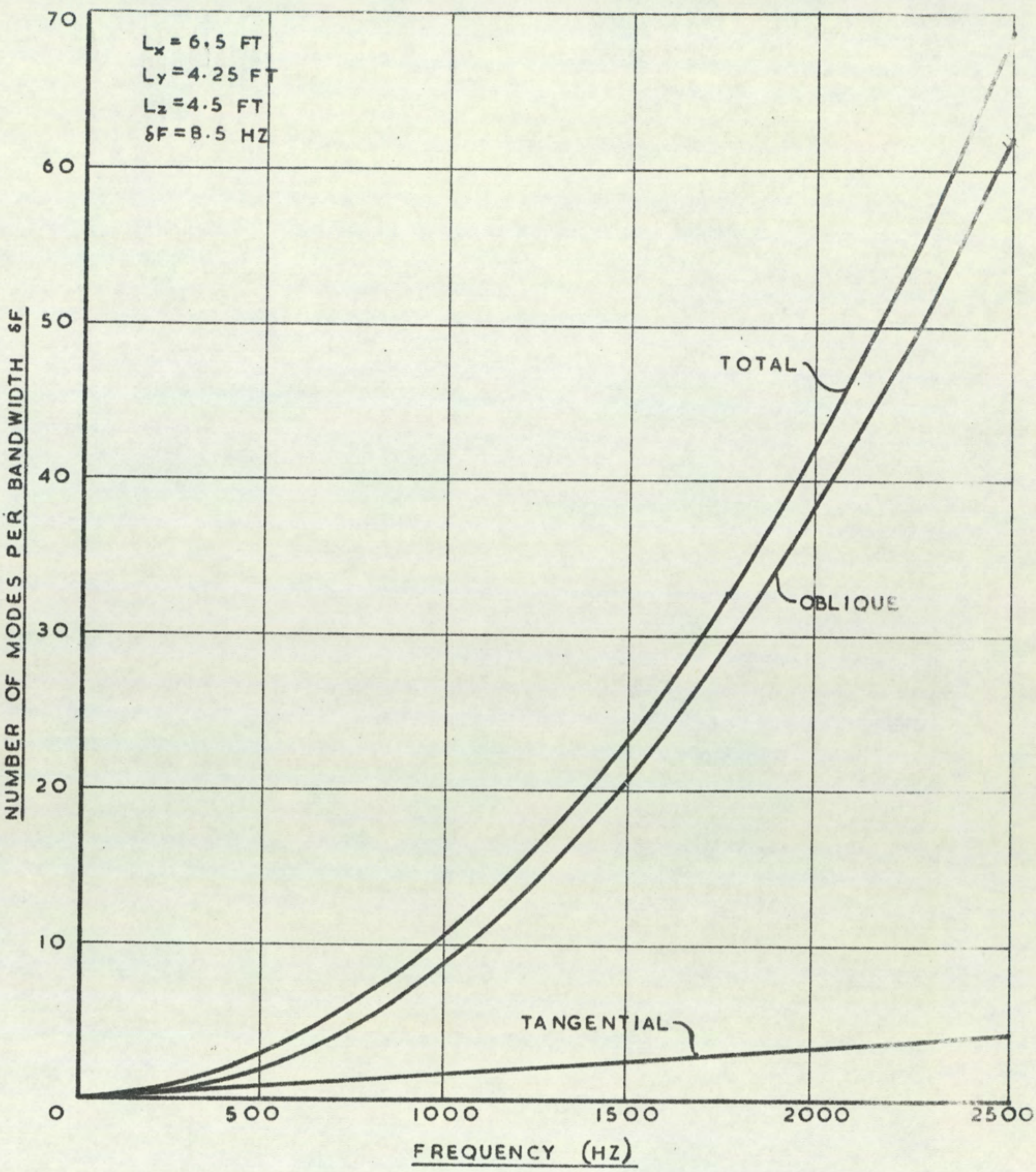
The microphone signal recordings were subjected to power spectral density analysis over the frequency range 400 to 600 Hz, and some typical spectra are shown in fig. 8-7.

The response of the compartment varied considerably with position due to the small number of modes active in this frequency range. As shown in fig. 8-7, there was only slight evidence of a peak in the response at 440 Hz, i.e. the frequency at which the major gear meshing noise was found to exist. This did not rule out the acoustics of the passenger compartment as a prime amplifier of the rear axle gear noise, since strong coupling of one or more of the modes of vibration of the enclosing pannels with acoustic modes could have increased the influence of those acoustic modes in the overall response.



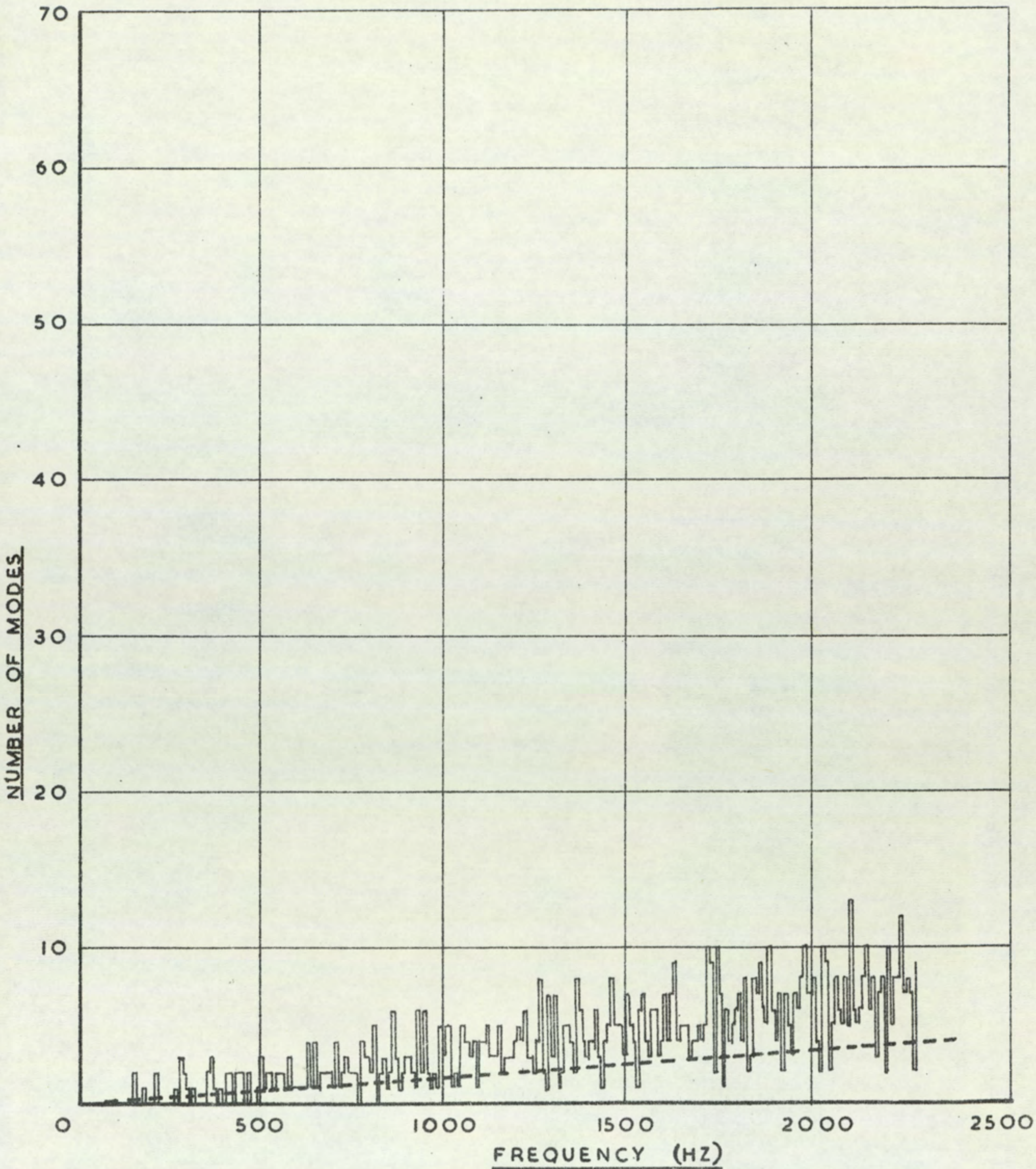
"FREQUENCY SPACE" LATTICE FOR THE NATURAL FREQUENCIES OF A RECTANGULAR ENCLOSURE

FIG 8-1



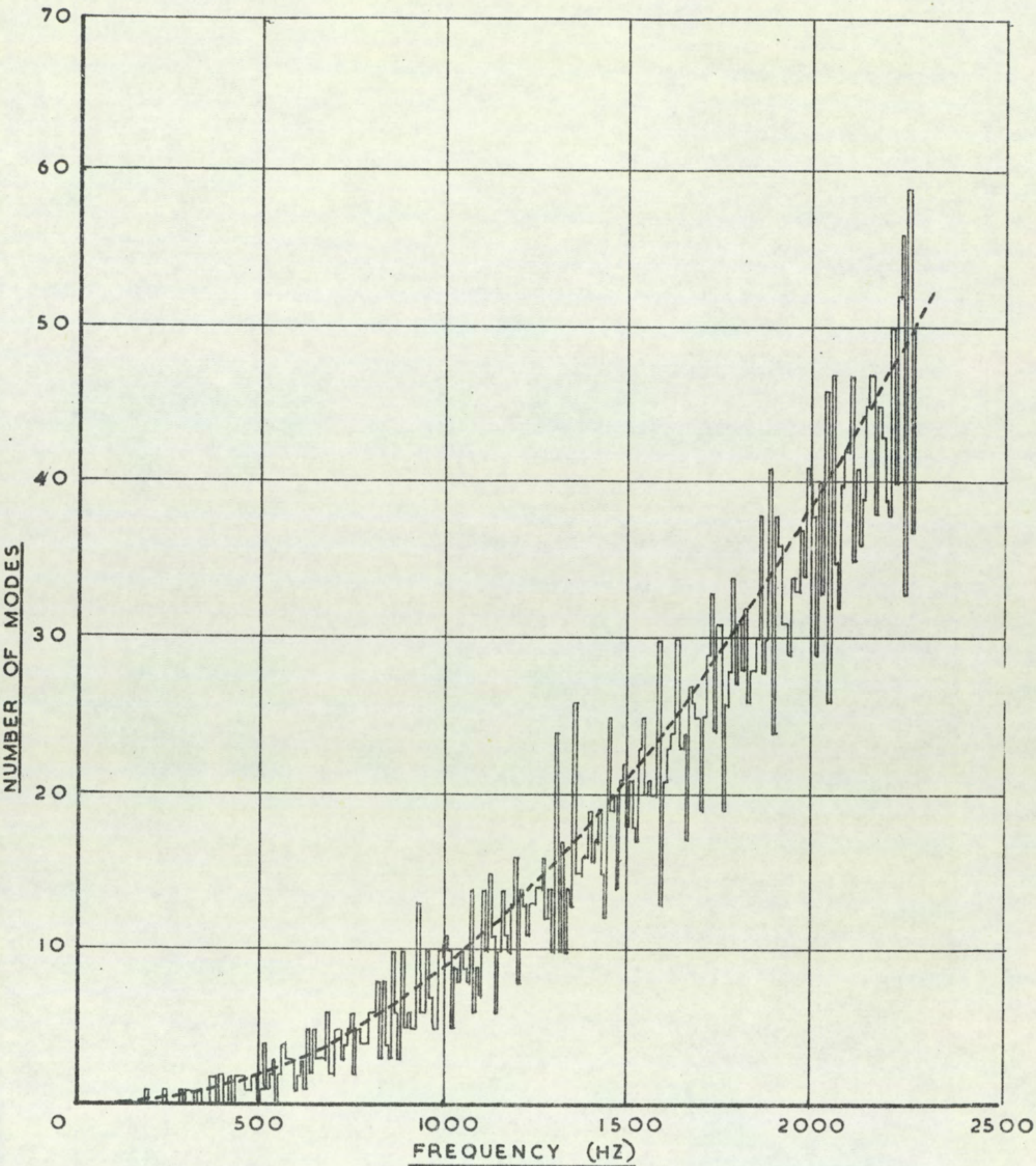
MODAL DENSITY CURVES FOR A RECTANGULAR ENCLOSURE GIVEN BY STATISTICAL THEORY

FIG 8-2



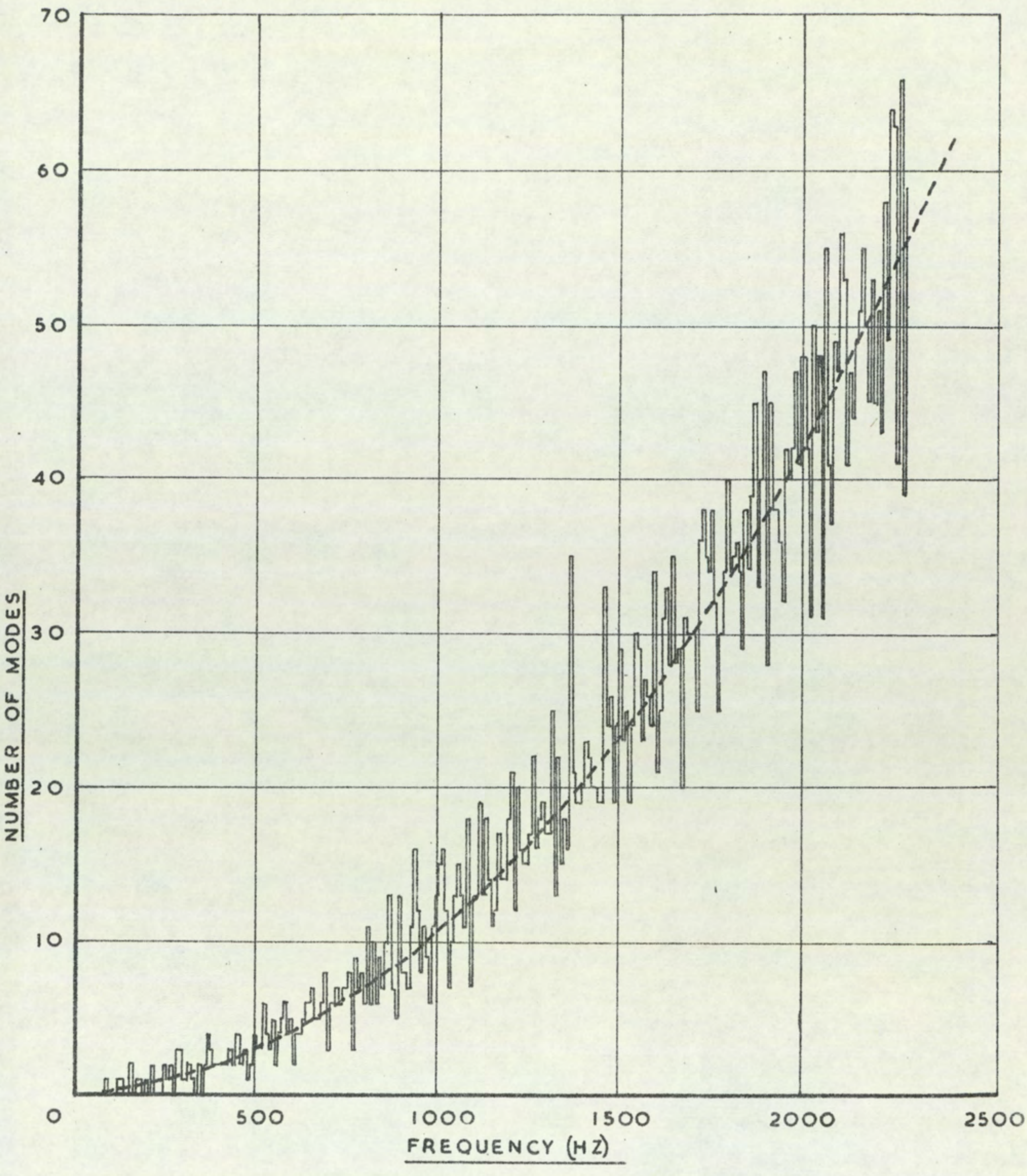
COMPARISON OF COMPUTED AND ESTIMATED MODAL DENSITY (TANGENTIAL MODES)

FIG 8-3



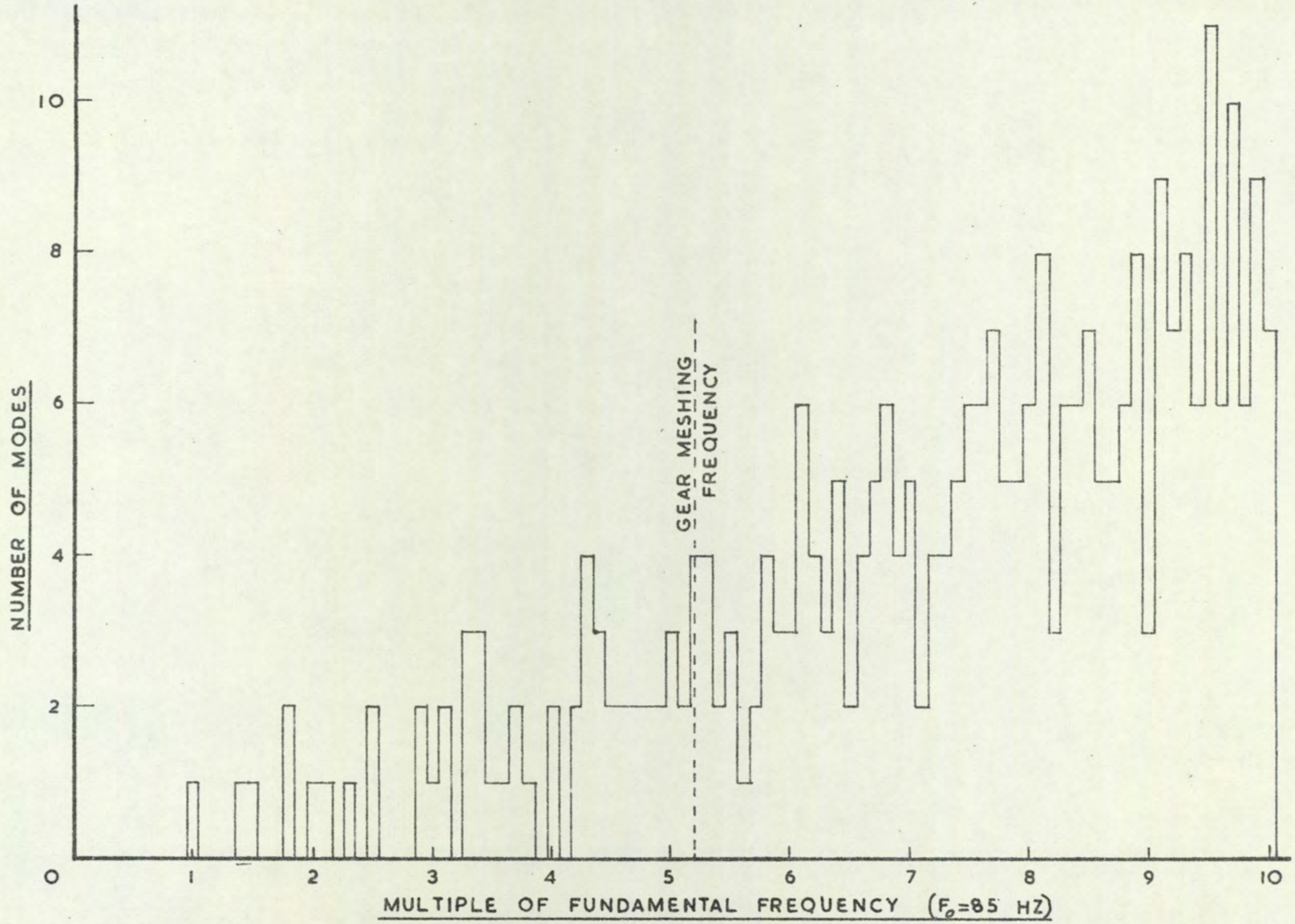
COMPARISON OF COMPUTED AND ESTIMATED MODAL DENSITY (OBLIQUE MODES)

FIG 8-4



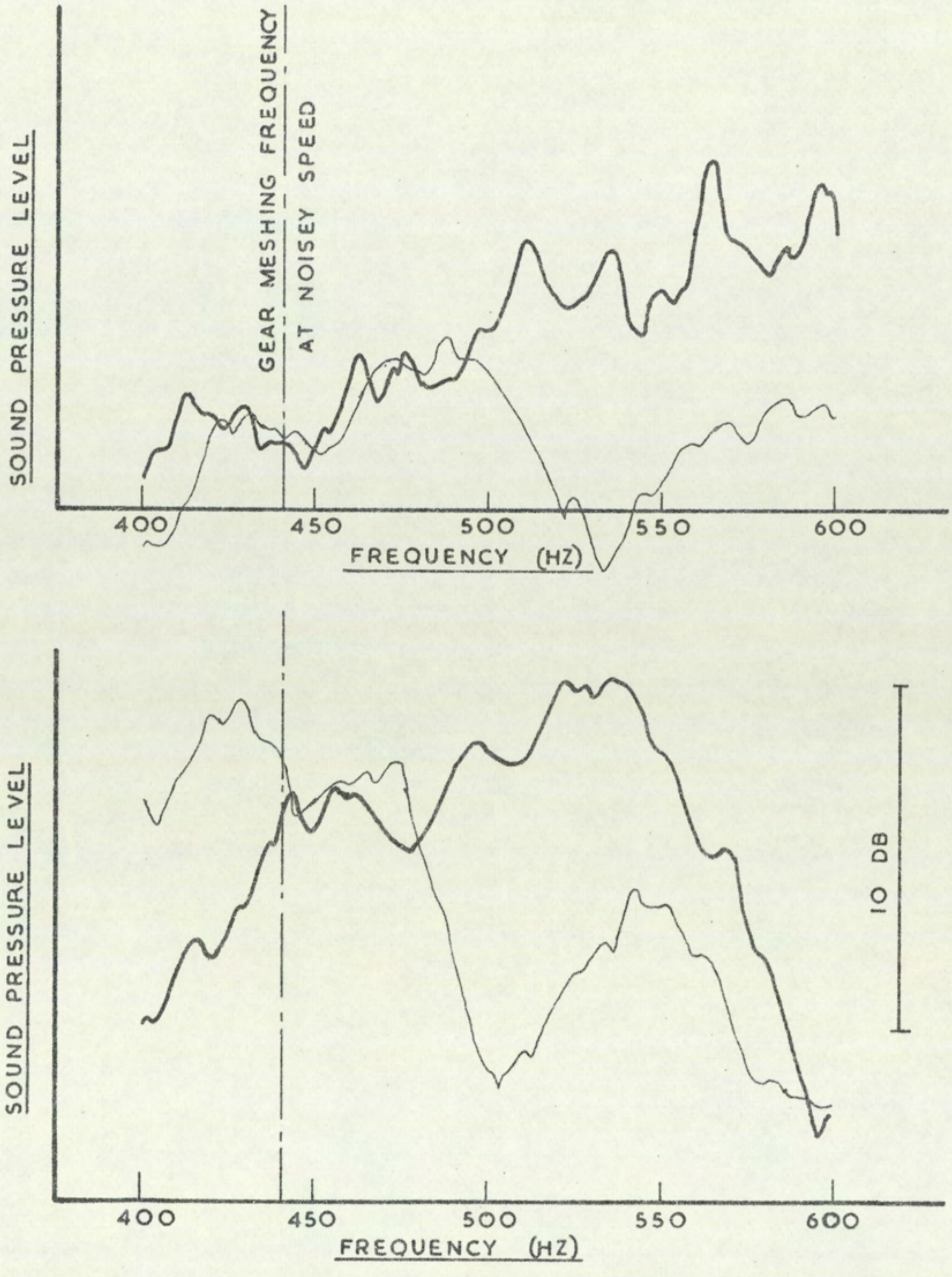
COMPARISON OF COMPUTED AND ESTIMATED MODAL DENSITY (TOTAL MODES)

FIG 8-5



LOW FREQUENCY END OF MODAL DENSITY PLOT (TOTAL MODES)

FIG 8-6



ACOUSTIC RESPONSE OF PASSENGER COMPARTMENT TO BROAD BAND RANDOM NOISE

FIG 8-7

CHAPTER 9 CONCLUSIONS.

9-1) The General Characteristics of Noise and Vibration Associated with the Power Transmission Systems of Passenger Automobiles.

Noise and vibration problems, created by the internal or external mechanical excitation of the power transmission system, were found to be whole vehicle problems. The reason for this was that the rubber bushes by which the transmission system was ultimately attached to the vehicle body did not act as isolators due to the low mechanical impedence of the unit construction body shell. Typical of the erroneous information which can be obtained by treating the transmission system in isolation was the incorrect hypothesis drawn from the preliminary investigations discussed in Chapter 2.

A second major characteristic discovered was that the live noises and vibrations, generated by the power transmission system while the vehicle was in motion, were random processes. Even disturbances associated with sources, such as rear axle gear meshing, which were intuitively assumed to be periodic were found to conform to this characteristic. The root cause of this characteristic is the random environment in which the vehicle must operate.

9-2) Rear axle gear meshing noise.

Both the major characteristics discussed in the previous section applied to the rear axle gear noise problem, together with the particular characteristics for the vehicle tested as follows:-

i) The audible noise in the passenger compartment was found to be most intense at 47 mile/hr. speedometer rading (45 mile/hr. actual speed).

ii) At this speed of 47 mile/hr. the noise was found to be a narrow band random process centred at 440 Hz.

These characteristics were concluded from the results, both subjective and objective, of the final set of road tests discussed in Chapter 7. Though the subjective assessments were essential in establishing the speed at which the whine was most audible, they were of no value in establishing the other characteristics of the noise. Objective analysis, using analog random process analysis techniques, enabled an identification to be made of the component of the general noise spectrum which was responsible for the audible noise. Once this component had been identified in objective analyses, it was possible to confirm that it did result from the vibrations generated by the meshing of the rear axle gears, and it was also possible to establish the other characteristics detailed above.

First stage amplification of the disturbance was considered to be due to a mechanical resonant condition and to be due to one or more of the internal components. This was inferred from the result of the various static response tests which showed the response at the rear spring rear-axle interface to be relatively low, but in these tests the main vibration was of the outer casing of the axle. Proof of such a resonant amplification of the internal components of the rear axle was not attempted due to the difficulties of simulating the loading of the components if they were removed from the axle, and the difficulties of access to the components in their normal environment.

No evidence of mechanical amplification of the vibrations in the region of 440 Hz during transmission through the rear suspension system was found. It was clear therefore, that any further amplification of the vibrations took place within the vehicle body or the passenger compartment.

9-3) Transmission of Transmission System noise and vibration to the vehicle body and the passenger compartment.

Two modes of transmission of power transmission system disturbances were initially assumed to be possible, i) direct radiation of acoustic energy from the outer surfaces of the transmission system or ii) mechanical transmission of the energy via the various suspension and mounting elements.

Though the first method of transmission applied to the low frequency disturbances such as engine noise, this was not found to be the case at higher frequencies. This conclusion was drawn from the lack of correlation between the noise spectra in the passenger compartment, and the noise spectra close to the transmission system. The reason for this result was assumed to be the efficiency of the damping materials applied to the floor pannels of the body.

Later investigations showed that for the rear axle gear noise the major mode of transmission of the disturbance was mechanical via the rear suspension systems. It would seem reasonable to assume that this was the general case for all the higher frequency vibrations of the rear axle. After transmission to the body of the vehicle, the vibrations excite the body pannels into vibration, and the pannels in turn excite the acoustic response of the passenger compartment, hence giving rise to the audible noise.

The obvious method of reducing the mechanical transmission of vibrations via the rear suspension system is to introduce some element into the suspension system which has a low transmissibility, but if this course of action is taken, care must be exercised to ensure that the ride and handling of the vehicle are not compromised.

9-4) Influence of the passenger compartment acoustics.

In any noise problem within a passenger vehicle, the acoustics of the passenger compartment will play a major role. This situation is created by a number of factors of which the following are the major:- i) Acoustic resonances are nearly always high Q resonances giving high amplifications. ii) The modes are closely spaced in the frequency domain, hence increasing the probability of a particular vibration exciting an acoustic mode. iii) There is a high probability of strong coupling between acoustic mode and the mechanical modes of vibration of the enclosing pannels which will increase the effect of low level excitation from the pannels.

For large enclosures it is possible to predict the acoustic response with a reasonable degree of confidence by the application of a statistical technique of modal density analysis. This method of prediction was found to be inadequate for small enclosures, and particularly for enclosures such as a passenger compartment of a motor vehicle, due to its irregular shape. It was found that for a rectangular enclosure of approximately the same size as a passenger compartment, quite high modal densities were obtained in the mid audio frequency range which was the type of condition which ould result in high audibility of particular frequency disturbances.

Some attempts were made at experimental measurement of the acoustic response of the passenger compartment of the test vehicle, but they were inconclusive, due to the difficulties of obtaining a suitable sound source.

9-5) Possible cures for rear axle gear noise

From the above discussion it is clear that there are a number of sections of the total vehicle system to which attention could be directed in attempting to reduce the audible rear axle gear noise to a more acceptable level.

a) Source of disturbance. This was confirmed to be the vibration caused by the meshing of the crownwheel and pinion gear teeth. It is doubtful if much reduction could be achieved in this source disturbance since the machining of the hypoid teeth used for this gear pair, has been developed to a high degree of sophistication.

b) The internal resonant components of the rear axle. If such components do exist as hypothesised, then some degree of cure could be achieved here by either increasing the damping or tuning the components. This course of action would be a long term solution due to the experimental work required to identify the components.

c) Isolation of the rear axle from the body. Definitely the most promising course of action since it may be possible by suitable redesign of the rubber bushes in the rear suspension system.

d) Mechanical vibration of the body pannels. If the resonant response of the body pannels was confirmed to be a major factor in the amplification of the vibrations, then two possible cures are available, i) tuning the pannels by the addition of suitable stiffening ribs or, ii) damping the pannels by the application of a viscous surface treatment.

e) Acoustics of the passenger compartment. The response of the acoustics of the passenger compartment would be very difficult to control, other than by the use of expensive sound absorbant material.

In establishing a positive recommendation from the above, the following factors were considered:-

i) Cost. This is a threefold factor covering project cost to establish the required design changes, production costs for tooling etc., and material costs.

ii) Time. This covers the length of time required to achieve the changes.

iii) Feasibility. This mainly covers technical feasibility, but also includes customer acceptance of the resulting changes.

Item a) was ruled out on all three counts, and would almost certainly be the least productive course of action.

Item c) would not lead to any large reduction in the noise intensity, and can be ruled out due to high material cost and possible customer rejection.

Item b) would be feasible, but it would be likely to incurr high project costs, and the time to achieve the desired results could be extensive.

Item d) would achieve results in a very short time, but at the expense of high production and material costs.

Item e) could produce good results, and would have a reasonable chance of meeting the criteria set out above.

The above considerations lead to the recommendation that the best course of action would be a study of the rear suspension system with particular regard to the redesign of the rubber mounting bushes. A further long term study of the dynamics of the internal components of the rear axle would also be likely to lead to positive reductions in the audible noise.

9-6) Influence of the propeller shaft on the dynamics of the transmission system.

From the static response tests it was confirmed that the propeller shaft was the least damped and consequently, dynamically the most active part of the power transmission system. Other than in the vicinity of the frequency of the first bending mode of vibration of the propeller shaft, it was found that the propeller shaft had only a minimal influence on the response of other parts of the power transmission system. Conversely, the response of the propeller shaft was found to be strongly influenced by the presence of the other components as was shown by the significant shifts of the frequencies at which the bending modes of vibration occurred compared to the theoretical frequencies for a pin ended beam of the same dimensions.

Measurements taken during the road tests showed that the frequencies at which the bending modes of the propeller shaft were occurring were more in agreement with the pin ended beam frequencies than was the case for the static tests. It was concluded from this that under dynamic conditions the universal joints were acting in a manner more akin to a pin joint, probably due to the relief of friction. Also, it was obvious that if this were the case the vibration of the propeller shaft would be well isolated from the other parts of the power transmission system.

9-6) Comparison of Static response measurements and road test measurements.

These comparisons were restricted to the resonant frequencies since no information was available for the actual forcing functions for the road tests.

It was quite clear even from this restricted comparison that considerable changes took place between the static and dynamic conditions. These changes were due to the difference in loading on the bearings, and gears resulting in different end conditions and changes in damping in the resonant modes.

Many of the characteristics deduced from the static response tests were found to be useful in interpreting the frequency domain analyses of the road test measurements in retrospect. However, use of the results of static response test results to predict the dynamic behaviour of the power transmission system during normal operation is considered to be rather hazardous procedure.

Static response tests do still have a use in investigating the absolute source of amplifing components once positive identification of the resonant response has been made.

9-7) Analytical calculations

The mathematical investigations included in this research programme showed that it was possible to make useful predictions of the natural frequencies and the associated modal shapes of systems like the power transmission system of an automobile. This was found to be the case even when the assumptions made in forming the mathematical modal were quite gross.

The transfer matrix method of evaluation of the natural frequencies and modal shapes was found to be well suited to this type of system. This technique was found to be very flexible in respect of extensions to the system being analysed, and required very little change of data for parameter changes of the components of the system. The main disadvantage of the method was its long computing time, but this could be improved by taking larger sections of the systems in forming the lumped parameter model and using somewhat more elaborate transfer matrices.

One of the main assets of the computed predictions was found to be the insight gained into the interaction of the various components of the power transmission system. This information augmented the information obtained from the experimental measurements which was rather imprecise due to the influence of the non-linearities in the actual system.

9-9) Experimental techniques

9-9-1) Static Response Tests

Static response tests of an automobile power transmission system should if possible, be performed with the system mounted in its normal environment. This is necessary due to the low mechanical impedance of the modern unit construction automobile body.

Very little can be done to improve the actual experimental measurements other than those changes brought about by the introduction of new testing equipment. One particular advance in the field of response testing equipment which will be of great value in tests of this type of system, were back lash type non-linearities exist, is the development of frequency tracking filters. This type of filter makes it possible to perform automatic sweep tests at high forcing levels since the increase in harmonic distortion is no longer an embarassment.

9-9-2) Road Tests.

The major problem to be overcome in road testing is the obtaining of a test run having stationary conditions for a sufficient duration to enable a reasonable statistical accuracy to be achieved in the data analysis. Some improvement of stationarity of the road test data was achieved by the use of a more sensitive speed indicator, than the normal speedometer, placed so that it was within the drivers normal field of view. It may be possible to improve still further on stationarity by the use of a servo control of the vehicle speed.

Truely ergodic conditions between tests cannot be achieved due to uncontrollable factors such as windage. These factors can be minimised by the careful selection of the test road. It is recommended that the test road should be reasonably sheltered, and not a section of the public highway, in order to remove the noises from passing vehicles.

The major difficulty in instrumenting a vehicle for road test is obtaining portable battery powered equipment which is compact enough and light enough to be carried in the vehicle under test. The most urgent area for improvement in this respect is the development of portable magnetic tape recording equipment having a performance more compatible with laboratory magnetic tape equipment.

9-10) Road test data analysis techniques.

Wave analysis techniques employing constant percentage bandwidth filters were found to be quite unsuited to the analysis of power transmission system noise, and vibration data obtained from road tests. There were two reasons for this a) the data was made up of a considerable number of narrow band components plus a broad band component which tended to swamp the ouput from the filter and, b) the data was basically a random process due to the random nature of most of the functions which controlled the level of the forces which generated the vibrations.

Of the various techniques available for the analysis of random processes, the power spectral density analysis is the most informative for this type of problem. Analog evaluation of this function is considered to be the best form of processing from the point of view of capital cost of equipment. Filtering equipment used in such an analysis system should have a good absolute rejection, since it was found that the data has a high background noise level.

Use of cross-correlation techniques for tracing the transmission paths for noise and vibration was not investigated, due to lack of time, but this technique might well be useful for the investigation of parallel transmission paths such as exist in the rear suspension system.

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