PRECISION CRYSTAL OSCILLATOR DESIGN

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DOCTOR OF PHILOSOPHY

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SUMMARY

The University of Aston in Birmingham

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This thesis describes a joint research project between the University of Aston and the Quartz Crystal Unit of STC Components. The project is concerned with the technical innovation of a new temperature compensated quartz crystal oscillator and details the development of the device from initial concept through to the start of volume production. The basis of the project is an integrated circuit which contains the majority of the circuit elements necessary for a temperature compensated crystal oscillator. This device uses a new method of compensation and synthesizes the control voltage of a voltage controlled crystal oscillator by summing voltages which represent the power series components of the control voltage which would give perfect compensation.

The initial work was directed towards an analysis of the company and the market in which it operates, to determine the requirements of a new oscillator and ensure any development was in line with customer needs. The literature was examined and previous methods of temperature compensation evaluated. It was established that the initial concept was a viable approach.

Some voltage controlled oscillators were measured and the control voltage variation with temperature analysed into a sum of the first four polynomials in temperature. From this analysis it was concluded that the voltages to represent these polynomial terms could be generated and summed together with sufficient accuracy. A bipolar integrated circuit was then designed. All component blocks of this circuit including voltage stabilizer, voltage generator, summing amplifier, oscillator circuit and output stages are described. The variation of the output voltages from the three generators are similar in shape to the Chebyshev polynomials $T_1(x)$,$T_2(x)$ and $T_3(x)$. A breadboard model was built and tested and full results of some sample integrated circuits are also given. These oscillators satisfy all aspects of the requirements derived during the market analysis and most significantly have a smaller frequency tolerance, a smaller size and a lower power consumption than oscillators in current production.

Keywords: temperature compensation, crystal oscillator

CONTENTS

LIST OF FIGURES, TABLES AND COMPUTER PROGRAMS

SYMBOLS

ppm	-	parts per million
K	-	unit of temperature (degree kelvin)
ϵ_o	-	absolute permittivity
L_1	-	motional arm inductance
C_1	-	motional arm capacitance
R_1	-	motional arm resistance
C_o	-	static capacitance across crystal
f_s	-	series resonant frequency of a crystal
$T_n(x)$	-	nth Chebyshev polynomial
V_T	-	electron volt equivalent of temperature
		$= \frac{kT}{q}$ where k = Boltzmann's constant T = temperature in degrees kelvin q = charge on an electron in coulombs
I_s	-	reverse saturation current of a transistor base-emitter junction
I_E	-	emitter current of a transistor
r_{po}	-	recombination coefficient for electrons
r_{no}	-	recombination coefficient for holes
DIL	-	dual in line package type
τ_e	-	minority carrier lifetime of electrons
τ_h	-	minority carrier lifetime of holes

ACKNOWLEDGEMENTS

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CHAPTER 1

INTRODUCTION

Quartz crystal oscillators are used where there is a requirement for a signal source of a specific frequency. Quartz crystals are mechanical resonators and their frequency stability greatly exceeds that of any electrical resonators using inductors and capacitors. They are extensively used in radio systems and are a key element in such systems. Indeed, the shape of the quartz crystal industry has been largely determined by the requirements of radio communications.

As technology has developed, the increasing use of radio has led to a better use of the available frequency spectrum. This has called for crystal oscillators with better and better frequency stability. In many cases it is no longer sufficient to use a simple quartz crystal oscillator. The sophisticated communication and navigation systems in use today require oscillators with a higher degree of frequency stability than can be provided by a simple oscillator. The major cause of frequency instability in such oscillators is temperature and precision oscillators may be temperature-controlled or temperature-compensated to reduce this effect. It is the continued development of these precision oscillators which forms the basis of this research project.

1.1 Frequency Stability

The radio frequency spectrum extends from about 15kHz to about 100GHz and over this entire range of frequencies it is essential that the frequency be accurately controlled to avoid interference between users. The transmitter carrier frequency is generated using a resonant circuit and the ability of a resonant circuit to oscillate at an accurately defined frequency depends on (a) the Q factor of the resonator, (b) the temperature coefficients of the frequency-determining components and (c) the ageing of these components.

Conventional LC resonant circuits have Qs limited to a few hundred. They have a long term drift in frequency due to component ageing of 100 parts per million (ppm) per year, and a drift in frequency with temperature of a few tens of ppm/ K. By contrast the Q of a resonator such as a quartz crystal is of the order of 300 000, has typically a long term drift of 2 ppm/year and a drift with temperature of about 1 ppm/K. Resonant circuits using a quartz crystal represent a significant advance in frequency control over other forms of resonant circuit. Historically this advance was first used in 1926 when radio station WEAF in New York became the first station in the world to use a crystal controlled transmitter.

In the 1930s when international comparisons of frequency were first carried out, accuracies of 100 ppm were measured. This improved to 1 ppm during World War II and by 1952 had reduced further to 1 part in 10^8. In the following decades short-term stability improved to a few parts in 10^{10} by 1960, several parts in 10^{12} by 1970 and today is a few parts in 10^{14}. In radio systems, accuracies such as these are not required at present. Most radio systems have requirements of a few ppm for medium-term stability and a few parts in 10^{11} for short-term stability. Quartz crystals are ideally suited for these applications.

1.2 Quartz Crystals

Quartz is a crystalline material and is piezoelectric. This results in a coupling between its mechanical and electrical properties. It is this coupling which enables the properties of mechanical resonance to be utilized in an electrical system. A frequency range of nearly a million to one, from audio frequencies to UHF, is covered by a variety of resonator designs. Various modes of vibration are used including thickness flexure, width flexure, extensional, face shear and thickness shear.

In every design the period of resonance is determined by the time of propagation of an acoustic wave through the resonator element. The most commonly used crystal cut is called the AT-cut which is suitable for oscillators with frequencies between 500kHz and 200MHz. The crystal operates in thickness shear mode where the frequency is proportional to the thickness of the crystal. Most crystals are circular in shape with electrodes of silver or gold evaporated onto the surfaces as shown in figure 1.1. This is called a blank and is typically 7.4mm in size. The blank is mounted in a metal or glass holder for protection. The AT-cut crystal has a small frequency/temperature coefficient and the frequency will drift approximately 50ppm over the temperature band 105^{o}C to -55^{o}C.

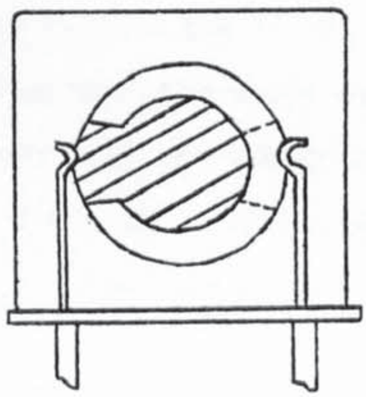

Figure 1.1 - Quartz Crystal

1.3 Crystal Oscillators

Crystal oscillators contain a quartz crystal and an oscillator circuit. They are divided into three categories depending on the tolerance required.

(1) Simple Packaged Crystal Oscillators

These contain a quartz crystal, a simple oscillator circuit and a buffer circuit. They are used mainly as clock oscillators in data processing and microprocessor systems and the frequencies used range from 250kHz to 90 MHz. The frequency tolerance is from 25ppm to 1000ppm over the temperature band 70^{o}C to 0^{o}C.

The package size is approximately equivalent to a 14 pin DIL integrated circuit.

(2) Temperature-Compensated Crystal Oscillator (TCXO)

This is the most common method used to reduce the frequency deviation with temperature. This deviation is dominated by the frequency/temperature coefficient of the crystal. The oscillator maintaining circuit has a freqeuncy/temperature characteristic which is the inverse of the frequency/temperature characteristic of the crystal. As the ambient temperature varies, the resonant frequency of the crystal will change. The electrical characteristics of the oscillator maintaining circuit also change and counteracts the frequency change of the crystal. This compensation is achieved by placing a variable capacitance diode in the oscillator circuit. This d.c. voltage is generated by a temperature dependant network of resistors and thermistors. Present TCXOs are compensated to ± 1ppm over the temperature band $70^{o}C$ to $0^{o}C$ and to ± 3ppm over $90^{o}C$ to $-55^{o}C$. Power consumption is typically 100 mW and package volume approximately 14 cubic centimetres.

(3) Temperature-Controlled Crystal Oscillators (OCXO)

A further increase in accuracy is achieved by using a temperature-controlled quartz crystal oscillator. The crystal is enclosed in an oven in which the temperature is kept constant and this reduces the frequency drift with ambient temperature. Present OCXOs have a tolerance of 0.1ppm over the temperature band $70^{o}C$ to $-20^{o}C$, consume 2.5 W of power typically and have a package volume of approximately 25 cubic centimetres.

1.4 An Improved Design of TCXO

An improved method for the design of a TCXO has been developed and forms the subject of this research project. Most of the work was carried out at the Quartz Crystal Unit of STC Components in Harlow, Essex.

STC Components are a major division within the framework of Standard Telephone and Cables and within STC Components, the Quartz Crystal Unit forms a specialized section supplying a major part of the quartz crystal products to the electronics industry in the UK and Europe. The work was also carried out as a project within the Total Technology option of the Interdisciplinary Higher Degrees Scheme at the University of Aston. This scheme promotes research projects of an applied character in real problems of design, development and manufacture which are encountered in industrial organisations.

Although there are three categories of crystal oscillator, only OCXOs and TCXOs are of interest in the field of precision oscillators. OCXOs have limited applications in radio communications and only form a small part of the overall production. The majority of applications require a TCXO. However, in recent years changes have occurred in radio communications which have required changes in the design of a TCXO. These may be summarised as:

(a) A reduction in the channel spacing due to the increased use of the frequency spectrum which has required oscillators of a higher stability.

(b) A reduction in the size of the associated equipment by the use of integrated circuits which has required oscillators of a smaller size.

(c) This size reduction has made lightweight manpack radio sets available and since these are battery powered, oscillators with lower power consumption are required.

The existing TCXOs in the present product line are capable of satisfying present customer demands, but it is more and more apparent that changes are needed to satisfy future customer needs.

The three trends which are summarised above can be clearly seen from a study of the specifications of oscillators for new equipment. These trends also appear likely to continue.

The technology used in the design and manufacture of TCXOs at present uses discrete components mounted on a printed circuit board. The compensation system using resistor-thermistor networks has remained almost unaltered since STC entered the market in 1969 and it is not capable of being improved. The key element in a TCXO is the compensation system, and the higher degree of frequency stability needed in the future requires a more complex system of compensation. This will require the design of an integrated circuit containing both the compensation system and the oscillator. The integrated circuit will be part of a hybrid thick-film module which, together with a quartz crystal, would form a complete TCXO. It is the objective of this research project to produce a TCXO using this approach.

The exact requirements of the oscillator must be determined by customer needs. A market orientation is essential since there is no standard product when considering TCXOs and every customer has his own special requirements. These requirements will be analyzed during an evaluation of market research information and used to produce a product specification. The initial work is therefore directed toward an analysis of the market place.

CHAPTER 2

COMMERCIAL BACKGROUND

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CHAPTER 2

COMMERCIAL BACKGROUND

2.1 STC Quartz Crystal Unit

Engineers in every aspect of engineering are involved in the commercial implications of their decisions. Products must be designed, developed, manufactured and sold in the face of severe competition. Hence, in order to conduct his work in an effective and profitable manner, the engineer must be keenly aware of his company's market position and viability.

Standard Telephone and Cables is split into four main business sections. These are business systems, marine and cable, switching and transmission, and components. STC Components has factories at nine locations within the United Kingdom and manufactures a variety of components ranging from thermistors and capacitors to fibre optics and liquid crystals. The Quartz Crystal Unit at Harlow is one of three factories within the Micro Devices Division. The unit has been established as a major supplier of quartz crystal products for many years and supplies products in three main groups:

(i) quartz crystals - these are loose crystal blanks suitably encapsulated which are used as components in circuits such as oscillators.

(ii) packaged oscillators- the crystal blanks are used with other components to form a packaged oscillator unit such as a DIL oscillator, a temperature compensated oscillator (TCXO) or a temperature controlled oscillator (OCXO).

(iii) filters - these are supplied as filter units with a specific response containing from 1 to 5 crystals with terminating impedances.

The Unit grows its own quartz bars from which the crystal blanks are produced. The Optical Shop in the factory processes these bars and cuts and laps the crystal blanks to the correct diameter and thickness. The Electrical Shop takes these blanks and plates electrodes onto both sides of the blank which is then mounted in a suitable holder. These units may be sold as loose crystals or used as components by the Oscillator and Filter shops in the production of oscillators and filters. Each section of the factory deals with work in batch system since the variety of product is large.
The specific business area which the unit serves is the low-to-medium volume, high technology area of crystal products. The product range of oscillators can be divided into custom products normally of high precision and specification (TCXO), low volume and high value (OCXO), and commodity products of low specification and high volume (DIL oscillator). The unit does not compete in the high volume, low cost crystal products such as TV crystals for consumer and commercial applications.

The business objective of the unit is to design, market and manufacture electronic components based on crystal technology serving sectors of the market in which a greater premium is placed on technological skills. In 1982 the unit had a revenue of £6.1M. £1.3M was derived from TCXOs and represents 21% of the total revenue.

2.2 Market Structure

STC primarily serves the military and professional radio communications market segments with the industrial market an important third segment. The market for TCXOs is a stable and durable one and the total market size is approximately £20M in Western Europe and a further £20M worldwide. Consumption in the UK is around £5M of which STC has a 25% share. The two main competitors in the UK are SEI and Cathodeon with shares of 25% and 5% respectively.

The rest of the UK market is served by foreign manufacturers such as CEPE of France, Oscilloquartz of Switzerland, Toyocom of Japan and Bulova and McCoy of the USA. The total market size is limited by the requirements of the major OEMs (original equipment manufacturer) since TCXOs are only one component within a system which is then sold to another customer. The large share of the market held by imported products is related to the technical merits of foreign products and the need for having more than one supplier. Products from the USA have been traditionally state-of-the-art products due to the large involvement of the military and space related activities in the USA electronics industry. The main world markets in geographical terms are Western Europe and the USA and the particular export markets for STC are France, Italy, Germany, Sweden and the USA. The main growth opportunities for STC lie in supplying high technology products imported from France and USA.

2.3 Product Range and Profile

TCXOs are used in applications requiring a stable frequency source. The uses are diverse but they are mainly used in radio communication systems. The product has remained similar in appearance for a number of years. This is due to the frequency tolerance specification remaining static. Competitor's products are similar to those of STC and are manufactured in a similar manner. The only significant difference in the technology used is the use by some companies of metal can crystals as opposed to the glass holder crystals used by STC. Metal cans are cheaper but are dirtier giving inferior ageing performance when compared to glass holders. Glass holder crystals can also be laser-adjusted to the correct frequency eliminating the need for select-on-test components in the oscillator circuit.

The profit margin on a TCXO has always been high and the market is insensitive to the price of the product. Quality and specification are of primary importance. Although STC prices are in general higher than most competitors, the company has a reputation for high quality which offsets the price difference. The high margin is a factor in favour of new competitors entering the market but the need for high capital investment to start manufacture acts as a disadvantage. The present product profile for oscillators covering the most widely used temperature range of 85°C to -40°C is shown in figure 2.1.

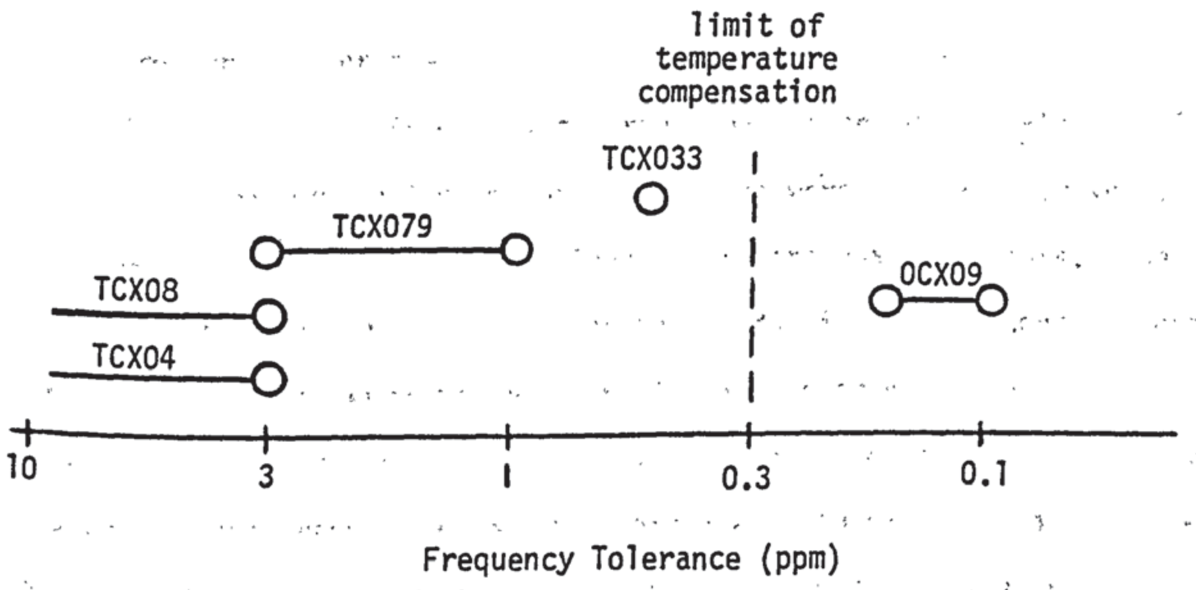


Figure 2.1 - Product Profile

The TCX04 and TCX08 are the "bread and butter" products. There are many variations of these oscillators and they represent approximately 75% of the total production. A TCX079 is a low power, low profile oscillator which can meet ±1ppm frequency tolerance. These units generally have to be selected from a large batch which meets ±3ppm tolerance. The TCX033 is a large size TCXO which meets ±0.5ppm tolerance. These units are difficult to make and give severe manufacturing problems. They use the same system of compensation as all other TCXOs and it is difficult to meet the ±0.5ppm frequency tolerance using this system of compensation. ±0.3ppm is the limit of temperature compensation at the present time and for frequency tolerances equal to or less than this figure, OCXOs have to be used.

2.4 New Product Requirements

There is no standard product when considering TCXOs and there are many variations. Indeed within the range of TCXO4 oscillators there are 45 variations. The normal method of selling is by direct representation to customers. The unit has always been seen as a product specialist and most orders come from specific customer requirements for specific systems. An examination of the specifications received by the company for new products over the last year shows several clear trends in the areas of:

(i) Frequency Tolerance

Figures for the frequency tolerance have changed from ±5ppm applicable 5 and 10 years ago to figures of ±2ppm which were typical last year. This seems likely to reduce to ±1ppm during the next few years. The major change in the user industry causing this reduction in frequency tolerance is the use of frequency synthesizers in radio communications. The most advanced communication systems use synthesizers and many are frequency hopping. These require a precision oscillator to set the channel spacing. The demand for this type of product should increase over the next few years due to the increased use of VHF/UHF communication sets using synthesis and single sideband transmission. The expansion of cellular radio is seen as providing a large stimulus for this.

(ii) Size

The demand for packaged oscillators is increasing generally and with the size reductions being made in systems through the use of integrated circuits it is inevitable that the size of packaged oscillators will decrease. Mobile radio sets in many cases have to be portable and often the TCXO is the largest component in a receiver. The size of a TCXO has become an important parameter with emphasis being placed on low profile devices to reduce the PCB board spacing required in complex systems.

Oscillators for hand-portable radio systems require smaller length and width dimensions although these are not as important in manpack military radio systems.

(iii)Power Consumption

As the size of components has reduced so also has the power consumption. In radio systems the requirement of a portable receiver is an important consideration and since this is battery powered there is a requirement for an oscillator of low power consumption.

2.5 New Product Specification

It is clear that a major change is required in the product range of TCXOs manufactured by STC to enable the company to maintain its position in the market. The requirement of TCXOs in the next five years will be for oscillators with a tighter frequency tolerance, a smaller size and a lower power consumption compared with those manufactured today. The actual specification of a new product cannot be fully defined since each product is different.

What can be stated is that there is a need for the ability to compensate an oscillator to within $\pm$1ppm over the temperature range of 85^{o}C to -40^{o}C. The size of the oscillator will need to be the equivalent to the TCXO79 which is the smallest oscillator manufactured at the present time or smaller. The current consumption of the oscillator is required to be less than 1mA so that the total current consumption of the compensated oscillator and output stage is only a few milliamperes.

The sales forecasts for units which conform in general to this specification are shown in Table 2.1 . They were obtained from the Marketing Department of Quartz Crystal Unit.

Year	Quantity	STC Share	Total Sales Value (£K)	STC Share
1984	50 000	9000	2200	440
1985	60 000	9000	2700	420
1986	60 000	9000	2700	430
1987	60 000	9000	2700	440

Table 2.1 - Sales Forecast

In 1982 STC made a total of 32000 oscillators and so the market for this new oscillator represents a substantial amount of business to the unit. As stated earlier, the unit receives orders for specific devices to be used in specific projects. Key customers can be identified and their future needs identified. The major customers identified at the present time are given in Table 2.2.

The requirements of each oscillator have been identified in terms of the three parameters: frequency tolerance, size and current consumption. It is seen that although each project has a different set of requirements, if a system can be developed which meets the specification identified earlier then it could be used to develop a suitable product for each of these key customers.

Project	Potential Quantity 1984	1985	Future	Frequency Tolerance		Size	Current Consumption
SINCGARS	15000	15000	15000 in 1986	± 3 ppm	$95^{o}C$ to $-55^{o}C$	TCX079	5 mA
SCIMITAR	3000	5000	5000/year until 1994	± 1 ppm	$85^{o}C$ to $-40^{o}C$	-	2.5mA
RAVEN	100	1000	1000/year until 1987	± 1 ppm	$70^{o}C$ to $-40^{o}C$	-	-
SYSTEM 4000	-	1000	?	± 1.5ppm	$70^{o}C$ to $-30^{o}C$	smaller than TCX079	1.5mA
PTR 420	1000	3000	?	± 1 ppm	$70^{o}C$ to $-40^{o}C$	-	-
EAS AVTX/RX	200	600	?	± 1 ppm	$105^{o}C$ to $-55^{o}C$	-	-
BATES	400	400	?	± 3 ppm	$125^{o}C$ to $-55^{o}C$	TCX079	-
MEL MOBILE	-	2000	?	± 0.5ppm	$75^{o}C$ to $-35^{o}C$	smaller than TCX079	8 mA
ELMER AVTX/RX	300	400	?	± 1 ppm	$70^{o}C$ to $-40^{o}C$	TCX079	-
JAGUAR U	500	500	?	± 1 ppm	$85^{o}C$ to $-40^{o}C$	TCX079	5 mA

Table 2.2 - Key Customers

CHAPTER 3

QUARTZ CRYSTALS AND OSCILLATORS

3.1 Piezoelectricity

Piezoelectricity is a phenomenon which occurs in certain crystals and results in a connection between the electrical and mechanical properties of the crystal. The word piezoelectricity literally means pressure electricity. It is defined as "the electric polarisation produced by mechanical strain in crystals belonging to certain classes, the polarisation being proportional to the strain and changing direction with it." The production of an electric polarisation by mechanically inducing a strain in a crystal is called the direct piezoelectric effect. The converse effect whereby a mechanical strain is produced in a crystal by a polarising electric field also exists. Both these effects were first demonstrated by Jacques and Pierre Curie in the early 1880s. This work was developed mathematically by Voigt who related the piezoelectric effect to the atomic structure of the crystal. The first engineering application was in ultrasonics. Langevin used a steel and quartz sandwich structure to generate high frequency sound waves in water for detecting submerged objects such as submarines. At this early stage, the existence of piezoelectricity was predicted and observed in materials other than quartz. However, most of these crystals were much less stable than quartz and none was as well understood.

The origin of piezoelectricity is seen by considering the charge distribution in a 2-dimensional model. Suppose six charges are located at the corners of a regular hexagon as shown in figure 3.1. If the potential at a point P is calculated, where OP=r and r>>a, then the contribution due to q_1 is :

$$V_{q1} = \frac{4\pi\varepsilon_0 q}{(r-a/2)}$$

The contributions from q_2 and q_3 are each given by :

$$V_{q2}, V_{q3} = \frac{4\pi\varepsilon_0 q}{(r+a/4)}$$

The potential at P due to the three positive charges is :

$$V+ = 4\pi\varepsilon_0(q/(r-a/2) + 2q/(r+a/4))$$

$$= 4\pi\varepsilon_0 \frac{3q}{r} \quad \text{neglecting a term } a^2/8$$

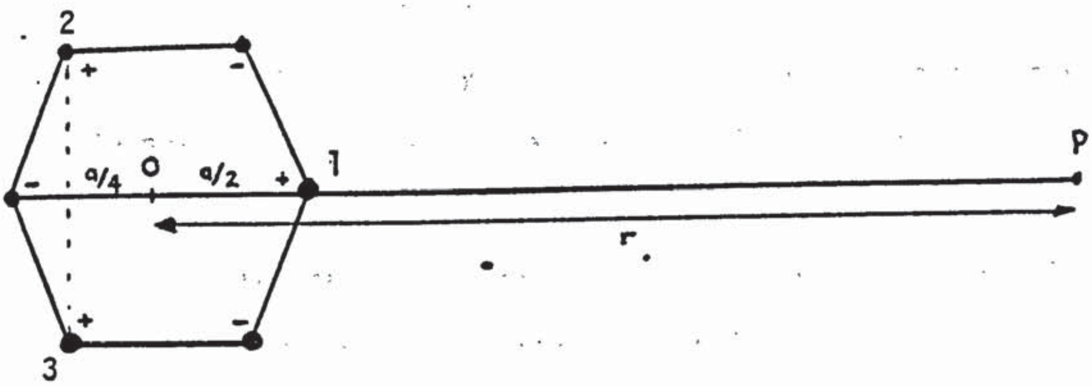


Figure 3.1 - Potential of six charges

This is exactly the potential due to a charge of +3q located at O. The potential and also the field intensity at a distant point for the three charges is the same as for three coincident charges at the "centre of gravity". The same result holds for the three negative charges, so the potential and field at P are zero because the contributions cancel.

If a force is applied as shown in figure 3.2 causing the system of charges to be distorted, the centre of gravity of the three positive charges moves to the left and that of the three negative charges moves to the right. A dipole has been created, and in a crystal having a large number of these molecules a surface charge on the external faces of the crystal will result. A reversal of the force causes a reversal of the dipoles and a charge reversal on the faces. A shear force also causes a dipole to be formed (figure 3.3).

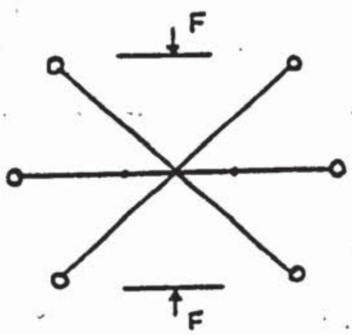


Figure 3.2
Dipole formed by tensile force

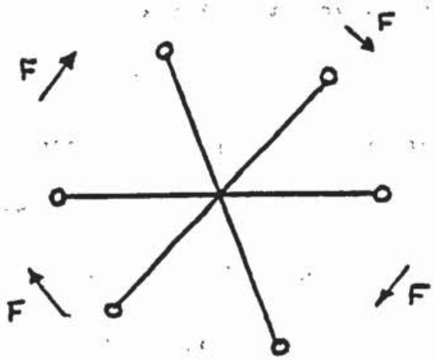


Figure 3.3
Dipole formed by shear force

3.2 Piezoelectric materials and equations [1]

Of the thirty-two different crystal classes, twenty are piezoelectric. Quartz is often considered to be the archetypal piezoelectric material although a great many different materials are used today. Other common materials are lithium niobate and barium titanate. Piezoelectricity is not a particularly rare phenomenon, but in practice few materials are very useful and quartz is almost the only material having the necessary combination of mechanical, electrical, chemical and thermal properties for making piezoelectric elements for the electrical communication field.

The relationship between the electrical and mechanical properties may be written as :

$$D = dT + \varepsilon^T E$$

$$S = s^E T + d^1 E$$

where E = electric field intensity

D = electric displacement

S = strain

T = stress

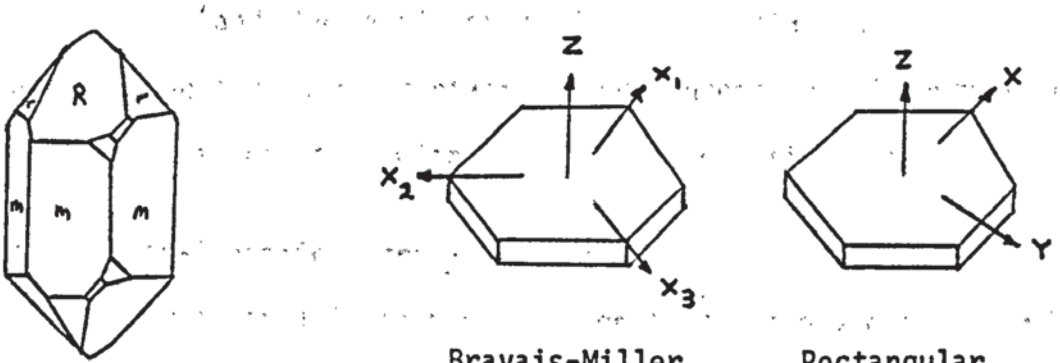
ε^T = permittivity at constant stress

s^E = compliance at constant electric field

d , d^1 = piezoelectric coefficients

There are a further six alternative equations which may be used involving the stiffness c (= 1/compliance) and the piezoelectric constants g (piezoelectric voltage constant), e (piezoelectric stress constant) and h (h-constant) to link E,D,S and T. Since a piezoelectric material has no centre of symmetry the properties of the material are anisotropic and the parameters ε,d,s etc. are dependent on the direction of measurement. To describe the anisotropic properties of quartz the conventional system of Bravais-Miller axes may be used. Quartz has trigonal symmetry about the Z-axis as shown in figure 3.4. It is sometimes easier to use a set of orthogonal axes and this system will be used here.

The principal axis is the Z-axis. The X-axis is the only polar axis in quartz and a polarisation is produced in this direction if the crystal is subject to a mechanical strain in the same direction. The Y-axis is not a polar axis and no polarisation occurs due to an extensional strain in the Y-direction. However, a face shear applied about the Y-axis does result in a electric polarisation in the Y-direction and this is used in a number of important types of quartz resonator.



Quartz Crystallography

Bravais-Miller axes

Rectangular axes

Figure 3.4 - Crystal Axes

3.2.1 Stress and Strain Relationship

In any general stress and strain relationship, the most general strain can be resolved into a linear combination of an extensional strain and a shear strain. The extensional strain resolves into three orthogonal components and the shear strain into three shear components about those orthogonal axes. To describe the stress-strain relationship of a crystalline solid any component of stress is considered capable of producing any component of strain. In the equation $T = cS$ (stress=stiffness x strain) c is a 6x6 matrix relating the six components of stress to the six components of strain. The number of coefficients is reduced by the principle of conservation of energy which shows that $c_{ij} = c_{ji}$ giving only 21 independant coefficients in the most general case.

3.2.2 Electric Displacement and Electric Field Relationship

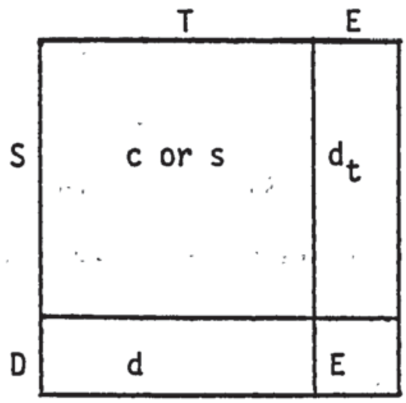
The electric field and the electric displacement are related by the equation $D = \varepsilon E$. ε is a 3x3 matrix relating the components along the orthogonal axes. Energy considerations show that ε is symmetrical i.e. $\varepsilon_{21} = \varepsilon_{12}$, $\varepsilon_{31} = \varepsilon_{13}$ and $\varepsilon_{32} = \varepsilon_{23}$. The maximum number of independent coefficients is reduced to six.

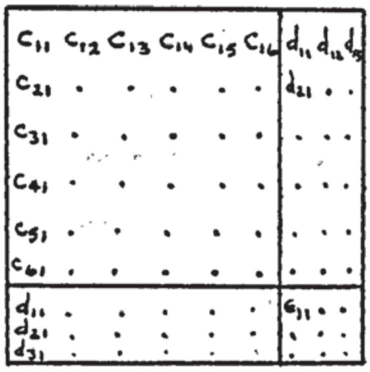
3.2.3 Interaction between Electrical and Mechanical Properties

An electric field is capable of producing both extensional and shear strains and the matrix of coefficients relating the electric field and the strain may contain 18 independent coefficients (6 elastic coefficients interacting with 3 dielectric coefficients).

3.3 Elasto-Electric Matrix for Quartz [2]

To summarise all the physical constants involved in describing a piezoelectric material, an elasto-electric matrix or domino table is a convenient form to use. The domino table gives all the coefficients required in the piezoelectric equations and has the form :





where d_t = transpose of the d matrix

For quartz

$$c = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & 0 & 0 \\ c_{12} & c_{11} & c_{13} & -c_{14} & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ c_{14} & -c_{14} & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 2c_{14} \\ 0 & 0 & 0 & 0 & 2c_{14} & 2(c_{11}-c_{12}) \end{bmatrix}$$

The s matrix has the same independent coefficients as the c matrix and s can be substituted for c in this matrix.

$$d = \begin{bmatrix} d_{11} & 0 & 0 \\ -d_{11} & 0 & 0 \\ 0 & 0 & 0 \\ d_{14} & 0 & 0 \\ 0 & -d_{14} & 0 \\ 0 & -2d_{11} & 0 \end{bmatrix}$$

$$\varepsilon = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix}$$

All these constants have been measured and recorded.

3.4 Vibrating Plates [3]

Quartz resonators are examples of mechanical resonators. Mechanical vibrations can be excited in almost any solid elastic material and the piezoelectric effect provides a simple and convenient method of exciting these vibrations. It is possible to set a solid body vibrating in many different modes and it will have many resonant frequencies.

3.4.1 One - dimensional vibrating systems

The simplest case of a one dimensional vibrating system is a long thin bar. The resonant frequencies are found by solving the wave equation :

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2}$$

where u = displacement of a point on the bar

v = velocity of wave propagation

x, t = space, time co-ordinates

By imposing boundary conditions such as the midpoint of the bar being a node, it can be shown that ;

frequency of vibration $f_n = \frac{nv}{2L}$ $\quad n = 1,3,5 \ldots$

L = length of bar

If the length of the bar is large compared with its width and thickness this equation is a very good approximation to the actual resonant frequencies. In quartz, thickness-flexural and length-extensional mode vibrations are used in low frequency crystals (figure 3.5). Table 3.1 summarises these modes of vibration.

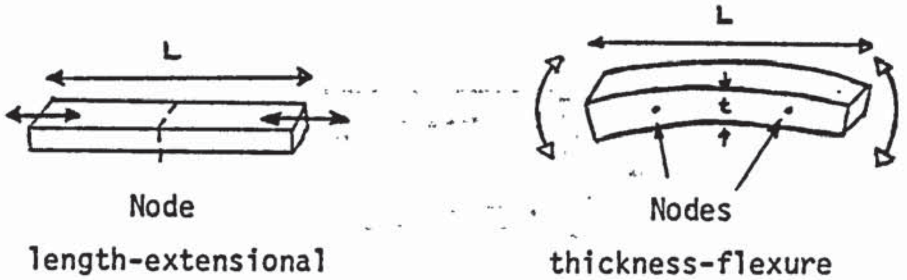


Figure 3.5 - One-dimensional vibrations

Mode	Cut	Frequency Range
length-width flexure	NT +5° X XY	4 → 100 kHz 10 → 50 kHz 1 → 35 kHz
length extensional	MT +5° X	50 → 500 kHz 60 → 250 kHz
thickness extensional	X	350 kHz → 20 MHz

Table 3.1 - Crystal Cuts (1-D)

These modes of vibration are due to the coefficients d_{11} and d_{12} in the piezoelectric coefficient matrix d whereby a field in the X direction causes longitudinal strain in the X and Y directions.

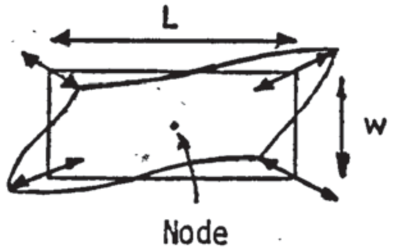
3.4.2 Two-dimensional vibrating systems

The CT-cut and DT-cut family of resonators consist of either a square or a circular plate of quartz vibrating in a face shear mode. The centre of the plate is a nodal point. Such resonators approximate to two-dimensional vibrating systems. The resonant frequencies are found by solving the wave equation in two dimensions and applying the appropriate boundary conditions.

$$\text{frequency of vibration } f_{nm} = \frac{v}{2}\left(\frac{n^2}{a^2} + \frac{m^2}{b^2}\right)^{\frac{1}{2}}$$

n,m = 1,3,5 ...

a,b = plate dimensions



face shear

Figure 3.6 - Two-dimensional vibrations

Note that for circular plates involving polar coordinates, Bessel's functions are required in the solution. Two-dimensional systems have many more modes of vibration which are not harmonically related as in a one-dimensional system. Table 3.2 shows the relative frequencies of various modes for a plate of length 3 units and width 2 units.

Mode	Relative Frequency
11	0.601
31	1.118
13	1.536
33	1.803
55	3.005

Table 3.2 - Frequency comparison

Table 3.3 summarises the two face shear cuts CT and DT. Figure 3.6 shows the mode of vibration and figure 3.7 the orientation of such plates with regard to the crystal axes.

Mode	Cut	Frequency Range
face	CT	250 kHz → 1 MHz
shear	DT	60 kHz → 500 MHz

Table 3.3 - Crystal Cuts (2-D)

These modes of vibration exist in quartz due to the coefficients d_{14} and d_{25} of the piezoelectric matrix d which relates fields in the X and Y directions to shear strains around the X and Y axes.

Figure 3.7 - Crystal Cuts

3.4.3 Three-dimensional vibrating systems

Three-dimensional vibrating systems are characterised by thickness shear vibrations (figure 3.8). The resonant frequencies can be found by solving the wave equation in three dimensions

$$\frac{\partial^2 u}{\partial t^2} = v^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\text{or} \qquad \frac{\partial^2 u}{\partial t^2} = v^2 \, \frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial \phi^2} \quad \text{in polar co-ordinates}$$

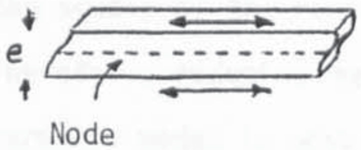
The frequencies of vibration are given by

$$f_{nmp} = \frac{v}{2}\left(\frac{n^2}{e^2} + \frac{m^2}{l^2} + \frac{p^2}{w^2}\right)^{\frac{1}{2}} \qquad n,m,p = 1,3,5....$$

e = thickness

l = length

w = width



thickness shear

Figure 3.8 - Three-dimensional vibrations

If the thickness is very small compared to the length and the width then

$$f = \frac{nv}{2e}$$

The overtone frequencies of an infinite plate would therefore be odd integer multiples of the fundamental. However, plates of a finite size have a large number of inharmonic modes (spurious modes) which depend on the dimensions of the plate. A typical frequency spectrum is shown in figure 3.9.

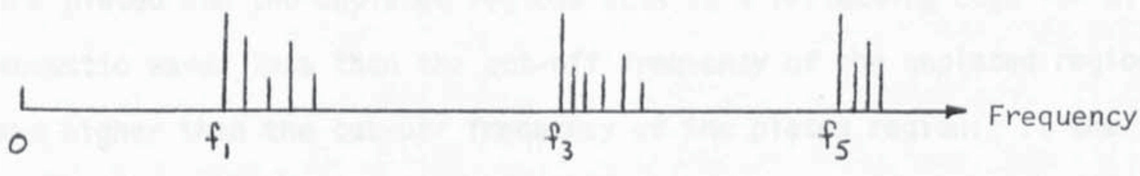


Figure 3.9 - Frequency spectrum

When three dimensional circular resonating systems are considered the solutions again involve Bessel functions. By idealising boundary conditions the frequencies are approximately given by

$$f_n = \frac{v}{2}\left[\frac{n^2\ \pi^2}{e^2} + \frac{\Psi^2 mk}{a^2}\right]^{\frac{1}{2}}$$

where a = radius of the plate

Ψmk = k^{th} root of Bessel funtion of order m

The overtone spectrum still contains many inharmonic modes but their effect is reduced by :

1. Contouring

The flat plate may have one or both sides contoured to give the major surfaces a spherical shape. The equation shows that the frequency is proportional to the square of the radius i.e. the area of the vibrating region. Therefore, reducing the size of the vibrating area should cause the inharmonic modes to move to higher frequencies and allow the overtone spectrum to become more dispersed. This method is commonly used on thick low frequency plates.

2. Mode Trapping

Waveguides have a cut-off frequency below which waves will not propagate. This cut-off frequency depends on the waveguide dimensions. A similar case exists with resonators. For a given thickness of resonator there is a frequency (the fundamental) below which resonance is not possible. If the resonant plate has electrodes deposited only in the centre of the element then the unplated region will have a higher cut-off frequency than the plated region. The boundary between the plated and the unplated regions acts as a reflecting edge for all acoustic waves less than the cut-off frequency of the unplated region and higher than the cut-off frequency of the plated region. If the cut-off frequency of the unplated region is less than the spurious mode of lowest frequency then the plated region can be excited into resonance only at its own fundamental frequency.

By controlling the thickness of the electrodes these spurious modes can be eliminated.

Table 3.4 summarises thickness shear cuts.

Mode	Cut	Frequency Range
thickness shear	AT	500 kHz → 350 MHz
	BT	1 kHz → 75 MHz
	Y	500 kHz → 25 MHz

Table 3.4 - Crystal Cuts (3-D)

The cuts given in tables 3.1, 3.2, 3.3 and 3.4 are not a complete list of all cuts but are representatives of those in common use. Above 1 MHz AT-cut crystals are almost universally used. Circular plates are most common, since in these designs the inharmonic modes can be suppressed more easily than in square plates.

One major class of cut gaining importance is the double rotated cut. From figure 3.7 it is seen that most cuts are single rotated and involve a single rotation about the X or Y axis. They present no problems of accurately defining the orientation of the crystal in large scale manufacture. Double rotated cuts involve a rotation about both X and Y axes and although they have a smaller frequency deviation with temperature, the problems presented in cutting a quartz element from a large quartz block have impeded the use of these cuts at the present time.

3.5 Frequency/temperature coefficient of a quartz crystal

The main reason for the large number of crystal cuts and the main factor in deciding which cut to use is the frequency/temperature coefficient of the cut. The most important factor in designing precision oscillators is the minimization of the effect of temperature. In the 1920-1930s when crystal cuts such as the X-and Y-cut were used, large frequency drifts with temperature were encountered. A Y-cut crystal has a frequency/temperature coefficient of 100 ppm/K.

By rotating the Y-cut crystal about the X-axis two positions where the frequency is almost independent of temperature are found. These positions are the AT- and BT-cuts. Most cuts have a parabolic frequency deviation with temperature but the AT-cut is unusual in having a cubic variation with temperature. The frequency of a resonator depends on its thickness, its density and its elastic constants and since these all change with temperature, the frequency becomes temperature dependent. The frequency/temperature coefficient itself is temperature dependent, since the constants which are involved in determining the coefficient, are temperature dependent. Figure 3.10 shows the frequency deviation versus temperature for several cuts.

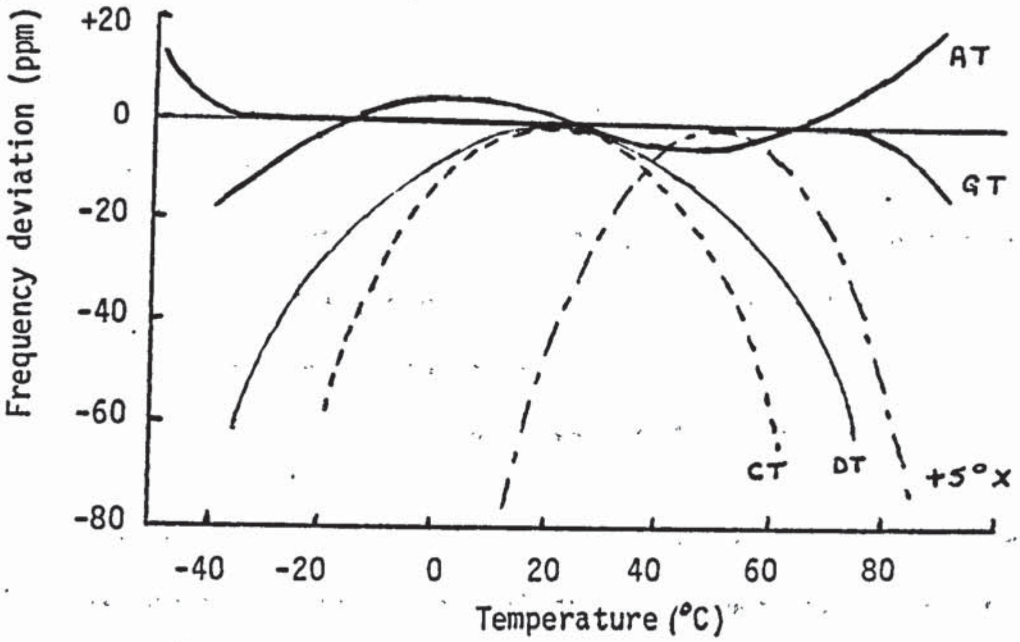


Figure 3.10 - Frequency/temperature coefficients

The small frequency deviation of the AT-cut crystal has led to its adoption as the preferred cut. Although the GT-cut crystal has an even smaller frequency deviation it is a double rotated cut with a limited frequency range of 100 to 500 kHz.

Since the AT-cut is found by rotating a Y-cut plate about the Z-axis, its frequency/temperature coefficient depends on the angle of rotation. The angle at which the coefficient is zero at 26°C depends also on the frequency, harmonic used, diameter/thickness ratio, electrode size and contour. It has never been possible to define any differences in the shape of the curves of frequency deviation versus temperature as a function of the shape, frequency, harmonic or of any other variable. Consequently one set of curves can be employed to cover every situation. There are no changes in the shape of the curves but a different reference angle is used for the interpretation of the curves (figure 3.11).[4]

The curves are cubic in nature and can be described by a third order polynomial :

$$\frac{\Delta f}{f} = a(t-t_i) + b(t-t_i)^2 + c(t-t_i)^3$$

where

$$a = -0.08583\ (\theta-\theta_i)$$
$$b = 0.39 \times 10^{-3} - 0.07833\ (\theta-\theta_i) \times 10^{-3}$$
$$c = 109.5 \times 10^{-6} - 0.033\ (\theta-\theta_i) \times 10^{-6}$$

θ = angle of cut

θ_i = reference angle i.e. zero temperature coefficient angle

$(\theta-\theta_i)$ = value assigned to curve in figure 3.11

t = ambient temperature

t_i = inflection temperature (26°C)

The values of the constants in the equation were found from measurements of actual crystal units. Crystal design is very empirical in nature and many of the design rules are based solely on measured data. The reference angle varies greatly and depends on many factors whose effect cannot be calculated.

The variation of frequency with temperature does not always occur in a smooth fashion as indicated by the curves in figure 3.11. One problem which is encountered, is a crystal which jumps from one frequency to another.

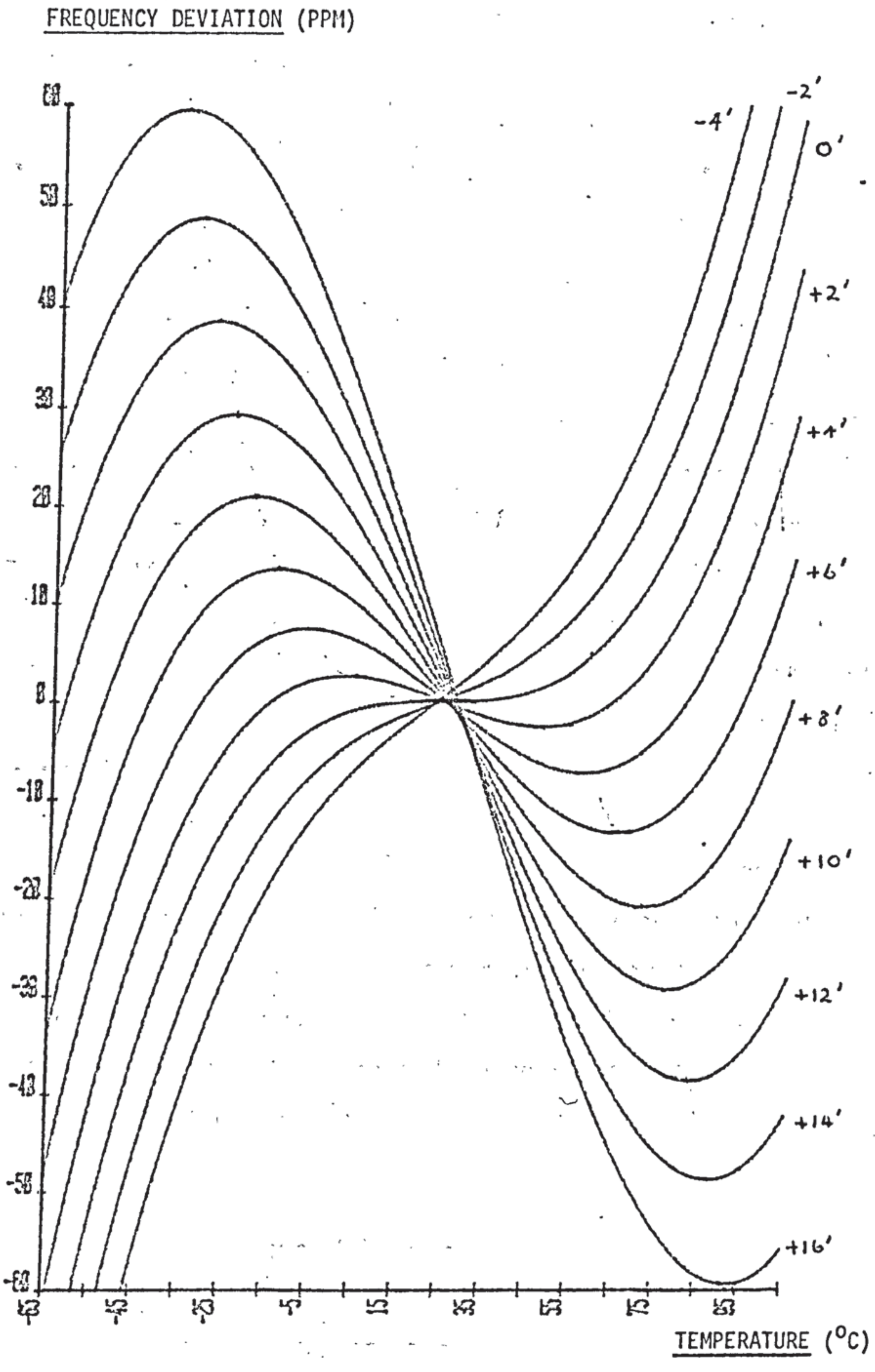


Figure 3.11 - Frequency/Temperature curves.

All vibrating modes do not have the same frequency/temperature coefficients and at some temperatures spurious modes may have a stronger response than the desired mode and the frequency jumps. Degradation of the main response is usually a more common effect and results in an activity dip. Coupling to other modes either elastically or electrically results in a drop in the activity of the main response giving an abrupt break in the smooth frequency/temperature characteristics. The dips occur at specific temperatures and are referred to as bandbreaks.

Coupling by electrical excitation is possible due to the coefficients in the piezoelectric strain matrix d. For an electric field in the Y-direction, coefficient d_{25} produces a shear strain in the XZ plane. If the electric field varies at the correct frequency the plate may be set into vibration in one of its many face shear modes. Similarly for mechanical coupling, the coefficient C_{56} in the stiffness matrix c couples a face shear mode to the thickness shear mode in the AT- and BT-cut plates.

3.6 Equivalent Circuit of an AT-cut quartz crystal

Equivalent circuit representations of crystal oscillators serve to translate certain properties of the piezoelectric elastic body into electrical network terminology. The simplest representation is the Butterworth-Van Dyke equivalent circuit[5] as shown in figure 3.12. It is valid only in the neighbourhood of a single mode resonance and is therefore applicable in crystal oscillator design.

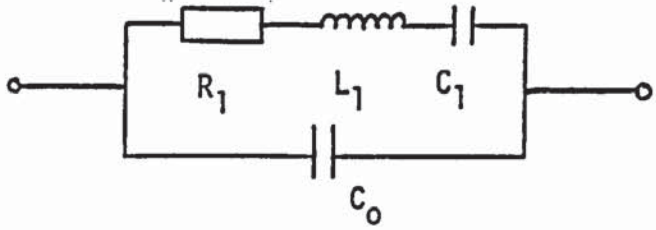


Figure 3.12 - Equivalent Circuit of a Crystal

It was first observed by Butterworth that any mechanical vibrating device driven by a periodic voltage across a capacitor appears to the driving source as though it consisted of a circuit as in figure 3.12. The inductance L_1 is a measure of the vibrating mass, the capacitance C_1 is a measure of the spring constant (i.e. the effective elastic constant) and the resistance R_1 is a measure of the energy that is converted into heat. Accordingly the crystal can be thought of as a lumped mass, ideal spring and dashpot arrangement. C_0 is the electrical capacitance of the electrodes and holder. Coupling to other modes may be simulated by the addition of other LCR arms to the model. The values of L, C and R (the motional parameters) are such that a discrete circuit with these values would be difficult if not impossibe to build.

The motional parameters are easily and accurately measured and the range of values for L_1, C_1, R_1 and C_0 are given in table 3.5.

Mode	Frequency (MHz)	L_1 (mH)	C_1 (fF)	R_1 (ohm)	C_0 (pF)
fundamental	1	1000→4000	6→25	100→2000	1.5→7
	10	3→30	7→70	10→100	2.0→4
	20	2→10	6→30	10→60	2.0→11
3rd overtone	20	50→200	0.3→1.2	15→80	2.0→7
	40	6→25	0.6→2.6	15→80	2→7
	60	2→7	1→4	15→100	2→7
5th overtone	60	8→30	0.2→0.8	20→100	2→7
	80	4→14	0.3→1.0	20→100	2→7
	100	2→7	0.4→1.4	20→100	2→7

Table 3.5 - Motional Parameters

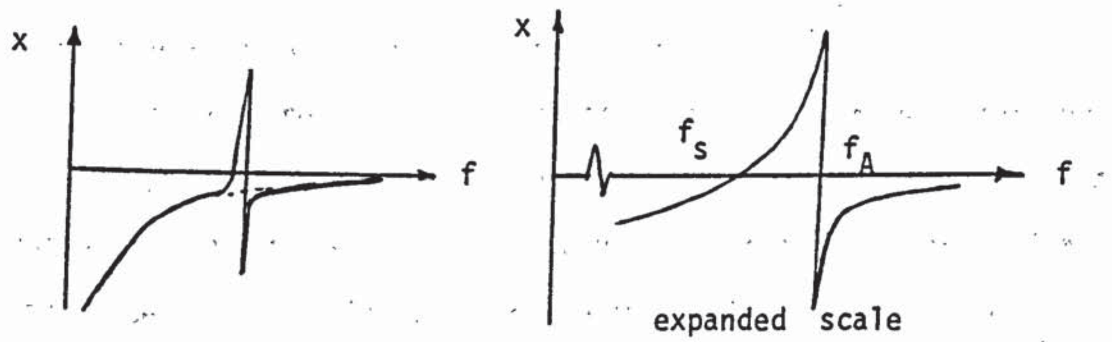


Figure 3.13 - Reactance of a crystal

If the reactance of the crystal unit is plotted as a function of frequency several important frequencies are noted (figure 3.13).

(1) Series Resonance f_s

This is the frequency at which the crystal is series resonant i.e. L_1 and C_1 form a series resonant circuit. The frequency of oscillation is given by $f_s = \frac{1}{2\pi\sqrt{L_1C_1}}$

The impedance of the crystal is given by R_1 in parallel with C_o.

(2) Parallel Resonance f_A

This is the frequency at which the crystal is slightly inductive and forms a parallel resonant circuit with C_o. The frequency of oscillation is given by $f_A = \frac{1}{2\pi}\sqrt{\frac{1}{L_1C_1} + \frac{1}{L_1C_o}}$

$$f_A = \frac{1}{2\pi\sqrt{L_1C}} \qquad \text{where } C = \frac{C_oC_1}{C_o+C_1}$$

The impedance of the crystal is given by $\frac{L_1}{C\ R_1}$

Due to the extremely high Q of the crystal the difference between the series resonant frequency f_s and the anti-resonant (parallel) resonant frequency f_A is extremely small and is of the order of a few tenths of a percent. In the derivation of crystal equations, frequencies other than f_s or f_A are calculated. Often these frequencies are the frequencies of zero phase when the crystal appears resistive.

There are also two other frequencies which can be calculated. These are where the impedance of the crystal is a minimum and a maximum.

In practice the series resonant frequency, the frequency of zero phase with the resistance of a low value, and the frequency of minimum impedance are very close. Likewise the anti-resonant frequency, the frequency of zero phase with the resistance of a high value, and the frequency where the crystal impedance is a maximum are very close. For an AT-cut crystal the differences are about one part in 10^8. The crystal normally operates at a frequency between its series resonant frequency and its anti-resonant frequency. This operating frequency depends on the external circuit capacitance in parallel with C_o and the frequency of resonance can be controlled by this external load capacitance. By definition, the crystal is operating at a load capacitance C_L when it is inductive and resonant with C_L. Crystal manufacturers define C_L (usually 20 pF) so that the frequency at which the crystal is manufactured is the same as when it is used in an oscillator circuit.

The expressions for f_s and f_A are :

$$f_s = \frac{1}{2\pi}\sqrt{\frac{1}{L_1C_1}}$$

$$f_A = \frac{1}{2\pi}\sqrt{\frac{1}{L_1C_1} + \frac{1}{L_1C_o}}$$

Taking $f_A = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{L_1C_1}\left(1+\dfrac{C_1}{C_o}\right)}$

but C_1/C_o <<1 and using the binomial theorem $(1 + x)^n = 1 + nx$ if $x<<1$

then $$f_A = \frac{1}{2\pi}\sqrt{\frac{1}{L_1C_1}}\left(1+\frac{C_1}{2C_o}\right) = f_s\left(1+\frac{C_1}{2C_o}\right)$$

$$\Delta f = f_A - f_s \quad \text{then} \quad \Delta f = \frac{C_1}{2C_o} f_s$$

The frequency at any load capacitance C_L can be calculated by making C_0 in this equation equal to the holder capacitance plus the load capacitance. Then $\frac{\Delta f}{f_s} = \frac{C_1}{2(C_0 + C_L)}$

The frequency of the resonator can be altered from its series resonant frequency (C_L = very large value) to the anti-resonant frequency ($C_L = 0$). The total frequency change available from a crystal is of the order of 200 ppm. The frequency change or "pulling" available at various values of load capacitance is given in table 3.6.

Mode	Pulling in ppm/pF at nominal C_L		
	10 pF	20 pF	30 pF
Frequency	$20 \rightarrow 45$	$7 \rightarrow 25$	$3 \rightarrow 16$
3rd overtone	$1.2 \rightarrow 7.0$	$0.3 \rightarrow 2.8$	$0.2 \rightarrow 1.5$
5th overtone	$0.8 \rightarrow 2.5$	$0.2 \rightarrow 1.0$	$0.1 \rightarrow 0.5$

Table 3.6 - Frequency change with C_L

Most crystals have either the series resonant frequency quoted or the frequency with 20 pF load capacitance. For overtone crystals only the series resonant frequency is quoted due to the small frequency changes with load capacitance.

Impedance diagrams are often used as a graphical means of showing the changes in crystal impedance as a function of frequency. If a plot of the admittance Y is plotted with B and G on two orthogonal axis and frequency on the third, the characteristic in figure 3.14 is obtained.[6]

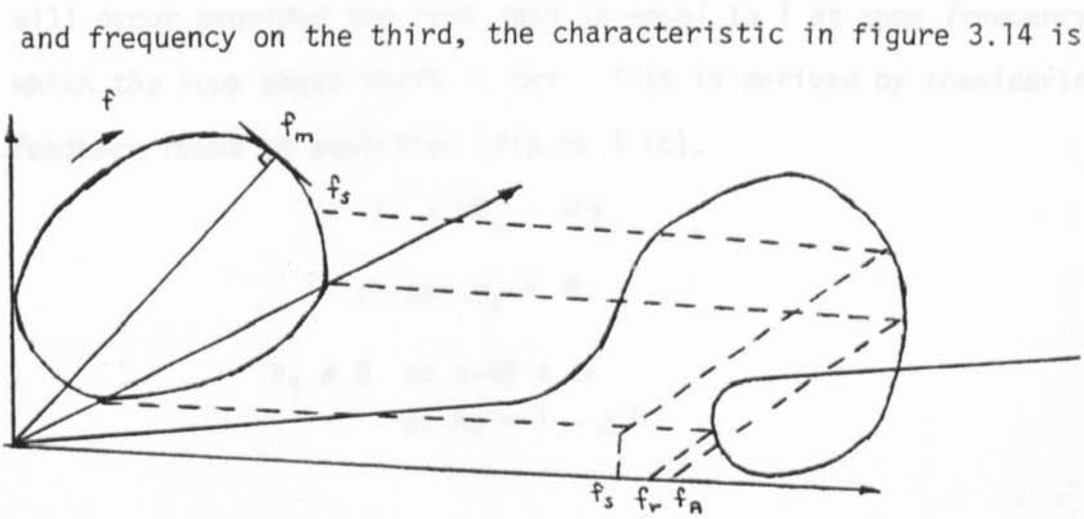


Figure 3.14 - Impedance diagram (3-D)

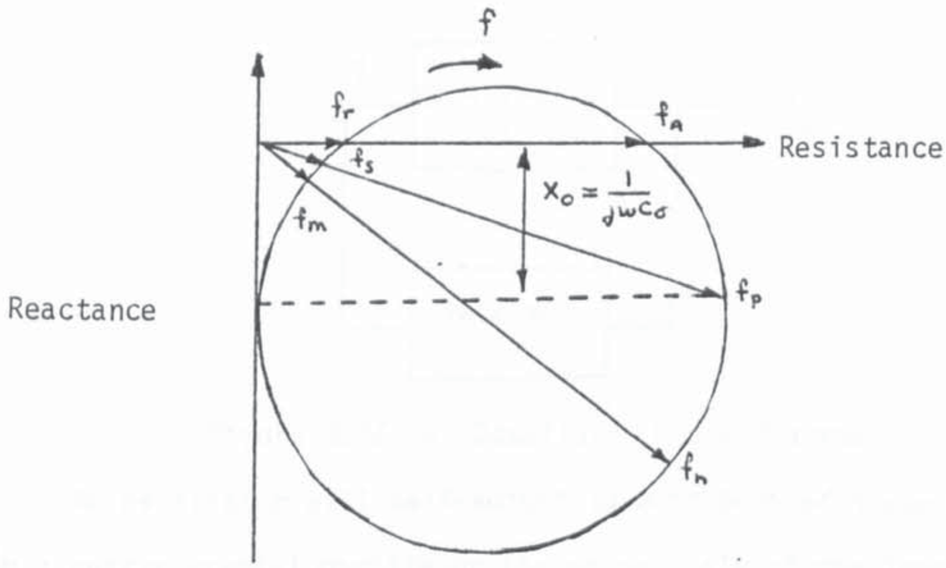


Figure 3.15 - Impedance diagram (2-D)

Normally two dimensional diagrams as shown in figure 3.15 are used. The six critical frequencies can be seen. f_n is the frequency at which the impedance is a maximum. Likewise f_m is the frequency where the impedance is a minimum. f_a is the frequency of zero phase where the impedance is resistive and of a high value and f_r is the frequency where the impedance is resistive and of a low value. At f_p, the parallel resonant frequency, the crystal has maximum resistance. f_s is the frequency of series resonance.

3.7 Oscillators

In very general terms a crystal-controlled oscillator can be described as consisting of an amplifier, or gain circuit, with a feedback network that contains the piezoelectric crystal unit. All crystal phase shift oscillators obey the Barkhausen criteria. This states self-oscillation will occur provided the loop gain is equal to 1 at some frequency for which the loop phase shift is $2n\pi$. This is derived by considering positive feedback round an amplifier (figure 3.16).

$$V_i = \beta V_o = A\beta V_i$$

$$(1-A\beta)\, V_i = 0$$

$$V_i \neq 0 \quad \text{so } 1-A\beta = 0$$
$$\text{or } A\beta = 1 + j\,0$$

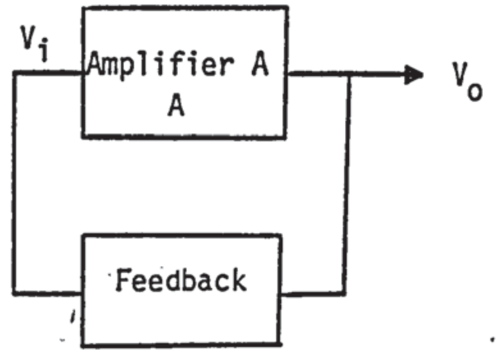


Figure 3.16 - Oscillator block diagram

An oscillator will self-adjust to meet both of these requirements. In a quartz crystal oscillator the phase angle of the impedance of the crystal changes rapidly with frequency, and changes in the circuit phase shift can be compensated for by small changes in the operating frequency, giving high stability. The stable level of oscillating signal is determined by the self-limiting characteristics of the loop or by the action of external AGC acting on the circuit.

Crystal oscillators can be divided into two main groups :

(a) those which operate the crystal unit in parallel resonance at high impedance

(b) those which operate the crystal unit in series resonance in a low impedance mode.

A typical parallel mode oscillator is shown in figure 3.17. The crystal acts as an inductive element resonating in parallel with the equivalent parallel capacitance.

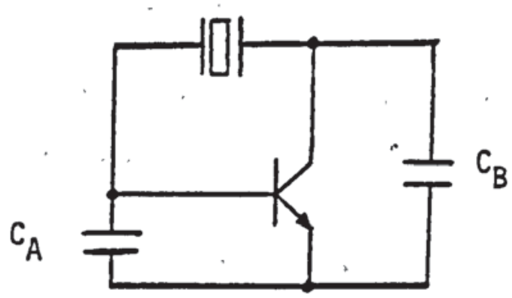


Figure 3.17 - Parallel mode oscillator

The equivalent parallel capacitance is

$$C_L = \frac{C_A C_B}{C_A + C_B}$$

The frequency of oscillation is

$$f = \frac{1}{2\pi\left(L_1 \dfrac{C_1 C_{LP}}{C_1 + C_{LP}}\right)^{\frac{1}{2}}}$$

The drive level of the crystal i.e. the power dissipated in the crystal, can be calculated by measuring the voltage across the crystal and using the formulae

$$\text{drive level} = \frac{V^2}{\text{E.P.R.}}$$

E.P.R. is the effective parallel resistance and is a measure of the activity of the crystal. The E.S.R. (effective series resistance) is more often used and both terms depend on the load capacitance C_L.

$$\text{E.P.R.} = \frac{1}{W^2\left(C_o + C_L\right)^2 R_1}$$

$$\text{E.S.R.} = R_1\left(1 + \frac{C_L}{C_o}\right)^2$$

A typical series mode oscillator is shown in figure 3.18. The crystal acts as a series tuned circuit. The series inductor lowers the crystal frequency to the series resonance. However, oscillation of L_s with C_0 can occur under certain conditions and the crystal may require a parallel inductor across it to cancel C_0. The frequency of oscillation is

$$f_s = \frac{1}{2\pi\ \left(L_s C_{LP}\right)^{\frac{1}{2}}}$$

where C_{LP} is the equivalent parallel capacitance as defined for the parallel mode oscillator.

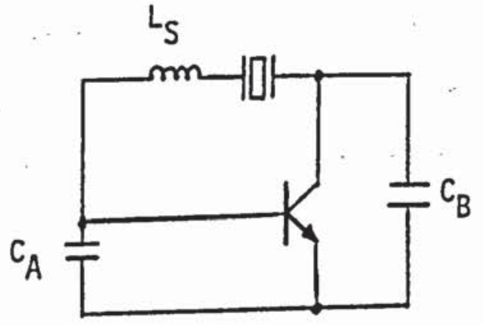


Figure 3.18 - Series mode oscillator

The drive level is found to be measuring the RF voltage across the crystal and using the formulae drive level = $\frac{V^2}{E.S.R.}$

Even though crystals offer a high degree of stability compared with LC oscillators, the factors causing instability are still important in the design of precision oscillators. There are three areas of instability:

1. Long term variations

These variations are caused by component ageing and are dominated by the crystal. A crystal will age logarithmically with figures of 2 ppm in the first year and 1 ppm per year after. It is caused primarily by a gradual transfer of mass to or from the crystal blank and by a relaxation of stresses. The rate of ageing is temperature dependent and a significant improvement is obtained by a high temperature baking of the crystal before use. The rate of ageing is also a function of drive level,being best at low drive levels. Ageing of cold-weld crystals and glass-holder crystals is slower than in solder-seal crystals since they can be kept cleaner.

2. Medium term variations

These variations, measured over minutes, are caused by:

(a) Environmental conditions such as vibration, shock, acceleration and humidity. They cause frequency variations of up to 1 ppm.

(b) Temperature: temperature effects other than that caused by the frequency/temperature coefficient of the crystal are small and may be minimized by using components of low temperature coefficient or by choosing components with temperature coefficients which cancel.

(c) Supply voltage; this causes changes of up to 0.5 ppm/volt and can be minimized by using a voltage stabilizer. Changes in supply voltage cause changes in the drive level of the crystal and cause the frequency to change.

(d) Load; this is minimized by incorporating a buffer amplifier between the oscillator and the load.

3. Short term variations

These variations are caused by noise in the oscillating loop and are inversely proportional to the circuit Q. The interactions may be simple superposition, amplitude modulation, frequency modulation, phase modulation or any combination thereof. However, only in the case of FM and PM is there a change of frequency. The short term stability may be defined in either the frequency or time domain. In the frequency domain the sideband level below the carrier is specified (in dB) at various offset frequencies from the carrier. In the time domain it is given as the rms fractional frequency deviation for a specified measurement time.

In a crystal-controlled oscillator the most significant parameter contributing to instability is the frequency/temperature coefficient of the crystal. This overshadows all other effects by an order of magnitude or more and in the search for higher accuracy, the reduction of the effect of temperature is the most important consideration.

CHAPTER 4

PRECISION OSCILLATORS

4.1 Temperature-Controlled Crystal Oscillators

One way of reducing the effect of temperature on a quartz crystal is to enclose the crystal in a chamber which is kept at a constant temperature. A crystal is normally kept in an oven at a high temperature and frequency stabilities of a few parts in 10^7 are obtained. These oscillators are called oven-controlled crystal oscillators (OCXOs). A single stage oven will have a temperature stability of $0.1^{o}C$ and the operating temperature of the oven must be higher (usually $10\text{-}15^{o}C$) than the highest ambient temperature of operation. The crystal used in an OCXO has a frequency/temperature curve as shown in figure 4.1. The operating temperature of the oven is at the upper turnover point of the crystal where the frequency change with temperature is small. Since crystals made by the same process do not have exactly the same turnover point due to the tolerance in the angle of cut, the operating temperature must be adjusted to correspond to the turnover point of the crystal used with that oven.

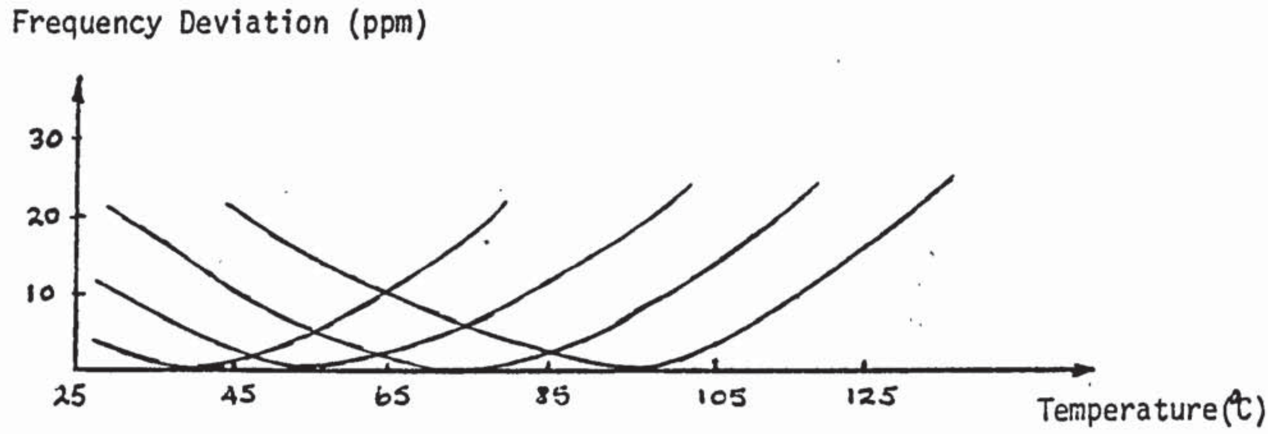


Figure 4.1 - Frequency/temperature curve

Insulation is used around the components in the oven to minimize the heat loss and to minimize temperature gradients within the oven. The best insulation is a vacuum. Normally foam material is used as a less expensive and more rugged alternative to a double wall Dewar flask.

For the highest stability a double oven system is used.

Crystal ovens have several disadvantages which limit their use in some applications. These are:

1. A warm-up time is required.
2. The volume of the unit is large.
3. The power consumption is high.

4.2 Temperature-Compensated Crystal Oscillators

To obtain a better frequency stability than is possible with a simple crystal oscillator, a temperature-compensated crystal oscillator (TCXO) is used. The stability is not as good as can be obtained with an ovened oscillator but a TCXO has a shorter warm-up time, is smaller in size, and has a much lower power consumption.

Temperature compensation is achieved by placing a variable capacitance diode in series with the crystal. Since the frequency of oscillation is affected by the effective load capacitance presented to the crystal, the frequency of oscillation is controlled by the d.c. voltage across the variable capacitance diode. The oscillator can always be "pulled" on to its nominal operating frequency by altering this d.c. voltage. Normally, a resistor-thermistor network is used to generate a temperature dependent voltage which is fed to the variable capacitance diode. Since the frequency/temperature coefficient of the crystal is the primary cause of the drift of frequency with temperature, the resistor-thermistor network generates a voltage with the inverse characteristics of the crystal's frequency/temperature coefficient(Fig. 4.2).

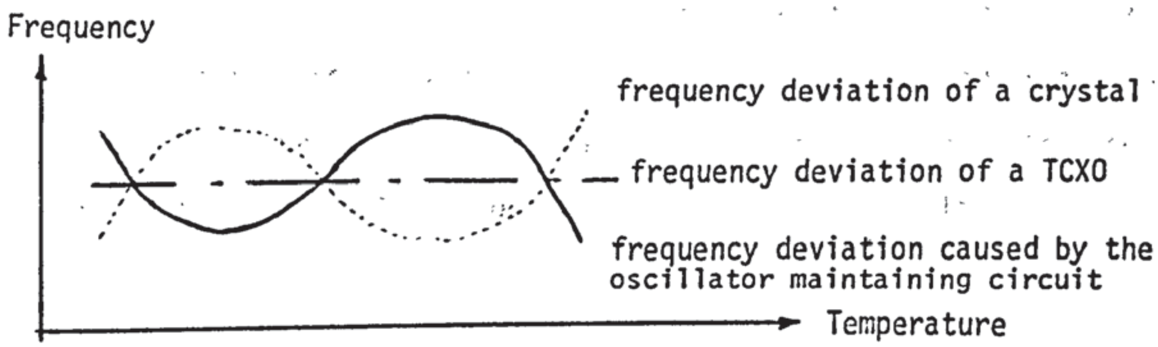


Figure 4.2 - Principle of temperature compensation

The best angle-of-cut for the crystal depends on the stability required and the operating temperature range. A compromise is reached between a crystal with a more linear frequency/temperature behaviour between the turnover points and greater frequency change, and a crystal with a smaller frequency change but a more non-linear behaviour. The more linear the slope, the easier the compensation. Normally crystals are cut at such an angle as to minimize the total frequency deviation which occurs. This angle is the optimum angle.

Figure 4.3 shows a TCXO which has been in production at STC for several years. The choice of values of the components in the resistor-thermistor network is complicated and requires a computer program using optimisation techniques to determine the values. Each device is individually compensated since the variation between units is large. The data for the computer program are measured during a compensation run. The unit is placed in an oven and measurements taken at a number of temperatures across the operating temperature range. The voltage required to pull the oscillator to the correct frequency is measured along with the resistance of the thermistors to be used in that device. This data is then used in the computer program to determine the values of the select-on-test resistors. (RA,RB,RC,R1, R2 in figure 4.3). These resistors are values from an E192 series and have a 1% tolerance. After fitting the resistors, the unit is then measured over the temperature range again to verify its frequency drift with temperature.

4.3 Alternative Compensation Systems

Resistor-thermistor networks have successfully competed with other methods of compensation for over twenty years and are still dominant today. The network is simple, requires only a few components, and is suitable for large volume production. To assess the alternative

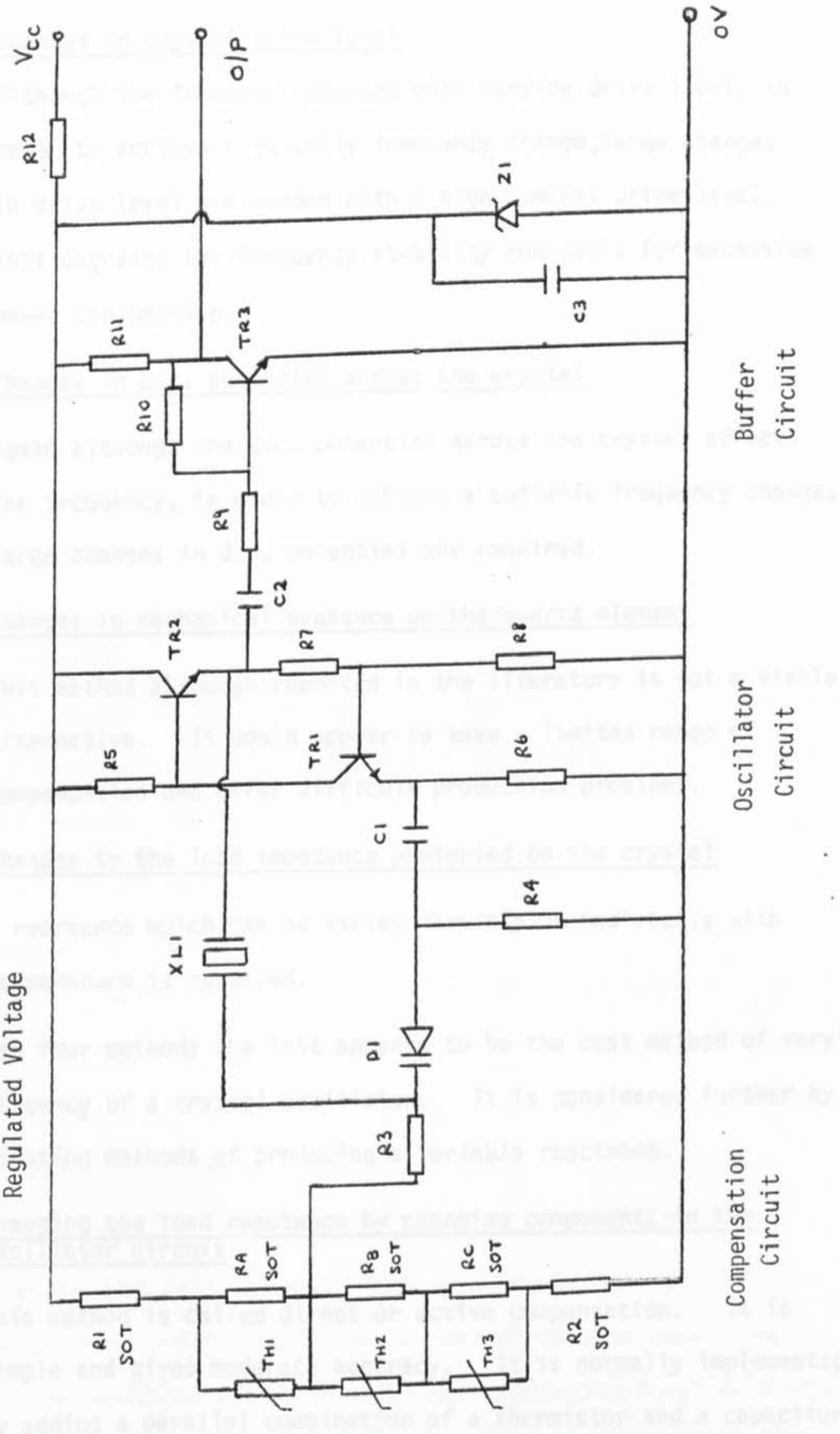


Figure 4.3 - TCXO circuit diagram

compensation systems it is necessary to consider the various methods of altering the frequency of a crystal oscillator.

(1) Changes in crystal drive level

Although the frequency changes with varying drive level, in order to achieve a suitable frequency change, large changes in drive level are needed with a high nominal drive level. This degrades the frequency stability and calls for excessive power consumption.

(2) Changes in d.c. potential across the crystal

Again although the d.c. potential across the crystal affects the frequency, in order to achieve a suitable frequency change, large changes in d.c. potential are required.

(3) Changes in mechanical pressure on the quartz element

This method although reported in the literature is not a viable alternative. It would appear to have a limited range of compensation and offer difficult production problems.

(4) Changes in the load impedance presented to the crystal

A reactance which can be varied directly or indirectly with temperature is required.

Of these four methods the last appears to be the best method of varying the frequency of a crystal oscillator. It is considered further by investigating methods of producing a variable reactance.

(a) Changing the load reactance by changing components in the oscillator circuit

This method is called direct or active compensation. It is simple and gives moderate accuracy. It is normally implemented by adding a parallel combination of a thermistor and a capacitor in series with the crystal.

(b) <u>Making use of the collector-base capacitance of the oscillator transistor</u>

The collector-base capacitance forms part of the load capacitance. Experiments indicate that the range of adjustment available using typical integrated circuit transistors is too small to allow compensation over a wide temperature range.

(c) <u>Using a diode and capacitor connected in parallel</u>

A reverse biased diode and fixed capacitor in parallel could be used as a variable capacitance in series with the crystal. Experiments show that the variation of capacitance with respect to the d.c. bias across the diode is too small to allow compensation over a wide temperature range.

(d) <u>Using a variable capacitance diode connected in series with the crystal</u>

This method is almost universally used to control the frequency of an oscillator since variable capacitance diodes are well characterised and readily available.

Method (d) is the most suitable method for realising a variable reactance. The frequency of an oscillator is controlled by a d.c. voltage and requires only a resistor and a variable capacitance diode for its implementation.

It was the availability of these diodes in the early 1960s which led to the expansion in the manufacture of TCXOs. Some attempts at compensating crystal oscillators had been made before this.

Koerner[7] of Bell Telephone Labs. reported as early as 1956 on temperature compensation employing thermistor shunted capacitors (active compensation) but the first system employing a variable capacitance diode to compensate for frequency drifts did not appear until 1961.

An oscillator was made having a frequency stability of ± 1 ppm over the temperature range 70^{o}C to 40^{o}C and used two reverse biased diodes as a variable capacitance.[8] Further work[9,10] firmly established variable capacitance diodes as the most suitable variable reactance element. The voltage controlled oscillator, of course, is only one part of a TCXO. The other is a circuit which generates a temperature varying control voltage for the voltage controlled oscillator. TCXOs at this time used a resistor-thermistor network and much work was carried out on this type of circuit to find better networks and to develop algorithms for computer programs which could be used to solve for the values of the fixed resistors required by the network. When STC entered the market in 1969, large scale production of TCXOs using resistor-thermistor compensation was in progress and research work became more and more focused on developing new systems of generating the control voltage.

Thermally controlled reactances were suggested in 1968[11] and involved a temperature sensitive capacitor which was used to compensate an oscillator to within ± 1 ppm. However, higher stabilities could be achieved using a conventional variable capacitance controlled oscillator and synthesizing the control voltage by a series of straight line segments.[12] Novelle achieved ±0.3 ppm over the temperature range 70^{o}C to -40^{o}C using this method. This was also suitable for an automatic production system but appears never to have reached large scale use.

In 1971 Bulova patented impedance switching.[13] Impedances in the form of capacitors were switched by analogue switches in and out of the oscillator circuit, changing the load capacitance presented to the crystal.

This was followed in 1973 by hybrid compensation[14] which had a coarse analogue compensation system using a resistor-thermistor network with fine compensation using a digital system. The digital circuit comprised a temperature dependent RC oscillator which was used to drive a counter. The output from the counter was used to address a programmable read only memory (PROM). The output from the PROM fed into a digital-to-analogue converter. The output from the converter altered the d.c. voltage across the variable capacitance diode. Tolerances of ± 0.4 ppm were recorded over the temperature range $80^{o}C$ to $-40^{o}C$. A miniature version was produced three years later. In 1977 Motorola produced a simpler digital TCXO.[15] The output from a temperature sensor was converted to a digital word by an analogue-to-digital convertor. This was used to address a PROM containing the values of the compensation voltage. The output from the PROM was converted to an analogue voltage by a digital-to-analogue converter and fed to the variable capacitance diode. Tolerances of ± 0.5 ppm were achieved over the temperature range $80^{o}C$ to $-30^{o}C$.

Microprocessors were also suggested as a method of compensation.[16] The microprocessor calculated the compensation voltage required using linear interpolation from a few measured data points. Analogue compensation returned in 1980[17] when Motorola reported using a custom designed integrated circuit which generated three linear curves each of which simulated the portion of the compensation curve in the cold, middle and hot regions. A curve approximation based on three straight line segments is generated by summing these together. This TCXO also used thick-film technology to achieve small size. Tolerances of ± 2 ppm over the temperature range $90^{o}C$ to $-40^{o}C$ were recorded.

Although there appear to be many diverse methods of compensating crystal oscillators, resistor-thermistor networks are still dominant today. Few alternative methods have been developed into commercial products. Some form of digital compensation is usually used with its inherent advantages and disadvantages. The accuracy of compensation is good with $\pm$0.5ppm attainable without difficulty but at the cost of medium current consumption (typically up to 10mA) and large size (typical case size is 50mm x 50mm x 10mm). The main problem with many systems of compensation is that the system synthesizes the required control voltage in segments. The maximum error occurs at the changeover points between the segments. For example, each thermistor in a resistor-thermistor network is effective in a different portion of the temperature band and the network has difficulty in maintaining a smooth transition between the three portions of the curve.

4.4 A New Method of Compensation

The compensation voltage is dominated by the frequency/temperature behaviour of the crystal and is similar to the cubic equation in temperature described in section 3.5. The compensation voltage is a continuous curve in a finite interval and, using the Weierstrass theorem,[18] can be expressed as:

$$V(t) = a_0 + a_1(t-t_i) + a_2(t-t_i)^2 + a_3(t-t_i)^3$$

where a_n = weighting coefficient

t = ambient temperature

t_i = reference temperature (26°C)

$V(t)$ = compensation voltage

$V(t)$ is an approximation to the desired compensation voltage and the accuracy of fit is determined by the number of terms included in the series. Higher order terms can be added if necessary.

If voltages representing each of these terms are generated and summed together with the appropriate weighting coefficients a compensation voltage is generated. A constant voltage can be generated using a voltage stabilizer and a linear voltage variation with temperature generated using the base-emitter junction voltage of a transistor. The higher order terms are generated using analogue multipliers. The linear voltage is multiplied by another linear voltage to generate a quadratically varying voltage with temperature. The quadratically varying voltage is then multiplied by the linear varying voltage to generate a cubic varying voltage with temperature. This process can be repeated to give any order of term required. The voltages are then summed together using an operational amplifier to give the compensation voltage V(t). See figure 4.4.

This method of compensation is purely analogue in nature and a linear integrated circuit containing the circuit elements of the compensation system; oscillator and output stage(s) can be designed. Using this method of compensation and other design techniques, the current consumption can be reduced to that specified and by having the majority of the circuit elements within the integrated circuit the desired size can be obtained. All the voltages are continuous functions of temperature and the voltage V(t) will be a smooth curve and should be a better fit to the desired control voltage. This should give a smaller frequency tolerance.

The only references to this method in the literature are in two UK patents. Patent 2046047[19] describes the integrated circuit developed by Motorola for compensating crystal oscillators and contains the following paragraph:

"Some prior circuits have created a cubic law temperature varying voltage by twice multiplying a linearly varying voltage but such systems are extremely complex and cannot be adequately and easily adjusted to fit the compensating voltage versus temperature curve which is required by

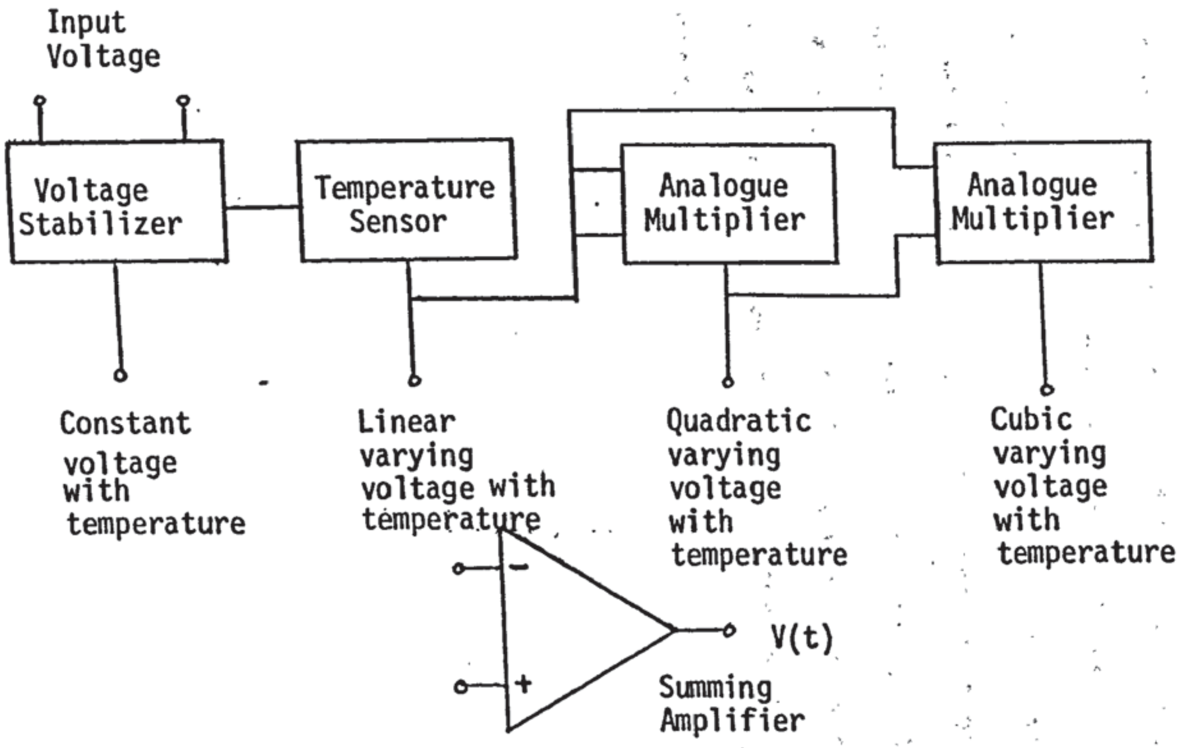


Figure 4.4 - Block diagram of Integrated circuit

any one crystal oscillator."

Patent 1535416[20] describes a circuit which multiplies a linear voltage and a quadratic voltage in an analogue multiplier to produce a cubic varying voltage with temperature. The linear and the quadratic voltages are generated by resistor-thermistor networks. Oscillators have been compensated to within $\pm$ 1 ppm over the temperature range $60^{\circ}C$ to $-20^{\circ}C$.

Compensation using this method is simple in concept requiring simple circuits for its implementation. Adjustment for individual crystals is easily accomplished by changing the resistors associated with the summing amplifier. However, the tolerance on these gains may be very small and be impossible to realise in practice. An analysis of the curves of compensation voltage is now required to determine the precision and ease with which an oscillator can be compensated using this method.

CHAPTER 5

COMPENSATION VOLTAGE CURVE ANALYSIS

5.1 Frequency/Temperature Coefficient of an AT-cut crystal

In Section 3.5 it was stated that the frequency behaviour of a quartz crystal with respect to temperature can be written as a three term power series. If the frequency of oscillation is f_0 at temperature t_i (usually 26°C) then with $\Delta t = t - t_i$

$$\frac{f_o - f_c}{f_o} = a\ \Delta t + b\,\Delta t^2 + c\ \Delta t^3$$

where a,b,c have values as in section 3.5.

A number of crystals were analysed and the coefficients a,b and c calculated. Three batches of crystals of frequencies 4 MHz, 6.4 MHz and 10 MHz were used. Each batch contained 15 crystals and is representative of those crystals used in TCXOs. A computer-controlled oven system was used to measure the frequency deviation of the crystals as the ambient temperature was varied. This machine can measure the frequency deviation of a large number of crystals automatically. The instrument also calculates the best fitting polynomial curve of third order to this data using a least squares error criterion.

Table 5.1 shows the frequency error of the curve fit at each measured temperature. Crystals 1 to 15 are 4MHz, crystals 16 to 30 are 6.4 MHz and crystals 31 to 45 are 10 MHz. The errors are small having a maximum value of 0.8ppm. Some crystals show a very good fit whilst others have a significant error. Part of the error is due to the frequency resolution of the oven system. The frequency of oscillation of a crystal can only be measured to the nearest hertz and for crystals 1 to 15 whose nominal frequency is 4MHz, the frequency deviation can only be measured in 0.25 ppm steps.

Table 5.2 gives the coefficients a,b and c in the equation of frequency deviation with temperature for these crystals. The coefficients are in agreement with the accepted values given in section 3.5. From section 3.5

$$a = -0.085\ (\theta - \theta_i)$$

$$b = 0.45 \times 10^{-3}\ -0.078 \times 10^{-3}(\theta - \theta_i)$$

$$c = 109 \times 10^{-6}(\theta - \theta_i)$$

for these crystals $(\theta - \theta_i)$ is approximately 2.5 and so

$$a = 0.224$$

$$b = -0.656 \times 10^{-3}$$

$$c = 109 \times 10^{-6}$$

The mean values of the coefficients given in Table 5.2 are

$$a = 0.233$$

$$b = -1.003 \times 10^{-3}$$

$$c = 97.5 \times 10^{-6}$$

The difference in the coefficient values is partly caused by a shift in inflection temperature. The inflection temperature has been previously taken as 26°C. It is found that when contoured crystals are measured, the inflection temperature is higher and is a function of the contour of the crystals. An inflection temperature of 30° is used with contoured crystals. There are also large changes in coefficient a which is a measure of the spread in the angle-of-cut of the crystal. Table 5.2 also shows the inflection temperature of each crystal and the offset angle which is the difference between the angle of zero temperature coefficient at 26°C and the actual angle-of-cut. It is equal to $(\theta - \theta_i)$. The mean offset angle is 2.64 minutes of arc with a standard deviation of 0.97. Internal company data gives the tolerance on the angle-of-cut as ±1 minute of arc. These figures indicate this tolerance is typically ±2 minutes of arc.

Crystal	Frequency Deviation (ppm)								
	87°C	75°C	60°C	45°C	25°C	5°C	-10°C	-25°C	42°C
1	0.16	-0.13	-0.24	0.05	0.31	-0.02	-0.14	-0.11	0.10
2	0.35	-0.43	-0.23	0.14	0.36	-0.10	-0.15	-0.21	0.17
3	0.15	-0.19	0.22	0.67	0.66	0.06	-0.12	-0.33	0.02
4	-0.13	-0.15	-0.14	0.44	0.10	-0.06	-0.04	-0.03	-0.04
5	0.34	-0.51	0.08	0.29	0.90	-0.18	0.27	-0.37	0.15
6	0.32	-0.45	-0.13	0.21	0.25	-0.14	-0.90	-0.02	0.06
7	0.26	-0.29	-0.21	0.13	0.20	0.20	-0.31	-0.11	0.13
8	0.40	-0.57	-0.15	0.26	0.28	-0.01	-0.39	-0.14	0.04
9	0.16	-0.22	-0.09	0.13	0.09	-0.01	-0.05	-0.08	0.06
10	0.21	-0.02	-0.53	-0.13	0.80	0.07	-0.35	-0.29	0.26
11	0.22	-0.29	-0.07	-0.01	0.26	0.09	-0.27	-0.03	0.08
12	0.21	-0.15	-0.34	0.09	0.22	0.32	-0.28	-0.31	0.22
13	0.31	-0.41	-0.15	0.21	0.18	0.00	-0.07	-0.18	0.12
14	0.17	-0.37	0.20	0.09	-0.13	0.04	-0.05	-0.06	-0.02
15	0.21	-0.34	0.06	0.84	-0.03	0.14	-0.11	-0.07	0.06
16	0.17	-0.16	-0.20	0.06	0.24	0.02	-0.07	-0.19	0.13
17	0.17	-0.17	-0.11	0.12	0.15	0.04	-0.07	-0.16	0.11
18	0.17	-0.32	0.20	-0.06	0.03	0.06	-0.10	0.00	0.02
19	0.22	-0.26	-0.14	0.06	0.21	0.06	-0.13	-0.16	0.12
20	0.12	-0.15	-0.10	0.10	0.17	-0.26	0.29	-0.25	0.08
21	0.11	-0.26	-0.05	0.04	0.15	0.06	-0.14	-0.08	0.08
22	0.20	-0.19	0.24	0.08	0.28	0.05	-0.18	-0.14	0.12
23	0.19	-0.18	-0.22	0.07	0.28	0.03	-0.14	0.15	0.12
24	0.21	-0.22	-0.27	0.23	0.14	0.08	-0.15	-0.14	0.12
25	0.18	-0.14	-0.30	0.13	0.28	-0.07	0.05	-0.30	0.16
26	0.22	-0.25	-0.17	-0.08	0.20	0.11	-0.15	-0.19	0.14
27	0.18	-0.20	-0.15	0.08	0.14	0.07	-0.55	-0.20	0.12
28	0.21	-0.18	-0.26	0.00	0.38	0.08	-0.20	-0.21	0.17
29	0.16	-0.17	-0.15	0.57	0.23	-0.01	-0.14	-0.05	0.07
30	0.16	-0.15	-0.20	0.05	0.25	0.05	-0.19	-0.08	0.09
31	0.24	-0.39	-0.09	0.34	0.09	-0.25	-0.21	-0.38	-0.12
32	0.12	-0.13	0.09	-0.40	0.25	0.52	0.43	-0.24	-0.20
33	0.45	-0.68	-0.05	-0.15	0.63	-0.79	0.39	-0.18	0.07
34	0.18	-0.35	-0.08	0.05	0.69	-0.81	-0.34	-0.90	-0.31
35	0.24	-0.43	-0.07	0.47	-0.04	-0.17	-0.54	0.78	-0.25
36	0.09	-0.24	-0.05	0.42	-0.36	-0.06	-0.18	0.48	-0.20
37	0.20	-0.28	-0.16	0.27	0.04	0.01	0.17	0.04	0.02
38	0.21	-0.43	0.06	0.28	0.09	-0.34	-0.36	0.72	-0.25
39	0.10	0.04	-0.53	0.33	0.23	-0.05	-0.22	0.05	0.05
40	0.19	-0.29	0.01	0.03	0.18	-0.03	-0.20	0.10	0.00
41	0.04	0.02	-0.25	0.06	0.18	0.08	-0.39	0.24	-0.03
42	0.32	-0.25	-0.41	0.04	0.27	0.77	-0.77	-0.29	0.30
43	0.15	-0.25	-0.06	0.06	-0.08	0.20	-0.08	-0.12	0.08
44	0.29	-0.42	0.19	-0.26	0.00	0.66	-0.08	-0.76	0.37
45	0.26	-0.27	-0.25	-0.14	0.18	0.23	-0.23	-0.24	0.18

Table 5.1 - Frequency Deviations

Crystal	Coefficients			Inflection	Offset Angle
	$a \times 10^{-1}$	$b \times 10^{-4}$	$c \times 10^{-5}$	Temp. (^{o}C)	(arc-min)
1	-1.01	-7.68	9.85	28.6	1.2
2	-2.46	-8.85	9.83	29.0	2.8
3	-2.83	-8.55	10.3	28.8	3.2
4	-4.86	-12.2	9.59	30.2	5.6
5	-2.27	-9.08	10.0	29.0	2.6
6	-3.08	-9.66	9.86	29.3	3.5
7	-2.42	-9.22	9.66	29.2	2.8
8	-2.27	-8.64	10.0	28.9	2.6
9	-2.72	-9.45	9.75	29.2	3.1
10	-3.04	-9.40	9.83	29.2	3.5
11	-1.31	-6.91	9.62	28.4	1.5
12	-2.63	-9.59	9.70	29.3	3.0
13	-2.31	-9.32	9.58	29.2	2.6
14	-4.91	-12.5	9.94	30.2	5.6
15	-4.55	-12.3	9.61	30.3	5.2
16	-2.66	-7.55	10.1	28.5	3.0
17	-1.66	-5.95	9.86	28.0	1.9
18	-1.51	-6.63	9.72	28.3	1.7
19	-2.82	-7.51	9.97	28.5	3.2
20	-2.30	-6.32	10.2	28.0	2.6
21	-2.89	-7.51	9.97	28.5	3.3
22	-2.41	-7.23	10.0	28.4	2.8
23	-2.46	-6.61	10.0	28.2	2.8
24	-2.36	-6.73	9.96	28.3	2.7
25	-1.60	-6.43	10.1	28.1	1.8
26	-1.62	-5.89	10.1	27.9	1.9
27	-2.78	-7.00	10.1	28.3	3.2
28	-1.68	-6.48	9.89	28.2	1.9
29	-2.37	-7.21	10.1	28.4	2.7
30	-1.93	-6.35	10.0	28.1	2.2
31	-1.46	-13.6	8.93	31.1	1.7
32	-2.01	-13.8	9.10	31.1	2.3
33	-2.80	-16.2	9.07	32.0	3.2
34	-1.49	-13.3	9.48	30.7	1.7
35	-1.87	-13.7	9.55	30.8	2.1
36	-2.12	-12.3	9.50	30.3	2.4
37	-1.02	-11.4	9.67	29.9	1.2
38	-1.61	-13.3	9.50	30.7	1.8
39	-1.52	-12.7	9.40	30.5	1.7
40	-2.77	-14.3	9.52	31.0	3.2
41	-1.80	-13.4	9.62	30.6	2.0
42	-1.57	-14.6	9.47	31.1	1.8
43	-2.92	-14.0	9.40	31.0	3.3
44	-1.91	-17.4	9.69	32.0	2.2
45	-1.37	-12.7	9.82	30.3	1.6

Table 5.2 - Crystal Data

5.2 Oscillator Non-linearities

When the crystal is placed in an oscillator circuit non-linearities in the frequency deviation with temperature are introduced by the load capacitance. If the load capacitance has a non-linear temperature coefficient it can distort the crystal's frequency deviation with temperature. Most non-linearities are introduced by the variable capacitance diode. The capacitance of the diode varies with bias voltage as shown in Figure 5.1. This graph describes a Ferranti ZC824 diode and the equation relating the capacitance and the bias voltage is -

$$C_d = \frac{K}{(V_o + V_r)^n}$$

where $K = 75 \times 10^{-12}$

$V_o = 0.6$

$n \doteq \frac{1}{2}$

V_r = reverse bias voltage for the ZC824 diode

The sensitivity of the crystal to changes in load capacitance is also non-linear and is described by the equation:

$$\frac{f - f_s}{f_s} = \frac{C_1}{2(C_o + C_L)}$$

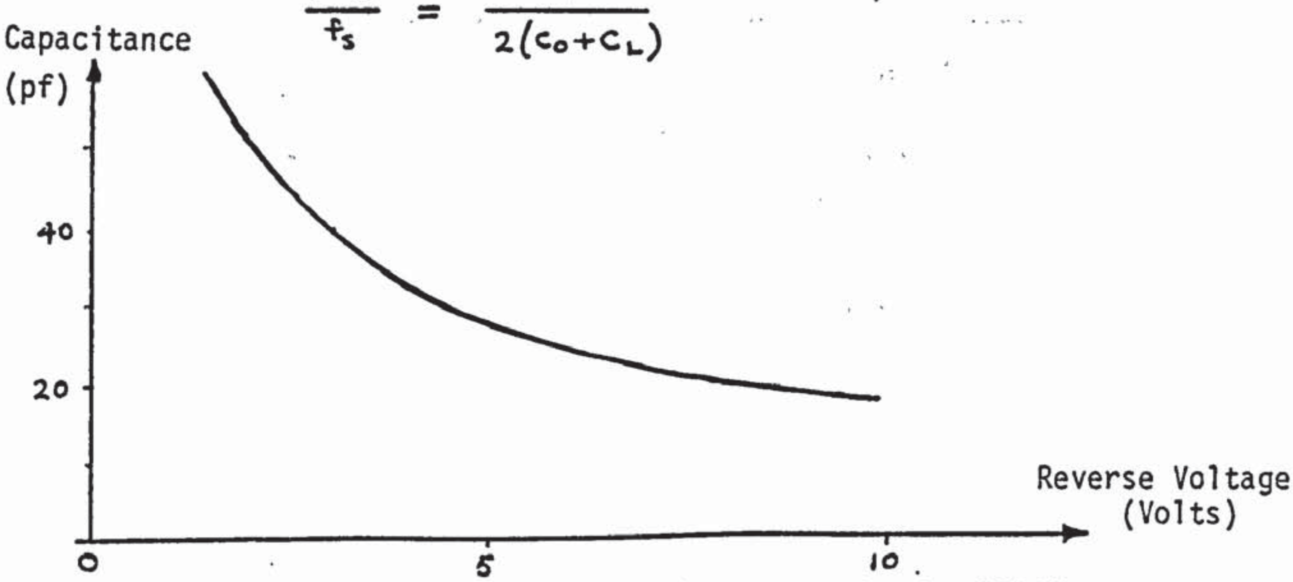


Figure 5.1 - Voltage/capacitance graph of a ZC824

Over the operating range of the oscillator these two effects tend to cancel each other out and it is possible to have a substantially linear frequency deviation with voltage in the voltage controlled oscillator. The compensation voltage of a TCXO should be adequately described by a third order power series and any distortion, which appears as higher order terms, should be small.

As an example take an oscillator with a ZC824 diode operating at an equivalent parallel capacitance of C_{LP} = 40 pF. If the crystal has C_1 = 6fF and C_o = 2pF then

$$\frac{f-f_s}{f_s} = \frac{C_1(K + C_{LP}(V_o + V_r)^{\frac{1}{2}})}{2(C_o(K + C_{LP}(V_o + V_r)^{\frac{1}{2}}) + KC_{LP})}$$

Let the nominal frequency of oscillation be 6.4 MHz and so the actual frequency of oscillation is

$$f = \left[1 + \frac{C_1(K + C_{LP}(V_o + V_r)^{\frac{1}{2}})}{2(C_o(K + C_{LP}(V_o + V_r)^{\frac{1}{2}} + K\,C_{LP})}\right] f_s$$

The variation of frequency with respect to the diode bias voltage V_r is given in Table 5.3 and in figure 5.2.

Reverse bias (volts)	Frequency (Hz)
1.0	6400741
1.5	6400781
2.0	6400816
2.5	6400848
3.0	6400877
3.5	6400904
4.0	6400929
4.5	6400953
5.0	6400975

Table 5.3 - Frequency deviation

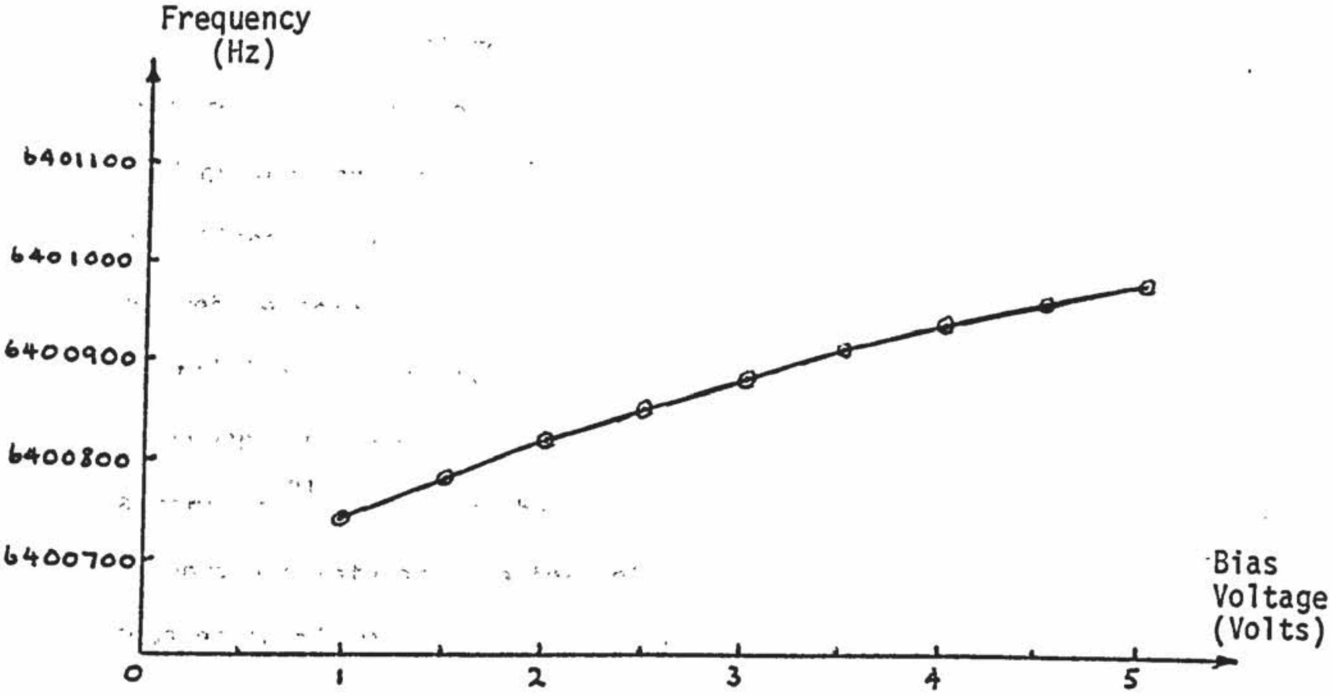


Figure 5.2 - Frequency deviation versus voltage

The graph contains a plot of frequency versus bias voltage using calculated values of diode capacitance. The value of n in the equation relating diode capacitance and bias voltage is not exactly ½ but this does not change the basic shape of the graph. Although a non-linearity does exist it should not introduce significant higher order terms.

5.3 Computer Curve Analysis

In Section 4.4 it was shown that the compensation voltage can be expressed as a sum of polynomial terms

$$V(t) = a_0 + a_1(t-t_i) + a_2(t-t_i)^2 + a_3(t-t_i)^3$$

where a_n = weighting coefficient

t = ambient temperature

t_i = compensation voltage

Some standard TCXOs were now analysed to determine the coefficients required, and the sensitivity of the coefficients. Two computer programs were written for this purpose.

The simplest method of curve fitting is the method of least squares. This method involves solving a set of simultaneous equations and conventional techniques use matrix inversion to solve these equations. This leads to a well-known phenomenon of ill-conditioning whereby extremely small errors in the data give large errors in the coefficients which are calculated. This can be overcome using orthogonal polynomials. The computer programs use Forsythe's algorithm [21] which can be easily implemented using a digital computer, is very accurate and the form of the error can be changed from least squares to minimax. See Appendix 1.

The form of the error is important in curve fitting. Normally the best fitting curve is developed by minimizing the total sum of the squares of the errors between the data points and the approximating curve. This is known as the least squares method. For our purposes a better method of curve fitting is necessary, minimizing the maximum error between the data points and the approximating curve. It is known as the minimax method.

An analysis of twelve TCXO4 units was made. A TCXO4 is a general purpose oscillator using three thermistors for compensation over the temperature range 85^{o}C to -40^{o}C. The compensation voltages had been measured at a number of temperatures (9 in this case) and the temperature and compensation voltage recorded. The computer program DAFIT1 was used and the curves fitted such that the minimax error was less than 50mV. This corresponds to ±2 ppm for a compensation voltage sensitivity of 40 ppm/volt. A further analysis was done with the minimax error of 12.5mV, which is equivalent to ±0.5 ppm. The results are given in Table 5.4.

Oscillator	a_0	a_1	a_2	a_3	Minimax error (mV)
1	0.16	-2.09	6.65	401	16
2	0.33	-2.24	9.56	421	29
3	0.44	-0.85	0.78	446	49
4	0.17	-2.45	6.19	441	28
5	0.09	-1.52	8.82	431	41
6	0.22	-2.86	9.27	412	21
7	0.32	-2.92	10.62	420	26
8	0.17	-1.77	6.11	406	19
9	0.19	-1.36	3.02	473	26
10	0.08	-2.33	8.66	413	11
11	0.07	-2.22	6.84	435	22
12	0.19	-1.76	6.24	460	29
	Order of equation needed to meet 12.5 mV error				
1	4th				8
2	5th				9
3	5th				8
4	4th				8
5	7th				8
6	4th				11
7	4th				10
8	4th				7
9	4th				11
10	3rd				11
11	4th				8
12	4th				11

Table 5.4 - Weighting coefficients

The temperature was not used directly in the curve fitting program. The integrated circuit used to generate the terms $(t-t_i)^n$ will use analogue multipliers based on a differential amplifier structure. These multiply differential voltages and a better choice of variable representing a linear variation of voltage with temperature is the difference in base -emitter junction voltage of a transistor at room temperature t_i and at the ambient temperature t. Analysis of some curves using the value of the base-emitter voltage directly indicated that large coefficients are necessary. They were sensitive to changes in their values since large quantities are being subtracted to give a small quantity.

Table 5.4 shows that the compensation voltage of all twelve oscillators is generated to within 50 mV using third order equations. The range of values of each coefficient is small. To meet the error bound of 12.5 mV, one unit requires a third order equation, eight require a fourth order equation and two require a fifth order equation. Only one unit requires an equation higher than fifth order. Another analysis was done which looked at the error at each data point as the order of the equation of V(t) was increased. Using a least squares error which will show up any single points at which there is a large error, it was apparent that there is usually one point at which the error exceeds that of all the other points. This is a characteristic of activity dips or bandbreaks and in these oscillators is responsible for the variation in degree of equation required for compensation using a 12.5 mV error bound.

5.4 Coefficient Sensitivity

Another computer program was written to evaluate the sensitivity of the coefficients which have been calculated in section 5.3. Program DAFIT2 was used to give the results in Table 5.5.

Error bound 50 mV

Oscillator	Change in value of coefficient in percent							
	a_o		a_1		a_2		a_3	
	+	-	+	-	+	-	+	-
1	9.5	9.5	6.4	5.9	13.6	23.0	1.9	3.2
2	3.8	3.8	4.9	4.2	7.4	12.6	1.4	2.0
3	1.9	1.9	14.4	12.0	40.0	68.0	1.0	1.5
4	6.8	6.8	4.2	3.9	11.4	18.6	1.3	1.8
5	14.7	14.7	6.8	4.2	7.1	21.9	1.4	1.9
6	6.6	6.6	4.3	4.1	9.2	15.0	1.7	2.3
7	4.0	4.0	3.9	3.5	7.8	11.3	1.5	2.1
8	8.5	8.5	6.9	6.4	14.8	25.0	1.8	2.4
9	6.0	6.0	7.1	6.5	24.0	39.0	1.2	1.7
10	21.0	21.0	5.9	5.7	12.1	19.5	2.0	2.5
11	19.0	19.0	5.2	4.8	11.6	19.5	1.5	2.0
12	5.8	5.8	5.3	5.1	11.2	17.9	1.2	1.6

Error bound 12.5 mV

Oscillator	Change in value of coefficient in percent											
	a_o		a_1		a_2		a_3		a_4		a_5	
	+	-	+	-	+	-	+	-	+	-	+	-
10	0.2	0.2	0.2	0.1	0.0	0.1	0.0	0.0				
1	0.8	0.8	0.9	0.9	1.3	1.2	0.7	0.3	2.3	2.3		
4	0.8	0.8	0.6	0.6	0.8	0.6	0.5	0.2	0.8	1.3		
6	0.4	0.4	0.4	0.5	0.8	0.5	0.8	0.1	1.0	2.0		
7	0.3	0.3	0.5	0.4	0.6	0.8	0.5	0.2	2.2	1.4		
8	0.9	0.9	1.7	0.9	1.3	1.5	0.4	0.4	3.1	3.7		
9	0.2	0.2	0.5	0.3	0.4	0.5	0.2	0.1	0.6	0.6		
11	2.3	2.3	0.6	0.8	0.8	0.8	0.6	0.2	1.2	1.5		
12	0.6	0.6	0.7	0.7	0.6	0.5	0.5	0.1	0.8	1.0		
2	0.1	0.1	0.3	0.3	0.2	0.2	0.0	0.0	0.5	0.2	1.0	0.3
3	0.0	0.0	0.0	0.1	0.1	0.4	0.0	0.0	1.1	0.4	0.7	1.1

Table 5.5 - Coefficient Changes

These results show that using 1% tolerance resistors it is possible to generate a compensation voltage to within ±50 mV. Once the error bound decreases to 12.5 mV the tolerance on the resistors makes synthesis of the curve impossible to realize in practice. The results also show the relaxed tolerances on the fourth order coefficients.

5.5 Conclusions

The amount of information which can be collected from this analysis is limited. It is easy to spend a lot of time analysing data without gaining any more insight into the problem. What can be concluded is that it seems feasible to proceed and design an integrated circuit which can be used to compensate a crystal oscillator. A voltage can be generated which will vary according to a known curve and the error will be less than 50mV if 1% tolerance resistors are used in the summing amplifier. If a voltage controlled oscillator with a compensation voltage sensitivity of 20 ppm/volt or less is used, then an oscillator can be compensated to within ±1 ppm using a third order polymonial. Although this is not as accurate as can be obtained with digital systems, it is as good as can be obtained with the best analogue systems and is adequate for a large percentage of customers. The accuracy can be improved by adding higher order terms of course.

Two other aspects were highlighted. Firstly the compensation voltage depends on the direction of the temperature change. Normally oscillators are measured starting at the hot end of the temperature range and reducing the temperature. If measurements are taken starting at the cold end of the temperature range and increasing the temperature different compensation voltages are measured. These differences are about 0.1 to 0.2 ppm. Secondly there is a limit to the order of equation which should be used. Beyond fifth order the minimax error decreases much more slowly as the order of the equation increases and it would seem practical to limit the order of the terms used to five. Using a fifth order curve a minimax error of ± 15 mV can be reliably obtained and this corresponds to ± 0.3 ppm for a compensation voltage sensitivity of 20 ppm/volt. For tolerances less than this the hysteresis in the crystal oscillator compensation voltage becomes the limiting factor.

The design of a circuit which will generate suitable voltages can be carried out.

CHAPTER 6

COMPENSATION SYSTEM DESIGN

6.1 Semi-custom Integrated Circuits

The aim of this research project is to design a specialized integrated circuit which contains a large proportion of the circuits necessary for a temperature compensated crystal oscillator. The use of specialized integrated circuits has not been extensive in the past due to the high cost of development, the long development times and the large risk involved. This has been changed by the development of the component array. A series of standard integrated arrays are used which contain a large number of components in fixed locations. These integrated arrays are designed so that the components can be interconnected in many different ways. To create a specific circuit, a single interconnection pattern only is required. The advantages of this method are significant in that only one metallisation mask is required for the integrated circuit, which reduces the cost and the development time. It is possible to simulate the performance of the integrated circuit using a breadboard or a computer model thereby ensuring the circuit design is correct.

An analogue component array was used in this project. These arrays contain NPN transistors, PNP transistors and diffused resistors. In the UK both Ferranti and Plessey market these arrays and for this development the Ferranti "Monochip" was chosen. Ferranti have a large selection of sizes of array and have over 10 years expertise in their manufacture. They are made using a standard bipolar process commonly used for producing operational amplifiers, comparators and other analogue devices. Ferranti also supply "kit parts" which are 14 pin DIL integrated circuits containing typical transistors and resistors. A breadboard model can be built using these, and the circuit performance evaluated. This also provides a low cost method of initially evaluating this system of compensation.

6.2 Initial Design Comments

The design work was carried out in two stages. The compensation system was developed first and sample integrated circuits obtained. These circuits were used with oscillators built using discrete components and provided an interim solution.

Any amendments and improvements to this compensation system were added at the second stage of development when the oscillator and output stages were designed. Any external components were kept to a minimum and comprised mostly resistors and capacitors which could not be placed inside the integrated circuit.

Other general points regarding the design are given. The maximum operating temperature range should be $105^{\circ}C$ to $-55^{\circ}C$. The circuit should operate from a supply voltage of 4.5V to 15V. 4.5V represents the lowest supply voltage of a TTL circuit and 15V the highest supply voltage of a CMOS circuit. The current consumption should be about 700 μA since the total current consumption of a compensated oscillator is required to be less than 1mA.

The elements within the compensation system are arranged as shown in figure 4.4.

The voltage stabilizer has an output voltage of 3.6V so that the integrated circuit can operate at supply voltages as low as 4.5V. This allows for a 0.9V drop in the stabilizer.

The temperature coefficient of the output voltage should be zero and the line and load regulation small. The maximum output current required is about 1mA.

The temperature sensor should be contained within the integrated circuit and it is possible to use either the h_{fe} of a transistor or the base-emitter junction voltage of a transistor. The variation of h_{fe} between transistors is large and is not a dependable property. The base-emitter junction voltage is well characterised and can be used to generate a temperature dependent current.

The first multiplier takes the temperature varying current from the temperature sensor and multiplies it by itself to produce a quadratic varying current with temperature. This quantity is then multiplied by the output from the temperature sensor in the second multiplier to produce a cubic varying current with temperature.

The summing amplifier is a high gain, poor frequency response amplifier which is used to sum the four output voltages from the voltage stabilizer, temperature sensor, first multiplier and second multiplier. The output from the amplifier is then fed to a voltage controlled crystal oscillator.

6.3 Chebyshev Polynomials

The analysis in chapter 5 considered the summation of terms of the form $(t - t_i)^n$ where t_i could be taken as the value of parameter t at 26^{o}C. The voltages corresponding to each of the terms $(t - t_i)^n$ n=1,2,3 appear as plotted on the graph of figure 6.1.

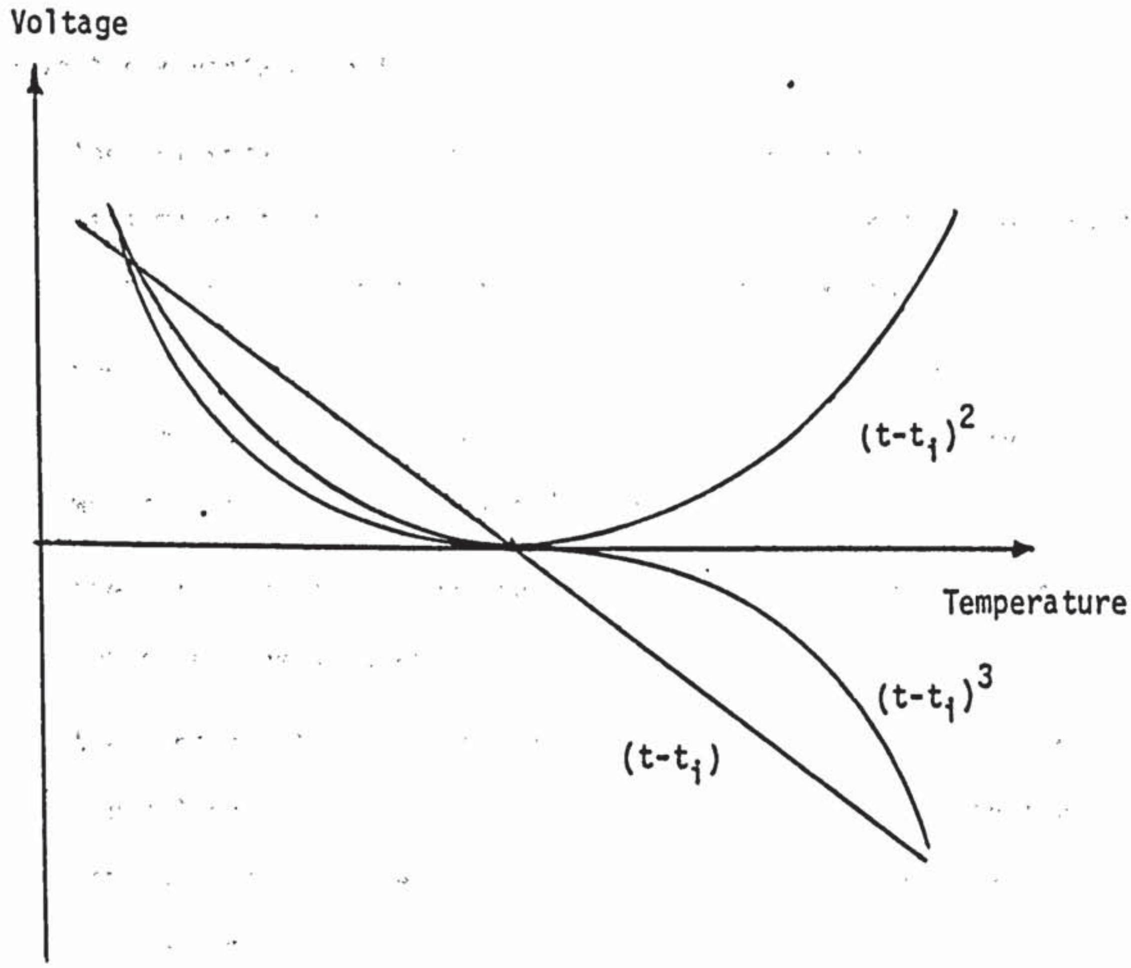


Figure 6.1 - Graphs of voltages

It is voltages of this form which should be generated by the integrated circuit. However, several advantages are gained by generating a set of curves known as the Chebyshev polynomials. The first four of these are shown in figure 6.2 and have equations:

$$T_0(x) = 1$$
$$T_1(x) = x$$
$$T_2(x) = 2x^2-1$$
$$T_3(x) = 4x^3-3x$$

These have the advantage that

(i) the compensation voltage for an optimum angle crystal is similar to the Chebyshev polynomial $T_3(x)$ and this polynomial is a good approximation to the compensation voltage.

(ii) the Chebyshev curves possess an equi-oscillation property making it easier to generate a compensation voltage curve which also has an equi-oscillation property.

(iii) the curves make maximum use of the output voltage range and the curves can be accurately measured.

(iv) the sensitivity of the coefficients should be smaller since the polynomial $T_3(x)$ represents a good approximation to the compensation voltage. The size of the coefficients will also be smaller.

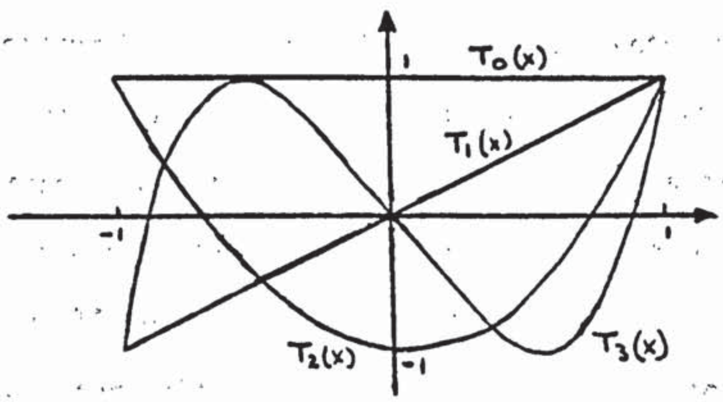


Figure 6.2 - Chebyshev polynomials

It would have been possible to return to the analysis of chapter 5 and repeat the work done. The size of the coefficients will change as will the sensitivity of the coefficients but the general conclusions will not.

The analysis concludes that it is possible to use this system of compensation and by generating Chebyshev-like polynomials the tolerances on the componentsshould increase. It is also not important that Chebyshev polynomials are generated exactly according to the equations given but that curves of a similar shape are generated. The breadboard model described generates Chebyshev-like curves.

6.4 Breadboard Model Design

6.4.1 Voltage Stabilizer

The voltage stabilizer uses a bandgap reference[22] to generate a temperature invariant voltage. The bandgap reference is the only convenient method of generating a temperature invariant voltage at low supply voltages. It can be easily designed using the components available in the "Monochip" array. The circuit is shown in Figure 6.3. Transistors 8,9,10,11,12,13 are considered as one single transistor. Transistors 104 and 105 form a current mirror so that the collector current of transistor 8-13 is one third of the collector current of transistor 14. When analyzing analogue circuits current mirrors are often seen. If two transistors of identical characteristics have the same base-emitter voltage then their emitter currents are the same. Transistors 13 and 14 operate at a current density of 1:18 and so have different base-emitter voltages. This difference can be measured across R6.

$$\Delta V_{BE} = \frac{kT}{q} \ln \frac{I_{14}}{I_{13}} = 75\text{mV at } 25^{o}\text{C}$$

This voltage has a temperature coefficient of $\frac{\partial \Delta VBE}{\partial T} = \frac{\Delta VBE}{T} = +0.252$ mV/K

Since the current in R6 is ⅓ of the collector current in transistor 14, the voltage across R7 is

$$\frac{\Delta V_{BE}}{R6} \cdot (3+1) \cdot R7 = 0.600V$$

The temperature coefficient of this voltage is

$$\frac{\Delta V_{BE}}{T} \cdot (3+1) \cdot \frac{R7}{R6} = +2.02 \text{ mV/K}$$

The temperature coefficient of the base-emitter junction voltage of a transistor is derived from the equation

$$V = \frac{kT}{q} \ln I/I_s$$

Now

$$\left(\frac{dV}{dT}\right)_{I \text{ constant}} = \frac{k}{q}\left(\ln I - \ln I_s\right) + \frac{kT}{q}\left(-\frac{1}{I_s} \cdot \frac{dI_s}{dT}\right)$$

$$I_s = A \exp\left(\frac{-E_G}{kT}\right)$$

where A = constant

E_G = forbidden energy gap of a semiconductor in electron volts

and
$$\left(\frac{dV}{dT}\right)_{I \text{ constant}} = \frac{V}{T} - \frac{kT}{q} \cdot \frac{1}{I_s} \cdot \frac{I_s E_G}{kT^2}$$

$$= \frac{V}{T} - \frac{E_G}{qT} = \frac{1}{T}\left(V - \frac{E_G}{q}\right)$$

Now $E_G = E_{GO} - T$

and for silicon = $1.21 - 3.6 \times 10^4 T$ electron volts[43]

Both V and E_G are proportional to temperature and so

$\left(\frac{dV}{dT}\right)_{I \text{ constant}}$ is independent of temperature

For silicon $\frac{E_G}{q}$ = 1.2V and from design data supplied by Ferranti[36] the base-emitter junction voltage of transistor 14 at 25°C is 0.61V

$$\therefore \left(\frac{dV}{dT}\right)_{I \text{ constant}} = \frac{1}{300}\left(0.61 - 1.2\right) = -1.97 \text{ mV/K}$$

In practice, the current in the diode changes and the diode voltage alters slightly. The current will vary linearly with temperature and since the logarithm of the current I/I_s is taken, this gives a slight variation of voltage from an exact linear variation. This is not significant in the multiplier circuits which will be described. Figure 6.13 shows the variation of current with temperature of two linear currents I_A and I_B which are generated by the temperature sensor. The derivation of the value of the temperature coefficient of the base-emitter voltage is important in the performance of the temperature sensor.

The voltage at the base of transistor 14 is therefore 1.21V with a temperature coefficient of 0.05mV/K. To achieve a better tolerance on the output voltage, the ratio of R6/R7, the scale factor of current mirror T104 and T105 and the number of transistors in the compound transistor 8-13 must be integers. These requirements are met using the circuit given.

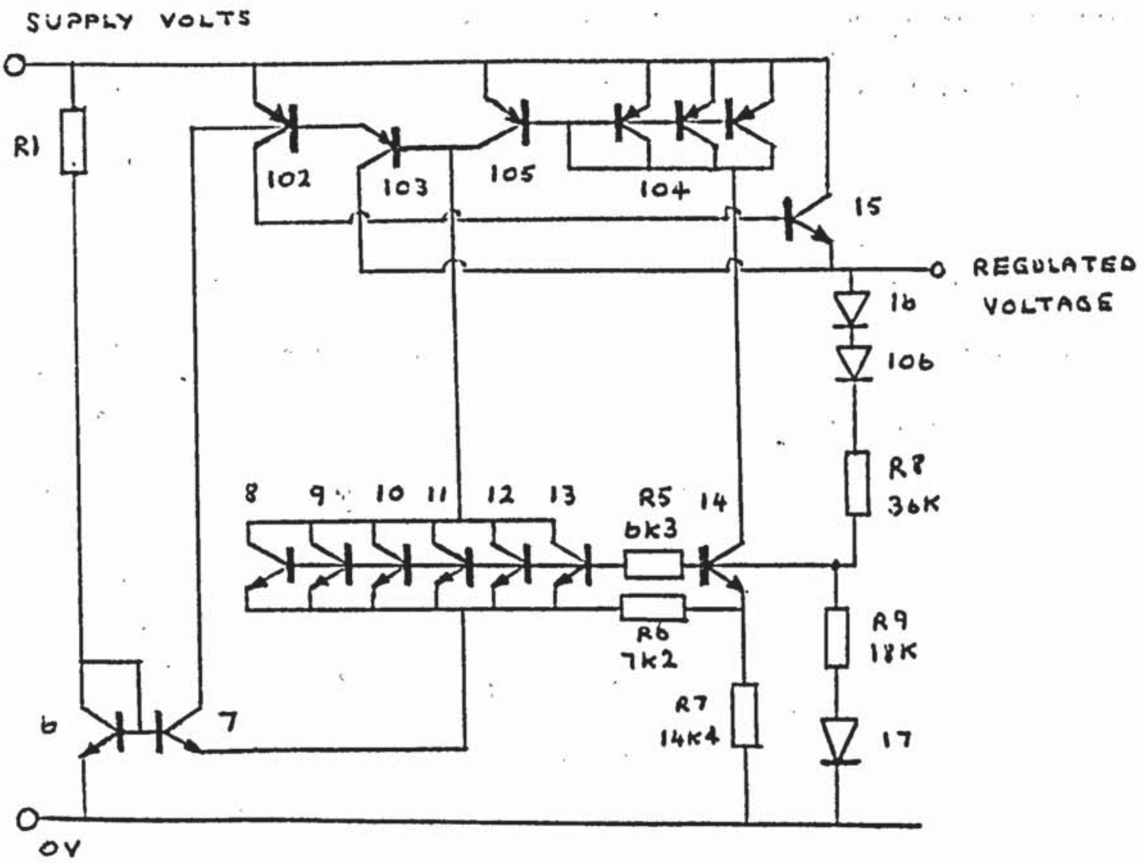


Figure 6.3 - Voltage Stabilizer circuit diagram

Transistors 102 and 103 are a Darlington pair which drive the NPN pass transistor 15. The 1.21V output from the base of transistor 14 is scaled up by x3 using R8,R9 and transistors 16,17 and 106. Transistors 6 and 7 form a start-up circuit since the regulator is stable when the REGULATED VOLTAGE is zero. When the supply voltage is turned on transistor 7 supplies base current for transistor 102 which turns on transistor 15. As the REGULATED VOLTAGE increases the bandgap reference turns on and transistor 7 is turned off. The bandgap reference is incorporated in a feedback amplifier. The base of transistor 102 is the inverting input and the base of transistor 14 (reference voltage) the non-inverting input.

R5 is used as a base current error correction resistor. In the circuit of figure 6.3 the base current for transistor 8-14 flows through R8 and transistors 16 and 106. This causes the output voltage to rise above its proper level. If E is taken as the value of the REGULATED VOLTAGE in the absence of base current for transistors 8-14 then E' resulting from considering R5 and the two base currents is given by

$$E' = E + R8\,(i_{b14} + i_{b8-13}) + i_{b8-13} \cdot R5 \cdot (3+1) \cdot \frac{R7}{R6}\left(1 + \frac{R8}{R9}\right)$$

This relation contains a term due to the base currents through R8 and an offsetting term due to the reduction of ΔV_{BE} by base current through R5. If E' is set equal to E, an expression for R5 can be found

$$R5 = R8\left(1 + \frac{i_{b14}}{i_{b8-13}}\right)\frac{1}{3+1} \cdot \frac{R6}{R7}\left(\frac{R9}{R8+R9}\right)$$

In this case

$$R5 = 6K\Omega$$

Layout considerations set this resistor at 6K3.

The regulator can supply 1mA output current.

The base current of transistor 103

$$= \frac{1mA}{\text{min. gain of T15} \times \text{min gain of T102} \times \text{min. gain of T103}}$$

$$= \frac{1mA}{80 \times 15 \times 80} = 0.1\mu A \text{ maximum}$$

This compares with a minimum current of 10μA flowing in the collector of transistor 105 and causes no serious mismatch in the current mirror. The regulator consumes typically 150μA of current although this is largely dependent on the supply voltage.

The tolerance on the output voltage can be calculated from the tolerance on the voltage across R7 and the tolerance on the base-emitter voltage of transistor 14. The voltage across R7 will vary due to the tolerance on the ratio of R7/R6 and this is 3%. The base-emitter voltage of transistor 14 varies by ±40mV and this gives a maximum temperature coefficient of ±0.16mV/K on the REGULATED VOLTAGE. Over the temperature range 105°C to -55°C this voltage could change by 30mV.

The output voltage is

$$(1.21 \pm (0.6 \times 0.03 + 0.05)) \times 3 \times \pm 1.03$$

$$= 3.63 \pm 0.21 \ \text{V}$$

The minimum supply voltage is the sum of:-

regulated voltage	= 3.63±0.21
V_{BE} of transistor 15 at -55°C	= 0.86±0.025
V_{CE} of transistor 102 at -55°C	= 0.07
	= 4.56±0.235

The maximum value of the minimum supply voltage is 4.795V and although this is not as low as 4.5V, it is acceptable. Most TCXOs are required to operate with a 5% supply voltage variation and for a TTL system the minimum supply voltage would be 4.75V. Units which operate from a supply voltage of 5V± 5% may not work correctly in some cases.

6.4.2 Temperature Sensor

The temperature sensor generates two currents I_A and I_B. The former has a negative temperature coefficient and is used to give an output voltage with a linear variation with temperature. The latter has a positive temperature coefficient. The circuit diagram is shown in figure 6.4.

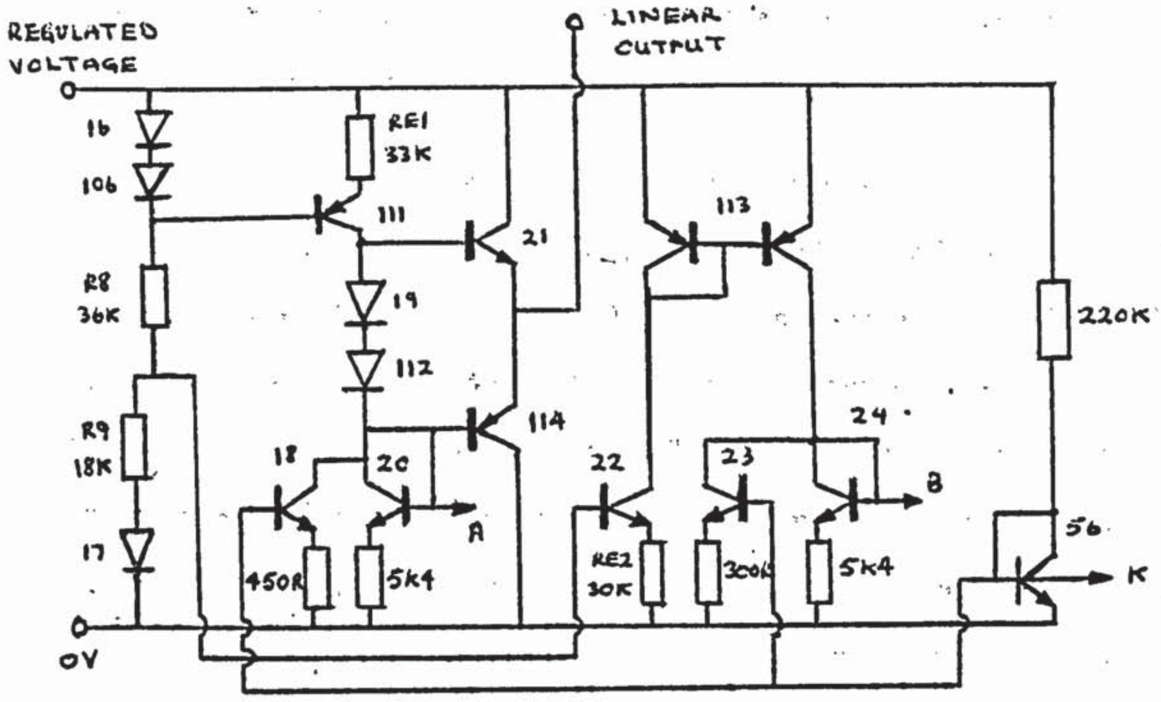


Figure 6.4 - Temperature sensor circuit diagram

A base-emitter junction voltage is applied across RE1 using transistors 16,106 and 111. The collector current of transistor 111 has a negative temperature coefficient. A constant current flows through transistors 16, 106, 17 and R8, R9. A constant current flows through transistor 18 and a temperature varying current through transistor 20 (I_A). Transistors 21, 19, 112 and 114 form a class AB output stage driven by transistor 20. The LINEAR OUTPUT voltage is approximately equal to two transistor base-emitter junction voltages. The current having a positive temperature coefficient is generated using transistor 22. The voltage at the base of transistor 22 is constant and the collector current of this transistor has a positive temperature coefficient. Transistor 113 acts as a current mirror and the emitter current of transistor 24 (I_B) is equal to the collector current of transistor 22 minus the constant current flowing in transistor 23. Transistors 20 and 24 are the driving transistors of a current mirror and are used to generate the two currents with negative and positive temperature coefficients at various points in the circuit.

Transistors 18 and 23 reduce the currents generated by RE1 and RE2 respectively, by a constant amount, and the effect of temperature on the currents flowing in transistors 20 and 24 is increased. The 450R resistor in the emitter leg of transistor 18 reduces the collector current of the transistor and there is not a 1:1 current mirror relationship between the driving transistor collector current (from transistor 56) and the driven transistor collector current[23]. The resistor is used to generate any value of collector current and forms a current divider of any ratio. The 300R resistor in the emitter leg of transistor 23 is used for this purpose. With reference to figure 6.5 the ratio of the currents is

$$\frac{I_{c1}}{I_{c2}} = e^{-\left(\frac{I_{c2} R}{V_T}\right)} \quad \text{where} \quad V_T = \frac{kT}{q}$$

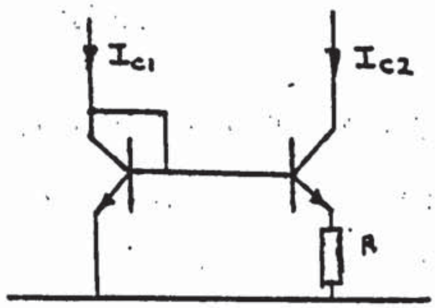


Figure 6.5 - Current mirror

The 5K4 resistors in the emitters of transistors 20 and 24 are matching resistors and improve the current mirror matching. The current ratio in the current mirror depends chiefly on the tolerance of the resistors and not critically on the matching of the base-emitter junction voltages.
Without the resistors a 5mV mismatch of the base-emitter junction voltages

causes a 20% current mismatch. With the resistors present the current mismatch is reduced to 4%.

6.4.3 First Multiplier Stage

The multiplier stages consist of conventional four transistor cross-coupled structures[24] as shown in figure 6.6.

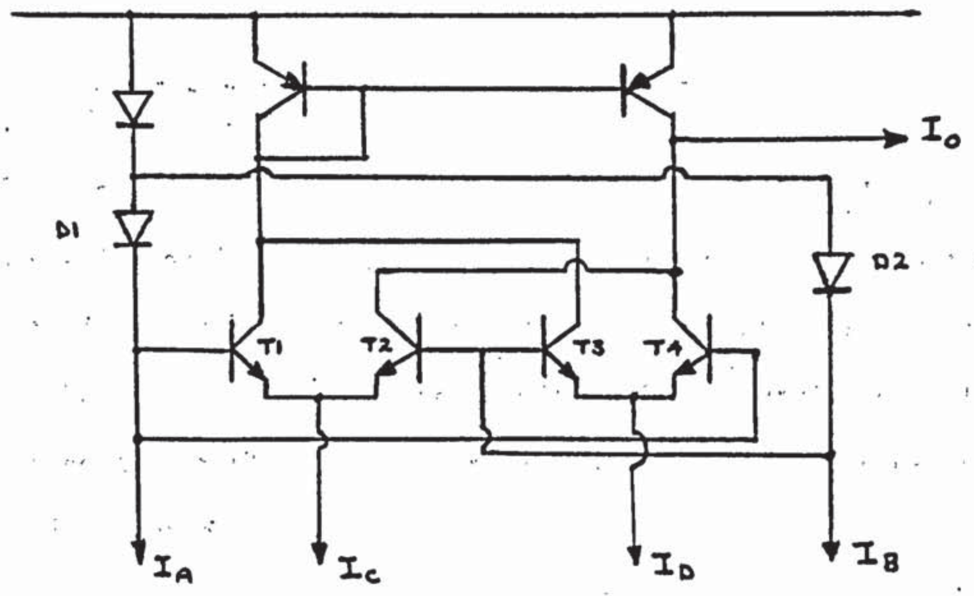


Fig 6.6 - Multiplier stage

An analysis of this circuit shows:

$$I_A = I_s\, e^{V_{D1}/V_T}$$

$$I_B = I_s\, e^{V_{D2}/V_T}$$

where I_s = reverse saturation current of diodes D1 and D2 and neglecting the base currents of T1, T2, T3 and T4

Let $V_i = V_{D1} - V_{D2} = V_T \ln I_A/I_B$

$$i_1 = \frac{I_c}{1 + e^{+V_i/V_T}} \qquad i_2 = \frac{I_c}{1 + e^{-V_i/V_T}}$$

$$\frac{i_1}{i_2} = \frac{1 + e^{-V_i/V_T}}{1 + e^{+V_i/V_T}} = \frac{I_B}{I_A} = \frac{i_4}{i_3}$$

$$i_1 = I_B/I_A \cdot i_2 \qquad i_3 = I_A/I_B \cdot i_4$$

$$i_2 = I_A/I_B \cdot i_1 \qquad i_4 = I_B/I_A \cdot i_3$$

$$i_1 + i_2 = I_c \qquad i_3 + i_4 = I_D$$

$$I_o = i_1 + i_3 - i_2 - i_4$$

$$= \frac{I_c}{1+I_A/I_B} + \frac{I_D}{1+I_B/I_A} - \frac{I_c}{1+I_B/I_A} - \frac{I_D}{1+I_A/I_B}$$

$$= \frac{I_B I_c}{I_A+I_B} + \frac{I_A I_D}{I_A+I_B} - \frac{I_c I_A}{I_A+I_B} - \frac{I_B I_D}{I_A+I_B}$$

$$= \frac{I_B(I_c-I_D) - I_A(I_c-I_D)}{I_A+I_B}$$

$$= \frac{(I_B-I_A)(I_c-I_D)}{I_A+I_B}.$$

Thus in the case of the first multiplier stage let $I_C - I_D = I_B - I_A$. Since I_A and I_B have equal and opposite temperature coefficients, the sum of I_A and I_B is constant and so the output current from this stage is the product of two currents with a linear variation with temperature giving a current with a quadratic variation with temperature.

I_0 is given by

$$I_o = I_2 = -\frac{(I_B-I_A)^2}{I_A+I_B} = k(I_B-I_A)^2$$

Figure 6.7 shows the circuit diagram of the first multiplier stage.

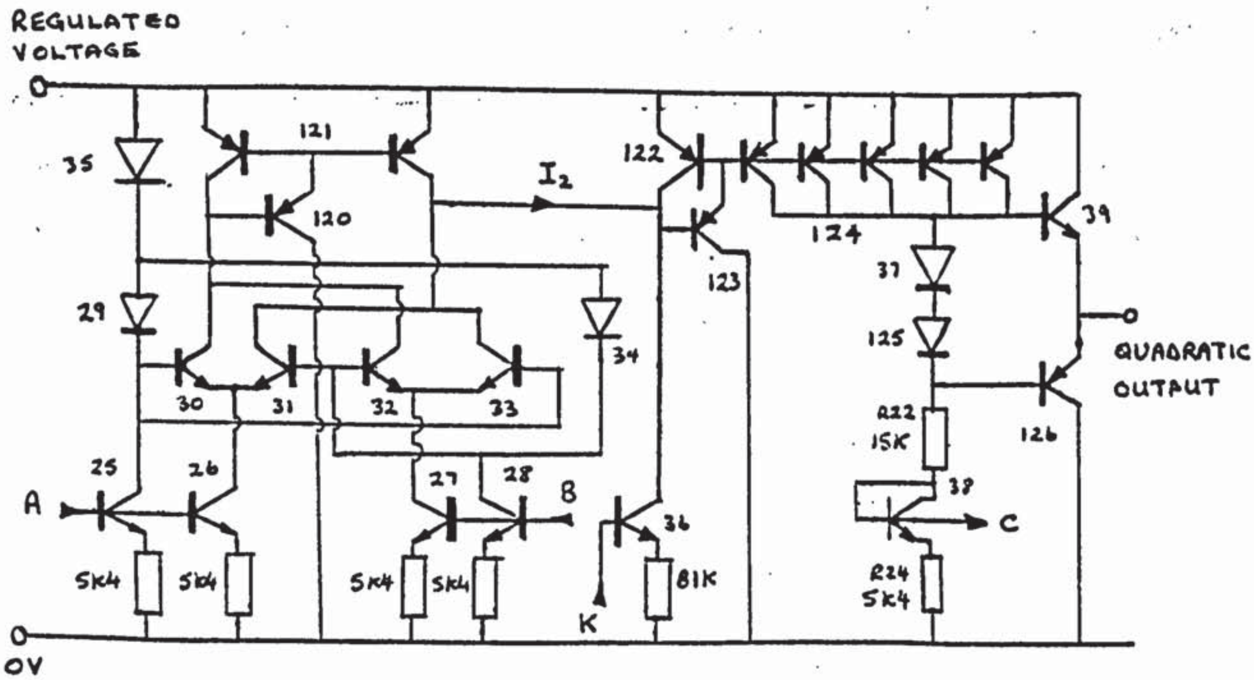


Figure 6.7 - First Multiplier circuit diagram

The multiplier stage consists of transistors 30,31,32 and 33. Transistors 25,26,27 and 28 are used as the driving transistors of the current mirrors supplying the multiplier stage. In this stage the bases of transistors 25 and 26 are connected to transistor 20 and the bases of transistors 27 and 28 are connected to transistor 24. Transistors 122, 123 and 124 form a current scaler of x5. Transistor 36 draws a small constant current through the current scaler. Transistors 37, 39, 125 and 126 form a class AB output stage driven by the voltage generated across R22, R24 and transistor 38. Transistor 38 and R24 are used to supply a quadratic varying current to the second multiplier stage.

6.4.4 Second Multiplier Stage

The second multiplier stage operates in a similar manner to the first stage. I_A and I_B are the same currents as used in the first stage. The quadratic varying current from the first stage is connected using transistor 41. Transistors 42, 43 and 44 supply a constant current such that when I_C-I_D is multiplied in this stage the output current I_3 has the form of a Chebyshev polynomial $T_3(x)$. A linear quantity multiplied by a (quadratic quantity - constant) yields a cubic quantity and a linear quantity. The output current from the multiplier is added to the constant current from transistor 52 and scaled by x3 in transistors 132, 133 and 134. The output voltage is derived from the voltage across R36, R37 and transistor 55. Figure 6.8 shows the circuit diagram of the second multiplier stage.

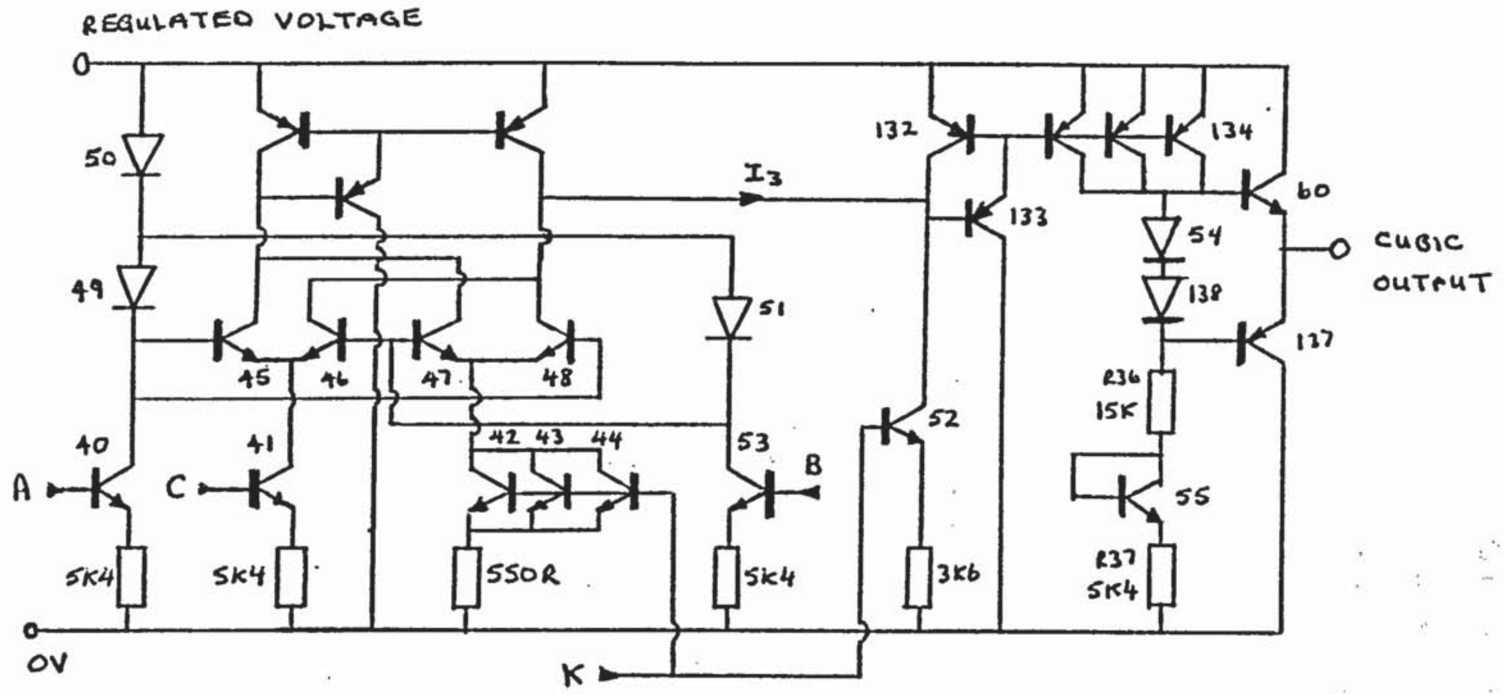


Figure 6.8 - Second Multiplier circuit diagram

The output impedance of the two multiplier stages can be calculated. The actual value of the output impedance depends on which transistor is switched on and upon how much current is being drawn by the load. However, a good estimate can be obtained. The impedance of the circuit configuration as shown in figure 6.8 is the output impedance of the emitter follower stage which is supplying current to the load. The output impedance of an emitter follower is given by [25]:

$$R_o = \frac{R_s + r_\pi}{1+\beta}$$

where R_s is the output impedance of the previous stage.
In this instance R_s is the load impedance of the transistor 134.
The resistance of the diodes is given by

$$r_d = \frac{V_T}{I\ (\text{in mA})} = \frac{0.026}{10\times10^{-3}} \doteqdot 2k6$$

$$\therefore\ R_s \doteqdot (2.6\times10^3 \times 3) + 15\times10^3 + 5.4\times10^3$$

$$\doteqdot 28k2$$

$$r_\pi = \beta/g_m \doteqdot \frac{100}{38\times0.02} \doteqdot 132\,\Omega$$

Hence

$$R_o \doteqdot \frac{28.2\times10^{-3} + 132}{101} \doteqdot 280\,\Omega$$

The values of the coefficient-setting resistors will typically be of a much higher value. An output impedance of 280Ω will not appreciably affect the output voltage.

6.4.5 Summing Amplifier

The summing amplifier is shown in figure 6.9. The input stage of the amplifier is a differential amplifier. Transistors 702 and 703 are the input transistors with transistors 71, 701 and 704 supplying an emitter current of 2.3 μA. Transistors 72, 73 and 74 are a current mirror which supplies a current to the output stage. This consists of a common emitter amplifier (transistor 77) and a class AB output stage (transistors 78, 79, 707 and 709). The other transistors are current mirrors supplying the bias currents to the output stage.

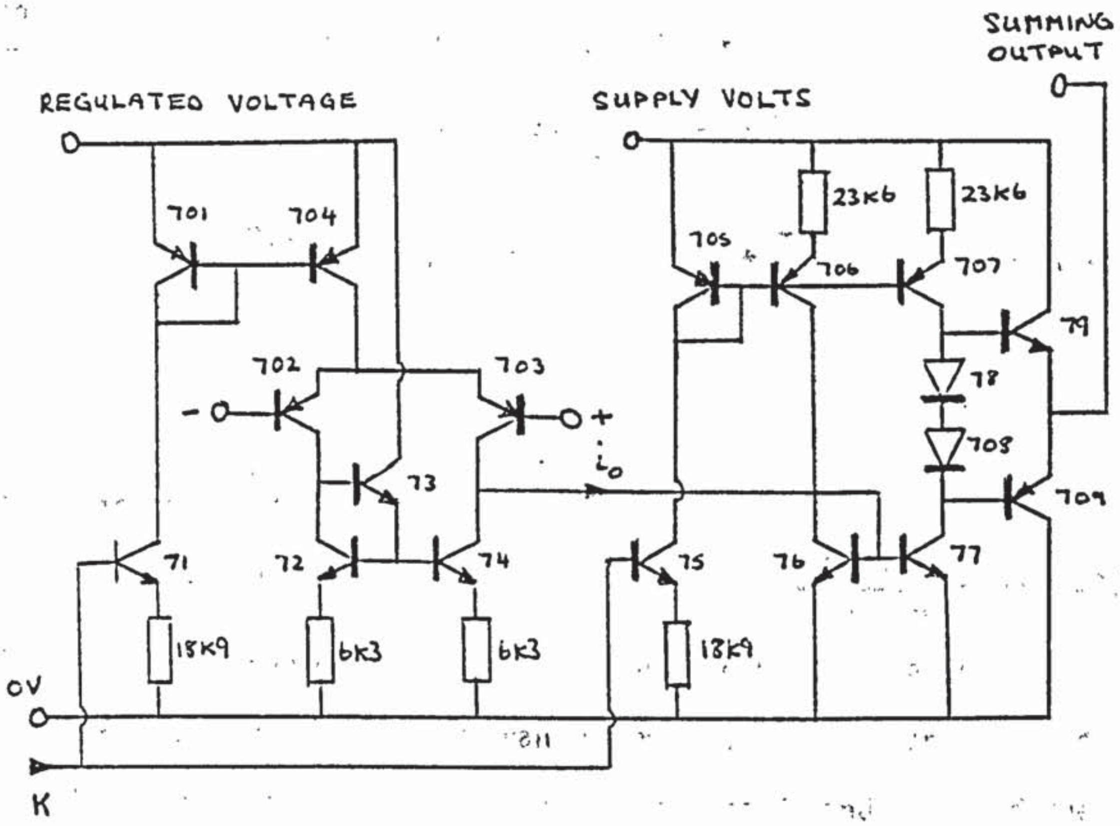


Figure 6.9 - Summing amplifier circuit diagram

The transconductance of the input stage can be calculated:

if i_0 is the output current flowing into the output stage then

$$i_o = I_{C703} - I_{C702}$$

$$= I_{EO}\left(e^{V_{BE703}/V_T} - e^{V_{BE702}/V_T}\right)$$

Let

$$V_{BE703} = V + \delta V_1$$

$$V_{BE702} = V - \delta V_2$$

so

$$V_i = (V + \delta V_1) - (V - \delta V_2) = \delta V_1 + \delta V_2$$

$$i_o = I_{EO}\left(e^{V/V_T} e^{\delta V_1/V_T} - e^{V/V_T} e^{-\delta V_2/V_T}\right)$$

$$= \frac{I_{C704}}{2}\left(1 + \frac{\delta V_1}{V_T} + \ldots - 1 + \frac{\delta V_2}{V_T} + \ldots\right) \text{ using a power series}$$

$$= \frac{I_{C704}}{2} \frac{V_i}{V_T}$$

I_{C702} is set at $2.3\mu A$ and so

transconductance $g_m = \frac{i_o}{V_i} = \frac{I_{C704}}{2V_T} = \frac{2.3\times10^{-6}}{2\times0.026} = 45\mu A/V$

The output impedance of the output stage can be calculated:

the current mirror formed by transistors 75, 705, 706 and 707 and the two 23K6 resistors supply $1\mu A$ to transistors 76 and 78. If the current from the input stage i_0 is zero the summing output is at its midpoint voltage. The output stage supply voltage is not connected to the REGULATED VOLTAGE so that a larger range of compensation voltages is possible.

The feedback resistor is nominally 150K and the loading effect of the voltage controlled oscillator is negligible.

The current i_0 flows into transistor 76 and is mirrored by transistor 77. The load resistance of transistor 77 is the output impedance of the current mirror (transistors 705/707) and the impedance of the emitter followers (transistors 79 and 709). The ac schematic of the output stage is given in figure 6.10.

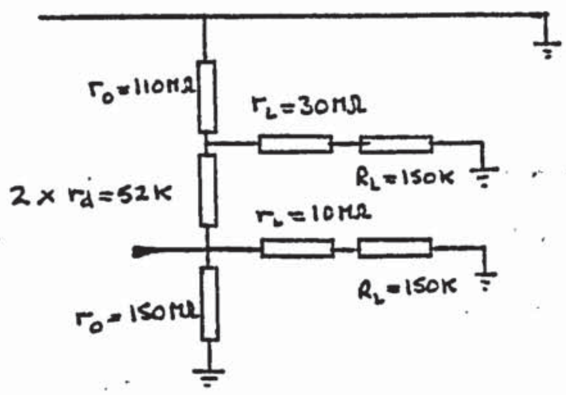


Figure 6.10 - Output stage impedances

for a current mirror[26] $r_o = \dfrac{\text{early voltage}}{\text{current through mirror}} = \dfrac{V_A}{I}$

and so for PNP current mirror 705, 707 $r_0 = \dfrac{110}{10^{-6}} = 110M\Omega$

and for NPN current mirror 76, 77 $r_0 = \dfrac{150}{10^{-6}} = 150M\Omega$

for an emitter follower[25] $r_i = \dfrac{\beta}{g_m} + (\beta+1)R_L$

and so for NPN transistor 79 $r_i = \dfrac{150}{38\times10^{-6}} + 151 \times 150\times10^3 = 30M\Omega$

and for PNP transistor 709 $r_i = \dfrac{50}{38\times10^{-6}} + 61 \times 150\times10^3 = 10M\Omega$

for a diode the impedance is $r_d = \dfrac{V_T}{I}$

$$= \frac{0.026}{10^{-6}} = 26K$$

The impedance of the output stage depends on whether transistor 79 or 709 is conducting. If transistor 79 is conducting

Z = 150MΩ in parallel with 110MΩ in parallel with 30MΩ

= 20.3MΩ

If transistor 709 is conducting

Z = 150MΩ in parallel with 110MΩ in parallel with 10.3MΩ

= 8.85M

The overall gain of the output stage is approximately $\frac{20.3 + 8.85}{2} \times 10^6$

and the gain of the amplifier is $45\times10^{-6} \times \frac{20.3+8.85}{2}\times10^6 = 655$

for a summing amplifier acting as an inverting amplifier[27]

$$\frac{V_o}{V_i} = \frac{-Rf/R_1}{1+\frac{1}{A}\left(\frac{Rf}{R_1}+1\right)}$$

If A=655, $Rf/R_1 = 2$

$$\frac{V_o}{V_i} = \frac{-2}{1+3/655} = -1.9908$$

This is an error of 0.9%

for a summing amplifier acting as a non-inverting amplifier[27]

$$\frac{V_o}{V_i} = \frac{A}{1+A\frac{R_1}{R_1+Rf}}$$

If A=655, $R_1/R_1+Rf = 1/2$

$$\frac{V_o}{V_i} = \frac{655}{1+655/2} = 1.9939$$

This is an error of 0.6%.

With the nominal gain of approximately 655, the error due to a finite gain A is 0.9% maximum. The resistors used to set the gain of each coefficient are of 2% tolerance and this value of amplifier gain is adequate. The loss in accuracy due to the finite gain can be offset by adjusting the value of the feedback resistor in the summing amplifier since the weighting coefficients of both positive and negative terms are reduced.

6.5 Calculation of the Weighting Coefficients

The tolerances on the components used in the integrated circuit. are such that the output voltages which represent the constant, linear, quadratic and cubic curves require to be measured at various temperatures across the operating temperature range to determine their value. The variation in crystals is such that the compensation voltage of a crystal oscillator also requires to be measured at various temperatures across the operating temperature range. The data from which the weighting coefficients are calculated, consists of

(a) compensation voltage of the oscillator

(b) output voltage from the voltage stabilizer labelled REGULATED VOLTAGE

(c) output voltage labelled LINEAR OUTPUT

(d) output voltage labelled QUADRATIC OUTPUT

(e) output voltage labelled CUBIC OUTPUT

If V is the compensation voltage and V_R, V_L, V_Q and V_C the output voltages representing the four components of the power series then the equation

$$a_o V_{Rn} + a_1 V_{Ln} + a_2 V_{Qn} + a_3 V_{Cn} = V_n + e_n$$

is true at any temperature n. e_n represents the error between the synthesized compensation voltage and the desired compensation voltage and has the Chebyshev equi-oscillation property. Using multiple linear regression a set of coefficients can be found which minimize the error according to a least squares criterion. Altering the desired compensation voltages causes the coefficients to change and the errors can be forced to have the equi-oscillation property. This process is the same as that used in the computer program DAFIT1. A better approach is to use the exchange algorithms.[28,29]

The algorithm involves four steps:

1. Choose any group of n+2 points: the degree of the minimax polynomial is n
2. Find the Chebyshevline for this group of points. The Chebyshev is the equal error line of alternating minima and maxima of value h.
3. Compute the errors at all data points for the Chebyshev line. Call the largest of these h_i values (in absolute value) H. If $|h| = H$, then the Chebyshev line is the minimax polynomial for the entire set of points. If $|h| < H$ proceed to step 4.
4. Choose a new group of points as follows. Add to the old group a data point at which the greatest error H occurs. Discard one of the former points in such a way that the remaining group has errors of alternating sign. Return to step 2.

A computer program called COEFFS was written to evaluate the coefficients from the measured input data using the exchange algorithm. The program also calculates the required resistor values and gives which value should be fitted to an oscillator during manufacture. See Appendix 2 for a full description of the program.

A typical configuration of resistors and summing amplifier is shown in figure 6.11.

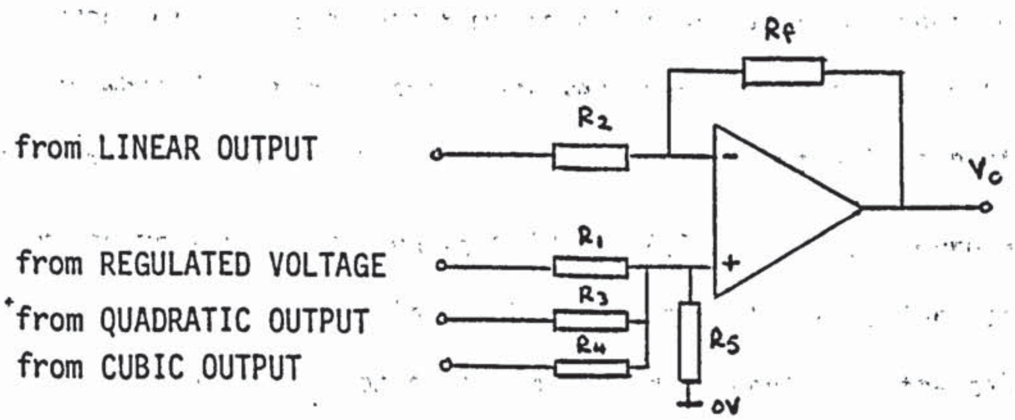


Figure 6.11 - Summing amplifier

The values of the gains are as follows:

-ve terms e.g. $\frac{R_f}{R_2}$

+ve terms e.g. $\frac{R_f}{R_1}$, $\frac{R_f}{R_3}$, $\frac{R_f}{R_4}$

given that

$$\frac{1}{R_S} = \frac{1}{R_f} + \frac{1}{R_2} - \frac{1}{R_1} - \frac{1}{R_3} - \frac{1}{R_4}$$

See Appendix 3.

6.6 Breadboard Model Evaluation

A complete breadboard model as shown in figure 6.12 was built. This model was used to verify the performance of the design. Figures 6.13 and 6.14 show the variation with temperature of the currents labelled I_A, I_B, I_2 and I_3. These are the currents flowing in resistors RA, RB, R2 and R3. Figure 6.15 shows the variation with temperature of the voltages labelled REGULATED VOLTAGE, LINEAR OUTPUT, QUADRATIC OUTPUT and CUBIC OUTPUT. The last two voltages are tilted since the PNP transistor and the diode connected transistor in the output stage add two base-emitter voltages to the voltage generated across the 15K and the 5K4 resistors. These graphs of the voltage variations were consistent over a number of seperate measurements and there were no significant variations when different transistors were used. These process variations in the integrated circuit are simulated by changing the "kit parts" used.

The currents measured for figures 6.13 and 6.14 show good correlation with the currents which should be obtained using the equations of 6.4.3. Table 6.1 gives a comparison of the measured currents and the calculated currents.

temperature (^oC)	I_A measured (μA)	I_B measured (μA)	I_2 measured (μA)	I_2 calculated (μA)	I_3 measured (μA)	I_3 calculated (μA)
120	2.5	12.0	38.0	35.0	8.7	1.7
100	3.7	10.5	23.0	20.5	33.0	26.7
85	4.4	9.6	15.3	13.9	38.1	32.0
70	5.2	8.7	9.6	9.6	36.6	31.0
60	5.7	8.0	6.8	6.3	32.6	28.1
50	6.1	7.3	5.1	4.8	27.1	23.4
40	6.6	6.8	4.4	4.4	21.3	18.2
30	6.9	6.2	4.5	4.4	15.9	13.8
20	7.4	5.5	5.5	5.6	10.0	7.9
10	7.9	5.0	7.3	7.5	5.9	4.2
0	8.2	4.3	9.2	10.3	3.1	1.8
-10	8.7	3.6	14.0	14.6	3.8	3.3
-20	9.1	3.0	18.1	19.6	7.1	6.4
-35	9.7	2.3	25.5	27.0	17.2	17.4
-55	10.9	1.3	39.4	42.0	40.1	35.4

Table 6.1 - Accuracy of computer model

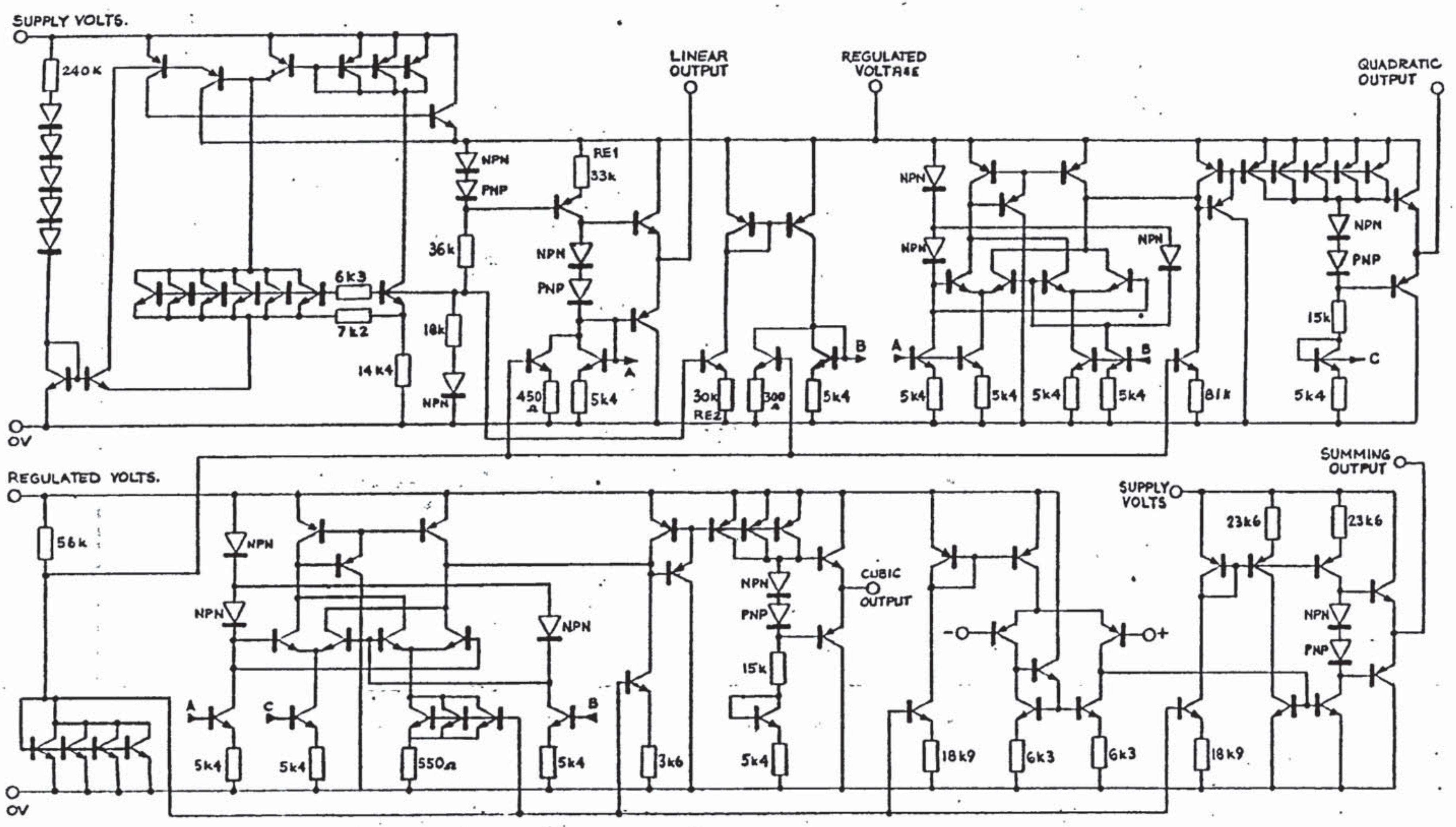


Figure 6.12 - Compensation system circuit diagram

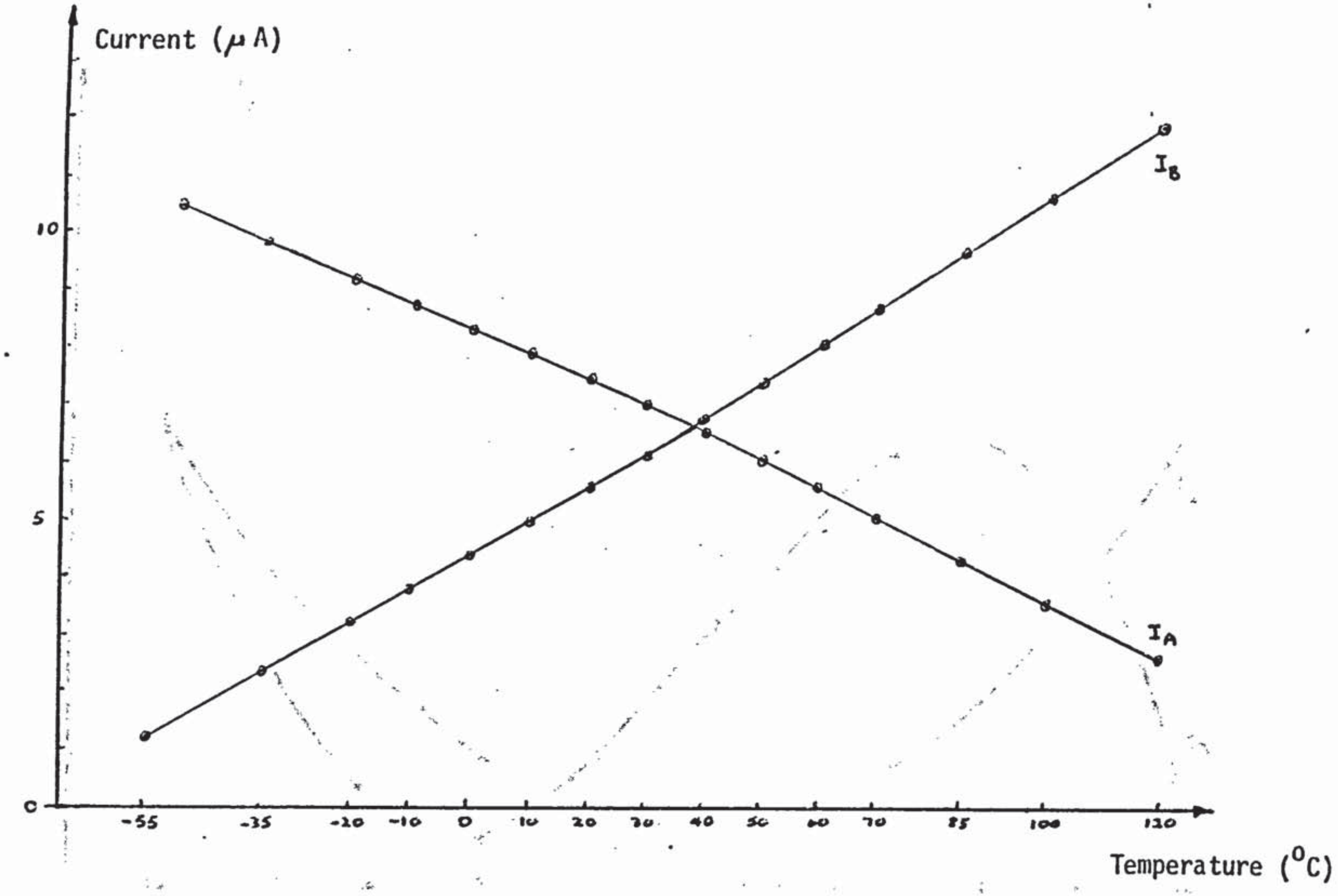


Figure 6.13 - Variation of current with temperature 1

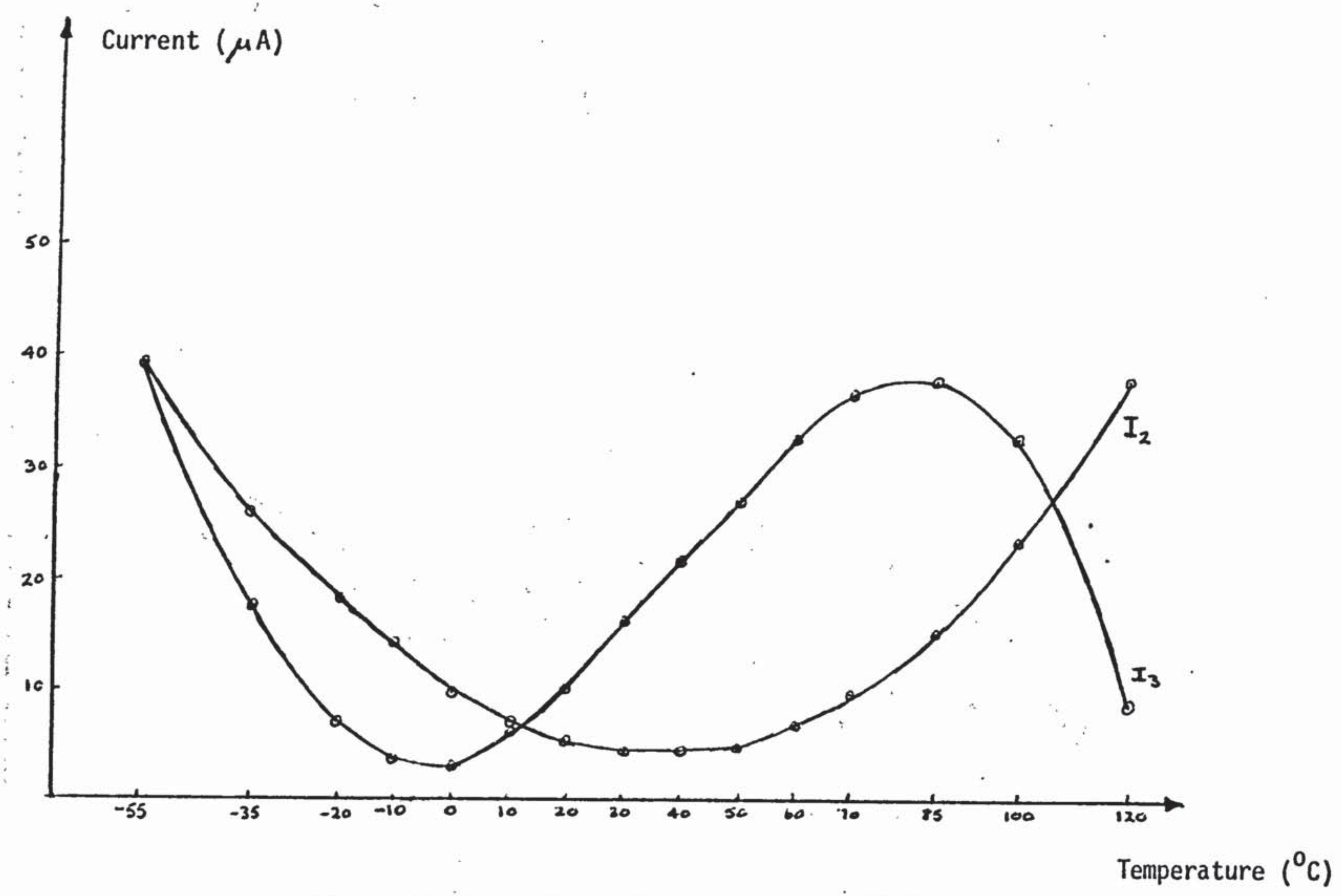


Figure 6.14 - Variation of current with temperature 2

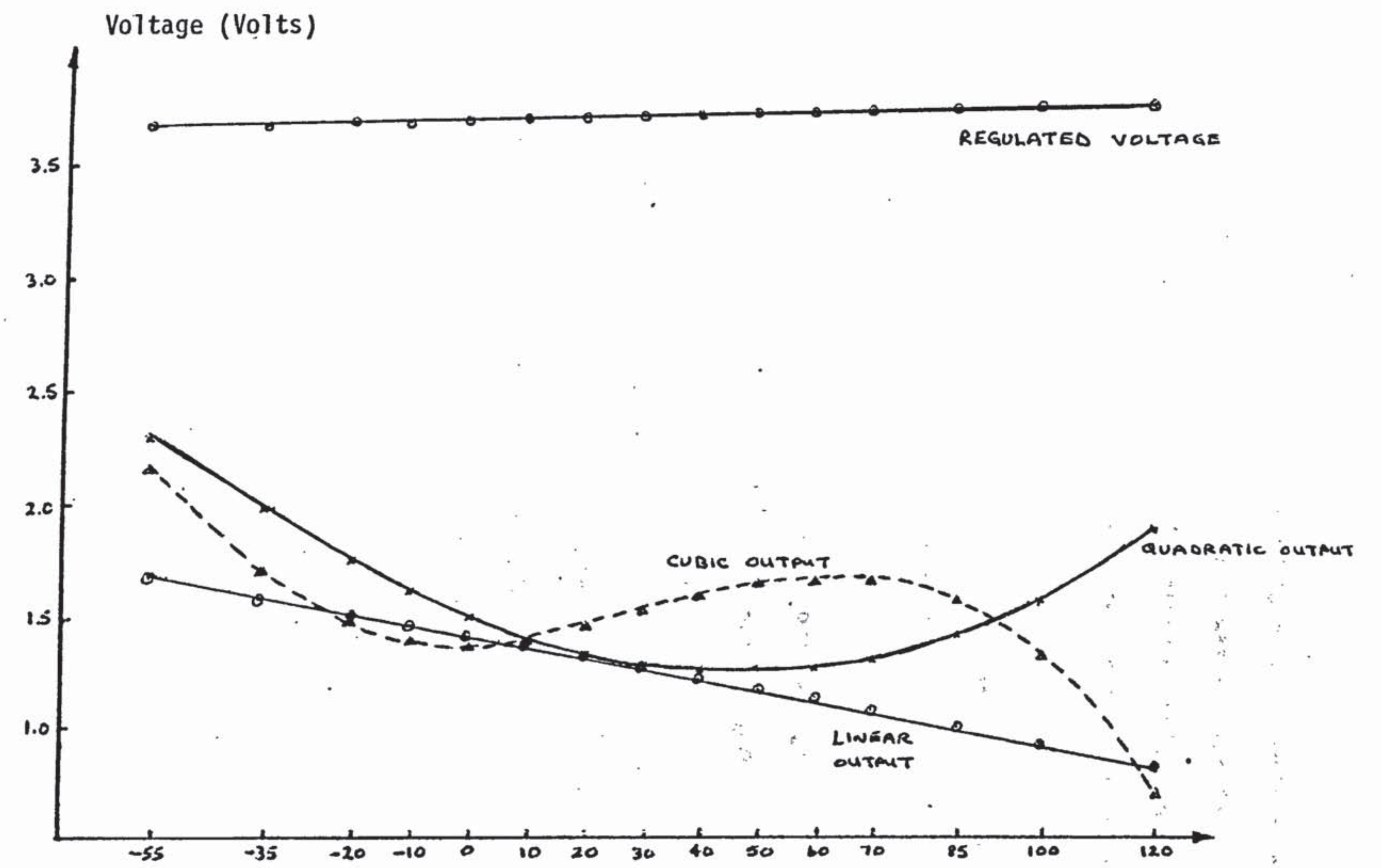


Figure 6.15 - Variation of voltage with temperature 1

The variations between the measured and the calculated values are due to small errors in the values of I_A and I_B which produce large errors in I_3 through the multiplication. There are also errors in the multiplication itself and in the gain of the current mirrors which become significant.

The equations can be extended to give a complete computer model of the compensation system. A program was written for a Hewlett-Packard HP-97 programmable calculator. A listing of this program is given in Appendix 4. A list of the constants required by the program is also given in the Appendix.

Table 6.2 shows the results of some analysis using this model. The format of each group of six answers is:

current flowing in RE1 in μA
current flowing in RE1 in μA
current I_A in μA
current I_B in μA
current I_2 in μA
current I_3 in μA

Temperature (^{o}C)	First Analysis	Second Analysis	Third Analysis	Fourth Analysis
120	15.4	13.9	13.6	17.5
	26.3	26.0	24.8	26.3
	4.56	3.13	3.13	6.70
	14.9	14.6	13.4	14.9
	31.9	41.4	36.0	19.9
	9.62	-10.5	0.64	25.5
85	17.9	16.4	16.4	20.0
	24.0	23.7	22.5	24.0
	7.06	5.63	5.63	9.20
	12.6	12.3	11.1	12.6
	12.1	16.6	13.3	6.9
	30.0	29.0	30.9	26.8
30	21.8	20.4	20.4	23.9
	20.3	20.0	19.0	20.3
	11.0	9.56	9.56	13.1
	8.93	8.60	7.65	8.93
	5.31	4.50	5.31	8.24
	10.7	13.8	10.2	6.70
0	23.9	22.5	22.5	26.1
	18.3	18.0	17.1	18.3
	13.1	11.7	11.7	15.3
	6.93	6.60	5.74	6.93
	13.8	11.4	14.4	19.9
	5.19	4.32	4.51	9.41
-55	27.9	26.4	26.4	30.0
	14.7	14.3	13.7	14.7
	17.1	15.6	15.6	19.2
	3.27	2.93	2.25	3.27
	51.0	47.0	54.3	60.7
	66.3	59.8	78.7	89.2

Table 6.2 - Computer model analysis 1

Temperature (^oC)	First Analysis	Second Analysis	Third Analysis	Fourth Analysis
120	15.2 26.6. 4.41 15.2 34.0 5.89	15.5 26.1 4.71 14.7 29.8 13.0	15.4 26.3 4.76 15.1 31.3 9.73	15.4 26.3 4.36 14.7 32.5 9.3
85	17.7 24.2 6.88 12.8 13.3 29.9	18.0 23.8 7.24 12.4 11.0 30.0	17.9 24.0 7.26 12.8 11.9 29.1	17.9 24.0 6.86 12.4 12.3 30.7
30	21.6 20.5 10.8 9.14 4.92 12.0	22.0 20.1 11.2 8.73 5.78 9.50	218 20.3 11.2 9.13 5.29 10.3	218 20.3 10.8 8.73 5.43 10.8
0	23.7 18.5 12.9 7.12 12.6 4.92	24.2 18.2 13.4 6.75 15.1 5.69	23.9 18.3 13.3 7.13 13.6 4.84	23.9 18.3 12.9 6.73 14.1 5.14
-55	27.6 14.8 16.8 3.41 48.5 60.1	28.1 14.5 17.3 3.12 53.6 72.9	27.9 14.7 17.3 3.47 50.1 63.4	27.9 14.7 16.9 3.07 52.1 68.9

Table 6.3 - Computer model analysis 2

The first analysis shows the values of the currents using nominal values of parameters.

The second analysis has the base-emitter voltage at its lower limit of 0.58V and the bandgap reference voltage at 1.17V. Although current I_3 is negative at 120^oC this can be changed to a positive value by altering RE1 and RE2. In the third analysis RE2 has been changed to 31k5. The fourth analysis has the base-emitter voltage set at its upper limit of 0.68V and the bandgap voltage at 1.28V. RE2 is set at 30k. RE1 and RE2 are external resistors since the internal resistors which are available in the integrated circuit have non-linear temperature coefficients. Their resistance changes by -5% between ambient temperatures of $25^{\circ}C$ and $-55^{\circ}C$ and by +15% between $25^{\circ}C$ and $125^{\circ}C$. These resistors also have to be set to different values on each integrated circuit due to component tolerances. There is also a 56K resistor which is external to the integrated circuit due to layout problems. The tolerance on these components can be investigated using the computer model. Table 6.3 gives the results of this analysis. In the first analysis RE1 is 1% high in value and RE2 is 1% low in value. The second analysis gives the currents if RE1 is 1% low in value and RE2 is 1% high in value. The third analysis studies variations in the 56k resistor. The resistor is 2% high in value in the third analysis and 2% low in value in the fourth analysis. The tolerance on this resistor is not critical. The tolerance of 1% for RE1 and RE2 is needed since the resistors which will be fitted to an oscillator will be slightly different to those used to determine the correct values of RE1 and RE2.

The values of RE1 and RE2 are found by replacing RE1 and RE2 by variable resistors of the appropriate values. The ends of these resistors are not connected to REGULATED VOLTAGE and OV but are connected to triangular waves of a suitable amplitude such as to simulate the variation of voltage across RE1 and RE2 which results from the variation of ambient temperature. Values of RE1 and RE2 are found such that the integrated circuit always generates a suitable set of voltages over the operating temperature range.

6.6.1 Compensation of a prototype oscillator

The breadboard model was now used to compensate a prototype oscillator. The four output voltages from the breadboard model were measured at 15 temperatures from 120°C to -55°C. The compensation voltage of a voltage controlled crystal oscillator was measured at these temperatures and this group of measurements used by the computer program COEFFS. The measurements are given in Table 6.4 and the results from the computer in Table 6.5.

Temperature (°C)	Compensation Voltage (volts)	Regulated Volts (volts)	Linear Output (volts)	Quadratic Output (volts)	Cubic Output (volts)
120	0.876	3.760	0.797	1.877	0.649
100	1.467	3.751	0.918	1.554	1.307
85	1.689	3.742	0.996	1.400	1.522
70	1.752	3.734	1.072	1.291	1.649
60	1.719	3.729	1.125	1.240	1.653
50	1.644	3.724	1.178	1.212	1.620
40	1.555	3.718	1.226	1.209	1.568
30	1.464	3.713	1.269	1.229	1.510
20	1.365	3.707	1.321	1.286	1.440
10	1.295	3.702	1.365	1.361	1.387
0	1.250	3.696	1.416	1.473	1.348
-10	1.253	3.691	1.467	1.602	1.369
-20	1.309	3.685	1.511	1.734	1.458
-35	1.530	3.678	1.579	1.959	1.696
-55	2.196	3.669	1.670	2.283	2.148

Table 6.4 - Prototype oscillator voltage measurements

The coefficients are -

constant term 0.06981

linear term -0.6887

quadratic term 0.1586

cubic term 1.243

Using a 150K feedback resistor in the summing amplifier the values of the resistors used were:

R1 1M8 + 330K

R2 200K + 18K

R3 820K + 130K

R4 120K

R5 680K + 10K

Compensation Voltage (volts)	Synthesized Voltage (volts)	Error (mV)	Measured Voltage (volts)	Error from compensation voltage (mV)
0.876	0.818	-58	0.817	-59
1.467	1.501	34	1.527	60
1.689	1.747	58	1.753	64
1.752	1.777	25	1.798	46
1.719	1.737	18	1.758	39
1.644	1.655	11	1.671	27
1.555	1.556	1	1.573	18
1.464	1.457	-7	1.471	7
1.365	1.343	-22	1.351	-14
1.295	1.258	-37	1.259	-36
1.250	1.192	-58	1.181	-69
1.253	1.203	-50	1.183	-70
1.309	1.304	-5	1.279	-30
1.530	1.588	58	1.560	30
2.196	2.138	-58	2.129	-67

Table 6.5 - Accuracy of compensation 1

Table 6.5 also includes the compensation voltage which was generated by the model when the coefficient-setting resistors had been fitted. The error between the compensation voltage which was generated and the desired compensation voltage is given graphically in figure 6.16 with the theoretical error. The curves are very similar in shape and the small differences between the curves are due to the inaccuracies in the values of the resistors which are used to set the coefficients and to the finite value of the gain of the summing amplifier. The accuracy of the voltages which are generated by the model and the accuracy of the summation of the voltages was rechecked several times, and no problems were encountered. It is intended to use a voltage controlled oscillator with a sensitivity of 10ppm/volt in the majority of applications. A minimax error of 70mV is therefore equivalent to a frequency tolerance of $\pm$0.7ppm.

6.7 Stability of Transistor Parameters

The performance of the compensation system in the long term relies on the stability of the base-emitter junction voltage of a transistor and on the stability of the resistors used to set the coefficients.

Some information is available on the stability of resistors. It is difficult however to obtain information on the stability of a resistor when used in the particular environment of a TCXO. Two important figures are the effect of temperature cycling and the long term endurance.

The oscillators will eventually use chip resistors mounted on a hybrid substrate and the stability of both these components is determined by the stability of a thick-film resistor. The long term stability of a thick-film resistor is typically less than $\pm$ 0.25% after 5000hrs at 125°C at maximum rated power. The power levels of the

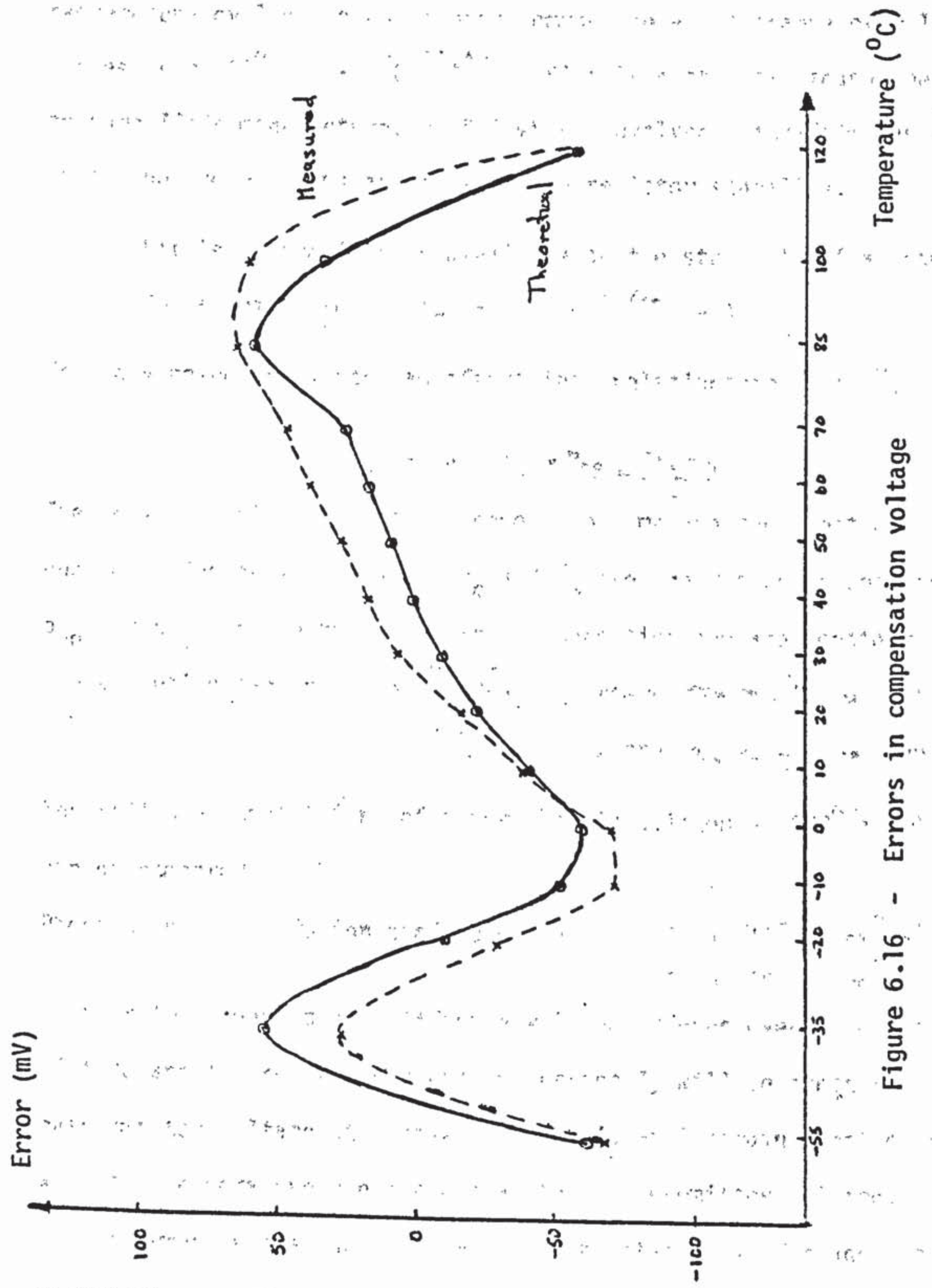


Figure 6.16 - Errors in compensation voltage

resistors used in the oscillator will be well within the maximum rated conditions resulting in even better stability. Slow temperature cycling is not detrimental to most resistor materials and only soldered adhesion is likely to be a problem. Typical figures for rapid temperature cycling which is most common are ±0.3% change over 100 cycles from 120°C to -55°C.[41,42] Finally historical measurements on some TCXOs manufactured by STC which involved thick-film resistors have shown no problems associated with resistor stability.

Little information is available on the stability of a transistor.

For a transistor $I_E = I_S (e^{V_{BE}/V_T} - 1)$

The only component which can affect this relationship is I_s[30]. Now

$$I_s = eA \left(\frac{D_p p_{no}}{L_p} + \frac{D_n n_{po}}{L_n} \right)$$

The only variables which can change with time are the junction area A due to diffusion with time. D_p and D_n are the diffusion constants. p_{no} and n_{po} are the minority carrier densities and are constant.

The diffusion length $L_p = \sqrt{D_n \tau_e}$ where $\tau_e = \frac{1}{r_{po}}$ = constant and

$L_n = \sqrt{D_p \tau_h}$ where $\tau_h = \frac{1}{r_{no}}$ = constant

The diffusion coefficient of impurities in silicon at 100° and 1000°C can be compared e.g.

Phosphorus - diffusion coefficient at 1000°C $= 10^{-14}$ cm^2 s^{-1}

100 °C $= 10^{-23}$ cm^2 s^{-1}.

Thus at 100°C negligible diffusion will occur even over a long period of time and the reverse saturation current I_s will be constant. The base emitter voltage at a fixed current I_E will remain constant as will the temperature coefficient at that base-emitter voltage. This is confirmed by the published work on the stability of transistor parameters. Three references are given (31,32,33) and all three papers state that the primary electrical characteristics show no discernable change with time.

6.8 Increasing the Accuracy of Compensation

Although the existing design generates only the first four terms of the power series given in section 4.4, there is no reason why additional high order terms cannot be added to increase the accuracy of compensation. The second breadboard model was extended to generate voltages with a fourth and fifth order variation with temperature.

6.8.1 Third Multiplier Stage

The present design can easily be extended to generate a voltage with a fourth order variation with temperature. The circuit is similar in configuration to the circuit used to generate a voltage with a cubic variation with temperature. Figure 6.17 shows the circuit used. The operation of the circuit is exactly the same as the operation of the second multiplier stage. The driving transistor in the output of the second multiplier (transistor 55 in figure 6.8) is connected to point C with currents I_A and I_B flowing in the transistors at points A and B. Figure 6.18 shows the variation of output voltage with temperature.

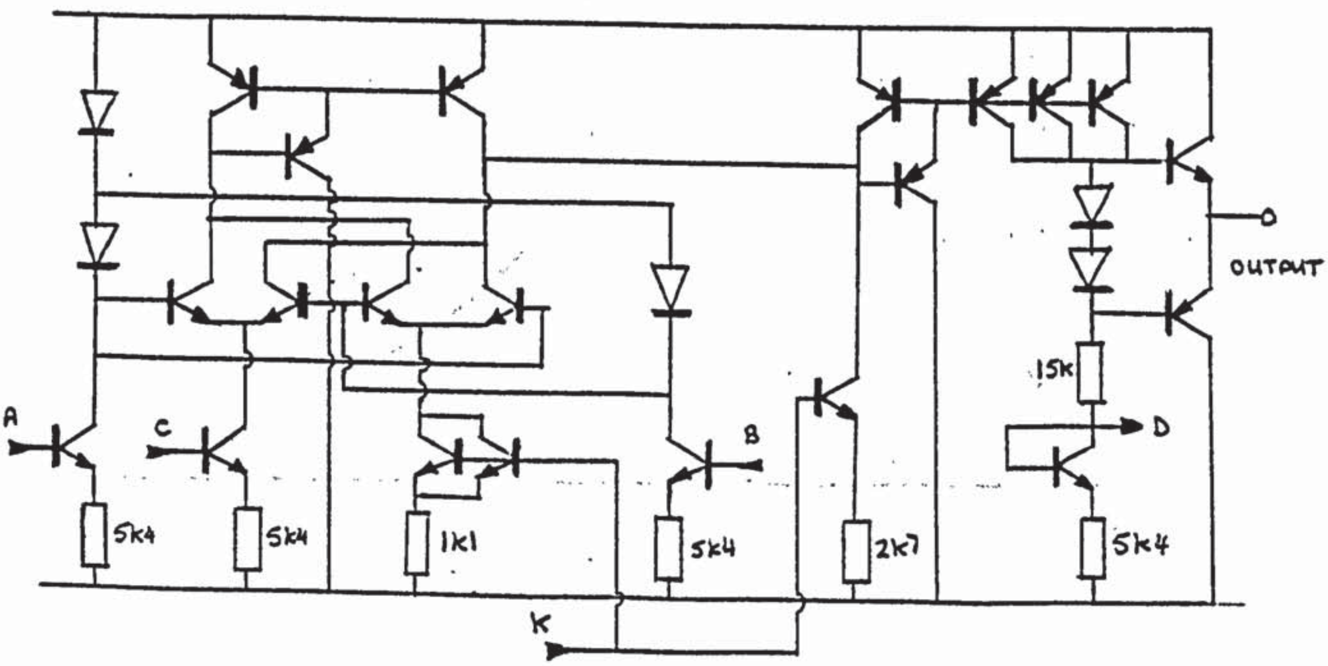


Figure 6.17 - Third Multiplier circuit diagram

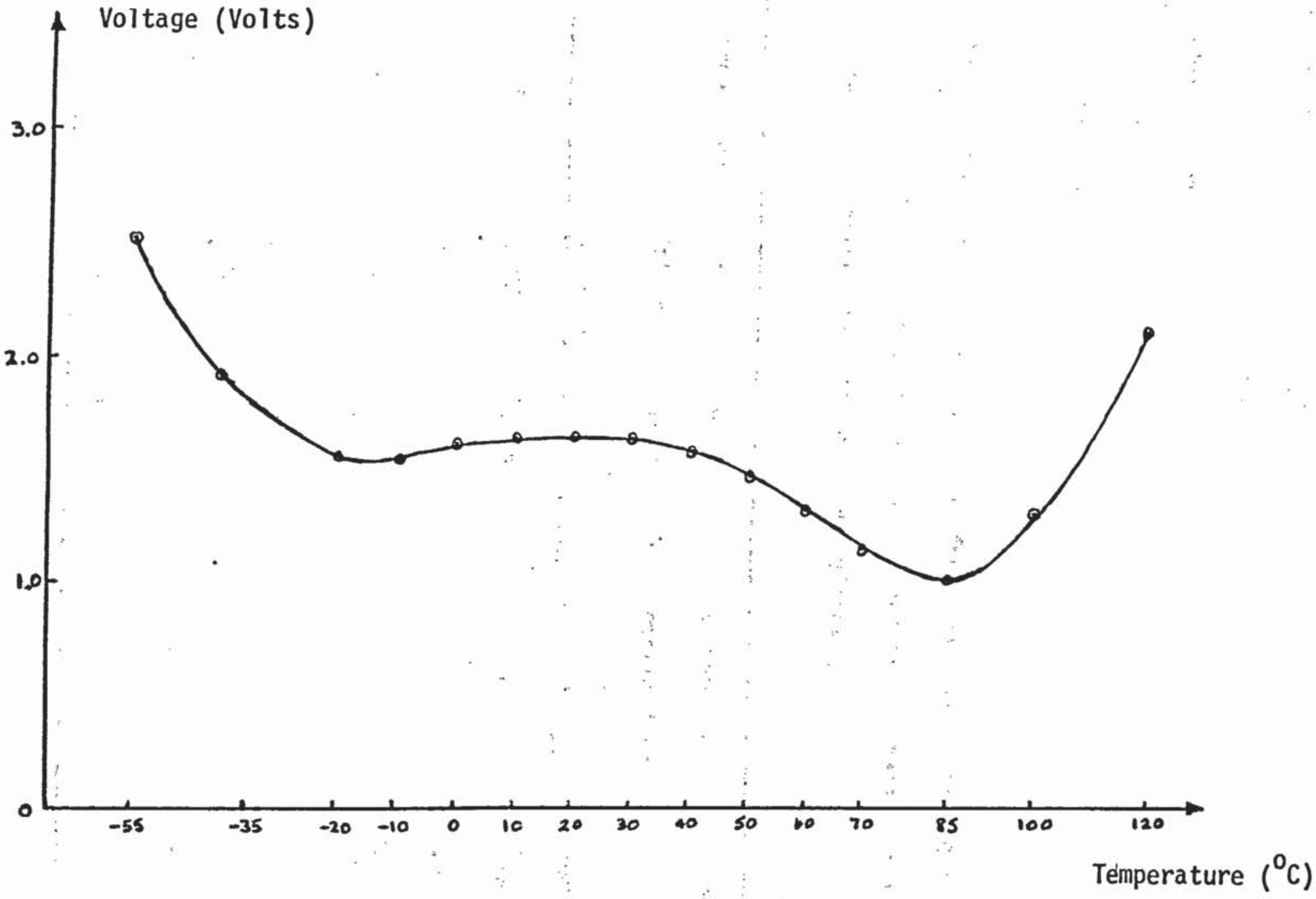


Figure 6.18 - Variation of voltage with temperature 2

The improvement in accuracy was determined by calculating the coefficients of the minimax polynomial.

The coefficients are -

constant term	0.07642
linear term	-0.6327
quadratic term	0.03541
cubic term	1.140
fourth order term	0.1639

Table 6.6 shows the measured voltages from the circuit of figure 6.17, the desired compensation voltage, the synthesized compensation voltage and the error between the compensation voltages.

Temperature (^{o}C)	Output Voltage (volts)	Desired Compensation Voltage (volts)	Synthesized Compensation Voltage (volts)	Error (mV)
120	2.082	0.876	0.915	39
100	1.284	1.467	1.452	-15
85	1.054	1.689	1.655	-34
70	1.157	1.752	1.713	-39
60	1.308	1.719	1.707	-12
50	1.454	1.644	1.659	15
40	1.557	1.555	1.586	31
30	1.615	1.464	1.503	39
20	1.634	1.365	1.396	31
10	1.611	1.295	1.307	12
0	1.609	1.250	1.224	-26
-10	1.517	1.253	1.214	-39
-20	1.536	1.309	1.295	-14
-35	1.763	1.530	1.569	39
-55	2.493	2.196	2.157	-39

Table 6.6 - Accuracy of compensation 2

These results show that the minimax error has been reduced from 58mV to 39mV.

6.8.2 Fourth Multiplier Stage

The breadboard model was extended to generate a voltage with a fifth order variation with temperature. This circuit is similar to the second and third multiplier stages and is shown in figure 6.19. The further improvement in accuracy was determined by calculation of the coefficients of the minimax polynomial.

The coefficients are -

constant term	0.07306
linear term	-0.6967
quadratic term	0.09392
cubic term	0.9947
fourth order term	0.1310
fifth order term	0.1384

Table 6.7 shows the measured voltage output from the circuit of figure 6.19, the desired compensation voltage, the synthesized compensation voltage and the error between these compensation voltages. Figure 6.20 shows a graph of the output voltage from the fourth multiplier stage versus temperature.

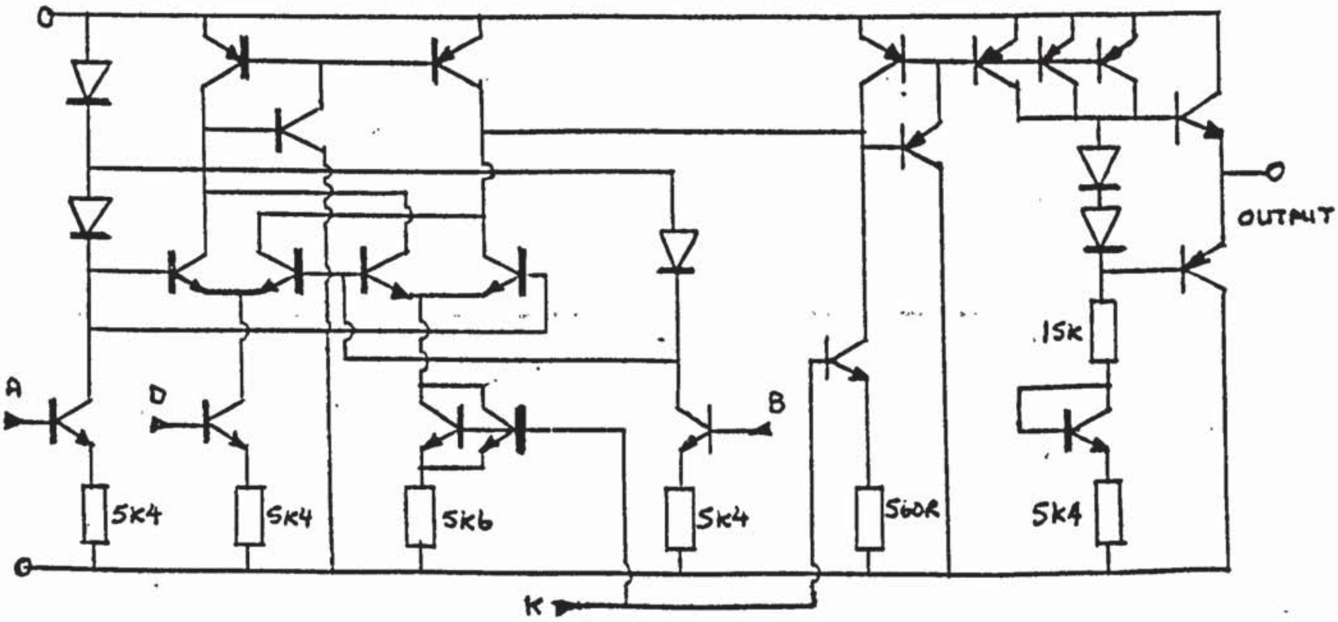


Figure 6.19 - Fourth Multiplier circuit diagram

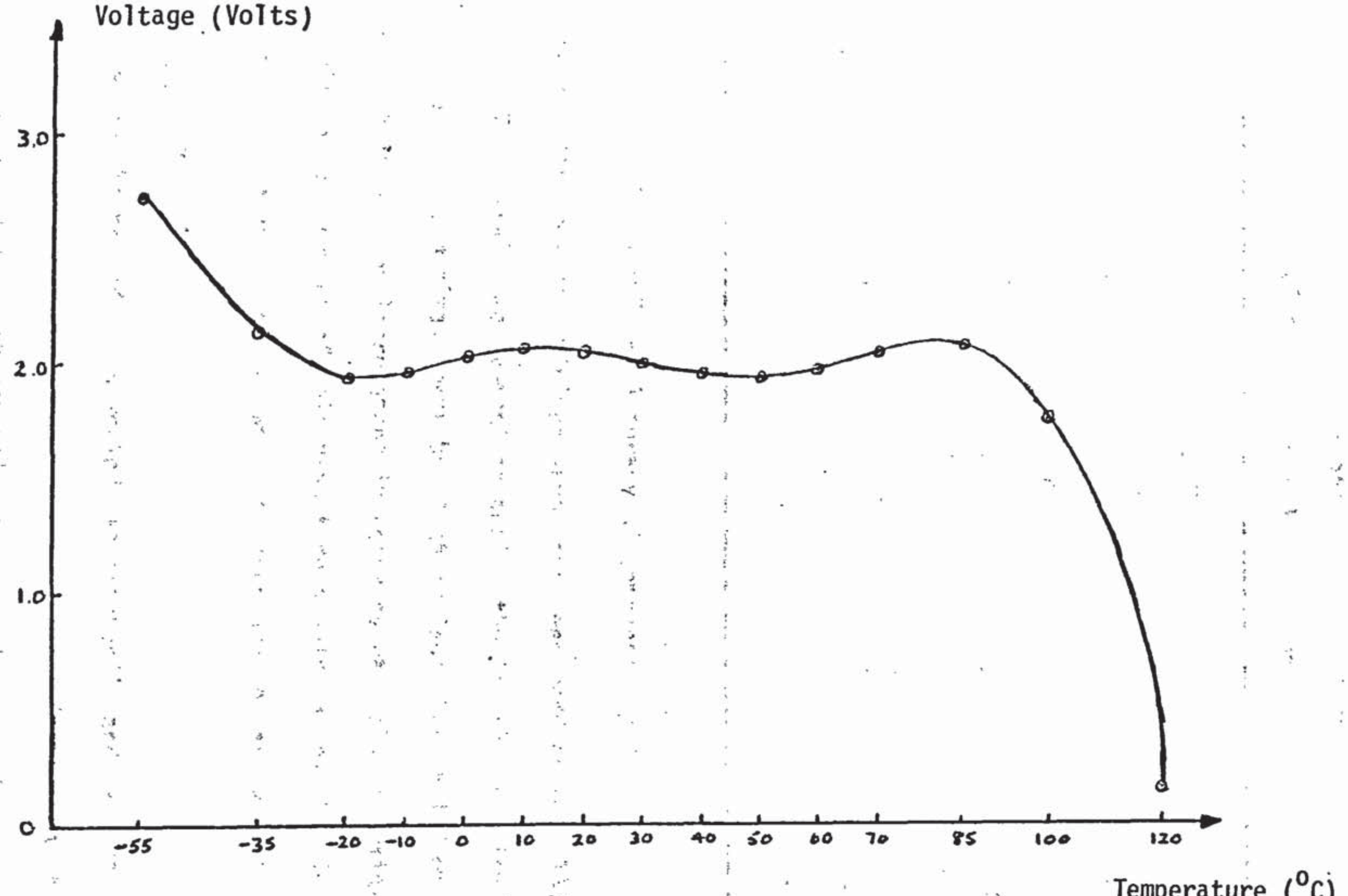


Figure 6.20 - Variation of voltage with temperature 3

Temperature (°C)	Output Voltage (volts)	Desired Compensation Voltage (volts)	Synthesized Compensation Voltage (volts)	Error (mV)
120	0.159	0.876	0.844	-31
100	1.764	1.467	1.498	31
85	2.081	1.689	1.683	- 6
70	2.038	1.752	1.721	-31
60	1.959	1.719	1.692	-27
50	1.928	1.644	1.633	-11
40	1.942	1.555	1.563	8
30	1.979	1.464	1.489	25
20	2.024	1.365	1.396	31
10	2.039	1.295	1.317	22
0	2.021	1.250	1.281	31
-10	1.949	1.253	1.222	-31
-20	1.934	1.309	1.295	-14
-35	2.136	1.530	1.561	31
-55	2.783	2.196	2.165	-31

Table 6.7 - Accuracy of compensation 3

These results show the minimax error has been reduced from 39mV to 31mV. The overall improvement in adding the fourth and fifth orders is to reduce the minimax error by slightly less than half from 58mV to 31mV. Although the improvement in accuracy was only determined in one example an improvement of this magnitude should be possible in the majority of cases. Further work is necessary to verify this.

6.9 Conclusions

The breadboard model has worked satisfactorily and has proved that it is possible to design an integrated circuit which generates voltages corresponding to the Chebshev polynomials. The circuit requires no precise measurements. The tolerances on RE1, RE2 and the integrated circuit components can be disregarded since each output voltage from the integrated circuit is measured at each temperature across the operating temperature range. The ambient temperature is not required to be known exactly. The only requirement is for a lengthy stabilization time (40-50 mins.) to ensure there are no thermal gradients inside the integrated circuit and between the circuit and the crystal. The long-term stability of these voltages is high. The stability of the coefficient-setting resistors is high ensuring that the gain associated with each component of the power series is constant. The differences in the temperature coefficient of the resistors adds a small error to these gains.

After the circuit design had been finalised, a proper layout of the integrated circuit was done. Ferranti supply layout sheets on which the circuit metallisation pattern can be drawn in coloured pencil. From this drawing a metallisation mask will be made and used to produce an initial 50 sample integrated circuits. In this project 25 samples were taken and used as prototypes. The test results in this chapter refer to the compensation system and include test results of 12 prototype oscillators which were built and compensated.

7.1 Static Tests

A test program had been supplied to Ferranti previously and had been used to test the integrated circuits at the manufacturing stage. All 25 samples were tested satisfactorily at Harlow.

7.1.1 Voltage Stabilizer

The three external resistors were set to their nominal values and the performance of the voltage stabilizer measured. The value of the REGULATED VOLTAGE, the current consumption (at 10V supply) and the lowest useable supply voltage were measured.

Unit	Regulated Voltage (volts)	Lowest supply voltage (volts)	Current consumption (μA)
1	3.659	4.39	880
2	3.667	4.38	823
3	3.635	4.36	837
4	3.677	4.39	869
5	3.666	4.45	829
6	3.741	4.48	899
7	3.685	4.41	840
8	3.649	4.38	840
9	3.644	4.35	880
10	3.658	4.36	860
11	3.684	4.39	870
12	3.627	4.33	810
13	3.639	4.37	880
14	3.701	4.42	830
15	3.638	4.35	870
16	3.651	4.35	840
17	3.610	4.33	840
18	3.676	4.39	850
19	3.664	4.38	880
20	3.645	4.37	850
21	3.664	4.38	890
22	3.687	4.41	860
23	3.680	4.41	870
24	3.656	4.37	860
25	3.667	4.38	840

Table 7.1 - Voltage stabilizer test results

For the REGULATED VOLTAGE the mean value is 3.663V and the tolerance on this is +0.078V, -0.053V which is within the design tolerance of 3.630V ±0.210V. The lowest supply voltage appears to be low but this is the room temperature value. 0.16V must be added to give the true value. This gives a mean value of 4.54V which is close to the design value of 4.56V. The line and load regulation were checked and had typical values of 0.3mV/ V and 13 mV/mA respectively. The temperature coefficient of the REGULATED VOLTAGE was determined during the temperature tests and typically measured 0.39mV/K.

7.1.2 Voltage Generators

The values of RE1 and RE2 are not fixed but vary according to the integrated circuit used and these resistors optimise the slope of the curves which are generated. RE2 is set at 30K and RE1 is a select-on-test resistor. The variation of temperature is simulated by varying the voltage across RE1 and RE2. If the ends of these resistors are not connected to REGULATED VOLTAGE and OV but to triangular waves as shown in figure 7.1 then the LINEAR OUTPUT, QUADRATIC OUTPUT and CUBIC OUTPUT vary in voltage similar in manner to the variation which results from a variation of the ambient temperature.

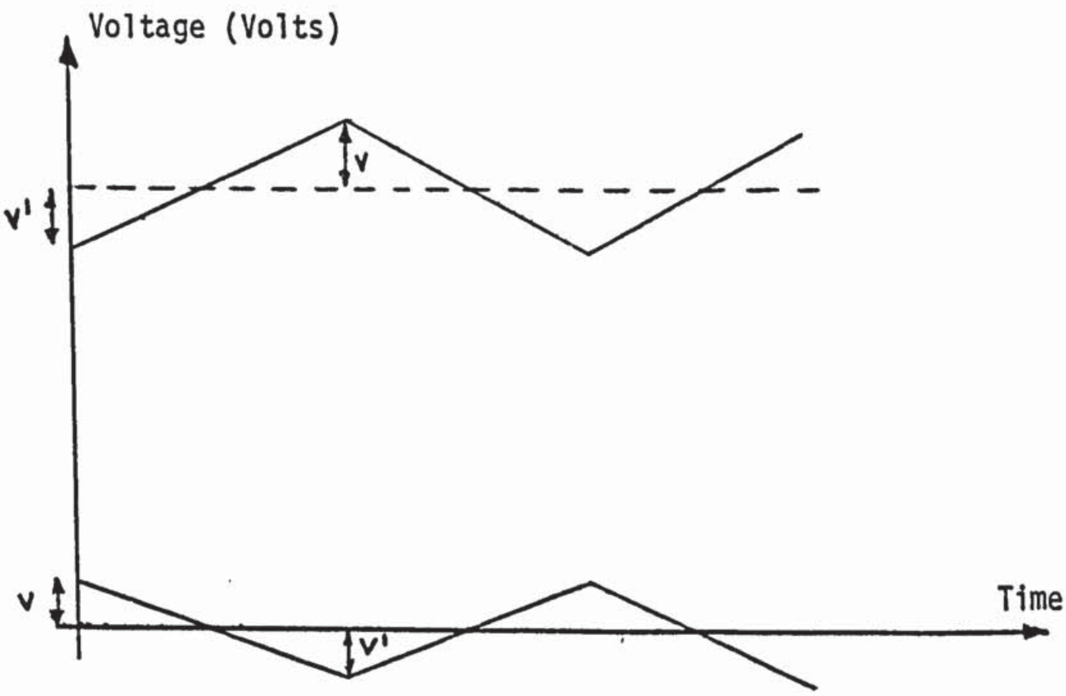


Figure 7.1 - Temperature simulation voltages

The voltage V is equal to twice the maximum positive temperature excursion from 25°C and the voltage V' is equal to twice the maximum negative temperature excursion from 25°C. The 25 sample circuits were tested and the values of RE1 determined. V and V' were equal to 150mV which represents a temperature excursion of 100°C to -50°C. The values of RE1 are given in Table 7.2. In all cases the variation of the output voltages was similar to that shown in figure 7.2.

Unit	Value of RE1 (kΩ)
1	29.3
2	29.3
3	31.3
4	28.1
5	29.1
6	23.9
7	27.8
8	29.5
9	29.5
10	29.5
11	27.8
12	32.1
13	30.6
14	26.6
15	29.8
16	29.9
17	32.5
18	28.1
19	28.1
20	30.1
21	28.2
22	27.6
23	27.6
24	29.0
25	29.0

Table 7.2 - Values of RE1

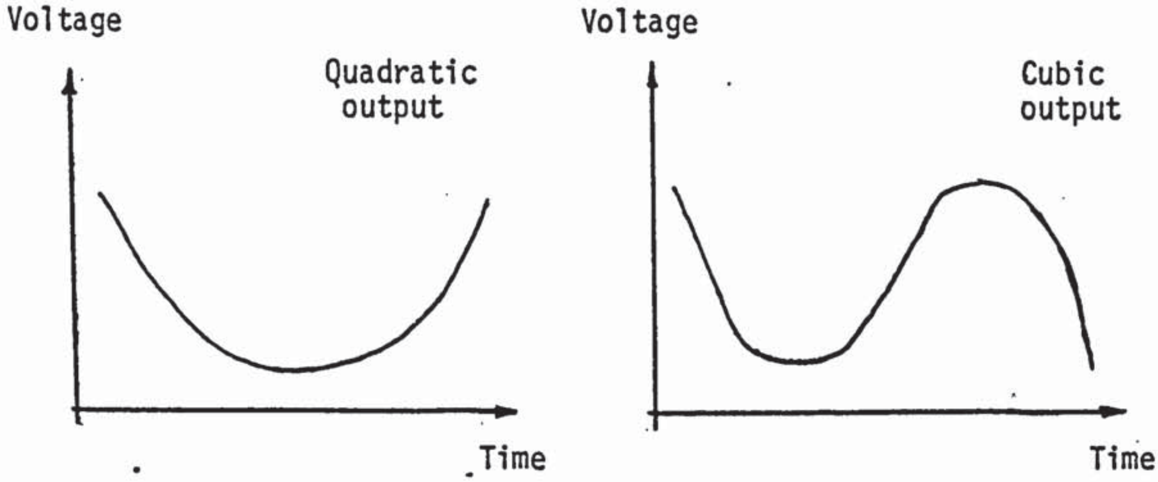


Figure 7.2 - Shapes of curves

7.1.3 Summing amplifier

The open loop gain of the amplifier was measured and found to vary from 600 to 850. The expected value of gain is 655.

7.2 Temperature tests

Twelve integrated circuits were used to build prototype oscillators. The oscillators were compensated over the temperature range 85°C to -40°C. The four output voltages were measured at nine temperatures and these measurements used to calculate the coefficient-setting resistors using the computer program COEFFS. The set of voltages measured on each device were similar and Table 7.3 gives one set of voltages. Figure 7.3 shows the variation of voltages with temperatures in graphical form.

Temperature (°C)	REGULATED VOLTAGE (volts)	LINEAR OUTPUT (volts)	QUADRATIC-OUTPUT (volts)	CUBIC OUTPUT (volts)
87	3.713	0.992	1.322	1.566
75	3.708	1.053	1.242	1.621
60	3.703	1.129	1.192	1.605
45	3.697	1.199	1.200	1.544
25	3.690	1.298	1.310	1.458
5	3.680	1.394	1.509	1.462
-10	3.673	1.460	1.684	1.571
-25	3.666	1.533	1.902	1.801
-40	3.658	1.599	2.123	2.105

Table 7.3 - Voltage measurements

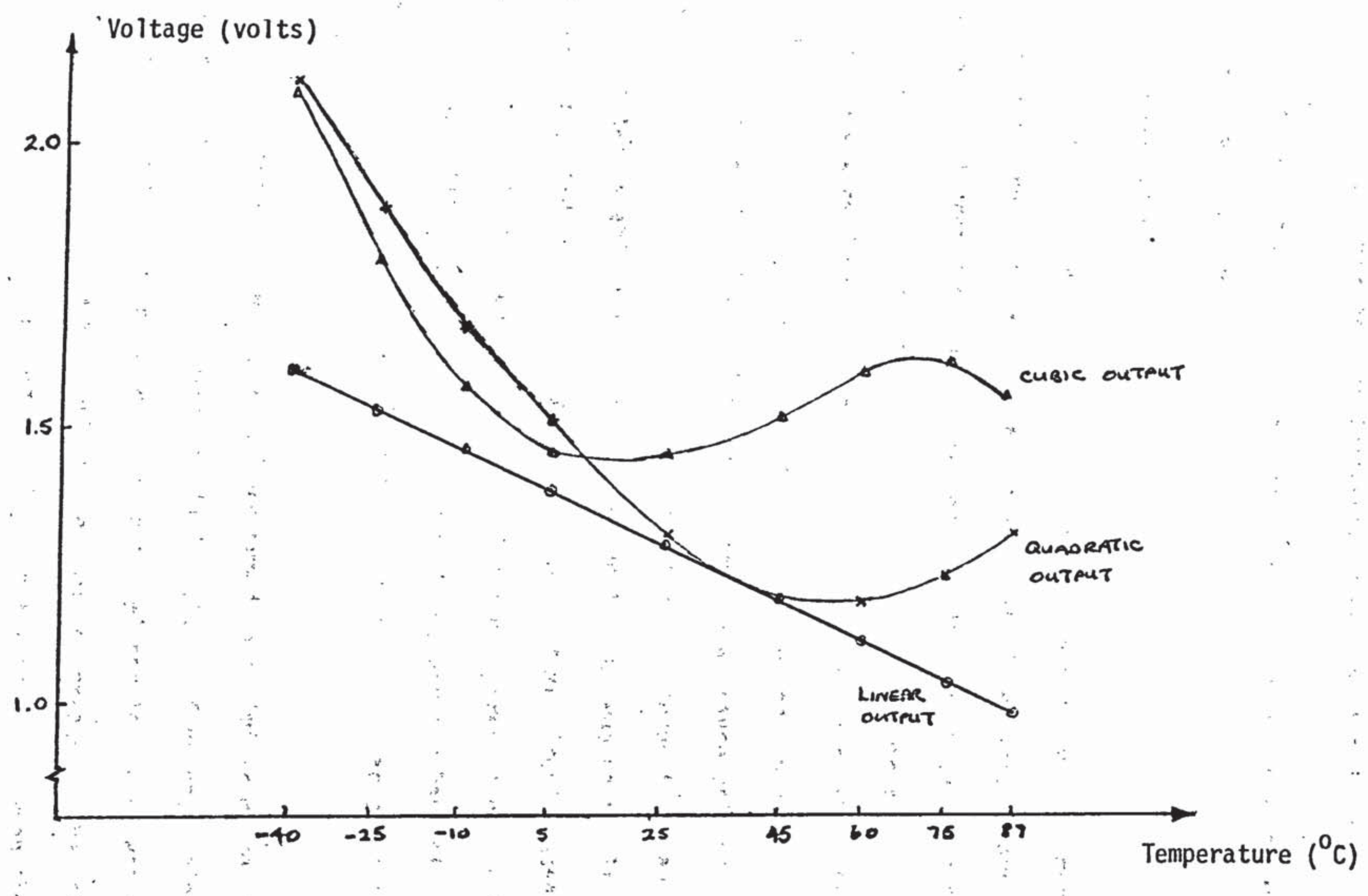


Figure 7.3 - Variation of voltage with temperature 4

The coefficients required by each oscillator are summarised in Table 7.4. This table also includes the theoretical minimax error, the measured minimax error and the frequency tolerance of the oscillator.

Figure 7.4 shows the variation of the theoretical minimax error and the actual minimax error for unit 5.

7.3 Conclusions

Each of the prototype oscillators was tested several times and no problems were found in correlating each set of results. Some of the sample integrated circuits were kept at 125°C for 9 days and then remeasured. No significant changes in the voltages were recorded. Two problems were found; the first was an oscillation of the REGULATED VOLTAGE output. This could be cured by a decoupling capacitor from this output to 0V. The difference in stray capacitance between the breadboard model and the integrated circuit has altered the stability of the voltage stabilizer. The second problem was due to the layout of the integrated circuit. Part of the temperature circuit passes close to the output stages and some oscillation is present on the QUADRATIC VOLTAGE and CUBIC VOLTAGE output. Again decoupling the base of transistor 20 (figure 6.4) which is taken to an output pad cures the problem.

The compensation system has performed in a similar manner to the breadboard model. With reference to Table 7.4, the frequency tolerances are variable with some units well within the ±1 ppm tolerance whilst others of a similar type are just on the limit or greater. The angle-of-cut of some crystals is not perfect and there is a mismatch between the

Unit	Frequency tolerance $\pm$ ppm	Output	Coefficients constant	linear	quadratic	cubic	Minimax Error theoretical (mV)	measured (mV)
1	1.4	TTL	-0.265	0.168	-1.990	3.905	84	90
2	1.0	TTL	-0.456	0.879	-1.712	3.583	49	66
3	0.6	TTL	-0.577	0.405	-0.735	3.303	27	35
4	0.88	TTL	0.274	-0.407	0.236	4.125	56	63
5	0.77	5V CMOS	-0.866	0.568	-0.511	3.565	50	50
6	0.78	5V CMOS	-0.791	0.680	-1.270	4.089	56	54
7	0.76	9V CMOS	-1.110	4.317	-3.776	4.492	70	75
8	0.55	9V CMOS	-0.649	-3.360	-0.262	3.685	45	40
9	0.50	9V CMOS	-0.654	-2.347	1.371	3.785	34	39
10	1.1	9V CMOS	-1.538	3.300	-2.016	4.540	70	100
11	1.5	9V CMOS	-1.057	1.967	-2.518	5.220	115	130
12	1.0	9V CMOS	-1.318	2.027	-2.008	5.236	53	86

Table 7.4 - Compensation results

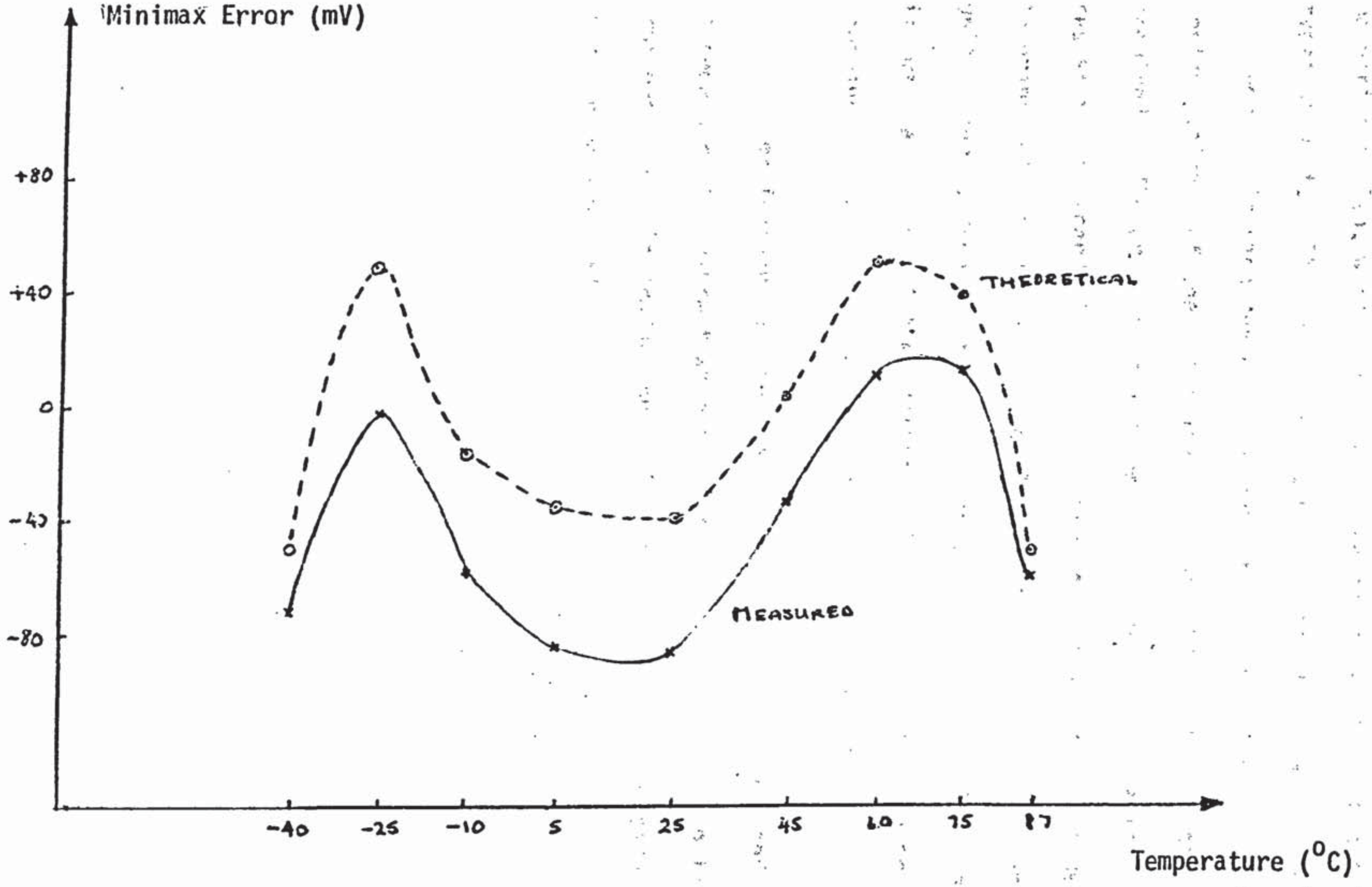


Figure 7.4 - Variation of control voltage error

control voltage of the oscillator versus temperature and the CUBIC OUTPUT voltage versus temperature. The coefficients tend to be variable also, with changes of value and polarity within similar types of unit. The cubic coefficient is consistent with a gain of 3-4 required in each device. A more careful match between the control voltage of the oscillator and the CUBIC OUTPUT voltage may be required. The drift in the value of the coefficient-setting resistors is dependent on the differences of the actual frequency tolerance of the device and the ±1 ppm limit. If this is large e.g. units 8 and 9, then the resistors can drift by 5% but if the difference is small as in unit 4 then drifts of 1% only are needed before the frequency tolerance exceeds the specified value of ±1 ppm.

Some further work on the evaluation of the integrated circuit is necessary to ensure that a satisfactory performance is obtained from every unit in production. The initial results indicate that this can be achieved.

CHAPTER 8

OSCILLATOR AND OUTPUT STAGE DESIGN

8.1 Oscillator Circuits [34]

The literature contains a great many different oscillator circuits. Yet despite the long history of crystal oscillators, the successful design of an oscillator depends largely on the skill and experience of the designer and usually proceeds in a "build and try" fashion. This has been greatly influenced by the well-known fact that it is not difficult to assemble a configuration that will oscillate somehow, since oscillations are produced over a wide range of parameters.

There has been little progress toward achieving a practical systematic and qualitative design procedure. Evaluation of the literature shows that it falls into certain categories:

(i) a general qualitative and quantitive description of oscillator theory with little attention paid to its practical application to producing oscillators.

(ii) qualitative descriptions of oscillator theory with specific circuit diagrams for particular designs with some general statements about the performance of these designs but with no clue as to how the circuits were derived.

In spite of these difficulties an attempt was made to determine in a logical manner the type of oscillator circuit best suited for incorporation within the integrated circuit. It is possible to classify oscillator designs into two very broad classes with very few exceptions. These are one-port and two-port oscillators.

A one-port oscillator consists of a single port circuit element that exhibits a negative resistance in a portion of its operating range, shunted by a resonant circuit. A two-port oscillator consists of an amplifying device and some form of frequency-sensitive two-port passive network that couples a portion of the amplifier output back to the input in the proper phase and magnitude to ensure oscillation at a frequency determined by the network.

One- and two-port oscillators are usually called negative resistance and feedback oscillators respectively. However, these terms are slightly misleading in that some negative resistance circuits involve regenerative feedback of some sort and some feedback oscillators exhibit negative resistance at one or more points in the circuit. Typical examples of one-port oscillators are integrated circuit oscillators comprising a multi-stage transistor amplifier with the crystal as a frequency selective bypass or feedback element. The negative resistance characteristic is dependent on the transistor parameters and on the stability of the operating points of the transistors. This characteristic will also change with supply voltage, loading and temperature and will affect the frequency and amplitude stability of the oscillator. This limits their use to medium precision applications. Furthermore, negative resistance oscillators will have at least two transistors within the frequency determinating loop which will give a poor short term stability due to noise from the transistor junctions. It is also difficult to design these oscillators since the equations for oscillation tend to be complex. A large number of possible circuit configurations are possible making a general design principle hard to apply.

Two-port oscillators are the most widely used class of oscillator. They are composed of two interconnected two-port networks. One contains the active element and the other the passive feedback network. The feedback network may take three forms:

(i) null network
(ii) π -impedance network(z-type oscillator)
(iii) T -impedance network(y-type oscillator)

The Meacham bridge[35] is an example of a null network oscillator; other configurations used are parallel-tee and bridged-tee networks. A major disadvantage of the Meacham bridge circuit is the need for a high gain amplifier operating at many megahertz. Amplitude stability is also difficult. The amplitude is stabilized by a temperature dependent resistor in the bridge network. Positive or negative temperature coefficient resistors are bulky and there is such a small variation in the power dissipation in the resistance that the value of the resistance is unaffected.

The z- and y-type oscillators are often encountered since they offer good frequency and amplitude stabilities, are easy to design and are insensitive to component tolerances. The simplest form of these oscillators is shown in figure 8.1 and their derivation is given in Appendix 5.

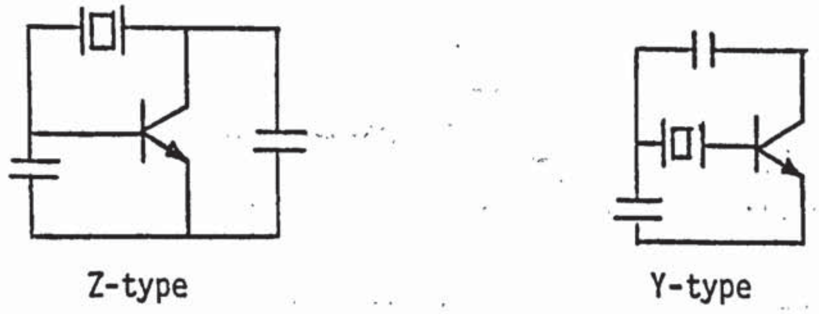


Figure 8.1 - Z and Y-type oscillators

For each of these two ac schematics, the ac ground can be placed at three points giving six variants (figure 8.2).

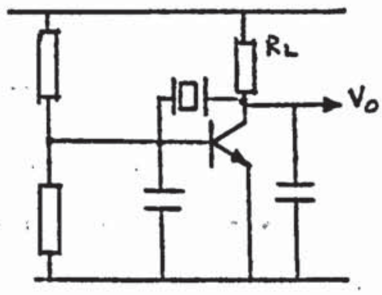
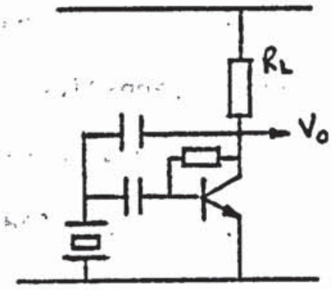
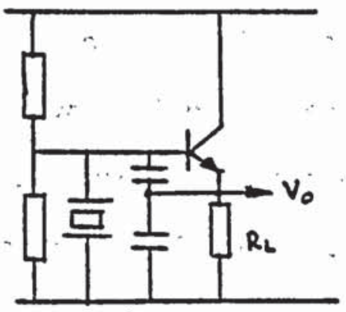
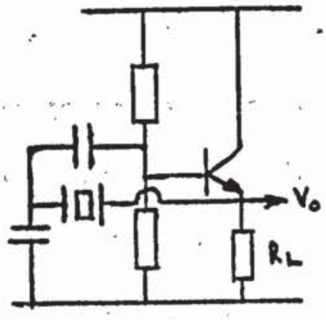
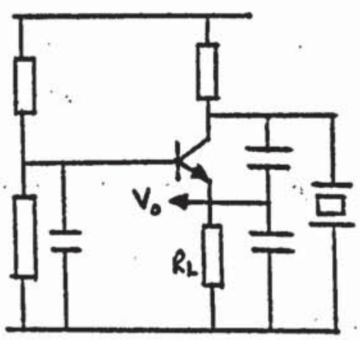
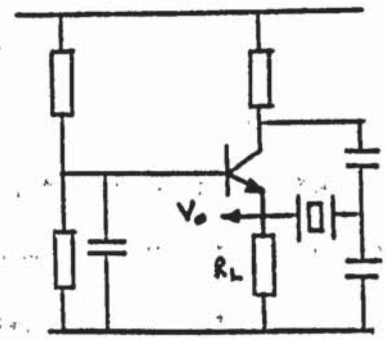
AC ground	z-type	y-type
Emitter	 Pierce oscillator	 Miller oscillator
Collector	 Colpitts oscillator	
Base	 Clapp oscillator	 Grounded base oscillator

Figure 8.2 - Oscillator circuits

The best circuit to use is one which has one leg of the crystal connected to ground. To enable the crystal oscillator to be voltage controlled a variable capacitance diode is added in series with the crystal. If the diode has one connection to ground the adjustment of the bias on the diode to compensate for crystal ageing is more easily achieved. Experiments with the Miller oscillator indicates that it is difficult to optimise the circuit components to operate over a wide frequency range of 1 to 20MHz. The Clapp oscillator is generally regarded as being more suitable for higher frequencies and has the disadvantage that it is unsuitable for low supply voltages. The best circuit to use is the Colpitts oscillator. Experience with this design in an existing product confirms its suitability.

8.2 Initial Design Comments

The design of the oscillator and output stages is now considered. The type of oscillator to be used is a Colpitts and this will supply a sinewave signal which is ac coupled to the appropriate output stage. Two output stages are required, one giving a TTL logic output and the other a CMOS output. It is also possible to use the CMOS output stage to supply a clipped sinewave signal. Due to a layout restriction in the number of bonding pads available in the integrated circuit a separate output stage could not be used.

8.3 Oscillator Design

The circuit diagram of figure 8.3 shows the layout of the oscillator circuit. The circuit operates from the REGULATED VOLTAGE supply to minimize the change of frequency with supply voltage. Transistor 1 is a low noise NPN transistor which serves as the amplifying device in the oscillator. The d.c. bias for this transitor is supplied using transistors 2,3,4 and resistors R1, R2 and R3. R4 is the load resistor and RG is used to control the signal level of the oscillations and maintain a 0.5V p-p ac signal across the load resistor irrespective of frequency. The signal level of the RF output is altered by adjusting the value of this resistor. C1 and C2 are

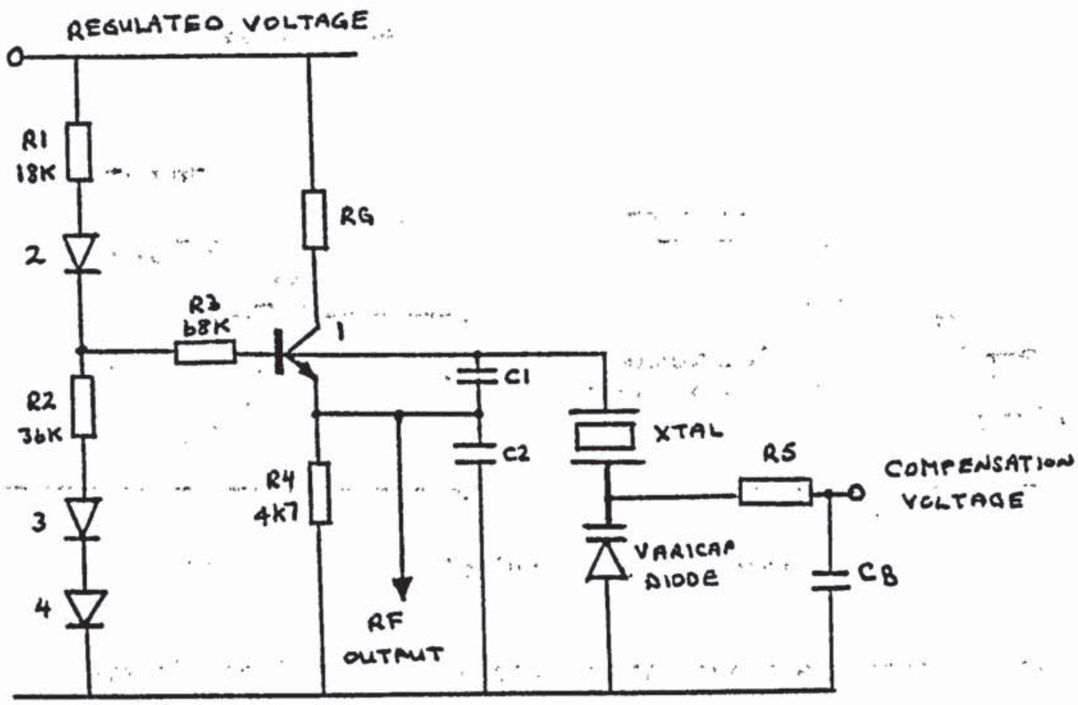


Figure 8.3 - Oscillator circuit diagram 1

the fixed capacitors which form part of the frequency determining loop. The crystal and the variable capacitance diode are connected in series and the bias to the diode is supplied via R5. C_B is a decoupling capacitor which removes any ac ripple from the compensation voltage and improves the short term stability of the oscillator. The oscillator frequency can be adjusted to compensate for ageing and this is achieved using a variable resistor or a variable capacitor. Figure 8.4 shows the components necessary for this. The variable resistor alters the bias voltage on the variable capacitance diode and the variable capacitor alters the load capacitance seen by the crystal.

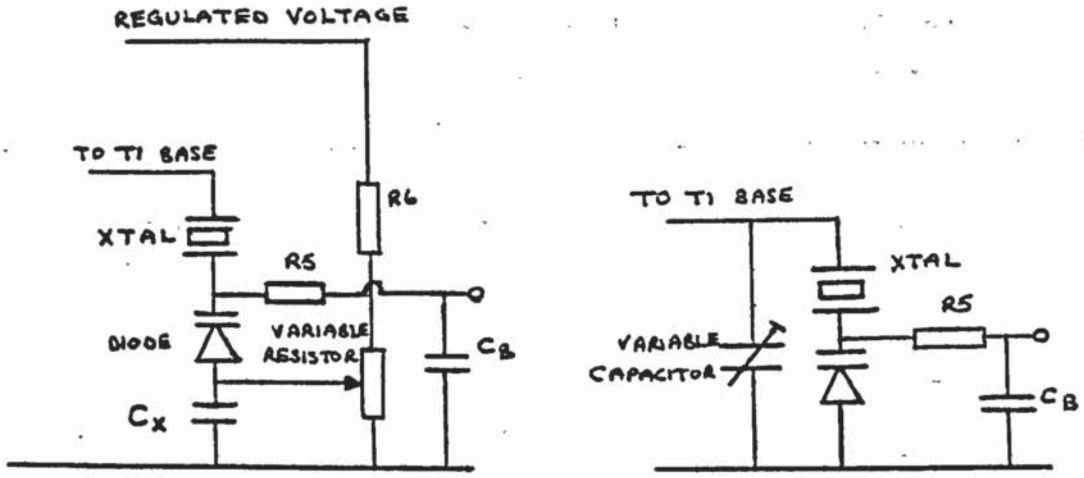


Figure 8.4 - Ageing adjustment

Several oscillators were constructed and their performance evaluated. The frequencies ranged from 1 to 20MHz and the sensitivity of the voltage control was set at 10ppm/volt and 20ppm/volt. The following aspects of the design were investigated.

(i) variation of the output voltage with transistor parameters - no significant change was found

(ii) variation of output voltage with crystal E.S.R - a 5% change in output level was typically observed when the crystal E.S.R was changed from its maximum value to half of that value

(iii) variation of output level with temperature - 0.1V changes were observed at all frequencies due to the temperature being varied from 120°C to -55°C.

(iv) variation of output voltage with bias voltage on the variable capacitance diode - no changes were observed

The linearity of the frequency with compensation voltage was measured. Typical results for two oscillators are given in Table 8.1 and the circuit diagrams are given in figure 8.5.

Compensation voltage (volts)	Frequency change oscillator 1 (ppm)	Frequency change oscillator 2 (ppm)
1.0 → 1.5	10.3	-
1.5 → 2.0	10.2	6.1
2.0 → 2.5	10.9	5.6
2.5 → 3.0	11.7	5.9
3.0 → 3.5	13.1	5.5
3.5 → 4.0	-	5.75
4.0 → 4.5	-	5.9
4.5 → 5.0	-	5.6
5.0 → 5.5	-	6.0
	Total = 56.2 = 22ppm/volt	Total = 46.35 = 11.6ppm/volt

Table 8.1 - Linearity of oscillator

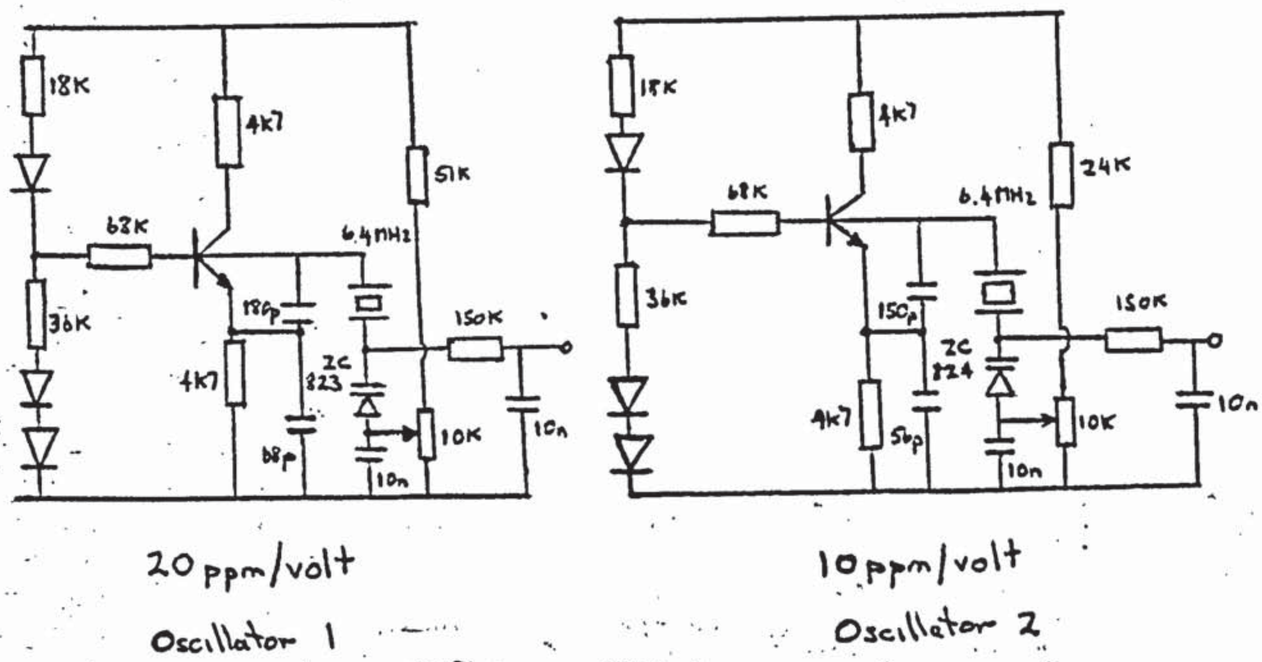


Figure 8.5 - Oscillator circuit diagram 2

The sensitivity of these oscillators was theoretically 20ppm/volt and 10ppm/volt. The values of sensitivity measured were independent of temperature and of the crystal used.

An optimum angle crystal will deviate in frequency by ±20ppm over the temperature range 120°C to -55°C. Using a 4. 5V supply voltage for the integrated circuit the range of compensation voltage is from 1V to 3.5V. This enables a crystal to be compensated over the temperature range 120°C to -55°C even when operated from a 4.5V supply voltage. In the majority of units a higher supply voltage is used and a voltage controlled oscillator with a sensitivity of 10ppm/volt can be used. The normal range of compensation voltage is 1V to 5.5V in these oscillators.

The ageing adjustment for an oscillator is ±5ppm. This is sufficient for 10 years ageing. There is a minimum d.c. bias of 0.5V on the variable capacitance diode and the diode is never forward biased even with a 0.5V p-p ac signal across it.

The short term stability of these breadboard models has not been evaluated. The Colpitts oscillator has an inherently good noise performance and little can be done to improve this. Again, experience of this oscillator in another product indicates that good noise figures are easily obtained.

8.4 Output Stage Design

8.4.1 TTL logic output stage

This output stage is required to convert a 0.5V p-p ac signal to a square wave corresponding to the limits of TTL logic. The circuit operates from a supply voltage of 4.5V to 5.5V and will drive five low power Schottky loads from 1 to 13MHz. The mark:space ratio of the output is 50:50 ±20% and the power consumption is less than 7.5mA. The power consumption is equivalent to one 54S00 integrated circuit.

The circuit diagram is shown in figure 8.6.

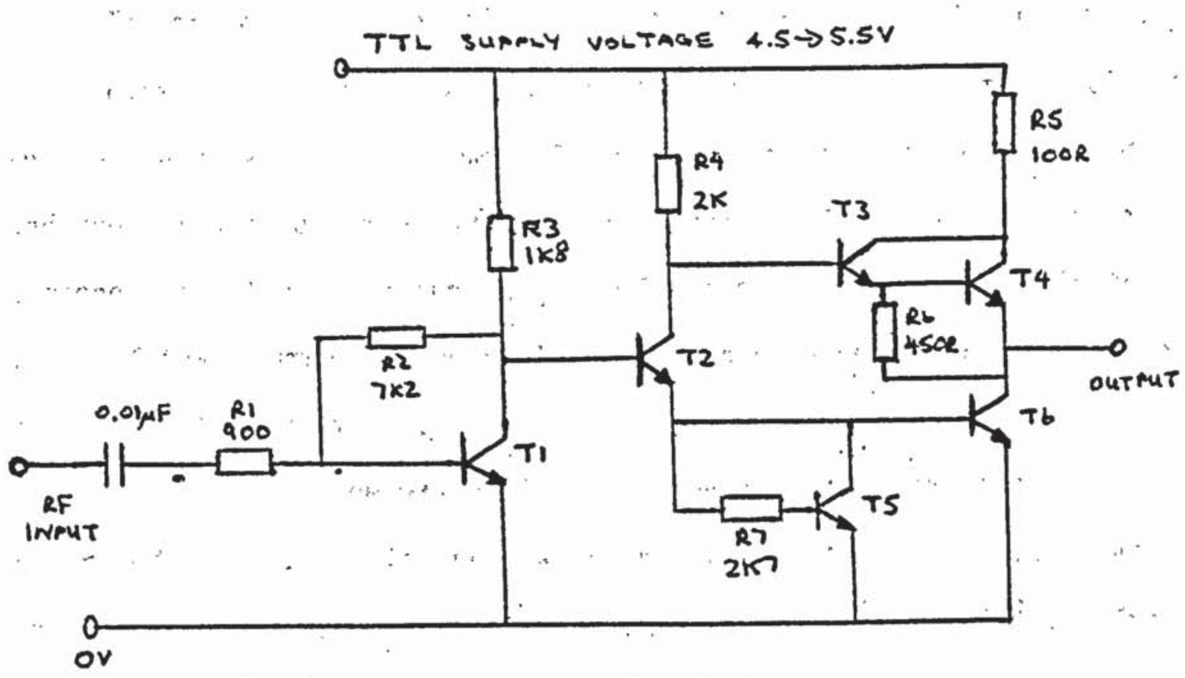


Figure 8.6 - TTL Output stage circuit diagram

Transistor 1 is a common emitter amplifier stage which amplifies the input waveform and provides a suitable waveform for driving transistor 2. This transistor operates as a phase splitter and drives the output transistors. Transistors 3 and 4 provide an active pull-up and transistors 5 and 6 provide an active pull-down. This output configuration is used for low power Schottky circuits. Transistors 3 and 4 are a Darlington pair with R5 acting as a current limiting resistor and R6 assisting in turning off transistor 3. The fall time of the output waveform is decreased using this. Transistor 5 is arranged in a squaring network. This transistor prevents transistor 6 conducting until the input voltage rises high enough to allow transistor 2 to supply current to transistor 6. Transistor 5 also turns transistor 6 off faster and decreases the propogation delay of the circuit.

The method of design of this output stage was "build and try". It is difficult to evaluate the time constants at points in the circuits and so the rise and fall times of the waveforms have to be measured and the circuit optimised by trial and error. Once the value of a component is fixed, the sensitivity of the circuit to changes in the value of that component are evaluated by using a component of a higher and lower standard value.

The output stage was evaluated using an oscillator circuit as described in 8.3. This oscillator and output stage were evaluated at various frequencies over the temperature range 120°C to -55°C. The output logic levels vary between 0.1V and 4V and the mark:space ratio is maintained to within 50:50±20%. The rise and fall times of the output are typically 20ns and 5ns at 4MHz and 12ns and 6ns at 10MHz. The current consumption varies between 3.5mA at 1MHz and 7.5mA at 13MHz.

8.4.2 CMOS logic output stage

This output stage is required to convert a 0.5Vp-p ac signal to a square wave corresponding to the limits of CMOS logic. The circuit operates from a supply voltage of 5V to 15V and is capable of driving one CMOS gate. The mark :space ratio is required to be 50:50 ±20% and the current consumption is frequency and voltage dependent with typical figures of

5V	supply voltage at a frequency of		6MHz	- 2mA current
10V	"	"	8MHz	- 4mA current
15V	"	"	10MHz	- 6mA current

The figures for current consumption represent tbe equivalent current consumption of a CD4011 integrated circuit.

The circuit diagram is shown in figure 8.7. A simple common emitter amplifier stage is sufficient to give a suitable waveform for driving a CMOS gate. The value of the resistor R1 is not fixed and depends on the supply voltage and frequency used. Typical values are 51K at 5V supply voltage and 1MHz frequency rising to 220K at 15V supply voltage and 10MHz frequency. The output stage was also evaluated using the oscillator described in 8.3 and the combination were evaluated at a number of frequencies over the temperature band 120°C to -55°C. The output logic levels were within 1V of the supply rails and the mark:space ratio maintained to within 50:50 ±20% over this temperature range. The rise and fall times of the output waveform were 35-40ns.

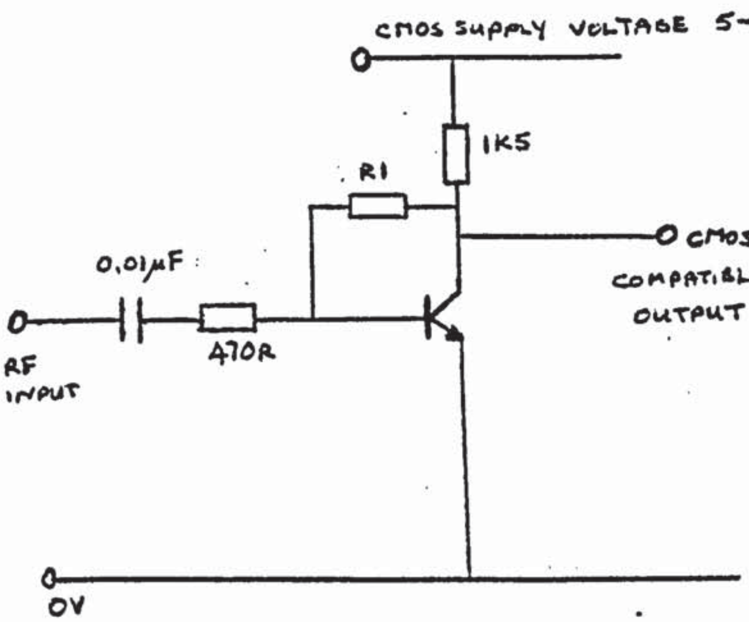


Figure 8.7 - CMOS output stage circuit diagram

This stage can be used as an open-collector output. If the supply voltage is not connected to the integrated circuit then the output can be connected to a voltage via an external resistor. The stage can also be used as a clipped sinewave output.

CHAPTER 9

OSCILLATOR AND OUTPUT STAGES : TEST RESULTS

The 25 sample integrated circuits also contain an oscillator circuit, TTL logic output stage and a CMOS logic output stage. The results of the evaluation of these circuits is described.

9.1 Oscillator circuit

The oscillator is capable of working with crystals from 4 to 20MHz and a simple test using the circuit of figure 9.1 confirmed this. The signal level varied from 4.5V at 4MHz to 0.9V at 20MHz.

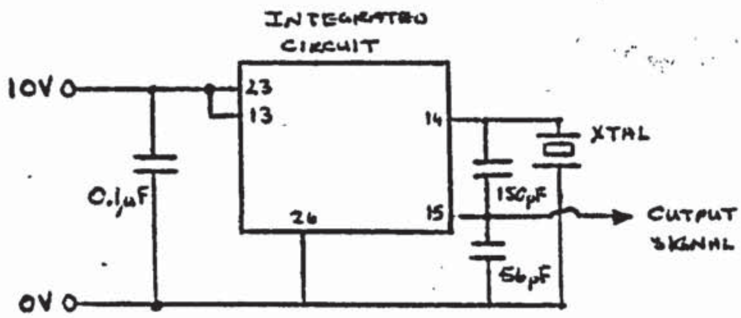


Figure 9.1 - Oscillator circuit

The circuits were used in the production of 12 prototype oscillators and some tests were carried out on each of these units. One important parameter is the linearity of the oscillator. The 12 units were divided into two groups; one had a control voltage sensitivity of about 10ppm/volt and the other a sensitivity of 20ppm/volt. The two graphs of figure 9.2 were typical of the results obtained with all 12 units and show good linearity over the operating voltage range. The circuits were also tested for deadbands. These are particular voltages at which the oscillator will not work. In every case no failures were found even at the extremes of the temperature range (120^{o}C and -55^{o}C).

9.2 TTL logic output stage

An oscillator circuit was connected to a TTL output stage and the performance of the output stage evaluated. The circuit used is shown in figure 9.3.

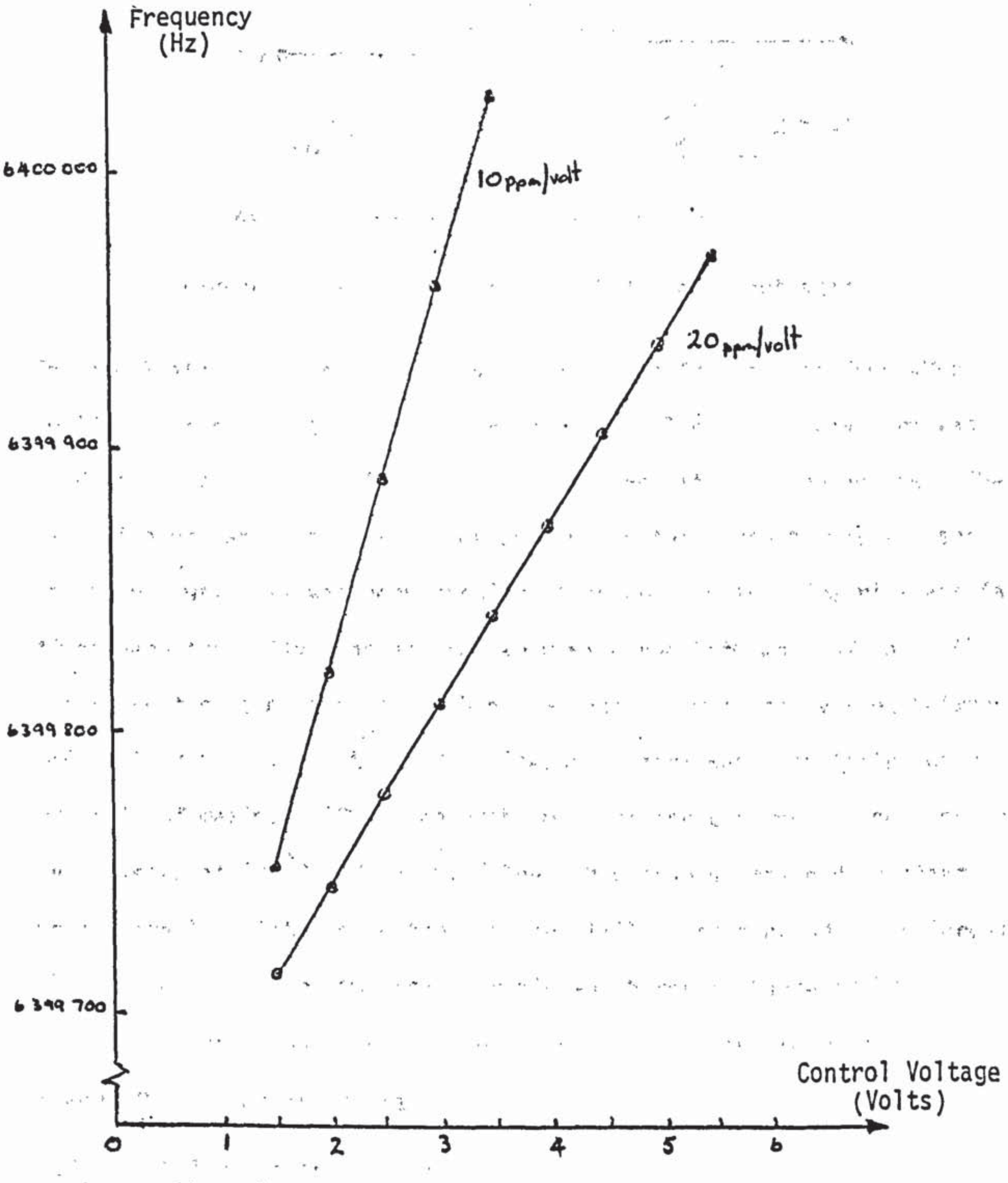


Figure 9.2 - Linearity of oscillator

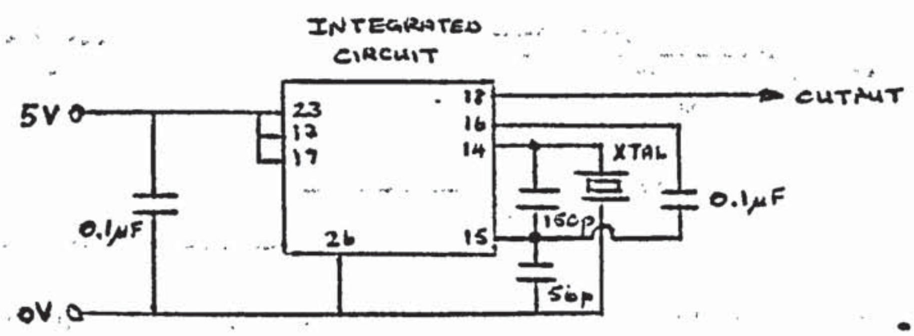


Figure 9.3 - Oscillator and TTL logic output stage

The oscillator and output stage were tested at frequencies from 4MHz to 13MHz, and gave satisfactory performance. The output waveform was good with logic levels of 0V and 4.4V obtained at all frequencies. The variation of mark:space ratio between integrated circuits was ±2% and the output waveform was very consistent between units. The rise and fall times were typically 10ns and 5ns when measured between 0.4V and 2.4V. The current consumption was within the design limits and varied between 5mA at 4MHz and 7.5mA at 13MHz. The oscillator was also tested with crystals of maximum ESR and no problems were encountered. Some devices were tested at 120^{o}C and -55^{o}C. The logic levels remained constant whilst small variations in the rise and fall times occured. The largest change occured in the mark:space ratio which could change by 5%. The oscillator output signal changes in amplitude and results in a change to the mark:space ratio.

9.3 CMOS logic output stage

An oscillator circuit was connected to a CMOS output stage and the performance of the output stage evaluated. The circuit used is shown in figure 9.4.

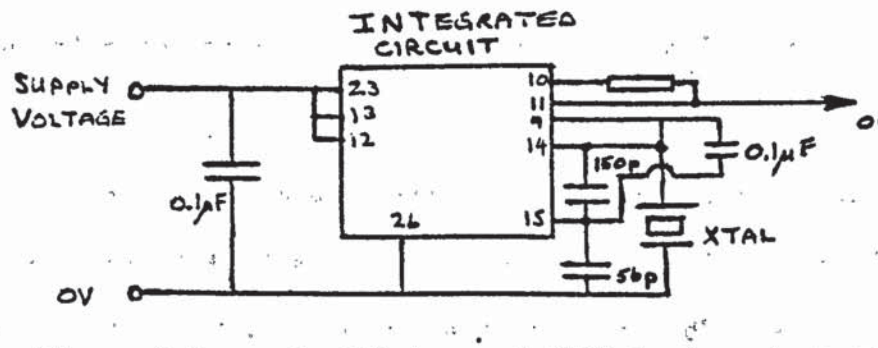


Figure 9.4 - Oscillator and CMOS logic output stage

The frequency range of the output stage was tested at supply voltages of 5, 10 and 15V and the maximum frequency of operation found to be as specific in section 4.2. The logic levels of the output waveforms were typically within 10% of the supply voltage with rise and fall times ranging from 20ns and 10ns at 4MHz and 5V supply to 40ns and 30ns at 10MHz and 15V supply. The variation of mark:space ratio between different integrated circuits was ±2% and the variation of output waveform small. The oscillator was also tested with crystals of maximum ESR and no problems encountered. Some devices were tested at 85^{o}C and -40^{o}C. The logic levels and the rise and fall times changed slightly but the mark:space ratio varied by 10%.

9.4 Conclusions

The performance of the oscillator and output stages is similar to that obtained using the breadboard model. No problems were encountered with the oscillator starting indicating the oscillator bias components in the integrated circuit are correct. The linearity of the oscillator at both sensitivities is good and no significant higher order distortion components should be introduced into the control voltage. The performance of both output stages is within specification and the only difficulty is the relatively large change in the mark:ratio of the CMOS output stage. At this time there appears to be no technical problems with introducing this oscillator into volume production.

The phase noise of three units was measured using a Hewlett-Packard 5390A Frequency Stability Analyzer. The noise level at 10Hz from the carrier was about -94dB which gives a margin of 14dB over typical quoted figures of -80dB.

CHAPTER 10

CONCLUSIONS, IMPLICATIONS AND FUTURE WORK

10.1 Conclusions

The objective of this work was the development of an integrated circuit which could be the nucleus of a temperature compensated crystal oscillator. A large proportion of the research and development work has been concluded and has led to the design of a new oscillator known as a TCXO70. At the present time two orders have been received and four other major customers will be supplied samples in the near future. The TCXO70 satisfies all the requirements identified in chapter 2 and much effort is being devoted at STC in transferring the device from development into production.

The development of this system of compensation represents an advance in the art of temperature compensation and the principle has been patented in the UK, Europe, USA and Japan.[39] Much interest was shown at the 37th Annual Frequency Control Symposium in June 1983 in Philadelphia USA when a paper was presented outlining this new method of compensation.[40] The results of a survey of TCXOs was reported at this conference and it was stated that ±0.5ppm devices represented the state-of-the-art. The prototype oscillators, although not as good as this, are a significant advance in the devices manufactured by STC. An improvement in frequency tolerance can be made by adding a second integrated circuit containing the third and fourth multiplier stages and should give the company the capability to make units with a frequency tolerance of ±0.5ppm, a similar current consumption to a TCXO70 but without the manufacturing problems of a TCXO33.

Temperature compensation at the present time is limited to ±0.3ppm. A device of this frequency tolerance may indeed be only ±0.5ppm accurate due to the effect of crystal hysteresis which can add up to ±0.2ppm to the frequency tolerance. The device must be verified over the temperature range starting at the cold end and increasing the temperature as well as starting at the hot end and decreasing the temperature. Crystal hysteresis is the subject of research at the present time and until the mechanism is fully understood, oscillators with a frequency tolerance of ±0.5ppm will remain as state-of-the-art.

There is no comparable device to the TCX070 of which STC is aware. Competitive devices are relying on improvements to resistor/thermistor networks to obtain frequency tolerances of ±1ppm. A few digital compensation systems have been developed. SEI have been approved to supply oscillators for SCIMITAR and their device comprises two packages; one is a 40-pin integrated circuit package containing a hybrid circuit with a second 18-pin package containing select-on-test resistors.

Apart from reference 19, no other references have been found which describes the generation of a temperature dependent voltage (or current) which has to vary according to a known mathematical function. The system of compensation could be used in other applications. One possible application within the division is the compensation of accelerometers which use SAW oscillators to measure acceleration. A SAW oscillator has a parabolic temperature coefficient and this system of compensation can be readily used with these devices.

10.2 Implications

There is some similarity in the method of manufacture between the TCX070 and existing TCXOs. Both require a compensation run during which the compensation voltage required to keep the oscillator at nominal frequency, and some d.c. voltages are measured at several temperatures across the operating temperature range. Both require the fitting of select-on-test resistors and a verify run during which the frequency deviation of the oscillator is measured across the operating temperature range.

Figure 10.1 shows the circuit diagram of a TCX070. The components used in the construction of an oscillator will all be surface mounting. The integrated circuit will be in a 28-pin chip carrier and all the resistors, capacitors and diodes will be chip components. This gives three problems of handling, identification and soldering. The chip components are small and are difficult to handle except with tweezers. They have no identification on them and can be easily mixed. Their size also means they are difficult to hand-solder. To achieve a smaller size than a TCX079 a hybrid circuit will be used. The use of surface mounting components is new to the division and their introduction will cause some difficulties. Hybrid circuits have been used in DIL oscillators and are familiar as components. Some investment in manufacturing equipment will almost certainly be necessary. It has been found that the oven system for the compensation and verify runs is inadequate for measuring the frequency to sufficient accuracy.

The TCX070 costs about £32 to make and although this cost is higher than other oscillators made by STC the cost should be reduced by using a hybrid substrate. At this price a large profit margin is maintained since the anticipated selling price is £60-150.

It is hoped that a hybrid circuit will be developed and will form the basis of a standard product although the term "standard" still has little meaning as each customer specifies his own product.

If an approach can be made early enough in the development cycle of a customer then it should be possible to sell an STC preferred product rather than have to adapt a product to meet an application. One benefit or perhaps a disadvantage is that the integrated circuit cannot be changed and this may be significant in reducing the product variety.

10.3 Future Work

The evaluation of the integrated circuit has been limited due to the time available. Some further evaluation is necessary to ensure that successful compensation of an oscillator is obtained in production. A close liaison will be necessary with the production engineers to ensure a successful transition from development to production.

Further work will be carried out on the third and fourth multiplier stages and the layout of a second integrated circuit will follow from this. It is hoped that the circuit will enable devices of $\pm$0.5ppm frequency tolerance over the temperature range 85^{o}C to -40^{o}C to be built.

Many of the types of oscillators built in production will simply be variations of the theme and little original development work will be needed. The main problems will be mechanical in nature and involve packaging.

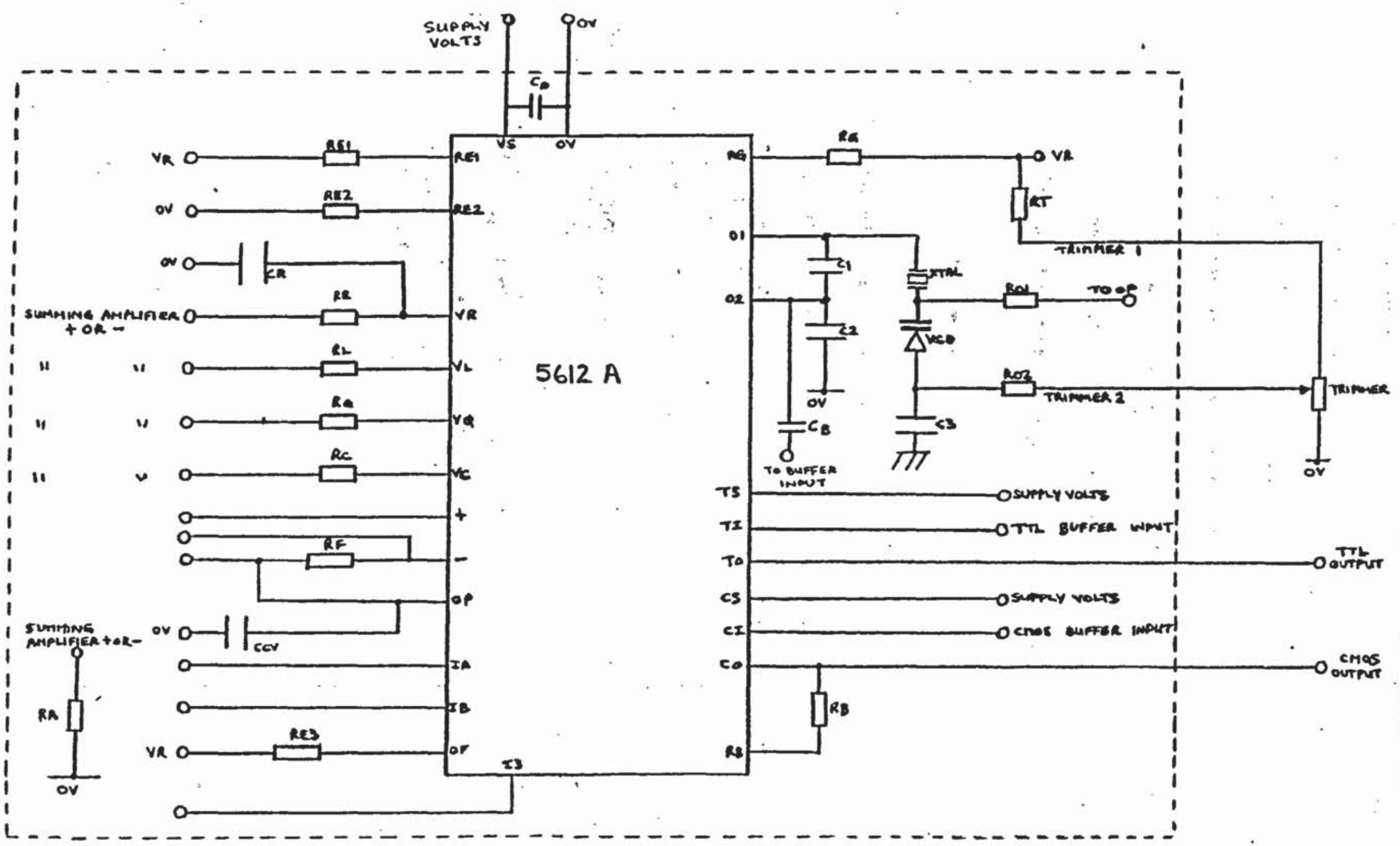


Figure 10.1 - TCXO70 Circuit diagram

APPENDIX 1 - CURVE FITTING USING ORTHOGONAL POLYNOMIALS

Curve fitting techniques using polynomials give the coefficients t_k, $k=1 \to n$, of the polynomial equation $y_k(x)$ of degree k :

$$y_k(x) = t_0 + t_1x + \dots\dots + t_kx^k$$

such that for a set of m points $(x_1,f_1) \dots (x_m,f_m)$

$$u_h = \left| f_h - y_k(x_h) \right| \text{ is small for each } h=1 \to m.$$

Normally the method of least squares is used in simple cases. This method minimises the total square of the error at each data point i.e.

$$\sum_{h=1}^{m} \mu_h^2 = \text{minimum}$$

The solution of the coefficients t_k involves solving a set of sumultaneous equations which is solved using matrix algebra.

Let $F = \sum_{h=1}^{m} \mu_h^2 = \sum_{h=1}^{m} \left(f_h - \sum_{j=1}^{k} t_jx_h^j \right)^2$

F is a function of t_j and for a least squares error, the partial derivatives $\frac{\partial F}{\partial t_j}$ are zero.

Hence

$$\frac{\partial F}{\partial t_j} = 2\sum_{h=1}^{m} \left(f_h - \sum_{j=1}^{k} t_j x_h^j\right)\left(-x_h^i\right) = 0 \qquad i=0 \to k$$

$$\Rightarrow \quad \sum_{h=1}^{m} f_h x_h^i = \sum_{j=1}^{k} \sum_{h=1}^{m} t_j x_h^j \cdot x_h^i$$

$$\Rightarrow \quad \sum_{j=1}^{k} t_j \sum_{h=1}^{m} x_h^j x_h^i = \sum_{h=1}^{m} f_h x_h^i$$

This is a set of linear equations in t_j. There are k partial derivatives which form k simultaneous equations. The equations can be defined as matrices:

$$TX = F$$

$$T = FX^{-1}$$

As k increases, the accuracy of the coefficient t_k tends to decrease. This well-known phenomenon of ill-conditioning, requires the points (x_m, f_m) to be accurately measured, since extremely small errors in these points cause large errors in the coefficients which are calculated. This effect can be overcome using orthogonal polynomials:

$$y_k(x) = t_o + t_1 p_1(x) + \dots + t_k p_j(x)$$

where $p_j(x), j=0 \rightarrow k$, are a set of orthogonal polynomials. These have the property that

$$\sum_{i=1}^{n} p_j(x_i) p_k(x_i) = 0 \quad \text{if } j \neq k.$$

In the matrix X all off-diagonal terms are zero and the solution of the coefficients t_k is trivial.

The orthogonal polynomials are generated using a standard recurrence relation giving

$$p_o(x) = 1$$
$$p_1(x) = (x-\alpha_1)$$
$$p_2(x) = xp_1(x) - \alpha_2 p_1(x) - \beta_1 p_o(x)$$
$$p_{i+1}(x) = xp_i(x) - \alpha_{i+1} p_i(x) - \beta_i p_{i-1}(x)$$

It can be shown that:

$$\alpha_1 = \frac{\sum_{i=1}^{m} x_i}{m}$$

$$\alpha_{i+1} = \frac{\sum_{\mu=1}^{m} x_\mu [p_i(x_\mu)]^2}{\sum_{\mu=1}^{m} p_i(x_\mu)^2}$$

$$\beta_i = \frac{\sum_{\mu=1}^{m} p_i(x_\mu)^2}{\sum_{\mu=1}^{m} p_{i-1}(x_\mu)^2}$$

If the required least squares approximation is

$$y_k(x) = \sum_{j=0}^{k} b_j\, p_j(x)$$

then

$$b_j = \frac{\sum_{i=1}^{m} f_i\, p_j(x_i)}{\sum_{i=1}^{m} p_j(x_i)^2}$$

A complete proof of this method is given in reference 21.

COMPUTER PROGRAM DAFIT1

This program was written in FORTRAN and referring to the program listing:

lines 30 → 40 : reserve space for all arrays

70 → 110 : read the input data

N = number of data points

X(I) = x-value of input data point

F(I) = y-value of input data point

lines 120 → 170 : copies F(I) into W(I) and calculates α_1 (ALPHA(1))

210 → 540 : calculates α_i, β_i and b_j according to the algorithm

α_i = ALPHA(I)

β_i = BETA(I)

b_j = B(J)

The values of $p_i(x_m)$ are stored in array P as $P(i, x_m)$.

The computer checks the error at each data point and calculates the coefficients t_k for the degree of approximating polynomial at which these errors are smaller than the specified error bound E. This degree is called IDWW (lines 750→770). The coefficients are printed by the instructions in lines 990→1010.

The program reduces the degree of the approximating equation by 1 and calculates a new set of coefficients based on a minimax error. The first set of coefficients give a least squares approximation. A more useful error form is a minimax approximation which minimizes the maximum error at all data points:

$$|\mu_h|_{max} = \text{minimum}$$

The error function for a minimax polynomial approximation of degree n to a set of data points, must have at least n+2 alternating maxima and minima of equal modulus ε_n, where ε_n is the error bound for the approximation. This is known as the Chebyshev Equioscillation Theorem.[37]

The program alters the y-value of the data point at which the biggest least squares error occurs until there are two errors of the same absolute value. 1030→1390 are the relevant program lines. The values of b_j for the minimax polynomial are stored in array BESTB. The program then prints the y-value of the data point, the value of minimax polynomial at that point and the error between these two quantities. The sum of orthogonal polynomials is then converted to a sum of powers of x in lines 790→970 and the coefficients printed (lines 1570→1670). The degree of the approximating polynomial is reduced by one and the minimax polynomial of that degree found. If the minimax error exceeds the specified error bound the program stops at that degree of polynomial.

COMPUTER PROGRAM : DAFIT1

```
C     CURVE FITTING PROGRAM V1.2 16-03-81
C     J. S. WILSON ITT COMPONENTS GCD
      DIMENSION X(15),F(15),W(15),C(15)
      DIMENSION ALPHA(15),BETA(15),B(15),P(15,15)
      DIMENSION POW(15,15,3),COEFFS(15),BESTB(15),Y(15)
      SMAXDY=5.0

C     READ INPUT DATA
      READ(1,10)N
   10 FORMAT(I3)
      READ(1,20)(X(I),F(I),I=1,N)
   20 FORMAT(2F7.3)
      DO 30 I=1,N
      W(I)=F(I)
      ALPHA(1)=ALPHA(1)+X(I)
      P(1,I)=1.0
   30 CONTINUE
      ALPHA(1)=ALPHA(1)/N
      READ(1,40)E
   40 FORMAT(F8.4)
C     CALCULATES ALPHA'S BETA'S AND NUMERICAL P'S
      DO 50 I=1,N
      P(2,I)=(X(I)-ALPHA(1))*P(1,I)
      ANUM=ANUM+P(2,I)**2
   50 CONTINUE
      BETA(1)=ANUM/N
      DO 60 J=2,9
      PP1=0.0
      PP2=0.0
      DO 70  I=1,N
      PP2=PP2+P(J,I)**2
      PP1=PP1+X(I)*P(J,I)**2
   70 CONTINUE
      ALPHA(J)=PP1/PP2
      BP1=0.0
      BP2=0.0
      DO 80 I=1,N
      P(J+1,I)=(X(I)-ALPHA(J))*P(J,I)-BETA(J-1)*P(J-1,I)
      BP1=BP1+P(J+1,I)**2
      BP2=BP2+P(J,I)**2
   80 CONTINUE
      BETA(J)=BP1/BP2
   60 CONTINUE


C     CALCULATES SIGMA'S OMEGA'S AND B'S
      DO 100 J=1,9
      SP1=0.0
      OP1=0.0
      DO 90 I=1,N
      OP1=OP1+W(I)*P(J,I)
      SP1=SP1+P(J,I)**2
   90 CONTINUE
      B(J)=OP1/SP1
  100 CONTINUE
      IF(KOUNT.GT.0)GOTO 170
C     CALCULATES UNWEIGHTED DEGREE
      WRITE(7,130)
  130 FORMAT(1H ,40HPOINT BY POINT ERROR IN INCREASING ORDER)
```

```
      DO 110 I=1,N
      DO 190 J=1,N
      C(J)=0.0
      DO 120 K=1,I
      C(J)=C(J)+B(K)*P(K,J)
  120 CONTINUE
  190 CONTINUE
      WRITE(7,140)I-1,(F(J)-C(J),J=1,N)
  140 FORMAT(1H ,I3,3X,15F7.3)
      DO 150 J=1,N
      IF(ABS(F(J)-C(J)).LE.E)GOTO 150
      GOTO 110
  150 CONTINUE
      GOTO 160
  110 CONTINUE
  160 IDWW=I-1
      WRITE(7,180)IDWW
  180 FORMAT(1H ,26HDEGREE WITHOUT WEIGHTING =,I3)
C     CALCULATES COEFFICIENTS

  170 POW(1,1,1)=B(1)
      POW(1,2,1)=(-B(2)*ALPHA(1))
      POW(2,2,1)=B(2)
      DO 200 J=3,IDWW+1
      DO 210 I=1,IDWW+1
      DO 220 K=1,3
      POW(I+1,J,1)=POW(I+1,J,1)+POW(I,J-1,K)*B(J)/B(J-1)
      POW(I,J,2)=POW(I,J,2)+POW(I,J-1,K)*(-ALPHA(J-1))*B(J)/B(J-1)
      POW(I,J,3)=POW(I,J,3)+POW(I,J-2,K)*(-BETA(J-2))*B(J)/B(J-2)
  220 CONTINUE
  210 CONTINUE
  200 CONTINUE
      DO 230 J=1,IDWW+1
      DO 240 I=1,IDWW+1
      DO 250 K=1,3
      COEFFS(J)=COEFFS(J)+POW(J,I,K)
  250 CONTINUE
  240 CONTINUE
  230 CONTINUE
      IF(IFLAG.EQ.1)GOTO 560
      WRITE(7,260)
  260 FORMAT(1H ,6HDEGREE,5X,11HCOEFFICIENT)
      WRITE(7,270)(J-1,COEFFS(J),J=1,IDWW+1)
  270 FORMAT(1H ,I3,9X,1PE11.4)
  280 KOUNT=KOUNT+1
C     ALTERS WEIGHTING AND CLCS B'S
C      ONLY CHANGES BIGGEST ERROR 0.001V PER ITERATION
      I1=0
      TMDSF=0.0
      DO 300 I=1,N
      C(I)=0.0
      DO 310 J=1,IDWW
      C(I)=C(I)+B(J)*P(J,I)
  310 CONTINUE
      IF(ABS(F(I)-C(I)).LE.E.OR.ABS(F(I)-C(I)).LT.TMDSF)GOTO 300
      TMDSF=ABS(F(I)-C(I))
      I1=I
  300 CONTINUE
      IF(SMAXDV.LT.TMDSF)GOTO 320
```

```
      SMAXDV=TMDSF
      DO 330 I=1,N
      BESTB(I)=B(I)
  330 CONTINUE
  320 IF(KOUNT.GT.10000)GOTO 500
      IF(I1.NE.0)GOTO 350
      DO 340 I=1,N
      BESTB(I)=B(I)
  340 CONTINUE
      GOTO 500
  350 IF(F(I1)-C(I1).GT.E)W(I1)=W(I1)+0.001
      IF(F(I1)-C(I1).LT.(-E))W(I1)=W(I1)-0.001
      DO 360  J=1,7
      OP1=0.0
      SP1=0.0
      DO 370 I=1,N
      OP1=OP1+W(I)*P(J,I)
      SP1=SP1+P(J,I)**2
  370 CONTINUE
      B(J)=OP1/SP1
  360 CONTINUE
      GOTO 280
C       OUTPUTS NEXT BEST FIT
  500 WRITE(7,510)
  510 FORMAT(1H ,17HBEST WEIGHTED FIT)
      WRITE(7,520)
  520 FORMAT(1H ,5X,4HDATA,6X,5HCURVE,6X,5HERROR)
      DO 530 I=1,N
      DO 540 J=1,N
      DO 550 K=1,3
      POW(I,J,K)=0.0
      COEFFS(I)=0.0
      B(I)=BESTB(I)
  550 CONTINUE
  540 CONTINUE
  530 CONTINUE
      IFLAG=1
      IDWW=IDWW-1
      GOTO 170
  560 DO 570 I=1,N
      Y(I)=COEFFS(1)
      DO 580 J=2,IDWW+1
      IF(ABS(X(I)).LT.0.001)GOTO 580
      Y(I)=Y(I)+COEFFS(J)*X(I)**(J-1)
  580 CONTINUE
      WRITE(7,590)F(I),Y(I),F(I)-Y(I)
  590 FORMAT(1H ,1P3E10.3)
  570 CONTINUE
      WRITE(7,260)
      WRITE(7,270)(J-1,COEFFS(J),J=1,IDWW+1)
      IF(KOUNT.GT.10000)STOP
      WRITE(7,600)
  600 FORMAT(1H ,28HATTEMPTING NEXT LOWER DEGREE)
      SMAXDV=5.0
      KOUNT=0
      GOTO 280

      END
```

COMPUTER PROGRAM DAFIT2

This program was written in FORTRAN and is similar to DAFIT1 except this program also calculates the sensitivity of the coefficients. In DAFIT2 the same input data is required with the addition of the degree of approximating polynomial. In lines 120 → 850 the least squares polynomial of the specified degree is calculated and in lines 890 → 1280 the minimax polynomial of the same degree is calculated. In lines 1290 → 1366 the values of the y-value of the input data, the value of the minimax polynomial and the error between these two quantities are printed. The value of the coefficients of the minimax polynomial are altered (lines 1370 → 2020) until the error in the coefficient contributes an error in the polynomial equal to 1/(degree of polynomial +1) of the difference between the specified error bound and the minimax error. This simulates a worst case error drift of all coefficients such that each coefficient contributes equally to the error of the approximating minimax polynomial. The lower limit, nominal and upper limit values of each coefficient are then printed (lines 2030 → 3010).

COMPUTER PROGRAM : DAFIT2

```
C      CURVE FITTING PROGRAM V1.3 16-03-81
C      J. S. WILSON ITT COMPONENTS QCD
       DIMENSION X(15),F(15),W(15),P(15,15),ALPHA(15),BETA(15),C(15),APLU(10)
       DIMENSION POW(15,15,3),COEFFS(15),SQ(15,15),AMIN(10),Y(15),B(15)
       DIMENSION A(15),AA(15)

C      READ INPUT DATA
       READ(1,10)N
    10 FORMAT(I3)
       READ(1,20)(X(I),F(I),I=1,N)
    20 FORMAT(2F7.3)
       DO 30 I=1,N
       W(I)=F(I)
       ALPHA(1)=ALPHA(1)+X(I)
       P(1,I)=1.0
       SQ(1,I)=1.0
    30 CONTINUE
       ALPHA(1)=ALPHA(1)/N
       READ(1,40)E
    40 FORMAT(F8.4)
       READ(1,45)IDWW
    45 FORMAT(I3)
C      CALCULATES ALPHA'S BETA'S AND NUMERICAL P'S
       DO 50 I=1,N
       P(2,I)=(X(I)-ALPHA(1))*P(1,I)
       ANUM=ANUM+P(2,I)**2
    50 CONTINUE
       BETA(1)=ANUM/N
        DO 60 J=2,8
      PP1=0.0
       PP2=0.0
       DO 70  I=1,N
       PP2=PP2+P(J,I)**2
      PP1=PP1+X(I)*P(J,I)**2
    70 CONTINUE
       ALPHA(J)=PP1/PP2
       BP1=0.0
      BP2=0.0
      DO 80 I=1,N
      P(J+1,I)=(X(I)-ALPHA(J))*P(J,I)-BETA(J-1)*P(J-1,I)
      BP1=BP1+P(J+1,I)**2
       BP2=BP2+P(J,I)**2
    80 CONTINUE
      BETA(J)=BP1/BP2
    60 CONTINUE


C      CALCULATES SIGMA'S OMEGA'S AND B'S
      DO 100 J=1,8
      SP1=0.0
      OP1=0.0
       DO 90 I=1,N
       OP1=OP1+W(I)*P(J,I)
      SP1=SP1+P(J,I)**2
    90 CONTINUE
       B(J)=OP1/SP1
   100 CONTINUE
       DO 140 I=2,N
       DO 150 J=1,N
```

```
      SQ(I,J)=SQ(I-1,J)*X(J)
  150 CONTINUE
  140 CONTINUE
      WRITE(7,180)IDWW
  180 FORMAT(1H ,26HDEGREE WITHOUT WEIGHTING =,I3)
C     CALCULATES COEFFICIENTS

  170 POW(1,1,1)=B(1)
      POW(1,2,1)=(-B(2)*ALPHA(1))
      POW(2,2,1)=B(2)
      DO 200 J=3,IDWW+1
      DO 210 I=1,IDWW+1
      DO 220 K=1,3
      POW(I+1,J,1)=POW(I+1,J,1)+POW(I,J-1,K)*B(J)/B(J-1)
      POW(I,J,2)=POW(I,J,2)+POW(I,J-1,K)*(-ALPHA(J-1))*B(J)/B(J-1)
      POW(I,J,3)=POW(I,J,3)+POW(I,J-2,K)*(-BETA(J-2))*B(J)/B(J-2)
  220 CONTINUE
  210 CONTINUE
  200 CONTINUE
      DO 230 J=1,IDWW+1
      DO 240 I=1,IDWW+1
      DO 250 K=1,3
      COEFFS(J)=COEFFS(J)+POW(J,I,K)
  250 CONTINUE
  240 CONTINUE
  230 CONTINUE
      IF(IFLAG.EQ.1)GOTO 560
C     ALTERS WEIGHTING AND CLCS B'S
C      ONLY CHANGES BIGGEST ERROR 0.001V PER ITERATION
  280 I1=0
      TMDSF=0.0
      DO 300 I=1,N
      C(I)=0.0
      DO 310 J=1,IDWW+1
      C(I)=C(I)+B(J)*P(J,I)
  310 CONTINUE
      IF(ABS(F(I)-C(I)).LT.TMDSF)GOTO 300
      TMDSF=ABS(F(I)-C(I))
      I1=I
  300 CONTINUE
      DO 120 I=1,N
      EBOUND=0.001
      IF(ABS(TMDSF-ABS(F(I)-C(I))).LT.EBOUND)KE=KE+1
  120 CONTINUE
      IF(KE.GE.IDWW+2) GOTO 500
      KE=0
  350 IF(F(I1)-C(I1).GT.0.001)W(I1)=W(I1)+0.001
      IF(F(I1)-C(I1).LT.(-0.001))W(I1)=W(I1)-0.001
      DO 360  J=1,IDWW+1
      OP1=0.0
      SP1=0.0
      DO 370 I=1,N
      OP1=OP1+W(I)*P(J,I)
      SP1=SP1+P(J,I)**2
  370 CONTINUE
      B(J)=OP1/SP1
  360 CONTINUE
      GOTO 280
C      OUTPUTS NEXT BEST FIT
```

```
  500 DO 530 I=1,N
      DO 540 J=1,N
      DO 550 K=1,3
      POW(I,J,K)=0.0
      COEFFS(I)=0.0
  550 CONTINUE
  540 CONTINUE
  530 CONTINUE
      IFLAG=1
      GOTO 170
  560 DO 570 I=1,N
      Y(I)=COEFFS(1)
      DO 580 J=2,IDWW+1
      IF(ABS(X(I)).LT.0.001)GOTO 580
      Y(I)=Y(I)+COEFFS(J)*X(I)**(J-1)
  580 CONTINUE
      AA(I)=F(I)-Y(I)
  570 CONTINUE
      WRITE(7,290)(F(I),Y(I),AA(I),I=1,N)
  290 FORMAT(1H ,1P3E10.3)
      DO 600 J=1,IDWW+1
      DO 610 I=1,N
       A(I)=COEFFS(I)
      C(I)=AA(I)
  610 CONTINUE
      STEP=0.001*COEFFS(J)
  630 A(J)=A(J)-STEP
       DO 620 I=1,N
      C(I)=C(I)-STEP*SQ(J,I)
      IF(ABS(C(I)).GT.ABS(TMDSF)+(E-ABS(TMDSF))/(IDWW+1))GOTO 635
  620 CONTINUE
         GOTO 630
  635 AMIN(J)=A(J)+STEP
     DO 640 I=1,N
       A(I)=COEFFS(I)
      C(I)=AA(I)
  640 CONTINUE
  650 A(J)=A(J)+STEP
      DO 660 I=1,N
      C(I)=C(I)+STEP*SQ(J,I)
      IF(ABS(C(I)).GT.ABS(TMDSF)+(E-ABS(TMDSF))/(IDWW+1))GOTO 670
  660 CONTINUE
        GOTO 650
  670 APLU(J)=A(J)-STEP
  600 CONTINUE
      WRITE(7,700)
  700 FORMAT(1H ,22HCOEFFICIENT TOLERANCES)
      WRITE(7,710)
  710 FORMAT(1H ,9HLOW VALUE,4X,6HMIDDLE,4X,10HHIGH VALUE)
       WRITE(7,720)(AMIN(J),COEFFS(J),APLU(J),J=1,IDWW+1)
  720 FORMAT(1H ,1P3E12.4)
      READ(1,20)(X(I),F(I),I=1,N)
      DO 800 I=1,N
      Y(I)=COEFFS(1)
      DO 810 J=2,IDWW+1
      IF(ABS(X(I)).LT.0.001)GOTO 810
      Y(I)=Y(I)+COEFFS(J)*X(I)**(J-1)
  810 CONTINUE
      AA(I)=F(I)-Y(I)
      WRITE(7,290)F(I),Y(I),AA(I)
  800 CONTINUE
        STOP
      END
```

APPENDIX 2 - COMPUTER PROGRAM COEFFS

This program was written in FORTRAN and the computer program is described by referring to the following line numbers which refer to the instructions in those lines.

30-90 : reserves memory space for all arrays

50-100 : values of the E24 series of resistors are placed in array RES. The values are from 1K to 1M.

110 : N= number of data points used

120 : M= degree of minimax equation + 1

130-140 : the input data is read. The FORMAT statement expects a tape input from a FACIT tape punch. The format of the data is (a) compensation voltage V

(b) voltage stabilizer output V_R

(c) linear voltage output V_L

(d) quadratic voltage output V_Q

(e) cubic voltage output V_C

for each temperature

170-180 : each input data value is divided by 1000 to give the actual voltage. The values of the compensation voltage are placed in array O and the values of the curve voltages in array V.

210-220 : the value of the feedback resistor in the summing amplifier is read

240-280 array NV contains the numbers of the temperature points which form the exchange group. For a third order polynomial the number of points is five.

290-460 : array W is the array used in the Gauss-Jordon matrix inversion.[38] For the five points defined in array NV if the Chebyshev line is

$$V_n = a_o V_{Rn} + a_1 V_{Ln} + a_2 V_{Qn} + a_3 V_{Cn}$$

then

$$a_o V_{R1} + a_1 V_{L1} + a_2 V_{Q1} + a_3 V_{C1} + h = V_1$$

$$a_o V_{R1} + a_1 V_{L2} + a_2 V_{Q2} + a_3 V_{C2} - h = V_2$$

$$a_o V_{R3} + a_1 V_{L3} + a_2 V_{Q3} + a_3 V_{C3} + h = V_3$$

$$a_o V_{R4} + a_1 V_{L4} + a_2 V_{Q4} + a_3 V_{C4} - h = V_4$$

$$a_c V_{R5} + a_1 V_{L5} + a_2 V_{Q5} + a_3 V_{C5} + h = V_5$$

Solving this set of simultaneous equations gives the Chebyshev coefficients for the group. Array W has the form:

$$\begin{array}{cccccccccc}
V_{R1} & V_{L1} & V_{Q1} & V_{C1} & +1 & 1 & 0 & 0 & 0 & 0 \\
V_{R2} & V_{L2} & V_{Q2} & V_{C2} & -1 & 0 & 1 & 0 & 0 & 0 \\
V_{R3} & V_{L3} & V_{Q3} & V_{C3} & +1 & 0 & 0 & 1 & 0 & 0 \\
V_{R4} & V_{L4} & V_{Q4} & V_{C4} & -1 & 0 & 0 & 0 & 1 & 0 \\
V_{R5} & V_{L5} & V_{Q5} & V_{C5} & +1 & 0 & 0 & 0 & 0 & 1
\end{array}$$

Using the Gauss-Jordon method this array is changed to:

$$\begin{array}{cccccccccc}
1 & 0 & 0 & 0 & 0 & k_{11} & k_{12} & k_{13} & k_{14} & k_{15} \\
0 & 1 & 0 & 0 & 0 & k_{21} & k_{22} & k_{23} & k_{24} & k_{25} \\
0 & 0 & 1 & 0 & 0 & k_{31} & k_{32} & k_{33} & k_{34} & k_{35} \\
0 & 0 & 0 & 1 & 0 & k_{41} & k_{42} & k_{43} & k_{44} & k_{45} \\
0 & 0 & 0 & 0 & 1 & k_{51} & k_{52} & k_{53} & k_{54} & k_{55}
\end{array}$$

490-810 : the row operations are performed

820-860 : matrix multiplications of the 5x5 array of k_{ij} values by the column matrix WA(I) which containes the values of the compensation voltages V_n gives the coefficients a_0, a_1, a_2, a_3, h which are stored in array C. Array WA is formed in line 430.

880-940 : the synthesized compensation voltage is found for all points and the errors between this voltage and the desired compensation voltage calculated and stored in array E.

960-1020 : the biggest error in array E is found and stored as BECUR. I1 is the number of the temperature at which this error is found. If the difference between BECUR and the minimax error of the group of points under consideration, h, is less than 1mV, the program jumps to line 1510. The Chebshev line for the group of points under consideration is the minimax polynomial for all points. A new group of points is calculated if the errors are not within 1mV.

1040-1090 : the maximum and minimum points in array NV are found and stored as MAXP and MINP.

1110-1130 : if the point of biggest error is such that one of the end points of the group is replaced, the lines 1140-1230 are used. The sign of the errors are evaluated and variable IS set to the number of the point to be replaced. The program then jumps to line 1430. IS in array NV is replaced by I1 (lines 1490-1510) and the array NV sorted into an ascending series of numbers using the function subroutine SORT (line 1520). The program has a new group and jumps to line 290 and the Chebyshev line for this new group found. If the point of biggest error is such that one of the middle points is replaced, then lines 1240-1470 are used. Firstly the two points in the existing group which are on either side of I1 are found and stored as IU (point above) and IL (point below). The signs of the errors at IU, IL and I1 are compared and variable IS set to the point that is to be replaced. This point is added to array NV and replaces point IS. The program returns to line 90 and finds the Chebyshev line for this new group.

1550-1670 : the coefficients are printed and the error between the synthesized compensation voltage and the desired compensation voltage calculated and printed.

1680-1890 : the coeffients are converted to resistor values. The feedback resistor is known and the resistor values calculated by dividing the feedback resistor by the coefficient. The resistor R5 is used to equalise the parallel resistance from each terminal of the summing amplifier. RX is the resistance from the inverting terminal and RY the resistance from the non-inverting terminal.

1900-2280 : each coefficient-setting resistor is evaluated as the sum of two of the resistance values stored in array RES. These values are printed out and gives the value of the resistors which should be fitted to an oscillator.

COMPUTER PROGRAM : COEFFS

```
C     MINIMAX TRIPLE EXCHANGE ALGORITHM
C      J S WILSON  ITTCG-QCD  17-12-81
      DIMENSION IV(15,6),V(15,6),O(15),W(5,10),WA(5),C(5),E(15),NV(5),RESIS(5)
      DIMENSION RES(73)
      DATA (RES(I),I=1,24),RES(73) / 1.0,1.1,1.2,1.3,1.5,1.6,1.8,2.0,2.2
     &,2.4,2.7,3.0,3.3,3.6,3.9,4.3,4.7,5.1,5.6,6.2,6.8,7.5,8.2,9.9,1000.0/
      DO 6 I=1,24
      RES(I+24)=RES(I)*10.0
      RES(I+48)=RES(I)*100.0
    6 CONTINUE
      N=9
      M=4
      READ(1,20)((IV(I,J),J=1,5),I=1,9)
   20 FORMAT(I5,2X,I5,2X,I5,2X,I5,2X,I5)
      DO 25 I=1,N
      DO 26 J=1,M
      V(I,J)=IV(I,J+1)/1000.0
      O(I)=IV(I,1)/1000.0
   26 CONTINUE
   25 CONTINUE
      READ(1,500)RF
  500 FORMAT(F9.1)
C     FORM WORKING MATRIX
      NV(1)=1
      NV(2)=3
      NV(3)=5
      NV(4)=7
      NV(5)=9
   35 DO 40 I=1,M+1
      DO 50 J=1,M
      DO 56 K=6,10
      W(I,J)=V(NV(I),J)
      W(I,K)=0
   56 CONTINUE
   50 CONTINUE
   40 CONTINUE
      W(1,5)=1
      W(2,5)=-1
      W(3,5)=1
      W(4,5)=-1
      W(5,5)=1
      DO 60 I=1,M+1
      WA(I)=O(NV(I))
      C(I)=0
      J=I+M+1
      W(I,J)=1
   60 CONTINUE
C     INVERT MATRIX W
      DO 70 K=1,M+1
      K1=K+1
      IF (K.EQ.M+1) GOTO 80
      L=K
      DO 90 I=K1,M+1
      IF (ABS(W(I,K)).GT.ABS(W(L,K)))  L=I
   90 CONTINUE
      IF (L.EQ.K)  GOTO 80
      DO 100 J=K,2*M+2
      T=W(K,J)
      W(K,J)=W(L,J)
```

```
      W(L,J)=T
  100 CONTINUE
   80 D1=W(K,K)
      DO 110 J=K,2*M+2
      W(K,J)=W(K,J)/D1
  110 CONTINUE
      IF (K.EQ.1)  GOTO 120
      K2=K-1
      DO 130 I=1,K2
      D2=W(I,K)
      DO 135 J=K,2*M+2
      W(I,J)=W(I,J)-D2*W(K,J)
  135 CONTINUE
  130 CONTINUE
      IF (K.EQ.M+1) GOTO 70
  120 DO 340 I=K1,M+1
      D3=W(I,K)
      DO 350 J=K,2*M+2
      W(I,J)=W(I,J)-D3*W(K,J)
  350 CONTINUE
  340 CONTINUE
   70 CONTINUE
      DO 140 I=1,M+1
      DO 150 J=1,M+1
      C(I)=C(I)+W(I,J+5)*WA(J)
  150 CONTINUE
  140 CONTINUE
C     COMPUTE ERRORS
      DO 160 I=1,N
      A=0
       DO 170 J=1,M
       A=A+C(J)*V(I,J)
  170 CONTINUE
      E(I)=O(I)-A
  160 CONTINUE
C     FIND LARGEST ERROR
      BECUR=0
      DO 180 I=1,N
      IF (ABS(E(I)).LE.ABS(BECUR))  GOTO 180
      BECUR=E(I)
      I1=I
  180 CONTINUE
      IF ((ABS(BECUR)-ABS(C(5))).LT.0.001) GOTO 190
C     FORM NEW QUINTET
      MINP=15
      MAXP=0
      DO 210 I=1,M+1
      IF (NV(I).LT.MINP) MINP=NV(I)
      IF (NV(I).GT.MAXP) MAXP=NV(I)
  210 CONTINUE
  710 FORMAT(1H ,3I3)
      IF (I1.LT.MINP) GOTO 400
      IF (I1.GT.MAXP) GOTO 410
      GOTO 420
  400 IF (E(I1).LT.0.AND.E(MINP).LT.0) IS=MINP
      IF(E(I1).GT.0.AND.E(MINP).LT.0) IS=MAXP
      IF (E(I1).LT.0.AND.E(MINP).GT.0) IS=MAXP
      IF (E(I1).GT.0.AND.E(MINP).GT.0) IS=MINP
      GOTO 430
```

```
 410 IF (E(I1).LT.0.AND.E(MAXP).LT.0) IS=MAXP
     IF (E(I1).GT.0.AND.E(MAXP).LT.0) IS=MINP
     IF (E(I1).LT.0.AND.E(MAXP).GT.0) IS=MINP
     IF(E(I1).GT.0.AND.E(MAXP).GT.0) IS=MAXP
     GOTO 430
 420 ICOPY=I1
     IL=0
     IU=0
     DO 220 I=1,N
     ICOPY=ICOPY+1
     DO 230 J=1,M+1
     IF (NV(J)-ICOPY.EQ.0)  GOTO 240
 230 CONTINUE
 220 CONTINUE
 240 IU=ICOPY
     DO 250 I=1,N
     ICOPY=ICOPY-1
     DO 260 J=1,M+1
     IF (NV(J)-ICOPY.EQ.0)  GOTO 270
 260 CONTINUE
 250 CONTINUE
 270 IL=ICOPY
     IF (E(IL).LT.0.AND.E(I1).LT.0) IS=IL
     IF (E(IU).GT.0.AND.E(I1).GT.0) IS=IU
     IF (E(IL).GT.0.AND.E(I1).GT.0) IS=IL
     IF (E(IU).LT.0.AND.E(I1).LT.0) IS=IU
     DO 360 I=1,M+1
     IF (NV(I).EQ.IS) NV(I)=I1
 360 CONTINUE
     GOTO 35
 430 DO 370 I=1,M+1
     IF (NV(I).EQ.IS) NV(I)=I1
 370 CONTINUE
     CALL SORT(NV,5,1,0)
     GOTO 35
 190 WRITE(7,310)
 310 FORMAT(1H ,16HCOEFFICIENTS ARE)
     WRITE(7,320)(C(I),I=1,M)
 320 FORMAT(1H ,1PE10.3)
     WRITE(7,280)
 280 FORMAT(1H ,5X,4HDATA,6X,5HCURVE,6X,5HERROR)
     DO 290 I=1,N
     A=0
     DO 330 J=1,M
     A=A+C(J)*V(I,J)
 330 CONTINUE
     WRITE(7,300)O(I),A,O(I)-A
 300 FORMAT(1H ,1P3E11.3)
 290 CONTINUE
     RX=1/RF
     RY=0
     DO 510 I=1,M
     RESIS(I)=RF/ABS(C(I))
 510 CONTINUE
     DO 520 I=1,M
     IF(C(I).GT.0) GOTO 520
     RX=RX+1/RESIS(I)
 520 CONTINUE
     DO 550 I=1,M
```

```
      IF(C(I).LT.0) GOTO 550
      RY=RY+1/RESIS(I)
  550 CONTINUE
      R5=1/(RX-RY)
      WRITE(7,540)RF
  540 FORMAT(1H ,18HFEEDBACK RESISTOR=,1PE11.3)
      WRITE(7,530)R5
  530 FORMAT(1H ,18HIMPEDANCE ADJUST= ,1PE11.3)
      WRITE(7,580)
  580 FORMAT(1H ,18HGAIN RESISTORS ARE)
      WRITE(7,590)(RESIS(I),I=1,M)
  590 FORMAT(1H ,1PE11.3)
      RESIS(5)=ABS(R5)
      C(5)=R5
      WRITE(7,600)
  600 FORMAT(1H ,6X,2HR1,7X,2HR2)
      DO 700 I=1,5
      INDEX1=0
      R=RESIS(I)
  705 R=R/1000.0
      INDEX1=INDEX1+1
      IF(R.GT.1000.0)GOTO 705
      DO 720 J=1,73
      I1=J
      IF(R.LT.RES(J))GOTO  730
  720 CONTINUE
  730 R1=RES(I1-1)
      R=R-R1
      IF(R.GT.1.0) GOTO 740
  735 R=R*1000.0
      INDEX1=INDEX1+2
      IF(R.LT.1.0)GOTO 735
  740 DO 760 J=1,73
      I1=J
      IF(R.LT.RES(J))GOTO 770
  760 CONTINUE
  770 IF((R-RES(I1-1)).GT.(RES(I1)-R)) R2=RES(I1)
       IF((R-RES(I1-1)).LT.(RES(I1)-R)) R2=RES(I1-1)
      IF(INDEX1.EQ.1)WRITE(7,800)R1,R2
      IF(INDEX1.EQ.3)WRITE(7,810)R1,R2
      IF(INDEX1.EQ.4)WRITE(7,820)R1,R2
      IF(INDEX1.EQ.6)WRITE(7,830)R1,R2
  800 FORMAT(2H  ,F7.1,3HK  ,F7.1,1HK)
  810 FORMAT(2H  ,F7.1,3HK  ,F7.1,1HR)
  820 FORMAT(2H  ,F7.1,3HM  ,F7.1,1HK)
  830 FORMAT(2H  ,F7.1,3HM  ,F7.1,1HR)
      IF(C(I).GE.0.0)WRITE(7,840)
      IF(C(I).LT.0.0)WRITE(7,850)
  840 FORMAT(1H ,4HSIGN,2X,1H+)
  850 FORMAT(1H ,4HSIGN,2X,1H-)
  700 CONTINUE
      STOP
      END
```

For an operational amplifier as shown in figure A3.1 the following analysis can be performed.

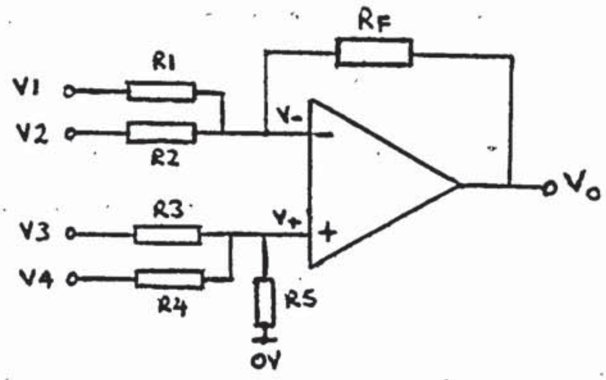


Figure A3.1 - Summing amplifier

$$\frac{V_1 - V_-}{R1} + \frac{V_2 - V_-}{R2} + \frac{V_o - V_-}{RF} = 0$$

$$\frac{V_3 - V_+}{R3} + \frac{V_4 - V_+}{R4} + \frac{0 - V_+}{R5} = 0$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_o}{R_f} = V_-\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_F}\right] = \frac{V_-}{R_x}$$

$$\frac{V_3}{R3} + \frac{V_4}{R_4} = V_+\left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right] = \frac{V_+}{R_y}$$

If the gain of the amplifier is large $V_+ = V_-$

$$V_o = -\frac{R_F}{R_1} \cdot \frac{R_y}{R_x} \cdot V_1 - \frac{R_F}{R_2} \cdot \frac{R_y}{R_x} \cdot V_2 + \frac{R_F}{R_3} \cdot \frac{R_y}{R_x} \cdot V_3 + \frac{R_F}{R_4} \cdot \frac{R_y}{R_x} \cdot V_4$$

$$= -\frac{R_F}{R_1} \cdot V_1 - \frac{R_F}{R_2} \cdot V_2 + \frac{R_F}{R_3} V_3 + \frac{R_F}{R_4} \cdot V_4 \qquad \text{if } R_y = R_x$$

For a fixed value of R_F, each gain can be individually set. R_5 is used to make $R_x = R_y$

$$\frac{1}{R_5} = \frac{1}{R_F} + \frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_3} - \frac{1}{R_4}$$

The listing of the HP-97 program is given in figure A4.1. The constants which require to be stored in the storage registers are given in figure A4.2. These values are the nominal values of the parameters and will give the values of the currents shown in the first analysis of Table 6.2.

The program operates as follows:

For a given value of temperature, the value of the base-emitter voltage of a transistor is calculated. The nominal value of this voltage at 25°C is stored in register 1. The currents flowing in the external resistors RE1 and RE2 (resistor values are stored in registers 3 and 4 respectively) are calculated and printed. The bandgap voltage from the voltage stabilizer must be given (register 2). The constant currents which are subtracted from the two currents flowing in RE1 and RE2 are stored in registers 0 and A and using these the currents I_A and I_B are calculated and printed. The current I_2 is calculated using the formulae

$$I_2 = \left[\frac{(I_B - I_A)^2}{I_A + I_B} + \underset{\text{(register 5)}}{\text{constant current in transistor 36}} \right] \times \underset{\text{(register 6)}}{\text{scale factor of transistor 124}}$$

This current is printed. The current I_3 is calculated using the formulae

$$I_3 = \left[\frac{(I_B - I_A)}{I_A + I_B} . \left(I_2 - \underset{\text{42, 43 and 44 (register 7)}}{\text{constant current in transistors}}\right) + \underset{\text{(register 8)}}{\underset{\text{transistor 52}}{\text{constant current in}}} \right]$$

multiplied by scale factor of transistor 134 (register 9)

```
001  *LBLA   21 11
002      2      02
003      5      05
004      -     -45
005   STOB   35 12
006      0      00
007      .     -62
008      0      00
009      0      00
010      2      02
011      x     -35
012    CHS     -22
013   RCL1   36 01
014      +     -55
015   RCL3   36 03
016            -24
017   STOC   35 13
018    ENG     -13
019   PRTX     -14
020   RCLB   36 12
021    ENT     -21
022      0      00
023      .     -62
024      0      00
025      0      00
026      2      02
027      x     -35
028   RCL1   35 01
029      -     -45
030   RCL2   36 02
031      +     -55
032   RCL4   36 04
033            -24
034   STOD   35 14
035    ENG     -13
036   PRTX     -14
037   RCLD   36 13
038   RCL0   36 00
039      -     -45
040   STOD   35 13
041    ENG     -13
042   PRIX     -14
043   RCLD   36 14
044   RCLA   36 11
045      -     -45
046   STOD   35 14
047    ENG     -13
048   PRTX     -14
049   RCLC   36 13
050   ENT↑     -21
051   RCLD   36 14
052      -     -45
053     X²      53
054   RCLC   36 13
055   ENT↑     -21
056   RCLD   36 14
057      +     -55
058      ÷     -24
059   RCL5   36 05
060      +     -55
061   RCL6   36 06
062      x     -35
063   STOE   35 15
064    ENG     -13
065   PRTX     -14
066   RCLC   36 13
067   ENT↑     -21
068   RCLD   36 14
069      -     -45
070   RCLC   36 13
071    ENT     -21
072   RCLD   36 14
073      +     -55
074      ÷     -24
075   RCLE   36 15
076   ENT↑     -21
077   RCL7   36 07
078      -     -45
079      x     -35
080   RCL8   36 08
081      +     -55
082   RCL9   36 09
083      x     -35
084    ENG     -13
085   PRTX     -14
086    R/S      51
```

Figure A4.1 - Program listing

620.-03	1
1.22+00	2
28.0+03	3
30.0+03	4
850.-09	5
5.00+00	6
27.0-06	7
5.80-06	8
3.00+00	9
10.8-06	0
11.5-06	A

Figure A4.2 - Constants

APPENDIX 5 - DERIVATION OF Z AND Y TYPE OSCILLATORS

(i) Z-Type

A z-type oscillator consists of an amplifier as one two port network and a π arrangement of three impedances as the other two-port network. Such an oscillator is shown in figure A5.1.

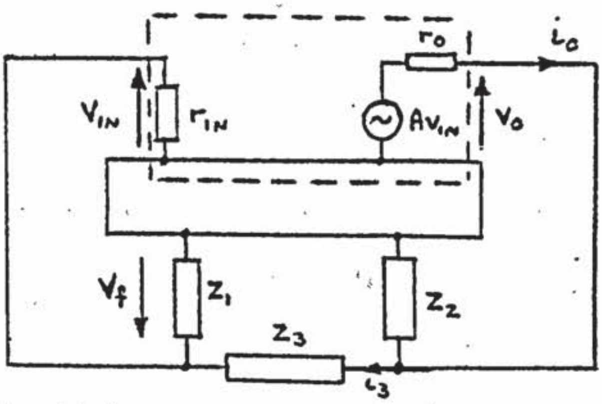


Figure A5.1 - Z-type oscillator

Assume that r_{in} is high and i_{in} can be neglected. This is valid since Z_1 is a reactance and r_{in} makes little difference to the magnitude or phase of the impedance of Z_1 and r_{in} in parallel.

For oscillation

$$V_f = V_{IN}$$

$$V_o = AV_{IN} - i_o r_o = Z_2(i_o - i_3)$$

$$\therefore \quad V_o - V_f = i_3 Z_3 - \frac{V_f Z_3}{Z_1} = V_f\left[1 + \frac{Z_3}{Z_1}\right]$$

$$i_o = \frac{V_o}{Z_2} + i_3 = \frac{V_o}{Z_2} + \frac{V_f}{Z_1}$$

$$V_o = AV_{IN} - r_o\left[\frac{V_o}{Z_2} + \frac{V_f}{Z_1}\right]$$

$$V_f\left[1 + \frac{Z_3}{Z_1}\right] = AV_{IN} - r_o\frac{V_f}{Z_2}\left[1 + \frac{Z_3}{Z_1}\right] - r_o\frac{V_f}{Z_1}$$

$$\Rightarrow \quad \frac{V_f}{V_{IN}} = \frac{AZ_1Z_2}{Z_2(Z_1+Z_3) + r_o(Z_1+Z_2+Z_3)} = 1.$$

Since A is real, AZ_1Z_2 is real and Z_1 and Z_2 are reactances.

$Z_1+Z_2+Z_3 = 0$ and so Z_3 must be a reactance.

$$\therefore \quad \frac{V_f}{V_{IN}} = \frac{AZ_1}{Z_1+Z_3}$$

$$\Rightarrow \quad A = \frac{Z_1+Z_3}{Z_1} = -\frac{Z_2}{Z_1}$$

Z_1 and Z_2 must be reactances of the same type and Z_3 a reactance of the opposite type. The simplest implementation is to let Z_1 and Z_2 be capacitive reactances and let Z_3 which is a quartz crystal be an inductive reactance.

(ii) <u>Y-type</u>

A y-type oscillator consists of an amplifier as one two-port network and a T-arrangement of three impedances as the other two-port network. Such an oscillator is shown in figure A5.2.

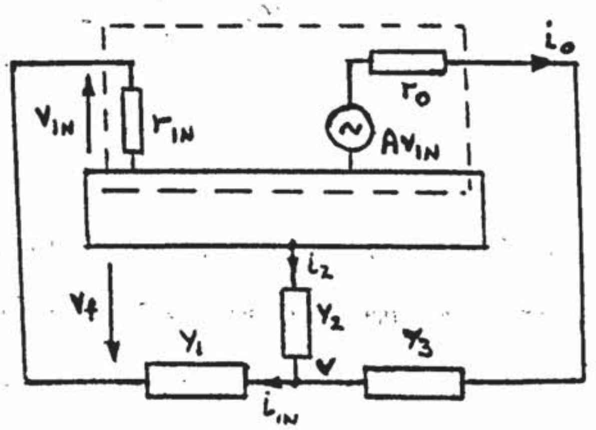


Figure A5.2 - Y-type oscillator

Assume that r_0 is small and can be neglected.

For oscillation $V_f = V_{IN}$

$$V = \frac{1/Y_1 + r_{IN}}{r_{IN}} \cdot V_f = \frac{1 + Y_1 r_{IN}}{Y_1 r_{IN}} \cdot V_f$$

$$(V_f - V)Y_1 + (0 - V)Y_2 + (V_o - V)Y_3 = 0$$

$$\therefore \quad V_f Y_1 + V_o Y_3 = V(Y_1 + Y_2 + Y_3)$$

$$V_f - \frac{Y_1 r_{IN}}{1 + Y_1 r_{IN}} V = \frac{Y_1 r_{IN}}{1 + Y_1 r_{IN}} \left[\frac{V_f Y_1}{Y_1 + Y_2 + Y_3} + \frac{V_o Y_3}{Y_1 + Y_2 + Y_3} \right]$$

$$\frac{V_f}{V_{IN}} = \frac{A Y_1 Y_3}{Y_1 r_{IN}(Y_1 + Y_2 + Y_3) + Y_1(Y_1 + Y_3)}$$

Since A is real, AY_1Y_3 is real and Y_1 and Y_3 are reactances.

$Y_1+Y_2+Y_3 = 0$ and so Y_2 is a reactance.

$$\frac{V_f}{V_{IN}} = \frac{A Y_3}{Y_1 + Y_3}$$

$$A = \frac{-Y_2}{Y_3}$$

Y_1 and Y_3 must be of the same type and Y_2 must be a reactance of the opposite type. The simplest implementation is to let Y_1 and Y_3 be capacitive reactances and Y_2 which is a crystal be an inductive reactance.

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