AN INTERACTIVE INVENTORY SIMULATION MODEL

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SUMMARY

The interactive inventory simulation model described in this thesis was developed in BASIC language in early 1978. It is given a coded name GIPSI- a General-purpose Inventory Policy Simulation Package, designed to be used on a Hewlett-Packard Access 2000 machine. The package occupies about 600 blocks or 0.3 M-bytes of storage.

GIPSI allows the user to simulate four commonly used, single-item inventory policies under varying demand and lead-time situations to produce various measures of effectiveness. The four inventory policies offered are: reorder level policy, reorder cycle policy, reorder level policy with periodic reviews and (s, S) policy.

The following facilities are incorporated in the package to enhance greater flexibility and utility of GIPSI:-

(i) Analysis of input data including goodness of fit test for both demand per unit time and lead-time data;

(ii) Sample display of simulation results;

(iii) Automatic optimization procedure in locating the optimal or near-optimal net revenue.

Furthermore, great effort has been taken in the design of GIPSI to ensure that simulation can be carried out interactively by users with little or no computer background.

So far GIPSI has shown to be particularly useful in the following areas:-

(i) As a teaching aid to students specialising in inventory control via interactive simulation;

(ii) As a tool for analysing certain stock situations encountered in industry;

(iii) As a research program for studying certain characteristics of inventory policies.

* * * * * * * * *

KEY WORDS: Inventory Policy - Simulation - Demand - Leadtime

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CHAPTER ONE

AN INTRODUCTION TO INTERACTIVE INVENTORY SIMULATION MODEL

1.1 Introduction

This thesis is about the development and application of GIPSI, an interactive inventory simulation model, designed to be used on a Hewlett-Packard Access 2000 machine. GIPSI is programmed in BASIC and allows the user to simulate four principal, single-item inventory policies under varying demand and lead-time situations. The four principal inventory policies are: reorder level policy, reorder cycle policy, reorder level policy subject to periodic review and (s, S) policy.

Early development of GIPSI was based on the extended work of the author's M.Sc. project in 1976 concerning the design of an inventory model capable of simulating three commonly used inventory policies under simple demand and lead-time situations. Subsequent work on GIPSI over a period of eighteen months (excluding three months spent on data collection) has resulted in the development of a more practical, general-purpose package which further incorporates a number of useful facilities designed to provide greater utility to the user. The facilities include the initial data analysis, goodness of fit test, an automatic optimization procedure in locating optimal or near-optimal net revenue, and an option for the sample display of the simulated results. Great effort has been taken in the design of GIPSI to ensure that simulation of inventory policy can be carried out interactively by users with little or no computer background.

A case study of a Malaysian firm, Tasek Cement Ltd., is presented to illustrate how GIPSI can be effectively used as a practical tool for analysing certain stock situations encountered in industry. In preparing this case study, collection of actual data was carried out in Malaysia over a period of three months and analysis of the inventory situations was done via interactive simulation using GIPSI.

Finally, it is noted that GIPSI has been used as a research program for studying certain characteristics of the inventory policies. A number of experiments have been carried out via interactive simulation using GIPSI and conclusions are drawn from the corresponding observations.

The ultimate aim is to develop GIPSI into one of the commonly used software packages in inventory control for the following uses:-

a) as a teaching aid for students specializing in inventory control via interactive simulations;
b) as a practical tool for analysing certain stock control situations encountered in industry;
c) as a research program for studying certain characteristics of the inventory policies.

1.2 Limitations of analytical approach

As early as 1918, Wilson[78] proposed the concept of "Economic Order Quantity" as a basis of calculating the optimum replenishment order size. Since then, analytical works on inventory management have resulted in the development of more and more complex inventory models. This trend has undoubtedly contributed to a better understanding about the nature of inventory systems in the real-world situation. But at the same time, it is also apparent that as the analytical model becomes more and more complex, solutions may not be readily available unless suitable assumptions are made. Hence, this analytical approach has given rise to yet another problem concerning the validity of the models. It is clear that usefulness of the analytical models will not be realized unless sensible solutions can be obtained to reflect the real-world situation. Very often when suitable assumptions are made in the process of simplification, the solutions produced may not represent the true situation. And even if a complex model can be solved analytically, its potential application is still dependent on the type of users. For managers who are not mathematically oriented, the use of a set of complex inventory formulae can become a formidable task. Thus, model builders are faced with the dilemma of building a highly complex model in an attempt to relate to the actual inventory problem, but then having to simplify its content to produce a solution which may or may not be useful in the final analysis.

1.3 Effects of computer on simulation

The advent of computer technology has had a tremendous impact on the progress of business, economics, engineering and applied sciences. Because of its flexibility, capacity and speed, the level of achievement has been able to be raised from simple model structuring to highly sophisticated problem solving and evaluation.

Among the various available techniques used in problem-solving, simulation is gaining wide-spread recognition as one of the best means of studying the stochastic nature of problems. This technique is widely used despite the inherent shortcomings such as the relatively high operating cost and the long waiting time before results can be obtained. A recent survey by

Shannon and Biles[65] indicated that the simulation technique was ranked as one of the most important techniques used in problem-solving and evaluation. This finding is further reinforced by Weston[73], Eilon et al[25], Lonnstedt[45] and Marinoff[48].

Undoubtedly, early developments in digital simulation techniques have been constrained by limitations on the computer size, difficulties in communicating with the computer and restrictions of computer accessibility. These constraints have been somewhat relaxed by the development of faster computers with larger storage facilities and more efficient operating systems. Such development has progressively reduced the operating cost and thus enabled simulation to become a feasible technique in problem-solving. The improvement of programming languages has resulted in the emergence of the general-purpose languages which are more natural and simpler to use. Examples of the common general-purpose languages are FORTRAN, ALGOL, COBOL, BASIC etc. These programming languages have provided a base for the most recent development of the special-purpose simulation languages which are designed as software packages for special purposes. Examples of such languages are briefly outlined below:-

GPSS	-	Developed in FORTRAN language and maintained by IBM as General Purpose System Simulator.
DYNAMO	-	Developed in AED (an ALGOL-type language) by Massachusetts Institute of Technology to simulate industrial systems dynamics.
SIMSCRIPT	-	Developed in FORTRAN language by Rand Corporation for extensive reporting facilities in its output.
CSL	-	Developed in FORTRAN language by Buxton & Laski of IBM United Kingdom Ltd., and Esso Petroleum as Control and Simulation Language.

A fairly comprehensive list of the classification of simulation languages is given by Shannon[63].

Man-machine interaction was made possible by the recent development of the less expensive modular computers and special-purpose peripheral equipment. The development of visual aids such as the Visual Display Unit has greatly reduced the interaction time and thus provide a level of communication which is more natural for the user. At the same time, the development of time-sharing systems has greatly enhanced computer accessibility through remote terminals. This remote time-sharing system has removed the restrictions imposed by the batch-process system. Batch processing usually involves submitting the program to the computer in the form of punch cards or in other similar medium, which then waits its turn before being run and output produced. In the time-sharing (now more commonly referred to as interactive) system, however, many users can have access to a single computer simultaneously. Thus, the computer works sequentially for a short period of time on each of the problems submitted to it. As a result, the response to the user at a remote terminal is almost immediate. In this way, the time taken to write, to debug and to run the program is substantially reduced. Thus, interactive simulation allows the researcher to play an active role in the simulation process as it progresses. Such interaction also permits the user to communicate with the machine whilst programs are running and thereby upgrade the utility of simulation.

1.4 Need for an Interactive Inventory Simulation Model

It is generally accepted that capital tied up in inventory forms an important part of a company's assets. Thus, it is desirable to have an efficient system of managing the inventory policy of an organization. Although inventory control methods can be theoretically used to determine the type of systems used and the controlling parameters needed to regulate the expected performance of the policy, in practice, however, an inventory system is never a static model. Variability of demand and lead-time durations often gives rise to a more complex inventory situation often too difficult to be analysed using analytical methods. Hence, there is an obvious need in developing GIPSI into a general-purpose, inventory control simulation program such that simulation can be run with inputs of different inventory parameters under varying demand and lead-time situations.

So far simulation study has been confined to designing and implementing a computer program on an individual basis. This means that up to now simulation programs on inventory policies such as the ICL SCAN System 3[32] have been designed to suit a certain inventory situation. This approach has generally restricted the flexibility of the program. Usually, simulation of different inventory policies requires the input of different parameters. As such, a simulation program designed for one particular policy cannot be effectively applied to another unless the program is restructured to suit yet another need. Hence, it is desirable to have a general-purpose program capable of performing simulation of the commonly used inventory policies.

There is no doubt that improvements of computer facilities and higher level programming languages have facilitated the programming of simulation studies. However, a user still needs time to learn the language in order to write a meaningful program for the simulation model. This involves the initial investment cost of learning the language and the subsequent costs of writing, debugging and running the program. Thus, the stress on economy of learning and operating costs is by no means trivial. In view of this, there is a need to develop GIPSI into a general-purpose simulation program such that the user can perform simulation by merely selecting proper input options to produce the various measures of effectiveness for a particular inventory policy.

Interactive simulation is recommended so that the users can easily communicate with the computer via remote terminals to produce the simulation output.

Summarizing, the framework of GIPSI is designed to include the following general features:

a) The program should be of general-purpose such that simulation of different inventory policies can be run by merely selecting different options without changing the structure of the program.

b) Economy in the learning and operating costs.

c) The program should be simple and easy to use, and able to produce simulation results within the acceptable limits of accuracy.

1.5 Outline of the Thesis

The main structure of this thesis consists of twelve chapters, of which the first seven Chapters are concerned with the tactical aspects related to the development of GIPSI. The remaining Chapters cover the operation, application and experimentation using GIPSI including recommendation for future developments and finally the conclusion.

Thus Chapter one introduces the development of GIPSI in the light of the limitation of analytical methods in dealing with complex inventory models and the growing importance of computer simulation techniques in evaluating complex inventory situations.

Chapter two outlines the research methodology and the general layout of GIPSI. A review of inventory policy theory is contained in Chapter three.

Chapter four discusses various aspects of demand and lead-time information including goodness of fit test for input data. An outline of the principal costs (ie. ordering, holding and stockout costs) is also contained in this chapter.

Chapter five describes various tactical aspects related to the design of an inventory simulation model.

An outline of the simulation process of inventory policies with inflation is contained in Chapter six. A brief discussion of the effect of inflation on the optimal reorder level policy is included in this chapter.

Chapter seven describes the design of an automatic optimization procedure in maximizing net revenue or minimizing net loss in operating a particular inventory system.

A user's guide to the operation of GIPSI is contained in Chapter eight.

Chapter nine presents a case study of Tasek Cement Ltd. involving an initial collection of industrial data and the subsequent analysis of inventory situation via interactive simulation using GIPSI.

In Chapter ten, an attempt is made to study certain characteristics of the inventory policies using GIPSI.

Finally, Chapter eleven and twelve cover the recommendation for future work and conclusion respectively.

1.6 Concluding Remarks

The use of Monte-Carlo sampling techniques in inventory policy simulation is not a new field of research. A number of books and articles have already covered this topic at various levels. As an example of simulation of inventory situations, the ICL SCAN System 3[32] has been successfully designed and implemented for the operation of production and inventory control systems. This package contains a simulation program with options of reorder level and time-based reorder cycle policies using inputs of forecast demand per unit time and fixed lead-time. However, flexibility of this program is somewhat restricted by the assumption of a constant lead-time and the limited choices of inventory policy.

The continuous improvement in computer technology has progressively reduced the operating cost using a computer. This phenomenon enables simulation to become a feasible and attractive technique in problem-solving, especially in a situation where an analytical approach fails to do so.

Interactive simulation has somewhat relaxed the rigidity of the simulation languages. Such relaxation has encouraged even the non-experienced users to run the simulation program interactively without undue worry over the input of proper formats. Thus, the ultimate aim of GIPSI is to make it as a general-purpose simulation program for inventory policy simulation, similar to the software packages such as the Interactive Forecasting package 'SYBIL'[47] already available.

CHAPTER TWO

DEVELOPMENT OF THE INTERACTIVE INVENTORY SIMULATION MODEL

2.1 Introduction

This chapter outlines the research methodology and the general layout of the Interactive Inventory Simulation Model (or in short, GIPSI). It is noted that a detailed discussion on theoretical assumptions leading to the development of GIPSI is too involved to be contained in one chapter. As such, the research methodology only outlines the basic approach in formulating, designing and constructing the computer model. More detailed discussions especially that related to the literature survey and the theoretical development relevant to the simulation program are contained in subsequent chapters.

2.2 Research Methodology

The research methodology in developing GIPSI could be broadly divided into two major stages:

(i) Literature survey

(ii) Research planning and design of the program.

2.2.1 Remarks on Literature Survey

A review of the existing literature was undertaken to find out the research development of certain topics relevant to this project to ensure that the computer program developed was broadly based on the established theories. Although the literature survey normally preceded the stage of research planning and development of the simulation model, no rigid rule was enforced in developing GIPSI. This was necessary because the program was too complicated to be completed at one time. Indeed, the full program could be viewed as one source program linked with a number of options. Each option could be quite independently developed and tested before chaining to the source program. Thus, development of the full program involved a number of stages in developing the options and other important subroutines. Where appropriate, each stage involved the normal approach in research methodology, ie. a literature search was carried out prior to the research planning and development.

At this point in the thesis, it is difficult to specify the particular sources or references that supported the construction of the program. Undoubtedly, quite a number of books and articles have been covered and hence contributed to the completion of this thesis. However, only those articles or books which were directly related to the model development are quoted as references.

2.2.2 Research Planning & Design of GIPSI

Treating simulation as a methodology in problem-solving, Mize and Cox[52] suggest the following procedure in developing a computer simulation model:

(i) Problem Formulation

- (a) Purpose of the study
- (b) System description
- (c) Recognition of assumptions

(ii) Design of Simulation Experiment

- (a) Formulation of a mathematical model
- (b) Data for simulation experiments
- (c) Sampling consideration
- (d) Model validation

(iii) Constructing the computer model

(a) Starting condition and equilibrium
(b) Time-flow mechanism
(c) Process generators
(d) Parameter changes and alternative
(e) Record keeping and generation of statistics
(f) Computer model validation

(iv) Analysis of simulation data

(a) Statistical tests
(b) Interpretation of results

A number of books such as those written by Shannon[64] , Naylor et al[55] have covered the above procedures to varying depths although some of the above terms may be defined differently. It must be stressed that the above approach only serves as a general guideline in developing a computer simulation model.

The interactive inventory simulation model, GIPSI, was developed using the above guidelines. As mentioned earlier, this simulation program was not a simple model, such as one concerned with the simulation of a single inventory policy with definite inputs of demand and lead-time information. It was meant to be general-purpose, in that simulation of different inventory policies could be run with a wide range of options concerning the inputs of demand and lead-time information. Thus, this program could be viewed as one source program linked with a number of subroutines and options. Each subroutine or option was regarded as a sub-model so that a normal methodological approach could be applied to develop it into a workable sub-program. Furthermore, validation of each sub-program was found to be simpler using this approach.

Debugging of the overall simulation program was conducted mainly via a visual display unit which had a distinct advantage of reducing the interaction time. Thus, in the process of model development, errors were easily detected and corrected via the visual display unit. Hence, a final workable program was made possible as a result of cumulative effort in formulating, designing and constructing the model inclusive of a series of tests and corrections.

2.3 General Layout of GIPSI

GIPSI is designed using "BASIC" as the programming language. It is based on the Monte-Carlo sampling technique with random demand and lead-time values being generated as inputs to produce various measures of effectiveness for a particular inventory policy. Four commonly used inventory policies are recommended as options to be selected by the user. These are: reorder level policy, reorder cycle policy, reorder level policy subject to periodic review and (s, S) policy.

A number of options are provided for the input of demand per unit time values so that the user may use any specific form of demand information which he may have available. The options are:

(a) Input of a series of successive demand per unit time (p.u.t.) values;

(b) Input of a set of ordered demand p.u.t. values together with their associated probability of occurrence, ie. as a relative frequency table;

(c) Input of a constant demand per unit time value;

(d) Input of mean and standard deviation of demand p.u.t. value to generate an approximate distribution such as Normal, Gamma, etc. for the purpose of simulation.

Similarly, options for the selection of the forms of lead-time information are as follows:

(a) Input of a series of successive lead-time durations;

(b) Input of a set of ordered lead-time durations and their corresponding probability of occurrence;

(c) Input of a series of order and receipt dates which will be automatically analysed and set-up as lead-time distribution;

(d) Input of a fixed lead-time value;

(e) Input of mean and standard deviation of the lead-time to generate an approximate distribution such as Normal, Gamma, Poisson, etc. for the purpose of simulation.

Additional facilities of data analysis and goodness of fit test are incorporated in GIPSI for analysing input data of demand and lead-time information. The problem of holidays and weekends affecting the lead-time durations will also be analysed to produce a more realistic lead-time distribution.

Evaluation of cost performance for a particular inventory system requires the following information:

(a) Selling price of the item;

(b) Cost price of the item;

(c) Purchase or prime cost of the item;

(d) Cost of placing a replenishment order or set-up cost per batch;

(e) Cost of backordering (per occasion backordering is initiated);

(f) Holding interest rate, expressed as % of prime cost;

(g) Annual inflation rates of selling price, cost price and purchase cost.

The simulation program is designed to account for the effect of inflation on inventory policy. It is noted that a complete evaluation of cost performance based on simulated results can only be produced with the inputs of demand, lead-time and cost information. If the cost information is not given, GIPSI can still be run; but the final results will not include the important cost output.

The following options are incorporated to provide greater flexibility and utility of the program:

(a) Options of backordering;

(b) Automatic optimization procedure in searching for an optimal or near-optimal net revenue including either a sensitivity or a ridge analysis at the optimal region for a particular inventory system.

The general layout of GIPSI is shown in Figure 2.1 and the detailed flow-diagrams are contained in Appendix A.

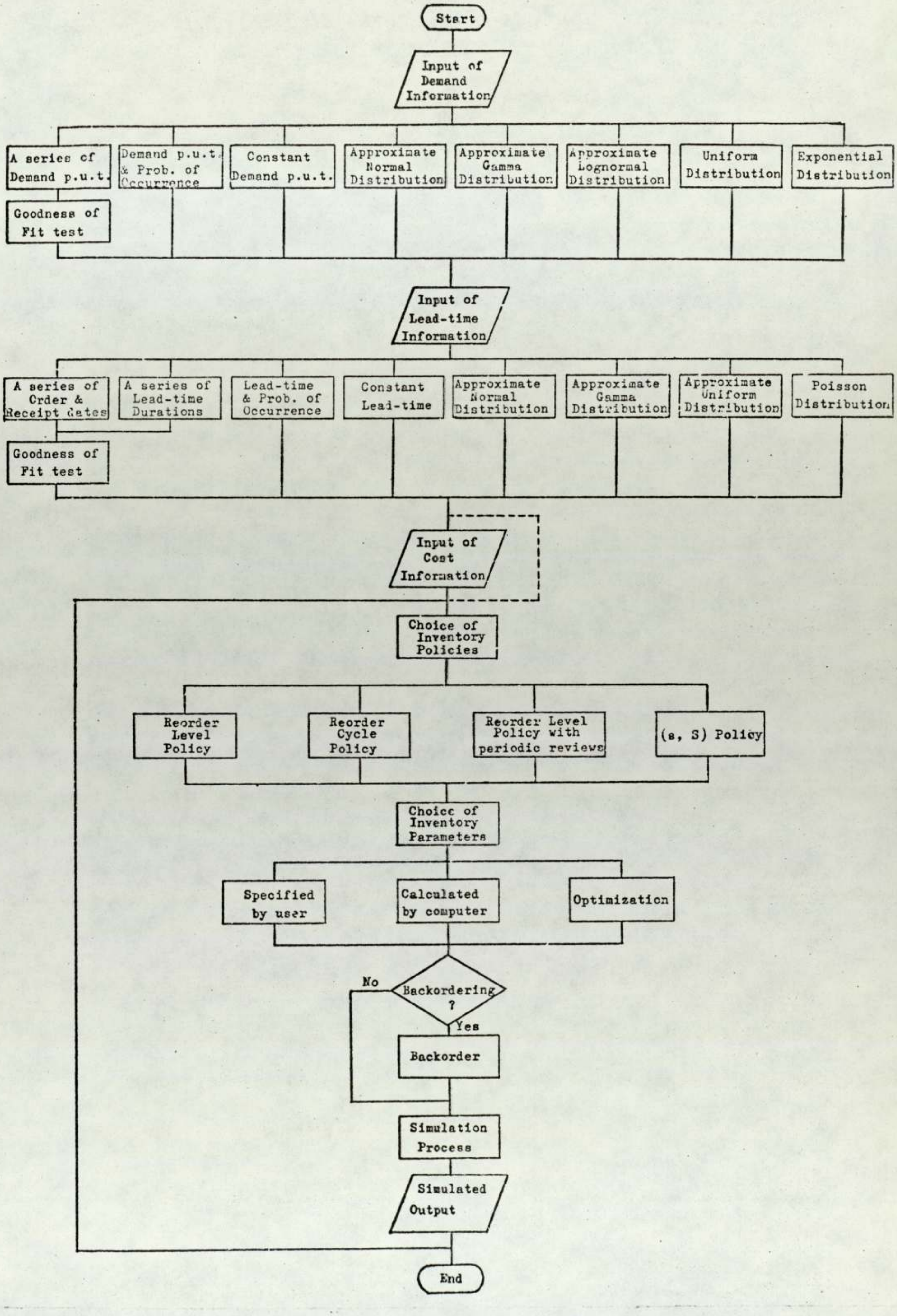


Figure 2.1: General layout of GIPSI

2.4 Assumptions

Most simulation models which are large enough to be of practical value will involve some kind of assumptions about certain aspects of the models. Thus, in developing GIPSI into a practical program, a number of assumptions and approximations have been used to streamline the general framework of the model, and where necessary, to simplify certain complex problems into workable routines without introducing too great an error. A summary of the important assumptions and approximations used in designing GIPSI is listed as follows:

Areas	Assumptions & Approximations
(1) Program	Suitable for simulation of a single-product inventory system in a stationary demand situation.
(2) Optimization	Valid for unconstrained optimization.
(3) Generating random variates of demand p.u.t. value	Approximate distribution for: (a) Normal: based on the Central Limit Theorem approach using Teichroew's modified method. (b) Gamma: based on an approximate Weibull Distribution using Ramberg & Tadikamalla's method.

Areas	Assumptions & Approximations
(4) Generating random variates of lead-time duration	(a) Approximate distributions for: Normal - based on the Central Limit Theorem. Gamma - based on an approximate Weibull Distribution using Ramberg & Tadikamalla's method. (b) Assume discrete leadtime. (c) Negative leadtime not admissible, thus implying a negatively truncated distribution. (d) For cyclical policies, leadtimes greater than 6 times the review cycle are inadmissible, thus implying a slight positively truncated distribution.
(5) Time unit	3 basic calendar units of time are specified: Month, Week and Day - any of which in practice can be regarded as an accounting period.
(6) Cost of stockout	(a) In situations where backordering is not allowed, stockout cost is the loss of profit and also the internal expenditure incurred per occasion of stockout. (b) Where backordering is allowed, stockout cost is the backordering cost and the internal expenditure incurred per occasion of stockout.
(7) Inflation	Effect of inflation to be spread uniformly throughout the year.

CHAPTER THREE

A REVIEW OF INVENTORY POLICY THEORY

3.1 Overview

It is possible to classify inventories into two major categories:-

(i) Manufacturing Inventories which usually contain dependent demand items* such as can occur with raw materials or semi-finished products to be processed. Although raw materials are often theoretically dependent, in practice, they are often independent depending on the nature of demand.

(ii) Distribution Inventories which usually contain independent demand items such as the finished products (or even raw materials) stored before shipment to client.

According to the literature, there are two principal methods of attempting to solve inventory problems in industry:-

(a) Statistical Inventory Control, which is part-oriented and ignores the dependency between the demands for the various items. This system is particularly suitable in operating distribution inventories.

(b) Material Requirement Planning (MRP)[58], which is product-oriented and treats the manufacturing inventories as a collection of dependent demand items.

In the pre-computer era, the vast amount of data processing required to convert the gross material requirement into net material requirement had made MRP a formidable task. Thus, the use of statistical inventory control techniques were preferred, even though they were primarily meant to be used in the distribution inventory system. With the introduction of more

* Demand is considered "dependent" when it is directly related to, or derives from, the demand for other items or end products. A detailed distinction between dependent and independent demand is discussed in Chapter 4.

efficient computers, however, MRP has gradually become a widely used and efficient method of handling the manufacturing inventories.

The first attempts to employ analytical techniques in studying inventory problems are to be found at the beginning of the twentieth century. In 1915, Harris[62] derived what is often called the "simple lot-size formula". The same formula was developed by Wilson[78] in 1918 and used as a theoretical basis of calculating the "Economic Order Quantity" for replenishment. Raymond[62] wrote the first full-length book covering various aspects of inventory problems in 1931 explaining how various extensions of the simple lot-size model could be used in practice. However, this reference does not contain any explicit theory or derivation[80].

After World War II, increasing attention was focused on inventory problems, particularly in the emerging management sciences and in operational research.

The fundamental concept of Statistical Inventory Control was put forward by Arrow, Harris and Marschak[4] who discussed the Wilson's lot-size formula, the (s, S) policy and other inventory policies in great detail . A similar approach was undertaken by Dvoretzky, Kiefer and Wolfowitz[20,21,22] who analysed inventory problems with a high degree of mathematical rigour.

The book by Arrow et al[5] (1958) provides a "second great stimulus"[33]. It contains detailed studies of optimal inventory policies, both deterministic and stochastic as well as covering the operating characteristics of the inventory policies.

Aggarwal[2] attempts to analyse inventory problems using a systems approach and covers various aspects of adaptive systems for inventory control in relation to the rapidly changing environment in modern industry.

For a quick orientation, ignoring mathematical details, Lewis[39,40] has provided an excellent treatment on reorder level policy (with continuous review and with periodic reviews), the reorder cycle policy and the (s, S) policy. It is difficult to list out all the books and the articles dealing with the basic concepts and the analytical treatment of Statistical Inventory Control. The following books provide an excellent treatment on inventory management: Hadley and Whitin (1963)[27], Magee and Boodman (1967)[46], Brown (1967)[11], Johnson and Montgomery (1974)[35].

A comprehensive review of inventory literature is contained in a review article by Fortuin (1977)[26].

3.2 Review of principal inventory policies

Most of the basic problems of inventory management are concerned with the decisions of "how much" and "when" to replenish. Thus, a company's stock-holding policy is determined by a series of rules which fix how and when such decisions concerning the holding of stocks should be made. This series of rules is known as an "inventory policy".

There are two basic types of inventory policy, ie. the reorder level and the reorder cycle policies. Within these two categories, a number of variants can be formed. However, only four common types of inventory policy are included as options of the Interactive Inventory Simulation Program, GIPSI. These are:-

(a) Reorder level policy
(b) Reorder cycle policy
(c) Reorder level policy with periodic reviews
(d) (s, S) policy.

3.2.1 Reorder level policy

In the reorder level system, a fixed replenishment order quantity is placed when the stock on-hand equals, or falls below, a fixed level which is referred to as reorder level. The stock on-hand includes the physical stock held plus any outstanding replenishment orders less any committed stock. Thus, the inventory situation is reviewed continuously and the effectiveness of the policy is regulated by two inventory parameters, ie. i) Reorder level, and (ii) Replenishment order quantity.

(i) Reorder level

The reorder level is generally determined in such a way as to provide sufficient stock to meet the average demand during the lead-time plus an additional amount of safety stock, held in order to reduce the probability of stockout.

In an inventory control system, the problem of stockout is a major concern to an organization. While it may be advantageous to increase stock levels in order to provide the system with greater protection against the probability of stockout, it is equally desirable to cut down stock levels in order to economize on the cost of holding stock. Therefore, the concept of a service level is introduced as one of several measures in assessing the effectiveness of an inventory policy.

There are a number of ways of defining service levels, each appropriate to the particular circumstances. Two of the most useful definitions of service levels are the Vendor Service Level and the Customer Service Level. The vendor service level is defined as the probability of not running out of stock per occasion such a stockout could occur, ie. subsequent to a replenishment order being placed. This in practice is a measure of the supplier's internal efficiency and is the definition used by most commercial packages. However, this particular service level does not indicate how successfully the customer demand is met. Thus, the concept of a customer service level has been introduced to evaluate the efficiency of fulfilling the customer demand, and is defined as the proportion of annual demand met ex-stock. Another useful measure of service level is the number of item-months (or weeks) of stock shortage per annum (see Lampkin[37]). This concept is particularly useful for captive demand eg. specialist spares and internal component stocks, but it is difficult to set standards.

Calculation of reorder level depends on the nature of the variability of demand and lead-time distributions. For example, if the demand distribution is assumed to be stationary and Normal, the reorder level for a fixed lead-time situation is estimated using the following formula:

$$M = \bar{D}L + K\,\sigma_d\sqrt{L} \qquad \text{where}$$

M = reorder level
$\bar{D}$ = average demand per unit time
L = fixed lead-time
K = normal deviate
σ_d = standard deviation of demand per unit time

The vendor service level is theoretically determined by the value of normal deviate, K used. Thus for K = 1.96, the vendor service level is estimated to be 97.5%.

The situation becomes more complicated when both demand and lead-time vary. This may lead to a situation where the demand during the leadtime can be much higher or lower than the average. Lewis[39] recommends the following analytical methods in estimating the reorder level and the levels of service:

(a) An iterative method assuming normality of demand per unit time only;

(b) Method using assumptions of normality and independence for both demand and leadtime distributions;

(c) Simplified method assuming normality and independence for both demand and leadtime distributions. Here, a modified form of normal deviate, K, is used.

In a reorder level system, a fixed replenishment order quantity will be placed as soon as the stock on-hand reaches the reorder level. However, in actual situations, the stock level tends to fall below the reorder level before action on placing the replenishment quantity is taken. This amount of overshoot* is to be estimated to give an appropriate adjustment to the reorder level.

Although it is often assumed that the demand distribution is normally distributed, in practice, the demand information may not conform to a normal probabilistic function. Even if it does, it is erroneous to assume

* The formula for an average overshoot derived by Lampkin[37] is given as Av. Overshoot = $\frac{1}{2}(\bar{D} - 1 + \sigma d^2/\bar{D})$ where $\bar{D}$ and σd refer, strictly speaking, to individual order sizes rather than demand per unit time.

that negative demand could exist in a practical system. A negative demand indicates that stock is returned to the stores instead of being removed. Thus, the effect of negative demand in the normal distribution effectively reduces the reorder level and hence the required level of service would be affected adversely.

The severity of this situation depends on the nature of the normal distribution of the demand values. If the average demand is high and the standard deviation is relatively low, then the proportion of the negative demand orders is small enough not to cause too great an inaccuracy in the required reorder level. However, in a situation of low average demand and a relatively high standard deviation, the negative effect of the demand orders could result in an erroneous level of service. In this case, the calculated reorder level has to be raised to account for the negative effect of the demand orders in order to achieve the required service level.

There are, of course, other patterns of demand distribution which may be more suitable than the assumed normal distribution. Burgin[13,14] favours the use of Gamma demand distribution in the inventory control systems. This view is supported by Johnson and Milne[34]. ICI have found that the Log-normal demand distribution is often more appropriate than the normal distribution. Hence, whatever demand model is to be used, it is important to recognize its deficiency in order to provide some form of compensation in estimating the required service level.

(ii) <u>Replenishment order quantity</u>

In a typical purchasing situation, the objective is to establish a replenishment policy to achieve optimum inventory costs. Ignoring the effect of inflation and costs of stockout, it can be shown that the optimum replenishment order quantity to achieve minimum inventory operating costs is as follows:-

$$Q_o = \sqrt{\frac{2CoA}{iCm}} \quad \text{where}$$

Q_o = optimum replenishment order quantity
Co = ordering cost per order
A = annual usage
i = holding interest rate, expressed as a fraction of the works prime cost
Cm = works prime cost

Q_o is referred to as Economic Order Quantity (EOQ) which was first put forward by Wilson (1918) as a simple replenishment order quantity model.

In situations where back-ordering is allowed, the optimum replenishment order quantity is modified [66] as:

$$Q_o' = \sqrt{\frac{2CoA}{iCm}} \sqrt{\frac{iCm+B}{B}} \quad \text{where}$$

Q_o' = optimum replenishment order quantity allowing back-ordering
B = cost of backordering per occasion out-of-stock

The concept of EOQ has caused a lot of discussion and criticism among the inventory control theorists. An obvious criticism is that the cost of stockout is not often taken into account in deriving the EOQ model. However, the nature of stockout cost is very complicated and consequently, development of the inventory cost model is very much dependent on the assumptions made by the theorists in deriving the optimum replenishment order quantity.

Hence, it is not surprising to find that a series of EOQ variants have been derived, each suited to the particular circumstances.

Eilon[23] proposed another approach in estimating a replenishment order quantity by means of maximizing the profit rather than minimizing the annual inventory operating cost. It can be shown that the replenishment order quantity derived by Eilon is greater than the EOQ and thus a greater stock capital would be employed.

Tate[69] has shown that to maximize profit per replenishment did not necessarily maximize profit per unit time which was the main objective of profit maximization and that the concept of EOQ is still useful in this respect.

Quantity discounts are often offered by suppliers as an incentive to bigger bulk purchase. The effect of price reduction per item because of discount causes sudden price breaks in the purchasing costs. Thus, in deciding the replenishment order quantity, the effect of price breaks should be taken into account in order to achieve cost optimization. In the case of minimizing the total inventory operating cost, the replenishment order quantity should be chosen by considering both the EOQ (or some modified order size) and the advantage of price breaks.

3.2.2 Reorder cycle policy

A reorder cycle policy is a time-based inventory policy with two parameters, ie. a fixed review period and a fixed maximum stock level. The stock situation is reviewed at regular intervals and a replenishment order placed at every review. Unlike the reorder level system in which the replenishment order size is fixed, the order quantity under the reorder cycle system is of a variable size and is evaluated as the maximum stock level, S, less the stock on-hand at review.

(i) Review Period

In the reorder level system, the EOQ is estimated by minimizing the inventory operating cost excluding the cost of stockout. By the same concept, it can be shown that the formula for the Economic Review Period (ERP) is as follows:

$$ERP = \sqrt{\frac{AiCm}{2Co}}$$

In practice, the review period is chosen to be a convenient interval such as one week, two months, etc., for the inspection clerks to review the stock situation of a group of items at fixed intervals.

(ii) Maximum stock level, S

In a reorder cycle system, there is a period of uncertainty concerning the possibility of stockout after placing a particular replenishment order. This uncertainty continues even after receipt of that particular order quantity until the next replenishment order quantity is received into stores. Thus, the period of uncertainty for placing a replenishment order is equal to its corresponding lead-time, L, plus the preceding review period, R.

If we consider that there is a continuous risk of running out of stock during the period of uncertainty, ie. R + L, the maximum stock level, S, is somewhat analogous to the reorder level of a reorder level policy. As an example, if we assume that the demand is stationary and normally distributed in a fixed lead-time situation, S can be estimated as follows:

$$S = \bar{D}(R + \bar{L}) + K\sigma d\sqrt{R + \bar{L}} \quad \text{where}$$

S = maximum stock level
$\bar{D}$ = average demand per unit time
R = review period
$\bar{L}$ = fixed (or approximately fixed) lead-time
K = normal deviate
σd = standard deviation of demand per unit time

In situations where the lead-time is not constant, an iterative method assuming normality of demand distribution may be used to estimate the level of service. Alternatively, a simulation method using a digital computer will be a practical approach to dealing with stochastic demand and lead-time distributions.

It is noted that when the review period is shorter than the lead-time, the replenishment order size is equal to the maximum stock level, S, less the stock on-hand. In this case, the stock on-hand refers to the physical stock level held plus any outstanding replenishment order yet to receive minus the committed stock such as backorders.

3.2.3 Reorder level policy with periodic reviews

In this system, the stock situation is reviewed at regular intervals. If the stock on-hand reaches or falls below the reorder level at review, then a fixed order quantity is placed. However, if the stock on-hand is still above the reorder level at review, no order is made until the next review occurs. Thus, this inventory control system can be regarded as a periodic review system superimposed on a reorder level policy. Hence, effectiveness of this policy can be regulated by three parameters, ie. :

(a) Review period which is normally chosen to be a convenient interval;

(b) Reorder level;

(c) Fixed replenishment order quantity.

Although the reorder level policy with periodic reviews is a time-based inventory control system, it is not possible to relate the frequency of stockouts with their probability of occurring as was possible with the reorder cycle policy. This is because orders for replenishment are not necessarily placed at every review as in the case of a reorder cycle policy. Therefore, it is possible that the stock level may fall substantially below the reorder level before the need of replenishment is detected at the next review. On the average, it is estimated that a replenishment order will be placed after a delay equal to half the review period. Thus, in the case of a normal stationary demand distribution in a fixed lead-time situation, the value of the reorder level required to provide a certain level of service is estimated as follows:

$$M = \bar{D}(R/2 + \bar{L}) + K\sigma d\sqrt{(R/2 + \bar{L})} \quad \text{where}$$

M = reorder level
$\bar{D}$ = mean demand per unit time
R = review period
$\bar{L}$ = fixed (or approximately fixed) lead-time
K = normal deviate
σd = standard deviation of demand per unit time

In a variable lead-time situation, as with other policies, values of the reorder level for a certain service requirement can be found using either an iterative process or simulation techniques in conjunction with a computer.

A reorder level policy subject to very frequent review tends to behave more as a reorder level policy than as a reorder cycle policy. On the other hand, if the review periods are relatively long, the stock on-hand may invariably fall below the reorder level each time a review takes place and, therefore, replenishment orders are placed at every review. Hence, the dominant effect of reorder level as a trigger point diminishes and thus the same policy tends to act more as a reorder cycle policy than as a reorder level policy.

At this stage, it is difficult to relate the three parameters into a tractable model such that exact solutions for an optimal policy can be obtained. Perhaps the most plausible approach in evaluating the effectiveness of the policy for a given set of parameters, is by means of simulation using a computer.

3.2.4 <u>(s, S) policy</u>

There are three parameters in a (s, S) policy:

(a) the review period, R;
(b) the equivalent reorder level, s, and
(c) the maximum stock level, S.

In the (s, S) system, when the stock on-hand falls to, or below, the level s at review, a replenishment order is placed. The order size is variable and is estimated as the level S less the stock on-hand at review. However, if the stock on-hand is above the level s, replenishment is not required.

In practice, the equivalent reorder level s is chosen large enough so that the system is provided with greater protection against the probability of stockout over a period equal to the lead-time plus the review period.

The average stock on-hand is approximately equal to the safety stock plus one-half the average quantity ordered. Since the inventory operating cost (holding, ordering costs, etc.) depends on the duration of the review period, the review period, R, should be chosen to minimize this cost. Thus, fixing the length of the review period sets the equivalent reorder level, s. Magee[46] considers that an average order placed will be approximately equal to the difference S-s, plus one-half the usage or demand during a review cycle. Using this approximation, it is possible to develop the following formula in estimating the maximum stock level, S:

$$S = \sqrt{\frac{2ACo}{iCm}} - s + \frac{AR}{2} \qquad \text{where}$$

S = maximum stock level
s = equivalent reorder level
A = annual demand
Co = ordering cost
Cm = works prime cost (or purchasing cost)
R = review period, expressed as fraction of a year

It is noted that the above approach in estimating the inventory parameters is very approximate. Arrow et al[5] have considered the (s, S) policy using a fixed cost of ordering in a constant lead-time situation, and found that the parameters s and S were related by two complex simultaneous integral equations.

Alfandary-Alexander[3] has studied the inventory situation through extensive digital simulation. He put forward his findings:

(i) Except for very low s cases, the fraction of lost sales was not too sensitive to changes in S.

(ii) By keeping S constant, relatively small variations in the level of s could induce large improvement (or deterioration) in the customer service level.

Lewis[38] has carried out a series of simulation experiments to evaluate the inventory operating cost of the (s, S) policy for a range of assumed costs of holding, ordering and stockout. He observed that the (s, S) policy did demonstrate the ability of a much lower minimum operating cost than that of either the optimal reorder level or reorder cycle policies through proper adjustment of the three parameters. In general, it was observed that the (s, S) policy was more sensitive to changes in s than changes in S.

3.3 Comparison of inventory policies

So far, four principal inventory policies have been discussed. It is therefore of great interest to examine the relative strengths and weaknesses of various inventory policies in the light of the research and development work done so far.

Naddor[54] examines the various principal policies, ie. reorder level, reorder cycle and (s, S) policies, and concludes as follows:

(i) The minimum inventory operating cost for the reorder cycle policy is larger than the minimum cost for the reorder level policy, which in turn is larger than the minimum cost for the (s, S) policy.

(ii) The sensitivity of cost to changes in the controllable variables is about the same in the optimal regions of the three policies.

(iii) In situations where several items are ordered jointly, the reorder cycle policy appears to be the best policy to achieve an overall minimum cost.

Hadley and Whitin[27] have studied the effect of review and ordering costs on cyclical policies. They concluded that the reorder level policy with periodic reviews would generally yield a higher average annual cost than the (s, S) policy; however, the cost difference was expected to be rather small.

In practice, the reorder level policy (which is often implemented as a two-bin system) serves well in situations where it is possible to have some form of continuous monitoring of the stock situation. This is made possible by ensuring that the physical stock is easily checked, as is the case in a two-bin system or when maintaining a perpetual stock recording system. Thus, this policy is particularly useful in managing inventories of low unit value and high annual usage such as bolts, nuts, etc., purchased in large quantities. However, this policy may not be suitable if the demand of an item is subject to strong seasonal variation.

A reorder cycle policy is useful where tighter and more frequent control is needed because of the relatively high unit value of the items. This policy is particularly suitable in situations where a large number of items are to be ordered jointly from the same supplier. This scheme permits each item to be shipped in smaller lots more frequently while still getting freight advantages on large total shipments.

The intermediate policies, such as the reorder level policy with periodic reviews and the (s, S) policy, are useful in controlling items of moderate usage and medium unit value.

Thomas[71] provides a good treatment on the relative merits of different inventory policies as implemented in the practical stock control systems together with suggested fields of application. His recommendations provide a quick reference, especially to those users who are not so mathematically oriented, in selecting the appropriate inventory policy for an inventory control system.

3.4 Computer-based inventory control systems

The advent of computer has gradually shifted manual inventory control to a more efficient and refined computer-based control system especially for organizations dealing with multi-item inventory situations. Special packaged programs for forecasting, parts explosion, order-quantity calculation, stock record-keeping and a number of inventory analysis routines are offered by some equipment manufacturers or consulting firms to retail, distribution and manufacturing firms. Examples of such software packages which are already available are shown below:

(i) PRINCE[31] - introduced by IBM as Production Requirements and

Inventory Cost Evaluation to include stock and order control, material requirement planning and other cost evaluation reports.

(ii) IMPACT[36] - developed by IBM as Inventory Management Program and Control Techniques for single and joint replenishment including a simulation program.

(iii) SCAN System 3[32] - developed by ICL to provide basic inventory analysis routines for various inventory control systems including a simulation program for reorder level and reorder cycle policies.

The use of software packages is appealing in that it avoids the cost and time of writing, debugging and validating the program. However, there are limitations and deficiencies regarding the use of a packaged program. Lewis[41] has observed the following technical deficiencies of most British reorder point (reorder level) software:

(i) That replenishment orders are in fact, rarely placed when stocks are equal to the reorder level;

(ii) That EOQ in their simplest and most basic form are rarely appropriate;

(iii) That all demand patterns need not be Normally distributed.

Hence, it is important that the needs for a computer-based inventory control system and the facilities offered by the software packages should be thoroughly studied before implementing the packaged system.

3.5 Summary

Undoubtedly, there are a number of different inventory policies, each distinguished by the way in which the need for a replenishment order is signalled. However, the basic element common to them is the requirement to handle the uncertainty as efficiently as possible while achieving a minimum operating cost. Ideally, it is desirable to have one inventory policy which can best fit all inventory situations. However, such an objective is normally difficult to achieve because each inventory policy is primarily designed to suit a particular inventory situation. Misapplication of inventory policy may give rise to either overstocking or high frequency of stockout occurrence. In practice, selection of an inventory system can be a difficult process involving a detailed study of the actual circumstances surrounding the inventory problem, quite separate from the policy itself. Thus, a proper selection of the inventory system depends on the nature of costs involved, pattern of demand, sources of supply and nature of control required.

The introduction of a computer-based inventory control system together with the appropriate software packages has reduced the previously regarded formidable task of data processing and analysis to a simple routine procedure. However, it is found that most of the commercial software packages for inventory control do not include simulation programs for the principal inventory policies under varying demand and lead-time situations. Hence, the introduction of GIPSI, which is a General Purpose Inventory Policy Simulation Package, is designed to allow users to simulate the four principal, single-item inventory policies under varying demand and lead-time situations.

CHAPTER FOUR

INPUT INFORMATION

4.1 Introduction

This chapter describes the three basic types of input data which are relevant to the simulation of an inventory policy, ie.:

(i) Demand information

(ii) Lead-time information

(iii) Inventory operating costs.

The design of a stochastic simulation model always involves a choice of whether to use empirical data directly in the model or to use theoretical probability distributions. The use of raw data implies that simulation is based on the past performance. This approach is useful provided that the basic form of the distribution remains unchanged with time.

The use of a theoretical probability distribution is appealing when the characteristic of input information is known to behave as that of a theoretical distribution. Using this method, random variates based on the appropriate distribution can be effectively generated for the process of simulation using a computer. This approach is usually less elaborate, and hence less expensive, as compared to the actual data collection and processing. However, it is important that new data must be collected and tested from time to time to update the controlling parameters for better estimation of an actual distribution.

Thus, decisions regarding the data to be used, their validity, form, and goodness of fit to theoretical distributions are all critical to the success of the simulation experiment, and far from being merely academic exercises.

Sometimes complete data are not easily available, in which case the input parameters have to be estimated either based on suitable assumptions or using a similar set of data from another process which is believed to follow a similar pattern.

During the compilation of numerical data, it is important that units of measurement should be consistent and compatible with each other in order to produce a meaningful simulation output.

4.2 Input of Demand Information

4.2.1 Characteristics of demand

In a stock control system, demand for a given inventory item is considered "dependent" when such demand is directly related to or derived from demand of other items or products. Thus, dependent demand can be determined from the demand for those items to which it is a component.

Independent demand is that where demand at one level is not related to the demand at a higher level. Examples of such items are finished products or the highest level of assemblies before shipment to customers.

In deciding the type of inventory system to be eventually used in stock control, it is important to identify the type of demand such that the appropriate inventory policy can be effectively applied. As mentioned in the earlier chapter, Material Requirement Planning is particularly useful in manufacturing inventories for planning the requirements of dependent demand items. However, for an independent demand situation, Statistical Inventory Control techniques are still the predominant and effective methods of controlling stock. Although raw materials are theoretically considered dependent, in practice demand at this level often exhibits characteristics of independency.

Where demand at the various levels of production is not interdependent, a practical approach is to study past records of demand at the appropriate level of production, whether that be raw materials,semi-finished products or finished goods. Such study normally reveals the pattern of demand. If it can be assumed that past demand patterns will be continued into the future, then extrapolation of those trends can be used to predict the future demand. The identification of such trends and the development of predictive models based on them is termed Forecasting.

Many authors[12,40] have dealt with forecasting of demand in great detail . Basically, there are three types of forecasting, ie. Short-term, Medium-term and Long-term forecasting, each suited to the particular requirement. Of these three, the short-term forecasting, especially that based on the exponential smoothing methods, is particularly useful in estimating the immediate demand for the purpose of stock control. A useful survey on demand forecasting methods is contained in Lewis (1975)[40] .

It is assumed that the demand distribution used for simulation in GIPSI is stationary over time.

4.2.2 Demand analysis

Usually, it is simpler to measure "demand" in demand quantity per unit time rather than demand in a lead-time. This is because historical data on demand per unit time can be directly obtained from the sales records, whereas a special recording system has to be designed for estimating demand in a lead-time. The unit of time used may vary considerably from perhaps a year for slow moving items such as spare parts for capital equipment to a day for fast moving stock items such as perishables.

When analysing the customer demand per unit time, the following factors are important in determining the pattern of demand:

(i) Average demand per unit time;
(ii) Standard deviation of demand per unit time
(iii) The type of probability distribution.

The values of average demand per unit time and its associated standard deviation can be obtained from a forecasting model. It is desirable to have these parameters updated each time a new forecast is made on the demand situation. This is easily achieved in a computerized inventory control system which automatically analyses and updates the control parameters of demand data. However, in a manual stock control system using stock record or bin cards, there is limitation to the degree of control. Unless there is a significant change in the demand pattern, it is unlikely that the parameters are to be updated as frequently as is in the computerized system.

Although the mean and its associated standard deviation give an indication of the central tendency of the value of demand per unit time and the spread of values about that central value respectively, for any statistical analysis of demand data to be complete, it is necessary to specify the type of probability distribution which the demand data are likely to conform. Hence, statistical "goodness of fit" tests are most appropriate to test if the demand sample is likely to be fitted to any of the mathematical probability distributions.

4.2.3 Goodness of Fit tests

A statistical "goodness of fit" test is used to test the degree of agreement between the sample distribution and some specified theoretical distribution. Several techniques or types of tests have been developed for such statistical analysis (see Lindgren[42]). Of these, the Chi-square, Kolmogorov-Smirnov, Cramer-Von Mises and Moments goodness of fit tests are commonly used. Phillips[60] has developed a "Goodness of Fit" package based on Fortran which allows users to test the sample distribution against ten common theoretical probability density functions using standard tests.

In theory, it is desirable to have a large sample size such that goodness of fit tests can be applied with less degree of bias. In practice, however, the pattern of demand may change over a relatively long period of time, and thus, a very large sample may not be a representative sample relevant to the current use. Usually, the Chi-square test is very powerful for large sample sizes greater than 30. Kolmogorov-Smirnov test is suitable for medium sizes between 10 and 100. For sample sizes under 10, the Cramer-Von Mises test appears to be most appropriate to use.

Thus, the Kolmogorov-Smirnov (on grouped data) and the Cramer-Von Mises tests are included as options in GIPSI for goodness of fit tests of input demand data (as well as lead-time information) against the following theoretical probability distributions:

(i) Normal
(ii) Gamma
(iii) Negative Exponential
(iv) Uniform
(v) Lognormal
(vi) Poisson

4.2.4 Generation of Demand Random Variates

The following theoretical probability distributions are included as options for the input of demand information for GIPSI:

(i) Normal distribution
(ii) Gamma distribution
(iii) Lognormal distribution
(iv) Uniform distribution
(v) Negative Exponential distribution

It is possible that a particular demand pattern may not fit any of the above probability distributions or any formal mathematical probability functions at all. In this case, simulation of inventory policy should be based on an actual sample distribution. For practical purposes, the above theoretical probability distributions are commonly used to provide approximate fits to the demand distributions in stock control.

There are two important problems in generating random variates from a specific probability distribution to be used eventually in a simulation process. Firstly, it is not practical to store all the data based on the established mathematical tables of known probability functions. Secondly, an exact random variate generator may not be available for certain types of mathematical probability distributions such as Normal, Lognormal, Gamma distribution etc. Thus, an approximate generator is used to generate random variates in situations where an exact method of such generation is not possible. An outline of random variate generation is contained in Appendix B.

4.2.5 Alternative means to collect demand information

Sometimes it is possible that records of historical data are not available or irrelevant because of the following reasons:

(i) There is no formal system of recording;

(ii) A sudden change in the market conditions may cause a significant change in the demand pattern of the existing product;

(iii) There is uncertainty concerning the demand pattern in launching a new product.

Indeed, any of the above factors or other causes may lead to uncertainty about the future demand pattern.

The following methods are suggested as alternative means to collect the relevant input information, both demand and lead-time distributions, for the simulation model:

(i) Market research especially when a new product is launched;

(ii) Intuitive judgement.

It is preferable that the estimated demand values should be associated with their corresponding probabilities of occurrence before feeding into the simulation package.

4.3 Input of Lead-time Information

Very often, delivery times of purchased items are quoted as constant periods by suppliers. In practice, delivery times (and hence lead-times) are seldom fixed and are often subject to some forms of variation mainly caused by internal factors such as delay in compiling replenishment orders in the purchase department, as well as external factors such as postal delay. The more variability or uncertainty there is associated with lead-times, the more safety stock will be required to provide the inventory system with greater protection against the probability of stockout occurring. Hence, an accurate knowledge of lead-times is needed for most forms of inventory control.

4.3.1 Definition of Lead-time

Lead-time is defined as the interval between making the decision that a replenishment is needed and the time when goods are available from stores. Delivery time is used to indicate the interval between placing a replenishment order and its subsequent receipt into stores. Thus, lead-time includes delivery time plus an additional time taken internally to generate a replenishment order and to receive goods into stores available for use. In practice, if the delivery time is very much greater than the time taken to initiate replenishment orders and to receive goods into stores, then the lead-time is approximately equal to the delivery time.

The above definition of lead-time does not take into account multiple deliveries of the replenishment order quantity. In practice, order items are often subject to multiple deliveries, possibly caused by limitation in transportation and packaging facilities for bulk delivery, or by the supplier who, knowing his inability to supply the whole consignment order, deliberately supplies a certain amount of goods at the agreed delivery date in order to relieve the pressure from expeditors.

The problems of multiple deliveries have given rise to the controversy concerning the exact definition of lead-time. Two questions are of particular importance:

(a) Should the delivery date of the first partial shipment be taken as a basis of estimating the lead-time?

(b) Should lead-time be defined as the interval between initiating a replenishment order and receiving the final shipment of the order quantity?

Fitzgerald and Harrison[43] defined lead-time as the time between placing the order and delivery of 70% of the order quantity. Surely, the figure of 70% is somewhat arbitrary? If delivery of the first partial shipment had sufficient quantity to prevent a stockout occurring, then that delivery date should be taken as a basis of evaluating lead-time. On the other hand, delivery of 70% of the order may not guarantee full protection against the probability of stockout occurring which might have been prevented if a full order quantity was delivered on the agreed delivery date. In practice, the definition of lead-time in a multiple-delivery situation depends very much on how severe the multiple-delivery scheme is affecting the performance of an inventory control system. Hence, whatever definition of lead-time is adopted, it is important that the definition should be consistent and that the lead-time should reflect the actual situation of delivery problems and the inventory systems.

4.3.2 Lead-time characteristics

Lead-time is usually measured in convenient time units such as 4 weeks, 3 months, etc. In practice, a lead-time duration is normally based on the quoted delivery time given by the supplier plus an additional contingency time allowed for unforeseen delays. Thus, the validity of lead-time estimates depends very much on various philosophies behind the quotation given by different suppliers, such as:

(i) A supplier may give a quotation which he knows will be acceptable to the customer to ensure obtaining the business.

(ii) A supplier may vary his quotation depending on the length of his order book, which in turn is a function of the economic "climate" of industrial activities.

(iii) Some suppliers may give standard quotations on delivery times.

Different strategies or policies on delivery time quotations may lead to either inflated or artificially reduced lead-times. Inflated lead-times result in an increase in paper volume, number of change orders issued, stock level, and the magnitude of forecasting errors. On the other hand, placing an order based on too short a lead-time may eventually result in the purchase department expediting this order. Expediting orders by exerting pressure on the supplier in the form of progress chasing, is costly. Moreover, it may not guarantee delivery of the order quantity at the previously agreed delivery date, and cannot do so if action is only taken when a failure to deliver on time occurs.

There is no doubt that the supplier has primary control over lead-times for bought out items. However, in reality, certain delays could well be attributed to the customer's purchase department, mainly because of the administrative delays involved in compiling and placing orders, and also administrative delays in receiving goods and booking them into stores. Thus, an accurate study of lead-time variation should be based on continuous monitoring of both the performance of supplier regarding his ability to supply the purchased items at the agreed time, and the efficiency of the purchase department in operating the purchasing and receiving systems.

4.3.3. Lead-time Analysis

The main objective of lead-time analysis for purchased items is to provide the buyer with information that will ultimately improve the lead-time estimates for a better inventory control system.

Collier[17] recommended three techniques in estimating the real lead-times, ie.:

(i) Constant interval technique;
(ii) Non-directional t-Test technique;
(iii) Cumulative sum technique.

The above techniques have been tested to produce fairly consistent results. Thus, any of such techniques can be built into a computerized inventory control system to update lead-time estimates.

Exponential smoothing, which has been extensively used in demand forecasting is another potential lead-time analysis technique. However, most of these techniques used in lead-time analysis merely provide the basic parameter estimates such as mean value and perhaps its associated standard deviation, but do not provide information about the pattern of lead-time variation. Such deficiency can be remedied by a statistical "goodness of fit" test to determine the type of theoretical distribution which the sample lead-time distribution is likely to be a reasonable fit to.

It is noted that the sample sizes of lead-time data are usually small because relatively few orders are placed per year in a well established inventory control system. Thus, Kolmogorov-Smirnov and Cramer-Von Mises tests are most appropriate to test if the given lead-time sample distribution is likely to be fitted to any of the theoretical probability distributions. The computer subroutine of "goodness of fit" tests for testing the lead-time sample is similar to that used for testing a demand distribution (see Section 4.2.3).

4.3.4 Effect of holidays and weekends on lead-time

The effect of holidays and weekends is to reduce the number of working days in a lead-time, and this may possibly lead to a delay of the delivery date. Therefore, a practical approach in estimating the real lead-time would be to exclude holidays and weekends within a lead-time duration .

A subroutine is built into GIPSI allowing a user to input a series of order and receipt dates which will be automatically analysed and set-up as a lead-time distribution. In this case, it is assumed that the lead-time is approximately equal to the delivery time and holidays can be excluded. An example of such analysis is given in the following computer print-out.

*** TABULATION OF INPUT DATA ***

DATES OF PLACING ORDERS	RECEIVING GOODS	NO.OF. CALENDAR DAYS	WEEKENDS AND HOLIDAYS	NO.OF. WORKING DAYS
12- 1-1976	3- 3-1976	50	15	35
15- 2-1976	14- 4-1976	58	17	41
8- 4-1976	5- 6-1976	58	17	41
17- 5-1976	12- 7-1976	56	18	38
26- 6-1976	10- 8-1976	45	16	29
1- 8-1976	14- 9-1976	44	15	29
12- 9-1976	23-11-1976	72	23	49
27-11-1976	3- 1-1977	37	17	20
20-12-1976	2- 2-1977	44	17	27
11- 1-1977	15- 3-1977	63	23	40

DO YOU AGREE WITH THE ABOVE TABULATION ? ANSWER 'YES' OR 'NO'.
?YES
DO YOU WANT TO PRINT OUT LEAD-TIME ANALYSIS ('YES'OR'NO')?YES

*** LEAD-TIME DISTRIBUTION ***

SAMPLE SIZE= 10
MEAN VALUE = 6.98 WEEKS
STD DEVIATION = 1.62837 WEEKS

RANGE OF LEAD-TIME	MID-PT LEAD-TIME	FREQ	PROB	CUM PROB
3.5- 4.5	4	1	0.100	0.100
4.5- 5.5	5	1	0.100	0.200
5.5- 6.5	6	2	0.200	0.400
6.5- 7.5	7	1	0.100	0.500
7.5- 8.5	8	4	0.400	0.900
8.5- 9.5	9	0	0.000	0.900
9.5- 10.5	10	1	0.100	1.000

4.3.5 Generation of Lead-time Variates

The following theoretical probability distributions are included as options for the input of lead-time information for GIPSI:

(i) Normal distribution
(ii) Gamma distribution
(iii) Uniform distribution
(iv) Poisson distribution

Methods of generating lead-time random variates based on the above distributions are described in Appendix B.

The following practical points are important for the simulation of inventory policy:

(a) Discrete lead-times are necessary in a simulation process. Thus, random variates generated (except in the case of a Poisson generator) are rounded-off to integers.

(b) Negative lead-times generated by the Normal variates generator are not admissible in a simulation process.

(c) A Poisson variate generator is inefficient for generating lead-time values greater than 20.

4.4 Inventory Operating Costs

There are three principal costs involved in operating an inventory system, ie.:

(i) Cost of ordering;
(ii) Cost of holding stock, and
(iii) Costs of stockout.

4.4.1 Cost of Ordering

Ordering cost is the total administrative cost per order for bought out items or set-up cost per batch for internally produced items. Hence, this cost could include the following cost components:

(a) All purchase department costs could be included as part of the ordering cost if replenishment order quantities are obtained from outside. Such costs are usually apportioned across all stock items ordered through the department, so that the cost of ordering is generally assumed to be the same for all items irrespective of their value. Where replenishment orders are obtained from within the organization, the ordering cost (best known as set-up cost) should include the cost of initiating works orders and also any set-up cost incurred.

(b) For bought-out items, the cost of receiving goods, including any transport costs incurred, could be included in the ordering cost.

(c) All quality control costs incurred in checking incoming materials (for bought-out items or internally produced goods) should be included in the cost of ordering.

(d) Where replenishment orders for purchased items are overdue or where internally manufactured items are behind schedule, the cost of expediting such overdue orders should be included in the ordering cost.

It is generally assumed that the ordering cost is independent of the size of replenishment order purchased or batch produced.

At current UK prices in 1978, a manufacturer's ordering cost of less than

*£15 is unrealistic. The increasing cost of ordering highlights the advantages of multiple replenishment orders. High costs of ordering also explain why certain manufacturing firms specify minimum quantities below which they are not prepared to trade, and why cheaper prices are often quoted for cash transactions which can bypass much of the paperwork involved and their associated costs.

4.4.2 Holding Cost

It is assumed that the cost of storing goods or inventories is proportional to the purchase cost of those goods. The purchase cost refers to the "boughtout" cost to the company if goods are purchased from outside. For internally manufacturing items, the purchase cost refers to the works prime cost (material + labour + works overheads).

Holding cost is usually expressed as a percentage of the purchase cost and is made up as follows:

(a) The opportunity cost of capital invested in stock (10%-15%).

(b) All costs directly associated with storing goods, ie. storemen's wages, rates, heating and lighting, store's transport, racking and palletisation, protective clothing, weighing equipment etc. (2%-6%).

(c) Deterioration costs, including costs incurred in preventing deterioration (1%-4%).

* Estimated cost based on Lewis (pg 134) in "Operational Research for Managers", edited by S.C. Littlechild[43] .

(d) Cost of pilferage which depends very much of the type of industry. eg. supermarkets have been quoted as budgeting for a loss of 1% to 2%.

(e) Obsolescence costs, including possible rework or scrapping (4%-7%).

(f) Insurance (1½%).

The holding interest rate, based on the above cost factors, is of the order of 17½% to 34½%; a value of 26% corresponds to ½p a £ per week.

It is remarked that the above listing does not include the cost of inflation on prices of goods. In situations where a high rate of inflation is anticipated, the expected rate of increase in stock prices may be deducted from the cost of holding stock, thereby promoting a tendency to "buy now rather than later".

4.4.3 Costs of Stockout

The costs of being out of stock are most difficult to assess and to incorporate in mathematical inventory models. This is because the concept of shortage or stockout costs is difficult to grasp and any attempts to define such a cost are generally rough attempts based on opinion and judgement. Perhaps the best way to define stockout cost is to examine the wide range of interpretations that can be applied. Some of these interpretations are as follows:

(a) In a retail store, a stockout cost would be incurred if a customer could not obtain the product. The stockout cost would be the loss of profit of that particular item. There might also be a loss of customer's goodwill which could possibly result in the loss of future sales of that particular item as well as other products.

(b) If a production process is forced to stop for lack of a particular raw material, the stockout cost would include the internal expenditure incurred during the period of stockout occurrence plus other costs (possibly cost of lost sales) incurred. If re-scheduling is possible in order to avoid idle time in the production process, there is still a re-scheduling cost that would be incurred.

(c) A customer ordering a particular item may be persuaded to take a substitute. In this case, the stockout cost would be the cost of substitution. In a situation where a customer is persuaded to wait until the item is available, the stockout cost would include a backordering cost plus other costs incurred during the period of stockout occurrence.

The problem of whether stockout costs should be computed on a unit basis, time basis or a combination of unit and time basis, adds to the above variety of interpretations. The following bases are commonly suggested for evaluating the costs of stockout:

(i) Cost per stockout occurrence;

(ii) Cost per unit time of stockout;

(iii) Cost per stocked unit out of stock per unit time.

The following methods have been selected to evaluate stockout costs for GIPSI:

(i) <u>Backordering prohibited</u>

When backordering is not permitted, the cost of incurring a stockout is evaluated as the lost potential profit plus the administrative overheads not recovered as the result of lost sales.

STOCKOUT COST = UNITS OF POTENTIAL SALES * (SELLING PRICE - PURCHASE COST)

(ii) Backordering permitted

When backordering is permitted, ie. accepting demand orders when no stock is available, it is assumed that there is no eventual loss in profit. However, there is an assumed cost of administrative overheads which are not recoverable. Thus, the stockout cost is evaluated as the cost of backordering (this being a fixed penalty cost per stockout occasion) plus the unrecovered administrative overheads.

STOCKOUT COST = BACKORDERING COST + BACKORDERED UNITS *
(COST PRICE - PURCHASE COST)

4.5 Concluding Remarks

Perhaps one of the most difficult aspects of inventory control is the collection of input data. These include the inputs of demand and lead-time information, and the inventory operating costs such as inventory carrying charges, ordering costs, backordering costs, lost sales costs and other important data provided by a continual monitoring of inventory control systems. Most of these may require a sizable expenditure or effort to obtain.

Input of demand information may be based on the forecast results using forecasting techniques such as exponential smoothing, together with the assumed probability distribution such that random variates can be generated for simulation. Alternatively, a demand distribution can be set-up based on either a representative sample of historical data or intuitive judgement with appropriate assumptions.

Fairly similar techniques are used for the analysis and input of lead-time information, except in this case, lead-time variates so generated should be positive and discrete.

The cost data are not generally available from accounting data. In some cases, however, such as profit, holding interest rate and ordering cost, the accounting data may be a starting point. The evaluation of stockout costs, on the other hand, would be mainly based on assumed models using suitable assumptions.

Having decided on the type of inventory policies, one is left with a choice whether to backorder or not when a stockout occurs. In situations where backordering is permitted, one still has to decide on the proportion of backorders allowed, and such a decision usually depends on the actual inventory situations. However, to simplify the process of simulation using GIPSI, a user is given an option either to allow 100% of the back-orders or not to allow backorders at all when a stockout occurs.

CHAPTER FIVE

DESIGN OF SIMULATION MODELS

5.1 Introduction

Shannon[64] defines simulation as "a process of designing a model of a real system and constructing experiments with this model for the purpose either of understanding the behaviour of the system or of evaluating various strategies (within the limits imposed by a criterion or set of criteria) for the operation of the system". Thus, the process of simulation includes both the construction of a model and the subsequent experimentation with it to produce results reflecting a real-world situation. A system is a set of objects united by some form of interaction or interdependence. A model represents a group of objects, a set of variables or even ideas existed in some form other than that of the entity itself. Thus, a simulation model may be manipulated in ways impossible or impractical to perform on the system being represented. Hence, to simulate is to duplicate the essence of a real system without actually attaining reality itself.

It is important to note that the above definition of simulation is extremely broad, and may include operations such as military war games, business management games, various electrical analog devices, testing of iconic models, manipulation of mathematical models, etc. A much narrower but useful definition of simulation by Naylor et al[56] is as follows:

> "Simulation is a numerical technique for conducting experiments on a digital computer which involves certain types of mathematical and logical models that describe the behaviour of a business or economic system (or some component thereof) over extended period of real time"

This definition is sometimes more appropriate when it is necessary to streamline the design of a simulation model used in conjunction with a computer.

Tocher[72] attributed the techniques of simulation as originating from three sources, ie.:

(i) The first and most respectable origin lies in the theory of mathematical statistics.

(ii) The second origin lies in the demands of applied mathematicians for methods of solving problems involving partial differential equations.

(iii) The third origin lies in the science of operational research.

With the development and advances of computers, the techniques of simulation especially in Monte-Carlo experimentation, have proven more successful than before.

Monte-Carlo* refers the technique of selecting numbers randomly from one or more probability distributions for use in a particular trial or run in a simulation study. Monte-Carlo sampling was first developed by Neumann and Ulam[50] in an attempt to study the random behaviour of a neutron diffusion problem. This technique was later used in solving stochastic and even deterministic models which could not be solved with analytical methods. Today, this technique is being applied to many kinds of problem, ranging from the highly mathematical to those almost totally lacking in mathematical rigour. It is also used as a practical tool in forecasting, planning and decision-making both in the strategic and tactical management levels of an organization.

* The term "Monte-Carlo method" is also used for techniques of variance reduction through sampling process (See Hammersley and Handscomb[28]).

5.2 Basic Concept of Inventory Policy Simulation

The Monte-Carlo sampling technique is used to select a number randomly from a probability distribution for use in a simulation study. It is noted that much of the subsequent analysis and accuracy depend very much on the nature of input data. Therefore, unless the input information of demand distribution, lead-time durations and others are reliable, it is of no use to expect great accuracy in the final simulation analysis.

As a simple illustration, consider that the input data consist of demand and lead-time information whose distributions are tabulated in Table 5.1 and Table 5.2 respectively.

Table 5.1: Probability Distribution of Demand Values

Number of Units Demanded per week	Probability of Occurrence	Cumulative Probability of Occurrence	Range of Random Numbers Allocated
20	.10	.10	00-09
30	.20	.30	10-29
40	.30	.60	30-59
50	.25	.85	60-84
60	.15	1.00	85-99

Table 5.2: Probability Distribution of Lead-time

Lead-time (weeks)	Probability of Occurrence	Cum. Prob. of Occurrence	Range of Random Numbers Allocated
4	.20	.20	00-19
5	.30	.50	20-49
6	.35	.85	50-84
7	.15	1.00	85-99

It is assumed that demand per unit time and lead-time values are discrete. The probability of occurrence may be derived from a known mathematical function or it may be obtained from historical data. It is further assumed that a series of random numbers between 00 and 99 are generated with all numbers having an equal probability of occurring. Thus, for a demand value of 40 units per week, a range of random numbers from 30 to 59 is allocated. Suppose a random number of, say, 43 is selected, this means that the corresponding demand value would be 40 units per week and this value would be used in the subsequent simulation.

In a similar process, a random lead-time duration would be obtained for the purpose of simulation.

A sample printout of Reorder Level Policy simulation with backordering prohibited based on the above distributions for a period of 50 time units is shown in Table 5.3. Additional information used in this simulation example is as follows:-

Reorder level	=	350 units
Reorder quantity	=	300 units
Initial inventory level	=	403 units

Note that the probabilities of occurrence in the first 50 demand values in this short simulation run are:

Demand per unit time	Theoretical probability of occurrence	Actual prob. of occurrence in first 50 demand values
20	.1	.06
30	.2	.24
40	.3	.38
50	.25	.20
60	.15	.12

A summary of the simulated results based on this short run assuming 50 time units a year is shown below:-

Annual demand	=	2040 units
Total replenishment quantity	=	2100 units
No. of orders/year	=	7
Average stock level	=	294 units
No. of stockout	=	0
Backorder quantity per year	=	0

Table 5.3: A sample printout of Reorder Level Policy Simulation

Reorder level = 350 units
Replenishment order qty. = 300 units

Period	Demand per week	Current stock level	Stock-out count	No. of stock-outs	Order qty	Remaining lead-time	No. of orders
1	30	403				0	
2	50	353				0	
3	50	303			300	4	1
4	30	273				3	
5	40	233				2	
6	50	183				1	
7	60	423				0	
8	40	483				0	
9	40	343			300	6	2
10	40	303				5	
11	40	263				4	
12	40	223				3	
13	30	193				2	
14	30	163				1	
15	30	433				0	
16	60	373				0	
17	40	333			300	4	3
18	40	293				3	
19	60	233				2	
20	20	213				1	
21	60	453				0	
22	40	413				0	
23	30	383				0	
24	40	343			300	6	4
25	30	313				5	
26	20	263				4	
27	40	223				3	
28	20	203				2	
29	30	173				1	
30	60	413				0	
31	60	353				0	
32	30	323			300	5	5
33	30	293				4	
34	50	243				3	
35	40	203				2	
36	30	173				1	
37	40	433				0	
38	40	393				0	
39	50	343			300	6	6
40	50	293				5	
41	40	253				4	
42	40	213				3	
43	20	193				2	
44	50	143				1	
45	40	403				0	
46	50	353				0	
47	40	313			300	7	7
48	50	263				6	
49	30	233				5	
50	40	193				4	

Although it is possible to perform manual simulation for a limited number of tedious runs such as the sample printout in Table 5.3, the results are far from being convincing and conclusive. Thus the use of computer simulation is required such that the final results obtained will be within the specific limits of acceptance. Nevertheless, it is important to recognize that the essential use of a computer arises not out of any conceptual advantage that the computer gives, but merely because the nature of simulation is such that a great many calculations are involved and repetitive, and these would be too tedious and time-consuming without the automation provided by the computer.

5.3 Random Number Generation

Randomness is associated with equally likely outcomes of a sample space. The idea is to consider each outcome of an experiment as equally likely to occur, independent of other trials of the same experiment. Thus, the key characteristics of random numbers are the nature of population distribution of such numbers and the independence associated with the sequence of occurrence of the numbers. The probability of occurrence of a number should always be the same - not affected by past occurrence. Thus, in generating true random numbers, there is no rule for prediction from one selection to the next. In addition, one cannot even completely and unequivocably test for randomness after a set of numbers has been obtained. This is because any repeatable procedure cannot produce truly random numbers.

In theory, it would be possible to design a random number generator to generate an infinite set of truly random numbers. An example of such generator is a low frequency random step generator designed by Wilkins[77]. In practice, however, imperfection of generators may cause a certain degree of bias to the generation of random numbers. Furthermore, cost is also a very important factor. Sophistication and refinement of any machinery require higher building and maintenance costs. Surely, other than in the pursuance of academic interest, the benefits derived from having too high a "randomness" in random number generation is not justified in view of the escalating cost factor. Furthermore, in a complex sampling experiment, it is always useful to be able to repeat a series of calculations with similar input conditions inclusive of similar sets of random numbers in order to check the accuracy (or errors) of the final results. Thus, theoretically a set of random numbers that can be regenerated is self-contradictory to the basic definition of random numbers. However, a practical approach is to generate a finite set of random numbers that are sufficiently random for the purpose of simulation, without incurring too great an error in the random number generation. This finite set of numbers is generated in such a way that any "reasonable" statistical test will show no significant departure from randomness. Such a generated set is referred to as being "Pseudo-random". The numbers (usually between 0 and 1) so generated are referred to as pseudo-random numbers, and they are used as if they were truly random.

There are several sources* from which pseudorandom numbers can be obtained for the purpose of simulation. Some of these are outlined below:

* A fairly comprehensive development of pseudorandom number generation is outlined in Tocher (Chapter 4)[72] and Naylor et al (Chapter 3)[55].

(i) Random Number Table - for small scale experimentation.

(ii) Electronic Calculators - for small scale experimentation.

(iii) Mathematical Formulae:

(a) Mid-product method - for medium simulation works.

(b) Lehmer Congruence method - for long simulation works.

(c) Second-order Recurrence Process - for long simulation works.

Most modern computer systems have a random number generator as part of their scientific subroutines to generate pseudorandom numbers based on the appropriate mathematical formulae. Thus, it can be seen that pseudorandom numbers can be easily generated for the purpose of simulation using a computer. Furthermore, these pseudorandom numbers are used to generate random variates from the theoretical probability distributions. Random variates are also used in simulation studies where theoretical probability distributions are used as input data. An outline of such generation from some of the theoretical probability distributions is contained in Appendix B.

5.4 Estimation of Simulation Runs

An inventory policy simulation can be viewed as a sampling process to evaluate certain measures of effectiveness for a particular inventory system. Very often, experimental evaluation is subject to a certain degree of uncertainty. There are two basic reasons why simulated results do not reflect an actual inventory situation. These are:

(a) The model itself may not be an accurate representation of the real system.

(b) The estimates obtained from the simulation runs of a truly representative model may not be precise enough to describe the model.

In order to eliminate the error due to inaccurate modelling, verification and validation are necessary. However, the procedures of verification and validation of simulation models can be very complicated and involved. Mihram[51] has reviewed the philosophical questions of model validity and described the need for practical tests of the adequacy and representation of computerized models. Nevertheless, in order to simplify the procedures of validation, the following steps are listed:

(a) Simulation runs are conducted to uncover the defects in order to determine whether the model is internally correct in a logical and programming sense.

(b) In the case of a stochastic model, a statistical test is necessary to see if the simulation model represents the real-world phenomena. However, because of the number of possibilities, it is not practical to test all measures of parameters which might be obtained from the validation runs of the simulation model against comparable measured parameters from the real-world. Thus, the model-builder has to use his judgement in selecting certain key parameters and distributions for the purpose of validity test. In the case of a deterministic model, a test for validity can be easily conducted by comparing the simulation results against the possible analytical solutions.

Regarding the degree of uncertainty caused by an insufficient number of simulation runs, it is therefore desirable to fix a suitable length of simulation in order to produce simulated results within the acceptable limits of confidence. A practical approach is to break a total simulation length into a number of sub-periods which are normally known as simulation runs. The simulated results obtained in each run are regarded as sampling estimates to be used in measuring the characteristics of the whole population. The whole population refers to a simulation experiment with infinite runs. Hence, the general problem is then to determine the number of simulation runs such that the estimates are statistically acceptable within a specified level of confidence. The following factors are related to the degree of accuracy in simulation:

(i) Possibility of correlation between each successive run;

(ii) Size of each simulation run;

(iii) Number of simulation runs.

By choosing an appropriate length of a simulation run, it can be assumed that the effect of auto-correlation between successive runs is small enough not to cause serious error in the final simulated results. In designing this simulation model, the length of each run is arbitrarily chosen to be 100 time units. In deciding the number of simulation runs for the required precision, cost of running the simulation program is another important factor to be considered. A higher level of confidence in the simulated results is associated with a higher cost in running the simulation program. Thus, a practical approach is to produce the simulated results with sufficient accuracy at a reasonable running cost. A good reference covering various methods in estimating the length and number of simulation runs can be found in Shannon[64] and Mize and Cox[52] .

For practical purposes, estimation of simulation runs in this model is based on the Central Limit Theorem using the estimated average stock level and its associated standard error. From statistical theory, it can be shown that for a normal or approximately normal distribution in the absence of auto-correlation, the number of simulation runs is given as:

$$N = \left(\frac{\sigma Z}{d}\right)^2 \qquad \text{where}$$

N = Number of simulation runs

σ = Standard deviation of the sample (also known as standard error)

d = specified limit

Z = Normal deviate (determined by the level of confidence required)

In designing this simulation model, Z is set at 1.96 corresponding to a 95% level of confidence, and d is set at a limit of at least $\pm$ 5% of the mean stock level. Therefore, the value of σ remains an unknown to be solved in order to determine the value of N. A practical approach is to conduct a series of simulation runs in order to estimate the mean stock level and its standard deviation, σ. Values of the limit, d, are continuously updated according to the number of runs N. As soon as d falls to, or below, 5% of the mean value, the simulation process is automatically terminated. A minimum number of simulation runs N is 5 to ensure that the experimental result is within a reasonable limit of acceptance. A simplified flow-diagram of an inventory policy simulation using automatic stopping rules is shown in Figure 5.1.

Although the mean stock level is taken as the reference parameter in estimating the number required of simulation runs, there are other parameters

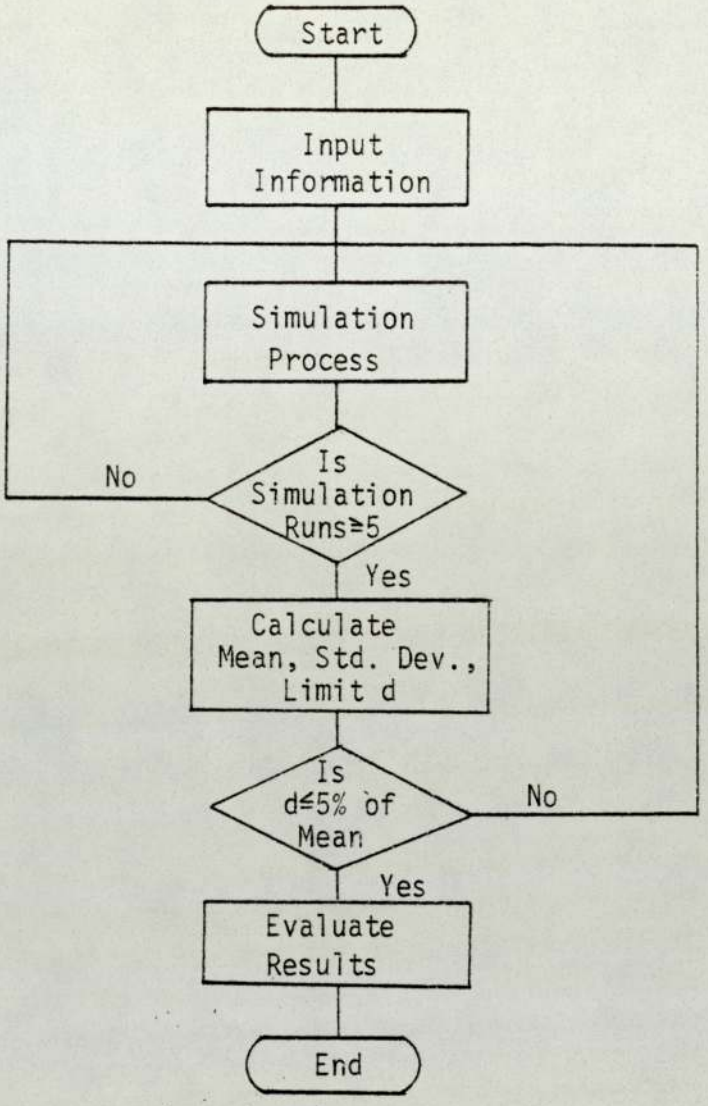


Figure 5.1: A simplified flow-diagram of an inventory policy simulation using Automatic Stopping Rules

such as the probability of stockout, annual inventory operating costs, net revenue, etc., which may be more suitable to use. Ideally the net revenue could be taken as a reference parameter in estimating the sample size. However, this approach is not applicable in a situation where simulation involves only demand and lead-time distributions without input of cost factors. Thus, the suitability of using a particular parameter in estimating a sample size depends very much on the nature of the inventory problem. Since this inventory simulation model is designed to be general-purpose, the method of estimating simulation runs is conveniently based on the mean stock level and its associated standard error derived from a series of simulation experiments.

5.5 Variance Reduction

Variance reduction refers to the reduction of variance between the true or population mean and the simulated mean. Several variance reduction techniques have been developed either to improve the precision of estimates for a fixed number of simulation runs or to decrease the number of simulation runs required to obtain a fixed degree of precision. Some of the commonly used techniques are:

(a) Antithetic variate method
(b) Stratified sampling method
(c) Importance sampling method
(d) Russian roulette and splitting technique
(e) Correlated sampling method.

Hammersley and Handscomb[28] have evaluated the relative performance of some of the variance reduction techniques and found that the antithetic variate method is one of the most efficient techniques in variance reduction. Furthermore, this technique can be easily applied to the existing simulation program without major structural changes. Thus, the antithetic variate method has been used in this Interactive Inventory Simulation model, GIPSI, to reduce the number of simulation runs required to obtain a certain level of precision with a specified degree of confidence.

The underlying principle of antithetic variate technique is to perform two identical simulation runs with two different sets of pseudorandom numbers, and to average the simulated results. The two sets of pseudorandom numbers are selected in such a way that they have a strong negative coefficient of correlation. This means that the two runs will tend to produce results on opposite sides of the population mean. An average of these two estimates will thus give a result closer to the mean than would be likely otherwise.

Although antithetic variates can be generated in different ways, one of the easiest is to use the following procedure:-

(a) Generate a random number x_i and use it to select the corresponding value y_i from its distribution.

(b) Find $(1 - x_i)$ and use it to select y'_i from the same distribution.

(c) Repeat the above process for n times. It is possible to produce two sets of values y and y' based on the corresponding sets of pseudorandom numbers x_i and $(1 - x_i)$ respectively.

It is clear that y' will have the same mean and variance as y, and will also be negatively correlated with it. The effect of this is to force values to be drawn from opposite ends of the distribution in two simulation runs, so that the results tend to be negatively correlated, thus achieving the desired improvement in estimation.

5.6 Starting Conditions

A simulation run represents the operation of a system from a given starting point for a period of time. Very often, this starting condition may induce an initial bias or transient condition which is not typical of steady-state conditions, owing to the fact that it takes some time for a simulation process to overcome an artificial situation created at the beginning of an operation. Thus, the effect of starting conditions can be significant in influencing the accuracy of the final results obtained from a simulation model.

There are at least three ways of reducing the bias caused by initial starting conditions:

(i) Use long enough computer runs such that the data from the transient period are insignificant relative to the data from steady-state conditions.

(ii) Exclude some initial simulation runs from the overall simulation period.

(iii) Choose initial starting conditions that are more typical of steady-state conditions and thus reduce the biased effects of transient period.

Each of these options creates problems in terms of implementation. In general, the first two approaches incur a certain amount of wastage in computer time. Furthermore, it is difficult to define the term "steady-state condition" in such a way that simple procedures can be applied to locate such a state. Shannon[64] summarized various heuristic rules (by several authors) concerning the location of steady-state conditions. However, there is still no completely satisfactory method of deciding when equilibrium has been achieved.

The third approach, ie. loading the simulation model initially with a representative set of data, is recommended in the design of GIPSI. In particular, the following initial values of inventory level are arbitrarily fixed according to the inventory policies used:

Inventory policy	Initial inventory level
(a) Reorder level policy	Reorder level plus twice the average demand per unit time.
(b) Reorder cycle policy	Maximum stock level minus twice the average demand per unit time.
(c) Reorder level subject to periodic review	Reorder level plus twice the average demand per unit time.
(d) (s, S) policy	Maximum stock level minus twice the average demand per unit time.

Although the choice of the above initial values may be sometimes outside the range of reasonable starting conditions, the gravity of this situation in affecting the accuracy of the final results is very much reduced by the design of automatic stopping rules. Thus, if the initial stock level is fixed atypical to a steady-state condition, the automatic stopping rule is applied in such a way that relatively long simulation runs are necessary to negate or reduce significantly the effects of initial conditions.

5.7 Concluding Remarks

The preceding discussion outlined various tactical aspects concerning the design of an inventory simulation model. Various tactical factors which are relevant to the design and improvement of the simulation package, GIPSI, are considered and implemented. These include:

(i) Start-up condition

(ii) Estimation of simulation runs

(iii) Design of automatic stopping rules

(iv) Improving simulation efficiency through variance reduction techniques.

Although there are theoretical bases and rational arguments in the design of a simulation model, much is still based on the experience and judgement of the experimenter.

CHAPTER SIX

DESIGN OF SIMULATION PROCESS TO COPE WITH INFLATION

6.1 Introduction

The classical EOQ formula assumes that all relevant costs and prices remain constant over time. However, with inflation rates currently running between 8% and 20% per annum in most Western countries, the impact of inflation on inventory policies now has to be examined such that various modifications can be applied to project more realistic results used in forecasting and planning under inflationary conditions.

Buzacott[15] examines the EOQ model with inflation under various pricing policies, and concludes that with inflation the EOQ formula should be modified so that the inventory holding rate is chosen in a way appropriate to the pricing policy used by the company. A brief recast of EOQ with uniform rate of inflation of inventory operating costs is outlined in section 6.2.

In reality, it is noted that an analysis of the effect of inflation is complicated by different cost factors often being subject to different rates and modes of inflationary pressure within and without the organization. Thus, no attempt is made here to cover the topic of inflation in all inventory policies in great detail. Only those points that are related to the development of the simulation models are discussed in subsequent sections.

Finally, an attempt is made to analyse both theoretically as well as experimentally the optimal characteristics of reorder level policy subject to inflation in a stationary demand situation. The experimental results obtained via interactive simulation using the subsequent inventory simulation program GIPSI are used as a comparison with the theoretical results derived from Buzacott's EOQ model with inflation.

6.2 Theoretical derivation of EOQ model with inflation

(i) Nomenclature:

d = average demand per unit time

L = average lead-time

q = replenishment order quantity (or = AT)

A = annual demand

T = interval between successive replenishment orders, expressed as fraction of a year.

n = number of replenishment orders per year (or = A/q)

Co = ordering cost per replenishment

C_m = purchase cost per unit

C_h = inventory holding cost (or = iC_m)

i = holding interest rate

r = uniform rate of inflation

(ii) Assumptions:

The following assumptions are made in order to derive the EOQ model with inflation:

(a) Price and cost are subject to the same rate of inflation, r spread uniformly throughout the year. Thus, if C indicates the cost at time zero, then the cost at time t with a uniform rate of inflation of r £/£/unit time becomes Ce^{rt}.

(b) Replenishment orders are placed at regular or fairly regular intervals.

(c) Costs of stockout are excluded.

(d) All cost factors are measured or taken at time zero.

(iii) Theoretical Derivation

The objective is to minimize the total cost, C_T, which is assumed to consist of ordering, purchase and storage costs. A simple EOQ model with zero safety stock and constant (or approximately constant) demand is shown in Figure 6.1.

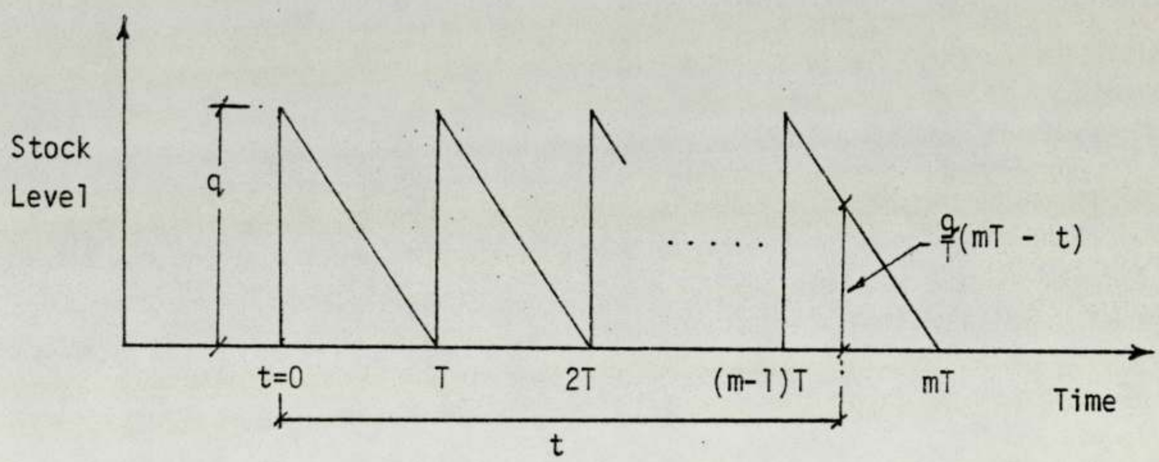


Figure 6.1: Inventory balances of a simple EOQ model

$$\text{Annual ordering cost} = Co(o) + Co(T) + \cdots Co\,(n-1)T$$

$$= Co\frac{e^r - 1}{e^{rT} - 1}$$

$$\text{Annual purchase cost} = qC_m(o) + qC_m(T) + \ldots. qC_m\,(n-1)T$$

$$= qC_m \frac{e^r - 1}{e^{rT} - 1}$$

$$\text{Annual storage cost} = \frac{1}{n}\sum_{m=1}^{m=n} \frac{1}{T}\int_{(m-1)}^{mT} \frac{q}{T}(mT-t)\ iC_m\ e^{rt}dt$$

$$= \frac{qiC_m}{(r^2)T} \cdot \frac{(e^r-1)\ (e^{rT}-rT-1)}{(e^{rT}-1)}$$

Hence, C_T = ordering cost + purchasing cost + storage cost

$$= \frac{e^r\ -1}{e^{rT}\ -1}\left[Co + ATC_m + \frac{AiC_m}{r^2}\ (e^{rT} - rT - 1)\right]$$

using approximation $e^{rT} = 1 + rT + \frac{r^2T^2}{2} + \ ....$

(ignore terms higher than 2nd order), and differentiating C_T with respect to T, the optimum T* is found to be:

$$T^* = \sqrt{\frac{2C_0\ (1+rT^*)}{AC_m\ (i-r)}}$$

This is the Buzacott's EOQ model with inflation in which the price and cost are subject to the same rate of inflation. When r = 0, $T^* = \sqrt{\frac{2C_0}{AiC_m}}$ which is the classical Economic Review Period (ERP) of the reorder cycle policy.

For most values of r and T*, $\sqrt{1 + rT^*}$ is relatively small and thus has only a minor effect on T*. For example, if replenishment orders are placed

5 times a year (ie. T*=.2) and the rate of inflation is assumed to be 20%,then $\sqrt{1 + rT^*}$ is approximately equal to 1.02. Hence, for practical purposes, T* can be evaluated from the following approximate relationship:

$$T^* = \sqrt{\frac{2C_o}{AiC_m(1-\frac{r}{i})}} \qquad \text{for } i > r$$

$$\text{or} \quad T^* = \frac{ERP}{\sqrt{1 - \frac{r}{i}}}$$

Thus, the effect of inflation tends to increase the duration of ERP if an optimum cost is to be achieved.

Using Q* = A x T*, the following relationship can be derived:

$$Q = \sqrt{\frac{2AC_o(1+rT^*)}{Cm(i-r)}}$$

when r = 0, $Q^* = \sqrt{\frac{2AC_o}{iC_m}}$ which is the classical EOQ formula.

By the same argument, the following approximate relationship for a EOQ model with inflation can be derived:

$$Q^* = \sqrt{\frac{2AC_o}{iC_m(1-\frac{r}{i})}} \qquad \text{for } i > r$$

$$\text{or } Q^* = \frac{EOQ}{\sqrt{(1-\frac{r}{i})}}$$

Again, the effect of inflation is to increase the EOQ.

6.3 Outline of inventory policy simulation with inflation

From the preceding discussion, it is noted that with appropriate assumptions of cost factors and inflation rates, a modified EOQ (or ERP) model with inflation can be derived and applied in the reorder level and reorder cycle policies. In reality, most of the cost factors are often subject to different rates and modes of inflation. Because of this, it is difficult to derive exact analytical models with inflation such that optimal characteristics of the particular inventory system can be effectively studied. Thus simulation is used in this package to overcome this difficulty by incorporating the assumed effects of inflation in the simulation process such that more realistic results can be produced to reflect the real-world situation.

In general, a cost factor can be regarded as being comprised of material, labour and overheads. An exact composition of these cost elements may vary from organization to organization and is normally difficult to generalize. The increase in material price may be linked to a retail price index or some other form of index such as a consumer price index or even a GNP deflator. However, it is important that the index number used should relate to the activities of the business organization, and not to general consumer goods and services. Similarly, historical records of salaries and wages can be used as a basis to project the inflation rate of labour cost.

Although it is possible to evaluate the rates of inflation of materials and labour cost without too much difficulty, estimation of inflation rate concerning overheads is not an easy task. This is because overheads generally consist of various cost compositions of materials, services, labour charges etc., and measurement of the relative effects of inflation related to these cost elements could be very difficult. Furthermore, it is rather confusing for a user to input a certain rate of inflation for overheads without fully understanding the term and nature of overheads in the business concerned. Hence, a practical approach is to narrow the term "inflation" such that users will not be confused over the input of various inflation rates, while at the same time, reasonable results can be produced via interactive simulation. The following assumptions are made regarding the nature of inflation:-

(i) all cost factors are taken at time zero of a simulation process. This means that the "First-in, First-out" method* in evaluating costs is more appropriate in a simulation process when dealing with inflationary situations.

(ii) the rate of inflation is assumed to spread uniformly throughout the year for all relevant cost factors concerned.

(iii) it is assumed that the impact of inflation on the inventory systems is mainly due to changes in selling price, cost price and purchase cost.

Based on the above assumptions, the rates of inflation of three main cost factors may be related to the following sources:-

1) Rate of inflation of Selling Price - possibly linked to a retail price index.

* Other methods such as averaging, standardizing, LIFO and Forecasting may be available (See Lockyer[44], Chapter 7).

2) Rate of inflation of Cost Price - possibly linked to the inflation rate associated with the purchase cost, with, perhaps, an added small wage inflation element.

3) Rate of inflation of Purchase Cost - mainly linked to wage and material inflation.

The approximations which take into account the effects of inflation used in GIPSI are briefly outlined in subsequent discussions. A list of nomenclature used is as follows:-

A = Annual demand

B = Level of Buffer Stock

C_b = Cost of backordering per stockout occasion

C_c = Cost price

C_o = Ordering cost

C_p = Purchase cost per unit

C_s = Selling price

i = Holding interest rate

K_b = Rate of inflation of backordering cost (assumed equal to K_c)

K_o = Rate of inflation of ordering cost (assumed equal to K_c)

K_c = Rate of inflation of cost price

K_p = Rate of inflation of purchase cost

K_s = Rate of inflation of selling price

Q = Total replenishment quantity

Q_b = Backordered quantity or loss of potential sales

T = Average ordering interval

T_1 = Average stockout interval

(i) Sales

The following assumptions are used to derive a formula for the approximate gross revenue subject to inflation:-

(a) Constant rate of sales;

(b) Rate of inflation, K_s is assumed to spread uniformly throughout the year.

Based on the above assumptions, it can be shown that the formula for the approximate gross revenue derived from sales is given as:-

$$\int_0^1 QC_s e^{k_s t} dt = \frac{QC_s(e^{k_s} - 1)}{K_s}$$

(ii) Ordering cost

The following assumptions are used to derive an approximation to take into account the effect of inflation on the ordering cost:-

(a) Replenishment orders are placed at regular intervals, T;

(b) Rate of inflation, K_o is assumed to spread uniformly throughout the year.

The formula for the approximate annual ordering cost is shown to be:-

$$C_o \frac{e^{k_o} - 1}{e^{k_o T} - 1}$$

(iii) Inventory holding cost

In general, inventory consists of both active and safety stocks. Thus, the following assumptions are used to simplify the analysis which takes into account the effect of inflation on both active and safety stocks:-

(a) Treating the usage of active stock as a simple saw tooth shape;

(b) Assuming constant safety stock, B throughout the year.

Based on the above assumptions, it can be shown that the formula for the approximate inventory holding cost subject to inflation, K_p is given as:-

$$iC_p \left[\frac{AT^2}{2} \frac{(e^{k_p} - 1)}{(e^{k_p T} - 1)} + \frac{B}{K_p} (e^{k_p} - 1) \right]$$

or alternatively, a much simplified expression assuming an equivalent "constant stock level" throughout the year can be used to evaluate the approximate inventory holding cost, and is given as

$$(\text{Average stock level}) \cdot \frac{iC_p}{K_p} (e^{K_p} - 1)$$

The latter approximation is easier to apply since the simulated average stock level is readily available from the simulation output.

(iv) Purchase cost

The following assumptions are used to derive a formula for the approximate annual purchase cost subject to inflation:-

(a) Replenishment order quantity is of a fixed size, Q and is delivered at regular intervals, T;

(b) Rate of inflation, K_p is assumed to spread uniformly throughout the year.

The formula for the approximate annual purchase cost is shown to be:-

$$QC_p \frac{e^{K_p} - 1}{e^{K_p T} - 1}$$

(v) Cost of stockout

It is difficult to determine the exact time at which a stockout could occur. A practical approach is to assume the number of stockout occasions spread evenly throughout the year. Also, depending on the nature of backordering, the cost of stockout may or may not include backordering cost. Thus, if backordering is prohibited, the cost of incurring a stockout is assumed to consist of the potential loss of profit plus the administrative overheads not recovered as a result of the lost sales. Taking into account the effect of inflation assumed to spread uniformly throughout the year, the approximate stockout cost is evaluated using the following formula:-

Stockout cost (backordering prohibited)

$$= Q_b \left[\frac{C_s e^{K_s T_1} (e^{K_s} - 1)}{e^{K_s T_1} - 1} - \frac{C_p e^{K_p T_1} (e^{K_p} - 1)}{e^{K_p T_1} - 1} \right]$$

When backordering is permitted, it is assumed that there is no loss of profit; but there is an assumed loss of administrative overheads plus the cost of backordering incurred to initiate backorders. Thus, the approximate stockout cost is estimated using the following formula:-

Stockout cost (backordering permitted)

$$= \frac{C_b e^{K_p T_1}(e^{K_b} - 1)}{e^{K_b T_1} - 1}$$

$$+ \; Q_b \left[\frac{C_c e^{K_c T_1}(e^{K_c} - 1)}{e^{K_c T_1} - 1} - \frac{C_p e^{K_p T_1}(e^{K_p} - 1)}{e^{K_p T_1} - 1} \right]$$

6.4 Effect of inflation on Reorder Level Policy

Having derived the modified EOQ model with inflation, it is interesting to examine the characteristics arising from the use of such model in the reorder level policy.

In the reorder level policy, when the stock on-hand falls to, or below, a specified reorder level, M, a replenishment order for a fixed quantity, q, is placed. For example, the reorder level, M, in a stationary normal demand situation is given as:

$$M = \bar{D}L + k\,\sigma d\sqrt{L}$$

Note: Although $\sigma d\sqrt{L}$ theoretically represents the standard deviation of demand in a lead-time, in practice due to autocorrelation effects, a more accurate evaluation might be given by $(0.659 + 0.341L)\,\sigma d$ (See Brown[11], pg 144).

where $\bar{D}$ = average demand per unit time

L = constant (or approximately constant) lead-time

k = normal deviate

σd = standard deviation of demand per unit time

The vendor service level which is defined as the probability of not running out of stock per occasion a stockout could occur, is determined by the value of k.

The customer service level which is defined as the proportion of annual demand met ex-stock, can be evaluated from the following relationship:

$$p' = 1 - \frac{E(k)\,\sigma d\sqrt{L}}{q}$$

where p' = customer service level

$E(k)$ = second definite integral of the normal probability density function from the reorder point to infinity of the probability density function of demand during the lead-time.

q = replenishment order quantity

If the inventory system is to operate at an optimal condition at which the inventory operating cost is to be minimized, the EOQ or modified EOQ model can be used as a theoretical basis of calculating the replenishment order quantity. With inflation, if the EOQ were to be increased by $\frac{1}{\sqrt{1 - \frac{r}{i}}}$ to meet inflationary conditions without any compensatory drop in the reorder level, such an increase in the replenishment order quantity would invariably raise the customer service level, whilst the vendor service level would remain unaltered.

It is noted that Buzacott's EOQ model is developed on the basis of maximizing net revenue (by Buzacott himself), or alternatively, it is based on the criterion of minimizing purchase cost and inventory operating cost under inflationary conditions as discussed in Section 6.2. The main criticisms of using such a model in a reorder level policy are:

(a) The cost of stockout and cost of holding the safety stock under inflationary conditions are not taken into consideration;

(b) The assumed criterion of maximizing net revenue (by Buzacott) may produce a higher inventory operating cost under inflationary conditions;

(c) The replenishment order quantity is evaluated in isolation of the reorder level instead of using a more realistic method for the joint calculation of replenishment order quantity and reorder level.

A series of tests have been carried out using GIPSI to investigate if Buzacott's EOQ model with inflation can be used as a basis to evaluate an optimum replenishment order quantity for the reorder level policy subject to different rates of inflation. The following data were used for such tests:-

(i) <u>Demand information</u>

Mean = 50 units per week (Gamma distributed)

Standard deviation = 15 units per week

(ii) <u>Lead-time information</u>

Lead-time = 3 weeks

(iii) Cost data

Selling price = £2 per unit

Cost price = £1.5 per unit

Purchase cost = £1 per unit

Ordering cost = £2 per order

Inventory holding rate, i = 24%

Inflation rate: assumed 0%, 12% and 20% applied uniformly to all cost factors.

(iv) Option of backordering

Backordering prohibited

The simulated results for different rates of inflation at the minimum-cost condition are shown in Table 6.1.

Table 6.1 : Simulated results of reorder level policy subject to different rates of inflation at minimum-cost conditions

Inflation rate (%)	Reorder level (units)	Replt. order qty. (units)	Inventory holding cost (£)	Ordering cost (£)	Stockout cost (£)	Annual inventory operating cost (£)	Net revenue (£)
0	190	200	33.7	24.9	2.9	61.5	2426
12	190	200	35.7	26.4	3.5	65.6	2582
20	190	200	37.1	27.5	4.5	69.1	2703

The optimum replenishment order quantity has been found experimentally to be 200 units which is close to the theoretical EOQ of 204 units. Treating this experimental minimum-cost order quantity as "EOQ", and using the Buzacott's EOQ model with 12% and later 20% inflation, two replenishment order quantities are estimated as 283 and 490 units respectively. These values were used for experimentation using GIPSI and a summary of the simulated results is contained in Table 6.2.

Table 6.2 : Simulated results of reorder level policy using replenishment order quantity derived from Buzacott's EOQ model with inflation

Inflation rate (%)	Reorder level (units)	Replt. order qty. (units)	Inventory holding cost (£)	Ordering cost (£)	Stockout cost (£)	Annual inventory operating cost (£)	Net revenue (£)
0	190	200	33.7	24.9	2.9	61.5	2426
12	190	283	46.3	18.7	2	67.0	2594
20	190	490	75.8	11.2	3	90.0	2722

A full discussion of the experimental results concerning the optimal characteristics of reorder level policy subject to different rates of inflation is contained in Chapter 10 (Section 4). Results of the finding are summarized below:-

(a) The annual inventory operating cost and net revenue are generally raised as a result of inflation being applied uniformly to all cost factors.

(b) The optimum values of reorder level and replenishment order quantity at minimum-cost conditionsare not affected by inflation.

(c) The Buzacott's EOQ model with inflation usually produces a bigger replenishment order quantity for the reorder level policy which often leads to a higher net revenue and a higher inventory operating cost than the corresponding values at the minimum-cost condition.

Note that a minimum-cost condition refers to the condition giving a minimum annual inventory operating cost derived from operating a particular inventory system.

An interesting situation arises regarding the interpretation of using the Buzacott's EOQ model with inflation in the reorder level polciy. The traditional EOQ is often used for an inventory system giving a minimum inventory operating cost. However, the use of an optimum replenishment order quantity derived from the Buzacott's EOQ model often produces a higher net revenue rather than a minimum inventory operating cost in an inflationary situation.

Thus the Buzacott's EOQ model with inflation should be used as a basis to evaluate an optimum order size for the reorder level policy to achieve an optimum net revenue under inflationary conditions. If a minimum inventory operating cost is sought, a simple EOQ or some modified optimum order size is more appropriate to use in an inflationary situation. In this case, the choice of optimal inventory parameters (ie. reorder level and replenishment order quantity) for the reorder level policy does not appear to be affected by the impact of inflation.

6.5 Conclusion

A general outline of the simulation process designed to cope with the effects of inflation has been discussed. Several assumptions regarding the nature of inflation are made to allow users to input different rates of inflation interactively to the inventory simulation program "GIPSI" without much computation required. This approach aims to produce simulated results of an inventory system under inflationary conditions good enough for decision making and planning.

A recast of Buzacott's EOQ with inflation is briefly outlined. Using Buzacott's EOQ model with inflation as a basis to evaluate the replenishment order quantity for the reorder level policy, the EOQ is shown to be theoretically increased by $1/\sqrt{1 - \frac{r}{i}}$ to achieve an optimum net revenue under inflationary conditions. The values of r and i refer to the rate of inflation applied uniformly to all cost factors and the inventory holding rate respectively.

Experimental results obtained via interactive simulation using GIPSI have shown that the increased replenishment order quantity derived from Buzacott's EOQ model has produced a higher net revenue and a higher inventory operating cost than the corresponding values of the reorder level policy at minimum-cost conditions. It is observed that values of the optimal inventory parameters (ie. reorder level and replenishment order quantity) appear not to be affected by the impact of inflation.

CHAPTER SEVEN

OPTIMIZATION

7.1 Introduction

In simulation models, there are rarely simple functional relationships that can be determined by analytical methods to obtain optimum values of the decision variables. Thus, optimization involves some form of sequential search for optimum responses through a series of small experiments. This process of experimentation and searching is normally referred to Response Surface Methodology.

Response surface methodology was first proposed by Box and Wilson[10] in 1951. The underlying philosophy and application of response surface methodology is well expounded in a number of books, including Davies[18], Myers[53], and Cochran and Cox[16].

In practice, two major stages of experimentation are involved in the search for optimum values. The first stage is a sequential search to move from the existing experimental region to the next so as to come closer to the optimal point on the underlying response surface. The second stage is to locate the optimal point and to study the nature of the underlying response surface, once an optimum or near-optimum condition is achieved. A number of techniques, sometimes called optimum-seeking methods, response surface techniques or techniques of evolutionary operation, have been developed for use in the response surface methodology. A fairly comprehensive discussion of these techniques is given by Wilde[75], and Wilde and Beightler[76]. In particular, the following methods are commonly used in sequential response surface exploration:

(i) Factor-at-a-time method

(ii) Simplex method

(iii) Steepest Ascent (or Descent) method.

It is possible to use the above methods as a basis either for a manually controlled search or for a computerized automatic search involving the use of an optimum-seeking program interfaced with the simulation model. In a manually controlled search, the process of optimization involves a series of stop-go procedures, ie. stopping the simulation run, interpreting the results and deciding new values of the decision variables for the next simulation run. This process entails a great amount of analyst effort and computer time. Hence, it is desirable to develop an optimization program interfaced with a particular simulation model such that the process of optimization can be automatically carried out until an optimum or near-optimum condition is achieved.

Of the various optimum-seeking techniques used in simulation studies, Simplex and Steepest-ascent methods are considered to be the most efficient methods.

A number of books, especially those written by Davis, Cochran and Cox, and Myers, have covered the Steepest-ascent method in great detail . The best guides to the journal literature on the steepest-ascent method and simulation designs are contained in reviews by Hill and Hunter (1966)[29] , and Shannon (1975)[63] . A modular program has been developed by Smith[67] based on Fortran IV using First order and Second order designs for constrained and unconstrained optimum-seeking in conjunction with deterministic or Monte-Carlo simulation.

The Simplex technique* was first propounded by Spendley, Hext and Himsworth using a sequence of experimental designs each in the form of a regular (or irregular) simplex. The optimization procedure is simply a process of forming new simplices by reflecting one point (which has the worst response) in the hyperplane of the remaining points until an optimum condition is attained.

Nedler and Mead[56] put forward an adaptive simplex technique using operations of reflection, contraction and expansion for the process of optimization. An optimum condition is said to have attained when the "standard error" of the simplex responses falls below a certain pre-set value.

Different versions of simplex techniques are also available. Examples of these versions are: Simplex technique using accelerated sequential blocks by Biles (1973)[6], and Self-regenerative simplex by Akitt (1975)[1].

7.2 Outline of Optimization Procedure

The following procedures have been chosen as a basis in designing the optimum-seeking program chained to the Interactive Inventory Simulation model:

(i) Nedler and Mead adaptive simplex method;

(ii) Second order designs and regression analysis at the optimum or near-optimum region;

* This "Simplex" search technique is different from the simplex method of linear programming, which also derives its name from the geometric configuration called a simplex.

(iii) Canonical analysis;

(iv) Ridge analysis at a minimax or saddle condition.

Each of these steps is discussed in the following sections. It is noted that the Nedler and Mead Simplex method is chosen because it requires fewer experimental points than the first-order design of the Steepest Ascent method in its sequential search for an optimum response. The use of this simplex method is confined to searching for unconstrained optimum or near-optimum values of the decision variables. Although it is possible to incorporate constraints into the interactive program, development of a constrained optimum-seeking program requires further research and programming work to be done.Thus, a constrained optimum-seeking program is excluded at this stage of development.

The chosen objective of optimization in GIPSI is to maximize net revenue or minimize net loss in operating a particular inventory system. The net revenue is evaluated in the following basis:-

NET REVENUE = SALES - PURCHASES - INVENTORY OPERATING COSTS

Treating cost factors as fixed input values, the inventory parameters will be the decision variables in a simulation model. Thus, optimization in conjunction with a particular inventory policy simulation model, is to search for the values of inventory parameters which give a maximum net revenue (or minimum net loss) in operating that particular inventory system.

7.3 Nedler and Mead Simplex Method

A simplex is a geometric figure formed by a set of (n + 1) points in n - dimensional space. n refers to the number of decision variables in a simulation model. Thus, in the case where n = 2, such as simulation of a reorder level policy, the simplex is a triangle. The principal idea of this method is that a new simplex can be easily formed by reflecting one point in the hyperplane spanned by the remaining points.

Three basic operations are used in the Nelder and Mead adaptive simplex method:

(i) Reflection
(ii) Expansion
(iii) Contraction

These operations enable the simplex searching technique to become more adaptive to the characteristics of the response surface. A simplified flow-diagram showing the basic operations of simplex optimization is shown in Figure 7.1. A more detailed simplex algorithm for an optimum-seeking process is contained in Nedler and Mead[56], and Olsson[57].

The stopping criterion suggested by Nedler and Mead is concerned with the variation of simplex response. Standard error or Root Mean Square (RMS) value of the responses is used to indicate the degree of such variation. In practice, a certain pre-set value is assigned before starting the simplex optimization. As soon as the experimental standard error of simplex responses falls to or below a pre-set value, the optimization process stops. This stopping criterion is particularly useful in exploring

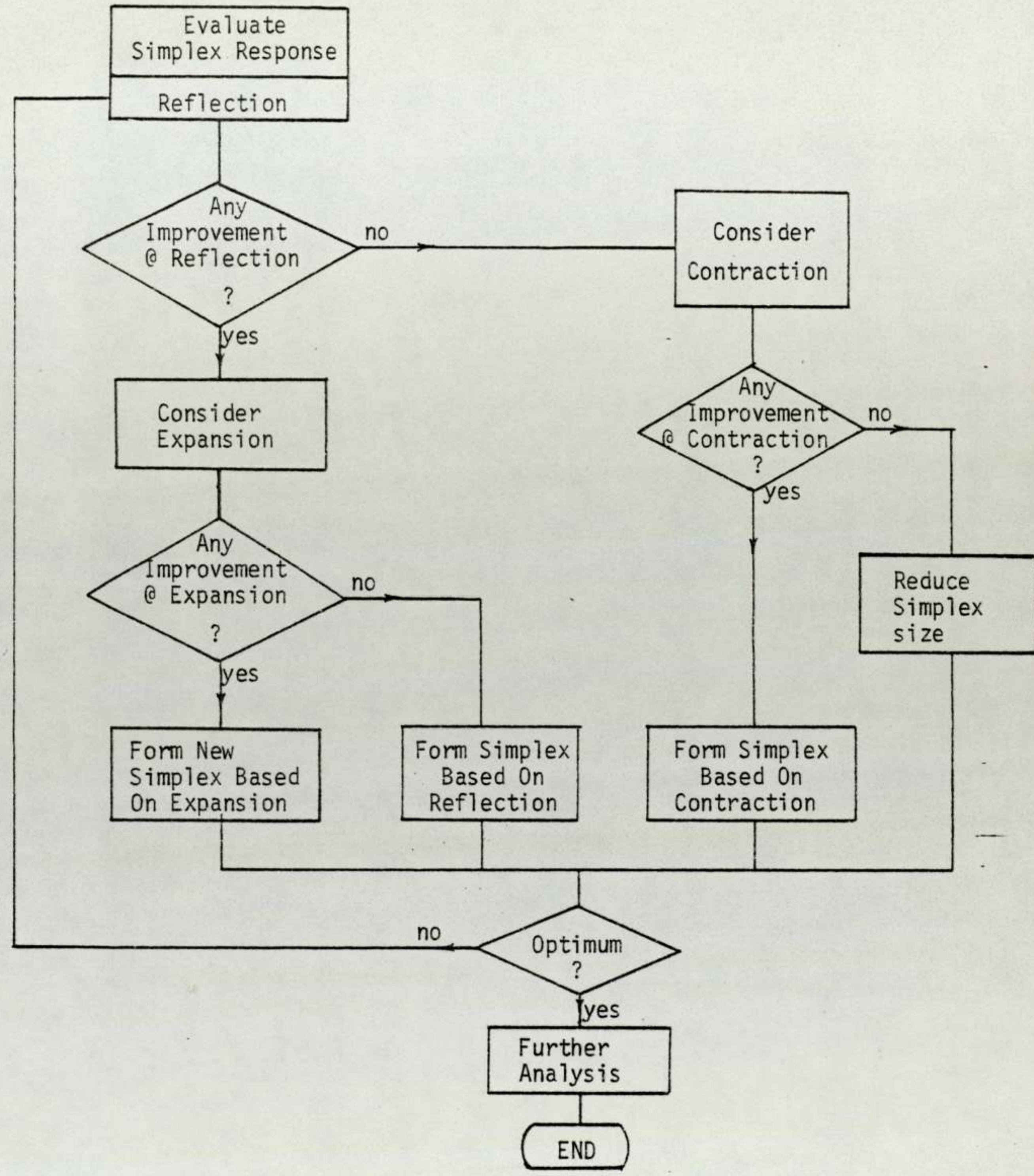


Figure 7.1: A simplified flow-diagram showing operations of Simplex optimization.

deterministic models in which the experimental standard error of the responses is progressively and consistently reduced to a pre-set value.

However, this halting criterion may not be effective to simulation models because of the presence of random elements. Two problems are generally recognized. Firstly, it is difficult, if not impossible, to assign a single RMS value suitable to all optimization processes regardless of the magnitude of input data and the choice of inventory simulation models. Secondly, convergence of the response output becomes exceedingly slow as the simplex approaches the optimum or near-optimum region. This problem is common in most optimum-seeking methods. Hence, a different stopping criterion is used in order to improve the efficiency of Simplex optimization search.

The stopping criterion used in the design of this program is based on the assumption that an optimum or near-optimum region is reached when the basic operations of reflection, expansion and contraction fail to produce any improved response. A repeated simplex process is carried out at this "optimum region" to ensure that an optimum condition is really attained.

7.4 Second order Design and Regression Analysis

Having reached an optimum or near-optimum region, it is desirable to evaluate the following results:

(i) Exact location of the optimal point;

(ii) Nature of the response surface.

A second order regression analysis is most appropriate in locating the optimal point and in studying the characteristics of the response surface at the optimum condition.

Undoubtedly, the final simplex at the optimum region can be used as a base for further points to be evaluated such that an approximate second order polynomial can be fitted to the response surface of the simplex. However, there could be a risk of singularity or near-singularity present in the simplex design. In this case, it is impossible to set up a second order polynomial in a singular situation. A detailed discussion of singularity and non-singular designs is contained in Box and Hunter[8], and Box and Behnken[7].

A practical approach to experimental design is to adopt a central composite design formed by full factorials. (Fractional factorial designs may be used for experiment with number of factors usually greater than 4 in order to reduce the size of experimentation). The units of measurement are chosen such that the levels of factors are coded as ± 2, ± 1, 0. The origin 0 is taken as the mid-point of the design.

Rotatable designs such as the basic "cube" plus "star" design can provide an additional sophistication to the technique of fitting a response surface approximation to the experimental data. An experimental design is said to be rotatable if the variance of the estimated response at some point depends on the distance from that point to the design centre and not on the direction. However, it is noted that more experimental points are normally required in the rotatable designs than in a central composite design. This involves more computer time on simulation. Hence, central composite designs with full factorials are used in the second-order experimentation and analysis.

The general form of a quadratic polynomial for n factors is given by:

$$\hat{y} = a_0 + \sum_{1}^{n} a_i X_i + \sum_{\substack{i,j=1 \\ i \leq j}}^{n} a_{ij} X_i X_j$$

where $\hat{y}$ = predicted response

a_o, a_i, a_j = regression coefficients

X_i, X_j = decision variables or factors

n = number of factors

The above equation may be rewritten in matrix notation as:

$$\hat{y} = a_o + \underline{X}' \, \underline{a} + \underline{X}' \, \underline{A} \, \underline{X}$$

where

$$\underline{a} = \begin{bmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_n \end{bmatrix}, \quad \underline{X} = \begin{bmatrix} X_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ X_n \end{bmatrix}, \quad \underline{A} = \tfrac{1}{2} \cdot \begin{bmatrix} 2a_{11} & a_{12} \cdots\cdots & a_{1n} \\ a_{12} & 2a_{22} \cdots\cdot & a_{2n} \\ \cdot & \ddots & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{1n} & a_{2n} \cdots\cdots & 2a_{nn} \end{bmatrix}$$

and $\underline{X}'$ = Transpose of $\underline{X}$

$$= \begin{bmatrix} X_1 & \cdots\cdots\cdots & X_n \end{bmatrix}$$

If it is assumed that a second-order polynomial can be fitted to the response surface at the stationary region, then the regression coefficients (ie. a_0, a_i, a_{ij}) can be estimated by the least-square method using data provided by the central composite design.

The stationary point for the response function is found by solving

$$\frac{\partial \hat{y}}{\partial \underline{X}} = 0$$

from which a stationary point, $\underline{X}_s$ is given by

$$\underline{X}_s = -\underline{A}^{-1} \cdot \underline{a} / 2$$

Thus, the predicted response, $\hat{y}_s$ at this point is given as

$$\hat{y}_s = a_0 + \underline{X}_s' \cdot \underline{a} / 2$$

7.5 Canonical Analysis

Having located the stationary point, $\hat{y}_s$, it is necessary to determine the nature of this point in relation to the response system. A canonical analysis is particularly useful in transforming the response surface polynomial to the following equation which is commonly known as Canonical Form:

$$\hat{y} = \hat{y}_s + \sum_{1}^{n} \lambda_i W_i^2$$

where λ_i's are the eigenvalues of matrix $\underline{A}$ and W_i's are the new axes corresponding to the principal axes of the contour system. In short, the origin (X_i's = 0) has been translated to the stationary centre of the response system, and axes of new variables (W_i's) are formed at this translated centre. This is illustrated for two variables in Figure 7.2. A rigorous Canonical analysis is contained in Myers[53] (Chapter 5).

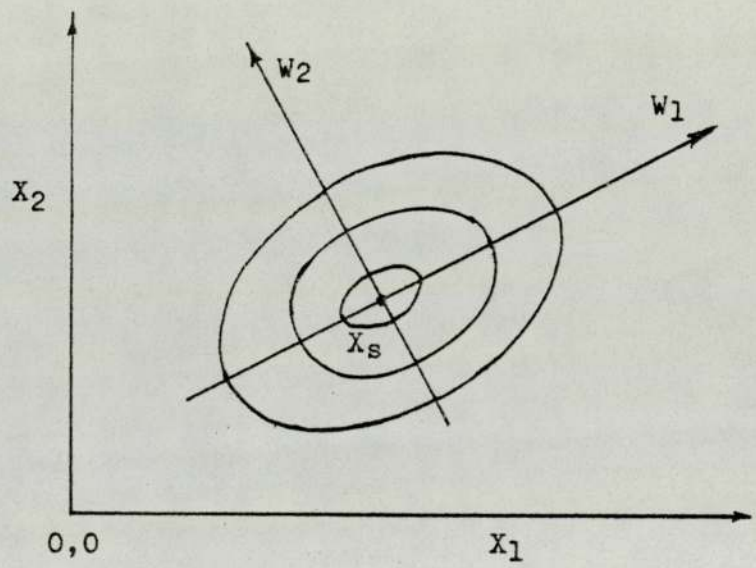


Figure 7.2: Illustration of Canonical form for a Response Surface in Two factors

The following interpretations can be deduced by observing the "sign" of the eigenvalues:

(i) If all eigenvalues are negative, the stationary point, $\underline{X}_s$ represents a point corresponding to the predicted Maximum response.

(ii) If all eigenvalues are positive, $\underline{X}_s$ defines a point of predicted Minimum response.

(iii) If eigenvalues differ in sign, $\underline{X}_s$ defines a saddle point or minimax response. In this case, a Ridge analysis is applied to locate the most favourable predicted response within the experimental region.

(iv) If one of the eigenvalues is zero (or near-zero), and the remaining negative, $\underline{X}_s$ defines a system of Stationary Ridge.

(v) If all eigenvalues are negative, but the estimated maximum lies outside the region covered by the experiment, then the predicted response surface is a Rising Ridge. Further experimentation is recommended along the path of increasing response.

The magnitude of eigenvalues defines the shapes of the predicted response surface. For example, in the case where n = 2, λ_1 and λ_2 are both negative and where $|\lambda_2|$ is considerably greater than $|\lambda_1|$, the shape of the predicted response surface is an elongated ellipse with the maximum located at the centre of the predicted response system. The direction of elongation is along the W_1 axis (See Fig. 7.2).

When a maximum condition is achieved, a user may want to know the effect caused by the deviations of inventory parameters from the optimal point, on the performance of a particular inventory system. In this case, a sensitivity test is applied to evaluate the relative performance of the inventory system with reference to the predicted optimum response by varying a particular decision variable and holding the remaining constant. It is remarked that inferences can only be made about the experimental region. Any attempt to draw conclusions about the surface outside the experimental region would be subject to unrealistic and misleading results.

7.6 Ridge Analysis

Ridge analysis, a term coined by Hoerl[30], was rigorously developed by Draper[19] to provide techniques for the experimenter to formulate a n-variable response surface problem in two dimensions.

A brief recast of this method is as follows:

Suppose the objective is to determine a point (X , ..., X_n) on a hypersphere of given radius R about the design centre point, which maximizes

$$\hat{y} = a_0 + \underline{X}'\, \underline{a} + \underline{X}'\underline{A}\, \underline{X}$$

The coordinate of this point can be found with the use of Lagrange Multipliers. Thus, we have the following function:

$$F = \hat{y} - \mu(\underline{X}'\underline{X} - R^2)$$

where
- F = a function to be maximized
- $\hat{y}$ = predicted response
- μ = Lagrange multiplier
- $\underline{X}$ = a system of variables
- $\underline{X}'$ = transpose of $\underline{X}$
- R = radius

Differentiating F with respect to $\underline{X}$ and equating the results to zero, we have the following systems of equation:

$$(A - \mu \underline{I})\ \underline{X} = -\ \underline{a}/2$$

and $$\underline{X}'\ \underline{X} = R^2$$

The above system of equations can only be solved by iteration.

It is noted that radius R depends on the configuration of the second-order design. For example, in a central composite design with levels of factors at coded distance $\pm$ 2, $\pm$ 1 and 0, maximum R is 2.

The following iterative procedures are recommended for a Ridge Analysis:

(i) Obtain the characteristic roots of $\underline{A}$ matrix.

(ii) Choose values of μ greater than the largest characteristic root (in the case where a maximum is sought), and evaluate the predicted values, $\underline{X}$, for different values of μ .

(iii) Compute radius R and the corresponding response $\hat{y}$ with different values of X. A predicted optimum response in a minimax situation is located at the maximum design radius R.

7.7 Concluding Remarks

Although the optimum-seeking program provides an automatic search for an optimum response of a particular inventory policy simulation opted by the user, there are two basic limitations regarding the use of each technique. Firstly, the use of an optimum-seeking program, regardless of whatever searching techniques are being employed, does not guarantee an optimum solution. Undoubtedly, an additional sophistication involving the design of more experimental points and the use of statistical methods, may produce a more refined solution. This involves more simulation time and hence a higher process cost for optimization. Even then, there is still no guarantee that a global optimum solution is really obtained. Thus, one could question whether it is worth the additional cost incurred in producing a more refined solution (which may or may not be a global optimum), when the input data are often subject to some form of uncertainty and inaccuracy. The second limitation is that inferences should be made about the experimental region. Any conclusion drawn outside the experimental design would be unrealistic.

Despite the above inherent pitfalls, the optimum-seeking program provides optimum or near-optimum results good enough for decision making. Furthermore, this optimum-seeking program is automatically interfaced with the various options of inventory policy simulation such that no additional programming effort is required to run the option of optimization.

The computer printout of an optimization process is displayed under the following headings:

INVENTORY PARAMETERS
INVENTORY OPERATING COSTS + PURCHASES
TOTAL SALES
NET REVENUE
VENDOR SERVICE LEVEL

The detailed printout offers as an alternative to a user in selecting different sets of inventory parameters to satisfy certain requirements such as, say, 95% Vendor service level.

CHAPTER EIGHT

USER'S GUIDE TO INTERACTIVE INVENTORY SIMULATION

8.1 Outline of the Package

GIPSI is an interactive general purpose, inventory control simulation package designed to be used by persons with no computer background. This package is programmed in BASIC and was originally designed to be used on a Hewlett-Packard Access 2000 machine. The package occupies about 600 blocks or .3 M-bytes of storage.

Further details can be referred to the Handbook "GIPSI - A General Purpose Inventory Policy Simulation Package".

8.2 Input Option for Demand Data

The package offers the user eight options for inputting demand data, which are:

1. Input of a series of successive demand per unit time (p.u.t.) values (maximum of 100 values allowed). Alternatively the user can call a prepared data file of successive demand per unit time values as stored by $FINPUT.

2. Input of a set of ordered demand p.u.t. values together with their associated probabilities of occurrence (maximum of 20 classes allowed).

3. Input of a fixed or constant value of demand per unit time

4,5,6,7,8. Input of an approximately Normal (4), Gamma (5), Lognormal (6), Uniform (7) or Negative exponential (8) distribution of demand per unit time.

DATA ANALYSIS

If the user opts to input demand data as a series of successive demand p.u.t. values (1), the package offers the user an analysis of the user's demand data and provides:

Sample size

Mean

Standard deviation

Class intervals, mid values frequency,
probability and cumulative probability values

The option of plotting the histogram of demand data

GOODNESS OF FIT TEST

As further backup to the Data Analysis section, the user entering demand data as a series of demand p.u.t. values can check whether the data entered is likely to be a reasonable fit to a Normal (1), Gamma (2), Negative Exponential (3), Uniform (4), Lognormal (5), or Poisson (6) distribution.

For sample sizes greater than ten (10) a Kolmogorov-Smirnov goodness of fit test is used and for sample sizes below this a Cramer-von Mises test. Either (or both) a summary or a more detailed analysis of the goodness of fit procedure is available. The goodness of fit test concludes the demand input stage. An example of a user deciding to input a series of successive demand p.u.t. values (option 1) and requesting a full data analysis and goodness of fit test procedure is shown in the accompanying four pages of printout (ie. pages 121 through to 124).

```
GET-GIPSI
RUN
GIPSI

*** GENERAL-PURPOSE INVENTORY POLICY SIMULATION ***

THIS IS AN INVENTORY POLICY SIMULATION PROGRAM WITH OPTIONS OF :
        (1) REORDER LEVEL POLICY
        (2) REORDER CYCLE POLICY
        (3) REORDER LEVEL POLICY SUBJECT TO PERIODIC REVIEW
        (4) (s, S) POLICY .

DO YOU NEED HELP ?
ANSWER 'YES' OR 'NO'
BE SURE TO PRESS THE 'RETURN KEY' AFTER EACH ANSWER .

YOUR REPLY ?NO

*** INPUT OF DEMAND QTY PER UNIT TIME INFORMATION ***

LIMIT TO MAX 6 CHARACTERS - SPECIFY UNIT OF MEASUREMENT FOR DEMAND QTY
E.G.: TON, UNIT, ETC.

YOUR REPLY ?UNIT

YOU HAVE THE FOLLOWING OPTIONS TO ENTER THE DEMAND QTY PER
UNIT TIME INFORMATION :
        1--INPUT OF ACTUAL DATA, I.E.,A NUMBER OF SUCCESSIVE RECENT
           DEMAND PER UNIT TIME ;
        2--INPUT OF A SET OF DEMAND P.U.T. & THE CORRESPONDING PROB. ;
           E.G.:
           DEMAND, PROB ? 123, 0.25
        3--INPUT OF A CONSTANT DEMAND P.U.T. ;
        4--GENERATING AN APPROX NORMAL DEMAND P.U.T. ;
        5--    ''      ''  ''   GAMMA     ''    ''    ;
        6--    ''      ''  ''   LOGNORMAL ''    ''    ;
        7--    ''      ''  ''   UNIFORM   ''    ''    ;
        8--    ''      ''  ''   NEGATIVE EXPONENTIAL DEMAND P.U.T.

YOUR OPTION (1,2,.....,8) ?1

PLEASE NOTE THAT A MAX. OF 100 VALUES ARE ALLOWED TO BE ENTERED.

NUMBER OF SUCCESSIVE DEMAND QTY /UNIT TIME ?15

TYPE IN THE  15   SUCCESSIVE VALUES AFTER EACH QUES.MARK.

?250
?297
?312
?340
?355
?365
?378
?389
?412
?456
?510
?352
?386
?339
?377
```

DO YOU WANT TO PRINT OUT ANALYSIS OF YOUR INPUT DATA ?

ANSWER 'YES' OR 'NO' ?YES

*** DEMAND PER UNIT TIME DISTRIBUTION ***

SAMPLE SIZE = 15
MEAN VALUE = 367.867 UNITS
STD DEVIATION = 60.5253 UNITS

RANGE	MID VALUE	FREQ	PROB	CUM PROB
2.500E+02 - 3.020E+02	2.760E+02	2.000E+00	1.333E-01	1.333E-01
3.020E+02 - 3.540E+02	3.280E+02	4.000E+00	2.667E-01	4.000E-01
3.540E+02 - 4.060E+02	3.800E+02	6.000E+00	4.000E-01	8.000E-01
4.060E+02 - 4.580E+02	4.320E+02	2.000E+00	1.333E-01	9.333E-01
4.580E+02 - 5.100E+02	4.840E+02	1.000E+00	6.667E-02	1.000E+00

DO YOU WANT TO PLOT HISTOGRAM & CUM PROB CURVE ('YES'OR'NO') ?YES

** HISTOGRAM **

```
            PROB FROM 0% TO 100%

            0%        20%       40%       60%       80%       100%
            +---------+---------+---------+---------+---------+

276         I*******
328         I*************
380         I********************
432         I*******
484         I***
```

** CUMULATIVE PROBABILITY CURVE **

```
            CUM PROB FROM 0% TO 100%

            0%        20%       40%       60%       80%       100%
            +---------+---------+---------+---------+---------+

276         I      *
328         I                  *
380         I                                       *
432         I                                            *
484         I                                               *
```

DO YOU WANT 'GOODNESS OF FIT' TESTS ('YES' OR 'NO') ?YES

*** GOODNESS OF FIT TEST ***

PLEASE NOTE :
KOLMOGOROV-SMIRNOV TEST (FOR SAMPLE SIZE GREATER THAN 10)
OR CRAMER-VON MISES TEST (FOR SAMPLE SIZE LESS THAN 10) IS
TO BE APPLIED TO TEST IF YOUR INPUT DATA CAN BE FITTED TO ANY
OF THE FOLLOWING DISTRIBUTIONS :

1--NORMAL DIST. ;
2--GAMMA DIST. ;
3--NEG. EXPONENTIAL DIST. ;
4--UNIFORM DIST. ;
5--LOGNORMAL DIST. ;
6--POISSON DIST.

DO YOU WANT TO PRINT OUT DETAILED ANALYSIS ('YES' OR 'NO') ?YES

** SUMMARY OF KOLMOGOROV-SMIRNOV TEST **

(1) TEST AGAINST NORMAL DIST.:

CELL NO	RANGE FROM	TO	OBSERVED FREQ	OBSERVED CUM PROB	EXPECTED CUM PROB	K-S DEVIATION
1	2.50E+02 -	3.02E+02	2.00E+00	1.33E-01	2.57E-02	-.49E-02
2	3.02E+02 -	3.54E+02	4.00E+00	4.00E-01	1.38E-01	-.94E-02
3	3.54E+02 -	4.06E+02	6.00E+00	8.00E-01	4.09E-01	6.43E-02
4	4.06E+02 -	4.58E+02	2.00E+00	9.33E-01	7.36E-01	1.55E-03
5	4.58E+02 -	5.10E+02	1.00E+00	1.00E+00	9.32E-01	9.43E-03

MAX ABS K-S DEVIATION ....= 6.43343E-02
CRITICAL VALUE @ 95% SIGNIFICANT LEVEL = .338

*RESULT :
YOUR INPUT SAMPLE IS LIKELY TO BE NORMAL DISTRIBUTED
WITH MEAN = 367.867 & STD DEV = 60.5253

(2) TEST AGAINST GAMMA DIST. :

CELL NO	RANGE FROM	TO	OBSERVED FREQ	OBSERVED CUM PROB	EXPECTED CUM PROB	K-S DEVIATION
1	2.50E+02 -	3.02E+02	2.00E+00	1.33E-01	1.19E-01	1.46E-02
2	3.02E+02 -	3.54E+02	4.00E+00	4.00E-01	4.14E-01	-.14E-01
3	3.54E+02 -	4.06E+02	6.00E+00	8.00E-01	7.31E-01	6.92E-02
4	4.06E+02 -	4.58E+02	2.00E+00	9.33E-01	9.09E-01	2.46E-02
5	4.58E+02 -	5.10E+02	1.00E+00	1.00E+00	9.69E-01	3.15E-02

MAX ABS K-S DEVIATION ....= 6.91971E-02
CRITICAL VALUE @ 95% SIGNIFICANT LEVEL = .338

*RESULT :
YOUR INPUT SAMPLE IS LIKELY TO BE GAMMA DISTRIBUTED
WITH MOD = 36.9408 & SCALE PARAMETER = .100419

(3) TEST AGAINST NEG. EXPONENTIAL DIST. :

CELL NO	RANGE FROM	TO	OBSERVED FREQ	OBSERVED CUM PROB	EXPECTED CUM PROB	K-S DEVIATION
1	2.50E+02	- 3.02E+02	2.00E+00	1.33E-01	5.60E-01	-.43E+00
2	3.02E+02	- 3.54E+02	4.00E+00	4.00E-01	6.18E-01	-.22E+00
3	3.54E+02	- 4.06E+02	6.00E+00	8.00E-01	6.68E-01	1.32E-01
4	4.06E+02	- 4.58E+02	2.00E+00	9.33E-01	7.12E-01	2.21E-01
5	4.58E+02	- 5.10E+02	1.00E+00	1.00E+00	7.50E-01	2.50E-01

MAX ABS K-S DEVIATION ....= .426653
CRITICAL VALUE @ 95% SIGNIFICANT LEVEL = .338

*RESULT :
YOUR INPUT SAMPLE IS NOT LIKELY TO BE NEG. EXPONENTIALLY DISTRIBUTED WITH PARAMETER (MEAN) = 367.867

(4) TEST AGAINST UNIFORM DISTRIBUTION :

CELL NO	RANGE FROM	TO	OBSERVED FREQ	OBSERVED CUM PROB	EXPECTED CUM PROB	K-S DEVIATION
1	2.50E+02	- 3.02E+02	2.00E+00	1.33E-01	0.00E+00	1.33E-01
2	3.02E+02	- 3.54E+02	4.00E+00	4.00E-01	2.50E-01	1.50E-01
3	3.54E+02	- 4.06E+02	6.00E+00	8.00E-01	5.00E-01	3.00E-01
4	4.06E+02	- 4.58E+02	2.00E+00	9.33E-01	7.50E-01	1.83E-01
5	4.58E+02	- 5.10E+02	1.00E+00	1.00E+00	1.00E+00	0.00E+00

MAX ABS K-S DEVIATION ....= .3
CRITICAL VALUE @ 95% SIGNIFICANT LEVEL = .338

*RESULT :
YOUR INPUT SAMPLE IS LIKELY TO BE UNIFORMLY DISTRIBUTED WITH MAX = 484 & MIN = 276

(5) TEST AGAINST LOGNORMAL DIST. :

CELL NO	RANGE FROM	TO	OBSERVED FREQ	OBSERVED CUM PROB	EXPECTED CUM PROB	K-S DEVIATION
1	2.50E+02	- 3.02E+02	2.00E+00	1.33E-01	1.13E-02	3.14E-03
2	3.02E+02	- 3.54E+02	4.00E+00	4.00E-01	1.30E-01	-.39E-01
3	3.54E+02	- 4.06E+02	6.00E+00	8.00E-01	4.39E-01	4.66E-02
4	4.06E+02	- 4.58E+02	2.00E+00	9.33E-01	7.53E-01	1.08E-02
5	4.58E+02	- 5.10E+02	1.00E+00	1.00E+00	9.23E-01	1.87E-02

MAX ABS K-S DEVIATION ....= 4.66012E-02
CRITICAL VALUE @ 95% SIGNIFICANT LEVEL = .338

*RESULT :
YOUR INPUT SAMPLE IS LIKELY TO BE LOGNORMAL DISTRIBUTED WITH MEAN = 367.867 & STD DEV = 60.5253

** COMMENT **

BASED ON THE ABOVE ANALYSES, IT IS LIKELY THAT YOUR INPUT SAMPLE IS BEST FITTED TO A LOGNORMAL DISTRIBUTION.

8.3 Input options of lead-time data

The package offers the user eight options on inputting lead-time information, which are:

1.	Input of a series of order and receipt dates
2.	Input of a series of successive lead-time durations
3.	Input of a set of ordered lead-time durations and their corresponding probability of occurrence
4.	Input of a constant lead-time value
5,6,7 & 8.	Input of an approximately Normal (5), Gamma (6), Uniform (7), or Poisson (8) distribution of lead time values or durations.

If the user decides to input a series of order and receipt dates, the package can evaluate:

The intervening number of calendar days

The intervening number of weekends and holidays, assuming

(a) Christmas to be 25th and 26th December
(b) New Year's Day to be 1st January
(c) Other holidays are entered by the user

The intervening number of working days, this being the difference between calendar days and holidays.

LEAD-TIME ANALYSIS

If the user opts to input lead-time data either as a series of order and receipt dates (1) or as a series of successive lead-time durations (2) or as successive lead-time durations stored in a data file using $FINPUT, the package offers an analysis of the user's lead-time data providing similar facilities to that offered for the analysis of demand data (see program). An example of lead-time data entered as a series of order and receipt dates is shown on the accompanying printout (ie. pages 126 through to 128).

```
*** INPUT OF LEAD-TIME INFORMATION ***

PLEASE SPECIFY UNIT OF MEASURING LEAD-TIME DURATIONS ('DAY','WEEK',
'MONTH')

YOUR REPLY ?WEEK

YOU HAVE THE FOLLOWING OPTIONS TO ENTER THE LEAD-TIME INFORMATION :
        1--INPUT OF ORDER & RECEIPT DATES ;
        2--INPUT OF ACTUAL DATA, I.E., A NUMBER OF SUCCESSIVE
           RECENT LEAD-TIME DURATIONS ;
        3--INPUT OF A SET OF LEAD-TIME DURATIONS & THE CORRESPONDING
           PROB. E.G. :
            LEADTIME, PROB ? 4, 0.25
        4--INPUT OF A CONSTANT LEAD-TIME ;
        5--GENERATING AN APPROX NORMAL LEAD-TIME DIST. ;
        6--    ,,     ,,  ,,   GAMMA     ,,     ,,   ;
        7--    ,,     ,,  ,,   UNIFORM   ,,     ,,   ;
        8--    ,,     A  POISSON LEAD-TIME DIST.

YOUR OPTION (1,2,.....,8) ?1

NO. OF VALUES TO BE ENTERED ?10

PLEASE NOTE THAT IT IS IMPORTANT TO ENTER CORRECT DATES ACCORDING TO
THE SEQUENCE OF 'DAY', 'MONTH', 'YEAR'. DATES OF PLACING REPLENISHMENT
ORDER ARE TO BE ENTERED FIRST, FOLLOWED BY DATES OF RECEIVING GOODS
ORDERED.  E.G.

ORD ? 23,4,1975
              REC ? 9,6,1975

THIS MEANS THAT THE DATE OF PLACING AN ORDER IS 23RD APRIL,1975 AND DATE
OF RECEIVING IS 9TH JUNE,1975.
```

```
ORD ?12,1,1976
               REC ?3,3,1976
ORD ?15,2,1976
               REC ?14,4,1976
ORD ?8,4,1976
               REC ?5,6,1976
ORD ?17,5,1976
               REC ?12,7,1976
ORD ?26,6,1976
               REC ?10,8,1976
ORD ?1,8,1976
               REC ?14,9,1976
ORD ?12,9,1976
               REC ?23,11,1976
ORD ?27,11,1976
               REC ?3,1,1977
ORD ?20,12,1976
               REC ?2,2,1977
ORD ?11,1,1977
               REC ?15,3,1977

THE FOLLOWING DAYS ARE FIXED AS HOLIDAYS IN A YEAR :

X'MAS : NORMALLY 25TH & 26TH OF DEC.
NEW YEAR : NORMALLY 1ST OF JAN.
EASTER, BANK'S & OTHER HOLIDAYS : EXACT DATES TO BE ENTERED BY USER

NO. OF HOLIDAYS (EXCLUDING X'MAS & NEW YEAR) YOU WISH TO ENTER.
IF NONE, TYPE '0'.
?2

TYPE IN EACH OF THE  2   DATE(S) AFTER THE QUESTION MARK IN THE ORDER
OF 'DAY', 'MONTH' & 'YEAR'.

?23,6,1976
?6,9,1976
```

*** TABULATION OF INPUT DATA ***

DATES OF PLACING ORDERS	RECEIVING GOODS	NO.OF. CALENDAR DAYS	WEEKENDS AND HOLIDAYS	NO.OF. WORKING DAYS
12- 1-1976	3- 3-1976	50	15	35
15- 2-1976	14- 4-1976	58	17	41
8- 4-1976	5- 6-1976	58	17	41
17- 5-1976	12- 7-1976	56	18	38
26- 6-1976	10- 8-1976	45	16	29
1- 8-1976	14- 9-1976	44	15	29
12- 9-1976	23-11-1976	72	23	49
27-11-1976	3- 1-1977	37	17	20
20-12-1976	2- 2-1977	44	17	27
11- 1-1977	15- 3-1977	63	23	40

DO YOU AGREE WITH THE ABOVE TABULATION ? ANSWER 'YES' OR 'NO'.
?YES
DO YOU WANT TO PRINT OUT LEAD-TIME ANALYSIS ('YES'OR'NO')?YES

*** LEAD-TIME DISTRIBUTION ***

SAMPLE SIZE= 10
MEAN VALUE = 6.98 WEEKS
STD DEVIATION = 1.62837 WEEKS

RANGE OF LEAD-TIME	MID-PT LEAD-TIME	FREQ	PROB	CUM PROB
3.5- 4.5	4	1	0.100	0.100
4.5- 5.5	5	1	0.100	0.200
5.5- 6.5	6	2	0.200	0.400
6.5- 7.5	7	1	0.100	0.500
7.5- 8.5	8	4	0.400	0.900
8.5- 9.5	9	0	0.000	0.900
9.5- 10.5	10	1	0.100	1.000

8.4 Inventory Costs

The following cost information is required to produce simulated results with cost output and to perform the optimization procedure:

(i) Selling price of the stocked item

(ii) Cost price

(iii) Purchase (or Works prime) cost

(iv) Cost of placing a replenishment order (or set-up cost per batch)

(v) Cost of backordering (per occasion backordering is initiated)

(vi) Holding interest rate or storage cost expressed as a percentage of the purchase cost

(vii) Rates of inflation of

- (a) Selling price
- (b) Cost price
- (c) Purchase cost

Page 130 demonstrates the cost acquisition of this package.

```
*** INPUT OF COST INFORMATION ***

GIVE THE FOLLOWING COST INFORMATION TO THE COMPUTER (OMIT
UNITS OF MEASUREMENT SUCH AS $, TON, ETC.).
TYPE '0' IF COST IS NOT AVAILABLE ( FOR HELP, TYPE '-1' ).

SELLING PRICE / UNIT ?2

COST PRICE / UNIT ?1.5

PURCHASE (OR WORKS PRIME) COST / UNIT ?1

COST OF PLACING A REPLENISHMENT ORDER (OR SET-UP COST / BATCH)
?2

COST OF BACKORDERING PER STOCKOUT OCCASION
?10

HOLDING INTEREST RATE, EXPRESSED AS % OF THE PURCHASE OR WORKS
PRIME COST (USUALLY 20 - 30%)
?24

ANNUAL RATES OF INFLATION (IN %) OF :
SELLING PRICE ?8
PURCHASE COST ?8
COST PRICE ?10
```

8.5 Choice of inventory policies

The package offers the user four commonly used inventory policies which can be simulated using the demand, lead-time and/or cost data previously entered.

The policies so offered are:

(a) Reorder level policy

(b) Reorder cycle policy

(c) Reorder level policy subject to periodic review

(d) (s, S) policy.

With all inventory policies the user has the option of allowing back-ordering (ie. allowing inventory balances to go negative) or not.

8.6 Simulation Choice

When proceeding to the simulation section of the package, the user is allowed three options with respect to the control of inventory parameters, which are:

1. The user can specify the values of the controlling inventory parameters.

2. The user can request the package to evaluate the controlling inventory parameters on the basis of orthodox stock control theory.

3. The user can request the package to evaluate optimal or near-optimal inventory parameters based on a criterion of maximising net revenue.

The subsequent simulation produces the values of the relevant controlling inventory parameters and produces a summary of results broken down into:

1. GENERAL INFORMATION - covering

   Number of simulation runs
   Average stock level
   Annual demand per annum
   Annual replenishments acquired
   Average number of replenishments p.a.
   Average number of stockouts p.a.
   Average number of time units of stockout p.a.
   Probability of stockout per occasion
   Average back-order quantity p.a.

2. SERVICE LEVELS - covering

   Vendor Service Level
   Customer Service Level

3. COST INFORMATION

   The simulation procedure produces the following sales and cost information both with and without inflation, covering -

   Total sales p.a.
   Annual purchases
   Average inventory holding or storage costs p.a.
   Average cost of placing orders p.a. (or setting up)
   Average stockout cost p.a.
   Total inventory operating costs p.a.
   Net revenue p.a.

An example of these facilities and the subsequent information offered when simulating a re-order cycle inventory policy is shown on the accompanying pages (pages 133 & 134).

```
*** OPTIONS OF INVENTORY POLICY ***

YOU HAVE THE FOLLOWING OPTIONS TO CHOOSE THE INVENTORY POLICY :
        1--REORDER LEVEL POLICY
        2--REORDER CYCLE POLICY
        3--REORDER LEVEL POLICY SUBJECT TO PERIODIC REVIEW
        4--(S, S) POLICY
        5--ENDING THE PROGRAM.

YOUR OPTION (1,2,....,5) ?2


DO YOU ALLOW BACK-ORDERING (I.E. DO YOU ACCEPT DEMAND ORDER WHEN
NO STOCK IS AVAILABLE & THUS IMPLYING NEGATIVE STOCK) ?

YOUR REPLY ('YES' OR 'NO') ?YES

YOU HAVE THE FOLLOWING OPTIONS TO SELECT THE INVENTORY PARAMETERS :
        1--TO BE SPECIFIED BY YOURSELF ;
        2--TO BE CALCULATED BY THE COMPUTER AS A WORKING EXAMPLE ;
        3--TO BE SEARCHED BY THE COMPUTER TO OBTAIN MAXIMUM OR
           NEAR-MAXIMUM NET REVENUE.

YOUR OPTION (1,2,3) ?2



*** REORDER CYCLE POLICY SIMULATION ***

MAX STOCK LEVEL = 685  UNITS
REVIEW CYCLE .. = 6    WEEKS

SIMULATION IN PROCESS
```

** SIMULATION SUMMARY **

(1) GENERAL INFORMATION

```
NO.OF SIMULATION RUNS ........ =  5
AV.STOCK LEVEL ............... =  259.139
ANNUAL DEMAND ................ =  2501.04
ANNUAL REPLENISHMENT QTY ..... =  2475.44
AV.REPLENISHMENT ORDERS/YR ... =  8.4
AV.NO.OF STOCKOUT OCCASIONS/YR =  .9
AV.NO.OF WEEKS OF STOCKOUT/YR  =  1.45
PROB.OF STOCKOUT ............. =  .107143
AV.BACKORDER QTY/YR .......... =  46.2008
```

(2) SERVICE LEVEL

```
VENDOR SERVICE LEVEL (I.E. PROB.OF NOT
       RUNNING OUT OF STOCK) ......... =  89.2857    %
CUSTOMER SERVICE LEVEL (I.E. PROPORTION OF
       ANNUAL DEMAND MET EX-STOCK) .... =  98.1527    %
```

(3) COST INFORMATION

	WITHOUT INFLATION	ANNUAL INFLATION SALES @ 8.0% PURCHASES @ 8.0% COST PRICE @ 10.0%
TOTAL SALES/YR .... =	4950.88	5154.2
ANNUAL PURCHASES .... =	2475.44	2564.87
AV.INVENTORY HOLDING COST/YR .... =	25.9139	26.9781
AV.ORDER COST (OR SET-UP COST)/YR =	16.8	17.5636
AV.STOCKOUT COST/YR .... =	32.1004	34.0835
TOTAL INVENTORY OPERATING COST/YR =	74.8143	78.6252
NET REVENUE .... =	2400.63	2510.7

8.7 OPTIMISING PROCEDURE

If the user requests that the controlling inventory parameters be optimised on a criterion of maximising the net revenue of the inventory system, a summary of results generated during the optimising procedure are generated which include:

The values of the controlling inventory parameters
Total costs
Total sales
Net revenue
Vendor Service Level

Finally optimal values of the above are produced together with one of the following analyses:

1. Sensitivity analysis of up to $\pm$ 5% on the optimal values of the controlling inventory parameters in a maximum condition.

2. Ridge analysis within the central composite design of the controlling inventory parameters in a minimax or saddle condition.

3. A direct search method in the case where a maximum or minimax has not been located.

Examples of some of these facilities are shown on pages 136 through to 138 .

*** OPTIONS OF INVENTORY POLICY ***

YOU HAVE THE FOLLOWING OPTIONS TO CHOOSE THE INVENTORY POLICY :

1--REORDER LEVEL POLICY
2--REORDER CYCLE POLICY
3--REORDER LEVEL POLICY SUBJECT TO PERIODIC REVIEW
4--(S, S) POLICY
5--ENDING THE PROGRAM.

YOUR OPTION (1,2,....,5) ?1

DO YOU ALLOW BACK-ORDERING (I.E. DO YOU ACCEPT DEMAND ORDER WHEN NO STOCK IS AVAILABLE & THUS IMPLYING NEGATIVE STOCK) ?

YOUR REPLY ('YES' OR 'NO') ?NO

YOU HAVE THE FOLLOWING OPTIONS TO SELECT THE INVENTORY PARAMETERS :

1--TO BE SPECIFIED BY YOURSELF ;
2--TO BE CALCULATED BY THE COMPUTER AS A WORKING EXAMPLE ;
3--TO BE SEARCHED BY THE COMPUTER TO OBTAIN MAXIMUM OR NEAR-MAXIMUM NET REVENUE.

YOUR OPTION (1,2,3) ?3

*** REORDER LEVEL POLICY SIMULATION ***

** SIMPLEX OPTIMIZATION **

NO.	REORDER LEVEL	REPLT QTY	TOTAL COST	SALES	NET REVENUE	VENDOR SERVICE LEVEL(%)
1	3.210E+02	2.220E+02	2.609E+03	4.593E+03	1.984E+03	24.7
	3.360E+02	2.040E+02	2.610E+03	4.224E+03	1.614E+03	8.6
	3.050E+02	2.040E+02	2.622E+03	4.231E+03	1.610E+03	9.0
2	3.210E+02	2.220E+02	2.609E+03	4.593E+03	1.984E+03	24.7
	3.520E+02	2.220E+02	2.629E+03	4.609E+03	1.980E+03	29.3
	3.360E+02	2.040E+02	2.610E+03	4.224E+03	1.614E+03	8.6

3	3.375E+02	2.580E+02	2.634E+03	5.130E+03	2.496E+03	91.7
	3.210E+02	2.220E+02	2.609E+03	4.593E+03	1.984E+03	24.7
	3.520E+02	2.220E+02	2.629E+03	4.609E+03	1.980E+03	29.3
4	2.837E+02	2.760E+02	2.640E+03	5.143E+03	2.504E+03	86.7
	3.375E+02	2.580E+02	2.634E+03	5.130E+03	2.496E+03	91.7
	3.210E+02	2.220E+02	2.609E+03	4.593E+03	1.984E+03	24.7
5	2.899E+02	3.570E+02	2.648E+03	5.166E+03	2.518E+03	95.7
	2.837E+02	2.760E+02	2.640E+03	5.143E+03	2.504E+03	86.7
	3.375E+02	2.580E+02	2.634E+03	5.130E+03	2.496E+03	91.7
10	3.279E+02	3.131E+02	2.654E+03	5.183E+03	2.529E+03	100.0
	3.122E+02	2.872E+02	2.645E+03	5.174E+03	2.529E+03	98.9
	3.192E+02	3.342E+02	2.658E+03	5.184E+03	2.526E+03	99.3
11	3.279E+02	3.131E+02	2.654E+03	5.183E+03	2.529E+03	100.0
	3.122E+02	2.872E+02	2.645E+03	5.174E+03	2.529E+03	98.9
	3.196E+02	3.172E+02	2.656E+03	5.185E+03	2.528E+03	98.1

REPEAT SIMPLEX

12	3.122E+02	2.872E+02	2.645E+03	5.174E+03	2.529E+03	100.0
	3.196E+02	3.172E+02	2.660E+03	5.185E+03	2.526E+03	99.4
	3.279E+02	3.131E+02	2.638E+03	5.151E+03	2.521E+03	100.0
13	3.039E+02	2.913E+02	2.650E+03	5.186E+03	2.537E+03	97.7
	3.122E+02	2.872E+02	2.645E+03	5.174E+03	2.529E+03	100.0
	3.196E+02	3.172E+02	2.660E+03	5.185E+03	2.526E+03	99.4
14	3.039E+02	2.913E+02	2.650E+03	5.186E+03	2.537E+03	97.7
	3.138E+02	3.032E+02	2.647E+03	5.177E+03	2.530E+03	99.4
	3.122E+02	2.872E+02	2.645E+03	5.174E+03	2.529E+03	100.0
15	3.039E+02	2.913E+02	2.650E+03	5.186E+03	2.537E+03	97.7
	3.055E+02	3.073E+02	2.653E+03	5.183E+03	2.530E+03	98.2
	3.138E+02	3.032E+02	2.647E+03	5.177E+03	2.530E+03	99.4
16	3.039E+02	2.913E+02	2.650E+03	5.186E+03	2.537E+03	97.7
	3.092E+02	3.013E+02	2.645E+03	5.175E+03	2.530E+03	99.4
	3.055E+02	3.073E+02	2.653E+03	5.183E+03	2.530E+03	98.2
17	3.039E+02	2.913E+02	2.650E+03	5.186E+03	2.537E+03	97.7
	3.060E+02	3.018E+02	2.652E+03	5.184E+03	2.533E+03	97.0
	3.092E+02	3.013E+02	2.645E+03	5.175E+03	2.530E+03	99.4

TERMINATE SIMPLEX & PROCEED TO LOCATE OPTIMAL VALUES.

18	3.092E+02	3.018E+02	2.652E+03	5.184E+03	2.533E+03	98.2
	2.985E+02	3.018E+02	2.634E+03	5.153E+03	2.519E+03	97.6
	2.985E+02	2.808E+02	2.646E+03	5.175E+03	2.529E+03	95.5
	3.092E+02	2.808E+02	2.652E+03	5.175E+03	2.523E+03	93.3
	3.146E+02	2.913E+02	2.638E+03	5.156E+03	2.518E+03	99.4
	2.931E+02	2.913E+02	2.618E+03	5.126E+03	2.507E+03	97.1
	3.039E+02	3.123E+02	2.627E+03	5.137E+03	2.510E+03	97.5
	3.039E+02	2.704E+02	2.632E+03	5.122E+03	2.490E+03	89.7
	3.039E+02	2.913E+02	2.650E+03	5.186E+03	2.537E+03	97.7

MAX CONDITION ACHIEVED

SEARCH TERMINATED

** RESULTS **

OPTIMAL VALUES :
* REORDER LEVEL ... = 305.266 UNITS
* REPLENISHMENT QTY = 293.778 UNITS
* NET REVENUE ..... = 2541.1
** SENSITIVITY ANALYSIS **

DECISION VARIABLE	% DRIFT OF DECISION VARIABLE	% CHANGE IN NET REVENUE
1 REORDER LEVEL	(A) 1% (3.00E+00)	-0.085 %
	(B) -1% (-.40E+01)	-0.086 %
	(C) 5% (1.50E+01)	-2.137 %
	(D) -5% (-.16E+02)	-2.137 %
2 REPLT. QTY.	(A) 1% (3.00E+00)	-0.030 %
	(B) -1% (-.40E+01)	-0.030 %
	(C) 5% (1.50E+01)	-0.757 %
	(D) -5% (-.16E+02)	-0.757 %

DONE

8.8 Additional Features

(i) Sample Output of Simulation Run

At the user's request the package can produce a sample of the simulated stock control situation as shown on page 140 for a Reorder level policy.

(ii) Graphical Display of Inventory Balances

A particularly useful feature of the package for teaching purposes is that the user can plot the sample of the simulated run as a pictorial representation of the inventory balances. This is demonstrated for a Reorder level policy on page 141.

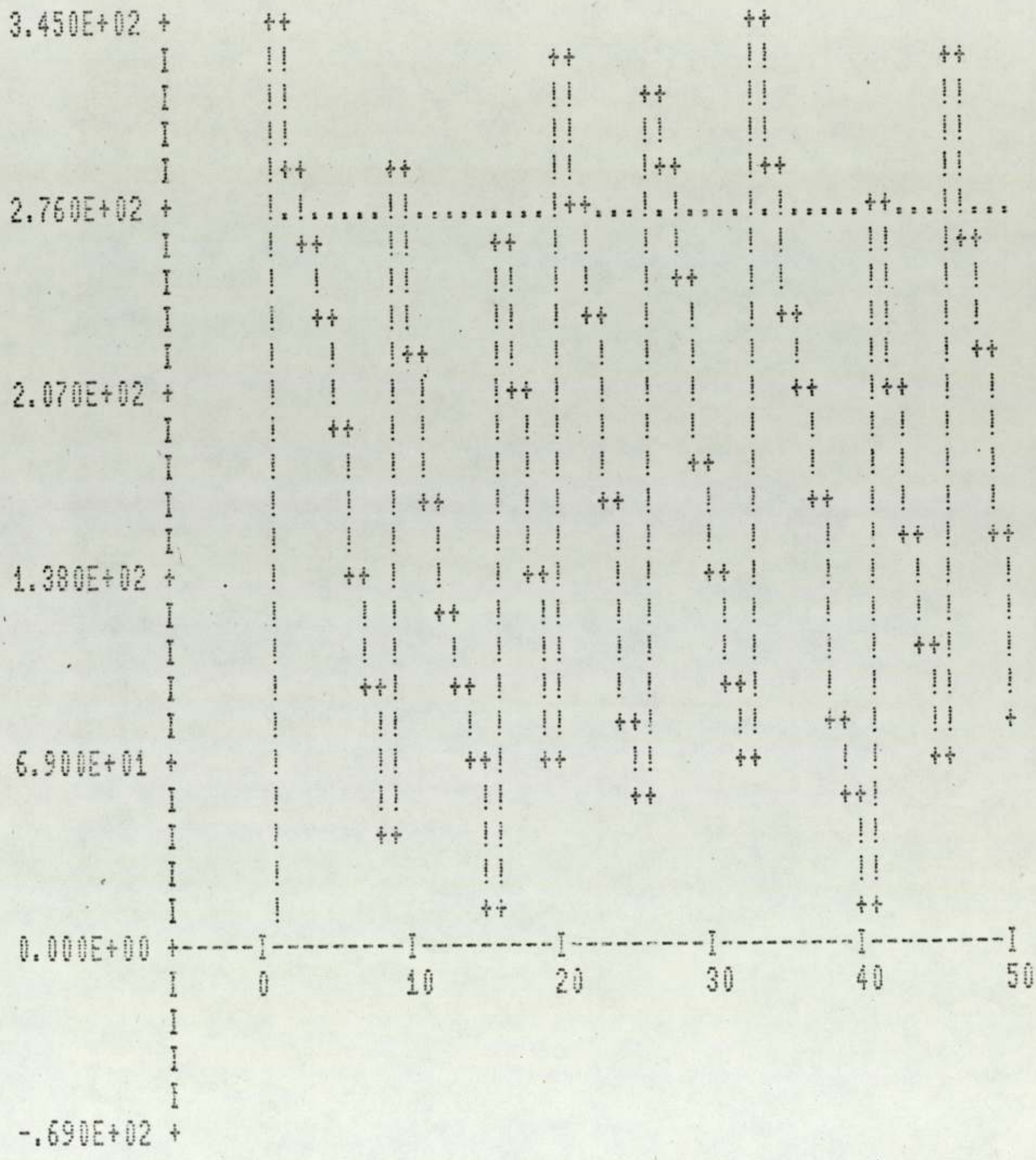
DO YOU WANT TO PRINT OUT A SPECIMEN SAMPLE OF 50 PERIODS ?

YOUR REPLY ('YES' OR 'NO') ?YES

PERIOD	DEMAND PER WEEK	CURRENT STOCK LEVEL	STOCK-OUT COUNT	NO.OF STOCK-OUTS	ORDER QTY	REMAINING LEAD-TIME	NO.OF ORDERS
1	3.459E+01	3.454E+02				0	
2	6.063E+01	2.848E+02				0	
3	2.473E+01	2.600E+02			3.000E+02	6	1
4	2.160E+01	2.384E+02				5	
5	5.037E+01	1.881E+02				4	
6	5.045E+01	1.376E+02				3	
7	3.812E+01	9.951E+01				2	
8	5.608E+01	4.342E+01				1	
9	6.029E+01	2.831E+02				0	
10	5.581E+01	2.273E+02			3.000E+02	6	2
11	5.714E+01	1.702E+02				5	
12	3.951E+01	1.307E+02				4	
13	3.709E+01	9.358E+01				3	
14	3.114E+01	6.244E+01				2	
15	4.652E+01	1.593E+01				1	
16	6.024E+01	2.557E+02			3.000E+02	4	3
17	4.315E+01	2.125E+02				3	
18	7.499E+01	1.376E+02				2	
19	7.196E+01	6.559E+01				1	
20	4.069E+01	3.249E+02				0	
21	4.911E+01	2.758E+02			3.000E+02	5	4
22	3.531E+01	2.405E+02				4	
23	6.991E+01	1.706E+02				3	
24	8.246E+01	8.810E+01				2	
25	2.675E+01	6.136E+01				1	
26	4.041E+01	3.209E+02				0	
27	2.698E+01	2.940E+02				0	
28	4.754E+01	2.464E+02			3.000E+02	5	5
29	6.017E+01	1.863E+02				4	
30	5.388E+01	1.324E+02				3	
31	3.350E+01	9.888E+01				2	
32	2.719E+01	7.169E+01				1	
33	3.203E+01	3.397E+02				0	
34	5.109E+01	2.886E+02				0	
35	4.996E+01	2.386E+02			3.000E+02	6	6
36	3.397E+01	2.046E+02				5	
37	4.385E+01	1.608E+02				4	
38	7.137E+01	8.942E+01				3	
39	4.034E+01	4.909E+01				2	
40	3.735E+01	1.174E+01				1	
41	3.415E+01	2.776E+02			3.000E+02	5	7
42	6.964E+01	2.079E+02				4	
43	5.184E+01	1.561E+02				3	
44	5.160E+01	1.045E+02				2	
45	3.666E+01	6.784E+01				1	
46	3.477E+01	3.331E+02				0	
47	6.930E+01	2.638E+02			3.000E+02	7	8
48	4.422E+01	2.195E+02				6	
49	6.921E+01	1.503E+02				5	
50	7.313E+01	7.721E+01				4	

DO YOU WANT TO PLOT INVENTORY BALANCES FOR THE SPECIMEN SAMPLE ?
ANSWER 'YES' OR 'NO' ?YES

** GRAPHICAL DISPLAY OF REORDER LEVEL POLICY **



CHAPTER NINE

INDUSTRIAL APPLICATION - A CASE STUDY OF TASEK CEMENT LTD.

9.1 Introduction

This chapter describes the industrial application of GIPSI which is used to evaluate certain inventory situations encountered in industry. Two cases have so far been analysed using GIPSI and results have proved encouraging. These include:

(a) Tasek Cement Ltd. (Malaysia) - a full analysis of the inventory situation was carried out via interactive simulation using GIPSI.

(b) Compair Industrial Ltd. (UK) - analysis of demand and lead-time data was carried out by Mr. D.I. Peckett using GIPSI.

The case study of Tasek Cement Ltd. is presented in subsequent discussion to illustrate how GIPSI can be effectively used as a practical tool for analysing certain inventory situations encountered in industry.

9.2 Background of Tasek Cement Ltd.

Tasek Cement Limited Company was established as a private limited company in 1962 with an authorized capital of M$20* million to manufacture cement. It is located in an industrial zone, 6 miles away from Ipoh which is the capital of Perak state of Malaysia.

In 1963, the company was converted into a public limited company and listed on the stock exchanges of Malaysia and Singapore. In 1965, the authorized

* Exchange rate of Malaysian dollar (M$): £1 : M$ 4.4

capital was raised to M$50 million. The initial production capacity of one kiln was limited to 250,000 metric tons annually. Owing to increased demand, the output capacity was raised to 500,000 metric tons by installing an additional kiln. Again in 1975/76, the production capacity was further raised to over one million metric tons annually after completion of the third kiln project.

In 1976, the authorized share capital was raised to M$100 million.

From 1971 through 1974, Tasek Cement experienced a tremendous growth in its domestic market. Since 1975, the impact of a general economic recession has slowed down the pace of building and construction activities, resulting in reduction of demand for cement. The company is still the major cement manufacturer supplying about 40% of cement in West Malaysia.

Prior to 1976, the dispatch system was a simple one by which cement was filled directly from the silo and immediately loaded onto lorries waiting below the filling platform. The storage silos were maintained at a level equivalent to about 4 to 5 days supply and the average loading time for a medium-sized lorry was approximately 20 minutes. Thus, it was not surprising to find that hundreds of customers' lorries were queuing for a matter of hours to get the freshly filled and packed cement. The service given to customers was considered to be appalling. However, since the completion of the new silo and an automatic filling platform in 1976, the dispatch system has much improved, although the true level of service has yet to be determined.

9.3 Lead-time Information

The normal Portland cement is made from a mixture of about 80% carbonate of lime (such as limestone) with 20% clay and a small amount of iron ore. After mixing, the materials are finely ground by a wet or dry process, and then calcined in kilns to a clinker. When cool, this clinker is ground to fine powder. During the process of grinding, a small amount of gypsum is added to regulate the setting of cement. Finally, the finely ground cement is conveyed by pneumatic means to the storage silos ready to be packed in standard bags or dispatched in bulk quantity.

As the process of making cement involves a number of steps ranging from supply of raw materials to filling and packing of cement, the lead-time information is thus difficult to determine with great accuracy. Thus, the appropriate lead-time duration has to be estimated based on the following components:-

(a) the time taken to notify the supplier of raw materials and the supplier's delivery time prior to final receipt into stores;

(b) Storage time of the raw materials before processing;

(c) Production time;

(d) time allowed for filling, packing and transportation of cement.

(i) Raw materials supply

In the following discussion, attempts are made to estimate the delivery time of raw materials:-

(a) Limestone:

The source of limestone supply is located in a hill, about half a mile from the Factory. Contract delivery at a price of M$4.50/metric ton is undertaken by a contracting firm which blasts the limestone rocks into the required size and then delivers them to the factory mills. The estimated delivery time is 5 days.

(b) Clay:

The source of clay is located 6 miles from the Factory. Delivery of clay is undertaken by a contracting firm at a price of M$3.50/metric ton. The estimated delivery time is 5 days.

(c) Gypsum:

Two types of gypsum, ie. the synthetic and natural gypsum, are used to mix clinker to become cement. Synthetic gypsum is obtained locally and the estimated delivery time is less than a week. Natural gypsum is obtained from Thailand and is currently used by Tasek Cement. The estimated delivery time is 6 days.

(d) Iron ore:

Iron ore is obtained locally and the estimated delivery time is 5 days.

From the preceding discussion, it is noted that the lead-time durations of various raw materials do not vary greatly. Hence, the average lead-time of the raw materials supply is estimated to be 5 days.

(ii) Storage time of Raw Materials

Raw materials are stored in the storage locations called storage halls.

Particulars of the storage capacities are as follows:-

	Raw Material	Storage Capacity
(i)	Limestone	27,000 metric tons
(ii)	Clay	5,000 metric tons
(iii)	Gypsum	1,500 metric tons

The incoming raw materials are regularly checked to ensure consistency of a high quality. Generally, raw materials are stored until a sufficient quantity is available for feeding into the raw mill mixing silo for processing. This is done to ensure that the batch of raw meal (ie. the mixture of finely ground limestone and clay) is of a suitable size to be processed in the raw mill itself. The storage time, depending on the supply of the raw materials, is about 2 days.

(iii) Production time

There are two raw mills to a rotary kiln in a single processing system. Particulars of the raw mills are shown below:-

Raw mill	No. off	Year built	Capacity/batch	Grinding Process
Old mill	2	1962/63	500 metric tons	semi-dry
Old mill	2	1965/66	800 metric tons	semi-dry
New mill	2	1975/76	1,500 metric tons	dry

When the raw materials have been mixed and ground by means of either a dry or a semi-dry process in the raw mill, the semi-finished product is called raw meal. Normally, the raw meal has to be transferred to the raw meal silo before feeding into the rotary kiln where it is heated to the required temperature and becomes clinker. There are altogether six raw meal storage silos, each having a holding capacity of 3,000 metric tons. Two silos are designed for each processing system to ensure that the raw meal will be continuously fed to the rotary kiln.

Clinker is stored in the clinker silo for about 2 days to allow for cooling. Finally, clinker and gypsum are mixed and ground to become cement at a rate of 90 metric tons per hour. The finished product is stored in the cement storage silos, ready to be filled and packed.

Since the whole process involves a lot of waiting, transportation and storage times, the production time is estimated as follows:-

	Process (inclusive of waiting, storing, transportation etc.)	Estimated time
a)	Mixing & grinding of raw materials	1½ days
b)	Raw meal storing	1½ - 4½ days
c)	Preheating	½ day
d)	Burning in kiln	½ day
e)	Clinker storing	2 - 6 days
f)	Clinker grinding	1 day
		7 -14 days

(iv) Filling, Packing & Transportation

Filling and packing are carried out by the automatic filling and packing machine. The holding capacity of the standard-size bag is equivalent to

50 kg. (or 110 lbs.) of cement. After sealing, the bags are discharged onto the waiting lorry below the filling platform. Two labourers are required to stack and arrange the bags on the lorry. The normal loading time for a medium-size lorry varies from 10 to 20 minutes.

(v) Estimated lead-time of packed cement

The lead-time of the packed cement is estimated to be between 14 days and 21 days (or 2 weeks - 3 weeks based on 7 days a week).

9.4 Demand of Cement

The recorded sales (excluding Government contract and exports) compiled by the Statistical Department are as follows:-

Month	Monthly Sales (Metric tons)	Average Weekly Sales (Metric tons)
1974-Nov.	28896	6761
Dec.	28503	6455
1975-Jan.	29081	6585
Feb.	23531	5899
March	27967	6333
April	30361	7104
May	31089	7043
June	31093	7275
July	28811	6524
Aug.	26041	5897
Sept.	28439	6635
Oct.	27646	6261
Nov.	21519	6368
Dec.	29411	6660
1976-Jan.	27571	6243
Feb.	25145	6303
March	37270	8440
April	39995	9357
May	42548	9635
June	40734	9531
July	48661	11019

Month	Monthly Sales (Metric tons)	Average Weekly Sales (Metric tons)
1976-Aug.	49829	11283
Sept.	45223	10581
Oct.	51437	11648
Nov.	53353	12483
Dec.	51903	11753
1977-Jan.	46823	10602
Feb.	45810	11484
March	51532 (projected)	11669 (projected)
April	44547 (")	10423 (")
May	47944 (")	10857 (")

From the above information, it is apparent that until March 1976, production was geared to a maximum capacity in order to fulfill customers' demand and other contractual commitments such as Government contract and exports which normally formed one third of overall sales.

The boom in the building and construction industry in 1973 and 1974 had induced practically all cement manufacturing companies, inclusive of Tasek Cement, to expand their production capacities. Subsequent to the commissioning of the third kiln expansion in March 1976, although monthly domestic sales had increased to above 35,000 metric tons, since the maximum capacity of the factory was designed new at 100,000 metric tons per month, a situation arose of under-utilisation of plant capacity. Two major factors could account for under-utilisation ie.

(a) the expected higher demand for cement could not be realized because of the gradual decline in building and construction activities as a result of the general economic recession as well as uncertainty facing the transitional period between the end of 2nd Malaysian Plan (1971-1975) and the beginning of 3rd Malaysian Plan (1976-1980).

(b) increased competition among the cement manufacturing companies.

Increased competition among the cement manufacturing companies has also prompted the management to look into the important question of service to customers.

9.5 Inventory System

The present cement storage system consists of 14 units of storage silos and 3 outstation depots capable of storing a maximum of 44,000 metric tons of cement which is equivalent to about 15 days of normal supply. Particulars of the storage capacities are as follows:-

Storage System	No. of Units	Capacity per unit (metric tons)	Total Storage capacity (metric tons)
New Silo	4	6,000	24,000
Old Silo	2	3,000	6,000
Old Silo	8	500	4,000
Outstation Depot	3		10,000
			44,000

As the whole production system is a high-volume flow process, clinker in the Clinker Silo can be considered as part of the buffer stock in the case of emergency supply. This is particularly useful as clinker can be stored for a long time without any hardening effect. A maximum amount of clinker equivalent to 50,000 metric tons of cement can be held in the Clinker Silo and the minimum time in processing to become cement is two days.

The present inventory control system is based on the two -bin method with a reorder level of 28,000 metric tons and an approximate replenishment batch quantity of 50,000 metric tons.

9.6 Cost Information

Cost data are acquired from either actual costs incurred in 1976 or estimates based on the information available. The relevant cost information is outlined as follows:-

(i) Selling Price

Cement is a controlled item and the price is fixed by the Government at M$100 per metric ton ex-factory exclusive of freight or insurance charges.

(ii) Cost of Cement

Based on actual costing in 1976, the works prime cost of cement is M$58 per metric ton. Details of the cost are:-

	M$/metric ton
Direct material cost:	13 *
Direct labour cost:	5
Overhead:	40
Works prime cost:	58
Adm. cost:	10
Selling & Distribution:	10
Cost of cement:	78

* inclusive of cost of packing material at M$7

(iii) Estimated holding interest rate

Inventory holding costs consist of the following components:-

(a) Cost of holding the semi-finished product before packing, ie. cost incurred as a result of depreciation of storage silos, rent etc.

(b) Maintenance cost such as repairs, routine check-up, lighting etc.

(c) Operating cost in handling, reporting, checking, recording, quality control etc.

(d) Cost of obsolescence, damage, pilferage etc.

(e) Opportunity cost of holding the inventory.

Estimates of the above items are as follows:-

(a) Holding cost:

(i)	Estimated depreciation of cement storage silos for 1977	M$ 50,000
(ii)	Estimated rent, insurance etc ..	M$ 10,000

(b) Maintenance cost:-

(i)	Annual repairs	M$ 30,000
(ii)	Routine check-up & maintenance ..	M$ 10,000
(iii)	Lighting, water supply etc. ..	M$ 5,000
		M$ 45,000

(c) Operating cost:-

(i)	Quality control	M$ 20,000
(ii)	Routine checking, recording etc. ..	M$ 10,000
(iii)	Handling of incoming materials ..	M$ 50,000
		M$ 80,000

(d) Cost of obsolescence, damage & pilferage:-

(i)	Obsolescence, recycling etc.	..	M$ 5,000
(ii)	Damage	.. ..	M$ 10,000
(iii)	Pilferage	.. ..	M$ 5,000
			M$ 20,000

Hence, total inventory holding costs	..	M$205,000
Capacity of cement storage silos	..	34,000 metric tons
Unpacked cement cost	.. ..	M$ 51/metric ton
Therefore, max. cost of cement in silos		M$ 1,734,000

Thus, Physical holding rate $= \frac{205,000}{1,734,000} \times 100\%$

$= 11.8\%$

Assuming opportunity cost of holding inventory = 10%

Therefore, estimated rate of inventory holding, expressed as a percentage of works prime cost is 21.8%.

(iv) Set-up cost per batch

The following assumptions are made to evaluate the set-up cost of cement:-

(i) The continuous flow process would be interrupted by either allowing a periodic routine maintenance or scheduling production of other commitments.

(ii) The present review period is assumed to be one month.

(iii) Evaluation is based on cost estimates for 1977.

Based on the above assumptions, the set-up cost is estimated as follows:-

Labour:	M$ 1,000
Materials:	M$ 2,000
Preheating:	M$ 8,000
Wastage, etc.:	M$ 2,000
Mechanical adjustment:	M$ 1,000
Miscellaneous:	M$ 1,000
	M$ 15,000

(v) Cost of Backordering

Backordering cost is the assumed internal cost incurred in holding customers orders when a stockout has occurred, and also informing customers when the backordered goods can be collected.

The estimated expenditure allowed for backordering is M$10,000 a year. Assuming that the number of stockout occasions is 10, the estimated cost of backordering per stockout occasion is M$1,000.

(vi) Rates of inflation

An average of 5% per year is assumed for the rates of inflation of selling price, cost price and works prime cost.

9.7 Analyses

(i) Demand per unit time

The demand pattern (November 1974 - May 1977) is plotted in Fig. 9.1.

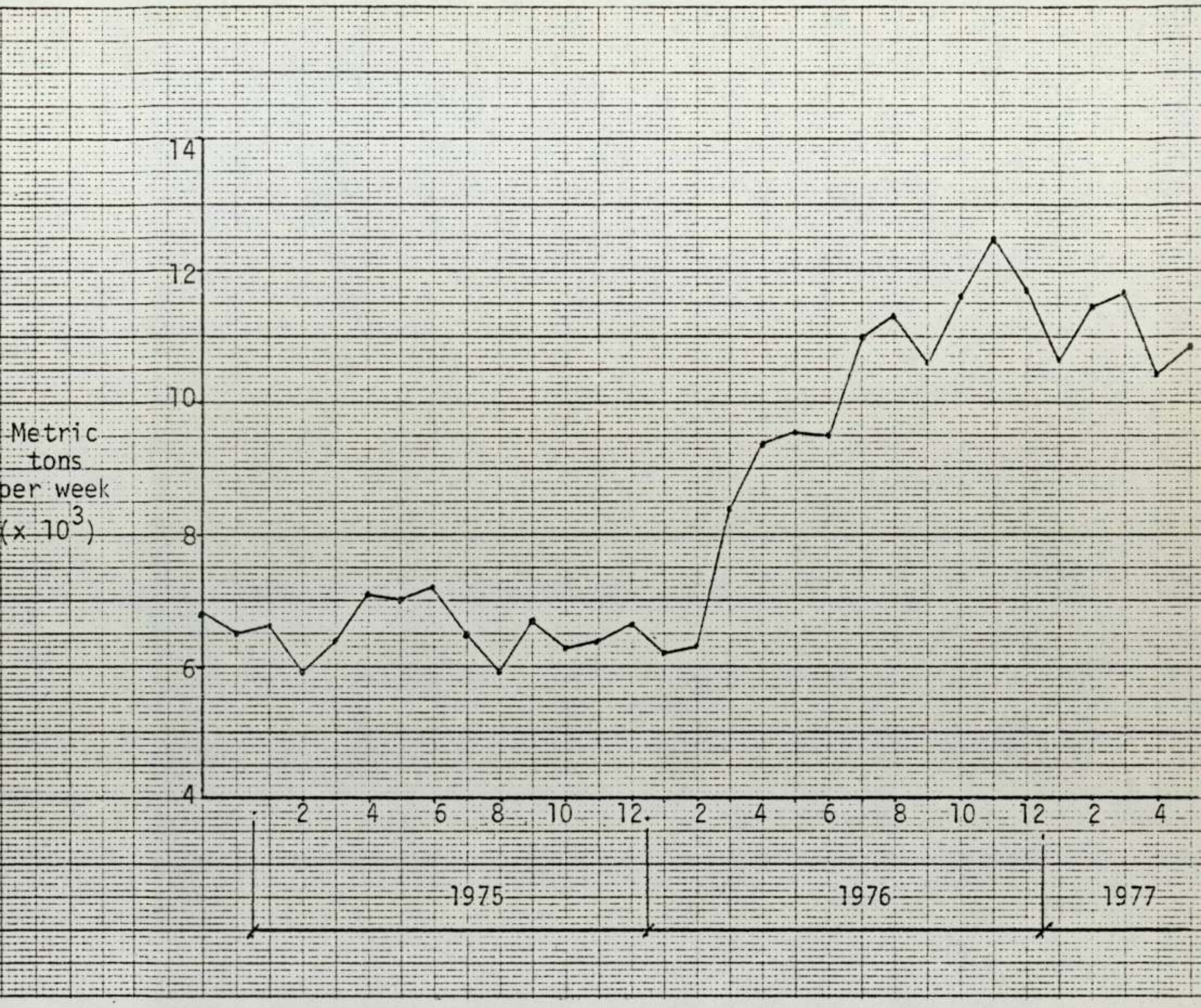


Figure 9.1: Demand Pattern of Cement

Two different levels of demand per unit time information can be identified in Fig. 9.1. These are:-

(a) Until February 1976, the demand pattern is fairly stationary with approximate mean of 6620 metric tons per week and standard deviation of 560 metric tons per week. When this demand distribution is tested against standard probabilistic distributions, it is found that the demand is best fitted to a Lognormal distribution.

(b) The most recent stationary demand pattern occurs from July 1976 to May 1977, with approximate mean of 11250 metric tons per week and standard deviation of 600 metric tons per week. It is found that this demand pattern is best fitted to a normal distribution.

The most recent demand data are used in studying the effectiveness of the existing inventory system.

(ii) Economic Batch Quantity

Using $EBQ = \sqrt{\dfrac{2ACo}{iCm(1-\frac{d}{p})}}$ the

Economic Batch Quantity is found to be approximately 51,100 metric tons.

(iii) Experimental results using GIPSI

Detailed experimental results obtained via interactive simulation using GIPSI are tabulated and plotted in Appendix C (See Tables C.1 to C.4 for tabulation, and Figures C.1 to C.2 for plotting). A summary of such results is shown below:-

(i) Based on current operating condition:-

Lead-time (weeks)	Reorder level (metric tons)	Replenishment batch qty. (metric tons)	Annual inventory operating cost ($M\$ \times 10^6$)	Net Revenue ($M\$ \times 10^6$)
2	28,000	50,000	.56$^+$	23.08$^+$
			.56*	23.75*
3	28,000	50,000	Poor results	

+ Indicates condition allowing backordering.

* Indicates condition with backordering prohibited.

(ii) For the optimal condition giving the minimum annual inventory operating cost:-

Lead-time (weeks)	Approx. Reorder level (Metric ton)	Approx. Replenishment Batch Qty. (Metric ton)	Approx. Min. Cost ($M\$ \times 10^6$)
2	23,000	35,000	0.48
3	35,000	40,000	0.49

(iii) For the optimal condition giving maximum net revenue:-

Lead-time (weeks)	Approx. Reorder level (Metric ton)	Approx. range of Replenishment Batch Qty. (Metric ton)	Approx. Max. revenue ($M\$ \times 10^6$)
2	23,000	25,000 - 70,000	23.9 @ Replenishment Batch Qty. 45,000
3	35,000	35,000 - 80,000	23.85 @ Replenishment Batch Qty. 55,000.

It is found that backordering has little effect on the inventory system operating at optimal conditions in which either the inventory operating cost is to be minimized or the net revenue is maximized. However, for a policy not operating at optimal conditions, backordering would improve the financial position of the company. Furthermore, the optimal surface underlying the maximum revenue condition is found to be rather flat (See Figure 9.2), and this partly explains why the optimization process in maximizing the net revenue takes a relatively long time in locating the optimal or near-optimal point.

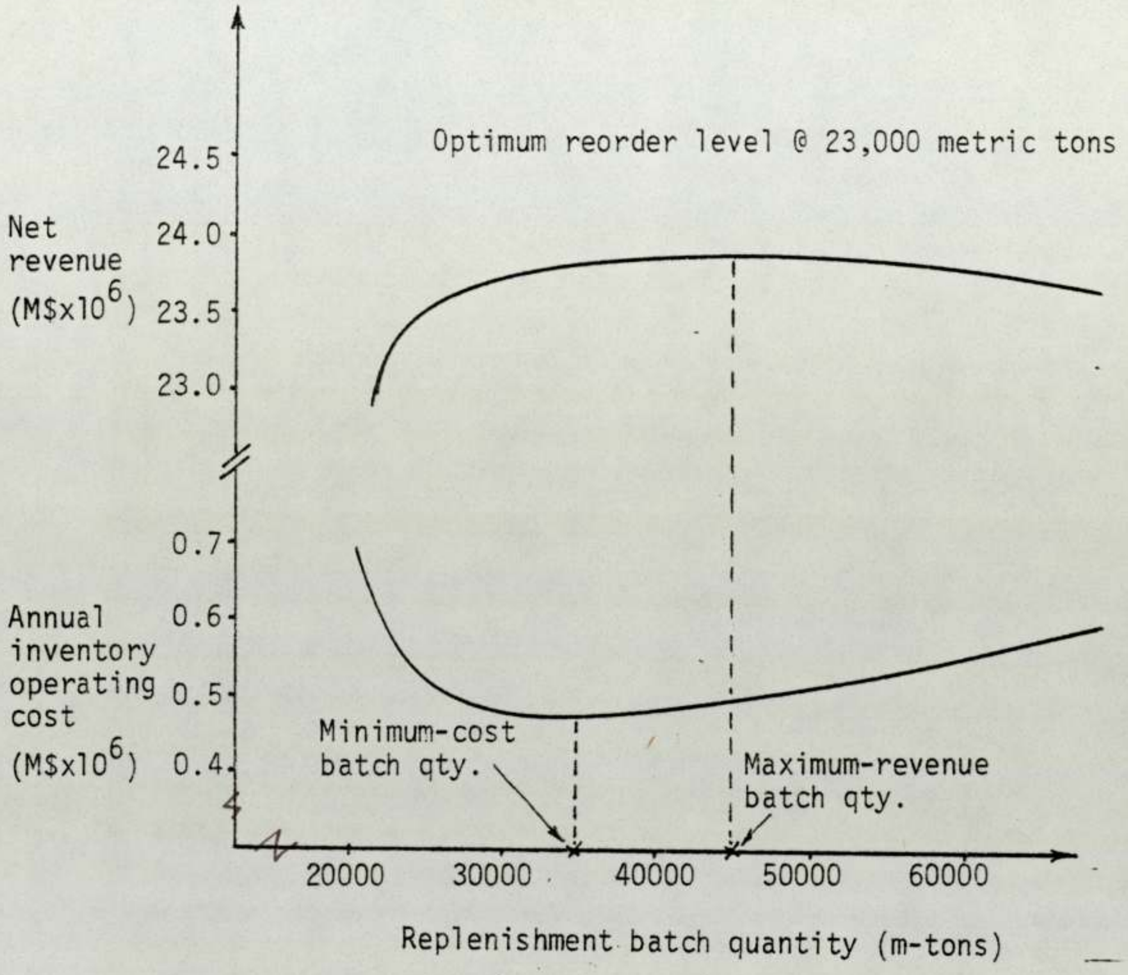


Figure 9.2: Optimal Characteristic of Reorder Level Policy in a two-week lead-time situation with backordering prohibited

9.8 Conclusion

From the preceding analyses, it is noted that in maximizing net revenue, an approximate reorder level of 23,000 metric tons and replenishment batch quantity of 45,000 metric tons would be required in running the reorder level inventory system in a two-week lead-time situation. Further analysis shows the following results:

Lead-time (week)	Reorder level (Metric ton)	Replenishment batch qty. (Metric ton)	Net revenue (M$ x 10^6)	*Estimated gain in net revenue (M$ x 10^6)
2	23,000	45,000	23.9	0.1
3	35,000	55,000	23.85	0.05

* When the result is compared with the current operating condition of reorder level = 28,000 metric tons and replenishment batch quantity = 50,000 metric tons.

Although the existing inventory control system with reorder level of 28,000 metric tons and approximate batch quantity of 50,000 metric tons is good enough to serve the domestic market satisfactorily, it is not operating at the optimal condition. Hence, it is recommended that the reorder level policy should be operated as a two-bin system with the reorder level set at 23,000 metric tons and the replenishment batch quantity at 45,000 metric tons, together with a tight control of manufacturing lead-time of two weeks. This would probably benefit the company with an estimated gain of M$100,000 in net revenue in 1978.

CHAPTER TEN

EXPERIMENTAL RESULTS

10.1 Introduction

One of the basic objectives of developing GIPSI has been to investigate certain characteristics of inventory policies via interactive simulation. Particular areas of research interest that have been investigated in some depth and which are described in this chapter are:

(a) Characteristics of overshoot within the reorder level policy;

(b) Comparison of customer service level and vendor service level;

(c) Effects of inflation on the reorder level policy.

10.2 Characteristics of overshoot

10.2.1 Introduction

In a reorder level policy it is often assumed by the underlying theory that replenishment orders are placed when the stock-on-hand exactly equals the reorder level. Closer examination of this assumption reveals that such an outcome will, in fact, rarely occur. Only when individual demand orders are all for single units will the stock-on-hand always equal the reorder level when a replenishment order is placed. In reality, the stock-on-hand will often fall below the reorder level when an order for replenishment is initiated, and the amount by which the reorder level is broken is known as the "overshoot". The effect such overshoot has on the operation of the policy is naturally to lower service levels.

Figure 10.1 shows a typical inventory balance situation for a reorder level policy where zero, medium and large overshoots are illustrated.

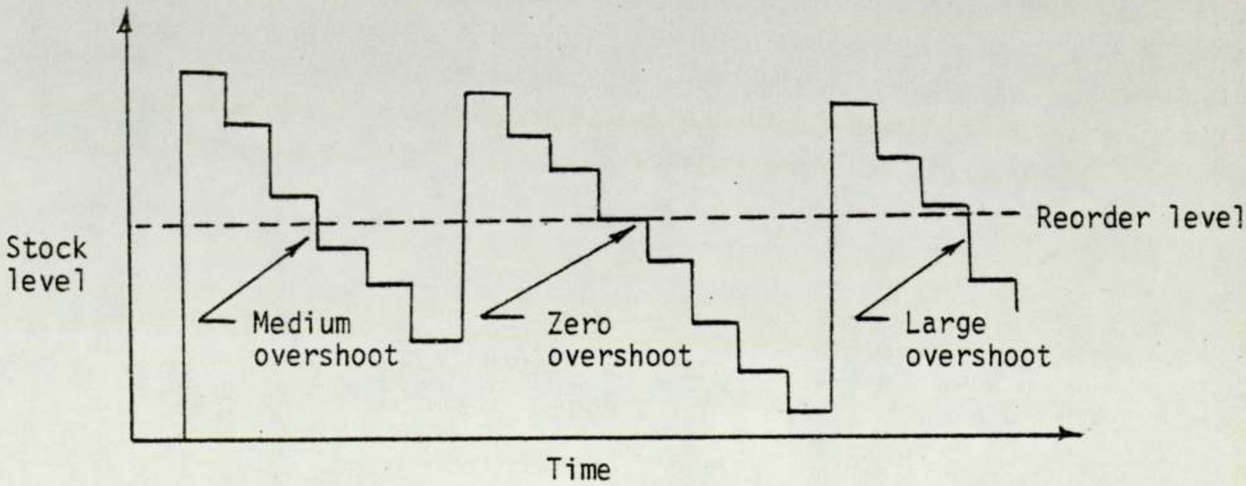


Figure 10.1: Typical inventory balances for a reorder level policy indicating differing degrees of overshoot

The formula for an average overshoot, $\bar{k}$, of the reorder level policy derived by Lampkin[37] is given by

$$\bar{k} = \tfrac{1}{2}(\bar{g} - 1 + \frac{\sigma g^2}{\bar{g}})$$

where $\bar{g}$ and σg are the mean and standard deviation respectively of individual demand order sizes.

In practice, one rarely analyses individual demand order sizes and thus, demand per unit time and its associated standard deviation (ie. $\bar{D}$ and σd) are sometimes used as subsitutes for $\bar{g}$ and σg respectively. This gives rise to the following approximation:-

$$\bar{k} = \tfrac{1}{2}(\bar{D} - 1 + \frac{\sigma d^2}{\bar{D}})$$

The distribution of overshoot tends to be rather an awkwardshape, being truncated at both ends.

10.2.2 Purpose of experimentation

The purpose of this experimental exercise is to study certain characteristics of overshoot distribution of a reorder level policy. For this purpose, the following areas are specifically covered:-

(a) To evaluate the experimental mean overshoot via interactive simulation and to use it to compare with the theoretical mean overshoot of a reorder level policy.

(b) To conduct goodness of fit tests on the experimental overshoot distribution against standard probability distributions.

10.2.3 Outline of experimentation

(a) Experimental mean overshoot investigation

Although GIPSI in its standard form could have been used to evaluate the experimental mean overshoot of a reorder level policy for a given set of input demand and lead-time information, to obtain sufficient information to draw meaningful conclusions such an approach would necessitate many manual calculations. Hence, a modified version of GIPSI was designed to speed up the process of experimentation to evaluate the simulated mean overshoot and its associated standard deviation. In general, the following steps of experimentation were taken:

(i) Generate 500 values of the experimental overshoot of a reorder level policy for a given set of demand and lead-time values.

(ii) Compute the mean experimental overshoot and its associated standard deviation, as well as the theoretical mean overshoot.

(iii) Compare the experimental and theoretical mean overshoot using a statistical "two-tail test "*.

(iv) Accept the hypothesis that the experimental mean overshoot is a good estimate for the theoretical mean overshoot if the difference between the two values is within the acceptance limit. Otherwise reject the above hypothesis.

(v) Repeat the above procedure for different sets of demand and lead-time values. (Different sets of lead-times were used only to represent reality, since the lead-time duration has no effect on overshoot).

* A detailed discussion of this method is contained in Yeoman[79], "Statistics for the Social Scientist : 2 - applied statistics" (Chapter 2).

(b) Shape of the overshoot distribution

An attempt was made, using the Chi-square test, to determine if the experimental overshoot distribution of a reorder level policy could be fitted to any of the commonly used probability distributions. A computer program "OSHOOT" was designed based on the original version of GIPSI to speed up the process of experimentation. In general, the following stages of experimentation were involved:-

(i) Generate 500 values of the experimental overshoot of a reorder level policy for a given set of demand and lead-time values.

(ii) Group the 500 values into 10 classes.

(iii) Conduct a Chi-square test for the grouped data against the standard Normal, Gamma, Uniform, Poisson, Lognormal and Negative Exponential Distributions.

(iv) Repeat the above procedure for different sets of demand and lead-time values which assume the use of the following information:-

(a) Demand information: Normal, Gamma and Uniform distributions.

(b) Lead-time information: Normal, Gamma and Poisson distributions.

It is noted that the Chi-square test was not used for goodness of fit test against the Poisson distribution for values of mean overshoot greater than 20.

10.2.4 Results and Observations

(i) Experimental mean overshoot

Although it is possible to estimate the theoretical average overshoot when the average demand per unit time and its associated standard deviation are given, an experimental average overshoot can also be produced via interactive simulation using GIPSI. Here, the statistical "two-tail test" is used to test if the simulated mean overshoot can be accepted as a reasonable estimate for the theoretical mean overshoot. The formula for testing such a hypothesis is given by

$$Z_{(calculated)} = \frac{\bar{x} - u}{\frac{s}{\sqrt{N}}}$$

where $\bar{x}$ = simulated mean overshoot

u = universal mean, assumed to be theoretical mean overshoot

s = standard deviation of simulated mean overshoot

N = number of samples taken.

$Z_{(calculated)}$ is determined from the experimental sample and used to compare with $Z_{(.05)}$ (ie. 1.96) which corresponds to 95% level of significance. If $|Z_{(cal)}|$ is less than $Z_{0.05}$, then it is likely that the experimental mean overshoot can be taken as a good estimate for the theoretical mean overshoot. A specimen sample of such experimental results is contained in Table 10.1*. The abreviations used in this table refer to the following descriptions:

NOR = NORMAL   POI = POISSON
GAM = GAMMA   UNI = UNIFORM

* Full results can be found in Appendix E.

Table 10.1 : Experimental results of average overshoot of reorder level policy by simulation

(Summarised from Appendix E)

Demand p.u.t.			Lead-time			Theo	Expt	$Z_{(.05)}$	$Z_{(cal)}$
Type	Mean	S.Dev	Type	Mean	S.Dev	o/shoot	o/shoot		
NOR	400	120	NOR	6	1.8	217.5	215.6	1.96	-0.38
NOR	400	120	NOR	6	3.0	217.5	219.2	1.96	0.51
NOR	400	120	GAM	6	1.8	217.5	216.9	1.96	-0.11
NOR	400	120	GAM	6	3.0	217.5	218.5	1.96	0.19
NOR	400	120	POI	6	-	217.5	218.9	1.96	0.45
NOR	400	200	NOR	6	3.0	249.5	245.6	1.96	-0.51
NOR	400	200	GAM	6	3.0	249.5	247.3	1.96	-0.37
NOR	400	200	POI	6	-	249.5	256.2	1.96	1.58
GAM	400	120	NOR	6	3.0	217.5	223.1	1.96	0.81
GAM	400	120	GAM	6	1.8	217.5	211.4	1.96	-0.98
GAM	400	120	POI	6	-	217.5	223.8	1.96	1.63
UNI	400	120	NOR	6	1.8	217.5	227.0	1.96	1.82
UNI	400	120	GAM	4	2.0	217.5	217.4	1.96	-0.03
UNI	400	120	POI	5	-	217.5	228.1	1.96	1.33
GAM	400	200	NOR	6	3.0	249.5	251.0	1.96	0.18
GAM	400	200	GAM	6	1.8	249.5	248.9	1.96	-0.07
GAM	400	200	POI	6	-	249.5	240.1	1.96	-1.31
UNI	400	200	NOR	6	1.8	249.5	247.8	1.96	-0.25
UNI	400	200	GAM	6	3.0	249.5	253.8	1.96	0.49
UNI	400	200	POI	4	-	249.5	258.7	1.96	1.75

Although it is possible that the absolute $Z_{(cal)}$ could sometimes be greater than $Z_{(0.05)}$, such a case could not immediately invalidate the use of theoretical mean overshoot as the universal mean overshoot of the experiment. When such a situation occurred, a further test was carried out using a different but larger sample size. So far results have indicated that the simulated mean overshoot could be accepted as a good estimate for the theoretical mean overshoot. It can also be seen from Table 10.1 that overshoot is in no way related to the lead-time distribution.

(ii) Shape of an overshoot distribution

The experimental data of an overshoot distribution were grouped and tested against the standard Normal, Gamma, Uniform, Poisson, Lognormal and Negative Exponential distributions using a Chi-square test. Summary of such results from a particular test using a normal demand and Gamma lead-time using "OSHOOT" is shown below. (For full results see Appendix E - page 241).

NORMAL DEMAND PER UNIT TIME:

MEAN	=	20 units
STD DEV	=	5 units

GAMMA LEAD-TIME DURATION:

MEAN	=	5 weeks
STD DEV	=	1.5 weeks

EXPT AV O/SHOOT	=	11.40 units
EXPT STD DEV	=	6.71 units
THEO AV O/SHOOT	=	10.12 units
% ERROR	=	11.2

CHI-SQUARE TEST FOR A NORMAL DISTRIBUTION

NO	RANGE FROM		RANGE TO	OBSERVED FREQ	EXPECTED FREQ	$(OBS-EXP)^2/EXP$
1	5.493E-03	-	3.121E+00	6.900E+01	3.191E+01	4.312E+01
2	3.121E+00	-	6.236E+00	6.300E+01	5.604E+01	8.644E-01
3	6.236E+00	-	9.352E+00	6.700E+01	7.963E+01	2.004E+00
4	9.352E+00	-	1.247E+01	7.600E+01	9.156E+01	2.644E+00
5	1.247E+01	-	1.558E+01	9.000E+01	8.518E+01	2.730E-01
6	1.558E+01	-	1.870E+01	6.100E+01	6.412E+01	1.514E-01
7	1.870E+01	-	2.181E+01	3.800E+01	3.905E+01	2.814E-02
8	2.181E+01	-	2.493E+01	2.800E+01	1.924E+01	3.987E+00
9	2.493E+01	-	2.804E+01	5.000E+00	7.670E+00	9.295E-01
10	2.804E+01	-	3.116E+01	3.000E+00	2.474E+00	1.120E-01
						54.1121

DEGREE OF FREEDOM = 7

CRITICAL VALUE @ 5% LEVEL OF CONFIDENCE = 14.1

The purpose of performing a Chi-square test on the experimental overshoot distribution is to determine if any of the commonly used standard probability distributions can be a reasonable fit to the sample distributions. In particular, the above example shows the result of a Chi-square test on the experimental overshoot distribution against a normal distribution. Based on the above result, the 'observed' and 'expected' probabilities of an overshoot distribution are plotted in Figure 10.2 to show the shape of an experimental overshoot distribution against that of a normal distribution.

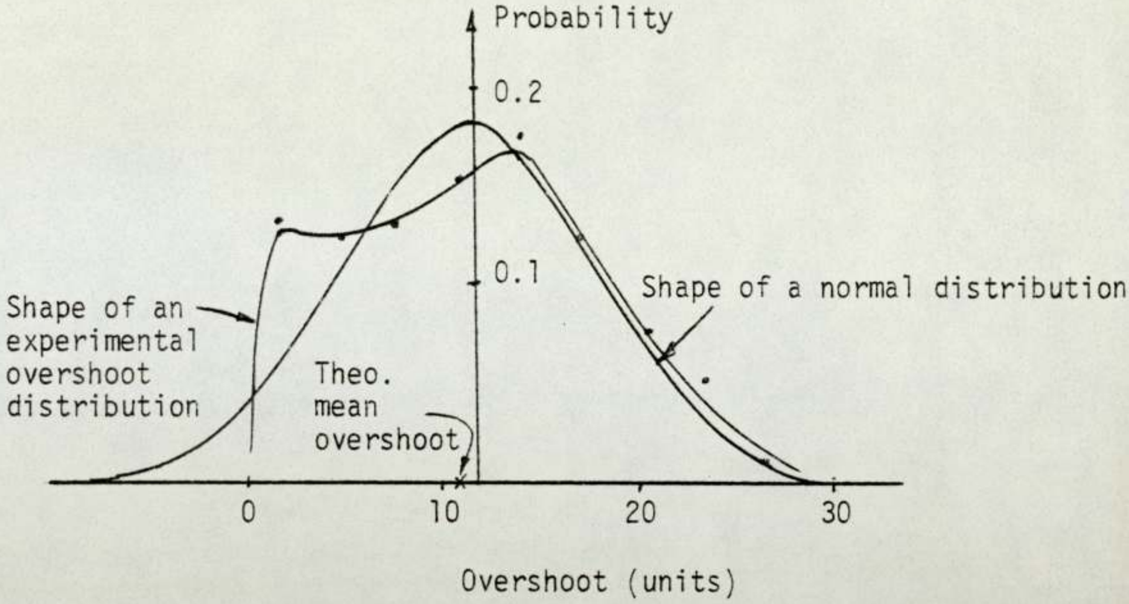


Figure 10.2: Shape of an experimental overshoot distribution

In this particular example, it is observed that the shape of an overshoot distribution in Figure 10.2 is skewed very much to the left and this shows ~~that~~ a high incidence of small overshoots close to the reorder level. This feature is caused by the sudden truncation of the overshoot distribution at the reorder level above which conceptually negative overshoots are not permitted. Consequently, the probability of getting small overshoots is much greater than that of getting large overshoots.

A summary of a series of Chi-square tests on the experimental overshoot distributions under varying demand and lead-time situations against six commonly used standard probability distributions is contained in Appendix E (Tables E1 to E6). From the results in those tables, so far none of the six standard probability distributions has been shown to be a reasonable fit to the overshoot distribution of a reorder level policy. The principal reason why no such fit can be found is due to high probability of occurrence of small values of overshoot.

10.2.5 Conclusion

The following conclusions have been drawn from the experimental results derived from the overshoot investigation:-

(i) The simulated mean overshoot can be accepted as a good estimate for the theoretical mean overshoot,

(ii) The truncated shape of a typical overshoot distribution does not provide a reasonable fit to any of the commonly used probability distributions such as the Normal, Gamma, Uniform, Poisson, Lognormal or Negative Exponential distributions.

10.3 Comparison of service levels

10.3.1 Introduction

There are a number of ways of defining service levels, each suited to the particular circumstances. Two of the most common definitions of service levels are the vendor service level and the customer service level. The vendor service level is defined as the probability of not running out of stock subsequent to a replenishment order being placed. This in practice is a measure of the supplier's internal efficiency and is the definition commonly used by most commercial packages. The customer service level is defined as the proportion of annual demand met ex-stock. Such a definition permits a customer to allow for an annual shortfall in his demand requirement.

It has been observed by Lampkin[37] and Lewis[40] that the customer service level is invariably higher than the vendor service level. Thus, it is interesting to conduct a series of tests using GIPSI to show the relative performance of the two service levels under varying demand and lead-time situations.

10.3.2 Purpose of experimentation

The purpose of this experimental exercise using GIPSI is:-

(a) to evaluate the relative performance of the customer service level and the vendor service level in situations where backordering is prohibited; and

(b) to compare the performance of the above service levels when backordering is allowed.

10.3.3 Outline of experimentation

The detailed experimental procedure in using GIPSI can be referred to the brochure "GIPSI - A General Purpose Inventory Policy Simulation Package". In general, the following experimental steps were taken:-

(i) Evaluate the values of vendor service level and customer service level for a given set of demand and lead-time values in situations where backordering is not allowed.

(ii) Four sets of the above results are to be obtained to give an average value of the required service level.

(iii) Repeat the above process of evaluation for a similar set of demand and lead-time information in situations where backordering is permitted.

It is noted that a large number of experiments could be carried out interactively by inputting various options of demand and lead-time values using GIPSI. Again each of these options could be possibly studied with inputs of a wide range of inventory parameters for a particular inventory policy. However, at this stage of experimentation using GIPSI, no attempt was made to cover the four inventory policies using all the input options of demand and lead-time information. Only a relatively few experiments were actually carried out in an attempt to analyse the relative performance of the vendor and the customer service levels. Values of the input control parameters for a particular inventory policy were fixed at certain arbitrary levels or values to produce an approximate range of customer service level between 80% and 100%.

The full experimental results are contained in Appendix F.

10.3.4 Results and Observations

It is generally observed from the experimental results that the customer service level is higher than the vendor service level under varying demand and lead-time situations regardless of whether backordering is allowed or not. However, when the inventory system is provided with sufficient stock in order to avoid any possibility of stockout, both service levels equal 100%. Appendix F contains the detailed experimental results and shows comparisons of both service levels for the four inventory policies (ie. reorder level policy, reorder cycle policy, reorder level policy subject to periodic review and (s, S) policy).

Values of the service levels are observed to increase with a decrease of the standard deviation of demand per unit time for all the four inventory policies. This can be illustrated in Table 10.2 (as well as in Figure 10.2a) which shows a specimen sample of the experimental results for a reorder level policy for which various service levels are tabulated (or plotted) against the varying standard deviations of demand per unit time under a normal demand and normal lead-time situation.

Table 10.2: Comparisons of service levels of the reorder level policy in a normal demand and normal lead-time situation.

Demand p.u.t.		Lead-time		Back-order	Inventory Parameters		Vendor service level (%)	Customer service level (%)
Mean	S.Dev	Mean	S.Dev		ROL	QTY		
100	40	5	1	NO	670	700	94.11	99.53
100	30	5	1	NO	670	700	95.28	99.69
100	20	5	1	NO	670	700	96.85	99.71
100	10	5	1	NO	670	700	97.20	99.91
100	40	5	1	YES	670	700	91.82	99.20
100	30	5	1	YES	670	700	94.25	99.41
100	20	5	1	YES	670	700	95.63	99.70
100	10	5	1	YES	670	700	97.01	99.90

Such an observation is in fact in accordance to the established inventory theory which shows that an inventory system with less variable demand values often gives rise to a higher vendor service level (and hence a higher customer service level) whilst holding other decision variables such as the lead-time, control parameters etc. constant.

The effect of allowing backorders in a reorder level system (with or without periodic reviews) appears to lower the service levels (see Figure 10.2a) This observation can be explained by the fact that an inventory system allowing backorders attempts to fulfil more demand orders than a system

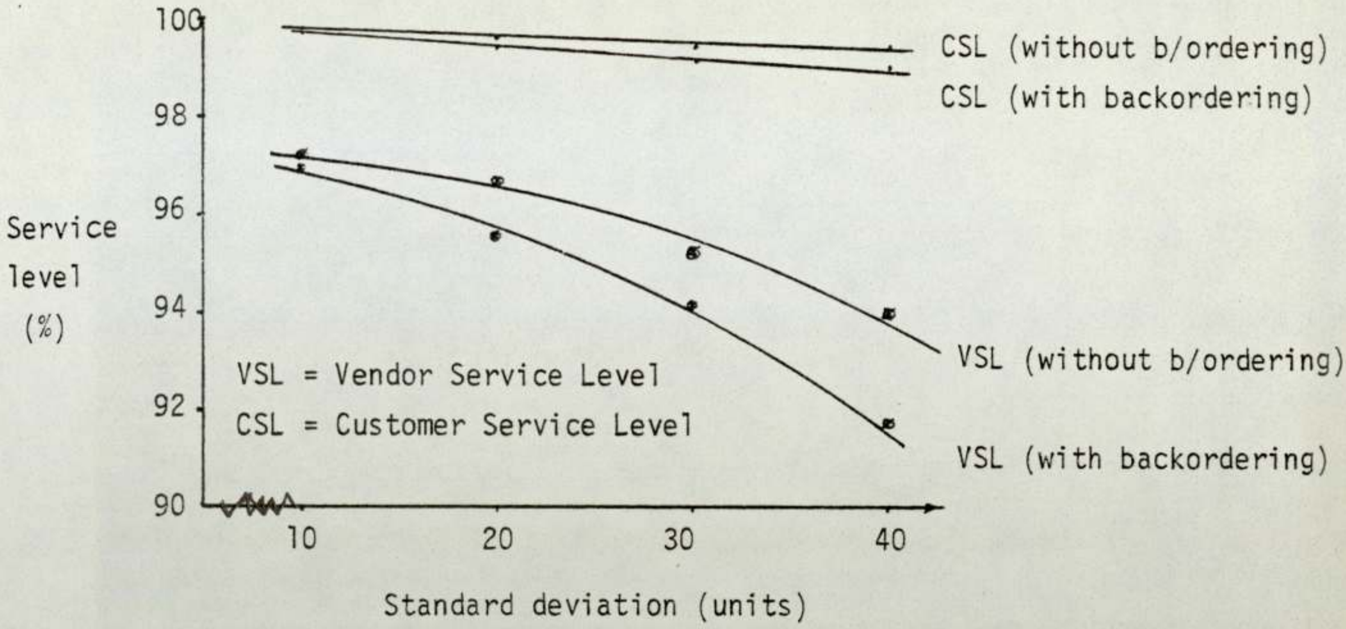


Figure 10.2a: Comparisons of service levels of the reorder level policy in a normal demand and normal lead-time situation.

with backordering prohibited, and if there is no compensatory increase in the fixed replenishment order quantity for a reorder level system allowing backorders, the probability of a stockout occurring in that system will be greater than that occurring in a system with backordering prohibited. Indeed, it was found that when the reorder level of a reorder level system allowing backorders was set at too low a level to cope with the expected demand orders, there would be a gradual depletion of stock and the vendor service level would be very low.

In a time-based inventory system such as the reorder cycle or (s, S) policy, the replenishment order quantity is variable and is determined by the actual inventory situation together with other control parameters. Such a system may provide a certain amount of compensatory increase in the replenishment order quantity if backordering is allowed. It has been found that the effect of allowing backorders to a time-based inventory system can cause certain changes in the service levels. So far the experimental results have shown that the nature of change in the service levels due to backordering could be positive, negative or even zero (ie. negligible). An improvement in the service levels could be explained by the fact that an increase of the total replenishment order quantity due to the effect of backordering was more than the amount just to overcome the backorder quantity. On the other hand, a deterioration in the service levels could be caused by having the backorder quantity able to outstrip the increase of the total replenishment order quantity. Finally, if an increase of the total replenishment order quantity due to the effect of backordering was able to compensate the backorder quantity, the net effect on service levels could be negligible in that inventory system.

10.3.5 Conclusion

The following conclusions are drawn from the experimental results obtained via interactive simulation using GIPSI:-

(i) The customer service level (defined as the proportion of annual demand met ex-stock) is greater than or at least equal to the vendor service level which is defined as the probability of not running out of stock subsequent to a replenishment order being placed.

(ii) The effect of allowing backorders in a reorder level system (with or without periodic reviews) appears to lower both the vendor service level and the customer service level.

(iii) The effect of allowing backorders in a reorder cycle or a (s, S) policy may induce certain changes in the service levels. However, the nature of change in the service levels may finally depend on the control parameters and the factor of allowing backorders.

10.4 Effect of inflation on reorder level policy

10.4.1 Introduction

In a reorder level policy, values of the control parameters, ie. reorder level and replenishment order quantity, can be adjusted to achieve an optimal operating condition. Without considering the possible effect of inflation and cost of stockout, a simple EOQ model can be used as a theoretical basis to evaluate an optimum replenishment order size. With inflation, however, Buzacott[15] has shown that the EOQ has to be increased by an inflationary factor of $\frac{1}{\sqrt{1-\frac{r}{i}}}$ to achieve an optimal operating condition. The values of r and i refer to the rate of inflation applied uniformly to all cost factors and the inventory holding rate respectively. As an experimental exercise it was felt that it could be interesting to investigate the following areas related to the operation of reorder level policy subject to inflation via interactive simulation using GIPSI:-

(a) The effect of inflation on the net revenue and inventory operating cost of the inventory system;

(b) The characteristics of an optimal reorder level policy subject to different rates of inflation at the minimum-cost condition;

(c) The effectiveness of the inventory system using the replenishment order quantity derived from Buzacott's EOQ model with inflation.

10.4.2 Data used for experimentation

The following data were used to investigate the possible effects of inflation on the reorder level policy using GIPSI:-

(a) Demand information

Mean = 50 units per week (Gamma distributed)
Standard deviation = 15 units per week

(b) Lead-time information

Fixed lead-time = 3 weeks

(c) Cost information

Selling price = £2 per unit
Cost price = £1.5 per unit
Purchase cost = £1 per unit
Ordering cost = £2 per order
Cost of stockout = £10 per stockout occasion
Inventory holding rate, i = 24%
Inflation rates (@ 12% and 20%) assumed to spread uniformly throughout the year and apply uniformly to all cost factors.

(d) Option of backordering

Backordering prohibited.

10.4.3 Outline of experimentation using GIPSI

The following stages of experimentation were involved:-

(a) Determine the optimum values of reorder level and replenishment order quantity for zero inflation at a minimum inventory operating cost condition.

(b) Increase the optimum replenishment order size by $1/\sqrt{1 - \frac{r}{i}}$, and use this increased order quantity to evaluate the performance of the inventory system for different values of the reorder level subject to 0%, 12% and 20% inflation.

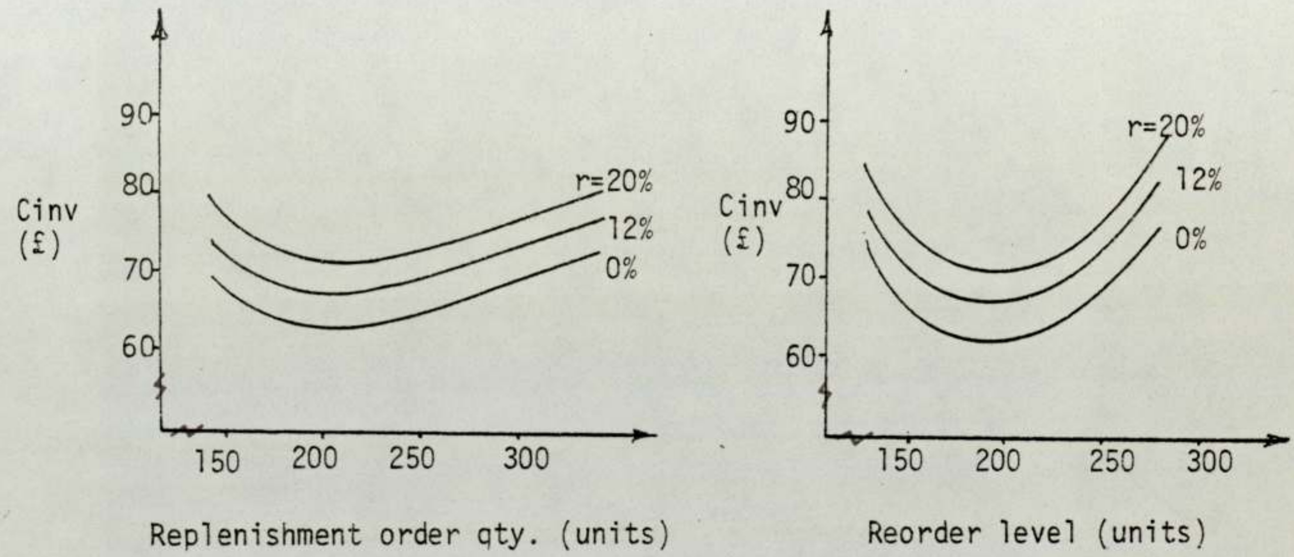
(c) Determine the optimum reorder level for each inflation rate specified in (b). Again using each of these optimum reorder levels, evaluate the performance of the inventory system for different values of the replenishment order quantity subject to 0%, 12% and 20% inflation.

It is noted that the above experimentation using GIPSI involved a series of iterative processes in determining the optimum values of the control parameters subject to 0%, 12% and 20% inflation.

10.4.4 Experimental results & observations

Detailed experimental results are contained in Appendix D. It is observed that the approximate optimum values of reorder level and replenishment order quantity are found to be 190 and 200 units respectively. (Note that the theoretical reorder level at 95% vendor service level and the EOQ are separately estimated as 195 and 204 units respectively).

It is also observed that the annual inventory operating cost (holding, ordering and stockout costs) and net revenue are generally raised as a result of inflation being applied uniformly to all cost factors. Both the optimum values of reorder level and replenishment order quantity at the minimum-cost condition are shown not to be affected by inflation (see Figure 10.3).



(a) Holding optimum reorder level at 190 units and varying replenishment order quantity for different inflation rates

(b) Holding optimum replenishment order quantity at 200 units and varying reorder level for different inflation rates

Figure 10.3 : Optimal characteristics of reorder level policy subject to different rates of inflation

Note: r = rate of inflation

Cinv = annual inventory operating cost

The simulated results at this optimal condition giving minimum inventory operating costs for different rates of inflation are shown in Table 10.3. It is noted that the simulated results of inventory holding cost, ordering cost and cost of stockout are obtained from the relevant graphs in Appendix D.

Table 10.3 : Simulated results of reorder level policy subject to different rates of inflation at minimum-cost conditions

Inflation rate (%)	Reorder level (units)	Replt. order qty. (units)	Inventory holding cost (£)	Ordering cost (£)	Stockout cost (£)	Annual inventory operating cost (£)	Net revenue (£)
0	190	200	33.7	24.9	2.9	61.5	2426
12	190	200	35.7	26.4	3.5	65.6	2582
20	190	200	37.1	27.5	4.5	69.1	2703

The optimum replenishment order quantity has been found experimentally to be 200 units which is close to the theoretical EOQ of 204 units. Treating this experimental order size as "EOQ", two additional replenishment order quantities were estimated using the Buzacott's EOQ model assuming 12% and 20% inflation being applied uniformly to all cost factors. The estimates were found to be 283 and 490 units, and were used for further simulation work using GIPSI whilst holding the optimum reorder level constant. The simulated results based on the above values are shown in Table 10.4.

Table 10.4 : Simulated results of reorder level policy using replenishment order quantity derived from Buzacott's EOQ model with inflation

Inflation rate (%)	Reorder level (units)	Replt. order qty. (units)	Inventory holding cost (£)	Ordering cost (£)	Stockout cost (£)	Annual inventory operating cost (£)	Net revenue (£)
0	190	200	33.7	24.9	2.9	61.5	2426
12	190	283	46.3	18.7	2	67.0	2594
20	190	490	75.8	11.2	3	90.0	2722

By comparing the results shown in Table 10.3 and 10.4, it is observed that optimum values of the inventory parameters (ie. reorder level and replenishment order quantity) at minimum-cost conditions are not affected by inflation. A bigger replenishment order size derived from Buzacott's EOQ model with inflation has produced a higher inventory operating cost. However, this finding does not invalidate the Buzacott's EOQ model with inflation since the Buzacott's model assumes the criterion of maximizing net revenue rather than minimizing the inventory holding operating cost under an inflationary situation. In fact, the simulated results based on the replenishment order quantity derived from Buzacott's EOQ model have produced higher net revenues than the corresponding values at minimum-cost conditions. Thus, the Buzacott's EOQ model with inflation should be used in an inflationary situation in which the net revenue of the inventory system is to be optimized. In the case where a minimum inventory operating cost is sought, a simple EOQ model (or other relevant models) is more appropriate to use in an inflationary situation.

10.4.5 Conclusion

The following conclusions are drawn from the experimental results obtained via interactive simulation using GIPSI:-

(a) The annual inventory operating cost and net revenue are generally raised as a result of inflation being applied uniformly to all cost factors.

(b) The optimum values of reorder level and replenishment order quantity at minimum-cost conditions are not affected by inflation.

(c) The Buzacott's EOQ model with inflation usually produces a bigger replenishment order quantity for the reorder level policy which often leads to a higher net revenue and a higher inventory operating cost than the corresponding values at minimum-cost conditions.

CHAPTER ELEVEN

USERS' EXPERIENCE AND PROPOSED FUTURE DEVELOPMENTS

11.1 Users' Experience

The first version of GIPSI was made available for trial to a group of postgraduate students specializing in the use of interactive packages for problem solving in March 1978. So far the response has been favourable and the package has demonstrated the following useful features:-

(a) As a teaching aid for postgraduate students specializing in inventory control.

(b) As a tool for analysing certain stock control situations encountered in industry via interactive simulation. An example of such industrial application is illustrated in the case study of Tasek Cement Ltd. in Chapter 9.

(c) As a research program for studying certain characteristics of inventory policies. This is illustrated in Chapter 10.

However, there are useful comments made to improve the existing package. Some of these are listed as follows:-

(i) For demonstration purposes, reduce the convergence criterion from 5% to larger values, say, 20% to reduce the simulation time.

(ii) Allow an input of zero values of selling price and cost price for the user to opt for optimizing the inventory operating cost plus the purchase cost.

(iii) Include a prepared data file of successive demand per unit time values as stored by $FINPUT.

(iv) Further explanation is required to input the cost of back-ordering per stockout occasion such that it can be realistically entered.

Subsequent to the comments raised during the trial period between March and May 1978, the above recommendations are gradually incorporated into the package.

11.2 Proposed future developments

Future developments currently under consideration are:-

(i) The optimization of annual inventory operating cost;

(ii) An analysis of demand during a lead-time including goodness of fit testing;

(iii) Incorporation of facilities to allow cross-correlation of demand and lead-time;

(iv) Price break structuring of replenishments;

(v) Alternative stockout cost formulation.

A brief discussion regarding the proposed developments of the above topics are summarized as follows:-

(a) Optimization of inventory operating cost

The first version of GIPSI allows a user to opt for maximizing the net revenue of a particular inventory policy chosen with all cost factors previosuly entered. It is felt that an additional option in minimizing the annual inventory operating cost which is more related to the orthodox inventory control theory should be included in the package.

(b) Analysis of demand during a lead-time

Although the demand per unit time distribution is used for inventory policy simulation in this package, it is common to find that the demand during a lead-time distribution is extensively used in the inventory control theory. Thus it is interesting to include an option in analysing the characteristics of the simulated results of the demand during a lead-time distribution including goodness of fit testing.

(c) Correlation of demand and lead-time

Inventory policy simulation in the first version of GIPSI assumes independence of demand per unit time and lead-time durations. In practice, the demand per unit time and the length of lead-time may not be independent of each other. This is particularly true both in a situation of high demand in a trade which is often typified by longer than usual lead-times due to the general increase in market activity, and in a low demand situation which often produces shorter than usual lead-times. When such conditions do occur there is obviously a strong correlation (ie. statistical dependence) between demand per unit time and lead-time durations.

Thus, future work should take into account the effect of correlation between demand per unit time and lead-time durations in evaluating the effectiveness of a particular inventory system via interactive simulation.

(d) Price breaks

The first version of GIPSI only allows inputs of net purchase price without consideration of possible price reduction through bulk purchase. In a situation where a price reduction per item is offered by a supplier for purchases over and above a certain quantity, the effect of sudden price breaks in the material costs of stock items (ie. purchase cost or works prime cost) must be taken into account. Thus, it is recommended that future work on GIPSI should include the analysis of price breaks reduction through bulk purchase and its effect on a particular inventory system via interactive simulation.

(e) Alternative stockout cost formulation

The current package assumes the following method in evaluating the stockout cost:-

(i) Backordering prohibited

Stockout cost = Loss of potential sales + Internal expenditure incurred.

(ii) Backordering permitted

Stockout cost = Internal expenditure incurred + Backordering cost.

Cost of backordering is the assumed internal cost (or fixed penalty cost) per stockout occasion when backordering is allowed in the inventory system. Thus, it is assumed that backordering cost is independent of the quantity being backordered when a stockout occurs.

In general, costs of stockout can be evaluated on several bases such as:-

(i) Cost per stockout occurrence;
(ii) Cost per unit time of stockout;
(iii) Cost per stocked unit out of stock per unit time.

Thus, it is suggested that an alternative stockout cost model can be formulated to allow users to opt for more appropriate methods in evaluating stockout costs.

CHAPTER TWELVE

CONCLUSION

12. Conclusion

The interactive inventory simulation model was developed in BASIC language in early 1978 by the author under the supervision of Professor C.D. Lewis. It is given the coded name, GIPSI - a General-Purpose Inventory Policy Simulation Package, originally designed to be used on a Hewlett-Packard Access 2000 machine. The package occupies about 600 blocks or .3 M-bytes of storage.

GIPSI allows the user to simulate four commonly used, single-item inventory policies under varying demand and lead-time situations to produce various measures of effectiveness. The four inventory policies offered are: reorder level policy, reorder cycle policy, reorder level policy subject to periodic review and (s, S) policy.

Additional facilities are incorporated into the package to enhance greater flexibility and usefulness of GIPSI. These include :-

(a) An analysis of input data including goodness of fit test for both demand and lead-time data;

(b) A sample display of simulation results;

(c) An automatic optimization procedure in locating the optimal or near-optimal net revenue for a particular inventory system.

In addition to the above facilities, GIPSI has been programmed in simple language and hence, simulation can be interactively carried out by users with little or no computer background. Thus, interactive simulation using GIPSI encourages the user to communicate with the machine and thereby upgrade the utility of simulation. Such interaction also allows the researcher to play an active role in simulation as it progresses.

Following its development, GIPSI was made available for testing in March 1978 by a group of postgraduate students. So far the response has been favourable and GIPSI has demonstrated its potential to become gradually a computerized teaching program for students studying inventory control via interactive simulation.

In preparing a case study of Tasek Cement Ltd., the collection of actual industrial data was carried out and analysis of the inventory situation was done via interactive simulation using GIPSI. The simulated results can be used as a general guidance by the management in adjusting the existing inventory situation to attain an optimal operating condition. Thus, using the Tasek Cement case as a typical example for firms having similar inventory problems, GIPSI can be effectively used as a practical tool in guiding the management managing and planning inventory control through interactive simulation.

Finally, GIPSI has been used as a research program for studying certain characteristics of inventory policies. Several areas related to the operation of certain inventory policies were examined via interactive simulation, and the results are summarized as follows:-

(i) <u>Characteristics of overshoot of the reorder level policy</u>

(a) The simulated mean overshoot can be accepted as a good estimate for the theoretical mean overshoot.

(b) The truncated shape of a typical overshoot distribution does not provide a reasonable fit to any of the commonly used probability distributions such as the Normal, Gamma, Uniform, Poisson, Lognormal or Negative Exponential distributions.

(ii) Comparison of service levels

(a) The customer service level (defined as the proportion of annual demand met ex-stock) is greater than or at least equal to the vendor service level which is defined as the probability of not running out of stock subsequent to a replenishment order being placed.

(b) The effect of allowing backorders in a reorder level system (with or without periodic reviews) appears to lower both the vendor service level and the customer service level.

(c) The effect of allowing backorders in a reorder cycle or a (s, S) policy may induce certain changes in the service levels. However, the nature of change in the service levels may finally depend on the control parameters and the factor of allowing backorders.

(iii) Effect of inflation on reorder level policy

(a) The annual inventory operating cost and net revenue are generally raised as a result of inflation being applied uniformly to all cost factors.

(b) The optimum values of reorder level and replenishment order quantity at minimum-cost conditions are not affected by inflation.

(c) The Buzacott's EOQ model with inflation usually produces a bigger replenishment order quantity for the reorder level policy which often leads to a higher net revenue and a higher inventory operating cost than the corresponding values at minimum-cost conditions.

Summarizing, it is observed that GIPSI has been particularly useful in the following areas:-

(a) As a teaching aid for students specializing in inventory management;

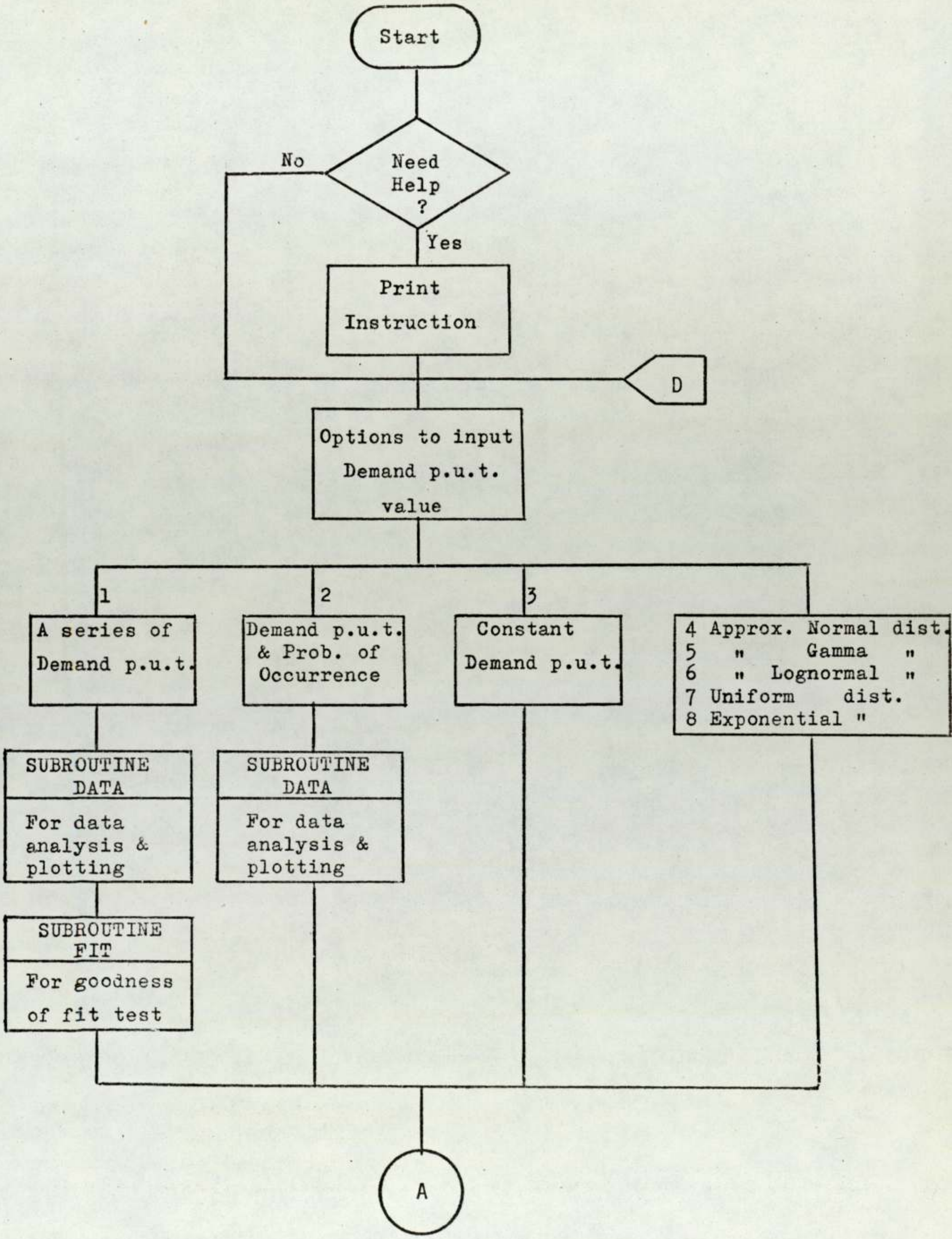
(b) As a tool for analysing certain stock situations encountered in industry;

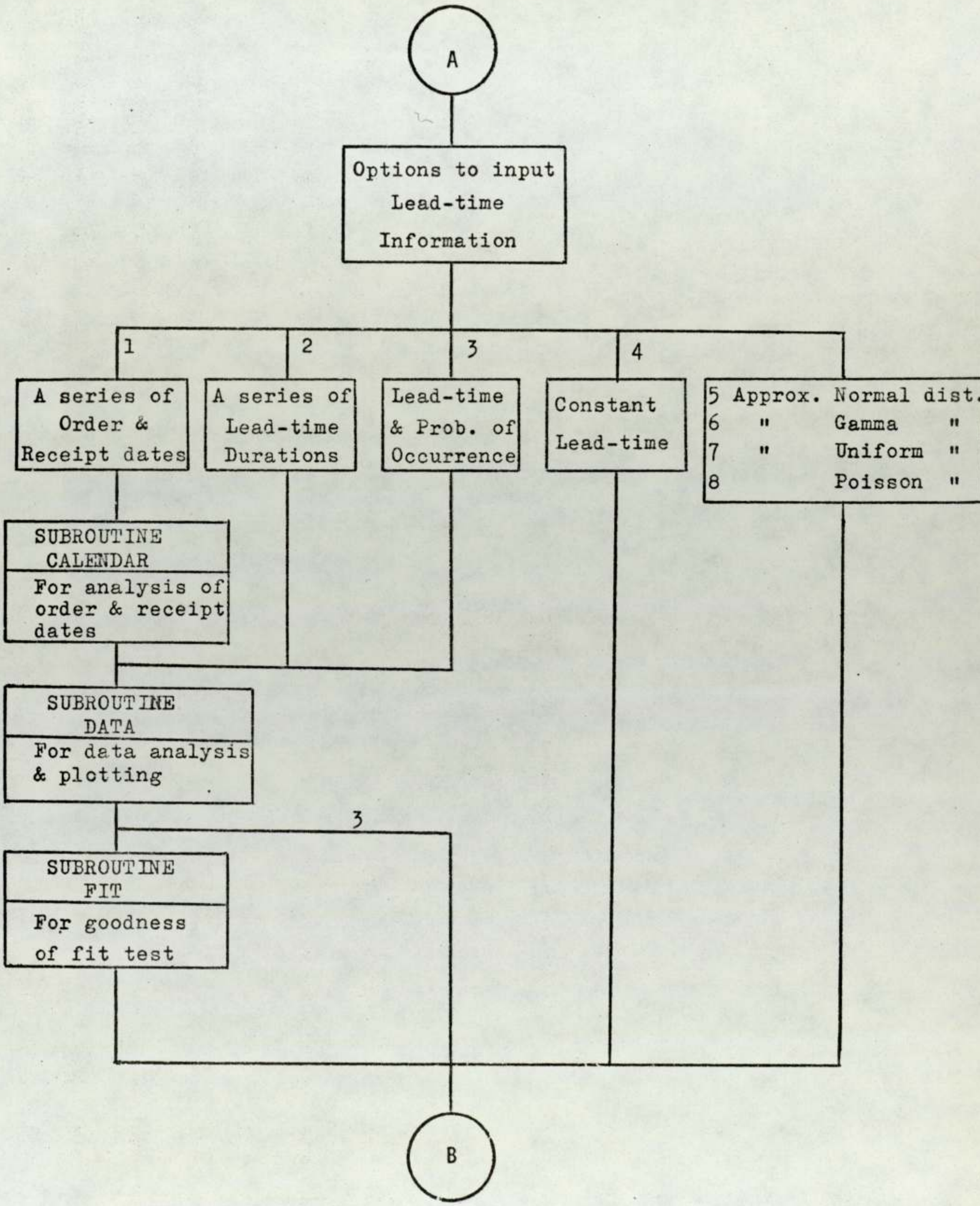
(c) As a research program for studying certain characteristics of inventory policies.

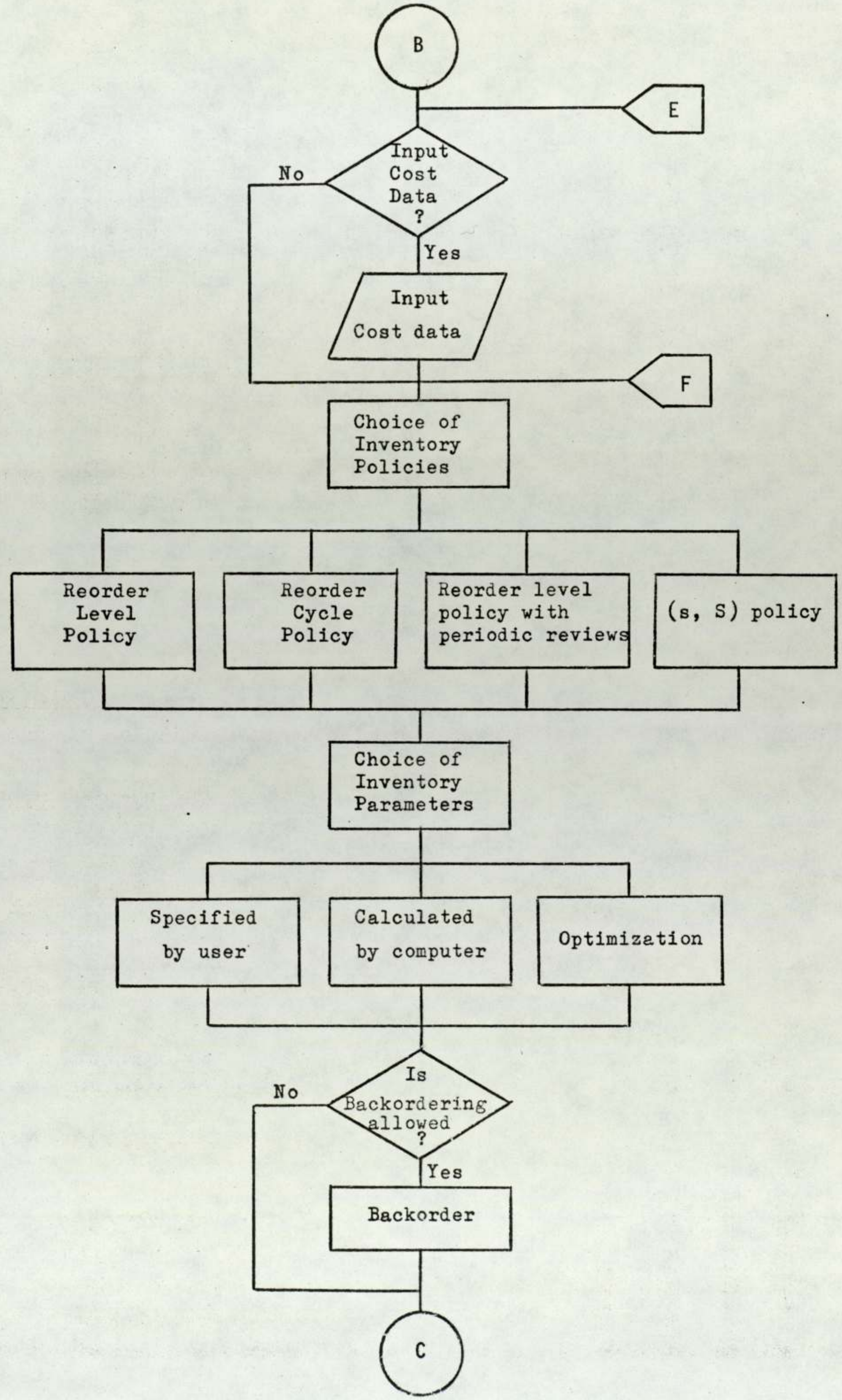
Finally, it is noted that although GIPSI has so far been successfully developed and tested with satisfaction, there is still room for future developments. It is, therefore, aimed to develop GIPSI into one of the commonly used software packages in inventory control both for educational as well as for industrial applications.

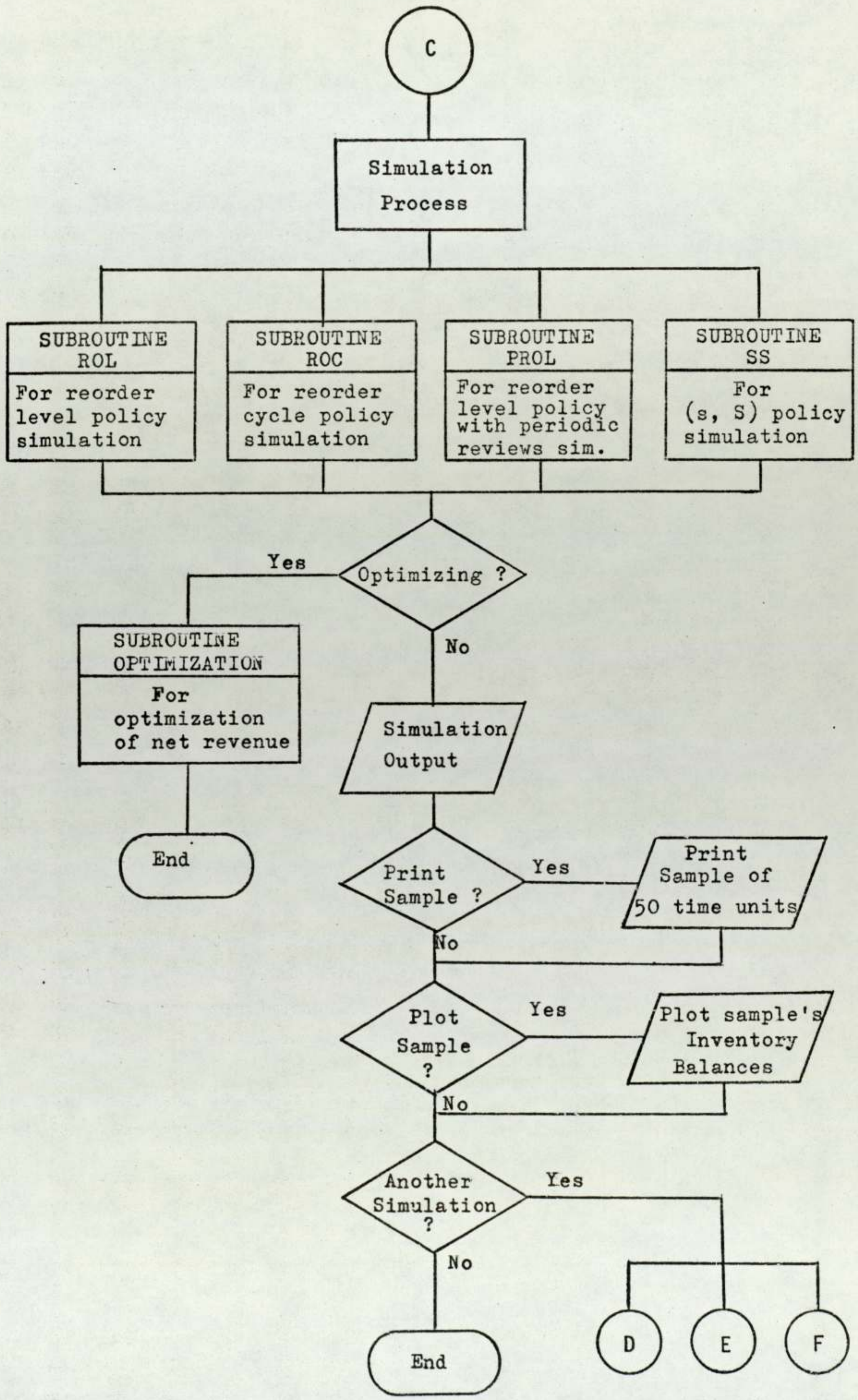
APPENDIX A

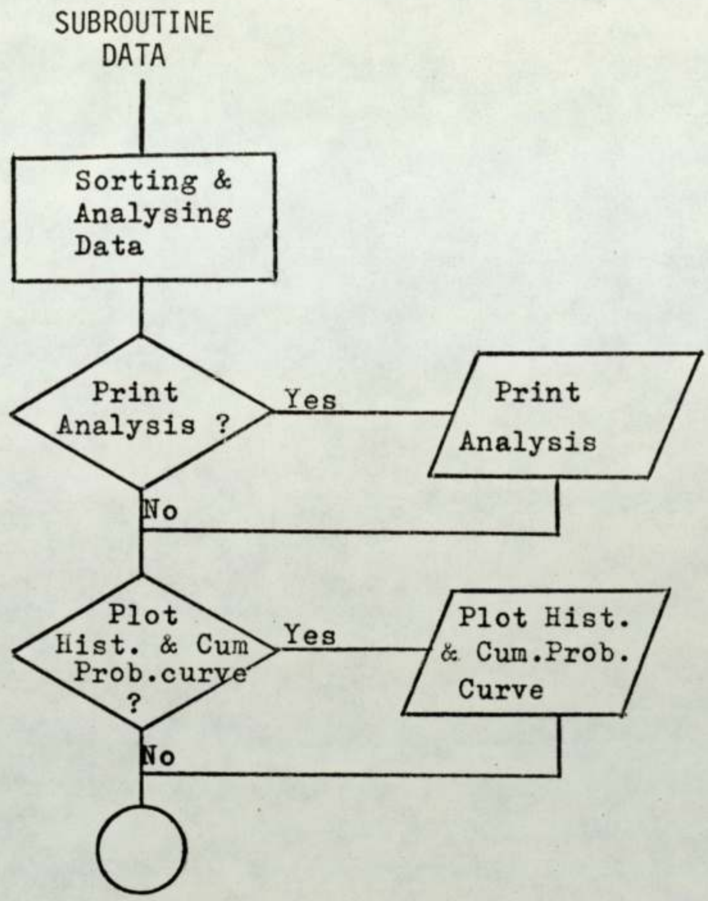
FLOW-DIAGRAMS OF GIPSI

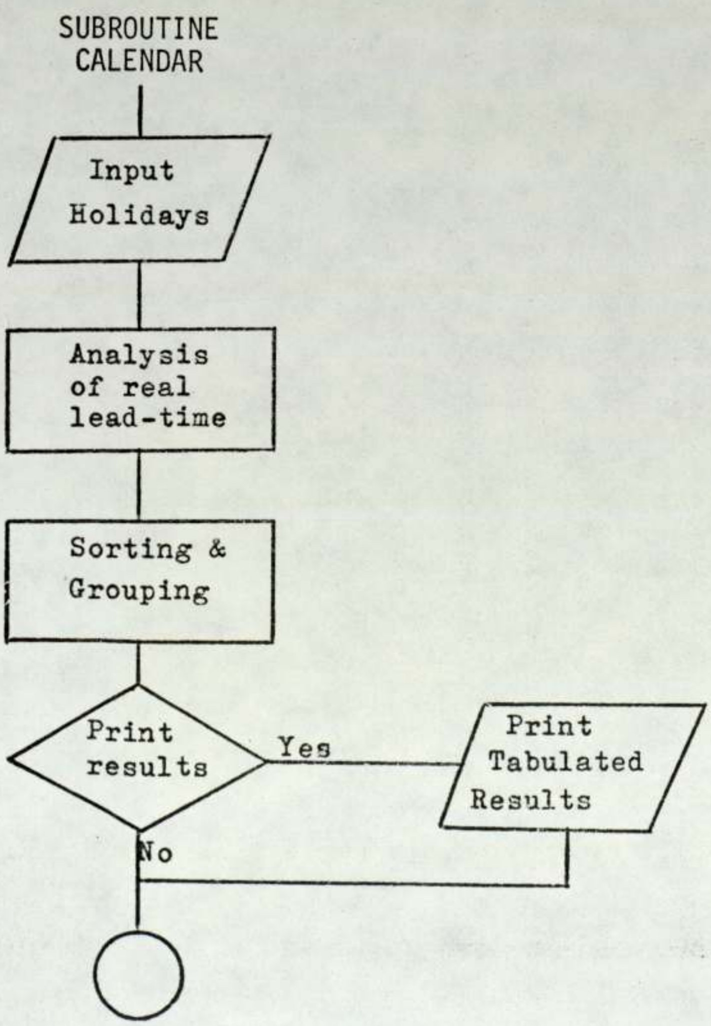

Start
Need Help ?
No
Yes
Print Instruction
D
Options to input Demand p.u.t. value
1
2
3
A series of Demand p.u.t.
Demand p.u.t. & Prob. of Occurrence
Constant Demand p.u.t.
4 Approx. Normal dist.
5 " Gamma "
6 " Lognormal "
7 Uniform dist.
8 Exponential "
SUBROUTINE DATA
For data analysis & plotting
SUBROUTINE DATA
For data analysis & plotting
SUBROUTINE FIT
For goodness of fit test
A

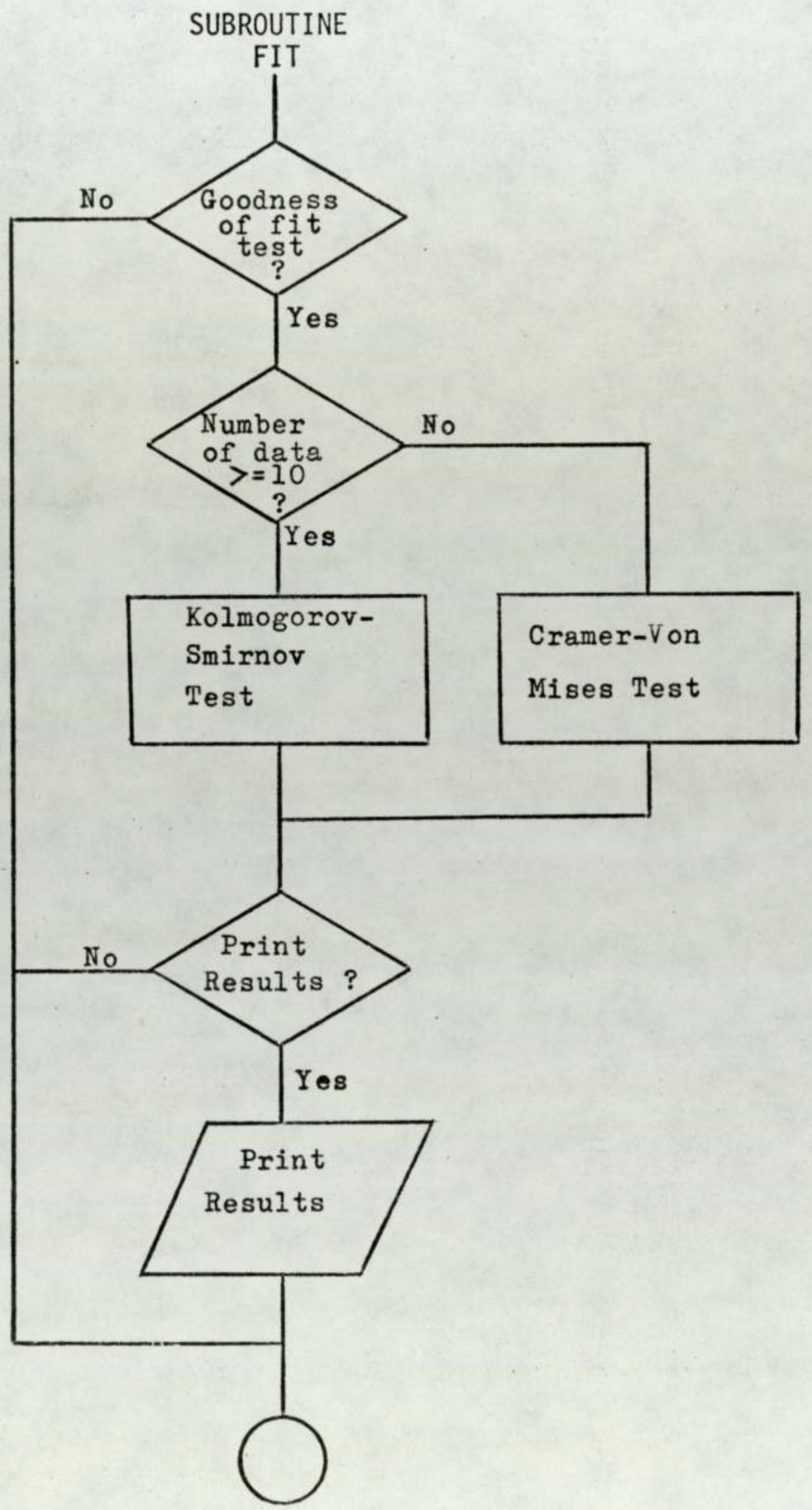

A
Options to input Lead-time Information
1
2
3
4
A series of Order & Receipt dates
A series of Lead-time Durations
Lead-time & Prob. of Occurrence
Constant Lead-time
5 Approx. Normal dist.
6 " Gamma "
7 " Uniform "
8 Poisson "
SUBROUTINE CALENDAR
For analysis of order & receipt dates
SUBROUTINE DATA
For data analysis & plotting
3
SUBROUTINE FIT
For goodness of fit test
B

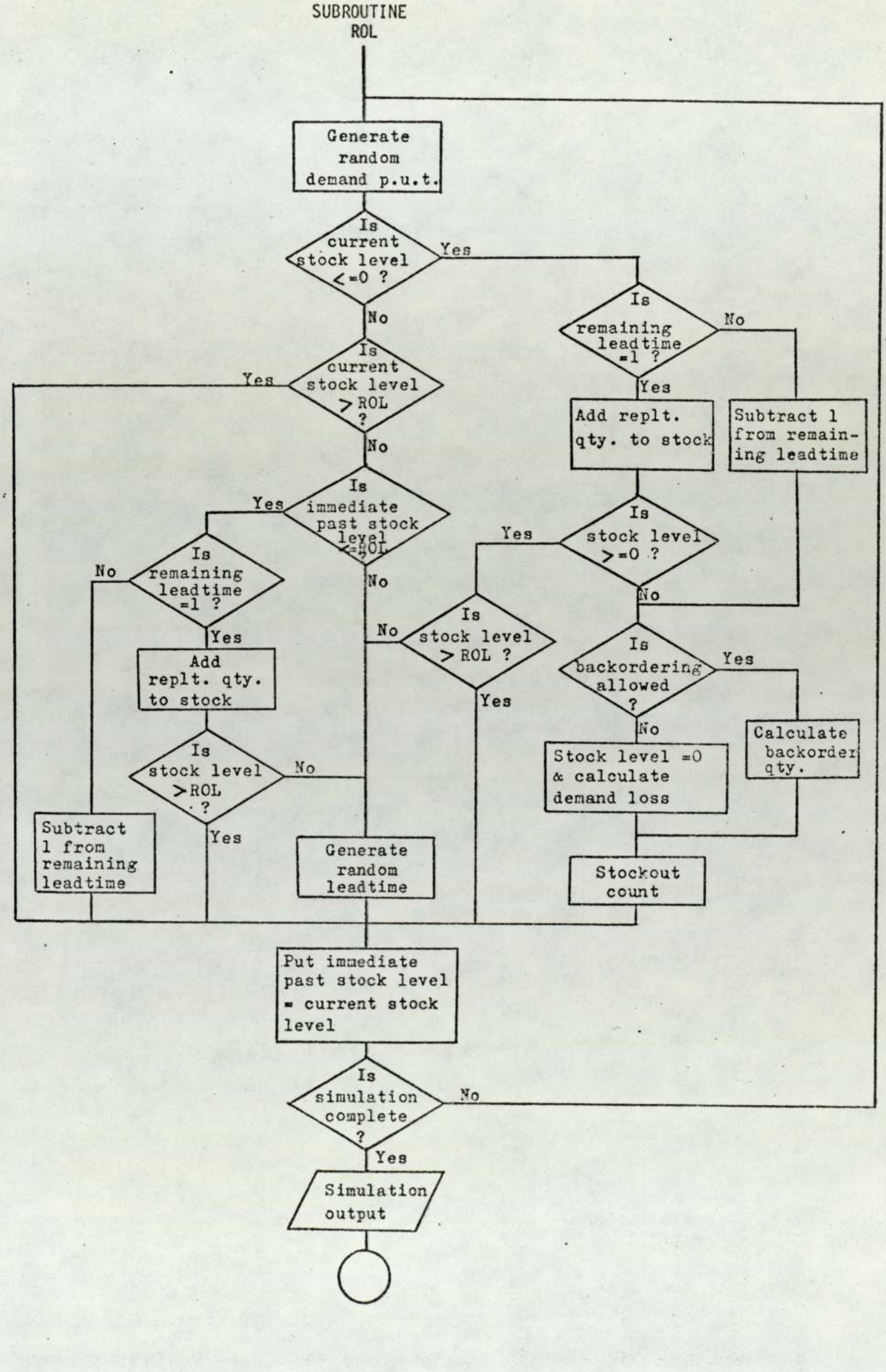

B
E
Input
Cost
Data
?
No
Yes
Input
Cost data
F
Choice of
Inventory
Policies
Reorder
Level
Policy
Reorder
Cycle
Policy
Reorder level
policy with
periodic reviews
(s, S) policy
Choice of
Inventory
Parameters
Specified
by user
Calculated
by computer
Optimization
Is
Backordering
allowed
?
No
Yes
Backorder
C

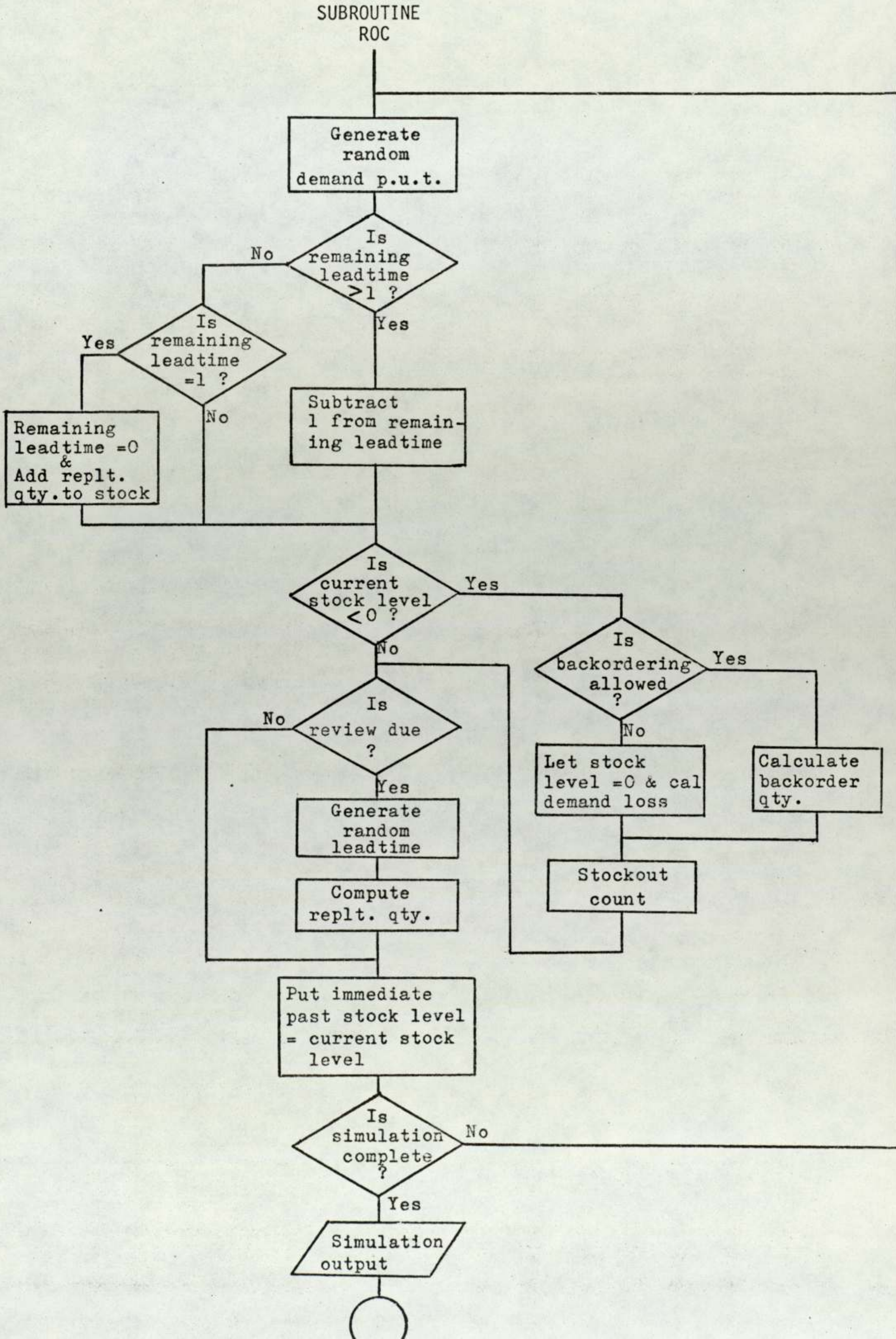

C
Simulation Process
SUBROUTINE ROL
For reorder level policy simulation
SUBROUTINE ROC
For reorder cycle policy simulation
SUBROUTINE PROL
For reorder level policy with periodic reviews sim.
SUBROUTINE SS
For (s, S) policy simulation
Optimizing ?
Yes
No
SUBROUTINE OPTIMIZATION
For optimization of net revenue
End
Simulation Output
Print Sample ?
Yes
Print Sample of 50 time units
No
Plot Sample ?
Yes
Plot sample's Inventory Balances
No
Another Simulation ?
Yes
No
End
D
E
F

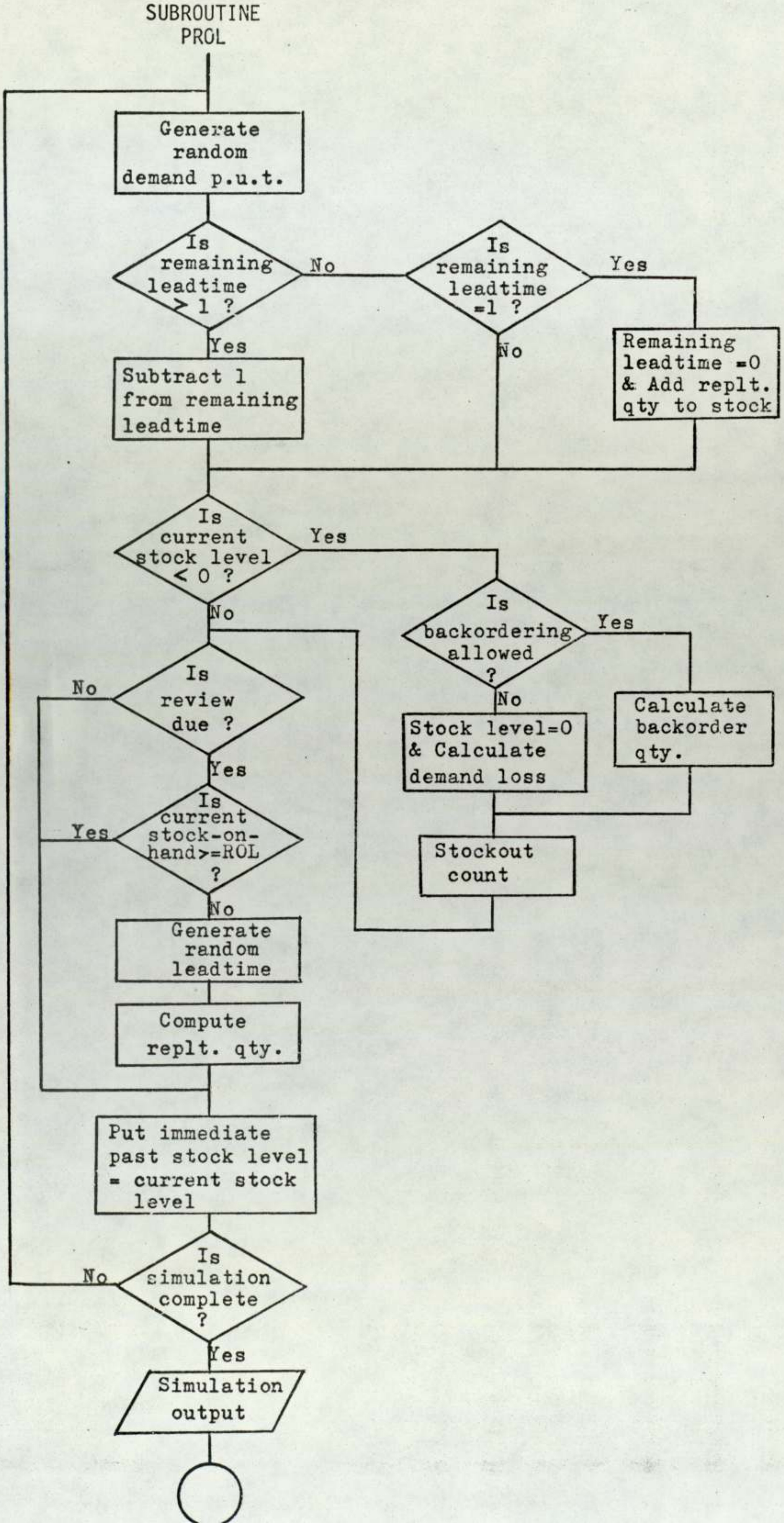

SUBROUTINE
DATA
Sorting & Analysing Data
Print Analysis ?
Yes
Print Analysis
No
Plot Hist. & Cum Prob.curve ?
Yes
Plot Hist. & Cum.Prob. Curve
No

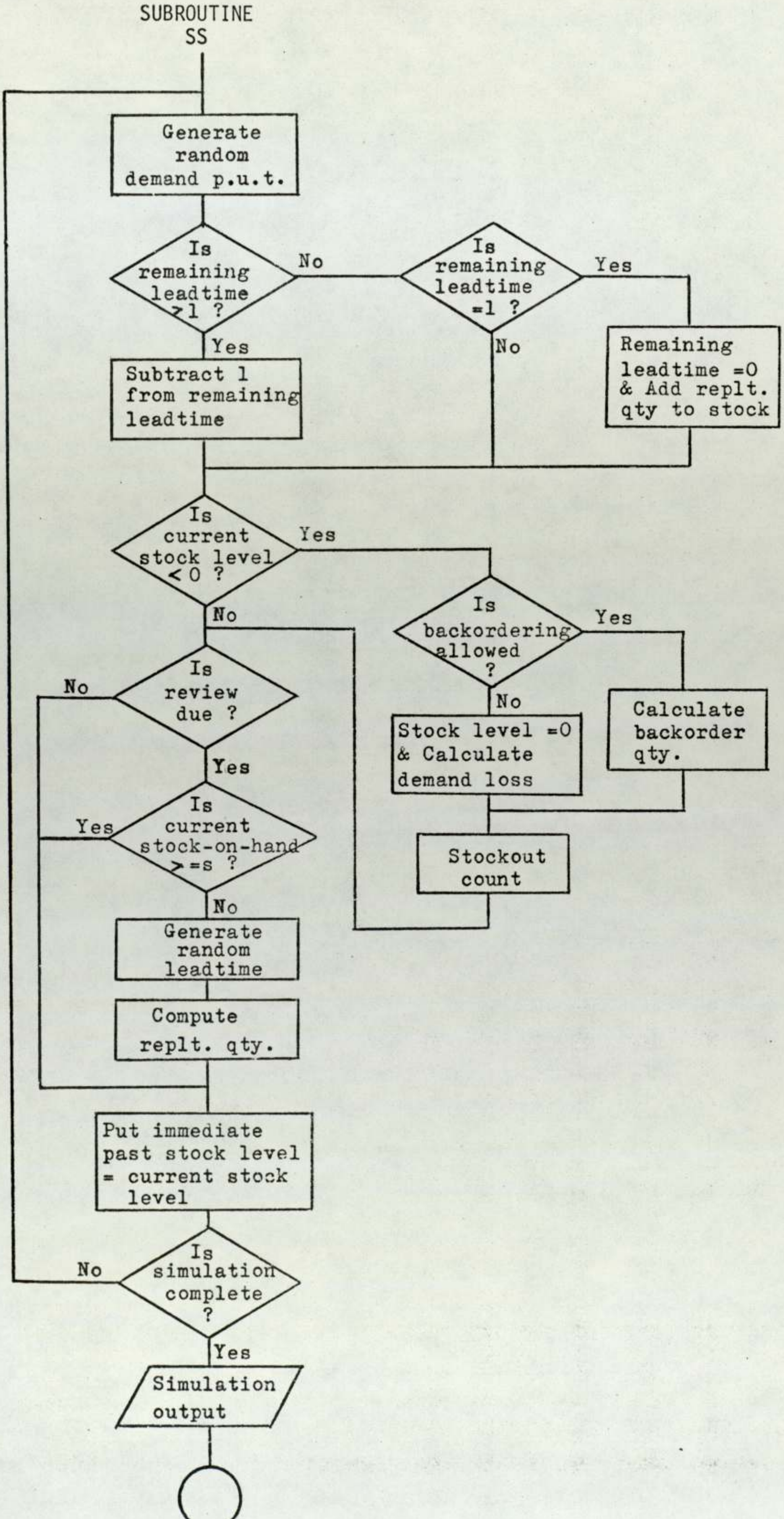

SUBROUTINE
CALENDAR
Input Holidays
Analysis of real lead-time
Sorting & Grouping
Print results
Yes
Print Tabulated Results
No

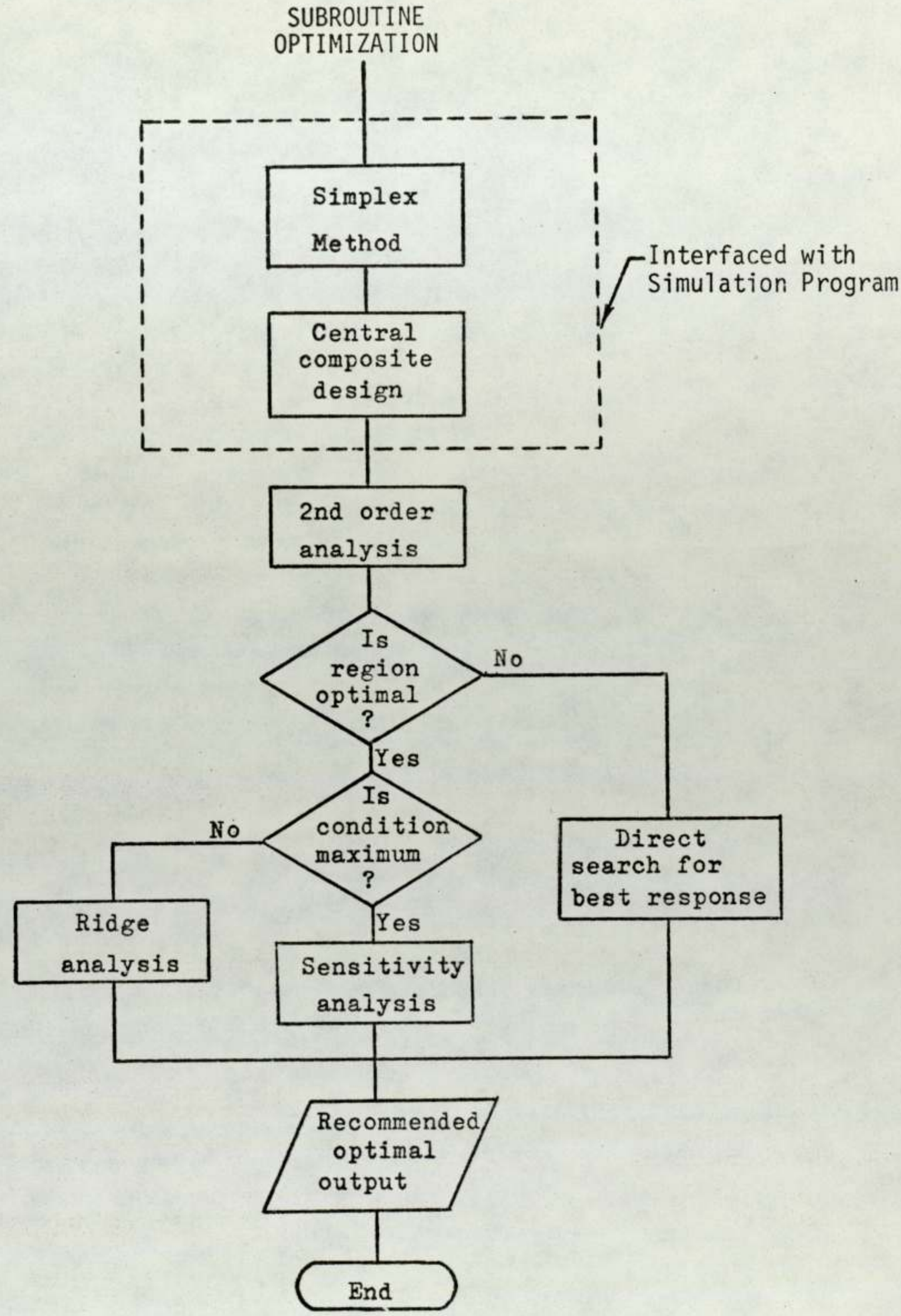

SUBROUTINE FIT
Goodness of fit test ?
No
Yes
Number of data >=10 ?
No
Yes
Kolmogorov-Smirnov Test
Cramer-Von Mises Test
Print Results ?
No
Yes
Print Results


SUBROUTINE ROL
Generate random demand p.u.t.
Is current stock level <=0 ?
Yes
No
Is remaining leadtime =1 ?
No
Yes
Is current stock level > ROL ?
Yes
No
Add replt. qty. to stock
Subtract 1 from remaining leadtime
Is immediate past stock level <=ROL
Yes
No
Is stock level >=0 ?
Yes
No
Is remaining leadtime =1 ?
No
Yes
Is stock level > ROL ?
No
Yes
Add replt. qty. to stock
Is backordering allowed ?
Yes
No
Is stock level > ROL ?
No
Yes
Calculate backorder qty.
Stock level =0 & calculate demand loss
Subtract 1 from remaining leadtime
Generate random leadtime
Stockout count
Put immediate past stock level = current stock level
Is simulation complete ?
No
Yes
Simulation output


SUBROUTINE ROC
Generate random demand p.u.t.
Is remaining leadtime >1 ?
No
Yes
Is remaining leadtime =1 ?
Yes
No
Remaining leadtime =0 & Add replt. qty.to stock
Subtract 1 from remaining leadtime
Is current stock level <0 ?
Yes
No
Is backordering allowed ?
Yes
No
Let stock level =0 & cal demand loss
Calculate backorder qty.
Stockout count
Is review due ?
No
Yes
Generate random leadtime
Compute replt. qty.
Put immediate past stock level = current stock level
Is simulation complete ?
No
Yes
Simulation output


SUBROUTINE PROL
Generate random demand p.u.t.
Is remaining leadtime >1 ?
No
Yes
Is remaining leadtime =1 ?
Yes
No
Subtract 1 from remaining leadtime
Remaining leadtime =0 & Add replt. qty to stock
Is current stock level < 0 ?
Yes
No
Is backordering allowed ?
Yes
No
Is review due ?
No
Yes
Stock level=0 & Calculate demand loss
Calculate backorder qty.
Is current stock-on-hand>=ROL ?
Yes
No
Stockout count
Generate random leadtime
Compute replt. qty.
Put immediate past stock level = current stock level
Is simulation complete ?
No
Yes
Simulation output


SUBROUTINE SS
Generate random demand p.u.t.
Is remaining leadtime >1 ?
No
Yes
Is remaining leadtime =1 ?
Yes
No
Subtract 1 from remaining leadtime
Remaining leadtime =0 & Add replt. qty to stock
Is current stock level <0 ?
Yes
No
Is backordering allowed ?
Yes
No
Stock level =0 & Calculate demand loss
Calculate backorder qty.
Stockout count
Is review due ?
No
Yes
Is current stock-on-hand >=s ?
Yes
No
Generate random leadtime
Compute replt. qty.
Put immediate past stock level = current stock level
Is simulation complete ?
No
Yes
Simulation output



Note: The following subroutines are used for the optimization of various inventory policies:-

OPTM - for Reorder Level Policy

OPTM2 - for Reorder Cycle Policy

OPTM3 - for Reorder Level Policy with periodic reviews and (s, S) Policy.

APPENDIX B

RANDOM VARIATE GENERATION

The following random variates are generated for the simulation process used in GIPSI:-

(a) Uniform variates

(b) Poisson variates

(c) Negative exponential variates

(d) Normal variates

(e) Lognormal variates

(f) Gamma variates

The symbols used in the subsequent generation of the above random variates refer to the following terms:-

$f(x)$ = Probability density function

$F(x)$ = Cumulative distribution function

u = Mean

σ = Standard deviation

σ^2 = Variance

R = Random number ($0 \leq R \leq 1$)

B.1 Generation of Uniform Variates

The uniform distribution is a continuous probability density function, which is constant over the interval from a to b and zero otherwise (see Figure B.1.1.

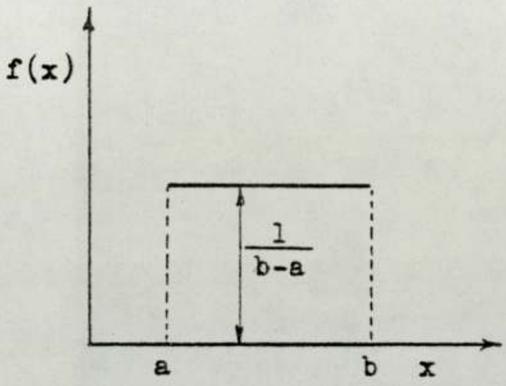


Figure B.1.1: Density function of a uniform distribution

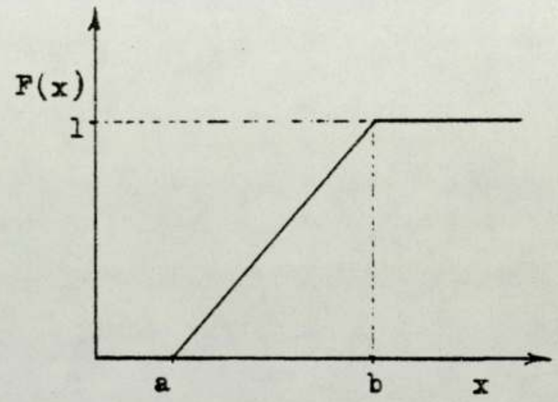


Figure B.1.2: Cumulative distribution function of a uniform distribution

Density: $f(x) = \frac{1}{b - a}$

Mean: $u = \frac{1}{2}(b + a)$

Variance: $\sigma^2 = (1/12)(b - a)^2$

The cumulative distribution function F(x) (see Figure B.1.2) for a uniformly distributed random variate x is derived as

$$F(x) = \frac{x - a}{b - a} \qquad 0 \leq F(x) \leq 1$$

To simulate a uniform distribution over the range from a to b, we use the inverse transformation of the density function:-

$$R = \frac{x - a}{b - a}$$

$$\text{or} \quad x = a + (b - a)R \qquad 0 \leq R \leq 1$$

The value of x is the uniform variate generated.

B.2 Generation of Poisson Variates

A Poisson distribution can be described by the following density function:-

Density: $f(x) = \dfrac{\lambda^x e^{-\lambda}}{x!}$

Mean: $u = \lambda$

Variance: $\sigma^2 = \lambda$

The shape of a Poisson density distribution is shown in Figure B.2.1.

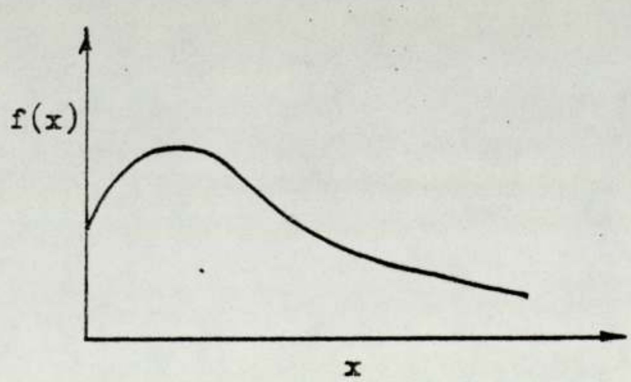


Figure B.2.1: Poisson distribution

It is noted that the Poisson distribution is a discrete distribution (ie. the variable can take on only integers including zero) with both mean and variance equal to λ. Lambda (λ) can have any positive value and need not be an integer.

The cumulative probability distribution, F(x) is shown to be

$$F(x) = e^{-\lambda} \sum_{0}^{x} \frac{\lambda^{x}}{x!}$$

One method of generating a Poisson random variate, x is based on the fact that the random number generated, R should be less than or equal to F(x) for all positive discrete values of x. Thus, we have the following relationship:-

$$R \leq e^{-\lambda} \sum_{0}^{x} \frac{\lambda^{x}}{x!} \qquad \text{for } x = 0, 1, 2, \ldots, x$$

This method is conveniently used to generate an antithetic variate, x' based on the previously generated random number, R using the following relationship:-

$$1 - R \leq e^{-\lambda} \sum_{0}^{x'} \frac{\lambda^{x'}}{x'} \qquad \text{for } x' = 0, 1, 2, \ldots, x'$$

A faster method of generating Poisson variates x is presented by Tocher[72] based on generating random numbers, Ri(0, 1) until the following relationship holds:-

$$\prod_{i=0}^{x} R_i \geq e^{-\lambda} > \prod_{i=0}^{x+1} R_i$$

It is noted that a Poisson variate generator is very inefficient for large values of mean (ie. λ). However, it can be shown that for λ to be greater than 10, a normal variate generator with both mean and variance equal to λ can be used to generate approximate Poisson variates.

B.3 Generation of Negative Exponential Variates

The density function f(x) and the cumulative distribution function F(x) of a negative exponential distribution are given as:-

$$f(x) = \lambda e^{-\lambda x} \quad \text{for } \lambda > 0 \text{ and } x \geq 0$$

$$F(x) = 1 - e^{-\lambda x}$$

The shapes of f(x) and F(x) of an exponential distribution are shown in Figure B.3.1 and B.3.2 respectively.

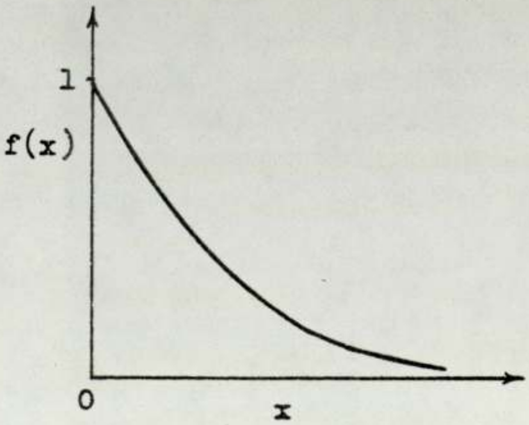


Figure B.3.1: Density function of an exponential distribution

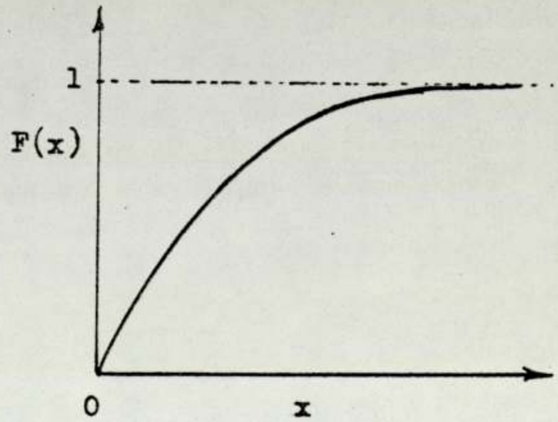


Figure B.3.2: Cumulative distribution function of an exponential distribution

The mean u and variance σ^2 of an exponential distribution can be derived as follows:-

$$u = \frac{1}{\lambda}$$

$$\sigma^2 = \frac{1}{\lambda^2}$$

Generation of exponential random variates, x can be accomplished by the inverse transformation technique. Thus we have

$$R = 1 - e^{-\lambda x}$$

where R is the random number (0, 1) generated. Furthermore, it is noted that R and (1 - R) are interchangeable because the random numbers generated are uniformly distributed. Hence, we have the following relationship:-

$$R = e^{-\lambda x}$$

$$\text{or } x = -\frac{1}{\lambda} \log R$$

B.4 Generation of Normal Variates

The normal distribution is a continuous distribution and is symmetrical about its mean value u (See Figure B.4.1). A number of different methods of generating normal variates have been reported. In general, all these methods utilize the transformation $z = (x - u)/\sigma$ to produce a standard normal distribution with mean equal 0 and standard deviation 1.

$$\text{Density:} \quad f(z) = \frac{1}{2\pi} e^{-\frac{z^2}{2}}$$

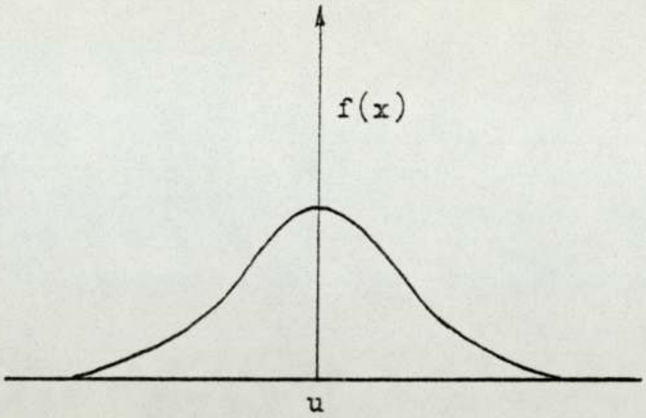


Figure B.4.1: Normal distribution

The normal variate generator used in GIPSI is based on the central limit theorem. A detailed discussion of this method is contained in Naylor et al[55] (Chapter 4). Using this method, we can calculate a randomly distributed normal variate Y with u = 0 and $\sigma^2 = 1$ by

$$Y = \sum_{i=1}^{12} R_i - 6$$

However, this particular generator has the following disadvantages:-

(i) It takes 12 random numbers to produce one normal variate Y;

(ii) This method does very poorly in generating the tails of a normal distribution.

In order to attain a higher accuracy, Teichroew's approximation technique[70] may be considered to improve the accuracy of tail probabilities obtained by the central limit approach. This modified approach is as follows:-

(a) Compute

$$y = \tfrac{1}{4}\left(\sum_{i=1}^{12} R_i - 6\right)$$

(b) Calculate the standard normal variate Z using the following polynomial:-

$$Z = a_1 y + a_3 y^3 + a_5 y^5 + a_7 y^7 + a_9 y^9$$

where $a_1 = 3.949846138$

$a_3 = 0.252408784$

$a_5 = 0.076542912$

$a_7 = 0.008355968$

$a_9 = 0.029899776$

(c) Calculate the normal deviate x as follows:-

$$x = u + Z\sigma$$

Other methods of generating normal variates include

(i) Box and Muller's inverse method[9]

(ii) Marsaglia and Bray's method[49].

B.5 <u>Generation of Lognormal Variates</u>

A random variate x is said to be lognormally distributed when the logarithm of that random variate (ie. logx) exhibits the characteristics of a normal distribution. The effect of logarithm transformation compresses the distribution at higher levels and stretching it at lower levels. Thus this type of transformation changes a positively skewed distribution into an approximately symmetrical distribution.

Consider the relationship $x = e^y$, if x is lognormally distributed, then y which is equal to log x will be a normal density function. Thus we have

$$f(y) = \frac{1}{\sigma_y\sqrt{2\pi}} \text{Exp}\left[- \frac{1}{2\,\sigma_y}(y - u_y) \right]$$

Where u_y and σ_y are the mean and standard deviation of y respectively.

The values of u_y and σ_y can be derived from the following formulae:-

$$u_y = \log(u_x) - \tfrac{1}{2}\,\sigma^2$$

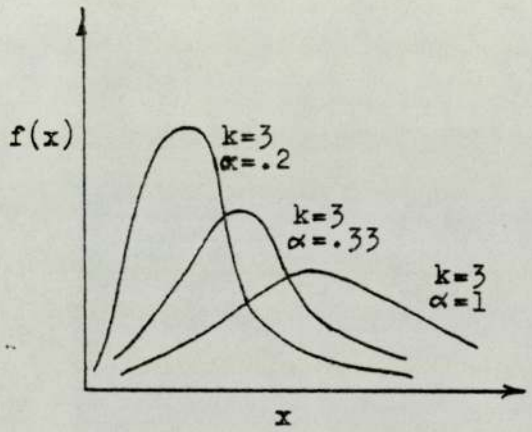
$$\sigma_y = \log\left[(\sigma_x/u_x)^2 + 1 \right]$$

Where u_x and σ_x are the mean and standard deviation of x respectively, and are normally given or estimated.

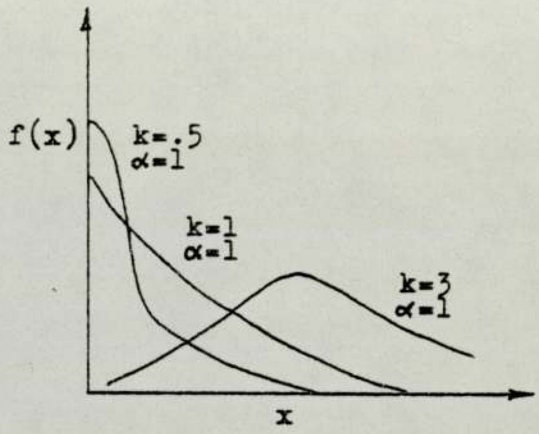
Hence, the function of f(y) can be transformed into a standard normal distribution from which a normal variate y is generated using method(s) discussed in Section B.4. Knowing the value of y and using the relationship $x = e^y$, the lognormal variate x can be obtained.

B.6 Generation of Gamma Variates

The gamma distribution is defined by two parameters k and α, where k is the modulus which determines the shape of the distribution and α is the scale parameter. As the two parameters are varied, the gamma function can assume a wide variety of shapes (See Figure B.6.1).



(a) Constant k



(b) Constant α

Figure B.6.1: Gamma distribution

The density function f(x) of a gamma distribution is given as:-

$$f(x) = \frac{\alpha^k x^{k-1} e^{-\alpha x}}{\Gamma(k)} \qquad \text{for } k>0, \alpha>0 \text{ and } x \geq 0$$

where $\Gamma(k)$ = complete gamma function

$$= \int_0^\infty \alpha^k x^{k-1} e^{-\alpha x} dx$$

The mean u and variance σ^2 of a gamma distribution can be derived as follows:-

$$u = k/\alpha$$

$$\sigma^2 = k/\alpha^2$$

Ramberg and Tadikamalla[61] have proposed an approximate two-parameter gamma generator by matching the gamma density function with a Weibull distribution. A brief outline of this method is as follows:-

(i) <u>Weibull distribution</u>

Density function:

$$f(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} \mathrm{Exp}\left(-\left(\frac{x-a}{b}\right)^{c}\right)$$

for $x > a$, $b > 0$ and $c > 0$

where a = location parameter

b = scale parameter

c = shape parameter

The cumulative distribution is found to be

$$F(x) = 1 - \mathrm{Exp}\left[-\left(\frac{x-a}{b}\right)^{c}\right]$$

The mean (u^*), the variance (σ^{*2}) and the standardized third moment (α_3^*) of x can be derived as follows:-

$$u^* = a + b\,\Gamma(1 + 1/c)$$

$$\sigma^{*2} = b^2\left[\Gamma(1 + 2/c) - \Gamma(1 + 1/c)^2\right]$$

$$\alpha_3^* = \frac{\Gamma(1 + 3/c) - 3\Gamma(1 + 2/c)\Gamma(1 + 1/c) + 2\Gamma(1 + 1/c)^3}{\left[\Gamma(1 + 2/c) - \Gamma(1 + 1/c)^2\right]^{3/2}}$$

Weibull variates x can be generated by a direct inverse transformation as

$$x = a + b\left[- \log R\right]^{1/c}$$

where R is the random number (0, 1) generated.

(ii) Gamma distribution

The mean (u), the variance (σ^2) and the third standardized moment (α_3) can be derived as

$$u = k/\alpha$$

$$\sigma^2 = K/\alpha^2$$

$$\alpha_3 = 2/\sqrt{K}$$

(iii) Method of approximation

Matching u, σ^2 and α_3 of the gamma distribution with u*, σ^{*2} and α_3^* of the Weibull distribution, the values of a, b and c can be determined. Hence, an approximate gamma variate, x can be generated based on the inverse transformation of a Weibull distribution.

Other methods of generating approximate gamma variates include:-

(i) Phillips' method[59]

(ii) Wheeler's Burr approximation[74].

APPENDIX C

SIMULATED RESULTS OF REORDER LEVEL POLICY

(A CASE STUDY OF TASEK CEMENT LTD.)

Table C.1: Simulated results of reorder level policy in a two-week lead-time situation with backordering prohibited

Reorder level (m-ton)	Replenishment batch qty. (m-ton)	Vendor service level (%)	Customer service level (%)	Annual inventory operating cost (M$x10^6)	Net revenue (M$x10^6)
	20,000	50.5	88.60	3.219	18.24
	30,000	92.5	99.05	0.689	23.32
	35,000	95.9	99.94	0.599	23.58
20,000	40,000	94.2	99.32	0.613	23.56
	45,000	92.9	99.35	0.588	23.51
	50,000	94.3	99.18	0.666	23.42
	60,000	95.4	99.37	0.659	23.59
	25,000	97.1	99.81	0.545	23.61
	30,000	97.2	99.84	0.515	23.69
	35,000	98.2	99.90	0.490	23.75
22,000	40,000	98.3	99.89	0.493	23.77
	45,000	98.4	99.89	0.490	23.80
	50,000	98.5	99.91	0.512	23.79
	60,000	98.9	99.95	0.539	23.84
	23,000	99.3	99.98	0.540	23.67
	25,000	99.1	99.96	0.525	23.74
	30,000	99.7	99.99	0.494	23.78
	35,000	99.6	99.98	0.482	23.84
23,000	40,000	99.7	99.99	0.487	23.78
	45,000	99.7	99.99	0.507	23.79
	50,000	99.8	100	0.505	23.79
	60,000	99.6	100	0.539	23.84
	20,000	50.4	88.59	3.223	18.26
	25,000	100	100	0.528	23.68
	30,000	100	100	0.503	23.77
	35,000	100	100	0.492	23.75
24,000	40,000	100	100	0.497	23.85
	45,000	100	100	0.516	23.87
	50,000	100	100	0.514	23.89
	60,000	100	100	0.552	23.83
	20,000	50.42	88.56	3.23	18.28
	25,000	100	100	0.578	23.68
	30,000	100	100	0.553	23.72
	35,000	100	100	0.541	23.71
28,000	40,000	100	100	0.546	23.80
	45,000	100	100	0.546	23.75
	50,000	100	100	0.566	23.81
	60,000	100	100	0.595	23.72

Table C.2: Simulated results of reorder level policy in a two-week lead-time situation allowing backorders

Reorder level (m-ton)	Replenishment batch qty. (m-ton)	Vendor service level (%)	Customer service level (%)	Annual inventory operating cost ($M \$ x10^6$)	Net revenue ($M \$ x10^6$)
20,000	25,000	89.9	98.85	0.616	23.59
	30,000	91.2	98.98	0.574	23.63
	35,000	93.2	99.28	0.532	23.79
	40,000	93.3	99.22	0.535	23.73
	45,000	94.9	99.28	0.548	23.74
	50,000	95.0	99.33	0.544	23.76
	60,000	96.2	99.54	0.556	23.82
	70,000	95.8	99.49	0.606	23.77
23,000	25,000	99.3	99.98	0.517	23.75
	30,000	99.6	99.97	0.495	23.76
	35,000	99.5	99.98	0.479	23.84
	40,000	99.6	99.99	0.486	23.78
	45,000	99.9	100	0.498	23.79
	50,000	99.6	99.99	0.503	23.80
	60,000	100	100	0.537	23.84
28,000	25,000	100	100	0.579	23.63
	30,000	100	100	0.556	23.71
	35,000	100	100	0.544	23.70
	40,000	100	100	0.551	23.80
	45,000	100	100	0.557	23.74
	50,000	100	100	0.569	23.84
	60,000	100	100	0.603	23.78

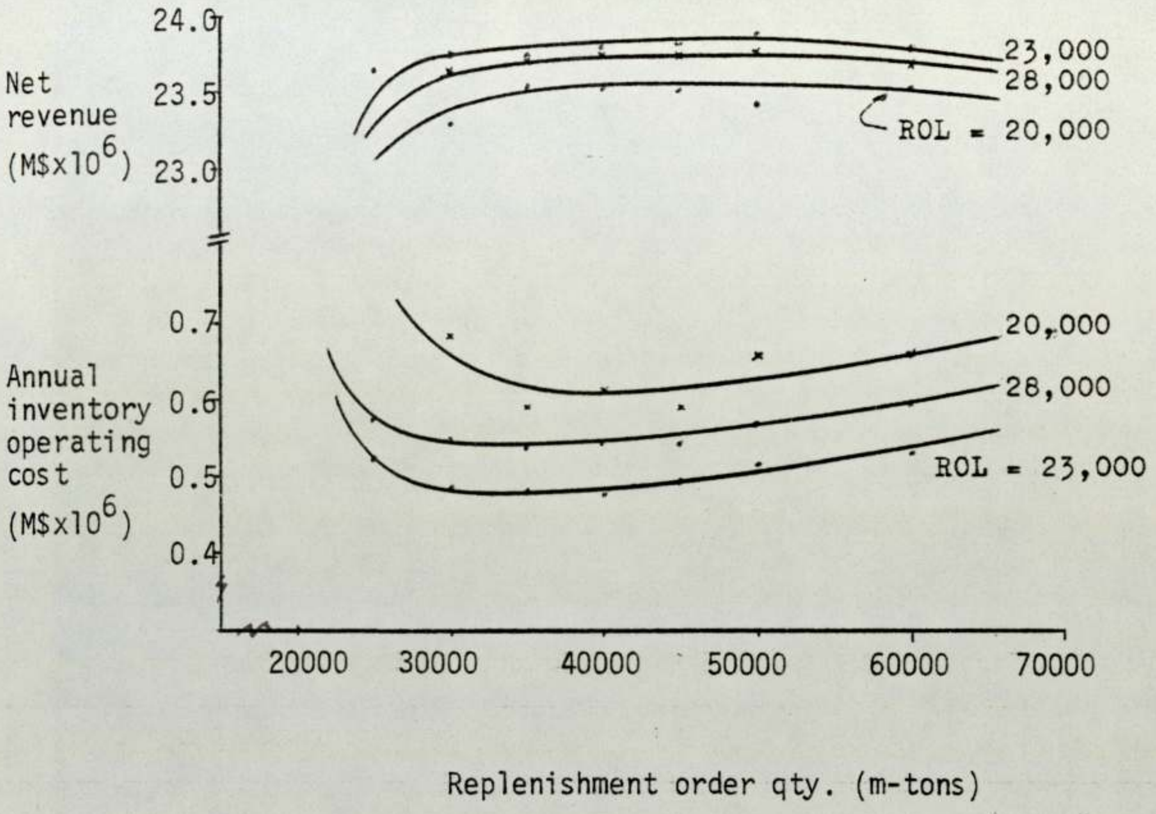
Table C.3: Simulated results of reorder level policy with a three-week lead-time situation with backordering prohibited

Reorder level (m-ton)	Replenishment batch qty. (m-ton)	Vendor service level (%)	Customer service level (%)	Annual inventory operating cost (M$x10^6)	Net revenue (M$x10^6)
32,000	30,000	67	88.64	3.140	18.29
	35,000	96.9	99.66	0.544	23.63
	40,000	96.0	99.62	0.550	23.54
	45,000	94.4	99.53	0.549	23.55
	50,000	95.1	99.49	0.596	23.60
	60,000	97.0	99.75	0.572	23.68
	70,000	97.4	99.74	0.622	23.60
35,000	30,000	67.25	88.66	3.136	18.33
	35,000	99.9	99.99	0.494	23.75
	40,000	99.8	100	0.491	23.77
	45,000	99.8	99.99	0.491	23.80
	50,000	99.9	99.99	0.512	23.68
	60,000	99.8	100	0.547	23.83
	70,000	100	100	0.592	23.78
40,000	30,000	67.19	88.67	3.133	18.35
	35,000	100	100	0.549	23.70
	40,000	100	100	0.559	23.70
	45,000	100	100	0.559	23.64
	50,000	100	100	0.577	23.61
	60,000	100	100	0.610	23.77
	70,000	100	100	0.652	23.72
	80,000	100	100	0.704	23.70

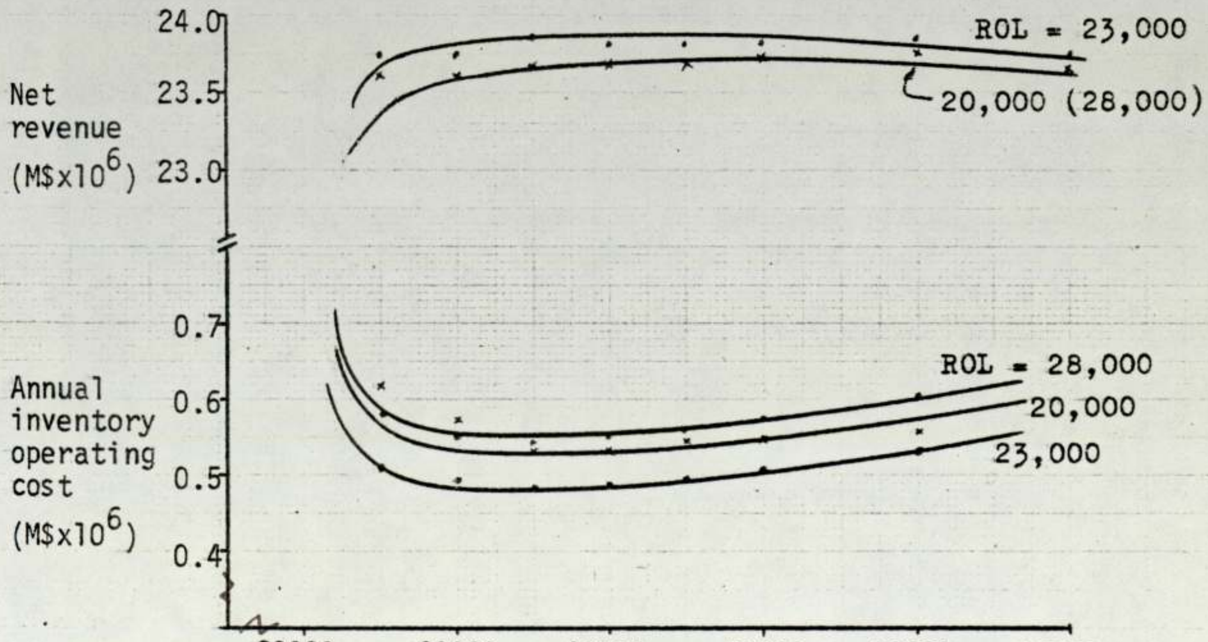
Table C.4: Simulated results of reorder level policy in a three-week lead-time situation allowing backorders

Reorder level (m-ton)	Replenishment batch qty. (m-ton)	Vendor service level (%)	Customer service level (%)	Annual inventory operating cost ($M\$ \times 10^6$)	Net revenue ($M\$ \times 10^6$)
32,000	32,000	Stock Depletion			
	35,000	93.2	99.08	0.555	23.61
	40,000	95.6	99.52	0.509	23.67
	45,000	95.2	99.48	0.528	23.76
	50,000	96.7	99.61	0.520	23.78
	60,000	97.4	99.71	0.547	23.83
	70,000	97.8	99.74	0.587	23.63
	80,000	97.4	99.67	0.637	23.77
35,000	32,000	Stock Depletion			
	35,000	99.8	99.99	0.489	23.76
	40,000	99.8	99.99	0.489	23.77
	45,000	99.9	99.99	0.491	23.80
	50,000	99.9	100	0.509	23.68
	60,000	99.9	100	0.545	23.70
	70,000	100	100	0.592	23.78
	80,000	100	100	0.647	23.76
40,000	32,000	Stock Depletion			
	35,000	100	100	0.551	23.69
	40,000	100	100	0.555	23.71
	45,000	100	100	0.562	23.73
	50,000	100	100	0.577	23.61
	60,000	100	100	0.613	23.64
	70,000	100	100	0.656	23.72
	80,000	100	100	0.715	23.69

Figure C.1: Characteristics of reorder level policy in a two-week lead-time situation (Tasek Cement Case)



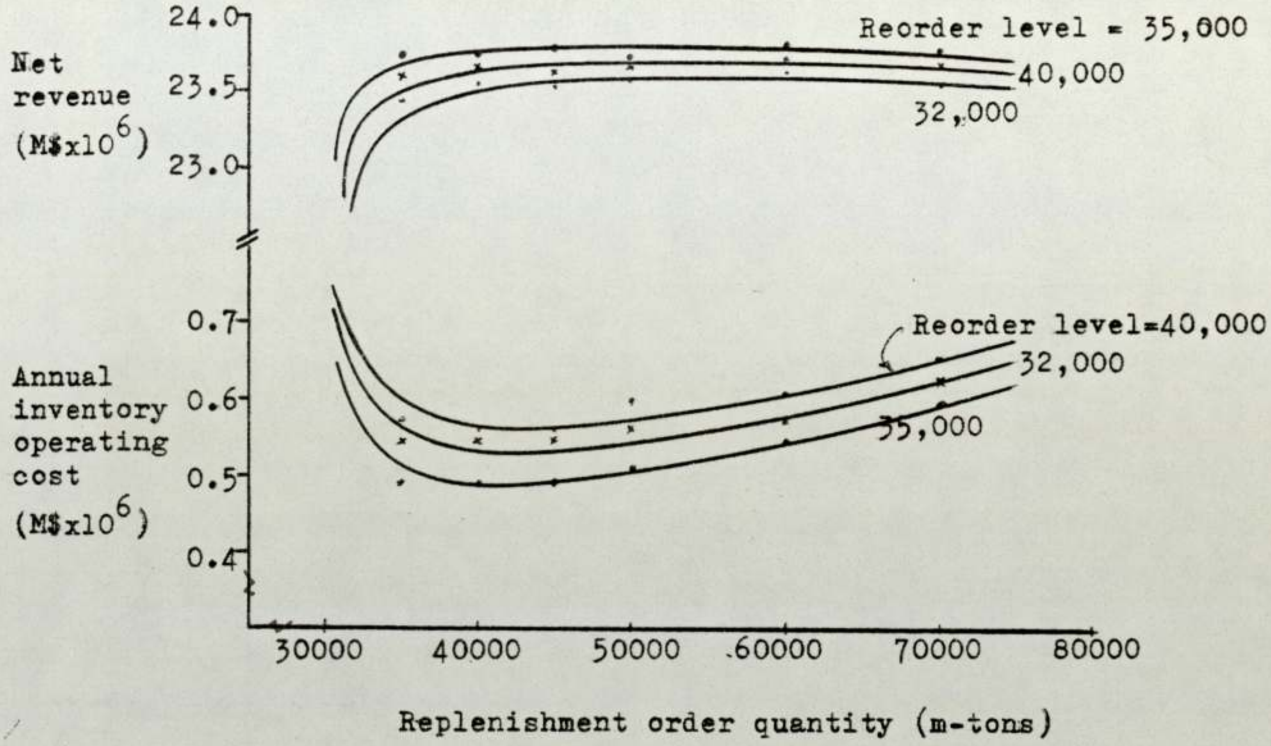
(a) Backordering prohibited



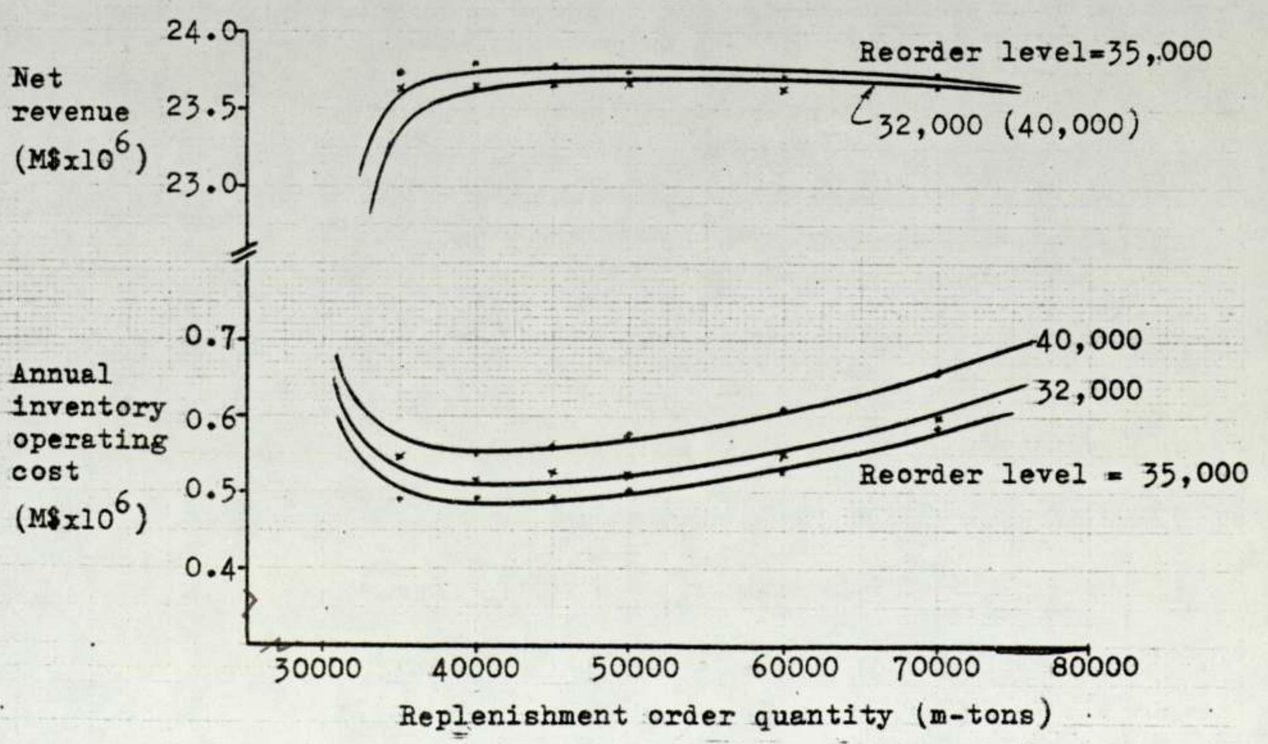
(b) Backordering permitted

Note: ROL = Reorder level in metric tons

Figure C.2 : Characteristics of reorder level policy in a three-week lead-time situation (Tasek Cement Case)



(a) Backordering prohibited



(b) Backordering permitted

Note: Reorder level is measured in metric tons

APPENDIX D

SIMULATED RESULTS OF REORDER LEVEL POLICY SUBJECT TO INFLATION

Input data for GIPSI

(1) Demand Information

Mean = 50 units per week (Gamma distributed)

Standard deviation = 15 units per week

(2) Lead-time Information

Lead-time = 3 weeks (fixed)

(3) Cost data

Selling price	=	£2 per unit
Cost price	=	£1.5 per unit
Purchase cost	=	£1 per unit
Ordering cost	=	£2 per order
Backordering cost	=	£10 per stockout occasion
Inventory holding rate	=	24%

Rates of inflation: 0%, 12%, 20%.

(4) Option of backordering

Backordering prohibited

Table D.1: Simulated results of reorder level policy without inflation

Reorder level (units)	Replenishment order qty. (units)	Vendor service level (%)	Customer service level (%)	Annual inventory operating cost (£)	Net revenue (£)
150	100	1.93	66.86	866	796
	150	69.28	95.93	150	2227
	200	75.10	97.95	100	2330
	250	80.81	98.87	78.9	2384
	300	81.71	98.78	83.5	2376
	350	80.39	98.17	88.3	2385
	400	78.23	98.85	89.4	2391
200	150	81.85	97.67	113	2317
	200	98.8	99.89	64	2416
	250	99	99.91	64.9	2423
	300	99.4	99.98	64.8	2425
	350	99.3	99.99	68.9	2416
	400	100	100	72.6	2407
250	150	90.90	98.93	88.1	2379
	200	100	100	73.1	2407
	250	99.5	99.98	74.5	2400
	300	100	100	76.9	2413
	350	100	100	80.0	2422
	400	100	100	84.1	2416

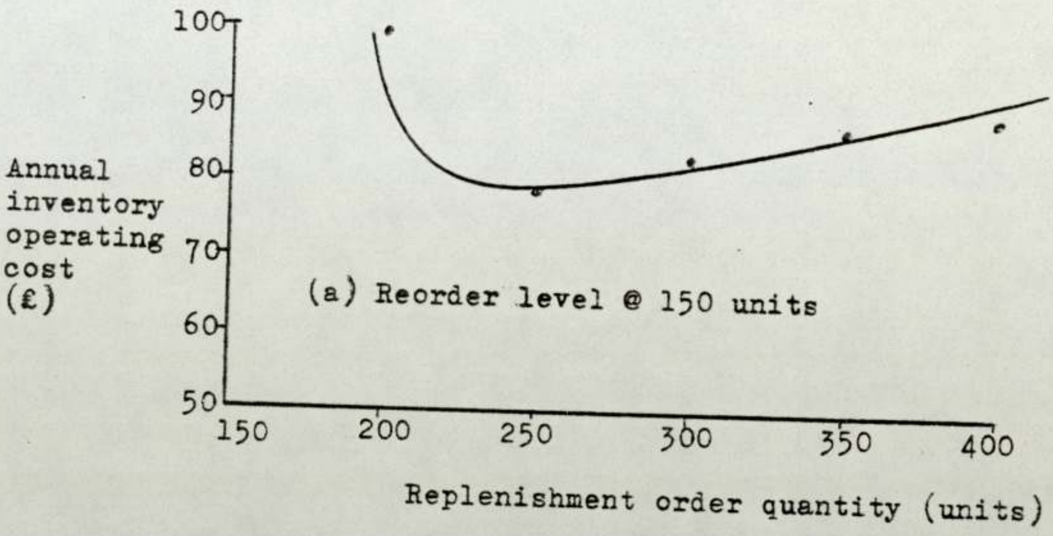
Table D.2: Simulated results of reorder level policy subject to inflation

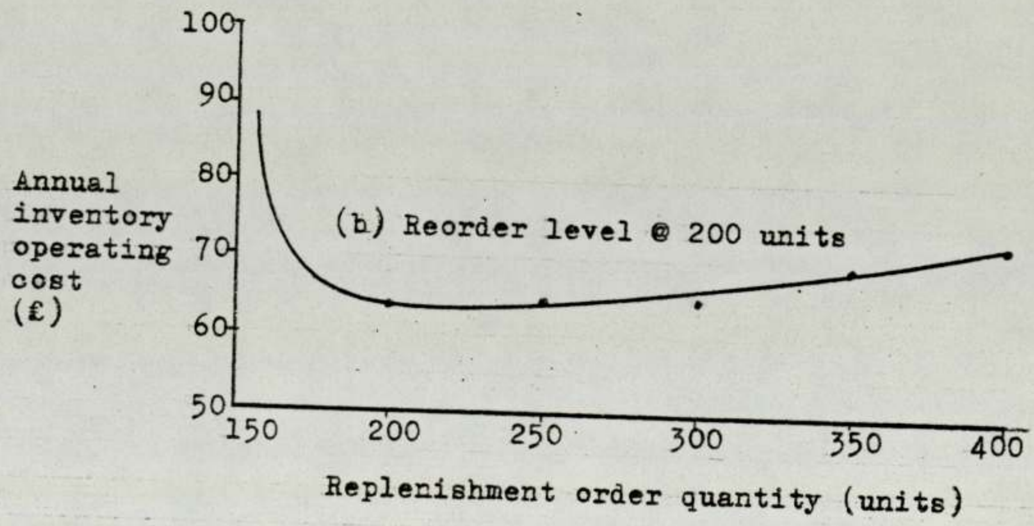
Inflation rate (%)	Replt. order qty. (units)	Reorder level (units)	Vendor service level (%)	Customer service level (%)	Annual sales (£)	Annual Purchases (£)	Annual inventory holding cost (£)	Annual ordering cost (£)	Stockout cost (£)	Annual inventory operating cost (£)	Net revenue
		150	80.61	98.19	4887	2443	24.8	24.6	45.2	94.6	2349
		180	96.25	99.73	4947	2473	31.3	24.9	6.7	62.8	2410
		190	97.83	99.89	4972	2486	33.7	24.9	2.9	61.5	2426
0	200	200	99.33	99.96	4973	2487	36.0	25.0	1.0	62.0	2425
		220	100	100	4973	2483	41.1	25.0	0	66.1	2421
		260	100	100	4967	2483	50.5	25.0	0	75.5	2408
		300	100	100	4960	2480	60.1	25.0	0	85.0	2395
		120	40.08	94.75	5021	2493	30.7	17.7	141.3	189.7	2338
		150	80.04	98.93	5231	2598	36.7	18.5	29.3	84.5	2549
		180	96.23	99.81	5272	2618	43.9	18.7	5.8	68.4	2585
12	283	190	97.63	99.88	5286	2625	46.3	18.7	4.1	69.1	2591
		200	100	100	5282	2623	49.1	18.7	0	67.7	2591
		220	99.62	99.99	5292	2628	54.2	18.6	0.4	73.2	2590
		260	100	100	5292	2628	69.5	18.7	0	88.2	2575
		280	100	100	5282	2623	69.3	18.6	0	87.8	2571
		120	44.16	97.11	5386	2640	59.3	10.8	82.6	152.7	2593
		150	80.39	99.28	5495	2695	65.5	11.1	22.0	98.6	2701
		170	93.17	99.81	5542	2718	70.6	11.1	7.1	88.7	2735
20	490	180	97.07	99.95	5531	2713	72.6	11.1	2.9	86.6	2732
		190	98.11	99.96	5520	2708	75.8	11.2	1.4	88.3	2724
		200	99.35	99.98	5549	2722	78.2	11.1	2.8	92.0	2735
		300	100	100	5531	2713	104.7	11.2	0	115.9	2702
		400	100	100	5549	2722	131.1	11.2	0	142.3	2685

Table D.3: Simulated results of reorder level policy subject to inflation (holding optimum reorder level constant).

Inflation rate (%)	Replt. order qty. (units)	Reorder level (units)	Vendor service level (%)	Customer service level (%)	Annual sales (£)	Annual Purchases (£)	Annual inventory holding cost (£)	Annual ordering cost (£)	Stockout cost (£)	Annual inventory operating cost (£)	Net revenue (£)
0	190	150	81.15	97.53	4838	2419	21.0	32.4	61.7	115.1	2304
		160	94.63	99.53	4933	2467	26.0	31.0	11.7	68.7	2397
		180	96.75	99.77	4956	2478	30.6	27.7	5.8	64.1	2414
		200	98.93	99.95	4960	2480	33.6	25.0	1.3	59.9	2420
		220	97.8	99.90	4965	2482	36.1	22.7	2.4	61.2	2422
		240	98.41	99.94	4968	2484	38.4	20.9	1.5	60.8	2423
		260	97.75	99.91	4975	2487	40.7	19.2	2.2	62.1	2426
		300	98.60	99.97	4960	2480	45.8	16.7	0.8	63.3	2425
		350	97.91	99.95	4970	2485	51.7	14.4	1.2	67.3	2418
		400	97.07	99.89	5000	2500	57.8	12.5	2.9	73.2	2427
12	190	150	81.15	97.53	5139	2560	22.3	34.3	66.8	123.4	2456
		160	94.63	99.53	5242	2611	27.6	32.8	13.4	73.8	2557
		180	96.75	99.77	5265	2621	32.5	29.3	7.1	68.9	2575
		200	98.93	99.95	5270	2622	35.7	26.4	2.1	64.2	2584
		220	97.8	99.90	5274	2623	38.3	24.0	3.4	65.7	2585
		240	98.41	99.94	5278	2624	40.8	22.1	2.2	65.1	2589
		260	97.75	99.91	5285	2626	43.3	20.3	2.9	66.5	2593
		300	98.60	99.97	5270	2613	48.7	17.6	1.4	67.7	2589
		350	97.91	99.95	5280	2618	54.9	15.2	1.9	72.0	2590
		400	97.07	99.89	5312	2631	61.4	13.2	4.2	78.8	2602
20	190	160	94.04	99.41	5458	2712	28.8	34.1	18.2	81.1	2665
		180	98.10	99.83	5483	2723	34.4	30.5	6.2	71.1	2689
		200	97.8	99.91	5495	2723	37.1	27.5	3.7	68.3	2704
		220	98.02	99.87	5490	2722	39.9	24.9	5.4	70.2	2698
		240	98.08	99.92	5498	2724	42.7	22.9	3.9	69.5	2704
		260	98.44	99.96	5510	2727	45.1	21.0	2.3	68.4	2715
		280	98.74	99.94	5508	2724	47.9	19.6	3.3	70.8	2713
		300	98.20	99.95	5511	2723	50.6	18.3	2.6	71.5	2716
		350	97.47	99.95	5500	2713	57.0	15.7	2.7	75.4	2712
		400	97.87	99.91	5518	2720	63.8	13.6	4.8	82.2	2716
		450	99.10	99.99	5528	2715	70.5	12.1	1.1	83.7	2729

Figure D.1 : Characteristics of reorder level policy at zero inflation rate (to locate approximate optimum replenishment order quantity)





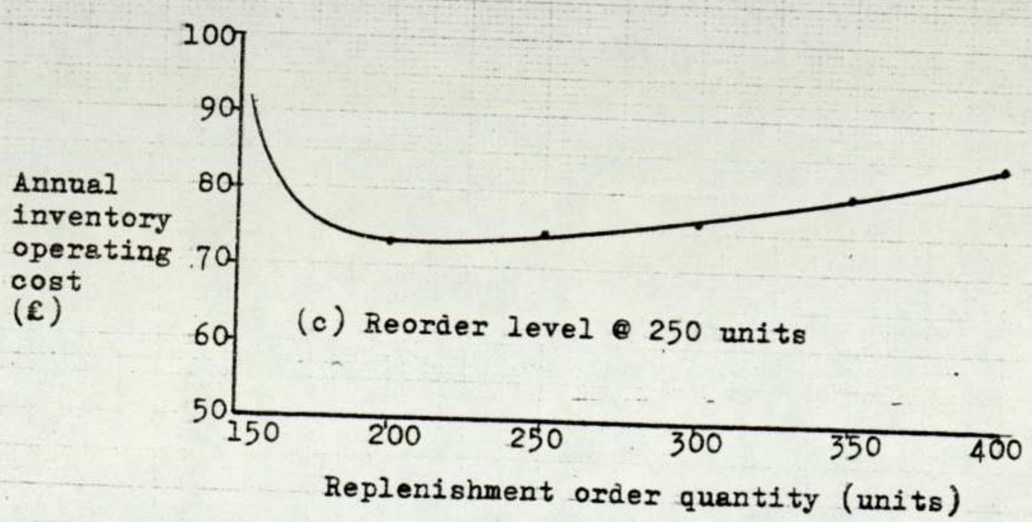
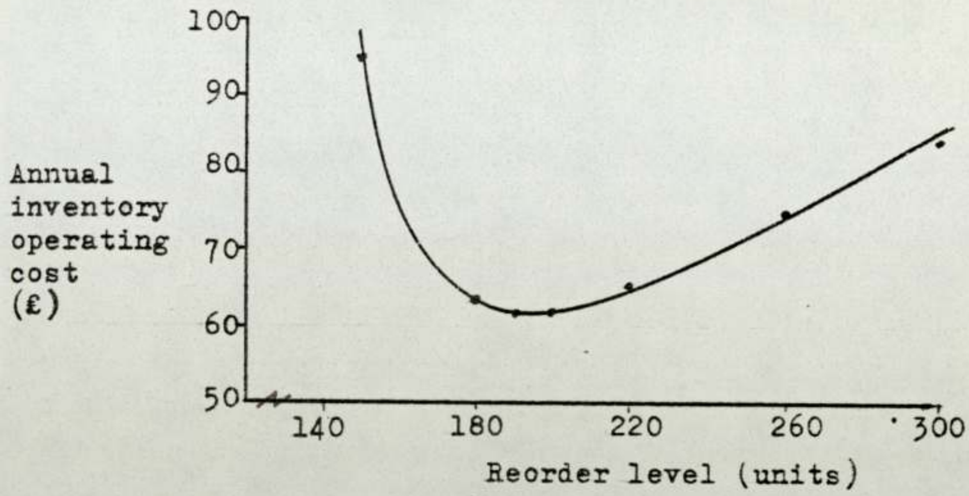
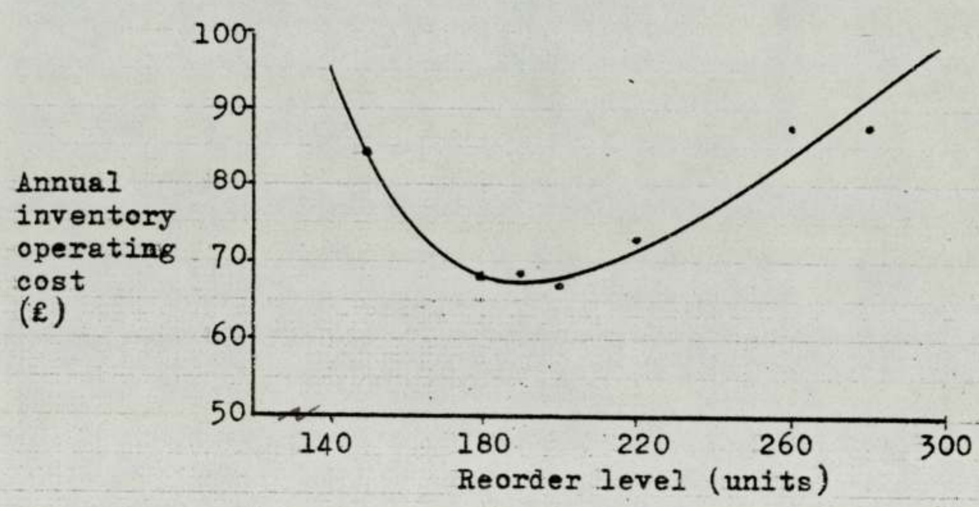


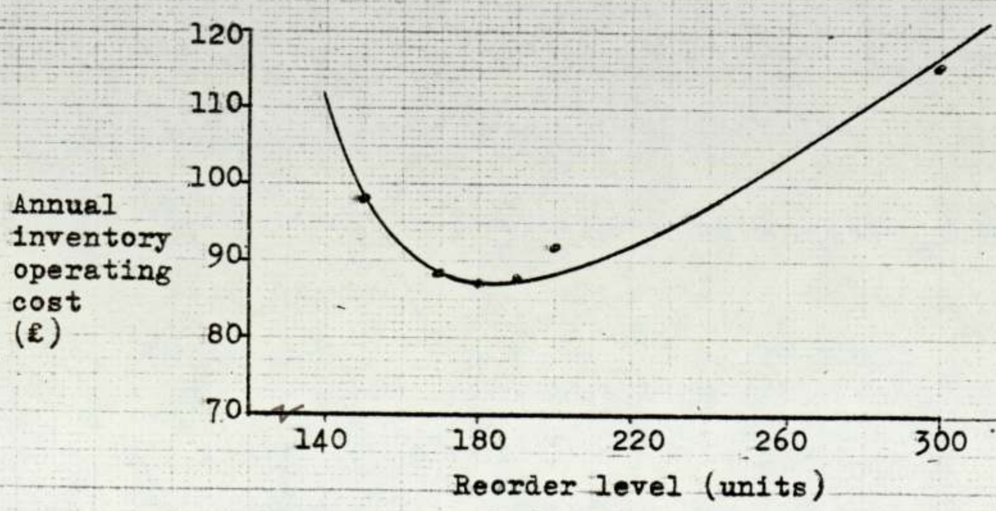
Figure D.2 : Characteristics of reorder level policy subject to inflation for replenishment order quantity fixed at 200, 283 and 490 units



(a) Replenishment order quantity at 200 units @ 0% inflation

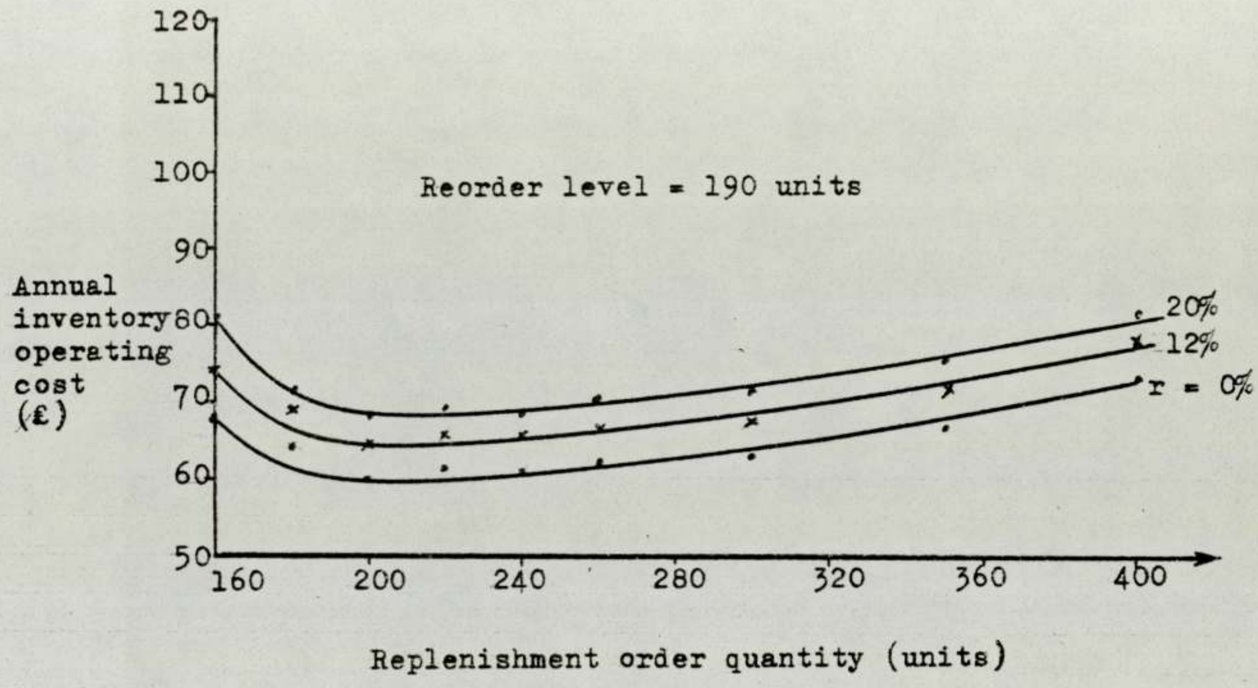


(b) Replenishment order quantity at 283 units @ 12% inflation



(c) Replenishment order quantity at 490 units @ 20% inflation

Figure D.3 : Characteristics of reorder level policy subject to inflation (holding optimum reorder level constant)



r = Rate of inflation

Figure D.4 : Characteristics of inventory holding cost subject to inflation

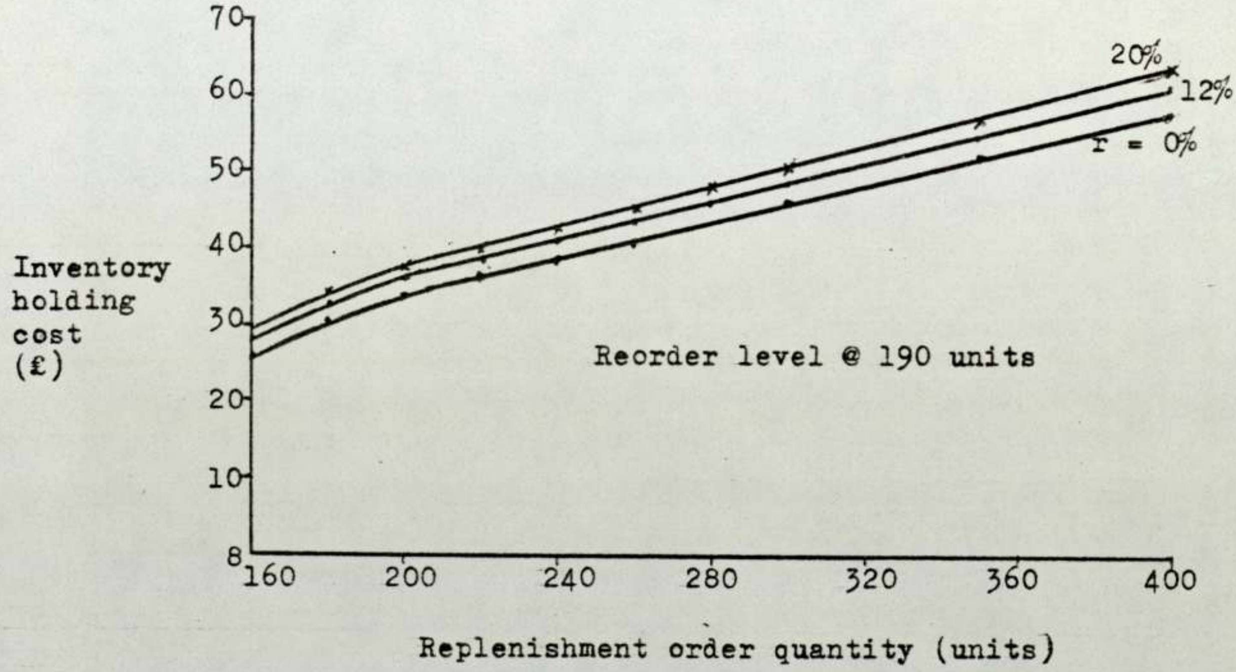


Figure D.5 : Comparison of inventory holding cost and cost of holding active stock at zero inflation rate

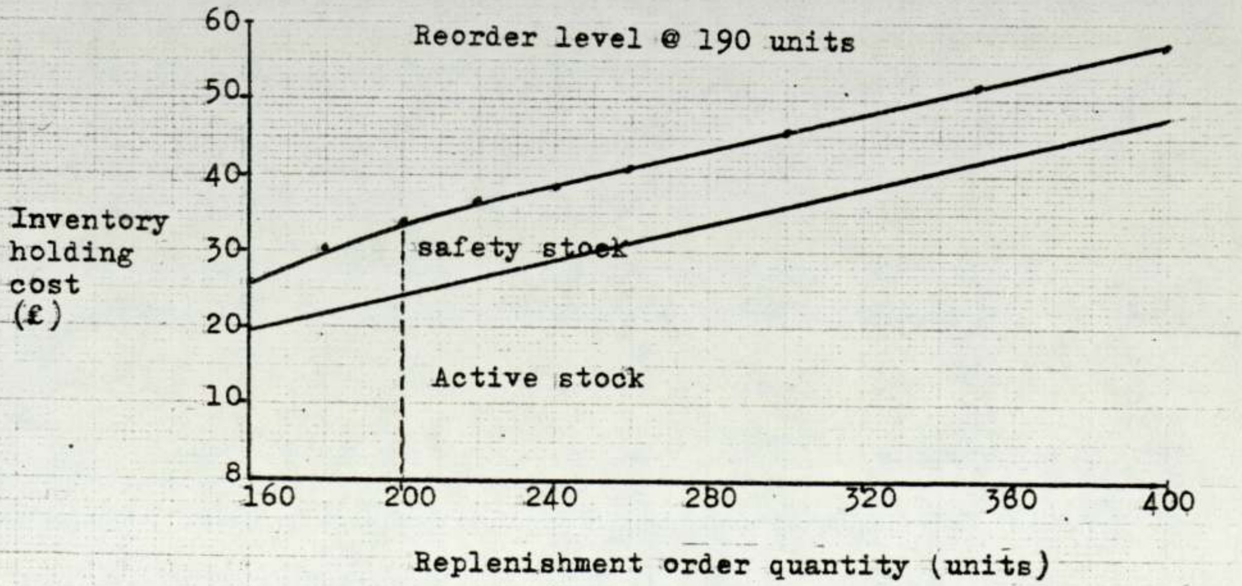


Figure D.6 : Characteristics of cost of stockout subject to inflation

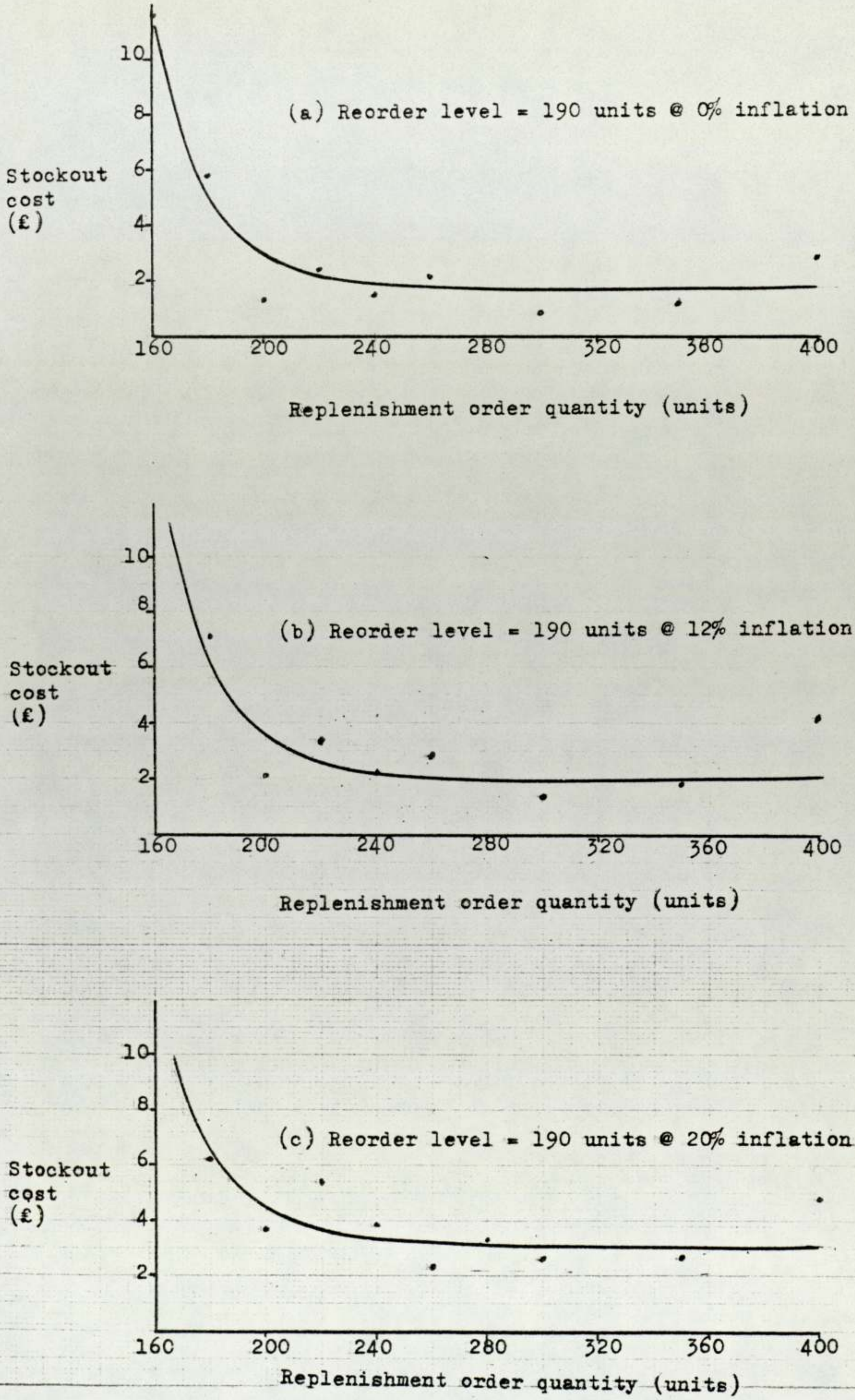
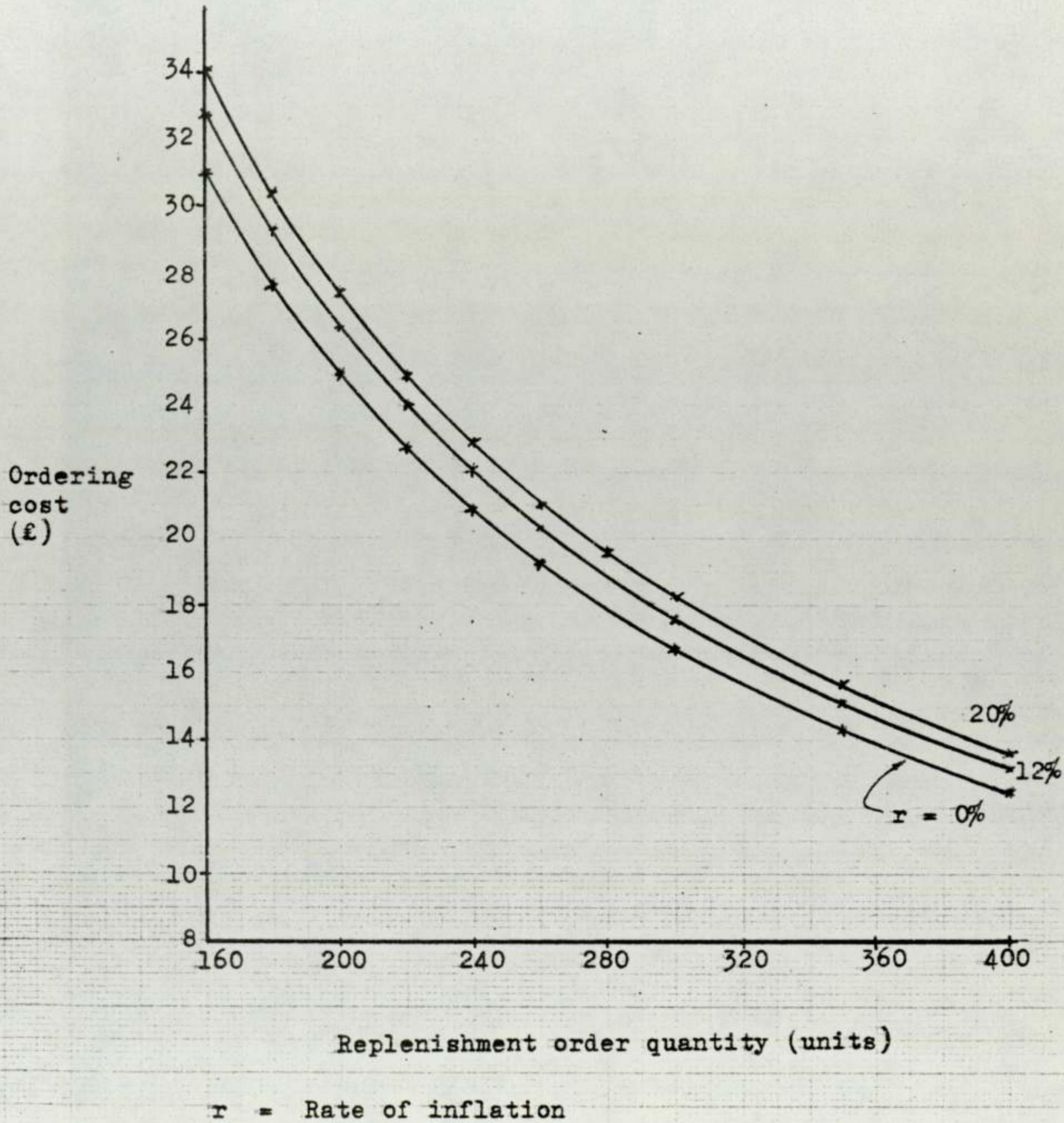


Figure D.7 : Characteristics of ordering cost subject to inflation



APPENDIX E

RESULTS OF THE CHI-SQUARE TEST ON OVERSHOOT DISTRIBUTION OF A REORDER LEVEL POLICY

(i) Computer printout of the Chi-square test on a specimen sample of overshoot distribution of a reorder level policy (pages 243 through to 247).

```
A SIMULATION PROGRAM TO STUDY OVERSHOOT CHARACTERISTICS
OF A REORDER LEVEL POLICY



DEMAND INF (1=NORMAL, 2=GAMMA, 3=UNIFORM)
?1
DEMAND P.U.T., STD DEV ?20
??5
LEADTIME INF (1=NORMAL, 2=GAMMA, 3=POISSON)
?2
LEADTIME, STD DEV ?5,1.5

NORMAL DEMAND PER UNIT TIME :
        MEAN = 20
        STD DEV = 5

GAMMA LEAD-TIME DURATION :
        MEAN = 5
        STD DEV = 1.5

EXPT AV O/SHOOT = 11.4044
EXPT STD DEV = 6.70874
THEO AV O/SHOOT = 10.125
% ERROR = 11.2181
```

```
DO YOU WANT TO PRINT OUT ANALYSIS OF YOUR INPUT DATA ('YES' OR 'NO')
?Y

** DATA ANALYSIS **

SAMPLE SIZE   = 500
MEAN VALUE    = 11.4044
STD DEVIATION = 6.70874
```

RANGE	MID VALUE	FREQ	PROB	CUM PROB
5.493E-03 - 3.121E+00	1.563E+00	6.900E+01	1.380E-01	1.380E-01
3.121E+00 - 6.236E+00	4.679E+00	6.300E+01	1.260E-01	2.640E-01
6.236E+00 - 9.352E+00	7.794E+00	6.700E+01	1.340E-01	3.980E-01
9.352E+00 - 1.247E+01	1.091E+01	7.600E+01	1.520E-01	5.500E-01
1.247E+01 - 1.558E+01	1.402E+01	9.000E+01	1.800E-01	7.300E-01
1.558E+01 - 1.870E+01	1.714E+01	6.100E+01	1.220E-01	8.520E-01
1.870E+01 - 2.181E+01	2.026E+01	3.800E+01	7.600E-02	9.280E-01
2.181E+01 - 2.493E+01	2.337E+01	2.800E+01	5.600E-02	9.840E-01
2.493E+01 - 2.804E+01	2.649E+01	5.000E+00	1.000E-02	9.940E-01
2.804E+01 - 3.116E+01	2.960E+01	3.000E+00	6.000E-03	1.000E+00

```
DO YOU WANT TO PLOT HISTOGRAM & CUM PROB CURVE ('YES'OR'NO') ?Y


** HISTOGRAM **

            PROB FROM 0% TO 100%

            0%        20%        40%        60%        80%       100%
            +---------+----------+----------+----------+----------+

 1.56319    I*******
 4.67858    I******
 7.79397    I*******
 10.9094    I********
 14.0248    I*********
 17.1401    I******
 20.2555    I****
 23.3709    I***
 26.4863    I*
 29.6017    I
```

```
** CUMULATIVE PROBABILITY CURVE **

          CUM PROB FROM 0% TO 100%

          0%        20%       40%       60%       80%       100%
          +---------+---------+---------+---------+---------+

 1.56319  I      *
 4.67858  I            *
 7.79397  I                   *
 10.9094  I                           *
 14.0248  I                                    *
 17.1401  I                                          *
 20.2555  I                                             *
 23.3709  I                                                *
 26.4863  I                                                 *
 29.6017  I                                                 *


DO YOU WANT 'GOODNESS OF FIT' TESTS ('YES' OR 'NO') ?Y

*** GOODNESS OF FIT TEST ***

YOU HAVE THE FOLLOWING DISTRIBUTIONS TO SELECT FOR THE GOODNESS
OF FIT TEST :
      (1) NORMAL      DISTRIBUTION
      (2) GAMMA            ''
      (3) EXPONENTIAL      ''
      (4) UNIFORM          ''
      (5) LOGNORMAL        ''
      (6) POISSON          ''
      (7) ALL DISTRIBUTIONS AS LISTED ABOVE

YOUR OPTION (1,2,3,....,7) ?7

PLEASE NOTE THAT THE FOLLOWING TEST(S) ARE USED TO TEST IF YOUR
INPUT DATA CAN BE FITTED TO THE DISTRIBUTION(S) YOU HAVE SELECTED :
   (A) CHI-SQUARE TEST (FOR SAMPLE SIZE GREATER THAN OR
       EQUAL TO 30) ;
   (B) KOLMOGOROV-SMIRNOV TEST (FOR SAMPLE SIZE GREATER
       THAN 10 BUT LESS THAN 100) ;
   (C) CRAMER-VON MISES TEST (FOR SAMPLE SIZE LESS THAN
       OR EQUAL TO 10).
```

** SUMMARY OF CHI-SQUARE TEST **

(1) TEST AGAINST NORMAL DIST.:

NO	RANGE FROM		TO	OBSERVED FREQ	EXPECTED FREQ	$(OBS-EXP)^2/EXP$
1	5.493E-03	-	3.121E+00	6.900E+01	3.191E+01	4.312E+01
2	3.121E+00	-	6.236E+00	6.300E+01	5.604E+01	8.644E-01
3	6.236E+00	-	9.352E+00	6.700E+01	7.963E+01	2.004E+00
4	9.352E+00	-	1.247E+01	7.600E+01	9.156E+01	2.644E+00
5	1.247E+01	-	1.558E+01	9.000E+01	8.518E+01	2.730E-01
6	1.558E+01	-	1.870E+01	6.100E+01	6.412E+01	1.514E-01
7	1.870E+01	-	2.181E+01	3.800E+01	3.905E+01	2.814E-02
8	2.181E+01	-	2.493E+01	2.800E+01	1.924E+01	3.987E+00
9	2.493E+01	-	2.804E+01	5.000E+00	7.670E+00	9.295E-01
10	2.804E+01	-	3.116E+01	3.000E+00	2.474E+00	1.120E-01
						54.1121

(2) TEST AGAINST GAMMA DIST.:

NO	RANGE FROM		TO	OBSERVED FREQ	EXPECTED FREQ	$(OBS-EXP)^2/EXP$
1	5.493E-03	-	3.121E+00	6.900E+01	2.722E+01	6.412E+01
2	3.121E+00	-	6.236E+00	6.300E+01	8.932E+01	7.754E+00
3	6.236E+00	-	9.352E+00	6.700E+01	1.078E+02	1.544E+01
4	9.352E+00	-	1.247E+01	7.600E+01	9.322E+01	3.181E+00
5	1.247E+01	-	1.558E+01	9.000E+01	6.844E+01	6.794E+00
6	1.558E+01	-	1.870E+01	6.100E+01	4.558E+01	5.216E+00
7	1.870E+01	-	2.181E+01	3.800E+01	2.846E+01	3.198E+00
8	2.181E+01	-	2.493E+01	2.800E+01	1.697E+01	7.165E+00
9	2.493E+01	-	2.804E+01	5.000E+00	9.780E+00	2.336E+00
10	2.804E+01	-	3.116E+01	3.000E+00	5.487E+00	1.127E+00
						116.327

(3) TEST AGAINST NEG. EXPONENTIAL DIST.:

NO	RANGE FROM		TO	OBSERVED FREQ	EXPECTED FREQ	$(OBS-EXP)^2/EXP$
1	5.493E-03	-	3.121E+00	6.900E+01	1.197E+02	2.148E+01
2	3.121E+00	-	6.236E+00	6.300E+01	9.091E+01	8.567E+00
3	6.236E+00	-	9.352E+00	6.700E+01	6.918E+01	6.845E-02
4	9.352E+00	-	1.247E+01	7.600E+01	5.264E+01	1.037E+01
5	1.247E+01	-	1.558E+01	9.000E+01	4.006E+01	6.227E+01
6	1.558E+01	-	1.870E+01	6.100E+01	3.048E+01	3.055E+01
7	1.870E+01	-	2.181E+01	3.800E+01	2.320E+01	9.449E+00
8	2.181E+01	-	2.493E+01	2.800E+01	1.765E+01	6.068E+00
9	2.493E+01	-	2.804E+01	5.000E+00	1.343E+01	5.293E+00
10	2.804E+01	-	3.116E+01	3.000E+00	1.022E+01	5.101E+00
						159.214

(4) TEST AGAINST UNIFORM DIST.:

NO	RANGE FROM		TO	OBSERVED FREQ	EXPECTED FREQ	$(OBS-EXP)^2/EXP$
1	5.493E-03	-	3.121E+00	6.900E+01	5.000E+01	7.220E+00
2	3.121E+00	-	6.236E+00	6.300E+01	5.000E+01	3.380E+00
3	6.236E+00	-	9.352E+00	6.700E+01	5.000E+01	5.780E+00
4	9.352E+00	-	1.247E+01	7.600E+01	5.000E+01	1.352E+01
5	1.247E+01	-	1.558E+01	9.000E+01	5.000E+01	3.200E+01
6	1.558E+01	-	1.870E+01	6.100E+01	5.000E+01	2.420E+00
7	1.870E+01	-	2.181E+01	3.800E+01	5.000E+01	2.880E+00
8	2.181E+01	-	2.493E+01	2.800E+01	5.000E+01	9.680E+00
9	2.493E+01	-	2.804E+01	5.000E+00	5.000E+01	4.050E+01
10	2.804E+01	-	3.116E+01	3.000E+00	5.000E+01	4.418E+01
						161.56

(5) TEST AGAINST LOGNORMAL DIST.:

NO	RANGE FROM	TO	OBSERVED FREQ	EXPECTED FREQ	$(OBS-EXP)^2/EXP$
1	5.493E-03	- 3.121E+00	6.900E+01	8.832E+00	4.099E+02
2	3.121E+00	- 6.236E+00	6.300E+01	9.214E+01	9.216E+00
3	6.236E+00	- 9.352E+00	6.700E+01	1.308E+02	3.113E+01
4	9.352E+00	- 1.247E+01	7.600E+01	1.025E+02	6.855E+00
5	1.247E+01	- 1.558E+01	9.000E+01	6.621E+01	8.552E+00
6	1.558E+01	- 1.870E+01	6.100E+01	3.996E+01	1.108E+01
7	1.870E+01	- 2.181E+01	3.800E+01	2.363E+01	8.743E+00
8	2.181E+01	- 2.493E+01	2.800E+01	1.397E+01	1.409E+01
9	2.493E+01	- 2.804E+01	5.000E+00	8.334E+00	1.334E+00
10	2.804E+01	- 3.116E+01	3.000E+00	5.038E+00	8.242E-01
					501.737

(6) TEST AGAINST POISSON DIST.:

NO	RANGE FROM	TO	OBSERVED FREQ	EXPECTED FREQ	$(OBS-EXP)^2/EXP$
1	5.493E-03	- 3.121E+00	6.900E+01	1.809E+00	2.495E+03
2	3.121E+00	- 6.236E+00	6.300E+01	2.992E+01	3.658E+01
3	6.236E+00	- 9.352E+00	6.700E+01	1.174E+02	2.165E+01
4	9.352E+00	- 1.247E+01	7.600E+01	1.727E+02	5.417E+01
5	1.247E+01	- 1.558E+01	9.000E+01	1.202E+02	7.608E+00
6	1.558E+01	- 1.870E+01	6.100E+01	4.571E+01	5.115E+00
7	1.870E+01	- 2.181E+01	3.800E+01	1.046E+01	7.252E+01
8	2.181E+01	- 2.493E+01	3.600E+01	1.714E+00	6.859E+02
9	2.493E+01	- 2.804E+01			
10	2.804E+01	- 3.116E+01			
					3378.64

DONE

(ii) Summary of the Chi-square test on overshoot distribution (pages 249 through to 254).

The abreviations used in Tables E.1 through to E.6 refer to the following terms:-

NOR	=	Normal distribution
GAM	=	Gamma distribution
POI	=	Poisson distribution
UNI	=	Uniform distribution
S.Dev	=	Standard deviation
Chi-sq	=	Chi-square
LOC	=	Level of confidence

Table E.1: Chi-square test for a Normal Distribution

Demand p.u.t.			Lead-time			Chi-sq	Degree of Freedom	Critical Value @ 5% LOC	Comment
Type	Mean	S.Dev	Type	Mean	S.Dev				
NOR	100	20	NOR	5	1.5	51.0	6	12.6	Reject H_0
NOR	100	10	GAM	5	2	32.3	7	14.1	Reject H_0
NOR	20	5	POI	6	-	77.5	7	14.1	Reject H_0
GAM	100	20	NOR	6	2	71.8	7	14.1	Reject H_0
GAM	100	10	GAM	5	2	37.6	7	14.1	Reject H_0
GAM	20	5	POI	5	-	101.9	7	14.1	Reject H_0
UNI	100	28.9	NOR	5	2	88.6	7	14.1	Reject H_0
UNI	100	11.5	GAM	6	2	139.1	7	14.1	Reject H_0
UNI	20	5.8	POI	4	-	67.2	7	14.1	Reject H_0
UNI	20	1	GAM	5	1	54.3	7	14.1	Reject H_0
GAM	10	4	GAM	5	2	95.8	7	14.1	Reject H_0
NOR	20	5	GAM	5	1.5	54.1	7	14.1	Reject H_0

Note: H_0 denotes the hypothesis that the sample is drawn from a Normal population.

Table E.2: Chi-square test for a Gamma Distribution

Demand p.u.t.			Lead-time			Chi-sq	Degree of Freedom	Critical Value @ 5% LOC	Comment
Type	Mean	S.Dev	Type	Mean	S.Dev				
NOR	100	20	NOR	5	1.5	60.6	7	14.1	Reject H_0
NOR	100	10	GAM	5	2	108.3	7	14.1	Reject H_0
NOR	20	5	POI	6	-	75.3	7	14.1	Reject H_0
GAM	100	20	NOR	6	2	115.8	7	14.1	Reject H_0
GAM	100	10	GAM	5	2	151.9	7	14.1	Reject H_0
GAM	20	5	POI	1	5	90.8	7	14.1	Reject H_0
UNI	100	28.9	NOR	5	2.	80.2	7	14.1	Reject H_0
UNI	100	11.5	GAM	6	2	226.4	7	14.1	Reject H_0
UNI	20	5.8	POI	4	-	91.1	7	14.1	Reject H_0
UNI	20	1	GAM	5	1	300.0	7	14.1	Reject H_0
GAM	10	4	GAM	5	2	41.1	7	14.1	Reject H_0
NOR	20	5	GAM	5	1.5	116.3	7	14.1	Reject H_0

Note: H_0 denotes the hypothesis that the sample is drawn from a Gamma population.

Table E.3: Chi-square test for a Uniform Distribution

Demand p.u.t.			Lead-time			Chi-sq	Degree of Freedom	Critical Value @ 5% LOC	Comment
Type	Mean	S.Dev	Type	Mean	S.Dev				
NOR	100	20	NOR	5	1.5	254.4	7	14.1	Reject H_0
NOR	100	10	GAM	5	2	149.2	7	14.1	Reject H_0
NOR	20	5	POI	6	-	198.6	7	14.1	Reject H_0
GAM	100	20	NOR	6	2	202.1	7	14.1	Reject H_0
GAM	100	10	GAM	5	2	98.6	7	14.1	Reject H_0
GAM	20	5	POI	5	-	189.7	7	14.1	Reject H_0
UNI	100	28.9	NOR	5	2	108.9	7	14.1	Reject H_0
UNI	100	11.5	GAM	6	2	55.3	7	14.1	Reject H_0
UNI	20	5.8	POI	4	-	90.6	7	14.1	Reject H_0
UNI	20	1	GAM	5	1	70.4	7	14.1	Reject H_0
GAM	10	4	GAM	5	2	208.3	7	14.1	Reject H_0
NOR	20	5	GAM	5	1.5	161.6	7	14.1	Reject H_0

Note: H_0 denotes the hypothesis that the sample is drawn from a Uniform population.

Table E.4: Chi-square test for a Poisson Distribution

Demand p.u.t.			Lead-time			Chi-sq	Degree of Freedom	Critical Value @ 5% LOC	Comment
Type	Mean	S.Dev	Type	Mean	S.Dev				
NOR	20	10	NOR	5	2	1902	3	7.8	Reject H_0
NOR	30	5	GAM	5	2	1813	4	9.5	Reject H_0
NOR	20	5	POI	6	-	4711	5	11.1	Reject H_0
GAM	20	10	NOR	5	2	3213	3	7.8	Reject H_0
GAM	10	4	GAM	5	2	1523	6	12.6	Reject H_0
GAM	20	5	POI	5	-	4012	5	11.1	Reject H_0
UNI	20	5.8	NOR	5	2	2341	5	11.1	Reject H_0
UNI	20	2.9	GAM	5	2	1354	6	12.6	Reject H_0
UNI	20	5.8	POI	4	-	4160	6	12.6	Reject H_0
UNI	20	1	GAM	5	1	728	7	14.1	Reject H_0
GAM	30	10	POI	5	-	2641	3	7.8	Reject H_0
NOR	20	5	GAM	5	1.5	3379	6	12.6	Reject H_0

Note: H_0 denotes the hypothesis that the sample is drawn from a Poisson population.

Table E.5: Chi-square test for a Lognormal Distribution

Demand p.u.t.			Lead-time			Chi-sq	Degree of Freedom	Critical Value @ 5% LOC	Comment
Type	Mean	S.Dev	Type	Mean	S.Dev				
NOR	100	20	NOR	5	1.5	233.0	7	14.1	Reject H_0
NOR	100	10	GAM	5	2	558.6	7	14.1	Reject H_0
NOR	20	5	POI	6	-	269.7	7	14.1	Reject H_0
GAM	100	20	NOR	6	2	385.8	7	14.1	Reject H_0
GAM	100	10	GAM	5	2	1061.3	7	14.1	Reject H_0
GAM	20	5	POI	5	-	317.1	7	14.1	Reject H_0
UNI	100	28.9	NOR	5	2	352.7	7	14.1	Reject H_0
UNI	100	11.5	GAM	6	2	1011.3	7	14.1	Reject H_0
UNI	20	5.8	POI	4	-	452.3	7	14.1	Reject H_0
UNI	20	1	GAM	5	1	199.0	7	14.1	Reject H_0
GAM	10	4	GAM	5	2	192.9	7	14.1	Reject H_0
NOR	20	5	GAM	5	1.5	501.7	7	14.1	Reject H_0

Note: H_0 denotes the hypothesis that the sample is drawn from a Lognormal population.

Table E.6: Chi-square test for a Negative Exponential Distribution

Demand p.u.t.			Lead-time			Chi-sq	Degree of Freedom	Critical Value @ 5% LOC	Comment
Type	Mean	S.Dev	Type	Mean	S.Dev				
NOR	100	20	NOR	5	1.5	126.8	8	15.5	Reject H_0
NOR	100	10	GAM	5	2	213.0	8	15.5	Reject H_0
NOR	20	5	POI	6	-	94.0	8	15.5	Reject H_0
GAM	100	20	NOR	6	2	144.0	8	15.5	Reject H_0
GAM	100	10	GAM	5	2	192.2	8	15.5	Reject H_0
GAM	20	5	POI	5	-	90.6	8	15.5	Reject H_0
UNI	100	28.9	NOR	5	2	80.9	8	15.5	Reject H_0
UNI	100	11.5	GAM	6	2	169.3	8	15.5	Reject H_0
UNI	20	5.8	POI	4	-	106.2	8	15.5	Reject H_0
UNI	20	1	GAM	5	1	273.0	8	15.5	Reject H_0
GAM	10	4	GAM	5	2	56.1	8	15.5	Reject H_0
NOR	20	5	GAM	5	1.5	159.2	8	15.5	Reject H_0

Note: H_0 denotes the hypothesis that the sample is drawn from a Negative Exponential population.

Table E.7 : Experimental results of average overshoot of reorder level policy by simulation

Demand p.u.t.			Lead-time			Theo	Expt	$Z_{(.05)}$	$Z_{(cal)}$
Type	Mean	S.Dev	Type	Mean	S.Dev	o/shoot	o/shoot		
NOR	400	120	NOR	6	1.8	217.5	215.6	1.96	-0.38
NOR	400	120	NOR	6	3.0	217.5	219.2	1.96	0.51
NOR	400	120	GAM	6	1.8	217.5	216.9	1.96	-0.11
NOR	400	120	GAM	6	3.0	217.5	218.5	1.96	0.19
NOR	400	120	POI	6	-	217.5	218.9	1.96	0.45
NOR	400	200	NOR	6	3.0	249.5	245.6	1.96	-0.51
NOR	400	200	GAM	6	3.0	249.5	247.3	1.96	-0.37
NOR	400	200	POI	6	-	249.5	256.2	1.96	1.58
GAM	400	120	NOR	6	3.0	217.5	223.1	1.96	0.81
GAM	400	120	GAM	6	1.8	217.5	211.4	1.96	-0.98
GAM	400	120	POI	6	-	217.5	223.8	1.96	1.63
UNI	400	120	NOR	6	1.8	217.5	227.0	1.96	1.82
UNI	400	120	GAM	4	2.0	217.5	217.4	1.96	-0.03
UNI	400	120	POI	5	-	217.5	228.1	1.96	1.33
GAM	400	200	NOR	6	3.0	249.5	251.0	1.96	0.18
GAM	400	200	GAM	6	1.8	249.5	248.9	1.96	-0.07
GAM	400	200	POI	6	-	249.5	240.1	1.96	-1.31
UNI	400	200	NOR	6	1.8	249.5	247.8	1.96	-0.25
UNI	400	200	GAM	6	3.0	249.5	253.8	1.96	0.49
UNI	400	200	POI	4	-	249.5	258.7	1.96	1.75

Cont'd from page 255

Demand p.u.t.			Lead-time			Theo	Expt	$Z_{(.05)}$	$Z_{(cal)}$
Type	Mean	S.Dev	Type	Mean	S.Dev	o/shoot	o/shoot		
NOR	100	20	NOR	5	1	51.5	51.3	1.96	-0.05
NOR	100	20	NOR	5	1.5	51.5	52.1	1.96	0.82
NOR	100	20	NOR	5	2	51.5	53.0	1.96	1.14
NOR	100	20	GAM	5	1	51.5	50.4	1.96	-0.75
NOR	100	20	GAM	5	1.5	51.5	52.4	1.96	0.96
NOR	100	20	GAM	5	2	51.5	50.6	1.96	-1.02
NOR	100	20	POI	5	-	51.5	52.6	1.96	1.08
GAM	100	20	NOR	5	1	51.5	51.0	1.96	-0.10
GAM	100	20	NOR	5	2	51.5	52.5	1.96	1.01
GAM	100	20	GAM	5	1	51.5	50.0	1.96	-1.13
GAM	100	20	GAM	5	2	51.5	53.1	1.96	1.31
GAM	100	20	POI	5	-	51.5	51.7	1.96	0.02
UNI	100	20	NOR	5	1	51.5	52.3	1.96	1.04
UNI	100	20	NOR	5	2	51.5	52.9	1.96	1.13
UNI	100	20	GAM	5	1	51.5	50.9	1.96	-0.85
UNI	100	20	GAM	5	2	51.5	51.0	1.96	-0.12
UNI	100	20	POI	5	-	51.5	52.1	1.96	1.02
NOR	100	20	NOR	6	1	51.5	53.1	1.96	1.65
GAM	100	20	GAM	6	1	51.5	52.7	1.96	1.27
UNI	100	20	POI	6	-	51.5	50.8	1.96	-0.94

Cont'd from page 256

Demand p.u.t.			Lead-time			Theo	Expt	$Z_{(.05)}$	$Z_{(cal)}$
Type	Mean	S.Dev	Type	Mean	S.Dev	o/shoot	o/shoot		
NOR	30	10	NOR	5	1	16.2	16.4	1.96	0.04
NOR	30	10	NOR	5	2	16.2	15.9	1.96	-0.21
NOR	30	10	GAM	5	1	16.2	15.2	1.96	-1.13
NOR	30	10	GAM	5	2	16.2	15.8	1.96	-0.52
NOR	30	10	POI	5	-	16.2	16.9	1.96	0.72
GAM	30	10	NOR	5	1	16.2	15.7	1.96	-0.87
GAM	30	10	NOR	5	2	16.2	16.7	1.96	0.21
GAM	30	10	GAM	5	1	16.2	17.0	1.96	1.65
GAM	30	10	GAM	5	2	16.2	16.8	1.96	0.08
GAM	30	10	POI	5	-	16.2	16.3	1.96	0.01
UNI	30	10	NOR	5	2	16.2	16.9	1.96	0.10
UNI	30	10	GAM	5	2	16.2	15.7	1.96	-0.91
UNI	30	10	POI	5	-	16.2	15.4	1.96	-1.03
NOR	20	10	NOR	5	2	12.0	12.9	1.96	1.20
NOR	20	10	GAM	5	2	12.0	12.5	1.96	0.78
NOR	20	10	POI	5	-	12.0	11.7	1.96	-0.65
GAM	20	10	NOR	5	2	12.0	11.8	1.96	-0.70
GAM	20	10	POI	5	-	12.0	12.3	1.96	0.08
UNI	20	10	GAM	5	2	12.0	12.9	1.96	1.01
UNI	20	2.9	NOR	5	2	9.7	10.5	1.96	1.38
GAM	20	2.9	GAM	5	2	9.7	9.3	1.96	-0.30

APPENDIX F

COMPARISONS OF SERVICE LEVELS

Comparisons of service levels

The abreviations used in Tables F.1 through to F.4 refer to the following descriptions:-

NOR	=	Normal distribution
GAM	=	Gamma distribution
LOGNOR	=	Lognormal distribution
EXPO	=	Exponential distribution
POI	=	Poisson distribution
UNI	=	Uniform distribution
CONST	=	Constant value
ROL	=	Reorder level
S.DEV	=	Standard deviation
VSL	=	Vendor service level
CSL	=	Customer service level

The demand and lead-time values used for experimentation are measured in "units" and "weeks" respectively.

Table F.1: Comparisons of customer service level and vendor service level of the reorder level policy

Demand p.u.t.			Lead-time			Back-	Inventory Parameters		VSL	CSL
Type	Mean	S.Dev	Type	Mean	S.Dev	order	ROL	Replt. Qty	(%)	(%)
NOR	100	40	NOR	5	1	NO	670	700	94.11	99.53
NOR	100	30	NOR	5	1	NO	670	700	95.28	99.69
NOR	100	20	NOR	5	1	NO	670	700	96.85	99.71
NOR	100	10	NOR	5	1	NO	670	700	97.2	99.91
NOR	100	40	NOR	5	1	YES	670	700	91.82	99.20
NOR	100	30	NOR	5	1	YES	670	700	94.25	99.41
NOR	100	20	NOR	5	1	YES	670	700	95.63	99.70
NOR	100	10	NOR	5	1	YES	670	700	97.01	99.90
GAM	100	30	NOR	5	1	NO	680	700	95.97	99.69
GAM	100	30	NOR	5	1	YES	680	700	95.80	99.56
LOGNOR	100	20	GAM	6	1.5	NO	800	900	92.57	99.45
LOGNOR	100	20	GAM	6	1.5	YES	800	900	92.50	99.34
UNI	100	28.9	POI	5	-	NO	600	600	75.16	93.62
UNI	100	28.9	POI	5	-	YES	600	600	60.73	83.94
EXPO	100	-	CONST	5	-	NO	600	600	75.10	93.51
EXPO	100	-	CONST	5	-	YES	600	600	50.13	75.64

Table F.2: Comparisons of customer service level and vendor service level of the reorder cycle policy

Demand p.u.t.			Lead-time			Back-order	Inventory Parameters		VSL	CSL
Type	Mean	S.Dev	Type	Mean	S.Dev		Period	S(max)	(%)	(%)
NOR	100	30	NOR	5	1	NO	8	1450	93.85	99.55
NOR	100	20	NOR	5	1	NO	8	1450	98.22	99.91
NOR	100	10	NOR	5	1	NO	8	1450	99.01	99.96
NOR	100	30	NOR	5	1	YES	8	1450	95.04	99.70
NOR	100	20	NOR	5	1	YES	8	1450	98.42	99.91
NOR	100	10	NOR	5	1	YES	8	1450	99.44	99.92
GAM	100	30	NOR	5	1	NO	8	1470	97.62	99.79
GAM	100	30	NOR	5	1	YES	8	1470	95.84	99.72
LOGNOR	100	20	GAM	6	1.5	NO	8	1400	77.14	96.78
LOGNOR	100	20	GAM	6	1.5	YES	8	1400	72.02	96.36
UNI	100	28.9	POI	5	-	NO	8	1450	84.72	97.20
UNI	100	28.9	POI	5	-	YES	8	1450	85.52	97.64
EXPO	100	-	CONST	5	-	NO	6	1400	89.73	96.85
EXPO	100	-	CONST	5	-	YES	6	1400	89.59	95.87

Table F.3: Comparisons of customer service level and vendor service level of the reorder level policy with periodic reviews

Demand p.u.t.			Lead-time			Back-	Inventory Parameters			VSL	CSL
Type	Mean	S.Dev	Type	Mean	S.Dev	order	ROL	Period	Qty.	(%)	(%)
NOR	100	30	NOR	5	1	NO	1200	8	1200	93.63	99.40
NOR	100	20	NOR	5	1	NO	1200	8	1200	93.78	99.69
NOR	100	10	NOR	5	1	NO	1200	8	1200	94.66	99.77
NOR	100	30	NOR	5	1	YES	1200	8	1200	93.51	99.37
NOR	100	20	NOR	5	1	YES	1200	8	1200	93.63	99.65
NOR	100	10	NOR	5	1	YES	1200	8	1200	94.14	99.68
GAM	100	30	NOR	5	1	NO	1000	6	1000	91.22	99.14
GAM	100	30	NOR	5	1	YES	1000	6	1000	89.31	99.03
LOGNOR	100	20	GAM	6	1.5	NO	1200	8	1400	83.86	98.63
LOGNOR	100	20	GAM	6	1.5	YES	1200	8	1400	83.68	98.60
UNI	100	28.9	POI	5	-	NO	1100	8	1100	83.00	97.06
UNI	100	28.9	POI	5	-	YES	1100	8	1100	80.63	96.40
EXPO	100	-	CONST	5	-	NO	1000	8	1200	72.12	93.16
EXPO	100	-	CONST	5	-	YES	1000	8	1200	66.03	91.12

Table F.4: Comparisons of customer service level and vendor service level of the (s, S) policy

Demand p.u.t.			Lead-time			Back-order	Inventory Parameters			VSL	CSL
Type	Mean	S.Dev	Type	Mean	S.Dev		s	S	Period	(%)	(%)
NOR	100	30	NOR	5	1	NO	1050	1500	8	97.58	99.73
NOR	100	20	NOR	5	1	NO	1050	1500	8	98.79	99.90
NOR	100	10	NOR	5	1	NO	1050	1500	8	98.79	99.91
NOR	100	30	NOR	5	1	YES	1050	1500	8	97.98	99.75
NOR	100	20	NOR	5	1	YES	1050	1500	8	98.39	99.83
NOR	100	10	NOR	5	1	YES	1050	1500	8	98.99	99.94
GAM	100	30	NOR	5	1	NO	1000	1400	6	99.40	99.93
GAM	100	30	NOR	5	1	YES	1000	1400	6	98.95	99.90
LOGNOR	100	20	GAM	6	1.5	NO	1000	1800	8	41.89	91.08
LOGNOR	100	20	GAM	6	1.5	YES	1000	1800	8	53.12	89.66
UNI	100	28.9	POI	5	-	NO	1100	1500	8	88.91	98.49
UNI	100	28.9	POI	5	-	YES	1100	1500	8	89.11	98.17
EXPO	100	-	CONST	5	-	NO	1400	2000	8	93.17	98.34
EXPO	100	-	CONST	5	-	YES	1400	2000	8	92.71	98.46

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