COMPUTER MODELLING OF THE ACTIONS OF EXTRAOCULAR MUSCLES

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Thesis submitted for the Degree of

Doctor of Philosophy

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February 1985

THE UNIVERSITY OF ASTON IN BIRMINGHAM

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SUMMARY

The mechanical basis of the actions of the extraocular muscles is examined. A number of assumptions, which are required to specify the mechanical constraints on the actions of the muscles, are described. Different assumptions about the way in which the extraocular muscles act are evaluated by comparing a set of models of extraocular muscle cooperation, each of which differ by just one assumption, against clinical data from patients with isolated nerve palsies.

Three applications of the model based on the most appropriate assumptions, are described. Firstly, the Hess screen test of oculomotility is investigated by calculating the mechanical actions of the muscles along the lines of the Hess chart. Secondly, alternative theories concerning the muscular factors underlying A and V syndromes are compared by using the model to predict the deviation that should occur according to each theory. Thirdly, a computer based ophthalmotrope is used to demonstrate geometrical limitations on the amount of recession and transposition surgery that can be performed on the muscle insertions.

Computer modelling; extraocular muscles;
Hess charts; A and V syndromes;
Ophthalmotropes

ACKNOWLEDGEMENTS

I would like to thank Professor G.F.A. Harding for his guidance throughout the work presented in this thesis, and for actively encouraging me to undertake the work. Mr Vernon Smith kindly obtained permission for me to consult patient records at the Orthoptic Department of the Birmingham and Midland Eye Hospital. I am especially grateful to Mrs A. Howrie, who is Head of the Orthoptic Department at the Birmingham and Midland Eye Hospital for many helpful discussions and it is a pleasure to acknowledge her collaboration on the study of A and V syndromes. Finally, I would like to thank Mrs Geddes for her excellent typing.

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INTRODUCTION

During the nineteenth century, Helmholtz and Hering pioneered the application of mechanical concepts to the understanding of the actions of the extraocular muscles. Whilst their work has been appreciated in Europe, much of it has only recently been assimilated into the mainstream of American investigations of the problem. This is probably because of the language barrier, since Helmholtz's Treatise on Physiological Optics was not translated until 1924, whilst Hering's Spatial Sense and Movements of the Eyes was translated in 1942 and his Binocular Vision as late as 1977.

By the time that these translations had appeared, the 'classical description' of the actions of the extraocular muscles had already been formulated by Duane (1896). He argued that the horizontal recti are responsible for movements of adduction and abduction and that the vertical recti and the obliques are responsible for movements of elevation and depression, with the vertical recti more effective in positions of abduction and the obliques more effective in positions of adduction.

Interest in the analytical approach to the problem of the actions of the extraocular muscles was rekindled by the work of Krewson (1950), who produced quantitative estimates of the relationships between rotations of the globe and actions of the muscles. He assumed that the muscles took the mechanical shortest path and calculated the corresponding axes of rotation. This enabled him to clarify the main actions of the individual muscles. However, whilst his approach revealed much about the actions of individual muscles, it was not so informative about how they co-operate. Also, because of the number of calculations involved, he only considered eye movements in the horizontal plane.

Boeder (1961) emphasized certain physiologically important results. In particular, he calculated the length changes that occur when the eye rotates through thirty degrees of elevation in accordance with Listings law, from any gaze position in the horizontal plane and found that the superior rectus and inferior oblique shortened by amounts that held to a fairly constant ratio of three to two respectively. He pointed out that this contradicted the view that the inferior oblique is the most important elevator in adduction. He attempted to provide a more realistic measure of the force exerted by each of the muscles by multiplying their changes in length by their respective cross sectional areas. When this was done, he found that the contribution of the inferior oblique to elevation was approximately half that of the superior rectus.

Boeder (1962) went on to consider the length changes and axes about which each muscle works, when acting along the shortest path in a sixty degree by sixty degree field of eye movements. He considered movements from a point A to a point B and found that, in general, the shortening and lengthening muscles did not act around the same axes, so that the movement was reversible only if both shortening and lengthening muscles were actively involved.

More recently, the purely geometrical calculations of the axes of rotation and changes in lengths of the muscles, have been put into matrix notation by Solomons (1978), who has calculated the adduction/abduction, elevation/depression and torsional action of each of the muscles in primary, secondary and tertiary positions of gaze. One interesting result which these calculations have demonstrated, is the balanced nature of torsional effects within pairs of antagonistic muscles. In general, however, whilst this approach has simplified the calculations, it has not by itself revealed anything further about the way in which the muscles co-operate. This last criticism is especially true if one tries to compute what will happen if some of the muscles are paretic. To be able to do this, the problem of muscle actions in different gaze directions should be approached by way of consideration of the mechanics of the situation. This approach was initiated by Robinson (1975) who formulated a computer model of the mechanical actions of the extraocular muscles.

The mechanical requirement that must be satisfied if the eye is to remain in a particular gaze direction, is that the moments on the eye caused by the forces exerted by the muscles about their respective axes of rotation must sum to a moment equivalent to that caused by the passive forces acting on the eye. The situation is complicated by the fact that the force exerted by a muscle depends both on its level of innervation and on its length. If M denotes the passive moment on the eye, fi (e,l) denotes the tension developed by the i th muscle when it has an innervation level e and a length l, and Ri denotes the axis of rotation of the i th muscle then the mechanical requirement can be expressed succintly by the equation:

$$M=\sum_{i=1}^{6} f_i\ (e,l)R_i$$

The description of the various alternative models given in the first Chapter has been organised around this equation, beginning with a description of the kinematics of the eyes. The orientation of the eye determines the paths of the muscles and these in turn determine the lengths and axes of rotation of the muscles. Together with the description of the length - tension characteristics of the muscles and their innervation levels, this Chapter specifies all of the quantities included in the equation.

Investigation of the assumptions underlying the model revealed that alternative assumptions were possible concerning the quantities in the equation. The second Chapter describes a procedure for determining the set of assumptions that provides the best model. The procedure involves setting up five alternative versions of Robinson's (1975) model, each of which differs from the original in just one assumption, and then comparing how well the behaviours of the alternative models compares with clinical data from patients with isolated nerve palsies. This procedure was also used to decide upon a binocular version of the model.

The third Chapter describes some applications of a working model of the actions of the extraocular muscles. The applications have been deliberately restricted to areas where the mechanical assumptions of the model have been tested. The first application involves using knowledge of the forces produced by the extraocular muscles in relation to screen tests for diagnosis of muscle palsies. The second application is concerned with using a model of the normal extraocular muscles as a testbed for investigating possible muscular causes of the A and V syndromes. The third application is a purely geometrical study of the amount by which the insertions of the muscles can be moved over the eye without the paths of the muscles interfering with each other.

CHAPTER 1

MECHANICS OF THE EXTRAOCULAR MUSCLES

1.1 Constraints on rotations of the eye

The initial investigations of the constraints on eye movements were made by studying the location of an afterimage as the eye was rotated. Helmholtz (1910) summarised the original discoveries in terms of two laws. The first law governing eye rotations, referred to as Donder's law, states that the orientation of the eye depends only on the final direction of the line of fixation and is independent of the initial direction. This law implies that the eye is constrained in the rotations it may make around the line of fixation. The nature of this constraint is specified by the second law, referred to as Listing's law which states that if the eye rotates about a centre O so that the line of fixation moves away from the primary position OA to another position OB, then the displacement of the eye is equivalent in rotating it around an axis perpendicular to the plane AOB. This law implies that the eye does not make any torsional movements at all, although an afterimage of a vertical line will appear to tilt with respect to a vertical line on a tangent screen, because of the geometry of the projection. The tilt of an afterimage with respect to a line on a tangent screen is referred to as false torsion.

Moses (1950) initiated an alternative approach to the experimental investigation of eye torsion. He placed a camera so that its optical axis lay along the primary direction of the line of fixation and photographed the appearance of the eye with various gaze directions. He placed ink marks on the upper and lower limbus so that the orientation of the eye could be determined from the photographs and also included a plumbline in them, so that true vertical was specified. He found that extorsion occurred with movements of the eye involving both elevation and abduction and both depression and adduction and intorsion occurred in the other two quadrants of the gaze field. It should be noted that he took torsion to mean any deviation of the vertical meridian of the eye from the plumbline, so that his concept of torsion corresponds to false torsion in the afterimage experiments. When this feature is taken into consideration, the results are entirely consistent with Listing's law.

Belcher (1964) carried out a systematic experimental investigation into the question of false torsion, photographing the eyes of his subjects in various directions of gaze, together with a plumbline, by means of a camera held along the line of fixation. He measured the rotation of the eye by tracking marks on the iris and his results confirmed that Listing's law holds.

1.2 Description of the extraocular muscles

The shape of the orbit is approximately that of a square based pyramid. At the apex of the pyramid are two holes, the optic foramen, through which passes the optic nerve and ophthalmic artery and the supra-orbital fissure through which passes all the oculomotor nerves, autonomic nerves and the ophthalmic vein.

Around the optic foramen, the membrane covering the bones of the orbit, referred to as the periorbita, thickens into a ring called the annulus of Zinn. The recti muscles are attached to the orbit at the annulus of Zinn and the membrane covering each muscle, referred to as its fascial sheath, is continuous with the annulus of Zinn. The fascial sheaths of the recti muscles join one another and together with the recti muscles themselves form what is known as the muscle cone. The muscles insert into the sclera of the eyeball and the fascial sheaths are continuous with Tenon's capsule, which is the layer of connective tissue that covers the eyeball and rotates with it. Sheaths of fibroelastic tissue, referred to as check ligaments, spread out to the orbital walls. Together with the fibrous connections between the levator palpebra and superior rectus, the check ligaments form a superior transverse ligament, while together with the fibrous connections between rectus and inferior oblique they form the ligament of Lockwood.

The gross anatomy of the extraocular muscles of the right eye is shown in Figure 1.2.1. Some of the more pertinent measurements of the extraocular muscles are given by Whitnall (1921), from whom the following values are taken. The lateral rectus has a split origin located at the part of the annulus of Zinn that surrounds the superior orbital fissure. It has an average length of 40.6 millimetres, tendon length of 8.8 millimetres and line of insertion which is 9.2 millimetres broad. The medial rectus arises from

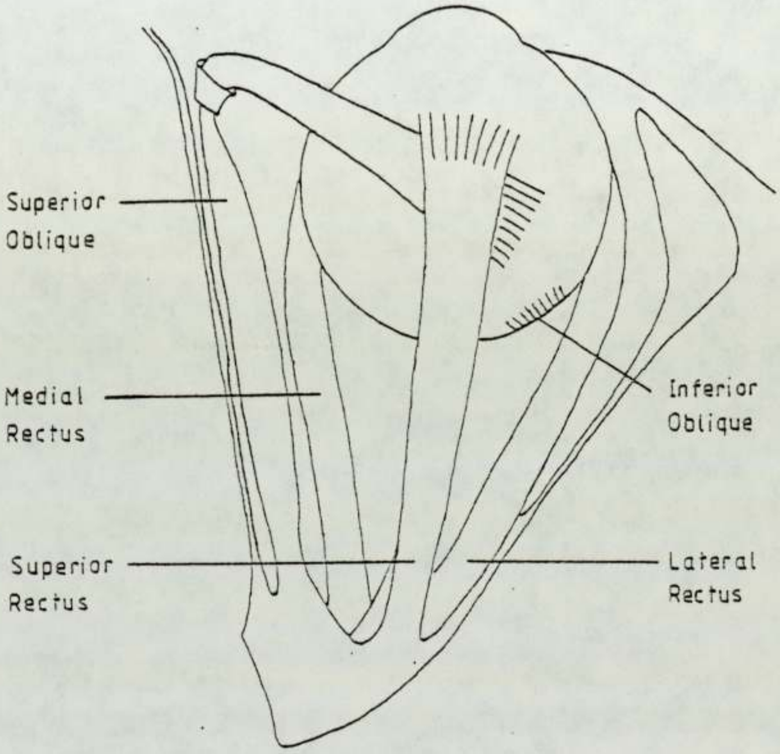

FIGURE 1.2.1

THE GROSS ANATOMY OF THE EXTRA-
OCULAR MUSCLES OF THE LEFT EYE

the medial part of the annulus of Zinn. It has an average length of 40.8 millimetres, tendon length of 3.7 millimetres and line of insertion which is 10.3 millimetres broad. The superior rectus arises from the upper part of the annulus of Zinn. It has an average length of 41.8 millimetres, tendon length of 5.8 millimetres and line of insertion which is 10.8 millimetres broad. The inferior rectus arises from the lower part of the annulus of Zinn. It has an average length of 40 millimetres, tendon length of 5.5 millimetres and line of insertion which is 9.8 millimetres broad. The superior oblique arises from the upper medial region of the apex of the orbit. It passes forward, through a loop of cartilage referred to as the ~~tochlea~~ trochlea, which acts as a pulley. It has the longest tendon, about 30 millimetres in length, which begins 10 millimetres before the trochlea. The average length of its line of insertion is 10.7 millimetres. The inferior oblique arises from an anterior portion of the floor of the orbit. It passes around the outside of the eyeball and inferior rectus and inserts beneath the medial rectus. Its line of insertion has an average length of 9.4 millimetres.

In the mechanical models described later each individual muscle is replaced by a single fibre which runs from the midpoint of the muscle insertion to its origin. The single fibre representations of the extraocular muscles of the right eye are shown in Figure 1.2.2.

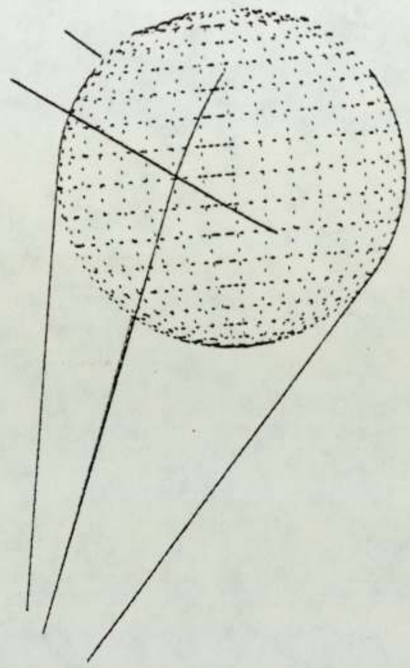

FIGURE 1.2.2

SINGLE FIBRE REPRESENTATIONS OF THE
EXTRAOCULAR MUSCLES OF THE RIGHT EYE

The co-ordinates for these points are provided by the data of Volkmann (1869), and are reproduced in Appendix 1.2. These are average measurements and examples of the variability of the insertions have been given by Howe (1906) and Fink (1946-7) who both point out the greater variability of the insertions of the obliques. The mechanical effect of the fan out of the muscle insertions will be considered in the next section.

1.3 Axes of rotation of the muscles

The analytical study of the axes of rotation of the muscles was initiated by Hering (1868) who calculated, for each pair of antagonistic muscles, the path that the line of fixation would traverse if rotated away from the primary position around the axis perpendicular to the plane of the muscles.

Helmholtz (1910) drew attention to the mechanical effect of the fan out of the tendons at the muscle insertion. He discussed the way in which elevation would stretch the lower tendons of the horizontal recti, so that the muscles would effectively continue to rotate the eye about an axis perpendicular to the visual plane.

Jampel (1970, 1975) has worked out the implications of the fan out of the muscle insertions of the obliques. These have wide insertions which allow the effective point of insertion to change its location to counter movements of the eye in the horizontal plane. Thus in adduction the inner fibres of the oblique muscles are stretched, while in abduction the outer fibres of the oblique muscles are stretched.

As evidence that the inferior oblique in man acts as if it is rotating the eye around a fixed axis, Jampel (1970) described the case of a sixty-three year old diabetic and hypertensive woman who had a sudden onset of paralysis of the right levator palpebrae and right superior rectus muscle. This case provided an instance where the inferior oblique was the only elevator and to investigate it he placed a piece of eggshell membrane on the patient's right cornea under topical anaesthesia and photographed her eye movements, both when she was exerting maximum effort to look upwards in a range of angles of adduction and abduction and, when she tried to look from a down position to an up position with about 40 degrees of adduction.

The locations of the markers on the eye as she looked from right to left while making maximum effort to elevate the eye were consistent with the eye being rotated about an axis, fixed with respect to the orbit, which was located in the horizontal plane and formed an angle of approximately sixty degrees with the primary direction of the line of fixation.

Jampel (1975) also gave evidence that the superior oblique in man acts about the same fixed axis as the inferior oblique. He states that in cases of total paralysis of the oculomotor nerve the eyes deviate outward (as opposed to downward and outward) and when the patient is instructed to look downwards the eye rotates about an axis located in the horizontal plane, 60 degrees temporally from the primary direction of the line of fixation. In patients who have paralysis of the abducens as well, the eye remains in the primary position and the nature of the rotation when the patient is asked to look down is even clearer. The same effect was observed in patients with paralysis of the oculomotor nerve only, in whom the paralysed eye had been returned to the primary position mechanically, by means of a corneoscleral limbal suture.

1.4 Paths of the muscles

The simplest assumption about the paths taken by the muscles is that they follow the shortest path. However, Robinson (1975) argued that the muscle tendon is too rigid a structure to allow the muscle to follow its shortest path.

Robinson (1975) assumed that each muscle left its line of insertion along a path that lay between the shortest path and the path perpendicular to the line of insertion. Two criteria were outlined that should be satisfied by a reasonable assumption as to the angle of twist away from the perpendicular path. The first of these was if its line of insertion stays perpendicular to the primary plane of the muscle, then the twist angle should be zero. This limits the path of each muscle as the direction of its insertion vector becomes directly opposite to its origin vector, whereupon slight movements of the eye cause extreme changes in the shortest path. The second criterion was that the twist angle should depend on the sideways force at the insertion. A satisfactory assumption was made by letting the twist angle depend on the cosine of the angle between the vector along the line of insertion of the muscle and the vector to its origin. In the primary plane of the muscle, this function is always zero and so there is no twist at the insertion.

A further assumption is needed to ensure that there is no abrupt change of direction when the muscle leaves the eyeball. This was achieved by assuming that the path of the muscle over the globe lay in a plane containing the vector corresponding to the direction in which the muscle leaves its insertion and the origin of the muscle. The intersection of this plane with the spherical globe is a circle, so that this assumption implies that the muscle makes contact with the globe along the arc of a circle.

In order to reveal the differerences between the shortest path assumption and the alternative assumption produced by Robinson (1975), a computerised ophthalmotrope was devised by Clement (1984). The paths of the fibres were traced out in two stages. Firstly, the coordinates of the point of insertion were rotated in 0.05 radian steps around the orientation of the muscle plane, through the angle of contact of the muscle. Secondly, the remaining straight section of the path of the muscle fibre was plotted out in steps of 0.02 of its overall length. Calculation of these intervening points made it possible to remove the segments of the fibres which were hidden by the globe.

As the portions of the fibres which were hidden by the globe depend on the viewpoint, it was decided to view the eyes from directly above since this allows at least some of all six muscles to be visible. The plane perpendicular to this viewing direction corresponds to the IK plane in the coordinate system used in the ~~claculations~~ calculations. No point on the muscle fibre was plotted if it lay within the circle defined by the globe and had a coordinate along the J direction that was negative.

With the point of fixation located at a meridional angle of 170 degrees and an eccentricity of 30 degrees, the muscles following the shortest path assumption appear as in Figure 1.4.1., while those following Robinson's assumption appear as in Figure 1.4.2. The most noticable difference is that the path of the lateral rectus of the right eye moves over the globe when following the shortest path assumption. Indeed, Robinson (1975) pointed out that if more than 36.3 degrees of adduction were possible then the lateral rectus should flip over the globe and pass round the nasal side of the eye, according to the shortest path assumption. Whilst this is a striking argument against the shortest path assumption, it does not prove the correctness of Robinson's alternative path. In particular, Figure 1.4.3. shows the paths of the muscles, according to Robinson's assumption, when the eyes are turned downwards through thirty degrees and are viewed from directly in front. It is clear that the superior rectus passes underneath the superior oblique, which is anatomically incorrect.

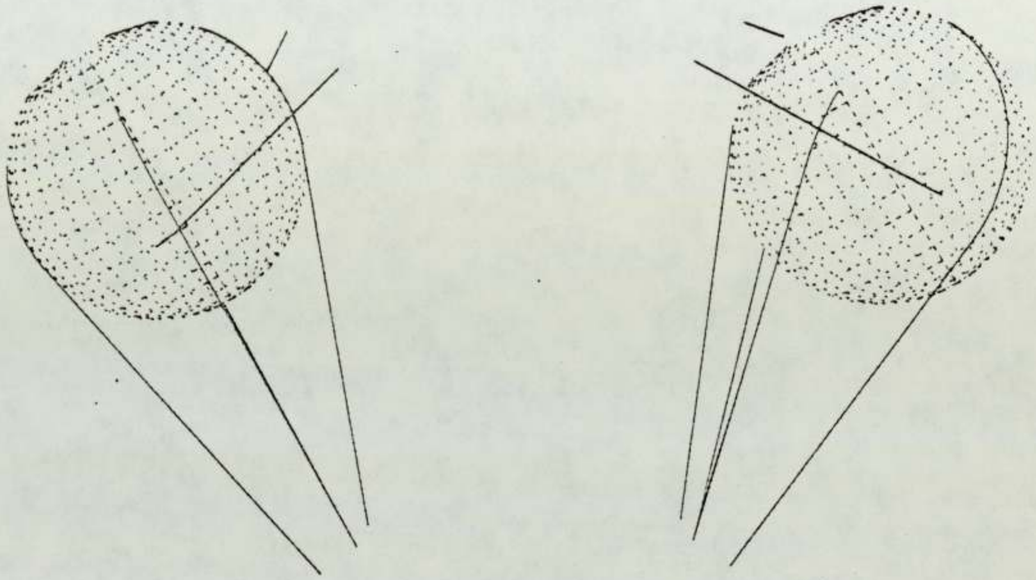

FIGURE 1.4.1

MUSCLES FOLLOWING THE SHORTEST PATH

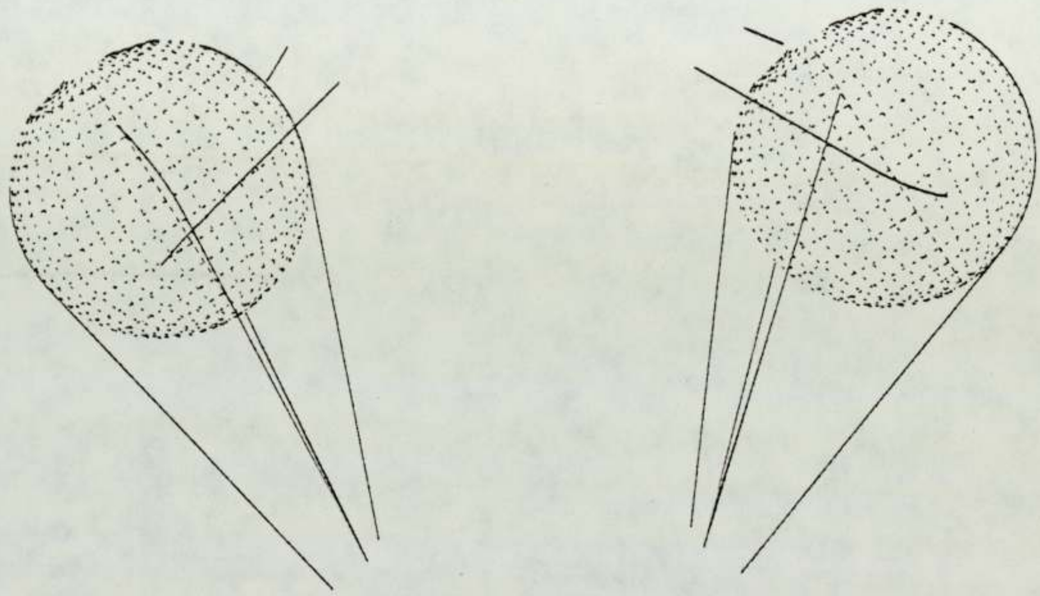

FIGURE 1.4.2.

MUSCLES FOLLOWING THE PATH DESCRIBED

by ROBINSON (1975)

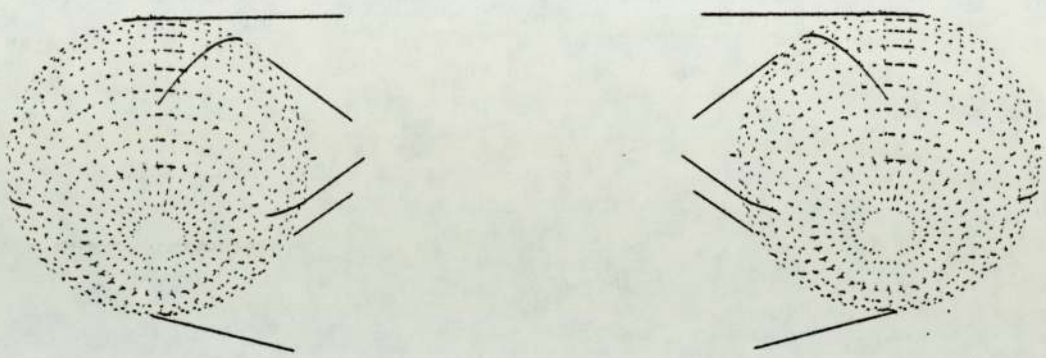

FIGURE 1.4.3

MUSCLES FOLLOWING THE PATH DESCRIBED
BY ROBINSON (1975) VIEWED FROM IN FRONT

1.5 Forces acting on the globe

The same experimental procedures have been used by Robinson et al. (1969); Collins (1971) and Scott (1971) to determine the static forces on the eye and it is their findings which are summarised here.

They investigated patients undergoing extraocular muscle surgery. The patient's head was supported by a vacuum sandcushion which was moulded to fit the contours of his head. Either the stump of a severed muscle insertion or the freed end of a muscle, according to what was being investigated, was connected by a silk suture to a strain gauge. The strain gauge was mounted on the end of a micromanipulator so that the length of the muscle could be varied. The lengths used were the length in the primary position and that length +/- 2,5 and 8 millimetres. Innervation was controlled by asking the patient to fixate, with the eye not being operated on, lights spaced 15 degrees apart, with the centre light coinciding with the primary direction of the line of fixation.

To measure the passive restraining forces on the globe, both the horizontal recti were detached and a strain gauge was connected to the stump of the insertion of one of the detached muscles while eye movements in the horizontal plane were made. Their overall finding was that there was an approximately linear relationship between the restraining force rotation of the eye away from the primary position of 0.5 grams per degree which held for up to 30 degrees of adduction or abduction.

Robinson (1975) described the passive forces on the globe in terms of the following equation, which expresses the passive force in grams in terms of the angle of deviation of the eye (β) in degrees.

$$\text{Passive force} = 0.48*\beta+(1.56*10**-4)+\beta**3$$

Robinson (1975) also included a force described by the same equation as the passive force on the globe, except that the angle was replaced by the angle of torsion. This force was presumed to act around the line of fixation in the opposite direction to the torsion. It appears to have been introduced in order to limit the deviation from Listing's law that can occur with his model when muscle paresis is simulated. It is not mentioned in any of the experimental studies.

To determine the passive force exerted by a muscle, the innervation to the muscle under investigation was reduced to a minimum, either by asking the patient to look as far as possible out of the muscle's field of action, or by working when the patient was under deep anaesthesia. They measured the length tension curve and with both procedures found that its shape was of the same form as when active contraction was present.

The length-tension curves found with various levels of innervation were of the same shape, the effect of innervation was to shift the curve along the length axis. Robinson (1975) found that the simplest relationship was given by plotting the muscle tension against change in length of the muscle expressed as a percentage of its length in the primary position, a variable which he referred to as Δl. The length-tension curves so formed could be described by an hyperbola which was shifted up or down the length change axis as the innervation was changed. For this reason a variable e was introduced so that the hyperbolas could be shifted to the left (e+) or to the right (e-). The equation that Robinson (1975) used to describe the relationship was:

$$\text{Force}=0.9\ (\Delta l+e)+\text{SQRT}(38.9376+0.81(\overset{\Delta l+e}{\cancel{\Delta l}})\ast\ast 2)$$

By considering a 1 millimetre change of muscle length to be equivalent to a 5 degree eye movement, the forces developed by the muscle when the eye is held in the position corresponding to the level of innervation can be read off the length tension curves. Collins (1971) did this and found that the resulting curve of normal muscle force was described by the equation $T=0.017\ (\beta-15)\ast\ast 2+16$ where T is the tension of the muscle and is as specified above. Thus the muscle exerts its minimum force 15 degree out of its field of action.

Collins et al. (1975) confirmed that this finding held in unrestrained eye movements. They used a force transducer which consisted of a split ring of aluminium, 2 millimetres in diameter and 1 millimetre high which had a foil resistance strain gauge mounted on it opposite the gap in the ring. Holes were drilled in the ring near the gap through which sutures could be threaded. In this way tension between sutures opened up the ring, with the maximum strain opposite the gap. The lateral and medial rectus tendon were sectioned near the insertions and the strain gauge was sutured in series. The other end of the transducer was then sutured to the sclera, slightly in advance of the muscle stump so that the effective length of the muscle was unchanged. The zero calibration of each transducer was made by unloading all force off it. This was done by rotating the globe passively so that the muscle was slack and by asking the patient to fixate with his other eye a target completely out of the muscles field of action. To calibrate the transducer, another suture was attached to the globe and connected to a preclibrated strain gauge. The globe was then pulled so that the muscle was stretched tight and its antagonist was slack. The patient then looked at an horizontal row of fixation lights with the other eye and the various levels of innervation caused the muscle to exert a range of forces which were measured by both the precalibrated strain gauge and the split ring transducer. They found the tension varied from a minimum of 8 to 12 grams at 15 degrees out of the muscles field of action to a maximum of 28 - 44 grams at 45 degrees into the muscles field of action.

Robinson (1975) gives the e values which are supposed to correspond to the levels of innervation at each of the fixation points. The corresponding length-tension curves are shown in Figure 1.5.1. Also shown in the Figure, as a dotted line, is the normal tension of the muscle. What is interesting is that this curve reaches a minimum at the primary position and not fifteen degrees out of the field of action of the muscle. For this reason, different e values were selected which give a normal tension curve which reaches a minimum fifteen degrees out of the field of action of the muscle and has the parabolic form described by Collins (1971). The resulting length-tension curves are shown in Figure 1.5.2.

As all the experimental work described was performed on the horizontal recti, the problem still remains as to what forces the other extraocular muscles exert. Robinson (1975) followed the approach of Boeder (1961) and multiplied the force exerted by each muscle, as calculated from the standard equation, by a scale factor corresponding to their relative cross sectional areas, which were taken from the data of Volkmann (1869). The actual values are as shown:

LR	MR	SR	IR	SO	IO
1.0	1.04	0.68	0.95	0.5	0.47

Since it was suggested by Clement (1982) that alterations in the relative strengths of the muscles should directly

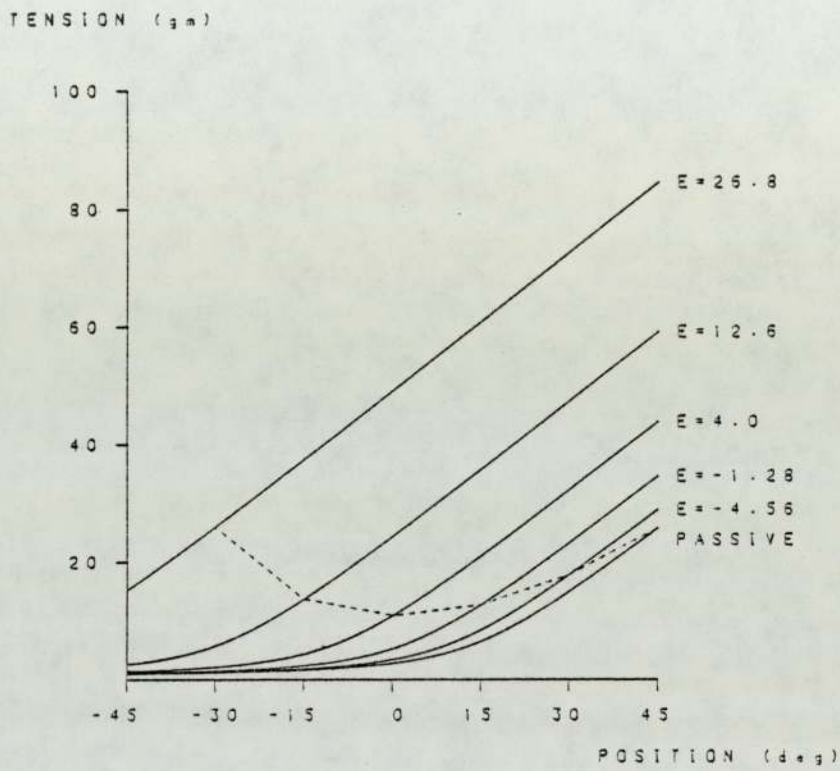

FIGURE 1.5.1

MUSCLE LENGTH-TENSION CURVES WITH THE INNERVATION LEVELS GIVEN BY ROBINSON (1975)

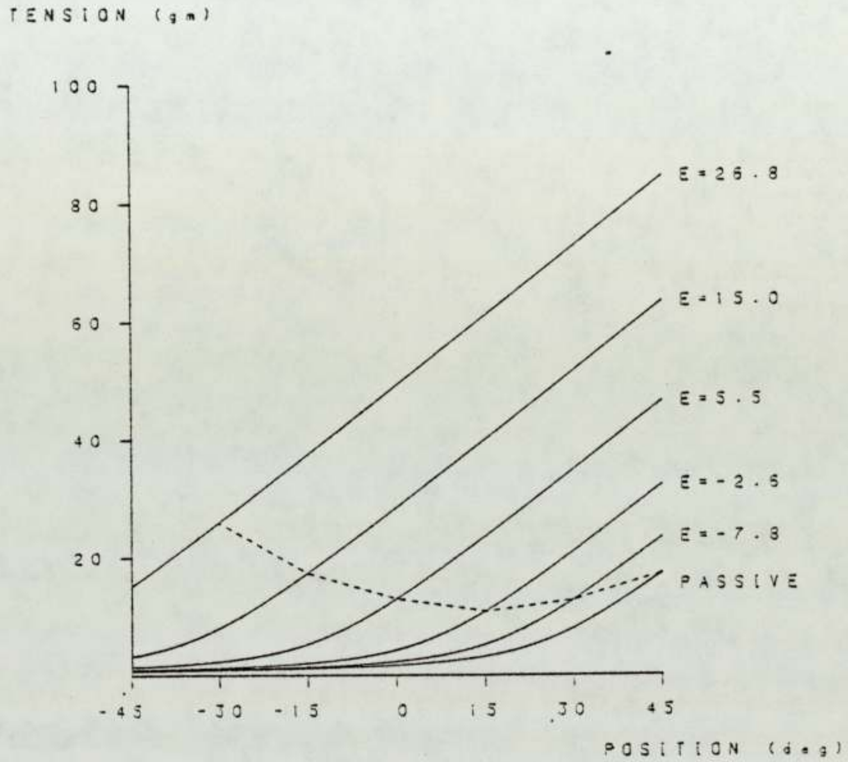

FIGURE 1.5.2

MUSCLE LENGTH-TENSION CURVES WITH THE INNERVATION LEVELS PROPOSED IN THE TEXT affect the mechanical relations between them, it was considered important to investigate this assumption. An alternative approach is to assume that the relative strengths are proportional to muscle volume rather than cross-sectional area. In order to follow up this approach, the volume of each muscle was calculated by approximating each muscle by a set of cylinders. The diameter of each cylinder was given by the average of the diameters of the muscle in successive anatomical sections and the length of each cylinder was given by the distance between successive sections. The anatomical data of Nakagawa (1965) was used. The relative strengths of the muscles obtained by this procedure are as shown:

LR	MR	SR	IR	SO	IO
1.0	0.95	0.69	0.81	0.32	0.35

1.6 Innervation of the extraocular muscles

Experimental evidence about the nature of the innervation of the extraocular muscles cannot be obtained directly, but in normal subjects is reflected by the excitation of the muscle cells, which can be recorded electromyographically. It was initially showh by Bjork and Kugelberg (1953) that the horizontal recti are active in the primary position and that their activity increases as the eye moves into their field of action. Breinin and Moldaver (1955) emphasized that the electromyograms of the horizontal recti showed considerable activity in the primary position and became silent only with movements well out of their field of action. They also found direct evidence for the reciprocal innervation relationship between direct and contralateral antagonists. For example, the right lateral rectus and left medial rectus both showed increasing activity with a gaze movement to the right, whilst the right medial rectus and left lateral rectus both showed decreased activity.

Independent of the reciprocal innervation relation, is the overall level of innervation and it would be possible for the eye to carry out certain movements by co-contraction of a pair of antagonistic muscles. Tamler, Marg and Jampolsky (1959) recorded the electromyogram from four muscles simultaneously while the eye was making slow following movements over a fifty degree range, centred on the primary position. This enabled them to determine what changes in innervation were occurring in co-contracting muscles. With vertical movements in the sagittal plane the horizontal recti showed no change in their electromyograms. Similarly, horizontal movements did not in general result in changes in the electromyograms of the vertical recti or the obliques. They concluded that around the primary position, at least, co-contraction was not occurring, though they admitted that their traces were not particularly accurate and probably would not show a discernible change for a following movement of less than eight degrees.

Tamler, Jampolsky and Marg (1959) went on to record the electromyogram of four muscles simultaneously while the eye was execeuting movements between teriary positions of gaze. This enabled them to discover that the obliques are most strongly innervated in positions of adduction while the recti are more strongly innervated in positions of abduction. They could find no changes in the way that the horizontal recti were innervated during vertical movements not through the primary position that held consistently between subjects.

Robinson (1975) embodied these innervational constraints by adopting the approach that the innervation of an agonist muscle should be equal and opposite to its antagonist muscle, so that with respect to the length-tension curves formulated in the previous section, if an innervation corresponding to plus fifteen degrees is given to the lateral rectus, then an innervation corresponding to minus fifteen degrees will be given to the medial rectus. Using this relationship for each level of innervation, one may plot the innervation of the agonist against the innervation of the antagonist. Robinson (1975) found that the curve so formed was described by the equation:

    (e(agonist)+9.7 (e(antagonist)+9.7)=(4.0+9.7)**2

i.e.

    e(antagonist)=((4.0+9.7)**2=/(e(agonist)+9.7))-9.7

This curve is shown in Figure 1.6.1. The corresponding curve for the alternative set of innervation values described in the previous section is shown in Figure 1.6.2. This curve is described by the equation:

e(antagonist)=((5.5+90.0)**2/(e(agonist)+90.0))-90.0

1.7 Models of extraocular muscle co-operation

The mechanics of the extraocular muscles and the globe require that if the globe is to stay in any given position then the sum of the moments acting around the centre of rotation must be zero in that position. Given the constraints described in the previous sections one arrives at what Robinson (1975) has referred to as the innervation problem, namely, if the position of the eye is given, what are the innervation values required to hold the eye in that position? The complementary problem, which he referred to as the position problem, is that given a set of innervation values, what will be the position adopted by the eye? Robinson (1975) solved both of these problems and so ~~mansged~~ managed to build a working model of extraocular muscle co-operation.

The approach adopted in this study is that if only one assumpton is changed in the model of Robinson (1975), then any changes in the behaviour of the model can be attributed to this assumption. Individual assumptions were only changed when there was either theoretical or experimental evidence for an alternative and this criterion resulted

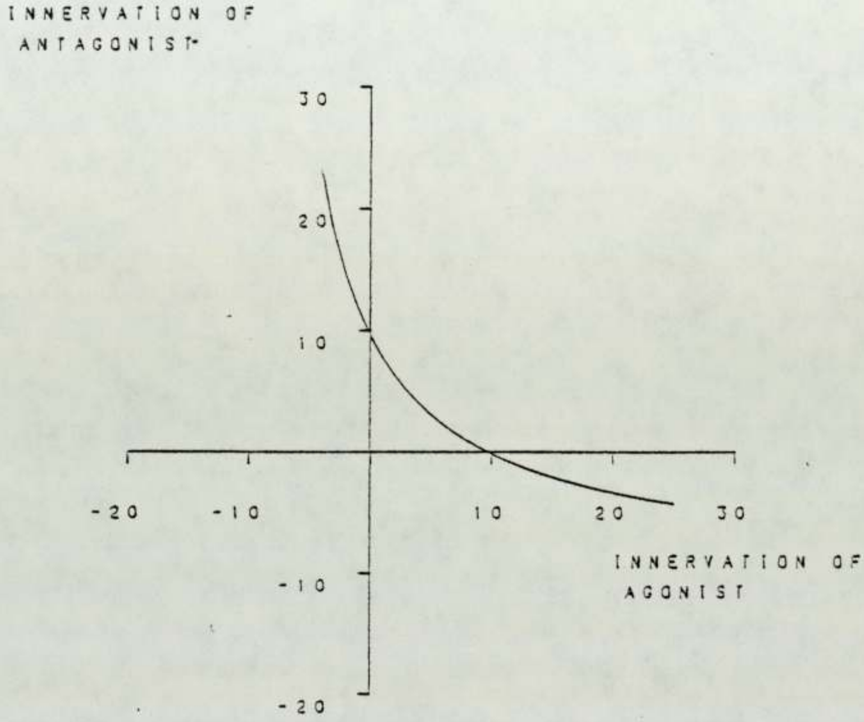

FIGURE 1.6.1

RECIPORICAL INNERVATION RELATIONSHIP WITH THE INNERVATION LEVELS GIVEN BY ROBINSON (1975)

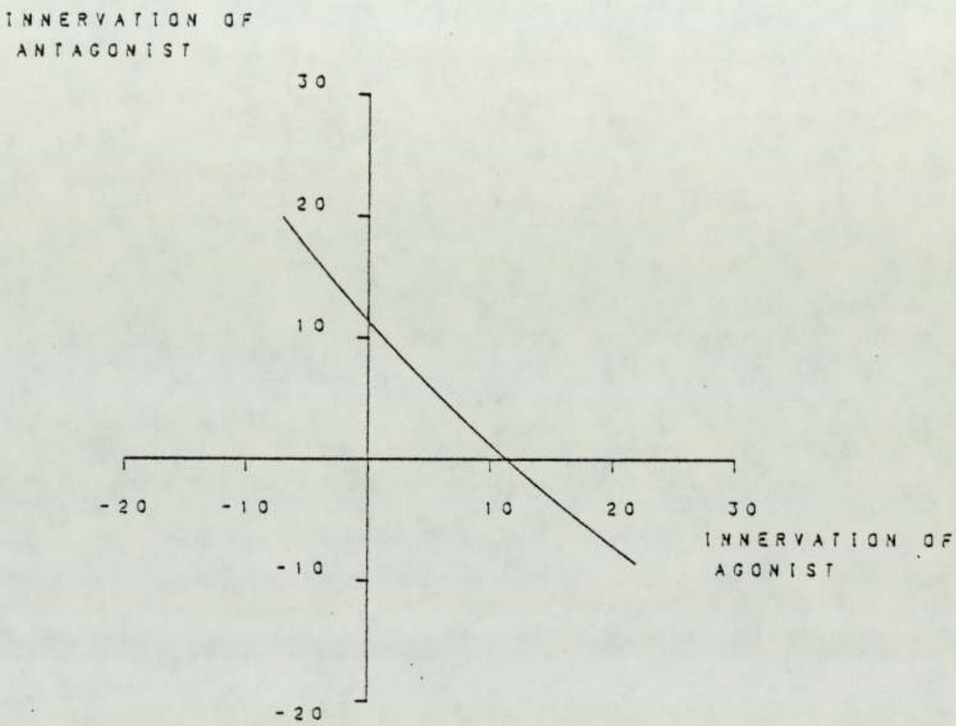

FIGURE 1.6.2

RECIPORICAL INNERVATION RELATIONSHIP

WITH THE INNERVATION LEVELS PROPOSED

IN THE TEXT

in six versions of the model which are as follows:

Model 1

The model defined by Robinson (1975)

Model 2

The same as model 1 with the exception that each muscle follows its mechanical shortest path and acts around the corresponding axis.

Model 3

The same as model 1 with the exception that each muscle acts around an axis fixed with respect to the head. The axis was taken to be perpendicular to the muscle plane with the eye in its primary position.

Model 4

The same as model 1 except that the alternative set of relative muscle strengths, derived in Section 1.5, were used.

Model 5

The same as model 1 except that the alternative reciprocal innervation function, derived in Section 1.6, was used.

Model 6

The same as model 1 except that there was no passive force assumed to be acting against the torsional movements of the eyes.

In the next Chapter these models will be evaluated by comparing their predictions against clinical data from patients with isolated nerve pareses.

CHAPTER 2

COMPARISON OF MODELS OF EXTRAOCULAR MUSCLE CO-OPERATION

2.1 Clinical tests of oculomotility

The most striking feature of the binocular co-ordination of eye movements is that the eyes act as though they are linked and that in almost every case, a movement of one eye only occurs in association with a movement of the other eye. Hering (1868) encapsulated this linkage in his law of equal innervation, which states that each eye receives equal innervation.

The constraint implied by Hering's law forms the basis of the screen tests of oculomotility. In these tests, binocular vision is dissociated so that one eye may be treated as a fixating eye and the other eye may be treated as a non-fixating eye. The interpretation of the relative gaze directions of the two eyes rests on the assumption that they are both receiving the same levels of innervation.

The Hess screen test utilises a tangent screen with the points spaced 5 degrees apart. Dissociation of binocular vision is achieved by means of the patient wearing red-green goggles with red over the fixating eye and green over the non-fixating eye. The experimenter specifies the point of fixation with a red light and the patient specifies the point of fixation of the non-fixating eye with a green light.

The Lees screen test utilises the same tangent screen as the Hess test, but dissociation is achieved by a mirror arrangement. A double sided mirror is aligned so that it points to the corner where two screens intersect at a right angle. The patient sits with his nose close to the mirror and sees the two screens superimposed in the non-fixating eye, so that one screen can be used to stimulate the fixating eye and one screen can be used to record the position of the non-fixating eye.

Both tests produce results of the same form. The result of the test consists of a pair of charts, the points marked on them showing the positions adopted by the non-fixating eye when the fixating eye is directed at the standard test positions. By convention, the chart on the left shows the positions adopted by the left eye when the right eye is fixating and the chart on the right shows the positions adopted by the right eye when the ~~right~~ left eye is fixating.

The interpretation of the test relies on knowledge of the geometry of the insertions and origins of the extraocular muscles. The lateral rectus is maximally effective in movements of pure abduction, the medial rectus in movements of pure adduction. The vertical recti are more effective ~~eith~~ with abduction and the obliques with adduction.

Because of the method involved in recording the position of the non-fixating eye, both the Hess and Lees screen tests rely on the patients having normal retinal correspondence.

2.2 Clinical data

Patient records were provided by the Orthoptic Department at the Birmingham and West Midlands Eye Hospital. Mr V. Smith, a Consultant at the Eye Hospital obtained permission from the hospital Ethical Committee for the records to be consulted. A student at the Orthoptic Department, Miss T. Smellie, kindly provided a list of patients with isolated nerve palsies of late onset, which she had used for her final year project. Throughout the study, the Head of the Orthoptic Department, Mrs A. Howrie, was very helpful in explaining details of the case histories. Since the records could not be removed from the department, the co-ordinates of the points on the Hess charts were measured in the department, and later transferred onto the computer.

Case histories were chosen in which the patients had suffered a paresis of either the IV or VI nerve which had subsequently recovered. By selecting patients who subsequently recovered it was hoped to avoid the complications of contracture of the ipsilateral antagonist and subsequent underaction of the contralateral antagonist. In order to exclude the possibility that the muscle tissue itself was different from normal, any patients who had been diagnosed as diabetic were excluded from the study. In all, 15 patients were used, 5 of whom had IV nerve pareses and 10 of whom had VI nerve pareses. The patient group had an age range of 34 to 80 (mean = 65.6) and was comprised of 3 females and 12 males. All the Hess charts had been measured within 2 to 20 days of the attack. The Hess charts of the patients with IV nerve paresis are shown in Figure 2.2.1 and the Hess charts of the patients with VI nerve paresis are shown in Figure 2.2.2.

2.3 Binocular models

France and Burbank (1978) have proposed the following computational scheme, which involves passing the innervation indirectly:

1) Calculate the innervation values required by the ACTUAL fixating eye (which may or may not be paretic).

2) Calculate the position adopted by the NORMAL fixating eye with these innervation values.

3) Calculate the innervation values required by the NORMAL non-fixating eye to reach this position.

4) Calculate the position adopted by the ACTUAL non-fixating eye with these innervation values.

Whilst this scheme is computationally effective it is open to the criticism that it does not reflect Hering's law of equal innervation. In particular, the scheme requires that when the fixating eye is paretic then it feeds abnormal

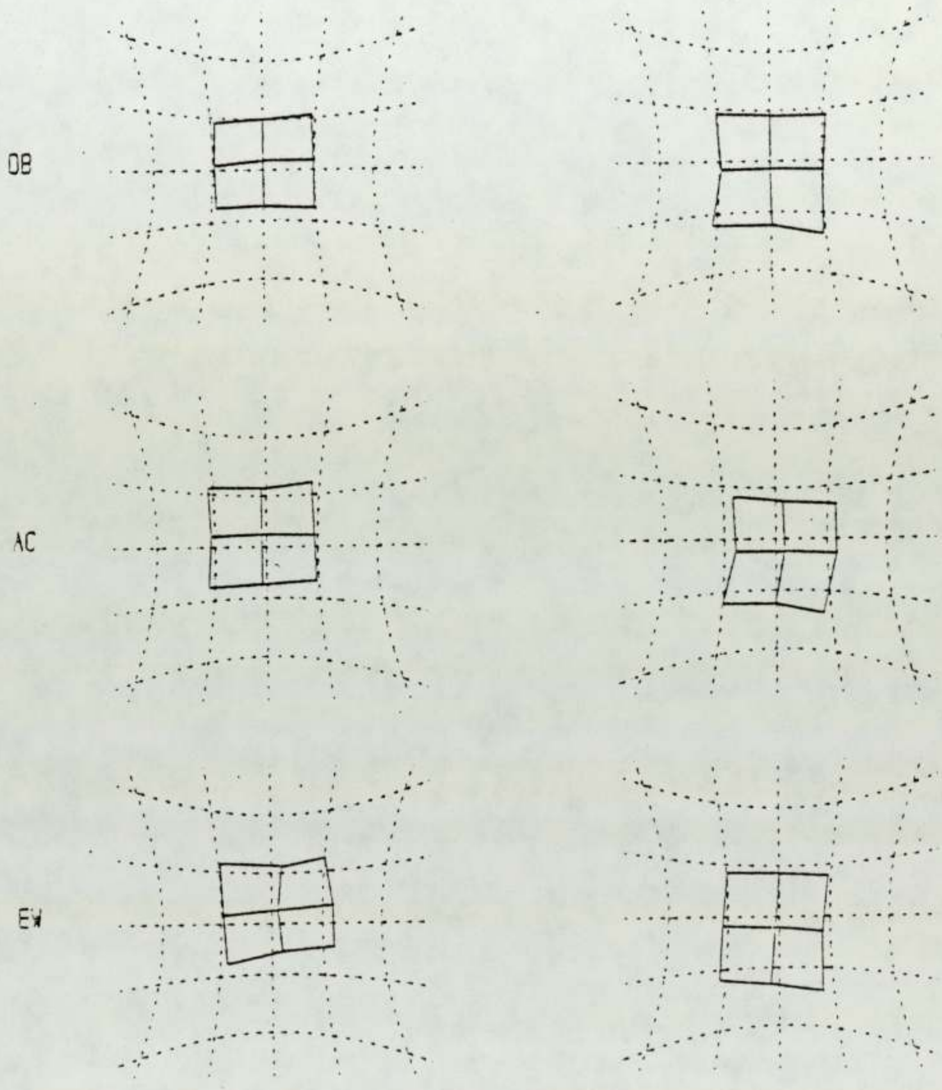

FIGURE 2.2.1

HESS CHARTS OF PATIENTS WITH

PARESIS OF THE IV NERVE

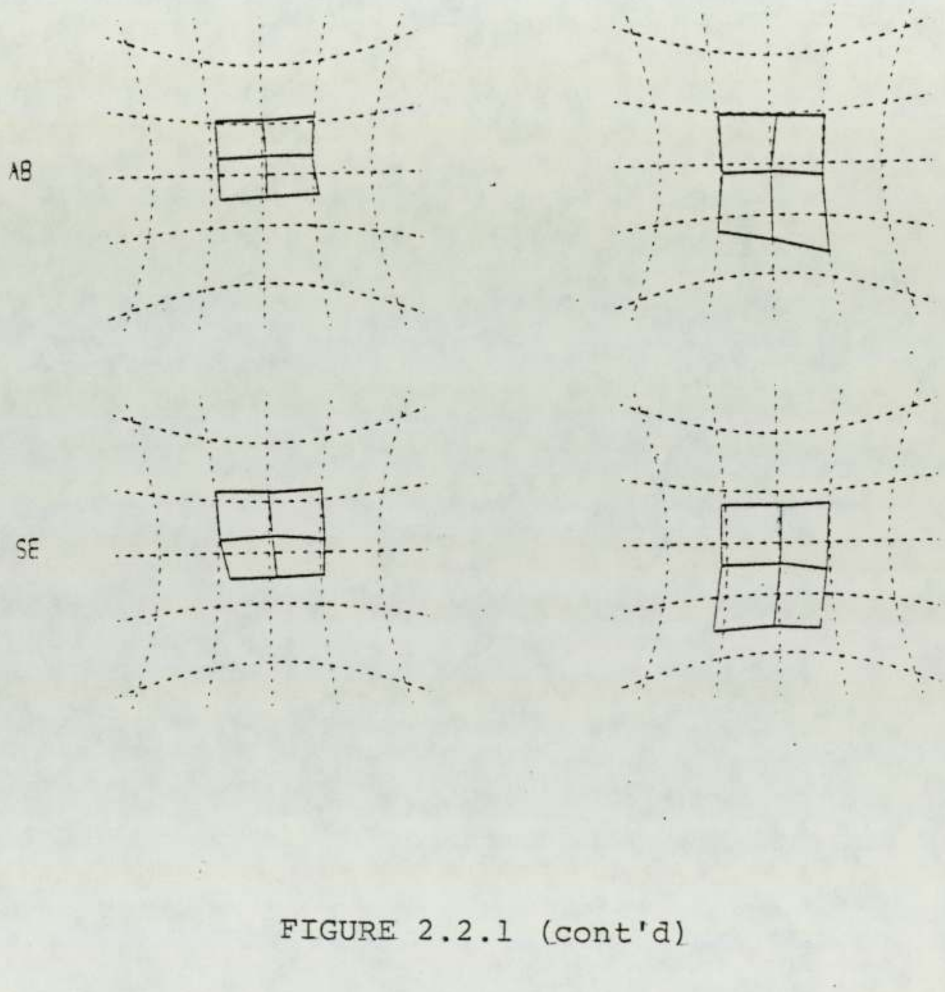

FIGURE 2.2.1 (cont'd)

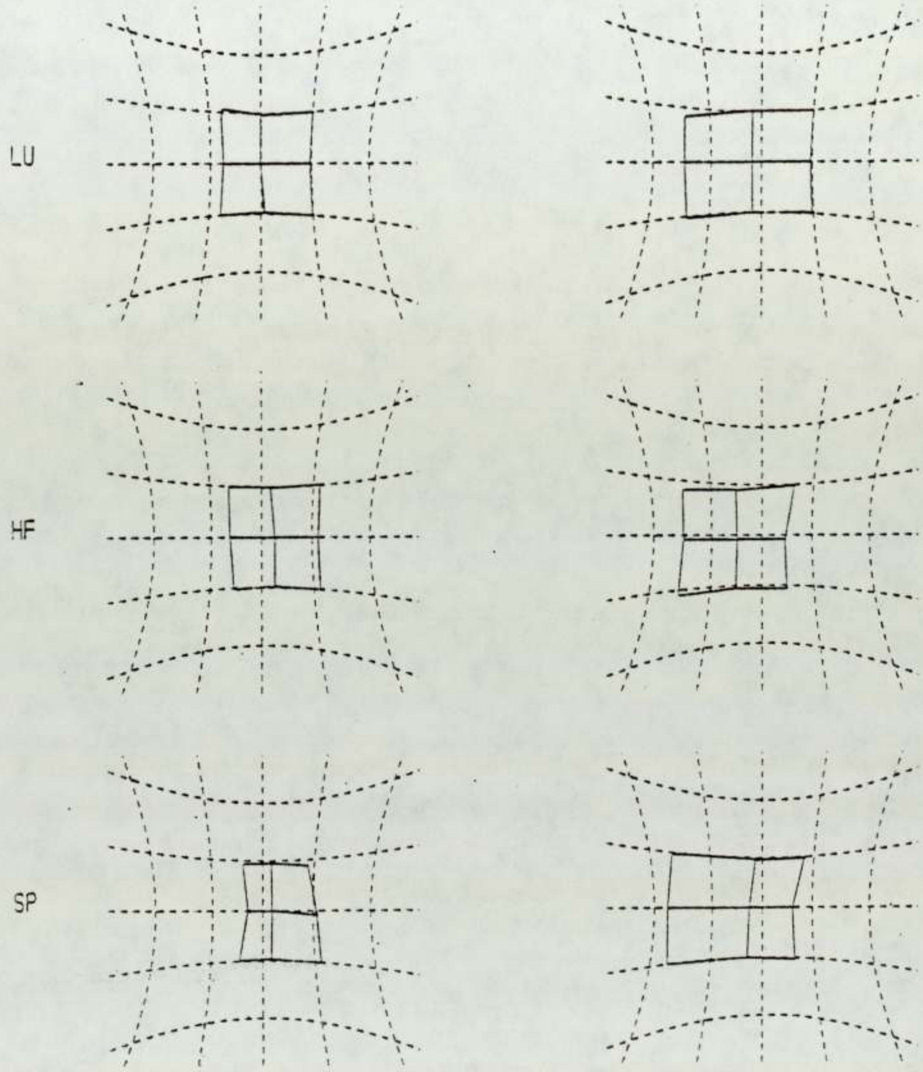

FIGURE 2.2.2

HESS CHARTS OF PATIENTS WITH
PARESIS OF THE VI NERVE

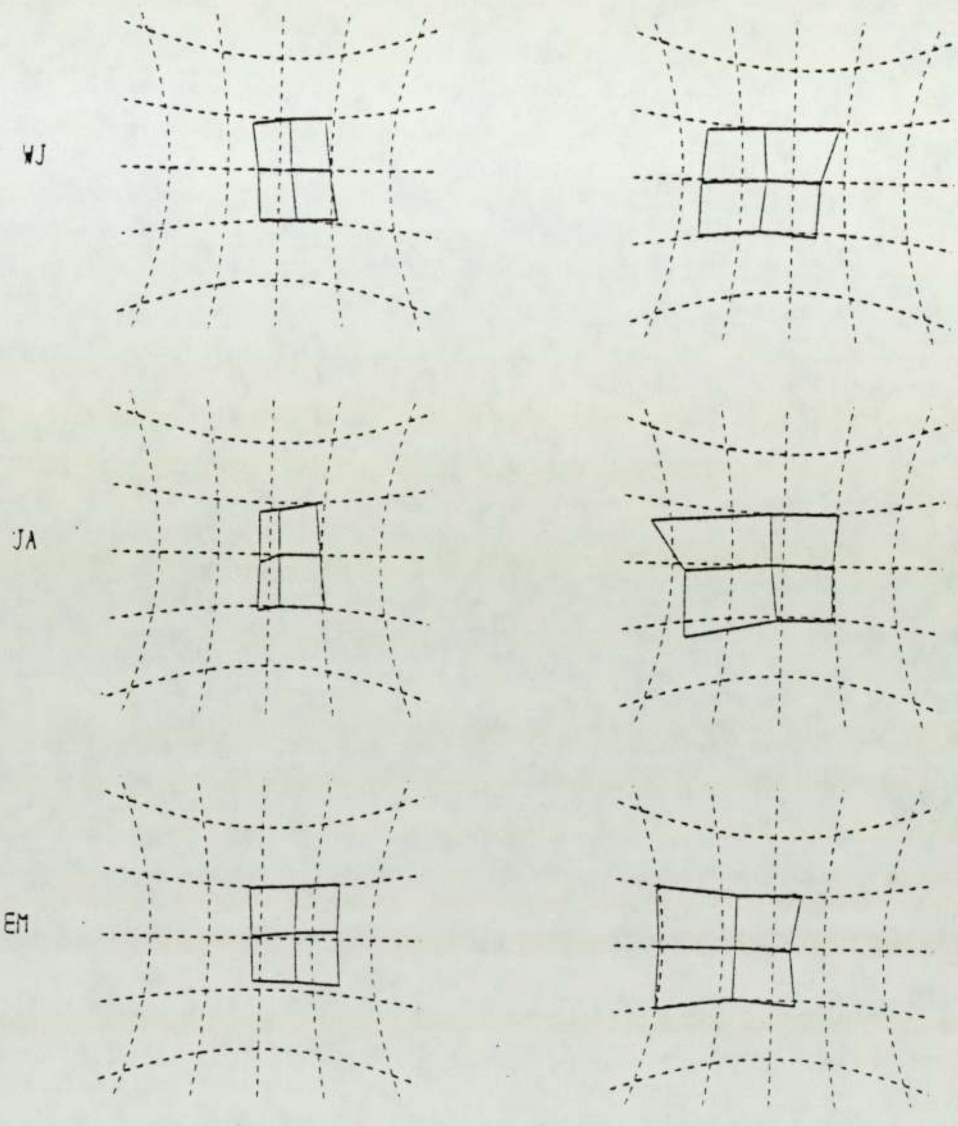

FIGURE 2.2.2 (cont'd)

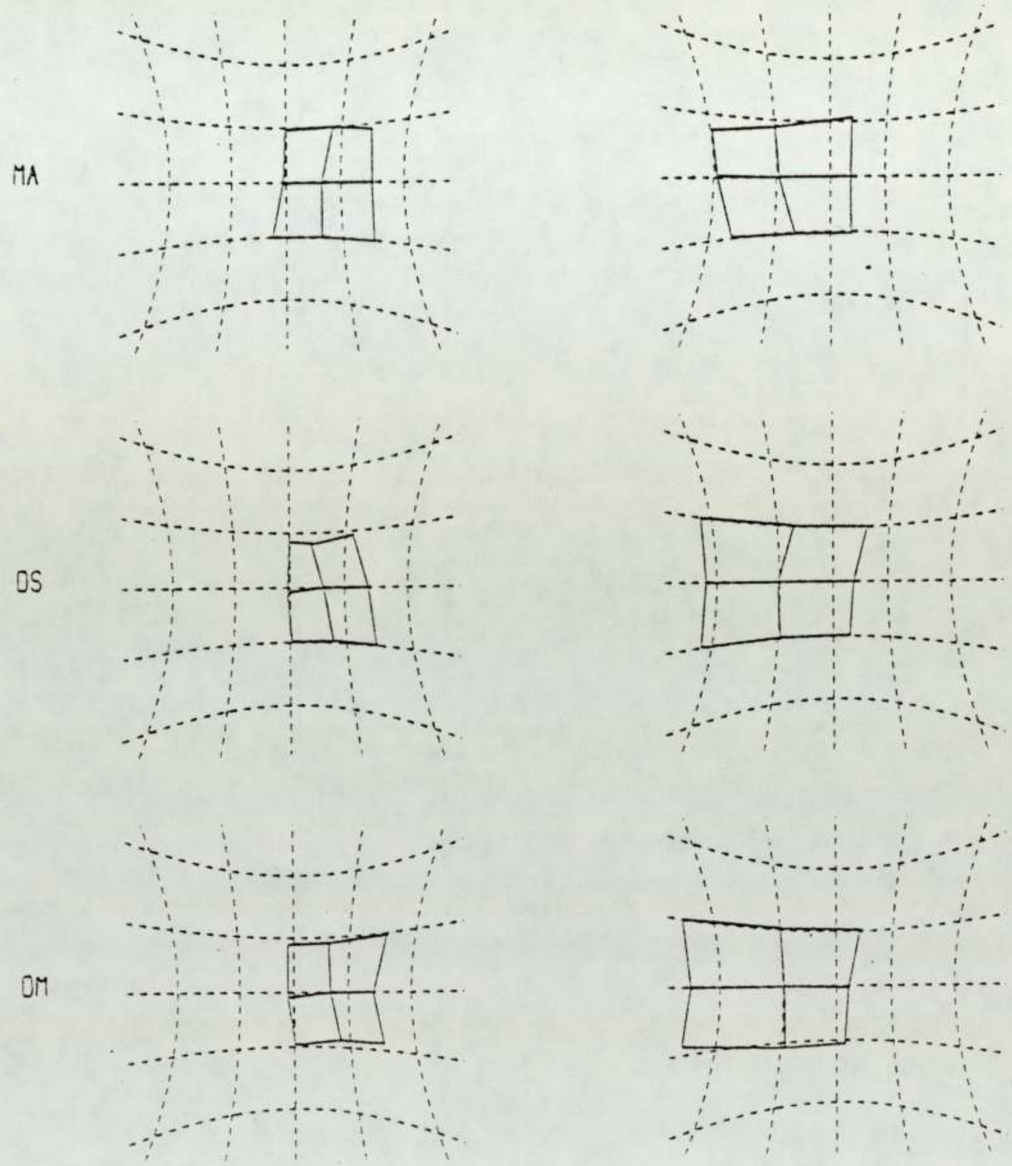

FIGURE 2.2.2 (cont'd)

PR

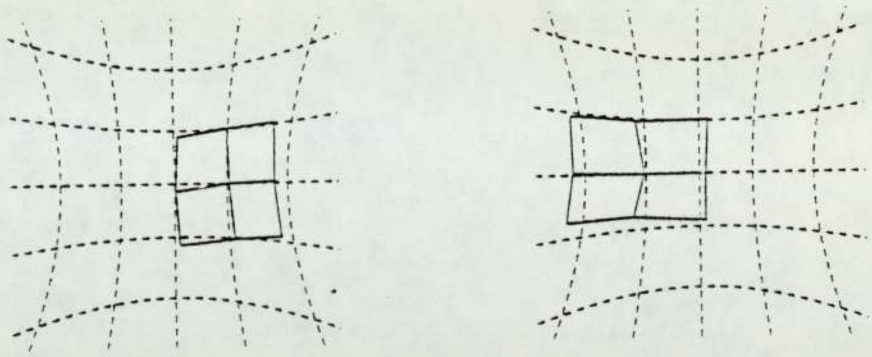

FIGURE 2.2.2 (cont'd)

innervation values into the non-fixating eye. It would be expected from the concept of the binoculus, as discussed by Hering (1868), that if the fixating eye did not reach a required position then the binoculus would supply the appropriate innervation to turn a normal eye in the direction in which the underaction is occurring. Consideration of this approach leads to an alternative computational scheme for a binocular model as follows:

1) Calculate the innervation values required by the NORMAL fixating eye (i.e. binoculus) for the specified gaze direction.

2) Calculate the position adopted by the ACTUAL fixating eye.

3) If the actual fixating eye has not reached the specified gaze direction, change the gaze direction of the binoculus, calculate the corresponding increase in innervation levels and repeat stage 2.

4) When the actual fixating eye has reached the specified gaze direction, then the innervation values for the non-fixating eye are given by the innervation values required by the normal non-fixating eye for the gaze direction of the binoculus.

The two schemes will be referred to as scheme A and scheme B respectively.

Examples of the behaviour of the six models are shown in Figure 2.3.1 and Figure 2.3.2. In both cases the pair of Hess charts showing both the primary and secondary deviations have been calculated using scheme A. Figure 2.3.1 shows the predictions of the models when the superior oblique of the left eye is producing only 30% of its normal active tension, whilst Figure 2.3.2 shows the predictions when the lateral rectus of the left eye is producing only 50% of its normal tension.

With respect to palsy of the lateral rectus it can be seen from the results of model 2 that if the muscles act around the shortest path then the ensuing pattern of eye movements differs from that with model 1. The predictions of models 3, 4 and 6 are virtually identical with those of model 1. However, model 5 shows that the change in the reciprocal innervation function results in deviations of eye positions which occur both in and out of the field of action of the muscle. This is in contrast to model 1 in which the deviation occurs almost totally in the field of action of the muscle.

With palsy of the superior oblique, it was found that models 1 to 4 gave virtually identical results. Again model 5 was different in that it showed a deviation over a wider range of eye positions than did model 1. Not surprisingly, model 6 showed greater torsion than did model 1.

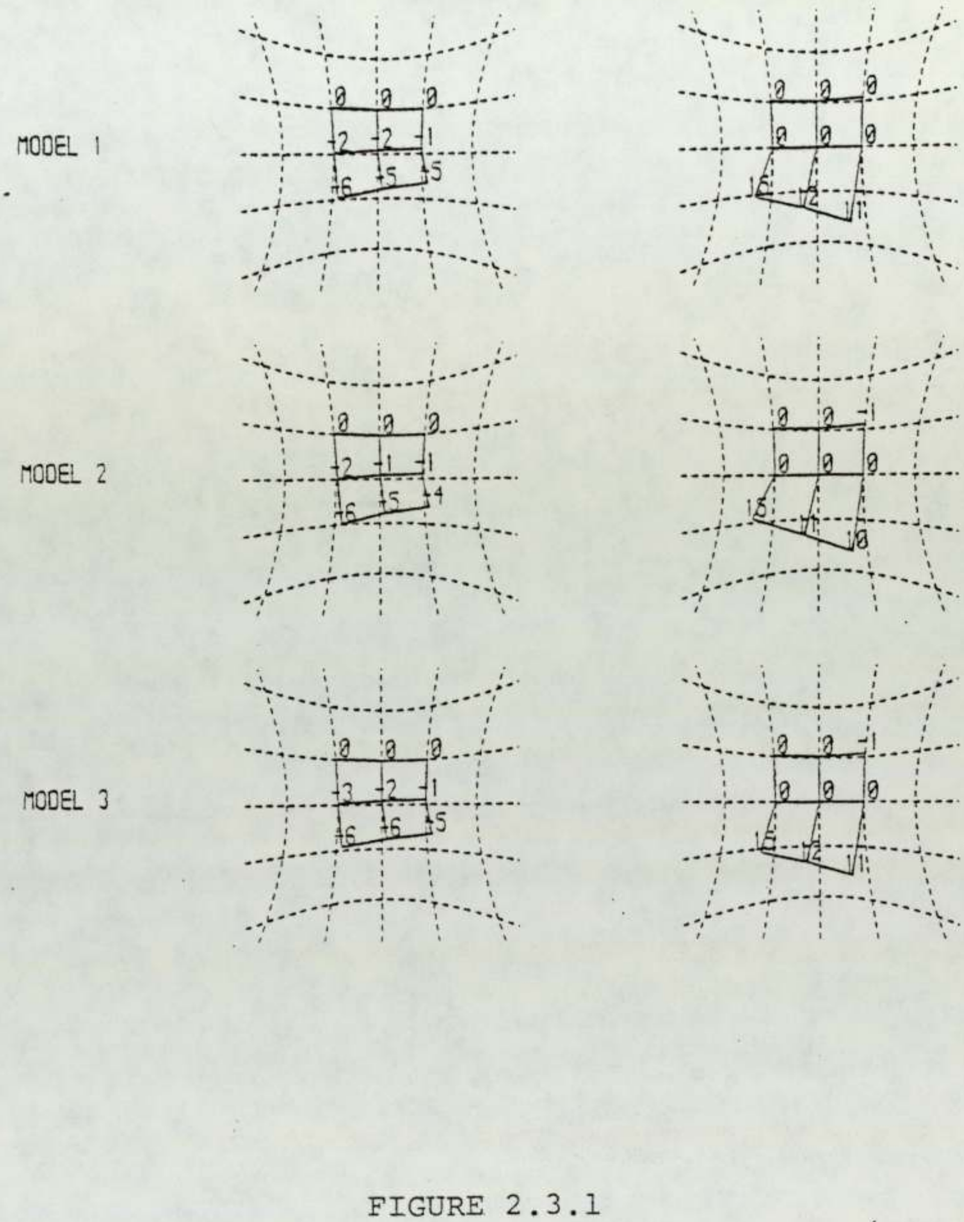

FIGURE 2.3.1

HESS CHARTS PREDICTED BY MODELS
WITH PARESIS OF THE IV NERVE

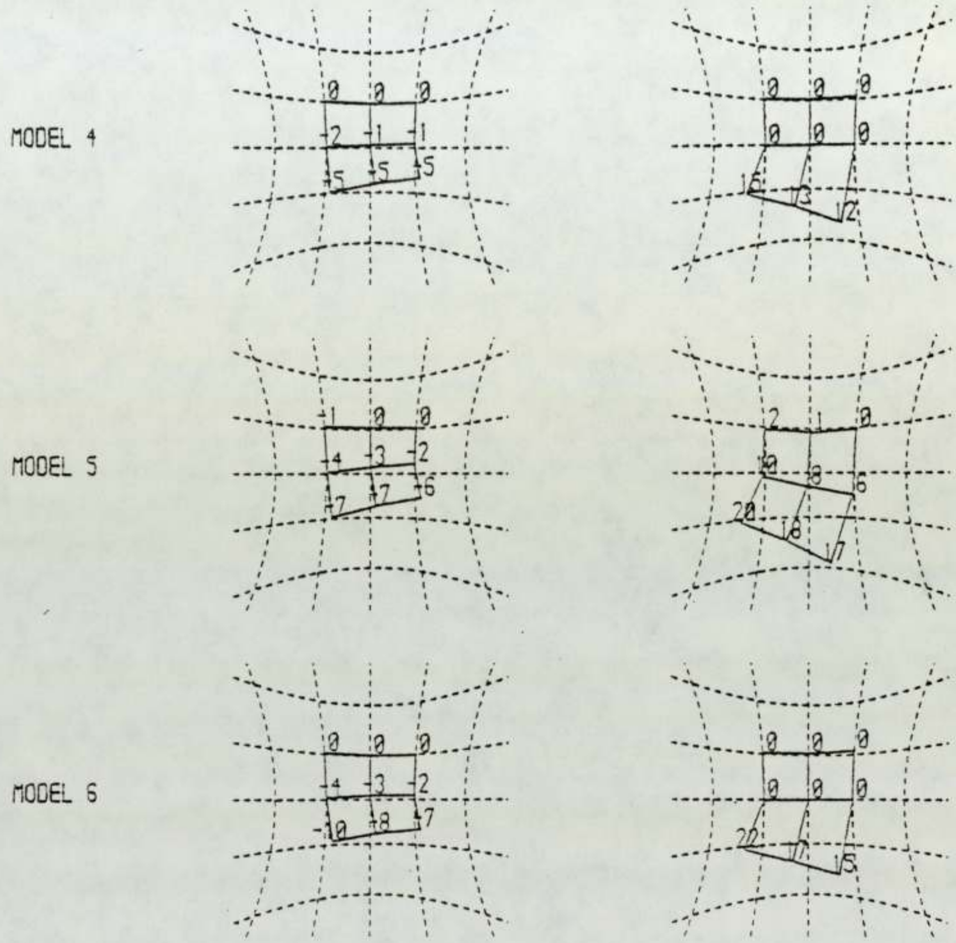

FIGURE 2.3.1 (cont'd)

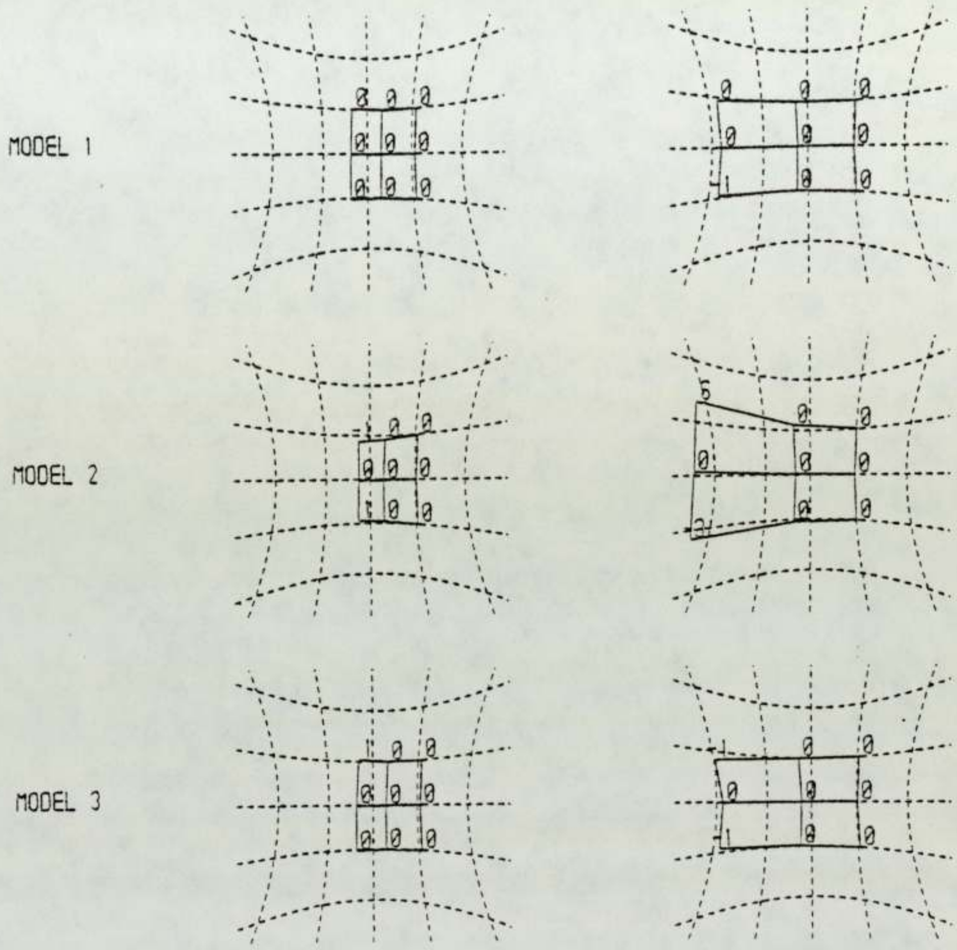

FIGURE 2.3.2

HESS CHARTS PREDICTED BY
MODELS WITH PARESIS OF THE
VI NERVE

MODEL 4

MODEL 5

MODEL 6

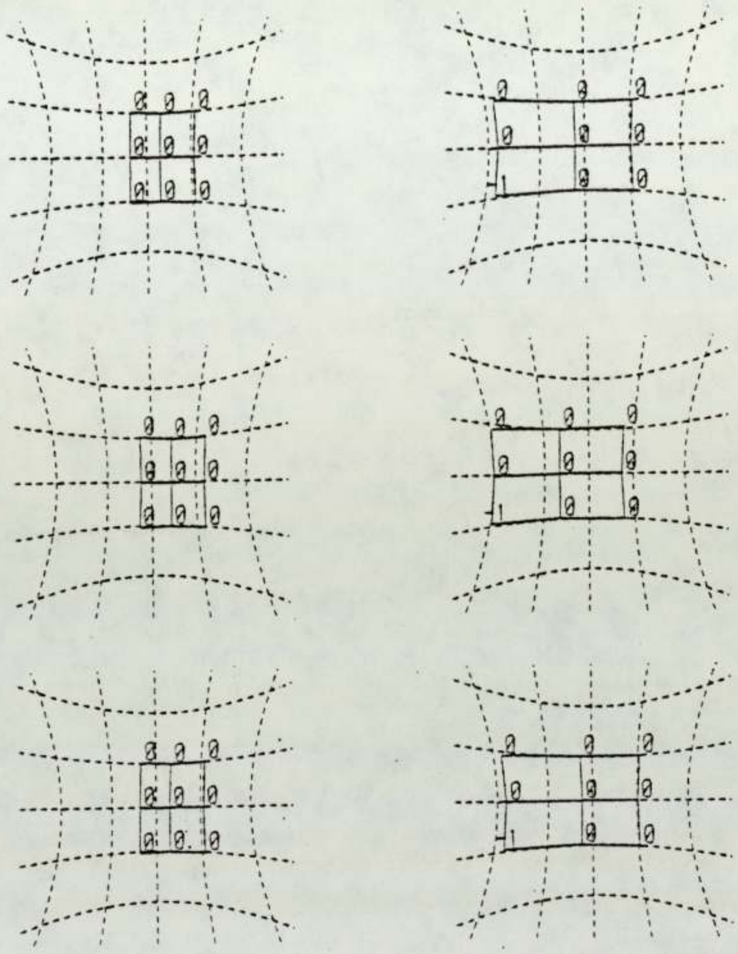

FIGURE 2.3.2 (cont'd)

2.4 Comparison of the predictions of the models with clinical data

The procedure for testing each model against the clinical data of each patient consisted of decreasing the muscle strength in steps of 10% and calculating a measure of deviation from the clinical data at each level of paresis. The measure of deviation that was used consisted of the square of the angle, in degrees, between the gaze direction predicted by the computer and the gaze direction recorded on the patient's Hess chart. In order to provide a quantitative estimate of the match over all 9 positions recorded for each eye, the calculated values for the measure of deviation at each position were summed, and these values have been tabulated.

The deviations of the predictions of each model from the clinical data are shown in Table 2.4.1. Initially, only the primary deviations were considered because of the additional assumptions needed to produce a binocular model. For both types of pareses, model 5 shows a considerable improvement over model 1. Model 6 performs as well as model 1 with sixth nerve palsy and somewhat better with fourth nerve palsy. Interestingly, model 3 performed better than model 1 with fourth nerve palsy, although it did not perform as well with sixth nerve palsy. Models 2 and 4 both performed worse than model 1.

TABLE 2.4.1

DEVIATIONS OF THE PREDICTIONS OF THE SIX MODELS FROM THE CLINICAL DATA

	MODEL					
LR Paresis	1	2	3	4	5	6
LU	8.6	10.7	8.2	8.2	13.9	8.6
HF	9.6	15.8	10.4	10.2	4.1	9.7
SP	34.1	21.7	32.1	33.9	49.4	33.9
WJ	32.4	22.5	32.8	31.7	50.2	32.5
JA	59.3	36.6	57.8	56.1	100.4	58.1
EM	167.9	195.2	174.0	172.5	98.5	169.2
MA	165.6	215.1	170.5	174.3	85.6	164.9
DS	100.6	109.9	108.4	106.5	63.4	100.7
DM	190.0	217.8	199.7	198.4	109.0	190.2
PR	503.7	596.2	522.8	515.0	414.6	504.1
Means	127.2	144.2	131.7	130.7	98.9	127.2
SO Paresis						
DB	57.0	58.9	54.0	59.1	28.9	54.7
AC	82.0	86.4	78.9	83.8	73.4	83.4
EW	70.7	76.0	67.9	76.6	51.9	69.3
AB	115.3	119.2	104.1	124.2	94.1	104.3
SE	103.9	109.9	92.6	111.4	86.2	89.9
Means	85.8	90.1	79.5	91.0	66.9	80.3

On the basis of these results it was decided that an improved version of Robinson's (1975) model could be produced by replacing the reciprocal innervation function used by him with the one tried out in model 5, and by removing the passive anti-torsion force, as tried in model 6. In order to decide between the two computational schemes for modelling both the primary and secondary deviation, described in the theory section, two versions of the model were produced, each utilising one of the alternative computational schemes.

The deviations of the predictions of each of the two computational schemes from the clinical data are shown in Table 2.4.2. For both the sixth and fourth nerve pareses scheme A performed better than scheme B, though the difference was most marked with fourth nerve pareses, where the match to the clinical data with scheme B was worse for the secondary deviation than with scheme A.

The validity of the final version of the model may be appreciated by noting that the measure of deviation of the clinical data from the standard Hess chart positions varied from 159.0 to 725.0 for the fourth nerve pareses and from 198.8 to 4272.1 for the sixth nerve pareses. The Hess charts produced by the final version of the model, which best matched the Hess charts of the patients with fourth nerve pareses and sixth nerve pareses, are shown in Figures 2.4.1 and 2.4.2 respectively. Also shown in the

TABLE 2.4.2

DEVIATIONS OF THE PREDICTIONS OF THE TWO BINOCULAR SCHEMES FROM THE CLINICAL DATA

	SCHEME A			SCHEME B		
	LE	RE	TOTAL	LE	RE	TOTAL
LR Paresis	1	2	3	4	5	6
LU	19.7	28.7	48.4	64.6	18.6	83.2
HF	33.4	99.7	133.1	33.4	103.1	136.5
SP	49.3	60.7	110.0	49.3	56.7	106.0
WJ	50.3	58.5	118.8	50.3	74.8	125.1
JA	99.5	181.1	280.6	99.5	182.2	281.7
EM	123.0	92.5	215.5	122.3	105.7	228.0
MA	142.5	168.4	310.9	142.5	186.0	328.5
DS	63.0	119.9	182.9	63.0	108.1	171.1
DM	109.0	149.5	258.5	109.0	157.9	266.9
PR	411.7	296.7	708.4	411.7	304.9	716.6
Means	110.1	126.6	236.7	114.6	129.8	244.4
SO Paresis						
DB	30.1	28.7	53.8	27.7	59.1	86.8
AC	73.0	113.3	186.3	73.0	169.5	242.5
EW	97.6	33.9	131.5	52.9	79.0	131.9
AB	154.5	60.5	215.0	72.1	104.7	176.9
SE	107.9	151.4	259.2	223.6	323.7	547.4
Means	92.6	77.6	169.2	89.9	147.2	237.1

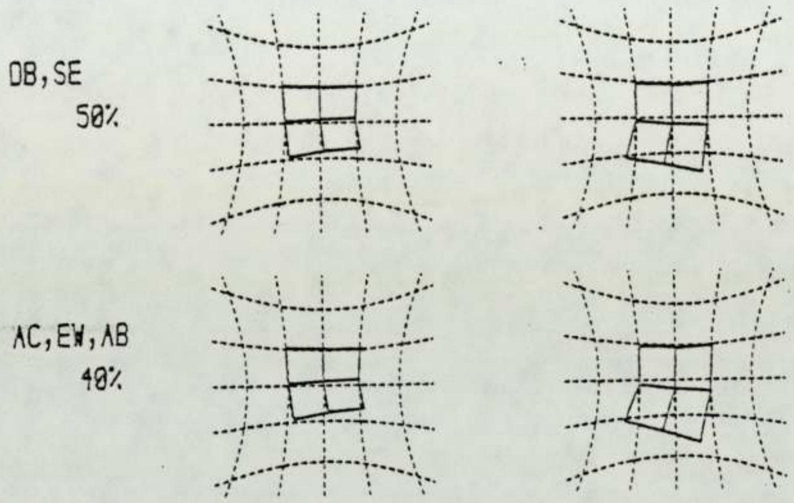

FIGURE 2.4.1

BEST FIT OF SCHEME A TO THE HESS
CHARTS OF PATIENTS WITH PARESIS
OF THE IV NERVE

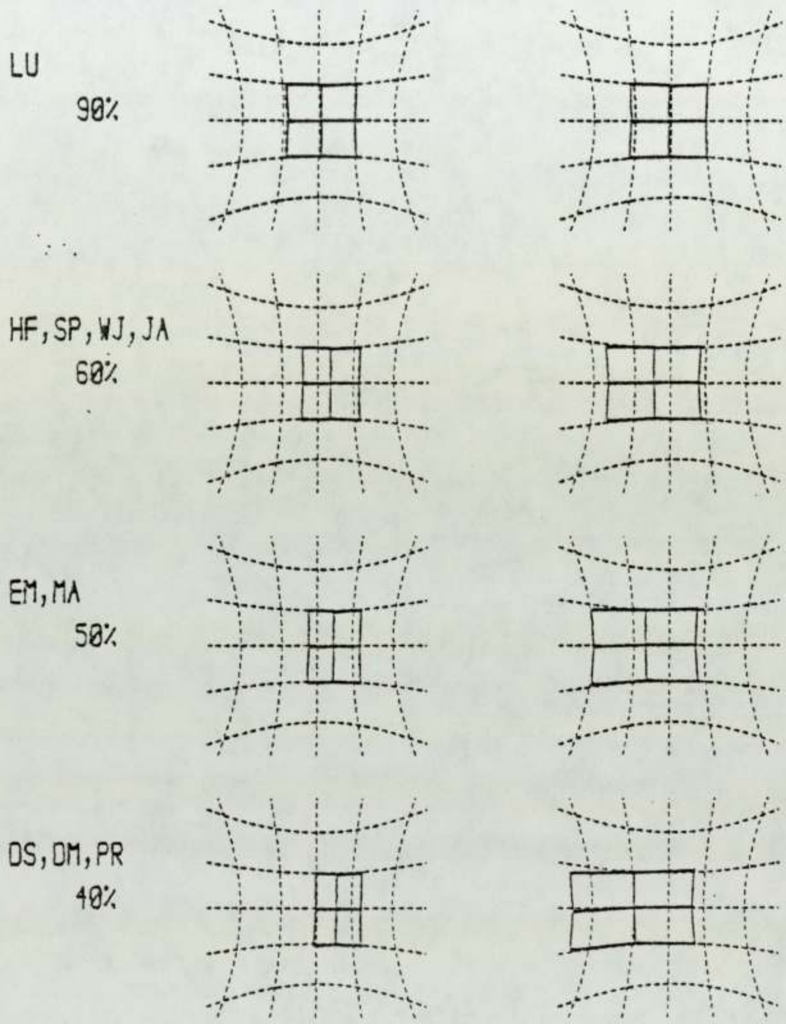

FIGURE 2.4.2

BEST FIT OF SCHEME A TO THE HESS CHARTS OF PATIENTS WITH PARESIS OF THE VI NERVE figures is the level of paresis which produced the best fit.

2.5 Selection of the best model

The most surprising finding in the comparison of the six different versions of the model was how similar their behaviour was. This makes the task of modelling extraocular muscle co-operation easier in that the predictions of the model remain valid despite variations of the parameters in different individuals. Thus a change in the relative strengths of the muscles, as might reasonably be expected in different individuals, did not produce radically different predictions. It was also interesting to note that the fixed axis assumption which has been argued for by Jampel (1970, 1975) provided a close match with the clinical data. From a purely computational viewpoint, this assumption simplifies the programming of the calculations.

For the computation of both primary and secondary deviations, scheme A produced a better match to the clinical data than did scheme B. However, this data did not test the difference between the way in which the two schemes pass cyclotorsion from one eye to the other. With a IV nerve palsy, both schemes predict excyclotorsion in the paretic eye when the non-paretic eye is fixating, but scheme A predicts greater excyclotorsion in the non-paretic eye, whilst scheme B predicts zero excyclotorsion in the non-paretic eye, when the paretic eye is fixating. Nakayama (1983) determined photographically the torsion in the eyes of a patient with Brown's syndrome and found that only the affected eye showed excyclotorsion, a finding which supports Scheme B in preference to Scheme A.

Another physiologically unrealistic feature of Scheme A is that if the paretic eye is fixating then the procedure requires that abnormal innervation values are generated to hold the eye in position. If the nervous system is capable of this computation, it is not apparent why muscle paresis cannot be compensated for. The reason why Scheme B does not produce as good a model as Scheme A is clear from Figure 2.5.1 which shows the Hess charts predicted Schemes A and B with the superior oblique of the left eye producing only 30% of its normal active tension. Because the torsional action of the muscle is not transferred, the secondary deviation is not larger than the primary deviation.

In the next chapter some applications of the model with the alternative reciprocal innervation function and no passive anti-torsional force will be described. Only the second application requires a binocular model and Scheme B was used for this application because of the arguments against Scheme A which have just been described.

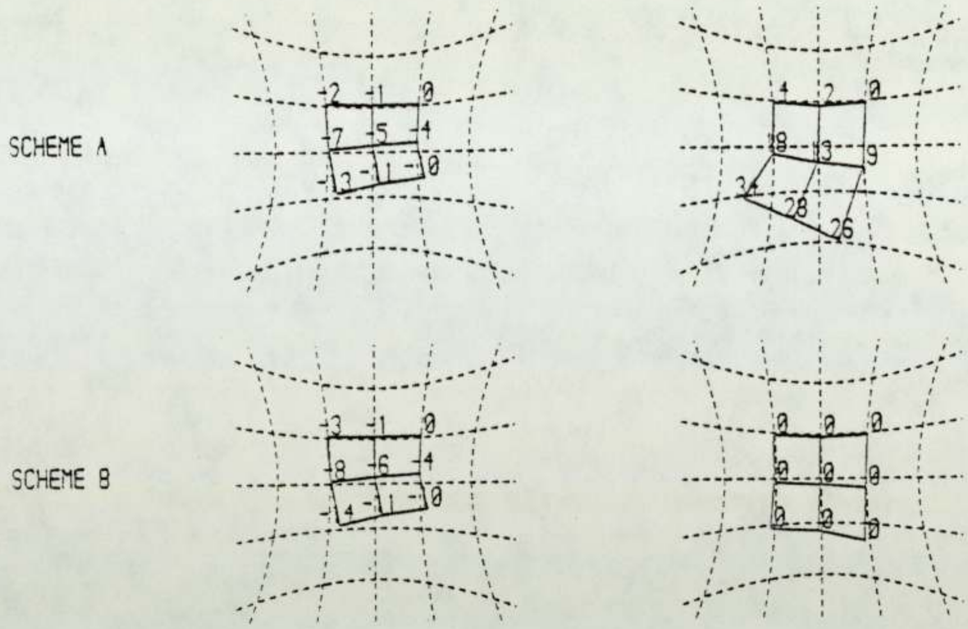

FIGURE 2.5.1

HESS CHARTS PREDICTED BY SCHEME A AND SCHEME B WHEN THE SUPERIOR OBLIQUE OF THE LEFT EYE PRODUCES ONLY 30% OF ITS NORMAL ACTIVE TENSION

CHAPTER 3

APPLICATIONS OF THE MODEL

3.1 Optimal use of the Hess screen test

On the standard Hess charts, are marked the directions in which each muscle is presumed to be maximally active. These directions are based on geometrical arguments from the anatomy of the muscles, and one application of a model of the mechanics of the extraocular muscles is to update these arguments to include muscle forces.

Although palsy of a single muscle can cause horizontal, vertical and torsional deviations, in order to accentuate the deviation recorded, a scheme of computation was devised with the model such that the horizontal deviation associated with palsy of the horizontal recti was maximised and so that the vertical deviation associated with palsy of the vertical recti and obliques was maximised, in the direction of the lines of the chart.

In order to describe the scheme, the lines on the Hess chart which correspond to rotating the line of fixation around a fixed horizontal axis will be designated as lines of iso-azimuth, and the lines which correspond to rotating the line of fixation around a vertical axis will be designated as lines of iso-latitude.

For the horizontal recti the scheme was as follows. First, the gaze direction associated with each point along a line of iso-azimuth was specified. Second, the axis of rotation which would move the line of fixation along the line of iso-latitude that passed through the point of fixation was determined. This involved calculating the cross product of the unit vectors corresponding to lines of fixation directed at points along the line of iso-latitude, but one degree on either side of the actual point of fixation. Finally, the muscle force directed along the line of iso-latitude was obtained by multiplying the force calculated by the model with the dot product of the axis of rotation of the muscle and the axis of rotation which would move the line of fixation along the line of iso-latitude.

For each angle of azimuth the angle of latitude was varied from 30 degrees of elevation to 30 degrees of depression in 5 degree steps, and the angle of elevation at which the muscle was maximally effective was recorded. The angle of azimuth was varied from -45 degrees of adduction to 45 degrees of abduction in 5 degree steps and the locus of points of maximal effectiveness of the horizontal recti of the right eye are shown in Figure 3.1.1. The procedure for the vertically acting muscles was identical except that the lines of iso-azimuth and iso-latitude were reversed in the sequence of calculations. The results for these muscles in the right eye are also shown in Figure 3.1.1.

The results conform with expectations in that the horizontal recti are most effective in the horizontal plane, although the medial rectus becomes more effective with depressed gaze in convergence, and in that the obliques

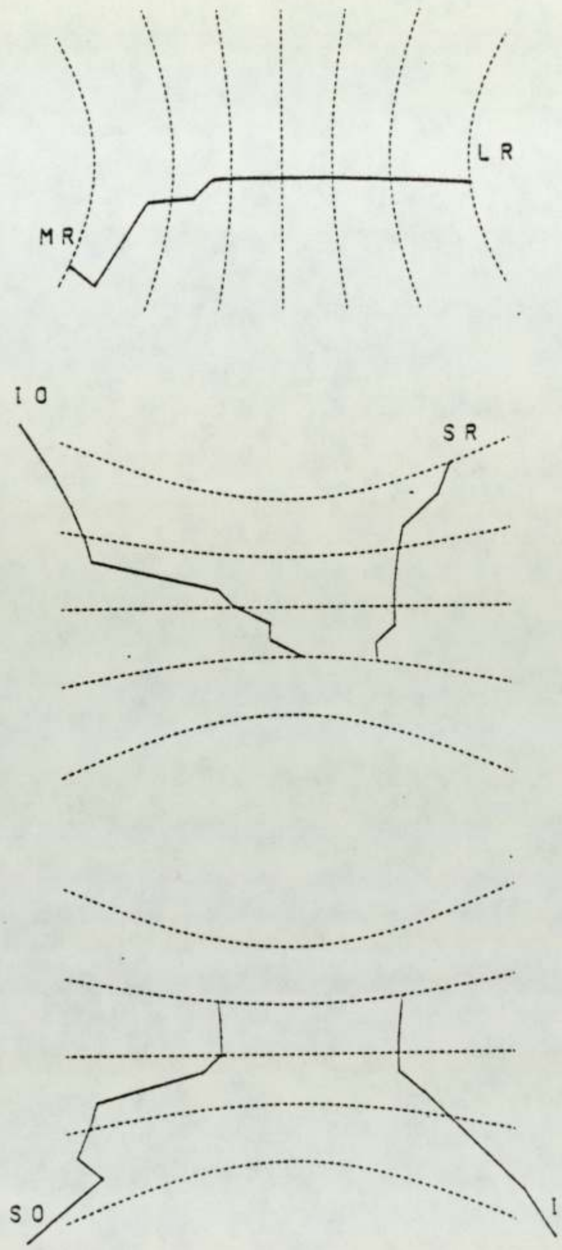

FIGURE 3.1.1

DIRECTIONS OF THE HESS CHART ALONG
WHICH MUSCLES ARE MAXIMALLY EFFECTIVE

are more effective in adduction whilst the vertical recti are more effective in abduction. From the point of view of improving the test however, the results of the calculations are disappointing in that no particular set of test positions, such as the central 15 degree positions or outer 30 degree positions on the Hess chart, is highlighted as being of diagnostic value. Indeed the inferior oblique was found to be maximally effective at the limit of 45 degrees of adduction, above 10 degrees of elevation, so that the gaze directions where it would be maximally effective according to the model are probably not within the field of view.

3.2 Muscular factors involved in the aetiology of A and V syndromes

This application was investigated in collaboration with Mrs Howrie, the Head of the Orthoptics Department at the Birmingham and West Midlands Eye Hospital. A and V phenomena are an interesting subject for investigation with the model because a number of possible muscular defects have been proposed as a cause for them and the mechanical effect of these defects can be tested with the model.

The A and V syndromes are forms of strabismus in which the horizontal deviation varies according to whether the eyes are looking up or down. In conjunction with exotropia, where the eyes are divergent, or esotropia, where the eyes are convergent, the following four patterns arise:

A EXO Eyes more divergent in DOWN gaze
A ESO Eyes more convergent in UP gaze
V EXO Eyes more divergent in UP gaze
V ESO Eyes more convergent in DOWN gaze

Urist (1958) proposed that A and V phenomena may be secondary to underaction or overaction of the horizontal recti since the medial recti are more active in convergence with depressed gaze, whilst the lateral recti are more active in divergence with elevated gaze. The Hess charts produced by the model with bilateral 50% changes in the effectiveness of the horizontal recti are shown in Figure 3.2.1. Both bilateral underaction and bilateral overaction of the horizontal recti produce no evidence of A or V patterns.

The most direct cause of A and V patterns appears to be a bilateral weakness in one of the vertically acting muscles, as demonstrated by the Hess charts shown in Figure 3.2.2, which were produced by the model with 50% underaction of the vertically acting muscles. The results clearly associate a V EXO pattern with bilateral underaction of the superior rectus, an A EXO pattern with bilateral underaction of the inferior rectus, a V ESO pattern with bilateral underaction of the superior oblique and an A ESO pattern with bilateral underaction of the inferior oblique. These results are in keeping with the view summarised by

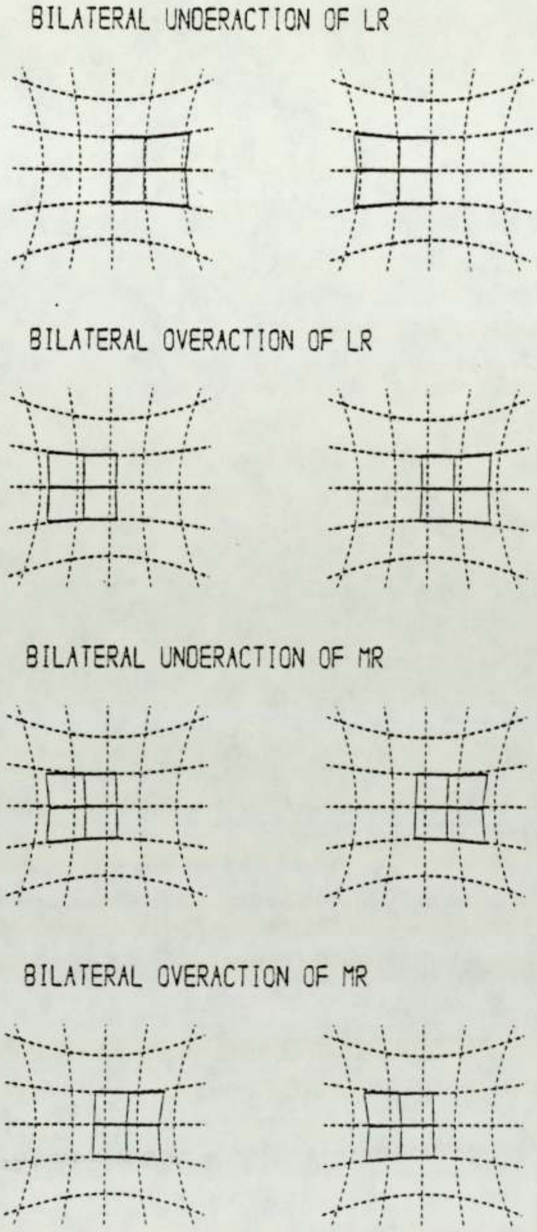

FIGURE 3.2.1

EFFECTS OF CHANGES IN THE STRENGTHS

OF THE HORIZONTALLY ACTING MUSCLES

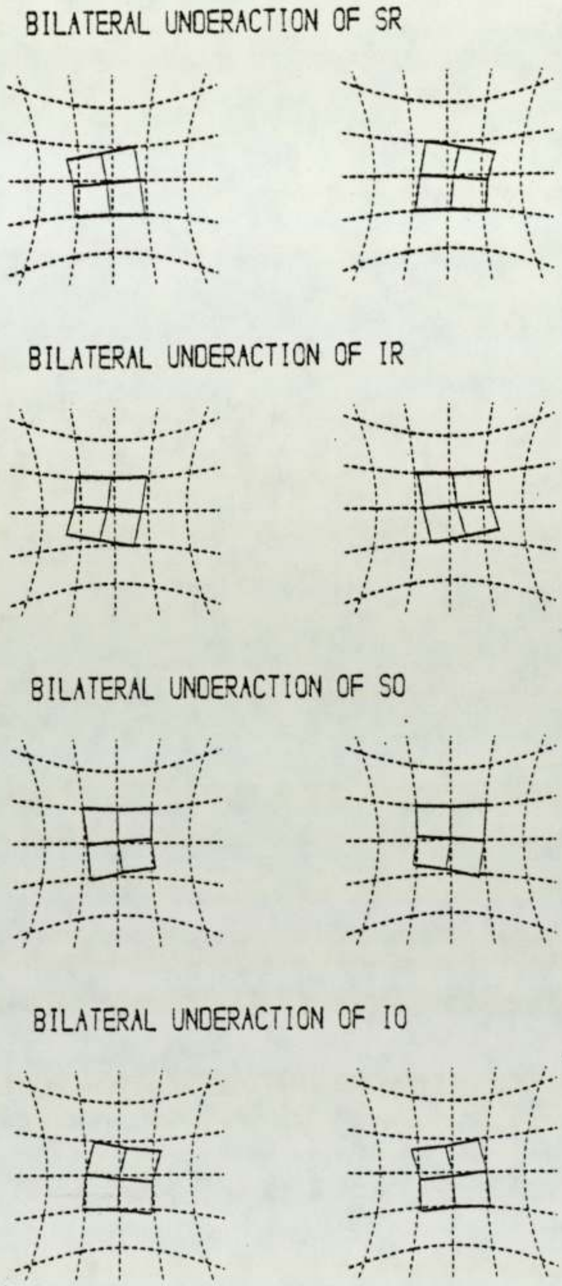

FIGURE 3.2.2

EFFECTS OF CHANGES IN THE STRENGTHS

OF THE VERTICALLY ACTING MUSCLES

Wesson (1960).

Gobin (1968) has proposed that if an oblique muscle is sagittalised (i.e. makes a smaller angle with the line of fixation when the eyes is in its primary position than does its antagonist) then it is relatively less effective at producing torsional movement, hence the pair of obliques active together produce a cyclophoria and that the A or V pattern arises as a result of compensatory actions by the other muscles for this cyclophoria.

The effect of sagittalisation was investigated with the model by posteropositioning the obliques. The Hess charts produced with bilateral 5 millimetre posteropositioning of the obliques are shown in Figure 3.2.3A. A bilateral 5 millimetre change in the location of the insertions of the superior and of the inferior obliques, produced little cyclotorsion, which suggests that the subsequent compensatory actions of the other muscles required by Gobin's theory would not be elicited. Similarly, it was found that anteropositioning of the superior obliques, recommended for surgical correction of the A pattern and anteropositioning of the inferior obliques, recommended for surgical correction of the V pattern produced little change according to the model.

Postic (1965) has suggested that vertical displacement of the insertions of the horizontal recti may be a cause of the A and V syndromes. This opinion was borne out by the model which demonstrated clear A and V patterns

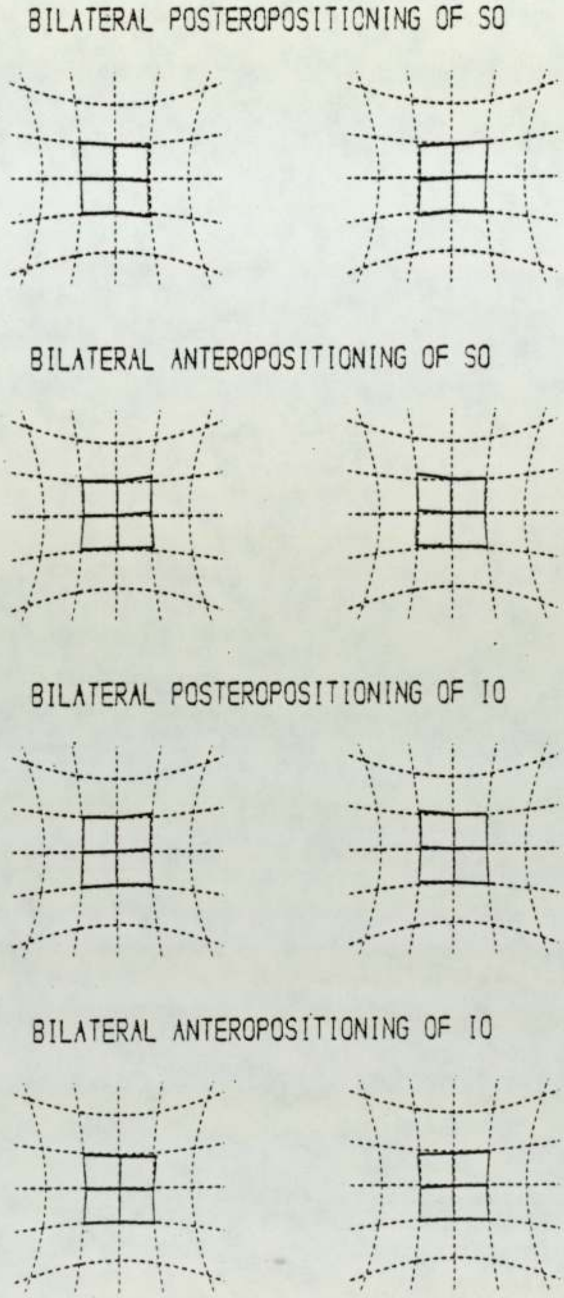

FIGURE 3.2.3

EFFECTS OF CHANGES IN THE LOCATIONS OF THE INSERTIONS OF THE OBLIQUES with bilateral 5 millimetre shifts of the insertions, as shown in Figure 3.2.4. The only similar calculations appear to be those of Crone (1973) who describes the A and V patterns that should be obtained with a displacement of the insertions through 45 degrees. His results are qualitatively similar to those given here, although he did not ~~tske~~ take into account the forces exerted by the individual muscles.

The model confirms that the most likely cause for an A or V syndrome is an underaction of one of the vertically acting muscles, although alteration of the height of the insertions of one of the horizontal recti will also result in an A or V pattern.

3.3 Geometric constraints on muscle surgery

The final application of the model is more speculative than the previous two since the results depend on the paths taken by the muscles according to the model, which have not been experimentally substantiated and indeed, are definitely incorrect in some gaze directions, as pointed out in Section 1.4. However, the application has been included because it illustrates another facet of the model which may be developed if more information becomes available, as suggested, by Clement (1984).

In order to investigate purely physical constraints on the amount of surgical repositioning of the insertions of the muscles that can be performed, it is appropriate

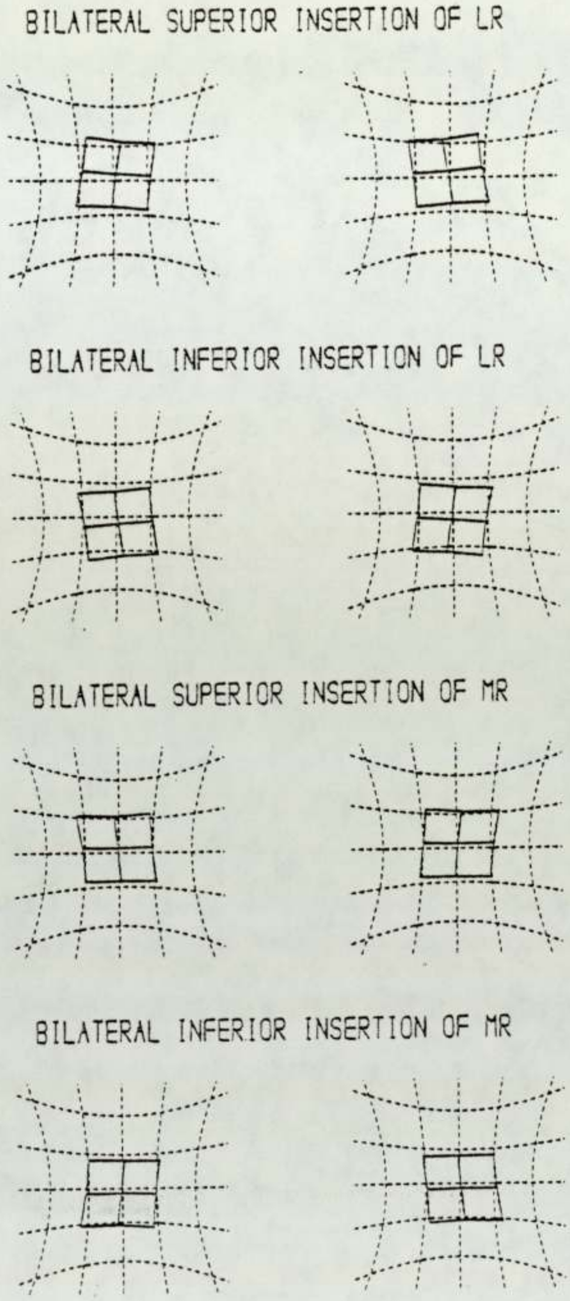

FIGURE 3.2.4

EFFECTS OF CHANGES IN THE LOCATIONS

OF THE INSERTIONS OF THE HORIZONTAL

RECTI

to replace each single muscle fibre in the model by a band of ten fibres. In this way the wide insertions of the muscles can be incorporated in the model and a computerised ophthalmotrope can be used to examine how far the insertions of the muscles can be moved before they interfere with each others movement.

To provide the co-ordinates of points along a line of insertion approximately 10 millimetres broad, the co-ordinates of the midpoints of the muscle insertions, taken from the data of Volkmann (1869), were rotated in fixed angular steps of 5.1 degrees around an axis which lay in the muscle plane but was perpendicular to the line of insertion. So as to ensure that the width of the line of insertion was the same for each muscle, the length of each of the insertion vectors was adjusted to their average value of 12.43 millimetres. In order to represent each muscle by a band of parallel fibres, the exit paths of each of the ten fibres were made equivalent to that of one of the central pair of fibres.

An example of the use of the ophthalmotrope is provided by Scott (1978) who recommended a maximum of 10 millimetres of recession of the inferior oblique, since the lateral border of the inferior oblique is about 10 millimetres from the normal path of the inferior rectus. The computer based ophthalmotrope was used to produce Figure 3.3.1 which shows the eyes, as seen from directly beneath them, after the inferior oblique of the right eye has been recessed by 10 millimetres. This figure implies that a recession of up to 15 millimetres could be carried out without the inferior oblique interfering with the path of the inferior rectus.

Another example of geometrical considerations in extra-ocular muscle surgery arises in the anteropositioning of the obliques. Figure 3.3.2 shows the eyes as seen from directly above, after the superior oblique has been anteropositioned by 10 millimetres and it can be seen that, according to the model, the insertion of the superior rectus is now located in the tendon of the superior oblique. Similarly, Figure 3.3.3 shows the normal left eye, viewed from the left hand side and Figure 3.3.4 shows the effect of anteropositioning the inferior oblique by 10 millimetres. The geometrical effect of this operation is to cause the insertion of the lateral rectus to be located in the tendon of the inferior oblique.

At present, these conclusions are speculative and in order for their mechanical ~~implicstions~~ implications to be clarified, some method of calculating the way in which the interference of the paths of the muscles alters their effective axes of rotation, will have to be formulated. In order to calculate the amount by which muscles can be deformed, a geometric model such as has been described in this Section, is insufficient. What is required is a mechanical model which incorporates the coupling between

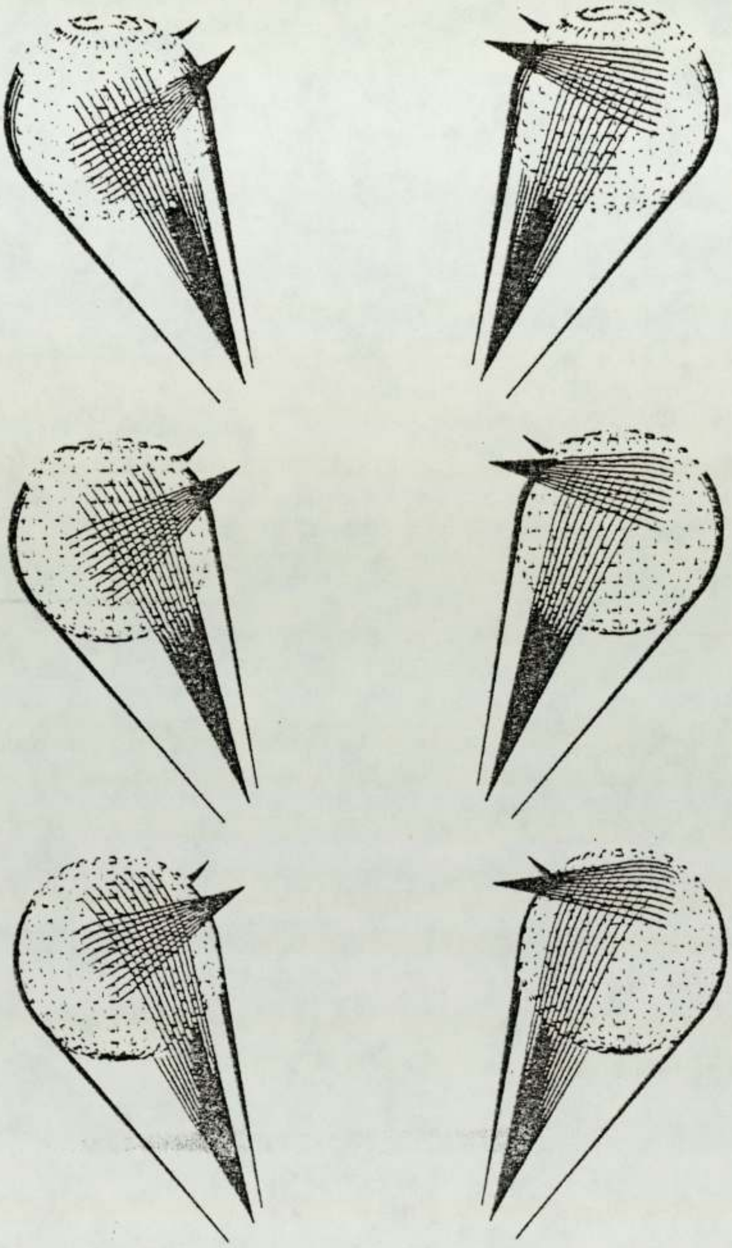

FIGURE 3.3.1

GEOMETRIC EFFECT OF ANTEROPOSITIONING

THE SUPERIOR OBLIQUE OF THE RIGHT EYE

BY 10 mm

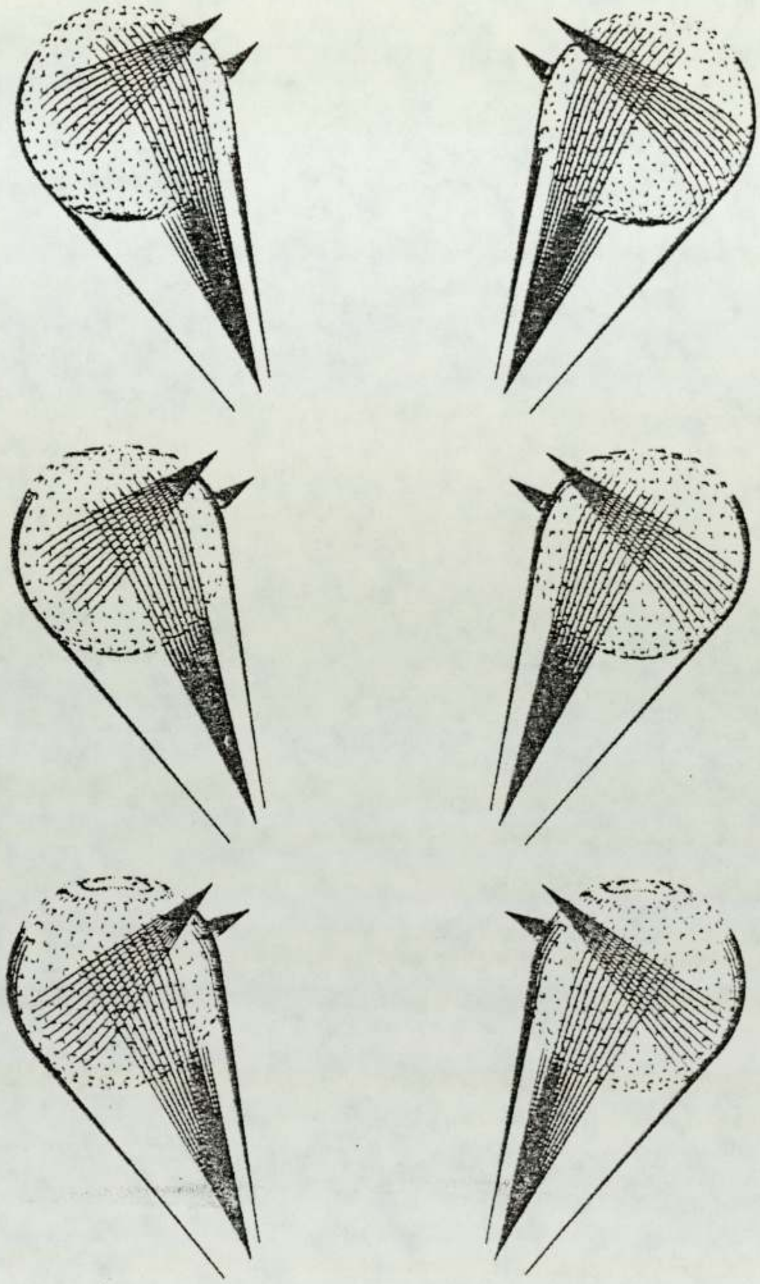

FIGURE 3.3.2

GEOMETRIC EFFECT OF RECESSION OF

THE INFERIOR OBLIQUE OF THE RIGHT EYE

BY 10mm

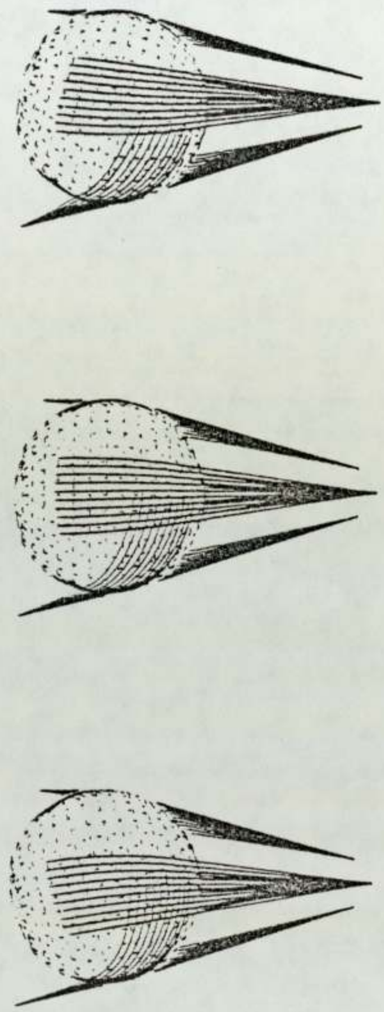

FIGURE 3.3.3

NORMAL LOCATION OF THE INFERIOR

OBLIQUE OF THE LEFT EYE

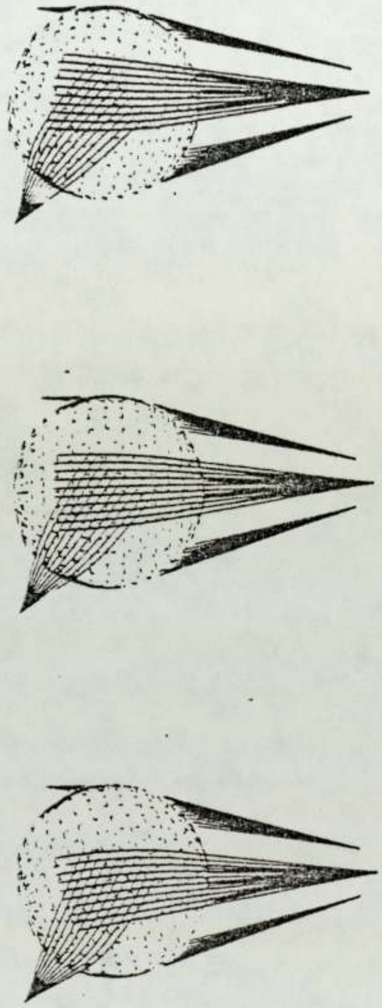

FIGURE 3.3.4

GEOMETRIC EFFECT OF ANTEROPOSITIONING THE INFERIOR OBLIQUE OF THE LEFT EYE BY 10mm individual fibres. Until experimental data on the mechanical coupling between fibres is available, realistic assumptions cannot be made.

CONCLUSION

Initially, Robinson's (1975) model was heralded as being an embodiment of the mechanics of the extraocular muscles and that it was only a short step from the model to solving all strabismus surgery calculations. This view is encapsulated in the following quotation from the concluding section of Carpenter's book "Movements of the Eyes", which surveyed the state of knowledge around 1977, two years after the model had been published. He writes that "Our knowledge of the kinematics and dynamics of the eye has progressed steadily since the first conceptions of such men as Fick and Helmholtz, a little over a century ago, to the point where we can now simulate the relationship between the activity in the six muscles and the resultant position of the eye with accuracy, and predict in advance what the effect will be of particular ophthalmic surgical procedures such as artificial lengthening or shortening of individual muscles to correct squint. Perhaps the main block of the acceptance of quantitative evaluation of such procedures is the novelty of the notion that such preoperative calculations can provide more than an approximate indication of the actual outcome.

The more modest goal which has been pursued in this study is to produce a model which can describe the simplest case of abnormal co-operation of the extraocular muscles, namely, the isolated nerve palsy. What became clear from the study was that with the reciprocal innervation function chosen by Robinson there was no possibility of his model showing the type of flexibility needed to cope with differing amounts of palsy. There is no real prospect of modelling strabismus surgery whilst the model cannot properly describe the initial underaction of the muscle, as in any longstanding strabismus the situation will be made even more uncertain by contracture of the direct antagonist and possibly also underaction of the contralateral antagonist.

At this outset, there was no way of knowing that the assumptions in the model were not highly interactive, in which case the procedure of comparing models each of which differ by just one assumption would not have worked. The effectiveness of the procedure in this instance suggests that it is a valid approach in the modelling of biological systems to assume that the assumptions are not highly interconnected. The most immediate continuation of the work would be to introduce possible assumptions concerning the effect of fibrosis of the muscles and to test their validity against clinical data. The problem arises in the selection of the clinical data, since it is not easy to be sure that only one muscle is fibrotic, unlike the situation with isolated nerve palsies. Hence this next advance will probably not be possible until a sufficient number of direct measurements of contracture have been made, during muscle surgery.

Given that the present model can be developed to incorporate fibrosis of the muscles, the question then arises as to whether or not it would be applicable to strabismus surgery. One further difficulty which occurs is that of coping with individual variations, which are especially prominent in muscle surgery. Individual eyes and muscles have different shapes and sizes and muscles can look the same whilst having different strengths. Furthermore, individual surgeons have different techniques and the effect of an operation such as recession will depend on how close to the line of insertion the surgeon cuts the muscle and on how small scar tissue is formed. For a model to be effective in dealing with individual cases, methods will have to be devised for estimating parameters such as muscle strength and amount of contracture. An advance in this direction has been made by Collins and Jampolsky (1982) in surgery of the horizontal recti. They have used a simple spring model of the forces on the globe and have measured the tension in the muscles during the operation. The effectiveness of their procedure is evidence that models of the co-operation of the extraocular muscles should be kept as simple as possible, with a minimum of assumptions which can be checked directly. One further advantage of this policy is that the model is more likely to have widespread use if it does not require large amounts of time on a mainframe computer, as do the models described in this thesis.

For any model of extraocular muscle co-operation to play a role in the investigation of disorders of the extraocular muscles it must be accepted as valid by the orthoptic and ophthalmic community. This problem of psychological acceptances commonly occurs when users are becoming acquainted with a computer programme and Weinberg (1971) has pointed out that the choice of appropriate test data on which to validate the programme is crucial to its acceptance, for if a programme fails on the particular data with which a user is familiar, it will usually be rejected in its entirety. Given that surgery of an extraocular muscle would not be carried out on the basis of the Hess chart alone, but on the basis of a battery of orthoptic investigations such as the cover test, the diplopia field, the binocular field of single vision and the position of the head, it would appear advantageous for the model to simulate the results of these tests as well as the Hess chart.

This study has shown that it is possible to produce a binocular model of the actions of the extraocular muscles which will automatically generate an accurate description of the behaviour of the extraocular muscles of a patient with a IV or VI nerve palsy. It is proposed that the utility of such models can be increased by simplfying the assumptions (for example, by using axes of rotation fixed with respect to the head, length changes based on the shortest path assumption and possibly linear length-tension curves) and by increasing the range of orthoptic tests which the model emulates. There is then a very real possibility of a model of the co-operation of the extraocular muscles playing a central role in orthoptic investigations and in surgical correction of strabismus.

APPENDIX 1

MATHEMATICAL DESCRIPTION OF THE MODELS

I.1 Specification of the orientation of the eye

To describe the position of the eye one may consider a set of Cartesian basis vectors I, J, K fixed in the orbit and a set I', J', K' fixed with respect to the eyeball, such that in the primary position they coincide with the K and K' vectors directed along the line of fixation. When they do not coincide the position of the eye may be specified in terms of the transformation required to change the IJK system into the I'J'K' system of base vectors.

To carry out a transformation from one set of Cartesian axes to another set with a common origin one can perform three successive rotations in a specific sequence. The following is such a sequence:

R1) Rotate I, J, K through an angle α clockwise around K to obtain I1,J1,K1.

$$\begin{bmatrix} I1 \\ J1 \\ K1 \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ J \\ K \end{bmatrix}$$

R2) Rotate I1, J1, K1 through an angle β clockwise around I1 to obtain I2,J2,K2.

$$\begin{bmatrix} I2 \\ J2 \\ K2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} I1 \\ J1 \\ K1 \end{bmatrix}$$

R3) Rotate I2, J2, K2 through an angle α clockwise around K2 to obtain I',J',K'.

$$\begin{bmatrix} I' \\ J' \\ K' \end{bmatrix} = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I2 \\ J2 \\ K2 \end{bmatrix}$$

The individual rotations can be combined to produce a matrix Q=R3R2R1 of the whole sequence of rotations. The components of the matrix Q are as follows:

$$Q= \begin{bmatrix} \cos\alpha\ \cos\gamma - \sin\alpha\ \cos\beta\ \sin\gamma & -\sin\alpha\ \cos\gamma - \cos\alpha\ \cos\beta \sin\gamma & \sin\beta\ \sin\gamma \\ \cos\alpha\ \sin\gamma + \sin\alpha\ \cos\beta\ \cos\gamma & -\sin\alpha\ \sin\gamma + \cos\alpha\ \cos\beta \cos\gamma & -\sin\beta\ \cos\gamma \\ \sin\alpha\ \sin\beta & \cos\alpha\ \sin\beta & \cos\beta \end{bmatrix}$$

Hence if x,y,z are the co-ordinates of a point in the I,J,K system and x',y',z' are the corresponding co-ordinates in the I',J',K' system then:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q^T \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

If Listings law is obeyed then the rotation angles must obey the constraint $\gamma - \alpha$

This system of orientation angles has the advantage over Fick's system used by Robinson (1975) that the passive force on the globe can be calculated directly.

I.2 Specification of the origins and insertions of the muscles

The origin and midpoint of the insertion of each muscle are specified by the vectors A and B respectively. The co-ordinates of these vectors for the muscles of the right eye are given in Table I.2.1, where the units are millimetres. The effect of rotations of the globe on the co-ordinates of the insertions may be described by using the matrix equation $B' = Q^T B$. An example of the vectors is given in Figure I.2.1 which shows the origin and insertion vectors of the lateral rectus of the right eye when it is elevated through 30 degrees.

I.3 Exit paths of muscles

Given the locations of the origins of the muscles and their insertions in the primary position, one must next specify the paths taken by the muscles in other positions of the eyeball. This problem is complicated by the fact that in the primary position the muscles fan out at their insertion so as to be attached along a line of insertion

TABLE I.2.1

CO-ORDINATES OF THE ORIGINS AND INSERTIONS OF
THE MUSCLES OF THE RIGHT EYE

		LR	MR	SR	IR	SO	IO
	I	-13. 0	-17. 0	-16. 0	-16. 0	-15.27	-11. 1
A	J	0. 6	0. 6	3. 6	- 2. 4	12.25	-15.46
	K	-34. 0	-30. 0	-31.76	-31.76	8.24	11.34
	I	10.08	- 9.65	0. 0	0. 0	2. 9	8. 7
B	J	0. 0	0. 0	10.48	-10.24	11.05	0. 0
	K	6. 5	8.84	7.63	8.02	- 4.41	- 7.18

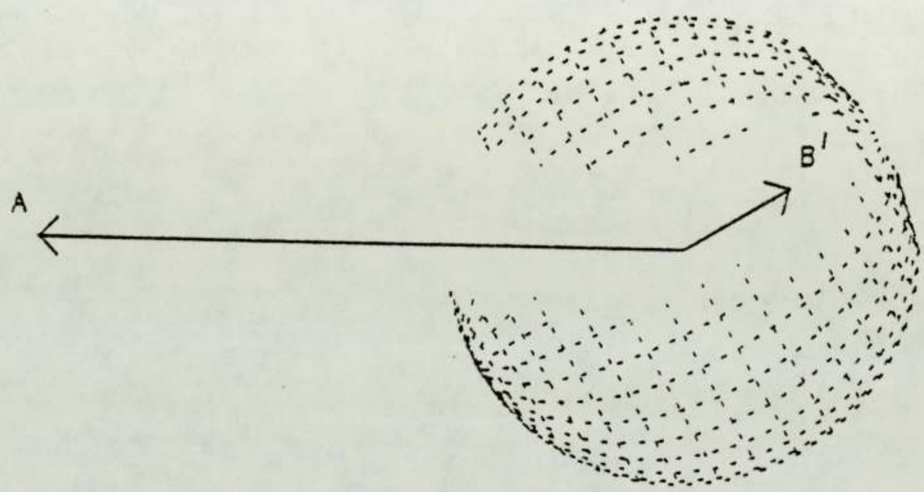

FIGURE I.2.1

ORIGIN AND INSERTION VECTORS
OF THE LATERAL RECTUS OF THE
RIGHT EYE

whose direction is given by a unit vector C (see Figure I.3.1) where:

$$\hat{C} = (A \wedge B) \; / \; |A \wedge B|$$

which is normal to the path of the muscle. Since the stiffer tendon fibres are involved at the insertion, when the eyeball is moved there will still be a tendency for the exit path of the muscle to be perpendicular to the line of insertion.

On the other hand, the shortest path for the muscle will no longer be perpendicular to the line of insertion in positions of the eyeball other than the primary one, but will lie in a plane (see Figure I.3.2), spanned by A and B' with a unit orientation vector D where:

$$\hat{D} = (A \wedge B') / \; |A \wedge B'|$$

The angle between $\hat{C}'$ and $\hat{D}$ is referred to as the 'twist' angle and one has that:

$$\cos(\text{twist}) = \hat{C}' . \hat{D}$$

To obtain the sign of the twist angle the following procedure may be used. First compute the perpendicular exit path F where:

$$\hat{F} = (B' \wedge \hat{C}') / |B' \wedge \hat{C}'|$$

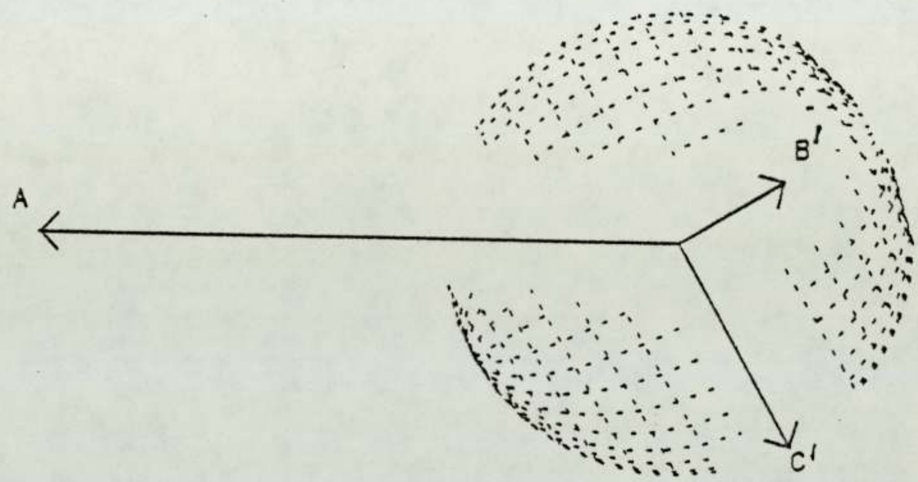

FIGURE I.3.1

THE VECTOR C' WHICH LIES ALONG
THE DIRECTION OF THE LINE OF
INSERTION

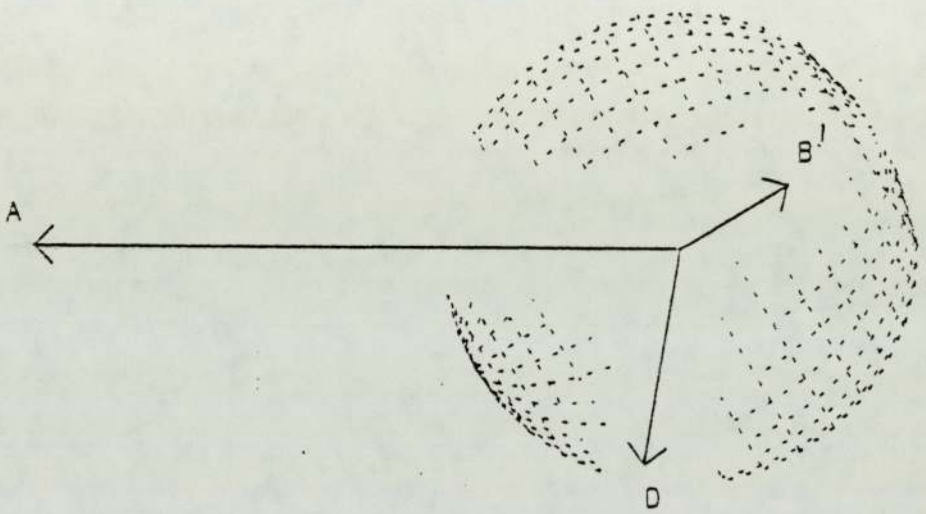

FIGURE I.3.2

THE VECTOR D WHICH IS PERPENDICULAR TO THE SHORTEST PATH PLANE

Next compute the cosine of the angle between F and D which is given F.D, and let the sign of this cosine function be the sign of the twist angle.

Robinson (1975) decided that the actual twist angle should be proportional to the cosine of the angle 'tilt' which is formed between the vector representing the line of insertion $\hat{C}'$ and the origin vector A:

$$\cos(\text{tilt}) = (\hat{C}'.A) / \left| A \right|$$

one may then define the actual twist angle (atwist) to be given by:

$$\text{atwist} = \cos(\text{tilt}) \times \text{twist}$$

Note that in the primary plane cos (tilt) is always zero so there is no twist at the insertion. The actual exit path F is then given by:

$$\hat{F} = -\sin(\text{atwist})\hat{C}' + \cos(\text{atwist})\hat{F}$$

I.4 Paths of the muscles over the eyeball

Even with the exit path of the muscle known it is still an open question as to what path the muscle follows away from the location of its insertion.

Robinson (1975) made the assumption that the muscle made contact with the eyeball along the arc of a circle.

Before determining this circle one has to check that no muscle has lost tangency to the eyeball, as for example occurs with the medial rectus in the normal eye when there is an adduction of greater than thirty-four degrees, for when a muscle loses tangency it automatically takes the shortest path. For a muscle to lose tangency the angle ang1 between A and B' must be less than the angle ang2 between A and B' when the muscle is tangential to the eyeball. These two angles may be computed for comparison according to the formula:

$$\cos(\text{ang1}) = A.B'/(\,|A|\,|B|)$$

and

$$\cos(\text{ang2}) = |B'|\;/\;|A|$$

If a muscle has lost tangency, the unit action vector $\hat{R}$ of the muscle is given by the orientation vector $\hat{D}$ of the shortest path plane (i.e. $\hat{R}=\hat{D}$). Since the muscle force is no longer acting tangentially it must be reduced, which can be achieved by scaling down the unit action vector by a constant given by:

$$\text{const}=|A| \ \sin(\text{ang1})/ \ |A-B'|$$

which is the perpendicular component of the force acting along A-B' divided by $|A-B'|$ to normalise it.

The change in the length of the muscle when it has lost tangency may be computed by means of the formula:

$$\Delta l= |B'| \ (\text{ang1}-\text{ang2})-l_o$$

where l_o is the length of the contact arc in the primary position.

When specifying the circle of contact an important consideration is that there should be no sharp changes in the direction of the muscle path at the point where it leaves the eyeball. The locus of such points forms a tangent circle orthogonal to the origin vector A and there will be no sharp changes in the direction of the muscle path if the circle of contact is orthogonal to the tangent circle, which is ensured if the plane of the contact arc is taken to contain both $\hat{F}$

located at B' and the origin of the muscle A. Thus the plane is spanned by two vectors $\hat{F}$ and G where G=A=B'. These vectors are shown in Figure I.4.1.

To specify the actual contact circle one may consider the vectors H1 and H2 from the centre of the contact circle to the point of insertion and the point where the muscle leaves the eyeball, respectively. The vector H1 may be expressed as a linear combination of $\hat{F}$ and G. To do this, first compute the angle I1 between $\hat{F}$ and G given by:

$$\cos(I1)=G.\hat{F}/\left|G\right|$$

an appropriate linear combination is of the form:

$$\sin(I1)\hat{H1}=\cos(I1)\hat{F}-\hat{G}$$

hence

$$H1=(\cos(I1)\hat{F}-G/\left|G\right|)/\sin(I1)$$

To find the length of H1 consider a vertical plane passing through both $\hat{H1}$ and B' from which it follows that the angle I2 between them is given by:

$$\cos(I2)=\hat{H1}.B'/\left|B'\right|$$

and so $\left|H1\right|$ =radius of eyeball x cos(I2) and $H1=\left|H1\right| x\hat{H1}$.

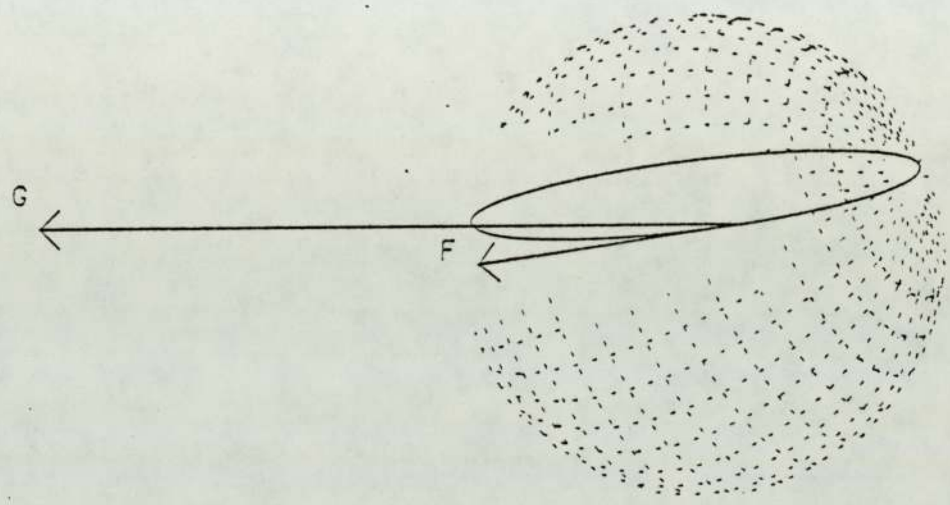

FIGURE I.4.1

THE VECTORS G AND F WHICH
SPAN THE MUSCLE PLANE

Next, form H2 by finding the angle m3 between H1 and H2 (see Figure I.4.2). To do this, consider the vector H3 given by H1+G. The angle between H1 and H3 is given by:

$$\cos(m1)=H1.H3/\ |H1|\ |H3|$$

whilst the angle m2 between H2 and H3 is given by:

$$\cos(m2)=|H2|\ /|H3|=|H1|\ /|H3|$$

since $|H1|=|H2|$

and $m3=m1-m2$

Hence the direction of the vector H2 to the point where the muscle leaves the eyeball is given by:

$$\hat{H2}=\cos(m3)\hat{H1}+\sin(m3)\hat{F}$$

so

$$H2=|H1|\ \hat{H2}$$

To form the unit action vector for each of the muscles, one must first find the point where the muscle leaves the eyeball. This point is specified by the vector P where:

$$P=B'-H1+H2$$

and the unit action vector is given by the orientation

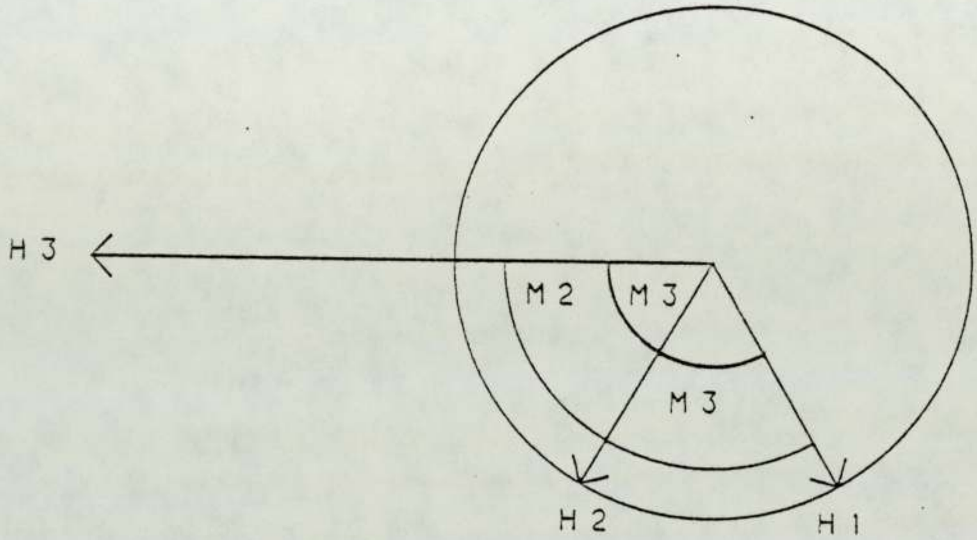

FIGURE I.4.2

THE VECTORS INVOLVED IN THE
CALCULATION OF THE LOSS OF
TANGENCY VECTOR H2

vector R of the plane spanned by A and P:

$$\hat{R}=A \wedge P/|A\wedge P|$$

To find the length changes of the muscles in any given position, consider the angle m3 of the contact arc of the muscle, from which the length Δl of the contact arc follows as $|Hl|$ m3. This variable may be expressed as:

$$\Delta l=l-l_o$$

The percentage change from the muscle length in the primary position is given by:

$$\Delta l'=\Delta l/l_p \times 100$$

where l_p is the length of the muscle in the primary position.

I.5 The innervation problem

Effectively there are only three independent variables e(1),e(3) and e(5) since e(2),e(4) and e(6) are determined by the innervation values with odd numbered indices. The actual values of e(1),e(3)and e(5) are found by an iterative procedure as follows:

1) Guess at e(1),e(3) and e(5) and use these values to calculate a first approximation to the total amount Mo:

i.e. $$Mo=T+\sum_{i=1}^{6} f(i)R(i)$$

2) At equilibrium the total moment M should be zero. A linear approximation to M around Mo is given by:

i.e. $$M=Mo+\sum_{i=1}^{6} M/\ e(i)*deltae(i)$$

$$=Mo+\sum_{i=1}^{6} (\partial f(i)/\partial e(i)*deltae(i))R(i)$$

and if M is zero this implies that:

$$-Mo=\sum_{i=1}^{6}(\partial f(i)/\partial e(i)*deltae(i))R(i)$$

$$=\sum_{i=1}^{6}(\partial f(2i-1)/\partial e(2i-1)R(2i-1)$$

$$+\ f(2i)deltae(2i)*de(2i)/de(2i-1)R(2i))deltae(2i-1)$$

3) Let $p(i)=\partial f(i)/\partial e(i)$ and $d(i)=de(2i)/de(2i-1)$. Then the above relation can be written as a matrix equation:

$$-Mo=A*DELTAE$$

where

DELTAE (1) = deltae (1)

DELTAE (2) = deltae (2)

DELTAE (3) = deltae (3)

and the components of the matrix A are as follows:

Similarly, the reciporical innervation relationship:

e(2i)=(9120.25/(e(2i-1)+90))-90

implies that

de(2i)/de(2i-1)=-9120.25/(e(2i-1)+90)**2

I.6 The position problem

The solution to the position problem used by Robinson (1975) involves decomposing the axis of the moment on the eye into rotations around Fick's axes, and then using the angles of rotation to estimate more appropriate orientation angles. This procedure is unstable because it does not take into account the order in which the rotations occur. Hence a more exact procedure was developed in which the eye is rotated around the axis of the moment and the new orientation angles are calculated directly. The details of the procedure are as follows:

Given an axis of rotation S and an angle or rotation p, with current orientation angles α, β and γ then the orientation angles after rotation α', β' and γ' may be calculated as follows:

1) Set up unit vectors X,Y and Z in the directions of the base vectors I,J and K in the head based co-ordinates.

```
A(1,1)=p(1)*R(1,1)+p(2)*d(1)*R(1,2)
A(1,2)=p(3)*R(1,3)+p(4)*d(2)*R(1,4)
A(1,3)=p(5)*R(1,5)+p(6)*d(3)*R(1,6)
A(2,1)=p(1)*R(2,1)+p(2)*d(1)*R(2,2)
A(2,2)=p(3)*R(2,3)+p(4)*d(2)*R(2,4)
A(2,3)=p(5)*R(2,5)+p(5)*d(3)*R(2,6)
A(3,1)=p(1)*R(3,1)+p(2)*d(1)*R(3,2)
A(3,2)=p(3)*R(3,3)+p(4)*d(2)*R(3,4)
A(3,3)=p(5)*R(3,5)+p(6)*d(3)*R(3,6)
```

4) This matrix equation may be solved for DELTAE and an improved guess at the innervation values is given by e(1)+DELTAE(1),e(3)+DELTAE(2) and e(5)+DELTAE(3). This procedure is repeated until Mo is made sufficiently small.

The required derivatives are straight forward to derive since:

```
f(i)=stren(i)*(passive force+palsy(i)*active force)
 =stren(i)*(passive force+palsy(i)*(total force-passive force))
```

and as the passive force does not change with the level of innervation:

```
∂f(l)/∂e(i)=∂(stren(i)*palsy(i)*total force)/∂e(i)
    =stren(i)*palsy(i)*(0.9+0.81(deltal(i)+e(i))/
                 sqrt(38.9376+0.81(deltal(i)+e(i))**2))
```

2) Rotate the vectors around the axis S through the angle p to obtain the vectors X',Y' and Z'.

3) Calculate the new orientation angle α', which is given by:

$$\alpha'=\mathrm{acos}((Z(1),Z(2),0).(0,1,0))$$

with

sign of α'=sign of acos ((Z(1),Z(2),0).(1,0,0))

4) Calculate the new orientation angle β' which is given by

$$\beta'=\mathrm{acos}(Z.(0,0,1))$$

5) Set up unit vectors U' and V' in the direction of the base vectors I and J in head based co-ordinates after rotation as specified by the orientation angles α', β' and $-\alpha'$.

6) Calculate the angle of torsion e given by:

$$e=\mathrm{acos}(Y,V')$$

and put

sign of e=sign of acos(Y,U')

7) Obtain γ' from the relation:

$$\gamma'=(\alpha'+e)$$

APPENDIX II

COMPUTATIONAL DESCRIPTION

OF THE MODELS

II.1 Description of the programmes

The programmes have been written in FORTRAN IV and graphical output has been produced by means of the routines in the GINO graphics library. The programmes have been implemented on the CDC 7600 at the University of Manchester Regional Computer Centre and on the ICL 1904s at the University of Aston Computer Centre. The majority of the programme runs were carried out at Aston University and the computations required from 400 to 2400 seconds of machine time for a pair of Hess charts, depending on the degree of paresis. Although the programmes will run on a Z80 based microcomputer running Microsoft Fortran 80 under the CP/M operating system, it was found that the time taken was prohibitively long, each run taking several hours.

The programmes listed in this section of the Appendix correspond to the two binocular schemes, scheme A and scheme B, and are followed by an example of their output. In order to facilitate portability of the programmes the graphics routines have been removed in these versions of the programmes.

II.2 Description of functions and subroutines

This section of the Appendix gives descriptions of the computations carried out by the functions and subroutines used in the programme. The descriptions are followed by complete listings of the routines.

FUNCTION ACOS(X) Computes the value of arcos x.

FUNCTION ANGLE(A,B) Computes the angle between the vectors A and B

SUBROUTINE COMPON (X,Y,Z) Accepts the orientation angles alpha, beta and gamma in x,y and z respectively, and sets up the components of the rotation matrix A.

SUBROUTINE CROSS (A,B,C) Accepts two vectors A and B and forms their normalised cross product.

FUNCTION DOT (X,Y) Calculates the scalar product of two vectors X and Y.

SUBROUTINE EXIT (A,B,TURN,D,F) Accepts the origin and insertion vectors of the muscles A and B, rotates the insertion vectors in accordance with the eye position characterised by the matrix TURN and returns the shortest path vector D and the exit vector F.

SUBROUTINE GYRO (P,T,X) Rotates the vector X about the unit axis vector P through the angle T.

SUBROUTINE LEVEL (THETA, DELTAL, R,S,T,E,PALSY) Starts with a guess at the innervation values E and proceeds iteratively to the correct solution.

SUBROUTINE MOMENT (T, DELTA,E,PALSY,STREN,R,S,CONST) Calculates the overall moment on the eyeball and returns its axis S and its size const.

SUBROUTINE PATH (A,B,D,F,LENGTH,CONTAC,R,DELTAL) Determines the paths of the muscles over the eye and computes the action vectors R and percentage length changes DELTAL that result.

SUBROUTINE PICKUP (A,N,X) Accepts the 3x6 matrix A and the variable n which specifies the required column of the matrix and places this column in the vector X.

SUBROUTINE PLUS (X,Y) Adds the vectors X and Y and returns the result in X.

SUBROUTINE PUTBAK (A,N,X) Accepts the 3x6 matrix A, the variable n which specifies the required column of the matrix and the vector X which is to be placed in the matrix at the specified column.

SUBROUTINE ROTAT (A,X) Rotates the vector X according to the rotation matrix A.

SUBROUTINE SETUP (ALPHA,BETA,GAMMA,PRIMO,A,PRIMI,B, TURN,T,EY) Sets up the muscle origin and insertion vectors A and B, the rotation matrix TURN and corresponding passive moment T, for the orientation angles specified by alpha, beta and gamma.

FUNCTION SIZE (X) Computes the length of the vector X.

SUBROUTINE SOL (A,Y,X) Accepts the matrix equation AX=Y and solves the equation by application of the inverse matrix, formed according to Cramer's rule.

SUBROUTINE SWIVEL (ALPHA,BETA,GAMMA,S,CONST) Rotates the eye, initially in the position specified by the orientation angles alpha, beta and gamma, about the vector S by an amount const, and returns the new orientation angles.

SUBROUTINE TIMES (A,X) Multiplies the vector X by the scalar A.

SUBROUTINE TRANSP(A) Forms the transpose of the rotation matrix A.

```
      PROGRAM SCHEMA
C
C MODELS THE EFFECT OF PALSIES ACCORDING TO SCHEME A
C
      REAL LENGTH
C
C DECLARE THE CONSTANT ARRAYS OF THE MUSCLE ORIGINS PRIMOL AND
C PRIMOR AND THE MUSCLE INSERTIONS PRIMIL AND PRIMIR OF THE
C NORMAL LEFT AND RIGHT EYES RESPECTIVELY
C
      DIMENSION PRIMOL(3,6),PRIMOR(3,6),PRIMIL(3,6),PRIMIR(3,6)
C
C DECLARE THE ARRAYS OF STANDARD MUSCLE LENGTHS LENGTH,CONTACT
C ARCS CONTAC AND CROSS SECTIONAL AREA STREN
C
      DIMENSION LENGTH(6),CONTAC(6),STREN(6)
C
C DECLARE THE CONSTANT ARRAY OF STANDARD EYE POSITIONS ORIENT AND
C VARIABLE ARRAY OF POSITIONS ACTUALLY ASSUMED BY THE EYES CHART
C
      DIMENSION ORIENT (3,9),CHART(3,18)
C
C DECLARE THE CONSTANT ARRAYS OF INNERVATION PALSY FACTORS PALSYL,
C PALSYR AND PALSY
C
      DIMENSION PALSYL(6),PALSYR(6),PALSY(6)
C
C DECLARE THE VARIABLE ARRAYS USED DURING THE CALCULATIONS.THESE ARE
C AS FOLLOWS:THE ARRAY OF ORIGINS OF THE MUSCLES A,THE ARRAY OF
C INSERTIONS OF THE MUSCLES B,THE ROTATION MATRIX TURN USED TO MOVE
C THE EYE,THE ARRAY OF ORIENTATION VECTORS OF THE SHORTEST PATH PLANE
C D,USED WHENEVER A MUSCLE LOSES TANGENCY,THE ARRAY OF ACTUAL EXIT
C PATHS OF THE MUSCLES F,THE ARRAY OF MUSCLE ACTION VECTORS R,THE ARRAY
C OF PERCENTAGE LENGTH CHANGES DELTA,THE ARRAY OF INNERVATION VALUES
C E ASSOCIATED WITH A GIVEN POSITION OF GAZE,THE ARRAY CONTAINING THE
C PASSIVE MOMENT T AND FINALLY,THE ARRAY CONTAINING THE OVERALL MOMENT
C S
C
      DIMENSION A(3,6),B(3,6),TURN(3,3)
      DIMENSION D(3,6),F(3,6),R(3,6)
      DIMENSION DELTA(6),E(6),T(3),S(3)
C
C SET UP ARRAY OF STANDARD EYE POSITIONS
C
      DATA ORIENT / 0.0    , 0.0    , 0.0    ,
     1              0.0    , 0.2618 , 0.0    ,
     2              0.7854 , 0.3752 ,-0.7854 ,
     3              1.5708 , 0.2618 ,-1.5708 ,
     4              2.3562 , 0.3752 ,-2.3562 ,
     5              3.1416 , 0.2618 ,-3.1416 ,
     6             -2.3562 , 0.3752 , 2.3562 ,
     7             -1.5708 , 0.2618 , 1.5708 ,
     8             -0.7854 , 0.3752 , 0.7854 /
C
C SET UP ARRAYS OF MUSCLE ORIGINS AND INSERTIONS
C
```

```
      DATA PRIMOL / 13.0  ,  0.6  ,-34.0  ,
     1              17.0  ,  0.6  ,-30.0  ,
     2              16.0  ,  3.6  ,-31.76 ,
     3              16.0  , -2.4  ,-31.76 ,
     4              15.27 , 12.25 ,  8.24 ,
     5              11.1  ,-15.43 , 11.34 /
C
      DATA PRIMOR /-13.0  ,  0.6  ,-34.0  ,
     1             -17.0  ,  0.6  ,-30.0  ,
     2             -16.0  ,  3.6  ,-31.76 ,
     3             -16.0  , -2.4  ,-31.76 ,
     4             -15.27 , 12.25 ,  8.24 ,
     5             -11.1  ,-15.43 , 11.34 /
C
      DATA PRIMIL /-10.08 ,  0.0  ,  6.5  ,
     1               9.65 ,  0.0  ,  8.84 ,
     2               0.0  , 10.48 ,  7.63 ,
     3               0.0  ,-10.24 ,  8.02 ,
     4              -2.9  , 11.05 , -4.41 ,
     5              -8.7  ,  0.0  , -7.18 /
C
      DATA PRIMIR / 10.08 ,  0.0  ,  6.5  ,
     1              -9.65 ,  0.0  ,  8.84 ,
     2               0.0  , 10.48 ,  7.63 ,
     3               0.0  ,-10.24 ,  8.02 ,
     4               2.9  , 11.05 , -4.41 ,
     5               8.7  ,  0.0  , -7.18 /
C
C SET UP THE ARRAYS OF MUSCLE LENGTHS,CONTACT ARCS,CROSS SECTIONAL
C AREAS AND INNERVATION SCALE FACTORS
C
      DATA LENGTH / 49.11,38.51,41.96,42.49,22.28,35.35 /
      DATA CONTAC / 15.94, 7.6 ,10.23,11.02, 5.01,17.13 /
      DATA STREN  /  1.0 , 1.04, 0.68, 0.95, 0.5 , 0.47 /
      DATA PALSY  /  1.0 , 1.0 , 1.0 , 1.0 , 1.0 , 1.0  /
      DATA PALSYL /  1.0 , 1.0 , 1.0 , 1.0 , 1.0 , 1.0  /
      DATA PALSYR /  1.0 , 1.0 , 1.0 , 1.0 , 1.0 , 1.0  /
C
C CALCULATE POSITIONS ADOPTED BY THE RIGHT EYE WHEN THE LEFT EYE IS
C FIXATING
C
      N=1
200   ALPHA=ORIENT(1,N)
      BETA=ORIENT(2,N)
      GAMMA=ORIENT(3,N)
C
C FIRSTLY CALCULATE THE INNERVATION VALUES REQUIRED BY THE PARETIC
C LEFT EYE TO MAINTAIN THE POSITION OF FIXATION
C
      CALL SETUP(ALPHA,BETA,GAMMA,PRIMOL,A,PRIMIL,B,TURN,T,1.0)
      CALL EXIT(A,B,TURN,D,F)
      CALL PATH(A,B,D,F,LENGTH,CONTAC,R,DELTA)
      DO 210 I=1,6
210   E(I)=5.5
      CALL LEVEL(STREN,DELTA,R,S,T,E,PALSYL)
```

```
C
C SECONDLY CALCULATE THE POSITION ADOPTED BY THE NORMAL LEFT EYE WITH
C THESE LEVELS OF INNERVATION
C
300     CALL SETUP(ALPHA,BETA,GAMMA,PRIMOL,A,PRIMIL,B,TURN,T,1.0)
        CALL EXIT(A,B,TURN,D,F)
        CALL PATH(A,B,D,F,LENGTH,CONTAC,R,DELTA)
        CALL MOMENT(T,DELTA,E,PALSY,STREN,R,S,CONST)
        IF(CONST.LT.0.1) GOTO 400
        CONST=CONST/200.0
        CALL SWIVEL(ALPHA,BETA,GAMMA,S,CONST)
        GOTO 300
C
C THIRDLY CALCULATE THE INNERVATION LEVELS REQUIRED BY THE NORMAL
C RIGHT EYE TO MAINTAIN THIS POSITION
C
400     CALL SETUP(ALPHA,BETA,GAMMA,PRIMOR,A,PRIMIR,B,TURN,T,-1.0)
        CALL EXIT(A,B,TURN,D,F)
        CALL PATH(A,B,D,F,LENGTH,CONTAC,R,DELTA)
        DO 410 I=1,6
410     E(I)=5.5
        CALL LEVEL(STREN,DELTA,R,S,T,E,PALSY)
C
C FINALLY CALCULATE THE POSITIONS ADOPTED BY THE PARETIC RIGHT EYE
C UNDER THESE INNERVATION VALUES
C
        ALPHA=ORIENT(1,N)
        BETA=ORIENT(2,N)
        GAMMA=ORIENT(3,N)
500     CALL SETUP(ALPHA,BETA,GAMMA,PRIMOR,A,PRIMIR,B,TURN,T,-1.0)
        CALL EXIT(A,B,TURN,D,F)
        CALL PATH(A,B,D,F,LENGTH,CONTAC,R,DELTA)
        CALL MOMENT(T,DELTA,E,PALSYR,STREN,R,S,CONST)
        IF(CONST.LT.0.5) GOTO 600
        CONST=CONST/100.0
        CALL SWIVEL(ALPHA,BETA,GAMMA,S,CONST)
        GOTO 500
C
C MOVE ON TO NEXT POSITION
C
600     CHART(1,N+9)=ALPHA
        CHART(2,N+9)=BETA
        CHART(3,N+9)=GAMMA
        N=N+1
        IF(N.GT.9) GOTO 700
        GOTO 200
C
C CALCULATE POSITIONS ADOPTED BY THE LEFT EYE WHEN THE RIGHT EYE IS
C FIXATING
C
700     N=1
710     ALPHA=ORIENT(1,N)
        BETA=ORIENT(2,N)
        GAMMA=ORIENT(3,N)
```

```
C
C FIRSTLY CALCULATE THE INNERVATION VALUES REQUIRED BY THE PARETIC
C RIGHT EYE TO MAINTAIN THE POSITION OF FIXATION
C
      CALL SETUP(ALPHA,BETA,GAMMA,PRIMOR,A,PRIMIR,B,TURN,T,-1.0)
      CALL EXIT(A,B,TURN,D,F)
      CALL PATH(A,B,D,F,LENGTH,CONTAC,R,DELTA)
      DO 720 I=1,6
720   E(I)=5.5
      CALL LEVEL(STREN,DELTA,R,S,T,E,PALSYR)
C
C SECONDLY CALCULATE THE POSITION ADOPTED BY THE NORMAL RIGHT EYE
C WITH THESE LEVELS OF INNERVATION
C
800   CALL SETUP(ALPHA,BETA,GAMMA,PRIMOR,A,PRIMIR,B,TURN,T,-1.0)
      CALL EXIT(A,B,TURN,D,F)
      CALL PATH(A,B,D,F,LENGTH,CONTAC,R,DELTA)
      CALL MOMENT(T,DELTA,E,PALSY,STREN,R,S,CONST)
      IF(CONST.LT.0.1) GOTO 900
      CONST=CONST/200.0
      CALL SWIVEL(ALPHA,BETA,GAMMA,S,CONST)
      GOTO 800
C
C THIRDLY CALCULATE THE INNERVATION LEVELS REQUIRED BY THE NORMAL
C LEFT EYE TO MAINTAIN THIS POSITION
C
900   CALL SETUP(ALPHA,BETA,GAMMA,PRIMOL,A,PRIMIL,B,TURN,T,1.0)
      CALL EXIT(A,B,TURN,D,F)
      CALL PATH(A,B,D,F,LENGTH,CONTAC,R,DELTA)
      DO 910 I=1,6
910   E(I)=5.5
      CALL LEVEL(STREN,DELTA,R,S,T,E,PALSY)
C
C FINALLY CALCULATE THE POSITIONS ADOPTED BY THE PARETIC LEFT EYE
C UNDER THESE INNERVATION VALUES
C
      ALPHA=ORIENT(1,N)
      BETA=ORIENT(2,N)
      GAMMA=ORIENT(3,N)
1000  CALL SETUP(ALPHA,BETA,GAMMA,PRIMOL,A,PRIMIL,B,TURN,T,1.0)
      CALL EXIT(A,B,TURN,D,F)
      CALL PATH(A,B,D,F,LENGTH,CONTAC,R,DELTA)
      CALL MOMENT(T,DELTA,E,PALSYL,STREN,R,S,CONST)
      IF(CONST.LT.0.5) GOTO 1100
      CONST=CONST/200.0
      CALL SWIVEL(ALPHA,BETA,GAMMA,S,CONST)
      GOTO 1000
C
C MOVE ON TO NEXT POSITION
C
1100  CHART(1,N)=ALPHA
      CHART(2,N)=BETA
      CHART(3,N)=GAMMA
      N=N+1
      IF(N.GT.9) GOTO 1200
      GOTO 710
```

```
C
C WRITE OUT THE POSITIONS ASSUMED
C
1200    WRITE(2,2000)
2000    FORMAT(1H0,'POSITIONS ASSUMED BY LEFT EYE')
2010    FORMAT(1H0,'POSITIONS ASSUMED BY RIGHT EYE')
        WRITE(2,2020)
2020    FORMAT(1H0,'ADDUCTION-',10X,'ELEVATION-',10X,'EXTORSION-')
        WRITE(2,2030)
2030    FORMAT(1H ,' ABDUCTION',10X,' DEPRESSION',10X,' INTORSION')
C
        DO 2100 N=1,18
        ALPHA=CHART(1,N)
        BETA=CHART(2,N)
        GAMMA=CHART(3,N)
        CONST1=SQRT(COS(BETA)**2+(COS(ALPHA)**2)*(SIN(BETA)**2))
        IF(CONST1.GT.0.9999) CONST1=0.9999
        IF(CONST1.LT.-0.9999) CONST1=-0.9999
        CONST1=ACOS(CONST1)
        CONST1=CONST1*57.3
        IF(ALPHA.LT.0.0) CONST1=-CONST1
        CONST2=SQRT(COS(BETA)**2+(SIN(ALPHA)**2)*(SIN(BETA)**2))
        IF(CONST2.GT.0.9999) CONST2=0.9999
        IF(CONST2.LT.-0.9999) CONST2=-0.9999
        CONST2=ACOS(CONST2)
        CONST2=CONST2*57.3
        IF(ABS(ALPHA).GT.1.5708) CONST2=-CONST2
        CONST3=(ALPHA+GAMMA)*57.3
        IF(N.EQ.10) WRITE(2,2010)
        IF(N.EQ.10) WRITE(2,2020)
        IF(N.EQ.10) WRITE(2,2030)
        WRITE(2,2040) CONST1,CONST2,CONST3
2040    FORMAT(1H ,3(F10.1,10X))
2100    CONTINUE
C
        END
```

```
      PROGRAM SCHEMB
C
C MODELS THE EFFECT OF PALSIES ACCORDING TO SCHEME B
C
      REAL LENGTH
C
C DECLARE THE CONSTANT ARRAYS OF THE MUSCLE ORIGINS PRIMOL AND
C PRIMOR AND THE MUSCLE INSERTIONS PRIMIL AND PRIMIR OF THE
C NORMAL LEFT AND RIGHT EYES RESPECTIVELY
C
      DIMENSION PRIMOL(3,6),PRIMOR(3,6),PRIMIL(3,6),PRIMIR(3,6)
C
C DECLARE THE ARRAYS OF STANDARD MUSCLE LENGTHS LENGTH,CONTACT
C ARCS CONTAC AND CROSS SECTIONAL AREA STREN
C
      DIMENSION LENGTH(6),CONTAC(6),STREN(6)
C
C DECLARE THE CONSTANT ARRAY OF STANDARD EYE POSITIONS ORIENT AND
C VARIABLE ARRAY OF POSITIONS ACTUALLY ASSUMED BY THE EYES CHART
C
      DIMENSION ORIENT (3,9),CHART(3,18)
C
C DECLARE THE CONSTANT ARRAYS OF INNERVATION PALSY FACTORS PALSYL,
C PALSYR AND PALSY
C
      DIMENSION PALSYL(6),PALSYR(6),PALSY(6)
C
C DECLARE THE VARIABLE ARRAYS USED DURING THE CALCULATIONS.THESE ARE
C AS FOLLOWS:THE ARRAY OF ORIGINS OF THE MUSCLES A,THE ARRAY OF
C INSERTIONS OF THE MUSCLES B,THE ROTATION MATRIX TURN USED TO MOVE
C THE EYE,THE ARRAY OF ORIENTATION VECTORS OF THE SHORTEST PATH PLANE
C D,USED WHENEVER A MUSCLE LOSES TANGENCY,THE ARRAY OF ACTUAL EXIT
C PATHS OF THE MUSCLES F,THE ARRAY OF MUSCLE ACTION VECTORS R,THE ARRAY
C OF PERCENTAGE LENGTH CHANGES DELTA,THE ARRAY OF INNERVATION VALUES
C E ASSOCIATED WITH A GIVEN POSITION OF GAZE,THE ARRAY CONTAINING THE
C PASSIVE MOMENT T AND FINALLY,THE ARRAY CONTAINING THE OVERALL MOMENT
C S
C
      DIMENSION A(3,6),B(3,6),TURN(3,3),XT(3)
      DIMENSION D(3,6),F(3,6),R(3,6),TR(3,6)
      DIMENSION DELTA(6),E(6),T(3),S(3),TDELTA(6),TT(3)
C
C SET UP ARRAY OF STANDARD EYE POSITIONS
C
      DATA ORIENT / 0.0    , 0.0    , 0.0    ,
     1              0.0    , 0.2618 , 0.0    ,
     2              0.7854 , 0.3752 ,-0.7854 ,
     3              1.5708 , 0.2618 ,-1.5708 ,
     4              2.3562 , 0.3752 ,-2.3562 ,
     5              3.1416 , 0.2618 ,-3.1416 ,
     6             -2.3562 , 0.3752 , 2.3562 ,
     7             -1.5708 , 0.2618 , 1.5708 ,
     8             -0.7854 , 0.3752 , 0.7854 /
C
C SET UP ARRAYS OF MUSCLE ORIGINS AND INSERTIONS
C
```

```
      DATA PRIMOL / 13.0  ,  0.6  ,-34.0  ,
     1              17.0  ,  0.6  ,-30.0  ,
     2              16.0  ,  3.6  ,-31.76 ,
     3              16.0  , -2.4  ,-31.76 ,
     4              15.27 , 12.25 ,  8.24 ,
     5              11.1  ,-15.43 , 11.34 /
C
      DATA PRIMOR /-13.0  ,  0.6  ,-34.0  ,
     1             -17.0  ,  0.6  ,-30.0  ,
     2             -16.0  ,  3.6  ,-31.76 ,
     3             -16.0  , -2.4  ,-31.76 ,
     4             -15.27 , 12.25 ,  8.24 ,
     5             -11.1  ,-15.43 , 11.34 /
C
      DATA PRIMIL /-10.08 ,  0.0  ,  6.5  ,
     1               9.65 ,  0.0  ,  8.84 ,
     2               0.0  , 10.48 ,  7.63 ,
     3               0.0  ,-10.24 ,  8.02 ,
     4              -2.9  , 11.05 , -4.41 ,
     5              -8.7  ,  0.0  , -7.18 /
C
      DATA PRIMIR / 10.08 ,  0.0  ,  6.5  ,
     1              -9.65 ,  0.0  ,  8.84 ,
     2               0.0  , 10.48 ,  7.63 ,
     3               0.0  ,-10.24 ,  8.02 ,
     4               2.9  , 11.05 , -4.41 ,
     5               8.7  ,  0.0  , -7.18 /
C
C SET UP THE ARRAYS OF MUSCLE LENGTHS,CONTACT ARCS,CROSS SECTIONAL
C AREAS AND INNERVATION SCALE FACTORS
C
      DATA LENGTH / 49.11,38.51,41.96,42.49,22.28,35.33 /
      DATA CONTAC / 15.94, 7.6 ,10.23,11.02, 5.01,17.13 /
      DATA STREN  /  1.0 , 1.04, 0.68, 0.95, 0.5 , 0.47 /
      DATA PALSY  /  1.0 , 1.0 , 1.0 , 1.0 , 1.0 , 1.0  /
      DATA PALSYL /  1.0 , 1.0 , 1.0 , 1.0 , 1.0 , 1.0  /
      DATA PALSYR /  1.0 , 1.0 , 1.0 , 1.0 , 1.0 , 1.0  /
C
C CALCULATE POSITIONS ADOPTED BY THE RIGHT EYE WHEN THE LEFT EYE IS
C FIXATING
C
      N=1
200   ALPHA=ORIENT(1,N)
      BETA=ORIENT(2,N)
      GAMMA=ORIENT(3,N)
C
C FIRSTLY CALCULATE THE INNERVATION VALUES REQUIRED BY THE PARETIC
C LEFT EYE TO MAINTAIN THE POSITION OF FIXATION
C
      XT(1)=0.0
      XT(2)=0.0
      XT(3)=1.0
      CALL COMPON(ALPHA,BETA,GAMMA,TURN)
      CALL TRANSP(TURN)
      CALL ROTAT(TURN,XT)
```

```
      CALL SETUP(ALPHA,BETA,GAMMA,PRIMOL,A,PRIMIL,B,TURN,TT,1.0)
      CALL EXIT(A,B,TURN,D,F)
      CALL PATH(A,B,D,F,LENGTH,CONTAC,TR,TDELTA)
      NT=0
205   NT=NT+1
C
      CALL SETUP(ALPHA,BETA,GAMMA,PRIMOL,A,PRIMIL,B,TURN,T,1.0)
      CALL EXIT(A,B,TURN,D,F)
      CALL PATH(A,B,D,F,LENGTH,CONTAC,R,DELTA)
      DO 210 I=1,6
210   E(I)=5.5
      CALL LEVEL(STREN,DELTA,R,S,T,E,PALSYL)
      CALL MOMENT(TT,TDELTA,E,PALSYL,STREN,TR,S,CONST)
      XC=CONST/400.0
      TCONST=CONST
      IF(CONST.LT.0.5) GOTO 405
      IF(ABS(DOT(XT,S)).GT.0.95) GOTO 405
      CALL TIMES(-1.0,S)
      CALL SWIVEL(ALPHA,BETA,GAMMA,S,XC)
      GAMMA=-ALPHA
      GOTO 205
405   C1=0.0
C
C SECONDLY CALCULATE THE INNERVATION LEVELS REQUIRED BY THE NORMAL
C RIGHT EYE TO MAINTAIN THIS POSITION
C
400   CALL SETUP(ALPHA,BETA,GAMMA,PRIMOR,A,PRIMIR,B,TURN,T,-1.0)
      CALL EXIT(A,B,TURN,D,F)
      CALL PATH(A,B,D,F,LENGTH,CONTAC,R,DELTA)
      DO 410 I=1,6
410   E(I)=5.5
      CALL LEVEL(STREN,DELTA,R,S,T,E,PALSY)
C
C FINALLY CALCULATE THE POSITIONS ADOPTED BY THE PARETIC RIGHT EYE
C UNDER THESE INNERVATION VALUES
C
      ALPHA=ORIENT(1,N)
      BETA=ORIENT(2,N)
      GAMMA=ORIENT(3,N)
500   CALL SETUP(ALPHA,BETA,GAMMA,PRIMOR,A,PRIMIR,B,TURN,T,-1.0)
      CALL EXIT(A,B,TURN,D,F)
      CALL PATH(A,B,D,F,LENGTH,CONTAC,R,DELTA)
      CALL MOMENT(T,DELTA,E,PALSYR,STREN,R,S,CONST)
      IF(CONST.LT.0.5) GOTO 600
      CONST=CONST/100.0
      CALL SWIVEL(ALPHA,BETA,GAMMA,S,CONST)
      GOTO 500
C
C MOVE ON TO NEXT POSITION
C
600   CHART(1,N+9)=ALPHA
      CHART(2,N+9)=BETA
      CHART(3,N+9)=GAMMA
      N=N+1
      IF(N.GT.9) GOTO 700
      GOTO 200
```

```
C
C CALCULATE POSITIONS ADOPTED BY THE LEFT EYE WHEN THE RIGHT EYE IS
C FIXATING
C
700     N=1
710     ALPHA=ORIENT(1,N)
        BETA=ORIENT(2,N)
        GAMMA=ORIENT(3,N)
C
C FIRSTLY CALCULATE THE INNERVATION VALUES REQUIRED BY THE PARETIC
C RIGHT EYE TO MAINTAIN THE POSITION OF FIXATION
C
        XT(1)=0.0
        XT(2)=0.0
        XT(3)=1.0
        CALL COMPON(ALPHA,BETA,GAMMA,TURN)
        CALL TRANSP(TURN)
        CALL ROTAT(TURN,XT)
        CALL SETUP(ALPHA,BETA,GAMMA,PRIMOR,A,PRIMIR,B,TURN,TT,-1.0)
        CALL EXIT(A,B,TURN,D,F)
        CALL PATH(A,B,D,F,LENGTH,CONTAC,TR,TDELTA)
        NT=0
715     NT=NT+1
C
        CALL SETUP(ALPHA,BETA,GAMMA,PRIMOR,A,PRIMIR,B,TURN,T,-1.0)
        CALL EXIT(A,B,TURN,D,F)
        CALL PATH(A,B,D,F,LENGTH,CONTAC,R,DELTA)
        DO 720 I=1,6
720     E(I)=5.5
        CALL LEVEL(STREN,DELTA,R,S,T,E,PALSYR)
        CALL MOMENT(TT,TDELTA,E,PALSYR,STREN,TR,S,CONST)
        IF(CONST.LT.0.5) GOTO 750
        XC=CONST/400.0
        TCONST=CONST
        IF(ABS(DOT(XT,S)).GT.0.9) GOTO 750
        CALL TIMES(-1.0,S)
        CALL SWIVEL(ALPHA,BETA,GAMMA,S,XC)
        GAMMA=-ALPHA
        GOTO 715
750     C1=0.0
C
C SECONDLY CALCULATE THE INNERVATION LEVELS REQUIRED BY THE NORMAL
C LEFT EYE TO MAINTAIN THIS POSITION
C
900     CALL SETUP(ALPHA,BETA,GAMMA,PRIMOL,A,PRIMIL,B,TURN,T,1.0)
        CALL EXIT(A,B,TURN,D,F)
        CALL PATH(A,B,D,F,LENGTH,CONTAC,R,DELTA)
        DO 910 I=1,6
910     E(I)=5.5
        CALL LEVEL(STREN,DELTA,R,S,T,E,PALSY)
C
C FINALLY CALCULATE THE POSITIONS ADOPTED BY THE PARETIC LEFT EYE
C UNDER THESE INNERVATION VALUES
C
```

```
      ALPHA=ORIENT(1,N)
      BETA=ORIENT(2,N)
      GAMMA=ORIENT(3,N)
1000  CALL SETUP(ALPHA,BETA,GAMMA,PRIMOL,A,PRIMIL,B,TURN,T,1.0)
      CALL EXIT(A,B,TURN,D,F)
      CALL PATH(A,B,D,F,LENGTH,CONTAC,R,DELTA)
      CALL MOMENT(T,DELTA,E,PALSYL,STREN,R,S,CONST)
      IF(CONST.LT.0.5) GOTO 1100
      CONST=CONST/200.0
      CALL SWIVEL(ALPHA,BETA,GAMMA,S,CONST)
      GOTO 1000
C
C MOVE ON TO NEXT POSITION
C
1100  CHART(1,N)=ALPHA
      CHART(2,N)=BETA
      CHART(3,N)=GAMMA
      N=N+1
      IF(N.GT.9) GOTO 1200
      GOTO 710
C
C WRITE OUT THE POSITIONS ASSUMED
C
1200  WRITE(2,2000)
2000  FORMAT(1H0,'POSITIONS ASSUMED BY LEFT EYE')
2010  FORMAT(1H0,'POSITIONS ASSUMED BY RIGHT EYE')
      WRITE(2,2020)
2020  FORMAT(1H0,'ADDUCTION-',10X,'ELEVATION-',10X,'EXTORSION-')
      WRITE(2,2030)
2030  FORMAT(1H ,' ABDUCTION',10X,' DEPRESSION',10X,' INTORSION')
C
      DO 2100 N=1,18
      ALPHA=CHART(1,N)
      BETA=CHART(2,N)
      GAMMA=CHART(3,N)
      CONST1=SQRT(COS(BETA)**2+(COS(ALPHA)**2)*(SIN(BETA)**2))
      IF(CONST1.GT.0.9999) CONST1=0.9999
      IF(CONST1.LT.-0.9999) CONST1=-0.9999
      CONST1=ACOS(CONST1)
      CONST1=CONST1*57.3
      IF(ALPHA.LT.0.0) CONST1=-CONST1
      CONST2=SQRT(COS(BETA)**2+(SIN(ALPHA)**2)*(SIN(BETA)**2))
      IF(CONST2.GT.0.9999) CONST2=0.9999
      IF(CONST2.LT.-0.9999) CONST2=-0.9999
      CONST2=ACOS(CONST2)
      CONST2=CONST2*57.3
      IF(ABS(ALPHA).GT.1.5708) CONST2=-CONST2
      CONST3=(ALPHA+GAMMA)*57.3
      IF(N.EQ.10) WRITE(2,2010)
      IF(N.EQ.10) WRITE(2,2020)
      IF(N.EQ.10) WRITE(2,2030)
      WRITE(2,2040) CONST1,CONST2,CONST3
2040  FORMAT(1H ,3(F10.1,10X))
2100  CONTINUE
C
      END
```

POSITIONS ASSUMED BY LEFT EYE

ADDUCTION-ABDUCTION	ELEVATION-DEPRESSION	EXTORSION-INTORSION
.8	.8	0.0
.8	15.0	0.0
15.0	15.0	0.0
15.0	.8	0.0
15.0	-15.0	0.0
.8	-15.0	0.0
-15.0	-15.0	0.0
-15.0	.8	0.0
-15.0	15.0	0.0

POSITIONS ASSUMED BY RIGHT EYE

ADDUCTION-ABDUCTION	ELEVATION-DEPRESSION	EXTORSION-INTORSION
.8	.8	0.0
.8	15.0	0.0
15.0	15.0	0.0
15.0	.8	0.0
15.0	-15.0	0.0
.8	-15.0	0.0
-15.0	-15.0	0.0
-15.0	.8	0.0
-15.0	15.0	0.0

```
      FUNCTION ACOS(X)
C
C COMPUTES THE VALUE OF ACOS X
C
      PI=3.1416
      IF(X.GT.0.0) ACOS=ATAN((SQRT(1-X**2))/X)
      IF(X.LT.0.0) ACOS=ATAN((SQRT(1-X**2))/X)+PI
      IF(X.EQ.0.0) ACOS=PI/2.0
      RETURN
      END
```

```
      FUNCTION ANGLE(A,B)
C
C COMPUTES THE ANGLE BETWEEN VECTORS A AND B
C
      DIMENSION A(3),B(3)
C
      X=A(1)*A(1)+A(2)*A(2)+A(3)*A(3)
      X=SQRT(X)
      Y=B(1)*B(1)+B(2)*B(2)+B(3)*B(3)
      Y=SQRT(Y)
      X=X*Y
      Y=A(1)*B(1)+A(2)*B(2)+A(3)*B(3)
      IF(ABS(X-Y).LT.0.00001) GOTO 10
      Z=Y/X
10    IF(ABS(X-Y).LT.0.00001)Z=1.0
      ANGLE=ACOS(Z)
C
      RETURN
      END
```

```
      SUBROUTINE COMPON(X,Y,Z,A)
C
C ACCEPTS THE ANGLES ALPHA,BETA AND GAMMA IN X,Y AND Z
C RESPECTIVELY AND SETS UP THE COMPONENTS OF THE ROTATION MATRIX A
C
      DIMENSION A(3,3)
C
      A(1,1)=COS(X)*COS(Z)-SIN(X)*COS(Y)*SIN(Z)
      A(1,2)=-(SIN(X)*COS(Z)+COS(X)*COS(Y)*SIN(Z))
      A(1,3)=SIN(Y)*SIN(Z)
C
      A(2,1)=COS(X)*SIN(Z)+SIN(X)*COS(Y)*COS(Z)
      A(2,2)=-(SIN(X)*SIN(Z))+COS(X)*COS(Y)*COS(Z)
      A(2,3)=-(SIN(Y)*COS(Z))
C
      A(3,1)=SIN(X)*SIN(Y)
      A(3,2)=COS(X)*SIN(Y)
      A(3,3)=COS(Y)
C
      RETURN
      END
```

```
      SUBROUTINE CROSS(A,B,C)
C
C ACCEPTS TWO VECTORS A AND B AND FORMS THEIR NORMAL IN C
C
      DIMENSION A(3),B(3),C(3)
C
      C(1)=A(2)*B(3)-A(3)*B(2)
      C(2)=A(3)*B(1)-A(1)*B(3)
      C(3)=A(1)*B(2)-A(2)*B(1)
      AREA=C(1)**2+C(2)**2+C(3)**2
      AREA=SQRT(AREA)
      C(1)=C(1)/AREA
      C(2)=C(2)/AREA
      C(3)=C(3)/AREA
C
      RETURN
      END
```

```
      FUNCTION DOT(X,Y)
C
C CALCULATES THE SCALAR PRODUCT OF THE
C TWO VECTORS X AND Y
C
      DIMENSION X(3),Y(3)
C
      DOT=X(1)*Y(1)+X(2)*Y(2)+X(3)*Y(3)
C
      RETURN
      END
```

```
      SUBROUTINE EXIT(A,B,TURN,D,F)
C
C ACCEPTS THE ORIGIN AND INSERTION VECTORS OF THE MUSCLES A AND
C B,ROTATES THE INSERTION VECTORS IN ACCORDANCE WITH THE EYE
C POSITION CHARACTERISED BY THE MATRIX TURN AND RETURNS THE
C SHORTEST PATH VECTOR D AND EXIT VECTOR F.
C
      DIMENSION A(3,6),B(3,6),TURN(3,3),D(3,6),F(3,6)
      DIMENSION C(3,6),X(3),Y(3),Z(3),U(3),V(3)
C
C PROCEED THROUGH THE ROUTINE,DOING EACH MUSCLE IN TURN
C
      DO 100 I=1,6
C
C COMPUTE ORIENTATION VECTOR C OF THE PLANE PERPENDICULAR TO THE
C MUSCLE INSERTION
C
      CALL PICKUP(A,I,X)
      CALL PICKUP(B,I,Y)
      CALL CROSS(X,Y,Z)
      CALL PUTBAK(C,I,Z)
C
C COMPUTE ORIENTATION OF THE INSERTION VECTOR (B) AND OF THE
C ORIENTATION VECTOR (C) OF THE PERPENDICULAR PLANE AFTER
C ROTATION OF THE EYEBALL
C
      CALL PICKUP(B,I,X)
      CALL ROTAT(TURN,X)
      CALL PUTBAK(B,I,X)
      CALL PICKUP(C,I,X)
      CALL ROTAT(TURN,X)
      CALL PUTBAK(C,I,X)
C
C COMPUTE ORIENTATION VECTOR OF THE SHORTEST PATH PLANE (D)
C
      CALL PICKUP(A,I,X)
      CALL PICKUP(B,I,Y)
      CALL CROSS(X,Y,Z)
      CALL PUTBAK(D,I,Z)
C
C COMPUTE THE TWIST ANGLE (TWIST) AND ITS SIGN
C
      CALL PICKUP(C,I,X)
      CALL PICKUP(D,I,Y)
      TWIST=ANGLE(X,Y)
      CALL PICKUP(B,I,U)
      CALL CROSS(U,X,V)
      CALL PUTBAK(F,I,V)
      ATWIST=DOT(V,Y)
      IF(ATWIST.LT.0.0) TWIST=-TWIST
C
C COMPUTE THE TILT ANGLE (TILT)
C
      CALL PICKUP(A,I,X)
      CALL PICKUP(C,I,Y)
      CONST1=SIZE(X)
      CONST2=DOT(X,Y)
      TILT=ACOS(CONST2/CONST1)
```

```
C
C AND HENCE FORM THE ACTUAL TWIST ANGLE (ATWIST)
C
        CONST=COS(TILT)
        IF (CONST.LT.0.0) CONST=(-CONST)
        ATWIST=CONST*TWIST
C
C COMPUTE THE ACTUAL EXIT PATH F ACCORDING TO THE FORMULA:
C F=(-SIN(ATWIST))*C+(COS(ATWIST))*F
C
        CONST=-SIN(ATWIST)
        CALL PICKUP(C,I,X)
C       CALL TIMES(CONST,X)

        CONST=COS(ATWIST)
        CALL PICKUP(F,I,Y)
        CALL TIMES(CONST,Y)
C
        CALL PLUS(X,Y)
        CALL PUTBAK(F,I,X)
C
100     CONTINUE
C
        RETURN
        END
```

```
      SUBROUTINE GYRO(P,T,X)
C
C ROTATES A VECTOR X ABOUT A UNIT
C AXIS P THROUGH AN ANGLE T
C
      DIMENSION P(3),X(3),Y(3)
C
      S=SIN(T)
      C=COS(T)
C
      C1=(1.0-P(1)**2)*C+P(1)**2
      C2=P(1)*P(2)*(1.0-C)+P(3)*S
      C3=P(1)*P(3)*(1.0-C)-P(2)*S
      Y(1)=C1*X(1)+C2*X(2)+C3*X(3)
C
      C1=P(1)*P(2)*(1.0-C)-P(3)*S
      C2=(1.0-P(2)**2)*C+P(2)**2
      C3=P(2)*P(3)*(1.0-C)+P(1)*S
      Y(2)=C1*X(1)+C2*X(2)+C3*X(3)
C
      C1=P(1)*P(3)*(1.0-C)+P(2)*S
      C2=P(2)*P(3)*(1.0-C)-P(1)*S
      C3=(1.0-P(3)**2)*C+P(3)**2
      Y(3)=C1*X(1)+C2*X(2)+C3*X(3)
C
      X(1)=Y(1)
      X(2)=Y(2)
      X(3)=Y(3)
C
      RETURN
      END
```

```
      SUBROUTINE LEVEL(THETA,DELTAL,R,S,T,E,PALSY)
C
C STARTS WITH A GUESS AT THE INNERVATION VALUES E,AND
C PROCEEDS ITERATIVELY TO THE CORRECT SOLUTION
C
      REAL LASTMO,LASTE
C
      DIMENSION THETA(6),DELTAL(6),R(3,6),S(3),T(3),E(6),PALSY(6)
      DIMENSION PDERIV(6),DERIV(3),COMP(3,3),LASTE(6)
      DIMENSION FORCE(6),X(3),Y(3)
C
C INITIALISE COMPARISON VALUE OF LAST MOMENT TO A HIGH ENOUGH VALUE TO
C ENSURE THAT THE ITERATION GETS STARTED
C
      LASTMO=1000000.0
      N=0
C
C COMPUTE MOMENT ON EYEBALL ACCORDING TO THE FORMULA:
C M=T+SUM OVER ALL THE MUSCLES OF FORCE*R
C
50    S(1)=T(1)
      S(2)=T(2)
      S(3)=T(3)
C
      DO 100 I=1,6
C
      CONST0=0.0
      CONST1=DELTAL(I)+E(I)
      CONST2=0.9*CONST1
      CONST3=SQRT(38.9376+0.81*(CONST1**2))
      FORCE(I)=THETA(I)*(CONST0+CONST2+CONST3)
C
      CONST1=DELTAL(I)-11.5
      CONST2=0.9*CONST1
      CONST3=SQRT(38.9376+0.81*(CONST1**2))
      CONST4=THETA(I)*(CONST2+CONST3)
      CONST5=FORCE(I)-CONST4
      FORCE(I)=PALSY(I)*CONST5+CONST4
C
      CALL PICKUP(R,I,X)
      CALL TIMES(FORCE(I),X)
      CALL PLUS(S,X)
C
100   CONTINUE
C
C IF THE MOMENT CANNOT BE MADE SMALLER,HALT THE ITERATION
C
      CONST=SIZE(S)
      IF(CONST.GE.LASTMO) GO TO 500
      LASTMO=CONST
      DO 110 I=1,6
      LASTE(I)=E(I)
110   CONTINUE
C
C COMPUTE PARTIAL DERIVATIVE OF THE MUSCLE FORCES WITH RESPECT TO
C CHANGES IN INNERVATION
C
```

```
      DO 200 I=1,6
      CONST1=DELTAL(I)+E(I)
      CONST2=0.81*CONST1
      CONST3=SQRT(38.9376+0.81*(CONST1**2))
      PDERIV(I)=THETA(I)*(0.9+CONST2/CONST3)*PALSY(I)
200   CONTINUE
C
C COMPUTE DERIVATIVE OF EVEN NUMBERED INNERVATION VALUES WITH RESPECT TO
C ODD NUMBERED INNERVATION VALUES
C
      DO 300 I=1,3
      J=2*I-1
      DERIV(I)=(-(5.5+90.0)**2)/((E(J)+90.0)**2)
300   CONTINUE
C
C FORM THE ELEMENTS OF THE MATRIX COMP WHICH HAS AN I TH COLUMN GIVEN
C BY THE SUM:
C PDERIV(2*I-1)*CORRESPONDING UNIT ACTION VECTOR +
C PDERIV(2*I)*DERIV(I)*CORRESPONDING UNIT ACTION VECTOR
C
      DO 400 I=1,3
      J=2*I-1
      CALL PICKUP(R,J,X)
      CALL TIMES(PDERIV(J),X)
      K=2*I
      CALL PICKUP(R,K,Y)
      CALL TIMES(PDERIV(K),Y)
      CALL TIMES(DERIV(I),Y)
      CALL PLUS(X,Y)
      COMP(1,I)=X(1)
      COMP(2,I)=X(2)
      COMP(3,I)=X(3)
400   CONTINUE
C
C FINALLY SOLVE THE MATRIX EQUATION:
C ERROR IN INNERVATION = COMP*(-OVERALL MOMENT)
C AND SET UP NEW INNERVATION VALUES BY ADDING
C ON THIS COMPONENT
C
      CONST=-1.0
      CALL TIMES(CONST,S)
      CALL SOL(COMP,S,X)
      E(1)=E(1)+0.1*X(1)
      E(3)=E(3)+0.1*X(2)
      E(5)=E(5)+0.1*X(3)
      E(2)=((5.5+90.0)**2/(E(1)+90.0))-90.0
      E(4)=((5.5+90.0)**2/(E(3)+90.0))-90.0
      E(6)=((5.5+90.0)**2/(E(5)+90.0))-90.0
      N=N+1
      GOTO 50
C
500   E(1)=LASTE(1)
      E(2)=LASTE(2)
      E(3)=LASTE(3)
      E(4)=LASTE(4)
      E(5)=LASTE(5)
      E(6)=LASTE(6)
```

```
C
      RETURN
      END
```

```
      SUBROUTINE MOMENT(T,DELTA,E,PALSY,THETA,R,S,CONST)
C
C CALCULATES THE AXIS S AND SIZE CONST OF THE OVERALL MOMENT
C ON THE EYE
C
      DIMENSION S(3),T(3),DELTA(6),E(6),PALSY(6),THETA(6)
      DIMENSION R(3,6),FORCE(6),X(3)
C
      S(1)=T(1)
      S(2)=T(2)
      S(3)=T(3)
C
      DO 100 I=1,6
      CONST0=0.0
      CONST1=DELTA(I)+E(I)
      CONST2=0.9*CONST1
      CONST3=SQRT(38.9376+0.81*(CONST1**2))
      FORCE(I)=THETA(I)*(CONST0+CONST2+CONST3)
C
      CONST1=DELTA(I)-11.5
      CONST2=0.9*CONST1
      CONST3=SQRT(38.9376+0.81*(CONST1**2))
      CONST4=THETA(I)*(CONST2+CONST3)
      CONST5=FORCE(I)-CONST4
      FORCE(I)=PALSY(I)*CONST5+CONST4
      CALL PICKUP(R,I,X)
      CALL TIMES(FORCE(I),X)
      CALL PLUS(S,X)
100   CONTINUE
C
      CONST=SIZE(S)
      IF(CONST.LT.0.01) RETURN
      S(1)=S(1)/CONST
      S(2)=S(2)/CONST
      S(3)=S(3)/CONST
C
      RETURN
      END
```

```
      SUBROUTINE PATH(A,B,D,F,LENGTH,CONTAC,R,DELTAL)
C
C DETERMINES THE PATH OF THE MUSCLES OVER THE EYEBALL,
C AND COMPUTES THE ACTION VECTORS R AND PERCENT LENGTH CHANGES
C DELTAL THAT RESULT
C
      REAL L1,L2,M1,M2,M3,LENGTH
C
      DIMENSION A(3,6),B(3,6),D(3,6),F(3,6),LENGTH(6),CONTAC(6),R(3,6)
      DIMENSION DELTAL(6)
      DIMENSION G(3,6),H1(3,6),H2(3,6),H3(3,6)
      DIMENSION X(3),Y(3),Z(3)
C
C PROCEED THROUGH THE ROUTINE DOING EACH MUSCLE IN TURN
C
      DO 200 I=1,6
C
C TEST IF THE THE MUSCLE HAS LOST TANGENCY.IF IT HAS THEN CARRY OUT THE
C CODE IMMEDIATELY FOLLOWING,OTHERWISE CARRY OUT THE CODE BEGINNING AT
C 100
C
      CALL PICKUP(A,I,X)
      CALL PICKUP(B,I,Y)
      ANG1=ANGLE(X,Y)
      CONST1=SIZE(X)
      CONST2=SIZE(Y)
      ANG2=ACOS(CONST2/CONST1)
      IF(ANG1.GT.ANG2) GO TO 100
C
C SET THE UNIT ACTION VECTOR R TO BE EQUAL TO THE ORIENTATION VECTOR
C D OF THE SHORTEST PATH PLANE,SCALED BY THE FACTOR IAI*SIN(ANG1)/IA-BI
C
      CALL PICKUP(D,I,X)
C
      CALL PICKUP(A,I,Y)
      CONST1=SIZE(Y)
      CONST2=SIN(ANG1)
      CALL PICKUP(B,I,Z)
      CONST=-1.0
      CALL TIMES(CONST,Z)
      CALL PLUS(Y,Z)
      CONST3=SIZE(Y)
      CONST=(CONST1*CONST2)/CONST3
C
      CALL TIMES(CONST,X)
      CALL PUTBAK(R,I,X)
C
C COMPUTE THE PERCENTAGE LENGTH CHANGES USING THE CHANGE IN LENGTH
C GIVEN BY IBI*(ANG1-ANG2)-CONTAC
C
      CALL PICKUP(B,I,X)
      CONST=SIZE(X)
      DELTAL(I)=CONST*(ANG1-ANG2)- CONTAC(I)
      DELTAL(I)=(DELTAL(I)/LENGTH(I))*100.0
      GOTO 200
```

```
C
C COMPUTE THE SECOND VECTOR G IN THE PLANE OF THE CONTACT CIRCLE AS
C GIVEN BY:
C G=A-B
C
100     CONST=-1.0
        CALL PICKUP(A,I,X)
        CALL PICKUP(B,I,Y)
        CALL TIMES(CONST,Y)
        CALL PLUS(X,Y)
        CALL PUTBAK(G,I,X)
C
C COMPUTE THE ANGLE L1 BETWEEN F AND G AS GIVEN BY:
C COS(L1)=F.G/IGI
C
        CALL PICKUP(F,I,X)
        CALL PICKUP(G,I,Y)
        L1=ANGLE(X,Y)
C
C COMPUTE THE VECTOR H1 GIVEN BY:
C H1=COS(L1)*F-G/IGI ALL DIVIDED BY SIN(L1)
C
        CONST1=COS(L1)
        CONST2=-1.0/SIZE(Y)
        CONST3=1.0/SIN(L1)
        CALL TIMES(CONST1,X)
        CALL TIMES(CONST2,Y)
        CALL PLUS(X,Y)
        CALL TIMES(CONST3,X)
C
C COMPUTE THE LENGTH OF H1 ACCORDING TO THE FORMULA:
C IHI=RAD*COS(L2) WHERE L2 IS THE ANGLE BETWEEN H1 AND B
C
        CALL PICKUP(B,I,Y)
        L2=ANGLE(X,Y)
        CONST1=COS(L2)
        CONST2=SIZE(Y)
        CONST3=CONST1*CONST2
        CALL TIMES(CONST3,X)
        CALL PUTBAK(H1,I,X)
C
C COMPUTE H3 GIVEN BY H1+G
C
        CALL PICKUP(G,I,Y)
        CALL PLUS(Y,X)
        CALL PUTBAK(H3,I,Y)
C
C COMPUTE THE ANGLE M1 BETWEEN H1 AND H3 GIVEN BY:
C COS(M1)=(H1.H3)/(IH1I*IH3I)
C
        M1=ANGLE(X,Y)
C
C COMPUTE THE ANGLE M2 BETWEEN H2 AND H3 GIVEN BY:
C COS(M2)=IH1I/IH3I
C
```

```
      CONST1=SIZE(X)
      CONST2=SIZE(Y)
      M2=ACOS(CONST1/CONST2)
C
C COMPUTE THE ANGLE M3 BETWEEN H1 AND H2 GIVEN BY:
C M3=M1-M2
C
      M3=M1-M2
C
C COMPUTE THE VECTOR H2 BY FIRST COMPUTING A UNIT VECTOR IN THE
C DIRECTION OF H2 BY MEANS OF A LINEAR COMBINATION OF A UNIT VECTOR IN
C THE DIRECTION OF H1 AND THE UNIT VECTOR F ACCORDING TO THE FORMULA:
C H2=COS(M3)*H1+SIN(M3)*F
C
      CONST1=COS(M3)
      CONST2=SIZE(X)
      CONST3=CONST1/CONST2
      CALL TIMES(CONST3,X)
C
      CONST1=SIN(M3)
      CALL PICKUP(F,I,Y)
      CALL TIMES(CONST1,Y)
      CALL PLUS(X,Y)
C
      CALL TIMES(CONST2,X)
      CALL PUTBAK(H2,I,X)
C
C COMPUTE THE UNIT ACTION VECTOR R BY FIRST COMPUTING THE VECTOR
C TO THE POINT WHERE THE MUSCLE LEAVES THE EYEBALL,GIVEN BY B-H1+H2
C
      CALL PICKUP(B,I,X)
      CONST=-1.0
      CALL PICKUP(H1,I,Y)
      CALL TIMES(CONST,Y)
      CALL PICKUP(H2,I,Z)
      CALL PLUS(X,Y)
      CALL PLUS(X,Z)
C
C NEXT FORM THE NORMALISED CROSS PRODUCT OF A WITH THIS VECTOR
C
      CALL PICKUP(A,I,Y)
      CALL CROSS(Y,X,Z)
      CALL PUTBAK(R,I,Z)
C
C COMPUTE THE LENGTH CHANGES
C
      CALL PICKUP(H1,I,X)
      CONST=SIZE(X)
      DELTAL(I)=CONST*M3-CONTAC(I)
      DELTAL(I)=(DELTAL(I)/LENGTH(I))*100.0
C
200   CONTINUE
C
      RETURN
      END
```

```
      SUBROUTINE PICKUP(A,N,X)
C
C ACCEPTS A MATRIX A AND A VARIABLE N WHICH
C SPECIFIES THE REQUIRED COLUMN OF THE MATRIX AND PLACES THIS COLUMN
C IN THE VECTOR X
C
      DIMENSION A(3,6),X(3)

      X(1)=A(1,N)
      X(2)=A(2,N)
      X(3)=A(3,N)

      RETURN
      END
```

```
      SUBROUTINE PLUS(X,Y)
C
C ADDS THE VECTORS X AND Y AND RETURNS THE RESULT IN X
C
      DIMENSION X(3),Y(3)
C
      X(1)=X(1)+Y(1)
      X(2)=X(2)+Y(2)
      X(3)=X(3)+Y(3)
C
      RETURN
      END
```

```
      SUBROUTINE PUTBAK(A,N,X)
C
C ACCEPTS A MATRIX A,A VARIABLE N,WHICH
C SPECIFIES THE REQUIRED COLUMN OF THE MATRIX AND A VECTOR X
C WHICH IS TO BE PLACED  IN THE MATRIX AT THE SPECIFIED COLUMN.

      DIMENSION A(3,6),X(3)

      A(1,N)=X(1)
      A(2,N)=X(2)
      A(3,N)=X(3)

      RETURN
      END
```

```
      SUBROUTINE ROTAT(A,X)
C
C ROTATES THE VECTOR X ACCORDING TO THE ROTATION MATRIX A
C
      DIMENSION X(3),Y(3),A(3,3)
C
      Y(1)=A(1,1)*X(1)+A(1,2)*X(2)+A(1,3)*X(3)
      Y(2)=A(2,1)*X(1)+A(2,2)*X(2)+A(2,3)*X(3)
      Y(3)=A(3,1)*X(1)+A(3,2)*X(2)+A(3,3)*X(3)
C
      X(1)=Y(1)
      X(2)=Y(2)
      X(3)=Y(3)
C
      RETURN
      END
```

```
      SUBROUTINE SETUP(ALPHA,BETA,GAMMA,PRIMO,A,PRIMI,B,TURN,T,EY)
C
C SETS UP THE MUSCLE ORIGIN AND INSERTION VECTORS A AND B,
C THE ROTATION MATRIX TURN AND CORRESPONDING PASSIVE
C MOMENT T,FOR THE ORIENTATION SPECIFIED BY ALPHA,BETA AND
C GAMMA
C
      DIMENSION PRIMO(3,6),PRIMI(3,6)
      DIMENSION A(3,6),B(3,6),TURN(3,3),T(3)
C
C INITIALISE MUSCLE VECTORS
C
      DO 100 I=1,6
C
      A(1,I)=PRIMO(1,I)
      A(2,I)=PRIMO(2,I)
      A(3,I)=PRIMO(3,I)
C
      B(1,I)=PRIMI(1,I)
      B(2,I)=PRIMI(2,I)
      B(3,I)=PRIMI(3,I)
C
100   CONTINUE
C
C SET UP ROTATION MATRIX
C
      CALL COMPON(ALPHA,BETA,GAMMA,TURN)
      CALL TRANSP(TURN)
C
C SET UP PASSIVE MOMENT
C
      CONST=57.296*BETA
      PFORCE=0.48*CONST+0.000156*CONST**3
      CONST=57.296*(-(ALPHA+GAMMA))
      SFORCE=0.48*CONST+0.000156*CONST**3
      SFORCE=0.0
      T(1)=-(PFORCE*COS(ALPHA))+SFORCE*SIN(ALPHA)*COS(BETA)
      T(2)=PFORCE*SIN(ALPHA)+SFORCE*COS(ALPHA)*SIN(BETA)+EY*3.6
      T(3)=SFORCE*COS(BETA)
C
      RETURN
      END
```

```
      FUNCTION SIZE(X)
C
C COMPUTES THE LENGTH OF THE VECTOR X
C
      DIMENSION X(3)
C
      SIZE=X(1)**2+X(2)**2+X(3)**2
      SIZE=SQRT(SIZE)
C
      RETURN
      END
```

```
      SUBROUTINE SOL(A,Y,X)
C
C ACCEPTS A MATRIX EQUATION AX=Y AND FORMS AN
C INVERSE MATRIX BY MEANS OF CRAMERS RULE,WHICH
C IS USED TO SOLVE THE EQUATION
C
      DIMENSION A(3,3),AINV(3,3),Y(3),X(3)
C
      DO 100 I=1,3
      DO 100 J=1,3
      AINV(I,J)=0.0
100   CONTINUE
C
C COMPUTE THE DETERMINANT OF A
C
      DET1=A(1,1)*A(2,2)*A(3,3)
      DET2=A(2,1)*A(3,2)*A(1,3)
      DET3=A(3,1)*A(1,2)*A(2,3)
      DET4=A(1,1)*A(3,2)*A(2,3)
      DET5=A(2,1)*A(1,2)*A(3,3)
      DET6=A(3,1)*A(2,2)*A(1,3)
      DET=DET1+DET2+DET3-DET4-DET5-DET6
C
C COMPUTE INVERSE MATRIX
C
      AINV(1,1)=((A(2,2)*A(3,3))-(A(3,2)*A(2,3)))/DET
      AINV(1,2)=-((A(1,2)*A(3,3))-(A(3,2)*A(1,3)))/DET
      AINV(1,3)=((A(1,2)*A(2,3))-(A(2,2)*A(1,3)))/DET
      AINV(2,1)=-((A(2,1)*A(3,3))-(A(3,1)*A(2,3)))/DET
      AINV(2,2)=((A(1,1)*A(3,3))-(A(3,1)*A(1,3)))/DET
      AINV(2,3)=-((A(1,1)*A(2,3))-(A(2,1)*A(1,3)))/DET
      AINV(3,1)=((A(2,1)*A(3,2))-(A(3,1)*A(2,2)))/DET
      AINV(3,2)=-((A(1,1)*A(3,2))-(A(3,1)*A(1,2)))/DET
      AINV(3,3)=((A(1,1)*A(2,2))-(A(2,1)*A(1,2)))/DET
C
C COMPUTE SOLUTION
C
      X(1)=AINV(1,1)*Y(1)+AINV(1,2)*Y(2)+AINV(1,3)*Y(3)
      X(2)=AINV(2,1)*Y(1)+AINV(2,2)*Y(2)+AINV(2,3)*Y(3)
      X(3)=AINV(3,1)*Y(1)+AINV(3,2)*Y(2)+AINV(3,3)*Y(3)
C
      RETURN
      END
```

```
      SUBROUTINE SWIVEL(ALPHA,BETA,GAMMA,S,CONST)
C
C ROTATES THE EYE,INITIALLY IN THE POSITION
C SPECIFIED BY THE ORIENTATION ANGLES ALPHA,BETA AND GAMMA,
C ABOUT THE VECTOR S BY AN AMOUNT CONST AND RETURNS THE
C NEW ORIENTATION ANGLES
C
      DIMENSION S(3),A(3,3),X(3),Y(3),Z(3),U(3),V(3)
C
C FIRST FIND THE COORDINATES OF THE X,Y AND Z AXES AFTER ROTATION
C THROUGH THE CURRENT ORIENTATION ANGLES ALPHA,BETA AND GAMMA
C
      X(1)=1.0
      X(2)=0.0
      X(3)=0.0
C
      Y(1)=0.0
      Y(2)=1.0
      Y(3)=0.0
C
      Z(1)=0.0
      Z(2)=0.0
      Z(3)=1.0
C
      CALL COMPON(ALPHA,BETA,GAMMA,A)
      CALL TRANSP(A)
      CALL ROTAT(A,X)
      CALL ROTAT(A,Y)
      CALL ROTAT(A,Z)
C
C NEXT FIND THE COORDINATES OF THE X,Y AND Z AXES AFTER A FURTHER
C ROTATION ABOUT THE AXIS S OF THE OVERALL MOMENT
C
      CALL GYRO(S,CONST,X)
      CALL GYRO(S,CONST,Y)
      CALL GYRO(S,CONST,Z)
C
C FINALLY COMPUTE THE NEW ORIENTATION ANGLES
C
      U(1)=Z(1)
      U(2)=Z(2)
      U(3)=0.0
C
      V(1)=0.0
      V(2)=1.0
      V(3)=0.0
C
      ALPHA=ANGLE(U,V)
C
      V(1)=1.0
      V(2)=0.0
      V(3)=0.0
C
      IF(DOT(U,V).LT.0.0) ALPHA=-ALPHA
C
```

```
      V(1)=0.0
      V(2)=0.0
      V(3)=1.0
C
      BETA=ANGLE(V,Z)
C
      U(1)=0.0
      U(2)=1.0
      U(3)=0.0
C
      V(1)=1.0
      V(2)=0.0
      V(3)=0.0
C
      CALL COMPON(ALPHA,BETA,-ALPHA,A)
      CALL TRANSP(A)
      CALL ROTAT(A,U)
      CALL ROTAT(A,V)
C
      CONST1=ANGLE(U,Y)
      CONST2=DOT(V,Y)
      IF(CONST2.LT.0.0) GAMMA=-ALPHA-CONST1
      IF(CONST2.GE.0.0) GAMMA=-ALPHA+CONST1
C
      RETURN
      END
```

```
      SUBROUTINE TIMES(A,X)
C
C MULTIPLIES THE VECTOR X BY THE SCALAR A
C
      DIMENSION X(3)
C
      X(1)=A*X(1)
      X(2)=A*X(2)
      X(3)=A*X(3)
C
      RETURN
      END
```

```
      SUBROUTINE TRANSP(A)
C
C FORMS THE TRANSPOSE OF THE MATRIX A
C
      DIMENSION A(3,3),B(3,3)
C
      DO 100 I=1,3
      DO 50  J=1,3
      B(I,J)=A(J,I)
50    CONTINUE
100   CONTINUE
C
      DO 200 I=1,3
      DO 150 J=1,3
      A(I,J)=B(I,J)
150   CONTINUE
200   CONTINUE
C
      RETURN
      END
```

APPENDIX III

DESCRIPTION OF PATIENT DATA

PATIENT DB

Positions assumed by left eye

ISO-AZIMUTH	ISO-LATITUDE
0.0	2.9
0.0	15.1
13.8	15.9
14.8	2.9
14.7	-11.0
0.2	-10.8
-13.7	-11.1
-14.5	1.3
-13.9	13.9

Positions assumed by right eye

ISO-AZIMUTH	ISO-LATITUDE
0.0	- 2.3
0.0	13.5
15.8	13.2
15.4	- 2.7
14.2	-20.5
0.3	-18.8
-16.2	-18.1
-14.3	- 2.5
-15.3	13.8

PATIENT AC

Positions assumed by left eye

ISO-AZIMUTH	ISO-LATITUDE
- 1.4	3.8
- 1.5	17.2
12.7	18.3
14.1	3.2
14.3	-10.0
- 1.4	-11.2
-16.6	-11.5
-16.2	3.1
-16.2	16.8

Positions assumed by right eye

ISO-AZIMUTH	ISO-LATITUDE
2.6	- 3.7
2.1	11.1
17.1	9.8
17.5	- 4.3
12.9	-20.9
- 0.3	-18.8
-15.0	-18.0
-11.9	- 3.6
-12.3	12.3

PATIENT EW

Positions assumed by left eye

ISO-AZIMUTH	ISO-LATITUDE
1.9	4.1
2.9	17.0
15.0	18.6
17.8	5.4
18.0	- 6.8
3.1	- 8.5
-13.1	-11.8
-14.3	2.5
-14.5	17.3

Positions assumed by right eye

ISO-AZIMUTH	ISO-LATITUDE
- 1.2	- 3.2
0.0	13.0
14.0	11.8
12.7	- 4.6
12.1	-21.2
- 2.4	-19.6
-16.6	-17.8
-16.5	- 2.6
-14.8	12.8

PATIENT AB

Positions assumed by left eye

ISO-AZIMUTH	ISO-LATITUDE
1.3	5.6
0.3	16.2
14.7	16.4
14.7	5.3
16.4	- 5.9
1.2	- 6.7
-12.6	- 7.2
-13.2	4.8
-13.2	15.8

Positions assumed by right eye

ISO-AZIMUTH	ISO-LATITUDE
0.0	2.3
2.0	14.4
15.6	13.5
15.0	- 3.4
14.9	-24.3
0.0	21.8
-15.1	-18.7
-14.3	- 2.5
-14.5	14.5

PATIENT SE

Positions assumed by left eye

ISO-AZIMUTH	ISO-LATITUDE
1.1	5.0
0.0	17.7
15.0	18.0
16.3	3.7
15.8	- 6.6
2.1	- 7.4
-11.7	- 7.6
-14.6	3.9
-14.7	17.6

Positions assumed by right eye

ISO-AZIMUTH	ISO-LATITUDE
0.5	- 6.3
1.0	11.3
14.7	11.4
14.0	- 8.0
10.9	-23.9
- 1.6	-23.7
-17.6	-23.6
-16.5	- 6.3
-16.1	11.2

PATIENT LU

Positions assumed by left eye

ISO-AZIMUTH	ISO-LATITUDE
0.0	0.0
0.0	14.6
15.0	15.0
14.6	0.0
14.9	-14.9
1.0	-14.5
-11.7	-15.1
-11.3	0.0
-11.5	16.0

Positions assumed by right eye

ISO-AZIMUTH	ISO-LATITUDE
- 2.9	0.0
- 2.7	15.3
14.9	14.9
14.6	0.0
14.6	-14.6
- 2.7	-15.2
-20.9	-15.6
-22.3	0.0
-21.7	12.8

PATIENT HF

Positions assumed by left eye

ISO-AZIMUTH	ISO-ELEVATION
4.0	0.0
2.6	14.9
16.6	15.2
16.7	0.0
16.7	-15.0
3.9	-14.6
- 8.2	-15.5
- 9.6	0.2
- 9.7	15.0

Positions assumed by right eye

ISO-AZIMUTH	ISO-ELEVATION
- 7.5	- 1.3
- 7.7	13.5
9.6	14.8
7.0	- 1.2
7.4	-15.9
- 7.7	-15.9
-23.1	-16.5
-22.8	- 1.2
-22.4	12.7

PATIENT SP

Positions assumed by left eye

ISO-AZIMUTH	ISO-ELEVATION
2.9	0.0
2.5	14.3
13.2	13.2
16.1	- 1.4
16.7	-15.0
2.5	-14.3
- 6.9	-15.0
- 4.6	0.0
- 6.0	14.2

Positions assumed by right eye

ISO-AZIMUTH	ISO-ELEVATION
- 4.0	0.0
- 2.0	14.4
12.3	14.7
9.2	0.2
9.6	-15.4
- 4.5	-14.9
-26.0	-15.2
-26.8	0.0
-24.9	14.7

PATIENT WJ

Positions assumed by left eye

ISO-AZIMUTH	ISO-ELEVATION
3.5	0.0
2.6	15.0
12.6	15.0
14.6	0.0
16.5	-13.8
5.3	-14.7
- 5.3	-14.7
- 6.4	0.0
- 8.0	13.5

Positions assumed by right eye

ISO-AZIMUTH	ISO-ELEVATION
- 8.0	0.0
- 8.7	15.1
12.9	15.4
8.1	- 0.7
6.8	-16.2
- 9.4	-14.6
-25.1	-14.8
-25.6	- 0.9
-23.8	13.5

PATIENT JA

Positions assumed by left eye

ISO-AZIMUTH	ISO-ELEVATION
2.9	0.0
2.3	13.3
12.8	14.7
14.6	0.0
15.2	-15.2
3.2	-15.0
- 3.1	-16.4
- 3.0	- 2.5
- 3.3	12.5

Positions assumed by right eye

ISO-AZIMUTH	ISO-ELEVATION
- 3.5	0.0
- 3.9	14.9
15.4	14.3
14.6	- 0.8
14.2	-15.5
- 1.4	-16.1
-25.4	-18.9
-27.1	- 1.8
-34.1	10.5

PATIENT EM

Positions assumed by left eye

ISO-AZIMUTH	ISO-ELEVATION
10.0	1.8
9.9	15.3
21.1	15.2
21.7	2.2
21.9	-12.5
10.4	-12.5
- 2.2	-12.5
- 2.8	0.6
- 3.4	14.7

Positions assumed by right eye

ISO-AZIMUTH	ISO-ELEVATION
-11.3	0.0
-10.9	15.1
7.8	15.5
5.1	- 0.9
6.6	-16.5
-11.4	-14.7
-30.2	-15.5
-30.9	0.0
-30.5	15.0

PATIENT MA

Positions assumed by left eye

ISO-AZIMUTH	ISO-ELEVATION
9.9	0.0
12.6	15.0
22.1	13.6
22.8	0.0
22.5	-14.9
9.6	-14.8
- 3.7	-15.2
- 0.9	0.0
- 0.3	14.6

Positions assumed by left eye

ISO-AZIMUTH	ISO-ELEVATION
-14.3	0.0
-14.9	13.4
5.8	16.2
5.7	0.0
5.6	-15.5
- 9.6	-15.4
-25.1	-14.8
-29.2	0.5
-29.6	11.5

PATIENT DS

Positions assumed by left eye

ISO-AZIMUTH	ISO-ELEVATION
9.4	0.0
5.9	12.1
16.7	14.0
21.0	0.0
22.1	-14.7
11.6	-14.3
0.5	-14.9
- 0.7	- 1.3
0.0	12.7

Positions assumed by right eye

ISO-AZIMUTH	ISO-ELEVATION
-15.4	0.0
-10.6	15.3
9.3	15.0
5.7	0.0
4.5	-14.7
-14.4	-14.9
-32.0	-15.0
-32.2	0.0
-31.9	15.0

PATIENT DM

Positions assumed by left eye

ISO-AZIMUTH	ISO-ELEVATION
10.2	0.0
9.5	13.6
23.8	14.6
21.3	0.0
23.2	-13.1
12.6	-13.0
0.3	-14.6
- 1.8	- 1.5
- 1.6	13.4

Positions assumed by right eye

ISO-AZIMUTH	ISO-ELEVATION
-14.6	0.0
-15.0	15.5
6.4	15.3
3.4	- 0.6
2.2	-16.0
-13.8	-16.5
-36.3	-13.1
-35.8	0.0
-35.7	15.1

PATIENT PR

Positions assumed by left eye

ISO-AZIMUTH	ISO-ELEVATION
15.6	0.0
14.6	15.2
26.2	15.4
27.0	0.5
27.6	-13.7
16.5	-15.3
1.5	-17.4
0.4	- 2.3
1.3	15.6

Positions assumed by right eye

ISO-AZIMUTH	ISO-ELEVATION
-15.5	0.0
-17.3	14.4
2.4	15.4
1.8	0.0
1.6	-13.4
-18.2	-11.7
-33.3	-11.5
-32.6	0.5
-31.9	14.3

APPENDIX IV

PUBLICATIONS

Ophthal. Physiol. Opt., Vol. 2, No. 2, pp. 107 – 117, 1982.
Printed in Great Britain.

0275 – 5408/82/020107 – 11 $03.00/0
Pergamon Press Ltd.

COMPUTER SIMULATION OF EXTRAOCULAR MUSCLE CO-OPERATION: AN EVALUATION

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(*Received* 10 *August* 1981, *in revised form* 28 *September* 1981)

Abstract—This paper is concerned with specifying how extraocular muscles co-operate in moving the eye. A set of assumptions is described which enable this to be done with enough precision for a computer model of the actions of the extraocular muscles to be set up. The behaviour of the model and its validity are then evaluated.

INTRODUCTION

Krewson (1950) was the first person to produce quantitative estimates of the relationships between rotations of the globe and actions of the muscles. He assumed that they took the mechanical shortest path and calculated their corresponding axes of rotation. To obtain some idea of the mode of action of each of the muscles he considered the projections of the axes of rotation into an eye-centred system of Cartesian axes. As has been conventional since the work of Helmholtz, the system of axes was such that one axis lay along the line of fixation in the primary position and one of the remaining two axes coincided with the line between the centres of rotation of the two eyes. He then considered that the projection onto these axes represented the amount of the forces exerted by each of the muscles that was devoted variously to adduction/abduction, elevation/depression and torsional movements. Because of the number of calculations involved he only considered movements in the horizontal plane. This enabled him to clarify the main actions of the individual muscles. However, his approach, whilst it revealed much about individual muscles, was not so informative about how they co-operate.

Boeder (1961) approached the analysis of extraocular muscle co-operation by calculating the length changes that occur when the muscles follow the shortest path around the globe. He also attempted to provide a more realistic measure of the forces exerted by each of the muscles by multiplying their changes in length by their respective cross-sectional areas. One important conclusion that he formed was that, while the inferior oblique is more contracted than the superior rectus in adduction, it is still the latter which exerts the larger force.

Boeder (1962) went on to consider positions of gaze within a 60 by 60° range. As well as computing the length changes that occur when muscles follow the shortest path over the globe, he also determined the direction in which each muscle would turn the line of fixation in terms of adduction/abduction and elevation/depression rotations. This enabled him to make a number of judicious observations about how the extraocular muscles co-operate. In particular, he considered whether or not it is only the contracting muscles that move the globe. He compared movements from A to B with the return movements from B to A and noticed that the direction of action of the chief shortening muscles was not necessarily the same as that of the chief lengthening muscles, so, if only the contracting muscles moved the eyeball, the movement would be irreversible.

More recently, the purely geometrical calculations of the axes of rotation and changes in lengths of the muscles, have been put into matrix notation by Solomons (1978), who has calculated the adduction/abduction, elevation/depression and torsional action of each of the muscles in primary, secondary and tertiary positions of gaze. The results of these calculations have brought out, *inter alia,* the balanced nature of the torsional effects within pairs of antagonistic muscles. In general, however, whilst this approach has simplified the calculations, it has not by itself revealed anything further about the way in which the muscles co-operate.

This last criticism is especially true if one tries to compute what will happen if some of the muscles are diseased. To be able to do this, the problem of muscle actions during rotations of the eye should be approached by way of consideration of the mechanics of the movement, which require that if the globe is to stay in any given position then the sum of the moments around the centre of rotation must be zero in that position. Robinson (1975) has formulated a model which incorporates this mechanical constraint, but to do so he had to make a number of more or less justifiable assumptions which are described in the next section.

THEORY

A complete description of the model is given in Robinson (1975) and what follows here will consist only of a statement of the main assumptions underlying his model so that they can be evaluated.

The first of these assumptions was that the origins and insertions of the muscles in the normal eye are adequately described by the data of Volkmann (1869) who used a co-ordinate system with the origin placed at the centre of rotation of the eye, which he judged to be 1.29 mm posterior to the geometric centre of the eye. If one shifts his origin forward by 1.29 mm along the primary direction of the line of fixation, one makes his co-ordinates directly comparable with those of Ruete and Fick, cited in Helmholtz (1911). This has been done by Von Kries and the results are given in an appendix in Helmholtz (1911). One may test whether or not the insertions of the muscles are consistent with the concept of a spherical globe by calculating the distances between the points of insertion of the muscles and the centre of rotation of the eye, which should all be equal with a spherical globe. In terms of the model, this corresponds to calculating the lengths of the insertion vectors of the muscles and the results of such a calculation are shown in Table 1. Considering the difficulty of making the measurements, the agreement is reasonable, although in order to set up the model it was assumed that the centre of rotation and geometrical centre of the eye are identical, which is not usually true.

The next two assumptions were concerned with specifying the shape of each muscle in any given position of the globe. The second assumption specified how the muscle was placed in relation to its insertion, a problem which is complicated by the fact that the muscles fan out at their insertion. Previous investigators had always selected the obvious assumption of the shortest path despite the mechanical restrictions at the insertions, but in this case it was proposed that the actual path lies somewhere between the shortest path and the path perpendicular to the line of insertion. Two criteria were outlined that should be satisfied by a reasonable assumption as to the angle of twist away from the perpendicular path. The first of these was that if the line of insertion stays perpendicular to the primary plane of the muscle, then the twist angle should be zero. This limits the

Table 1. Lengths of the insertion vectors of the extraocular muscles (mm) according to the various investigators

	LR	MR	SR	IR	SO	IO
REUTE	11.9	11.6	11.7	11.8	11.6	12.0
FICK	12.0	12.0	12.0	12.0	11.3	12.0
VOLKMANN (AFTER VON KRIES)	11.4	12.3	12.3	12.3	12.8	12.1
VOLKMANN (AFTER KREWSON)	12.0	13.1	13.0	13.0	12.2	11.3

path of each eye muscle as the direction of the insertion vector becomes directly opposite to the direction of the origin vector, whereupon slight movements of the eye cause extreme changes in the shortest path. The second criterion was that the twist angle should depend on the sideways force at the insertion. A satisfactory assumption was made by letting the twist angle depend on the cosine of the angle between the vector along the line of insertion of the muscle and the vector to its origin. In the primary plane of the muscle, this function is always zero and so there is no twist at the insertion.

The third assumption specified the path of the muscle away from its insertion. This assumption was directed towards ensuring that there is no abrupt change of direction when the muscle leaves the eyeball. This was achieved by assuming that the path of the muscle over the globe lay in a plane containing the vector corresponding to the direction in which the muscle leaves its insertion and the origin of the muscle. The intersection of this plane with the spherical globe is a circle, so that this assumption implies that the muscle makes contact with the globe along an arc of a circle.

As well as specifying the shapes of the muscles it is also necessary to specify the forces that they exert in the different orientations of the eye and the fourth and fifth assumptions were concerned with this aspect of the problem. The steady-state force exerted by a muscle is a function of its length and its innervation level. Innervation cannot be measured directly but it can be manipulated by asking a patient undergoing extraocular muscle surgery on the horizontal recti of one eye to look with the other eye at targets located in the horizontal plane at known angles with the primary direction of the line of fixation and measuring the force changes in the detached recti of the eye being operated on. The fourth assumption, then, consisted of a function describing the force exerted by a muscle in accordance with its length and its innervation which was based on the experimental data of Collins and O'Meara cited in Robinson (1975). It was found that if muscle tension was plotted against extension (ΔL), calculated as a percentage of the length in the primary position, then the function was a portion of an hyperbola. Furthermore, the effect of a change in the innervation level was to shift the curve along the muscle extension axis and this shift could be characterized by incorporating a factor (E) to reflect the level of innervation. The equation actually specified by Robinson (1975), after substitution of parameters, takes the form:

$$\text{Force (g)} = 0.9 \times (\Delta L + E) + \sqrt{38.94 + 0.81 \times (\Delta L + E)^2}.$$

The fifth assumption was that the force functions of the other muscles were identical to that of the lateral rectus, except for a multiplicative factor corresponding to their cross-sectional area, relative to that of the lateral rectus. The actual values for this factor were based on the data of Volkmann (1869) and were as follows:

LR	MR	SR	IR	SO	IO
1.0	1.04	0.68	0.95	0.5	0.47

As well as the active forces of the muscles, there are also passive forces due to check ligaments and other orbital structures which restrain the eye in movements away from the primary position. A function was developed to describe the way in which the passive force varied with the angle (beta) between the primary position and the line of fixation, based on the experimental results of Robinson *et al.* (1969) and Scott (1971). When the angle beta is given in degrees, the function specified by Robinson (1975), with the parameters inserted, takes the form:

$$\text{Passive force (g)} = 0.48 \times \text{beta} + 0.000156 \times \text{beta}^3.$$

The sixth assumption was that the passive force in any position could be described by this function and acted around the axis specified by Listing's law. A constant moment was added which made the resting point deviate 7.5° temporally which is consistent with the abduction seen in deep anaesthesia.

Given these assumptions, the problem of simulating extraocular muscle co-operation breaks down into two halves, which can be referred to as the innervation problem and the position problem. The innervation problem arises when the position of the eye is given and one has to determine the appropriate levels of innervation for each of the muscles. This involves finding the innervation values which result in the overall moment on the eyeball in that position being zero. Since there are six muscles and only 3 df for the globe, if each muscle is independently innervated there will be an infinite number of solutions to this problem. Hence, the law of reciprocal innervation was invoked and the seventh assumption was made, namely, that the innervation of the antagonist muscle was reciprocal to that of the agonist muscle for each of the three muscle pairs. The actual equation specifying the innervation of the antagonist in terms of that of the agonist as given in Robinson (1975) becomes, after insertion of the parameter values:

$$E(\text{antagonist}) = \{187.69/[E(\text{agonist}) + 9.7]\} - 9.7.$$

The position problem arises when one has determined the innervation values, but does not know what position the globe will take up to achieve mechanical equilibrium. Up to this point it has been assumed that the eye rotates in accordance with Listing's law, but with diseased eyes this need no longer be so. Therefore the final assumption involved setting up an additional passive force, governed by the same function as the original passive force, except that the torsion angle was substituted for the angle of deviation from the primary position, which opposed any torsional movements of the eye. This allowed some torsion in diseased eyes, but resulted in zero torsion in normal eyes.

RESULTS

Action of the individual muscles

Given the parameters of the model one can use it to gain some idea of the actual forces exerted by each of the muscles in any particular direction of gaze. To do this, the force exerted by each muscle is calculated by inserting the values for the extension and innervation of the muscle into the force equation specified earlier. Then, as was done by Krewson (1950) the axis around which the force of the muscle acts may be decomposed into an eye-centred system of Cartesian axes to obtain the relative amounts of the muscle force devoted to the various types of movement. These calculations have been done for nine central gaze positions, and the results are shown in Figs 1 – 3. It must be emphasized that the results will only be valid over this limited range of movements.

With respect to forces acting around the adduction/abduction axis (shown in Fig. 1), there are three points which are noteworthy. The first is that the horizontal recti develop the main forces, with the lateral rectus exerting the largest force of any muscle for any type of rotation. The second is that the vertical recti always adduct. The third point is that the obliques do not contribute anything significant to movements of abduction and adduction. Recordings of the actual muscle tensions in the lateral and medial recti during unrestrained eye movements of patients with strabismus have been made by Collins *et al.* (1975) and the agreement of the model with their results is good. They found that the minimum tension of each of the horizontal recti did not normally fall below 8 – 12 g and that the minimum tension of each muscle usually occurred 15° out of their field of action. These findings are matched by the predictions of the model, except for the location of the minimum tension of the medial rectus, which, as can be seen in Fig. 1, achieves its minimum tension in the primary position, instead of with 15° of abduction.

As regards the forces acting around the elevation/depression axis (shown in Fig. 2), it was found that the superior rectus exerts the dominant force in elevation and the inferior rectus exerts the dominant force in depression. However, the relative participation of the vertical recti and the obliques does change in accordance with the classical picture with the superior rectus acting more in abduction and the inferior oblique acting more in adduction. The model was also tried using the shortest-path assumption and the only consistent difference was found in connection with this component of the force, since it was found that with the shortest-path assumption both the horizontal recti elevated the eye with elevated gaze and depressed it with depressed gaze.

The torsional forces (shown in Fig. 3) are of especial interest since any imbalance in them leads to a deviation from Listing's law. Perhaps the most striking feature of these forces is that each muscle pair exerts opposing forces about the line of fixation, so that the muscle actions are predisposed towards a zero torsion equilibrium. Overall, the forces are smaller than for the other rotations so any passive moment about the axis will provide effective constraint against torsional movements.

Concerning the actions of the individual muscles, the forces exerted are as formulated in the classical description with the superior rectus and oblique both acting as intorters while the inferior rectus and oblique both act as extorters. Surprisingly, though, the horizontal recti produce a not inconsiderable amount of torsion. This result must be placed within the context of the assumption of the relative strengths of the muscles, for although they may project less than the other muscles onto the torsion axis, the horizontal recti counter this factor by exerting larger forces. This type of consideration will be overlooked by the purely geometrical analyses of Boeder (1962) and Solomons (1978).

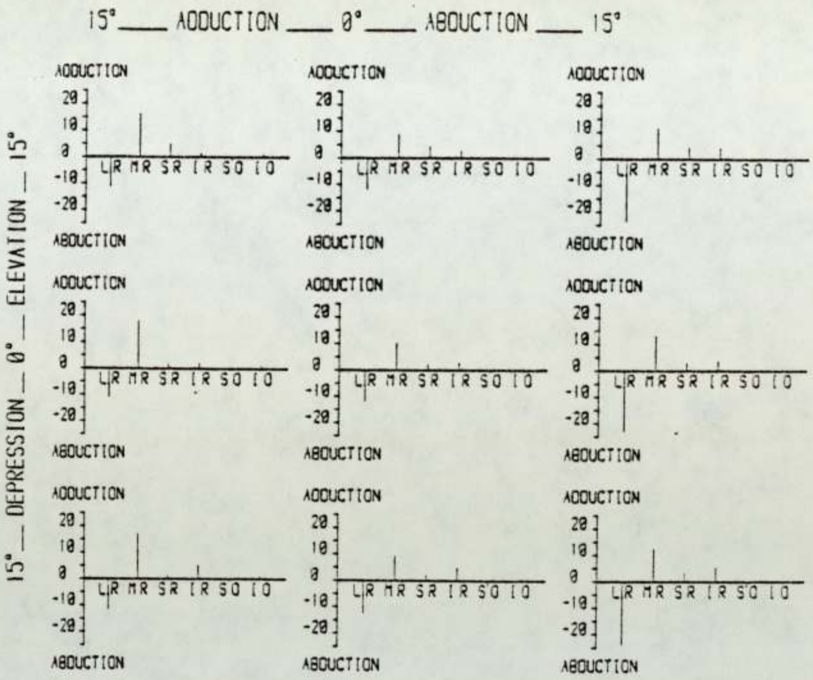

Fig. 1. Forces exerted by each of the muscles around the adduction – abduction axis in nine positions of gaze. For this and the following two graphs the conventions are as follows. The forces in each position are represented by a separate graph. The forces of individual muscles are distinguished by the letters LR for lateral rectus, MR for medial rectus, SR for superior rectus, IR for inferior rectus, SO for superior oblique, and IO for inferior oblique. The vertical axis of each graph gives the direction in which the muscle force acts and is calibrated in grams.

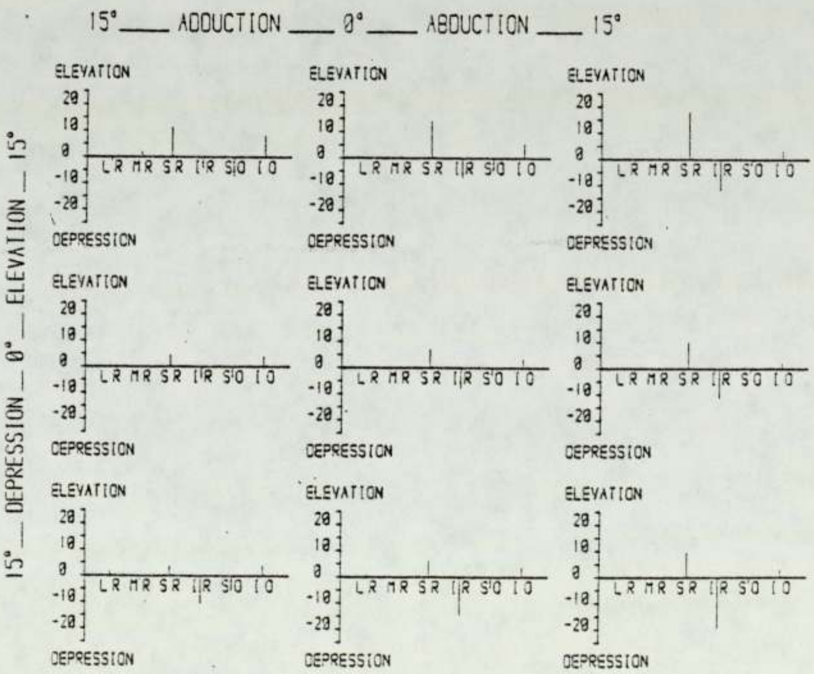

Fig. 2. Forces exerted by each of the muscles around the elevation – depression axis in nine positions of gaze.

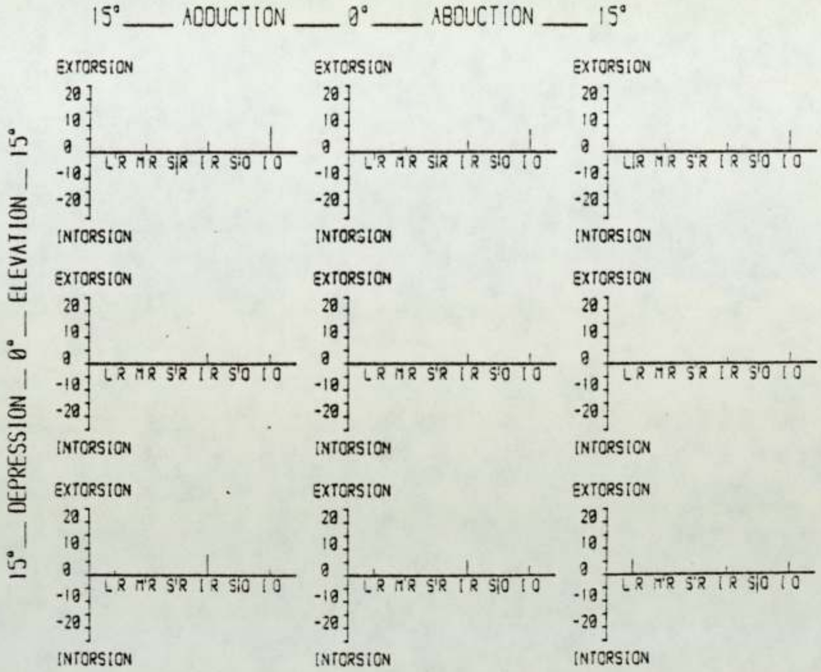

Fig. 3. Forces exerted by each of the muscles around the torsion axis in nine positions of gaze.

Effects of muscle paresis

Following the lead of France and Burbank (1979), the model has been used to simulate the effects of oculomotor nerve palsies. The model, as it stands, is essentially monocular, but by using two versions of the model in combination, the effect of lesions of individual oculomotor nerves of the right eye and the resulting Hess screen projections could be determined.

The Hess chart for the right eye shows the position adopted by the right eye when the left eye is fixating. In terms of the model, this involves solving the position problem for the right eye, given the normal innervation values. The chart for the left eye shows the position adopted to the left eye when the right eye is fixating. In terms of the model this involves first computing the innervation values needed to maintain the fixation of the affected right eye and then solving the position problem for the normal eye, given these innervation values.

The effects of damage to the third, fourth and sixth nerves are shown in Figs 4 – 6 respectively. The palsies were modelled by reducing the innervation level to the muscle or muscles supplied by the nerve to half their normal levels. These projections are reasonably consistent with those found in actual isolated nerve lesions. The numbers at each position in the figures give the predicted angle of torsion in degrees, with a positive number signifying a clockwise rotation about the line of fixation.

In order to investigate the role of the final assumption of a counter-torsional force, which was introduced to control the deviation from Listing's law in clinical conditions, the Hess chart corresponding to damage to the fourth nerve, which shows the most torsion, was repeated with the counter-torsional force halved and with it doubled. These changes made no appreciable difference to the shape of the resulting Hess chart, but with

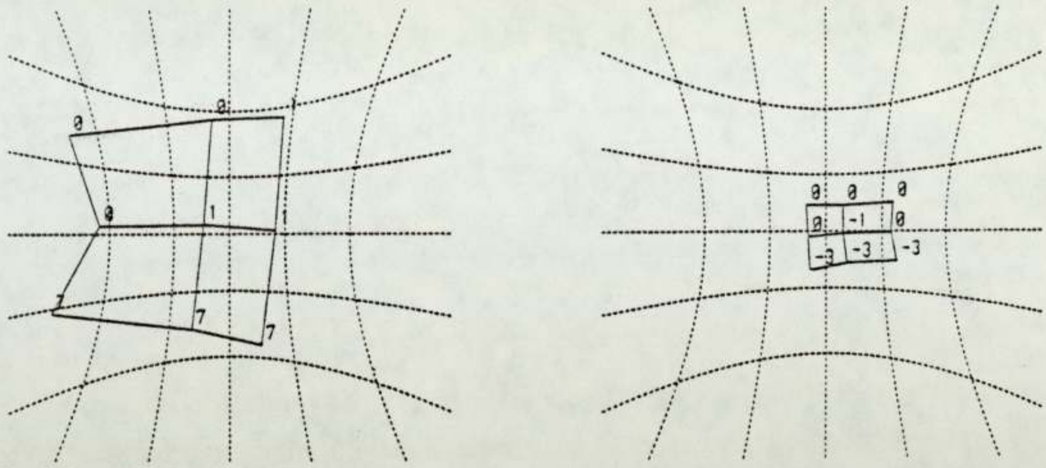

Fig. 4. Simulated Hess chart with damage to the third nerve. In this and all subsequent figures, the left half gives the Hess screen projection of the left eye and the right half gives the Hess screen projection of the right eye.

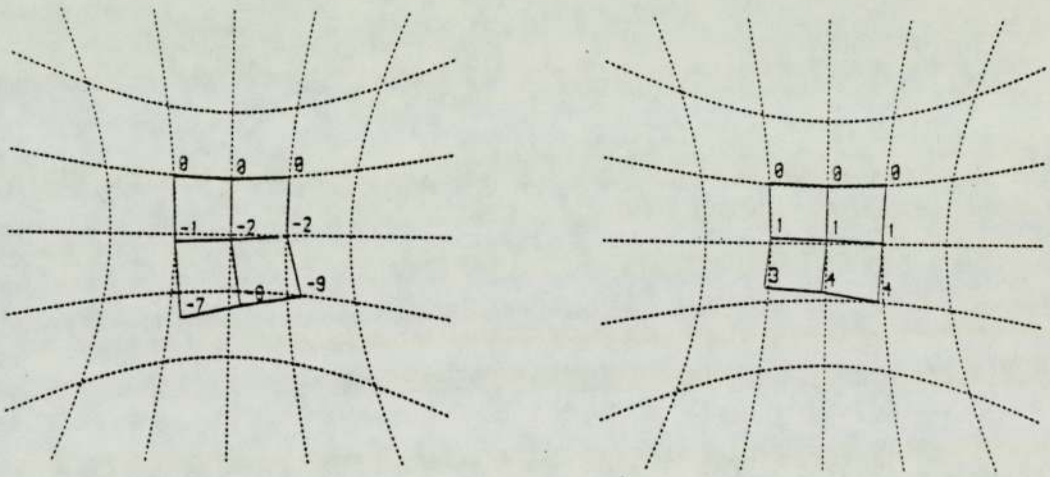

Fig. 5. Simulated Hess chart with damage to the fourth nerve.

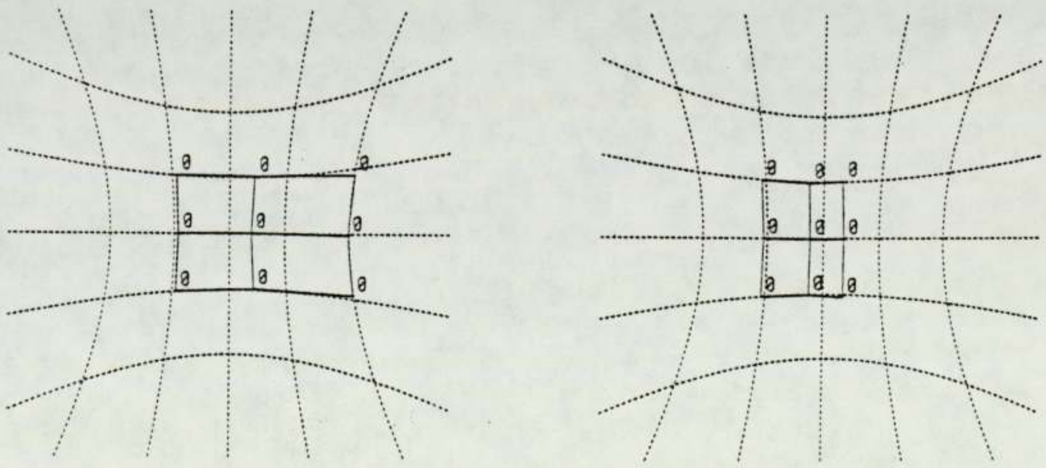

Fig. 6. Simulated Hess chart with damage to the sixth nerve.

a reduced counter-torsional force, more torsion occurred and with an increased counter-torsional force, less torsion occurred. The change in torsion in both cases was not large, being around 1° and occurring in the depressed-gaze positions. These results are closely related to the characterization of the force exerted by each muscle as being in part due to its extension and in part due to its innervation. For instance, if one considered the example of the sixth nerve lesion then it is noticeable that the movements to the left are relatively unaffected, because the force exerted by the lateral rectus of the right eye in these positions of gaze is mainly due to extension of the muscle rather than its level of innervation. Instead of altering the levels of innervation one could alter the muscle strength factor, to reproduce the effect of a diseased muscle as opposed to a diseased nerve. This has been done for the superior oblique and lateral rectus of the right eye using the same 50% reduction as with the nerve lesions and the results are shown in Figs 7 and 8 respectively.

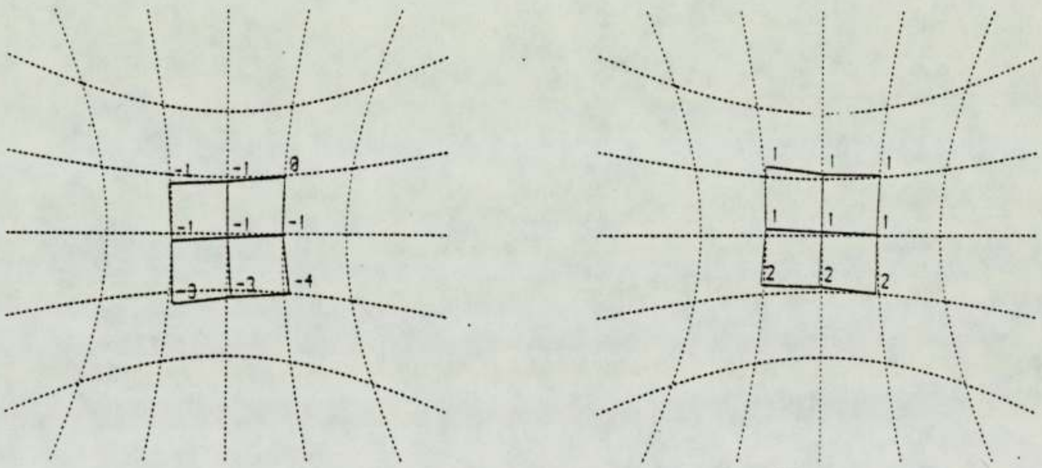

Fig. 7. Simulated Hess chart with paresis of the superior oblique.

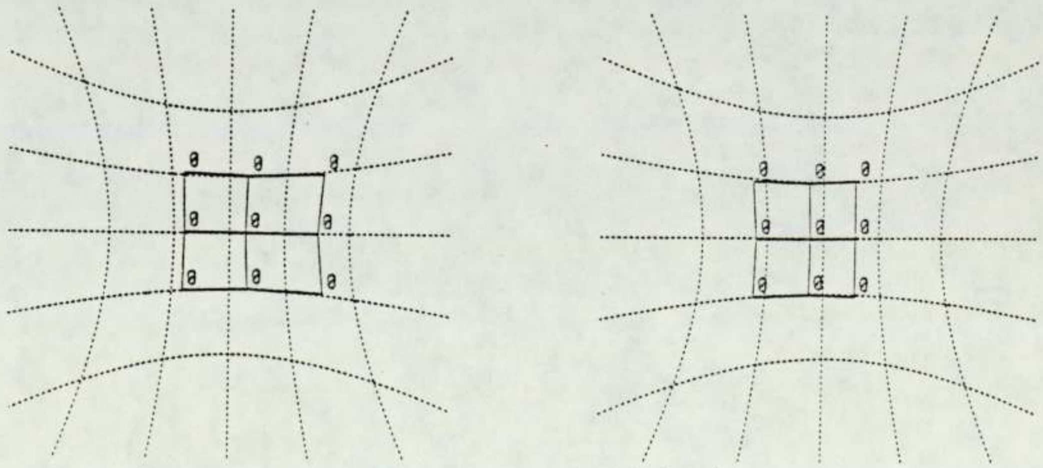

Fig. 8. Simulated Hess chart with paresis of the lateral rectus.

CONCLUSION

Overall, the model seems to provide a promising approach to understanding the mechanisms underlying some forms of squint. However, it is clear that it rests on a number of assumptions, which must be kept in mind if it is not going to be misleading. Obviously the assumptions are not equally valid and in order to isolate the more tentative ones an attempt has been made to assess the relative soundness of the assumptions.

The muscle insertions are not consistent with a spherical eyeball and it would be preferable if they were scaled so that the insertions were all the same distance from the centre of the eye, since the calculations of the paths of the muscles over the globe are based on the geometry of a spherical eye. In general, the model seems fairly robust with respect to the assumptions about the positions and shapes of the muscles, as demonstrated by the limited effects of switching to the shortest-path assumption.

The fourth assumption of the equation governing the relationship between the force exerted by the muscle and its length change and innervation, and also the sixth assumption of the passive-force equation are both based directly on experimental investigations and need only be changed to incorporate additional experimental results. The fifth assumption of the relative muscle strengths is a dominant one in that changes in this assumption will significantly alter the simulations produced by the model. Since it is based on the anatomical measurements of Volkmann (1869) rather than on actual measurements of relative force, it should be treated with caution.

On a methodological level, the question arises as to how much confidence one can put in the solution to the innervation problem. Fry (1978) has emphasized the point that an infinite variety of patterns of tension could be holding the eye in any given position, and whilst the reciprocal innervation assumption leads to unique solution, its formulation may not be correct in detail. This question is also pertinent to the origin of the tendency to adhere to Listing's law, which may be due to neural constraints on the pattern of innervation, but which in the model requires the assumption of a counter-torsional force. Fortunately it was found that alterations to the size of the assumed counter-rotational force did not markedly affect the positions adopted by the eye, only its angle of torsion, so the results produced by the model are relatively independent of this assumption.

As regards the future of such computer models, whilst there are enough parameters in the model for it to be flexible enough to simulate several types of squint and their surgical treatment, it will not be predictive until the parameter changes corresponding to such modifications as palsy, contracture, recession and resection have been isolated. Even as it stands, however, it provides a useful embodiment of much of our current knowledge of the actions of the extraocular muscles and its purely educational value pointed out by Robinson (1975) should not be overlooked.

Acknowledgements—I would like to thank Dr J. M. Miller of the Wilmer Institute, Baltimore, for helpfully clarifying some of the details of the model, and Mr N. Drasdo and Mr P. B. Linfield for discussions and useful suggestions. The figures were produced on the microfilm device at the University of Manchester Computer Centre and I would also like to thank Dr P. D. Mallinson for his advice on implementing the program at Manchester.

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0093-7002/84/6105-0338$02.00/0
AMERICAN JOURNAL OF OPTOMETRY & PHYSIOLOGICAL OPTICS

Vol. 61, No. 5, pp. 338–339
Printed in U.S.A.

short report

Computer-Based Ophthalmotropes

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ABSTRACT

The versatility of a computer-based display of the extraocular muscles is demonstrated by comparing two different assumptions as to the paths of the muscles over the globe.

Key Words: ophthalmotrope, computer simulation, extraocular muscles

Since the description by Ruete[1] of his ophthalmotrope, numerous other ophthalmotropes have been devised. The widespread availability of digital computers provides an alternative medium to the mechanical one in which to construct an ophthalmotrope. The advantage of a computer-based ophthalmotrope is that different assumptions about the shapes of the muscles can be tested unencumbered by mechanical constraints. This enables one to build more realistic models of the extraocular muscles than the single fibers models currently in use. In the ophthalmotrope described here, each muscle will be modeled by 10 fibers. A complete listing of the program is available from the author upon request.

The locations of the origins and midpoints of the insertions of the muscles were taken from the data of Volkmann.[2] To provide the coordinates of points along a line of insertion approximately 7.5 mm broad, the coordinates of the midpoint were rotated in 3.5° steps, around the axis passing through the center of rotation of the eye and the origin of the muscle. This procedure was repeated to give 10 points along the line of insertion of each muscle, so that each muscle was comprised of 10 individual muscle fibers. The actual calculation of the orientation of the plane of each muscle was done by the methods described by Robinson.[3]

The paths of the fibers were traced out in two stages. First, the coordinates of the point of insertion were rotated in 0.05 radian steps around the orientation of the muscle plane, through the angle of contact of the muscle. Second, the remaining straight section of the path of the muscle was plotted out in steps of 0.02 of its overall length. Calculation of these intervening points made it possible to remove the segments of the fibers which were hidden by the globe.

As the portions of the fibers which are hidden by the globe depend on the viewpoint, it was decided to view the eyes from directly above because this allows at least some of all six muscles to be visible. The plane perpendicular to this viewing direction corresponds to the XZ plane in the coordinate system used in the calculations. No point on the muscle fiber was plotted if it lay within the circle defined by the globe and had a Y coordinate that was negative.

At present, there are two theoretical descriptions of the paths that the muscles follow over the globe. The simplest assumption is that they follow the shortest path, but this has been criticized by Robinson,[3] who has introduced an alternative assumption that incorporates the stiffness of the tendons at the muscle insertions. With a gaze direction 30° to the left in the horizontal plane, the muscles following the shortest path appear as in Fig. 1, while those following Robinson's assumption appear as in Fig. 2.

Robinson pointed out that if the muscles followed the shortest path assumption, then with the eye adducted through more than 36.3° and slightly elevated, the lateral rectus should flip over to the other side of the globe. With the broad insertion used here it can be seen that with only 30° of adduction the fibers of the lateral rectus of the right eye are already spread

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Received September 8, 1983; revision received January 24, 1984.
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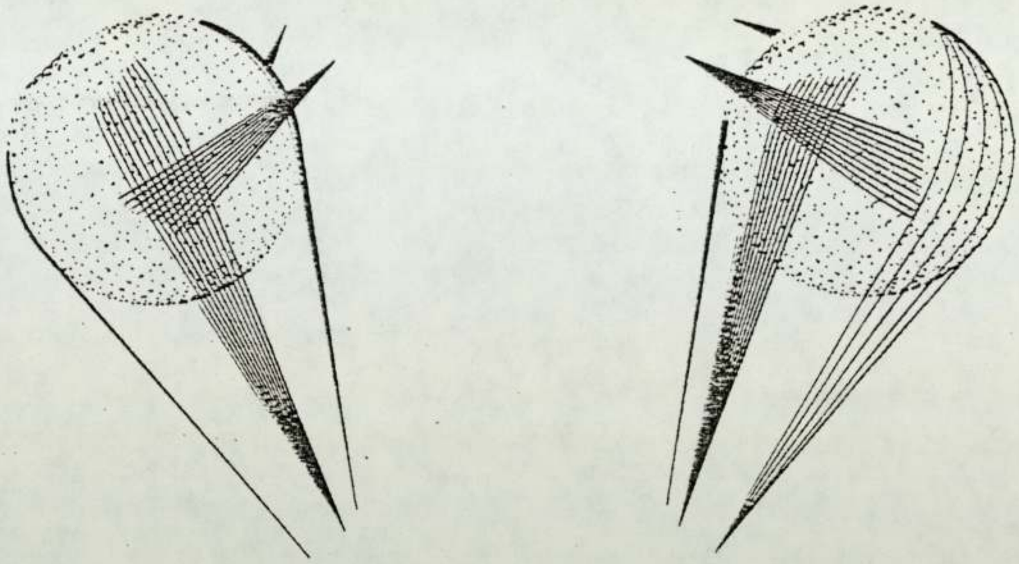

FIG. 1. Paths of the muscles when following the shortest path assumption.

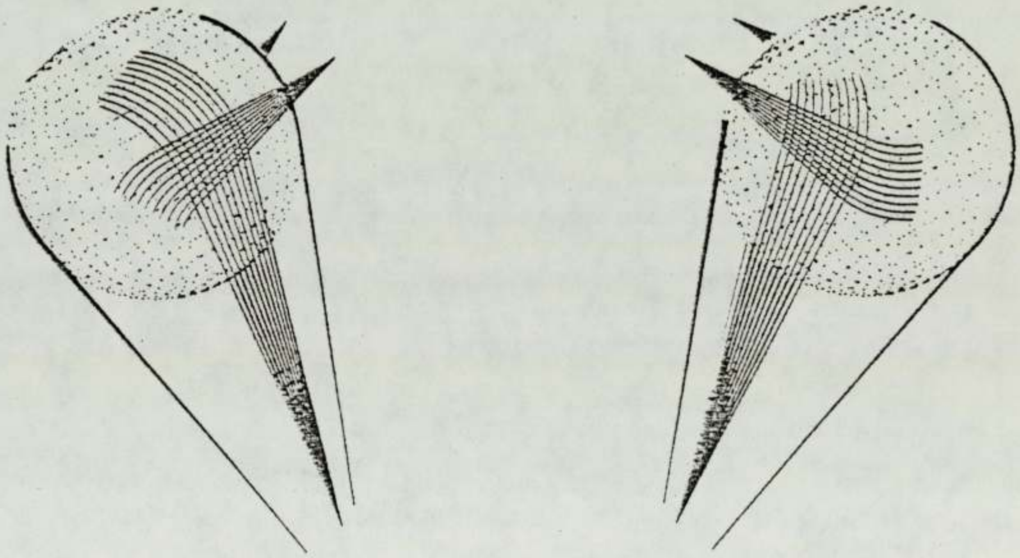

FIG. 2. Paths of the muscles when following the path assumed by Robinson.

over the globe, when they follow the shortest path assumption.

Both of these assumptions are theoretical and there is no experimental evidence to support either one or the other of them. If such data can be produced, then an accurate model of the shapes of the muscles should prove useful for determining how much recession or transposition can be carried out during strabismus surgery without the muscles subsequently interfering with each other's movements.

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