A Theoretical Investigation into the Ultimate Strength in Bending and Torsion of Plain and Reinforced Concrete Members

by

L.H. Martin BSc. C.Eng. M.I.C.E.

A thesis submitted for the degree of Doctor of Philosophy

Department of Civil Engineering,
The University of Aston in Birmingham

APRIL 1973

Synopsis

Presented in this thesis are original theoretical solutions for the determination of the ultimate strength in bending and torsion for:-

(a) Plain concrete members.

(b) Concrete members reinforced with longitudinal steel only.

(c) Concrete members reinforced with longitudinal and transverse steel at yield.

(d) Concrete members reinforced with longitudinal and transverse steel, where partial yielding and non yielding occurs.

The theories are compared with available experimental results and show reasonable agreement.

Acknowledgements

The author is indebted to Professor Holmes, B.Sc., Ph.D., C.Eng., F.I.C.E., F.I.Struct.E., F.I.Mun.E., Head of the Department of Civil Engineering at the University of Aston in Birmingham for his supervision, encouragement and for the opportunity to carry out this work.

The author would also like to thank Miss J. Stanley for typing the thesis, and Miss P.C. Page for preparation of the drawings.

The Author

The author graduated from the department of Civil Engineering at Leeds in 1952. Training was undertaken with British Railways from 1952 to qualify as a chartered engineer in 1956. Further industrial experience with British Railways was obtained from 1955 to 1959. From 1959 the author was employed as a lecturer in the then College of Advanced Technology in Birmingham. In 1966 the College obtained its charter and became the University of Aston in Birmingham. In 1969 the author was appointed as a senior lecturer in the Department of Civil Engineering.

No part of this work has been submitted in support of an application for another degree or qualification.

Notation

Subscripts 1 and 2 refer to the different modes of failure

u refers to ultimate strength

s refers to steel

c refers to concrete

v refers to shear stress

y refers to yield

A	cross sectional area of longitudinal tension steel in the beam
b	breadth of section
b_1	effective breadth of section
b_n	depth of the compression zone for mode 2
d	depth of a section
d_n	depth of compression zone
d_1	effective depth of a section
D_ℓ	dowel force at right angles to the longitudinal steel in the tension zone
E_c	Young's modulus for concrete
E_s	Young's modulus for steel
fc'	uniaxial cylinder compressive strength of concrete
f_{cm}	maximum direct stress in the concrete due to the bending moment
f_{cmi}	maximum direct stress due to bending and torsion on a skew failure plane
f_{cv}	maximum shear stress in the concrete due to the torsional moment
f_r	modulus of rupture of concrete
f_{sm}	axial stress in the tensile steel
f_{sv}	shear stress in the longitudinal tensile steel due to the dowel force
f_s	axial stress in the stirrups
f_y	yield stress in the longitudinal steel

G_s	modulus of rigidity or shear modulus for the steel
k_{cm}	bending direct stress distribution constant for the concrete
k_{cmi}	direct stress distribution constant for the concrete on the skew failure plane
k_{cv}	torsional shear stress distribution constant for the concrete
k_ℓ	constant associated with the lever arm
l_a	the lever arm
$m = \frac{E_s}{E_c}$	modular ratio
M	bending moment applied to a member
$P_1 = \frac{A}{bd_1}$	ratio of area of tensile steel to concrete area for mode 1 type of failure
P'_1	modified value of P_1
T	torsional moment applied to a member
u_w	uniaxial cube compressive strength of concrete
α	crack angle in the concrete
ε_c	strain in the concrete at the top of member due to bending for mode 1 type of failure
ε_{ci}	maximum compressive strain, perpendicular to the skew failure plane
ε_{cs}	strain in the concrete at the level of the steel due to bending for mode 1 type of failure
ε_{csi}	strain in the concrete adjacent to the tensile steel, perpendicular to the skew failure plane
ε_s	strain in the steel
θ	angle of the compression hinge in the concrete
θ'	" " " " " " " " associated with T_u'
λ_s	constant used in the yield criteria expression for steel
T_u'	modified ultimate torsional moment of resistance of a plain concrete member

f_t uniaxial tensile strength of concrete

L maximum depth of the trapezoidal failure plane in plain concrete

L_1 minimum depth of the trapezoidal failure plane in plain concrete

L_1' equivalent projected length of L_1

Z elastic section modulus

D diameter of circular cross section

A_s cross sectional area of one leg of a stirrup

f_{sy} yield stress for the stirrup steel

S longitudinal spacing of stirrups

b' minimum distance between legs of stirrups centre to centre

d' maximum " " " " " " " "

$$r_{1y} = \frac{A_s f_{sy} b'}{S A_1 f_{1y}}$$

$$r_{2y} = \frac{A_s f_{sy} d'}{S A_2 f_{2y}}$$

$$r_{3y} = \frac{A_s f_{sy} b'}{S A_3 f_{3y}}$$

$$\rho_2 = \frac{A_2}{db_1}$$

$$\rho_3 = \frac{A_3}{bd_3}$$

β angle to the horizontal of the straight line failure envelope for concrete.

$$r_2 = \frac{A_s d'}{S A_2}$$

$$r_1 = \frac{A_s b'}{S A_1}$$

$$r_{12} = \frac{A_s b'}{S A_2}$$

P_ℓ volume percentage of longitudinal bars

P_s volume percentage of stirrups

P_t volume percentage of total reinforcement

r_1' critical value of r

$$m' = \frac{A_2 \ S}{A_s \ (b'+d')}$$

P_{tb} volume percentage of total reinforcement at the limit of partial yielding

I second moment of area of a cross section about the neutral axis

h length of sloping side of trapezoidal cross section in plain concrete.

CONTENTS

page number

APPENDIX

CHAPTER 1 - Review of Previous Work on the Ultimate Strength in Torsion and Bending

1.1 Introduction

Modern methods of designing and constructing monolithic reinforced concrete structures tend to introduce torsional moments into members, which cannot be ignored in design. At the same time many codes of practice for reinforced concrete for various countries are due for revision, and now need to include appropriate clauses on torsion. The poor state of knowledge on torsion requirements in design in 1964 was shown in a review of codes of practice by Fisher and Zia[1] for 22 countries. Only 16 specified torsion design requirements and only half of these gave permissible stresses. Many codes which have adopted an ultimate strength approach for design are still based on the classic elastic theory developed by St. Venant in 1853[2].

Recently investigations in this field have received a new theoretical impetus, based on an experimental observation by Lessig[3] for reinforced concrete members that the failure mechanism is that of bending on a skew failure plane. This mode of failure has also been shown to occur in plain concrete beams by Hsu[4] using high speed photography. These experimental observations have provided the basis for the development of theory by many investigaters including the author.

Before developing the theory however it is proposed to review the state of knowledge of the ultimate strength of members subject to bending and torsion, in the fields studied by the author.

1.2 Review of Plain Concrete Members

In 1853[2] Saint-Venant presented his memoir on torsion to the French Academy of Science. He assumed a homogeneous elastic body and obtained an expression for a rectangular section subject to torsion of

$$T = (\text{factor})\ db^2\ (\text{max. shear stress})$$

The factor is a function of $^b/_d$ varying between $^1/_5$ to $^1/_3$. This expression was initially applied to concrete at ultimate load. The maximum shear stress was interpreted as the maximum principal tensile stress, which is approximately equal to the uniaxial tensile strength of the concrete. Experimental results however showed that the torsional strength was greater than predicted by the St. Venant equation.

It was assumed therefore by Turner and Davies[5] in 1934 that the excess of torsional resistance was due to plastic behaviour of concrete in tension. Although this assumption gave better agreement with experimental results, tensile tests on concrete showed little or no plasticity. Further tests on plain concrete by Marshall and Tembe[6] in 1941 produced the conclusion that concrete possesses both elastic and plastic properties.

In 1950 Fisher[7] for his Ph.D. thesis conducted an experimental investigation into the interaction in bending and torsion for circular cross sections. This work provided results for a full range of $^M/_T$ ratios and illustrated that the relationship between bending and torsion was probably an ellipse. Fisher appreciated the importance of a standard tensile test to relate to his results and provided uniaxial tensile strengths and modulus of rupture for a 6" deep beam.

In 1953 Cowan[8], again assuming plastic behaviour in the concrete, developed a theory which considered the interaction of torsion and bending, using the principal tensile stress as the failure criteria for concrete. The general non dimensional interaction equation connecting torsion and bending was

$$\left(\frac{T}{To}\right)^2 + \frac{M}{Mo} = 1$$

where $$To = \frac{1}{2} b^2 \left(d - {}^b/_3\right) f_t$$

and $$Mo = \frac{bd^2}{4.23} f_t$$

The expression for Mo was derived by assuming a parabolic distribution of tensile stress and a linear distribution for compressive stress in bending. The expression for To is the one developed by Nadai[9] assuming full plasticity of the concrete. Despite the controversial assumption of plastic behaviour of concrete in tension, the form of the general interaction equation is of considerable importance, since a similar comparable expression can be obtained using the "skew bending" approach and assuming elastic behaviour as shown in this thesis.

Cowan[10] seems to have made little or no attempt to obtain or produce sufficient evidence to corroborate his theoretical approach. This was however attempted by Walsh et al [11] in 1966 who conducted experiments to determine whether the concrete behaved elastically or plastically. The conclusion was that the concrete behaved plastically.

Experiments by Hsu[4] in 1966 using high speed photography showed that failure of a rectangular section in pure torsion occurred in bending on a skewed failure plane with the neutral axis parallel to the longer side. This mode of failure had earlier been shown to occur by Lessig [3] for reinforced sections subject to torsion and bending. The photographic evidence led Hsu to develop a theory for pure torsion for a failure plane at a variable angle θ to the longitudinal axis of the member. A minimum value of T occurred when $\theta = 45^{o}$ in which case

$$T_2 = (\text{factor}) \frac{db^2}{3} f_{r2}$$

Hsu assumed and verified by strain readings that the value of the tensile strength (f_{r2}) was related to the modulus of rupture, which varied with the breadth of the section and the strength of the concrete. He developed an expression to determine f_{r2} related to the uniaxial tensile strength f_t, the uniaxial compressive strength and the standard modulus of rupture specimen. It was however necessary to modify f_{r2} by the factor of 0.85 to allow for the fact that the tensile stress was produced in torsion.

In an appendix to his paper Hsu constructed an equation connecting self weight bending moment and torsional bending moment involving a variable angle θ for the failure plane. This produced a solution which necessitated determining the angle θ and then determining T using this angle of θ. Constructing the equation in a slightly different manner would have produced a more elegant solution which does not involve the variable angle θ as shown by the author later in this thesis. Hsu made no attempt to relate his equation to available experimental results in torsion and bending.

1.3 Review of Concrete Members Containing Longitudinal Steel Only

This particular area has attracted little interest for research in the past. Anderson[12] concluded in 1937 that longitudinal steel cannot be relied on to increase the pure torsional resistance of rectangular sections.

The first theoretical solution was presented by Nylander[13] in 1945. He assumed that the torque is partly resisted by the uncracked portion of the beam and partly by the shear forces in the steel acting approximately about the mid point of the section. The shear stress and the bending stress in the steel were combined according to the Huber-Beltramis criterion of yield to the steel. Since this theory assumes a yield condition for the steel it is not applicable in conditions when failure is dependent on the strength of the concrete.

Gesund and Boston[14] in 1964 tested 10 beams with longitudinal steel. From the appearance of the failure mechanism they developed a theory which introduced a dowel force. The effect of the lateral dowel force was to break out small concrete pyramids on the side of the beam which produced failure in the beam. They experienced difficulty in determining the length of the side of the pyramid, and the tensile strength involved. The member was assumed to rotate at failure about a point at the top of the section, or at the mid point of the side. Generally the method gives conservative results, but the approach deserves considerable attention because it is a mode of failure that will obviously occur if the cover to the steel is insufficient. Walsh et al [11] concluded in 1966 that "no satisfactory method has been proposed for calculating the ultimate strength of beams of this type under combined bending and torsion." After a series of tests they concluded that "it has not been possible to evolve a rational method of calculating the failure loads of beams with longitudinal steel only subject to bending and torsion." Failing to suggest a relationship connecting M and T, Walsh et al recommended an empirical value of the torsional strength of a member To which does not depend on M where

$$To = 1.75 b^2 (d - {}^{b}/_{3}) \sqrt{f_c'}$$

In 1966 Iyengar and Rangan[15] concluded after a few tests in pure torsion that the longitudinal steel, even if increased to 6%, increases the torsional strength of plain concrete by only 10%. This conclusion is the same as made by Anderson[12] in 1937.

A further review of test results and theory by Hsu[16] in 1968 again produced conservative equation of an empirical nature in non dimensional form

$$^{T}/_{Tuo} = 1 \qquad \text{for } ^{M}/_{Muo} \leq 0.5$$

$$^{T}/_{Tuo} = 1.7 - 1.40\ (^{M}/_{Muo}) \quad \text{for } 0.5 < {}^{M}/_{Muo} \leq 1.0$$

$$^{M}/_{Muo} = 1 \qquad \text{for } ^{T}/_{Tuo} < 0.3$$

$$\text{where } Tuo = 6(b^2+10)\ d\ (fc')^{1/3} \qquad \text{for } b > 4'' \text{ imperial units.}$$

In 1969 Mirza and McCutcheon[17] tested 107 quarter scale model beams in torsion bending and shear. The models were too small to take strain readings on the concrete or the steel, but the large number of results did enable them to draw the 3 dimensional, non dimensional interaction diagrams for torsion shear and bending. These diagrams also provide indications of the effects of increasing longitudinal reinforcement for a fixed $^d/_b$ ratio of 1.5 and subject to torsion and bending. Their graphs show an increase in strength from pure torsion to an $^M/_T$ ratio of approximately 4. Since in this region the theoretical mode of failure is by the formation of the compression hinge at the side of the member, then it must be assumed that this increase occurs due to a transition from mode 2 to mode 1, where the compression hinge is at the top of the section.

Since these test results were for small scale beams using crushed quartz sand and high early strength concrete, they will no doubt have to be repeated for normal concrete mixes, since there is always the possibility that there is some difference in behaviour.

1.4 Review of Concrete Members Containing Longitudinal and Transverse Reinforcement

An accumulation of experimental observations over the last 50 years has enabled members which are reinforced longitudinally and transversely to be broadly divided into three cases at ultimate load conditions:-

Case 1 - All the steel intercepting the failure zone yields.

Case 2 - Part of the steel intercepting the failed zone yields.

Case 3 - None of the steel intercepting the failure zone yields.

Early investigators concentrated on theory and experiment for members subject to pure torsion. Rausch[18] in 1929 employed the space truss analogy where the steel bars behave as tension members and the concrete as compression members. Assuming that the steel yielded for a member reinforced with closed hoops and an equal amount of longitudinal steel

$$T = \frac{2A_s f_{sy}}{s} b'd'$$

Anderson[12] in 1937 developed an expression for the torsioned strength of a circular cross section, and then modified this for a rectangular cross section by assuming that only the bars at the centre of the wider face reached yield stress.

Cowan[19] in 1950 adopted a distribution of stress based on the St. Venant Theory, and considered that the strength of the member was that for plain concrete plus the strength from the stirrups.

$$T = Tc + 1.6 \frac{A_s f_{sy}}{s} b'd'$$

This form of expression is considered important since it has been developed by later investigators.

Cowan [8] then in 1953 was the first to consider the action of combined bending and torsion on a member reinforced longitudinally and transversely. He also followed the same general approach by assuming that the applied torsional moment was resisted part by the concrete and part by the torsonal reinforcement. The torsional resistance of the concrete was expressed as the plastic torsional resistance for a plain concrete member, and the stress in the adjacent torsional steel was expressed in terms of a concrete stress assuming elastic behaviour and using a modular ratio. The torsional stress in the concrete could therefore be expressed in terms of the applied torsional moment and parameters such as section size and reinforcement.

The applied bending moment was assumed to act on the member as in pure bending, producing a crack at the bottom of the section which extended to the neutral axis, creating a compression zone at the top of the section. This compression zone was then considered to be subjected to a direct stress due to bending. This direct stress due to bending was then expressed in terms of the applied bending moment, size of section and reinforcement.

Where torsion predominated Cowan combined the torsional stress and the direct stress due to bending using the principal tensile stress criterion or the principal strain criterion for failure. Where bending predominated these stresses were combined using the Coulomb-Mohr internal friction theory. The application of these failure criteria enabled Cowan to form equations connecting the bending and torsional moments.

Cowan[10] showed in a later paper that this theory tended to be conservative. The assumption of cracks existing in the case of bending stress, and not existing in the case of torsion can be criticised in relation to experimental evidence. Cowan also failed to notice, as others did later, that the compression zone does not always remain at the top of the section, and may be at the side or at the bottom.

Lessig [3] in 1959 significantly changed theoretical approaches by basing a theory for longitudinally and tranversely reinforced beams at yield on a mode of failure observed in experiments. She observed that at ultimate load when bending predominated, failure occurred by rotation of the beam about a compression zone which was at an angle to the longitudinal axis. When failure occurred in pure bending this compression zone was at 90° to the longitudinal axis. She formed equilibrium equations by equating moments of forces about an axis perpendicular to the neutral axis, and by equating forces perpendicular to the plane containing the neutral axis. Subsequently this approach was modifed by Gvozdev to give the same answers based on the method of equating the work of external and internal forces with their virtual displacement.

This approach produced equations connecting bending moment, torsional moment, reinforcement and shape parameters, but unfortunately they were complicated and difficult to use. The theory was developed for mode 1 where the compression zone is at the top of the section, and also for mode 2 where the compression zone is at the side of the section. She failed however to identify mode 3 where the compression zone is at the bottom of the beam.

Later experiments by Lessig defined the condition for yielding of the steel as

$$\frac{A_s f_{sy}}{s A_1 f_{1y}} = \frac{1}{1 + \frac{2M}{T}\sqrt{\frac{1}{1 + {}^{2d}/_b}}}$$

Further experiments yielded an expression for the depth of the compression zone as

$$\frac{d_n}{d_1} = \frac{A_1 f_{1y} - A_{1c} f_{1cy}}{f_c' \, bd_1 \, (1+5\, {}^T/_M)}$$

This introduces the notion that the depth of the compression zone is reduced by the increase in torsional moment. Experiments which increased the area of steel until it did not yield produced an expression for the depth of the compression zone for an over-reinforced beam which again shows a decrease for an increase in torsional moment.

$$\frac{dn}{d_1} = 0.55 - 0.7\sqrt{{}^T/_M}$$

These experiments also lead to an expression for the torsional resistance of an over reinforced member.

$$\frac{M}{db^2 fc'} = 0.07 \text{ to } 0.12$$

The variation in limits was not explained except by the statistical variation in the strength of the concrete. Further experiments including shear showed that the value did not vary beyond these same limits.

Chinekov[21] in 1959 conducted further tests to substantiate Lessig's[3] theory. A series of beams with longitudinal reinforcement and closed vertical stirrups showed that the theory was reasonably accurate. He also took strain readings on the steel to show that the steel intercepting the failure zone did in fact reach yield.

Further tests with vertical stirrups on the side faces only and not connected in a closed hoop, showed that as soon as the concrete cracked the member split longitudinally and the stirrups were ineffective.

The crack angle on the sides of the beam was also investigated by Chinekov who concluded that it varied between being vertical in pure bending to 45° in pure torsion. He also concluded that although it was not correct to assume the crack was in a straight line when shown on a developed plan, the variation was small and did not substantially affect the theory. The maximum skew angle for the compression zones was a line joining the cracks at 45° on the other faces.

In 1959 Iyalin[22] also conducted a series of tests on members reinforced longitudinally and transversely and subject to varying ratios of bending, torsion and shear.

One of the major objectives with the tests was to determine the ratio of transverse to longitudinal reinforcement to produce yield at the failure zones. From the tests he produced a table giving empirical values of $\frac{A_s f_{sy} d}{S A_l f_{ly}}$ for varying $\frac{T}{M}$ ratios. These values were not exactly defined since he was not always certain when yield occurred despite having taken strain gauge readings. His main problem was that having decided to fix strain gauges and a certain section, there was no guarantee failure would occur there. Other conclusions from the tests were that compression failures could occur where the longitudinal steel and transverse steel did not reach yield. Also partial yield could occur, where the longitudinal steel reached yield and the transverse stirrups did not, or the transverse stirrups reached yield and the longitudinal steel did not.

It is assumed that the new design method of calculation referred to in the paper is that of Lessig , and values of torsional resistance calculated using this method are too high when compression failures, or partial yielding occur.

Yudin[23] in 1962 formed theoretical equations by taking moments of forces about the longitudinal and transverse axes of the member. He also assumed that the crack angle on the bottom and sides of the member was 45°. This produced two equations one of which connected the bending moment and the torsional moment linearly. Experimental results however showed that this approach was conservative.

Gesund et al[24] theory in 1964 was the first attempt to include dowel action of the longitudinal steel. The failure zone was assumed to be bounded by crack angles of 45^{o} on the side of the number, and a variable angle θ on the bottom. The axis of rotation was assumed to be at the centre of the extreme fibres of the compression zone and parallel to the longitudinal axis of the beam.

Two types of failure were recognised, a "bending failure" occurring when the longitudinal steel yields before collapse, and a torsional failure if it does not. The value of θ to be used was based on experimental data, 90^{o} for $^{T}/_{M} < 0.25$, and 63.5^{o} if $^{T}/_{M} \geq 0.25$.

The solution for the bending failure was similar to that of Yudin, linear between pure bending and the torsional failure with a discontinuity at $^{T}/_{M} = 0.25$. Two expressions were given for the "torsional failure, one based on the yielding of the stirrups, and the other based on dowel action of the longitudinal bars. The one that is applicable is the one which gives the larger value.

In 1965 Evans & Sarkar[25] published experimental and theoretical work on 18 hollow rectangular beams reinforced longitudinally and transversely. The theoretical equations were formed by taking moments of forces about a skew compression fulcrum assuming the steel intercepting the failure section was at yield. The compression fulcrum was assumed to be at 45^{o} to the main axis, and indeed a change to 60^{o} showed very little change in the theoretical solutions. The equation so formed included a crack angle α on the bottom and sides of the beam. This was related to the cracking of the member assuming it to be plain concrete. The approach on this point is in direct contrast to Lessig[3] who assumed that this angle is governed by the steel intercepting the failure section. Evans and Sarker assumed that for the bending and torsional stresses in the concrete before cracking, there was a plastic distribution of stress. These stresses were then combined using the principal tensile stress criterion. The resulting equation showed that the crack angle varied from 45° in pure torsion to 0° for pure bending, as would be expected. It was however dependent for intermediate values on the $^{d}/_{b}$ ratio.

The equation also involved the depth of the skew compression zone which was obtained by equating forces perpendicular to the skew failure plane and thus includes the stirrups in the tensile zone at the bottom of the beam.

It also involved the cylinder crushing strength of the concrete in the compression zone, although it was recognised that it may be less than this, due to the presence of torsional stresses.

No guidance was given on under reinforced of over reinforced members, or on the ideal ratio of transverse to longitudinal steel. The theory was confined to mode 1 type of failure, although a torsional failure presumably mode 2 was recognised. The theory when applied to experimental results was reasonably accurate with their own results and other investigations.

Walsh et al [26] in 1966 used the same theoretical model as Lessig[3] but formed only one equilibrium equation about a skew axis of bending. This equation involved the variable θ, the angle of the failure plane. The value of θ was then determined for which the torsional resistance was an optimum. This approach gave approximately the same solution as Lessig but in a simpler form due to approximations.

The theoretical expressions were produced for modes 1 and 2 and also for a new mode 3. In mode 3 the compression zone formed at the bottom of the member which was confirmed by experimental behaviour. Mode 3 form of failure had not been identified by Lessig.

Walsh et al also obtained a ratio of transverse to longitudinal steel r_{y} which optimised the volume of reinforcement and the torsional resistance of the member. This value of r_{1y} also provided a limit to the accuracy of the yield theory but the reason for this seemed to be based on experimental evidence. No theory was presented for results that lay beyond this limit except to say that for a few results the yield theory appeared conservative. It is however not conservative for other experimental results.

In 1968 the American Concrete Institute published papers on "Torsion of Structural Concrete", and four of these papers were based on the presentation made at the 62nd Annual ACI Convention in Philadelphia in March 1966.

A paper by Hsu[27] gave experimental results of 53 members reinforced longitudinally and transversely and subject to pure torsion. The variables investigated were, strength of concrete, breadth to depth ratio, scale and ratio of transverse to longitudinal steel. Generally however $r_{2y} = \frac{A_s f_y d_1}{S A_2 f_{2y}} = \frac{1}{1+b/d}$ which produced equal volumes of longitudinal and transverse steel. From extensive strain measurements on the concrete and the steel and torque/angle of twist curves, Hsu concluded that with increasing areas of reinforcement a member changed from under reinforced, to partially over reinforced, to completely over reinforced. He also presented an empirical formula to determine the torsional resistance of a member over the linear portion of the torque/torsional reinforcement relationship. This empirical formula states in the simplest terms that the torsional resistance is partly dependent on the concrete and partly on the steel.

$$T = T\ \text{concrete} + T\ \text{steel}$$

This tends to conflict with the subdivision of underreinforced where the torsional resistance would be expected to be entirely dependent on the steel. The torque/angle of twist experimental curves clearly show that yield at the failure section only occurs at small areas of longitudinal and transverse steel.

The empirical expression is however useful and implies that a theoretical expression should also follow the same lines. The completely over reinforced members confirm the conclusion reached by Lessig[19] that at the limit $\frac{M}{bd^2 f_c'} = 0.07 \rightarrow 0.12$.

Maximum compressive strain measurements on the concrete indicated that failure occurred with a small amount of reinforcing steel at a strain of approximately .01, and when over-reinforced this value reached approximately .04. This indicated that failure could occur before the uniaxial compressive strength of the concrete had been reached. His results also show that theories which assume that all steel yields which intercept the failure plane, overestimate the torsional strength of a member.

A further paper by Hsu[27a] in 1968 gave a theoretical interpretation of the empirical expression from the previous paper, but at various stages in the analysis he was forced to adopt empirical values.

The proposed theoretical failure surface he assumed was a plane perpendicular to the wider face and inclined at 45° to the axis of the beam. This surface is therefore parallel to the shorter legs of stirrups and therefore ignores those forces. Experimental stresses were in fact very low in these legs.

The basic equation for torsional resistance was then formed by taking moments about a longitudinal axis of the beam. This included yield stress in the long leg of the stirrups, dowel forces on the longitudinal steel and compressive and shear stresses in a compression zone.

He applied the theoretical/empirical expression to his own test beams with considerable success since empirical values adopted were based on these beams. He also applied the expression to other test beams but not with the same degree of agreement.

A paper by Goode and Helmy[28] published in 1968 provided further experimental evidence of non-yielding of the reinforcing steel. The experimental results for 27 beams covered bending and torsion with varying ratios of traverse to longitudinal reinforcement, including ties at 45° to the axis of the beam. Goode and Helmy confirmed that modes 1, 2 and 3 existed and also that a member may be completely over reinforced. In addition to these they included partial yielding cases where either the longitudinal steel yielded or the transverse steel yielded.

The theory developed in the paper combined equilibrium equations formed by taking moments of forces about an axis through the centre of the compression zone and parallel to the skew neutral axis, and also about an axis perpendicular to this. For partial yielding the crack angle was at 45°, the average stress in the compression block was taken as .85 f_c', and the compression reinforcement ignored. Only fair agreement was obtained between the theory and the experimental results. In one test series considerable discrepancy between theory and experiment was explained by the fact that the theory ignored the dowel action of the longitudinal bars.

Also in 1968 at the same ACI conference. Iyengar and Rangan[15] presented further test beam results and theory. The theory applied to members with or without transverse reinforcement, and was based on the assumptions that the member was cracked, the transverse steel yielded, the contribution of the horizontal (shorter legs) of the stirrups to the torsional resistance was neglected and the dowel action was also neglected.

Like Hsu[27] and others they assumed that for torsional failures the torsional resistance was related to the concrete and to the steel, and may be expressed as:

$$T_u = T \text{ concrete} + T \text{ steel}$$

The torsional resistance based on the concrete assumed that the compression zone was subject to direct stress and torsional stress. Failure was controlled by the Krishnaswamy[29] failure criteria for concrete

$$\frac{\sigma_1}{f_c'} + \left(\frac{\sigma_2}{f_t}\right)^2 = 1$$

where σ_1 = the principal compressive stress

and σ_2 = " " tensile stress.

The advantage of this criterion was that it was a continuous function from compressive to tensile failures.

The combined bending and torsion failures were also based on the failure criterion, the direct stress being produced by bending and dependent on lever arm and neutral axis depth factors. The torsional shear stress depended on assuming plastic shear stress distribution due to the application of the torsional moment.

The method of necessity introduced many constants which were combined into a single factor λ. The comparison of theory with experimental results was good since failure was controlled by the concrete, but it was unfortunate that so many arbitrary factors had to be introduced into the theory. The approach in this paper is comparable with that of Hsu[27] since both assumed that the longitudinal steel and transverse steel were not at yield.

In 1968 Zia and Cardenas[30] tested model plaster beams reinforced longitudinally and transversely. The advantage of the plaster was that it could be tested two hours after casting.

The disadvantages were that the tensile strength of the plaster was slightly higher than concrete, the modulus of elasticity lower than concrete, and due to size it was not possible to take strain readings.

Experimental results were compared with Lessig's[3] theory and although in general the theory was conservative for mode 1, it tended to overestimate the strength for mode 2, suggesting that the steel was not at yield.

Measurements of crack angles were made and in general crack angles on opposite faces of the beam were not the same. The values of the angle of the crack were also more inclined from the vertical than for plain concrete, suggesting that the equilibrium of forces in the longitudinal steel and stirrups affected the cracking for members reinforced longitudinally and traversely.

In 1968 Pandit and Warwarak[31] conducted further tests on members reinforced longitudinally and transversely and developed a theory to explain the behaviour.

The basis of the theory was expressed as

$$T_u = T_{concrete} + T_{steel}$$

The resistance of the concrete depended on the compression zone which was clearly defined for pure bending as existing at the top of the section. For some torsion however it was a central area of the cross section. This assumption Hsu[27] had shown experimentally to be incorrect, since if a hollow member was tested which effectively removesthis area, the effect on torsional distance was negligible. For combined bending and torsion the position of the compression zone was intermediate between pure bending and pure torsion.

This compression zone was assumed to be one quarter of the cross sectional area and subject to a biaxial stress due to a direct stress due to bending and a plastic torsional shear stress. These stresses were combined using the Cowan[8] failure criterion.

Pandit and Warwarak seemed uncertain of the torsional shear stress to be applied for a predominantly torsional failure and therefore expressed their solution as lying between an upper and lower bound limit. For this reason the comparison between theory and experimental results varied considerably.

In 1969 Fairburn and Davies[32] attempted to improve the theoretical approach of Evans and Sarkar[25]. Evans & Sarkar had kept the crack angle α in the concrete and the angle of the compression zone θ independent of each other, and in fact fixed θ at 45^o. Fairburn and Davies however extended the crack angle to the compression zone and the connecting line then formed the angle θ. Since the crack angle α was determined by the $^M/_T$ ratio, when T was zero the crack was vertical and θ was at right angles to the member. This seems more logical than the Evans and Sarkar approach when it appeared that if T was zero β was still at 45^o to the main axis of the beam.

The equation of equilibrium was formed in the same way as Evans and Sarkar by taking moments about the skewed compression zone, assuming the steel intercepting the failure zone was at yield. The strength of the concrete in the compression zone was still assumed to be the full compressive strength in bending.

Comparison of the theory with experimental results was fairly accurate for beams manufactured and tested by the authors and other beams tested by Evans and Sarkar, and Chinekov[21].

Swan[33] in 1970 organised his experimental work to form the basis for a design method for rectangular reinforced concrete beams in torsion. The main aim of the work was to obtain a reliable ultimate load method of design for rectangular reinforced concrete sections in torsion combined with bending and shear. Secondary to this was the task of obtaining information on stiffness and crack width.

After reviewing and classifying torsion theories he decided that the ultimate torsional strength could be separated from bending and shear and expressed as

$$T_u = T_c + \phi_s \frac{b'd' A_s f_{sy}}{S}$$

where $T_c = 0$

and ϕ_s is a constant factor to be determined from experiment. The value of T_c was put at zero because of the conflict between those proposing this type of expression, and because he believed that it was being confused with the torsional strength of plain concrete.

From experimental work Swan concluded that $\phi_s = 1.2$. He realised however that ϕ_s probably depended on the shape of the section, cube strength, yield stress of the steel and that it also varied as it approached the over-reinforced state. The report does not give the relation of the theory to the experimental results which make it difficult to assess. He also does not give any restraint to the use of the formula. If it is applied to member R5I tested in torsion and bending ($M/T \simeq 22$) by Iyengar and Rangan[15] $T_{expt.}/T_{theo.} = .45$. This does not appear to be satisfactory for a design expression even if a materials factor is introduced.

A design expression such as this is in fact likely to be nonconservative as the M/T ratio increases. The statement by Swan that torsion may be considered separately from shear and bending at ultimate strength is in direct contrast to the theories of Lessig[3] and Walsh et al [26]. In these theories torsion and bending are combined to form an interaction curve and may not be considered separately.

Jackson and Estañero[34] in 1971 investigated the plastic flow law for reinforced concrete beams under combined flexure and torsion. Their 80 members were reinforced to produce yielding although with present knowledge it is difficult to decide how to achieve this, since most investigators report non yielding in the stirrups at the side of the beam.

The readings for rotations about the longitudinal and transverse axes lead to the conclusion that the term "axis of rotation" which is used by many investigators is incorrect. This statement seems to be a criticism of the Lessig skew bending approach, which many investigators have used with a reasonable degree of success.

It is particularly clear from the axes of rotation readings that for a mode 2 form of failure rotation is largely about the longitudinal axis of the member. In general there are considerable differences between the axis of rotation for the Lessig skew bending theory and the values obtained in this paper. This suggests that present theoretical approaches need closer examination and the assumption that all the steel yields at the critical section is not valid. It is also particularly interesting to note that the interaction curve approximated to an ellipse, whereas the Lessig skew bending approach and the space truss theory resulted in a parabola.

They also confirmed the plastic potential flow law by determining that the incremental plastic rotation vectors at all points on the associated interaction curve are in the direction of the outward normal to the curve.

Kuyt [35] in 1971 made a comparison of the Lessig equilibrium method considering bending on a skew failure plane, and the truss theory which is normally accepted in Western Europe. He considered only the equations for ultimate strength in pure torsion for a member reinforced longitudinally and transversely and based on yield of the steel.

He concluded that for the case where there were four longitudinal bars, one in each carrier, and two extra on the long side, the two methods gave the same solution. For the case where there were only four longitudinal bars then it was incorrect to assume that the crack angle on the two sides of the member were the same. It was also incorrect to assume that the stresses in the stirrups on the two sides were the same.

From examination of photographs of specimens by Hsu [27] it is difficult to challenge the crack angle statement. From examination of the stresses in the stirrups however Kuyt claims that the results by Hsu[27] confirm that the stresses in the side stirrups are reduced in the proportion of the dimensions of the stirrup. Certainly they are less but not necessarily in that ratio, and they are still less than yield when Hsu used six longitudinal bars.

Kuyt[36] in a later paper in 1972 applied the truss theory to available experimental results and plotted them graphically on non dimensional interaction diagrams.

1.5 Conclusions from the Review

It was concluded from the review that further work was required in the following fields.

(1) For plain concrete to compare the skew bending theoretical solution with that obtained by Cowan, and to relate the theory to experimental data.

(2) To develop a theory for concrete members with longitudinal steel and compare this with experimental data.

(3) To consider and improve existing yield theories for members with longitudinal and transverse steel, and to determine the boundary conditions.

(4) To develop partial yield theories for members with longitudinal and transverse steel, and compare with experimental data.

(5) To consider and improve existing theories for over-reinforced members, minimum reinforcement and for optimum torsional strength and optimum reinforcement.

CHAPTER 2 - Ultimate Strength of Plain Concrete Members

2.1 Introduction

The analysis of plain rectangular concrete sections subject to torsion has until fairly recently been based on the Saint Venant[2] theory of 1853. This assumes that the material is homogeneous and elastic, and that the failure condition is reached when the maximum principal tensile stress is equal to the direct tensile strength of the concrete. Tests show that the ultimate strength is greater than that predicted by the St. Venant theory and it was therefore modified on the assumption that the concrete behaved plastically at failure. This modification to the theory produced better agreement with the experimental results, but gave no indication how the theory should be modified for reinforced concrete sections, where the material is not homogeneous and the concrete cracks prior to failure.

In 1959 Lessig[3] introduced the idea of considering the failure mechanism and the equilibrium of forces in bending on a "skewed" failure plane for reinforced concrete sections. In 1966 Hsu[4] verified by the use of high speed photography that the same "skewed" mechanism of failure occurred in plane concrete in pure torsion. Since failure occurred in bending on the "skewed" failure plane Hsu used the modulus of rupture of the concrete as the critical stress at failure. This approach gives good agreement between theory and experiment for Hsu's results, and Kemp[37] by regression analysis showed that it appeared to give better correlation with available test data. If the theory for the "skewed" bending failure plane mechanism is applied to cases where bending and torsion co exist, then it is necessary to consider two different modes of failure. The failure surfaces are initially considered as plane rectangles for ease in developing the theory, although photographs by Hsu[4] of a specimen in pure torsion show that the actual failure plane is more distorted.

2.2 Theory for the Ultimate Strength of Plain Concrete Members Subject to Bending and Torsion-Mode 1

This form of failure occurs when the $^{M}/_{T}$ ratio is high. Hsu[4] showed experimentally that for mode 2 the form of failure appeared to be bending about a skew axis with the compression zone at the side of the section. It is therefore assumed that the skew bending will alternatively occur with the compression zone at the top of the section as shown in fig. 2.2.1. It is important to note that at this stage the failure plane is not distorted. Taking moments of forces about the neutral axis of the skew failure plane

$$T_1 \sin \theta_1 + M_1 \cos \theta_1 = \frac{\left(^{bd^2}/_6\right) f_{r1}}{\cos \theta_1} \qquad \text{..... 2.2.1}$$

(note: a similar equation has been formed by Hsu[4], but the choice of angle and subsequent development and solution is different.)

Equation 2.2.1 is an equilibrium equation including a compatibility condition of linear strain in bending, and a linear stress/ strain relationship. Rearranging equation 2.1.1 and substituting

$$M_{u1} = \frac{bd^2}{6} \cdot f_{r1} = z_1 f_{r1} \qquad \text{..... 2.2.2}$$

$$\frac{T_1}{M_{u1}} = \frac{1 + \tan^2\theta_1}{\tan \theta_1 + {}^{M_1}/_{T_1}} \qquad \text{..... 2.2.3}$$

For a minimum value of T_1, $^{dT_1}/_{d\theta_1} = 0$

If $\frac{T_1}{M_{u1}} = {}^{u}/_{v}$, then for a minimum value of T_1

$$\frac{u}{v} = \frac{^{du}/_{d\theta}}{^{dv}/_{d\theta}}$$

$$\frac{1 + \tan^2\theta_1}{\tan\theta_1 + {}^{M_1}/_{T_1}} = \frac{2 \tan\theta_1 \left(^{1}/_{\cos^2\theta_1}\right)}{^{1}/_{\cos^2\theta_1}}$$

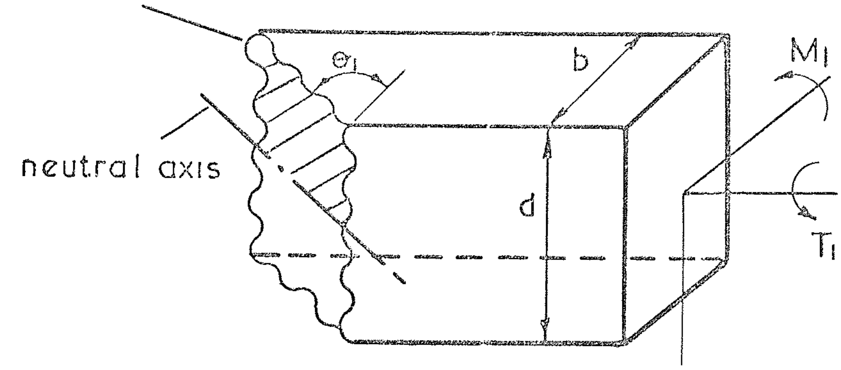


Fig 2.2.1 Mode I type of failure in plain concrete (no distortion)

rearranging

$$1 + \tan^2\theta_1 = 2\tan^2\theta_1 + 2\tan\theta_1\left({}^{M_1}/_{T_1}\right)$$

$$\tfrac{1}{2}\tan^2\theta_1 + \tan\theta_1\left({}^{M_1}/_{T_1}\right) - \tfrac{1}{2} = 0$$

$$\tan\theta_1 = \pm\sqrt{\left({}^{M_1}/_{T_1}\right)^2 + 1} - \left({}^{M_1}/_{T_1}\right)$$

The appropriate value for this mode of failure is

$$\tan\theta_1 = +\sqrt{\left({}^{M_1}/_{T_1}\right)^2 + 1} - \left({}^{M_1}/_{T_1}\right) \qquad \ldots\ldots\ 2.2.4$$

substituting equation 2.2.4 in equation 2.2.3

$$\frac{T_1}{Mu_1} = \frac{1 + \left\{\sqrt{\left({}^{M_1}/_{T_1}\right)^2 + 1} - {}^{M_1}/_{T_1}\right\}^2}{\sqrt{\left({}^{M_1}/_{T_1}\right)^2 + 1}}$$

$$\frac{T_1}{Mu_1} = \frac{1 + \left({}^{M_1}/_{T_1}\right)^2 + 1 - 2\left({}^{M_1}/_{T_1}\right)\sqrt{\left({}^{M_1}/_{T_1}\right)^2 + 1} + \left({}^{M_1}/_{T_1}\right)^2}{\sqrt{\left({}^{M_1}/_{T_1}\right)^2 + 1}}$$

$$\frac{T_1}{2Mu_1} = \frac{1 + \left({}^{M_1}/_{T_1}\right)^2 - {}^{M_1}/_{T_1}\sqrt{\left({}^{M_1}/_{T_1}\right)^2 + 1}}{\sqrt{\left({}^{M_1}/_{T_1}\right)^2 + 1}}$$

$$\frac{T}{2Mu_1} = \sqrt{\left({}^{M_1}/_{T_1}\right)^2 + 1} - \left({}^{M_1}/_{T_1}\right) \qquad \ldots\ldots\ 2.2.5$$

When ${}^{M_1}/_{T_1} = 0$, $T = 2Mu_1 = Tu_1$ 2.2.6

equation 2.2.5 may be rearranged.

$$\left\{\frac{T_1}{Tu_1} + \frac{M_1}{T_1}\right\}^2 = \left(\frac{M_1}{T_1}\right)^2 + 1$$

$$\left(\frac{T_1}{Tu_1}\right)^2 + \frac{2M_1}{Tu_1} = 1$$

and since from equation 2.2.6 $Mu_1 = {Tu_1}/{2}$

$$\left(\frac{T_1}{Tu_1}\right)^2 + \frac{M_1}{Mu_1} = 1 \qquad \ldots\ldots 2.2.7$$

Equation 2.2.7 is in the general non dimensional form connecting T_1 and M_1. This form of the equation is particularly useful for design purposes and for the comparison of experimental results for differing section shapes, sizes and strength of concrete.

2.3 Theory for Mode 2 Form of Failure

This form of failure occurs when the $^{M}/_{T}$ ratio is low. The compression zone forms at the side of the section and the basic equilibrium - compatability - stress/strain equation is formed by taking moments of forces about the neutral axis of the failure plane (see fig. 2.3.1.).

$$T_2 \sin\theta_2 = \frac{\left(db^2/6\right) f_{r2}}{\cos\theta_2} \qquad \ldots\ldots 2.3.1$$

Rearranging and substituting $Mu_2 = \left(db^2/6\right) f_{r2} = Z_2\ f_{r2}$ 2.3.2

$$\frac{T_2}{Mu_2} = \frac{1 + \tan^2\theta_2}{\tan\theta_2} \qquad \ldots\ldots 2.3.3.$$

This is now seen as a special form of equation 2.2.3 where the $^{M}/_{T}$ ratio is omitted. The value of θ_2 when T_2 is a minimum must also be a special case when $^{M}/_{T}$ is zero. In which case from equation 2.2.4

$$\tan\theta_2 = \pm 1 \qquad \ldots\ldots 2.3.4$$

Using the positive value of $\tan\theta_2$ and substituting in equation 2.3.3

$$\frac{T_2}{2Mu_2} = \frac{T_2}{Tu_2} = 1 \qquad \ldots\ldots 2.3.5$$

Comparing equation 2.3.5 with equation 2.2.7, shows that equation 2.3.5 is a special case of equation 2.2.7, where Tu_2 replaces Tu_1.

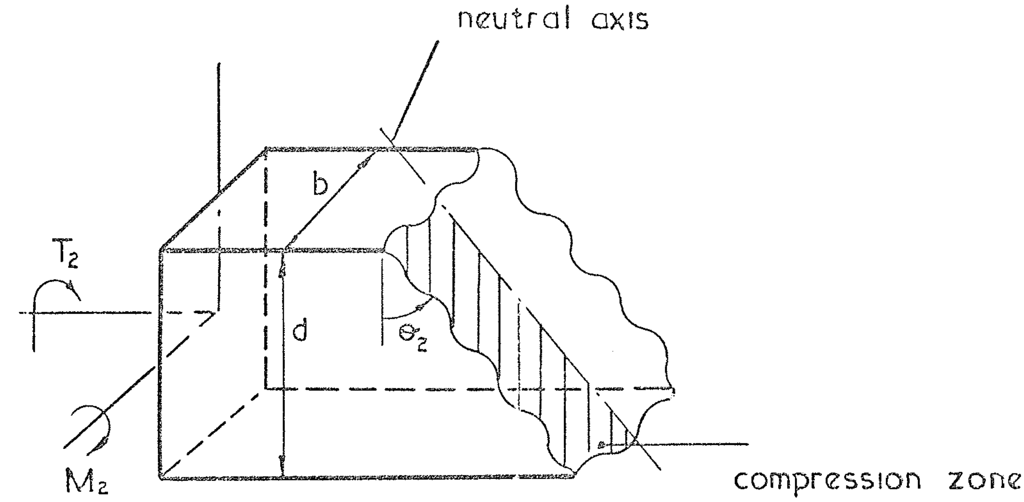


Fig 2.3.1 Mode 2 Type of failure in plain concrete (no distortion)

2.4 Theory for Mode 3 Form of Failure

This form of failure is theoretically possible when M/T ratios are low, and there is a difference of the section modulae for bending about an axis, e.g. a trapezium (see fig. 2.4.1). The compression zone now forms at the bottom of the section and the basic equibrium - compatability - stress/strain equation is formed by taking moments of forces about the neutral axis of the failure plane

$$T_3 \sin \theta_3 - M_3 \cos \theta_3 = \frac{z_3 f_{r3}}{\cos \theta_3} \qquad \ldots\ldots 2.4.1$$

rearranging

$$\frac{T_3}{Mu3} = \frac{1 + \tan^2\theta_3}{\tan\theta_3 - \left(M_3/T_3\right)} \qquad \ldots\ldots 2.4.2$$

as previously shown in 2.2 is a minimum when

$$\tan\theta_3 = \pm \sqrt{\left(M_3/T_3\right)^2 + 1} + M_3/T_3 \qquad \ldots\ldots 2.4.3$$

substituting equation 2.4.3 in equation 2.4.2

$$\frac{T_3}{2Mu_3} = \sqrt{\left(M_3/T_3\right)^2 + 1} + M_3/T_3 \qquad \ldots\ldots 2.4.4$$

rearranging

$$\left(\frac{T_3}{Tu_3}\right)^2 - \frac{M_3}{Mu_3} = 1 \qquad \ldots\ldots 2.4.5$$

where $Tu_3 = 2Mu_3$ 2.4.6

The three possible theoretical modes are related generally as shown in fig. 2.4.2. The axes of the graph are bending moment and torsional moment, and are not expressed in non-dimensional because of the variation in the values of Tu_1, Tu_2 and Tu_3. For a circular section however there is only mode 1 form of failure since $Mu_1 = Mu_2 = Mu_3$ and $Tu_1 = Tu_2 = Tu_3$.

Generally the relationship between mode 1 and mode 2 is established by results given by Walsh et al [11] for a rectangular cross section, and by Fisher[7] for a circular cross section. No results are

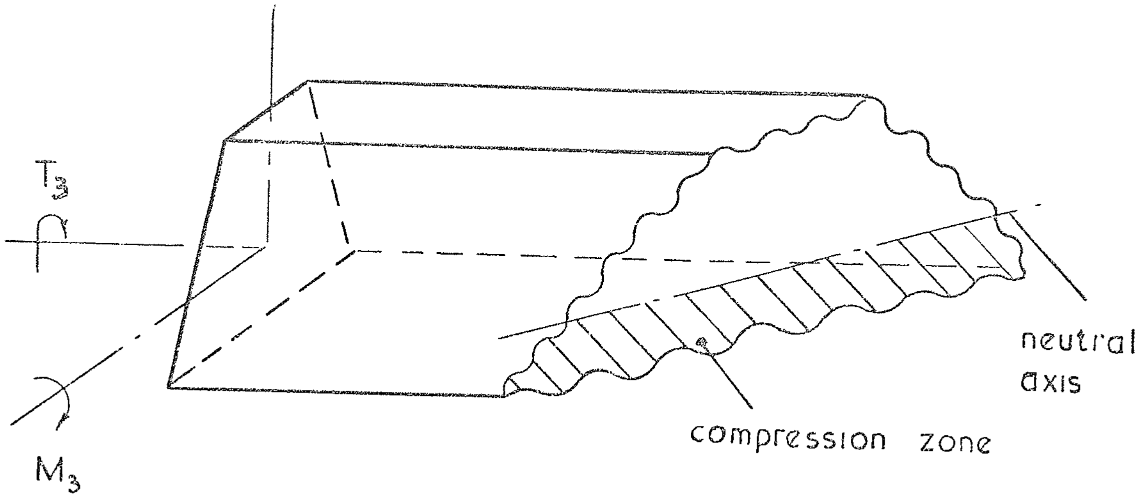


Fig.2.4.1 Mode 3 type of failure in plain concrete for a trapezoidal section (no distortion)

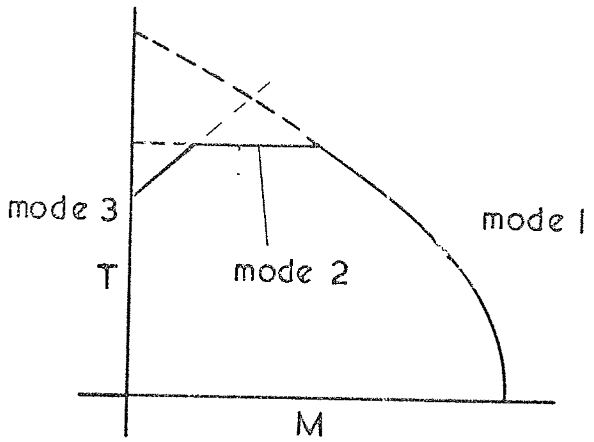


Fig 2.4.2. Theoretical relationship between modes 1. 2. and 3 for plain concrete

available to substantiate mode 3.

Cowan[8] derived similar a equation to mode 1 in 1953 but did not appear to appreciate modes 2 and 3. His approach was based on an elastic-plastic distribution of stress in bending and the St. Venant[2] elastic distribution of torsional shear stress. He combined these stresses using the maximum principal tensile stress failure criterion for concrete. The corresponding values of Tu and Mu obtained by Cowan although of the same form were not exactly the same values. It is important however that both approaches produce essentially the same solution, and with present knowledge Cowan's approach can be adapted for modes 2 and 3. It does however reintroduce the problem of whether the concrete behaves elastically or plastically in torsion, since the "skew bending" theory is essentially on elastic approach, and the Cowan approach is plastic or semi-plastic.

2.5 Modified Values of Ultimate Strength in Pure Torsion - Rectangular Cross Section

It is apparent from the preceding theory which results in a general non dimensional interaction formula, that the values of Mu and Tu are particularly important. The value of Mu is the value in pure bending mode 1 which is in fact a standard modulus of rupture test if a 4" x 4" or 6" x 6" section is used. If Mu = (factor) $bd^2 f_{r1}$ then the factor may be fixed at $^1/_6$ assuming a linear strain distribution and linear stress/strain relationship. The value of the modulus of rupture f_{r1} was then found by Wright[38] and Hsu[4] to vary with the depth of the section. A higher value was obtained for a smaller depth of section, assuming other possible variables remained constant.

An alternative approach favoured by Cowan[8] is that the tensile strength is fixed at the uniaxial tensile strength of concrete. The factor then varies depending on the depth of the section. Since both

methods can produce the same answer then it may be argued that the difference in approach is not important. Hsu[4] produced an empirical expression relating the modulus of rupture to the uniaxial tensile strength of concrete as the beam depth varies.
In imperial units these were expressed as:-

$$f_r = 7.17\left(1 + \frac{10}{x^2}\right) f_t^{2/3} \quad \text{for } x > 4" \qquad \ldots\ldots 2.5.1$$

$$f_r = 7.17\left(\frac{2.4}{x^{1/3}}\right) f_t^{2/3} \quad \text{for } 2" < x \leqslant 4" \qquad \ldots\ldots 2.5.2$$

where x = d for mode 1 and x = b for mode 2 for a rectangular cross section.

To produce these expressions Hsu had results of his own but also related them to work by other investigators. This introduced other variables involved in the different mixes of concrete.

Many investigators in this field in the past had expressed their strength of concrete as a uniaxial crushing strength and Hsu therefore related his modulus of rupture to this once again using empirical relations in imperial units.

$$f_t = 5\sqrt{f_c'} \qquad \ldots\ldots 2.5.3$$

The constant 5 used by Hsu varies for other investigators from 3.5 to 6.5. The variation in this constant gives some indication of the error involved in using this expression, and is confirmed by experimental results later in this chapter.

The value of Tu is more difficult to obtain than Mu. From the preceding theory $Tu_1 = 2Mu_1$, $Tu_2 = 2Mu_2$ and $Tu_3 = 2Mu_3$. Experimental readings of the maximum tensile strain for a torsion specimen lead Hsu to conclude that the tensile strength was related to the modulus of rupture but that it was necessary to introduce a factor of 0.85 such that

$$Tu_2' = \frac{db^2}{3}(0.85\, f_{r2}) \qquad \ldots\ldots 2.5.4$$

The factor was introduced based on the Mohr failure theory which reduces the modulus of rupture when produced in conditions of pure torsion. Hsu assumed that the theoretical failure plane was not distorted but his own photographs showed that this was not correct, and this distortion may account for the necessity to introduce the factor of 0.85.

If it is assumed that the failure plane for a rectangular cross section is a distorted trapezium as shown in fig. 2.5.1 then the preceding theory requires modification. The introduction of this distortion over the full range of $^M/_T$ values produces equations which are difficult to solve, and for simplicity therefore it is proposed to only modify the values of T_{u1}, T_{u2} and T_{u3}, and assume that the general non dimensional interaction formula is approximately correct. For the case of pure torsion mode 2 failure, taking moments of forces about the neutral axis and using a projected plane area to allow for the tensile stresses being at varying angles to the neutral axis (see figs. 2.5.1 and 2.5.2).

$$T'_{u2} \sin \theta'_2 = z_2 . f_{r2} \qquad \text{..... 2.5.5}$$

The form of fracture produces a distorted trapezium where the elastic section modulus

$$z_2 = \frac{b^2}{12} \frac{(L^2 + 4LL'_1 + L'_1{}^2)}{(2L + L'_1)} \qquad \text{..... 2.5.6}$$

$$L = \frac{d}{\cos\theta'_2} \qquad \text{..... 2.5.7}$$

and to allow for tensile stresses not being in the same plane as the neutral axis, L_1 projected into the same plane as L is (see Fig. 2.5.2).

$$L' = \frac{d}{\cos\theta'_2} - \frac{2b \tan\theta'_2 \ \sin\theta'_2}{(1 + \frac{2b}{d})}$$

rearranging

$$L'_1 = \frac{d\ (1 + {}^{2b}/d) - 2b \sin^2\theta'_2}{(1 + {}^{2b}/d)\ \cos\theta'_2}$$

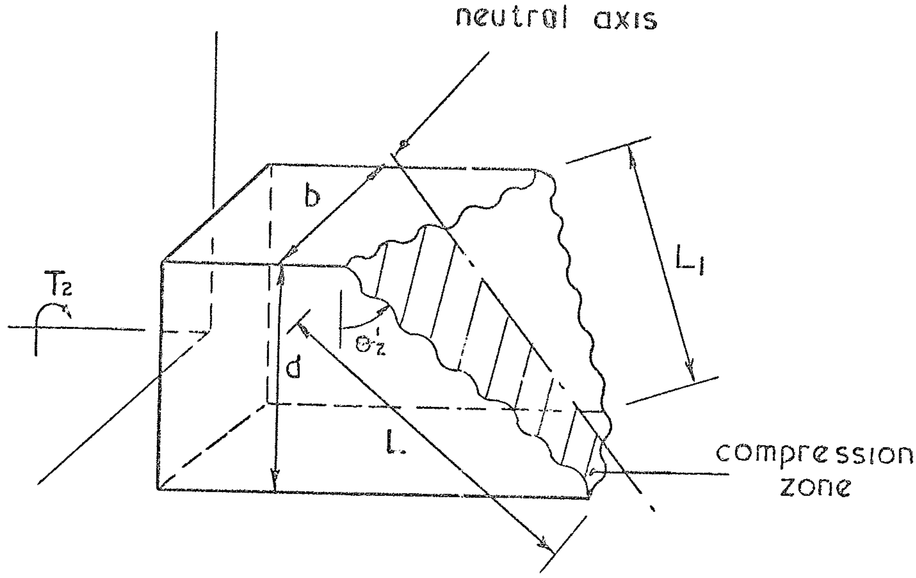


Fig. 2.5.1 Mode 2 type of failure in plain concrete (distorted failure plane) rectangular cross section

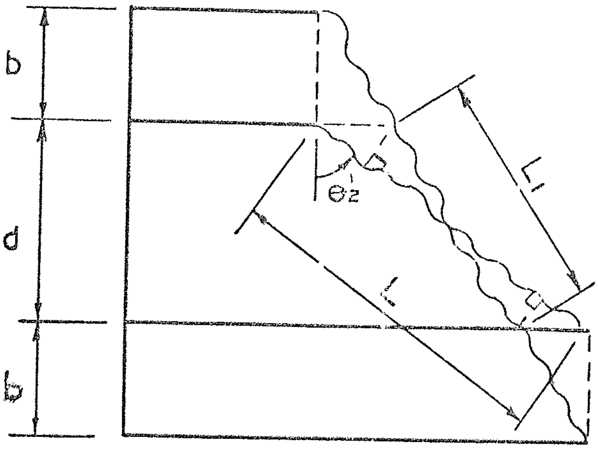


Fig . 2.5.2 Developed distorted failure plane for plain concrete – rectangular cross section

$$L_1' = \frac{d + 2b - 2b\ (1 - \cos^2\theta_2')}{(1 + {}^{2b}/_{d})\ \cos\ \theta_2'}$$

$$L_1' = \frac{d\ (1 + {}^{2b}/d\ \cos^2\theta_2')}{(1 + {}^{2b}/d)\ \cos\theta_2'} \qquad \text{..... 2.5.9}$$

Substituting equations 2.5.6 through to 2.5.9 into equation 2.5.5 and numerically obtaining the value of θ_2' for which Tu_2' is a minimum (using the computer and the interval halving method) produces

$$T'_{u2} \simeq \frac{1}{3 + (\frac{b}{d})^{\frac{1}{2}}}\ db^2 f_{r2} \qquad \text{..... 2.5.10}$$

The value of the modulus of rupture should theoretically be based on the distance from the extreme fibres to the neutral axis but since this error is relatively small and because of previous assumptions it has been ignored.

This expression may now be compared graphically (see Fig. 2.5.3) with the St. Venant elastic theory, the plastic theory, and the Hsu[4] theory mode 2, since in general form they may all be expressed as

$$Tu_2' = \text{(factor) } db^2 \text{ (tensile strength).} \qquad \text{..... 2.5.11}$$

It should be noted that Hsu's theory maintains that the factor is independent of the $\frac{b}{d}$ ratio. It should also be noted that the expression given in equation 2.5.10 produces a line close to the St. Venant expression. This suggests that the St. Venant theory is correct and that all that is required is an appropriate value of the tensile strength.

A similar expression to equation 2.5.10 may be developed for mode 1 value of Tu_1' by exchanging b for d.

$$Tu_1' = \frac{1}{3 + (\frac{d}{b})^{\frac{1}{2}}}\ bd^2\ f_{r1} \qquad \text{..... 2.5.12}$$

There is no direct experimental evidence that this occurs except for a square section, since if b < d then it fails in mode 2 in pure torsion. The value is required however if the general interaction equation is used for mode 1.

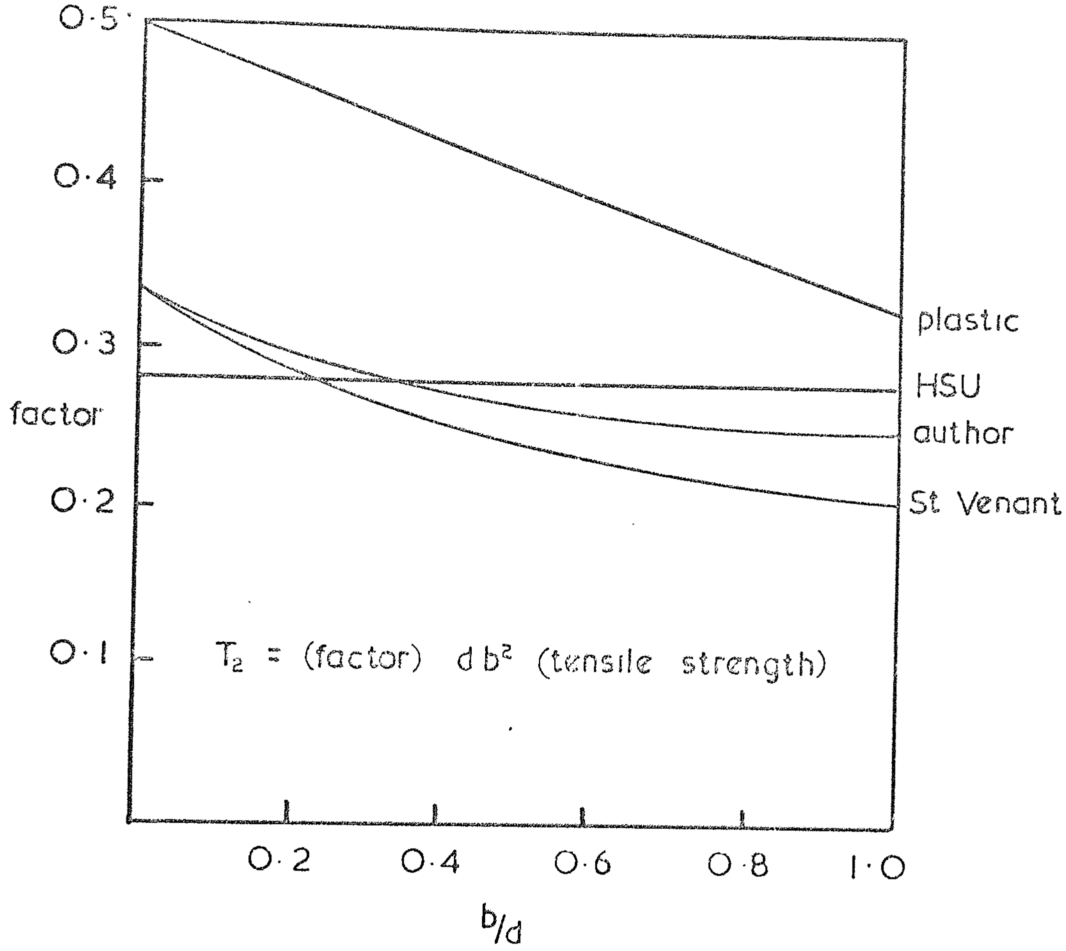


Fig. 2 5. 3 Comparison of theoretical factors for plain concrete in torsion, failure mode 2 — rectangular section

For a rectangular cross section there is no value of T_{u3}' since it will always fail at a lower failing load in mode 2.

2.6 Modified Values of the Ultimate Strength in Pure Torsion - Circular Cross Section

Experimental results are available for a circular cross section and it is therefore necessary to consider the distorted cross section once again. Hsu[4] found it necessary to introduce 0.85 once again, but the projected shape of the distorted failure plane is shown in fig. 2.6.1. This has been idealised so that half an ellipse bounds the compression zone and a parabola bounds the tensile zone.

It is necessary to determine the properties of this cross section. If it is assumed that the neutral axis lies at the centroid of the cross section then the position can be determined by taking moments of areas about the centroid of the cross section.

moment of area of the half ellipse = moment of area of the parabola

$$\left(\frac{1}{2}\pi\ bc\right) \times \frac{4c}{3\pi} = \frac{4}{3}ab \times \frac{2}{5}a$$

rearranging

$$c^2 = \frac{4}{5}a^2 \qquad \ldots\ldots\ 2.6.1$$

also

$$a + c = D \qquad \ldots\ldots\ 2.6.2$$

combining equations 2.6.1 and 2.6.2

$$c^2 = \frac{4}{5}(D-c)^2$$

rearranging

$$\frac{c^2}{2} + 4Dc - 2D^2 = 0$$

solving for c

$$c = -\ 4D \pm \sqrt{(4D)^2 + 4D^2}$$

$$c = 0.4721D \qquad \ldots\ldots\ 2.6.3$$

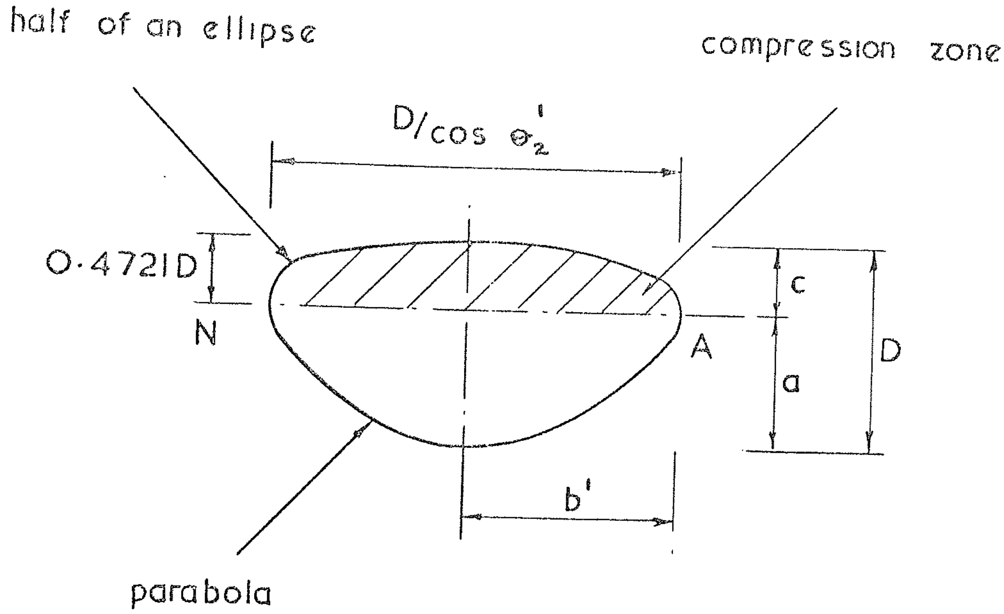


Fig. 2.6.1. Equivalent theoretical shape of projected failure plane, mode 2 failure – circular cross section

The second moment of area about the neutral axis is determined as follows

$$I_{NA} = I_{NA} (\tfrac{1}{2} \text{ ellipse}) + I_{NA} (\text{parabola})$$

$$= \frac{\pi}{8} c^3 b' + \frac{32}{105} a^3 b' \qquad \ldots\ldots 2.6.4$$

where ½× breadth of the section b' on the skew plane is

$$b' = {}^{D}/2 \cos\theta_2' \qquad \ldots\ldots 2.6.5$$

combining equations 2.6.3, 2.6.4, and 2.6.5

$$I_{NA} = \frac{D^4}{\cos\theta_2'} (0.04295) \qquad \ldots\ldots 2.6.6$$

$$Z_{2\,bottom} = \frac{D^3}{\cos\theta_2'} (0.0813) \qquad \ldots\ldots 2.6.7$$

For the case of pure torsion mode 2 failure, the basic equilibrium - compatability - stress/strain equation is formed by taking moments of forces about the neutral axis of the failure plane

$$Tu_2' \sin\theta_2' = z_2 f_{r2} \qquad \ldots\ldots 2.6.8$$

combining equations 2.6.7 and 2.6.8

$$Tu_2' = (0.0813) \frac{D^3}{\sin\theta_2' \cos\theta_2'} f_{r2} \qquad \ldots\ldots 2.6.9$$

for a minimum value of Tu_2', $\frac{d(Tu_2')}{d\theta} = 0$

$$\frac{(-\sin^2\theta_2' + \cos^2\theta_2')}{(\sin\theta_2' \cos\theta_2')^2} = 0$$

$$\tan\theta_2' = 1, \quad \theta_2' = 45^\circ \qquad \ldots\ldots 2.6.10$$

substituting equation 2.6.10 in equation 2.6.9

$$Tu_2' = (0.1626)\, D^3 f_{r2}$$

or $$Tu_2' = (.83) \frac{\pi D^3}{16} f_{r2} \qquad \ldots\ldots 2.6.11$$

The value of ${}^{\pi D^3}/_{16}$ is the elastic section modulus for a circular cross section, and this is to be multiplied by the factor 0.83. The correspon-ding figure that Hsu[4] adopted based on the Mohr combined stress failure criterion was 0.85. The value of f_{r2} for a circular cross section is assumed to be based on the diameter, and not on twice the value of the distance from the neutral axis to the extreme fibres in tension. The error involved in this assumption is assumed to be small.

2.7 Modified Values of the Ultimate Strength in Pure Torsion - Trapezoidal Cross Section

The distortion of the rectangular and circular cross section produced a reduced torsional resistance at ultimate load. It seems reasonable to assume therefore that this would also occur for a trapezoidal cross section. Consider the case of pure torsion mode 3 form of failure as shown in fig. 2.7.1. Taking moments of forces about the neutral axis.

$$Tu_3' \sin \theta_3' = Z_3 f_{r3} \qquad \text{..... 2.7.1}$$

The cross section of the member is a trapezium, but it is also assumed that this will be further distorted but still form a trapezium.

The elastic section modulus

$$z_3 = \frac{b^2}{12} \frac{(L^2 + 4LL_1' + L_1'^2)}{(2L + L_1')} \qquad \text{..... 2.7.2}$$

and $$L = \frac{D}{\cos \theta_3'} \qquad \text{..... 2.7.3}$$

To allow for tensile stresses being at varying angles to the neutral axis L_1 is projected into the same plane as L where

$$L_1' = \frac{d}{\cos \theta_3'} - \frac{2\,t \tan \theta_3' \sin \theta_3'}{(D/d + 2t/d)} \qquad \text{..... 2.7.4}$$

rearranging

$$L_1' = D\left(\frac{d}{D}\right) \frac{\left[D/d + \left(2t/d\right) \cos^2 \theta_3'\right]}{(D/d + 2t/d) \cos \theta_3'} \qquad \text{..... 2.7.5}$$

where $$2t = d + 2h - D \qquad \text{..... 2.7.6}$$

$$h = \sqrt{b^2 + \left(\frac{D-d}{2}\right)^2} \qquad \text{..... 2.7.7}$$

and $$\frac{2t}{d} = 1 + \frac{2h}{d} - \frac{D}{d}$$

$$= 1 + \sqrt{4\left(\frac{b}{d}\right)^2 + \left(\frac{D}{d} - 1\right)^2} - \frac{D}{d} \qquad \text{..... 2.7.8}$$

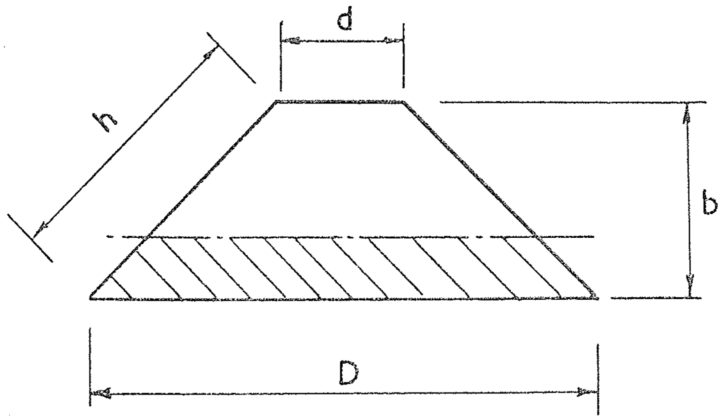


Fig 2.7.1. Cross section of trapezoidal member

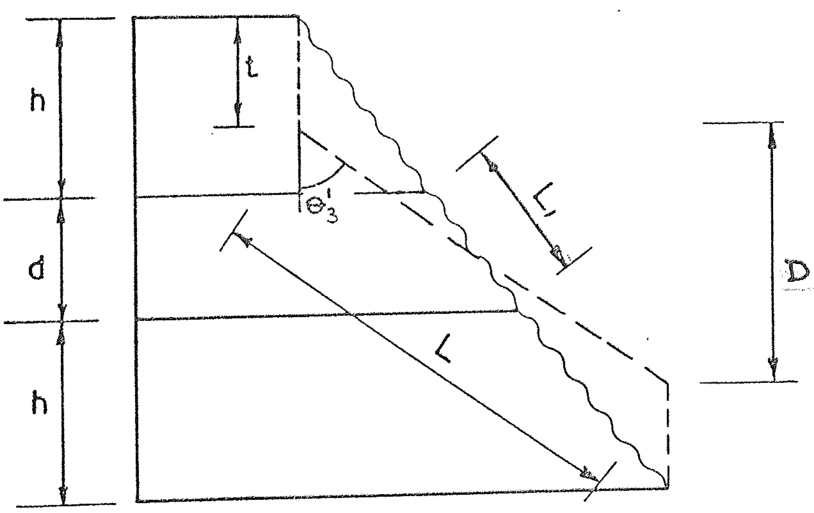


Fig. 2.7.2 Developed distorted failure plane for plain concrete — trapezoidal cross section.

Substituting equations 2.7.2 through 2.7.8 into equation 2.7.1 and numerically obtaining the value of θ_3' for which Tu' is a minimum (using the computer and the interval halving method)

$$Tu_3' = (\text{factor})\, Db^2 f_{r3} \qquad \ldots\ldots 2.7.9$$

The factor is a function of $^d/_D$ and $^b/_d$ and values are plotted graphically as shown in fig. 2.7.3. When $^d/_D = 1$ then the values are the same as a rectangular cross section. When the cross section forms a triangle, i.e. $^d/_D = 0$, the factor is a constant for all values of $^b/_d$.

2.8 Modification of the General Interaction Equations

In the previous chapters the theoretical values of the torsional resistance for a rectangular cross section, circular cross section and trapezoidal cross section have been modified.

If this is also carried out for every change in bending moment then the general interaction formula as expressed in 2.2.7, 2.3.5, and 2.4.5 would need revision. To avoid losing the simplicity of the parabolic interaction equation it is proposed therefore to retain it but introduce the modified values of the torsional resistance

$$Tu_1',\ Tu_2' \text{ and } Tu_3'.$$

Equation 2.2.7 for mode 1 form of failure now becomes

$$\left(\frac{T_1}{Tu_1'}\right)^2 + \frac{M_1}{Mu_1} = 1 \qquad \ldots\ldots 2.8.1$$

alternatively this may be expressed using an $^M/T$ ratio

$$T_1 = Tu_1' \left\{\sqrt{\left(^{M_1}/_{T_1}\right)^2 + 1} - \left(^{M_1}/_{T_1}\right)\right\} \qquad \ldots\ldots 2.8.2$$

where $$Tu_1' = \frac{1}{3 + \left(^d/_b\right)^{\frac{1}{2}}}\, bd^2 f_{r1} \qquad \ldots\ldots 2.8.3$$

for a rectangular cross section

and $$Tu_1' = 0.83\left(\frac{\pi D^3}{16}\right) f_{r1} \qquad \ldots\ldots 2.8.4$$

for a circular cross section.

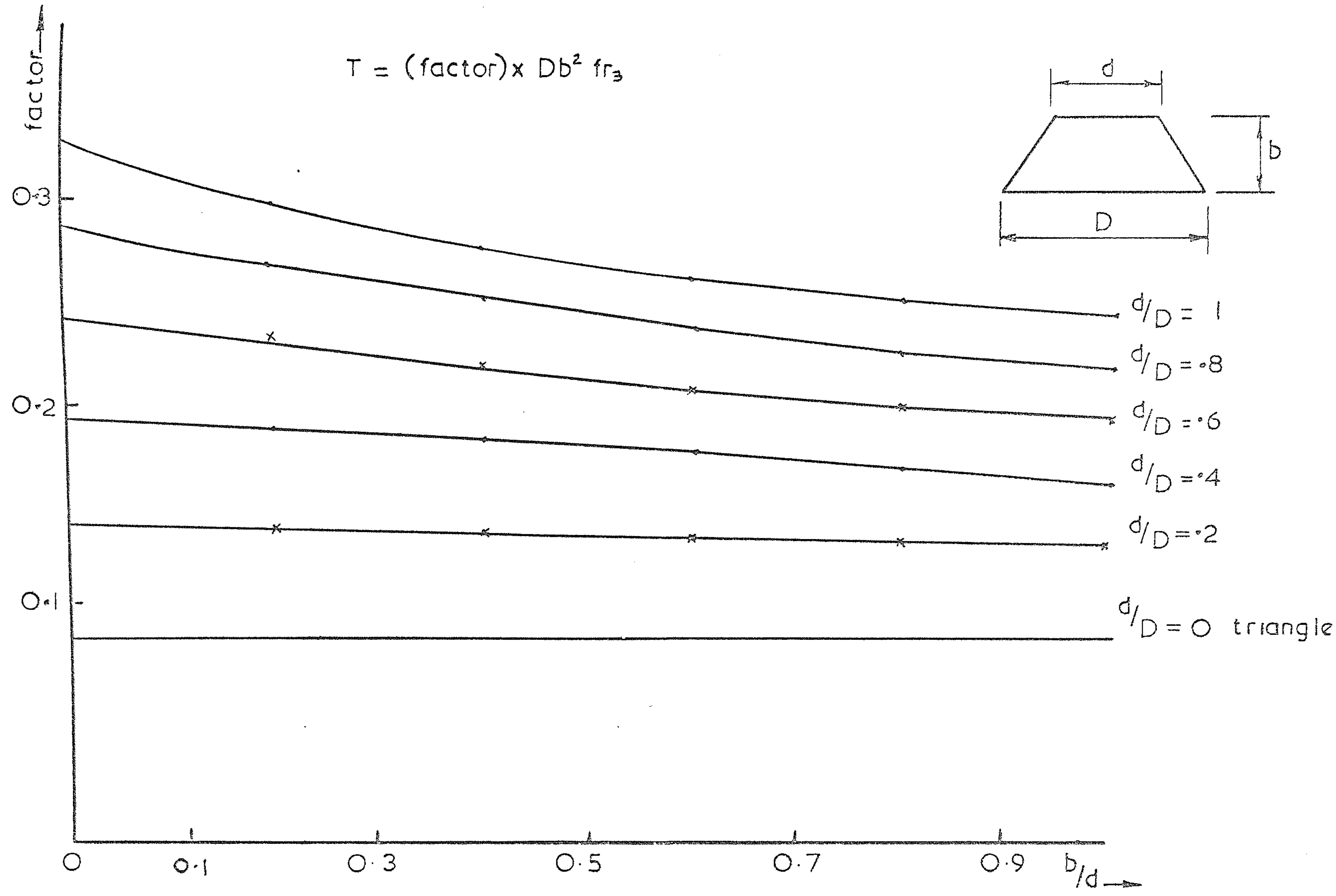


Fig. 2.7.3. Theoretical factors for plain concrete in torsion – trapezoidal cross section (distorted failure plane)

Equation 2.3.5 for mode 2 form of failure now becomes

$$\frac{T_2}{Tu_2'} = 1 \qquad \text{..... 2.8.5}$$

where $$Tu_2' = \frac{1}{3 + \left(b/_d\right)^{\frac{1}{2}}} db^2 f_{r2} \qquad \text{..... 2.8.6}$$

and Tu_2' for a circular cross section is the same as for Tu_1' for a mode 1 form of failure.

Equation 2.4.5 for mode 3 form of failure now becomes

$$\left(\frac{T_3}{Tu_3'}\right)^2 - \frac{M_3}{Mu_3} = 1 \qquad \text{..... 2.8.7}$$

alternatively this may be expressed using $^M/T$ ratio

$$T_3 = Tu_3' \left\{\sqrt{\left(^{M_3}/_{T_3}\right)^2 + 1} + \left(^{M_3}/T_3\right)\right\} \qquad \text{..... 2.8.8}$$

where $$Tu_3' = \left(\begin{matrix}\text{factor from}\\ \text{graph Fig. 2.7.3}\end{matrix}\right) Db^2 f_{r3} \qquad \text{..... 2.8.9}$$

2.9 Experimental Results - Rectangular Cross Section

There are not sufficient experimental results available to completely substantiate the preceding theory. Although there may be a series of results for differing $^M/_T$ ratios for one investigation, e.g. Walsh et al [11], not all the possible variables such as section size, mix, curing condition, aggregate size etc, have been investigated.

In many cases also the only indication of the strength of the concrete is a uniaxial crushing strength, whereas one of the standard tensile strengths is preferred to incorporate in the preceding theory.

22 results by Hsu[4] and Marshall & Tembe[6] give a basic uniaxial tensile strength and this has been converted to a modulus of rupture using the Hsu[4] equations 2.5.1 and 2.5.2. Detailed values and properties are given in table 2.9.1 for mode 1 and table 2.9.2 for mode 2 (see appendix). The calculated theoretical value of T_1 has been determined

using equation 2.8.2 which incorporates an $^M/_T$ ratio. If the corresponding theoretical value of M_1 is required then it is determined using this $^M/_T$ ratio. This ensures that there is the same percentage error in T_1 as in M_1. The alternative is to use the general non dimensional interaction formula where M_1 needs to be known to determine T_1. The error is then confined to the value of T_1.

The general non dimensional interaction formula equation 2.8.1 has however been used to plot experimental results graphically for mode 1 as shown in fig. 2.9.1. Results may also be plotted graphically using the $^M/_T$ ratio as shown in fig. 2.9.2. Both of these graphs are non dimensional which enables all experimental results of varying cross section and strength to be plotted on the same graph.

Further results for rectangular cross sections by Walsh et al [11], Cowan and Armstrong[10], Iyengar and Ranyan[15], and Zia[39] are also given in tables 2.9.1 and 2.9.2 (see appendix) and plotted on graphs figs 2.9.1 and 2.9.2. It is clear from these results that where the tensile strength is based on the crushing strength, the theory is not so accurate.

For these results the expression for the modulus of rupture given by Hsu[4] has been modified so that $f_t = 6\sqrt{f_c'}$ this leads to an expression connecting the modulus of rupture to the crushing strength.

In imperial units

$$f_r = 24\left(1 + {}^{10}/_{\chi^2}\right)(f_c')^{1/3} \quad \text{for } \chi > 4"$$

or

$$f_r = 24\left(\frac{2.4}{\chi^{1/3}}\right)\left(f_c'\right)^{1/3} \quad \text{for } 2" < \chi \leq 4" \quad \ldots\ldots \; 2.9.2$$

where $\chi = d$ for mode 1, and $\chi = b$ for mode 2. A general summary of the results for different investigations is given in table 2.9.3.

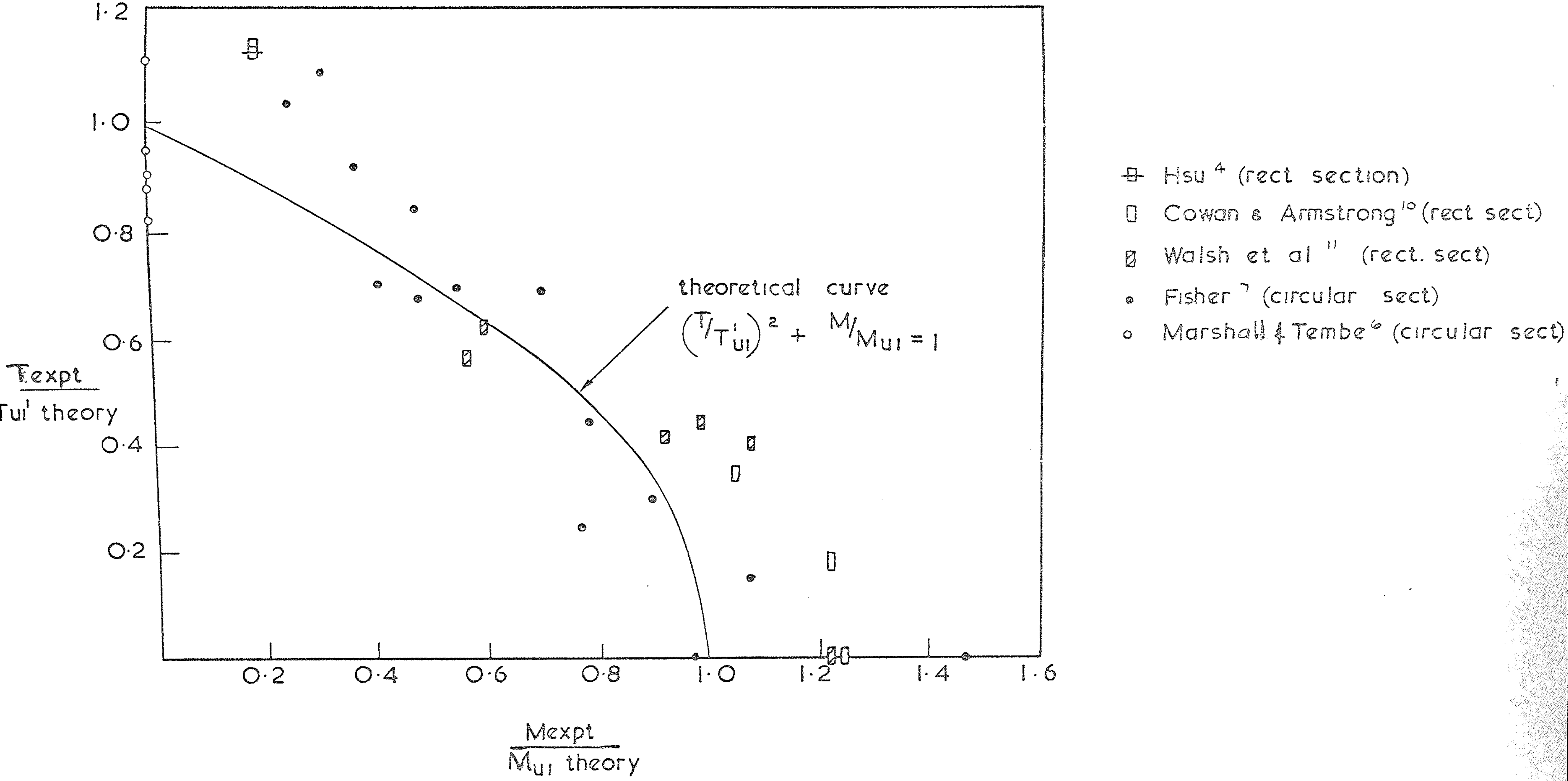
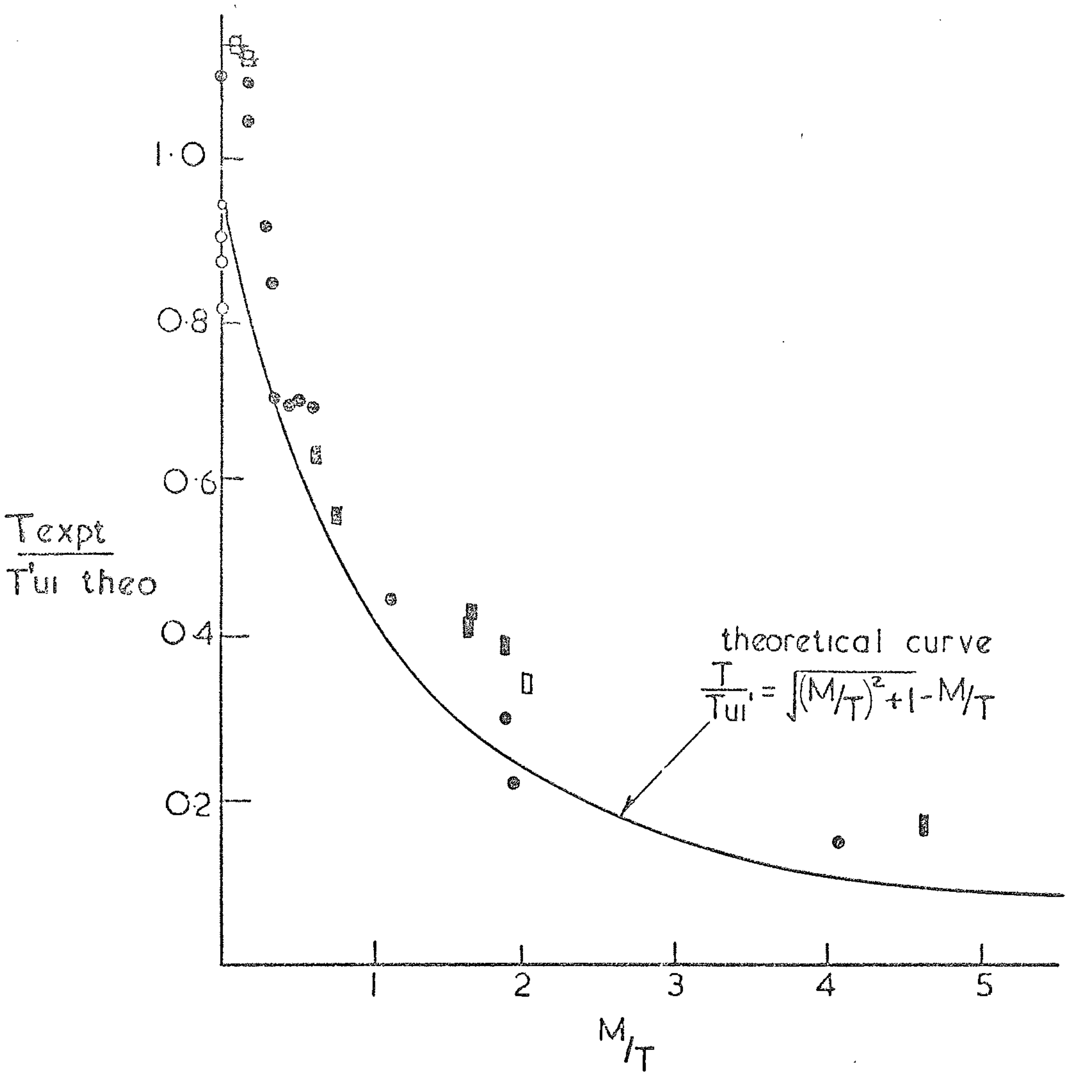


Fig. 2.9.1 Torsion and bending of plain concrete, mode I failures



- Hsu[4] (rect section)
- Cowan & Armstrong[10] (rect. section)
- Walsh et al[11] (rect. section)
- Fisher[7] (circ. section)
- Marshall & Tembe[6] (circ. section)

Fig. 2.9.2. Torsion and bending of plain concrete, mode I failures

Table 2.9.3 - Summary of Experimental Results

Investigator	Number of Beams	Shape of Sections	Theoretical modulus at rupture based on	Mode of Failure	Mean $\frac{T_{expt}}{T_{theory}}$	% coeff. of variation
Hsu[4]	2	rectangle	f_t	1	1.13	-
Hsu[4]	8	rectangle	f_t	2	1.06	5
Marshall and Tembe[6]	12	rectangle	f_t	2	0.91	4
Fisher[7]	13	circle	f_r	1	1.12	9
Marshall and Tembe[6]	4	circle	f_t	1	0.89	7
	39	Results based on tensile strength			1.02	12
Walsh et al[11]	6	rectangle	f_c'	1	1.41	17
Cowan and Armstrong[10]	1	rectangle	u_w	1	1.54	-
Walsh et al[11]	8	rectangle	f_c'	2	1.09	21
Iyengar and Rangan[15]	3	rectangle	u_w	2	0.85	-

Table 2.9.3 (continued) - Summary of Experimental Results

Investigator	Number of Beams	Shape of Sections	Theoretical Modulus at rupture based on	Mode of Failure	Mean $\frac{T_{expt}}{T_{theory}}$	% coeff. of variation
Cowan and Armstrong[10]	1	rectangle	u_w	2	1.22	-
Zia[39]	9	rectangle	f_c'	2	0.78	8
	28	Results based on compressive strength			1.06	29
	67	Results based on tensile and compressive strengths			1.04	21

2.10 Experimental Results - Circular Cross Section

Fisher[7] obtained results for a circular cross section subject to a full range of $^M/T$ values. This is particularly useful for in this case there is no mode 2, since $Tu_1' = Tu_2'$.

Fisher's experimental results (see table 2.10.1) give an average modulus of rupture of 444 lb/in² (3.06 N/mm²) for a 6" (152.4 mm) deep modulus of rupture beam. Since the circular section was 7.5" (190.5 mm) deep, the modulus of rupture has been revised on the Hsu[4] approach using imperial units.

$$f_r = 444 \frac{(1 + \frac{10}{7.5^2})}{(1 + \frac{10}{(6)^2})} = 409 \text{ lbf/in}^2 \ (2.82 \text{N/mm}^2) \quad \ldots\ldots \ 2.10.1$$

The mean ratio of $\frac{T_{expt}}{T_{theory}} = 1.12$ should be closer to 1.0 since a modulus of rupture has been used, but part of this discrepancy is probably due to the fact that only Fisher's preferred experimental results have been quoted.

Results by Marshall and Tembe[6] are also included in Table 2.10.1 (appendix) with the theoretical results which are based on the average tensile strength of the concrete. These results provide an opportunity of comparing experimentally and theoretically the torsional resistance of a rectangular cross section with a circular cross section in dimensionless form for the same strength concrete (see Table 2.10.2).

Marshall and Tembe[6] experiments in pure torsion for varying size rectangular sections and a single circular cross section enables a ratio of (Trect.expt/Tcirc.expt) to be determined. From the previous theory this ratio is

$$\frac{Tu_2' \text{ rct. theo.}}{Tu_2' \text{ circ theo.}} = \frac{\left(\frac{1}{3 + (b/d)^{1/2}}\right) db^2 f_{r2}}{(0.83)\left(\frac{\pi}{16} D^2\right) f_{r2}}$$

Table 2.10.2 - Comparison of Torsional Resistance of Rectangular and Circular Cross Sections - Experimental Results by Marshall and Tembe[6]

Nominal size and shape of member in (mm)	Number of test beams	T_{expt} average kipf in (KNm)	T_{theory} average kipf in (KNm)	$\frac{T_{expt}\ rect}{T_{expt}\ circ.}$	$\frac{T_{theory}\ rect}{T_{theory}\ circ.}$
4.85 (123.2) dia circ.	4	7.37 (0.83)	8.32 (0.94)	-	-
2 x 6 (50.8 x 152.4) rect.	2	3.34 (0.38)	3.62 (0.41)	0.45	0.44
3 x 6 (76.2 x 152.4) rect.	2	6.67 (0.75)	7.31 (0.83)	0.90	0.88
4 x 4 (101.6 x 101.6) rect.	2	6.96 (0.79)	7.76 (0.88)	0.94	0.93
3.5 x 6 (88.9 x 152.4) rect.	2	9.37 (1.06)	9.80 (1.11)	1.27	1.18
4 x 6 (101.6 x 152.4) rect.	4	10.15 (1.15)	11.44 (1.29)	1.38	1.38

Since the size of the circular section is constant this ratio varies with the size of the rectangular cross section.

The experimental and theoretical ratios are compared in columns 5 and 6 in table 2.10.2. These are favourable except for one value for a 3.5" x 6" (88.9 x 152.4) rectangular cross section, but even for this one the error is only of the order of 7%.

Table 2.9.3 gives a summary of all experiental results quoted in this Chapter including mean values of $\frac{T_{expt}}{T_{theory}}$ and the % coefficient of variation. Although the mean value for all results is 1.04 with a 21% coefficient of variation, individual investigators' values range from 0.78 to 1.54 as a mean of 4% to 21% for the coefficient of variation. The table shows that the greatest variations occur where the modulus of rupture is based on the compressive strength of the concrete, and indicates that this relationship requires further investigation. The mean value of $\frac{T_{expt}}{T_{theory}}$ based on the compressive strength is 1.06 with a coefficient of variation of 29%. When based on the tensile strength the values are 1.02 and 12%.

CHAPTER 3 - The Ultimate Strength of Concrete Members Reinforced With Longitudinal Steel Only

3.1 Introduction

Collins et al [40] concluded from a critical review of published works on torsion that no satisfactory method has been proposed for calculating the ultimate strength of concrete beams containing longitudinal steel and subject to bending and torsion. After further tests Walsh et al [11] concluded that to date it has not been possible to evolve a rational method of calculating the failure loads of these beams. Since a number of members in practice are reinforced longitudinally and subject to bending and torsional moments it is necessary to be able to determine the moment of resistance at ultimate load. The theoretical solution to this case is also related to the case of plain concrete members and to the case of members reinforced longitudinally and transversely.

Walsh et al [11] described two types of failure based on experimental observations. When bending predominates ($^M/T > 2$ approximately) failure was gradual, with spiral tensile cracks extending into a skewed compression hinge. Failure occurred when the compression zone crushed similar to the case of pure bending. When torsion predominates ($^M/T < 2$ approximately) a more sudden brittle fracture occurs with diagonal tension cracks visible only just before failure.

These descriptions provide the basis for the following theoretical approach, considering two modes of failure.

3.2 Theory for Mode 1 Form of Failure (controlled by failure of the Concrete)

This occurs when the $^M/_T$ ratio is high. Tensile cracks are present in the bottom of the member since the action of the bending moment predominates, and the steel at the bottom of the member is in tension. The concrete in the compression zone is subject to direct stresses due to the bending moment, and shear stresses due to the torsional moment. The direct stresses are assumed to be distributed as for a member subjected to bending moment alone, and the torsional shear stresses are distributed as shown in Figs 3.2.1 and 3.2.2. To balance the shear stresses in the compression zone a dowel force is assumed to act at right angles to the tension steel.

Taking moments of forces about an axis through the centroid of the tension steel and perpendicular to the length of the member

$$M_1 = k_{CM}\, f_{CM}\, bd_{n1}\, l_{a1} \qquad \text{..... 3.2.1}$$

Taking moments of forces about an axis through the centroid of the tension steel and parallel to the length of the member.

$$T_1 = k_{cv} . f_{cv} . bd_{n1}\, l_{a1} \qquad \text{..... 3.2.2}$$

Equation 3.2.2 is formed assuming that the lever arm l_{a1} is the same for the torsional moment of resistance as for the moment of resistance in bending. The error involved in this assumption is assumed to be small. Also steel and dowel forces in the compression zone are ignored.

The shear stress f_{cv} and the bending stress f_{CM} in the concrete can now be combined by the use of a failure criterion. Cowan[8] suggested that more complicated failure envelopes for the failure of concrete in compression could be replaced by a straight line tangent to the uniaxial compressive strength as shown in Fig. 3.2.3.

From the geometry of the diagram

$$\sin\beta = \frac{\sqrt{\left(f_{cv}\right)^2 + \left(f_{CM}/2\right)^2}}{\left(\frac{1}{\sin\beta} - 1\right)\frac{fc'}{2} + \frac{f_{CM}}{2}} \qquad \text{..... 3.2.3}$$

rearranging

$$\frac{f_{cv}}{fc'} = \frac{\left(\frac{1-\sin\beta}{2}\right)}{\sqrt{1+\left(\frac{f_{CM}}{2f_{cv}}\right)^2} - \frac{f_{CM}}{2f_{cv}}\sin\beta} \qquad \text{..... 3.2.4}$$

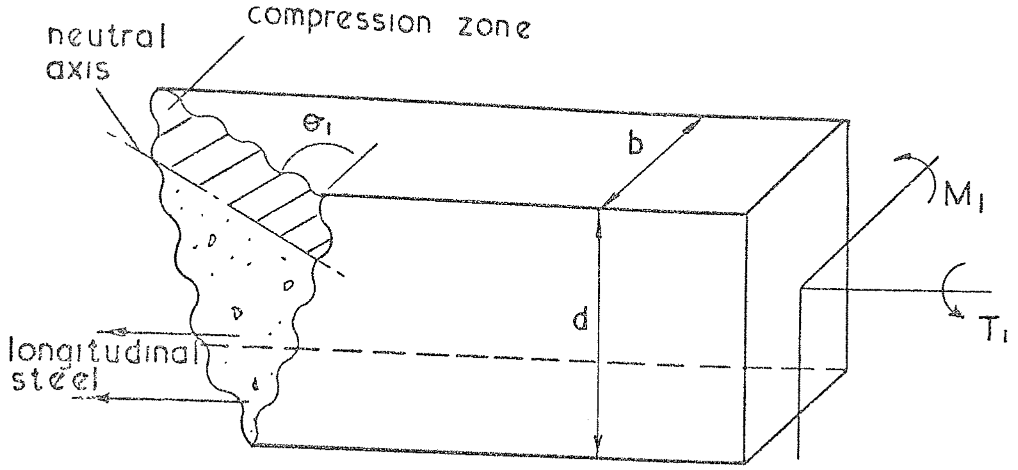


Fig. 3.2.1. Mode I type of failure for concrete members with longitudinal reinforcement only

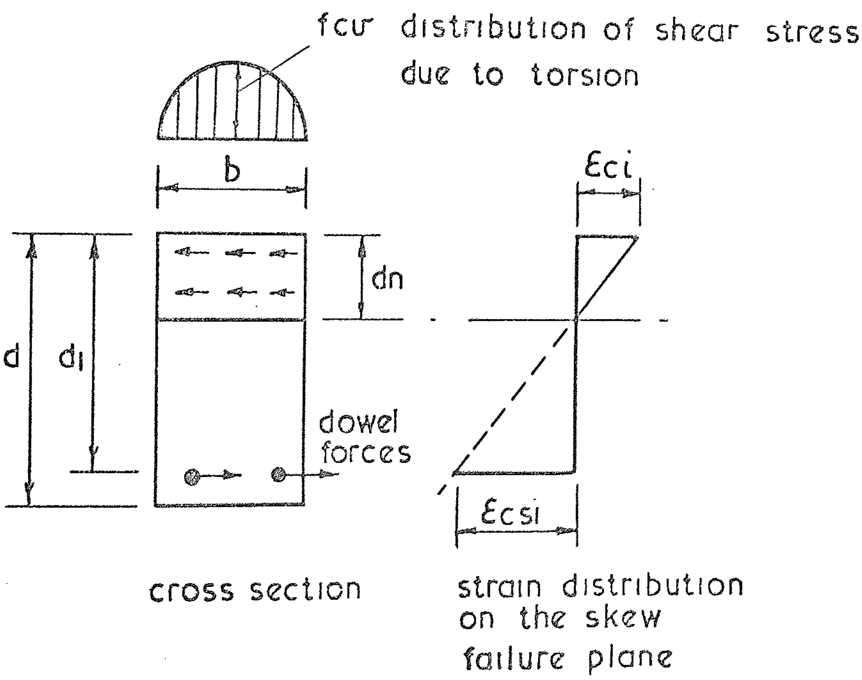


Fig. 3.2.2. Distribution of shear stress due to torsion

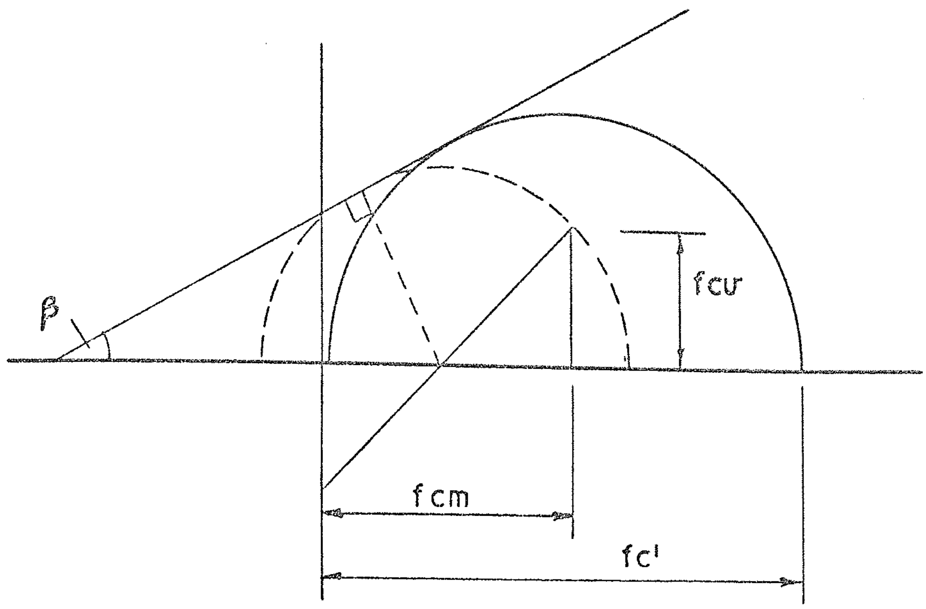


Fig. 3.2.3 Failure criteria for concrete, envelope tangent to the stress circle for uniaxial compression

The values of f_{CM} and f_{cv} obtained in equations 3.2.1 and 3.2.2 may be substituted in equation 3.2.4 to give

$$T_{\prime} = k_{cv} bd_{n\prime} l_{a\prime} f_c' \left\{ \frac{(1 - \sin\beta)/2}{\sqrt{1 + \left(\frac{k_{cv} M}{2k_{CM} T}\right)^2} - \frac{k_{cv} M}{2k_{CM} T} \sin\beta} \right\} \quad \ldots\ldots 3.2.5$$

Equation 3.2.5 gives a value of $T_{\prime}$ involving the $^{M}/_{T}$ ratio. An alternative form is the general interaction equation obtained by rearranging equation 3.2.3 to form

$$\frac{4}{(1-\sin\beta)^2}\left(\frac{f_{cv}}{f_c'}\right)^2 + \frac{(1 + \sin\beta)}{(1 - \sin\beta)}\left(\frac{f_{CM}}{f_c'}\right)^2 - \frac{2\sin\beta}{(1-\sin\beta)}\left(\frac{f_{CM}}{f_c'}\right) = 1 \quad \ldots\ldots 3.2.6$$

substituting in equation 3.2.6 the values of f_{CM} and f_{cv} obtained in equations 3.2.1 and 3.2.2

$$\left(\frac{T_{\prime}}{T_{cu\prime}}\right)^2 + \frac{(1+\sin\beta)}{(1-\sin\beta)}\left(\frac{M_{\prime}}{M_{cu\prime}}\right)^2 - \frac{2\sin\beta}{(1-\sin\beta)}\left(\frac{M_{\prime}}{M_{cu\prime}}\right) = 1 \quad \ldots\ldots 3.2.7$$

where $$T_{cu\prime} = \frac{(1-\sin\beta)}{2} k_{cv} bd_{n\prime} l_{a\prime} f_c' \quad \ldots\ldots 3.2.8$$

and $$M_{cu\prime} = k_{cm} bd_{n\prime} l_{a\prime} f_c' \quad \ldots\ldots 3.2.9$$

Involved in equations 3.2.8 and 3.2.9 is the depth of the compression zone $d_{n\prime}$. Since it has been assumed that the longitudinal tensile steel has not reached yield the value of $d_{n\prime}$ is determined using an elastic analysis. This involves the equilibrium of forces on a skew failure plane, a linear strain distribution over the depth of the section, and a linear stress/strain relationship for the steel and concrete. If it is assumed that there is a dowel force D_ℓ acting at right angles to the longitudinal tensile steel.

Taking moments about a longitudinal axis through the centroid of the compression zone

$$T_{\prime} = D_\ell . l_{a\prime} \quad \ldots\ldots 3.2.10$$

Equating the compressive force in the concrete to the tensile force and dowel force acting on the steel, in a direction perpendicular to the skew compression zone.

$$k_{CMi}\frac{b}{\cos\theta_1}\, d_{n1}\, f_{CMi} = A_1\, f_{sm1} \cos\theta_1 + D_\ell \sin\theta_1 \qquad \ldots\ldots \; 3.2.11$$

combining equations 3.2.10 and 3.2.11

$$k_{CMi}\frac{b}{\cos\theta_1}\, d_{n1}\, f_{CMi} = A_{s1}\, f_{sM1} \cos\theta_1 + \frac{T_1}{l_{a1}} \sin\theta_1 \qquad \ldots\ldots \; 3.2.12$$

Assuming a linear strain distribution over the depth of the skew cross section

$$\frac{\varepsilon_{ci}}{d_{n1}} = \frac{\varepsilon_{csi}}{(d_1 - d_{n1})} \qquad \ldots\ldots \; 3.2.13$$

If the stress/strain relationship for concrete is linear

$$\varepsilon_{ci} = \frac{f_{cMi}}{E_c} \qquad \ldots\ldots \; 3.2.14$$

and if it is also assumed linear for steel, then the strain in the concrete adjacent to the tensile steel in an a direction perpendicular to the skew failure plane is

$$\varepsilon_{csi} = \left(\frac{f_{sm1}}{E_s}\right) \cos^2\theta_1 + \left(\frac{T_1}{l_{a1}\, A_{s1}\, G_s}\right) \sin\theta_1 \cos\theta_1 \qquad \ldots\ldots \; 3.2.15$$

This assumes that the strain in the concrete is the same as in the adjacent steel. If there is a breakdown in bond between the steel and the concrete then a bond slip factor should be introduced.

Experimental evidence exists for this in prestressed concrete.

Combining equations 3.2.13, 3.2.14 and 3.2.15

$$\frac{\left(f_{cM1}/E_c\right)}{d_{n1}} = \frac{\left(f_{sm1}/E_s\right) \cos^2\theta_1 + \left(T_1/l_{a1} A_s f_{1s}\right) \sin\theta_1 \cos\theta_1}{(d_1 - dn_1)} \qquad \ldots\ldots \; 3.2.16$$

combining equations 3.2.11 and 3.2.16 to eliminate f_{sM1}

$$\frac{k_{cMi}\, bd_{n1}}{\cos\theta_1} = \frac{m\, A_{s1}}{\cos\theta_1}\, \frac{(d_1 - d_{n1})}{d_{n1}} - \frac{T_1}{l_{a1}}\, \frac{\sin\theta_1}{f_{cMi}} \left(E_s/G_s - 1\right) \qquad \ldots\ldots \; 3.2.17$$

Taking moments of forces about the tension steel for the skew failure plane

$$M_1 \cos\theta_1 + T_1 \sin\theta_1 = k_{cMi} \frac{b}{\cos\theta_1} d_{n1} f_{dMi} \, la_1 \qquad \text{..... 3.2.18}$$

Combining equations 3.2.17 and 3.2.18 to eliminate f_{cmi}.

$$k_{cMi}\left(\frac{d_{n1}}{d_1}\right)^2 + mp_1'\left(\frac{d_{n1}}{d_1}\right) - mp_1' = 0 \qquad \text{..... 3.2.19}$$

$$\text{when } P_1' = \left(\frac{M_1/T_1 + \tan\theta_1}{M_1/T_1 + \left(E_s/G_s\right)\tan\theta_1}\right)\frac{A_1}{bd_1} \qquad \text{..... 3.2.20}$$

Equation 3.2.19 is a quadratic in $^{d}n/d_1$ which gives

$$\frac{d_{n1}}{d_1} = \frac{mP_1'}{2k_{cMi}} + \sqrt{\left(\frac{mP_1'}{2k_{cMi}}\right)^2 + \frac{mP_1'}{k_{cMi}}} \qquad \text{..... 3.2.21}$$

This expression is comparable to the case of pure bending in the elastic stage of analysis, where P_1 is replaced by P_1'. From consideration of equation 3.2.20 it is apparent that in pure bending, i.e. when $^{M}/T \rightarrow \alpha$, $P_1 = P_1'$. In pure torsion $^{M}/T = 0$ and $P_1' = \left(^{Gs}/E_s\right)\left(^{A_1}/bd_1\right)$. The effect of this change is that $^{d_{n1}}/d_1$ is a maximum in pure bending and a minimum in pure torsion.

The value of the lever arm ratio is $\frac{la_1}{d_1} = 1 - k_\ell' \frac{d_{n1}}{d_1}$ 3.2.22

The value of P_1' (equation 3.2.20) involves the angle of the skew failure plane θ_1. When the bending and torsional moments are applied to the member, the first cracks appear in the extreme fibres in the tension zone at the bottom of the section. In pure bending this crack is zero degrees to an axis perpendicular to the longitudinal axis of the member. As the $^{M}/_{T}$ decreases the angle θ_1 increases until in pure torsion it is approximately 45°. Experimental values for this angle θ_1 are plotted in fig. 3.2.4 for varying $^{M}/_{T}$ ratios.

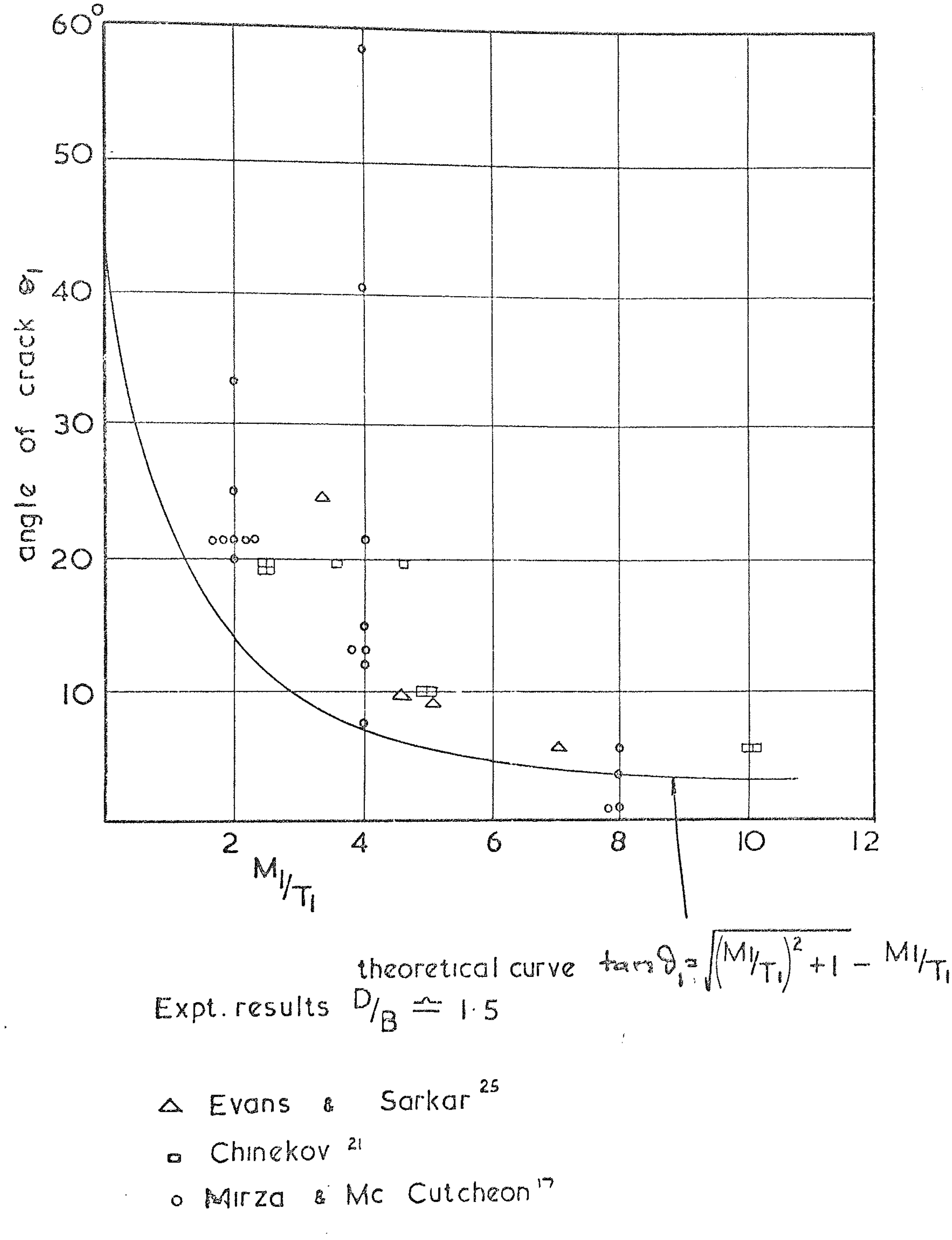


Fig. 3.2.4. Variation of angle of crack in the concrete with M_1/T_1 ratio for mode I

A theoretical solution to these experimental results can be obtained by considering a simple skew failure plane for plain concrete as shown previously in fig. 2.2.1.

Taking moments of the forces about the neutral axis on the skew failure plane

$$M_1 \cos\theta_1 + T_1 \sin\theta_1 = \frac{b}{\cos\theta_1} d^2 f_{r1} \qquad \text{..... 3.2.23}$$

rearranging

$$T_1 = \frac{bd^2 f_{r1}}{6} \frac{(1 + \tan^2\theta_1)}{(M_1/T_1 + \tan\theta_1)} \qquad \text{..... 3.2.24}$$

The minimum value of T_1 occurs when

$$\tan\theta_1 = \sqrt{\left(\frac{M_1}{T_1}\right)^2 + 1} - \frac{M_1}{T_1} \qquad \text{..... 3.2.25}$$

This curve is plotted on fig. 3.2.4 to enable a comparison to be made with the experimental results. It is probable that this theoretical value of tan θ_1 is too small since available experimental values for the crack on the tensile face are greater, and also cracks on the side of the member are not vertical.

3.3 Theory for Mode 1 Form of Failure (based on yielding of the steel)

This is an alternative form of behaviour in mode 1 and will occur when the steel has a definite yield point. As previously for mode 1 the stresses are distributed as shown in Fig. 3.2.2. In particular it should be noted that the tension steel is subjected to a direct stress due to bending and a shear stress produced by the dowel forces. These stresses are combined using a yield failure criterion.

Taking moments of forces about an axis through the centroid of the compression force and perpendicular to the length of the member.

$$M_1 = A_1 f_{sM1} l_{as1} \qquad \text{..... 3.3.1}$$

Taking moments of forces about an axis through the centroid of the compression force and parallel to the length of the member

$$T_1 = A_1 \; f_{sv1} \; l_{as1} \qquad \ldots\ldots 3.3.2$$

The shear stress f_{sv1} and the bending stress f_{sM1} in the steel can now be combined by the use of a failure criterion. The shear distortion strain energy expression is widely accepted as a yield criteria for steel and in this case is expressed as

$$\sqrt{f_{sM1}^2 + \lambda_s \; f_{sv1}^2} = f_{1y} \qquad \ldots\ldots 3.3.3$$

substituting equations 3.3.1 and 3.3.2 in 3.3.3 and rearranging T_1 and M_1 may be expressed using an $^M/_T$ ratio.

$$T_1 = \frac{M_{su1}}{\sqrt{\left(M_1/T_1\right)^2 + \lambda_s}} \qquad \ldots\ldots 3.3.4$$

where $M_{us1} = A_1 \; f_{1y} \; l_{a1y}$ 3.3.5

or using a $^T/_M$ ratio

$$M_1 = \frac{M_{su1}}{\sqrt{1 + \lambda_s \left(T_1/M_1\right)^2}} \qquad \ldots\ldots 3.3.6$$

Alternatively it may be expressed in the general non dimensional interaction form as

$$\left(\frac{T_1}{T_{su1}}\right)^2 + \left(\frac{M_1}{M_{su1}}\right)^2 = 1 \qquad \ldots\ldots 3.3.7$$

where $T_{su1} = M_{su1}/\sqrt{\lambda_s}$ 3.3.8

The previous expressions include the lever arm l_{as1} which involves a value of d_{ns1}. This value may be calculated by combining the failure criterion for concrete and steel.

Equating equations 3.2.5 and 3.3.4 and rearranging

$$\frac{d_{ns_1}}{d_1} = \frac{P_1 f_{1y}}{k_{cM} f_c'} \left[\frac{\sqrt{\left(k_{cM}/k_{cv}\right)^2 + \left(M_1/2T_1\right)^2} - \left(M_1/2T_1\right) \sin\beta}{\frac{1}{2}(1 - \sin\beta)\sqrt{\left(M_1/T_1\right)^2 + \lambda s}} \right] \quad \ldots\ldots 3.3.9$$

The lever arm ratio is

$$\frac{l_{as1}}{d_1} = 1 - k_\ell \frac{d_{ns1}}{d_1} \quad \ldots\ldots 3.3.10$$

There is the possibility of a third form of failure in mode 1. If the concrete cover to the longitudinal steel is inadequate then the dowel force at right angles to the steel will fracture the concrete and failure will occur. This mode of failure has been described previously by Gesund and Boston[14], and also dealt with theoretically. It appears that this form of failure is more applicable to mode 2, and relies on empirical values which make it conservative. Using present experimental test results a reasonable accuracy can be obtained by ignoring it. If the cover to the steel were however reduced to values considerably less than used in normal practice it may prove to be critical.

3.4 Theory for Mode 2 Form of Failure (controlled by the tensile strength of the concrete)

This mode of failure occurs at lower values of the M/T ratio. The first crack appears in the side of the beam. The load does not increase beyond the value at the first crack but gradually decreases as more cracks develop. It seems reasonable to assume that the member behaves as a plain concrete member to determine the ultimate strength. This conclusion was reached by Collins et al [40], and confirmed experimentally more recently by Iyengar and Rangan[15]. The theoretical equation to the ultimate strength in torsion is therefore assumed to be the same as for plain concrete failing in mode 2.

The theory for this has been presented previously in Chapter 2 and may be expressed as

$$T'_{u2} \simeq \frac{1}{3 + \left(^{b}/_{d}\right)^{1/2}} \, db^2 \, f_{r2} \quad \text{(see equation 2.5.10)}$$

The general relationship between modes 1 and 2 for plain concrete and concrete with longitudinal steel, is shown in fig. 3.4.1.

3.5 Comparison of Theory with Experimental Results Mode 1 - based on failure of the concrete.

The application of the theory necessitates the adoption of values for the bending stress factor k_{cM} & k_{cMi} the lever arm factor k_ℓ the shear stress factor k_{cv} & k_{cvi} and the angle β for the concrete failure envelope.

The value of k_{cM} & k_{cMi} varies between ½ for a triangular stress distribution to ⅔ for a parabolic stress distribution. Since mode 1 type of failures occur at high $^M/_T$ ratios a value of ⅔ has been adopted with the corresponding value of k_ℓ = ⅜. Since the shear stress distribution is probably parabolic k_{cv} is also assumed to be ⅔. The value of β recommended by Cowan[8] is 37°.

Substituting these values in equation 3.2.5

$$T_1 = \frac{2}{3} \, bdn_1 \, la_1 \, f_c' \left(\frac{0.2}{\sqrt{1 + \left(^{M_1}/_{2T_1}\right)^2} - 0.3\,\frac{M_1}{T_1}} \right) \quad \text{..... 3.5.1}$$

This equation has been used to determine the theoretical values given in table 3.5.1 (see appendix). The comparison between experimental and theoretical result is expressed as a ratio of $^{Texpt}/Ttheory$ which ideally should be unity. The mean value of this ratio for the five available experimental results for reinforced concrete members is 1.01 with a coefficient of variation of 5%.

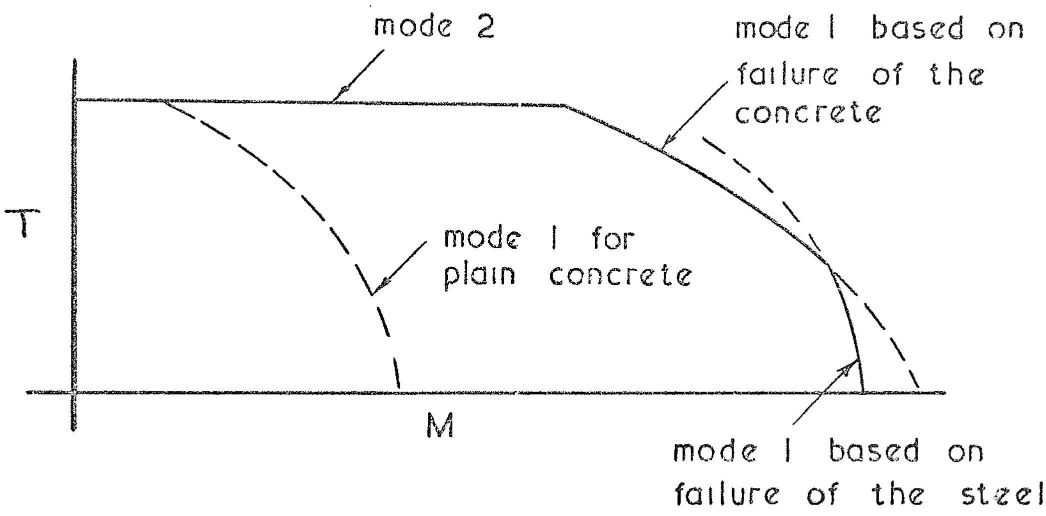


Fig. 3.4.1. Theoretical relationship between modes I and 2 for concrete members with longitudinal steel

The general interaction equation 3.2.7 is particularly useful for a graphical representation of the experimental results see fig. 3.5.1 and 3.5.2. If the value of the constants k_M, k_ℓ, k_{cv}, and β are substituted in equation 3.2.7

$$\left(\frac{T_1}{T_{cu1}}\right)^2 + \left(\frac{2M_1}{M_{cu1}}\right)^2 - \frac{3M_1}{M_{cu1}} = 1 \qquad \ldots\ldots 3.5.2$$

where

$$T_{cu} = 0.2 \times \frac{2}{3} bd_{n1} \, l_{a1} \, f_c' \text{ and} \qquad \ldots\ldots 3.5.3$$

$$M_{cu} = \frac{2}{3} bd_{n1} \, l_{a1} \, f_c' \qquad \ldots\ldots 3.5.4$$

The use of equation 3.5.1 and 3.5.2 include evaluating the depth of the compression zone d_{n1} using equation 3.2.21. This involves a value of the ratio E_s/G_s which has been taken as 2.5. The value of the modular ratio for steel and concrete is also involved. It has been assumed that Young's modulus for steel = E_s = 29 x 10^6 lbf/in^2 (200 kN/mm^2) and the secant modulus for concrete E_c = 45,000 $\sqrt{f_c'}$ lbf/in^2 (3737 $\sqrt{f_c'}$ N/mm^2).

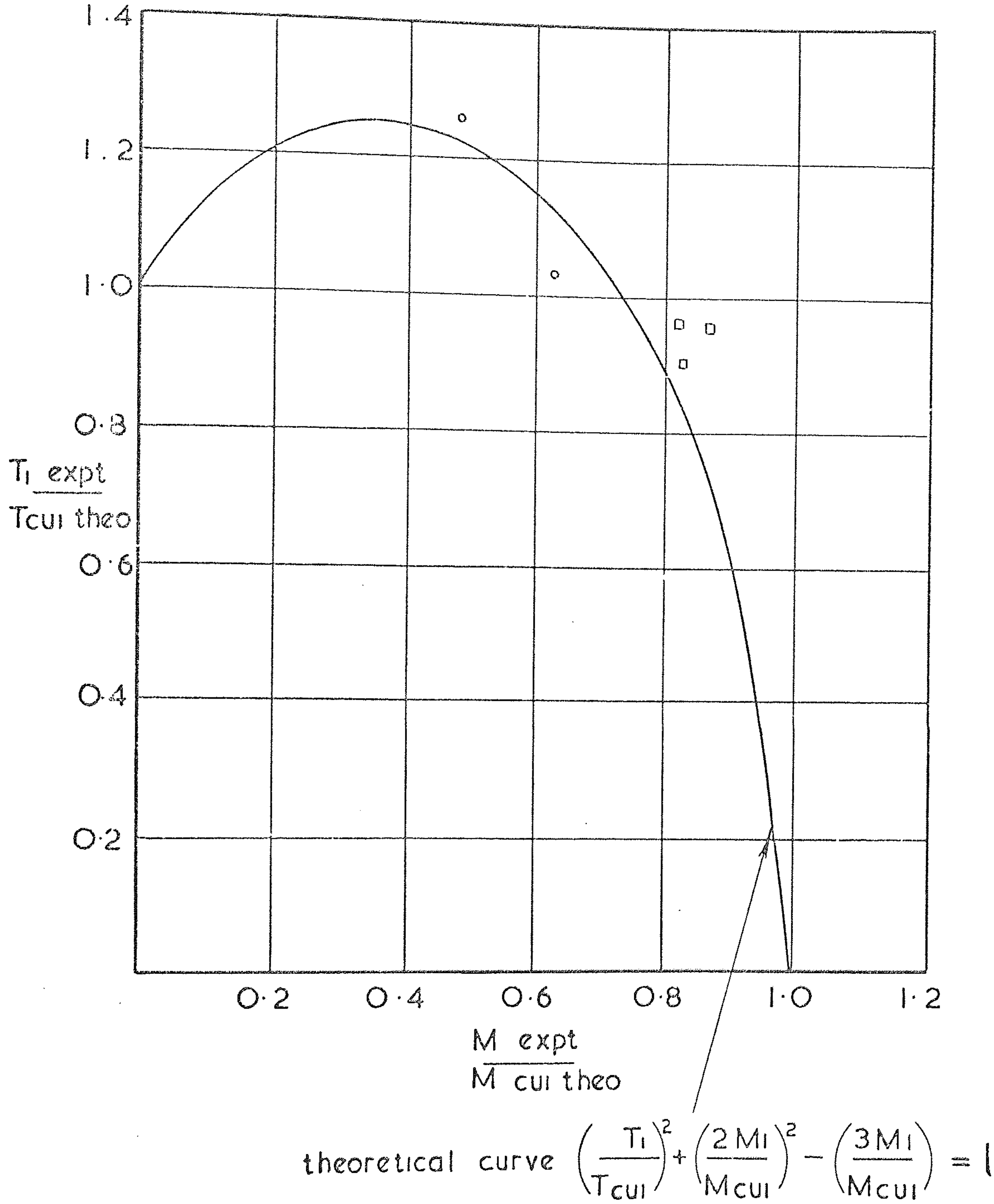
3.6 Comparison of Theoretical with Experimental Results Mode 1 - based on yielding of the steel

The values of k_{cM}, k_ℓ, k_{cv} and β are also involved in this mode, and since they apply over the same range of M_1/T_1 values the same values have been adopted as for failure in mode 1 based on failure of the concrete.

The value of the constant λ_s = 3 for the Hüber-von Mises-Hencky shear distortion strain energy expression. Substituting this value in equation 3.3.4.

$$T_1 = \frac{M_{us1}}{\sqrt{\left(M_1/T_1\right)^2 + 3}} \qquad \ldots\ldots 3.6.1$$

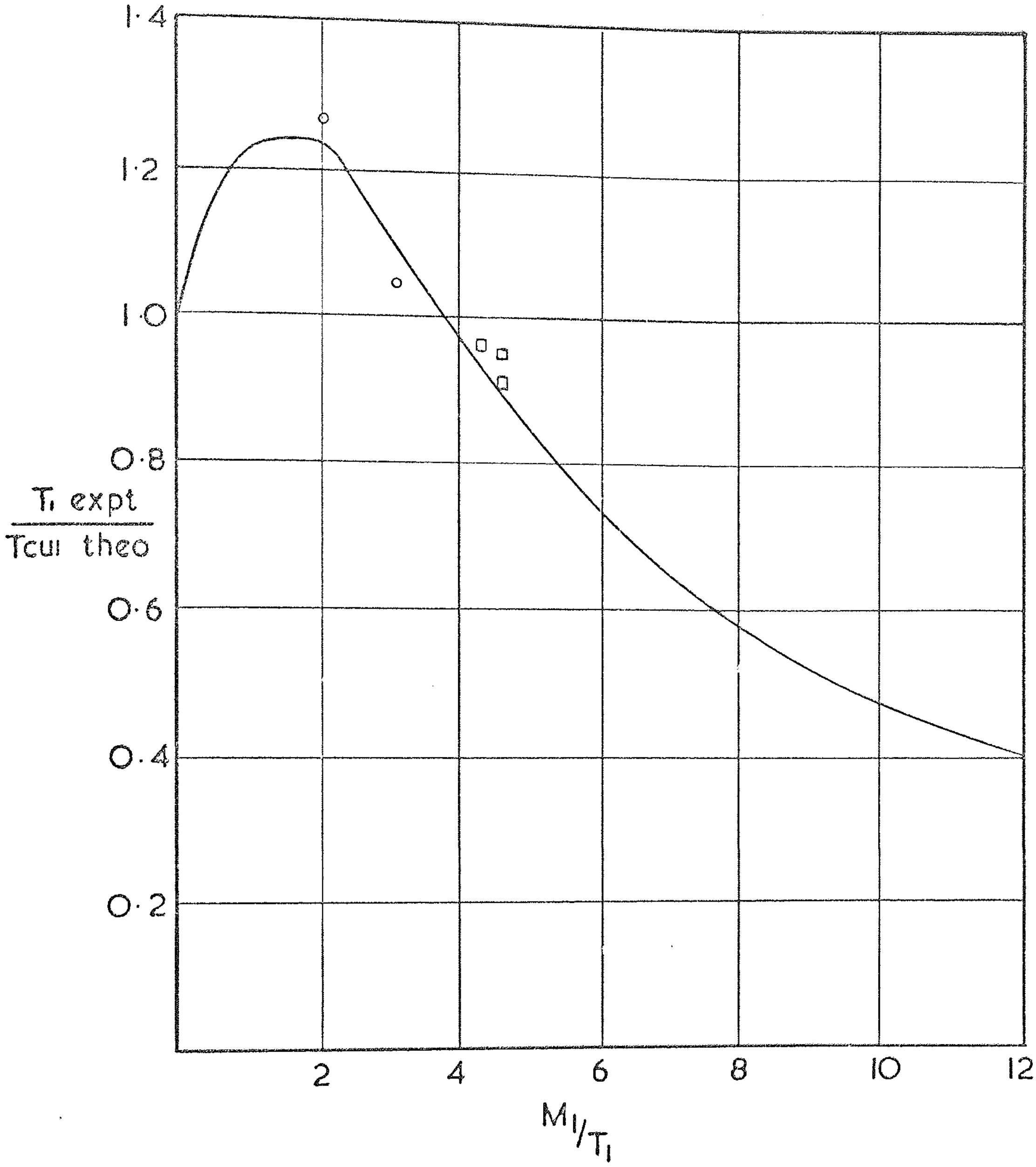
This equation has been used to determine the theoretical values given in table 3.6.1 (see appendix). The comparison between experimental and theoretical result is expressed as a ratio of Texpt/Ttheory which ideally



○ Gesund & Boston [14]

□ Ramakrishnan & Vijayarangan [41]

Fig. 3.5.1 Torsion and bending of concrete beams reinforced with longitudinal steel only, mode 1 based on failure of the concrete



o Gesund & Boston [14]
□ Ramakrishnan & Vijarangan [41]

Fig 3.5.2 Torsion and bending of concrete beams reinforced with longitudinal steel only, mode 1 based on failure of the concrete should be unity. The mean value for the fourteen available experimental results for reinforced concrete containing steel with a pronounced yield point is 1.17 with a coefficient of variation of 11%.

The results may also be expressed graphically using the general interaction equation 3.3.7 as shown in fig.3.6.1

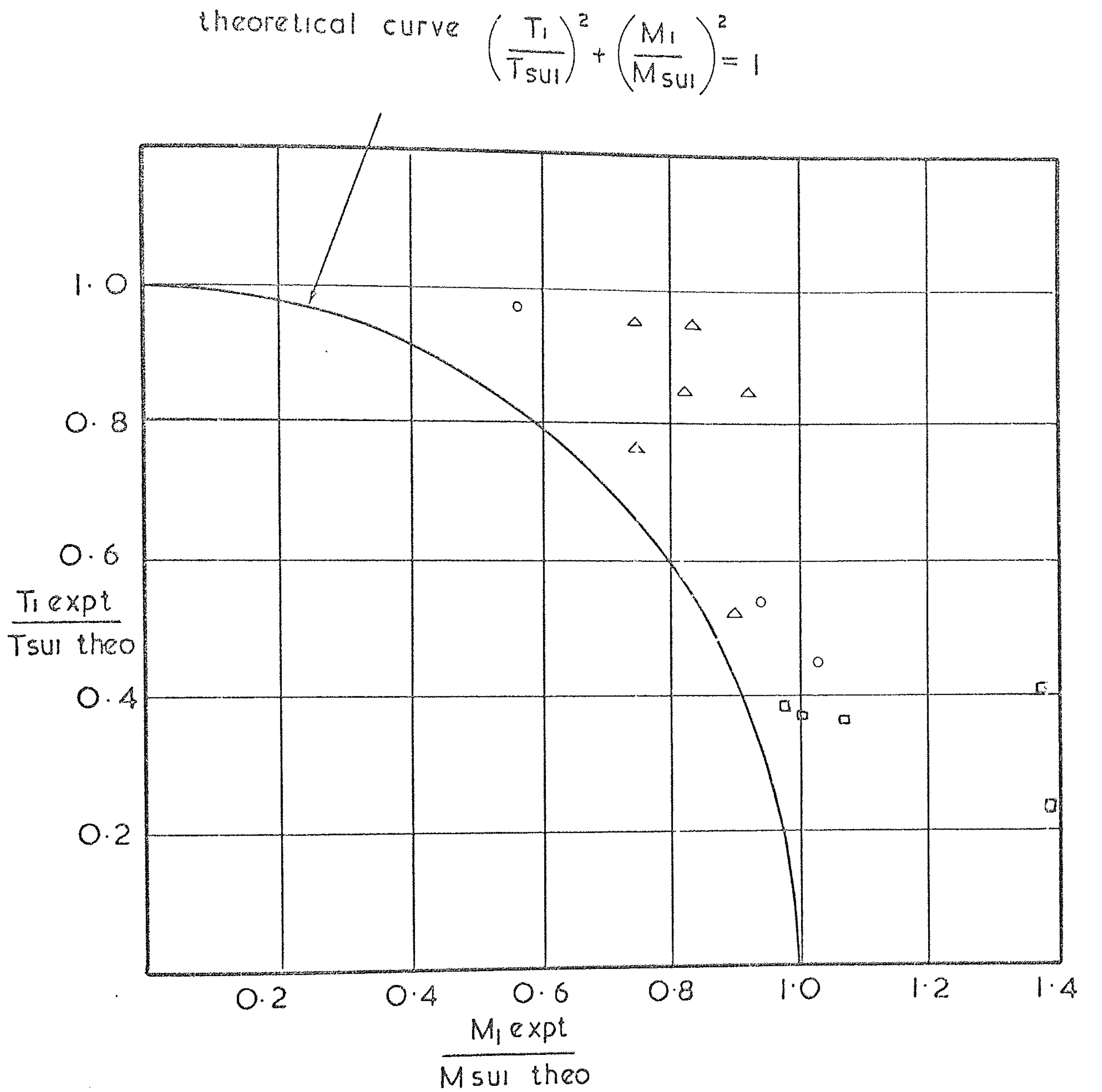
3.7 Comparison of Theory with Experimental Results Mode 2

The experimental results have been divided into two sections. Where the M/T ratio is equal to zero see table 3.7.1 (appendix). Where the M/T ratio is greater than zero see table 3.7.2 (appendix). The value of the modulus of rupture of the concrete is generally not available, and a value has been obtained indirectly based on an empirical expression developed by Hsu[4] which incorporates the cylinder crushing strength. In imperial units these expressions are:

$$f_r = 24\ (1 + 10/b^2)\ (f_c')^{1/3} \quad \text{for } b > 4"$$

$$f_r = 24\ (2.4/b^{1/3})\ (f_c')^{1/3} \quad \text{for } 2" < b \leq 4"$$

The use of the cylinder crushing strength to obtain a modulus of rupture introduces further inaccuracies as shown previously in chapter 2. The accuracy of the theory for modes 1 and 2 is shown in the summary table 3.7.3. For comparison the table includes values for plain concrete failing in mode 2, the theory being related to the cylinder crushing strength.



△ Nylander [13]
○ Gesund & Boston [14]
□ Ramakrishnan & Vijayarangan [41]

Fig. 3.6.1 Torsion and bending of concrete beams reinforced with longitudinal steel only, mode 1 based on failure of steel.

Table 3.7.3

Summary of experimental results for concrete members with longitudinal steel only.

Type of Cross Section	Number of Investigaters	Number of Beams	Mode of failure	Mean $\frac{T\ expt}{T\ theory}$	% Coeff of Variation
Longitudinal reinforcement M/T = 0	11	58	2	0.99	23
Longitudinal reinforcement M/T > 0	4	18	2	0.98	15
Plain concrete (for comparison)	6	28	2	1.06	29
Longitudinal reinforcement M/T > 0 based on failure of the concrete	2	5	1	1.01	5
Longitudinal reinforcement M/T > 0 based on failure of the steel	3	14	1	1.17	14

Total 95 Mean 1.02 C.V. 20% (excluding plain concrete results)

CHAPTER 4 - The Ultimate Strength in Bending and Torsion of Concrete Members Reinforced with Longitudinal and Transverse Steel - All steel at Yield.

4.1 Introduction

This case is of most interest practically since a designer, from lack of knowledge, tends to add stirrups to take the torsional moment, hoping that they will yield and therefore give adequate warning of collapse.

Experimental evidence of strains measured in the steel by Lessig[20], Hsu[27] and others, indicate that it is very rare for all the steel at the failure sections to yield. This is also complicated by the fact that strain readings are often not recorded at the failure section of the beam. The latest experimental work by Jackson and Estanero[34] in 1971 however suggests that yield theories may work successfully providing the reinforcement limits are defined.

In 1959 Lessig[3] developed a theory based on yielding of the steel which was complicated and not particularly suitable for design purposes. Later work[20] produced an empirical expression for the depth of the compression zone and over-reinforcing. In 1965 Evans & Sarkar[25] made a similar approach but based on the assumption that the cracking of the concrete controlled the steel that was effective at the failure section. Lessig had assumed that it was controlled by the equilibrium of the forces. The paper also included a theoretical expression for the depth of the compression zone based on the assumption that torsional shear stresses did not reduce the strength of the concrete below the cylinder crushing strength.

In 1966 Walsh et al [26] simplified the Lessig theory and identified mode 3 form of failure. They did not however form an expression for the depth of the compression zone, but assumed the lever arm remained reasonably constant.

The truss theory as presented by Kuyt [36] in 1972 also assumes yielding of the steel, but found difficulty in defining the area resisting the compressive forces.

The author's approach differs in particular by attempting to, theoretically, define the depth of the compression zone using the Cowan failure criterion for concrete. It is similar to the theoretical approach made by Yudin [23] by taking moments of forces about the longitudinal and transverse axes of the member. Yudin however used a constant crack angle of 45^{o}, whereas this angle is determined by the author from the equilibrium of forces.

4.2 Theory for Mode 1 Form of Failure

This form of failure occurs when the $^{M}/_{T}$ ratio is high. The concrete in the compression zone is subject to direct stresses due to the bending moment, and shear stresses due to the torsional moment. The transverse steel and longitudinal steel are at yield at the failure section (see fig. 4.2.1).

Taking moments of forces about a transverse axis through the centroid of the compression zone and perpendicular to the length of the member

$$M_1 = A_1 f_{1y} l_{a1} - \frac{A_s f_{sy}}{S} (d_1 - d_{n1})(b' + d_1 - d_{n1}) \tan^2\alpha_1 \;\; ..... \;\; 4.2.1$$

Taking moments of forces about a longitudinal axis through the centroid of the compression zone (see fig. 4.2.2).

$$T_1 = \frac{A_s f_{sy}}{S} (d_1 - d_{n1}) \tan \alpha_1 \, b' + \frac{A_s f_{sy}}{S} b' \tan \alpha_1 \, l_{a1}$$

rearranging

$$T_1 = \frac{A_s f_{sy}}{S} b' \tan \alpha_1 \, (d_1 - d_{n1} + l_{a1}) \;\; ..... \;\; 4.2.2$$

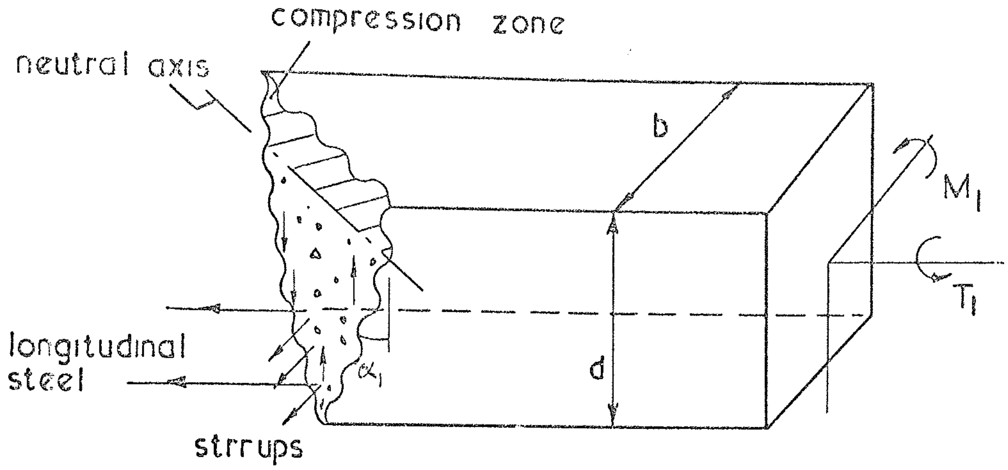


Fig. 4.2.1 Mode I type of failure for concrete members reinforced longitudinally and transversely

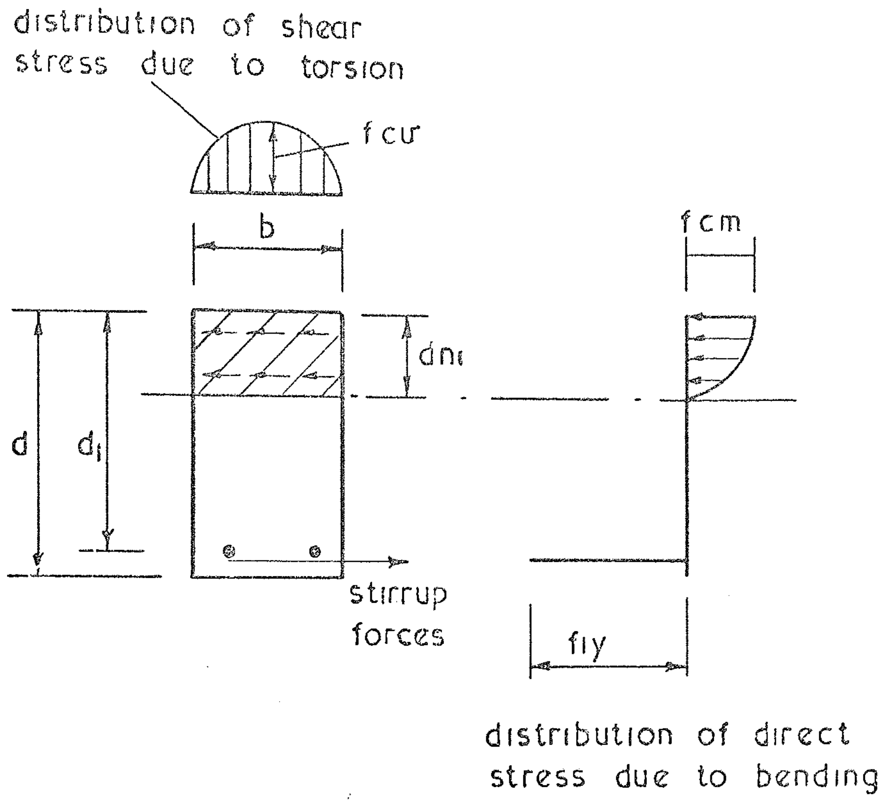


Fig 4.2.2. Distribution of shear stress due to torsion and direct stress due to bending for mode 1

Combining equations 4.2.1 and 4.2.2 to eliminate α_1

$$M_1 = A_1 f_{1y} l_{a1} - \frac{A_s f_{sy}}{S} (d_1 - d_{n1})(b' + d_1 - d_{n1}) \frac{T_1^2}{\left[\frac{A_s f_{sy}}{S} b' (d_1 - d_{n1} + l_{a1})\right]^2}$$

if by definition $M_{u1} = A_1 f_{1y} l_{a1}$, by rearranging

$$\frac{M_1}{M_{u1}} = 1 - \frac{T_1^2}{\frac{A_s f_{sy}}{S} \frac{(d_1 - d_{n1} + l_{a1})^2 (b')^2 A_1 f_{1y} l_{a1}}{(d_1 - d_{n1})(b' + d_1 - d_{n1})}}$$

and also if by definition $r_{1y} = \frac{A_s f_{sy} b'}{S A_1 f_{1y}}$

$$\frac{M_1}{M_{u1}} = 1 - \left(\frac{T_1}{M_{u1} \sqrt{\frac{r_{1y}}{\frac{(d_1 - d_{n1})(b' + d_1 - d_{n1}) l_{a1}}{(d_1 - d_{n1} + l_{a1})^2 b'}}}} \right)^2 \quad \ldots\ldots 4.2.3$$

where d_{n1} is small in relation to d_1

equation 4.2.3 can be further simplified by the approximation

$$\frac{(d_1 - d_{n1})(b' + d_1 - d_{n1}) l_{a1}}{(d_1 - d_{n1} + l_{a1})^2 b'} \simeq \frac{1}{4} (1 + {}^{d}/_{b}) \quad \ldots\ldots 4.2.4$$

The inaccuracy involved in this approximation is shown in Fig.4.2.3. As ${}^{d_{n1}}/_{d_1}$ increases so the error increases, but since in practice it is assumed that ${}^{d_{n1}}/_{d_1}$ will be small for under reinforced members then the factor is more applicable to small values of ${}^{d_{n1}}/d_1$. Also since $T \propto {}^{1}/\text{factor}$ then the solution is conservative. Walsh et al [26] chose a value of $\frac{1}{4} (1 + {}^{2d}/b)$ which is more in error, but is also more conservative.

Substituting equation 4.2.4 in equation 4.2.3 and rearranging

$$\left(\frac{T_1}{2M_{u1} \sqrt{\frac{r_{1y}}{(1 + {}^{d}/_{b})}}} \right)^2 + \frac{M_1}{M_{u1}} = 1 \quad \ldots\ldots 4.2.5$$

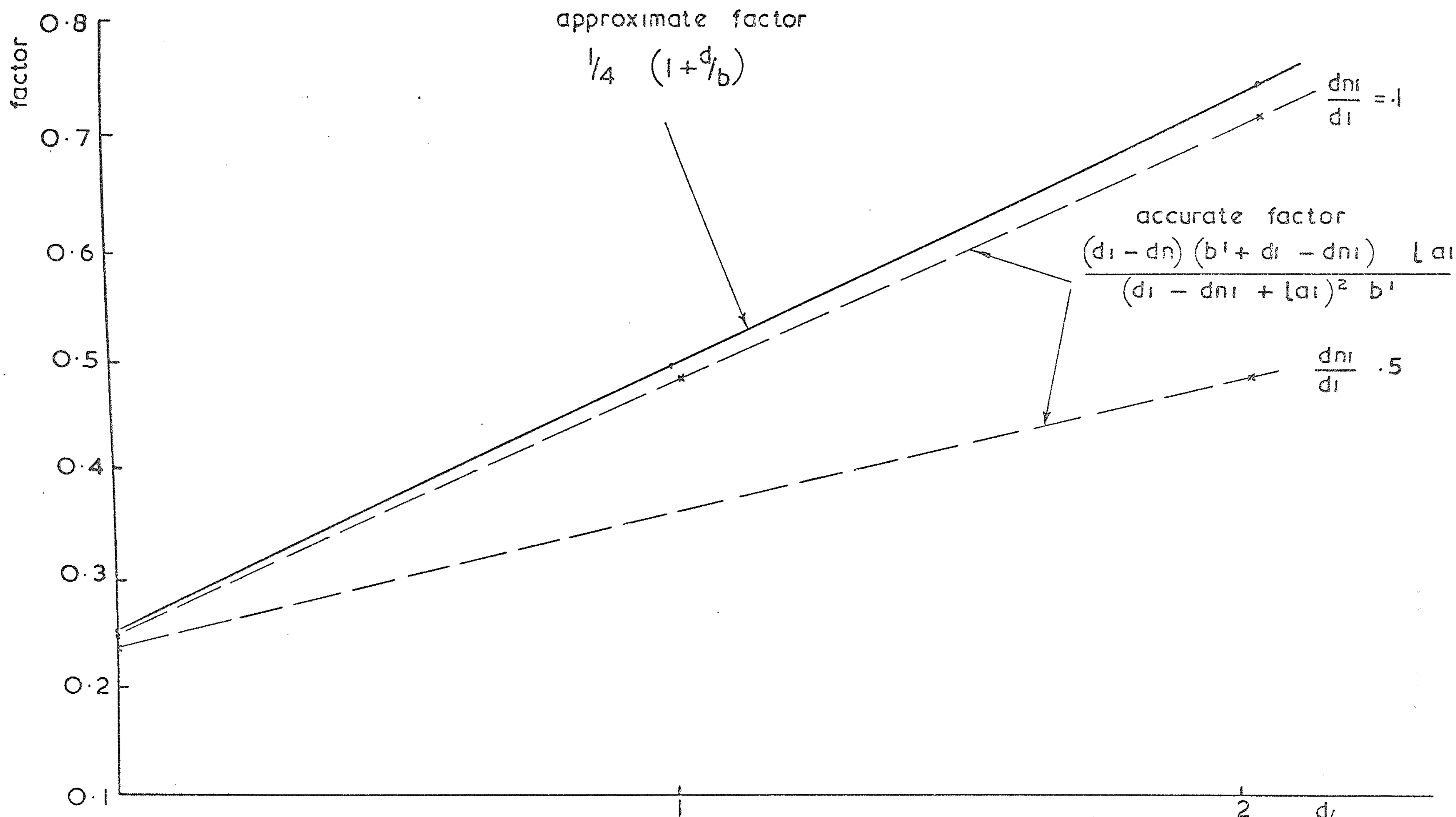


Fig. 4.2.3 Accuracy of factor for yielding of the steel

This is now in the general interaction form and may be expressed as

$$\left(\frac{T_1}{T_{u1}}\right)^2 + \frac{M_1}{M_u} = 1 \qquad \text{..... 4.2.6}$$

if

$$T_{u1} = 2M_{u1}\sqrt{\frac{r_{1y}}{(1 + d/b)}} \qquad \text{..... 4.2.7}$$

Alternatively by rearranging equation 4.2.5

$$\frac{1}{2}T_1^{\,2} + 2M_{u1}\frac{r_{1y}}{(1 + d/b)}\left(\frac{M_1}{T_1}\right)T_1 - 2(M_{u1})^2\frac{r_{1y}}{(1 + d/b)} = 0$$

Solving this equation for T_1

$$T_1 = 2M_{u1}\frac{r_{1y}}{(1 + d/b)}\left[\sqrt{\left(\frac{M_1}{T_1}\right)^2 + \frac{(1 + d/b)}{r_{1y}}} - \frac{M_1}{T_1}\right] \qquad \text{..... 4.2.8}$$

This form of the equation has been produced previously by Walsh et al [26] but using theory based on a skew bending approach. The constant of $(1 + d/b)$ given here was given by Walsh et al as $(1 + 2d/b)$.

From consideration of the general interaction equation 4.2.6 it is apparent that M_{u1} and T_{u1} are important. Both of these contain a value of the lever arm l_{a1}, which involves determining a value of the depth of the compression zone d_{n1}.

The compression zone is subject to a direct stress f_{cm} due to bending and a shear stress f_{cv} due to torsion as shown in fig. 4.2.2. These can be related to the steel areas and dimensions of the section as follows.

Resolving forces parallel to the longitudinal axis of the member and ignoring the compression reinforcement

$$A_1 f_{1y} = k_{cm}\, b d_{n1}\, f_{cm} \qquad \text{..... 4.2.9}$$

Resolving forces at right angles to the longitudinal axis of the member and ignoring any dowel effects in the tension or compression zone.

$$\frac{A_s f_{sy}}{S} b' \tan\alpha_1 = k_{cv}\, b d_{n1}\, f_{cv} \qquad \text{..... 4.2.10}$$

The direct stress f_{cm} and the shear stress f_{cv} can now be related using a failure criterion for concrete. Cowan[8] suggested that more complicated failure envelopes for the failure of concrete could be replaced by a straight line tangent to the uniaxial compressive strength as shown in fig. 4.2.4.

From the geometry of the diagram

$$\sin\beta = \frac{\sqrt{\left(f_{cv}\right)^2 + \left(f_{cm}/2\right)^2}}{\left(\frac{1}{\sin\beta} - 1\right)\frac{f_c'}{2} + \frac{f_{cm}}{2}} \qquad \ldots\ldots 4.2.11$$

rearranging equation 4.2.11

$$\frac{f_{cv}}{f_c'} = \frac{\frac{1-\sin\beta}{2}}{\sqrt{1 + \left(\frac{f_{cm}}{2f_{cv}}\right)^2} - \left(\frac{f_{cm}}{2f_{cv}}\right)\sin\beta} \qquad \ldots\ldots 4.2.12$$

substituting equations 4.2.9 and 4.2.10 in equation 4.2.12

$$\frac{A_s f_{sy}\, b' \tan\alpha_1}{S\, k_{cv}\, bd_{n1}\, f_c'} = \frac{\frac{1-\sin\beta}{2}}{\sqrt{1 + \left(\frac{k_{cv}}{2k_{cm} r_{1y} \tan\alpha_1}\right)^2} - \frac{k_{cv}\sin\beta}{2k_{cm} r_{1y}\tan\alpha_1}}$$

rearranging

$$\frac{d_{n1}}{d_1} = \frac{p_1 f_{1y}}{(1-\sin\beta)\, k_{cm} f_c'} \left\{\sqrt{1 + \left(\frac{2k_{cm} r_{1y} \tan\alpha_1}{k_{cv}}\right)^2} - \sin\beta\right\} \qquad \ldots\ldots 4.2.13$$

Equation 4.2.13 contains $\tan\alpha_1$ which may be evaluated as follows

Dividing equation 4.2.1 by equation 4.2.2

$$\frac{M_1}{T_1} = \frac{A_1 f_{1y} l_{a1} - \left(A_s f_{sy}/S\right)(d_1 - d_{n1})(b' + d_1 - d_{n1})\tan^2\alpha_1}{\left(A_s f_{sy}/S\right) b' \left(d_1 - d_{n1} + l_{a1}\right)\tan\alpha_1}$$

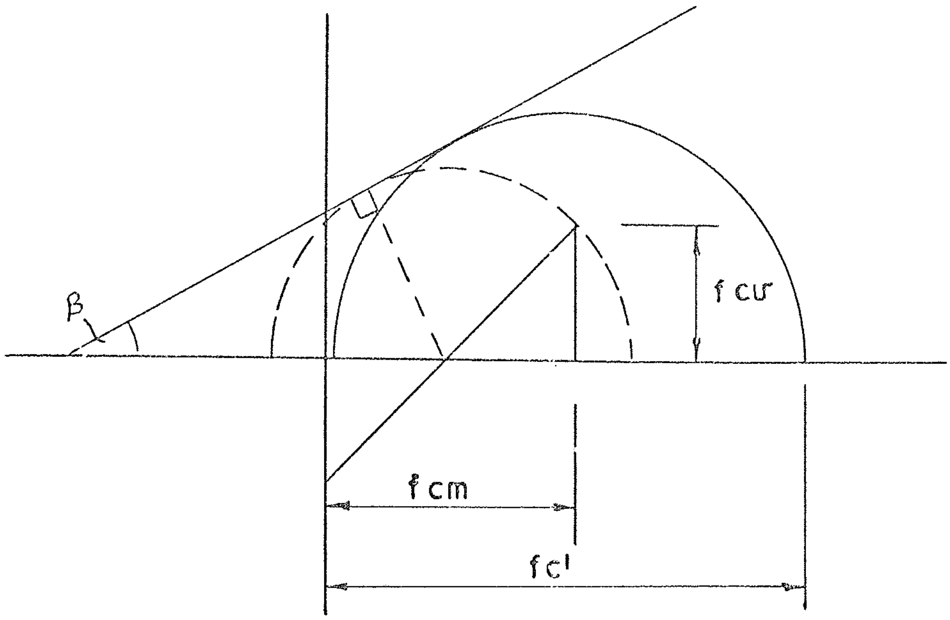


Fig. 4.2.4. Failure criteria for concrete - envelope tangent to the stress circle for uniaxial compression.

rearranging and using the approximation given in equation 4.2.4

$$\frac{1}{2}\tan^2\alpha_1 + \left(\frac{M_1}{T_1}\right)\frac{\tan\alpha_1}{(1+{}^d/_b)} - \frac{1}{2r_{1y}\,(1+{}^d/_b)} = 0$$

solving for $\tan\alpha_1$

$$\tan\alpha_1 = \frac{1}{(1+{}^d/_b)}\left\{\sqrt{\left(\frac{M_1}{T_1}\right)^2 + \frac{(1+{}^d/_b)}{r_{1y}}} - \frac{M_1}{T_1}\right\} \qquad \ldots\ldots 4.2.14$$

The expression for the lever arm factor is

$$\frac{l_{a1}}{d_1} = 1 - k_\ell \frac{d_{n1}}{d_1} \qquad \ldots\ldots 4.2.15$$

The expression for ${}^{d}{}_{n1}/d_1$ would be reduced by any compression steel that exists, but if this is taken into account theoretically it complicates the expressions.

4.3 Theory for Mode 2 Form of Failure

An alternative failure mode, first identified by Lessig[3], is when the ${}^M/_T$ values are low or zero. The compression hinge at failure occurs on the side of the member as shown in fig. 4.3.1. The theory again assumes that the steel yields and the development is similar to mode 1.

Taking moments of forces about a transverse axis through the centroid of the compression zone and perpendicular to the longitudinal axis of the beams

$$A_2 f_{2y} l_{a2} = \frac{A_s f_{sy}}{S}(b_1 - b_n)(d' + b_1 - b_n)\tan^2\alpha_2 \qquad \ldots\ldots 4.3.1$$

Taking moments of forces about a longitudinal axis through the centroid of the compression zone

$$T_2 = \frac{A_s f_{sy}}{S}(b_1 - b_n)\tan\alpha_2 d' + \frac{A_s f_{sy}}{S} d' \tan\alpha_2\, l_{a2} \qquad \ldots\ldots 4.3.2$$

These two equations are comparable with equations 4.2.1 and 4.2.2 and reduce finally to

$$\frac{T_2}{T_{u2}} = 1 \qquad \ldots\ldots 4.3.3$$

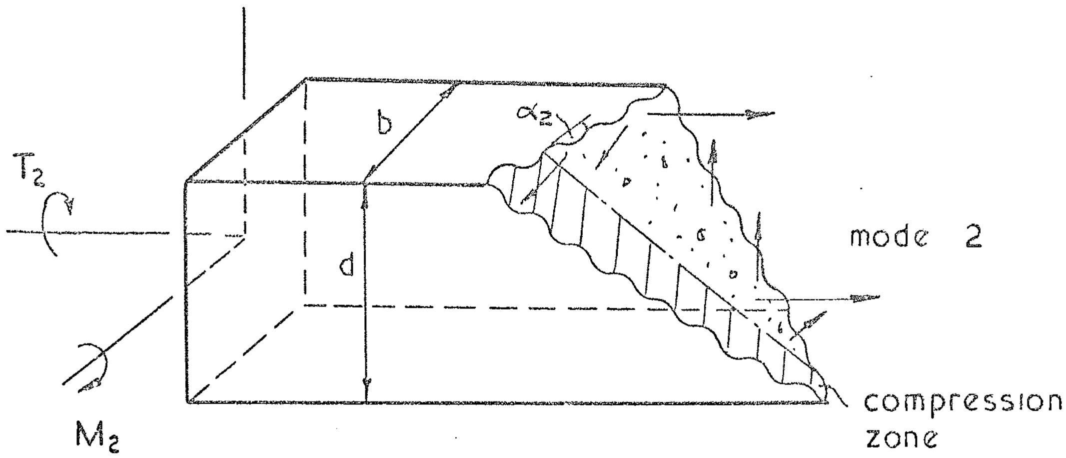


Fig 4.3.1 Mode 2 type of failure for concrete members reinforced longitudinally and transversely.

where $$T_{u2} = 2\, M_{u2} \sqrt{\frac{r_{2y}}{(1 + {}^{b}/_{d})}}$$ 4.3.4

and $$M_{u2} = A_2\, f_{2y}\, l_{a2}$$ 4.3.5

The depth of the compression zone b_n may be obtained using the Cowan[8] compressive stress criterion as with mode 1. The expression for d_{n_1} is modified by exchanging b for d (or b_1 for d_1) and the ratio ${}^{M}/_{T}$ does not appear in the equations.

$$\frac{b_n}{b_1} = \frac{p_2 f_{2y}}{(1-\sin\beta) k_{cm} f_c{}'} \left(\sqrt{1 + \left(\frac{2k_{cm}}{k_{cv}}\, r_{2y} \tan\alpha_2\right)^2} - \sin\beta \right)$$ 4.3.6

where $$\tan \alpha_2 = \frac{1}{\sqrt{r_{2y}\ (1+{}^{b}/_{d})}}$$ 4.3.7

and $$\frac{l_{a2}}{b_1} = 1 - k_\ell \frac{b_n}{b_1}$$ 4.3.8

4.4 Theory for Mode 3 Form of Failure

A final alternative mode of failure, first identified by Walsh et al [26], also occurs when the ${}^{M}/_{T}$ ratio is low or zero. This gives a lower torsional failure moment in members where the longitudinal steel at the top of the section is less than the steel at the bottom. The cross section at failure is shown in fig.4.4.1 and to develop the theory it is assumed that the steel is at yield.

Taking moments of forces about a transverse axis through the centroid of the compression zone and perpendicular to the longitudinal axis of the beam

$$M_3 = -\ A_3 f_{3y} l_{a3} + \frac{A_s f_{sy}}{S} (d_3 - d_{n3})(b' + d_3 - d_{n3}) \tan^2\alpha_3$$ 4.4.1

Taking moments of forces about a longitudinal axis through the centroid of the compression zone

$$T_3 = \frac{A_s f_{sy}}{S} (d_3 - d_{n3}) \tan\alpha_3\, b' + \frac{A_s f_{sy}}{S} b' \tan\alpha_3\, l_{a3}$$ 4.4.2

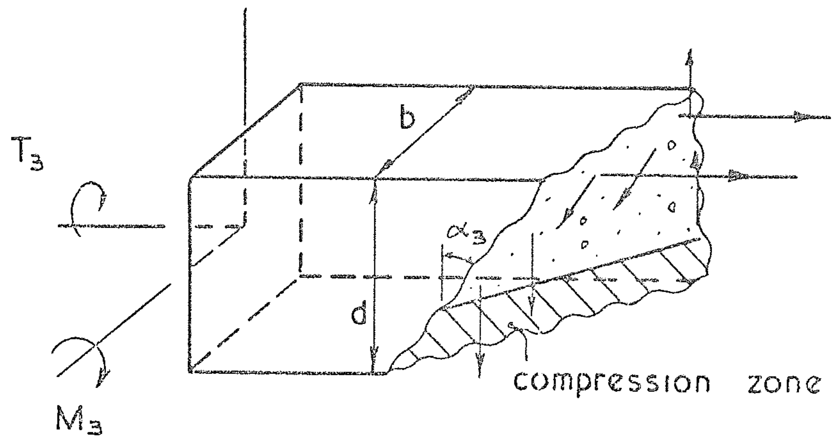


Fig. 4.4.1. Mode 3. type of failure for concrete members reinforced longitudinally and transversely

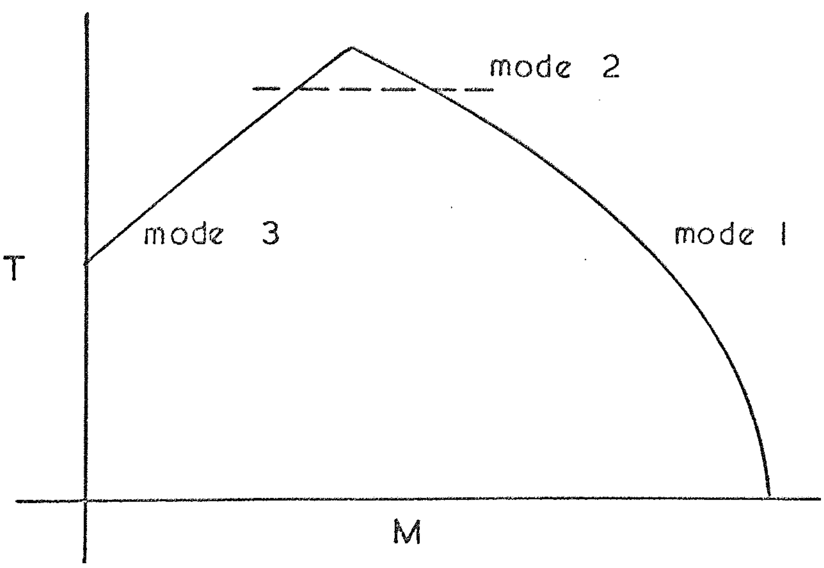


Fig. 4.4.2. Theoretical relationship between modes of failure for a member reinforced longitudinally and transversely

These equations are similar to those for mode 1 and when rearranged and simplified produce the general interaction form of

$$\left(\frac{T_3}{T_{u3}}\right)^2 - \frac{M_3}{M_{u3}} = 1 \qquad \text{..... 4.4.3}$$

where

$$T_{u3} = 2M_{u3}\sqrt{\frac{r_{3y}}{1 + {}^{d}/_{b}}} \qquad \text{..... 4.4.4}$$

and

$$M_{u3} = A_3 f_{3y}\, l_{a3} \qquad \text{..... 4.4.5}$$

or

$$T_3 = 2M_{u3}\frac{r_{3y}}{(1+{}^{d}/_{b})}\left[\sqrt{\left(\frac{M_3}{T_3}\right)^2 + \frac{1+{}^{d}/_{b}}{r_{3y}}} + \frac{M_3}{T_3}\right] \qquad \text{..... 4.4.6}$$

The values of ${}^{d_{n3}}/_{d_3}$, $\tan\alpha_3$ and ${}^{l_{a3}}/_{d_3}$ comparable to those in modes 1 and 2 and formed on the same theoretical approach are

$$\frac{d_{n3}}{d_3} = \frac{p_3 f_{3y}}{(1-\sin\beta)k_{cm}f_c'}\left\{\sqrt{1 + \left(\frac{2k_{cm}r_{3y}}{k_{cv}}\tan\alpha_3\right)^2} - \sin\beta\right\} \qquad \text{..... 4.4.7}$$

where $$\tan\alpha_3 = \frac{1}{(1+{}^{d}/_{b})}\left[\sqrt{\left(\frac{M_3}{T_3}\right)^2 + \frac{(1+{}^{d}/_{b})}{r_{3y}}} + \frac{M_3}{T_3}\right] \qquad \text{..... 4.4.8}$$

and $$\frac{l_{a3}}{d_3} = 1 - k_\ell \frac{d_{n3}}{d_3} \qquad \text{..... 4.4.9}$$

Three possible modes of failure for an under reinforced member have now been considered theoretically and a particular member which is subjected to varying ${}^{M}/_{T}$ ratios will fail in the mode which gives the lowest torsional failure moment. Fig. 4.4.2 shows the general relationship between the three failure modes. As shown in the diagram mode 2 values are generally only slightly less than the mode 1 or mode 3 value at the same point. For this reason mode 2 form of failure is often ignored.

4.5 Comparison of Theoretical and Experimental Results for Mode 1 Form of Failure

The application of the theory necessitates the adoption of values for the bending stress factor k_{cm}, the shear stress factor k_{cv}, and the angle β for the failure envelope.

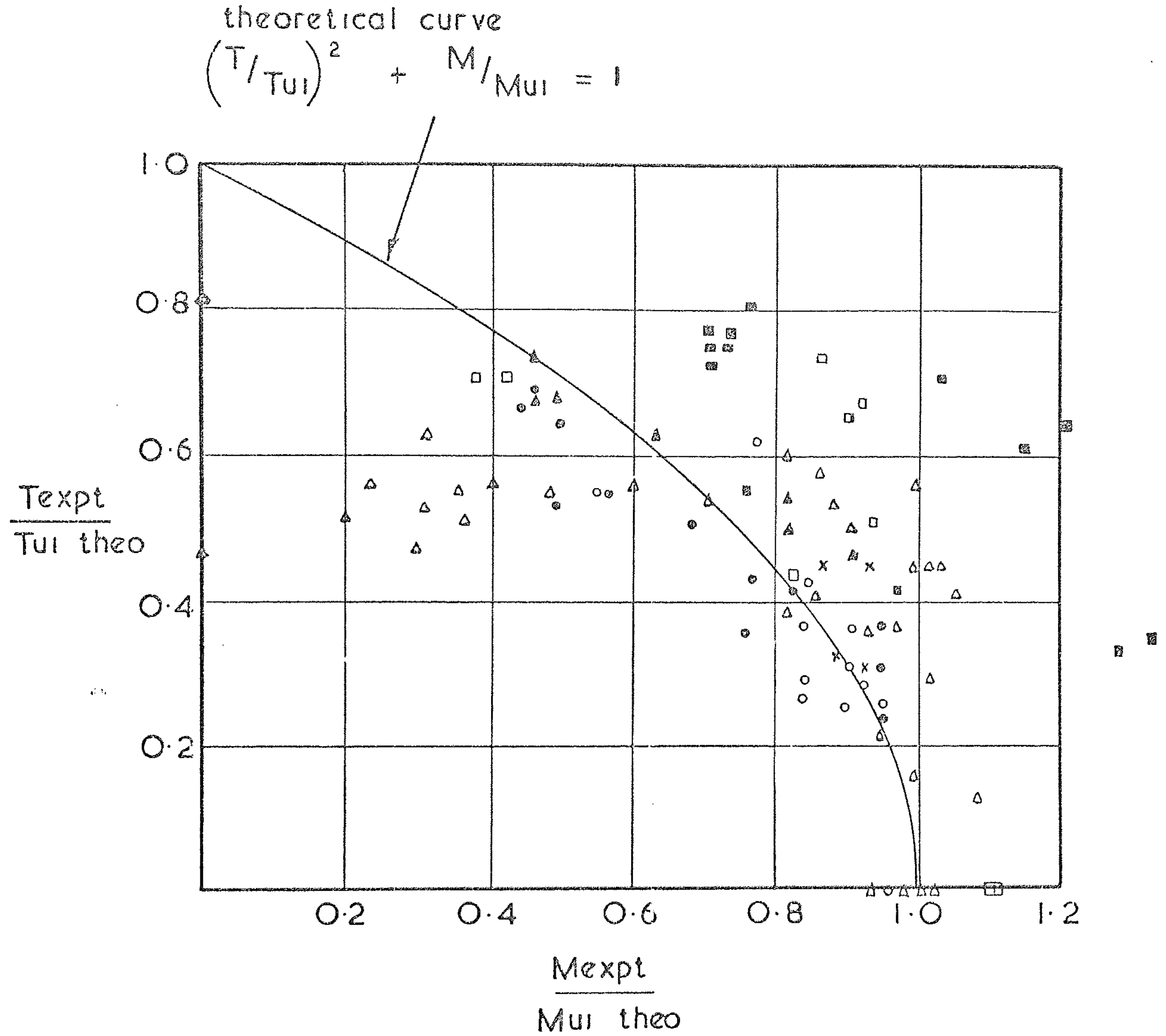
Since the member fails when the bending moment is high k_{cm} is taken as $\frac{2}{3}$. The distribution of torsional shear stress is assumed to be parabolic and k_{cv} is also taken as $\frac{2}{3}$. The value k_{ℓ} which corresponds with $k_{cm} = \frac{2}{3}$ is $k_{\ell} = \frac{3}{8}$. The value of β recommended by Cowan[8] is 37^{o}.

The experimental results of Evans and Sarkar[25], Gesind et al [24], Goode and Helmy[28], Fairburn[32], Iyengar and Rangan[15], and Jackson and Estanero[34], are tabulated in Table 4.5.1,(see appendix) and expressed graphically using the general equation in fig.4.5.1. All available results have been used even where it is suspected that not all the steel yields.

It is apparent from the graph and from examination of the tables that a theory where all the steel yields applies to very few cases. Generally for mode 1 the higher the $^{M}/_{T}$ ratio the more successful the theory. These conclusions agree with the experimental readings and comments of many investigations.

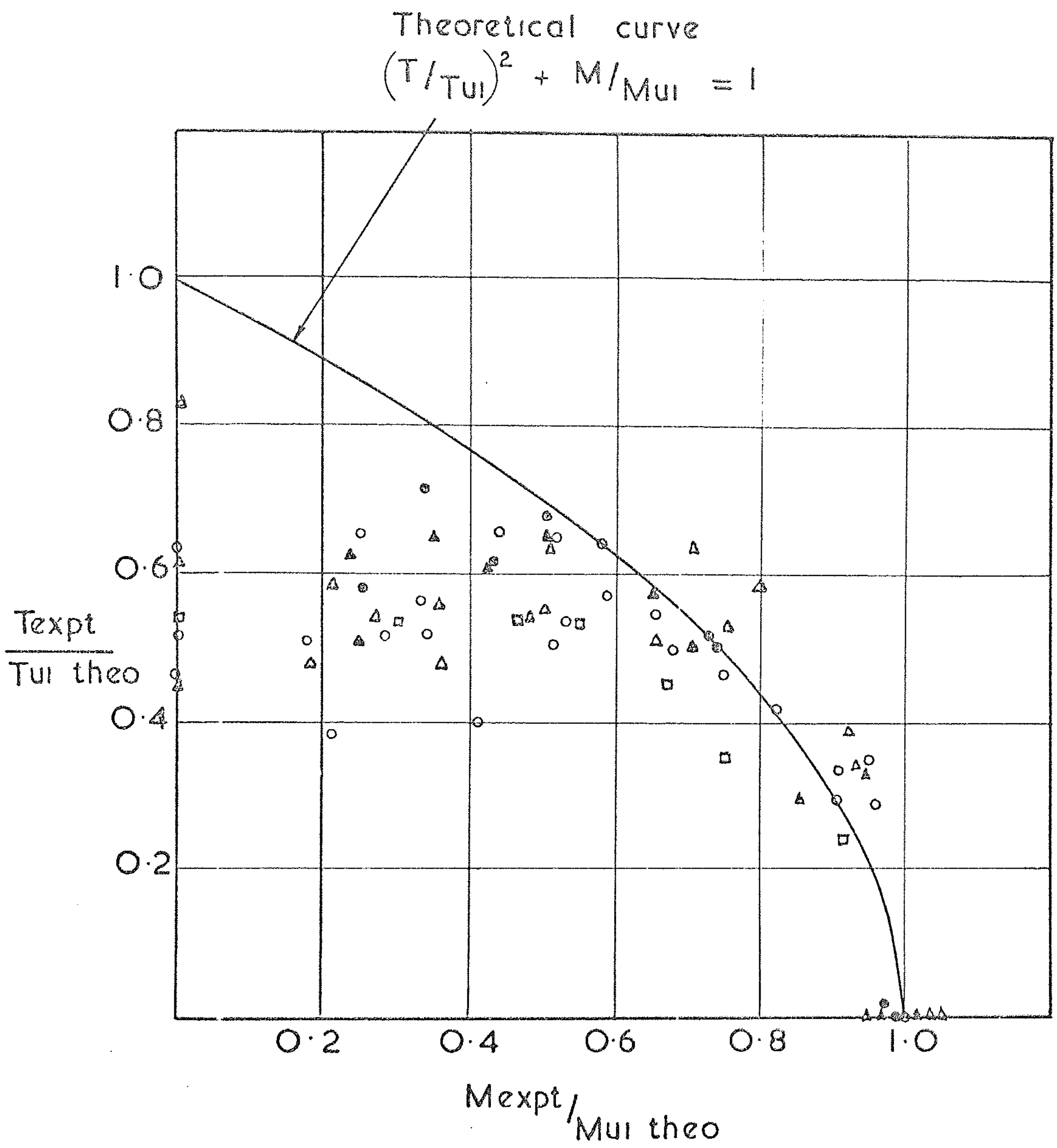
Graphs by Jackson and Estanero[34] plotting torque against angle of twist show that the graph is only bilinear in a few cases such as beams G2-3, EU-2, EU-4, and CU-2. Results for these particular members do show a good correlation with the theory, $^{Ttest}/Ttheory$ = 1.06, 1.04, 1.03 and 0.95 respectively. Other graphs of G2-8, C4-8, C4-3, and CU-4, show no horizontal line when yielding occurs, but continue to rise to form a parabola. The correlation with a yield theory is therefore not as good, $^{Ttest}/Ttheory$ = 0.84, 0.62, 0.92, and 1.00.

Detailed experimental results where partial yielding of the stirrups has occurred are indicated in table 4.5.1 (see appendix). These have been isolated by use of the theory given in Chapter 5.2 The



▲ Pandit & Warwaruk[31]
▪ Chinekor[21]
△ Iyengar & Rangan[15]
□ Goode & Helmy[28]
○ Evans & Sarkar[25]
• Gesund et al[24]
× Fairburn[32]

Fig. 4.5.1 Torsion and bending of concrete beams containing longitudinal steel and stirrups, mode I failures (including partial yield results)



Jackson & Estañero[34]

▵ A5, C4, G2.
○ B4 D4 E4
□ F3
▲ AU CU GU
● DU EU

Fig 4.5.1. Torsion and bending of concrete beams containing longitudinal steel steel and stirrups, mode I failures (including partial yield results) remaining 77 results give a mean of $^{Texpt}/_{Ttheory}$ = 1.05 with a coefficient of variation of 11.6%, as shown in the summary table 4.7.2. The selected results are also plotted graphically in fig. 4.5.2.

4.6 Comparison of Theoretical and Experimental Results for Mode 2 form of failure

The same factors have been adopted for the constants as in mode 1, i.e. $k_{cm} = \frac{2}{3}$, $k_{cv} = \frac{2}{3}$, $k_{\ell} = \frac{3}{8}$ and $\beta = 37^{o}$.

Most of these results are by Hsu[27] who conducted tests on 53 beams, with varying cross sections and strength of concrete. Most of the members had a fixed ratio of the area of transverse steel to longitudinal steel, since Hsu considered this the ideal ratio to ensure simultaneous yielding of the steel.

As with mode 1 form of failure detailed results where partial yielding is considered to occur, have been indicated in Table 4.6.1 (see appendix).

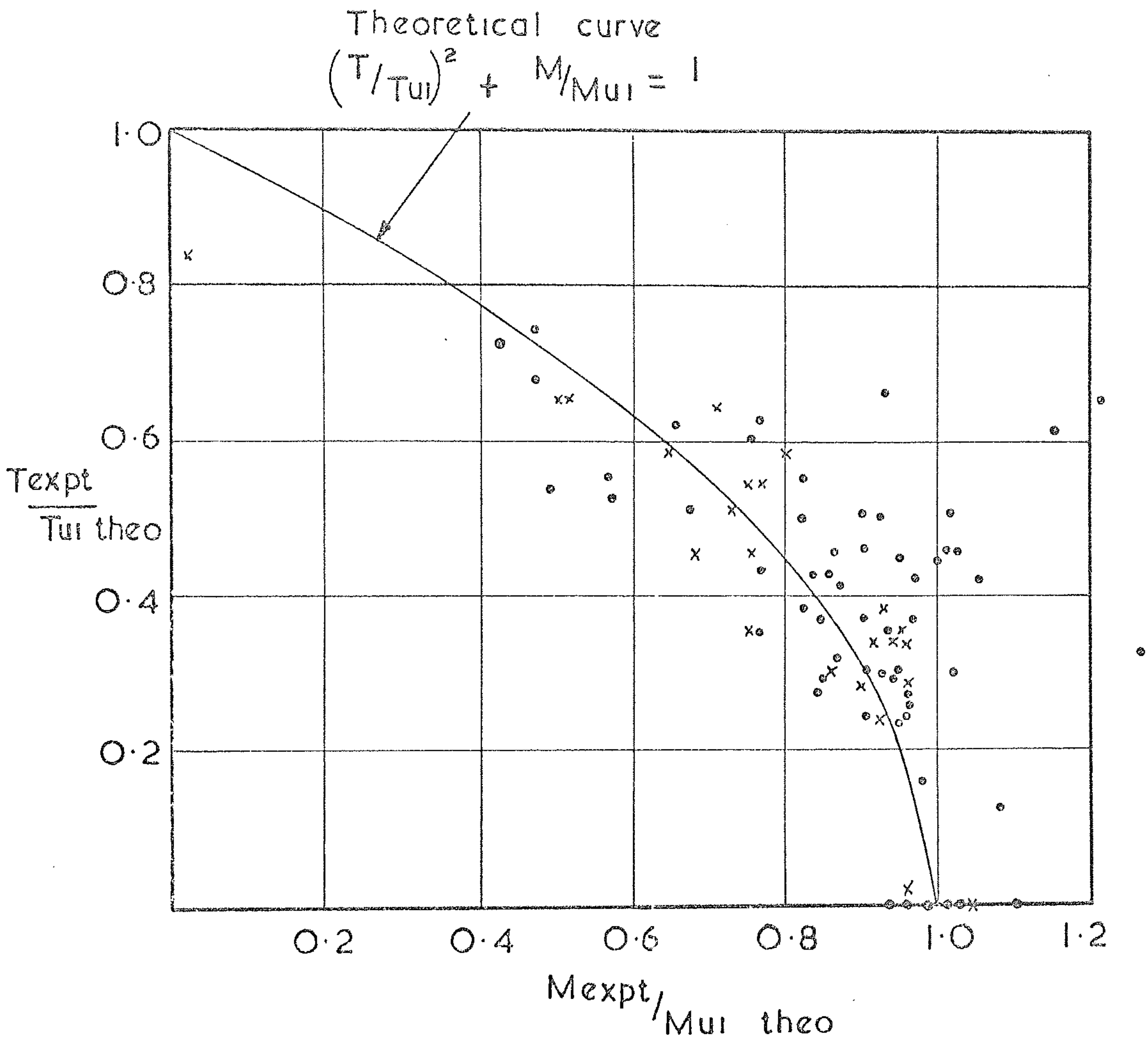
The results also include over-reinforced members and these have been indicated in accordance with chapter 5.8

If partial yielding and over reinforced members are excluded from the mode 2 failures then for 38 results the mean value of $^{Texpt}/_{Ttheory}$ = 0.97 with a coefficient of variation of 10.2%, as shown in the summary table 4.7.2.

4.7 Comparison of Theoretical and Experimental Results for Mode 3 Form of Failure

The same constant has been adopted for k_{cm}, k_{cv}, k_{ℓ} and β as in modes 1 and 2.

There are four investigations producing results for this mode but again the results for yielding are few once the partial yield cases have been removed, as shown in the detailed results of table 4.7.1. (see



x Jackson and Estañero [34]
• Iyengar and Rangan [15]
Goode and Helmy [28]
Evans and Sarkar [25]
Gesund et al [24]
Fairburn [32]
Pandit and Warwaruk [31]
Chinekov [21]

Fig 4.5.2 Torsion and bending of concrete beams containing longitudinal steel and stirrups mode 1 — $r_{iy} \geq 1/(1 + d/b + 2M/T)$

appendix). All results are plotted graphically in fig. 4.7.1.

For 6 experimental results where yielding occurs the mean value of $^{Texpt}/_{Ttheory}$ = 0.95 with a coefficient of variation of 14.8% as shown in the summary table 4.7.2.

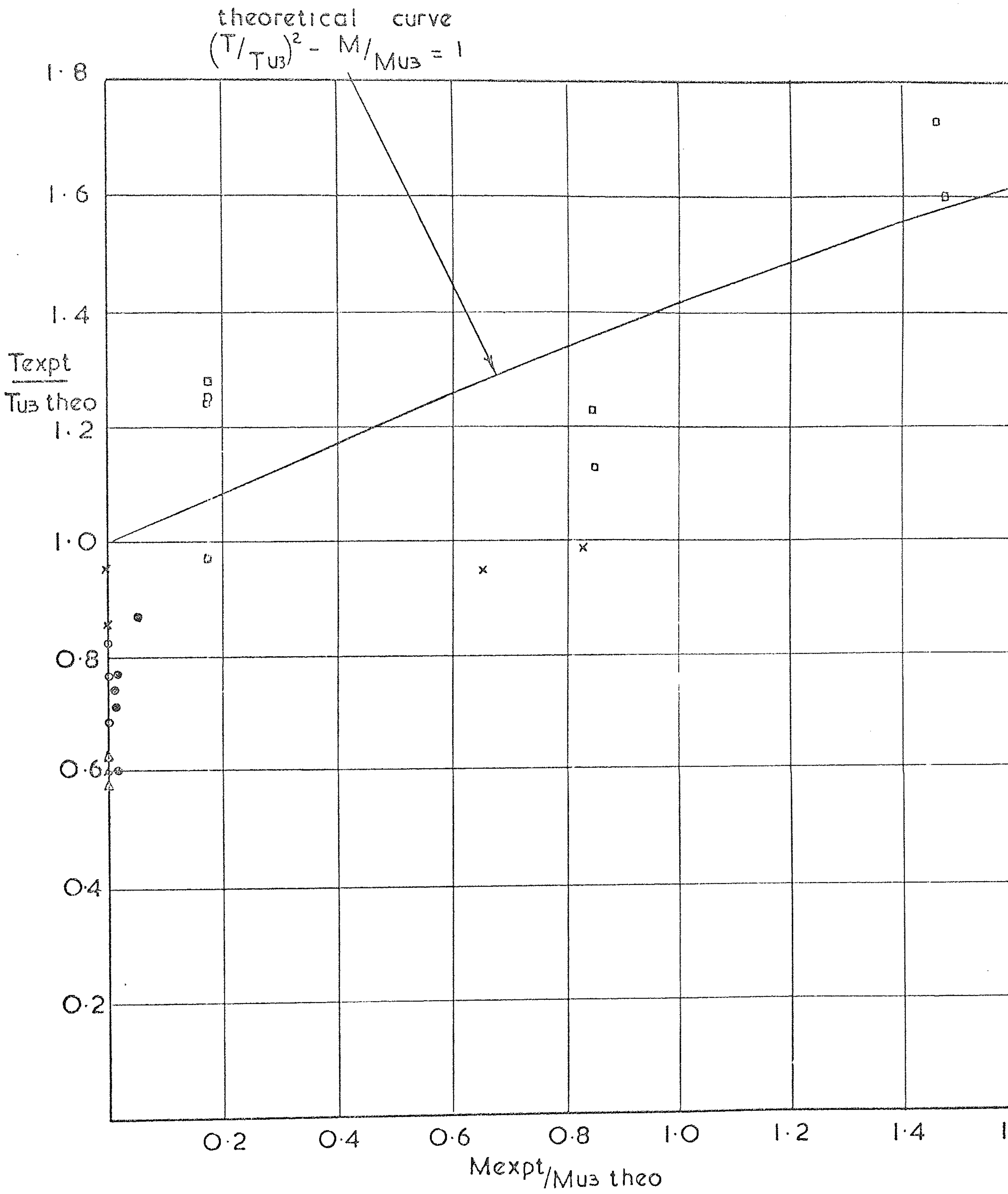
4.8 The Ratio of Transverse to Longitudinal Steel for the Optimum Value of the Torsional Resistance

It is of interest theoretically to obtain the ratio of transverse to longitudinal steel for the optimum value of the torsional resistance, assuming that all the steel yields. It will also be useful in design to have knowledge of this ideal ratio, although in practice it is not likely to be achieved. In practice the longitudinal steel is often determined from the section at the maximum bending moment and carried through the member. The transverse steel is also likely to be fixed from the section of maximum torsional moment, and carried through the member. If this method is used then the ideal ratio may be obtained at only one section, unless the designer is willing to spend a considerable time varying the spacing of the stirrups and curtailing the longitudinal reinforcement. The Australian code of practice committee is however considering recommending an optimum value of the ratio of transverse to longitudinal steel, and incorporating it into their design formula.

Walsh et al [26] considered a similar problem previously but they determined the ratio of transverse to longitudinal steel, to optimise the volume of reinforcement. They also commenced with a different expression for the torsional resistance.

The torsional resistance for mode 1 assuming all the steel yields is given in equation 4.2.8. This has been modified slightly to form

$$T_1 = \frac{2A_s f_{sy}\, b'\, l_{a1}}{s(1+{}^{d}/_{b})} \left(\sqrt{\left(\frac{M_1}{T_1}\right)^2 + \frac{(1+{}^{d}/_{b})}{r_{1y}}} - \frac{M_1}{T_1} \right) \qquad \ldots\ldots 4.8.1$$



□ Goode and Helmy [28]
△ Iyengar and Rangan [15]
○ Evans and Sarkar [25]
× Pandit and Warwaruk [31]
● Jackson and Estañero [34]

Fig. 4.7.1 Torsion and bending of concrete beams containing longitudinal steel and stirrups mode 3 failures (including partial yield results)

Table 4.7.2 SUMMARY TABLE OF ALL YIELD RESULTS

$$r > \frac{1}{1 + {}^{d}/_{b} \pm {}^{2M}/_{T}}$$

Investigator	Number of Beams	Mode of Failure	Mean Texpt / Ttheo	% Coeff. of Variation
Evans & Sarkar [25]	12	1	0.99	6.5
Gesund et al [24]	9	1	0.96	7.9
Goode & Helmy [28]	3	1	1.12	14.4
Fairburn [32]	4	1	1.04	6.2
Iyengar & Rangan [15]	15	1	1.10	6.7
Chinekov [21]	6	1	1.31	13.0
Pandit & Warawaruk [31]	7	1	1.03	5.3
Jackson & Estanero [34]	21	1	1.00	6.7
Total for mode 1	77	1	1.05	11.6

Table 4.7.2 SUMMARY TABLE OF ALL YIELD RESULTS (continued)

$$r \gtrless \frac{1}{1 + {}^{d}/_{b} \pm {}^{2M}/_{T}}$$

Investigator	Number of Beams	Mode of Failure	Mean $\frac{T_{expt}}{T_{theo}}$	% Coeff. of Variation
Hsu[27]	35	2	0.98	9.7
Ernst[42]	2	2	0.82	–
Pandit & Warwaruk[31]	1	2	0.89	–
Total for mode 2	38	2	0.97	10.2
Goode & Helmy[28]	4	3	0.98	17.0
Pandit & Warwaruk[31]	1	3	0.96	–
Jackson & Estanero[34]	1	3	0.84	–
Total for mode 3	6	3	0.95	14.8
Grand Total	121	1/2/3	1.02	11.8

The ratio of the total volume of steel (in the form of closed rectangular stirrups), to the volume of concrete is

$$p_t = \frac{2A_s b'}{bd\, s} \left[\left(1 + {}^{d'}/_{b'}\right) + {}^{F_1}/_{r_1} \right] \quad \ldots\ldots 4.8.2$$

where $$F_1 = \frac{1}{2}\left(1 + {}^{A_3}/_{A_1}\right) \quad \ldots\ldots 4.8.3$$

and $$r_1 = \frac{A_s b'}{S\, A_1} \quad \ldots\ldots 4.8.4$$

r_1 is not to be confused with

$$r_{1y} = \frac{A_s f_{sy}\, b'}{S\, A_1\, f_{1y}}$$

which contains the yield stresses.

Equation 4.8.2 is combined with equation 4.8.1 to eliminate A_s and to express T_1 in terms of r_1

$$T_1 = \frac{l_{a1}\, f_{sy}\, p_t\, bd}{(1+{}^{d}/_{b})\left[(1+{}^{d}/_{b}) + {}^{F_1}/_{r_1}\right]} \left(\sqrt{\left(\frac{M_1}{T_1}\right)^2 + \frac{(1+{}^{d}/_{b})}{r_1\left({}^{f_{sy}}/f_{1y}\right)}} - \frac{M_1}{T_1} \right) \quad \ldots\ldots 4.8.5$$

Providing the stirrups are closed, rectangular, single, and of the same ratio ${}^{d}/_{b}$ as ${}^{d'}/_{b'}$, then it is assumed in equation 4.8.5. that

$$(1 + {}^{d}/_{b}) \simeq (1 + {}^{d'}/_{b'})$$

The value of r_1 for which T_1 is an optimum may be obtained by differentiating T_1 with respect to r_1 and equating to zero.

This gives

$$\frac{1}{\left[(1+{}^{d}/_{b}) + {}^{F_1}/_{r_1}\right]} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{\left(\frac{M_1}{T_1}\right)^2 + \frac{(1+{}^{d}/_{b})}{r_1\left({}^{f_{sy}}/_{f_{1y}}\right)}}} \cdot \frac{-(1+{}^{d}/_{b})}{(r_1)^2\left({}^{f_{sy}}/_{f_{1y}}\right)} +$$

$$\left(\sqrt{\left(\frac{M_1}{T_1}\right)^2 + \frac{(1+{}^{d}/_{b})}{r_1\left({}^{f_{sy}}/_{f_{1y}}\right)}} - \frac{M_1}{T_1} \right) \frac{-\left(-{}^{F_1}/_{(r_1)^2}\right)}{\left[\left(1+{}^{d}/_{b}\right) + {}^{F_1}/_{r_1}\right]^2} = 0$$

rearranging

$$-\frac{(1+{}^d/_b)^2}{2\left({}^{f_{sy}}/f_{1y}\right)} + F_1\left(\frac{M_1}{T_1}\right)^2 + F_1\frac{(1+{}^d/_b)}{2r_1\left({}^{f_{sy}}/f_{1y}\right)} = F_1\left(\frac{M_1}{T_1}\right)\sqrt{\left(\frac{M_1}{T_1}\right)^2 + \frac{(1+{}^d/_b)}{r_1\left({}^{f_{sy}}/f_{1y}\right)}}$$

squaring both sides

$$\frac{1}{4}\frac{(1+{}^d/_b)^4}{\left({}^{f_{sy}}/f_{1y}\right)^2} - F_1\left(\frac{M_1}{T_1}\right)^2\frac{(1+{}^d/_b)^2}{\left({}^{f_{sy}}/f_{1y}\right)} - \frac{F_1}{2r_1}\frac{(1+{}^d/_b)^3}{\left({}^{f_{sy}}/f_{1y}\right)^2} + \left(\frac{M_1}{T_1}\right)^4 F_1^2$$

$$+\left(\frac{M_1}{T_1}\right)^2\frac{\left(1+{}^d/_b\right)F_1^2}{r_1\left({}^{f_{sy}}/f_{1y}\right)} + \frac{\left(1+{}^d/_b\right)^2 F_1^2}{4(r_1)^2\left({}^{f_{sy}}/f_{1y}\right)} =$$

$$\left(\frac{M_1}{T_1}\right)^4 F_1^2 + \left(\frac{M_1}{T_1}\right)^2\frac{(1+{}^d/_b)F_1^2}{r_1\left({}^{f_{sy}}/f_{1y}\right)}$$

Collecting terms and rearranging to form a quadratic equation in r_1

$$\left[\left(1+{}^d/_b\right)^2 + 4F_1\left(\frac{M_1}{T_1}\right)^2\left({}^{f_{sy}}/f_{1y}\right)\right](r_1)^2 - 2F_1\left(1+{}^d/_b\right)r_1 + F_1^2 = 0 \qquad \text{..... 4.8.6}$$

The value of r_1 that satisfies this equation for mode 1 form of failure is

$$r_1' = \frac{F_1}{1+{}^d/_b + 2\left(\frac{M_1}{T_1}\right)\sqrt{\frac{f_{sy}}{f_{1y}}}\sqrt{F_1}} \qquad \text{..... 4.8.7}$$

It should be pointed out that the same value of r_1 would be obtained for an optimum value of the volume of steel, and therefore is of particular importance.

A special case of the above expression which is likely to occur often in practice is where $A_1 = A_3$ and $f_{sy} = f_{1y}$, in which case

$$r_1' = \frac{1}{1+{}^d/_b + 2\left(\frac{M_1}{T_1}\right)} \qquad \text{..... 4.8.8}$$

The comparable expression to equation 4.8.7 obtained by Walsh et al [26] by optimising the volume of steel is, with some rearrangement,

$$r'_1 = \frac{F_1}{1 + {}^{d}/_{b} + 2\left(\frac{M_1}{T_1}\right)\sqrt{\frac{(1+{}^{d}/_{b})}{(1+{}^{2d}/_{b})}}\sqrt{\frac{f_{sy}}{f_{1y}}}\sqrt{F_1}} \qquad \ldots\ldots 4.8.9$$

An approximate design expression recommended by Walsh et al [26] is

$$r'_1 = \frac{0.25\left({}^{f_{1y}}/_{f_{sy}}\right)}{1 + \dfrac{\left({}^{M_1}/T_1\right)}{\sqrt{1 + {}^{2d}/_{b}}}} \qquad \ldots\ldots 4.8.10$$

The Russian code of practice (Ni Tu 123-55 code) incorporates an alternative expression for r'_1 to ensure simultaneous yielding of the transverse and longitudinal steel. This is interpreted to mean optimising the value of T_1

$$r'_1 = \frac{0.8\left({}^{f_{1y}}/_{f_{sy}}\right)}{1 + \dfrac{2\left({}^{M_1}/T_1\right)}{\sqrt{1 + {}^{2d}/_{b}}}} \qquad \ldots\ldots 4.8.11$$

The above expression incorporates the assumption that $b' \simeq 0.8b$. There is a considerable difference between equation 4.8.11 and 4.8.7 which requires further explanation. According to Lessig [20] this expression has been derived by optimising T_1 as previously for a fixed volume of reinforcement. The volume of reinforcement considered is however on the tension face of the member only. This is in contrast to the approach of Walsh et al [26] and the author where the volume of reinforcement considered is the total volume on all four faces. To confirm this the author repeated the process from equation 4.8.1 through 4.8.7 using the Lessig assumption. The resulting expression is

$$r'_1 = \frac{1}{1 + 2\left(\frac{M_1}{T_1}\right)\sqrt{\dfrac{\left({}^{f_{sy}}/_{f_{1y}}\right)}{(1 + {}^{d}/_{b})}}} \qquad \ldots\ldots 4.8.12$$

This expression gives approximately the same solution as 4.8.11.

The same approach used for optimising T_2 for mode 2 form of failure yields

$$r_2' = \frac{A_s d'}{S\,A_2} = \frac{1}{1 + {}^{b}/_{d}} \qquad \text{..... 4.8.13}$$

or alternatively

$$r_1' = r_2' \times \frac{b'}{d'} = \frac{1}{1 + {}^{d}/_{b}} \qquad \text{..... 4.8.14}$$

A factor to allow for unequal areas of longitudinal steel on opposite faces is not included since it does not generally occur. This case was not considered separately by Walsh et al since they considered a combination of modes 1 and 3. Equation 4.8.13 however does agree with the German and Australian codes of practice which recommend that

$$m' = \frac{A_2 S}{A_s (b'+d')} = 1 \qquad \text{..... 4.8.15}$$

rearranging and putting in terms of r_2

$$r_2' = \frac{1}{1 + {}^{b'}/_{d'}} \simeq \frac{1}{1 + {}^{b}/_{d}} \qquad \text{..... 4.8.16}$$

Hsu[27] also recommends $m' = 1$ to ensure plastic behaviour in the transverse and longitudinal steel at failure.

The same approach used for mode 3 produces the value of

$$r_3' = \frac{F_3}{1 + {}^{d}/_{b} - 2\left({}^{M_3}/_{T_3}\right)\sqrt{{}^{f_{sy}}/_{f_{3y}}}\sqrt{F_3}} \qquad \text{..... 4.8.17}$$

where $F_3 = \frac{1}{2}\left(1 + {}^{A_l}/_{A_3}\right)$.

There is no comparable expression in other literature.

The theoretical variation of r_1' with the ${}^{M_1}/_{T_1}$ ratio is shown graphically in Fig. 4.8.1. To make the comparison it is necessary to adopt a value of the ${}^{d}/_{b}$ ratio, to assume all steel yield stresses are equal and to assume the longitudinal steel is evenly distributed.

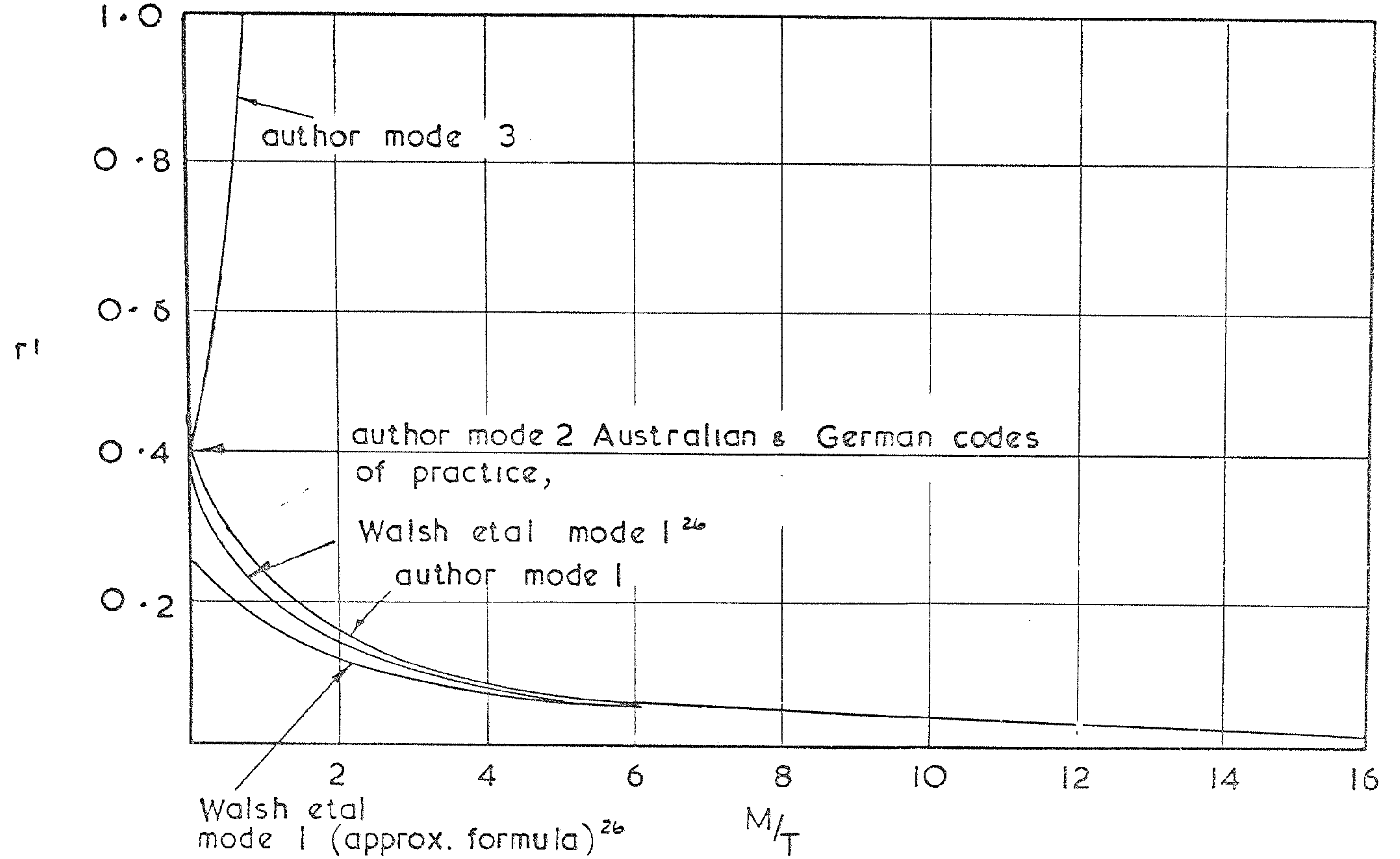


Fig. 4.8.1. Values of r' for the optimum value of the torsional resistance ($d/b = 1\cdot5$, $f_{sy_1} = f_{1y_1} = f_{3y}$, $A_1 = A_2 = A_3$)

The latter assumption of course excludes mode 3 failure, but it has been shown for comparison purposes, and the lines for r_1' and r_3' intersect when $^M/_T$ is zero.

When $A_1 \neq A_3$ then mode 2 is eliminated and the r_1' and r_3' lines intersect where $^M/_T > 0$.

To verify the values of r_1' experimentally requires that members are tested at each $^M/_T$ ratio, with varying r_1 values, and fixed volume of reinforcement. Hsu[27] produced a few results for mode 2 where the $^M/_T$ is zero. The results are shown in table 4.8.1 and plotted in fig. 4.8.2. They are not sufficient to be convincing and show irregularity due to variation of the volume of reinforcement and strength of the materials. The experimental results show the same trend as the theoretical line on the graph. The theoretical line was constructed using p_{tb} = 3.2%, $f_{sy} = f_y$ = 47,000 lbf/in^2 (324 N/mm^2), f_c' = 4000 lbf/in^2 (27.56N/mm^2), b = 10" (254 mm), d = 15" (381 mm). Generally however the theory gives higher values of the torsional resistance than the experimental values, since from experimental observations some of the steel did not yield. This will be considered in Chapter 5.

No direct experimental evidence for the value of r_1' is available for mode 1, but Lyalin[22] from experimental observations of yielding constructed a table relating r_1' to the $^M/_T$ ratios. These values were not continuous and r_1' varied for a given $^M/_{T_1}$ ratio. The values are plotted in fig. 4.8.3 and compared with theoretical results. The theoretical lines of Walsh et al [36] and the author agree reasonably well. The design expression recommended by Walsh et al [26] produces considerable error at low $^M/_T$ ratios, and would be even more inaccurate if $A_3 \neq A_1$. The Russian code of practice gives r_1' values which are too high and will only give acceptable agreement at $^M/_T$ ratios > 4. This is due to optimising the torsional resistance of the section based on the

Table 4.8.1 EXPERIMENTAL RESULTS

b = 10" (254mm), d = 15" (381mm), b' = 8.5" (215.9), d' = 13.5" (342.9mm)

Investigator	Beam	T_{expt} kipf in (kNm)	f_c' lbf/in^2 (N/mm^2)	Ratio r_1	f_{sy} lbf/in^2 (N/mm^2)	P_{tb} %
Hsu[27]	B10	304.0	3840	.078	49,600	3.21
	M3	388.0	3880	.252	47,300	2.67
	M4	439.0	3850	.257	47,400	3.53
	B4	419.0	4430	.382	46,900	3.21
	B8	288.0	3880	1.889	46,400	3.14

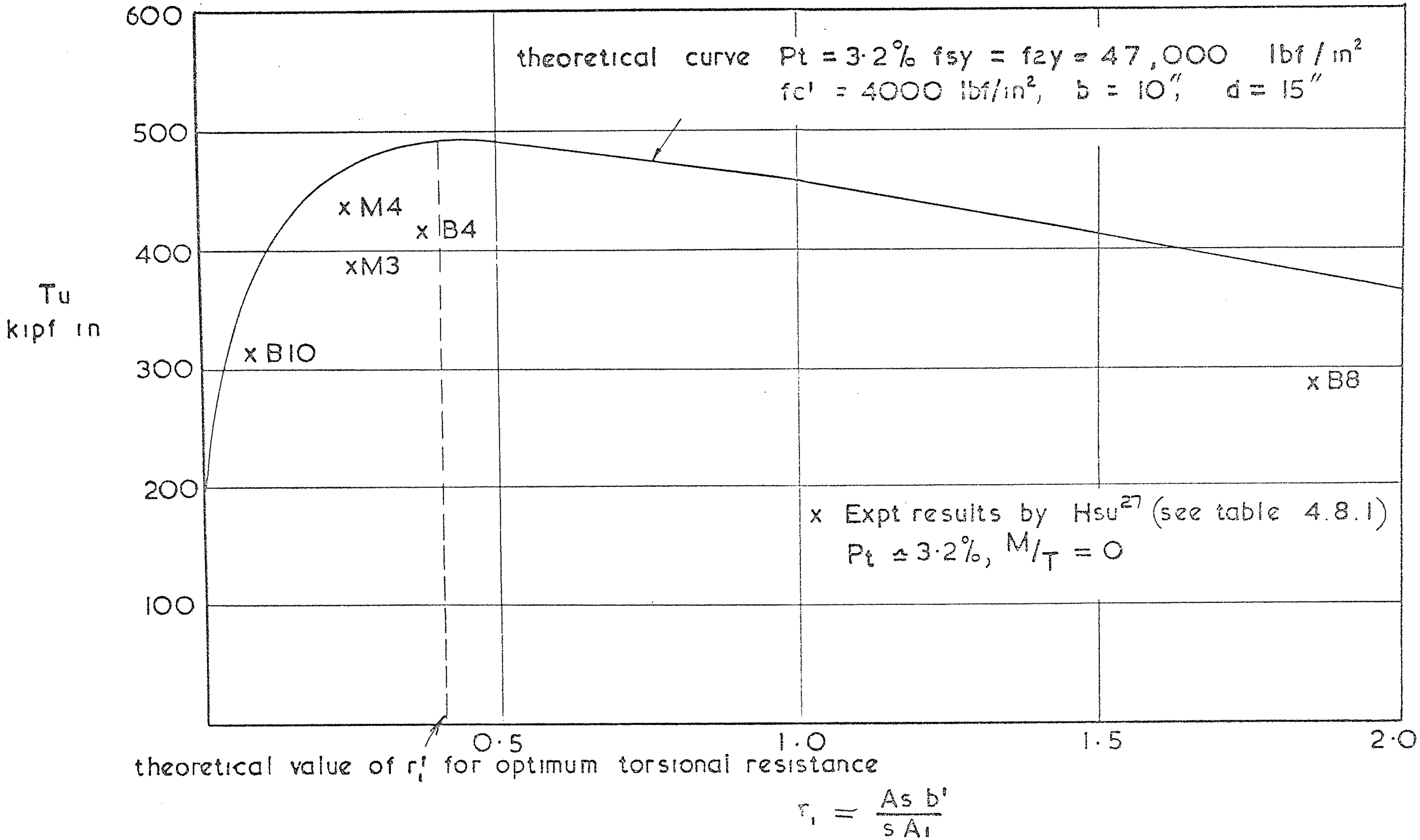


Fig 4.8.2. The value of the ratio of the transverse to the longitudinal steel for the optimum torsional resistance

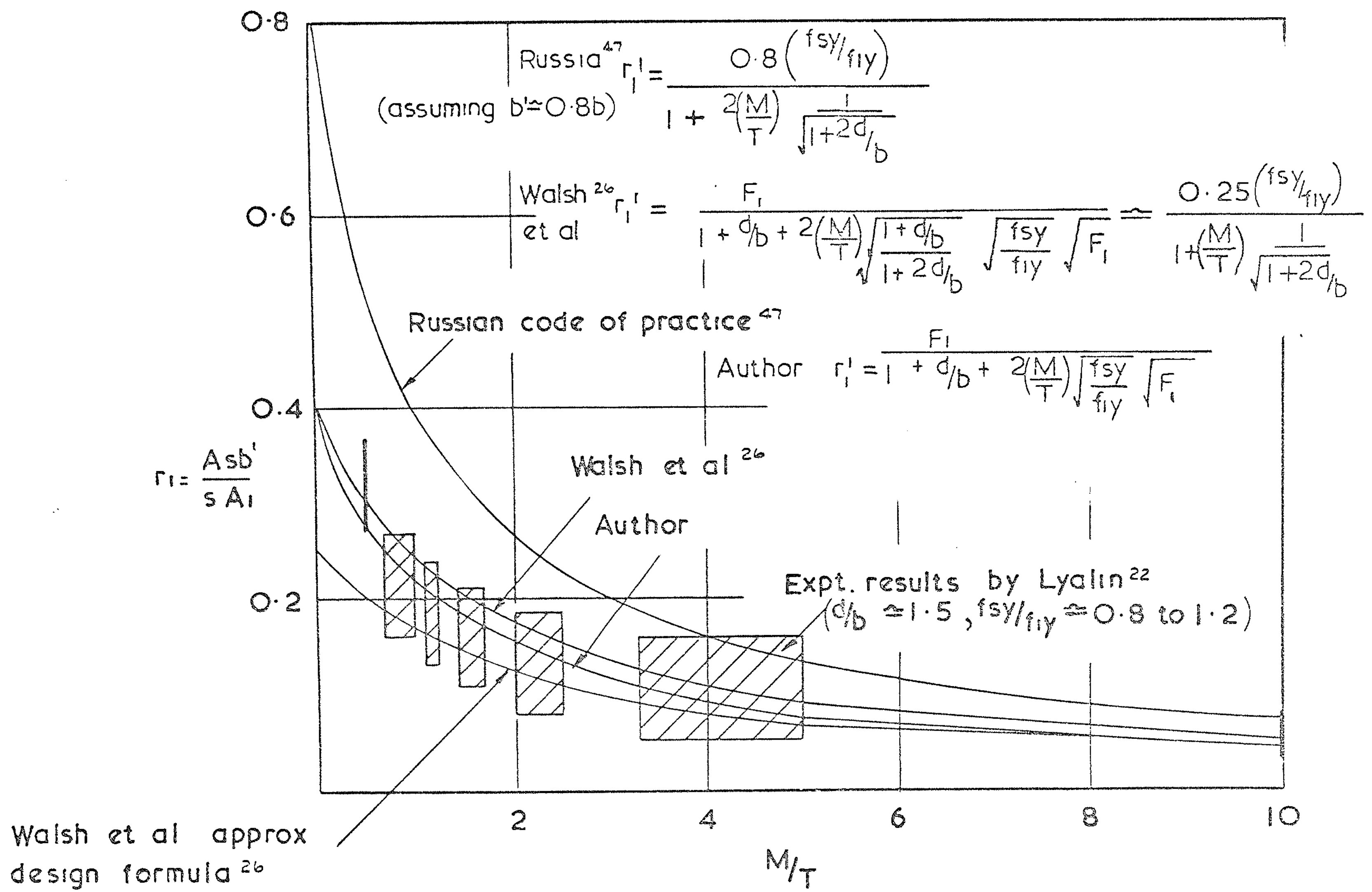


Fig. 4.8.3. Compraison of theoretical and experimental values of r_1' for the optimum value of the torsional resistance

volume of reinforcement on one face as explained previously. Further work is required here, since the size of section and strength of the materials varied in the tests by Lyalin[22].

CHAPTER 5 - The Ultimate Strength of Partially Over-Reinforced and Over-Reinforced Concrete Members Reinforced with Longitudinal and Transverse Steel.

5.1 Introduction

There are many cases in experimental work of longitudinal and/ or transverse steel not reaching yield stress at failure. The inaccuracy in the preceding yield theory given in chapter 4 also shows that the assumption that all steel yields is not correct. Hsu[27] has shown clearly in tests in pure torsion that three cases of non yielding of the steel need to be considered.

(a) where stirrups yield and the longitudinal steel is in the elastic stage

(b) where longitudinal steel yields and the stirrups are in the elastic stage

(c) where longitudinal and transverse steel are both still in the elastic stage at failure.

5.2 Limit for Yielding of the Longitudinal and Transverse Steel Simultaneously

Attempts have been made by various investigators, and in codes of practice, to place empirical limits on the ratio of transverse to longitudinal steel. Walsh et al [26] compared the accuracy of their yield theory with the value of $r_{,y}$ for mode 1. Large discrepancies occurred where

$$r_{,y} \leqslant \frac{1}{4\left(1 + \dfrac{{}^{M}/_{T}}{1 + {}^{2d}/_{b}}\right)} \qquad 5.2.1.$$

This value of $r_{,y}$ was obtained by optimising the theoretical volume of reinforcement of a member for a fixed torsional resistance. This is also the value of r_{1y} required for the optimum value of the torsional resistance of the member. Values of r_{1y} below this critical value lead to conservative theoretical estimations of the torsional strength of the member, according to the results produced by Walsh et al.

The comparable values of r_{1y} by Lessig[20], the German and Australian codes of practice and the author, for the optimum torsional resistance, have been considered previously in Chapter 4.8. These values of r_{1y} however have not been related directly to the accuracy of the all yield theory.

Lessig[20] placed a restraint on the angle of the failure plane in the compression zone. In terms of the author's theory this means that the crack angle on the side of the beam should be equal to or less than 45°.

For mode 1

$$\tan \alpha_1 \leqslant 1$$

Using the value of $\tan \alpha_1$ developed in previous theory (equation 4.2.14)

$$\frac{1}{(1 + {}^{d}/_{b})} \left[\sqrt{\left(\frac{M_1}{T_1}\right)^2 + \frac{(1 + {}^{d}/_{b})}{r_{1y}}} - \frac{M_1}{T_1} \right] \leqslant 1$$

rearranging and squaring both sides of the equation

$$\left(\frac{M_1}{T_1}\right)^2 + \frac{1 + {}^{d}/_{b}}{r_{1y}} \leqslant \left(1 + {}^{d}/_{b} + {}^{M_1}/_{T_1}\right)^2$$

rearranging

$$r'_{1y} \geqslant \frac{1}{1 + {}^{d}/_{b} + 2\,{}^{M_1}/_{T_1}} \qquad 5.2.2$$

This expression is very similar to the full yield expression developed for the optimum torsional resistance of the section (see equation 4.8.7). The values will be identical if $f_{sy} = f_{1y}$ and $A_1 = A_3$ (see equation 4.8.8).

The comparable expression for mode 2 is

$$r'_{2y} \leq \frac{1}{1 + {}^{b}/_{d}} \qquad 5.2.3$$

and for mode 3 is

$$r'_{3y} \leq \frac{1}{1 + {}^{d}/_{b} - 2\,{}^{M_3}/_{T_2}} \qquad 5.2.4$$

(note: these critical values of r_{1y}, r_{2y} and r_{3y} are denoted by r'_{1y}, r'_{2y} and r'_{3y}).

To check whether the limits given in equation 5.2.2, 5.2.3 and 5.2.4 are acceptable, graphs plotting ${}^{T_{test}}/T_{theory}$ against ${}^{r_{1y}}/{r'_{1y}}$ are shown in fig. 5.2.1. It is apparent that when ${}^{r_{1y}}/{r'_{1y}} < 1$ that the accuracy of the yield theory is affected. Fig. 5.2.2. is the comparable graph for mode 3 failure. In this case accuracy of the all yield theory is affected when ${}^{r_{3y}}/{r'_{3y}} < 1$. This is the basis for the exclusion of the partial yield results in Chapter 4.

5.3 Partial Yield Theory A

The results excluded from the yield theory i.e. where $r_{1y}/r'_{1y} < 1$ are those that are likely to occur in practice and therefore require a solution. If the crack angle is fixed at 45°, then the angle is not governed by the simultaneous yielding of the longitudinal steel and transverse steel. Since the area of transverse steel is low then this steel will most likely be at yield, while the longitudinal steel is in the elastic stage.

Taking moments of forces about a longitudinal axis through the centroid of the compression zone assumed to be at the side of the cross section

$$T_1 = \frac{A_s f_{sy}}{S} b' \tan\alpha_1 (\ell_{a1} + d_1 - d_{n1}) \qquad 5.3.1.$$

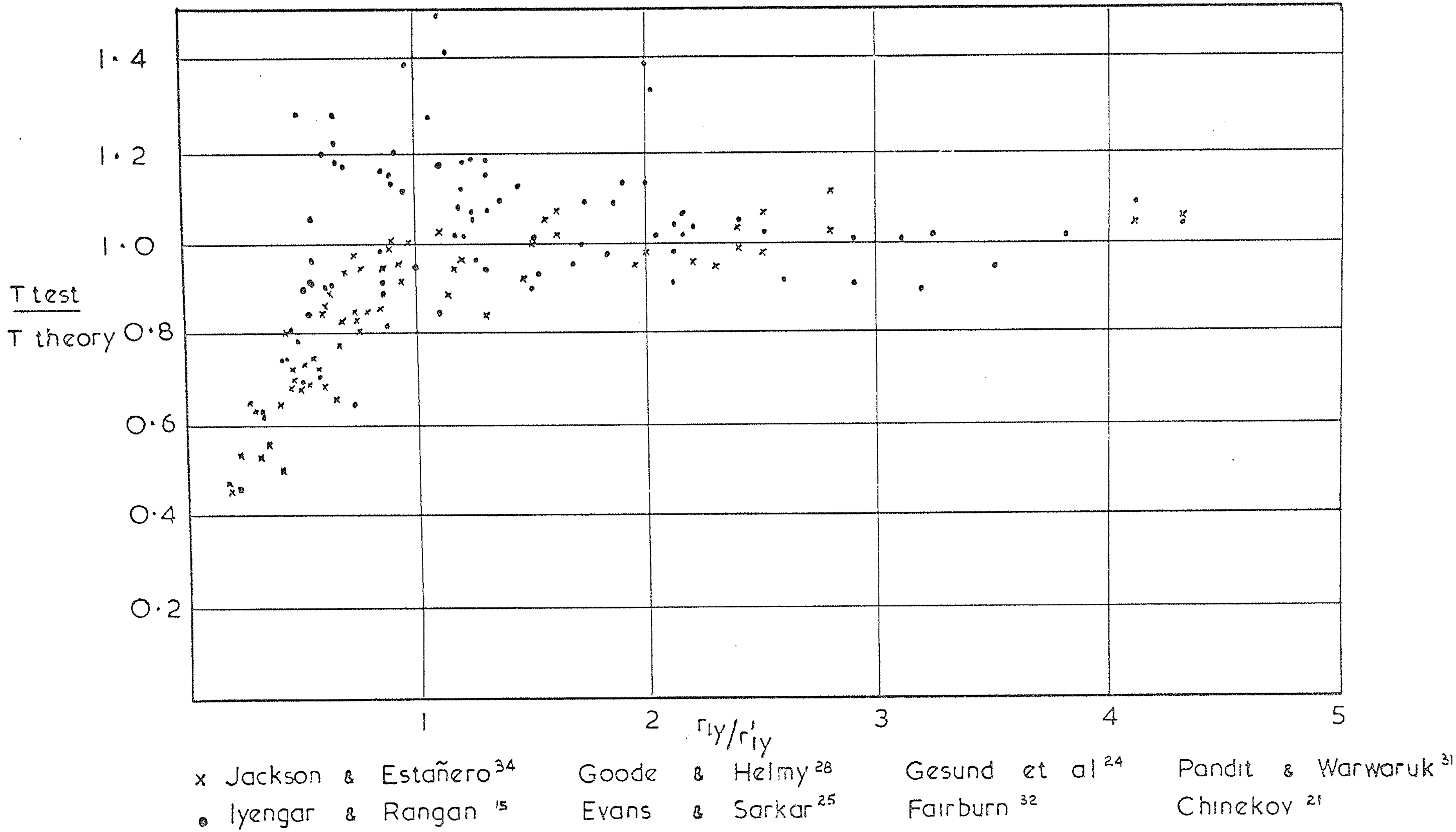
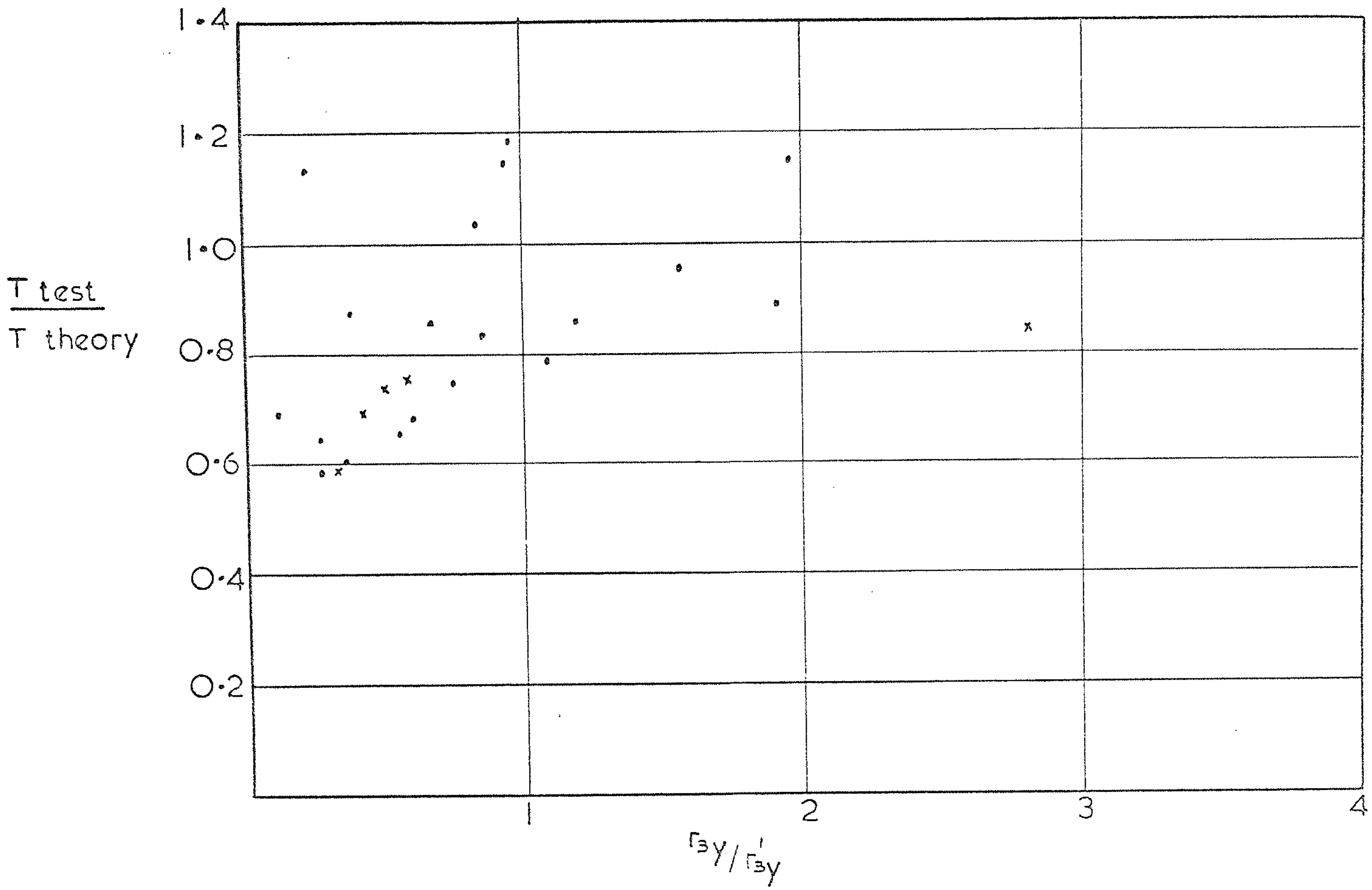


Fig. 5.2.1 Limit for yielding mode I $r_{Iy}/r'_{Iy} \geq 1$



x Jackson & Estañero[34]   Iyengar & Rangan[15]   Pandit & Warwaruk[31]

• Goode & Helmy[28]   Evans & Sarkar[25]

Fig. 5.2.2. Limit for yielding mode 3 $r_{3y}/r'_{3y} \geq 1$

Since $\alpha_1 = 45^\circ$, $\tan\alpha_1 = 1$ and to avoid developing a complicated expression for the lever arm ℓ_a it is assumed that $\ell_{a1} \simeq d_1 - d_{n1} \simeq d'$

Hence
$$T_1 = \frac{2A_s\, f_{sy}\, b'd'}{S} \qquad 5.3.2$$

This expression is the same for modes 1, 2 and 3, which is a further advantage. It is also the same as that produced by Rausch[18] in 1929 for experiments in pure torsion.

To test the accuracy of equation 5.3.2 experimental results where ${}^{r_{1y}}/_{r'_{1y}} < 1$ are tabulated in tables 5.3.1, 5.3.2 and 5.3.3 (see appendix) and shown graphically in fig.5.3.1. The 120 results are summarised in table 5.3.4 and produce a mean value of ${}^{T_{expt}}/_{T_{theory}} = 1.10$ with a coefficient of variation of 22.9%. The result is conservative and requires further work to improve its accuracy. Equation 5.3.2 is however interesting because of a reasonably rational basis and its simplicity.

5.4 Partial Yield Theory B - Draft British Unified Code of Practice for Reinforced Concrete April 1972.

The general form of the expression for partial yielding of the stirrups as derived in Chapter 5.3 is important. Swan[33] in 1970 suggested from empirical considerations that the ultimate torsional resistance could be expressed as $T = 1.2\, A_s f_{sy} b'd'/s$ for the purposes of design. The suitability of this expression for design or analysis will not be considered here since it is similar to the one in the draft unified code of practice.

The draft code of practice includes expressions for shear, torsion and bending that can be considered separately and independently of each other within certain limits. It is of interest to relate the one on torsion to the available experimental results. The expression for the ultimate strength of a member in torsion is

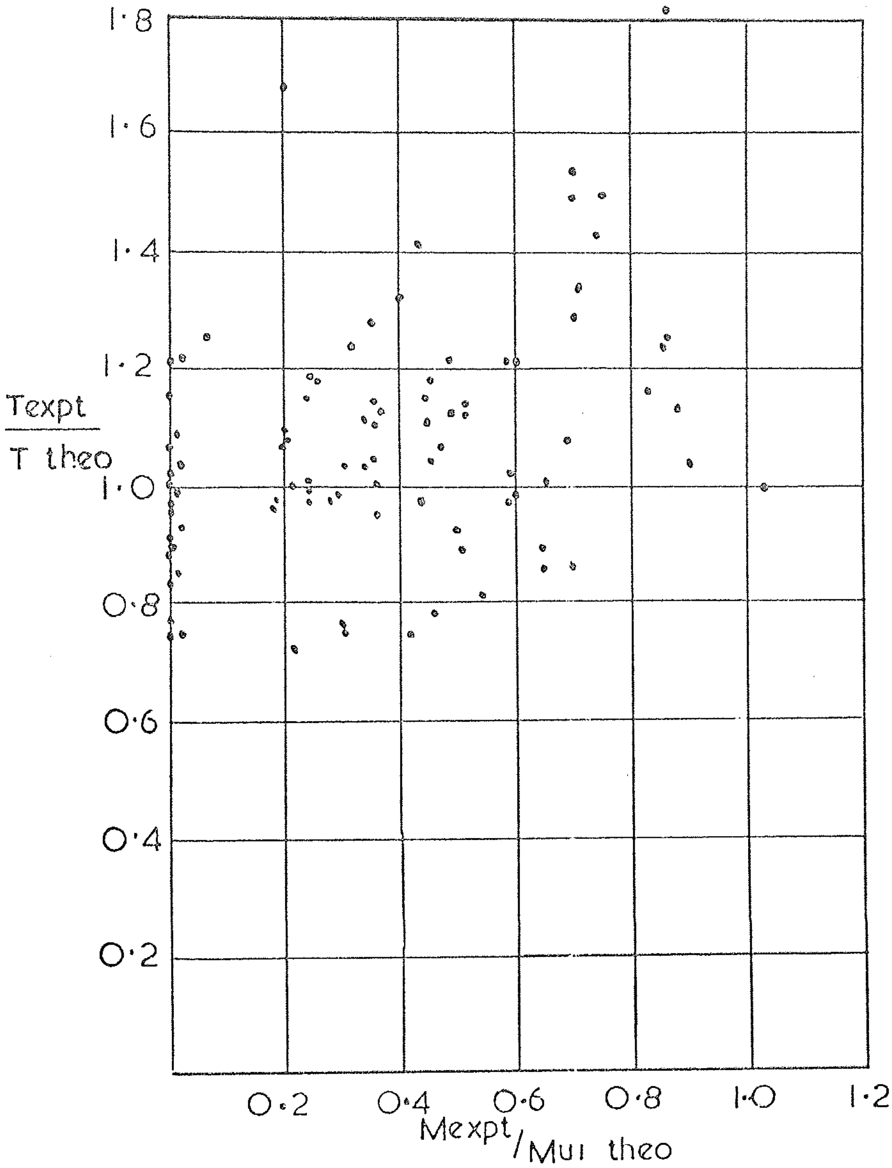
$$T = 2(0.8)\, \frac{A_s}{S}\, f_{sy}\, b'd' \qquad 5.4.1$$

TABLE 5.3.4

Summary Table of Partial Yield Theory A

$$T = \frac{2A_s f_{sy} b'd'}{S}$$

Investigator	Number of Beams	Mean $\frac{T_{expt}}{T_{theo}}$	% Coeff. of Variation
Goode & Helmy [28]	9	1.32	20.5
Gesund et al [24]	3	1.03	15.5
Iyengar & Rangan [15]	24	1.14	12.1
Pandit & Warwaruk [31]	5	0.96	16.5
Jackson & Estanero [34]	44	0.97	11.3
Chinekov [21]	7	1.38	13.9
Hsu [27]	7	1.09	22.8
Ernst [42]	8	1.42	36.4
Evans & Sarkar [25]	3	0.89	2.5
Grand Total	120	1.10	22.9



Expt results by

Iyengar & Rangan [15]

Goode & Helmy [28]

Pandit & Warwaruk [31]

Gesund et al [24]

Chinekov [21]

Jackson & Estañero [34]

$$T = 2\frac{A_s}{S} f_{sy}\ b'd'$$

Fig. 5.3.1. Torsion and bending of concrete beams containing longitudinal and transverse steel $r_{1y} < 1/(1 + d/b + 2M/T)$ or $r_{3y} < 1/(1 + d/b - 2M/T)$

This expression omits the materials factor of 0.87 for steel so that comparisons can be made with other partial yield theories. If this expression is used longitudinal steel must be added to balance the forces in the stirrups. This is additional to that required for the bending moment at the cross section. The total tension steel A_1 required is approximately

$$A_1 \geq \frac{M}{\ell_{a1} f_{1y}} + \frac{A_s f_{sy} b'}{S f_{1y}} (1 + d/b) \qquad 5.4.2$$

since $$r_{1y} = \frac{A_s f_{sy} b'}{S A_1 f_{1y}}$$

$$1 \geq \frac{M S r_{1y}}{l_{a1} A_s f_{sy} b'} + r_{1y} (1 + d/b)$$

also since $$T \simeq 1.6 \frac{A_s f_{sy}}{S} b' l_{a1}$$

$$1 \geq 1.6 \left(\frac{M}{T}\right) r_{1y} + r_{1y} (1 + d/b)$$

$$r_{1y} \leq \frac{1}{1 + d/b + 1.6 (M/T)} \qquad 5.4.3$$

This expression is similar to the equation produced by the author (equation 5.2.2) based on a limit of 45° for the crack angle on the side of the member. The equivalent angle for the draft code of practice is obtained as follows:-

From equation 4.2.14

$$\tan \alpha_1 = \frac{1}{(1 + d/b)} \left[\sqrt{\left(\frac{M_1}{T_1}\right)^2 + \frac{(1 + d/b)}{r_{1y}}} - \frac{M_1}{T_1} \right]$$

rearranging and squaring both sides of the equation

$$\left(\frac{M_1}{T_1}\right)^2 + \frac{(1 + d/b)}{r_{1y}} = \left[(1 + d/b) \tan \alpha_1 + \frac{M_1}{T_1}\right]^2$$

rearranging

$$r_{1y} = \frac{1}{(1 + {}^{d}/_{b})\ \tan^2\alpha_1 + 2\left(\frac{M_1}{T_1}\right)\tan\alpha_1} \qquad 5.4.4$$

equating equation 5.4.3 with 5.4.4 produces

$$(1 + {}^{d}/_{b})\ \tan^2\alpha_1 + 2\left(\frac{M_1}{T_1}\right)\tan\alpha_1 = 1 + {}^{d}/_{b} + 1.6\left(\frac{M_1}{T_1}\right)$$

rearranging

$$\tan\alpha_1 = \sqrt{\frac{\left({}^{M_1}/_{T_1}\right)^2}{(1 + {}^{d}/_{b})^2} + 1.6\ \frac{M_1}{T_1} + 1} - \frac{\left({}^{M_1}/_{T_1}\right)}{(1 + {}^{d}/_{b})} \qquad 5.4.5$$

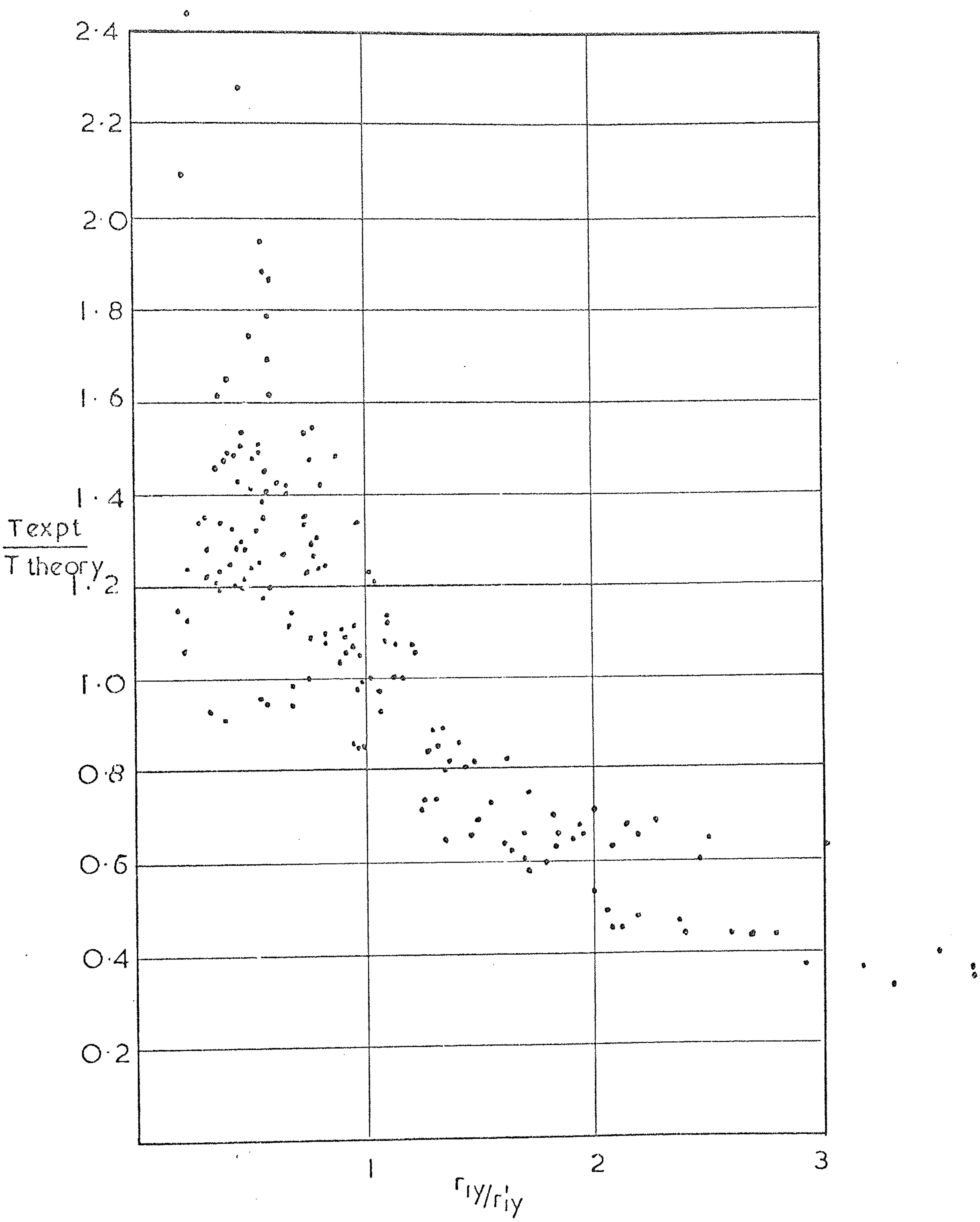
This expression does not always give a value of $\alpha_1 = 45^o$ as previously. Values for d/b = 1.5 vary with the M/T ratio as follows:-

M/T	0	1	2	3	4	6	8	10
$\tan\alpha_1$	1.00	1.26	1.40	1.49	1.56	1.64	1.70	1.74

It should be emphasised that the draft code of practice is recommending that $r_{1y} < r_{1y}'$, which classes it as a partial yield theory. The presentation in the code however leads the reader into believing it to be an all yield theory, since the term partial yield is not mentioned.

The variation in accuracy of the theory in relation to the ratio of transverse steel to longitudinal steel is shown in fig. 5.4.1. T_{expt}/T_{theo} is plotted against r_{1y}/r_{1y}'. Values of T_{expt}/T_{theo} when $r_y/r_y' > 1$ are not acceptable.

The results that are acceptable are replotted in fig. 5.4.2 with $M_{expt}/M_{u1\ theo}$ as the abscissa. This clearly illustrates the scatter that occurs which is confirmed by the values given in table 5.4.1 where the coefficients of variation are listed for each investigator. For the 127 experimental results listed the mean value of $T_{expt}/T_{theo} = 1.34$ with a coefficient of variation of 25.1%. The conservative nature of the theory is no doubt justified by the fact that it is the first time that a clause on torsion has been included in a British code of practice on reinforced concrete. The accuracy of the theory leaves room for

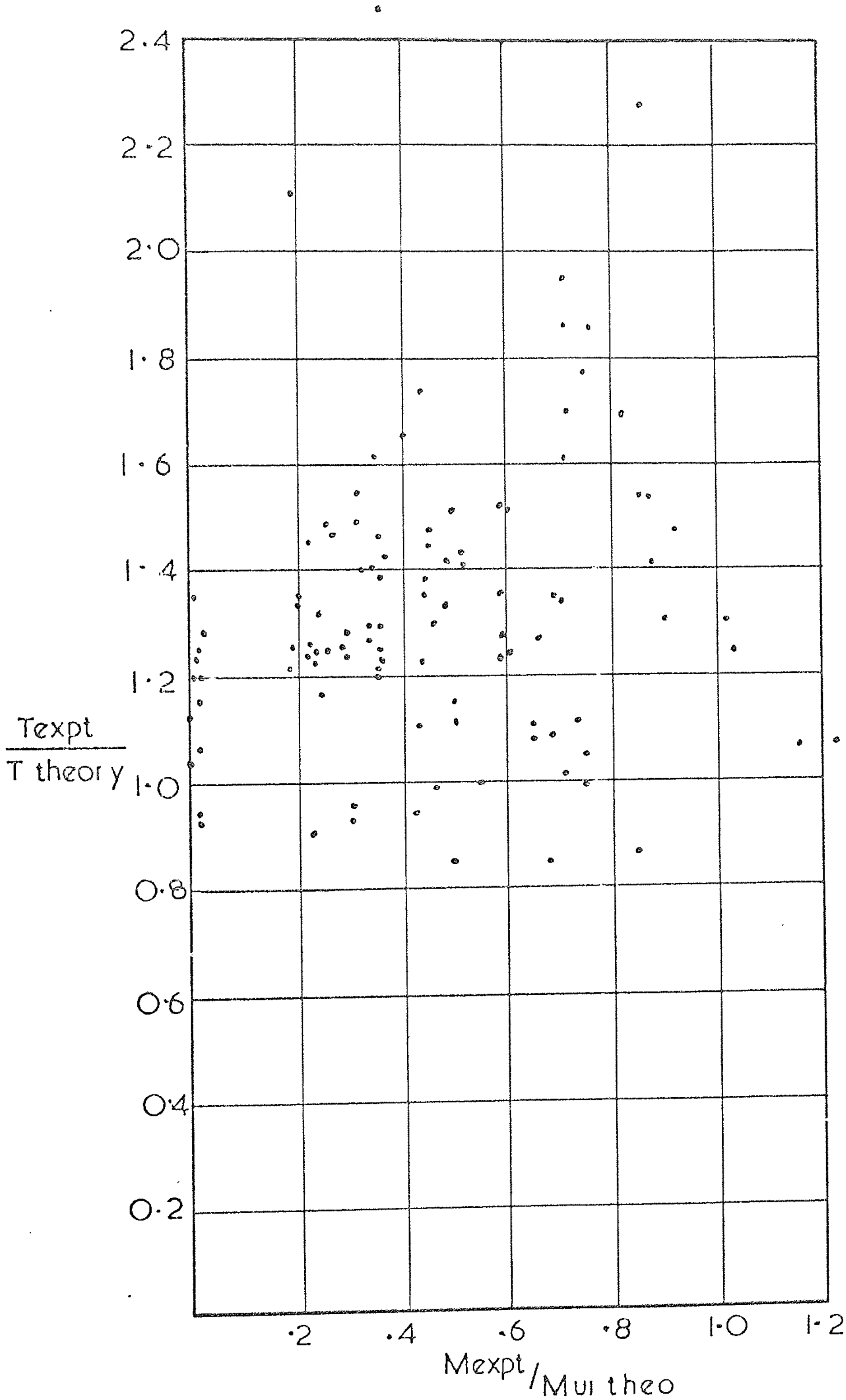


$$T = 1{\cdot}6 \frac{A_s f_{sy} b' d'}{S}$$

Expt. results by

Iyengar & Rangan [15]
Goode & Helmy [28]
Evans & Sarkar [25]
Gesund et al [24]
Fairburn [32]
Pandit & Warwaruk [31]
Chinekov [21]
Jackson & Estañero [34]

Fig. 5.4.1 Draft British unified code of practice 1972



$$T = 1{\cdot}6 \; \frac{A_s f_{sy}}{s} \; b'd'$$

$$r_{1y} \leq 1/(1 + d/b + 1{\cdot}6\, M/T)$$

Iyengar & Rangan [15]
Goode & Helmy [28]
Evans & Sarkar [25]
Gesund et al [24]
Fairburn [32]
Pandit & Warwaruk [31]
Chinekov [21]
Jackson & Estañero [34]

Fig. 5.4.2 Draft British unified code of practice 1972

TABLE 5.4.1

Summary Table of Partial Yield Case B

(Draft British Unified Code of Practice $T = \dfrac{1.6\, A_s f_{sy} b'd'}{S}$,

$$r_{ty} \leq \frac{1}{1 + {}^{d}/_{b} + 1.6\, {}^{M}/_{T}})$$

Investigator	Number or Beams	Mean $\frac{T_{expt}}{T_{theo}}$	% Coeff of Variation
Goode & Helmy [28]	15	1.51	30.9
Gesund et al [24]	4	1.18	23.4
Iyengar & Rangan [15]	25	1.41	12.7
Pandit & Warwaruk [31]	7	1.12	19.4
Jackson & Estanero [34]	49	1.20	12.4
Chenekov [21]	9	1.58	22.6
Hsu[27]	7	1.37	22.8
Ernst [42]	8	1.78	36.5
Evans & Sarkar [25]	3	1.11	2.7
Grand Total	127	1.34	25.1

improvement but is reasonable in relation to the simplicity of theory. The main criticism is that the code excludes values where $r_{y} > r_{y}'$. These values could have been included by using an all yield theory as given in Chapter 4.

5.5 Partial Yield Theory C - Empirical Method

An alternative approach to the method given in 5.3 for partial yield is based on experimental work by Hsu[27]. Hsu concentrated on experiments in pure torsion for reinforced concrete cross sections, where the ratio of transverse steel to longitudinal steel was related by

$$m' = \frac{A_2 \; S}{A_s \; (b'+d')} \simeq 1.0 \qquad 5.5.1$$

He also gave a few results where this value of m' varied from 0.2 to 5. These few results provide the basis for an empirical approach for the solution of members where the longitudinal or transverse steel does not yield.

Experimental results by Hsu for members B4, B7, B8, B9 & B10, show that as m' decreases then yield changes from stirrups (longitudinal steel not at yield) to longitudinal steel at yield (stirrups not at yield). The case where the stirrups are at yield is in fact the partial yield condition considered in 5.3. Hsu[27a] gave a semi-empirical expression for the ultimate strength as

$$T = \frac{2.4db^2\sqrt{f_c'}}{\sqrt{b}} + \left[0.66m' + 0.33\left(\frac{d'}{b'}\right)\right] \frac{A_s f_{sy} b'd'}{S} \qquad 5.5.2$$

provided $0.7 < m' < 1.5$ and

$\frac{d'}{b'} = 2.6$ when $\frac{d'}{b'} > 2.6$

This expression was primarily intended for members where $m' \simeq 1$ and was not intended to be accurate for the condition where m' varies between 0.2 to 5. An empirical expression developed by the author based on the experimental results of B4, 7, 8, 9 and 10 by Hsu is

$$T = 0.75\, db^2 \sqrt{f_c'} + \frac{A_s f_{sy} b'd'/s}{\left[0.25 + 1.20\,(r_{12})^{2/3}\right]} \qquad 5.5.3$$

provided $f_{sy} \simeq f_{sy}$ and $.08 < r_{12} < 1.9$.

Table 5.5.1 gives a comparison of the accuracy of the Hsu[27] expression and the author's empirical formula by listing $^{Ttest}/_{Ttheory}$ for both cases for members B4, B7, B8, B9 and B10.

Equation 5.5.3 is also capable of a solution for the other members tested by Hsu where $m' \simeq 1$, as shown in table 5.5.2. (see appendix) Some members are shown as over reinforced these will be discussed in Chapter 5.8

Although this expression is satisfactory for the steel and concrete tested by Hsu it requires to be applied to results by other investigators. Unfortunately results do not exist over the full range in pure torsion, but a few results for work by Ernst[42], Evans & Sarkar[25], Pandit and Warwarak[31] and Iyergar and Rangan[15] are shown in table 5.5.3 (see appendix), and summarised in table 5.5.6. The mean value of $^{Ttest}/_{Ttheory}$ for each investigator is less than one but the % coefficient of variation is acceptable. It seems probable therefore that different factors need to be applied to different concrete.

So far only results in pure torsion have been considered. Hsu reported that in these cases the crack angle was approximately 45°. This is also the condition for the partial yield of the stirrups in chapter 5.3

where $\quad r_{1y} \leqslant \dfrac{1}{1 + {}^{d}/_{b} + {}^{2M_1}/_{T_1}}$

or where $\quad r_{3y} \leqslant \dfrac{1}{1 + {}^{d}/_{b} - {}^{2M_3}/_{T_3}}$ and it therefore seems reasonable to expect that this empirical expression to apply. Tables 5.5.4 and 5.5.5

TABLE 5.5.1 - Partial Yield Experimental Results

Investigator	Beam	T_{expt} kpf in (kNm)	m'	r_{f2}	Ratio T_{test}/T_{theo} Hsu 27 equ. 5.5.2	Ratio T_{test}/T_{theo} empirical method equ. 5.5.3
Hsu[27]	B4	419.0 (47.35)	0.990	0.386	0.99	1.04
	B7	238.0 (26.89)	0.456	0.850	0.99	1.05
	B8	288 (32.54)	0.205	1.889	0.77	0.98
	B9	264 (29.83)	2.180	0.176	0.94	1.10
	B10	304 (34.35)	4.970	0.078	0.65	1.04
Mean % coeff. of variation					0.87 17.5	1.04 4.2

TABLE 5.5.6

Summary Table of Partial Yield Theory C

Empirical Method

$$T = 0.75\ db^2 \sqrt{f_c'} + \frac{1}{\left[0.25 + 1.20\ (r_{12})^{2/3}\right]} \frac{A_s f_{sy} b'd'}{S}$$

Investigator	Number of Beams	Mean $\frac{T_{expt}}{T_{theo}}$	% Coeff of Variation
Goode & Helmy [28]	10	1.05	18.7
Gesund et al [24]	3	0.82	23.9
Iyengar & Rangan [15]	24	0.82	9.1
Pandit & Warwaruk [31]	5	0.89	12.8
Jackson & Estanero [34]	44	0.85	13.9
Chinekov [21]	7	1.13	9.7
Hsu [27]	43	1.09	7.1
Ernst [42]	10	0.93	7.0
Evans & Sarkar [25]	3	0.90	13.2
Grand Total	149	0.95	16.6

(see appendix) therefore list all the members to which the expression should be applicable. These results include members where the longitudinal steel is unsymmetrical. Since this case was not considered by Hsu it is assumed that $A_2 = \frac{A_1 + A_3}{2}$. This value is required for use in the empirical expression given in equation 5.5.3 which includes the value

$$r_{12} = \frac{A_s b'}{S A_2}$$

The results are summarised in table 5.5.6.

As will be seen the mean value of $^{Ttest}/Ttheory$ varies as in the case of pure torsion, but the coefficient of variation is low and implies that the empirical expressions may be of the correct form.

The three partial yield theories may be compared using tables 5.3.4, 5.4.1 and 5.5.6. The unified code method B is conservative and is less accurate than method A as presented by the author. The empirical method C is the most accurate. Generally the coefficient of variation for the results of individual investigators is low for the empirical method C, and suggests that if this could be improved based on further experimental results, that this may be preferred in the future.

5.6 Optimum Value of the Ultimate Torsional Resistance for Partial Yield Cases

The solution for the optimum value of the ultimate torsional resistance for full yield of the steel was presented in Chapter 4. The partial yield case will now be considered. The empirical expression given in equation 5.5.3 expresses the ultimate torsional resistance of a section which varies with the ratio of transverse to longitudinal steel (r_{12}).

$$T_u = 0.75\ db^2\ \sqrt{f_c'} + \frac{A_s f_{sy} b'd'/S}{\left[0.25 + 1.20\ (r_{12})^{2/3}\right]} \qquad 5.6.1\ (\text{see } 5.5.3)$$

The ratio of the total volume of steel reinforcement for a given volume of concrete bds is

$$P_t = \frac{2\,A_s\,(b' + d') + 2A_2S}{bds}$$

rearranging and assuming $d'/_{b'} \approx d/_b$

$$P_t = \frac{2A_s b'}{bds}\left[(1 + d/_b) + 1/_{r_{12}}\right] \qquad 5.6.2$$

Substituting equation 5.6.2 in 5.6.1 to eliminate A_s and express T_u in terms of r

$$T_u = 0.75\,db^2\sqrt{f_c'} + \frac{1}{\left[0.25 + 1.20\,(r_{12})^{2/3}\right]}\;\frac{P_t\;bd\,d'\,f_{sy}}{2\left[(1+d/_b) + 1/_{r_{12}}\right]}$$

5.6.3

For a given fixed value of the volume of reinforcment P_t, the optimum value of T_u is obtained by differentiating T_u with respect to r_{12} and equating to zero.

$$\frac{dT_u}{dr_{12}} = \left[0.25 + 1.20\,(r_{12})^{2/3}\right]\left[-1/_{r_{12}^{\,2}}\right] +$$

$$\left[(1 + d/_b) + 1/_{r_{12}}\right]\left[1.20 \times \tfrac{2}{3} \times 1/_{r_{12}^{\,1/3}}\right] = 0$$

rearranging

$$0.25 + 0.4\,r_{12}^{\,2/3} - 0.8\,(1+d/_b)\,r_{12}^{\,5/3} = 0 \qquad 5.6.3$$

The approximate solution to equation 5.6.3 is

$$r_{12} = \frac{A_s\,b'}{S\,A_2} \approx \frac{1}{1 + d/_b} \qquad 5.6.4$$

this may be alternatively expressed as

$$r_2 = \frac{A_s\,d'}{S\,A_2} \approx \frac{1}{1 + b/_d} \qquad 5.6.5$$

$$\text{or} \quad m' = \frac{A_2\,S}{A_s\,(b' + d')} \approx 1.0 \qquad 5.6.6$$

Equation 5.6.6 is the value recommended by Hsu based on experimental evidence to ensure plastic behaviour of the steel at collapse. The original empirical equation given by Hsu[27] (see equation 5.5.2) does not have an optimum value.

There are very few experimental results to determine the optimum value of T_u as r_{12} varies. Hsu[27] however tested a few beams which may be used for this purpose, and these are listed in table 5.6.1. They have been chosen, as far as is practical, for the same size and strength, and a fixed volume of reinforcement. The variable is the ratio of transverse to longitudinal steel r_{12}.

These results have been plotted in relation to the theoretical lines based on b = 10" (254 mm), d = 15" (381 mm), b' = 8.5 (215.9 mm), d' = 13.5" (342.9 mm), f_c' = 4000 lbf/in^2 (27.58 N/mm^2), f_{sy} = 47,000 lbf/in^2 (324 N/mm^2), P_t = 1.7% and 3.2%. (see fig. 5.6.1)

The values for the experimental results vary from these adopted theoretical values and will not therefore agree exactly. M4 for instance would be expected to be high in relation to the theoretical line since P_t = 3.53% and not 3.2%.

The theoretical value of T_u does not reduce greatly when $r_{12} > \frac{1}{1+d/b}$. This is the case when the longitudinal steel yields and the stirrups are elastic. This portion of the curve requires further experimental investigation since it is based on the experimental results of Hsu[27] where the yield stress of the stirrups and the longitudinal steel were approximately equal.

The value of T_u is more sensitive to r_{12} when $r_{12} < 1/(1+d/b)$. This is the case which is more likely to occur in practice, when the stirrups yield and the longitudinal steel is elastic.

The theoretical lines in fig. 5.6.1 also illustrate why the theories which assume all the steel yields are successful for high r_{12}

TABLE 5.6.1 Partial Yielding Experimental Results

b = 10" (254mm), d = 15" (381mm),

b' = 8.5" (215.9mm), d' = 13.5" (342.9mm).

Investigator	Beam	T_{expt} kpf in (kNm)	f_c' lbf in (N/mm^2)	Ratio r_{12}	f_{sy} lbf/in^2 (Nmm^2)	p_t %
Hsu[27]	B10	304.0 (34.35)	3840 (26.48)	.078	49,600 (342)	3.21
	M3	388.0 (43.84)	3880 (26.75)	0.252	47,300 (326)	2.67
	M4	439.0 (49.61)	3850 (26.55)	0.257	47,400 (327)	3.53
	B4	419.0 (47.35)	4430 (30.54)	0.382	46,900 (323)	3.21
	B8	288.0 (32.54)	3880 (26.75)	1.889	46,400 (320)	3.14
	B9	264.0 (29.83)	4180 (28.82)	0.176	49,700 (343)	1.71
	M1	269.0 (30.40)	4330 (29.85)	0.260	51,200 (353)	1.38
	B2	259.0 (29.27)	4150 (28.61)	0.382	46,400 (320)	1.65
	B7	238.0 (26.89)	3770 (25.99)	0.850	46,200 (319)	1.70

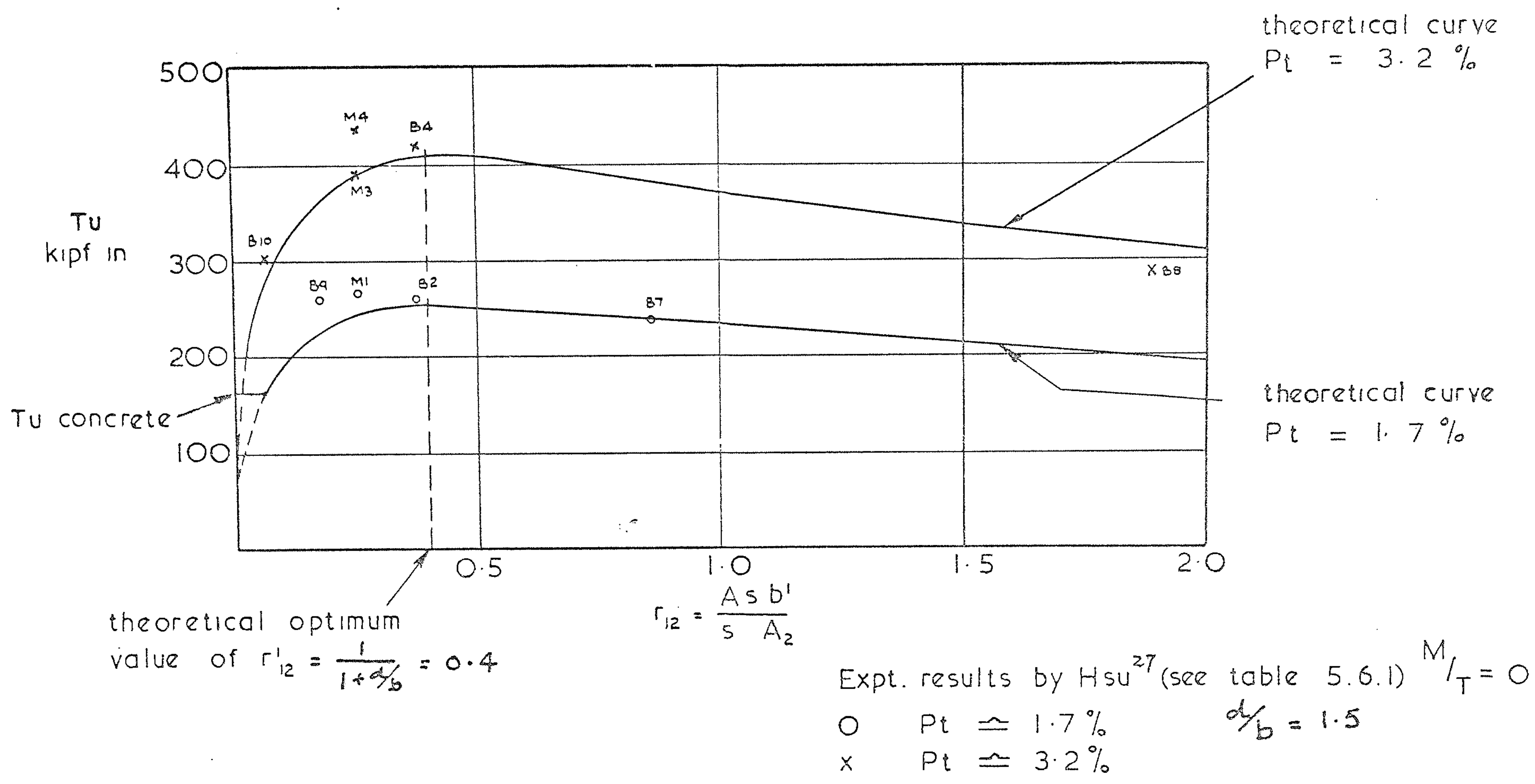


Fig. 5.6.1. The value of the ratio of transverse to longitudinal steel for the optimum torsional resistance

ratios. Over the range of 0.5 to 2.0 the reduction from the optimum value is small. The "all yield theories" show their greatest inaccuracies when the r_{12} ratio is small, when the fall from the optimum value is 50% for very small r_{12} values.

The curves in fig. 5.6.2 with their limited experimental confirmation, also illustrate that the partial yield theories given in Chapters 5.3 and 5.4 require a restraint. These theories give no optimum value but become progressively more inaccurate as r_{12} exceeds the optimum. The value of P_t = 3.2% may appear to be too high to be of practical value, but it should be remembered that P_t includes the longitudinal steel on both faces and the stirrups. It is therefore likely to be a typical value occuring in practice.

5.7 The Minimum Percentage of Steel

If the volume of reinforcing steel is too low then the torsional resistance of reinforced cross section will be less than that for plain concrete. This is more likely to occur where the $^M/_T$ ratio is low, and is well illustrated by a few experimental results by Ernst [42] as shown in table 5.7.1. These results are for pure torsion where with low % of steel the strength is approximately equal to that for the plain concrete member.

One theoretical solution to this problem is to determine the torsional resistance of the plain concrete section using equation 2.5.10 and to compare this with the value obtained for a reinforced cross section as given in equation 5.5.3. If the torsional resistance for the reinforced cross section is too low then more steel may be added.

It should be pointed out that it is not also necessary to consider the singly reinforced concrete member for low $^M/_T$ ratios, since it has been shown to be approximately equal to that for plain concrete in Chapter 3.

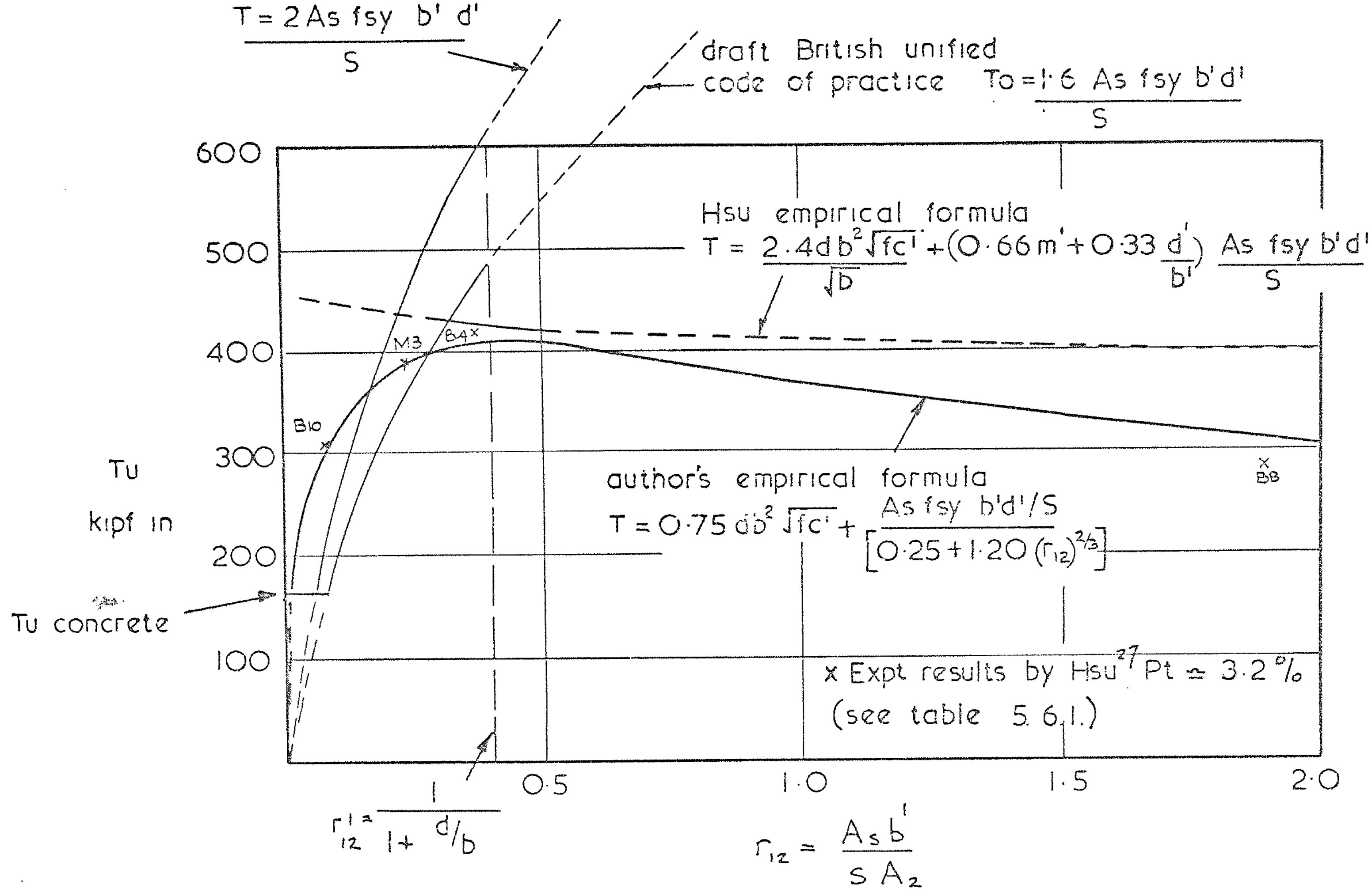


Fig. 5.6.2 Comparison of partial yield theories ($Pt = 3{\cdot}2\%$)

TABLE 5.7.1 Experimental Results

Investigator	Beam	Texpt kip£ in (kNm)	r_{12}	P_s% expt	P_s% min theo equn 5.7.6	$\frac{P_s \text{ expt}}{P_s \text{ min theo}}$
Ernst[42]	3TRO	37.6 (4.25)	-	-	-	-
	3TR1	35.0 (3.96)	0.036	0.071	0.162	0.438
	3TR3	34.3 (3.88)	0.072	0.141	0.194	0.727
	3TR7	49.7 (5.62)	0.143	0.282	0.244	1.156
	3TR15	61.7 (6.97)	0.251	0.494	0.305	1.620
	3TR30	76.0 (8.59)	0.501	0.987	0.420	2.349
	4TRO	34.4 (3.89)	-	-	-	-
	4TR1	32.1 (3.63)	0.020	0.071	0.145	0.485
	4TR3	35.0 (3.96)	0.040	0.141	0.167	0.845
	4TR7	54.8 (6.19)	0.080	0.282	0.201	1.403
	4TR15	74.0 (8.36)	0.141	0.494	0.242	2.039
	4TR30	85.0 (9.61)	0.281	0.987	0.321	3.078
	5TRO	33.8 (3.82)	-	-	-	-
	5TR1	33.4 (3.77)	0.013	0.071	0.136	0.522
	5TR3	43.0 (4.86)	0.026	0.141	0.152	0.929
	5TR7	59.7 (6.75)	0.051	0.282	0.177	1.592
	5TR15	76.5 (8.64)	0.090	0.494	0.208	2.375
	5TR30	92.6 (10.46)	0.180	0.987	0.266	3.707

An alternative approach is to try to arrive at a minimum percentage of stirrup reinforcement. This has been attempted previously by Hsu[27] but the expression he produced was not applicable to the full range of ratios of transverse to longitudinal steel. It was also complicated . The torsional resistance of a reinforced concrete section as expressed by the empirical formula is

$$T_2 = 0.75\, db^2 \sqrt{f_c'} + \frac{A_s f_{sy} b'd'/S}{\left[0.25 + 1.20 (r_{12})^{2/3}\right]} \qquad 5.7.1 \text{ (see originally 5.5.3)}$$

The first term in this expression $.75\, db^2 \sqrt{f_c'}$ is approximately half that of plain concrete. The torsional resistance of plain concrete is given in Chapter 2.

$$T_2 = \frac{1}{3 + (b/d)^{1/2}} \cdot db^2\, 24 \,(1 + 10/d^2)\, (f_c')^{1/3} \qquad 5.7.2$$

this can be further approximated to

$$T_2 = \frac{1}{3} db^2\, 24\, (f_c')^{1/3} \qquad 5.7.3$$

If half this value is substituted in equation 5.7.1 then

$$\frac{1}{6} db^2\, 24\, (f_c')^{1/3} = \frac{A_s f_{sy} b'd'/S}{\left[0.25 + 1.20 (r_{12})^{2/3}\right]} \qquad 5.7.4$$

The ratio of the volume of stirrups in a volume of concrete bds is

$$P_s = \frac{2A_s (b'+d')}{bds} \qquad 5.7.5$$

Combining equations 5.7.4 and 5.7.5 and assuming $b' \simeq .8b$

$$P_s = \frac{5\, (f_c')^{1/3} \left[0.25 + 1.20 (r_{12})^{2/3}\right] \left[2 (1 + b'/d')\right]}{f_{sy}} \qquad 5.7.6$$

The minimum volume of reinforcement could also be expressed as a ratio of the longitudinal reinforcement A_2 by substituting $r_{12} = \frac{A_s b'}{S A_2}$ into equation 5.7.4.

This produces

$$4\,db^2\,(f_c')^{2/3} = \frac{d'\,f_{sy}\,r_{12}\,A_2}{\left[0.25 + 1.20\,(r_{12})^{2/3}\right]} \qquad 5.7.7$$

and since by definition

$$P_\ell = 2A_2/bd \qquad 5.7.8$$

substituting equation 5.7.8 in equation 5.7.6 and assuming b' ≃ 0.8b

$$P_\ell = \frac{10\,(f_c')^{1/3}\left[0.25 + 1.20\,(r_{12})^{2/3}\right]\,(b'/d')}{r_{12}\,f_{sy}} \qquad 5.7.9$$

A further alternative is to express the minimum volume of reinforcement as a ratio of the total reinforcement (transverse steel and longitudinal steel) to the volume of concrete.

$$P_t = \frac{2\,A_s\,b'}{bds}\left[(1 + d'/b') + 1/r_{12}\right] \qquad 5.7.10$$

substituting equation 5.7.10 in equation 5.7.4

$$\frac{1}{6}\,db^2\,24(f_c')^{1/3} = \frac{1}{\left[0.25 + 1.20\,(r_{12})^{2/3}\right]}\;\frac{P_t\,bd\,d'\,f_{sy}}{2\left[(1 + d'/b') + 1/r_{12}\right]}$$

rearranging

$$P_t = 10\,\frac{(f_c')^{1/3}}{f_{sy}}\left[0.25 + 1.20\,(r_{12})^{2/3}\right]\left[(1+d'/b') + 1/r_{12}\right]\,(b'/d') \qquad 5.7.11$$

Fig. 5.7.1 illustrates theoretically the variation of P_s, P_ℓ and P_t with r_{12} for a given d/b ratio, strength of steel, and strength of concrete. As r_{12} increases the volume of steel in the stirrups (P_s) increases and the volume of longitudinal steel (P_ℓ) decreases. The total volume of steel P_t, however, has a minimum value at $r = 1/(1+d'/b')$. This value of r_{12} is also the value for the maximum torsional resistance of the section as shown in Chapter 5.6. For this particular case Hsu[27] gave $P_s \simeq 0.5\%$ and $P_t \simeq 1.0\%$. These values agree with those obtained from Fig. 5.7.1. Hsu did not consider the full range of r_{12} values.

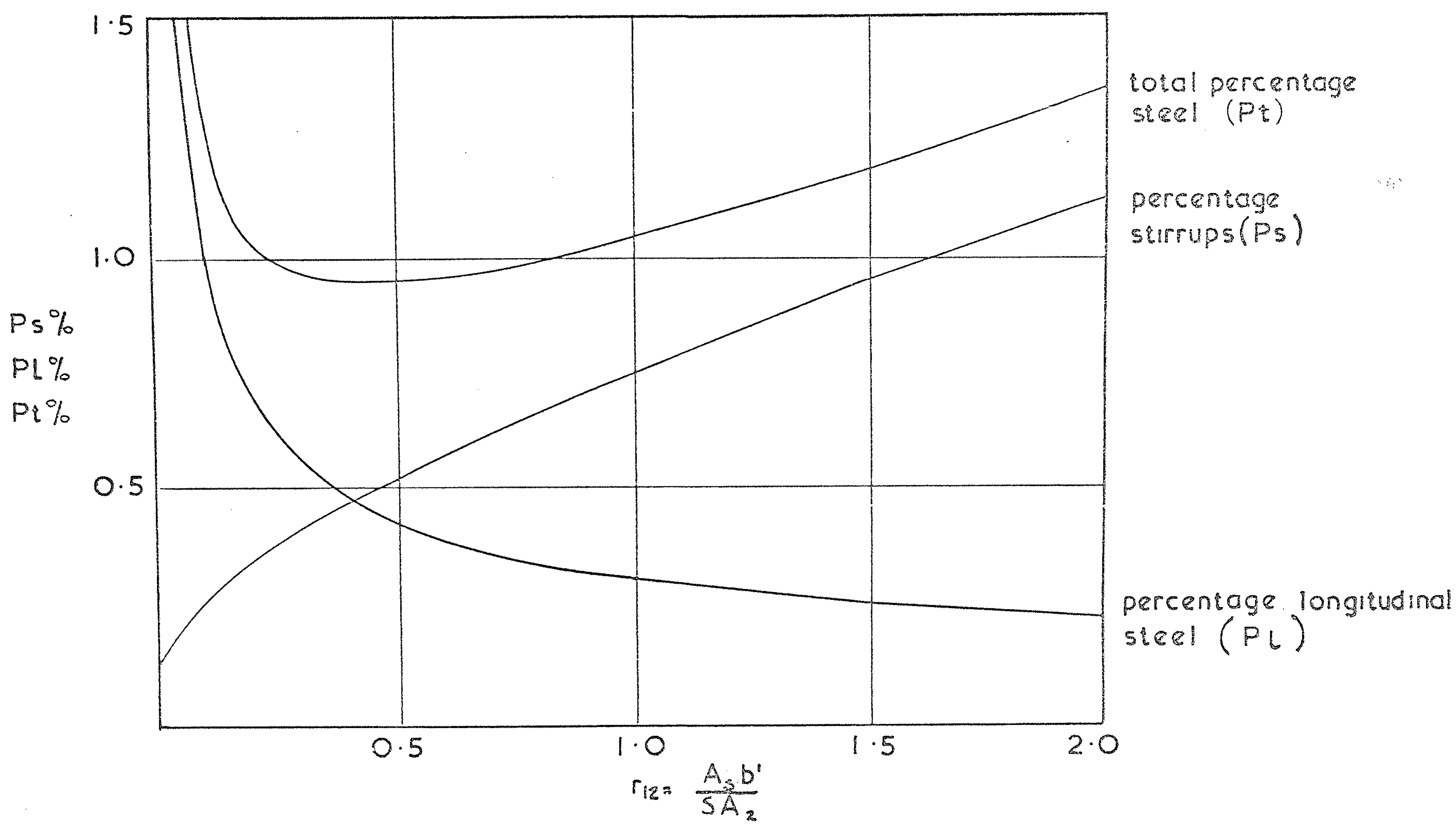


Fig. 5.7.1. Minimum reinforcement (theoretically)
[$f_c' = 4000$ lbf/in² (27·58 N/mm²), $f_{sy} = 50{,}000$ lbf/in² (344·7 N/mm²)
$b'/d' = 2/3$]

Ernst[42] conducted 3 series of tests which indicate the minimum percentage of reinforcement as shown in Table 5.7.1. Commencing with plain concrete he added reinforcement until the ultimate strength showed an increase above the plain concrete ultimate strength in torsion. The theory expressed in equation 5.7.6 enables the minimum theoretical percentage of stirrups reinforcement to be determined. This is related to the actual percentage stirrup reinforcement by a ratio of (p_s experimental/ p_s minimum). When this value is greater than one the strength of the member should be greater than that for plain concrete. As will be seen from the table the expression is reasonably accurate.

5.8 Over-Reinforced Members

To be completely over-reinforced requires that none of the steel yields. In experiments this would require a large number of strain gauges to be certain, since yielding could occur at any section or in any leg of the stirrup. It is possible therefore that experimental results reported as over-reinforced are in fact partial yielding cases. No satisfactory theoretical approach is available for over-reinforced members, and investigators have concentrated on simple empirical equations. The first group of these relates the torsional strength to the size of the section and the strength of concrete and excludes the reinforcement.

From experiment Lessig[20] produced the equation

$$T = k\, db^2\, f_c' \qquad 5.8.1$$

where k varies between 0.07 and .12. No explanation is given for the range of k values, but it was noted that further work was required in connection with $^d/_b$ ratios.

Walsh et al[26] gave an equation which is of the same form

$$T = 5\, db^2 \sqrt{f_c'} \qquad 5.8.2$$

The second group of empirical equations to present failures by over-reinforcing members, are related to the reinforcement. Walsh et al[24] recommended limiting the tensile longitudinal steel for mode 1 form of failure to

$$\frac{p_l f_c'}{f_{ly}} \not> 0.04 \qquad 5.8.3$$

There appears to be no direct experimental evidence for suggesting this, except in pure bending.

Based on experiments in pure torsion where $r_2 = {}^{1}/_{(1+{}^{b}/_{d})}$ Hsu[27] recommended

$$p_{tb} > \frac{2400 \sqrt{f_c'}}{f_{sy}} \qquad 5.8.4$$

The experimental results by Hsu[27] in pure torsion provide a means of comparing equations 5.8.1, 5.8.2 and 5.8.4. From the strain gauge readings it is possible to select members where no yielding occurs. This may of course lead to error since at another cross section yielding may have occurred. The results are given in table 5.8.1 and compared with the above equations. The expression given by Walsh et al[26] appears to improve the Lessig[20] expression, by relating T to $\sqrt{f_c'}$. Both appear not to allow for variation in the ${}^{b}/_{d}$ ratio. This criticism also applies to the expression given by Hsu[27].

For these particular experimental results an improvement on the Walsh et al[26] equation to allow for the variation in ${}^{b}/_{d}$ is

$$T = 6 \; (1.5 - {}^{b}/_{d}) \; db^2 \sqrt{f_c'} \qquad 5.8.5$$

This expression is also compared with the experimental results in table 5.8.1.

The value of P_t recommended by Hsu was based on $r_2 = {}^{1}/_{(1+{}^{b}/_{d})}$. The value of P_t may vary for other values of r_2. If however P_t remains reasonably constant then when substituted into empirical partial yielding expression (see equation 5.5.3).

$$T = 0.75 \; db^2 \sqrt{f_c'} + \frac{A_s f_{sy} \; b'd'/S}{\left[0.25 + 1.20 \; (r_{12})^{2/3}\right]}$$

TABLE 5.8.1 Over-reinforced Members

Investigator	Beam	Ratio $^{b}/_{d}$	Texpt kip in kNm	Ratio $\frac{Texpt}{Ttheo}$ Lessig[20] eqn. 5.8.1	Ratio $\frac{Texpt}{Ttheo}$ Walsh et al[26] eqn. 5.8.2	Ratio $\frac{Texpt}{Ttheo}$ Hsu[27] eqn. 5.8.4	Ratio $\frac{Texpt}{Ttheo}$ Modification of Walsh et al eqn. 5.8.5
Hsu[27]	B6	$^{2}/_{3}$	546.0 (61.70)	1.24	1.13	1.16	1.13
	M6	$^{2}/_{3}$	532.0 (55.71)	1.19	1.09	1.15	1.09
	I6	$^{2}/_{3}$	679.0 (76.73)	0.97	1.11	0.95	1.11
	J4	$^{2}/_{3}$	360.0 (40.68)	1.41	0.97	0.97	0.97
	G5	$^{1}/_{2}$	637.0 (71.98)	1.17	1.02	0.75	0.85
	K4	.31	310.0 (35.03)	1.52	1.37	1.07	0.96
	C3	1	177.0 (20.00)	0.65	0.57	0.58	0.95
Mean				1.16	1.04	0.95	1.01
% coeff of variation				24.7	23.3	22.7	10.4

and combined with

$$P_t = \frac{2A_s b'}{bds} \left[(1 + d'/b') + 1/r_{12}\right]$$

and the Hsu[27] limit for over reinforcing given in equation 5.8.4 produces

$$T = .75\ db^2 \sqrt{f_c'} + \frac{1}{\left[0.25 + 1.20\ (r_{12})^{2/3}\right] 2} \ \frac{2400 \sqrt{f_c'}\ bd\ d'}{\left[(1+d'/b') + 1/r_{12}\right]}$$

This expression could be put in the form of $T = k\ db^2 \sqrt{f_c'}$ as previously suggested by Walsh et al[26]. The value of k would then depend on the value of the ratio of transverse steel to longitudinal steel r_{12}, and the d/b ratio. Further experimental work is required to verify this.

5.9 Summary of Chapter 5.

The cases considered in this chapter are partial yielding of the stirrups and over-reinforcing the member.

Three formula to determine the ultimate strength of a member have been considered and their comparative accuracy for the same group of results is

	Expression	Mean	% Coefficient of Variation
A	$T = 2\ A_s\ f_{sy}\ b'd'/S$	1.10	22.9
B	$T = 1.6\ A_s\ f_{sy}\ b'd'/S$	1.34	25.1
C	$T = .75\ db^2 \sqrt{f_c'} + \dfrac{A_s\ f_{sy}\ b'd'/S}{\left[0.25 + 1.20\ (r_{12})^{2/3}\right]}$ empirical.	0.95	16.6

Equations A and B in the table are functions which produce no value of the ratio of transverse to longitudinal steel for which the torsional value is an optimum. Empirical formula C gives a value of $r_{12} \simeq 1/(1+d/b)$ which agrees with experimental results.

The third empirical method C is also suitable to obtain a minimum volume of reinforcement to produce a torsional resistance greater than a member of plain concrete. This is in reasonable agreement with experimental results.

Over reinforcing has also been considered but no firm conclusions are presented due to a lack of experimental results.

CHAPTER 6 Conclusions

6.1 Conclusions for Plain Concrete Members

From the presented theory and its correlation with experimental results the following conclusions are drawn for a plain concrete member subject to bending and torsion:

6.1.1 When the $^M/_T$ ratio is in the middle or high range, failure is premdominantly a bending failure about a skew axis, but the failure bending moment is less than that is pure bending. The magnitude of the moments at failure are related by the general non-dimensional interaction equation mode 1

$$\left(\frac{T_1}{T'_{u1}}\right)^2 + \frac{M_1}{M_u} = 1$$

where for a rectangular section,

$$T'_{u1} \simeq \frac{1}{3 + (\frac{d}{b})^{\frac{1}{2}}} bd^2 f_{r1}, \text{ and } M_{u1} = \frac{bd^2}{6} f_{r1}$$

and for a circular cross section

$$T'_{u1} \simeq (0.83) \frac{\pi D^3}{16} f_{r1} \text{ and } M_{u1} = \frac{\pi D^3}{32} f_{r1}.$$

Alternatively the general interaction equation may be expressed using a $^M/_T$ ratio as

$$T_1 = T'_{u1} \left[\sqrt{\left(\frac{M_1}{T_1}\right)^2 + 1} - \frac{M_1}{T_1} \right]$$

6.1.2 When the $^M/_T$ ratio is low failure is predominantly a torsional failure about a skew axis of bending. The magnitude of the torsional moment at failure can be determined from the general non dimensional interaction equation mode 2.

$$\frac{T_2}{T'_{u2}} = 1$$

where

$$T'_{u2} = \frac{1}{3 + (\frac{b}{d})^{\frac{1}{2}}} db^2 f_{r2}$$

for a rectangular cross section.

6.1.3 Mode 3 form of failure is theoretically possible for say a trapezium cross section. The general interaction form of equation is

$$\left(\frac{T_3}{T'_{u3}}\right)^2 - \frac{M_3}{M_{u3}} = 1$$

where $T'_{u3} = \begin{pmatrix}\text{factor see}\\ \text{fig 2.7.3}\end{pmatrix} Db^2 f_{r3}$

$$M_{u3} = \frac{b^2}{12} \frac{(D^2 + 4dD + d^2)}{(2D + d)} f_{r3}$$

alternatively this may be expressed using the $^M/_T$ ratio

$$T_3 = T'_{u3} \left[\sqrt{\left(\frac{M_3}{T_3}\right)^2 + 1} + \frac{M_3}{T_3}\right]$$

No experimental results are available to substantiate this part of the theory.

6.1.4 Available experimental evidence from various investigators shows that the general interaction equation is reasonably accurate for determining the failure moments for a plain concrete beam of rectangular or circular cross section when subject to torsion and bending. The mean ratio of $\frac{\text{Texpt}}{\text{Ttheory}} = 1.04$ with a coefficient of variation of 21%. Where investigators have quoted a tensile strength however the mean ratio of $\frac{\text{Texpt}}{\text{Ttheory}} = 1.02$ and 12% coefficient of variation.

6.1.5 The establishment of a reasonably accurate method of determining the failure moment in torsion and bending for plain concrete sections provides a means of assessing the effectiveness of any reinforcement which may be added.

6.1.6 The theoretical approach for plain concrete adopted in this thesis results in a general non-dimensional interaction equation which is useful for analysis and design. Cowan[8] showed that the same form of equation can be obtained by forming equilibrium equations about the longitudinal and transverse axes and combining these using the principal tensile stress criterion.

6.2 Conclusions for Concrete Members with Longitudinal Steel Only

From the presented theory and its correlation with experimental results the following conclusions are drawn for a concrete member of rectangular cross section reinforced with longitudinal steel only and subject to torsion and bending.

6.2.1 When the M/T_1 ratio is high failure is predominantly a bending failure about a skew axis, but the failure moment in bending is less than that in pure bending. The magnitude of the moments at failure are related by the general non dimensional interaction equation based on the Cowan[8] failure criteria for concrete

$$\left(\frac{T_1}{T_{cu1}}\right)^2 + \left(2\,\frac{M_1}{M_{cu1}}\right)^2 - 3\,\frac{M_1}{M_{cu}} = 1$$

where

$$T_{cu1} = \frac{2}{15}\, bd_{n1}\, l_{a1}\, f_c'$$

and

$$M_{cu1} = \frac{2}{3}\, bd_{n1}\, l_a\, f_c'$$

Alternatively this general interaction equation may be expressed using an M/T_1 ratio as

$$T_1 = T_{cu1}\,\frac{1}{\sqrt{1 + \left(M/2T_1\right)^2 - 0.3\, M/T_1}}$$

6.2.2 Alternatively when the M/T_1 ratio is high, the magnitude of the moments at failure are related by the general non dimensional interaction equation based on the Huber-Von Mises-Hencky failure criteria for steel.

$$\left(\frac{T_1}{T_{su1}}\right)^2 + \left(\frac{M_1}{M_{su1}}\right)^2 = 1$$

where

$$M_{su1} = A_1\, f_{1y}\, l_{as1}$$

and

$$T_{su1} = \frac{M_{su1}}{\sqrt{3}}$$

Alternatively the general interaction equation may be expressed using an $^{M}/T_1$ ratio as

$$T_1 = \frac{M_{su1}}{\sqrt{\left(^{M}/_{T_1}\right)^2 + 3}}$$

6.2.3 When the $^{M}/_{T}$ ratio is low failure is predominantly a torsional failure about a skew axis of bending based on the tensile strength of concrete. The magnitude of the moments at failure are related by the general interaction formula developed for plain concrete

$$\frac{T_2}{T'_{u2}} = 1$$

where

$$T'_{u2} \simeq \frac{1}{3 + (^{b}/_{d})^{\frac{1}{2}}} \, db^2 \, f_{r2}$$

6.2.4 Available experimental evidence for various investigators show reasonable correlation with the theory. The mean ratio of $^{Ttest}/_{Ttheory}$ for 95 experimental results is 1.02 with a coefficient of variation of 20% for all modes of failure.

6.2.5 The establishment of a reasonably accurate method of determining the failure moment in torsion and bending for members with longitudinal steel only, provides a means of assessing the effectiveness of adding stirrups.

6.3 Conclusions for Members with Transverse Steel and Longitudinal Steel at Yield.

From the presented theory and its correlation with experimental results the following conclusions are drawn for a concrete member of rectangular cross section reinforced longitudinally and transversely with single closed stirrups and subject to torsion and bending.

6.3.1 Where the $^{M}/_{T}$ ratio is high failure is predominantly a bending failure with the "skew" compression hinge at the top of the section. The failure moment in bending is less than in pure bending and is related to the torsional moment by the general non dimensional interaction equation

$$\left(\frac{T_1}{T_{u1}}\right)^2 + \frac{M_1}{M_{u1}} = 1 \quad \text{(mode 1)}$$

where

$$T_{u1} = 2M_{u1}\sqrt{\frac{r_{1y}}{1 + d/b}} \quad \text{and } M_{u1} = A_1 f_{1y}\, l_{a1}$$

Alternatively the general interaction equation may be expressed using an M/T ratio as

$$T_1 = 2M_{u1}\frac{r_{1y}}{(1+d/b)}\left\{\sqrt{\left(\frac{M_1}{T_1}\right)^2 + \frac{(1+d/b)}{r_{1y}}} - \frac{M_1}{T_1}\right\}$$

provided $r_{1y} \geqslant \dfrac{1}{1 + d/b + 2M/T}$ and that the member is not over reinforced.

6.3.2 When the M/T ratio is low and the area of the top longitudinal steel equals the area of the bottom steel failure is predominantly a torsional failure about a "skew" axis with the compression hinge located at the side of the member. The magnitude of the moments at failure are related by the general non dimensional interaction equation

$$\frac{T_2}{T_{u2}} = 1 \quad \text{(mode 2)}$$

where $T_{u2} = 2M_{u2}\sqrt{\dfrac{r_2}{(1+b/d)}}$ and $M_{u2} = A_2 f_{2y}\, l_{a2}$

Provided $r_{2y} \geqslant \dfrac{1}{1+b/d}$ and that the member is not over-reinforced.

6.3.3 When the M/T is low and the area of the top longitudinal steel is less than the area of the bottom longitudinal steel, failure is predominantly a torsional failure about a "skew" axis with the compression hinge located at the bottom of the section. The magnitude of the moments at failure are related by the general non dimensional interaction equation

$$\left(\frac{T_3}{T_{u3}}\right)^2 - \frac{M_3}{M_{u3}} = 1 \text{ (mode 3)}$$

where $T_{u3} = 2M_{u3}\sqrt{\dfrac{r_{3y}}{(1+{}^{d}/_{b})}}$ and $M_{u3} = A_3 f_{3y} l_{a3}$

Alternatively the general interaction equation may be expressed using an ${}^{M}/_{T}$ ratio as

$$T_{u3} = 2M_{u3}\frac{r_{3y}}{(1+{}^{d}/_{b})}\left(\sqrt{\frac{M_3}{T_3} + \frac{(1+{}^{d}/_{b})}{r_{3y}}} + \frac{M_3}{T_3}\right)$$

Provided $r_{3y} \geqslant \dfrac{1}{1 + {}^{d}/_{b} - {}^{2M}/_{T}}$ and that the member is not over-reinforced.

6.3.4 The general non-dimensional interaction equations can be produced theoretically in three ways:-

(a) By optimising the value of the torsional resistance on a skew bending failure plane

(b) By forming equations of equilibrium about two perpendicular axes

(c) By using the space truss analogy.

In each case the equations are of parabolic form and differ only in detail. The equations are suitable for analysis or design.

6.3.5 Available experimental evidence from various investigators shows that the interaction equations are reasonably accurate. The mean ratio of Texpt/Ttheo = 1.02 with a coefficient of variation of 11.8% for 121 experimental results.

6.3.6 The theoretical optimum value of the torsional resistance of a member occurs when

$$r_1' = \frac{F_1}{1 + {}^{d}/_{b} + \dfrac{2}{}\dfrac{(M_1}{T_1}\sqrt{\dfrac{f_{sy}}{f_{1y}}}\sqrt{F_1}} \quad \text{for mode 1}$$

and

$$r_3' = \frac{F}{1 + {}^{d}/_{b} - \dfrac{2\,M_3}{T_3}\sqrt{\dfrac{f_{sy}}{f_{y}}}\sqrt{F_3}} \quad \text{for mode 3.}$$

There are insufficient experimental results to confirm this. This value agrees approximately with the theoretical value obtained by Walsh et al [11], but not with the value obtained by Lessig[20].

6.4 Conclusions for Partially Over Reinforced and Over Reinforced Members.

6.4.1 The accuracy of the all yield theories becomes progressively worse as $r_{1y} < \frac{1}{1 + {}^{d}/_{b} + {}^{2M}/_{T}}$ for mode 1 and $r_{3y} < \frac{1}{1 + {}^{d}/_{b} - \frac{2M}{T}}$ for mode 3. These values are related to a 45° crack angle constraint.

6.4.2 The simple assumption that only the stirrups yield and that the cracks form at 45°, when r_{7} is less than the values given in the conclusion 6.4.1 produces the theoretical expression

$$T = \frac{2\ A_s\ f_{sy}\ b'd'}{S}$$

6.4.3 This expression produces a mean value of Texpt/Ttheory = 1.10 and a coefficient of variation of 22.9% for 120 experimental results.

6.4.4 The British Draft Unified Code of Practice for Reinforced Concrete includes a similar partial yield expression and constraint namely

$$T = \frac{1.6\ A_s\ f_y\ b'd'}{S}$$

and

$$r_{1y} < \frac{1}{1 + {}^{d}/_{b} + 1.6\ \frac{M}{T}}$$

6.4.5 This expression gives a mean value of ${}^{Texpt}/Ttheory = 1.34$ and a coefficient of variation of 25.1% for 127 experimental results.

6.4.6 An alternative to the partial yield expressions given in clauses 6.4.1 through 6.4.5 is an empirical formula.

$$T = 0.75\ db^2\ \sqrt{f_c'} + \frac{A_s\ f_{sy}\ b'd'/S}{\left[0.25 + 1.20\ (r_{12})^{2/3}\right]}$$

provided $f_{sy} \simeq f_{ly}$, $.08 < r_{12} < 1.9$, and subject to the limits given in 6.4.1. This expression was formed based on a few experimental results by Hsu[27] in pure torsion.

6.4.7 The empirical expression gives a mean $^{Texpt}/_{Ttheo} = 0.95$ and a coefficient of variation of 16.6% for 149 experimental results from 9 investigators.

6.4.8 The empirical expression for partial yielding given in clause 6.4.6 is continuous for the partial yielding of the stirrups and of the longitudinal steel in pure torsion. It is therefore suitable for determining the value of r_2 for which the torsional resistance is an optimum. The value of $r_2 \simeq \frac{1}{1+^b/_d}$, and agrees with the experimental value obtained by Hsu[27].

6.4.9 By equating the theoretical ultimate strength of a plain concrete section to the empirical formula for partial yielding it is possible to form an expression for the minimum percentage of steel. If this is arranged in terms of the total volume of reinforcement then

$$P_t > \frac{10(f_c')^{1/3}}{f_{sy}} \left[0.25 + 1.20\, (r_{12})^{2/3} \right] \left[(1+{}^{d'}/b') + {}^{1}/_{r_{12}} \right] ({}^{b'}/d')$$

It can also be expressed in terms of the volume of stirrups or of the volume of longitudinal steel. The values compare reasonably well with available experimental results.

6.4.10 There is insufficient available experimental and theoretical work on over reinforced members to produce a definite conclusion. Empirical formulas by Lessig[20], Walsh et al [26] and by Hsu[27] give reasonable agreement with the few experimental results. They may need to be modified to allow for the ratio of breadth to depth of a section, and to include the ratio of transverse steel to longitudinal steel.

6.5 Recommendations for Future Work.

6.5.1 Further experimental work required for plain concrete is to relate cube crushing strengths and cylinder crushing strengths, to the uniaxial tensile strength, the modulus of rupture and the tensile strength in bending and torsion.

6.5.2 Further experimental work for plain concrete to correctly establish the factor in pure torsion, either as 0.85 based on the Mohr failure criteria or as given by the St. Venant theory.

6.5.3 Experiments on a plain concrete trapezoidal cross section to establish the mode 3 form of failure.

6.5.4 Further experiments for members with longitudinal reinforcement in mode 1 form of failure, to establish the theory based on failure of the concrete.

6.5.5 Experiments to establish the dowel force action in members with longitudinal steel only.

6.5.6 Further experiments to confirm the yield of the steel type of failure for members with longitudinal steel only.

6.5.7 Experiments with members containing transverse and longitudinal steel at fixed $^M/_T$ ratios and varying ratios of transverse to longitudinal steel, to determine the variation in torsional resistance.

6.5.8 Existing yield and partial yield theories to be related to 6.5.7 and modified as required.

6.5.9 Systematic experiments on members with longitudinal and transverse steel to consider the variation produced by concrete strength and strength of longitudinal and transverse steel.

6.5.10 Further experiments on members with longitudinal and transverse steel in the partial yield condition. Results to be related to partial yield theories A and C as given in this thesis, with a view to improving the accuracy.

6.5.11 Further theoretical and experimental work to determine the conditions for over-reinforced members.

6.5.12 Theoretical and experimental work on combined, bending, torsion and shear in plain, reinforced, and prestressed concrete members.

References

1. Fisher G.P. and Zia P. - "Feview of Code Requirements for Torsion Design" - A.C.I.J., v.61, No.1. Jan 1964 pp. 1-44.

2. St. Venant - Mém. sav étrangers, vol. 14, 1855 (Todhunter and Pearson's History of the Theory of Elasticity, Cambridge, Vol. 2, 1893, p. 312).

3. Lessig N.N. - "The Determination of the Load Bearing Capacity of Rectangular Reinforced Concrete Sections subject to Combined Torsion and Bending", Study No.5, Concrete and Reinforced Concrete Institute (Moscow), 1959, pp. 5-28.

4. Hsu, Thomas T.C. - "Torsion of Structural Concrete - Plain Concrete Rectangular Sections," Torsion of Structural Concrete, Special Publication No. 18-8, American Concrete Institute, Detroit, 1966, pp. 203-238.

5. Turner L. and Davies V.C. - "Plain and Reinforced Concrete in Torsion", Selected Engineering Paper, No. 165, Institution of Civil Engineers, 1934.

6. Marshall W.T. and Tembe N.R. - "Experiments on Plain and Reinforced Concrete in Torsion", Structural Engineer, V.19, No.11, Nov. 1941, pp. 177-191.

7. Fisher D. - "The Strength of Concrete in Combined Bending and Torsion", Ph.D. Thesis, University of London, 1950.

8. Cowan H.J. - "The Strength of Plain, Reinforced and Prestressed Concrete Under the Action of Combined Stresses with Particular Reference to the Combined Bending and Torsion of Rectangular Sections", Magazine of Concrete Research, V.5, No.14, Dec 1953, pp. 75-86.

9. Nadai A. - "Plasticity", McGraw-Hill (New York), 1931.

10. Cowan H.J. and Armstrong S. - "Experiments on the Strength of Reinforced and Prestressed Concrete Beams and of Concrete-Encased Steel Joists in Combined Bending and Torsion", Magazine of Concrete Research, V.7, No. 19, March 1955, pp. 3-20.

11. Walsh P.F., Collins M.P., Archer F.E., and Hall A.S. - "Experiments on the Strength of Concrete Beams in Combined Flexure and Torsion", University of New South Wales, UNICIV report No. R15 Feb 1966.

12. Anderson P. - "Rectangular Concrete Sections Under Torsion" A.C.I.J. Proc., V.34, No.1, Sept-Oct 1937, pp. 1-11.

13. Nylander H. - "Vridning och Vridningsinspanning vid Betong Konstruktioner", (Torsion and Torsional Restraint by Concrete Structures), Statens Kommittee för Byggnadsforskning, Stockholm, Bulletin No.3, 1945.

14. Gesund H. and Boston L.A. - "Ultimate Strength in Combined Bending and Torsion of Concrete Beams Containing Only Longitudinal Reinforcement", A.C.I.J., V.61, No. 11, Nov 1964, pp. 1453-71.

15. Iyengar K.T., Sundara Raja and Rangan B. Vijaya - "Strength and Stiffness of Reinforced Concrete Beams Under Combined Bending Torsion", Special Publication No. 18-15, American Concrete Institute, Detroit, 1966, pp. 403-440.

16. Hsu T.T.C. - "Torsion of Structural Concrete - Interaction Surface for Combined Torsion, Shear and Bending in Beams without Stirrups", A.C.I.J., Jan 1968, Vol.65, pp. 51-60.

17. Mirza M.S. and McCutcheon J.O. - "The Behaviour of Reinforced Concrete Beams Under Combined Bending, Shear and Torsion", A.C.I.J. Proc, V.66, No.5, May 1969, pp. 421-427.

18. Ransch E. - "Berechnung des Eisenbetons gegen Verdrehung", Julius Springer, Berlin, 1929.

19. Cowan H.J. - "Elastic Theory for the Torsional Strength of Rectangular Reinforced Beams", Magazine of Concrete Research, Vol.2, No. 4, July 1950.

20. Lessig N.N. - "Study of Cases of Failure of Concrete in Reinforced Concrete Elements with Rectangular Cross-section Subjects to Combined Flexure and Torsion", Moscow 1961, (partial translation by Margaret Corbin, Foreign Literature Study No. 398 for the Portland Cement Association, Skokie, Ill.).

21. Chinekov Ju. V. - "Study of the Behaviour of Reinforced Concrete Elements in Combined Flexure and Torsion", Institut Betana i Zhelezobetona, Study No.5, Moscow, 1959 (translation by Margaret Corbin, Foreign Literature Study No. 370 for the Portland Cement Association, Skokie, Ill.).

22. Lyalin I.M. - "Experimental Studies of the Behaviour of Reinforced Concrete Beams with Rectangular Cross Section subjected to the Combined Action of Transverse Force, Flexural and Torsional Moment", Institut Betona i Zhelezobetona, Study No.5, 1959, PP. 54-77. (translation by Margaret Corbin, Foreign Literature Study No.402 for the Portland Cement Association, Skokie, Ill.).

23. Yudin V.K. - "Determination of the Load-Bearing Capacity of Reinforced Concrete Elements of Rectangular Cross-section Under Combined Torsion and Bending", Beton i Zhelezobeton, No.6, June 1962, pp. 265-268.

24. Gesund H., Shuette F.J., Buchanan G.R. and Gray G.A. - "Ultimate Strength in Combined Bending and Torsion of Concrete Beams Containing Both Longitudinal and Transverse Reinforcement", A.C.I.J., Proc. V.61, No.12, Dec 1964 pp. 1509-1521.

25. Evans R.H. and Sarkar S. - "A method of Ultimate Strength Design of Reinforced Concrete Beams in Combined Bending and Torsion", Structural Engineer, V.43, No.10, October 1965, pp. 337-344.

26. Walsh P.E., Collins M.P., Archer F.E., and Hall A.S. - "The Ultimate Strength Design of Rectangular Reinforced Concrete Beams Subjected to Combined Torsion, Bending and Shear", Transactions of the Institution of Civil Engineers, Australia, October 1966, pp. 143-157.

27. Hsu T.T.C. - "Torsion of Structural Concrete, Behaviour of Reinforced Rectangular Members", Special Publication No. 18-10, American Concrete Institute, Detroit, 1968, pp. 261-306.

27a Hsu T.T.C. - "Ultimate Torque of Reinforced Rectangular Beams", Am. Soc. of C.E., Struct. Div., Feb 1968, pp. 485-510.

28. Goode C.D. and Helmy M.A. - "Ultimate Strength of Reinforced Concrete Beams in Combined Bending and Torsion", Special Publication No. 18-13, American Concrete Institute, Detroit, 1968 pp. 357-377.

29. Krishnaswamy K.T. - "Some Studies of the Biaxial Strength of Concrete and Mortar". Thesis submitted for M.Sc. Degree, Indian Institute of Science, Bangalore, 1964.

30. Zia P. and Cardenas R. - "Combined Bending and Torsion of Reinforced Plaster Model Beams", Special Publication No. 18-12, American Concrete Institute, Detroit, 1968 pp. 337-356.

31. Pandit G.S. and Warwaruk J. - "Reinforced Concrete Beams in Combined Bending and Torsion", Special Publication No. 18-5, American Concrete Institute, Detroit, 1968 pp. 133-163.

32. Fairburn D.R. and Davies S.R. - "Combined Bending and Torsion in Reinforced Concrete Beams", Structural Engineer, April 1969, Vol. 47, No.4, pp. 151-156.

33. Swann R.A. - "Experimental Basis for a Design Method for Rectangular Reinforced Concrete Beams In Torsion", London, Cement and Concrete Association, Technical Report 42.452, 1970, pp 38.

34. Jackson N. and Estañero R.A. - "The Plastic Flow Law for Reinforced Concrete Beams Under Combined Flexure and Torsion." Magazine of Concrete Research, Vol. 23, No. 77, Dec. 1971, pp. 169-179.

35. Kuyt B. - "A Theoretical Investigation of Ultimate Torque as Calculated by Truss Theory and by the Russian Equilibrium Method", Magazine of Concrete Research, Vol. 23, No. 77, Dec 1971, pp. 155-160.

36. Kuyt B. - "A Method for Ultimate Strength Design of Rectangular Reinforced Concrete Beams in Combined Torsion, Bending and Shear", Magazine of Concrete Research, Vol. 24, No. 78, March 1972, pp. 15-24.

37. Kemp E.L. - "Behaviour of Concrete Members Subject to Torsion and to Combined Torsion, Bending and Shear", Special Publication, No. 18-7, American Concrete Institute, Detroit, 1966, pp. 179-201.

38. Wright, P.J.F., - "The Effect of the Method of Test on the Flexural Strength of Concrete", Mag. of Conc. Res., No.11, Oct 1952, pp. 67-76.

39. Zia P. - "Torsional Strength of Prestressed Concrete Members", A.C.I.J., Proc. V.57, No. 10, April 1961, pp. 1337-1359.

40. Collins M.P., Walsh P.F., Archer F.E. and Hall A.S. - "Reinforced Concrete Beams Subjected to Combined Torsion Bending and Shear", University of New South Wales, UNICIV Report No. R-14, October 1965.

41. Ramakrishnan V. and Vijaya R.B. - "The Influence of Combined Bending and Torsion on Rectangular Beams without Web Reinforcement", Indian Concrete Journal Vol 37, No. 11, Nov 1963.

42. Ernst G.C. - "Ultimate Torsional Properties of Rectangular Reinforced Concrete Beams", A.C.I.J. October 1957.

43. Bach C. and Graf O. - "Tests on the Resistance of Plain and Reinforced Concrete to Torsion", Deutscher Ausschuss fur Eisenbeton, Berlin, Heft 16, 1912.

44. Young C.R., Sagar W.L. and Hughes C.A. - "Torsional Strength of Rectangular Sections of Concrete, Plain and Reinforced", University of Toronto, School of Engineering. Bulletin No.3, 1922.

45. Turner L. and Davies V.C. - "Plain and Reinforced Concrete in Torsion with Particular Reference to Reinforced Concrete Beams", Institution of Civil Engineers, London, Selected Engineering Papers, No. 165, 1934.

46. Cowan H.J. - "Elastic Theory for the Torsional Strength of Rectangular Reinforced Beams", Magazine of Concrete Research, Vol.2, No.4, July 1950.

47. Gvozdev A.A., Lessig N.N. and Rulle L.K. - "Research in Reinforced Concrete Beams Under Combined Bending and Torsion in the Soviet Union", Special Publication No. 18-11 American Concrete Institute, Detroit, 1968 pp 307-336.

A P P E N D I X

Table 2.9.1 - Detailed Results for Plain Concrete of Rectangular Cross Section Failing in Mode 1

Investigator	Beam	Dimensions b x d in (mm)	f_c' lbf/in^2 (N/mm^2)	Experimental		Theory equ^n 2.8.2 $T_,$ kipf in (KNm)	Ratio $\frac{Texpt}{T_, theo}$	Ratio $\frac{Texpt}{T'_{u,} theo}$	Ratio $\frac{Mexpt}{M_, theo}$	$(\frac{Texpt}{T'_{u,} theo})^2 + \frac{Mexpt}{M_{u,} theo}$ equ^n 2.8.1
				T kipf in (KNm)	M kipf in (KNm)					
Walsh et al [11]	P1	6 x 9 (152.4 x (228.6)	7100 (48.95)	37.4 (4.23)	24.7 (2.79)	32.1 (3.63)	1.17	0.63	0.59	0.98
	P2		6350 (43.78)	25.2 (2.85)	39.6 (4.47)	16.7 (1.89)	1.51	0.44	0.98	1.17
	P5		7450 (51.37)	-	51.3 (5.80)	-	-	-	1.21	1.21
	P6		7400 (51.02)	11.1 (1.25)	51.4 (5.81)	6.5 (0.73)	1.72	0.19	1.21	1.24
	CPR1		6300 (43.44)	23.3 (2.63)	36.6 (4.14)	16.7 (1.89)	1.40	0.41	0.91	1.07
	REP2		4600 (31.72)	29.6 (3.34)	20.3 (2.29)	27.2 (3.07)	1.09	0.57	0.56	0.89
	RUP2		3680 (25.37)	19.0 (2.15)	35.7 (4.03)	11.9 (1.34)	1.59	0.40	1.06	1.22

Table 2.9.1 -(continued) - Detailed Results for Plain Concrete of Rectangular Cross Section Failing in Mode 1

Investigator	Beam	Dimensions b x d in (mm)	f_t lbf/in^2 (N/mm^2)	Experimental		Theory equ^n 2.8.2 $T_{,}$ kipf in (KNm)	Ratio $\frac{Texpt}{T_{,}\ theo}$	Ratio $\frac{Texpt}{T'_{u,}\ theo}$	Ratio $\frac{Mexpt}{M_{u,}}$	$\left(\frac{Texpt}{T'_{u,}\ theo}\right)^2 + \frac{Mexpt}{M_{u,}\ theo}$ equ^n 2.8.1
				T kipf in (KNm)	M kipf in (KNm)					
Hsu[4]	A3	10 x 10 (254.0 x 254.0)	367 (2.53)	102 (11.53)	12.7 (1.44)	89.2 (10.08)	1.14	1.14	0.19	1.50
	A4			100 (11.30)	12.7 (1.44)	89.2 (10.08)	1.12	0.12	1.19	1.45
			u_w lbf/in^2 (N/mm^2)							
Cowan and Armstrong[10]	T2	6 x 9 (152.4 x 228.6)	6620 (45.64)	-	47.0 (5.31)	38.1 (4.31)	-	-	1.23	1.23
	T3		8240 (56.81)	21.2 (2.40)	42.2 (4.77)	13.7 (1.55)	1.54	0.36	1.03	1.16

$f_c' = 0.8\ u_w$

Table 2.9.2 - Detailed Results for Plain Concrete of Rectangular Cross Section Failing in Mode 2

Investigator	Beam	Dimensions b x d in (mm)	f_c' lbf/in^2 (N/mm^2)	Experimental		Theory equn 2.8.6 T'_{u2} kipf in (KNm)	Ratio $\frac{T_{expt}}{T'_{u2}\ theo}$
				T kipf in (KNm)	M kipf in (KNm)		
Walsh et al [11]	P3	6 x 9 (152.4 x 228.6)	6550 (45.16)	47.6 (5.38)	5.5 (0.62)	48.7 (5.50)	0.98
	P4		6363 (43.87)	43.0 (4.86)	5.5 (0.62)	48.2 (5.45)	0.89
	R1	4 x 6 (101.6 x 152.4)	7200 (49.64)	13.4 (1.51)	-	17.6 (1.99)	0.76
	REP4	6.5 x 10 (165.1 x 254.0)	4600 (31.72)	50.5 (5.71)	-	54.8 (6.19)	0.92
	RUP4		3680 (25.37)	66.3 (7.49)	-	50.9 (5.75)	1.30
	W1	6.8 x 10.4 (172.7 x 264.2)	4630 (31.92)	73.2 (8.27)	-	61.4 (6.94)	1.19
	W2	6.5 x 10 (165.1 x 254.0)	4320 (29.79)	74.5 (8.42)	-	53.6 (6.06)	1.39
	W3	6.9 x 10.2 (175.3 x 259.1)	4050 (27.92)	77.4 (8.75)	-	58.8 (6.64)	1.32

Table 2.9.2 -(continued)- Detailed Results for Plain Concrete in Rectangular Cross Section Failing in Mode 2

Investigator	Beam	Dimensions b x d in (mm)	U_w lbf/in^2 (N/mm^2)	Experimental T kipf in (KNm)	Experimental M kipf in (KNm)	Theory equn 2.8.6 T'_{u2} kipf in (KNm)	Ratio $\frac{T_{expt}}{T'_{u2}\ theo}$
Iyengar and Rangan [15]	A1	5 x 8 (127.0 x 203.2)	3980 (27.44)	23.0 (2.60)	-	26.1 (2.95)	0.88
	A2		3785 (26.10)	22.0 (2.49)	-	25.6 (2.89)	0.86
	A3		4460 (30.75)	22.0 (2.49)	-	27.0 (3.05)	0.82
Cowan and Armstrong [10]	T1	6 x 9 (152.4 x 228.6)	7720 (53.23)	58.1 (6.57)	-	47.8 (5.40)	1.22
			f_c' lbf/in^2 (N/mm^2)				
Zia [39]	RP1	4 x 12 (101.6 x 304.8)	6000 (41.37)	28.55 (3.23)	-	35.39 (4.00)	0.81

$f_c' = 0.8\ uw$

Table 2.9.2 (continued) - Detailed Results for Plain Concrete of Rectangular Cross Section Failing in Mode 2

Investigator	Beam	Dimensions	f_c' lbf/in^2 (N/mm^2)	Experimental		Theory equn 2.8.6 T'_{u2} kipf in (kNm)	Ratio $\frac{T_{expt}}{T'_{u2}\ theo}$
				T kipf in (KNm)	M kipf in (KNm)		
Zia[39]	RP2	4 x 12 (101.6 x 304.8)	6200 (42.75)	26.36 (2.98)	-	35.78 (4.04)	0.74
	RP3		6350 (43.78)	24.24 (2.74)	-	36.06 (4.07)	0.67
	RP4		6350 (43.78)	27.44 (3.10)	-	36.06 (4.07)	0.76
	RP5		6400 (44.13)	28.04 (3.17)	-	36.16 (4.09)	0.78
	RP6		6400 (44.13)	32.04 (3.62)	-	36.16 (4.09)	0.89
	RP7		7180 (49.50)	28.24 (3.19)	-	37.57 (4.25)	0.75
	RP8		6500 (44.82)	29.76 (3.36)	-	36.35 (4.11)	0.82
	RP9		6950 (47.92)	31.24 (3.53)	-	37.17 (4.20)	0.84

Table 2.9.2 -
continued - Detailed Results for Plain Concrete of Rectangular Cross Section Failing in Mode 2

Investigator	Beam	Dimensions b x d in (mm)	f_t lbf/in^2 (N/mm^2)	Experimental		Theory equn 2.8.6 T'_{u2} kipf in (KNm)	Ratio $\frac{T_{expt}}{T'_{u2} theo}$
				T kipf in (KNm)	M kipf in (KNm)		
Hsu[4]	A1	10 x 15 (254.0 x 381.0)	354* (2.44)	162 (18.31)	-	155.7 (17.59)	1.04
	A2		354* (2.44)	169 (19.10)	-	155.7 (17.59)	1.09
	A5	10 x 20 (254.0 x 508.0)	343 (2.36)	216 (24.41)	-	208.5 (23.56)	1.04
	A6		346 (2.39)	216 (24.41)	-	209.7 (23.70)	1.03
	A7	6 x 11 (152.4 x 279.4)	354* (2.44)	54 (6.10)	-	48.5 (5.48)	1.11
	A8		354* (2.44)	56.5 (6.38)	-	48.5 (5.48)	1.16
	A9	6 x 19.5 (152.4 x 495.3)	361 (2.49)	101 (11.41)	-	91.7 (10.36)	1.10
	A10		338 (2.33)	85 (9.61)	-	87.8 (9.92)	0.97

* average value of f_t

Table 2.9.2 (continued) - Detailed Results for Plain Concrete of Rectangular Cross Section Failing in Mode 2

Investigator	Beam	Dimensions b x d in (mm)	f_t lbf/in^2 (N/mm^2)	Experimental		Theory equn 2.8.6 T'_{u2} kipf in (Knm)	Ratio $\frac{T_{expt}}{T'_{u2}\,theo}$
				T kipf in (KNm)	M kipf in (KNm)		
Marshall and Tembe [6]	A1	4 x 6 (101.6 x 152.4)	282 (1.94)	10.25 (1.16)	-	11.73 (1.33)	0.87
	A2	3.9 x 6 (99.1 x 152.4)		9.74 (1.10)	-	11.27 (1.27)	0.86
	A3	3.9 x 6 (99.1 x 154.9)		10.25 (1.16)	-	11.48 (1.30)	0.89
	A4	3.9 x 6 (99.1 x 152.4)		10.35 (1.17)	-	11.27 (1.27)	0.92
	A5	1.9 x 6.1 (48.3 x 154.9)		3.59 (0.41)	-	3.70 (0.42)	0.97
	A6	1.85 x 6.1 (47.0 x 154.9)		3.08 (0.35)	-	3.54 (0.40)	0.87

Table 2.9.2 (continued) - Detailed Results for Plain Concrete of Rectangular Cross Section Failing in Mode 2

Investigator	Beam	Dimensions	f_t lbf/in^2 (N/mm^2)	Experimental		Theory equn 2.8.6 T'_{uz} kipf in (Knm)	Ratio $\frac{Texpt}{T'_{uz} theo}$
				T kipf in (KNM)	M kipf in (KNm)		
Marshall and Tembe [6]	A7	2.8 x 6.1 (71.1 x 154.9)	282 (1.94)	6.15 (0.69)	-	6.83 (0.77)	0.90
	A8	3.05 x 6.08 (77.5 x 154.4)		7.18 (0.81)	-	7.78 (0.88)	0.92
	A9	3.55 x 6.1 (90.2 x 154.9)		9.52 (1.08)	-	9.91 (1.12)	0.96
	A10	3.5 x 6.1 (88.9 x 154.9)		9.22 (1.04)	-	9.69 (1.09)	0.95
	A11	3.95 x 4.1 (100.3 x 104.1)		6.86 (0.78)	-	7.52 (0.85)	0.91
	A12	4.0 x 4.25 (101.6 x 108.0)		7.06 (0.80)	-	7.99 (0.90)	0.88

Table 2.10.1 - Detailed Results for Plain Concrete of Circular Cross Section Failing in Mode 1

Investigator	Beam	Dimensions diameter in (mm)	$f_{r7.5}$ lbf/in^2 (N/mm^2)	Experimental		Theory equn 2.8.2 T_1 kipf in (KNm)	Ratio $\frac{Texpt}{T_1\ theo}$	Ratio $\frac{Texpt}{T_{u1}\ theo}$	Ratio $\frac{Mexpt}{M_{u1}\ theo}$	$\left(\frac{Texpt}{T_{u1}\ theo}\right)^2 + \frac{Mexpt}{M_{u1}\ theo}$ equn 2.8.1
				T kipf in (KNm)	M kipf in (kNm)					
Fisher[7] (preferred results)	5	7.5 (190.5)	409 (2.82)	30.25 (3.42)	5.20 (0.59)	23.72 (2.68)	1.28	1.08	0.31	1.46
	7			29.00 (3.28)	3.90 (0.44)	24.59 (2.78)	1.18	1.03	0.23	1.29
	8			25.87 (2.92)	6.50 (0.73)	21.92 (2.48)	1.06	0.92	0.38	1.23
	9			19.30 (2.18)	7.80 (0.88)	18.96 (2.14)	1.02	0.69	0.46	0.93
	10			19.70 (2.23)	9.10 (1.03)	17.99 (2.03)	1.10	0.70	0.54	1.03
	11			19.50 (2.20)	11.70 (1.32)	15.93 (1.80)	1.22	0.69	0.69	1.17
	15			30.90 (3.49)	-	28.14 (3.18)	1.10	1.10	-	1.21
	16			-	16.50 (1.86)	-	-	-	0.97	0.97

Table 2.10.1 (continued) - Detailed Results for Plain Concrete of Circular Cross Section Failing in Mode 1

Investigator	Beam	Dimensions diameter in (mm)	$f_{r7.5}$ lbf/in² (N/mm²)	Experimental		Theory equn 2.8.2 $T_{\prime}$ kipf in (KNm)	Ratio $\frac{Texpt}{T_{\prime}\ theo}$	Ratio $\frac{Texpt}{T_{u\prime}\ theo}$	Ratio $\frac{Mexpt}{M_{u\prime}\ theo}$	$\left(\frac{Texpt}{T_{u\prime}\ theo}\right)^2 + \frac{Mexpt}{M_{u\prime}\ theo}$ equn 2.8.1
				T kipf in (KNm)	M kipf in (KNm)					
Fisher[7] (preferred results)	17	7.5 (190.5)	409 (2.82)	6.50 (0.73)	12.75 (1.44)	6.76 (0.76)	0.96	0.22	0.76	0.81
	19			12.00 (1.36)	13.00 (1.47)	11.00 (1.24)	1.09	0.43	0.77	0.95
	20			4.45 (0.50)	18.00 (2.03)	3.42 (0.39)	1.30	0.16	1.06	1.09
	23			8.13 (0.92)	15.16 (1.71)	7.06 (0.80)	1.15	0.29	0.89	0.98
	24A			20.00 (2.26)	6.75 (0.76)	20.20 (2.28)	0.99	0.71	0.40	0.90
	24B			-	24.6 (2.78)	-	-	-	1.45	1.45
	27			23.80 (2.69)	7.80 (0.88)	20.39 (2.30)	1.17	0.85	0.46	1.18

Table 2.10.1 (continued) - Detailed Results for Plain Concrete of Circular Cross Section Failing in Mode 1

Investigator	Beam	Dimensions diameter in (mm)	f_t lbf/in^2 (N/mm^2)	Experimental		Theory equn 2.6.11 T'_{u2} lbf in (KNm)	Ratio $\frac{T_{expt}}{T'_{u2}\ theo}$
				T kipf in (KNm)	M kipf in (KNm)		
Marshall and Tembe[6]	01	4.80 (121.9)	282 (1.94)	7.59 (0.86)	-	7.97 (0.90)	0.95
	02	4.88 (124.0)		7.28 (0.82)	-	8.29 (0.94)	0.88
	03	4.94 (125.5)		6.91 (0.78)	-	8.54 (0.97)	0.81
	04	4.92 (125.0)		7.69 (0.87)	-	8.46 (0.96)	0.91

Table 3.5.1 - Detailed Results for Members with Longitudinal Steel Only Failing in Model 1
(Based on failure of the concrete)

Investigator	Dimensions b x d in (mm)	d, in (mm)	A, in^2 (mm^2)	f_c' lbf/in^2 (N/mm^2)	Experimental T, kipf in (kNm)	Experimental M, kipf in (kNm)	Theory equn 3.2.5 T, kipf in (kNm)	Ratio $\frac{T, expt}{T, theory}$	Ratio $\frac{T, expt}{T_{cu}, theory}$	Ratio $\frac{M, expt}{M_{cu}, theory}$
Gesund & Boston [14]	8 x 8 (203.2 x 203.2)	6.75 (171.5)	0.59 (380.6)	2320 (16.00)	43.0 (4.86)	86.0 (9.72)	42.2 (4.77)	1.02	1.25	0.50
					36.0 (4.07)	108.0 (12.20)	39.0 (4.41)	0.92	1.02	0.61
Ramakrishnan & Vijayarangan [41]	5 x 8 (127.0 x 203.2)	6.75 (171.5)	0.58 (374.2)	1970 (13.58)	20.1 (2.27)	90.7 (10.25)	20.0 (2.26)	1.01	0.90	0.82
				2200 (15.17)	23.2 (2.62)	105.0 (11.87)	21.9 (2.47)	1.06	0.95	0.86
				2000 (13.79)	21.7 (2.45)	90.7 (10.25)	21.2 (2.40)	1.02	0.96	0.81

Table 3.6.1 - Detailed Results for Members with Longitudinal Steel Only and Failing in Mode 1 (based on failure of the steel)

Investigator	Dimensions b x d in (mm)	d, in (mm)	A, in^2 (mm^2)	f_y $kipf/in^2$ (N/mm^2)	f_c' lbf/in^2 (N/mm^2)	Experimental T, kipf in (kNm)	Experimental M, kipf in (kNm)	Theory equn 3.3.4 T, kipf in (kNm)	Ratio $T_{,expt}$ / $T_{,theory}$	Ratio $T_{,expt}$ / $T_{su,theory}$	Ratio $M_{,expt}$ / $M_{su,theory}$
Nylander 13	7.875 x 7.875 (200.0 x 200.0)	7.08 (179.8)	0.244 (157.4)	43.25 (298.2)	3330 (22.96)	39.0 (4.41)	52.1 (5.89)	32.2 (3.64)	1.21	0.96	0.74
						31.2 (3.53)	52.1 (5.89)	29.4 (3.32)	1.06	0.77	0.74
				43.70 (301.3)		39.1 (4.42)	58.0 (6.55)	31.2 (3.53)	1.25	0.95	0.82
						35.2 (3.98)	58.0 (6.55)	29.9 (3.38)	1.18	0.85	0.81
				38.65 (266.5)	3390 (23.37)	31.3 (3.54)	58.0 (6.55)	25.2 (2.85)	1.25	0.85	0.91
				36.95 (254.8)		19.5 (2.20)	58.0 (6.55)	18.8 (2.12)	1.04	0.52	0.90
Gesund & Boston 14	8 x 8 (203.2 x 203.2)	6.81 (173.0)	0.33 (212.9)	50.00 (344.7)	4360 (30.06)	58.0 (6.55)	58.0 (6.55)	51.7 (5.84)	1.12	0.97	0.56
			0.58 (374.2)	31.20 (215.1)	2180 (15.03)	23.2 (2.62)	100.8 (11.39)	21.9 (2.47)	1.06	0.39	0.98
		6.75 (171.5)	0.59 (380.6)	51.00 (351.6)	5590 (38.54)	59.0 (6.67)	177.0 (20.0)	54.3 (6.14)	1.09	0.54	0.94
						49.0 (5.54)	195.0 (22.0)	43.7 (4.94)	1.12	0.47	1.08
Ramakrishnan & Vijayarangan 41	5 x 8 (127.0 x 203.2)	6.75 (171.5)	0.39 (251.6)	29.60 (204.1)	3160 (21.79)	17.1 (1.93)	99.0 (11.19)	12.0 (1.36)	1.42	0.41	1.36
			0.39 (251.6)	32.40 (223.4)	2420 (16.69)	10.7 (1.21)	108.0 (12.20)	7.6 (0.86)	1.41	0.24	1.38
			0.58 (374.2)	31.20 (215.2)	2150 (14.82)	21.7 (2.45)	111.0 (12.54)	19.1 (2.16)	1.14	0.37	1.07
			0.39 (251.6)	32.40 (223.4)	2420 (16.69)	23.2 (2.62)	108.0 (12.20)	21.8 (2.46)	1.07	0.37	1.00

Table 3.7.1 - Detailed Results for Members with Longitudinal Steel Only Subject to Torsion and Failing in Mode 2.

Investigator	Dimensions b x d in (mm)		f_c' lbf/in^2 (N/mm^2)	Expt T_2 kipf in (kNm)	Theory T_{u2}' equn 2.5.10 kipf in (kNm)	Ratio $\frac{T_{expt}}{T_{u2}'\ theo}$
Young Sagar & Hughes 44	5.0 x 5.0 (127.0 x 127.0)		1700 (11.72)	14.0 (1.58)	12.6 (1.42)	1.12
	5.0 x 7.5 (127.0 x 190.5)			22.7 (2.57)	19.7 (2.23)	1.15
	5.0 x 10.0 (127.0 x 254.0)			36.7 (4.15)	27.0 (3.05)	1.36
Turner and Davies 45	5.0 x 5.0 (127.0 x 127.0)		2500 (17.24)	12.5 (1.41)	14.2 (1.60)	0.88
	4.0 x 8.0 (101.6 x 203.2)			12.0 (1.36)	17.0 (1.92)	0.71
Anderson 12	10.0 x 10.0 (254.0 x 254.0)		2100 (14.48)	69.8 (7.89)	85.0 (9.61)	0.83
			2250 (15.51)	77.1 (8.71)	86.5 (9.77)	0.89
				80.1 (9.05)	86.5 (9.77)	0.93
			3600 (24.82)	84.5 (9.55)	101.1 (11.42)	0.84
				88.5 (10.00)	101.1 (11.42)	0.87
			3680 (25.37)	97.6 (11.03)	101.9 (11.51)	0.96
			5000 (34.47)	105.1 (11.88)	112.8 (12.75)	0.93
				109.2 (12.34)	112.8 (12.75)	0.97
			5200 (35.85)	119.9 (13.55)	114.3 (12.92)	1.05
	8.0 x 8.0 (203.2 x 203.2)		3900 (26.89)	50.0 (5.65)	55.9 (6.32)	0.90
				73.0 (8.25)	55.9 (6.32)	1.30
				90.0 (10.17)	55.9 (6.32)	1.61
			7000 (48.26)	62.0 (7.01)	67.9 (7.67)	0.91
				98.0 (11.07)	67.9 (7.67)	1.44
				122.0 (13.79)	67.9 (7.67)	1.80
Nylander 13	3.7 x 7.9 (94.0 x 200.7)		2550 (17.58)	13.0 (1.47)	14.9 (1.68)	0.87
				13.0 (1.47)	14.9 (1.68)	0.87
			3080 (21.24)	15.6 (1.76)	15.9 (1.80)	0.98
				14.7 (1.66)	15.9 (1.80)	0.92
Cowan 46	6.0 x 10.0 (152.4 x 254.0)		3600 (24.82)	36.0 (4.07)	44.8 (5.06)	0.80

Table 3.7.1 (continued) - Detailed Results for Members with Longitudinal Steel Only Subject to Torsion and Failing in Mode 2.

Investigator	Dimensions b x d in (mm)		f_c' lbf/in² (N/mm²)	Expt T_2 kipf in (kNm)	Theory T_{u2}' equn 2.5.10 kipf in (kNm)	Ratio $\frac{T_{expt}}{T_{u2}' \text{ theo}}$
Ernst [42]	6.0 x 12.0 (152.4 x 304.8)		3923 (27.05)	37.6 (4.25)	56.3 (6.36)	0.67
				34.4 (3.89)	56.3 (6.36)	0.61
				33.8 (3.82)	56.3 (6.36)	0.60
Marshall & Tembe 6	4.0 x 6.0 (101.6 x 152.4)		2720 (18.75)	11.8 (1.33)	12.7 (1.44)	0.93
				11.3 (1.28)	12.7 (1.44)	0.89
				11.8 (1.33)	12.7 (1.44)	0.93
				11.3 (1.28)	12.7 (1.44)	0.89
				11.9 (1.34)	12.7 (1.44)	0.93
				10.8 (1.22)	12.7 (1.44)	0.85
Bach and Graf [43]	11.8 x 11.8 (299.7 x 299.7)		3000 (20.68)	156.0 (17.63)	152.4 (17.22)	1.02
				173.4 (19.59)	152.4 (17.22)	1.14
				160.2 (18.10)	152.4 (17.22)	1.05
				160.2 (18.10)	152.4 (17.22)	1.05
				180.0 (20.34)	152.4 (17.22)	1.16
				173.4 (19.59)	152.4 (17.22)	1.14
	8.3 x 16.5 (210.8 x 419.1)			130.0 (14.69)	121.4 (13.72)	1.07
				136.8 (15.46)	121.4 (13.72)	1.13
				136.8 (15.46)	121.4 (13.72)	1.13
				141.0 (15.93)	121.4 (13.72)	1.16
				130.0 (14.69)	121.4 (13.72)	1.07
				141.0 (15.93)	121.4 (13.72)	1.16
Nylander [13]	3.74 x 7.875 (95.0 x 200.0)		2860 (19.72)	13.0 (1.47)	15.7 (1.77)	0.83
				13.0 (1.47)	15.7 (1.77)	0.83
			3000 (20.68)	13.0 (1.47)	16.0 (1.81)	0.81
				13.0 (1.47)	16.0 (1.81)	

Table 3.7.1 (continued) - Detailed Results for Members with Longitudinal Steel Only Subject to Torsion and Failing in Mode 2.

Investigator	Dimensions b x d in (mm)	Area of Steel A_z	f_c' lbf/in² (N/mm²)	Expt T_2 kipf in (kNm)	Theory T'_{u2} equn 2.5.10 kipf in (kNm)	Ratio $\frac{T_{expt}}{T'_{u2}\ theo}$
Gesund & Boston 14	8.0 x 8.0 (203.2 x 203.2)		4380 (30.2)	36.0 (4.07)	58.1 (6.57)	0.62
				39.0 (4.41)	58.1 (6.57)	0.67
Walsh et al 26	6.0 x 9.0 (152.4 x 228.6)		7225 (49.81)	61.0 (6.89)	50.3 (5.68)	1.21
Ramakrishnan & Vijayarangan 41	5.0 x 8.0 (127.0 x 203.2)		3120 (21.51)	28.8 (3.25)	25.9 (2.93)	1.11
			3100 (21.37)	28.8 (3.25)	25.9 (2.93)	1.11
			2640 (18.20)	26.1 (2.95)	24.5 (2.77)	1.07
			2180 (15.03)	23.2 (2.62)	22.9 (2.59)	1.01
			2000 (13.79)	21.7 (2.45)	22.3 (2.52)	0.97

Table 3.7.2 - Detailed Results for Members with Longitudinal Steel Only Subject to Torsion and Bending and Failing in Mode 2

Investigator	Dimensions b x d in (mm)	f ' lbf/in² (N/mm²)	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory T'_{u2} equn 2.5.10 kipf in (kNm)	Ratio Texpt / T'_{u2} theo
Nylander [13]	3.74 x 7.875 (95.0 x 200.0)	3000 (20.68)	13.0 (1.47)	9.25 (1.05)	16.0 (1.81)	0.81
			13.0 (1.47)	9.25 (1.05)	16.0 (1.81)	0.81
		2870 (19.79)	14.3 (1.62)	48.40 (5.47)	15.7 (1.77)	0.91
			13.9 (1.57)	48.40 (5.47)	15.7 (1.77)	0.89
		2740 (18.39)	16.5 (1.86)	72.6 (8.20)	15.5 (1.75)	1.06
			15.6 (1.76)	72.6 (8.20)	15.5 (1.75)	1.01
	7.875 x 7.875 (200.0 x 200.0)	3180 (21.93)	54.7 (6.18)	75.5 (8.53)	50.0 (5.65)	1.09
			50.7 (5.73)	75.5 (8.53)	50.00 (5.65)	1.01
		3480 (23.99)	50.7 (5.73)	110.0 (12.43)	51.6 (5.83)	0.98
			54.7 (6.18)	110.0 (12.43)	51.6 (5.83)	1.06
Gesund and Boston 14	8.0 x 8.0 (203.2 x 203.2)	4360 (30.06)	64.0 (7.23)	64.0 (7.23)	58.0 (6.55)	1.10
		4700 (32.41)	42.0 (4.75)	83.0 (9.38)	59.8 (6.76)	0.70
		2800 (19.31)	39.0 (4.41)	156.0 (17.63)	50.4 (5.70)	0.77
Walsh et al 26	6.0 x 9.0 (152.4 x 228.6)	6360 (43.85)	57.0 (6.44)	32.2 (3.64)	48.2 (5.45)	1.18
		6100 (42.06)	47.8 (5.40)	80.5 (9.10)	47.6 (5.38)	1.00
		6580 (45.37)	60.1 (6.79)	205.7 (23.24)	48.8 (5.51)	1.23
		6470 (44.61)	49.6 (5.60)	263.5 (29.78)	48.5 (5.48)	1.02
Ramakrishnan & Vijayarangan 41	5.0 x 8.0 (127.0 x 203.2)	2330 (16.06)	24.8 (2.80)	45.4 (5.13)	23.5 (2.66)	1.06

Table 4.5.1 - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 1.

$$* \; r_{,y} > \frac{1}{1 + {}^{d}/_{b} + {}^{2M}/_{T}}$$

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory T_{t} equn 4.2.8 kipf in (kNm)	Ratio $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{T_{expt}}{T_{u, theo}}$	Ratio $\frac{M_{expt}}{M_{u, theo}}$	Ratio equn 4.2.6
Evans and Sarkar 25	*HB2	33.9 (3.83)	66.8 (7.55)	30.3 (3.42)	1.12	0.62	0.77	1.16
	*HB3	20.4 (2.31)	75.3 (8.51)	19.8 (2.24)	1.03	0.38	0.89	1.03
	*HB4	15.7 (1.77)	81.6 (9.22)	15.7 (1.77)	1.00	0.28	0.92	1.00
	*HB5	13.2 (1.49)	81.5 (9.21)	13.0 (1.47)	1.02	0.25	0.96	1.02
	HB6	-	84.5 (9.55)	-	-	-	0.96	0.96
	*HB8	21.4 (2.42)	79.6 (8.99)	21.9 (2.47)	0.98	0.38	0.83	0.97
	*HB9	18.3 (2.07)	85.1 (9.62)	19.8 (2.24)	0.92	0.30	0.83	0.92
	*HB10	17.3 (1.96)	91.3 (10.32)	19.0 (2.15)	0.91	0.27	0.83	0.91
	*HB11	14.1 (1.59)	94.0 (10.62)	14.7 (1.66)	0.96	0.23	0.90	0.96
	*HB12	-	105.6 (11.93)	-	-	-	1.10	1.10
	*HB14	41.7 (4.71)	82.1 (9.28)	46.7 (5.28)	0.89	0.54	0.57	0.86
	*HB15	29.9 (3.38)	111.0 (12.54)	29.4 (3.32)	1.02	0.42	0.84	1.02
	*HB16	23.5 (2.66)	129.0 (14.58)	23.6 (2.67)	1.00	0.30	0.90	1.00
	*HB17	19.4 (2.19)	137.0 (15.48)	19.1 (2.16)	1.02	0.25	0.96	1.02
	HB18	-	147.0 (16.61)	-	-	-	1.11	1.11

Table 4.5.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 1.

$$* \; r_{y} > \frac{1}{1 + {}^{d}/_{b} + {}^{2M}/_{T}}$$

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory T, equn 4.2.8 kipf in (kNm)	Ratio $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{T_{expt}}{T_{u,theo}}$	Ratio $\frac{M_{expt}}{M_{u,theo}}$	Ratio equn 4.2.6
Gesund et al 24	1	79.0 (8.93)	79.0 (8.93)	85.1 (9.62)	0.93	0.67	0.44	0.89
	* 2	102.0 (11.53)	102.0 (11.53)	112.0 (12.66)	0.91	0.55	0.57	0.88
	* 3	61.0 (6.89)	122.0 (13.79)	64.2 (7.25)	0.95	0.51	0.67	0.94
	* 4	67.0 (7.57)	134.0 (15.14)	74.8 (8.45)	0.90	0.36	0.75	0.88
	* 5	49.0 (5.54)	147.0 (16.61)	49.1 (5.55)	1.00	0.42	0.82	1.00
	* 6	56.0 (6.33)	168.0 (18.98)	54.1 (6.11)	1.04	0.30	0.95	1.04
	* 7	43.0 (4.86)	173.0 (19.55)	40.2 (4.54)	1.07	0.36	0.95	1.08
	* 8	44.0 (4.97)	176.0 (19.89)	43.4 (4.90)	1.01	0.23	0.96	1.02
	9	120.0 (13.56)	60.0 (6.78)	65.9 (7.45)	0.91	0.66	0.43	0.87
	10	176.0 (19.89)	44.0 (4.97)	48.7 (5.50)	0.90	0.49	0.63	0.87
	*11	138.0 (15.59)	68.0 (7.68)	81.7 (9.23)	0.83	0.53	0.49	0.78
	*12	53.0 (5.99)	213.0 (24.07)	55.4 (6.26)	0.96	0.42	0.77	0.95

Table 4.5.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 1.

$$* \ r_{ty} > \frac{1}{1 + d/b + 2M/T}$$

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory T_t equn 4.2.8 kipf in (kNm)	Ratio $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{T_{expt}}{T_{ut\ theo}}$	Ratio $\frac{M_{expt}}{M_{ut\ theo}}$	Ratio equn 4.2.6
Goode & Helmy 28	III-4	36.8 (4.16)	180.0 (20.34)	28.2 (3.19)	1.31	0.76	0.86	1.44
	IV-3	46.0 (5.20)	75.4 (8.52)	50.7 (5.73)	0.91	0.70	0.37	0.86
	IV-4	42.5 (4.80)	180.0 (20.34)	34.5 (3.90)	1.23	0.65	0.90	1.31
	V-3	28.5 (3.22)	40.7 (4.60)	27.1 (3.06)	1.05	0.82	0.42	1.09
	*V-4	24.0 (2.71)	92.8 (10.49)	18.9 (2.14)	1.27	0.67	0.92	1.36
	*VI-3	25.0 (2.83)	92.8 (10.49)	21.8 (2.46)	1.15	0.50	0.93	1.18
	*VI-4	36.0 (4.07)	40.7 (4.60)	37.9 (4.28)	0.95	0.72	0.41	0.92
Fairburn [32]	*D2-3	2.53 (0.286)	15.05 (1.701)	2.26 (0.255)	1.12	0.44	0.94	1.14
	*D2-5	1.96 (0.221)	13.86 (1.566)	2.01 (0.227)	0.97	0.32	0.87	0.97
	*D2-6	2.87 (0.324)	13.40 (1.514)	2.71 (0.306)	1.06	0.46	0.86	1.07
	*D2-R	1.76 (0.199)	14.25 (1.610)	1.75 (0.198)	1.01	0.29	0.93	1.01

Table 4.5.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 1.

$$* \; r_{iy} > \frac{1}{1 + d/b + 2M/T}$$

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory T_t equn 4.2.8 kipf in (kNm)	Ratio $\frac{Texpt}{T\ theo}$	Ratio $\frac{Texpt}{Tu_t\ theo}$	Ratio $\frac{Mexpt}{M_{ut}\ theo}$	Ratio equn 4.2.6
Iyengar & Rangan 15	V1	41.0 (4.63)	55.0 (6.22)	51.9 (5.87)	0.79	0.62	0.31	0.69
	V2	36.0 (4.07)	121.0 (13.67)	36.4 (4.11)	0.99	0.55	0.69	0.99
	*V3	25.0 (2.83)	140.0 (15.82)	25.9 (2.93)	0.97	0.39	0.81	0.96
	V4II	-	173.0 (19.55)	-	-	-	0.98	0.98
	V5	39.0 (4.41)	142.0 (16.05)	34.5 (3.90)	1.13	0.60	0.81	1.17
	V6	34.0 (3.84)	52.0 (5.88)	48.4 (5.47)	0.70	0.53	0.30	0.58
	S1I	-	154.0 (17.40)	-	-	-	1.02	1.02
	*S1II	24.0 (2.71	154.0 (17.40)	20.1 (2.27)	1.19	0.46	1.02	1.23
	*S2I	16.0 (1.81)	154.0 (17.40)	14.7 (1.66)	1.09	0.30	1.01	1.10
	S2II	31.0 (3.50)	130.0 (14.69)	27.0 (3.05)	1.15	0.58	0.85	1.19
	S3	29.0 (3.28)	32.0 (3.62)	43.0 (4.86)	0.67	0.56	0.22	0.53
	*S4I	12.0 (1.36)	148.0 (16.72)	11.8 (1.33)	1.01	0.22	0.96	1.01
	S4II	31.0 (3.50)	132.0 (14.92)	26.9 (3.04)	1.15	0.58	0.86	1.20
	*S5I	20.0 (2.26)	148.0 (16.72)	18.2 (2.06)	1.10	0.38	0.97	1.11

Table 4.5.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 1.

$$* \; r_l y > \frac{1}{1 + d/b + 2M/T}$$

Investigation	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory T_1 equn 4.2.8 kipf in (kNm)	Ratio $\frac{Texpt}{T\ theo}$	Ratio $\frac{Texpt}{T_{u1}\ theo}$	Ratio $\frac{Mexpt}{M_{u1}\ theo}$	Ratio equn 4.2.6
Iyenger & Rangan 15	S5II	28.0 (3.16)	132.0 (14.92)	25.1 (2.84)	1.12	0.53	0.87	1.15
	R1	25.0 (2.83)	35.0 (3.96)	38.3 (4.33)	0.65	0.48	0.30	0.53
	*R2	28.0 (3.16)	111.0 (12.54)	24.9 (2.81)	1.13	0.50	0.90	1.15
	R3I	-	185.0 (20.90)	-	-	-	1.01	1.01
	*R3II	30.0 (3.39)	185.0 (20.91)	25.3 (2.86)	1.19	0.45	1.01	1.22
	R4	30.0 (3.39)	-	64.6 (7.30)	0.46	0.46	-	0.22
	*R5I	9.0 (1.02)	200.0 (22.60)	8.2 (0.93)	1.09	0.13	1.08	1.09
	*R5II	30.0 (3.39)	185.0 (20.91)	25.7 (2.90)	1.17	0.44	1.00	1.20
	R6	33.0 (3.73)	64.0 (7.23)	46.2 (5.22)	0.71	0.51	0.36	0.61
	L1-2I	-	169.0 (19.10)	-	-	-	0.93	0.93
	L1-2II	32.0 (3.62)	60.0 (6.78)	42.3 (4.78)	0.76	0.56	0.34	0.65
	*L2-1I	10.0 (1.13)	181.0 (20.45)	9.8 (1.11)	1.02	0.17	0.99	1.02
	L2-1II	33.0 (3.73)	72.0 (8.14)	41.1 (4.64)	0.80	0.57	0.40	0.72
	*L2-2I	10.0 (1.13)	181.0 (20.45)	9.8 (1.11)	1.02	0.17	0.99	1.02
	L2-2II	33.0 (3.73)	72.0 (8.14)	41.1 (4.64)	0.80	0.57	0.40	0.72

Table 4.5.1 - (continued) Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 1.

$$* \; r_{y} > \frac{1}{1 + {}^{d}/_{b} + {}^{2M}/_{T}}$$

Investigation	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory T_{1} equn 4.2.8 kipf in (kNm)	Ratio $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{T_{expt}}{T_{u1\,theo}}$	Ratio $\frac{M_{expt}}{M_{u1\,theo}}$	Ratio equn 4.2.6
Iyengar & Rangan 15	*L3-1I	20.0 (2.26)	163.0 (18.42)	19.0 (2.15)	1.06	0.36	0.93	1.06
	L3-1II	30.0 (3.39)	105.0 (11.87)	32.5 (3.67)	0.92	0.55	0.60	0.90
	*L3-2I	20.0 (2.26)	163.0 (18.42)	19.0 (2.15)	1.06	0.36	0.93	1.06
	L3-2II	30.0 (3.39)	102.0 (11.53)	32.9 (3.72)	0.91	0.55	0.59	0.88
	L4-1	30.0 (3.39)	85.0 (9.61)	36.2 (4.09)	0.83	0.54	0.48	0.77
	L4-2	27.0 (3.05)	34.0 (3.84)	43.9 (4.96)	0.62	0.51	0.20	0.45
	L5-1	27.0 (3.05)	33.0 (3.73)	43.6 (4.93)	0.62	0.51	0.20	0.46
	L5-2	29.0 (3.28)	58.0 (6.55)	39.0 (4.41)	0.75	0.55	0.35	0.64
	*L6-1I	20.0 (2.26)	163.0 (18.42)	16.8 (1.90)	1.19	0.41	1.05	1.22
	*L6-2I	22.0 (2.49)	156.0 (17.63)	18.7 (2.11)	1.18	0.45	1.01	1.21

Table 4.5.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 1.

$$* \; r_y > 1 + \frac{1}{d/b + 2M/T}$$

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory T_t equn 4.2.8 kipf in (kNm)	Ratio $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{T_{expt}}{T_{u,theo}}$	Ratio $\frac{M_{expt}}{M_{u,theo}}$	Ratio equn 4.2.6
Chinekov [21]	*B-2-8-01	48.6 (5.49)	486.1 (54.93)	34.5 (3.90)	1.41	0.35	1.32	1.45
	*B-2-9-01a	46.9 (5.30)	468.7 (52.96)	35.1 (3.97)	1.34	0.33	1.26	1.36
	*B-2-8-02	83.3 (9.41)	416.6 (47.08)	55.9 (6.32)	1.49	0.65	1.21	1.63
	*B-2-8-02a	83.3 (9.41)	416.6 (47.08)	58.9 (6.66)	1.42	0.61	1.15	1.52
	B-2-8-04b	97.2 (10.98)	347.2 (39.23)	69.9 (7.90)	1.39	0.72	1.02	1.53
	B-2-8-04	145.8 (16.48)	364.5 (41.19)	113.9 (12.87)	1.28	0.82	0.76	1.43
	B-2-8-04a	138.9 (15.70)	347.2 (39.23)	113.8 (12.86)	1.22	0.78	0.73	1.33
	B-2-8-04b	145.8 (16.48)	364.5 (41.19)	123.9 (14.00)	1.18	0.75	0.70	1.26
	B-2-8-04c	152.8 (17.27)	381.9 (43.16)	127.6 (14.42)	1.20	0.78	0.70	1.30
	B-2-8-04d	125.0 (14.13)	312.5 (35.31)	107.0 (12.09)	1.17	0.74	0.70	1.25
	B-2-8-04e	131.9 (14.91)	329.8 (37.27)	109.7 (12.40)	1.20	0.77	0.71	1.30
	*B-2-8-04f	138.9 (15.70)	347.2 (39.23)	130.3 (14.72)	1.07	0.57	0.76	1.09
	*B-2-8-04g	97.2 (10.98)	434.0 (49.04)	85.9 (9.71)	1.13	0.42	0.97	1.15

Table 4.5.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 1.

$$* \; r_{y} > \frac{1}{1 + {}^{d}/_{b} + {}^{2M}/_{T}}$$

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory T_1 equn 4.2.8 kipf in (kNm)	Ratio $\frac{Texpt}{T\ theo}$	Ratio $\frac{Texpt}{T_{u1}\ theo}$	Ratio $\frac{Mexpt}{M_{u1}\ theo}$	Ratio equn 4.2.6
Pandit & Warwaruk 31	*B2	72.0 (8.14)	195.0 (22.04)	66.0 (7.46)	1.09	0.55	0.81	1.12
	*B3	95.0 (10.74)	110.0 (12.43)	94.9 (10.72)	1.00	0.74	0.46	1.00
	*C1	78.0 (8.81)	280.0 (31.64)	71.5 (8.08)	1.09	0.46	0.90	1.11
	*C2	105.0 (11.87)	195.0 (22.04)	104.2 (11.78)	1.01	0.62	0.63	1.01
	*D1	99.0 (11.19)	637.0 (71.98)	98.4 (11.12)	1.01	0.41	0.84	1.01
	D2	164.0 (18.53)	365.0 (41.25)	170.4 (19.26)	0.96	0.68	0.48	0.94
	*E1	76.0 (8.59)	195.0 (22.04)	72.3 (8.17)	1.05	0.50	0.81	1.06
	*E2	101.0 (11.41)	110.0 (12.43)	107.7 (12.17)	0.94	0.67	0.46	0.91
	E3	121.0 (13.67)	-	148.9 (16.83)	0.81	0.81	-	0.66

Table 4.5.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 1.

$$* \; r_y > \frac{1}{1 + d/b + 2M/T}$$

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory T, equn 4.2.8 kipf in (kNm)	Ratio $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{T_{expt}}{T_{u\,theo}}$	Ratio $\frac{M_{expt}}{M_{u\,theo}}$	Ratio equn 4.2.6
Jackson & Estañero 34	AU1	-	140.7 (15.90)	-	-	-	0.95	0.95
	AU3	24.6 (2.78)	69.5 (7.85)	29.6 (3.35)	0.83	0.55	0.47	0.77
	AU5	22.5 (2.54)	32.4 (3.66)	35.3 (3.99)	0.64	0.51	0.23	0.49
	CU1	-	151.5 (17.12)	-	-	-	0.98	0.98
	*CU2	24.7 (2.79)	133.1 (15.04)	25.8 (2.92)	0.95	0.31	0.85	0.95
	*CU3	39.6 (4.48)	110.1 (12.44)	40.7 (4.60)	0.98	0.51	0.71	0.97
	CU4	48.2 (5.45)	91.9 (10.38)	48.5 (5.48)	1.00	0.63	0.60	0.99
	CU5	46.0 (5.20)	63.3 (7.15)	54.1 (6.11)	0.85	0.61	0.42	0.79
	CU6	47.6 (5.38)	53.5 (6.04)	55.5 (6.27)	0.85	0.66	0.35	0.79
	CU7	46.0 (5.20)	34.4 (3.89)	63.7 (7.01)	0.74	0.62	0.23	0.61
	DU1	-	107.0 (12.09)	-	-	-	0.99	0.99
	*DU2	19.1 (2.16)	102.7 (11.60)	18.0 (2.03)	1.07	0.35	0.95	1.07
	*DU3	28.0 (3.16)	78.7 (8.89)	27.4 (3.21)	0.99	0.49	0.74	0.98
	DU4	32.4 (3.66)	63.2 (7.14)	34.5 (3.90)	0.94	0.57	0.59	0.92

Table 4.5.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 1.

$$* \; r_{y} > \frac{1}{1 + {}^{d}/_{b} + {}^{2M}/_{T}}$$

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory T_1 equn 4.2.8 kipf in (kNm)	Ratio $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{T_{expt}}{T_{u1\,theo}}$	Ratio $\frac{M_{expt}}{M_{u1\,theo}}$	Ratio equn 4.2.6
Jackson & Estañero 34	DU-5	33.3 (3.76)	46.1 (5.21)	38.7 (4.37)	0.86	0.61	0.43	0.80
	DU-6	30.9 (3.49)	35.6 (4.02)	40.6 (4.59)	0.76	0.57	0.33	0.66
	DU-7	32.0 (3.62)	24.3 (2.75)	45.4 (5.13)	0.71	0.58	0.23	0.57
	EU-1	-	107.0 (12.09)	-	-	-	0.97	0.97
	*EU-2	19.8 (2.24)	108.3 (12.24)	19.0 (2.15)	1.04	0.27	0.97	1.05
	*EU-3	31.6 (3.57)	89.9 (10.16)	31.5 (3.56)	1.00	0.42	0.82	1.00
	*EU-4	41.0 (4.63)	79.7 (9.01)	39.6 (4.48)	1.03	0.56	0.73	1.04
	EU-5	46.4 (5.24)	63.7 (7.20)	46.7 (5.28)	0.99	0.64	0.59	0.99
	EU-6	48.2 (5.45)	54.0 (6.10)	49.4 (5.58)	0.98	0.68	0.50	0.96
	EU-7	51.2 (5.79)	36.5 (4.13)	57.1 (6.45)	0.90	0.71	0.34	0.84
	GU-1	-	68.0 (7.68)	-	-	-	1.02	1.02
	*GU-3	21.4 (2.42)	64.5 (7.29)	20.2 (2.28)	1.06	0.34	0.95	1.07
	*GU-5	36.2 (4.09)	51.0 (5.76)	32.4 (3.66)	1.12	0.59	0.80	1.15
	*GU-6	36.4 (4.11)	42.3 (4.78)	36.7 (4.15)	0.99	0.59	0.64	0.99
	*GU-7	40.0 (4.52)	30.4 (3.44)	42.0 (4.75)	0.95	0.66	0.50	0.93

Table 4.5.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 1.

$$* \; r_{1y} > \frac{1}{1 + {}^{d}/_{b} + {}^{2M}/_{T}}$$

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory T_1 equn 4.2.8 kipf in (kNm)	Ratio $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{T_{expt}}{T_{u1\,theo}}$	Ratio $\frac{M_{expt}}{M_{u1\,theo}}$	Ratio equn 4.2.6
Jackson & Estañero 34	A5-1	-	179.0 (20.23)	-	-	-	1.05	1.05
	A5-3	21.3 (2.41)	61.1 (6.92)	31.6 (3.57)	0.68	0.46	0.36	0.57
	A5-5	22.5 (2.54)	32.4 (3.66)	39.2 (4.43)	0.57	0.47	0.19	0.41
	A5-8	21.4 (2.42)	1.1 (0.12)	47.3 (5.34)	0.45	0.45	0.01	0.21
	B4-1	-	115.0 (13.00)	-	-	-	1.00	1.00
	B4-3	15.4 (1.74)	44.9 (5.07)	23.6 (2.67)	0.65	0.40	0.41	0.57
	B4-5	14.8 (1.67)	21.9 (2.48)	29.3 (3.31)	0.50	0.39	0.21	0.36
	B4-8	16.8 (1.90)	1.0 (0.09)	37.3 (4.21)	0.45	0.45	0.01	0.21
	C4-1	-	188.0 (21.24)	-	-	-	1.04	1.04
	*C4-2	31.6 (3.57)	166.1 (18.77)	29.7 (3.36)	1.06	0.39	0.92	1.07

Table 4.5.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 1.

$$* \; r_{y} > \frac{1}{1 + d/b + 2M/T}$$

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory $T_{'}$ equn 4.2.8 kipf in (kNm)	Ratio $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{T_{expt}}{T_{u', theo}}$	Ratio $\frac{M_{expt}}{M_{u', theo}}$	Ratio equn 4.2.6
Jackson & Estañero 34	C4-3	41.4 (4.68)	114.6 (12.95)	45.0 (5.09)	0.92	0.51	0.64	0.90
	C4-4	44.0 (4.97)	84.9 (9.59)	52:2 (5.90)	0.84	0.55	0.49	0.79
	C4-5	45.9 (5.19)	63.0 (7.12)	58.7 (6.63)	0.78	0.57	0.37	0.69
	C4-6	40.2 (4.54)	46.1 (5.21)	58.0 (6.55)	0.69	0.54	0.27	0.56
	C4-7	47.0 (5.31)	35.0 (3.96)	67.0 (7.57)	0.70	0.59	0.21	0.55
	C4-8	48.4 (5.47)	1.6 (0.18)	78.2 (8.84)	0.62	0.61	0.01	0.39
	D4-1	-	115.0 (13.00)	-	-	-	0.96	0.96
	*D4-2	20.5 (2.32)	109.7 (12.40)	20.2 (2.28)	1.02	0.34	0.91	1.02
	D4-3	29.2 (3.30)	81.7 (9.23)	31.4 (3.55)	0.93	0.48	0.68	0.91
	D4-4	30.3 (3.42)	59.6 (6.74)	36.8 (4.16)	0.82	0.51	0.51	0.77
	D4-5	29.5 (3.33)	41.4 (4.68)	40.1 (4.53)	0.73	0.53	0.35	0.63
	D4-6	28.0 (3.16)	32.7 (3.70)	41.2 (4.66)	0.68	0.52	0.28	0.55
	D4-7	27.4 (3.10)	21.2 (2.40)	45.5 (5.14)	0.60	0.51	0.18	0.44
	D4-8	28.0 (3.16)	1.2 (0.13)	52.8 (5.97)	0.53	0.52	0.01	0.29

Table 4.5.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 1.

$* \; r_{iy} > \dfrac{1}{1 + d/b + 2M/T}$

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory T_i equn 4.2.8 kipf in (kNm)	Ratio $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{T_{expt}}{T_{u_i\,theo}}$	Ratio $\frac{M_{expt}}{M_{u_i\,theo}}$	Ratio equn 4.2.6
Jackson & Estañero 34	E4-1	-	126.0 (14.24)	-	-	-	1.01	1.01
	*E4-2	22.4 (2.53)	111.3 (12.58)	22.9 (2.59)	0.98	0.28	0.90	0.98
	*E4-3	31.4 (3.55)	91.9 (10.38)	32.9 (3.72)	0.95	0.44	0.75	0.94
	E4-4	40.4 (4.56)	78.8 (8.90)	42.1 (4.76)	0.96	0.55	0.64	0.94
	E4-5	45.2 (5.11)	62.2 (7.03)	48.1 (5.44)	0.94	0.63	0.51	0.91
	E4-6	46.0 (5.20)	51.9 (5.86)	51.2 (5.79)	0.90	0.65	0.43	0.85
	E4-7	46.8 (5.29)	29.4 (3.32)	58.9 (6.65)	0.80	0.66	0.25	0.68
	E4-8	43.0 (4.86)	1.6 (0.18)	68.8 (7.77)	0.63	0.62	0.01	0.40
	F3-1	-	68.0 (7.68)	-	-	-	0.97	0.97
	*F3-2	11.8 (1.33)	65.1 (7.36)	12.0 (1.36)	0.98	0.23	0.92	0.98
	*F3-3	18.8 (2.12)	54.7 (6.18)	20.4 (2.31)	0.92	0.36	0.77	0.90
	*F3-4	21.8 (2.46)	44.6 (5.04)	24.3 (2.75)	0.89	0.45	0.67	0.87
	F3-5	26.0 (2.94)	36.6 (4.14)	30.2 (3.41)	0.86	0.53	0.54	0.82
	F3-6	25.5 (2.88)	29.7 (3.36)	31.9 (3.61)	0.80	0.53	0.45	0.73

Table 4.5.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 1.

$$* \; r_{y} > \frac{1}{1 + {}^{d}/_{b} + {}^{2M}/_{T}}$$

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory T_t equn 4.2.8 kipf in (kNm)	Ratio $\frac{T expt}{T\ theo}$	Ratio $\frac{T expt}{T_{u}\ theo}$	Ratio $\frac{M\ expt}{M_{u}\ theo}$	Ratio equn 4.2.6
Jackson & Estañero 34	F3-7	24.9 (2.81)	19.3 (2.18)	35.8 (4.05)	0.69	0.52	0.30	0.57
	F3-8	24.0 (2.71)	1.1 (0.12)	45.0 (5.08)	0.53	0.53	0.02	0.29
	G2-1	-	68.5 (7.74)	-	-	-	1.03	1.03
	*G2-3	20.8 (2.35)	63.0 (7.12)	19.6 (2.22)	1.06	0.34	0.95	1.07
	*G2-5	32.9 (3.72)	47.0 (5.31)	31.9 (3.60)	1.03	0.54	0.75	1.04
	*G2-6	38.0 (4.29)	43.9 (4.96)	35.5 (4.01)	1.07	0.63	0.70	1.09
	*G2-7	41.6 (4.70)	31.5 (3.56)	43.6 (4.93)	0.95	0.65	0.50	0.93
	*G2-8	51.2 (5.79)	1.6 (0.18)	61.1 (6.90)	0.84	0.83	0.03	0.71

Table 4.6.1 - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 2

$$* \; r_{2y} > \frac{1}{1 + {}^{b}/_{d}}$$

Investigator	Beam	Texpt kipf in (kNm)	Ratio Texpt/Ttheo equn 4.3.4	
Hsu [27]	*B1	197.0 (22.26)	1.03	
	*B2	259.0 (29.27)	0.94	
	*B3	332.0 (37.52)	0.87	
	*B4	419.0 (47.35)	0.84	
	*B5	497.0 (56.16)	0.80	OR
	*B6	546.0 (61.70)	0.76	OR
	*B7	238.0 (26.89)	0.90	
	*B8	288.0 (32.54)	0.76	
	B9	264.0 (29.83)	0.95	
	B10	304.0 (34.35)	0.79	
	*D1	198.0 (22.37)	1.01	
	*D2	245.0 (27.69)	0.87	
	*D3	346.0 (39.10)	0.86	
	*D4	424.0 (47.91)	0.82	
	M1	269.0 (30.40)	1.09	
	M2	359.0 (40.57)	1.04	
	M3	388.0 (43.84)	0.94	
	M4	439.0 (49.61)	0.86	
	M5	493.0 (55.71)	0.78	
	M6	532.0 (60.12)	0.72	OR

Table 4.6.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 2

$$* \; r_{2y} > \frac{1}{1 + b/d}$$

Investigator	Beam	Texpt kipf in (kNm)	Ratio Texpt / Ttheo equn 4.3.4
Hsu[27]	*I2	319.0 (36.05)	1.04
	*I3	404.0 (45.65)	0.96
	*I4	514.0 (58.08)	0.98
	*I5	626.0 (70.74)	0.95
	*I6	679.0 (76.73)	0.84
	*J1	190.0 (21.47)	1.02
	*J2	258.0 (29.15)	0.96
	*J3	312.0 (35.26)	0.85
	*J4	360.0 (40.68)	0.81
	*G1	237.0 (26.78)	1.09
	*G2	357.0 (40.34)	1.09
	*G3	439.0 (49.61)	0.99
	*G4	574.0 (64.86)	1.01
	*G5	637.0 (71.98)	0.89
	*G6	346.0 (39.10)	1.05
	*G7	466.0 (52.66)	1.01
	*G8	650.0 (73.45)	1.04

OR

OR

OR

Table 4.6.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 2

$$* \; r_{zy} > \frac{1}{1 + {}^{b}/_{d}}$$

Investigator	Beam	T_{expt} kipf in (kNm)	Ratio $\frac{T_{expt}}{T_{theo}}$ equn 4.3.4	
Hsu [27]	*N1	80.5 (9.10)	1.09	
	*N1a	79.6 (8.99)	1.09	
	*N2	128.0 (14.46)	1.07	
	*N2a	117.0 (13.22)	0.97	
	*N3	108.0 (12.20)	1.02	
	*N4	139.0 (15.71)	0.94	
	*K1	136.0 (15.37)	1.15	
	*K2	210.0 (23.73)	1.10	
	*K3	252.0 (28.48)	1.01	
	*K4	310.0 (35.03)	0.95	OR
	*C1	100.0 (11.30)	1.07	
	*C2	135.0 (15.26)	0.84	
	*C3	177.0 (20.00)	0.78	OR
	*C4	224.0 (25.31)	0.72	OR
	*C5	263.0 (29.72)	0.68	OR
	*C6	303.0 (34.24)	0.65	OR

Table 4.6.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 2

$* \; r_{2y} > \dfrac{1}{1 + {}^{b}/_{d}}$

Investigator	Beam	Texpt kipf in (kNm)	Ratio $\frac{Texpt}{Ttheo}$ equn 4.3.4
Ernst [42]	3TR7	49.7 (5.62)	0.96
	3TR15	61.7 (6.97)	0.92
	*3TR30	76.0 (8.59)	0.83
	4TR7	54.8 (6.19)	0.93
	4TR15	74.0 (8.36)	0.97
	*4TR30	85.0 (9.61)	0.82
	5TR3	43.0 (4.86)	0.79
	5TR7	59.7 (6.75)	0.79
	5TR15	76.5 (8.64)	0.77
	5TR30	92.6 (10.46)	0.70
Pandit & Warwaruk 31	*E3	121.0 (13.67)	0.89
Iyengar & Rangan 15	R4	30.0 (3.39)	0.52

Table 4.7.1 - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 3

$$* \; r_{3y} > \frac{1}{1 + {}^{2d}/_{b} - {}^{2M}/_{T}}$$

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory equn 4.4.6 T kipf in (kNm)	Ratio $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{T_{expt}}{T_{u3\,theo}}$	Ratio $\frac{M_{expt}}{M_{u3\,theo}}$	Ratio equn 4.4.3
Goode & Helmy 28	III 1	24.5 (2.77)	4.8 (0.54)	21.2 (2.40)	1.16	1.24	0.17	1.37
	*III 2	39.5 (4.46)	75.4 (8.52)	36.5 (4.13)	1.08	2.02	2.68	1.40
	III 3	34.0 (3.84)	40.7 (4.60)	29.5 (3.33)	1.15	1.73	1.44	1.55
	* IV 1	34.3 (3.88)	4.8 (0.54)	29.8 (3.37)	1.15	1.23	0.17	1.34
	IV 2	43.5 (4.92)	40.7 (4.60)	42.5 (4.80)	1.03	1.60	1.47	1.09
	V 1	25.5 (2.88)	4.8 (0.54)	21.3 (2.41)	1.20	1.28	0.17	1.47
	V 2	24.0 (2.71)	23.4 (2.64)	27.4 (3.10)	0.88	1.22	0.83	0.66
	* VI 1	27.0 (3.05)	4.8 (0.54)	30.2 (3.41)	0.89	0.98	0.17	0.79
	* VI 2	31.0 (3.50)	23.4 (2.64)	39.4 (4.45)	0.79	1.13	0.84	0.44
Iyengar & Rangan 15	V4-1	33.0 (3.73)	-	54.9 (6.20)	0.60	0.60	-	0.36
	S6	30.0 (3.39)	-	47.6 (5.38)	0.63	0.63	-	0.40
	L1-1	29.0 (3.28)	-	46.1 (5.21)	0.63	0.63	-	0.40
	L6-1II	25.0 (2.83)	-	42.5 (4.80)	0.59	0.59	-	0.35

Table 4.7.1 continued - Detailed Results for Members with Longitudinal and Transverse Steel at Yield Failing in Mode 3

$$* \; r_{3y} > \frac{1}{1 + {}^{2d}/_{b} - {}^{2M}/_{T}}$$

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Theory equn 4.4.6 T kipf in (kNm)	Ratio $\frac{Texpt}{T\ theo}$	Ratio $\frac{Texpt}{T_u\ theo}$	Ratio $\frac{Mexpt}{M_u\ theo}$	Ratio equn 4.4.3
Evans & Sarkar 25	HB1	44.1 (4.98)	-	53.7 (6.07)	0.82	0.82	-	0.67
	HB7	36.1 (4.08)	-	52.8 (5.97)	0.68	0.68	-	0.46
	HB13	51.3 (5.80)	-	66.5 (7.52)	0.77	0.77	-	0.59
Pandit & Warwaruk 31	B4	85.0 (9.61)	-	98.4 (11.12)	0.86	0.86	-	0.74
	C3	111.0 (12.54)	111.0 (12.43)	167.7 (18.95)	0.66	0.99	0.83	0.15
	*C4	111.0 (12.54)	-	116.3 (13.14)	0.96	0.96	-	0.92
	D3	156.0 (17.63)	195.0 (22.04)	230.7 (26.07)	0.63	0.95	0.65	0.25
	D4	146.0 (16.50)	-	168.9 (19.09)	0.86	0.86	-	0.74
Jackson & Estanero 34	AU8	22.0 (2.49)	1.1 (0.12)	37.1 (4.19)	0.59	0.60	0.01	0.35
	CU8	43.5 (4.92)	1.6 (0.18)	58.9 (6.65)	0.74	0.75	0.02	0.54
	DU8	31.6 (3.57)	1.2 (0.13)	45.0 (5.09)	0.70	0.71	0.02	0.48
	EU8	45.2 (5.11)	1.6 (0.18)	59.6 (6.74)	0.76	0.77	0.02	0.57
	*GU8	36.4 (4.11)	1.6 (0.18)	43.4 (4.90)	0.84	0.87	0.06	0.70

Table 5.3.1 - Detailed Results for Members with Longitudinal and Transverse Steel

Partial Yield Theory A ($T = 2\,A_s\,f_{sy}\,b'd'/s$, $r_{ly} < \frac{1}{1 + d/b + 2M/T}$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.3.2 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u,theo}}$	Ratio r_{ly}/r'_{ly}
Goode & Helmy 28	III4	36.8 (4.16)	180.0 (20.34)	1.82	0.86	0.52
	IV3	46.0 (5.20)	75.4 (8.52)	1.14	0.37	0.51
	IV4	42.5 (4.80)	180.0 (20.34)	1.05	0.90	0.93
	V3	28.5 (3.22)	40.7 (4.60)	1.41	0.42	0.56
Gesund et al 24	1	79.0 (8.93)	79.0 (8.93)	1.04	0.44	0.86
	9	120.0 (13.56)	60.0 (6.78)	1.19	0.43	0.54
	10	176.0 (19.89)	44.0 (4.97)	0.87	0.63	0.85
Iyengar & Rangan 15	V1	41.0 (4.63)	55.0 (6.22)	1.24	0.31	0.43
	V2	36.0 (4.07)	121.0 (13.67)	1.09	0.69	0.84
	V5	39.0 (4.41)	142.0 (16.05)	1.18	0.81	0.89
	V6	34.0 (3.84)	52.0 (5.88)	1.03	0.30	0.51

Table 5.3.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel

Partial Yield Theory A ($T = 2 A_s f_{sy} b'd'/s$, $r_{\psi y} < \frac{1}{1 + d/b + 2M/T}$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.3.2 Texpt/Ttheo	Ratio Mexpt/$M_{u\psi}$ theo	Ratio $r_{\psi y}/r'_{\psi y}$
Iyengar & Rangan 15	S2II	31.0 (3.50)	130.0 (14.69)	1.25	0.85	0.87
	S3	29.0 (3.28)	32.0 (3.62)	1.17	0.22	0.38
	S4II	31.0 (3.50)	132.0 (14.92)	1.25	0.86	0.88
	S5II	28.0 (3.16)	132.0 (14.92)	1.13	0.87	0.95
	R1	25.0 (2.83)	35.0 (3.96)	0.75	0.30	0.73
	R4	30.0 (3.39)	-	0.90	-	0.23
	R6	33.0 (3.37)	64.0 (7.23)	1.00	0.36	0.56
	L1-2II	32.0 (3.62)	60.0 (6.78)	1.29	0.34	0.41
	L2-1II	33.0 (3.73)	72.0 (8.14)	1.33	0.40	0.45
	L2-2II	33.0 (3.73)	72.0 (8.14)	1.33	0.40	0.45
	L3-1II	30.0 (3.39)	105.0 (11.87)	1.21	0.60	0.63
	L3-2II	30.0 (3.39)	102.0 (11.53)	1.21	0.59	0.61
	L4-1	30.0 (3.39)	85.0 (9.61)	1.21	0.48	0.54
	L4-2	27.0 (3.05)	34.0 (3.84)	1.09	0.20	0.33
	L5-I	27.0 (3.05)	33.0 (3.73)	1.09	0.20	0.33

Table 5.3.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel

Partial Yield Theory A ($T = 2 A_s f_{sy} b'd'/s$, $r_{'y} < \frac{1}{1 + d/b + 2M/T}$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.3.2 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u'} theo}$	Ratio $r_{'y}/r'_{'y}$
Iyengar & Rangan 15	L5-2	29.0 (3.28)	58.0 (6.55)	1.17	0.35	0.43
Pandit & Warwaruk 31	D2	164.0 (18.53)	365.0 (41.25)	1.13	0.48	0.55
	E3	121.0 (13.67)	-	0.83	-	0.88
Jackson & Estanero 34	AU3	24.6 (2.78)	69.5 (7.85)	1.07	0.47	0.61
	AU5	22.5 (2.54)	32.4 (3.66)	0.98	0.23	0.41
	CU4	48.2 (5.45)	91.9 (10.38)	0.99	0.60	0.92
	CU5	46.0 (5.20)	63.3 (7.15)	0.98	0.42	0.73
	CU6	47.6 (5.38)	53.5 (6.04)	1.11	0.35	0.60
	CU7	46.0 (5.20)	34.4 (3.89)	1.01	0.23	0.53
	DU4	32.4 (3.66)	63.2 (7.14)	0.98	0.59	0.84

Table 5.3.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel

Partial Yield Theory A ($T = 2 A_s f_{sy} b'd'/s$, $r_{\,y} < \frac{1}{1 + d/b + 2M/T}$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.3.2 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u\,theo}}$	Ratio $r_{\,y}/r'_{\,y}$
Jackson & Estanero 34	DU5	33.3 (3.76)	46.1 (5.21)	1.10	0.43	0.62
	DU6	30.9 (3.49)	35.6 (4.02)	1.02	0.33	0.56
	DU7	32.0 (3.62)	24.3 (2.75)	0.99	0.23	0.49
	EU5	46.4 (5.24)	63.7 (7.20)	1.02	0.59	0.88
	EU6	48.2 (5.45)	54.0 (6.10)	1.12	0.50	0.73
	EU7	51.2 (5.79)	36.5 (4.13)	1.12	0.34	0.62
	A5-3	21.3 (2.41)	61.1 (6.92)	0.97	0.36	0.49
	A5-5	22.5 (2.54)	32.4 (3.66)	0.98	0.19	0.35
	A5-8	21.4 (2.42)	1.1 (0.12)	0.93	0.01	0.18
	B4-3	15.4 (1.74)	44.9 (5.07)	0.76	0.41	0.65
	B4-5	14.8 (1.67)	21.9 (2.48)	0.73	0.21	0.43
	B4-8	16.8 (1.90)	1.0 (0.09)	0.85	0.01	0.20
	C4-3	41.4 (4.68)	114.6 (12.95)	0.88	0.64	0.96
	C4-4	44.0 (4.97)	84.9 (9.59)	0.93	0.49	0.78

Table 5.3.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel

Partial Yield Theory A ($T = 2 A_s f_{sy} b'd'/s$, $r_{\backslash y} < \frac{1}{1 + {}^{d}/_{b} + {}^{2M}/_{T}}$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.3.2 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u, theo}}$	Ratio $r_{\backslash y}/r'_{\backslash y}$
Jackson & Estanero 34	C4-5	45.9 (5.19)	63.0 (7.12)	0.97	0.37	0.64
	C4-6	40.2 (4.54)	46.1 (5.21)	0.98	0.27	0.51
	C4-7	47.0 (5.31)	35.0 (3.96)	1.00	0.21	0.49
	C4-8	48.4 (5.47)	1.6 (0.18)	1.03	0.01	0.31
	D4-3	29.2 (3.30)	81.7 (9.23)	0.87	0.68	0.96
	D4-4	30.3 (3.42)	59.6 (6.74)	0.90	0.51	0.75
	D4-5	29.5 (3.33)	41.4 (4.68)	1.04	0.35	0.51
	D4-6	28.0 (3.16)	32.7 (3.70)	0.99	0.28	0.46
	D4-7	27.4 (3.10)	21.2 (2.40)	0.97	0.18	0.38
	D4-8	28.0 (3.16)	1.2 (0.13)	0.99	0.01	0.22
	E4-6	40.4 (4.56)	78.8 (8.90)	1.01	0.64	0.86
	E4-5	45.2 (5.11)	62.2 (7.03)	1.14	0.51	0.68
	E4-6	46.0 (5.20)	51.9 (5.86)	1.16	0.43	0.61
	E4-7	46.8 (5.29)	29.4 (3.32)	1.18	0.25	0.45

Table 5.3.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel

Partial Yield Theory A ($T = 2 A_s f_{sy} b'd'/s$, $r_{\prime y} < \frac{1}{1 + d/b + 2M/T}$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.3.2 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u\prime}\ theo}$	Ratio $r_{\prime y}/r_{\prime y}'$
Jackson & Estanero 34	E4-8	43.0 (4.86)	1.6 (0.18)	1.08	0.01	0.28
	F3-5	26.0 (2.94)	36.6 (4.14)	0.81	0.54	0.86
	F3-6	25.5 (2.88)	29.7 (3.36)	0.79	0.45	0.77
	F3-7	24.9 (2.81)	19.3 (2.18)	0.77	0.30	0.61
	F3-8	24.0 (2.71)	1.1 (0.12)	0.75	0.02	0.32
Chinekov 21	B-2-8.04b	97.2 (10.98)	347.2 (39.23)	1.00	1.02	0.96
	B-2-8-04	145.8 (16.48)	364.5 (41.19)	1.50	0.76	0.65
	B-2-8-04a	138.9 (15.70)	347.2 (39.23)	1.43	0.73	0.66
	B-2-8-04b	145.8 (16.48)	364.5 (41.19)	1.50	0.70	0.66
	B-2-8-04c	152.8 (17.27)	381.9 (43.16)	1.57	0.70	0.61
	B-2-8-04d	125.0 (14.13)	312.5 (35.31)	1.29	0.70	0.68
	B-2-8-04e	131.9 (14.91)	329.8 (37.27)	1.36	0.71	0.66

Table 5.3.2 - Detailed Results for Members with Longitudinal and Transverse Steel.
Partial Yield Theory A ($T = 2 A_s f_{sy} b'd'/s$, $r_{2y} < \frac{1}{1 + b/d}$)

Investigator	Beam	Texpt kipf in (kNm)	Ratio Texpt / Ttheo equn 5.3.2	Ratio r_{2y}/r_{2y}'
Hsu 27	B9	264.0 (29.83)	1.26	0.50
	B10	304.0 (34.35)	1.46	0.21
	M1	269.0 (30.40)	1.22	0.75
	M2	359.0 (40.57)	1.13	0.74
	M3	388.0 (43.84)	1.00	0.68
	M4	439.0 (49.61)	0.85	0.70
	M5	493.0 (55.50)	0.74	0.67
	M6	532.0 (55.71)	0.66	0.73
Ernst 42	3TR7	49.7 (5.62)	1.42	0.49
	3TR15	61.7 (6.97)	1.01	0.87
	4TR7	54.8 (6.19)	1.57	0.36
	4TR15	74.0 (8.36)	1.21	0.64
	5TR3	43.0 (4.86)	2.46	0.10
	5TR7	59.7 (6.75)	1.71	0.20
	5TR15	76.5 (8.64)	1.23	0.34
	5TR30	92.6 (10.46)	0.76	0.69

OR

Table 5.3.3 - Detailed Results for Members with Longitudinal and Transverse Steel

Partial Yield Theory A $(T = 2 A_s f_{sy} b'd'/s,\ r_{3y} < \frac{1}{1 + d/b - 2M/T})$

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.3.2 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u'}\ theo}$	Ratio r_{3y}/r'_{3y}
Goode & Helmy 28	III 1	24.5 (2.77)	4.8 (0.54)	1.21	.02	0.95
	III 3	34.0 (3.84)	40.7 (4.60)	1.68	.20	0.22
	IV 2	43.5 (4.92)	40.7 (4.60)	1.08	.20	0.82
	V 1	25.5 (2.88)	4.8 (0.54)	1.26	.05	0.95
	V 2	24.0 (2.71)	23.4 (2.64)	1.19	.24	0.38
Iyengar & Rangan 15	V4-1	33.0 (3.73)	-	1.00	-	0.35
	S6	30.0 (3.39)	-	1.21	-	0.26
	L1-1	29.0 (3.28)	-	1.17	-	0.26
	L6-1II	25.0 (2.83)	-	1.01	-	0.26
Evans & Sarkar 25	HB1	44.1 (4.98)	-	0.89	-	0.86
	HB7	36.1 (4.08)	-	0.87	-	0.60
	HB13	51.3 (5.80)	-	0.90	-	0.72

Table 5.3.3 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel

Partial Yield Theory A ($T = 2 A_s f_{sy} b'd'/s$, $r_{3y} < \frac{1}{1 + {}^{d}/_{b} - {}^{2M}/_{T}}$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.3.2 $\frac{Texpt}{Ttheo}$	Ratio $\frac{Mexpt}{M_{u1}\ theo}$	Ratio r_{3y}/r'_{3y}
Pandit & Warwaruk 31	C3	111.0 (12.54)	110.0 (12.43)	0.76	0.35	0.54
	D3	156.0 (17.63)	195.0 (22.04)	1.07	0.26	0.10
	D4	146.0 (16.50)	-	1.00	-	0.66
Jackson & Estanero 34	AU8	22.0 (2.49)	1.1 (0.12)	0.96	0.01	0.33
	CU8	43.5 (4.92)	1.6 (0.18)	1.00	0.01	0.50
	DU8	31.6 (3.57)	1.2 (0.13)	0.97	0.01	0.44
	EU8	45.2 (5.11)	1.6 (0.18)	0.96	0.01	0.56

Table 5.4.1 - Detailed Results for Members with Longitudinal and Transverse Steel.

Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T = 1 \cdot 6 A_s f_{sy} b'd'/s$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.4.1 Texpt/Ttheo	Ratio Mexpt/M_u theo	Ratio r_y/r'_y
Evans & Sarkar 25	HB1	44.1 (4.98)	-	1.11	-	0.65
	HB2	33.9 (3.83)	66.8 (7.55)	1.07	0.77	1.22
	HB3	20.4 (2.31)	75.3 (8.51)	0.67	0.89	1.82
	HB4	15.7 (1.77)	81.6 (9.22)	0.49	0.92	2.39
	HB5	13.2 (1.49)	81.5 (9.21)	0.43	0.96	2.70
	HB6	-	84.5 (9.55)	-	0.96	-
	HB7	36.1 (4.08)	-	1.08	-	0.53
	HB8	21.4 (2.42)	79.6 (8.99)	0.64	0.83	1.81
	HB9	18.3 (2.07)	85.1 (9.62)	0.49	0.83	2.21
	HB10	17.3 (1.96)	91.3 (10.32)	0.45	0.83	2.44
	HB11	14.1 (1.59)	94.0 (10.62)	0.38	0.90	2.93
	HB12	-	105.6 (11.93)	-	1.10	-
	HB13	51.3 (5.80)	-	1.13	-	0.64

Table 5.4.1 - (continued) Detailed Results for Members with Longitudinal and Transverse Steel.

Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T = 1{\cdot}6\ A_s\ f_{sy}\ b'd'/s$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.4.1 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u'\,theo}}$	Ratio $r_{'y}/r_{'y}'$
Evans and Sarkar 25	HB14	41.7 (4.71)	82.1 (9.28)	0.81	0.57	1.37
	HB15	29.9 (3.38)	111.0 (12.54)	0.66	0.84	1.91
	HB16	23.5 (2.66)	129.0 (14.58)	0.45	0.90	2.63
	HB17	19.4 (2.19)	137.0 (15.48)	0.38	0.96	3.19
	HB18	-	147.0 (16.61)	-	1.11	-
Goode & Helmy 28	III 1	24.5 (2.77)	4.8 (0.54)	1.51	0.02	0.13
	III 2	39.5 (4.46)	75.4 (8.52)	2.44	0.36	0.24
	III 3	34.0 (3.84)	40.7 (4.60)	2.10	0.20	0.20
	III 4	36.8 (4.16)	180.0 (20.34)	2.28	0.86	0.44
	IV 1	34.3 (3.88)	4.8 (0.54)	1.06	0.02	0.26
	IV 2	43.5 (4.92)	40.7 (4.60)	1.35	0.20	0.36
	IV 3	46.0 (5.20)	15.4 (8.52)	1.42	0.37	0.45
	IV 4	42.5 (4.80)	180.0 (20.34)	1.31	0.90	0.79

Table 5.4.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel.

Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T = 1{\cdot}6\ A_s\ f_{sy}\ b'd'/s$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.4.1 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u,theo}}$	Ratio $r_{,y}/r'_{,y}$
Goode & Helmy 28	V 1	25.5 (2.88)	4.8 (0.54)	1.58	0.05	0.32
	V 2	24.0 (2.71)	23.4 (2.64)	1.48	0.24	0.44
	V 3	28.6 (3.22)	40.7 (4.60)	1.76	0.41	0.51
	V 4	24.0 (2.71)	92.8 (10.49)	1.48	0.92	0.88
	VI1	27.0 (3.05)	4.8 (0.54)	0.84	0.05	0.63
	VI2	31.0 (3.50)	23.4 (2.64)	0.96	0.23	0.80
	VI3	25.0 (2.83)	92.8 (10.49)	0.77	0.93	1.71
	VI4	36.0 (4.07)	40.7 (4.60)	1.11	0.40	0.92
Fairburn 32	D2-3	2.53 (0.286)	15.05 (1.701)	1.22	0.94	1.02
	D2-5	1.96 (0.221)	13.86 (1.566)	0.74	0.87	1.53
	D2-6	2.87 (0.324)	13.40 (1.514)	1.09	0.86	1.10
	D2-R	1.76 (0.199)	14.25 (1.610)	0.67	0.93	1.69

Table 5.4.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel.

Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T \doteq 1{\cdot}6\ A_s\ f_{sy}\ b'd'/s$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.4.1 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u}\ theo}$	Ratio r'_y/r'_y
Gesund et al 24	1	79.0 (8.93)	79.0 (8.93)	1.30	0.44	0.78
	2	102.0 (11.53)	102.0 (11.53)	0.67	0.57	1.94
	3	61.0 (6.89)	122.0 (13.79)	1.00	0.67	1.12
	4	67.0 (7.57)	134.0 (15.14)	0.44	0.75	2.80
	5	49.0 (5.54)	147.0 (16.61)	0.81	0.82	1.47
	6	56.0 (6.33)	168.0 (18.98)	0.37	0.95	3.66
	7	43.0 (4.86)	173.0 (19.55)	0.71	0.95	1.82
	8	44.0 (4.97)	176.0 (19.89)	0.29	0.96	4.53
	9	120.0 (13.56)	60.0 (6.78)	1.48	0.43	0.48
	10	176.0 (19.89)	44.0 (4.97)	1.09	0.63	0.73
	11	138.0 (15.59)	68.0 (7.68)	0.84	0.49	0.97
	12	53.0 (5.99)	213.0 (24.07)	0.66	0.77	1.44

Table 5.4.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel.

Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T = 1{\cdot}6\ A_s\ f_{sy}\ b'd'/s$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.4.1 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u,theo}}$	Ratio r_{y}/r'_{y}
Iyengar & Rangan 15	V1	41.0 (4.63)	55.0 (6.22)	1.55	0.31	0.43
	V2	36.0 (4.07)	121.0 (13.67)	1.36	0.69	0.72
	V3	25.0 (2.83)	140.0 (15.82)	0.94	0.81	1.04
	V4-1	33.0 (3.73)	-	1.24	-	0.24
	V4II	-	173.0 (19.55)	-	0.98	-
	V5	39.0 (4.41)	142.0 (16.05)	1.47	0.81	0.76
	V6	34.0 (3.84)	52.0 (5.88)	1.28	0.30	0.46
	S1I	-	154.0 (17.40)	-	1.02	-
	S1II	24.0 (2.71)	154.0 (17.40)	1.21	1.02	1.02
	S2I	16.0 (1.81)	154.0 (17.40)	0.80	1.01	1.43
	S2II	31.0 (3.50)	130.0 (14.69)	1.56	0.85	0.74
	S3	29.0 (3.28)	32.0 (3.62)	1.46	0.21	0.35
	S4I	12.0 (1.36)	148.0 (16.72)	0.60	0.96	1.77
	S4II	31.0 (3.50)	132.0 (14.92)	1.56	0.86	0.75
	S5I	20.0 (2.26)	148.0 (16.72)	1.01	0.97	1.15

Table 5.4.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel.

Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T = 1{\cdot}6\ A_s\ f_{sy}\ b'd'/s$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.4.1 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u,theo}}$	Ratio $r_{'y}/r'_{'y}$
Iyengar & Rangan 15	S5II	28.0 (3.16)	132.0 (14.92)	1.41	0.87	0.80
	S6	30.0 (3.39)	-	1.51	-	0.21
	R1	25.0 (2.83)	35.0 (3.96)	0.94	0.30	0.66
	R2	28.0 (3.16)	111.0 (12.54)	1.06	0.90	1.21
	R3I	-	185.0 (20.90)	-	1.01	-
	R3II	30.0 (3.39)	185.0 (20.91)	1.13	1.01	1.08
	R4	30.0 (3.39)	-	1.13	-	0.23
	R5I	9.0 (1.02)	200.0 (22.60)	0.34	1.08	3.31
	R5II	30.0 (3.39)	185.0 (20.91)	1.13	1.00	1.08
	R6	33.0 (3.73)	64.0 (7.23)	1.24	0.36	0.50
	L1-1	29.0 (3.28)	-	1.46	-	0.17
	L1-2I	-	169.0 (19.10)	-	0.93	-
	L1-2II	32.0 (3.62)	60.0 (6.78)	1.61	0.33	0.36

Table 5.4.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel.

Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T = 1{\cdot}6\ A_s\ f_{sy}\ b'd'/s$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.4.1 Texpt/Ttheo	Ratio Mexpt/M_{u1} theo	Ratio r_{1y}/r'_{1y}
Iyengar & Rangan 15	L2-1I	10.0 (1.13)	181.0 (20.45)	0.50	0.99	2.05
	L2-1II	33.0 (3.73)	72.0 (8.14)	1.66	0.40	0.40
	L2-2I	10.0 (1.13)	181.0 (20.45)	0.50	0.99	2.05
	L2-2II	33.0 (3.73)	72.0 (8.14)	1.66	0.40	0.40
	L3-1I	20.0 (2.26)	163.0 (18.42)	1.01	0.93	1.02
	L3-1II	30.0 (3.39)	105.0 (11.87)	1.51	0.60	0.53
	L3-2I	20.0 (2.26)	163.0 (18.42)	1.01	0.93	1.02
	L3-2II	30.0 (3.39)	102.0 (11.53)	1.51	0.59	0.52
	L4-1	30.0 (3.39)	85.0 (9.61)	1.51	0.48	0.46
	L4-2	27.0 (3.05)	34.0 (3.84)	1.36	0.20	0.30
	L5-1	27.0 (3.05)	33.0 (3.73)	1.36	0.19	0.30
	L5-2	29.0 (3.28)	58.0 (6.55)	1.46	0.34	0.38
	L6-1I	20.0 (2.26)	163.0 (18.42)	1.01	1.05	1.02
	L6-1II	25.0 (2.83)	-	1.26	-	0.17
	L6-2I	22.0 (2.49)	156.0 (17.63)	1.11	1.01	0.91

Table 5.4.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel.
Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T = 1{\cdot}6\ A_s\ f_{sy}\ b'd'/s$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.4.1 Texpt / Ttheo	Ratio Mexpt / $M_{u\,theo}$	Ratio r_{y}/r'_{y}
Chinekov 21	B-2-8-01	48.6 (5.49)	486.1 (54.93)	0.63	1.32	1.65
	B-2-9-01a	46.9 (5.30)	468.7 (52.96)	0.60	1.26	1.68
	B-2-8-02	83.3 (9.41)	416.6 (47.08)	1.07	1.21	0.93
	B-2-8-02a	83.3 (9.41)	416.6 (47.08)	1.07	1.15	0.95
	B-2-8-04b	97.2 (10.98)	347.2 (39.23)	1.25	1.02	0.82
	B-2-8-04	145.8 (16.48)	364.5 (41.19)	1.88	0.76	0.57
	B-2-8-4a	138.9 (15.70)	347.2 (39.23)	1.79	0.73	0.57
	B-2-8-04b	145.8 (16.48)	364.5 (41.19)	1.88	0.70	0.57
	B-2-8-04c	152.8 (17.27)	381.9 (43.16)	1.97	0.70	0.53
	B-2-8-04d	125.0 (14.13)	312.5 (35.31)	1.61	0.70	0.59
	B-2-8-04e	131.9 (14.91)	329.8 (37.27)	1.70	0.71	0.57
	B-2-8-04f	138.9 (15.70)	347.2 (39.23)	0.98	0.76	1.02
	B-2-8-04g	97.2 (10.98)	434.0 (49.04)	0.69	0.97	1.59

Table 5.4.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel.

Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T = 1{\cdot}6,\ A_s\ f_{sy}\ b'd'/s$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.4.1 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u',theo}}$	Ratio $r_{'y}/r'_{'y}$
Pandit & Warwaruk 31	B2	72.0 (8.14)	195.0 (22.04)	0.82	0.81	1.62
	B3	95.0 (10.74)	110.0 (12.43)	1.09	0.46	1.08
	B4	85.0 (9.61)	-	0.97	-	0.66
	C1	78.0 (8.81)	280.0 (31.64)	0.67	0.90	1.95
	C2	105.0 (11.87)	195.0 (22.04)	0.90	0.62	1.33
	C3	111.0 (12.54)	110.0 (12.43)	0.95	0.35	1.02
	C4	111.0 (12.54)	-	0.95	-	0.64
	D1	99.0 (11.19)	637.0 (71.98)	0.85	0.84	0.99
	D2	164.0 (18.53)	365.0 (41.25)	1.41	0.47	0.49
	D3	156.0 (17.63)	195.0 (22.04)	1.34	0.26	0.37
	D4	146.0 (16.50)	-	1.25	-	0.22
	E1	76.0 (8.59)	195.0 (22.04)	0.65	0.81	2.09
	E2	101.0 (11.41)	110.0 (12.43)	0.87	0.46	1.39
	E3	121.0 (13.67)	-	1.04	-	0.88

Table 5.4.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel.

Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T = 1{\cdot}6\ A_s\ f_{sy}\ b'd'/s$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.4.1 $\frac{Texpt}{Ttheo}$	Ratio $\frac{Mexpt}{M_{u1}\ theo}$	Ratio r_{1y}/r'_{1y}
Jackson & Estanero 34	AU1	-	140.7 (15.90)	-	0.95	-
	AU3	24.6 (2.78)	69.5 (7.85)	1.34	0.47	0.53
	AU5	22.5 (2.54)	32.4 (3.66)	1.23	0.22	0.37
	AU8	22.0 (2.49)	1.1 (0.12)	1.20	0.01	0.22
	CU1	-	151.5 (17.12)	-	0.98	-
	CU2	24.7 (2.79)	133.1 (15.04)	0.63	0.85	1.63
	CU3	39.6 (4.48)	110.1 (12.44)	1.05	0.71	0.98
	CU4	48.2 (5.45)	91.9 (10.38)	1.24	0.60	0.80
	CU5	46.0 (5.20)	63.3 (7.15)	1.22	0.42	0.65
	CU6	47.6 (5.38)	53.5 (6.04)	1.39	0.35	0.54
	CU7	46.0 (5.20)	34.4 (3.89)	1.26	0.22	0.49
	CU8	43.5 (4.95)	1.6 (0.18)	1.25	0.01	0.32
	DU1	-	107.0 (12.09)	-	0.99	-
	DU2	19.1 (2.16)	102.7 (11.60)	0.80	0.95	1.37
	DU3	28.0 (3.16)	78.7 (8.89)	1.06	0.74	0.92

Table 5.4.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel.
Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T = 1{\cdot}6\ A_s\ f_{sy}\ b'd'/s$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.4.1 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u,theo}}$	Ratio $r_{'y}/r'_{'y}$
Jackson & Estanero 34	DU4	32.4 (3.66)	63.2 (7.14)	1.22	0.59	0.73
	DU5	33.3 (3.76)	46.1 (5.21)	1.37	0.43	0.55
	DU6	30.9 (3.49)	35.6 (4.02)	1.27	0.33	0.50
	DU7	32.0 (3.62)	24.3 (2.75)	1.23	0.23	0.45
	DU8	31.6 (3.57)	1.2 (0.13)	1.21	0.01	0.29
	EU1	-	107.0 (12.09)	-	0.97	-
	EU2	19.8 (2.24)	108.3 (12.24)	0.54	0.97	2.04
	EU3	31.6 (3.57)	89.9 (10.16)	0.84	0.82	1.27
	EU4	41.0 (4.63)	79.7 (9.01)	1.12	0.73	0.95
	EU5	46.4 (5.24)	63.7 (7.20)	1.27	0.59	0.77
	EU6	48.2 (5.45)	54.0 (6.10)	1.41	0.50	0.65
	EU7	51.2 (5.79)	36.5 (4.13)	1.40	0.33	0.57
	EU8	45.2 (5.11)	1.6 (0.18)	1.20	0.01	0.37

Table 5.4.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel. Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T = 1\cdot6\ A_s\ f_{sy}\ b'd'/s$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.4.1 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u\,theo}}$	Ratio r'_y/r'_y
Jackson & Estanero 34	GU-1	-	68.0 (7.68)	-	1.02	-
	GU-3	21.4 (2.42)	64.5 (7.29)	0.38	0.95	3.68
	GU-5	36.2 (4.09)	51.0 (5.76)	0.65	0.80	2.5
	GU-6	36.4 (4.11)	42.3 (4.78)	0.66	0.80	2.5
	GU-7	40.0 (4.52)	30.4 (3.44)	0.70	0.50	2.05
	GU-8	36.4 (4.11)	1.6 (0.18)	0.63	0.02	1.25
	A5-1	-	179.0 (20.23)	-	1.05	-
	A5-3	21.3 (2.41)	61.1 (6.92)	1.21	0.36	0.43
	A5-5	22.5 (2.54)	32.4 (3.66)	1.23	0.36	0.43
	A5-8	21.4 (2.42)	1.1 (0.12)	1.17	0.01	0.18
	B4-1	-	115.0 (13.00)	-	1.00	-
	B4-3	15.4 (1.74)	44.9 (5.07)	0.95	0.41	0.56
	B4-5	14.8 (1.67)	21.9 (2.48)	0.91	0.21	0.38
	B4-8	16.8 (1.90)	1.0 (0.09)	1.06	0.01	0.20
	C4-1	-	188.0 (21.24)	-	1.04	-
	C4-2	31.6 (3.57)	166.1 (18.77)	0.84	0.92	1.30

Table 5.4.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel.
Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T = 1{\cdot}6\ A_s\ f_{sy}\ b'd'/s$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.4.1 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u\prime}\ theo}$	Ratio $r_{\prime y}/r'_{\prime y}$
Jackson & Estanero 34	C4-3	41.4 (4.68)	114.6 (12.95)	1.10	0.64	0.82
	C4-4	44.0 (4.97)	84.9 (9.59)	1.17	0.49	0.68
	C4-5	45.9 (5.19)	63.0 (7.12)	1.22	0.37	0.57
	C4-6	40.2 (4.54)	46.1 (5.21)	1.23	0.27	0.46
	C4-7	47.0 (5.31)	35.0 (3.96)	1.24	0.21	0.45
	C4-8	48.4 (5.47)	1.6 (0.18)	1.28	0.01	0.31
	D4-1	-	115.0 (13.00)	-	0.96	-
	*D4-2	20.5 (2.32)	109.7 (12.40)	0.76	0.91	1.33
	D4-3	29.2 (3.30)	81.7 (9.23)	1.09	0.68	0.82
	D4-4	30.3 (3.42)	59.6 (6.74)	1.12	0.51	0.65
	D4-5	29.5 (3.33)	41.4 (4.68)	1.30	0.35	0.45
	D4-6	28.0 (3.16)	32.7 (3.70)	1.23	0.28	0.41
	D4-7	27.4 (3.10)	21.2 (2.40)	1.21	0.18	0.34
	D4-8	28.0 (3.16)	1.2 (0.13)	1.23	0.01	0.22

Table 5.4.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel.

Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T = 1{\cdot}6\ A_s\ f_{sy}\ b'd'/s$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.4.1 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u'theo}}$	Ratio r'_y/r'_y
Jackson & Estanero 34	E4-1	-	126.0 (14.24)	-	1.01	-
	E4-2	22.4 (2.53)	111.3 (12.58)	0.59	0.90	1.72
	E4-3	31.4 (3.55)	91.9 (10.38)	0.99	0.75	0.97
	E4-4	40.4 (4.56)	78.8 (8.90)	1.27	0.64	0.74
	E4-5	45.2 (5.11)	62.2 (7.03)	1.42	0.51	0.60
	E4-6	46.0 (5.20)	51.9 (5.86)	1.45	0.43	0.54
	E4-7	46.8 (5.29)	29.4 (3.32)	1.47	0.25	0.42
	E4-8	43.0 (4.86)	1.6 (0.18)	1.35	0.01	0.27
	F3-1	-	68.0 (7.68)	-	0.97	-
	F3-2	11.8 (1.33)	65.1 (7.36)	0.46	0.92	2.07
	F3-3	18.8 (2.12)	54.7 (6.18)	0.73	0.77	1.23
	F3-4	21.8 (2.46)	44.6 (5.04)	0.84	0.67	0.96
	F3-5	26.0 (2.94)	36.6 (4.14)	1.01	0.54	0.75
	F3-6	25.5 (2.88)	29.7 (3.36)	0.99	0.45	0.67

Table 5.4.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel.

Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T = 1{\cdot}6\ A_s\ f_{sy}\ b'd'/s$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.4.1 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u\prime}\ theo}$	Ratio $r_{\prime y}/r'_{\prime y}$
Jackson & Estanero 34	F3-7	24.9 (2.81)	19.3 (2.18)	0.96	0.03	0.32
	F3-8	24.0 (2.71)	1.1 (0.12)	0.93	0.02	0.32
	G2-1	-	68.5 (7.74)	-	1.03	-
	G2-3	20.8 (2.35)	63.0 (7.12)	0.40	0.95	3.53
	G2-5	32.9 (3.72)	47.0 (5.31)	0.60	0.75	2.50
	G2-6	38.0 (4.29)	43.9 (4.96)	0.69	0.70	2.26
	G2-7	41.6 (4.70)	31.5 (3.56)	0.68	0.50	2.14
	G2-8	51.2 (5.79)	1.6 (0.18)	0.89	0.03	1.29

Table 5.4.1 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel. Partial Yield Theory B (Draft British Unified Code of Practice 1972, $T = 1.6 A_s f_{sy} b'd'/s$)

Investigator	Beam	Texpt kipf in (kNm)	Ratio equn 5.4.1 Texpt / Ttheo	Ratio r_{2y}/r'_{2y}
Hsu 27	B9	264.0 (29.83)	1.58	0.50
	B10	304.0 (34.35)	1.83	0.21
	M1	269.0 (30.40)	1.53	0.75
	M2	359.0 (40.57)	1.41	0.74
	M3	388.0 (43.84)	1.25	0.68
	M4	439.0 (49.61)	1.06	0.70
	M5	493.0 (55.50)	0.93	0.67
	M6	532.0 (60.00)	0.83	0.73
Ernst 42	3TR7	49.7 (5.62)	1.78	0.49
	3TR15	61.7 (6.97)	1.26	0.87
	4TR7	54.8 (6.19)	1.96	0.36
	4TR15	74.0 (8.36)	1.51	0.64
	5TR3	43.0 (4.86)	3.08	0.10
	5TR7	59.7 (6.75)	2.14	0.20
	5TR15	76.5 (8.64)	1.54	0.34
	5TR30	92.6 (10.46)	0.95	0.69
Evans & Sarkar 25	HB1	44.1 (4.98)	1.40	0.53
	HB7	36.1 (4.08)	1.09	0.56
	HB13	51.3 (5.80)	1.13	0.70

Table 5.5.2 - Detailed Results for Members with Longitudinal and Transverse Steel - Partial Yield Theory C - Empirical Method

$$(r_{2y} < \frac{1}{1+{}^{b}/_{d}})$$

Investigator	Beam	Texpt kipf in (kNm)	r_{12}	Ratio $\frac{\text{Texpt}}{\text{Ttheo}}$ equn 5.5.2	Ratio $\frac{\text{Texpt}}{\text{Ttheo}}$ equn 5.5.3	
Hsu[27]	B1	197.0 (22.26)	0.398	1.02	1.05	
	B2	259.0 (29.27)	0.382	1.05	1.09	
	B3	332.0 (37.52)	0.377	1.03	1.07	
	B4	419.0 (47.35)	0.382	0.99	1.04	
	B5	497.0 (56.16)	0.386	0.95	0.99	
	B6	546.0 (61.70)	0.372	0.86	0.90	O.R.
	B7	238.0 (26.89)	0.850	0.99	1.05	
	B8	288.0 (32.54)	0.889	0.77	0.98	
	B9	264.0 (29.83)	0.176	0.94	1.10	
	B10	304.0 (34.35)	0.078	0.65	1.04	
	D1	198.0 (22.37)	0.398	1.04	1.07	
	D2	245.0 (27.69)	0.382	0.98	1.02	
	D3	346.0 (39.10)	0.377	1.04	1.08	
	D4	424.0 (47.91)	0.382	0.98	1.02	
	M1	269.0 (30.40)	0.260	1.12	1.21	
	M2	359.0 (40.57)	0.256	1.13	1.24	
	M3	388.0 (43.84)	0.252	1.05	1.16	
	M4	439.0 (49.61)	0.257	0.95	1.04	
	M5	493.0 (55.71)	0.258	0.86	0.94	
	M6	532.0 (60.12)	0.257	0.78	0.85	O.R.

O.R. denotes over reinforced.

Table 5.5.2 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel - Partial Yield Theory C - Empirical Method ($r_{2y} < \frac{1}{1+b/d}$)

Investigator	Beam	Texpt kipf in (kNm)	r_{12}	Ratio Texpt/Ttheo equn 5.5.2	Ratio Texpt/Ttheo equn 5.5.3	
Hsu[27]	I2	319.0 (36.05)	0.394	1.12	1.16	
	I3	404.0 (45.65)	0.377	1.14	1.19	
	I4	514.0 (58.08)	0.382	1.16	1.21	
	I5	626.0 (70.74)	0.386	1.14	1.19	
	I6	679.0 (76.73)	0.372	1.02	1.06	O.R
	J1	190.0 (21.47)	0.398	1.09	1.13	
	J2	258.0 (29.15)	0.394	1.07	1.12	
	J3	312.0 (35.26)	0.377	0.97	1.02	
	J4	360.0 (40.68)	0.382	0.88	0.91	O.R.
	G1	237.0 (26.78)	0.323	0.92	0.99	
	G2	357.0 (40.34)	0.322	1.04	1.13	
	G3	439.0 (49.61)	0.308	1.03	1.12	
	G4	574.0 (64.86)	0.308	1.06	1.16	
	G5	637.0 (71.98)	0.314	0.92	1.01	O.R.
	G6	346.0 (39.10)	0.318	1.02	1.10	
	G7	466.0 (52.66)	0.316	1.04	1.13	
	G8	650.0 (73.45)	0.305	1.09	1.19	

Table 5.5.2 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel - Partial Yield Theory C - Empirical Method ($r_{2y} < \frac{1}{1+b/d}$)

Investigator	Beam	Texpt kipf in (kNm)	r_{12}	Ratio Texpt/Ttheo equn 5.5.2	Ratio Texpt/Ttheo equn 5.5.3	
Hsu[27]	N1	80.5 (9.10)	0.316	1.00	1.17	
	N1a	79.6 (8.99)	0.316	0.99	1.16	
	N2	128.0 (14.46)	0.321	1.05	1.21	
	N2a	117.0 (13.22)	0.320	0.92	1.06	
	N3	108.0 (12.20)	0.305	1.01	1.17	
	N4	139.0 (15.71)	0.321	0.91	1.04	
	K1	136.0 (15.37)	0.200	0.99	1.07	
	K2	210.0 (23.73)	0.204	1.02	1.08	
	K3	252.0 (28.48)	0.197	0.91	0.95	
	K4	310.0 (35.03)	0.197	0.78	0.80	O.R.
	C1	100.0 (11.30)	0.500	1.07	1.08	
	C2	135.0 (15.20)	0.516	1.04	1.03	
	C3	177.0 (20.00)	0.495	1.04	1.04	O.R.
	C4	224.0 (25.31)	0.486	1.01	1.01	O.R.
	C5	263.0 (29.72)	0.482	0.92	0.92	O.R.
	C6	303.0 (34.24)	0.499	0.84	0.84	O.R.

Table 5.5.3 – Detailed Results for Members with Longitudinal and Transverse Steel – Partial Yield Theory C – Empirical Method ($r_{2y} < \frac{1}{1+b/d}$)

Investigator	Beam	Texpt kipf in (kNm)	r_{1z}	Ratio Texpt/Ttheo equn 5.5.2	Ratio Texpt/Ttheo equn 5.5.3
Ernst 42	3TR7	49.7 (5.62)	0.143	0.77	0.98
	3TR15	61.7 (6.97)	0.251	0.83	0.99
	3TR30	76.0 (8.57)	0.501	0.79	0.94
	4TR7	54.8 (6.19)	0.080	0.65	0.95
	4TR15	74.0 (8.36)	0.141	0.80	1.00
	4TR30	85.0 (9.61)	0.281	0.73	0.85
	5TR3	43.0 (4.86)	0.026	0.42	0.95
	5TR7	59.7 (6.75)	0.051	0.55	0.96
	5TR15	76.5 (8.64)	0.090	0.63	0.91
	5TR30	92.6 (10.46)	0.180	0.66	0.79
Evans & Sarkar 25	HB1	44.1 (4.98)	0.289	0.87	1.03
	HB7	36.1 (4.08)	0.285	0.69	0.81
	HB13	51.3 (5.80)	0.285	0.71	0.85
Pandit & Warwaruk 31	E3	121.0 (13.67)	0.300	0.90	1.04
Iyengar & Rangan 15	R4	30.0 (3.39)	0.087	0.44	0.68

Table 5.5.4 - Detailed Results for Members with Longitudinal and Transverse Steel

Partial Yield Theory C - Empirical method ($r'_y < \frac{1}{1 + d/b + 2M/T}$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.5.3 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u\,theo}}$	Ratio r'_y/r'_y
Goode & Helmy 28	III4	36.8 (4.16)	180.0 (20.34)	1.28	0.86	0.52
	IV3	46.0 (5.20)	75.4 (8.52)	1.11	0.37	0.51
	IV4	42.5 (4.80)	180.0 (20.34)	1.02	0.90	0.93
	V3	28.5 (3.22)	40.7 (4.60)	1.22	0.42	0.56
	III2	39.5 (4.46)	75.4 (8.52)	1.37	0.36	0.28
Gesund et al 24	1	79.0 (8.93)	79.0 (8.93)	1.01	0.44	0.86
	9	120.0 (13.56)	60.0 (6.78)	0.82	0.43	0.54
	10	176.0 (19.89)	44.0 (4.97)	0.62	0.63	0.85
Iyengar & Rangan 15	V1	41.0 (4.63)	55.0 (6.22)	0.96	0.31	0.48
	V2	36.0 (4.07)	121.0 (13.67)	0.86	0.69	0.84
	V5	39.0 (4.41)	142.0 (16.05)	0.94	0.81	0.89
	V6	34.0 (3.84)	52.0 (5.88)	0.82	0.30	0.51

Table 5.5.4 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel

Partial Yield Theory C - Empirical method ($r_{,y} < \frac{1}{1 + {}^{d}/_{b} + {}^{2M}/_{T}}$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.5.3 $\frac{Texpt}{Ttheo}$	Ratio $\frac{Mexpt}{M_{u,} theo}$	Ratio $r_{,y}/r'_{,y}$
Iyengar & Rangan 15	S2II	31.0 (3.50)	13C.0 (14.69)	0.88	0.85	0.87
	S3	29.0 (3.28)	32.0 (3.62)	0.84	0.22	0.38
	S4II	31.0 (3.50)	132.0 (14.92)	0.87	0.86	0.88
	S5II	28.0 (3.16)	132.0 (14.92)	0.80	0.87	0.95
	R1	25.0 (2.83)	35.0 (3.96)	0.67	0.30	0.73
	R4	30.0 (3.39)	-	0.68	-	0.23
	R6	33.0 (3.37)	64.0 (7.23)	0.75	0.36	0.56
	L1-2II	32.0 (3.62)	60.0 (6.78)	0.89	0.34	0.41
	L2-1II	33.0 (3.73)	72.0 (8.14)	0.90	0.40	0.45
	L2-2II	33.0 (3.73)	72.0 (8.14)	0.90	0.40	0.45
	L3-1II	30.0 (3.39)	105.0 (11.87)	0.85	0.60	0.63
	L3-2II	30.0 (3.39)	102.0 (11.53)	0.85	0.59	0.61
	L4-1	30.0 (3.39)	85.0 (9.61)	0.84	0.48	0.54
	L4-2	27.0 (3.05)	34.0 (3.84)	0.78	0.20	0.33
	L5-I	27.0 (3.05)	33.0 (3.73)	0.78	0.20	0.33

Table 5.5.4 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel

Partial Yield Theory C - Empirical method ($r_{ty} < \frac{1}{1 + d/b + 2M/T}$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.5.3 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u\,theo}}$	Ratio r_{ty}/r'_{ty}
Iyengar & Rangan 15	L5-2	29.0 (3.28)	58.0 (6.55)	0.84	0.35	0.43
Pandit & Warwaruk 31	D2	164.0 (18.53)	365.0 (41.25)	0.97	0.48	0.55
	E3	121.0 (13.67)	-	1.04	-	0.88
Jackson & Estanero 34	AU3	24.6 (2.78)	69.5 (7.85)	0.86	0.47	0.61
	AU5	22.5 (2.54)	32.4 (3.66)	0.80	0.23	0.41
	CU4	48.2 (5.45)	91.9 (10.38)	0.96	0.60	0.92
	CU5	46.0 (5.20)	63.3 (7.15)	0.94	0.42	0.73
	CU6	47.6 (5.38)	53.5 (6.04)	1.04	0.35	0.60
	CU7	46.0 (5.20)	34.4 (3.89)	0.96	0.23	0.53
	DU4	32.4 (3.66)	63.2 (7.14)	0.91	0.59	0.84

Table 5.5.4 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel

Partial Yield Theory C - Empirical method ($r_{ly} < \frac{1}{1 + d/b + 2M/T}$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.5.3 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{ul\ theo}}$	Ratio r_{ly}/r'_{ly}
Jackson & Estanero 34	DU5	33.3 (3.76)	46.1 (5.21)	0.99	0.43	0.62
	DU6	30.9 (3.49)	35.6 (4.02)	0.92	0.33	0.56
	DU7	32.0 (3.62)	24.3 (2.75)	0.92	0.23	0.49
	EU5	46.4 (5.24)	63.7 (7.20)	0.96	0.59	0.88
	EU6	48.2 (5.45)	54.0 (6.10)	1.04	0.50	0.73
	EU7	51.2 (5.79)	36.5 (4.13)	1.06	0.34	0.62
	A5-3	21.3 (2.41)	61.1 (6.92)	0.72	0.36	0.49
	A5-5	22.5 (2.54)	32.4 (3.66)	0.73	0.19	0.35
	A5-8	21.4 (2.42)	1.1 (0.12)	0.69	0.01	0.18
	B4-3	15.4 (1.74)	44.9 (5.07)	0.61	0.41	0.65
	B4-5	14.8 (1.67)	21.9 (2.48)	0.59	0.21	0.43
	B4-8	16.8 (1.90)	1.0 (0.09)	0.68	0.01	0.20
	C4-3	41.4 (4.68)	114.6 (12.95)	0.78	0.64	0.96
	C4-4	44.0 (4.97)	84.9 (9.59)	0.82	0.49	0.78

Table 5.5.4 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel

Partial Yield Theory C - Empirical method ($r_{y} < \frac{1}{1 + d/b + 2M/T}$)

Investigation	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.5.3 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u,theo}}$	Ratio r_{y}/r'_{y}
Jackson & Estanero 34	C4-5	45.9 (5.19)	63.0 (7.12)	0.86	0.37	0.64
	C4-6	40.2 (4.54)	46.1 (5.21)	0.83	0.27	0.51
	C4-7	47.0 (5.31)	35.0 (3.96)	0.88	0.21	0.49
	C4-8	48.4 (5.47)	1.6 (0.18)	0.91	0.01	0.31
	D4-3	29.2 (3.30)	81.7 (9.23)	0.75	0.68	0.96
	D4-4	30.3 (3.42)	59.6 (6.74)	0.79	0.51	0.75
	D4-5	29.5 (3.33)	41.4 (4.68)	0.86	0.35	0.51
	D4-6	28.0 (3.16)	32.7 (3.70)	0.83	0.28	0.46
	D4-7	27.4 (3.10)	21.2 (2.40)	0.81	0.18	0.38
	D4-8	28.0 (3.16)	1.2 (0.13)	0.83	0.01	0.22
	E4-4	40.4 (4.56)	78.8 (8.90)	0.83	0.64	0.86
	E4-5	45.2 (5.11)	62.2 (7.03)	0.96	0.51	0.68
	E4-6	46.0 (5.20)	51.9 (5.86)	0.98	0.43	0.61
	E4-7	46.8 (5.29)	29.4 (3.32)	0.99	0.25	0.45

Table 5.5.4 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel

Partial Yield Theory C - Empirical method ($r_{'y} < \frac{1}{1 + d/b + 2M/T}$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.5.3 Texpt/Ttheo	Ratio Mexpt/$M_{u'}$ theo	Ratio $r_{'y}/r_{'y}'$
Jackson & Estanero 34	E4-8	43.0 (4.86)	1.6 (0.18)	0.92	0.01	0.28
	F3-5	26.0 (2.94)	36.6 (4.14)	0.74	0.54	0.86
	F3-6	25.5 (2.88)	29.7 (3.36)	0.73	0.45	0.77
	F3-7	24.9 (2.81)	19.3 (2.18)	0.71	0.30	0.61
	F3-8	24.0 (2.71)	1.1 (0.12)	0.68	0.02	0.32
Chinekov 21	B-2-8.04b	97.2 (10.98)	347.2 (39.23)	0.90	1.02	0.96
	B-2-8-04	145.8 (16.48)	364.5 (41.19)	1.22	0.76	0.65
	B-2-8-04a	138.9 (15.70)	347.2 (39.23)	1.17	0.73	0.66
	B-2-8-04b	145.8 (16.48)	364.5 (41.19)	1.16	0.70	0.66
	B-2-8-04c	152.8 (17.27)	381.9 (43.16)	1.21	0.70	0.61
	B-2-8-04d	125.0 (14.13)	312.5 (35.31)	1.09	0.70	0.68
	B-2-8-04e	131.9 (14.91)	329.8 (37.27)	1.13	0.71	0.66

Table 5.5.5 - Detailed Results for Members with Longitudinal and Transverse Steel Partial Yield Theory C - Empirical Method, ($r_{3y} < \frac{1}{1 + {}^{2d}/_{b} - {}^{2M}/_{T}}$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.5.3 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u,theo}}$	Ratio r_{3y}/r_{3y}'
Goode & Helmy 28	III1	24.5 (2.77)	4.8 (0.54)	0.76	.02	0.95
	III3	34.0 (3.84)	40.7 (4.60)	1.04	.20	0.22
	IV2	43.5 (4.92)	40.7 (4.60)	0.87	.20	0.82
	V1	25.5 (2.88)	4.8 (0.54)	0.92	.05	0.95
	V2	24.0 (2.71)	23.4 (2.64)	0.89	.24	0.38
Iyengar & Rangan 15	V4-1	33.0 (3.73)	-	0.75	-	0.35
	S6	30.0 (3.39)	-	0.83	-	0.26
	L1-1	29.0 (3.28)	-	0.79	-	0.26
	L6-1II	25.0 (2.83)	-	0.72	-	0.26
Evans & Sarkar 25	HB1	44.1 (4.98)	-	0.96	-	0.86
	HB7	36.1 (4.08)	-	0.81	-	0.60
	HB13	51.3 (5.80)	-	0.85	-	0.72

Table 5.5.5 (continued) - Detailed Results for Members with Longitudinal and Transverse Steel
Partial Yield Theory C - Empirical Method, ($r_{3y} < \frac{1}{1 + {}^{2d}/_{b} - {}^{2M}/_{T}}$)

Investigator	Beam	Experimental T kipf in (kNm)	Experimental M kipf in (kNm)	Ratio equn 5.5.3 $\frac{T_{expt}}{T_{theo}}$	Ratio $\frac{M_{expt}}{M_{u,theo}}$	Ratio r_{3y}/r_{3y}'
Pandit & Ivarwaruk 31	C3	111.0 (12.54)	110.0 (12.43)	0.81	0.35	0.54
	D3	156.0 (17.63)	195.0 (22.04)	0.83	0.26	0.10
	D4	146.0 (16.50)	-	0.78	-	0.66
Jackson & Estanero 34	AU8	22.0 (2.49)	1.1 (0.12)	0.72	0.01	0.33
	CU8	43.5 (4.92)	1.6 (0.18)	0.88	0.01	0.50
	DU8	31.6 (3.57)	1.2 (0.13)	0.86	0.01	0.44
	EU8	45.2 (5.11)	1.6 (0.18)	0.92	0.01	0.56