# NON-CONVENTIONAL TYPES OF VARIFOCAL SPECTACLE

LENS

Evangelos Pateras

Master of Philosophy

## THE UNIVERSITY OF ASTON IN BIRMINGHAM

November 1989

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#### The University of Aston in Birmingham

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The condition known as presbyopia is a normal physiological process, which begins to cause problems with near vision in middle-age. Unfortunately, although there are many varifocal lenses available which give some form of continuously variable vision, as in the pre-presbyopic eye, all current designs are compromise approaches to the problem, Sullivan and Fowler (1989) reporting a 12.5 % failure rate with one type. The aim therefore of this project was to investigate three different approaches to the one normally used, these being liquid crystal lenses, gradient index optics, and deformable lenses.

In the case of liquid crystals, an extensive literature survey was carried out, and a theoretical analysis of performance. In the case of gradient index optics and deformable lenses, a literature review was made and practical measurements were carried out, on experimental lenses constructed in the laboratory.

The liquid crystal lenses would seem to offer the best prospects for the long term, although there are severe difficulties to be overcome before they can be mass- produced. Gradient index lenses still require the development of a suitably stable lens material, and even then would not seem to offer many advantages over the existing lenses. Deformable lenses can be readily constructed, but are difficult to mass- produce, and suffer from leakage of the hydraulic control system

KEY WORDS: varifocal, spectacle lens, gradient index, liquid crystal lenses, deformable lenses

To my parents and my uncle John, who are always there when I need them

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List of contents	
List of Contents	5
List of Figures	11
List of Tables	18

# Chapter 1. Introduction

1.1.	Presbyopia; need for correction	22
1.2.	Overview of the attempts to correct	
	presbyopia with ophthalmic lenses	28
1.2.1.	Bifocal lenses; the first step	29
1.2.1.1.	History of bifocals	29
1.2.1.2.	Optical characteristics of bifocals	34
1.2.2.	Trifocal lenses; a better approach	
	to the problem	36
1.2.2.1.	History of trifocals	36
1.2.2.2.	Optical characteristics and performance	
	of trifocal lenses	37
1.2.3.	Progressive addition and variable focus	
	lenses; the alternative to bifocal and	
	trifocal lenses	39
1.2.3.1.	Progressive addition lenses review	40
1.3.	"Varilux" lens, a commercially successful	
	ophthalmic lens for presbyopia correction.	

Its history and optical performance,

and the necessity for other

approaches to correct presbyopia. 47

# Chapter 2. Liquid crystal lens-cell

2.1.	The history and nature of liquid crystals	55
2.2.	Applications of liquid crystals	60
2.3.	Liquid crystal lens-cell operation	61
2.3.1.	Liquid crystal lens-cells	61
2.3.2.	Magneto-optical and electro-optical	
	effects	61
2.3.3.	Characteristics of nematic structure	63
2.3.3.1.	Dielectric and diamagnetic anisotropy	65
2.3.3.2.	Birefringence. The key in liquid crystal	
	lens operation	67
2.3.3.3.	Selection of suitable materials to be used	73
2.3.3.4.	Alignment; an important process in liquid	
	crystal lens-cell	79
2.4.	Liquid crystal lens-cell evolution	82
2.5.	Conclusions-Discussion	106

# Chapter 3. Gradient-index lens with radial index distribution

3.1.	Gradient-index optics. Their applications	122
3.2.	The use of gradient-index optics in	
	ophthalmic lenses	128
3.3.	Review of the techniques used to create	
	a gradient	130
3.4.	Theory of concentric gradient-power	
	varifocal lenses	140
3.5.	Parameters and aberrations of gradient-	
	index lenses	150

# Chapter 4. Gradient-index varifocals. Experimental work

4.1.	Construction of "non-homogenous" lenses,	
	convex and concave according to Wood'	
	method	158
4.2.	The dioptric behavior of "non-homogenous"	
	or "pseudo" lenses	165
4.3.	The method followed to construct	
	progressive addition lenses with radial	
	index distribution	172
4.4.	The dioptric behavior of progressive	
	addition lenses with gradient-index	
	distribution Experimental data	176

		Page
4.5.	Conclusions related to the construction	
	and dioptric behavior of the	
	progressive addition lenses with	
	gradient-index distribution	178

# Chapter 5. Deformable lenses

5.1.	Literature review; different approaches	196
5.2.	Objectives of the research on deformable	
	lenses	216

# Chapter 6. Deformable lenses. Experimental

6.1.	Method of construction	219
6.2.	Experimental data related to deformable	
	lenses constructed	223
6.3.	Conclusions-Discussion	227
6.6.	Aspheric surfaces	252
6.5.	The usage of aspheric surfaces in	
	ophthalmic lenses	256
6.6.	Correlation of the aspheric deformable	
	surfaces with the related conicoids	257
6.7.	Assessment of the optical performance	
	of the deformable lenses	260

Chapter 7. Summary and comparison of the three researched		
types	. Recommendations for the future	
7.1.	Summary	280
7.2.	Comparison	283
7.3.	Recommendations for the future	286
Appendix A <sub>1</sub>	Changes of colour in cholesteric liquid	
	crystals	288
Appendix A <sub>2</sub>	The principle types of deformation in a	
	nematic liquid crystal	289
Appendix A <sub>3</sub>	The thickness problem in liquid crystal	
	lenses	290
Appendix B <sub>1</sub>	The results of Whitney's method	292
Appendix B <sub>2</sub>	Explanations on Marchand equations	293
Appendix C <sub>1</sub>	The dioptric behavior of Wood's concave	
	and convex lenses with 12 mm diameter	
	and 20 minutes diffusion time	294
Appendix C <sub>2</sub>	The dioptric behavior of Wood's concave	
	and convex lenses with 12 mm diameter	
	and 40 minutes diffusion time	295
Appendix C <sub>3</sub>	The dioptric behavior of Wood's concave	
	and convex lenses with 22 mm diameter	
	and one and a half hours diffusion time	296

Appendix C <sub>4</sub>	The dioptric behavior of the varifocal	
	lenses constructed following Wood's	
	modified method	297
Appendix D <sub>1</sub>	The dioptric behavior of deformable	
	lenses	310
Appendix D <sub>2</sub>	Computer scheme for assessing the	
	(p) value of the deformable lenses	
	aspheric surface	327
Appendix D <sub>3</sub>	Computer scheme for assessing the optical	
	performance of deformable lenses	329
Appendix E	Suporting publications	334
References		335

# List of Figures

		Page
1.1.	Cross-section of the human eye	24
1.2.	Schematic representation of the human	
	crystalline lens	25
1.3.	The accommodation function	26
1.4.	Types of bifocal lenses	31
1.5.	The effect of "image jump"	35
1.6.	Types of trifocal lenses	38
1.7.	Ave's design	41
1.8.	Spread of astigmatism	43
1.9.	The power change and the zones of	
	indistinct vision in the Omnifocal lens	45
1.10.	Power and radius change in the	
	"Varilux-1" lens, and its zones of indistinct vision	49
1.11.	Gradual change of the conic sections in "Varilux-2"	51
1.12.	Difference in power change between the	
	"V.M.D." and any tranditional "mono-design" lens	52
2.1.	The three categories of thermotropic liquid crystals	58
2.2.	Molecular arrangement in a nematic liquid crystal	64
2.3.	Dielectric susceptibilities variation	66
2.4.	Arrangement of nematic molecules when a	
	field is applied	70
2.5.	Schematic representation of birefringence	72

2.6.	Arrangement of Janning's device	81
2.7.	The lens made by Skaupy	83
2.8.	The lens made by Kosanke and Kulcke	85
2.9.	The device made by Sprokel	87
2.10.	The lens made by Bricot et al	89
2.11.	Molecular arrangement in Fray's device	91
2.12.	The lens made by Courtney et al	92
2.13.	The lens made by Berreman	94
2.14.	The lens made by Berreman (second embodiment)	95
2.15.	The lens structure proposed by Sato. The action of	
	a TN lens-cell	98
2.16.	The lens made by Okada at al with detecting and	
	compensating means	100
2.17.	Two types of lens-cells proposed by Okada et al	102
2.18.	Three types of liquid crystal lenses incorporating	
	a Fresnel type plate	105
2.19.	Effectiveness of acrylic resin with Nd added	108
2.20.	Molecular arrangement of a negative nematic material	
	in a) the quiescent state, b and c) when a field is applied	110
2.21.	Birefringence response and recovery times	112
2.22.	Proposed lens-cells to tackle the problem of thickness	115
2.23.	Okada et al lens-cell proposed to tackle the problem	
	of field uniformity	116

2.24.	The refractive index variation	118
2.25.	The system for operating a liquid crystal lens	120
2.26.	Liquid crystal lens-cell	291
3.1.	The three types of gradients	124
3.2.	Bifocal and varifocal lens proposed by Charman	131
3.3.	Sinai's apparatus	133
3.4.	Relation of diffusion depth with time and temperature	
	needed. Relation of diffusion depth and change in the	
	refractive index	136
3.5.	Lens undergone the ion exchange method	137
3.6.	Different apparatus used in ion exchange method	138
3.7.	The lens made by Bausch and Lomb	142
3.8.	Beach concentric varifocal	144
3.9.	Concentric type of varifocal with the astigmatism it	
	presents	145
3.10.	Concentric type of varifocal	146
3.11.	Relation of diffusion depth and change of the refractive	
	index	151
3.12.	Spherical aberration and meridional coma in a Wood	
	lens	155
3.13.	Meridional and sagittal curvature in a Wood lens	157
4.1.	Comparison of a Wood converging lens with a	
	conventional converging lens	159

4.2.	Wood's gradient-index lens-disk	162
4.3.	Relation of thickness and dioptric power in	
	Wood's lenses	167
4.4.	Relation of thickness, diffusion time and dioptric	
	power in Wood's lenses	169
4.5.	The lens-cell used in the construction of a varifocal	173
4.6.	The spherical change in a positive gradient-index	
	varifocal constructed with two hours diffusion time	180
4.7.	The spherical change in a positive gradient-index	
	varifocal constructed with two hours diffusion time	181
4.8.	The spherical change in a positive gradient-index	
	varifocal constructed with one and a half hours	
	diffusion time	182
4.9.	The spherical change in a negative gradient-index	
	varifocal constructed with two hours diffusion time	183
4.10.	The spherical change in a negative gradient-index	
	varifocal constructed with two hours diffusion time	184
4.11.	The spherical change in a negative gradient-index	
	varifocal constructed with one and a half hours	
	diffusion time	185
4.12.	The cylindrical change in the lens of Figure 4.6.	186
4.13.	The cylindrical change in the lens of Figure 4.7.	187
4.14.	The cylindrical change in the lens of Figure 4.8.	188

4.15.	The cylindrical change in the lens of Figure 4.9.	190
4.16.	The cylindrical change in the lens of Figure 4.10.	191
4.17.	The cylindrical change in the lens of Figure 4.11.	192
4.18.	The spherical and cylindrical change in a positive	
	gradient-index lens after 10 days of construction	195
5.1.	Cusco's deformable lens system	199
5.2.	Gordon's deformable lens system	201
5.3.	Mitchell's deformable lens	203
5.4.	Mitchell's spectacle pair of deformable lenses	205
5.5.	Graham's lens	207
5.6.	Fragioli's deformable lens system	210
5.7.	Wright's deformable spectacle pair	212
5.8.	Wylde's deformable lens	214
5.9.	A piezoelectric deformable lens system	215
6.1.	The deformable lens system constructed for this	
	experimental research	220
6.2.	A deformable spectacle pair constructed for this	
	experimental research	224
6.3.	Mean power change of deformable lens of Table 6.1.	236
6.4.	Mean power change of deformable lens of Table 6.2.	237
6.5.	Mean power change of deformable lens of Table 6.3.	238
6.6.	Mean power change of deformable lens of Table 6.4.	239
6.7.	Mean power change of deformable lens of Table 6.5.	240

6.8.	Mean power change of deformable lens of Table 6.6.	241
6.9.	Mean power change of deformable lens of Table 6.7.	242
6.10.	Mean power change of deformable lens of Table 6.8.	243
6.11.	Mean power change of deformable lens of Table 6.9.	244
6.12.	Mean power change of deformable lens of Table 6.10.	245
6.13.	Mean power change of deformable lens of Table 6.11.	246
6.14.	Mean power change of deformable lens of Table 6.12.	247
6.15.	Mean power change of deformable lens of Table 6.13.	248
6.16.	Mean power change of deformable lens of Table 6.14.	249
6.17.	Mean power change of deformable lenses of a	
	spectacle pair (First lens)	250
6.18	Mean power change of the other deformable	
	lens of the spectacle pair	251
6.19.	Conic sections and the conicoids they represent	253
6.20.	Conic sections with a common axis of symmetry	255
6.21.	The different optical correction that an eye	
	experiences from a conventional lens at	
	different angles of gaze	261
6.22.	The optical performance of the lens of Table 6.1.	268
6.23.	The optical performance of the lenses of Tables	
	6.2. and 6.13.	269
6.24.	The optical performance of the lens of Table 6.3.	270

6.25.	The optical performance of the lenses of Tables	
	6.4. and 6.5.	271
6.26.	The optical performance of the lens of Table 6.6.	272
6.27.	The optical performance of the lens of Table 6.7.	273
6.28.	The optical performance of the lens of Table 6.8.	274
6.29.	The optical performance of the lens of Table 6.9.	275
6.30.	The optical performance of the lens of Table 6.10.	276
6.31.	The optical performance of the lens of Table 6.11.	277
6.32.	The optical performance of the lens of Table 6.12.	278
6.33.	The optical performance of the lens of Table 6.14.	279

# List of Tables

1.1.	The colour changes of a cholesteric liquid crystal	288
3.1.	Astigmatism presented by a conventional lens at	
	different angles of gaze	292
3.2.	Astigmatism presented by a lens undergone	
	Whitney's method at different angles of gaze	292
4.1.	Dioptric behavior of Wood's concave lenses	
	with 12 mm diameter and 20 minutes diffusion	
	time	294
4.2.	Dioptric behavior of Wood's convex lenses	
	with 12 mm diameter and 20 minutes diffusion	
	time	294
4.3.	Dioptric behavior of Wood's concave lenses	
	with 12 mm diameter and 40 minutes diffusion	
	time	295
4.4.	Dioptric behavior of Wood's convex lenses	
	with 12 mm diameter and 40 minutes diffusion	
	time	295
4.5.	Dioptric behavior of Wood's concave lenses	
	with 22 mm diameter and 11/2 hours diffusion	
	time	296
4.6.	Dioptric behavior of Wood's convex lenses	
	with 22 mm diameter and 11/2 hours diffusion	
	time	296

4.7.	Gradient-index varifocal with two hours diffusion	
	time and 15 mm thickness	298
4.8.	Gradient-index varifocal with two hours diffusion	
	time and 15 mm thickness	299
4.9.	Gradient-index varifocal with two hours diffusion	
	time and 18 mm thickness	300
4.10.	Gradient-index varifocal with 11/2 hours diffusion	
	time and 15 mm thickness	301
4.11.	Gradient-index varifocal with 11/2 hours diffusion	
	time and 15 mm thickness	302
4.12.	Gradient-index varifocal with four hours diffusion	
	time and 15 mm thickness	303
4.13.	Gradient-index varifocal with 45 minutes diffusion	
	time and 15 mm thickness	304
4.14.	Gradient-index varifocal with two hours diffusion	
	time and 15 mm thickness	305
4.15.	Gradient-index varifocal with two hours diffusion	
	time and 15 mm thickness	306
4.16.	Gradient-index varifocal with 11/2 hours diffusion	
	time and 15 mm thickness	307
4.17.	Gradient-index varifocal with 45 minutes diffusion	
	time and 15 mm thickness	308

4.18.	Gradient-index varifocal of Table 4.10. measured	
	after 10 days of construction	309
6.1.	Deformable lens with 40 mm effective aperture,	
	1 mm deformable surface and 1 mm spacer	311
6.2.	Deformable lens with 40 mm effective aperture,	
	1 mm deformable surface and 2 mm spacer	312
6.3.	Deformable lens with 40 mm effective aperture,	
	1.5 mm deformable surface and 1 mm spacer	313
6.4.	Deformable lens with 40 mm effective aperture,	
	1 mm deformable surface and 2 mm spacer	314
6.5.	Deformable lens with 40 mm effective aperture,	
	1 mm deformable surface and 2 mm spacer	315
6.6.	Deformable lens with 40 mm effective aperture,	
	1.5 mm deformable surface and 2 mm spacer	316
6.7.	Deformable lens with 40 mm effective aperture,	
	1 mm deformable surface and 1 mm spacer	317
6.8.	Deformable lens with 40 mm effective aparture,	
	1.5 mm deformable surface and 3 mm spacer	318
6.9.	Deformable lens with 30 mm effective aperture,	
	0.5 mm deformable surface and 3 mm spacer	319
6 10.	Deformable lens with 40 mm effective aperture,	
	2.5 mm deformable surface and 2 mm spacer	320

6.11.	Deformable lens with 55 mm effective aperture,	
	2.5 mm deformable surface and 3 mm spacer	321
6.12.	Deformable lens with 60 mm effective aperture,	
	1 mm deformable surface and 3 mm spacer	322
6.13.	Deformable lens with 65 mm effective aperture,	-
	1 mm deformable surface and 3 mm spacer	323
6.14.	Deformable lens with 60 mm effective aperture,	
	2.5 mm deformable surface and 3 mm spacer	325
6.16.	The deformable lenses constructed with the (p)	
	value of their deformed surface	259

# **CHAPTER 1**

## INTRODUCTION

## 1.1. Presbyopia ; need for correction

The human body has sense organs through which it senses its surroundings and the eye is one of them. More than any other sense organ, the eyes feed the human brain with very detailed information. By converting the light rays into a coded neural activity, the human body visualises the surrounding world. Light waves, which are propagated in all directions, from every point of a visible object, pass through the optical system of the eye and are focused upon the retina. The retina is a thin layer of neural tissue and lines the back of the eyeball. At the retina, the light-sensitive receptor cells are located, where the light rays are converted into nerve impulses.

The optical system of the eye, is responsible for the focusing of the image of an object upon the retina. The most important elements of the optical system of the eye are the cornea and the lens. Of these two the cornea plays a larger role in focusing the incoming light rays. The shape of the cornea and the lens combined with the length of the eyeball determine where the light rays, entering the eye, will be focused. For an emmetropic eye this point should be on the retina.

Although the cornea performs the greater part of the focusing effect, the adjustments for the distance and near vision are attributed to the lens of the

eye. The lens, by adjusting its shape, brings into focus objects which are near or far from the eye. This remarkable automatic faculty, that allows objects at various distances to be clearly focused, is known as the accommodation of the eye. Figure 1.1. shows a cross-section of the human eye.

The human lens consists of elements which differ in shape, thickness and refractive index. These are the capsule, which is homogeneous and elastic, the epithelium and the nucleus, which consists of lens fibres. Figure 1.2. shows a schematic representation of the human crystalline lens and its composing elements. The lens is a biconvex, transparent optical system, which lacks blood vessels, otherwise the vessels would interfere with the lens transparency. The posterior pole of the lens is more curved than the anterior. Its shape is controlled by the ciliary muscle, which applies tension to the suspensory ligaments of the lens. The lens is flattened for distant objects to be focused on the retina, while it thickens and obtains a more spherical shape when the eye is trying to focus on near objects. Figure 1.3. represents the accommodation mechanism of the eye for near and distant vision.

The ciliary muscle is controlled by autonomic nerve fibres. More precisely, the sympathetic system relaxesthe ciliary muscle and the ligaments are tighten ed, resulting in the flattening of the lens. The parasympathetic system contributes to the contraction of the ciliary muscle resulting in the relaxation







Figure 1.2. Schematic representation of the human crystalline lens. After Peyman et al (1980).

•



Figure 1.3. Cross-section of the human illustrating the accommodation function. a) the lens is flattened to focus a distant object, b) the lens is thickened to focus clearly a near object. After Vander et al (1985).

of the lens suspensory ligaments, allowing the lens to thicken and become more convex for near focusing.

The crystalline lens continues to grow throughout life and more cell layers are added to the outer surface underneath the lens capsule. The older layers, which are located at the centre of the lens, are trapped in the nucleus where they become compressed. These cells die quicker than those at the outer surface due to the fact that they are away from the nutrient fluid which bathes the outside of the lens. After their death they become swollen and stiff. Consequently, the lens loses its elasticity and its ability to be deformed in order to accommodate for far and distant vision. This impairment of accommodation is known as presbyopia.

Presbyopia is a process which goes through out life, but only becomes a problem in middle-age. At that point some people start to find it difficult to focus clearly for near vision tasks and it becomes a necessity the assistance of a lens with a positive power in addition to any other correction required for distant vision. It should be mentioned that presbyopia is a physiological condition and does not induce any changes to the already existing refractive errors of the eye, but it is a dynamic condition which gradually increases through the passage of years and is completed nearly at the age of 70.

It is apparent that the usage of a plus lens will compensate for this loss of accommodation. The power of this plus lens is known as the near addition.

This additional assistance is essential when the patient's occupation involves near vision tasks. The first solution to presbyopia was the usage of two pairs of spectacle lenses, one for distance vision and another auxiliary one for near correction. The inconvenience of having to carry all the time an additional pair of spectacles and the fact that the patient has to change spectacles every time when near vision is needed led to the thought of one pair of spectacle but with lenses which would have more than one focal powers, multifocal lenses.

# 1.2. Overview of the attempts to correct presbyopia with ophthalmic lenses

The attempts that so far have been made in presbyopia correction consist of bifocal lenses, trifocals and progressive addition and variable focus lenses. Each of the attempts presents advantages and disadvantages which have to be considered.

#### 1.2.1. Bifocal lenses; the first step

## 1.2.1.1. History of bifocals

The first attempt to correct presbyopia, without having to carry two pairs of spectacles, is attributed to Benjamin Franklin (Emsley and Swaine, 1949). He was credited with the invention of the split bifocal lenses. In 1785, Franklin described a lens arrangement which was the originator of the earliest types of bifocals. He cut two lenses in half and mounted each half in the same frame. The first two original uncut lenses were for distance and near vision respectively. The bifocal lens constructed had excellent optical properties with very sharp images over the whole reading area. The optical centres could be placed as desired and there was an insignificant difference in the chromatic aberration presented by the two parts, as they were from the same material. Its main disadvantages were concentrated in the dividing line, where dirt and dust was collected and it produced unacceptable reflections to the eye. Besides that, the structure of the lens could easily come apart due to the fact that the two parts of the lens were held together only by the force exerted by the rim of the frame. Franklin's bifocal lens is very similar, in appearance with the "Executive" bifocal which is available commercially today and is a one-part bifocal. The front surface of the "Executive" bifocal has a more convex curvature in the lower half portion of the lens, producing in this way the necessary addition for near vision. The "Executive" bifocal was developed in 1954 by the American Optical Company.

In 1838, Schnaitmann introduced the "Solid Upcurve" bifocal, which was a one-piece bifocal lens (Sterling, 1935). The upper portion of the back surface of the lens had a less convex curvature than the lower part and it was used for distant vision. Morck, in 1888, invented the "Perfection" bifocal (Fannin and Grosvenor, 1987), which was a modified Franklin bifocal. In the same year Morck introduced another type of bifocal lens the "Cemented " bifocal, where at the back of the main lens a smaller segment of positive power was cemented, producing the addition. The adhesive used to cement the two parts together was Canada Balsam. The "Cemented Kryptok" bifocal, which was invented by Borsch in 1889 (Emsley and Swaine, 1949), consisted of the main lens and a button of higher refractive index, which was cemented into a depression in the back surface of the main lens. The back surface then was covered by a plano lens, which was cemented. In 1908, J.Borsch Jr. introduced the first fused bifocal (Emsley and Swaine, 1949). On the front surface of the main lens a button of higher refractive index glass was fused under high temperature. It should be mentioned that the first solid bifocal, commercially produced, was the "Ultex" bifocal in 1910 and the actual manufacturing process for producing it was proposed by Conner (Sterling, 1935). The addition was obtained by a change in the curvature of the back surface of a large lens blank. Figure 1.4. shows cross-sections and face views of the previously mentioned types of bifocals.

Figure 1.4. Schematic representation of the bifocals mentioned at 1.2.1.1.



a) Franklin's split bifocal - Face view and cross-section





b) the Solid Upcurve bifocal - Face view and cross-section





c) the Perfection lens - Face view and cross-section

Figure 1.4.a



d) the cemented bifocal - Face view and cross-section





e) the cemented Kryptok bifocal - Face view and cross-section



6

f) the fused bifocal - Face view and cross-section

Figure 1.4.b

## 1.2.1.2. Optical characteristics of bifocals

Although bifocals were a better solution in correcting presbyopia than the usage of two pairs of spectacles, problems arose related to the optical performance of a bifocal lens when it was placed in front of a presbyopic eye. These problems were related to prismatic effects presented by the lens and chromatic aberrations due to its design.

When the eye moves downwards from the distant portion of the lens to the near segment it experiences prismatic effects with a gradual or sudden change. As the eye moves away from the optical centre for distant vision a gradual increase in prism power is presented due to the main lens, being base up if the main lens is positive and base down if negative. When the visual axis of the eye reaches the dividing line a sudden change in the prismatic effect is introduced due to the near portion alone. This sudden introduction of additional prismatic power disturbs the vision and the objects viewed through that part of the lens instantaneously change position. This phenomenon, which is known as the "image jump" depends upon two factors, the distance of the dividing line to the optical centre for near vision and the power of the addition. This prismatic effect is always base down due to the fact that the addition is always positive. Such an effect will result to the loss of visual field. Figure 1.5. shows how the human eye is influenced by the "image jump" effect.

In bifocals with high plus power for distant vision, transverse chromatic



Figure 1.5. The effect of the "image jump" due to the dividing line of the segment. (After Jalie, 1988).

aberration might be a problem. Although the total transverse chromatic aberration for any point within the segment region might be considered negligible, the aberrations at the segment's periphery might be a problem. Some bifocal wearers complain of seeing colour fringes when viewing through the lower or upper edge of the segment. It should be noted that in both cases the transverse chromatic aberration, for a lens of +3.00 dioptres for distant vision, would be less than 1 prism dioptre (Jalie, 1988).

Although bifocals can be considered successful in correcting presbyopia, there is a point in the physiology of presbyopia which urges for another approach. That point is related to intermediate vision. The bifocal wearer is able to focus objects which are about 15 to 20 cm away or objects which are in a long distance from the eye, but it is almost impossible to see clearly objects that are 60 to 90 cm away. Occupations, involving intermediate distance vision as well as near vision, require a lens which would have more than two focal powers. These are trifocal and varifocal lenses.

## 1.2.2. Trifocal lenses; a better approach to the problem

#### 1.2.2.1. History of trifocals

The idea of trifocal lenses was first mentioned by J.I.Hawkins in 1826 (Sterling, 1935). In the description he gave, the trifocal lens consisted of three separate pieces of glass. The designing idea was relevant to
Franklin's split bifocal, where the three pieces were mounted together on the rim of the frame. In 1910, Conner introduced the first curved-top "Ultex" trifocal (Sterling, 1935). The first commercially available one piece trifocal was attributed to V.Hancock in 1932 (Fannin and Grosvenor, 1987) and it was a round-top fused trifocal. Hancock also developed the first straight-top fused trifocal in 1936 (Fannin and Grosvenor, 1987). After producing the "Executive" bifocal, American Optical Company introduced in 1954 the "Executive" trifocal. Figure 1.6. is a face view representation of the above mentioned trifocals.

# 1.2.2.2. Optical characteristics and performance of trifocal lenses

Although trifocals where a better approach to the problem of presbyopia they still do not provide the best solution. Their optical performance presents the same problems that the bifocals have and they also do not give progressive power change that the eye needs for intermediate vision. The wearer is able to focus clearly objects which are at a distance or near to the eye, but the intermediate vision is limited. Objects, which are in a distance between the distance and intermediate focal lengths or the intermediate and near focal lengths, will not be focused clearly. Also, the abrupt power change, which exists in bifocals, is present in trifocals too. So, the search for another type of lens had to be concentrated on creating a progressive power zone for the intermediate vision.



a) Round-top fused trifocal



b) straight-top fused trifocal





c) the Ultex one-piece trifocal

d) the Executive one-piece trifocal

Figure 1.6. Face view of the four types of trifocals mentioned at 1.2.2.1.

1.2.3. Progressive addition and variable focus lenses; the alternative to bifocal and trifocal lenses

Progressive lenses are those in which there is a gradual change in their radius of curvature. The aspheric lenses are also progressive where the changes in curvature are similar in all radial meridians. In progressive addition lenses the changes are limited to the front surface on the lens. They appeared in the 1950's and their popularity increased through that time due to the fact that the public demanded ophthalmic lenses of excellent quality. Unlike the bifocals and trifocal lenses their transition, from the distance to the near portion, was smooth and the fact that they had an "invisible" appearance made a large percentage of presbyopes turn to progressive addition wearers.

On the other hand, variable focus lens systems are those in which the power change can be adjusted over the whole effective aperture of the lens. According to Bennett (1973), these can be divided into systems in which the power alters a) after variably separating the component lenses, b) by an adjustable sliding contact between the components of the lens and c) by deforming the surfaces of the lens after exerting some kind of a pressure (deformable lenses).

#### 1.2.3.1. Progressive addition lenses review

Although this type of lenses became commercially available in the 1950's, their first appearance was in 1907. Aves (1907), that year was granted a patent on a progressive addition lens design, which was based on the combined effect of two surfaces. The front surface could be characterised as a convex cylinder with its axis horizontal and a profile which resembled the lower half of an ellipse having its major axis vertical. The second surface could be characterised as a cone with its axis vertically placed and its apex downwards. Figure 1.7. illustrates the power variation produced by this design. It is seen that there is a power increase from the top to the bottom of the lens, while at the central part the lens is free of astigmatism and is practically spherical. The main disadvantage lies in the fact that the cylindrical correction, if needed, for distance vision cannot be incorporated and that it is limited to hypermetropic presbyopes.

Gowlland (1914) patented a progressive addition lens which was based on an aspheric surface of revolution. The front surface of the lens was a toroidal surface, while the back was paraboloidal. In 1920, Poulain and Cornet proposed a progressive addition lens in which the progressive surface ( the curvature change took place on a single surface) had a vertical umbilical, which according to Bennett (1973) resembled an "elephant's trunk". Its advantage was that the correction for distance vision could be incorporated on the back surface of the lens even if a cylindrical correction was needed. This design did not present a surface with an axial symmetry,



Figure 1.7. Schematic representation of the effective power produced by Aves's design. A and B are the two elements of the lens, and they are positioned at 45° and 135° respectively. At the central portion of the lens (median meridian) a spherical increase of power occurs. (After Sullivan and Fowler, 1988).

which would have characterized it as a surface of revolution. Such a design was introduced, in 1946, by Beach, where the lens could be characterized as a blended bifocal. In 1924, Bugbee had an alternative approach in producing a progressive addition lens. He proposed the usage of gradient-index optics. With a suitable process a gradient of refractive index could be produced in a lens element, which would result to a gradual change of the focal power on a portion of the lens element.

Through the years many other researchers proposed new ideas on progressive addition lenses, but none of them proved to be the appropriate one. Commercial success was achieved in 1959, when Essel Optical of France introduced a progressive addition lens, which was developed by Bernard Maitenaz (1966). This lens gained popularity and it was the originator of a whole series of copies and improved versions, which are now available in the market. The trade mark of this lens is the well known "Varilux". Maitenaz in 1972, was the first to introduce the terms "dynamic" and "static" vision related to progressive addition lenses. The "dynamic" vision, which is the movement of the patient's eye relating to the object viewed, is correlated to a "soft" lens design. In a "soft" lens design, the astigmatism presented is spread over a large area of the lens, occupying most of lens surface. In contrast, the "static" vision is correlated to a "hard" lens design, where astigmatism is concentrated onto a smaller area than the one in "soft" lens design. The astigmatism presented in a "hard" lens design is of a much higher value than in "soft" lens design. Figure 1.8. shows the



Static / hard lens design



Dynamic / soft lens design

Figure 1.8. Schematic representation of the "hard" and "soft" lens designs. In the hard lens design astigmatism is spread in a smaller area than in the "soft" lens design, and consequently is of a higher degree. After Sullivan and Fowler, 1988. difference between these two designs as represented by Sullivan and Fowler (1988). Considering the technical characteristics of these designs, in the "hard" lens design the progression is short, while in "soft" lenses the progression is larger. Besides the "Varilux" lens there are other lenses which were launched in the market, with variable commercial success. First there was the "Omnifocal" lens, invented by Volk and Weinberg (1962). It was a bitoric lens with a front aspherical surface. On the front surface, where the progression took place, the radius of curvature gradually decreased from the upper part of the lens to the lowest point of the lens in the vertical meridian. while in the horizontal meridian the radius was kept constant. The gradual change of the spherical power was accompanied by a constant increase of astigmatism. This is the reason that a back toric surface was needed to compensate for the astigmatism presented on the front surface. The "Omnifocal" lens could be classified under the "soft" lens design since the progression takes place on the entire lens aperture. Figure 1.9. shows the progression surface and the zones of indistinct vision in the "Omnifocal" lens, which is no longer available in the market.

Another design was proposed by Guilino and Barth (1980), and it was introduced by Rodenstock under the trade name "Progressiv R" in 1984. The lens is available in CR-39, Crown glass and high index glass. The lens was based on the "Varilux" principle and can be classified under the "hard" lens design. Barken and Sklar (1987) produce a patent on a lens, with a linear progression in power along the vertical meridian from the distance to the near portions of the lens. This lens could be classified somewhere between



a) The power gradual change from the top to the bottom of the Omnifocal lens



b) The Omnifocal lens surface with the indistinct zones of vision

Figure 1.9. Schematic representation of a) the power change, and b) the zones of indistinct vision of the "Omnifocal" lens. After Jalie (1988).

the "hard" and "soft" lens design, having a large distance area and a relatively large near area, with the unwanted astigmatism spread widely in the periphery. The lens was launched, in 1984, by Sola Optical under the trade name "VIP" or "Graduate" and it is available in CR-39, high index (1.6) and Crown glass.

Progressive addition lenses are now numerous and so are the patents dealing with them. Sullivan and Fowler (1988) reviwed the literature, relating to such lenses as a sequel to that of Bennett (1973).

3. "Varilux" lens, a commercially successful ophthalmic lens for presbyopia correction; Its history and optical performance, and the necessity for other approaches to correct presbyopia

Although the progressive addition lenses available in the market now are quite enough to cover the public need, the lens of this kind that has gained the most commercial success is the "Varilux" lens and its successors.

The original design was developed by Maitenaz (1966) and is known as the "Varilux-1". The difference between "Varilux-1" and the "Omnifocal" lens is that the upper half portion of the lens in "Varilux-1" presented a constant power with no progression on it as in the "Omnifocal". This part of the "Varilux-1" lens was the largest of the three zones making up the lens. The progression zone, which was worked on the front surface, was confined to the centre of the lens, having a depth of about 12 mm. The third zone, after the progressive portion, had a constant power of the maximum addition required. The progressive corridor of the lens had a surface which was free of astigmatism having approximately a 5 mm width. Outside this area the amount of unwanted astigmatism increased going downwards to the bottom of the lens. Increase of the power of the add resulted to the decrease of the usable width of the progressive corridor. This amount of unwanted astigmatism presented at the periphery of the progressive corridor of "Varilux-1" sometimes could cause a sensation of blurring and vertigo to the wearer. To this is added the variation of the astigmatic axis, which resulted in

the intolerance of the lens by the wearer. Figure 1.10.a illustrates the power and radius of curvature change in "Varilux-1" related to the three zones of the lens. At the distance zone the lens consisted of a conventional spherical surface with a constant power. At the progressive zone, the radius of curvature decreased and gradually the power increased. In the reading portion the radius became constant again, reaching the maximum addition needed. The indistinct areas of vision on the lens surface are shown in Figure 1.10.b.

The design of the original "Varilux-1" evolved over a period of time and took place in four principle stages before the lens was commercially produced with a stabilised vision in both the distance and near zones. Such a design can be classified under the "hard" lens design due to its short progression zone. The "Varilux" lens should also be noted that it was not produced exactly the same for the right and left eye. There was an inclination of the line of symmetry in the progression zone, producing an inset of the near segment of 2.5 mm nasally. This provided a uniform power change for both eyes.

The sequel to "Varilux-1" was the "Varilux-2" patented again by Maitenaz (1974) and introduced in the market by Essel Optical in 1973. This time the aspherical design was not confined to the progression zone but over a much larger area on the front surface of the lens. The curves that formed the front surface belong to the conic sections family. The upper part of the



(b)

Figure 1.10. Schematic representation of a) the power change in the original "Varilux-1", related to the change in the radius of curvature at the intermediate zone. After Fannin and Grosvenor (1987), and b) the zones of indinstinct vision. After Jalie, 1988.

progression consisted of oblate ellipses while approaching the optical centre for distance vision they were transformed to circles. Below the optical centre for distance the surface was formed by prolate ellipses, which gradually turned into parabolas and finally hyperbolas. Figure 1.11. shows the gradual change in the conic sections of the front surface of "Varilux-2" related to the lens front surface aperture. With such a long progression the surface astigmatism and distortion, present in the periphery of the lens, were reduced in value compared to that of "Varilux-1". Therefore this lens is classified as a "soft" lens design.

Essilor of France, which is essentially the same company as the previously mentioned Essel Optical launched in 1988 the sequel to "Varilux-2". It was introduced under the trade name "V.M.D.", which stands for Varilux Multi-Design. The main difference compared with any other traditional "mono-design" is that the progression continues also through the near vision zone of the lens and does not suddenly stop just before the near vision zone. It is also a "soft" lens design, with a larger intermediate zone, providing greater visual comfort and width of view for the near area as its manufacturers claim. Figure 1.12. illustrates the difference between a traditional "mono-design" lens and the new "V.M.D." lens, in power. The diagram shows that the "V.M.D." has a much more soft power increase from the distance zone to the near zone, which increases and in the near zone.



Figure 1.11. Schematic representation of the gradual change of the conic sections consisting the front surface of "Varilux-2". From the top to the bottom of the lens surface (1), (2), and (3) are oblate ellipses, which gradually become a circle (4). (5) and (6) are prolate ellipses, while (7), (8), (9), and (10) are parabolas and hyperbolas. From Essilor Optical.



Figure 1.12. Diagram illustrating the difference between a traditional mono-disign varifocal and "V.M.D." lately introduced to the market by Essilor Optical (From Essilor Optical).

Considering the optical performance of the "Varilux" series, especially of "Varilux-2" (the "V.M.D." lens has been launched so recently in the market so not much information on its optical performance is available). It is apparent that "Varilux-2" suffers, as all the other progressive addition lenses of a substantial unwanted astigmatism with an increase value towards the peripheral areas of the lens. Although the astigmatic area is spread on a larger area on the lens surface, a "Varilux-2" lens of a +3.00 addition will suffer 3.00 cylindrical dioptres near the periphery of the lens (Sullivan and Fowler, 1988), producing blurred vision and distortion. Also, by increasing the area over which the astigmatism is spread, the width of the progressive corridor and the near portion, which are free of astigmatism , is smaller than in a "hard" lens design and it becomes even smaller by increasing the addition of the lens.

Besides the above problems, there is also the effect of the pupil size. Consider a wearer looking through the progressive corridor of the lens, which has an essentially spherical power. The rays, passing through the upper part of the pupil's aperture, will have a different convergence than the rays passing through the lower part of the pupil. It has been calculated that this difference, for a lens with a +2.00 D addition and a 6 mm pupil aperture, will be 1 dioptre (Fannin and Grosvenor, 1987).

Patients, who are nervous and impatient, will not cope easily with progressive addition lenses. They will have to learn to move their heads

rather than their eyes, in order to focus as clearly as possible through the progression zone. Their efforts should be concentrated in overcoming the initial discomfort presented by such lenses, due to the problems previously mentioned. According to Sullivan and Fowler (1989), there is a 12.5 % of presbyopes, who failed to adapt in wearing progressive addition lenses, while another 8 % needed a period of 10 weeks or more to get used to their lenses. It should be admitted that the progressive addition lenses are a compromise in presbyopia correction. The previously mentioned drawbacks and the results of the analysis of Sullivan and Fowler (1989) indicate that a better solution must be found, in order to satisfy all presbyopes, even the 12.5 % who failed to adapt to the existing lens designs. Also, the fact that , according to late demographic analysis the population of middle and old aged people will increase and especially in all the developed countries due to the reduced rate of births, is urging for research in other fields and for other approaches, which will result to an ideal solution in presbyopia correction. The intention of this project is to research on three fields that might reveal this ideal lens of the future in correcting presbyopia. These fields that were researched are a) Liquid crystals and their possible usage for creating a variable refractive index lens, b) the Gradient-index optics and c) the deformable lenses.

# **CHAPTER 2**

# LIQUID CRYSTAL LENS-CELL

#### 2.1. The history and nature of liquid crystals

The term liquid crystals refers to a state of matter that lies between the crystalline solid state and the amorphous liquid state, known also as the isotropic state (Chandrasekhar, 1977). This intermediate state is also known as "mesophase", according to the nomenclature quoted by Friedel (1922). The term derives from the Greek words "mesos", which means the intermediate and "phasis", which denotes state. Liquid crystals are in fact organic compounds, where according to Gray (1962) in every two hundred organic compounds, on average, one is liquid crystalline.

According to Kelker (1972), the history of liquid crystals can be divided into four periods of time. The first period starts from 1837, when the first report about them was made by the author Edgar Allan Poe in one of his stories (Poe, 1966). Poe made an alusion to a liquid crystal which he could not classify under the solid or the isotropic state. This period ends in 1908 and comprises the discovery of liquid crystals. The actual discovery of "mesophase" is attributed to Reinitzer in 1888 and Lehmann in 1890 (Lehmann, 1904,1909).

The second period starts in 1908 and ends in 1922, where the first theoretical work and nomenclature took place. The third period, from 1922 to

1933, comprises the development in the investigation of the effects presented by liquid crystals. The fourth period, between 1933 and 1945, consists of the new experimental work in comprehending the chemistry and morphology of liquid crystals. To these four periods, a fifth period should be added, from 1945 up to date, which comprises the vast applications of liquid crystals in todays technology.

Liquid crystals are divided into two major categories (Gray, 1962). These are the thermotropic and the lyotropics. The thermotropic liquid crystals are organic compounds in which the transition from the solid to the isotropic state takes place through thermal process. It is known that under certain conditions of temperature and pressure, the matter could be in three possible states, the solid, the isotropic or the gas state. The solid state is either crystalline or amorphous. The majority of the compounds have a crystalline structure in the solid state, where the molecules are arranged in a certain pattern of a lattice. In this state the mobility of the individual molecules, comprising the lattice, is restricted by cohesion forces. It is true that the crystalline lattice of some compounds under variable temperature and pressure conditions could be altered and a lattice modification could be presented (dimorphism, polymorphism), but the restriction of movements of molecules in the lattice still exists. In some organic compounds, when the crystalline lattice is heated and reaches a certain temperature threshold, due to the increase of thermal vibrations (Gray, 1962), the molecules cannot maintain their original arrangement within the lattice. For a certain

temperature range, this generally disorganised arrangement exists and a limited freedom is given to the molecules. Such behaviour does not comply to the solid crystalline properties nor to the totally molecular disorganised properties of the isotropic state. This anisotropy presented by the compounds is known as the "thermotropic mesomorphism". The lyotropic liquid crystals (lyotropic mesomorphism) transform from the solid state to the mesophase and then to the isotropic state through the influence of solvents. The lyotropics consist of more than two components, which are usually amphiphile and water. An example of such a composition is soap (sodium dodecyl sulphate) and water.

The thermotropic liquid crystals are divided into three categories according to their molecular arrangement (Saupe, 1969), a) the nematic, b) the smectic and c) the cholesteric. The nematic liquid crystals have no molecular arrangement but a common orientation in certain regions of the layer. The smectics have a stratified structure of molecules with a variety of molecular arrangements within each stratification. There are many smectic structures which differ in their molecular arrangements, but three are the most commonly met, the smectic A,B and C. The cholesterics are derivatives of cholesterol or any sterol systems. They are characterised from their helical aggregates of molecules. Figure 2.1. illustrates the molecular arrangement in these three categories.



The Nematic structure

The smectic A structure



The cholesteric structure



The smectic B structure



The smectic C structure

Figure 2.1. The molecular arrangement in the three categories of thermotropic liquid crystals

It should be noted that an organic compound exhibiting mesomorphism could obtain both the smectic and the nematic structure. The sequence that these changes present is illustrated below (Gray, 1962).

 T1
 T2
 T3

 Crystals ------ Smectic ------ Nematic ------ Isotropic

 mesophase
 mesophase
 liquid

These transitions occur by heating or generally increasing the temperature in any of these states. The sequence of these transitions is also reversible and takes place by supercooling of the compounds. The temperatures for the reverse route are the same as when the compound had undergone the increase of temperature. Transitions and changes such as the above described are associated with enantiotropic meso morphic

compounds. On the other hand, in monotropic mesomorphic compounds, when the crystalline solid is heated and the temperature reaches a transition temperature it transforms to an isotropic liquid, without exhibiting mesomorphism. When the isotropic liquid is supercooled and the temperature falls to less than the transition temperature T<sub>1</sub>, mesophase occurs before the recrystallisation of the compound. These transitions take place as illustrated below (Gray, 1962).

T<sub>1</sub>

Crystals ----- Isotropic liquid Mesophase

## 2.2. Applications of liquid crystals

Although considerable research had been done by the end of the fourth period of the liquid crystals history (Kelker, 1972), it was not until the last 25 years that this materials were considered to be appropriate for certain kinds of commercial applications. One of the major applications was in indicating skin temperature as temperature sensors (Selawry et al, 1966). The idea was based on the change in the colour of cholesteric liquid crystals according to small variations in temperature. Table 1. in Appendix A1 shows the colour changes of a cholesteric liquid crystal according to temperature variations. Also, cholesterics proved valuable to medicine. By observing the skin temperature changes the sympathetic system can be scanned and any open neurologic or vascular pathway is determined (Brown, 1969).

Besides their contribution to medicine liquid crystals are used in a numerous electronic displays such as watches, clocks, calculators, electronic games, car dashboards and screens of pocket televisions (Saeva, 1979). In addition to these, they are also used as anisotropic solvents in order to study compounds by nuclear magnetic resonance (Snyder and Meiboom, 1969), (Dong and Forbes, 1972), (Luckhurst, 1973). The liquid crystal applications and their future outlook are described in detail by Saeva (1979).

#### 2.3. Liquid crystal lens-cells operation

#### 2.3.1. Liquid crystal lens-cells

Liquid crystal lens-cells consist of a cell, where a layer of nematic liquid crystal is introduced into the cavity. These devices have been recently suggested as spectacle lenses used correcting presbyopia (Okada et al , 1986 a, 1986 b), (Sato, 1979). These lens-cells exhibit a variation in their focal length, which can provide, theoretically, the necessary addition for near vision. The variation in their focal length is related to the variable refractive index presented by the nematic layer. By applying an electric or magnetic field through the nematic layer, its refractive index changes from a minimum to a maximum value contributing to the change of focal length.

#### 2.3.2. Magneto-optical and electro-optical effects

The electromagnetic character of light is clearly demonstrated by a group of optical effects known as magneto-optics and electro-optics, depending whether a magnetic or electric field is used. In addition, these effects demonstrate the interaction between light waves and matter when the latter is under the influence of a strong external magnetic or electric field. According to Jenkins and White (1976), these effects can be classified in the following order :

Magneto-optics Zeeman effect Inverse Zeeman effect Voigt effect Cotton-Mouton effect Faraday effect Kerr magneto-optic effect *Electro-optics* Stark effect Inverse Stark effect Electric double refraction Kerr electro-optic effect

From the above mentioned effects, those which are of particular interest in liquid crystal lens-cells operation are the Cotton-Mouton effect and the Kerr electro-optic effect. In 1907, Cotton and Mouton discovered that when a liquid is placed in a transverse magnetic field the light passing through it was double refracted (Jenkins and White, 1976). This double refraction of light passing through pure liquids like nitrobenzene, which are under the influence of a magnetic field, is much greater than the one observed in the Voig effect. The mechanism of the effect is attributed to the alignment of the applied field. The alignment will be observed whether the magnetic dipole moments of the substance's molecules are permanent or induced by the applied field.

The Kerr electro-optic effect is the counterpart of the Cotton-Mouton effect with an electric applied field instead of a magnetic one. When a glass plate

is under the influence of a strong electric field it becomes double refracting. This effect is observed in many liquids and can even be exhibited by gases. By placing a liquid in an electric field, this will behave as a uniaxial crystal having its optical axis parallel to the direction of the field applied. The most suitable liquid, for observing the Kerr effect, is nitrobenzene due to its large Kerr constant (2.44 x 10  $^{-12}$ ).

# 2.3.3. Characteristics of nematic structure

In order to understand the mechanism of the liquid crystal lens-cells, some of the properties of the nematic structure must be stated. The molecules of nematic liquid crystals are either rod-shaped or disk-shaped (Berreman, 1978). In liquid crystal lenses the nematic material used consists of molecules which obtain a rod-shape structure. As mentioned previously, in the nematic structure there is no molecular arrangement between the neighbouring molecules except an average orientation of the long axis of the molecules in a specified region. This common orientation is designated as the director of that region, which corresponds to the direction of the optical axis of the layer. For the disk-shaped molecules the director of the region is designated as the direction perpendicular to the plane defined by the disk-shaped molecules. Figure 2.2. shows the molecular structure of a nematic material when this is in contact with a glass plate.



Figure 2.2. Schematic representation of the arrangement that the molecules of a nematic liquid crystal attain when they are in contact with a flat glass plate. It is clear that there is no specific order in their arrangement, only a common orientation of their long axes.

## 2.3.3.1. Dielectric and diamagnetic anisotropy

The rod-shaped molecules of the nematic structure exhibit an anisotropy in their dielectric and diamagnetic constant. The dielectric constant, also known as relative permittivity, is the ratio of the permittivity of a substance to that of a vacuum ( $e = e_0/e_s$ ), while the diamagnetic constant, also known as the relative permeability, is the ratio of the permeability of a medium to that of free space ( $\mu = \mu_m/\mu_o$ ).

Nematic molecules can be characterised as dipoles, having  $e_{\perp} \neq e_{\prime\prime}$  and  $\mu_{\perp} \neq \mu_{\prime\prime}$ , where  $e_{\perp}$  and  $\mu_{\perp}$  are the dielectric and diamagnetic susceptibilities perpendicular to the long axis of the molecules, while  $e_{\prime\prime}$  and  $\mu_{\prime\prime}$  are the dielectric and diamagnetic susceptibilities parallel to the long axis of the molecules. This anisotropy is determined by two factors a) the polarizability anisotropy and b) the dipole orientation effect (Chandrasekhar, 1977). It should be mentioned that the value of the dielectric and diamagnetic susceptibilities in every medium is dependent on the nature of the medium and the temperature. Figure 2.3. shows how the parallel and perpendicular dielectric susceptibilities are affected by temperature of the liquid crystal material n (4'-ethoxy- -benzylidene) 4-aminobenzonitrile (Schadt , 1972). It is clear that the dielectric susceptibility along the long axis of the molecule is affected more than the one perpendicular. When an electric or magnetic field is applied in a nematic liquid crystal layer, due to this anisotropy, the



Figure 2.3. The dielectric susceptibilities variation, according to temperature variatiobs of the n(4'-ethoxybenylidene) 4-aminobenzonitrile.  $e_1$  is the dielectric susceptibility parallel to the optic axis of the nematic material, while  $e_2$  is the one perpendicular, and  $e_3$  is the dielectric susceptibility in the isotropic state.

molecules will be affected and tend to align themselves in a different orientation than the one initially have. The new orientation of the molecules' long axes will be such as their dielectric or diamagnetic susceptibility, which is larger in value, will be align to the direction that the electric or magnetic field has (Saeva, 1979).

It should also be noted that nematic liquid crystals, related to the value of their dielectric and diamagnetic susceptibilities, exhibit either a positive or a negative dielectric and diamagnetic anisotropy. When  $e_{//} > e_{\perp}$  or  $\mu_{//} > \mu_{\perp}$  then the nematic material is characterised as positive. The directors of such a nematic layer will tend to align parallel to the field's direction applied. If  $e_{//} < e_{\perp}$  or  $\mu_{//} < \mu_{\perp}$  then the material is characterised as negative and its directors will align perpendicular to the field's direction (Saeva, 1979).

# 2.3.3.2. Birefringence. The key in liquid crystal lens operation

Besides the dielectric and diamagnetic anisotropy, nematic liquid crystals exhibit optical anisotropy. This anisotropy is presented as an anisotropy in the refractive index, and it is known as birefringence.

According to ray optics, birefringence refers to the splitting of a light ray into two components when it enters a solid uniaxial crystal like calcite and quartz. The two new components have orthogonal polarisations and are known as the ordinary and the extraordinary rays. The refractive index of the ordinary ray is independent of the orientation of the initial ray entering the crystal, while the index of the extraordinary ray strongly depends upon the angle that the initial ray and the orientation of the optic axis of the crystal make. Consequently, when the crystal is tilted and the optic axis of the crystal has changedorientation, while the orientation of the initial ray is kept constant, a variable refractive index is presented by the crystal, which is related to the extraordinary ray.

In liquid crystals, the tilting of the optic axis, which has the same orientation as the director of the region, is obtained by applying an electric or magnetic field. With a variable voltage field the changes in the orientation of the director is controlled and consequently the index presented by the nematic material will vary between the ordinary index  $n_0$  ( the index of the ordinary ray ) and the extraordinary index  $n_e$  ( the index of the extraordinary ray ). Chandrasekhar et al (1969) give a detailed analysis of the birefringence theory in the nematic liquid crystals. The theory is developed taking into account the intermolecular potential energy due to dipole-dipole, anisotropic dispersion repulsion and induction interactions. The birefringence is evaluated in terms of the Boltzmann distribution of the oriented molecules.

The electrically induced birefringence (Soref and Rafuse, 1972), (Soref, 1973), (Kahn, 1972) can be explained as follows. When a liquid crystal with

a positive dielectric anisotropy is placed in a cell comprising two transparent flat glass plates and a spacer to create the cavity, where the liquid crystal will be introduced, the long axis of the molecules will orientate in a direction Z parallel to the plates. This orientation is attained after a suitable preparation of the inner surface of the plates (alignment process). Figure 2.4.a illustrates the above mentioned configuration. The optic axis of the nematic molecules corresponds to the orientation of the long axis of the molecules and is related to the ordinary ray. The incident rays enter perpendicular to the plates and consequently to the optic axis of the material. If now, the entering rays are polarised in the direction Z, then the index presented by the layer will be the extraordinary index ne. If the inner surface of the plates are covered with transparent electrodes, an electric field can be applied by means of a voltage source. Due to the nematic material's dielectric anisotropy, the molecules will rotate and their long axes will obtain a new orientation, which will be parallel to the applied field E. When the electric field reaches a sufficient intensity the molecules' long axes rotate 90°. At that point, the index presented will be the ordinary index no. Related to the field's intensity, it is possible to obtain orientations of the molecules' long axes, which will be between the orientation where the ordinary index no and the orientation where the extraordinary index ne is presented. Consequently, the refractive index presented could take values which are between a minimum (ordinary index no) and a maximum (extraordinary index ne). When the field is removed, the molecules reorientate and obtain their initial orientation, where the extraordinary ne is presented.



(a)



(b)

Figure 2.4. The arrangement of the nematic molecules in a cell, comprising flat glass plates, when a) the field is off, and b) the field is on. After Bricot et al 1977.

If, in a liquid crystal lens-cell with the same configuration as the one in Figure 2.4., a nematic material with negative dielectric anisotropy is placed, then the opposite effects will be observed. It should be mentioned that in a negative nematic material, the molecules' long axis is perpendicular to the direction Z. When light rays, which are polarised in a direction Z again , enter the cell the index will correspond to the ordinary index  $n_0$ , while when the field is on the extraordinary index  $n_e$  will appear. Figure 2.5. illustrates birefringence on a molecular level of a positive nematic material. For plane polarised light entering perpendicular the cell plates, the birefringence presented by the nematic layer is given by the Kerr equation (Beevers, 1975)

$$\Delta n = n_e - n_o = B \cdot \lambda \cdot E^2$$

where B is the Kerr constant of the nematic material,  $\lambda$  is the wavelength of the incident ray and E is the applied field. It is clear that birefringence is proportional to the square of the field's strength. Sheridan and Giallorenzi (1974) stated that the value of the electric field needed to produce a start in the reorientation of the molecules is given by the equation

$$E_{c} = (\pi / d)(K / e_{a})$$

where d is the thickness of the liquid crystal film, K is an average Frank



Figure 2.5. Schematic representation of birefringence in a liquid crystal lens at molecular level. When the field is off the component of light which is related to the extraordinary index passes through the liquid crystal layer. The opposite takes place when the field is on.
elastic constant for bend and splay and  $e_a = e_{//} - e_{\perp}$ . The whole bulk of the liquid crystal molecules needs several times stronger field than  $E_c$ , in order to reach the final new orientation. In addition, the threshold voltage for starting a change in the initial orientation of the molecules is given to be (Labrunie and Robert, 1973)

$$V_{th} = 2\pi (\pi \cdot K_{33} / |e_a|)^{1/2}$$

where  $K_{33}$  is the elastic constant of bend curvature in Frank's notation and  $e_a = e_{//} - e_{\perp}$ . More information about K and  $K_{33}$  are given in Appendix A<sub>2</sub>.

# 2.3.3.3. Selection of suitable materials to be used in liquid crystal lens-cells

The selection of the nematic material, suitable for use in liquid crystal lenses, is based upon a number of properties that these material have. The properties, which are of great interest and importance are described below. a) The temperature range in which the material obtains the nematic structure. Materials, which are nematic at room temperature, are preferred.

b) The response of the directors to the changes of the applied field.

c) The threshold of the field's strength, which is needed for inducing the change in the orientation of the molecules.

d) The resistivity of the material. High resistant materials are particularly suited, otherwise turbidity will prevail at elevated field strengths. The

resistivity should be more than  $10^{10} \Omega$ /cm (Saeva , 1979). Generally the nematic materials used in electro-optical effects belong to the categories described below (Saeva, 1979).

## A. Benzylideneanilines

These are derivatives of aromatic anil and have the general type



Benzylideneanilines, although are easily synthesised from the substituted benzaldehyde and aniline derivatives, are chemically unstable. It has been found that in the presence of acid, even trace of acid and moisture, these compounds easily hydrolize. Labes et al (1973) found that by introducing an ortho-hydroxy group in the starting benzaldehyde derivative, the anil does not hydrolyse.

#### B. Esters and Carbonates derivatives

Esters and carbonates derivatives are more stable, chemically, than the benzylideneanilines described above. Replacement of oxygen by sulphur occurs in a terminal substitution ( $-OCH_3 - --- - SCH_3$ ). This is due to the possible presence of sulphur compounds inside the lens-cell, which results in the destabilisation of the nematic state. These compounds have the following general types



where R, R' = alkoxy



where R, R' = alkoxy

X in a) is usually -C=C-

#### C. Cyanobiphenyl derivatives

These type of compounds, which exhibit the nematic have the following general structure



where R = n-alkyl.

These compounds are chemically and photochemically more stable than the two categories mentioned previously A and B. They are also colourless and their nematic range covers room temperature. They possess desirable properties related to electro-optical effects, such as low threshold voltage and fast switching times. Consequently, these materials, which were discovered at Hull University in 1972 by Gray et al (1972), are preferably used in liquid crystal displays.

In some of the patents and papers related to liquid crystal cells, the materials used for presenting the electro-optical effects are :

a) Materials such as the "Kodak" (trade mark, catalogue # 11900) nematic mixture (Sprokel, 1977), with positive dielectric anisotropy. The "Kodak" mixture has  $e_{1/}$  about 8.5 and  $e_{\perp}$  around 7.

b) The mixture consisting of (Gerritsma et al, 1975)



This mixture has a nematic range between 20° and 92° C and its dielectric anisotropy changes sign according to changes in the frequency of the

applied field. At low frequencies (125 Hz) the material is positive, while at high frequencies (50 KHz) it is negative. The threshold voltage at the lower frequency is  $V_{th} = 1.95$  V.

c) The nematic liquid crystal N4 , which consists of

60 % 
$$CH_3O \longrightarrow N(O) = N \longrightarrow C_4H_9$$
  
40 %  $CH_3O \longrightarrow N = N(O) \longrightarrow C_4H_9$ 

It has a resistivity of about 5 x 10<sup>10</sup>  $\Omega$  / cm , showing negative dielectric anisotropy

d) Para-methoxybenzylidene para-n-butylaniline (M.B.B.A.) (Sato, 1979), which has a negative dielectric anisotropy and an isotropic-nematic state transition temperature ranging between 41° to 47° C depending upon impurities. Its refractive index takes values from  $n_0 = 1.545$  to  $n_e = 1.755$ . M.B.B.A is easy to prepare by mixing equimolar amounts of p-anisaldehyde and para-n-butylaniline with a small amount of absolute ethanol in a flask fitted with a reflux condenser. It is formed rapidly on heating. The ethanol-water azeotrope has to be removed by distillation. After that, M.B.B.A. has to undergone many distillations again in order to get rid of the impurities, until a constant isotropic-nematic state transition temperature is obtained.

e) The nematogen 4,4-n-heptyl-cyanobiphenyl, with positive dielectric anisotropy.

f) M.E.E. is a mixture of M.B.B.A., E.B.B.A. (p-ethoxybenzylidene-p-n-bytil aniline) and E.B.A.B. (p-ethoxybenzylidene-p-aminobenzonitrile) in a

weigth ratio of 2:2:1 (Sato, 1979)

g) P.C.B. (p-n-pentyl-p-cyanobiphenyl ) with a positive dielectric anisotropy of  $\Delta e = +11$ .

P.C.B. has no absorption band in the visible range and is more transparent than M.B.B.A., but it is more difficult to prepare than M.B.B.A. P.C.B.'s variation in the refractive index is between  $n_0 = 1.53$  and  $n_e = 1.74$ .

After the production of liquid crystals, the chemists efforts were directed towards producing liquid crystals which had properties suitable for displays for commercial production. Advances in the synthesis of liquid crystals (Gray, 1969) were concentrated on improving the quality and generally the properties of liquid crystals. Lately, formulation of new liquid crystal mixtures has occured. B.D.H. (1988) has formed such mixtures, with particular properties, for specific applications. These mixtures are known as 2-bottle system and 3-bottle system There is a class of B.D.H. products, which has a nematic range between  $-9^{\circ}$  to  $+60^{\circ}$  C, and their birefringence is very near to 0.2, some of them have a  $\Delta n$  which is greater than 0.2.

Generally, a variety of liquid crystals now exists, having a unique combination of desirable properties like low voltage operation, low power consumption, variation in colour, large temperature nematic range and large optical anisotropy.

2.3.3.4. Alignment; an important process in liquid crystal lens-cells

The alignment process refers to the preferential orientation of the nematic molecules, in order to orientate their long axis in a preferred and the same direction. This process can be divided into a) the initial alignment, which the molecules obtain in the quiescent state (field off) and b) the induced alignment, which is due to the applied field

Because the optical transparency of the liquid crystal lens-cells is dependent on this process, the alignment should be perfect and most of all uniform. It should be noted that the molecules are already orientated in a certain direction, parallel or perpendicular to the supporting walls, but this orientation is not totally uniform. The reason must be sought in a special kind of interaction between the molecules and the material of the walls. A number of techniques have been reported to produce this initial uniform orientation of the molecules. The simplest method has been reported by Chatelain (1954). This technique was based upon rubbing the inner surfaces of the plates with a piece of cotton or cloth, producing a quite uniform alignment along the direction of rubbing. Berreman (1973) explained that the uniformity was achieved due to groove formation along the inner surface of the plates urging the molecules to orientate their long axis parallel to the produced grooves.

Janning (1972) proposed another technique. He deposited a thin film of silicon or gold monoxide (SiO, AuO) on the inner surfaces of the cell at an angle by evaporation. The process took place in a vacuum system under 10<sup>-5</sup> Torr pressure and the film thickness was near 70 A. The angle he used was approximately 85° to the normal. Figure 2.6. is a representation of the system that Janning used for such a process. According to his results, parallel alignment was induced when the liquid crystal molecules were in touch with the walls of the cell. He, also, mentioned that the system was not affected through normal processing, cleaning and reprocessing circles. According to Saeva (1979), when the angle of evaporation takes values between O° and 45° no preferential orientation was obtained. For angles between 45° and 80°, perpendicular alignment was produced, while from 80° to 90° parallel alignment was presented. This technique is the most commonly used to-date. It gives better results than Chatelain's method, due to the fact the rubbing process did not produce the same uniform alignment when the lens-cell undergoing this process was used more than one time.

Suitable additive materials, such as the long chain alkyl pyridinium or quaternary ammonium salts, were found to provide parallel alignment (Sprokel, 1977). Also, surface-active agents, such as lecithin (Creagh and Kmetz, 1973) and polymerized films of hexamethyldisiloxane (Dubois et al, 1974), can provide an orientation effect perpendicular to the supporting plates. Perpendicular alignment can also be obtained by treating the inner surfaces of the plates with sulphuric acid and then rinsing with distilled water (Saeva, 1979).



Figure 2.6. Schematic representation of the arrangement that Janning used in order to produce initial alignment of the nematic molecules. The control substrated was placed in the centre of the Janning's device. After Janning 1972.

### 4. Liquid crystal lens-cells evolution.

As mentioned previously the electro-optical and magneto-optical effects, which are exhibited by certain classes of matter, were known by the beginning of this century. The first device, which has been reported to utilize these effects was introduced by Skaupy in 1933. It was an apparatus for deflecting light rays and intended to be used in picture telegraphy, television or any other relevant use. The apparatus consisted of a cell comprising four rectangular plane glass plates, which were filled with nitrobenzene. Inside the cell, three condenser plates  $P_1$ ,  $P_2$ ,  $P_3$  form with the surrounding liquid medium three deflecting prisms. When light passes through the cell in a direction as the one shown in Figure 2.7., the rays are deflected after applying an electric field upon the prisms. The final deflection of the rays is the sum of the three similar deflections produced by each of the three prisms.

It was only after twenty four years, when a U.K. patent was granted to a Swiss Body (No 808981) related to an optical system for amplifying the intensity of light of an optically projected image, that electro-optical effects were utilized again. The system consisted of a multi-layer, which comprised a light-sensitive layer combined with a layer of a medium which presented the Kerr effect and an interposed electrically non-conducting mirror layer. The electrodes, which were used to apply the electric field to the Kerr effect medium consisted of a thin layer of platinium.







Another type of electro-optic adjustable focus lens was introduced by Kosanke and Kulcke (1967). They proposed such a device to be used in controlling and directing electromagnetic radiation such as light and particularly to focus the radiation at preselected points. The electrode array for applying the field had a Fresnel type appearence, while the Kerr-effect substance used was a solid plate of potassium dihydrogen phosphate (KH<sub>2</sub>PO<sub>4</sub>). The actual desired result of such a lens was to change the state of polarization of the entering radiation. In the patent a table is given with the changes of the focal length for different wavelenghts of radiation. Figure 2.8. illustrates a face view of this lens. Two years later, Lotspeich (1969) produced an electro-optic lens with a variable focal length, utilizing two identical arrays of cylindrical electrode-rods, which were arranged in tandem and immersed into a Kerr-effect medium such as nitrobenzene. Each array consisted of four electrode-rods, which were evenly spaced, creating a circle and were charged alternatively positive and negative. One of the arrays was rotated 45° with respect to the other. The variable focal length lens described above was negative in power. A variable focal length positive lens could be produced by placing a conventional positive lens at the exit of the rays. Lotspeich gave also a detailed analysis of the related theory.

In 1972, Haas et al produced an image system by utilizing an electro-optic system again. Different types of materials were suggested to be used as supporting walls of the cell such as glass, polyethylene, polyvinylchloride, Teflon or a polytetrafluoroethylene. Suitable electrical conductors, having



Figure 2.8. Face view of the lens produced by Kosanke and Kulcke. The electrode's Fresnel configuration is shown, which is attached to the Kerr-effect plate. After Kosanke and Kulcke, 1967.

electrical resistivities in a range between  $10^{-8}$  to  $10^{-3} \Omega$  / cm, were also suggested such as aluminium, copper, silver, gold or tin oxide.. Their thickness should not be greater than 250 µm. As for the liquid crystal substance, a list of nematic materials was included in the patent such as M.B.B.A., p-azoxyanisole, p-azoxyphenetole, anisol-p-amino-azo-benzene and others.

An explanation of the theory of birefringence in nematic liquid crystals and the use of it in electro-optic information systems was given by Sprokel (1975). He suggested that the ideal thickness of the nematic layer should not exceed 20 µm. In his other two patents in 1977, Sprokel gave information on the liquid crystal cells operation and sugested the "Kodak" mixture to be used as the nematic layer, which is a homeotropically aligned positive nematic material. Figure 2.9. shows how Sprokel (1977) utilized the electro-optical properties of liquid crystals to produce a diaphragm, based upon the electrical exitation of the nematic layer when light passed through the system. The system consisted of a polarizer, a liquid crystal cell and an analyzer. When the field was off no light was transmitted through the system, while when it was on light passed through the system.

Bricot et al (1977) produced a liquid crystal lens-cell, with a variable refractive index such as the one illustrated in Figure 2.10.. It was suggested for use in an optical information system. According to them, the nematic material should have a dielectric anisotropy, which changes sign depending

Figure 2.9. The device that Sprokel used to produce a diaphragm. When the field is on light passes through the system, while when it is off the light does not pass. After Sprokel, 1977.





Figure 2.10. Schematic representation of the lens introduced by Bricot et al, 1977. (1) is a flat glass plate used as supporting wall of the lens-cell, (2) are electrodes used to apply the electric field, (3) annular spacers, (4) *opening through which the liquid crystal material will be introduced into the lens-cell*, (5) the nematic material, (6) convex glass plate used as supporting wall of the lens-cell. After Bricot et al, 1977.

on the frequency of the applied field. Below a reference frequency  $f_r$  these nematic materials have an anisotropy which is positive, while above the frequency  $f_r$  their anisotropy becomes negative. The change in their refractive index is about  $\Delta n = 0.2$  for a change in frequency  $\Delta f = 2$  kHz.. The response time of such a material to the changes of the frequency is 5 msec. In 1978, Fray et al, contributed a very clever approach to the utilization of liquid crystals. They introduced a liquid crystal lens-cell, electrically switchable, with a gradient index through its aperture. The gradient was produced by changing the polarity in one of the four electrodes used, incorporated in the cell, to apply the field. The molecules of the positive nematic material attained the structure shown in Figure 2.11. It is clear that the graduel change in the orientation of the molecules, through the aperture of the lens-cell, produced the gradient of index.

Courtney et al (1978) proposed a lens-cell configuration as the one shown in Figure 2.12., comprising a liquid crystal layer to be used in altering the state of polarization of light when an electric field is applied to the layer. It consisted of a linear polarizer and an analyzer, where in between a nematic layer was placed. In addition , it incorporates positive and negative optical elements, crown and flint, to form a lens. In the same year and in 1980, Berreman proposed a liquid crystal lens-cell with a different configuration than in the previously mentioned lens-cells. It was proposed to be used in cameras, telescopes, binoculars, projectors and spectacles. It is probably the first time that such a lens-cell was proposed for spectacle use.



Figure 2.11. The molecular arrangement in Fray et al's device. The production of the gradient is obtained by changing the polarity of one of the four electrodes used. After Fray et al, 1978.



Figure 2.12. Schematic representation of the component elements of Courtney's et al lens. After Courtney et al, 1978.

Berreman's lens-cell is shown in Figure 2.13. The cell consisted of a polarizer (1) orientated in a direction X. This is also the direction of the director of the nematic material. Layers (2) are for inducing the initial preferred orientation of the molecules (alignment layers). Electrodes (3) are used for the application of the field produced by the source (4), while (5) are the supporting walls of the cell. Figure 2.14. illustrates a second lens-cell proposed by Berreman. It comprised of two single variable refractive index lens-cells, each one producing a variable focusing effect on perpendicularly polarized components of light entering the lens in a direction Z. The proposed nematic material in such a lens-cell was the mixture (b) with the variable sign dielectric anisotropy mentioned previously in 2.3.3.3. Again in 1978, Hilsum and Raynes introduced a liquid crystal displayhaving voltage variation to compensate for the temperature variation, which influences and alters the optical performance of the cell. The nematic material layer, in the Hilsum and Raynes patent had a thickness of 12 µm. The cell had sensing electrodes, which were connected in series with a reference resistance. Such a unit produced a variable voltage across the sensing electrodes capacitance, which was used to control the voltage applied to the nematic layer and compensate for the temperature variations. Another cell with thermal compensation means was described by Portmann (1981). The compensation means consisted of a measuring capacitor and a reference capacitor. The reference capacitor was formed by two constituent reference capacitor 5. All of the previously mentioned capacitors were connected in series.



Figure 2.13. Schematic representation of the lens constructed by Berreman. After Berreman, 1980.



Figure 2.14. Schematic representation of the second embodiment in Berreman's patent. After Berreman, 1980.

In 1979 Sato proposed the usage of a plano-convex or plano-concave liquid crystal lens prepared with M.B.B.A. or P.C.B. or M.E.E., which is the mixture consist of M.B.B.A., E.B.B.A. and E.B.A.B.. The lens according to Sato could be used in presbyopia correction and even by aphakic patients. Figure 2.15.a illustrates the structure of such lenses. He also proposed the usage of a TN (twisted nematic) cell in front of a conventional convex lens. Such a configuration would have the same effects as the previously mentioned plano-convex lens. Figure 2.15.b shows this proposed construction. It should be mentioned that a polarizer is needed in both constructions. Sato analyzed the problems related to the optical performance of such lense, due to the thickness of the nematic layer, the response and recovery times, the temperature dependence of the nematic material's refractive index and the dynamic scattering effect, which is exhibited by negative materials. In a later paper Sato et al (1985) proposed the usage of a convex Fresnel lens to be one of the cell's walls. Such a configuration will improve the performance of the lens and specifically the response and the recovery times of the nematic layer. They suggested that such a lens could be used for spectacle wear. The incorporation of a Fresnel glass plate will improve the transparency of the lens-cell and increase the transmission according to their experimental results up to 80 % or even more. The actual proposed lens consisted of a double-layered structure of two identical Fresnel cells with mutually orthogonal optic axes. The nematic materials proposed had a positive dielectric anisotropy and a layer thickness of 100 to 250 µm.

Figure 2.15. Schematic repesentation of the lens structure proposed by Sato. (a) and (b) are liquid crystal lenses plano-convex and plano-concave respectively. (c) is a system comprising a TN lens, for altering the value of the refractive index of the entering ray from the extraordinary to the ordinary index and vise versa, according to the variable voltage applied to the TN cell. After Sato, 1979.



The most advanced liquid crystal lens-cell for spectacle wear has beeen described by Okada et al. Two patents, produced by Okada et al in 1986 and one in 1987, deal with the construction and the problems encountered by such a lens. In the first patent Okada et al (1986a) introduced a liquid crystal lens with variable refractive index due to an applied variable voltage source and means for detecting and compensating for any temperature change. With such means the refractive index of the nematic layer will be independent of any temperature variation. Figure 2.16 illustrates a cross-section of this proposed system. The liquid crystal lens comprises two plano-convex lens-cells A and B. The nematic molecules of the cell A are orientated in a direction designated by the arrows in the drawing and are at 90° to the direction of the molecules in the layer B (their orientation is perpendicular to the plane of the drawing). The transparent glass plates (7) create with the flat glass plate (8) two cavities due to spacers (9), where the nematic molecules of A and B are placed. The field is applied through the electrodes (10). The detecting and compensating unit consists of a temperature sensor (1), a temperature detecting and processing unit (2), a comparator (3), a corection unit (4), a variable voltage circuit (5) and a DC/AC converter (6). The voltage source V provides the current for the reorientation of the molecules, while source Vs produces the current which will compensate for any temperature change. In the same patent, further more sophisticated detecting and compensating units are described but their basic constructive idea and operation is the same as the one mentioned above. In one of these embodiments a solar cell is incorporated (mounted



Figure 2.16. Schematic representation of Okada et al liquid crystal lens with detecting and compensating means for the refractive index variation. After Okada et al, 1986a. on the fornt surface of the lens frame), for charging the power supply utilized by the detecting and compensating means. In another embodiment an arrangement of anti-misting means is incorporated to be used in such lens-cells.

The second patent (Okada et al ,1986b) deals with the protection of the nematic layer. At least part of the lens-cell is formed of a photochromic glass material combined with a thin film of an acrylic resin with added neodymium (Nd). This acts as a barrier to any harmful radiation, especially to ultra-violet rays, which will degrade the optical performance of the lens due to photochemical reactions. According to Okada et al (1986b) such a structure will also protect the eye of the wearer. Figure 2.17. illustrates a two lens configuration proposed in this patent. The first drawing (a) is a liquid crystal lens with a photochromic convex glass plate (1), electrodes (2), spacers (3), a flat glass plate (4), which is not necessarily of photochromic glass and the nematic material A (5). The molecules of the nematic layer have a preferred orientation indicated by the arrow. The field, produced by the source, passes through the DC/AC converter (6) and the variable resistor (7) and then is applied to the nematic layer through the electrodes (2). Such a lens needsa polarizing plate in combination with the photochromic glass (1). The second drawing (b) is a lens comprising two plano-convex lenses, such as the above mentioned, with their molecular orientation mutually orthogonal. In the third patent, Okada et al (1987) proposed the utilization of liquid crystal lenses which have one of their walls consisting of a Fresnel type lens. Such

Figure 2.17. Schematic representation of the liquid crystal lenses proposed by Okada et al. The lens-cells consist at least of one photochromic plate covered with acrylic resin with Nd added. A and B indicate the orientation of the nematic molecules's long axes. a) is a single liquid crystal lens-cell, while b) is a liquid crystal lens consisting of two single lens-cells, where a polariser is not needed. After Okada et al, 1986b.



(a)



a shape will reduce the thickness of the layer and the performance of the lens should be improved. Figure 2.18. illustrates the three proposed Fresnel types of liquid crystal lens-cells.



(a)



(6)



Figure 2.18. The three types of liquid crystal lenses proposed by Okada et al. One of the supporting walls of each lens-cell consists of a Fresnel type plate. After Okada et al 1987.

#### 5.Conclusions-Discussion

The production of a liquid crystal lens for practical and commercial use involves a number of problems. Almost all of these problems are related to the nematic material and its characteristics. For the construction of a high quality and high performance liquid crystal lens, the nematic material first of all should have a large birefringence. In order to achieve a maximum addition of three dioptres the birefringence of the material should be near the value of 0.2. Not all liquid crystal materials comply with this demand, but a reasonable number exist, where the birefringence is near the desired value. The other limitation in the selection of the nematic layer as discussed in 2.3.3.3, lies in their nematic temperature range. A material with a nematic range between -10° to +60 or +70° C should be suitable for such devices. It should be pointed out that in liquid crystals birefringence exists also in the isotropic state in temperatures very near the clearing point (transition point), but its value is insignificant and cannot be used in liquid crystal lenses.

The next important factor is the purity of the samples. The nematic layer should be perfectly purified in order to perform as desired. For this reason these materials have to undergo six or more distillations. A method to assess the purity of liquid crystals was described by Gray (1962). According to Gray (1962) the transition temperature from nematic to isotropic state is reversible. When the nematic material is heated enough to reach the clearing point, the material changes from the nematic to the isotropic state. By cooling down the isotropic liquid to the clearing point, it transforms to a nematic liquid

crystal. When the two temperatures are not the same the purity of the material must be suspected. Degradation of nematic liquid crystals can also occur by the exposure of the material to high intensity light. Ultra-violet rays and some of the visible spectrum of light are responsible for the degradation of the material's performance due to photochemical changes resulting in their unsuitability for use in liquid crystal lens-cells. The solution to the problem was given by Okada et al (1986b), where the cell consists of a photochromic glass covered with a thin film of acrylic resin with added neodymium (Nd). Figure 2.19. is a diagram showing the effectiveness of the acrylic resin with neodymium (Nd) against the light spectrum.

It was mentioned in 2.3.3.4. that the initial alignment produced in the nematic molecules was either parallel or perpendicular. The truth is that this alignment is statistically parallel or perpendicular to the supporting walls. This is due to thermal motion that the molecules exhibit as reported by DeGennes (1969). The degree of the parallel alignment is given to be (Chen et al, 1969)

$$S = 1/2 (3\cos\theta - 1)$$

where S is the order parameter,  $\theta$  is the angle between the long molecular axis and some external reference (boundary surface). For perfect parallel alignment to the external reference S=1, for a random distribution (isotropic state) S=0 and for a perfect perpendicular alignment S=1/2.



Figure 2.19 Diagram illustrating the effectiveness of acrylic resin with Nd added. After Okada et al, 1986b.
A similar effect is presented when the field is on. Due to boundary tension the molecules near the boundary surface do not reorientate at the same rate as the molecules in the centre of the layer, creating a lack of uniformity through the bulk of the layer. Figure 2.20. illustrates the two above mentioned effects, which will affect the performance of the liquid crystal lens.

Another serious problem that these lenses might present is the Dynamic Scattering Effect (D.S.E.). This effect was first discussed by Heilmeier et al (1968), and takes place when the field is on. Such an effect has been observed in nematic layers which have a negative dielectric anisotropy. When the field is applied a strong turbulence takes place and a large number of light scattering centres are formed. That means that although in the quiescent state the apearence of the nematic layer will be clear and transparent, in the state where the field is applied, the layer will be cloudy. D.S.E. needs both conduction and displacement current, compared to birefringence which needs only displacement current. The D.S.E. ceases above a certain frequency threshold (Shoref and Rafuse 1973). Consequently, birefringence has a wider frequency response than the D.S.E.. The upper frequency limit  $f_c$  for D.S.E. is approximately (Born and Wolf, 1964)

$$f_c = (\zeta^2 - 1)^{1/2} / 2\pi\tau$$

where  $\tau$  is the space-charge relaxation time and  $\zeta$  is the Helfrich parameter.



Figure 2.20. The molecules of a negative liquid crystal in a) the quiescent state, b) just after the field reaches a certain threhold voltage  $V_{th}$  and the molecules are starting to change the orientation of their long axes, c) when the applied voltage the maximum value necessary to reorientated the molecules' long axes.

This should be born in mind when negative nematic materials are used in liquid crystal lenses.

Two other properties of liquid crystal lenses, which must be taken into account, are the response and recovery times of the nematic material. In practice, the response time relates to the ease with which the molecules change from the quiescent state to the induced reorientation state, while the recovery time is the time needed for the molecules to attain their initial alignment after the field is off. Preferentially, these two times should be the same when it is necessary to reduce and then increase the focal length of the lens several times. Both these properties are found to be strongly dependent on the applied voltage (Labrunie and Robert, 1973). Theoretical values of the response and recovery times of M.B.B.A., related to the voltage applied are given in Figure 2.21.. It is obvious that the response time is proportional to the voltage applied and much shorter than the recovery time. They are also dependent on the thickness of the nematic layer (Sato et al, 1985). By decreasing the thickness of the layer these times are improved, becoming quicker and more sharp. This improvement can be achieved by adopting the structure of a Fresnel lens for the one or both supporting walls of the cell (Sato et al, 1985).

The most serious factor in the performance of liquid crystal lenses is the thickness of the nematic layer. Firstly, as mentioned above it affects the response and recovery times of the lens. The thicker the layer is the larger





Thickness of the cell =  $21.5 \,\mu m$ Liquid crystal used = MBBA

Figure 2.21. Birefringence response and recovery times. Approximate theoretical curves. After Labrunie and Robert, 1973.

these times become. But most important is the influence that it has upon the transmittance of the liquid crystal. The transmission of light beams entering the lens is expected to decrease since the absorption by the liquid crystal material increases and light scattering centres are formed, due to the degradation of the order in molecular alignment. By increasing the thickness of the nematic layer, the molecules in the centre of the cell will not align in the same direction as those near the boundaries and obtained a random orientation in space when the field is on or off. In the case of a plano-convex lens-cell with M.B.B.A., the centre of the lens will become milky-yellowish-white, while in a plano-concave lens the periphery is tinged. In most of the research work carried out up to now, the optimum thickness of the nematic layer is found to be 10 to 100 µm (preferably 50 µm ), in order for the cell to be transparent and perform ideally. But this is contradictory to the practical side of liquid crystal operation. With such a thickness the power change will be insignificant. For a lens with 40 mm diameter, having a plano-convex configuration and filled with M.B.B.A. of 25 µm thickness at the centre of the cell, the power change , which in the case of varifocal liquid crystal lenses will be the addition for the near vision, will be 0.02445 dioptres, which is insignificant. The analytical equations are described in Appendix A<sub>3</sub>. This is due to the very small sag of the liquid crystal layer. It is obvious that the effective power change that these lense exhibit is strongly dependent upon the sag of the liquid crystal lens. The smaller the sag the more insignificant the power change (addition ) becomes.

Two solutions have been proposed to handle the problem. Either by using a lens-cell with a Fresnel type structure (Okada et al, 1987) or by utilizing a multi-layered liquid crystal lens-cell. According to Okada et al (1987) the response and recovery times will be improved and the effective power change will be unaffected. The second proposed solution comprises the usage of more than one nematic layers. Probably 6 or more layers sandwiched together could provide the desired results. The layers of the liquid crystal material will be separated by electrodes formed in very thin transparent films reducing the thickness of the individual layers without reducing the actual thickness of the whole liquid crystal lens. Figure 2.22. shows such lens configurations.

Commenting on the plano-convex and plano-concave configurations proposed by a number of researchers, to be used as liquid crystal lens-cells, it is apparent that the lenses will suffer lack of uniformity of the applied field. This is due to the fact that the thickness at the centre of the lens is not the same as at the periphery. Consequently, the field applied at the centre of the lens-cell will be weaker than the one at the periphery, for a lens with a plano-convex configuration. The opposite will occur to a plano-concave lens-cell. How much this will affect the lens performance has not yet been estimated. Okada et al (1986a) proposed a configuration like the one shown in Figure 2.23 in order to overcome this problem.



Figure 2.22. Two proposed lens-cells in order to tackle the problem of thickness. (1) the supporting glass walls of the cells, (2) annular spacers, (3) transparent films of conductive material, (4) nematic material layers. In (A) the conductive layers have a convex configuration, while in (B) the layers are flat films.



Figure 2.23. Schematic representation of the liquid crystal lens proposed by Okada et al to tackle the problem of uniformity in the applied field. After Okada et al, 1986a.

The last and probably the most serious drawback that the liquid crystal lenses present is the temperature dependence of the nematic material's refractive index. The refractive index variation according to temperature changes has been pointed out by a number of researchers. An example of the index variation of H.C.B. related to temperature variation is given in Figure 2.24. (Davies et al, 1976). According to the diagram it is clear that the extraordinary index ne is more affected by the temperature variation than the ordinary index no. Above the clearing point of the material, birefringence disappears and a third refractive index n; is presented. Consequently, with any environmental changes the optical performance of the liquid crystal lens will alter. Okada et al (1986a) proposed a system of temperature detecting and compensating means in order to avoid this unwanted variation of the refractive index. This is probably the best approach to the problem among the previous attempts by other researchers to tackle the problem. The temperature sensor (1) could be a thermistor, or a thermocouple and it is mounted around the liquid crystal lens cell as shown in Figure 2.16. If the sensor is a thermistor it should be connected in series with a reference resistor across a constant voltage source of the temperature detecting and processing unit (2). This unit derives a voltage signal in the form of a resistance variation with temperature. In the case of a thermocouple, as a temperature sensor, the output temperature is obtained as a differential signal against a compensated cold contact at the unit (2). In both cases the output from the unit (2) is send to the comparator (3), where it is compared with the voltage  $V_{\text{S}}$  , which represents a given focal length. The output of the



Figure 2.24. The diagram illustrating the refractive index variation of PCB, according to temperature variations. (a) the variation of the extraordinary index  $n_e$ , (b) the variation of the ordinary index  $n_o$  and (c) the refractive index in the isotropic state  $n_i$ . After Davies et al, 1976.

comparator (3), which represents a deviation from the given temperature goes through the correction unit (4) and the variable voltage circuit (5) and is compensated for any temperature changes. This takes place by reducing or increasing the voltage level\_applied by the voltage source V. Instead of a DC/AC converter (6) an oscillator could also be used.

The preselected focal lengths of such a lens can be manually established as shown in Figure 2.25. In this case three selected focal lengths (distance, intermediate and near vision), are established. The switch S allows different voltages to be applied on the liquid crystal layer, from the variable voltage circuit (5) at a given temperature, while to maintain the selected focal length the correction unit (4) is co-operatively connected through the switch S to compensate for any temperature change. The actual number of focal lengths presented by a liquid crystal lens of such configuration is not limited to three, and a preselected number of focal lengths can be attained with a similar configuration to that of Figure 2.25.

Attempting an evaluation of the drawbacks of the liquid crystal lenses, it can be concluded that although problems exist are not unsurmountable. The only problem that seem difficult to handle is the one related to the thickness of the nematic layer, and consequently to its transparency. This is probably the main obstacle to liquid crystal lens production. It is my belief that there is a promising future in such an approach and perhaps in the next decade the manufacture of such alens might become a reality. Further experimental



Figure 2.25. Schematic representation of the detecting and compensating means of the lens configuration proposed by Okada et al. (1) a temperature sensor, (2) a temperature detecting and processing unit, (3) a comparator, (4) a correction unit, (5) a variable voltage unit. The switch S allows different voltages to be applied by manually changing the position of the switch S. Here, three preselected focal lengths are provided by such a unit. After Okada et al, 1986a.

work will need to be carried out, both in terms of the lens optics and effects upon visual perception, before such lenses could become useful devices for the correction of presbyopia.

# **CHAPTER 3**

# GRADIENT-INDEX LENS WITH RADIAL INDEX DISTRIBUTION

# 3.1. Gradient-index optics. Their applications

Although the concept of gradient-index optics has been known for a long time, it is only in the last few years that applications of such theory have been considered. These applications have come about as a result of new materials and new techniques being developed. It is well known that in a convential lens-element the index of refraction is considered to be homogenous, meaning that the refractive index is constant through the whole material. The only changes that can be applied to such a lens-element are, variations in the curvature of the front or the back surface or in both surfaces, and variations in the thickness of such a lens-element.

In gradient-index optics the lens-element manufactured has an index of refraction, which changes continuously through the whole aperture of the lens-element's material. There are three main theoretical types of refractive index gradients (Moore, 1980). These are a) the cylindrical gradient, b) the axial gradient, and c) the spherical gradient. Only the first two types have been developed, while the last one remains a theoretical example, in optical applications.

In the cylindrical gradient the index of refraction varies from the optical axis to the edges of the optical medium in a gradual and continuous way, creating cylindrical surfaces with constant refractive index. Examples of such a gradient are the optical fibres used in telecommunications, having a length greater than their diameters. Another example of this type of gradient is the Wood's "non-homogenous cylinders" or "pseudo-lenses" (Wood, 1905). The second type, the axial gradient, has an index of refraction, which varies continuously along the optical axis of the lens-element. This will create plane surfaces, which are parallel to one another but perpendicular to the optical axis, with constant refractive index. An example of such a gradient is the axial gradient collimator (Moore, 1977). It is suggested that this kind of gradient could be used in replacing aspheric surfaces producing similar or even better effects. The last type, the spherical gradient, has an index of refraction, which is symmetrical around a certain point. This centre of symmetry does not necessarily coincide with the centre of curvature of the surface of the lens-element. Compared with the two previously mentioned types of gradients, the spherical gradient has surfaces with constant refractive index, which are spheres. As mentioned previously no practical optical application of such a gradient has ever been developed. Examples of such a theoretical gradient are the Maxwell "fish-eye" (Maxwell, 1890), and the "Luneburg lens" (Luneburg, 1966), which theoretical aspects having useful applications in antenna design. Figure 3.1. illustrates the three types of gradient mentioned above.



Spherical gradient

Figure 3.1. The three types of gradients. The stippling indicates portions of higher refractive index, while the dashed lines represent the surfaces with constant refractive index. After Charman (1981).

Gradient-index optics currently have two major practical applications nowadays. These are, firstly in telecommunications (Jenkins and White, 1976) with fibre optics, and secondly in imaging systems. In telecommunications, a very long fibre with a gradient index is used to transmit light pulses over long distances. The idea of fibre optics is based on the principle that when light propagating in an optically dense medium approaches the boundary of a less dense medium at a certain angle  $\theta$ , which is greater than a critical angle  $\theta_c$ , then that light is totally reflected. It is apparent that the gradient-index fibres have a greater refractive index at the centre than at the edges. More precisely, in telecommunications an ordered array of very small transparent fibres is used. These fibres have a diameter of approximately 2 µm and are arranged in a certain order, otherwise, if they were randomly interwoven, the emerging light pulses at the end of the array would be meaningless.

The first gradient-index lens was proposed and developed by Berreman (1964) and it was actually a gas lens. According to Berreman (1964), in transmission light systems, losses were observed due to scattering and absorption. If gases are used as the media, which will guide light radiation, losses as the above mentioned are minimized. Berreman's gas lens, utilized gradients of refractive index, which were caused by temperature gradients without an axial symmetry about a certain point. The gases used in such a lens were air, carbon dioxide ( $CO_2$ ) and propane gas ( $C_3H_8$ ). According to Berreman, such a lens is aberrational free and even the amount of

astigmatism measured was surprisingly small. The other major application of gradient-index optics, besides telecommunications, involves the design (Moore, 1970) and analysis of the optical system used for image focusing. The analysis is carried out either by the third-order aberration theory (Sands, 1970a, Moore and Sands, 1971) or by ray tracing (Moore, 1974, Marchand, 1969 and 1972).

More recently, a glass rod with a radially gradient-index of refraction was developed by Pearson et al (1969). The gradient was produced by an exchange of alkali metal ions in the glass with alkali metal ions of a different size, from a fused salt bath. In practice, it was the replacement of lithium (Li<sup>+</sup>) ions in a lithium aluminosilicate glass with sodium (Na<sup>+</sup>) ions from a salt bath. That created a decrease in the refractive index of the glass, which was dependent upon the temperature of the glass and the exchange time. According to their results, the gradient-index profile of the glass was approximately parabolic, where the refractive index at any distance from the optical axis was given to be (Miller, 1965)

$$N(r) = N_A \cdot (1 - 2\pi^2 r^2 / L^2)$$

where N (r) is the refractive index at a distance from the optical axis,  $N_A$  is the index at the optical axis r is the distance from the optical axis and L is the length of the rod. The next lens-like glass, with a parabolic variation in its refractive index, was the SELFOC lens, which stands for self-focusing lens. It was produced in 1969 and was introduced in the market for commercial applications by Nippon Electric Company in Japan. This gradient-index lens was developed by means of an ion exchange technique (Kitano et al, 1969) and its optical characteristics were determined by Uchida et al (1970). It was intended to be used in optical communication systems, optical data processing and other optical instuments. Its refractive index variation was given to be (Marcuse and Miller, 1964)

$$n(r) = n_a(1 - 1/2 \cdot a_2 \cdot r^2)$$

where r is the distance from the optical axis,  $n_a$  is the refractive index at the optical axis, n (r) is the refractive index at a distance r from the axis, and  $a_2$  is a positive constant.

It has been recently reported (Nishizawa and Kitano, 1982) that SELFOC lenses are used as a) endoscopes with a diameter less than 2 mm, providing painless inspection of patients, b) compact optical systems in copy machines, c) optical couplers and wavelength multiplexers, having a relatively large diameter. Besides SELFOC lenses, it has also been reported (Iga, 1980) that gradient-index lens-elements were proposed for use as flight focuser for video disk players.

#### 3.2. The use of gradient-index optics in ophthalmic lenses.

It was mentioned previously that gradient-index optics theory had been applied in imaging systems, resulting in lens-elements like the SELFOC lenses. The gradient-index concept has also been utilized in the field of ophthalmic lenses. Such an application is clearly illustrated in a patent produced by Whitney (1977), where after the base curve of an ophthalmic lens was selected a gradient of refractive index was introduced to the lens to replace and obviate the usage of aspheric curvature, in order to correct off-axis defects. The base curvature was chosen so as to minimize the oblique astigmatism, while the refractive index gradation was used to minimize the curvature field error. As Whitney (1977) suggested, it is sometimes better to use aspheric surface design in combination with carefully selected base curve and a gradient of index, in order to minimize distortion besides the two off-axis aberrations mentioned above. In this case, base curve selection is used to minimize distortion, whether it is of the barrel or pinckshion type. The surface asphericity is employed to minimize oblique astigmatism and the refractive index gradient is utilized to reduce the curvature of field (power error).

The Tables 3.1. and 3.2. ,in Appendix  $B_1$ , are chosen from Whitney's patent results to show the difference between a conventional +3.00 dioptres lens at various angles of gaze a +3.00 dioptres lens, which has a gradient of refraction through its whole aperture. As Table 3.2. shows, at an angle of

35°, oblique astigmatism and field curvature have been reduced to nearly zero compared with the results in Table 3.1.. The total change in the index of refraction in the lens of Table 3.2. was 0.03 with steps of 0.005 at every 5° from the centre of the lens towards the edges. The gradient (cylindrical) was produced by an ion exchange technique. The lens in Table 3.1. had a front surface curvature of +8.64 dioptres, while the back surface curvature had a value of -6.00 dioptres. In Table 3.2. the lens had a concave base curve of -6.25 dioptres with the gradient spread along its entire surface. It is apparent that, according to Whitney's results, the oblique astigmatism and field curvature presented by the lens were minimized and for a viewing angle of 30° they became nearly zero.

Besides the above mentioned patent, Sinai (1971) reported, that the neutron irradiation technique he proposed, could be utilized to correct spherical aberration in ophthalmic lenses. Using a Twyman-Green interferometer he observed the fringe pattern obtained for three lenses which were neutron irradiated. According to Sinai the correction was clearly seen by comparing the fringe pattern of the irradiated lenses with the fringe pattern of the three lenses before they had undergone the process of neutron irradiation.

Charman (1982b) suggested that a theoretical Wood's type of lens could be utilized in three optometric applications, in lenticular lenses, bifocal and varifocal lenses, and in contact lenses as a bifocal contact lens. All these three applications are critically dependent, according to Charman, upon the

development of techniques for producing large geometry and index gradients. Figure 3.2. illustrates the theoretical approach of a bifocal and a varifocal lens utilizing Wood's principle idea.

### 3.3. Review of the techniques used to create a gradient

There are two important factors in the manufacture of gradient-index systems. These two factors are a) the depth of the refractive index variation and b) the magnitude of the change in the refractive index. Several techniques have been reported to produce a gradient in an optical element, but the most important can be classified under six major categories, which are a) the neutron irradiation technique (Sinai, 1971), b) the chemical vapor deposition technique (Keck and Olshansky, 1975), c) the partial polymerization technique (Moore, 1973), d) the ion exchange technique (Hamblen, 1969), e) the ion stuffing technique (Mohr et al, 1979), and f) the crystal growth technique (Moore, 1980).

In the first method (Sinai, 1971), the index variation is produced after bombarding the material with thermal neutrons. Boron-silicate glass has been suggested for use in an optical element undergoing this method. The neutrons are produced from a triton reactor. It was found that by varying the irradiation along the surface of the optical element it was possible to create



# (b) Varifocal lens

Figure 3.2. (a) Gradient-index bifocal, where the distance portion consists of a material with constant refractive index and the near portion comprises a gradient-index Wood lens. After Charman (1982b). (b) Varifocal lens with a distance portion of a material with constant refractive index, an intermediate zone comprising a gradient-index material and a near portion consisting of a Wood lens. After Charman (1982b).

and additionally adjust the change in the refractive index induced in the material. Figure 3.3. illustrates the principle of the experimental set up used by Sinai (1971) to reach the desired result. The neutron flux enters the device at the point (1) and travels along the collimator (2), which produces a directional effect upon the neutron flux. Afterwards the neutron flux passes through the absorbing screens (3), which induce the necessary modulations in the neutron flux providing the desired index variation. The collimator was made of cadmium and had lengths of 5 cm and 0.7 cm, while the collimator's diameters were 3 cm and 2.3 cm. It has been reported that lens-elements irradiated with thermal neutrons present a certain amount of unwanted coloring. This can be corrected, according to Sinai (1971), by bleaching the lens-elements under ultraviolet (U.V.) irradiation, which will not affect the induced change in the refractive index. Bleaching can be accelerated by raising the temperature, however, the temperature must not exceed 300-400° C. In lens-elemnts containing alkali metals, the coloring is fairly stable and the bleaching process, becomes guite difficult. The main drawbacks of such a technique are the possibility that the gradient might not be permanent, and the large number of neutrons required. The refractive index change attained with this method is around  $\Delta n = 0.02$ . Sinai suggested that the method could be used in correcting the spherical aberration of any optical system.

The chemical vapour deposition technique (Keck and Olshansky, 1974) is used widely in the field of telecommunications. It based on the deposition of



Figure 3.3. The principle of the device that Sinai used for neutron irradiation. (1) are the neutrons entering the device from the Triton reactor, (2) is the cadmium collimator, (3) are the absorbing screens and (4) is the lens element undergoing this treatment. After Sinai (1971). glass material of a given refractive index on the inside or the outside of a glass or a quartz tube and then depositing another layer of the same glass material previously used, but with a very small difference in its chemical composition. This process will continue until sufficient steps of refractive index are produced. The layers and the initial rod used are heated, consisting the final optical element produced. The layers are smaller than the wavelength of light and consequently the gradient of refractive index appears to be continuous. The change induced in the refractive index by applying this technique in an optical element such as optical fibres, will be around  $\Delta n = 0.01$ . The limitations presented by such method lie in the fact that this technique cannot be applied in ophthalmic lenses, but only in optical elements with very small diameters around 100  $\mu$ m.

The third technique of partial polymerization is the one employed in plastic (R.S. Moore, 1973). This takes place by partially polymerizing the monomer of the organic material employed, with ultraviolet (U.V.) irradiation. This technique can be applied to large geometry optical elements and the gradient produced can have an arbitrary profile. It should be noted that such technique remains a theoretical approach and has never been put into practice due to the thermal properties of plastics materials, which do not allow such a treatment to be employed.

The fourth technique and probably one of the most successful methods in producing a gradient, is that of ion exchange introduced by Hamblen (1969).

It is based on a process, where solid optical matrices of glass are treated in such a way as to produce increments of refractive index on the lens surface. It is actually a diffusion process, which takes place when the surface of the glass is brought in contact with a salt material in a bath. It depends upon thermal migration, which produces a refractive index gradient inwardly to the material surface. More particularly, a borate or alkali silicate glass matrix is brought in contact with a molten salt containing silver or thallium cations. An ion exchange process takes place, where the silver ion (Ag) replaces the alkali or boron ion within the glass matrix. There is no actual change in the composition of the glass matrix, only an ion exchange occurs, where the silver ion (Ag) (replacement ion) has a large degree of polarization. The distribution of the ions diffusing into the glass matrix is predictable and the gradient can be approximately linear. Due to the uniform diffusion of the ions inside the glass matrix, after the diffusion is finished, some portions of the processed surface of the glass matrix are ground more than others, resulting in a lens-element with a non-homogenous refractive index. The limit in the usefulness of this technique is related to the depth of the diffusion. This depends upon the time and the temperature that the diffusion takes place and also upon the glass matrix material. Figure 3.4.a illustrates the relation between the diffusion depth and the time and temperature needed. The change in the refractive index related to the depth of diffusion is given in Figure 3.4.b. Normally, the maximum depth of diffusion is around 10 mm linearly. Figure 3.5. shows a lens before and after the whole process, described above, is finished, while Figure 3.6. illustrates different types of



Diffusion depth (linear units)

Figure 3.4. Diagrams showing the relation between a) the diffusion depth and the time and temperature needed, b) the diffusion depth and the change in the refractive index. After Hamblen (1969).



Figure 3.5. The three stages of a lens undergone the ion exchange technique, a) lens untreated, b) lens undergone the ion exchange technique, c) lens ground more at the periphery than at the centre in order to produce the gradient in the refractive index.





Figure 3.6. Schematic representation of different apparatus used for the ion exchange technique. After Hamblen (1969).

apparatus in which the diffusion and ion exchange is carried out. In all of these, K is the solid optical matrix, which has undergone this treatment.

Ion stuffing is another technique for producing a gradient of refractive index in an optical element (Mohr et al, 1979) and is based on a molecular diffusion process. In such a technique a specific type of glass is selected and heated. This type of glass is characterized by the fact that after it is heated and the temperautre reaches a certain point a phase seperation occurs within the glass. If at that point the glass is brought in contact with an acid, one of the phases, which is soluble in acid is taken away from the glass. When the remaining glass is brought into a bath a molecular diffusion starts, which creates a gradient of the index of refraction in the glass. The diffusion then must be stopped and the glass recondensed by a process of heating, when the final gradient will be established. The process of diffusion should be carefully controlled in order to avoid the diffusion of a larger or smaller number of molecules entering the glass. This technique suffers from the fact that the glasses which are currently manufactured are not phase separable.

The last of the aforementioned methods is that of crystal growth (Moore, 1980). It is based on the exchange of sodium chloride and silver chloride in a bath. A crystal is formed of sodium chloride, which is pulled out of the bath. The crystal growth continues up to a point, where the concentration of silver chloride in the bath is larger than that of sodium chloride, so the crystal is forced to take up a small amount of silver chloride. As this process continues.

more silver chloride is forced into the crystal, where such a combination produces a gradient. According to Moore (1980), combinations of silicon and germanium can be utilized for infrared transmitting gradients. It is obvious that non of the above mentioned techniques is ideal in producing a gradient, having a refractive index change adequate for a varifocal, which will provide the necessary progression in power and cover the range of additions required by presbyopic patents.

## 3.4. Theory of concentric gradient-power varifocal lenses.

The first patent for multifocal lens, utilizing a gradient-index of refraction, was that of Bugbee (1924). The lens he proposed was a fused bifocal, where the near segment consisted of at least three different glass elements of different refractive indices. These different glass elements were arranged in a concentric configuration, with the glass element of the highest refractive index at the centre. With the fused process these glass elements were made indistinguishable.

The other patent, which describes a lens construction very relevant to Bugbee's lens, was the one by Bausch and Lomb (1966). According to the patent a round fused bifocal is used again, where the refractive index at the near segment varies from 1.523 (at the top of the segment) to 1.665 (at the centre of the segment). After that point the refractive index remains constant having a value of 1.665 to provide the necessary maximum addition. The method that was utilized to produce the gradient in this patent was the one proposed by Spiegel (1950). It involves the composition of a glass element consisting of a number of glass materials slightly different to one another, through thermal treatment, which will produce the variation in the index of refraction of the composed glass element. According to Spiegel, the change in the refractive index is around 0.012 over a length of 0.24 mm. This change is not enough to provide the necessary addition for near work. Figure 3.7. illustrates the lens proposed by Bausch and Lomb (1966) and the refractive index change related to the distance from the top down to the bottom of the lens.

According to Charman (1982a) gradient -index varifocals are expected to have nearly the same characteristics as the concentric design homogenous lenses involving materials of uniform refractive index and aspheric surfaces of revolution. Such a lens is the Beach "blended bifocal". Beach (1946) in his patent introduced a type of multifocal lens, which is characterized as concentric. It was a lens comprising a piece of glass matrix, having a uniform index of refraction through its entire material. One of the surfaces (usually the front surface) has the reading portion and an outer distance portion, which are in a circular configuration. An intermediate zone, between the two portions mentioned above, has a plurality of powers, progressively increasing from the distance to the reading portion. This intermediate zone





Figure 3.7. (a) The lens proposed by Bausch and Lomb Co., while (b) is the change in the refractive index of such lens related to the distance from the top to the bottom of the lens. After Bausch and Lomb (1966).

consists of numerous curved areas with relatively narrow width. These areas are arranged side by side and adjoin one another. In this way a progression in power is created from the distance to the near portion. All these three zones have a circular configuration, where the outer and the inner zones for distance and near vision have spherical curvature, which gives a constant power through their apertures. Figure 3.8. illustrates a cross-section and face view of the Beach lens. As can be seen the lens consists of a plano lens blank (1) with an area (2) for the distance prescription, an annular portion (5), which is concentric around an axis X, which passes through the centre of the reading portion (4). This reading portion (4) has a constant power to provide the necessary maximum addition. The Beach lens is a typical example of what is called "concentric varifocal" lens with a homogenity in its refractive index.

The theoretical aspects of such a lens are much the same with a radial gradient-index lens, having a cylindrical symmetry about an axis. There has been a theoretical study of this kind of concentric lenses by Charman (1981b) and experimental measurements have been taken by Knoll (1952). These investigations showed that the main drawback of this type of lens is that it presents substantial amounts of unwanted oblique astigmatism in the transition zone, where the progressive increase of power takes place. This problem puts a limit to their use. Figure 3.9. shows in simplified way how this unwanted oblique astigmatism is presented by this type of lens.



(a) Cross-section



Figure 3.8. Cross-section and face view of Beach concentric varifocal. (1) is the lens blank, (2) is the area for the distance prescription, (3) is the back surface of the lens, where the cylinder can be incorporated if needed, (4) is the reading portion, (5) is the annular progressive power portion. After Hamblen (1946).


Figure 3.9. Concentric type of varifocal lens with the astigmatism presented at the annular transition zone. After Charman (1982a).

Consider equidistant points, which have a distance y from the axis C of the lens. The annular power at these points will be Fs (sagittal power). It is also necessary to take into account the power FT (tangential power) in each local area of the lens transition zone. This is in the perpendicular (tangential) section of the lens transition zone and it is defined by the meridian, where the main ray of the bundle enters the lens surface and the axis C. In such a lens, where a symmetry exists around an axis, the sagittal and tangential powers at any distance y from the axis C will be different. This is the reason for the resulting unwanted astigmatism locally at each point. This astigmatism will be the same in magnitude at all points which are equidistant from the axis C, due to the fact that the sagittal and tangential powers at any of these points are the same. Although the magnitude of the astigmatism will be the same at these points the axis direction of the cylindrical power will differ. According to Charman's calculations and assuming a concentric varifocal lens like the one in Figure 3.10., then the sagittal power at a radius y from the axis C will be given by the equation

$$F_{S} = F(y_{2} - y) / (y_{2} - y_{1})$$

where  $y_1$  is the radius of the reading portion,  $y_2$  is the radius of the distance portion and F is the power at the reading portion. It is assumed that the distance portion power has not yet been incorporated on the lens and the power for that portion is taken to be zero.



Figure 3.10. The representation of the concentric varifocal related to the distance from the centre of the lens. After Charman (1982a).

The sagittal power all over the three portion of the lens is considered to be

a) when 
$$y \le y_1$$
 then  $F_s = F$  b) when  $y_2 \le y$  then  $F_s = 0$ 

c) when  $y_1 \le y \le y_2$  then  $F_s = F(y_2 - y)/(y_2 - y_1)$ 

For the tangential power at any point at a radius y from the centre C the equations are

a) when 
$$y \le y_1$$
 then  $F_T = F$  b) when  $y_2 \le y$  then  $F_T = 0$ 

c) when  $y_1 \le y \le y_2$  then  $F_T = F(y_2 - 2y)/(y_2 - y_1)$ 

It is clear that while the distance and the reading portion will be free of astigmatism, the intermediate zone will suffer from unwanted cylindrical power, which will be the difference between the sagittal and the tangential powers at each point on the zone. The magnitude of the astigmatism presented will be given by the equation (Charman, 1982a)

$$A = F_{s} - F_{T} = F(y/y_{2} - y_{1})$$

According to Charman the only way to minimize the cylindrical power will be by having a very small value of y and a large value of the difference ( $y_2 - y_1$ ), which is actually the width of the annular intermediate zone. The fact that y takes values up to a maximum of  $y_2$  makes it obvious that it is almost impossible to obviate this unwanted astigmatism and that the maximum amount of it will be

 $A = F(1 - y_2 / y_1)$ 

when  $y = y_2$ , and must always have a relation  $A \ge F$  with the power of the reading portion. Charman also mentioned that the astigmatism in any concentric miltifocal could be given by the general equation

$$A = F_s - F_T = -y (dF_s / dy)$$

where  $dF_s / dy$  is the sagittal power variation, according to the radius of the intermediate zone. As it is almost impossible to eliminate the astigmatism, the best solution according to Charman is to keep it at least constant and independent of the radius of the zone. This can be achieved by selecting a sagittal power in the transition zone which will be

$$F_s = F[\log_e(y_2 - y) / \log_e(y_2 - y_1)]$$

then the astigmatism will become

 $A = F_{s} - F_{T} = F / \log_{e} (y_{2} - y_{1})$ 

which makes it constant and free of changes related to any change of the radius y.

#### 3.5. Parameters and aberrations of gradient-index lenses

The parameters which have to be defined in gradient-index lenses are a) the refractive index profile, which is the most important factor of these gradients. There are several techniques now known, that can provide accurate measurments of the index profile like modified interferometry (Moore and Ryan, 1978) and moire patterns (Oster et al, 1964). In modified interferometry, an interferometer of one of the standard types, such as the Mach-Zehnder or the Twyman-Green, is used. This is coupled with an opto-electronic system, which provides an electronic signal proportional to the index of refraction at the particular point measured. Moore and Ryan (1978) proposed the usage of the Mach-Zehnder interferometer due to the fact that it was found to be better and more useful for this purpose. Figure 3.11. shows the index of refraction profile for a glass fabricated by Bausch and Lomb and treated with the ion exchange technique of silver-sodium

exchange at a wavelength of  $\lambda = 0.5145 \ \mu m$ .

b) the chromatic variation that characterizes the gradient. Gradients of rotational symmetry around an axis present paraxial chromatic aberration in the visible part of the light spectrum. If such a gradient is used in a colour-correction system, these aberrations have to be known beforehand.
c) the maximum and the minimum slope of the gradient. Their value derived from the refractive index profile of the gradient.

d) the transition of the gradient. The materials should be chosen accordingly, having the highest transparency possible in the visible part of the light



Distance into the sample (cm x  $10^{2}$ )



spectrum.

e) the thermal and mechanical properties of the glass before and after the gradient is introduced. The thermal change of the gradient, produced either by glass or plastic, is very difficult to measure.

A number of papers have appeared in the past twenty five years on the aberrations of inhomogenous lenses (Sands, 1970a, 1970b, 1970c, 1971, 1973), (Moore and Sands, 1971), (Marachand, 1976). All these papers are concerned with the gradient having a cylindrical distribution of refractive index. In a few, there is also reference to a specific example of such gradient, the Wood's "photographic" lens. The term "photographic" derives from the fact that this gradient-index lens had photographic gelatine as its basic element.

In any gradient-index media, with a concentric symmetry, the following differential equation applies (Born and Wolf, 1964)

 $d/ds(n \cdot dr/ds) = grad n$ 

where r (x,y,z) is the vector position of a typical point on the ray, s is the arc length of the ray from a fixed point on the ray and n(x,y,z) is the refractive index function. This equation applies to any differentiable index function.

Instead of the above equation it is possible to emply the principle of Fermat

(Born and Wolf, 1964), in order to derive the differential equations directly in the coordinates selected. According to Fermat's principle, which is also known as the princible of the shortest optical path, the optical length of a light ray between any two points  $P_0$  and  $P_1$  is given by the integral

$$P_1 = \int n \, ds$$

$$P_0$$

this is stationary relative to infitesimal variations of the integration path joining the points  $P_0$  and  $P_1$ .

The refractive index function in a "photographic" Wood's lens is assumed to have the form

 $n = N_0 + N_1 r^2 + N_2 r^4 + \dots$ 

where  $r^2 = x^2 + y^2$ , while N<sub>0</sub>, N<sub>1</sub>, N<sub>2</sub> are constants.

According to Marchand (1976), the third-order aberration formulas for an object that is at infinity, in a Wood's "photographic" lens are found to be a) transverse spherical aberration

$$S_T = c h^3 N$$

b) longitudial spherical aberration

$$S_L = f' h^2 N$$

Figure 3.12a. illustrates the spherical aberration in a Wood's lens

c) for the meridional coma the equation is given to be

$$C_{m} = 1/2 (y_{c}' + y_{b}') - y_{p}'$$

Figure 3.12b. illustrates schematically the meridional coma presented by the lens. The ray (p) passes through the centre of a diaphragm A, which is in front of the lens and lies at a distance L from the front surface of the lens. Rays (c) and (b) are meridional rays, parallel to the ray (p) and equidistant from the centre of the diaphragm A, passing through at the upper and lower part of it respectively. In order for the sagittal coma to be found, the same method is used only this time the rays (c) and (b) are skew rays. In the equation of the meridional coma  $y_p'$ ,  $y_c'$ ,  $y_b'$  are the heights from the optic axis, where the rays (p),(c) and (b) intersect the image plane.

d) the distortion of the lens is given to be

$$D = 100 (y_p' - y_o') / y_o'$$

where  $y_p$ ' is the height where the ray (p) intersectes the image plane and  $y_o$ ' is the height of a meridional ray, parallel to ray (p), with the image plane from the optic axis .

e) definition of the meridional field of curvature  $Z_m$  is given by the equation

$$Z_{m} = \lim (y' - y_{p'}) / (u' - u_{p'})$$

where (p) is a paraxial ray intersecting the image plane at a height  $y_p$ ' and having an inclination angle o with the optic axis z, (a) is a meridional ray parallel to (p) at a distance h from ray (p), having a height y' at the point, where the ray intersects the image plane, from the optic axis. u' and  $u_p$ ' are given by the equations u' = - tan o', while  $u_p$ ' = - tan  $o_p$ ', where o' and  $o_p$ '



(a) Spherical aberration of Wood's lens



(b) Meridional coma in a Wood lens

Figure 3.12. Spherical aberration and Meridional coma in a Wood lens. After Marchand (1976).

are the angle that the rays (a) and (p) respectively, have with the optic axis at the image plane. Figure 3.13a. shows how these two rays are refracted, resulting in meridional field curvature  $Z_m$ . In order to find the sagittal field curvature  $Z_s$ , instead of the meridional ray (a) a skew ray should be considered. Figure 3.13b. shows how the sagittal field of curvature is presented by the lens.

f) the astigmatism in such a lens is given to be

$$A = 1/2 (Z_m - Z_s)$$

g) the Petzval curvature is given to be

$$P = Z_m - 3Z_s$$

Analysis of the equations related to the transverse and longitudial spherical aberrations are given in Appendix B<sub>2</sub>.



(c) Meridional curvature in Wood's lens



#### (d) Sagittal curvature in Wood's lens

Figure 3.13. Meridional and Sagittal curvature in a Wood lens. After Marchand (1976).

### **CHAPTER 4**

GRADIENT-INDEX VARIFOCALS. EXPERIMENTAL WORK.

# 4.1. Construction of "non-homogenous" lenses, convex and concave according to Wood's method

Wood (1905) in his book "Physical Optics" makes reference to a lens, which he characterizes as "non-homogenous cylinder" or "pseudo" lens. According to Wood's idea it is possible to construct cylindrical rods in which the refractive index presents a regular variation, where the maximum value is at the surface of the rods or alternatively at the axis. These "pseudo" lenses act as convex lenses when the maximum value of the refractive index is at the axis of the rods or as concave, when the maximum value of the index of refraction is at the surface.

The lenses have a radial variation in their refractive index and provide in this way equi-index layers, which are coaxial cylinders. The procedure of constructing such lenses is quite straightforward, and is described by Wood. For a convergent Wood's lens, having parallel surfaces and a radial gradient of index, symmetrical to its axis, the transmitted wavefront must emerge as a portion of a sphere (Charman, 1981). Figure 4.1. shows the action of a Wood's lens on an incident plane wavefront compared to a conventional converging lens. According to Charman (1981), the power of a Wood's converging lens is given to be







(b) Wood's converging lens

Figure 4.1. Schematic representation of the action of a converging Wood's lens on an incident plane wavefront, compared to the action of a conventional converging lens. After Charman (1981).

where d is the thickness of the lens and K is given to be  $1 / 2 \cdot f \cdot d$ , where f' corresponds to the second focal length of the lens. Charman (1981) also gave the equation for the change in the refractive index of such convergent lens. This is given to be

$$\Delta n = n_h - n_o = - K \cdot h^2$$

where  $n_h$  is the refractive index at a perpendicular distance h from the axis of the lens and  $n_o$  is the index at the axis. It should be mentioned that for a diverging Wood's lens the index change is given to be

$$\Delta n = n_h - n_o = K \cdot h^2$$

The first part of the experimental work was concerned with preparing lenses in exactly the way that Wood had described. A handful of gelatine powder was left to soak in water for approximately 20 minutes, until it became soft. Then the excessive water was poured away and the remaining mass of gelatine and water was heated with a "GALLENKAMP MAGNETIC STIRRER REGULATOR HOTPLATE" in a Pyrex flask. The heating process lasted for about 5 to 10 minutes, If the hotplate was operated at its maximum or 10 to 15 minutes if it was midway. Then half of the quantity of the produced fluid was poured into a Pyrex test-tube (A), after it was filtered through a funnel with an absorbing paper (Whatman). Sometimes gelatine did not run through the funnel easily and a small amount of boiling water was necessary to pour through the gelatine-water fluid. The remaining gelatine with water was heated again until it obtained the consistency of a syrup. As Wood suggests one third of gelatine-water original volume should remain in the Pyrex flask. The heating process took place for about 30 to 40 minutes. After this process was finished, an equal amount of glycerol was added and the mixture was poured into another Pyrex test-tube (B). The two mixtures in test-tubes (A) and (B), were left to set and become nearly solid. The mixtures were left for about two and a half hours at room temperature conditions and then the resulting cylindrical rods were taken out of the Pyrex test-tubes. The rods actually had a jelly-like structure rahter than solid. The process of taking the jelly-rods out of the test-tubes was not an easy one, and heat had to be employed in order to separate the rods' outer surface from the inner surface of the test-tubes.

Afterwards, the resulting rods (A) and (B), were cut into small disks with a diameter of 12 mm and in a variety of thicknesses from 2 mm up to 5 mm. The disks produced had their front and back surfaces covered with perspex square plates to prevent diffusion along the axis of the disks, and obtain clear and flat surfaces without irregularities. The mounting of the disks between the two square perspex plates was achieved by warming the plates. Figure 4.2. illustrates the resulting lens-disk of Wood's. The cylinders of gelatine and water (A) were then immersed into glycerol, while the other rods in which glycerol was added (B) were immersed into cold water. Immediately, a diffusion started in both cases, where glycerol gradually replaced the water in the lens-rods (A) and water replaced glycerol in the



Figure 4.2. Schematic representation of Wood's gradient-index lens-disk.

lens-rods (B). Glycerol should be stirred occasionally in the bath with lens-rods (A), so that the layers in contact with the rods outer surface take away the water forced out of the rods and allow glycerol to diffuse into the rods. Both disks of type (A) and (B) were left in the two baths for 20 minutes, and then were taken out, stopping in this way the diffusion of glycerol in the first bath of disks (A) and water in the other bath of disks (B). The resulting lens-disks (A) act as diverging lenses, while the lens-disks (B) act as converging lenses.

According to Wood, their focal lengths were about 8 to 10 cm in his resulting lenses. Wood also mentioned that similar lenses were constructed by Otto Schott, which were based on differential cooling. In his attempt, Schott poured molten glass into iron tubes and then by sudden cooling of the outer layers of the rods a tension was produced in the cylindrical rods glass material, which coresponded to a variation of their refractive index. The resulting lenses, after the rods were cut into lens-disks, had a radial index variation and act as divergent lenses.

In the first experiment 48 lenses were constructed, which had undergone the diffusion process for 20 minutes. 24 of them were made after following the method for lens-disks (A) mentioned above and act as concave lenses, while the remaining 24 lenses were made after the method of lens-disks (B) described above again, acting as convex lenses. The resulting dioptric behavior of the above constructed lenses is given in Appendix  $C_1$ .

The next step, in this experimental section, was to repeat the experiment again, but change the time for which the lenses (A) and (B) were immersed into the two baths. The lenses created this time were left in the glycerol and water baths, respectively, for about 40 minutes, which is nearly double the time the first lenses were left in the baths (20 min.). The purpose of this move was to assess the relation between the time of the diffusion and the dioptric behavior of such lenses. The lenses finally constructed were 36, 18 of them acting as concave lenses (A), while the other 18 were convex lenses. Their resulting dioptric behavior is given in Appendix  $C_2$ 

The final part of the experimental work on the construction of Wood's "pseudo" lenses, was to create "pseudo" lenses again according to Wood's basic constructive idea but with a larger diameter. This time, Pyrex test-tubes with a diameter of 22 mm were used in which to pour the two mixtures (A) and (B). The amount of gelatine soaked into water now was doubled. The softened gelatine was heated for 15 to 20 minutes and then gelatine was filtered in the same way as previously mentioned, before it was poured into the new test-tube (A'). The remainder was evaporated (heating process for about 50 min. ) and the syrup produced was added to an equal volume of glycerol. The mixture then waspoured into another test-tube (B') also of 22 mm diameter and both mixtures (A') and (B') were left for two and a half hours to set. The two type of lens-rods were cut into disks with a thickness which varied between 4 and 7 mm. They were covered with square perspex plates and immersed into glycerol, (lens-disks (A')), and

water, (lens-disks (B')). The first constructed lenses were left in the baths for 20 minutes but after the diffusion process was finished they did not show any dioptric behavior, their powers being insignificant. This was probably due to the fact that the time was insufficient for the diffusion to take place. The critical time for having a measurable dioptric effect, which could be characterized as significant was assessed to be 40 minutes. The lens-disks constructed were 36, 18 were of type (A') and the rest of type (B'). The resulting dioptic behavior of such lenses is given in Appendix  $C_3$ . It should be noted that the diffusion time of these lenses was one and a half hours.

## 4.2. The dioptric behavior of "non-homogenous" or "pseudo" lenses.

From the total number of 120 lenses constructed, following the procedure that Wood (1905) proposed, only in 16 lenses did we fail to get any measurements, related to their dioptric behavior. That seemed to be promising, related to the validy of such process in creating lenses with flat surfaces, in which their focusing effect is dependent upon the radial change in their refractive index. The first thing that was noticed from the results of the dioptric power measurements (Appendix  $C_1$ ), was that the increase in thickness of the lens-disks created was acompanied by an increase in the dioptric power. The power presented by the concave lenses (A) increased from -0.50 dioptres, for a thickness of 2 mm, and reached up to -2.50 dioptres, at a thickness of 5 mm, when the diffusion time was 20 minutes. For the convex lenses (B), the power starts from +0.50 dioptres, at 2 mm thickness, and reached up to +1.75 dioptres. It is apparent, that the increase of power in the increase of the power in the convex lenses (B). Figure 4.3. illustrates the relation between the thickness and the power of the concave lenses (a) and the convex lenses (b).

The next aspect of interest with these lenses (Appendix C<sub>1</sub>) is that although the experimental procedure was the same for lenses, which had the same thickness, the finished lenses had variable refractive power. For example, lens-disks (A) with 2.5 mm thickness, according to the results in Appendix C<sub>1</sub>, have a dioptric power, which varied from +0.25 up to +1.00 dioptres. This raised the question about the reproducibility of such lenses. Apart from that, a few lenses, especially those for which no measurement could be obtained, exhibited the phenomenon of birefringence. This was probably due to the fact that the plates where the lenses were mounted did not adhere perfectly to the lens-disks surfaces, creating bubbles on the surface of the lenses, which probably were responsible for the birefringence noted.



(a)

(b)



Figure 4.3. The relation between the thickness and the dioptric power of a) concave Wood lenses, b) convex Wood lenses

Comparing the results given in Appendix  $C_1$  to those of the lenses in Appendix  $C_2$ , it is apparent that there is an increase in the dioptric power of the "pseudo" lenses, when the diffusion time increases from 20 to 40 minutes. An increase in the power, which is proportional to the increase in the diffusion time, takes place in both type of lens-disks (A) and (B) as illustrated in Figure 4.4.

The results in Appendix  $C_3$  show that there is also an increase in power presented by the lens-disks constructed with a 22 mm diameter, but this increase is probably not due to the fact that the diameter of the lens-disks was increased. It is believed that the increase is due to the fact that the lens-disks constructed have a thickness which is larger than the lenses in Appendices  $C_1$  and  $C_2$  and also due to the increase of the diffusion time. The increase of the diameter has an inverse effect on the dioptric power of such lenses, because when the thickness was kept constant and the diffusion time was from 15 to 40 minutes, no dioptric effect was presented by the lens-disks. The need to increase the diffusion time was apparent at that time, and it is the combined effect of the increase in the diffusion time and thickness that gave the resulting increase in the power, as illustrated in Appendix  $C_3$ . Figure 4.4. The relation between the thickness and a) the power of the lens-disks type (A), b) the power of the lens-disks type (B).







Thickness of the gradient Wood's lens in mm

b)

4.3. The method followed to construct progressive addition lenses with radial index distribution.

The procedure introduced by Wood (1905) was modified, in order to produce varifocal lenses having a gradient-index distribution. The idea was to create lenses, whose half upper portion would present a constant refractive index, whils the lower part would present an index variation, which would provide a smooth and continuous transition from the distance portion to the near reading portion. The gradual change of power would take place from the centre of the lens and be continuous downwards to the bottom of the lens resembling a "soft" lens design related to the progression. This was rendered possible by immersing only the lower half portion of the lens-disks constructed. The materials again used were a) gelatine powder prod. 44045 produced by B.D.H., b) glycerol with index of refraction ranging between 1.471 to 1.473 also produced by B.D.H..

Because of the problem arising from the irregularities of the lens-disk surfaces when the lens-disks were mounted on the perspex plates, a type of lens-cell was introduced. Figure 4.5. illustrates the lens-cell, which was used for such purpose. It consisted of two square plates of perspex, where in between a cylindrical spacer of a prearranged height was interposed, also made of perspex. These three parts of the lens-cell were cemented in the configuration shown in Figure 4.5. A small hole was made at the top of the lens-cell at the cylindrical spacer, in order that the mixtures could be poured

Figure 4.5. The lens-cell constructed to produce a varifocal lens according to Wood's modified method. It is also illustrated the way the diffusion took place.

a) Face view of the lens-cell used for the experiment



into the lens-cell. The cylindrical spacer, interposed between the two perspex plates, had a mobile part. As shown in Figure 4.5., half of the cylindrical spacer was mobile in order to allow diffusion of glycerol, in lens-disks (A), and water, in lens-disks (B), to take place only in the lower half part of the lens. The varifocal lens-cells were constructed after following the same technique as the one followed for lens-disks (A) and (B) mentioned previously. Consequently, the lenses produced had an increase in their negative power, for the (A") lens-cells produced, or in their positive power, for the (B") lens-cells produced. The experimental procedure was repeated several times with different diffusion times and different thicknesses. This was accomplished by changing the height of the cylindrical spacer of the lens-cell. The resulting lenses were measured with a NIDEK electronic focimeter and the measurements were concentrated in the lower half portion of the lens-cell, where the gradient was produced by diffusion. 4.4. The dioptric behavior of progressive addition lenses with gradient-index distribution. Experimental data.

A large number of lenses were constructed by the above technique, but the results presented relate to seven lenses of the (B") type with an increase in their positive spherical power and four lenses of type (A") with an increase in their negative power. The parameter which seemed to have most effect on the performance of these lenses, was the time of the diffusion or more precisely the exchange of water with glycerol for both type of lenses.

The relationship of the dioptric power to the thickness of the constructed lenses could not be assessed with accuracy, due to the fact that measurements of the power of such lenses could be made only when the lenses had a thickness which ranged between 15 and 18 mm. Below or above such thickness range the results were insignificant or inaccurate, especially for thicknesses above 25 mm, where the transparency of such lenses was reduced.

One day after the lenses were constructed, the lenses were measured with a NIDEK electronic focimeter. The points measured were selected in order to have a 6 mm distance between them, in the horizontal and 4 mm distance in the vertical. The distance in the vertical, of 4 mm was selected in order to obtain a more detailed analysis in that direction about the increase in the dioptric power of the lenses, due to the fact that this is the direction which the

eye of a presbyope will rotate in order to focus near distance objects. The Tables giving the dioptric power change, in the lower half portion of the constructed lenses, can be found in Appendix  $C_4$ . It is seen from the Tables in Appendix  $C_4$  that three lenses out of the seven of the lens type (B") were immersed into water for two hours, and had a thickness of 15 mm (Tables 4.7., 4.8.) or 18 mm (Table 4.9.). Their diameter was 75 mm, while the diffusion took place under room temperature conditions. Two lenses (Tables 4.10., 4,11.)had a diffusion time of one and a half hours, with a thickness of 15 mm and a diameter of 75 mm. One of these lenses (Table 4.12.) had a diffusion time of four hours, a thickness of 15 mm and a diameter again of 75 mm, while the last one (Table 4.13.) had a diffusion time of forty five minutes, a thickness of 15 mm and diameter of 75 mm. Below the critical time of forty five minutes, the lenses constructed did not show any significant dioptric behavior.

For the lenses with an increase in negative power (A"), which were immersed into glycerol, four lenses gave useful information. Two had a diffusion time of two hours (Tables 4.14., 4.15.), a thickness of 15 mm and a diameter of 75 mm. One was immersed into glycerol for one and a half hours (Table 4.16.), had a thickness of 15 mm and a diameter of 75 mm, while the final lens (Table 4.17.) had forty five minutes diffusion time, thickness 15 mm and a diameter again of 75 mm.

4.5. Conclusions related to the construction and dioptric behavior of the progressive addition lenses with gradient-index distribution

Judging from the results of the Tables in Appendix C<sub>4</sub>, it is apparent that the objective of the experimental method introduced seems to have been accomplished. Especially, in Tables 4.7. to 4.13. an increase in the spherical power of the lenses produced can be observed. In Tables 4.14. to 4.17., the lenses (A") produced an increase in negative spherical power. These types of lenses could be utilized as an up-curve varifocal lens with a slight different configuration to that constructed or by utilizing only the half portion of the lens, turning it upside down, so that the central part of the original lens will be inverted when looking downwards, and incorporating a positive conventional lens at the back surface of the gradient-index lens. It will again give a varifocal lens with a progression through its entire aperture, due to the fact that the lens will actually present an increase in its positive spherical power.

The results in Tables 4.7. to 4.13. indicate that these lenses have an increase in their positive spherical power, which reaches a maximum at a distance which is nearly midway from the centre to the bottom part of the lenses. This power change starts usually at +0.25 dioptres at the centre of the lens and reaches a maximum at the previously mentioned point, which has a power nearly of +2.75 dioptres in some of the lenses. Such an

addition is quite efficient to cover the addition needed for presbyopes. After that point of the maximum addition, the spherical power decreases resulting in negative powers. Figures 4.6.,4.7.,4.8. illustrate the change in the spherical power of three representative lenses of type (B") (Tables 4.7.,4.8.,4.10.) through the entire half portion of these lenses. In the lenses of type (A") the change in the negative power is more regular and smoother than the one in the lenses of type (B"). The spherical power changes from -0.50 dioptres reaching up to -3.00 or -4.00 dioptres. Figures 4.9.,4.10,4.11. illustrate the change of the spherical power of three representative lenses of type (A") (Tables 4.14.,4.15.,4.16.).

According to the results in the Tables of Appendix  $C_4$ , it is apparent that a substantial amount of unwanted astigmatism is present in the gradient-index varifocal lenses, produced after following the modified Wood's method, which is high enough to put a limit to their use as varifocal lenses. This is actually true for almost every point measured at the lower half portion of these lenses. Figures 4.12.,4.13.,4.14. illustrate the change in the cylindrical power and the axis of the cylinder through the lower portion of the three representative lenses of type (B") (Tables 4.7.,4.8,.4.10.). It is clear that there is an increase in the cylindrical power, from the centre going towards the edges of the lenses. The increase of the cylinder is due to the increase of the difference between the sagittal and tangential powers at any point on that part of the lenses, where the diffusion took place. This cylindrical power reaches in almost all the lenses a very high value, sometimes going up to 5.00 dioptres cylindrical, which is unacceptable to the eye of a patient.



Figure 4.6. Schematic representation of the spherical increase of the lens of Table 4.7. measured at predetermined points. The white circles indicate that the power at that point is positive, while the black circles indicate that the power is negative.


Figure 4.7. Schematic representation of the spherical increase of the lens of Table 4.8. measured at predetermined points. The white circles indicate that the power at that point is positive, while the black circles indicate that the power is negative.



Figure 4.8. Schematic representation of the spherical increase of the lens of Table 4.10. measured at predetermined points. The white circles indicate that the power at that point is positive, while the black circles indicate that the power is negative.



Figure 4.9. Schematic representation of the spherical increase of the lens of Table 4.14. measured at predetermined points. The black circles indicate that the power is negative.



1.00 D.

Figure 4.10. Schematic representation of the increase in the spherical power of the lens of Table 4.15. measured at predetermined points. The black circles indicate that the power is negative at that point.



Figure 4.11. Schematic representation of the spherical increase of the lens of Table 4.16. measured at predetermined points. The black circles indicate that the power at that point is negative.



Figure 4.12. Schematic representation of the cylindrical increase of the lens of Table 4.7. measured at predetermined points.



Figure 4.13. Schematic representation of the cylindrical increase of the lens of Table 4.8. measured at predetermined points.



Figure 4.14. Schematic representation of the cylindrical increase of the lens of Table 4.10. measured at predetermined points.

Looking at the central part of the lenses, the amount of the cylindrical power is relatively insignificant or nearly zero. As the eye rotates from the central part of the lenses the amount of cylinder increases rapidly, so that where spherical addition is a maximum, a very high value is reached. It can be observed that after the maximum spherical addition is reached, and a decrease in the spherical power occurs, the cylinder also decreases in most of the lenses of type (B"), nearly half the value of the previous cylindrical power accompanying the maximum spherical addition. For example, In Table 4.7. at the fourth column, fifth row, when the spherical power reaches a maximum of +2.50 dioptres, the cylindrical power at that point is 2.00 dioptres. The next point at the sixth row shows a decrease in the spherical power from +2.50 to +2.25 dioptres, which is accompanied by a decrease in the cylindrical power from 2.00 to 1.50 dioptres. This is probably due to the odd diffusion of water into the lens, replacing glycerol. For the lens-cells of type (A"), the cylindrical power change is more regular than the one in the lens-cells of type (B") mentioned previously. Figures 4.15.,4.16.,4.17. illustrate the cylindrical power change in the entire lower half portion of these lens-cells (Tables 4.14., 4.15., 4.16.).

Trying to estimate the relation between the diffusion time and the spherical power change of lens-cells of type (B") according to the results we can conclude that an increase in the diffusion time will result in an increase in the maximum spherical power of these lenses. Observing the results related to the lens, which had a diffusion time of 45 minutes (Table 4.13.) and those

1.00 D

Figure 4.15. Schematic representation of the cylindrical increase of the lens of Table 4.14. measured at predetermined points.

I 1 1 - 1 - 1. 1  $\begin{array}{c} & & & & & \\ & & & \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ &$ 1.00 D /

Figure 4.16. Schematic representation of the cylindrical increase of the lens of Table 4.15. measured at predetermined points.

1.00 D /

Figure 4.17. Schematic representation of the cylindrical increase of the lens of Table 4.16. measured at predetermined points.

related to the lenses, which had diffusion times of two or one and a half hours (Tables 4.7.,4.8.,4.9.,4.10) this increase of the maximum spherical power is apparent. This is not true for the lens that had a diffusion time of four hours (Table 4.12.). Concluding, it can be stated that approximately an increase of the diffusion time up to a certain point results in an increase of the maximum spherical power of these lenses. In the lens-cells of type (A") the change of the diffusion time did not show any significant change in the maximum spherical power presented, so this relation for such lenses cannot be successfully assessed.

The other notable fact that is present in such lenses is the existence of a symmetry at the axis of the cylindrical power at any point. It is clear, as illustrated in Figures 4.12. to 4.17. that the axis of the cylinder has a symmetry around the centre or near the centre of the lenses. Probably this is due to the radial distribution of the refractive index, during the diffusion time, and was something that was expected. Besides the great amount of unwanted astigmatism presented by the lenses, their other major drawback related to their manufacture, is that the gradient induced to the lenses is not reproducible. This means that a lens constructed under certain conditions cannot be made again under exactly the same conditions and following the same technique. For example, the lenses of type (B") in Tables 4.7.,4.8. present different dioptric behavior although they were produced under the same conditions, following the same method and have the same diffusion times. This non-reproducibility presented by these lenses is due to the fact

that the gradient has an unpredictable character, it is not possible to predict the index distribution pattern with accuracy, and that the diffusion process can not be controlled, in order to have the same resulting lens every time.

The last and the most important drawback of the gradient-index varifocals produced is that the distribution pattern of the refractive index at the lower half portion of the lens is not permanent. The index distribution pattern is not static or constant and changes as time passes. Figure 4.18. illustrates the spherical and the cylindrical power distribution of the lens of type (B") in Table 4.10. after 10 days of its construction (the data related to this lens is given in Table 4.18). It is apparent that there is a significant change in the index distribution, which resulted to negative spherical powers. For the lenses of type (A") this change also occurrs but it is not the same in magnitude as in the lenses of type (B").

Concluding our observations and bearing in mind of the aforementioned drawbacks, it is obvious that the utilization of gradient-index optics, in creating a varifocal lens with an ideal performance, is probably limited at the present time using this type of glycerol gel. However, it is possible that a more permanent lens could be produced by using a plastics material.



Figure 4.18. Schematic representation of the spherical and cylindrical power increase of the lens of Table 4.10. after 10 days of its construction.

## **CHAPTER 5**

## DEFORMABLE LENSES

## 5.1. Literature review; different approaches

As it was pointed out in the introduction, the ophthalmic spectacle lens systems utilized in the correction of presbyopia can be divided into the progressive addition lenses and the variable focus lens systems. The variable focus lens systems present a power change which is adjustable over the whole effective aperture of the lens (Bennett, 1973). This category of lens systems comprises a kind of lens, which is known as "deformable lens", where by deforming its surfaces, after exerting some kind of pressure, a variable focal length can be achieved.

The construction concept of such lenses was based upon the human crystalline lens and its accommodative faculty. The crystalline lens deforms due to the pressure exerted by the ciliary muscle, producing a change in its refractive state and consequently accommodation for near vision. This action is clearly shown in Figure 1.3 of Chapter 1. Bearing in mind the human crystalline lens and its accommodative faculty, it is easy to envisage how a deformable lens would appear.

Through the years a number of deformable lens designs have been presented, and all of them had two common features. These common characteristics were the flexible lens walls and the fluid material, which was

196

captive inside the cavity produced by the flexible lens walls with such lens designs, either both of the lens walls were able to be deformed by exerting pressure or one of the lens walls was constructed flexible and liable to be deformed after pressure was exerted, having the other wall constant to provide a firm basis for the distance correction to be incorporated. The latter configuration seemed to be preferable due to the fact that any prescribed astigmatism can also be incorporated into the firm lens wall. It would be a very complicated case, if both walls were flexible and a distance correction incorporating astigmatism was needed. In such a case an auxiliary lens would be needed to provide for the distance correction.

The deformation imposed on the lens wall was achieved either by changing the volume of the fluid in cavity or by changing the curvature of the lens wall after exerting pressure directly to the lens wall though some kind of mechanical means. The volume of the fluid in cavity, in the first case, was controlled by an external source, which usually was an airtight pumping device.

The first deformable lens reported, was constructed by an instrument maker named Grummert from Dresden in 1747 (Bennett, 1973). It was used in a telescope, having a very long focus. Nearly one hundred years later, in 1877, Jung from Carlsruhe tried to construct a deformable lens, which would represent the human crystalline lens and its accommodative action, used as a theoretical model (Bennett, 1973). In 1879, Cusco from Paris, constructed

197

a deformable lens, which he intended to use as a binocular optometer to study the physiology of accommodation. The lens consisted of a plano-convex lens made of glass and having a power of around +3.00 dioptres. That was combined with a very thin glass plano lens, which was the flexible wall of the deformable lens-cell. These two elements were joined together, having a brass collar as a spacer to provide the cavity for the fluid, which in this case was water.

The internal pressure of water was controlled by a pumping system, and the increase of power with such a system reached up to +1.00 dioptres. Figure 5.1. illustrates Cusco's deformable lens system for which he received a patent in the same year. The manometer shown in Figure 5.1. at the side of the binocular lens system of Cusco's was calibrated to additionally record the lens powers with an accuracy of 0.01 dioptres as Cusco claimed. The first patented deformable lens in United States, was the one produced by Ohmart in 1893. It consisted of a lens-cell, with a fluid filled cavity. The lens-cell consisted of two flexible plates, where their surface curvature varied by changing the internal pressure of the fluid, after pumping more fluid into the lens-cell's cavity. The plates of the lens-cell consisted of a curvature varied by changing the internal pressure of the fluid, after pumping more fluid into the lens-cell's cavity. The plates of the lens-cell consisted of a curvature of the fluid, after pumping more fluid into the lens-cell's cavity. The plates of the lens-cell consisted of a curvature is cavity in the plates of the lens-cell consisted of a curvature is cavity. The plates of the lens-cell consisted of a curvature is cavity. The plates of the lens-cell consisted of a curvature is cavity. The plates of the lens-cell consisted of a curvature is curvature is cavity. The plates of the lens-cell consisted of a curvature is curvature is curvature.

In Russia, Rudin (1912) constructed a deformable lens, which consisted of a single element providing a firm basis for the distance vision and an elastic



Figure 5.1. (a) Cusco's deformable lens and (b) a pair of deformable lenses with the control system that Cusco used. After Cusco (1879).

membrane. The membrane was attached to the rim of the frame where the lens system was mounted. The fluid was inserted into the cavity of the lens-cell produced through a small inlet, while its pressure was controlled by a hydraulic system consisting of a bulb and a lengthy rubber tube, which connected the bulb with the cavity of the lens system. Rudin also gave a brief reference to the various practical problems that such a design presents. In Germany, Schaller (1913) produced a deformable lens, which was intended to be used in a photographic apparatus, with a view-finding prism and a focusing arrangement, which were connected with the mechanical system for deforming the lens walls.

Gordon, in 1918, had a different approach in constructing deformable lens systems. The lens that he produced was made by fastening two thin glass plates (spheres) together in a frame. The space provided by the two glass plates was then fiiled with a fluid and the frame contracted to squeeze the lens system with a constricting or sphincter-like action. This action induced a pressure at the periphery of the glass plates, which altered their curvature. Figure 5.2. illustrates Gordon's lens system, showing that the pressure was exerted by tightening up the rims of the frame. It should be mentioned that the initial diameter of the glass plates making up the deformable lens were made intentionally larger than it should be in order to fit perfectly into the rim of the frame. In 1924, Pfleegor had the idea of utilizing a small hole, which was drilled into and not through a plate of glass. After polishing the bottom part of the hole, glycerine was added. By increasing or decreasing the





Figure 5.2. Gordon's deformable lens and the spactacle pair he used to incorporate his lenses. . It is clear that by tightening the rims the lens deform. After Gordon (1918)

amount of glycerine from that it was required to completely fill the hole, a small convex or concave lens was formed. The disadvantage of such an approach was that such lenses can only be used in a horizontal position, due to the fact that gravity at any other position will cause the fluid surfaces to bulge asymmetrically. It should be noted that these lenses did not have any means for producing any specific dioptric power at will.

Mitchell (1926) made a deformable lens, which he proposed would have a useful application in cameras for producing zoom lenses. Another proposed used was in a microscopic projection apparatus, where a variable magnification is required for objects with different size. According to Mitchell, the most useful application of such a lens system would be in spectacles, to provide lenses for both distance and near vision as required. Figure 5.3. shows a cross-section of Mitchell's deformable lens. It consisted of a magnifying lens M and a plain glass cover N, which had such an arrangement in order to act as guards to the two flexible walls B and C. This arrangement was proposed by Mitchell to prevent the very thin walls B and C being overstressed. Inside the chamber produced by the flexible walls B and C, glycerine was inserted, where the volume being regulated by means of a pressure bulb or a piston connected with the chamber through the duct D. In the case that the walls B and C were made of glass, the proposed thickness was 1/200 of an inch. Mitchell (1926) also made a pair of spectacles where two of his deformable lenses were mounted in a frame. The two lenses were connected in such way that the fluid could pass

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Figure 5.3. Cross-section of Mitchell's deformable lens. After Mitchell (1926)

through from one lens to the other. That was achieved by introducing a frame, which consisted of a small tube through which the fluid could freely flow. One of the frame rims was sealed, while the other was connected to a bulb for controlling the volume of the fluid. Figure 5.4.(a) shows this proposed arrangement, with (p) being the bulb for controlling the volume of the fluid, (o) is the rubber tube, (1) is one of the ends of the tubular frame while (4) is the other end of the tubular frame, where a plug is fitted. When the fluid is inserted into the tubular frame through the opening (1), the plug at the opening (4) is temporarily removed in order to let the air out. The small holes (2) are opened inwards into the chamber of the deformable lenses to provide a passage for the fluid entering from the opening (1). The other small holes (3) are made at the top of the deformable lenses to also allow the air from the two chambers to get out passing to the upper tubular part of the frame and afterwards through the opening (4). Instead of using the bulb (p) for controlling the volume of the fluid, Mitchell had the idea of using an apparatus as the one shown in Figure 5.4.(b). This consisted of the same rubber tube (o), where one of its ends (o') was firmly closed and the other was attached to one of the deformable lenses. The rubber tube (o) passed between two pressure rollers (q) and (q'), which could be turned by the hundle (h) and consequently winding the rubber tube (o) in or out. That action could provide more or less pressure inside the deformable lenses' chamber and adjust their focal length.

Ritzmann (1936), in Germany, had a totally diferent approach than the



(a)

Figure 5.4. Mitchell's pair of spectacles. (Specially made frame to incorporate deformable lenses). After Mitchell (1926)

previously mentioned attempts. His deformable lens was based on the usage of a crossed cylindrical lens form, instead of the spherical ones used so far. In the Ritzmann device, the edges of the plane glass surface used were inserted into slots cut in a pair of metal rods. By applying torsion (twisting the rods in opposite directions) the plane glass surface obtained a bi-cylindrical form with a convex bending. By rotating the rods in the reverse direction instead of a convex bending a concave one was produced. The actual device consisted of a combination of two systems like the one aforementioned, having their axes mutually perpendicular. A spherical refractive effect could be obtained with such a system, when the two cylinders produced were of the same value and obviously had their axes at right angles. The advantage of Ritzmann's device was that it produced spherical, cylindrical and sphero-cylindrical effects and in variable amounts. Its drawbacks lie in the fact that the mechanism for applying the torsion was large related to the effective aperture of the lens, and also that the means for retaining the produced powers did, not exist.

Graham (1938), in the United States, produced a similar deformable lens to the one produced by Ritzmann, only this time the cylinders were not produced by torsion but by forcing the two opposite edges of a plane surface slightly towards to each other. Actually, the plane surface become elliptical rather than cylindrical when it was flexed in this way. The actual refracting unit of Graham's deformable lens consisted of two plane surfaces of glass, with between them a fluid sealed in by a rubber spacer, which joined the



Figure 5.5. Graham's deformable lens. After Graham (1938).

edges of the two glass surfaces. Distilled water could be used as the fluid between the two glass surfaces due to the fact that it is inert. He also proposed the usage not only of fluids but also gases or gels, provided they were sufficiently transparent and of suitable refractive index. The proposed glass surfaces had a diameter of 34 mm and their optimum thickness was found to be around 0.007 mm. With such extremely thin glass a sufficient change in its refractive action was obtained by exerting only moderate pressure. The effective aperture of such a lens was 18 mm, while the surface, due to the mechanical means used to induce the deformation, remained opaque. The fluid was injected into the chamber and the air was let out through the rubber spacer by means of a hypodermic syringe. According to Graham one of his finished lenses presented a range of plus 3.50 to minus 3.50 dioptres. Figure 5.5. shows Graham lens structure.

In 1951, Wagner introduced a lens having both surfaces flexible. The material he used for his lens-cell was Plexiglass and this is probably the first time that plastic materials were proposed for such a purpose. In his arrangement he also used a bulb to control the volume of the fluid in cavity, produced by the two flexible walls. His theoretical model could have a variable power ranging from zero to +6.00 dioptres as he claimed. Fagioli (1955) from Italy, made a deformable lens using a similar idea to Gordon's (1918). The lens consisted of two flexible wall-plates and an annular ring interposed between. The annular ring had a T shaped cross-section and was used as a fulcrum in order to bend the walls outwards. The lens

arrangement is described in Figure 5.6. It is apparent from the diagrams that depending on the position that the T cross-section annular ring obtains, the lenses could behave as positive or negative lenses with variable powers.

Grunberg, in 1956, constructed a deformable lens with only one flexible surface, where the other was rigid (made of thick glass) and used it as a hand magnifier. When he incorporated his lens in a spectacle pair, he used a piston, one for each lens separately, to exert pressure on the fluid in the chamber. Whether having a different piston to control the pressure in each of the lenses of the spectacle pair would help to get better results is doubtful. It seems almost impossible to be able to have equal deformation imposed on the lenses surfaces with such an arrangement.

The last two deformable lenses produced were based on a similar arrangement and were introduced by Wright (1971) and Wylde (1972). They were both proposed to be suitable for ophthalmic use and more specifically in correcting presbyopia. Wright's lens was a sandwich lens comprising three components, which were laminated together. The front component was a conventional spectacle lens, having a back curvature of about 2.00 dioptres, while its front surface could be used to incorporate any correction for distance vision. The intermediate component was made from polyvinyl butyral, which was used as a spacer to provide the cavity, where the fluid would be fed. The actual diameter of the cavity was 25 mm and it was centered upon a point displaced 2 mm vertically downwards and 2 mm

209



Figure 5.6. Fagioli's deformable lens construction. (cross-section). After Fagioli (1955).

horizontally inwards with respect to the optical centre of the front component. The rear component was a thin glass of 0.15 mm thickness. This rear component was the deformable wall of the lens-cell, which was made to bulge after exerting pressure through a piston assembly located at the side of the frame, where the lenses were mounted. Wright followed Mitchell's idea, having one piston for both lenses, which were connected through a duct. The fluid was a saturated aqueous solution of calcium bromide and glycerol. Figure 5.7. illustrates Wright's variable focus lenses mounted on a specific kind of frame. Unfortunately, the lens could only have a power change of about 1.00 doptre, due to the fact that the rear component was not flexible enough as it was made of glass (Sullivan and Fowler, 1988). That limited its usage in presbyopia correction. In another embodiment of Wright's lens, the variable focus construction was incorporated into a fixed focus lens replacing one component of a conventional bifocal lens. That provided distance and near vision zones with a variable focus intermediate zone.

Wylde's design was very similar to the previously mentioned lens. It had front rigid component, where the distance correction could be incorporated and a rear component which centrally was made narrower than at the periphery. The two components were made of a synthetic plastics material and had been bonded together. The chamber produced at the narrow part of the rear component was filled with a fluid, which according to a private communication with Wylde was xyline. He also mentioned that medicinal paraffin could be used due to the fact it is quite inert. The pressure exerted



Figure 5.7. Wright's spectacles incorporating deformable lenses. A special kind of frame is used for this purpose with a duct running through it and two deformable lenses with effective aperture of 25 mm. After Wright (1971).

on the fluid was produced by a piston assembly connected with the chamber through a duct drilled at the rear component of the lens. Wylde also mentioned at the private communication we had, that the front and the rear component of the lens should be made of CR 39 (he used it in several of his prototypes), which would give better results and it was safer than glass. Figure 5.8. shows a cross-section of Wylde's deformable lens.

More recently, a very novel type of deformable lens has been reported (Quantel ..., 1981), a more advanced type of the previously mentioned deformable lenses. This is based on the piezoelectric effect, which is responsible for changing the refracting action of such lenses. Figure 5.9. shows a cross-section of such a lens. It consists of a container (1), which is formed of a transparent material. This is connected through a seal (2) with a wall (3) also made from the same transparent material. Adhered to the wall (3) a multilayer structure (4) of a piezoelectric material is formed, consisting of two elements (5) and (6). These are separately connected to terminals (7) and (8) respectively, while a terminal (9) is connected to the surface of separation between the elements (5) and (6). A window (10) is also formed at the central part of the multilayer structure (4) in order to permit light passing through the wall (3). Between the elements (5) and (6) of the multilayer structure (4) a strip of a transparent flexible material (11) is placed. The container (1) is filled with a refracting fluid (12) e.g. silicon oil, and it also has an expansion chamber (13) communicating with the container (1). When voltage is applied to the terminals (7) and (8), the multilayer structure (4) is







Figure 5.9. Deformable lens based upon piezoelectric effects. After Quantel S.A. (1981).

deformed producing a change in the refractive action of the strip (11). This change in the focal length of the whole lens unit varies according to the potential difference at the terminals. The expansion chamber (13) is used to accept the excessive fluid when no voltage is applied to the terminals (7) and (8). The proposed piezoelectric materials suitable for the above described optical lens-cell element are

> Quartz Seignette salt Potassium monophosphate Rubidium monophosphate Ceasium monophosphate

The lens is not limited to a plano-convex configuration but it could have a bi-convex, plano-concave, bi-concave or miniscus form. Unfortunately, no information is given about the actual change of the lens focal length, consequently, it is difficult to say if such a lens could be utilized in correcting presbyopia.

## 5.2. Objectives of the research on deformable lenses

Looking through the literature review carried out on the deformable lenses it is apparent that such a field is of great interest in the efforts made for
providing an ideal multifocal lens, which could be utilized in presbyopia correction. The deformable lenses' advantage, in contrast to the progressive addition lenses widely used nowadays, lies in the fact that the distance, near and intermediate corrections can occupy nearly the whole aperture of the lens. In these cases the field of view through such a lens would be wider compared to the progressive addition lenses, where the distance and near corrections of constant power occupy portion of the lens, while the intermediate zone is limited to a narrow corridor a few millimetres in length.

Besides that, if what Cusco (1879) claimed is true, and the variable power of the deformable lenses can be controlled with accuracy, then such a lens-system can be used in presbyopia correction providing the steps of power needed. The objectives of this research in this area were concentrated on the following:

a) To construct a lens system, which will be deformable using a sandwiched lens, a refracting fluid and a system, which will exert pressure on the refracting fluid resulting in the deformation of the walls of the lens-cell.

b) To try and estimate what would be the ideal constructive configuration for such a lens system. Most of the previously mentioned attempts had two common features, the flexible walls and the refracting fluid. That would be also used as the basic constituents in our attempt.

c) To try and find out which would be the ideal materials to be used for such an attempt.

(what type of fluid is preferable, what kind of material should be used for the

lens walls and what kind of cementing gives the best sealing).

d) After constructing the lens, try to measure and estimate what would be the maximum addition that can be provided by a deformable lens. So far, no one apart from Cusco (1879) has ever given such an information.

e) To try and assess what kind of deformation is achieved when pressure is exerted upon the fluid and consequently upon the flexible walls of the cell. It should be clarified whether the deformation is spherical or not, and if not, the actual type of deformation should be defined. Again this information is lacking in the literature.

f) When all the above have been assessed, research has to be carried out on how would the resulting lenses behave, if a very wide effective lens aperture is used. Up to now, all the deformable lenses introduced had an effective aperture of 15 to 25 mm the maximum. That is also the major objective of this part of the experimental work.

g) To try and answer the question related to their reproducibility. Is it possible to construct two lenses having the same features and act the same way?

h) The last objective is related to the optical performance of deformable lenses. An assessment has to be done on whether such lens-systems are suitable for any ophthalmic use and more specifically on how these  $\underset{\substack{\omega \in \mathcal{U}\\ \omega \in \mathcal{U}}}{\underset{\substack{\omega \in \mathcal{U}\\ \omega \in \mathcal{U}}}}{\underset{\substack{\omega \in \mathcal{U}\\ \omega \in \mathcal{U}}}{\underset{\substack{\omega \in$ 

218

# **CHAPTER 6**

## DEFORMABLE LENSES-EXPERIMENTAL

#### 6.1. Method of construction

The construction concept of deformable lenses is based on the theory related to fluids flow and hydrostatics. It is well known that by exerting pressure on a fluid, which is captured in an enclosed area created by elastic walls, this pressure will transpose to the elastic walls. The same effect is presented when the volume of a fluid completely filling the cavity, created by two elastic walls, is changed and more or less fluid is pushed into the cavity. It is apparent that the above mentioned effect is present only when the cell created by the flexible walls is airtight, otherwise the fluid will run out of the lens-cell through a possible small opening that the lens-cell might have.

The manufacturing procedure that was followed consisted of making a lens-cell, having a cavity, centrally placed, which will accept the fluid. Figure 6.1. illustrates a cross-section and a face view of such a configuration made for this purpose. It consisted of two walls (1) and (2), where one or both of them were flexible. They were plake surfaces with no dioptric power. (3) was an annular spacer with a predetermined height and width, which was interposed between the two plain surfaces (1) and (2). At a certain point in the annular spacer (3), preferably at the top part of the lens-cell, two small holes were drilled, having a distance of about 10 mm between them. In one of these (4), a syringe (6) was cemented, in order to provide an airtight lens-cell. The syringe had a maximum volume of 5 ml.



Figure 6.1. A cross-section of the deformable construction used in this experimental work.

The plane surfaces (1) and (2) were also cemented to the annular spacer (3) using the same adhe sive material. The second hole (5) was left open for the time being. When the above mentioned lens-cell was ready, and the cementing procedure had finished, a fluid (7) was inserted into the lens- cell by pushing it through the syringe (6) into the cavity produced. When the whole cavity was filled with the fluid (7), the second hole (5) in the annular spacer (3) was also sealed using the same adhesive material as previously mentioned, providing a lens-cell, which was airtight. By pushing more fluid into the cavity through the syringe (6), pressure was exerted upon the walls (1) and (2) resulting in the walls bulging outwards. The lenses constructed had only one flexible surface (1), due to the fact that the other one (2) should provide a firm basis to incorporate the distance correction, especially if an astigmatic correction is prescribed. Although most of the lenses produced had only one flexible surface, a few lenses were also created with both surfaces bulging in order to compare which of the two configurations would give the better results. The surface (1) could have a curved form but in this experimental part a flat surface (1) was used.

The thickness of the surface (1) varied between 0.5 to 2.5 mm, while the thickness of surface (2) was kept constant (4 mm). The spacer's height also varied taking values between 1 to 3 mm. The pressure exerted upon the fluid (7) and consequently upon the flexible wall (1) of the cell by the syringe (6) was measured in terms of millilitres of extra fluid pushed into the cavity after the whole cavity was filled. The diameter of the cavity, which was produced

221

by the annular spacer (3) and the plane surfaces (1) and (2) ranged between 30 to 65 mm. The diameter of the cavity was the effective aperture of the lens-cell.

The materials used to construct such a lens-system, in our experimental work, consisted of :

a) Perspex plastic (Polymethyl methacrylate) with refractive index 1.491, for the plake surfaces of the lens-cell (1) and (2) having different thickness. The annular spacer was also made from perspex plastic and had a predetermined height, which varied. Glass material was not considered to be a good solution due to the fact that it is more difficult to bend than plastics material unless very thin sheets are used. The use of CR 39 would seem to be another solution for this construction.

b) Tensol No 12 Dichloromethane mixture, containing methylmethacrylate, was used for cementing the constituent parts of the lens-cell, as an initial trial. Subsequently the lens-cells had as the cementing material epoxy resin (Araldite), which gave better sealing results.

c) Glycerol (prod. 44045 of B.D.H.) as the refracting fluid injected into the produced cavity, having a refractive index of 1.472, which is very close to the refractive index of the Perspex material used for the walls. This was used in the majority of the lens-cells constructed although some attempts were made using xyline, water and medicinal paraffin.

The second part of the experimental work consisted of constructing a pair of

222

deformable spectacle lenses. Figure 6.2. is a face view of the pair constructed for this purpose. (1) and (2) are the deformable lenses made after following the previously described method, where a plastic tube (4) was placed in such way that the two chambers of the deformable lenses are connected, in order that the fluid can easily flow from one chamber to the other. Only one pumping assembly (3) was used to exert pressure on the fluid in both chambers. The small opening (5) was drilled through the annular spacer of the deformable lens which was not connected directly to the pumping assembly (3), in order the air trapped in the two chambers could be released when fluid is inserted through the pumping assembly (3), which consisted of a syringe. After the fluid was inserted into the two chambers passing through the plastic tube (4) filling them completely, the opening (5) was closed using the previously mentioned adhesive material creating an airtight system. By pushing more or less fluid into the chambers a deformation occurs in both deformable lenses. It should be made clear that only one of the two surfaces of each deformable lens was flexible, while the other surface consisted of a rigid plane wall.

#### 6.2. Experimental data

The deformable lenses constructed following the aforementioned method had the common feature of only one deformable surface. This was essential





in order to assess the kind of deformation occuring on the surface of such lenses when pressure was exerted on the fluid contained in the chamber of the lens. A few lenses were also constructed with both surfaces flexible to provide the necessary information for comparing the two structures and assessing which is best for such kind of lenses. The data for these lenses were quite irregular so it is not presented here but it is only used for the conclusions derived generally for these lenses.

Fourteen satisfactory lenses were produced and their dioptric behavior is given in Appendix  $D_1$ . Tables 6.1. to 6.14. present the maximum addition obtained by the deformable lenses constructed and additionally the behavior of the entire front surface of the measured lenses at predetermined points on their deformable surface. These predetermined points had a 5 mm distance from each other vertically and horizontally.

From these fourteen deformable lenses ten had an overall diameter of 60 mm and are presented in Tables 6.1. to 6.10., one had a diameter of 65 mm (Table 6.11.), two had a diameter of 75 mm (Tables 6.12. and 6.13.), while the last one presented in Table 6.14. had an overall diameter of 80 mm. The actual effective aperture, which was the surface where the deformation took place was varied taking the following values :

a) one lens had an affective aperture of 30 mm (Table 6.10.)

b) nine lenses had effective apertures of 40 mm (Tables 6.1. to 6.9.)

c) one lens had an effective aperture of 55 mm (Table 6.11.)
d) two lenses had effective apertures of 60 mm (Tables 6.12. and 6.14.)
e) and finally one lens had an effective aperture of 65 mm (Table 6.13.)
More lenses were constructed having an effective aperture of 40 mm,
because that seemed to be the optimum aperture, which will provide enough field of view to a presbyopic patient, and is suitable to be mounted in a conventional fashion frame.

The pressure, exerted upon the fluid (glycerol) contained by these lens-cells, was measured in millilitres of glycerol added into the chamber of the lenses, after the chamber was completely filled. In the lenses presented in Appendix  $D_1$  the pressure was varied from 0.25 to 2.00 ml of glycerol. The annular spacer used had a predetermined thickness, which varied as follows

a) three lenses had a 1 mm annular spacer (Tables 6.1.,6.3.,6.7.)

b) five lenses had a 2 mm annular spacer (Tables 6.2.,6.4.,6.5.,6.6.,6.10.)

c) six lenses had a 3 mm annular spacer (Tables 6.8.,6.9.,6.11.,6.12.,6.13.,

6.14.)

From the resulting powers of the maximum addition presented by the lenses, measured at the predetermined points which lie on the vertical line passing through the centre of the lenses, the mean power at these points was calculated and a graphical representation of its variation is presented in

226

Figures 6.3. to 6.16. coresponding to the lenses of Tables 6.1. to 6.14.. This graphical representation of the mean power variation of all the aforementioned lenses will be used later to assess the kind of deformation occuring in the deformable lenses constructed.

Additionally, the resulting maximum power change for a pair of deformable spectacle lenses constructed, was measured and their mean power variation at pretedermined points on the vertical line passing through their centre is given in the graphical representation of Figures 6.17 and 6.18.

### 6.3. Conclusions-Discussion

From the resulting lenses and their measured dioptric behavior, when the maximum addition is obtained, it is apparent that such a lens-system can provide the necessary addition needed to correct the near vision of a presbyopic patient. So far no one has yet proved that these lenses can actually be made and used for presbyopia correction, based on experimental data. All the research carried out in the past did not provide sufficient evidence, that these lenses can be a reality, apart from Cusco

(1879). All the previous research was concerned with the theoretical aspects of deformable lenses, trying to assess the best configuration that these should have.

According to our results, the best configuration for these lens-cells is the one described previously in the section "Method of construction". It must be pointed out that only one surface should be made flexible and liable to deform, while the other should be made rigid enough in order to obtain the best possible results. Although, according to the results obtained, the lenses which had both their surfaces deformed, due to the pressure exerted, presented greater values of addition, their dioptric behavior through their entire effective aperture was not ideal and a greater amount of astigmatism was observed, compared to the amount observed at the lenses which had only one of their surfaces being deformed. This observetion and the fact that the distance correction could not be incorporated resulted in the rejection of such a configuration.

The spherical change obtained by deforming only one surface of these types of lens-cells ranged between +0.25 to +3.50 dioptres spherical, which is enough to cover the needs of most presbyopic patients. This maximum addition of 3.50 dioptres spherical was obtained with a lens-cell having a deformable surface of 0.5 mm (Table 6.9.) although the effective deformable surface had a diameter of 30 mm. It is clear from the results that by increasing the thickness of the deformed surface, the addition obtained is reduced. This is something that was expected due to the fact that a thicker surface does not have the same flexibility as a thin one and more pressure has to be exerted upon the containing fluid in order to deform such a surface to the same degree as a thin one. Besides the above, high additions ranging between 2.00 to 3.00 dioptres spherical, are also obtained by increasing the effective aperture of the lens-cell up to 55 mm (Table 6.11.), 60 mm (Table 6.12.) or 65 mm (Table 6.13.). For most of the deformable lenses constructed the maximum addition obtained ranged between 1.50 to 2.00 dioptres spherical, where the dioptric behavior of these lenses was nearly optimum.

Observing the data collected, it apparent that this lens type presents a relatively small amount of unwanted astigmatism through its whole aperture, especially near the edges. This amount of unwanted astigmatism decreased going towards the centre of the lens, where in few of the constructed lenses the power, presented at the centre and the neighbouring measured predetermined points, is actually spherical and free of any unwanted astigmatism.

The astigmatism presented at the lens periphery, after pressure was exerted, ranged between -0.25 to -0.50 dioptres cylindrical. That was observed for the lenses, where the maximum addition obtained was near 1.75 dioptres (Tables 6.4.,6.5.). Such an amount of astigmatism can be considered, generally, acceptable by the eye and will not affect the vision that much, although the change of its axis through the effective aperture of

the lens might create some problems, especially when an astigmatic distance correction is prescribed and occupies the rear surface of the lens-cell. In the lens-cells where the maximum dioptric change obtained in the spherical component reached up to 2.00 or 3.50 dioptres, the respective astigmatism at the periphery of the lens-cells ranged between -1.50 to -1.75 dioptres cylindrical (e.g. Tables 6..8.,6.9.). It is clear that when the maximum addition obtained by these lenses increases and reaches relatively high values a proportional increase of the values of the presented astigmatism occurs, which undoubtly affects the performance of the lens. This increase is probably also related to the increase of the effective aperture of the lens-cell.

The experimental data also showed the reproducibility of these lenses. This is shown by observing the results of two lenses (Tables 6.4.,6.5.), which were constructed in exactly the same way and the pressure exerted in both cases was again the same. It can be seen that although their dioptric behavior is not exactly the same, through their effective aperture, its very similar and there is a possibility that the minor differences observed was probably due to manufacturing error. It must be admitted that with such a complecated structure it might be almost impossible to have lenses with identical dioptric behavior, even though they are constructed under the same conditions and exactly the same pressure is exerted. The above might be proved a serious obstacle if these lenses are considered for mass-production, due to the uncertainty related to their reproducibility.

230

The other problem related to deformable lenses, lies in the fact that the response and recovery times of these lenses are very slow. Glycerol, due to its large viscosity, does not respond quickly to the pressure exerted, taking nearly 15 to 20 minutes to reach an equilibrium, where it is completely homogenous (glycerol moves in layers). The same is valid when the excessive amount of glycerol is taken out of the lens' chamber (recovery time). This is a major drawback and it is hard to envisage a presbyope having to wait such a long time, in order to focus clearly again objects at a different distance. A solution to this problem is to use another type of fluid, which will also have nearly the same refractive index as the material of the lens walls are made, but with a significantly smaller value of viscosity. Wylde, in a private communication, mentioned that the usage of xyline would give better response and recovery times than glycerol, due to its low value of viscosity, in combination with a lens-cell made of CR 39. According to our experiments, the lens-cells constructed using xyline as the refracting fluid, when they were under pressure, came apart due to the fact that the cementing material used (epoxy resin or Tensol No 12) was chemically influenced by xyline.

The other drawback related again to the usage of glycerol, is that the lens does not recover completely, reaching its initial state, and present zero additional power when the excessive glycerol is taken out of the lens chamber. When the power of the lens of Table 6.4. was again measured at the predetermined points of its front deformable surface, its resulting dioptric behavior is given in Table 6.15.. It is clearly seen that a residual value of spherical and cylindrical powers remain through the whole effective aperture of the lens. For the spherical component this value varied between +0.25 to +0.50 dioptres, while the value of the cylindrical component ranged between zero to -0.25 dioptres. The possible explanations for this phenomenon are two. It is either the fact that gravity affects glycerol and a small amount of it cannot be pushed out of the lens chamber, or the elastic wall of the lens-cell is deformed to a certain degree, which is larger than the one it could accept, and consequently cannot recover completely. This permanent induced change results in a permanent change in the raduis of curvature of this surface. The former explanation seems to be the more possible one.

Another drawback related to the manufacturing of deformable lenses is that the cementing of the two wall plates of the lens-cell with the interposed spacer is not always perfect. After a certain period of use, the cementing material breaks down probably due to chemical influnce of glycerol. This was also observed with xyline or medicinal paraffin as previously mentioned. None of the researchers, who have worked in this field, ever mentioned that a perfect cementing, after a cycle of use, was obtained by using a certain method or a certain adh *e* sive material. That might also proved to be a very serious problem if deformable lenses were considered to be launched in the market. It should also be noted that the spacer, interposed between the two plain plates of the lens-cell, should have a circular configuration, otherwise the change in the curvature would not be spherical. It would actually be very irregular and the resulting power through the whole effective aperture would present significantly greater amounts of astigmatism compared with the resulting astigmatism of the lenses presented in Tables 6.1. to 6.14.

By representing the mean power change at predetermined points on a vertical line passing through the centre of the deformable lenses constructed, the diagrams obtained in Figures 6.3. to 6.16. show clearly the aspherical character of such a change. It is obvious that the mean power increases going towards the centre of the lens-cell, where it also reaches its maximum value, while due to the increase of astigmatism, the mean power decreases rapidly at the periphery of the lens, resulting even to negative powers. Observing the diagrams of the mean power change in a pair of deformable lenses (Figures 6.17. and 6.18), a similar variation is presented for both lenses. These results reinforced the hypothesis, that these lenses should be connected together and only one pumping assembly used.

Concluding, it has to be accepted that deformable lenses are feasible to make and a solution to presbyopia correction but not the ideal one. They present drawbacks, which for the time being, are difficult to overcome, such as the cementing problem, the amount of astigmatism they present at high addition, and the problems related to the fluid used. In addition to the previously mentioned problems, the fact that their operational procedure is

233

inconvenient to a presbyopic patient, have put limits on their use, although wider fields of view can be obtained with them, compared to the conventional varifocals used nowadays. Figures 6.3. to 6.18. illustrate the mean power variation at a vertical line passing from the centre of the deformable lenses. The predetermined points measured have a 5 mm distance from each other. The mean power variation is presented in relation to the distance of the above mention points. The curves obtained are approximately fitted to the above mentioned points in order to obtain the graphical representation of the mean power variation for each lens.

# Deformable lens divided into columns and rows indicating the points measured

#### Columns



Note : Depending on the diameter and the effective aperture of the deformable lens, the number of rows and columns will vary, and consequently the column that is used in the following diagrams, in order to represent the power change of the deformable lens, will also vary.



Figure 6.3. Graphical representation of the mean power variation of the lens of Table 6.1.



Figure 6.4. Graphical representation of the mean power variation of the lens of Table 6.2.



Figure 6.5. Graphical representation of the mean power variation of the lens of Table 6.3.



Figure 6.6. Graphical representation of the mean power variation of the lens of Table 6.4.



Figure 6.7. Graphical representation of the mean power variation of the lens of Table 6.5.



Figure 6.8. Graphical representation of the mean power variation of the lens of Table 6.6.



Figure 6.9. Graphical representation of the mean power change of the lens of Table 6.7.



Figure 6.10. Graphical representation of the mean power variation of the lens of Table 6.8.



Figure 6.11. Graphical representation of the mean power variation of the lens of Table 6.9.



Figure 6.12. Graphical representation of the mean power variation of the lens of Table 6.10.



Figure 6.13. Graphical representation of the mean power variation of the lens of Table 6.11.



Figure 6.14. Graphical representation of the mean power variation of the lens of Table 6.12.



Figure 6.15. Graphical representation of the mean power variation of the lens of Table 6.13.



Figure 6.16. Graphical representation of the mean power variation of the lens of Table 6.14.



Figure 6.17. Graphical representation of the mean power variation of one of the deformable spectacle lens pair.



Figure 6.18. Graphical representation of the mean power variation of the other lens of the deformable spectacle lens pair.
#### 6.4. Aspheric surfaces

It is well known that a spherical surface is obtained by rotating a circle about its diameter. The common feature of all the spherical surfaces is the constant radius of curvature at all its points. Nearly all the ophthalmic lenses commonly used to correct eye deffects such as myopia or hyperopia consist of spherical surfaces.

Any other kind of surface, which departures from the sphere produced by the rotation of a circle can be characterised as aspheric. In a strict sense, that could include surfaces, which are either cylidrical or toroidal, but actually the term addresses surfaces which are produced by rotating a symmetrical non-circular curve about its axis of symmetry. The symmetrical but not circular curve, used for this purpose, belongs to a certain class of curves known as conic sections. The circle also belongs to the previously mentioned class, which includes the ellipse, the parabola and the hyperbola. (Volk, 1958). The surfaces produced by the rotation of the above mentioned curves are known as the conicoids. Figure 6.19. illustrates these conic sections and the conicoids they represent. It should be made clear that all these surfaces can have the same radius of curvature at their vertex, where their major axis pass through, but at any other point the radius of curvature will be different.

252



Figure 6.19. Schematic representation of the conic sections consisting conicoid surfaces. After Volk (1958).

A convenient mathematical representation of a conic section is given to be (Jalie, 1988)

$$y^2 = 2r_0 x - px^2$$

where  $(r_0)$  is the radius of curvature at the vertex of the conic section, (x) is the sag of the conic curve and 2y is its diameter. The quantity (p), known as the asphericity, specifies the type, the shape and the degree of asphericity of a conic section. Knowing the value of (p), the type of the conic section and consequently the type of the conicoid surface consisted of it can be deduced, according to the following (Jalie, 1988)

a) if p = 1 then the conic section is a *circle* and the surface *spherical*b) if p > 1 then the conic section is an *oblate ellipse* and the surface *ellipsoidal*

c) if 0 prolate ellipse and the surface<sup>•</sup> *ellipsoidal* 

d) if p = 0 then the conic section is a *parabola* and the surface *paraboloidal*e) if p < 0 then the conic section is a *hyperbola* and the surface *hyperboloidal*

Figure 6.20. illustrates the different conic sections, having a common axis of symmetry. It is apparent that as (p) takes smaller values the conic section and consequently the conicoid become flatter going away from the axis of symmetry. The flattest conic conicoid is obviously the hyperboloidal.



Figure 6.20. Different conic sections with a common axis. After Jalie (1988).

#### 6.5. The usage of aspheric surfaces in ophthalmic lenses

In relatively recent years, lenses utilizing aspheric surfaces have been successfully made. One familiar use of aspheric surfaces is that related to progressive addition lenses, where part of the front surface of the lens is aspheric, in order to produce the steps of power needed for intermediate vision. One of the first patents related to sing/e vision aspheric lens design was produced by von Rohr in 1910, where he described two aspheric lenses one of high positive power and another of high negative power. After that a lot of aspheric lens designs have appeared like the Volk lens (Volk, 1966a, 1966b), the Essel Atoral lens (Societe des Lunetiers, 1968, 1970) or the Jalie lens (Jalie, 1972). These lenses consist of conic section surfaces, and more particularly ellipses. Lately, aspheric lenses, having polynomial surface curves, have been introduced such as Jeffree's design (Jeffree, 1963) or Bechtold's design (Bechtold, 1973). These designs are more complex than the previously described conicoids

The advantages that the aspheric surfaces present are concentrated in minimising the optical aberrations (Katz, 1982) and produce a lens having a better off-axis performance than the equivalent conventional spherical or toroidal lenses. At first, they were used in high power spectacle lenses, and more specifically, were intented to be used for aphakic patients. In such case, besides the fact that they present a better off-axis performance than their counterpart conventional lenses made of spherical surfaces, the

256

resulting lenses are thinner and lighter. That is more evident if one of the lens surface is a conicoid, which is either a paraboloidal or a hyperboloidal and generally has a (p) value which is much less than unity. Today aspheric lenses are also available in low positive and negative powers.

# 6.6.. Correlation of the aspheric deformable surfaces with the related conicoids

The resulting deformable lenses, experimentally produced following the method previously mentioned, were found to have their deformable surface aspheric. This is proven to be true after observing the Figures 6.3. to 6.16., where their mean power change is graphically represented in relation to the distance from the centre of the surface. These graphical representations were used in order to correlate these surfaces with the respective conicoid surfaces having a certain (p) value. By correlating the graphical representations of the deformable lenses mean power change (Figures 6.3. to 6.16.) with the graphical representations of the mean power change of known aspheric surfaces with certain (p) values calculated with the computer scheme in Appendix  $D_2$ , the conclusions made about the type of the conicoids resembling these surfaces and their (p) values were as follows

a) all the deformable lenses constructed had their bulging surface resembling a hyperboloidal surface, where the (p) value differed from one lens to the other.

257

b) the range of (p) value for these lenses was between -25 to -1800, which indicates that these surfaces were extremely flat related to the dioptric power they presented at their optical centre. The detailed results for each of thse lenses, related to the (p) value of the correlated aspheric surface, are given in the Table 6.16.

c) besides the deformable surface, which had a (p) value of -1800, the other lenses had a (p) value ranging between -25 to -500.

d) It is apparent from the results given in Table 6.16., that the lenses (Tables 6.4. and 6.5.), which were constructed in exactly the same way, have the same (p) value of -0.25.

e) it is apparent that by increasing the pressure on the fluid causing larger deformation on the front surface of these lenses, an increase of the (p) value of these surfaces occurs, but in an arbitrary way. (This statement is made after excluding the lens presented in Table 6.1., where the large value of (p) does not probably represent the actual type of deformation that would possibly occur for such an addition)

Summarising the above, it can be concluded that the deformation occuring on one of the surfaces of a deformable lens, when pressure is exerted, resembles a hyperboloidal surface having (p) values with quite a wide range (from -25 to -500.) The reason, that such a wide range of (p) values is presented, is probably do to the fact that the lenses constructed had different effective apertures, a front surface of different thickness, and finally different heights for the annular spacer. For the lens where the (p) value falls far outside this range a manufacturing error is possible.

### Table 6.16.

Deformable lens surface (p) value, according to mean power

Table No		Mean power		(p) value
6.1.	;	1.125	:	-1800
6.2.	:	1.75	1	-440
6.3.	:	1.375	;	-100
6.4.	1	1.625	:	-25
6.5.	1	1.625	:	-25
6.6.	:	1.625	:	-60
6.7.	;	2.125	:	-110
6.8.	:	2.875		-70
6.9.	:	3.00		-200
6.10.	1	1.50	:	-500
6.11.		2.375	:	-70
6.12.	:	2.25	:	-260
6.13.	:	1.75	:	-440
6.14.	:	1.75	•	-180

#### 6.6. Assessment of the optical performance of deformable lenses

This part of the project concerns the assessment of the optical performance of the deformable lenses constructed after following the method described in Chapter 6.. Although their dioptric behavior has been estimated, no information was given on how these lenses will perform infront of a presbyopic eye and what would be their actual optical effects.

It is well known that as the eye rotates around its axis, viewing points away from the optical centre of a conventional lens, especially such a lens has a high power, optical aberrations appear, which affect the performance of the lens. Figure 6.21. illustrates the rotation of the eye away from the optical axis of the lens placed infront of it, where the optical correction presented to the eye at the vertex (V) of the lens is not the same as the one at a point (C), which is at a distance (h) from the vertex (V). These aberrations usually cause problems for eye rotations of more than 20°.

Using the ray trace computer scheme given in Appendix  $D_3$ , the actual optical performance of the deformable lenses was caclulated for rotating angles of the eye ranging between 0° to 35° degrees, and assuming that the lenses are at a distance of 27 mm away from the centre of the eye rotation. The centre thickness is assumed to be 4 mm for all the deformable lenses. The resulting graphical representations of the optical performance of the deformable lenses 6.1. to 6.14.) are given in Figures

260



Figure 6.21. The eye is rotated at an angle (A) viewing through the point (P). The optical correction at (P) is not the same as at the point (V) at the vertex of the lens. 6.22. to 6.33.. These diagrams illustate the astigmatism presented to the eye, when it is viewing at an angle to the optic axis of the lens placed in front of it. These diagrams are based on the assumptions previously stated and the information that they present related to the maximum astigmatism that such lens-systems will present to the eye has as follows

a) for the first lens (Table 6.1.), which has a mean power at its optical centre of +1.125 dioptres and a front surface with a (p) value of -1800, the astigmatism presented to the eye at different angles is illustrated in Figure 6.22. According to the graph the maximum astigmatism, presented by the lens-system at an angle of  $35^{\circ}$  away from its optic axis, has a value of -0.401 b) the lens of Table 6.2., with a mean power at its optical centre of +1.75 dioptres and a (p) value for the front aspheric surface of -440, presents to the eye, again at an angle of  $35^{\circ}$  degrees away from the optical centre, -0.6 dioptres astigmatism as Figure 6.23. shows

c) the lens of Table 6.3., with a mean power at its optival centre of +1.325 dioptres and a (p) value for the front aspheric surface of -100, presents to the eye, at an agnle of 35° degrees away from the optical centre, +0.122 dioptres astigmatism as Figure 6.24. shows

d) for the lenses of Tables 6.4. and 6.5., with a mean power at their optical centre of +1.625 dioptres and a (p) value for the front aspheric surface of -25, the astigmatism, presented to the eye at an angle of 35° degrees away from the optical centre, has a

value of +0.478 dioptres as Figures 6.25. shows

e) the lens of Table 6.6., with a mean power at its optical centre of +1.625

dioptres and a (p) value for the front aspheric surface of -60, presents to the eye, at an angle of  $35^{\circ}$  degrees away from the optical centre, +0.205 dioptres as Figure 6.26. shows

f) the lens of Table 6.7., with a mean power at its optical centre of +2.125 dioptres and a (p) value for the front aspheric surface of -110, presents a maximum astigmatism, at 35° degrees away from the optical centre, of -0.382 as Figure 6.27. shows

g) the lens of Table 6.8., with a mean power at its optical centre of +2.875 dioptres and

a (p) value front its front aspheric surface of -70, presents to the eye an astigmatism of -0.633 dioptres, at an angle of 35° degrees away from the optical centre of the lens as Figure 6.28. shows

k) the lens of Table 6.9., with a mean power of +3.00 dioptres at its optical centre and a (p) value for the front aspheric surface of -200, presents to the eye at an angle of 35° degrees away from the optical centre, an astigmatism of -1.065 dioptres as Figure 6.29. shows

I) the lens of Table 6.10., with a mean power at the optical centre of +1.50 dioptres and a (p) value for the front aspheric surface of -500, presents to the eye an astigmatism of -0.49 dioptres as Figure 6.30. shows, at an angle of  $35^{\circ}$  degrees away from the optical centre

m) the lens of Table 6.11., with a mean power at its optical centre of +2.375 dioptres and a (p) value for the front aspheric surface of -70, presents to the eye, at an anlge of 35° degrees away from the optical centre, an astigmatism of -0.297 dioptres as Figure 6.31. shows

n) the lens of Table 6.12., with a mean power at its optical centre of +2.25 dioptres and a (p) value for the front aspheric surface of -260, presents to the eye, at an agnle of 35° degrees away from the optical centre, an astigmatism of -0.768 dioptres as Figure 6.32. shows

o) the lens of Table 6.13., with a mean power of +1.75 dioptres at its optical centre and a (p) value for the front aspheric surface of -440, presents to the eye, at an angle of  $35^{\circ}$  degrees away from the optical centre, an astigmatism of -0.6 dioptres as Figure 6.23. shows

p) The last lens of Table 6.14., with a mean power at its optical centre of +1.75 dioptres and a (p) value for thr front aspheric surface of -180, presents to the eye, at an angle of  $35^{\circ}$  degrees away from the optical centre, an astigmatism of -0.353 dioptres as Figure 6.33. shows

In all the above mentioned lenses the second surface of the lens-cell was considered to be flat, in the ray trace computer scheme, having a power F = -0.001 dioptres, and the refractive index was taken to be n = 1.5.

It is clear that for all the above mentioned lenses, the astigmatism presented to the eye, when the lenses are placed at a distance of 27 mm away from the centre of the eye rotation, is always less than the one measured with the focimeter. Especially, in the case of deformable lenses where the maximum addition obtained was 2.00 to 3.50 dioptres, the astigmatism presented was nearly half than that measured with the focimeter. For the lenses, which had a mean power of +1.625 dioptres at their optical centre and a (p) value ranging between -25 to -60, the value of astigmatism presented to the eye

was nearly the same but of the opposite sign in contrast to the one measured with the focimeter. In both lenses the astigmatism presented was minimum. The maximum value of astigmatism, calculated by using the computer scheme in Appendix  $D_3$ , at angle of  $35^\circ$  degrees away from the optic axis, was found to correspond to the maximum addition obtained by the lens of Table 6.9.. The mean power of that lens, at the optical centre, was +3.00 dioptres and the astigmatism -1.065. The conclusion drawn from these result indicates that by increasing the addition presented by deformable lenses, an increase of the astigmatism presented to the eye will occur, but of less value than the one actually measured with the focimeter. This was something that it was expected. The value of asphericity of the deformable lenses mentioned previously indicated that such lenses will perform in such a way. That is a characteristic that aspheric surfaces have and the advandage they have compared to conventional spherical surfaces.

It has to be admitted that the amount of astigmatism presented to a presbyopic eye, when a deformable lens is placed in front of it at a certain distance from the eye rotation centre, was found to be within acceptable limits, especially for low additions (up to 1.75 dioptres). When higher addition were obtained the astigmatism was increased but again within the acceptable limits. it should be made clear that this conclusion is made after the results obtained, using the ray trace computer scheme and after using the approximations mentioned previously and especially using an optimum thickness of 4 mm for each deformable lens, which is much less than the real

265

one. In reality, these lenses will perform slightly different than the calculated results, due to the approximations made. Assuming that the results obtained by using the ray trace computer scheme is what realy happens in practice, deformable lenses can be considered to have a fairly good optical performance and would not present major problems to the wearer.

Figures 6.22. to 6.33. illustrate the optical performance of the deformable lenses constructed for the purpose of this experimental work. The above mentioned Figures present the sagittal and tangential powers error and the astigmatism presented to the eye at different viewing angles. The results obtained are theoretical assumptions due to the approximations made (centre thickness of these lenses 4 mm and distance that the lens is placed from the centre of the eye rotation 27 mm).



Figure 6.22. The optical performance of the lens of Table 6.1.







Figure 6.24. The optical performance of the lens of Table 6.3.



Figure 6.25. The optical performance of the lenses of Tables 6.4. and 6.5.



Figure 6.26. The optical performance of the lens of Table 6.6.



Figure 6.27. The optical performance of the lens of Table 6.7.



Figure 6.28. The optical performance of the lens of Table 6.8.



Figure 6.29. The optical performance of the lens of Table 6.9.



B.V.P: 1.5 F2: -.001 t(mm): 4 n: 1.5

Figure 6.30. The optical performance of the lens of Table 6.10.



6.31. The optical performance of the lens of Table 6.11.



Figure 6.32. The optical performance of the lens of Table 6.12.





## **CHAPTER 7**

SUMMARY AND COMPARISON OF THE THREE RESEARCHED TYPES OF LENS. RECOMENDATIONS FOR THE FUTURE.

#### 7.1. Summary

Summarising the results of the research for the three types of ophthalmic lenses, proposed to be used in presbyopia correction, it can be concluded that for

#### a) the liquid crystal lenses

The most important factor for their ideal performance is the selection of a suitable nematic material, having a large birefringence in order to achieve the maximum addition as possible. The other serious factor in the performance of liquid crystal lenses is the thickness that the nematic layer should have. The proposed solutions to the problem are on a theoretical basis and it has not yet been proved experimentally how much these will improve their performance. Besides the above the most serious factor is the one related to the refractive index temperature dependence of the nematic material. These materials are very sensitive to even minor changes of the environmental temperature. The solution proposed by Okada et al (1986a) seems to be satisfactory in overcoming such a problem.

#### b) the gradient-index lens

This lens is actually a new proposed structure and is based upon the theory

related to gradient-index optics. Its constructive idea is more specifically based on Wood's (1905) method, modified for this purpose. It is a diffusion process where a gradient is produced at the lower half portion of the lens. The data collected after constructing such lenses proved that the initial objective set, of creating a lens with increased spherical power resulting in a maximum addition, was successful. But unfortunately, this increase of the spherical power was accompanied by the presence of a cylindrical power also increasing with the same or in some lenses even twice the rate of the spherical power.

The presence and additionally the increase of the cylindrical power renders this type of lens unacceptable, according to the standards set. Besides the above, the reproducibility problem presented by these lenses was apparent from the experimental results obtained. The most serious drawback of gradient-index lenses created after following a modified Wood's method was related to the gradient produced. This has been proven, experimentally, to be unstable and not permanent, probably due to the nature of the materials used.

#### c) the deformable lenses

This type of lens was proven to be able to provide the necessary additions, covering the range of presbyopic patients. The maximum addition obtained, after deforming one of the lens surfaces was +3.00 dioptres. Such lenses preferably to have only one surface flexible, while the other is a rigid one

providing a basis for incorporating any distance correction needed, if necessary. It has also been found that they actually perform quite satisfactorily, when their effective aperture is large, ranging from 30 to 65 mm. According to the data obtained, the deformable lenses present an amount of astigmatism through their whole effective aperture, which increased going towards the lens periphery. This finding indicated that the front flexible surface of the lens-cell constructed deform aspherically, and more precisely, the deformed surface resembles a hyperboloidal surface.

By using a ray trace computer scheme, the optical performance of such lenses was assessed, when these deformable lenses wer placed infront of a presbyopic eye rotating about the optical axis at a maximum of 35° degrees, using steps of 5° degrees. Approximations have been made on the lens-cell thickness and the distance that the lens-cell was placed in front of the eye. The calculated astigmatism was found to be less or much less, in lenses with high addition, than the one measured with the focimeter, which was expected due to the degree of asphericity the lenses present. The major drawbacks of such a lens type lie on the poor cementing of its constituent parts and on the large response and recovery times of the fluid used.

#### 7.2. Comparison of the three researched types of lens

A comparison has been attempted for the three types of ophthalmic lenses researched in this project and conclusions are made on which of them provides the best solution in presbyopia correction. Observing the summary of each type, it can be stated that the liquid crystal and deformable lenses have the advantage compared with the gradient-index varifocals constructed, on the fact that they provide a wider field of view to the presbyopic patient. Actually, the entire aperture of the lens, in both cases, is used to *provide* the correction for distance, near and intermediate vision. In contrast, the gradient-index lenses have their distance, intermediate and near corrections confined to smaller regions on the lens surface, resembling the existing varifocals (progressive addition lenses).

The construction of the gradient-index lenses seems more straightforward than the construction of liquid crystal and deformable lenses. The latter have a very complicated construction and require a very sophisticated technique. Their technique will also be time consuming and probably more expensive to the manufacturer than the varifocals which have already gained commercial success. Additionally, it is hard to envisage a presbyope wearing one of these type of lens, due to the fact that these lenses, are accompanied by an auxiliary unit, which either provides the voltage applied or the pressure upon the fluid, respectively. The whole configuration, in both cases, would apear as an alien device, aesthetically unacceptable according to the standards set nowadays.

283

The other problem related to the two aforementioned types, is that it appears difficult or even impossible for the wearer to estimate, which of the predetermined focal lengths should be utilized, in order to focus clearly objects at different distances. Such a problem might confuse the presbyope, due to the inconvenient manual operation of the auxiliary unit, which in both cases alters the lens dioptric power, and finally may result in the rejection of such lenses. The sudden change of the liquid crystal lens appearance, when a different voltage is applied to the nematic layer, or the loss of transparency for a certain amount of time, due to the large viscosity of glycerol in deformable lenses, might probably add more confusion to the wearer. It has to be admitted, that all the three types are thicker and heavier than the lenses currently used in presbyopia correction. This factor might also prove to be a great disadvantage if any of these lenses were considered to be launched in the market.

Commenting on their optical performance, it is apparent that a comparison can be attempted only for the gradient-index and deformable lenses, due to the fact that only for these two types experimental data has been collected. For the liquid crystal lenses, no evidence exists about their dioptric behavior and only theoretical speculation can be provided for their optical performance. According to the experimental results, the deformable lens apears to have a much better optical performance to that of the gradient-index. The latter presents a substantial amount of unwanted astigmatism, which reaches up to very high powers near the edges of the lens in the lower half portion. This astigmatism will affect the wearer's vision resulting in the possible rejection of the lens. Apart from that, the maximum addition obtained with the gradient-index lens was +2.75 dioptres, which is affected by the large cylindrical component presented in every point on the lower half portion of them. The deformable lenses cover successfully the range of necessary additions in presbyopia correction, reaching up to +3.00 dioptres without major problems, related to vision, providing steps of 0.25 dioptres and can be easily and accurately controlled. By contrast the addition presented by the gradient-index lens is quite arbitrary and steps of 0.25 dioptres cannot be accurately obtained with such a method of construction.

Generally, the conclusions reached for the three ophthalmic lenses researched, can be summarised as follows. The liquid crystal lens provides a solution to presbyopia correction, but due to the currently existing problems (theoretically assessed) its usage remains limited. Certainly, the future seems to be very promising and such a lens can be considered as one of the possible spectacle lenses of the future to replace existing varifocals. The gradient-index lens, investigated in this project, did not present the desired results expected and its usage is severely limited, due to the major drawbacks it presents, especially the one related to the gradient produced, where the index distribution pattern is not permanent. The third type of ophthalmic lens, the deformable, has a quite satisfactory optical performance and could provide a solution to a presbyope. The problem related to its construction and its inconvenient operation, impose limits to its utilization and demote its advantages.

#### 7.3. Recommendations for the future

Although, at the present time, the usage of the three types of ophthalmic lenses researched appears to be limited, future research and improvements can be applied to them.

For the liquid crystal lens, further research should be carried out on selecting the most suitable nematic material, which will give the desired results. In addition its dioptric behavior has to be assessed with accura cy, in order to provide sufficient evidence on its suitability for presbyopia correction. With the advance of technology, the problem of the nematic material index temperature dependence should be solved and its bulky appearance should be reduced, in order to be practically and aesthetically acceptable. Its usage should not be limited in ophthalmic lenses and alternatively can be used as a mechanical diaphragm in other optical instruments like cameras. In the future a liquid crystal lens also might be considered in replacing the human crystalline lens in cases such as cataract as an implant. This might sound odd, but with the advance of science, an application such as the one mentioned above might become true. In the case where these lenses are intended to be used as multifocals, a system, which would provide an autofocus on objects at different distances might proved to be essential for their successful use.

The gradient-index lens although unsuitable as a multifocal lens in the form constructed here, should benefit from research on better materials, which will provide gradients of refractive index more permanent and unaffected by external factors (environmental changes). Such a development, especially in plastic materials might provide a lens with a much better optical performance than the one investigated in this project.

For the last type, the deformable lens, improvements can be made in the future providing a better cementing process is used for the constituent parts of the lens-cell. Such a develop m ent will increase the useful life of the lens, after a cycle of use and probably increase the value of the maximum addition obtained. Besides the above, the selection of a more suitable fluid might reduce its drawbacks to a minimum. Additionally, as in liquid crystal lenses, an autofocusing system would exceptionally improve the overall performance of such a lens.
## APPENDIX A1

The changes in the colour of cholesteric liquid crystals with changes in temperature

Table 1.1

Human body	r temperature	36.9° C
Colorless	red	39.5° C
Red	_ orange	39.7° C
Orange	yellow	40.0° C
Yellow	green	40.2° C
Green	cyan	40.6° C
Cyan	blue	41.6° C
Blue	violet	42.0° C
Violet	colorless	43.0° C
	•	

According to Frank (1958), the principle types of deformation in a nematic liquid crystal are a) the splay, b) the twist, c) the bend. If the preferred orientation of the liquid crystal molecules is changed under the influence of an electric or magnetic field then one or two or all of the above deformations will take place depending on the material.

The constants related to each of the above deformations are a)  $K_{11}$  for splay, b)  $K_{22}$  for twist and c)  $K_{33}$  for bend. The constant K used in the equation of the critical electric field  $E_c$ , is an average constant for bend and splay and depends on the nature of the nematic material.

## APPENDIX A3

An example is given of the problem related to the thickness of the liquid crystal lens and its effective power change. If a lens such as the one in Figure A<sub>3</sub> 1. is produced with a central thickness of 25  $\mu$ m, this means that the sag of the lens will be also 25  $\mu$ m. The radius of curvature for such a lens will be

Approximate formula of sag 
$$s = y^2 / 2r_2 \supset r_2 = y^2 / 2s$$
 (1)

where  $r_2$  is the radius of curvature of the second surface 2, s is the segment of the lens-cell and 2y is the diameter of the lens. Supposing that this lens has a diameter of 40 mm, then y = 20 mm. According to equation (1) the radius of curvature of such a lens will be  $r_2 = 8$  m. The power of the second syrface of the lens-cell  $F_2$  is given by the equation

$$F_2 = n_0 - n' / r_2$$
 (2)

where  $n_0$  is the ordinary refractive index of the nematic layer, n' is the refractive index of glass. If now M.B.B.A. is used as the nematic layer then  $F_2 = +2.75 \times 10^{-3}$  D., which is insignificant. The power  $F_2$  for the extraordinary index  $n_e$ , presented by the layer when the field is applied

$$F_2 = n_e - n' / r_2$$
 (3)

will be  $F_2 = +0.027$  D., which is also insignificant. It is clear that with such a small thickness the effective power change is not of any use.



- (1) is the refracting surface with constant power  $F_1$
- (2) is the refracting surface with the variable power  $F_2$
- (3) is the nematic layer of M.B.B.A.
- (4) is a flat surface with zero power  $F_3$

Figure 2.26. An exemplary liquid crystal lens cell.

## **APPENDIX B1**

## Table 3.1

Angle A°	Sagittal Power	Tangential Power	S Error	T Error	Astigmatism Power
5	3.00	3.00	-0.00	0.00	0.01
10	2.99	3.01	-0.01	0.02	0.02
15	2.98	3.03	-0.02	0.04	0.05
20	2.97	3.06	-0.03	0.06	0.09
25	2.95	3.08	-0.05	0.09	0.14
30	2.92	3.11	-0.09	0.09	0.19
35	2.88	3.12	-0.12	0.12	0.24

Index = 1.56 Centre of rotation distance = 28.5 mm Axial power = + 3.00 Dioptres.

## Table 3.2

Angle A°	Sagittal Power	Tangential Power	S Error	T Error	Astigmatism Power
5	3.02	3.02	0.02	0.03	0.00
10	3.04	3.04	0.04	0.05	0.01
15	3.05	3.06	0.05	0.06	0.01
20	3.04	3.06	0.04	0.06	0.02
25	3.03	3.05	0.03	0.05	0.02
30	3.00	. 3.02	-0.00	0.02	0.02
35	2.96	2.97	-0.04	-0.03	0.01

Index = 1.56Index step = 0.005Centre of rotation distance = 28.5 mm Axial power = + 3.00 Dioptres. In order to understand the equations of Marchand (1976) relating to the transverse and longitudial spherical aberrations, some explanation must be given about the notation used. The equations which will be useful are

 $b = (2 | N_1 | N_0)^{1/2}$   $c = \cos [(b / N_0) d]$   $s = \sin [(b / N_0) d]$  $N' = -[(2 N_2 / N_1) + (N_1 / N_0)]$ 

 $N = \{ (N_1 b d / N_0^2 c s) - 1/2 f^2 + (1/8)(N') [2c^2 + 3(1 + (b d / N_0 c s))] \}$ 

a) in the transverse spherical aberration equation  $S_T = c h^3 N$  the notation used can be explained by the above equations, where d is the thickness of the lens, h is the height of the ray (p) from the optic axis z, when the ray enters the lens front surface.

b) in the longitudial spherical aberration equation  $S_L = f' h^2 N$ , where N is given by the equations above, h is the height of the ray (p) from the optic axis z, when the ray enters the front surface of the lens, and f' is the distance of the focal plane from the second surface of the lens.

## APPENDIX C1

Table 4.1. illustrates the dioptric behavior of 24 lens-disks of type (A). These lenses have a 12 mm diameter and had undergone the diffusion process in glycerol, which replaced the water in these lenses, for 20 minutes.

Table 4.1.

	Dioptric pow	ver			Thickness in (mm)
1)	-1.00	-0.50	-0.50	-0.75	2
2)	-1.50	-1.00	-1.25	-1.00	2.5
3)	-1.75	-1.25	-1.25	-1.50	3
4)	-1.75	-1.75	-1.50		3.5
5)	-2.00		-1.75	-2.00	4
6)	-2.50	-2.00	-2.00	-2.25	5
	Rod 1	Rod 2	Rod 3	Rod 4	

Measurements of the lens disks are arranged in columns to show which lenses were produced from each rod. Where there is a missing measurement this indicates that the lens power could not be read on the NIDEK electronic focimeter.

Table 4.2. illustrates the dioptric behavior of 24 lens-disks of type (B). These lenses have a 12 mm diameter and had undergone the diffusion process in water, which replaced the glycerol in these lenses, for 20 minutes.

			Table 4	.2.	
	Dioptric p	ower			Thickness in (mm)
1)	+0.75	-0.25	+0.50	+0.25	2
2)	+1.00	+0.25	+0.50	+0.75	2.5
3)	+1.25	+1.00	+0.75	+1.25	3
4)	+1.50		+1.50		3.5
5)	+1.75		+1.50	+1.50	4
6)		+1.00		+1.75	5
	Rod 1	Rod 2	Rod 3	Rod 4	

Measurements of the lens disks are arranged in columns to show which lenses were produced from each rod. Where there is a missing measurement this indicates that the lens power could not be read on the NIDEK electronic focimeter.

## APPENDIX C2

Table 4.3. illustrates the dioptric behavior of 18 lens-disks of type (A). These lenses have a 12 mm diameter and had undergone the diffusion process in glycerol, which replaced the water in these lenses, for 40 minutes.

## Table 4.3

Diopt	ric power		Thickness in (mm)
1) -2.50	-2.25	-1.75	2
2) -3.00	-2.50	-2.25	2.5
3) -3.25	-3.00	-2.75	3
4) -3.75	-3.25	-3.50	3.5
5) -4.00	-3.75	-3.50	4
6)	-4.00	-4.50	5
Rod 1	Rod 2	Rod 3	

Measurements of the lens disks are arranged in columns to show which lenses were produced from each rod. Where there is a missing measurement this indicates that the lens power could not be read on the NIDEK electronic focimeter.

Table 4.4. illustrates the dioptric behavior of 18 lens-disks of type (B). These lenses have a 12 mm diameter and had undergone the diffusion process in water, which replaced the glycerol in these lenses, for 40 minutes.

### Table 4.4.

	Dioptric b	ehavior		Thickness in (mm)
1)	+1.75	+1.25	+1.50	2
2)	+1.75	+2.25	+1.75	2.5
3)	+2.00	+2.25	+2.00	3
4)	+2.25	+2.75	+2.50	3.5
5)		+2.75	+2.75	4
6)	+2.75		+2.75	5
	Rod 1	Rod 2	Rod 3	

Measurements of the lens disks are arranged in columns to show which lenses were produced from each lens. Where there is a missing measurement this indicates that the lens power could not be read on the NIDEK electronic focimeter. Table 4.5. illustrates the dioptric behavior of 18 lens-disks of type (A'). Theses lenses have a 22 mm diameter and had undergone the diffusion process in glycerol, which replaced the water in these lenses, for one and a half hours.

			Table 4.5.	
	Dioptric po	ower		Thickness in (mm)
1)	-2.75	-3.25	-2.50	4
2)	-3.00	-3.50	-3.50	4.5
3)	-4.00	-4.50	-3.50	5
4)	-4.50	-5.25	-4.25	5.5
5)	-5.00	-5.00	-4.75	6
6)	-5.50		-5.25	7
	Rod 1	Rod 2	Rod 2	

Measurements of the lens disks are arranged in columns to show which lenses were produced from each rod. Where there is a missing measurement this indicates that the lens power could not be read on the NIDEK electronic focimeter.

Table 4.6. illustrates the dioptric behavior of 18 lens-disks of type (B'). These lenses have a 22 mm diameter and had undergone the diffusion process in water, which replaced the glycerol in these lenses, for one and a half hours.

			Table 4.6	
	Dioptric pow	<u>/er</u>		Thickness in (mm).
1)	+1.75	+2.00	+1.75	4
2)	+2.25	+2.50	+1.75	4.5
3)	+2.25	+2.75	+2.25	5
4)		+2.75	+2.50	5.5
5)	+2.50			6
6)		+3.00	+2.50	7
	Rod 1	Rod 2	Rod 3	

Measurements of the lens disks are arranged in columns to show which lenses were produced from each rod. Where there is a missing measurement this indicates that the lens power could not be read on the NIDEK electronic focimeter.

# APPENDIX C4

## Table 4.7.

Gradient-index varifocal lens with 75 mm diameter, diffusion time in water two hours and 15 mm thickness

#### Dioptric power

1 1) +1.00 DS / -1.25 DC x 65° 2) +1.25 DS / -2.25 DC x 65° 3) 0.00 DS / -1.75 DC x 75° 4) -1.00 DS / -3.50 DC x 105° 5) -1.00 DS / -5.00 DC x 120° 6) 7) 3)	2 +0.50 DS / -0.75 DC × 50° +1.50 DS / -1.75 DC × 45° +2.25 DS / -3.00 DC × 45° +1.75 DS / -2.25 DC × 45° +0.50 DS / -1.00 DC × 135° +0.50 DS / -3.75 DC × 135°	3 -0.25 DS +0.25 DS / -0.50 DC × 45° +1.25 DS / -1.25 DC × 30° +2.25 DS / -2.00 DC × 35° +2.75 DS / -2.00 DC × 45° +1.00 DS / -1.25 DC × 135° 0.00 DS / -4.00 DC × 145° +0.25 DS / -4.75 DC × 145°
4 1) -0.25 DS 2) 0.00 DS 3) 0.00 DS 4) +1.00 DS / -1.00 DC × 55° 5) +2.50 DS / -2.00 DC × 55° 6) +2.25 DS / -1.50 DC × 65° 7) +0.50 DS / -1.25 DC × 155° 8) +0.50 DS / -2.25 DC × 155° 9) +0.25 DS / -3.25 DC × 160°	5 -0.25 DS -0.25 DS 0.00 DS +0.50 DS / -0.50 DC x 70° +1.50 DS / -1.25 DC x 70° +2.50 DS / -2.00 DC x 75° +0.75 DS / -1.00 DC x 180° +0.50 DS / -3.25 DC x170°	<u>6</u> -0.25 DS -0.25 DS -0.25 DS -0.25 DS 0.00 DS +1.25 DS / -1.25 DC × 100° +2.50 DS / -1.75 DC × 100° +1.00 DS / -0.75 DC × 180° +1.00 DS / -3.00 DC × 180° +0.75 DS / -4.25 DC × 180°
Z 1) -0.25 DS 2) 0.00 DS / -0.25 DC x 140° 3)+0.25 DS / -0.50 DC x 140° 4)+0.75 DS / -1.00 DC x 140° 5)+2.25 DS / -2.00 DC x 125° 6)+2.75 DS / -2.00 DC x 120° 7)+1.00 DS / -0.50 DC x 10° 8)+1.00 DS / -3.00 DC x 10°	8 -0.25 DS +0.50 DS / -0.75 DC x 145° +1.00 DS / -1.25 DC x 145° +2.25 DS / -2.25 DC x 145° +3.00 DS / -2.50 DC x 140° +1.00 DS / -0.75 DC x 10° +0.75 DS / -2.00 DC x 25°	9 +0.25 DS / -0.75 DC × 135° +1.75 DS / -2.25 DC × 140° +2.75 DS / -3.25 DC × 145° +2.75 DS / -3.25 DC × 145° +0.50 DS / -1.00 DC × 55° +0.50 DS / -2.75 DC × 40°
<u>10</u> 1) +1.00 DS / -1.50 DC x 2) +2.00 DS / -3.00 DC x 3) +0.50 DS / -2.50 DC x 4) +0.75 DS / -3.00 DC x 5) +0.75 DS / -3.50 DC x	120° 125° 125° 75° 60°	11 +0.75 DS / -1.50 DC x 125° +1.00 DS / - 2.00 DC x 125° 0.00 DS / - 2.50 DC x 120° - 1.00 DS / - 3.00 DC x 80° - 1.00 DS / - 3.50 DC x 80°

## Table 4.8.

Gradient-index varifocal lens with 75 mm diameter, diffusion time in water two hours, and thickness 15 mm.

#### Dioptric power

1 1) +0.75 DS / -1.75 DC x 80° 2) +1.00 DS / -3.25 DC x 95° 3) -1.00 DS / -1.25 DC x 90° 4) -0.75 DS / -1.75 DC x 70° 5) -1.00 DS / -2.00 DC x 60° 6) 7)	2 +0.75 DS / -1.25 DC x 103° +1.25 DS / -2.25 DC x 103° +0.50 DS / -1.75 DC x 10° - 0.25 DS / -1.00 DC x 72° - 0.50 DS / -1.75 DC x 55° - 0.50 DS / -2.50 DC x 35°	3 +0.75 DS / -1.00 DC x 120° +1.00 DS / -1.50 DC x 122° +1.25 DS / -1.50 DC x 120° +0.50 DS / -0.75 DC x 85° +0.25 DS / -1.25 DC x 45° 0.00 DS / -2.25 DC x 35° - 0.25 DS / - 3.00 DC x 34°
<u>4</u> 1) +0.50 DS / -0.75 DC x 120° 2) +1.00 DS / -1.25 DC x 123° 3) +1.50 DS / -1.50 DC x 123° 4) +1.25 DS / -0.75 DC x 118° 5) +0.50 DS / -0.50 DC x 118° 6) - 1.50 DS / -1.00 DC x 115° 7) 0.00 DS / -2.50 DC x 110° 8) 9)	5 0.00 DS +0.50 DS / -0.50 DC x 125° +1.25 DS / -0.75 DC x 110° +1.50 DS / -0.75 DC x 100° +0.75 DS / -0.50 DC x 15° +0.50 DS / -1.50 DC x 10 0.00 DS / - 2.50 DC x 10° -0.25 DS / - 3.00 DC x 10°	<u>6</u> 0.00 DS +0.50 DS / -0.50 DC x 60° +1.25 DS / -0.75 DC x 70° +1.50 DS / -0.50 DC x 75° +1.00 DS / -0.50 DC x 165° +0.75 DS / -1.75 DC x 170° +0.50 DS / -1.75 DC x 170° 0.00 DS / -2.25 DC x 170°
<u>Z</u> 1) +0.50 DS / -0.75 DC x 35° 2) +1.00 DS / -1.00 DC x 35° 3) +1.25 DS / -1.00 DC x 50° 4)+ 1.25 DS / -0.75 DC x 60° 5) +0.75 DS / -1.25 DC x 150° 6) +0.25 DS / -1.75 DC x 155° 7) 0.00 DS / -2.50 DC x 160°	8 +0.75 DS/ -1.00 DC x 55° +1.00 DS / -0.75 DC x 55° +1.00 DS / -0.75 DC x 55° +0.50 DS / -0.50 DC x 80° +0.25 DS / -1.50 DC x 135° 0.00 DS / -2.50 DC x 145°	9 +0.50 DS / -1.00 DC x 55° +0.75 DS / -1.25 DC x 65° -1.00 DS / - 1.50 DC x 157° +0.75 DS / -1.00 DC x 105° -0.25 DS / -1.50 DC x 125°
10 1) 10 50 DS ( 1 75 DC x	1009	11

1) +0.50 DS / -1.75 DC x 100° 2) +1.25 DS / -2.50 DC x 90° 3) +0.50 DS / -1.00 DC x 105° 4) - 0.25 DS / -1.50 DC x 110° 5) - 0.25 DS / -2.25 DC x 110°	+0.75 DS /-1.50 DC x 100° +0.75 DS / -2.00 DC x 105° - 0.50 DS / -1.25 DC x 110° - 0.75 DS / -1.50 DC x 115°
$(-0.25 \text{ US} / -2.25 \text{ UC} \times 110^{\circ})$	

### Table 4.9.

Gradient-index varifocal lens with 75 mm diameter, difusion time in water two hours, and thickness 18 mm

1) - 0.25 DS / -0.50 DC x 45°

2) - 0.25 DS / -0.75 DC x 40°

3) +0.50 DS / -1.00 DC x 36°

4) +1.50 DS / -1.50 DC x 34°

5) +2.25 DS / -2.00 DC x 42°

6) +1.00 DS / -1.25 DC x 136° 7) +0.25 DS / -0.50 DC x 139°

8) +0.25 DS / -2.00 DC x 145°

1) - 0.25 DS / -0.25 DC x 110°

2) - 0.25 DS / -0.50 DC x 122°

3)+0.25 DS / -0.75 DC x 128°

4)+1.00 DS / -1.25 DC x 129°

5)+1.75 DS / -1.50 DC x 135°

6)+2.00 DS / -2.25 DC x 125°

7)+1.00 DS / -1.75 DC x 30°

8)+0.75 DS / -3.00 DC x 25°

7

#### Dioptric power

1	2	3
1) +1.00 DS / -1.50 DC x 75°	+0.50 DS / -1.00 DC x 60°	+0.25 DS / -0.50 DC x 50°
2) +1.00 DS / -2.50 DC x 78°	+0.50 DS / -1.50 DC x 55°	+0.75 DS / -1.25 DC x 50°
3) +0.25 DS / -3.00 DC x 85°	+2.00 DS / -2.50 DC x 52°	+1.50 DS / -1.75 DC x 46°
4) - 0.50 DS / -3.75 DC x 68°	+1.00 DS / -1.25 DC x 50°	+2.00 DS / -1.75 DC x 40°
5) - 0.75 DS / -4.00 DC x 58°	+0.25 DS / -2.00 DC x 125°	+0.75 DS / -0.75 DC x 120°
6)	- 0.25 DS / -2.75 DC x 129°	+0.25 DS / -1.50 DC x 125°
7)		- 0.25 DS / -2.75 DC x 140°

5

- 0.25 DS / -0.50 DC x 80°

+ 0.50 DS / -0.75 DC x 94°

+ 0.75 DS / -0.75 DC x 90°

+1.25 DS / -1.00 DC x 75°

+1.75 DS / -2.00 DC x 160°

+0.75 DS / -1.50 DC x 150°

+0.50 DS / -2.50 DC x 157°

0.00 DS ( -1.00 DC x 140°

+0.75 DS / -1.50 DC x 130°

+1.50 DS / -1.75 DC x 150°

+2.00 DS / -2.25 DC x 150°

+2.50 DS / -2.75 DC x 30°

+1.00 DS / -3.00 DC x 35°

+1.00 DS / -3.50 DC x 15°

8

0.00 DS / -0.75 DC x 82°

6 0.00 DS / -0.25 DC x 95° 0.00 DS / -0.25 DC x 97° - 0.25 DS / -0.50 DC x 102° +0.75 DS / -0.50 DC x 110° +1.50 DS / -0.75 DC x 115° +2.25 DS / -1.50 DC x 170° +0.75 DS / -1.75 DC x 172° +0.25 DS / -2.75 DC x 179°

9	
+0.50 DS / -1.25	5 DC x 132°
+0.75 DS / -2.00	DC x 120°
+1.50 DS / -2.25	5 DC x 130°
+2.25 DS / -3.00	DC x 134°
+2.25 DS / -3.25	5 DC x 50°
+0.50 DS / -3.50	DC x 40°

10	11
1) $+0.75 \text{ DS}/-1.00 \text{ DC} \times 140^{\circ}$	+1.00 DS / -1.25 DC x 110°
2) +1.50 DS / -2.00 DC x 123°	+1.25 DS / -2.50 DC x 110°
3) +2.25 DS / -3.50 DC x 120°	+0.50 DS / -3.00 DC x 106°
4) +0.50 DS / -3.50 DC x 43°	- 1.00 DS / -3.00 DC x 70°
5) 0.00 DS / - 4.00 DC x 14°	- 1.25 DS / -3.50 DC x 65°

#### Table 4.10.

Gradient-index varifocal lens with 75 mm diameter, difusion time in water one and a half hours, and thickness 15 mm

2

#### Dioptric power

1 1)+1.00 DS / -2.75 DC × 55° 2) 0.00 DS / -2.25 DC × 50° 3) - 0.50 DS / -2.50 DC × 95° 4) 0.00 DS / -3.00 DC × 110° 5) 0.00 DS / -4.50 DC × 110° 6) 7)	2 +2.00 DS / -3.50 DC x 40° +2.00 DS / -2.75 DC x 30° +1.25 DS / -1.25 DC x 30° +0.50 DS / -1.75 DC x 30° +0.50 DS / -2.25 DC x 30° +0.25 DS / -3.75 DC x 30°	3 0.00 DS +0.50 DS / -1.00 DC × 25° +2.00 DS / -2.00 DC × 25° +2.50 DS / -2.25 DC × 35° +2.25 DS / -1.50 DC × 50° +0.75 DS / -1.75 DC × 140° 0.00 DS / -4.50 DC× 140°
<u>4</u> 1)+0.25 DS / -0.50 DC × 60° 2)- 0.50 DS 3)+0.25 DS / -0.75 DC × 30° 4)+1.25 DS / -1.50 DC × 50° 5)+2.50 DS / -2.00 DC × 60° 6)+0.75 DS / -0.50 DC × 150° 7)+0.25 DS / -2.75 DC × 155° 8)+0.25 DS / -3.00 DC × 155°	5 -0.75 DS -0.50 DS -0.25 DS / -0.25 DC x 90° 0.00 DS / -0.50 DC x 80° +2.25 DS / -2.25 DC x 80° +2.75 DS / -2.50 DC x 80° +1.00 DS / -0.75 DC x 80° +0.25 DS / -3.25 DC x 170°	<u>6</u> -0.25 DS -0.25 DS / -0.50 DC × 30° -0.25 DS / -0.50 DC × 30° -0.25 DS / -0.50 DC × 30° -0.00 DS / -0.25 DC × 90° +1.75 DS / -2.00 DC × 90° +2.50 DS / -2.75 DC × 180° -0.00 DS / -3.00 DC × 180°
<u>Z</u> 1)- 0.75 DS 2)- 0.25 DS 3)- 0.25 DS 4)+0.25 DS / -0.50 DC x 110° 5)+2.00 DS / -1.75 DC x 110° 6)+2.25 DS / -1.50 DC x 110° 7)+0.75 DS / -2.00 DC x 20° 8)+0.50 DS / -3.00 DC x 20°	8 - 0.25 DS - 0.50 DS +0.25 DS / -0.75 DC x 145° +2.00 DS / -2.00 DC x 135° +2.25 DS / -1.75 DC x 125° +1.25 DS / -1.50 DC x 115° +0.50 DS / -1.00 DC x 35° +0.50 DS / -2.00 DC x 35°	9 0.00 DS +0.50 Ds / -1.00 DC x 115° +2.00 DS / -1.75 DC x 145° +2.75 DS / -2.50 DC x 150° +2.50 DS / -1.25 DC x 150° +1.25 DS / -1.50 DC x 40° +0.25 DS / -2.75 DC x 40°

	-
0.75 DS	- 0.25 DS
0.25 DS	- 0.50 DS
0.25 DS	+0.25 DS / -0.75 DC x 145°
-0.25 DS / -0.50 DC x 110°	+2.00 DS / -2.00 DC x 135°
+2.00 DS / -1.75 DC x 110°	+2.25 DS / -1.75 DC x 125°
+2.25 DS / -1.50 DC x 110°	+1.25 DS / -1.50 DC x 115°
+0.75 DS / -2.00 DC x 20°	+0.50 DS / -1.00 DC x 35°
+0.50 DS / -3.00 DC x 20°	+0.50 DS / -2.00 DC x 35°

10 1)+2.00 DS / -2.50 DC x 140° 2)+2.25 DS / -2.25 DC x 145° 3)+1.50 DS / -1.50 DC x 145° 4)+0.75 DS / -1.75 DC x 145° 5)+0.50 DS / -1.50 DC x 145° 6)+0.50 DS / -2.75 DC x 150°

11 +0.75 DS / -2.25 DC x 130° 0.00 DS / -200 DC x 130° -0.75 DS / -1.75 DC x 75° 0.00 DS / -2.50 DC x 65° -0.25 DS / -3.00 DC x 65°

## Table 4.11.

Gradient-index varifocal lens with 75 mm diameter,diffusion time in water one and a half hours, thickness 15 mm

## Dioptric power

1 1)- 0.25 DS / -1.50 DC x 75° 2)+1.00 DS / -2.50 DC x 72° 3)+1.25 DS / -3.00 DC x 78° 4) 0.00 DS / -3.50 DC x 130° 5)- 0.25 DS / -4.25 DC x 135° 6) 7)	2 +1.50 DS / -2.50 0 +1.50 DS / -2.00 0 +1.75 DS / -3.00 0 +0.25 DS / -3.25 0 +0.25 DS / -3.25 0 0.00 DS / -3.50 0	DC x 68° DC x 65° DC x 65° DC x 60° DC x 110° DC x 115°	<u>3</u> +0.50 DS / -1.75 DC x 55° +0.75 DS / -2.50 DC x 51° +1.50 DS / -2.75 DC x 42° +2.50 DS / -3.50 DC x 50° +1.75 DS / -2.00 DC x 45° +1.00 DS / -3.75 DC x 120° +0.50 DS / -3.50 DC x 135°
4 1)=0.25 DS / -0.25 DC x 60° 2)+0.25 DS / -0.75 DC x 66° 3)+1.00 DS / -0.75 DC x 70° 4)+1.50 DS / -1.50 DC x 75° 5)+2.00 DS / -2.50 DC x 70° 6)+2.75 DS / -1.00 DC x 140° 7)+0.50 DS / -2.00 DC x 145° 8)+0.25 DS / -2.75 DC x 150° 9)	5 - 0.50 DS - 0.25 DS 0.00 DS +1.75 DS / -0.50 D +2.50 DS / -2.00 D +2.75 DS / -2.50 D +0.25 DS / -3.00 D 0.00 DS / -3.25 D - 0.25 DS / -3.50 D	DC x 86° DC x 90° DC x 80° DC x 90° DC x 158° DC x 165°	<u>6</u> - 0.50 DS 0.00 DS 0.00 DS +1.50 DS / -0.50 DC x 60° +2.00 DS / -2.25 DC x 50° +2.50 DS / -2.75 DC x 78° +0.25 DS / -3.00 DC x 85° 0.00 DS / -3.50 DC x 170° 0.00 DS / -3.75 DC x 174°
<u>Z</u> 1)- 0.50 DS 2)- 0.50 DS 3)- 0.25 DS 4)+0.75 DS 5)+1.75 DS / -1.00 DC x 140° 6)+2.75 DS / -2.00 DC x 135° 7)+1.00 DS / -2.50 DC x 130° 8)+0.25 DS / -2.50 DC x 40° 9)- 0.25 DS / -3.25 DC x 25°	<u>8</u> - 0.50 DS / -1.00 [ - 0.25 DS / -1.50 [ +0.75 DS / -1.50 [ +1.50 DS / -2.00 [ +2.50 DS / -2.50 [ +0.75 DS / -1.00 [ +0.50 DS / -3.00 [	DC x 145° DC x 140° DC x 140° DC x 138° DC x 138° DC x 120° DC x 34° DC x 20°	<u>9</u> - 0.75 DS / -1.00 DC x 130° - 0.25 DS / -1.25 DC x 128° +1.50 DS / -1.75 DC x 125° +2.25 DS / -2.50 DC x 125° +2.75 DS / -1.00 DC x 110° +1.00 DS / -2.00 DC x 20° +0.25 DS / -2.50 DC x 18°
<u>10</u> 1)+0.50 DS / -1.50 DC x 1 2)+1.50 DS / -2.25 DC x 1 3)+1.75 DS / -2.75 DC x 1 4)+2.50 DS / -1.75 DC x 1 5)+0.75 DS / -2.50 DC x 1 6)+0.25 DS / -3.25 DC x 1	20° 25° 30° 18° 0° 80°	<u>11</u> +0.50 DS / -2.00 C +1.25 DS / -2.50 C +0.25 DS / -1.50 C - 0.50 DS / -3.00 C - 0.25 DS / -3.00 C	DC x 110° DC x 108° DC x 110° DC x 110° DC x 100° DC x 15°

## Table 4.12.

Gradient-index varifocal lens with 75 mm diameter, diffusion time in water four hours, and thickness 15 mm

6)+0.50 DS / -2.00 DC x 25°

Dioptric power

1 1)+0.25 DS / -1.00 DC x 70° 2)- 0.25 DS / -2.25 DC x 105° 3)- 0.25 DS / -3.00 Dc x 107° 4)- 0.50 DS / -3.50 DC x 115° 5) 6) 7)	2 +1.00 DS / -1.50 DC x 52° +0.75 DS / -1.25 DC x 52° +0.25 DS / -1.00 DC x 60° - 0.25 DS / -1.00 DC x 110° 0.00 DS / -2.25 DC x 125° - 0.25 DS / -3.25 DC x 130°	<u>3</u> +0.75 DS / -1.00 DC x 130° +1.25 DS / -1.50 DC x 130° +1.00 DS / -0.75 DC x 130° +0.50 DS / -0.50 DC x 40° +0.50 DS / -1.50 DC x 140° +0.50 DS / -2.75 DC x 140° +0.25 DS / -3.50 DC x 150°
<u>4</u> 1)+0.25 DS / -0.50 DC x 25° 2)+0.75 DS / -0.75 DC x 30° 3)+1.25 DS / -1.00 DC x 120° 4)+1.50 DS / -0.75 DC x 35° 5)+1.25 DS / -1.00 Dc x 150° 6)+0.75 DS / -2.75 DC x 150° 7)+0.50 DS / -3.00 DC x 155°	5. +0.25 DS / -0.50 DC x 150° +0.50 DS / -0.50 DC x 165° +1.00 DS / -0.50 DC x 165° +1.50 DS / -0.25 DC x 95° +1.75 DS / -1.00 DC x 175° +1.25 DS / -2.25 DC x 180° +0.75 DS / -2.75 DC x 175°	<u>6</u> +0.50 DS / -0.75 DC x 145° +1.25 DS / -1.50 DC x 145° +1.75 DS / -1.75 DC x 145° +0.75 DS / -0.25 DC x 55° +1.25 DS / -0.75 DC x 10° +1.00 DS / -1.75 DC x 10° +0.50 DS / -2.50 DC x 10°
Z 1)+1.00 DS / -1.50 DC x 135° 2)+1.50 DS / -2.00 DC x 135° 3)+1.25 DS / -1.75 DC x 135° 4)+0.50 DS / -0.75 DC x 135° 5)+0.50 DS / -1.00 DC x 45°	8 +1.25 DS / -1.75 DC x 125° +1.00 DS / -1.75 DC x 125°	9 +1.25 DS / -1.75 DC x 115°

## Table 4.13.

Gradient-index varifocal lens with 75 mm diameter, diffusion time in water forty five minutes, and thickness 15 mm

#### Dioptric power

1	2	3
1)+0.25 DS / -0.50 DC x 100°	+0.25 DS / -0.50 DC x 140°	0.00 DS/-0.25 DC x 140°
2)+0.50 DS / -1.00 DC x 100°	+0.50 DS / -1.00 DC x 140°	+0.25 DS / -0.50 DC x 150°
3)+0.50 DS / -1.75 DC x 115°	+1.00 DS / -1.25 DC x 140°	+1.00 DS / -1.00 DC x 145°
4)- 0.75 DS / -2.50 DC x 104°	- 1.50 DS / -1.00 DC x 140°	+1.50 DS / -1.25 DC x 130°
5)- 0.25 DS / -3.25 DC x 110°	- 0.50 DS / -3.00 DC x 115°	+1.00 DS / -1.25 DC x 130°
6)		+0.25 DS / -0.50 DC x 115°
7)		0.00 DS / -2 00 DC x 138°

4	5
1)- 0.25 DS	0.00 DS
2)- 0.25 DS	0.00 DS
3) 0.00 DS / -0.25 DC x 165°	-0.25 DS
4)+0.25 DS / -0.50 DC x 140°	0.00 DS
5)+1.00 DS / -1.00 DC x 125°	+0.50 DS / -0.50 DC x 90°
6)+1.50 DS / -1.25 DC x 125°	+1.25 DS / -1.00 DC x 95°
7)+0.50 DS / -1.75 DC x 10°	+0.50 DS / -1.50 DC x 175°
8)+0.25 DS / -1.50 DC x 154°	+0.25 DS / -1.75 DC x 95°

Z	8
1) 0.00 DS	0.00 DS/-0.25 DC x 25°
2) 0.00 DS / -0.25 DC x 5°	+0.25 DS / -0.50 DC x 100°
3) 0.00 DS / -0.50 DC x 25°	+0.75 DS / -0.75 DC x 10°
4)+0.50 DS / -0.50 DC x 40°	+0.75 DS / -0.50 DC x 40°
5)+1.00 DS / -1.25 DC x 55°	+0.75 DS / -0.75 DC x 40°
6)+0.25 DS / -1.00 DC x 145°	0.00 DS / -1.50 DC x 135°
7)+0.25 DS / -2.25 DC x 145°	

6 - 0.25 DS - 0.25 DS 0.00 DS / -0.25 DC x 180° +0.25 DS / -0.50 DC x 130° +0.75 DS / -0.75 DC x 60° +1.25 DS / -1.00 DC x 70°

+0.25 DS / -1.75 DC x 160°

	Z			
+0.25	DS /	-0.50	DC x	25°
+0.50	DS /	-0.75	DC x	30°
+0.75	DS /	-1.00	DC x	30°
+1.00	DS /	-1.50	DCx	45°

### Table 4.14

Gradient-index varifocal lens with 75 mm diameter, diffusion time in glycerol two hours, and thickness 15 mm

#### Dioptric power

<u>1</u>	<u>2</u>	<u>3</u>
1)- 0.75 DS / +1.00 DC x 65	-1.00 DS / +0.75 DC x 51	-0.75 DS / +0.25 DC x 25
2)-0.50 DS / +2.50 DC x 128	-1.00 DS / +1.00 DC x 41	-1.00 DS / +0.75 DC x 27
3)-0.50 DS / +3.00 DC x 76	-1.00 DS / +0.75 DC x 48	-1.25 DS / +0.75 DC x 33
4)-0.25 DS / +3.75 DC x 104	-1.25 DS / +0.75 DC x 69	-1.50 DS / +0.75 DC x 51
5)-0.50 DS / +7.75 DC X 115	-1.50 DS / +1.25 DC x 74	-1.75 DS / +1.00 DC x 60

6

9

-0.75 DS / +0.25 DC x 126

-1.25 DS / +0.50 DC x 109

-1.50 DS / +0.75 DC x 105

-2.00 DS / +0.75 DC x 109 -2.25 DS / +1.00 DC x 108

-0.75 DS/+0.50 DC x 140

-1.00 DS / +0.50 DC x 135

-1.25 DS / +1.00 DC x 123

-1.50 DS / +1.25 DC x 112

-2.25 DS / +2.00 DC x 94

-2.25 DS / +2.50 DC x 107

-0.50 DS -0.50 DS

4	5
1)-0.50 DS / +0.25 DC x 15	-0.25 DS
2)-0.75 DS / +0.25 DC x 20	-0.50 DS
3)-1.00 DS / +0.25 DC x 32	-0.75 DS
4)-1.50 DS / +0.75 DC x 53	-1.00 DS / +0.25 DC x 73
5)-1.75 DS / +0.75 DC x 61	-1.50 DS / +0.50 DC x 74
6)-2.00 DS / +0.75 DC x 68	-1.75 DS / +0.50 DC x 81
7)	

Z	8
1)-0.50 DS / +0.25 DC x 154	-0.50 DS/+0.25 DC x 148
2)-0.75 DS / +0.25 DC x 142	-0.75 DS / +0.50 DC x 138
3)-1.00 DS / +0.50 DC x 129	-1.25 DS / +0.75 DC x 127
4)-1.25 DS / +0.75 DC x 119	-1.50 DS / +1.00 DC x 119
5)-2.00 DS / +1.00 DC x 114	-2.00 DS / +1.25 DC x 113
6)-2.25 DS / +1.25 DC x 115	-2.75 DS / +2.00 DC x 113
7)-2.50 DS / +1.25 DC x 117	

<u>10</u>	11
1)-0.75 DS / +0.50 DC x 121	-0.50 DS / +0.75 DC x 90
2)-1.00 DS / +0.50 DC x 121	-0.75 DS / +1.50 DC x 93
3)-1.25 DS / +1.00 DC x 108	-0.50 DS / +1.75 DC x 83
4)-1.25 DS / +1.75 DC x 91	-0.75 DS / +1.00 DC x 69
5)-2.00 DS / +2.25 DC x 113	-2.00 DS / +4.50 DC x 81

## Table 4.15.

Gradient-index varifocal lens with 75 mm diameter, diffusion time in glycerol two hours, and thickness 15 mm

#### Dioptric power

1	2	<u>3</u>
1)-1.25 DS / +1.50 DC x 65	-1.25 DS / +1.00 DC x 44	-0.75 DS / +0.50 DC x 21
2)-0.25 DS / +2.25 DC x 127	-1.50 DS / +1.25 DC x 31	-1.25 DS / +1.00 DC x 21
3)-0.75 DS / +4.50 DC x 75	-1.50 DS / +1.00 DC x 32	-1.50 DS / +1.00 DC x 21
4)-1.25 DS / +6.00 DC x 102	-1.75 DS / +1.00 DC x 59	-2.00 DS / +1.00 DC x 46
5)-1.50 DS / +7.00 DC x 106	-2.50 DS / +2.00 DC x 67	-2.50 DS / +1.50 DC x 55
4	5	6
1)-0.50 DS / +0.25 DC x 10	-0.25 DS	-0.25 DS
2)-0.50 DS / +0.25 DC x 15	-0.50 DS	-0.50 DS
3)-0.75 DS / +0.50 DC x 31	-0.75 DS	-0.75 DS / +0.25 DC x 120
4)-1.50 DS / +0.75 DC x 51	-1.00 DS / +0.50 DC x 72	-1.00 DS / +0.50 DC x 102
5)-2.00 DS / +1.00 DC x 58	-1.75 DS / +0.75 DC x 80	-1.75 DS / +1.00 DC x 97
6)-2.25 DS / +1.00 DC x 60	-2.25 DS / +1.00 DC x 87	-2.50 DS / +1.25 DC x 104
7)	-2.50 DS / +1.00 DC x 87	-3.00 DS / +1.50 DC x 106
Z 1)-0.25 DS 2)-0.50 DS / +0.25 DC x 148 3)-0.75 DS / +0.50 DC x 132 4)-1.50 DS / +0.75 DC x 120 5)-2.25 DS / +1.50 DC x 109 6)-3.00 DS / +1.75 DC x 114 7)-3.25 DS / +1.75 DC x 113	8 -0.50 DS / +0.25 DC x 151 -0.50 DS / +0.25 DC x 145 -1.25 DS / +0.75 DC x 130 -1.75 DS / +1.25 DC x 115 -2.50 DS / +1.75 DC x 117 -3.75 DS / +3.00 DC x 115	9 -0.75 DS / +0.50 DC × 136 -1.00 DS / +0.75 DC × 137 -1.50 DS / +1.00 DC × 128 -2.50 DS / +2.75 DC × 55 -3.00 DS / +3.25 DC × 66 -3.50 DS / +3.00 DC × 111
<u>10</u> 1)-0.75 DS / +0.50 DC ×	127 -0.50 DS / +0.75	DC x 102

10	- 11 ,
1)-0.75 DS / +0.50 DC x 127	-0.50 DS / +0.75 DC x 102
2)-1.25 DS / +1.00 DC x 122	-0.75 DS / +1.75 DC x 99
3)-2.25 DS / +3.00 DC x 53	-1.00 DS / +5.00 DC x 58
4)-1.50 DS / +1.75 DC x 98	-1.50 DS / +2.50 DC x 54

### Table 4.16.

Gradient-index lens with 75 mm diameter, diffusion time in glycerol one and a half hours, and thickness 15 mm

	Dioptric power	
1 1)-0.50 DS / +0.25 DC × 134 2)-0.75 DS / +0.25 DC × 163 3)-1.00 DS / +0.50 DC × 163 4)-1.25 DS / +0.75 DC × 155 5)-2.00 DS / +1.25 DC × 149	2 -0.50 DS -0.75 DS -1.00 DS / +0.25 DC x 149 -1.75 DS / +1.00 DC x 131 -2.25 DS / +1.75 DC x 130 -3.00 DS / +2.50 DC x 127	3 -0.50 DS -0.75 DS -1.00 DS / +0.50 DC x 116 -1.50 DS / +0.75 DC x 115 -1.75 DS / +1.25 DC x 111 -2.50 DS / +1.75 DC x 101
4	-5.00 D37 +2.50 D0 x 127	-3.00 DS / +2.50 DC x 110

2

-0.50 DS -0.50 DS -0.75 DS

1)-0.50	DS				
2)-0.50	DS/	+0.25	DC	x	180
3)-0.75	DS /	+0.25	DC	x	117
4)-1.00	DS /	+0.50	DC	x	117
5)-1.50	DS /	+1.00	DC	x	111
6)-2.00	DS /	+1.50	DC	x	107
7)-2.25	DS /	+1.75	DC	x	110
8)-2.75	DS /	+2.00	DC	x	107

Z 1)-0.50 DS 2)-0.50 DS 3)-0.75 DS / +0.50 DC x 30 4)-1.00 DS / +0.50 DC x 35 5)-1.75 DS / +1.00 DC x 50 6)-2.50 DS / +2.00 DC x 50 7)-2.75 DS / +2.25 DC x 55

8 -0.50 DS -0.75 DS -1.25 DS / +0.50 DC x 54 -1.75 DS / +1.00 DC x 60

-2.50 DS / +1.75 DC x 60

-0.75 DS / +0.25 DC x 170 -1.00 DS / +0.25 DC x 50 -1.25 DS / +0.50 DC x 70 -1.75 DS / +1.00 DC x 75 -2.25 DS / +1.50 DC x 70

6	
-0.50 DS	
-0.50 DS	
-0.75 DS / +0.25 DC x 13	
-0.75 DS / +0.50 DC x 35	
-1.00 DS / +0.50 DC x 46	
-1.75 DS / +1.00 DC x 54	
-2.25 DS / +1.50 DC x 60	
-2.50 DS / +2.00 DC x 63	

9 -0.50 DS -1.00 DS / +0.25 DC x 30 -1.25 DS / +0.50 DC x 35 -1.75 DS / +1.00 DC x 40 -2.25 DS / +1.50 DC x 45

#### Table 4.17.

Gradient-index lens with 75 mm diameter, diffusion time in glycerol forty five minutes, and thickness 15 mm

#### Dioptric power

1	2	3
1)-1.25 DS / +1.25 DC x 125	-1.00 DS / +0.75 DC x 155	-1.00 DS / +0.75 DC x 155
2)-1.25 DS / +1.00 DC x 135	-1.25 DS / +1.00 DC x 155	-1.00 DS / +0.75 DC x 155
3)-1.25 DS / +1.25 DC x 150	-1.25 DS / +0.75 DC x 155	-1.00 DS / +0.50 DC x 155
4)-1.25 DS / +1.00 DC x 150	-1.50 DS / +0.75 DC x 155	-1.25 DS / +0.50 DC x 145
5)	-1.25 DS	-1.50 DS / +0.50 DC x 120
6)		-1.25 DS
7)		-1.25 DS

4 1)-0.50 DS / +0.25 DC x 160 2)-0.75 DS / +0.50 DC x 160 3)-1.00 DS / +0.50 DC x 150 4)-1.25 DS / +0.50 DC x 120 5)-1.50 DS / +0.75 DC x 115 6)-1.25 DS / +0.50 DC x 120 7)-1.75 DS / +1.00 DC x 100 8) 5 -0.50 DS / +0.25 DC x 175 -0.50 DS -0.75 DS / +0.25 DC x 175 -1.25 DS / +0.25 DC x 120 -1.00 DS / +0.50 DC x 105 -0.75 DS / +0.50 DC x 150 -2.50 DS / +1.25 DC x 105 -1.25 DS <u>6</u> -0.50 DS / +0.25 DC x 175 -0.75 DS / +0.25 DC x 125 -1.00 DS / +0.50 DC x 40 -1.00 DS / +0.25 DC x 55 -1.50 DS / +0.50 DC x 75 -1.25 DS -1.75 DS / +0.75 DC x 125 -2.25 DS / +1.00 DC x 50

<u>7</u> 1)-0.75 DS / +0.50 DC x 20 2)-1.00 DS / +0.50 DC x 30 3)-1.25 DS / +0.75 DC x 35 4)-1.50 DS / +0.75 DC x 35 5)-1.75 DS / +0.75 DC x 35 6)-1.25 DS 7)-2.00 DS / +1.00 DC x 60 8)-2.25 DS / +1.25 DC x 80 8 -1.25 DS / +0.50 DC x 45 -1.50 DS / +1.00 DC x 35 -1.75 DS / +1.00 DC x 35 -2.00 DS / +1.00 DC x 35 -2.25 DS / +1.00 DC x 35 -2.50 DS / +1.25 DC x 35

<u>9</u> -1.50 DS / +1.50 DC × 55 -2.00 DS / +2.00 DC × 55 -2.50 DS / +2.25 DC × 55 -2.75 DS / +2.25 DC × 55

## Table 4.18

Gradient-index varifocal lens with 75 mm, diffusion time in water one and a half hours, thickness 15 mm , measured 10 days after construction

#### Dioptric power

1	2	3
1) 0.00 DS / -0.50 DC x 110	0.00 DS / -0.50 DC x 115	0.00 DS / -0.50 DC x 125
2)-0.25 DS / -0.75 DC x 105	-0.25 DS / -0.50 DC x 120	0.00 DS / -0.50 DC x 125
3)-0.25 DS / -1.50 DC x 115	-0.25 DS / -0.50 DC x 120	-0.25 DS / -0.50 DC x 130
4)-0.25 DS / -2.25 DC x 115	-0.25 DS / -0.75 DC x 120	-0.25 DS / -0.50 DC x 130
5)	-0.50 DS / -0.75 DC x 120	-0.25 DS / -0.50 DC x 130
6)	-0.75 DS / -2.25 DC x 130	-0.25 DS / -1.00 DC x 140
7)		-0.25 DS / -1.75 DC x 140

5	6
-0.25 DS	-0.25 DS
0.00 DS / -0.25 DC x 130	-0.25 DS
0.00 DS / -0.25 DC x 130	-0.25 DS
-0.25 DS / -0.50 DC x 130	-0.25 DS
-0.25 DS / -0.50 DC x 140	-0.50 DS
-0.25 DS / -0.50 DC x 140	-0.50 DS
-0.50 DS / -0.25 DC x 160	-0.50 DS
-0.50 DS / -0.75 DC x 160	-0.50 DS / -0.75 DC x 180
	5 -0.25 DS 0.00 DS / -0.25 DC x 130 0.00 DS / -0.25 DC x 130 -0.25 DS / -0.50 DC x 130 -0.25 DS / -0.50 DC x 140 -0.25 DS / -0.50 DC x 140 -0.50 DS / -0.25 DC x 160 -0.50 DS / -0.75 DC x 160

Z	8	9
1) 0.00 DS / -0.25 DC x 105	0.00 DS / -0.25 DC x 90	-0.25 DS / -0.50 DC x 75
2)-0.25 DS	0.00 DS / -0.25 DC x 90	-0.25 DS / -0.50 DC x 90
3)-0.25 DS	-0.25 DS	-0.25 DS / -0.50 DC x 90
4)-0.25 DS	-0.25 DS	-0.50 DS
5)-0.25 DS	-0.25 DS	-0.25 DS / -0.75 DC x 40
6)-0.50 DS	-0.25 DS / -0.50 DC x 35	-0.25 DS / -1.00 DC x 35
7)-0.25 DS / -0.75 DC x 25	-0.25 DS / -1.25 DC x 30	
8)-0.50 DS / -0.75 DC x 20		

10	11
1) 0.00 DS / -0.75 DC x 75	-0.25 DS / -0.75 DC x 70
2) 0.00 DS / -0.75 DC x 90	-0.25 DS / -0.75 DC x 75
3)-0.25 DS / -0.50 DC x 90	-0.25 DS / -1.25 DC x 75
4)-0.50 DS / -0.75 DC x 75	-0.50 DS / -1.75 DC x 70
5)-0.50 DS / -1.00 DC x 60	

## APPENDIX D1

## Table 6.1.

Deformable lens contstructed with perspex plastic and glycerol as the fluid Overall diameter : 60 mm Effective aperture : 40 mm Back surface of : 4 mm Front deformable surface of : 1 mm Annular spacer of : 1 mm Maximum pressure exerted : 0.75 ml of glycerol

#### Dioptric power

7) +0.50 DS/ -1.00 DC x 14°

1	2	3
1) +0.50 DS/ -1.00 DC x 41°	+0.50 DS/ -0.75 DC x 20°	+0.75 DS/ -0.75 DC x 10°
2) +0.50 DS/ -0.75 DC x 50°	+0.50 DS/ -0.75 DC x 20°	+1.00 DS/ -0.50 DC x 12°
3) +0.50 DS/ -0.50 DC x 90°	+0.75 DS	+1.00 DS/ -0.25 DC x 10°
4) +0.50 DS/ -0.50 DC x 102°	+1.00 DS/ -0.25 DC x 114°	+1.25 DS/ -0.25 DC x 172°
5) +0.75 DS/ -0.75 DC x 120°	+1.00 DS/ -0.50 DC x 130°	+1.00 DS/ -0.25 DC x 160°
6) +0.50 DS/ -1.00 DC x 132°	+0.75 DS/ -0.75 DC x 153°	+0.75 DS/ -0.75 DC x 172°
7)	+0.75 DS/ -1.25 DC x 160°	+0.50 DS/ -1.00 DC x 176°
	-	
4	2	5
1) +0.75 DS/ -0.75 DC x 169°	+0.75 DS/ -0.50 DC x 153°	+0.25 DS/ -0.75 DC x 145°
2) +1.00 DS/ -0.50 DC x 169°	+0.75 DS/ -0.50 DC x 140°	+0.25 DS/ -0.25 DC x 130°
3) +1.00 DS/ -0.25 DC x 166°	+0.50 DS/ -0.25 DC x 100°	+0.75 DS/ -0.50 DC x 100°
4) +1.25 DS/ -0.25 DC x 45°	+0.75 DS	+0.75 DS/ -0.50 DC x 78°
5) +1.00 DS/ -0.25 DC x 5°	+0.75 DS	+0.25 DS/ -0.50 DC x 74°
6) +0.75 DS/ -0.75 DC x 10°	+0.50 DS/ -0.25 DC x 170°	+0.25 DS/ -0.75 DC x 68°

## Table 6.2.

Deformable lens contstructed with perspex plastic and glycerol as the fluid Overall diameter : 60 mm Effective aperture : 40 mm Back surface of : 4 mm Front deformable surface of : 1 mm Annular spacer of : 2 mm Maximum pressure exerted : 1.00 ml of glycerol

#### Dioptric power

1	2	3
1) +0.75 DS/ -1.25 DC x 48°	+1.00 DS/ -1.25 DC x 30°	+1.00 DS/ -0.75 DC x 115°
2) +1.00 DS/ -0.75 DC x 60°	+1.50 DS/ -0.75 DC x 47°	+1.50 DS/ -0.25 DC x 26°
3) +1.25 DS/ -0.75 DC x 78°	+1.50 DS/ -0.25 DC x 80°	+1.75 DS/ -0.25 DC x 70°
4) +1.25 DS/ -1.25 DC x 112°	+1.50 DS/ -0.50 DC x 110°	+1.75 DS
5) +1.00 DS/ -1.50 DC x 127°	+1.50 DS/ -0.75 DC x 140°	+1.50 DS/ -0.25 DC x 165°
6) +0.75 DS/ -1.75 DC x 129°	+0.75 DS/ -1.25 DC x 147°	+1.00 DS/ -1.00 DC x 9°
7)		+1.00 DS/ -1.00 DC x 15°
4	5	6
4	5	<u>6</u>
<u>4</u> 1) +1.25 DS/ -1.00 DC x 165°	5 +1.00 DS/ -1.25 DC x 150°	<u>6</u> +0.75 DS/ -1.50 DC x 128°
<u>4</u> 1) +1.25 DS/ -1.00 DC x 165° 2) +1.50 DS/ -0.25 DC x 112°	5 +1.00 DS/ -1.25 DC x 150° +1.25 DS/ -0.50 DC x 133°	<u>6</u> +0.75 DS/ -1.50 DC x 128° +1.00 DS/ -1.00 DC x 113°
<u>4</u> 1) +1.25 DS/ -1.00 DC x 165° 2) +1.50 DS/ -0.25 DC x 112° 3) +1.75 DS	5 +1.00 DS/ -1.25 DC x 150° +1.25 DS/ -0.50 DC x 133° +1.75 DS/ -0.50 DC x 117°	<u>6</u> +0.75 DS/ -1.50 DC x 128° +1.00 DS/ -1.00 DC x 113° +1.25 DS/ -1.00 DC x 99°
<u>4</u> 1) +1.25 DS/ -1.00 DC x 165° 2) +1.50 DS/ -0.25 DC x 112° 3) +1.75 DS 4) +1.75 DS	5 +1.00 DS/ -1.25 DC × 150° +1.25 DS/ -0.50 DC × 133° +1.75 DS/ -0.50 DC × 117° +1.25 DS/ -0.25 DC × 73°	<u>6</u> +0.75 DS/ -1.50 DC x 128° +1.00 DS/ -1.00 DC x 113° +1.25 DS/ -1.00 DC x 99° +0.50 DS/ -0.25 DC x 21°
<u>4</u> 1) +1.25 DS/ -1.00 DC x 165° 2) +1.50 DS/ -0.25 DC x 112° 3) +1.75 DS 4) +1.75 DS 5) +1.50 DS/ -0.50 DC x 14°	5 +1.00 DS/ -1.25 DC x 150° +1.25 DS/ -0.50 DC x 133° +1.75 DS/ -0.50 DC x 117° +1.25 DS/ -0.25 DC x 73° +1.25 DS/ -0.50 DC x 40°	<u>6</u> +0.75 DS/ -1.50 DC x 128° +1.00 DS/ -1.00 DC x 113° +1.25 DS/ -1.00 DC x 99° +0.50 DS/ -0.25 DC x 21° +1.00 DS/ -1.00 DC x 55°
<u>4</u> 1) +1.25 DS/ -1.00 DC x 165° 2) +1.50 DS/ -0.25 DC x 112° 3) +1.75 DS 4) +1.75 DS 5) +1.50 DS/ -0.50 DC x 14° 6) +1.25 DS/ -1.25 DC x 10°	5 +1.00 DS/ -1.25 DC x 150° +1.25 DS/ -0.50 DC x 133° +1.75 DS/ -0.50 DC x 117° +1.25 DS/ -0.25 DC x 73° +1.25 DS/ -0.50 DC x 40° +1.00 DS/ -1.50 DC x 28°	<u>6</u> +0.75 DS/ -1.50 DC x 128° +1.00 DS/ -1.00 DC x 113° +1.25 DS/ -1.00 DC x 99° +0.50 DS/ -0.25 DC x 21° +1.00 DS/ -1.00 DC x 55° +0.75 DS/ -1.25 DC x 50°

## Table 6.3.

Deformable lens contstructed with perspex plastic and glycerol as the fluid Overall diameter : 60 mm Effective aperture : 40 mm Back surface of : 4 mm Front deformable surface of : 1.5 mm Annular spacer of : 1 mm Maximum pressure exerted : 1.25 ml of glycerol

### Dioptric power

1	2	3
1) +1.25 DS 2) +1.25 DS/ -0.25 DC x 103° 3) +1.25 DS/ -0.25 DC x 120° 4) +1.25 DS 5) 6) 7)	+1.25 DS/ -0.25 DC x 50° +1.25 DS +1.50 DS +1.25 DS +1.25 DS/ -0.25 DC x 134°	+1.50 DS/ -0.25 DC x 17° +1.50 DS/ -0.25 DC x 21° +1.50 DS/ -0.25 DC x 36° +1.50 DS/ -0.25 DC x 18° +1.50 DS +1.25 DS +1.25 DS/ -0.25 DC x 167°
4	5	<u>6</u>
1) +1.50 DS/ -0.25 DC x 12°	+1.25 DS/ -0.25 DC x 180°	+1.25 DS/ -0.25 DC x 43°
2) +1.50 DS/ -0.25 DC x 35°	+1.50 DS/ -0.25 DC x 40°	+1.25 DS/ -0.25 DC x 44°
3) +1.50 DS/ -0.25 DC x 37	+1.50 DS/ -0.25 DC x 45°	+1.50 DS/ -0.25 DC x 57°
4) +1.50 DS/ -0.25 DC x 40°	+1.50 DS/ -0.25 DC x 46°	+1.50 DS/ -0.25 DC x 65°
5) +1.50 DS/ -0.25 DC x 40°	+1.50 DS/ -0.25 DC x 37°	+1.50 DS/ -0.25 DC x 57°
6) +1.50 DS/ -0.25 DC x 37°	+1:50 DS/ -0.25 DC x 39°	+1.25 DS/ -0.25 DC x 48°
7) +1.25 DS/ -0.25 DC x 19°	+1.25 DS/ -0.25 DC x 20°	+1.25 DS/ -0.25 DC x 30°
Z	8	

1) +1.25 DS/ -0.25 DC x 90° 2) +1.25 DS/ -0.25 DC x 78° 3) +1.50 DS/ -0.25 DC x 68° 4) +1.50 DS/ -0.50 DC x 67° 5) +1.25 DS/ -0.25 DC x 59° +1.25 DS/ -0.50 DC x 100° +1.25 DS/ -0.25 DC x 92° +1.25 DS/ -0.25 DC x 86° +1.25 DS/ -0.25 DC x 81°

## Table 6.4.

Deformable lens contstructed with perspex plastic and glycerol as the fluid Overall diameter : 60 mm Effective aperture : 40 mm Back surface of : 4 mm Front deformable surface of : 1 mm Annular spacer of : 2 mm Maximum pressure exerted : 1.25 ml of glycerol

#### Dioptric power

1	2	3
1) +1.75 DS 2) +1.50 DS/ -0.25 DC x 120° 3) +1.50 DS/ -0.25 DC x 144° 4) +1.75 DS/ -0.25 DC x 160° 5) 6) 7)	+1.75 DS/ -0.25 DC x 168° +1.75 DS/ -0.25 DC x 180° +1.75 DS +1.50 DS/ -0.25 DC x 148° +1.50 DS/ -0.25 DC x 122°	+1.75 DS/ -0.25 DC x 170° +1.75 DS/ -0.25 DC x 29° +1.75 DS/ -0.25 DC x 25° +1.75 DS +1.50 DS/ -0.25 DC x 35° +1.50 DS +1.50 DS/ -0.25 DC x 180°
<u>4</u>	5	<u>6</u>
1) +1.75 DS	+1.50 DS	+1.50 DS/ -0.25 DC x 152°
2) +1.75 DS/ -0.25 DC x 38°	+1.75 DS/ -0.25 DC x 32°	+1.50 DS/ -0.25 DC x 36°
3) +1.75 DS/ -0.25 DC x 35°	+1.75 DS/ -0.25 DC x 39°	+1.50 DS/ -0.25 DC x 51°
4) +1.75 DS/ -0.25 DC x 36°	+1.75 DS/ -0.25 DC x 42°	+1.75 DS/ -0.25 DC x 51°
5) +1.75 DS/ -0.25 DC x 43°	+1.75 DS/ -0.25 DC x 46°	+1.75 DS/ -0.50 DC x 50°
6) +1.50 DS/ -0.25 DC x 30°	+1.75 DS/ -0.25 DC x 31°	+1.50 DS/ -0.25 DC x 40°
7) +1.50 DS/ -0.25 DC x 20°	+1.50 DS/ -0.50 DC x 20°	+1.50 DS/ -0.50 DC x 30°
<i>L</i> 1) +1.25 DS/ -0.25 DC x 77° 2) +1.50 DS/ -0.25 DC x 65° 3) +1.50 DS/ -0.50 DC x 62° 4) +1.50 DS/ -0.50 DC x 60°	<u>×</u> +1.50 DS/ -0.25 DC x 90° +1.50 DS/ -0.50 DC x 92° +1.50 DS/ -0.25 DC x 103° +1.50 DS/ -0.50 DC x 111°	

Each point measured on the front deformable surface had a 5 mm distance vertically and horizontally from any other neighbouring one

5) +1.50 DS/ -0.50 DC x 55°

314

## Table 6.5.

Deformable lens contstructed with perspex plastic and glycerol as the fluid Overall diameter : 60 mm Effective aperture : 40 mm Back surface of : 4 mm Front deformable surface of : 1 mm Annular spacer of : 2 mm Maximum pressure exerted : 1.25 ml of glycerol

#### Dioptric power

1	2	3
1) +1.50 DS/ -0.75 DC x 86° 2) +1.25 DS/ -0.50 DC x 99° 3) +1.50 DS/ -0.50 DC x 102° 4) 5) 6) 7)	+1.75 DS/ -0.25 DC x 60° +1.50 DS/ -0.25 DC x 60° +1.50 DS +1.50 DS/ -0.25 DC x 138° +1.75 DS/ -0.50 DC x 123°	+1.75 DS +1.75 DS +1.75 DS/ -0.25 DC x 12° +1.75 DS/ -0.25 DC x 170° +1.75 DS/ -0.25 DC x 152° +1.75 DS/ -0.25 DC x 145° +1.75 DS/ -0.50 DC x 140°
4	. 5	6
1) +1.75 DS/ -0.25 DC x 180° 2) +1.75 DS/ -0.25 DC x 2° 3) +1.75 DS/ -0.50 DC x 180° 4) +1.75 DS/ -0.25 DC x 178° 5) +1.75 DS/ -0.25 DC x 165° 6) +1.50 DS/ -0.25 DC x 154° 7) +1.50 DS/ -0.50 DC x 150°	+1.75 DS/ -0.25 DC x 171° +1.75 DS/ -0.25 DC x 180° +1.75 DS/ -0.50 DC x 180° +1.75 DS/ -0.25 DC x 180° +1.75 DS/ -0.25 DC x 177° +1.75 DS/ -0.25 DC x 160° +1.75 DS/ -0.50 DC x 161°	+1.75 DS/ -0.25 DC x 163° +1.75 DS/ -0.25 DC x 180° +1.75 DS +1.75 DS +1.75 DS +1.75 DS +1.75 DS +1.75 DS +1.75 DS +1.50 DS/ -0.50 DC x 170°
Z	<u>8</u>	
1) +1.75 DS/ -0.25 DC x 130° 2) +1.50 DS 3) +1.75 DS 4) +1.75 DS/ -0.25 DC x 162 5) +1.75 DS/ -0.25 DC x 63°	+1.50 DS/ -0.25 DC x 105° +1.50 DS/ -0.50 DC x 76° +1.75 DS/ -0.25 DC x 70°	

Each point measured on the front deformable surface had a 5 mm distance vertically and horizontally from any other neighbouring one

9

## Table 6.6.

Deformable lens contstructed with perspex plastic and glycerol as the fluid Overall diameter : 60 mm Effective aperture : 40 mm Back surface of : 4 mm Front deformable surface of : 1.50 mm Annular spacer of : 2 mm Maximum pressure exerted : 1.50 ml of glycerol

#### Dioptric power

#### 2

-		*
1) +1.50 DS/ -0.50 DC x 70° 2) +1.50 DS/ -0.50 DC x 82° 3) +1.50 DS/ -0.50 DC x 86° 4) 5) 6) 7)	+1.50 DS/ -0.50 DC x 75° +1.50 DS/ -0.50 DC x 70° +1.50 DS/ -0.25 DC x 60° +1.50 DS/ -0.50 DC x 110° +1.50 DS/ -0.50 DC x 106°	+1.75 DS/ -0.25 DC x 5° +1.75 DS/ -0.25 DC x 7° +1.75 DS +1.75 DS +1.75 DS/ -0.25 DC x 170° +1.75 DS/ -0.25 DC x 167° +1.75 DS/ -0.25 DC x 165°
4	5	6
1) +1.50 DS/ -0.25 DC x 5° 2) +1.75 DS/ -0.25 DC x 7° 3) +1.75 DS/ -0.25 DC x 180° 4) +1.75 DS/ -0.25 DC x 169° 5) +1.75 DS/ -0.25 DC x 160° 6) +1.75 DS/ -0.50 DC x 130° 7) +1.50 DS/ -0.50 DC x 127°	+1.75 DS/ -0.50 DC x 110° +1.75 DS/ -0.25 DC x 108° +1.75 DS/ -0.25 DC x 102° +1.75 DS/ -0.50 DC x 102° +1.75 DS/ -0.25 DC x 91° +1.75 DS/ -0.25 DC x 115° +1.50 DS/ -0.50 DC x 111°	+1.75 DS/ -0.50 DC x 123° +1.75 DS/ -0.25 DC x 112° +1.75 DS/ -0.25 DC x 112° +1.75 DS/ -0.25 DC x 104° +1.75 DS/ -0.25 DC x 110° +1.75 DS/ -0.25 DC x 115° +1.75 DS/ -0.25 DC x 115°

3

#### 7

1) +1.75 DS/ -0.50 DC x 125° 2) +1.75 DS 3) +1.75 DS/ -0.25 DC x 142° 4) +1.50 DS/ -0.25 DC x 148° 5) +1.50 DS/ -0.50 DC x 160°

#### 8

+1.75 DS/ -0.50 DC x 117° +1.50 DS/ -0.25 DC x 108° +1.50 DS/ -0.50 DC x 98°

#### Table 6.7.

Deformable lens contstructed with perspex plastic and glycerol as the fluid Overall diameter : 60 mm Effective aperture : 40 mm Back surface of : 4 mm Front deformable surface of : 1 mm Annular spacer of : 1 mm Maximum pressure exerted : 1.75 ml of glycerol

Dioptric power

2

1) +2.00 DS/ -0.50 DC x 106° 2) +1.75 DS/ -0.25 DC x 109° 3) +1.75 DS/ -0.75 DC x 118° 4) +1.75 DS/ -0.75 DC x 130° 5) 6) 7)

1

1) +1.75 DS/ -0.75 DC x 104° 2) +2.00 DS/ -0.75 DC x 98° 3) +2.25 DS/ -0.50 DC x 85° 4) +2.25 DS/ -0.25 DC x 40° 5) +2.25 DS/ -0.25 DC x 37° 6) +2.25 DS/ -0.50 DC x 33° 7) +1.75 DS/ -0.75 DC x 26°

7

4

1) +1.75 DS/ -1.00 DC x 97° 2) +2.00 DS/ -0.50 DC x 92° 3) +2.00 DS/ -0.50 DC x 81° 4) +1.75 DS/ -0.50 DC x 77° 5) +1.75 DS/ -0.75 DC x 61° +1.75 DS/ -0.75 DC x 66° +1.75 DS/ -0.75 DC x 72° +2.00 DS/ -0.50 DC x 74° +2.25 DS/ -0.25 DC x 78° +2.25 DS/ -0.25 DC x 51° +2.00 DS/ -0.75 DC x 50° +1.75 DS/ -0.75 DC x 35°

+2.00 DS/ -0.75 DC x 143°

+2.00 DS/ -0.25 DC x 162°

+2.25 DS/ -0.25 DC x 165°

+2.00 DS/ -0.50 DC x 155°

+2.00 DS/ -0.75 DC x 138°

#### 8

5

+1.75 DS/ -1.00 DC x 93° +1.75 DS/ -0.75 DC x 86° +1.75 DS/ -0.50 DC x 110° +1.50 DS/ -0.75 DC x 124° 3

+2.00 DS/ -0.75 DC x 160° +2.25 DS/ -0.75 DC x 152° +2.25 DS/ -0.50 DC x 102° +2.25 DS/ -0.25 DC x 100° +2.25 DS/ -0.75 DC x 85° +2.00 DS/ -0.75 DC x 63° +1.75 DS/ -0.75 DC x 90°

#### 6

+1.75 DS/ -0.75 DC x 122° +1.75 DS/ -0.50 DC x 72° +2.00 DS/ -0.50 DC x 68° +2.25 DS/ -0.25 DC x 65° +2.00 DS/ -0.25 DC x 59° +2.00 DS/ -0.25 DC x 53° +1.75 DS/ -0.75 DC x 41°

## Table 6.8.

Deformable lens contstructed with perspex plastic and glycerol as the fluid Overall diameter : 60 mm Effective aperture : 40 mm Back surface of : 4 mm Front deformable surface of : 1.5 mm Annular spacer of : 3 mm Maximum pressure exerted : 2.00 ml of glycerol

#### Dioptric power

### 2

54° 60°
62° 60° 113° 148° 151°
x 11°
x 11° x 90°
x 11° x 90° x 84°
x 11° x 90° x 84° x 80°
x 11° x 90° x 84° x 80° x 50°
x 11° x 90° x 84° x 80° x 50° x 50° x 10°

3

<u>Z</u> 1) +2.25 DS/ -1.25 DC x 123° 2) +3.25 DS/ -1.50 DC x 107° 3) +3.25 DS/ -1.00 DC x 102° 4) +3.25 DS/ -0.75 DC x 94° 5) +2.75 DS/ -0.25 DC x 90° 6) +2.75 DS/ -0.50 DC x 16° 8.

+2.75 DS/ -1.75 DC x 115° +3.00 DS/ -1.50 DC x 104° +3.00 DS/ -1.25 DC x 99° +2.75 DS/ -1.00 DC x 90°

## Table 6.9.

Deformable lens contstructed with perspex plastic and glycerol as the fluid Overall diameter : 60 mm Effective aperture : 30 mm Back surface of : 4 mm Front deformable surface of : 0.50 mm Annular spacer of : 3 mm Maximum pressure exerted : 1.00 ml of glycerol

#### Dioptric power

1	2	3
1) +1.50 DS/ -1.50 DC x 27° 2) +2.00 DS/ -1.25 DC x 86° 3) +1.50 DS/ -1.50 DC x 127° 4) 5) 6)	+2.00 DS/ -1.50 DC x 30° +3.00 DS/ -1.25 DC x 17° +3.00 DS/ -0.75 DC x 1° +2.50 DS/ -0.75 DC x 174° +2.50 DS/ -1.50 DC x 160°	+2.50 DS/ -1.75 DC x 6° +3.50 DS/ -1.75 DC x 24° +3.00 DS/ -0.75 DC x 171° +3.50 DS/ -1.00 DC x 21° +3.25 DS/ -1.25 DC x 30° +2.25 DS/ -1.75 DC x 5°
4	5	<u>6</u>
1) +2.50 DS/ -1.50 DC x 147° 2) +2.75 DS/ -0.75 DC x 174°	+1.75 DS/ -1.75 DC x 147° +2.50 DS/ -1.00 DC x 161°	+1.25 DS/ -1.75 DC x 120° +1.75 DS/ -0.75 DC x 118°
3) +3.25 DS/ -0.75 DC x 13° 4) +2.75 DS/ -1.25 DC x 20° 5) +2.25 DS/ -1.50 DC x 17°	+3.25 DS/ -0.75 DC x 170° +2.50 DS/ -1.00 DC x 11° +2.50 DS/ -1.75 DC x 34°	+1.50 DS/ -1.50 DC x 73°

## Table 6.10.

Deformable lens contstructed with perspex plastic and glycerol as the fluid Overall diameter : 60 mm Effective aperture : 40 mm Back surface of : 4 mm Front deformable surface of : 2.50 mm Annular spacer of : 2 mm Maximum pressure exerted : 1.00 ml of glycerol

#### Dioptric power

1	2	3
1) +0.75 DS/ -1.00 DC x 52°	+1.00 DS/ -1.00 DC x 35°	+1.00 DS/ -1.00 DC x 9°
2) +1.00 DS/ -0.75 DC x 70°	+1.25 DS/ -0.50 DC x 50°	+1.25 DS/ -0.25 DC x 39°
3) +1.00 DS/ -0.75 DC x 96°	+1.25 DS/ -0.25 DC x 85°	+1.50 DS
4) +1.00 DS/ -1.00 DC x 114°	+1.25 DS/ -0.25 DC x 107°	+1.50 DS
5) +0.75 DS/ -1.00 DC x 127°	+1.00 DS/ -0.50 DC x 130°	+1.25 DS/ -0.25 DC x 167°
6)	+1.00 DS/ -0.75 DC x 149°	+1.25 DS/ -0.75 DC x 167°
4	5	6
1) 1 05 001 0 75 00 1070	-	<u>v</u>
1) +1.25 $DS/ -0.75 DC \times 167^{\circ}$	+0.75 DS/ -0.75 DC x 148°	+1.00 DS/ -1.00 DC x 120°
2) +1.25 DS/ -0.25 DC x 156°	+1.25 DS/ -0.50 DC x 122°	+1.25 DS/ -0.75 DC x 105°
3) +1.50 DS	+1.25 DS/ -0.25 DC x 105°	+1.25 DS/ -0.75 DC x 29°
4) +1.50 DS	+1.50 DS/ -0.25 DC x 77°	+1.25 DS/ -0.75 DC x 59°
5) +1.50 DS/ -0.50 DC x 19°	+1.00 DS/ -0.50 DC x 34°	+0.75 DS/ -1.00 DC x 62°
6) +1.25 DS/ -0.755 DC x 18°	+1.00 DS/ -1.00 DC x 28°	

### Table 6.11.

Deformable lens contstructed with perspex plastic and glycerol as the fluid Overall diameter : 65 mm Effective aperture : 55 mm Back surface of : 4 mm Front deformable surface of : 2.50 mm Annular spacer of : 3 mm Maximum pressure exerted : 1.75 ml of glycerol

#### Dioptric power

2	
<u> </u>	

1) +1.50 DS/ -0.50 DC x 51°	+2.00 DS/ -0.75 DC x 30°	+2.25 DS/ -1.00 DC x 4°
2) +1.75 DS/ -0.75 DC x 60°	+2.00 DS/ -0.50 DC x 48°	+2.50 DS/ -0.75 DC x 18°
3) +1.75 DS/ -0.25 DC x 67°	+2.00 DS/ -0.25 DC x 30°	+2.50 DS/ -0.25 DC x 16°
4) +1.75 DS/ -0.25 DC x 110°	+2.00 DS	+2.50DS/ -0.25 DC x 180°
5) +1.75 DS/ -0.50 DC x 140°	+2.00 DS/ -0.50 DC x 179°	+2.25 DS/ -0.25 DC x 173°
6)	+1.75 DS/ -0.50DC x 160°	+2.25 DS/ -0.25 DC x 157°
7)	+2.00 DS/ -1.25DC x 168°	+2.25 DS/ -0.50 DC x 160°
8)		+2.25 DS/ -1.25 DC x 162°
9)		+2.25 DS/ -1.50 DC x 161°

#### 4 1) +2.25 DS/ -1.25 DC x 4° 2) +2.50 DS/ -1.00 DC x 180° 3) +2.50 DS/ -0.75 DC x 6° 4) +2.25 DS/ -0.50 DC x 172° 5) +2.50 DS/ -0.50 DC x 176° 6) +2.50 DS/ -0.50 DC x 169° 7) +2.50 DS/ -0.50 DC x 156° 8) +2.25 DS/ -0.75 DC x 157° 9) +2.25 DS/ -1.00 DC x 160° 10)+2.25 DS/ -1.50 DC x 165°

1

<u>Z</u> 1) +1.75 DS/ -0.75 DC x 137° 2) +2.00 DS/ -0.50 DC x 142° 3) +2.25 DS/ -0.75 DC x 111° 4) +2.00 DS/ -0.25 DC x 113° 5) +2.25 DS/ -0.25 DC x 104° 6) +2.00 DS/ -0.25 DC x 70° 7) +1.75 DS/ -0.25 DC x 57° 8) +1.50 DS/ -0.50 DC x 29° 9) +1.50 DS/ -0.75 DC x 26° +2.00 DS/ -1.25 DC x 10° +2.50 DS/ -0.50 DC x 4° +2.25 DS/ -0.25 DC x 168° +2.25 DS/ -0.25 DC x 147° +2.50 DS/ -0.25 DC x 155° +2.25 DS/ -0.25 DC x 161° +2.25 DS/ -0.50 DC x 160° +2.00 DS/ -0.75 DC x 166° +1.75 DS/ -1.00 DC x 172° +2.00 DS/ -1.25 DC x 175°

#### 8

5

+2.00 DS/ -1.00 DC x 131° +2.00 DS/ -0.75 DC x 121° +2.00 DS/ -0.50 DC x 114° +2.00 DS/ -0.50 DC x 90° +1.75 DS/ -0.50 DC x 72° +1.75 DS/ -0.50 DC x 58° +1.50 DS/ -0.75 DC x 36°

#### 6

3

+2.00 DS/ -1.00 DC x 153° +2.25 DS/ -0.50 DC x 157° +2.25 DS/ -0.25 DC x 159° +2.25 DS/ -0.25 DC x 159° +2.00 DS +2.00 DS +2.00 DS/ -0.25 DC x 174° +1.75 DS/ -0.50 DC x 178° +1.75 DS/ -1.00 DC x 180° +1.75 DS/ -1.25 DC x 171°

#### 9

+1.75 DS/ -1.25 DC x 124° +1.75 DS/ -1.00 DC x 113° +1.75 DS/ -1.00 DC x 94° +1.75 DS/ -1.00 DC x 81° +1.75 DS/ -1.25 DC x 75°

Table 6.12.

Deformable lens contstructed with perspex plastic and glycerol as the fluid Overall diameter : 75 mm Effective aperture : 60 mm Back surface of : 4 mm Front deformable surface of : 1 mm Annular spacer of : 3 mm Maximum pressure exerted : 1.50 ml of glycerol

Dioptric power

1	2	3
1) +1.75 DS/ -1.25 DC x 105° 2) +2.00 DS/ -1.50 DC x 113° 3) +2.25 DS/ -1.50 DC x 122° 4) +1.75 DS/ -1.50 DC x 130° 5) +1.50 DS/ -1.75.DC x 140° 6) 7) 8) 9)	+1.25 DS/ -0.75 DC x 10° +1.75 DS/ -0.50 DC x 117° +2.25 DS/ -1.00 DC x 126° +2.25 DS/ -1.25 DC x 130° +2.00 DS/ -1.25 DC x 137° +1.50 DS/ -1.25 DC x 149° +1.25 DS/ -1.50 DC x 157°	+1.50 DS/ -1.25 DC x 123° +1.75 DS/ -1.00 DC x 127° +2.00 DS/ -0.75 DC x 130° +2.25 DS/ -0.50 DC x 130° +2.50 DS/ -0.75 DC x 137° +2.25 DS/ -0.75 DC x 134° +2.00 DS/ -1.00 DC x 142° +1.50 DS/ -1.00 DC x 154° +1.25 DS/ -1.50 DC x 165°
4	5	6
1) +1.75 DS/ -1.25 DC x 153° 2) +1.75 DS/ -1.00 DC x 152° 3) +2.25 DS/ -0.75 DC x 150° 4) +2.75 DS/ -0.75 DC x 145° 5) +2.25 DS/ -0.75 DC x 144° 6) +2.25 DS/ -0.75 DC x 149° 7) +2.00 DS/ -0.75 DC x 160° 8) +1.75 DS/ -1.00 DC x 173° 9) +1.50 DS/ -1.25 DC x 180°	+1.50 DS/ -1.25 DC x 163° +1.75 DS/ -1.00 DC x 161° +2.00 DS/ -0.75 DC x 159° +2.50 DS/ -0.50 DC x 145° +2.50 DS/ -0.50 DC x 151° +2.25 DS/ -0.50 DC x 151° +2.25 DS/ -0.50 DC x 155° +2.00 DS/ -0.75 DC x 171° +1.75 DS/ -1.25 DC x 180° +1.75 DS/ -1.50 DC x 175° +1.50 DS/ -1.50 DC x 11°	+1.50 DS/ -1.50 DC x 165° +1.75 DS/ -1.00 DC x 161° +2.00 DS/ -0.75 DC x 154° +2.50 DS/ -0.75 DC x 137° +2.50 DS/ -0.50 DC x 140° +2.25 DS/ -0.50 DC x 149° +1.75 DS/ -0.75 DC x 8° +1.75 DS/ -1.25 DC x 11° +1.50 DS/ -1.25 DC x 12° +1.25 DS/ -1.50 DC x 13°
Z	· <u>8</u> ·	9
1) +1.25 DS/ -1.25 DC x 149° 2) +1.50 DS/ -1.25 DC x 147° 3) +1.75 DS/ -0.75 DC x 140° 4) +2.00 DS/ -0.75 DC x 136° 5) +2.25 DS/ -0.75 DC x 120° 6) +2.00 DS/ -0.50 DC x 121° 7) +1.50 DS 8) +1.50 DS/ -1.00 DC x 23° 9) +1 25 DS/ -1 50 DC x 19°	+1.25 DS/ -1.50 DC x 114° +1.50 DS/ -1.25 DC x 106° +2.00 DS/ -1.00 DC x 100° +2.00 DS/ -1.00 DC x 98° +1.75 DS/ -0.75 DC x 94° +1.25 DS/ -0.50 DC x 60° +1.00 DS/ -1.50 DC x 37°	+1.50 DS/ -1.25 DC x 100° +1.50 DS/ -1.25 DC x 90° +1.75 DS/ -1.00 DC x 104° +1.50 DS/ -1.50 DC x 107° +1.25 DS/ -1.50 DC x 118°

## Table 6.13.

Deformable lens contstructed with perspex plastic and glycerol as the fluid Overall diameter : 75 mm Effective aperture : 65 mm Back surface of : 4 mm Front deformable surface of : 1 mm Annular spacer of : 3 mm Maximum pressure exerted : 1.50 ml of glycerol

Dioptric power

1	2	3
1) +0.50 DS/ -1.25 DC x 19°	+0.50 DS/ -1.50 DC x 40°	+0.75 DS/ -1.50 DC x 55°
2) +0.75 DS/ -1.50 DC x 88°	+0.75 DS/ -1.50 DC x 20°	+0.75 DS/ -0.75 DC x 10°
3) +1.25 DS/ -1.50 DC x 86°	+0.75 DS/ -1.50 DC x 98°	+1.50 DS/ -0.75 DC x 92°
4) +1.25 DS/ -1.50 DC x 81°	+1.75 DS/ -0.75 DC x 74°	+1.75 DS/ -0.75 DC x 90°
5) +1.25 DS/ -1.75 DC x 67°	+1.75 DS/ -0.75 DC x 74°	+1.75 DS/ -0.75 DC x 60°
6) +1.00 DS/ -1.25 DC x 58°	+1.50 DS/ -0.75 DC x 62°	+1.75 DS/ -0.75 DC x 62°
7)	+1.00 DS/ -1.00 DC x 55°	+1.25 DS/ -1.00 DC x 52°
8)	+1.00 DS/ -1.25 DC x 51°	+0.75 DS/ -1.25 DC x 40°
9) 10)	+0.75 DS/ -1.50 DC x 48°	+0.75 DS/ -1.50 DC x 38° +0.50 DS/ -1.50 DC x 36°
4	5	6
1) +0.50 DS/ -1.25 DC x 70°	+0.75 DS/ -1.50 DC x 93°	+1.25 DS/ -1.50 DC x 94°
2) +1.00 DS/ -0.50 DC x 74°	+0.75 DS/ -1.50 DC x 90°	+1.75 DS/ -1.00 DC x 98°
3) +1.75 DS/ -0.50 DC x 74°	+1.00 DS/ -1.50 DC x 96°	+2.00 DS/ -0.50 DC x 28°
4) +2.00 DS/ -0.25 DC x 70°	+1.00 DS/ -1.25 DC x 90°	+2.00 DS/ -0.25 DC × 50°
5) +2.00 DS/ -0.25 DC x 70°	+1.75 DS	+2.00 DS/ -0.25 DC × 50°
6) +2.00 DS/ -0.75 DC x 64°	+2.25 DS/ -0.50 DC x 60°	+1.75 DS/ -0.25 DC x 50°
7) +1.50 DS/ -0.75 DC x 40°	+2.00 DS/ -0.50 DC x 68°	+1.50 DS/ -0.50 DC x 180°
8) +1.25 DS/ -1.00 DC x 36°	+2.00 DS/ -0.50 DC x 48°	+1.25 DS/ -1.00 DC x 8°
9) +1.00 DS/ -1.50 DC x 36°	+1.50 DS/ -0.75 DC x 40°	+0.75 DS/ -1.25 DC x 3°
10)+0.75 DS/ -1.50 DC x 35°	+1.25 DS/ -0.75 DC x 22°	+0.75 DS/ -1.50 DC x 180°
11)+0.75 DS/ -1.50 DC x 31°	+0.50 DS/ -1.25 DC x 18°	+0.50 DS/ -1.50 DC x 5°
Z	8	9
1)+1.00 DS/ -1.50 DC x 104°	+0.75 DS/ -1.50 DC x 104°	+0.50 DS/ -1.50 DC x 116°
2)+1.50 DS/ -1.25 DC x 106°	+1.25 DS/ -0.75 DC x 109°	+1.25 DS/ -1.00 DC x 117°
3)+1.75 DS/ -0.50 DC x 28°	+1.50 DS/ -0.50 DC x 20°	+1.25 DS/ -0.75 DC x 35°
4) +2.00 DS/ -0.25 DC x 50° 5) +1.75 DS/ -0.25 DC x 60° 6) +1.75 DS/ -0.50 DC x 55°	+1.75 DS/ -0.25 DC x 18° +1.50 DS	+1.25 DS/ -0.25 DC x 100° +1.75 DS/ -0.50 DC x 112° -1.50 DS/ -0.50 DC x 107°
7) +1.75 DS/ -0.25 DC x 170°	+1.50 DS/ -0.75 DC x 120°	+1.25 DS/ -0.75 DC x 10/
8) +1.75 DS/ -0.75 DC x 168°	+1.50 DS/ -1.00 DC x 138°	+1.00 DS/ -1.00 DC x 130°
9)+1.25 DS/ -1.00 DC x 168° 10)+1.00 DS/ -1.50 DC x 170° 11)+0.75 DS/ -1.50 DC x 173°	+1.00 DS/ -1.25 DC x 140° +0.50 DS/ -1.50 DC x 150°	+0.75 DS/ -1.50 DC x 140°
10)

1) +0.75 DS/ -1.25 DC x 128° 2) +1.25 DS/ -1.25 DC x 70° 3) +1.25 DS/ -1.00 DC x 105° 4) +1.50 DS/ -1.00 DC x 112° 5) +1.25 DS/ -1.50 DC x 113° 6) +0.75 DS/ -1.50 DC x 120°

Each point measured on the front deformable surface had a 5 mm distance vertically and horizontally from any other nesighbouring one

## Table 6.14.

Deformable lens contstructed with perspex plastic and glycerol as the fluid Overall diameter : 80 mm Effective aperture : 60 mm Back surface of : 4 mm Front deformable surface of : 2.5 mm Annular spacer of : 3 mm Maximum pressure exerted : 1.50 ml of glycerol

#### Dioptric power

1	2	3
1) +1.50 DS/ -1.25 DC x 140° 2) +1.00 DS/ -0.25 DC x 115°	+1.50 DS/ -1.00 DC x 132° +1.50 DS/ -0.75 DC x 130°	+1.50 DS/ -1.25 DC x 110° +1.50 DS/ -0.75 DC x 127° +1.50 DS/ 0.50 DC x 127°
3) +0.75 DS 4) +1.00 DS/-0.75 DC × 118°	+1.50 DS/ -0.50 DC x 117°	+1.50 DS/-0.50 DC x 123*
5) +1.50 DS/ -1.25 DC x 114°	+1.50 DS/ -1.00 DC x 115°	+1.75 DS/ -0.25 DC x 118°
6) +1.50 DS/ -1.50 DC x 115°	+1.50 DS/ -1.00 DC x 119°	+1.75 DS/ -0.50 DC x 121°
7)	+1.50 DS/ -1.25 DC x 123°	+1.75 DS/ -0.75 DC x 122°
8)	+1.50 DS/ -1.75 DC x 100°	+1.50 DS/ -1.00 DC x 125"
10)		+1.00 DS/ -1.25 DC x 141°
4	5	<u>6</u>
1) +1.50 DS/ -1.50 DC x 105°	+1.50 DS/ -1.00 DC x 102°	+1.25 DS/ -0.75 DC x 91°
2) +1.50 DS/ -1.00 DC x 110°	+1.75 DS/ -0.50 DC x 110°	+1.50 DS/ -0.25 DC x 10/°
3) +1.75 DS/ -0.75 DC x 117° 4) +1.75 DS/ -0.50 DC x 115°	+1.75 DS/ -0.25 DC x 120	+1.75 DS/ -0.25 DC x 45°
5) +1.75 DS/ -0.25 DC x 25°	+1.75 DS	+1.75 DS
6) +1.75 DS	+1.75 DS	+1.75 DS
7) +1.75 DS/ -0.25 DC x 120°	+1.75 DS	+1.75 DS
8) +1.75 DS/ -0.50 DC x 128°	+1.75 DS/ -0.25 DC x 128°	+1.25 DS/ -0.25 DC x 160
(10) + 1.50  DS/ - 0.30  DC x 133	+1.25 DS/ -0.50 DC x 155°	+1.25 DS/ -0.75 DC x 172°
11)+1.25 DS/ -1.00 DC x 154°	+1.25 DS/ -1.00 DC x 165°	+1.25 DS/ -0.75 DC x 179°

### 8

Z

+1.00 DS/ -1.50 DC x 104°	+1.50 DS/ -1.50 DC x 64°	+1.00 DS/ -0.75 DC x 170°
+1.25 DS/ -1.00 DC x 106°	+1.50 DS/ -1.00 DC x 67°	+1.00 DS/ -1.00 DC x 174°
+1.50 DS/ -0.50 DC x 28°	+1.50 DS/ -0.50 DC x 2073°	+1.50 DS/ -0.75 DC x 60°
+1.50 DS	+1.50 DS	+1.50 DS/ -1.00 DC x 83°
+1.75 DS	+1.75 DS/ -0.25 DC x 60°	+1.50 DS
+1.75 DS	+2.00 DS/ -0.25 DC x 64°	+1.75 DS/ -0.50 DC x 60°
+2.00 DS/ -0.25 DC x 60°	+2.00 DS/ -0.50 DC x 60°	+2.00 DS/ -0.75 DC x 60°
+2.00 DS/ -0.25 DC x 64°	+1.75 DS/ -0.50 DC x 57°	+1.75 DS/ -0.75 DC x 56°
+1.75 DS/ -0.25 DC x 60°	+1.75 DS/ -0.75 DC x 51°	+1.75 DS/ -1.25 DC x 50°
+1.50 DS/ -0.25 DC x 50°	+1.50 DS/ -1.00 DC x 46°	
+1 25 DS/ -0.25 DC x 50°	+1.00 DS/ -1.50 DC x 45°	
	+1.00 DS/ -1.50 DC x 104° +1.25 DS/ -1.00 DC x 106° +1.50 DS/ -0.50 DC x 28° +1.50 DS +1.75 DS +1.75 DS +2.00 DS/ -0.25 DC x 60° +2.00 DS/ -0.25 DC x 64° +1.75 DS/ -0.25 DC x 60° +1.50 DS/ -0.25 DC x 50°	+1.00 DS/ -1.50 DC x 104° +1.25 DS/ -1.00 DC x 106° +1.50 DS/ -0.50 DC x 28° +1.50 DS +1.75 DS +1.00 DC x 46° +1.00 DS +1.00 DC x 46° +1.00 DS +1.50 DS +1.5

9

Each point measured on the front deformable surface had a 5 mm distance vertically and horizontally from any other neighbouring one

### 10)

1) +1.25 DS/ -0.50 DC x 67° 2) +1.25 DS/ -0.25 DC x 70° 3) +1.25 DS 4) +1.50 DS/ -0.50 DC x 60° 5) +1.75 DS/ -1.00 DC x 57° 6) +1.75 DS/ -1.25 DC x 56°

Each point measured on the front deformable surface had a 5 mm distance vertically and horizontally from other nesighbouring one

APPENDIX D2

```
10 INPUT "F ";f
20 INPUT "n ";n
30 r = 1000*(n-1)/f
40 LPRINT " y Mean Power Fs
                                         Ft
                                                    Ast"
50 FOR p= 21 TO -19 STEP -5
55 LPRINT
56 LPRINT "p ";p
60 FOR y=5 TO 30 STEP 5
70 rs=SQR(r^2+(1-p)*y^2)
80 Fs=1000*(n-1)/rs
90 rt=rs^3/r^2
100 Ft=1000*(n-1)/rt
105 Ast=Ft-Fs
110 LPRINT USING " ##.##
                            ";y;(Fs+Ft)/2;Fs;FT;Ast
120 NEXT y
130 NEXT p
140 LPRINT:LPRINT
```

The program was written in collaboration with Dr. C.W. Fowler.

# APPENDIX D<sub>3</sub>

```
5 PRINT"Aspheric Ray Trace"
15 INPUT "F'v ":v
16 INPUT "F2 ";f2
17 INPUT"t (mm) ";d
18 INPUT "z (mm) ";z2
19 INPUT " n ";n
26 INPUT "Numerical (1) or Graphical (2) output"; question
29 INPUT "Incident vergence ";11
30 LOCATE 1,51
32 INPUT "scale ";scale
33 LOCATE 2.51
34 INPUT "p2";p2
35 LOCATE 3,51
36 INPUT "p1";p1
37 IF question = 2 THEN CLS:GOSUB 3000
55 f1 = (v-f2)/(1+d/1000/n^*(v-f2))
56 r2=1000*(1-n)/f2
57 r1 = 1000*(n-1)/f1
80 IF question = 2 THEN GOTO 110
100 LPRINT:LPRINT
103 CLS
106 LPRINT"F'v ";v;" F2 ";f2;" t(mm) ";d
107 LPRINT"n ";n;" z(mm) ";z2;" L1 ";l1
LPRINT "F1 ";f1;" r1 ";r1;" r2 ";r2
108 LPRINT"p1=";p1;" p2=";p2:LPRINT
109 LPRINT " Angle S'v T'v Ast. M.O.P. Dist.(%)
S(error) T(error)"
110 FOR q= 5 TO 35 STEP 5
120 t9=.017453*a
125 t8 = TAN(t9)
130 t=t8:p=p2:r=r2:z=z2
140 GOSUB 1000
150 \times 2 = x : y 2 = y
160 b3 = y2/(r2 - p2 x2)
180 \ s9 = ATN(b3) - ATN(t8)
```

```
185 s8 = SIN(s9)
```

```
190 r8 = s8/n
195 r7 = SQR(1 - r8^{2})
200 r9 = ATN(r8/r7)
205 t7=ATN(b3)-r9
210 t6 = TAN(t7)
215 w = y^2/t6
220 z1 = w + x2 + d
225 t=t6:p=p1:r=r1:z=z1
226 GOSUB 1000
227 x1=x:y1=y
230 b1 = y1/(r1 - p1 * x1)
235 q9=ATN(b1)-ATN(t6)
240 q8 = SIN(q9)
245 p8=n*q8
250 p7=SQR(1-p8^2)
255 p9=ATN(p8/p7)
260 t5=ATN(b1)-p9
265 g = SQR((x^2+d-x^1)^2+(y^1-y^2)^2)
270 r_3=SQR(r_1^2+(1-p_1)^*y_1^2)
275 r4=r3^3/r1^2
280 r5 = SQR(r2^{2}+(1-p2)^{*}y2^{2})
285 r6=r5^3/r2^2
286 IF I1=0 THEN s1=0 ELSE s1=1000/((1000/I1-x1)/COS(t5))
288 t1=s1
290 f6=1000*(n*COS(q9)-COS(p9))/r3+s1
291 c8=COS(q9)^2
295 f7=(1000*(n*COS(q9)-COS(p9))/r4+COS(p9)^2*t1)/c8
300 s2=1000/(1000/f6-g/n)
310 t_{2}=1000/(1000/f_{7}-g/n)
320 f8=1000*(COS(s9)-n*COS(r9))/r5
330 f9=1000*(COS(s9)-n*COS(r9))/r6/COS(s9)^2
340 s3=s2+f8
350 t3=(COS(r9)/COS(s9))^2*t2+f9
360 v_1 = y_2/SIN(t_9) - z_2
370 v2=1000/(1000/s3-v1)
380 v3=1000/(1000/t3-v1)
385 t4 = TAN(t5)
390 m = (v2 + v3)/2
395 IF 11<>0 THEN GOTO 2000
400 l=1/((1-z2/1000*v)*(1-d/1000/n*f1))
```

```
410 t4=TAN(t5)
```

```
420 b9=t8/t4
430 d2=100*(b9-1)/l
440 a2=v3-v2
441 IF question=2 THEN GOSUB 4200
443 IF question=2 THEN GOTO 990
445 LPRINT TAB (4);
450 LPRINT USING "## ";q;
460 LPRINT USING ###.###
                             ";v2;v3;a2;m;d2;
490 LPRINT USING"###.### ";v2-v;v3-v
990 NEXT a
991 IF question=2 THEN GOTO 30
995 LPRINT:INPUT "next p1 ";p1: GOTO 108
999 END
1000 REM subroutine
1005 a = (t^2 + p)
1010 b = (-2^*z^*t^2-2^*r)
1020 c = (z^2t^2)
1030 x=(-b-SQR(b^2-4*a*c))/(2*a)
1040 y = (z - x)^{*}t
1050 RETURN
2000 REM Near vision subroutine
2001 |3=|1+f1
2002 |2=|3/(1-d/1000/n*|3)
2004 15=12+f2
2006 g1=1-z2/1000*15
2010 h=1-x1/1000*11
2020 d2=(13*g1*t8)/(12*(-y1*11/1000+h*t4))
2025 d2=100*(d2-1)
2030 GOTO 440
3000 REM graphics subroutine
3001 FOR a=-1 TO 1 STEP 1
3002 LOCATE (8-3*a),7:PRINT USING "+#.##";(2*a*scale)
3003 NEXT a
3005 REM draw x axis
3010 LINE(100,210)-(450,210)
3020 FOR c =100 TO 450 STEP 50
3025 PSET (c,120):IF c<450 THEN PSET (c+25,120)
3030 LINE(c,210)-(c,215)
```

```
3040 NEXT c
3045 REM draw y axis
3050 LINE(100,45)-(100,195)
3060 FOR c=45 TO 195 STEP 25
3070 LINE(100,c)-(95,c)
3080 NEXT c
3085 LOCATE 4,14:PRINT"Sagittal: ";"o"
3086 LOCATE 5,14:PRINT"Tangential":"+"
3090 LOCATE 15,13
4000 PRINT"0
                 5 .
                         10
                              15
                                       20
                                               25
30
       35"
4030 PRINT " B.V.P: ";v;" F2 : ";f2;" t(mm): ";d;" n: ";n
4040 PRINT "
                 z(mm): ";z2;" p1: ";p1;" p2: ";p2;"
Incident Vergence: ":11
4045 LOCATE 6.2:PRINT"E"
4050 LOCATE 7,2:PRINT"r"
4060 LOCATE 8,2:PRINT"r"
4070 LOCATE 9,2:PRINT"o"
4080 LOCATE 10,2 :PRINT"r"
4090 LOCATE 12,1:PRINT"(D)
4105 RETURN
       plotting subroutine*********
REM
4200 IF I1=0 THEN ose=v2-v ELSE ose=I5-v2
4205 IF |1 =0 THEN ote =v3-v ELSE ote=15-v3
4206 ose=120-(ose/scale*25)
4207 ote=120-(ote/scale*25)
4210 xc=100+a*10
4300 CIRCLE (xc,ose),3
4500 CIRCLE (xc,ote),3,....0001
4600 CIRCLE(xc,ote),3,...10000
5000 RETURN
```

The program was written in collaboration with Dr. C.W. Fowler.

# APPENDIX E

## SUPPORTING PUBLICATIONS

1) C.W. Fowler and E.S. Pateras "Liquid crystal lens review" (accepted in press) Ophthalmic and Physiological Optics, 1990.

2) C.W. Fowler and E.S. Pateras "A gradient-index ophthalmic lens based on Wood's convex pseudo-lens" (accepted in press) Ophthalmic and Physiological Optics, 1990.

### REFERENCES

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