EVALUATION OF PUBLIC HOUSING STRATEGIES IN MEXICO CITY: AN EXPLORATORY GOAL PROGRAMMING APPROACH

ALVARO DE GARAY

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SUMMARY

This thesis describes the development and use of a goal programming methodology for the evaluation of public housing strategies in Mexico City. The methodology responds to the need to incorporate the location, size and densities of housing projects on the one hand, and 'external' constraints such as the ability of low income families to pay for housing, and the amounts of capital and land available, on the other.

The provision of low cost housing by public housing agencies in Mexico City is becoming increasingly difficult because there are so many constraints to be met and overcome, the most important of which is the ability of families to pay for housing. Other important limiting factors are the availability of capital and of land plots of the right size in desired locations.

The location of public housing projects is significant because it determines the cost and pattern of work trips, which in a metropolitan area such as Mexico City are of considerable importance to both planners and potential house owners. In addition, since the price of land is closely related to its location, the last factor is also significant in determining the price of the total housing package. Consequently there is a major trade-off between a housing strategy based on the provision of housing at locations close to employment, and the opposite one based on the provision of housing at locations where employment accessibility is poorer but housing can be provided at a lower price.

The goal programming evaluation methodology presented in this thesis was developed to aid housing planners to evaluate housing strategies which incorporate the issues raised above.

Key Words

Housing
Planning
Location
Goal - programming
Employment - accessibility

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CHAPTER 1

INTRODUCTION

1.1. Overview

Over the last decades Mexico City has experienced a tremendous growth of its population. Between 1940 and 1970 its population more than quadrupled and it has been estimated that if present trends continue Mexico City could have over thirty million people by the year 2000.

The rapid expansion of Mexico City has generated considerable demands for land, housing, services, transport, etc., which are becoming increasingly difficult to meet given the growing gap between needs and resources. In this context the need for planning requires little if any justification. In the area of housing policy, efficiency in the allocation of public resources is crucial; housing goals must be met as closely as possible at the lowest possible cost.

The purpose of this thesis is to make a contribution to the housing policy area by the development and testing of a goal programming evaluation methodology through which the implications of alternative policy decisions may be determined and assessed.

To show the real possibilities of the model as a policy assessment tool an actual case has been fully analysed in relation to Mexico City. It is for this reason that several chapters have centered on a rather extensive presentation of the main features of Mexico City's housing situation and the remaining ones on the development and testing of the goal programming model.

1.2 The Research Problem

It is often argued that public housing projects should be located near employment centres so that low income families do not have to spend too big a share of their incomes on transportation and too much time on the journey to work. In big metropolitan areas such as Mexico City where employment, specially service employment, is highly concentrated this objective can only be achieved at a high cost due to the high price of land in such locations.

On the other hand the option of locating public housing projects at the periphery is costly too. Because of the big size of the city a work trip from the periphery to the city centre can easily take more than two hours, even by car.

The time spent on the journey to work has a very high cost for the worker in terms of lost wages. Not to mention the cost of providing the necessary infrastructure which increases more than proportionally as the city expands.

In the presence of these conditions public housing agencies in Mexico City are finding it increasingly difficult to evaluate decisions regarding the size, location and design of housing projects. The problem is aggravated by the fact that capital is limited and the characteristics of the plots of land available in the city for public housing developments seldom adequate to meet the requirements of size and location.

The goal programming model presented in this thesis is aimed at aiding housing planners to answer questions related to the location, size and design of public housing projects in relation to the characteristics of the families that are to be housed, especially their levels of income, and also in relation to the constraints that are known to be determinant of the range of feasible options.

1.3 Social and Economic Framework

One of the most prominent characteristics of Mexico City's expansion has been the increase in the size and number of slums and of squatter settlements. In the early 1930's when Mexico City's population was 1.2 million, half a million people lived in slums but very few in squatter settlements. By the early 1970's the population living in slums was estimated to be around 2.0 million people, and that living in squatter settlements of over 2.5 million people; together comprising roughly sixty per cent of the population of Mexico City's entire urban area, which in 1970 contained eight million people.

Although rapid population growth has contributed a great deal to aggravate the housing situation one must not lose sight of the importance of economic factors. City unemployment and underemployment is undoubtedly a very important cause of the problem. Mexico City has traditionally been the most important provider of urban employment in Mexico. Over 17% of the country's labour force is concentrated in the capital city and it accounts for about 12% of the nation's G.D.P.

However, the proportion of Mexico City's labour force engaged in marginal (non formal) occupations is not less than 25% (Muñoz, 1970) and their corresponding incomes very low. Since income is the most important determinant of any household's ability to pay housing, one cannot expect housing standards to be high when the employment problem is so acute.

Moreover the inability of households to pay for good housing is aggravated by problems relating to the supply side of the housing market. Of these the provision of land is one of the greatest problems. At the present time (1980) land cost can account for as much as 55% of the total cost of a low cost single housing unit at the periphery of

Mexico City, (this compares with approximately 25% - 30% for low cost housing units in the U.K) .

This situation has forced many poor families to acquire illegally plots of land in many areas of the city but predominantly at peripheral locations and to build in very hard conditions dwellings made of scrap materials and with no access to water, electricity, sewage, etc.

1.4. Institutional Framework

In Mexico City the public sector has been for a long time involved in the provision of low cost housing for low income families. Currently there are several housing agencies working in all the activities related to public housing such as financing, construction, planning and so forth. By far the most important of these agencies is the National Housing Workers Fund (INFONAVIT) created in 1972.

INFONAVIT resources come from over 240,000 firms from all over the country in which more than four million persons work. These firms are required to make a monthly contribution to the fund equal to 50% of the amount paid in wages and salaries to their employees. One of the most important decisions INFONAVIT has to make concerns, on the one hand the allocation of credits among applicants and on the other the location, size and design of the developments.

A typical INFONAVIT development called 'conjunto habitacional', generally takes the form of blocks of flats of two, three and even five stories. Due to the high cost of urbanized land the conjuntos are often located at the periphery of the city, where land is relatively cheap.

The allocation of credits among applicants as well as the location size and design of the conjuntos are two important and related decisions for which no formal evaluation methodology is currently used by INFONAVIT.

In spite of the big amount of resources that the government dedicates to low cost housing construction in Mexico and Mexico City, the contribution of this sector is limited, considering that it accounts for less than 20% of total residential construction. The contribution of the private sector is even less significant given that its participation is just over 17%. This means that the so called 'popular sector' is responsible for the construction of more than 60% of all the houses that are built in the country and probably of a higher proportion in Mexico City.

This situation has lead many housing experts to conclude that traditional approaches such as public housing are totally inadequate to deal with the housing problem.

It is further argued by this school that the new approach should be based on a 'self - help' housing policy in which the government does not build houses but simply organizes, finances and plans the process of self - construction.

This view, however, is still very much at issue. The fact that so much people in Mexico City have been forced to house themselves in very hard circumstances, does not necessarily imply that self - construction should become a housing policy.

It is true that many public housing projects are unsuitable to the needs and requirements of the families to whom such programs are directed. However the causes of the failure are in many cases known and by no means impossible to eliminate. Among others the most important one is the setting of very high standards for the construction of many of the misnamed 'low cost' housing projects.

1.5 The Modelling Approach

The use of mathematical models in urban planning is in no way new. Models have been developed since the early fifties to aid planners to understand complex systems such as the urban economic system and to design better policies and take better decisions. There are many generic types of models. For example, there are econometric models, optimization models and dynamic - simulation models. Econometric models are generally used to determine the degree of correlation among a set of variables and to forecast the probable future value of a variable if other related variables change. Dynamic simulation models of the Forrester type (see Forrester, 1969) are generally used to understand complex systems. For instance the phenomenon of migration can be modelled with a Forrester simulation type model very successfully since by introducing the feed - back concept it can be stated that migration from city A to city B is not only a function of, say, employment growth rate differences between the two cities but also in the long run, a function of the diminishing gap between such growth rates, caused by the very migration process which eventually would tend to decrease. Optimization models are concerned with allocation problems. A linear programming model for example is generally developed to find optimal ways in which scarce resources can be allocated to maximize or minimize an objective function. The most typical example in practice is given by a firm that wishes to maximize profits given certain financial or man power constraints, but there are also many theoretical applications such as the economic text book case of a household wishing to maximize a utility function subject to an income constraint.

When the modelling field is first being considered as a means of providing an analytical tool, the first question that needs to be asked is, what is the model wanted for?; the answer indicates the kind of model to be used. When the present project on housing was started there was no interest in developing a model of any sort. The original idea was to research the main causes of the housing problem of Mexico City in order to identify and evaluate a number of housing policies for low income groups.

However it soon became apparent that an ex-post evaluation of housing problems would be of limited utility in terms of ex-ante prescriptions. Thus basic questions soon emerged such as how do we know a policy is going to have the desired effect; moreover, how do we measure the effects?. Another important problem was the existence of trade-offs between policies. This kind of questions suggested that a modelling technique would not only be useful in the evaluation of housing policies but one probably essential to the development of future strategies.

Although the full development of this field would probably require a combination of techniques and models it was hoped that this initial contribution would be able to develop the most suitable type of model given the policy issues outlined briefly above.

Two approaches were considered in depth; the systems dynamic approach and the optimization approach. The former, as pointed out earlier is very useful, because it enables the user to trace not only primary direct policy impacts but long run secondary effects. There is, however, an important limitation with this approach, namely that it tells us nothing about efficiency in the allocation of resources. One has to judge from the obtained results how 'good' or 'bad' a policy turns out to be. Since the central problem is the allocation of scarce resources to try and achieve a number of housing policy objectives, the optimization approach was adopted.

There are many types of optimization models, linear, dynamic, etc. Eventually, a linear goal programming type of model was built to evaluate housing strategies. This approach was considered suitable for the following reasons. Firstly linear relationships are easier to handle than non linear ones. Secondly goal programming, as opposed to conventional linear programming is a powerful technique for dealing with multiobjective programming problems. Basically the aim of any goal programming model is to determine the most efficient way of meeting as closely as possible a number of goals, not just one single objective, and it therefore responds to the issues under consideration.

1.6 Structure of the Model

A central element in the proposed goal programming model is the existence of a trade-off between a housing strategy based on the provision of housing in areas far from work places where land is cheaper, and the opposite housing strategy of providing housing in areas close to work places. A trade-off between these two strategies exists because the former implies more housing, or less capital investment but higher transport costs, whereas the latter implies the reverse. Both strategies can be represented as goals in the goal programming model, one seeking to satisfy a housing cost minimization objective and the other a transport minimization objective. The theory behind this model is not new.

Since the early sixties several so called trade-off accessibility models have been developed to represent the locational behaviour of households (see Muth 1969; Alonso 1965; Evans 1974). The idea of the research is to make such theoretical analysis applicable to real problems within the context of public housing provision. We wish to provide the existing public housing agencies of Mexico with a useful technique for the assessed housing strategies. In point of fact the proposed model was partially developed with the aid of the National Housing Workers Fund, in Mexico City.

1.7 Structure of the Thesis

Chapters 2, 3 and 5 are contextual. They were written to frame the housing problem of Mexico City within the national economic and demographic context.

Chapter 2 describes the main features of the housing situation of Mexico City. It does so by looking at the changes in occupancy per housing unit since 1950 and other indicators of housing conditions. This chapter also examines the composition and growth of the low income housing stock.

Chapter 3 contains a brief discussion of the main demographic and economic factors underlying the housing problem of Mexico City. It analyses the process of urbanization, population growth, underemployment and low incomes in relation to the housing problem.

Chapter 4 is intended to provide theoretical as well as empirical support to the basic assumption upon which the housing goal programming model rests i.e. that the trade - off between housing and accessibility is relevant to the case of Mexico City.

Chapter 5 considers the role of the government in relation to the housing problem. It firstly discusses the performance and achievement of the several agencies set up by the government to deal with Mexico City's housing situation. This is followed by a discussion of "Self - Help" as a housing option, and a brief description of legislation affecting the housing sector is also included in this chapter.

Chapter 6 is introductory to Chapter 7. It contains an outline of basic linear programming concepts followed by a review of Linear programming models used in the context of urban planning.

Chapter 7 presents the goal programming housing model and finally Chapter 8 contains an application of the proposed model.

The conclusions emerging from the study are set out in Chapter 9.

CHAPTER 2

MAIN FEATURES OF THE HOUSING SITUATION OF MEXICO CITY

This chapter is intended to provide a general picture of Mexico City's housing situation. Section 2.1 takes a look at housing conditions referring to conventional indicators such as the rate of occupancy and the availability of services. In the following section it is explained how the low income housing stock is organized and how it has grown over the years. Some concluding remarks are made in the final section.

2.1. INDICATORS OF HOUSING CONDITIONS IN MEXICO CITY

2.1.1 Occupancy per Housing Unit and Room.

According to official statistics[1] between 1950 and 1960 Mexico City's population increased from 3.1 million to 4.8 million people, an increase of 55%. Over the same period of time the number of housing units went up only 44%; from 626 to 902 thousand housing units. Between 1960 and 1970 population rose from 4.8 to 6.9 million while the housing stock reached the figure of 1.2 million units; an increase in population and the number of units of 43% and 35% respectively. This means that over the years the average number of persons per housing unit has increased from 4.87 in 1950 to 5.39 in 1960 and to 5.72 in 1970, (table 2.1). The same trend is observed at the national level, for the number of persons per housing unit went up from 4.90 in 1950 to 5.48 in 1960 and to 5.83 in 1970.

1 Censos Generales de Población. Dirección General de Estadística. 1950, 1960 and 1970.

TABLE 2.1

GROWTH OF MEXICO CITY'S POPULATION AND HOUSING STOCK

YEAR	(1) POPULATION (MILLIONS)	% INCREASE	(2) HOUSING UNITS	% INCREASE	(1)/(2) OCCUPANTS PER HOUSING UNITS
1950	3 050		626		4.87
1960	4 870	60	902	44	5.39
1970	6 967	43	1 219	35	5.72

SOURCE: Censos Generales de Población 1950 - 1970.

These figures can be interpreted as an indication of deterioration of the housing situation of Mexico City as they suggest that crowding has increased over the years. On the other hand, however, it could be argued that an average of 5.7 persons per unit is not very high considering that that is the average size of the Mexican family. Yet a complete different picture emerges if one considers the composition of the housing stock. In 1970 of the 1.2 million units that existed 351 thousand had only one room, 311 thousand two rooms, 203 thousand five or more rooms (including the kitchen). In other words, over 54% of Mexico City's housing stock in 1970 was made up of two rooms houses, that is to say houses with one bedroom, kitchen and toilet. Approximately 3.5 million people were living in these one and two rooms houses which represent over the half the number of occupants registered by the census (table 2.1). Additionaly, if we divide the total number of rooms by the number of occupants we obtain a rate of occupancy of 1.95 occupants per room which is very high in comparison with other countries. The same rates are for La Havana (Cuba) 1.20; for Panama City 1.92; for Brazilia 1.10, and for London and Paris 0.60 and

1.0 respectively[2]. Of course it could be argued that international comparisons of housing indicators are often misleading because the conditions and attitudes vary enormously from one country to another, and therefore people in different countries attach different importance to factors like overcrowding. Yet given the composition of the housing stock it is difficult to deny that the high rate of occupancy observed in Mexico City is an indicator of very low levels of housing living conditions.

TABLE 2.2

HOUSING: NUMBER OF UNITS, ROOMS AND OCCUPANTS. MEXICO CITY (1970)

No. ROOMS	UNITS	%	% (ACCUM)	OCCUPANTS	%	% (ACCUM)
1	351	28.79	28.79	1 794	26.09	26.09
2	311	25.51	54.31	1 733	25.21	51.30
3	203	16.65	70.95	1 143	16.63	67.9
4	144	11.81	82.77	820	11.93	79.86
5	210	17.23	100.00	1 385	20.14	100.00

SOURCE: Censos Generales de Población. 1970

2 Global Review of Human Settlements. United Nations 1976

2.1.2 Facilities

One important indicator of housing conditions is the extent to which dwellings are provided with facilities such as water supply, bathroom, and electricity. The available information on this aspect of the housing stock indicates that even though the percentage of dwellings with running water increased from 50% to 62% between 1960 and 1970 there were still 456 thousand dwellings without running water in 1970. (See table 2.3)

As regards to the number of dwellings with bathroom, the situation improved very moderately between 1960 and 1970, for the percentage of dwellings with bathroom only increased from 53% to 57% during that period. The number of dwellings without bathroom was approximately 500 thousand in 1970.

TABLE 2.3

HOUSING: FACILITIES AVAILABILITY. MEXICO CITY (1970)
(Percentage)

YEAR	RUNNING WATER	ELECTRICITY OR GAS	BATHROOM
1960	50	23	53
1970	62	79	57

SOURCE: Censos Generales de Población 1960 - 1970.

The number of dwellings with electricity or gas has risen considerably since 1960. In 1960 only 23% had electricity or gas, by 1970 this figure had increased to 79%. Nevertheless there were still 250 thousand dwellings without electricity or gas in 1970. Thus it can be said that even though conditions have improved relatively, the improvement has been rather moderate and that the levels reached are still quite low, particularly as regards to water and bathroom facilities.

2.2. ORGANIZATION AND GROWTH OF THE LOW INCOME HOUSING STOCK.

2.2.1 Organization

Field research by SUDRA and TURNER (1972) and WARD (1973) suggests that the low income housing stock of the entire urban area of Mexico City is organized into a series of subsystems which can be differentiated according to their locational, structural and tenurial properties. The main basic subsystems identified by SUDRA and TURNER are:

1o. "Vecindades", (literally neighbourhoods) or slums.

2o. "Colonias Proletarias", (literally Proletarian Neighbourhoods) or squatter settlements.

3o. "Ciudades Perdidas", (literally lost cities) or shanty towns.

4o. "Conjuntos subsidiados" or public housing projects.

2.2.1.1. Location

The majority of the "vecindades" are located in the centre of the city but in some cases they can also be found in the periphery and the intermediate ring. "Ciudades Perdidas" are typically found in the intermediate ring. "Colonias Proletarias" and "Conjuntos Subsidiados" are generally located at the periphery.

2.2.1.2. Tenure

Most of the "vecindades" located in the central area consist of rented units many of them subject to rent control since 1948. In contrast the majority of the dwellings of the "Colonias Proletarias" are "owned" by the occupier illegally; either because they were acquired through a self appointed landowner who has no legal title to the land or because the vendor (subdivider) defaults in the provision of services. Such is the case of the so - called "Fraccionamientos clandestinos" (illegal subdivisions) a variety of the Colonias Proletarias subsystem.

In other cases "owners" are aware of their illegal situation when they have acquired the land through straightforward land squatting. This is the case of so - called "Colonias Paracaidistas" another variety of Colonias Proletarias.

Housing units of both "Ciudades Perdidas" and "Conjuntos Subsidiados" are generally rented although many of the units of the "conjuntos" are often owned.

2.2.1.3. Structure and Services

All the dwellings of the "Vecindades" have access to all services but quite often they are shared. The classic "Vecindades" in the central city very often come from old buildings which are adapted as dwellings. Overcrowding is particularly high due to the fact that they are located in a very priviledged area of the city with access to employment and therefore such dwellings are very much in demand.

The typical housing unit of the "Colonias Proletarias" is relatively more roomy than any other low income housing unit. This housing unit is generally built of provisional material by the occupants themselves and in some cases improved over the years. It generally lacks one or all of the following services: drainage, paving, water, refuse collection.

The "Conjuntos Subsidiados" built by government agencies are generally of a very high standard and have access to all services.

2.2.2. Expansion of the Low Income Housing Stock.

"Colonias Proletarias "

The expansion of the "Colonias Proletarias" since the late 1940's has been rather dramatic. Between 1950 and 1964 the population living in "Colonias Proletarias" increased from 420 thousand to 1.5 million and it has been estimated that in 1972 about 3 to 3.5 million people were living there. This means an average annual growth of 9.24% between 1950 and 1972.

"Vecindades"

The expansion of the "Vecindades" of the central city has been relatively slower. Between 1935 and 1952 the population of "Vecindades" increased from half a million to nearly a million. In 1972 the estimated people of the classic "Vecindades" in the centre of the city and that living in the "Vecindades" at the periphery was of 2 million. That is, an average annual growth of 3.84% between 1935 and 1972.

"Ciudades Perdidas"

The population of the "Ciudades Perdidas" has decreased since 1952. In that year the population of "Ciudades Perdidas" was about 315 thousand, by 1972 it had decreased to 112 000 - 200 000. The factor sometimes mentioned to explain this decrease in the population of "Ciudades Perdidas" is a more efficient police surveillance which makes squatting more difficult.

The expansion of "Colonias Proletarias," "Vecindades" and "Ciudades Perdidas" is sumarized in Table 2.4.

TABLE 2.4

EXPANSION OF THE LOW INCOME HOUSING STOCK (VECINDADES, COLONIAS PROLETARIAS AND CIUDADES PERDIDAS POPULATION)

1935 - 1972

YEAR	VECINDADES	CIUDADES PERDIDAS	COLONIAS PROLETARIAS
1935	500 000[1]	N.A.	N.A.
1947	N.A.	100 000[1]	N.A.
1952	993 000[1]	315 000[1]	420 000[1]
1955	N.A.	N.A.	750 000[2]
1964	N.A.	N.A.	1 500 000[3]
1972	2 000 000[4]	112 000 - 200 000[4]	3-3 500 000[4]

SOURCE: 1. Banco Nacional Hipotecario y de Obras Públicas. Estudio No. 6. El Problema de la Habitación en la Ciudad de México. (1952)

2. Instituto Nacional de la Vivienda. Herradura de Tugurios: Problemas y Soluciones Mexico D.F. 1958.

3. Plan Regulador del Distrito Federal 1964.

4. TURNER (1972), WARD (1977).

N.A. Not Available

2.3. CONCLUSION

Some points can be concluded from the preceding sections. Mexico City's housing problem is a very serious one considering the high number of housing units lacking basic facilities, the fact that overcrowding has increased over time and also the size and staggering growth of squatter settlements. In the following chapter some of the economic and demographic factors underlying this situation will be analyzed.

CHAPTER 3

DEMOGRAPHIC AND ECONOMIC FACTORS UNDERLYING THE HOUSING PROBLEM

This chapter is about the economic and demographic factors underlying the housing situation of Mexico City.

Section 3.1 provides a brief description of Mexico's intense process of urbanization and of Mexico City's rapid growth since 1940 followed by a short analysis of the rural sector and of the rural to urban migration process. Section 3.2 deals with the relationship between housing and urban growth. Several hypotheses about the relationship between demographic growth and the housing problem are tested for correlation. The following section deals with some aspects of the employment situation in Mexico City, providing background information for section 3.5 which looks at the relationship between family income and housing costs. Some conclusions are then drawn in the final section.

3.1. THE PROCESS OF URBANIZATION

Mexico, like most developing countries in the world, has been undergoing a rapid process of urbanization consequence of the political, economic, and technological changes that have been taking place since the beginning of this century. It can be said that the process of urbanization has gone through two phases, a relatively slow one from 1900 to 1940 and a rapid one from 1940 to 1970.

During the first phase the nation's urban population increased from 1.4 million to nearly 4.0 million. Consequently, the proportion of the population living in urban areas increased from 10.5% to 20% during that period. In almost dramatic contrast, during the second phase of urbanization the nation's urban population increased by 18.1 million people; that is over seven times as much as the increase registered in the first phase which is ten years longer.

Between 1940 and 1970 the proportion of the population living in urban areas increased from 20% to 45%. (See table 3.1)

Looking at individual city growth it is interesting to observe that social growth[1] has accounted for an important proportion of most cities population increase. For example, cities like Acapulco and Tijuana had average rates of social growth of 6.4% and 3.7% during the period 1960 - 1970. (See table 3.2)

However, it should be pointed out that natural population growth has been very high and that the overall increase at city level can be generally accounted for more in terms of natural growth than in terms of social growth. (See table 3.2)

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1 Social Growth is defined as the net difference between inmigration and emigration over a given period of time.

TABLE 3.1

MEXICO'S TOTAL AND URBAN POPULATION* (1960 - 1970)

	TOTAL (1) POP.	URBAN (2) POP.	(2) / (1)
1900	13,607,000	1,434,000	10.54
1921	14,335,000	2,100,000	14.65
1930	16,553,000	2,891,000	17.47
1940	19,649,000	3,928,000	19.99
1950	25,779,000	7,210,000	27.97
1960	34,923,000	12,747,000	36.50
1970	48,382,000	21,004,000	43.41

SOURCE: L. UNIKEL et al, "El Desarrollo Urbano de México" El Colegio de México, 1976.

*Centres of 15,000 or more people.

TABLE 3.2

AVERAGE ANNUAL RATES OF GROWTH (TOTAL, NATURAL AND SOCIAL) OF FORTY MEXICAN CITIES

1940 - 1970

CITY	1940 - 1950			1950 - 1960			1960 - 1970		
	Total	Natural	Social	Total	Natural	Social	Total	Natural	Social
1. Aguascalientes	1.3	2.6	-1.3	3.0	3.4	-0.4	3.7	4.0	-0.3
2. Mexicali	11.0	2.8	8.3	9.1	4.2	4.9	4.2	3.8	0.5
3. Tijuana	11.4	2.3	9.1	8.7	3.9	4.8	7.6	3.9	3.7
4. La Paz	2.3	3.2	-0.9	6.0	3.8	2.2	6.4	3.9	2.5
5. Campeche	2.9	3.0	-0.1	3.4	3.6	-0.2	4.7	3.4	1.3
6. Torreon	5.2	3.1	2.1	3.2	3.5	-0.3	2.3	3.9	-1.6
7. Saltillo	3.4	3.7	-0.3	3.4	3.3	0.1	5.0	4.1	0.9
8. Matamoros	-	-	-	-	-	-	1.0	4.0	-3.0
9. Colima	2.8	2.4	0.4	3.9	5.0	-1.1	3.8	3.2	0.6
10. Manzanillo	6.2	2.0	4.2	4.2	3.2	1.0	0.4	4.3	3.9
11. Tuxtla Gtez.	5.6	2.6	3.0	3.7	3.0	0.7	4.9	3.5	1.4
12. Venustiano C.	-	-	-	-	-	-	7.7	2.1	5.6
13. Cd. Juarez	8.6	1.9	6.7	7.3	2.8	4.4	4.5	3.0	1.5
14. Chihuahua	4.2	2.2	2.0	5.7	2.9	2.8	5.0	3.5	1.5
15. Cd. Mexico	5.4	1.7	3.7	4.9	3.0	1.9	5.2	3.0	2.2
16. Leon	4.9	2.9	2.0	5.2	3.0	2.2	5.6	2.7	2.8
17. Irapuato	4.2	3.0	1.2	5.1	3.4	1.7	3.4	3.6	-0.2
18. Guanajuato	-0.1	2.0	-2.1	1.9	1.9	0.0	2.7	3.3	-0.6
19. Acapulco	9.6	3.4	6.2	5.3	5.0	0.3	11.4	5.0	6.4

TABLE 3.2

(Continue)

CITY	1940 - 1950			1950 - 1960			1960 - 1970		
	Total	Natural	Social	Total	Natural	Social	Total	Natural	Social
20. Pachuca	1.0	2.7	1.7	1.1	3.3	2.2	2.6	4.0	-1.4
21. Guadalajara	4.9	1.9	3.0	6.4	2.7	3.7	5.2	3.2	2.0
22. Toluca	2.0	2.4	-0.4	4.1	3.0	1.1	5.1	4.6	0.5
23. Morelia	3.6	1.6	2.0	4.7	3.0	1.7	4.5	3.7	0.8
24. Cuernavaca	8.1	1.9	6.2	4.0	2.3	1.7	7.4	3.0	4.5
25. Tepic	3.3	2.6	0.7	7.5	3.3	4.2	5.0	3.1	1.9
26. Tuxpan	1.7	2.7	-1.0	3.8	2.6	1.2	3.8	2.9	0.9
27. Monterrey	6.0	2.4	3.6	6.3	3.1	3.2	4.4	3.7	0.7
28. Puebla	4.5	0.9	3.6	2.6	1.8	0.8	5.1	3.0	2.1
29. Queretaro	3.8	2.1	1.7	3.1	2.7	0.4	5.1	3.5	1.7
30. Sn. Luis P.	4.9	2.3	2.6	2.3	3.2	-0.9	3.8	4.3	-0.5
31. Mazatlàn	2.6	2.1	0.5	5.8	2.8	3.0	5.0	3.6	1.4
32. Los Mochis	5.0	3.3	1.7	5.6	3.7	1.9	5.8	4.3	1.4
33. Hermosillo	8.0	3.0	5.0	7.5	3.8	3.7	6.1	3.8	2.3
34. Guaymas	7.3	2.8	4.5	5.9	4.1	1.8	5.1	4.8	0.3
35. Villahermosa	3.2	1.9	1.3	4.4	2.8	1.6	6.1	3.3	2.8
36. Tampico	2.0	2.0	0.0	2.6	2.9	-0.3	4.4	3.1	1.3
37. Nvo. Laredo	6.6	2.0	4.6	4.6	3.0	1.6	4.9	2.8	2.0
38. Veracruz	3.4	1.2	2.2	3.7	1.6	2.1	4.1	2.6	1.4
39. Merida	3.8	1.9	1.9	1.8	2.5	-0.7	2.3	2.6	-0.3
40. Zacatecas	1.0	2.4	-1.4	2.7	3.4	-0.7	4.8	3.9	0.9

SOURCE: L. UNIKEL et al "El Desarrollo Urbano de México". El Colegio de México (1976)

3.1.1. Expansion of Mexico City

Before saying anything about Mexico City's urban expansion it is important to make clear what is understood by the term "Mexico City".

According to the 1970 "Ley Orgânica"[2], Mexico City is the geographical area (having an approximate area of 1500 Km^2.) made up of the following sixteen administrative units called "delegaciones": Azcapotzalco, Coyoacan, Ixtapalapa, La Magdalena Contreras, Milpa Alta, Alvaro Obregon Tlahuac, Tlalpan, Xochimilco, Cuauhtemoc, Venustiano Carranza, Miguel Hidalgo and Benito Juarez. (See Table 3.3) Since the political limits of Mexico City (or Federal District as it is sometimes referred to) do not correspond with the limits of the built up area, mainly because those of the latter have extended beyond the northern limits of the former, Mexico City's built up area is generally identified as Mexico City's urban area. (Map. 3.1)

Thus in order to avoid confusion we shall adopt the official definition of Mexico City (M.C.) given above and refer to its entire built up area as Mexico City's Urban Area (M.C.U.A.).

It is generally accepted that Mexico City has gone through three stages of urban growth. The first one covers the period between 1900 - 1930; the second stage the period 1930 - 1950, and the last one from 1950 to 1970. (UNIKEL, 1976)

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2 Ley Orgânica del Departamento del Distrito Federal, Editorial popular del DDF, México 1971.

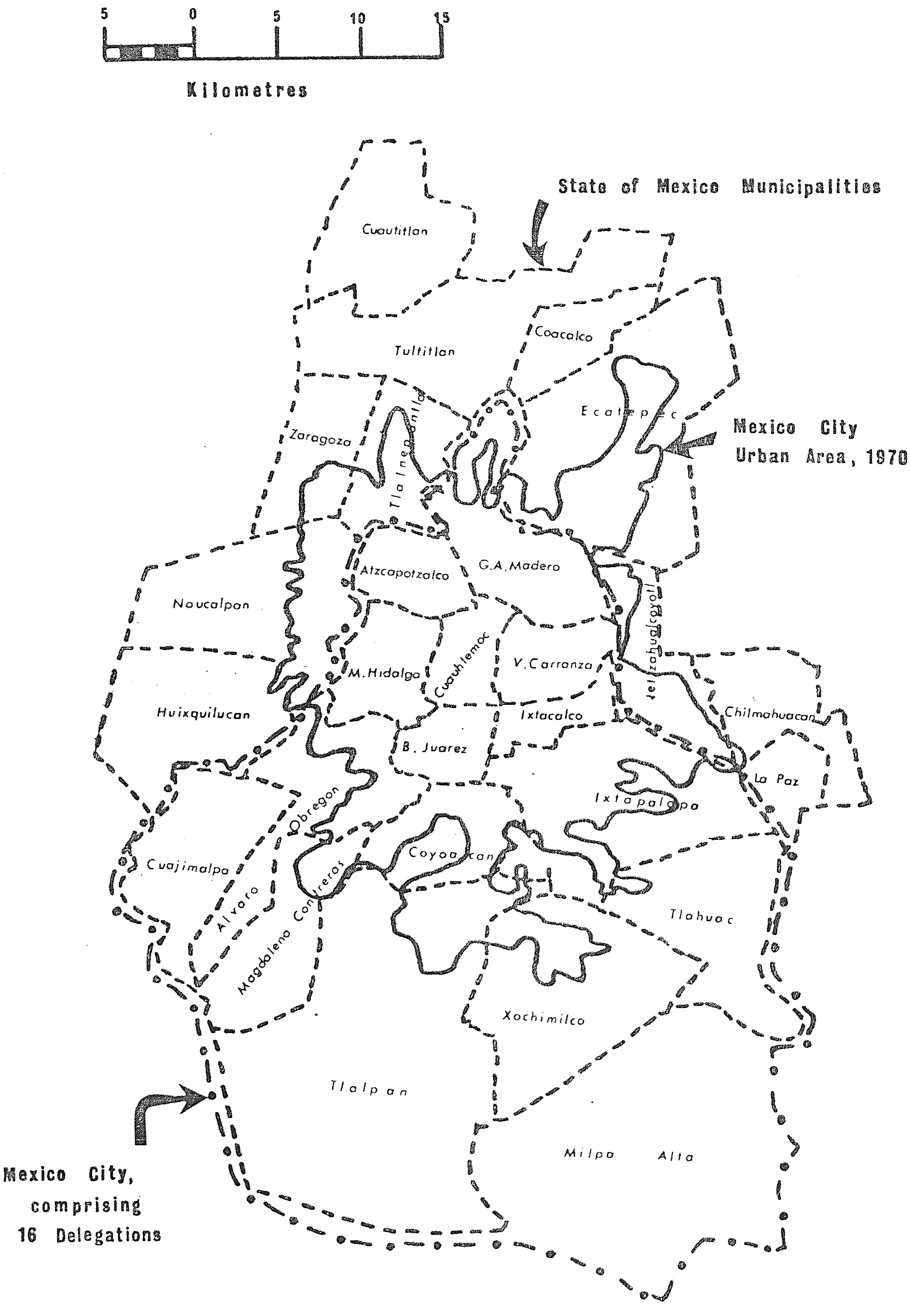
TABLE 3.3

MEXICO CITY: POPULATION OF DELEGATIONS
(1970)

DELEGATIONS	POPULATION
Azcapotzalco	542 994
Coyoacan	319 794
Cuajimalpa	37 210
Gustavo A. Madero	1 224 536
Iztacalco	480 412
Iztapalapa	555 980
Magdalena Contreras	99 881
Milpa Alta	34 172
Alvaro Obregon	501 856
Tlahuac	64 454
Tlalpan	149 335
Xochimilco	119 079
Benito Juarez	589 867
Cuauhtemoc	925 752
Miguel Hidalgo	604 623
V. Carranza	747 513
TOTAL	6 997 458

SOURCE: Censo General de Población 1970, and "Estudio demográfico del Distrito Federal", vol. 1. Centro de Estudios Económicos y Demográficos de el Colegio de México. (1970)

Map.3.1 MEXICO CITY BOUNDARIES



During the first stage population growth was relatively slow with an increase of less than a million people in 30 years. In 1900 the population of Mexico City was 542 thousand, by 1930 it had increased to 1,229 thousand.

In the second stage population growth was due more to social growth than to natural growth - it was the result of rural urban migration especially in the period 1940 - 1950. It is in this period that some important changes begin to take place such as the process of suburbanization and a rapid expansion of the CBD. Between 1930 and 1950 population rose from 1229 to 3050 thousand, almost three times the increase registered in the first stage of growth.

Over the last period (1950 - 1970) Mexico City had an staggering increase of 3,917,000 people. This means that in 1970 Mexico City had a population of approximately 7 million people. (See table 3.4)

TABLE 3.4

MEXICO CITY POPULATION GROWTH 1900 - 1970

(Thousands)

YEAR	POPULATION	ABSOLUTE INCREASE

SOURCE: "Censos Generales de Población" and "Dinámica de la Población de México, El Colegio de México" (1970).

If we look at the expansion of the MCUA it is interesting to observe that since 1900 a considerable proportion of the nation's urban growth has been generated there. For example, during the period 1940-1950, 40% of the nation's urban population increase took place in the MCUA. In the following decade this proportion decreased to 36.8% but went up again to 37.2% between 1960 and 1970. (See table 3.5)

TABLE 3.5

URBAN GROWTH OF MEXICO AND MEXICO CITY'S URBAN AREA

1900 - 1970

PERIOD	(1) URBAN POP. INCREASE	(2) MCUA'S INCREASE	(2) / (1) %

SOURCE: L. UNIKEL et al "El Desarrollo Urbano de México". El Colegio de México 1976.

It is also interesting to observe that between 1940 and 1970 no less than 38.0% of the MCUA growth was due to immigration, mainly rural immigration.

In the period 1940 - 1950, one of intensive rural urban migration 68.9% of the MCUA'S growth was accounted for by immigration. This percentage fell during the following decade to 38.3 but went up again to 43.2% in the period 1960 - 1970. (Table 3.6)

TABLE 3.6

MEXICO CITY'S URBAN AREA: NATURAL AND SOCIAL GROWTH

SOURCE: L. UNIKEL et al, Op. Cit.

3.2. RURAL BACKWARDNESS AND THE DRIFT TO CITIES

We have seen in section III.1 that between 1940 and 1970 no less than 36% of the UAMC's growth was accounted for by immigration. Since it is widely known that the main component of the migrant flow is rural, then a relevant question to ask is what are the main causes behind this phenomenon. This question is the subject of considerable debate[3]. However, a superficial examination of the agricultural sector of Mexico suggests that rural to urban migration must be highly affected by the low levels of labour productivity and the corresponding low levels of income generated in that sector.

The agricultural sector of Mexico is made up of two main sectors. One highly modernized in which the use of sophisticated capital intensive technology is widespread. This sector produces about 45% of total agricultural production on 20% of the total crop land. Not more than 7% of the existing total number of farm units can be counted as belonging to the modern sector.

In contrast there is the traditional sector which can be divided into two : the traditional semicommercial and the subsistence sectors. The former is relatively more commercialy orientated than the latter and uses

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3 For a good discusion and summary of the literature on the subject see SINCLAIR (1978).

some of the technology that is taken for granted in the modern sector. Levels of productivity, however, can be as low as five times below average levels in the modern sector. About 40.5% of the existing farm units can be found in this group.

The subsistence sector which accounts for 52.4% of the existing farm units is made up of small farms cultivated by rural families mainly concerned with producing enough corn and beans for themselves. Only a fraction of their production is channelled to the market place. About 55% of the total agricultural production is produced between the semicommercial and the subsistence sectors on 80% of crop land. Underemployment in these sectors is quite considerable. It is estimated that of the 5.1 million people making up the total labor force in the agricultural sector about 3.5 million are underemployed; that is over 68% of the labour force of that sector (WELLHAUSEN, 1976).

This situation obviously affects the prospects of a young rural worker who finds himself unemployed for long periods of time and has to share most of what he produces with members of his generally large family. Underemployment, however, is not enough to explain the decision to migrate because underemployment in a rural area can be more tolerable than unemployment in a big city. The answer to this question suggested by TODARO (1970) is that in the long term the benefits of migration exceed the cost. TODARO argues that the decision to migrate is a function of the expected long term income gain of an urban job weighted by the probability of finding such a job. Thus, if the prospects of a rural job are poor in terms of the expected incomes over a given horizon and those of an urban job better over that same time horizon, then the decision to migrate is sensible even if it involves being unemployed for some time after the arrival to the city.

This theory has not been tested for Mexico City yet. However, the fact that the majority of the newcomers to the city are young people willing to put up with the adversities of urban life can be seen as an indication of the fact that they feel confident -perhaps through the experience of other rural migrants- that in the long run they are able to succeed and find a permanent job in the so called "formal" sector of the economy and therefore secure for themselves a higher income than they would have obtained had they stayed in the rural sector.

3.3. HOUSING CITY SIZE AND URBAN GROWTH

It is clear from the preceding sections that Mexico City in common with most of the important cities of Mexico has grown very rapidly in recent years. In this section we look at the relationship between rapid urban growth, city size and the housing problem.

It is quite often argued that city growth due to natural population growth and rural to urban migration is one of the main causes of the housing problem of Mexico City and other cities in developing countries. The demographic argument suggests that houses cannot be built at the same rate as the rate of household increases over time. From this situation, it is said, an increasing gap between the number of housing units that exist and those needed arises.

Along with this argument the 'too big a city' argument is also put forward. It is said that housing problems are more acute in big cities than in smaller ones, because housing needs are greater in the former than in the latter. How valid are these arguments? In order to test such arguments several hypotheses were tested for correlation.

Before talking about the results of the correlation test it may be important to explain what will be here understood by the term "housing problem". To some the magnitude of the housing problem can be simply measured in terms of standards. For example, it is argued that if it is assumed that there must be a housing unit for each family then an indication of the magnitude of the housing problem would be given by the difference between the number of houses that exist and the number of families. Others argue that the use of such arbitrary standards can be very misleading since housing is a concept embracing a great deal more than the housing unit itself. It has to do with other factors as well, such as location, tenure, etc.

Although there is a great deal of truth in the second view it seems practical as a first approximation to use rough indicators such as a housing deficit to measure the magnitude of the housing problem, and that is how the problem will be approached here. However, the subject will be taken up later on in the final chapter when dealing with the problem of developing housing policies.

A recent estimate of the housing deficit of Mexico City and another 172 Mexican cities has been carried out by Garza et al (1978). The estimated deficit for 40 cities are presented in Table 3.7. Now the question to be asked is this, to what extent has this situation been created by rapid population growth?

In order to answer this question four hypotheses were tested for correlation.

HYPOTHESIS I. Two samples of data, the average rate of social growth of forty Mexican cities (1960 - 1970) and their corresponding housing deficit rates in 1970, were tested for correlation.

HYPOTHESIS II. Instead of average rates of social growth, global rates of population growth for each city were used.

HYPOTHESIS III. The population of the same 40 Mexican cities in 1970 and their estimated deficit rates in the same year were tested for correlation.

HYPOTHESIS IV. The percentage of city population of 35 cities from developing countries (including Mexico city) living in slums and squatter settlements (1970) and the average rate of population growth that each of these cities had during 1960 - 1970, were also tested for correlation. The data used for the analysis is presented in table 3.8.

The results of the correlation analysis are presented in table 3.9. The following conclusion can be drawn from the analysis:

a) Given the obtained low values for r and r^2 and the negative sign of the regression coefficient the hypothesis (I) that high rates of social growth are the cause of high housing deficits cannot be accepted.

TABLE 3.7

POPULATION, HOUSING DEFICIT RATES AND AVERAGE RATES (SOCIAL AND TOTAL) OF POPULATION GRWOTH (1970)

CITY	TOTAL POPULATION	HOUSING DEFICIT	RATE OF SOC. GROWTH	TOTAL RATE OF POPULATION GROWTH

TABLE 3.7

(Continue)

CITY	TOTAL POPULATION	HOUSING DEFICIT	RATE OF SOC. GROWTH	TOTAL RATE OF POPULATION GROWTH

SOURCE: Garza, et.al. "La Acción Habitacional del Estado en Mexico" 1978.

TABLE 3.8

PERCENTAGE OF VARIOUS CITIES' POPULATION LIVING IN SLUMS AND SQUATTER SETTLEMENTS AND THEIR POPULATION GROWTH
1960 - 1970

City	Rate of Growth	Percent of City population in Slums and Uncontrolled settlements

SOURCE : Global Review of Human Settlements, United Nations, 1976

b) For the same reasons the hypothesis (II) that it is rapid population growth the cause of a large housing deficit can neither be accepted.

c) The hypothesis (III) that city size, as measured by population is the cause of a large housing deficit has also to be rejected on basically the same grounds.

d) As regards the last hypothesis (IV) the obtained values for r and r2 indicate that correlation is not high enough to support the argument that the formation of slums and squatter settlements is a direct consequence of rapid population growth.

For a better appreciation of the non correlation between the variables indicated above scatter diagrams are presented in figures (3.1, 3.2, 3.3 and 3.4).

The results obtained are not altogether surprising considering that Mexico City -the largest city in the country and one which absorbed nearly 60% of the country's total migration during the period 1960-1970- had in 1970 a housing deficit of 43.6% (according to Garza's own estimate) which is considerably lower than the estimated deficit for smaller cities which had during the same period lower rates of either social growth or population growth. (See table 3.9)

TABLE 3.9

REGRESSION EQUATIONS OF FOUR HYPOTHESES CONCERNING THE RELATIONSHIP BETWEEN HOUSING DEFICIT RATES AND CITY SIZE AND GROWTH

HYPOTHESIS	EQUATION	r	r^2
I	HDR = 47,8342 - 0.3129 ARSG (30.154) (-0.449)	-0.0726	0.0053
II	HDR = 48,295 - 0.1781 ARCGM (13.364) (-0.245)	-0.0398	0.0016
III	HDR = 45,4772 + 0.00000011 CPOP (74.74) (0.122)	0.0093	0.0001
IV	SLUMP = 26,2244 + 2.35277 ARCG (2.953) (1.514)	0.2549	0.0650

HDR = HOUSING DEFICIT RATE
ARSG = AVERAGE RATE OF SOCIAL GROWTH
ARCGM= AVERAGE RATE OF CITY GROWTH
CPOP = CITY POPULATION
SLUMP= PERCENTAGE OF CITY POPULATION LIVING IN SLUM AND SQUATTER SETTLEMENTS. (Different Countries)
ARCG = AVERAGE RATE OF CITY GROWTH (Different Countries)

Figures in parenthesis correspond to the t values of the parameters.

fig.3.1 HOUSING DEFICIT RATE VERSUS THE RATE OF SOCIAL GROWTH OF FOURTY MEXICAN CITIES. 1970

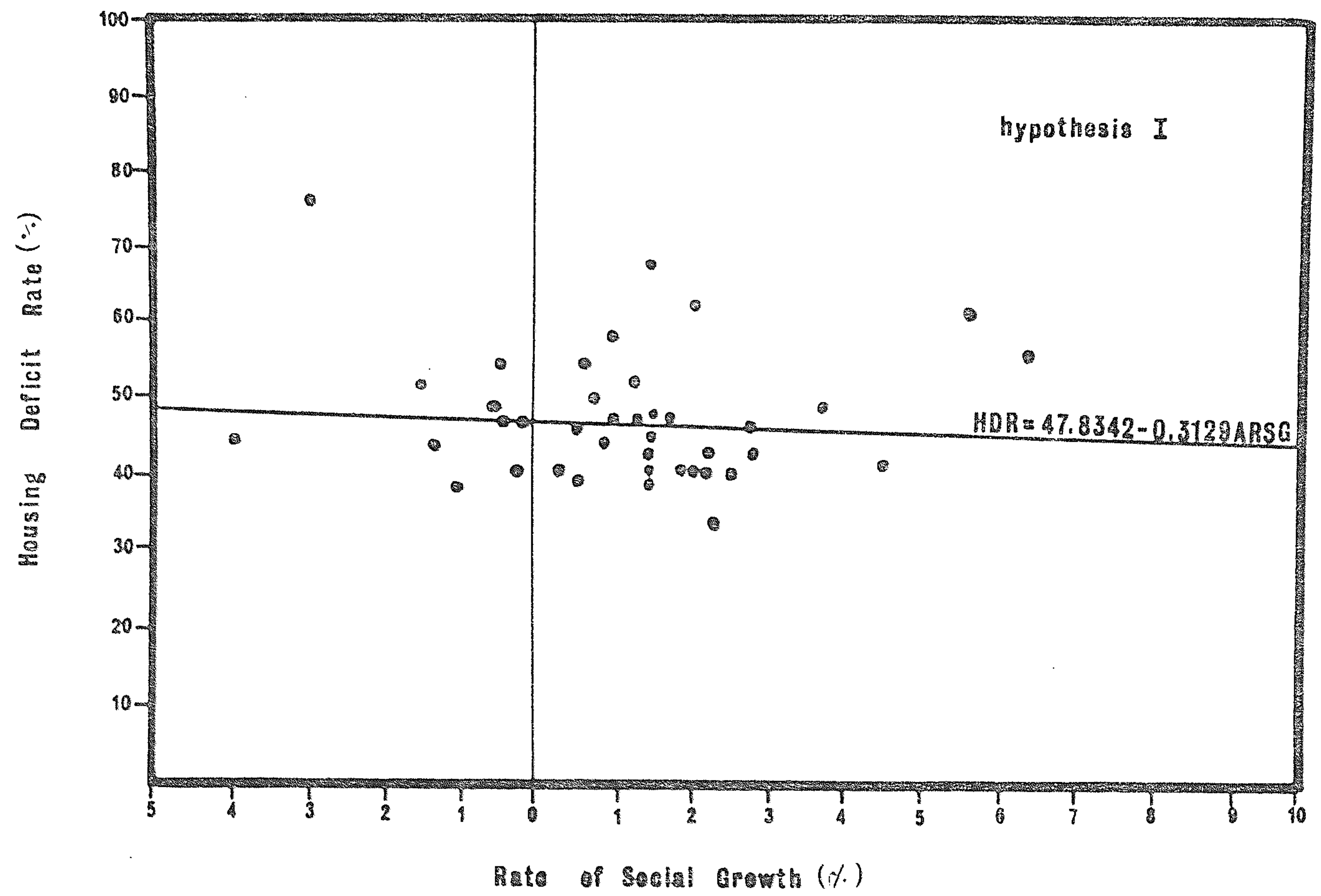


fig. 3.2 HOSING DEFICIT RATE VERSUS THE RATE OF POPULATION GROWTH OF FOURTY MEXICAN CITIES, 1970

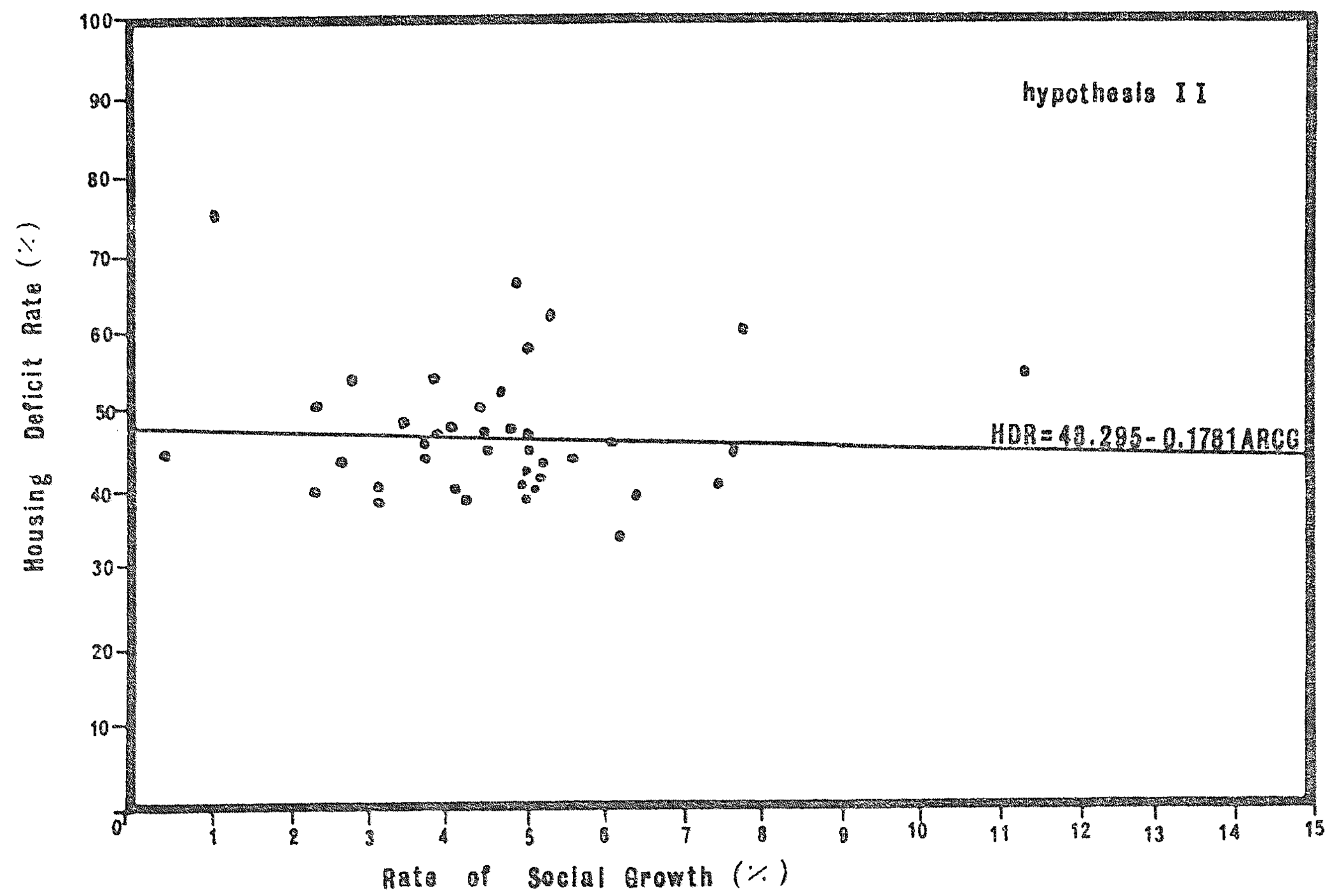


fig.3.3 HOSING DEFICIT RATE VERSUS TOTAL POPULATION OF FOURTY MEXICAN CITIES, 1970

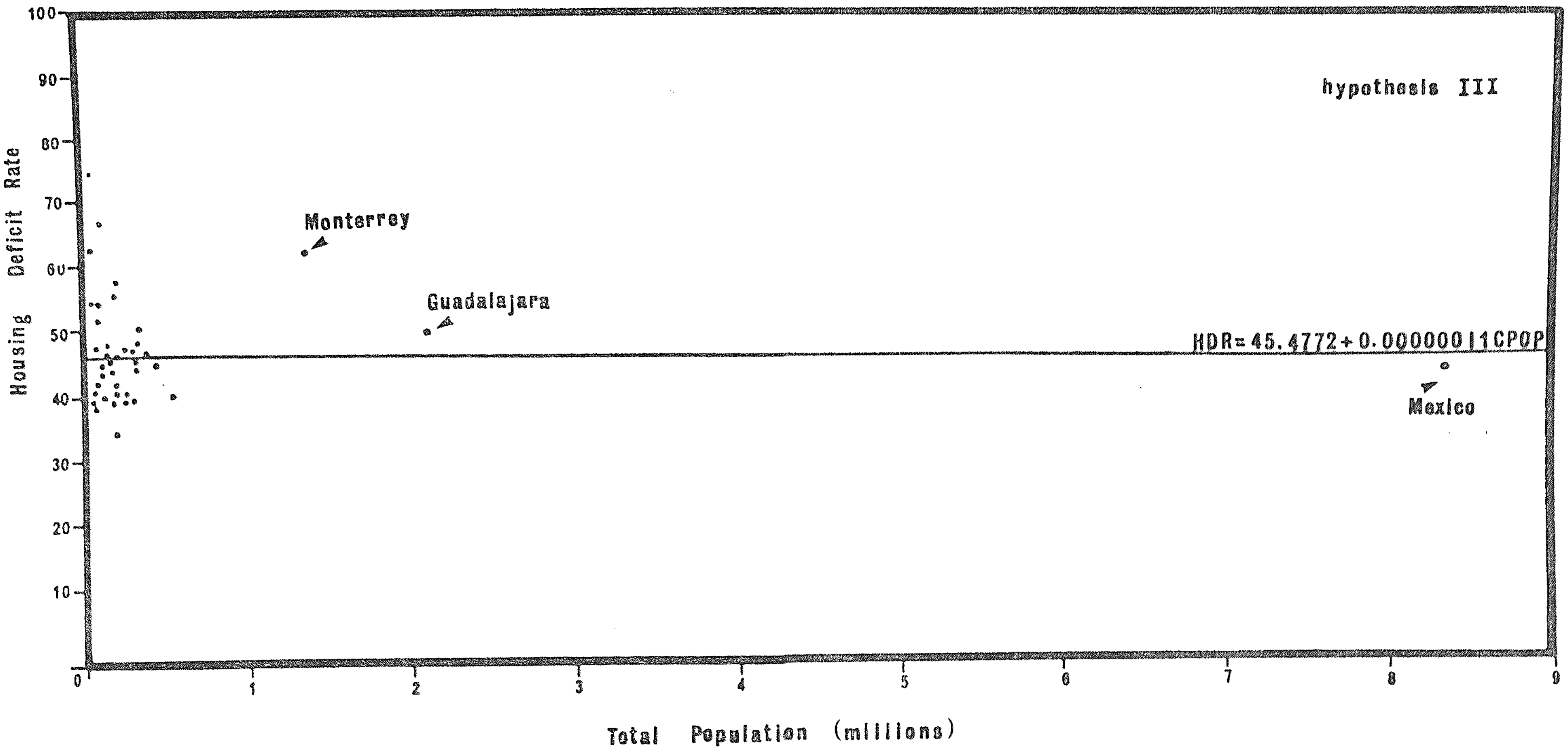
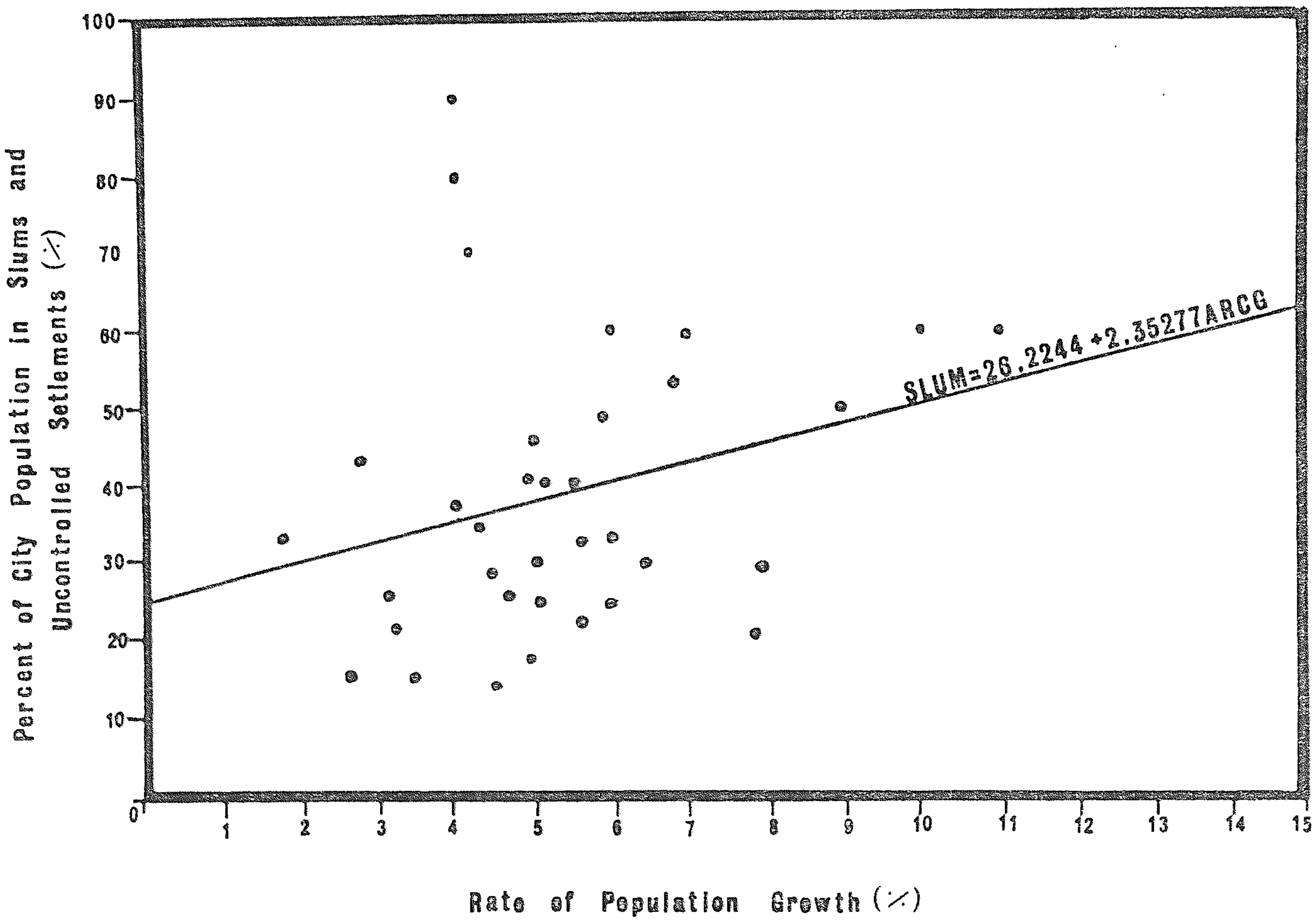


fig. 3.4 PERCENTAGE OF CITY POPULATION IN SLUMS AND UNCONTROLLED SETTLEMENTS AND THEIR POPULATION GROWTH. 1960-1970



Given the disproportionate size of Mexico City with respect to the size of the remaining Mexican cities considered for the analysis it was thought that correlation between variables in hypoteses I, II and III might improve by carrying out the analysis without including Mexico City. As can be seen from table 3.10 the estimated coefficients of correlation did not improve in fact the opposite happened. Only in the case of the third hypothesis did the correlation coefficients improve slightly.

Thus the preceding analysis suggests that neither population growth nor city size can really account for either large housing deficits or the formation of slums and squatter settlements. These results however must be taken with some reserve as they rely heavily on the kind of data used for the analysis of correlation. A housing deficit 'rate' is no doubt too crude a figure to represent the multidimensionality of something concisely called 'the housing problem'. As regards the existence of slums and squatter settlements and its relation to population growth, problems of definition and quantification cannot be ruled out, considering that the information about their size and nature was generated in each country.

If neither population growth nor city size can provide any convincing explanation of the housing situation faced by Mexico and other Mexican cities, what is then the key variable? Many economists have argued that low family incomes are really the major cause of a housing problem (GRIMES, 1976). In the following section this point will be considered but first some aspects of the employment situation of Mexico City will be analysed.

TABLE 3.10

REGRESSION EQUATIONS OF THREE HYPOTHESIS CONCERNING THE RELATIONSHIP BETWEEN HOUSING DEFICIT RATES AND CITY SIZE AND GROWTH.(EXCLUDES MEXICO CITY)

HYPOTHESIS	EQUATION	r	r^2
I	HDR = 47,8971 - 0.2880 ARSG (29.726) (-0.457)	-0.0668	0.0045
II	HDR = 48,318 - 0.16202 ARCG (10.226) (-0.221)	-0.0362	0.0013
III	HDR = 46.6163 + 0.0000038 CPOP (26.285) (0.750)	0.1224	0.0150

HDR = HOUSING DEFICIT RATE

ARSG = AVERAGE RATE OF SOCIAL GROWTH

ARCG = AVERAGE RATE OF CITY GROWTH

CPOP = CITY POPULATION

3.4. THE EMPLOYMENT PROBLEM IN MEXICO CITY

3.4.1. Changes in the Employment Structure of Mexico City (1940 - 1970)

Over the years the importance of Mexico City as a national centre of employment has increased considerably. In 1940 only 10% of the nation's labour force was concentrated in Mexico City. By 1960 this proportion had increased to 16%, and to 17% by 1970. The capital's contribution to the nation's output, however decreased from 34% in 1940 to 31% in 1950 and from 37% in 1960 to 36% in 1970. (See table 3.11)

TABLE 3.11

MEXICO CITY'S PARTICIPATION IN THE NATION'S GROSS DOMESTIC PRODUCT AND ACTIVE POPULATION (1940 - 1970)

SOURCE: APPENDINI, KIRSTEN A de, Producto bruto interno por entidades federativas, El Colegio de México, and Censos Generales de Población. 1940 - 1970

This rapid increase in the size of Mexico City's labour force has been accompanied by important changes in the structure of employment, especially in the period from 1960 to 1970.

In 1940 approximately 32% of the labour force was engaged in industrial activities and 62% in service activities. As industrialization began to take momentum during the 50's, employment in the industrial sector increased to peak of 39% by 1960 and decreased to 58% in the service sector. By 1970 however, the percentage of the labour force engaged in industry had decreased slightly to 38% while that engaged in service activities had increased again to 59%. (See table 3.12) During 1960-1970 the labour force of the industrial sector rose by 23.7% whereas that of the service sector increased by nearly 30%.

TABLE 3.12

PERCENTAGE OF THE LABOUR FORCE EMPLOYED IN THE AGRICULTURAL, INDUSTRIAL AND SERVICE SECTORS (1940 - 1970)

SOURCE : Censos Generales de Población 1940 - 1970.

This recent relative decline in industrial employment can to a large extent be attributed to the introduction of labour saving technologies and to institutional factors. The increase in service employment on the other hand is a reflection of the fact that many of the unskilled workers who migrate from rural areas into the city tend to look for jobs in the service sector where skills are not so important.

3.4.2. Industrialization and Employment

During the 1950's and 1960's there was a belief that the only cure to the problems of rural stagnation and urban unemployment was industrialization.

In order to achieve industrialization, high public investments were carried out in Mexico City and other important urban centres in Mexico. Also considerable fiscal incentives were given to private investors. For example, the Law on New and Necessary Industries (revised in 1955) was introduced in 1941. Under this law fiscal exemptions of 5,7 and 10 years are granted to private investors who carry out investments on what the government defines as "new" and "necessary" industries. Although the law applies nationwide, between 1940 and 1960 over 70% of the firms benefited were located in the metropolitan area of Mexico City (LAVELL, 1972).

As part of the industrialization programme too, in 1953 a fund was created to stimulate medium and small firms (Fondo de Garantía y Fomento a la Pequeña Industria). Although in theory industries located outside the Metropolitan Area of Mexico City are given priority in the allocation of credits, between 1953 and 1961 50% of the credits went to industries located in Mexico City. In 1970 this figure decreased to 32% (LAVELL, 1972).

The results of this industrialization policy proved very disappointing as far as employment is concerned.

TABLE 3.13

GROWTH OF INDUSTRIAL EMPLOYMENT AND OUTPUT MEXICO CITY (1940 - 1970)

YEAR	Industrial Employment (Annual growth rate)	Industrial Output (Annual growth rate)

SOURCE: APPENDINI, KIRSTEN A. de, Op. cit.

During the period from 1950 to 1960 industrial output grew at the very high rate of 11.2 per cent per year, whereas employment in that sector only increased at an annual average rate of 5.2%. In the following decade employment increased by 2.2 per cent and industrial output by 4.2 per cent. (Table 3.13)

Why has rapid industrial output growth failed to generate correspondingly rapid rates of employment growth? One very important factor has been labour productivity growth, which to a large extent can be attributed to the substitution of capital for labour in production. Industrialization in Mexico as well as in many other developing nations has been largely associated with the development of industries where mechanization and the use of modern "western" technology has been widespread. This situation in turn has lead to a gradual increase in productivity which has meant that less people are needed to produce the same level of output.

Some economists have argued that an important reason why capital intensive technologies are preferred in many developing nations where labour is relatively abundant is that the price of this latter factor of production exceeds its true market value. ELKAN (1974) has argued that minimum wage legislation can have an important effect on the levels of labour sector, this suggests that minimum wages in Mexico City have probably had an important impact on the utilization of labour in that sector, given that from 1954 to 1974 the labour cost index increased more than twice as much as did the material cost index. (Table 3.14)

TABLE 3.14

BUILDING COST AND PRICE INDEXES IN MEXICO CITY
1954 - 1974

SOURCE : Secretaría de Industria y Comercio

This conclusion, however, cannot be emphasized too strongly as it is well known that in many cases less than the minimum wage is paid to workers.

3.4.3. Unemployment and Underemployment: Nature and Magnitude of the Problem

Both the industrial and service sector of the economy of Mexico City (and of most cities of developing countries) can be said to be made up of two subsectors the "modern" industrial and service sector and the "informal"[4] industrial and service sector. The former provides regular employment to more or less skilled workers and pays them regular wages and salaries. The informal sector on the other hand provides irregular employment to workers who are generally newcomers in the city and have little or no skills. Incomes in this sector are very low and irregular. Jobs in the informal sector may include selling fruits at the corner of a busy street, shoe cleaning, or working as "velador" (care-taker) in a building site during the night. Those workers in the informal sector can be underemployed in the sense that they work less (weekly, monthly, etc.) than they would like to work due to the irregularity of job opportunities.

How many people can be said to depend on the informal sector in Mexico City? This is a rather difficult question to answer because official statistics do not provide any information which could lead to the determination of this figure. However, a recent study carried out in the Metropolitan Area of Mexico City by MUÑOZ et al (1970)

4 The concept of the "informal" sector is not accepted by all development economists. For a critique of the concept see SINCLAIR (1978).

revealed that about 24% of the labour force was engaged in "marginal activities" which can be said to correspond to those of the informal sector described above. If we add this figure to the unemployment figure provided by the National Census of 1970 the proportion of the labour force unemployed and underemployed was in that year, over 30%. This gives us an idea of the magnitude of the employment problem in Mexico City.

This study by MUÑOZ et al provided other interesting findings worthy of mention. For example, there seems to be a strong relationship between period of residence in the Metropolitan Area of Mexico City and employment status. Those earning the lowest incomes in the informal sector (in marginal occupations) were predominantly people with less than 10 years of residence. There also appears to be an important relationship between age and employment situation. Over 40 per cent of all those working in the informal sector are between 21 and 30 years of age.

Thus in conclusion it can be said that unemployment and underemployment in Mexico City are probably in the order of 25 to 30 per cent of the labour force, and that many of those unable to find employment in the modern sectors of the economy are young new comers to the city.[5]

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5 Another figure that gives an idea of the magnitude of the underemployment situation in Mexico City is the number of people reported in the population Census of 1970 as earning less than the minimum legal wage. This population amounted to 35% of the economically active population in 1970.

3.5. INCOME AND HOUSING COST

3.5.1. Household Income levels and distribution

Family income levels in Mexico City are rather low. According to the last Household Income Survey (Encuesta Nacional de Ingresos y Gastos de los Hogares 1977), 28.34 per cent of all families living in Mexico's Metropolitan Area, earned in 1977 an income equivalent or inferior to the legal minimum wage[6], and 57.08 per cent of the families earned only two times the minimum wage or less (Table 3.15).

TABLE 3.15

FAMILY INCOME DISTRIBUTION IN MEXICO CITY METROPOLITANA AREA

(1 9 7 7)

Income Range (pesos earned in six months)	Number of Families	%	Cumulated
	2 313 253	100.0	

SOURCE: Encuesta Nacional de Ingresos y Gastos de los Hogares. Primera Observación, Sria. de Programación y Presupuesto. 1977.

6 The minimum wage for Mexico's Metropolitan Area was 106.4 pesos pesos day, in 1977.

As regards the distribution of family incomes it can be shown that 21.75 per cent of the families in the upper income groups account for as much as 47.72 per cent of total family income, or to put it differently 78.25 per cent of all families share 52.28 per cent of total family income (Table 3.16)

TABLE 3.16

INCOME DISTRIBUTION IN THE METROPOLITAN AREA

OF MEXICO CITY (1977)

Income Range (pesos)	Percent of Families		Income Share	
1 - 10800	5.30		0.61	
10801 - 25800	29.49	78.25	11.50	52.28
25801 - 60900	43.46		40.17	
60901 - 108000	14.62	21.75	26.31	47.72
108000 - –	7.13		21.41	

SOURCE: Adapted from Encuesta Nacional de Ingresos y Gastos de los hogares. Primera observación; Sria. de Programación y Presupuesto 1977.

In relative terms Mexico City's situation is not so bad if compared with other Mexican cities. For example the per cent of families earning 25 800 pesos or less (over a period of six months) is smaller in Mexico City than in Guadalajara and Monterrey, the second and third most important cities of Mexico, (in terms of population size). (See table 3.17)

TABLE 3.17
PERCENT OF FAMILIES EARNING 25,800 PESOS OR LESS (OVER SIX MONTHS)

City	PERCENT OF FAMILIES
Mexico	34.84
Guadalajara	39.68
Monterrey	38.43

It is also interesting to note that in terms of average household incomes Mexico figures together with Monterrey and Guadalajara as having one of the largest average household incomes in the country[7]. Besides it may be worth pointing out that in general average household incomes increase with city size, something that surely explains why housing deficits are not necessarily greater in big fast growing cities than in small slow growing ones. (Table 3.18)

7 Presumably differences in real incomes are not so great since the cost of living is generally lower in smaller cities.

TABLE 3.18

AVERAGE HOUSEHOLD INCOME ACCORDING TO CITY SIZE

(MEXICO 1977)

City Size (Population)	Average household monetary income (over a six months period)
Up to 10 000	11 809
10 000 to 100 000	15 293
more than 100 000	35 209
Metropolitan Areas of Mexico, Guadalajara and Monterrey	46 368
Country	27 740

SOURCE: Adapted from "Encuesta Nacional de ingresos y gastos de los hogares". (1977) Secretaria de Programacion y Presupuesto.

3.5.2. Households Ability To Pay For Housing

What is the magnitude of the gap between household's incomes and housing cost in Mexico City? One way of answering this question is by determining the proportion of households unable to afford the cheapest new housing unit available in the market.

According to GRIMES (1978) in 1970 the cheapest new housing unit available in the market had a cost of 3 005 dollars. He estimated that 55 per cent of all families living in Mexico were unable to afford it assuming no down payment and a twenty - five - year repayment period, and interest rates of 10 per cent. If the rate of interest is assumed to be 15 per cent, GRIMES points out, the percentage of household unable to afford it increases to 66 per cent.

Now if the 1970 estimated cost of the cheapest new housing unit is expressed in 1977 price using the construction price index,[8] so as to determine the percentage of families unable to afford it in 1977 (when the last household survey was carried out) the following result emerges.

The housing unit estimated cost in Mexican pesos is 233 747 (9728 dollars). Now assuming the same payment conditions (10 per cent interest rate, 25 years repayment period, no down payment) the percentage of families unable to afford it turns out to be over 87 per cent, assuming as GRIMES does that 15 per cent, of household income is devoted to housing. This is so because the monthly income required is 13 555 pesos and over 87% of families were below this income level, according to the 1977 Household Income Survey.[9]

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8 Indice de Precios del Producto Interno Bruto, a precios de mercado por tipo de actividad económica. Informe del Banco de México 1979.

9 Such a figure is not directly comparable with the figure estimated by GRIMES based on the 1968 Household Survey since the 1977 Household Survey was developed on a very different methodology. (Encuesta Nal. de Ingresos y Gastos de los Hogares. Informe Metodológico, 1977).

As a result of this situation families have probably been forced to spend a larger proportion of their incomes on housing. Thus if it is assumed that households devote 30 per cent of their incomes to housing instead of 15 per cent, the percentage of households unable to afford the cheapest housing unit decreases to 60 per cent but this is still higher than GRIMES 59% figure at 15% for 1970. Moreover the previous assumptions regarding interest rates and repayment periods are undoubtedly too optimistic in present conditions. Thus the unavoidable conclusion is that the proportion of Mexico City's households unable to afford the cheapest housing unit available in the market, is very large indeed. This situation no doubt explains why over 40 per cent of the MCUA population lives in squatter settlements, shanty towns and slums.

3.5.3 Supply Factors

One must not however, place all the emphasis on low incomes as the only cause of Mexico City's housing problem. Supply factors have also contributed to put decent housing far from the reach of most low income families.

Building costs for example have been increasing quite rapidly in recent years. Since 1972 they have increased at an average rate of 24.2% per year. By constrast the workers cost of living has increased at an average rate of 19.9 per cent per year

The rapid increase in land prices that has been observed in Mexico City over the last decades has also played an important role in creating a housing problem.

In Mexico City like in many other cities of developing countries land investment represents a very good way of protecting an individual's savings against inflation and of making considerably large profits by just buying a plot of land and selling it a few years later. This situation has given rise to land speculation and consequently limited the supply of land that is available in the market for its development. As a direct implication of this phenomenon, land prices have been increasing sharply over the last decades. For example, between 1930 and 1958 peak commercial land values rose from 120 pesos m^2 to 7000 m^2 (Avenida Juarez in the city centre). Peak residential prices on the other hand increased from 5 - 10 pesos m^2 to 500 - 600 m^2 in the same period (Lomas de Chapultepec at the periphery) (FLORES, 1959). According to a United Nations study (1976) peak commercial values in 1970 were around 21,000 pesos m^2 and residential peak values around 2,250 pesos m^2.

These remarkable increases in land prices have undoubtedly had a dramatic impact on housing costs which has hit poor families most. For example, land servicing and raw land costs can account for as much as 56.1% of the total cost of a single family unit and for as much as 20.1% of the cost of a multifamily unit located at the periphery of the city. For a moderate income housing single and multifamily unit these figures are 41.2% and 19.9% respectively. (See table 3.19)

TABLE 3.19

COST OF BASIC CONSTRUCTION LAND SERVICING AND RAW LAND AS PERCENTAGE OF TOTAL HOUSING COST FOR LOW AND MODERATE INCOME HOUSING

Ho Ty	w nd
Singl	.4
Multi	,2

SOURCE : O.F. GRIMES, "Housing for low Income Urban Families", World Bank, 1976 and C. Arau et al, "Studies on Employment in the mexican Housing Industry". (OECD,1973)

3.6. CONCLUSION

For more than three decades Mexico City has grown very rapidly partly because a large proportion of the country's migration flow has been towards the capital city and partly because natural population growth rates have been very high. Rural - to - urban migration has made an important contribution to Mexico City's population increase, but natural growth has had the largest impact.

City size or population growth however do not seem to be key variables in explaining bad housing conditions in one particular city as the correlation test revealed through low coefficients of correlation and determination. A more reasonable conclusion would be that both factors (population growth and city size) aggravate the problem but are not its cause.

Immediate causes relate to problems on the demand and supply sides of the housing market. On the demand side the key issue is underemployment and low levels of family income. Over 30% of Mexico City's labour force is engaged in so called 'marginal occupations' which explains why family income levels are so low. According to the last Household Income Survey in Mexico City 57.08 per cent of the families earn monthly incomes equivalent to twice the minimum legal wage or less. This implies that over 87% of the families are unable to afford the cheapest housing unit available in the market.

On the supply side of the housing market the factor which has probably contributed more than any other to the inflation in housing prices has been land price increases which have soared over the last decades. At the periphery of the city (where land prices are generally lower) land can still account for as much as 55.1 per cent of overall housing cost.

In the following chapter the relationships between transport cost, land cost and household location are therefore examined in order to provide a basis for the examination of housing policies in Chapter 5 and to explore the possible contribution that modelling techniques could make to the development and assessment of alternative housing strategies (chapters 6 to 9 inclusive).

CHAPTER 4

HOUSING AND EMPLOYMENT ACCESSIBILITY

The purpose of this chapter is to discuss some aspects of the relationship that exists between employment accessibility and housing. The reason for dealing with this subject is to provide a thecrical justification for the housing goal programming model presented in chapter 7 which is based on the principle that housing agencies in Mexico should design their housing policies taking into consideration accessibility as a very important dimension of the housing activity.

The first section considers the main theoretical approaches to residential location. The following one discusses the relevance of these approaches to the Mexican case as well as some of the main planning implications for the location of low income families.

4.1. ACCESSIBILITY AND RESIDENTIAL LOCATION

Several models of residential location have been developed since the early sixties. We can however, say that two important and conflicting approaches are the "Behavioural" model, developed by RICHARDSON (1970) and the more traditional "Accessibility - space trade - off" models developed by ALONSO (1964), MUTH (1969) and EVANS (1974). We consider the latter approach first.

4.1.1. The Accessibility - Space Trade - Off Model.

According to this theory a household's location within a city is greatly influenced by the trade-off that exists between living close to the CBD - facing a relatively low cost of transportation but a relatively high cost of housing - and living at other more distant locations from the CBD - involving a relatively higher cost of transportation but a relatively lower cost of housing.

The most important assumptions of this model are:

1. All employment is located at the centre of the city.
2. The city is located on a flat featureless plain.
3. The city is large enough so as to make the cost of transportation an important consideration.
4. Land can be bought and sold freely in the urban land market.
5. The cost of land declines at a decreasing rate with distance from the city centre.
6. Transport costs per mile are uniform whatever the direction of travel. However, transport costs increase at a decreasing rate with distance from the city centre.

In MUTH's model of residential location a household's optimum location within a city is determined as follows.

Assume that a household has a utility function $U = u(x, q)$ where q is consumption of housing and x is pounds expenditure on all commodities except housing and transportation, which it wishes to maximize subject to a budget constraint.

$$y = x + p(k)q + T(k,y)$$

and that

$$g = x + p(k)q + T(k,y) - y = 0$$

Where

$p(k)$ = price per unit of housing a function of location or distance k from the CBD.

T = cost per trip, a function of location and income y, for a given number of trips from the CBD for the household.

Then the first order conditions for household equilibrium are found by equating the partial derivatives of the Lagrangian function $L = U(x,q) - \lambda(g)$ to zero.

$$\partial L/\partial x = U_x - \lambda = 0 \qquad (1)$$

$$\partial L/\partial q = U_q - p = 0 \qquad (2)$$

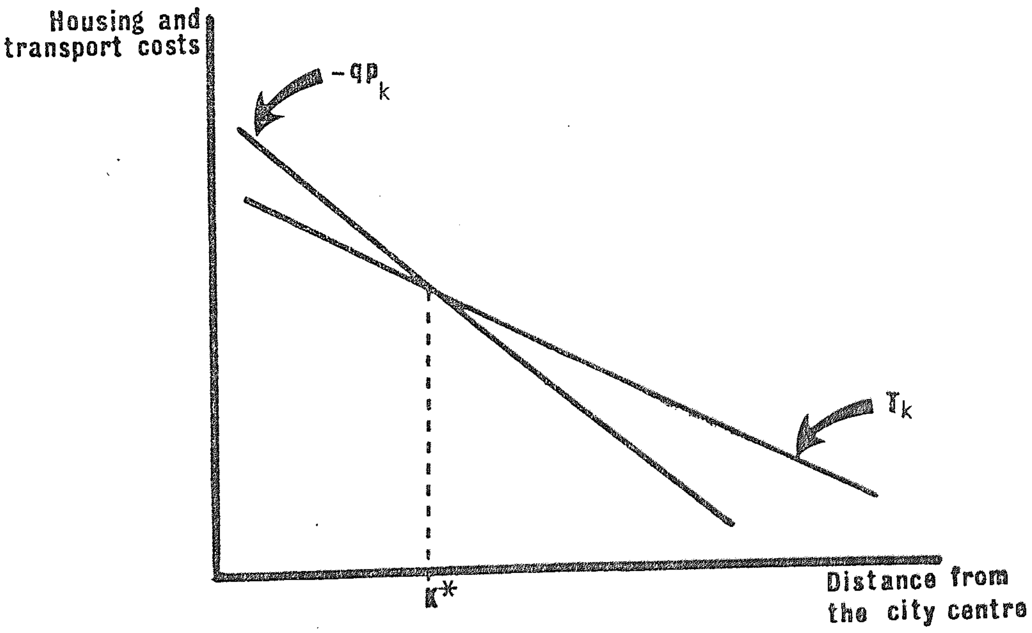
$$\partial L/\partial k = -(qp_k - T_k) = 0 \qquad (3)$$

$$\partial L/\partial \lambda = y - \{x + p(k)_q + T(k,y)\} = 0 \qquad (4)$$

Equation (3) states that the optimal location for the household is that point in the urban area where housing cost decreases (increases) resulting from a small movement away from (or towards) the city centre equal transport increases (decreases) resulting from the same movement i.e. that point where marginal housing cost equals marginal transport cost $- qp_k = T_k$.

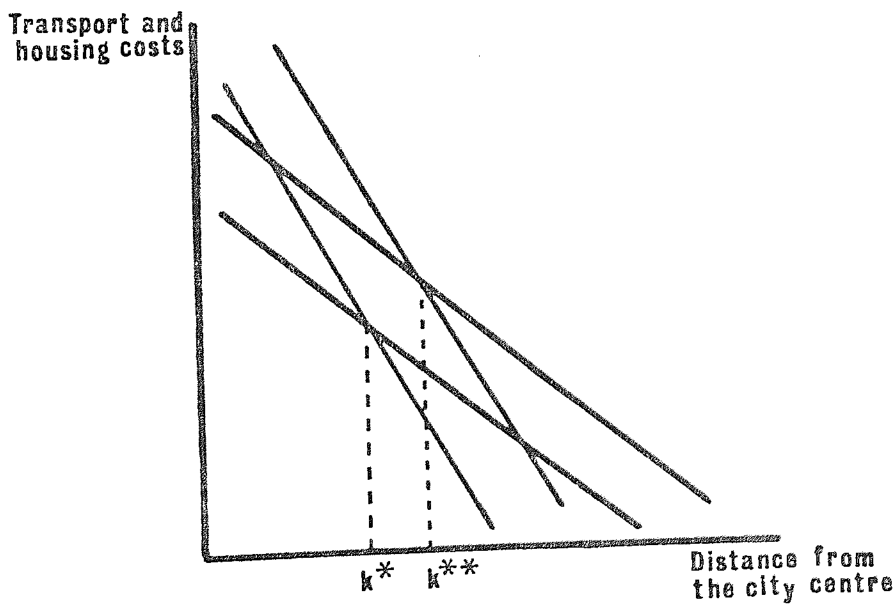
This can be illustrated diagrammatically. In fig. 4.1 $-qp_k$ represents the marginal housing cost function and T_k the marginal transport cost function. Since both are determined by distance k, it is at k* miles from the CBD that $-qp_k = T_k$. If the household moves further from the CBD its savings on housing cost are not enough to offset the corresponding increase in transportation cost. Inversely if it moves closer to the city centre its savings on transportation cost will not be enough to offset the corresponding increase in housing cost.

fig. 4.1



How is the equilibrium position affected if the household's income increases? This depends on how its demand for housing and accessibility is income related, i.e. on the values of the income elasticities of housing and accessibility. If the household's demand for housing happens to have an income elasticity greater than one and for accessibility an income elasticity inferior to one the household new optimum equilibrium location would be at a greater distance from the city centre than the original optimum location was. Thus in fig. 4.2 income increases, ceteris paribus, would shift T_k and $-qp$ upward to the right k^{**} becoming the new equilibrium optimum location. If the demand for housing of all households living in a city is assumed to be income elastic a general increase in incomes leads to an increase in housing consumption on the part of all households and the city spreads out. The population will distribute itself so that the poorest households with the smallest demands for housing are located near the city centre and the richest households with the largest housing demands are located at the periphery.

fig. 4.2



The trade-off model also makes predictions regarding the supply of housing. It predicts that if certain empirical relations hold the optimum amount of housing provided at each location i.e. density, is an inverse function of distance from the CBD. Following EVANS, this point can be explained as follows.

If it is assumed that a developer wishes to maximize profits and that the level of profits is a function of the density of development then if:

$$\pi = P_k d - g - c(d)$$

where π is profits; P_k rent payable per dwelling by the future occupants, which is assumed to be function of distance k from the CBD; d is density of development; g is land cost, and c building cost a function of density of development. Then the first order condition for profit maximization is found by equating the first derivative of the profit function respect to density of development to zero.

Thus $$\frac{d\pi}{dd} = P_k - c'(d) = 0 \qquad (1)$$

and $$P_k = c'(d) \qquad (2)$$

Equation (2) states that a necessary condition for profit maximization is that the rent payable per dwelling at location k must equal marginal development cost.

Condition (2) is a necessary but not sufficient one for profit maximization. If c(d) happens to be a linear function there is no density development at which profit maximization can be maximized. Therefore, the second order condition for profit maximization is that:

$$c''(d) > 0 \qquad (3)$$

That is, the second derivative of the cost function must be greater than zero, i.e. marginal building cost must increase at a decreasing rate.

This can be illustrated graphically. In fig. 4.3 CM represents the graph of total costs and OR the graph of total revenues. Both, revenues and costs, are a function of density. Thus the density at which profits are maximized is the density at which the difference between costs and revenues is maximum. This happens at 0 d since it is at that point where the vertical distance between CM and OR is greater.

An interesting result emerges from this analysis. If R_k changes d has to change as well if profits are to be maximized. If the expected revenue increases, density development must increase too.

fig.4.3

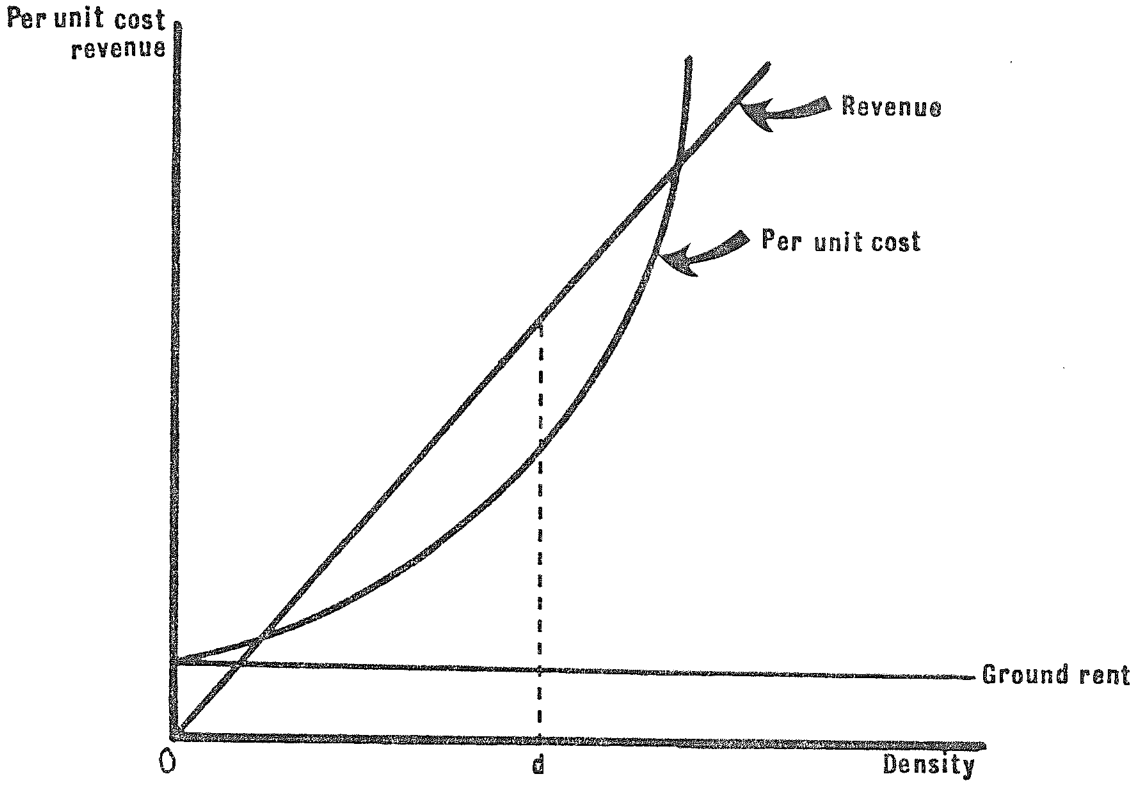
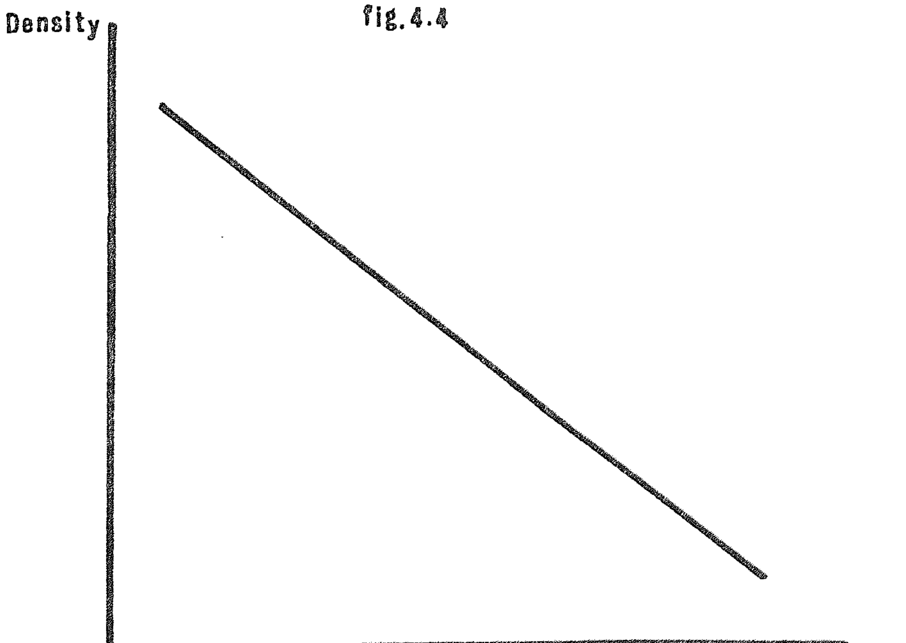

Per unit cost
revenue
Revenue
Per unit cost
Ground rent
0
d
Density

fig.4.4


Density
0
Distance from
city centre

Since expected revenues are directly related to land values and it is assumed that land values decrease with distance from the city centre it follows that densities will also decrease with distance from the city centre. Therefore, population densities within an urban area will be relatively high in central areas and correspondingly low in peripheral ones. See fig. 4.4

It is worth repeating that a fundamental assumption behind the "trade - off" model is that rents or land values decrease with distance from the city centre. This idea has its origins in the work of VON THUNEN (1936) who laid out the foundations of modern rent theory. Very briefly, the theory can be explained as follows.

Any buyer of urban land is able to derive a certain utility from every site in the urban area and the greater the utility to be derived from a site the greater the rent an individual is prepared to pay. If it is assumed that intra-urban accessibility is the most important fa factor determining site utility, then intra-urban accessibility is maximized at the city centre, and declines with distance from it. Thus if accessibility determines utility and utility determines rent, then rent can be equally expected to decline outward from the city centre.

Obviously if accessibility into the CBD improves due to better transportation or if employment becomes decentralized, the assumption that land values decrease with distance from the CBD becomes more difficult to sustain. Since in many cities this is what in fact has been happening over the years, a number of urban students have begun to question the relevance of trade-off models of residential location arguing that households do not in fact give so much consideration to accessibility when making their choice of residence site.

4.1.2 The "Behavioural" Model of Residential Location

According to the "behavioural" model a household's location within a city is chiefly affected by the following factors:

a) The quality of the environment

b) The household's own housing requirements, (type of house, number bedrooms, existence of garage and garden, preferred area, etc).

The main constraint is mortgage availability. The journey to work according to the environment model is not an important factor as long as the house chosen lies within maximum commuting limits.

According to RICHARDSON (1970), the housing preference function takes the form:

$$D_{H_i} = f(n_i, q_i)$$

Where n_i is the quality of the environment of the area at location i and q_i the quantity of housing purchased. The household will locate at that site which most satisfactorily meets its environmental and size preferences subject to the following constraints:

$$a_i \quad Y_i = J_{n_m} e^{-kd_{o}m} q_m$$

and

$$d_o \ m \leq dx$$

Where:

a_i = maximum mortgage /income ratio that a building society is willing to lend to households of income class i.

Y_i = income of household i.

n_m = index of environment quality at location m.

e = neperian logarithm.

q_m = quantity of housing purchased.

d_{om} = distance between the city centre o and location m.

J and k are constants.

d_x = maximum commuting distance.

RICHARDSON (1976) argues that the trade-off model is of little relevance since accessibility to the CBD is no longer an important factor influencing most household's choice of location, because the decentralization of urban motorways and the increase in the level of car ownership have made suburban locations just as accessible as those in the centre of the city.

RICHARDSON has also criticised the trade - off model for assuming that both buyers and sellers have a perfect knowledge of the market.

4.2. THE RELEVANT APPROACH

Here we shall briefly discuss the relevance of both approaches -the trade - off model and the behavioural model - to the case of Mexico City. Because of the lack of empirical evidence our analysis is rather superficial. This however, seems more appropriate than simply assuming that a theory "works" and that it can be used to make all sorts of predictions and policy recommendations.

It can be argued that the trade - off model has probably more to offer than the behavioural model has as an explanatory theory of residential locations for a city like Mexico. Several reasons can be given to support this view.

Firstly the assumption in the trade-off model that employment is centralized seems to be justified in the case of Mexico City: over 62 per cent of the total land devoted to service activities (shops and offices) is concentrated in delegation Cuauhtemoc. This is 940.6 hectares out of the reported 1513.5 hectares devoted to service activities in the whole of Mexico City. (Estudio Demográfico, 1970). The second largest concentration of service activities as measured by the space used for such activities is found in the contiguous delegation of Benito Juarez. Nearly 13 per cent of service floor space is concentrated in this delegation amounting to over 195 hectares. (See table 4.1)

The greatest concentration of industrial floor space is found in delegation Azcapotzalco, where over 24 per cent of all industrial floor space is concentrated. The following largest concentration of industrial floor space is found in delegation V.A. Obregon which accounts for over 16 per cent of total industrial floor space. Taken together industrial and service floor space can be concentrated in only three of the sixteen delegations that form Mexico City. (See Map 4.1)

TABLE 4.1

INDUSTRIAL AND SERVICE FLOOR SPACE IN MEXICO CITY

1 9 7 0 (Hectares)

DELEGATION	SERVICE	%	INDUSTRIAL	%
1. M. Hidalgo	102.4	6.77	255.8	7.52
2. Cuauhtemoc	940.6	62.21	203.1	5.97
3. B. Juarez	195.2	12.90	23.4	0.69
4. V. Carranza	24.3	1.61	46.6	1.37
5. Azcapotzalco	30.9	2.04	824.6	24.23
6. Coyoacan	10.3	0.68	124.4	3.66
7. Cuajimalpa	1.2	0.00	136.7	4.02
8. G.A. Madero	29.8	1.97	341.2	10.03
9. Ixtacalco	13.7	0.91	318.6	9.36
10. Ixtapalapa	29.1	1.92	462.5	13.59
11. M. Contreras	6.9	0.46	1.6	0.05
12. M. Alta	NA	--	NA	--
13. V. Obregon	108.5	7.17	548.7	16.12
14. Tlahuac	2.1	0.14	30.2	0.89
15. Tlalpan	10.5	0.69	66.8	1.96
16. Xochimilco	8.0	0.53	18.7	0.55
TOTAL	1 513.5	100.0	3 402.9	100.0

SOURCE: Adapted from "Estudio Demográfico del Distrito Federal", El Colegio de México. 1970

An additional factor giving support to the assumption of concentration is the distribution of the population throughout Mexico City. In the two central delegations of Cuauhtemoc and V. Carranza, population residential densities are higher than anywhere else in the city. In delegation Cuauhtemoc where the concentration of service floor space is greater population residential densities are 31,231.30 inhabitants per Km^2, which is really twice the city's average. (See Table 4.2 and Map 4.2)

It should be pointed out however, that since 1950 the population of the central delegations of Mexico City B. Juarez, Cuauhtemoc, M. Hidalgo and V. Carranza has increased very slowly and in some cases it has decreased. For example, between 1950 and 1970 the population of delegation Cuauhtemoc decreased from 990,572 to 925,752 people. (See table 4.3)

TABLE 4.2

RESIDENTIAL DENSITIES *

DELEGATION	POP. / Km 2	%
1. M. Hidalgo	21,800.01	7.29
2. Cuauhtemoc	31,231.30	23.26
3. B. Juarez	23,653.44	4.45
4. V. Carranza	34,133.27	1.44
5. Azcapotzalco	25,656.70	17.40
6. Coyoacan	12,722.17	2.74
7. Cuajimalpa	6,593.84	2.80
8. G.A. Madero	26,416.51	7.55
9. Ixtacalco	33,995.46	6.76
10. Ixtapalapa	11,157.33	10.00
11. M. Contreras	6,232.18	0.17
12. M. Alta	NA	NA
13. V. Obregon	18,147.52	13.37
14. Tlahuac	6,219.74	0.66
15. Tlalpan	6,766.82	1.57
16. Xochimilco	8,432.52	0.54

SOURCE: Adapted from "Estudio Demográfico del Distrito Federal". El Colegio de México, 1970.

* Residential Density = Population of Delegation/Land used for residential activities.

Map.4.1 RESIDENTIAL DENSITIES MEXICO CITY, 1970

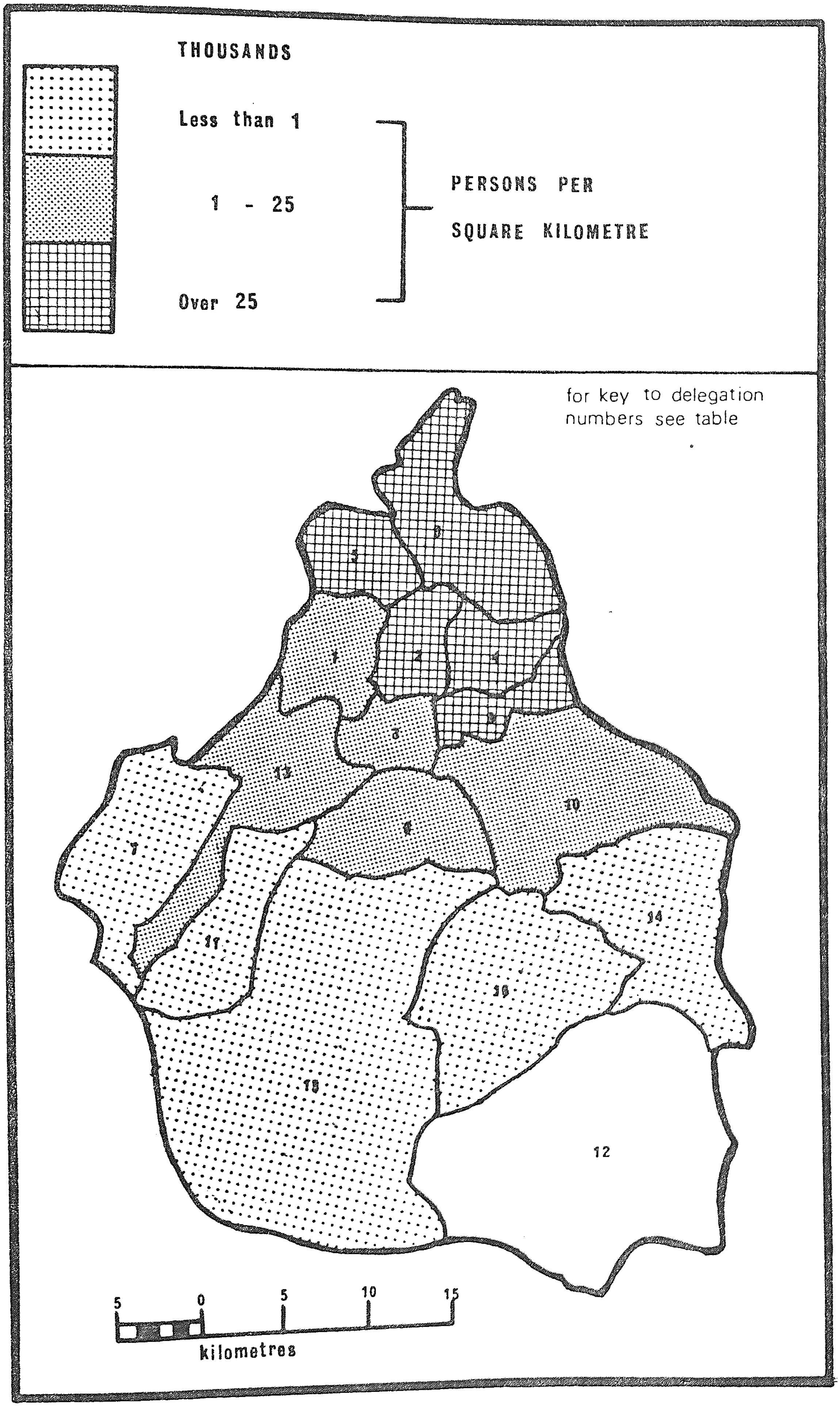


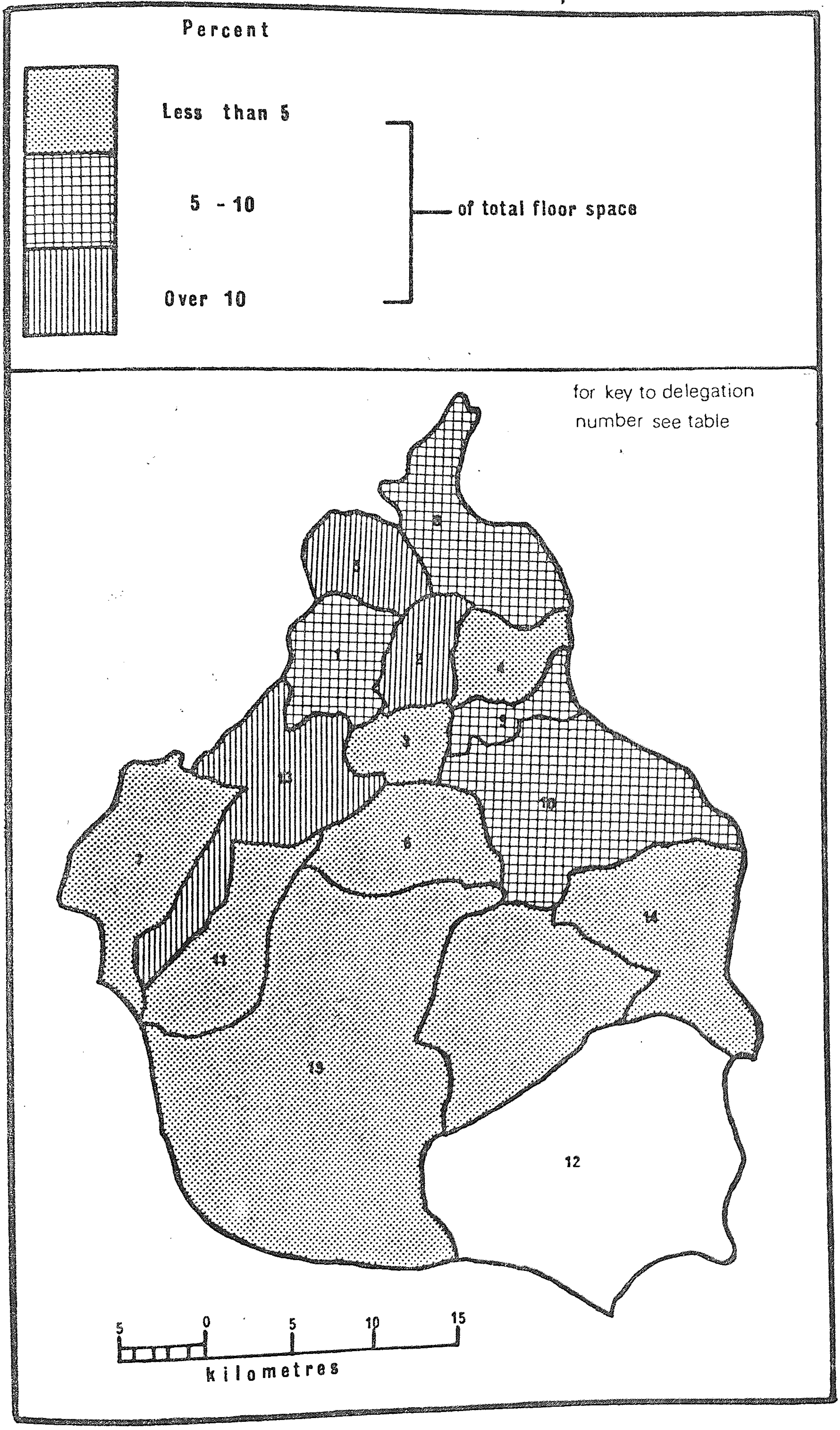
TABLE 4.3

MEXICO CITY: POPULATION IN DELEGATIONS IN DIFFERENT YEARS

16. Xochimilco	47,206	70,352	119,079

SOURCE : Estudio Demográfico del Distrito Federal. El Colegio de México. 1970

MAP. 4.2 INDUSTRIAL & OFFICE FLOOR SPACE, MEXICO CITY 1970



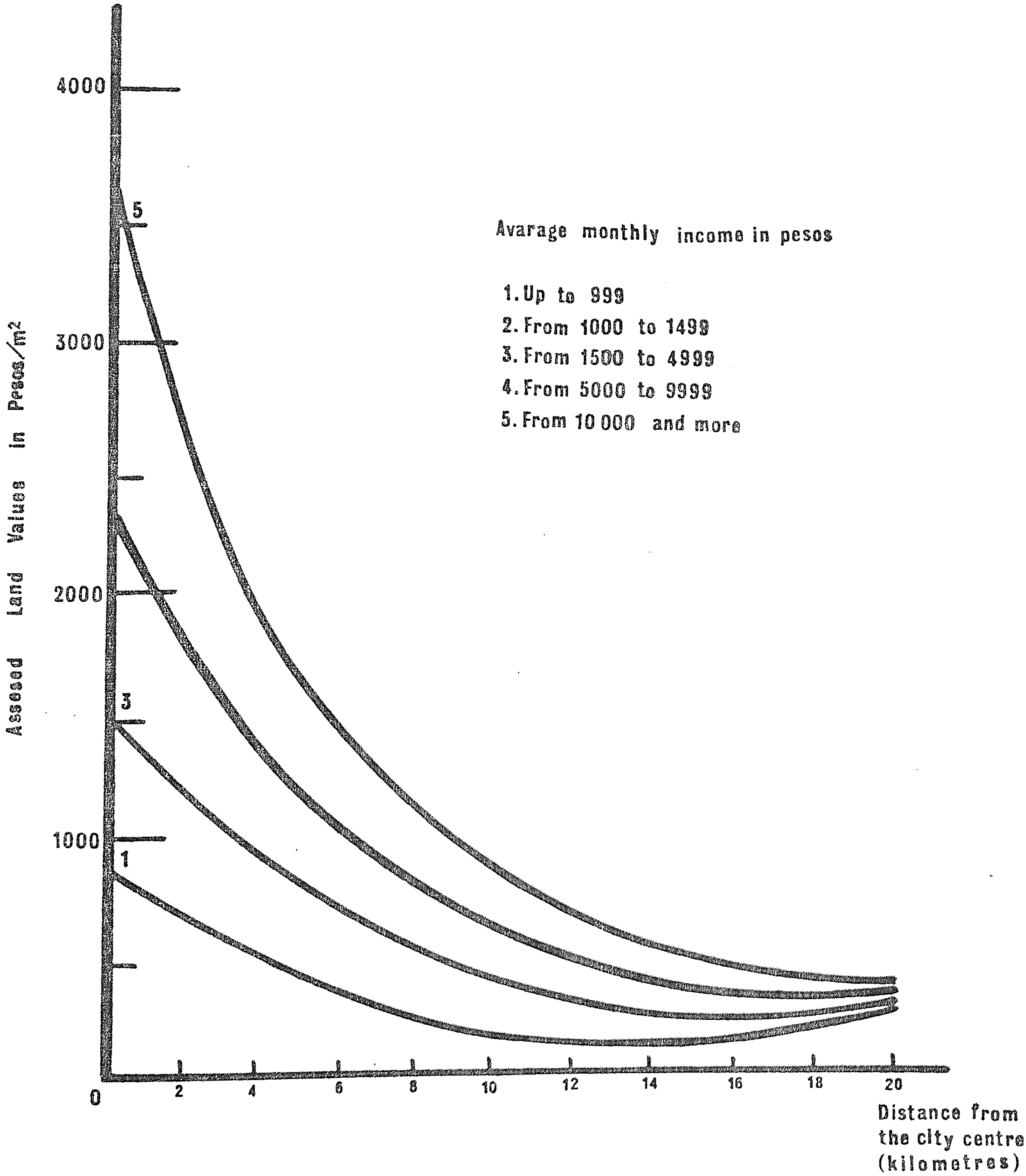
Thirdly, the assumption of exponentialy decreasing land values from the city centre upon which the trade-off model rests so heavily is fully supported by a study of land values in Mexico City carried out by SORDO (1975). By taking average land values of five income-areas all around Mexico City, SORDO was able to fit a curve of the type $y = ae^{-bx}$ to each district category. His results are presented in table 4.4 and illustrated in fig. 4.5.

TABLE 4.4

EXPONENTIAL RELATION BETWEEN LAND VALUES AND DISTANCE FROM THE CITY CENTRE FOR FIVE INCOME - AREAS

SOURCE: J.R. SORDO 'El Sistema de Valores de la Tierra y la Organización del Espacio Urbano en la Ciudad de México' INFONAVIT, Boletín Informativo número 24, 1975.

fig.4.5 EXPONENTIAL RELATIONSHIP BETWEEN LAND VALUES AND DISTANCE FROM THE CITY CENTRE FOR FIVE INCOME GROUPS. (MEXICO,1970)



As many case studies show low income families give greater priority to factors such as proximity to work centre than they do to the physical of the housing unit itself. (MANGIN, 1967; TURNER, 1968; FLINN, 1968).

One question seems relevant at this point though. If proximity to work is so important for low income groups why then a substantial proportion of Mexico City's low income households locate at the periphery in the so called "Colonias Proletarias" far away from the city centre where most service employment is concentrated?

It can be said that the formation of squatter settlements at the periphery of Mexico City can be attributed to the following factors.

Firstly, the explosive growth of Mexico City has brought about a rapid increase in land prices which has encouraged extensive redevelopment of many parts of the central area (CORTES,1969). This has meant that quite a few of the old dwellings that used to provide accommodation for many low income families have been torn down and replaced by either modern apartments for middle income families or, as in most cases, by shops and offices.

Secondly, because the size of the middle class is still relatively small, the suburbanization by this group does not ensure that the volume of old dwellings filtered down is big enough to meet the demands of the lower income groups, as has happened in many cities of developed countries (ALONSO, 1964).

Thirdly, rent control on a considerable number of old dwellings in the central area since 1948 has reduced mobility greatly. Many families or even single individuals who benefit from rent control find it very advantageous to stay in these units even if they could afford suburban locations. These results in an even shorter supply of cheap housing.

As a consequence of all these factors a substantial number of poor families has located in the periphery on cheap unserviced land,facing a long journeys to work.

The trade-off model appears then to be suitable for analysing residential location decision in a context such as that of Mexico City.

However, one must be cautious when applying the model to predict actual patterns of residential location in Mexico City and perhaps in most cities of developing countries, since some of the assumptions embodied in the model are quite unsuitable for these cities. For example, the assumption of a free land market is not completely justified. Given the lack of an efficient capital market, high inflation and tradition, people in developing countries tend to regard land investment as a very secure form of investment. This results in considerable land withholding which prevents land from being provided at the right location in the right time and in the required amount, therefore making it difficult for the developer to meet housing demands satisfactorily.

Forthly, the assumption in the behavioural model that intra-urban transportation is easier and cheaper because more people own cars and urban motor-ways have made transportation from one point of the city to another quicker is only partially true in the case of Mexico City. Although the number of cars in circulation has increased over the years and the transportation network has extended, the benefits of these changes have accrued mainly to the middle classes who represent a relatively small proportion of the population. Furthermore, intra-urban movements of people and goods in Mexico are costly in terms of time. The average speed on the main thoroughfares is three times lower than in cities like London and Paris (BATAILLON, 1970).

An indication of how important accessibility is is provided by the fact that land prices vary a great deal within small areas mainly as a function of proximity to main streets and transportation routes. For example, according to a land survey carried out in Mexico City, the assessed value of a plot of land at the corner of Avenida Madero and San Juan de Letran (a busy corner in the central city) was 4,500 pesos m^2 but it was ten times lower six blocks from that site because accessibility to transport was not so good (United Nations, 1970).

A fifth reason why the behavioural model has less relevance than the trade-off model is that in a city where low income and very low income households predominate such as Mexico, people are not likely to give much importance to the qualitative aspects of housing.

Also the assumption of perfect knowledge on the part of house buyers is unwarranted. The cost of information regarding housing opportunities in different parts of the city is a very important factor limiting the choice of households and in particular of low income households.

Having said that, we can conclude this section by saying that the basic predictions of the trade-off model are relevant mainly because employment is still highly centralized and therefore accessibility still plays an important rule in households location decisions.

4.3. PLANNING IMPLICATIONS

To understand why people locate where they do is of considerable importance to the urban planner given that housing is the most important type of urban land use. RICHARDSON (1970) points out:

"The implications for planning of how households decide where to live cannot be overemphasized, especially in an age characterized by increasing job mobility and growing overspill problems. In a situation where jobs cannot be decentralized far enough, then the appropriate prescription for transportation planning from travel-cost minimization models is to improve speeds and reduce costs, and for the planning of housing to build at very high densities as centrally as possible. On the other hand if households locate primarily to satisfy housing preferences, then more emphasis should be given to supplying the type of houses people want in the kind of environment and area they desire;

this might be more important than marginal reductions in travel time...

..."Knowing why people choose to live at particular locations and understanding the role played by the journey to work in these decisions is of fundamental importance to planners, helping them to answer such questions as: How far is it necessary to encourage decentralization of employment? What types of housing should be built and where? How should the allocation of investment priorities between, say, improving suburban environments and making the transportation network to the central city more efficient be determined? ..." (p.28)

Our analysis suggests that in order to mitigate Mexico City's housing problem it would be necessary to provide as much low cost housing as possible at central locations since it is there that most employment concentrate. Is this feasible? This is a difficult question to answer because obviously there are many factors to consider.

In chapter 6 we have developed a goal programming model which can be very helpful in providing answers to this question since the proposed model deals with the situation where two conflicting objectives exist: The objective of providing housing at the lowest cost to ensure maximum dwelling space subject to the limit imposed by the ability to pay of individual households; and on the other hand the objective of locating households as close as possible to the city centre where employment is concentrated so as to minimize household's transport cost. Therefore a conflict exists between these two goals because the former goal would imply the promotion of housing construction at peripheral locations where the cost is relatively lower.

CHAPTER 5

THE GOVERNMENT AND THE HOUSING PROBLEM

The purpose of this chapter is to briefly describe the activities of the main public agencies set up throughout the years by successive governments to cope with Mexico's growing housing problem, as well as to discuss the argument that a new approach such as 'self help' may have to be instituted in order to ameliorate Mexico's housing problem.

Looking into the structure and performance of the agencies which directly or indirectly promote the construction of low income housing in Mexico and Mexico City is necessary and relevant because the goal programming methodology presented in Chapter 7 has been precisely developed to be used by agencies such as the existing ones to assess alternative housing strategies. In point of fact the methodology was tested using data provided by INFONAVIT, currently the largest public housing agency that exists in the country.

It could be argued that by developing such a methodology an implicit acceptance exists of conventional policies towards housing the poor i.e. public housing. This is not necessarely so since the methodology can be easily adapted to be applicable in a different policy or institutional context as long as it is recognised that there is a need for an 'optimal' allocation of scarce (public or private) resources and also the importance of making an optimal use of the urban space within a metropolitan area, such as Mexico City.

Having said that it seems relevant enough to make a brief but critical comparison of the public housing approach with the so called 'self-help' approach, currently the most popular alternative proposed for dealing with the housing problem in developing countries, in order to emphasise the fact that the former approach can be a viable alternative if some more realistic actions are adopted and that the latter is by no means free of problems and unfavorable side effects.

5.1. LOW COST HOUSING BUILDING AND FINANCE

It can be said that over the years the participation of the government in the construction of low cost housing has increased considerably particularly since the early 1970's. During the forties public construction in the country amounted to no more than 2,500 units per year compared to 19,500 units built annually by the private sector. In the 1960's public construction increased to just over 6,000 units per year compared to 52,000 units built by the private sector.

This situation changed quite dramatically in the early 1970's as the volume of public construction increased to 18,352 units per year. (see table 5.1).

Not withstanding the growing participation of government in the house building process, the largest amount of new housing units is still being supplied by the so called 'popular sector', that is by low income families who build their houses themselves. It has been estimated that since 1950 the Popular Sector share in housing construction has been over 60% (table5.2). This means that the private, as well as the public sector, is still some way from playing a key role in housing construction. Nevertheless it is relevant and interesting to make a quick survey of the performance of public agencies in this field. The functions of these agencies are briefly as follows.

a) The National Housing Institute (Instituto Nacional de la Vivienda) The Institute was set up in 1954 to promote and organize all housing programmes. In ten years (1954 - 1964) it built only 10,000 low cost dwellings and, as many observers have pointed out (GERMIDIS, 1972) it failed arguably to satisfactorily coordinate the housing programmes it was supposed to coordinate in accordance with a National housing policy. Over the years its importance has diminished considerably mostly due to political changes. In recent years the I.N.V. has tended to concentrate its efforts in the rehabilitation of deteriorated dwellings rather than in housing construction.

TABLE 5.1

PUBLIC AND PRIVATE RESIDENTIAL CONSTRUCTION (1940 - 1970)

(UNITS BUILT PER YEAR)

PERIOD	PUBLIC	PRIVATE

SOURCE: Secretaría de la Presidencia. Subcomisión de la Vivienda y Banco de México (1974).

TABLE 5.2

MEXICO : HOUSING CONSTRUCTION BY SECTORS

1950 - 1974

(Percentages)

	1950 - 1960	1961 - 1970	1970 - 1974

ADAPTED FROM: "La Acción Habitacional del Estado de México" Gustavo Garza et al. El Colegio de México 1978; and Hugh Evans, 'Towards a Policy for Housing Low Income Families in Mexico', Department of Architecture Cambrige University, England, PhD thesis 1974.

b) The National Bank of Public Works and Services. (Banco Nacional de Obras y Servicios Públicos). This Bank was established in 1953. Initially it had only advisory functions concerning low cost planning and construction. After a reform in 1966 its functions embraced the construction and finance of low-cost housing units. Since the Bank was established, about 22 000 low cost dwellings have been built, 12,000 of which are concentrated in the Nonoalco Project carried out in Mexico City.

Over 90% of the Bank's resources have been chanalled to Mexico City where most of those benefitting have been middle income families. (Table 5.3)

c) The Institute of Social Security and Services for State Workers. (Instituto de Seguridad y Servicios Sociales de los Trabajadores del Estado). The Institute was set up in 1925 as the Office of Civil Pensions and Retirement. From 1925 to 1972 approximately 35,000 dwellings were financed by I.S.S.T.E. Since 1972 the housing activities of I.S.S.T.E. are carried out through a trust fund called F.O.V.I S.S.T.E. In 1973 the trust fund financed approximately 4,000 dwellings, nearly 30% of which were built in Mexico City.

d) Department of the Federal District. (Departamento del Distrito Federal, DDF). The Department's main functions are those concerning land use planning and control, the provision of urban infrastructure as well as the administration of real state tax revenues and rates, generated within Mexico City. Most of the houses built by the Department are

TABLE 5.3

INCOME OF FAMILIES AND PERCENTAGE BENEFITED BY HOUSING PROJECTS BUILT BY BNSOPSA

1967 - 1970

9
8
5
8
1
9
)

SOURCE: Banco Nacional de Obras y Servicios Públicos S.A., Conjuntos Habitacionales en la Ciudad de Mexico, Programa Buena Vivienda, Mexico 1970.

either to accommodate DDF employees or to relocate persons displaced by redevelopment programmes Between 1970 and 1973 the D.D.F. promoted the construction of 27,080 units in Mexico City. This is 36 per cent of the total number cf dwellings built by government agencies in the whole nation during that period.

It is worth pointing out that within the D.D.F. a specific department exists called General Direction of Popular Housing (DGHP) aimed at building very low cost housing projects.

The sort of houses built by the DGHP are both cheap and flexible in that families can increase the size of the house, which occupies a relatively small proportion of the plot of land on which it is first built. Many of the families benefited through this program are those who generally are not reached by conventional housing programs. (e.g. those living in 'ciudades perdidas' -shanty towns-). However the number of families benefited is proportionately very low considering the size of this group of families. (see Chapter 1). Between 1972 and 1975 over 34 thousand dwellings were built by the DGHP, 47.8% of which benefited families living in 'ciudades perdidas'. (Table 5.A)

e) The Mexican Institute of Social Security. (Instituto Mexicano del Seguro Social). It was set up in 1943. Its participation in the overall supply of low cost housing was never important since this is not what the Institute was set up for. However, it was responsible for the construction of 9,467 units between 1952 and 1962. After 1962 further construction was halted.

TABLE 5.4

ORIGIN OF FAMILIES LIVING IN HOUSES BUILT BY THE DGHP

1972 - 1975

	YEAR
	6
Und	.50
Ciu	.80
Relo wor	.10
Relo	.60
TO	.0

SOURCE: Dirección General de la Habitación Popular del D.D.F. from "La Acción Habitacional . . ." op.ut 1978.

N.A. = Not Available

In spite of the activities of these public agencies, it was clear by the early 1960's that their achievements were having very little impact on Mexico's housing problem. Between 1950 and 1960 Mexico City's population increased by 60 per cent while the housing stock increased by only 44 per cent. Of the total increase of 276,000 units during this period, only 19,000 new units were built by public agencies. (FRIDEN 1965).

As a result in 1962/1963 a series of legislative reforms were introduced aimed at promoting and implementing low-cost housing programmes. These reforms included:

a) Incentives to private banks in order to involve them in the finance of low cost homes. Private banks for example, were allowed to increase the proportion of their loans to 80 per cent of the dwelling's value. Financial institutions were authorized to channel 30 per cent of their new mortgage bond issues as credit for low-cost housing construction.

b) The creation of the Operational and Bank Discount Housing Fund (Fondo de Operación y Descuento Bancario a la Vivienda -FOVI) and the Fund for Housing Loans (Fondo de Garantía y Apoyo a los Créditos para la Vivienda - FOGA).

FOVI's main functions are to provide financial support complementary to that provided by private banks as well as the supervision of programmes.

The FOGA was set up to assist private banks when buyers are unable to advance the required down payments (20 per cent of house with a maximum value of 117000 in 1975), or in cases of default.

c) Another innovation was the establishment of a compulsory system of life, disability and property risk insurance.

The impact of these reforms has been neligible in the whole country as well as in Mexico City where most of the credits have been granted. Between 1964 and 1974 the FOVI/FOGA programme accounted for less than ten per cent of the total number of dwellings built during that period. In view of the mediocre results obtained under the FOVI/FOGA programmes, in the early 70's three Trust Funds were created to promote low cost housing construction in the country. These are the National Housing Workers Fund (INFONAVIT), the National Housing State Workers Fund (FOVISSSTE), and the National Housing Militars Fund (FOVIMI).

Given that the functioning of these three funds is very similar and that the INFONAVIT fund is the most important in terms of the volume of construction, only the activities of this fund will be described in detail presently.

The creation of these funds was born from the need to make employers fulfil what under article 123 of the Mexican Constitution represent their obligation to provide decent housing for their employees.

TABLE 5.5

CONTRIBUTION FROM AND DISTRIBUTION BY GEOGRAPHICAL REGION OF INFONAVIT RESOURCES 1974

Content has been removed for copyright reasons

SOURCE: 'La Acción Habitacional . . .' op. cit pg.155

The National Housing Workers Fund concentrates on the construction of low cost housing for all kind of workers except state workers or militars whose housing needs are looked after by the National Housing State Workers Fund and the National Housing Militars Fund respectively.

The National Housing Workers Fund, operates basically as follows.

INFONAVIT resources come from over 240,000 firms from all over the country in which more than four million persons work. These firms are required to make a monthly contribution to the fund equal to 5% of the amount paid in wages and salaries to their workers.

These contributions are formally delivered by the employers to the Exchequer. The funds are transferred to INFONAVIT after an individual account for each worker has been opened.

INFONAVIT is responsible for managing the funds through a Board of Trustees in which three groups are represented: the workers, generally by trade union leaders, employers and government officials. All policy decisions have to be approved by this Board.

INFONAVIT operates through two programmes, internal and external programmes. The former implies a direct involvement of INFONAVIT in the actual construction of the developments. The latter, on the other hand implies that INFONAVIT supervises the construction of the houses or even acquires them after they have been built by a private developer. Over 80% of the houses provided by INFONAVIT correspond to internal programmes, mainly because in this way the profits that have to be paid to the private developer are eliminated.

With respect to the origin of resources it should be pointed out that in 1974 over 50% of INFONAVIT's resources came from the Mexico City region which together with the contribution of the State of Mexico amounted to what thirty other states were able to contribute in the same year. On the other hand, however, only 49% of the credits were granted in Mexico City and the state of Mexico. This reflects INFONAVIT's policy to favour backwarded regions most and also to discourage migration from those areas into Mexico City. (Table 5.5)

One of the most important decisions INFONAVIT has to make concerns the allocation of credits among applicants. This decision is based on an analysis of the 'information cards'. This cards contain information about the age and occupation of the applicant, its income, number or dependents, and so forth. Priority is given to workers earning the minimum legal wage. In 1974 and 1975, 50% and 52.6% of the credits were given to families earning between 1.0 and 1.5 times the minimum wage[1]. (Table 5.6)

When a credit is granted to a worker by INFONAVIT, 40% of the deposits made by his employer on his account (up to that time) are taken as down payment. Since the employer has to carry on making the contributions, after the credit has been granted these are taken as part of the payment the worker has to make to repay the credit.

1 One of the most relevant aspects of the goal programming methodology presented in Chapter 7 concerns the possibility of exploring the implications of adopting an strategy based on the provision of houses for the lower income group of applicants against the opposit strategy of favoring the higher income groups of applicants, or indeed the compromise or intermediate alternative. Each policy option has different implications in terms of the amount of subsidies that have to be paid, depending on the type of families that are benefited by a particular programme.

TABLE 5.6

INFONAVIT, CREDITS ALLOCATED BY WAGE LEVEL.

1974 - 1975

	WAGE GROUP				
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
%	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

SOURCE: "La Acción Habitacional del Estado . . . ." op ut pg. 157

The amount that each worker has to pay to INFONAVIT monthly depends on its wage. For example a worker earning between 1 and 1.25 times the minimum wage has to make a payment equal to 14% its wage. A worker earning more than 1.25 times the minimum wage has to pay an amount equal to 18% its wage. The rate of interest is general and fluctates around 4%.

Since the creation of INFONAVIT (1972) the annual volume of construction has been of roughly 25,250 units. Nearly 40% of these units have been built in Mexico City.

The typical form of construction that INFONAVIT builds are 'conjuntos habitacionales'. These are blocks of flats of two, three and even five stories. Because of the high cost of urbanized land the 'conjuntos' are generally located at the periphery of the city, where land is cheaper, specially when it has not been urbanized.[2]

2 For example the cost per square metre of urbanized land in the central area of Mexico City is of $900.00 (mexican pesos) whereas in the periphery the cost per square metre of non urbanized land can be of $25.5 .

The allocation of credits among applicants as well as the location of the conjuntos are two important and related decisions that INFONAVIT has to take. These however are not the product of what might be termed a general strategy based on an efficient allocation of resources. Each project is evaluated individually, generally in terms of its cost which tends to favor low density kind of developments at locations where land is relatively cheaper.[3]

Will the INFONAVIT programme have a real impact on Mexico City's housing situation? Some people think that it will, some that it will not. Now if public housing is not the answer to the country's housing problem and particularly to Mexico City's housing problem, what are the alternatives?

5.2. SELF HELP

Different solutions have been proposed to improve the effectiveness of housing policies. Some people have called for more housing investment and heavier subsidies. Others have called for a more extensive use of modern building technology like prefabrication. But by far the most controversial approach to housing the poor is SELF - HELP HOUSING, that is that people should be directly involved in the actual construction of their dwellings. It is claimed by the supporters of this policy that the

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3 This is another important aspect of the housing problem which is considered in the evaluation methodology presented in Chapter 7 i.e. The trade off that exists between housing families at points where land is relatively cheaper but transport costs are higher and the alternative option of housing them at points where land is relatively costly but transport cost are lower.

effectiveness of public housing has been and will continue to be very limited because of the magnitude and nature of the housing problem. The volume of resources required to meet the housing needs of most developing countries in the conventional way i.e. public housing, it is argued, is beyond the economic possibilities of most developing countries. The answer to the so-called 'housing crisis' is in the hands of the dwellers themselves and the government role should be limited to the organization and planning of squatter settlements. Thus PAYNE (1977) observes:

"The Third World does not in fact have a housing problem at all, since the mass of the people have always housed themselves and are perfectly capable of still doing so even in the changed environment of large cities. What does exist is a problem of land use and resource planning - in short a settlement problem".

Clearly the key to the discussion rests in how one defines the housing problem. One may or may not be concerned about the fact that many of the dwellings that are built by squatters are very vulnerable to fire flooding and earthquakes, and that they do not provide a healthy environment for family development, since the dwellings generally consist of a single room in which all the family has to carry out its activities.

The fact that forty per cent of Mexico City's built up area is made up of districts in which self-help building activity is the normal method of house construction gives considerable support to the point that people are able to house themselves. However, one must consider the implications of a self-help housing policy more closely because the fact that people are being forced to house themselves in very hard circumstances does not necessarily mean that it should become a housing policy.

According to TURNER (1969) who has been leading the self-help school for several years the advantages of a self-help policy are many, but a very important one is that it gives people the freedom to build their houses gradually as their incomes rise, and become more stable. TURNER argues that low income households are divided into two groups with different housing priorities. On the one hand there are the 'bridgeheaders' who are unemployed or underemployed and therefore have a very low income. On the other hand there are the "consolidators" who have a regular employment and have higher incomes. According to TURNER the former group's housing priorities are, in order of importance: location, tenure, and shelter. Since the bridgeheader's main problem is finding regular employment proximity to work is crucial. Tenure is important only in the short term and the material standards of shelter and services are at that stage of little importance. The low income consolidator on the other hand, puts tenure above location but still places shelter at the bottom of the list. However, TURNER argues, once tenure is secured the consolidator begins to improve his dwelling and eventually turns it into a good standard unit.

The direct policy implications of TURNER's argument are that the government should provide bridgeheaders with cheap land at very accessible locations, and provide consolidators with cheap serviced land so as to enable them to consolidate without having to worry about the legal aspect of landownership. The term given to this sort of approach is "sites and services" because the government only provides the site and the services but the construction of the dwelling is the responsibility of the household.

TURNER argues that public housing forces most low income households to maximize their expenditure on housing and that it very often forces them to live in unsuitable locations.

Using one of his case studies from Mexico City, TURNER (1976) describes the impact of a public housing project in the following way:

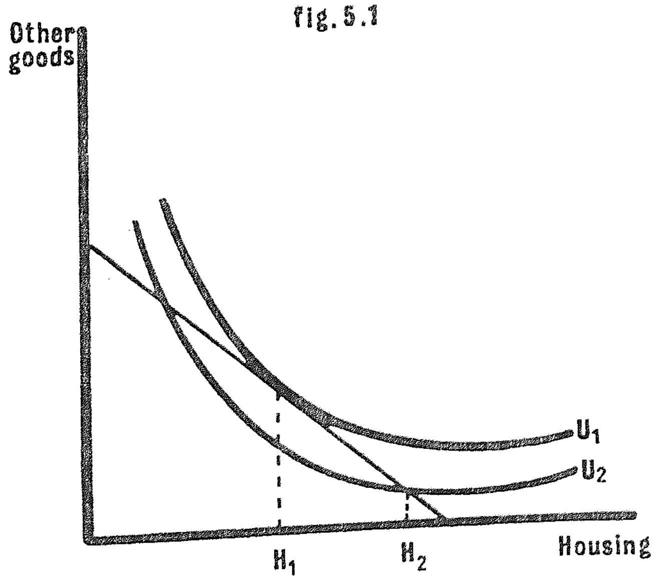
"Before relocation in the Vicente Guerrero public housing project the family (previously resident in a shanty town) supported itself from a small shop serving tourists and from the elderly husband's irregular employment as a semi-skilled mason. The family had a low income but with low housing and transportation expenditures it was able to eat reasonably well and maintain a fair level of health..."

This family now lives in a vastly improved modern house equipped with basic modern services and conveniences ... Incredibly, the family is required to spend 55 per cent of its total income to meet the rent-purchase and utility payments. On top of this, the working members must pay another 5 per cent for public transportation to work making a total of at least 60 per cent of a reduced income on housing services alone. Before the family was spending 5 per cent of a large income on their housing and journeys to work combined, and they could both eat well and save a proportion of their total income".

TURNER's point can be illustrated using some of the theoretical concepts introduced in the previous chapter.

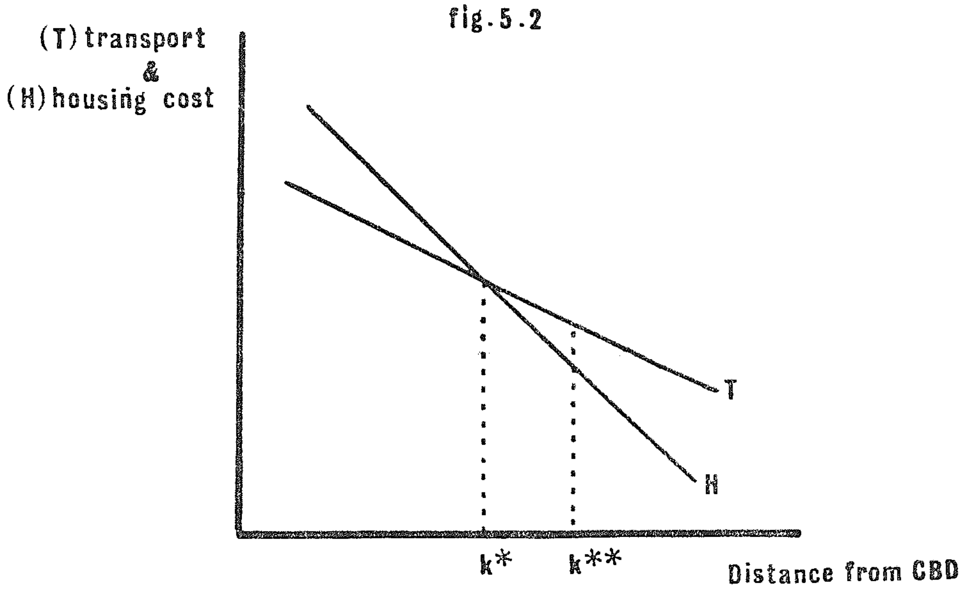
Figure 5.1 depicts a situation where a low income household has been forced to spend more than it is willing to spend on housing. Originally when living in the shanty town the household was consuming OH_1 units of housing. This was the optimum level of housing consuption given the household's income and preferences. After rehousing takes place the household is worse off since it is being forced to spend a very large proportion of its income on housing and very little on other goods. Its new welfare level is represented by indifference curve V_2.

fig.5.1



Moreover since relocation meant an increase in the cost of transport to work the decrease in welfare is even greater. This is illustrated in fig. 5.2.

fig.5.2



The optimum location for the low income household is at k* miles from its work place. After rehousing it is forced to live k** miles away from its work-place in a clearly suboptimal location.

The conclusion would then be that public housing gives rise to a situation where the household's welfare decreases inas much as it is forced to consume more housing than it is willing to consume giving its income. This conclusion is valid as far as it goes, however, if we examine the problem more closely the case against public housing becomes less powerful.

Firstly, even if the amount of housing consumed by a household is said to be optimal from the households own point of view that does not mean that it is socially optimal. Thus underconsuption of housing is normally associated with health hazards and social problems both of which generally have high social costs.

This argument in the view of the self-help school has relevance only in the short run, because they would argue, as consolidation takes place the major sources of these negative external effects are eliminated. A relevant question at this point is, do all squatter households improve their dwellings?

WARD's (1978) investigation of the levels of consolidation in three typical squatter settlements in Mexico City one of which was formed in 1947, led him to the following conclusion:

"TURNER and others have argued that squatter housing provides an opportunity for many families to overcome some of the structural aspects of poverty. While it may provide an environment that ameliorates their poor socioeconomic position it does not provide them with a vehicle of upward socio-economic mobility. Not all squatter households improve. There remains a significant proportion of whom potential improvement via self-help is severely restricted. For them, the structural nature of poverty that operates at a national level - access to education and stable employ- ment, reasonable wage levels, social security benefits and so forth - intervenes to inhibit the potential success of self-help housing. It does this by preventing the creation of an investment surplus".

If WARD is correct in this view, as it relates to self-help families in general then the referred external effects are not really eliminated, even after long periods of time.

Secondly, and of crucial importance to the policy prescription of this work it is arguable that public housing would not have such a disastrous effect if housing projects were suitable designed. In this regard a comparison of the housing experience of two developing nations, Venezuela and Hong Kong is very illustrative of the importance of setting realistic standards. In both countries the policy undertaken during the 1950's to deal with the housing problem was to build high rise flats on a massive scale. However, whereas in Venezuela most of the flats built were self-contained with three or more bedrooms, in Hong Kong most of the flats consisted of a single room with communal washing and toilet facilities. It is well known that the Venezuelan experience ended up in failure and that of Hong Kong in a moderate success (DWEYER,1975).

In the former case the flats were occupied by middle and upper income families after many of the low income families for whom they were built left them because of their inability to make the payments even though a high subsidy had been introduced. In the Hong Kong case, the flats were occupied by mostly low and very low income families. Furthermore the amount of land taken up by the developments was considerably less than what the squatters were occupying prior to their rehousing.

In Mexico City all the so-called low cost housing projects consist of self contained flats or houses with two or three bedrooms and individual facilities. Besides, since the locational aspects are seldom considered the majority of these projects have been located at the periphery of the city. In other words the government is building or promoting the construction of housing units of an unrealistic high standard which are beyond the reach of low and very low income families. Furthermore many of the developments built by the government do not even offer the advantage of a location with good accessibility to jobs and services.

Moreover cheaper housing could be provided if private developers were allowed to supply the market with housing of a lower standard. GRIMES (1976) estimates that a drop in building standards[4] could mean that 86% to 96% of the households of Mexico City could be able to afford the cheapest housing unit available in the market, and not only 45% as at present.

Thirdly, self-help has the limitation that only very low density kind of developments can be built. This has two important implications. One is that the city expands more rapidly and therefore utility costs tend to be higher (Real State Research Corporation, 1974). The other is that transportation from one place to another becomes more costly in terms of time specially when the transportation system cannot be improved and/or employment decentralized fast enough.

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4 This may include: a reduction in interior livable space; an increase in gross densities; the provision of shared as opposed to individual facilities such as cooking and toilet.

As pointed out earlier, in Mexico City land prices have been soaring over the last two or three decades. This obviously makes a low density solution less attractive since it is not very economic tc provide low income households with very expensive land which is going to be occupied by a few families in a reduced space when a greater number of families could be housed on the same plot at a proportionally lower cost.

Fourthly the income and the employment effects that public housing programmes have are seldom considered It has been estimated that in countries such as Kores, Mexico, Pakistan and India for every US $10,000 invested in construction about fourteen additional jobs are created. (GRIMES 1978 and STRASSMAN, 1979).

Fifthly, the filtering dcwn effects of new housing construction are also important. As Strassman (1977) points out a trade-off exists between building expensive good quality dwellings which have a high 'filtering effect' and building low quality dwellings which are cheaper to build but generate little or no filtering. Thus even though public housing construction may nct be suitable for low income families, the filtering effect that it has, benefits, albeit indirectly, low income groups[5].

One final criticism to self-help is ideological. Self-help may seem a very inexpensive way of "solving" the housing problem but there is very little real sccial justification for its existence. It is worth pointing out that to a large extent the housing problem is the result of the little

5 For an enalysis of the filtering effect in the Mexican case, see Yañez (1976).

importance that successive governments have given to it in the past. During the 1950's and 1960's the prevailing philosophy was in favour of those investments with a high capital-output ratio and therefore against housing which consumes too much of the scarce available resources and creates so little in terms of measurable output. (ABRAMS 1966) This very narrow criterion still pervades the minds of many politicians and for that reason the self-help movement is running the risk of becoming another "academic" justification of poverty and bad housing. (HARMS,1976)

Also it is important to recall that during the 1950's and 1960's as well, there was a prevaling argument concerning the trade - offs between an equitable distribution of income and growth. It was argued that developing countries could either have high incomes or high growth but not both, because since capital investment came from profits and profits were partially constrained by the size of the wage bill one way of ensuring rapid growth was by keeping wages as low as possible (SEN, 1969)

Today not many economists accept this argument (MYRDAL,1976) mainly because the true social cost of such a policy is now beginning to emerge. Bad housing is arguably one of the more visible effects of the "growth philosophy" at work.

5.3. REHABILITATION OF EXISTING SQUATTER SETTLEMENTS.

In discussing the housing problem of Mexico City in the context of housing policies it is important to distinguish between policies towards existing squatter settlements and policies towards new settlements. Policies towards new settlements can take many forms. As pointed out earlier households can be housed by the government in public built houses or be encouraged to build their own houses.. In our view the case in favour of the latter approach is not very convincing for the reasons given above. On the other hand policies towards existing squatter settlements can as well take many forms. It can be said that basically there are two kinds of policies those which fight squatters and those which assist squatter settlements. In the latter case the government provides the basic urban infraestructure, grants the land to the invaders and in some cases assists them financially to improve their dwellings. In the former case the government involves itself in a battle against squatters erradicating them whenever possible offering them no alternative.

It can be said that in Mexico City the policy towards existing squatter settlements has been one of assistance[6] albeit confrontations between government and squatters occur from time to time.

Is this the correct approach? In our view the answer is yes for two reasons. One is that to rehouse all those living in the squatter

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6 The main body responsible for the control of squatters is the Department of the Federal District which has a department called Oficina de Colonias Proletarias which deals with all the problems affecting people living in the Colonias Proletarias.

settlements would be impossible considering that over 3 million people live there. The other is that in those colonias (districts) where consolidation has been relatively successful rehabilitation is cheaper and more effective than rehousing.

In conclusion we think that housing policies should try to minimize the expansion of squatter settlements but at the same time assist those squatter settlements which have achieved an acceptable level of consolidation. In the case of shanty towns (Ciudades Perdidas) and other slum areas where consolidation is non-existent, rehousing (preferably in sito) should be the policy to follow.

5.4. LAND USE PLANNING AND REGIONAL PLANNING

Before concluding this chapter it seems important to comment briefly on two pieces of legislation which have been recently introduced by the government to ease the urban problems of Mexico City and other important urban centres of Mexico, which we consider will affect directly Mexico City's housing situation.

The first one is the Planning Law of the Federal District, introduced in 1976 which replaced the 1953 Planning Law. The new law has been designed to give the planning authority i.e. the Department of the Federal District greater power to control Mexico City's urban development. Like the planning law of 1953 the new law has as its main intrument for controlling land uses the subdivision and building permision. However, under the new law the rejection or approval of an application is going to depend on how the proposed development "fits" in what is now called the "Director Plan". The plan (equivalent perhaps to Britain's Structure Plans) will be generated taking into account economic, social and demographic factors and the expected short, medium and long term land requirements of different activities[7].

As far as housing is concerned, one important innovation has been the introduction of a system of subsidies where by those applying for development permision can get a reduction in the "develpment fee" if land is used for the provision of low cost residential developments. (Article 63) For example the development fee will be reduced by 50% when the selling price for square metre is not greater than six times the minimum wage.

Another interesting point is that the law explicitly states that the promotion, improvement and construction of low cost housing should be closely related to employment accessibility considerations. (Article 77)

The new law also reflects the government preoccupation regarding land speculation. To this effect the Department of the Federal District will exercise its powers to acquire land in order to secure the availability of cheap land for low cost residential developments.

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7 The Department of the Federal District has already published the 'Plan de Desarrollo Urbano del Distrito Federal' in which both policy goals and means to achieve them are described in detail. (Plan de Desarrollo Urbano D.F. 1980)

The 1976 Law, if implemented, will no doubt have an important effect on Mexico City's housing situation. The expansion of the city will no longer be as anarchic as it has been in the past and urban efficiency will be greater.

The General Law of Human Settlements (Ley General de Asentamientos Humanos) introduced in 1977 will also have important implications for Mexico City's housing situation. This law requires in its 45th article the elaboration by the government of the National Plan of Urban Development. The Plan published in 1978 contains the governments goal regarding urban national and regional planning.

Specific goals of the National Plan include among other the slow down of Mexico City Metropolitan Area's growth through the promotion of urban and economic growth in other cities in different parts of the country; the introduction of controls to regulate effectively the location of new industries and the decentralization of established ones; descentralization of administrative power, etc. It is hoped that if the plan works effectively in this respect, Mexico City will have a population of 20 million people in the year 2000 and not the expected 32 to 40 million people. Thus if the plan is successful housing needs in Mexico City are likely to become less urgent than they have been so far.

The plan's goal concerning housing is to benefit 789,000 families through several improvement and construction programmes.

It is estimated that to achieve this goal the government will have to spend about 76 million pesos (nearly 2 million pounds), between 1978 and 1982. (PLAN NACIONAL DE DESARROLLO URBANO, pag. 26)

Given Mexico's lack of experience in the area of urban and regional planning one cannot be very optimistic about the results the National Plan may bring about; nevertheless, it is significant that these important problems are beginning to be tackled seriously and a much broader approach than before is being taken.

5.5. CONCLUSION

It can be said that for several decades an important proportion of Mexico's population has been forced to house itself in view of the fact that traditional market and public mechanisms have failed to supply the right amount and the right type of housing, that low income families need. For more than 20 years the public and the private sector together have supplied not more than 30% of what is needed leaving the so called 'popular sector' fill the gap.

Several housing agencies have been set up to cope with the housing problem but because of the lack of enough resources as well as the lack of effective coordination the official response to the housing problem has been very limited.

Radical policies such as self-help have been proposed to deal with the problem arguing that this is the only realistic approach considering the magnitude of the housing problem. We argue that what is needed is a revision of present policies particulary with regards to standars and also to take into consideration the indirect effects such as employment creation and filtering down that a well designed housing policy can have.

CHAPTER 6

LINEAR PROGRAMMING MODELS IN URBAN PLANNING

The aim of this chapter is twofold. Firstly to introduce the reader not familiar with linear programming to the basic concepts in order to facilitate the reading of section 6.2, which contains a review of Linear Programming Models developed as urban planning tools. Secondly, to set an appropriate framework for the assessment of the housing goal programming model proposed in the following chapter.

A detailed discussion of goal programming, which deals with linear multiobjective programming problems is set out in the first part of Chapter 7. It is intended that this will facilitate understanding of the housing goal programming model which forms the basis of this dissertation set out in the latter part of Chapter 7 and developed more fully in Chapter 8.

6.1. SOME BASIC LINEAR PROGRAMMING CONCEPTS

6.1.1. Optimization and Linear Programming

Problems which seek to maximize or minimize a numerical function of a number of variables, with the variables subject to one or more constraints are called optimization problems. Programming problems are a special class of optimization problems. They are concerned with determining optimal allocations of limited resources to meet given objectives. They deal with situations where a number of resources, such as land, capital, and man - hours, are available and are to be combined to yield one or more products. Linear programming is concerned with those kind of programming problems for which all relationships among the variables are linear. The relations must be linear both in the constraints and in the functions to be optimized. The general linear programming problem can be described as follows: given a set of n linear equations or inequalities in n variables, we wish to find non-negative values of these variables which can satisfy the constraints and maximize or minimize some linear functions of the variables. (HADLEY, 1974).

6.1.2. A Simple Linear Programming Problem

Assume a group of planners faced with the problem of deciding whether house type A or house type B should be built on a given site. For the sake of simplicity assume that only two resources, capital and land, are needed for the construction of both house types. House type A requires 15000 units of capital and 250 units of land whereas house type B requires 10000 units of capital and 400 units of land. The local authority, who is responsible for the development, has available 150000 units of capital and 5000 units of land. Additionally it is known that house type B is preferred to house type A, by the future occupants, because houses type B have bigger gardens than houses type A. The planners have been able to estimate a measure of the subjective utility generated by each house type. House type A generates an estimated utility of 10 units and house type B an estimated utility of 12 units. The planner whishes to know which mixture of house types should be built so as to provide the maximum overall utility, given the available resources of capital and land.

This is a typical case in which linear programming can be used to solve the problem. The problem is formulated as follows:

(utility maximization)	MAX $Z = 10x_1 + 12x_2$	(1)
	Subject to:	
(capital constraint)	$15000x_1 + 10000x_2 \leq 150000$	(2)
(land constraint)	$250x_1 + 400x_2 \leq 5000$	(3)
(non-negativity constraint)	$x_1, x_2 \geq 0$	(4)

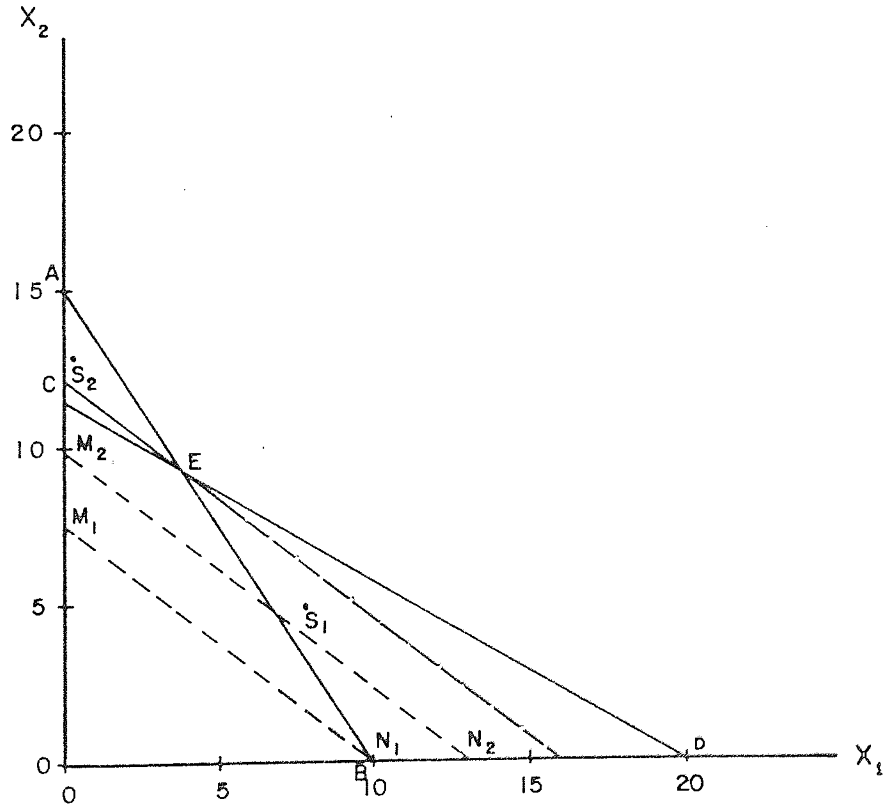
(1) is the objective function of the linear programming problem which is to be maximized. (2) and (3) are respectively the capital and land constraints. (4) is a non-negativity constraint which requires all values for x_1 and x_2 to correspond to the number of dwelling types A and B that can be built. Any set of values which satisfies the constraints is called a solution to the linear programming problem. Any solution which satisfies the non-negativity restriction is called a feasible solution. Any feasible solution which optimizes the objective function is called an optimal feasible solution. The problem presented above is very simple and can be solved using the graphical method. However, linear programming problems involving several variables and constraints are generally solved by a technique known as the simplex method. (DANTZING 1963 , HADLEY 1974 , WAGNER 1970)

The graphical solution of the problem is presented in fig. 6.1 (p.139). AB represents the capital constraint. It is worked out by giving x_1 a value of zero to obtain the value of x_2 on the vertical axis and then giving x_2 a value of zero to obtain the corresponding value of the variable x_1 on the horizontal axis. Once these points have been determined they can be joined to obtain the line AB. The same operation is carried out to obtain line CD, which represents the land constraint. Only points within the area OCEB are said to be feasible solutions to the linear programming problem. Points such as s_1 or s_2 are not feasible solutions because s_1 violates the capital constraint and s_2 violates the land constraint. To solve the linear programming problem we must find the point in the region of feasible solutions which gives the largest value of the objective function.

For any fixed value of Z, $Z = 10x_1 + 12x_2$ is a straight line. For each different value of Z we obtain a different line. We wish to find the line with the largest value of Z which has at least one point in common with the region of feasible solution OCBE. Line $M_1 N_1$ in figure 1 is obtained by giving Z an arbitrary value of 100. Line $M_2 N_2$ is obtained by giving Z an arbitrary value of 120. Both lines are parallel to each other which is not surprising because they are derived from the same equation. The further these lines are from the origin, the greater the value of Z is and the greater the level of utility. It is easy to see in fig. 1, that it is at point E where the value of the objective function is maximized since no other point is further from the origin while satisfying the constraints.

The objective function running through point E is actually $157 = 10x_1 + 12x_2$ (3), which by solution of equation (2) and (3) would be achieved by building 2.80 houses of type A and 10.75 houses of type B.

FIGURE No. 6.1



6.1.3. Sensitivity Analysis

In many practical problems the planner or policy maker wishes not only to find the optimal solution of a linear programming problem but also to determine the sensitivity of the optimal solution to changes in the system. He would like to determine the effects of such changes without having to solve a new problem or a series of new problems. Changes in the system can be divided into the following five categories:

1. Changes in the amount of available resources
2. Changes in the objective functions
3. Change in the coefficients
4. Addition of new variables
5. Addition of new constraints

This makes linear programming more attractive for planning purposes for the planner can, for instance, and with reference to the example given previously, see how the optimal solution (i.e. that which maximizes the community's utility) is affected if land resources are increased, say from 5000 units to 5500 or, to see what is the effect of assuming a different level of utility for each house type or an increase in the cost of construction of one house type, etc.

6.2. PROBLEMS AND LIMITATIONS OF LINEAR PROGRAMMING MODELS

a) The requirements of linearity. As pointed out earlier both the objective function and the constraints must be linear. In the context of urban modelling this may be an important limiting requirement because there are quite a few instances where the relationship between inputs and outputs is not linear. For instance, in the example given earlier it may be unrealistic to assume that the average cost per dwelling remains the same regardless of the size of the development if it is known that substantial economies of scale can be obtained with bigger kinds of developments.

b) The non-discrete nature of the variables. An additional limitation of linear programming is the need for continuous rather than discrete values for the variables. In some cases when the values can be rounded off without creating important discrepancies between the discrete and the continuous optimal solution, this may not be a serious disadvantage. For instance in the example presented above it seems of little importance to round off the resulting values of the variables $x_1 = 2.80$ and $x_2 = 10.75$ since by taking $x_1 = 3$ and $x_2 = 11$ the optimum value i.e. the level of utility, increases from 157.00 to 162.0. However, in other cases the discrepancy may be considerable. For instance if the values of the variables correspond to the optimum number of industrial plants to be located in a region, to say that 2.4 industrial plants are approximately the same as 2 or 3 may have a rather important effect on the optimal solution.

c) <u>The "objectivity" of the objective function</u>. Choosing the objective function is a very important stage of the model building process. It requires a clear identification of planning goals and their incorporation into the objective function. This in turn requires the quantification of goals which can be a rather difficult task especially when goals of a social nature are being considered. In regard to this point LEWIS (1969) observes:

"The choice of aim is itself subjective: but even if we have chosen to minimize the journey to work we must ask how to measure it? Do we do it in time, distance, cost, some mixture of these, or what? Do we take mean time along a route or model time? Do we pay more attention to time spent by certain kinds of workers? . . . It is possible to go on listing more questions of this type, what we have to realise is that whether we ask them or not, we do in fact imply answers to all of them by our very choice of objective function. We simplify away the question but not the answer. And as long as any one of the implied questions remains incapable of being answered objectively, so the so-called "objective function" is objective only in the sense that it states an objective not in the sense that is has been objectively chosen. Furthermore, even if and when all of those questions can be answered objectively, that we have chosen to consider journey to work rather than, say, rateable value, is itself probably a subjective decision". (p. 39)

This problem, however, can be partially solved by adopting a goal programming approach. Goal programming is about solving linear programming problems where several objective functions are considered simultaneously. In Chapter 7 of this thesis we explain the basic characteristics of a goal programming model and show how it can be used in the context of housing planning.

6.3. REVIEW OF PROGRAMMING MODELS USED IN URBAN PLANNING

In order to organize this review we classify models into three main categories: 1. Land use and development; 2. Housing and improvement models; and 3. Transportation and Land use.

6.3.1. Land Use and Development

One of the earliest attempts at modelling land use and development was the Penn-Jersey model proposed by HERBERT and STEVENS (1960). Originally it included all land use activities but due to several practical and conceptual difficulties the model concentrated on the simulation of residential location. The model requires an enormous amount of data. It requires an inventory of households classified by income, patterns of consumption and of daily movement plus an inventory of all residential sites in the region classified by size, type and quality of structure, location characteristics and neighbourhood amenities. It also requires the estimation of the transportation costs generated from each site. The linear programming formulation has the following form:

(1) MAX $Z = \sum_k \sum_i \sum_h x_i^k \; (b_{ih} - c_{ih}^k)$

Subject to:

(2) $s_{ih} \; x_{ih}^k \leq L^k$

(3) $x_{ih}^k = N_i$

and all $x \geq 0$

Where :

b_{ih} = residential budget of household of group i using accommodation type h.

c_{in}^k = cost to a household of group i using accommodation type h in zone k.

s_{ih} = land for household group i using residential accommodation type h.

L^k = land available in zone k.

N_i = total number of households of group i.

x_{ih}^k = household of group i using accommodation type h in zone k.

The objective function (equation 1) refers to the maximization of rent-paying ability by all households or "savings" as HERBERT and STEVENS call it. Constraint equation (2) prevents the consumption of land in each area from exceeding the land available. Constraint (3) requires the model to locate the projected number of households of each group. The model distributes households in an optimal configuration from the point of view of all households because since all households try to maximize rent – paying ability no household can move to increase its rent-paying ability without reducing the rent-paying ability of some other households and simultaneously reducing aggregate paying-ability i.e. the value of Z in equation (1).

The Penn-Jersey model was never implemented due to practical problems. The main problem was to assess the housing preferences of each income group (one for which some ingenious solutions have been proposed by HARRIS et al, 1966). The model also requires an enormous amount of a priori knowledge. One needs to know where the services shops, schools, etc., are located to calculate transport requirements, workplaces, preference structures and so on. (ECHENIQUE, 1974)

A more pragmatic linear programming model has been proposed by SCHLAGER (1965). He argues that the problem of the designer of urban form is:

1. "Given design requirements expressed as:

   a) A set of design standards in terms of restrictrions on land use relationships that may exist in the plan.

   b) A set of needs or demands for each type of land use based on forecasts of future activity.

2. Synthesize a land use plan design that satisfies both the land use demands and design standards considering the current state of both natural and man-made land characteristics, at a minimal combination of public and private costs". (p. 107)

SCHLAGER then proposes a linear programming model which generates a complete land use plan which minimizes total public and private investment costs subject to a number of design constraints.

The objective function is to minimize :

$$c_t = c_1 x_1 + c_2 x_2 + \ldots + c_i x_i$$

Subject to :

$$d_1 x_1 + d_2 x_2 + \ldots + d_n x_n = E_k$$

$$x_1 + x_2 \ldots x_n = F_m$$

$$x_n = Gx_m$$

$$x_n \succeq 0$$

Where:

c_t = total cost of development for period t.

c_i = cost coefficient.

x_i = type of land use.

E_k = demand for each land use.

d_i = service coefficients for supporting development such as streets, utilities, and improvements.

F_m = limit on land use n in zone m.

G = ratio of land use m in the same or different zones for the area.

The model has been applied to metropolitan areas in South East Wisconsin by the Regional Planning Commission. Experience with 30 zones could yield as many as 400 decision variables and 60 constraints.

One can run the model several times under different assumptions regarding design standards and different estimates of land use demands. This allows the planner to determine the sensitivity of the plan to forecast inaccuracies as well as to determine the effect of each design standard on the form and cost of the plan.

SCHLAGER's model, however, had to be modified by the South Eastern Wisconsin Regional Planning Commission because the discrete nature of the location of activities was not considered to be adequately reflected in the linear form of the model. Also the estimated cost of development was thought to be underestimating the true cost which involves not only the locational cost but also the linkage cost. (See South Eastern Wisconsin Regional Planning Commission, 1965).

Another linear programming model of interest is that of BEN-SHAHAR et al (1969) for allocating residential and employment land uses and determining transport investments in urban development planning.

Because of the complexity of the notation it would be tedious to detail the model here as we have done with the models reviewed previously. However, we can summarize its structure verbally as follows.

Objective function :

Maximize: Total demand prices for all buildings, <u>less</u> construction and demolition cost for new ones, <u>less</u> all costs associated with the provision and use of communications infrastructure and capital.

Constraints:

The set of constraints includes the following:

1. Equality between the number of dwellings and the number of household of each income group in each time period.

2. Equality between land supply and land demand in each time period.

3. Balance between work and non-work trip attraction and trips generation in each period.

The model has very useful policy implications which are better seen once the dual problem is derived from the primal problem. "The dual problem provides vital information to policy makers for making rational decisions. It does so by determining the shadow price of each constraint . . . The shadow price of a constraint is a measure which indicates by how much the total value of the goal function increases as a result of a marginal relaxation of these constraints". (BEN - SHAHAR, p. 111). Two examples are the marginal output of land and the size of population. The shadow price of land in any zone indicates by how much the total present value of the plan could have risen (declined) had the total land available in the zone increased (decreased) by one acre. By the same token the shadow price of the population derived from the primal population constraint, measures the marginal alternative benefit.

Additional useful information for rational decision making is derived by sensitivity analysis. Sensitivity analysis can be use to determine the effects of risk and uncertainty upon the optimal solution. Sensitivity analysis can tell the planner by how much the total value of the plan will diminish if any estimated variable deviates from the expectation. The plan having a greater sensitivity of total value to such deviations involves greater risk.

The model has been applied in Israel and other countries including the U.K (See P.A. Management Consultants and Israel Institute of Urban Studies, 1969).

BOADEN (1977) has developed a model aimed at cases where the planner is seeking to establish a land use mix which will maximize some measure of the profitability of a project. BOADEN points out, however, that the profitability goal can be modified or overriden by the use of suitable constraints.

The model comprises two submodels, a Discounted Cash Flow model and a Linear Programming model. The Discounted Cash Flow model provides the Net Present Values of the fixed cash flows and the variable per unit cash flow which are used in the Linear Programming model in order to obtain the optimum mix.

BOADEN argues that the development of a plot of land may be looked upon as the production and eventual sale of a number of commodities. For example, the production and sale of different house types, apartments and an area of land set aside for shopping. The costs or revenues generated by each type of development, BOADEN points out, are of two types, variable and fixed. The former refers to those costs or revenues which are a function of the quantity of each type of commodity (or land uses) produced; for example construction costs or sales revenues. Fixed costs or revenues on the other hand are not a function of quantity produced; for example the price paid for the raw land or compensation received for a national road.

The objective function of the Linear Programming submodel is to maximize :

$$Z = \sum_{j=1}^{n} V_j \, X_j + F$$

Where:

V_j = net present value of variable cash flows per unit of land use j.

X_j = number of units of land use j.

F = net present value of fixed cash flows.

n = number of land uses

V_j and F are the output parameters of the DCF submodel, where:

$$V_j = \sum_{t=1}^{m} G_{jt} (1 + i/100)^{-t} \quad \text{for } j = 1, 2, \ldots n.$$

Where:

G_{jt} = the present value of variable cash inflows minus present value of variable cash outflow per unit of land use j occurring in year t.

H_t = the present value of fixed cash inflows minus present value of fixed cash outflows occurring in year t.

m = investment period.

BOADEN then points out that the analytical value of the model lies in the user's ability to test the sensitivity of the optimal solution to changes in the input data.

The model has been applied in a number of studies and the results have been satisfactory (see BOADEN 1976). A slightly different version of this model is the one proposed by ORNE et al (1975) which differs from the BOADEN model in that the former places less emphasis on the profitability aspect of the problem. In spite of the simplicity of the BOADEN model, the actual estimation of the cash flows must be quite difficult since it requires a very good knowledge of future conditions. There is also the problem concerning the choice of discount rate which has a very important effect on the results.

DICKEY et al (1973) have developed a model to evaluate various land use schemes. They point out:

> "The development of land use plans for an urban area usually is a time-consuming and expensive process. As a result the planner often is limited to investigating only a few alternate land use development schemes, and these investigations generally are rather quick and rough . . . The planner would be greatly aided if he had a fairly rapid technique, that with a given set of data would generate and determine some of the consequences of various land use schemes". (p.39)

The model proposed by DICKEY et al seeks to minimize the combination of overall travel costs and the establishment costs minus benefits, subject to land use demand constraints. More formally:

$$\text{MIN } Z = \sum_i \sum_j C_{ij} X_{ij} + \sum_j \sum_k K_{jk} PR_i \,(X_{ij} + E_{ij})\; \frac{\sum AT_i \,(X_{ik}+E_{ik}) \frac{1}{T_{jk}^2}}{\sum_n \sum_i AT_i \,(X_{in}+E_{in}) \frac{1}{T_{jk}^2}}$$

Subject to :

$$\sum_j X_{ij} = A_i, \text{ all } i$$

$$\sum_i X_{ij} = B_j, \text{ all } j$$

$$\sum_i A_i = \sum_j B_j$$

$$X_{ij} \geq 0, \text{ all } i, j$$

Where:

X_{ij} = amount of activity i allocated to zone j, acres.

E_{ij} = existing amount of activity i in zone j.

A_i = future amount of activity to be allocated

B_j = area available for development in zone j.

C_{ij} = total establishment costs-benefits for locating activity i in zone j.

PR_i = trip production rate for activity i.

AT_i = attraction rate for activity i.

K_{jk} = cost of daily trip from j to k.

The model has been used to test a proposed land use scheme in Blacksbury, Australia. A prediction was made of how much would be needed by 1985 for high and low density residential land and industrial land. The model then is employed to determine where to allocate the needed land use areas so as to minimize the public service and travel cost.

As the authors point out the model has several drawbacks many of which could be eliminated by modifying some of the assumptions. For example, in the estimation of C_{ij} (the cost-benefit coefficient) the authors assume that land prices in each zone equate benefits. This is not correct because what a person pays for a site is not necessarily what he is willing to pay. Benefits therfore, should be derived from a demand curve.

6.3.2. Improvement and Housing Models

An interesting model has been proposed by STUART (1969) to aid planners in the evaluation of alternative urban plans and improvement programs.

Using the Model Cities Programme as an example, STUART developed a simple linear programming model built around a matrix of relative effectiveness coefficients, a set of performance standards and appropriate budget programmes.

The Model's function is to allocate dollars among various programme alternatives either minimizing the total budget spent or maximizing the achievement of specific objectives. The objectives are expressed as percentage impacts upon various socioeconomic conditions within a model neighbourhood.

Thus the linear programming model can be stated as either:

$$\text{MINIMIZE} \sum_{i} x_i \quad \text{or} \quad \text{MAXIMIZE} \sum_{i} \sum_{j} d_j \, a_{ij} \, x_i$$

Subject to :

$$\sum_{i} a_{ij} x_i \geq y_j$$

$$\sum_{j} x_i \leq m_j$$

$$\sum_{i} x_i \geq n_j$$

Subject to :

$$\sum a_{ij} x_i \geq y_j$$

$$\sum_{i} x_i \leq b$$

$$\sum_{i} x_i \leq m_j$$

$$\sum_{i} x_i \geq n_j$$

Where:

x_i = model city program alternatives, in dollars.

y_j = performance standards for each objective.

a_{ij} = relative effectiveness coefficients.

d_j = relative weights for each objective (optimal).

b = total budget available.

m_j = maximum program budgets.

n_j = minimum program budgets.

In spite of the model's simple structure a number of problems have to be satisfactorily solved before it can be used. There is the problem of identifying objectives and alternative programmes of action and also the problem of predicting and measuring effectiveness. STUART concluded then from his experiment that much work remains to be done before viable urban improvement programming models can actually be developed.

Another improvement programming model of interest is that proposed by STEGER (1965). The objective function is to maximize the number of sound housing units to be added by public and private resources subject to a budget constraint.

The city is divided into n areas and for each one the planner determines (a priori) whether rehabilitation/redevelopment or code enforcement will be the policy to follow in order to eliminate blight. The model determines which fraction of each area should be treated so as to provide the maximum number of sound units per period.

The model is formally expressed as:

Maximize : $h_{1t}\ x_1 + h_{2t}\ x_2 + h_{nt}\ x_n$ (1)

Subject to: $c_{1t}\ x_1 + c_{2t}\ x_2 + c_{nt}\ x_n + s_t = B_t$ (2)

$c_1\ (t+1)\ x_2 + c_2\ (t+2)\ x_2 + c_n\ (t=1)\ x_n + s\ (t+1) =$

$B(t+1)$ (3)

$$x_1 + q_1 = 1 \qquad (4)$$

$$x_2 + q_2 = 1 \qquad (5)$$

$$x_n + q_n = 1 \qquad (n)$$

Where:

x_n $(0 \geq x_n \leq 1)$ = the fraction of area n treated

q_n $(0 \geq q_n \leq 1)$ = the fraction of area n untreated

c_{nt} = the cost of applying the chosen treatment to the entire area in year t.

h_{nt} = number of sound units that each treatment would provide in period t.

s_t = is a slack variable which represents the amount of funds not used in period t which can be used in any following period.

The main limitation of the model is obviously that it assumes that if 3/4 of a treatment is selected, 3/4 of the total cost of treating the entire area will be incurred. This in the light of fixed cost and economies of scale is unwarranted. However, as STAGER points out any mathematical representation of reality cannot incorporate all the features of the actual situation.

A somewhat similar housing model has been proposed by SILVERS and SLOAN (1965). The model allocates public funds so as to maximize the number of low cost dwellings which generate the greatest level of welfare. Welfare or utility is a measure derived from the planner's assessment of the priorities of different households groups.

Obviously the greatest disadvantage of this model is the inclusion of the subjective concept of utility. However, as CAMINA (1969) points out, in spite of the current crudity and inevitable subjectivity of such judgements, it may be better for the basis of decisions to be made clear and not to be allowed to remain subconcious and suppressed.

STRASSMAN (1977) has proposed a linear programming model to investigate the trade-off that exists between building expensive good quality dwellings which has a high "filtering effect" and building low quality dwellings which are cheaper to build but generate little or no filtering. In STRASSMAN's words:

> "The planner's allocation of funds among housing types should reflect the fact that an expensive house potentially generates more filtering (or prevents more raiding), while many more cheap dwellings could be built at the same cost. There is a filtering volume trade-off. The problem can be specified mathematically and be solved with linear programming methods for specific cases and with a variety of additional constraints". (p.315)

The objective function takes the following form:

$$\text{Maximize } Z = \sum_{j=1}^{n} T_j + \sum_{j=1}^{n} D_j$$

Subject to:

$$T_j \leq H_{jo} - R_j \qquad (1)$$

$$\sum_{j=1}^{n} c_j D_j \leq I_g \qquad (2)$$

Where:

T_j = number of dwellings that income group i transfers to i - 1.

D_j = new construction of housing type j.

c_j = cost of constructing a dwelling type j.

H_{jo} = the initial housing stock of category j.

R_j = housing of type j that will be removed from the new stock during the planning period.

I_g = proportion of the city gross product devoted to housing investment.

Like may other models STRASSMAN's model is based on a number of simplifying assumptions, one of which is that the cost of a new dwelling is the cost of construction, which means that neither land cost nor transport costs are taken into account.

The model has been applied to two countries Tunisia (FERCHIOU, 1975) and Mexico (YAÑEZ, 1976).

6.3.3. Transportation and Land Use

An interesting application of linear programming to transportation planning has been proposed by HAMBURG (1966). HAMBURG's interest is to explore the magnitude of the influence of the work trip on the residential location decision. HAMBURG does this by comparing the actual travel time involved in the journey to work with what the travel time might be under some ideal conditions.

The minimum time spent commuting is computed by the use of the transportation problem.

$$\text{MIN} \quad T_M = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \; x_{ij}$$

Subject to :

$$x_{ij} = W_i$$

$$x_{ij} = H_j$$

$$x_{ij} \geq 0$$

Where:

T_M = minimum total travel time.

a_{ij} = travel time from zone i to zone j.

x_{ij} = trips from zone i to zone j.

m = number of work zones.

W_i = work places in zone i.

H_j = homes in zone j.

In order to compare the optimal time spent in commuting with the actual time, HAMBURG developed an index of indifference, expressed as:

$$I_S = \frac{T_A - T_M}{T_P - T_M}$$

Where:

I_s = index of indifference for group s.

T_A = actual travel time.

T_P = probable travel time for complete indifference.

$$T_A = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \ x_{ij}$$

The probable travel time, if commuters were assumed to be totally indifferent to time, was found by assuming that workers were allocated to homes on a proportional basis, so that:

$$T_P = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \ x_{ij}$$

Where:

$$x_{ij} = \frac{W_i \quad H_j}{\sum_j \ H_j}$$

The index I has a minimum value of zero and a maximum value of one. The higher the value of I the more indifferent people can be said to be towards time spent in commuting to work.

In his analysis of the Buffalo, New York area, HAMBURG found that lower and middle income groups were more sensitive to travel time than the high-income groups. HAMBURG also concluded from his analysis that white drivers were more indifferent than black drivers towards time spent in commuting. Perhaps the most important result of HAMBURG's analysis was to show that commuters are not indifferent to time spent in commuting as has been suggested by some theorists.

A rather complex and ambitious combined land use and transportation model has been proposed by OCHS (1969). The model is an hybrid of the Penn-Jersey model and the standard transportation problem.

Because of the complexity of the notation we shall simply describe its main features verbally.

Objective function: Minimize the total costs that the population of an urban area pays for housing and travel to work.

The constraints are:

a) A traffic flow capacity constraint, which holds that the traffic over a traffic lane cannot exceed the capacity of the traffic lane.

b) A land demand constraint which holds that the total land required for residential use in any given tract cannot exceed the land available for residential use in the tract.

c) A labour demand constraint which holds that the employment requirements of each plant are satisfied.

Due to its complexity this model is more realistic than many others including the HERBERT - STEVENS model, which assumes that the cost of transportation between any origin and any destination is a known value independent of the flow of traffic over routes which connect those two points. By contrast in OCH's model the average cost of travel between two points is a value determined by the model itself.

It can be said that in spite of its theoretical value, the enormous data requirements of OCH's model makes it rather difficult and probably very expensive to use for land use plan evaluation.

CONCLUSION

In this chapter we have included but a few examples of the numerous linear programming models developed to help planners in their evaluation of urban plans. Something that characterises these models is that only one objective can be considered. This may be an important limitation because more often than not in the real world planners have to deal with situations where several conflicting objectives can be identified. This has lead to the development of new approaches like goal programming which allows the possibility of modeling, analysing and solving decisions which involve the consideration of multiple and conflicting objectives. This will be the subject matter of Chapter 7.

CHAPTER 7

A GOAL PROGRAMMING HOUSING MODEL

It was pointed out in Chapter 4 that in order to improve Mexico City's housing problem the government has taken in recent years the main following measures:

a) Introduced a new planning law (1976) as well as an Urban Development Plan for Mexico City both of which give the government greater powers to control urban growth and therefore more scope to plan and organise the settlements of low-income groups.

b) Set up a number of public agencies aimed at providing low income households with credit facilities and to finance public housing projects or rehabilitation schemes.

c) Begun the acquisition of privately owned land for urban residential reserves.

These are important developments which no doubt can help to make the task of improving Mexico City's housing situation less arduous. Yet the following question needs to be asked: How can the government know that these sort of measures are going to have the desired effect upon Mexico City's housing situation? These are many ways in which the government can make use of the resources available to improve housing conditions. The government's task is to find out "the best" way of making use of the resources at its disposal to achieve the desired objectives.

This is a typical allocation problem in which the best allocation is that which maximizes the achievement or impact of certain policies. Linear goal programming is a technique having a great deal to offer in this respect since it is concerned with making "optimum" allocation of resources given certain conflicting objectives. One can then build a goal programming model to explore the impact of different allocation strategies upon certain objective functions or goals.

The purpose of this chapter is to present a housing goal programming model which could be used as a planning tool by any of the housing agencies set up by the government to deal with the allocation of public funds in the construction and or rehabilitation of low income housing.

The proposed model is presented in two versions; one for what we call the monocentric case and one for the polycentric case. The former refers to the case where employment in the city is highly concentrated in the CBD and the latter to the case where the opposite occurs i.e. an important share of total employment is found in the suburbs of the city.

In Chapter 3 it was argued that Mexico City falls more in the monocentric than in the polycentric case because employment is highly concentrated in one delegation, Delegacion Cuauhtemoc, and also because land prices tend to decrease exponentially with distance from the city centre. This is, as we have previously discussed one of the predicted results in a city where employment is highly concentrated.

The proposed goal programming model therefore uses for the monocentric version actual data from Mexico City, developed with the aid of the Planning Department of the National Worker's Housing Fund.

For the polycentric version hypothetical data was used. The idea of developing the second version was to show that the model can also be applied when employment is scattered throughout the urban area. Also for the second version the possibility of a rehabilitation - redevelopment alternative was introduced.

Before the full formulation of the models is given a short description is given of goal programming, and its major advantages.

7.1. GOAL PROGRAMMING *

'Goal programming is just one approach that has been proposed for dealing with the modelling solution and analysis of decisions which involve the consideration of multiple and conflicting objectives.. the type of problem so typically encountered in actual practice' (IGNIZIO 1978)

There are certain concepts that play an important role in the structure of the goal programming model. A few of these concepts are defined below.

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*This section draws heavily on IGNIZIO (1978) Other books and articles consulted were CHARNES and COOPER (1961), LEE(1971, COURTNEY (1972) and PRICE (1974).

Objective	An objective is a rather general statement reflecting the desires of the decision maker. For example, minimize housing subsidies, maximize housing construction.
Aspiration Level	An aspiration level is an specific target value which it is considered an objective should reach.
Goal	An objective in conjunction with an aspiration level is termed a goal. For example 'achieve at least X units of houses built'.
Goal Deviation	Not all aspirations can be achieved and not all restrictions may be satisfied. Deviation from the problem goals will therefore be encountered. In a typical problem one seeks to minimize these deviations.
Achievement Function	The achievement function amounts to what in linear programming is known as the objective function. In a goal programming model however choice variables are not included. Only the deviation variables of each goal are considered. The achievement function represents the optimal compromise and its measure.

The mathematical formulation of a goal programming model can then be stated as follows.

Letting

$f_i(x) =$ The mathematical representation of objective i as a function of the decision variables $\bar{x} = X_1, X_2, \ldots X_1^1$.

T_i = The aspiration level associated with goal i.

d_i = The negative deviation associated with goal i.

e_i = The positive deviation associated with goal i.

Then goal i is represented as

$$f_i(\bar{x}) + d_i - e_i = T_i$$

The achievement of a given goal may be represented solely in terms of the deviation variable values, therefore the achievement function is given strictly in terms of d_i and e_i, the goal deviation variables.

The specific deviation variable to be minimized will depend on the goal type. We then

MINIMIZE $Z_i = e_i$ if we have a goal of the type $t(x) \leq T_i$

MINIMIZE $Z_i = d_i$ if we have a goal of the type $t_i(x) \geq T_i$

MINIMIZE $Z_i = e_i + t_i$ if we have a goal of the type $t_i(\bar{x}) = T_i$

7.1.1 Weighted Linear Goal Programming

The term goal programming was coined by CHARNES and COOPER in the early sixties (CHARNES and COOPER, 1960). The sort of goal programming proposed by CHARNES and COOPER deals strictly with linear multiobjective models. The central idea is the use of the goal deviation variables. By adding these deviation variables to conflicting objectives, and by specifying aspiration levels the linear multiobjective problem is transformed into a conventional single objective linear programming problem. The CHARNES and COOPER approach is often referred to as 'Weighted linear goal programming'.

Example :

Let us suppose that three goals have been set in a given planning situation.

Goal (1) $30X_1 + 25X_2 + d_1 - e_1 \geq 750$

Goal (2) $75X_1 + 35X_2 + d_2 - e_2 \geq 1,950$

Goal (3) $4X_1 + 12X_2 + d_2 - e_2 \geq 400$

Subject to

$$X_1 + 2X_2 \leq 40$$

$$1.5X_1 + .5X_2 \leq 30$$

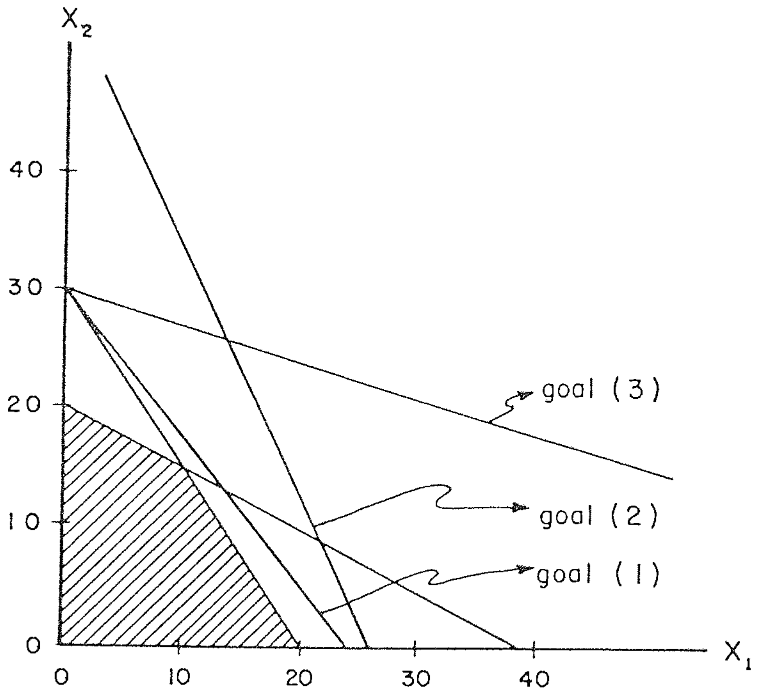
$$X_i \geq 0$$

Since goals (1), (2) and (3) are of the type $\geq T$ then the achievement function must be formulated as follows:

MIN $Z = d_1 + d_2 + d_3$

The graphical solution to this particular problem is shown in fig. 7.1.

FIG. 7.1



There is one problem with this approach and it is that there normally exist what is known as dominance among goals; that is, there is one goal which dominates the others and therefore determines the problem's optimal solution. In this particular example goal 2 dominates the others because the problem's solution is the same whether we minimize $d_1 + d_2 + d_3$ or minimize only d_2.

We can easily alter the dominance among goals by giving weights to d_1, d_2 and d_3. Thus if we give d_1 a weight of 4 in the achievement function and minimize $4\,d_1 + d_2 + d_3$, goal 1 dominates.

Since the weights have such an important impact upon the problem's optimal solution we shall discuss in the following section a method sometimes used to estimate such values.

7.1.2 A Method for Calculating Suitable Weights for the Achievement Function.

Several methods are available for determining the decision maker's judgement policy concerning competing goals. An interesting and simple method has been suggested by HAMMOND and BOYLE (1971) known as "Policy Capture Method". (See also GREEN 1968, HAMMOND 1965, HAMMOND & BREHEMER, 1973)

Basically the idea is to ask the decision maker to indicate the attractiveness of a number of outcomes that are considered achievable. Using regression analysis his preference function is inferred.

For example, suppose we wish to 'capture' the policy judgement of the decision maker regarding goals (1), (2) and (3). According to the policy capture method we first define a range of options representing the possible combination of levels of achievements of goal 1, 2 and 3.

This information is condensed in Table 7.1. Thus for example, in the first row we see that one possibility is to build 5000 dwellings, rehabilitate 200 and generate a transport cost of $ 950,800. Another possibility (second row) is to build 4800 dwellings and rehabilitate 465 with a transport cost of $870,500, and so on. Each option can be given a number for easy identification.

We would then ask the decision maker the following question. How desirable would you find achievement of option 1 ? His answer would consist of a numerical rating between perhaps 1 and 30, with 30 indicating a very desirable evaluation and 1 indicating very undesirable.

TABLE 7.1

Rating	Option No.	Goal (1) Houses Built	Goal (2) Houses Rehabilitated	Goal (3) Transport Cost Generated ($)
25	1	5 000	200	950 800
30	2	4 800	465	870 500
20	3	4 500	900	850,300
15	4	4 200	1 500	830 400
17	5	3 900	1 950	800 300
18	6	3 800	2 220	789 400
14	7	3 500	2 500	706 400
12	8	3 450	2 900	687 500
10	9	2 990	3 500	650 000
8	10	2 500	4 200	630 400

In providing an answer the decision maker may take into consideration that for example house building has an employment effect greater than the housing rehabilitation. He may also take into account the social pressures coming from different sectors of the community, etc.

Having completed the "test" the information can be used to derive a 'preference function' using regression analysis.

With the decision maker's evaluative rating as dependent variable and the goal's aspiration levels as independent variables, a set of coefficients can be estimated through multiple regression analysis. The interpretation of these coefficients is of great interest to us because they tell us how marginal changes in the level of achievement of one goal affect the decision maker's satisfaction. Thus for example in the following equation:

$$Y = A + B_1 X_1 + B_2 X_2 - B_3 X_3$$

Where:

X_1 is the level of achievement of goal i

$X2$ is the level of achievement of goal ii.

X_3 is the level of achievement of goal iii.

Y is the decision taker's evaluative rating.

B_1 represents the marginal change in the decision maker's satisfaction as the number of houses is increased by one unit; B_2 marginal change in the decision maker's satisfaction as an extra unit is rehabilitated and B_3 the marginal contribution of the subsidy level to the decision maker's satisfaction (or dissatisfaction).

Coefficients B_1, B_2 and B_3 can be used to weigh the deviation variables in the achievement function. Where B_1 corresponds to the weight given to goal 1, B_2 to the weight given to goal 2, and so on.

If $B_2 \gg B_1$, the model will make an allocation giving priority to goal 2 over goal 1. Because B_3 has a positive sign, goal 3 will be given the last priority.

Thus, by using the policy capturing method we can ensure that the achievement function reflects the decision maker's actual intent.

7.1.3. Lexicographic Goal Programming

In contrast with 'weighted goal programming' lexicographic goal programming employs no weights but instead simply ranks each objective according to preference. The solution that provides the lexicographic minimum to this ranking is then considered optimal.

Example:*

Let $Y_1 = -2x_1 + X_2$ (1)

$Y_2 = -X_1 + 2X_2$ (2)

* Taken from IGNIZIO (1979).

be two conflicting objectives subject to

$$- X_1 + 3X_2 \leq 21$$

$$X_1 + 3X_2 \leq 27$$

$$4X_1 + 3X_2 \leq 45$$

$$3X_1 + X_2 \leq 30$$

$$X_1 , X_2 \geq 0$$

If objective one (Y_1) is preemptively preferred to objective (Y_2), then the Lexicographic formulation becomes:

Minimize $\bar{a} = (e_1 + e_2 + e_3 + e_4), (d_5), (d_6)$

Such that

$$-X_1 + 3X_2 + d_1 - e_1 = 21$$

$$X_1 + 3X_2 + d_2 - e_2 = 27$$

$$4X_1 + 3X_2 + d_3 - e_3 = 45$$

$$3X_1 + X_2 + d_4 - e_4 = 30$$

$$Y_1 \ (\bar{X}) : 2X_1 + X_2 + d_5 - e_5 = 40$$

$$Y_2 \ (\bar{X}) : -X_1 + 2X_2 + d_6 - e_6 = 20$$

$$\bar{x} , \bar{n}, \bar{p} - \bar{0}$$

As can be seen all rigid constraints are treated in the same manner as the objectives i.e negative and positive deviation variables have been added. Since priority one is associated with the satisfaction of these rigid constraints finding a feasible solution by minimizing P_1 , P_2 , P_3 and P_4 becomes priority number one.

The solution to the problem is

$\bar{X} = (9,3)$

$a = (0, 19, 23)$

Both weighted linear goal programming and lexicographic goal programming have advantages and disadvantages. The former has one major advantage, namely that no special goal programming programme is required since it can be treated like a conventional linear programming problem, and solved using the simplex method. Its major disadvantage is that finding suitable weights for the achievement function can be a very difficult problem. This problem , as pointed out earlier, can be partially overcome using the 'policy capturing method'.

For some purposes (or people) lexicographic goal programming is considered preferable because it is not based on weights, and therefore it places very few demands on the decision maker. However a major disadvantage of the technique is that a specific goal programming programme, based on the multiphase simplex method or the sequential linear goal programming method is required. This, in practice, can be an important limitation since this specialised type of programme is difficult to obtain and/or develop as demonstrated by my experience with the National Workers Housing Fund.

For the proposed Housing Goal Programming model which is presented in the following section it was decided to adopt the Weighted Goal Programming approach for the reasons given above and also because it was considered more suitable for the specific problem to what the research is directed in Mexico City.

7.1.4 Goal Programming Applications

Goal Programming is one of the available techniques developed to deal with multi-objective programming problems*. As pointed out earlier it was developed in the early sixties by CHARNES and COOPER. Since then goal programming has been applied in several fields particularly in the business management area (LEE, 1972). Not many goal programming models have been developed to deal with multiobjective programming problems in the area of urban planning and more specifically for the design of public housing policies. Some authors have argued that this has been so due to the fact that goal programming and similar methods such as linear programming are unsuited for local authority application. With regard to this point GEARY (1978) observes:

'Goal programming and weighting seem eminently unsuited for local authority application since they require information which is unlikely to be available and which more over may be inconceivable. How can relative weights be assigned to such diverse activities as house building and school construction? How can goals be set for home help provision in a way which differs from a simple maximize provision objective?'

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*For a review of multiobjectives programming techniques see COHON and MARKS (1975) and LOUCKS (1975).

However such a generalisation is highly misleading for two reasons. Firstly not all goal programming models have to be designed to deal with the problem of allocating resources among activities as diverse as housing and education. There are often situations where the problem is one of allocating resources to one single activitie e.q. housing, but with the possibility of aiming at two or more conflicting objectives simultaneously. In other words the suitability of goal programming depends very much on the type of problem that is being considered. Secondly, goal programming is not a 'one shot' operation since several weights can be used to test the implications and then generate a set of feasible - desirable solutions. These in turn can be presented to the decision taker to explore his or her preferences. The policy capturing method can be very helpful in this respect.

In short the real problem is due to the fact that not many planners are aware of the potential uses of goal programming, and therefore little time and effort has been devoted to the development of suitable goal programming urban models.

7.2. DEVELOPMENT OF THE MONOCENTRIC CASE GOAL PROGRAMMING MODEL.

The previous sections have outlined the essentials of the goal programming approach, have pointed to its potential in the field of urban planning, and have also suggested that it is an underutilised tool largely because of limitations of knowledge and understanding of the technique on the part of urban planners.

This section describes the development of a practical application of goal programming in Mexico City developed with the cooperation of INFONAVIT. It will be recalled that one of the prime functions of the technique is to encourage decision makers to explore their preference functions relating to the achievement of multiple goals. However, the actual development of the model was the sole responsability of the author, as are the conclusions drawn from the use of the model.

For any public housing agency the need for an 'optimal' use of scarce resources is easy to understand. A rather more difficult concept to define is the concept of 'optimal' itself. Optimal for whom? For potential house owners? For those who have a political obligation to some sectors of the community? For society as a whole? These are questions difficult to answer not only because they imply value judgements but also because there are not clear cut methods for measuring these alternatives.

When INFONAVIT was first approached to develop the goal programming model similar questions arose, and many answers emerged, during the objectives identification process. After several working sessions it was suggested that potential house owner's priorities should be incorporated as objectives in the goal programming model, and the means which the Institute has available to satisfy such priorities, as constraints.

What are then the priorities of the families that may occupy a house built with INFONAVIT resources? According to the economic model of residential location presented in Chapter 4 households are utility maximizers subject to income and time constraints. This principle can be taken to mean that what low income families seek is to be able to afford the cost of a housing unit and to live close to their work place, in order to save time and money. However a clear conflict exists between these two objectives, for living close to the work place in a city where employment is centralized means spending more on housing albeit less on transport. This is why the model is called the housing - accessibility trade-off model.

From this we can derive two important conflicting policy objectives. To minimize 'aggregate housing subsidies' and to minimize ' aggregate transport costs'. These objectives can be formally stated as

$$\text{Goal 1} \quad \sum_{i}\sum_{j}\sum_{t} (h_{ijt} - c_{ut})\, x_{ijt} + d_1 - e_1 = T_1$$

$$\text{Goal 2} \quad \sum_{u}\sum_{j}\sum_{t} t_{cujt}\, x_{ijt} + d_2 - e_2 = T_2$$

Where:

h_{ijt} is the cost of a house type i in zone j in period t.

C_{ut} is the credit capacity or paying ability of family u in period t.

d_1 corresponds to the negative deviation variable associated with goal one (the subsidy minimization goal).

e is the positive deviation variable associated with goal one.

tc is the generalized transport cost of family u living in zone j in period t.

d_2 is the negative deviation variable associated with goal two (the transport minimization goal).

e is the positive deviation variable associated with goal two.

T_1 is the achievement level of goal 1.

T_2 is the achievement level of goal 2.

x_{ijt} is the number of houses of type i in zone j in period t given by the model after an optimal solution is found.

In words, Goal 1 seeks to minimize the difference between what families are able to pay and what they have to pay for a house type i in zone j built by the Housing Institute in period t.

On the other hand Goal 2 seeks to minimize the amount of time and money that a family type u would have to spend if it was located in zone j in period t and had to travel for work in the city centre.

The next step was to ascertain INFONAVIT's main constraints. There are two basic constraints, land and capital. Finding suitable land plots is not easy in Mexico City given that either they are very expensive or simply because they are not on sale. Large plots of land are typically available in the city's periphery, most of them are not urbanized and connected to the city (if connected) by a poor transport system. The other important constraint is capital. This is a universal constraint since there is no housing institution in the world which does not suffer from a lack of sufficient capital funds to carry out its programmes. The inclusion of this constraint therefore needs little if any justification.

Finally it was necessary to include demand constraints. Given that INFONAVIT grants credits to workers whose daily incomes range from 1 to 4 times the minimum wage it was thought necessary to include a set of demand constraints corresponding to each income group.

We can now formulate the constraints as follows:

Land constraint $$\sum_i \sum_j \sum_t l_{ijt} \; x_{ijt} \leq L_{jt}$$

Capital constraint $$\sum_i \sum_j \sum_t h_{ijt} \; x_{ijt} \leq K_t$$

Demand constraint $$\sum_i \sum_j \sum_t x_{ijt} \leq F_{ut}$$

Where:

l_{ijf} is the amount of land taken up by house type i in zone j in period t.

L_{jt} is the amount of land available in zone j in time t.

K_t corresponds to money capital resources, available in period t.

F_{ut} is the number of houses that should be built for family type u in period t.

Having identified goals and constraints we can now state the achievement function. Since what we seek is to minimize the value of the deviation variables we have that

$$MIN \ Z = d_1 + e_1 + d_2 + e_2$$

$$MIN \ Z = \alpha_2 (d_1 + e_1) + \alpha_2 (d_2 + e_2)$$

Now this formulation implies that both goals are of the type $f_i(\bar{x}) = T_i$. However this does not have to be less than or equal to a given aspiration level and also the transport cost minimization goal to be less than or equal to a given aspiration level then the correct formulation would be:

$$MIN \ Z = e_1 + e_2$$

Where e_1 and e_2 are the positive overachievement deviation variables of Goal 1 and Goal 2. (α_1 and α_2 are the weights given to each goal).

7.2.1. Summary

Given the above formulation of the model it should now be possible to evaluate the trade offs implied by a particular housing strategy. For example, the trade - off that exists between a housing strategy based on housing low income families in areas with good employment accessibility and low transport costs which at the same time implies heavier subsidies due to higher housing costs against the alternative strategy of housing them in peripheral locations(where costs are higher but housing cost are lower).

Another interesting aspect of the housing problem that can be investigated are the implications of housing mainly families earning 1 to 1.5 times the minimum wage who demand heavier subsidies against housing families earning 3 to 4 times the minimum wage that require little or no subsidy.

The model can also be used as a budget programming tool. By letting both land and capital constraints be equal to or greater than zero and by specifying demand restrictions about the type and number of families to be housed, we can explore what the land and capital requirements would be. A number of feasible strategies can be generated and presented to the decision maker allowing him to see clearly the implication of each housing strategy with respect to capital and land requirements as well as the transport cost and subsidy implications.

7.3. DEVELOPMENT OF THE POLYCENTRIC CASE GOAL PROGRAMMING MODEL.

The polycentric version of the proposed housing goal programming model was developed to show that the model can be easily modified if the assumption of monocentricity i.e. employment concentration in one single point, is not sustainable and also when other alternatives such as redevelopment and rehabilitation are incorporated. This second version is conceptually similar to version one in that two goals, a housing and a transport goal are identified.

Before discussing the model it would be convenient to define some of the terms we shall be using in order to avoid confusion.

1. Potential Areas for Residential Development are those areas which the planning authority considers have characteristics favourable for the development of low cost residential developments.

2. Target Areas are those areas occupied by squatters of groups of low income households which the planning authority is considering upgrading or redeveloping.

3. Employment Centers These are well defined areas where employment concentrates. A minimum size can be imposed upon these areas to simplify the problem. The size of an employment centre is measurable in either floor space or number of people employed.

The Trade-off between Transport Costs and Housing Costs

There are assumed to be a number of potential areas for residential development. There are as well a number of employment centres each having a different level of attractiveness determining the probability of movement of the household head from the potential area to the employment centre. Those areas close to employment centres with a high attractiveness will offer the greatest opportunity to save on transport costs. The model will, therefore, allocate as many dwellings as possible in those areas in order to meet the transport minimization objective. Furthermore since it is assumed that dwellings differ in their per unit consumption of land and building costs the model will allocate those dwelling types which consume less of both resources (land and capital) into the best locations. On average densities will tend to be higher in those areas where the locational advantage is greater and relatively lower in those areas where the locational advantage is small.

In a target area the model will compare the cost of revedelopment against the cost of improvement. It will also consider the possible savings in transportation which might be obtained if by redeveloping, a greater number of dwellings than the existing number could be "fit" into an area with an important locational advantage.

One of the objectives is to minimize costs. Since total costs increase directly with the cost of land which to a large extent is a direct function of its proximity to employment centres, it can be predicted that housing costs will be greater in those areas where the locational advantage is greater. In order to meet the cost minimization objective the model will allocate the maximum number of cheap dwellings i.e. those with a low construction /improvement cost, in those areas where land is cheapest. This allocation criteria conflicts with the transport minimization criteria and therefore a trade-off is created between cost minimization and transport cost minimization objectives.

The allocation of capital and land resources can be said to be efficient when "aggregate transport costs" can only be reduced at the cost of increasing "aggregate housing costs" and viceversa.

7.3.1. The Full Model

We can now express the goal programming problem as follows.

$$\text{MIN } Z = \alpha_1 (d_1 + e_1) + \alpha_2 (d_2 + e_2)$$

$$\sum B_{ij} X_{ij} + \sum R_{ir} X_{ir} + d_1 - e_1 = C$$

$$\sum W_i X_{ij} + \sum W_i X_{ij} + d_2 - e_2 = T$$

$$\sum l_j X_{ij} + \sum l_r X_{ir} = L_i$$

$$X_{ij} = D_{ij}$$

$$X_{ir} \geqq D_{ir}$$

Where:

B_{ij} = building cost of dwelling type j in (potential or target) area i.

R_{ir} = rehabilitation cost of dwelling type r in (target) area i.

W_i = transport cost index of (potential or target) area i.

l_j = land requirement of dwelling type j.

l_r = land requirement of dwelling type r (dwellings of target areas that can be rehabilitated).

d_1 = negative deviation variable of building cost minimization goal.

e_1 = positive deviation variable of building cost minimization.

d_2 = negative deviation variable of transport cost minimization.

e_2 = positive deviation variable of transport cost minimization goal.

C = achievement level of building cost minimization goal.

T = achievement level of transport cost minimization goal.

L_i = land available in (potential or target) area i.

D_{ij} = demand for dwellings type j in area i.

D_{ir} = number of dwellings type r to be rehabilitated in (target) area i.

α_1 = weight given to building minimization goal.

α_2 = weight given to transport minimization goal.

X_{ij} = number of new units type j allocated by the model in (potential or target) area i.

X_{ir} = number of units rehabilitated in target area i.

W_i is a transport cost index which is estimated as follows. It is assumed that the probability of a resident (the household head) of area i having a job in employment centre j and therefore commuting to that point is given by:

$$P_{ij} = \frac{E_j \quad t_{ij}^{-\lambda}}{\sum_{j=1}^{n} E_j \quad t_{ij}^{-\lambda}}$$

Where:

P_{ij} = probability of movement by a household head from i to j.

E_j = employment in employment centre j.

t_{ij} = travel time from i to j.

λ = constrant derived on calibration.

Thus if there were N households living in area i; N (P_{ij}) individuals would commute from i to j. If we assume that the cost per trip is c then the total cost of transportation generated from area i would be:

$$TC_i = NP_{ij} \times C_{ij}$$

$$= N (P_{ij} \times C_{ij})$$

Where: $P_{ij} \times C_{ij} = W_i$

7.4. SENSITIVITY ANALYSIS

It would be a mistake to think that the solution generated by the model is really an optimum one in the wide sense of the word and that the decision taker should necessarily adopt a policy based on the model's results. A model is a simplification of the real world and therefore the quality of the information generated is only as good as the information and assumptions with which we "feed" the model. Perhaps the model's greatest advantage is not to produce an optimum solution but to enable the policy maker to explore the implications of different housing strategies.

Several strategies can be tested in order to explore their effects on the model's optimum solution. A strategy is defined by a set of constraints and parameter values.

There are a nomber of parameter values we can change to test their effect on the model's optimum solution. These include:

1. The coefficients in the achievement function.
2. The coefficients in the goal constraints.
3. The coefficients in the resource constraints.
4. The availability of land in each area.
5. The availability of capital finance.

Change in the coefficients of the achievement function would mean that we want to know what would be the effect on the model's optimum solution if the decision taker's views regarding the importance of each one of the goals changed.

Changes in the coefficients of the transport minimization goal constraint would be useful in order to see what would happen to the model's optimum solution if the cost imposed on trips were modified. Alternatively we could change the relative attractiveness of employment centres by varying the size of employment centres.

This would provide the policy maker with some useful information regarding the possible effects of employment decentralization on transport cost and the composition and spatial distribution of the housing stock.

We can also test the sensitivity of the optimum solution to changes in the coefficients of the land constraints. So, for example, we can explore the implications of changes in the density ratios allowed in each area. Additionally we can explore the implications of increasing or decreasing building costs and /or rehabilitation costs.

Finally one can test the sensitivity of the optimum solution to changes in the availability of land in each area, and to changes in the housing budget.

7.5. ASSUMPTIONS AND LIMITATIONS

In the polycentric version for the estimation of the transport index it is assumed that for each dwelling there is a household and that every household has a household head who generates a number of work trips per period which is the same for all heads of household. The only difference is the cost of the trips which is assumed to be a function of the distance travelled and the route used. Thus if household heads A and B generate 300 work trips a year, but A travels a distance greater than the distance B travels, A's cost of transportation is also greater. If both travel the same distance but use different routes the cost of transportation may be different. Further it is assumed that all household heads use the same mode of transportation work.

This limitation however is easy to eliminate if household types are introduced, as in the monocentric version. By doing so we can specify both trip probabilities and trip costs between two points for a given household type. For example:

$$P_{ij}^{h} = \frac{E_j^{h} \quad t_{ijh}^{-\lambda}}{\sum_{j=1}^{h} E_j^{h} \quad t_{ijh}^{-\lambda}}$$

Where:

P_{ij}^{h} = probability of movement by a household type h from i to j

E_j^{h} = employment type h in employment centre j

t_{ij}^{h} = travel time of household h from i to j

and the trip cost would be

$$TC_i^{h} = NP_{ij}^{h} \quad C_{ij}^{h}$$

TC_i^{h} = is the cost of transportation of household type h

C_{ij}^{h} = is the cost per trip of household type h

This may seem a rather strong assumption, however, considering that we are talking about a fairly homogeneous socio - economic group i.e. low income families, the assumption is not so unreal.

A greater problem on the transport front is the prediction of the distribution of work trips which is carried out using a gravity model. Unfortunately there is very little experience in developing countries regarding this technique. However, the experience of developed countries indicates that in many, if not most cases the results are more than adequate for the purposes of modelling exercises.

For the model as a whole the single greatest limitation is probably the assumption of linearity. This means, as it was pointed out in Chapter V, that we are assuming that, say, the average cost of rehabilitation is the same whether 100 or 200 dwellings are rehabilitated. This is not very realistic since generally there are economies of scale which tend to reduce costs. The same points apply to the construction of dwellings.

This drawback, however, is the price one has to pay for the efficiency and relative simplicity of linear programming models, whether we use them in urban planning or in economic planning. Thus input-output models of the economy are widely used in many countries in spite of the limits imposed by linearity because of the very useful information yielded by such models for policy makers.

CHAPTER 8

RESULTS OF THE HOUSING GOAL PROGRAMMING MODEL

This Chapter presents the results obtained after the model was tested for both the monocentric and the polycentric case. As pointed out in chapter 7 for the monocentric case actual data from Mexico City was used. The assumption of monocentricity is supported by the evidence presented in Chapter 4 concerning the relative high concentration of employment in the central area of Mexico City and also the existence of exponentially decreasing land values and population densities with distance from the city centre. It should be pointed out that the data on which such evidence is based is not recent since it refers to the 1970 period. This however does not invalidates the assumption of monocentricity since other more recent - albeit - preliminary - investigations have revealed that over the last decade employment concentration in the city centre (specially service employment) has increased. (Plan de Desarrollo Urbano del Distrito Federal 1980, p.49)

The point about monocentricity is important because it implies that the cost of transportation depends on distance and commuting time from some point within the urban area to the city centre, where most work trips are supposed to end. Something that must be understood is that monocentricity represents, from a computational point of view, the simplest possible case since there is no problem concerning the distribution of work trips among a set of possible work places, given that there is only one major work place i.e. the CBD.

The interest in developing a polycentric-case-version steams from the need to show that when employment is scattered throughout the urban area the model does not have to be altered conceptually. What it requires is a more elaborated method for the estimation of transport costs, which depend on the pattern of work trips. The use of a gravity model is proposed for solving this problem, although other methods may well be used to estimate the distribution of work trips. The important point is that even in this case (polycentricity) the trade-off between work accessibility and housing costs can still be assessed.

Now it could be argued that since in Mexico City monocentricity is what actually holds the development of a polycentric-case-version lacks relevance from a policy point of view. This however, is not the case since the polycentric-case-version could be very useful in exploring the implications of proposed urban plans. For example the effects on transport cost of promoting the development of additional major employment centres in different points of the urban area might be assessed. In point of fact an essential strategic element of the Urban Plan of Mexico City recently published (1980) is based on the promotion of economic subcentres in different parts of the city in order to desconcentrate employment from the central area . (Plan de Desarrollo Urbano del D.F. 1980)

Another important extensions included in the polycentric-case-version are the possibilities of redeveloping and or rehabilitating run down areas. Again this is just an extension of the monocentric-case-version. These two alternatives were not included in that version simply because INFONAVIT the housing agency which provides the institutional context, is not currently involved in neither of these activities. However it would not alter the central idea of the model to include them although it may well require some changes in its present formulation.

One major advantage of keeping separate (into two goals) transport and housing costs is that decision takers can visualize more easily the implications that one particular housing strategy can have upon these two components of housing . Thus if work accessibility is considered of no importance or having a secundary importance both the achievement level in the transport goal and the weights associated to the transport goal in the achievement function (see Chapter 7) can be set so as to represent the unimportance of this aspect of the problem. Conversely the transportation problem may be given the greatest importance by imposing a certain level of achievement and penalizing with a high value (or cost) any deviations from the specified level in the achievement function.

8.1 Evaluation of housing strategies: The Monocentric case .

8.1.1. Summary of Conditions

The results obtained from the model will be presented in the following section. These results are derived from a range of strategies each of which is defined in terms of a set of basic conditions.

Before discussing at lenght the results obtained the conditions defining each strategy will be outlined presently.

For the monocentric case -Mexico City case- six strategies were analysed. These are based on the conditions summarised in table 8.1. The meaning of these conditions are summarised verbally in table 8.2.

As indicated in Chapter 7, there are two goals in the model which are considered to be conflicting in nature: a transport goal, which seeks to achieve as close as possible a given aspiration level with respect to the cost of transportation i.e. what the potential house owners are required to spend on transportation over the planning horizon (5 years), and a housing or subsidy goal seeking to achieve a given aspiration level with respect to the amount paid in subsidies by the housing agency in this case INFONAVIT (National Workers Housing Fund).

Basically what is done for the generation of these six strategies is to change in each case the priority of the subsidy and transport goals under different demand conditions. The amounts of Land and capital in the corresponding constraints are not modified throughout the exercise.

In the case of land, it is assumed that no more than 50 hectares of Land can be taken up in the intermediate zone, for housing development.

With respect to money capital, it is assummed that what policy makers are interested in is in satisfying a given demand (that of the applicants asking for credit from INFONAVIT) and checking whether this demand may or may not be satisfied considering the amount of capital funds available. Thus in order for the solution not to be constrained by the amount of capital this is assumed limitless(≥ 0), leaving the model to determine the optimum required.

The assumptions concerning Land and capital are later on, in a sensitivity analysis excersise changed , so as to investigate the resulting implications.

Thus the interest in running the model derives from the possibility of exploring through six strategies , the "best" possible way of housing a number of low income families so that the difference between the <u>actual</u> cost of building the houses and what the benefited families are able to pay is kept within the limits of a predetermined level, and that what families will have to pay on transportation to work will also be kept within certain limits i.e. close to the aspiration level set for the transportation goal.

By introducing Land and capital constraints additional to demand constraints, the "feasability area" is reduced and therefore the possibilities of fully satisfying both goals become even more difficult.

This however does not mean that a goal will always have to be underachieved . As pointed out in chapter 6 a goal may very well be overachieved , since any deviation from the aspiration level negative or positive may be required for the solution to be an optimum one. For instance a solution for the model can be such that the resulting cost of transportation is less than that considered to be acceptable. By the nature of the model this means that the transport goal has been overachieved given that a positive deviation from the desired level exists.

Before discussing the results of the strategies developed for the monocentric case , it is important to explain with some detail the sort of input data that is used to feed the model. This is done in the following section.

8.1.2. Data Inputs for the Monocentric Case

The required data to operate the model can be classified as follows:

a) Transportation cost data

b) Housing cost data

c) Household credit capacity

d) Number of Areas, land available, and density ratios.

e) Aspiration level of subsidy goal

f) Aspiration level of transport goal

g) Housing demands by household type

h) Capital available for housing construction

i) Number of periods

j) Weights in the achievement function

a) T r a n s p o r t a t i o n

By definition transport costs are a function of distance to the city centre, where a high proportion of total employment is assumed to be concentrated. The cost of transportation of family type u in period t is computed as follows.

$$TC_{ut} = dc_t + Tc_t$$

Where dc_t = direct cost which is estimated as follows

dc_t = bus fare in period t from point i to the city centre X number of estimated yearly work trips X5

and

T_{c_t} = time cost, which is estimated as follows

Tc_t = average time spent on journey to work from point i to the city centre in period t X the workers average wage X number of estimated yearly work trips X 5.

The actual transportation data used for this case is contained in Annex I.

b) House Types and Housing Costs

The National Housing Workers Fund allows the construction of the following types of houses: single two storey houses, two storey flats, three storey flats, four storey flats, and five storey flats. However the most typical types that are built are single houses, three storey flats and five storey flats. These were the types considered.

Housing costs are dependent on direct and indirect costs. Direct costs are dependent on urbanization or development cost plus building costs, plus land costs. Indirect costs are mainly INFONAVIT administrative costs.

To estimate the cost of a housing unit at different locations , its cost at a peripheral location was estimated first. Then its cost was increased using the rent gradient presented in chapter 4 to determine the cost of the same housing unit at intermediate and central locations.

c) Household Types and Household's credit capacity

Infonavit's policy is to grant credits to households earning between 1 to 4 times the minimum wage. For this example we considered three household types:

Type 1 earning 1-2 times the minimum wage

Type 2 earning 2.1 - 3 times the minimim wage

Type 3 earning 3.1 - 4 times the minimum wage

The credit capacity of each one of these income groups is computed as follows:

$$CC_{ut} = DMW_{ut} \times 30 \times 12 \times 5$$

Where : DMW is the daily minimum wage of family type u in period t.

d) Number of areas, land available in each area, and density ratios.

For this example it was assumed that development could take place in three different areas :

i A central area located at a distance of 2 Km from the city centre ii An intermediate area located at a distance of 8 Km from the city centre

iii A peripherical area located at a distance of 16 Km or more from the city centre

Three density levels were considered one for each house type :

Two storey single housing unit = 50 units/10,000 m^2

Three storey flats = 60 units/10,000 m^2

Five storey flats = 70 units/10,000 m^2

e) Aspiration level of Subsidy minimization Goal

An achievement level of '0' was set, implying that INFONAVIT should not pay subsidies.

f) Aspiration level of transport cost minimization Goal

Here also the achievement level was set equal to '0' implying that families should spend as little as possible on transportation to work.

g) Housing demands by Household type

As pointed out earlier three income types of households are considered.

It is also assumed that within each household type two subgroups of households may be identified.

i Those with strong locational accessibility needs

ii Those with strong space needs

In the first group we can count young couples without children, old couples , etc. In the second group we can find middle age couples with children, etc.

Thus we can establish demands not only for each income group but also for each one of the subgroups mentioned.

h) Capital available for Housing Construction

As with the amount of land available in each area it was initially assumed that the amount of capital available for housing construction was ≥ 0 in order to allow the model to indicate the optimum required. In a further run a different capital amount was assumed to test the implications.

i) Number of Periods

For this example only three periods were considered. Housing costs, transportation costs, and household's credit capacities are different in each period.

TABLE 8.1. CONDITIONS SET FOR STRATEGIES

C O N D I T I O N S	STRATEGIES S I	S II	S III	S IV	S V	S VI
1.- Weights given to transportation deviation variables in the achievement function.	$_1 = 1$	$_1 = 10$	$_1 = 1$	$_1 = 10$	$_1 = 10$	$_1 = 1$
2.- Weights given to subsidy deviation variables in the achievement function.	$_2 = 10$	$_2 = 10$	$_2 = 10$	$_2 = 1$	$_2 = 1$	$_2 = 10$
3.- Aspiration level of subsidy goal.	$T_1 = 0$	Same as S - I	Same as S - I	Same as S - III	Same as S - IV	Same as S - V
4.- Aspiration level of transport goal .	$T_L = 0$	"	"	"	"	"
5.- Land available in each area and time period. 5.1.- AREA 1 t_1 t_2 t_3	$L_{1,1} \geq 0$ $L_{2,1} \geq 0$ $L_{1,3} \geq 0$	"	"	"	"	"

TABLE 8.1. (Continue)

	S I	S II	S III	S IV	S V	S VI
5.2.- AREA 2 t_1 t_2 t_3	$L_{2,1} \leq 50$ $L_{2,2} \leq 50$ $L_{2,3} \leq 50$	Same as S - I	Same as S - I	Same as S - III	Same as S - IV	Same as S - V
5.3.- AREA 3 t_1 t_2 t_3	$K_1 \geq 0$ $K_2 \geq 0$ $K_3 \geq 0$	"	"	"	"	"
6.- Demands by household type in each period. 6.1.- TYPE 1 (earning between 3.1-4 times the minimum wage) t_1 t_2 t_3	$D_{1,1} \geq 0$ $D_{1,2} \geq 0$ $D_{1,3} \geq 10500$	"	≥ 0 ≥ 0 $= 2625$	"	≥ 0 ≥ 0 ≥ 2625	"

TABLE 8.1. (Continue)

	S I	S II	S III	S IV	S V	S VI
6.1.1.- TYPE 14 (with strong accesibility preferences)		Same as S - I		Same as S - III		Same as S - V
t_1	$D^A_{11} \geq 0$		≥ 0		≥ 0	
t_2	$D^A_{12} \geq 0$		≥ 0		≥ 0	
t_3	$D^A_{13} \geq 2_1100$		$= 525$		≥ 525	
6.1.2.- TYPE 18 (with strong space preferences)		"		"		"
t_1	$D^B_{11} \geq 0$		≥ 0		≥ 0	
t_2	$D^B_{12} \geq 0$		≥ 0		≥ 0	
t_3	$D^B_{13} \geq 2,100$		$= 525$		≥ 525	
6.2.- TYPE 2 (earning between 2.1→3 times the minimum wage)		"		"		"
t_1	$D_{2,1} \geq 0$		≥ 0		≥ 0	
t_2	$D_{2,2} \geq 0$		≥ 0		≥ 0	
t_3	$D_{2,3} \geq 2625$		$= 10,500$		≥ 2625	

TABLE 8.1. (Continue)

	S I	S II	S III	S IV	S V	S VI
6.2.1 TYPE 2A (with strong accesibility preferences) t_1 t_2 t_3	$D^A_{21} \geq 0$ $D^A_{22} \geq 0$ $D^A_{23} \geq 525$	Same as S - I	≥ 0 ≥ 0 $= 2,100$	Same as S - III	≥ 0 ≥ 0 ≥ 2625	Same as S - V
6.2.2.- TYPE 2B (with strong space preferences) t_1 t_2 t_3	$D^B_{21} \geq 0$ $D^B_{22} \geq 0$ $D^B_{23} \geq 525$	"	≥ 0 ≥ 0 $= 2,100$	"	≥ 0 ≥ 0 $\geq 2,625$	"
6.3.- TYPE 3 (earning between 3.1 to 4 times the minimum wage) t_1 t_2 t_3	$D_{31} \geq 0$ $D_{32} \geq 0$ $D_{33} \geq 2,625$	"	≥ 0 ≥ 0 $= 2,625$	"	≥ 0 ≥ 0 $= 10,500$	"

TABLE 8.1. (Continue)

	S I	S II	S III	S IV	S V	S VI
6.3.1.- TYPE 3 A (with strong accessibility preferences)		Same as S - I		Same as S - III		Same as S - V
t_1	$D^A_{31} \geq 0$		≥ 0		≥ 0	
t_2	$D^A_{32} \geq 0$		≥ 0		≥ 0	
t_3	$D^A_{33} \geq 525$		= 2,625		= 2,100	
6.3.2.- TYPE 3 B (with strong space preferences)		"		"		"
t_1	$D^B_{31} \geq 0$		≥ 0		≥ 0	
t_2	$D^B_{32} \geq 0$		≥ 0		≥ 0	
t_3	$D^B_{33} \geq 525$		= 2,625		= 2,100	
7.- Capital available in each period		"	Same as S-I		Same as S-I	
t_1	$K_1 \geq 0$					
t_2	$K_2 \geq 0$					
t_3	$K_3 \geq 0$					

TABLE 8.2. SUMMARY OF CONDITIONS SET FOR STRATEGIES

	Transport Goal	Subsidy Goal	Demand	Land	Capital
S - I	Not very important	Very important	All groups are equally benefited	No more than 50 hectares can be taken up in the intermediate area.	The optimum required is determined by the model.
S - II	Very important	Not very important	"	"	"
S-III	Not very important	Very important	Mainly Benefits groups earning between 2.1-3 times the minimum wage.	"	"
S-IV	Very important	Not very important	"	"	"
S - V	"	"	Mainly benefits groups earning between 3.1-4 times the minimum wage.	"	"
S-VI	Not very important	Very important	"	"	"

8.1.3. Results of housing strategies for the monocentric case (Mexico City)

S t r a t e g y I

Under Strategy I, it is assumed that goal 1, the housing goal, is more important to satisfy than goal 2, the transport one. As regards demands it is required that families earning between 3.1 to 4 times the minimum wage should receive 10,500 units from the housing program, of which at least 2,100 should be located in the central area of Mexico City and at least 2,100 should be two storey single houses. By doing this the housing agency is making sure that the demands of those applicants with strong locational needs such as young couples without children and both working, old couples without a car, etc., as well as the demands of those families with strong space needs (middle age couples with children, etc.) are satisfied .

Families earning between 2.1 to 3 times the minimum wage should get at least 2,625 houses, of which 525 should be located in the central area and at least other 525 should be two storey single houses. Similar conditions apply to families earning between 1 to 2 times the minimum wage.

The main implications of Strategy I are presented in table 8.3.

TABLE 8.3 RESULTS OF STRATEGY I
(Monocentric Case)

1. Amount paid in subsidies	0.0
2. Transport cost	$ 2,586,028.8
3. Land required	227
4. Capital required	$ 8,899,161.9

House Types	LOCATION		
	Central	Interme-diate	Peri-pheral
Single Houses			3,150
Three storey flats			
Five storey flats	3,150		9,450

By mostly benefiting families of the upper income group, the housing agency has no need to subsidize housing provision. This however is achieved by locating 12,600 out of the programmed 15,750 at the periphery which gives rise to a cost of transportation of 2,586,028 pesos. The required amount of Land for these developments is 145 hectares.

It appears to be economic to build mostly five storey flats , since it provides the lowest cost alternative for satisfying the demands of the major group being benefited.

It should be remembered that these results were reached under the requirement that decision makers were mostly concerned about achieving the subsidy goal and not so the transport goal.

S t r a t e g y II

For Strategy II all conditions of Strategy I hold except that the weights given to the deviation variables in the achievement function are inverted so that goal 2 (the transport goal) becomes more important than goal 1 .

From the comparison presented in table 8.4 it can be concluded that by giving priority to transportation over subsidies, more than 7.0 million pesos would have to be paid in subsidies to house the same number of families. On the other hand transport costs are reduced by more than 2.0 million pesos, an amount not comparable with the increase in subsidies. Consequently, capital needs increase from 8.9 to 14.3 million pesos to finance this particular scheme. The required amount of Land is the same under both strategies.

TABLE 8.4 COMPARISON OF STRATEGIES I AND II

	STRATEGY I	STRATEGY II
1. Amount paid in subsidies	0.0	7,062,300.0
2. Transport cost	2,586,028.8	360,150.0
3. Land required	227.0	227.0
4. Capital required	8,899,161.9	14,252,279.0

Strategy III

Under this strategy it is assumed that the major family group benefited is that earning between 2.1 to 3 times the minimum wage. It is further assumed that decision makers give greater priority to the subsidy goal than to the transport one.

The results obtained from the strategy are presented in table 8.5. By mostly benefiting this family income group there is a need to pay subsidies, ($939,750). Because of the little importance given to the transport goal the amount spent on transportation by all the benefited families is considerably high, although not as high as in strategy I, since the cost of transportation of the major group benefited is lower, due to a lower wage rate which is used to compute its work trip costs.

TABLE 8.5 RESULTS OF STRATEGY III

1. Amount paid in subsidies	$ 939,750.0
2. Transport cost	$ 2,173,500.0
3. Land required	226.0 hectares
4. Capital required	$ 8,904,104.0

House Types	LOCATION		
	Central	Intermediate	Peripheral
Single Houses	525		3150
Three storey flats			
Five storey flats	2625		9450

The amount of money capital required is almost the same as that required to finance Strategy I, and so the amount of land required (226.0 hectares). The house mix pattern, however, differs in some important way since the construction of 525 single houses in the central area appears to be economic.in this case. In neither case (strategy I and II) though the construction of three storey flats turns out to be economic, since marginal savings on land by the addition of one storey are not enough to compensate the increase in building costs.

S t r a t e g y IV

The only conditions that are changed to generate this strategy are the weights in the achievement function so as to give the transport goal priority over goal 1 (the subsidy goal). Otherwise demand land and capital constraints are the same as that of strategy III.

A comparison between these two strategies is presented below in table 8.6.

TABLE 8.6. COMPARISON OF STRATEGIES III AND IV RESULTS.

	STRATEGY III	STRATEGY IV
1. Amount paid in subsidies	$ 939,750.0.	$ 7,788,375.0
2. Transport Cost	$2,173,500.0	$ 311,325.0
3. Land required	226.0 hectares	227.0 hectares
4. Capital required	$8,904,104.0	$14,252,279.0

Again if the transport goal is given priority over the subsidy goal, the reduction obtained in transport cost is not comparable with the increase in the amount that had to be paid in subsidies.

With respect to the house mix, produced by Strategy IV, nearly all the houses are built in the central area (table 8.7).

TABLE 8.7. HOUSE MIX CORRESPONDING TO STRATEGY IV

	Central	Intermediate	Peripheral
Single Houses	4,725	525	
Three storey flats			
Five storey flats	10,500		

This result is in no way surprising since by given a large weight to the transport goal coeficients of the achievement function, the model allocates as many housing units as possible in the central area in order to keep transport costs as low as possible i.e. close to zero, which is the aspiration level set for this goal.

It is worth that for the solution to be optimum only single houses and five storey flats should be built. Three storey flats are a non economic option, not even at the central area. This of course reflects the cost structure that prevails in Mexico , corresponding to a particular design. It might be possible that by some modifications to the prototype, building cost could be lowered and by so doing make the three-storey-flat solution feasible.

Another thing which may be important pointing out is the fact that the number of single houses located at the intermediate zone is 525 . This clearly indicates that the model is making an allocation which seeks among other things to save on land in the intermediate zone, where an upper limit in the amount of land that can be taken up has been imposed. Later on this condition will be modified to investigate the implications.

S t r a t e g y V

Strategy V is based on a different demand structure. It is now assumed that families earning between 1 and 2 times the minimum wage are the major group benefited by the construction programme.

The implications of this strategy are summarized in table 8.6.

TABLE 8.8 RESULTS OF STRATEGY V

1.	Amount paid in subsidies	$ 1,684,725.0
2.	Transport cost	$ 1,683,675.0
3.	Land required	227.0 hectares
4.	Capital required	$ 8,325,764.9

The adoption of this strategy implies that the amount paid in subsidies by the housing agency (INFONAVIT) would be almost equivalent to what the whole group of families benefited (mostly those with the lowest incomes) would have to pay for transportation to work. This amounts to saying that each peso spent by the families on transportation is saved on housing , since they are not being required to pay the full cost of the houses they are getting from the housing agency. This is a well balanced strategy since both components of housing, transport and the provision of the housing unit itself, are in equilibrium.

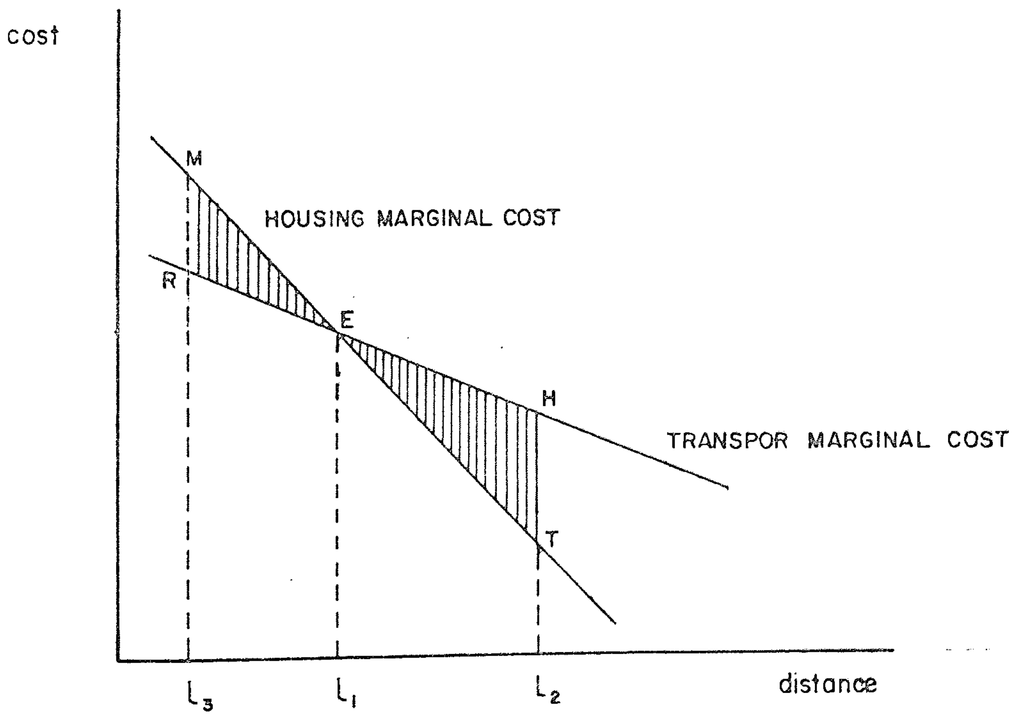
The findings of Strategy V provide a useful guide for the assessment of a subsidy policy. This is that a housing strategy based on the construction of public housing projects at the periphery of the city , could be considered to be justified when the size of the subsidy introduced is at least equivalent for the cost of transportation generated. Put in a different way, low income families will seek optimum locations to live in (close to employment centres). However they will accept suboptimal ones if they receive a compensation in the form of a housing subsidy, equivalent to what they have to pay in transportation to work. Of course the higher the subsidy the better off families will be, but the minimum level of such a subsidy should be given by what families are required to spend on transportation from the suboptimal location.

More generally it can be said that the provision of a housing unit by a public agency at the periphery of the city or at any other non optimal location within the city will be justified when a subsidy is introduced and the size of it is at least equivalent to the difference between the additional cost of transportation and the additional housing cost resulting from moving the household away from its optimal location.

This point can be illustrated using the graphical aparatus presented in chapter 6 to illustrate the trade-off accessibility theory of residential location.

In fig. 8.1 l_1 represents a household's optimum location and l_2 the location at which the household is to be located by a public housing agency. For that household to be as better off as before, a compensation in the form a subsidy should be allowed and the size of that compensation should be at least equivalent to the shaded area ETH. Also if the house was built up at location l_3 the required subsidy should be REM.

FIG. 8.1



Of course this analysis ignores the positive externality that is generated to the household by the provition of a new house healthier and safer than the one it had before. However the actual net effect of this positive externality is difficult to assess since as it was discussed in chapter

4 it is still at issue the extent to which low income families enjoy the benefit of leaving in a public housing unit. Since the acquisition of a public unit generally requires them to spend less on other goods, therefore leaving their level of welfare unchanged, or possibly at a lower level.

The point is then that if the house provided by the housing agency produces a benefit to the household , at least equal to that cost, due to a reduction in its consumption of other goods, then in order to make sure that the household is not made worse-off, when located far from employment centres, the size of the subsidy should be at least equivalent to the cost of transportation generated.

The housing mix produced by Strategy V is presented in Table 8.7. It takes the form of mostly five storey flats & single two storey houses located some (3150) in the central zone and the rest (12,600) at the periphery.

TABLE 8.9. STRATEGY V HOUSE MIX

	LOCATION		
	Central	Intermediate	Peripheral
Single House	525		3,150
Three storey flat			
Five storey flat	2,625		9,450

Strategy VI

As with strategies II and IV it will now be assumed that the main interest on the part of decision takers is the provision of the housing units at locations close to where employment concentrates the central zone. This implies giving the transport goal a heavier weight in the achievement function.

The results are summarized in table 8.9., where they are compared with those of Strategy V.

TABLE 8.9. COMPARISON OF STRATEGIES V AND VI RESULTS

		STRATEGY V	STRATEGY VI
1.	Amount paid in subsidies	$ 1,684,725.0	$ 7,706,475.0
2.	Transport Cost	$ 1,683,675.0	$ 345,975.0
3.	Land required	227.0 ha.	227.0 ha.
4.	Capital reouired	$ 6,325,764.0	$13,702,604.0

As in previous cases the strategy of housing families very close to the city centre where employment opportunities are greatest, is a very costly one indeed. It implies paying higher subsidies which in turn calls for greater capital outlays. Transport costs are reduced nearly by 80%, however in absolute terms such as reduction does not compensate the amount that would have to be paid in subsidies, since these increase by 6.02 million pesos , compared with the transport cost reduction of 1.3 million pesos.

8.1.4. Sensitivity of the solution to changes in the weights of the achievement function.

In the previous section six different strategies were analysed to investigate the implications that a particular housing scheme would have if such a scheme was to benefit mainly very low income families or alternatively not so low income families; and on the location front, to ascertain the trade-offs involved when housing is provided at locations where employment accessibility is high against the case when it is provided (at a lower subsidy cost) where employment accessibility is low(as measured by a high cost of transportation). This two possibilities are extreme. In fact a weight of 10 was used in the achievement function to give priority to goal 1 first (subsidy goal) and then to goal 2 (the transport goal).

The purpose in this section is to test different weight values in the achievement function to see how sensitive the model's solution can be to such changes. This are not new strategies since the problem of benefiting one family group instead of any other is no longer of interest nor it is the housing mix or the resulting locational pattern. The interest is simply to determine the model's sensitivity to changes in the parameters and in particular those associated with the achievement function. To carry out the excersise the conditions set for strategy V will be taken as the relevant ones.

As pointed out in chapter 7 it is common when dealing with goal programming models to find a problem called dominance. This implies that one of the goals that are being considered, dominates the others in the sense that the model's solution depends on the dominant goal. This problem is eliminated by the use of weights to give the remaining goals a competitive role. The problem is that the solution may be highly sensitive to the weight structure used. Therefore it is neccesary to carry out a sensitivity analysis to ascertain which goal dominates and how sensitive the solution may be to the use of certain weights.

The first step was to make the achievement function weights equal in order to determine goal dominance. By comparing the results obtained with those of strategy V where it was assumed that goal 1 was 10 times more important than goal 2, we ascertain whether, and if so, to what extent the results were similar. This comparison enabled us to determine goal dominance, as it showed that goal 1 dominates goal 2, since the obtained results (transport cost and the subsidy) were similar to those obtained when goal 1 was given a weight of 10, in strategy V.

The second step was then to test the impact of assuming higher weights for the non dominant goal (transport) in the achievement function. A summary of the obtained results is presented in table 8.10.

TABLE 8.10. SENSITIVITY ANALYSIS (ACHIEVEMENT FUNCTION'S WEIGHTS)

RUN	WEIGHT GIVEN TO GOAL 1 (SUBSIDY)	WEIGHT GIVEN TO GOAL 2 (TRANSPORT)	SUBSIDY (PESOS)	TRANSPORT COST (PESOS)
1	1	1	1,684,725	1,683,675
2	1	1.5	1,684,725	1,683,675
3	1	2.5	1,684,725	1,683,675
4	1	4.5	3,559,767	978,065
5	1	5.5	7,706,475	345,975
6	1	10.0	7,706,475	345,975

The conclusion of this analysis is that when a weight of 4.5 is given to the transport goal , a sort of balance between the two goals is achieved since the amount paid in subsidies is not as low as when the subsidy goal dominates , but neither as high as when the transport goal dominates. This is confirmed by the fact that in all cases, except run 4, the amount of subsidies and the cost of transportation are exactly the same as those obtained for strategies V (runs 1, 2, and 3) and for strategy VI (runs 5 and 6).

From a policy point of view this is a relevant finding since it provides insights into what can be regarded as a compromise strategy i.e. one for which equal importance is given by the decision takers to both the transport and subsidy goals.

The implication resulting from this compromise strategy in terms of the house mix and the locational pattern can be appreciated in table 8.11.

TABLE 8.11. HOUSE MIX AND LOCATIONAL PATTERN COMPROMISE STRATEGY.

HOUSE TYPE	L O C A T I O N		
	Central	Intermediate	Peripheral
Two storey houses	525		2,625
Three storey flats			
Five storey flats	4,725	3,846	4,029

Never before (strategies I through VI) the construction of either houses or flats in the intermediate zone had turned out to be recomendable . In this case, however, it is an acceptable possibility since by making both goals equaly important, decision makers are willing to accept a certain level of subsidization and transport cost.

8.1.5. Sensitivity Analysis : Changes in capital and land constraints

Until now it has been assumed, that the amount of land (in the central and peripheral zones) and the amount of capital required to achieve the construction of specific house types is an output information provided by the model once an optimum solution has been found. Here the effects of setting specific upper land and capital constraints will be investigated. The new conditions are given below (table 8.12.)

TABLE 8.12. SENSITIVITY ANALYSIS: MODIFIED CAPITAL AND LAND CONDITIONS.

1. Land Availability		New condition	Previous Condition
Area 1 (central zone)	t_1	$L_{1,1} \leq 5$	≥ 0
	t_2	$L_{2,1} \leq 5$	≥ 0
	t_3	$L_{1,3} \leq 5$	≥ 0
Area 2 (Intermediate zone)	t_1	$L_{2,1} \leq 10$	≤ 50
	t_2	$L_{2,2} \leq 10$	≤ 50
	t_3	$L_{2,3} \leq 10$	≤ 50
2. Capital Availability			
	t_1	$K_1 \leq 2,629,000$	$K_1 \geq 0$
	t_2	$K_2 \leq 3,980,000$	$K_2 \geq 0$
	t_3	$K_3 \leq 7,940,000$	$K_3 \geq 0$

The remaining values such as aspiration levels for both the subsidy goal and the transport goal as well as demand conditions were left as under strategy V. This is because strategy V represents a feasible and desirable housing strategy in that it mostly benefits very low income families (among all those applying for housing credit to INFONAVIT) and the size of the subsidy (granted to all families benefited by the scheme) can be justified in terms of the amount families have to pay in transport to work, over the planning horizon (which is assumed to be of five years), which is equivalent to what the housing agency pays in subsidies (see section 8.1.3). Of course different strategies might be considered to explore the effects of changing land and capital constraints. However it would be combersome to explore the implications derived from a change in capital and land constraints for all the strategies.

The modifications presented in table 8.13. imply that the possibility of building houses and flats in the central and specially the intermediate zone is very restricted. It is also being assumed that there is a given annual budget as indicated by the different amounts of capital-money available at each time period.

As with the six strategies generated previously, the results corresponding to the case where the subsidy goal dominates and when the transport goal dominates, are presented in table 8.14.

TABLE 8.13. RESULTS OF STRATEGY V WITH A RESTRICTED BUDGET AND A LIMIT TO THE AMOUNT LAND THAT CAN BE DEVELOPED.

	Amount of Subsidy	Transport Cost
Goal 1 dominates	2,686,598.8	1,393,764.7
Goal 2 dominates	8,439,275.0	275,575.0

A comparison of these results with those obtained for strategy V (the unrestricted case) is shown in table 8.14.

TABLE 8.14. COMPARISON OF STRATEGY V FOR RESTRICTED AND UNRESTRICTED LAND AND BUDGET

	Condition	Amount of Subsidy (Pesos)	Transport Cost (Pesos)
Subsidy Goal	Unrestricted	1,684,725.0	1,683,675.0
(Goal 1) Dominates	Restricted	1,686,598.8	1,393,764.7
Transport Goal	Unrestricted	7,706,475.0	345,975.0
(Goal 2) Dominates	Restricted	8,439,275.0	275,575.0

Under the restricted situation, where the subsidy goal dominates , two results can be observed. One is that the amount paid in subsidies increases a little and the other that transport cost decreases albeit by a greater proportion. This is due to the fact that a few more housing units are located at the intermediate and central zones, basically because the relative proportions of capital and land available have changed, making land relatively scarce.

Also under the restricted case, where the transport goal dominates, the same result can be observed, than in the unrestricted case, favoring therefore higher density type of developments.

The results of both the restricted and the unrestricted cases are not that different , which indicates that for this particular case the kind of constraints introduced had no much of an impact on the model's optimum solution. Of course many more changes in the constraints could be introduced in order to explore the implications.

8.1.6. Conclusions of monocentric case analysis

a.) Among the three different types of households considered-type 1 (earning 1 to 2 times the minimum wage); type 2 (earning 2.1 to 3 times the minimum wage); and type 3 (earning 3.1 to 4 times the minimum wage) - it is type 1 the only one for which a non subsidized housing program could be undertaken. However transportation costs would be very high in relative terms, since a non subsidized program would imply the location of mostly, five storey flats at the periphery. On the other hand a subsidized program for this type of families would be extremely costly and would be far from being compensated by savings on transportation.

b.) If the housing program was to be chiefly directed to benefit families type 2, it would be necessary a subsidy of at least $ 939,750. , which means locating families at the periphery. Such a subsidy could be as high as $ 7,063,300 if families were located close to the city centre i.e. close to employment. In this case transport cost would be reduced considerably from $2,173,500 to $ 311,325. This however will be far from compensating the increase in the subsidy.

c.) If the housing program is to benefit mainly families earning 1 to 2 times the minimum wage, the subsidy amount increases to $ 1,684,725 and almost by the same amount the cost of transportation i.e. $ 1,683,675. This happens when the housing or subsidy goal is given priority over the transport goal..In the opposite case (when transport has the priority) subsidies increase considerably to $ 7,706,475 although transport cost decreases to $ 345,975. The former case is of interest since the model gives a solution in which what is paid in subsidies is comparable with what families -mostly type 1- have to pay for transportation to work. As pointed out earlier this solution can be considered of guidance to decision takers in the sense that they can argue that when families are located far from employment (as they often are) they benefit not only from receiving a better house , but because they enjoy a subsidy which is equivalent to what they pay for transportation to work.

d.) The results given by the model are to an important extent sensitive to the weights used in the achievement function.

The analysis carried out to test the sensibility of the model's solution to different weights revealed that the housing (subsidy) goal dominates over the transport goal, when equal weights are allocated to both goals. It also revealed that a compromise solution - one in which neither the housing nor the transport goal dominates - is achieved when a weight value of 4.5 is allocated to the transport goal and a value of 1 allocated to the subsidy goal. In this case the amount paid in subsidies by the housing agency is not as high as when the transport goal dominates, and subsidization is not as low as when the housing goal is dominant. The resulting pattern of house types as well as their location also differs with respect to extreme cases in that an important proportion of the required construction takes the form of five storey flats, many of which are located at the intermediate zone.

e.) With regard to land and capital, it appears that when either is increased the solution is not altered in any significant fashion, as long as enough land is available for an optimum allocation of housing units.

f.) Two additional points can be made in relation to both the modelling technique itself and the policy implications of the results obtained. As regards the former point , it can be said that goal programming provides an efficient technique to assess public housing strategies in that the major elements of a public housing program - transport (or work accessibility) and subsidization- can be evaluated separately in relation to both aspiration levels and constraints.

In relation to the results themselves and their relevance for the design of public housing policies it can be said that perhaps for the formulation of a more detailed 'package' of policies it would be necessary to include a greater number of housing types and also to consider the possibility of various and not just one transport mode (bus) which has been assumed.

8.2. The Polycentric Case

The aim in developing a polycentric case is twofold Firstly to show (using hypothetical data) that public housing policies can be assessed by means of a goal programming model when employment is not highly concentrated in the city centre, as it is in Mexico City.

Secondly to show that alternative possibilities such as redevelopment and rehabilitation can be incorporated to the model as an extension of the simplest case when only new construction on undeveloped land can take place.

Conceptually the model is based on the same principle i.e. that a trade-off exists between housing families at locations with a poor accessibility to work places, but offering instead the advantage of low land costs; and the alternative possibility of housing them at locations with good employment accesibility but at a higher cost, due to higher land costs.

The only important difference rests on how transport cost are estimated, since they are dependent on the pattern of work trips, which is something difficult to determine in practice. The approach suggested by the author for dealing with this problem is the use of a gravity type of model so much used in these cases by urban planners.

Before presenting and discussing the implications of the results obtained , a number of points will be made clear in the following section.

8.2.1. Structure of the model

The structure of the polycentric case version of the model has already been described formally in chapter 7. It seems however necessary to state the fact that the polycentric version has been simplified in two aspects. Firstly there is no longer a subsidy goal representing the difference between housing costs and what a group of families are able to pay, but simply a budget goal whose aim is to satisfy as close as possible an aspiration level corresponding to an acceptable level of investment in housing construction. Secondly, time periods are not considered.

As regards the transport goal it remains as in the monocentric case, although the coefficients have (as was explained in chapter 7) a different meaning , since they no longer represent the cost per trip but rather a transport cost index.

8.2.2. Data Inputs for the Polycentric Case

For the polycentric version of the proposed housing goal programming model the following data is required.

a) Transport data

b) Housing building costs, redevelopment costs, and rehabilitation costs

c) Land available and density ratios

d) Aspiration level of housing goal

e) Aspiration level of transport goal

f) Weight for the achievement functions

a) Transportation data

It is assumed that there are two major employment centres and three minor ones located as shown in fig. 1 Annex III. The data concerning the size of employment centres is presented in table 1 annex III.

There are five potential areas and two target areas. The main roads connecting potential areas and target areas with employment centres have links of the same lenght but different travel speeds. Vehicular travel costs are the same on all links.

Table 2 in annex III shows the distance in miles from potential areas and target areas to employment centres. Data regarding the average speed over the 42 links is presented in table 3 of annex III. Table 4 in annex II shows the travel times from potential and target areas to employment centres. These values (travel distances and times) were calculated by adding the link distances and times respectively over a path which is predetermined e.g. a bus route.

To work out the distances we have that

$$D_{ij} = \sum_{a=1}^{42} L_a$$

Where :

D_{ij} = distance over bus route path from target/potential area i, to employment centre j.

L_e = Length of link e. By assumption all links are 1 mile long and to compute travel times.

$$T_{ij} = 60 \sum L_l / S_l$$

Where :

T_{ij} = bus travel time from target/potential area i to employment centre j.

S_l = speed over link l, mph

Travel cost from target/potential area i to employment centre j is calculated as follows.

$$R_{ij} = m\ y \sum_{l=1}^{42} q_l \quad L_l$$

Where :

R_{ij} = travel cost from target/potential area i to employment centre j.

m = number of repetitions of daily trips in a year, which is assumed to be 200.

y = length of planning horizon. It is assumed equal to 20 years.

q_l = cost of travelling by bus over link l.

A gravity model is used to compute the expected proportion of trips from each potential / target area to each employment centre. Finally with all the information generated we compute the transport cost index through the following expression.

$$C_i = \sum_{j=1}^{S} \frac{(E_j T_i^{-X})}{\sum_{j=1} E_j t_{it}^{-X}} R_{ij} \qquad i = 1, 2, \ldots 7$$

Where.

- C_i = transport cost index for target/potential area i
- E_j = size of employment centre j
- t_{ij} = travel time from target/potential area i to employment centre j
- R_{ij} = travel cost from target/potential area i to employment centre j
- X = is a constraint whose value is assumed to be Z.

The travel cost indexes are used as coefficients in the transport minimization goal. As the model allocates dwelling types in each area an overall travel cost is generated given by the product between the number of dwellings allocated (of whatever type) and the transport cost index.

The transport cost index for the seven areas is presented in table 7 annex II.

b) Housing building costs, redevelopment costs and rehabilitation costs.

It is assumed that only the following dwelling types can be built.

two storey houses
three storey flats
four storey flats
eight storey flats
twelve storey flats

Land and building costs of each dwelling type are presented in table 8 and 9 annex II.

c) Land available and density ratios.

Densities are assumed to be :

two - storey houses	11.5 u/acre
three - storey flats	30.0 u/acre
four - storey flats	35.0 u/acre
eight - storey flats	47.0 u/acre
twelve - storey flats	50.0 u/acre

These figures together with the land and building costs are based on STONE (1962), STONE (1960) and NEEDLEMAN (1962).

It is assumed that redevelopment costs equals development cost plus demolition cost. Temporary relocation costs are ignored. (see annex II).

Land availability in each area is assumed to be as follows :

AREA	ACRES OF LAND
Potential Area 1	4
Potential Area 2	3
Potential Area 3	11
Potential Area 4	30
Potential Area 5	10
Target Area 1	5
Target Area 2	24

d) Aspiration level of housing goal. An aspiration level of £2,500,000 was set for this goal.

e) Aspiration level of transport goal. An aspiration level of £200,000 was set for this goal.

f) Weights for the achievement function. Initially weights of 15 for Goal 2 and 1 for Goal 1 were assumed.

8.2.3. Results

What will be presented in this section are the results of eleven runs each of which corresponds to a set of conditions which are summarized in table 8. 15.

TABLE 8.15. SUMMARY OF RUNS

	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6	RUN 7	RUN 8	RUN 9	RUN 10	RUN 11	RUN 12
housing 1)	1	1	1	1	15	10	5	5	15	1	1	15
transport 2)	15	10	5	1	1	1	1	1	1	15	15	1
level of al	2500000	← *	←	←	←	←	←	←	←	←	←	←
level of goal	200000	←	←	←	←	←	←	←	←	←	←	←
ability land)												
al areas 1	≤ 16	←	←	←	←	←	←	←	←	←	←	←
2	≤ 13	←	←	←	←	←	←	←	←	50	13	←
3	≤ 21	←	←	←	←	←	←	←	←	←	←	←
4	≤ 30	←	←	←	←	←	←	←	←	←	←	←
5	≤ 80	←	←	←	←	←	←	100	100	80	←	←
areas 1	≤ 25	←	←	←	←	←	←	←	←	←	←	←
2	≤ 30	←	←	←	←	←	←	←	←	←	←	←
s for abilita- develop- rnet areas												
1	≤ 20	←	←	←	←	←	←	←	←	←	100	←
2	≤ 50	←	←	←	←	←	←	←	←	←	300	←
uction potential	= 2000	←	←	←	←	←	←	←	←	←	←	←

[illegible]plies same figure as in the left.

TABLE 8.15. SUMMARY OF RUNS

	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6	RUN 7	RUN 8	RUN 9	RUN 10	RUN
1. Weight of housing goal (goal 1)	1	1	1	1	15	10	5	5	15	1	1
2. Weight of transport goal (goal 2)	15	10	5	1	1	1	1	1	1	15	15
3. Aspiration level of housing goal	2500000	← *	←	←	←	←	←	←	←	←	←
4. Aspiration level of transport goal	200000	←	←	←	←	←	←	←	←	←	←
5. Land availability (acres of land)											
5.1. Potential areas											
Area 1	≤ 16	←	←	←	←	←	←	←	←	←	←
Area 2	≤ 13	←	←	←	←	←	←	←	←	50	13
Area 3	≤ 21	←	←	←	←	←	←	←	←	←	←
Area 4	≤ 30	←	←	←	←	←	←	←	←	←	←
Area 5	≤ 80	←	←	←	←	←	←	100	100	80	←
5.2. Target areas											
Area 1	≤ 25	←	←	←	←	←	←	←	←	←	←
Area 2	≤ 30	←	←	←	←	←	←	←	←	←	←
6. Lower limits for housing rehabilitation or redevelopment in target areas											
Area 1	≤ 20	←	←	←	←	←	←	←	←	←	10[illegible]
Area 2	≤ 50	←	←	←	←	←	←	←	←	←	30[illegible]
7. DEMAND : New construction desired in potential areas	= 2000	←	←	←	←	←	←	←	←	←	←

(←) * Implies same figure as in the left.

R U N 1

The conditions set for this run imply that the transport goal dominates over the housing goal i.e. the transport goal must be satisfied as closely as possible, even if it implies overachieving or underachieving goal one (the housing goal).

It is also required that al least 20 units in target area 1 and a further 50 in target area 2 should be rehabilitated or if it is economic to do so replaced by new units.

One final requirement is that at least 2000 units should be built in potential areas.

The results of this run are presented in table 8.16.

TABLE 8.16. RESULTS OF RUN 1

Achievement of goal 1	112 %						
Achievement of goal 2	112 %						
HOUSE MIX	L O C A T I O N						
	P1	P2	P3	P4	P5	T1	T2
Two storey houses	33				920		
Three storey flats	397	650					
Four storey flats							
Eight storey flats							
Twelve storey flats							
Units rehabilited						20	50

As can be observed both goals have been overachieved by the same percentage amount i.e. 112 %. The model also indicates that about 50% of all new construction should take the form of two storey houses located in potential area 5 and the rest three storey flats located in potential areas 1 and 2 , which correspond to those areas closest to the major employment centres. It is worth pointing out that all the land available in potential areas 1 and 2 has been taken up which means that if there has been more land available a greater number of flats would have been located there.

In the following six runs the effects of changing the weight values in the achievement function will be examined.

R U N S 2, 3 and 4

Three runs were carried out to test the effects of assuming three different weight values, corresponding to the transport goal in the achievement function. The results are presented in table 8.17.

TABLE 8.17. RESULTS OF RUNS 2, 3 and 4 : LEVELS OF ACHIEVEMENT OF GOALS.

	Housing Goal Weight	Trans Goal Weight	Overachievement of Goals	
			Housing	Transport
RUN 2	1	10	68%	156%
RUN 3	1	5	68%	156%
RUN 4	1	1	62%	177%

The results indicated in table 8.18 reveal that as the importance given to the transport goal is diminished, its level of achievement or rather of overachievement increases. Because of the nature of the problem, an overachievement of either the housing goal or the transport goal means a non desired deviation from what is considered to be the acceptable level. In other words transport cost exceeds its acceptable level by 156% in Run 2, by the same percentage in Run 3 and by 177% in Run 4. On the other hand the level of achievement of the housing goal decreases from 68% to 62%.

Thus the more the transport goal is overachieved , the less the level of overachievement of the housing goal is. However, the decrease in the percentage of overachievement of the housing goal is less than proportional to the increase in the percentage of overachievement of the transport goal (6 points decrease against 21 points increase).

The results, corresponding to the housing mix and the locational pattern of Runs 2 and 4 are presented in tables 8.18. and 8.19.

TABLE 8.18. RESULTS OF RUN 2 ✻ (HIGH WEIGHT GIVEN TO TRANSPORT GOAL)

HOUSE MIX	LOCATION						
	P1	P2	P3	P4	P5	T1	T2
Two storey houses			241	345	914		
Three storey flats	485						
Five storey flats							
Eight storey flats							
Twelve storey flats							
Rehabilited units						20	50

✻ The results of Run 3 are equal to those of Run 2

TABLE 8.19. RESULTS OF RUN 4 (EQUAL VALUES GIVEN TO THE TRANSPORT AND HOUSING GOAL)

HOUSE MIX	LOCATION						
	P1	P2	P3	P4	P5	T1	T2
Two storey houses					556		
Three storey flats	485				960		
Five storey flats							
Eight storey flats							
Twelve storey flats							
Rehabilited Units						20	50

From table 8.18. it can be concluded that as the transport goal is given the less importance, the construction of two storey houses in potential areas 3, 4 and 5 is recommended. Also the construction of three storey flats in potential area 7 - that with the highest accesibility to the main employment centre - is recommended. There is as well a requirement to rehabilitate 70 units in target areas 1 and 2. It is interesting that the construction of five, eight and twelve storey flats is not recommended.

The results presented in table 8.19. reveal that as the importance of the transport goal is further diminished, the construction of two storey houses and three storey flats at potential area 5 is recommended, although it is still feasible and desirable building three storey flats in potential area 1. Also as required 70 units are rehabilitated at target areas 1 and 2.

R U N S 5, 6 and 7

Runs 5, 6 and 7 were made to test the implications of given weight values greater than one to the deviation variables of the housing goal in the achievement function. Three different weight values were tested. They are presented in the table below:

	WEIGHT
RUN 5	15.0
RUN 6	10.0
RUN 7	5.0

The results obtained were similar in each case and for those previously obtained in RUN 3 where weight values of 1 were allocated to both goal 1 and goal 2. The percentage of overachievement in all three cases is 62% , which indicates that with the given constraints the minimum level by which goal 1 can be overachieved is 62% and goal 2, 112%. This conclusion suggests that if land available in certain key areas was increased , a better solution might be obtained. This possibility is explored in the following section.

R U N S 8 and 9 : Changing the amount of land available.

So far no change has been made in the constraints. In this section the effects of increasing the amount of land available firstly in potential areas five and secondly in potential area 2, will be considered . This is to see whether levels of achievement of either Goal 1 or Goal 2 can be increased by allowing more construction to take place in the mentioned areas.

To increase the levels of achievement means to reduce either underachievement or overachievement i.e. to satisfy as close as possible the aspiration levels set for both the housing and transport goals.

In previous runs aspiration levels have been over-achieved considerably mainly because such levels are difficult to satisfy given the demand constraints imposed. Thus an overachievement of say 60% of the housing goal implies that, to satisfy the demand would require a 60% increase in the budget. On the other hand an underachievement of 60% would imply that the budget is more than enough to satisfy the demand. The same applies when the transport goal is either over-achieved or underachieved.

Something that constraints the achievement of both goals is the amount of land available in each area. In relation to transport, the more households can be located at potential area 2 - the area closest to the main employment centre - the lower the overall cost of transportation will be and therefore the easier it will be to satisfy the aspiration level set for that goal. On the other hand if the amount of land available in potential area 5 - where land is cheapest - is increased the possibility of building many housing units at a relatively low cost, is also raised, and therefore the possibility of satisfying the aspiration level set for the housing goal.

The effects of increasing the amount of land available in potential area 5 from 80 acres to 100, in terms of the percentages of overachievement for goal 1 and goal 2 turned out to be :

RUN 8	Overachievement of Goal 1	60%
	Overachievement of Goal 2	178%

Since the improvement with respect to previous runs was not all significant, the weight value given to the housing goal (Goal 1) which was five, was increased to 15 to give Goal 1 even more importance over Goal 2 (Run 9) . The result however did not change at all. Thus the minimum level of overachievement for Goal 1 is 60%.

It can be concluded then that in order to fully satisfy the demand a 60% increase in the budget will be required .

Having explored the effect of increasing land availa-bility in potential area 5 , the effect of increasing land availability in potential area 2 which is the one closest to the major employment centre was tested. This was done to see if the level of achievement of Goal 2 (the transport goal) could be improved by allowing more high density construction to take place in potential area 1. The result is presented below.

RUN 10	Overachievement of Goal 1	112%
	Overachievement of Goal 2	112%

As can be seen , both goals are overachieved by the same percentage as in run 1. This indicates that inspite of the increase in the amount of land available - from 13 to 50 acres - in potential area 2, the transport goal is still overachieved by a 112%. This result implies that it is just not economic to build up more units in potential area 2 even though some savings on transport costs could be obtained, and that the lowest level of overachievement for goal 2 is 112% i.e. Households would have to spend on transportation to work more than double the amount considered acceptable.

R U N 10 : Increasing the minimum number of units to be rehabilitated.

Until now the lower limit for housing rehabilitation has been set equal to 20 units for target area 1 and 50 units for target area 2. This section analyses the effects of increasing these levels, to 100 and 300 units respectively in target areas .

The resulting levels of overachievement for Goal 1 and Goal 2 giving Goal 2 a weight value of 15, are as follows :

RUN 11	Overachievement of Goal 1	123%
	Overachievement of Goal 2	165%

Levels of overachievement have now increased for both goals. This is not surprising since a greater number of units have been rehabilitated , which necessary implies greater investments and also greater transport costs. However it is worth pointing out that a greater number of families is being benefited.

The increases in the levels of overachievement imply - with respect to the housing goal (goal 1)- that to satisfy the demand including new rehabilitation requirements, the budget would have to be increased by 123%.

With respect to transport costs, the accepted level (aspiration level), would be exceeded by 165%.

The resulting house mix for this run is presented in table 8. 20 .

TABLE 8.20. RUN 11 : INCREASING REHABILITATION REQUIREMENTS.

HOUSE MIX	LOCATION						
	P1	P2	P3	P4	P5	T1	T2
Two storey houses	33				920		
Three storey flats	397						43
Four storey flats							
Eight storey flats							
Twelve storey flats		650					
Units rehabilitated						100	307

The model's solution recommends building 920 two storey houses in potential area 5; 397 three storey flats in potential area 1 and 650 twelve storey flats in potential area 2.. A few two storey houses and three storey flats are also indicated for potential area 1 and target area 2 respectively.

These results were obtained giving a weight value of 15 to goal 2. In a further run (Run 12) the effects of giving a weight value of 15 to Goal 1 were examined. The results are presented below:

RUN 12	Overachievement of Goal 1	73 %
	Overachievement of Goal 2	185 %

As expected, by giving Goal 1 priority over Goal 2 the level of overachievement of Goal 1 is reduced, from 123% to 73%, and the levels of overachievement of Goal 2 increased from 165% to 185%.

8.2.4. A comparison of outcomes : conclusion

A comparison can now be made of the obtained results in order to draw some relevant conclusions. The results of the twelve runs, regarding levels of achievement are summarized in table 8.21

TABLE 8.21. SUMMARY OF OUTCOMES (ACHIEVEMENT LEVELS) OF RUNS 1 TO 12

R U N	W E I G H T		OVERACHIEVEMENT (%)	
	Goal 1	Goal 2	Goal 1	Goal 2
1	1	15	112	112
2	1	10	68	156
3	1	5	68	156
4	1	1	62	177
5	15	1	62	112
6	10	1	62	112
7	5	1	62	112
8	5	1	60	178
9	15	1	60	178
10	1	15	112	112
11	1	15	123	165
12	15	1	73	185

a) Giving priority to either the housing goal or the transport one do have very different implications in terms of th e resulting levels of achievement of each goal. It is clear that when goal 1 (the housing goal) is given priority over the transport goal its minimum level of overachievement is 60%. On the other hand when priority is placed on goal 2 (the transport goal) its minimum level of overachievement turns out to be 112%. In other words aspiration levels are exceeded at least by 60% and 112% in each case.

b) To overachieve goal 1 by 60% implies that the available budget for housing construction would have to be increased by 60% to fully satisfy the demand of 2000 new units and the rehabilitation of at least 70 units.

c) On the other hand, overachieving goal 2 by 112% implies that families - even if located at sites close to employment centres - would spend more than double (112%) the level considered acceptable, i.e. £ 200,000 (for all families over the planning horizon).

d) When goal 2 (the transport goal) is given priority, housing construction takes the form of two storey houses and three storey flats. The former are mainly located at potential area five, far from the major employment centres. The latter, instead, are located at potential area 1 and 2 close to them.

e) When goal 1 is given priority, housing construction also takes the form of two storey houses and three storey flats. However the pattern of location changes in a rather important fashion.

More than 1500 units out of the 2000 which are located at potential area five, far from the major employment centres.

f) A compromise solution between satisfying goal 1 and goal 2 can be achieved when goal one is given a weight of 1 and goal 2 a weight of 10. This solution implies that goal 1 is overachieved by 68% and goal 2 by 156%. It also implies the construction of mainly two storey houses distributed in potential areas 3, 4 and 5 and not just in the last one.

g) By increasing the amount of land available in potential area 5, the housing goal is not improved in terms of a significant reduction in the percentage of overachievement of this goal. On the other hand an increase of the available land in potential area 1 does not either contributes to a significant reduction of the percentage of overachievement of the transport goal.

8.3 Final Conclusion

In this chapter several housing strategies have been assessed with the aid of a goal programming model. Such a modelling approach has proven useful for an efficient assessment of the implication of a particular housing strategy with respect to two of its most important elements : The cost of the housing unit in relation to family income and the cost of transportation to work.

The monocentric version of the model was tested using real data from Mexico City. The results obtained have been already discussed and summarized in section 8.1.6. However it can be emphasized that due to the fact that land prices in Mexico City are so high particularly at central locations, The possibility of providing housing units for low income families there, where employment accessibility is greater, seem quite unfeasible since it would imply the introduction of very heavy subsidies. On the other hand housing provision at the intermediate zone or even at the periphery seems more feasible, so long as what families are required to spend on transportation does not take to big a share of their incomes. In fact it was suggested as a rule of thumb that the size of the housing subsidy should be at least as great as what a household spends on transportation to work.

As regards the polycentric version the main conclusion is that the assessment of housing strategies is still possible using the sort of modelling approach proposed when employment is scatered throughout an urban area. The specific results obtained have already been summarized in section 8.2.4.

It should be added however that the model could still be made even more complex in many respects. Many more house types and locations could be considered.Also the number of household types could be increased. In the monocentric case only three household types are considered because these are the main income groups toward whom the National Housing Institute (INFONAVIT) directs its housing programms. However as it was pointed out in Chapter 4, there are several public housing agencies in Mexico which at present operate in a rather independent way. For public housing planning purposes not only for Mexico City but other important cities as well a model such as the one proposed could be of great value in providing answers to questions related to land, subsidies, transport cost, densities, investment costs, development size and location.

CHAPTER 9

CONCLUSIONS

9.1. General Conclusions

Mexico City's housing problem is very serious because of the high number of housing units lacking basic facilities, the fact that overcrowding has increased over time and also because of the size and staggering growth of squatter settlements. (Chapter 2)

Although Mexico City attracted more than 60% of the country's migrants during the period 1960 - 1970 and the natural growth of its population is very high, the fundamental causes of the prevailing housing situation can be considered to be of an economic nature. To support this point the rates of population growth of forty mexican cities over the period 1960 - 1970 were correlated with the housing deficit rates prevailing in those cities in the year 1970. The resulting correlation and determination coefficients turned out to be very low, thus indicating a lack of correlation between the variables. (Chapter 3).

Further analyses were carried out to ascertain the extent to which population growth causes high housing deficits in a city. However similar results were obtained.

Additional analysis in respect of city size suggests that this does not seem to be the cause of high deficit rates either as indicated by the correlation exercise in Chapter 3.

This is not surprising considering that there are many cities in Mexico smaller than Mexico City, such as Acapulco, Cuernavaca, Queretaro, Tampico and many more where slums and squatter settlements exist almost in the same proportion.

Immediate causes relate to problems on the demand and supply sides of the housing market. On the demand side the key issues are underemployment and low levels of family income. Over 30% of Mexico City's labour force is engaged in unproductive marginal occupations which explains why family income levels are so low.

On the supply side an important factor is the price of land, which has soared over the last decade. At the periphery of the city, land can account for as much as 55.1 per cent of overall housing cost. (Chapter 3)

One important characteristic of Mexico City is the relatively high concentration of employment, specially service employment in the central part of the city. In 1970 more than 60 per cent of total land devoted to service activities (shops and offices) was concentrated in the central area of Mexico City (Delegacion Cuauhtemoc).

As a result of this, land values decrease exponentially with distance from the city centre an so do population densities. (Chapter 4)

In contrast to what is observed in most British and American cities poor families can be found not only in the ring adjacent to the CBD but also at the periphery, on large plots of land which they have occupied illegally. (Chapter 2 and 5)

Over the last decades several housing agencies have been set up to cope with the housing problem but because of the lack of resources as well as the lack of effective coordination the official response has been very limited. (Chapter 5)

In view of this many housing experts have called for a drastic change of approach favoring a 'self - help' housing policy. According to this school the role of the government should limit itself to the finance, organization and planning of 'self-construction'. (Chapter 5)

One of the most important housing agencies in Mexico is the National Housing W orkers Fund created in the early 1970's (INFONAVIT).

Public housing provision by INFONAVIT in Mexico City is constrained by many factors the most important of which are families ability to pay for 'good' housing, the availability of land plots of the right size and in the right locations and the amount of funds available to carry out housing projects. (Chapter 5)

This poses INFONAVIT many important questions relating to the allocation of credits among applicants, the location size and densities of housing projects.

These decisions must be evaluated in terms of families' needs and income levels on the one hand, and in terms of the resources the housing agency can use to carry out the construction or promotion of residential developments on the other.

Family needs comprise locational as well as space needs which in turn depend on family size, age composition, occupations and so forth. (Chapters 5 and 7)

The goal programming methodology presented in this thesis was developed to facilitate the assessment of different public housing strategies in response to the issues outlined above.

The model comprises two goals, a housing or subsidy goal and a transport goal. The objective of the former is to achieve an aspiration level which is the maximum amount the housing agency is able to pay in the form of subsidies to the benefitted families.

The transport goal's objective is to achieve an aspiration level which corresponds to the maximum accepted level of what families should pay for transportation to work, where the cost includes direct bus fares plus lost wages.

Clearly a conflict exists between these two goals because the closer to employment centres families are housed the greater is the subsidy needed because land prices are higher there. The cost of transportation to work however will be lower, and viceversa. (Chapter 7)

Many linear programming models have been developed to serve as analytical tools in the evaluation of urban and housing plans. The goal programming model presented in this thesis represents an improvement with respect to those models in that it allows the possibility of defining not just one policy objective, which is somehow incorporated in the objective function, but several even conflicting policy objectives which are expressed as goals. Priority can be given to particular goals according to the decision taker preferences, using suitable weights in the achievement function or grant objective function. (Chapters 6 and 7)

Two versions of the model were developed and tested, the monocentric and the polycentric. The former was used to evaluate housing strategies in a case where employment was highly concentrated (Mexico City). The latter was used to illustrate the use of the model when employment is concentrated in more than one single point. (Hypothetical case), (Chapters 7 and 8)

9.2 The Model's Results

The results obtained from the model are relevant not only because they tend to confirm what would be expected to be the logical effect of a certain policy but also because they provide precise quantitative results which allow the assessment of the trade-offs involved.

In this respect a number of conclusions can be drawn from the analysis.

Of the three different types of families considered - type 1 (earning 1 to 2 times the minimum wage); type 2 (earning 2.1 to 3 times the minimum wage); and type 3 (earning 3.1 to 4 times the minimum wage) - type 3 is the only one for which a non-subsidized housing programme could be undertaken. However transportation costs would be very high in relative terms since this programme would require the location of mostly five storey flats at the periphery of Mexico City.

If the housing programme is to benefit mainly families earning 1 to 2 times the minimum wage the amount of the subsidy would be $ 1 684 725 and the cost of transportation $ 1 683 675. This occurs when the housing or subsidy goal is given priority over the transport goal. In the opposite case (when employment accessibility has the priority) subsidies increase considerably to $ 7 706 475 although transport cost decreases to $ 345 975 since almost all housing construction takes the form of five storey flats located near the central area of Mexico City.

The main conclusion of the analysis is that, however desired a housing strategy based on the provision of low cost housing near employment centres may be the cost of such an strategy is very high because of the high cost of land in the central area of Mexico City.

Transport cost savings are just not enough to justify a solution of that kind. It should be pointed out however that this conclusion is based on an assumption of standard densities of dwelling unit per hectare. Varying densities might well produce a different conclusion and thus this is clearly an area for further work.

Because of the high cost of land throughout Mexico City it appears that a sound housing strategy should be oriented toward the provision of high density kind of developments at intermediate and peripheral areas. This strategy offers several advantages. It allows the provision of a greater number of units and therefore the possibility of benefiting a greater number of families.

The amount of the subsidy is not as great as if a similar number of units were built in the central area. Finally, the probability of finding the required number of plots of the right size is greater.

The major disadvantage of this kind of strategy is that it implies higher transport costs. This problem however can be lessened in the short run by a well designed subsidy policy.

The fact that even at the periphery the model's solution indicates that housing provision should take the form of five storey flats is significant since it suggests that a low density housing policy such as self-help would be non economical and that to be so it would have to be carried out at locations even further from the city outskirts. It could of course be argued that self-help is not an economic optimum market solution to the housing problem but rather an organised social response to an economic problem, and that that is better than no solution at all.

It can be said, however, that the provision of public services and utilities is an aspect of the housing problem as important as the very construction of dwellings, and that there is enough empirical evidence (Real State Corporation 1974) to support the view that the lower residential densities are the higher the cost of providing public services and utilities will be. This implies that even if the point about whether densities are economic in relation to the cost of land is ignored the social cost implied by residential sprawl most be considered.

The goal programming model was modified so as to enable the assessment of housing strategies when employment is scattered throughout the urban area. For this purpose a polycentric version of the model was developed and tested using hypothetical data.

The basic problem in this case was the estimation of transport costs, since they depend on the pattern of work trips. A gravity type of model was used to estimate the probability of a journey taking place from a particular point within the urban area to the employment centre. (Chapters 7 and 8)

The polycentric version of the model is not of academic interest only since as pointed out in Chapter 4, the local government of Mexico City recently (1980) published its urban plan for Mexico City which states as one of its basic objectives the creation of employment subcentres throughout Mexico City in order to eliminate the existing pattern characterized by the overconcentration of economic activity in one major area of the city (Delegacion Cuauhtemoc).

The results obtained with the polycentric version indicate that when employment is scattered within an urban area the possibility of low density residential developments becomes a feasible and economic one. (Chapter 8)

This is a result which is consistent with the theory presented in Chapter 4 i.e. the trade-off accessibility theory of residential location according to which household's location is a function of their preferences in relation to housing space (densities) and employment accessibility subject to their level of income. This implies (ceteris paribus) that as transportation to work becomes less costly, people become less interested in bidding for plots of land and houses located near the city centre and more interested in housing space.

The direct consequence of this is that the slope of the rent gradient falls and so do residential land prices in the inner areas, therefore making the construction of low cost residential developments in the central area more feasible.

One further point which can be made is that if land prices fall in the inner areas a filtering process would be set into motion as high and medium income families move further out to improve their welfare at optimal locations. This would make the housing process of low income families a lot easier since not a great deal of either land or capital would be required. The problem with a process of this sort is that it can take several years to have any significant effect since there are other forces at work, such as the increase in real incomes, which would also have an important impact on the slope of the rent gradient, and consequently on house prices.

Thus it can be said that if the government of Mexico City pursues a policy of employment decentralization, the possibilities of implementing a self-help housing policy could be increased for the problem of transportation, which at present is of importance, could be given less attention, concentrating more on the actual process of housing construction, or even housing rehabilitation.

Finally, it is evident that the work presented in this study is but one step towards the development of more elaborate and powerful tools to assess housing strategies (Chapter 1, pg. 9). However the use and development of the model in the real world situation afforded by Infonavit data has shown the inherent value of this approach in terms of both identifying and measuring the effects of policies, and the resulting trade offs between these policies. Hence, the value of such models in the development of future housing strategies is clearly demonstrated. Moreover, in addition to the potential benefits to be realised by policy makers in the housing field (e.g. in terms of information, reduced uncertainty, lowered resource costs etc), there are also significant benefits to be potentially realised by the recipients of housing subsidies in terms of improved levels of welfare. However, in order to fully maximise these benefits, further research is needed, particularly in respect of the sort of data that is used and also the kind of models that are used to generate certain information. For example, in both the monocentric and the polycentric versions of the model a more refined technique should be developed for the estimation of transport costs. Furthermore, the modification or relaxation of certain assumptions which are implicit in the model is also necessary. For instance it is assumed that household's transportation to work is done by bus, which a priori may seem a sensible assumption to make given that they are all low income households, but because of the lack of empirical evidence it is difficult to support or even to modify in any precise way.

In relation to the modelling approach itself, i.e. goal programming, it should be said that, as pointed out in Chapter 7, there are two forms in which a model of this kind can be conceived. One, used for this study, is the weighted goal programming approach and the other the lexicographic approach. In the view of some experts the former is rather limited and frecuently inadecuate to solve problems where two or more conflicting objectives are to be satisfied in an optimum fashion, and that, therefore, the lexicographic approach is of greater applicability.

For reasons given in Chapter 7, the weighted goal programming approach was adopted, however for a more solid justification it would be of great value to develop a lexicographic version in order to be able to contrast the results so as to objectively determine which one yields more relevant information for the design of housing policies.

ANNEX 1

INPUT DATA OF MONOCENTRIC CASE.

Table 1. Credit Capacity and Subsidy of Income Group 3.

AREA	PERIOD	VARIABLE	COST ($)	CREDIT CAPACITY	SUBSIDY *
Two Storey House					
Central	t_1	X_1	1,158.3	464.0	694.3
	t_2	X_2	1,389.9	534.0	855.9
	t_3	X_3	1,667.9	631.0	1,037.0
Interm.	t_1	X_4	693.3	464.0	229.3
	t_2	X_5	832.0	534.0	368.0
	t_3	X_6	998.2	631.0	534.2
Periphe.	t_1	X_7	442.2	464.0	-21.8
	t_2	X_8	530.6	534.0	-70.0
	t_3	X_9	636.6	631.0	- 5.7
Three Storey Flats					
Central	t_1	X_{10}	910.2	464.0	446.2
	t_2	X_{11}	1,092.0	534.0	558.3
	t_3	X_{12}	1,310.7	631.0	679.7
Interm.	t_1	X_{13}	544.8	464.0	80.8
	t_2	X_{14}	653.8	534.0	119.8
	t_3	X_{15}	784.5	631.0	153.5
Periphe.	t_1	X_{16}	347.5	464.0	-116.5
	t_2	X_{17}	416.9	534.0	-117.1
	t_3	X_{18}	500.4	631.0	-130.6

* Subsidy = Cost - Credit Capacity

Table 1. (Continued)

Five Storey Flats

AREA	PERIOD	VARIABLE	COST($)	CREDIT CAPACITY	SUBSIDY*
Central	t_1	X_{19}	728.7	464.0	264.7
	t_2	X_{20}	874.5	534.0	340.5
	t_3	X_{21}	1,049.4	631.0	418.4
Interm.	t_1	X_{22}	436.2	464.0	-27.8
	t_2	X_{23}	523.4	534.0	-10.6
	t_3	X_{24}	628.1	631.0	- 2.9
Periphe.	t_1	X_{25}	278.2	464.0	-185.8
	t_2	X_{26}	333.8	534.0	-200.2
	t_3	X_{27}	400.6	631.0	-230.4

Table 2. Credit Capacity and Subsidy of Income Group 2.

AREA	PERIOD	VARIABLE	COST ($)	CREDIT CAPACITY	SUBSIDY *
Two Storey House					
Central	t_1	X_{28}	1,158.3	378.0	780
	t_2	X_{29}	1,389.9	435.0	955.0
	t_3	X_{30}	1,667.9	514.0	1,153.9
Interm.	t_1	X_{31}	693.3	378.0	315.3
	t_2	X_{32}	832.0	435.0	397.0
	t_3	X_{33}	998.2	514.0	484.2
Periphe.	t_1	X_{34}	442.2	378.0	64.2
	t_2	X_{35}	530.6	435.0	95.6
	t_3	X_{36}	636.6	514.0	122.7
Three Storey Flats					
Central	t_1	X_{37}	910.2	378.0	532.2
	t_2	X_{38}	1,092.0	435.0	657.3
	t_3	X_{39}	1,310.7	514.0	796.7
Interm.	t_1	X_{40}	544.8	378.0	166.8
	t_2	X_{41}	653.8	435.0	218.8
	t_3	X_{42}	784.5	514.0	270.5
Periphe.	t_1	X_{43}	347.5	378.0	-30.5
	t_2	X_{44}	416.9	435.0	-18.1
	t_3	X_{45}	500.4	514.0	-13.6

* Subsidy = Cost - Credit Capacity

Table 2. (Continued)

Five Storey Flats

AREA	PERIOD	VARIABLE	COST($)	CREDIT CAPACITY	SUBSIDY*
Central	t_1	X_{46}	728.7	378.0	350.7
	t_2	X_{47}	874.5	435.0	439.5
	t_3	X_{48}	1,049.4	514.0	535.4
Interm.	t_1	X_{49}	436.2	378.0	58.2
	t_2	X_{50}	523.4	435.0	88.4
	t_3	X_{51}	628.1	514.0	114.1
Periphe.	t_1	X_{52}	278.2	378.0	-99.8
	t_2	X_{53}	333.8	435.0	-101.2
	t_3	X_{54}	400.6	514.0	-113.4

Table 3. Credit Capacity and Subsidy of Income Group 1.

AREA	PERIOD	VARIABLE	COST ($)	CREDIT CAPACITY	SUBSIDY *
Two Storey House					
Central	t_1	X_{55}	1,158.3	302.0	856.3
	t_2	X_{56}	1,389.9	348.0	1,041.9
	t_3	X_{57}	1,667.9	411.0	1,256.9
Interm.	t_1	X_{58}	693.3	302.0	391.3
	t_2	X_{59}	832.0	348.0	484.0
	t_3	X_{60}	998.2	411.0	587.2
Periphe.	t_1	X_{61}	442.2	302.0	140.2
	t_2	X_{62}	530.6	348.0	182.6
	t_3	X_{63}	636.6	411.0	225.7
Three Storey Flats					
Central	t_1	X_{64}	910.2	302.0	608.2
	t_2	X_{65}	1,092.0	348.0	744.3
	t_3	X_{66}	1,310.7	411.0	899.7
Interm.	t_1	X_{67}	544.8	302.0	242.8
	t_2	X_{68}	653.8	348.0	305.8
	t_3	X_{69}	784.5	411.0	373.5
Periphe.	t_1	X_{70}	347.5	302.0	45.5
	t_2	X_{71}	416.9	348.0	68.9
	t_3	X_{72}	500.4	411.0	89.4

* Subsidy = Cost - Credit Capacity

Table 3. (Continued)

Five Storey Flats

AREA	PERIOD	VARIABLE	COST($)	CREDIT CAPACITY	SUBSIDY*
Central	t_1	X_{73}	728.7	302.0	426.7
	t_2	X_{74}	874.5	348.0	526.5
	t_3	X_{75}	1,049.4	411.0	638.4
Interm.	t_1	X_{76}	436.2	302.0	134.2
	t_2	X_{77}	523.4	348.0	175.4
	t_3	X_{78}	628.1	411.0	217.1
Periphe.	t_1	X_{79}	278.2	302.0	-23.8
	t_2	X_{80}	333.8	348.0	-14.2
	t_3	X_{81}	400.6	411.0	-10.4

Table 4. Time and Total Transport Costs of Income Group 3.

AREA	PERIOD	VARIABLE	t_{ct}	TC_{ut}
Central	t_1	X_1	18,375	22,575
	t_2	X_2	21,131	25,331
	t_3	X_3	24,977	29,177
Interm.	t_1	X_4	82,740	86,940
	t_2	X_5	95,151	99,351
	t_3	X_6	112,469	116,669
Periphe.	t_1	X_7	165,375	173,775
	t_2	X_8	190,181	198,581
	t_3	X_9	224,794	233,194
Central	t_1	X_{10}	18,375	22,575
	t_2	X_{11}	21,131	25,331
	t_3	X_{12}	24,977	29,177
Interm.	t_1	X_{13}	82,740	86,940
	t_2	X_{14}	95,151	99,351
	t_3	X_{15}	112,469	116,669
Periphe.	t_1	X_{16}	165,375	173,775
	t_2	X_{17}	190,181	198,581
	t_3	X_{18}	224,794	233,194

Table 4 . (Continued)

AREA	PERIOD	VARIABLE	tc_t	TC_{ut}
Central	t_1	X_{19}	18,375	22,575
	t_2	X_{20}	21,131	25,331
	t3	X_{21}	24,977	29,177
Interm.	t_1	X22	82,740	86,940
	t_2	X_{23}	95,151	99.351
	t_3	X24	112,469	116,669
Periphe.	t_1	X_{25}	165,375	173,775
	t_2	X_{26}	190,181	198,581
	t_3	X_{27}	224,794	233,194

Table 5. Time and Total Transport Costs of Income Group 2.

AREA	PERIOD	VARIABLE	tc_t	TC_{ut}
Central	t_1	X_{28}	13,125	17,325
	t_2	X_{29}	15,094	19,294
	t_3	X_{30}	17,841	22,041
Interm.	t_1	X_{31}	59,073	63,273
	t_2	X_{32}	67,934	72,134
	t_3	X_{33}	80,298	84,498
Periphe.	t_1	X_{34}	118,125	126,525
	t_2	X_{35}	135,844	144,244
	t_3	X_{36}	160,568	168,968
Central	t_1	X_{37}	13,125	17,325
	t_2	X_{38}	15,094	19,294
	t_3	X_{39}	17,841	22,041
Interm.	t_1	X_{40}	59,073	63,273
	t_2	X_{41}	67,934	72,134
	t_3	X_{42}	80,298	84,498
Periphe.	t_1	X_{43}	118,125	126,525
	t_2	X_{44}	135,844	144,244
	t_3	X_{45}	160,568	168,968

Table 5. (Continued)

A R E A	PERIOD	VARIABLE	tc_t	TC_{ut}
Central	t_1	X_{46}	13,125	17,325
	t_2	X_{47}	15,094	19,294
	t_3	X_{48}	17,841	22,041
Interm.	t_1	X_{49}	59,073	63,273
	t_2	X_{50}	67,934	72,134
	t_3	X_{51}	80,298	84,498
Periphe.	t_1	X_{52}	118,125	126,525
	t_2	X_{53}	135,844	144,244
	t_3	X_{54}	160,568	168,968

Table 6. Time and Total Transport Costs of Income Group 1.

AREA	PERIOD	VARIABLE	tc_t	TC_{ut}
Central	t_1	X_{55}	8,568	12,768
	t_2	X_{56}	9,853	14,053
	t_3	X_{57}	11,646	15,846
Interm.	t_1	X_{58}	38,640	42,840
	t_2	X_{59}	44,436	48,636
	t_3	X_{60}	52,523	56,723
Periphe.	t_1	X_{61}	77,175	85,575
	t_2	X_{62}	88,751	97,151
	t_3	X_{63}	104,904	113,304
Central	t_1	X_{64}	8,568	12,768
	t_2	X_{65}	9,853	14,053
	t_3	X_{66}	11,646	15,846
Interm.	t_1	X_{67}	38,540	42,840
	t_2	X_{68}	44,436	48,636
	t_3	X_{69}	52,523	56,723
Periphe.	t_1	X_{70}	77,175	85,575
	t_2	X_{71}	88,751	97,151
	t_3	X_{72}	104,904	113,304

Table 6. (Continued)

AREA	PERIOD	VARIABLE	tc_t	TC_{ut}
Central	t_1	X_{73}	8,568	12,768
	t_2	X_{74}	9,853	14,053
	t_3	X_{75}	11,646	15,846
Interm.	t_1	X_{76}	38,640	42,840
	t_2	X_{77}	44,436	48,636
	t_3	X_{78}	52,523	56,723
Periphe.	t_1	X_{79}	77,175	85,575
	t_2	X_{80}	88,751	97,151
	t_3	X_{81}	104,904	113,304

ANNEX 2

OUTPUT DATA OF MONOCENTRIC CASE

```
SETUP   TEST

TIME =    0.02

    SCALE


MATRIX1  ASSIGNED TO MATRIX1
MATRIX2  ASSIGNED TO MATRIX2
MATRIX3  ASSIGNED TO MATRIX3
MATRIX4  ASSIGNED TO MATRIX4

ETA1     ASSIGNED TO ETA1
ETA2     ASSIGNED TO ETA2
ETA3     ASSIGNED TO ETA3
ETA4     ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE   7

NO CYCLING

     POOLS           NUMBER    SIZE     CORE
H.REG-BITS MAP                           184
WORK    REGIONS        9        360     3240
MATRIX BUFFERS         5       2216    11080
ETA     BUFFERS        6       5736    34416

                         TOTAL  NORMAL  .FREE.   FIXED BOUNDED
ROWS     (LOG.VAR.)        42      36       1       5       0
COLUMNS  (STR.VAR.)        85      85       0       0       0

     751 ELEMENTS - DENSITY =  14.07 -     5 MATRIX RECORDS (WITHOUT RHS'S)


PRIMAL       OBJ = GDF          RHS = RH

TIME =     0.02  MINS.       PRICING    7
 SCALE =      .

   ITER   NUMBER  VECTOR  VECTOR  REDUCED       SUM
  NUMBER  INFEAS    OUT     IN      COST       INFEAS
M     1     15        2     72    3.00195-   69763.3
      2               3    122    3.58115-   69763.3
      3              72     43    2.07239-   69763.3
M     4     15      122     69   14.0353-    69763.3
      5              69     67     .84789-   69763.3
M     6     11       18    127    2.83590-   45529.5
      7              67    125     .04399-   26623.9
      8              15    115     .05994-   23315.9
      9              21     88     .08774-   20514.8
     10              26     61     .00680-   20297.6
```

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
	11		7	64	.00971-	20254.2
M	12	5	42	99	1.46139-	7030.45
	13		36	78	1.00547-	5321.72
	14		14	72	.76807-	4894.41
	15		41	105	1.32540-	4894.41
	16		99	117	.02047-	4729.69
M	17	3	78	15	8.42295-	3210.82
	18		115	122	.48047-	3037.21
	19		13	95	.13752-	2903.17
M	20	1	30	45	1.12590-	356.163
	21		24	123	.76106-	.

FEASIBLE SOLUTION

PRIMAL   OBJ = GDF   RHS = RH

TIME =   0.03 MINS.   PRICING   7
SCALE =   1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	22	20	123	121	15.8691-	.32E+08
	23		29	51	190.016-	.32E+08
	24		35	78	93.9349-	.32E+08
M	25	21	45	63	13.5522-	.31E+08
	26		72	90	8.80645-	.31E+08
	27		95	50	4.53720-	.31E+08
	28		64	118	4.46343-	.30E+08
	29		25	109	.73882-	.30E+08
	30		8	47	62.6416-	.30E+08
M	31	5	109	7	10.3232-	.30E+08
	32		122	95	73.0840-	.28E+08
M	33	15	118	67	5.79165-	.28E+08
	34		43	49	.66775-	.28E+08
M	35	3	16	94	1.22307-	.28E+08
M	36	1	67	25	.10086-	.28E+08

OPTIMAL SOLUTION

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	28259750.1968	28259750.1968-	NONE	NONE	1.00000
2	G1	EQ	.	.	.	.	10.00000
3	G2	EQ	.	.	.	.	1.00000
4	LC1	BS	14.71951	9.71951-	5.00000	NONE	.
5	LC2	BS	14.71951	9.71951-	5.00000	NONE	.
6	LC3	BS	55.66951	50.66951-	5.00000	NONE	.
7	LC4	BS	.	10.00000	NONE	10.00000	.
8	LC5	UL	10.00000	.	NONE	10.00000	136272.29462
9	LC6	BS	10.00000	.	NONE	10.00000	.
10	LC7	BS	84.33714	84.33714-	.	NONE	.
11	LC8	BS	109.15549	109.15549-	.	NONE	.
12	LC9	BS	172.15549	172.15549-	.	NONE	.
13	KC1	LL	2629899.99999	.	2629899.99999	NONE	1.60567-
14	KC2	LL	3979999.99999	.	3979999.99999	NONE	8.16105-
15	KC3	BS	9291214.99996	1341214.99997-	7949999.99998	NONE	.
16	DC1	LL	.	.	.	NONE	387.09477-
17	DC2	BS	1575.00000	1575.00000-	.	NONE	.
18	DC3	EQ	2625.00000	.	2625.00000	2625.00000	4831.25348
19	DC4	BS	1319.74236	1319.74236-	.	NONE	.
20	DC5	BS	1575.00000	1575.00000-	.	NONE	.
21	DC6	EQ	2625.00000	.	2625.00000	2625.00000	3590.15871
22	DC7	BS	6300.00000	6300.00000-	.	NONE	.
23	DC8	BS	6300.00000	6300.00000-	.	NONE	.
24	DC9	EQ	10500.00000	.	10500.00000	10500.00000	2871.15871
25	D1AA	BS	.	.	.	NONE	.
26	D1AS	LL	.	.	.	NONE	79.22420-
27	D2AA	BS	.	.	.	NONE	.
28	D2AS	BS	1075.00000	1075.00000-	.	NONE	.
29	D3AA	LL	525.00000	.	525.00000	NONE	9040.25348-
30	D3AS	LL	525.00000	.	525.00000	NONE	5004.25348-
31	D1BA	BS	1132.27009	1132.27009-	.	NONE	.
32	D1BS	BS	.	.	.	NONE	.
33	D2BA	BS	.	.	.	NONE	.
34	D2BS	BS	.	.	.	NONE	.
35	D3BA	LL	525.00000	.	525.00000	NONE	8962.15871-
36	D3BS	LL	525.00000	.	525.00000	NONE	4989.15871-
37	D1CA	BS	.	.	.	NONE	.
38	D1CS	BS	.	.	.	NONE	.
39	D2CA	BS	.	.	.	NONE	.
40	D2CS	BS	.	.	.	NONE	.
41	D3CA	LL	2100.00000	.	2100.00000	NONE	9267.15871-
42	D3CS	LL	2100.00000	.	2100.00000	NONE	5244.15871-

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
43	X1	LL	.	.	.	NONE	14.90455
44	X2	LL	.	.	.	NONE	2073.20916
45	X3	LL	.	.	.	NONE	1165.74652
46	X4	LL	.	.	.	NONE	2695.97086
47	X5	BS	500.00000	.	.	NONE	.
48	X6	LL	.	.	.	NONE	5284.00000
49	X7	BS	.	.	.	NONE	.
50	X8	BS	1075.00000	.	.	NONE	.
51	X9	BS	525.00000	.	.	NONE	.
52	X10	LL	.	.	.	NONE	37.30300
53	X11	LL	.	.	.	NONE	1521.93784
54	X12	LL	.	.	.	NONE	2620.00000
55	X13	LL	.	.	.	NONE	19098.25867
56	X14	LL	.	.	.	NONE	2702.37066
57	X15	LL	.	.	.	NONE	6488.25348
58	X16	LL	.	.	.	NONE	54.15205
59	X17	LL	.	.	.	NONE	449.75041
60	X18	LL	.	.	.	NONE	3754.25348
61	X19	BS	.	.	.	NONE	.
62	X20	LL	.	.	.	NONE	1129.41468
63	X21	BS	525.00000	.	.	NONE	.
64	X22	LL	.	.	.	NONE	1762.36558
65	X23	LL	.	.	.	NONE	2320.29939
66	X24	LL	.	.	.	NONE	4918.25348
67	X25	LL	.	.	.	NONE	41.00000
68	X26	LL	.	.	.	NONE	306.09477
69	X27	LL	.	.	.	NONE	2764.25348
70	X28	LL	.	.	.	NONE	94.12875
71	X29	LL	.	.	.	NONE	1823.45934
72	X30	LL	.	.	.	NONE	1200.84129
73	X31	LL	.	.	.	NONE	2757.19506
74	X32	LL	.	.	.	NONE	3567.61045
75	X33	LL	.	.	.	NONE	3536.00000
76	X34	LL	.	.	.	NONE	38.22420
77	X35	LL	.	.	.	NONE	363.90523
78	X36	BS	525.00000	.	.	NONE	.
79	X37	LL	.	.	.	NONE	37.30300
80	X38	LL	.	.	.	NONE	1264.84307
81	X39	LL	.	.	.	NONE	2620.00000
82	X40	LL	.	.	.	NONE	1909.94975
83	X41	LL	.	.	.	NONE	2424.27591
84	X42	LL	.	.	.	NONE	6385.15871
85	X43	LL	.	.	.	NONE	13.15205
86	X44	LL	.	.	.	NONE	143.65564
87	X45	LL	.	.	.	NONE	3619.15871
88	X46	BS	1132.27009	.	.	NONE	.
89	X47	LL	.	.	.	NONE	872.31990
90	X48	BS	525.00000	.	.	NONE	.
91	X49	LL	.	.	.	NONE	1744.36558

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
92	X50	LL	.	.	.	NONE	2042.20462
93	X51	LL	.	.	.	NONE	4815.15871
94	X52	BS	187.47227	.	.	NONE	.
95	X53	BS	255.25764	.	.	NONE	.
96	X54	LL	.	.	.	NONE	2629.15871
97	X55	LL	.	.	.	NONE	131.12875
98	X56	LL	.	.	.	NONE	1969.45934
99	X57	LL	.	.	.	NONE	935.84129
100	X58	LL	.	.	.	NONE	2778.19506
101	X59	LL	.	.	.	NONE	3695.61045
102	X60	LL	.	.	.	NONE	3554.00000
103	X61	LL	.	.	.	NONE	38.22420
104	X62	LL	.	.	.	NONE	467.90523
105	X63	BS	2100.00000	.	.	NONE	.
106	X64	LL	.	.	.	NONE	74.30300
107	X65	LL	.	.	.	NONE	1410.84307
108	X66	LL	.	.	.	NONE	2620.00000
109	X67	LL	.	.	.	NONE	1920.94975
110	X68	LL	.	.	.	NONE	2552.27591
111	X69	LL	.	.	.	NONE	6668.15871
112	X70	LL	.	.	.	NONE	581.17112
113	X71	LL	.	.	.	NONE	247.65564
114	X72	LL	.	.	.	NONE	3874.15871
115	X73	LL	.	.	.	NONE	37.00000
116	X74	LL	.	.	.	NONE	1018.31990
117	X75	BS	2100.00000	.	.	NONE	.
118	X76	LL	.	.	.	NONE	1765.36558
119	X77	LL	.	.	.	NONE	2170.20462
120	X78	LL	.	.	.	NONE	5098.15871
121	X79	BS	6300.00000	.	.	NONE	.
122	X80	LL	.	.	.	NONE	104.00000
123	X81	LL	.	.	.	NONE	2884.15871
124	X82	LL	.	10.00000	.	NONE	20.00000
125	X83	BS	2686598.55268	10.00000	.	NONE	.
126	X84	LL	.	1.00000	.	NONE	2.00000
127	X85	BS	1393764.67003	1.00000	.	NONE	.

SETUP TEST

TIME = 0.02

SCALE

MATRIX1 ASSIGNED TO MATRIX1
MATRIX2 ASSIGNED TO MATRIX2
MATRIX3 ASSIGNED TO MATRIX3
MATRIX4 ASSIGNED TO MATRIX4

ETA1 ASSIGNED TO ETA1
ETA2 ASSIGNED TO ETA2
ETA3 ASSIGNED TO ETA3
ETA4 ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE 7

NO CYCLING

POOLS	NUMBER	SIZE	CORE
H.REG-BITS MAP			184
WORK REGIONS	9	360	3240
MATRIX BUFFERS	5	2216	11080
ETA BUFFERS	6	5736	34416

	TOTAL	NORMAL	.FREE.	FIXED	BOUNDED
ROWS (LOG.VAR.)	42	36	1	5	0
COLUMNS (STR.VAR.)	85	85	0	0	0

751 ELEMENTS - DENSITY = 14.07 - 5 MATRIX RECORDS (¤ITHOUT RHS'S)

PRIMAL OBJ = GDF RHS = RH

TIME = 0.02 MINS. PRICING 7
SCALE = .

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
M	1	15	2	72	3.00195-	69763.3
	2		3	122	3.58115-	69763.3
	3		72	43	2.07239-	69763.3
M	4	15	122	69	14.0353-	69763.3
	5		69	67	.A4789-	69763.3
M	6	11	18	127	.18641-	45529.5
	7		10	125	.28918-	26623.9
	8		15	115	.05994-	23315.9
	9		21	88	.08774-	20514.8
	10		26	61	.00680-	20297.6

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
	11		7	64	.00971-	20254.2
M	12	5	42	99	1.46139-	7030.45
	13		36	78	1.00547-	5321.72
	14		14	72	.76807-	4894.41
	15		41	105	1.32540-	4894.41
	16		99	117	.02047-	4729.69
M	17	3	78	15	8.42295-	3210.82
	18		115	122	.48047-	3037.21
	19		13	10	1.45343-	2789.88
M	20	1	30	45	1.09333-	316.554
	21		24	123	.76106-	.

FEASIBLE SOLUTION

PRIMAL OBJ = GDF RHS = RH

TIME = 0.03 MINS. PRICING 7
SCALE = 1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	22	48	123	14	55.3657-	.15E+08
	23		9	66	58.7271-	.15E+08
	24		8	92	14.5131-	.15E+08
	25		66	93	8.63244-	.15E+08
M	26	33	67	62	26.2771-	.14E+08
M	27	25	122	112	16.2137-	.14E+08
	28		93	120	26.2898-	.14E+08
	29		92	119	20.1728-	.14E+08
	30		120	102	.27376-	.14E+08
M	31	15	22	116	3.12427-	.13E+08
	32		64	7	2.43522-	.12E+08
	33		11	118	2.21615-	.12E+08
M	34	6	105	99	.74998-	.12E+08
	35		119	8	3.79173-	.12E+08
	36		62	13	.48542-	.12E+08
	37		8	22	6.42087-	.12E+08
	38		112	121	.70101-	.12E+08
M	39	1	116	115	.41168-	.11E+08
M	40	2	118	8	.87322-	.11E+08
	41		35	75	.29528-	.11E+08

OPTIMAL SOLUTION

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	11195025.0001	11195025.0001-	NONE	NONE	1.00000
2	G1	EQ	.	.	.	.	1.00000
3	G2	EQ	.	.	.	.	10.00000
4	LC1	BS	157.30000	152.30000-	5.00000	NONE	.
5	LC2	BS	157.30000	152.30000-	5.00000	NONE	.
6	LC3	BS	216.80000	211.80000-	5.00000	NONE	.
7	LC4	BS	.	10.00000	NONE	10.00000	.
8	LC5	BS	.	10.00000	NONE	10.00000	.
9	LC6	UL	10.00000	.	NONE	10.00000	900.00000
10	LC7	BS	.	.	.	NONE	.
11	LC8	LL	.	.	.	NONE	21461.53846-
12	LC9	BS	.	.	.	NONE	.
13	KC1	BS	8817270.00002	6187370.00002-	2629899.99999	NONE	.
14	KC2	BS	8817269.99994	4837269.99995-	3979999.99999	NONE	.
15	KC3	BS	14261004.9999	6311004.99994-	7949999.99998	NONE	.
16	DC1	BS	2100.00000	2100.00000-	.	NONE	.
17	DC2	BS	2100.00000	2100.00000-	.	NONE	.
18	DC3	EQ	2625.00000	.	2625.00000	2625.00000	495.00000-
19	DC4	BS	2100.00000	2100.00000-	.	NONE	.
20	DC5	BS	2100.00000	2100.00000-	.	NONE	.
21	DC6	EQ	2625.00000	.	2625.00000	2625.00000	521.00000-
22	DC7	BS	7900.00000	7900.00000-	.	NONE	.
23	DC8	BS	7900.00000	7900.00000-	.	NONE	.
24	DC9	EQ	10500.00000	.	10500.00000	10500.00000	557.00000-
25	D1AA	BS	2100.00000	2100.00000-	.	NONE	.
26	D1AS	LL	.	.	.	NONE	429.00000-
27	D2AA	BS	.	.	.	NONE	.
28	D2AS	BS	.	.	.	NONE	.
29	D3AA	BS	525.00000	.	525.00000	NONE	.
30	D3AS	LL	525.00000	.	525.00000	NONE	832.00000-
31	D1BA	BS	2100.00000	2100.00000-	.	NONE	.
32	D1BS	BS	.	.	.	NONE	.
33	D2BA	BS	.	.	.	NONE	.
34	D2BS	BS	.	.	.	NONE	.
35	D3BA	LL	525.00000	.	525.00000	NONE	21.00000-
36	D3BS	LL	525.00000	.	525.00000	NONE	632.00000-
37	D1CA	BS	7900.00000	7900.00000-	.	NONE	.
38	D1CS	BS	.	.	.	NONE	.
39	D2CA	BS	.	.	.	NONE	.
40	D2CS	BS	.	.	.	NONE	.
41	D3CA	LL	2100.00000	.	2100.00000	NONE	241.00000-
42	D3CS	LL	2100.00000	.	2100.00000	NONE	616.00000-

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
43	X1	BS	.	.	.	NONE	.
44	X2	LL	.	.	.	NONE	611.00000
45	X3	BS	525.00000	.	.	NONE	.
46	X4	LL	.	.	.	NONE	193.00000
47	X5	LL	.	.	.	NONE	881.00000
48	X6	LL	.	.	.	NONE	395.00000
49	X7	LL	.	.	.	NONE	364.76923
50	X8	LL	.	.	.	NONE	995.76923
51	X9	LL	.	.	.	NONE	997.00000
52	X10	LL	.	.	.	NONE	181.00000
53	X11	LL	.	.	.	NONE	313.00000
54	X12	LL	.	.	.	NONE	475.00000
55	X13	LL	.	.	.	NONE	468.60000
56	X14	LL	.	.	.	NONE	627.60000
57	X15	LL	.	.	.	NONE	841.60000
58	X16	LL	.	.	.	NONE	827.53846
59	X17	LL	.	.	.	NONE	1077.53846
60	X18	LL	.	.	.	NONE	1704.00000
61	X19	BS	2100.00000	.	.	NONE	.
62	X20	LL	.	.	.	NONE	96.00000
63	X21	LL	.	.	.	NONE	213.00000
64	X22	LL	.	.	.	NONE	358.70000
65	X23	LL	.	.	.	NONE	495.70000
66	X24	LL	.	.	.	NONE	683.70000
67	X25	LL	.	.	.	NONE	780.00000
68	X26	LL	.	.	.	NONE	1016.00000
69	X27	LL	.	.	.	NONE	1605.00000
70	X28	LL	.	.	.	NONE	429.00000
71	X29	LL	.	.	.	NONE	624.00000
72	X30	BS	525.00000	.	.	NONE	.
73	X31	LL	.	.	.	NONE	442.00000
74	X32	LL	.	.	.	NONE	614.00000
75	X33	BS	.	.	.	NONE	.
76	X34	LL	.	.	.	NONE	512.53846
77	X35	LL	.	.	.	NONE	585.76923
78	X36	LL	.	.	.	NONE	460.00000
79	X37	LL	.	.	.	NONE	181.00000
80	X38	LL	.	.	.	NONE	326.00000
81	X39	LL	.	.	.	NONE	475.00000
82	X40	LL	.	.	.	NONE	288.60000
83	X41	LL	.	.	.	NONE	430.60000
84	X42	LL	.	.	.	NONE	612.60000
85	X43	LL	.	.	.	NONE	417.53846
86	X44	LL	.	.	.	NONE	600.53846
87	X45	LL	.	.	.	NONE	1155.00000
88	X46	BS	2100.00000	.	.	NONE	.
89	X47	LL	.	.	.	NONE	109.00000
90	X48	LL	.	.	.	NONE	213.00000
91	X49	LL	.	.	.	NONE	178.70000

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
92	X50	LL	.	.	.	NONE	298.70000
93	X51	LL	.	.	.	NONE	454.70000
94	X52	LL	.	.	.	NONE	370.00000
95	X53	LL	.	.	.	NONE	539.00000
96	X54	LL	.	.	.	NONE	1056.00000
97	X55	LL	.	.	.	NONE	429.00000
98	X56	LL	.	.	.	NONE	625.00000
99	X57	BS	1600.00000	.	.	NONE	.
100	X58	LL	.	.	.	NONE	282.00000
101	X59	LL	.	.	.	NONE	435.00000
102	X60	BS	500.00000	.	.	NONE	.
103	X61	LL	.	.	.	NONE	142.53846
104	X62	LL	.	.	.	NONE	166.76923
105	X63	LL	.	.	.	NONE	181.00000
106	X64	LL	.	.	.	NONE	181.00000
107	X65	LL	.	.	.	NONE	327.00000
108	X66	LL	.	.	.	NONE	262.00000
109	X67	LL	.	.	.	NONE	127.60000
110	X68	LL	.	.	.	NONE	251.60000
111	X69	LL	.	.	.	NONE	399.60000
112	X70	LL	.	.	.	NONE	48.53846
113	X71	LL	.	.	.	NONE	181.53846
114	X72	LL	.	.	.	NONE	662.00000
115	X73	BS	7900.00000	.	.	NONE	.
116	X74	LL	.	.	.	NONE	110.00000
117	X75	BS	500.00000	.	.	NONE	.
118	X76	LL	.	.	.	NONE	18.70000
119	X77	LL	.	.	.	NONE	119.70000
120	X78	LL	.	.	.	NONE	241.70000
121	X79	BS	.	.	.	NONE	.
122	X80	LL	.	.	.	NONE	120.00000
123	X81	LL	.	.	.	NONE	563.00000
124	X82	LL	.	1.00000	.	NONE	2.00000
125	X83	BS	8439275.00010	1.00000	.	NONE	.
126	X84	LL	.	10.00000	.	NONE	20.00000
127	X85	BS	275575.00000	10.00000	.	NONE	.

SETUP TEST

TIME = 0.02

SCALE

```
MATRIX1  ASSIGNED TO MATRIX1
MATRIX2  ASSIGNED TO MATRIX2
MATRIX3  ASSIGNED TO MATRIX3
MATRIX4  ASSIGNED TO MATRIX4

ETA1     ASSIGNED TO ETA1
ETA2     ASSIGNED TO ETA2
ETA3     ASSIGNED TO ETA3
ETA4     ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2
```

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE 7

NO CYCLING

POOLS	NUMBER	SIZE	CORE
H.REG-BITS MAP			184
WORK REGIONS	9	360	3240
MATRIX BUFFERS	5	2216	11080
ETA BUFFERS	6	5736	34416

	TOTAL	NORMAL	.FREE.	FIXED	BOUNDED
ROWS (LOG.VAR.)	42	36	1	5	0
COLUMNS (STR.VAR.)	85	85	0	0	0

751 ELEMENTS - DENSITY = 14.07 - 5 MATRIX RECORDS (WITHOUT RHS'S)

PRIMAL OBJ = GDF RHS = RH

TIME = 0.02 MINS. PRICING 7
SCALE = .

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
M	1	9	2	72	1.84560-	26379.3
	2		3	51	1.52675-	26379.3
	3		51	78	.56823-	26379.3
M	4	9	72	90	.22065-	26379.3
	5		78	125	.11185-	26379.3
	6		35	127	.54685-	23903.6
	7		29	63	1.26017-	21852.0
M	8	5	42	99	1.53594-	7953.59
	9		36	78	1.37301-	5620.27
	10		30	51	1.50107-	4351.39

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
	11		24	117	.03664-	3172.40
M	12	1	21	92	1.00000-	1270.69
	13		18	66	1.00000-	.

FEASIBLE SOLUTION

PRIMAL OBJ = GDF RHS = RH

TIME = 0.03 MINS. PRICING 7
SCALE = 1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	14	32	117	112	105.106-	.11E+08
	15		9	119	5.44710-	.11E+08
	16		41	105	13.3681-	.11E+08
M	17	16	99	117	5.38542-	.10E+08
	18		8	61	1.89570-	.97E+07
	19		112	121	1.45165-	.94E+07
M	20	12	63	45	4.38292-	.92E+07
	21		66	9	21.9952-	.92E+07
	22		51	48	4.28647-	.92E+07
	23		9	75	2.20165-	.92E+07
M	24	7	119	118	.90883-	.91E+07
	25		7	88	.69148-	.89E+07
	26		92	8	7.64056-	.89E+07
	27		48	29	1.86156-	.89E+07

OPTIMAL SOLUTION

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	8878489.42306	8878489.42306-	NONE	NONE	1.00000
2	G1	EQ	.	.	.	.	1.00000
3	G2	EQ	.	.	.	.	5.50000
4	LC1	BS	47.77500	47.77500-	.	NONE	.
5	LC2	BS	47.77500	47.77500-	.	NONE	.
6	LC3	BS	92.40000	92.40000-	.	NONE	.
7	LC4	UL	50.00000	.	NONE	50.00000	1038.46154
8	LC5	BS	50.00000	.	NONE	50.00000	.
9	LC6	UL	50.00000	.	NONE	50.00000	5000.00000
10	LC7	BS	31.90000	31.90000-	.	NONE	.
11	LC8	BS	31.90000	31.90000-	.	NONE	.
12	LC9	BS	84.40000	84.40000-	.	NONE	.
13	KC1	BS	5038324.80770	5038324.80770-	.	NONE	.
14	KC2	BS	5038324.80764	5038324.80764-	.	NONE	.
15	KC3	BS	10339984.8076	10339984.8076-	.	NONE	.
16	DC1	BS	2100.00000	2100.00000-	.	NONE	.
17	DC2	BS	2100.00000	2100.00000-	.	NONE	.
18	DC3	EQ	2625.00000	.	2625.00000	2625.00000	391.50000-
19	DC4	BS	1575.00000	1575.00000-	.	NONE	.
20	DC5	BS	1575.00000	1575.00000-	.	NONE	.
21	DC6	EQ	2625.00000	.	2625.00000	2625.00000	444.50000-
22	DC7	BS	6300.00000	6300.00000-	.	NONE	.
23	DC8	BS	6300.00000	6300.00000-	.	NONE	.
24	DC9	EQ	10500.00000	.	10500.00000	10500.00000	449.00000-
25	D1AA	BS	2100.00000	2100.00000-	.	NONE	.
26	D1AS	BS	.	.	.	NONE	.
27	D2AA	BS	.	.	.	NONE	.
28	D2AS	BS	.	.	.	NONE	.
29	D3AA	BS	525.00000	.	525.00000	NONE	.
30	D3AS	LL	525.00000	.	525.00000	NONE	805.00000-
31	D1BA	BS	1575.00000	1575.00000-	.	NONE	.
32	D1BS	BS	.	.	.	NONE	.
33	D2BA	BS	.	.	.	NONE	.
34	D2BS	BS	.	.	.	NONE	.
35	D3BA	LL	525.00000	.	525.00000	NONE	211.50000-
36	D3BS	LL	525.00000	.	525.00000	NONE	608.00000-
37	D1CA	BS	.	.	.	NONE	.
38	D1CS	BS	.	.	.	NONE	.
39	D2CA	BS	.	.	.	NONE	.
40	D2CS	BS	.	.	.	NONE	.
41	D3CA	LL	2100.00000	.	2100.00000	NONE	277.00000-
42	D3CS	LL	2100.00000	.	2100.00000	NONE	398.50000-

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
43	X1	LL	.	.	.	NONE	429.00000
44	X2	LL	.	.	.	NONE	602.00000
45	X3	BS	525.00000	.	.	NONE	.
46	X4	LL	.	.	.	NONE	436.76923
47	X5	LL	.	.	.	NONE	621.00000
48	X6	LL	.	.	.	NONE	81.00000
49	X7	LL	.	.	.	NONE	543.50000
50	X8	LL	.	.	.	NONE	633.00000
51	X9	LL	.	.	.	NONE	79.00000
52	X10	LL	.	.	.	NONE	181.00000
53	X11	LL	.	.	.	NONE	304.00000
54	X12	LL	.	.	.	NONE	448.00000
55	X13	LL	.	.	.	NONE	252.53846
56	X14	LL	.	.	.	NONE	343.00000
57	X15	LL	.	.	.	NONE	476.00000
58	X16	LL	.	.	.	NONE	448.50000
59	X17	LL	.	.	.	NONE	586.00000
60	X18	LL	.	.	.	NONE	759.00000
61	X19	BS	2100.00000	.	.	NONE	.
62	X20	LL	.	.	.	NONE	87.00000
63	X21	LL	.	.	.	NONE	186.00000
64	X22	LL	.	.	.	NONE	137.50000
65	X23	LL	.	.	.	NONE	207.00000
66	X24	LL	.	.	.	NONE	314.00000
67	X25	LL	.	.	.	NONE	379.50000
68	X26	LL	.	.	.	NONE	503.00000
69	X27	LL	.	.	.	NONE	660.00000
70	X28	LL	.	.	.	NONE	429.00000
71	X29	LL	.	.	.	NONE	615.00000
72	X30	LL	.	.	.	NONE	11.00000
73	X31	LL	.	.	.	NONE	337.76923
74	X32	LL	.	.	.	NONE	448.50000
75	X33	BS	.	.	.	NONE	.
76	X34	LL	.	.	.	NONE	318.00000
77	X35	LL	.	.	.	NONE	443.50000
78	X36	BS	525.00000	.	.	NONE	.
79	X37	LL	.	.	.	NONE	181.00000
80	X38	LL	.	.	.	NONE	317.00000
81	X39	LL	.	.	.	NONE	262.00000
82	X40	LL	.	.	.	NONE	153.53846
83	X41	LL	.	.	.	NONE	240.50000
84	X42	LL	.	.	.	NONE	364.00000
85	X43	LL	.	.	.	NONE	223.00000
86	X44	LL	.	.	.	NONE	329.50000
87	X45	LL	.	.	.	NONE	471.00000
88	X46	BS	1575.00000	.	.	NONE	.
89	X47	LL	.	.	.	NONE	100.00000
90	X48	BS	525.00000	.	.	NONE	.
91	X49	LL	.	.	.	NONE	38.50000

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
92	X50	LL	.	.	.	NONE	104.50000
93	X51	LL	.	.	.	NONE	202.00000
94	X52	LL	.	.	.	NONE	154.00000
95	X53	LL	.	.	.	NONE	246.50000
96	X54	LL	.	.	.	NONE	372.00000
97	X55	LL	.	.	.	NONE	478.50000
98	X56	LL	.	.	.	NONE	670.00000
99	X57	LL	.	.	.	NONE	219.50000
100	X58	LL	.	.	.	NONE	299.26923
101	X59	LL	.	.	.	NONE	404.50000
102	X60	LL	.	.	.	NONE	153.00000
103	X61	LL	.	.	.	NONE	164.00000
104	X62	LL	.	.	.	NONE	267.50000
105	X63	BS	2100.00000	.	.	NONE	.
106	X64	LL	.	.	.	NONE	230.50000
107	X65	LL	.	.	.	NONE	372.00000
108	X66	LL	.	.	.	NONE	262.00000
109	X67	LL	.	.	.	NONE	114.03846
110	X68	LL	.	.	.	NONE	196.50000
111	X69	LL	.	.	.	NONE	308.50000
112	X70	LL	.	.	.	NONE	70.00000
113	X71	LL	.	.	.	NONE	153.50000
114	X72	LL	.	.	.	NONE	261.50000
115	X73	LL	.	.	.	NONE	49.50000
116	X74	LL	.	.	.	NONE	155.00000
117	X75	BS	2100.00000	.	.	NONE	.
118	X76	BS	3846.15385	.	.	NONE	.
119	X77	LL	.	.	.	NONE	60.50000
120	X78	LL	.	.	.	NONE	146.50000
121	X79	BS	2453.84615	.	.	NONE	.
122	X80	LL	.	.	.	NONE	70.50000
123	X81	LL	.	.	.	NONE	162.50000
124	X82	LL	.	1.00000	.	NONE	2.00000
125	X83	BS	4270092.30768	1.00000	.	NONE	.
126	X84	LL	.	5.50000	.	NONE	11.00000
127	X85	BS	837890.38462	5.50000	.	NONE	.

```
SETUP   TEST

TIME =    0.02

    SCALE


MATRIX1   ASSIGNED TO MATRIX1
MATRIX2   ASSIGNED TO MATRIX2
MATRIX3   ASSIGNED TO MATRIX3
MATRIX4   ASSIGNED TO MATRIX4

ETA1      ASSIGNED TO ETA1
ETA2      ASSIGNED TO ETA2
ETA3      ASSIGNED TO ETA3
ETA4      ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE   7

NO CYCLING

     POOLS          NUMBER    SIZE     CORE
H.REG-BITS MAP                          184
WORK    REGIONS        9       360     3240
MATRIX BUFFERS         5      2216    11080
ETA    BUFFERS         6      5736    34416

                            TOTAL  NORMAL  .FREE.  FIXED BOUNDED
ROWS     (LOG.VAR.)           42     36       1       5      0
COLUMNS  (STR.VAR.)           85     85       0       0      0

     751 ELEMENTS - DENSITY =  14.07 -    5 MATRIX RECORDS (WITHOUT RHS'S)


PRIMAL        OBJ = GDF           RHS = RH

TIME =     0.03  MINS.        PRICING    7
 SCALE =     .

    ITER   NUMBER  VECTOR  VECTOR  REDUCED      SUM
  NUMBER   INFEAS    OUT      IN     COST      INFEAS
M      1      9        2      72   1.84560-   26379.3
       2               3      51   1.52675-   26379.3
       3              51      78    .56823-   26379.3
M      4      9       72      90    .22065-   26379.3
       5              90     127    .06368-   26379.3
       6              36     125    .28068-   24046.0
       7              29      63   1.26017-   21994.4
M      8      5       42      99   1.53594-   8095.98
       9              35      90   1.34417-   5620.27
      10              30      51   1.50107-   4351.39
```

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
	11		24	117	.03664-	3172.40
M	12	1	21	92	1.00000-	1270.69
	13		18	66	1.00000-	.

FEASIBLE SOLUTION

PRIMAL OBJ = GDF RHS = RH

TIME = 0.03 MINS. PRICING 7
SCALE = 1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	14	27	117	122	365.536-	.67E+07
M	15	9	122	121	1.40479-	.64E+07
	16		66	64	2.21855-	.62E+07
	17		92	91	1.16474-	.61E+07
	18		41	105	38.3018-	.61E+07
M	19	2	99	117	10.0979-	.55E+07

OPTIMAL SOLUTION

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	5489137.49996	5489137.49996-	NONE	NONE	1.00000
2	G1	EQ	.	.	.	.	1.00000
3	G2	EQ	.	.	.	.	2.50000
4	LC1	BS	.	.	.	NONE	.
5	LC2	BS	.	.	.	NONE	.
6	LC3	BS	40.95000	40.95000-	.	NONE	.
7	LC4	BS	40.95000	9.05000	NONE	50.00000	.
8	LC5	BS	40.95000	9.05000	NONE	50.00000	.
9	LC6	BS	40.95000	9.05000	NONE	50.00000	.
10	LC7	BS	81.90000	81.90000-	.	NONE	.
11	LC8	BS	81.90000	81.90000-	.	NONE	.
12	LC9	BS	144.90000	144.90000-	.	NONE	.
13	KC1	BS	3126689.99999	3126689.99999-	.	NONE	.
14	KC2	BS	3126689.99998	3126689.99998-	.	NONE	.
15	KC3	BS	8437904.99994	8437904.99994-	.	NONE	.
16	DC1	BS	1575.00000	1575.00000-	.	NONE	.
17	DC2	BS	1575.00000	1575.00000-	.	NONE	.
18	DC3	EQ	2625.00000	.	2625.00000	2625.00000	189.50000-
19	DC4	BS	1575.00000	1575.00000-	.	NONE	.
20	DC5	BS	1575.00000	1575.00000-	.	NONE	.
21	DC6	EQ	2625.00000	.	2625.00000	2625.00000	215.50000-
22	DC7	BS	6300.00000	6300.00000-	.	NONE	.
23	DC8	BS	6300.00000	6300.00000-	.	NONE	.
24	DC9	EQ	10500.00000	.	10500.00000	10500.00000	191.00000-
25	D1AA	BS	.	.	.	NONE	.
26	D1AS	BS	.	.	.	NONE	.
27	D2AA	BS	.	.	.	NONE	.
28	D2AS	BS	.	.	.	NONE	.
29	D3AA	LL	525.00000	.	525.00000	NONE	301.00000-
30	D3AS	LL	525.00000	.	525.00000	NONE	387.00000-
31	D1BA	BS	.	.	.	NONE	.
32	D1BS	BS	.	.	.	NONE	.
33	D2BA	BS	.	.	.	NONE	.
34	D2BS	BS	.	.	.	NONE	.
35	D3BA	LL	525.00000	.	525.00000	NONE	374.50000-
36	D3BS	LL	525.00000	.	525.00000	NONE	330.00000-
37	D1CA	BS	.	.	.	NONE	.
38	D1CS	BS	.	.	.	NONE	.
39	D2CA	BS	.	.	.	NONE	.
40	D2CS	BS	.	.	.	NONE	.
41	D3CA	LL	2100.00000	.	2100.00000	NONE	487.00000-
42	D3CS	LL	2100.00000	.	2100.00000	NONE	317.50000-

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
43	X1	LL	.	.	.	NONE	562.00000
44	X2	LL	.	.	.	NONE	729.00000
45	X3	LL	.	.	.	NONE	232.00000
46	X4	LL	.	.	.	NONE	257.00000
47	X5	LL	.	.	.	NONE	426.00000
48	X6	LL	.	.	.	NONE	250.00000
49	X7	LL	.	.	.	NONE	223.50000
50	X8	LL	.	.	.	NONE	238.00000
51	X9	BS	525.00000	.	.	NONE	.
52	X10	LL	.	.	.	NONE	314.00000
53	X11	LL	.	.	.	NONE	431.00000
54	X12	LL	.	.	.	NONE	262.00000
55	X13	LL	.	.	.	NONE	109.00000
56	X14	LL	.	.	.	NONE	178.00000
57	X15	LL	.	.	.	NONE	257.00000
58	X16	LL	.	.	.	NONE	129.50000
59	X17	LL	.	.	.	NONE	191.00000
60	X18	LL	.	.	.	NONE	262.00000
61	X19	LL	.	.	.	NONE	133.00000
62	X20	LL	.	.	.	NONE	214.00000
63	X21	BS	525.00000	.	.	NONE	.
64	X22	BS	1575.00000	.	.	NONE	.
65	X23	LL	.	.	.	NONE	47.00000
66	X24	LL	.	.	.	NONE	100.00000
67	X25	LL	.	.	.	NONE	59.50000
68	X26	LL	.	.	.	NONE	108.00000
69	X27	LL	.	.	.	NONE	163.00000
70	X28	LL	.	.	.	NONE	607.00000
71	X29	LL	.	.	.	NONE	787.00000
72	X30	LL	.	.	.	NONE	289.00000
73	X31	LL	.	.	.	NONE	257.00000
74	X32	LL	.	.	.	NONE	361.50000
75	X33	LL	.	.	.	NONE	152.00000
76	X34	LL	.	.	.	NONE	166.00000
77	X35	LL	.	.	.	NONE	240.50000
78	X36	BS	525.00000	.	.	NONE	.
79	X37	LL	.	.	.	NONE	359.00000
80	X38	LL	.	.	.	NONE	489.00000
81	X39	LL	.	.	.	NONE	262.00000
82	X40	LL	.	.	.	NONE	109.00000
83	X41	LL	.	.	.	NONE	183.50000
84	X42	LL	.	.	.	NONE	268.00000
85	X43	LL	.	.	.	NONE	71.00000
86	X44	LL	.	.	.	NONE	126.50000
87	X45	LL	.	.	.	NONE	193.00000
88	X46	LL	.	.	.	NONE	178.00000
89	X47	LL	.	.	.	NONE	272.00000
90	X48	BS	525.00000	.	.	NONE	.
91	X49	BS	1575.00000	.	.	NONE	.

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
92	X50	LL	.	.	.	NONE	52.50000
93	X51	LL	.	.	.	NONE	111.00000
94	X52	LL	.	.	.	NONE	2.00000
95	X53	LL	.	.	.	NONE	43.50000
96	X54	LL	.	.	.	NONE	94.00000
97	X55	LL	.	.	.	NONE	697.50000
98	X56	LL	.	.	.	NONE	886.00000
99	X57	LL	.	.	.	NONE	300.50000
100	X58	LL	.	.	.	NONE	307.50000
101	X59	LL	.	.	.	NONE	415.50000
102	X60	LL	.	.	.	NONE	221.00000
103	X61	LL	.	.	.	NONE	164.00000
104	X62	LL	.	.	.	NONE	234.50000
105	X63	BS	2100.00000	.	.	NONE	.
106	X64	LL	.	.	.	NONE	449.50000
107	X65	LL	.	.	.	NONE	588.00000
108	X66	LL	.	.	.	NONE	262.00000
109	X67	LL	.	.	.	NONE	158.50000
110	X68	LL	.	.	.	NONE	237.50000
111	X69	LL	.	.	.	NONE	325.50000
112	X70	LL	.	.	.	NONE	70.00000
113	X71	LL	.	.	.	NONE	120.50000
114	X72	LL	.	.	.	NONE	180.50000
115	X73	LL	.	.	.	NONE	268.50000
116	X74	LL	.	.	.	NONE	371.00000
117	X75	BS	2100.00000	.	.	NONE	.
118	X76	LL	.	.	.	NONE	50.50000
119	X77	LL	.	.	.	NONE	106.50000
120	X78	LL	.	.	.	NONE	168.50000
121	X79	BS	6300.00000	.	.	NONE	.
122	X80	LL	.	.	.	NONE	37.50000
123	X81	LL	.	.	.	NONE	81.50000
124	X82	LL	.	1.00000	.	NONE	2.00000
125	X83	BS	2272199.99999	1.00000	.	NONE	.
126	X84	LL	.	2.50000	.	NONE	5.00000
127	X85	BS	1286775.00000	2.50000	.	NONE	.

SETUP TEST

TIME = 0.02

SCALE

```
MATRIX1   ASSIGNED TO MATRIX1
MATRIX2   ASSIGNED TO MATRIX2
MATRIX3   ASSIGNED TO MATRIX3
MATRIX4   ASSIGNED TO MATRIX4

ETA1      ASSIGNED TO ETA1
ETA2      ASSIGNED TO ETA2
ETA3      ASSIGNED TO ETA3
ETA4      ASSIGNED TO ETA4

SCRATCH1  ASSIGNED TO SCRATCH1
SCRATCH2  ASSIGNED TO SCRATCH2
```

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE 7

NO CYCLING

POOLS	NUMBER	SIZE	CORE
H.REG-BITS MAP			184
WORK REGIONS	9	360	3240
MATRIX BUFFERS	5	2216	11080
ETA BUFFERS	6	5736	34416

	TOTAL	NORMAL	.FREE.	FIXED	BOUNDED
ROWS (LOG.VAR.)	42	36	1	5	0
COLUMNS (STR.VAR.)	85	85	0	0	0

751 ELEMENTS - DENSITY = 14.07 - 5 MATRIX RECORDS (WITHOUT RHS'S)

PRIMAL OBJ = GDF RHS = RH

TIME = 0.03 MINS. PRICING 7
SCALE = .

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
M	1	9	2	72	1.84560-	26379.3
	2		3	51	1.52675-	26379.3
	3		51	78	.56823-	26379.3
M	4	9	72	90	.22065-	26379.3
	5		90	127	.12183-	26379.3
	6		36	125	.21477-	24046.0
	7		29	63	1.26017-	21994.4
M	8	5	42	99	1.53594-	8095.98
	9		35	90	1.34417-	5620.27
	10		30	51	1.50107-	4351.39

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
	11		24	117	.03664-	3172.40
M	12	1	21	92	1.00000-	1270.69
	13		18	66	1.00000-	.

FEASIBLE SOLUTION

PRIMAL OBJ = GDF RHS = RH

TIME = 0.03 MINS. PRICING 7
SCALE = 1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	14	36	117	122	606.844-	.46E+07
M	15	18	92	95	16.3031-	.44E+07
	16		66	68	13.6656-	.42E+07
	17		41	105	59.3049-	.42E+07
M	18	5	99	117	15.8978-	.35E+07
	19		122	121	1.02611-	.33E+07
	20		95	94	.50221-	.33E+07
	21		68	67	.34527-	.33E+07

OPTIMAL SOLUTION

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	3299100.00000	3299100.00000-	NONE	NONE	1.00000
2	G1	EQ	.	.	.	.	1.00000
3	G2	EQ	.	.	.	.	1.00000
4	LC1	BS	.	.	.	NONE	.
5	LC2	BS	.	.	.	NONE	.
6	LC3	BS	40.95000	40.95000-	.	NONE	.
7	LC4	BS	.	50.00000	NONE	50.00000	.
8	LC5	BS	.	50.00000	NONE	50.00000	.
9	LC6	BS	.	50.00000	NONE	50.00000	.
10	LC7	BS	122.85000	122.85000-	.	NONE	.
11	LC8	BS	122.85000	122.85000-	.	NONE	.
12	LC9	BS	185.85000	185.85000-	.	NONE	.
13	KC1	BS	2628989.99999	2628989.99999-	.	NONE	.
14	KC2	BS	2628989.99998	2628989.99998-	.	NONE	.
15	KC3	BS	7940204.99994	7940204.99994-	.	NONE	.
16	DC1	BS	1575.00000	1575.00000-	.	NONE	.
17	DC2	BS	1575.00000	1575.00000-	.	NONE	.
18	DC3	EQ	2625.00000	.	2625.00000	2625.00000	12.00000
19	DC4	BS	1575.00000	1575.00000-	.	NONE	.
20	DC5	BS	1575.00000	1575.00000-	.	NONE	.
21	DC6	EQ	2625.00000	.	2625.00000	2625.00000	27.00000-
22	DC7	BS	6300.00000	6300.00000-	.	NONE	.
23	DC8	BS	6300.00000	6300.00000-	.	NONE	.
24	DC9	EQ	10500.00000	.	10500.00000	10500.00000	62.00000-
25	D1AA	BS	.	.	.	NONE	.
26	D1AS	BS	.	.	.	NONE	.
27	D2AA	BS	.	.	.	NONE	.
28	D2AS	BS	.	.	.	NONE	.
29	D3AA	LL	525.00000	.	525.00000	NONE	459.00000-
30	D3AS	LL	525.00000	.	525.00000	NONE	239.00000-
31	D1BA	BS	.	.	.	NONE	.
32	D1BS	BS	.	.	.	NONE	.
33	D2BA	BS	.	.	.	NONE	.
34	D2BS	BS	.	.	.	NONE	.
35	D3BA	LL	525.00000	.	525.00000	NONE	530.00000-
36	D3BS	LL	525.00000	.	525.00000	NONE	265.00000-
37	D1CA	BS	.	.	.	NONE	.
38	D1CS	BS	.	.	.	NONE	.
39	D2CA	BS	.	.	.	NONE	.
40	D2CS	BS	.	.	.	NONE	.
41	D3CA	LL	2100.00000	.	2100.00000	NONE	592.00000-
42	D3CS	LL	2100.00000	.	2100.00000	NONE	277.00000-

2629
3970
1342

2682

7940

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
43	X1	LL	.	.	.	NONE	729.00000
44	X2	LL	.	.	.	NONE	893.00000
45	X3	LL	.	.	.	NONE	380.00000
46	X4	LL	.	.	.	NONE	328.00000
47	X5	LL	.	.	.	NONE	479.00000
48	X6	LL	.	.	.	NONE	424.00000
49	X7	LL	.	.	.	NONE	164.00000
50	X8	LL	.	.	.	NONE	141.00000
51	X9	BS	525.00000	.	.	NONE	.
52	X10	LL	.	.	.	NONE	481.00000
53	X11	LL	.	.	.	NONE	595.00000
54	X12	LL	.	.	.	NONE	262.00000
55	X13	LL	.	.	.	NONE	180.00000
56	X14	LL	.	.	.	NONE	231.00000
57	X15	LL	.	.	.	NONE	283.00000
58	X16	LL	.	.	.	NONE	69.00000
59	X17	LL	.	.	.	NONE	94.00000
60	X18	LL	.	.	.	NONE	114.00000
61	X19	LL	.	.	.	NONE	300.00000
62	X20	LL	.	.	.	NONE	378.00000
63	X21	BS	525.00000	.	.	NONE	.
64	X22	LL	.	.	.	NONE	71.00000
65	X23	LL	.	.	.	NONE	100.00000
66	X24	LL	.	.	.	NONE	126.00000
67	X25	BS	1575.00000	.	.	NONE	.
68	X26	LL	.	.	.	NONE	11.00000
69	X27	LL	.	.	.	NONE	15.00000
70	X28	LL	.	.	.	NONE	770.00000
71	X29	LL	.	.	.	NONE	947.00000
72	X30	LL	.	.	.	NONE	354.00000
73	X31	LL	.	.	.	NONE	351.00000
74	X32	LL	.	.	.	NONE	442.00000
75	X33	LL	.	.	.	NONE	278.00000
76	X34	LL	.	.	.	NONE	164.00000
77	X35	LL	.	.	.	NONE	213.00000
78	X36	BS	525.00000	.	.	NONE	.
79	X37	LL	.	.	.	NONE	522.00000
80	X38	LL	.	.	.	NONE	649.00000
81	X39	LL	.	.	.	NONE	262.00000
82	X40	LL	.	.	.	NONE	203.00000
83	X41	LL	.	.	.	NONE	264.00000
84	X42	LL	.	.	.	NONE	329.00000
85	X43	LL	.	.	.	NONE	69.00000
86	X44	LL	.	.	.	NONE	99.00000
87	X45	LL	.	.	.	NONE	128.00000
88	X46	LL	.	.	.	NONE	341.00000
89	X47	LL	.	.	.	NONE	432.00000
90	X48	BS	525.00000	.	.	NONE	.
91	X49	LL	.	.	.	NONE	94.00000

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
92	X50	LL	.	.	.	NONE	133.00000
93	X51	LL	.	.	.	NONE	172.00000
94	X52	BS	1575.00000	.	.	NONE	.
95	X53	LL	.	.	.	NONE	15.00000
96	X54	LL	.	.	.	NONE	29.00000
97	X55	LL	.	.	.	NONE	807.00000
98	X56	LL	.	.	.	NONE	994.00000
99	X57	LL	.	.	.	NONE	341.00000
100	X58	LL	.	.	.	NONE	372.00000
101	X59	LL	.	.	.	NONE	471.00000
102	X60	LL	.	.	.	NONE	305.00000
103	X61	LL	.	.	.	NONE	164.00000
104	X62	LL	.	.	.	NONE	218.00000
105	X63	BS	2100.00000	.	.	NONE	.
106	X64	LL	.	.	.	NONE	559.00000
107	X65	LL	.	.	.	NONE	696.00000
108	X66	LL	.	.	.	NONE	262.00000
109	X67	LL	.	.	.	NONE	223.00000
110	X68	LL	.	.	.	NONE	293.00000
111	X69	LL	.	.	.	NONE	369.00000
112	X70	LL	.	.	.	NONE	70.00000
113	X71	LL	.	.	.	NONE	104.00000
114	X72	LL	.	.	.	NONE	140.00000
115	X73	LL	.	.	.	NONE	378.00000
116	X74	LL	.	.	.	NONE	479.00000
117	X75	BS	2100.00000	.	.	NONE	.
118	X76	LL	.	.	.	NONE	115.00000
119	X77	LL	.	.	.	NONE	162.00000
120	X78	LL	.	.	.	NONE	212.00000
121	X79	BS	6300.00000	.	.	NONE	.
122	X80	LL	.	.	.	NONE	21.00000
123	X81	LL	.	.	.	NONE	41.00000
124	X82	LL	.	1.00000	.	NONE	2.00000
125	X83	BS	1774500.00000	1.00000	.	NONE	.
126	X84	LL	.	1.00000	.	NONE	2.00000
127	X85	BS	1524600.00001	1.00000	.	NONE	.

```
SETUP  TEST

TIME =    0.01

    SCALE


MATRIX1   ASSIGNED TO MATRIX1
MATRIX2   ASSIGNED TO MATRIX2
MATRIX3   ASSIGNED TO MATRIX3
MATRIX4   ASSIGNED TO MATRIX4

ETA1      ASSIGNED TO ETA1
ETA2      ASSIGNED TO ETA2
ETA3      ASSIGNED TO ETA3
ETA4      ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE   7

NO CYCLING

    POOLS          NUMBER   SIZE    CORE
H.REG-BITS MAP                       184
WORK    REGIONS      9       360    3240
MATRIX BUFFERS       5      2216   11080
ETA     BUFFERS      6      5736   34416

                          TOTAL  NORMAL  .FREE.   FIXED BOUNDED
ROWS     (LOG.VAR.)         42      36       1       5       0
COLUMNS  (STR.VAR.)         85      85       0       0       0

     751 ELEMENTS - DENSITY =  14.07 -     5 MATRIX RECORDS (WITHOUT RHS'S)


PRIMAL       OBJ = GDF          RHS = RH

TIME =    0.02  MINS.        PRICING    7
 SCALE =     .

    ITER   NUMBER  VECTOR  VECTOR  REDUCED      SUM
  NUMBER   INFEAS    OUT      IN     COST     INFEAS
M      1      9        2      72  1.84560-   26379.3
       2               3      51  1.52675-   26379.3
       3              51      78   .56823-   26379.3
M      4      9       72      90   .22065-   26379.3
       5              90     127   .08988-   26379.3
       6              36     125   .23768-   24046.0
       7              29      63  1.26017-   21994.4
M      8      5       42      99  1.53594-   8095.98
       9              35      90  1.34417-   5620.27
      10              30      51  1.50107-   4351.39
```

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
	11		24	117	.03664-	3172.40
M	12	1	21	92	1.00000-	1270.69
	13		18	66	1.00000-	.

FEASIBLE SOLUTION

PRIMAL OBJ = GDF RHS = RH

TIME = 0.03 MINS. PRICING 7
SCALE = 1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	14	32	117	122	509.452-	.53E+07
M	15	14	92	95	10.1987-	.52E+07
	16		66	68	7.94586-	.50E+07
	17		41	105	50.8024-	.50E+07
M	18	5	99	117	14.0478-	.43E+07
	19		122	121	1.17232-	.41E+07
	20		95	94	.69487-	.41E+07
	21		68	67	.66651-	.41E+07

OPTIMAL SOLUTION

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	4061399.99997	4061399.99997-	NONE	NONE	1.00000
2	G1	EQ	.	.	.	.	1.00000
3	G2	EQ	.	.	.	.	1.50000
4	LC1	BS	.	.	.	NONE	.
5	LC2	BS	.	.	.	NONE	.
6	LC3	BS	40.95000	40.95000-	.	NONE	.
7	LC4	BS	.	50.00000	NONE	50.00000	.
8	LC5	BS	.	50.00000	NONE	50.00000	.
9	LC6	BS	.	50.00000	NONE	50.00000	.
10	LC7	BS	122.85000	122.85000-	.	NONE	.
11	LC8	BS	122.85000	122.85000-	.	NONE	.
12	LC9	BS	185.85000	185.85000-	.	NONE	.
13	KC1	BS	2628989.99999	2628989.99999-	.	NONE	.
14	KC2	BS	2628989.99998	2628989.99998-	.	NONE	.
15	KC3	BS	7940204.99994	7940204.99994-	.	NONE	.
16	DC1	BS	1575.00000	1575.00000-	.	NONE	.
17	DC2	BS	1575.00000	1575.00000-	.	NONE	.
18	DC3	EQ	2625.00000	.	2625.00000	2625.00000	75.00000-
19	DC4	BS	1575.00000	1575.00000-	.	NONE	.
20	DC5	BS	1575.00000	1575.00000-	.	NONE	.
21	DC6	EQ	2625.00000	.	2625.00000	2625.00000	90.50000-
22	DC7	BS	6300.00000	6300.00000-	.	NONE	.
23	DC8	BS	6300.00000	6300.00000-	.	NONE	.
24	DC9	EQ	10500.00000	.	10500.00000	10500.00000	105.00000-
25	D1AA	BS	.	.	.	NONE	.
26	D1AS	BS	.	.	.	NONE	.
27	D2AA	BS	.	.	.	NONE	.
28	D2AS	BS	.	.	.	NONE	.
29	D3AA	LL	525.00000	.	525.00000	NONE	386.50000-
30	D3AS	LL	525.00000	.	525.00000	NONE	268.50000-
31	D1BA	BS	.	.	.	NONE	.
32	D1BS	BS	.	.	.	NONE	.
33	D2BA	BS	.	.	.	NONE	.
34	D2BS	BS	.	.	.	NONE	.
35	D3BA	LL	525.00000	.	525.00000	NONE	477.50000-
36	D3BS	LL	525.00000	.	525.00000	NONE	286.00000-
37	D1CA	BS	.	.	.	NONE	.
38	D1CS	BS	.	.	.	NONE	.
39	D2CA	BS	.	.	.	NONE	.
40	D2CS	BS	.	.	.	NONE	.
41	D3CA	LL	2100.00000	.	2100.00000	NONE	557.00000-
42	D3CS	LL	2100.00000	.	2100.00000	NONE	290.50000-

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
43	X1	LL	.	.	.	NONE	653.50000
44	X2	LL	.	.	.	NONE	818.50000
45	X3	LL	.	.	.	NONE	350.50000
46	X4	LL	.	.	.	NONE	284.50000
47	X5	LL	.	.	.	NONE	441.50000
48	X6	LL	.	.	.	NONE	366.00000
49	X7	LL	.	.	.	NONE	164.00000
50	X8	LL	.	.	.	NONE	153.50000
51	X9	BS	525.00000	.	.	NONE	.
52	X10	LL	.	.	.	NONE	405.50000
53	X11	LL	.	.	.	NONE	520.50000
54	X12	LL	.	.	.	NONE	262.00000
55	X13	LL	.	.	.	NONE	136.50000
56	X14	LL	.	.	.	NONE	193.50000
57	X15	LL	.	.	.	NONE	254.50000
58	X16	LL	.	.	.	NONE	69.00000
59	X17	LL	.	.	.	NONE	106.50000
60	X18	LL	.	.	.	NONE	143.50000
61	X19	LL	.	.	.	NONE	224.50000
62	X20	LL	.	.	.	NONE	303.50000
63	X21	BS	525.00000	.	.	NONE	.
64	X22	LL	.	.	.	NONE	27.50000
65	X23	LL	.	.	.	NONE	62.50000
66	X24	LL	.	.	.	NONE	97.50000
67	X25	BS	1575.00000	.	.	NONE	.
68	X26	LL	.	.	.	NONE	23.50000
69	X27	LL	.	.	.	NONE	44.50000
70	X28	LL	.	.	.	NONE	715.00000
71	X29	LL	.	.	.	NONE	893.00000
72	X30	LL	.	.	.	NONE	333.00000
73	X31	LL	.	.	.	NONE	319.00000
74	X32	LL	.	.	.	NONE	414.50000
75	X33	LL	.	.	.	NONE	236.00000
76	X34	LL	.	.	.	NONE	164.00000
77	X35	LL	.	.	.	NONE	221.50000
78	X36	BS	525.00000	.	.	NONE	.
79	X37	LL	.	.	.	NONE	467.00000
80	X38	LL	.	.	.	NONE	595.00000
81	X39	LL	.	.	.	NONE	262.00000
82	X40	LL	.	.	.	NONE	171.00000
83	X41	LL	.	.	.	NONE	236.50000
84	X42	LL	.	.	.	NONE	308.00000
85	X43	LL	.	.	.	NONE	69.00000
86	X44	LL	.	.	.	NONE	107.50000
87	X45	LL	.	.	.	NONE	149.00000
88	X46	LL	.	.	.	NONE	286.00000
89	X47	LL	.	.	.	NONE	378.00000
90	X48	BS	525.00000	.	.	NONE	.
91	X49	LL	.	.	.	NONE	62.00000

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
92	X50	LL	.	.	.	NONE	105.50000
93	X51	LL	.	.	.	NONE	151.00000
94	X52	BS	1575.00000	.	.	NONE	.
95	X53	LL	.	.	.	NONE	24.50000
96	X54	LL	.	.	.	NONE	50.00000
97	X55	LL	.	.	.	NONE	770.50000
98	X56	LL	.	.	.	NONE	958.00000
99	X57	LL	.	.	.	NONE	327.50000
100	X58	LL	.	.	.	NONE	350.50000
101	X59	LL	.	.	.	NONE	452.50000
102	X60	LL	.	.	.	NONE	277.00000
103	X61	LL	.	.	.	NONE	164.00000
104	X62	LL	.	.	.	NONE	223.50000
105	X63	BS	2100.00000	.	.	NONE	.
106	X64	LL	.	.	.	NONE	522.50000
107	X65	LL	.	.	.	NONE	660.00000
108	X66	LL	.	.	.	NONE	262.00000
109	X67	LL	.	.	.	NONE	201.50000
110	X68	LL	.	.	.	NONE	274.50000
111	X69	LL	.	.	.	NONE	354.50000
112	X70	LL	.	.	.	NONE	70.00000
113	X71	LL	.	.	.	NONE	109.50000
114	X72	LL	.	.	.	NONE	153.50000
115	X73	LL	.	.	.	NONE	341.50000
116	X74	LL	.	.	.	NONE	443.00000
117	X75	BS	2100.00000	.	.	NONE	.
118	X76	LL	.	.	.	NONE	93.50000
119	X77	LL	.	.	.	NONE	143.50000
120	X78	LL	.	.	.	NONE	197.50000
121	X79	BS	6300.00000	.	.	NONE	.
122	X80	LL	.	.	.	NONE	26.50000
123	X81	LL	.	.	.	NONE	54.50000
124	X82	LL	.	1.00000	.	NONE	2.00000
125	X83	BS	1774499.99999	1.00000	.	NONE	.
126	X84	LL	.	1.50000	.	NONE	3.00000
127	X85	BS	1524600.00000	1.50000	.	NONE	.

```
SETUP   TEST

TIME =    0.02

    SCALE


MATRIX1  ASSIGNED TO MATRIX1
MATRIX2  ASSIGNED TO MATRIX2
MATRIX3  ASSIGNED TO MATRIX3
MATRIX4  ASSIGNED TO MATRIX4

ETA1     ASSIGNED TO ETA1
ETA2     ASSIGNED TO ETA2
ETA3     ASSIGNED TO ETA3
ETA4     ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE    7

NO CYCLING

    POOLS          NUMBER   SIZE     CORE
H.REG-BITS MAP                        184
WORK   REGIONS       9       360     3240
MATRIX BUFFERS       5      2216    11080
ETA    BUFFERS       6      5736    34416

                        TOTAL  NORMAL  .FREE.   FIXED BOUNDED
ROWS    (LOG.VAR.)        42      36       1        5       0
COLUMNS (STR.VAR.)        85      85       0        0       0

     751 ELEMENTS - DENSITY =  14.07 -     5 MATRIX RECORDS (WITHOUT RHS'S)


PRIMAL        OBJ = GDF          RHS = RH

TIME =     0.02  MINS.        PRICING    7
 SCALE =       .

   ITER   NUMBER  VECTOR  VECTOR  REDUCED      SUM
  NUMBER  INFEAS   OUT      IN     COST       INFEAS
M    1      9        2      72   1.84560-    26379.3
     2               3      51   1.52675-    26379.3
     3              51      78    .56823-    26379.3
M    4      9       72      90    .22065-    26379.3
     5              78     125    .09622-    26379.3
     6              35     127    .57498-    23903.6
     7              29      63   1.26017-    21852.0
M    8      5       42      99   1.53594-    7953.59
     9              36      78   1.37301-    5620.27
    10              30      51   1.50107-    4351.39
```

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
	11		24	117	.03664-	3172.40
M	12	1	21	92	1.00000-	1270.69
	13		18	66	1.00000-	.

FEASIBLE SOLUTION

PRIMAL OBJ = GDF RHS = RH

TIME = 0.03 MINS. PRICING 7
SCALE = 1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	14	28	117	122	150.447-	.94E+07
	15		9	119	2.29475-	.94E+07
M	16	13	122	121	1.43431-	.90E+07
	17		8	64	2.28420-	.87E+07
	18		41	102	24.4328-	.87E+07
M	19	10	66	9	19.7661-	.87E+07
	20		9	117	2.97328-	.87E+07
M	21	6	99	105	7.71492-	.82E+07
	22		92	91	1.00648-	.81E+07
	23		63	45	1.77837-	.80E+07
	24		119	118	.97079-	.80E+07
M	25	4	64	61	.20776-	.80E+07
	26		102	9	7.27496-	.80E+07
	27		9	75	.40948-	.80E+07

OPTIMAL SOLUTION

SECTION 1 – ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	7961061.53845	7961061.53845-	NONE	NONE	1.00000
2	G1	EQ	.	.	.	.	1.00000
3	G2	EQ	.	.	.	.	4.50000
4	LC1	BS	27.30000	27.30000-	.	NONE	.
5	LC2	BS	27.30000	27.30000-	.	NONE	.
6	LC3	BS	71.92500	71.92500-	.	NONE	.
7	LC4	BS	50.00000	.	NONE	50.00000	.
8	LC5	UL	50.00000	.	NONE	50.00000	1930.76923
9	LC6	UL	50.00000	.	NONE	50.00000	800.00000
10	LC7	BS	52.37500	52.37500-	.	NONE	.
11	LC8	BS	52.37500	52.37500-	.	NONE	.
12	LC9	BS	104.87500	104.87500-	.	NONE	.
13	KC1	BS	4328787.30769	4328787.30769-	.	NONE	.
14	KC2	BS	4328787.30766	4328787.30766-	.	NONE	.
15	KC3	BS	9630447.30762	9630447.30762-	.	NONE	.
16	DC1	BS	2100.00000	2100.00000-	.	NONE	.
17	DC2	BS	2100.00000	2100.00000-	.	NONE	.
18	DC3	EQ	2625.00000	.	2625.00000	2625.00000	368.50000-
19	DC4	BS	1575.00000	1575.00000-	.	NONE	.
20	DC5	BS	1575.00000	1575.00000-	.	NONE	.
21	DC6	EQ	2625.00000	.	2625.00000	2625.00000	377.00000-
22	DC7	BS	6300.00000	6300.00000-	.	NONE	.
23	DC8	BS	6300.00000	6300.00000-	.	NONE	.
24	DC9	EQ	10500.00000	.	10500.00000	10500.00000	363.00000-
25	D1AA	BS	2100.00000	2100.00000-	.	NONE	.
26	D1AS	BS	.	.	.	NONE	.
27	D2AA	BS	.	.	.	NONE	.
28	D2AS	BS	.	.	.	NONE	.
29	D3AA	LL	525.00000	.	525.00000	NONE	125.00000-
30	D3AS	LL	525.00000	.	525.00000	NONE	674.00000-
31	D1BA	BS	.	.	.	NONE	.
32	D1BS	BS	.	.	.	NONE	.
33	D2BA	BS	.	.	.	NONE	.
34	D2BS	BS	.	.	.	NONE	.
35	D3BA	LL	525.00000	.	525.00000	NONE	257.00000-
36	D3BS	LL	525.00000	.	525.00000	NONE	506.50000-
37	D1CA	BS	.	.	.	NONE	.
38	D1CS	BS	.	.	.	NONE	.
39	D2CA	BS	.	.	.	NONE	.
40	D2CS	BS	.	.	.	NONE	.
41	D3CA	LL	2100.00000	.	2100.00000	NONE	347.00000-
42	D3CS	LL	2100.00000	.	2100.00000	NONE	371.50000-

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
43	X1	LL	.	.	.	NONE	429.00000
44	X2	LL	.	.	.	NONE	600.00000
45	X3	BS	525.00000	.	.	NONE	.
46	X4	LL	.	.	.	NONE	306.61538
47	X5	LL	.	.	.	NONE	499.61538
48	X6	LL	.	.	.	NONE	34.00000
49	X7	LL	.	.	.	NONE	392.50000
50	X8	LL	.	.	.	NONE	457.00000
51	X9	BS	.	.	.	NONE	.
52	X10	LL	.	.	.	NONE	181.00000
53	X11	LL	.	.	.	NONE	302.00000
54	X12	LL	.	.	.	NONE	317.00000
55	X13	LL	.	.	.	NONE	385.50769
56	X14	LL	.	.	.	NONE	235.23077
57	X15	LL	.	.	.	NONE	323.20000
58	X16	LL	.	.	.	NONE	297.50000
59	X17	LL	.	.	.	NONE	410.00000
60	X18	LL	.	.	.	NONE	549.00000
61	X19	BS	2100.00000	.	.	NONE	.
62	X20	LL	.	.	.	NONE	85.00000
63	X21	LL	.	.	.	NONE	55.00000
64	X22	LL	.	.	.	NONE	30.50000
65	X23	LL	.	.	.	NONE	101.50000
66	X24	LL	.	.	.	NONE	165.40000
67	X25	LL	.	.	.	NONE	228.50000
68	X26	LL	.	.	.	NONE	327.00000
69	X27	LL	.	.	.	NONE	450.00000
70	X28	LL	.	.	.	NONE	479.50000
71	X29	LL	.	.	.	NONE	663.50000
72	X30	LL	.	.	.	NONE	112.50000
73	X31	LL	.	.	.	NONE	276.11538
74	X32	LL	.	.	.	NONE	398.61538
75	X33	BS	.	.	.	NONE	.
76	X34	LL	.	.	.	NONE	258.50000
77	X35	LL	.	.	.	NONE	367.00000
78	X36	BS	525.00000	.	.	NONE	.
79	X37	LL	.	.	.	NONE	231.50000
80	X38	LL	.	.	.	NONE	365.50000
81	X39	LL	.	.	.	NONE	262.00000
82	X40	LL	.	.	.	NONE	111.73077
83	X41	LL	.	.	.	NONE	204.23077
84	X42	LL	.	.	.	NONE	287.70000
85	X43	LL	.	.	.	NONE	163.50000
86	X44	LL	.	.	.	NONE	253.00000
87	X45	LL	.	.	.	NONE	369.50000
88	X46	LL	.	.	.	NONE	50.50000
89	X47	LL	.	.	.	NONE	148.50000
90	X48	BS	525.00000	.	.	NONE	.
91	X49	BS	1575.00000	.	.	NONE	.

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
92	X50	LL	.	.	.	NONE	70.50000
93	X51	LL	.	.	.	NONE	129.90000
94	X52	LL	.	.	.	NONE	94.50000
95	X53	LL	.	.	.	NONE	170.00000
96	X54	LL	.	.	.	NONE	270.50000
97	X55	LL	.	.	.	NONE	551.50000
98	X56	LL	.	.	.	NONE	742.00000
99	X57	LL	.	.	.	NONE	246.50000
100	X58	LL	.	.	.	NONE	276.11538
101	X59	LL	.	.	.	NONE	396.11538
102	X60	LL	.	.	.	NONE	125.00000
103	X61	LL	.	.	.	NONE	164.00000
104	X62	LL	.	.	.	NONE	256.50000
105	X63	BS	2100.00000	.	.	NONE	.
106	X64	LL	.	.	.	NONE	303.50000
107	X65	LL	.	.	.	NONE	444.00000
108	X66	LL	.	.	.	NONE	262.00000
109	X67	LL	.	.	.	NONE	110.73077
110	X68	LL	.	.	.	NONE	201.73077
111	X69	LL	.	.	.	NONE	278.70000
112	X70	LL	.	.	.	NONE	70.00000
113	X71	LL	.	.	.	NONE	142.50000
114	X72	LL	.	.	.	NONE	234.50000
115	X73	LL	.	.	.	NONE	122.50000
116	X74	LL	.	.	.	NONE	227.00000
117	X75	BS	2100.00000	.	.	NONE	.
118	X76	BS	2271.15385	.	.	NONE	.
119	X77	LL	.	.	.	NONE	68.00000
120	X78	LL	.	.	.	NONE	120.90000
121	X79	BS	4028.84615	.	.	NONE	.
122	X80	LL	.	.	.	NONE	59.50000
123	X81	LL	.	.	.	NONE	135.50000
124	X82	LL	.	1.00000	.	NONE	2.00000
125	X83	BS	3559767.30769	1.00000	.	NONE	.
126	X84	LL	.	4.50000	.	NONE	9.00000
127	X85	BS	978065.38462	4.50000	.	NONE	.

```
SETUP   TEST

TIME =    0.02

    SCALE


MATRIX1   ASSIGNED TO MATRIX1
MATRIX2   ASSIGNED TO MATRIX2
MATRIX3   ASSIGNED TO MATRIX3
MATRIX4   ASSIGNED TO MATRIX4

ETA1      ASSIGNED TO ETA1
ETA2      ASSIGNED TO ETA2
ETA3      ASSIGNED TO ETA3
ETA4      ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE    7

NO CYCLING

    POOLS          NUMBER    SIZE     CORE
H.REG-BITS MAP                         184
WORK    REGIONS       9       360     3240
MATRIX  BUFFERS       5      2216    11080
ETA     BUFFERS       6      5736    34416

                         TOTAL  NORMAL  .FREE.   FIXED BOUNDED
ROWS     (LOG.VAR.)        42      36       1       5       0
COLUMNS  (STR.VAR.)        85      85       0       0       0

    751 ELEMENTS - DENSITY =  14.07 -     5 MATRIX RECORDS (WITHOUT RHS'S)


PRIMAL        OBJ = GDF          RHS = RH

TIME =     0.02  MINS.        PRICING    7
 SCALE =      .

   ITER    NUMBER  VECTOR  VECTOR  REDUCED      SUM
  NUMBER   INFEAS    OUT      IN     COST      INFEAS
M     1       9        2      72  1.84560-   28844.7
      2                3      51  1.52675-   28844.7
      3               51      78   .56823-   28844.7
M     4       9       72     127   .72136-   28844.7
      5               36     125   .12077-   26511.4
      6               29      63  1.26017-   18305.1
      7               35      90  1.34417-   15829.4
M     8       6       42      99  1.53594-   12354.8
      9               30      51  1.50107-   7279.23
     10               24     108   .03664-   6984.48
```

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
M	11	1	18	69	1.00000-	1901.70
	12		21	92	1.00000-	.

FEASIBLE SOLUTION

PRIMAL OBJ = GDF RHS = RH

TIME = 0.03 MINS. PRICING 7
SCALE = 1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	13	32	108	122	1712.07-	.77E+07
M	14	11	20	96	47.4022-	.47E+07
	15		41	105	115.329-	.47E+07
	16		92	95	45.0482-	.47E+07
	17		95	94	.03907-	.47E+07
M	18	3	99	117	29.3953-	.28E+07
	19		125	121	.96637-	.26E+07
	20		122	108	2.25072-	.26E+07
M	21	56	108	65	2.83163-	.26E+07
M	22	4	17	20	.84362-	.26E+07
	23		65	68	10.7901-	.26E+07

OPTIMAL SOLUTION

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	2586028.84615	2586028.84615-	NONE	NONE	1.00000
2	G1	EQ	.	.	.	.	3.23077
3	G2	EQ	.	.	.	.	1.00000
4	LC1	BS	.	.	.	NONE	.
5	LC2	BS	.	.	.	NONE	.
6	LC3	BS	40.95000	40.95000-	.	NONE	.
7	LC4	BS	.	50.00000	NONE	50.00000	.
8	LC5	BS	.	50.00000	NONE	50.00000	.
9	LC6	BS	.	50.00000	NONE	50.00000	.
10	LC7	BS	21.00000	21.00000-	.	NONE	.
11	LC8	BS	21.00000	21.00000-	.	NONE	.
12	LC9	BS	185.85000	185.85000-	.	NONE	.
13	KC1	BS	449399.99984	449399.99984-	.	NONE	.
14	KC2	BS	449399.99984	449399.99984-	.	NONE	.
15	KC3	BS	8899161.92306	8899161.92306-	.	NONE	.
16	DC1	BS	.	.	.	NONE	.
17	DC2	LL	.	.	.	NONE	62.92308-
18	DC3	EQ	10500.00000	.	10500.00000	10500.00000	510.07692
19	DC4	BS	40.38461	40.38461-	.	NONE	.
20	DC5	BS	40.38461	40.38461-	.	NONE	.
21	DC6	EQ	2625.00000	.	2625.00000	2625.00000	196.07692
22	DC7	BS	1575.00000	1575.00000-	.	NONE	.
23	DC8	BS	1575.00000	1575.00000-	.	NONE	.
24	DC9	EQ	2625.00000	.	2625.00000	2625.00000	8.46154-
25	D1AA	BS	.	.	.	NONE	.
26	D1AS	BS	.	.	.	NONE	.
27	D2AA	BS	.	.	.	NONE	.
28	D2AS	BS	.	.	.	NONE	.
29	D3AA	LL	2100.00000	.	2100.00000	NONE	1889.53846-
30	D3AS	LL	2100.00000	.	2100.00000	NONE	723.69231-
31	D1BA	BS	.	.	.	NONE	.
32	D1BS	BS	.	.	.	NONE	.
33	D2BA	BS	.	.	.	NONE	.
34	D2BS	BS	.	.	.	NONE	.
35	D3BA	LL	525.00000	.	525.00000	NONE	1946.53846-
36	D3BS	LL	525.00000	.	525.00000	NONE	762.46154-
37	D1CA	BS	.	.	.	NONE	.
38	D1CS	BS	.	.	.	NONE	.
39	D2CA	BS	.	.	.	NONE	.
40	D2CS	BS	.	.	.	NONE	.
41	D3CA	LL	525.00000	.	525.00000	NONE	2068.76923-
42	D3CS	LL	525.00000	.	525.00000	NONE	834.69231-

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
43	X1	LL	.	.	.	NONE	2712.30769
44	X2	LL	.	.	.	NONE	3237.69231
45	X3	LL	.	.	.	NONE	1276.15385
46	X4	LL	.	.	.	NONE	1274.00000
47	X5	LL	.	.	.	NONE	1735.07692
48	X6	LL	.	.	.	NONE	1628.61538
49	X7	LL	.	.	.	NONE	550.07692
50	X8	LL	.	.	.	NONE	420.00000
51	X9	BS	2100.00000	.	.	NONE	.
52	X10	LL	.	.	.	NONE	1911.07692
53	X11	LL	.	.	.	NONE	2274.92308
54	X12	LL	.	.	.	NONE	846.46154
55	X13	LL	.	.	.	NONE	795.84615
56	X14	LL	.	.	.	NONE	933.84615
57	X15	LL	.	.	.	NONE	1124.61538
58	X16	LL	.	.	.	NONE	243.15385
59	X17	LL	.	.	.	NONE	268.15385
60	X18	LL	.	.	.	NONE	319.84615
61	X19	LL	.	.	.	NONE	1326.30769
62	X20	LL	.	.	.	NONE	1573.84615
63	X21	BS	2100.00000	.	.	NONE	.
64	X22	LL	.	.	.	NONE	443.69231
65	X23	LL	.	.	.	NONE	510.61538
66	X24	LL	.	.	.	NONE	617.38462
67	X25	LL	.	.	.	NONE	20.23077
68	X26	BS	.	.	.	NONE	.
69	X27	BS	6300.00000	.	.	NONE	.
70	X28	LL	.	.	.	NONE	2733.07692
71	X29	LL	.	.	.	NONE	3300.46154
72	X30	LL	.	.	.	NONE	1237.38462
73	X31	LL	.	.	.	NONE	1276.76923
74	X32	LL	.	.	.	NONE	1550.69231
75	X33	LL	.	.	.	NONE	1085.53846
76	X34	LL	.	.	.	NONE	529.84615
77	X35	LL	.	.	.	NONE	650.23077
78	X36	BS	525.00000	.	.	NONE	.
79	X37	LL	.	.	.	NONE	1931.84615
80	X38	LL	.	.	.	NONE	2337.69231
81	X39	LL	.	.	.	NONE	846.46154
82	X40	LL	.	.	.	NONE	798.61538
83	X41	LL	.	.	.	NONE	975.61538
84	X42	LL	.	.	.	NONE	1156.61538
85	X43	LL	.	.	.	NONE	222.92308
86	X44	LL	.	.	.	NONE	281.92308
87	X45	LL	.	.	.	NONE	319.84615
88	X46	LL	.	.	.	NONE	1347.07692
89	X47	LL	.	.	.	NONE	1636.61538
90	X48	BS	525.00000	.	.	NONE	.
91	X49	LL	.	.	.	NONE	446.46154

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
92	X50	LL	.	.	.	NUNE	552.38462
93	X51	LL	.	.	.	NUNE	649.38462
94	X52	BS	40.38461	.	.	NUNE	.
95	X53	LL	.	.	.	NUNE	13.76923
96	X54	BS	1534.61539	.	.	NUNE	.
97	X55	LL	.	.	.	NUNE	2770.07692
98	X56	LL	.	.	.	NUNE	3372.00000
99	X57	LL	.	.	.	NUNE	1161.92308
100	X58	LL	.	.	.	NUNE	1297.76923
101	X59	LL	.	.	.	NUNE	1604.23077
102	X60	LL	.	.	.	NUNE	1110.30769
103	X61	LL	.	.	.	NUNE	529.84615
104	X62	LL	.	.	.	NUNE	679.76923
105	X63	BS	525.00000	.	.	NUNE	.
106	X64	LL	.	.	.	NUNE	1968.84615
107	X65	LL	.	.	.	NUNE	2409.23077
108	X66	LL	.	.	.	NUNE	846.46154
109	X67	LL	.	.	.	NUNE	816.38462
110	X68	LL	.	.	.	NUNE	1029.15385
111	X69	LL	.	.	.	NUNE	1256.84615
112	X70	LL	.	.	.	NUNE	226.15385
113	X71	LL	.	.	.	NUNE	311.46154
114	X72	LL	.	.	.	NUNE	392.07692
115	X73	LL	.	.	.	NUNE	1384.07692
116	X74	LL	.	.	.	NUNE	1708.15385
117	X75	BS	525.00000	.	.	NUNE	.
118	X76	LL	.	.	.	NUNE	467.46154
119	X77	LL	.	.	.	NUNE	605.92308
120	X78	LL	.	.	.	NUNE	749.61538
121	X79	BS	1575.00000	.	.	NUNE	.
122	X80	LL	.	.	.	NUNE	43.30769
123	X81	LL	.	.	.	NUNE	72.23077
124	X82	LL	.	10.00000	.	NUNE	13.23077
125	X83	LL	.	10.00000	.	NUNE	6.76923
126	X84	LL	.	1.00000	.	NUNE	2.00000
127	X85	BS	2586028.84618	1.00000	.	NUNE	.

```
SETUP   TEST

TIME =    0.02

    SCALE


MATRIX1  ASSIGNED TO MATRIX1
MATRIX2  ASSIGNED TO MATRIX2
MATRIX3  ASSIGNED TO MATRIX3
MATRIX4  ASSIGNED TO MATRIX4

ETA1     ASSIGNED TO ETA1
ETA2     ASSIGNED TO ETA2
ETA3     ASSIGNED TO ETA3
ETA4     ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE   7

NO CYCLING

    POOLS          NUMBER    SIZE     CORE
H.REG-BITS MAP                         184
WORK    REGIONS      9        360     3240
MATRIX BUFFERS       5       2216    11080
ETA     BUFFERS      6       5736    34416

                           TOTAL  NORMAL  .FREE.   FIXED BOUNDED
ROWS     (LOG.VAR.)          42      36       1       5       0
COLUMNS  (STR.VAR.)          85      85       0       0       0

     751 ELEMENTS - DENSITY =  14.07 -     5 MATRIX RECORDS (WITHOUT RHS'S)


PRIMAL        OBJ = GDF          RHS = RH

TIME =    0.02  MINS.        PRICING    7
 SCALE =     .

   ITER   NUMBER  VECTOR  VECTOR  REDUCED       SUM
  NUMBER  INFEAS    OUT      IN     COST       INFEAS
M     1      9        2      72   1.84560-   28844.7
      2               3      51   1.52675-   28844.7
      3              51      78    .56823-   28844.7
M     4      9       72      90    .22065-   28844.7
      5              78     125    .17514-   28844.7
      6              35     127    .47093-   26369.0
      7              29      63   1.26017-   18162.7
M     8      5       42      99   1.53594-   14688.1
      9              30      51   1.50107-   9612.55
     10              36      78   1.37301-   7279.23
```

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
	11		24	117	.03664-	6984.48
M	12	1	18	69	1.00000-	1901.70
	13		21	92	1.00000-	.

FEASIBLE SOLUTION

PRIMAL OBJ = GDF RHS = RH

TIME = 0.03 MINS. PRICING 7
SCALE = 1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	14	40	69	62	24.5225-	.15E+08
M	15	21	63	45	11.7672-	.13E+08
	16		92	89	2.54652-	.13E+08
	17		78	72	5.95756-	.12E+08
	18		41	119	6.21059-	.12E+08
	19		99	102	2.16668-	.12E+08
	20		51	48	17.3195-	.12E+08
M	21	5	62	61	.35928-	.11E+08
	22		33	88	.40794-	.11E+08
	23		119	115	.40419-	.11E+08
	24		48	29	5.53358-	.11E+08

OPTIMAL SOLUTION

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	10663800.0000	10663800.0000-	NONE	NONE	1.00000
2	G1	EQ	.	.	.	.	1.00000
3	G2	EQ	.	.	.	.	10.00000
4	LC1	BS	156.97500	156.97500-	.	NONE	.
5	LC2	BS	156.97500	156.97500-	.	NONE	.
6	LC3	BS	216.30000	216.30000-	.	NONE	.
7	LC4	BS	.	50.00000	NONE	50.00000	.
8	LC5	BS	.	50.00000	NONE	50.00000	.
9	LC6	BS	10.50000	39.50000	NONE	50.00000	.
10	LC7	BS	.	.	.	NONE	.
11	LC8	BS	.	.	.	NONE	.
12	LC9	BS	.	.	.	NONE	.
13	KC1	BS	8799052.50001	8799052.50001-	.	NONE	.
14	KC2	BS	8799052.49989	8799052.49989-	.	NONE	.
15	KC3	BS	14252279.9999	14252279.9999-	.	NONE	.
16	DC1	BS	8400.00000	8400.00000-	.	NONE	.
17	DC2	BS	8400.00000	8400.00000-	.	NONE	.
18	DC3	EQ	10500.00000	.	10500.00000	10500.00000	495.00000-
19	DC4	BS	2100.00000	2100.00000-	.	NONE	.
20	DC5	BS	2100.00000	2100.00000-	.	NONE	.
21	DC6	EQ	2625.00000	.	2625.00000	2625.00000	521.00000-
22	DC7	BS	1575.00000	1575.00000-	.	NONE	.
23	DC8	BS	1575.00000	1575.00000-	.	NONE	.
24	DC9	EQ	2625.00000	.	2625.00000	2625.00000	557.00000-
25	D1AA	BS	8400.00000	8400.00000-	.	NONE	.
26	D1AS	BS	.	.	.	NONE	.
27	D2AA	BS	.	.	.	NONE	.
28	D2AS	BS	.	.	.	NONE	.
29	D3AA	BS	2100.00000	.	2100.00000	NONE	.
30	D3AS	LL	2100.00000	.	2100.00000	NONE	832.00000-
31	D1BA	BS	2100.00000	2100.00000-	.	NONE	.
32	D1BS	BS	.	.	.	NONE	.
33	D2BA	LL	.	.	.	NONE	109.00000-
34	D2BS	BS	.	.	.	NONE	.
35	D3BA	LL	525.00000	.	525.00000	NONE	234.00000-
36	D3BS	LL	525.00000	.	525.00000	NONE	619.00000-
37	D1CA	BS	1575.00000	1575.00000-	.	NONE	.
38	D1CS	BS	.	.	.	NONE	.
39	D2CA	BS	.	.	.	NONE	.
40	D2CS	BS	.	.	.	NONE	.
41	D3CA	LL	525.00000	.	525.00000	NONE	241.00000-
42	D3CS	LL	525.00000	.	525.00000	NONE	600.00000-

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
43	X1	LL	.	.	.	NONE	429.00000
44	X2	LL	.	.	.	NONE	611.00000
45	X3	BS	2100.00000	.	.	NONE	.
46	X4	LL	.	.	.	NONE	604.00000
47	X5	LL	.	.	.	NONE	863.00000
48	X6	LL	.	.	.	NONE	377.00000
49	X7	LL	.	.	.	NONE	1223.00000
50	X8	LL	.	.	.	NONE	1425.00000
51	X9	LL	.	.	.	NONE	997.00000
52	X10	LL	.	.	.	NONE	181.00000
53	X11	LL	.	.	.	NONE	313.00000
54	X12	LL	.	.	.	NONE	475.00000
55	X13	LL	.	.	.	NONE	456.00000
56	X14	LL	.	.	.	NONE	615.00000
57	X15	LL	.	.	.	NONE	829.00000
58	X16	LL	.	.	.	NONE	1128.00000
59	X17	LL	.	.	.	NONE	1378.00000
60	X18	LL	.	.	.	NONE	1704.00000
61	X19	BS	8400.00000	.	.	NONE	.
62	X20	LL	.	.	.	NONE	96.00000
63	X21	LL	.	.	.	NONE	213.00000
64	X22	LL	.	.	.	NONE	347.00000
65	X23	LL	.	.	.	NONE	484.00000
66	X24	LL	.	.	.	NONE	672.00000
67	X25	LL	.	.	.	NONE	1059.00000
68	X26	LL	.	.	.	NONE	1295.00000
69	X27	LL	.	.	.	NONE	1605.00000
70	X28	LL	.	.	.	NONE	429.00000
71	X29	LL	.	.	.	NONE	515.00000
72	X30	BS	525.00000	.	.	NONE	.
73	X31	LL	.	.	.	NONE	424.00000
74	X32	LL	.	.	.	NONE	596.00000
75	X33	LL	.	.	.	NONE	195.00000
76	X34	LL	.	.	.	NONE	813.00000
77	X35	LL	.	.	.	NONE	1015.00000
78	X36	LL	.	.	.	NONE	673.00000
79	X37	LL	.	.	.	NONE	181.00000
80	X38	LL	.	.	.	NONE	217.00000
81	X39	LL	.	.	.	NONE	262.00000
82	X40	LL	.	.	.	NONE	276.00000
83	X41	LL	.	.	.	NONE	418.00000
84	X42	LL	.	.	.	NONE	600.00000
85	X43	LL	.	.	.	NONE	718.00000
86	X44	LL	.	.	.	NONE	901.00000
87	X45	LL	.	.	.	NONE	1155.00000
88	X46	BS	2100.00000	.	.	NONE	.
89	X47	BS	.	.	.	NONE	.
90	X48	BS	.	.	.	NONE	.
91	X49	LL	.	.	.	NONE	167.00000

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
92	X50	LL	.	.	.	NONE	287.00000
93	X51	LL	.	.	.	NONE	443.00000
94	X52	LL	.	.	.	NONE	649.00000
95	X53	LL	.	.	.	NONE	818.00000
96	X54	LL	.	.	.	NONE	1056.00000
97	X55	LL	.	.	.	NONE	429.00000
98	X56	LL	.	.	.	NONE	625.00000
99	X57	LL	.	.	.	NONE	18.00000
100	X58	LL	.	.	.	NONE	264.00000
101	X59	LL	.	.	.	NONE	417.00000
102	X60	BS	525.00000	.	.	NONE	.
103	X61	LL	.	.	.	NONE	443.00000
104	X62	LL	.	.	.	NONE	596.00000
105	X63	LL	.	.	.	NONE	199.00000
106	X64	LL	.	.	.	NONE	181.00000
107	X65	LL	.	.	.	NONE	327.00000
108	X66	LL	.	.	.	NONE	262.00000
109	X67	LL	.	.	.	NONE	115.00000
110	X68	LL	.	.	.	NONE	239.00000
111	X69	LL	.	.	.	NONE	387.00000
112	X70	LL	.	.	.	NONE	349.00000
113	X71	LL	.	.	.	NONE	482.00000
114	X72	LL	.	.	.	NONE	662.00000
115	X73	BS	1575.00000	.	.	NONE	.
116	X74	LL	.	.	.	NONE	110.00000
117	X75	BS	525.00000	.	.	NONE	.
118	X76	LL	.	.	.	NONE	7.00000
119	X77	LL	.	.	.	NONE	108.00000
120	X78	LL	.	.	.	NONE	230.00000
121	X79	LL	.	.	.	NONE	279.00000
122	X80	LL	.	.	.	NONE	399.00000
123	X81	LL	.	.	.	NONE	563.00000
124	X82	LL	.	1.00000	.	NONE	2.00000
125	X83	BS	7062300.00006	1.00000	.	NONE	.
126	X84	LL	.	10.00000	.	NONE	20.00000
127	X85	BS	360150.00000	10.00000	.	NONE	.

```
SETUP   TEST

TIME =    0.02

    SCALE


MATRIX1  ASSIGNED TO MATRIX1
MATRIX2  ASSIGNED TO MATRIX2
MATRIX3  ASSIGNED TO MATRIX3
MATRIX4  ASSIGNED TO MATRIX4

ETA1     ASSIGNED TO ETA1
ETA2     ASSIGNED TO ETA2
ETA3     ASSIGNED TO ETA3
ETA4     ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE   7

NO CYCLING

     POOLS        NUMBER   SIZE     CORE
H.REG-BITS MAP                       184
WORK   REGIONS      9       360     3240
MATRIX BUFFERS      5      2216    11080
ETA    BUFFERS      6      5736    34416

                        TOTAL  NORMAL  .FREE.   FIXED BOUNDED
ROWS     (LOG.VAR.)       42     36       1       5       0
COLUMNS  (STR.VAR.)       85     85       0       0       0

     751 ELEMENTS - DENSITY =  14.07 -     5 MATRIX RECORDS (WITHOUT RHS'S)


PRIMAL       OBJ = GDF          RHS = RH

TIME =     0.03  MINS.        PRICING    7
 SCALE =    .

   ITER    NUMBER  VECTOR  VECTOR   REDUCED      SUM
  NUMBER   INFEAS    OUT     IN      COST      INFEAS
M     1      9        2      72   1.84560-   35203.4
      2               3      51   1.52675-   35203.4
      3              51      78    .56823-   35203.4
M     4      9       72      90    .22065-   35203.4
      5              78     125    .17514-   35203.4
      6              35     127    .47093-   25300.6
      7              29      63   1.26017-   23249.0
M     8      5       42      99   1.53594-   19774.4
      9              36      78   1.37301-   10441.1
     10              30      51   1.50107-   9172.26
```

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
	11		24	117	.03664-	8877.51
M	12	1	21	96	1.00000-	1270.69
	13		18	66	1.00000-	.

FEASIBLE SOLUTION

PRIMAL OBJ = GDF RHS = RH

TIME = 0.03 MINS. PRICING 7
SCALE = 1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	14	46	96	89	13.5481-	.15E+08
	15		51	48	17.3195-	.15E+08
M	16	25	78	72	5.95756-	.14E+08
	17		66	62	9.36049-	.13E+08
	18		41	119	6.21059-	.13E+08
M	19	6	89	88	.40794-	.12E+08
	20		63	45	5.21615-	.11E+08
	21		119	115	.40419-	.11E+08
	22		62	61	.35928-	.11E+08
	23		99	102	.30953-	.11E+08
M	24	1	48	29	5.53358-	.11E+08

OPTIMAL SOLUTION

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	10901625.0000	10901625.0000-	NONE	NONE	1.00000
2	G1	EQ	.	.	.	.	1.00000
3	G2	EQ	.	.	.	.	10.00000
4	LC1	BS	156.97500	156.97500-	.	NONE	.
5	LC2	BS	156.97500	156.97500-	.	NONE	.
6	LC3	BS	216.30000	216.30000-	.	NONE	.
7	LC4	BS	.	50.00000	NONE	50.00000	.
8	LC5	BS	.	50.00000	NONE	50.00000	.
9	LC6	BS	10.50000	39.50000	NONE	50.00000	.
10	LC7	BS	.	.	.	NONE	.
11	LC8	BS	.	.	.	NONE	.
12	LC9	BS	.	.	.	NONE	.
13	KC1	BS	8799052.50005	8799052.50005-	.	NONE	.
14	KC2	BS	8799052.49993	8799052.49993-	.	NONE	.
15	KC3	BS	14252279.9999	14252279.9999-	.	NONE	.
16	DC1	BS	2100.00000	2100.00000-	.	NONE	.
17	DC2	BS	2100.00000	2100.00000-	.	NONE	.
18	DC3	EQ	2625.00000	.	2625.00000	2625.00000	495.00000-
19	DC4	BS	8400.00000	8400.00000-	.	NONE	.
20	DC5	BS	8400.00000	8400.00000-	.	NONE	.
21	DC6	EQ	10500.00000	.	10500.00000	10500.00000	521.00000-
22	DC7	BS	1575.00000	1575.00000-	.	NONE	.
23	DC8	BS	1575.00000	1575.00000-	.	NONE	.
24	DC9	EQ	2625.00000	.	2625.00000	2625.00000	557.00000-
25	D1AA	BS	2100.00000	2100.00000-	.	NONE	.
26	D1AS	BS	.	.	.	NONE	.
27	D2AA	BS	.	.	.	NONE	.
28	D2AS	BS	.	.	.	NONE	.
29	D3AA	BS	525.00000	.	525.00000	NONE	.
30	D3AS	LL	525.00000	.	525.00000	NONE	832.00000-
31	D1BA	BS	8400.00000	8400.00000-	.	NONE	.
32	D1BS	BS	.	.	.	NONE	.
33	D2BA	BS	.	.	.	NONE	.
34	D2BS	BS	.	.	.	NONE	.
35	D3BA	LL	2100.00000	.	2100.00000	NONE	234.00000-
36	D3BS	LL	2100.00000	.	2100.00000	NONE	619.00000-
37	D1CA	BS	1575.00000	1575.00000-	.	NONE	.
38	D1CS	BS	.	.	.	NONE	.
39	D2CA	BS	.	.	.	NONE	.
40	D2CS	BS	.	.	.	NONE	.
41	D3CA	LL	525.00000	.	525.00000	NONE	241.00000-
42	D3CS	LL	525.00000	.	525.00000	NONE	600.00000-

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
43	X1	LL	.	.	.	NONE	429.00000
44	X2	LL	.	.	.	NONE	611.00000
45	X3	BS	525.00000	.	.	NONE	.
46	X4	LL	.	.	.	NONE	604.00000
47	X5	LL	.	.	.	NONE	863.00000
48	X6	LL	.	.	.	NONE	377.00000
49	X7	LL	.	.	.	NONE	1223.00000
50	X8	LL	.	.	.	NONE	1425.00000
51	X9	LL	.	.	.	NONE	997.00000
52	X10	LL	.	.	.	NONE	181.00000
53	X11	LL	.	.	.	NONE	313.00000
54	X12	LL	.	.	.	NONE	475.00000
55	X13	LL	.	.	.	NONE	456.00000
56	X14	LL	.	.	.	NONE	615.00000
57	X15	LL	.	.	.	NONE	829.00000
58	X16	LL	.	.	.	NONE	1128.00000
59	X17	LL	.	.	.	NONE	1378.00000
60	X18	LL	.	.	.	NONE	1704.00000
61	X19	BS	2100.00000	.	.	NONE	.
62	X20	LL	.	.	.	NONE	96.00000
63	X21	LL	.	.	.	NONE	213.00000
64	X22	LL	.	.	.	NONE	347.00000
65	X23	LL	.	.	.	NONE	484.00000
66	X24	LL	.	.	.	NONE	672.00000
67	X25	LL	.	.	.	NONE	1059.00000
68	X26	LL	.	.	.	NONE	1295.00000
69	X27	LL	.	.	.	NONE	1605.00000
70	X28	LL	.	.	.	NONE	429.00000
71	X29	LL	.	.	.	NONE	624.00000
72	X30	BS	2100.00000	.	.	NONE	.
73	X31	LL	.	.	.	NONE	424.00000
74	X32	LL	.	.	.	NONE	596.00000
75	X33	LL	.	.	.	NONE	195.00000
76	X34	LL	.	.	.	NONE	813.00000
77	X35	LL	.	.	.	NONE	1015.00000
78	X36	LL	.	.	.	NONE	673.00000
79	X37	LL	.	.	.	NONE	181.00000
80	X38	LL	.	.	.	NONE	326.00000
81	X39	LL	.	.	.	NONE	262.00000
82	X40	LL	.	.	.	NONE	276.00000
83	X41	LL	.	.	.	NONE	418.00000
84	X42	LL	.	.	.	NONE	600.00000
85	X43	LL	.	.	.	NONE	718.00000
86	X44	LL	.	.	.	NONE	901.00000
87	X45	LL	.	.	.	NONE	1155.00000
88	X46	BS	8400.00000	.	.	NONE	.
89	X47	LL	.	.	.	NONE	109.00000
90	X48	BS	.	.	.	NONE	.
91	X49	LL	.	.	.	NONE	167.00000

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
92	X50	LL	.	.	.	NONE	287.00000
93	X51	LL	.	.	.	NONE	443.00000
94	X52	LL	.	.	.	NONE	649.00000
95	X53	LL	.	.	.	NONE	818.00000
96	X54	LL	.	.	.	NONE	1056.00000
97	X55	LL	.	.	.	NONE	429.00000
98	X56	LL	.	.	.	NONE	625.00000
99	X57	LL	.	.	.	NONE	18.00000
100	X58	LL	.	.	.	NONE	264.00000
101	X59	LL	.	.	.	NONE	417.00000
102	X60	BS	525.00000	.	.	NONE	.
103	X61	LL	.	.	.	NONE	443.00000
104	X62	LL	.	.	.	NONE	596.00000
105	X63	LL	.	.	.	NONE	199.00000
106	X64	LL	.	.	.	NONE	181.00000
107	X65	LL	.	.	.	NONE	327.00000
108	X66	LL	.	.	.	NONE	262.00000
109	X67	LL	.	.	.	NONE	115.00000
110	X68	LL	.	.	.	NONE	239.00000
111	X69	LL	.	.	.	NONE	387.00000
112	X70	LL	.	.	.	NONE	349.00000
113	X71	LL	.	.	.	NONE	482.00000
114	X72	LL	.	.	.	NONE	662.00000
115	X73	BS	1575.00000	.	.	NONE	.
116	X74	LL	.	.	.	NONE	110.00000
117	X75	BS	525.00000	.	.	NONE	.
118	X76	LL	.	.	.	NONE	7.00000
119	X77	LL	.	.	.	NONE	108.00000
120	X78	LL	.	.	.	NONE	230.00000
121	X79	LL	.	.	.	NONE	279.00000
122	X80	LL	.	.	.	NONE	399.00000
123	X81	LL	.	.	.	NONE	563.00000
124	X82	LL	.	1.00000	.	NONE	2.00000
125	X83	BS	7788375.00011	1.00000	.	NONE	.
126	X84	LL	.	10.00000	.	NONE	20.00000
127	X85	BS	311325.00000	10.00000	.	NONE	.

SETUP TEST

TIME = 0.02

SCALE

MATRIX1 ASSIGNED TO MATRIX1
MATRIX2 ASSIGNED TO MATRIX2
MATRIX3 ASSIGNED TO MATRIX3
MATRIX4 ASSIGNED TO MATRIX4

ETA1 ASSIGNED TO ETA1
ETA2 ASSIGNED TO ETA2
ETA3 ASSIGNED TO ETA3
ETA4 ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE 7

NO CYCLING

POOLS	NUMBER	SIZE	CORE
H.REG-BITS MAP			184
WORK REGIONS	9	360	3240
MATRIX BUFFERS	5	2216	11080
ETA BUFFERS	6	5736	34416

	TOTAL	NORMAL	.FREE.	FIXED	BOUNDED
ROWS (LOG.VAR.)	42	36	1	5	0
COLUMNS (STR.VAR.)	85	85	0	0	0

751 ELEMENTS - DENSITY = 14.07 - 5 MATRIX RECORDS (WITHOUT RHS'S)

PRIMAL OBJ = GDF RHS = RH

TIME = 0.02 MINS. PRICING 7
SCALE = .

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
M	1	9	2	72	1.84560-	35203.4
	2		3	51	1.52675-	35203.4
	3		51	78	.56823-	35203.4
M	4	9	72	127	.72136-	35203.4
	5		36	125	.12077-	25870.2
	6		35	90	1.34417-	15967.3
	7		29	63	1.26017-	13915.8
M	8	4	42	99	1.53594-	10441.1
	9		30	51	1.50107-	9172.26
	10		24	108	.03664-	8877.51

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
M	11	1	21	96	1.00000-	1270.69
	12		18	66	1.00000-	.

FEASIBLE SOLUTION

PRIMAL   OBJ = GDF   RHS = RH

TIME = 0.03 MINS.   PRICING 7
SCALE = 1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	13	34	108	122	1712.07-	.17E+08
M	14	13	66	69	79.8787-	.14E+08
	15		41	105	115.329-	.14E+08
M	16	3	99	117	29.3953-	.12E+08
	17		122	121	.96637-	.12E+08

OPTIMAL SOLUTION

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	11571000.0000	11571000.0000-	NONE	NONE	1.00000
2	G1	EQ	.	.	.	.	10.00000
3	G2	EQ	.	.	.	.	1.00000
4	LC1	BS	.	.	.	NONE	.
5	LC2	BS	.	.	.	NONE	.
6	LC3	BS	40.95000	40.95000-	.	NONE	.
7	LC4	BS	.	50.00000	NONE	50.00000	.
8	LC5	BS	.	50.00000	NONE	50.00000	.
9	LC6	BS	.	50.00000	NONE	50.00000	.
10	LC7	BS	20.47500	20.47500-	.	NONE	.
11	LC8	BS	20.47500	20.47500-	.	NONE	.
12	LC9	BS	185.85000	185.85000-	.	NONE	.
13	KC1	BS	438165.00000	438165.00000-	.	NONE	.
14	KC2	BS	438165.00000	438165.00000-	.	NONE	.
15	KC3	BS	8904104.99990	8904104.99990-	.	NONE	.
16	DC1	BS	.	.	.	NONE	.
17	DC2	BS	.	.	.	NONE	.
18	DC3	EQ	2625.00000	.	2625.00000	2625.00000	2067.00000
19	DC4	BS	.	.	.	NONE	.
20	DC5	BS	.	.	.	NONE	.
21	DC6	EQ	10500.00000	.	10500.00000	10500.00000	961.00000
22	DC7	BS	1575.00000	1575.00000-	.	NONE	.
23	DC8	BS	1575.00000	1575.00000-	.	NONE	.
24	DC9	EQ	2625.00000	.	2625.00000	2625.00000	154.00000
25	D1AA	BS	.	.	.	NONE	.
26	D1AS	BS	.	.	.	NONE	.
27	D2AA	BS	.	.	.	NONE	.
28	D2AS	BS	.	.	.	NONE	.
29	D3AA	LL	525.00000	.	525.00000	NONE	6276.00000-
30	D3AS	LL	525.00000	.	525.00000	NONE	2240.00000-
31	D1BA	BS	.	.	.	NONE	.
32	D1BS	BS	.	.	.	NONE	.
33	D2BA	BS	.	.	.	NONE	.
34	D2BS	BS	.	.	.	NONE	.
35	D3BA	LL	2100.00000	.	2100.00000	NONE	6333.00000-
36	D3BS	LL	2100.00000	.	2100.00000	NONE	2360.00000-
37	D1CA	BS	.	.	.	NONE	.
38	D1CS	BS	.	.	.	NONE	.
39	D2CA	BS	.	.	.	NONE	.
40	D2CS	BS	.	.	.	NONE	.
41	D3CA	LL	525.00000	.	525.00000	NONE	6550.00000-
42	D3CS	LL	525.00000	.	525.00000	NONE	2527.00000-

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
43	X1	LL	.	.	.	NONE	9030.00000
44	X2	LL	.	.	.	NONE	10652.00000
45	X3	LL	.	.	.	NONE	3950.00000
46	X4	LL	.	.	.	NONE	4444.00000
47	X5	LL	.	.	.	NONE	5846.00000
48	X6	LL	.	.	.	NONE	5284.00000
49	X7	LL	.	.	.	NONE	2021.00000
50	X8	LL	.	.	.	NONE	1566.00000
51	X9	BS	525.00000	.	.	NONE	.
52	X10	LL	.	.	.	NONE	6550.00000
53	X11	LL	.	.	.	NONE	7672.00000
54	X12	LL	.	.	.	NONE	2620.00000
55	X13	LL	.	.	.	NONE	2964.00000
56	X14	LL	.	.	.	NONE	3366.00000
57	X15	LL	.	.	.	NONE	3724.00000
58	X16	LL	.	.	.	NONE	1071.00000
59	X17	LL	.	.	.	NONE	1096.00000
60	X18	LL	.	.	.	NONE	990.00000
61	X19	LL	.	.	.	NONE	4740.00000
62	X20	LL	.	.	.	NONE	5502.00000
63	X21	BS	525.00000	.	.	NONE	.
64	X22	LL	.	.	.	NONE	1874.00000
65	X23	LL	.	.	.	NONE	2056.00000
66	X24	LL	.	.	.	NONE	2154.00000
67	X25	LL	.	.	.	NONE	381.00000
68	X26	LL	.	.	.	NONE	266.00000
69	X27	BS	1575.00000	.	.	NONE	.
70	X28	LL	.	.	.	NONE	8778.00000
71	X29	LL	.	.	.	NONE	10530.00000
72	X30	LL	.	.	.	NONE	3830.00000
73	X31	LL	.	.	.	NONE	4174.00000
74	X32	LL	.	.	.	NONE	5003.00000
75	X33	LL	.	.	.	NONE	3536.00000
76	X34	LL	.	.	.	NONE	1728.00000
77	X35	LL	.	.	.	NONE	2065.00000
78	X36	BS	2100.00000	.	.	NONE	.
79	X37	LL	.	.	.	NONE	6298.00000
80	X38	LL	.	.	.	NONE	7550.00000
81	X39	LL	.	.	.	NONE	2620.00000
82	X40	LL	.	.	.	NONE	2694.00000
83	X41	LL	.	.	.	NONE	3223.00000
84	X42	LL	.	.	.	NONE	3756.00000
85	X43	LL	.	.	.	NONE	778.00000
86	X44	LL	.	.	.	NONE	925.00000
87	X45	LL	.	.	.	NONE	990.00000
88	X46	LL	.	.	.	NONE	4488.00000
89	X47	LL	.	.	.	NONE	5380.00000
90	X48	BS	2100.00000	.	.	NONE	.
91	X49	LL	.	.	.	NONE	1604.00000

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
92	X50	LL	.	.	.	NONE	1913.00000
93	X51	LL	.	.	.	NONE	2186.00000
94	X52	LL	.	.	.	NONE	88.00000
95	X53	LL	.	.	.	NONE	95.00000
96	X54	BS	6300.00000	.	.	NONE	.
97	X55	LL	.	.	.	NONE	8727.00000
98	X56	LL	.	.	.	NONE	10588.00000
99	X57	LL	.	.	.	NONE	3653.00000
100	X58	LL	.	.	.	NONE	4107.00000
101	X59	LL	.	.	.	NONE	5043.00000
102	X60	LL	.	.	.	NONE	3554.00000
103	X61	LL	.	.	.	NONE	1640.00000
104	X62	LL	.	.	.	NONE	2081.00000
105	X63	BS	525.00000	.	.	NONE	.
106	X64	LL	.	.	.	NONE	6247.00000
107	X65	LL	.	.	.	NONE	7608.00000
108	X66	LL	.	.	.	NONE	2620.00000
109	X67	LL	.	.	.	NONE	2617.00000
110	X68	LL	.	.	.	NONE	3263.00000
111	X69	LL	.	.	.	NONE	3951.00000
112	X70	LL	.	.	.	NONE	700.00000
113	X71	LL	.	.	.	NONE	941.00000
114	X72	LL	.	.	.	NONE	1157.00000
115	X73	LL	.	.	.	NONE	4437.00000
116	X74	LL	.	.	.	NONE	5438.00000
117	X75	BS	525.00000	.	.	NONE	.
118	X76	LL	.	.	.	NONE	1537.00000
119	X77	LL	.	.	.	NONE	1953.00000
120	X78	LL	.	.	.	NONE	2381.00000
121	X79	BS	1575.00000	.	.	NONE	.
122	X80	LL	.	.	.	NONE	111.00000
123	X81	LL	.	.	.	NONE	167.00000
124	X82	LL	.	10.00000	.	NONE	20.00000
125	X83	BS	939750.00000	10.00000	.	NONE	.
126	X84	LL	.	1.00000	.	NONE	2.00000
127	X85	BS	2173500.00001	1.00000	.	NONE	.

```
SETUP  TEST

TIME =    0.02

   SCALE


MATRIX1  ASSIGNED TO MATRIX1
MATRIX2  ASSIGNED TO MATRIX2
MATRIX3  ASSIGNED TO MATRIX3
MATRIX4  ASSIGNED TO MATRIX4

ETA1     ASSIGNED TO ETA1
ETA2     ASSIGNED TO ETA2
ETA3     ASSIGNED TO ETA3
ETA4     ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE   7

NO CYCLING

    POOLS          NUMBER   SIZE    CORE
H.REG-BITS MAP                       184
WORK   REGIONS       9       360    3240
MATRIX BUFFERS       5      2216   11080
ETA    BUFFERS       6      5736   34416

                       TOTAL  NORMAL  .FREE.  FIXED BOUNDED
ROWS     (LOG.VAR.)      42      36      1       5      0
COLUMNS  (STR.VAR.)      85      85      0       0      0

    751 ELEMENTS - DENSITY =  14.07 -     5 MATRIX RECORDS (WITHOUT RHS'S)


PRIMAL       OBJ = GDF          RHS = RH

TIME =     0.02  MINS.        PRICING    7
 SCALE =      .

  ITER    NUMBER  VECTOR  VECTOR  REDUCED       SUM
 NUMBER   INFEAS    OUT      IN     COST       INFEAS
M     1      9        2      72  1.84560-   26379.3
      2               3      51  1.52675-   26379.3
      3              51      78   .56823-   26379.3
M     4      9       72     127   .72136-   26379.3
      5              36     125   .12077-   24046.0
      6              35      90  1.34417-   21570.3
      7              29      63  1.26017-   19518.7
M     8      4       42      99  1.53594-   5620.27
      9              30      51  1.50107-   4351.39
     10              24     108   .03664-   3172.40
```

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
M	11	1	21	92	1.00000-	1270.69
	12		18	66	1.00000-	.

FEASIBLE SOLUTION

PRIMAL OBJ = ODF RHS = RH

TIME = 0.03 MINS. PRICING 7
SCALE = 1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	13	42	108	122	1712.07-	.34E+08
M	14	21	66	69	79.8787-	.30E+08
	15		20	96	47.4022-	.27E+08
	16		41	105	115.329-	.27E+08
	17		92	95	45.0482-	.27E+08
M	18	4	99	117	29.3953-	.19E+08
	19		122	121	.96637-	.19E+08
	20		95	94	.03907-	.19E+08

OPTIMAL SOLUTION

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	18530924.9999	18530924.9999-	NONE	NONE	1.00000
2	G1	EQ	.	.	.	.	10.00000
3	G2	EQ	.	.	.	.	1.00000
4	LC1	BS	.	.	.	NONE	.
5	LC2	BS	.	.	.	NONE	.
6	LC3	BS	40.95000	40.95000-	.	NONE	.
7	LC4	BS	.	50.00000	NONE	50.00000	.
8	LC5	BS	.	50.00000	NONE	50.00000	.
9	LC6	BS	.	50.00000	NONE	50.00000	.
10	LC7	BS	81.90000	81.90000-	.	NONE	.
11	LC8	BS	81.90000	81.90000-	.	NONE	.
12	LC9	BS	185.85000	185.85000-	.	NONE	.
13	KC1	BS	1752659.99999	1752659.99999-	.	NONE	.
14	KC2	BS	1752659.99998	1752659.99998-	.	NONE	.
15	KC3	BS	8325764.99992	8325764.99992-	.	NONE	.
16	DC1	BS	.	.	.	NONE	.
17	DC2	BS	.	.	.	NONE	.
18	DC3	EQ	2625.00000	.	2625.00000	2625.00000	2067.00000
19	DC4	BS	.	.	.	NONE	.
20	DC5	LL	.	.	.	NONE	88.00000-
21	DC6	EQ	2625.00000	.	2625.00000	2625.00000	961.00000
22	DC7	BS	6300.00000	6300.00000-	.	NONE	.
23	DC8	BS	6300.00000	6300.00000-	.	NONE	.
24	DC9	EQ	10500.00000	.	10500.00000	10500.00000	154.00000
25	D1AA	BS	.	.	.	NONE	.
26	D1AS	BS	.	.	.	NONE	.
27	D2AA	BS	.	.	.	NONE	.
28	D2AS	BS	.	.	.	NONE	.
29	D3AA	LL	525.00000	.	525.00000	NONE	6276.00000-
30	D3AS	LL	525.00000	.	525.00000	NONE	2240.00000-
31	D1BA	BS	.	.	.	NONE	.
32	D1BS	BS	.	.	.	NONE	.
33	D2BA	BS	.	.	.	NONE	.
34	D2BS	BS	.	.	.	NONE	.
35	D3BA	LL	525.00000	.	525.00000	NONE	6333.00000-
36	D3BS	LL	525.00000	.	525.00000	NONE	2360.00000-
37	D1CA	BS	.	.	.	NONE	.
38	D1CS	BS	.	.	.	NONE	.
39	D2CA	BS	.	.	.	NONE	.
40	D2CS	BS	.	.	.	NONE	6550.00000-
41	D3CA	LL	2100.00000	.	2100.00000	NONE	2527.00000-
42	D3CS	LL	2100.00000	.	2100.00000	NONE	

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
43	X1	LL	.	.	.	NONE	9030.00000
44	X2	LL	.	.	.	NONE	10652.00000
45	X3	LL	.	.	.	NONE	3950.00000
46	X4	LL	.	.	.	NONE	4444.00000
47	X5	LL	.	.	.	NONE	5846.00000
48	X6	LL	.	.	.	NONE	5284.00000
49	X7	LL	.	.	.	NONE	2021.00000
50	X8	LL	.	.	.	NONE	1566.00000
51	X9	BS	525.00000	.	.	NONE	.
52	X10	LL	.	.	.	NONE	6550.00000
53	X11	LL	.	.	.	NONE	7672.00000
54	X12	LL	.	.	.	NONE	2620.00000
55	X13	LL	.	.	.	NONE	2964.00000
56	X14	LL	.	.	.	NONE	3366.00000
57	X15	LL	.	.	.	NONE	3724.00000
58	X16	LL	.	.	.	NONE	1071.00000
59	X17	LL	.	.	.	NONE	1096.00000
60	X18	LL	.	.	.	NONE	990.00000
61	X19	LL	.	.	.	NONE	4740.00000
62	X20	LL	.	.	.	NONE	5502.00000
63	X21	BS	525.00000	.	.	NONE	.
64	X22	LL	.	.	.	NONE	1874.00000
65	X23	LL	.	.	.	NONE	2056.00000
66	X24	LL	.	.	.	NONE	2154.00000
67	X25	LL	.	.	.	NONE	381.00000
68	X26	LL	.	.	.	NONE	266.00000
69	X27	BS	1575.00000	.	.	NONE	.
70	X28	LL	.	.	.	NONE	8690.00000
71	X29	LL	.	.	.	NONE	10442.00000
72	X30	LL	.	.	.	NONE	3830.00000
73	X31	LL	.	.	.	NONE	4086.00000
74	X32	LL	.	.	.	NONE	4915.00000
75	X33	LL	.	.	.	NONE	3536.00000
76	X34	LL	.	.	.	NONE	1640.00000
77	X35	LL	.	.	.	NONE	1977.00000
78	X36	BS	525.00000	.	.	NONE	.
79	X37	LL	.	.	.	NONE	6210.00000
80	X38	LL	.	.	.	NONE	7462.00000
81	X39	LL	.	.	.	NONE	2620.00000
82	X40	LL	.	.	.	NONE	2606.00000
83	X41	LL	.	.	.	NONE	3135.00000
84	X42	LL	.	.	.	NONE	3756.00000
85	X43	LL	.	.	.	NONE	690.00000
86	X44	LL	.	.	.	NONE	837.00000
87	X45	LL	.	.	.	NONE	990.00000
88	X46	LL	.	.	.	NONE	4490.00000
89	X47	LL	.	.	.	NONE	5292.00000
90	X48	BS	525.00000	.	.	NONE	.
91	X49	LL	.	.	.	NONE	1516.00000

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
92	X50	LL	.	.	.	NONE	1825.00000
93	X51	LL	.	.	.	NONE	2186.00000
94	X52	BS	.	.	.	NONE	.
95	X53	LL	.	.	.	NONE	7.00000
96	X54	BS	1575.00000	.	.	NONE	.
97	X55	LL	.	.	.	NONE	8727.00000
98	X56	LL	.	.	.	NONE	10588.00000
99	X57	LL	.	.	.	NONE	3653.00000
100	X58	LL	.	.	.	NONE	4107.00000
101	X59	LL	.	.	.	NONE	5043.00000
102	X60	LL	.	.	.	NONE	3554.00000
103	X61	LL	.	.	.	NONE	1640.00000
104	X62	LL	.	.	.	NONE	2081.00000
105	X63	BS	2100.00000	.	.	NONE	.
106	X64	LL	.	.	.	NONE	6247.00000
107	X65	LL	.	.	.	NONE	7608.00000
108	X66	LL	.	.	.	NONE	2620.00000
109	X67	LL	.	.	.	NONE	2617.00000
110	X68	LL	.	.	.	NONE	3263.00000
111	X69	LL	.	.	.	NONE	3951.00000
112	X70	LL	.	.	.	NONE	700.00000
113	X71	LL	.	.	.	NONE	941.00000
114	X72	LL	.	.	.	NONE	1157.00000
115	X73	LL	.	.	.	NONE	4437.00000
116	X74	LL	.	.	.	NONE	5438.00000
117	X75	BS	2100.00000	.	.	NONE	.
118	X76	LL	.	.	.	NONE	1537.00000
119	X77	LL	.	.	.	NONE	1953.00000
120	X78	LL	.	.	.	NONE	2381.00000
121	X79	BS	6300.00000	.	.	NONE	.
122	X80	LL	.	.	.	NONE	111.00000
123	X81	LL	.	.	.	NONE	167.00000
124	X82	LL	.	10.00000	.	NONE	20.00000
125	X83	BS	1684725.00000	10.00000	.	NONE	.
126	X84	LL	.	1.00000	.	NONE	2.00000
127	X85	BS	1683675.00000	1.00000	.	NONE	.

SETUP TEST

TIME = 0.01

SCALE

```
MATRIX1   ASSIGNED TO MATRIX1
MATRIX2   ASSIGNED TO MATRIX2
MATRIX3   ASSIGNED TO MATRIX3
MATRIX4   ASSIGNED TO MATRIX4

ETA1      ASSIGNED TO ETA1
ETA2      ASSIGNED TO ETA2
ETA3      ASSIGNED TO ETA3
ETA4      ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE   7

NO CYCLING
```

POOLS	NUMBER	SIZE	CORE
H.REG-BITS MAP			184
WORK REGIONS	9	360	3240
MATRIX BUFFERS	5	2216	11080
ETA BUFFERS	6	5736	34416

	TOTAL	NORMAL	.FREE.	FIXED	BOUNDED
ROWS (LOG.VAR.)	42	36	1	5	0
COLUMNS (STR.VAR.)	85	85	0	0	0

751 ELEMENTS - DENSITY = 14.07 - 5 MATRIX RECORDS (WITHOUT RHS'S)

PRIMAL   OBJ = GDF   RHS = RH

TIME = 0.02 MINS.   PRICING 7
SCALE = .

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
M	1	9	2	72	1.84560-	26379.3
	2		3	51	1.52675-	26379.3
	3		51	78	.56823-	26379.3
M	4	9	72	90	.22065-	26379.3
	5		78	125	.17514-	26379.3
	6		35	127	.47093-	23903.6
	7		29	63	1.26017-	21852.0
M	8	5	42	99	1.53594-	7953.59
	9		36	78	1.37301-	5620.27
	10		30	51	1.50107-	4351.39

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
	11		24	117	.03664-	3172.40
M	12	1	21	92	1.00000-	1270.69
	13		18	66	1.00000-	.

FEASIBLE SOLUTION

PRIMAL OBJ = GDF RHS = RH

TIME = 0.03 MINS. PRICING 7
SCALE = 1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	14	32	66	62	9.36049-	.15E+08
	15		63	45	11.7672-	.14E+08
	16		8	119	6.21059-	.14E+08
	17		51	48	17.3195-	.14E+08
	18		9	75	6.72121-	.14E+08
	19		48	29	12.5496-	.14E+08
M	20	16	117	116	1.71245-	.13E+08
	21		92	88	1.08160-	.13E+08
	22		78	72	7.10893-	.12E+08
M	23	7	116	115	.41168-	.12E+08
	24		119	118	.79222-	.11E+08
	25		62	61	.35928-	.11E+08
	26		41	102	7.80692-	.11E+08
	27		90	9	7.15069-	.11E+08
M	28	2	118	8	.32687-	.11E+08
	29		99	117	.21385-	.11E+08

OPTIMAL SOLUTION

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	11166225.0000	11166225.0000-	NONE	NONE	1.00000
2	G1	EQ	.	.	.	.	1.00000
3	G2	EQ	.	.	.	.	10.00000
4	LC1	BS	136.50000	136.50000-	.	NONE	.
5	LC2	BS	136.50000	136.50000-	.	NONE	.
6	LC3	BS	184.80000	184.80000-	.	NONE	.
7	LC4	BS	.	50.00000	NONE	50.00000	.
8	LC5	BS	.	50.00000	NONE	50.00000	.
9	LC6	BS	42.00000	8.00000	NONE	50.00000	.
10	LC7	BS	.	.	.	NONE	.
11	LC8	BS	.	.	.	NONE	.
12	LC9	BS	.	.	.	NONE	.
13	KC1	BS	7651350.00004	7651350.00004-	.	NONE	.
14	KC2	BS	7651349.99993	7651349.99993-	.	NONE	.
15	KC3	BS	13702604.9999	13702604.9999-	.	NONE	.
16	DC1	BS	2100.00000	2100.00000-	.	NONE	.
17	DC2	BS	2100.00000	2100.00000-	.	NONE	.
18	DC3	EQ	2625.00000	.	2625.00000	2625.00000	495.00000-
19	DC4	BS	2100.00000	2100.00000-	.	NONE	.
20	DC5	BS	2100.00000	2100.00000-	.	NONE	.
21	DC6	EQ	2625.00000	.	2625.00000	2625.00000	521.00000-
22	DC7	BS	6300.00000	6300.00000-	.	NONE	.
23	DC8	BS	6300.00000	6300.00000-	.	NONE	.
24	DC9	EQ	10500.00000	.	10500.00000	10500.00000	557.00000-
25	D1AA	BS	2100.00000	2100.00000-	.	NONE	.
26	D1AS	BS	.	.	.	NONE	.
27	D2AA	BS	.	.	.	NONE	.
28	D2AS	BS	.	.	.	NONE	.
29	D3AA	BS	525.00000	.	525.00000	NONE	.
30	D3AS	LL	525.00000	.	525.00000	NONE	832.00000-
31	D1BA	BS	2100.00000	2100.00000-	.	NONE	.
32	D1BS	BS	.	.	.	NONE	.
33	D2BA	BS	.	.	.	NONE	.
34	D2BS	BS	.	.	.	NONE	.
35	D3BA	LL	525.00000	.	525.00000	NONE	39.00000-
36	D3BS	LL	525.00000	.	525.00000	NONE	814.00000-
37	D1CA	BS	6300.00000	6300.00000-	.	NONE	.
38	D1CS	BS	.	.	.	NONE	.
39	D2CA	BS	.	.	.	NONE	.
40	D2CS	BS	.	.	.	NONE	.
41	D3CA	LL	2100.00000	.	2100.00000	NONE	241.00000-
42	D3CS	LL	2100.00000	.	2100.00000	NONE	600.00000-

SECTION 2 - COLUMNS

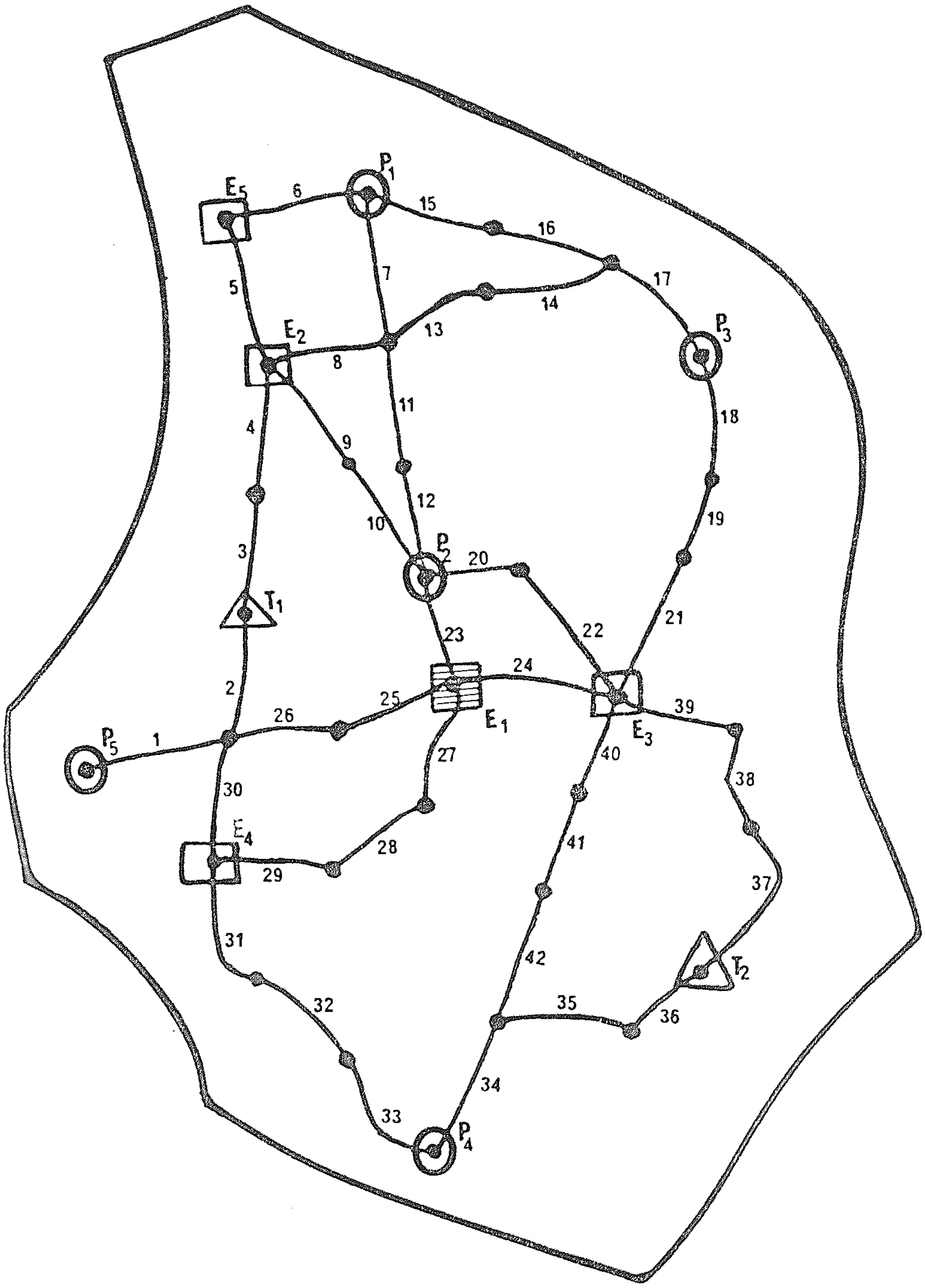
NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
43	X1	LL	.	.	.	NONE	429.00000
44	X2	LL	.	.	.	NONE	611.00000
45	X3	BS	525.00000	.	.	NONE	.
46	X4	LL	.	.	.	NONE	604.00000
47	X5	LL	.	.	.	NONE	863.00000
48	X6	LL	.	.	.	NONE	377.00000
49	X7	LL	.	.	.	NONE	1223.00000
50	X8	LL	.	.	.	NONE	1425.00000
51	X9	LL	.	.	.	NONE	997.00000
52	X10	LL	.	.	.	NONE	181.00000
53	X11	LL	.	.	.	NONE	313.00000
54	X12	LL	.	.	.	NONE	475.00000
55	X13	LL	.	.	.	NONE	456.00000
56	X14	LL	.	.	.	NONE	615.00000
57	X15	LL	.	.	.	NONE	829.00000
58	X16	LL	.	.	.	NONE	1128.00000
59	X17	LL	.	.	.	NONE	1378.00000
60	X18	LL	.	.	.	NONE	1704.00000
61	X19	BS	2100.00000	.	.	NONE	.
62	X20	LL	.	.	.	NONE	96.00000
63	X21	LL	.	.	.	NONE	213.00000
64	X22	LL	.	.	.	NONE	347.00000
65	X23	LL	.	.	.	NONE	484.00000
66	X24	LL	.	.	.	NONE	672.00000
67	X25	LL	.	.	.	NONE	1059.00000
68	X26	LL	.	.	.	NONE	1295.00000
69	X27	LL	.	.	.	NONE	1605.00000
70	X28	LL	.	.	.	NONE	429.00000
71	X29	LL	.	.	.	NONE	624.00000
72	X30	BS	525.00000	.	.	NONE	.
73	X31	LL	.	.	.	NONE	424.00000
74	X32	LL	.	.	.	NONE	596.00000
75	X33	BS	.	.	.	NONE	.
76	X34	LL	.	.	.	NONE	813.00000
77	X35	LL	.	.	.	NONE	1015.00000
78	X36	LL	.	.	.	NONE	478.00000
79	X37	LL	.	.	.	NONE	181.00000
80	X38	LL	.	.	.	NONE	326.00000
81	X39	LL	.	.	.	NONE	457.00000
82	X40	LL	.	.	.	NONE	276.00000
83	X41	LL	.	.	.	NONE	418.00000
84	X42	LL	.	.	.	NONE	600.00000
85	X43	LL	.	.	.	NONE	718.00000
86	X44	LL	.	.	.	NONE	901.00000
87	X45	LL	.	.	.	NONE	1155.00000
88	X46	BS	2100.00000	.	.	NONE	.
89	X47	LL	.	.	.	NONE	109.00000
90	X48	LL	.	.	.	NONE	195.00000
91	X49	LL	.	.	.	NONE	167.00000

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
92	X50	LL	.	.	.	NONE	287.00000
93	X51	LL	.	.	.	NONE	443.00000
94	X52	LL	.	.	.	NONE	649.00000
95	X53	LL	.	.	.	NONE	818.00000
96	X54	LL	.	.	.	NONE	1056.00000
97	X55	LL	.	.	.	NONE	429.00000
98	X56	LL	.	.	.	NONE	625.00000
99	X57	LL	.	.	.	NONE	18.00000
100	X58	LL	.	.	.	NONE	264.00000
101	X59	LL	.	.	.	NONE	417.00000
102	X60	BS	2100.00000	.	.	NONE	.
103	X61	LL	.	.	.	NONE	443.00000
104	X62	LL	.	.	.	NONE	596.00000
105	X63	LL	.	.	.	NONE	199.00000
106	X64	LL	.	.	.	NONE	181.00000
107	X65	LL	.	.	.	NONE	327.00000
108	X66	LL	.	.	.	NONE	262.00000
109	X67	LL	.	.	.	NONE	115.00000
110	X68	LL	.	.	.	NONE	239.00000
111	X69	LL	.	.	.	NONE	387.00000
112	X70	LL	.	.	.	NONE	349.00000
113	X71	LL	.	.	.	NONE	482.00000
114	X72	LL	.	.	.	NONE	662.00000
115	X73	BS	6300.00000	.	.	NONE	.
116	X74	LL	.	.	.	NONE	110.00000
117	X75	BS	2100.00000	.	.	NONE	.
118	X76	LL	.	.	.	NONE	7.00000
119	X77	LL	.	.	.	NONE	108.00000
120	X78	LL	.	.	.	NONE	230.00000
121	X79	LL	.	.	.	NONE	279.00000
122	X80	LL	.	.	.	NONE	399.00000
123	X81	LL	.	.	.	NONE	563.00000
124	X82	LL	.	1.00000	.	NONE	2.00000
125	X83	BS	7706475.00008	1.00000	.	NONE	.
126	X84	LL	.	10.00000	.	NONE	20.00000
127	X85	BS	345975.00000	10.00000	.	NONE	.

ANNEX 3

INPUT DATA OF POLYCENTRIC CASE

FIG.1 CITY EXAMPLE



MAIN EMPLOYMENT CENTRE

EMPLOYMENT CENTRES

POTENTIAL AREAS

TARGET AREAS

ONE MILE LINK

TABLE 8

BUILDING COST OF DIFFERENT DWELLING TYPES

DWELLING TYPE	BUILDING COST
(1) 2 storey house	1,507.
(2) 3 storey flat	1,831.
(3) 4 storey flat	2,212.
(4) 8 storey flat	2,586.
(5) 12 storey flat	2,677.

DWELLING TYPE	TARGET AREA **	
	1	2
(1)	2,351	1,925
(2)	2,233	2,070
(3)	2,576	2,436
(4)	2,892	2,788
(5)	2,973	2,875

** Land cost included

DWELLING TYPE	AREA Potential 1	2	3	4	5	Target 1	2
(1)	650	4,737	398	398	476	824	398
(2)	331	1,897	234	234	264	397	234
(3)	302	1,643	219	219	245	359	219
(4)	259	1,259	197	197	216	301	197
(5)	251	1,191	193	193	211	291	193

TABLE 10

BUILDING COSTS REDEVELOPMENT COSTS AND REHABILITATION COSTS

(POUNDS PER DWELLING)

Building Costs P. Area					
(1)	2,157	2,162	2,514	2,845	2,928
(2)	6,244	3,728	3,855	3,845	3,868
(3)	1,905	2,065	2,431	2,783	2,870
(4)	1,905	2,065	2,431	2,783	2,870
(5)	1,983	2,095	2,457	2,802	2,888
Redevelopment Costs T. Area					
1	2,351	2,233	2,576	2,892	2,973
2	1,925	2,070	2,436	2,788	2,875
Rehabilitation Costs			Dwelling Type		
T. Area		6		7	
1		640			
2				540	

TABLE 6

PROPORTION OF TRIPS MADE FROM POTENTIAL/TARGET AREAS TO EMPLOYMENT CENTRES

AREA	EMPLOYMENT CENTRE				
	1	2	3	4	5
P - 1	.19	.21	.06	.01	0.07
P - 2	.64	.14	.21	.01	0
P - 3	.35	.31	.29	.03	0.02
P - 4	.33	.03	.31	.33	0
P - 5	.54	.20	.08	.16	0
T - 6	.36	.25	.05	.34	0
T - 7	.42	.06	.49	.03	0

TABLE 7

TRANSPORT COST INDEX OF AREAS

AREA	T. C. I.
P - 1	240.0
P - 2	110.4
P - 3	305.4
P - 4	329.0
P - 5	249.0
T - 1	196.4
T - 2	302.4

TABLE 4

TRAVEL TIMES (BY BUS) FROM POTENTIAL AREAS AND TARGET AREAS TO EMPLOYMENT CENTRES (MINUTES)

POTENTIAL AREAS	EMPLOYMENT CENTRE 1	2	3	4	5
1	36	24	36	60	12
2	12	18	12	36	30
3	31	23	19	49	23
4	29	59	17	13	71
5	30	34	42	24	46
TARGET AREAS					
1	22	18	34	10	30
2	25	43	13	41	55

TABLE 5

TRAVEL COST (POUNDS).

AREA	EMPLOYMENT CENTRE 1	2	3	4	5
1	320	160	400	560	80
2	80	160	160	320	240
3	320	320	240	560	320
4	400	640	320	240	160
5	240	320	320	160	400
6	240	160	320	160	240
7	320	560	240	560	640

TABLE 3

AVERAGE SPEED OVER LINKS (mph).

Link	Average Speed	Link	Average Speed
(1)	15	(22)	10
(2)	15	(23)	5
(3)	10	(24)	5
(4)	5	(25)	5
(5)	5	(26)	10
(6)	5	(27)	5
(7)	5	(28)	10
(8)	5	(29)	10
(9)	5	(30)	10
(10)	10	(31)	10
(11)	10	(32)	15
(12)	10	(33)	15
(13)	15	(34)	15
(14)	15	(35)	15
(15)	15	(36)	15
(16)	15	(37)	15
(17)	15	(38)	15
(18)	15	(39)	10
(19)	15	(40)	10
(20)	10	(41)	15
(21)	5	(42)	15

TABLE 2

DISTANCE FROM POTENTIAL AND TARGET AREAS TO EMPLOYMENT CENTRES

(M i l e s)

POTENTIAL AREAS	EMPLOYMENT CENTRE				
	1	2	3	4	5
1	4	2	5	7	1
2	1	2	2	4	3
3	4	4	3	7	4
4	5	8	4	3	8
5	3	4	4	2	5
Target Areas					
1	3	2	4	2	3
2	4	7	3	7	8

Table 1. Employment Centres Size.
(People employed)

Employment Centre	Size (000)
1	250
2	120
3	80
4	50
5	10
Total	510

ANNEX 4

OUTPUT DATA OF POLYCENTRIC CASE

```
TIME =     0.01
    SCALE

MATRIX1   ASSIGNED TO MATRIX1
MATRIX2   ASSIGNED TO MATRIX2
MATRIX3   ASSIGNED TO MATRIX3
MATRIX4   ASSIGNED TO MATRIX4

ETA1      ASSIGNED TO ETA1
ETA2      ASSIGNED TO ETA2
ETA3      ASSIGNED TO ETA3
ETA4      ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE    7

NO CYCLING

     POOLS           NUMBER   SIZE    CORE
H.REG-BITS MAP                          64
WORK   REGIONS         9      128     1152
MATRIX BUFFERS         5      704     3520
ETA    BUFFERS         6      992     5952

                             TOTAL  NORMAL  .FREE.   FIXED  BOUNDED
ROWS    (LOG.VAR.)             13      9       1        3       0
COLUMNS (STR.VAR.)             41     41       0        0       0

     169 ELEMENTS - DENSITY =  24.07 -     4 MATRIX RECORDS (WITHOUT RHS'S)


PRIMAL        OBJ = SUE          RHS = RH

TIME =     0.02  MINS.        PRICING     7
 SCALE =      .

   ITER    NUMBER  VECTOR  VECTOR  REDUCED      SUM
  NUMBER   INFEAS    OUT     IN      COST      INFEAS
M     1       4        3     44   2.71495-   6685.68
M     2       3        2     54    .75802-   4964.97
M     3       2       12     50    .80352-   4833.79
      4               11     49    .71095-   4787.82
      5                7     24    .20049-   3719.52
      6               44     23    .20049-   3387.80
      7                6     17    .06585-   3320.73
M     8       1       17     52    .01147-   1904.37
      9               13     21   1.00000-      .
 FEASIBLE SOLUTION
```

PRIMAL   OBJ = SDF   RHS = PU

TIME =   0.02 MINS.   PRICING   7 .
SCALE =   1.00000

	ITER NUMBER	NUMBER INVERT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	10	5	21	18	5.95038-	.86E+07
	11		24	17	10.9758-	.80E+07
	12		8	16	10.7112-	.79E+07
	13		4	14	.60717-	.79E+07
	14		23	25	.26557-	.79E+07
M	15	1	25	21	.00473-	.79E+07

OPTIMAL SOLUTION

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
14	X1	BS	134.73819	.	.	NONE	.
15	X2	LL	.	.	.	NONE	20264.72222
16	X3	BS	241.37931	.	.	NONE	.
17	X4	BS	344.82759	.	.	NONE	.
18	X5	BS	1149.42529	.	.	NONE	.
19	X6	LL	.	.	.	NONE	7956.00000
20	X7	LL	.	.	.	NONE	6825.40000
21	X8	BS	129.62963	.	.	NONE	.
22	X9	LL	.	.	.	NONE	7686.12222
23	X10	LL	.	.	.	NONE	33.52414
24	X11	LL	.	.	.	NONE	48.17241
25	X12	LL	.	.	.	NONE	.58621
26	X13	LL	.	.	.	NONE	7940.40000
27	X14	LL	.	.	.	NONE	7625.00000
28	X15	LL	.	.	.	NONE	1758.14815
29	X16	LL	.	.	.	NONE	8320.12222
30	X17	LL	.	.	.	NONE	1806.74815
31	X18	LL	.	.	.	NONE	1822.48148
32	X19	LL	.	.	.	NONE	1769.14815
33	X20	LL	.	.	.	NONE	16026.00000
34	X21	LL	.	.	.	NONE	9455.00000
35	X22	LL	.	.	.	NONE	3394.44444
36	X23	LL	.	.	.	NONE	8271.12222
37	X24	LL	.	.	.	NONE	3513.19617
38	X25	LL	.	.	.	NONE	3471.09962
39	X26	LL	.	.	.	NONE	3411.27203
40	X27	LL	.	.	.	NONE	11235.40000
41	X28	LL	.	.	.	NONE	11215.00000
42	X29	LL	.	.	.	NONE	3823.98148
43	X30	LL	.	.	.	NONE	8385.12222
44	X31	LL	.	.	.	NONE	3874.00217
45	X32	LL	.	.	.	NONE	3920.02235
46	X33	LL	.	.	.	NONE	3930.91252
47	X34	LL	.	.	.	NONE	11640.40000
48	X35	LL	.	.	.	NONE	11650.40000
49	X36	BS	20.00000	.	.	NONE	.
50	X37	BS	50.00000	.	.	NONE	.
51	X38	LL	.	5.00000	.	NONE	10.00000
52	X39	BS	1506724.01020	5.00000	.	NONE	.
53	X40	LL	.	1.00000	.	NONE	2.00000
54	X41	BS	355840.68966	1.00000	.	NONE	.

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDP	BS	7889460.74060	7889460.74060-	NONE	NONE	1.00000
2	G1	EQ	2500000.00000	.	2500000.00000	2500000.00000	5.00000
3	G2	EQ	200000.00000	.	200000.00000	200000.00000	1.00000
4	P1	UL	16.00000	.	NONE	16.00000	462.96296
5	P2	BS	.	13.00000	NONE	13.00000	.
6	P3	UL	21.00000	.	NONE	21.00000	14193.49745
7	P4	UL	30.00000	.	NONE	30.00000	13422.73308
8	P5	UL	100.00000	.	NONE	100.00000	10350.51469
9	T6	BS	1.86000	23.14000	NONE	25.00000	.
10	T7	BS	4.65000	25.35000	NONE	30.00000	.
11	D1	LL	20.00000	.	20.00000	NONE	3396.00000-
12	D2	LL	50.00000	.	50.00000	NONE	3002.00000-
13	D3	EQ	2000.00000	.	2000.00000	2000.00000	11065.27778-

```
TIME =    0.01

    SCALE

MATRIX1   ASSIGNED TO MATRIX1
MATRIX2   ASSIGNED TO MATRIX2
MATRIX3   ASSIGNED TO MATRIX3
MATRIX4   ASSIGNED TO MATRIX4

ETA1      ASSIGNED TO ETA1
ETA2      ASSIGNED TO ETA2
ETA3      ASSIGNED TO ETA3
ETA4      ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE    7

NO CYCLING

    POOLS          NUMBER   SIZE    CORE
N.REG-BITS MAP                        64
WORK   REGIONS       9      128     1152
MATRIX BUFFERS       5      704     3520
ETA    BUFFERS       6      992     5952

                        TOTAL  NORMAL  .FREE.   FIXED BOUNDED
ROWS     (LOG.VAR.)      13       9       1       3      0
COLUMNS  (STR.VAR.)      41      41       0       0      0

    162 ELEMENTS - DENSITY =  24.07 -     4 MATRIX RECORDS (WITHOUT RHS'S)


PRIMAL        OBJ = GPF          RHS = RH

TIME =     0.02  MINS.       PRICING    7
 SCALE =     .

   ITER   NUMBER  VECTOR  VECTOR  REDUCED      SUM
  NUMBER  INFEAS    OUT     IN     COST       INFEAS
M    1       4       3      44   2.71495-   6685.69
M    2       3       2      22   1.28353-   5284.84
M    3       2      12      50    .53594-   5177.33
     4              11      49    .53723-   5142.60
     5              44      18    .22428-   4503.70
M    6       1       8      54    .02994-   3911.10
     7               5      52    .01628-   2876.25
M    8       0      13      38   1.00000-      .
 FEASIBLE SOLUTION
```

[illegible]

TIME = 0.02 MINS. PRICING 7
SCALE = 1.00000

	ITER NUMBER	NUMBER UNBORT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
*	9	16	6	23	13.7775-	.39E+07
	10		29	14	14.0966-	.37E+07
	11		4	16	7.62669-	.37E+07
	12		53	21	2.32162-	.35E+07
*	13	2	14	5	10.7623-	.36E+07
*	14	7	32	25	1.96639-	.33E+07
*	15	1	16	6	.22991-	.33E+07

OPTIMAL SOLUTION

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDP	BS	6153774.04217	6153774.04213-	NONE	NONE	1.00000
2	G1	EQ	2500000.00000	.	2500000.00000	2500000.00000	1.00000
3	G2	EQ	200000.00000	.	200000.00000	200000.00000	15.00000
4	P1	UL	16.00000	.	NONE	16.00000	92.59259
5	P2	UL	13.00000	.	NONE	13.00000	12052.77778
6	P3	BS	.	21.00000	NONE	21.00000	.
7	P4	BS	.	30.00000	NONE	30.00000	.
8	[illegible]	UL	[illegible]	.	NONE	[illegible]	[illegible]
9	T6	BS	1.86000	23.14000	NONE	25.00000	.
10	T7	BS	4.65000	25.35000	NONE	30.00000	.
11	D1	LL	20.00000	.	20.00000	NONE	3580.00000-
12	D2	LL	50.00000	.	50.00000	NONE	5070.00000-
13	D3	EQ	2000.00000	.	2000.00000	2000.00000	5765.05556-

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GPF	BS	3263344.44444	3263344.44444-	NONE	NONE	1.[illegible]
2	G1	EQ	2500000.00000	.	2500000.00000	2500000.00000	1.[illegible]
3	G2	EQ	[illegible]0000.00000	.	[illegible]500.00000	[illegible]	[illegible]
4	P1	UL	16.00000	.	NONE	16.00000	14[illegible].[illegible]
5	P2	BS	.	13.00000	NONE	13.00000	.
6	P3	BS	.	21.00000	NONE	21.00000	.
7	P4	BS	.	30.00000	NONE	30.00000	.
8	D5	UL	90.00000	.	NONE	90.00000	2076.17[illegible]7
9	T6	BS	1.86000	23.14000	NONE	25.00000	.
10	T7	BS	4.65000	25.35000	NONE	30.00000	.
11	D1	LL	20.00000	.	20.00000	NONE	1620.00000-
12	D2	LL	50.00000	.	50.00000	NONE	2050.00000-
13	D3	EQ	2000.00000	.	2000.00000	2000.00000	[illegible]-

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
14	X1	LL	.	.	.	NONE	71.[illegible]
15	X2	LL	.	.	.	NONE	3305.[illegible]
16	X3	LL	.	.	.	NONE	23.55556
17	X4	LL	.	.	.	NONE	141.5555[illegible]
18	X5	BS	555.55556	.	.	NONE	.
19	X6	LL	.	.	.	NONE	1596.00000
20	X7	LL	.	.	.	NONE	1367.00000
21	X8	BS	[illegible]	.	.	NONE	.
22	X9	LL	.	.	.	NONE	976.55556
23	X10	LL	.	.	.	NONE	183.55556
24	X11	LL	.	.	.	NONE	301.55556
25	X12	BS	959.59596	.	.	NONE	.
26	X13	LL	.	.	.	NONE	1590.00000
27	X14	LL	.	.	.	NONE	1525.00000
28	X15	LL	.	.	.	NONE	346.37037
29	X16	LL	.	.	.	NONE	901.55556
30	X17	LL	.	.	.	NONE	549.55556
31	X18	LL	.	.	.	NONE	667.55556
32	X19	LL	.	.	.	NONE	353.70370
33	X20	LL	.	.	.	NONE	3210.00000
34	X21	LL	.	.	.	NONE	1891.00000
35	X22	LL	.	.	.	NONE	663.11111
36	X23	LL	.	.	.	NONE	993.55556
37	X24	LL	.	.	.	NONE	1201.55556
38	X25	LL	.	.	.	NONE	1019.55556
39	X26	LL	.	.	.	NONE	692.11111
40	X27	LL	.	.	.	NONE	2242.00000
41	X28	LL	.	.	.	NONE	2243.00000
42	X29	LL	.	.	.	NONE	747.70370
43	X30	LL	.	.	.	NONE	1011.55556
44	X31	LL	.	.	.	NONE	988.55556
45	X32	LL	.	.	.	NONE	1106.55556
46	X33	LL	.	.	.	NONE	766.03704
47	X34	LL	.	.	.	NONE	2330.00000
48	X35	LL	.	.	.	NONE	2332.00000
49	X36	BS	20.00000	.	.	NONE	.
50	X37	BS	50.00000	.	.	NONE	.
51	X38	LL	.	1.00000	.	NONE	2.00000
52	X39	BS	1700062.62626	1.00000	.	NONE	.
53	X40	LL	.	5.00000	.	NONE	10.00000
54	X41	BS	312656.36364	5.00000	.	NONE	.

```
TIME =    0.01
    SCALE

MATRIX1  ASSIGNED TO MATRIX1
MATRIX2  ASSIGNED TO MATRIX2
MATRIX3  ASSIGNED TO MATRIX3
MATRIX4  ASSIGNED TO MATRIX4

ETA1     ASSIGNED TO ETA1
ETA2     ASSIGNED TO ETA2
ETA3     ASSIGNED TO ETA3
ETA4     ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE   7

NO CYCLING

     POOLS          NUMBER    SIZE     CORE
H.DEG-BITS MAP                          64
WORK    REGIONS       9        128     1152
MATRIX BUFFERS        5        704     3520
ETA    BUFFERS        6        992     5952

                        TOTAL   NORMAL   .FREE.    FIXED BOUNDED
ROWS     (LOG.VAR.)       13       9        1        3       0
COLUMNS  (STR.VAR.)       41      41        0        0       0

      169 ELEMENTS - DENSITY =  24.07 -   4 MATRIX RECORDS (WITHOUT RHSIS)

PRIMAL        OBJ = GPF          RHS = RH

TIME =    0.02  MINS.       PRICING   7
 SCALE =

    ITER    NUMBER   VECTOR   VECTOR   REDUCED      SUM
   NUMBER   INFEAS    OUT      IN       COST       INFEAS
 H    1        4        3       44    2.71495-   6685.68
 H    2        3        2       22    1.28353-   5284.84
 H    3        2       12       50     .53594-   5177.33
      4                11       49     .53723-   5142.60
      5                44       11     .22426-   4503.70
 H    6        1        [illegible]   54     .02275-   3811.10
      7                 [illegible]   52     .03710-   2876.25
 H    8        0       13       38    1.00000-
 FEASIBLE SOLUTION
```

TIME = 0.02 MINS. PRICING 7
SCALE = 1.00000

ITER NUMBER	NUMBER [illegible]	[illegible]	VECTOR IN	REDUCED COST	FUNCTION VALUE
M [illegible]	19	[illegible]	21	[illegible].1416-	.62[illegible]+07
10		[illegible]	14	.02[illegible]7-	.62[illegible]+07
M 11	2	22	43	.11[illegible]0-	.62E+07

OPTIMAL SOLUTION

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	6152848.11620	6152848.11620-	NONE	NONE	1.00000
2	S1	EQ	2500000.[illegible]	.	2500000.00000	2500000.00000	1.00000
3	G2	EQ	200000.[illegible]	.	200000.00000	200000.00000	15.00000
4	P1	UL	26.00000	.	NONE	26.00000	92.60259
5	P2	UL	13.00000	.	NONE	13.00000	12052.77778
6	P3	BS	.	21.00000	NONE	21.00000	.
7	P4	BS	.	30.00000	NONE	30.00000	.
[illegible]	[illegible]	UL	[illegible]	.	NONE	[illegible]	[illegible]
9	T6	BS	1.[illegible]	23.14000	NONE	25.00000	.
10	T7	BS	4.[illegible]5000	25.25000	NONE	30.00000	.
11	D1	LL	20.00000	.	20.00000	NONE	3580.00000-
12	D2	LL	50.00000	.	50.00000	NONE	6070.00000-
13	D3	EQ	2000.00000	.	2000.00000	2000.00000	6765.[illegible]-

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
14	X1	BS	218.42273	.	.	NONE	.
15	X2	LL	.	.	.	NONE	3177.53611
16	X3	LL	.	.	.	NONE	720.94444
17	X4	LL	.	.	.	NONE	1074.94444
18	X5	BS	919.54023	.	.	NONE	.
19	X6	LL	.	.	.	NONE	1606.00000
20	X7	LL	.	.	.	NONE	1371.00000
21	X8	BS	[illegible]	.	.	NONE	.
22	X9	LL	.	.	.	NONE	31.68611
23	X10	LL	.	.	.	NONE	880.94444
24	X11	LL	.	.	.	NONE	1234.94444
25	X12	LL	.	.	.	NONE	82.70310
26	X13	LL	.	.	.	NONE	1594.00000
27	X14	LL	.	.	.	NONE	1525.00000
28	X15	LL	.	.	.	NONE	351.62963
29	X16	LL	.	.	.	NONE	95.47500
30	X17	LL	.	.	.	NONE	1246.94444
31	X18	LL	.	.	.	NONE	1600.94444
32	X19	LL	.	.	.	NONE	442.62963
33	X20	LL	.	.	.	NONE	3220.00000
34	X21	LL	.	.	.	NONE	1891.00000
35	X22	LL	.	.	.	NONE	678.88889
36	X23	LL	.	.	.	NONE	4.05278
37	X24	LL	.	.	.	NONE	2498.94444
38	X25	LL	.	.	.	NONE	1952.94444
39	X26	LL	.	.	.	NONE	783.30268
40	X27	LL	.	.	.	NONE	2253.00000
41	X28	LL	.	.	.	NONE	2243.00000
42	X29	LL	.	.	.	NONE	764.79630
43	X30	BS	650.00000	.	.	NONE	.
44	X31	LL	.	.	.	NONE	1685.94444
45	X32	LL	.	.	.	NONE	2039.94444
46	X33	LL	.	.	.	NONE	868.76181
47	X34	LL	.	.	.	NONE	2334.00000
48	X35	LL	.	.	.	NONE	2336.00000
49	X36	BS	20.00000	.	.	NONE	.
50	X37	BS	50.00000	.	.	NONE	.
51	X38	LL	.	1.00000	.	NONE	2.00000
52	X39	BS	2907010.18518	1.00000	.	NONE	.
53	X40	LL	.	15.00000	.	NONE	30.00000
54	X41	BS	223055.86207	15.00000	.	NONE	.

```
TIME =     0.01
      SCALE

MATRIX1   ASSIGNED TO MATRIX1
MATRIX2   ASSIGNED TO MATRIX2
MATRIX3   ASSIGNED TO MATRIX3
MATRIX4   ASSIGNED TO MATRIX4

ETA1      ASSIGNED TO ETA1
ETA2      ASSIGNED TO ETA2
ETA3      ASSIGNED TO ETA3
ETA4      ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE    7

NO CYCLING

     POOLS          NUMBER    SIZE     CORE
H.REG-FITS MAP                           64
WORK    REGIONS        9       128     1152
MATRIX BUFFERS         5       704     3520
ETA     BUFFERS        6       992     5952

                           TOTAL  NORMAL  .FREE.   FIXED BOUNDED
ROWS     (LOG.VAR.)          13       9      1        3      0
COLUMNS  (STR.VAR.)          41      41      0        0      0

      169 ELEMENTS - DENSITY =  24.07 -      6 MATRIX RECORDS (WITHOUT RHS'S)


PRIMAL        OBJ = OBF          RHS = RH

TIME =     0.02  MINS.       PRICING    7
 SCALE =     .

   ITER    NUMBER   VECTOR   VECTOR   REDUCED       SUM
  NUMBER   INFEAS    OUT       IN       COST       INFEAS
M    1        4        3       44    2.71495-   6685.68
M    2        3        2       22    1.28353-   5284.84
M    3        2       12       50     .53594-   5177.33
     4                11       49     .53723-   5142.60
     5                44       10     .22428-   4503.70
M    6        1        8       56     .02275-   3811.10
     7.                5       52     .03710-   2876.25
M    8        0       13       39    1.00000-    .
 FEASIBLE SOLUTION
```

TIME = 0.02 MINS.   PRICING 7
SCALE = 1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	9	19	30	25	0.16512-	.63E+07
	10		4	21	.90646-	.62E+07
M	11	2	25	43	.44410-	.62E+07
M	12	3	22	16	.46059-	.62E+07

OPTIMAL SOLUTION

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
14	X1	BS	31.23[illegible]	.	.	NONE	.
15	X2	LL	.	.	.	NONE	3177.5[illegible]11
16	X3	LL	.	.	.	NONE	720.94444
17	X4	LL	.	.	.	NONE	1974.94444
18	X5	BS	919.54023	.	.	NONE	.
19	X6	LL	.	.	.	NONE	1606.00000
20	X7	LL	.	.	.	NONE	1371.00000
21	X8	BS	397.22222	.	.	NONE	.
22	X9	LL	.	.	.	NONE	31.88611
23	X10	LL	.	.	.	NONE	880.94444
24	X11	LL	.	.	.	NONE	1234.94444
25	X12	LL	.	.	.	NONE	82.79310
26	X13	LL	.	.	.	NONE	1594.00000
27	X14	LL	.	.	.	NONE	1525.00000
28	X15	LL	.	.	.	NONE	351.62963
29	X16	LL	.	.	.	NONE	95.47500
30	X17	LL	.	.	.	NONE	1246.94444
31	X18	LL	.	.	.	NONE	1600.94444
32	X19	LL	.	.	.	NONE	442.62963
33	X20	LL	.	.	.	NONE	3220.00000
34	X21	LL	.	.	.	NONE	1891.00000
35	X22	LL	.	.	.	NONE	679.88889
36	X23	LL	.	.	.	NONE	4.05278
37	X24	LL	.	.	.	NONE	2499.94444
38	X25	LL	.	.	.	NONE	1952.94444
39	X26	LL	.	.	.	NONE	783.30268
40	X27	LL	.	.	.	NONE	2253.00000
41	X28	LL	.	.	.	NONE	2243.00000
42	X29	LL	.	.	.	NONE	764.79630
43	X30	BS	650.00000	.	.	NONE	.
44	X31	LL	.	.	.	NONE	1685.94444
45	X32	LL	.	.	.	NONE	2039.94444
46	X33	LL	.	.	.	NONE	868.76161
47	X34	LL	.	.	.	NONE	2334.00000
48	X35	LL	.	.	.	NONE	2336.00000
49	X36	BS	20.00000	.	.	NONE	.
50	X37	BS	50.00000	.	.	NONE	.
51	X38	LL	.	1.00000	.	NONE	2.00000
52	X39	BS	2007936.11111	1.00000	.	NONE	.
53	X40	LL	.	15.00000	.	NONE	30.00000
54	X41	BS	223055.86207	15.00000	.	NONE	.

```
TIME =     0.01

    SCALE


MATRIX1   ASSIGNED TO MATRIX1
MATRIX2   ASSIGNED TO MATRIX2
MATRIX3   ASSIGNED TO MATRIX3
MATRIX4   ASSIGNED TO MATRIX4

ETA1      ASSIGNED TO ETA1
ETA2      ASSIGNED TO ETA2
ETA3      ASSIGNED TO ETA3
ETA4      ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE    7

NO CYCLING

    POOLS          NUMBER    SIZE     CORE
H.REG-BITS MAP                          64
WORK    REGIONS       9       128     1152
MATRIX BUFFERS        5       704     3520
ETA     BUFFERS       6       992     5952

                        TOTAL  NORMAL   .FREE.   FIXED BOUNDED
ROWS     (LOG.VAR.)       13       9       1        3       0
COLUMNS  (STR.VAR.)       41      41       0        0       0

    169 ELEMENTS - DENSITY =  24.07 -     4 MATRIX RECORDS (WITHOUT RHS'S)


PRIMAL        OBJ = COF          RHS = RH

TIME =      0.02  MINS.        PRICING     7
 SCALE =      .

  ITER    NUMBER   VECTOR   VECTOR   REDUCED       SUM
 NUMBER   INFEAS     OUT      IN       COST      INFEAS
M     1       4        3       44    2.71495-   6685.68
M     2       3        2       22    1.28353-   5284.84
M     3       2       12       50     .53594-   5177.33
      4               11       49     .53723-   5142.60
      5               44       19     .22420-   4503.70
M     6       1        8       54     .02517-   3811.10
      7                5       52     .02737-   2876.25
M     8       0       13       38    1.00000-      .
 FEASIBLE SOLUTION
```

TIME = 0.02 MINS. PRICING 7
SCALE = 1.00000

ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
9	19	38	25	8.63412-	.49E+07
10		4	21	.89608-	.49E+07
11	1	22	5	2.28660-	.48E+07

OPTIMAL SOLUTION

SECTION 1 - ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	4826626.26261	4826626.26261-	NONE	NONE	1.00000
2	G1	EQ	2500000.00000	.	2500000.00000	2500000.00000	1.00000
3	G2	EQ	200000.00000	.	200000.00000	200000.00000	10.00000
4	P1	UL	16.00000	.	NONE	16.00000	2771.04377
5	P2	BS	.	13.00000	NONE	13.00000	.
6	P3	BS	.	21.00000	NONE	21.00000	.
7	P4	BS	.	30.00000	NONE	30.00000	.
8	P5	UL	80.00000	.	NONE	80.00000	2074.07407
9	T6	BS	1.86000	23.14000	NONE	25.00000	.
10	T7	BS	4.65000	25.35000	NONE	30.00000	.
11	D1	LL	20.00000	.	20.00000	NONE	2600.00000-
12	D2	LL	50.00000	.	50.00000	NONE	3560.00000-
13	D3	EQ	2000.00000	.	2000.00000	2000.00000	4653.44444-

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
14	X1	LL	.	.	.	NONE	144.63736
15	X2	LL	.	.	.	NONE	2700.55556
16	X3	LL	.	.	.	NONE	305.55556
17	X4	LL	.	.	.	NONE	541.55556
18	X5	BS	555.55556	.	.	NONE	.
19	X6	LL	.	.	.	NONE	1401.00000
20	X7	LL	.	.	.	NONE	1369.00000
21	X8	BS	[illegible]	.	.	NONE	.
22	X9	LL	.	.	.	NONE	189.55556
23	X10	LL	.	.	.	NONE	455.55556
24	X11	LL	.	.	.	NONE	701.55556
25	X12	BS	950.59596	.	.	NONE	.
26	X13	LL	.	.	.	NONE	1502.00000
27	X14	LL	.	.	.	NONE	1525.00000
28	X15	LL	.	.	.	NONE	340.91512
29	X16	LL	.	.	.	NONE	305.55556
30	X17	LL	.	.	.	NONE	831.55556
31	X18	LL	.	.	.	NONE	1067.55556
32	X19	LL	.	.	.	NONE	353.70370
33	X20	LL	.	.	.	NONE	3215.00000
34	X21	LL	.	.	.	NONE	1891.00000
35	X22	LL	.	.	.	NONE	646.74747
36	X23	LL	.	.	.	NONE	305.55556
37	X24	LL	.	.	.	NONE	1783.55556
38	X25	LL	.	.	.	NONE	1419.55556
39	X26	LL	.	.	.	NONE	682.11111
40	X27	LL	.	.	.	NONE	2251.00000
41	X28	LL	.	.	.	NONE	2243.00000
42	X29	LL	.	.	.	NONE	729.97643
43	X30	LL	.	.	.	NONE	318.55556
44	X31	LL	.	.	.	NONE	1270.55556
45	X32	LL	.	.	.	NONE	1506.55556
46	X33	LL	.	.	.	NONE	766.03704
47	X34	LL	.	.	.	NONE	2332.00000
48	X35	LL	.	.	.	NONE	2334.00000
49	X36	BS	20.00000	.	.	NONE	.
50	X37	BS	50.00000	.	.	NONE	.
51	X38	LL	.	1.00000	.	NONE	2.00000
52	X39	BS	1700062.62626	1.00000	.	NONE	.
53	X40	LL	.	10.00000	.	NONE	20.00000
54	X41	BS	312656.36364	10.00000	.	NONE	.

```
TIME =    0.01

    SCALE

MATRIX1   ASSIGNED TO MATRIX1
MATRIX2   ASSIGNED TO MATRIX2
MATRIX3   ASSIGNED TO MATRIX3
MATRIX4   ASSIGNED TO MATRIX4

ETA1      ASSIGNED TO ETA1
ETA2      ASSIGNED TO ETA2
ETA3      ASSIGNED TO ETA3
ETA4      ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE    7

NO CYCLING

    POOLS          NUMBER   SIZE    CORE
M.REG-BITS MAP                        64
WORK    REGIONS       9      128    1152
MATRIX BUFFERS        5      704    3520
ETA    BUFFERS        6      992    5952

                        TOTAL  NORMAL  .FREE.   FIXED BOUNDED
ROWS     (LOG.VAR.)       13      9       1       3       0
COLUMNS  (STR.VAR.)       41     41       0       0       0

     169 ELEMENTS - DENSITY =  24.07 -     4 MATRIX RECORDS (WITHOUT RHS(S)


PRIMAL        OBJ = GDP          RHS = RH

TIME =     0.02  MINS.        PRICING  7
 SCALE =     .

  ITER    NUMBER  VECTOR  VECTOR  REDUCED      SUM
 NUMBER   INFEAS    OUT      IN     COST      INFEAS
M    1       4        3      44   2.71495-   6685.68
M    2       3        2      54   1.72792-   4994.97
M    3       2       12      50    .80352-   4833.79
     4               11      49    .71095-   4787.82
     5                7      24    .28049-   3719.52
     6               44      23    .28049-   3397.80
     7                6      17    .06585-   3320.73
M    8       1       17      52    .00871-   1904.37
     9               13      21   1.00000-      .
 FEASIBLE SOLUTION
```

```
TIME =     0.33  MINS.          PRICING     7
 SCALE =    1.00000

   ITER    NUMBER   VECTOR   VECTOR   REDUCED     FUNCTION
  NUMBER   INFEAS    OUT       IN      COST         VALUE
M    10       5       21       18    7.88418-    .25E+08
     11                8       17   13.6580-     .24E+08
     12               24       14     .49624-    .24E+08
     13                4       16     .74799-    .24E+08
     14               23       25     .10187-    .24E+08
 OPTIMAL SOLUTION
```

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GDF	BS	23577702.7581	23577702.7581-	NONE	NONE	1.00000
2	G1	EQ	2500000.00000	.	2500000.00000	2500000.00000	15.00000
3	G2	EQ	200000.00000	.	200000.00000	200000.00000	1.00000
4	P1	UL	16.00000	.	NONE	16.00000	1214.55039
5	P2	BS	.	13.00000	NONE	13.00000	.
6	P3	UL	21.00000	.	NONE	21.00000	43911.11111
7	P4	UL	30.00000	.	NONE	30.00000	43639.84675
8	P5	UL	80.00000	.	NONE	80.00000	31111.11111
9	T6	BS	1.86000	23.14000	NONE	25.00000	.
10	T7	BS	4.65000	25.35000	NONE	30.00000	.
11	D1	LL	20.00000	.	20.00000	NONE	9796.00000-
12	D2	LL	50.00000	.	50.00000	NONE	8402.00000-
13	D3	EQ	2000.00000	.	2000.00000	2000.00000	32790.66667-

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
14	X1	BS	[illegible]	.	.	NONE	.
15	X2	LL	.	.	.	NONE	61069.33333
16	X3	BS	241.37931	.	.	NONE	.
17	X4	BS	344.82759	.	.	NONE	.
18	X5	BS	729.88506	.	.	NONE	.
19	X6	LL	.	.	.	NONE	23866.00000
20	X7	LL	.	.	.	NONE	20475.40000
21	X8	LL	.	.	.	NONE	9.41379
22	X9	LL	.	.	.	NONE	23330.73333
23	X10	LL	.	.	.	NONE	28.80000
24	X11	LL	.	.	.	NONE	43.44828
25	X12	BS	500.00000	.	.	NONE	.
26	X13	LL	.	.	.	NONE	23820.40000
27	X14	LL	.	.	.	NONE	22475.00000
28	X15	LL	.	.	.	NONE	5284.55556
29	X16	LL	.	.	.	NONE	25234.73333
30	X17	LL	.	.	.	NONE	5343.15556
31	X18	LL	.	.	.	NONE	5358.88889
32	X19	LL	.	.	.	NONE	5305.55556
33	X20	LL	.	.	.	NONE	48076.00000
34	X21	LL	.	.	.	NONE	28365.00000
35	X22	LL	.	.	.	NONE	10194.82908
36	X23	LL	.	.	.	NONE	25085.73333
37	X24	LL	.	.	.	NONE	10331.86667
38	X25	LL	.	.	.	NONE	10289.77011
39	X26	LL	.	.	.	NONE	10231.66667
40	X27	LL	.	.	.	NONE	33705.40000
41	X28	LL	.	.	.	NONE	33645.00000
42	X29	LL	.	.	.	NONE	11483.62452
43	X30	LL	.	.	.	NONE	25429.73333
44	X31	LL	.	.	.	NONE	11532.95556
45	X32	LL	.	.	.	NONE	11630.46996
46	X33	LL	.	.	.	NONE	11490.55556
47	X34	LL	.	.	.	NONE	34920.40000
48	X35	LL	.	.	.	NONE	34950.40000
49	X36	BS	20.00000	.	.	NONE	.
50	X37	BS	50.00000	.	.	NONE	.
51	X38	LL	.	15.00000	.	NONE	30.00000
52	X39	BS	1540075.86205	15.00000	.	NONE	.
53	X40	LL	.	1.00000	.	NONE	2.00000
54	X41	BS	356564.82759	1.00000	.	NONE	.

```
TIME =    0.01

    SCALE


MATRIX1  ASSIGNED TO MATRIX1
MATRIX2  ASSIGNED TO MATRIX2
MATRIX3  ASSIGNED TO MATRIX3
MATRIX4  ASSIGNED TO MATRIX4

ETA1     ASSIGNED TO ETA1
ETA2     ASSIGNED TO ETA2
ETA3     ASSIGNED TO ETA3
ETA4     ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE   7

NO CYCLING

    POOLS          NUMBER   SIZE    CORE
H.REG-BITS MAP        .               64
WORK   REGIONS        9     128     1152
MATRIX BUFFERS        5     704     3520
ETA    BUFFERS        6     992     5952

                        TOTAL  NORMAL  .FREE.   FIXED BOUNDED
ROWS    (LOG.VAR.)        13      9      1        3      0
COLUMNS (STR.VAR.)        41     41      0        0      0

     169 ELEMENTS - DENSITY =  24.07 -     4 MATRIX RECORDS (WITHOUT RHS(S))


PRIMAL        OBJ = SDF          RHS = CH

TIME =    0.02  MINS.        PRICING   7
 SCALE =      .

   ITER    NUMBER  VECTOR  VECTOR  REDUCED      SUM
  NUMBER   INFEAS    OUT     IN     COST      INFEAS
M    1       4        3      44   2.71495-  6685.68
M    2       3        2      54   1.72792-  4994.97
M    3       2       12      50    .00352-  4833.79
     4               11      49    .71095-  4787.82
     5                7      24    .28049-  3719.52
     6               44      23    .28049-  3387.80
     7                6      17    .06585-  3320.73
M    8       1       17      52    .00871-  1904.37
     9               13      21   1.00000-      .
 FEASIBLE SOLUTION
```

TIME = 0.02 MINS. PRICING 7
SCALE = 1.00000

ITER NUMBER	NUMBER COMPAT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
* 10	5	31	18	7.88418-	.25E+08
11		24	17	11.7550-	.23E+08
12		8	15	13.5419-	.23E+08
13		4	14	.42958-	.23E+08
14		53	25	.19187-	.23E+08

OPTIMAL SOLUTION

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	CPF	BS	22955480.5358	22955480.5358-	NONE	NONE	1.00000
2	C1	EQ	2500000.00000	.	2500000.00000	2500000.00000	15.00000
3	C2	EQ	200000.00000	.	200000.00000	200000.00000	1.00000
4	P1	UL	16.00000	.	NONE	16.00000	1214.55039
5	P2	BS	.	13.00000	NONE	13.00000	.
6	P3	UL	21.00000	.	NONE	21.00000	43911.11111
7	P4	UL	30.00000	.	NONE	30.00000	43739.84775
8	P5	UL	120.00000	.	NONE	120.00000	41111.11111
9	T6	BS	1.86000	23.14000	NONE	25.00000	.
10	T7	BS	4.65000	25.35000	NONE	30.00000	.
11	D1	LL	20.00000	.	20.00000	NONE	9796.00000-
12	D2	LL	50.00000	.	50.00000	NONE	8402.00000-
13	D3	EQ	2000.00000	.	2000.00000	2000.00000	32700.66667-

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
14	X1	BS	[illegible]	.	.	NONE	.
15	X2	LL	.	.	.	NONE	[illegible]
16	X3	BS	241.37931	.	.	NONE	.
17	X4	BS	744.82759	.	.	NONE	.
18	X5	BS	1100.25543	.	.	NONE	.
19	X6	LL	.	.	.	NONE	23866.00000
20	X7	LL	.	.	.	NONE	20475.40000
21	X8	LL	.	.	.	NONE	9.41379
22	X9	LL	.	.	.	NONE	[illegible]
23	X10	LL	.	.	.	NONE	28.80000
24	X11	LL	.	.	.	NONE	43.44828
25	X12	BS	129.62963	.	.	NONE	.
26	X13	LL	.	.	.	NONE	23820.40000
27	X14	LL	.	.	.	NONE	22575.00000
28	X15	LL	.	.	.	NONE	[illegible]
29	X16	LL	.	.	.	NONE	25234.73333
30	X17	LL	.	.	.	NONE	5343.15556
31	X18	LL	.	.	.	NONE	5358.88889
32	X19	LL	.	.	.	NONE	5205.55556
33	X20	LL	.	.	.	NONE	48076.00000
34	X21	LL	.	.	.	NONE	28365.00000
35	X22	LL	.	.	.	NONE	10194.83908
36	X23	LL	.	.	.	NONE	25085.73333
37	X24	LL	.	.	.	NONE	10331.86667
38	X25	LL	.	.	.	NONE	10289.77011
39	X26	LL	.	.	.	NONE	10231.66667
40	X27	LL	.	.	.	NONE	33705.40000
41	X28	LL	.	.	.	NONE	33645.00000
42	X29	LL	.	.	.	NONE	11483.62452
43	X30	LL	.	.	.	NONE	25429.73333
44	X31	LL	.	.	.	NONE	11532.95556
45	X32	LL	.	.	.	NONE	11638.40996
46	X33	LL	.	.	.	NONE	11490.55556
47	X34	LL	.	.	.	NONE	34920.40000
48	X35	LL	.	.	.	NONE	34950.40000
49	X36	BS	20.00000	.	.	NONE	.
50	X37	BS	50.00000	.	.	NONE	.
51	X38	LL	.	15.00000	.	NONE	30.00000
52	X39	BS	1506594.38056	15.00000	.	NONE	.
53	X40	LL	.	1.00000	.	NONE	2.00000
54	X41	BS	356564.82758	1.00000	.	NONE	.

```
TIME =    0.01

    SCALE

MATRIX1  ASSIGNED TO MATRIX1
MATRIX2  ASSIGNED TO MATRIX2
MATRIX3  ASSIGNED TO MATRIX3
MATRIX4  ASSIGNED TO MATRIX4

ETA1     ASSIGNED TO ETA1
ETA2     ASSIGNED TO ETA2
ETA3     ASSIGNED TO ETA3
ETA4     ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING LOT REQUIRED - MAXIMUM POSSIBLE    7

NO CYCLING

    POOLS          NUMBER    SIZE    CORE
H.REG-BITS MAP                          64
WORK    REGIONS       9       128     1152
MATRIX BUFFERS        5       704     3520
ETA    BUFFERS        6       992     5952

                        TOTAL  NORMAL  .FREE.   FIXED BOUNDED
ROWS    (LOG.VAR.)        13       9       1       3       0
COLUMNS (STR.VAR.)        41      41       0       0       0

    169 ELEMENTS - DENSITY =  24.07 -    4 MATRIX RECORDS (WITHOUT RHS'S)

PRIMAL        OBJ = OBF          RHS = RH

TIME =    0.02  MINS.       PRICING    7
 SCALE =      .

   ITER   NUMBER  VECTOR  VECTOR  REDUCED      SUM
  NUMBER  INFEAS    OUT      IN     COST     INFEAS
M     1       4       3      44  2.71495-   6685.68
M     2       3       2      22  1.28353-   5284.84
M     3       2      12      50   .53594-   5177.33
      4              11      49   .53723-   5142.60
      5              44      18   .22428-   4503.70
M     6       1       8      54   .09150-   3811.10
      7               5      52   .00995-   2876.25
M     8       0      13      38  1.00000-      .
 FEASIBLE SOLUTION
```

```
TIME =     0.02  MINS.         PRICING    7
 SCALE =    1.00000
```

	ITER NUMBER	NUMBER INVERT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
*	9	13	33	24	19.3570-	.26E+07
	10		7	5	48.2239-	.22E+07
	11		32	16	37.3227-	.20E+07
	12		6	17	10.5493-	.19E+07
	13		4	21	2.42038-	.19E+07
	14		34	23	.39491-	.19E+07
*	15	1	23	25	.93262-	.19E+07

OPTIMAL SOLUTION

SECTION 1 - [illegible]

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	ODF	BS	1902617.07417	1902417.07417-	NONE	NONE	1.00000
2	C1	EQ	[illegible]	.	[illegible]	[illegible]	1.00000
3	C2	EQ	[illegible]	.	[illegible]	[illegible]	1.00000
4	D1	UL	15.[illegible]	.	NONE	15.00000	[illegible]
5	D2	BS	.	11.00000	NONE	11.00000	.
6	D3	UL	21.[illegible]	.	NONE	21.00000	2122.[illegible]
7	D4	UL	30.00000	.	NONE	30.00000	2051.[illegible]
8	D5	UL	40.[illegible]	.	NONE	40.00000	2074.97407
9	T6	BS	1.36000	23.14000	NONE	25.00000	.
10	T7	BS	4.65000	25.35000	NONE	30.00000	.
11	D1	LL	20.00000	.	20.00000	NONE	835.00000-
12	D2	LL	50.00000	.	50.00000	NONE	842.00000-
13	D3	EQ	2000.00000	.	2000.00000	2000.00000	2412.44444-

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
14	X1	LL	.	.	.	NONE	12.09091
15	X2	LL	.	.	.	NONE	3941.55556
16	X3	BS	241.37931	.	.	NONE	.
17	X4	BS	344.82759	.	.	NONE	.
18	X5	BS	913.79310	.	.	NONE	.
19	X6	LL	.	.	.	NONE	1592.00000
20	X7	LL	.	.	.	NONE	1765.40000
21	X8	BS	[illegible]	.	.	NONE	.
22	X9	LL	.	.	.	NONE	1424.95556
23	X10	LL	.	.	.	NONE	34.50310
24	X11	LL	.	.	.	NONE	49.24138
25	X12	BS	15.15152	.	.	NONE	.
26	X13	LL	.	.	.	NONE	1588.40000
27	X14	LL	.	.	.	NONE	1525.00000
28	X15	LL	.	.	.	NONE	350.77401
29	X16	LL	.	.	.	NONE	1552.95556
30	X17	LL	.	.	.	NONE	391.30370
31	X18	LL	.	.	.	NONE	407.03704
32	X19	LL	.	.	.	NONE	353.70370
33	X20	LL	.	.	.	NONE	3206.00000
34	X21	LL	.	.	.	NONE	1891.00000
35	X22	LL	.	.	.	NONE	676.20202
36	X23	LL	.	.	.	NONE	1543.95556
37	X24	LL	.	.	.	NONE	784.72490
38	X25	LL	.	.	.	NONE	742.62835
39	X26	LL	.	.	.	NONE	692.11111
40	X27	LL	.	.	.	NONE	2247.40000
41	X28	LL	.	.	.	NONE	2243.00000
42	X29	LL	.	.	.	NONE	761.88552
43	X30	LL	.	.	.	NONE	1565.95556
44	X31	LL	.	.	.	NONE	809.40255
45	X32	LL	.	.	.	NONE	631.67944
46	X33	LL	.	.	.	NONE	766.03704
47	X34	LL	.	.	.	NONE	2328.40000
48	X35	LL	.	.	.	NONE	2330.40000
49	X36	BS	20.00000	.	.	NONE	.
50	X37	BS	50.00000	.	.	NONE	.
51	X38	LL	.	1.00000	.	NONE	2.00000
52	X39	BS	1548560.71054	1.00000	.	NONE	.
53	X40	LL	.	1.00000	.	NONE	2.00000
54	X41	BS	353856.36364	1.00000	.	NONE	.

```
[illegible]  [illegible]       [illegible]    [illegible]    COST      [illegible]

M    11        0         12        36     .68817-      .
 FEASIBLE SOLUTION

[illegible]        [illegible] = [illegible]         [illegible] = [illegible]

TIME =     [illegible]                [illegible]   7
 SCALE =    1.[illegible]

   ITER    [illegible]   [illegible]   [illegible]   REDUCED    FUNCTION
  [illegible]  [illegible]   [illegible]     [illegible]     COST       VALUE
M     12        15        [illegible]    1[illegible]   12.0442-   .80E+07
M     13         3        3[illegible]    27   1.07990-   .80E+07
      14                   0       14    .44460-   .80E+07
 OPTIMAL SOLUTION
```

```
TIME =    0.01

    SCALE

MATRIX1   ASSIGNED TO MATRIX1
MATRIX2   ASSIGNED TO MATRIX2
MATRIX3   ASSIGNED TO MATRIX3
MATRIX4   ASSIGNED TO MATRIX4

ETA1      ASSIGNED TO ETA1
ETA2      ASSIGNED TO ETA2
ETA3      ASSIGNED TO ETA3
ETA4      ASSIGNED TO ETA4

SCRATCH1 ASSIGNED TO SCRATCH1
SCRATCH2 ASSIGNED TO SCRATCH2

MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE    7

NO CYCLING

     POOLS          NUMBER    SIZE     CORE
H.REG-BITS MAP                           64
WORK    REGIONS        9       128     1152
MATRIX  BUFFERS        5       704     3520
ETA     BUFFERS        6       992     5952

                          TOTAL  NORMAL  .FREE.   FIXED BOUNDED
ROWS     (LOG.VAR.)         13      9       1       3       0
COLUMNS  (STR.VAR.)         41     41       0       0       0

     169 ELEMENTS - DENSITY =  24.07 -     4 MATRIX RECORDS (WITHOUT RHS'S)

PRIMAL        OBJ = OBF         RHS = RH

TIME =    0.02  MINS.        PRICING  7
 SCALE =      .

   ITER    NUMBER   VECTOR   VECTOR   REDUCED       SUM
  NUMBER   INFEAS     OUT      IN      COST       INFEAS
M     1       4        3       44    2.71495-   8147.88
M     2       3        2       22    1.28351-   6747.03
M     3       2       11       49     .53723-   6573.39
      4               44       18     .22428-   5913.64
      5                5       50     .11234-   5904.70
      6               10       21     .00184-   5901.47
M     7       2       19       54     .02697-   5301.67
      8                4       52     .06380-   4807.44
      9               22       43     .33731-   3734.65
M    10       1       13       38    1.00000-    110.008
```

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	GNF	BS	8025086.54212	8025086.54212-	NONE	NONE	1.00000
2	G1	EQ	2500000.00000	.	2500000.00000	2500000.00000	1.00000
3	G2	EQ	200000.00000	.	200000.00000	200000.00000	15.00000
4	P1	UL	16.00000	.	NONE	16.00000	92.59259
5	P2	UL	13.00000	.	NONE	13.00000	12052.77778
6	P3	BS	.	21.00000	NONE	21.00000	.
7	P4	BS	.	30.00000	NONE	30.00000	.
8	P5	UL	80.00000	.	NONE	80.00000	540.86846
9	T6	BS	9.30000	15.70000	NONE	25.00000	.
10	T7	UL	30.00000	.	NONE	30.00000	25416.66667
11	D1	LL	100.00000	.	100.00000	NONE	3580.00000-
12	D2	LL	350.00000	.	350.00000	NONE	7433.75000-
13	D3	EQ	2000.00000	.	2000.00000	2000.00000	5765.05556-

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
14	X1	BS	33.23755	.	.	NONE	.
15	X2	LL	.	.	.	NONE	3177.53611
16	X3	LL	.	.	.	NONE	720.94444
17	X4	LL	.	.	.	NONE	1074.94444
18	X5	BS	919.54023	.	.	NONE	.
19	X6	LL	.	.	.	NONE	1606.00000
20	X7	LL	.	.	.	NONE	1218.50000
21	X8	BS	397.22222	.	.	NONE	.
22	X9	LL	.	.	.	NONE	31.68611
23	X10	LL	.	.	.	NONE	880.94444
24	X11	LL	.	.	.	NONE	1234.94444
25	X12	LL	.	.	.	NONE	82.79310
26	X13	LL	.	.	.	NONE	1594.00000
27	X14	BS	42.50000	.	.	NONE	.
28	X15	LL	.	.	.	NONE	351.62963
29	X16	LL	.	.	.	NONE	95.47500
30	X17	LL	.	.	.	NONE	1246.94444
31	X18	LL	.	.	.	NONE	1600.94444
32	X19	LL	.	.	.	NONE	442.62963
33	X20	LL	.	.	.	NONE	3220.00000
34	X21	LL	.	.	.	NONE	264.33333
35	X22	LL	.	.	.	NONE	678.88889
36	X23	LL	.	.	.	NONE	4.05278
37	X24	LL	.	.	.	NONE	2498.94444
38	X25	LL	.	.	.	NONE	1952.94444
39	X26	LL	.	.	.	NONE	783.30268
40	X27	LL	.	.	.	NONE	2253.00000
41	X28	LL	.	.	.	NONE	413.00000
42	X29	LL	.	.	.	NONE	764.74630
43	X30	BS	650.00000	.	.	NONE	.
44	X31	LL	.	.	.	NONE	1685.94444
45	X32	LL	.	.	.	NONE	2039.94444
46	X33	LL	.	.	.	NONE	868.76181
47	X34	LL	.	.	.	NONE	2334.00000
48	X35	LL	.	.	.	NONE	480.58333
49	X36	BS	100.00000	.	.	NONE	.
50	X37	BS	307.50000	.	.	NONE	.
51	X38	LL	.	1.00000	.	NONE	2.00000
52	X39	BS	3085948.61110	1.00000	.	NONE	.
53	X40	LL	.	15.00000	.	NONE	30.00000
54	X41	BS	329335.86207	15.00000	.	NONE	.

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