

$$E_{FWM} = i \frac{2\pi w}{nc} D \chi^3 E_p E_q E_r^* \times e^{\left(-\frac{\alpha+i\beta}{2}\right)L} \times \frac{1 - e^{\{-\alpha+i\Delta\beta\}L}}{\alpha - i\Delta\beta}, \quad (1)$$

where, E_{FWM} is the nonlinearly generated field, w is the angular frequency, E_p , E_q and E_r^* are signal components, n is the refractive index, L is the span length, χ^3 is the nonlinearity coefficient, degeneracy factor D is either three or six for degenerate and non-degenerate FWM and $\Delta\beta$ is the effective propagation constant difference. For a modulated signal whose total bandwidth exceeds the phase matching bandwidth of a single fibre span ($\sim 3\text{GHz}$ for standard single mode fibre) integration of Eq. (1) over the continuous power spectral density (PSD) is required, accounting for contributions from both strongly and weakly phase matched contributions.

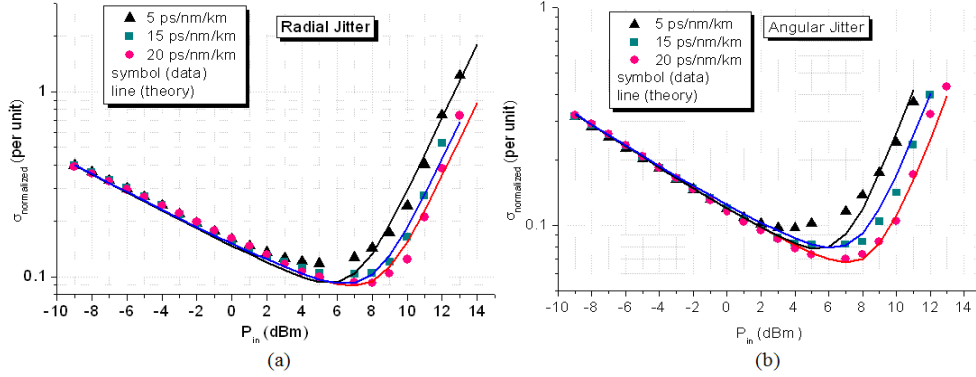


Fig. 3. Normalized standard deviation of the received field for various dispersion maps after DBP, as a function of launch power per channel per span for 112 Gb/s PM-QPSK transmission after 4,800 km (a) Radial jitter (b) Angular Jitter. Data (symbols), Lines (theory).

Recently closed-form expressions have been derived for the nonlinear interaction between spectral components of signals with continuous spectra where the effects of strongly and weakly phase matched conditions are treated separately [16]. Following the same approach to the derivation, but replacing one of the signal fields by the noise field, results in a solution for the parametric amplification of ASE from the N^{th} amplifier along the remainder of the link,

$$I_{SN-FWM} = I_{noise} \cdot I_{signal}^2 (C_1 + C_2), \quad (2)$$

$$C_1 = \frac{N\gamma^2 \ln(2\pi^2 B^2 |\beta_2|/\alpha)}{\pi |\beta_2| \alpha}, \quad C_2 = \frac{\gamma^2}{\pi \alpha |\beta_2|} \left(\frac{N}{\pi} + \frac{2}{\alpha L} \{N \log(N) - N + 1\} \right),$$

where, I_{SN-FWM} is the nonlinear noise power spectral density, I_{signal} is the signal power spectral density, I_{noise} is the noise power spectral density from a single amplifier. C_1 and C_2 represent weakly and strongly phase matched regimes, respectively. N is the number of spans after a given amplifier, B is signal bandwidth and β_2 the group velocity dispersion ($D = -2\pi c \beta_2 / \lambda^2$). From Eq. (2), treating the contribution from each amplifier as an independent random variable, with Gaussian statistics, the total nonlinear noise at the output of an M -span system is,

$$I_{Total}^2 = M^2 I_{noise}^2 + \sum_{N=1}^M (I_{SN-FWM})^2. \quad (3)$$

The solid lines in Fig. 3 show the result of Eq. (3) for our simulation conditions, and show an excellent match. In contrast to the four-wave mixing experienced by an ultra wide-band signal [16], for signal-ASE FWM the contributions from both strongly and weakly phase matched terms are significant for the given 28 Gbaud system with DCF free transmission.

Summing the independent contribution from each amplifier gives a length variation with significant terms up to $M^{3/2}$. Figure 4a shows the evolution of the noise σ_{norm} with length for a signal launch power of 5 dBm, close to the optimum shown in Fig. 3, and again shows an excellent agreement with analytical prediction of (2). Figure 4a also shows the expected evolution in σ_{norm} in the absence of nonlinearity, corresponding to noise loading at the receiver. Comparing the two curve fits reveals that the signal-ASE FWM process becomes dominant in this system after $\sim 5,000$ km for PM-QPSK.

In order to indicate the relative impact of the transmission limit originating from the signal-ASE FWM constraint, Fig. 4b plots the nonlinearity limited information spectral density (ISD) versus transmitted power density, following a similar analysis to [17]. The figure depicts that for the example reported here the effect of nonlinearities at high powers prevents indefinite growth in ISD. It can be seen that the effect of XPM becomes prominent at transmitted power densities beyond 0.01 W/THz, and a maximum ISD of 3 b/s/Hz/pol is predicted [18]. However, for a point to point transmission system such effects are compensable [6]. However, even with such compensation scheme, the transmission capacity remains limited, but by signal-ASE FWM. As can be seen in the figure, for this system configuration the maximum transmittable power spectral density may be only increased to 0.1 W/THz and the maximum ISD may only be increased by a factor of 2.

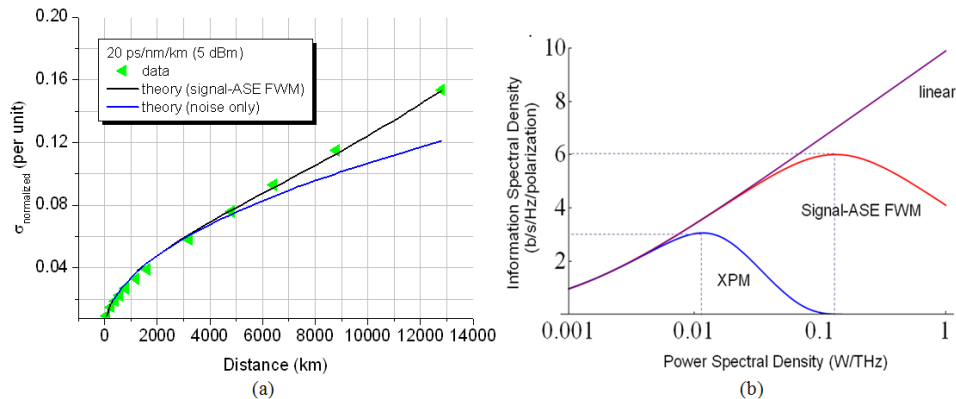


Fig. 4. (a) Normalized standard deviation (Angular Jitter) after DBP as a function of distance transmitted for 20 ps/nm/km dispersion at 5 dBm. Data (green triangles), analytical fit (black line) noise only (blue line), (b) Predicted information spectral density limits per polarization after 12,000 km for linear transmission (magenta curve), for non-linear transmission including XPM for a WDM system with 101, 50GHz spaced channels (blue curve) and signal-ASE FWM within one channel (red curve) (and all other parameters same as numerical simulations).

4. Conclusion

We have demonstrated the influence of four-wave mixing between signal and ASE on the transmission performance of a long-haul coherently-detected 112 Gb/s PM-QPSK system employing DBP. Our results show excellent agreement with analytical theory and suggest that the BER and the associated maximum reach remain ultimately limited even after digital back-propagation due to the signal-noise interactions, and that an optimum signal power exists even with DBP. Further investigation reveals that the evolution of the excess noise with signal power, dispersion and transmission length is fully consistent with the four-wave mixing between the signal and ASE, whereby the noise is parametrically amplified by the signal.

Acknowledgments

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