Design of a flat-top fiber Bragg filter via quasi-random modulation of the refractive index

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The statistics of the reflection spectrum of a short-correlated disordered fiber Bragg grating are studied. The averaged spectrum appears to be flat inside the bandgap and has significantly suppressed sidelobes compared to the uniform grating of the same bandwidth. This is due to the Anderson localization of the modes of a disordered grating. This observation prompts a new algorithm for designing passband reflection gratings. Using the stochastic invariant imbedding approach it is possible to obtain the probability distribution function for the random reflection coefficient inside the bandgap and obtain both the variance of the averaged reflectivity as well as the distribution of the time delay of the grating. © 2008 Optical Society of America

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Fiber Bragg gratings (FBGs) have found numerous applications in modern day telecommunications [1]. The information about optical properties of the FBG (including reflectivity, time delay, induced dispersion, etc.) is given by a single complex reflection coefficient r that defines what is called an FBG spectrum [2]. These spectra are uniquely determined by the induced effective refractive index change of the guided mode of interest. The inverse problem is, however, of equal importance. It is very often desirable to be able to fabricate a designer FBG with prescribed spectral characteristics. In particular for an FBG filter it is of great importance to produce reflectivity that has a square-topped profile, steep sides, low sidelobes, and flat dispersion. Different semianalytical techniques have been used to tackle this problem, an incomplete list of which includes layer peeling [3], one section design [4], optimized modulated Gaussian profiles [5], and the Gel'fand–Levitan–Marchenko equation approach [6,7]. An ingenious FBG synthesis algorithm was also suggested in [8]. Most of these methods, however, are quite sensitive to the accuracy of the numerical scheme employed for the profile reconstruction and can be difficult to implement in the practical fabrication process (with the possible exception of that of Poladian [8]).

So the question remains whether one can suggest a simple and practical alternative method for designing flat-top passband filters. In the present Letter we suggest a simple idea for such a filter based on an analogy with linear systems. Indeed if one is to achieve a flat average Fourier spectrum one has to look no further than a short correlated Gaussian signal. If L_c is the correlation length then all spatial harmonics corresponding to scales $z > L_c$ will be statistically independent and the spatial average spectrum will be flat within the bandwidth $B \sim L_c^{-1}$. Here we apply the same idea to the problem associated with the random corrugation of the fiber core $\delta n(z)$. We will make an important assumption in the process; i.e., the scale of random corrugation is large enough so that a coupled-mode theory is applicable. Moreover, inside the bandgap we shall make an additional assumption that the refractive index modulation can be approximated by a short-correlated Gaussian process (white noise approximation). This simplification allows us to obtain the exact statistics of the reflectivity inside the bandgap. Outside the bandgap we will resort to direct Monte Carlo simulations of the coupled-mode equations to obtain the average spectrum.

Note that the effects of stochastic perturbation on the FBG spectra have been studied before in a number of references [9–11]. However what distinguishes the proposed approach from what was studied before is the fact that most of the time the cited references deal with noise in the inscribed index change $\delta n(z)$ as a small spurious perturbation existing in the background of a uniform dc-index change $\overline{\delta n}$. In particular in [9] the authors studied small linear corrections to the spectrum of a dc-index profile and calculated their variances using linear perturbation theory. In the current Letter we propose an entirely different viewpoint on the stochastic variations of the refractive index. Instead of treating them as small undesired errors we want to study purely random corrugation of the fiber waveguide in the absence of a dc background and then see if rare fluctuations of the reflective index play any role and whether one can use randomness in a positive way, namely, in achieving flat average reflection spectra with small variance. Then one can generate a simple and easily scripted algorithm for inscribing such pseudorandom gratings and such an algorithm will be relatively easy to implement compared to elaborate and time-consuming inverse-problem techniques suggested earlier in [3,4,6,7].

We start from a standard coupled-wave formulation for the amplitudes of forward and backward propagating modes in the span of fiber containing an inscribed FBG [1,2,12],

$$\frac{\mathrm{d}\psi_1}{\mathrm{d}z} = i\,\delta\psi_1 + q^*(z)\psi_2,$$

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$$\frac{d\psi_2}{dz} = -i\delta\psi_2 + q(z)\psi_1, \tag{1}$$

where z is the coordinate along the fiber, $\psi_{1,2}$ are the complex components of the copropagating and contrapropagating waves, and the complex coefficient $q(z)$ is related to the ac component of the uniformly inscribed change of refractive index: $q(z) = \omega v n_{co} \delta n_{co}(z) I \exp[-i\phi(z)]$ [1,2]. We assume that the grating is produced by a uniform inscription in the core so that in the latter expression n_{co} stands for the effective core refractive index, $\delta n_{co}(z)$ stands for its induced change, $\phi(z)$ describes the chirp of the grating, I is the scalar product of the transverse modes of an ideal fiber, ω is the mode angular frequency, and v is the fringe visibility. We will assume that no dc variation of the refractive index is present or, alternatively, $v \gg 1$. Finally, spectral parameter δ describes the detuning of the wavelength from the Bragg resonance $\delta = \beta - \pi/\Lambda$, where β is the longitudinal component of the wave vector and Λ is the grating period. We will assume that $q(z)$ represents a complex Gaussian noise with zero average and the correlation function localized at short scale L_c. This can be achieved if within intervals of length L_c the index change δn_{co} is sampled from a Rayleigh distribution with given rms $\overline{\delta n}$ and the phase is sampled from a uniform distribution between 0 and 2π. This algorithm is particularly easy to implement if one uses a femtosecond laser inscription method that has become quite popular recently. In this case one simply has to script the movement of the table accordingly (which is allowed by existing experimental hardware). It is also feasible with more traditional phase mask techniques: one simply has to corrugate the silica phase mask respectively. Generally the technique is not that different from manufacturing a deterministically chirped grating. Inside the bandgap $q(z)$ can be approximated as white Gaussian noise with zero mean and the correlation function $\langle q(z) q^*(z') \rangle = 2D\delta(z-z')$ with $2D = \omega^2 v^2 n_{co}^2 \overline{\delta n}^2 I^2 L_c$. Unlike the authors of [9] we do not wish to decouple phase and amplitude fluctuations, since both are important for achieving an uncorrelated input.

Important remarks should be made about the applicability of the suggested model. First, for the short correlated approximation to be compatible with coupled-mode theory one must have the correlation length L_c to be much less than the detuning wavelength, $|\delta|^{-1}$. As mentioned earlier this criterion holds inside the bandgap and is sufficient for the analytical description of the statistics of the reflectivity. Outside the bandgap, the applicability criterion for the coupled-mode approximation is that the scales of variation of all parameters in Eq. (1) are much larger than the grating period Λ, because all fast oscillating terms with period $\sim\Lambda$ have already been averaged out [12]. This puts an upper limit on both spectral parameter and the inverse correlation length scale: $\max\{|\delta|, 1/L_c\} \ll 1/\Lambda$.

Let us now turn to the analytical description of the random reflectivity inside the reflection band. Spectral problem Eq. (1) and its stochastic counterpart are well known in different contexts, e.g., in theory of integrable systems [13]. In particular one can see that if we introduce a function $r(z) = \psi_1(z)/\psi_2(z)$ it satisfies the following Ricatti equation [6,13]:

$$\frac{dr}{dz} = 2i\delta r - qr^2 + q^*, \quad r(0) = 0. \tag{2}$$

If the grating has length L then the reflection coefficient r_L is restored via $r_L = r(z)\exp[-2i\delta]$. This constitutes the mathematical basis of the invariant imbedding method particularly useful for analyzing stochastic problems [13]. Mathematically, Eq. (2) is a stochastic differential equation with multiplicative white noise written here in a complex form. The statistics of the complex reflection coefficient can be studied via the standard approach of the Fokker–Planck equation (FPE) [13]. If we assume the following parametrization of the reflection coefficient $r = \tanh(\chi/2)\exp[i\theta]$, $0 \leqslant \chi < \infty$, $0 \leqslant \theta < 2\pi$ then the probability density function (PDF) $P(\chi, \theta; z)$ satisfies the following (Stratonovich) FPE:

$$\frac{\partial P}{\partial z} = -2\delta \frac{\partial P}{\partial \theta} + 2D \coth^2 \chi \frac{\partial^2 P}{\partial \theta^2} + 2D \frac{\partial}{\partial \chi}\left[\sinh \chi \frac{\partial}{\partial \chi}\left(\frac{P}{\sinh \chi}\right)\right]. \tag{3}$$

Using the periodic boundary condition in phase θ one can obtain the autonomous equation for the quantity $Y(\chi; z)$, which is related to the marginal PDF $P(\chi; z)$ via $P = Y \sinh \chi$,

$$\frac{\partial Y}{\partial z} = \frac{2D}{\sinh \chi} \frac{\partial}{\partial \chi}\left[\sinh \chi \frac{\partial Y}{\partial \chi}\right], \quad Y(\chi; 0) = \delta(\chi)/\sinh \chi. \tag{4}$$

One can immediately observe that the statistics of the reflectivity $R = |r_L|^2 = \tanh^2 \chi/2$ is independent on the spectral detuning parameter δ and the averaged reflectivity spectrum if flat $\langle R(\delta) \rangle = \bar{R}$ inside the reflective band, i.e., for $\delta \ll L_c^{-1}$. To estimate the average spectrum $\bar{R}$ and its standard deviation one needs to solve Eq. (4). This equation is known in the theory of disordered systems and its solution has the following explicit form [14]:

$$Y(\chi; z) = \frac{e^{-Dz/2}}{8\sqrt{\pi D^3 z^3}} \int_\chi^\infty \frac{\chi' \exp(-\chi'^2/8Dz)}{\sqrt{\cosh \chi' - \cosh \chi}} d\chi'. \tag{5}$$

Now it is easy to obtain the averaged reflectivity of a grating with length $z = L$,

$$\bar{R} = 1 - \frac{e^{-DL/2}}{4\sqrt{\pi (DL)^3}} \int_0^\infty \frac{\chi^2 \exp[-\chi^2/8DL]}{\sqrt{1 + \cosh \chi}} d\chi \simeq 1 - \frac{\pi^{5/2}}{\sqrt{32(DL)^3}} e^{-DL/2}, \quad DL \gg 1. \tag{6}$$

The last line applies to long gratings and gives exponential mode localization with the length of the system very much like the Anderson localization in linear disordered systems [14]. In the same limit of long grating one can obtain the expression for the relative error due to fluctuations around the average $\bar{R}$,

$$\eta = \frac{\sqrt{\langle (R-\bar{R})^2 \rangle}}{\bar{R}} \simeq \frac{\pi^{5/4}}{(512(DL)^3)^{1/4}} \exp[-DL/4]. \quad (7)$$

One can see that for long gratings the fluctuations of the reflectivity are exponentially small, which is important for practical applications.

Let us now discuss briefly the phase properties of the pseudorandom gratings. An important quantity that has significance in the telecom applications is the time delay introduced by the grating, which is defined as $\tau_p = \mathrm{d}\theta/\mathrm{d}\omega = (n_{\mathrm{eff}}/c)\mathrm{d}\theta/\mathrm{d}\delta$ [1,2] where n_{eff} is the effective refractive index. Using Ricatti equation (2) and complex parametrization for the reflection coefficient one can obtain an Ito stochastic differential equation for the phase derivative $\theta_\delta \equiv \mathrm{d}\theta/\mathrm{d}\delta$. Unfortunately the marginal PDF for the quantity θ_δ cannot be obtained in the closed form. However in the localization regime $DL \gg 1$ one can neglect the fluctuations of the reflectivity $R=|r|^2$ and obtain a simple closed equation $\mathrm{d}\theta_\delta/\mathrm{d}z = 2+2\theta_\delta\eta$ where $\eta(z)$ is a real Gaussain delta correlated noise with $\langle \eta(z)\eta(z')\rangle = D\delta(z-z')$. Within the same localization regime the statistics of the phase derivative (and hence the statistics of the time delay) is given by the stationary PDF that does not depend on L,

$$P(\theta_\delta) = \frac{1}{D\theta_\delta^2} \exp\left[-\frac{1}{D\theta_\delta}\right], \quad \theta_\delta > 0. \quad (8)$$

One can see that PDF Eq. (8) has a power law tail, i.e., the fluctuations are strong—much stronger in fact than those for the reflectivity. However as follows from Eq. (8) the probabilities of extreme outages (spikes) in the spectral dependence of the time delay still decrease with the increase of the parameter DL and additional numerical runs (not shown) confirm this.

The analytical results above apply only to the spectral region $|\delta| \ll 1/L_c$ far from the band edge of the filter, where white noise approximation is applicable. To describe the whole spectrum with the sidelobes we have to resort to numerical Monte Carlo simulation of the reflectivity from Eq. (2) with fixed finite correlation radius. Figure 1 presents the results of such simulations for $L_c/L = 0.1$, $DL = 4$. The solid curve represents a spectrum averaged over 10,000 Monte Carlo realizations of random samples. We compare the average reflectivity with that of the uniform grating of the same bandwidth (red dashed curve) and given rms magnitude of index variation $\bar{q} = \sqrt{2D/L_c}$ (green dotted curve). Note that in the latter case the bandwidth of the pseudorandom grating differs from that of the uniform grating of the same strength $\bar{q}$ by a factor $1/\sqrt{DL_c} \approx 1.6$. As seen from Fig. 1 the pseudorandom grating represents a good compromise in terms of bandwidth and variability of the sidelobes between the two uniform gratings with comparable bandwidth and strength, respectively.

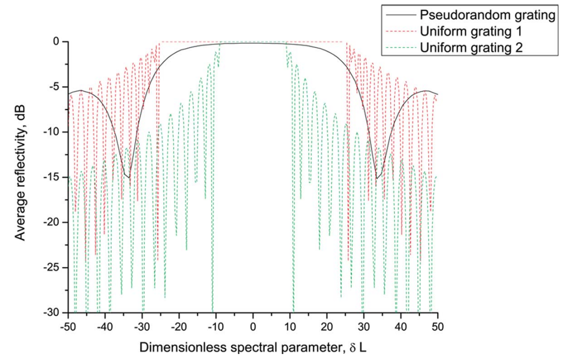

Fig. 1. (Color online) Results of Monte Carlo simulation of quasi-random FBG (solid curve) compared to the uniform grating of the same bandwidth (red dashed curve) and rms strength (green dotted curve).

To sum up, we have suggested relatively simple method of fabricating a flat-top filter by combining L/L_c pseudorandomly modulated segments sampled from normal distribution and showed that in the regime of strong localization (i.e., sufficiently long grating, or stronger rms index modulation) one can achieve exponentially small variability of the spectra inside the bandgap. Outside the bandgap the averaged spectra has fewer sidelobes than the spectrum of a uniform grating of the same bandwidth. We believe that the results of this short Letter show that the considered simple pseudorandom design of FBG filters can be used as a viable alternative to existing techniques relying on inverse scattering transform or layer peeling.

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