RELIABILITY EVALUATION AND PREDICTION
WITH SPECIAL REFERENCE TO
LIFE TESTING

by

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SUMMARY

This thesis is in three major parts.

In Part I the mathematics of the Weibull distribution are extensively explained and the validity of the Weibull model is demonstrated; of particular interest is the possibility of carrying out truncated tests with or without suspensions with a high degree of confidence.

In Part II, the investigation to prove such validity is part of the original experimental work carried out by the author and is an important contribution to, and confirmation of, the knowledge of the subject treated. Of particular interest is the demonstration of the sensitivity and discriminating powers of the Weibull analysis in revealing two or more failures distributions. Results are given and discussed.

In Part III the author deals with the extremely interesting topic of Reliability Prediction and explains in detail the original Computer Programme he has perfected to predict accurately and almost immediately characteristic life, mean life, reliability, failure rate, etc., by analysing but a few data. Such speed and accuracy are a major contribution to the advancement of knowledge on this subject, and are vitally important to all manufacturers and buyers interested in reliability.

A copy of the Computer Programme on disc or cassette can be obtained from the author on application.

Evaluation, prediction, reliability, failure, life.

DECLARATION

No part of the work described in this thesis has been submitted in support of an application for another degree or qualification of this or any other University or Institution of learning.

It gives an account of the author's own research work performed at the Department of Production Technology and Production Management of the University of Aston in Birmingham and field work undertaken around Birmingham, as described in the thesis.

It is a well-known fact that in any work of this nature the researcher must rely extensively upon the wide and ever growing body of already published and unpublished research evidence, and this work is no exception.

The material referred to in the text has been gathered from many sources and over a period of years - much from the author's experience in industry, some in visits to plants and discussions with technical staff, some from public and private institutions, and some from technical literature.

A list of references to publications and papers consulted during the course of the investigation, is given in the last Chapter of this thesis.

L. G. D. Petrucci.

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FOREWORD

It is recognised that, to achieve the highest degree of reliability in a component or assembly, its specification must be clearly defined in respect of its dimensional and material parameters and there must be a statement of its expected performance when tested under controlled conditions of load, speed, temperature and vibration.

What is not readily recognised is the need to accurately determine and define the degree of variability that exists within any specification, or indeed within any set of statistics.

The Weibull analytical method described and illustrated in this thesis is a proven procedure for obtaining this picture of variability and hence an understanding of the limitations of the specification and the penalties of working outside these limitations.

Many people in industry are still not fully aware that the intelligent application, preferably using computers, of these recently developed but well-proven reliability theories can bring appreciable reduction in the costs of defects and failures, improvements in operational reliability and utilisation, and reduction in repair time and maintenance costs.

Those who are aware of these applications sometimes find difficulty in choosing the most appropriate reliability technique to meet a given situation. Others tend to avoid an analytical approach because they expect to find difficulty with the mathematics or lack specific examples of applications to practical problems.

Increasingly, reliability is becoming recognised as a vital factor to be dealt with in a quantitative manner when tendering, in feasibility studies, and during development, production, operations and maintenance. Reliability as a separate function and a formal discipline as quality, is a relatively new development. However, it has always been a consideration, although most specifically by name.

Reliability when expressed in a qualitative manner is meaningless and when quoted quantitatively can be misleading unless interpreted correctly. Often it is simple to specify, difficult to achieve, harder still to predict and very expensive to demonstrate.

Specification - Reliability may be specified in terms of the probability of successful completion of a mission within the design performance envelope.

Prediction - Simple formulas have been developed into which a preponderance of empirical constants have been injected to enable one to arrive at a fairly plausible solution. In certain cases one is hampered still further by the paucity of reliability data (the empirical constants).

Achievement - The prescribed reliability may have to be achieved in a severe environment with complex hybrid equipment in which the failure of any single component may well cause failure of the whole. It may have to be brought about, too, where no human corrective action is possible and when "early life" failures may predominate.

Demonstration - Demonstration of reliability is required during Research and Development with a fair degree of engineering confidence on "one-shot" devices such as guided weapons, which have limited monitoring services during the mission, are without recovery after the mission, and may yield a total mission time of only a few minutes in an entire development firing programme of a few score rounds. Financial considerations preclude the demonstration of reliability with a high degree of statistical confidence on pilot or production runs, and the lower confidence level arrived at is to a large extent due to the limitations imposed by the restricted nature of evaluation trials rather than a reflection on the time reliability of the system.

Importance of Reliability Engineering - Reliability is thus not an easy matter to deal with. It is, however, becoming recognised as one of the most important single characteristics of many complex hybrid systems and has to be dealt with as such. It may take precedence in a weapon system, for example over requirements such as the resistance of the guidance system to interference and counter measures, strike effectiveness in terms of the accuracy of the missile, and killing capacity of the warhead.

The intrinsic reliability of any equipment is a function of both quality of design and quality of conformance to design. By quality is meant "fitness for purpose", and by reliability "continuing fitness for purpose". In aircraft, fitness for purpose may be related to two distinct cases: ground and flight.

Taking aircraft equipment or an aircraft system, minimum probability of success could be specified for:-

a) Period tests while in stock at depots

b) Field tests on installation

c) Check-out prior to take-off

d) Flight.

PART I - EVALUATION OF THE VALIDITY OF THE WEIBULL DISTRIBUTION

1 - INTRODUCTION

Reliability is generally defined as "the mathematical probability that a product will operate, without failure, under prescribed operating and environmental conditions, for a specified period of time (i.e. hours, cycles, miles, etc.)".

To arrive at a mathematical expression for reliability, certain theoretical functions may be combined with the known laws of probability and applied to the collected data.

Data analysis consists, in the main, of grouping together (in alphabetical groups, numerical ranks, or in some other order) the variables under consideration, and of examining the groups to see whether there are any significant relationships, simple or systematic, corresponding to the observed output. It may be possible to describe these relationships with a few symbols, but this is normally done by constructing some form of histogram or plotting a graph or curve. Data from tests or measurements can be plotted to show the frequency distribution against a selected independent variable such as weight, length, stress, cycles or time. If we can establish and describe these relationships, a study of the histogram or graph thus produced will usually give some indication of the distribution of the parent population from which the sample was drawn: it may show for example how many times each value of the variable is likely to occur; we may find that we have an exponential distribution, or the familiar normal distribution, or something else. If the distribution curve were known, then we could define mathematically the product reliability, quality or conformity and we can say that the analysis has added to the meaning and usefulness of the original data.

However, to arrive at a distribution type by this method entails the collection and plotting of a considerable number of data, in other words a fairly large sample size has to be drawn and, in the case of life testing, a lot of time and money is required.

Thus, in our society of high speed computers and automation, simplified statistical methods for the manual or computer-assisted solution of analytical problems still bring relief to overburdened engineers and statisticians.

Among the various statistical probability density functions used in reliability studies, the Weibull distribution is assuming increased prominence. In 1950 Wallodi Weibull[1], a Swedish professor engaged initially in the study of fatigue characteristics, arrived at a very useful statistical model whereby certain distribution types could be represented by a straight line law. Thus only a small number of results were required in order to locate this line, and from it useful conclusions could be drawn.

Whilst investigating this analysis Mr. A. Plait[2, 3], an American engineer, devised a method of presenting this very useful distribution by plotting the data in a Weibull line on specially constructed graph paper, now known as "Weibull Probability Paper". A few simple measurements then directly provide the shape and position parameters of the distribution curve. The scales on the Weibull Probability Paper are laid out to display the three parameters that define the distribution: the slope β, the characteristic life η, and the starting point of the curve, or minimum life t_o.

Direct measurements on the paper can be made to determine the reliability of a product.

Since that time further work[4] on this subject has been carried out so as to reduce and simplify the procedure involved with extracting the relevant information from this graphical presentation.

When we are carrying out an experiment and we find that the failure rate is not constant, the Weibull model could be used in appropriate circumstances. (See Ch. 6).

In Part I of this thesis the author deals mainly with the mechanics of the Weibull Distribution. Thus, Chapters 2 and 3 give the history, analytical expressions, properties, graphical constructions, and applications of the Weibull Distribution. In Chapter 4 the author offers theoretically accurate and approximate formulae for calculating the Median Ranks. These formulae are used as sub-routines in the Computer Programme of Chapter 10. The User can select at will the accurate but slow method, or the approximate but instantaneous method. This is an important contribution, since the User is now free from the need to consult Tables and to plot laboriously his data: all he has to do is to select either routine to obtain his data, nicely tabulated and plotted, and the relative extrapolations.

In Part II the author describes his experimental research to prove beyond doubt the validity of the Weibull Distribution when applied to life testing, and demonstrates its high capacity to discriminate between different failure modes. This is extremely important, in order to detect the influence of various parameters (in our case, slightly different welding speeds and different plates from the same steel). The results of this original investigation, carried out at the University of Aston, are reported in Chapter 7 and discussed in Chapter 8, which also gives ideas for further uses and applications of the Weibull Distribution.

Part III is dedicated to the problem of Reliability Prediction, which is the main purpose of this thesis and the major contribution by the author to enable private individuals or Companies to use the Weibull Distribution without any knowledge of the distribution itself or of computers. Chapter 10 describes in outline the Computer Programme which is an integral part of this thesis. All the user has to do is to load the programme (which comes in disc or cassette form) into his computer and follow the simple directions which appear on the screen. The programme is fully interactive: it offers choices of accurate but slow or approximate but fast routines, requests data, checks errors, calculates the best fitting line to the data, corrects curved plots, and offers the choice of a visual display or a print-out. Very few data are required (usually five or six) to give a reasonable accuracy, and all types of tests (completed or truncated, with or without suspensions) can be accommodated. Thus a life testing programme can be interrupted short, to give an idea of the projected life (B-10, B20, ..., mean, characteristic life) and of other parameters, and informs on whether more than one mode of failure is present. All this within little more than the time needed to enter the few data required. One can see immediately, therefore, the immense advantage of using this programme for anyone interested in life testing, such as buyers or manufacturers of equipment, stores, Armed Forces, railways, truck and aircraft companies, etc.

In this thesis, special attention is given to carefully conducted experiments and to industrial experience gained in using the technique considered. The application of this analytical technique to product reliability problems can provide better information for making decisions. This strategy is essential to efficient operation amd may enhance a Company's competitive position and business opportunities.

Sometimes, in fact, it is difficult to take decisions. The common reason seems to be that people do not like to risk the criticism which might follow a wrong decision. They feel that if an impending decision can be deferred, the need for making the decision will vanish. Also, if a decision can be delayed long enough, many feel, the appropriate information to make a proof decision will ultimately materialise. But these attitudes are unproductive.

The modern work environment is fast-paced and high-pressured. The rate of technological discovery and innovation, the political and economic pressures for continued business growth, and rising economic and social aspirations, all combine in requiring earlier decisions at each step of the research, development and production cycles. This faster pace results in less available time, and so research is accelerated, fewer experiments are conducted, and fewer pre-production models are built. These measures, in turn, yield less information and data from which to derive decisions.

Due to the large overall environmental pressure, decisions must be correct, or nearly so, as soon as possible. This stringent goal requires effective, fast and practical methods of feedback of the resulting conclusions and decisions. A method of computer-assisted graphical analysis, based on Weibull probability plots, which is presented in this thesis, meets these requirements. The relationship of probability analysis to research, development and production is shown in the flow chart of Fig. 1-1 on page 24 .

The mathematics describing the Weibull reliability equation and the method of construction of the special probability paper are presented in this thesis for the benefit of those wishing to obtain proof of the statistical model. However, this section may be immediately passed over without fear of being unable to understand and employ this distribution model to advantage. Some may even prefer to refer directly to the computer programme which, on entering the life data, provides immediately all information required and produces Weibull and failure plots, both on the screen and on a print-out.

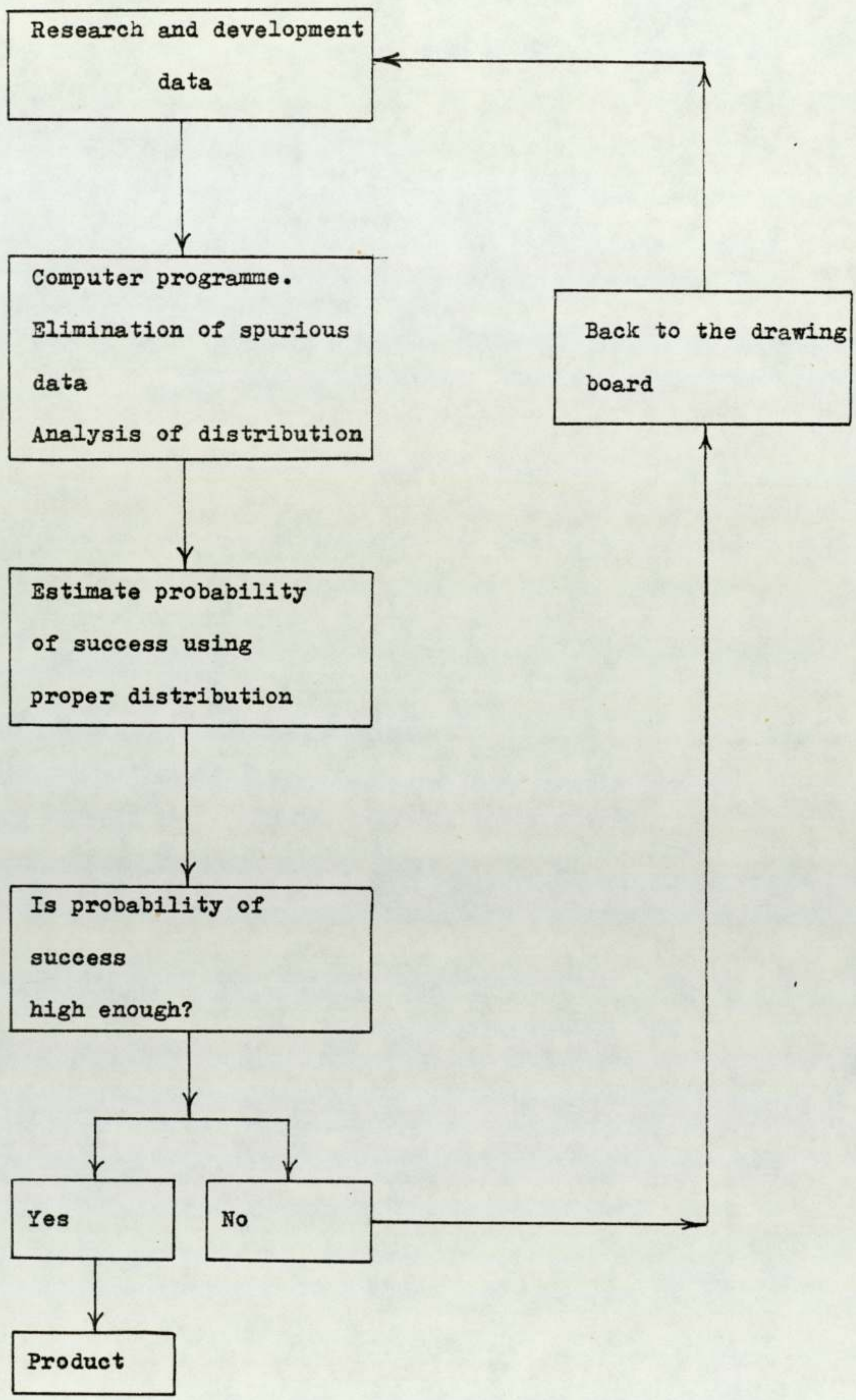


Fig. 1.1 - Graphic analysis for the determination of underlying distribution for research and development decisions.

2 - MATHEMATICAL MODEL

The Weibull system of analysis is based on a mathematical model known as the Weibull equation.

2.1 - Derivation of the reliability equation

$$R = \frac{S}{N} = \frac{N - F}{N} = 1 - \frac{F}{N}$$

where: R = Reliability

N = Number tested

S = Survivors

F = Failures.

Taking the derivative with respect to time of $R = 1 - \frac{F}{N}$:

$$\frac{dR}{dt} = \frac{d}{dt}(1 - \frac{F}{N}) = - \frac{1}{N}\frac{dF}{dt} \; ; \; \frac{dF}{dt} = - N\frac{dR}{dt} .$$

Dividing by S to obtain the failure rate per unit still being tested:

$$\frac{1}{S}\frac{dF}{dt} = - \frac{N}{S}\frac{dR}{dt} = - \frac{1}{R}\frac{dR}{dt} \text{ . Now } \frac{1}{S}\frac{dF}{dt} = Z(t) = \lambda$$

where λ is termed the instantaneous failure rate, or the hazard rate.

Substituting: $\lambda = - \frac{1}{R}\frac{dR}{dt} \; ; \; \frac{1}{R}\frac{dR}{dt} = - \lambda$.

Separating the variables: $\frac{dR}{R} = - \lambda \, dt$.

Integrating between t_o and t_i, and remembering that when $t_o = 0$, $R = 1$ and when $t = t_i$, $R = R_{t_i}$:

$$\int_1^{R_{t_i}} \frac{dR}{R} = - \int_{t_o}^{t_i} \lambda \, dt. \text{ Integrating:}$$

$$\ln R_{t_i} = - \int_{t_o}^{t_i} \lambda \, dt ,$$

$$R_{t_i} = e^{- \int_{t_o}^{t_i} \lambda \, dt} . \qquad (2.1\text{-}1)$$

If λ is constant, then

$$R_{t_i} = e^{- \int_{t_o}^{t_i} \lambda \, dt} = e^{- \lambda(t_i - t_o)} = \text{the exponential reliability expression} . \qquad (2.1)$$

2.2 - The Weibull distribution

In 1951 Weibull[1] suggested that the simplest empirical expression representing a great variety of actual data could be obtained by writing eq.(2.1-1) in the following manner:

$$R_t = e^{-\left(\frac{t-t_o}{\eta}\right)^{\beta}} \qquad (2.2\text{-}1)$$

where $\left(\frac{t-t_o}{\eta}\right)^{\beta} = \int_{t_o}^{t_i} \lambda dt$

and t_o = the starting point of the distribution

η = the characteristic life

β = the slope (or the underlying type of distribution).

This is the Weibull model.

It is readily seen that the Weibull distribution may be expressed in any of the following forms:

$$R(t) = e^{-\left(\frac{t-t_o}{\eta}\right)^{\beta}}$$

$$F(t) = 1 - R(t) = 1 - e^{-\left(\frac{t-t_o}{\eta}\right)^{\beta}} \quad \text{(Weibull equation)}$$

$$\lambda(t) = \frac{\beta}{\eta} \cdot \left(\frac{t-t_o}{\eta}\right)^{\beta - 1}$$

where R(t) = the cumulative probability of survival, or fraction surviving, or reliability

F(t) = the cumulative probability of failure (area under the distribution from t_o to t), or fraction failing

(t) = the instantaneous failure rate

t = the random variable (time, cycles, stress, size, etc., to failure)

t_o = origin of the distribution.

Multiplying R and F by 100 converts the fraction surviving or failing to the percent reliability or unreliability.

The probability density function f(t) is equal to $\frac{dF(t)}{dt}$, the ordinate of the frequency distribution:

$$f(t) = \frac{dF(t)}{dt} = \frac{\beta \cdot (t-t_o)^{\beta-1} \cdot e^{-\left(\frac{t-t_o}{\eta}\right)^{\beta}}}{\eta^{\beta}} .$$

Note that the constants t_o, η and β appearing in the Weibull expression can each be given a physical interpretation as discussed in Section 6-12 on page 95, Section 6-17 on page 166 and Section 6-18 on page 167.

It has since been found that by the appropriate choice of values for the three parameters t_o, β and η the Weibull equation can be used to represent a wide variety of distributions, including many actual failure distributions and both the practically important random and Normal distributions. Indeed, since failures tend to occur right from the moment the parts are put into operations, the minimum life parameter t_o is usually zero, leaving only the shape parameter β and the characteristic life η to be estimated. Consequently, the Weibull model has been widely used in statistical reliability.

Although experience has since shown that the Weibull model can be used for the vast majority of failure patterns, it is essential to note that it is an empirical function and may not be able to represent some particular distributions encountered in practice. Its importance lies in its simplicity and wide adaptability.

2.3 - Properties of the Weibull distribution

For $\beta = 3.4$, the Weibull distribution approximates a Normal distribution.

<u>Skewness</u>

In the Weibull distribution, the skewness is given by:

$$\frac{\Gamma(1+\frac{3}{\beta}) - 3\,\Gamma(1+\frac{1}{\beta})\,\Gamma(1+\frac{2}{\beta}) + 2\Gamma^3(1+\frac{1}{\beta})}{\left[(1+\frac{2}{\beta}) - \Gamma^2(1+\frac{1}{\beta})\right]^{3/2}} \,.$$

A Normal distribution has a skewness of zero and, as $\beta = 3{,}4$ for a normally distributed Weibull distribution, if one substitutes this value of β in the above formula, one will see that the top line will come to zero.

Values of the skewness for different values of β are shown here below:

β	Skewness
0.5	6.619
1.0	2.000
2.0	0.626
3.0	0.454
3.5	-0.026
4.0	-0.062
5.0	-0.333
6.0	-0.905
10.0	-1.000
20.0	-2.000

2.4 - Construction of the Weibull Probability Paper

From the Weibull model:

$$R = e^{-\left(\frac{t - t_o}{\eta}\right)^{\beta}}$$

and, letting $t_o = 0$;

$$R = e^{-(\frac{t}{\eta})^{\beta}}$$

and $\frac{1}{R} = e^{(\frac{t}{\eta})^{\beta}}$

Taking logarithms:

$\ln \frac{1}{R} = (\frac{t}{\eta})^{\beta}$, since $\ln e = 1$.

Taking logarithms again:

$\ln \ln \frac{1}{R} = \beta.\ln(\frac{t}{\eta}) = \beta.\ln t - \beta.\ln \eta$.

This is of the form: $y = m.x + n$ (Straight Line Law), where

$y = \ln \ln \frac{1}{R}$;

$x = \ln t$;

$m = \beta$ (slope)

and $n = - \beta \ln \eta$ (intercept).

The ordinate of the probability paper is constructed from $\ln \ln \frac{1}{R}$ or,

since $R = 1 - F$, from $\ln \ln \frac{1}{1-F}$.

(Remember that Reliability (R) plus Unreliability (F) equals 1).

2.5 - Graphical presentation of the Weibull Analysis

As stated in the introduction, the Weibull model gives means of expressing certain types of distributions as a straight line. Hence, if this line were to be presented graphically in some way, then the complications involved in analysing these distributions could be greatly reduced.

By taking logs of the basic Weibull model twice, the following expression is obtained:

$$\ln \ln \frac{1}{R} = \beta \ln(t-t_o) - \beta \ln \eta$$

which is in the form

$y = m.x + n$ (the straight line law).

Thus in order to present the Weibull distributions graphically it would be necessary to use graph paper which adopted a double log scale on one axis (the "y" axis or ordinate) and a single log scale on the other axis (the "x" axis or abscissa). This in fact is what Weibull Probability Paper does and, since the scales are non-linear, by merely repositioning a straight line on this paper the type of distribution represented may be radically altered.

Each line plotted on this paper may be considered as having three variables (as in the mathematical model) which may affect the distribution represented in different ways. Firstly, the slope (β) which determines the shape of the distribution; secondly, the characteristic life (η) which determines the "spread" of the distribution; and, thirdly, the value of t_o for this line, which determines the starting point of the distribution.

Fig. 2-1 on page 32 shows the relationship between distribution types and slope. The distribution types, ranging from exponential to highly skewed to the left, carry slope values from 0.5 to 5. A perfectly exponential distribution has a slope of 1, whilst a slope of 3.44 approximates to a Normal distribution.

Fig. 2-2 on page 33 shows the variations of the frequency distribution when η varies.

Fig. 2-3 on page 34 shows how the distribution is displaced along the horizontal axis when t_o varies.

The great advantage of using the Weibull analysis in graphical form is that a measure of a component's reliability may be obtained quickly and easily, with few samples.

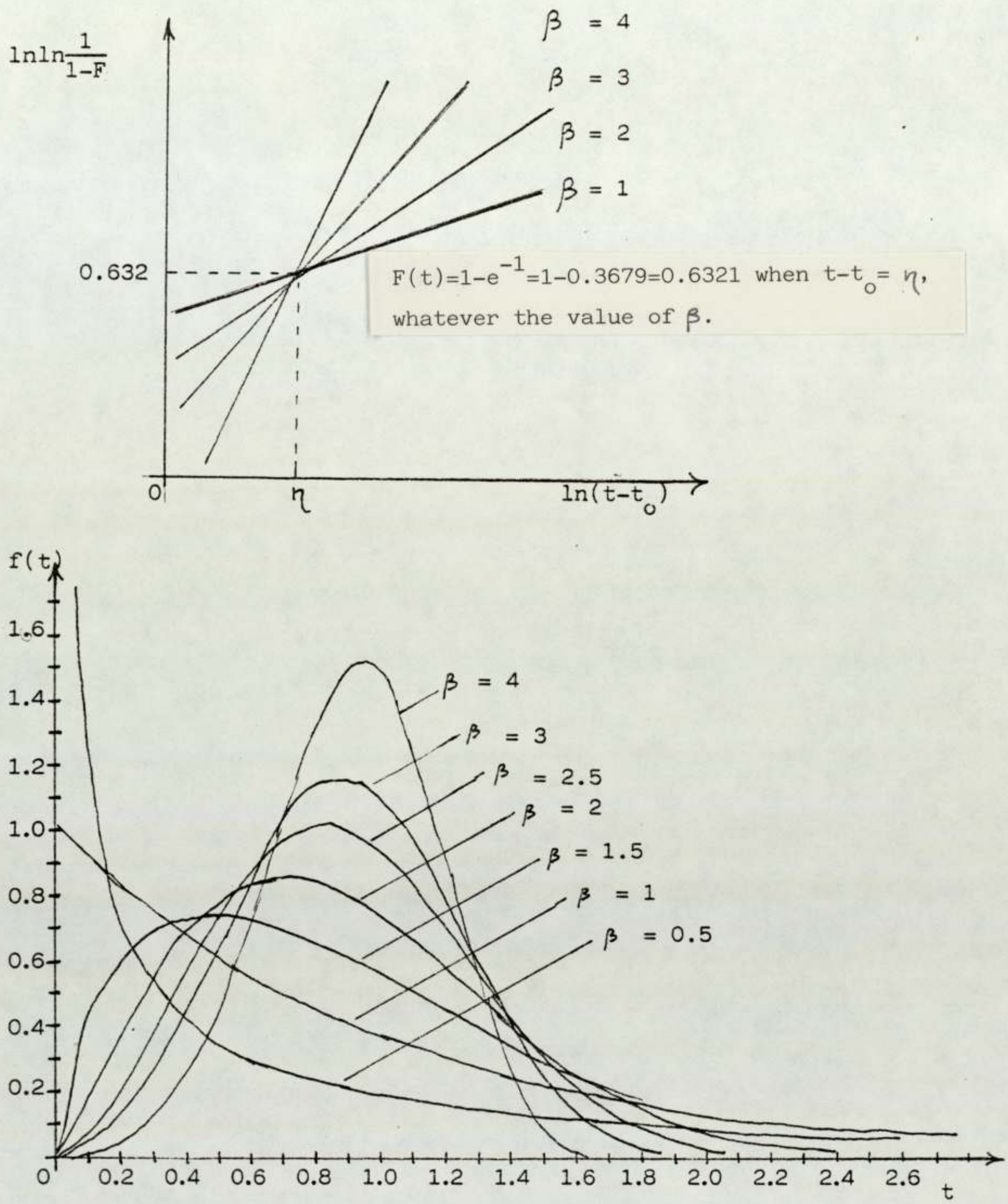


Fig. 2.1. Variations of the distribution for different values of the slope β.

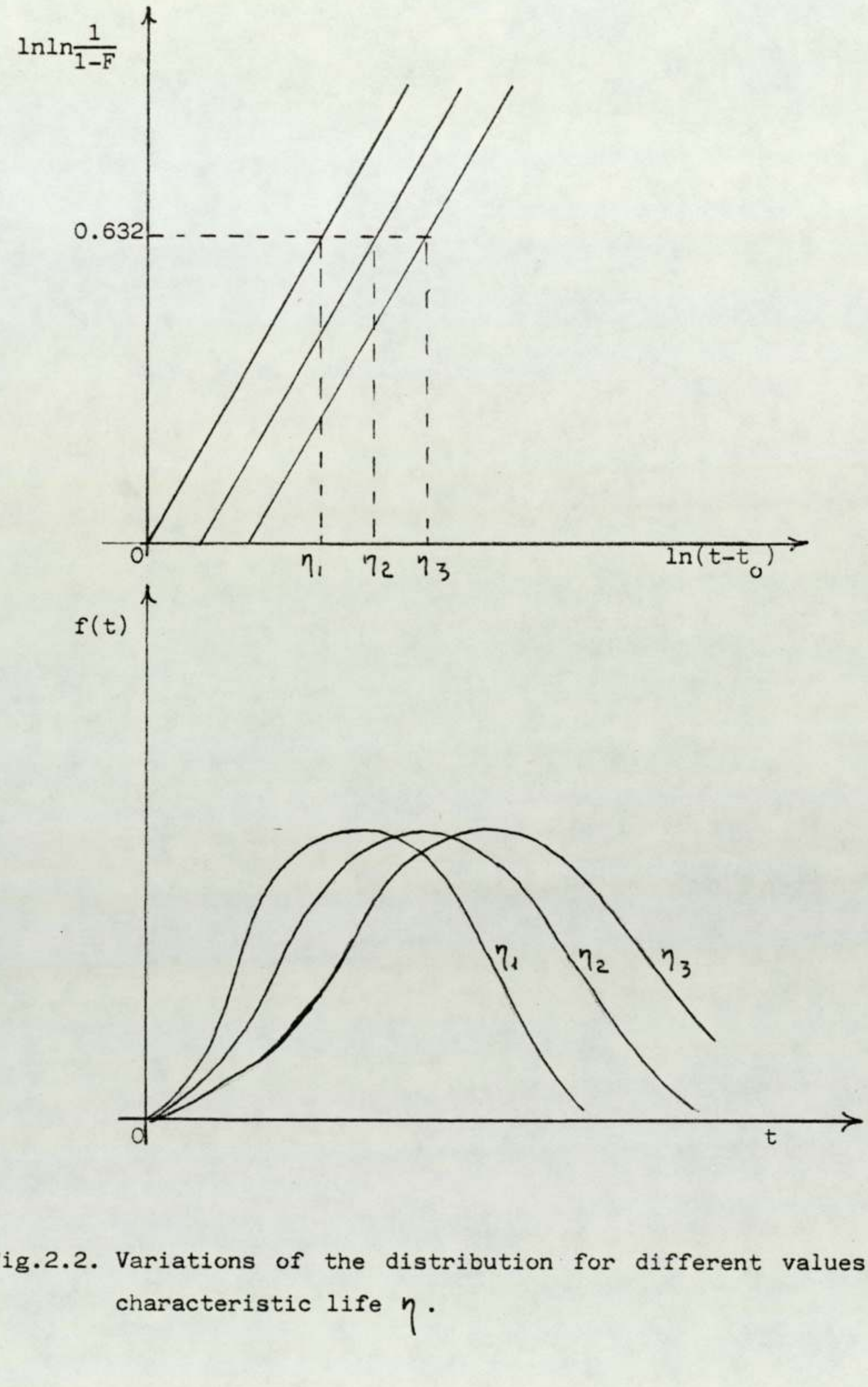


Fig.2.2. Variations of the distribution for different values of the characteristic life η.

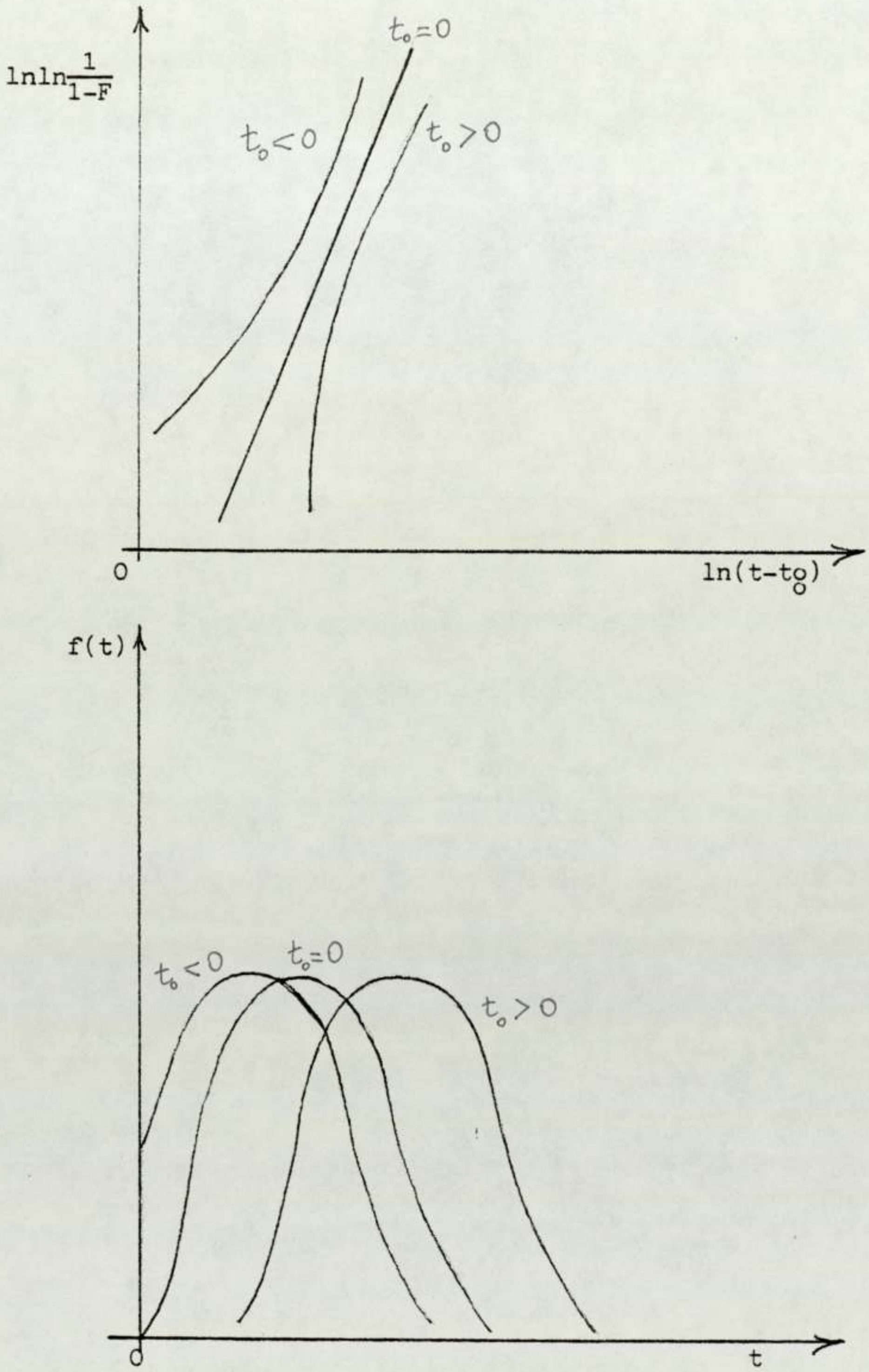


Fig. 2-3. Variations of the distribution for different values of the starting point t_o.

3 - APPLICATIONS OF THE WEIBULL ANALYSIS

The Weibull distribution function, equation 2.2-1 on page 26 can be applied to very widely differing fields and, in many cases, with quite satisfactory results.[17]

The statistical analysis of inspection data and test information can provide useful and quantitative information about a product. The analysis is particularly useful in evaluating product reliability, conformity to specification, or in comparing alternative materials, designs and fabrication procedures. The Weibull analysis is one statistical method of analysing data. It provides a graphical solution of statistical problems with a minimum of time and effort.

Before going on to describe the Weibull distribution, its graphical construction, usage and application to Reliability Engineering, the writer would like to point out the wide applicability of this distribution and the fact that it need not be confined to reliability alone. Professor Weibull has applied the distribution to a variety of problems. More recently others have made extensive use of the function in the study of inspection functions, stock control, and manpower planning. Many more applications could derive some benefit from this technique. To this end, examples dealing with non-reliability functions are presented in the final part of this thesis.

This thesis contains a discussion of the industrial applications of the Weibull model. It presents specific examples of situations where the Weibull technique is applicable. It offers a thorough computer programme that permits the elaboration of the few data required and produces and predicts all parameters and results that it is possible to extract from the Weibull model.

3.1 - Applications to Reliability Engineering

Since it is obviously more desirable to detect items with poor reliability before they reach the customer, emphasis is placed on the investigation of rig data, with graphical presentation of results.

When considering reliability testing, it must be remembered that the most convenient method of obtaining a measure of an item's reliability is to record and analyse its unreliability, i.e. its failures.

For example, consider the following approximately Normal distribution of failures of some items, against the number of hours of test time.

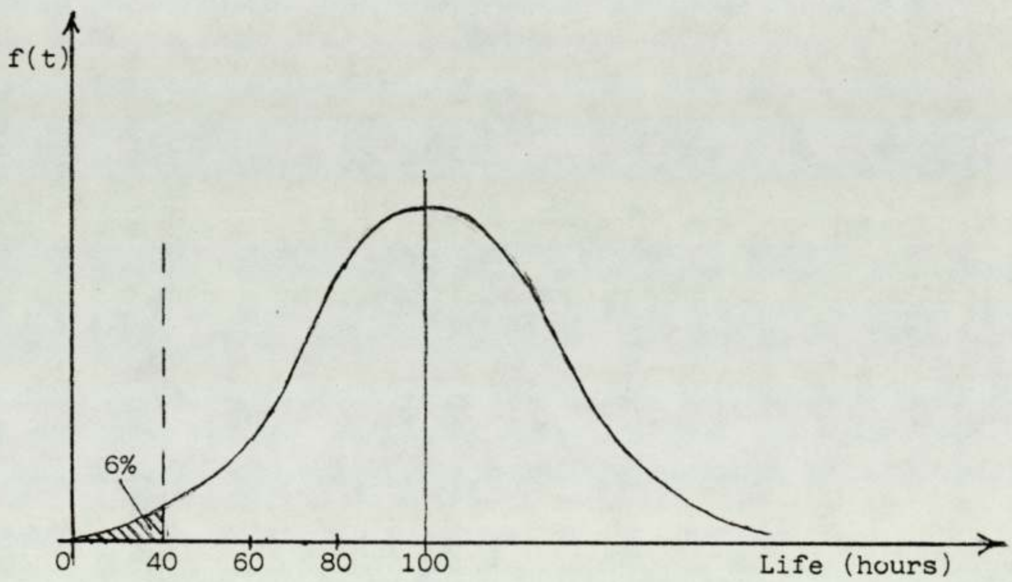


Fig. 3-1.

This diagram represents the total population and it is shown clearly that 6% of this population would not reach a required 40 hours in service; in other words, the item has a reliability of only 0.94. But how can we discover this, by sample testing?

Prior to the introduction of the Weibull analysis it was necessary to test a great many samples in order to establish a picture of this distribution, to find the mean, median, mode, etc., and hence to obtain a measure of reliability at the given life. Such a method is lengthy, tiresome and often complicated, and hence some engineers opted for a simple proof test on a limited sample size. This involved testing a sample of items to failure and observing for any which did not meet the minimum life requirements. The usefulness of this type of test is debatable particularly when the item considered is a new one. Consider the following figures obtained using elementary probability theory. If one sample item is taken from the population and life tested, the probability of it failing before 40 hours is only 6%. Also, if seven samples were life tested, the probability of having one or more failures below 40 hours is still $P(x \geqslant 1) = 1 - P(x = 0) = 1 - \binom{7}{0}.(0.94)^7 = 1 - 0.648477594$ or only about 35%. Thus again a large number of items would need to be tested in order to be confident of discovering the 6% of all items failing before 40 hours, i.e. in order to obtain a measure of the reliability of the items.

As an example, consider a light switch intended for use on motorcars by a car firm, and passed to their Reliability Department for assessment. It is specified that the switch shall have a minimum life of 50,000 cycles and a reliability level of 0.98. Since reliability plus unreliability must equal unity,

$F = 1 - R = 1 - 0.98 = 0.02$

or, by multiplying by 100, the failure level must not exceed 2%.

Seven switches are tested and the cycles to failure recorded.[4]

1st failure	72,000 cycles
2nd failure	115,000 cycles
3rd failure	150,000 cycles
4th failure	180,000 cycles
5th failure	210,000 cycles
6th failure	285,000 cycles
7th failure	320,000 cycles

It is apparent that each of the seven switches tested exceeded the minimum life requirement. To many engineers this could constitute an immediate pass; however, any decision to accept this switch would be totally wrong, as we shall see. Although the minimum life requirements appears to be met, what of the specified reliability level?

By plotting these failure ages on Weibull Probability Paper it is observed that the resulting line falls to the left of the specified acceptance mark, see Fig. 3-2 on page 40 , and that at a minimum life requirement of 50,000 cycles a failure level of some 5% can be expected. The corresponding distribution for this line is as shown by a continuous line in Fig. 3-3 on page 41 , and it can be seen from this and the Weibull line that the switch does not meet the specification and must be rejected.

Following the rejection of this switch, a re-design was called for and tests on the re-designed switches were carried out. The recorded failure ages were as follows:

1st failure	65,000 cycles
2nd failure	75,000 cycles
3rd failure	81,000 cycles
4th failure	88,000 cycles
5th failure	95,000 cycles
6th failure	100,000 cycles
7th failure	110,000 cycles

By plotting these results we see that this time the line passes exactly through the acceptance mark, see Fig. 3-2 on page 40. The frequency distribution for this line is shown by a dotted line in Fig. 3-3 on page 41. Thus these switches may now be accepted.

It is worth noting in this instance that the average "life to failure" of the second set of results is considerably lower than that for the first set and yet, unlike the original switch, the re-designed switch was accepted. This is because the second distribution was of a form giving less scatter to the results, thus containing the majority of the failures in a narrower band than in the first distribution.

It can be seen from the foregoing that each Weibull line represents a unique distribution and a very real danger exists of accepting unreliable components unless the scatter of this distribution is taken into account.

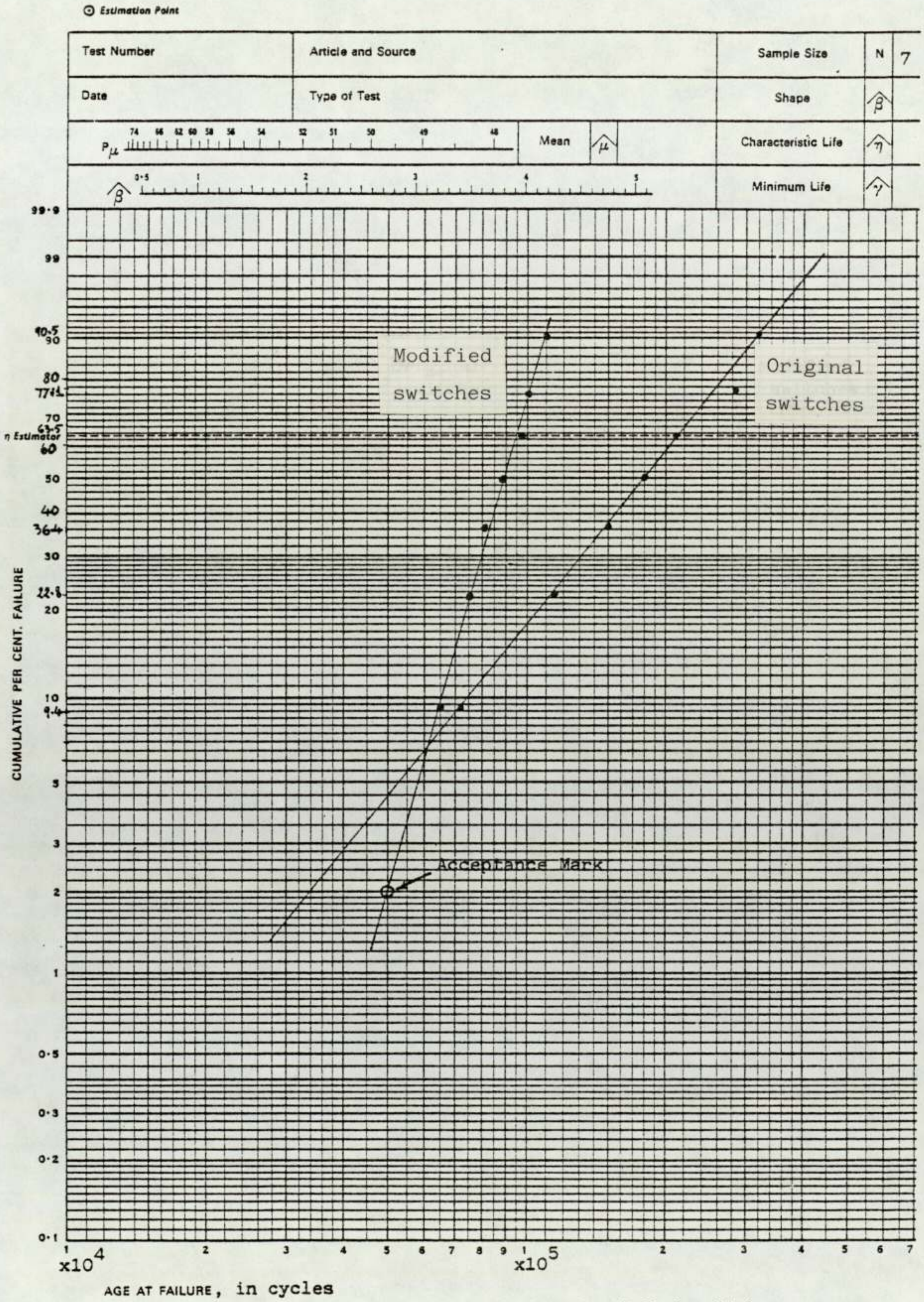


Fig.3-2. Weibull plot of original and modified switches.

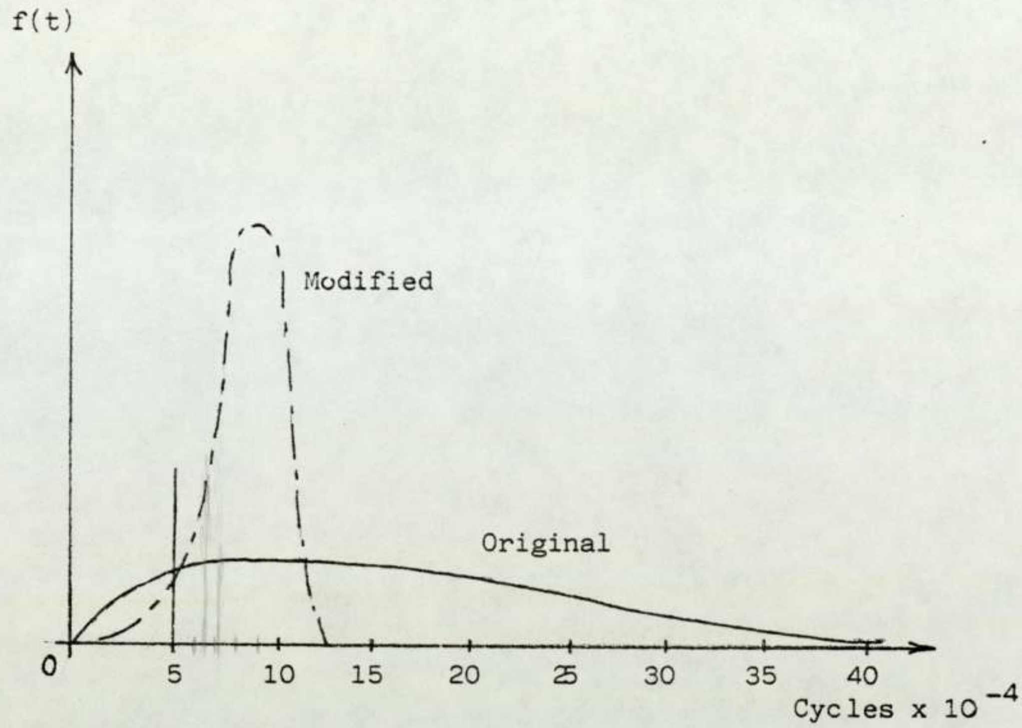


Fig. 3-3. Frequency Distribution of original and modified switches.

3.2 - Application to industry

The Weibull analysis is widely used in industry and it has a variety of interesting applications.

3.2-a - Time-to-failure distributions

Many industries use the Weibull analysis to estimate the frequency distribution of failures in components or assemblies, spalling or galling of various parts, etc.

3.2-b - B-10 life

Some industries are interested not only in the time-to-failure distribution, but also in the percent which failed at a given time. The time by which 10% have failed is called the 10% rated life or B-10 life. In Fig. 3-2 on page 40 this is shown as the 10% level of fraction failing. Referring to the data in Fig.3-2 on page 40, the B-10 life is estimated at 72,000 cycles for the original switch and at 65,000 cycles for the modified switch.

3.2-c - Differences in B-10 life

We have considered so far only one application of the Weibull analysis: estimating the frequency distribution. However, the Weibull approach can also be used to determine if a difference in B-10 life exists between (i) standard material and new or modified material, (ii) standard and new or modified designs, (iii) different vendors producing the same material, and (iv) materials used by different customers or subjected to different uses.

3.2-d - Analytical determination of rated life

By definition, B-1 is the time $(t-t_o)$ corresponding to a Median Rank $F(t-t_o) = 0.01$. From $y = a + b.x$, we have

$$x = \frac{y - a}{b} = \frac{\ln.\ln(1/1-F) - a}{b} \text{, and, since } x = \ln(t-t_o)$$

$$B\text{-}1 = (t-t_o) = e^x = e^{\frac{\ln.\ln(1/1-0.01) - a}{b}} \qquad \text{when } F = 0.01$$

Similarly:

$$B\text{-}10 = e^{\frac{\ln.\ln(1/1-0.10) - a}{b}} \qquad \text{when } F = 0.10$$

$$B\text{-}20 = e^{\frac{\ln.\ln(1/1-0.20) - a}{b}} \qquad \text{when } F = 0.20$$

$$B\text{-}50 = e^{\frac{\ln.\ln(1/1-0.50) - a}{b}} \qquad \text{when } F = 0.50$$

and so on.

3.2-e - Failure distributions with variables other than time on the abscissa

The Weibull technique can be used for a great number of types of distribution analyses. Variation in manufacturing processes is inevitable. Whether one is trying to control the dimension of a part or any other quality characteristic of a manufactured product, it is certain that the results will vary.

The reliability of operating devices such as switches, valves, cylinders, etc., is best described in terms of the number of cycles performed to failure, and in this case the Weibull plot will show cycles on the abscissa (see Fig. 3-4 A on page 45).(4)

Consider a Quality Control problem of part size where a given minimum per cent are expected to be within tolerance. A Weibull plot of a sample of part sizes would describe the distribution of sizes (see Fig. 3-4B on page 45)(4) and could be used to determine whether the manufacturing process was operating as well as expected.

In many industries, like the aircraft industry, normal life testing could take too long; for example, turbine blades might not fail for many thousands of hours under normal running conditions. In order to avoid excessively long testing times, testing can be accelerated by increasing the percent stress (Fig. 3-4C on page 45)[4] or the percent stress-time (Fig. 3-4D on page 45)[4]. In Fig. 3-4C on page 45 we are accelerating the detection of failure modes by gradually and continuously increasing the applied stress and determining the resultant failure distribution. Fig. 3-4D on page 45[4] incorporates the factor of time as well as that of stress increase. Such a procedure, aided by the Weibull distribution analysis, can accelerate the evaluation of safety margins of individual modes of failure.

3.2-f - The separation of two or more distributions

The test failure points from two or more distributions, usually will not plot on Weibull probability paper as a straight line. In bearing tests, for example, ball and inner race failures could occur. Each failure mode has its own distinct failure distribution and a special analysis is required to separate the different failure modes. An example of this type of analysis is seen in Section 6.9 and 6.16 on pages 90 and 149 respectively.

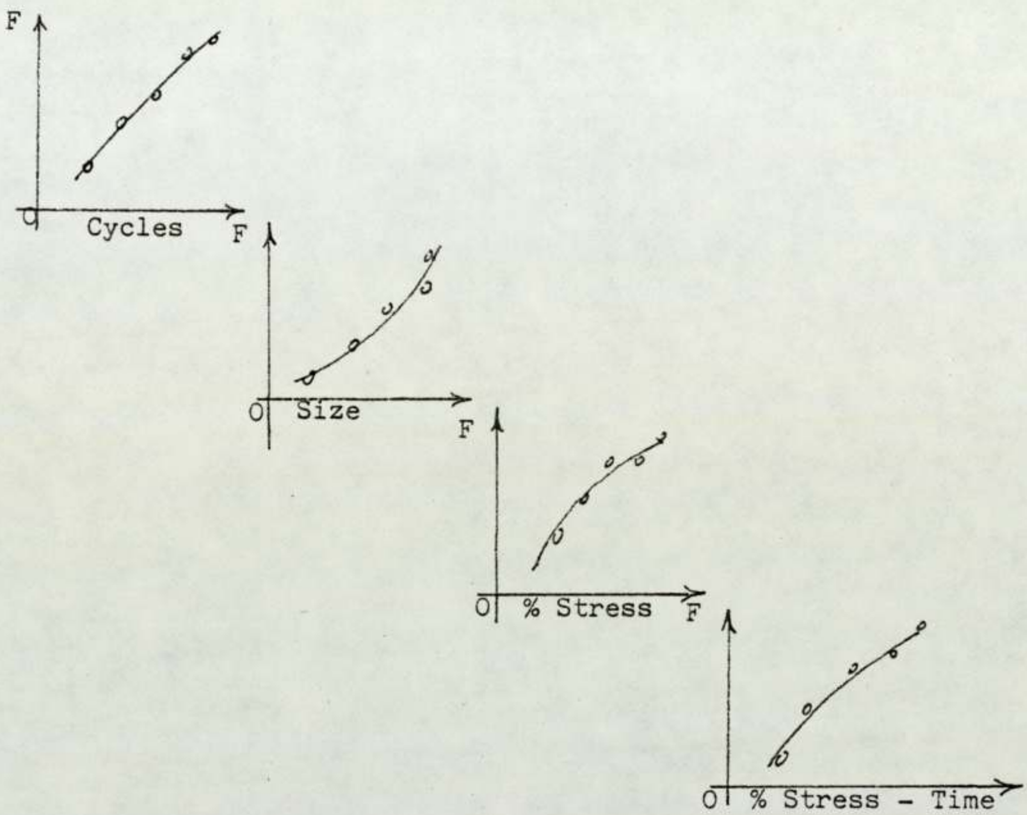


Fig. 3-4. Weibull plots of test data using cycles, size, stress and stress- time as the independent variable [4]

4 - MEDIAN RANKS

When plotting failure ages it is necessary to assign to each failure a failure rank. The first failure in a group of tested units will have a definite percentage of the total population failing before it. If this exact percentage is known, then this number is the true rank of the first failure.

Consider 5 imaginary units taken from a population of 5, i.e. the total population is tested to failure. Suppose the imaginary failure ages are:

4,800; 23,000; 13,000; 67,000; 39,000.

Assigning failure ranks and arranging the failures in ascending order, we have:

Order number	Failure age	Failure rank
1	4,800	0.2
2	13,000	0.4
3	23,000	0.6
4	39,000	0.8
5	67,000	1.0

Order No. 1 failed at 4,800 cycles and represents a 20% population failure rate; order No. 2 failed at 13,000 cycles and represents a 40% population failure rate;.....; order No. 5 failed at 67,000 cycles and, being the final failure, represents 100% or total population failure.

Now, while the above is true when, and only when total population failure is observed, it is not true when only sample failure is observed. Normally, one does not know the true rank.

Consider the 5 units tested as being but a small sample of a larger population. In this case it is incorrect to say that the first order number represents 20% of the total population, as it is highly probable that some failure, within the population as a whole, will have occurred prior to this observed sample failure. Similarly, the fifth cannot be said to represent 100% or total population failure, since it is highly probable that some items within the parent population will survive this observed sample failure.

To cater for such contingencies, one can use the "mean rank" or one can use an estimate that will be too high half the time and too low the other half of the time: the "median rank". These "median ranks" are simply statistical estimates of the failure rank, being such that negative errors are cancelled out by positive errors. While both estimates are statistically unbiassed, the mean rank will give more pessimistic results at low values of fraction failing (and more optimistic at high values) than will the median rank.

An advantage of using mean ranks is that they are easier to calculate, being given by:

$$\text{Mean Rank} = \frac{i}{N_o + 1}$$

where i = failure order number

N_o = sample size, i.e. the total number of tested units in the group.

When the sample is small, it is important to use a correction factor to relate the fraction of the sample which has failed at any given moment, to the fraction of the population which would have failed at the same moment. Various methods have been used, but it is now generally agreed that the Median Rank offers the most unbiassed prediction.

Also, the practitioners who have the Tables available generally prefer the consistency of using a uniform probability basis for confidence limits (*) as well as for the plotting of the points for the Weibull line:

Ranks

5%

50% = Median Ranks

95%

Median Ranks F are determined from the binomial expansion:

$$(R+F)^n = \left[(1-F)+F\right]^n = 0.50 = (1-F)^n + nF(1-F)^{n-1} + \frac{n(n-1)}{2}F^2(1-F)^{n-2} + \frac{n(n-1)(n-2)}{3.2}F^3(1-F)^{n-3} + \ldots\ldots\ldots$$

To determine the first median rank in a sample size of 10, we take the first term of the binomial expansion to the power of 10 and solve for F: $0.50 = (1-F)^n$

$$\ln(0.50) = 10\ln(1-F)$$

$$\ln(1-F) = \frac{\ln(0.50)}{10} = \frac{-0.6931471806}{10} = -0.06931471806$$

$$1-F = e^{-0.06931471806} = 0.9330329916$$

$$F = 1 - 0.9331329916 = 0.066967008 \text{ or } 6.7\%.$$

In general, to find the ith median rank, we expand the binomial to the power of n to i terms and equate this to 0.50, and then solve for F. However, solving this for more than i=1 is rather difficult.

The following formula by A. Bernard is easy to use and gives good approximation:

$$F_{0.50} = \frac{i-0.3}{n+0.4}, \text{ with } 1 \leqslant i \leqslant n$$

where n = number of parts on life test, or sample size

i = number of failures so far, or failure order number.

(*) See Section 6-15-a on Page 131.

Then, the best estimate of the percentage of the population failed so far is $\frac{i-0.3}{n+0.4}$. 100%.

Thus, if we put 10 parts on life test, when the first failure occurs, we predict that $\frac{1-0.3}{10+0.4}$. 100 = 6.73 per cent of the population has failed.

The two methods differ in the third decimal place when i=1 and n=10. Median ranks for sample sizes of one through fifty have been calculated from the binomial and are listed in Appendix A on pages 222 to 225 inclusive.

It will be seen in Chapter 10 on page 216 that the computer programme suggested calculates the median ranks and thus dispenses with the need for consulting Appendix A on page 222 .

Going back to the case on page 46, with the sample of 5 as tested it can be seen that:

Failure order number 1 occurred at 4,800 cycles and i=1 (first failure order) while N = 5 (sample size.

The Median Rank for this failure being:

$\frac{1-0.3}{5+0.4}$ = 0.1296 or 12.96% of the population.

Continuing in this manner for the full sample of 5 the results will appear as follows:

Rank Order Number	Failure Age	Median Rank
1	4,800	0.1296
2	13,000	0.3147
3	23,000	0.5000
4	39,000	0.6853
5	67,000	0.8706

It will be noted that the first order number failure represents 12.96% of the population and the last represents 87.06%. The first being as distant from 0% failure as the last is from 100%, while the third is equidistant between the two.

Having assigned Median Ranks it is now possible to construct the Weibull Plot.

Using Weibull Probability Paper proceed as follows: Failure age is represented on the x-axis, and the percentage of the population failed, or Median Ranks on the y-axis.

For the first rank order number plot 4,800 on the x-axis against 12.96 on the y-axis.

For the second rank order number plot 13,000 on the x-axis against 31.47 on the y-axis.

Continue until all five points are plotted.

If by inspection it appears that the points fall along a straight line then a line may be immediately fitted. In some instances a straight line may not be apparent, in this case a special technique, mentioned later will have to be employed.

A plot of the above results is shown in Fig. 4-1 on page 51 .

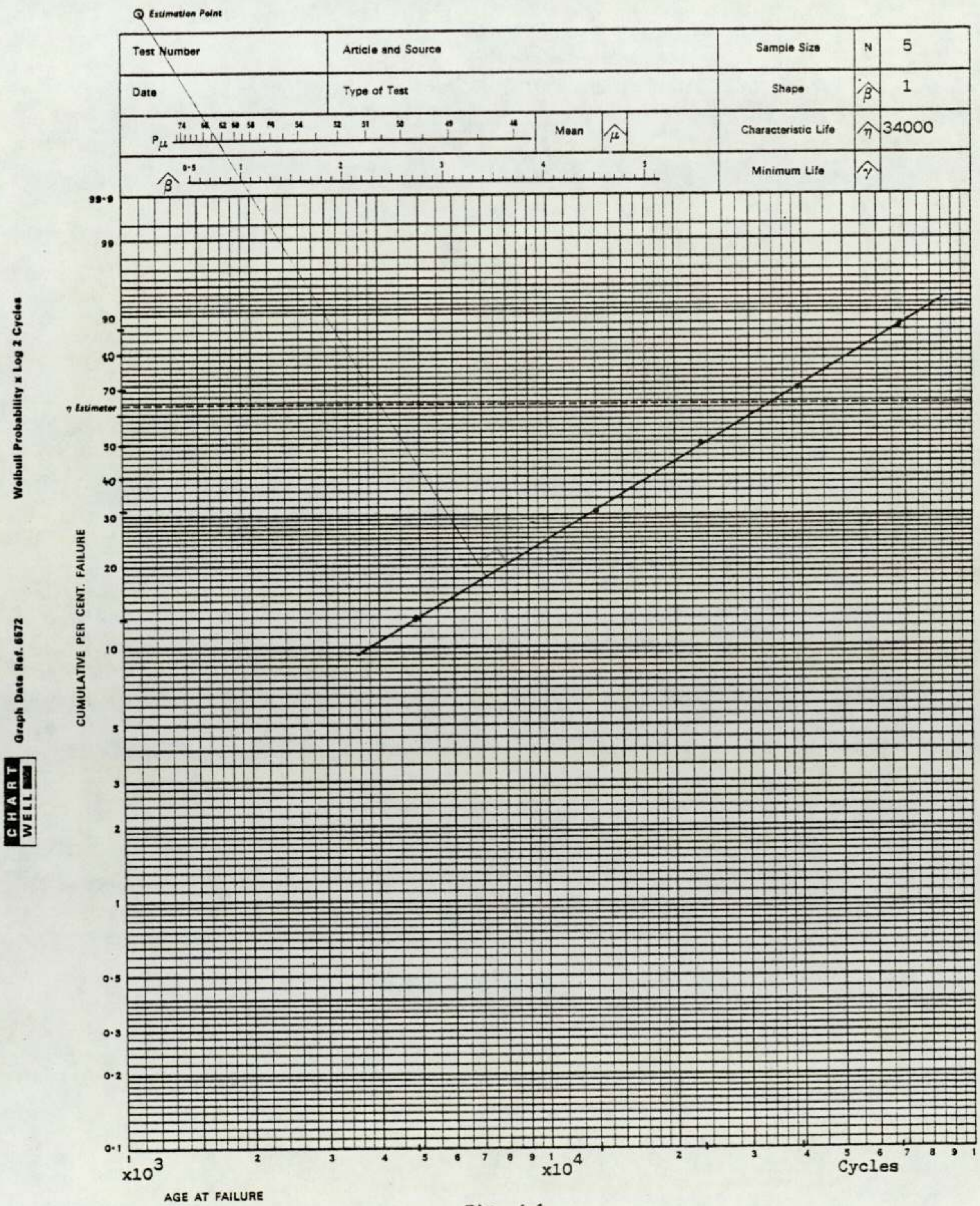

Estimation Point
Test Number
Article and Source
Sample Size
N
5
Date
Type of Test
Shape
β
1
Mean
μ
Characteristic Life
η
34000
Minimum Life
γ
η Estimator
CUMULATIVE PER CENT. FAILURE
Weibull Probability x Log 2 Cycles
Graph Data Ref. 6572
CHART WELL
x10^3
x10^4
Cycles
AGE AT FAILURE

Fig. 4-1.

5 - EXAMPLES IN PLOTTING

The following examples serve to demonstrate the method of obtaining the Weibull line for a particular set of sample failures and the type of information which may be obtained from this line. It also indicates the speed at which a plot may be produced when Median Rank tables are available. Using the computer programme discussed in Chapter 10 on page 214, the results would be nearly immediate.

5.1 - Completed tests without suspensions.

A completed test of magnitude N occurs when it is decided to test a sample of N items and all N items are tested to failure. The failure index N_o = N is the number of failures of the mode under study. The times, or cycles, to failure are plotted against the Median Ranks for a sample size N.

Example - The following data were obtained from BL Technology.

A new switch was proposed for production introduction. The Reliability Department, in order to arrive at an assessment, took 10 of these switches at random and carried out tests-to-failure. The results were as follows:

Cycles to Failure
12,800
34,500
52,000
82,500
102,000
145,000
180,000
222,000
300,000
490,000

PROCEDURE

1) Failure times are ranked in ascending order, as above.

2) Median Ranks are assigned using tables if available, or the general formula if not,

Order No.	Cycles to Failure x 10^{-4}	Median Ranks
1	1.28	6.6
2	3.45	16.2
3	5.20	25.8
4	8.25	35.5
5	10.20	45.1
6	14.50	54.8
7	18.00	64.4
8	22.20	74.1
9	30.00	83.7
10	49.00	93.3

3) Using Weibull Probability Paper, cycles to failure are plotted on the x-axis against Median Ranks on the y-axis.

4) Should the plotted points approximate to a straight line then a line may be immediately drawn through these points. This is the Weibull Line. In some instances it may be necessary to fit the "best line" by means of the "Method of Least-Squares", see section 6.13 on page 103.

Where it is obvious that a straight line does not fit the plotted points, or where a dichotomous plot is in evidence, it becomes necessary to apply special techniques which will be explained later. (See section 6.9 on page 90).

5) The slope of the Weibull Line is determined by projecting a line perpendicular to the Weibull Line passing through the "Estimator" point at the top left-hand corner of the paper. The slope is read off the "β" scale where it is cut by the projected line. For the definition and the analytical calculation of the slope, or shape parameter β, see section 6.18 on page 167.

Similarly a value may be read off where the perpendicular line to the Estimator point cuts the p_{μ} scale. This value gives the point on the "y" axis from which a line projected horizontally, to the Weibull Line and then vertically to the "x" axis gives the percentile mean in hours, cycles etc. For the definition and analytical calculation of the mean life μ, see section 6.19 on page 169.

The Characteristic Life, the point at which 63.2% of the population will have failed, is read from the x-axis immediately below the point where the Weibull Line cuts the horizontal Estimator Line. (Marked at 63.2% of the y-axis). For the definition and analytical calculation of the characteristic life η, see section 6.17 on page 166.

It should be noted here that the values for Mean Life and Characteristic Life are the same for this example. This occurs when the line slope is 1.

The Weibull Line which has been produced, Fig. 5-1 on page 55, may now be considered as giving an estimate of the failure level at any given failure age.

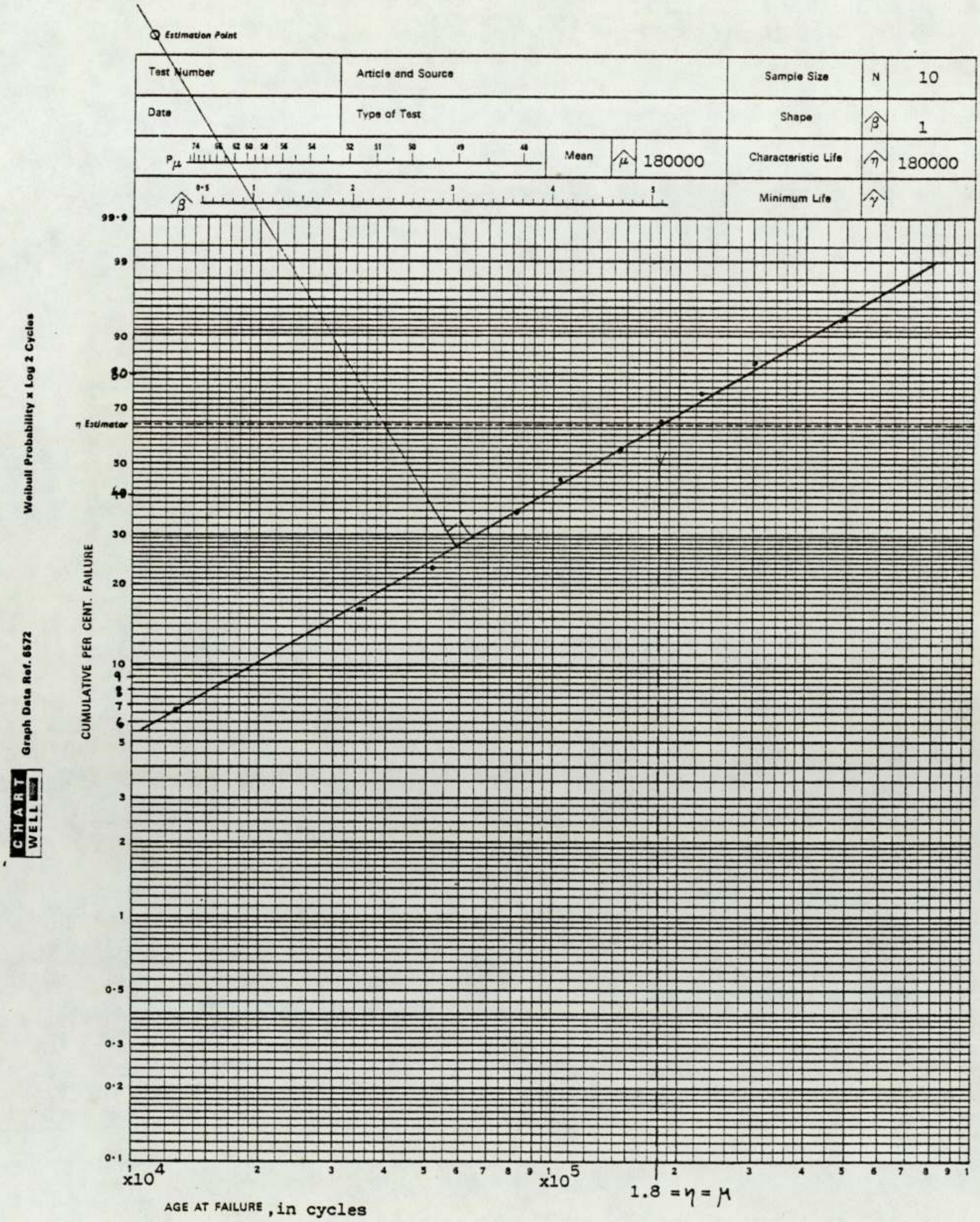

Estimation Point
Test Number
Article and Source
Sample Size
N
10
Date
Type of Test
Shape
β̂
1
Mean
μ̂
180000
Characteristic Life
η̂
180000
Minimum Life
γ̂
η Estimator
CUMULATIVE PER CENT. FAILURE
Weibull Probability x Log 2 Cycles
Graph Data Ref. 6572
CHART WELL
x10^4
x10^5
1.8 = η = μ
AGE AT FAILURE , in cycles

Fig. 5-1.

5.2 - Truncated tests without suspensions

This type of test obtains when it is decided to test a sample of N items, but the test is terminated when N_o items ($N_o \leq N$ is called the failure index) are tested to failure. ($N - N_o$) items survive. The times, or cycles, to failure of the N_o items are plotted against the Median Ranks calculated for a sample size N.

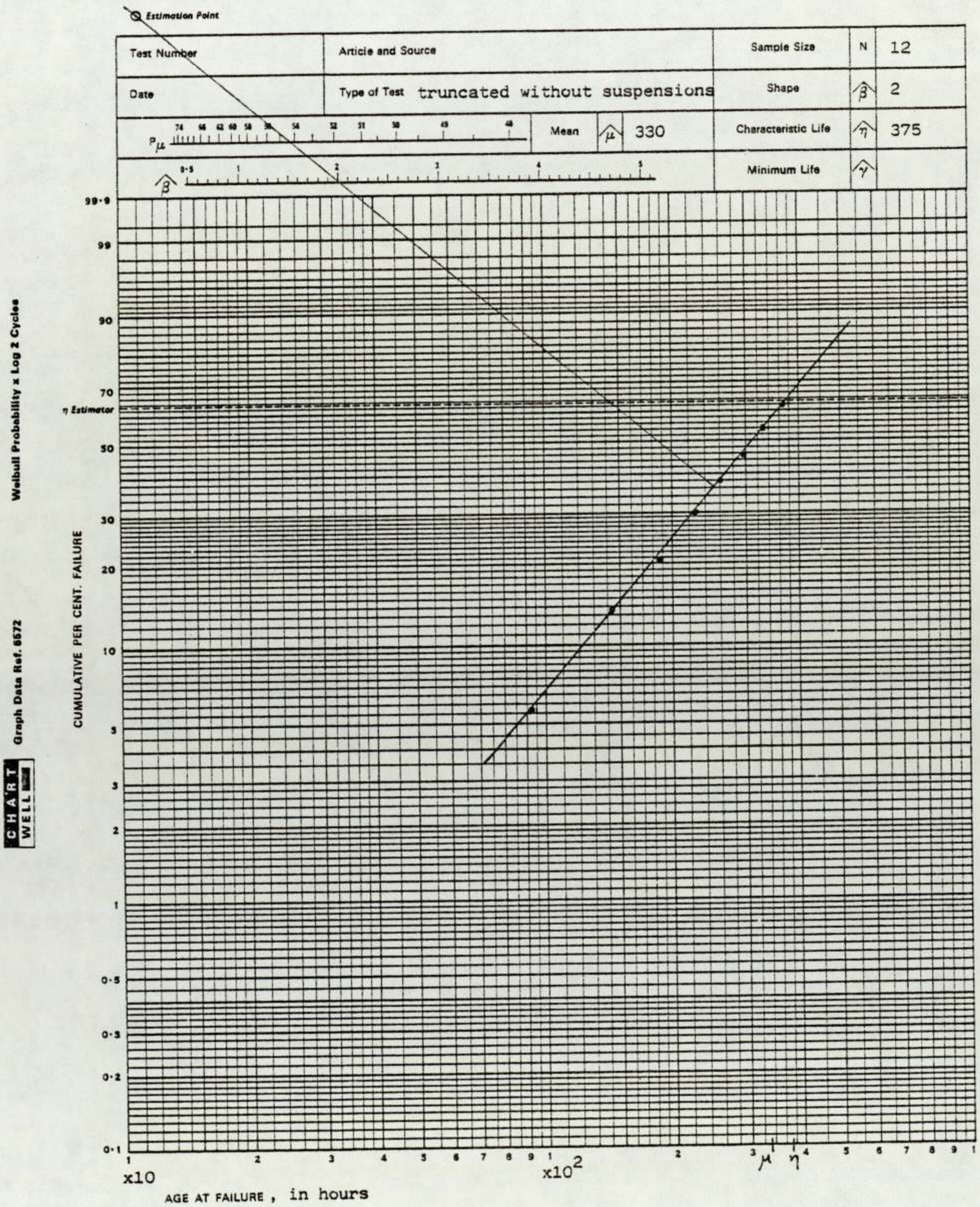
Example

The Reliability Department placed twelve components simultaneously on life test under conditions simulating those of actual use. The test was terminated after the failure of the first eight components. The times to failure of the eight components were: 92, 143, 186, 225, 295, 330 and 365 hours.

Procedure

1) The N_o failure items are ranked in ascending order.
2) Using tables or the general formula, the first N_o Median Ranks are assigned to the N_o items, <u>calculated for a sample size N</u>.

Order No.	Time to failure (hours)	Median Ranks (in %)
1	92	5.6
2	143	13.5
3	186	21.6
4	225	29.7
5	260	37.8
6	295	45.9
7	330	54.0
8	365	62.1
9		
10		
11		
12		



Estimation Point
Test Number
Article and Source
Sample Size
N 12
Date
Type of Test truncated without suspensions
Shape
β 2
Mean
μ 330
Characteristic Life
η 375
Minimum Life
η Estimator
CUMULATIVE PER CENT. FAILURE
Weibull Probability x Log 2 Cycles
Graph Data Ref. 6572
CHART WELL
x10
x10^2
AGE AT FAILURE , in hours

Fig. 5-2.

3) Using Weibull Probability Paper, the times to failure are plotted on the X-axis against Median Ranks on the Y-axis. See Fig. 5-2 on page 57 .

5.3 - Incomplete tests (tests with suspensions)

Suspensions are missing data in a life study programme. These missing data come from items which were prematurely removed from test, that is, before completion of that test, for any of a variety of reasons. Rig failures, failure modes different from the mode under investigation, accidents to items, curtailment or withdrawal of some of the items on test (others failing later), result in suspended data.

The test in which one or more items are withdrawn from test before they fail is called an incomplete test. The failure index N_o is less than the magnitude N in incomplete tests. Note that this is not the same as stopping a test after the first N_o items have failed in the same mode, there being no suspensions, and $N_o < N$. This is the truncated test discussed in Chapter 5.2 on page 56.

An incomplete test of magnitude N has suspensions and must be treated in a special manner. Suspensions cannot be considered failure data and at the same time they cannot be ignored. Each suspension has a chance to fail after the time of removal and before the end of the test. Moreover, the suspended item has successfully run up to the time of removal from test.

A way of handling incomplete test data (that is, data containing suspensions) is to assign to each observed failure of a given mode its correct rank order number. Remember, in the previous examples, in Chapter 5.1 on page 52 and in Chapter 5.2 on page 56, the data were ranked in ascending order and were assigned rank order numbers 1, 2, 3,.....,N_0. Then the Median Ranks were looked up in Appendix A on page 222, corresponding to those order numbers for a sample size of N. Median Ranks (or confidence limits) for an incomplete test cannot be assigned until first an order number is assigned to each failure in that failure mode.

In general, the rank order numbers of failures following the first suspension will no longer be integers, but because of the suspended item or items, they will be fractional values. The following example illustrates the reason for fractional values.

Consider the previous test, in Chapter 4 on page 46, as being incomplete, the following results being obtained:

Failed at 4,800 cycles

Suspended at 13,000 cycles

Failed at 23,000 cycles

Suspended at 39,000 cycles

Failed at 67,000 cycles

5.3-a - Interpolation of new rank order numbers

The order number 1 is assigned to the first failure since no suspension preceded it. With the second failure however it is not possible to assign an order number of 2 since the suspension preceding it may well, had it not been removed from test, have failed before 23,000 cycles. Similarly an order number of 3 cannot be assigned since the suspended item might well have survived beyond 23,000 cycles. The second failure therefore will require an assigned order number somewhere between 2 and 3.

Derived from considering every possible rank position of the suspended items, the general formula used to calculate a new increment of such rank order numbers is given by:

$$\frac{(N + 1) - (\text{previous rank order number})}{1 + (\text{number of items beyond present suspended items})} .$$

Referring to our example of five test items, the result was an incomplete test of magnitude $N = 5$ and a failure index $N_o = 3$.

Failure No. 1 has a rank order number of 1 since no suspensions occurred before it. For failure No. 2 we must calculate a new rank order amount to be added:

$$\frac{(5 + 1) - 1}{1 + 3} = 1.25$$

Therefore failure No. 2 has a rank order number $1 + 1.25 = 2.25$.

The new increment for failure No. 3 is

$$\frac{(5 + 1) - 2.25}{1 + 1} = 1.875 .$$

Thus, failure No. 3 has a rank order number equal to

$2.25 + 1.875 = 4.125$

From these results, the tabulation of the five items will be:

Cycles to failure or suspension	Order Number
4,800 failed	1
13,000 removed from test	-
23,000 failed	$1 + \frac{(5 + 1) - 1}{1 + 3} = 2.25$
39,000 removed from test	-
67,000 failed	$2.25 + \frac{(5+1)-2.25}{1+1} = 2.25 + 1.875 = 4.125$

A check may be carried out as follows:

The last order number doubled minus the previous order number must equal the number undergoing test plus 1.

Thus (4.125 x 2) - 2.25 = 5 + 1

8.250 - 2.25 = 6

6 = 6 Check .

Alternatively, the last increment calculation added to the last rank order number should equal N + 1. In this case,

1.875 + 4.125 = 6 = 5 + 1 Check.

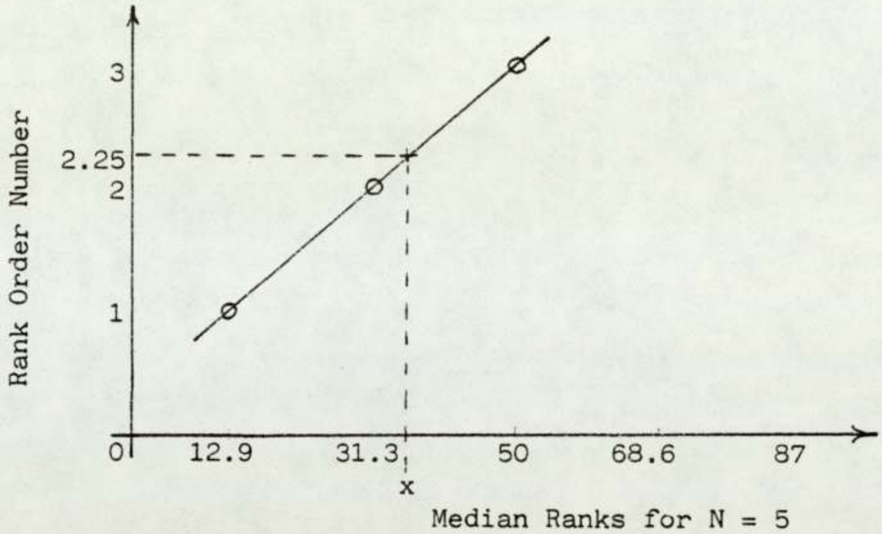
Median ranks and confidence limits can now be determined by referring to the column corresponding to a sample size of five in each table and interpolating between the integer rank order numbers.

5.3-b - Interpolation of new Median Ranks

Median Ranks may now be assigned to each rank order number in accordance with the previously mentioned procedure.

<u>Method I</u>

If a table of Median Ranks is available such as Appendix A on page 222 , new median ranks can be calculated by referring to the column corresponding to a sample size of N and interpolating between the integer rank order numbers as shown for item No. 2.25 in Fig. 5-3 on page 62 .



$$\frac{3-2}{50-31.3} = \frac{2.25-2}{x-31.3}$$

$$x-31.3 = \frac{(2.25-2)(50-31.3)}{3-2} = 4.675$$

$$x = 31.3 + 4.675 = 35.975$$

Fig. 5-3 Example of interpolation of Median Ranks for fractional rank order numbers

Method II

Obviously, without interpolation, which is undesirable, it is not possible to use the standard Median Rank Tables for order numbers which are not integers. However, the Median Ranks can also be calculated using the simple formula by A. Bernard mentioned in Chapter 4 on page 48:

$$\frac{i-0.3}{N+0.4} \qquad 1 \leqslant i \leqslant N$$

where i = rank order number

N = number in sample .

Thus, rank order number 4.125 gives:

$$\frac{4.125-0.3}{5+0.4} = \frac{3.825}{5.4} = 0.708\bar{3} \approx 70.8\bar{3}\ \%$$

Thus, the full set of results is as follows:

Cycles	Rank Order No.	Median Ranks (%)	
		Method I	Method II
4,800 failed	1	12.900	12.96
13,000 suspended	-	-	
23,000 failed	2.25	35.975	$36.\overline{11}$
39,000 suspended	-	-	
67,000 failed	4.125	70.900	$70.8\bar{3}$

Method III

This method consists of determining the Median Ranks directly from the binomial expansion:

$$(R+F)^N = \left[(1-F)+F\right]^N = 0.50 = \binom{N}{o}(1-F)^N+\binom{N}{1}(1-F)^{N-1}+\binom{N}{2}(1-F)^{N-2}+\cdots$$

as mentioned in Chapter 4 on page 48. This method, however, is rather difficult and is used only when a computer is available. See Chapter 10 on page 216.

Notice that there are small differences between the three methods. Method III is the most accurate. The plot of these results gives Fig. 5-4 on page 65.

The plotted points as shown in this figure may then be treated as a normal Weibull plot and a line fitted by the usual methods. In the case of this example only three failures occurred giving only three plotted points thus making it less obvious as to where the straight line should be fitted. This reduces the confidence we may have in the resulting line and for this reason it is necessary to aim for at least seven plotted points, which give a reasonable level of confidence, by making the line more representative of the plotted data.

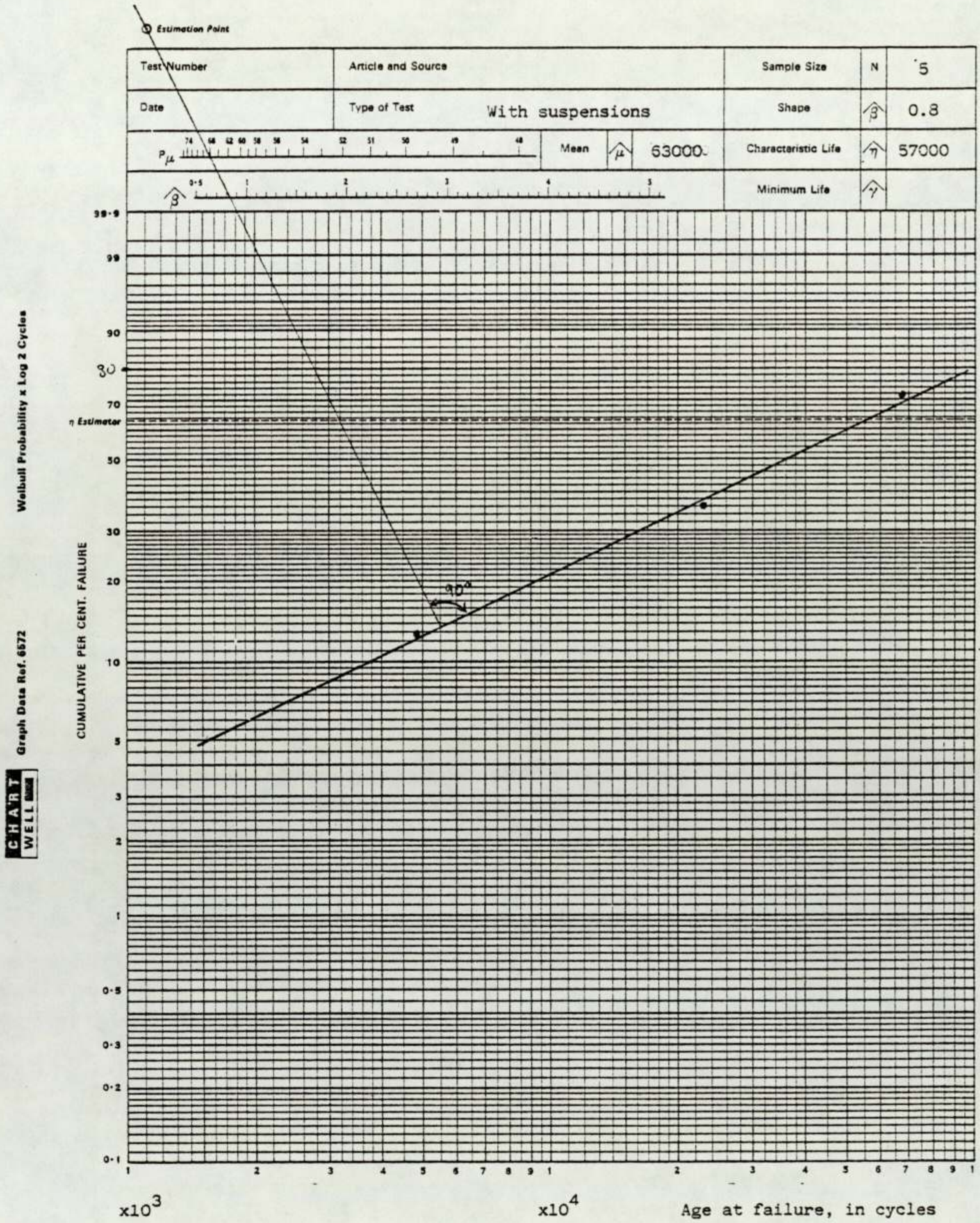

Estimation Point
Test Number
Article and Source
Sample Size
N
5
Date
Type of Test
With suspensions
Shape
0.8
Mean
63000
Characteristic Life
57000
Minimum Life
CUMULATIVE PER CENT. FAILURE
η Estimator
90°
Weibull Probability x Log 2 Cycles
Graph Data Ref. 6572
CHARTWELL
x10^3
x10^4
Age at failure, in cycles

Fig. 5-4.

PART II - EXPERIMENTAL PROOF

6 - PRELIMINARY INVESTIGATION, CARRIED OUT AT ASTON, TO TEST THE DISCRIMINATING POWERS OF THE WEIBULL ANALYSIS.

6.1 - Fatigue Testing of butt-welded joints

6.1.1 - Hystorical note

There has not been yet any work done on the application of the Weibull analysis to welded joints, but there is one particular example, set up by Weibull in his paper[1], which shows how the Weibull analysis can be applied to fatigue life results. This particular example is on the fatigue life of an ST-37 steel. The observed values are taken from Müller-Stock [15]. The frequency curve in figure 6-1 gives no impression of a complex distribution (two failure mode distributions) which, on the other hand, may easily be seen when using plottings in figure 6-2. The parameters are: Component No. 1: $t_o = 4.032$, $\beta = 5.956$; Component No. 2: $t_o = 4.484$. $\beta = 1.215$. Table 1 shows the close agreement between the observed and the calculated values.

Table 1 - Fatigue Life of ST-37

(Rotating-beam test at ± 32 kg/mm^2)

	N 10^3	Expected values n_1	n_2	n_{1+2}	Observed values n_{1+2}
1	17.5	4.6	-	4.6	4.6
2	22.5	47.4	-	47.4	47.4
3	27.5	125.1	-	125.1	125.1
4	32.5	161.2	8.1	169.3	169.2
5	37.5	164.9	28.0	192.9	192.7
6	42.5	165.0	41.9	206.9	207.3
7	47.5	165.0	51.0	216.0	215.9
8	52.5	165.0	57.0	222.2	222.2
9	57.5	165.0	61.0	226.0	225.9
10	62.5	165.0	63.7	228.7	228.7
11	67.5	165.0	65.6	230.6	230.5
12	72.5	165.0	66.9	231.9	231.9
13	77.5	165.0	67.9	232.9	232.9
14	82.5	165.0	68.6	233.6	233.5
15	87.5	165.0	69.1	234.1	233.9
16	92.5	165.0	70.0	235.0	235.0

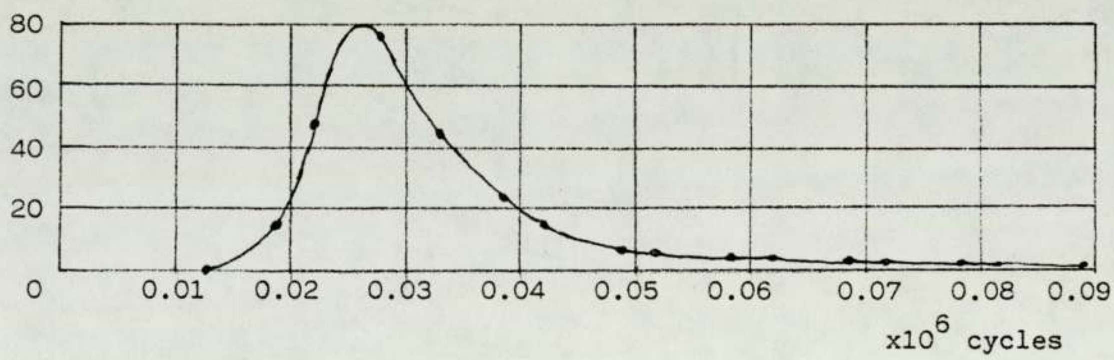
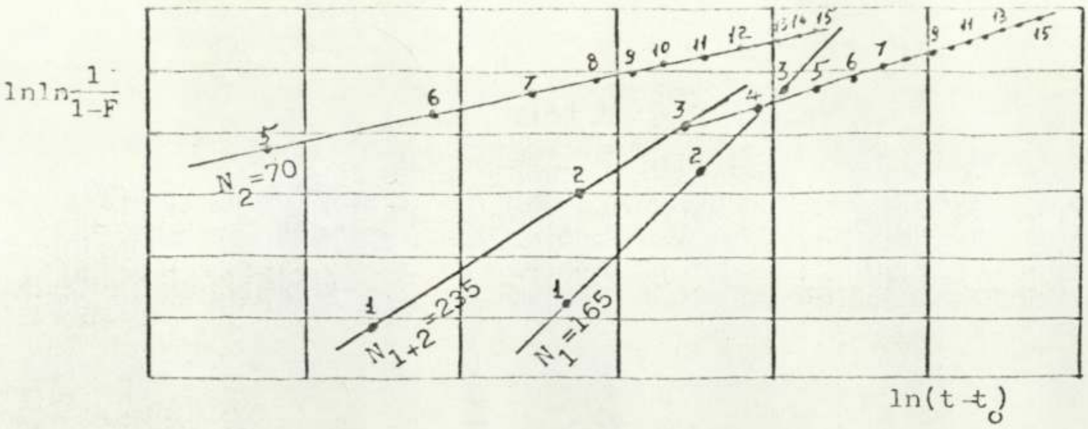


Fig. 6-1 Frequency curve of fatigue life of ST-37 steel
(Number of specimens versus number of stress cycles)



Component 1

$t_0 = 4.032, \ \eta = 10800, \ \beta = 5.956$

Component 2

$t_0 = 4.484, \ \eta = 30500, \ \beta = 1.215$

Fig. 6-2 Fatigue life of ST-37 steel

The real causes of this splitting up in two components may be found by examining the frequency curve of the yield strength of the same material, figure 6-3 on page 69. It is easy to see that the material, probably not being killed, is composed of two different kinds. If it is supposed that all the specimens with a yield strength of less than 25 kg/mm^2 belong to Component No. 1, we obtain 14 specimens out of 20, making 70%. Exactly the same proportion has been found by the statistical analysis, as $\frac{165}{235}$=70%.

The reason why this partition is so easily seen in figure 6-3 on page 69 and not at all in figure 6-1 on page 67 depends, of course, upon the much larger scatter in fatigue life than in yield strength.

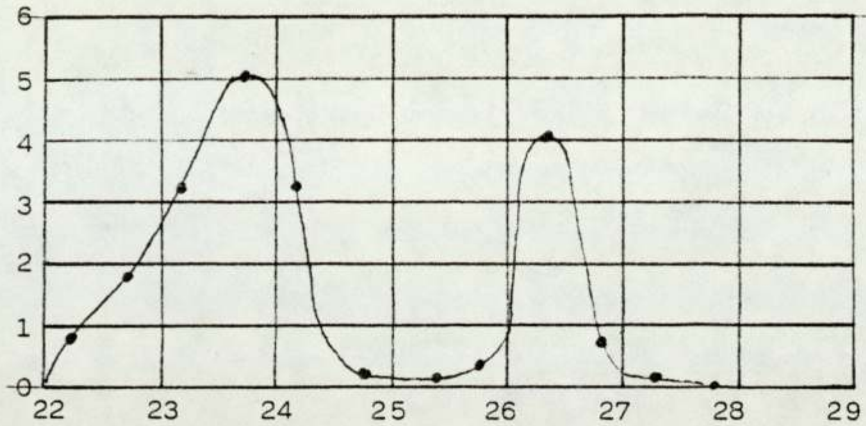


Fig. 6-3 Frequency curve of yield strength of ST-37 steel

(Number of specimens versus yield strength in kg/mm^2)

Although the Weibull distribution function has many practical applications in many fields, it is not always valid, and it is the purpose of this project to examine the applicability of the Weibull distribution function to the fatigue results of welded joints.

6.1.2 - Fatigue of welded joints

In order to determine the mean life and the reliability of a certain welded joint subjected to repeated loading, fatigue tests are carried out on a number of welded specimens to determine the time to failure. It is, therefore, of importance to have an understanding of the factors which influence the fatigue life of a welded joint.

Fatigue failure consists of the formation of a crack or cracks under the action of varying loads. If fatigue cracks do occur they are invariably initiated at stress concentrations. Since all joints are inevitably points of stress concentration, it is axiomatic that fatigue failures are likely to occur at joints and that the behaviour of a welded structure subjected to repeated loading will depend, to a very large extent, on detailed joint design.

The type of weld that is used in this investigation is a transverse butt weld in steel.

6.1.3-Factors influencing the fatigue strength of transverse butt welds

The transverse butt weld used as a means of joining together two plates, produces the least disturbance to stress flow and would therefore be expected to exhibit relatively good fatigue strength. In the absence of weld defects and with the weld reinforcement left in the as welded condition, the major stress concentration occurs at the weld toe and it is from here that fatigue failure invariably occurs.

The expectation of good fatigue strength has often been fulfilled, but the fatigue strength of transverse butt welds can still vary between wide limits. In recent years it has become apparent that the weld shape is the overriding factor in determining the fatigue strength of sound transverse butt joints, and the influence of many of the other factors is determined by their effects on the shape at the weld toe. Newman and Gurney[5] tested several types of butt welds, made by both manual and automatic welding, and obtained a wide range of fatigue strengths, which varied from 100.4 Nmm^{-2} (6.5 $tons/in^2$) to 177 Nmm^{-2} (11.5 tons/ sq. in) at 2×10^6 cycles under pulsating tension loading. (See Chapter 6.1.4 on page 78 for the definition of fatigue terms). As a quantitative measure of reinforcement shape the (obtuse) angle ϑ between the plate surface and the tangent to the reinforcement at its point of contact with the plate surface was used, see figure 6-4.

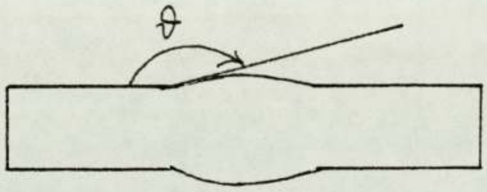


Figure 6-4 Reinforcement angle

Examination of the specimens revealed that this "reinforcement angle" varied along the length of a weld - particularly in manually welded joints - but that failure usually originated at the point of minimum angle. A few specimens of each test series were selected from those which gave fatigue test results, lying close to the relevant S-N curve. These were sectioned at the point of crack initiation and the angle measured with the aid of a projection microscope. The measured angles were plotted against the fatigue strength at 2×10^6 cycles of the particular test series from which the specimen originated, as shown in figure 6-5 on page 72.

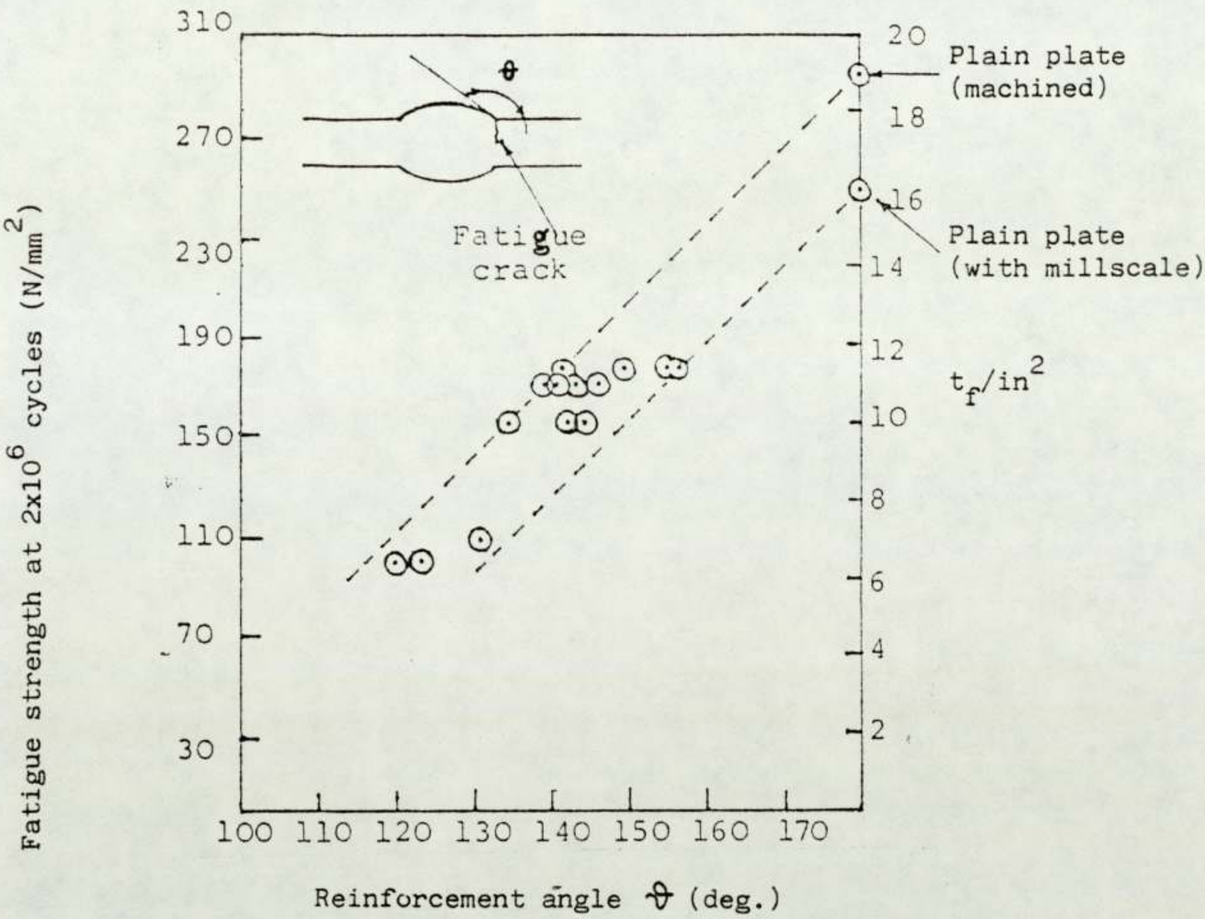


Fig. 6-5 The relationship between Reinforcement angle and fatigue strength of transverse butt welds, Newman and Gurney (5) .

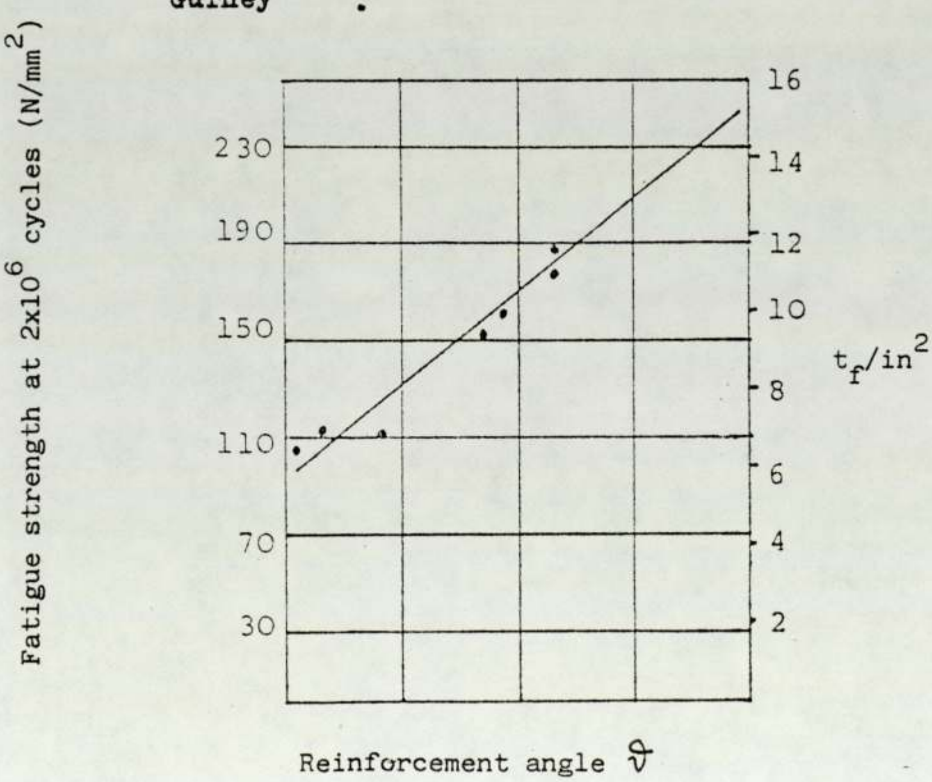


Fig. 6-6

For the manually welded series the scatter was about 15^0, but for the automatic welds it was somewhat less, but it can be seen that all the experimental points lie within a scatter band which can conveniently be located at its upper end by the strengths of plain plate with and without millscale. These results were confirmed by tests performed in the welding institute[6] which show that the fatigue strength of transverse butt welds at 2×10^6 cycles under pulsating tension can be anywhere between 100.4 Nmm^{-2} (6.5 t.f.s.i.) and 177 Nmm^{-2} (11.5 t.f.s.i.). The marked changes in the fatigue strength were produced by variations in the shape of the weld profile. All the welds were free from any internal defect and would normally be considered good quality welds. Under fatigue loading conditions the term "quality" must also refer to the shape of the excess weld metal. It can be seen from the results in Table 2 on page that the lower fatigue strengths are associated with welds having a poor shape, in that there is a very sudden change of section at the junction between parent plate and weld metal (ϑ small). Those welds having the minimum of excess metal and a smooth transition at the weld toe, (ϑ high), give the higher fatigue strengths.

Another way in which the effect of reinforcement shape can be expressed is related to its influence on the life of specimens tested at a given stress level. This was the approach of Fall et al[7], who graded the various specimens visually and found an approximately linear relationship between the grading and log endurance. Unfortunately such qualitative grading methods invite criticism and are of no assistance in helping to define the critical features of reinforcement shape, although it shows that a definition of "good" and "bad" shape may be possible by visual inspection.

Table 2

Type of transverse butt joint	Fatigue strength (f_{max}) at 2 x 10^6 cycles
0.5 in thick single-V butt weld, manual electrode	11.5 tonf/in^2 (178 N/mm^2)
0.5 in thick, close square butt weld, manual deep-penetration electrodes	7 tonf/in^2 (108.5 N/mm^2)
1.25 in thick, single-V butt weld, manual electrodes	9.5 tonf/in^2 (147 N/mm^2)
0.5 in thick, close square butt weld, submerged-arc	6.5 tonf/in^2 (100.8 N/mm^2)
0.5 in thick, close square butt weld, submerged-arc	7 tonf/in^2 (108.5 N/mm^2)
0.5 in thick, double-V butt weld, submerged-arc	11 tonf/in^2 (170.5 N/mm^2)
1.25 in thick, double-V butt weld, submerged-arc	10 tonf/in^2 (115 N/mm^2)

It would seem reasonable to attribute the influence on fatigue strength of many other factors, such as plate preparation, welding conditions, welding process and type of electrode to their effect on the shape of the weld toe. Evidence confirming this viewpoint as the cause of the reported poor fatigue performance of automatic welds compared with manual welds, is supplied by Newman and Gurney[5] in their investigation mentioned previously. In two series of tests on submerged arc welds in 12.7 mm (0.5 in) thick mild steel, fatigue strengths of 100.4 Nm^{-2} (6.5 tfsi) and 108.1 Nmm^{-2} (7.0 tfsi) at 2×10^6 cycles were obtained under pulsating tension loading. However, in tests on automatically welded joints with the reinforcement removed the fatigue strength was found, as with manual joints, to be the same as that of the parent material. The low fatigue strength of the automatic welds in the as welded condition cannot, therefore, be attributed to any adverse metallurgical factor as this would also have made itself apparent in the tests on machined welds. The possibility that residual stresses were the dominating influence was eliminated by testing the welds in the stress relieved condition. Two further series were also welded by the submerged arc process, but with the welding conditions adjusted to give an improved reinforcement shape. These were tested and resulted in fatigue strengths of 169.9 Nmm^{-2} (11.0 tfsi) and 154.4 Nmm^{-2} (10.0 tfsi) at 2×10^6 cycles.

Residual stresses have little effect on fatigue strength when the applied stress cycle is wholly tensile. Several investigations have been carried out in which the fatigue strength of "as-welded" and "stress-relieved" specimens have been compared under pulsating tension loads. Under such conditions, with axially loaded transverse buttwelds, the maximum increases in strength at 2×10^6 cycles that have been obtained are about 17% for welds with the reinforcement machined flush[8], and 12½% for welds with the reinforcement unmachined[9]. However, for both types, it has frequently been found that stress relieving had no effect at all on fatigue behaviour[10, 11, 5], while intermediate, and therefore obviously small, strength increases have also been recorded[12, 9].

Finally, the possible effect of variations in the static strength of the parent materials must be accounted for. Due to the critical dependence of the fatigue strength of transverse butt welds on the shape of the weld profile, fatigue test results on these welds do not provide a particularly consistent set of data for defining the effect of static strength of the parent material. Gurney[13] summarised the results relating to manual steel butt welds subjected to 2×10^6 cycles of pulsating tension loading, figure 6-7 on page 77. It can be deduced from this graph that the fatigue strength under these conditions is independent of static strength.

Thus any variations in the static strength of the proposed steel would be insignificant.

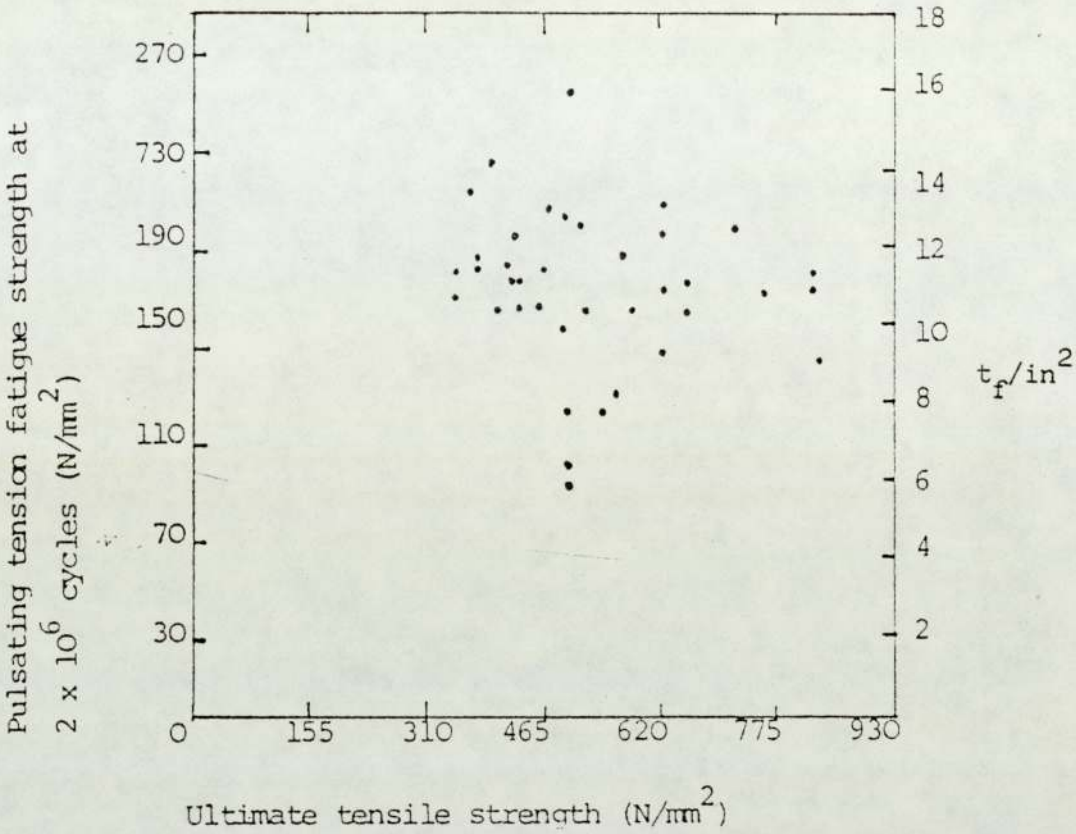


Fig. 6-7 Relationship between the pulsating tension fatigue strength at 2×10^6 cycles of transverse butt welds and the ultimate tensile strength of the material. Gurney(13).

6.1.4 - Fatigue-Testing Notation

The terms used in fatigue testing to describe the applied stress cycle can be defined as:

Minimum stress in the cycle	f_{min}
Maximum stress in the cycle	f_{max}
Stress ratio	$R = \dfrac{f_{min}}{f_{max}}$

Tensile stresses are considered positive, and compressive stresses are negative.

Fig. 6-8 shows the type of stress cycle used in fatigue testing in this project. See Chapter 6.6 on page 83.

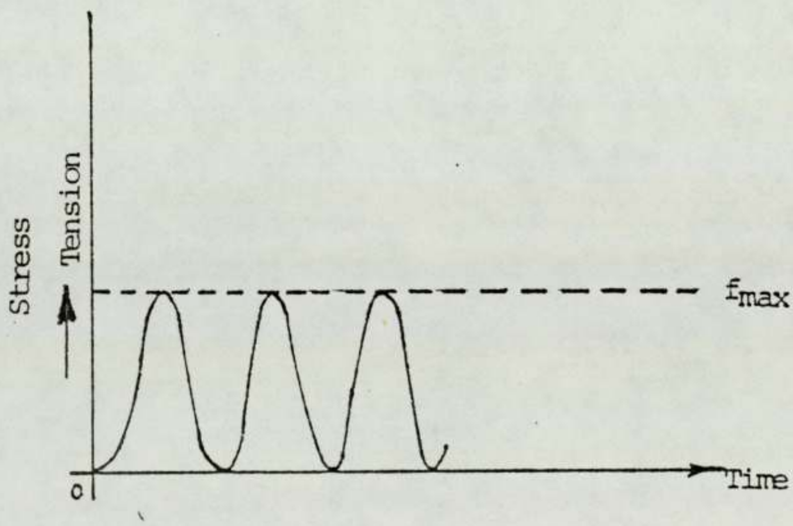


Fig. 6-8 Pulsating tension

6.2 - Objectives and Experimental Approach

The objective of one particular project was to apply the Weibull analysis to welded joints in order to determine their mean life, characteristic life, failure rate at any time t, and reliability.

The approach adopted was to fatigue test a number of welded specimens, at a given stress level, until failure. The stress cycle chosen for this purpose was pulsating tension , in which the stress varied between zero and maximum.

$$\left(\frac{\text{min. stress in the cycle } f_{min}}{\text{max. stress in the cycle } f_{max}} = 0\right).$$

The Weibull analysis was then applied to the failure test data, which consisted of the time to failure of each specimen.

Two sets of specimens were made using two different welding speeds, while all the remaining welding variables were held constant. Varying the welding speed should change the weld profile. Therefore, it would be expected to have two different fatigue lives, and hence, two different Weibull distributions with different mean lives, characteristic lives, failure rates and reliabilities.

6.3 - Testing equipment and specimen design

The fatigue testing equipment available for this project was an "Amsler Vibrophore" high frequency magnetic resonance fatigue machine fitted with a 99.6 kN (10 tonf) dynamometer. Based on the limitations on loading and the maximum dimensions of a plain sheet of material which could be accommodated in the grips of the fatigue machine, test pieces with the dimensions shown in figure 6-9 were used. Figure 6-10 shows the specimen used, which were manufactured in the Welding Laboratory of the P.T. & P.M. Department.

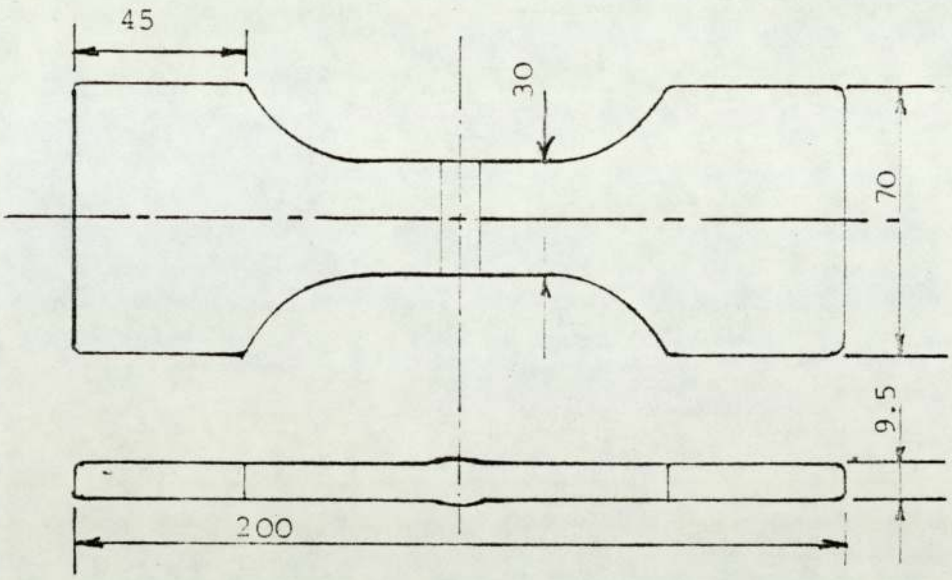


Fig. 6-9 Dimensions of test pieces used

It is important to mention that the reason for adopting the above specimen shape was not to ensure that failure would occur in the weld, since under fatigue loading fracture would certainly initiate from the weld and not from anywhere else, even if the specimen was rectangular. The main reason was that, in order to prevent the specimen slipping in the grips of the machine at high testing loads, the parts of the specimen that were located inside the grips should be wide enough to allow good gripping.

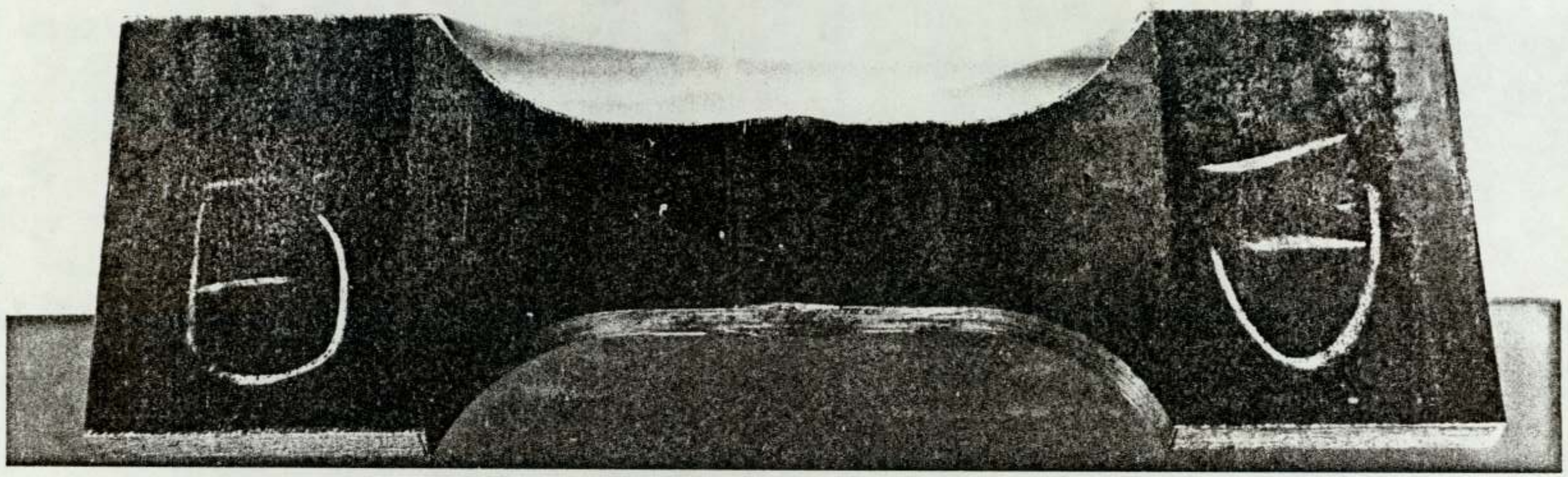


Fig. 6-10 The Welded Specimen Used.

6.4 - Material

The specimens were made of mild steel, because this material is readily available and easy to weld. The material used was in the form of Bright Drawn Mild Steel (BDMS) plates 9.525mm (3/8") thick. The specimens were obtained from two plates as above, 3.6576 m (12 ft.) long and 101.6 mm (4 in.) wide, which had been cut into strips approximately 550 mm (21 5/8 in.) long and 101.6 mm (4 in.) wide.

6.5 - Welding operations

A constant reinforcement angle was required on the specimens, and it was therefore necessary to automate the welding process. The welding was carried out using the Submerged Arc Welding process employing 1/8 in. diameter mild steel consumable electrode (No. 1 Unionmelt), and Unionmelt flux grade 50. Both the welding wire and flux were obtained from the British Oxygen Company Ltd. The Unionmelt No. 1 wire is suitable for either single- or multipass welding. It is used for making butt and fillet welds, where maximum ductility is required. The weld metal has a tensile strength of 434 to 465 N/mm^2 (28 to 30 tons/sq. in.). It is used in conjunction with 50 grade Unionmelt powder. This combination meets the mechanical requirement of BS 639: 1952.

Unionmelt powder grade 50 is very suitable for high speed, high quality welds in thin gauge steel, but also gives excellent results on heavier section requiring up to 1100 amp welding current. It is particularly satisfactory on surfaces which have been ineffectively cleaned and which have excessive amounts of mill scale, dirt and rust.

6.6 - Specimens preparation

Two plates, 550 mm long x 101.6 mm wide, were butt welded together along the 550 mm edge, with no edge preparation (square butt weld). Each plate was cleaned with an industrial solvent. The plates were placed in intimate contact and three tack welds were made at the centre and ends of the joint line. Also, two pieces of steel, a run-in, and a run-off, were tack welded to both ends of the plates. The plates were then welded from one side. Time was allowed for the plate to cool before commencing welding on the reverse side. Four plates were thus welded. For the first two plates, the welding conditions were taken from the American Welding Handbook[14], and were as follows:

Thickness t	1st side (backing pass)				2nd side (finishing pass)			
	Current amp	Voltage volt.	Speed mm per min	Electrode dia.	Current amp	Volt. volt.	Speed mm per min	Electrode dia.
9.5 mm (3/8 in.)	425	33	711 (28 in/min)	3.175 mm. (1/8 in.)	475	35	711 (28 in/min)	3.175 mm. (1/8 in.)

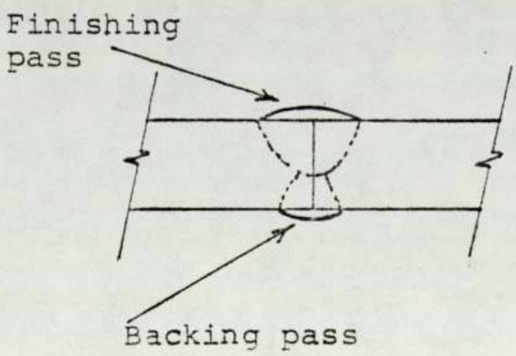


Table 3. Two-pass butt welds.

From those two plates, ten specimens were cut, at right angles to the weld and numbered from 1B to 10B. Specimens were then machined to the final dimensions. Because sufficient material was not available, specimens 8B, 9B and 10B were sectioned from a 300 x 200 mm welded plate, instead of a 550 x 200 mm plate

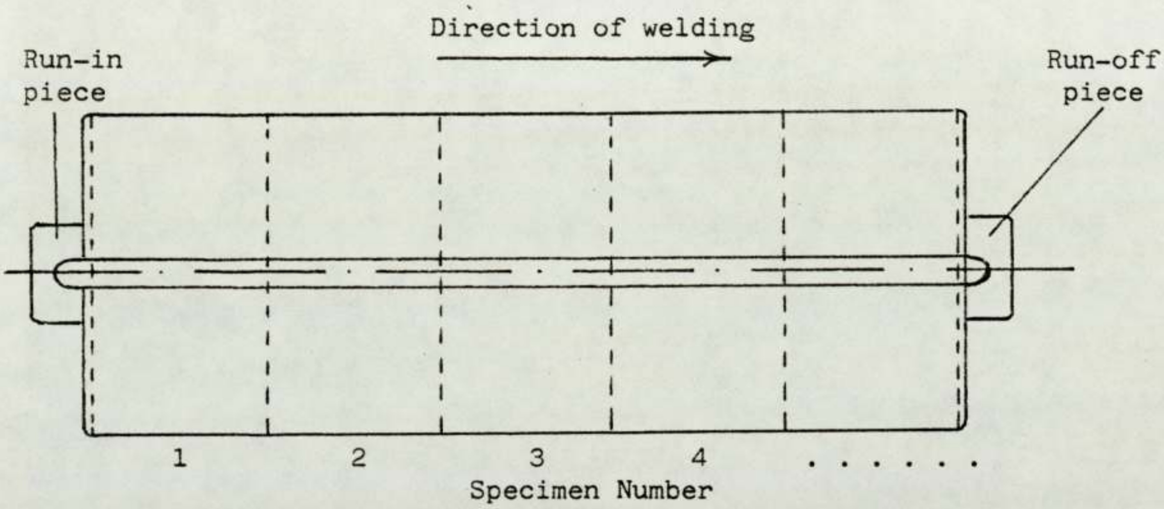


Fig. 6-11 Sectioning of butt welded plates

For the second two plates, the welding speed was changed, while keeping all the remaining welding variables constant. The welding speed was taken from recommendations made by the British Oxygen Company Ltd. The welding conditions were as follows:

Table 4. Two-Pass butt welds

Thickness t	1st side (backing pass)				2nd side (finishing pass)			
	Current amp	Voltage Volts	Speed mm per min.	Electrode dia.	Current amp	Voltage Volts	Speed mm per min.	Electrode dia.
9.5 mm (3/8 in)	425	33	508 (20 in/min)	3.175 mm (1/8 in.)	475	35	508 (20 in/min)	3.175 mm (1/8 in.)

Ten specimens were cut from the above two plates and numbered from 1C to 10C.

Before fatigue testing, all the specimen edges were finished by hand to a radius of approximately 2 mm. Some of the specimens were radiographed to make sure that there were no weld defects. No weld defects were detected.

6.7 - Fatigue testing

It was found that the alignment of the specimen in the grips of the Amsler was important, as any misalignment would now allow the machine to resonate properly. A vernier depth gauge placed against the sides of the grips was therefore used in positioning the specimen. Care was also taken to ensure the bolts clamping the specimen were tightened equally. This avoided the possibility of the specimen slipping in the grips during testing. Initial trials with the welded specimens gave the testing frequency of 233 C/S (HZ). Due to the large mass of the gripping heads, the dynamometer correction at high frequencies becomes large. That is why it is recommended to operate at frequencies between 150 and 200 HZ. The testing frequency was lowered by the use of weights attached to the top section of the Amsler. The testing frequency was thus lowered to 175 HZ (C/S).

Important features of this type of fatigue machine are the cut out relays which operate if the applied loads either exceed or fall below the pre-set values. Apart from guarding against possible fluctuations in the power supply, these relays can also represent a failure criterion. When a fatigue crack is produced the damping characteristics of the specimen are altered so that more energy is required to keep it resonating at the same frequency and stress level. Since the power requirements needed to allow the machine to fatigue test a sound specimen (i.e. uncracked) at a determined stress level have been preset, the presence of a fatigue crack decreases the stress on a specimen due to the greater energy requirement.

This causes the cut out relays to operate. Because the relays were sensitive to change and came into operation quickly, the machine stopped before the crack had propagated through the specimen completely. The operation of the relays was taken as the "failure point" of the specimens. Each specimen was tested until failure. At failure, the total number of applied load cycles was given by means of synchronised dials and counters mounted on the control unit.

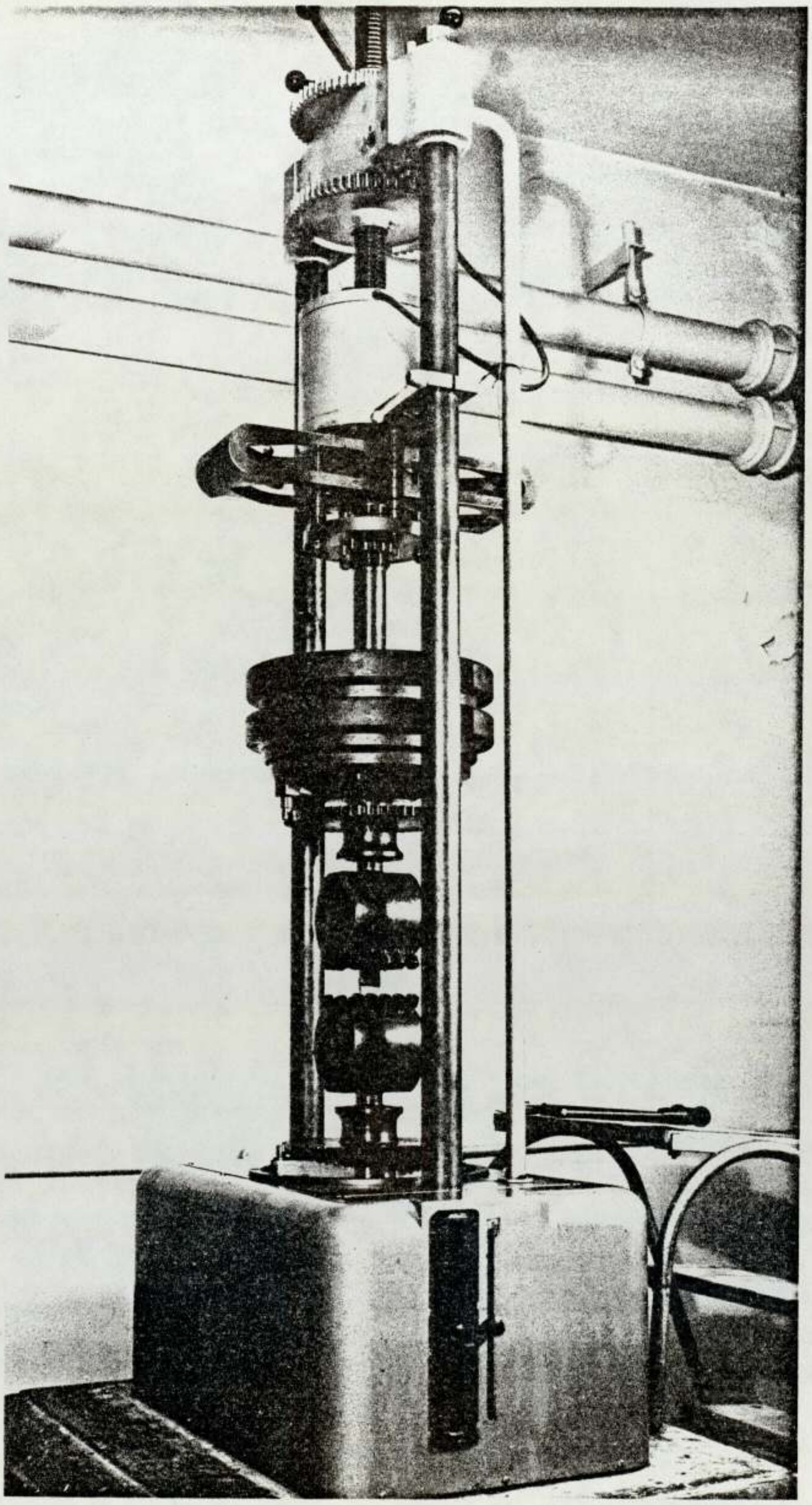


Fig. 6-12 The welded specimen located in the grips of the Amsler Vibrophore. The weights used to lower the testing frequency can be seen above the specimen.

6.8 - Results

The results obtained from the fatigue tests are summarised in Table 5.

Order No.	Cycles-to-failure x 10^5	Welding speed mm/sec	Median Rank*	Failure mode	Remarks
1	4.6	11.85 (28"/min)	3.4	B	Toe crack
2	5.32	8.5 (20"/min)	8.2	C	Toe crack
3	5.39	8.5 (20"/min)	13.1	C	Toe crack
4	5.84	8.5 (20"/min)	18.0	C	Toe crack
5	6.16	8.5 (20"/min)	22.9	C	Toe crack
6	6.68	8.5 (20"/min)	27.8	C	Toe crack
7	6.77	8.5 (20"/min)	32.7	C	Toe crack
8	7.91	8.5 (20"/min)	37.7	C	Toe crack
9	10.15	8.5 (20"/min)	42.6	C	Toe crack
10	10.53	11.85 (28"/min)	47.5	B	Toe crack
11	11.78	11.85 (28"/min)	52.4	B	Toe crack
12	11.94	8.5 (20"/min)	57.3	C	Toe crack
13	12.13	11.85 (28"/min)	62.2	B	Toe crack
14	13.65	11.85 (28"/min)	67.2	B	Toe crack
15	15.10	8.5 (20"/min)	72.1	C	Toe crack
16	15.51	11.85 (28"/min)	77.0	B	Toe crack
17	18.03	11.85 (28"/min)	81.9	B	Toe crack
18	18.08	11.85 (28"/min)	86.8	B	Toe crack
19	18.96	11.85 (28"/min)	91.7	B	Toe crack
20	18.98	11.85 (28"/min)	96.5	B	Toe crack

Table 5. Fatigue life results for transverse butt welds in mild steel (Pulsating pulling stress loading, $\frac{f_{min}}{f_{max}} = 0$, $f_{max} = 200\ N/mm^2$.

* Median Ranks were obtained from Appendix A on page

** See section 6.6 on page 83

A Weibull plot of these twenty data points is shown in Fig. 6-13 on page 89.

Obviously, the data did not describe a straight line on Weibull Probability Paper. A special analysis was necessary to separate the different failure modes.

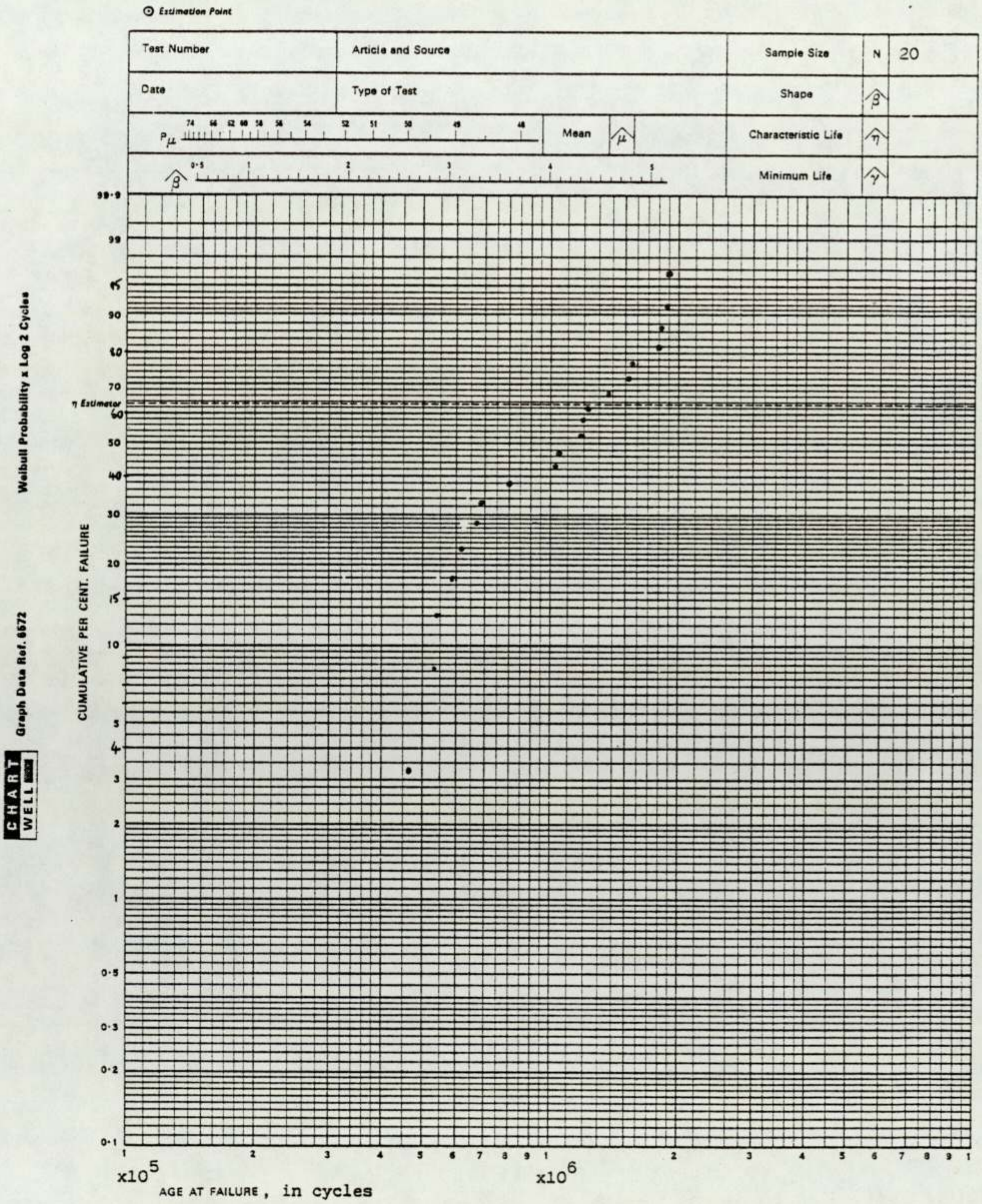


Fig. 6-13. Weibull plot of all data points.

6.9 - Analysis of two or more failure distributions - Separation of C failures.

If the plotted data form two straight lines having either different slopes or different characteristic lives, they are said to form a dichotomous plot and infer that a specification change has affected the units behaviour. In this case a re-inspection of the sample units must be made and the pre-modification and post-modification units identified. These two groups may then be re-plotted, with new Median Ranks, and analysed separately as two individual samples from two parent populations. The same procedure must be used when a dichotomous plot is given by a sample containing the products of two different manufactures.

In the case where two distinct failure modes exist a more lengthy analysis of the data is necessary for reasons which will become apparent.

Plotting failure data which contain two or more different modes of failure (e.g. components welded at different welding speeds) will not normally result in a straight (t_o=0) or smoothly curved ($t_o \neq 0$) Weibull line, yet with a special analysis we should be able to separate the failure distributions.

Consider the data in Table 5 on page 88. A Weibull plot of these twenty data is shown in fig. 6-13 on page 89. Obviously, the data do not describe a straight line on Weibull probability paper, and therefore a special analysis is necessary to separate the failure modes.

It may be noted at this stage that the actual readings from this dichotomous plot are meaningless since one mode of failure affects the other by "stealing" Median Rank numbers from it. The correct way of handling these data is to:

1. Identify the failure mode for each component, by technical inspection.
2. Separate the data by failure modes (B or C).
3. Replot the data for each mode by treating failures of the other mode as suspended items.

When replotting the data for C failures, B failures are considered suspensions (not failures). On the other hand, C failures are considered suspensions when plotting B failures (see Chapter 6.16 on page 149).

6.10 - Interpolation of new rank order numbers

If the first four results in Table 5 on page 88 are considered, then to separate the C failures the four results can be written as:

1. Suspension (B failure)
2. Failure at 5.32×10^5 cycles
3. Failure at 5.39×10^5 cycles
4. Failure at 5.84×10^5 cycles

The order number of the first failure is in doubt. It is not correct to assign a rank order number of 1 because the suspended item might have failed before 5.32×10^5 cycles. Neither it is possible to assign a mean order number of 2 to the first failure since the suspension could have lasted longer than 5.32×10^5 cycles (time of failure of a suspension is considered to be unknown). However, it is known that the first failure should be assigned a mean order number between 1 and 2.

Referring to our case and using the method discussed in Chapter 5.3-a on page 59, the results of separation of the C failures for the 20 items are as shown in Table 6 on page 92 .

Table 6. Calculation of new rank orders

Item	Failure mode	Cycles to failures x 10^5	New Rank Order Number of failure
1		4.6 Suspension	-
2	C	5.32 Failure	$0 + \frac{21-0}{1+19} = 1.05$
3	C	5.39 Failure	$1.05+\frac{21-1.05}{1+18} = 2.10$
4	C	5.84 Failure	$2.10+\frac{21-2.10}{1+17} = 3.15$
5	C	6.16 Failure	$3.15+\frac{21-3.15}{1+16} = 4.20$
6	C	6.68 Failure	$4.20+\frac{21-4.2}{1+15} = 5.25$
7	C	6.77 Failure	$5.25+\frac{21-5.25}{1+14} = 6.30$
8	C	7.91 Failure	$6.30+\frac{21-6.30}{1+13} = 7.35$
9	C	10.15 Failure	$7.35+\frac{21-7.35}{1+12} = 8.40$
10		10.53 Suspension	-
11		11.78 Suspension	-
12	C	11.94 Failure	$8.40+\frac{21-8.40}{1+9} = 9.66$
13		12.13 Suspension	-
14		13.65 Suspension	-
15	C	15.10 Failure	$9.66+\frac{21-9.66}{1+6} = 11.28$
16		15.51 Suspension	-
17		18.03 Suspension	-
18		18.08 Suspension	-
19		18.96 Suspension	-
20		18.98 Suspension	-

6.11 - Determination of new Median Ranks

After calculating the new rank order numbers, new Median Ranks can be determined using one of the methods discussed in Chapter 5.3-b on page 61, for the actual sample size N = 20.

The results can be seen in Table 7 on page 93 .

Order No. Old	Order No. New	Cycles to Failure $t \times 10^5$	New Rank Order No.	New Median Rank %
2	1	5.32	1.05	$\frac{(8.2-3.4)(1.05-1)}{2-1}+3.4=3.64$
3	2	5.39	2.10	$\frac{(13.1-8.2)(2.1-2)}{3-2}+8.2=8.69$
4	3	5.84	3.15	$\frac{(18-13.1)(3.15-3)}{4-3}+13.1=13.83$
5	4	6.16	4.20	$\frac{(22.9-18)(4.20-4)}{5-4}+18=18.98$
6	5	6.68	5.25	$\frac{(27.8-22.9)(5.25-5)}{6-5}+22.9=24.12$
7	6	6.77	6.30	$\frac{(32.7-27.8)(6.3-6)}{7-6}+27.8=29.27$
8	7	7.91	7.35	$\frac{(37.7-32.7)(7.35-7)}{8-7}+32.7=34.45$
9	8	10.15	8.40	$\frac{(42.6-37.7)(8.4-8)}{9-8}+37.7=39.66$
12	9	11.94	9.66	$\frac{(47.5-42.6)(9.66-9)}{10-9}+42.6=45.83$
15	10	15.10	11.28	$\frac{(57.3-52.4)(11.28-11)}{12-11}+52.4=53.77$

Table 7. Separation of C Failures with uncorrected value of t_o.

These results are also displayed on Weibull paper in Fig.6-14 on page 94 .

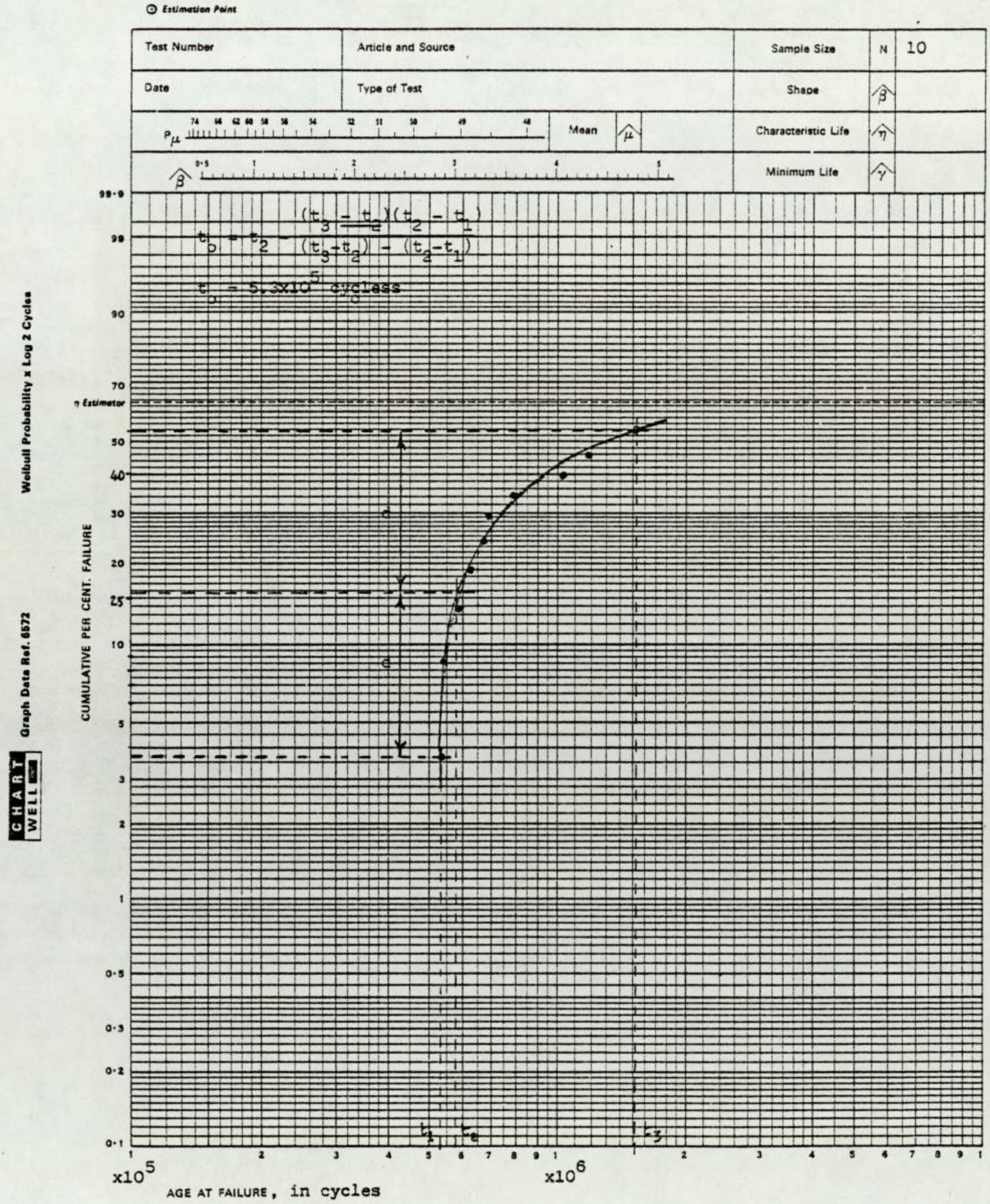


Fig. 6-14. Weibull plot of C-failures with uncorrected value of t_o

6.12 - Correction of a Curved Plot (correction for $t_o \neq 0$) *

Sometimes, the plotted data forms a smooth curve, concave to the left or right on the Weibull paper.

To appreciate the reason for this curvature it is necessary to study the diagrams of number failed against life at failure i.e. the failure distribution diagrams.

In the case of a curve concave to the left some failures have occurred before the start of the life test, e.g. components which have physically deteriorated since manufacture such as to prevent satisfactory operation and thus creating a failure level at zero life as shown in Fig. 6-15 b) on page 96.

If the curve is concave to the right no failures would be experienced until a certain test life is reached at which point a distribution pattern will commence. A good example of this type of failure would be that due to work hardening, which by its very nature would require a certain amount of test time to develop the conditions for failure, Fig. 6-16 b) on page 96.

The correction procedure intended to bring the plotted data into a straight line may be termed "Curvature Correction". This correction method applies equally well to curves concave to the left or right, as shown in Fig. 6-15 on page 96 and in Fig.6-16 on page 96 but is restricted to lines with a single curve, since a curve with the form of an "S bend" would suggest a more complicated distribution which could not be described by the Weibull analysis. See fig. 6-13 on page 89.

Basically this correction merely changes the scale of the x-axis by a constant amount (t_o) hence repositioning and straightening the line.

* See Appendix F on page 236 a.

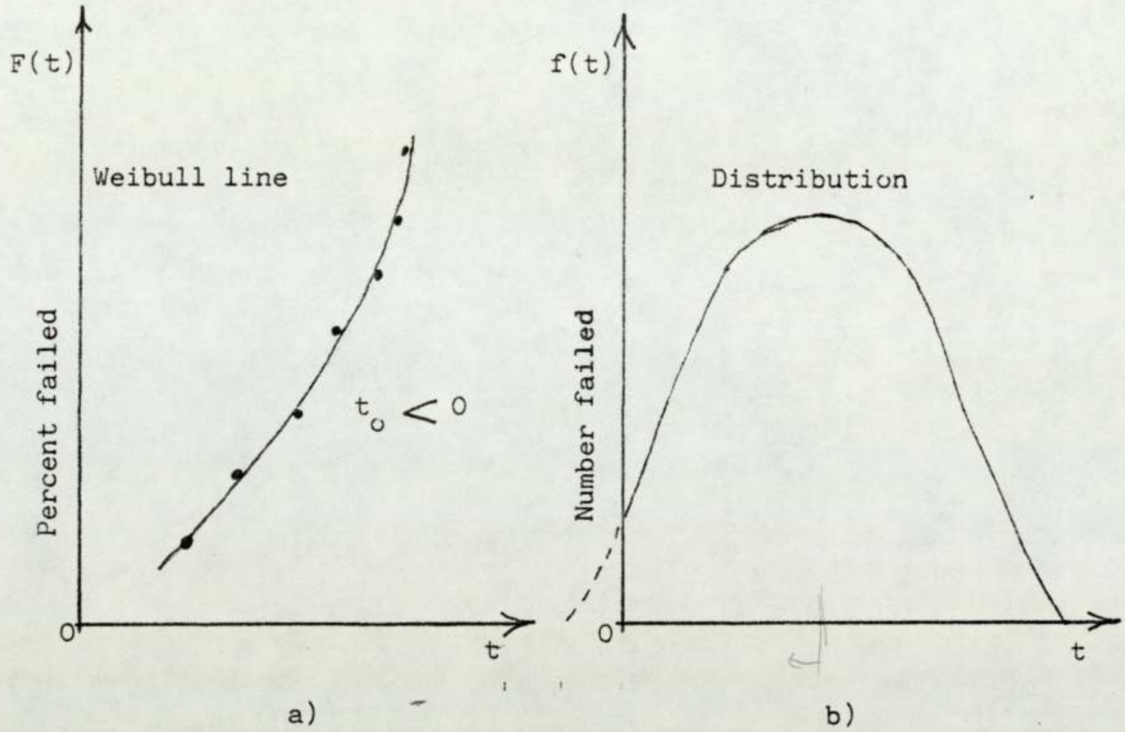


Fig. 6-15

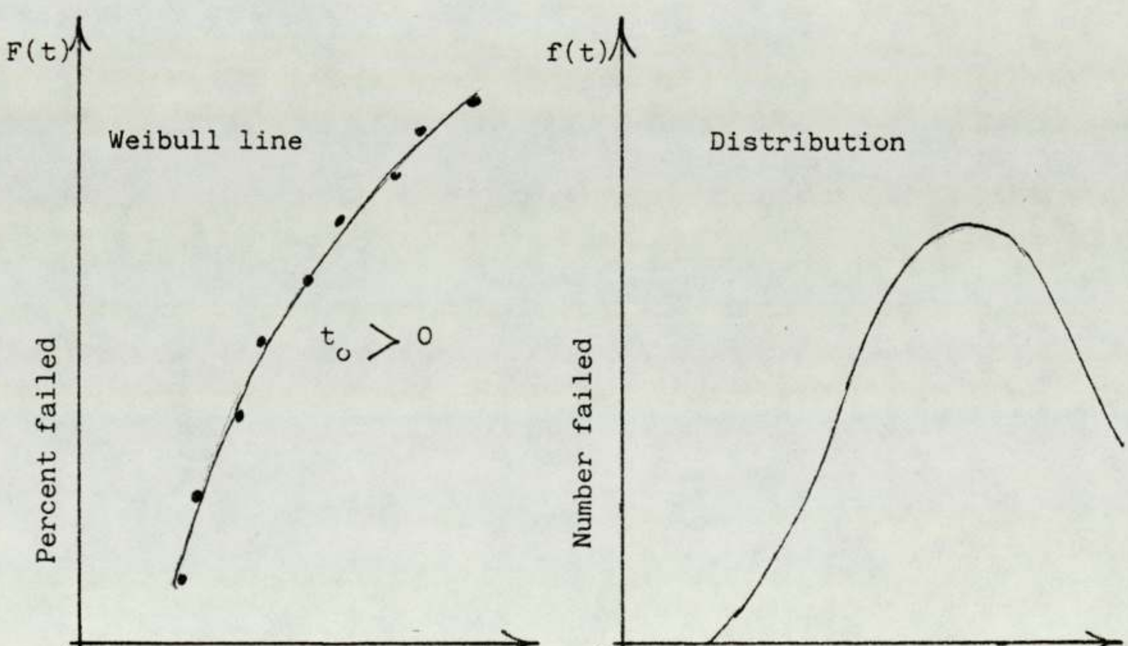


Fig. 6-16

The constant t_o is a locating constant defining the starting point or origin of the distribution.

When t_o does not equal zero, the distribution of failures will not lie on a straight line. When t_o is less than zero the curve will be concave to the left, as illustrated in fig. 6-15 on page 96 ; when t_o is greater than zero, the curve will be concave to the right, as illustrated in fig. 6-16 on page 96 .

To eliminate this curvature, a correction factor must be added to or subtracted from the abscissa. This correction factor is t_o and can be estimated wither graphically or analytically.

6.12-a - Graphical determination of the minimum life t_o

a) The data are plotted on Weibull paper and the "best curve" is fitted to these points.

b) Two parallel horizontal lines are drawn through the extreme failure points and a third horizontal line midway between the two, its position being determined with a linear scale, not using the y-axis scale). Alternatively, select an arbitrary point roughly in the middle of the curve. Two other points, both distant d on a linear scale in the Y direction, are selected and referenced subscript 1, 2 and 3 as illustrated in fig. 6-17 on page 98 .

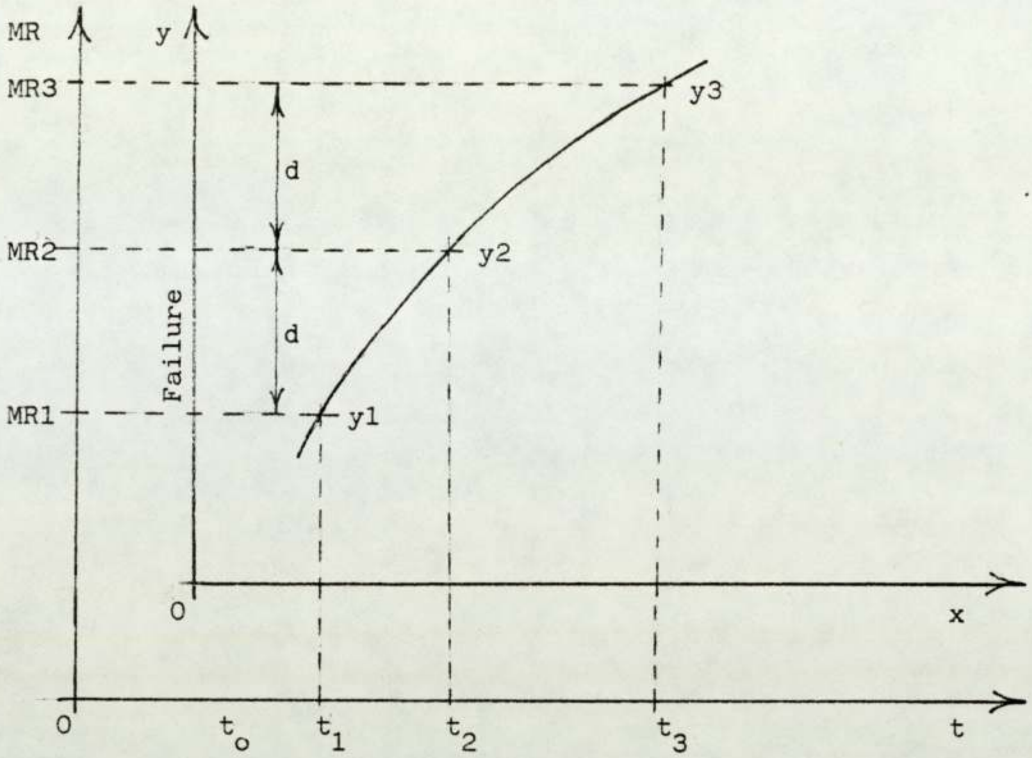


Fig. 6-17 Estimation of t_o

c) Three vertical lines are drawn through the points where the horizontal lines intersect with the curve. These values on the x-axis may be identified as T1, T2 and T3.

d) Since:

$$Y_2 - Y_1 = Y_3 - Y_2 ,$$

it follows from the linear equation of the Weibull line

$Y = \beta \cdot x + C$, that

$$X_2 - X_1 = X_3 - X_2$$

or

$$\ln (t_2 - t_o) - \ln (t_1 - t_o) = \ln (t_3 - t_o) - \ln (t_2 - t_o)$$

giving

$$(t_2 - t_o)^2 = (t_3 - t_o) \times (t_1 - t_o)$$

or

$$t_2^2 - 2t_2t_o + t_o^2 = t_3t_1 - t_1t_o - t_3t_o + t_o^2 .$$

Solving for t_o gives:

$$t_o = \frac{t_3 t_1 - t_2^2}{t_3 - 2t_2 + t_1} = t_2 - \frac{(t_3 - t_2) \times (t_2 - t_1)}{(t_3 - t_2) - (t_2 - t_1)}$$

e) t_o is used with its own algebraic sign. Alternatively, one could consider the magnitude of t_o and ignore its sign.

If the original curve was concave to the left then t_o is added to all values on the x-axis.

If the curve was concave to the right, then t_o is subtracted from all values on the x-axis.

f) The data are replotted using $(t - t_o)$ as the independent variable. If the data follow the Weibull distribution, the points will lie on a straight line.

From fig. 6-14 on page 94, $t_2 = 5.72 \times 10^5$ and:

$$t_o = 5.72 - \frac{(15.10-5.72)\ (5.72-5.32)}{(15.10-5.72)-(5.72-5.32)} \times 10^5 = (5.72 - \frac{9.38 \times 0.40}{9.38-0.40}) \times 10^5 =$$

$$= (5.72 - \frac{3.752}{8.980}) 10^5 = (5.72 - 0.42) 10^5 = 5.30 \times 10^5$$

Our data for the C failure are now displayed in Table 8 on page 99 and in fig. 6-18 on page 100.

Order No.	$(t-t_o)$ $\times 10^5$ cycles	Rank Order No.	Median Ranks %
1	0.02	1.05	3.64
2	0.09	2.10	8.69
3	0.54	3.15	13.835
4	0.86	4.20	18.980
5	1.38	5.25	24.125
6	1.47	6.30	29.270
7	2.61	7.35	34.450
8	4.85	8.40	39.660
9	6.64	9.66	45.834
10	9.80	11.78	53.772

Table 8. C failures with corrected values of $(t-t_o)$

($t_o = 5.3 \times 10^5$ cycles)

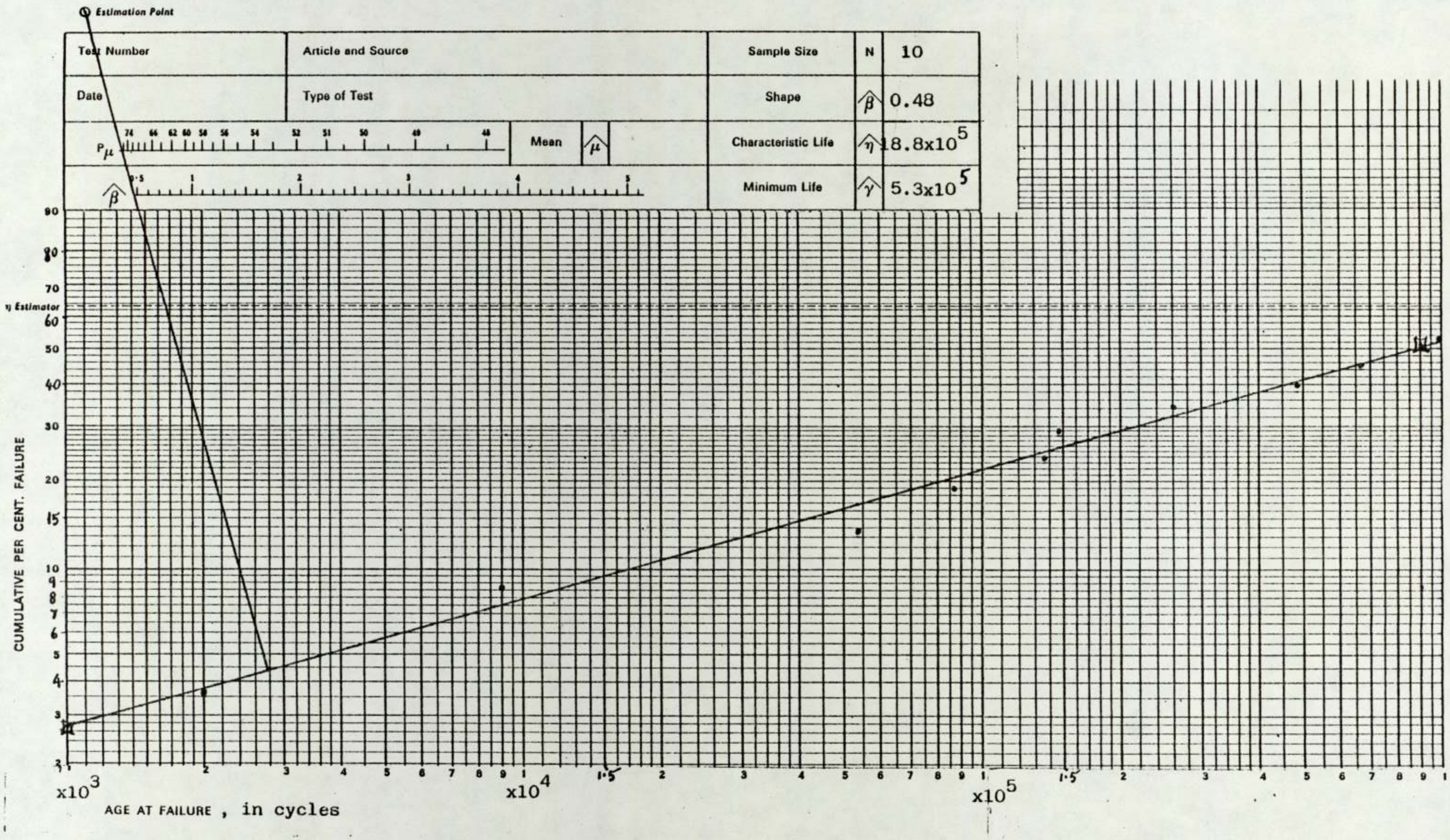


Fig. 6-18. C failures with corrected values of $(t-t_o)$ $\left[t_o = 5.3x10^5 \text{ cycles}\right]$

6.12-b - Analytical determination of the minimum life t_o

Let the curved Weibull plot be represented by a second-degree equation of the form:

$$y = ax^2 + bx + d . \qquad \text{eq. 6-2}$$

For every particular point (x_i , y_i) the vertical distance from such point to the Weibull plot is

$$d_i = y_i - (ax^2 + bx + d).$$

The constants a, b and d should be determined so that the vertical distances are as small as possible. Since each of the d_i cannot be minimised individually, it is best to minimise the sum of their squares . In other words, a, b and d should be chosen so as to minimise the function

$$D = \sum_{i=1}^{n} di^2 \quad \sum_{i=1}^{n} \left[y_i - (ax_i^2 + bx_i + d) \right]^2$$

where n is the number of points.

The function D is a minimum when

$$\frac{\partial D}{\partial a} = - 2 \sum x_i^2 (y_i - ax_i^2 - bx_i - d) = 0$$

$$\frac{\partial D}{\partial b} = - 2 \sum x_i (y_i - ax_i^2 - bx_i - d) = 0$$

$$\frac{\partial D}{\partial d} = - \sum (y_i - ax_i^2 - bx_i - d) = 0$$

These three requisites produce the simultaneous equations:

$$\sum x_i^2 y_i = a \sum x_i^4 + b \sum x_i^3 + d \sum x_i^2$$

$$\sum x_i y_i = a \sum x_i^3 + b \sum x_i^2 + d \sum x_i$$

$$\sum y_i = a \sum x_i^2 + b \sum x_i + d\, n$$

The values of a, b and d are given by:

$$a = \frac{\left|\begin{array}{ccc|cc} \sum x_i^2 y_i & \sum x_i^3 & \sum x_i^2 & \sum x_i^2 y_i & \sum x_i^3 \\ \sum x_i y_i & \sum x_i^2 & \sum x_i & \sum x_i y_i & \sum x_i^2 \\ \sum y_i & \sum x_i & n & \sum y_i & \sum x_i \end{array}\right|}{\left|\begin{array}{ccc|cc} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 & \sum x_i^4 & \sum x_i^3 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^2 & \sum x_i & n & \sum x_i^2 & \sum x_i \end{array}\right|} =$$

$$= \frac{n\sum x_i^2 y_i \sum x_i^2 + \sum x_i^3 \sum x_i \sum y_i + \sum x_i^2 \sum x_i y_i \sum x_i - \sum y_i (\sum x_i^2)^2 - (\sum x_i)^2 \sum x_i^2 y_i - n \sum x_i y_i \sum x_i^3}{n \sum x_i^4 x_i^2 + 2 \sum x_i^3 \sum x_i^2 \sum x_i - (\sum x_i^2)^3 - \sum x_i^4 (\sum x_i)^2 - n (\sum x_i^3)^2}$$

$$b = \frac{\left|\begin{array}{ccc|cc} \sum x_i^4 & \sum x_i^2 y_i & \sum x_i^2 & \sum x_i^4 & \sum x_i^2 y_i \\ \sum x_i^3 & \sum x_i y_i & \sum x_i & \sum x_i^3 & \sum x_i y_i \\ \sum x_i^2 & \sum y_i & n & \sum x_i^2 & \sum y_i \end{array}\right|}{\left|\begin{array}{ccc|cc} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 & \sum x_i^4 & \sum x_i^3 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^2 & \sum x_i & n & \sum x_i^2 & \sum x_i \end{array}\right|} =$$

$$= \frac{n \sum x_i^4 \sum x_i y_i + \sum x_i^2 y_i \sum x_i^2 \sum x_i + \sum x_i^3 \sum x_i^2 \sum y_i - (\sum x_i^2)^2 \sum x_i y_i - \sum x_i^4 \sum x_i \sum y_i - n \sum x_i^3 \sum x_i^2 y_i}{n \sum x_i^4 x_i^2 + 2 \sum x_i^3 \sum x_i^2 - (\sum x_i^2)^3 - \sum x_i^4 (\sum x_i)^2 - n (\sum x_i^3)^2}$$

$$c = \frac{\left|\begin{array}{ccc|cc} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 y_i & \sum x_i^4 & \sum x_i^3 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i y_i & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^2 & \sum x_i & \sum y_i & \sum x_i^2 & \sum x_i \end{array}\right|}{\left|\begin{array}{ccc|cc} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 & \sum x_i^4 & \sum x_i^3 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^2 & \sum x_i & n & \sum x_i^2 & \sum x_i \end{array}\right|} =$$

$$= \frac{\sum x_i^4 \sum x_i^2 \sum y_i + \sum x_i^3 \sum x_i^2 \sum x_i y_i + \sum x_i^3 \sum x_i^2 y_i \sum x_i - (\sum x_i^2)^2 \sum x_i^2 y_i - \sum x_i^4 \sum x_i y_i \sum x_i - (\sum x_i)^2 \sum y_i}{n \sum x_i^4 \sum x_i^2 + 2 \sum x_i^3 \sum x_i^2 \sum x_i - (\sum x_i^2)^3 - \sum x_i^4 (\sum x_i)^2 - n (\sum x_i^3)^2}$$

The ordinates of the first and last point on the Weibull plot are given by.

$$y_1 = \ln \ln \frac{1}{1 - MR_1} =$$

$$y_n = \ln \ln \frac{1}{1 - MR_n} =$$

The midway point will have the coordinate

$$y_m = \frac{y_1 + y_n}{2}$$

and on a linear scale.

$$m = 1 - \frac{1}{e^{e^{y_m}}} =$$

This value is substituted for y in eq. 6.11-2 on page 101 which, with the substitution d - m = c becomes

$$ax^2 + by + c = 0$$

The roots of this equation are:

$$\alpha = \frac{- b + \sqrt{b^2 - 4ac}}{2a} =$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a} =$$

One of these values is discarded as being obviously wrong and the other one gives the required value of t_2.

The value of t_o is then calculated using eq. 6-1 on page 99 , its value is added or subtracted to the life values t, and the data are replotted as $(t \pm t_o)$ as already discussed in Chapter 6.12-a on page 97 . This is the method used in the computer programme discussed in Chapter 10.

6.13 - Best fitting line

It has been found from experience, from representative rig testing, that most part failure data can be interpreted as a Weibull distribution, and appear as a set of points on a reasonably straight line. Indeed, since failures tend to occur right from the moment the parts are put into operation, the minimum life parameter t_o is usually zero, leaving only the shape parameter β and the characteristic life η to be estimated.

Once the points are plotted, we must determine the position of the best fitting straight line. (If the data should be non-linear, this indicates a non-zero value of the minimum life parameter t_o. The value of t_o must first be estimated, and then subtracted from each time t to failure in turn. Replotting $(t-t_o)$ almost always yields a straight line, as seen in Chapter 6.12 on page 100).

6.13-a - Drawing the best fitting line by eye

In most cases, a straight line may be fitted by eye.

As an example, ten parts were subjected to life test, and the following results were obtained:

Failure No.	Age at Failure (hours)	Median Ranks, %
1	500	6.6
2	1200	16.2
3	1650	25.8
4	2050	35.5
5	2650	45.1
6	3250	54.8
7	3750	64.4
8	4500	74.1
9	4950	83.7
10	7300	93.3

These data are plotted, and a straight line is drawn by eye, in fig. 6-19 on page 106. It will be seen that the data form a reasonably good straight line, only the first point A being apparently somewhat adrift. Taking account of the scales, however, reveals that this conclusion is over hasty. Since the actual time of each failure has been observed, suppose it is assumed that all errors in the data are attributable to our estimation of the cumulative percentage of the population which has failed. It is seen that, to comply with our line, the first failure should have occurred at 4.7 per cent instead of 6.6, so that its "error" in this sense is 1.9 per cent. However, after around 50 per cent have failed, owing to the contraction of the scale, the same displacement in millimeters would indicate an error of about 13 per cent. Hence, in fitting a line by eye, more importance must be attached to discrepancies in mid-life than to those early on. From this it follows that a life test should not be truncated too soon, particularly if the Weibull line is to be extrapolated to the left, in order to estimate the age at which some high value of reliability occurs. There must be enough data to determine the slope β of the line (see Chapter 6.18 on page 167) with acceptable accuracy.

Reliability engineers usually draw the best-fit line by eye, since its position tends to be fairly obvious. (The cramping of the y axis scale in middle life, and the x axis scale in later life, tends to mask any discrepancies!).

However, in certain cases, the plotted data tend to form a straight line but significant scatter is apparent. In such cases, any attempt to fit a straight line "by eye" would be basically incorrect or would give such poor accuracy as to render the exercise useless.

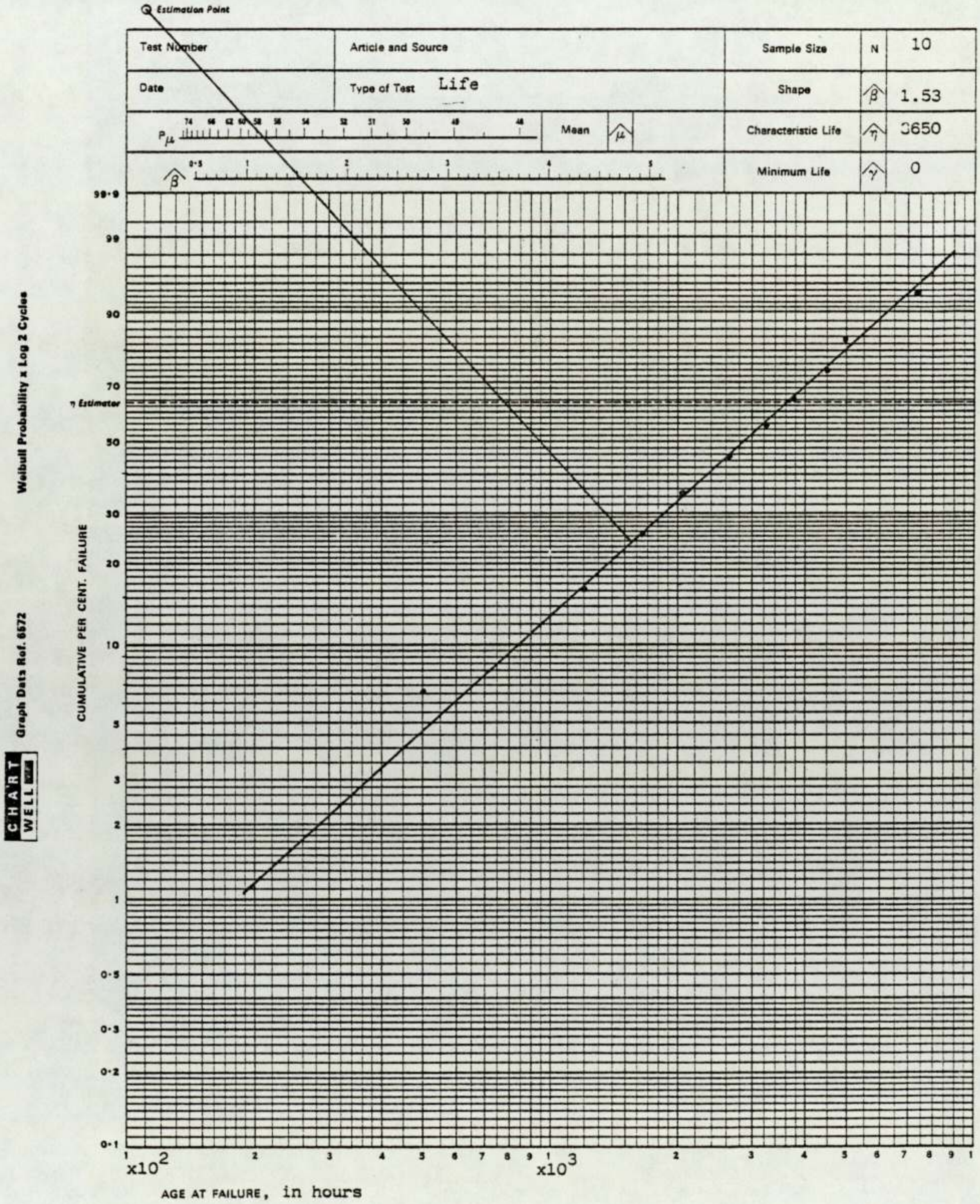

Estimation Point
Test Number
Article and Source
Sample Size
N
10
Date
Type of Test
Life
Shape
1.53
Mean
Characteristic Life
3650
Minimum Life
0
η Estimator
CUMULATIVE PER CENT. FAILURE
Weibull Probability x Log 2 Cycles
Graph Data Ref. 6572
CHART WELL
x10²
x10³
AGE AT FAILURE, in hours

Fig. 6-19.

6.13-b - Drawing the best-fitting line by the method of least squares

If the points should be so scattered that the position of the straight line is not easy to determine, it is necessary to find out what has gone wrong (see section 6.14 on page 120), and whether the data are worthy of being fitted by a more sophisticated method. If it is decided that they are, then regression analysis, suitably modified, is indicated.

If n paired observations (x, y) are available, for which it is reasonable to assume a linear relation between x and y and if it is required to determine the line (that is, the equation of the line) which in some sense provides the "best fit", then one way of doing that is to apply the method of least squares.

The equation of a straight line which is required to fit to the set of data which appears in fig. 6-18 on page 100 is

$$y = a + bx \qquad \text{eq. 6-3}$$

where a and b are constants, which have to be determined in order to get the best fitting line.

Of the infinite number of lines that could be drawn, which one fits the data best? Before this question would be answered satisfactorily, some criterion must be agreed upon by which to judge the suitability of any line that could be chosen. The most generally adopted criterion is based upon minimising the sum of the squares of the distances from the line to the data points. This may sound formidable but, as it will be seen, it is not so difficult.

The distance from the line to each point must now be defined. The minimum distance is, of course, along a line normal to the line $y = a + bx$ that it is required to find. In many cases this is the value to be used. In this case the input (time, cycles, etc.) is known quite precisely. It can be safely assumed that any deviation from linearity occurs because of variation of the quantity on the y-axis (median rank). Since x is known much better than y, it is reasonable to choose to minimise the vertical distance from the point to the line. For a particular data point (x_i, y_i) for instance, the vertical distance is given by

$$s_i = y_i - (a + bx_i)$$

The constants a and b should be determined so that the estimated vertical distances are as small as possible. Since each of the s_i cannot be minimised individually, their sum $\sum_{i=1}^{N} s_i$ should be made as close as possible to zero. However, since this sum can be made equal to zero by many choices of totally unsuitable straight lines for which the positive and negative errors cancel out, it is much better instead to minimise the sum of the squares of the vertical distances s_i. In other words, a and b should be chosen so as to minimise the function

$$S = \sum_{i=1}^{N} s_i^2 = \sum_{i=1}^{N} \left[y_i - (a + bx_i) \right]^2$$

where N is the number of points since a and b are the constants to be determined, the function S is minimum when

$$\frac{\partial S}{\partial a} = \frac{\partial}{\partial a} \sum_{i=1}^{N} \left[y_i - (a + bx_i) \right]^2 = 0 \qquad \text{eq. 6}$$

and

$$\frac{\partial S}{\partial b} = \frac{\partial}{\partial b} \sum_{i=1}^{N} \left[y_i - (a + bx_i) \right]^2 = 0 \qquad \text{eq. 6}$$

These two requisites produce the simultaneous equations:

$$\sum y_i = aN_o + b\sum x_i$$

$$\sum x_i y_i = a\sum x_i + b\sum x_i^2$$

where N is the sample size and x_i and y_i are the co-ordinates of the plotted data. Solving for a and b gives:

$$a = \frac{\begin{vmatrix} \sum y_i & \sum x_i \\ \sum x_i y_i & \sum x_i \end{vmatrix}}{\begin{vmatrix} N & \sum x_i \\ \sum x_i & \sum x_i^2 \end{vmatrix}} = \frac{\sum y_i \sum x_i - \sum x_i \sum x_i y_i}{N \sum x_i^2 - (\sum x_i)^2} \qquad \text{eq. 6-6}$$

$$b = \frac{\begin{vmatrix} N & \sum y_i \\ \sum x_i & \sum x_i y_i \end{vmatrix}}{\begin{vmatrix} N & \sum x_i \\ \sum x_i & \sum x_i^2 \end{vmatrix}} = \frac{N \sum x_i y_i - \sum x_i \sum y_i}{N \sum x_i^2 - (\sum x_i)^2} \qquad \text{eq. 6-7}$$

To obtain a least squared line on Weibull probability paper the following procedure may be observed.

6.13-b-I- Method I

Regression analysis cannot be used in its normal form, because of the non-linear Weibull scales. One suggestion is that the points should be traced onto ordinary equal-division graph paper, the best-fit line determined in terms of the linear scale, and that this should then be traced back onto the Weibull paper. The test data are plotted on Weibull paper with the appropriate Median Ranks. Each plotted point is then transposed onto a piece of linear graph paper placed over the Weibull paper. Axes are then drawn on this linear paper and any convenient scales given to these axes. The X (horizontal) and Y (vertical) scale readings for each point are then noted and tabulated along with their corresponding X^2 and X.Y values. Finally the X, Y, X^2 and X.Y columns are summated to give values which may be substituted into the above simultaneous equations.

The following example using six sample failures demonstrates the layout of a least squares calculation (see fig. 6.20 on page 110).

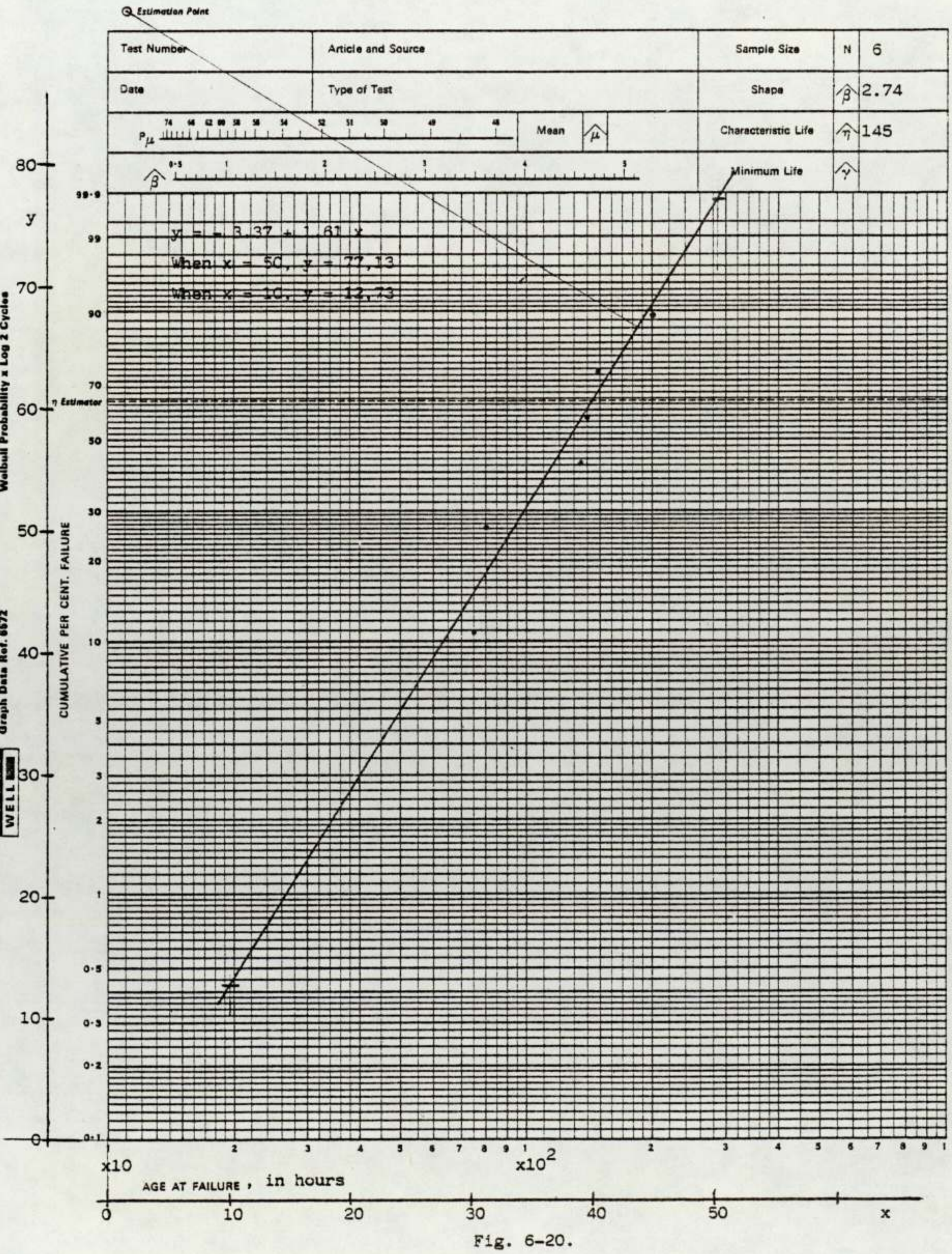

Estimation Point
Test Number
Article and Source
Sample Size
N
6
Date
Type of Test
Shape
2.74
Mean
Characteristic Life
145
Minimum Life
η Estimator
y = - 3.37 + 1.61 x
When x = 50, y = 77.13
When x = 10, y = 12.73
Weibull Probability x Log 2 Cycles
Graph Data Ref. 6572
CHART WELL
CUMULATIVE PER CENT. FAILURE
y
x10
x10^2
AGE AT FAILURE , in hours
x

Fig. 6-20.

	Order Number	Life (hrs.)	Median Ranks
	1	75	10.9
	2	80	26.4
	3	123	42.1
N = 6	4	130	57.8
	5	149	73.5
	6	200	89.0

Transposed from Weibull paper to linear paper with arbitrary scales gives:

X	Y	X^2	XY
30.1	42.3	906.0	1273.2
31.2	50	973.4	1560
37.4	55.2	1398.8	2064.5
38.4	59.3	1474.6	2277.1
40.3	63	1624.0	2538.9
44.6	67.6	1989.2	3015.0

$\sum X = 222.0$ $\sum Y = 337.4$ $\sum X^2 = 8366.02$ $\sum XY = 12728.7$

Thus:

$\sum Y = AN + B\sum X$ gives $337.5 = 6A + 222B$........(1)

$\sum XY = A\sum X + B\sum X^2$ $\quad 12728.7 = 222A + 8366B$.......(2)

Now (1) X 37 gives $12487.5 = 222A + 8214B$..........(3)

$12728.7 + 222A + 8366B$ ....(4)

and (4) - (3) gives $241.2 = 152\ B$

$$\text{therefore } B = \frac{241.2}{152} = 1.61\ .$$

Substituting B in (1) $A = \dfrac{337.5 - 357.6}{6} = \dfrac{-\ 20.1}{6} = -\ 3.37\ .$

Hence $Y = -3.37 + 1.61\ X$.

Thus a best fit line is constructed on the linear graph paper and transposed back onto the Weibull paper.

The objection to this is that the same importance is given to points which have the same linear displacement from the mean line, regardless of where they occur on the Weibull scale.

6.13-b-II - Method II

The objection to Method I can be overcome with the following method, which disperses with the need to transpose the data from Weibull paper to linear paper and allows to find directly on the Weibull paper two points which, when joined, will give the best fitting line.

In section 2.2 on page 26 it was shown that the Weibull equation can be written in the form:

$$R(t) = 1 - F(t) = e^{-\left(\frac{t-t_o}{\eta}\right)^{\beta}}$$

Perform the manipulations which are the basis of Weibull probability paper. Invert both sides and take natural logarithms twice.

Then: $- \ln \left[1 - F(t)\right] = \left(\frac{t-t_o}{\eta}\right)^{\beta} = \frac{(t-t_o)^{\beta}}{\eta^{\beta}}$

and $\ln \left\{-\ln \left[1 - F(t)\right]\right\} = \beta .\ln (t - t_o) - \beta .\ln \eta$.

This gives:

$$\ln \ln \left[\frac{1}{(1-F(t))}\right] = \beta \ln (t-t_o) - \beta \ln \eta$$

This can be expressed as the standard equation to a straight line,

$y = a + b.x$

where $y = \ln \left\{ - \ln \left[1 - F(t)\right]\right\} = \ln \ln \frac{1}{1-F(t)}$

$x = \ln (t - t_o)$

$a = - \beta .\ln \eta$ (this is constant) eq. 6-8

$b = \beta$ (also constant) . eq. 6-9

On Weibull graph paper, the scales are ingeniously devised to include the functions which are on the right-hand sides of the above equations. Hence reliability data almost always plot as a straight line. However, there is no reason why the data themselves should not be converted, so that they will plot as a straight line on ordinary graph paper. This method will be illustrated using the example in section 6.13-a on page 104, where it has already been determined that $t_o = 0$.

First convert the data as above. The figures thus obtained are shown in Table 9 on page 113.

t	x=ln(t)	F(t)	$y=\ln \ln \left(\frac{1}{1 \quad F(t)}\right)$
500	6.21	0.067	2.67
1200	7.09	0.163	1.72
1650	7.41	0.260	1.20
2050	7.63	0.356	0.82
2650	7.88	0.452	0.51
3250	8.09	0.548	0.23
3750	8.23	0.644	+0.03
4500	8.41	0.740	+0.30
4950	8.51	0.837	+0.59
7300	8.90	0.933	+0.99

Table 9.

If x and y are now plotted on ordinary graph paper, fig. 6-21 is obtained, showing that a straight line has been achieved. (Fig. 6-21 is included here for clarity; it is not necessary to plot it in order to find the equation of the best-fit straight line). Next, these data are used in the usual way, to find the regression of y on x. (It is assumed that all errors in the data arise from errors in estimation of the cumulative percentage of the population failed. The other regression line can of course be deduced if required).

This gives:

b = 1.42

a = -11.62

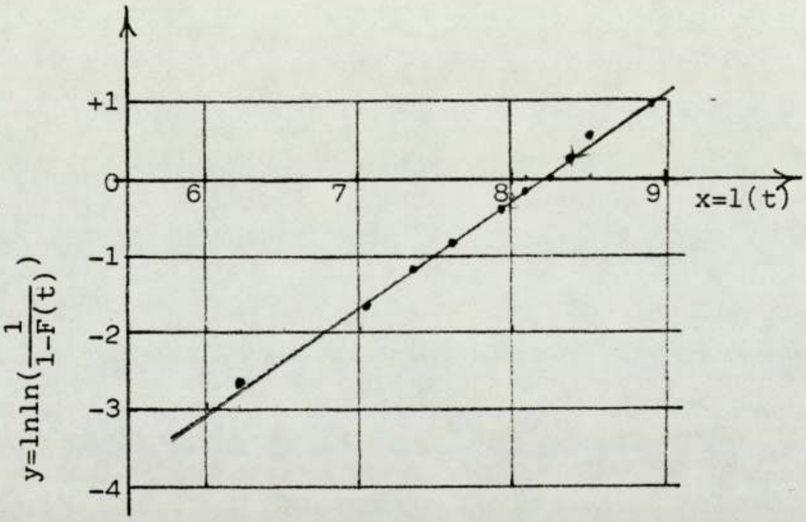


Fig. 6-21

These results must be converted back to the Weibull parameters.

From eq. 6-9 :

β = b = 1.42,

and from eq. 6-8 :

a = β ln (η) .

Therefore

$$\ln(\eta) = -\frac{a}{\beta} = \frac{11.62}{1.42} = 8.20 \ .$$

Therefore $\eta = 3658$ hours .

Hence, the predicted Weibull equation is:

$$R(t) = e^{\left(\frac{t}{3658}\right)^{1.42}} \ .$$

Comparing this with the constants deduced from the scales of the Weibull graph paper, gives the following results.

	Weibull paper	Regression line
β	1.53	1.42
η	3650	3658

Bearing in mind that a line fitted by eye usually lies between the two regression lines, it will be seen that the agreement is quite good.

6.13-c - The correlation coefficient

The product moment correlation coefficient produces an exact measurement of the correlation between the variables. Its calculation was a laborious arithmetic process until the advent of the modern calculators which produce it at the touch of a button.

The correlation coefficient is given by the following expression:

$$n_{xy} = \frac{S_{xy}}{S_x \ S_y}$$

where:
$$S_{xy} = \frac{\sum x_i y_i}{n} - \frac{\sum x_i}{n} \frac{\sum y_i}{n}$$

$$S_x = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$S_y = \sqrt{\frac{\sum y_i^2}{n} - \left(\frac{\sum y_i}{n}\right)^2}$$

6.13-d - Application

The data should appear as a straight line on Weibull probability paper. A visual inspection of the new Weibull plot for the C failures, fig. 6-18 on page 100, shows that this is the case, and it is therefore possible to proceed with the analysis.

The next step is to fit the best straight line through the graph obtained from Table 8 on page 99. This can be done "by eye" or by the more favourable methods of least squares. Method II will be applied to determine the equation of the Weibull line and the two points that will allow to draw the best fitting line.

Consider, as an example, the results taken from Table 8 on page 99, from which Table 10 on page 118 can be constructed. From equations 6-6 and 6-7 on page 109 :

$$a = \frac{33.968209 \times (-13.79619) - 0.3331916 \times 15.827753}{10 \times 33.968209 - 0.3331916^2} =$$

$$= -\frac{468.63187 - 5.2736743}{339.68209 - 0.1110166} = \frac{-473.90554}{339.57107} = -1.396$$

$$b = \frac{10 \times 15.327753 - 0.3331916\ (-13.79619)}{10 \times 33.968209 - 0{,}3331916^2} =$$

$$= \frac{158.27753 + 4.5967746}{339.68209 - 0.1110166} = \frac{162.8743}{339.57107} = 0.4797$$

Therefore the equation of the best fitting line is:

$y = -1.395{,}649{,}613 + 0.479{,}686{,}094{,}1\ x$

and $\beta = 0.479{,}686{,}049{,}1$.

The best fitting line can then be drawn by joining with a straight line two paired values of x_i and y_i. For example, when $(t-t_o) = 0.01\times10^5$, $x_1 = \ln(t-t_o) = -4.605{,}170{,}186$, $y_1 = -1.39560\ 5 + 0.4796472\ x_1 = -3.6044575$ and therefore $F(t-t_o) = 1-1/e^{e^{y1}} = 0.026835$; when $(t-t_o) = 9\times10^5$, $x_2 = 2.197224577$, $y_2 = -0.341707884$ and $F(t-t_o) = 0.5086289525$. This is done in fig. 6-22 on page 119.

The correlation coefficient is:

$$r_{xy} = \frac{\frac{\sum x_i y_i}{N} - \frac{\sum x_i}{N} \cdot \frac{\sum y_i}{N}}{\sqrt{\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2} \sqrt{\frac{\sum y_i^2}{N} - \left(\frac{\sum y_i}{N}\right)^2}} =$$

$$= \frac{\frac{15.8277}{10} - \frac{0.3331916}{10} \cdot \left(\frac{-13.7961891}{10}\right)}{\sqrt{\frac{33.968209}{10} - \left(\frac{0.3331916}{10}\right)^2} \sqrt{\frac{27.70007272}{10} - \left(\frac{-13.7961891}{10}\right)^2}} =$$

$= 0.9929490254.$

$F(t-t_o)$	$(t-t_o)$ $x10^5$	$x_i=\ln(t-t_o)$	$y_i=\ln\ln \frac{1}{1-F(t-t_o)}$	x_i^2	x_iy_i	y^2
0.03640	0.02	ln0.02 = -3.912023005	-3.294704285	15.3039240	+12.8889610	10.8550763
0.08690	0.09	ln0.09 = -2.407945609	-2.397886644	5.8030189	+ 5.7763784	5.749860357
0.13835	0.54	ln0.54 = -0.616186139	-1.904439217	0.3796853	+ 1.1734891	3.626888731
0.18980	0.86	ln0.86 = -0.150822890	-1.558392445	0.0227470	+ 0.2350413	2.428587013
0.24125	1.38	ln1.38 = 0.322083499	-1.287053963	0.1037378	- 0.4145388	1.656507904
0.29270	1.47	ln1.47 = 0.385262401	-1.060448745	0.1484271	- 0.4085510	1.124551541
0.34480	2.61	ln2.61 = 0.959350221	-0.861904408	0.9203528	- 0.8268682	0.742879209
0.39660	4.85	ln4.85 = 1.578978705	-0.682850470	2.4931737	- 1.0782066	0.466284764
0.45834	6.64	ln6.64 = 1.893111963	-0.489199854	3.5838730	- 0.9261100	0.2393164975
0.53772	9.80	ln9.80 = 2.282382386	-0.259309072	5.2092694	- 0.5918425	0.06724119493
		$\sum x_i$ = 0.334191532	$\sum y_i$ = -13.796189	$\sum x_i^2$ = 33.96339271	$\sum x_iy_i$ = +15.8253529	$\sum y^2$ = 6.95719354

Table 10. C Failures - Calculation of parameters for best fitting line.

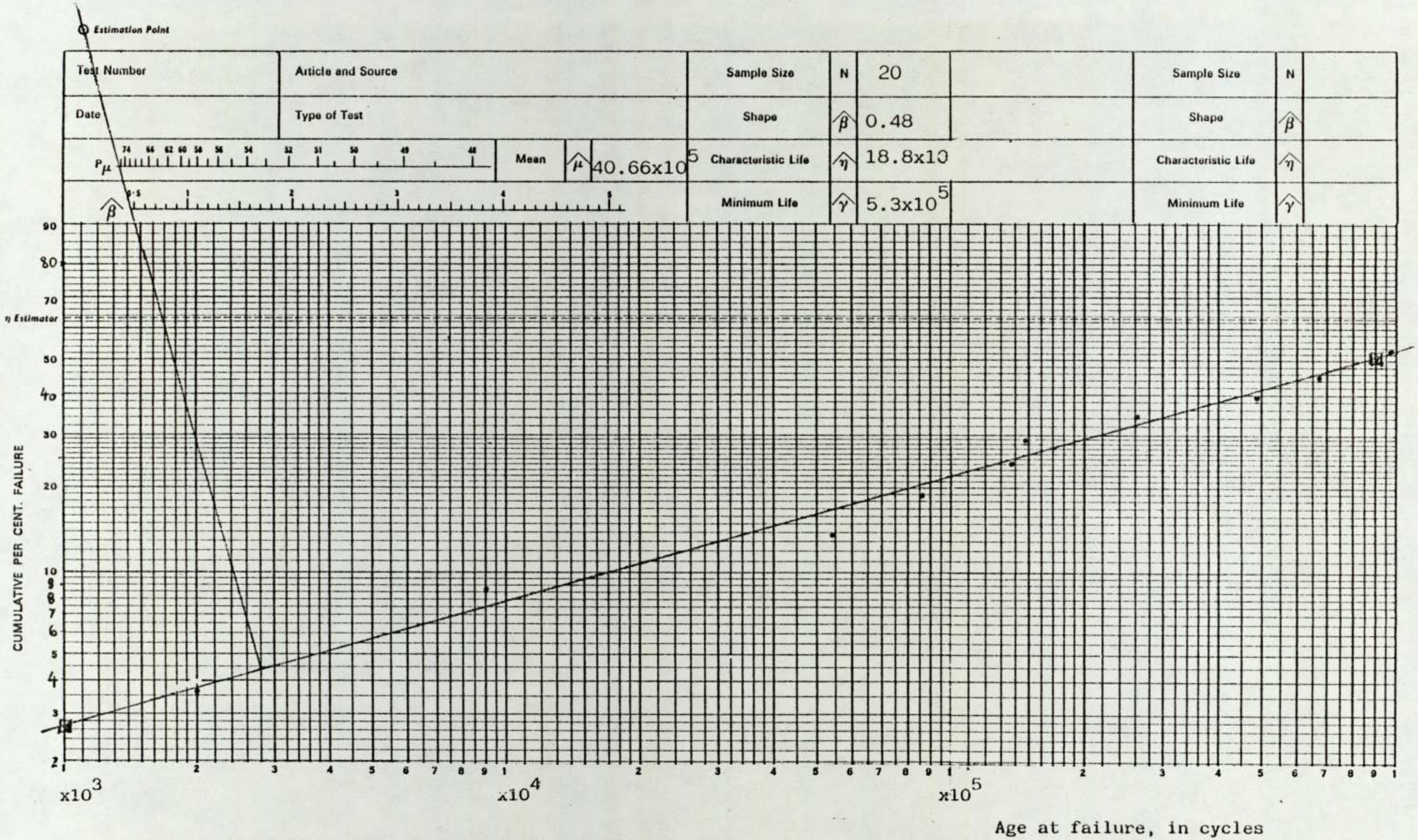


Fig. 6-22. Best fitting line for C data from Table 10.

6.14 - Dealing with spurious data[18]

When only a few data are available, care must be taken to extract as much information as possible from them. Points should not be discarded too readily. It is often possible to determine what probably occurred and hence why one or more points are out of position. Occasionally it is possible to show that a point is almost certainly spurious.

When the sample size is very small, spurious points present a real problem. For example, if a sample of 5 is considered, each point represents 20 per cent of our data. If two points are different from the other three, how is it known that the two are spurious and the three correct? It might be the other way round! Experience has led to be very wary about rejecting data which do not appear to fit some preconceived notion. The following examples illustrate this.

6.14-a - Glass manufacture

Fig. 6-23 shows a study which was made on the life of two moulds in a glass-processing factory. The production routine was to run one mould until its surface became unacceptable. It was then replaced by the other, and the rejected mould refurbished for further use. In this case a reasonable number of data were available, and two approximately Normal distributions, with means at M_1 = 15.0 and M_2 = 20.8 h, resulted. We therefore concluded that mould-surface deterioration was a straightforward wear-out situation. Mould 2, however, consistently gave a longer production run than M_1, and this presented the factory with an engineering problem to find out why, so that in future all moulds could be made like M_2.

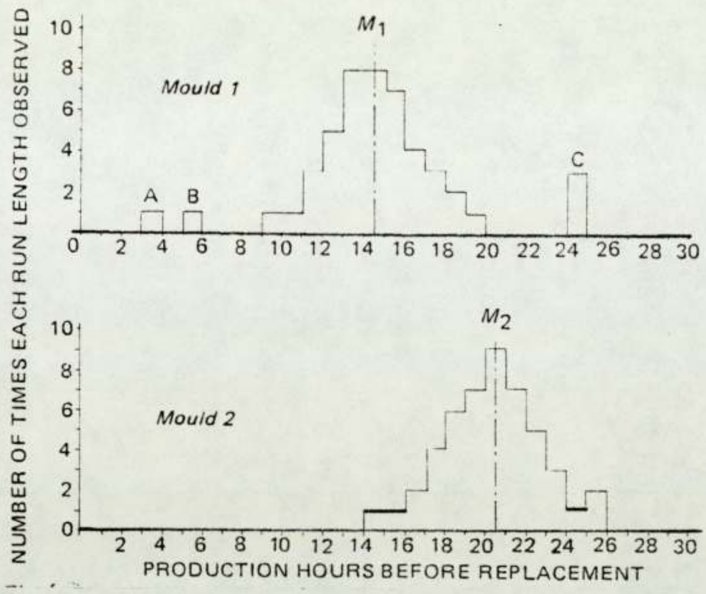


Fig. 6-23

In three places, the observed data were not to this pattern. Points A and B were easily explained, operator errors had caused premature damage to the mould surface, but C needed more thought. Notice how these failures occur.

a) They are over four standard deviations from the mean of mould 1, to which they purport to belong, and, for a Normal frequency distribution, the probability of an observation's being four or more standard deviations from mean is 0.0003.

b) All three failures at C due to mould 1 have exactly the same value of 25 h. Since the probability of three failures at 25 h or more is $(0.0003)^3 = 0.27 \times 10^{-10}$, the possibility that they belong to the main distribution can be discounted. There must be some large source of variation, contravening the conditions for a Normal distribution. The obvious possibility is that there is one very careful operator, who achieves, with mould 1, the sort of production runs that are observed for mould 2.

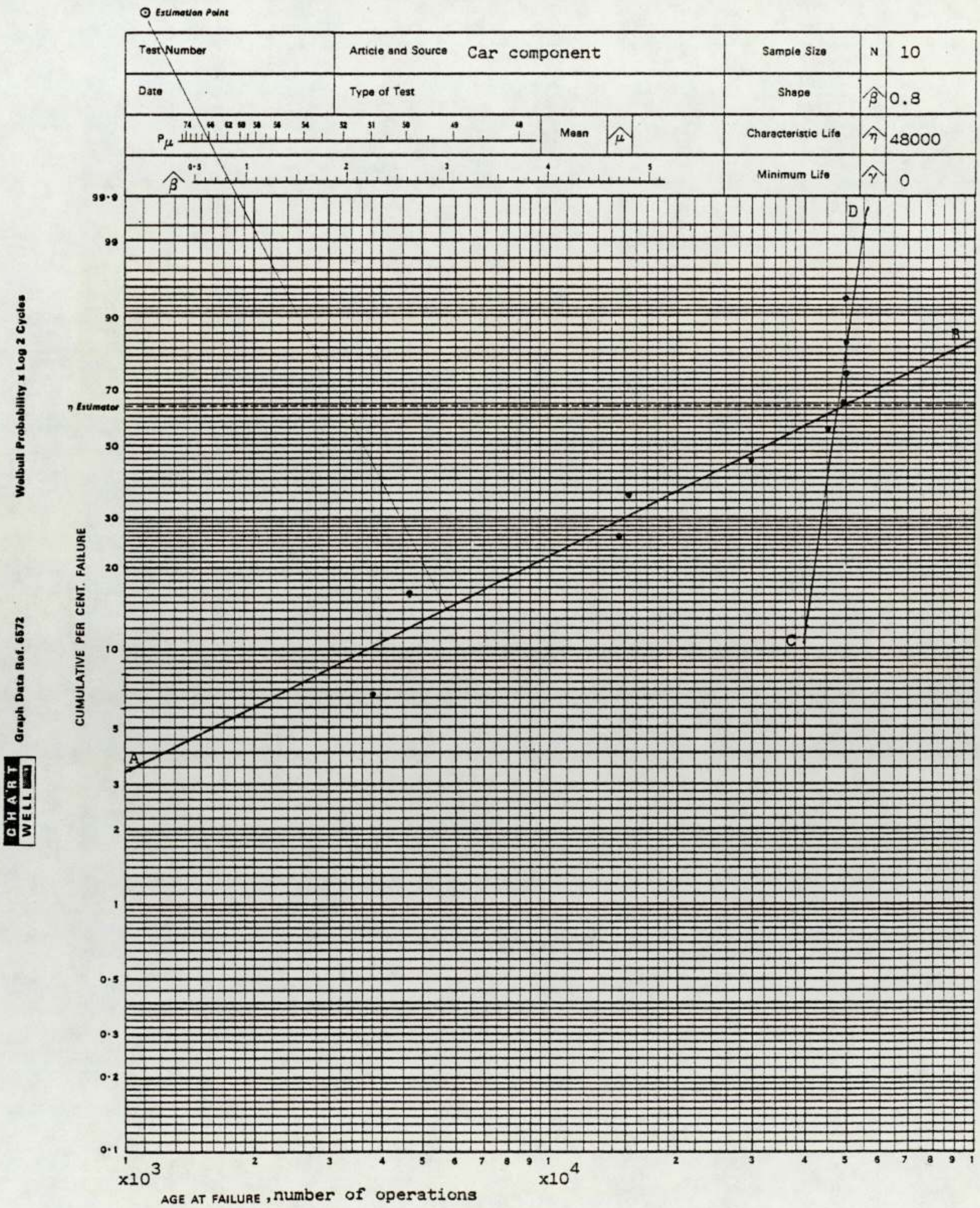
Suppose, however, that there was such an operator. His work would in principle form a third distribution curve, centred about his own mean run length. The probability of three runs with exactly 25 h, with no other neighbouring run length observed, is so small we discounted it. It did not fit the pattern of manning, anyway. Here it seems almost certain that these three observations originated from the bane of all reliability engineers, incorrect data. The possibility that these readings really refer to mould 2 is unlikely, because there is nothing significant missing from the M_2 data, either at that point or in total. Wherever these points should be, it seems quite certain that they were not three runs of exactly 25 h as reported.

6.14-b - Car components

This was a study of the number of operations a car component could withstand before failure. The Weibull plot for a sample of 10 is shown in fig. 6-24 on page 123. We can distinguish two straight sections AB and CD, from which it would appear that some dramatic change in mode and mechanism of failure took place where these sections intersect just below 50,000 cycles. However, consider the raw data which were as in Table 11.

Failure No.	Number of Operations
1	3805
2	4612
3	14560
4	15108
5	29950
6	45506
7	48575
8	50000
9	50000
10	50000

Table 11.

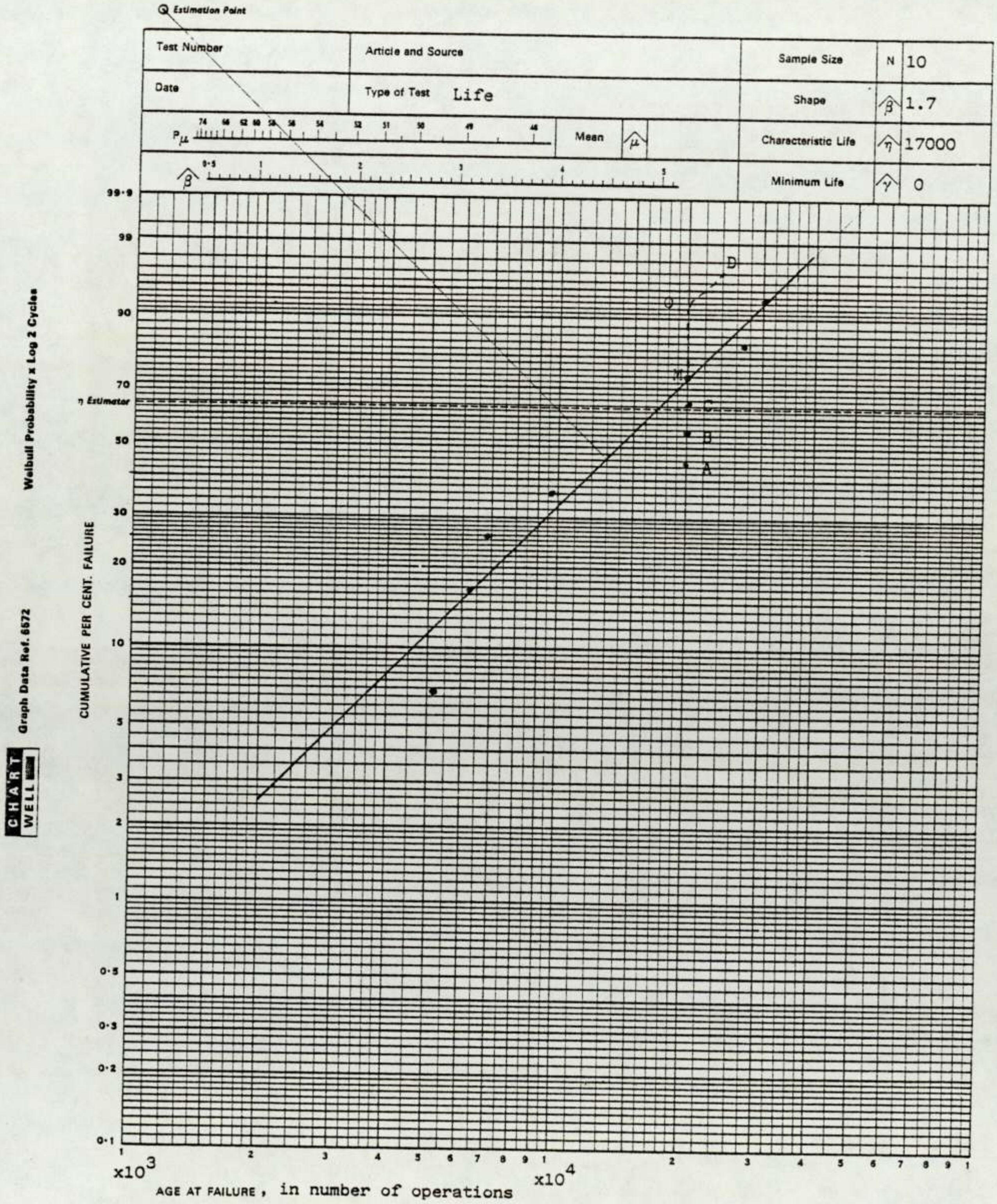


Estimation Point
Test Number
Article and Source Car component
Sample Size N 10
Date
Type of Test
Shape 0.8
Mean
Characteristic Life 48000
Minimum Life 0
η Estimator
CUMULATIVE PER CENT. FAILURE
Weibull Probability x Log 2 Cycles
Graph Data Ref. 6572
CHARTWELL
A
B
C
D
x10³
x10⁴
AGE AT FAILURE ,number of operations

Fig.6-24.

Now, over the period of the test, components can fail at any number of operations from 1 up to 50,000 or so. The probability that they would fail at a round number like 40,000 or 50,000 is (depending upon the shape of the failure probability density function) broadly the same as that they would fail at any other particular number. The average probability is 1 in 50,000, and the probability that three would fail together is 8×10^{-15}, which is so small it can be discounted. Reference back to the company revealed, as we suspected, that so-called "failures" numbers 8, 9 and 10, were not really failures at all. They merely survived when the test was truncated at 50,000 operations. Hence line AB of fig. 6-24 is correct; CD is spurious and must be deleted.

Fig. 6-25 shows the results of a life test on another car component. Here seven points form a reasonable straight line, and the problem is to interpret points A, B and C. Notice that these are not additional failures, since the line itself has three points missing in this region. (Had they been additional failures, the line would have displaced itself upwards, as indicated by MO, and then continued at about the same slope, as OD). Next, consider the size of the gap, which is roughly from 9,700 to 21,000 operations. Since one operation took approximately 4 s, this gap represents about 12.5 clock hours, which would be consistent with the test having been left unattended overnight. Further inquiries showed that this was indeed what happened. Failures, A, B and C were not observed to occur at these times, they were merely discovered in a failed condition next morning. Their exact moment of failure was not known.



Estimation Point
Test Number
Article and Source
Sample Size
N 10
Date
Type of Test Life
Shape
$\hat{\beta}$ 1.7
Mean
Characteristic Life
$\hat{\eta}$ 17000
Minimum Life
$\hat{\gamma}$ 0
η Estimator
CHART WELL
Graph Data Ref. 6572
Weibull Probability x Log 2 Cycles
CUMULATIVE PER CENT. FAILURE
D
Q
M
C
B
A
x10^3
x10^4
AGE AT FAILURE, in number of operations

Fig.6-25.

6.14-c - Flyover arm

Fig. 6-26 shows the results of a small life test on 10 flyover arms. At first sight the pattern is similar to fig. 6-24, and again the problem is to establish the true track of the Weibull line. In this case there seemed to be no evidence to suggest that the observed data were incorrect, nor was there any sign of a dramatic change in the mode of failure in the vicinity of the 5th point. Yet apparently two quite different Weibull lines could be drawn, depending upon whether we believed the first five points and discarded the other two (line AB), or believed the last four points and discarded the first three (line CD). About three points in the region of the intersection of AB and CD could be regarded as belonging to either line, as we pleased! Because it predicted superior reliability, there was a natural desire to conclude that CD was correct.

Consider the data, however, which are given in Table 12.

Failure Number	Number of operations at failure
1	147
2	402
3	633
4	2374
5	3159
6	3171
7	3412
Truncation	6500 with three arms surviving

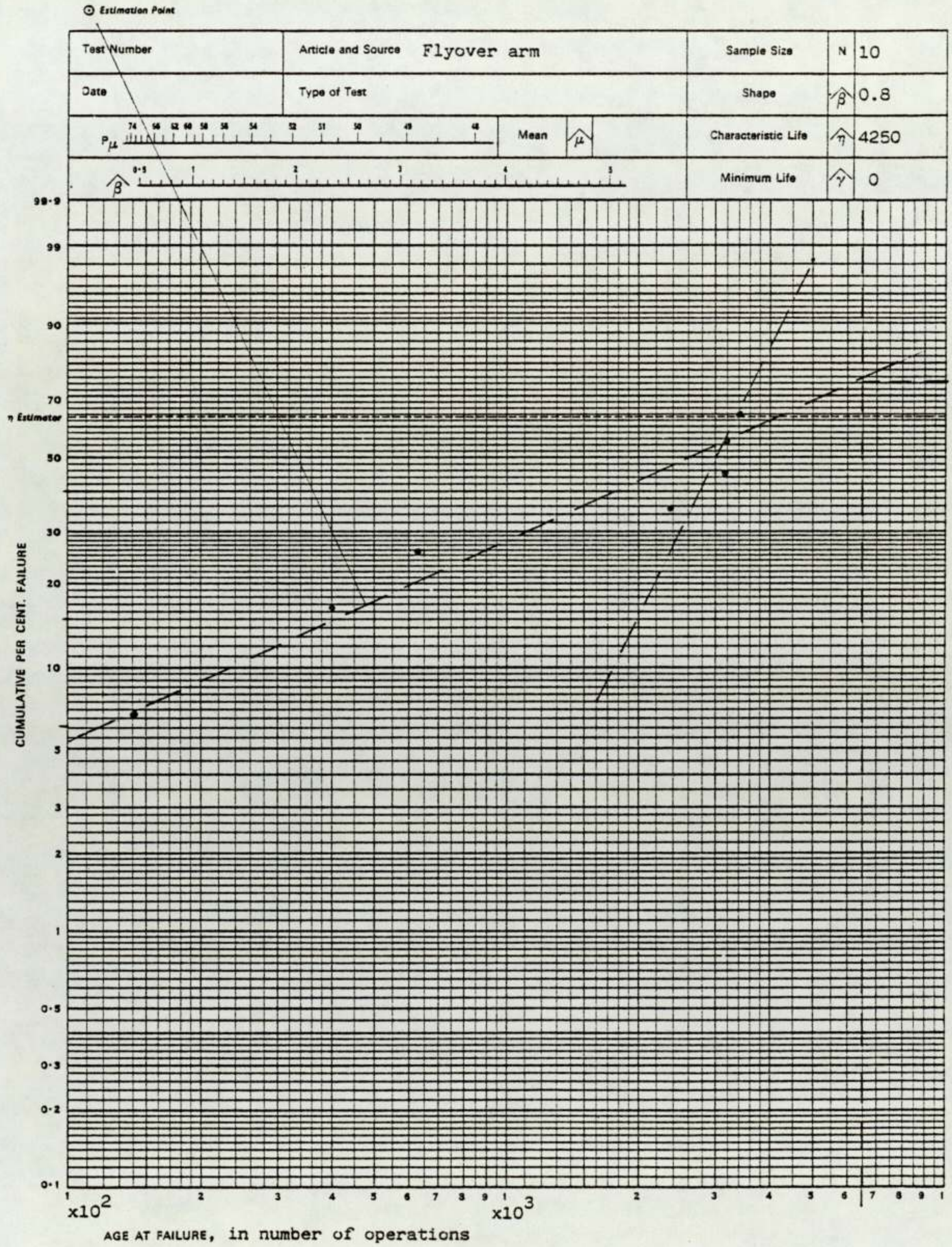
Table 12.

We found the key to the problem in the truncation data. Because the test had been run on to 6,500 operations, the next failure could not occur until after that. Since it would have been the eighth failure in 10, it would, using Bernard, be plotted at 74.0 cumulative per cent of population failed. Hence it must lie on line MN, somewhere to the right of M, and in the worst case would be almost on top of it. But M is almost on line AB produced. It is a long way from CD produced, and, the longer the 8th failure took to occur, the further away it would be. Reluctantly we concluded that line AB was correct and CD spurious.

In this case it was possible to test our theory, because the three surviving items could be returned to life test, without any great risk that the interruption would affect results. We were right, since the remaining failures occurred as follows.

Failure No.	Number of operations at failure
8	6985
9	9554

The 10th item survived 10,000 operations, when the test was again truncated.



Estimation Point
Test Number
Article and Source Flyover arm
Sample Size
N 10
Date
Type of Test
Shape
0.8
Mean
Characteristic Life
4250
Minimum Life
0
η Estimator
CUMULATIVE PER CENT. FAILURE
Weibull Probability x Log 2 Cycles
Graph Data Ref. 6572
CHART WELL
x10^2
x10^3
AGE AT FAILURE, in number of operations

Fig.6-26.

6.14-d - The assumption of a constant failure rate

From time to time there has been discussion about the validity of the practice, especially in electronics, of assuming a constant failure rate. In the strict statistical sense, this implies that the probability of failure is rigidly constant, and in no way influenced by the age of the items. This in turn suggests that the mode and mechanism of failure are exactly the same every time.

Experience shows that this is unlikely to be strictly correct. It is true that Weibull β values around unity are often observed, although the best-fit β may have a value of 1.1 or 1.2. The mechanism of failure, too, is seldom rigidly constant, and sometimes the mode varies as well. Having as far as possible eliminated all assignable causes of failure, we are often left with a loose scatter of miscellaneous failures which, particularly if data are scarce, we can only interpret as roughly constant with time. Usually there is no great risk in this, so long as the limitations of the data and of our analysis of them are borne in mind. An assumed constant failure rate does make reliability prediction much easier.

There is another danger inherent in using published failure-rate data, and this comes from the fact that our use conditions may be quite different from those under which the failure data were prepared. Anyone who doubts this should calculate the overall failure rate of any piece of equipment in which he or she is interested, using data from two or more sources. One prediction may exceed another by a ratio of more than two to one. If we are concerned only with comparing two alternative designs, this may not matter, but, if we need an absolute prediction of the failure rate, it is very serious.

6.14-e - Guidelines for predicting the reliability of parts from small samples

From the above it has been possible to draw up a set of guidelines which, although of general use, are particularly valuable when it is only possible to life-test a small sample.

1. Use median ranks (i.e. Bernard's formula) to estimate the cumulative percentage of the population failed.
2. Beware of rejecting apparently spurious points, when they represent a significant part of the available data. (Rejecting two "inconvenient" points out of, say, seven probably amounts to forcing the data to fit preconceived ideas!).

   Consider:

   a) Is there evidence that the failure mode might have been different when the spurious failure occurred?

   b) If points appear on the Weibull or other plot where we did not expect them, look to see whether they are also missing from a position where we did expect them. If so, are there valid reasons why they have turned up in the "wrong" place?

   c) Are there sound statistical or other reasons why the data concerned must be spurious? For example, did the tester admit to an error, or the test rig fail? Be careful, however, about accepting doubtful reasons, because we may be merely forcing the data to fit preconceived notions.

   d) When a test is truncated, the earliest time and percentage at which the next failure can occur should be inserted on the Weibull plot, as M in fig. 6-26. The correct line will pass below this point (or, in the limit, through it), but, on the available data, it cannot pass above it.

6.15 - Slope, estimates and confidence intervals

Before explaining the method of obtaining "confidence limits" for a Weibull analysis of test data, it is necessary to present a definition of confidence in general terms, and to discuss the various aspects which affect it.

In all statistics, when an analysis is carried out and some result obtained, it is necessary to qualify this result by stating how sure we are that the result is a true one, i.e. how confident we are.

Several factors may affect the level of confidence given to a result, depending upon the type of analysis carried out and the type of result required. Thus each statistical exercise must be studied to establish the types of errors which may occur and their cause.

In the case of Weibull analysis of random sample testing, for instance, we may make the following statements for this type of analysis. Firstly that the data obtained from a sample test is accurate for that particular sample. (This must be true since providing test conditions are correct, the recorded failure ages cannot be disputed). Secondly that manipulation of this test data may be considered accurate and in accordance with established theory. However, that the test data, although accurate, may <u>not</u> in itself be truly representative of the parent population from which it was drawn.

Nothing has been said about the use of confidence limits. These are valuable with small samples, if only to deflate any excessive personal confidence we may have in our results! The critical factor is usually the slope of the Weibull line denoted by β because, the sample being small, it will probably be necessary to extrapolate the line back to shorter working-life times and higher reliability. This magnifies any error in prediction arising from incorrect estimation of the slope, as fig. 6-27 illustrates. Here we have redrawn the Weibull line shown in fig. 6-19, and added confidence limits at a 90 per cent level of confidence. On the basis of the 10 observations available to us, WW represents the best estimate of the position of the true Weibull line. However, our sample was small, and if we were able to test another set of 10 items, it is unlikely that exactly the same prediction would result. We should then have another "best estimate", and so on for every repeat. Now the confidence limits mark the boundaries of the area within which we can be 90 per cent certain that the true Weibull line lies. We hope that the true line is not too different from our prediction, but we cannot be sure. Thus lines AB and CD show two extreme possibilities which still lie just within the confidence limits. Suppose the required working life for these items is 200 h. Our Weibull line WW predicts a reliability of (100 - 1.2) = 98.8 per cent, whereas AB predicts (100 - 7.2)=92.8 per cent, and CD something better than (100 - 0.1) = 99.9 per cent. Hence anywhere within the range 92.8 to 99.9 per cent could turn out to be correct. Indeed there is a 10 per cent risk that the true value is outside the confidence limits altogether!

Just how representative a sample may be considered to be, is dependent upon certain theories of probability, stemming from the sample size. A sample of ten units could be used to estimate the failure distribution but, actually, two points determine the parameters of the failure distribution if t_o is set equal to zero.

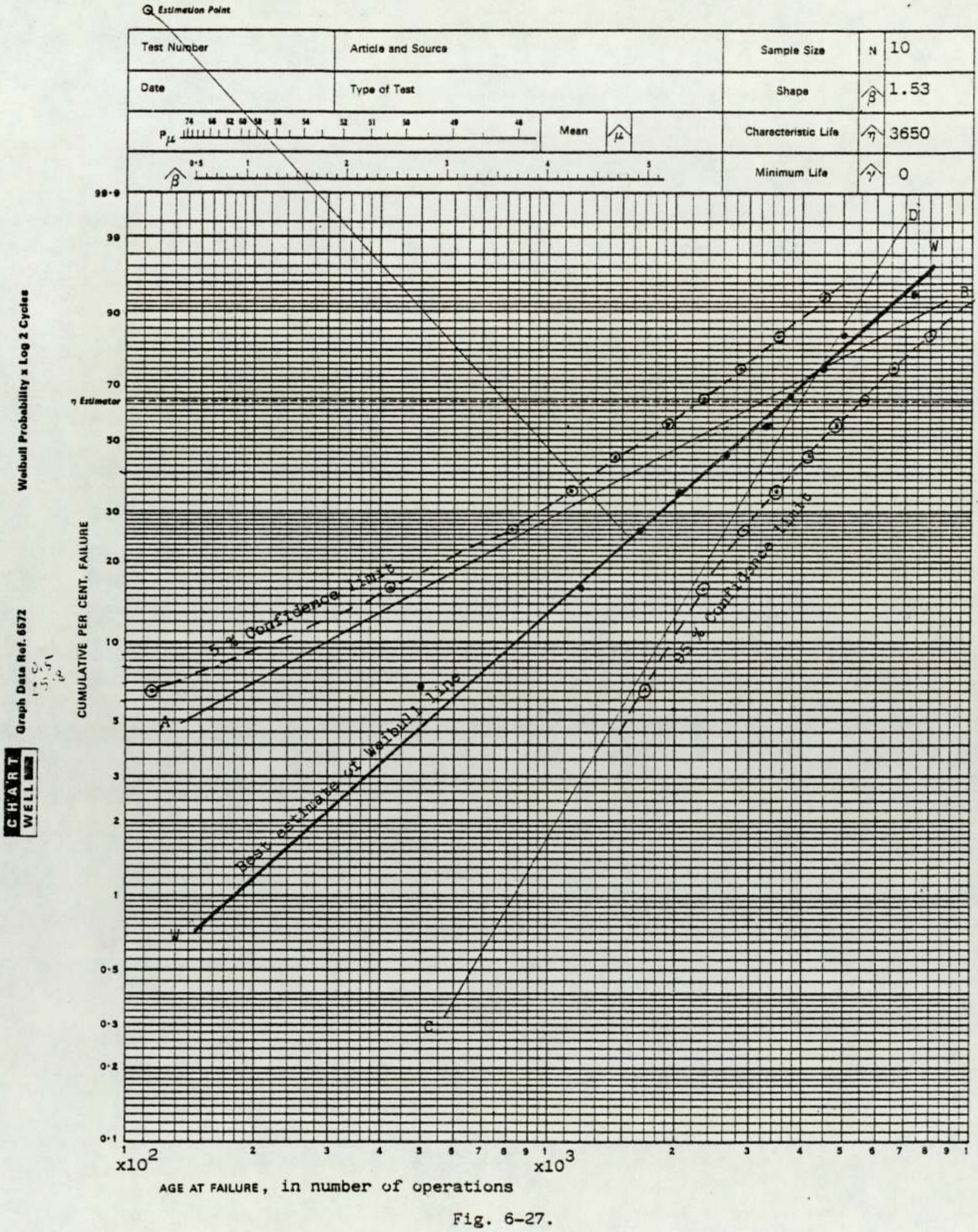
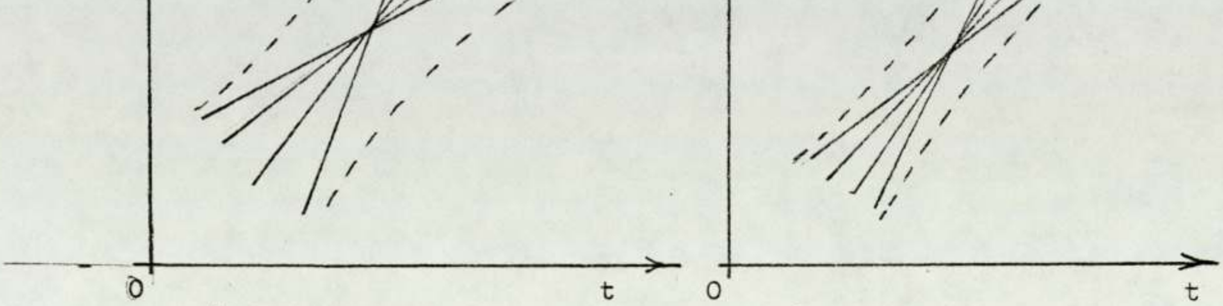

Estimation Point
Test Number
Article and Source
Sample Size
N 10
Date
Type of Test
Shape
1.53
Mean
Characteristic Life
3650
Minimum Life
0
η Estimator
Weibull Probability x Log 2 Cycles
Graph Data Ref. 6572
CHART WELL
CUMULATIVE PER CENT. FAILURE
5 % Confidence limit
Best estimate of Weibull line
95 % Confidence limit
A
B
C
D
W
x10^2
x10^3
AGE AT FAILURE, in number of operations

Fig. 6-27.

With a small sample, sampling error is larger than with a large sample. Thus, if a sample size is large, the probability of it being representative is high, and therefore a high level of confidence may be given to the results drawn from a statistical analysis.

In the Weibull analysis it has been shown that Median Ranks are employed to relate a sample of items to the parent population and that these Median Ranks allow positive errors to balance out with negative errors. Hence it is a question of selecting a median point, or balance point, in the estimate and thus the line which results on Weibull Probability Paper may be considered as a 50% confidence line.

At first sight this does not seem very encouraging but it can be shown that superimposing lines known as confidence limits, as shown in fig. 6-28 on page 134 effectively "widens" the 50% line to cover a larger area and hence increases our confidence level significantly.



a) Small number tested   b) Large number tested

Fig. 6-28 - Slope estimates and confidence intervals

The dashed lines of the funnel-shaped curves in fig. 6-28 on page 134 show the relative error associated with smaller and larger sample sizes. The true Weibull line can be expected to be anywhere between the dashed lines. In statistical terms, the dashed lines are referred to as confidence bands. Since the estimates of the slope, position and curvature of the fitted line do vary from sample to sample, some statement describing the degree of precision should be made. Confidence bands provide a method of describing this precision. The width of the confidence band is determined by (1) sample size, (2) the fitted Weibull slope, (3) selected confidence level and selected reliability level.

Confidence intervals are measurements of the precision when estimating a statistic. A ninety per cent confidence interval around an observed statistic is that proportion of such intervals which in the long run will contain the true value of the statistic. In the case of the 3-parameter Weibull distribution, a rigorous determination of such confidence boundaries would involve taking into account simultaneously the effects of the separate sampling errors of estimating β, η, and t_o.

The 5% and 95% ranks are often used to estimate the statistical error associated with the slope β, the characteristic life η, the B10 etc., life, and the minimum life t_o.

Thus if the confidence bands shown in fig. 6-28[a)] on page 134 are taken as being those for 95% confidence, then the space between these bands, known as the "confidence interval", would contain 95% of the Weibull lines produced, if the total population was tested as a series of samples. In other words we are 95% sure that the true Weibull line for the entire population lies somewhere within the confidence interval.

The width of this interval is affected by several factors, which may be summarised as follows; the selected confidence and reliability levels; the fitted Weibull slope; the sample size tested.

The first two factors usually tend to be preselected and hence are difficult to vary. However, the sample size may be increased easily (depending on economic considerations) and thus the confidence interval narrowed as much as desired. This is demonstrated well in fig. 6-28 a) and b) where two samples of different size would have been drawn from the same parent population and would have been fitted with confidence bands of the same level.

It can be shown that confidence bands may be very useful in reducing test time and cost since they may be used in circumstances which would normally require 100% testing to ensure the required quality and reliability was obtained.

For instance, suppose that emission control regulations call for certain standards to be achieved on all cars to be sold in a certain market. Assume that the requirements are as follows: It is necessary to show that we are 95% confident that 90% of vehicles will emit no more than 2% CO during a certain test.

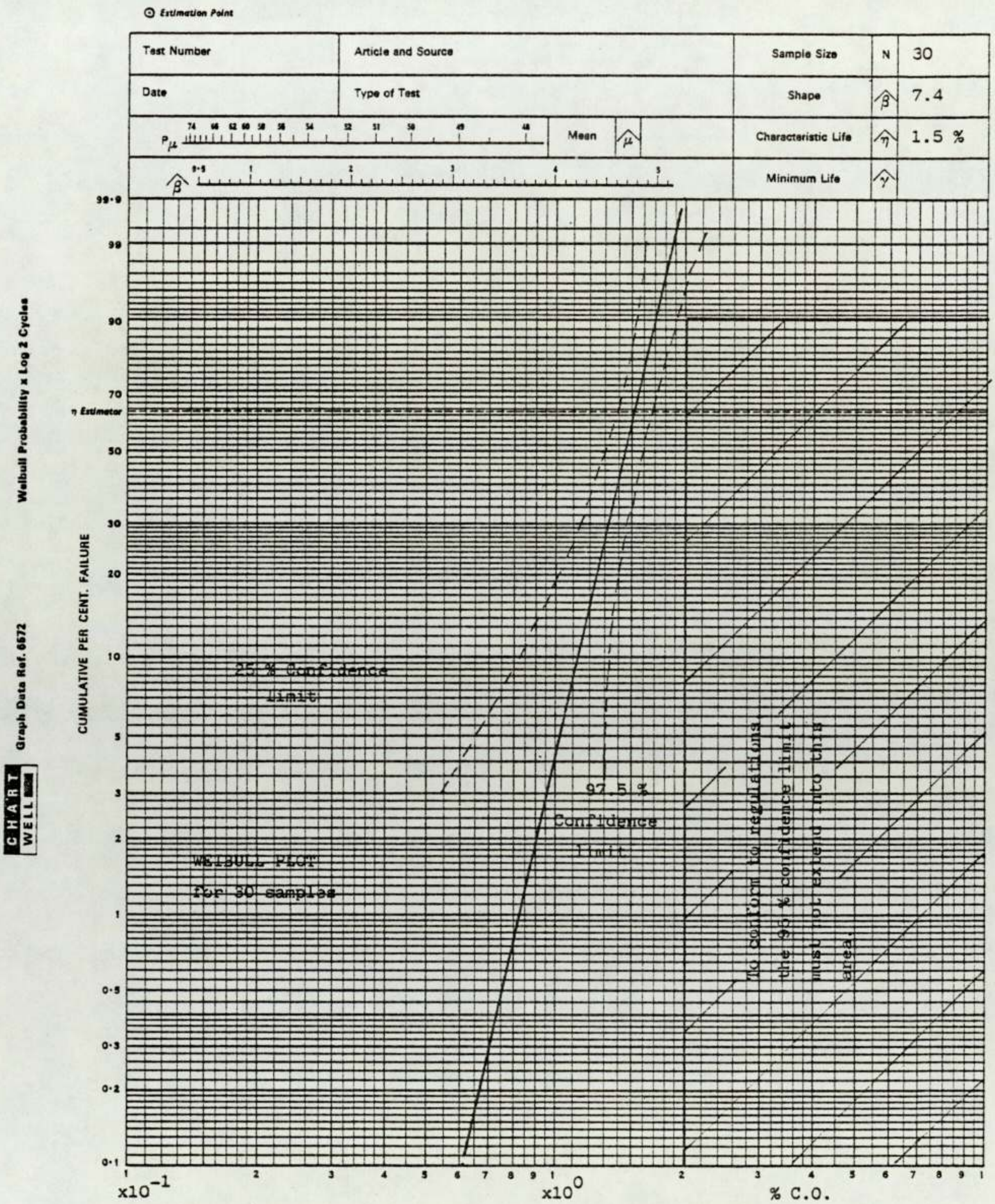
Normally it would be necessary to perform a 100% check on the vehicles and hence to find the exact figure which exceeds 2% CO. This method is obviously lengthy and expensive and in this case can be shown to be unnecessary.

The method of approach when using confidence bands is firstly to select a random sample of vehicles from the total population and to test them under the required conditions. This test information is then arranged such that it may be presented on Weibull Probability Paper, in this particular case the % CO is plotted on the x-axis and the Median Ranks for cumulative % population on the y-axis. By plotting the test data on these scales, the median, (or 50% confident) Weibull line is obtained as shown in fig. 6-29 on page 138. The area enclosed by less than 90% population and more than 2% CO, may be called the restricted area and represents two of the limits called for in the specification. Thus at this stage, since the Weibull line is well outside the restricted area, it is possible to be 50% confident that 90% of vehicles emit less than 2% CO.

In order to obtain the required confidence, the 95% confidence bands can now be fitted by the method explained later, and these bands can be studied in relation to the restricted area. If, as in fig. 6-29 on page 138, the 95% confidence interval does not enter the restricted area then it is possible to be 95% confident that over 90% of all vehicles emit less than 2% CO.

In the case where the confidence bands do enter the restricted area then two possibilities exist. Firstly, that the vehicles are not capable of meeting the requirements; hence a modification to the emission control equipment would be necessary. Secondly, that the sample of vehicles taken was not large enough to give sufficiently narrow confidence bands; hence further samples should be taken and the analysis repeated on the increased sample size. A quick check to establish which possibility exists, after the first set of samples, is to note whether or not the 50% confidence Weibull line enters the restricted area. If so, the first possibility could be accepted without fitting any confidence bands.

There are two commonly used methods for determining confidence intervals. One method uses the binomial distribution which fixes the observed failure time and places a$(1 - \alpha)$ interval about the median rank value associated with each t_i on the Weibull line.



Estimation Point
Test Number
Article and Source
Sample Size
N
30
Date
Type of Test
Shape
7.4
Mean
Characteristic Life
1.5 %
Minimum Life
Weibull Probability x Log 2 Cycles
Graph Data Ref. 6572
CHART WELL
CUMULATIVE PER CENT. FAILURE
η Estimator
25 % Confidence
limit
97.5 %
Confidence
limit
WEIBULL PLOT
for 30 samples
To conform to regulations
the 90 % confidence limit
must not extend into this
area.
x10^-1
x10^0
% C.O.

Fig. 6-29.

However, this method has a serious drawback in applied statistics. The lower confidence interval does not extend far enough to bound the estimated failure time for the first failure.

Engineering people are usually interested in a confidence interval for the time associated with the first failure. A method for establishing this interval was developed by Mr. Leonard Johnson of the General Motors Research Laboratories in 1959. His method employs the transformed binomial distribution rather than fixing the observed failure time to bound the high and low failure times for each failure. The procedure works well and gives useful answers. In addition, the accuracy of the results has been confirmed by a Monte Carlo computer simulation study. The computer study was performed by Mr. Lloyd Schlitzer of Pratt and Whitney Aircraft in 1966. His study showed that Johnson's method gives more conservative results. A comparison of confidence bands obtained by these methods is shown in fig. 6-30 on page 140.

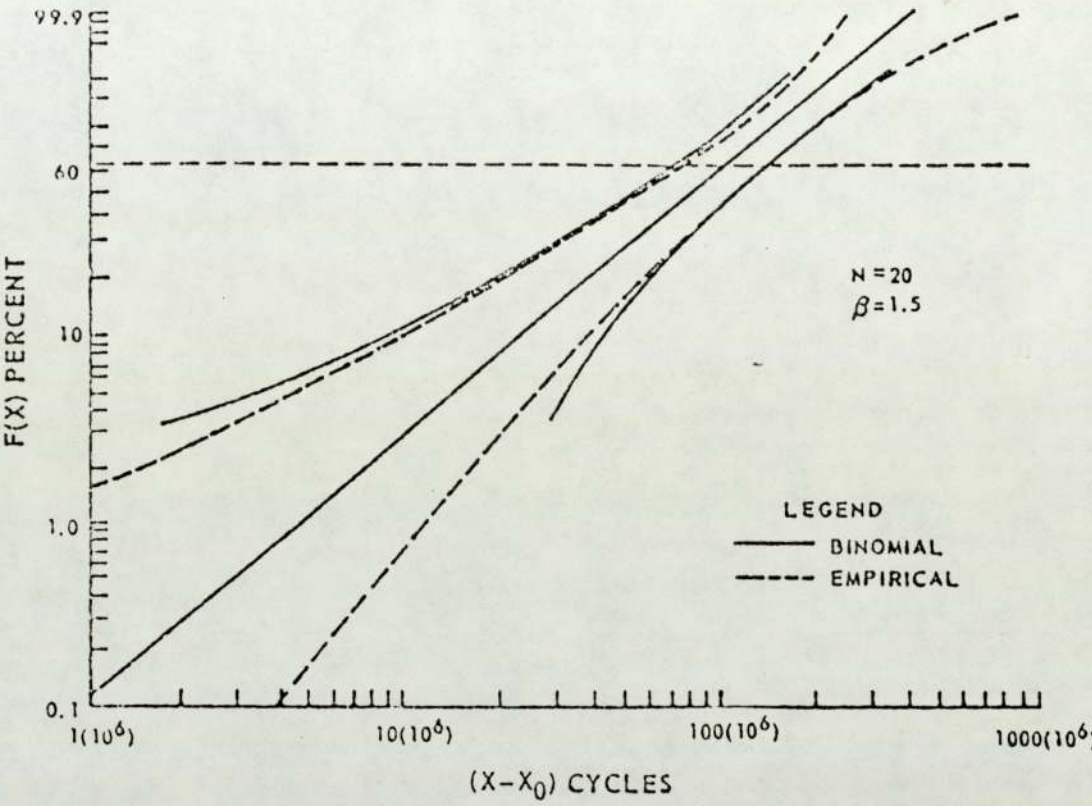


Fig. 6-30 Typical 90% Confidence Band. Determined Empirically and from Binomial Probability Function.
(Reproduced from the book by R. A. Mitchell, Ref. 4).

6.15-a - Fitting Confidence Bands (L. Johnson)

The method devised by Mr. Leonard Johnson of obtaining confidence bands is considered the most useful since unlike other methods, it is capable of giving the confidence interval for the time associated with the first failure. Although it is based on the rather lengthy theory of binomial expansion for each failure point, it may be reduced to a simple graphical construction by the use of standard ranking tables for the level of confidence required. These tables exist for most commonly used confidence levels, (e.g. 90%, 95% and 98%) but tables may be produced, by a simple computer programme, for any confidence level.

Johnson's procedure for fitting confidence bands to a Weibull line may be summarised as follows:

1 - Denote by $(1 - \alpha)$ the required confidence level, where is the risk of accepting an invalid statistic.

2 - Calculate the upper and lower confidence limits thus: $(1 - \frac{\alpha}{2})$ is the upper confidence limit and $\frac{\alpha}{2}$ is the lower confidence limit. Therefore a 90% confidence interval gives $\alpha = 0.10$; the upper limit is $(1 - \frac{0.10}{2}) = 0.95$ or 95%, and the lower limit is $\frac{0.10}{2} = 0.05$ or 5%. The confidence interval lies between these two limits.

3 - Determine the 5% and 95% ranks for all failure points.

To find the ith five per cent or ninety five per cent ranks, expand the binomial

$$(R+F)^N = [(1-F)+F]^N = \binom{N}{0}(1-F)^N + \binom{N}{1}F(1-F)^{N-1} + \binom{N}{2}F^2(1-F)^{N-2} + \ldots + \binom{N}{i-1}(1-F)^{N-i+1}$$

to i terms, equate it to 0.95 or to 0.05, and then solve for F. Repeat for each of the N_o failures in a sample of N. Notice that i can have a fractional value if there are suspensions, whereas this formula can be used only for integer i 's. Thus interpolation may be necessary. This binomial expansion is calculated by the computer programmes described in Chapter 10 on page 214 .

Alternatively, obtain the 5% and 95% ranks from the standard Appendices B and C on pages 226 and 230 respectively, for the particular sample size of N used and for integer values of i. The correct values of the 5% and 95% ranks can then be determined by interpolating linearly for the new fractional rank order numbers as we did for the median ranks (see Table 7 on page 93). These interpolated values of the 5% and 95% ranks are shown in Table 13 on page 143 .

Order No.	$(t-t_o)$ $x10^5$	Rank Order No.	0.05 Ranks % *	Median Ranks %	0.95 Ranks % *
1	0.02	1.05	$0.2+\frac{(1.8-0.2)(1.05-1)}{2-1}=0.28$	3.640	$13.9+\frac{(21.6-13.9)(1.05-1)}{2-1}=14.285$
2	0.09	2.10	$1.8+\frac{(4.2-1.8)(2.10-2)}{3-2}=2.04$	8.690	$21.6+\frac{(28.2-21.6)(2.10-2)}{3-2}=22.26$
3	0.54	3.15	$4.2+\frac{(7.1-4.2)(3.15-3)}{4-3}=4.635$	13.835	$28.2+\frac{(34.3-28.2)(3.15-3)}{4-3}=29.12$
4	0.86	4.20	$7.1+\frac{(10.4-7.1)(4.20-4)}{5-4}=7.760$	18.980	$34.3+\frac{(40.1-34.3)(9.20-4)}{5-4}=35.46$
5	1.38	5.25	$10.4+\frac{(13.9-10.4)(5.25-5)}{6-5}=11.275$	24.125	$40.1+\frac{(48.5-40.1)(5.25-5)}{6-5}=41.45$
6	1.47	6.30	$13.9+\frac{(17.7-13.9)(6.30-6)}{7-6}=15.040$	29.270	$45.5+\frac{(50.7-45.5)(6.30-6)}{7-6}=47.06$
7	2.61	7.35	$17.7+\frac{(21.7-17.7)(7.35-7)}{8-7}=19.100$	34.450	$50.7+\frac{(55.8-50.7)(7.35-7)}{8-7}=52.49$
8	4.85	8.40	$21.7+\frac{(25.8-21.7)(8.40-8)}{9-8}=23.340$	39.660	$55.8+\frac{(60.6-55.8)(8.40-8)}{9-8}=57.72$
9	6.64	9.66	$25.8+\frac{(30.1-25.8)(9.66-9)}{10-9}=28.638$	45.834	$60.6+\frac{(65.3-60.6)(9.66-9)}{10-9}=63.70$
10	9.80	11.28	$34.6+\frac{(39.3-34.6)(11.28-11)}{11-10}=35.916$	53.772	$65.3+\frac{(69.8-65.3)(11.28-10)}{11-10}=71.00$

Table 13. C failures with corrected values of t_o ($t_o = 5.3 \times 10^5$ cycles) and 90% confidence interval.

* Interpolated values from Appendices B & C on pages 226 and 230 respectively.

6.15-a-I - Analytical determination of confidence limits

4 - $t_i(0.05)$, the lower confidence limit for the i-th failure, is determined by solving the following Weibull equation for $t_i(0.05)$:

$$F_i(0.05)=1 - e^{-\left[\frac{t_i(0.05)}{\eta}\right]}\text{, which gives:}$$

$$t_i(0.05)=\eta \quad n\left[\frac{1}{1-F_i\ (0.05)}\right]^{1/\beta}.$$

5 - $t_i(0.95)$, the upper confidence limit for the i-th failure, is similarly determined:

$$t_i(0.95) =\eta \ n\left[\frac{1}{1-F_i\ (0.95)}\right]^{1/\beta}.$$

6 - The failure times t_i associated with the 5% and 95% ranks are calculated for all i failures (i=1, 2,...,n).

7 - A horizontal line is drawn through each failure plotted on the Weibull probability paper.

8 - The failure times associated with the 5% and 95% ranks are plotted for each t_i on the respective horizontal line determined from step 7.

The results are summarised in Table 14 on page 145. Comparing the graphical solution in fig. 6-31 on page 147 with the results obtained analytically, we see the answers are the same.

Order No.	Rank Order	$(t-t_o)$ $\times 10^5$ cycles	Median Rank	5% Ranks %	5% confidence limits $\eta = 18.8\times10^5$; $\beta=0.48$; $1/\beta = 2.08333$	0.95 Ranks %	95% confidence limits (in cycles)
1	1.05	0.02	3.64	0.28	$\eta \ln\left[\frac{1}{1-0.0028}\right]^{1/\beta} = 0.000090574356 \times 10^5$	14.285	$\eta \ln\left\vert\frac{1}{1-0.14285}\right\vert^{1/\beta} = 0.38223338 \times 10^5$
2	2.10	0.09	8.69	2.04	" $= 0.005779126584 \times 10^5$	22.26	$= 1.06257162 \times 10^5$
3	3.15	0.54	13.835	4.635	" $= 0.0328457871 \times 10^5$	29.12	$= 2.037665318 \times 10^5$
4	4.20	0.86	18.98	7.760	" $= 0.09946406481 \times 10^5$	35.46	$= 3.365048468 \times 10^5$
5	5.25	1.38	24.125	11.275	" $= 0.225$	41.45	$= 5.113$
6	6.30	1.47	29.270	15.040	" $= 0.429$	47.06	$= 7.323$
7	7.35	2.61	34.450	19.100	" $= 0.742$	52.49	$= 10.1597$
8	8.40	4.85	39.660	23.340	" $= 1.189$	57.72	$= 13.759$
9	9.66	6.64	45.834	28.638	" $= 1.954$	63.70	$= 19.327$
10	11.28	9.80	53.772	35.916	$\eta \ln\left\vert\frac{1}{1-0.35916}\right\vert^{1/\beta} = 3.4795\times10^5$	71.00	$\eta \ln\left\vert\frac{1}{1-0.71}\right\vert^{1/\beta} = 29.324730 \times 10^5$

Table 14. Analytical determination of confidence limits

6.15-a-II - Graphical determination of the confidence limits

Horizontal lines are drawn through each of the plotted points on the Weibull line.

Using the values obtained for the particular sample size N used, employing one of the methods explained in section 6.15-a on page 141, the first rank value for the lower confidence limit (5% in our case) is taken and projected horizontally from the y-axis to intersect the Weibull line. From this intersection point a vertical line is projected upwards until the horizontal line (median rank) for the first failure is reached. This intersection gives the first point on the lower confidence band.

Now the first rank value for the upper confidence limit (5% in our case) is taken and projected horizontally from the y-axis to intersect the Weibull line. From this intersection point a vertical line is projected downwards until the horizontal line (median rank) for the first failure is reached. This intersection gives the first point on the upper confidence band.

This procedure is repeated with a second rank from Table 13 on page 143 and with the horizontal line through the second failure, obtaining a second point on the lower confidence band and a second point on the upper confidence band. By doing this for the full number in the sample, two series of points are obtained through which the lower and upper confidence bands may be drawn.

The graphical determination of the confidence limits is explained in fig. 6-31 on page 147 for the second failure of Table 14 on page 145.

Rank order number	$(t-t_o)x10^5$	0.05 Rank	Median Rank	0.95 Rank
2.1	0.09	2.04	8.69	22.26

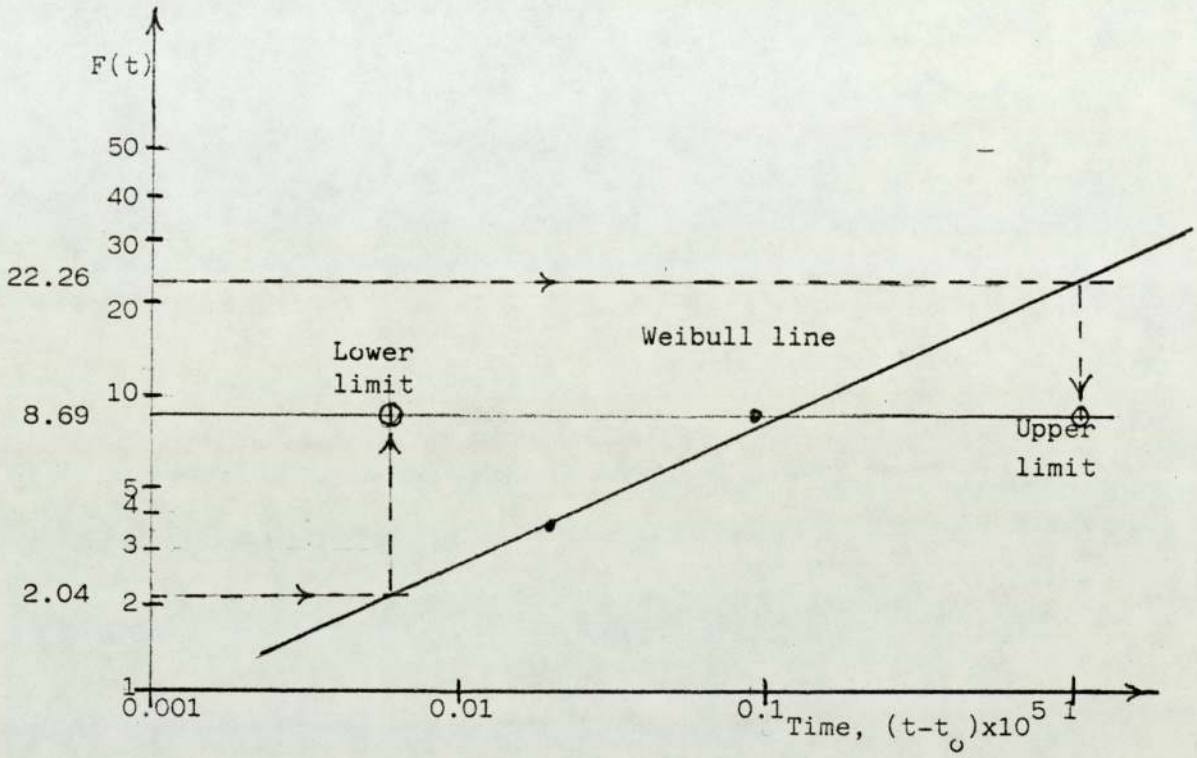


Fig. 6-31 Confidence Band Determination

The same procedure is applied to all failure points on the Weibull line. Two lines are drawn, one through the lower limit points and one through the upper limit points. These two lines define the lower and upper confidence intervals.

For 90% confidence and a sample size of 20, the 5% and 95% confidence limits thus obtained graphically are very similar to those given by Table 14 on page 145 and the location of the points for both 5% and 95% confidence bands, is demonstrated in fig. 6-32 on page 148.

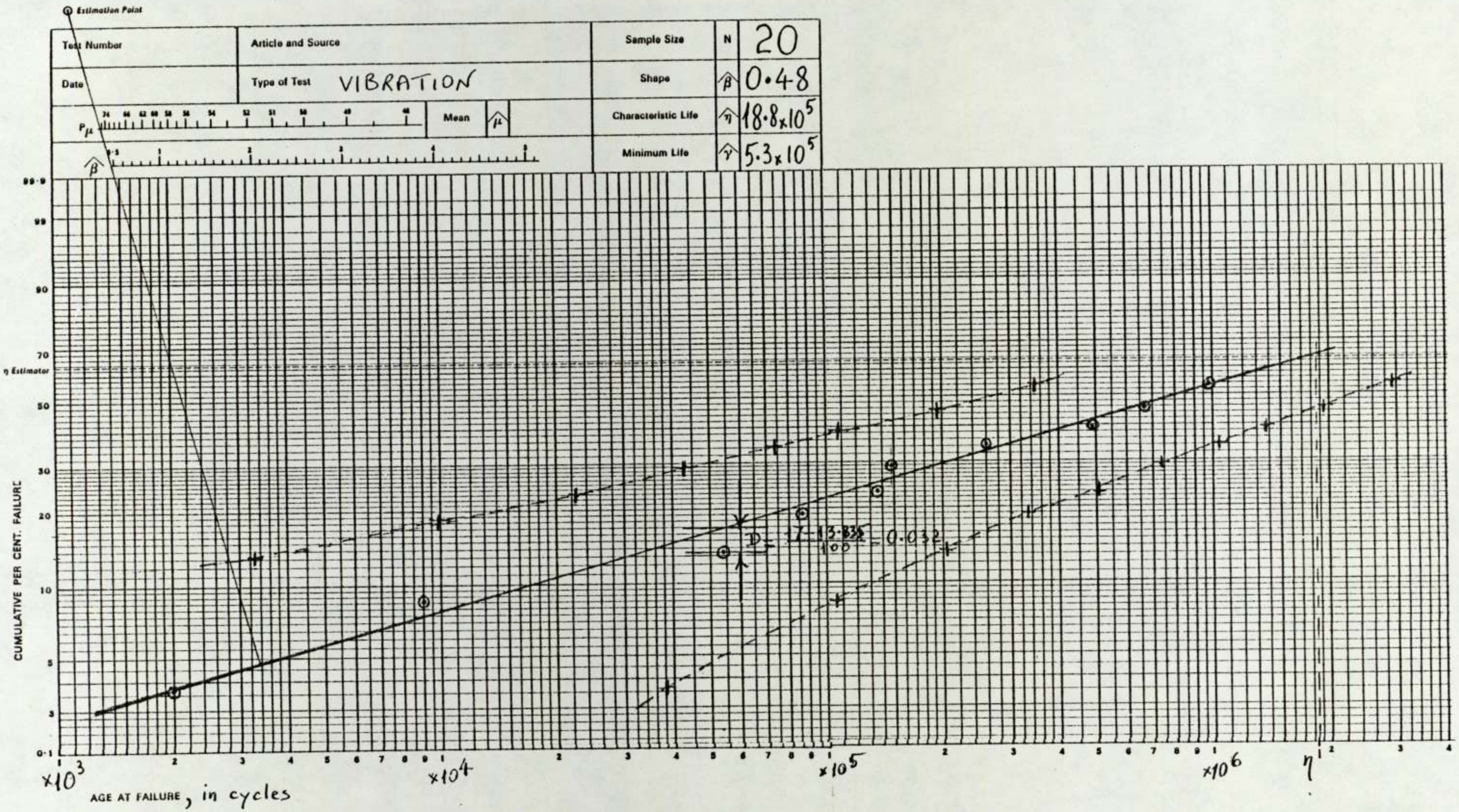


Fig. 6-32. Weibull plot of C-failures with corrected value of t_o.

6.16 - Separation of the B failures

The procedures described in section 6.9 on page 90 were repeated for the B failures:

The results are shown in Table 15 on page 150 and in fig. 6-33 on page 151.

Original Order No.	New Order No.	Cycles to failure x 10^5	New Rank Order Number	New Median Ranks	Failure Mode
-1	1	4.6	1	3.4	1B
10	2	10.53	$1+\frac{(20+1)-1}{1+11}=2.\dot{6}$	$(13.1-8.2)0.667+8.2=11.4\dot{6}$	1B
11	3	11.78	$2.6667+\frac{(20+1)-2.6667}{1+10}=4.\bar{3}$	$(22.9-18.0)0.833+18=19.6\bar{3}$	2B
13	4	12.13	$4.33+\frac{(20+1)-4.333}{1+8}=6.\overline{185}$	$(32.7-27.8)0.\overline{185}+27.8=28.71$	2B
14	5	13.65	$6.1851+\frac{(20+1)-6.1851}{1+7}=8.04$	$(42.6-37.7)0.037+37.7=37.88$	2B
16	6	15.51	$8.03696+\frac{(20+1)-8.03696}{1+5}=10.20$	$(52.4-47.5)0.1975+47.5=48.47$	2B
17	7	18.03	$10.1975+\frac{(20+1)-10.1975}{1+4}=12.36$	$(62.2-57.3)0.358+57.3=59.05$	2B
18	8	18.08	$12.358+\frac{(20+1)-12.358}{1+3}=14.52$	$(72.1-67.2)0.5185+67.2=69.74$	2B
19	9	18.96	$14.5185+\frac{(20+1)-14.5185}{1+2}=16.68$	$(81.9-77)0.679+77=80.33$	1B
20	10	18.98	$16.679+\frac{(20+1)-16.679}{1+1}=18.84$	$(91.7-86.8)+86.8=90.91$	2B

Table 15. Separation of B failures.

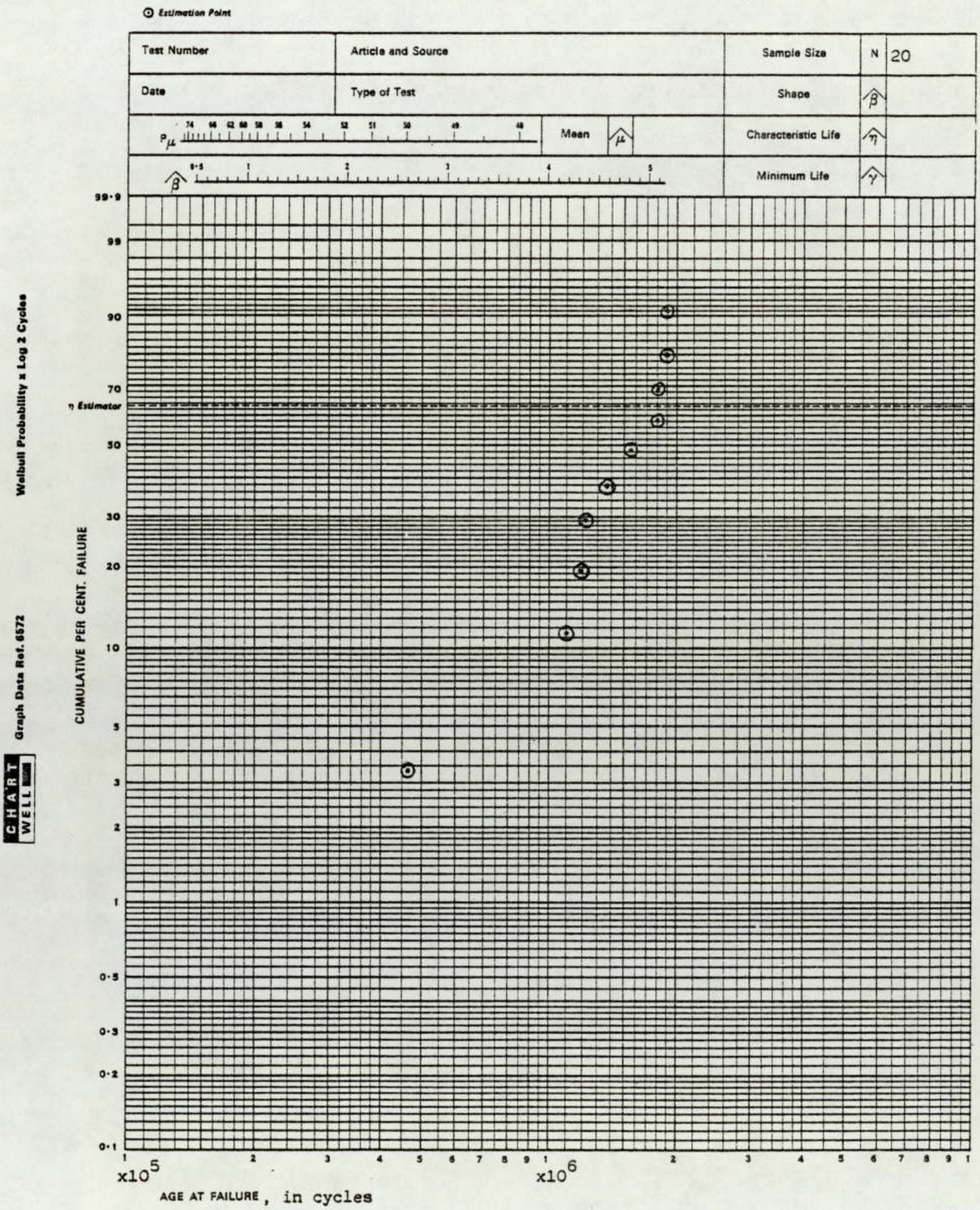

Estimation Point
Test Number
Article and Source
Sample Size
N 20
Date
Type of Test
Shape
Mean
Characteristic Life
Minimum Life
η Estimator
CUMULATIVE PER CENT. FAILURE
Weibull Probability x Log 2 Cycles
Graph Data Ref. 6572
CHART WELL
x10^5
x10^6
AGE AT FAILURE, in cycles

Fig. 6-33.

Fig. 6-33 on page 151 shows that the data do not describe either a straight or a smooth curve. This indicates more than one different mode of failures. Knowing that two types of plates were used, which we shall call 1B and 2B. The 1B failures are summarised in Tables 16 and 17 on pages 152 and 155 respectively and are shown in figures 6-34 and 6-35 on pages 153 and 157 respectively.

Old Order No.	New Order No.	Cycles to failure $tx10^5$	New Rank Order Number (n=20)	New Median Rank
1	1	4.60	1	3.4
10	2	10.53	$1+\frac{(20+1)}{1+11}=2.667$	(13.1-8.2)0.6667+8.2= 11.4668
19	3	18.96	$2.667+\frac{(20+1)-2.667}{1+2}=$ 8.778	(42.6-37.7)0.778+ 37.7=41.5112
			$8.\bar{7}+\frac{(20+1)-8.7}{1+1}=14.\bar{8}$	

Table 16 - Separation of 1B failures, with uncorrected value of t_o.

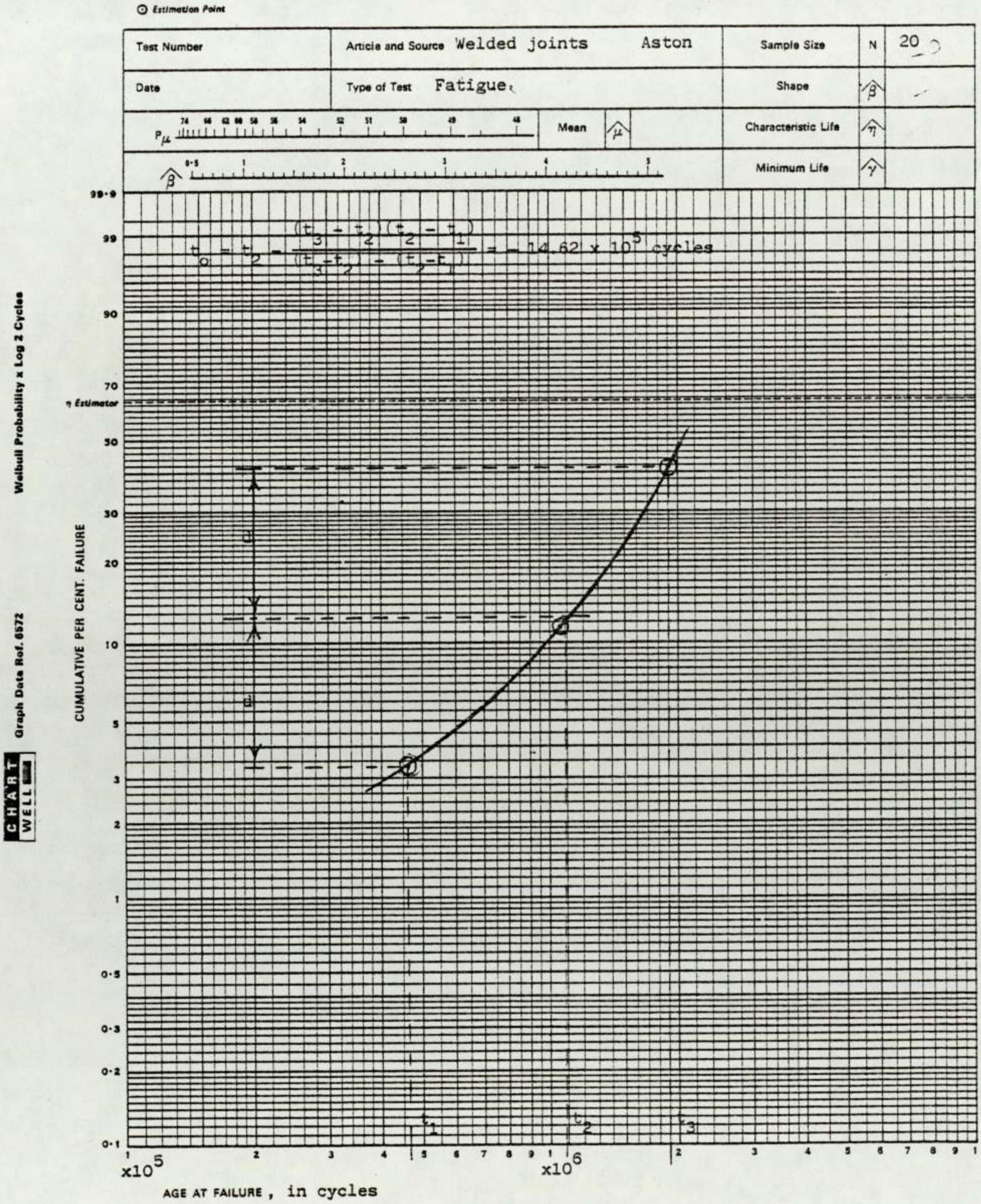


Fig. 6-34. Weibull plot of 1B-Failures with uncorrected value of t_o.

From fig. 6-34 on page 153 , $t_2=10.785x10^5$ cycles and:

$$t_o=10.785-\frac{(18.96-10.785)(10.785-4.60)}{(18.96-10.785)-(10.785-4.60)}=10.785-\frac{8.175x6.185}{8.175-6.185}=$$

$$=10.785-\frac{50.562375}{1.99}=10.785-25.408229=-14.623229x10^5 \text{ cycles.}$$

The 1B failures with corrected value of t_o are shown in Table 18 on page 156 and are displayed in fig. 6-35 on page 157.

Original Order No.	New Order No.	$(t-t_o)$ $\times 10^5$	New Rank Order No.	5% Ranks	Median Ranks F	95% Ranks	$x=\ln(t-t_o)$	$y=\ln\ln\frac{1}{1-F}$
1	1	19.22	1	0.2	0.0340	13.9	14.468877	-3.364149
10	2	25.15	$1+\frac{(20+1)-1}{1+11}=$ 2.67	(4.2-1.8)(2.67-2) +1.8=3.41	0.1147	(28.2-21.6)0.67+ 21.6=26.02	14.737783	-2.105139
19	3	33.58	$2.67+\frac{(20+1)-2.67}{1+2}=$ 8.78	(25.8-21.7)(8.78-8) +21.7=24.90	0.4151	(60.6-55.8)0.78+ 55.8=59.54	15.026856	-0.623035

$a = -74.50370807 \quad b = 4.915250228 \quad r = 0.9996596251$

$\eta = e^{-a/b} = 3{,}827273.838 = 38.\overline{27} \times 10^5$ cycles.

Table 17.

Original Order No.	Order No.	$(t-t_o)$ x10^5	New Rank Order No.	0.05 Ranks	Median Ranks	0.95 Ranks
1	1	19.22	1	0.2	3.4	13.9
10	2	25.15	$1+\frac{20+1-1}{1+11}=2.67$	(4.2-1.8)0.67+1.8=3.41	11.47	(28.2-21.6)0.67+21.6=26.02
19	3	33.58	$2.67+\frac{20+1-2.67}{1+2}=8.78$	(25.8-21.7)0.78+21.7=24.9	41.51	(60.6-55.8)0.78+55.8=59.54

Table 18 1B Failures with corrected value of t_o

(t_o = -14.62 x 10^5 cycles), and 90% confidence interval.

See also figure 6- 35

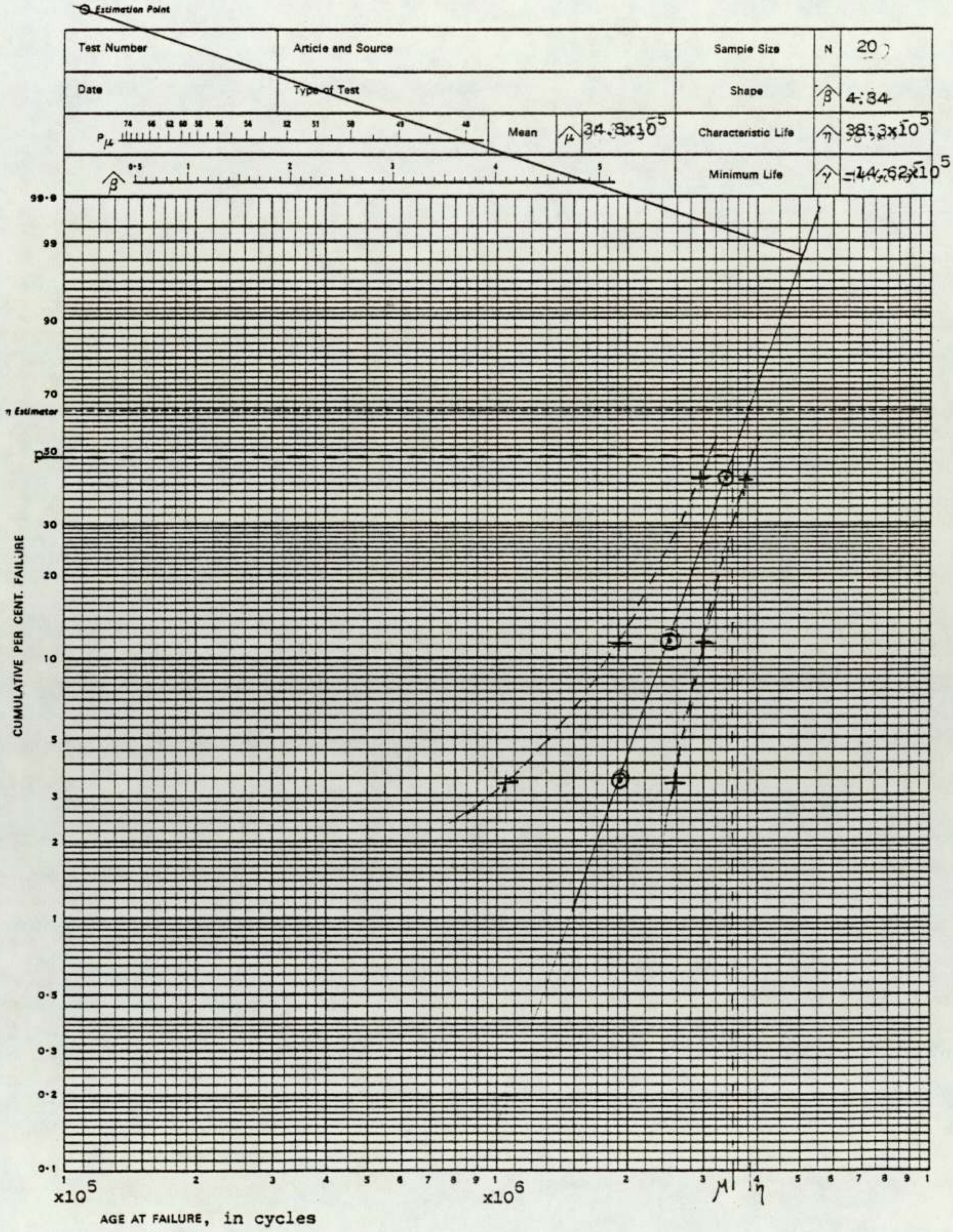


Fig. 6-35. Weibull plot of 1B-Failures with corrected values of to.

The 2B failures are summarised in the following.Tables 19 and 21 on pages 159 and 163 respectively, and are shown in figures 6-36 and 6-37 on pages 161 and 165 respectively.

Original Order No.	Order No.	Cycles to failure $t \times 10^5$	New Rank Order No.	New Median Rank
11	1	11.78	$0+\frac{20+1-0}{1+10}=0+1.909=1.91$	(8.2-3.4)0.90 = 7.76
13	2	12.13	$1.909+\frac{20+1-1.909}{1+8}=4.03$	(22.9-18)0.03+18 = 18.15
14	3	13,65	$4.03+\frac{20+1-4.03}{1+7} = 6.15$	(32.7-27.8)0.15+27.8 = 28.54
16	4	15.51	$6.151+\frac{20+1-6.151}{1+5} = 8.63$	(42.6-37.7)0.63+37.7 = 40.79
17	5	18.03	$8.626+\frac{20+1-8.626}{1+4} = 11.10$	(57.3-52.4)x0.10+52.4 = 52.89
18	6	18.08	$11.1008+\frac{20+1-11.1008}{1+3} = 13.58$	(67.2-62.2)0.58+62.2 = 65.10
20	7	18.98	$13.5756+\frac{20+1-13.5756}{1+1}=17.29$	(86.6-81.9)0.29+81.9 = 83.32

Table 19. Separation of 2B failures, with uncorrected value of t_o

$tx10^5$	F	x=lnt	$y=\ln\ln \frac{1}{1-F}$
11.78	0.0776	13.97932864	-2.516071549
12.13	0.1815	14.00860719	-1.608029494
13.65	0.2854	14.12666499	-1.090547893
15.51	0.4079	14.25441044	-0.646111432
18.03	0.5289	14.4049625	-0.284108607
18.08	0.6510	14.40773182	0.051342482
18.98	0.8332	14.45631126	0.582751671
		99.63801684	-5.510774822

a = -72.85066521 b = 5.062765174 r = 0.957665262

For $t = 11 \times 10^5$; x = 13.91082074 y = a+bx = -2.423446426

F = 0.084802768

For t 20 x 10^5 x = 14.50865774 y = 0.603261918

F = 0.839276874

Table 20.

Best fitting line from Table 19

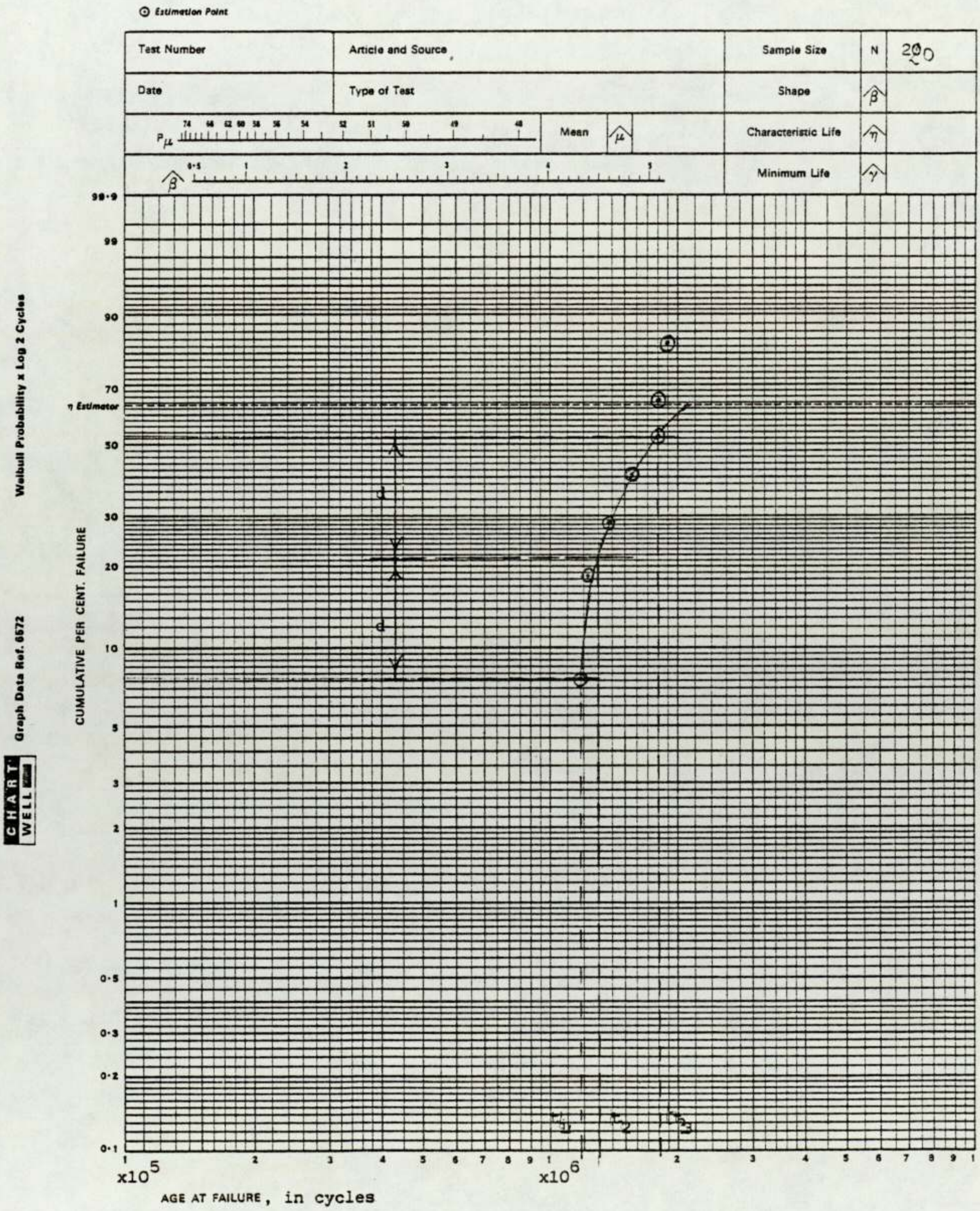


Fig. 6-36. Weibull plot of 2B-Failures with uncorrected value of t_o.

From fig. 6-36 on page 161, $t_2 = 12.8 \times 10^5$ cycles and:

$$t_o = 12.80 - \frac{(18.03-12.80)\times(12.80-11.78)}{(18.03-12.80)-(12.80-11.78)} = 12.80 - \frac{5.25\times1.02}{4.23} =$$

$$= 12.80 - 1.2659574 = 11.534 \times 10^5 \text{ cycles.}$$

The 2B failures with corrected value of t_o are shown in Table 21 on page 163 and in fig. 6-37 on page 165.

Order No.	$(t-t_o)$ $x10^5$ cycles	New Rank Order No.	0.05 Ranks	Median Rank	0.95 Ranks
1	0.25	1.91	(1.8-0.2)0.91+0.2=1.66	07.76	(21.6-13.9)0.91+13.9=20.91
2	0.60	4.03	(10.4-7.1)0.03+7.1=7.20	18.15	(40.1-34.3)0.03+34.3=34.47
3	2.12	6.15	(17.7-13.9)0.15+13.9=14.47	28.54	(50.7-45.5)0.15+45.5=46.28
4	3.98	8.63	(25.8-21.7)0.63+21.7=24.28	40.79	(60.6-55.8)0.63+55.8=58.82
5	6.50	11.10	(39.3-34.6)0.10+34.6=35.12	52.89	(74.1-69.8)0.10+69.8=70.23
6	6.55	13.58	(49.2-44.1)0.58+44.1=47.11	65.10	(82.2-78.2)0.58+78.2=80.52
7	7.45	17.29	(71.7-65.6)0.29+65.6=67.43	83.32	(95.7-92.8)0.29+92.8=93.64

Table 21. 2B Failures with corrected values of t_o

(t_o = 11.53 x 10^5 cycles), and 90% Confidence interval.

See also fig. 6-37 on page 165.

Order No.	$(t-t_o)$ $\times 10^5$ cycles	New Rank Order No.	5% Rank	Median Ranks F	95% Ranks	$x=\ln(t-t_o)$	$y=\ln\ln\frac{1}{1-F}$
1	0.25	1.91	(1.8-0.2)0.91+0.2=1.66	0.0776	(21.6-13.9)0.91+13.9=20.91	10.126631	-2.516072
2	0.60	4.03	(10.4-7.1)0.03+7.1=7.20	0.1815	(40.1-34.3)0.03+34.3=34.47	11.002100	-1.608030
3	2.12	6.15	(17.7-13.9)0.15+13.9=14.47	0.2854	(50.7-45.5)0.15+45.5=46.28	12.264342	-1.090548
4	3.98	8.63	(25.8-21.7)0.63+21.7=24.28	0.4079	(60.6-55.8)0.63+55.8=58.82	12.894207	-0.646111
5	6.50	11.10	(39.3-34.6)0.10+34.6=35.12	0.5289	(74.1-69.8)0.10+69.8=70.23	13.384728	-0.284109
6	6.55	13.58	(49.2-44.1)0.58+44.1=47.11	0.6510	(82.2-78.2)0.58+78.2=80.52	13.392391	0.051343
7	7.45	17.29	(71.7-65.6)0.29+65.6=67.43	0.8332	(95.7-92.8)0.29+92.8=93.64	13.521140	0.582752

$a = -10.19244392$ $b = 0.7603617716$ $r = 0.9625882589$

$\eta = e^{-a/b} = 6.63132 \times 10^5$ cycles

Table 22.

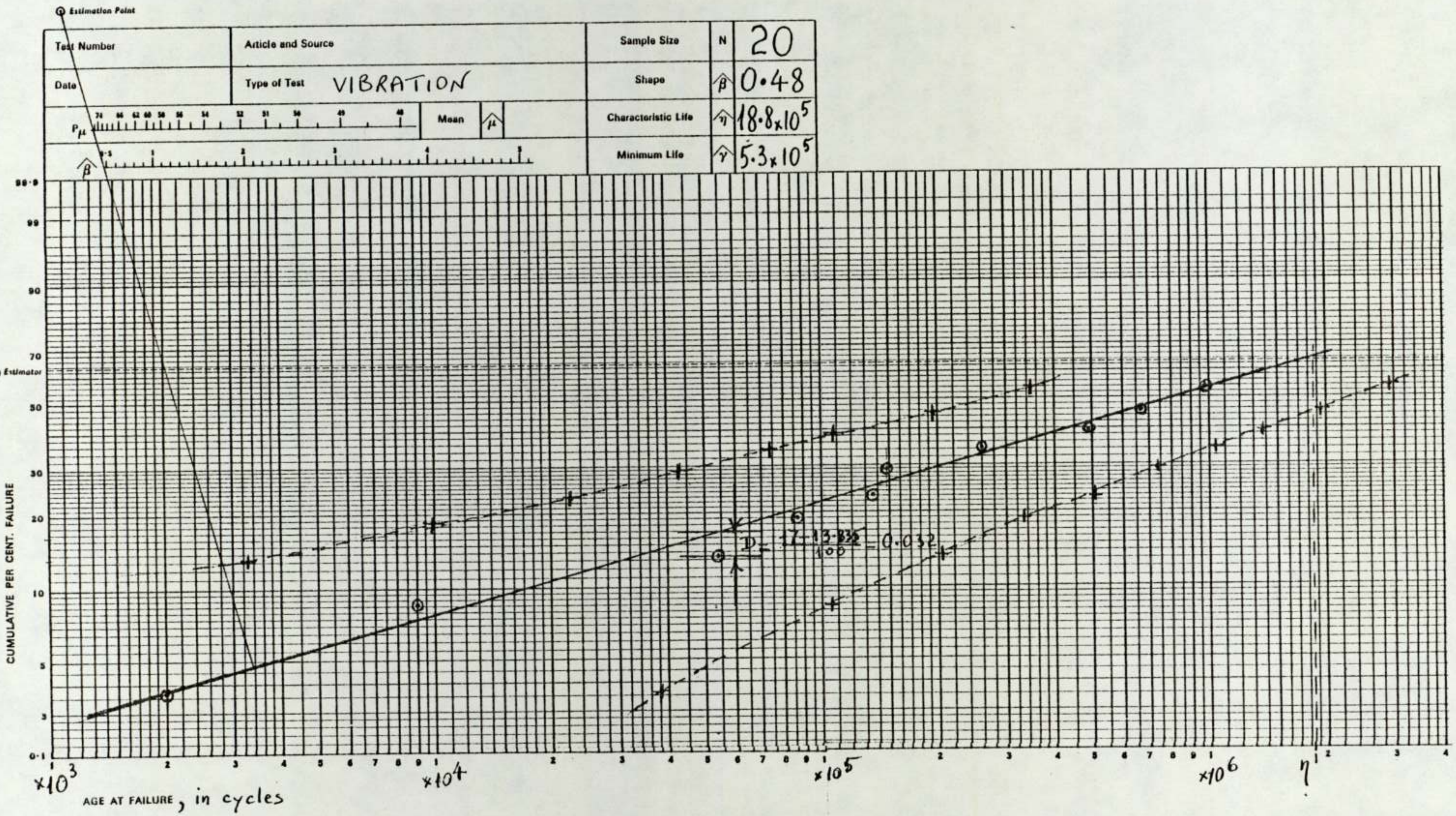


Fig. 6-37. Weibull plot of 2B-failures with corrected value of t_o

6.17 - The characteristic life, or 63.2 percentile,

6.17-a - Definition

The characteristic life η is a scaling constant, stretching the distribution along the time axis. Also, when $(t-t_o)$ is equal to η, the reliability is given by:

$R(t) = e^{-(1)} = e^{-1} = 0.3678794412$.

The constant η therefore represents the time, measured from $t_o=0$, by which 63.21206% of the population can be expected to fail, whatever value is assigned to β. For this reason it is often referred to as the "Characteristic Life".

6.17-b - Analytical calculation

By definition, η is the time $(t-t_o)$ corresponding to a Median Rank $F(t-t_o)=0.6321205588$. Since $1-F(t-t_o) = 0.3678794412 = e^{-1}$, we have that $y=\ln\ln\left[\frac{1}{1-F(t-t_o)}\right]=\ln\ln\left(\frac{1}{e^{-1}}\right)=\ln\ln(e)=\ln 1=0$.

Therefore, from $y = a+bx$, $x=\ln(t-t_o) = -\frac{a}{b}$ and $=(t-t_o)=e^{-a/b}$

For the 1B mode of failure (see page 155)

$$\eta = e^{-a/b} = 38.(27)\times 10^5 \text{ cycles .}$$

For the 2B mode of failure (see page 164)

$$\eta = e^{-a/b} = 6.63132\times 10^5 \text{ cycles .}$$

For the C mode of failure (see page 116)

$$\eta = e^{-a/b} = e\ \frac{1.396}{0.4796} = 18.36 \times 10^5 \text{cycles .}$$

6.17-c - Graphical determination

The value of the characteristic life η, and its 90% confidence interval can be estimated graphically. The estimate of the characteristic life η is that life corresponding to the intersection of the line fitted to the data and of the dashed line labelled "η estimator".

For the 1B mode of failure, fig. 6-35 on page 157

$$\eta = 38 \times 10^5 \text{ cycles} .$$

For the 2B mode of failure, fig. 6-37 on page 165

$$\eta = 6.6 \times 10^5 \text{ cycles} .$$

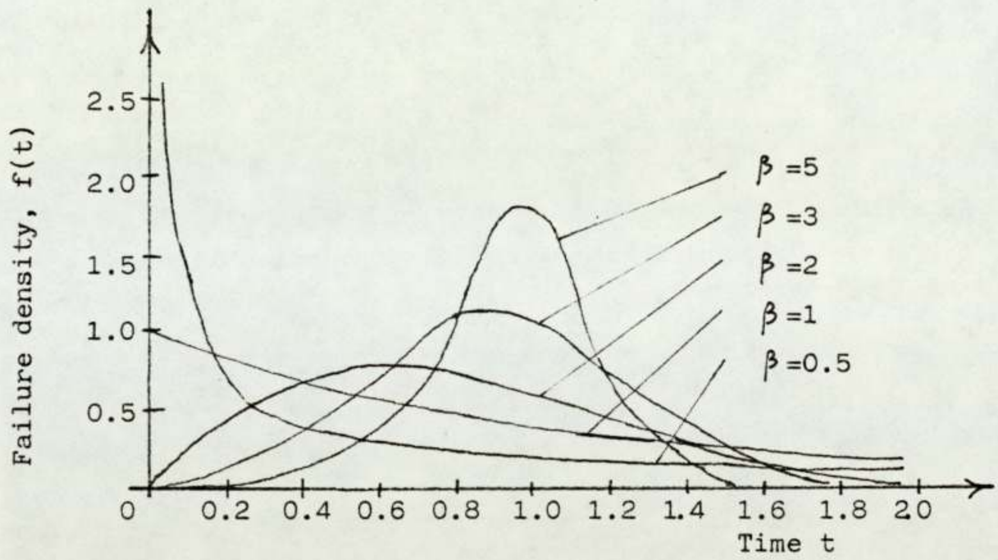
For the C mode of failure, fig. 6-32 on page 148

$$\eta = 18.4 \times 10^5 \text{ cycles} .$$

6.18 - The Weibull slope, or shape parameter β

6.18-a - Definition

The Weibull slope β is a shaping constant which primarily controls the shape of the curve. The failure density distribution and the failure rate are shown plotted against time in fig. 6-38 on page 168 for various values of β. For $\beta < 1$ the curves take on the shape associated with early life failures. For $\beta = 1$ the Weibull distribution reduces exactly to the exponential distribution ($Z(t)$=constant) and can thus represent random failures. For $\beta > 1$ the curve takes on the form associated with wear out of the various types; in particular, with $\beta = 3.44$, the Weibull distribution becomes an approximately normal distribution.



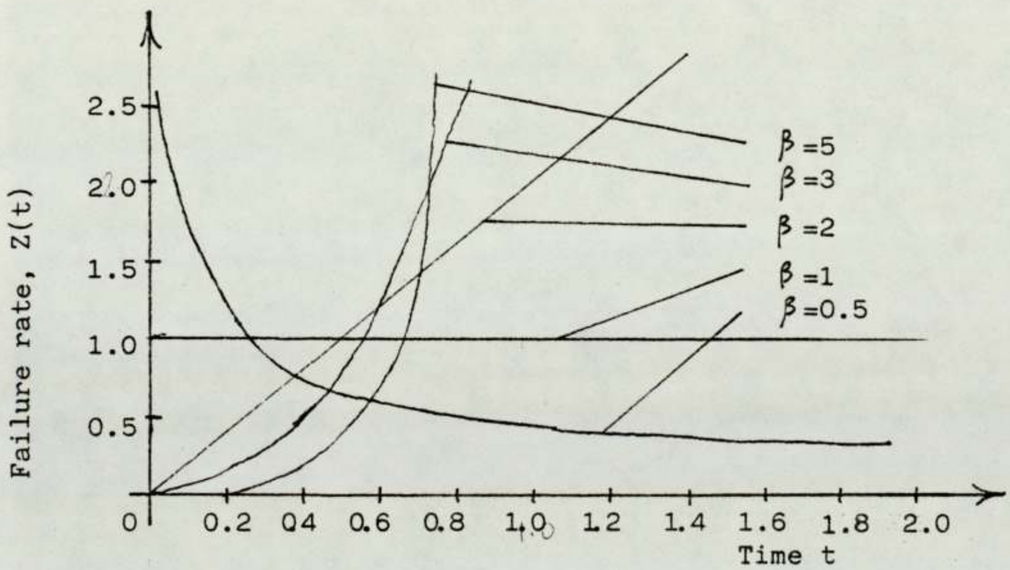


Fig. 6-38 Effect of β on failure density and failure rate for $t_o = 0$ and $\eta = 1.0$ in the Weibull distribution .

6.18-b - Graphical determination

The shape parameter β can be estimated graphically from the plot on Weibull paper in the following manner.

After a best-fitting line is drawn in, a draughtsman's set square is used to construct a line which is both perpendicular to the fitted line and passes through the Estimation Point at the top left hand corner of the paper. The estimate of β is read at the intersection of this line and the scale labelled β .

For the 1B mode of failure, fig. 6-35 on page 157, $\beta = 4.84$

For the 2B mode of failure, fig. 6-37 on page 165, $\beta = 0.76$

For the C mode of failure, fig. 6-22 on page 119, $\beta = 0.48$.

6.18-c - Analytical calculation

The slope parameter β can be calculated more accurately from equation 6-7 on page 109. We have;

For the 1B failures (see page 155), $\beta = 4.92$

For the 2B failures (see page 164), $\beta = 0.76$

For the C failures (see page 116), $\beta = 0.48$

6.19 - The mean life μ

6.19-a - Definition

Mean life is the arithmetic average of the lifetimes of all items considered. A "lifetime" may consist of time between malfunctions, time between repairs, time to removal of tubes or other parts, or any other desired interval of observation.

Mean life values have meaning only in relation to type of frequency distribution assumed by the data. For example, if a constant rate of malfunction is present in the system, the times between malfunctions will be exponentially distributed, and the mean life will occur at the point where there is a 36.78794% per cent probability of survival. However, if the times between malfunctions are normally distributed, the rate of malfunctions will increase with time and the mean life will occur at the point where there is a 50 per cent probability of survival.

6.19-b - Calculation

6.19-b-i - Mathematical calculation

The mean of the Weibull distribution having the parameters η and β may be calculated by evaluating the integral:

$$\mu = \int_0^\infty t . \frac{\beta}{\eta^\beta} t^{\beta-1} . e^{-\left(\frac{t}{\beta}\right)^\beta} . dt = \int_0^\infty t . \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\beta}\right)^\beta} . dt.$$

Making the change of variable $u = \left(\frac{t}{\eta}\right)^\beta$, we get:

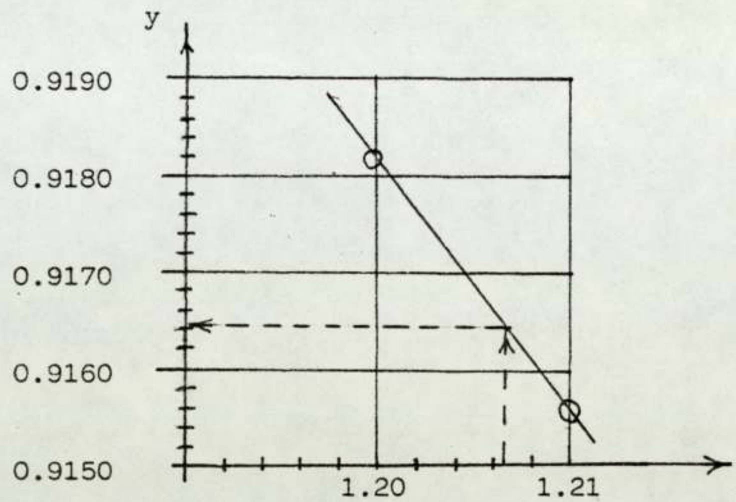
$$\mu = \eta \int_0^\infty u^{1/\beta} e.^{-u} \, du .$$

Recognising the integral as $\Gamma(1 + \frac{1}{\beta})$, namely, as a value of the gamma function which can be determined from mathematical tables, we find that the mean time to failure for the Weibull model is:

$\mu = \eta \Gamma (1 + \frac{1}{\beta})$,

which can be calculated using the Table in Appendix D on page 234.

For the 1B mode of failure we have:



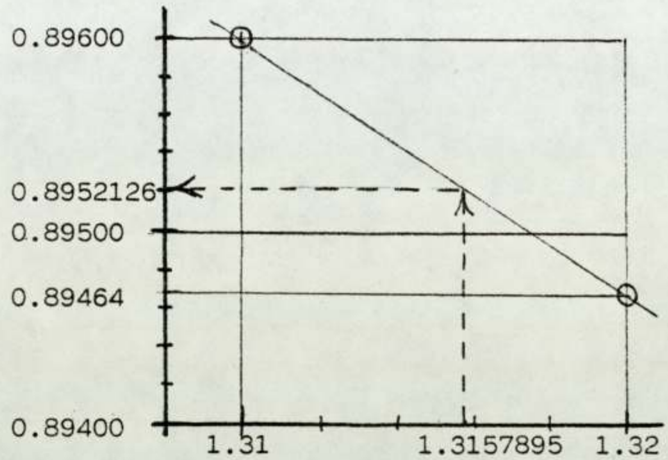
$$\frac{0.91817-0.91558}{1.20 - 1.21} = \frac{y - 0.91558}{1.2066116-1.21}$$

$$y=0.91558+\frac{0.00259x0.0033884}{0.01} = 0.9164576$$

and $\mu = 38.\overline{\overline{27}}x10^5 \Gamma(1+\frac{1}{4.84})=38x10^5 \Gamma(1.20661157)=$

$= 38x10^5 x0.9164576 = 34.825389 \ x \ 10^5$ cycles.

For the 2B mode of failure we have:



$$\frac{0.89600-0.89464}{1.31-1.32} = \frac{y-0.89464}{1.3157895-1.32}$$

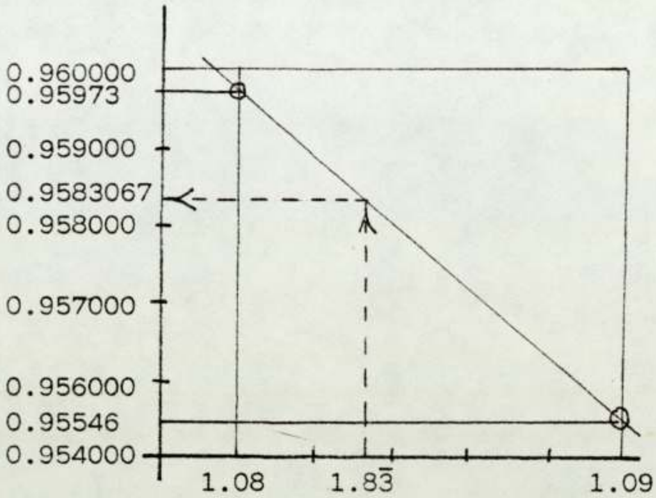
$$y = 0.89464 + \frac{0.00136 \times 0.0042105}{0.01} = 0.8952126$$

and $\mu = 6.63 \times 10^5 \Gamma(1 + \frac{1}{0.76}) = 6.6 \times 10^5 \times \frac{1}{0.76} \Gamma(\frac{1}{0.76}) =$

$= 8.6842105 \times 10^5 \Gamma(1.3157895) =$

$= 8.6842105 \times 10^5 \times 0.8952126 = 7.7742149 \times 10^5$ cycles

For the C mode of failure we have:



$$\frac{0.95973 - 0.95546}{1.08 - 1.09} = \frac{y - 0.95546}{1.0833333-1.09}$$

$$y = 0.95546 + \frac{0.00427x0.0066667}{0.01} = 0.9583067$$

and $\mu=18.36x10^5 \Gamma(1+\frac{1}{0.48})=18.8x10^5 x\frac{1}{0.48} \Gamma(\frac{1}{0.48})=$

$$= 39.166667 \text{ x } 10^5 \quad (2.0833333)=$$

$$=39.166667x10^5 \Gamma(1+1.0833333)=39.166667x10^5x1.0833333x(1.0833333)=$$

$$=42.430556x10^5x0.9583067=40.661485x10^5 \text{cycles} .$$

6.19-c - Graphical determination of μ

The mean life μ can also be estimated graphically from the Weibull plot. After a "best fitting" line is drawn in, a draughtsman's set square is used to construct a line which is both perpendicular to the best fitting line and passes through the Estimation point at the top left hand corner of the paper. The estimate of P_μ, the percentile of the mean, is read at the intersection of this line and the scale labelled P_μ. Mark P_μ on the ordinate scale, and draw a horizontal line to intersect the best-fitting line. From the intersection, draw the vertical to the abscissa scale to find μ. See figures 6-35, 6-37 and 6-22 on pages 157 , 165 and 119 respectively.

7 - RESULTS

7.1 - The best estimate of the actual mean life t

Since the plots in figures 6-35, 6-37 and 6-22 on pages 157, 165 and 119 respectively have been obtained starting from a time t_o, the best estimate of the mean life t is $\hat{t} = (\mu + t_o)$.

Thus

For the 1B mode of failure $\hat{t}=(34.825+(-14.62))x10^5= 20.205x10^5$ cycles

For the 2B mode of failure $\hat{t}=(7.774+11.53)x10^5=19.304x10^5$ cycles

For the C mode of failure $\hat{t}=(40.661+5.30)x10^5=45.961x10^5$ cycles.

7.2 - The goodness of fit of the Weibull Distribution as applied to the results of tests on welded specimens

It has been mentioned that if the test data follow the Weibull distribution, the points will lie on a straight line. Some scatter will exist, and the best straight line can be drawn in, either "by eye" or more accurately using the method of least squares. If a straight line is not obtained (even after considering that $t_o \neq 0$, or the possibility of two or more failure modes), the data cannot be represented by a Weibull distribution.

It is clearly seen in figures 6-35, 6-37 and 6-22 on pages 157, 165 and 119 respectively, that the test data follow nicely a straight line. Obviously some scatter exists which is normal but, to be sure, a test for the validity of the assumption of the Weibull distribution will have to be carried out on the failure data.

The validity of many statistical techniques used in the calculation, analysis, or prediction of reliability depends on the distribution of the failure times. Many techniques are based on specific assumptions about the probability distribution and are often sensitive to departures from the assumed distribution. That is, if the actual distribution differs from that assumed, these methods sometimes yield seriously wrong results. Therefore, in order to determine whether or not certain techniques are applicable to a particular situation, some judgement must be made as to the underlying probability distribution of the failure times. One technique only will be considered here and that is The Kolmogorov-Smirnov or "D-test" for goodness of fit. This is one of many tests designed for the purpose of testing whether or not the assumptions made about distributions of failure times are reasonable. The "D-test" is suitable for very small samples where other tests do not apply. It must be remembered that this test is used only with continuous distributions.

It is based on the maximum absolute difference D between the values of the cumulative distribution of a random sample of size n and a specified theoretical distribution. To determine whether this difference is larger than can reasonably be expected, we look up the critical value of D on page 236 . If the difference, D, is too large, the chance that the observations actually come from a population with the specified distribution is very small. This is evidence that the specified distribution is not the correct one.

As the maximum absolute difference or error, D, between the values of the actual cumulative distribution of data points and the Weibull distribution is less than the critical value of D listed in Appendix E on page 236 it is not unreasonable to assume that the Weibull distribution is correct for this kind of test data.

The reason for applying the "D-test", and not any other test, for goodness of fit, is that it is more efficient for small samples, which is the case in this project.

Failure mode	D	$D^*_{0.10}$	Remarks
1B	0	0.264	$D < D_{critical}$
2B	0.83-0.66=0.17	0.264	$D < D_{critical}$
C	0.29-0.25=0.04	0.264	$D < D_{critical}$

Table 23. The Kolmogorov-Smirnov or "d-test" for goodness of fit. D is the maximum absolute difference or error between the values of the cumulative distribution of data points and the Weibull distribution (D is measured from figures 6-32, 6-35 and 6-37 on pages 148, 157 and 165 respectively.

$*D_{0.1}$ is the critical value of D for n = 20 and α = 0.1. See Appendix E on page 235 .

7.3 - The sensitivity of the Weibull analysis

From the start of the project, it was intended to test the sensitivity of the Weibull analysis to changes in the welding procedure used, by varying one welding parameter and examining the effect on the Weibull distribution of the failure data. If the analysis is sensitive, it should detect that change by having two Weibull distributions with different mean lives, characteristic lives, reliabilities and failure rates.

The welding speed was changed between two sets of specimens, B and C, each consisting of 10 specimens. The Weibull analysis indicated, successfully, as shown in fig. 6-13 on page 89, that there is more than one failure mode. The data were separated into two families, B and C. Applying the Weibull analysis on family C resulted in a straight line, fig. 6-32 on page 148. But when it came to family B, the analysis showed two more failure modes, 1B and 2B, instead of one. Re-examination of the welding procedure used provided the answer to what caused the extra mode of failure. It was found that the two sub-families 1B and 2B were sectioned from two different welded plates with different sizes as shown in fig. 7-1 on page 178.

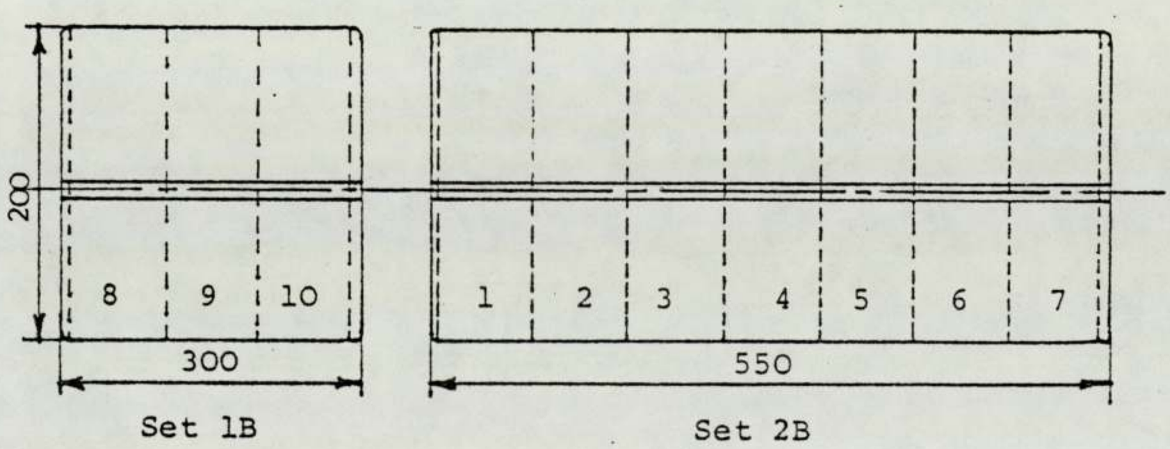


Fig. 7-1 Sectioning of butt welded plates

At first, it was thought that the difference in plate sizes would not have any effect on the fatigue life (and hence the failure mode) since fatigue testing was carried out under pulsating tension loads, but apparently it did have an effect. The difference in size caused a difference in residual stresses and that can explain the change in the failure mode between the two sets of specimens 1B and 2B.

According to a literature survey for transverse butt welds and under pulsating tension loads, residual stresses can be expected to have little effect on the fatigue strength (or fatigue life). There is contradicting evidence to that latter statement, because while some investigators had recorded increases in fatigue strength at 2×10^6 cycles of about 12½% for welds with the reinforcement unmachined[(9)] as a result of stress relieving, others like Ross[(10)], Newman and Gurney[(5)] found that stress relieving had no effect on the fatigue strength of transverse butt welds. However, in our case, there was a change in the fatigue life, recorded as a difference in mean life, as a result of differences in residual stresses. It is very difficult to reach conclusions without carrying out more experiments to investigate the effect of residual stresses on the fatigue life of welded specimens, which is outside the scope of this project.

However, one thing is certain, which is, that the Weibull analysis is very sensitive even to very small changes in the welding procedure used in producing the specimens. The difference between the three sets of specimens, C, 1B and 2B, came out as three different Weibull distributions, with different mean lives, characteristic lives, reliabilities and failure rates.

It is important to notice that it is not possible to compare the results of the two sets of specimens, C and 1B, since they differ in more than one variable (welding speed and welded plate sizes from which the specimens were sectioned). Hence, only the results of the two sets of specimens C and 2B will be compared with each other to see what effect changing the welding speed had on the Weibull distribution of both sets.

7.4 - Comparison between the different modes of failure

The three modes of failure 1B, 2B and C are compared in Table 24 on page 180.

Failure mode	1B	2B	C
t_o min.life parameter	-14.62×10^5 cycles	11.53×10^5 cycles	5.3×10^5 cycles
η 90% confidence interval	38×10^5 cycles 6 hours 34×10^5 42×10^5 cycles	6.6×10^5 cycles 1.05 hours 3.4 12 $\times10^5$ cycles	18.8×10^5 cycles 3 hours 5.6 47 $\times10^5$ cycles
$\hat{\eta} = \eta + t_o$	23.38×10^5 cycles	18.13×10^5 cycles	24.1×10^5 cycles
β	4.84	0.76	0.48
Mean value $\mu = \eta\Gamma(1+\frac{1}{\beta})$ 90% confidence interval	$34.8^2\times10^5$ cycles (5.53 hrs) 31×10^5 38.5×10^5 cycles	$7.77^4\times10^5$ cycles (1.23 hrs) 4×10^5 13×10^5 cycles	40.66×10^5 cycles (6.46 hrs) 13×10^5 100×10^5 cycles
Mean value of t $\hat{t}$=(mean life)+t_o	20.18×10^5 cycles (3.2 hrs)	19.304×10^5 cycles (3.06 hrs)	45.961×10^5 cycles (7.3 hrs)

Table 24. Comparison between the different modes of failure

7.4-a - Mean life

Mean life has been defined as the arithmetic average of the lifetimes of all items considered. The "lifetime" here, consists of the time to failure of each specimen tested. As there are three different Weibull distributions for the three sets of specimens 1B, 2B and C, we can expect to have three different mean lives. From Table 24 on page 180, it can be seen that the mean life $\hat{t}$ of set 1B is 3.2 hours, while for set 2B it is 3.06 hours. This slight change agrees with the fact that the two sets only differed in residual stresses, all other welding parameters being equal (welding, current, voltage, speed), and as previously discussed the change in residual stresses can be expected to have little effect on the fatigue life (and hence on the mean life).

The second main thing to notice in Table 24 on page 180 is the marked change in the mean life $\hat{t}$ between the two sets of specimens 2B and C due to difference in welding speed. The effect of welding speed on the fatigue life will be discussed later in section 7.5 on page 191, but at this stage it is sufficient to say that changing the welding speed affected the weld profile, and that explains the marked change in the mean life (or the fatigue life).

It is of importance to mention that, when mean life values, with no other information, are given as representative of component reliability, this is sometimes misinterpreted by the unitiated to mean that the component will operate failure-free for a period of time equal to the mean life. The fallacy of this conclusion is evident by examining fig. 7-2 on page 183, in which it is obvious that at the mean life of each set of specimens ($\hat{t}$ =20.18x10^5 for 1B, 19.304x10^5 for 2B and 45.96 x10^5 for C), 52% of the 1B specimens, 32.2% of the 2B specimens and 23.5% of C specimens could be expected to operate failure-free. This is borne out by the following calculations:

For the 1B specimens:

$$R(t)=e^{-\left(\frac{t-t_o}{\eta}\right)^{\beta}} = e^{-\left[\frac{(20.18+14.62)x10^5}{38 \times 10^5}\right]^{4.84}} = e^{-0.9157895^{4.84}} = e^{-0.6532673} = $$

$$= 0.5203429 .$$

For the 2B specimens:

$$R(t)=e^{-\left[\frac{(19.304-11.53)10^5}{6.6 \times 10^5}\right]^{0.76}} = e^{-1.1778788^{0.76}} = e^{-1.1324954} =0.3222282.$$

For the C specimens:

$$R(t)=e^{-\left[\frac{(45.96-5.3)10^5}{18.8x10^5}\right]^{0.48}} = e^{-2.162766^{0.48}} = e^{-1.4481201} =0.2350117 .$$

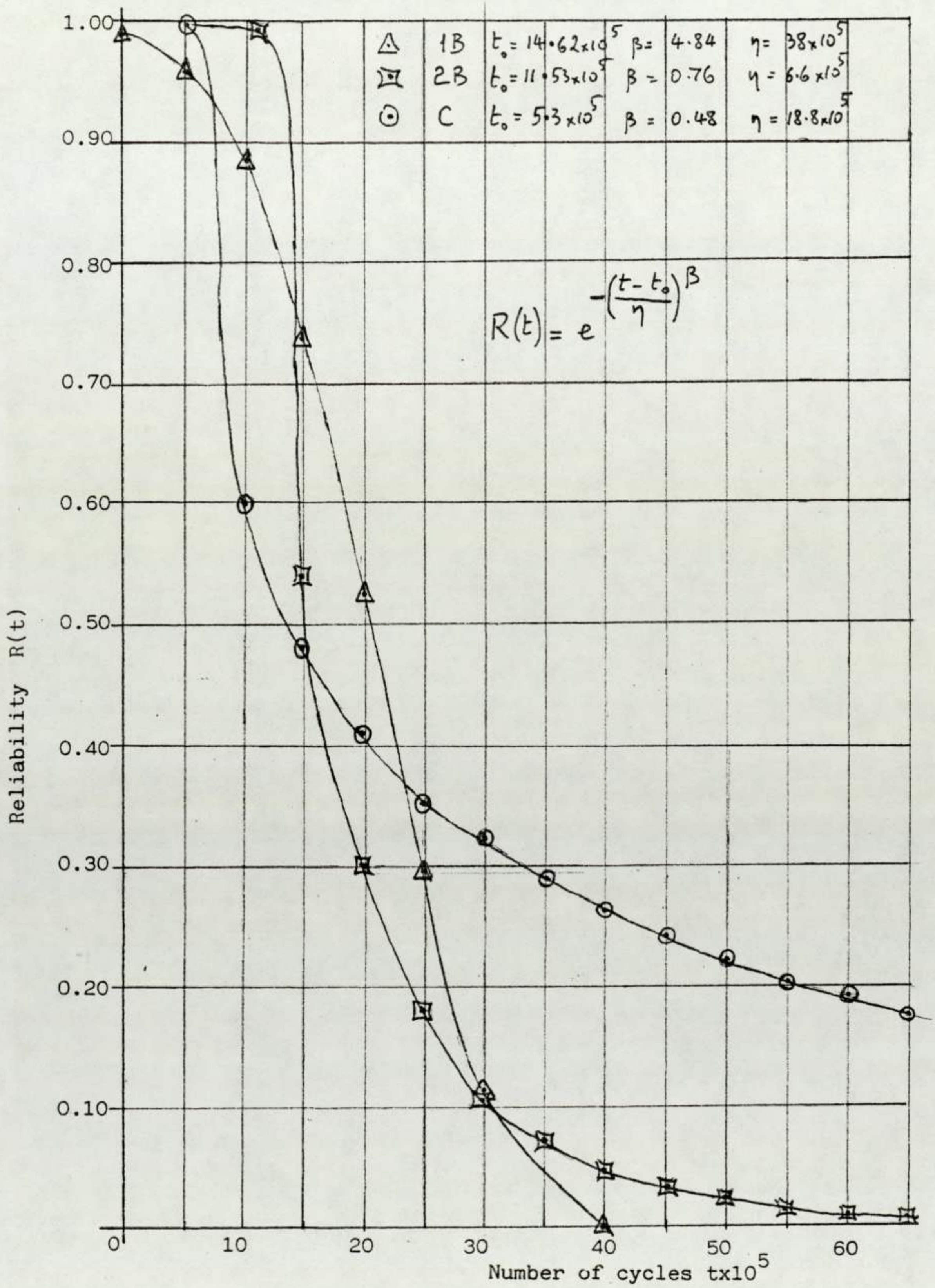


Fig. 7-2 Reliability distribution for the various modes of failure.

7.4-b - Reliability

Once the three Weibull parameters β, η and t_o are determined, the reliability can be regarded as a function of time, as could be seen from the equation of reliability:

$$R(t) = \exp\left| -\left(\frac{t-t_o}{\eta}\right)^{\beta} \right| .$$

Thus in specifying a component reliability, it is meaningless just to state the reliability figure, but the time at which it occurs must also be specified.

In fig. 7-3 on page 185, which shows the failure density distribution f(t), the welded specimens from sets 2B and C, cannot fail between time 0 and t_o, hence we can say that the reliability is 100% before 11.53×10^5 cycles for set 2B and before 5.3×10^5 cycles for set C. But since (t_o) has a negative value for set 1B, which means that failure can start before testing commences, the reliability does not reach 100%, as could be seen from fig. 7-2 on page 183. This negative value of t_o can occur in some products like car batteries which can fail before they are installed. One can only suggest that the reason for the negative value of t_o in our case, was the presence of too much residual stress. But again this is not certain and further investigation needs to be carried out on this matter.

From fig. 7-2 on page 183, the reliability generally decreases with time. At first, set C has the lowest reliability of all three sets, but at longer durations its reliability has the highest value. Normally, when we are comparing different components to see which is more reliable, reliability is calculated at a specific time. So, at 10×10^5 the reliability of each set of spec-

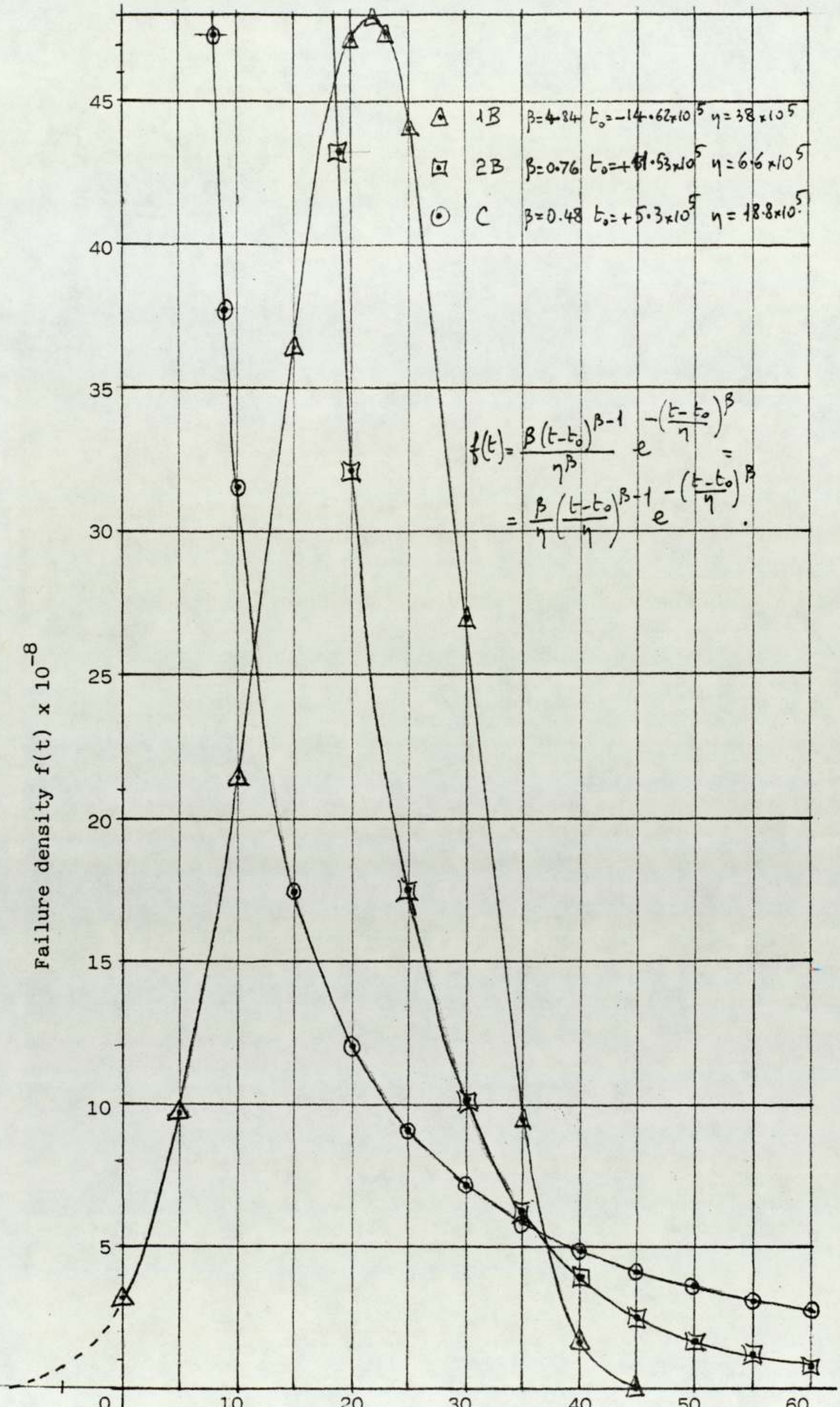


Fig. 7-3. Failure density distribution for the various modes of failure.

imens is:

$$R_{1B}(10x10^5)=e^{-(\frac{10+14.62}{38})^{4.84}}=e^{-0.1223719}=0.8848193= 88.48\%$$

$$R_{2B}(10x10^5)=100\%$$

$$R_C\ (10x10^5)=e^{-(\frac{10-5.3}{18.8})^{0.48}}=e^{-0.25^{0.48}}=e^{-0.5140569}=0.5980644=59.8\%$$

Thus we can see that set 2B is the most reliable in all sets for short durations (up to 15 x 10^5 cycles). At longer durations (above 10 x 10^5 cycles) the reliability of all sets 1B, 2B and C becomes low for any practical uses.

The fact that the mean life must be interpreted in relationship to the form of the distribution (and hence the reliability) on which it is based, is emphasised by considering the two sets of specimens 1B and 2B. The reliability at 20 x 10^5 cycles for the two sets may be determined from fig. 7-2 on page 183 as follows:

$$R_{1B}(20x10^5)=e^{-(\frac{20+14.62}{38})^{4.84}}=0.5288372 \simeq 52.88\%$$

$$R_{2B}(20x10^5)=e^{-(\frac{20-11.53}{6.6})^{0.76}}=e^{-1.2087542} \simeq 29.86\%$$

$$R_C(20x10^5)=e^{-(\frac{20-5.3}{18.8})^{0.4796472}}=e^{-0.8886981}=0.4111907 \text{ or } 41.12\%$$

Thus, it is seen that the probability of non-failure for 20 x 10^5 cycles is significantly higher for set 1B than it is for set 2B, although the mean life of set 1B is slightly higher than that of set 2B.

7.4-c - Failure rate

A failure rate that is typical for many manufactured items is shown in fig. 7-4.

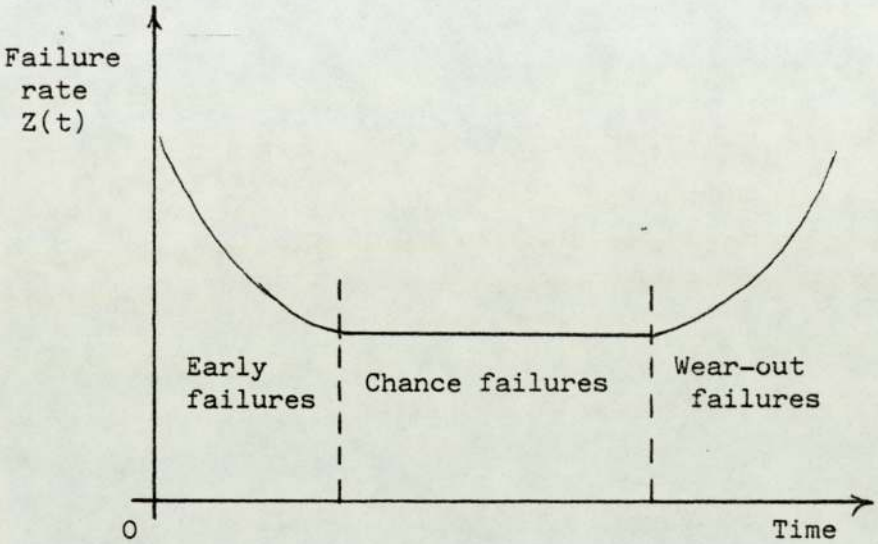


Fig. 7-4 Typical failure-rate curve

The curve is conveniently divided into three parts. The first part is characterised by a failure rate which decreases rapidly with time and represents the period of early failures, during which poorly manufactured items are weeded out. It is common in the electronics industry to "burn in" components prior to actual use in order to eliminate any early failures).

The second part, which is often characterised by a constant failure rate, is normally regarded as the period of useful life during which only chance or random failures occur.

The third part is characterised by a failure rate which increases rapidly with time and represents the period of wear-out failures during which components fail primarily because they are worn out.

Note that the same general failure rate curve is typical of human mortality, where the first part represents infant mortality and the third part corresponds to old-age mortality.

In fig. 7-5 on page 189, since β is less than one for the two sets of specimens 2B and C, the failure-rate curves take on the shape associated with early life failures. One conclusion can be drawn out by examining the two different failure-rate curves of sets 2B and C; decreasing the welding speed will decrease the failure-rate.

For set 1B, since β is more than one, the failure rate curve takes on the shape associated with wear out failures. This change in the failure pattern from set 2B can, again, be attributed to the presence of large residual stresses in the specimens.

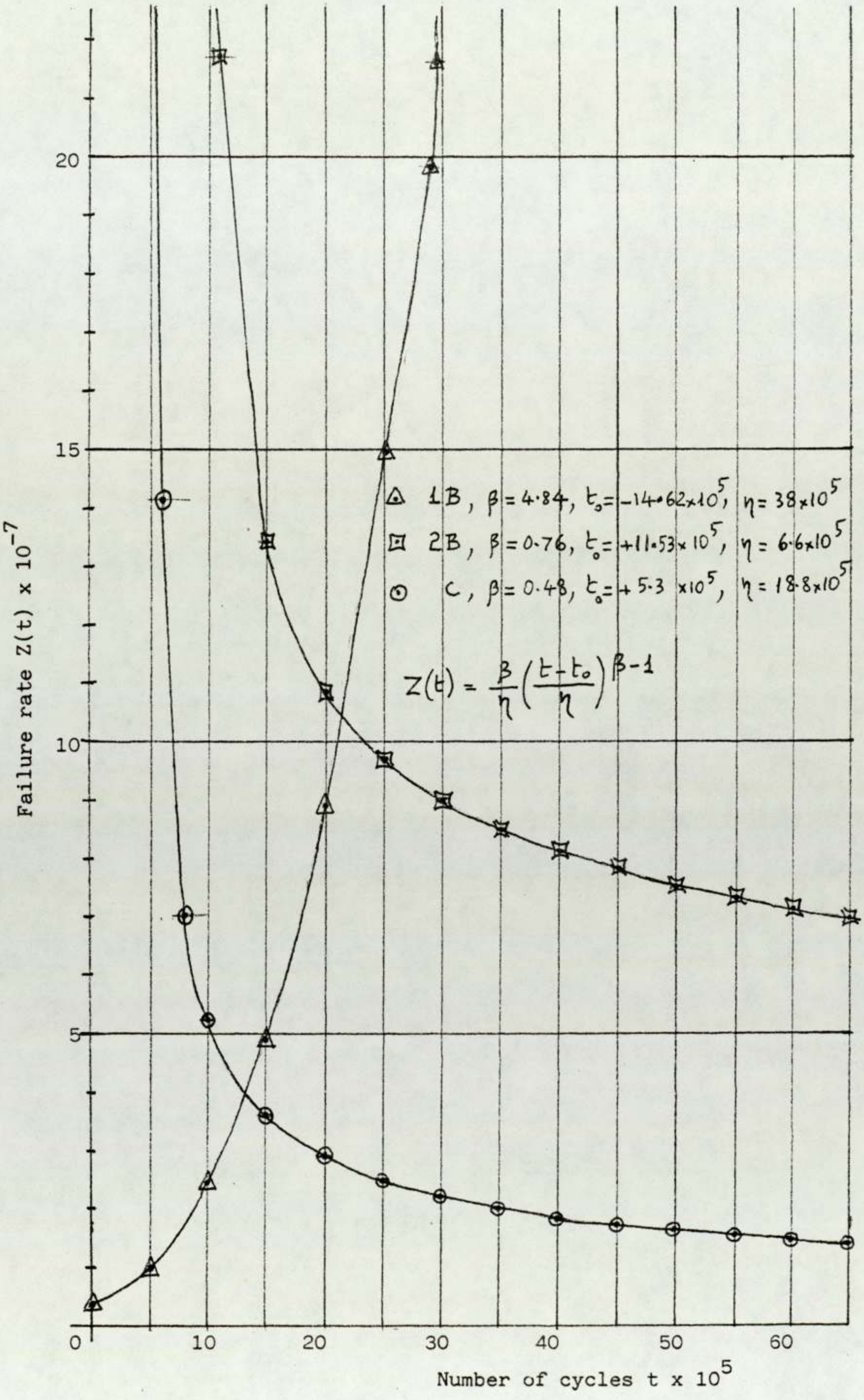


Fig. 7-5. Failure rate distribution for the various modes of failure.

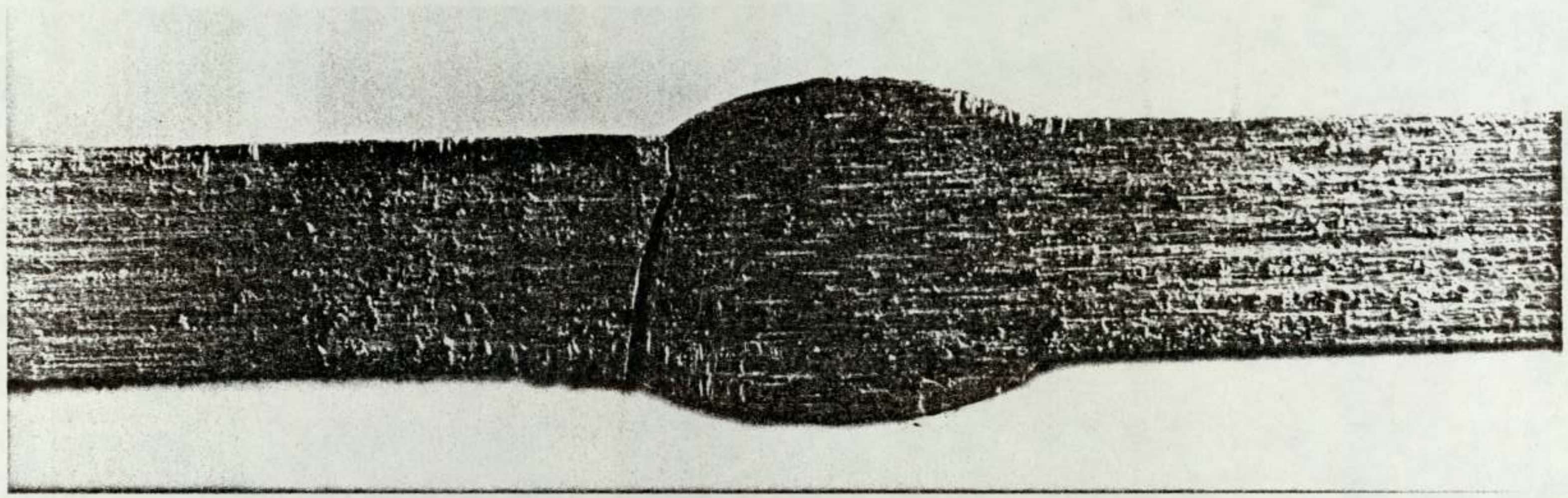


Fig. 7-6 Typical fatigue crack in the welded specimen.

7.5 - Effect of varying the welding speed

It is known, from the literature on the subject, that the weld shape, and in particular, the reinforcement angle ϑ between the plate surface and the tangent to the reinforcement at its point of contact with the plate surface, is the overriding factor in determining the fatigue strength of transverse butt joints. It has been found that the fatigue crack propagates from the weld toe at the point of minimum angle. Thus, it would seem reasonable to attribute the influence on fatigue strength (or fatigue life) of many other factors, such as plate preparation, welding conditions, welding process and type of electrode to their effect on the shape of the weld toe.

With any combination of welding current and voltage, the effects of changing the welding speed conform to a general pattern as follows:

If the welding speed is decreased: 1) power or heat input per length of weld is increased; 2) more welding wire is applied per unit length of weld, and 3) consequently, there is more weld reinforcement.

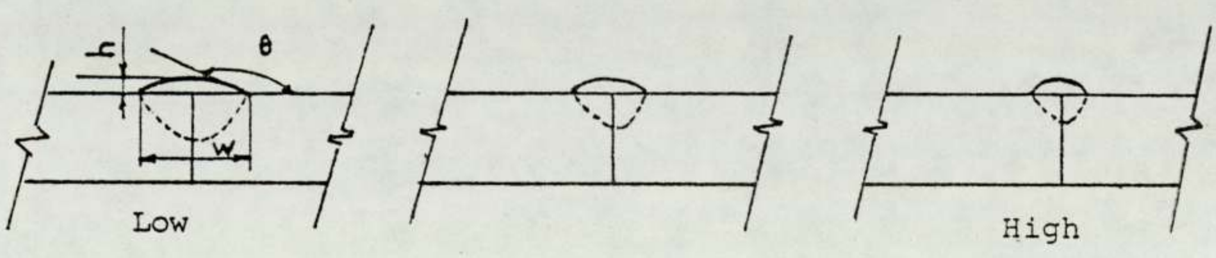


Fig. 7-7 Effect of varying the welding speed on weld shape

Increasing the weld reinforcement leads to a wider weld bead (w increases) as shown in fig. 7-7 on page 191. If the weld bead height (h) remains constant, then it can be said that decreasing the welding speed will result in a better weld profile. By that it is meant that the reinforcement angle ϑ will be increased, leading to a better fatigue life. But if h increases, the reinforcement angle ϑ can increase or decrease or remain constant, depending on the amount of increase in h.

Reinforcement angle measurements showed that for all specimens, fatigue cracks were initiated at the point of minimum angle, and that they were on the same side of the welded specimen (the specimens were welded by a two-pass weld, one from each side).

For set 2B, the average reinforcement angle at the point of crack initiation was 139.65°, while for set C it was 143.4°. This indicates that the fatigue life of set C should be higher than that of set 2B.

The fact that the reinforcement angle for set C is higher than that of set 2B, was assessed by comparing the two weld bead widths and heights. Results showed that, decreasing the welding speed led to a noticeable increase in bead width, and only a slight increase in weld bead height. This can mean that the reinforcement angle increases as the welding speed is decreased in our case.

From the above discussion, the application of the Weibull analysis to the fatigue results of the welded specimens 2B and C, should indicate that the mean life of set C is higher than that of set 2B.

From Table 24 on page 180, the Weibull analysis gives a mean life $\hat{t}$ of 7.3 hours for set C, while for set 2B it gives a mean life $\hat{t}$ of 3.06 hours, which agrees with the practical results.

It can be concluded that decreasing the welding speed will improve the fatigue life of the welded joint. As for the reliability of the joint, it will increase as a result of decreasing the welding speed, but only at long durations (above 15×10^5 cycles) as shown in fig.7-2 on page 183.

Failure mode	2B	C
Average reinforcement angle θ*	139°	143.4°
Average weld bead height (h)	2 mm	2.37 mm
Average weld bead width (w)	13.23 mm	16.22 mm
h/w	0.151	0.146

* θ is the average reinforcement angle at the point of crack initiation.

Table 25. Comparison between the weld profiles of the two sets of specimens 2B and C.

7.6 - Conclusions

1. The Weibull analysis can be applied successfully to the results of fatigue tests on welded joints.
2. The sensitivity of the Weibull analysis is so high that not only does it detect the variation in the welding speed, but also the difference in residual stresses resulting from a change in the welded plate size.
3. The analysis shows that, decreasing the welding speed, increases the mean life of the transversely butt welded specimen.

   Also, the reliability, at long durations, increased as a result of decreasing the welding speed.

8 - DISCUSSION

8.1 - Special techniques for reducing testing time

Three methods exist whereby testing time may be reduced.

1) Running simultaneously more specimens than we intend to fail eventually.
2) Sudden death testing.
3) Sequential analysis.

8.1-a - Running simultaneously more specimens than we intend to fail eventually

Should we require 8 failures it would result in a considerable time saving to test 16 items until 8 failures had occurred. In fact the time required to fail 8 items out of 16 is only 26% of the time required to fail 8 out of 8 (for a Weibull slope of 1). In this instance we have plotted the lowest 8 from 16 as opposed to all 8 from 8.

<u>Note:</u> The Median Ranks for these failures must be taken from the first 8 of those assigned to a sample size of 16. Generally for a Weibull slope b the time required to fail r out of n as opposed to n out of n is given by:

$$\log \left[\frac{1 - \left\{ r - 0.30685 - 0.3863 \ \left(\frac{r-1}{n-1}\right) \right\} \frac{1}{n}}{\log \ \left(\frac{0.69315}{r}\right)} \right]^{1/b}$$

This is particularly true for values of n greater than 20. For values of n less than 20 a more exact formula is used.

$$\log \left[\frac{\left\{ 1 - \left[1 - 2^{-1/n} + \frac{(r-1)}{(n-1)} \left((2^{\ (1-1/n)} - 1 \right) \right] \right\}}{\log \ \left(\frac{0.69315}{r}\right)} \right]^{1/b}$$

8.1-b - Sudden death testing

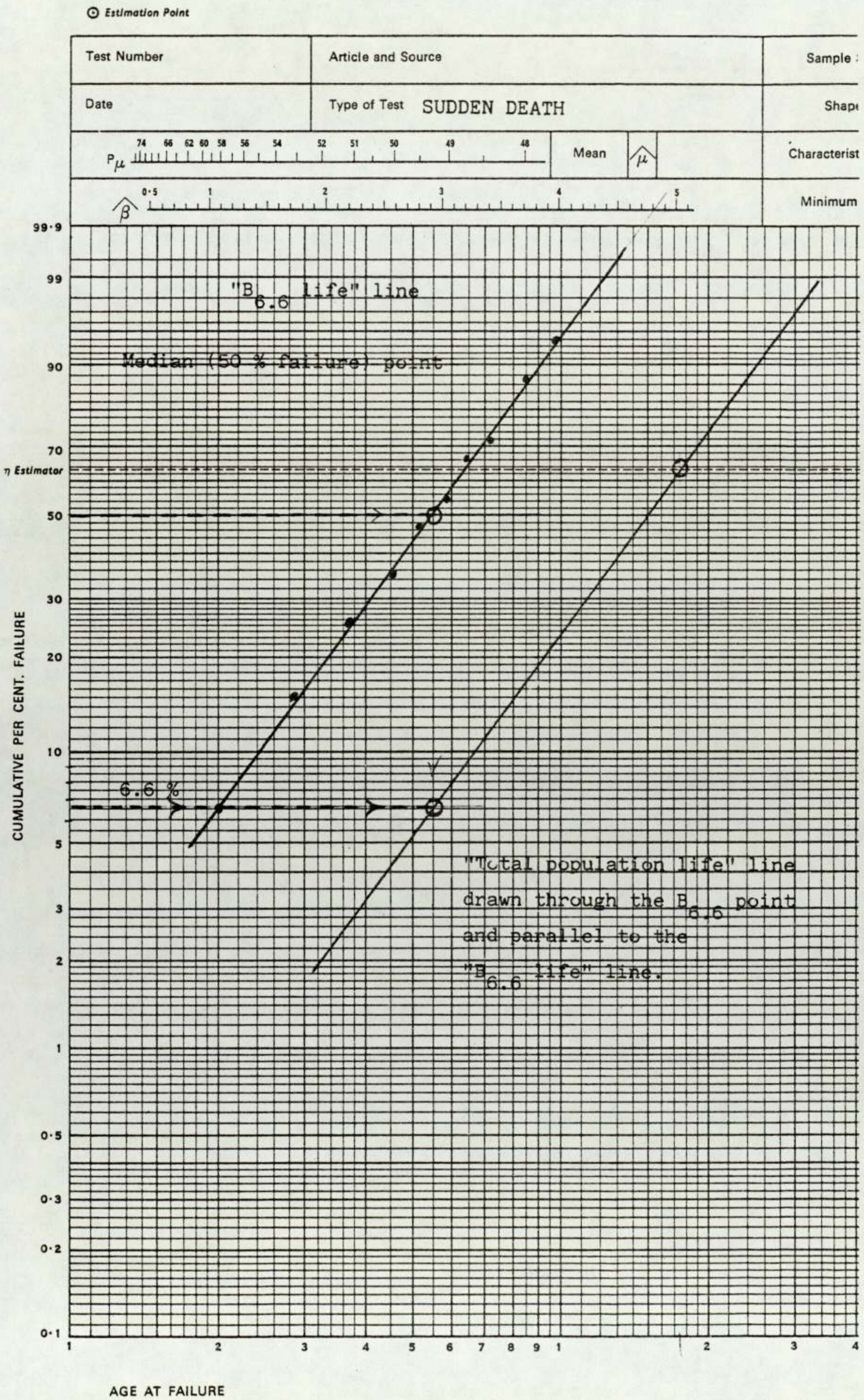
The following example is presented as an illustration of the use of Sudden Death Testing.

100 units are taken for reliability examination. These are divided randomly into 10 sets of 10, and each set tested until the first failure in each set is observed. (Note: All 10 items in each set must be tested simultaneously).

The test on each set will be terminated immediately the first failure in the set under test is observed. Thus, at the end of the test of all 10 sets, 10 failure values, one for each set, will be in evidence. These 10 failure values are arranged in ascending order and plotted in the usual manner on Weibull Probability Paper. The resulting line will be the best estimate of what is termed the "B6.6 Life", 6.6 being the lowest Median Rank for a test set of 10 items.

To relate these 10 B6.6 Life failures to the population as a whole it is necessary to determine the "Median Life" of the 10. This is obtained by reading the failure life at the 50% point. A vertical line is now dropped from the Median point to the 6.6% line and a line parallel to the B6.6 Life line, is drawn through this point. This second line, to the right of the first, describes the distribution for the population as a whole. See fig. 8-1.

The estimate obtained in this manner is equally as good as the estimate that would be obtained by testing 100 units singly, at the same time the testing time required is reduced considerably.



⊙ Estimation Point
Test Number
Article and Source
Sample
Date
Type of Test SUDDEN DEATH
Shape
Mean
Characterist
Minimum
"B6.6 life" line
Median (50 % failure) point
η Estimator
6.6 %
"Total population life" line
drawn through the B6.6 point
and parallel to the
"B6.6 life" line.
CUMULATIVE PER CENT. FAILURE
AGE AT FAILURE

In general, fraction reduction in test time can be evaluated from the following formula:

$$\frac{r\,(r-1)!\,\Gamma(1+1/b)}{(r+1/b)} \cdot \left[\frac{(r-1+\ln 2)\log 2}{s.\ \ln 2 \log \frac{r.s.}{\cdot 69315}}\right]^{1/b}$$

Where b = Weibull slope

r = Number of sets

s = Number in set

Γ = Gamma function.

For a Weibull Slope of 1 the time required to carry out a "Sudden Death" test on the 100 units mentioned would be only 19.44% of the time required to fail all 100 units singly.

Further if all 10 sets were run simultaneously the test time would be further reduced to 5.37% only of the time required to fail 100 singly.

Now if the sets can run simultaneously the following formula is used to calculate the fraction test time reduction.

$$\left[\frac{\log M\ R_n}{s \log M\ R_r}\right]^{1/b}$$

where $M\ R_n$ is the Median Rank for the 1st value in n = rs specimens and $M\ R_r$ is the Median Rank for the 1st value in r specimens.

8.1-c - Sequential analysis

Sequential analysis provides a method of assessing the effectiveness of a modification or change of specification of an existing item. It is best applied where the item being tested is an expensive one or where test facilities are limited.

It has been shown that according to Weibull any standard distribution may be described by three independent variables, a slope parameter β, a scaling parameter η, and a location parameter t_o. For convenience in sequential analysis it is assumed that only one of these variables change when a modification is carried out and that this variable is the scaling parameter, also known as "Characteristic Life", (life to fail 63.2% of the population). Thus the slope is assumed the same before and after the modification and the Weibull line is assumed to pass through the origin, ($t_o = 0$).

The basic concept of this analysis is to test, in sequence sample items incorporating the modification and to compare the results, after each test, with known information about the unmodified item. This means of comparison, which will be explained later, enables us to make one of three decisions, with a specified level of confidence.

These decisions are:

a) The modification gives a greater characteristic life.

b) The modification does not give a greater characteristic life.

c) There is insufficient evidence for either a) or b) hence a further test is required.

To illustrate the method employed in arriving at one of these decisions, take the following example:

Let us say that a gearbox mainshaft is known to have a Weibull slope of 2 and a characteristic life of 30 hours. This characteristic life (η) is thought to be inadequate and as a consequence shot peening is proposed in the hopes of increasing the value of η to something above 45 hours. Hence if 50 hours is a more reasonable estimate of the shot peening shaft characteristic life than 30 hours, then the true value must be above 45 hours. In this case the three decisions may be written as:

a) The new material gives $\eta 2$ = 60 hours (slope=2).

b) The new material gives $\eta 1$ = 30 hours (slope=2).

c) Neither a) nor b), hence a further test.

Before a decision may be made we must set confidence levels such that the risk of making the wrong decision is small and specified.

There are two risks involved:

First, the probability of accepting 60 hours when 30 hours is the true figure. (Denote this probability as α).

Second, the probability of accepting 30 hours when 60 hours is the true figure. (Denote this probability as β).

For this example let us make $\alpha = \beta = 0.05$, which means that we want to be wrong no more than 5 times in any 100 decisions. Thus we are 95% confident of any decision either for 30 hours or 60 hours.

To discover if $\eta 2$ = 60 hours is a more reasonable estimate of characteristic life than $\eta 1$ = 30 hours.

Decision a) η nearer $\eta 2$ than $\eta 1$

Decision b) η nearer $\eta 1$ than $\eta 2$.

Several tests carried out as required.

<u>FIRST TEST</u> $x_1 = 70$ hours

For decision a) $$x_1^b > \frac{\eta_2^b}{\gamma^b - 1}\left[b\ r\ \ln\gamma + \ln\left(\frac{1-\beta}{\alpha}\right)\right]$$

$$70^2 > \frac{60^2}{2^2-1}\left[2 \times 1 \times \ln 2 + \ln\left(\frac{1-0.05}{0.05}\right)\right]$$

$$4900 > 1200 \times (2 \times 0.6931 + 2.9445)$$

$4900 > 5196$ ---------- untrue, therefore reject decision a).

For decision b) $$x_1^b < \frac{2^b}{\gamma^b - 1}\left[b\ r\ \ln\gamma + \ln\left(\frac{\beta}{1-\alpha}\right)\right]$$

$$4900 < 1200 \cdot \left[2 \times 0.6931 + (-3.0549)\right]$$

$4900 < -2022$ ---------- untrue, therefore reject decision b).

Accept decision c).

<u>SECOND TEST</u> $x_2 = 40$ hours

For decision a) $70^2 + 40^2 > 1200\ (2 \times 2 \times 0.6931 + 2.9445)$

$6500 > 6860$ ---------- untrue, therefore reject decision a).

For decision b) $70^2 + 40^2 < 1200\ (2 \times 2 \times 0.6931 + (-3.0549)$

$6500 < -339$ ---------- untrue therefore reject decision b).

Accept decision c).

<u>THIRD TEST</u> $x_3 = 59$ hours

For decision a) $70^2 + 40^2 + 46^2 > 1200(2\text{x}3\text{x}0.6931 + 2.9445)$

$8616 > 8523.7$ ---------- TRUE, therefore accept decision a).

<u>Note</u>: When a decision is arrived at it does not mean that this gives the characteristic life of the modified item, but merely that is is nearer the true value than the other decision. Hence in the above example should decision a) be arrived at, we may be 95% confident that the characteristic life of the new material is nearer 60 hours than 30 hours, i.e. above 45 hours.

8.2 - Summary of requirements for Weibull analysis

8.2-a - Essential requirements

1) To ensure that the components in the sample under consideration are to specification, or are at least typical of the parent population.

2) To specify initially the mode of failure to be considered such that confusion or inaccuracy are not introduced due to failure for other reasons.

3) To study the component so that a degree of failure is decided upon, i.e. to decide at which point in a component's deterioration is it considered to have failed.

4) To test a number of components such that at least seven have failed due to the mode of failure under consideration. It has been found from experience that seven plotted points on Weibull Probability Paper is adequate to; fit a good line; to give a reasonable level of confidence and to hold the test cost down to the minimum which will give meaningful results.

5) To analyse these failures according to the Weibull method and to construct the straight line representation of the failure distribution.

8.2-b - Preference requirements

1) To obtain results which do not include suspended items thus reducing the calculations necessary to produce the Weibull plot.

2) To fit confidence bands to all Weibull plots so as to obtain a measure of confidence, whether requested or not.

3) When time reduction methods are employed, to obtain some measure of the saving, by using the given formulae.

8.3 - Reliability prediction from service information

It is generally agreed that reliability prediction is essential if customer hostility and warranty cost are to be kept to a minimum.

Representative rig testing may be considered the ideal method of obtaining and maintaining reliability levels since it is usually quick, cheap in terms of component cost and may be repeated at will. However there are exceptions, for which rig testing would prove impractical, either due to the type of test necessary or due to the subsequent analysis of the test results. Such exceptions are as follows:

When the considered items are in short supply or are very expensive.

When the item is incorporated in a large assembly and cannot be tested individually, thus making the analysis of the results lengthy and complicated.

When the rig test cannot be accelerated, thus making the collection of results a long process and creating a large time lag for repeated spot checks from production.

When it is found difficult to exactly simulate conditions experienced in service thus somewhat invalidating any rig tests which may be attempted.

In cases such as these or where a comparison is required with existing rig tests it is possible to implement a prediction system based on customer complaint or warranty returns from service.

Before any prediction can be made, it is necessary to know the number of failures and the number of non-failures occurring at any one time in service or in any particular mileage band. It is this determination of non-failures that offers the most resistance to the implementation of an effective system. However in the system proposed it is possible to determine not only the number of failures but also the number of non-failures.

Given that a computer suitably programmed and containing details of all warranty and production information is available it is possible to carry out a full performance analysis on any component in a very short space of time.

From the warranty claim form submitted by the repairing agent it should be possible to identify the chassis/unit number, the mileage at failure, the complaint item, the complaint and the date of repair. Similarly, production information should be available for each vehicle produced giving chassis/unit numbers, date of build etc. Obviously all this information will be stored within the computer memory banks.

The programme needs to be such that for any one component performance call up the computer will:

a) Sort by month of production all complaints received - this is achieved by identifying the unit date of build already on record.

b) Sort each failure into time-in-service bands e.g. 1.MIS 2.MIS etc. achieved by comparing the repair date with the build date.

c) Arrange the month-in-service complaints into assigned mileage bands.

d) Identify the total number of complaints in each band.

e) Construct population mileage distribution charts for each month in service - these can be constructed from warranty mileages and time-in-service information already stored.

f) Identify the number of non-failures in each mileage band for each month-in-service - obtained by multiplying the monthly production total by the percentage population within the mileage bands at the month-in-service point.

g) Using the last failure mileage in each band for each month in service and applying the "suspended set" technique for the number of non-failures in each band for each month in service, complete the Weibull plot.

h) From the plot provide the predicted failure levels at the end of the warranty period or any other time if required, give the minimum and mean life, the characteristic life, the distribution shape (slope) the B_{10} B_{50} etc. life with various confidence intervals.

Such a system will enable rapid assessment of any component in service, give a rapid feed back of modification action, enable comparison of test schedule severity to be measured, predict the complaint level at any point in time and consequently the warranty cost likely to be incurred. By comparing the performance of each month of production, problem cause can be readily identified, a peak of one month alone without any design change having occurred would indicate operator/machine fall down.

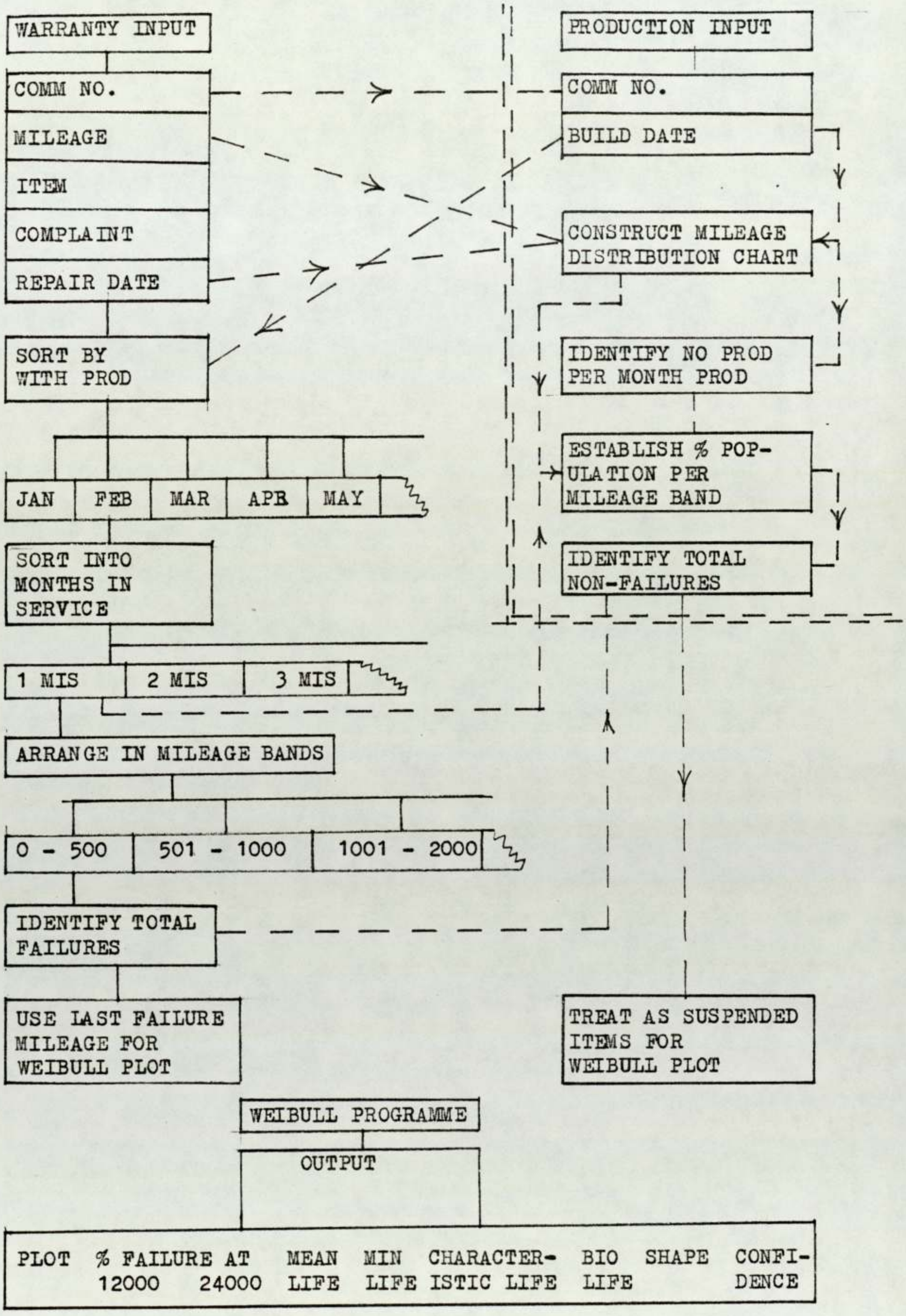

WARRANTY INPUT
COMM NO.
MILEAGE
ITEM
COMPLAINT
REPAIR DATE
SORT BY
WITH PROD
JAN
FEB
MAR
APR
MAY
SORT INTO
MONTHS IN
SERVICE
1 MIS
2 MIS
3 MIS
ARRANGE IN MILEAGE BANDS
0 - 500
501 - 1000
1001 - 2000
IDENTIFY TOTAL
FAILURES
USE LAST FAILURE
MILEAGE FOR
WEIBULL PLOT
PRODUCTION INPUT
COMM NO.
BUILD DATE
CONSTRUCT MILEAGE
DISTRIBUTION CHART
IDENTIFY NO PROD
PER MONTH PROD
ESTABLISH % POP-
ULATION PER
MILEAGE BAND
IDENTIFY TOTAL
NON-FAILURES
TREAT AS SUSPENDED
ITEMS FOR
WEIBULL PLOT
WEIBULL PROGRAMME
OUTPUT
PLOT
% FAILURE AT
12000
24000
MEAN
LIFE
MIN
LIFE
CHARACTER-
ISTIC LIFE
BIO
LIFE
SHAPE
CONFI-
DENCE

Fig. 8-2

8.4 - Further uses of and aids to Weibull analysis

It has been shown earlier in this thesis that the Weibull analysis may be applied specifically to reliability and that use of the special Probability Paper allows us to present clearly and simply the characteristics of most modes of failure.

In the following section certain systems are suggested concerning supplier liability and service stores information which using the basic Weibull graphical presentation would help to reduce the time lag between the introduction of an item with poor reliability, the discovery of this situation and the necessary corrective action.

Since the Weibull analysis is based on a mathematical model involving three independent variables, it can be seen that non-reliability functions may be dealt with by simply assigning the relevant parameters in the problem to the variables in the mathematical model.

The following brief examples are intended to demonstrate this point and to emphasise the broad scope to which the Weibull graphical presentation may be adopted.

8.4-a - Inspection functions (19)

By applying the Weibull graphical technique to relatively small samples, accurate assessments of production capabilities may be obtained.

Quick answers may be given to such questions as:

How much scrap is produced from a particular operation during production?

What proportion of this scrap lies above and below the tolerance band?

What effect does a particular production modification have on this scrap level?

What would be the most effective type of modification?

Fig. 8-3 shows a plot of 30 sample measurements of a particular characteristic, during an operation, and demonstrates how narrow is the confidence band which may be achieved for a sample of this size. It is obvious what sort of information may be obtained from such a plot.

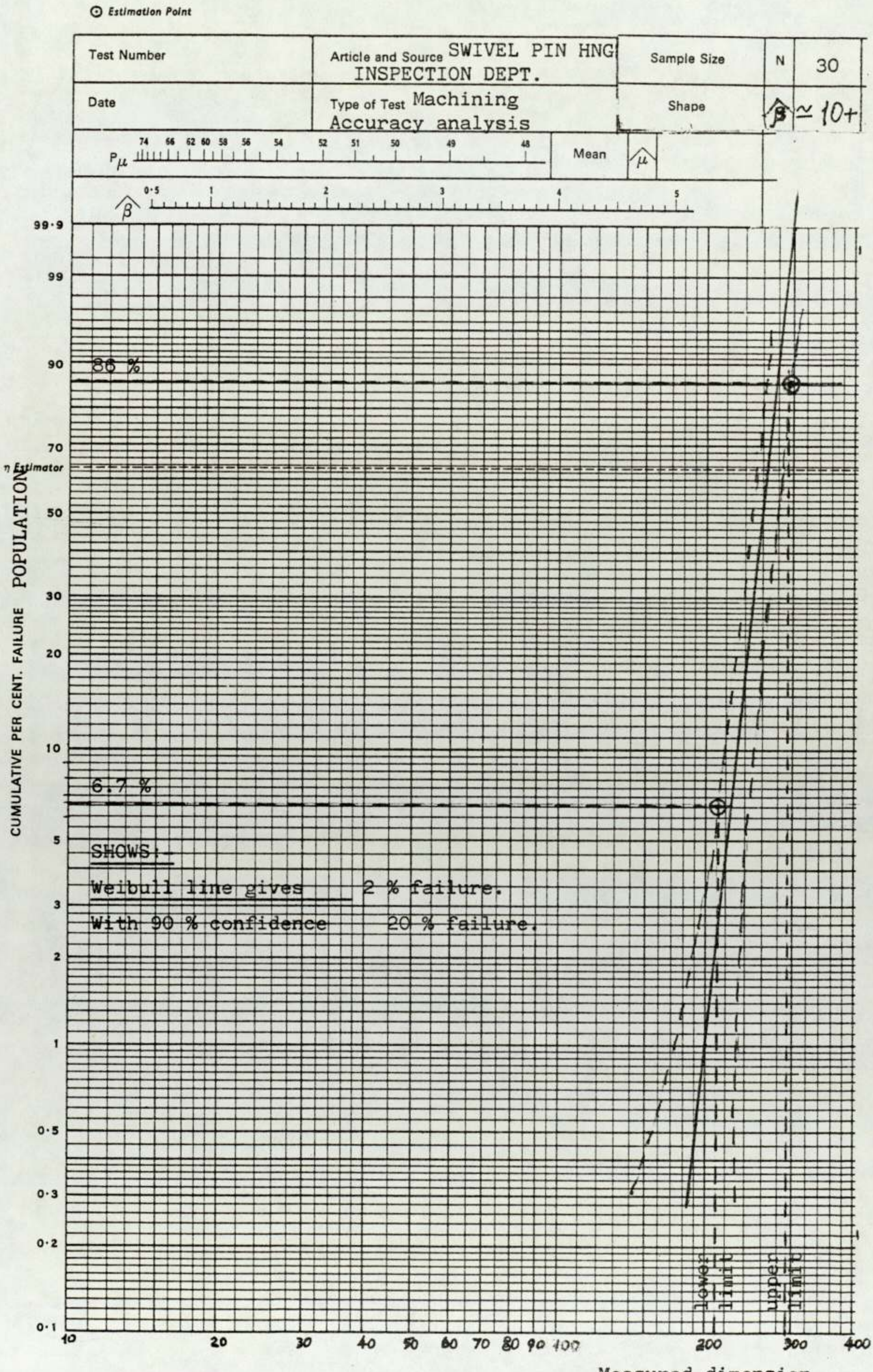
8.4-b - Manpower planning

It is always useful, when dealing with large numbers of personnel, to be prepared for any significant movement of manpower internal or external to an organisation. In many cases the distributions thrown-up by this type of movement may be described by means of the Weibull analysis.

Thus predictions may be made well ahead of the fact and compensatory action taken.

If for example it is required to know how many of a certain number of personnel, recently employed in an organisation will have left after say 50 weeks, then it would be a simple matter to predict this from the information given in the first few weeks. A table would be drawn up, similar to the one following, and a plot made of length of service in two weekly bands against the percentage with terminated service in these periods.

Length of Service	% Terminated Service
Up to 2 weeks	1.0
Up to 4 weeks	1.8
Up to 6 weeks	3.7
Up to 8 weeks	4.9
Up to 10 weeks	6.4


Estimation Point
Test Number
Article and Source SWIVEL PIN HNG INSPECTION DEPT.
Sample Size
N 30
Date
Type of Test Machining Accuracy analysis
Shape
β ≃ 10+
Mean
η Estimator
CUMULATIVE PER CENT. FAILURE POPULATION
86 %
6.7 %
SHOWS:-
Weibull line gives 2 % failure.
With 90 % confidence 20 % failure.
lower limit
upper limit
AGE AT FAILURE
Measured dimension

Hence predictions could be made with reasonable confidence if it is assumed that the distribution of personnel movement is a single one and that other aspects such as seasonal changes may be ignored or compensated for.

8.4-c - Stores planning

In order to predict how many days after ordering it is taking for delivery to be made several random sample observations could be made. From this information, when plotted, it would be possible to predict; what percentage of orders would have been delivered after 40 days; how long would it take for 90% of orders to be delivered, etc.

The random observations would be put in a tabular form as follows:

Time to delivery (days)	% Probability of Delivery
23	10.9
37	26.4
42	42.1
48	57.8
56	73.5
61	89.0

N.B. The percentage probability of delivery values are taken direct from the Median Rank values for this sample size.

8.5 - Further aids to quality and reliability functions

8.5-a - Reliability assurance

As a means of establishing and maintaining the level of reliability on a particular bought-out component it may be proposed that suppliers actually state a reliability level on their component detail drawing. This could be confirmed by a "Reliability Assurance Certificate" presented by the supplier to the customer which would state a specific value of life (cycles, hours etc.) accompanied by a reliability level at this life.

Requirements could be specified as to the suppliers method of assessing the reliability level, i.e. Weibull should be used, and hence spot checks could be made by a customer either as routine or when the components reliability fell into doubt. This of course would mean that the suppliers could be held responsible for maintaining not only quality but also reliability levels.

It could be said that reliability assurance would ensure that the original design of the component was satisfactory, and quality assurance would ensure that production components were manufactured according to this design. However once the design was accepted and production well established these two functions would work hand in hand.

Thus a Reliability Assurance Certificate would provide the customers Reliability Departments with a solid basis on which to place their expectations from their suppliers.

8.5-b - Reliability - Stores relationship

As demonstrated earlier in this report the gleaning of relevant information from warranty return claims proves to be quite difficult unless a comprehensive information feed-back system is employed.

One means of by-passing the necessity for this type of feed-back could be to study the day-to-day stock turnover in the stores which supply the replacement parts to service. Thus, since any failure epidemic occurring in service would be reflected almost immediately by the output from these stores, a rapid early warning system could be implemented based on information concerning this output.

Care would be required to ensure that a true picture was given by this information. For example not all items replaced in service are ultimately found to be defect, therefore it would be necessary to deduct the number returned undefective, from the total issued by the stores. This would give a figure for the number of items failed as opposed to those merely returned.

A further lag due to the storage capacity of the distributors who replace the failed items.

The reliability service stores relationship could also work the opposite way since it would be possible for the results of a reliability analysis to be related to the total population, thus giving a prediction of parts usage and an indication of the necessary stores capacity to any given item.

9 - GUIDELINES FOR PREDICTING THE RELIABILITY OF PRODUCTS FROM SMALL SAMPLES

It is now possible to draw up a set of guidelines which, although of general use, are particularly valuable when it is only possible to life-test a small sample.

1) Use median ranks to estimate the cumulative percentage of the population failed.

According to accuracy requirements and time available, instruct the computer to use either the exact but slow formula, or Bernard's formula, which is extremely fast but approximate. For small smaples (up to 6 or 7), the first method is preferable.

2) Beware of rejecting apparently spurious points, when they represent a significant part of the available data. (Rejecting two "inconvenient" points out of, say, seven probably amounts to forcing the data to fit preconceived ideas!).

10 - COMPUTER PROGRAMME

10.1 - Generalities

The computer programme has been written in BASIC for a Commodore PET computer 4032 and has been recorded on a standard tape cassette and on a "Minidisk".

The programme is suitable for truncated or completed tests, with or without suspensions, for the analysis of up to 50 items.

The data can be either typed-in directly, in any order, or can be recorded (again in any order) in a special portion of the programme.

The data and all input instructions, as well as the intermediate and final results, are displayed on the screen; if a printer is available, the final results can be plotted and tabulated on paper.

10.2 - Characteristics of the programme

I - Loading

The programme is loaded using either cassette or disc.

II - Print Heading 540-620

As soon as the command RUN is given, first the screen is cleared of all previous information and then the following words appear on the screen: "THIS PROGRAMME IS SUITABLE FOR TRUNCATED OR COMPLETED TESTS, WITH OR WITHOUT SUSPENSIONS, OF PR COMPONENTS.

I IS THE NUMBER AT WHICH THE TEST IS TRUNCATED;

MR IS THE MEDIAN RANK

R IS THE RELIABILITY

L IS THE FAILURE RATE

A ARE THE LIFE DATA".

III - Data Input 630-1010

Pre-recorded data can now be read by typing RE, or data can be input after typing IN. In the latter case, the following instructions appear on the screen: "GIVE 1st VALUE OF A (ENTER THE LIFE, FOLLOWED BY A COMMA AND THEN BY AN F FOR A FAILURE OR AN S FOR A SUSPENSION. TO END, ENTER 99999,)". The operator now enters the first datum available and presses the return key. The computer now instructs: "GIVE 2nd VALUE OF A ETC.". The operator enters second datum and presses the return key. This process is repeated until all data are entered, when the operator should enter the rogue value "99999," to get the programme going. Notice that the data can be entered in any order whatsoever, since the computer is programmed to rearrange all the data in order of increasing value.

IV - Data Sorting

The data are now sorted in increasing order.

V - Tabulation of Data

Item numbers, life data and failure mode are now tabulated on the screen in the correct order of increasing values.

VI - Alterations to Data

The data must now be checked and, if any mistakes were made, alterations can be typed in. The corrected data are again sorted in increasing order and displayed on the screen.

VII - Input No. of Items to be tested

The following words now appear on the screen;

"WHAT IS THE NUMBER PR OF ITEMS TO BE TESTED?"

The operator must now input the number PR .

VIII - New Rank Orders 1340-1470

If there are any suspensions, the computer calculates the New Rank Order.

Tabulation 1480-1580

The computer then tabulates four columns of data: the item number from 1 to I, the life data A, the failure mode (F or S), and the New Rank Order for all items from 1 to I. If there are no suspensions, the New Rank Order is the same as the item number.

IX - Choice of formula for calculating the Median Ranks

F 1590-1710

The computer now asks: "IF YOU WISH TO CALCULATE THE MEDIAN RANKS F(K) USING THE ACCURATE BUT VERY SLOW BINOMIAL FORMULA, TYPE B1. IF YOU WISH TO USE THE APPROXIMATE BUT VERY FAST BERNARD'S FORMULA, TYPE BE".

X - Median Ranks

As soon as the operator types in the required choice, the computer calculates and displays the normal Median Ranks F(K) for all the PR items, assuming there are no suspensions.

XI - New Median Ranks 2070-2250

The computer calculates the New Median Ranks, assuming there are some suspensions, and displays four columns of data: The item numbers, the life data, the new rank order and the new median ranks for those items only which are failures, ignoring suspensions.

XII - Various Calculations 2260-2480

The computer calculates X=ln A, $\sum x$, Cumulative reliability R=1-MR, failure rate L=(-ln(1-MR))/A, y=lnln(1/(1-MR)), $\sum Y$, $\sum x^2$, $\sum xy$, and $\sum y^2$.

XIII - Tabulation 3400-3470

The computer now tabulates life data A, new Median Ranks MR, $X=\ln A$, and $Y = \ln\ln \frac{1}{1-MR}$.

XIV - Weibull plot

The computer displays a Weibull plot of X against Y.

XV-Correction for $t_o \neq 0$(4970-5700)

The following words now appear on the screen: "IS THE WEIBULL PLOT A STRAIGHT LINE? TYPE YES OR NO".

If the points are approximately on a straight line, the operator types YES and the programme continues. See Section 10.2-XVI.

If the points are not on a straight line, the operator types NO. The computer calculates and displays: YM and its corresponding value M2; the equation of the best fitting curve; and the roots AX of the equation of the best fitting curve.

The computer now requests: "GIVE SUITABLE VALUE OF AX CORRESPONDING TO M2".

Remembering that a correct value of AX must be comprised between the smallest and the largest of the life data, the operator can easily discard the unsuitable value of AX and input the suitable value, corresponding to t_2.

The computer then calculates and displays t_o.

The following words now appear on the screen: "IS WEIBULL PLOT CONCAVE TO THE RIGHT OR TO THE LEFT?" The operator must type RIGHT or LEFT as need be. The computer then adds or subtracts t_o, as required, from all the original life data, and goes back to stage 10.2-VIII.

XVI - Display type of test

One of the following headings is displayed, as required:

Either:

TRUNCATED TEST WITHOUT SUSPENSIONS OF 1 OUT OF PR ITEMS

or:

COMPLETED TEST WITHOUT SUSPENSIONS OF PR ITEMS

or:

TRUNCATED TEST WITH SUSPENSIONS OF 1 OUT OF PR ITEMS

or:

COMPLETED TEST WITH SUSPENSIONS OF PR ITEMS

XVII - Tabulate A, MR, R, L.

Immediately under the heading, the computer now tabulates the life data t, the New Median Ranks MR, the reliability R and the failure rate L.

XVIII - Calculate M, N, CL, RC

The computer calculates the intercept M and the slope N of the best fitting line: Y = M + NX, the characteristic life CL and the Regression Coefficient RC.

XIX - Calculate Γ and μ

The computer calculates the gamma function G and the mean life μ = CL x G.

XX - Final display

The computer now displays heading (as at section 10.2-XVI), tabulates A, X, MR, Y, R and L, and displays M, N, shape parameter, correlation coefficient, characteristic life, mean life, B1 life, B10 life, B20 life, B50 life, YM, M2, AX, AO, PR, I.

XXI - Weibull plot

On depressing any one key, the computer now displays the Weibull plot of $y = \ln\ln \frac{1}{1-F(t)}$ on the ordinate against $x = \ln t$ on the abscissa.

XXII - Linear plot

On depressing any one key the computer now displays the plot of the median ranks F(t) on the ordinate against the life data t on the abscissa.

XXIII - Failure rate plot

On depressing any one key, the computer displays the plot of the failure life L on the ordinate against the life data t on the abscissa.

XXIV - Print-out

The following words now appear on the screen:

"DO YOU REQUIRE A PRINT OUT?

PLEASE ANSWER YES OR NO".

If the operator types NO, the programme ends. If the operator types YES, and if a printer is available,a printout is produced showing all items described in 10.2-XX to XXIII inclusive, and the programme ends.

A number of print-outs are collected in a special pocket at the end of this thesis. Some are based on imaginary data, and some on actual tests discussed in the previous Chapter. These print-outs are accompanied by tables and Weibull plots, obtained in the traditional manner, for comparison purposes and to show the large amount of labour and time that the computer programme can save.

11 - GENERAL CONCLUSIONS

Weibull probability plotting as a computer-assisted graphical method for data analysis is simple, fast and flexible. It is simple because it requires only a few easy operations that are easily learned: a) Enter the data in the computer programme, in any order; b) Analyse plots produced; c) If necessary, remove or shift spurious data (see Chapter 9); d) If plot is curved, ask computer to calculate t_o and to re-enter the data as $(t-t_o)$; e) Use answers in computer's print-out to take necessary decisions. It is fast because: a) Erroneous data are identified by specific shape patterns in the plots; b) On linear plots, the Weibull parameter values are obtained directly by a visual observation of the computer's print-out. It is flexible because: a) One picture is worth ten thousand words; b) The probability points, or percentiles, corresponding to many values of the variable of interest, may be determined (visually).

12 - RECOMMENDATIONS FOR FUTURE WORK

The Author thinks that the most valid improvement to the Computer Programme would be the addition of routines for calculating the 5% and 95° Ranks, and for plotting the confidence limits. The subject has been dealt with, analytically and graphically, in section 6.

APPENDIX A

MEDIAN RANKS (PER CENT)

SAMPLE SIZE

RANK ORDER	1	2	3	4	5	6	7	8	9	10
1	50.0	29.2	20.6	15.9	12.9	10.9	9.4	8.3	7.4	6.6
2		70.7	50.0	38.5	31.3	26.4	22.8	20.1	17.9	16.2
3			79.3	61.4	50.0	42.1	36.4	32.0	28.6	25.8
4				84.0	68.6	57.8	50.0	44.0	39.3	35.5
5					87.0	73.5	63.5	55.9	50.0	45.1
6						89.0	77.1	67.9	60.6	54.8
7							90.5	79.8	71.3	64.4
8								91.7	82.0	74.1
9									92.5	83.7
10										93.3

MEDIAN RANKS

SAMPLE SIZE

RANK ORDER	11	12	13	14	15	16	17	18	19	20
1	6.1	5.6	5.1	4.8	4.5	4.2	3.9	3.7	3.5	3.4
2	14.7	13.5	12.5	11.7	10.9	10.2	9.6	9.1	8.6	8.2
3	23.5	21.6	20.0	18.6	17.4	16.3	15.4	14.5	13.8	13.1
4	32.3	29.7	27.5	25.6	23.9	22.4	21.1	20.0	18.9	18.0
5	41.1	37.8	35.0	32.5	30.4	28.5	26.9	25.4	24.1	22.9
6	50.0	45.9	42.5	39.5	36.9	34.7	32.7	30.9	29.3	27.8
7	58.8	54.0	50.0	46.5	43.4	40.8	38.4	36.3	34.4	32.7
8	67.6	62.1	57.4	53.4	50.0	46.9	44.2	41.8	39.6	37.7
9	76.4	70.2	64.9	60.4	56.5	53.0	50.0	47.2	44.8	42.6
10	85.2	78.3	72.4	67.4	63.0	59.1	55.7	52.7	50.0	47.5
11	93.8	86.4	79.9	74.3	69.5	65.2	61.5	58.1	55.1	52.4
12		94.3	87.4	81.3	76.0	71.4	67.2	63.6	60.3	57.3
13			94.8	88.2	82.5	77.5	73.0	69.0	65.5	62.2
14				95.1	89.0	83.6	78.8	74.5	70.6	67.2
15					95.4	89.7	84.5	79.9	75.8	72.1
16						95.7	90.3	85.4	81.0	77.0
17							96.0	90.8	86.1	81.9
18								96.2	91.3	86.8
19									96.4	91.7
20										96.5

APPENDIX A

MEDIAN RANKS (PER CENT)

SAMPLE SIZE

RANK ORDER	21	22	23	24	25	26	27	28	29	30
1	3.2	3.1	2.9	2.8	2.7	2.6	2.5	2.4	2.3	2.2
2	7.8	7.5	7.1	6.8	6.6	6.3	6.1	5.9	5.7	5.5
3	12.5	11.9	11.4	10.9	10.5	10.1	9.7	9.4	9.1	8.8
4	17.2	16.4	15.7	15.0	14.4	13.9	13.4	12.9	12.5	12.1
5	21.8	20.9	20.0	19.1	18.4	17.7	17.0	16.4	15.9	15.3
6	26.5	25.3	24.2	23.2	22.3	21.5	20.7	20.0	19.3	18.6
7	30.2	29.8	28.5	27.4	26.3	25.3	24.3	23.5	22.7	21.9
8	35.9	34.3	32.8	31.5	30.2	29.1	28.0	27.0	26.1	25.2
9	40.8	38.8	37.1	35.8	34.2	32.9	31.7	30.5	29.5	28.5
10	45.3	43.2	41.4	39.7	38.1	36.7	35.3	34.1	32.9	31.8
11	50.0	47.7	45.7	43.8	42.1	40.5	39.0	37.6	36.3	35.1
12	54.6	52.2	50.0	47.9	46.0	44.3	42.6	41.1	39.7	38.4
13	59.3	56.7	54.2	52.0	50.0	48.1	46.3	44.7	43.1	41.7
14	64.0	61.1	58.5	56.1	53.9	51.8	50.0	48.2	46.5	45.0
15	68.7	65.8	62.8	60.2	57.8	55.6	53.6	51.7	50.0	48.3
16	73.4	70.1	67.1	64.3	61.8	59.4	57.3	55.2	53.4	51.6
17	78.1	74.6	71.4	68.4	65.7	63.2	60.9	58.8	56.8	54.9
18	82.7	79.0	75.7	72.5	69.7	67.0	64.6	62.3	60.2	58.2
19	87.4	83.5	79.9	76.7	73.6	70.8	68.2	65.8	63.6	61.5
20	92.1	88.0	84.2	80.8	77.6	74.6	71.9	69.4	67.0	64.8
21	96.7	92.4	88.5	84.9	81.5	78.4	75.6	72.9	70.4	68.1
22		96.8	92.8	89.0	85.5	82.2	79.2	76.4	73.8	71.4
23			97.0	93.1	89.4	86.0	82.9	79.9	77.2	74.7
24				97.1	93.3	89.8	86.5	83.5	80.6	78.0
25					97.2	93.6	90.2	87.0	84.0	81.3
26						97.3	93.8	90.5	87.4	84.6
27							97.4	94.0	90.8	87.8
28								97.5	94.2	91.1
29									97.6	94.4
30										97.7

APPENDIX A

MEDIAN RANKS (PER CENT)

SAMPLE SIZE

RANK ORDER	31	32	33	34	35	36	37	38	39	40
1	2.2	2.1	2.0	2.0	1.9	1.9	1.8	1.8	1.7	1.7
2	5.3	5.1	5.0	4.8	4.7	4.6	4.4	4.3	4.2	4.1
3	8.5	8.2	8.0	7.7	7.5	7.3	7.1	6.9	6.7	6.6
4	11.7	11.3	11.0	10.6	10.3	10.1	9.8	9.5	9.3	9.1
5	14.9	14.4	14.0	13.6	13.2	12.8	12.5	12.1	11.8	11.5
6	18.0	17.5	17.0	16.5	16.0	15.6	15.1	14.7	14.4	14.0
7	21.2	20.6	20.0	19.4	18.8	18.3	17.8	17.3	16.9	16.5
8	24.4	23.7	23.0	22.3	21.7	21.1	20.5	20.0	19.4	19.0
9	27.6	26.8	26.0	25.2	24.5	23.8	23.2	22.6	22.0	21.4
10	30.8	29.9	29.0	28.1	27.3	26.6	25.8	25.2	24.5	23.9
11	34.0	32.9	32.0	31.0	30.1	29.3	28.5	27.8	27.1	26.4
12	37.2	36.0	35.0	33.9	33.0	32.1	31.2	30.4	29.6	28.9
13	40.4	39.1	38.0	36.8	35.8	34.8	33.9	33.0	32.2	31.4
14	43.6	42.2	41.0	39.8	38.6	37.6	36.6	35.6	34.7	33.8
15	46.8	45.3	44.0	42.7	41.5	40.3	39.2	38.2	37.2	36.3
16	50.0	48.4	47.0	45.6	44.3	43.1	41.9	40.8	39.8	38.8
17	53.1	51.5	50.0	48.5	47.1	45.8	44.6	43.4	42.3	41.3
18	56.3	54.6	52.9	51.4	50.0	48.6	47.3	46.0	44.9	43.8
19	59.5	57.7	55.9	54.3	52.8	51.3	50.0	48.6	47.4	46.2
20	62.7	60.8	58.9	57.2	55.6	54.1	52.6	51.3	50.0	48.7
21	65.9	63.9	61.9	60.1	58.4	56.8	55.3	53.9	52.5	51.2
22	69.1	67.0	64.9	63.1	61.3	59.6	58.0	56.5	55.0	53.7
23	72.3	70.0	67.9	66.0	64.1	62.3	60.7	59.1	57.6	56.1
24	75.5	73.1	70.9	68.9	66.9	65.1	63.3	61.7	60.1	58.6
25	78.7	76.2	73.9	71.8	69.8	67.8	66.0	64.3	62.7	61.1
26	81.9	79.3	76.9	74.7	72.6	70.6	68.7	66.9	65.2	63.6
27	85.0	82.4	79.9	77.6	75.4	73.3	71.4	69.5	67.7	66.1
28	88.2	85.5	82.9	80.5	78.2	76.1	74.1	72.1	70.3	68.5
29	91.4	88.6	85.9	83.4	81.1	78.8	76.7	74.7	72.8	71.0
30	94.6	91.7	88.9	86.3	83.9	81.6	79.4	77.3	75.4	73.5
31	97.7	94.8	91.9	89.3	86.7	84.3	82.1	79.9	77.9	76.0
32		97.8	94.9	92.2	89.6	87.1	84.8	82.6	80.5	78.5
33			97.9	95.1	92.4	89.8	87.4	85.2	83.0	80.9
34				97.9	95.2	92.6	90.1	87.8	85.5	83.4
35					98.0	95.3	92.8	90.4	88.1	85.9
36						98.0	95.5	93.0	90.8	88.4
37							98.1	95.6	93.2	90.8
38								98.1	95.7	93.3
39									98.2	95.8
40										98.2

APPENDIX A

MEDIAN RANKS (PER CENT)
SAMPLE SIZE

RANK ORDER	41	42	43	44	45	46	47	48	49	50
1	1.6	1.6	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.3
2	4.0	3.9	3.8	3.7	3.7	3.6	3.5	3.4	3.4	3.3
3	6.4	6.3	6.1	6.0	5.8	5.7	5.6	5.5	5.4	5.3
4	8.8	8.6	8.4	8.2	8.0	7.9	7.7	7.5	7.4	7.2
5	11.3	11.0	10.7	10.5	10.3	10.0	9.8	9.6	9.4	9.2
6	13.7	13.3	13.0	12.7	12.5	12.2	11.9	11.7	11.4	11.2
7	16.1	15.7	15.3	15.0	14.7	14.3	14.0	13.7	13.5	13.2
8	18.5	18.1	17.6	17.2	16.9	16.5	16.2	15.8	15.5	15.2
9	20.9	20.4	20.0	19.5	19.1	18.7	18.3	17.9	17.5	17.2
10	23.3	22.8	22.3	21.8	21.3	20.8	20.4	20.0	19.5	19.2
11	25.8	25.2	24.6	24.0	23.5	23.0	22.5	22.0	21.6	21.1
12	28.2	27.5	26.9	26.3	25.7	25.1	24.6	24.1	23.6	23.1
13	30.6	29.9	29.2	28.5	27.9	27.3	26.7	26.2	25.6	25.1
14	33.0	32.2	31.5	30.8	30.1	29.4	28.8	28.2	27.7	27.1
15	35.4	34.6	33.8	33.0	32.3	31.6	30.9	30.3	29.7	29.1
16	37.9	37.0	36.1	35.3	34.5	33.8	33.1	32.4	31.7	31.1
17	40.3	39.3	38.4	37.5	36.7	35.9	35.2	34.4	33.7	33.1
18	42.7	41.7	40.7	39.8	38.9	38.1	37.3	36.5	35.8	35.1
19	45.1	44.0	43.0	42.1	41.1	40.2	39.4	38.6	37.8	37.0
20	47.5	46.4	45.3	44.3	43.3	42.4	41.5	40.6	39.8	39.0
21	50.0	48.8	47.6	46.6	45.5	44.6	43.6	42.7	41.8	41.0
22	52.4	51.1	50.0	48.8	47.7	46.7	45.7	44.8	43.9	43.0
23	54.8	53.5	52.3	51.1	50.0	48.9	47.8	46.8	45.9	45.0
24	57.2	55.9	54.6	53.3	52.2	51.0	50.0	48.9	47.9	47.0
25	59.6	58.2	56.9	55.6	54.4	53.2	52.1	51.0	50.0	49.0
26	62.0	60.6	59.2	57.8	56.6	55.3	54.2	53.1	52.0	50.9
27	64.5	62.9	61.5	60.1	58.8	57.5	56.3	55.1	54.0	52.9
28	66.9	65.3	63.8	62.4	61.0	59.7	58.4	57.2	56.0	54.9
29	69.3	67.7	66.1	64.6	63.2	61.8	60.5	59.3	58.1	56.9
30	71.7	70.0	68.4	66.9	65.4	64.0	62.6	61.3	60.1	58.9
31	74.1	72.4	70.7	69.1	67.6	66.1	64.7	63.4	62.1	60.9
32	76.6	74.8	73.0	71.4	69.8	68.3	66.9	65.5	64.1	62.9
33	79.0	77.1	75.3	73.6	72.0	70.5	69.0	67.5	66.2	64.8
34	81.4	79.5	77.6	75.9	74.2	72.6	71.1	69.6	68.2	66.8
35	83.8	81.8	79.9	78.1	76.4	74.8	73.2	71.7	70.2	68.8
36	86.2	84.2	82.3	80.4	78.6	76.9	75.3	73.7	72.2	70.8
37	88.7	86.6	84.6	82.7	80.8	79.1	77.4	75.8	74.3	72.8
38	91.1	88.9	86.9	84.9	83.0	81.2	79.5	77.9	76.3	74.8
39	93.5	91.3	89.2	87.2	85.2	83.4	81.6	79.9	78.3	76.8
40	95.9	93.6	91.5	89.4	87.4	85.6	83.7	82.0	80.4	78.8
41	98.3	96.0	93.8	91.7	89.6	87.7	85.9	84.1	82.4	80.7
42		98.3	96.1	93.9	91.9	89.9	88.0	86.2	84.4	82.7
43			98.4	96.2	94.1	92.0	90.1	88.2	86.4	84.7
44				98.4	96.2	94.2	92.2	90.3	88.5	86.7
45					98.4	96.3	94.3	92.4	90.5	88.7
46						98.5	96.4	94.4	92.5	90.7
47							98.5	96.5	94.5	92.7
48								98.5	96.5	94.6
49									98.5	96.6
50										98.6

APPENDIX B

FIVE PERCENT RANKS

SAMPLE SIZE

RANK ORDER	1	2	3	4	5	6	7	8	9	10
1	5.0	2.5	1.6	1.2	1.0	0.8	0.7	0.6	0.5	0.5
2		22.3	13.5	9.7	7.8	6.2	5.3	4.6	4.1	3.6
3			36.8	24.8	18.9	15.3	12.8	11.1	9.7	8.7
4				47.2	34.2	27.1	22.5	19.2	16.8	15.0
5					54.9	41.8	34.1	28.9	25.1	22.2
6						60.6	47.9	40.0	34.4	30.3
7							65.1	52.9	45.0	39.3
8								68.7	57.0	49.3
9									71.6	60.5
10										74.1

FIVE PERCENT RANKS

SAMPLE SIZE

RANK ORDER	11	12	13	14	15	16	17	18	19	20
1	0.4	0.4	0.3	0.3	0.3	0.3	0.3	0.2	0.2	0.2
2	3.3	3.0	2.8	2.6	2.4	2.2	2.1	2.0	1.9	1.8
3	7.8	7.1	6.6	6.1	5.6	5.3	4.9	4.7	4.4	4.2
4	13.5	12.2	11.2	10.4	9.6	9.0	8.4	7.9	7.5	7.1
5	19.9	18.1	16.5	15.2	14.1	13.2	12.3	11.6	10.9	10.4
6	27.1	24.5	22.3	20.6	19.0	17.7	16.6	15.6	14.7	13.9
7	34.9	31.5	28.7	26.3	24.3	22.6	21.1	19.8	18.7	17.7
8	43.5	39.0	35.4	32.5	29.9	27.8	26.0	24.3	22.9	21.7
9	52.9	47.2	42.7	39.0	35.9	33.3	31.0	29.1	27.3	25.8
10	63.5	56.1	50.5	45.9	42.2	39.1	36.4	34.0	32.0	30.1
11	76.1	66.1	58.9	53.4	48.9	45.1	41.9	39.2	36.8	34.6
12		77.9	68.8	61.4	56.0	51.5	47.8	44.9	41.8	39.3
13			79.4	70.3	63.6	58.3	53.9	50.2	47.0	44.1
14				80.7	72.0	65.6	60.4	56.1	52.4	49.2
15					81.8	73.6	67.3	62.3	58.0	54.4
16						82.9	74.9	68.9	64.0	59.8
17							83.8	76.2	70.4	65.6
18								84.6	77.3	71.7
19									85.4	78.3
20										86.0

APPENDIX B

FIVE PERCENT RANKS

SAMPLE SIZE

RANK ORDER	21	22	23	24	25	26	27	28	29	30
1	0.2	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.1
2	1.7	1.6	1.5	1.5	1.4	1.3	1.3	1.2	1.2	1.1
3	4.0	3.8	3.6	3.4	3.3	3.2	3.0	2.9	2.8	2.7
4	6.7	6.4	6.1	5.9	5.6	5.4	5.2	5.0	4.8	4.6
5	9.8	9.4	8.9	8.5	8.2	7.8	7.5	7.3	7.0	6.8
6	13.2	12.6	12.0	11.4	11.0	10.5	10.1	9.7	9.4	9.0
7	16.8	15.9	15.2	14.5	13.9	13.3	12.8	12.3	11.9	11.4
8	20.5	19.5	18.6	17.7	17.0	16.3	15.6	15.0	14.5	14.0
9	24.4	23.2	22.1	21.1	20.2	19.3	18.6	17.9	17.2	16.6
10	28.5	27.1	25.8	24.6	23.5	22.5	21.6	20.8	20.0	19.3
11	32.8	31.1	29.6	28.2	26.9	25.8	24.7	23.8	22.9	22.1
12	37.1	35.2	33.5	31.9	30.5	29.2	28.0	26.9	25.8	24.9
13	41.7	39.5	37.5	35.7	34.1	32.6	31.3	30.0	28.9	27.8
14	46.4	43.9	41.6	39.6	37.8	36.2	34.6	33.3	32.0	30.8
15	51.2	48.4	45.9	43.7	41.6	39.8	38.1	36.6	35.2	33.8
16	56.3	53.1	50.3	47.8	45.6	43.5	41.7	40.0	38.4	36.9
17	61.5	58.0	54.9	52.1	49.6	47.3	45.3	43.4	41.7	40.1
18	67.0	63.0	59.6	56.5	53.7	51.3	49.0	47.0	45.1	43.3
19	72.9	68.4	64.5	61.0	58.0	55.3	52.8	50.6	48.5	46.6
20	79.3	74.0	69.6	65.8	62.4	59.4	56.7	54.3	52.0	50.0
21	86.7	80.1	75.0	70.7	67.0	63.7	60.7	58.1	55.7	53.4
22		87.2	80.9	76.0	71.8	68.1	64.9	62.0	59.4	57.0
23			87.7	81.7	76.8	72.8	69.2	66.0	63.2	60.6
24				88.2	82.3	77.7	73.7	70.2	67.1	64.2
25					88.7	83.0	78.4	74.5	71.1	68.1
26						89.1	83.6	79.1	75.3	72.0
27							89.4	84.1	79.8	76.1
28								89.8	84.6	80.4
29									90.1	85.1
30										90.4

APPENDIX B

FIVE PERCENT RANKS
SAMPLE SIZE

RANK ORDER	31	32	33	34	35	36	37	38	39	40
1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
2	1.1	1.1	1.0	1.0	1.0	0.9	0.9	0.9	0.9	0.8
3	2.6	2.6	2.5	2.4	2.3	2.3	2.2	2.1	2.1	2.0
4	4.5	4.3	4.2	4.1	3.9	3.8	3.7	3.6	3.5	3.4
5	6.5	6.3	6.1	5.9	5.8	5.6	5.4	5.3	5.1	5.0
6	8.7	8.4	8.2	7.9	7.7	7.5	7.3	7.1	6.9	6.7
7	11.1	10.7	10.4	10.0	9.7	9.4	9.2	8.9	8.7	8.5
8	13.5	13.0	12.6	12.2	11.9	11.5	11.2	10.9	10.6	10.3
9	16.0	15.5	15.0	14.5	14.1	13.7	13.3	12.9	12.6	12.2
10	18.6	18.0	17.4	16.9	16.3	15.9	15.4	15.0	14.6	14.2
11	21.3	20.6	19.9	19.3	18.7	18.1	17.6	17.1	16.6	16.2
12	24.0	23.2	22.5	21.7	21.1	20.4	19.8	19.3	18.8	18.3
13	26.8	25.9	25.1	24.3	23.5	22.8	22.1	21.5	20.9	20.4
14	29.7	28.7	27.7	26.8	26.0	25.2	24.5	23.8	23.1	22.5
15	32.6	31.5	30.4	29.5	28.5	27.7	26.9	26.1	25.4	24.7
16	35.6	34.4	33.2	32.1	31.1	30.2	29.3	28.4	27.6	26.9
17	38.6	37.3	36.0	34.8	33.7	32.7	31.7	30.8	30.0	29.1
18	41.7	40.3	38.9	37.6	36.4	35.3	34.2	33.2	32.3	31.4
19	44.9	43.3	41.8	40.4	39.1	37.9	36.8	35.7	34.7	33.7
20	48.1	46.4	44.8	43.3	41.9	40.6	39.3	38.2	37.1	36.1
21	51.4	49.5	47.8	46.2	44.7	43.3	41.9	40.7	39.5	38.4
22	54.8	52.7	50.9	49.1	47.5	46.0	44.6	43.3	42.0	40.8
23	58.2	56.0	54.0	52.1	50.4	48.8	47.3	45.9	44.5	43.3
24	61.7	59.3	57.2	55.2	53.3	51.6	50.0	48.5	47.1	45.7
25	65.3	62.8	60.4	58.3	56.3	54.5	52.8	51.2	49.6	48.2
26	69.0	66.3	63.8	61.5	59.4	57.4	55.6	53.9	52.3	50.8
27	72.8	69.9	67.2	64.7	62.5	60.4	58.4	56.6	54.9	53.3
28	76.8	73.6	70.7	68.1	65.6	63.4	61.3	59.4	57.6	55.9
29	81.0	77.5	74.3	71.5	68.9	66.5	64.3	62.3	60.3	58.6
30	85.5	81.6	78.1	75.0	72.2	69.7	67.3	65.2	63.1	61.2
31	90.7	86.0	82.1	78.7	75.7	72.9	70.4	68.1	66.0	64.0
32		91.0	86.4	82.6	79.3	76.3	73.6	71.1	68.9	66.7
33			91.3	86.7	83.0	79.8	76.9	74.3	71.8	69.6
34				91.5	87.1	83.5	80.3	77.5	74.9	72.5
35					91.7	87.4	83.9	80.8	78.0	75.4
36						92.0	87.8	84.3	81.3	78.5
37							92.2	88.1	84.7	81.7
38								92.4	88.4	85.0
39									92.6	88.6
40										92.7

APPENDIX B

FIVE PERCENT RANKS

SAMPLE SIZE

RANK ORDER	41	42	43	44	45	46	47	48	49	50
1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
2	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	0.7	0.7
3	2.0	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.6	1.6
4	3.4	3.3	3.2	3.1	3.0	3.0	2.9	2.8	2.8	2.7
5	4.9	4.8	4.6	4.5	4.4	4.3	4.2	4.1	4.1	4.0
6	6.5	6.4	6.2	6.1	5.9	5.8	5.7	5.5	5.4	5.3
7	8.2	8.0	7.8	7.7	7.5	7.3	7.2	7.0	6.9	6.7
8	10.0	9.5	9.6	9.3	9.1	8.9	8.7	8.5	8.3	8.2
9	11.9	11.6	11.3	11.1	10.8	10.6	10.3	10.1	9.9	9.7
10	13.8	13.5	13.1	12.8	12.5	12.2	12.0	11.7	11.5	11.2
11	15.8	15.4	15.0	14.6	14.3	14.0	13.7	13.4	13.1	12.8
12	17.8	17.3	16.9	16.5	16.1	15.7	15.4	15.1	14.7	14.4
13	19.8	19.3	18.9	18.4	18.0	17.5	17.2	16.8	16.4	16.1
14	21.9	21.4	20.8	20.3	19.8	19.4	18.9	18.5	18.1	17.7
15	24.0	23.4	22.8	22.3	21.7	21.2	20.8	20.3	19.9	19.4
16	26.2	25.5	24.9	24.3	23.7	23.1	22.6	22.1	21.6	21.2
17	28.4	27.6	26.9	26.3	25.6	25.0	24.5	23.9	23.4	22.9
18	30.6	29.8	29.0	28.3	27.6	27.0	26.4	25.8	25.2	24.7
19	32.8	32.0	31.2	30.4	29.6	28.9	28.3	27.6	27.0	26.5
20	35.1	34.2	33.3	32.5	31.7	30.9	30.2	29.5	28.9	28.3
21	37.4	36.4	35.5	34.6	33.7	32.9	32.2	31.5	30.8	30.1
22	39.7	38.7	37.7	36.7	35.8	35.0	34.2	33.4	32.6	31.9
23	42.1	41.0	39.9	38.9	37.9	37.0	36.2	35.3	34.5	33.8
24	44.5	43.3	42.2	41.1	40.1	39.1	38.2	37.3	36.5	35.7
25	46.9	45.6	44.4	43.3	42.2	41.2	40.2	39.3	38.4	37.6
26	49.3	48.0	46.7	45.5	44.4	43.3	42.3	41.3	40.4	39.5
27	51.8	50.4	49.1	47.8	46.6	45.5	44.4	43.3	42.4	41.4
28	54.3	52.8	51.4	50.1	48.8	47.6	46.5	45.4	44.4	43.4
29	56.9	55.3	53.8	52.4	51.1	49.8	48.6	47.5	46.4	45.3
30	59.5	57.8	56.2	54.8	53.4	52.0	50.8	49.6	48.4	47.3
31	62.1	60.3	58.7	57.1	55.7	54.3	52.9	51.7	50.5	49.3
32	64.8	62.9	61.2	59.5	58.0	56.5	55.1	53.8	52.6	51.4
33	67.5	65.5	63.7	62.0	60.4	58.8	57.4	56.0	54.7	53.4
34	70.3	68.2	66.3	64.5	62.7	61.1	59.6	58.2	56.8	55.5
35	73.1	70.9	68.9	67.0	65.2	63.5	61.9	60.4	58.9	57.6
36	76.0	73.7	71.5	69.5	67.6	65.9	64.2	62.6	61.1	59.7
37	79.0	76.5	74.3	72.1	70.2	68.3	66.5	64.9	63.3	61.8
38	82.1	79.5	77.0	74.8	72.7	70.8	68.9	67.2	65.5	64.0
39	85.4	82.5	79.9	77.5	75.3	73.3	71.3	69.5	67.8	66.2
40	88.9	85.7	82.9	80.3	78.0	75.8	73.8	71.9	70.1	68.4
41	92.9	89.1	86.0	83.3	80.8	78.4	76.3	74.3	72.4	70.6
42		93.1	89.4	86.8	83.6	81.1	78.9	76.8	74.8	72.9
43			93.2	89.6	86.6	83.9	81.5	79.3	77.2	75.3
44				93.4	89.8	86.9	84.3	81.9	79.7	77.6
45					93.5	90.0	87.2	84.6	82.2	80.1
46						93.6	90.3	87.4	84.9	82.6
47							93.8	90.4	87.7	85.2
48								93.9	90.6	87.9
49									94.0	90.8
50										94.1

APPENDIX C

NINETY FIVE PERCENT RANKS

SAMPLE SIZE

RANK ORDER	1	2	3	4	5	6	7	8	9	10
1	95.0	77.6	63.1	52.7	45.0	39.3	34.8	31.2	28.3	25.8
2		97.4	86.4	75.1	65.7	58.1	52.0	47.0	42.9	39.4
3			98.3	90.2	81.0	72.8	65.8	59.9	54.9	50.6
4				98.7	92.3	84.6	77.4	71.0	65.5	60.6
5					98.9	93.7	87.1	80.7	74.8	69.6
6						99.1	94.6	88.8	83.1	77.7
7							99.2	95.3	90.2	84.9
8								99.3	95.8	91.2
9									99.4	96.3
10										99.4

NINETY FIVE PERCENT RANKS

SAMPLE SIZE

RANK ORDER	11	12	13	14	15	16	17	18	19	20
1	23.8	22.0	20.5	19.2	18.1	17.0	16.1	15.3	14.5	13.9
2	36.4	33.8	31.6	29.6	27.9	26.3	25.0	23.7	22.6	21.6
3	47.0	43.8	41.0	38.5	36.3	34.3	32.6	31.0	29.5	28.2
4	56.4	52.7	49.4	46.5	43.9	41.6	39.5	37.6	35.9	34.3
5	65.0	60.9	57.2	54.0	51.0	48.4	46.0	43.8	41.9	40.1
6	72.8	68.4	64.5	60.9	57.7	54.8	52.1	49.7	47.5	45.5
7	80.0	75.4	71.2	67.4	64.0	60.8	58.0	55.4	52.9	50.7
8	86.4	81.8	77.6	73.6	70.0	66.6	63.5	60.7	58.1	55.8
9	92.1	87.7	83.4	79.3	75.6	72.1	68.9	65.9	63.1	60.6
10	96.6	92.8	88.7	84.7	80.9	77.3	73.9	70.8	67.9	65.3
11	99.5	96.9	93.3	89.5	85.8	82.2	78.8	75.6	72.6	69.8
12		99.5	97.1	93.8	90.3	86.7	83.3	80.1	77.0	74.1
13			99.6	97.4	94.3	90.9	87.6	84.3	81.2	78.2
14				99.6	97.5	94.6	91.5	88.3	85.2	82.2
15					99.6	97.7	95.0	92.0	89.0	86.0
16						99.6	97.8	95.2	92.4	89.5
17							99.6	97.9	95.5	92.8
18								99.7	98.0	95.7
19									99.7	98.1
20										99.7

APPENDIX C

NINETY FIVE PERCENT RANKS

SAMPLE SIZE

RANK ORDER	21	22	23	24	25	26	27	28	29	30
1	13.2	12.7	12.2	11.7	11.2	10.8	10.5	10.1	9.8	9.5
2	20.6	19.8	19.0	18.2	17.6	16.9	16.3	15.8	15.3	14.8
3	27.0	25.9	24.9	23.9	23.1	22.2	21.5	20.8	20.1	19.5
4	32.9	31.5	30.3	29.2	28.1	27.1	26.2	25.4	24.6	23.8
5	38.4	36.9	35.4	34.1	32.9	31.8	30.7	29.7	28.8	27.9
6	43.6	41.9	40.3	38.9	37.5	36.2	35.0	33.9	32.8	31.8
7	48.7	46.8	45.0	43.4	41.9	40.5	39.2	37.9	36.8	35.7
8	53.5	51.5	49.6	47.8	46.2	44.6	43.2	41.8	40.5	39.3
9	58.2	56.0	54.0	52.1	50.3	48.7	47.1	45.6	44.2	42.9
10	62.8	60.4	58.3	56.2	54.3	52.6	50.9	49.3	47.9	46.5
11	67.1	64.7	62.4	60.3	58.3	56.4	54.6	52.9	51.4	49.9
12	71.4	68.8	66.4	64.2	62.1	60.1	58.2	56.5	54.8	53.3
13	75.5	72.8	70.3	68.0	65.8	63.7	61.8	59.9	58.2	56.6
14	79.4	76.7	74.1	71.7	69.4	67.3	65.3	63.3	61.5	59.8
15	83.1	80.4	77.8	75.3	73.0	70.7	68.6	66.6	64.7	63.0
16	86.7	84.0	81.3	78.8	76.4	74.1	71.9	69.9	67.9	66.1
17	90.1	87.3	84.7	82.2	79.7	77.4	75.2	73.0	71.0	69.1
18	93.2	90.5	87.9	85.4	82.9	80.6	78.3	76.1	74.1	72.1
19	95.9	93.5	91.0	88.5	86.0	83.6	81.3	79.1	77.0	75.0
20	98.2	96.1	93.8	91.4	88.9	86.6	84.3	82.0	79.9	77.8
21	99.7	98.3	96.3	94.0	91.7	89.4	87.1	84.9	82.7	80.6
22		99.7	98.4	96.5	94.3	92.1	89.8	87.6	85.4	83.3
23			99.7	98.4	96.6	94.5	92.4	90.2	88.0	85.9
24				99.7	98.5	96.7	94.7	92.6	90.5	88.5
25					99.7	98.6	96.9	94.9	92.9	90.9
26						99.8	98.6	97.0	95.1	93.1
27							99.8	98.7	97.1	95.3
28								99.8	98.7	97.2
29									99.8	98.8
30										99.8

APPENDIX C

NINETY FIVE PERCENT RANKS
SAMPLE SIZE

RANK ORDER	31	32	33	34	35	36	37	38	39	40
1	9.2	8.9	8.6	8.4	8.2	7.9	7.7	7.5	7.3	7.2
2	14.4	13.9	13.5	13.2	12.8	12.5	12.1	11.8	11.5	11.3
3	18.9	18.3	17.8	17.3	16.9	16.4	16.0	15.6	15.2	14.9
4	23.1	22.4	21.8	21.2	20.6	20.1	19.6	19.1	18.6	18.2
5	27.1	26.3	25.6	24.9	24.2	23.6	23.0	22.4	21.9	21.4
6	30.9	30.0	29.2	28.4	27.7	27.0	26.3	25.6	25.0	24.5
7	34.6	33.6	32.7	31.8	31.0	30.2	29.5	28.8	28.1	27.4
8	38.2	37.1	36.1	35.2	34.3	33.4	32.6	31.8	31.0	30.3
9	41.7	40.6	39.5	38.4	37.4	36.5	35.6	34.7	33.9	33.2
10	45.1	43.9	42.7	41.6	40.5	39.5	38.6	37.6	36.8	35.9
11	48.5	47.2	45.9	44.7	43.6	42.5	41.5	40.5	39.6	38.7
12	51.8	50.4	49.0	47.8	46.6	45.4	44.3	43.3	42.3	41.3
13	55.0	53.5	52.1	50.8	49.5	48.3	47.1	46.0	45.0	44.0
14	58.2	56.6	55.1	53.7	52.4	51.1	49.9	48.7	47.6	46.6
15	61.3	59.6	58.1	56.6	55.2	53.9	52.6	51.4	50.3	49.1
16	64.3	62.6	61.0	59.5	58.0	56.6	55.3	54.0	52.8	51.7
17	67.3	65.5	63.9	62.3	60.8	59.3	58.0	56.6	55.4	54.2
18	70.2	68.4	66.7	65.1	63.5	62.0	60.6	59.2	57.9	56.6
19	73.1	71.2	69.5	67.8	66.2	64.6	63.1	61.7	60.4	59.1
20	75.9	74.0	72.2	70.4	68.8	67.2	65.7	64.2	62.8	61.5
21	78.6	76.7	74.8	73.1	71.4	69.7	68.2	66.7	65.2	63.8
22	81.3	79.3	77.4	75.6	73.9	72.2	70.6	69.1	67.6	66.2
23	83.9	81.9	80.0	78.2	76.4	74.7	73.0	71.5	69.9	68.5
24	86.4	84.4	82.5	80.6	78.8	77.1	75.4	73.8	72.3	70.8
25	88.8	86.9	84.9	83.0	81.2	79.5	77.8	76.1	74.5	73.0
26	91.2	89.2	87.3	85.4	83.6	81.8	80.1	78.4	76.8	75.2
27	93.4	91.5	89.5	87.7	85.8	84.0	82.3	80.6	79.0	77.4
28	95.4	93.6	91.7	89.9	88.0	86.2	84.5	82.8	81.1	79.5
29	97.3	95.6	93.8	92.0	90.2	88.4	86.6	84.9	83.3	81.6
30	98.8	97.3	95.7	94.0	92.2	90.5	88.7	87.0	85.3	83.7
31	99.8	98.8	97.4	95.8	94.1	92.4	90.7	89.0	87.3	85.7
32		99.8	98.9	97.5	96.0	94.3	92.6	91.0	89.3	87.7
33			99.8	98.9	97.6	96.1	94.5	92.8	91.2	89.6
34				99.8	98.9	97.6	96.2	94.6	93.0	91.4
35					99.8	99.0	97.7	96.3	94.8	93.2
36						99.8	99.0	97.8	96.4	94.9
37							99.8	99.0	97.8	96.5
38								99.8	99.0	97.9
39									99.8	99.1
40										99.8

APPENDIX C

NINETY FIVE PERCENT RANKS
SAMPLE SIZE

RANK ORDER	41	42	43	44	45	46	47	48	49	50
1	7.0	6.8	6.7	6.5	6.4	6.3	6.1	6.0	5.9	5.8
2	11.0	10.8	10.5	10.3	10.1	9.9	9.7	9.5	9.3	9.1
3	14.5	14.2	13.9	13.6	13.3	13.0	12.7	12.5	12.2	12.0
4	17.8	17.4	17.0	16.6	16.3	16.0	15.6	15.3	15.0	14.7
5	20.9	20.4	20.0	19.6	19.1	18.8	18.4	18.0	17.7	17.3
6	23.9	23.4	22.9	22.4	21.9	21.5	21.0	20.6	20.2	19.8
7	26.8	26.2	25.6	25.1	24.6	24.1	23.6	23.1	22.7	22.3
8	29.6	29.0	28.4	27.8	27.2	26.6	26.1	25.6	25.1	24.6
9	32.4	31.7	31.0	30.4	29.7	29.1	28.6	28.0	27.5	27.0
10	35.1	34.4	33.6	32.9	32.3	31.6	31.0	30.4	29.8	29.3
11	37.8	37.0	36.2	35.4	34.7	34.0	33.4	32.7	32.1	31.5
12	40.4	39.6	38.7	37.9	37.2	36.4	35.7	35.0	34.4	33.7
13	43.0	42.1	41.2	40.4	39.5	38.8	38.0	37.3	36.6	35.9
14	45.6	44.6	43.7	42.8	41.9	41.1	40.3	39.5	38.8	38.1
15	48.1	47.1	46.1	45.1	44.2	43.4	42.5	41.7	41.0	40.2
16	50.6	49.5	48.5	47.5	46.5	45.6	44.8	43.9	43.1	42.3
17	53.0	51.9	50.8	49.8	48.8	47.9	47.0	46.1	45.2	44.4
18	55.4	54.3	53.2	52.1	51.1	50.1	49.1	48.2	47.3	46.5
19	57.8	56.6	55.5	54.4	53.3	52.3	51.3	50.3	49.4	48.5
20	60.2	58.9	57.7	56.6	55.5	54.4	53.4	52.4	51.5	50.6
21	62.5	61.2	60.0	58.8	57.7	56.6	55.5	54.5	53.5	52.6
22	64.8	63.5	62.2	61.0	59.8	58.7	57.6	56.6	55.5	54.6
23	67.1	65.7	64.4	63.2	62.0	60.8	59.7	58.6	57.5	56.5
24	69.3	67.9	66.6	65.3	64.1	62.9	61.7	60.6	59.5	58.5
25	71.5	70.1	68.8	67.4	66.2	64.9	63.7	62.6	61.5	60.4
26	73.7	72.3	70.9	69.5	68.2	67.0	65.7	64.6	63.4	62.3
27	75.9	74.4	73.0	71.6	70.3	69.0	67.7	66.5	65.4	64.2
28	78.0	76.5	75.0	73.6	72.3	71.0	69.7	68.5	67.3	66.1
29	80.1	78.5	77.1	75.6	74.3	72.9	71.6	70.4	69.1	68.0
30	82.1	80.6	79.1	77.6	76.2	74.9	73.5	72.3	71.0	69.8
31	84.1	82.6	81.0	79.6	78.2	76.8	75.4	74.1	72.9	71.6
32	86.1	84.5	83.0	81.5	80.1	78.7	77.3	76.0	74.7	73.4
33	88.0	86.4	84.9	83.4	81.9	80.5	79.1	77.8	76.5	75.2
34	89.9	88.3	86.8	85.3	83.8	82.4	81.0	79.6	78.3	77.0
35	91.7	90.1	88.6	87.1	85.6	84.2	82.7	81.4	80.0	78.7
36	93.4	91.9	90.3	88.8	87.4	85.9	84.5	83.1	81.8	80.5
37	95.0	93.5	92.1	90.6	89.1	87.7	86.2	84.8	83.5	82.2
38	96.5	95.1	93.7	92.2	90.8	89.3	87.9	86.5	85.2	83.8
39	97.9	96.6	95.3	93.8	92.4	91.0	89.6	88.2	86.8	85.5
40	99.1	98.0	96.7	95.4	94.0	92.6	91.2	89.8	88.4	87.1
41	99.8	99.1	98.0	96.8	95.5	94.1	92.7	91.4	90.0	88.7
42		99.8	99.1	98.1	96.9	95.6	94.2	92.9	91.6	90.2
43			99.8	99.1	98.1	96.9	95.7	94.4	93.0	91.7
44				99.8	99.2	98.1	97.0	95.8	94.5	93.2
45					99.8	99.2	98.2	97.1	95.3	94.6
46						99.8	99.2	98.2	97.1	95.9
47							99.8	99.2	98.3	97.2
48								99.8	99.2	98.3
49									99.8	99.2
50										99.8

APPENDIX D

THE GAMMA DISTRIBUTION [20]

A random variable X is said to be distributed as the Gamma Distribution if the density function is given by

$$f(x) = \frac{1}{\Gamma(\alpha+1)\beta^{\alpha+1}} x^{\alpha} e^{-\frac{x}{\beta}}, \quad 0 < x < \infty$$

where α and β are parameters with $\alpha > -1$ and $\beta > 0$.

Properties:

Mean $= \mu = \beta(\alpha+1)$

Variance $= \sigma^2 = \beta^2(\alpha+1)$

Standard Deviation $= \sigma = \beta\sqrt{\alpha+1}$

Moment Generating Function $= m_x(t) = (1 - \beta t)^{-(\alpha+1)}, \quad t < \frac{1}{\beta}$

GAMMA FUNCTION*

Values of $\Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1} dx; \quad \Gamma(n+1) = n\Gamma(n)$

n	$\Gamma(n)$	n	$\Gamma(n)$	n	$\Gamma(n)$	n	$\Gamma(n)$
1.00	1.00000	1.25	0.90640	1.50	0.88623	1.75	0.91906
1.01	0.99433	1.26	0.90440	1.51	0.88659	1.76	0.92137
1.02	0.98884	1.27	0.90250	1.52	0.88704	1.77	0.92376
1.03	0.98355	1.28	0.90072	1.53	0.88757	1.78	0.92623
1.04	0.97844	1.29	0.89904	1.54	0.88818	1.79	0.92877
1.05	0.97350	1.30	0.89747	1.55	0.88887	1.80	0.93138
1.06	0.96874	1.31	0.89600	1.56	0.88964	1.81	0.93408
1.07	0.96415	1.32	0.89464	1.57	0.89049	1.82	0.93685
1.08	0.95973	1.33	0.89338	1.58	0.89142	1.83	0.93969
1.09	0.95546	1.34	0.89222	1.59	0.89243	1.84	0.94261
1.10	0.95135	1.35	0.89115	1.60	0.89352	1.85	0.94561
1.11	0.94739	1.36	0.89018	1.61	0.89468	1.86	0.94869
1.12	0.94359	1.37	0.88931	1.62	0.89592	1.87	0.95184
1.13	0.93993	1.38	0.88854	1.63	0.89724	1.88	0.95507
1.14	0.93642	1.39	0.88785	1.64	0.89864	1.89	0.95838
1.15	0.93304	1.40	0.88726	1.65	0.90012	1.90	0.96177
1.16	0.92980	1.41	0.88676	1.66	0.90167	1.91	0.96523
1.17	0.92670	1.42	0.88636	1.67	0.90330	1.92	0.96878
1.18	0.92373	1.43	0.88604	1.68	0.90500	1.93	0.97240
1.19	0.92088	1.44	0.88580	1.69	0.90678	1.94	0.97610
1.20	0.91817	1.45	0.88565	1.70	0.90864	1.95	0.97988
1.21	0.91558	1.46	0.88560	1.71	0.91057	1.96	0.98374
1.22	0.91311	1.47	0.88563	1.72	0.91258	1.97	0.98768
1.23	0.91075	1.48	0.88575	1.73	0.91466	1.98	0.99171
1.24	0.90852	1.49	0.88595	1.74	0.91683	1.99	0.99581
						2.00	1.00000

*For large positive values of x, $\Gamma(x)$ approximates the asymptotic series

$$x^{n} e^{-x} \sqrt{\frac{2\pi}{x}} \left[1 + \frac{1}{12x} + \frac{1}{288x^2} \quad \frac{139}{51840x^3} - \frac{571}{2488320x^4} + \cdots\cdots\right]$$

APPENDIX E

Non-Parametric Statistics

CRITICAL VALUES FOR THE KOLMOGOROV-SMIRNOV ONE-SAMPLE STATISTIC

A sample of size n is drawn from a population with cumulative distribution function F(x). Define the empirical distribution function $F_n(x)$ to be the step function.

$$F_n(x) = \frac{k}{n} \quad \text{for} \quad x_{(i)} \leqslant x \leqslant x_{(i+1)},$$

where k is the number of observations not greater than x. $x_{(1)}, \ldots, x_{(n)}$ denote the sample values arranged in ascending order. Under the null hypothesis that the sample has been drawn from the specified distribution, $F_n(x)$ should be fairly close to F(x). Define

$$D = \max \left| F_n(x) - F(x) \right| .$$

For a two-tailed test this table gives critical values of the sampling distribution of D under the null hypothesis. Reject the hypothetical distribution if D exceeds the tabulated value. If n is over 35, determine the critical values of D by the divisions indicated in the table.

A one-tailed test is provided by the statistic

$$D^+ = \max \left| F_n(x) - F(x) \right| .$$

CRITICAL VALUES FOR THE KOLMOGOROV-SMIRNOV TEST OF GOODNESS OF FIT

Sample Size (n)	Significance Level 0.20	0.15	0.10	0.05	0.01
1	0.900	0.925	0.950	0.975	0.995
2	0.684	0.726	0.776	0.842	0.929
3	0.565	0.597	0.642	0.708	0.829
4	0.494	0.525	0.564	0.624	0.734
5	0.446	0.474	0.510	0.563	0.669
6	0.410	0.436	0.470	0.521	0.618
7	0.381	0.405	0.438	0.486	0.577
8	0.358	0.381	0.411	0.457	0.543
9	0.339	0.360	0.388	0.432	0.514
10	0.322	0.342	0.368	0.409	0.486
11	0.307	0.326	0.352	0.391	0.468
12	0.295	0.313	0.338	0.375	0.450
13	0.284	0.302	0.325	0.361	0.433
14	0.274	0.292	0.314	0.349	0.418
15	0.266	0.283	0.304	0.338	0.404
16	0.258	0.274	0.295	0.328	0.391
17	0.250	0.266	0.286	0.318	0.380
18	0.244	0.259	0.278	0.309	0.370
19	0.237	0.252	0.272	0.301	0.361
20	0.231	0.246	0.264	0.294	0.352
25	0.21	0.22	0.24	0.264	0.32
30	0.19	0.20	0.22	0.242	0.29
35	0.18	0.19	0.21	0.23	0.27
40				0.21	0.25
50				0.19	0.23
60				0.17	0.21
70				0.16	0.19
80				0.15	0.18
90				0.14	
100				0.14	
Asymptotic Formula:	$\frac{1.07}{\sqrt{n}}$	$\frac{1.14}{\sqrt{n}}$	$\frac{1.22}{\sqrt{n}}$	$\frac{1.36}{\sqrt{n}}$	$\frac{1.63}{\sqrt{n}}$

Reject the hypothetical distribution $F(x)$ if $D_n = \max \left| F_n(x) - F(x) \right|$ exceeds the tabulated value.
(For $\alpha = .01$ and $.05$, asymptotic formulas give values which are too high - by 1.5 per cent for $n = 80$.)

The significance level represents the risk of accepting an invalid assumption made about distributions of failure times.

APPENDIX F - Notes on minimum life parameters t_o

Failure-free time

The examples given so far all relate to cases where $t_o = 0$, i.e. there is no failure-free time. Of course this is not necessarily the case in reliability work.

Case 1

If an item has a finite (positive) failure-free time under test, e.g. a fatigue test specimen, the failure data will plot as a curve, seen convex from above, since the transformation to achieve the Weibull scales assumes that the data fit a two-parameter distribution. The effect of a finite life is to shift the age of failure to the left.

Case 2

It is possible to have an apparent negative value for t_o, for example if the items under test had accumulated unrecorded operating time before the start of the test. In this case the curve will appear concave from above - The effect of an apparent negative value for t_o is to shift the age of failure to the right.

Procedure

Either way, the data are re-plotted, with the value of t_o as calculated on page 99 subtracted elgebraically from each life value. The life parameters estimated from the plot must then have the value of t_o added algebraically to give the true life values (see page 175).

Discussion

Discretion must be used in interpreting data that do not plot as a straight line, since the cause of the non-linearity may be due to the existence of mixed distribution. It is quite likely to be due simply to the randomness or the periodicity in the sample. The failure mechanisms must be studied, and engineering judgement must be used, to ensure that the correct interpretations are made. It is a common error to assume that, because a straight line provides a reasonably good fit to the data, that there is no failure-free life. Therefore a value for t_o can sometimes be estimated from knowledge of the product and its application. Alternatively, the time to first failure is often a satsifactory estimate of t_o. In these cases the procedure described above is not necessary. Generally, data on several failure modes in a system are likely to fit a two-parameter distribution ($t_o = 0$), but single wear-out failure modes ($\beta > 3.43$) are more likely to have positive values of t_o.

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Weibull Analysis

Rover-Triumph Reliability Department.

THE UNIVERSITY OF ASTON IN BIRMINGHAM

RELIABILITY EVALUATION AND PREDICTION

WITH SPECIAL REFERENCE TO LIFE TESTING

by

L. G. D. Petrucci

Submitted in partial fulfilment of the Degree of Doctor of Philosophy

1983

Examples of print-outs, obtained with the Computer Programme described in Chapter 10, accompanied for comparison purposes by Tables and Weibull plots obtained in the conventional manner.

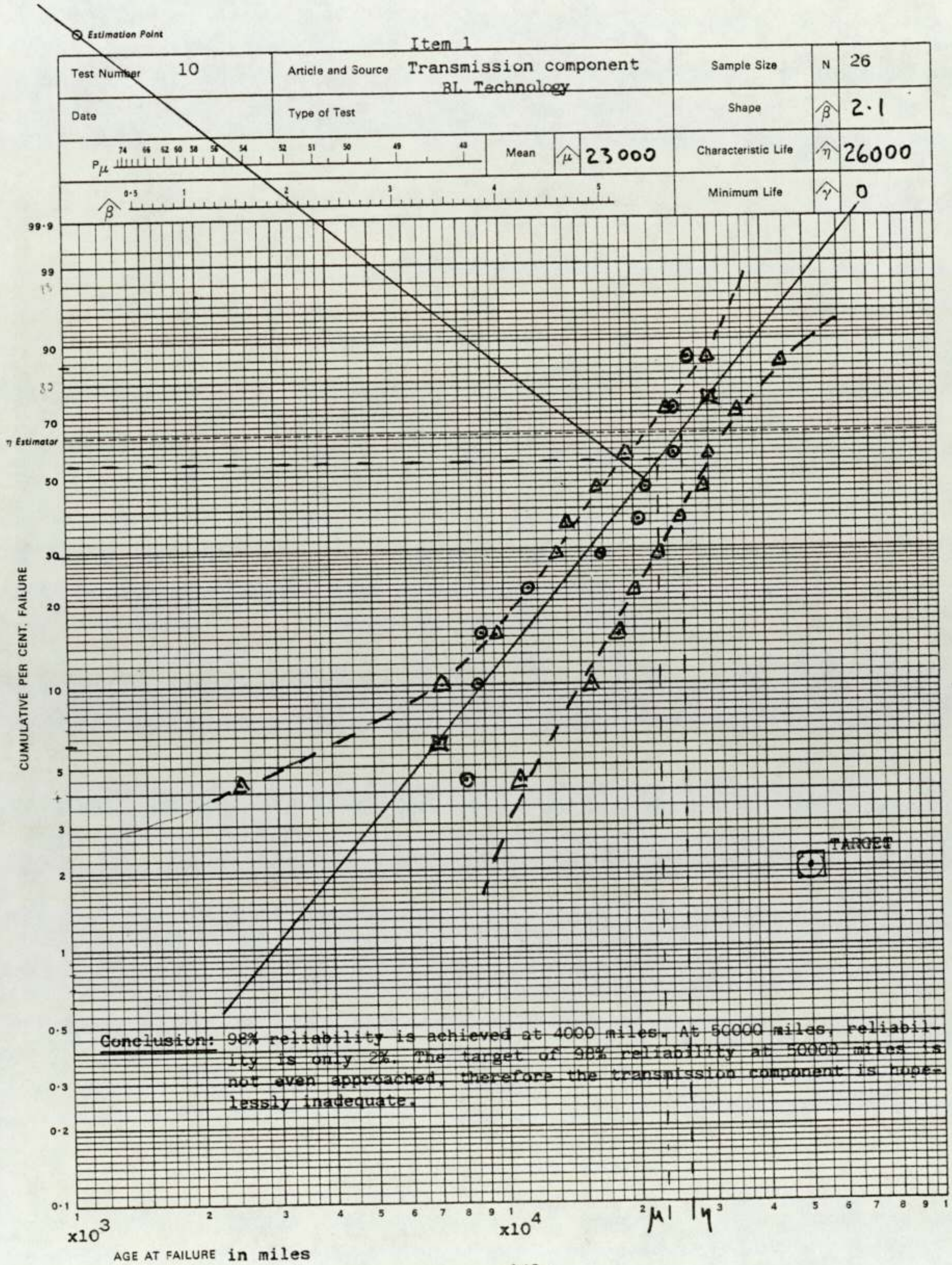
ITEM 1	SUSPENDED (MILES)	FAILED (MILES)
Transmission component	100	8,097
	1,333	8,700
	1,454	8,920
	2,838	11,500
	3,107	16,963
	5,346	20,315
	5,544	21,563
	6,367	24,767
	7,100	25,377
	11,247	27,321
	11,249	
	18,482	
	18,748	
	20,708	
	22,541	
	24,996	

Target: 0.98 Reliability at 50,000 miles

Sample size: 26

Item 1

Test Number	10	Article and Source	Transmission component BL Technology	Sample Size	N	26
Date		Type of Test		Shape	$\hat{\beta}$	2·1
P_μ		Mean	$\hat{\mu}$ 23000	Characteristic Life	$\hat{\eta}$	26000
$\hat{\beta}$				Minimum Life	$\hat{\gamma}$	0



Conclusion: 98% reliability is achieved at 4000 miles. At 50000 miles, reliability is only 2%. The target of 98% reliability at 50000 miles is not even approached, therefore the transmission component is hopelessly inadequate.

COMPLETED TEST. WITH SUSPENSIONS, OF 26 ITEMS

A-AO	X	MR	Y	R	L
8097	8.9997	0.0455	-3.0685	0.9550	5.74533969E-06.
8700	9.0715	0.1025	-2.2275	0.8962	1.24010301E-05
8920	9.0965	0.1595	-1.7535	0.8414	1.94250808E-05
11500	9.3506	0.2245	-1.3695	0.7758	2.21253208E-05
16963	9.7392	0.2905	-1.0715	0.7102	2.02078542E-05
20315	9.9196	0.3705	-0.7715	0.6301	2.2769978E-05
21563	9.9792	0.4615	-0.4795	0.5385	2.87406754E-05
24767	10.1177	0.5715	-0.1655	0.4286	3.42460681E-05
25377	10.1420	0.7185	0.2365	0.2821	4.9926921E-05
27321	10.2159	0.8645	0.6935	0.1356	7.32521445E-05

MINIMUM LIFE AO= 0

M=-22.7000225   N= 2.2460487   Y=M+N*X, X=LOG(A), F=1-1/EXPEXP(Y)

SHAPE PARAMETER BETA= 2.2460487

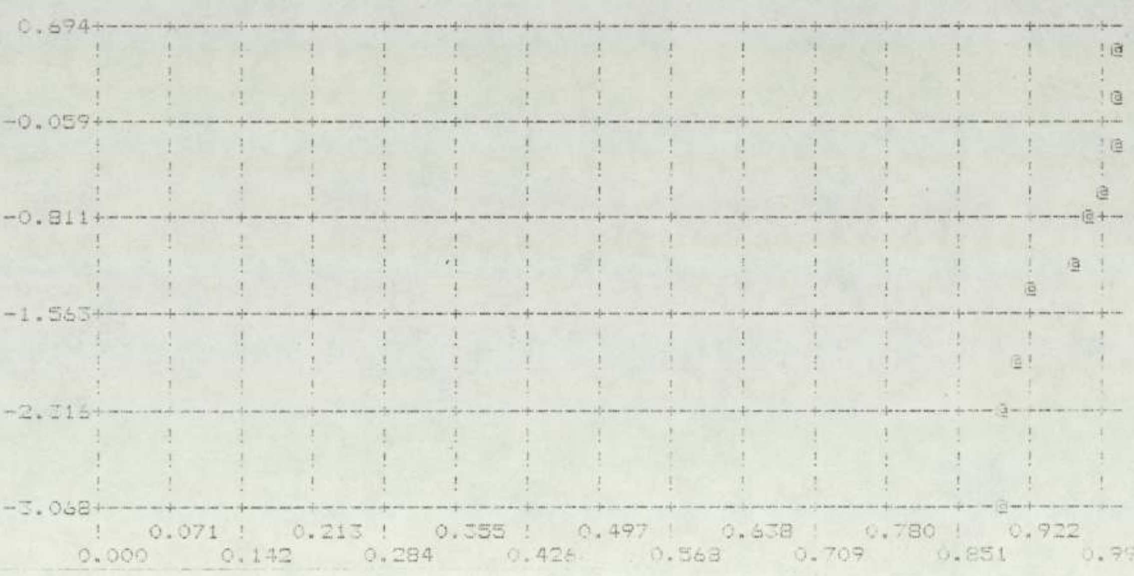
CORRELATION COEFFICIENT RC= .948072566

B1= 3160.67206 B10= 8997.70397 B20= 12567.0641 B50= 20615.7726

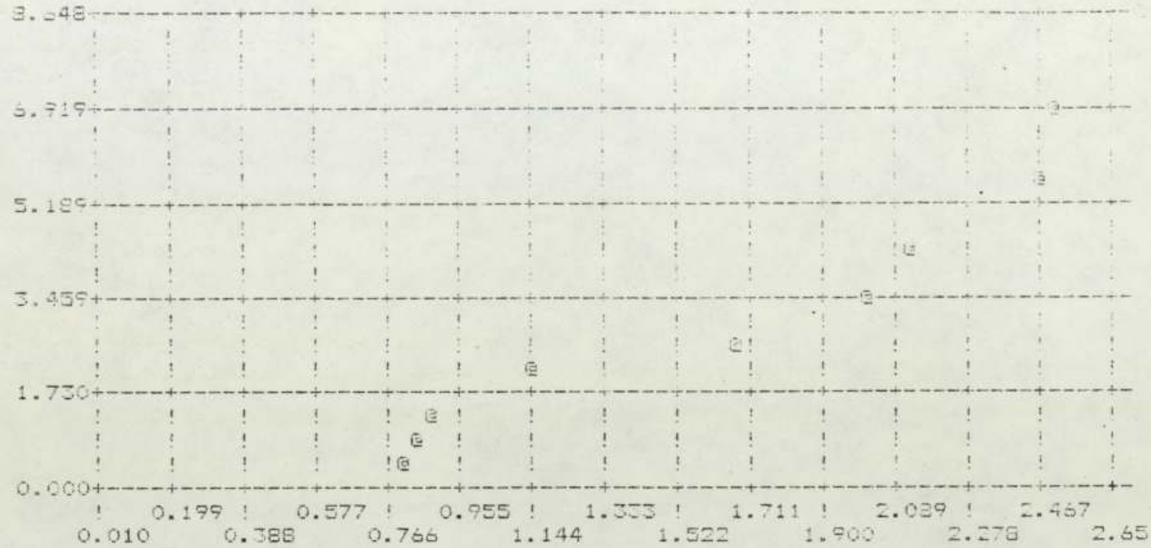
CHARACTERISTIC LIFE ETA= 24505.368

MEAN LIFE M= 21704.8696

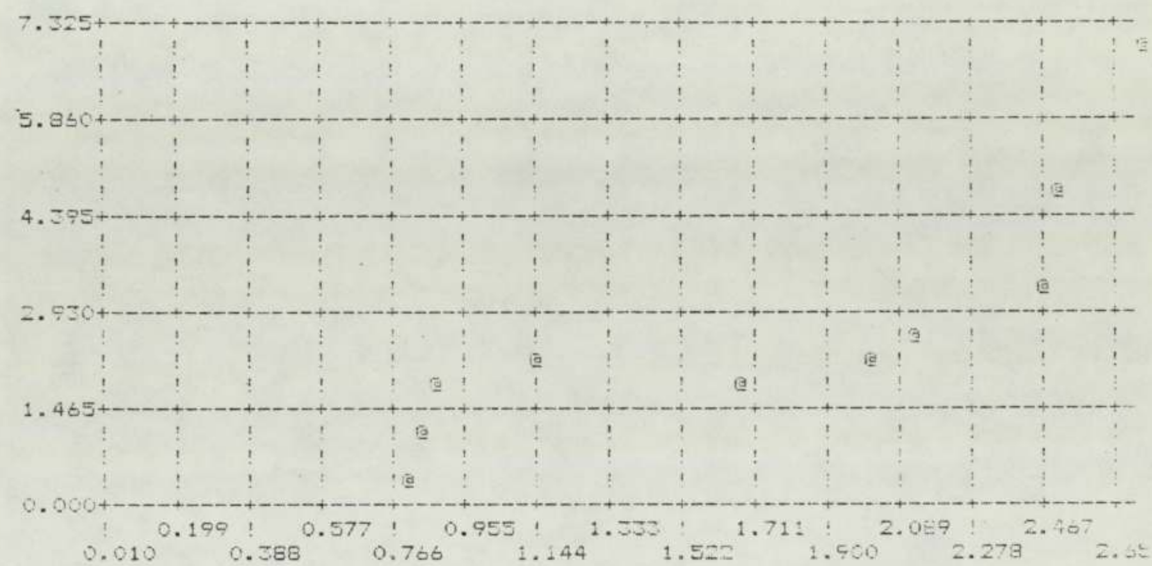
PR= 26   I= 26   R=1-MR



WEIBULL PLOT : X*10 AGAINST Y



WEIBULL PLOT : A*10000 AGAINST MR/10

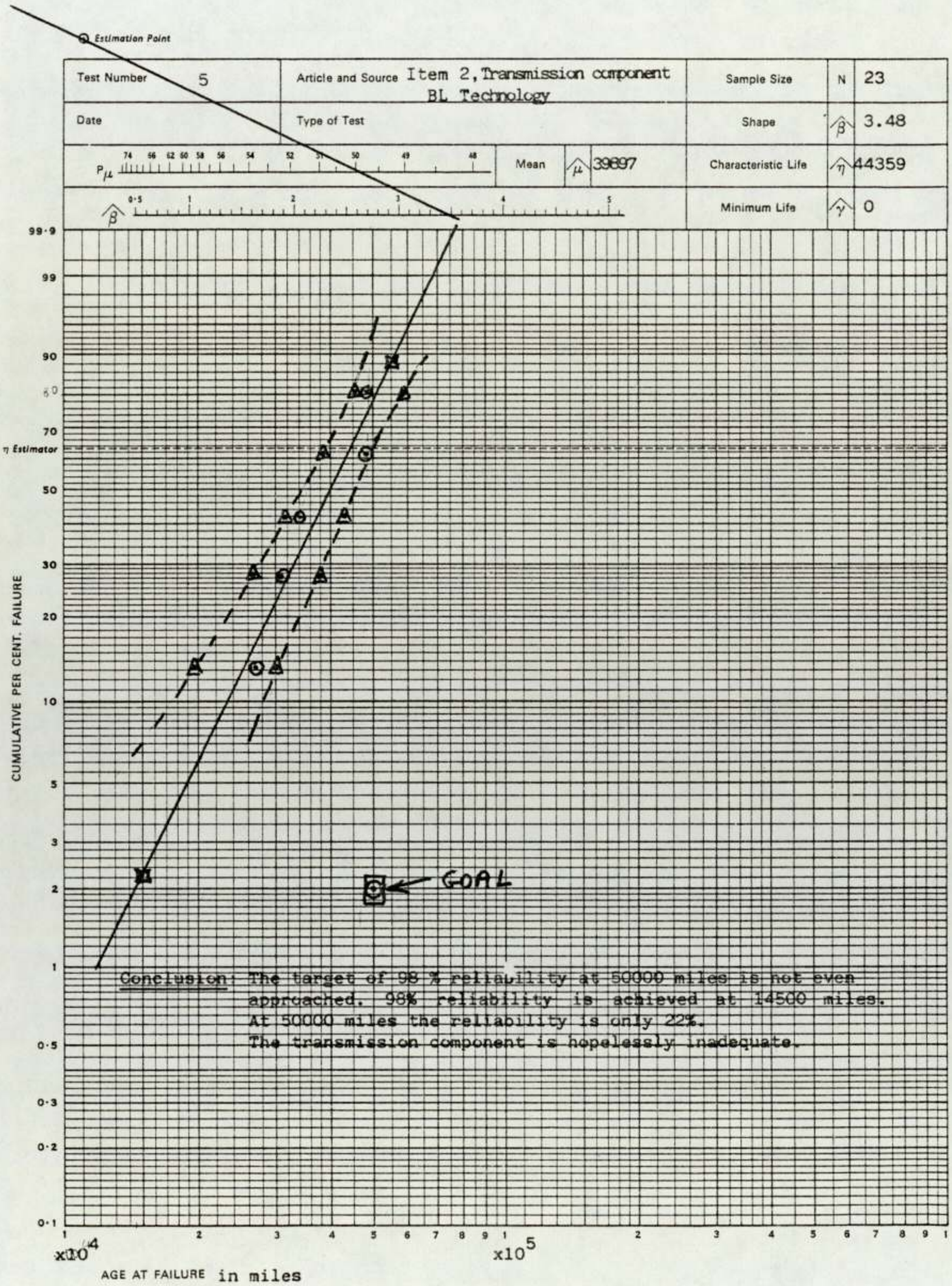


FAILURE RATE : A*10000 AGAINST L/100000

ITEM 2	SUSPENDED (MILES)	FAILED (MILES)
Transmission component	40	27,330
(same type as item 1 with modifications)	222	31,314
	420	34,333
	1,029	48,600
	2,392	48,800
	2,845	
	4,394	
	4,453	
	5,620	
	6,450	
	6,767	
	10,205	
	11,900	
	14,165	
	18,609	
	21,354	
	23,872	
	42,181	

Target: 0.98 Reliability at 50,000 miles

Batch size: 23


Estimation Point
Test Number 5
Article and Source Item 2, Transmission component BL Technology
Sample Size N 23
Date
Type of Test
Shape $\hat{\beta}$ 3.48
P_{μ}
Mean $\hat{\mu}$ 39897
Characteristic Life $\hat{\eta}$ 44359
$\hat{\beta}$
Minimum Life $\hat{\gamma}$ 0
η Estimator
CUMULATIVE PER CENT. FAILURE
GOAL
Conclusion: The target of 98 % reliability at 50000 miles is not even approached. 98% reliability is achieved at 14500 miles. At 50000 miles the reliability is only 22%. The transmission component is hopelessly inadequate.
x10^4
x10^5
AGE AT FAILURE in miles

COMPLETED TEST, WITH SUSPENSIONS, OF 23 ITEMS

A-AO	X	MR	Y	R	L
27330	10.2162	0.1335	-1.9425	0.8668	5.25150337E-06
31314	10.3523	0.2805	-1.1135	0.7202	1.05003948E-05
34333	10.4443	0.4265	-0.5875	0.5737	1.62064423E-05
48600	10.7918	0.6225	-0.0285	0.3783	2.00231686E-05
48800	10.7959	0.8175	0.5315	0.1830	3.48522068E-05

MINIMUM LIFE AO= 0

M=-37.1985589　　N= 3.4764796　　Y=M+N*X, X=LOG(A), F=1-1/EXPEXP(Y)

SHAPE PARAMETER BETA= 3.4764796

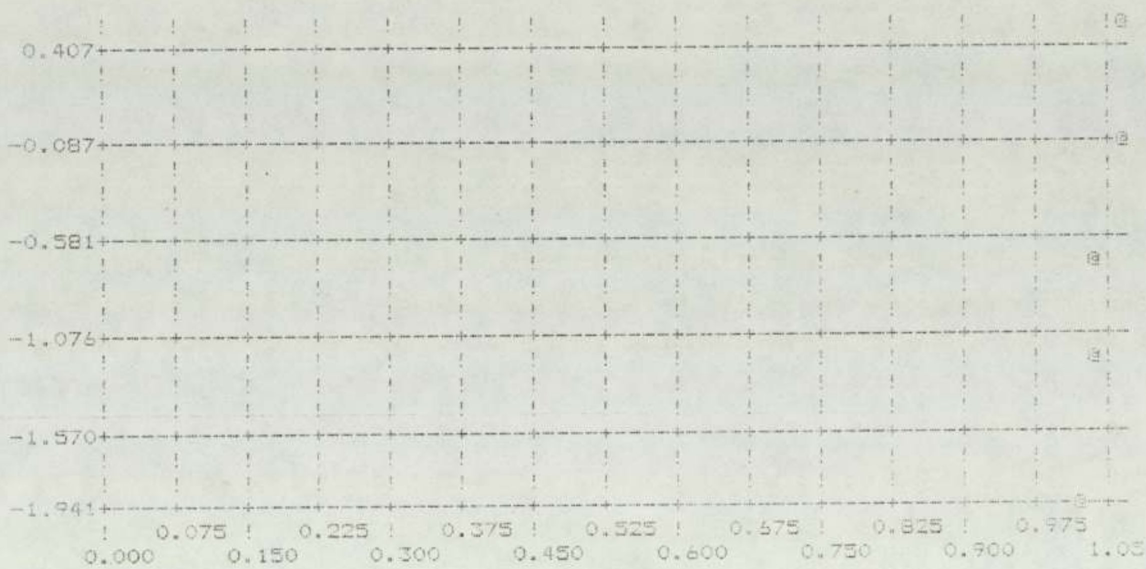
CORRELATION COEFFICIENT RC= .95431616

B1= 11811.6879 B10= 23219.6287 B20= 28813.8562 B50= 39920.2468

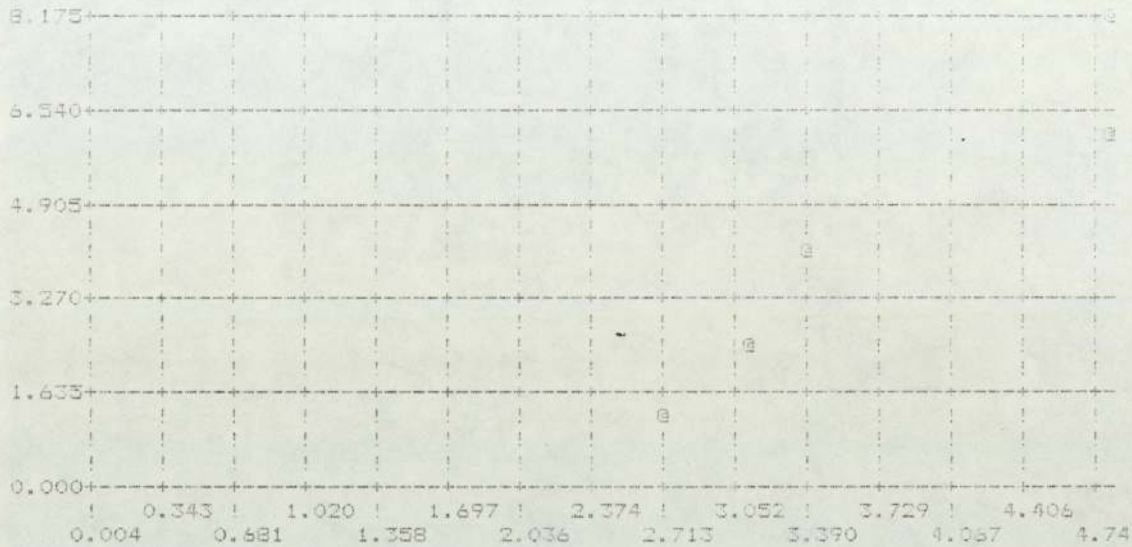
CHARACTERISTIC LIFE ETA= 44358.7547

MEAN LIFE M= 39897.4438

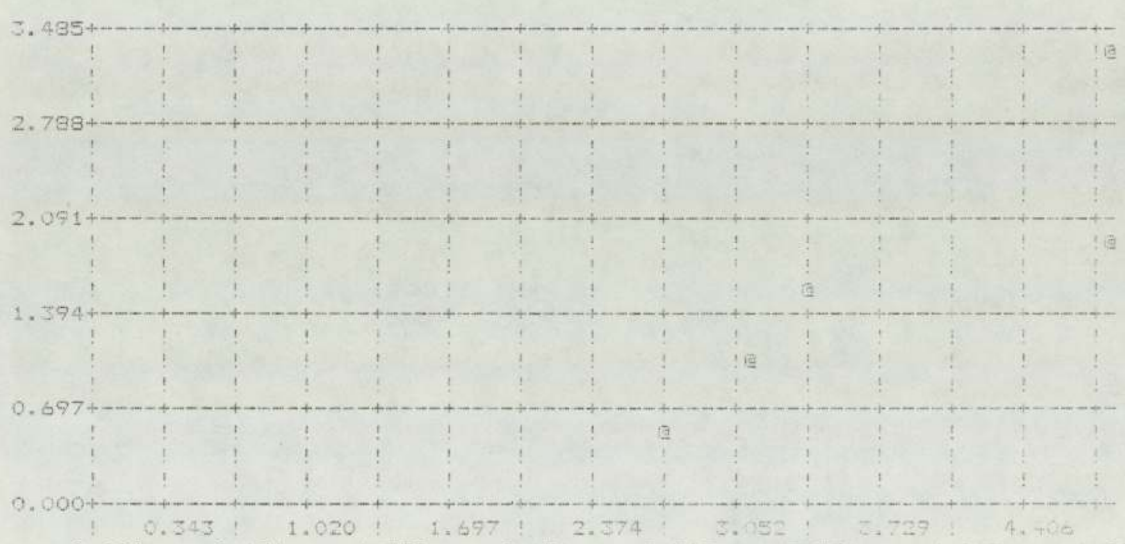
PR= 23　　I= 23　　R=1-MR



WEIBULL PLOT : X*10 AGAINST Y



WEIBULL PLOT : A*10000 AGAINST MR/10



FAILURE RATE : A*10000 AGAINST L/100000

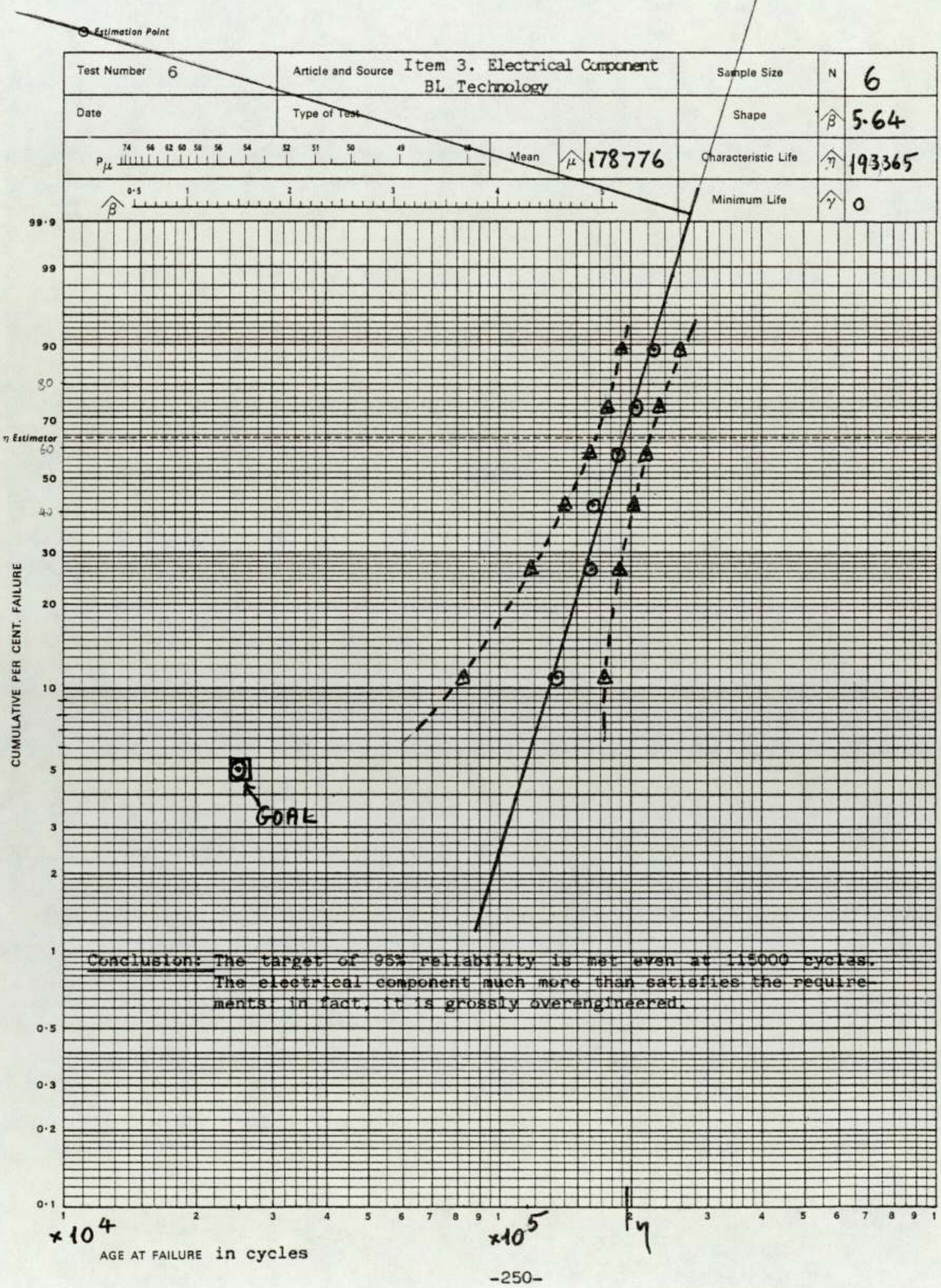
ITEM 3	CYCLES TO FAILURE
Electrical component	134,378 162,547 163,064 186,023 208,409 222,195

Target: 0.95 Reliability at 25,000 cycles

with 95% Confidence.

Batch size 6

Test Number 6	Article and Source Item 3. Electrical Component BL Technology		Sample Size	N	6
Date	Type of Test		Shape	$\hat{\beta}$	5·64
P_{μ}	Mean	$\hat{\mu}$ 178776	Characteristic Life	$\hat{\eta}$	193365
$\hat{\beta}$			Minimum Life	$\hat{\gamma}$	0



Conclusion: The target of 95% reliability is met even at 115000 cycles. The electrical component much more than satisfies the requirements: in fact, it is grossly overengineered.

COMPLETED TEST, WITHOUT SUSPENSIONS, OF 6 ITEMS

A-AO	X	MR	Y	R	L
134378	11.8089	0.1095	-2.1595	0.8913	8.59817845E-07
162547	11.9992	0.2645	-1.1815	0.7360	1.88946266E-06
163064	12.0023	0.4215	-0.6045	0.5790	3.35545349E-06
186023	12.1341	0.5785	-0.1465	0.4219	4.64344187E-06
208409	12.2477	0.7355	0.2855	0.2649	6.38231389E-06
222195	12.3118	0.8905	0.7955	0.1096	9.97020467E-06

MINIMUM LIFE AO= 0

M=-68.6993386 N= 5.64389165 Y=M+N*X, X=LOG(A), F=1-1/EXPEXP(Y)

SHAPE PARAMETER BETA= 5.64389165

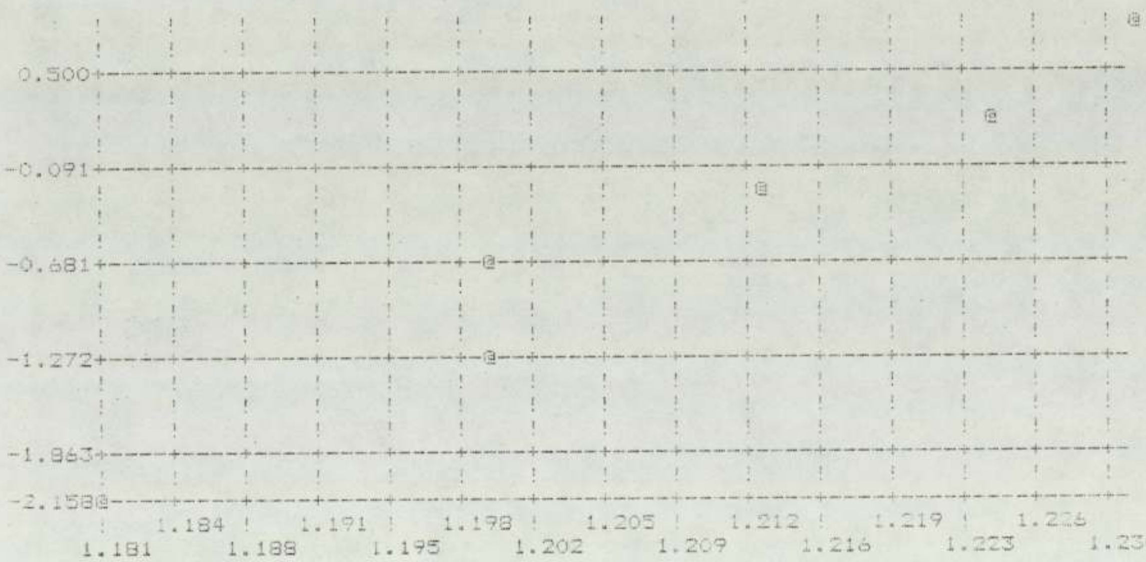
CORRELATION COEFFICIENT RC= .981736614

B1= 85585.22 B10= 129781.637 B20= 148237.552 B50= 181206.949

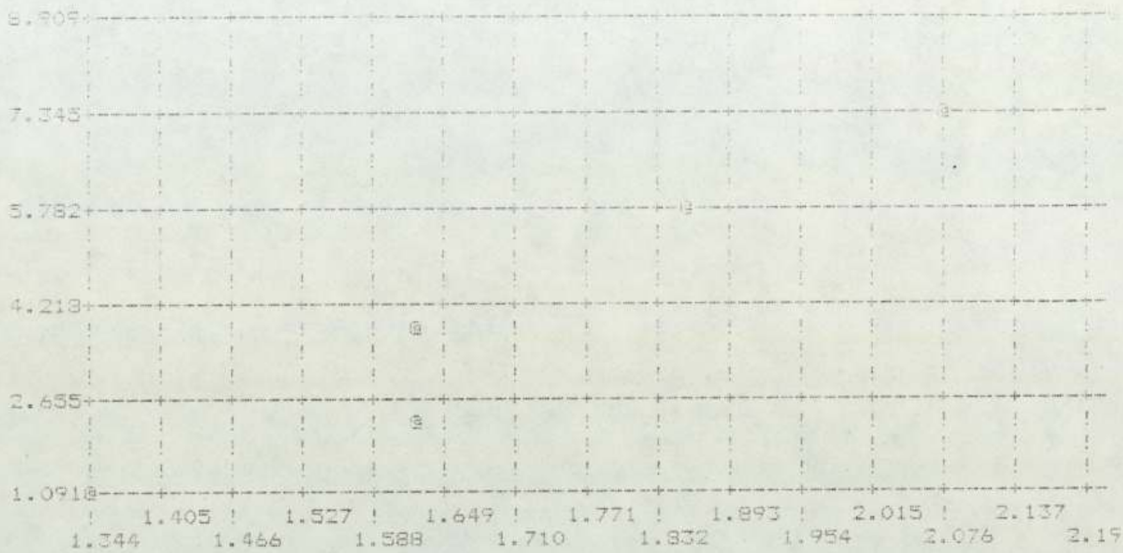
CHARACTERISTIC LIFE ETA= 193364.981

MEAN LIFE M= 178775.974

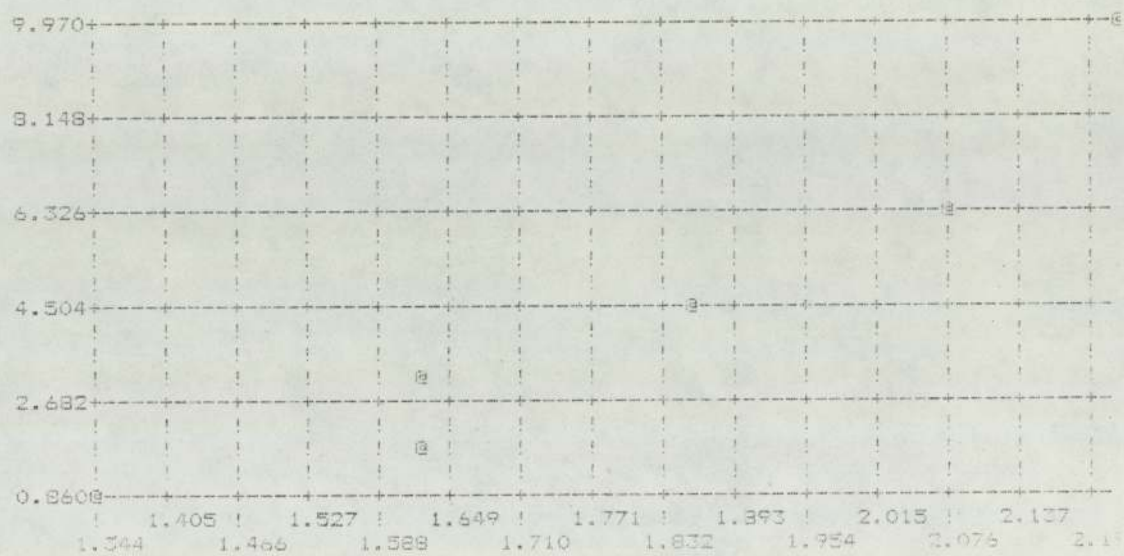
PR= 6 I= 6 R=1-MR



WEIBULL PLOT : X*10 AGAINST Y



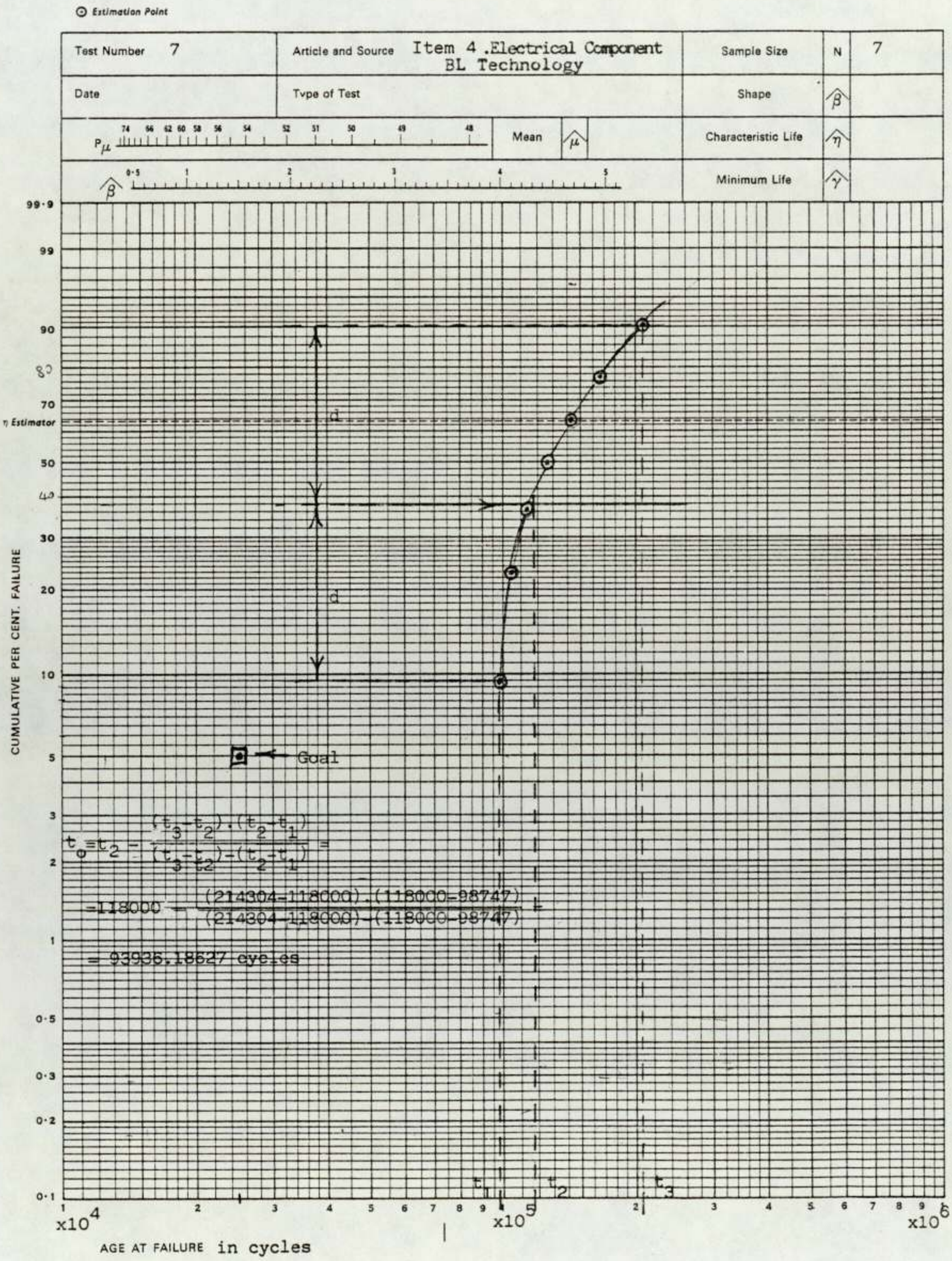
WEIBULL PLOT : A*100000 AGAINST MR/10

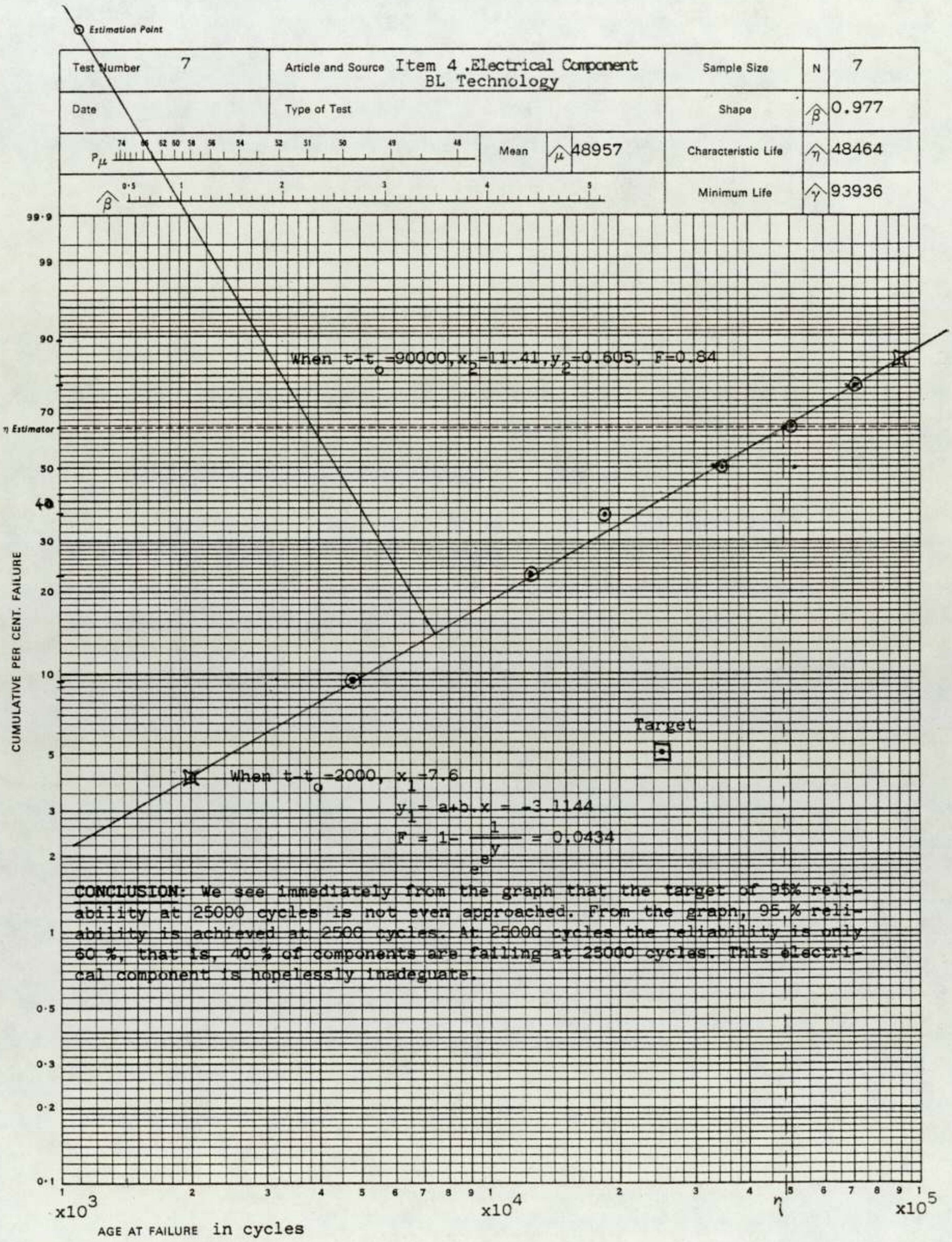


FAILURE RATE : A*100000 AGAINST L/1000000

ITEM 4	CYCLES TO FAILURE
Electrical component	98,747 106,440 112,538 128,655 144,108 164,197 214,304
Batch of 7	

Target: 0.95 Reliability at 25,000 cycles
with 95% Confidence


Estimation Point
Test Number 7
Article and Source Item 4 .Electrical Component BL Technology
Sample Size N 7
Date
Type of Test
Shape $\hat{\beta}$
P_{μ}
Mean $\hat{\mu}$
Characteristic Life $\hat{\eta}$
$\hat{\beta}$
Minimum Life $\hat{\gamma}$
η Estimator
CUMULATIVE PER CENT. FAILURE
d
d
Goal
$t_0 = t_2 - \frac{(t_3-t_2).(t_2-t_1)}{(t_3-t_2)-(t_2-t_1)} =$
$= 118000 - \frac{(214304-118000).(118000-98747)}{(214304-118000)-(118000-98747)}$
= 93936.18627 cycles
t_1
t_2
t_3
$x10^4$
$x10^5$
$x10^6$
AGE AT FAILURE in cycles



Estimation Point
Test Number 7
Article and Source Item 4 .Electrical Component BL Technology
Sample Size N 7
Date
Type of Test
Shape β̂ 0.977
Mean μ̂ 48957
Characteristic Life η̂ 48464
Minimum Life γ̂ 93936
η Estimator
CUMULATIVE PER CENT. FAILURE
When t-t$_o$=90000, x$_2$=11.41, y$_2$=0.605, F=0.84
Target
When t-t$_o$=2000, x$_1$=7.6
y$_1$= a+b.x = -3.1144
F = 1- 1/e$^{e^y}$ = 0.0434
CONCLUSION: We see immediately from the graph that the target of 95% reliability at 25000 cycles is not even approached. From the graph, 95 % reliability is achieved at 2500 cycles. At 25000 cycles the reliability is only 60 %, that is, 40 % of components are failing at 25000 cycles. This electrical component is hopelessly inadequate.
x10^3
x10^4
x10^5
η
AGE AT FAILURE in cycles

COMPLETED TEST, WITHOUT SUSPENSIONS, OF 7 ITEMS

A-AO	X	MR	Y	R	L
4810.81372	8.4791	0.0945	-2.3095	0.9059	2.065606635E-05
12505.8137	9.4342	0.2295	-1.3445	0.7707	2.08747372E-05
18601.8137	9.8315	0.3645	-0.7905	0.6356	2.44017867E-05
34718.8137	10.4555	0.5005	-0.3675	0.5005	1.99645986E-05
50171.8137	10.8237	0.6355	0.0085	0.3653	2.00955108E-05
70260.8137	11.1604	0.7705	0.3855	0.2302	2.0934169E-05
120367.814	11.6988	0.9055	0.8575	0.0950	1.95912418E-05

MINIMUM LIFE AO= 93936.1863

M=-10.5407702 N= .97703038 Y=M+N*X, X=LOG(A), F=1-1/EXPEXP(Y)

SHAPE PARAMETER BETA= .97703038

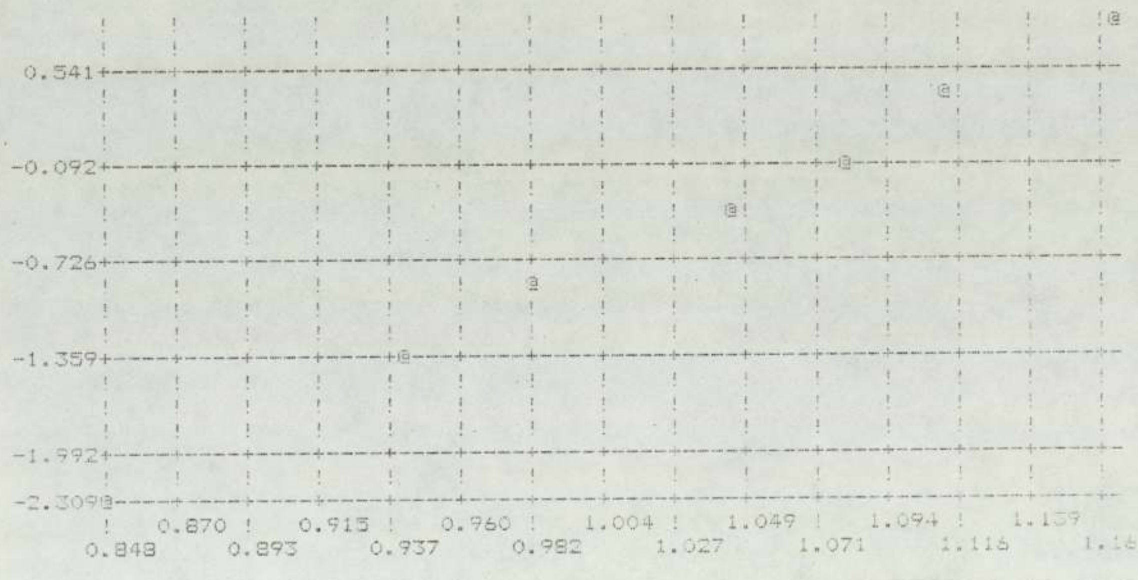
CORRELATION COEFFICIENT RC= .997981434

B1= 437.152789 B10= 4843.08483 B20= 10439.7589 B50= 33304.5825

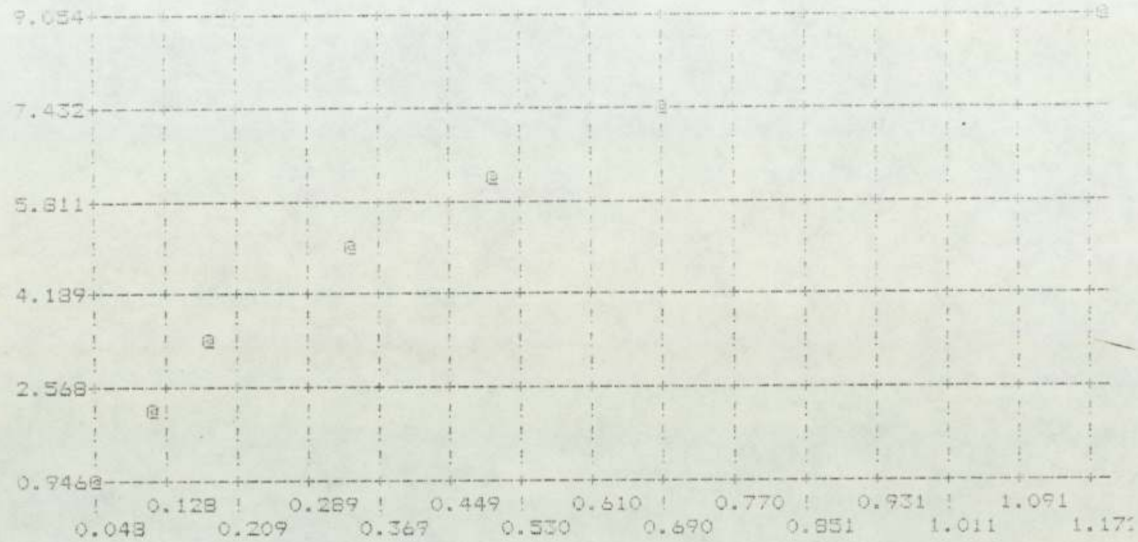
CHARACTERISTIC LIFE ETA= 48464.1575

MEAN LIFE M= 48956.9513

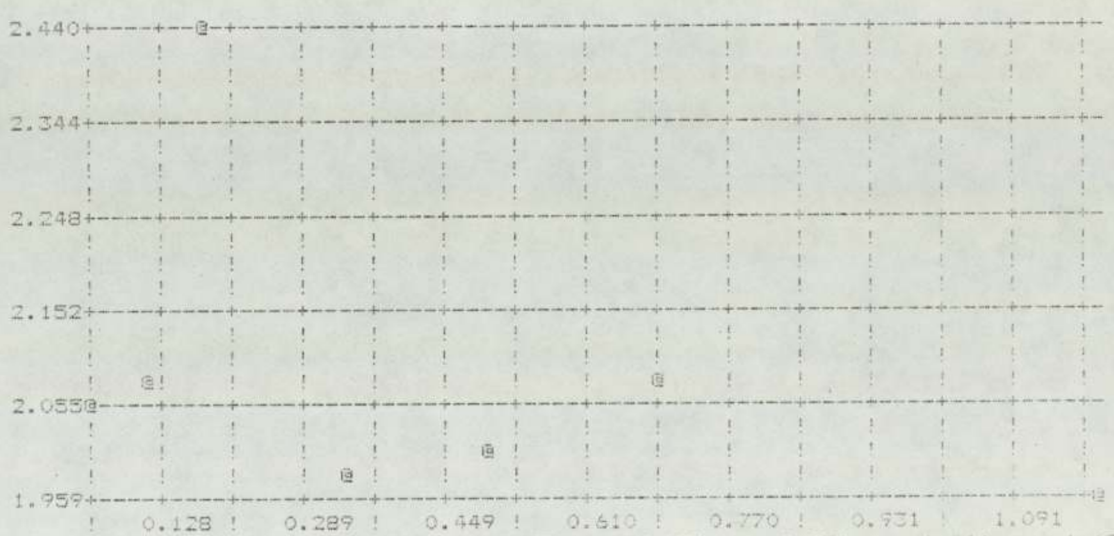
PR= 7 I= 7 R=1-MR



WEIBULL PLOT : X*10 AGAINST Y



WEIBULL PLOT : A*100000 AGAINST MR/10



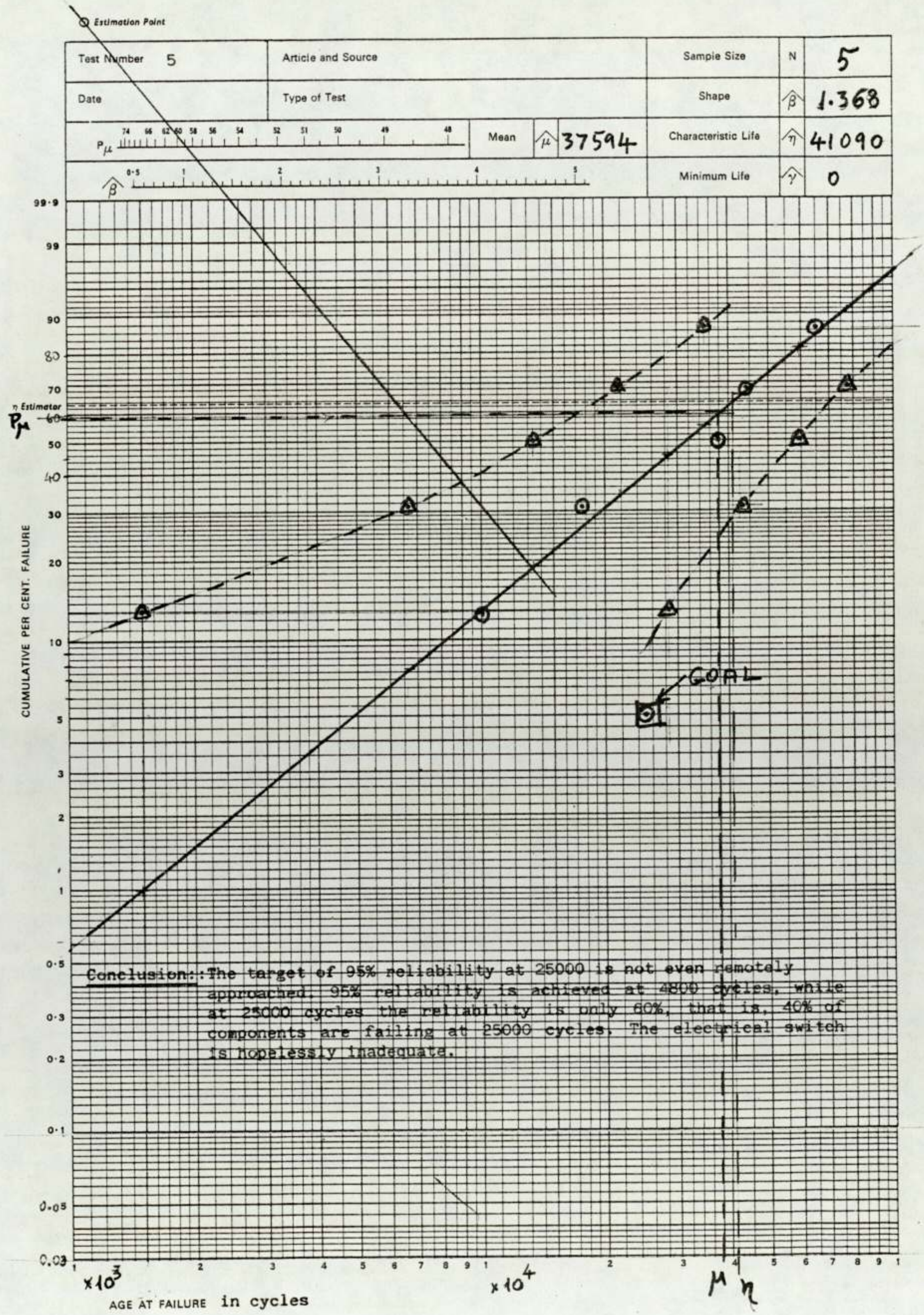
FAILURE RATE : A*100000 AGAINST L/100000

ITEM 5	CYCLES TO FAILURE
Electrical switch	10,054 17,860 37,903 43,895 65,058

Target: 0.95 Reliability at 25,000 cycles

with 95% Confidence

Batch size 5



Conclusion: The target of 95% reliability at 25000 is not even remotely approached. 95% reliability is achieved at 4800 cycles, while at 25000 cycles the reliability is only 60%, that is, 40% of components are failing at 25000 cycles. The electrical switch is hopelessly inadequate.

COMPLETED TEST, WITHOUT SUSPENSIONS, OF 5 ITEMS

A-AO	X	MR	Y	R	L
10054	9.2162	0.1295	-1.9765	0.8710	1.37887503E-05
17860	9.7908	0.3135	-0.9775	0.6867	2.10852056E-05
37903	10.5432	0.5005	-0.3675	0.5005	1.82873963E-05
43895	10.6900	-0.6865	0.1475	0.3142	2.64041237E-05
65058	11.0835	0.8705	0.7155	0.1299	3.14249906E-05

MINIMUM LIFE AO= 0

M=-14.528273　　　N= 1.3673575　　　Y=M+N*X, X=LOG(A), F=1-1/EXPEXP(Y)

SHAPE PARAMETER BETA= 1.3673575

CORRELATION COEFFICIENT RC= .98771409

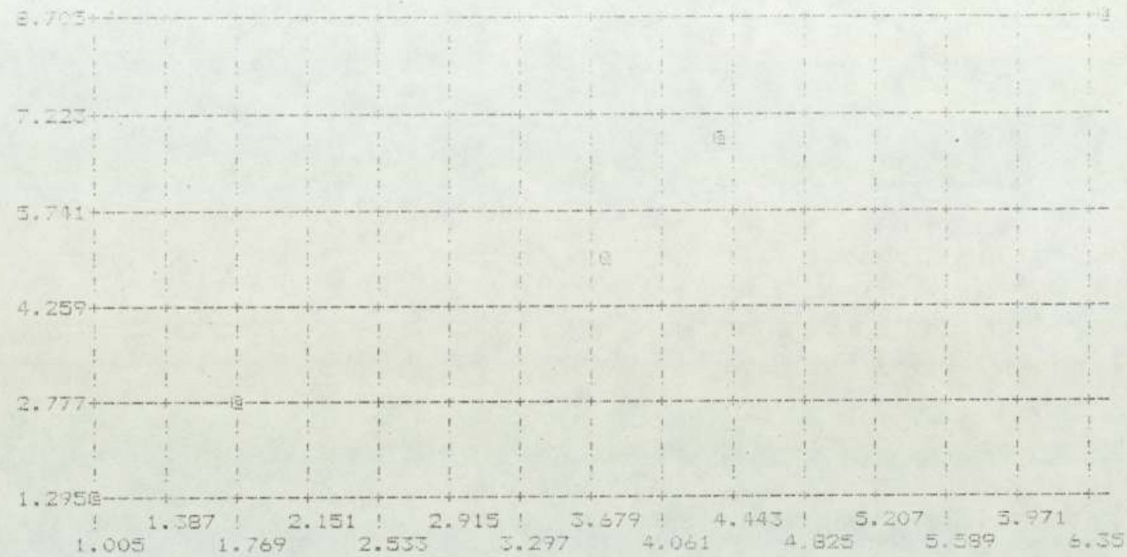
B1= 1421.89923 B10= 7926.61494 B20= 13721.4675 B50= 31429.8544

CHARACTERISTIC LIFE ETA= 41089.956

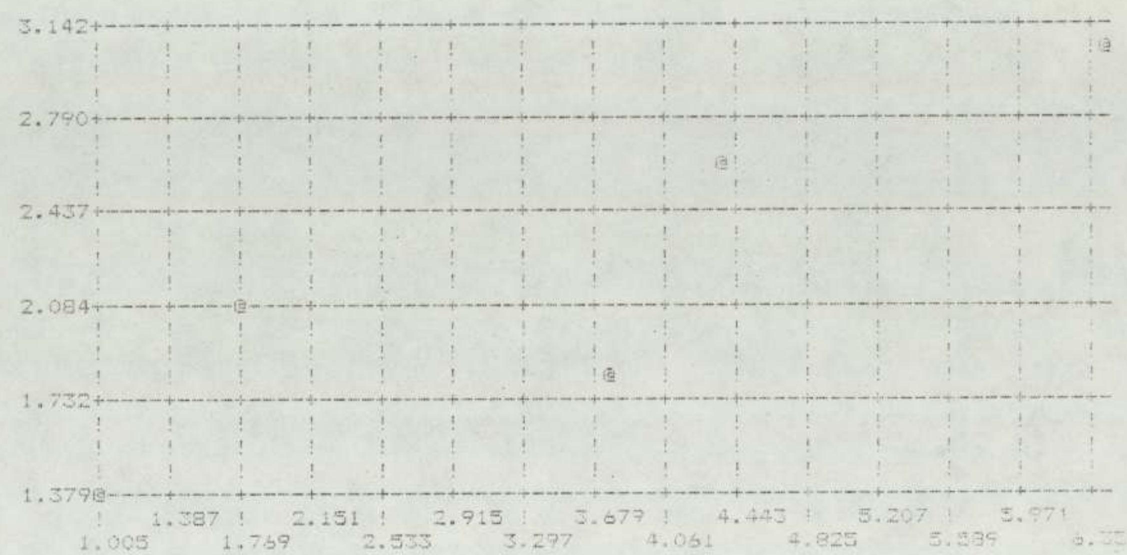
MEAN LIFE M= 37594.309

PR= 5　　　I= 5　　　R=1-MR

WEIBULL PLOT : X*10 AGAINST Y



WEIBULL PLOT : A*10000 AGAINST MR/10



FAILURE RATE : A*10000 AGAINST L/100000

GKN/UNIVERSITY OF ASTON

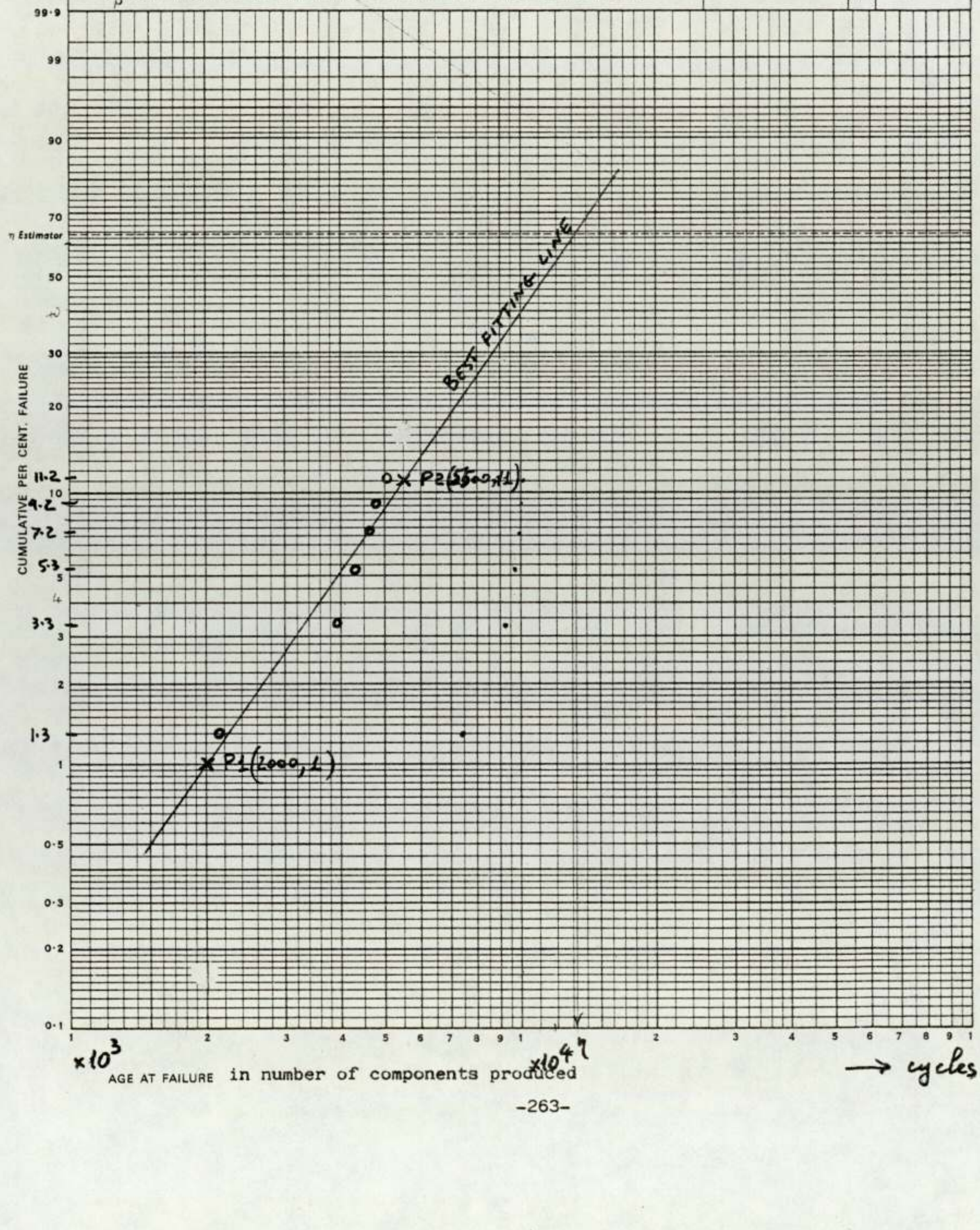
JOINT PROJECT

TOOL ANALYSIS

Machine No	Component Size	Tool Standard No	Supplier	Week No	No of Components Made
15/.3	3/8.	TS.2485/1	DENNER.	49	
		a	front chip		3888
		b	corner "		4.320
		c	corner "		4.608
		d	front "		2,160
		e	front "		5,040
		f	breakage		4752
					24768
	BATCH	OF 50			

⊙ *Estimation Point*

Test Number 6	Article and Source Punch GKN		Sample Size	N	50
Date Dec. 1981	Type of Test Failure		Shape	$\hat{\beta}$	2.416
P_{μ}		Mean $\hat{\mu}$ 11 863	Characteristic Life	$\hat{\eta}$	13381
$\hat{\beta}$			Minimum Life	$\hat{\gamma}$	0



TRUNCATED TEST, WITHOUT SUSPENSIONS, OF 6 OUT OF 50 ITEMS

A-AO	X	MR	Y	R	L
2160	7.6783	0.0143	-4.2705	0.9866	6.475112E-06
3888	8.2661	0.0342	-3.3735	0.9667	8.82514008E-06
4320	8.3715	0.0540	-2.9005	0.9469	1.27433188E-05
4608	8.4360	0.0739	-2.5745	0.9270	1.65466602E-05
4752	8.4668	0.0937	-2.3245	0.9072	2.06003529E-05
5040	8.5256	0.1135	-2.1215	0.8874	2.38130304E-05

MINIMUM LIFE AO= 0

M=-22.955731 N= 2.41599475 Y=M+N*X, X=LOG(A), F=1-1/EXPEXP(Y)

SHAPE PARAMETER BETA= 2.41599475

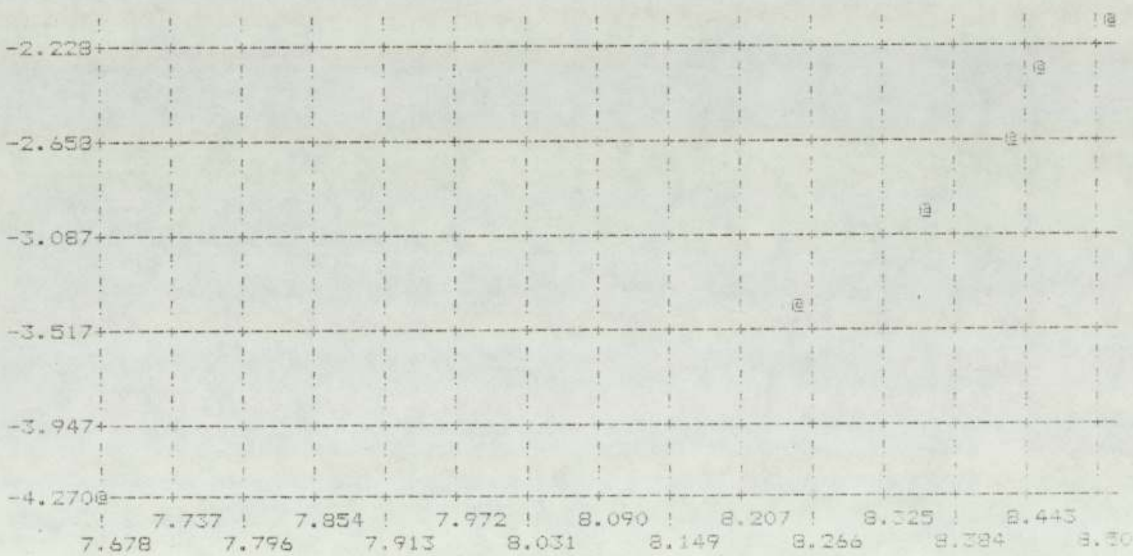
CORRELATION COEFFICIENT RC= .953642325

B1= 1993.25748 B10= 5271.75953 B20= 7192.02089 B50= 11497.2393

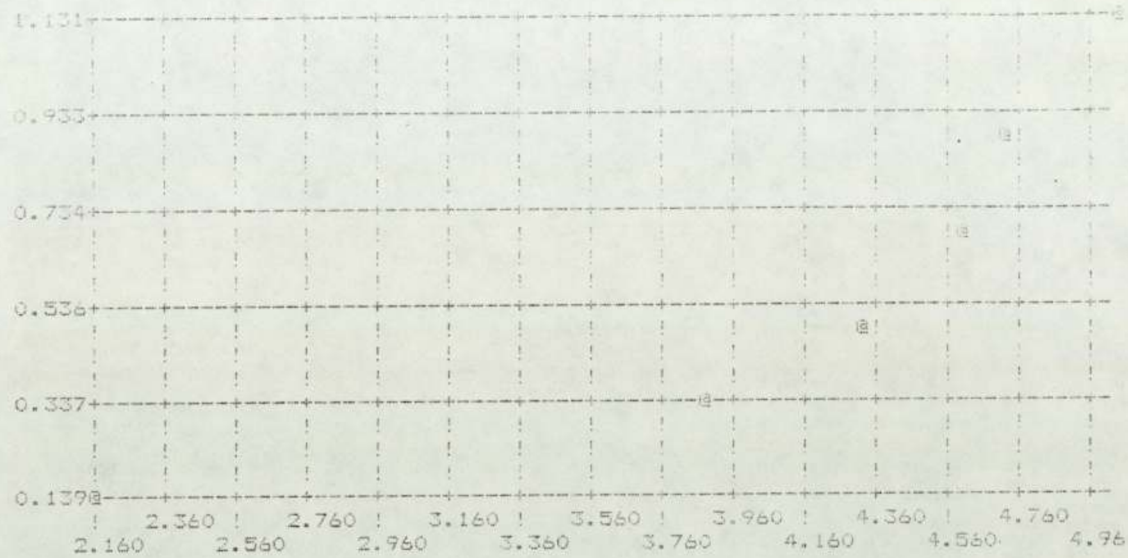
CHARACTERISTIC LIFE ETA= 13380.6501

MEAN LIFE M= 11863.2037

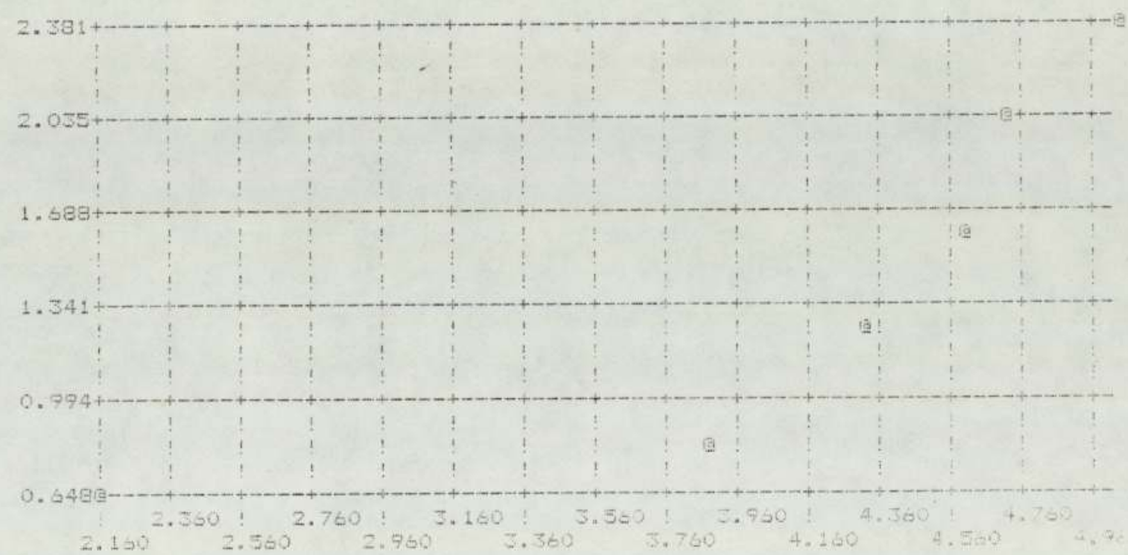
PR= 50 I= 6 R=1-MR



WEIBULL PLOT : X AGAINST Y



WEIBULL PLOT : A*1000 AGAINST MR/10



FAILURE RATE : A*1000 AGAINST L/100000

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