THE CHOICE OF THE MODEL IN INVENTORY CONTROL AND THE COST OF SOPHISTICATION

A thesis presented to the University of Aston in Birmingham for the degree of Doctor of Philosophy.

by

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A C K N O W L E D G E M E N T

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S Y N O P S I S

This thesis is concerned with the inventory control of items that can be considered independent of one another. The decisions when to order and in what quantity, are the controllable or independent variables in cost expressions which are minimised.

The four systems considered are referred to as (Q,R), (nQ,R,T), (M,T) and (M,R,T). With (Q,R) a fixed quantity Q is ordered each time the order cover (i.e. stock in hand plus on order) equals or falls below R, the re-order level. With the other three systems reviews are made only at intervals of T. With (nQ,R,T) an order for nQ is placed if on review the inventory cover is less than or equal to R, where n, which is an integer, is chosen at the time so that the new order cover just exceeds R. In (M,T) each order increases the order cover to M. Finally in (M,R,T) when on review, order cover does not exceed R, enough is ordered to increase it to M. The (Q,R) system is examined at several levels of complexity, so that the theoretical savings in inventory costs obtained with more exact models could be compared with the increases in computational costs. Since the exact model was preferable for the (Q,R) system only exact models were derived for theoretical systems for the other three.

Several methods of optimization were tried, but most were found inappropriate for the exact models because of non-convergence. However one method did work for each of the exact models.

Demand is considered continuous, and with one exception, the distribution assumed is the normal distribution truncated so that demand is never less than zero. Shortages are assumed to result in backorders, not lost sales. However, the shortage cost is a function of three items, one of which, the backorder cost, may be either a linear, quadratic or an exponential function of the length of time of a backorder, with or without period of grace.

Lead times are assumed constant or gamma distributed. Lastly, the actual supply quantity is allowed to be distributed. All the sets of equations were programmed for a KDF 9 computer and the computed performances of the four inventory control procedures are compared under each assumption.

VOLUME 1

CONTENTS

* Volumes 2 and 3 are bound separately.

CHAPTER 1.

INTRODUCTION

The introduction is split into four sections: the evaluation of inventory, the control systems chosen for study, 'Aspirations and Expectations', and finally the layout of the volume. In the evaluation of inventory, we give a brief background of inventory control and the variables that need controlling. We describe the various inventory control procedures chosen for study in the second section 'control systems chosen for study'. We give the scope and the various cases looked into in the thesis in the third section 'Expectations and Aspirations'. Finally, in section four we give the layout of this volume of the thesis.

Section 1.1 The Evaluation of Inventory.

This section is divided into three subsections; the first subsection gives the basic concepts of inventory control. The second subsection describes various crude ways of controlling inventory and introduces the exact form of a transactions reporting model. In the third subsection we describe the various cost factors that affect the choice of an inventory control procedure.

Section 1.1.1 Inventory Control

Inventory control is concerned with the control of stocks, stocks of money, fuel, consumer durables and countless other commodities.
Stocks act as a buffer between supply and demand and enable a more reliable or faster service to be given, or economies to be made in the replenishment process. Some supply functions, such as the creation of coal seams and oil fields, or the filling of reservoirs, are largely uncontrollable , and some demand functions are largely controllable, for example by television advertising, discounts, and other marketing strategies.

However, inventory control is usually concerned with the more common case, when demand is not known exactly, although forecasts of it may be made, and supply is controllable, in the sense that decisions may be made for any item, on when to order a replenishment, and how large the replenishment quantity should be.

A series of these two decisions (or the system generating them) not only determines the stock levels for given characteristics of the demand and supply functions (e.g. mean and variance of the expected demand during the lead time, between ordering and obtaining replenishment), but also the number of replenishments and the number and severity of shortages or stockouts. Thus, for given unit costs of stock holding, replenishment, and shortage, they determine what may be called the annual inventory costs of the system, i.e. the annual costs of holding stock, replenishing it and servicing stockouts. What this sum is for an individual item, or even for a firm, may be estimated. What it is for the country as a whole is not known, but it is certainly very large, possibly of the order of £150 million per annum, on the basis of stocks of £1.000 million.

Much of the stocks may be strategic, held against crises such as national strikes and wars, or speculative, held against possible upsurges in price or world demand. Much again may be obsolete, and awaiting scrapping. Inventory control should be concerned with such stocks, but most of the theoretical attention has been given to stock which can be replenished within a known lead time, at a constant cost per unit, and for most of this thesis the same conventions are observed.

A number of different procedures for controlling inventory have been devised and the decisions within each that appear to be optimal depend on what effects are expected to result from the decisions. These expectations are in turn derived from models of what happens. The problem is to determine what procedures, models, and decisions are appropriate. This thesis is concerned with this problem, for some of the inventory situations in which items may be considered independently of one another.

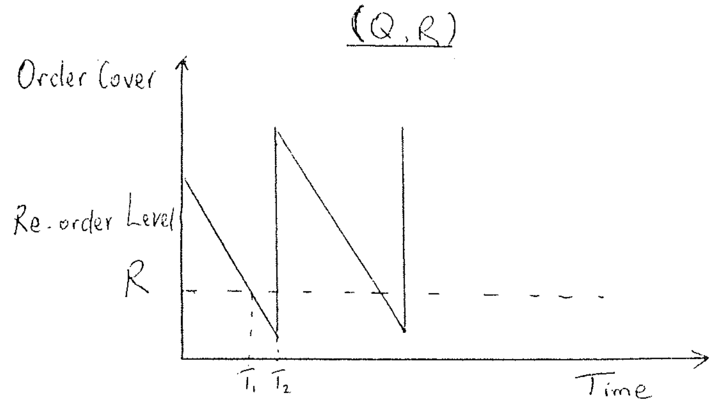
Independence of items is the simplest case, but it is so complex that it does not seem to have been adequately dealt with previously. For example, most advanced text books leave most of their equations unsolved. It will be shown that it is only with the discovery of some recent numerical methods that some of the equations have now become soluble. . Moreover some of the results should be of assistance in cases when items are not independent, as will be seen later.

But even if one were talking about improving the control of only 10% of stock by as little as 1%, the value of research could be £150,000 per annum. The actual benefit may well be larger, not only if the 10% and 1% proved to be underestimates, but also because by using an improved stock control system, there might be other unquantifiable benefits to the overall system.

1.1.2. MODELS

The kind of models we have been alluding to are mathematical ones in which the costs resulting from a particular control procedure are represented by a mathematical expression that contains the decisions to be made as variables. The optimal values of these variables are then to be found, i.e. the values that made the mathematical cost expression as small as possible. Let us take the most well known examples, the economic batch quantity and safety stock in a re-order level system, to indicate briefly, in anticipation of chapter 2 of vol.2. some of the different kinds of models that can be formulated.

A re-order level system, in which all transactions are reported and the stock level is examined after each transaction and compared with a re-order level, we designate a (Q,R) system.



A fixed quantity Q is ordered whenever the order cover falls to or below R, the reorder level. The quantity Q is received in stock after the elapse of a lead time L, equal to $T_2 - T_1$. During this lead time the stock falls, on average to some lower level, known as the safety stock, which we might call m.

The annual costs are usually modelled, at the crudest level as

Batch Quantity costs = Costs per order × Orders per annum
+ average Batch Stock × Stockholding cost per unit per annum.

and Safety Stock Costs = Cost per shortage × shortages per annum
+ Safety stock × Stockholding cost per annum

These may be represented as

$$C_1 = S \times \frac{D}{Q} + \frac{Q.hc}{2}$$

and $$C_2 = s \times \frac{D}{Q} F(m) + \frac{m}{2} \times hc$$

Where S = Cost per order; m= safety stock, Q = batch quantity, hc = holding cost.
s = Cost per shortage, D = demand, F(m) = Probability of stockout given m

C_1 may be minimised by differentiating with respect to Q, and the optimum value of Q is found to be $\sqrt{2DS/hc}$. The value of Q may then be used in C_2, to find similarly the optimum value of m, although graphical or numerical means may need to be employed, depending on the mathematical expression for F(m), the probability of stockout.

This, although more sophisticated and cost effective than the practice in many companies, is never the less crudest level of modelling, as was stated.

For example, the second equation, which contains Q was not considered when solving for Q. To be able to solve for Q and m simultaneously, we combine the first and second equations and write as the annual cost equation

$$C = (S + s\,F(m)) \times D/Q + (Q/2 + m) \times hc$$

This kind of model is designated the Heuristic model of (Q,R) in the thesis. However, it is sometimes difficult to solve for Q and m, when demand follows a normal distribution. When demand follows a normal distribution, Tate has suggested the following annual cost function.

$$C = (S + s.\exp(a-bk)) \times D/Q + (Q/2 + m) \times hc$$

where the safety stock m is expressed as k standard deviations of the forecasting error of demand over lead time.

This we call the exponential approximation of the heuristic model in the thesis. Thus Tate's exponential approximation is a device that ensures that Q and k can be obtained analytically. The correct formulation must consider the probability of a stockout from any starting stock position, not from the reorder level. If sales are lost in the event of a stockout then the expressions for the average stock will not be correct either, but in this thesis it is assumed that shortages are met later, i.e. backordered, and the shortage cost on each occasion is a constant plus a function of the number of backorders and of their durations.

However, even when the shortage cost is simply a constant on each occasion, as in the equation given earlier, the exact model is extremely complicated. Its complexity is of a mathematical type that is called non-convex, in which circumstances most of the numerical techniques fail.

By omitting two small terms from the exact model we can derive a fifth model, more accurate than the heuristic model, and, although slightly less accurate than the exact model, it is slightly easier to solve because it is convex. Thus, conceptually we have five (Q,R) models with a simple shortage cost function, the constant, s. Which should we choose ?

1.1.3. EVALUATION

The parameters that define an inventory control system such as the cost of holding stock, shortage costs, and the costs of ordering are not static over time. For example, the cost of ordering goods and the cost of holding stock increases as world prices increase or as inflation increases. In every competitive market the cost of shortage varies as the degree of competition varies. The cost of a shortage would also be expected to increase as inflation increases.

The cost of holding inventory depends upon many costs besides the cost of capital. These include insurance, taxes, breakages and pilferage at the storage site, and possibly warehouse rental rates and depreciation, lighting, heating, night watchman and storekeepers.

Also demand is continually changing. As a result a complete inventory control system should include adequate ways of taking account of this. In most systems including all those considered in the thesis, the demand rate is also unknown. As a result, a forecast of the demand rate has to be made.

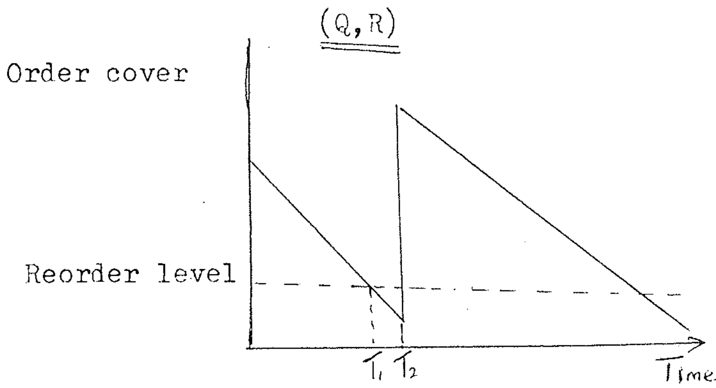
After a system has been installed, all the parameters have to be updated as they change or as new forecasts of their values are made. The cost of the system would depend upon how frequently the parameters are updated and the cost of each updating. In the extreme case, a new inventory system might have to be installed if the parameters change to such an extent as to make it uneconomical compared to another system. Some examples of this are defined in this thesis. Thus, the total costs of any system includes the cost of installation, updating, forecasting and the relevant inventory costs of the system.

Although the effectiveness of the system as a whole will depend on the frequency and effectiveness of updating parameter values as well as its cost, this interesting field for research has not been studied. In defence of this position it can be argued that most versions of the (Q,R) system, or other system to be defined later would derive equal costs and benefits from equal updating policies.

However, two potentially important ways in which the models differ are in their difficulty of comprehension by decision-makers and in the cost of solution of the equations. On the grounds that the decision makers will be chiefly interested in the cost effectiveness of their policies, the computational costs are computed, and set against the improved inventory costs resulting from the more sophisticated models. In this research it has not been possible to explore the question of difficulty of comprehension. To be able to explore the question of difficulty of comprehension, it is necessary to ask as many decision makers in as many firms as possible for their understanding and comprehension of the models. This would take a considerable length of time and the time for the completion of the thesis is limited. Hence we have not explored the question of difficulty of comprehension.

Section 1.2 CONTROL SYSTEM CHOSEN FOR STUDY

The version of (Q,R) described in 1.1.2 was an over simplification because it neglected the fact that more than one order might be out-standing at one time. There are also a number of alternatives to (Q,R) based on ordering and reviewing at fixed intervals, and also mixed systems. The four systems this thesis attempts to compare, chosen because they seem to be the most important ones, are described below. First we replaced the stock version of (Q,R) with an 'order cover' version.

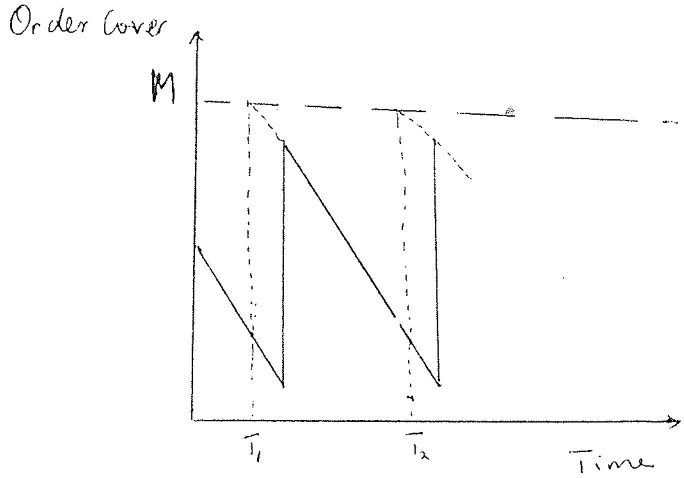


A fixed quantity Q is ordered whenever the order cover equals R, the reorder level. The quantity Q is received in stock after the elapse of a lead time L, equal to $T_2 - T_1$. The order breaking the reorder level, would normally overshoot it by a variable amount, but since we have taken a continuous model of demand this phenomenon does not occur and so the maximum order cover Q+R is always reached whenever the reorder level is broken. Thus, the system is equivalent to an inventory control procedure we designate (M,R), in which on breaking a reorder level an order is placed of a size to increase immediately the order cover to M. Therefore, we do not distinguish between these two systems which might well be slightly different in practice.

The second inventory control procedure is designated (M,T). Like (Q,R), (M,T) has two controllable variables but the cost of review may be included with the cost of an order unlike the (Q,R) system, which does not consider the cost of reviewing, in the derivation of the cost equations for (Q,R).

In (Q,R) reviewing is continuous and comes with every demand for an item, while it occurs after every period T in (M,T). In both systems the cost of an order can be regarded as the cost of replenishing stock in the warehouse.

(M, T)

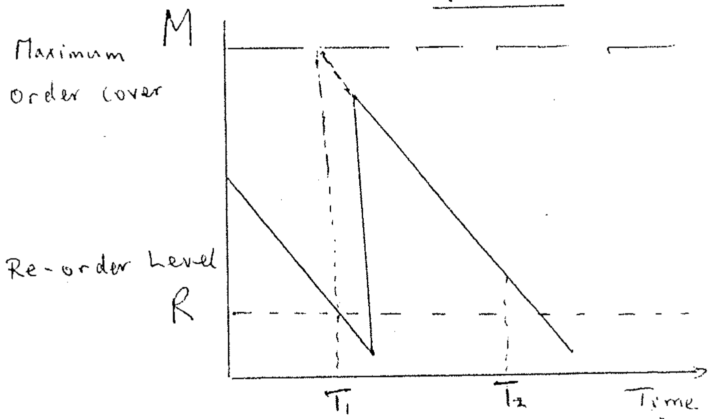


Review time $T = T_2 - T_1$. At each review time an order is always placed to bring the order cover to M.

The third control procedure is designated (M,R,T). Here a review is made after a period T, and an order is placed if the order cover is less than or equal to R at that time. The quantity ordered is such as to make the order cover M after every order.

This model is introduced because it takes into consideration three controllable variables unlike (M,T) and should be adaptable to more subtle combinations of parameter values than is (M,T).

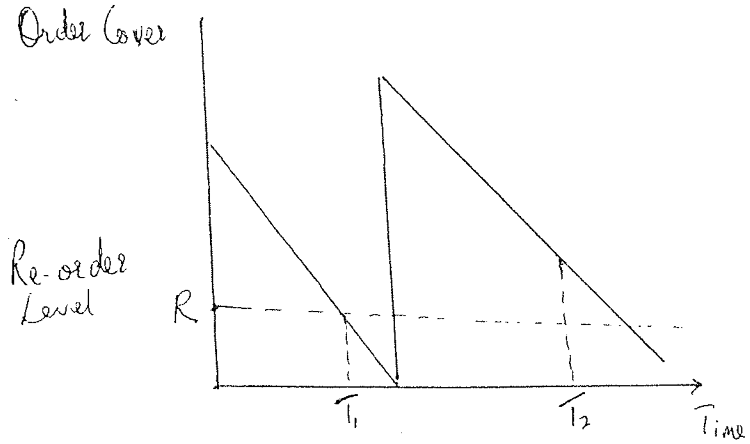
(M, R, T)



Review time is equal to $T_2 - T_1$. An order is placed at T_1 to bring the order cover to M and at T_2 no order is placed. The amount ordered when there is an order varies at every order.

The fourth inventory control procedure considered is designated (nQ,R,T). Hadley proposed this model which would be appropriate when the batch size must be a multiple of some quantity Q and which he speculates might sometimes be an improvement over (M,T).

(nQ, R, T)



A review is made after every period T. In the above diagram reviews are made at T_2 and T_1 and T equals $T_2 - T_1$.

An integer multiple of a fixed quantity Q is placed only when the order cover equals or falls below R at review time, n is the minimum integer such that the quantity ordered nQ, is such as to bring the order cover above R.

Section 1.3 ASPIRATIONS AND EXPECTATIONS

(Q,R), (nQ,R,T), (M,T) and (M,R,T) cover most of the inventory control procedures met with in practice in industry, and that have also been considered in the literature. However, authors who have considered relatively simple models (e.g. Hadley and Whitin ref.1.) have only considered relatively simple shortage costs and constant lead times. In the real world, neither are the lead times usually constant nor are the shortage costs simple. However, the exact models of even the simple cases have not previously been solved.

Shortage costs can have three components, one constant each time a stockout occurs, one varying with the number of backorders, and one varying with the length of time of backordering.

Three backorder cost functions chosen here, linear, quadratic and exponential form a good representation of most of the backorder cost functions met with in practice. In an additional set of functions, a time is allowed to elapse before a penalty is applied.

In the real world, lead times are not constant. People who use inventory control procedures often use an artificially increased value of the standard deviation of demand over the lead time in a model in which the lead time is held constant to compensate for this.

From the literature surveyed only T.A. Burgin (Inventory Control with normal demand and gamma lead times, Operational Research Quarterly Vol. 23. No. 1) has derived expressions for the behaviour of stock that take into consideration the distribution function of the lead time explicitly.

There are very few articles that compare the inventory cost performance of these inventory control procedures, under the assumptions considered in the thesis. Most of them consider constant lead times only and linear backorder costs. In Operational Research Vol. 10. No. 3. pp. 401-407, Naddor compares in our notation (M,T) and (M,R) analytically. He shows that (M,R) is always the better policy when replenishment costs are not common to several items. The reason (M,T) can be the better policy when replenishment costs are common to several items is that in (M,T) an order is always placed at every review and when the order cost is quite high (M,T) would benefit from sharing its high order costs with other items

Also in the Journal of the Royal Statistical Series B Vol 24, No. 1. 1962 pp 1-32, Thatcher compares the (M,R),(M,T) and (M,R,T). He assumes constant shortage costs per unit time and shows analytically that (M,R,T) is always the better policy amongst the periodic review models. (Q,R) is compared with the periodic review models in this thesis by adding the cost of a review to the cost of ordering.

Boothroyd and R.C. Tomlinson come to the conclusion that (Q,R) produces smaller inventory costs than (M,T) for discrete demand for spares, for a particular set of parameters. (Ref. 11)

The purpose of this thesis is to compare four inventory control procedures (Q,R), (nQ,R,T), (M,T) and (M,R,T) on the criterion of which gives the best inventory costs, when the variables entering each model are simultaneously optimised, and also taking into consideration the cost of computing for each model, for certain cases listed.

These cases are (1) constant lead times and the cost of a backorder is a linear function of the length of time of a backorder,

(2) constant lead times and the cost of a backorder is a quadratic function of the length of time of a backorder and

(3) constant lead times and the cost of a backorder is an exponential function of time.

Other assumptions are (4) that backorders do not incur a backorder cost until after a period of grace, with constant lead times for each of the backorder cost functions and (5) lead time is assumed continuous and gamma distributed, with no period of grace for each of the three backorder cost functions.

Finally (6) Supply is allowed to vary with constant lead times and continuous lead times respectively, with no period of grace for each of the three backorder cost functions.

Situations could arise where scrap is a distributed variable giving rise to an uncertain production quantity. (Ref. 6).

All combinations are examined, apart from the period of grace, for all the four control procedures we have defined. The Table below summaries the various combinations examined.

Backorder Cost Function

			Linear	Quadratic	Exponential
Fixed Supply	Lead Time	Constant	* ✓	* ✓	* ✓
		Variable	✓	✓	✓
Distributed Supply	Lead Time	Constant	✓	✓	✓
		Variable	✓	✓	✓

* Includes cases in which the 'period of grace' is taken into consideration.

Intuitively the (Q,R) control procedure always has lower average annual costs than a periodic review system, when review costs are ignored. The reason for this is that in the periodic review system, sufficient stock must be held to offer protection for a length of L + T (lead time plus review time), while for (Q,R) the lead time L is the relevant time. Thus the periodic review system will require higher safety stocks and have higher costs.

The question is how much better is it, and does this affect potential savings when items are not independent. To examine this we have calculated the exact savings for a whole range of combinations of parameters specified in the next chapter. Also intuitively the choice between (M,T) and (nQ,R,T) inventory control procedures would depend on the relative magnitudes of order cost and review cost. If the review costs are higher than order costs, it would be uneconomical to have a review without placing an order, hence (M,T) would yield less inventory costs than (nQ,R,T).

Similarly when order costs are higher than review costs (M,T) would yield higher inventory costs than (nQ,R,T) because it is uneconomical to order at every review as the (M,T) does. (M,R,T) is more flexible than both and so is likely to have lower inventory costs. But it is also likely to have higher computational costs, where computational cost is the cost of computing the optimum values of the control variables.

The normal distribution was taken as our basic distribution although it introduces some probability that demand could be negative, which is impossible in practice. Consequently we truncated the normal distribution although this introduces some approximation. We also illustrate some points, for the sake of simplicity, with the uniform distribution.

1.4. LAYOUT OF THESIS

In chapter 2 of this volume we describe briefly the methods of deriving the cost equations and the optimisation techniques, used to solve the equations.

In chapter 3 we give a sample of the results. The detailed results are in the volume 3 of the thesis.

In chapter 4 we give a discussion of the results.

In Volume 2 of the thesis we give the mathematical derivation of all the cost equations and we give the computational results in Volume 3.

This thesis has developed from Tate's joint calculation of reorder level and replenishment order quantity from his exponential approximation of the heuristic model of (Q,R) (Ref.5), which formed the basis of chapter 2 of volume 2.

Chapter 3 of volume 2 is also based on chapters 4 and 5 of Hadley and Whitin's Analysis of Inventory Systems (Ref.1.). Extensive use is made of T.A.Burgin's (Ref.6.) mathematical derivations on Normal demand with gamma lead times in chapter 8 of Volume 2.

C H A P T E R 2

M E T H O D

Introduction

In section 2.1 of this chapter we describe the two approaches of deriving the cost equations of the models, developed in volume 2 of the thesis. The methods of optimisation used to solve the cost equations are described briefly in Section 2.2. In section 2.3 we specify which optimisation technique is applied to each model, and we also give the difficulties encountered in the research.

In Section 2.4, we specify the ranges of parameters covered in the thesis during the programming of the cost equations.

Section 2.1 Equations.

2.1.1 Shortage Costs

The shortage cost is made up of three items; one is a constant per stock-out, the second is proportional to the number of units back-ordered and the third is related to the time for which an item has been backordered. When the backorder cost is a linear function of the length of time of a backorder, the approach employed is to calculate for a single period separately, the probability of a stockout, the expected number of backorders incurred per year and the expected number of backorders at any time, respectively. The probability of a stockout applies to the constant cost per stockout, and the expected number of backorders incurred per year applies to the cost proportional to the number of units backordered, and the expected backorders at any time applies to the cost proportional to the length of time of backorder.

The above approach can not be applied when the backorder cost function is more complicated than the linear form. The approach employed is to assume an initial level of order cover at time 0 and assume that in some time Z, before the lead time, L, the system is out of stock. Hence the length of time of a backorder that occurred at Z = L-Z. The cost of such a backorder that occurs at Z is obtained and the expected costs for the period is averaged over the states of Z.

When the period of grace is introduced, this approach is easily modified, for example if the period of grace is p (after which a backorder starts bearing a backorder cost) then the relevant length of time that bears a cost is L-Z-p ($Z+p<L$). The cost for a length of L-Z-p time is obtained

and averaged over the states of Z.

When the lead time is a continuous random variable, there are two ways of obtaining the annual inventory costs. One approach is to compute the various expected values using the marginal distribution of lead time demand rather than the lead time demand for a fixed lead time L. The second approach is to calculate the annual inventory costs for a fixed lead time L, and then to average the annual inventory costs obtained over the states of lead time L. The approach employed in the thesis is to calculate firstly the annual inventory costs for a fixed lead time L and average over the states of lead time.

This approach is chosen because we have available in the thesis the annual inventory costs, in the earlier chapters before the chapters that deal with continuous lead times. Hence at that stage of the thesis it becomes mathematically easier to average the inventory costs for fixed lead times over the states of lead time.

Similarly when the size of the batch quantities are distributed, the annual inventory costs are obtained by averaging the annual inventory costs for fixed batch quantities over its states.

2.1.2. Validity of Equations

The mathematics has been worked through several times in various notations. However, a number of checks are introduced into different sections of the thesis to check the validity of some of the equations. When the backorder duration cost is linear, the approach used for the more complicated backorder cost function can be reconciled with the simpler approach by putting the cost parameter, b_3 (for the quadratic term) or b_4 (for the exponential term) equal to zero. This can be done for the expected backorders incurred per year and for the number outstanding at anyone time as well as for the number of stockouts and total inventory costs.

In Section 2.6 of Volume 2 the exact version of the (Q,R) model for the linear case was derived by calculating the expected number of backorders per year and the expected backorders incurred at any one time.

In Section 4.2 of Volume 2, the (Q,R) model for the linear case was derived by using the duration of backordering explicitly and both cost equations are reconciled.

When the 'period of grace' p is set equal to zero in the cost equations of Chapters 6 and 7 of Volume 2, the corresponding cost equations that do not consider a period of grace can be obtained.

SECTION 2.2 METHODS OF OPTIMIZATION

In this section the methods of optimization used to solve the inventory models are described very briefly. The methods with the least computing time with respect to each inventory model are indicated in later sections.

The methods described are:-

(1) The Steepest Descent (Reference 2)

(2) Powell's method (Reference 3)

(3) Simplex technique (Reference 4)

(4) Complex method (Reference 7)

(5) Hadley & Whitin's iterative technique (Reference 1).

The Steepest Descent requires values of the first order derivatives of the cost expression with respect to the variables and at least the first order derivatives with respect to one of the variables must not be zero.

Powell's method is based on a quadratic approximation of the cost expressions. It uses n mutually conjugate directions in n stages to find the minimum where n is the number of variables.

The Simplex and Complex methods do not require values of the derivatives of the cost expressions.

2.2.1. STEEPEST DESCENT

Let C be the cost expression, where C must have at least one 1st derivative. C is a function of X_1, X_2, X_3 .... X_n variables. Let δC be a decrease in the objective function for a change in the variables. Let δs be the change in the point where δs is defined as

$$\delta s = \sum_{i=1}^{n} (\delta x_i)^2 \qquad 2.2.1.1.$$

$$\text{Now } \delta C = \frac{\partial C}{\partial X_1}\delta x_1 + \frac{\partial C}{\partial X_2}\delta x_2 + \frac{\partial C}{\partial X_3}\delta x_3 + \dots\dots\dots\dots$$

$$= \sum_{i=1}^{n} \left(\frac{\partial C}{\partial X_2}\right) , \delta x_i \qquad 2.2.1.2.$$

$$\text{Hence } \frac{\delta C}{\delta s} = \sum_{i=1}^{n} \left(\frac{\partial C}{\partial X_i}\right) \frac{\delta x_1}{\delta s} \qquad 2.2.1.3.$$

One particular set of displacements will make $\delta C/\delta s$ as small as possible (since we are minimising the function). This direction is the Steepest Descent. Minimise $\delta C/\delta s$ subject to constraint 2.2.1.1.

$$\text{Min} \quad \delta C/\delta s$$

$$\text{Such that} \qquad \delta s = \sum_{i=1}^{n} \left(\delta x_i\right)^2$$

Let F be the modified objective function where

$$F = \sum_{i=1}^{n} \left(\frac{\partial C}{\partial X_i}\right) \frac{\delta X_i}{\delta s} + \lambda\left(1 - \sum_{i=1}^{n} \left(\frac{\delta X_i}{\delta s}\right)^2\right) \qquad 2.2.1.4.$$

where λ is a Lagrangian multiplier.

Differentiate F with respect to $\delta x_i/\delta s$

$$\frac{\partial C}{\partial X_i} - 2\lambda \frac{\delta x_i}{\delta s} = 0$$

or $$\frac{\delta x_i}{\delta s} = \frac{1}{2\lambda} \frac{\partial C}{\partial X_i} \qquad i = 1 \ldots\ldots\ldots\ldots n$$

2.2.1.5.

This tells us that the direction of the Steepest Descent is proportional to the partial derivatives of C. The direction of steepest descent is parallel to the gradiant of the function.

Substitute 2.2.1.5. into 2.2.1.1. We obtain

$$1 = \sum_{i=1}^{n} \left(\frac{\delta x_i}{\delta s}\right)^2 = \frac{1}{4\lambda^2} \sum_{i=1}^{n} \left(\frac{\partial C}{\partial X_i}\right)^2$$

Solving for λ

$$\lambda = \pm \frac{1}{2} \sqrt{\sum_{i=1}^{n} \left(\frac{\partial C}{\partial X_i}\right)^2}$$

2.2.1.6.

ubstitute 2.2.1.5. into 2.2.1.3. We obtain

$$\frac{\delta C}{\delta s} = \sum_{i=1}^{n} \frac{1}{2\lambda} \left(\frac{\partial C}{\partial X_i}\right)^2$$

$$= \pm \sqrt{\sum_{i=1}^{n} \left(\frac{\partial C}{\partial X_i}\right)^2}$$

The positive sign gives the direction of the greatest increase in C while the negative sign gives the direction of the greatest decrease in C.

Let X_j^o, j = 1, ...n be the starting point provided at the point $\underline{X}^o$.

The next point $\underline{X}^1$ is obtained from

$$X^1_j = X_j^o + M_j . s$$

where $M_j = \dfrac{-\dfrac{\partial c}{\partial X_j}}{\sqrt{\sum_{i=1}^{n}\left(\dfrac{\partial c}{\partial X_i}\right)^2}}$ j = 1.......n.

2.2.1.7.

SECTION 2.2.2. POWELL'S METHOD

Let C be the cost expression which is a function of $X_1, X_2 \ldots\ldots\ldots X_n$ variables. C must have continuous first and second order derivatives.

Taking the Taylor series expansion at X^*

$$C(\underline{X}) = C(\underline{X}^*) + \sum_{i=1}^{n} \frac{\partial C}{\partial X_i} \cdot (X_i - X_i^*)$$

$$+ \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 C}{\partial X_i \partial X_j} (X_i - X_i^*)(X_j - X_j^*)$$

In rector form

$$C(\underline{X}) = C(\underline{X}^*) + A\underline{X}^* + \underline{X}' B\underline{X}^*$$

where B in such that $\underline{X}\ B\underline{X}$ is positive definite

Step 1.

Let the starting value of the K^{th} iteration be $\underline{X}o^{(k)}$

Let n linearly independent directions for the K^{th} step be $S_1^{(k)}, S_2^{(k)} \ldots\ldots\ldots\ldots S_n^{(k)}$.

Begin the search by finding the position of the optimum point along the line passing through $\underline{X}o^{(k)}$ which is parallel to $S_1^{(k)}$. At this optimum point $\underline{X}_1^{(k)}$, begins a second search from this new point in the $S_2^{(k)}$ direction.

Continues until all n search directions have been explored.

Step 2.

$$\text{Define } \Delta j = \left| C\left(\underline{X}_j^{(k)}\right) - C\left(\underline{X}_{j-1}^{(k)}\right) \right| \quad j=1\ldots\ldots n$$

where Δj is the improvement of the objective function over its previous value.

Let $\underline{X}m^{(k)}$ yield the largest Δj j=1....n, of any of the n moves.

$$\Delta m = \left| C\left(\underline{Xm}^{(k)}\right) - C\left(\underline{Xm-1}^{(k)}\right) \right| \text{is largest}$$

Determine the rector $\underline{U} = \underline{X}n^{(k)} - \underline{X}o^{(k)}$

Step 3

Determine $\underline{X}_t^{(k)}$

where $\underline{X}_t^{(k)} = 2\underline{X}_n^{(k)} - 2\underline{X}o^{(k)}$ and calculate $C\left(\underline{X}_t^{(k)}\right)$

Step 4.

If in seeking a minimum

$$C\left(\underline{X}_t^{(k)}\right) \geq C\left(\underline{X}o^{(k)}\right)$$

and/or $C(\underline{X}o^{(k)}) - 2C(\underline{X}n^{(k)}) + C(\underline{X}t^{(k)}) * (C(\underline{X}o^{(k)})-C(\underline{X}n^{(k)})$

$$- \Delta M)^2 \geq \frac{\Delta m}{2} (C(\underline{X}o^{(k)}) - C(\underline{X}i^{(k)}))^2$$

Then begin the search again, starting at the best point and using the same directions.

that is $\underline{X}o^{k+1} = \underline{X}n^k$

and $\underline{S}i^{k+1} = \underline{S}i^k \quad i = 1......n$

step 1 is repeated.

If neither of these inequations is satisfied, search along the direction U until the minimum is found. This point is defined as $\underline{X}o^{(k+1)}$ and the new search directions for the $(k+1)^{th}$ stage are

$$\underline{S}_i^{(k+1)} = \underline{S}_i^{(k)} \quad i = 1, 2 \ldots\ldots\ldots m-1$$

$$\underline{S}_i^{(k+1)} = \underline{S}_{i+1}^{k} \qquad i = m \ldots\ldots\ldots n-1$$

$$\underline{S}_n^{(k+1)} = \underline{U}$$

And repeat the entire process starting with step 1.

SECTION 2.2.3. SIMPLEX TECHNIQUE

The objective function is evaluated at n+1 points in the space spanned by n independent variables. The points form the vertices of a regular simplex.

Let the vertices of the Simplex be X_i and the corresponding function values be C_i $i = o, 1 \ldots\ldots\ldots n$.

Step 1

Let g, h, s be respectively the subscripts of the vertices possessing the largest, next largest and smallest function values.

Let $\bar{X}$ be the centroid of all vertices excluding Xg

$$\bar{X} = \sum_{\substack{i=1 \\ i \neq g}}^{n} \frac{X_i}{n}$$

Step 2

Xg is reflected in $\bar{X}$ to Xr

where $Xr = (1 + \alpha)\bar{X} - \alpha Xg$. $\alpha > o$ ($\alpha = 1$)

If $C_h > Cr > Cs$ then Xr replaces Xg then return to step 1.

Step 3

If $Cs > Cr$ then it is investigated whether a further step in this direction will also be successful. Therefore a new point Xe on the extended line Xg $\bar{X}$ Xr is calculated where

$$Xe = \gamma Xr + (1 - \gamma)\bar{X} \qquad \gamma > 1 \ (\gamma = 2)$$

If $Ce < Cr$ then the expansion has been successful and Xg is replaced by Xe. Otherwise Xg is replaced by Xr

Step 4

If $Cg > Cr > C_h$ then Xr replaces Xg, but not otherwise.

A new point Xc is calculated as follows

$$Xc = \beta\, Xg + (1 - \beta)\, \overline{X}$$

where $o < \beta < 1$, $(B = \frac{1}{2})$

If $Cc < Cg$ then Xc replaces Xg.

The basic process is repeated.

Step 5

If $Cc > Cg$ then contract the whole simplex.

$$Xi = \frac{1}{2}(Xs + Xi) \quad i = o,1 \ldots\ldots\ldots n$$

Repeat basic process.

Stop when the standard deviation of the function values at (n+1) vertices $< \varepsilon$.

i.e. $$\sqrt{\sum_{i=1}^{n+1} \frac{(Ci - \overline{C})^2}{n}} < \varepsilon$$

SECTION 2.2.4. COMPLEX METHOD

The Complex method is a modification of the Simplex method, designed to handle constrained problems.

Constraints are of the types $g_i(x) = 0$ and/or $(l_i \leq X_i \leq U_i)$ $i = 1, \ldots\ldots m$. Where the implicit constraints are $g_i(X) \leq 0$ and l_i and U_i are either constants or also functions of $X_1, X_2, \ldots\ldots X_n$ and are the explicit constraints.

Find an initial point that satisfies all constraints.

The further (k-1) points required to set up the initial Complex figure are found as follows.

A tentative trial point is generated with co-ordinates.

$$X_i = l_i + r_i(U_i - l_i) \quad i = 1, 2 \ldots\ldots n$$

where the random numbers r_i are pseudo-random rectangularly distributed deviates in the interval (0,1). If an implicit constraint is violated, the trial point is moved halfway in toward the centroid of the points already determined. Repeat until the complete set of k points is defined.

The objective function is evaluated at each point, and the point yielding the poorest value of the function is rejected and replaced by the Simplex technique described in Section 2.2.3. Various cases calling for different treatments, arise as follows.

(a) If the trial point does not satisfy some explicit constraint, that variable is reset just inside the appropriate boundary (by say 0.000001) to give a further trial point.

(b) If some implicit constraint is violated a further trial point is constructed by a move halfway back towards the centroid, this process being repeated as necessary.

$k > n+1$ points are necessary in order to prevent the configuration from collapsing pre-maturely into a sub-space.

SECTION 2.2.5. HADLEY'S AND WHITIN'S ITERATIVE TECHNIQUE

It is strictly applicable only to a function of two variables that can each be expressed explicitly in terms of the other. This rarely applies except in the case of the heuristic models referred to later in Chapter 2.

Let C be a function of X_1 and X_2.

Express X_1 as a function of X_2, $X_1 = g_1(X_2)$

" X_2 " " " " X_1, $X_2 = g_2(X_1)$

Start with a value for X_2 say X_2^o and obtain $X_1^{\,o} = g(X_2^{\,o})$ and obtain a new value for $X_2^{\,1} = g_2\ (X_1^{\,o})$.

Repeat until convergence is obtained.

Section 2.3. SPECIFIC OPTIMISATION TECHNIQUE APPLIED TO EACH MODEL

Hadley and Whitin's iterative technique described in Section 2.2.5. was used to solve the Heuristic model of (Q,R). Tate's exponential model was solved by calculating the inventory costs for each pair of values of a and b and the minimum inventory costs is chosen.

The Simplex, Powell's and Complex methods were used to solve the exact and inexact models of (Q,R), for the linear case but the results presented in the thesis are derived by the Complex method. The Complex method was found to be converging on all the sets of parameters considered unlike the Simplex technique. Powell's method converged for the sets of parameters considered for the linear case for both the inexact and exact models of (Q,R). However, for each of the periodic review models, only the Complex method converged for all the sets of parameters. As a result all the results presented in the thesis are obtained by the Complex Method.

Two major difficulties were encountered in the research. The first was the lack of convergence of most of the numerical methods tried, and the continual search for methods that would converge.

Secondly while still on chapter 4 of Volume 2 my computer expenditure £400 had exhausted more than my share of the computer budget and more than half of the computer budget for the department. As a result, I encountered some difficulties in having to rewrite programs to suit another computer which my supervisor had made special arrangements for me to use.

Section 2.4. Scope of Parameters

Owing to the cost and time of computation it has not been possible to consider all the possible ranges of the parameters. The standard deviation of demand over the lead time $\sqrt{\sigma^2 L}$ is expressed as a fraction of expected demand over the lead time DxL. This is done because the standard deviation of demand over the lead time, for the same demand rate and lead time, can be varied as desired by multiplying the expected lead time demand by a constant to attain the desired standard deviation of demand.

The stockout cost, when a stockout occurs is expressed as a fraction of the expected demand over the lead time. Since in practice the stockout cost would vary each occasion a stockout occurs, the stockout cost is averaged over the expected demand over the lead time. Also it makes it easy to vary the stockout cost by multiplying the expected lead time demand by a constant to achieve the desired value of stockout cost.

When considering the different versions of model (Q,R), (the EBQ - ROL, Tate's exponential, Heuristic, Inexact and Exact models), the following values were assigned to the parameters.

Annual Demand = 10, 100, 1000, 10000, 100000, 1000000

Order cost in £s. = 0.1, 1.0, 10

Holding Cost in £s=0.01, 0.1, 1.0

$\sqrt{\sigma^2 L}$ / DxL = 0.1, 1.0

Lead Time = 0.1, 0.2, 0.4 of a year

Stockout Cost =£0.005, 0.05 x Lead time demand per period.

The values assigned to the parameters cover most of the values that are met with in practice.

When a stockout occurs the cost may consist of the cost of telephone calls and paper work. Hence the values assigned to the stockout costs would cover the stockout costs met with in practice.

The lead time ranges from six weeks to twenty weeks. In practice the lead time could be a lot less than six weeks.

We now discuss the values assigned to the parameters for the exact models, (Q,R),(nQ,R,T), (M,T) and (M,R,T). When the shortage cost is more complicated than the simple stockout costs assumed above, the time and cost of computation increases. As a result, the values assigned to the parameters had to be limited.

A sample of values was assigned to the parameters and it was found that the difference in annual inventory costs did not vary much with changes in the values assigned to the parameters except those assigned to order costs, review costs and demand, hence some parameters are held constant. Parameters held constant are b_1, b_2, b_3, b_4 as can be seen below.

The backorder cost functions are as follows: exponential = $0.1 \times e^{b_4 t}$; quadratic = $b_1 + b_2 t + b_3 t^2$; linear = $b_1 + b_2 t$.

In cases where the values of b_1, b_2, b_3, b_4 are related to the cost of the item, it is only necessary to change their value in proportion to the cost of the item.

Values assigned are:

Demand, D		=10, 1000, 100000
Standard deviation of demand per year		=0.1 × D
Holding cost, hc		=£s 0.01, 0.1, 1.0
	b_1	=£1.0
	b_2	=£0.1
	b_3	=£1.0
	b_4	= 2.5
Review cost, R_c		=£0.01,0.1,1.0,10.0,100
Order cost , S		=£0.01,0.1,1.0,10.0,100.0,1000.0
Period of grace, p		=0.25 of lead time.
Lead time, L		=0.1 of a year

In the cases of continuous lead times and distributed supply, values assigned to the parameters of their gamma distributions, respectively, were such as to make the expected lead times and expected supply quantity equal to the values being used in earlier chapters.

Chapter 3

In this chapter we present a subset of the results. The detailed results can be found in volume 3 of the thesis. The diagram below shows the relationship of the subset to the whole results.

			Backorder Cost Function								
			Linear			Quadratic			Exponential		
			a	b	c	a	b	c	a	b	c
Fixed Supply	Lead Time	Constant	✓			✓	*✓		✓		
		Variable	✓			✓			✓		
Variable Supply	Lead Time	Constant	✓			✓			✓		
		Variable	✓			✓			✓		

* Includes period of grace. Holding cost =£0.01 for a; =£0.1 for b; =£1.0 for c.

The subset presented in this chapter is the results for the linear backorder cost function for all combinations and the results for constant lead times for exponential and quadratic backorder cost for fixed supply.

The subset is chosen because any conclusion drawn from it would be applicable to the quadratic and exponential backorder cost functions respectively, for all the assumptions, after inspection of the whole results.

We present a guide to the results on pages 41 to 75 of this chapter.

Different Versions of model (Q,R)

Demand = 10 -----10^6

Order cost = 0.1 -- 10.0

Holding cost = £0.01 --1.0

Standard deviation of demand over lead time = 0.1 $(\sqrt{\sigma^2 L/DL})$

Lead Time = 0.1 of a year

Stockout cost =£ 0.05xLead time demand

Model	Page
EBQ-ROL	41
Tate's Exponential	42
Heuristic	43
Inexact	44
Exact	45

On pages 41 and 43 it will be seen that the inventory costs decreases as order cost increases for high levels of demand. This is not due to the programming technique but due an error in the model itself. An explanation of the reason is given in chapter 2 of volume 2.

The results of (Q,R) given in page 45 can not be compared with the results of (Q,R) given in page 48 or later pages because the shortage costs taken into consideration are different. Page 45 considers a constant average total costs whenever there is a stockout while 48 considers a linear function of backorder cost per unit backordered per length of time of backorder.

Case 1 **Linear Backorder Costs and Constant Lead Times**

Demand	=	$10 - - - - - - 10^6$
Review Cost, Rc	=	$0.01 - - - - 10^2$
Order Cost, S	=	$0.01 - - - - 10^3$
Holding Cost, hc	=	0.01
Lead Time	=	0.1 of a year
$\sqrt{\sigma^2 L}/DL$	=	0.1
b_1	=	1.0
b_2	=	0.1

Quadratic Backorder Costs and Constant Lead Times

Case 2

Case 2 = Case 1 plus b_3

where $b_3 = 1.0$

Exponential Backorder Costs and Constant Lead Times

CASE 3: Case 3 = Case 1 deleting b_1 and b_2 and adding $b_4 = 0.5$ (where the backorder costs $= 0.1 \times e^{b_4 t}$ where t is the length of time of a backorder).

CASE 4: **Period Of Grace: Quadratic Backorder Costs**

Case 4 = Case 2 plus period of grace, p

where p = 0.25 of Lead Time

CASE 5: **Continuous Lead Times and Linear Backorder Costs**

Linear Backorder Costs And Constant Lead Times.

Variable Supply

CASE 6:

CASE 7: Continuous Lead Times and Variable Supply

Linear backorder costs

EBQ-REORDER LEVEL MODEL: ANNUAL INVENTORY COSTS

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND OVER LEAD TIME = 0.1 * D * SQRT(L)

STOCKOUT COST =0.05 * LEAD TIME DEMAND

	ANNUAL DEMAND	HC=0.01	HC=0.1	HC=1.0
S=0.1	10	0.147	0.474	1.492
	100	0.520	1.916	8.082
	1000	2.285	12.019	78.501
	10000	14.400	106.074	873.822
	100000	124.583	1039.545	9860.495
	1000000	1248.833	11185.830	99931.360
S=1.0	10	0.450	1.415	4.481
	100	1.469	4.745	14.922
	1000	5.202	19.163	80.815
	10000	22.849	120.193	785.010
	100000	144.004	1060.740	8738.224
	1000000	1245.830	10395.453	98604.950
S=10.0	10	1.414	4.472	14.144
	100	4.498	14.147	44.809
	1000	14.690	47.446	149.222
	10000	52.024	191.628	808.151
	100000	228.491	1201.935	7850.099
	1000000	1440.041	10607.404	87382.236

TATE'S EXPONENTIAL APPROXIMATION TO THE HEURISTIC MODEL

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND OVER LEAD TIME = 0.1*D *SQRT(L)

STOCKOUT COST = 0. 05* LEAD TIME DEMAND

	ANNUAL DEMAND	HC=0.01	HC=0.1	HC=1.0
S=0.1	10	0.147	0.472	1.473
	100	0.520	1.973	6.648
	1000	2.281	11.761	53.277
	10000	14.411	99.286	516.681
	100000	124.430	966.070	5151.179
	1000000	1212.890	9631.636	51496.055
S=1.0	10	0.450	1.415	4.484
	100	1.470	4.719	14.727
	1000	5.197	19.727	66.484
	10000	22.810	117.607	532.773
	100000	144.107	992.863	5166.905
	1000000	1244.300	9660.696	51511.794
S=10.	10	1.414	4.472	14.145
	100	4.497	14.149	44.835
	1000	14.697	47.191	147.291
	10000	51.966	197.276	664.836
	100000	228.096	1176.072	5322.733
	1000000	1441.073	9928.628	51669.051

HEURISTIC MODEL: ANNUAL INVENTORY COSTS

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND OVER LEAD TIME = 0.1 * D * SQRT(L)

STOCKOUT COST = 0.05 * LEAD TIME DEMAND

	ANNUAL DEMAND	HC=0.01	HC=0.1	HC=1.0
S=0.1	10	0.147	0.473	1.464
	100	0.520	1.962	6.575
	1000	2.286	11.757	52.905
	10000	14.304	99.874	573.179
	100000	121.660	972.805	5765.944
	1000000	1178.703	9700.249	57684.875
S=1.0	10	0.450	1.415	4.481
	100	1.469	4.730	14.640
	1000	5.200	19.621	65.751
	10000	22.856	117.572	529.049
	100000	143.045	998.740	5731.751
	1000000	1216.599	9728.052	57659.492
S=10.0	10	1.414	4.472	14.144
	100	4.498	14.148	44.811
	1000	14.689	47.300	146.397
	10000	52.002	196.206	657.510
	100000	228.557	1175.715	5290.492
	1000000	1430.446	9987.402	57317.446

INEXACT MODEL: ANNUAL INVENTORY COSTS

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND OVER LEAD TIME = 0.1 * D * SQRT(L)

STOCKOUT COST = 0.05 * LEAD TIME DEMAND

	ANNUAL DEMAND	HC=0.01	HC=0.1	HC=10.
	10	0.142	0.451	1.461
	100	0.476	1.663	6.537
	1000	2.071	10.437	52.818
S=0.1	10000	13.304	92.066	549.979
	100000	111.191	904.471	5128.916
	1000000	1036.974	9033.752	51266.953
	10	0.447	1.415	4.481
	100	1.418	4.509	14.613
	1000	4.757	16.633	65.366
S=1.0	10000	20.708	104.372	527.973
	100000	133.066	920.641	5141.736
	1000000	1116.211	9044.545	51294.349
	10	1.414	4.472	14.144
	100	4.473	14.472	44.809
	1000	14.175	45.087	146.133
S=10.0	10000	47.571	166.350	653.702
	100000	207.083	1043.723	5277.328
	1000000	1332.412	9205.967	51417.420

EXACT MODEL: ANNUAL INVENTORY COSTS

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND OVER LEAD TIME = 0.1 * D * SQRT(L)

STOCKOUT COST =0.[illegible]05 * LEADTIME DEMAND

	ANNUAL DEMAND	HC=0.01	HC=0.1	HC=1.0
	10	0.142	0.451	1.461
	100	0.476	1.663	6.528
	1000	2.071	10.436	52.503
S=0.1	10000	13.302	92.055	511.426
	100000	111.142	904.385	5098.044
	1000000	1035.889	9032.652	50959.148
	10	0.447	1.415	4.481
	100	1.418	4.509	14.612
	1000	4.757	16.632	65.282
S=1.0	10000	20.707	104.363	525.035
	100000	133.049	920.529	5111.343
	1000000	1115.777	9043.360	51002.757
	10	1.414	4.472	14.144
	100	4.473	14.147	44.809
	1000	14.175	45.087	146.124
S=10.0	10000	47.571	166.339	653.725
	100000	207.067	1043.629	5252.228
	1000000	1332.232	9205.515	51123.544

DEMAND FOLLOWS A NORMAL DISTRIBUTION

INEXACT MODEL OF (Q,R)

LEAD TIME IS 0.1 OF A YEAR

BACKORDER COST PER YEAR =£1.0

ORDER COSTS =£1.0 AND HOLDING COSTS =£0.1

BACKORDER COST PER UNIT BACKORDERED =£0.1

$0.1 \sqrt{\sigma^2 L/DL}$	D=10	D=100	D=1000
0.0000	1.415	4.501	14.645
0.0006	1.421	4.680	17.627
0.0012	1.429	4.880	20.690
0.0018	1.437	5.096	23.821
0.0024	1.447	5.323	27.005
0.0029	1.458	5.561	30.235
0.0035	1.470	5.808	33.502
0.0041	1.484	6.063	36.801
0.0047	1.498	6.324	40.127
0.0053	1.517	6.591	43.475
0.0059	1.531	6.863	46.843
0.0065	1.548	7.140	50.229
0.0071	1.567	7.422	53.627
0.0076	1.587	7.707	57.039
0.0082	1.607	7.996	60.462
0.0088	1.628	8.288	63.897
0.0094	1.650	8.583	67.337
0.0100	1.672	8.881	70.786

DEMAND FOLLOWS A NORMAL DISTRIBUTION

EXACT MODEL OF (Q,R)

LEAD TIME IS 0.1 OF A YEAR

BACKORDER COST PER YEAR = 1.0

ORDERING COSTS = 1.0 AND HOLDING COSTS = 0.1

BACKORDER COST PER UNIT BACKORDERED =0.10

$0.1 \times \sqrt{\sigma^2 L}/DL$	D=10	D=100	D=1000
0.0000	1.415	4.501	14.645
0.0006	1.421	4.680	17.627
0.0012	1.429	4.889	20.674
0.0018	1.437	5.096	23.800
0.0024	1.447	5.320	26.980
0.0029	1.458	5.558	30.203
0.0035	1.470	5.804	33.449
0.0041	1.484	6.058	36.705
0.0047	1.498	6.319	39.972
0.0053	1.517	6.586	43.246
0.0059	1.530	6.858	46.526
0.0065	1.547	7.135	49.808
0.0071	1.566	7.416	53.090
0.0076	1.586	7.700	56.379
0.0082	1.606	7.988	59.665
0.0088	1.627	8.279	62.952
0.0094	1.649	8.572	66.242
0.0100	1.670	8.868	69.531

MODEL (Q,R) :

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.049	0.113	0.337	1.041	3.224
0.10	0.113	0.152	0.352	1.045	3.226
1.00	0.337	0.352	0.472	1.090	3.240
10.00	1.041	1.045	1.090	1.462	3.379
100.00	3.224	3.226	3.240	3.379	4.532
1000.00	9.995	9.995	10.000	10.044	10.474
D= 1000.0					
0.01	3.141	7.259	21.577	66.596	206.356
0.10	7.259	9.737	22.501	66.889	206.447
1.00	21.577	22.501	30.185	69.755	207.357
10.00	66.596	66.889	69.755	93.573	216.239
100.00	206.356	206.447	207.357	216.239	290.075
1000.00	639.675	639.704	639.987	642.808	670.342
D=100000.0					
0.01	201.022	464.547	1380.943	4262.137	13206.788
0.10	464.547	623.168	1440.095	4280.924	13212.626
1.00	1380.943	1440.095	1931.822	4464.296	13270.865
10.00	4262.137	4280.924	4464.296	5988.648	13839.316
100.00	13206.788	13212.626	13270.865	13839.316	18564.808
1000.00	40939.232	40941.042	40959.141	41139.680	42901.881

MODEL(M,T)

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.095	0.215	0.620	1.852	5.554
0.10	0.215	0.286	0.646	1.860	5.557
1.00	0.620	0.646	0.859	1.937	5.580
10.00	1.852	1.860	1.937	2.577	5.812
100.00	5.554	5.557	5.580	5.812	7.731
1000.00	16.662	16.662	16.670	16.741	17.437
D= 1000.0					
0.01	6.108	13.777	39.682	118.540	355.467
0.10	13.777	18.325	41.332	119.047	355.619
1.00	39.682	41.332	54.975	123.996	357.141
10.00	118.540	119.047	123.996	164.924	371.987
100.00	355.467	355.619	357.141	371.987	494.773
1000.00	1066.354	1066.400	1066.857	1071.423	1115.960
D=100000.0					
0.01	390.932	881.746	2539.670	7586.542	22749.860
0.10	881.746	1172.795	2645.238	7619.010	22759.625
1.00	2539.670	2645.238	3518.385	7935.715	22857.031
10.00	7586.542	7619.010	7935.715	10555.154	23807.146
100.00	22749.860	22759.625	22857.031	23807.146	31665.461
1000.00	68246.649	68249.579	68278.876	68571.094	71421.438

MODEL (Q,R,T)

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.097	0.232	0.676	2.020	6.059
0.10	0.205	0.291	0.696	2.027	6.061
1.00	0.581	0.614	0.873	2.088	6.080
10.00	1.733	1.744	1.842	2.620	6.264
100.00	5.197	5.200	5.231	5.527	7.859
1000.00	15.591	15.592	15.601	15.693	16.582
D= 1000.0					
0.01	6.209	14.849	43.233	129.296	387.768
0.10	13.102	18.628	44.547	129.698	387.889
1.00	37.198	39.305	55.885	133.641	389.093
10.00	110.939	111.593	117.916	167.654	400.922
100.00	332.621	332.818	334.780	353.749	502.961
1000.00	997.805	997.864	998.454	1004.341	1061.248
D=100000.0					
0.01	397.401	950.334	2766.887	8274.961	24817.160
0.10	838.517	1192.204	2851.003	8300.661	24824.884
1.00	2380.660	2515.550	3576.611	8553.010	24901.982
10.00	7100.120	7141.980	7546.650	10729.832	25659.031
100.00	21287.758	21300.361	21425.941	22639.950	32189.496
1000.00	63859.493	63863.275	63901.083	64277.829	67919.850

MODEL (M,R,T)

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.
D= 10.0					
0.01	0.088	0.204	0.599	1.761	5.282
0.10	0.195	0.265	0.611	1.768	5.284
1.00	0.558	0.584	0.795	1.834	5.304
10.00	1.665	1.673	1.751	2.385	5.501
100.00	4.993	4.996	5.020	5.254	7.154
1000.00	14.979	14.980	14.987	15.059	15.761
D= 1000.0					
0.01	5.653	13.039	37.717	112.722	338.037
0.10	12.453	16.958	39.116	113.151	338.166
1.00	35.696	37.358	50.873	117.347	339.453
10.00	106.574	107.087	112.075	152.619	352.040
100.00	319.568	319.722	321.261	336.226	457.857
1000.00	958.658	958.705	959.167	963.784	1008.677
D=100000.0					
0.01	361.764	834.465	2413.889	7214.213	21634.385
0.10	796.979	1085.291	2503.394	7241.668	21642.640
1.00	2284.524	2390.938	3255.874	7510.183	21725.004
10.00	6820.746	6853.573	7172.815	9767.622	22530.549
100.00	20452.360	20462.239	20560.718	21518.446	29302.065
1000.00	61354.124	61357.088	61386.718	61682.155	64555.337

Quadratic Backorder Costs and Constant Lead Times

MODEL (Q,R)

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER SQUARE TIME BACKORDERED=1.00

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.071	0.154	0.421	1.196	3.407
0.10	0.154	0.202	0.438	1.201	3.408
1.00	0.421	0.438	0.575	1.248	3.422
10.00	1.196	1.201	1.248	1.638	3.557
100.00	3.407	3.408	3.422	3.557	4.669
1000.00	9.708	9.709	9.713	9.752	10.138
D= 1000.0					
0.01	5.113	11.103	30.437	86.394	246.122
0.10	11.103	14.572	31.642	86.746	246.222
1.00	30.437	31.642	41.530	90.180	247.227
10.00	86.394	86.746	90.180	118.361	257.014
100.00	246.122	246.222	247.227	257.014	337.328
1000.00	701.418	701.447	701.734	704.597	732.491
D=100000.0					
0.01	369.412	802.159	2199.094	6241.954	17782.291
0.10	802.159	1052.823	2286.154	6267.418	17789.568
1.00	2199.094	2286.154	3000.545	6515.539	17862.141
10.00	6241.954	6267.418	6515.539	8551.554	18569.286
100.00	17782.291	17789.568	17862.141	18569.286	24371.929
1000.00	50677.455	50679.529	50700.268	50907.102	52922.466

MODEL(M,T)

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER SQUARE TIME BACKORDERED=1.00

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.138	0.292	0.775	2.122	5.834
0.10	0.292	0.380	0.804	2.131	5.836
1.00	0.775	0.804	1.046	2.212	5.859
10.00	2.122	2.131	2.212	2.876	6.083
100.00	5.834	5.836	5.859	6.083	7.910
1000.00	16.042	16.043	16.049	16.112	16.728
D= 1000.0					
0.01	9.993	21.133	55.976	153.330	421.491
0.10	21.133	27.480	58.115	153.934	421.657
1.00	55.976	58.115	75.571	159.816	423.319
10.00	153.330	153.934	159.816	207.821	439.495
100.00	421.491	421.657	423.319	439.495	571.506
1000.00	1159.053	1159.099	1159.557	1164.126	1208.611
D=100000.0					
0.01	721.985	1526.841	4044.267	11078.084	30452.695
0.10	1526.841	1985.459	4198.812	11121.733	30464.731
1.00	4044.267	4198.812	5460.012	11546.733	30584.767
10.00	11078.084	11121.733	11546.733	15015.032	31753.515
100.00	30452.695	30464.731	30584.767	31753.515	41291.339
1000.00	83741.599	83744.911	83778.312	84108.108	87322.167

MODEL (Q,R,T)

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10.D

HOLDING COST = 0.01

BACKORDER COST PER SQUARE TIME BACKORDERED=1.00

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.140	0.313	0.838	2.299	6.320
0.10	0.279	0.386	0.862	2.305	6.322
1.00	0.730	0.768	1.062	2.370	6.340
10.00	1.997	2.007	2.112	2.920	6.517
100.00	5.488	5.491	5.520	5.808	8.031
1000.00	15.090	15.091	15.099	15.181	15.972
D= 1000.0					
0.01	10.145	22.642	60.572	166.097	456.635
0.10	20.177	27.899	62.265	166.572	456.766
1.00	52.741	55.486	76.722	171.229	458.072
10.00	144.254	145.037	152.587	210.985	470.880
100.00	396.482	396.699	398.852	419.615	580.210
1000.00	1090.267	1090.327	1090.921	1096.842	1153.942
D=100000.0					
0.01	732.980	1635.875	4376.294	12000.495	32991.905
0.10	1457.777	2015.694	4498.655	12034.008	33001.360
1.00	3810.516	4008.886	5543.159	12371.302	33095.723
10.00	10422.353	10478.920	11024.435	15243.688	34021.079
100.00	28645.855	28661.471	28817.030	30317.197	41920.143
1000.00	78771.807	78776.102	78819.044	79246.832	83372.292

MODEL (M,R,T)

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER SQUARE TIME BACKORDERED=1.00

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.129	0.278	0.739	2.026	5.570
0.10	0.267	0.354	0.765	2.033	5.572
1.00	0.703	0.733	0.974	2.103	5.591
10.00	1.924	1.933	2.015	2.678	5.782
100.00	5.289	5.291	5.315	5.542	7.365
1000.00	14.544	14.545	14.551	14.616	15.242
D= 1000.0					
0.01	9.304	20.087	53.418	146.387	402.423
0.10	19.255	25.587	55.239	146.900	402.565
1.00	50.776	52.950	70.363	151.908	403.975
10.00	139.019	139.635	145.613	193.500	417.748
100.00	382.133	382.303	383.997	400.437	532.124
1000.00	1050.819	1050.866	1051.333	1055.991	1101.202
D=100000.0					
0.01	672.233	1451.289	3859.464	10576.469	29075.075
0.10	1391.150	1848.640	3991.045	10613.526	29085.291
1.00	3668.597	3825.662	5083.760	10975.373	29187.197
10.00	10044.140	10088.642	10520.571	13980.339	30182.275
100.00	27609.108	27621.384	27743.767	28931.572	38445.932
1000.00	75921.670	75925.047	75958.907	76295.359	79561.622

Exponential Backorder Costs and Constant Lead Times

MODEL (Q,R)

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDERED COSTS = 0.1*EXP(2.5* TIME BACKORDERED)

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	BC=0.01	BC=0.1	BC=1.0	BC=10.0	BC=100.0
D= 10.0					
0.01	0.114	0.237	0.618	1.661	4.483
0.10	0.237	0.327	0.641	1.667	4.484
1.00	0.618	0.641	0.829	1.730	4.502
10.00	1.661	1.667	1.730	2.239	4.671
100.00	4.483	4.484	4.502	4.671	6.045
1000.00	12.103	12.103	12.108	12.155	12.611
D= 1000.0					
0.01	9.213	19.221	50.020	134.532	363.096
0.10	19.221	24.875	51.896	135.053	363.237
1.00	50.020	51.696	67.163	140.118	364.642
10.00	134.532	135.053	140.118	181.340	378.319
100.00	363.096	363.237	364.642	378.319	489.617
1000.00	980.322	980.360	980.740	984.534	1021.461
D=100000.0					
0.01	746.253	1556.869	4051.581	10897.112	29410.788
0.10	1556.869	2014.884	4203.545	10039.268	29422.202
1.00	4051.581	4203.545	5440.187	11349.572	29536.023
10.00	10897.112	10939.268	11349.572	14688.505	30643.844
100.00	29410.788	29422.202	29536.023	30643.844	39658.964
1000.00	79406.046	79409.129	79439.946	79747.262	82738.378

MODEL (M, T)

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDERED COSTS = 0.1*EXP(2.5* TIME BACKORDERED)

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.223	0.453	1.136	2.942	7.646
0.10	0.453	0.580	1.177	2.953	7.649
1.00	1.136	1.177	1.508	3.059	7.677
10.00	2.942	2.953	3.059	3.921	7.954
100.00	7.646	7.649	7.677	7.954	10.193
1000.00	19.878	19.879	19.886	19.960	20.680
D= 1000.0					
0.01	18.068	36.656	91.988	238.282	619.303
0.10	36.656	46.077	95.305	239.160	619.534
1.00	91.988	95.305	122.140	247.793	621.840
10.00	238.282	239.169	247.793	317.564	644.261
100.00	619.303	619.534	621.840	644.261	825.666
1000.00	1610.127	1610.188	1610.789	1616.783	1675.079
D=100000.0					
0.01	1463.512	2969.114	7451.036	19300.871	50163.536
0.10	2969.114	3805.130	7719.698	19372.694	50182.264
1.00	7451.036	7719.698	9893.338	20071.214	50369.006
10.00	19300.871	19372.694	20071.214	25722.679	52185.156
100.00	50163.536	50182.264	50369.006	52185.156	66878.966
1000.00	130420.323	130425.194	130473.887	130959.415	135681.405

MODEL (Q,R,T)

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDERED COSTS = 0.1*EXP(2.5* TIME BACKORDERED)

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.218	0.463	1.171	3.037	7.894
0.10	0.422	0.568	1.204	3.046	7.806
1.00	1.047	1.096	1.476	3.130	7.918
10.00	2.710	2.723	2.851	3.838	8.137
100.00	7.041	7.045	7.079	7.412	9.979
1000.00	18.307	18.308	18.317	18.406	19.271
D= 1000.0					
0.01	17.688	37.499	94.879	245.991	639.394
0.10	34.158	45.988	97.408	246.686	639.576
1.00	84.827	88.812	119.568	253.496	641.384
10.00	219.474	220.549	230.911	310.876	659.089
100.00	570.353	570.634	573.427	600.369	808.277
1000.00	1482.845	1482.918	1483.647	1490.911	1560.960
D=100000.0					
0.01	1432.689	3737.448	7685.225	19925.238	51790.938
0.10	2766.836	3724.992	7897.365	19981.584	51805.620
1.00	6870.947	7193.773	9684.980	20533.150	51952.119
10.00	17777.432	17864.463	18703.809	25180.249	53386.190
100.00	46198.609	46221.323	46447.603	48629.902	65470.467
1000.00	120110.477	120116.384	120175.441	120763.769	126437.745

MODEL (M,R,T)

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDERED COSTS = 0.1*EXP(2.5* TIME BACKORDERED)

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.289	0.431	1.087	2.816	7.319
0.10	0.414	0.542	1.122	2.825	7.321
1.00	1.036	1.078	1.410	2.916	7.345
10.00	2.682	2.693	2.802	3.665	7.582
100.00	6.970	6.972	7.002	7.284	9.529
1000.00	18.120	18.121	18.128	18.204	18.940
D= 1000.0					
0.01	16.890	34.940	88.013	228.078	592.807
0.10	33.571	43.913	90.844	228.833	593.003
1.00	83.896	87.284	114.174	236.195	594.966
10.00	217.220	218.129	226.939	296.853	614.107
100.00	564.534	564.771	567.135	590.043	771.819
1000.00	1467.727	1467.789	1468.406	1474.550	1534.111
D=100000.0					
0.01	1368.066	2830.148	7129.026	18474.333	48017.329
0.10	2719.245	3556.970	7358.385	18535.466	48033.265
1.00	6795.549	7070.036	9248.123	19131.800	48192.213
10.00	17594.800	17668.426	18382.094	24045.119	49742.680
100.00	45727.275	45746.480	45937.908	47793.444	62517.310
1000.00	118885.921	118890.916	118940.849	119438.560	124262.954

Period of Grace: Quadratic Backorder Costs

MODEL (Q,R)

LEAD TIME IS 0.1 OF A YEAR

PERIOD OF GRACE = 0.25 OF LEAD TIME

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

BACKORDER COST PER SQUARE TIME BACKORDERED=1.00

BACKORDER COST PER YEAR = 1.0 HOLDING COST = 0.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.053	0.120	0.350	1.059	3.217
0.10	0.120	0.160	0.364	1.063	3.218
1.00	0.350	0.364	0.486	1.108	3.232
10.00	1.059	1.063	1.108	1.479	3.368
100.00	3.217	3.218	3.232	3.368	4.496
1000.00	9.779	9.780	9.784	9.826	10.240
D= 1000.0					
0.01	3.635	8.279	24.153	73.108	222.152
0.10	8.279	11.050	25.169	73.425	222.249
1.00	24.153	25.169	33.592	76.514	223.211
10.00	73.108	73.425	76.514	102.121	232.604
100.00	222.152	222.249	223.211	232.604	310.448
1000.00	675.314	675.343	675.636	678.563	707.116
D=100000.0					
0.01	251.013	571.740	1667.905	5048.565	15340.971
0.10	571.740	763.081	1738.090	5070.432	15347.636
1.00	1667.905	1738.090	2319.766	5283.792	15414.114
10.00	5048.565	5070.432	5283.792	7052.088	16062.729
100.00	15340.971	15347.636	15414.114	16062.729	21438.348
1000.00	46634.527	46636.553	46656.314	46853.907	48830.696

MODEL(M,T)

LEAD TIME IS 0.1 OF A YEAR

PERIOD OF GRACE = 0.25 OF LEAD TIME

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

BACKORDER COST PER SQUARE TIME BACKORDERED=1.00

BACKORDER COST PER YEAR = 1.0 HOLDING COST = 0.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.102	0.228	0.643	1.883	5.534
0.10	0.228	0.301	0.669	1.891	5.537
1.00	0.643	0.669	0.886	1.968	5.560
10.00	1.883	1.891	1.968	2.604	5.787
100.00	5.534	5.537	5.560	5.787	7.656
1000.00	16.270	16.271	16.278	16.346	17.013
D= 1000.0					
0.01	7.077	15.725	44.419	130.046	382.174
0.10	15.725	20.806	46.231	130.592	382.335
1.00	44.419	46.231	61.169	135.919	383.941
10.00	130.046	130.592	135.919	179.837	399.601
100.00	382.174	382.335	383.941	399.601	528.721
1000.00	1123.543	1123.590	1124.064	1128.786	1174.827
D=100000.0					
0.01	488.697	1085.892	3067.409	8980.456	26391.419
0.10	1085.892	1436.769	3192.522	9018.182	26402.549
1.00	3067.409	3192.522	4224.101	9386.015	26513.454
10.00	8080.456	9018.182	9386.015	12418.858	27594.883
100.00	26391.419	26402.540	26513.454	27594.883	36511.443
1000.00	77587.502	77590.773	77623.467	77949.555	81128.757

MODEL (Q,R,T)

LEAD TIME IS 0.1 OF A YEAR

PERIOD OF GRACE = 0.25 OF LEAD TIME

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

BACKORDER COST PER SQUARE TIME BACKORDERED=1.00

BACKORDER COST PER YEAR = 1.0 HOLDING COST = 0.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.100	0.234	0.666	1.952	5.737
0.10	0.210	0.294	0.687	1.958	5.739
1.00	0.587	0.618	0.865	2.019	5.757
10.00	1.716	1.726	1.818	2.542	5.937
100.00	5.043	5.046	5.074	5.344	7.475
1000.00	14.826	14.827	14.835	14.917	15.710
D= 1000.0					
0.01	6.909	16.134	45.998	134.804	396.196
0.10	14.521	20.312	47.433	135.234	396.322
1.00	40.536	42.692	59.717	139.454	397.588
10.00	118.521	119.176	125.515	175.568	409.996
100.00	348.259	348.452	350.378	369.013	516.171
1000.00	1023.824	1023.881	1024.449	1030.111	1084.898
D=100000.0					
0.01	477.097	1114.139	3176.437	9309.007	27359.728
0.10	1002.771	1402.665	3275.570	9338.723	27368.48[illegible]
1.00	2799.266	2948.146	4123.835	9630.175	27455.84[illegible]
10.00	8184.605	8229.842	8667.548	12124.074	28312.71[illegible]
100.00	24049.394	24062.740	24195.734	28482.591	35644.77[illegible]
1000.00	70701.294	70705.219	70744.455	71135.458	74918.81[illegible]

MODEL (M,R,T)

LEAD TIME IS 0.1 OF A YEAR

PERIOD OF GRACE = 0.25 OF LEAD TIME

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

BACKORDER COST PER SQUARE TIME BACKORDERED=1.00

BACKORDER COST PER YEAR = 1.0 HOLDING COST = 0.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.095	0.216	0.612	1.792	5.268
0.10	0.206	0.279	0.634	1.799	5.270
1.00	0.580	0.606	0.821	1.865	5.289
10.00	1.696	1.704	1.782	2.413	5.482
100.00	4.985	4.987	5.011	5.240	7.095
1000.00	14.655	14.656	14.663	14.732	15.406
D= 1000.0					
0.01	6.558	14.897	42.259	123.778	363.771
0.10	14.240	19.281	43.796	124.240	363.907
1.00	40.034	41.864	56.686	128.761	365.267
10.00	117.148	117.701	123.081	166.657	378.557
100.00	344.251	344.415	346.042	361.857	489.971
1000.00	1012.051	1012.099	1012.579	1017.362	1063.860
D=100000.0					
0.01	452.880	1028.706	2918.216	8547.624	25120.605
0.10	983.326	1331.467	3024.395	8579.554	25130.014
1.00	2764.622	2890.977	3914.514	8891.721	25223.888
10.00	8089.771	8127.987	8499.473	11508.672	26141.660
100.00	23772.656	23783.927	23896.282	24988.452	33835.496
1000.00	69888.301	69891.615	69924.745	70255.071	73466.049

LEAD TIME IS GAMMA DISTRIBUTED

MODEL (Q,R)

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.
D= 10.0					
0.01	0.056	0.130	0.388	1.197	3.708
0.10	0.130	0.175	0.404	1.202	3.710
1.00	0.388	0.404	0.542	1.253	3.726
10.00	1.197	1.202	1.253	1.681	3.886
100.00	3.708	3.710	3.726	3.886	5.212
1000.00	11.494	11.495	11.500	11.550	12.045
D= 1000.0					
0.01	3.612	8.347	24.814	76.585	237.309
0.10	8.347	11.198	25.877	76.923	237.414
1.00	24.814	25.877	34.712	80.218	238.461
10.00	76.585	76.923	80.218	107.609	248.675
100.00	237.309	237.414	238.461	248.675	333.586
1000.00	735.627	735.659	735.985	739.229	770.893
D=100000.0					
0.01	231.175	534.229	1588.085	4901.458	15187.806
0.10	534.229	716.644	1656.110	4923.063	15194.520
1.00	1588.085	1656.110	2221.595	5133.940	15261.494
10.00	4901.458	4923.063	5133.940	6886.945	15915.214
100.00	15187.806	15194.520	15261.494	15915.214	21349.530
1000.00	47080.116	47082.198	47103.012	47310.632	49337.163

LEAD TIME IS GAMMA DISTRIBUTED

MODEL(M,T)

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.110	0.248	0.713	2.130	6.387
0.10	0.248	0.329	0.743	2.139	6.390
1.00	0.713	0.743	0.988	2.228	6.417
10.00	2.130	2.139	2.228	2.963	6.684
100.00	6.387	6.390	6.417	6.684	8.890
1000.00	19.161	19.162	19.170	19.252	20.052
D= 1000.0					
0.01	7.025	15.844	45.635	136.321	408.787
0.10	15.844	21.074	47.532	136.904	408.962
1.00	45.635	47.532	63.221	142.595	410.712
10.00	136.321	136.904	142.595	189.663	427.785
100.00	408.787	408.962	410.712	427.785	568.989
1000.00	1226.307	1226.360	1226.886	1232.137	1283.354
D=100000.0					
0.01	449.571	1014.008	2920.621	8724.523	26162.339
0.10	1014.008	1348.714	3042.024	8761.862	26173.569
1.00	2020.621	3042.024	4046.142	9126.073	26285.586
10.00	8724.523	8761.862	9126.073	12138.427	27378.218
100.00	26162.339	26173.569	26285.586	27378.218	36415.280
1000.00	78483.646	78487.016	78520.708	78856.758	82134.653

LEAD TIME IS GAMMA DISTRIBUTED

MODEL (nQ, R, T)

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.112	0.267	0.777	2.323	6.968
0.10	0.235	0.335	0.800	2.331	6.970
1.00	0.668	0.706	1.004	2.401	6.992
10.00	1.993	2.005	2.119	3.013	7.204
100.00	5.977	5.980	6.016	6.356	9.038
1000.00	17.929	17.930	17.941	18.047	19.069
D= 1000.0					
0.01	7.141	17.076	49.717	148.691	445.933
0.10	15.067	21.422	51.229	149.152	446.072
1.00	42.777	45.201	64.267	153.687	447.457
10.00	127.580	128.332	135.604	192.802	461.061
100.00	382.514	382.741	384.997	406.812	578.405
1000.00	1147.475	1147.543	1148.223	1154.992	1220.435
D=100000.0					
0.01	457.011	1092.885	3181.920	9516.206	28539.734
0.10	964.294	1371.034	3278.654	9545.760	28548.617
1.00	2737.759	2892.882	4113.102	9835.262	28637.280
10.00	8165.138	8213.277	8678.647	12339.307	29507.886
100.00	24480.922	24495.415	24639.832	26035.942	37017.920
1000.00	73438.417	73442.766	73486.246	73919.497	78107.827

LEAD TIME IS GAMMA DISTRIBUTED

MODEL (M,R,T)

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.102	0.234	0.678	2.025	6.074
0.10	0.224	0.305	0.703	2.033	6.076
1.00	0.641	0.671	0.914	2.109	6.100
10.00	1.915	1.924	2.014	2.742	6.326
100.00	5.742	5.745	5.773	6.042	8.227
1000.00	17.226	17.227	17.235	17.318	18.125
D= 1000.0					
0.01	6.500	14.994	43.375	129.630	386.743
0.10	14.321	19.501	44.983	130.124	388.891
1.00	41.050	42.962	58.504	134.949	390.371
10.00	122.560	123.150	128.887	175.512	404.846
100.00	367.503	367.681	369.450	386.660	526.536
1000.00	1102.457	1102.510	1103.043	1108.351	1159.979
D=100000.0					
0.01	416.028	959.635	2775.973	8296.345	24879.543
0.10	916.526	1248.085	2878.904	8327.918	24889.036
1.00	2627.203	2749.579	3744.255	8636.711	24983.755
10.00	7843.858	7881.609	8248.738	11232.765	25910.132
100.00	23520.217	23531.575	23644.826	24746.213	33698.295
1000.00	70557.243	70560.651	70594.725	70934.479	74238.638

SUPPLY FOLLOWS A GAMMA DISTRIBUTION

MODEL (Q,R)

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.067	0.149	0.421	1.236	3.637
0.10	0.149	0.197	0.439	1.241	3.639
1.00	0.421	0.439	0.581	1.292	3.654
10.00	1.236	1.241	1.292	1.710	3.804
100.00	3.637	3.639	3.654	3.804	5.035
1000.00	10.712	10.712	10.717	10.762	11.201
D= 1000.0					
0.01	5.665	12.604	35.662	104.584	307.871
0.10	12.604	16.684	37.119	105.024	308.001
1.00	35.662	37.119	49.135	109.315	309.297
10.00	104.584	105.024	109.315	144.701	321.933
100.00	307.871	308.001	309.297	321.933	426.145
1000.00	906.643	906.681	907.064	910.880	948.093
D=100000.0					
0.01	479.504	1066.806	3018.429	8852.028	26058.224
0.10	1066.806	1412.138	3141.742	8889.273	26069.222
1.00	3018.429	3141.742	4158.748	9252.431	26178.909
10.00	8852.028	8889.273	9252.431	12247.513	27248.409
100.00	26058.224	26069.222	26178.909	27248.409	36068.924
1000.00	76738.231	76741.470	76773.858	77096.886	80246.564

SUPPLY FOLLOWS A GAMMA DISTRIBUTION

MODEL(M,T)

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.
D= 10.0					
0.01	0.130	0.283	0.775	2.199	6.266
0.10	0.283	0.371	0.806	2.208	6.268
1.00	0.775	0.806	1.057	2.296	6.294
10.00	2.199	2.208	2.296	3.013	6.543
100.00	6.266	6.268	6.294	6.543	8.588
1000.00	17.857	17.857	17.865	17.938	18.648
D= 1000.0					
0.01	11.017	23.923	65.585	186.159	530.335
0.10	23.923	31.399	68.182	186.918	530.553
1.00	65.585	68.182	89.488	194.318	532.717
10.00	186.159	186.918	194.318	255.040	553.807
100.00	530.335	530.553	532.717	553.807	726.864
1000.00	1511.394	1511.456	1512.075	1518.243	1578.349
D=100000.0					
0.01	932.501	2024.880	5551.143	15756.479	44887.595
0.10	2024.880	2657.627	5770.908	15820.758	44905.964
1.00	5551.143	5770.908	7574.236	16447.087	45089.160
10.00	15756.479	15820.758	16447.087	21586.572	46874.197
100.00	44887.595	44905.964	45089.160	46874.197	61521.727
1000.00	127924.410	127929.647	127981.998	128504.106	133591.464

SUPPLY FOLLOWS A GAMMA DISTRIBUTION

MODEL (Q,R,T)

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.132	0.304	0.841	2.389	6.807
0.10	0.269	0.377	0.865	2.396	6.809
1.00	0.729	0.768	1.074	2.466	6.830
10.00	2.065	2.076	2.188	3.061	7.027
100.00	5.881	5.885	5.918	6.237	8.723
1000.00	16.761	16.762	16.771	16.865	17.775
D= 1000.0					
0.01	11.191	25.694	71.168	202.229	576.183
0.10	22.804	31.894	73.229	202.828	576.354
1.00	61.665	64.991	90.899	208.702	578.060
10.00	174.763	175.745	185.225	259.062	594.802
100.00	497.793	498.074	500.873	527.893	738.326
1000.00	1418.630	1418.710	1419.511	1427.488	1504.494
D=100000.0					
0.01	947.206	2174.770	6023.640	17116.701	48768.129
0.10	1930.131	2699.538	6198.093	17167.375	48782.599
1.00	5219.315	5500.872	7693.684	17664.566	48927.018
10.00	14791.923	14875.049	15677.486	21927.000	50344.013
100.00	42133.203	42156.982	42393.889	44680.835	62491.950
1000.00	120072.849	120079.629	120147.398	120822.583	127340.379

SUPPLY FOLLOWS A GAMMA DISTRIBUTION

MODEL (M,R,T)

LEAD TIME IS 0.1 OF A YEAR

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.121	0.268	0.738	2.096	5.973
0.10	0.257	0.345	0.764	2.104	5.975
1.00	0.700	0.732	0.982	2.178	5.996
10.00	1.987	1.996	2.085	2.799	6.208
100.00	5.661	5.664	5.690	5.942	7.976
1000.00	16.133	16.134	16.141	16.216	16.935
D= 1000.0					
0.01	10.232	22.699	62.485	177.439	505.517
0.10	21.726	29.162	64.692	178.083	505.701
1.00	59.289	61.919	83.111	184.372	507.536
10.00	168.201	168.973	176.466	236.867	525.461
100.00	479.153	479.373	481.573	502.934	675.071
1000.00	1365.523	1365.586	1366.214	1372.482	1433.363
D=100000.0					
0.01	866.055	1921.238	5288.743	15018.435	42786.975
0.10	1838.875	2468.258	5475.529	15072.916	42802.540
1.00	5018.197	5240.795	7034.535	15605.258	42957.812
10.00	14236.552	14301.862	14936.266	20048.424	44474.965
100.00	40555.503	40574.173	40760.307	42568.358	57138.008
1000.00	115577.860	115583.183	115636.392	116166.875	121319.81[illegible]

Continuous Lead Times and Variable Supply.

SUPPLY FOLLOWS A GAMMA DISTRIBUTION

LEAD TIME IS GAMMA DISTRIBUTED

MODEL (Q,R)

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.082	0.176	0.472	1.311	3.656
0.10	0.176	0.229	0.490	1.316	3.658
1.00	0.472	0.490	0.640	1.367	3.672
10.00	1.311	1.316	1.367	1.785	3.815
100.00	3.656	3.658	3.672	3.815	4.979
1000.00	10.201	10.201	10.205	10.246	10.643
D= 1000.0					
0.01	6.955	14.867	39.932	110.967	309.474
0.10	14.867	19.406	41.480	111.410	309.598
1.00	39.932	41.480	54.142	115.730	310.835
10.00	110.967	111.410	115.730	151.057	322.886
100.00	309.474	309.598	310.835	322.886	421.449
1000.00	863.397	863.432	863.778	867.230	900.852
D=100000.0					
0.01	588.713	1258.382	3379.851	9392.247	26193.868
0.10	1258.382	1642.510	3510.885	9429.783	26204.369
1.00	3379.851	3510.885	4582.602	9795.370	26309.094
10.00	9392.247	9429.783	9795.370	12785.458	27329.081
100.00	26193.868	26204.369	26309.094	27329.081	35671.429
1000.00	73077.960	73080.891	73110.189	73402.373	76248.136

SUPPLY FOLLOWS A GAMMA DISTRIBUTION

LEAD TIME IS GAMMA DISTRIBUTED

MODEL(M,T)

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.160	0.333	0.868	2.334	6.298
0.10	0.333	0.431	0.900	2.343	6.301
1.00	0.868	0.900	1.165	2.431	6.325
10.00	2.334	2.343	2.431	3.146	6.562
100.00	6.298	6.301	6.325	6.562	8.493
1000.00	17.005	17.006	17.012	17.078	17.719
D= 1000.0					
0.01	13.526	28.220	73.438	197.520	533.096
0.10	28.220	36.522	76.193	198.284	533.303
1.00	73.438	76.193	98.608	205.721	535.366
10.00	197.520	198.284	205.721	266.242	555.446
100.00	533.096	533.303	535.366	555.446	718.853
1000.00	1439.304	1439.359	1439.918	1445.488	1499.705
D=100000.0					
0.01	1144.882	2388.507	6215.828	16718.061	45121.253
0.10	2388.507	3091.182	6448.968	16782.735	45138.764
1.00	6215.828	6448.968	8346.191	17412.213	45313.385
10.00	16718.061	16782.735	17412.213	22534.716	47012.974
100.00	45121.253	45138.764	45313.385	47012.974	60843.733
1000.00	121822.654	121827.383	121874.663	122346.139	126935.030

SUPPLY FOLLOWS A GAMMA DISTRIBUTION

LEAD TIME IS GAMMA DISTRIBUTED

MODEL (Q,R,T)

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.162	0.357	0.938	2.524	6.814
0.10	0.319	0.438	0.963	2.531	6.816
1.00	0.818	0.860	1.182	2.601	6.835
10.00	2.198	2.210	2.323	3.193	7.022
100.00	5.931	5.934	5.966	6.271	8.620
1000.00	16.014	16.014	16.023	16.108	16.931
D= 1000.0					
0.01	13.729	30.197	79.354	213.656	576.708
0.10	26.966	37.068	81.532	214.256	576.871
1.00	69.269	72.808	100.082	220.136	578.490
10.00	186.034	187.025	196.580	270.222	594.367
100.00	502.023	502.291	504.968	530.767	729.600
1000.00	1355.389	1355.461	1356.187	1363.414	1433.071
D=100000.0					
0.01	1161.998	2555.873	6716.517	18083.827	48812.598
0.10	2282.382	3137.396	6900.858	18134.596	48826.333
1.00	5862.894	6162.431	8470.968	18632.317	48963.409
10.00	15745.907	15829.813	16638.564	22871.614	50307.257
100.00	42491.206	42513.948	42740.495	44924.122	61753.358
1000.00	114720.113	114726.256	114787.560	115399.335	121295.130

SUPPLY FOLLOWS A GAMMA DISTRIBUTION

LEAD TIME IS GAMMA DISTRIBUTED

MODEL (M,R,T)

STANDARD DEVIATION OF DEMAND PER YEAR=0.10*D

HOLDING COST = 0.01

BACKORDER COST PER YEAR = 1.0

BACKORDER COST PER UNIT BACKORDERED =0.10

STOCKOUT COST INDEPENDENT OF UNIT BACKORDERED =0.

ORDER COST	RC=0.01	RC=0.1	RC=1.0	RC=10.0	RC=100.0
D= 10.0					
0.01	0.149	0.317	0.829	2.230	6.019
0.10	0.304	0.402	0.856	2.238	6.021
1.00	0.788	0.822	1.086	2.312	6.041
10.00	2.120	2.129	2.218	2.933	6.243
100.00	5.720	5.723	5.748	5.989	7.918
1000.00	15.444	15.445	15.452	15.519	16.171
D= 1000.0					
0.01	12.611	26.848	70.142	188.735	509.407
0.10	25.755	34.049	72.489	189.384	509.583
1.00	66.735	69.539	91.932	195.721	511.336
10.00	179.403	180.184	187.755	248.216	528.448
100.00	484.177	484.389	486.496	506.939	670.184
1000.00	1307.221	1307.279	1307.849	1313.539	1368.734
D=100000.0					
0.01	1067.369	2272.408	5936.830	15974.489	43116.247
0.10	2179.916	2881.897	6135.503	16029.442	43131.121
1.00	5648.423	5885.774	7781.122	16565.857	43279.494
10.00	15184.686	15250.742	15891.590	21009.028	44727.813
100.00	40980.762	40998.653	41177.004	42907.292	56724.377
1000.00	110643.224	110648.057	110696.363	111177.912	115849.688

CHAPTER 4

Discussion of the results

Section 4.1. Introduction

Volume 3 contains the optimum inventory costs per annum for each of the systems and ranges of parameters described in section 2.4 of this volume. A complete analysis of these results would constitute a further major research effort. The purpose of this final chapter is to draw a number of broad conclusion.

A subset of the results given in volume 3 is given in chapter 3 of this volume so that most of the points made in this chapter can be illustrated by examples taken from this subset without referring to the full results given in volume 3.

In section 4.2. we discuss how to use the results. In section 4.3. we discuss the five different versions of the model (Q,R).
In section 4.4 we discuss the results obtained on models (Q,R),(M,T),(nQ,R,T) and (M,R,T) taking into consideration the introduction of period of grace, variable lead times and variable supply, as well as a more complete model of the costs incurred when there is a stockout.

In section 4.5 we consider how the inventory costs varies as model becomes more complicated.

Section 4.2. Usage of the results

The results given in volume 3 are volumin-ous and may be used as a source of reference. Four approaches are possible in cases where the range of parameters covered in the thesis does not cover the values desired.

A number of techniques such as that of Lagrangian interpolation formula, are available to interpolate or extrapolate for the values desired.
Obviously it is safer to interpolate than to extrapolate for the given lead times

0.1, 0.2, 0.4 of a year provided in the thesis. (i.e. It is safer to interpolate to 0.3 rather than to extrapolate to 0.02).

In this thesis the unit of time was conceptually one year. However nowhere was this unit of time defined and so the logic does not change if it is one day, a week or a month. If the inventory costs for a lead time of one week is desired, the unit of time can be expressed as one fifth of a year. Then the lowest range of lead time of 0.1 of a time unit provided in the thesis would correspond to 0.1 of one fifth of a year, i.e. one week.

All the time dependent parameters will also change, and this second technique for which an example is given below, will only work if the adjusted parameters fall within the set for which results are given in the thesis.

Suppose the minimum inventory cost on each of the (Q,R) models is required for an item with the following set of parameter values:

<u>Unit of time: 1 year</u>

Demand per year,D	=100000
Lead time,L	=1 week = 1/50 of unit time.
S. d. of demand over the lead time	=400
Stockout cost,s	=£25
Holding cost,hc	=£5 p.a.
Order cost,S	=£10

If the unit of time is changed to one fifth of a year we have the following values assigned to the parameters.

<u>Unit of time : 1/5 of a year</u>

Demand per unit time	=20000
Lead time,L	= 1 week = 1/10 of unit time
S.d of demand over lead time	=400
Stockout cost,s	=£25
Holding cost,hc	=£1 per unit time.
Order cost,S	=£10

Apart from the demand rate this set of parameters is given in the thesis, and interpolating between 10000 and 100000 for the demand is probably more accurate than extrapolating for the lead time. However the above examples illustrates the problem that will result in using a different unit of time that is not given in the thesis. A similar problem arises from using the fact that the cost of an item is undefined but assumed to be about £1. The set of cost parameters can therefore all be changed in proportion too, to the cost of the item. Another alternative will be to write one's own program using the equations in volume 2 and feed one's own values of the parameters to obtain the necessary 'optimum' inventory costs, and variables of the inventory system.

Section 4.3. Different Versions of (Q,R)

The (Q,R) system was examined at several levels of complexity with the stockout cost held constant. By stockout cost we mean the total costs incurred on each occasion the system goes out of stock. We anticipated that as the complexity of the models approached that of the exact model their inventory costs would decreas and computational costs would increase. Hence we investigated to see, for some ranges of the parameters, whether the extra cost of computing the exact model might outweigh the savings in inventory costs obtained by using the exact version of (Q,R).

The 'optimum' values of the batch quantity Q and reorder level R, for each of these approximate versions, were fed into the exact cost expression of (Q,R) to obtain the equivalent annual inventory costs in terms of the exact model. Ranking the different versions of (Q,R) in terms of the complexity and ease of understanding of the models we have the following table, starting with the simplest.

1. EBQ-ROL
2. Tate's Exponential Approximation
3. Heuristic
4. Inexact
5. Exact

Firstly we discuss the inventory costs obtained using the five models before discussing their computational costs. To illustrate the performance of the models we take an item with the following set of parameters.

Demand per year	=10000
Holding cost	=£0.1
Order cost	=£1.0
Lead time	=0.1 Of a year
Stockout cost	=£0.05 Lead time demand =£50
S.d. of demand over lead time	=0.1×D× SQRT(L)

The inventory costs are obtained from pages 41 to 45 of this volume. Ranking the models in the order of complexity suggested above we have

Versions of (Q,R)	Annual Inventory costs
EBQ-ROL	£106.074
Tate's Exponential App.	£99.286
Heuristic	£99.874
Inexact	£92.066
Exact	£92.055

In this specific example EBQ-ROL gives the highest inventory costs. This is true in general for all the range of parameters considered. Also in this example Tate's exponential approximation is unexpectedly better than the Heuristic model, despite it's lower complexity. This is also true in general and is discussed further in volume 2.

As expected the inexact model and exact model are superior to the three simpler models. In this example the error of the Inexact over the Exact model is trivial.In general the Exact model produces the least inventory costs but for certain demand levels, the savings in the Exact and Inexact models do not outweigh the extra cost of computing them compared to Tate's exponential approximation. This take's us into the discussion of their cost of computation.

Ranking the different versions of (Q,R) again in terms of complexity we have the following table.

Model	Computational costs
EBQ-ROL	£0.004
Tate's Exponential App.	£0.004
Heuristic	£0.006
Inexact	£0.02
Exact	£0.03

Firstly the computational costs apply to a KDF 9 computer. On a much bigger machine the computational costs might be considerably less, and conversely for a smaller machine.

It might be possible to compute these costs only once, and to update the parameters of items whose demand changed by means of some heuristic approach . The annual costs of computation for each model depending upon the frequency of computation are shown below.

Annual costs of computation

Model	No of computations per year			
	Once	5 times	10 times	50 times
EBQ-ROL	£0.004	£0.02	£0.04	£0.2
Tate'e Exponential App.	£0.004	£0.02	£0.04	£0.2
Heuristic	£0.006	£0.03	£0.06	£0.3
Inexact	£0.02	£0.1	£0.2	£1.0
Exact	£0.03	£0.15	£0.3	£1.5

From the above table we can see that the annual costs of computation are quite small compared to the inventory costs, even when computations are carried out more than once a year.

Since the computational costs of Tate's is lower than the EBQ-ROL model's and no dearer than the Heuristic, we therefore always prefer it to these two, providing the level of complexity was acceptable. It would therefore seem reasonable intuitively to favour one system, such as Tate's, as the best of the simple models, or the Exact model which gives the lowest inventory costs.

However it was the purpose of this thesis to compare the total costs, and this we proceed to do, at first on the basis of one calculation per year.

For the parameter ranges considered we illustrate in the diagram below cases for which the inventory costs plus computational costs of Tate's method was better than either the Exact or Inexact models.

Also when stockout cost was equal to £0.005 x Lead time demand, Tate's method was better than the Inexact and the Exact models for most of the ranges considered.

STOCKOUT COST = £0.05 LEAD TIME DEMAND

	D \ hc	0.01	0.1	1.0
S=0.1	10	*	*	*
	100	*	*	*
	1000			
S=1.0	10	*	*	*
	100	*	*	
	1000			
S=10.	10	*	*	*
	100	*		
	1000			

Key: For cells marked *, Tate's method gave lower total costs than the exact method.

The portion of the total results for which the savings of the exact and inexact models over the other models do not outweigh the extra cost of computation is only about eight percent.

An item, taken from this small area of the results for which the extra cost of computation of the exact or inexact does not outweigh the savings in inventory costs is as follows.

Holding costs	=£0.01
Lead time	=0.1 of a year
Demand	=10
S.d of demand over lead time	=0.1×D×SQRT(L)
Order cost	=£0.1

We shall assume that computations are done once a year.

From pages 42 to 45 of this volume we obtain the following table, noting that Total costs is defined as Annual Invento-ry costs plus computational costs.

Versions of (Q,R)	Annual Inventory costs	Computational costs	Total Co.
Tate's exponential App.	£0.147	£0.004	£0.151
Inexact	£0.142	£0.02	£0.162
Exact	£0.142	£0.03	£0.172

We can see immediately that the extra cost of computing the exact model or the inexact over Tate's exponential model outweighs economically the savings in inventory costs. As previously suggested, the advantage is trivial but in general, one would recommend Tate's exponential approximation for levels of demand of 100 a year or less.

Hence the only issue that remained to be solved for the class of models was which version to use, the exact or inexact for high demand rates.

The two most important variables turned out to be standard deviation of demand over the lead time and demand level. We take a sample of the results from pages 46-47 of this volume and present the inventory costs for the two models.

Exact (Inventory costs)

$\sqrt{\sigma^2 L}$/DL	100	1000	10000	100000
0.012	4.88	20.674	128.338	1094.099
0.047	6.319	39.972	345.273	3333.114
0.07	7.416	53.090	483.146	4728.465
0.094	8.572	66.242	618.974	6095.742

Inexact (Inventory costs)

$\sqrt{\sigma^2 L}$/DL	100	1000	10000	100000
0.012	4.88	20.69	128.519	1107.904
0.047	6.324	40.127	353.499	3469.551
0.07	7.422	53.627	499.458	4950.061
0.094	8.583	67.337	643.696	6403.643

Difference in costs (Inexact - Exact)

$\sqrt{\sigma^2 L}$/DL	100	1000	10000	100000
0.012	0.000	0.016	0.181	13.805
0.047	0.005	0.155	8.226	136.437
0.07	0.006	0.537	16.312	221.596
0.094	0.011	1.095	24.722	307.892

For demand levels of 10 and 100 we have already recommended that Tate's exponential approximation should be used. Hence we shall comment only on demands of 1000 and above. The difference in inventory costs is greater than the extra cost of computing the exact model over that of the inexact model, (which is £0.01 for one computation per year) for demands greater than 100 per year and all levels of standard deviation of demand considered.

Hence in general one would use the exact model instead of the inexact model. We illustrate below for completeness, cases where the computation is done more than once a year.

Difference in costs

$\sqrt{\sigma^2 L/DL}$	100	1000	10000	100000
0.012	0.000	0.016	0.181	13.805
0.047	0.005	0.0155	8.226	136.437
0.071	0.006	0.537	16.312	221.596
0.094	0.011	1.095	24.722	307.892

The above table shows the indifference curves for which the computations are carried out ten times and fifty times a year. In these cases the exact model would be appropriate for combinations of the parameters below the line and the Inexact for combinations above the line. The general conclusion is that if computations are done once a year it will cost less money to use the exact model in almost every case.

We now sum up the results in this section 4.3. A management system that uses the (Q,R) system as an inventory control procedure and that also has a shortage cost equal to the expected number of stockouts multiplied by the cost of a stockout, has five versions of (Q,R) model to choose from. What we have shown is that the model to use for low levels of demand such as 10 or 100 per year is Tate's exponential approximation and the model to use for higher levels of demand is the exact model. If only one model is allowed, one would choose the exact model, and the extra computational cost could not be great. The relative complexity of this model is thus worth mastering or accepting.

Section 4.4 (Q,R),(nQ,R,T),(M,T) and (M,R,T)

Because of the superiority of the exact model for (Q,R) it was decided to derive the exact models only for the models (nQ,R,T), (M,T) and (M,R,T). In (Q,R) systems a review occurs after every transaction. The optimum batch quantity and reorder level are independent of the cost of reviewing, and so the cost equations of (Q,R) do not take into consideration the cost of reviewing. However in the periodic review models, a review is made after every period T and the optimum values of the control variables depend partly on review costs. Hence the cost equations take into consideration directly the cost of reviewing in every period T in the periodic review models. Consequently the 'total costs' of the periodic review systems include an element which occurs in reorder level systems, but which is not included in the (Q,R) costs.

In practice the cost of reviewing in a (Q,R) model might be larger than the cost of reviewing in the periodic review models because the frequency of reviewing is more frequent in the (Q,R) system. However it might also be less, since only active items have their stock levels reviewed. We have to devise a way of incorporating the review cost into the (Q,R) system so as to make it comparable with the periodic review models. There are many possible ways of doing this. The method we have chosen in this thesis is to add the review cost for the periodic review models to the order cost of (Q,R). We choose this approach because in practice, the optimum interval between orders for (Q,R) is not much different from the optimum review period for (M,T); and in (M,T), review cost is directly added to the order cost.

We normally expect that (M,R,T) would produce the least inventory costs amongst the periodic review models ignoring the computational costs. We intend to show that for some combinations of the parameters the total cost of operating (M,R,T), where total cost is defined as inventory cost plus computational cost, may exceed that of the other review models.

In the comparisons that follow we shall assume that the solutions are computed once a year. The cost of computing each set of 'optimum' values of each model is given below. The models are ranked in order of complexity.

Models	Cost of computation	Extra cost of computing (M,R,T)
(Q,R)	£0.03	£0.47
(M,T)	£0.19	£0.31
(nQ,R,T)	£0.25	£0.25
(M,R,T)	£0.5	-

We first note that the rank order of computational cost, as we might expect, is again the same as that of complexity.

To illustrate our comparison of the four models, consider an item with the following set of parameters:

Demand per year	=1000
Lead time	=0.1 of a year
S.d. of demand per year	=100
b_1	=£1.0
b_2	=£0.1
Holding cost	=£0.01
Order cost	=£10
Review cost	=£1.0

We obtain the following table from pages 48 to 51 of this volume. Again ranking the models in order of complexity, we have the following table for the inventory costs. We are still assuming that computations are carried out once a year and thus Total cost is equal to Annual inventory costs plus Computational costs.

Models	Annual inventory costs	Computational costs	Total costs
(Q,R)	£1.09	£0.03	£1.12
(M,T)	£1.937	£0.19	£2.127
(nQ,R,T)	£1.842	£0.25	£2.092
(M,R,T)	£1.751	£0.5	£2.251

From the above table (Q,R) gives the least inventory costs and computational costs. In general, for all the different set of parameters considered (Q,R) gives the least inventory costs, as expected, as well as the lowest computational costs. However periodic models may be desirable for administrative convenience or to save set up costs, or for production scheduling reasons. As a result from now on we mainly compare these three models.

(M,R,T) gives the least inventory costs in all cases, although (nQ,R,T) gives the least total costs amongst the periodic review models for particular set of parameters chosen above.

The addition of computational costs to the annual costs of the models does not always change the ranking, when ranked in the order of their inventory costs.

For example:

Demand	=100000
Lead time	=0.1 of a year
S.d of demand per year	=10000
b_1	=£1.0
b_2	=£0.1
Holding cost	=£0.01
Order cost	=£10
Review cost	=£1.0

From pages 48 to 51 of this volume we obtain the following table.

Models	Annual inventory costs
(M,T)	£20071.24
(nQ,R,T)	£18703.809
(M,R,T)	£18382.094

Comparing the difference in inventory costs for (M,R,T) with the extra cost of computation we obtain the following table.

Model	Difference in inventory costs	Extra cost of computing (M,R,T)
(M,T)-(M,R,T)	£1689.12	£0.31
(nQ,R,T)-(M,R,T)	£ 321.715	£0.25

The extra cost of computation is extremely small compared to the difference in inventory costs. As a result the cost of computation could be ignored.

From the table, model (M,R,T) gives the lowest inventory costs amongst the periodic review models and for high demand rates the extra cost of computation can be ignored. The review costs depend on whether the system is manual or on a computer, and whether stock checks are involved, and they are thus very difficult to estimate.

We now give an example of how the difference in inventory costs and the cost of computation varies for high and low values of review costs.

Demand	=1000
Lead time	=0.1 of a year
S.d of demand per year	=100
b_1	=£1.0
b_2	=£0.1
Holding cost	=£0.01
Order cost	=£1.0
Review cost	=£1.0

From pages 48 to 51 we obtain the following table.

Review costs = £1.0

Models	Annual inventory costs	Difference in costs. Model-(M,R,T)
(M,T)	£122.14	£8.966
(nQ,R,T)	£119.568	£5.394
(M,R,T)	£114.174	-

When the review cost is changed to £10.0 we obtain the following table.

Review costs = £10.0

Models	Annual inventory costs	Difference in costs. Model-(M,R,T)
(M,T)	£247.793	£11.578
(nQ,R,T)	£253.496	£17.301
(M,R,T)	£236.195	-

Comparing the difference in inventory costs for both levels of review costs with the extra cost of computation we obtain the following table.

Difference in Inventory costs

Model	Review cost= £1.0	Review cost= £10.0	Cost of computation
(M,T)-(M,R,T)	£8.966	£11.578	£0.25
(nQ,R,T)-(M,R,T)	£5.394	£17.301	£0.31

For both levels of review costs illustrated we can see that the extra cost of computation can be ignored and model (M,R,T) produces the least inventory costs amongst the periodic review models. At the moment we are not comparing (M,T) with (nQ,R,T).

We now give an example of how the difference in inventory costs varies for high and low values of order costs.

Demand	=1000
Lead time	=0.1 of a year
S.d of demand per year	=100
b_1	=£1.0
b_2	=£0.1
Holding cost	=£1.0
Review cost	=£1.0

From pages 48 to 51 we obtain the following table

Order costs = £1.0

Model	Annual inventory costs	Difference in costs. Model-(M,R,T)
(M,T)	£122.14	£8.966
(nQ,R,T)	£119.568	£5.394
(M,R,T)	£114.174	-

When the order cost is changed to £10.0 we obtain the following table.

Order cost = £10.0

Model	Annual inventory costs	Difference in costs. Model-(M,R,T)
(M,T)	£123.996	£11.921
(nQ,R,T)	£117.916	£ 4.841
(M,R,T)	£112.075	-

Combining the difference in inventory costs for both levels of order costs with the extra cost of computing (M,R,T) we obtain.

Difference in costs

	Order cost = £1.0	Order cost =£10.0	Cost of computation
(M,T)-(M,R,T)	£8.966	£11.921	£0.25
(nQ,R,T)-(M,R,T)	£5.394	£ 4.841	£0.31

For both levels of order costs, the extra cost of computing (M,R,T) can be ignored, and (M,R,T) still produces the lowest total costs. If computations are done more than once a year, say 50 times a year, the extra cost of computing (M,R,T) could not be ignored. However we are doing our comparisons of the models on the basis of once a year.

Backorder costs

Now we show the difference in inventory costs when the backorder cost function is linear, quadratic and exponential. We choose the following set of parameters for the linear case.

Demand	=1000
Lead time	=0.1 of a year
S.d of demand per year	=100
b_1	=£1.0
b_2	=£0.1
Holding cost	=£0.01
Order cost	=£1.0
Review cost	=£1.0

From pages 48 to 59 we obtain the following table for the linear case.

Linear

Models	Annual inventory costs	Difference in costs. Model-(M,R,T)
(M,T)	£122.14	£8.966
(nQ,R,T)	£119.568	£5.394
(M,R,T)	£114.174	-

When the backorder cost function is quadratic, we retain the above set of parameters and add

b_3 =£1.0

From pages 48 to 59 we obtain the following table for the quadratic case.

Quadratic

Models	Annual inventory costs	Difference in costs. Model-(M,R,T)
(M,T)	£159.816	£14.203
(nQ,R,T)	£152.587	£ 6.974
(M,R,T)	£145.613	-

When the backorder cost function is exponential we replace b_1, b_2, b_3 in the above examples where applicable by the backorder cost = $0.1 \times e^{2.5t}$ where t is the length of time of a backorder for the exponential case.

From pages 48-59 we obtain the following table.

Exponential

Model	Annual inventory costs	Difference in costs. Model-(M,R,T)
(M,T)	£247.793	£20.854
(nQ,R,T)	£230.911	£13.772
(M,R,T)	£226.939	-

Combining the difference in inventory costs into a single table we have

	Backorder cost function		
Model	Linear	Quadratic	Exponential
(M,T)-(M,R,T)	£8.966	£14.203	£20.854
(nQ,R,T)-(M,R,T)	£5.394	£ 6.974	£13.772

Since the values assigned to the three backorder cost functions are different we are not at this stage relating the results with the backorder cost functions. An attempt is made to do this at a later stage for one model. From the example chosen above (M,R,T) is still best amongst the periodic review models for all the three backorder cost functions.

Period of grace

We now compare the performance of the three models when a period of grace is introduced. The computational costs of the models do not change when a period of grace is introduced.

We choose the following set of parameters.

Demand	=1000
Lead time	=0.1 of a year
S.d of demand per year	=100
	b_1=£1.0
	b_2=£0.1
	b_3=£1.0
Holding cost	=£0.01
Order cost	=£10.0
Review cost	=£10.0
Period of grace	=0.0

From pages 52-55 we obtain the following table.

Model	Annual inventory costs	Difference in costs. Model-(M,R,T)
(M,T)	£207.821	£14.321
(nQ,R,T)	£210.989	£17.489
(M,R,T)	£193.5	-

When the period of grace is 0.25 of a lead time, we obtain the following table from pages 60 to 63

Model	Annual inventory costs	Difference in costs. Model-(M,R,T)
(M,T)	£179.837	£13.18
(nQ,R,T)	£175.568	£ 8.911
(M,R,T)	£166.657	-

Combining both tables, with and without a period of grace, we obtain the following table.

Model	No Period of Grace	With Period of Grace
(M,T)-(M,R,T)	£14.321	£13.18
(nQ,R,T)-(M,R,T)	£17.489	£ 8.911

We find that model (M,R,T) produces the least inventory costs with or without a period of grace.

Continuous Lead Times

We next compare the performance of the three models when the lead time is a continuous variable, as opposed to a constant. (Sometimes we refer to the distributed lead time as continuous, sometimes as variable).

We choose the following set of parameters.

Demand =1000

S.d. of demand per year =100

Density function of Lead time $=\dfrac{e^{-\alpha L}\alpha^{k}L^{k-1}}{\Gamma(k)}$ $L > 0$

$\alpha = 20$

$k=2$

$b_1=£1.0$

$b_2=£0.1$

Holding cost =£0.01

Order cost =£10.0

Review cost =£1.0

From pages 64 to 67 we obtain the following table.

Model	Annual inventory costs	Difference in inventory costs
		Model-(M,R,T)
(M,T)	£142.595	£13.708
(nQ,R,T)	£135.604	£ 6.717
(M,R,T)	£128.887	-

The cost of computation and total costs for each model is given in the table below for continuous lead times

Model	Computation costs	Total costs
(M,T)	£0.43	£143.025
(nQ,R,T)	£0.61	£136.214
(M,R,T)	£0.89	£129.777

We also find that for this particular set of parameters that (M,R,T) gives the lowest total costs amongst the three models when lead time is continuous. We now show the increase in computational costs for continuous lead times.

Constant and Continuous Lead times: Computation costs

Model	Constant	Continuous	Increase
(M,T)	£0.19	£0.43	£0.24
(nQ,R,T)	£0.25	£0.61	£0.36
(M,R,T)	£0.5	£0.89	£0.49

The increase in computatonal costs as lead time is changed from constant to continuous lead times increases as the models increase in complexity. The increase for (M,R,T) is about twice the increase for (M,T), as shown in table.

Variable Supply

We now compare the models when supply is a distributed variable.

The set of parameters chosen is as follows.

Demand =1000

Lead time =0.1 of a year

S.d of demand per year =100

b_1 =£1.0

b_2 =£0.1

Holding cost =£0.01

Order cost =£10.0

Review cost =£1.0

Supply is gamma distributed

From pages 68 to 71 we obtain the following table

Model	Annual inventory costs	Difference in costs. Model-(M,R,T)
(M,T)	£194.318	£17.852
(nQ,R,T)	£185.225	£ 8.759
(M,R,T)	£176.466	-

For this particular case, model (M,R,T) still produces the lowest inventory costs amongst the three models.

In general, model (M,R,T) would yield considerable savings in inventory costs that outweigh the extra cost of computing it, compared to the other periodic review models.

(M,T) and (nQ,R,T)

To compare (M,T) and (nQ,R,T) we choose the following set of parameters.

Demand	=1000
Lead time	=0.1 of a year
S.d of demand per year	=100
b_1	=£1.0
b_2	=£0.1
Holding cost	=£0.01
Order cost	=£10.0
Review cost	=£1.0

From pages 48-51 we obtain the following table.

Model	Annual inventory cost	(M,T)-(nQ,R,T)
(M,T)	£123.996	£4.08
(nQ,R,T)	£117.91	-

When we change order cost to £1 and review cost to £10 we obtain the following table.

Model	Annual inventory cost	(M,T)-(nQ,R,T)
(M,T)	£123.996	-£9.645
(nQ,R,T)	£133.641	-

What we have shown is that theoretically the choice between (M,T) and (nQ,R,T) would depend upon the relative magnitude of order costs and review costs.

In practice review cost per item is almost certainly less than order costs. Consequently, in practice (nQ,R,T) would always yield less inventory costs than (M,T)

Difference in inventory costs for changes in some parameters

In this section, we show how the differences in inventory costs between (M,R,T) and the other models are related to the magnitudes of changes in some of the parameters.

The 'basic' set of parameters chosen is as follows.

Demand	=1000
Lead time	=0.1 of a year
S.d. of demand per year	=100
b_1	=£1.0
b_2	=£0.1
Holding cost	=£0.01
Order cost	=£10
Review cost	=£1.0

Difference in inventory costs

	Holding cost		Order cost		Review cost	
	Basic	x10	Basic	x10	Basic	x10
(M,T)-(M,R,T)	11.92	29.866	11.92	84.13	11.92	30.261
(nQ,R,T)-(M,R,T)	5.841	14.659	5.841	31.765	5.841	36.954

x/c/Basic

	Holding cost	Order cost	Review cost
(M,T)-(M,R,T)	2.5061	7.058	2.5386
(nQ,R,T)-(M,R,T)	2.5095	5.4413	6.3256

We see that the increase in inventory costs is more when order costs change than when holding costs change. Also the increase in inventory costs is more in (M,T) when order cost changes than when review cost changes and vice-versa for (nQ,R,T).

We extend the idea of changes in inventory costs as the model becomes more complicated in the next section.

Section 4.4 Variation in Inventory Costs for A Particular Model

We choose one model (Q,R) and show how the inventory costs vary for various combinations of the assumptions considered in the thesis.
We choose (Q,R) because it produced the lowest inventory costs and computational costs.

Owing to the different values that can be assigned to the parameters it is extremely difficult to compare the inventory costs for the different sets of assumptions. However in this section we shall be assuming that the values assigned to b_1, b_2, b_3, b_4, constant lead times, α and k of continuous lead times and variable supply are such as to make the different assumptions nearly equivalent.

The table below relates to the following set of parameters.

Demand		=10
Holding cost		=£10
Review cost		=£10
	b_1	=£1.0
	b_2	=£0.1
	b_3	=£1.0
Exponential cost		$=0.1 \times e^{b_4 t}$
where	b_4	=2.5
	t	= length of time of backorder
Lead time when constant		=0.1 of a year
Lead time when variable		$= \frac{e^{-\alpha L} L^{k-1} \alpha^k}{\Gamma(k)}$
where	α	= 20
	k	=2

From the above values, we can see that the expected lead time for the continuous case is 0.1 of a year.

This helps to preserve closeness with the value already assigned to the constant lead time.

At this stage we shall take our inventory costs from volume 3, pages 67,94, 121, 175, 184, 193, 202, 211, 220, 229, 238 and 247.

Inventory costs

Time related part of backorder cost

		Linear	Quadratic	Exponential
Fixed Supply	Constant Lead T.	1.462	1.638	2.239
	Variable Lead T.	1.681	1.884	2.575
Variable Supply	Constant Lead T.	1.71	1.916	2.618
	Variable Lead T.	1.785	2.000	2.733

The technique used for comparison is to compute the ratios of various costs in this table.

We firstly show the ratios of the inventory costs for continuous lead times to those of constant lead times for each of the three backorder cost functions and for both fixed to variable supply.

Ratio of Continuous Lead Times Inventory Costs to Constant Lead Times Inventory Costs.

	Linear	Quadratic	Exponential
Fixed Supply	1.1498	1.1503	1.1503
Variable Supply	1.0437	1.044	1.044

From the above table we can see that the increase in inventory costs for when lead time is continuous over when lead time is constant is less for variable supply than for fixed supply.

For both fixed supply and variable supply, the introduction of continuous lead times has about the same effect for the three backorder cost functions.
Also from the above table it is more significant to have continuous lead time with fixed supply than continuous lead time with variable supply.
However the increase in inventory cost of variable lead times over constant lead times is less than say 15%.
One could cater for variable lead time by increasing the variance of demand and then we could treat lead time as a constant.

Next we consider constant lead times and variable lead times and show how the inventory costs for variable supply and fixed supply varies.
The table below gives the inventory costs for variable supply as a percentage of fixed supply for each of constant and continuous lead times.

Variable Supply over Fixed Supply

	Linear	Quadratic	Exponential
Constant Lead Times	1.1709	1.1689	1.1685
Continuous Lead Times	1.06186	1.0617	1.0608

From the above table the inventory cost of variable supply as a percentage of fixed supply is almost the same for the three backorder cost functions for both constant and continuous lead times.

Concluding, the performance of the models, for each of the three backorder cost functions was the same, with (M,R,T) giving the lowest inventory costs amongst the review models for each of the three backorder cost functions.
When 'period of grace' was introduced, (M,R,T) still produced the lowest inventory costs amongst the review models.
The computational costs of the models increases as the complexity of the models increased.

(M,R,T) produces the lowest inventory costs when supply is a variable and also when lead time is a continuous variable.
It was found that it is more significant to have continuous lead time with fixed supply than to have continuous lead time with variable supply.

In general, for all the cases considered (Q,R) gave the lowest inventory costs amongst the four models (Q,R), (nQ,R,T), (M,T) and (M,R,T). (M,.R.T) gave the lowest inventory costs amongst (M,T), (nQ,R,T) and (M,R,T).
(nQ,R,T) was better than (M,T) unless order costs were much less than review costs.

SUGGESTIONS FOR FURTHER INVESTIGATIONS

Owing to the time limit for the thesis and the cost of computing, it has not been possible in this thesis to investigate with other demand distibutions. Performance of the models could be investigated assuming gamma distribution of demand. This would not involve truncation of the density function as was done for normal demand distribution. The normal distribution introduces some probability that demand could be negative, which does not happen in practice. In this sense the gamma distribution could be slightly more accurate than the normal distribution as a demand distribution.

It has not been possible to give a wide range of values to the parameters. Further investigations could be made by giving further values to the parameters. A savings function could then be obtained, such that with a given set of parameter values, the savings between any two inventory control procedures could be obtained by direct substitution into the savings function.

This thesis concentrated mainly on backorders. It would be worth carrying out some investigations on lost sales.

This thesis has looked at various backorder cost functions. It would be worth carrying out some investigations to determine their validity or to suggest alternative functions.

Definitions

Let D be the demand per year.

Let S be the set up or order costs.

Let Q be the batch quantity.

Let M be the maximum order cover.

Let R be the re-order level.

In chapter 2 R is expressed as k standard deviations of stock.

Let T be the review period.

Let L be the lead time.

Let σ be the standard deviation of demand per year.

Let hc be the annual holding cost of a unit of stock.

Let p be the period of grace for which a backorder does not incur a cost.

(p is also used briefly in chapter 2 to indicate a particular ratio).

Let s be the cost dependent only on the number of stockouts.

Let b_1 be the cost of a backorder, independent of the time for which a backorder exists.

Let b_2 be the cost per year relating to the time for which a backorder exists.

Let b_3 be the cost of a backorder per square time for which a backorder exists.

When the backorder cost is an exponential function, the cost of a backorder is $b_1\exp(b_4.z)$ where z is the length of time of the backorder.

Let $C_B(t)$ be the backorder cost function where t is the length of time for which a backorder has been backordered, including the cost proportional to number of backorders.

Let $\rho(x)$ be the steady state probability that the inventory position of the system is x.

Let POR be the probability of placing an order.

Let POUT be the probability of been out of stock.

Let H(L) be the probability density function of Lead time L

Let U(Q) be the probability density function of supply Q.

The probability density function of Normal demand x in a period t is defined

as

$$= \frac{1}{\sqrt{2\pi\sigma^2 t}} \exp -\frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2$$

$$\text{Let } g\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) = \frac{1}{\sqrt{2\pi}} \exp -\frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2$$

$$\text{Let } f(x,Dt) = \frac{1}{\sqrt{\sigma^2 t}}\, g\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)$$

$$\text{Let } F\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) = \int_{\frac{x-Dt}{\sqrt{\sigma^2 t}}}^{\infty} g(v)\, dv$$

The notation adopted for the inventory control procedures as defined in the intro-
duction to the thesis, in Vol. 1, are (Q,R), (nQ,R,T),(M,T) and (M,R,T).
Some authors use (z,Z) or (s,S) or other notations for (Q,R),or (s,S,T) for
(M,R,T), (Z,T) for (M,T). Mnemonic notations have been adopted in this thesis:
for example in (Q,R) Q stands for batch quantity and R stands for reorder level.
The notation adopted conveys the characteristics of the models better than not-
ations adopted by some authors. In (M,T) and (M,R,T) M stands for maximum order cover,
T stands for reorder time and R for reorder level.

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CONTENTS OF VOLUME 3

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