TECHNIQUES FOR EQUALISATION OF SPEECH CHANNELS FOR HIGH-SPEED DATA TRANSMISSION.

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Summary.

The main purpose of this project was to identify equalisation strategies which would facilitate efficient transmission of data signals over channels primarily intended for the transmission of analogue speech signals. Such channels include leased "private-wire" and switched telephone network speech paths and h.f. radio speech channels.

To this end, a comparative study was made of the relative merits of various transversal filter configurations as the basis for an automatically adjustable data channel equaliser. Automatic pre-set and continuously adaptive coefficient setting strategies were considered and their performances compared. Computer simulation was used wherever possible, thus avoiding much of the effort and expense involved in the construction of actual physical models.

The growing use of large-scale integration as a manufacturing process led to special consideration being given to modes of operation which could be implemented using almost entirely digital techniques. As a result of this work, a design has been produced for a highly efficient adaptive equaliser which converges rapidly to its optimum condition. The design uses digital techniques and is based on a modular concept. This makes it extremely flexible in application and yet convenient for implementation using L.S.I. techniques.

Acknowledgments.

The major part of the work described in this thesis was carried out whilst the author was with Standard Telecommunication Laboratories Ltd. Harlow.

Thanks are due to the management of S.T.L. for permission to submit this work for a higher degree, and especially to Mr.A.D.Odell, who acted as external supervisor for the project.

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The author is also grateful to Prof. J.E.Flood of the University of Aston in Birmingham for the encouragement and helpful guidance he has given throughout the project in his capacity as internal project supervisor.

TABLE OF SYMBOLS USED.

N = Theoretical maximum symbol transmission rate (Nyquist Rate).

A = Maximum alphabet size.

$\dot{R}$ = Theoretical maximum bit transmission rate.

D = Intersymbol interference.

D_0 = Unequalised channel intersymbol interference.

D_{min} = Minimum value of intersymbol interference attainable.

D_q = Intersymbol interference due to quantisation.

t = Time.

T = Symbol sampling interval.

m, n, x, i — indexing integers and variables.

$h(t)$ = Data channel single-pulse response.

$g(t)$ = Equaliser single-pulse response.

$y(t)$ = Overall single-pulse response of equaliser + channel.

$h'(t)$ = Intermediate single-pulse response.

$f_x(t)$ = Linear network response function.

K_x = Coefficient value.

j = Anticipatory dispersion of single-pulse response.

k = Trailing dispersion of single-pulse response.

L = Total number of equaliser taps.

p = Number of equaliser taps before reference tap.

q = Number of equaliser taps after reference tap.

Z = Unit advance operator.

s = Complex frequency transform variable.

α = Real part of s.

ω = Imaginary part of s.

$H(\omega)$ = Data channel frequency domain characteristic.

$G(\omega)$ = Equaliser frequency domain amplitude characteristic.

$\phi(\omega)$ = Equaliser frequency domain phase characteristic.

Δ = Increment step size.

C = Convergence coefficient.

e_i^2 = Intersymbol variance.

e_m = Mean error.

ω_b = Upper limit on band-width.

$X(t)$ = Data channel input signal.

$Y(t)$ = Equaliser input data signal.

$Z(t)$ = Equaliser output data signal.

$N(t)$ = Data channel additive noise signal.

$E(t)$ = Error signal.

P_s = Signal power.

P_n = Noise power.

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1. INTRODUCTION

1.1. Background.

There has been a rapid increase in recent years in the use of large, centrally-based computers to process data originated at locations remote from the main computing centre. The ability to obtain direct access to the computer from the remote location is thus a highly desirable feature; in fact the whole concept of centralised control depends on the availability of just such a facility[1.1]. Since the public telephone network already enables connections to be established to almost every part of the civilised world, it is not surprising that this is being widely utilised for the transmission of such data[1.2].

Furthermore, in remote areas, or at sea, where it is impossible to obtain direct telephone connection, it is still nevertheless possible to establish connection via radio-telephone. More recently, therefore, there has also been a growing interest in the possibilities of utilising radio-communication voice channels in a similar way for the transmission of data[1.3, 1.4].

For economical utilisation of both the transmission and the computer access facilities, it is desirable that the data transfer is effected as quickly as possible. Because of this, there is a great deal of interest in techniques for attaining the maximum possible data transmission rate over channels which are primarily intended for the transmission of analogue speech signals.

1.2. Project Motivation.

The origin of the work described in this thesis was the design of an automatic equaliser for use in conjunction with a vestigial side-band high-speed data modem for transmission over leased "data quality" telephone line connections[1.5]. This original design, which, because of commerical urgency, was largely based on existing techniques, is currently being engineered for quantity manufacture and commerical exploitation. It was acknowledged, however, that a number of deficiencies existed in this design and, as a result, a full study of automatic equalisation techniques was initiated.

1.3. Objectives of study.

The main purpose of this study was to identify those equalisation strategies which would give near optimum utilisation of the communication channel, whether leased data quality line[1.6], h.f. radio channel or a connection through the normal switched telephone network. However, concurrent with this study there have been considerable advances in technology which have made large-scale integration a commercially attractive method of implementation. Since large-scale integration is most applicable where signal processing is carried out using digital techniques, the emphasis of this work has changed during its course to consider more especially those modes of operation which offer the greatest advantages when implemented entirely digitally.

Because high data transmission rates are normally associated with a shorter channel occupancy, the time required to aquire and condition the channel becomes increasingly more significant as data rates are increased. It is therefore important that any study of techniques to achieve higher data rates also considers techniques for faster channel aquisition. These problems have thus been considered as complementary to one another throughout the course of this work.

1.4. Typical Channel Characteristics.

Extensive use has been made of computer simulation to assess the relative performances of the various equalisation strategies and equaliser configurations considered in the course of this work. A number of typical equivalent base-band channel characteristics have been computed for use in conjunction with the simulation programmes. These represent a wide variety of v.s.b. modem implementations used in conjunction with a selection of leased 'data quality' lines. The line characteristics used were actual measured characteristics obtained on lines supplied by the G.P.O. Details of each channel, together with its computed single-pulse response and the associated intersymbol interference D_0 (defined in section 2.3), are given in appendix 1. Details of the line characteristics and the method of computation are also given in this appendix. For ease of identity, each channel characteristic has been given a designatory code, which is used for reference purposes throughout the text of this thesis.

The results given in appendix 1 are summarised, for convenience, in table 1.1. The table indicates that binary transmission is possible without equalisation at 2400 symbols/s, but at higher transmission rates it is generally necessary to provide equalisation to support even binary transmission. A comparison of the results for 2400 symbols/s. transmission indicates that the expected increase in intersymbol interference, arising from the use of filters with steeper "roll-off" characteristics, is more than off-set by the attendant restriction of the line signal spectrum to the portion of the line characteristic having the least amount of group-delay distortion.

Because of lack of information, previous workers have frequently resorted to the use of "idealised line" characteristics[e.g 1.7]. Unfortunately, such a simplification can well lead to misleading results. By using actual characteristics, it has been possible, in this work, to obtain more reliable indications of expected performance. To avoid the possibility of drawing erroneous conclusions through lack of generality, a range of characteristics has been used, rather than a single "typical" characteristic.

SYMBOL RATE Symbols/s.	CARRIER FREQUENCY Hz.	SHAPING FILTER ROLL-OFF %	V.S.B. FILTER ROLL-OFF Hz.	LINE SIGNAL SPECTRUM BAND-WIDTH. Hz.	UNEQUALISED INTERSYMBOL INTERFERENCE D_o					MODEM PARAMETER DESIGNATION
					L1	L2	L3	L4	L5	
2400	2400	50	600	600 - 3000	0.5521*	0.2672*	0.4215*	0.3964*	0.7637*	A
2400	2200	25	200	700 - 2400	0.5307	—	0.4265	—	0.4136	B
3200	2600	25	300	600 - 2900	0.7230	—	—	—	1.6311	C
3200	2800	25	200	600 - 3000	—	—	—	—	2.4543	D
3600	2700	50	300	0 - 3000	1.4688	—	1.1597	1.2043	2.4353	E
3600	2550	25	300	300 - 2850	2.4484*	0.7120*	—	—	—	F

Details of Lines L1 - L5 and Modem Parameters A - F are given in appendix 1.

Table 1.1. Unequalised Intersymbol Interference for Various V.S.B. Modem Implementations.

(* Indicates that equivalent base-band characteristics are included in Appendix 1.)

2. HIGH-SPEED DATA TRANSMISSION - BASIC CONSIDERATIONS.

2.1. Theoretical Maximum Rate of Transmission.

The two factors which determine the upper bound to the rate of transmission are the channel transmission characteristics and the noise encountered on the channel. The theoretical maximum rate (N) at which data symbols may be transmitted over a channel is a function of the channel bandwidth and is known as the Nyquist Rate[2.1]. Each symbol, however, may be any one from an alphabet of symbols. The maximum size of alphabet that can be used depends on the noise present in the channel, which limits the discrimination of one symbol from another. If the maximum size alphabet contains A symbols and there is no redundancy in the bit-to-symbol encoding, the maximum rate of transmission of digital data is given by:

$$R = N \log_2 A \quad \text{bits/s.} \qquad \ldots\ldots\ldots\ldots (2.1)$$

In an amplitude-modulated system, the number of symbols in the alphabet corresponds to the number of amplitude levels distinguished in the transmitted signal. A similar correspondence exists for other modulation systems.

Nyquist[2.1] has shown that the theoretical maximum rate at which distortionless transmission can be achieved over a base-band channel is twice the cut-off frequency of the channel. In order to attain this theoretical limit, however, the channel has to be conditioned to have an "ideal filter" characteristic. This characteristic is not attainable in practice and the best approximation is the channel whose attenuation/frequency response possesses vestigial symmetry about a frequency equal to half the data symbol transmission rate and whose associated phase/frequency response is linear. This means that some excess bandwidth must be provided for the equivalent filter characteristic "roll-off".

The process of conditioning the available bandwidth to exhibit the required characteristics is the process known as equalisation.

In general, the smaller the excess bandwidth utilised the more complex the modem filters and the equaliser become. There is thus an economic compromise for a given channel between maximum rate of transmission and equipment complexity.

2.2. Modulation.

The frequency spectrum of a basic digital data stream normally extends to zero at its lower frequency bound. Since, in general, a telephone speech channel does not extend to zero frequency, but has a lower frequency cut-off in the region of 300 Hz., it is necessary to use modulation to enable the lower frequency components of the spectrum to be transmitted.

A detailed study of modulation schemes has not been carried out as part of this work, as this has already been carried out elsewhere[2.2]. Because of the differing side-bands generated by the various modulation processes, some modulation schemes yield better band-width utilisation than others. Two schemes are generally regarded as giving good bandwidth utilisation, namely, vestigial-sideband amplitude-modulation and phase-shift keying[2.3]. The choice between these two modulation schemes has been the subject of international discussion[2.4] and it is likely that both these schemes will be generally adopted for high-speed data transmission.

The phase-shift keying (p.s.k.) modulation scheme may be considered as equivalent to two independent double-sideband amplitude-modulated channels, one being modulated onto an in-phase carrier component and the other onto a quadrature carrier component[2.2]. Since both the vestigial side-band (v.s.b.) and double side-band (d.s.b.) modulation processes linearly transform the channel characteristics into the base-band frequency domain, it is possible to consider the modulation - demodulation equipments (Modems) and the transmission medium together in terms of its equivalent linear base-band channel as shown in fig. 2.1.

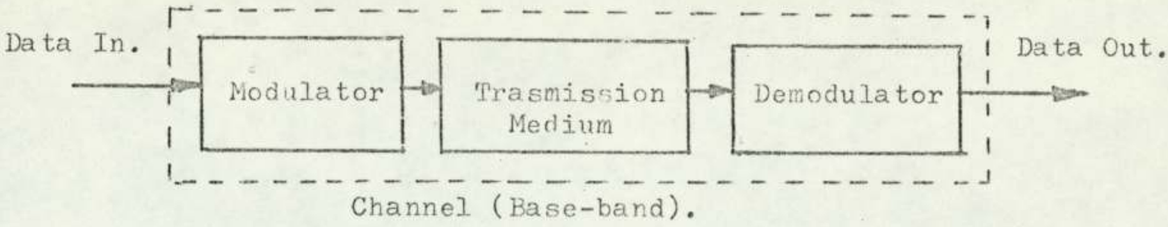


Fig. 2.1. Equivalent Base-band Channel.

The v.s.b. modulation scheme consists of a single equivalent base-band channel whereas the p.s.k. scheme consists of two identical equivalent base-band channels.* Because these equivalent base-band channels are linear, it is possible to carry out the channel conditioning (equalisation) at base-band using simple linear network techniques. This is of considerable advantage where automatic equalisation is required, as will be seen from the subsequent chapters.

2.3. Effect of Channel Characteristics on Transmitted Symbol.

If a data symbol is transmitted through a network, the shape of the symbol envelope is modified as a result of attenuation and phase-shift of the various frequency components of the transmitted signal. The extent of this distortion depends on the transmission characteristics of the network, but in general the distorted response is dispersed in time and may extend over a period equal to several symbol epochs. As consecutive data symbols are transmitted they will therefore overlap each other and intersymbol interference will arise. Because high-speed data is transmitted as a synchronous stream of data symbols, it is usual to examine the received signal on a time-sampled basis. It is thus the values of the data symbol response at the sampling instants of adjacent symbols that determines the quality of the channel for the transmission of digital data. The total

*Some cross-modulation will also be present.

intersymbol interference at the sampling instants of adjacent symbols would therefore seem to constitute a reasonable measure of the channel quality.

The intersymbol interference, D, may thus be defined as:

$$D = \frac{1}{h(0)} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} |h(nT)| \quad \ldots\ldots\ldots \quad (2.2)$$

where h(t) is the response to a single pulse representing one transmitted symbol, T is the sampling interval and the time origin is chosen so that t = 0 represents the instant at which h(t) is a maximum (See fig.2.2.).

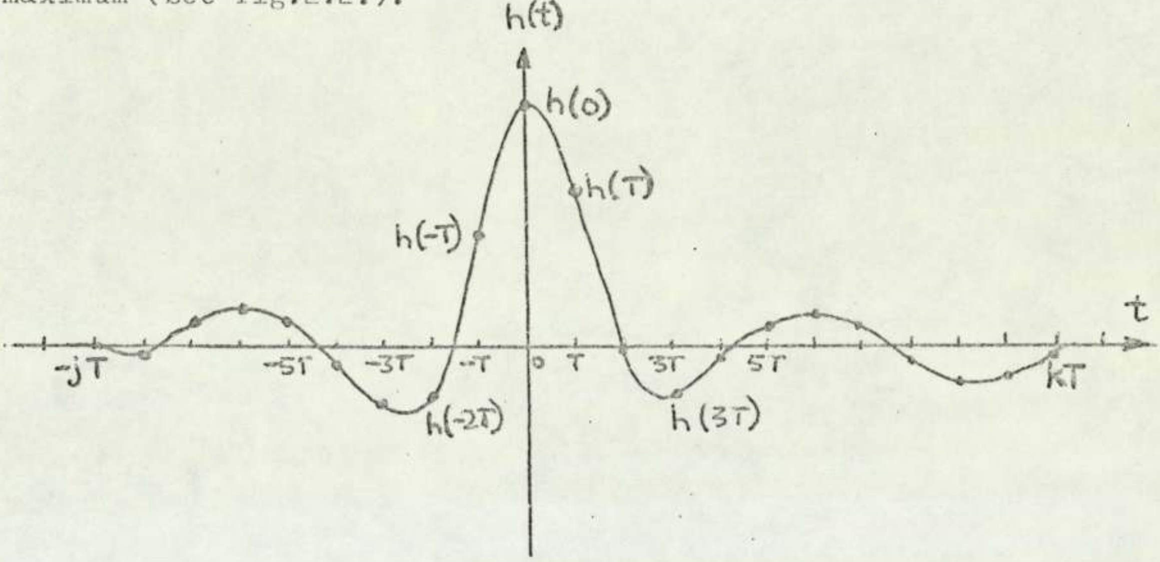


Fig. 2.2. Single-Pulse Response, showing values at Sampling Instants.

In practice, the response will be negligible for $-j > n > k$, whence (2.2) becomes the finite summation:

$$D = \frac{1}{h(0)} \sum_{\substack{n=-j \\ n\neq 0}}^{k} |h(nT)| \quad \ldots\ldots\ldots \quad (2.3)$$

The symbol D_0 is used throughout this thesis to denote the actual value of intersymbol interference of a channel before equalisation is applied.

The number of symbol periods over which the samples in the single-pulse response remains significant, (j+k), is referred to as the dispersion of the response.

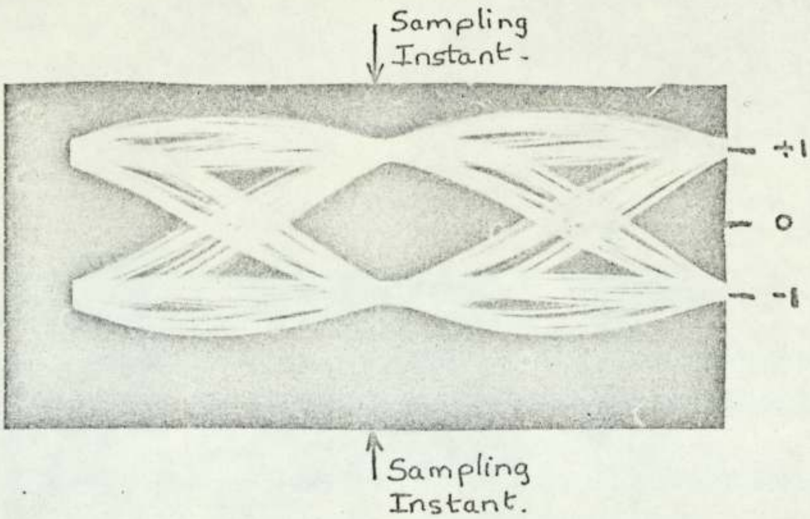
2.4. The Eye Pattern.

The eye pattern[2,5] is a convenient method of displaying on the oscilloscope the effect of intersymbol interference on the received data stream. The received data stream amplitude voltage is used to deflect the oscilloscope trace vertically and the horizontal sweep is synchronised to the data symbol rate. The resulting display for a random binary bipolar data stream is shown in fig.2.3(a). The optimum sampling instant in the symbol period occurs where the 'eye' is open widest.

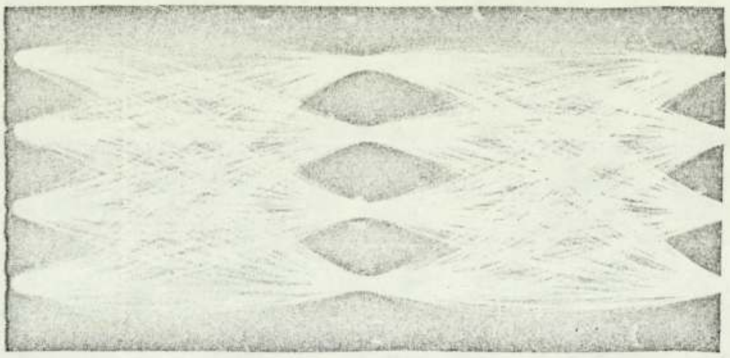
It can be seen that, in the absence of intersymbol interference or noise, all symbol amplitudes would pass through one of two single points at the symbol sampling instant. As the intersymbol interference increases, there is a widening of the band formed by the sample amplitudes at the sampling instant and the eye begins to close. The binary eye is just closed when, under worth case sequence conditions, there is a translation of the symbol amplitude at the sampling instant from its optimum value to zero level. This is equivalent to the condition $D = 1$. For $D > 1$, the binary eye is completely closed and the sampling instant transitions may now be of the opposite polarity to that of the associated data sample. Hence, it becomes impossible to decipher the data simply by slicing the signal about the zero level.

Figs.2.3(b) and 2.3(c) show 4-level amplitude-modulated data eye patterns for different values of D. The eye-closing effect of the larger value of D associated with fig.2.3(c) is evident from these photographs. The 4-level eye is completely closed for $D \geqslant 1/3$. In general, the eye pattern for an A-level amplitude-modulated system is closed for $D \geqslant 1/(A - 1)$.*

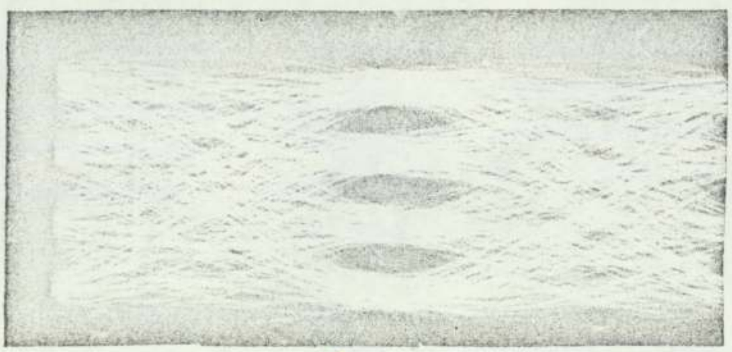
*assumes fixed peak signal power.



a) The Binary Eye Pattern.



b) 4-level Eye Pattern.



c) 4-level Eye Pattern, Increased D.

Fig.2.3. EYE PATTERNS.

The eye pattern is extremely useful for making practical assessments of the effect of intersymbol interference in working data transmission systems.

3. TRANSVERSAL EQUALISERS - GENERAL DESCRIPTION.

There is a wide variety of network configurations which can be used for the synthesis of equalisers for channels which have pre-determined and time-invariant transmission characteristics. In most cases, however, the interdependence of the parameters of these networks render them quite unsuitable for adjustable operation.

We have already seen that optimisation for synchronous data transmission requires the adjustment of the sampled single-pulse response in the time domain and the transversal filter[3.1] is eminently suitable for this type of adjustment. The basic transversal filter configuration is given in fig.3.1. It consists of a tapped delay-line with multipliers connected to each of the tapping points on the line. The outputs from the multipliers are added together in the summing network to form the output signal. The second inputs to the multipliers are the adjustable coefficients K_x, which determine the characteristics of the filter. Some variations on the basic transversal filter configuration are considered in the following section.

The mathematical analysis of the transversal equaliser is best carried out using the Z transform concept[3.2] and extensive use will be made of it in subsequent sections. The operator Z^{-1} ~~represents~~ corresponds to a unit delay T, so that the single-pulse response of fig.2.2. may be expressed in terms of Z as:

$$h(Z) = \sum_{n=-j}^{k} h(nT).Z^{-n} \quad \ldots\ldots\ldots\ldots (3.1)$$

The right-hand expression in equation (3.1) is thus the Z transform of h(t).

3.1. Transversal Filter Configurations.

The transversal equalisers considered in this chapter are

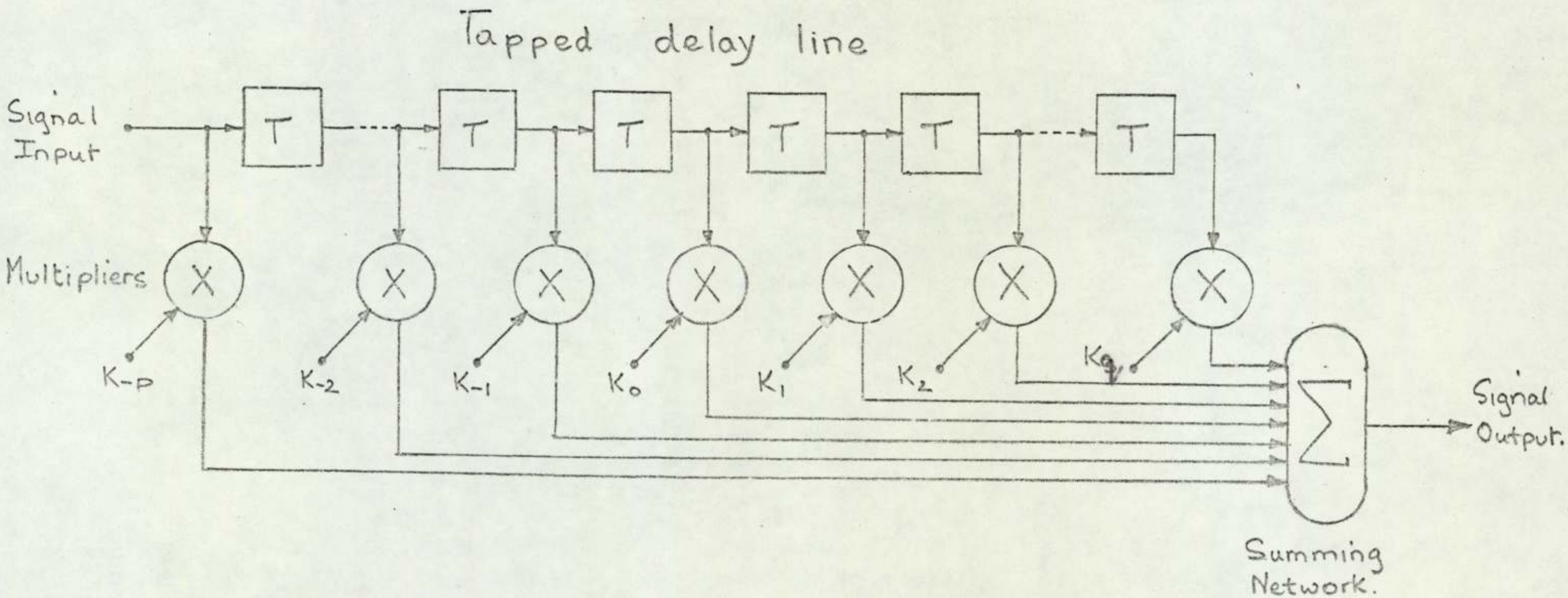


Fig 3.1. Transversal Equaliser – Basic Transversal Filter Configuration.

based on delay-lines, where the unit delay T is equal to one symbol period. The possibility of using delay-lines where the tapping points occur at other than unit symbol periods along the line are considered in chapter 6. The use of other networks as an alternative to pure delay is also considered in this later chapter.

There are basically four configurations in which the unit delay modules may be arranged.

a) <u>The Basic Transversal Filter. (Fig.3.1).</u> This is the simplest and the most widely used configuration. The sampled single-pulse response of the basic transversal filter, expressed as a function of Z, is:

$$g(Z) = \sum_{x=-p}^{q} K_x . Z^{-(x+p)} \quad . . . .(3.2)$$

The overall sampled single-pulse response of the channel + equaliser $\quad y(Z) = h(Z).g(Z)$

$$= \sum_{x=-p}^{q} K_x . Z^{-(x+p)} \sum_{n=-j}^{k} h(nT).Z^{-n}$$

$$= \sum_{m=-j}^{k+p+q} y(mT).Z^{-m} \quad . . . .(3.3)$$

For mathematical simplicity, the units of y(t) are normalised so that the equalised single-pulse amplitude is unity at the main sampling instant. Then, for minimum intersymbol interference D, we adjust the K's so that $y(pT) = 1$ and

$$\sum_{\substack{m=-j \\ m \neq p}}^{k+p+q} |y(mT)| = \text{minimum} \quad . . . .(3.4)$$

Lucky[3.3] has shown that, for $D_0 < 1$, this condition is satisfied when: $y(mT) = 0$ for $0 \leqslant m \leqslant p+q$, $m \neq p$

$y(mT) = 1$ for $m=p$. *

Hence, the values of the K's are given by the solution of the following set of simultaneous equations, where $h_x = h(xT)$:

$$\begin{bmatrix} h_0 & \cdots & h_{-p} & \cdots & h_{-p-q} \\ \vdots & \ddots & \vdots & & \vdots \\ h_{p-1} & & h_{-1} & & h_{-1-q} \\ h_p & \cdots & h_0 & \cdots & h_{-q} \\ h_{p+1} & & h_1 & & h_{1-q} \\ \vdots & & \vdots & \ddots & \vdots \\ h_{p+q} & \cdots & h_q & \cdots & h_0 \end{bmatrix} \cdot \begin{bmatrix} K_{-p} \\ \vdots \\ K_{-1} \\ K_0 \\ K_1 \\ \vdots \\ K_q \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad . . (3.5)$$

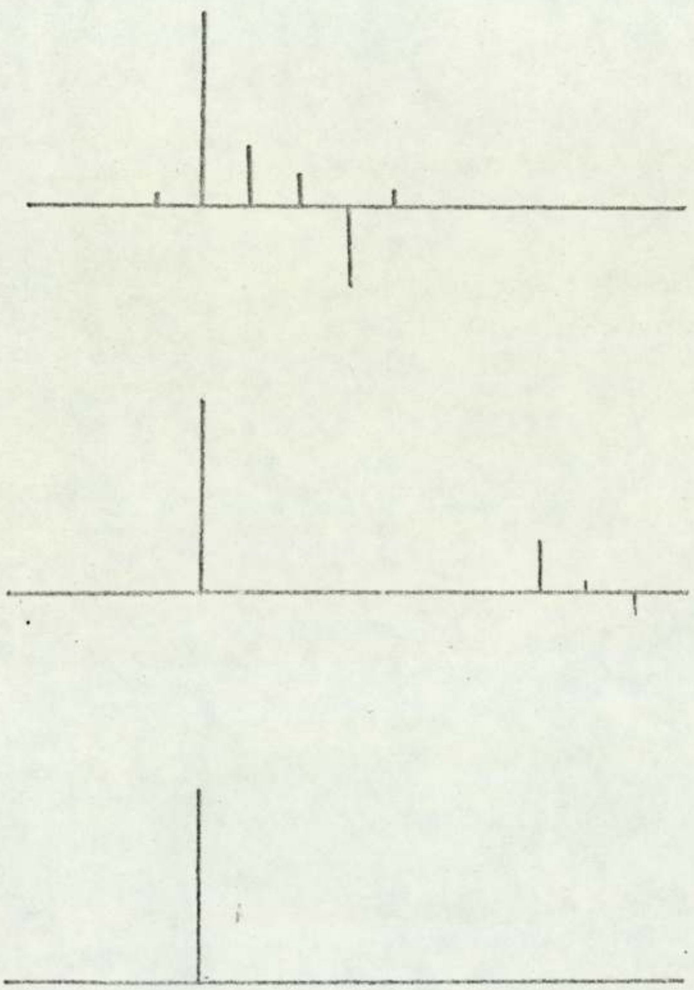
The basic transversal filter configuration is thus particularly useful for systems with small distortion ($D_0 < 1$) and limited dispersion, since, under these conditions, the multiplier coefficients are easily computed from these equations. Unfortunately, the dispersion in the equalised single-pulse response is increased by one unit symbol period for each delay module added to the equaliser. This can be seen from the range of the summation in equation 3.3, which shows that the dispersion of the single-pulse response is increased from $j+k$ to $j+k+p+q$ by the operation of an equaliser having $p+q$ delay modules. The magnitude of the distortion associated with this added dispersion can be very significant when the distortion D_0 and the dispersion of the single-pulse response to be equalised are initially large, and may decrease quite slowly as the number of delay modules is increased.

Thus, the number of delay modules, including the associated coefficient stores and multipliers, required

*This strategy is widely known as "Zero-forcing."

to achieve a given degree of equalisation increases quite rapidly with increase in distortion D_0 and in dispersion in the single-pulse response of the channel before equalisation. This is clearly demonstrated by the tests described later in section 5.3. To combat this disadvantage, the recursive[3.4] and decision feed-back[3.5] configurations have been suggested.

An example illustrating the advantage that can be gained by using the recursive configuration is given in fig.3.2.



a) Initial Response.

$D_0 = 0.98$

b) Feed-forward Equaliser.

$D = 0.45$

c) Recursive Equaliser.

$D = 0.0$

Fig.3.2. Equalisation of Typical Response (R1K) using 13 Tap Equaliser. ($p = q = 6$).

In each case a 13 tap equaliser is used to equalise a channel (R1K of appendix 1) having an unequalised intersymbol interference of 0.98. Using the normal feed-forward equaliser, the value of intersymbol interference attained is 0.45, whereas, with the recursive equaliser, the residual intersymbol interference is so small that it cannot be measured.

b) <u>The Recursive Transversal Filter.</u> In the basic transversal filter, the connection of the outputs from the coefficient multipliers into the summing network is made in what is normally referred to as the "feed-forward" arrangement. This title is suggested by the alternative "feed-back" arrangement which is shown in fig.3.3. The general recursive equaliser consists of both a feed-forward and a feed-back equaliser section in combination.

The sampled single-pulse response of the feed-back arrangement is:

$$g(Z) = \frac{K_0}{1 - \sum_{x=1}^{q} K_x . Z^{-x}} \quad \ldots\ldots\ldots (3.6)$$

and the overall single-pulse response of the channel + equaliser becomes: $y(Z) = h(Z).g(Z)$

$$= \frac{K_0}{1 - \sum_{x=1}^{q} K_x . Z^{-x}} \cdot \sum_{n=-j}^{k} h(nT).Z^{-n} \quad . . (3.7)$$

One the face of it, it would seem that, providing $j+k \leqslant q$, it it possible to select the K's so that the single-pulse response $y(Z)$ reduces to the single (negative) delay Z^j; that is, it is possible to obtain perfect equalisation. The difficulty lies in the fact that, with the feed-back arrangement, it is possible to obtain unstable networks.

The Shifting Theorem defines the relationship between the unit advance operator Z and the complex frequency transform variable s as: $Z = e^{sT}$ . . . .(3.8)

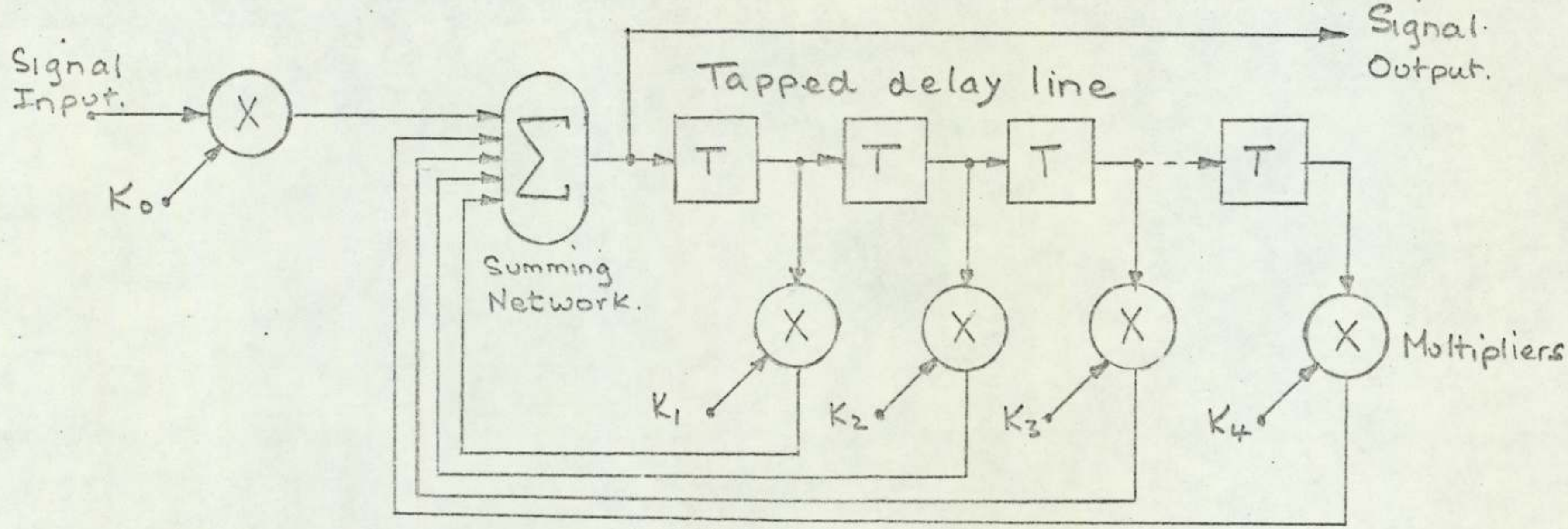


Fig 3.3. Feed-Back Transversal Filter Configuration.

The imaginary axis of the s plane thus maps conformally into the unit circle on the Z plane. The network stability criterion that all poles of $g(s)$ must lie in the left half-plane is therefore equivalent to the condition that all poles of $g(Z)$ must lie within the unit circle.

The fact that the value of any K_x, $1 \leqslant x \leqslant q$, in equation (3.6) is equal to or greater than unity means that the function $g(Z)$ ~~to~~ may have a pole outside the unit circle. Such a condition would arise if an attempt was made at perfect equalisation, except for the case where $h(-jT)$ is the largest sample in the function $h(nT)$. However, by definition, $h(0)$ is the largest sample in the function. It is thus only possible to obtain perfect equalisation where $j=0$, that is, where there are no anticipatory samples in the single-pulse response. In all other cases, an attempt at perfect equalisation would result in instability.

The feed-back filter can be used, however, to completely equalise the trailing response, providing $q \geqslant k$, by adjusting K_0 to normalise the amplitude of the sample $y(0)$ and K_x, $1 \leqslant x \leqslant q$, so as to force the response sample $y(xT)$ to zero. The remaining anticipatory response can then be separately equalised by means of an additional feed-forward filter, as shown in fig.3.4.

The single-pulse response of the combined feed-forward and feed-back network is: $g(Z) = g_f(Z).g_b(Z)$

$$= \frac{\sum_{x=-p}^{0} K_x . Z^{-(x+p)}}{1 - \sum_{y=1}^{q} K_y . Z^{-y}} \quad \ldots\ldots\ldots\ldots (3.9)$$

where $g_f(Z)$ is the response of the feed-forward network and $g_b(Z)$ is the response of the feed-back network. The

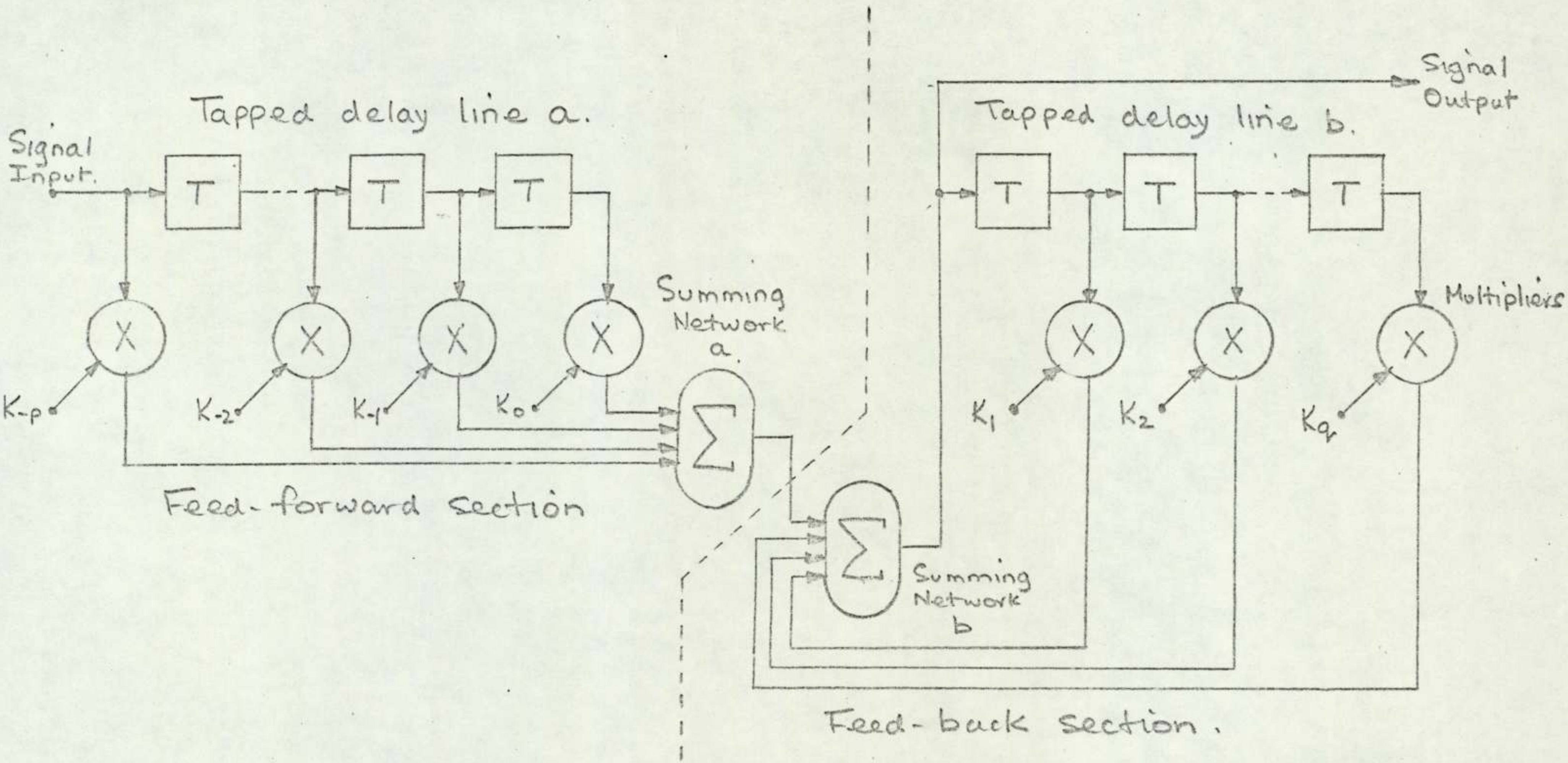


Fig 3.4. Recursive Filter Configuration A

overall response of the channel + equaliser then becomes:

$$y(Z) = \frac{\sum_{x=-p}^{0} K_x . Z^{-(x+p)}}{1 - \sum_{y=1}^{q} K_y . Z^{-y}} \sum_{n=-j}^{k} h(nT) . Z^{-n} \quad . . (3.10)$$

If $q \geqslant k$, optimum equalisation of the trailing response will reduce this equation to:

$$y(Z) = \sum_{x=-p}^{0} K_x . Z^{-(x+p)} \qquad \sum_{n=-j}^{0} h'(nT) . Z^{-n} \quad . . . (3.11)$$

$$= \sum_{m=-j}^{p} y(mT) . Z^{-m} \quad . . . . (3.12)$$

$$\text{If } \frac{1}{h'(0)} . \sum_{n=-j}^{-1} |h'(nT)| < 1 \quad . . . . (3.13)$$

the values of K_x in equation 3.11 can then be determined from equation 3.5. An examination of the responses given in appendix 1 shows that both the magnitude and the dispersion of the anticipatory response are generally much smaller than those of the associated trailing response. The necessary condition given by the inequality 3.13 is therefore satisfied in most cases encountered in practice. Alternative criteria for coefficient optimisation, which can be employed in cases where this condition is not satisfied, are discussed in section 3.3.

It can be seen from equation 3.10 that it is of little consequence whether the signal is operated on first by the feed-forward network or first by the feed-back network. The former arrangement, as shown in fig.3.4, bears the closest physical resemblance to the basic transversal filter configuration because the anticipatory response is equalised by the taps nearest the equaliser input. It is apparent from the diagram that, since the intermediate single-pulse response h'(nT) of equation 3.11 is not of any practical interest, the two summing networks can be combined to give the configuration as shown in fig.3.5. The similarity between this configuration and that given in fig.3.1 now becomes even more apparent.

Because of its ability to equalise completely the trailing response, the recursive transversal filter configuration is extremely useful where D_0 is high due to the existence of large magnitude samples in the trailing response. Such a case exists, for example, in h.f. radio channels[3.6], where multipath effects can result in secondary pulses in the trailing response which have amplitudes comparable to that of the main single-pulse response sample. An equaliser for this purpose, using the recursive transversal filter configuration, is described in the literature[3.4].

c) The Canonic Recursive Transversal Filter Configuration.

The alternative arrangement, where the feed-back network operates on the signal before the feed-forward network, is shown in fig.3.6. An interesting feature of this arrangement is that both the delay lines contain the same signal, together with the same delayed versions. This arrangement can therefore be reduced to the canonic configuration shown in fig.3.7, with a considerable saving in component requirements. The optimum settings for the coefficient values are identical to those required for the configuration descibed in the previous section and can therefore be computed in exactly the same manner. Although the equivalence of these two configurations is well known in connection with digital filter design[3.7], the economy afforded by the latter configuration does not seem to have been exploited in connection with adjustable transversal equalisers for data transmission channels.

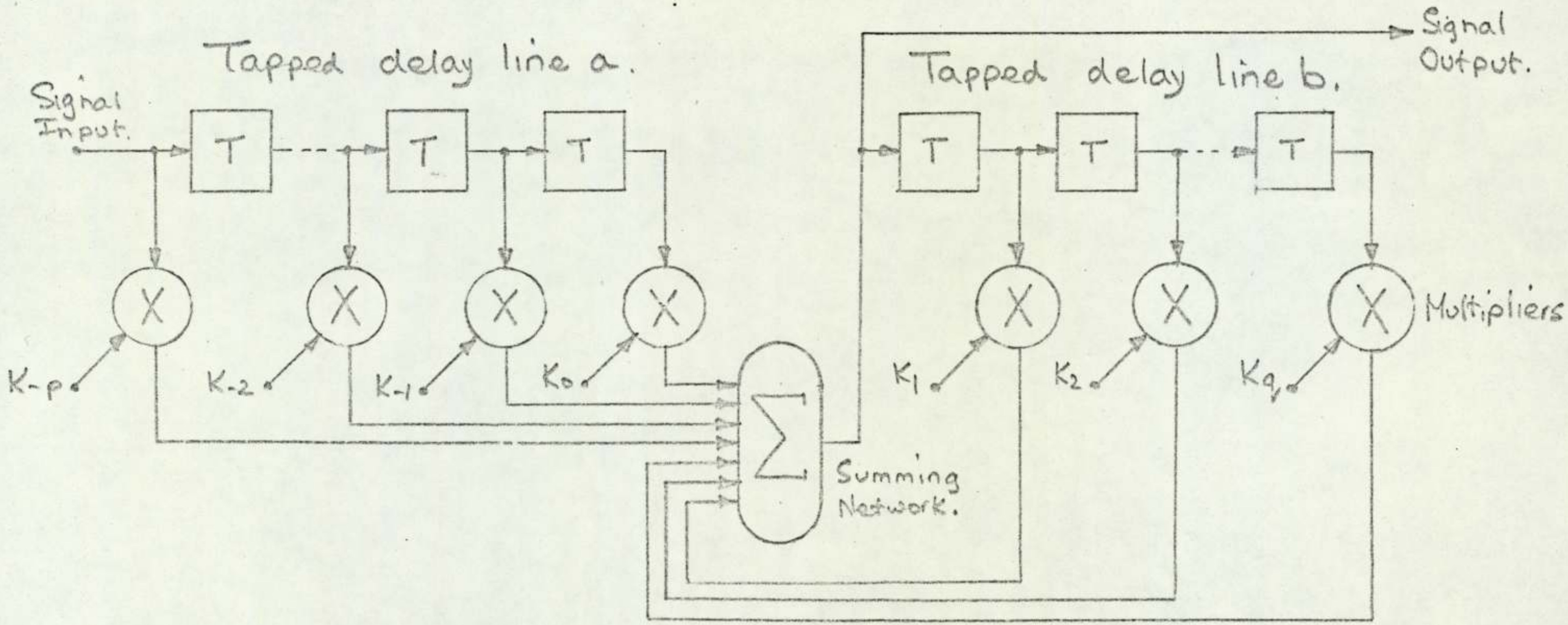


Fig. 3. 5. Transversal Equaliser — Recursive Filter Configuration A

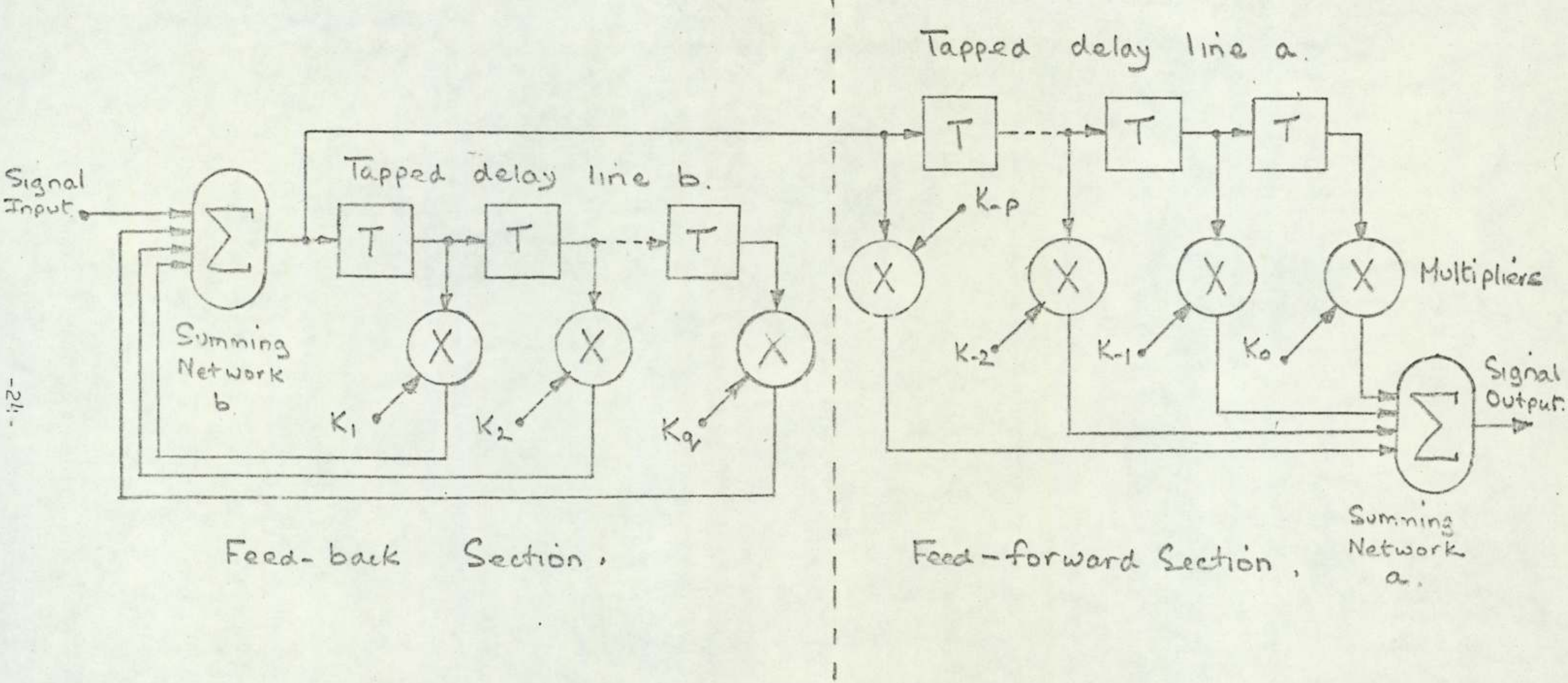


Fig. 3.6. Recursive Filter Configuration B

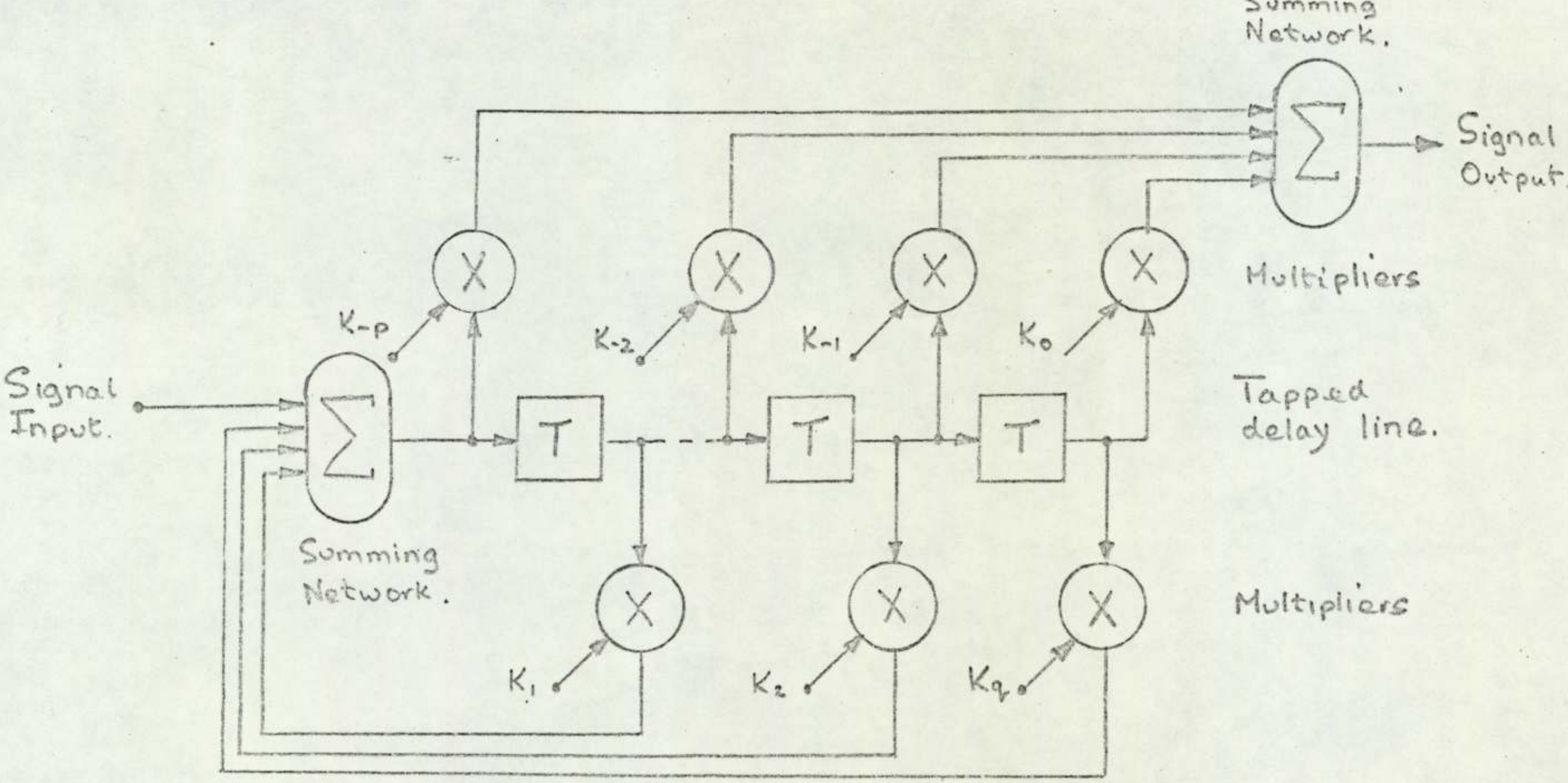


Fig. 3.7. Transversal Equaliser – Canonic Recursive Filter Configuration.

d) The Decision Feed-back Configuration. The schematic diagram of the decision feed-back equaliser[3.5] is shown in fig.3.8. It is a modification of the recursive filter configuration given in fig.3.5, the signal output being sampled and sliced in the signal detector circuit before it is fed into the second tapped delay line b. Since the decision feed-back equaliser is a non-linear system, it is less amenable to mathematical analysis than the equivalent recursive equaliser. In practice, however, the decision feed-back and the recursive equaliser are almost identical in behaviour once satisfactory equalisation has been attained because, under this condition, there is little difference between the signals at the input and the output of the signal detector.

3.2. Modes of Adjustment.

In the most elementary form of transversal equaliser, the coefficient multipliers consist simply of potentiometers which are adjusted manually to produce the required equaliser response[3.8]. Such an arrangement is satisfactory only where the same line and equipment are always used togther and the characteristics of the line and equipment are virtually time-invariant. In practice, such a situation is rare and the general requirement is for an automatically adjustable equaliser.

There are two possible basic modes of automatic adjustment for setting up the transversal equaliser multiplier coefficients. In the first mode, the coefficients are automatically pre-set to their correct value before the actual transmission of data begins[3.3]. In the second mode, the coefficient values are continuously adapted to their correct value throughout the course of the transmission of data[3.6]. This latter mode of operation is particularly useful where the channel characteristics vary significantly with time.

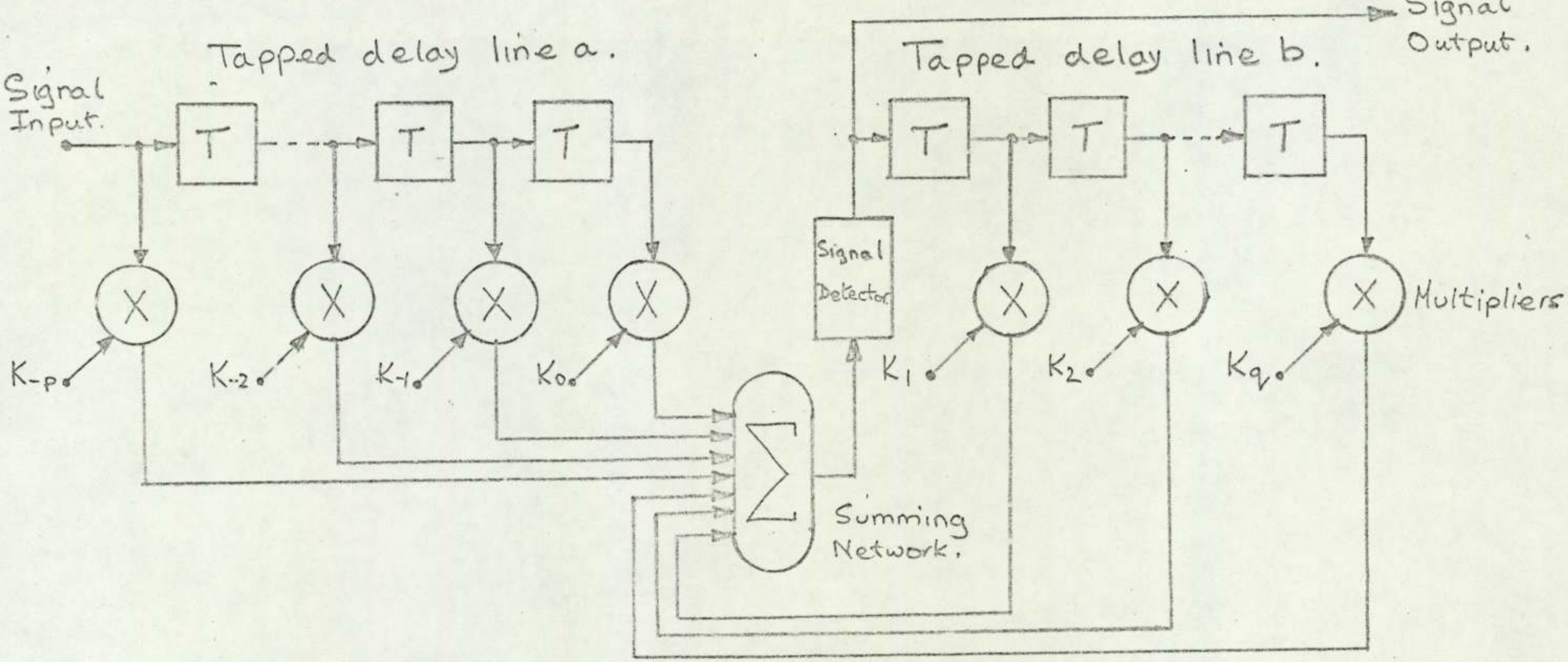


Fig. 3.8. Transversal Equaliser - Decision Feed-back Configuration

a) The Pre-set Mode of Operation. Fig.3.9 illustrates the automatic pre-set mode of operation, in which a training pattern is transmitted prior to the transmission of data. Since the training pattern is known a priori by the receiver, this knowledge can be used to enable the single-pulse response to be computed directly from the received signal. From the single-pulse response so obtained it is then possible either to calculate the tap coefficients directly or to use an iterative technique whereby the coefficients are successively incremented until an optimum value of inter-symbol interference is attained. Various strategies are considered in detail in chapter 6. A number of suitable training patterns exist and these are considered in detail in section 6.2.

b) The Adaptive Mode of Operation. The adaptive mode of operation is illustrated in fig.3.10. In this method, a measurement is made of the differences between the received signal and the receiver estimate of the transmitted signal. The error signal so obtained is correlated with the data stream to obtain estimates of the coefficient setting errors. The coefficients are then updated in accordance with these estimates so as to minimise the magnitude of the error signal. No training pattern is needed in setting-up, since no a priori knowledge of the transmitted data is assumed.

The advantage of this mode of operation is that the setting-up procedure is a continually adapting process and it is therefore capable of following and neutralising any changes in characteristic that may occur during the course of transmission.

The correctness of the error signal used to update the coefficients depends on the validity of the a posteriori estimate of the transmitted data at the output of the equaliser. It is therefore necessary for the received data at the output

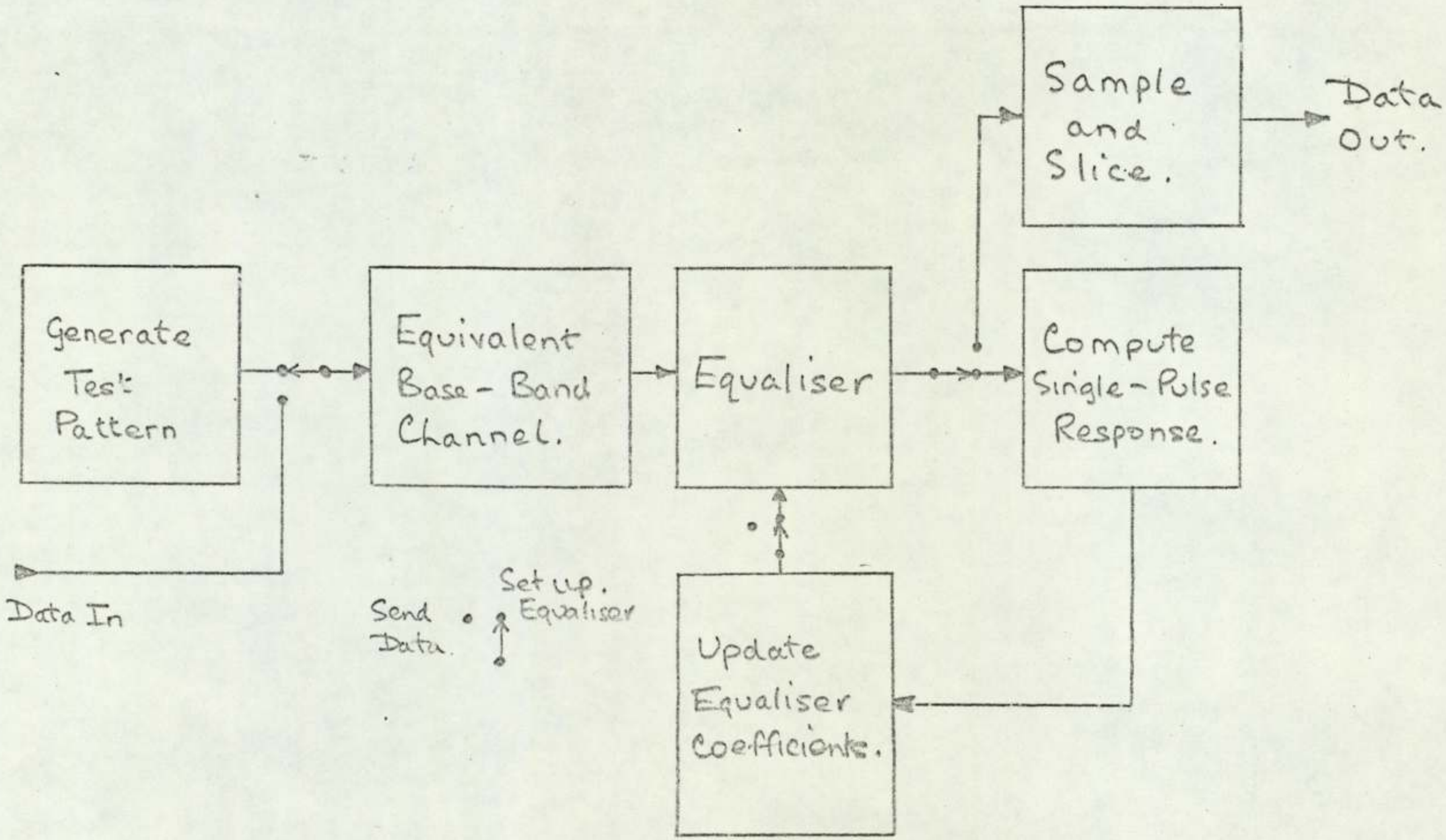


Fig. 3.9. Automatic Pre-Set Equaliser Strategy.

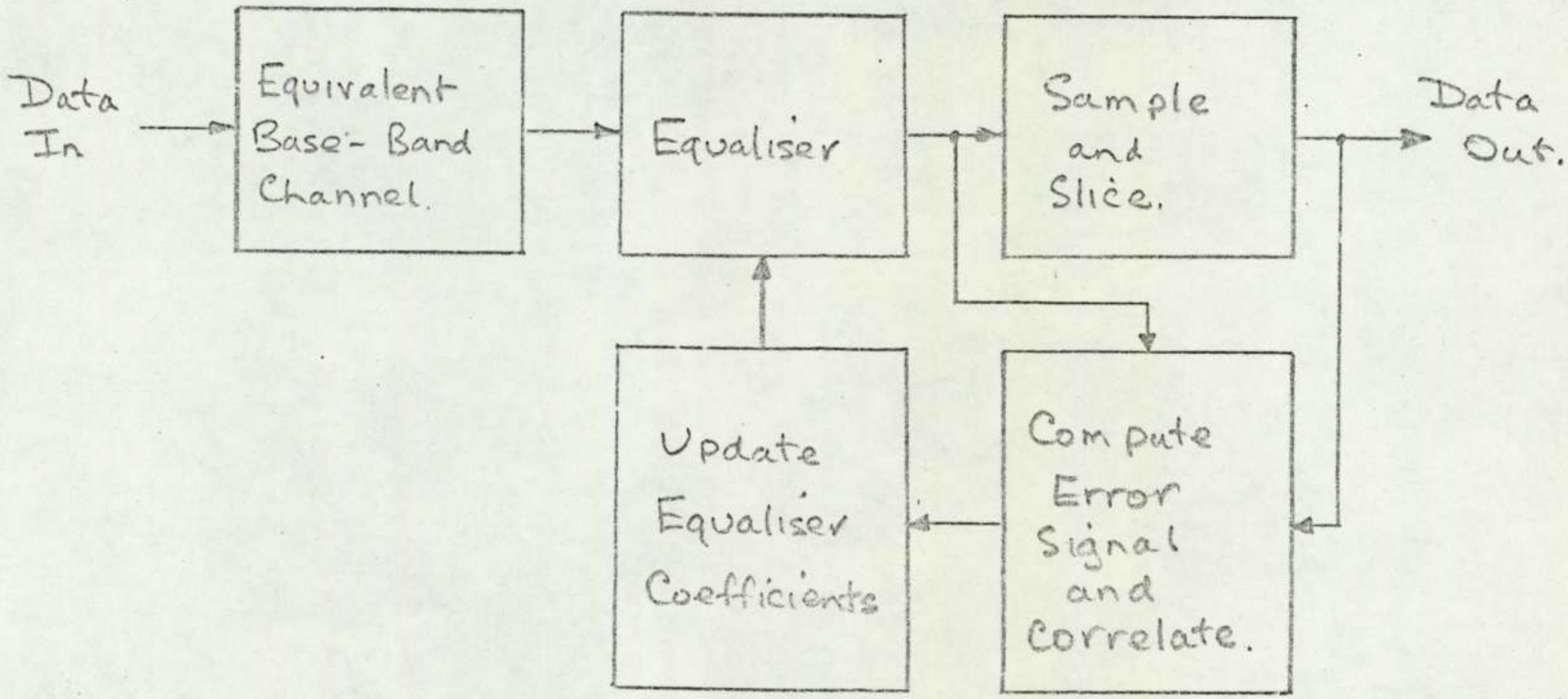


Fig. 3.10. Adaptive Equaliser Strategy.

of the equaliser to be substantially correct in order to be assured of the convergence of the adaptive strategy. The way in which this condition can be achieved depends on the amount of distortion present in the unequalised input signal. Where $D_0 < 1$, the channel is capable of supporting binary transmission and the initial equaliser setting can, if necessary, be obtained by simply commencing operation with a period of binary transmission with the equaliser coefficients initially set to zero, except the reference coefficient, which is set to unity. However, if $D_0 \geqslant 1$, then it is necessary to commence operation in the automatic pre-set mode until the channel is conditioned sufficiently to sustain substantially error-free transmission. The equaliser can then be operated in the adaptive mode so as to compensate for any changes in characteristic as they occur. A method for changing the mode of operation is described later in chapter 8.

3.3. Criteria of Adjustment.

The ultimate criterion we wish to achieve is the ability to transmit the data through the channel with the minimum of error. So far, it has been tacitly assumed that this criterion is achieved by minimisation of the intersymbol interference D. This assumption is not strictly true, however, for although in many cases it will yield the optimum condition, there are circumstances in which a smaller error rate could be achieved by optimisation to some other alternative criterion. The choice of criteria is, in fact, mainly dependent on the amount and character of the noise perturbations present in the system, although the choice may also be influenced somewhat by the unequalised channel characteristics.

However, in most cases the minimum D criterion gives a result that is so near to optimum that the comparative ease with which it can be implemented makes it the obvious choice. A brief summary of a selection of the criteria which are considered as being both useful and practicable is given here:

a) Minimum D. This is equivalent to maximum eye-opening and thus gives, under noise-free conditions, the maximum amount of tolerance in amplitude slicing level in the symbol decision circuit. Although the eye pattern may be open under noise-free conditions, it can be closed by the superimposition of noise perturbations on the data stream. Errors will occur under this condition and the noise-free minimum D criterion may not then necessarily give the conditions under which error rate will be a minimum. Also, although at the optimum sampling instant the eye may be widest open, the eye may close quite rapidly with mis-timing in the sampling. An alternative criterion may thus be more suitable where timing errors are likely to be encountered.

b) Minimisation of the Sum of the Squares of the Sample Values.

The multiplier coefficients are adjusted so that:

$$\sum_{\substack{m=-j \\ m\neq p}}^{k+p+q} y(mT)^2 = \text{minimum.} \quad \ldots\ldots\ldots \quad (3.14)$$

This equation should be compared with equation 3.4. The value given by the left-hand expression in equation 3.14 is sometimes referred to as the intersymbol variance[3.4] and is denoted by the symbol e_i^2. This criterion can be shown to give a lower error rate than would be obtained by minimum D in the presence of noise.

c) Minimisation of Mean-Square Error in Frequency Characteristic.

The multiplier coefficients are adjusted so as to minimise the mean-square error between the equalised channel frequency domain characteristic $H(\omega)$ and the ideal channel characteristic $G(\omega)$, that is[3.10]:

$$\int_{-\infty}^{\infty} \left| H(\omega) - G(\omega) \right|^2 d\omega = \text{minimum.}$$

For this criterion, it is not necessary for the delay of the delay-line sections to be a function of the symbol rate. It is therefore useful in the equalisation of non-synchronous systems. The ideal characteristic to which the channel is to be equalised must be specified to the equaliser. Since, for synchronous transmission, this ideal characteristic is a function of symbol rate, it is still necessary to re-equalise whenever this rate is altered.

It is only possible to approximate the ideal characteristic, because the number of coefficients limits the degrees of freedom in adjustment. The approximation will not in general yield the minimum D or minimum e_i^2 condition. It will not therefore give an optimally minimum error rate. It does, however, allow a general-purpose equaliser to be designed without the need for the ability to change the delay-line timing.

Adjustment Strategies.

A large variety of adjustment strategies exist for setting up the multiplier coefficients to achieve one or other of the adjustment criteria described in the previous section. In the automatic pre-set mode, these strategies may be classified into two broad classes:

a) Those which determine the channel single-pulse response and from this calculate directly the coefficients.

b) Those which determine the single-pulse response and increment the coefficient values on an iterative basis until the optimum condition is attained. Within this class a variety of methods of incrementing the coefficient values is possible. For instance, it is possible to increment the coefficients in pre-determined fixed steps. Alternatively, the step size can be variable in a way related to the significance of the error signal, so as to increase the rate of convergence of the adjustment strategy.

A number of strategies also exist for adaptive mode operation. Further consideration of the more important strategies is to be found in chapter 7.

The important features of an adjustment strategy are that it should be absolutely convergent and that it should converge with reasonable rapidity. For the adaptive equaliser, the rate of adaptation must be adequate to deal with changes in characteristic as they occur.

The convergence of some of the strategies that have been proposed in the literature has only been proved mathematically by assuming that the delay elements are ideal[e.g.3.3]. Such an assumption cannot always be justified in practice and some work has been carried out to determine the effect of various inadequacies in the delay elements on the convergence of certain strategies. Details of this work are given in chapter 6.

4. TRANSVERSAL EQUALISERS - SURVEY OF PUBLISHED WORK.

The transversal filter, patented by Blumhiem[4.1] in 1938, was first described in the literature by Kallmann[4.2] in 1940. Its first use as a time-domain variable equaliser was for video-frequency television signal correction and was described by Linke[4.3] as far back as 1952. Dr. Linke's work was carried out in the United Kingdom at the Post Office Research Station, Dollis Hill, London. Most of the difficulties in realisation described by Dr. Linke have been solved since that date by the vast progress that has occurred during the last decade in device technology.

In contrast to Linke's pioneering work, most of the significant published work on transversal equalisers for data transmission is of United States origin.

4.1. Work carried out at IBM Corporation.

One of the earliest records in the literature relating to the use of the transversal filter for the equalisation of digital data channels is a paper by Mohn and Stickler[4.4] published in October 1963. The paper describes an elementary digital transversal filter which can be used to achieve either pre- or post-transmission equalisation. The strategy described is a simple zero-forcing automatic algorithm which limits the usefulness of the equaliser to conditions where the initial distortion D_0 is less than unity. The same work was later reported in the IBM Journal of Research and Development, January 1965, in an article by Shreiner, Funk and Hopner.[4.5] All further work at IBM appears to be towards a computer software implementation of the basic automatic transversal filter concept. To this end, Meinster[4.6] published a paper in IBM Journal in July 1968 describing a fast mathematical algorithm for computing the optimal tap coefficient setting from the measured single-pulse response.

4.2. Work carried out at Bell Telephone Laboratories.

The majority of the literature relating to transversal equalisers is of Bell Laboratories origin. The first paper from Bell Laboratories appeared soon after the initial IBM paper and was published by Rappeport[4.7] in September 1964. The paper describes a computer simulation of an equaliser configuration similar to that proposed by IBM. A rather more sophisticated mathematical presentation is made than in the IBM papers. Subsequent to this publication, a number of papers have been published by Bell Laboratories, the names of R.W.Lucky and H.R.Rudin being associated with the most significant of these. The first paper by Lucky[4.8] is a very thorough treatment of the basic automatic preset transversal equaliser. Automatic iterative strategies are proposed for both the condition $D \leqslant 1$ and the condition $D \geqslant 1$. The absolute covergence of these strategies is proved analytically for the case where perfect delay-line sections are assumed.

In his next paper[4.9] Lucky describes an adaptive strategy for the basic transversal equaliser and again proves its convergence for the perfect delay-line case. Following this, Lucky and Rudin describe automatic pre-set[4.10,4.11] and adaptive[4.12] strategies for the basic transversal equaliser which minimise the mean-square error in the frequency-domain characteristic. This equaliser is thus suitable for non-synchronous data systems. At about the same time, Gersho[4.13] published a thorough mathematical treatment of an adaptive strategy for the basic transversal equaliser which minimises the mean-square intersymbol interference for synchronous data transmission.

The next two papers from Bell Laboratories[4.14,4.15] give details of physical realisations of the equalisers described theoretically in the earlier papers.

Work on transversal equalisers is still continuing at Bell Laboratories and more recently Chang has published several papers[4.16 to 4.18] which detail various improvements to the strategies described in the earlier work. The only paper to suggest that Bell have considered the use of any equaliser configuration other than the basic transversal filter is a brief contribution by Taylor[4.19]. In this paper, Taylor indicates a distinct preference for the non-recursive filter configuration for the particular application under consideration.

4.3. Work carried out at Cardion Electronics.

Several papers[4.20 to 4.22] have been published by Di Toro of Cardion Electronics. These papers discuss the recursive transversal equaliser and describe an automatic technique for optimisation. Although an adaptive strategy is claimed, it is in fact an automatic pre-set method which is reactivated at predetermined intervals. It appears that Cardion are especially interested in the equalisation of radio channels. This work has culminated in a commercial equipment marketed under the trade name ADAPTICOM.

4.4. Work carried out at GT&E - Lenkurt Electric Corporation.

Some interest in automatic/adaptive equalisers has been shown by GT&E - Lenkurt Electric Corporation[4.23 to 4.26]. The work appears to be directed towards the provision of a digitally implemented adaptive equaliser which would enable the commercially available 4800 Bits/s. Data Set 26D to be used on switched telephone network circuits. The Lenkurt proposals are similar in concept to those previously described in the Bell Laboratories publications.

4.5. Work carried out by Universities in the United States.

a) Massachusetts Institute of Technology.

Various research projects have been pursued at M.I.T. The most significant of these are:

(i) Austin.[4.27] A decision-feedback equaliser with manual adjustment optimisation.

(ii) Niessen et al.[4.28 & 4.29] An automatic pre-set optimisation strategy for the basic transversal equaliser.

b) Harvard University.

Tufts[4.30] has been working in co-operation with Aaron of Bell Laboratories on problems associated with the relationship of intersymbol interference and error probabilties.

c) University of Washington.

Lytle[4.31] has made a general mathematical study of convergence criteria for automatic pre-set strategy algorithms. As the work was published in Bell System Technical Journal, it is assumed that this work was sponsored by Bell Laboratories and forms part of their programme of research and development.

d) N.E. University, Boston.

Gonsalves[4.32 & 4.33] has carried out some theoretical study of maximum likelihood receivers, especially with regard to noisy channels. Some of this work was carried out in collaboration with Bell Laboratories.

4.6. Work carried out in Canada.

a) Communications Research Centre and Carleton University.

A number of papers have been published jointly by the Communications Research Centre (formerly DRTE) Ontario and Carleton University Ottawa. The first of these[4.34] describes a simple adaptive strategy for a basic transversal filter which is suitable for digital implementation. Other publications[4.35 to 4.37] describe various items of research, mainly concerned with adaptive decision feed-back equalisers.

b) McMaster University.

A recent publication[4.38] describes a research project caried out at the University into adaptive equalisers for digital communication. An adaptive strategy is proposed and the paper concludes with the results of a computer simulation of the strategy.

c) Northern Electric Company, Ottawa.

A paper[4.39] describes an adaptive equalser for television channels. The equaliser is based on the transversal filter concept.

4.7. Work carried out in Australia.

Potter[4.40] describes work on adaptive equalisation carried out at the University of Melbourne. The work is similar to that carried out concurrently at Bell Laboratories, USA.

4.8. Work published in Western Germany.

A paper by Rupprecht[4.41] suggests the possibility of the use of networks other than unit delays, but no rigorous mathematical treatment is given. Rupprecht is with the German Telephone Administration (DBP).

A paper by Möhrmann[4.42] of Siemens AG, Munich, describes the basic concepts of adaptive equalisation. Although it makes little contribution to the advancement of knowledge, it does indicate an interest in the subject of equalisation at Siemens.

There is no indication of any extensive research programme on equalisation being pursued in W.Germany.

4.9. Work in the United Kingdom.

It is known that various companies in the United Kingdom are interested in the problems of automatic pre-set and adaptive equalisation. There is no significant published papers, however, which indicate the lines of research and development which are being pursued by these companies.

A number of U.K. universities have projects concerned with the study of adaptive equalisation. The following publications have resulted from these projects:

a) Imperial College, London.

(i) Clark[4.43] describes an adaptive correlation detection process. The work was carried out as a Ph.D. project and was sponsored by the Plessey Co.

(ii) Lawrence and Bognor[4.44] suggest a set of orthogonal functions that can be used to approximate the channel single-pulse response. The practical aspects of implementation had not been considered, however, at the time of publication.

b) Loughborough University.

Tomlinson[4.45 & 4.46] describes an automatic equaliser which uses a feed-back transversal filter configuration employing modulo arithmetic. The equaliser operates on the signal prior to transmission. Since the channel single-pulse response is determined at the receiver, it is necessary to provide feed-back to the transmit terminal in order to optimise the equaliser coefficients.

c) City University, London.

De and Davies[4.47] propose an optimisation algorithm for the conventional feed-forward transversal equaliser which can give faster convergence than is generally obtained using the normal "steepest-decsent" algorithms.

d) Queens University of Belfast.

Boyd and Monds[4.48 & 4.49] describe a decision-feedback equaliser which is particularly suitable for signals with impairments arising from multipath transmissions.

In addition to the work carried out at the universities, other work in the UK was reported at the conference on "Digital processing of signals in communicaions" held at Loughborough University of Technology in April 1972. This work is not taken into account in other chapters of this thesis as it was published subsequent to the completion of the work reported therein. It is included here, however, to make this survey as comprehensive and up-to-date as possible. The work reported was as follows:

e) S.R.D.E. Christchurch.

A general review of known equalisation strategies has been made[4.50]. An experimental equipment[4.51] has also been constructed, which can be programmed for different layouts of equaliser, to cater for various conditions of channel distortion.

f) The Post Office Research Establishment, Dollis Hill, London.

Two contributions[4.52 & 4.53] indicate that the Post Office have considered an adaptive decision feed-back equaliser for use on switched telephone network circuits using v.s.b. amplitude modulation at 2400 Bauds. There is close agreement between results obtained by the Post Office and similar results reported in this thesis.

g) Racal-Milgo Limited.

The equaliser used by Racal-Milgo in their commercial high-speed data modem 5500/96 is described[4.54]. The development work for the equaliser was carried out by the Milgo Corporation in the United States.

The work described in chapter 9 of this thesis was presented by the author at the Loughborough conference[4.55].

5. ASSESSMENT OF TRANSVERSAL FILTER NETWORK CONFIGURATIONS.

5.1. Introduction.

In this chapter various filter configurations are compared and the factors governing the choice of the optimum configuration in any given application are considered. Consideration is also given to methods of implementing the tapped delay-line which forms the basis of the transversal filter. The various strategies for setting the filter coefficients are considered in detail in the following chapters.

5.2. Choice of Filter Configuration.

The most appropriate filter configuration, the number of delay line modules necessary to obtain satisfactory equalisation and the optimum disposition of these modules with respect to the reference tapping point (i.e. the tap associated with the coefficient K_0 in equation 3.2.) are all functions of the characteristics of the equivalent base-band channel to be equalised. Since the cost of any practical equaliser is almost directly proportional to the number of delay modules it contains, it is of obvious economic advantage to select the equaliser parameters so that the required degree of equalisation can be obtained using the least possible number of delay modules. Also, where multiplier coefficients are stored in quantised form, the provision of unnecessary stages simply leads to a deterioration in the degree of equalisation that can be achieved by the addition of further errors due to quantisation.

In general the detailed characteristics of the transmission channel are not known in advance and the most suitable choice of configuration is that which most adequately satisfies the range of characteristics that would normally be encountered under the specified operating conditions.

5.3. Consideration of Basic Transversal Filter Configuration.

In this section we consider the degree of equalisation that can be achieved for various v.s.b. modem designs operating over leased 'data quality' lines using the basic transversal filter configuration. The minimum value of intersymbol interference (D_{min}) that can be obtained with a given number of delay modules and a given disposition about the reference tap was computed for those equivalent base-band channels shown in table 1.1. which have an initial intersymbol interference $D_0 < 1$. This was done by solving the set of simultaneous equations (3.5.) to obtain the optimum values for the multiplier coefficient settings. From these coefficient values the residual intersymbol interference (D_{min}) remaining after equalisation was then computed for each case considered.

Graphs showing the variation of residual intersymbol interference D_{min} with number of delay modules L and with disposition about reference tap, where p is number of taps before reference tap and q is number of taps after reference tap (i.e. $L=p+q+1$), were plotted for each channel response considered. These are given in figs.5.1(a) to 5.1(e). It should be noted that the values of D_{min} indicated on these graphs are those that would be obtained under ideal conditions and does not take into account the deterioration in D resulting from quantisation and other errors that may occur in the coefficient settings. These factors are considered in more detail in section 5.4.

It will be seen that there is virtually no difference between the equaliser requirements for modems using the A and B parameter sets as detailed in appendix 1. The choice between these two parameter sets will therefore depend entirely upon their effect on the modem implementation.

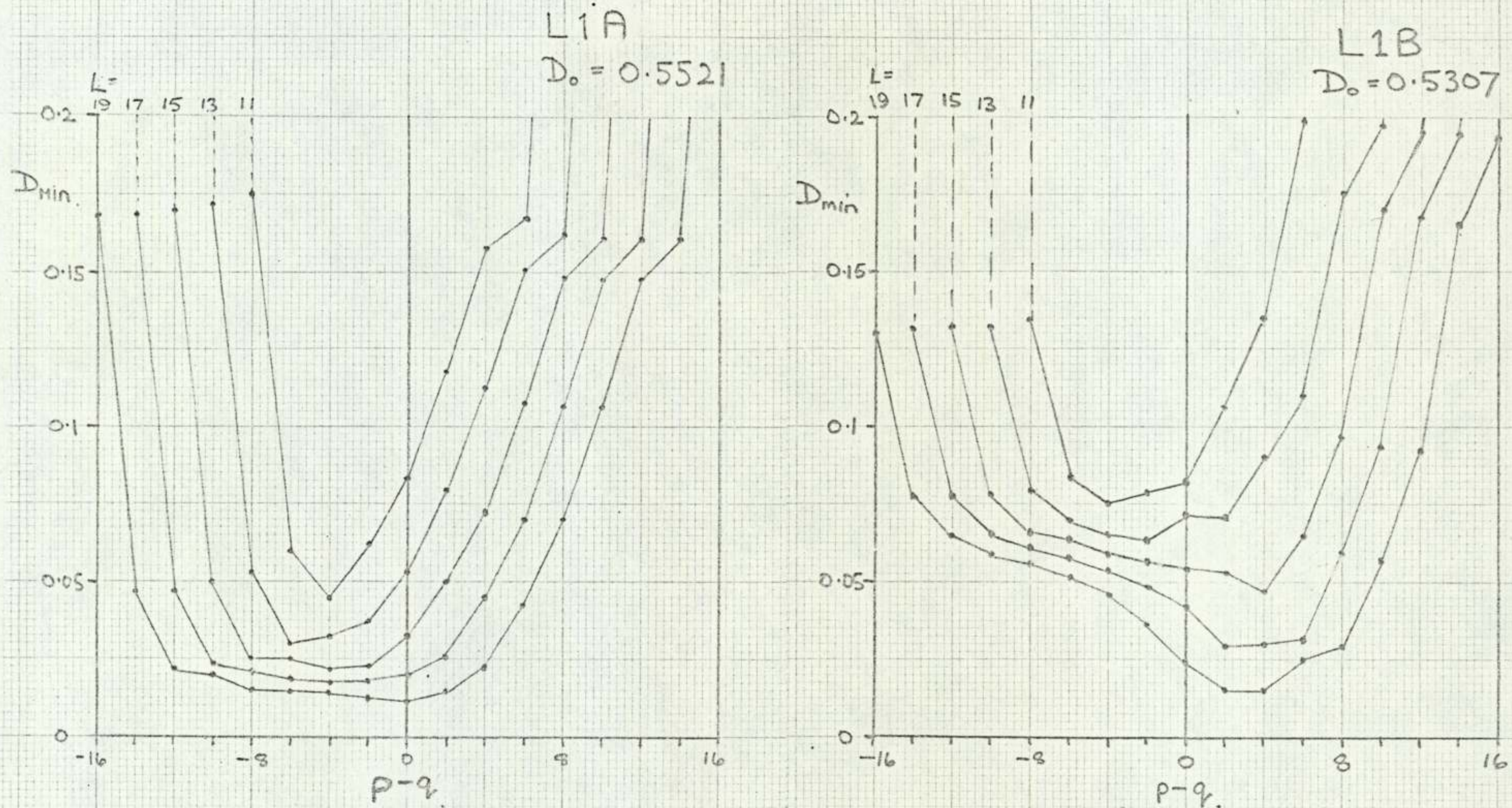


Fig 5.1 (a). Effect of position of reference top on residual distortion.

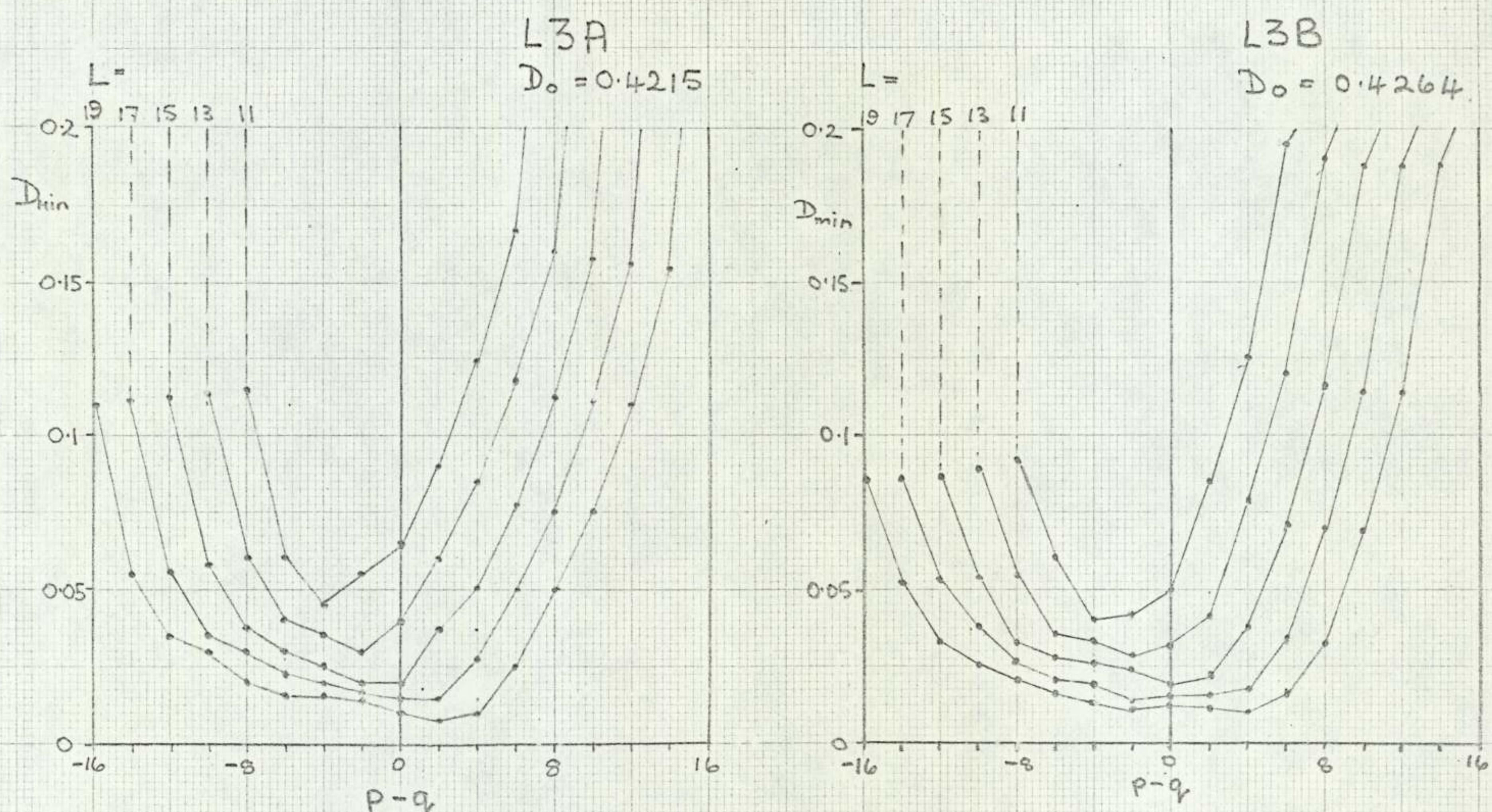


Fig. 5.1(b) Effect of position of reference tap on residual distortion

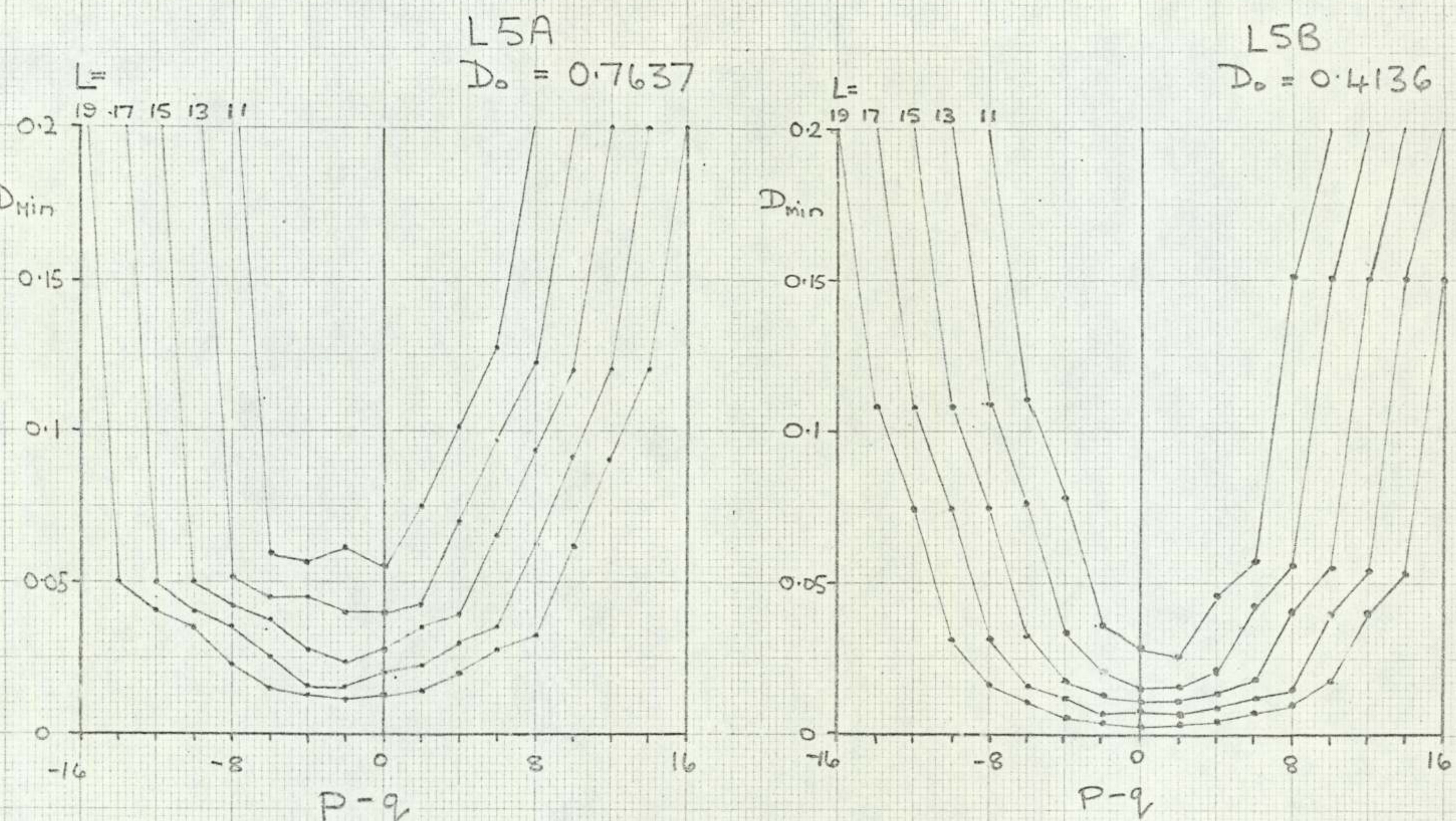


Fig. 5.1 (c) Effect of position of reference tap on residual distortion:

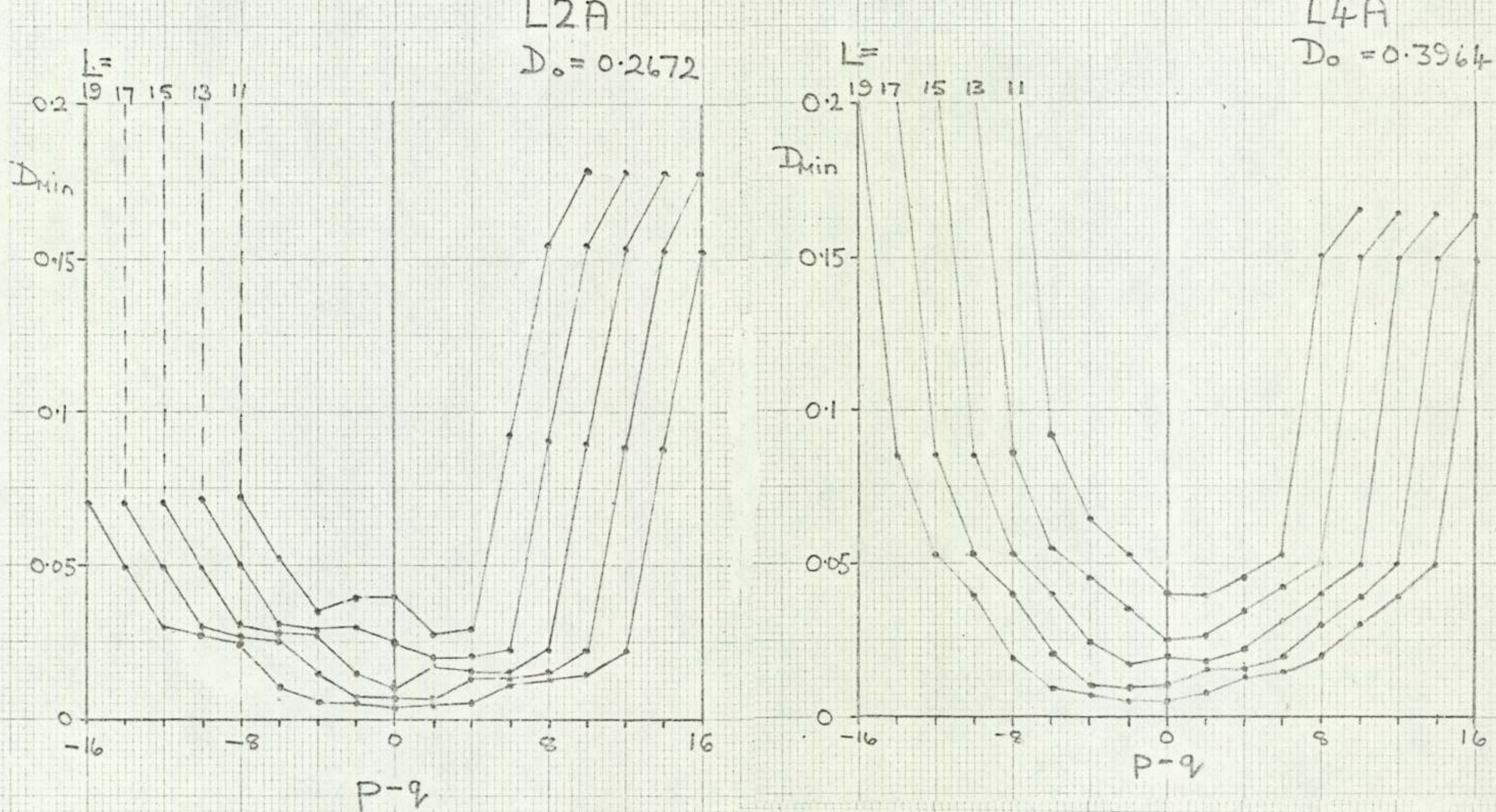


Fig. 5.1(d) Effect of position of reference tap on residual distortion.

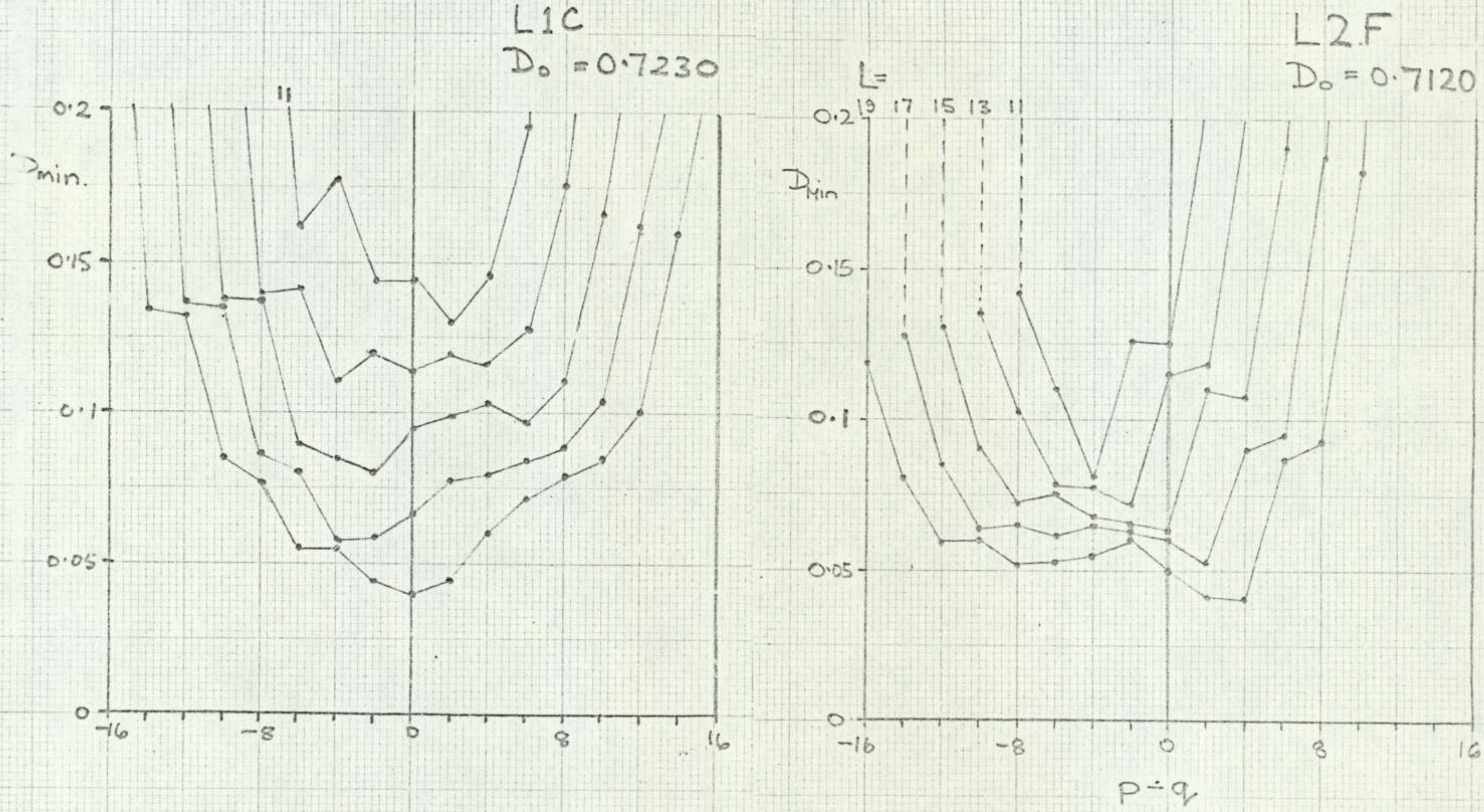


Fig. 5.1(e) Effect of position of reference tap on residual distortion.

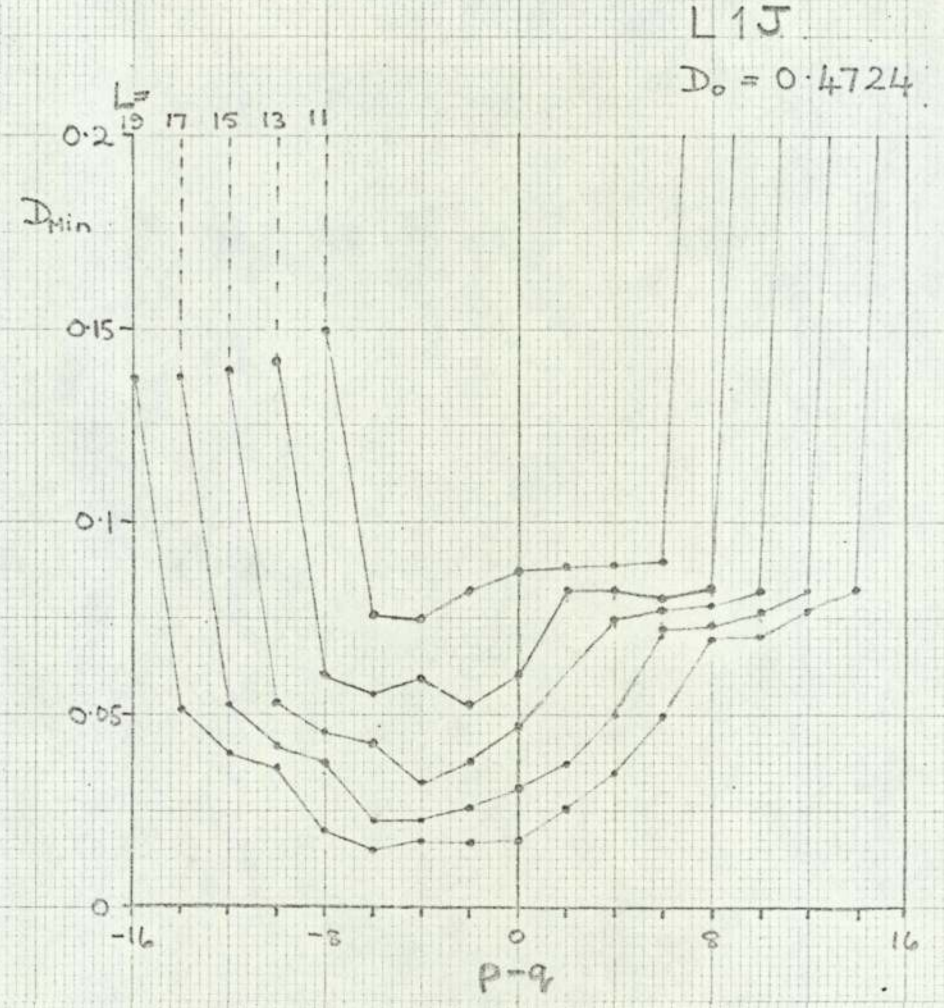


Fig 5.1.(f). Effect of position of reference tap on residual distortion.

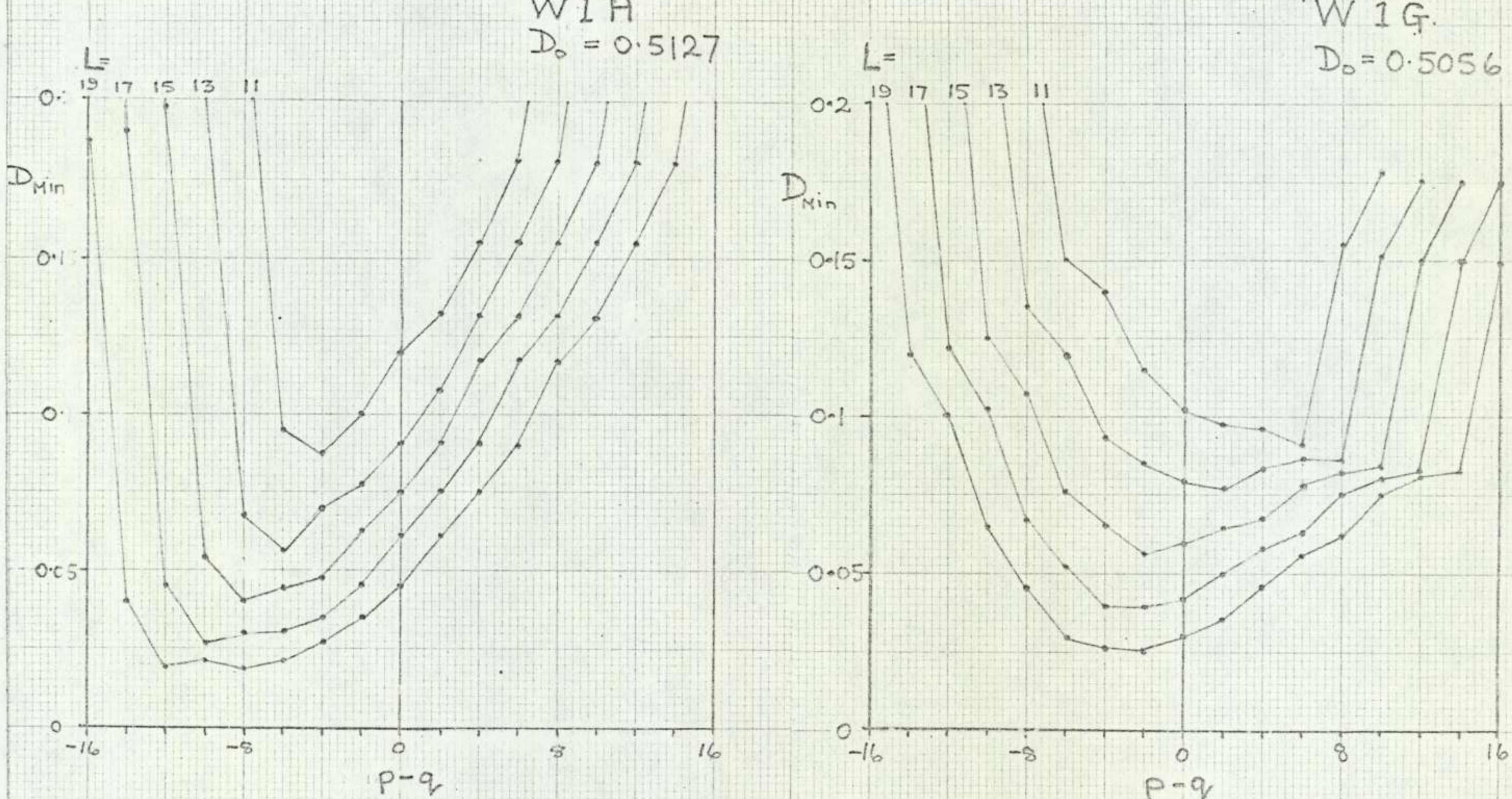


Fig. 5.1(g). Effect of position of reference tap on residual distortion

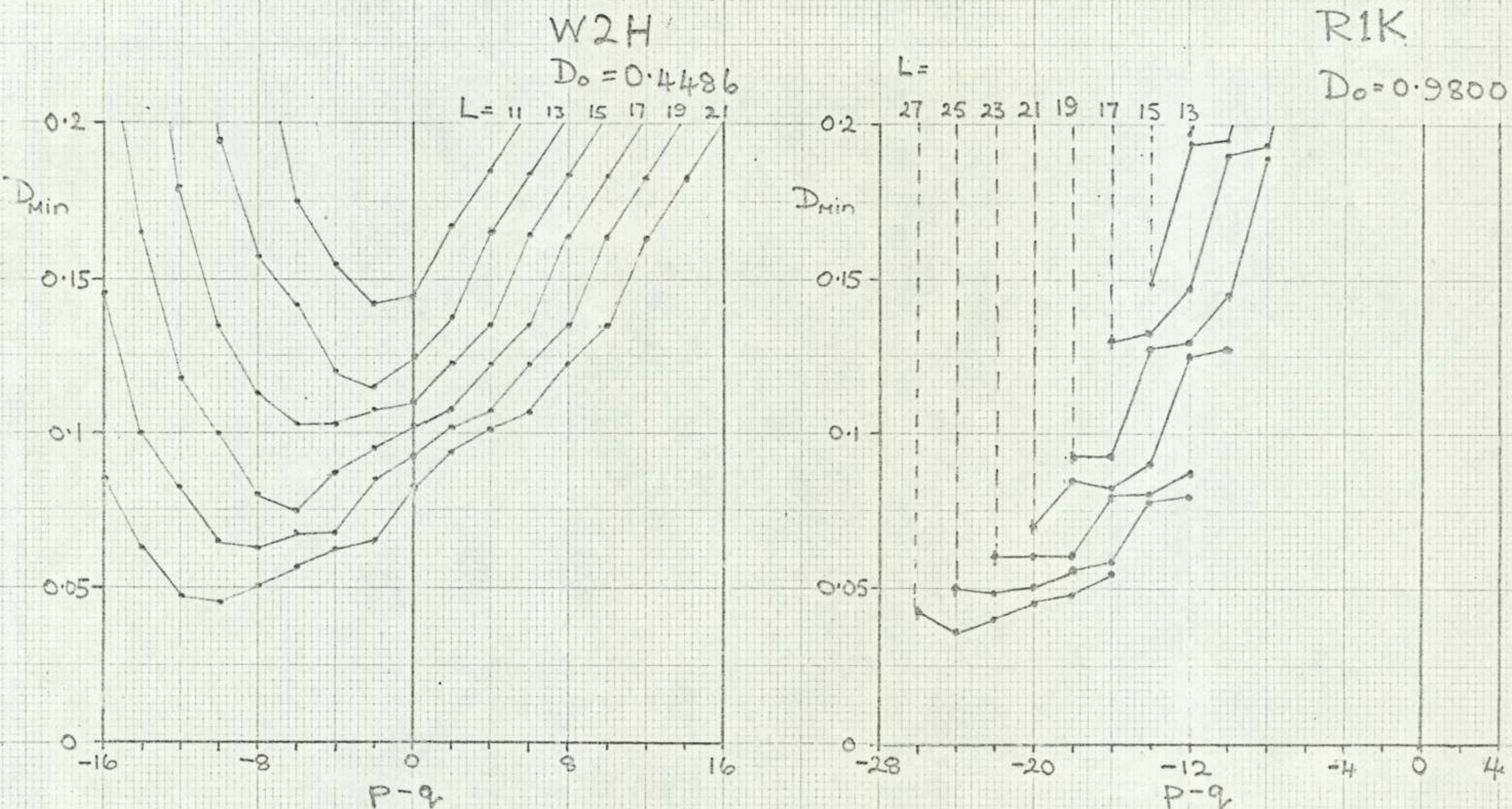


Fig. 5.1(h). Effect of position of reference tap on residual distortion.

The computations indicate that the most suitable tap disposition for the basic equaliser configuration used in this application is a symetrical distribution of coefficients about the reference tapping point. This is surprising, since a cursory glance at the time domain responses given in appendix 1 would indicate that the anticipatory response was generally less significant than the trailing response. With a total of 15 delay-line taps, all the 2400 symbols/s modem-line combinations can be equalised to better than a 0.05 residual intersymbol interference D_0. The figure is improved to better than 0.025 for a total of 19 delay-line taps.

The channel characteristic L1J is for the same line and modem combination as L1A but, whereas L1A uses the idealised approximation for the shaping and v.s.b. filter characteristics as described in appendix 1, L1J uses characteristics obtained by measurement of a practically implemented filter. A comparison of the results for these two channels characteristics, given in figs.5.1(a) and 5.1(f), indicates that the 'idealised' approximation gives results which do not differ significantly from these obtained using actual measured characteristics.

Similar computations were carried out for equivalent base-band responzes W1A, W1G, W2H, and R1K, as these are used for comparative purposes elsewhere in this thesis. The results of these computations are given in figs.5.1(g) and 5.1(h).

5.4. Consideration of Recursive Transversal Filter Configuration.

We now consider the degree of equalisation that can be obtained using the recursive equaliser configuration. The computation for this configuration is somewhat more complex than for the basic configuration, because the provision of insufficient feed-back sections may well cause the trailing dispersion to be extended indefinitely. On the other hand, since complete equalisation of the trailing response can be achieved with the number of feed-back sections equal to the number of symbol periods in the trailing response, there is no advantage to be gained by the provision of any further feed-back taps. We therefore assume that just sufficient feed-back sections are provided to fully equalise the trailing response and that all the additional sections are associated with the anticipatory response. The lower curves (marked "not quantised") in figs.5.2(a) and 5.2.(b) show the comparison between the degree of equalisation that can be achieved for a given number of sections in the recursive configuration using the disposition of sections described, with that obtained using the same number of sections in the basic configuration, assuming optimum selection of the disposition of the sections about the reference coefficient.

It will be seen that for leased telephone-line type channels there is no advantage in using the recursive configuration, except where a very high degree of equalisation is necessary requiring the provision of a large number of equaliser sections. However, the graphs for characteristic R1K indicate that the recursive configuration can be very advantageous when used in connection with channels characterised by the presence of large echoes. Such echoes are typical of h.f. and troposcatter radio channels[5.1], where they occur as the result of multipath transmission effects. They are also encountered on connections established through the normal switched telephone network[5.2].

5.5. Effect of Quantisation of Coefficient Values.

In most practical implementations of automatic equalisers it is desirable (and sometimes necessary) to store and update the coefficient values so that they have discrete step increments. This quantisation of the coefficient values means that the intersymbol interference cannot, in general be completely cancelled at each sampling instant. Instead,

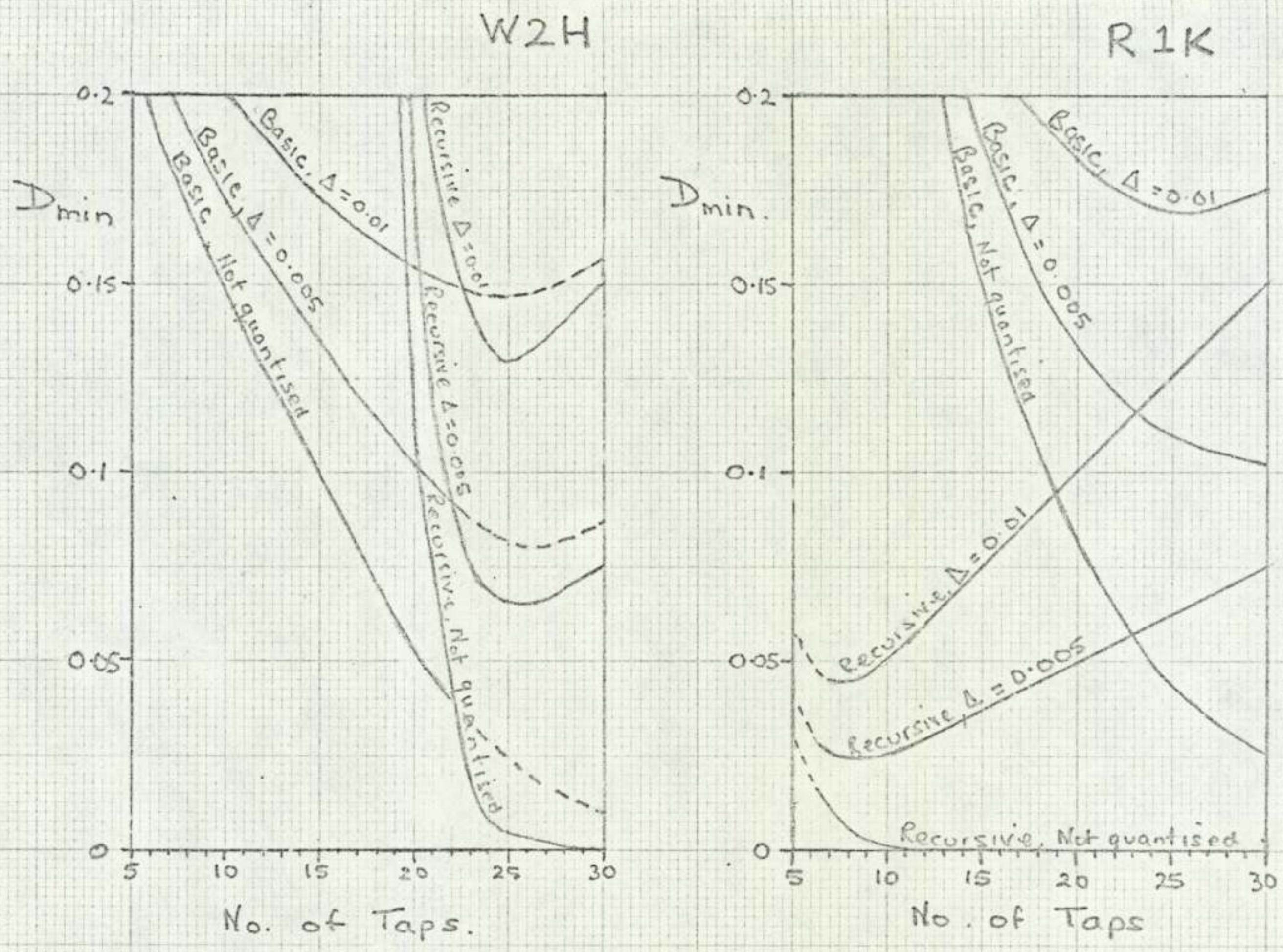


Fig. 5.2(a). Effect of coefficient quantisation on residual distortion.

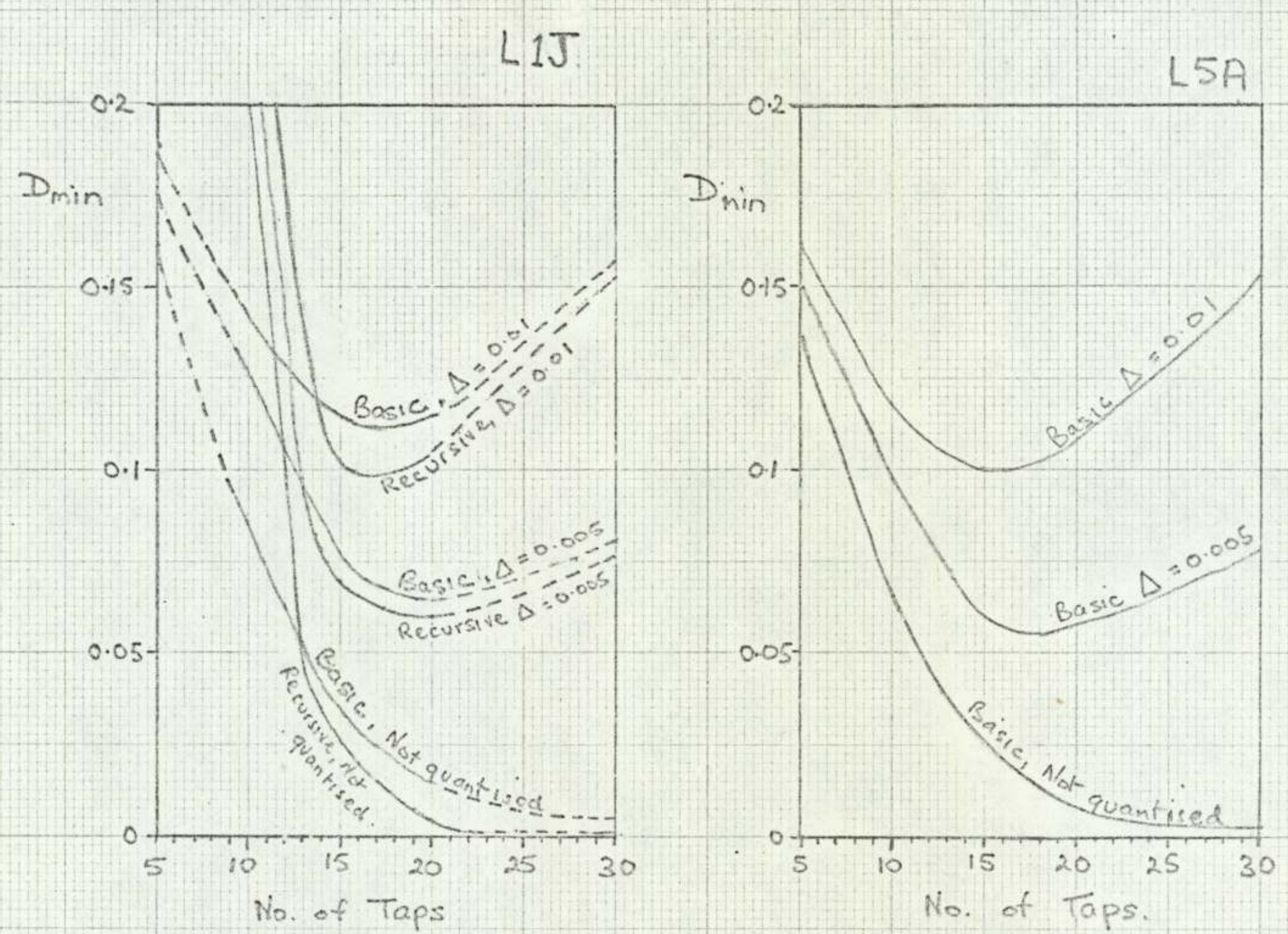


Fig. 5.2 (b). Effect of coefficient quantisation on residual distortion.

a small residue exists which causes a deterioration in the optimum condition that can be achieved for a given number of taps.

The effect of the quantisation of the coefficients is somewhat dependent on the strategy used to set them up. If the coefficients are calculated directly and rounded off, the quantised value chosen will be that nearest to the ideal value. If the increment step size is Δ, then the magnitude of the maximum error will be $\frac{1}{2}\Delta$. Since the probability distribution of the error magnitude will be uniform for values 0 to $\frac{1}{2}\Delta$, and zero outside this range, the mean error e_m will be $\frac{1}{4}\Delta$. On the other hand, with automatic and adaptive strategies, the coefficients will normally alternate between the quantised values either side of the ideal value and the mean error e_m then becomes $\frac{1}{2}\Delta$. The value e_m represents the average contribution to the intersymbol interference due to quantisation per tap of the equaliser. The total mean deterioration in intersymbol interference due to quantisation for the complete equaliser (D_q) is therefore given by;

$$D_q = e_m.L.$$

The effect of this deterioration is illustrated in figs.5.2(a) and 5.2(b), where the mean error e_m has been taken to be $\frac{1}{2}\Delta$. It will be seen that, whereas with optimally adjusted coefficients the intersymbol interference tends to zero as the number of taps tends to infinity, with quantised coefficient adjustments there is a minimum value of intersymbol interference, associated with an optimum size of equaliser, that can be attained for any given increment size. Further reduction in intersymbol interference can only be achieved by reducing the size of the coefficient increment. This has the effect of increasing the time required to set up the equaliser when certain iterative adjustment strategies are employed. This problem is considered further in chapter 7, where the operation of various adaptive equaliser adjustment strategies is studied in detail.

5.6. Delay Module Implementation.

One of the problems associated with the design of transversal equalisers is the choice of a suitable delay module implementation. The perfect delay-line is unattainable in practice and a compromise has to be made between choosing a delay element which causes little distortion of the applied signal and one which is not inordinately complicated or expensive to implement. The conventional lumped network delay-line constructed from LC elements does not lend itself well to this purpose since, in order to obtain the necessary quality, each section must consist of several elementary delay networks, each of which in turn consists of several expensive and bulky components.

Good timing accuracy is required to avoid the formation of cumulative delay errors along a line consisting of several delay sections. This demands accuracy in the components from which the delay-line is constructed.

5.7. The Active All-Pass Phase-Shift Network as a Delay Element.

One possibility for the construction of a suitable delay element is the active all-pass phase-shift network. A study was therefore carried out to assess the suitability of such networks for this application.

a) Requirements. The requirement is for a delay-line section which has a substantially constant delay characteristic over a frequency band from zero (d.c.), to an upper band limit ω_b, such that the delay-bandwidth product is of the order of 1.5π radians. The second order all-pass network appears to give the best compromise between meeting this requirement and the necessity of keeping the circuit configuration reasonably simple. This study is therefore based on the second order network.

b) The Second Order Network. The function describing the transfer characteristics of an all-pass network consists of the ratio of two nth order polynomials whose roots are in quadrantal symmetry. A second order all-pass function would have the form:

$$G(s) = \frac{(s - s_1)(s - s_1^*)}{(s + s_1)(s + s_1^*)} \quad \ldots\ldots\ldots\ldots (5.1.)$$

(See fig.5.3.)

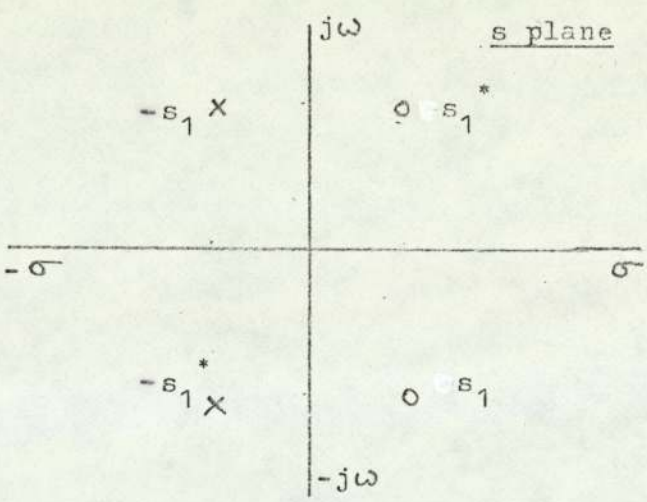


Fig.5.3. Singularities of Transfer Function of Second Order 'All-Pass' Network.

If we let $s_1 + s_1^* = \alpha$ and $s_1 s_1^* = \beta$

then (5.1.) becomes:

$$G(s) = \frac{s^2 - \alpha s + \beta}{s^2 + \alpha s + \beta} \quad \ldots\ldots\ldots\ldots (5.2.)$$

Again, let $s = j\omega$ whence (5.2.) becomes:

$$G(\omega) = \frac{(\beta - \omega^2) - j\omega\alpha}{(\beta - \omega^2) + j\omega\alpha}$$

Hence the phase characteristic $\phi(\omega) = \text{Arg.}G(\omega)$

$$= -2 \arctan \frac{\omega\alpha}{\beta - \omega^2}$$

and the delay $T(\omega) = -\frac{d\phi(\omega)}{d\omega}$

$$= 2\,\frac{(\beta + \omega^2)}{(\beta + \omega^2)^2 + (\omega\alpha)^2} \quad \ldots\ldots\ (5.3.)$$

Substituting in equation (5.3.) to eliminate a variable, we let $y = \frac{T(\omega)}{R}$, $x = \omega R$ and $k = \alpha R$, where $R = \frac{\alpha}{\beta}$:

Thence (5.3.) becomes:

$$y = \frac{2k(x^2 + k)}{x^4 + k(k-2)x^2 + k^2} \quad \ldots\ldots\ldots\ (5.4.)$$

Graphs of y against x for various values of k are given in fig.5.4. k = 3 is maximally flat.

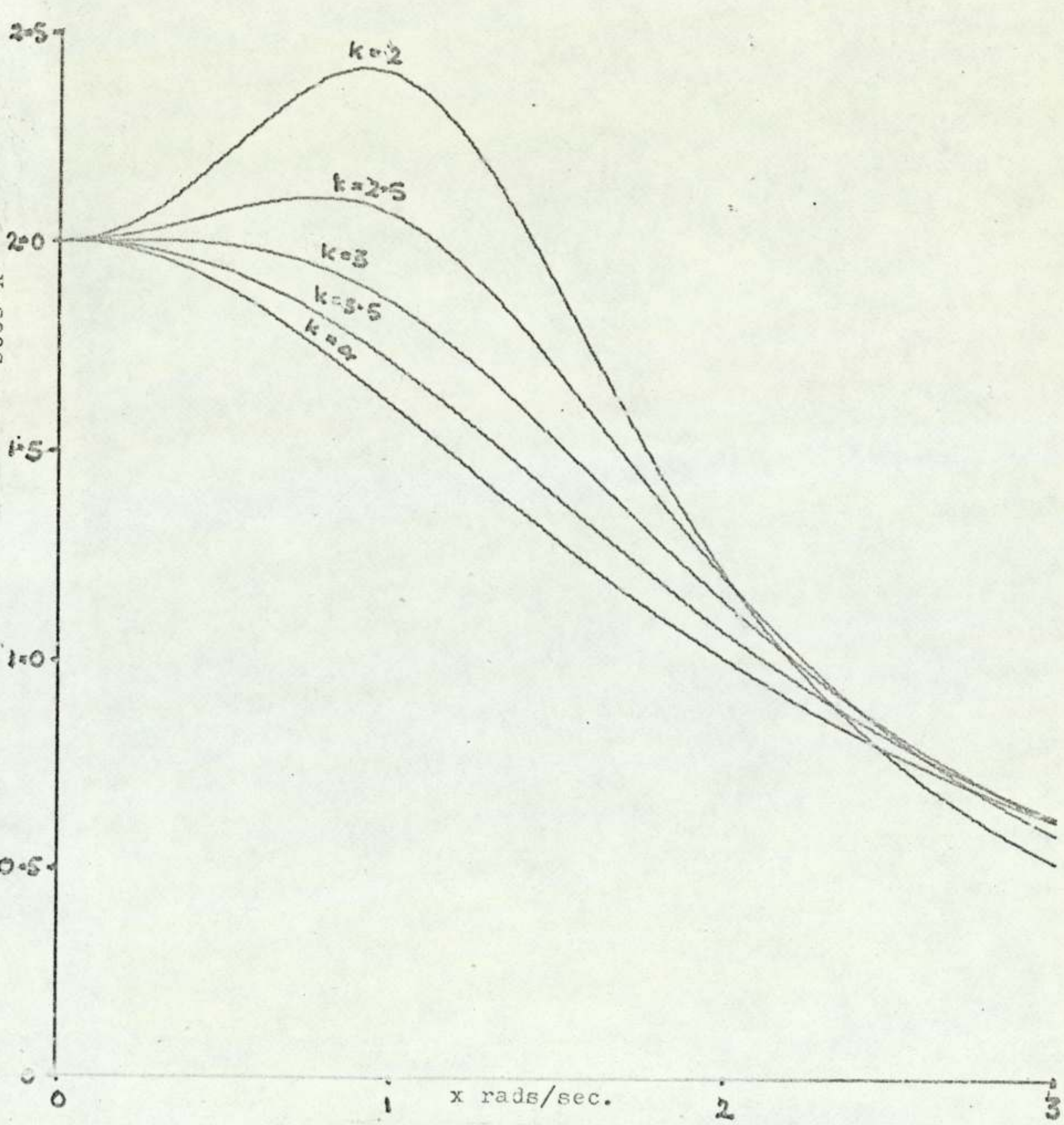


Fig.5.4. Normalised Frequency/Delay Characteristics for Second Order All-Pass Network.

Let the nominal delay = $T(o)$

$\therefore$ The delay-bandwidth product $= T(o) \times \omega_b$ (ω_b = bandwidth)

$= y(o) \times x_b$ (x_b = normalised bandwidth)

But $y(o) = 2$; thus for a delay-bandwidth product of 1.5π, $x_b = 0.75\pi = 2.36$.

More than one delay element may be used per delay line section. However, the delay-bandwidth product per element then becomes the delay-bandwidth product per section, divided by the number of elements per section, n. The normalised upper frequency bound x_n then becomes $x_n = \frac{x_b}{n}$. For each value of x_n there is an optimum value of k which minimises the variation in delay over the frequency band. For $n = 1$, the maximally flat curve, with $k = 3$, is optimum.

Fig.5.5 shows the deviation in delay ΔT, where

$$\Delta T = \frac{T_{(max)} - T_{(min)}}{T_{(nom)}}$$

against k, for n = 2, 3 and 4.

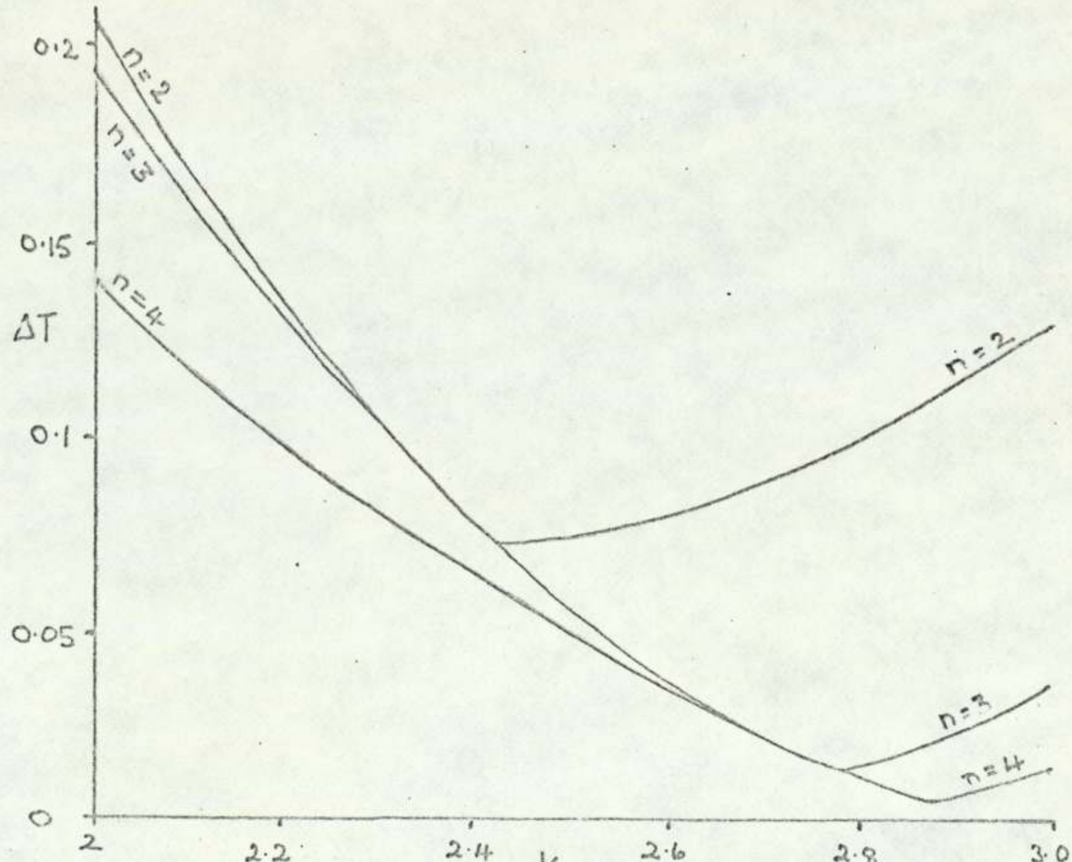


Fig.5.5. Deviation in Delay Factor versus k.

From these graphs the values of $k_{(opt)}$ and $T_{(min)}$ are taken. These are tabulated in the following table. $n\Delta T_{(min)}$ is the delay distortion per delay line section.

n	x_n	$T_{(min)}$	$k_{(opt)}$	$n\Delta T_{(min)}$
1	2.36	0.547	3	0.547
2	1.18	0.07	2.43	0.14
3	0.79	0.012	2.70	0.036
4	0.59	0.004	2.88	0.016

c) Estimate of Acceptable Distortion. In order to make a qualitative assessment of the degree of permissible distortion, computer simulation of the response of 1, 5 and 13 section delay lines to a 25% cosine roll-off band-limited single-symbol stimulus was carried out. The intersymbol interference D was also computed in each case in accordance with equation (2.3) The summation was restricted to 16 samples since the samples became insignificant outside this range. The values of D obtained are given in the following table. The 25% cosine roll-off band limiting filter itself gave an intersymbol interference of 0.013.

No. of Sections in delay-line / n	1 Sect.	5 Sects.	13 Sects.
1	0.408	0.750	2.407
2	0.129	0.235	0.642
3	0.043	0.126	0.131
4	0.080	0.065	0.033

The plotted response waveforms are given in figs.5.6(a) to (m), the sampling points being indicated on the waveforms.

Although the amount of distortion that can be tolerated is a function of the algorithm used to adjust the equaliser coefficients, it is unlikely that a binary channel could be equalised satisfactorily if the distortion contributed by the delay-line is in excess of 0.25. For a 5-section line, 2 elements per section may therefore be satisfactory but 3 elements per section would be required for a 13-section line. The number of sections required depends on the degree of equalisation it is necessary to provide on the data channel. For a multilevel channel, 4 elements per section would seem to be necessary for satisfactory operation.

d) Realisation of Second Order Network. A second order active network can be synthesised using the basic circuit configuration shown in fig.5.7.

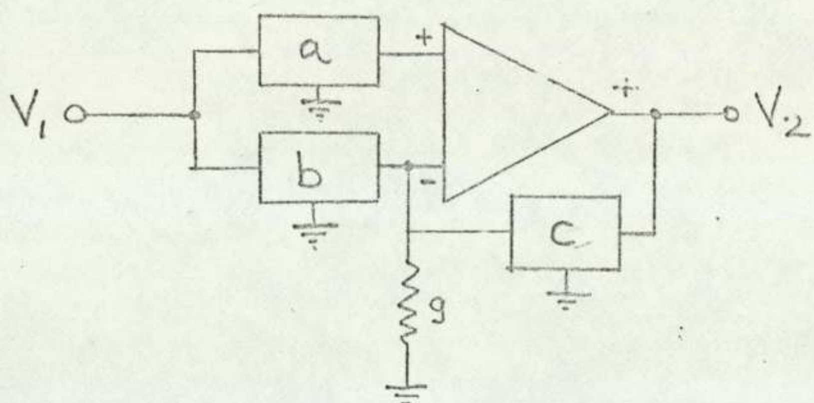


Fig.5.7. Basic Delay Element.

The overall transfer characteristic G_{12} is given by

$$G_{12} = \frac{1}{G_{12c}} \left[2G_{12a} - G_{12b} \right]$$

To realise the polynomial of the all-pass function

$$\frac{s^2 - ks + k}{s^2 + ks + k}$$

we can identify certain parts of the polynomial with the transfer characteristics of networks 'a', 'b' and 'c'.

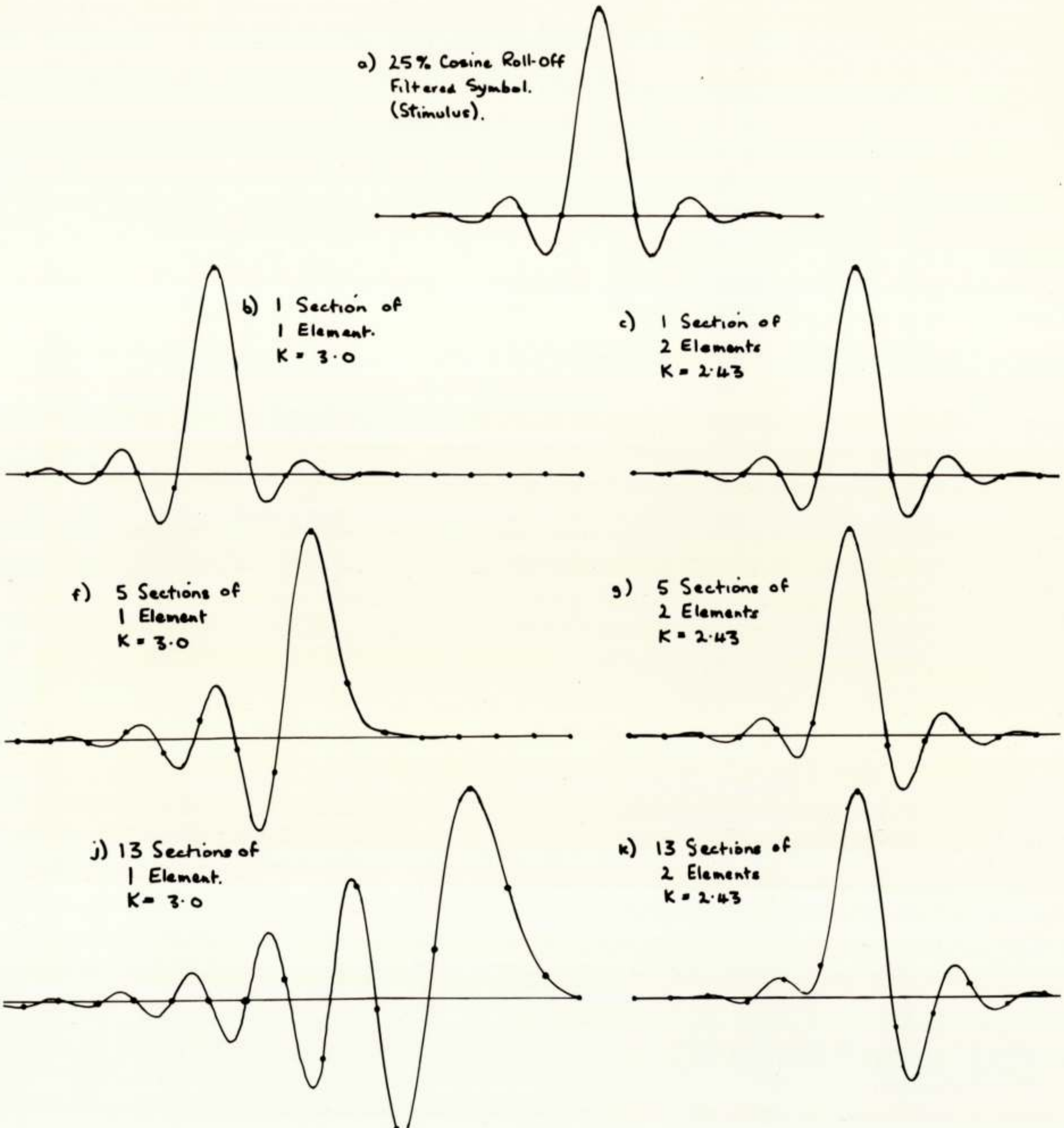


Fig. 5.6 (Part I) Single-Pulse Responses of All-Pass Network Delay-Lines.

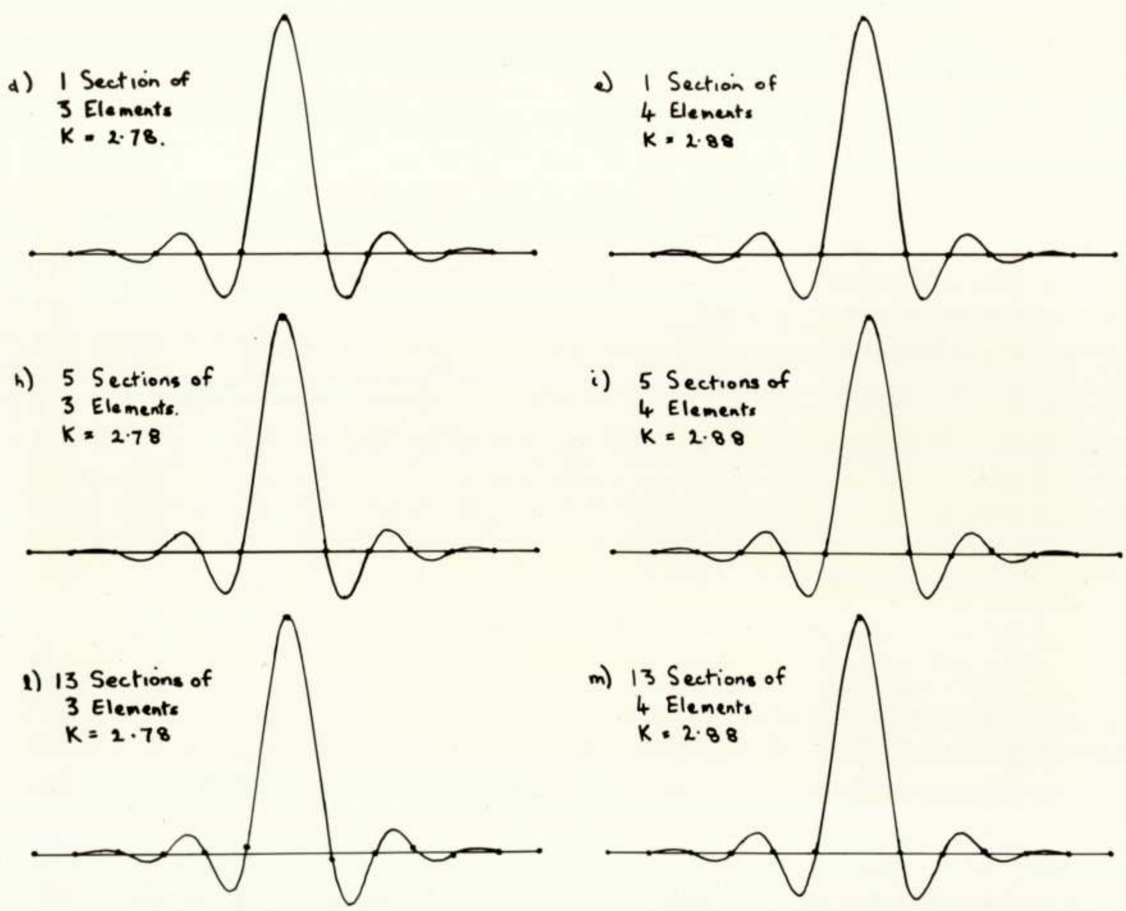


Fig. 5.6. (Part II) Single-Pulse Responses of All-Pass Network Delay-Lines.

One possibility is
$$G_{12a} = \frac{C(s^2 + k)}{M(s)}$$
$$G_{12b} = \frac{C(s^2 + ks\ k)}{M(s)}$$
$$G_{12c} = \frac{C(s^2 + ks + s)}{M(s)}$$

Thus the networks b and c can be identical. Realisation of G_{12a} requires a twin 'T' network to give real frequency transmission zeros. G_{12b} and G_{12c} may be realised with a bridged 'T' network. A complete circuit diagram for the delay element is given in fig.5.8.

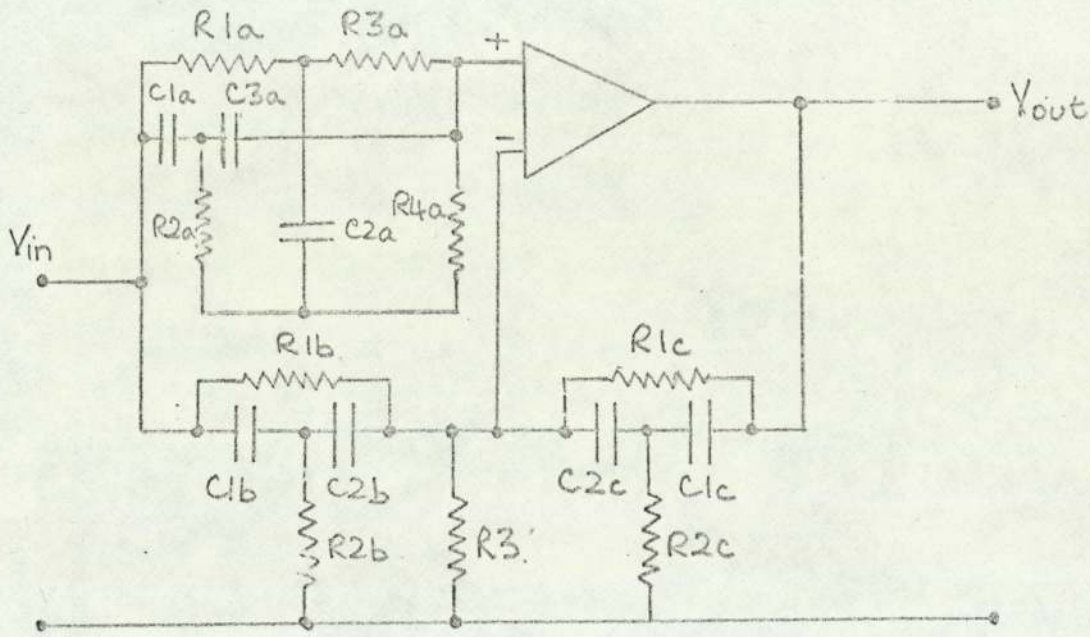


Fig.5.8. Circuit Diagram of Delay Element.

Normalised values of the components required for $k = 2.43$ and $k = 2.78$ are given in the following table

k	2.43	2.78	k	2.43	2.78
C1a	2	2	R1a	0.327	0.265
C2a	3.04	3.55	R2a	0.139	0.139
C3a	2	2	R3a	0.414	0.383
C1b	1	1	R4a	0.503	0.558
C2b	1	1	R1b	0.824	0.72
C1c	1	1	R2b	0.5	0.5
C2c	1	1	R1c	0.824	0.72
			R2c	0.5	0.5
			R3	0.278	0.311

The following table gives a set of component values for an equaliser operating on a data stream having a symbol rate of 2400 Bauds. The values given are suitable for realisation using thin-film techniques.

k	2.43	2.78	k	2.43	2.78
C1a	2000	2000	R1a	34.0	18.4
C2a	3040	3550	R2a	14.5	9.65
C3a	2000	2000	R3a	43.2	26.6
C1b	1000	1000	R4a	52.4	38.8
C2b	1000	1000	R1b	85.8	50.0
C1c	1000	1000	R2b	52.0	34.8
C2c	1000	1000	R1c	85.8	50.0
			R2c	52.0	34.8
			R3	29.2	22.6

Capacitors in pF. Resistors in kilohms.

Laboratory models of single delay-line sections were constructed using discrete components for values of k = 2.43 and k = 2.78. The amplifier used for these sections was the Fairchild differential operational amplifier type µA709. Tests on these sections confirmed the performance predictions given in figs.5.6(e) and 5.6(h).

5.8. Sampled Data Delay-Lines.

We have shown that it is possible to realise an active all-pass network delay-line element which can be manufactured using thin-film circuit techniques. The cost of such a delay-line, however, was still somewhat excessive and alternative methods of obtaining the delay function were therefore considered. Since the data is determined at the receiver by sampling and slicing the received signal at the symbol transmission rate, it would seem reasonable, provided the symbol timing can be extracted before equalisation, to carry out the sampling prior to equalisation and to perform the equalisation process on the resulting sampled signal.

Associated work on data modem design[5.3] indicated that, even when the intersymbol interference is such that the received data 'eye-pattern' is completely closed and it is thus impossible to decipher the actual data content without equalisation, it is in fact still quite feasible to extract the symbol timing from the signal polarity transitions of the unequalised signal. As the symbol timing can be recovered from the received signal before equalisation, it is thus possible to equalise the received signal following the sampling process. Since the equaliser is now only required to process the sample values, a full analogue delay-line is no longer a necessity. There are two possible ways of storing the time-sampled signals; either the sample amplitude is stored on an analogue basis, using, for example, capacitive storage, or the sample amplitude is also quantised and stored digitally using, for example, shift-register storage. At the time of this investigation large-scale integration technology was not generally available and so capacitive storage was given consideration.

5.9. Capacitive Storage.

There are two basic approaches to implementing a suitable capacitive store; one is to use a 'push-down' type store, where the information is stepped along a series of storage locations under the control of a clock pulse, and the alternative is to store samples in fixed locations and to access by means of a commutator. The commutation logic required, however, proved to be quite complex and only the former type of store was therefore considered.

A schematic diagram of the basic storage element is given in fig.5.9.

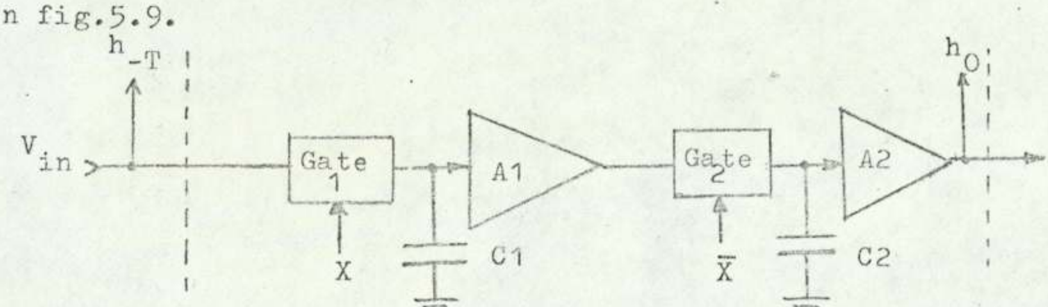


Fig.5.9. Basic Delay Line Element.

A master-slave storage arrangement is required to provide memory during the transfer operation. The clock waveform X is provided normally and inverted at the symbol transmission rate. The normal phase opens gate 1 for the first half of the clock period, thus charging storage capacitor C1 to the input voltage V_{in}. The inverted phase opens gate 2 for the second half period so that the storage capacitor C2 is charged to the same potential as C1. The gates are bi-directional analogue gates with low 'on' resistance, low leakage in the 'off' condition and low d.c. offset. The F.E.T. gate[5.4] is a suitable method of realising the required conditions. Unity gain, high input impedance, low output impedance, low d.c. offset amplifiers are required for the buffer amplifiers. Suitable low-cost operational amplifiers are available for this application.

If C1 is made very much larger than C2, so that C2 can be charged from C1 without significant loss in voltage, then it is possible to dispense with buffer amplifier A1. The difficulty with this approach is that C1 has to be kept sufficiently small to charge fully via the gate 'on' resistance during its charging period, whereas C2 has to be kept sufficiently large to prevent significant change in voltage due to leakage over the same period. By careful choice of the C1 charging period it is possible to obtain an optimum C1/C2 ratio, but additional logic is then required to obtain the optimum charging period from the standard symbol-rate clock available from the modem.

Another alternative is to commutate two capacitors for alternate symbol clock periods so that whilst one is charged from the previous stage, the other is, at the same time, holding the earlier sample at the input to its own stage.

The arrangement is illustrated in fig.5.10.

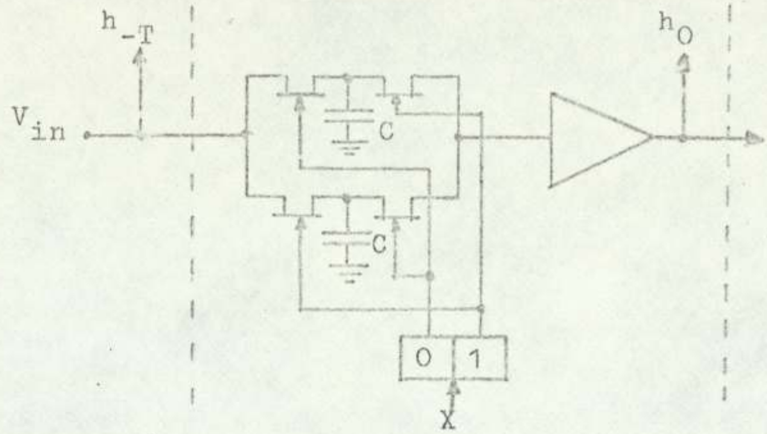


Fig.5.10. Gated-Capacitance-Store Delay Network.

Since the original study was carried out, this latter arrangement has been implemented elsewhere using thin-film circuit technology[5.5]. Even more recently, the basic design described in this section has been implemented using MOS large-scale integration[5.6] and a variation on the design using bi-polar large-scale integration[5.7].

5.10. Digital Storage.

Since large-scale integration is now commercially available, the implementation of the equaliser, including the delay-line function, can be carried out using entirely digital techniques. The delay-line function is shown schematically in fig.5.11.

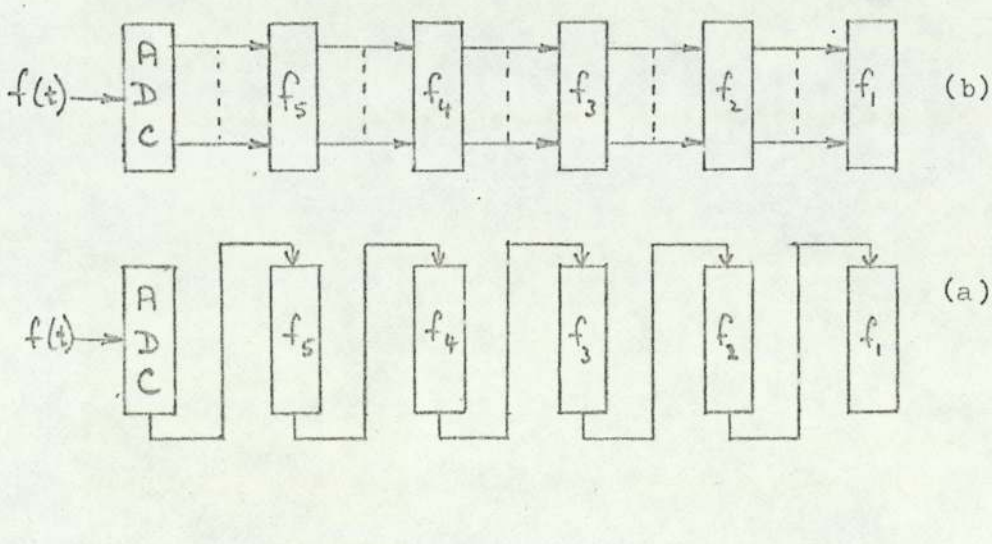


Fig.5.11. Digital Implementation of Delay-Line.

The input signal to the equaliser is quantised by means of the analogue-to-digital converter (ADC). The digitised sample values are then either shifted serially (fig.5.11(a)), or in parallel (fig.5.11(b)), along a shift-register arrangement, the sample values passing from one storage location to the next at the symbol transmission rate. The number of quantisation levels required for the line signal is calculated in chapter 9, where the design of an entirely digital equaliser is considered in detail.

5.11. Conclusions on Methods of Delay-Line Implementation.

A number of methods of obtaining the delay-line function for the transversal equaliser have been described in the preceeding sections. The choice of method used in any given situation will depend largely on the techniques used in the construction of the associated equipment. Where production quantities are large enough, the use of large-scale integration becomes an attractive possibility. In these circumstances, digital storage would almost certainly be used for the delay-line function. If, however, insufficient demand is foreseen to warrant the use of large-scale integration, then digital storage becomes much less economically attractive. In these circumstances the capacitive storage delay-line would generally seem to be the best solution. An experimental automatic/adaptive equaliser was built using this technique[5.8] and very satisfactory performance was obtained. This equaliser is currently being engineered for quantity production (see section 1.2).

6. AUTOMATIC PRE-SET ADJUSTMENT STRATEGIES

6.1. Introduction.

The first priority of this work was the provision of automatic equalisation for leased 'data quality' lines because these were already being widely used for the transmission of digital data. Since these lines have virtually time-invariant characteristics, it is not necessary for the equaliser to be continually adapting throughout the course of transmission of data. This means that the equaliser coefficients can be set up using an initial training pattern transmission, which is known a priori at the receive terminal as described in para. 2.2(a). As this yields more reliable updating information than can be obtained by the assessment of errors in an unkown signal, it is possible to obtain faster convergence rates than is possible without the use of a training pattern transmission.

There are a number of different automatic pre-set adjustment strategies which can be used to set up the coefficients of the transversal equaliser, the simplest of these, however, is limited in use to where the unequalised intersymbol interference D_0 is less than unity. Because of this it is desirable, wherever possible, to use modulation parameters which, together with the expected range of line characteristics, will generally yield an equivalent base-band channel which has an unequalised intersymbol interference D_0 which satisfies this condition. This may necessitate the use of a sub-optimal symbol transmission rate to avoid the line transmission-band limits which cause the greatest amount of signal distortion.

If higher symbol transmission rates are essential, then more complex equaliser strategies are available which will converge whatever value the unequalised intersymbol interference D_0 takes. It must be borne in mind, however, that it will not be possible to reduce the intersymbol interference to an acceptable value by means of the equaliser if the equivalent base-band channel band-width is inadequate to support the transmission according to Nyquist's criterion. The contravention of Nyquist's criterion always gives an intersymbol interference equal to or greater than unity.

In this chapter we consider first the problem of automatic pre-set equalisation for channels where D_0 is always expected to be less than unity. We then consider a strategy proposed by Lucky[6.1] for systems where the unequalised intersymbol interference D_0 can take on values equal to and greater than unity. Since the absolute convergence of both these strategies is dependent on the use of perfect delay-line modules, we also consider a strategy that can be used in conjunction with more general sets of networks than simple unit delay networks. The relative performances of these three strategies, used in conjunction with imperfect unit delay modules, is then demonstrated by computer simulation.

The use of sets of networks other than unit-delay modules is then considered, including the use of delay networks where the delay period is a fraction of the symbol period. The advantages and disadvantages of such networks are discussed.

6.2. An Iterative Pre-Set Equaliser Strategy for use when $D_0 < 1$.

We have stated in section 2.1 that for $D_0 < 1$ all that is required is to adjust the overall sampled single-pulse response $y(mT)$ so that:

$$y(mT) = 0 \text{ for } 0 \leqslant m \leqslant p+q,\ m \neq p,$$

$$y(mT) = 1 \text{ for } m=p.$$

Because the strategy requires the adjustment of the single-pulse response to give voltage zeros at the sampling instants, it is frequently referred to in the literature as the " Zero Forcing Strategy".

Since the coefficient adjustments are substantially independent for $D_0 < 1$, it is possible to simply increment the coefficients in steps, the sign of the increment being opposite to that of the corresponding sample in the single-pulse response, i.e. $\delta K_x = -\Delta \operatorname{sgn} y((x+p)T)$

where Δ is the increment step size.

This strategy was originally proposed by R.W.Lucky of Bell Laboratories[6.1] and has been implemented in the course of this work both by computer simulation and also in hardware. Lucky has shown that the algorithm is convergent for all conditions of $D_0 < 1$, assuming perfect delay-line modules are used.

In order to implement this strategy it is necessary to determine the single-pulse response of the system. To do this a pre-determined training pattern is transmitted by the sender terminal, the a priori knowledge of this pattern at the receiver being used to compute the single-pulse response from the received signal. In theory there is a very wide range of training patterns that could be used for this purpose. In practice, however, there are basically three types of pattern for which it is relatively easy to perform this computation.

a) Single-Pulse Pattern. The simplest form of training pattern consists of single pulses representing one transmitted symbol separated by a period that is longer than the time duration of the overall response of the system. This separation is necessary to avoid interaction between the tails of the responses of adjacent pulses. This pattern gives the single-pulse response of the system at the receiver terminal without further computation. Although it has been widely used because of this computational simplicity, it does, nevertheless, have a number of disadvantages.

Firstly, the pattern contains a d.c. component which cannot normally be transmitted through the modem, since these are generally a.c. coupled to prevent variations in the d.c. conditions being reflected into the output signal. This can be eliminated quite simply, however, by transmitting each alternate pulse of opposite polarity.

The second disadvantage is that the zero voltage condition which has to be sent between the pulses is not normally one of the discrete levels used in either binary or multilevel transmission. This arises because, for ease of encoding, it is usual to operate at 2^n levels which are then spaced symetrically about zero so that random signals do not produce a d.c. component in tne data stream. It means that special facilities have to be provided in the 'send' modem to provide this zero voltage condition. It also means that the transmission is no longer compatible with an equaliser functioning in the adaptive mode if dual mode operation, as described in section 8, is envisaged.

A further disadvantage is that a number of single-pulse patterns must be transmitted and the received signals averaged in order to eliminate any perturbations of the received signal caused by noise.

b) Reversals Training Pattern. Instead of sending single-pulses, alternate reversals from positive to negative and negative to positive are sent, the signal being maintained at the positive or negative level between reversals (See fig. 6.1). Then the response to alternate positive and negative single-pulses can be computed by delaying the equaliser output by one symbol period and subtracting the delayed signal from the direct output.

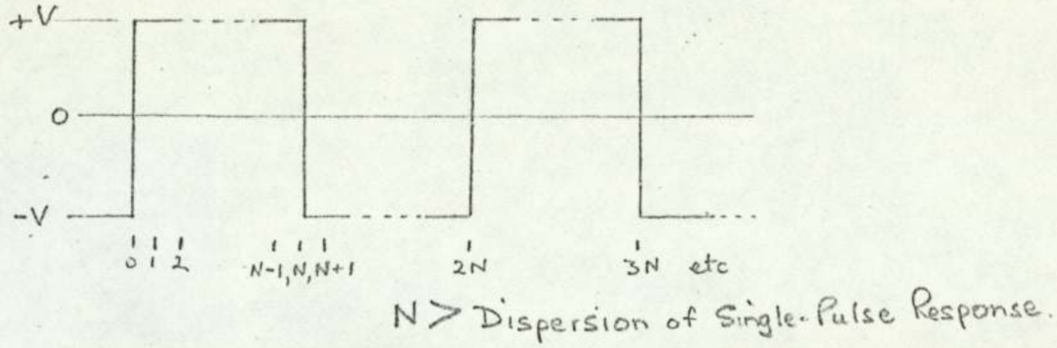


Fig.6.1. 'Reversals' Training Pattern.

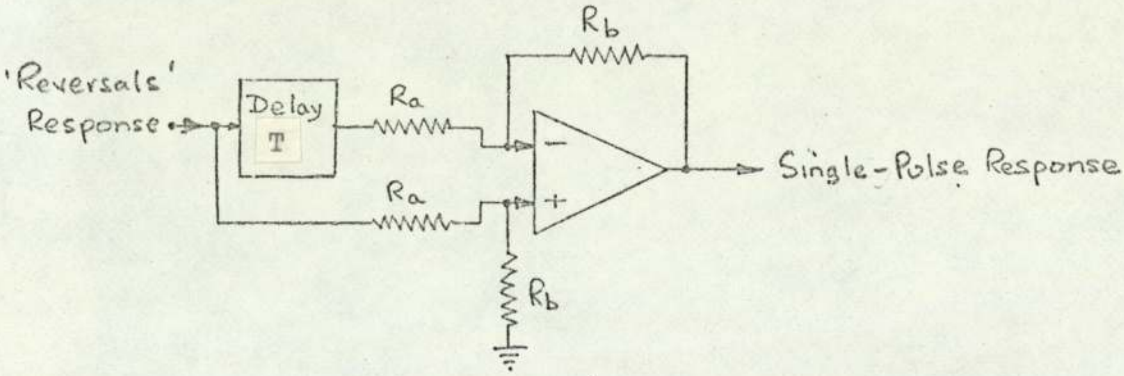


Fig.6.2. Circuit to Compute Single-Pulse Response from Response to 'Reversals' Pattern.

A schematic of the computational circuit is given in fig.6.2. This 'Reversals Pattern' has the advantages that there is no d.c. component in the signal and the transmitted levels can be standard multilevel transmission levels. The signal is therefore compatible with equalisers operating in the adaptive mode, though such operation will not generally be ideal since it is necessary for the transmitted signal to be random in nature for proper operation in this mode (See section 7).

As the computation of the single-pulse response involves the determination of small differences between two relatively large quantities, care has to be taken to ensure good accuracy in the computation circuits.

c) P.r.b.s. Training Pattern. The pseudo-random binary sequence (p.r.b.s.) is another useful training pattern. Because of the special properties of p.r.b.s.[6.2 to 6.4], the single-pulse response can be obtained by cross-correlation of the signal at the equaliser output with delayed versions of the same p.r.b.s. generated at the receive terminal.

Many modern modems incorporate a data scrambler[6.5] in order to give properties of randomness to the transmitted data stream. The transmitted sequence from such a modem in the 'rest' condition (i.e. an all-zero sequence input) will in fact be a p.r.b.s. The descrambler in the receive modem is readily usable to generate the delayed versions of the p.r.b.s. required by the equaliser. The pattern generation facilities required are therefore already incorporated in these modems.

Although a number of correlators are required, these in fact prove to be very simple as the multiplication function consists of simple gatings, one of the inputs taking only the values of +1 or -1. A schematic diagram of the correlator is given in fig.6.3.

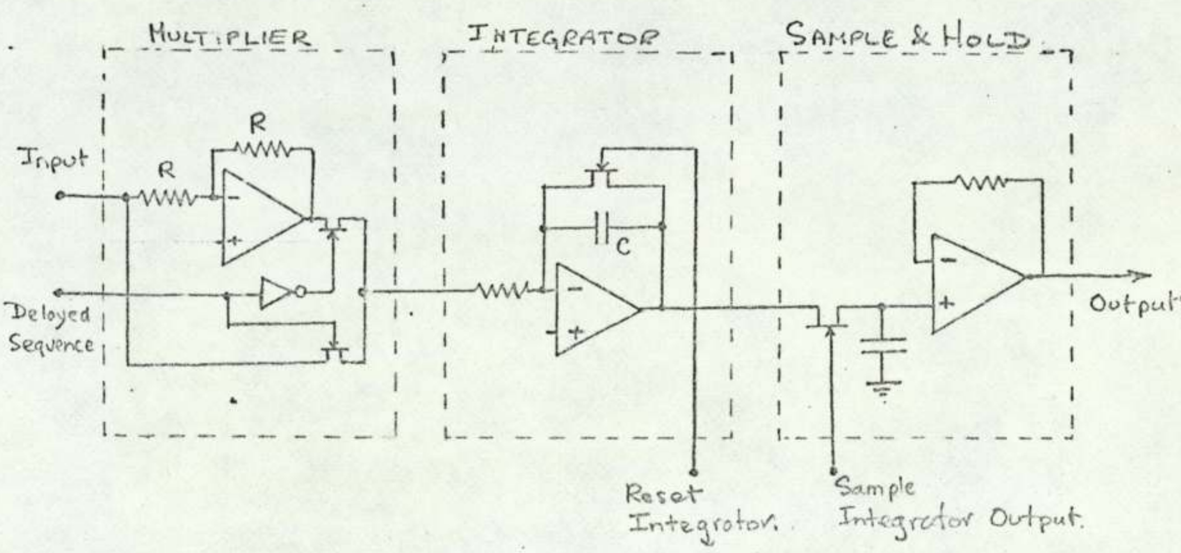


Fig.6.3. Correlator - Schematic Diagram.

The main advantage of the p.r.b.s. test pattern is that the signal is equally compatible with equalisers operating in the adaptive and automatic pre-set modes. The correlation process eliminates the effects of noise on the response determined over a single pattern transmission. It is therefore possible to achieve faster setting-up, a feature which is of interest where fast aquisition of the conditioned data channel is a prime requirement.

If sufficient computing power is availabe at the receive terminal, it is possible to calculate the coefficient values directly from the single-pulse response using the equations (2.5) given earlier. It would not generally be economical, however, to provide special-purpose computing circuits especially for this operation within the actual modem equipment.

6.3. Lucky's Iterative Strategy for $D_0 \geqslant 1$.

Lucky has shown[6.1] that the zero-forcing strategy described in the previous section will not guarantee a minimum D condition where $D_0 \geqslant 1$; in fact it may actually increase D to a value greater than the unequalised value D_0. He therefore proposes an alternative strategy for minimisation of D which can be used under all conditions. In this strategy the coefficients are adjusted by some constant amount in a direction opposite to the sign of the function

$$\frac{\partial D}{\partial K_i} = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} (h_{n-i} - h_n h_{-i}).\mathrm{sgn}\, y_n \quad . \; . \; . \; . \; . \; (6.1)$$

where $h_x = h(xT)$ and $y = y(xT)$.

Alternatively, the adjustment may be by an amount proportional to the derivative (Although see comments in section 6.4, which also apply here.)

To have available the necessary values of h_x to perform the computation, it is necessary to provide a delay-line which has ideal delay. The use of conventional delay-lines with this strategy is therefore impracticable for all but very small equalisers, since the cumulative distortion along the line can become very significant. To combat this, a strategy has been developed which enables use to be made of more general classes of network than the tapped delay-line of the conventional transversal equaliser. Since the strategy described in this section is simply a special case of the general network strategy, a separate mathematical treatment is not given here.

6.4. A General Network Strategy for $D_0 \geqslant 1$.

The mathematical treatment given in this section was suggested to the author by Dr.L.F.Turner of Imperial College of Science and Technology, London[6.6].

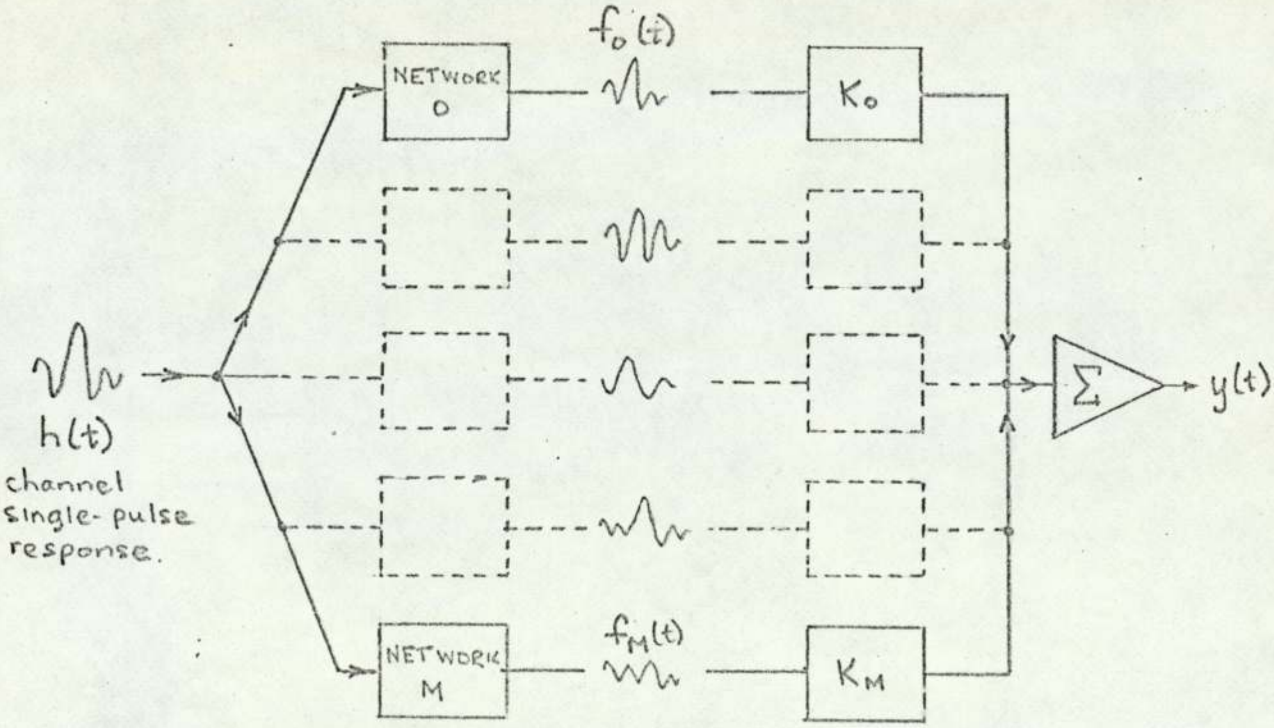


Fig.6.4. Equaliser Using a Parallel Bank of Linear Networks.

If the unequalised single-pulse response $h(t)$ is fed in parallel to M+1 arbitary linear networks whose outputs in response to the input function $h(t)$ are $f_0(t)$, $f_1(t)$ . . . $f_M(t)$ respectively, and these outputs are combined in a summing network as shown in fig.6.4, then the overall system single-pulse response $y(t) = K_0 f_0(t) + K_1 f_1(t) + \ldots\ldots K_M f_M(t)$.

The overall system output at time $t=nT$ in response to a single-pulse stimulus at the system input is then given by:

$$y(t=nT) = y_n = K_0 f_0(nT) + K_1 f_1(nT) + \ldots\ldots K_M f_M(nT)$$

$$= \sum_{i=0}^{M} K_i f_{in} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.2)$$

where $f_{in} = f_i(nT)$, is the sample value of $f_i(t)$ at the time $t = nT$.

The object of the equalisation process is to adjust the weighting coefficients K_0, K_1 . . . K_M, so as to minimise the intersymbol interference D. If, as in section 3.1, we normalise the units of $y(t)$ so that the equalised single-pulse response amplitude is unity at the main sampling instant pT, then the condition for minimum intersymbol interference is satisfied when the Ks are adjusted so that $y(pT) = 1$ and

$$D = \sum_{\substack{m=-\infty \\ m\neq p}}^{\infty} |y(mT)| \quad = \text{minimum} \ \ldots\ldots\ldots (6.3)$$

(Compare with equation 3.4.)

The only restrictions in the choice of networks is that they be linear and that their output functions $f_i(nT)$ be linearly independent.

In the following analysis it is assumed that $f_0(t)$ is normalised to have the value of unity at the reference instant $t = pT$. Using K_0 to satisfy the constraint on $y(t)$ at $t = pT$, it follows that:

$$y(pT) = y_p = 1 = K_0 f_{0p} + K_1 f_{1p} + \ldots\ldots K_M f_{Mp},$$

and since f_{0p} is normalised to unity, the value of K_0 satisfying the constraint condition on y_p is:

$$K_0 = 1 - \sum_{i=1}^{M} K_i f_{ip} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.4)$$

On substituting for K_0, equation 6.2 becomes

$$y_n = f_{0n}\left\{1 - \sum_{i=1}^{M} K_i f_{ip}\right\} + \sum_{i=1}^{M} K_i f_{in}$$

$$= f_{0n} + \sum_{i=1}^{M} K_i \left[f_{in} - f_{0n} f_{ip}\right]$$

and, on using this expression for y_n in equation 6.3, the intersymbol interference D can be written as:

$$D = \sum_{\substack{n=-\infty \\ n \neq p}}^{\infty} \left\{\left| f_{0n} + \sum_{i=1}^{M} K_i \left[f_{in} - f_{0n} f_{ip}\right]\right|\right\} \quad \ldots\ldots (6.5)$$

The object of equalisation is to minimise this expression by varying the weighting coefficients $K_1, K_2, \ldots K_M$, K_0 being given by equation 6.4.

If it can be shown that D is a function of the M variables $K_1, \ldots K_M$ having a single minimum value D_X, then it is possible to carry out the minimisation process on an iterative basis, with the coefficients being gradually adjusted to their optimum values.[6.7]

Let D_j ($j = 1, 2, \ldots M$) be the intersymbol interference when the weighting coefficients $K_1 \ldots\ldots K_M$ are set to $K_{j1} \ldots\ldots K_{jM}$ respectively, and let D_X be the intersymbol interference at any intermediate setting where the weighting coefficients have the values:

$$
\begin{aligned}
K_1 &= K_{X1} = a_1K_{11} + a_2K_{21} + \ldots\ldots + a_MK_{M1} \\
K_2 &= K_{X2} = a_1K_{12} + a_2K_{22} + \ldots\ldots + a_MK_{M2} \\
&\vdots \\
K_M &= K_{XM} = a_1K_{1M} + a_2K_{2M} + \ldots\ldots + a_MK_{MM}
\end{aligned}
\qquad \ldots\ldots (6.6)
$$

with $0 \leqslant a_i \leqslant 1$; $i = 1, 2, \ldots, M$, and $\sum_{i=1}^{M} a_i = 1$.

In order to show that the intersymbol interference D has a single minimum value, it is only necessary to show that D_X is less than or, at most, equal to any of $D_1, D_2, \ldots D_M$ at the M distinct settings of the equaliser. Thus it is only necessary to prove that:

$$D_X \leqslant \sum_{j=1}^{M} a_jD_j.$$

From equation 6.5 it can be seen that

$$D_X = \sum_{\substack{n=-\infty \\ n\neq p}}^{\infty} \left\{ \left| f_{On} + \sum_{i=1}^{M} K_{Xi} \left[f_{in} - f_{On}f_{ip} \right] \right. \right.$$

and, on substituting for K_{Xi} from 6.6, we get:

$$D_X = \sum_{\substack{n=-\infty \\ n\neq p}}^{\infty} \left\{ \left| f_{On} + \sum_{i=1}^{M} (a_1K_{1i} + \ldots + a_MK_{Mi}) \left[f_{in} - f_{On}f_{ip} \right] \right| \right\}$$

$$= \sum_{\substack{n=-\infty \\ n\neq p}}^{\infty} \left\{ \left| a_1 \left(f_{On} + \sum_{i=1}^{M} K_{1i} \left[f_{in} - f_{On}f_{ip} \right] \right) \right. \right.$$

$$\left. \left. + \ldots\ldots\ldots + a_M \left(f_{On} + \sum_{i=1}^{M} K_{Mi} \left[f_{in} - f_{On}f_{ip} \right] \right) \right| \right\}$$

Since $\left| x_1 + x_2 + \ldots + x_M \right| \leqslant \left| x_1 \right| + \ldots\ldots + \left| x_M \right|$, it is thus clear that:

$$D_X \leqslant \sum_{n=-\infty}^{\infty}\left\{\left|a_1\left(f_{0n} + \sum_{i=1}^{M} K_{1i}\left[f_{in} - f_{0n}f_{ip}\right]\right)\right|\right\}$$

$$+ \; . \; . \; . \; . \; . + \sum_{n=-\infty}^{\infty}\left\{\left|a_M\left(f_{0n} + \sum_{i=1}^{M} K_{Mi}\left[f_{in} - f_{0n}f_{ip}\right]\right)\right|\right\}$$

That is:

$$D_X \leqslant a_1D_1 + \; . \; . \; . \; . \; . \; . \; . \; . + a_MD_M,$$

which proves that D has a single minimum value D_X.

Since the intersymbol interference D has a single minimum value, an iterative technique can be used to minimise the value of D given by equation 6.5. An iterative strategy is therefore required to systematically adjust the weighting coefficients to effect this minimisation. One such method is the method of gradients[6.7]. In this method, the slope of the function with respect to each of the variables K_1, K_2, . . . K_M is derived and each variable is adjusted in a direction opposite to the sign of the derivative by some constant amount, or by an amount proportional to the derivative. In the present case, the method in which the variables are adjusted by amounts proportional to the derivatives is not very satisfactory, since equation 6.5 is piece-wise linear and its derivatives are therefore constant close to the minimum, and may be large.

The function D of equation 6.3 can be written as

$$D = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} |y_n| = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} y_n . \left\{\text{sign } y_n\right\}$$

$$= \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty}\left\{f_{0n} + \sum_{i=1}^{M} K_i(f_{in} - f_{0n}f_{ip})\right\}.\left\{\text{sign } y_n\right\} \quad . \; . \; . \; (6.7)$$

From equation 6.7, the partial derivative with respect to the i^{th} weighting coefficient K_i is found to be:

$$\frac{\partial D}{\partial K_i} = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \left\{ f_{in} - f_{On} f_{ip} \right\} \cdot \left\{ \text{sign } y_n \right\} \ldots (6.8)$$

The adjustment procedure therefore involves the computation of the sign of this function for each coefficient, which is then adjusted in a direction opposite to the sign of the function, either by a small constant change or a change proportional to the magnitude of the derivative.

If the function $f_O(t)$ is made the channel single-pulse response $h(t)$ and the functions $f_i(t)$ delayed versions of $h(t)$, such that $f_i(t) = h(t).Z^{-i}$, the main sampling instant of $h(t)$ being defined as $t = 0$, then the equation 6.8 becomes the equation 6.1 of section 6.3. The algorithm described in section 6.3 is thus the special case of the general algorithm when the networks are perfect delays of integer multiples of the unit-symbol period.

6.5. Computer Simulation Program for Automatic Pre-set Equaliser Strategies.

A computer program was written to enable simulation experiments to be carried out to determine the behaviour of the automatic pre-set equaliser strategies described in the preceding sections. A flow-chart of the simulation program is given in fig.6.5.

The equaliser specification and the single-pulse response of the channel to be equalised are provided to the computer as program input data. The equaliser specification details the number of delay-sections used, the position of the reference tap, the size of the coefficient increment and the strategy used to update the coefficients. It is also possible to

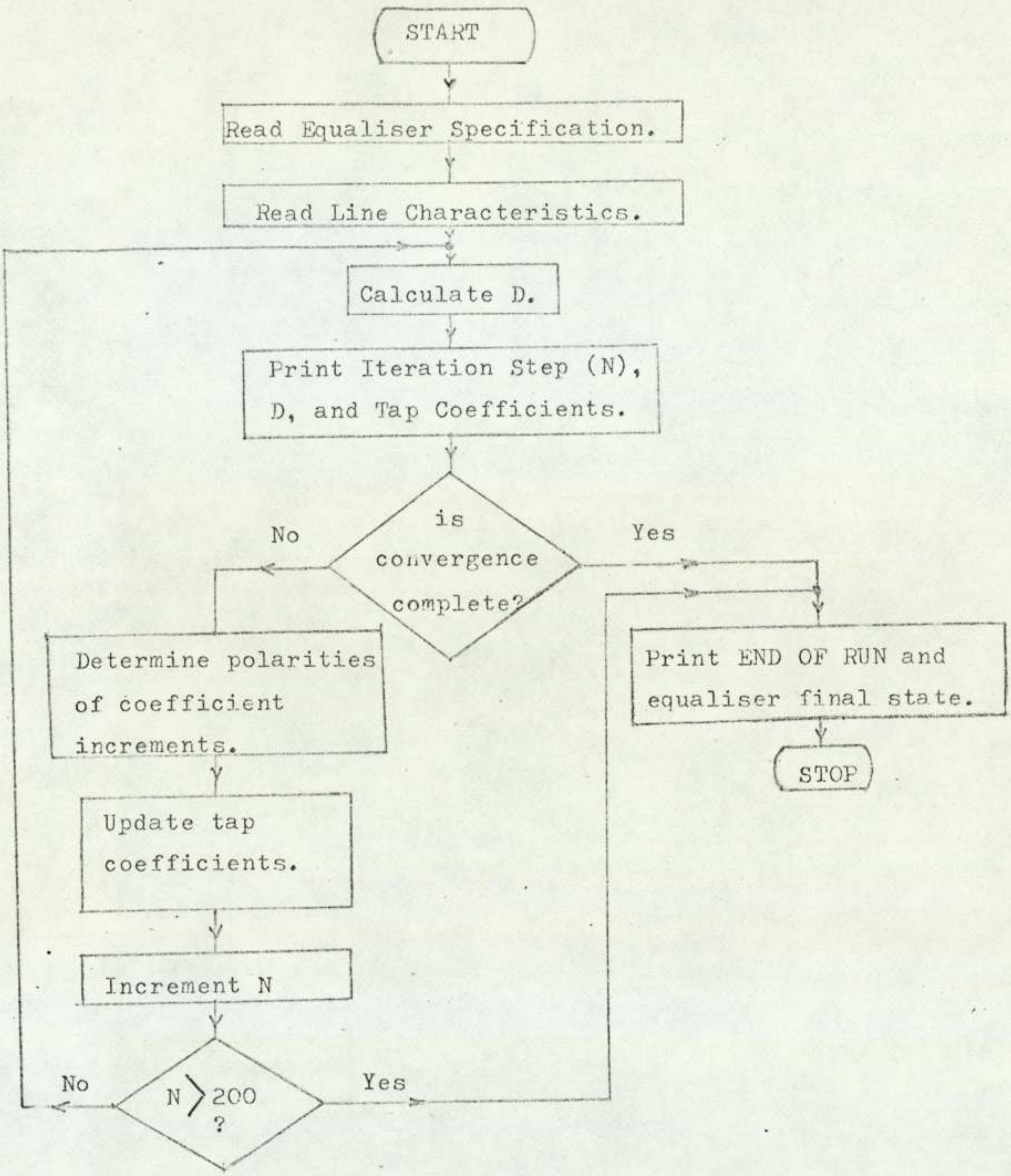


Fig.6.5. Flow-Chart for Automatic Pre-set Equaliser Program.

vary the delay-time per section from one unit-symbol period by a specified proportion, so that the effects of timing error can be studied. The sampling instants are assumed to be correctly defined at the reference tapping point.

The program prints out the initial conditions of the equaliser and the conditions following each iteration of the equalisation process. The polarities of the coefficient updates are computed according to the strategy specified in the input data and the coefficients are incremented in steps, the magnitudes of which are also specified in the input data. The program automatically terminates when all the coefficient values have converged to a condition of oscillation between two adjacent steps in the quantised value scale, or after 200 iterations, whichever occurs the earlier.

The program is written in FORTRAN IV language and the program card deck is compatible with the IBM System/360 Operating system.

6.6. Effect of Delay-Timing Errors on Equaliser Performance.

One of the major deficiencies in the use of conventional delay elements in the transversal filter is the accuracy to which the delay-time can be achieved. The computer program was therefore used to determine the effect of delay-timing errors on the performance of the various strategies described earlier in this chapter. For these tests, the delayed waveforms were taken to be exact replicas of the input waveform, but the delay period of each delay-line element (τ) was altered from the symbol period (T) by a fixed proportion Δ, such that $\tau = (1 + \Delta)T$. The sampling timing was arranged to be ideal at the reference output.

Table 6.1 gives the results of tests on five equivalent base-band channel responses, equalised by means of thirteen-tap, non-recursive transversal equalisers, the fifth tap being taken as the reference output. The coefficient increment was 0.01 and timing errors of ± 5 % and $\pm 20\%$ were introduced. For the first three channel responses, where D_0 is less than unity, both the strategies described in sections 6.2 and 6.3 were used, the results in each case being identical. However, since the responses X1 and X2 have $D_0 \geqslant 1$, only the strategy described in section 6.3 was applicable in these cases. It is seen that with the size

Channel No.	D_O	Range of Variation of D After Equalisation.				
		ΔT=0	ΔT=+5%	ΔT=-5%	ΔT=+20%	ΔT=-20%
W1A.	0.5127	0.1207 to 0.1458	0.1178 to 0.1430	0.1264 to 0.1328	N.C.	N.C.
W1G.	0.5056	0.1339 to 0.1804	0.1294 to 0.1595	0.1317 to 0.1545	N.C.	N.C.
W2H.	0.4486	0.1752 to 0.1770	0.1473 to 0.1877	0.1675 to 0.2178	N.C.	N.C.
X1.	2.0444	1.7038 to 1.7244	1.7378 to 1.7557	1.6869 to 1.7269	1.7343 to 1.7877	1.6064 to 1.6264
X2.	2.5214	1.1766 to 1.2334	1.3804 to 1.3852	1.1914 to 1.2233	N.C.	N.C.

ΔT = Timing Error. Coefficients before reference tap = 4

N.C. = Not convergent. Coefficients after reference tap = 8

Coefficient Increment = 0.01. Non-recursive configuration.

Table 6.1. Effect of Timing Errors in Delay-Line on Performance of Lucky's Algorithms.

Channel No.	D_O	Range of Variation of D After Equalisation.				
		ΔT=0	ΔT=+5%	ΔT=-5%	ΔT=+20%	ΔT=-20%
W2H.	0.4486	0.1752 to 0.1770	0.1473 to 0.1877	0.1675 to 0.2178	0.1695 to 0.1724	0.2002 to 0.2307
X1.	2.0444	1.7038 to 1.7244	1.7195 to 1.7590	1.6869 to 1.7269	1.6785 to 1.6851	1.6549 to 1.6850
X2.	2.5214	1.1766 to 1.2334	1.1794 to 1.2408	1.1587 to 1.2143	1.2017 to 1.2431	1.1931 to 1,2063

ΔT = Timing Error. Coefficients before reference tap = 4

N.C. = Not Convergent. Coefficients after reference tap = 8

Coefficient Increment = 0.01. Non-recursive configuration.

Table 6.2. Effect of Timing Errors in Delay-Line on Performance of General Network Algorithm.

and configuration of equaliser tested, a timing error of 5% is tolerable, but for errors of the order of 20%, the strategies are no longer convergent.

Three of these channel responses were then tested with identical equalisers but using the strategy described in section 6.4. The results of these tests are given in table 6.2, from which it will be seen that the performance of the strategy is unaffected by timing errors of the order of 20%.

6.7. Equalisers with Fractional Delay-Elements.

The fact that the proper operation of the strategy described in section 6.4 does not depend on the use of perfect unit-delay modules, suggests the possibility of using delay-elements whose delay-times are a fractional proportion of the unit symbol period. The advantage of such an arrangement over the conventional transversal equaliser is that the dispersion of the equalising waveforms no longer increases in proportion to the number of equaliser delay sections used. It therefore becomes possible, in theory, to equalise completely any finite response with a finite length equaliser[6.8].

Suppose the response extends from the reference point in one direction a period of $(n + \delta)T$ secs. $(\delta < 1)$. See fig.6.6.

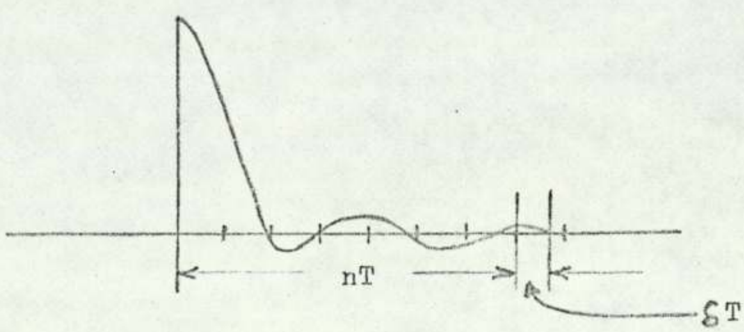


Fig.6.6. Typical Response Dispersion.

If we delay the response by CT secs. per stage of equalisation, and (n + k) stages of equalisation are provided, then the response is extended by (n + k)CT secs., as shown in fig.6.7.

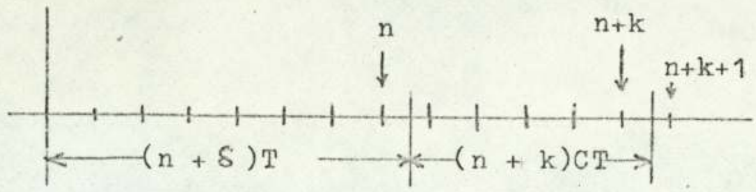


Fig.6.7. Increase in Dispersion due to Delay-Stages.

Now we have to satisfy n + k conditions to obtain complete equalisation. We therefore have to find the smallest integer k such that:

$$n + k \leqslant \frac{(n + \delta)T + (n + k)CT}{T} < n + k + 1.$$

i.e. $$k \leqslant (n + k)C + \delta < k + 1.$$

i.e. $$k \leqslant nC + \delta + kC$$

$$\therefore k(1 - C) \leqslant nC + \delta$$

$$\therefore k \leqslant \frac{(nC + \delta)}{1 - C}$$

& $$(n + k)C + \delta < k + 1$$

$$\therefore nC + kC + \delta < k + 1$$

$$\therefore nC - 1 + \delta < k(1 - C)$$

$$\therefore \frac{(nC - 1 + \delta)}{1 - C} < k$$

Thus we select the smallest k such that:

$$\frac{(nC - 1 + \delta)}{1 - C} < k \leqslant \frac{(nC + \delta)}{1 - C} \quad \ldots\ldots\ldots\ldots \quad (6.9)$$

For channel response W2H, the values of n and δ are as follows:

a) Anticipatory response: $n = 4$, $\delta = 0.3$

b) Trailing response: $n = 18$, $\delta = 0.6$

Thus, if $C = 0.5$, on substituting values in (6.9) we get:

a) $k = 3$ for the anticipatory response and

b) $k = 18$ for the trailing response;

which means that 7 variable coefficients before the reference tap and 36 after the reference tap, a total of 42 stages of equalisation, are required to completely equalise channel response W2H, using a delay-line with a delay-time per module of 0.5T. Similarly, for C = 0.2, we get k_a = 1 and k_b = 5 respectively, giving 5 coefficients before and 23 coefficents after the reference tap, a total of 29 stages, for complete equalisation.

Because of the large number of stages required for complete equalisation, a test was carried out using the same equaliser size as for the tests detailed in section 6.6 (i.e. 4 coefficients before and 8 coefficients after the reference tap), to determine the degree of equalisation obtainable with a more limited number of stages. The test was carried out on channel response W2H, using a delay-line with a delay-time per module τ = 0.5T. The equaliser converged to a value of D with a range of random variation from 0.2774 to 0.3136. Since this result is somewhat inferior to that obtained with the same size equaliser but with τ = T, the size of the equaliser was increased to 10 coefficients both before and after the reference tap (21 coefficients in all), and a further series of tests conducted, again on channel response W2H. The results of these tests are given in table 6.3.

Coefficient Increment.	Range of Variation of D After Equalisation.		
	C = 1.0	C = 0.5	C = 0.2
0.01	0.1764 to 0.1833	0.2213 to 0.2382	Not tested.
0.005	0.1345 to 0.1350	0.1906 to 0.1907	0.3125 to 0.3258

Table 6.3. Effect of Fractional Delays on Equaliser Performance.

It will be seen that the equaliser using fractional-delay elements still gave inferior results to that obtained using unit-delay elements.

A test was then conducted on an equaliser with a fractional-delay C = 0.2, using the optimum number of coefficients calculated above. It was found that this equaliser did converge, but the rate of convergence was extremely slow. Since complete equalisation was theoretically possible, the optimum values of the equaliser coefficients were computed by solving the set of simultaneous equations describing the equaliser operation. The values of these coefficients are given in table 6.4.

n	K_n	K_{n+1}	K_{n+2}	K_{n+3}
1	428.136	-1222.383	780.431	1803.356
5	-3246.726	-639.406	6180.863	-788.688
9	-12055.457	10636.856	4851.703	-9572.773
13	-3701.121	13206.488	-4823.383	-6575.625
17	2156.305	8256.773	-4574.410	-5572.098
21	5440.266	2441.695	-7801.117	6207.711
25	-92.344	-3421.043	1998.187	-79.813
29	-159.626	-	-	-

Table 6.4. Coefficients of Fractional-Delay Equaliser (C=0.2), for Channel Response W2F.

It will be seen that with an increment of 0.005 and an initial set of conditions where the coefficients start from zero, except the reference coefficient, which starts from unity, over $2\frac{1}{2}$ million iterations would be required to achieve optimum equalisation!

Because of the considerable interdependence between the coefficient settings, and because the optimum condition is achieved by minimisation of the differences between very large quantities, the descent on the D hypersurface from D_0 to D_{min} is found to be via a narrow, steep-sided "valley".

Since the D hypersurface is piece-wise linear, the steep gradients of the surface make it impracticable to weight the coefficient increments in proportion to their derivatives. (See section 6.4.) On the other hand, the coefficients need to be represented to an absolute accuracy of the order of 0.005 to ensure that the deterioration in intersymbol interference due to quantisation D_q is not greater than about 0.1. Thus, the final increment step size is also limited to about 0.005.

An experiment to investigate the possibility of commencing operation with large increments, later changing to smaller increments, was carried out. The same equaliser and channel response was used as in the previous experiment, but with an increment size of 10.0. If convergence were achieved, this would require about 1500 iterations to approach the optimum coefficient values. However, the step size was found to be too large to allow the algorithm to follow the valley in the hypersurface and the equaliser simply oscillated on alternate iterations between the value of $D_0 = 0.4486$ and a value of $D = 285.1$. It was therefore concluded that, although it is theoretically possible to acieve complete equalisation with fractional-delay elements in the equaliser implementation, there are a number of practical difficulties which make the technique unsuitable for use in a practical situation. These difficulties include:

a) To gain an advantage over the equaliser using unit-delay modules, a near-optimum number of coefficients must be used, and this optimum number is prohibitively large, except where C is small ($\not> 0.2$).

b) The unlimited range of values that can be taken by the coefficients as compared to the range of $-1 \leqslant K \leqslant 1$ applicable to the equaliser implemented using unit-delay modules.

c) The difficulty of representing the coefficient values to sufficient accuracy in view of the wide range of values possible and the need to maintain an absolute accuracy of the order of 0.005.

d) The impracticability of implementing a strategy that will converge to an optimum setting within a reasonable number of iterations.

6.8. Equalisers Using Simplified Delay-Networks.

The possiblity of using simplified delay-networks in conjunction with the general network equaliser strategy was then considered. For this test the delay-line sections were assumed to consist of the simplified network given in fig.6.8.

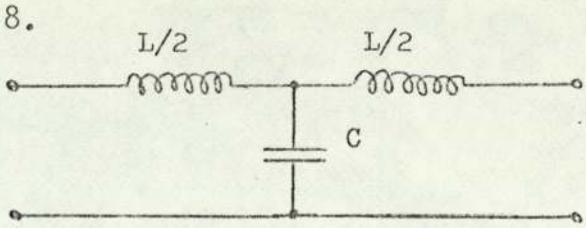


Fig.6.8 Simplified Delay-Network.

The values of L and C were chosen so that the delay of a single stage was equal to the symbol period T, the delay per stage (T_d) being given by $T_d = \sqrt{LC}$. Assuming the characteristic impedance $Z_0 = \frac{L}{C} = 600$ ohms and a symbol rate of 2400 symbols/sec., we have:

$$\frac{L}{2} = 125 \text{ mH.}$$

& $$C = 0.694 \text{ uF.}$$

Using 12 such delay sections, the final section being terminated with the characteristic impedance of 600 ohms, the single-symbol response at each of the 13 tapping points was computed. This was then convolved with the delay-line input waveform to give the corresponding waveforms at each tapping point. Equivalent base-band characteristic W2H was used and the fifth tap was taken as the reference output. The sampling timing was referred to the original input waveform.

The intersymbol interference generally increases along the line due to the nature of the transfer characteristics of the delay-line sections. The value of D at the reference output was therefore 4.6419. Using the strategy described in section 6.4, the equaliser converged to a value of D with a range of random variation between D = 2.2756 and D = 2.2857. Although this is a great improvement over the initial value of D at the reference tap, it is still much worse than the value of D_0 relating to the original channel response before equalisation. It is not generally possible, therefore, to obtain satisfactory equalisation using a simplified delay-network of the type under consideration.

6.9. Other Networks for use with General-Network Strategy.

The general network strategy described in section 6.4 does not necessarily require the use of delay-elements, in fact superior operation may possibly be obtainable with networks other than delay networks. The requirements for a suitable set of networks is that they shall be linear and their single-pulse responses shall be linearly independent. The number of networks required in the set is equal to the number of symbol periods in the overall time-period covered by the outputs from all the networks when stimulated by a signal equivalent to the single-pulse response of the channel to be equalised. The preferred networks are, therefore, those that do not significantly delay or disperse the signal input to the network. Since an arbitary selection of networks generally requires an unconstrained range of coefficient values, similar disadvantages apply as for fractional-delay networks.

There are possibly sets of networks which would suitably constrain the coefficient values. For instance, network sets are known which correspond to certain well-known sets of orthogonal functions, e.g. Legendre and Laguerre Functions[6.9]. However, their properties were not studied as part of this work because improvements in technology during the course of the work made sample-and-hold techniques a feasible method of delay-line implementation for voice-band channel equalisers. The sample-and-hold delay-line is equivalent to perfect delay-line sections as far as the optimisation strategies are concerned, and can therefore be used with Lucky's algorithms. Under circumstances where Lucky's algorithms can be used, it is advantageous to do so, since only one set of single-pulse response samples need be computed and stored (h_i), instead of one set per network (f_{ij}), in order to determine the coefficient updating increments.

6.10. Conclusions.

Because perfect delay characteristics are impossible to achieve in practice, using conventional delay-line implementations, it is possible that unsatisfactory optimisation will result from the use of the coefficient setting strategies proposed by Lucky and widely used in equaliser design. A general network strategy has therefore been described which gives satisfactory performance with non-ideal delay modules.

The use of the strategy with fractional-delay modules was investigated, since it is theoretically possible to achieve perfect equalisation with such an arrangement. Although it was shown that the strategy could be made to converge, the number of iterations required to reach convergence was generally too large to be of practical interest.

Advances in technology favour sample-and-hold techniques for delay-line implementation for voice-band channel equalisers and these can be used satifactorily with Lucky's algorithms (See section 5.8). The general network strategy may well have advantages, however, in other equaliser applications, especially where the disadvantages of additional sample storage can be overcome. One case where this may be possible is where the receive terminal is in fact a central computer, receiving data from a remote terminal, and it is thus possible to implement the equaliser within the computer in "soft-ware", rather than in special-purpose "hard-ware" within the receive modem.

7. ADAPTIVE ADJUSTMENT STRATEGIES.

7.1. Introduction.

Originally it had been thought that connections through the switched telephone network would have virtually time-invariant characteristics, much as those experienced on leased connections. The only differences expected were more restricted band-width and greater noise impairment, due, in the main, to switching crosstalk[7.1]. Experimental work carried out by the G.P.O.[7.2], however, showed that considerable variations in characteristic did occur, mainly as a result of changes in switch-contact resistance. These variations were particularly noticeable when observed as "listener echoes".

Because variations in characteristic were shown to exist, a requirement became apparent for adaptive equalisation of switched-network channels. A study of adaptive equaliser adjustment strategies was therefore carried out. This study happened to coincide with a developing interest in the use of h.f. radio voice channels for data transmission[7.3,7.4], which also requires the use of adaptive equalisation techniques.

The adaptive equaliser strategies considered in this chapter are based on the assessment of error signals obtained during the course of actual data transmission. The equaliser coefficents are thus continually corrected whilst the data is being transmitted. The error signal is obtained by measuring the differences between the equalised received signal and the 'a posteriori' estimate of the transmitted data by the receiver. This signal is correlated with the data stream to obtain estimates of the coefficient setting errors. The coefficients are then updated according to some criterion so as to minimise the magnitude of the error signal.

Various criteria are discussed later in this chapter.

The adaptive equaliser has two distinct advantages over the automatic pre-set equaliser. Firstly, it can be set up without the necessity for special training pattern transmission facilities and, secondly, it will adapt to compensate for any changes in the transmission channel characteristics as they occur. Against these advantages it has the disadvantage that under adverse conditions of intersymbol interference there is a possiblity that the adaptive strategy will not converge at all. There are also conditions under which the time taken to reach the optimum setting may become long in comparison to that required to set up the automatic pre-set equaliser. In this chapter a study is made of the conditions under which the advantages of the adaptive equaliser can be realised with no significant deterioration in performance to that obtained using the automatic pre-set mode of operation.

7.2. Basic Operation of Adaptive Strategies.

The operation of the adaptive strategies will be described in terms of the channel model given in fig.7.1.

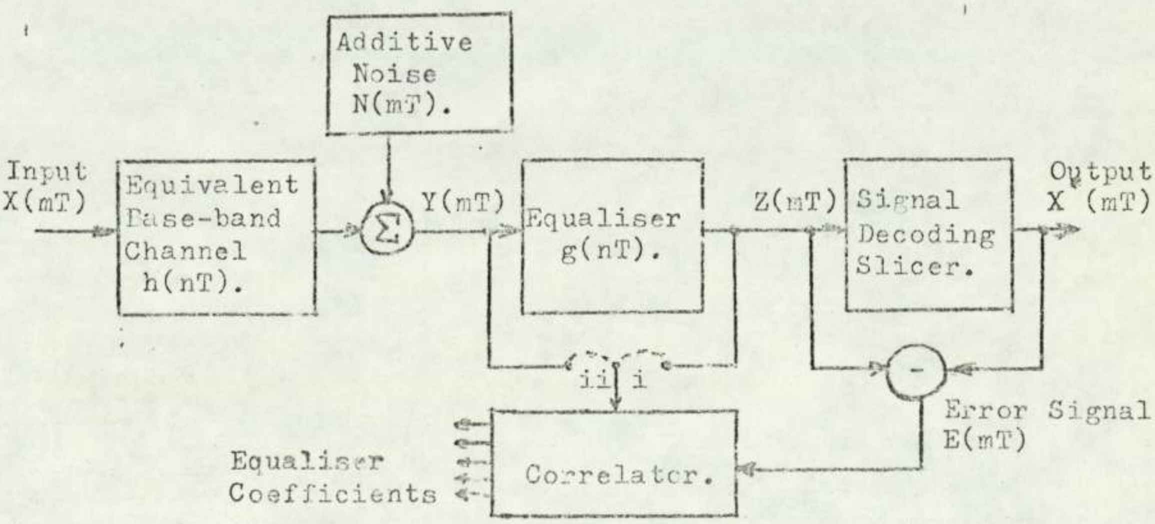


Fig.7.1 Channel Model.

We shall consider the system as a sampled-data system, with a sampling period T, equal to the symbol transmission interval. The input data is a multilevel digital signal $X(mT)$. This passes through the transmission channel, which has a single-pulse response $h(nT)$. Additive noise $N(mT)$ will be encountered in the course of transmission. The input to the equaliser $Y(mT)$ is thus given by:

$$Y(mT) = X(mT) * h(nT) + N(mT) \quad \ldots \ldots \ldots \ldots (7.1)$$

where * denotes convolution.

The equaliser single-pulse response is $g(nT)$, so that the equaliser output $Z(mT)$ is given by:

$$Z(mT) = Y(mT) * g(nT) \quad \ldots \ldots \ldots \ldots \ldots (7.2)$$

The signal is then 'sliced' by the output decoder to yield an estimate of the transmitted signal $X^*(mT)$.

The error signal $E(mT)$ is given by:

$$E(mT) = Z(mT) - X^*(mT) \quad \ldots \ldots \ldots \ldots \ldots (7.3)$$

There are two different strategies that can be used to update the tap weighting coefficients as applied to the basic transversal equaliser. In the first strategy, which is shown in more detail in fig.7.2, the error signal $E(t)$ is correlated with the equaliser output signal $Z(t)$ such that:

$$K_x(t_i) = \propto \int_0^{t_i} Z\left[t-(x+p)T\right].E(t-pT)\, dt \quad \ldots \ldots \ldots (7.4)$$

which, in the case of sampled data, may be written as:

$$K_x(m_iT) = \propto \sum_{m=0}^{m_i} Z\left[(m-x-p)T\right].E\left[(m-p)T\right]$$

$$\therefore \quad K_x(mT) = K_x\left[(m-1)T\right] + \propto.Z\left[(m-x-p)T\right].E\left[(m-p)T\right] \quad \ldots (7.5)$$

It has been shown[7.5] that this strategy optimises the channel characteristics to the same criterion (minimum D, $D_0 < 1$) as the zero-forcing strategy described in section 6.2 for

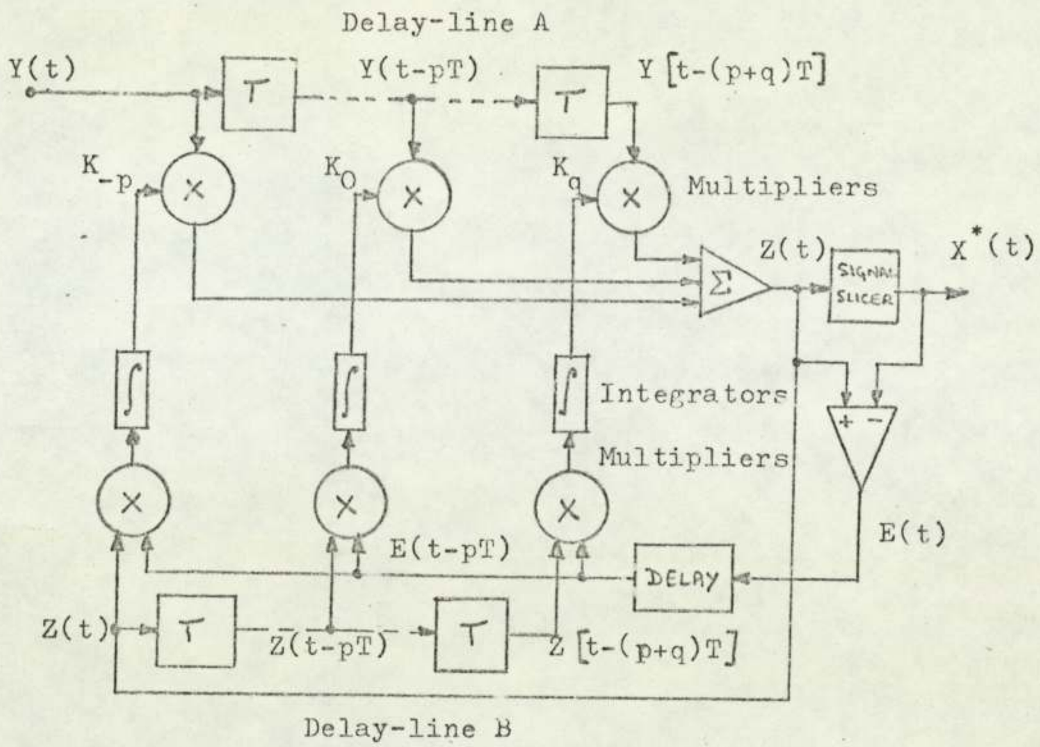


Fig.7.2. Basic Transversal Equaliser Adaptive Strategy I.

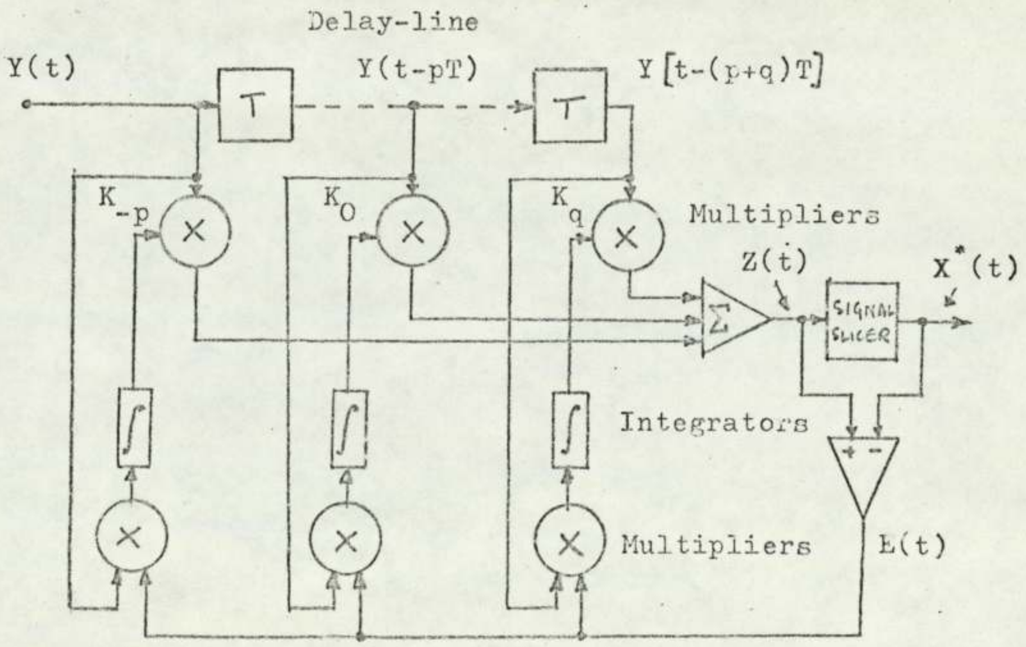


Fig.7.3. Basic Transversal Equaliser Adaptive Strategy II.

the automatic pre-set equaliser. Although this limits its use to where the initial unequalised intersymbol interference is less than unity, this is only a small disadvantage since it is in any case necessary to receive substantially correct data (i.e. $X^*(mT)$ is a reasonably good representation of $X(mT)$) before the strategy will converge satisfactorily. This requirement is discussed more fully later in this chapter.

In the alternative strategy, shown in fig.7.3, the error signal $E(t)$ is correlated with the equaliser input signal $Y(t)$ such that:

$$K_x(t_i) = \alpha \int_0^{t_i} Y\left[t-(x+p)T\right].E(t)\, dt \quad . \; . \; . \; . \; . \; . \; . \; . \quad (7.6)$$

which, in the case of sampled data, may be written as:

$$K_x(m_i T) = \alpha \sum_{m=0}^{m_i} Y\left[(m-x-p)T\right].E(mT)$$

$$\therefore \quad K_x(mT) = K_x\left[(m-1)T\right] + \alpha Y\left[(m-x-p)T\right].E(mT) \quad . \; . \; . \quad (7.7)$$

It has been shown[7.5] that this strategy minimises the mean-square value of the error signal $E(mT)$. This strategy therefore gives better results in the presence of noise $N(mT)$ than the 'zero-forcing' strategy described earlier. It may also be used where the initial unequalised intersymbol interference $D_0 \gg 1$, provided $X^*(mT)$ is a sufficiently good representation of $X(mT)$ to obtain convergence.

7.3. Use of Strategies with Recursive and Decision Feed-back Equalisers.

One of the disadvantages of the simple transversal equaliser is that it may require a large number of variable coefficients to obtain satisfactory equalisation. This is especially the case when the channel impairments take the form of substantial echoes delayed several sampling periods from the main signal. This is just the condition frequently encountered on switched-network connections. Since these echoes can best be dealt with using either the recursive or the decision-feedback equaliser, it would be of great advantage if the adaptive strategies could be applied to these configurations. A possible arrangement is shown in fig.7.4.

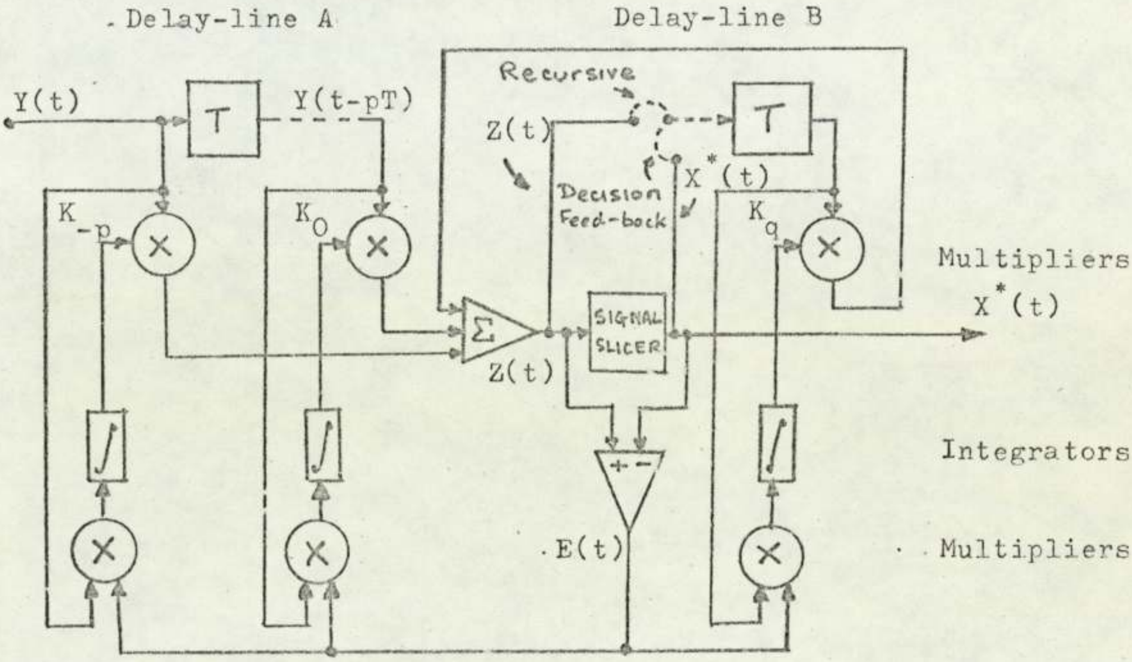


Fig.7.4. Adaptive Strategy for Recursive and Decision-Feedback Equaliser.

In this arrangement, the recursive or decision-feedback equaliser configuration is used in conjunction with the same weighting coefficient adaptation strategy as is given in fig.7.3 for the conventional feed-forward equaliser.

We shall show that such an arrangement converges absolutely to its optimum setting. The following assumptions are made, however, for the purposes of mathematical simplification:

a) The effects of noise on the equaliser performance is negligible.

b) That the equaliser is capable of cancelling completely the intersymbol interference present in the received signal.

c) That the equaliser output signal $X^*(mT)$ represents the transmitted signal without error.

The effects of departures from these assumptions have been determined, either theoretically, or experimentally by computer simulation. These are discussed in detail later in this chapter.

For any given sampling epoch m, the multiplier signal inputs derived from the delay-lines will comprise (p+1) samples of Y(t), namely Y(mT), Y (m-1)T ,.........Y (m-p)T, and q samples of either Z(t) or $X^*(t)$, namely Z (m-1)T , Z (m-2)T ,........Z (m-q)T, or X^* (m-1)T , X^* (m-2)T ,... ....X^* (m-q)T . These signal samples will be represented by an L dimensional vector $\underline{W}$, where L = p+q+1. The following proof is also applicable to the strategies described in the previous section, when the vector $\underline{W}$ will consist of either L samples of Z(t) for strategy I, or L samples of Y(t) for strategy II. The coefficient values K_{-p}....K_0....K_q will also be represented by an L dimensional vector $\underline{K}$. We shall refer to this vector $\underline{K}$ as the state vector of the equaliser. The equaliser output Z(mT) is then given by the scalar product of the signal vector and the state vector:

$$Z(mT) = \underline{W}.\underline{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7.8)$$

Assuming that the digital output of the signal slicer $X^*(t)$ is without error (i.e. it represents X(t) precisely), the error in the equaliser output is given by:

$$E(mT) = Z(mT) - X^*(mT) \quad \ldots\ldots\ldots\ldots (7.9)$$

This error is multiplied by each stored signal and the products are added to the coefficients by the integrators. If the corrections applied to the coefficients are represented by the vector $\underline{\Delta K}$, then:

$$\underline{\Delta K} = -\alpha E(mT)\underline{W} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7.10)$$

where α is a constant defining the rate of convergence of the equaliser strategy. (c.f. equations 7.5 and 7.7).

Since we have assumed that the equaliser is capable of cancelling completely the intersymbol interference in the received signal, there is a state $\underline{P}$ such that, when $\underline{K} = \underline{P}$, the error $E(mT)$ will be zero.

Thus:

$$\underline{W}.\ \underline{P}. = X^*(mT) \qquad (7.11)$$

Substituting (7.8) and (7.11) in (7.9) gives:

$$E(mT) = \underline{W}.\ \underline{K}. - \underline{W}.\ \underline{P} \qquad (7.12)$$

and substituting (7.12) in (7.10) gives:

$$\underline{\Delta K} = -\alpha \underline{W}\ (\underline{W}.\ (\underline{K} - \underline{P})) \qquad (7.13)$$

From this equation it will be seen that the change in state vector during the single sample epoch m is in the direction of the signal vector $\underline{W}$, but its magnitude depends on the scalar product of the signal vector and the state-error vector $(\underline{K} - \underline{P})$

Fig 7.5 is a diagram of the plane in the vector space which contains both the signal vector $\underline{W}$ and the state-error vector $(\underline{K} - \underline{P})$

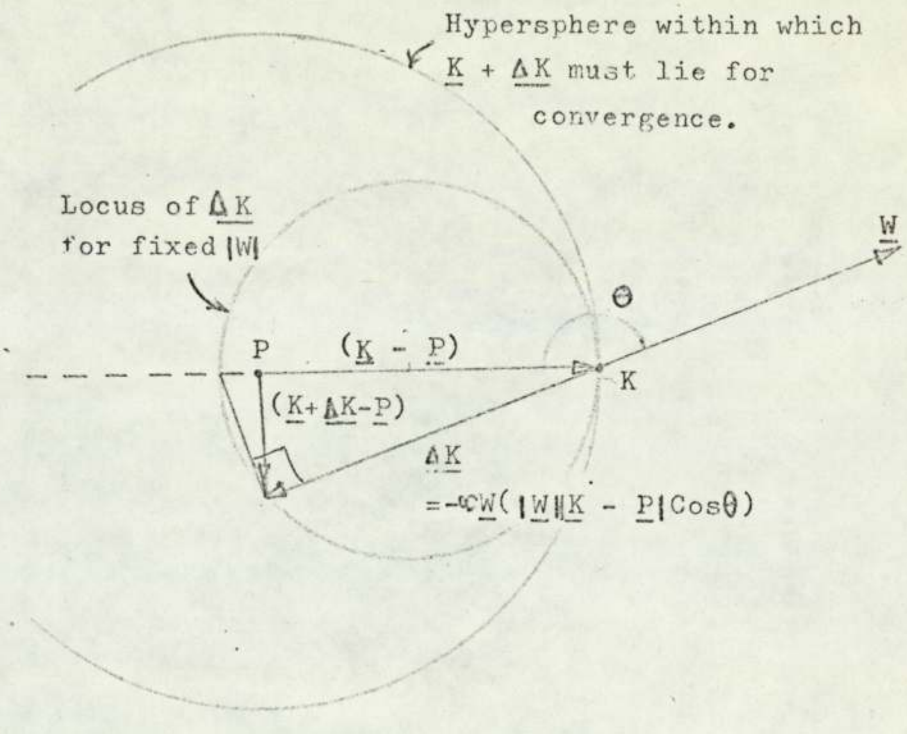


Fig.7.5. Section through state space of adaptive equaliser in the plane of $\underline{W}$ and $(\underline{K} - \underline{P})$ to illustrate constraint on α for convergence.

If the angle between the signal vector and state-error vector is θ, then

$$E(mT) = -|\underline{W}||\underline{K} - \underline{P}|\cos\theta \quad \ldots\ldots\ldots (7.14)$$

Substituting (7.14) in (7.10) gives:

$$\underline{\Delta K} = -\alpha \underline{W}(|\underline{W}||\underline{K} - \underline{P}|\cos\theta) \quad \ldots\ldots (7.15)$$

and hence the locus of the point $(\underline{K} + \underline{\Delta K})$ is, for a given magnitude of $\underline{W}$, a circle passing through $\underline{K}$, with its centre on $(\underline{K} - \underline{P})$ and diameter $\alpha |\underline{W}|^2 |\underline{K} - \underline{P}|$. All points whose distance from $\underline{P}$ is less than the distance between $\underline{P}$ and $\underline{K}$ are contained within the hypersphere centred on $\underline{P}$ and passing through $\underline{K}$. Therefore, if the diameter of the $(\underline{K} + \underline{\Delta K})$ circle is less than $2|\underline{K} - \underline{P}|$, any $(\underline{K} + \underline{\Delta K})$ will be nearer to $\underline{P}$ than $\underline{K}$ is.

Hence, if: $\alpha|\underline{W}|^2|\underline{K} - \underline{P}| \leqslant 2|\underline{K} - \underline{P}|$

i.e. $$\alpha \leqslant \frac{2}{|\underline{W}|^2}, \quad \ldots\ldots\ldots\ldots\ldots (7.16)$$

the system is stable because no change in $\underline{K}$ can occur which moves it further from $\underline{P}$.

7.4. Rate of Convergence of Adaptive Strategy.

One of the points of major interest in regard to the adaptive equaliser is the rate at which the strategy converges the coefficient values to their optimum settings. This parameter also governs the maximum rate of change of channel characterstics which the equaliser is capable of following and neutralising in the course of actual data transmission. The proof in the previous section has already set a limit on the constant of proportionality α that can be used if absolute convergence is to be achieved. In this section we shall consider the dynamic response of a single coefficient in terms of the same constant value. A functional model of the operation of a single coefficient selected from the array Kp...Ko...Kq is given in fig 7.6. This model is based on the mathematical analysis of the previous section.

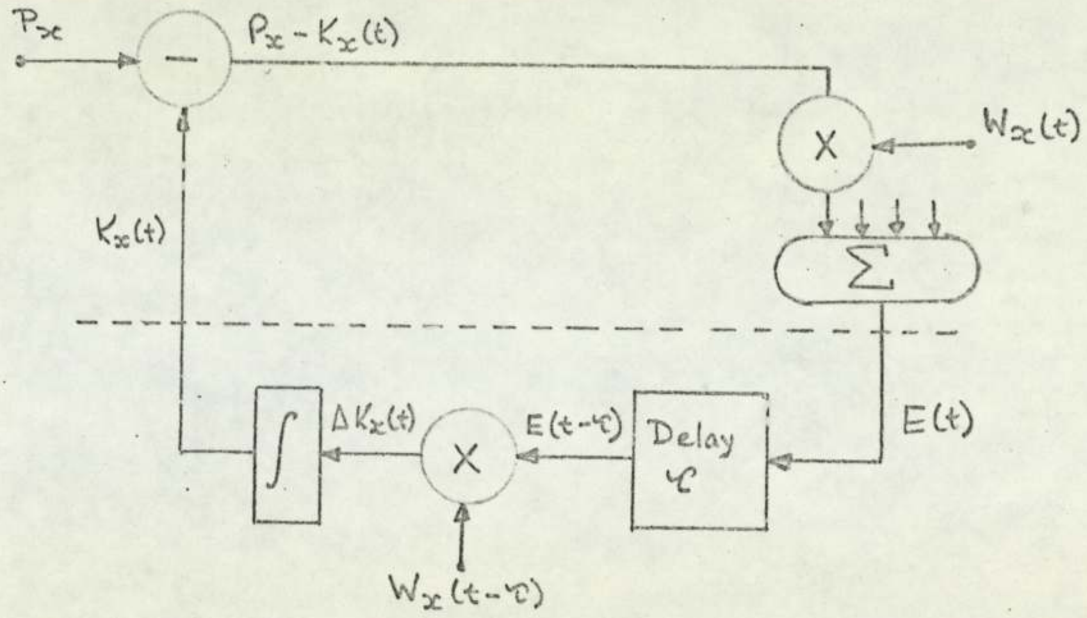
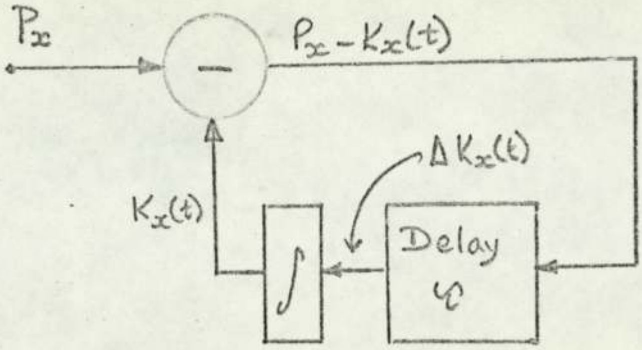


Fig.7.6, Functional Model of Coefficient Updating Strategy.

In the case of strategy II for the basic transversal equaliser and the strategy described for the recursive/decision-feedback equaliser, the delay τ is simply one symbol period T. This arises because the output at sampling instant m has to be determined before the appropriate coefficient correction sample can be calculated and applied. It is therefore not effective on the output until the next sampling operation takes place at instant (m+1). In the case of strategy I for the basic transversal equaliser, a delay of at least p sample epochs is necessary in order to obtain the appropriate samples together at the correlator multipliers. (See fig.7.2.).

The functions above the dotted line in fig.7.6. are implicitly embodied in the equaliser and the value P_x-$K_x(t)$ is thus not explicitly available. The correlation process effected by the multiplier and integrator shown below the dotted line extracts from the error signal E(t) that portion of the signal deriving from the error in the coefficient K_x, i.e. P_x-$K_x(t)$. The contributions to E(t) arising from the errors in the other coefficient settings can therefore be neglected in considering the effect of integration over a number of sample epochs. $\Delta K_x(t)$ is thus the best estimate of the setting error $P_x - K_x(t)$. Assuming that this estimate is satisfactory, we may simplify fig.7.6. as follows:



$$\Delta K_x(t) = P_x - K_x(t-\tau)$$

Fig.7.7. Simplified Model of Coefficient Updating Strategy.

Using this simplified model, then

$$K_x(t) = \beta \int \Delta K_x(t)dt.$$

$$= \beta \int P_x - K_x(t-\tau)dt.$$

Assuming $K_x(t) = 0$ for $t < 0$, we may plot $K_x(t)$ against t for various values of β. These plots are given in fig.7.8.

Various conditions have been assumed in order to simplify the analysis procedure. Departures from the conditions generally result in a random variation which is superimposed on the convergence characteristics as given in fig.7.8. The magnitude of this variation directly depends on the convergence factor β. It can therefore be made as small as desired at the cost of increasing the time required to converge the coefficients to their optimum settings.

Since any overshoot in the convergence characteristic could give rise to errors in the equaliser output, it is necessary to ensure that β does not exceed the value giving the maximally flat response. This value lies between 0.25 and 0.5. The value of $\beta = 0.25$ normally gives too great a random variation, however, and it is necessary in practice to use a rather smaller value for β. The actual effect of β on the magnitude of the

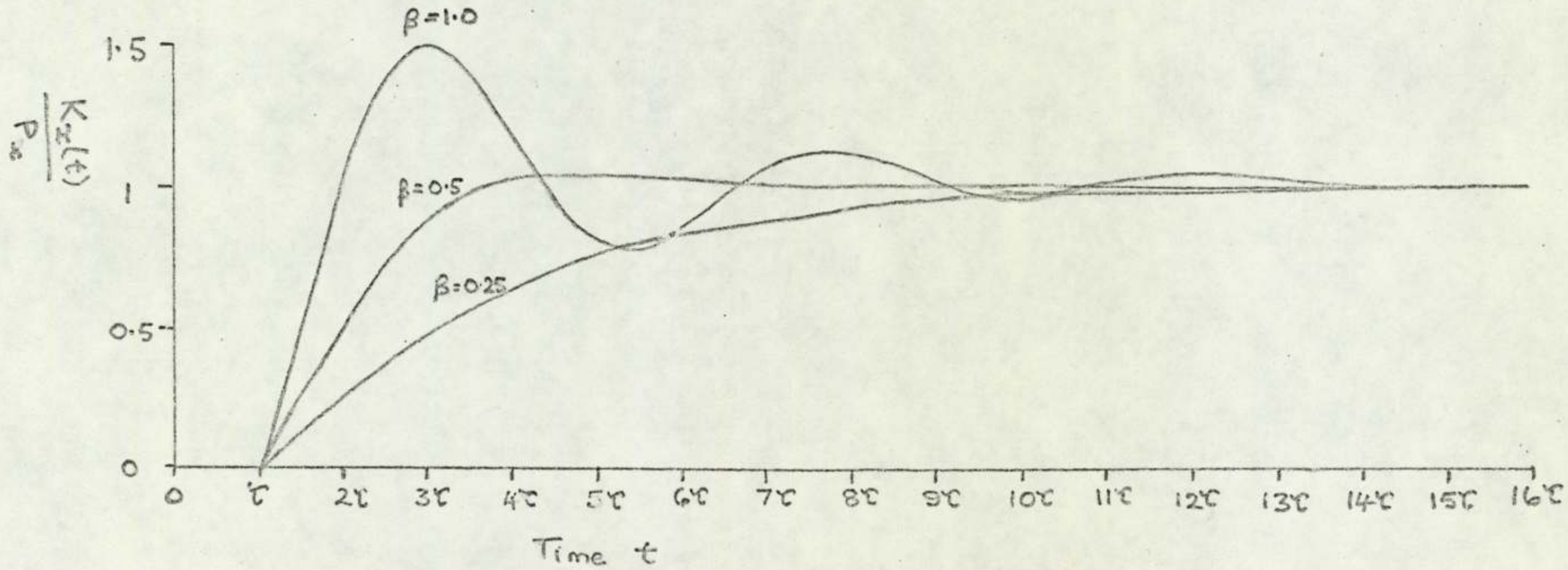


Fig.7.8. Rate of Convergence of Coefficient Value $K_x(t)$ to its Optimum Value P_x.

random variation was determined by computer simulation and is discussed in a later section.

7.5. Fixed Increment Strategies.

The strategies considered so far require a considerable amount of circuit hardware for implementation. To reduce these requirements it is possible to modify the strategies so that the coefficients are incremented in steps of constant magnitude. The error signal is therefore simply used to determine the polarity of the increment required to converge the coefficient towards the optimum setting. The functional model incorporating this modification is given in fig.7.9.

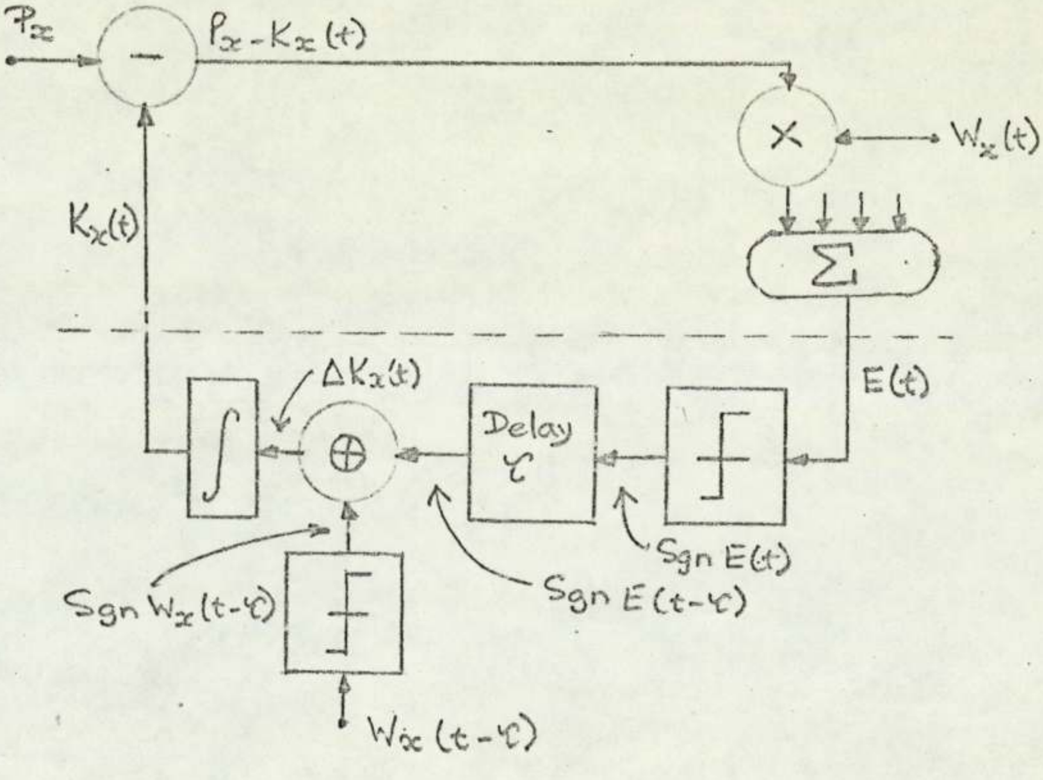
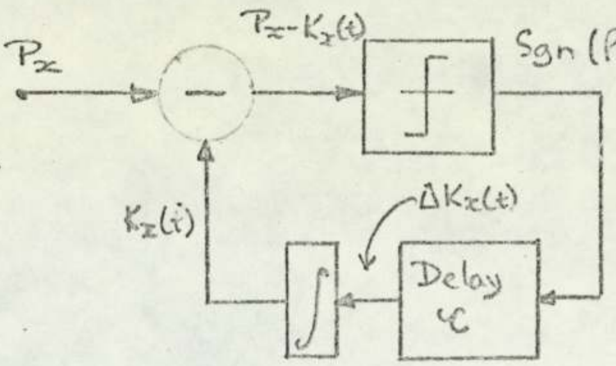


Fig.7.9. Functional Model of Modified Coefficient Updating Strategy.

This model simplifies to that shown in fig.7.10.



$$\Delta K_x(t) = Sgn\left[P_x - K_x(t-\tau)\right]$$

Fig.7.10. Simplified Model of Modified Coefficient Updating Strategy.

This model gives:

$$K_x(t) = \beta\int\Delta\ K_x(t)dt.$$

$$= \beta\int Sgn\ P_x - K_x(t-\tau)\ dt.$$

Assuming, as before, that $K_x(t) = 0$ for $t < 0$, we may plot $K_x(t)$ against t for various values of β. These plots are given in fig.7.11.

It will be seen that the coefficient value converges linearly towards its optimum value, the rate of convergence being directly proportional to the convergence factor β. Once the coefficient has reached its optimum value, however, it then assumes a systematic variation about the optimum value, the amplitude of the variation also being directly proportional to β. There is thus a direct choice between the accuracy with which the coefficient value may be represented and the period of time required to reach the optimum condition.

The convergence characteristics given in figs.7.8 and 7.11 have been plotted assuming that the updating of the coefficients is carried out continuously using an integrator process. Since, in practice, the error signal is normally determined on a time-sampled basis, it is usual to increment the coefficients on a similar basis by direct addition into the coefficient store.

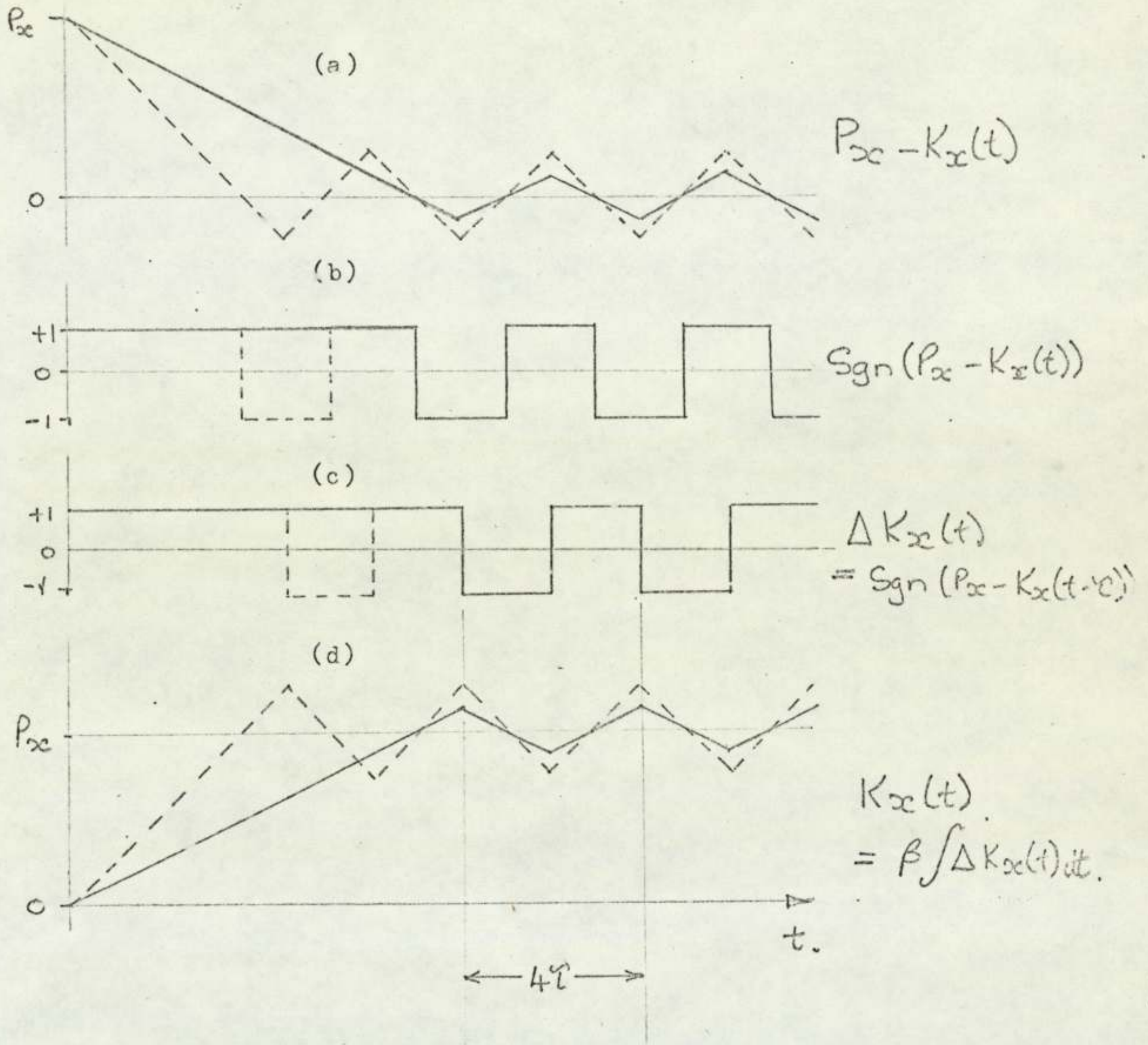


Fig.7.11. Rate of Convergence of Coefficient Value $K_x(t)$ to its Optimum Value P_x, Using Polarity Information Only.

a) Error signal.

b) Polarity of error signal.

c) Updating signal.

d) Coefficient value.

In this case, the convergence characteristics also become time-sampled and appear as a series of steps following the general shapes of the continuous curves given.

Generally, a random variation appears superimposed on the systematic variation of the coefficient value. This random variation arises from various sources as follows:

a) The presence of noise in the received signal.

b) Departures of the error signal from its mean value at specific instants of times.

c) False indications in the error signal arising from the temporary occurrence of short repetitive patterns in the data stream.

The random variation may be reduced by averaging the coefficient update decisions before actually applying a correction to the coefficient value. A suitable averager consists of an up-down counter which indicates the difference between the number of positive and negative coefficient increment indications. When this difference value exceeds a given threshold (averager excess value), the coefficient is updated accordingly and the counter reset to zero. This has repercussions, however, on the amplitude of the systematic variation and on the rate of convergence. In order to maintain the rate of convergence constant, it is necessary to increase the size of the increment step in proportion to the averager excess value used. Providing this value is less than the number of symbol epochs embraced by the error signal delay τ, the effect of averaging is simply to increase the duration and amplitude of the step approximation to the continuous update curve given in fig.7.11. The amplitude value that the step assumes is the nearest discrete increment level to the value indicated on the continuous update curve, in the direction of the update.

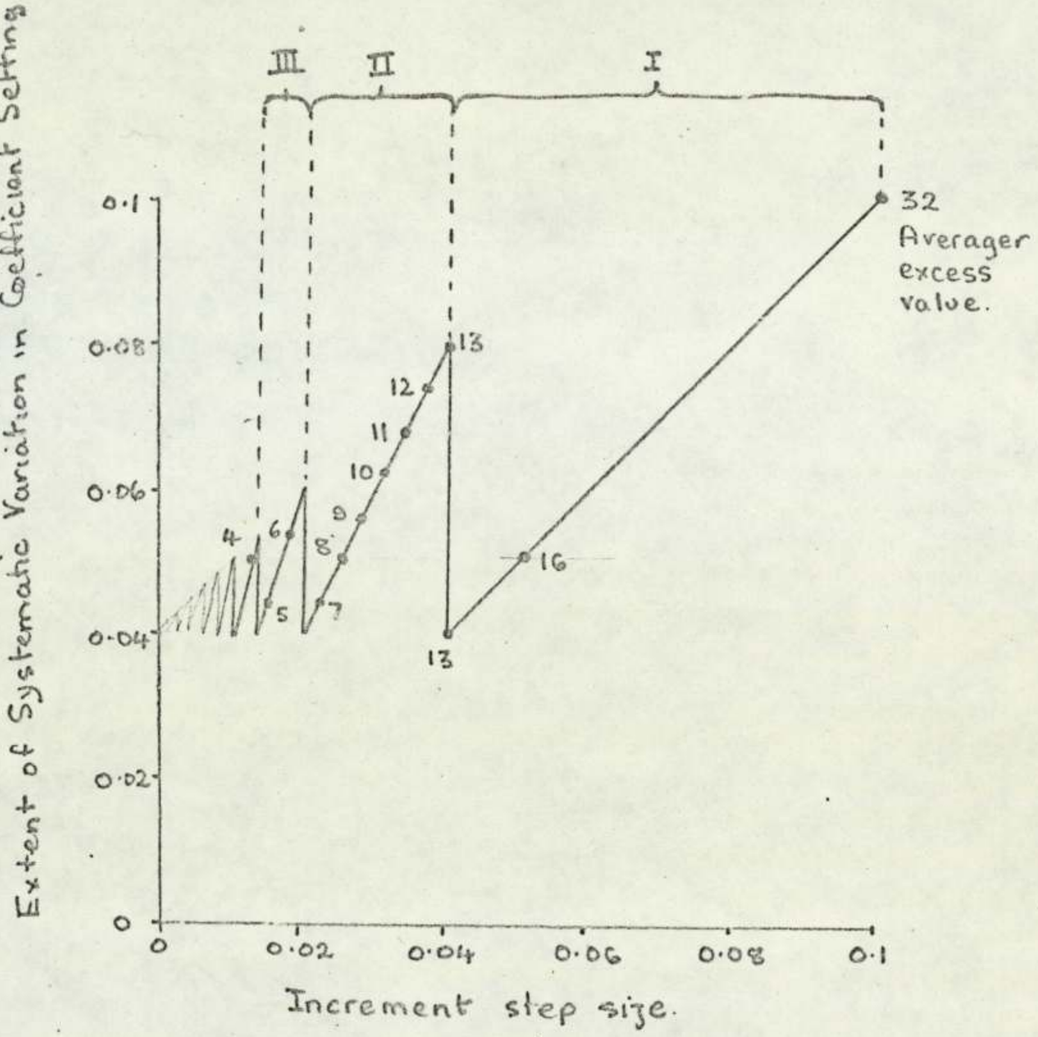


Fig.7.12. Effect of Averaging on Systematic Variation in Coefficient Setting.

Fig.7.12 shows the effect of averaging on the extent of the systematic variation in the coefficient setting. The example illustrated is for an equaliser coefficient which has a convergence rate which would have given a systematic variation of 0.04 in the coefficient value and in which the delay Υ is equal to 13 symbol sampling epochs. In the region I, where the averager excess value (M) is greater than the number of sybmol epochs embraced by the delay (Q), the systematic variation consists simply of single steps in each direction. Hence the magnitude of the variation is equal to one increment step. In the region II, however, where $Q/2 \leqslant M \leqslant Q$, two steps will be taken in each direction and the magnitude of the variation is two increment steps. Similarly, in region III the variation is three increment steps and so on, as shown in the diagram.

The effect of averaging on the random variations is most easily determined by computer simulation and this is described in a later section. The choice of averager excess value is dependent on the relative contribution to coefficient variation of the systematic and random components.

7.6. Computer Simulation Program.

A computer simulation program was written to facilitate the study of the effects of the various parameters and configurations on the performance of the adaptive equaliser.

A block diagram of the system simulated by the computer is given in fig.7.13.

The bit stream generator (a) consists of a feed-back shift-register with a single feed-back tapping point within the shift-register length. The length of the shift-register and the position of the tapping point are specified as program input data and these are normally selected to give the data in the form of a maximal-length pseudo-random binary sequence.

The multi-level encoder (b) takes the bit stream either as single bits or in groups of two, three or four, according to the requirements specified in the input data, and encodes them respectively into two, four, eight or sixteen-level amplitude-modulated symbols. The bit-to-symbol encoding is carried out using a cyclic (modified 'Gray') code so that a transition from one level into an adjacent level results in only a single error in the bit stream. The coding used is illustrated in fig.7.14.

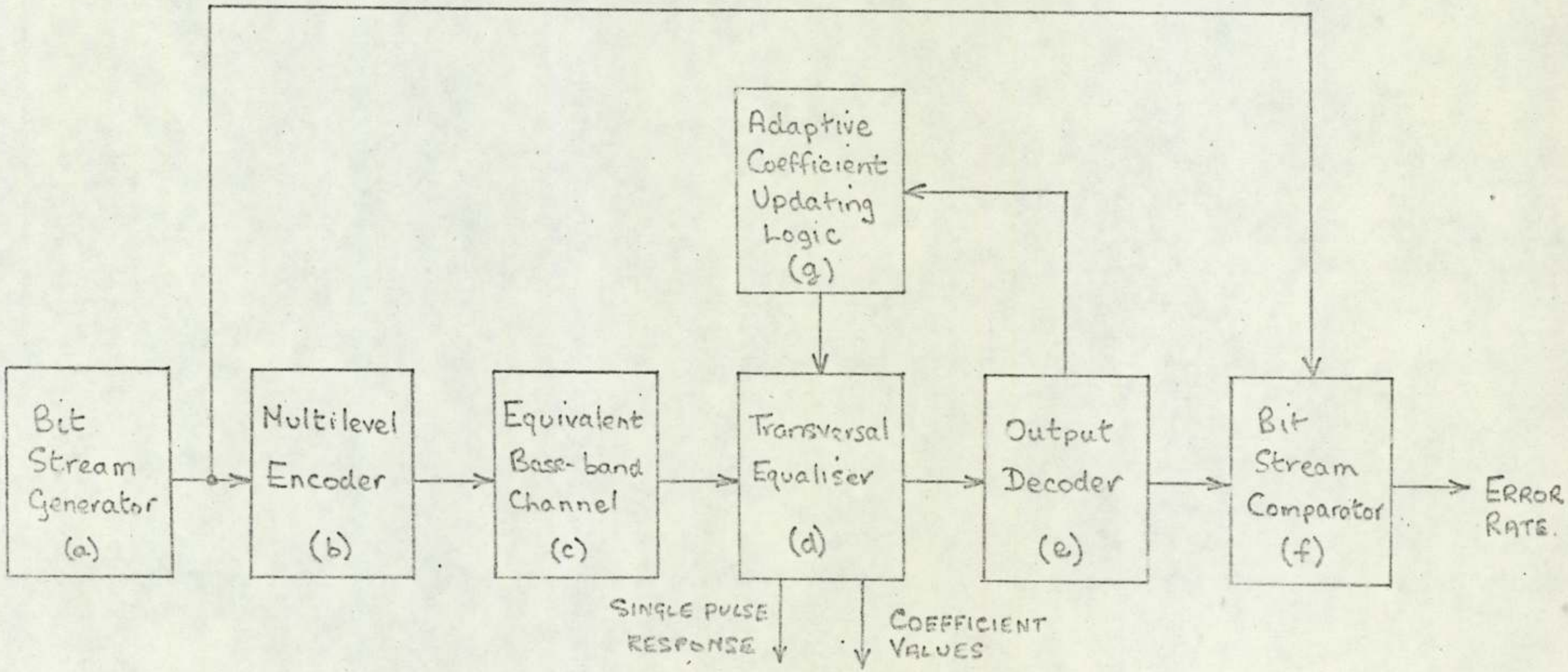


Fig. 7.13. Block Diagram of System Simulated by Computer.

16 level transmission.
8 level trans.
4 level.
2L.

Transmitted level.	Data bit.			
	A	B	C	D
15	0	0	0	1
13	0	1	0	1
11	0	1	1	1
9	0	0	1	1
7	0	0	1	0
5	0	1	1	0
3	0	1	0	0
1	0	0	0	0
-1	1	0	0	0
-3	1	1	0	0
-5	1	1	1	0
-7	1	0	1	0
-9	1	0	1	1
-11	1	1	1	1
-13	1	1	0	1
-15	1	0	0	1

Fig.7.14. Bit-to-Symbol Encoding for Multi-level Transmission.

The characteristics of an equivalent base-band channel are provided to the computer as part of the program input data. these characteristics are specified in the form of the single-pulse response of the equivalent base-band channel to be equalised. The operation of the equivalent base-band channel (c) on the multi-level symbol stream is computed by convolving the multi-level signal sequence with the equivalent channel single-pulse response. The resultant signal is that which would be observed at the input terminal of the receiver demodulator.

This signal is then operated on by the transversal equaliser (d). The number of delay sections and the position of the reference tap are specified in the program data. The option of either the basic feed-forward configuration or the recursive configuration is also selected in the data. The program enables the multiplier coefficient values and the intersymbol interference at the equaliser output to be printed out after each block of data has been generated and processed. The overall single-pulse response of the channel plus equaliser is printed out at the termination of the complete simulation run.

The output from the equaliser is then sliced and decoded in the output decoder (e). The decoding operation also yields the information necessary to update the equaliser multiplier coefficients.

The bit stream comparator (f) compares the output from the decoder with the generated data stream and thus computes the bit error rate. The bit error rate is printed out for each block of data that has been processed.

The adaptive coefficient updating logic (g) gives a choice of four updating strategies as described in the preceding sections. The choice of strategy is specified in the program data according to the following selection code:

1. Strategy I of section 7.2, but with fixed increment steps as described in section 7.5.
2. Strategy II of section 7.2, but with fixed increment steps as described in section 7.5.
3. Strategy I of section 7.2, using weighted coefficient increment steps.
4. Strategy II of section 7.2, using weighted coefficient increment steps.

In the first two strategies a facility is provided to enable the data in the increment signal to be averaged before being used to update the appropriate multiplier coefficients. The amount of averaging on these updating indications and the magnitude of the coefficient increment steps are also specified in the program input data.

A flow-chart for the simulation program is given in fig.7.15, and the program listing is given in appendix 4. The program is written in FORTRAN IV language and the program card deck is compatible with the IBM System/360 Operating System.

7.7. Program of Tests Carried Out Using Computer Simulation.

The program of tests was carried out in two phases. In the first phase the well known strategy of R.W.Lucky[7.6.] (Strategy No.1 of section 7.6) was examined and analysed in some detail. Its performance, when used in conjunction with the recursive equaliser configuration, was determined and compared with that using the basic feed-forward configuration. The physical implementation of this strategy is simplified by the fact that only polarity information is used in the coefficient updating logic circuits. The additional hardware required to implement the strategies using weighted increments is such that it makes their use economically prohibitive using conventional circuit technology. However, recent developments in the use of LSI technology have indicated that an economical implementation is now feasible.

A second study phase was therefore carried out to determine what advantages might be obtained by the use of the weighted increment strategies. Particular consideration was given to the likelihood of reducing the time required for the initial setting-up procedure and of reducing the random variation in the equaliser setting after convergence has been achieved.

7.8. Analysis of Tests using Computer Simulation, Phase I.

a) Effect of Multilevel Operation on Convergence of Optimisation Strategy. Fig.7.16 compares the rate of convergence of the optimisation strategy and the block error rate for a given channel response and equaliser operating with two,

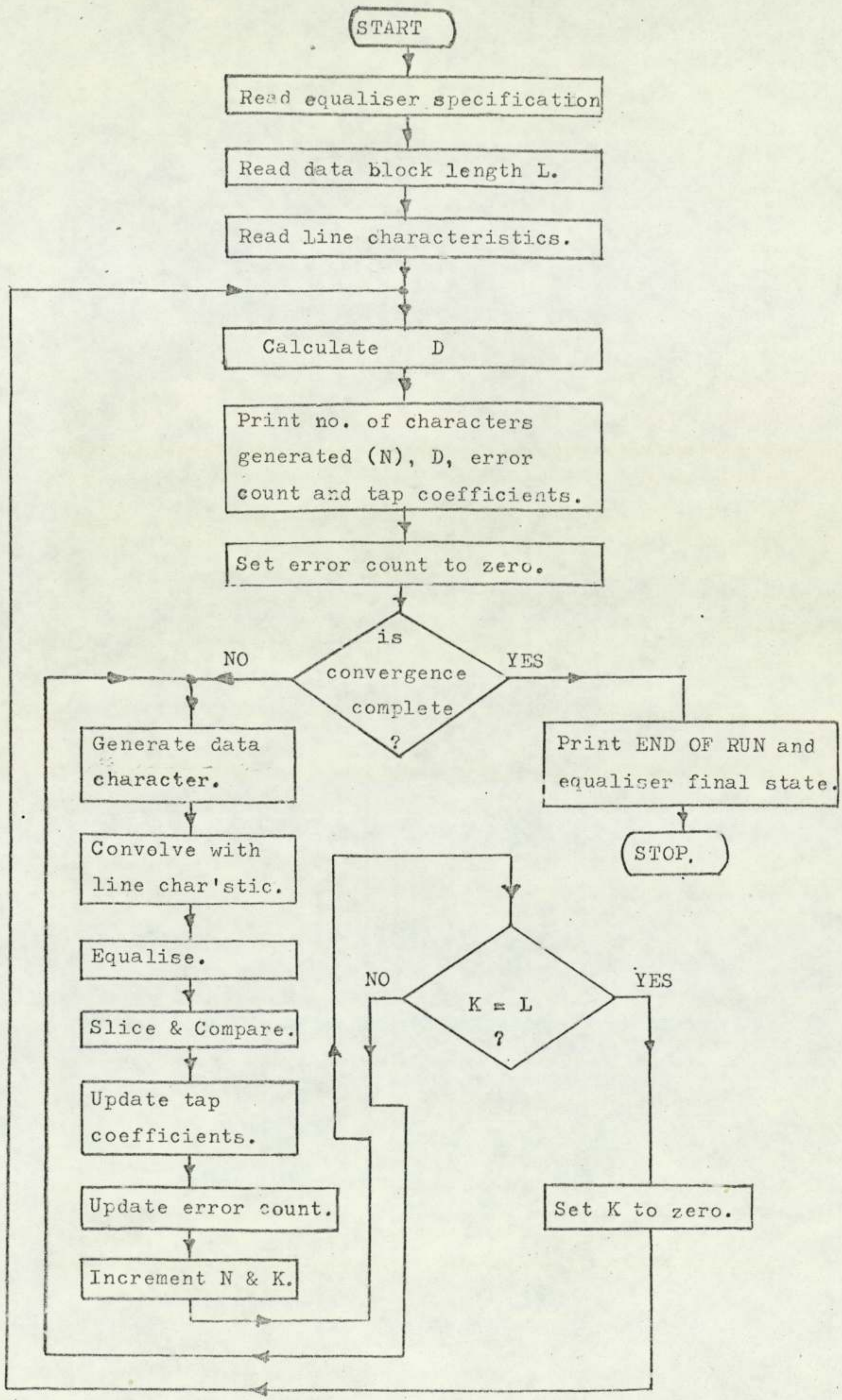


Fig.7.15. Flow-Chart For Adaptive Equaliser Program.

Rate of Convergence and Block Error Rates. - W2H

Basic Equaliser Configuration, $p=6$, $q=12$.
Averaging = 32 samples excess. Block Size = 127 symbols,
Coefficient Increment 0.01.

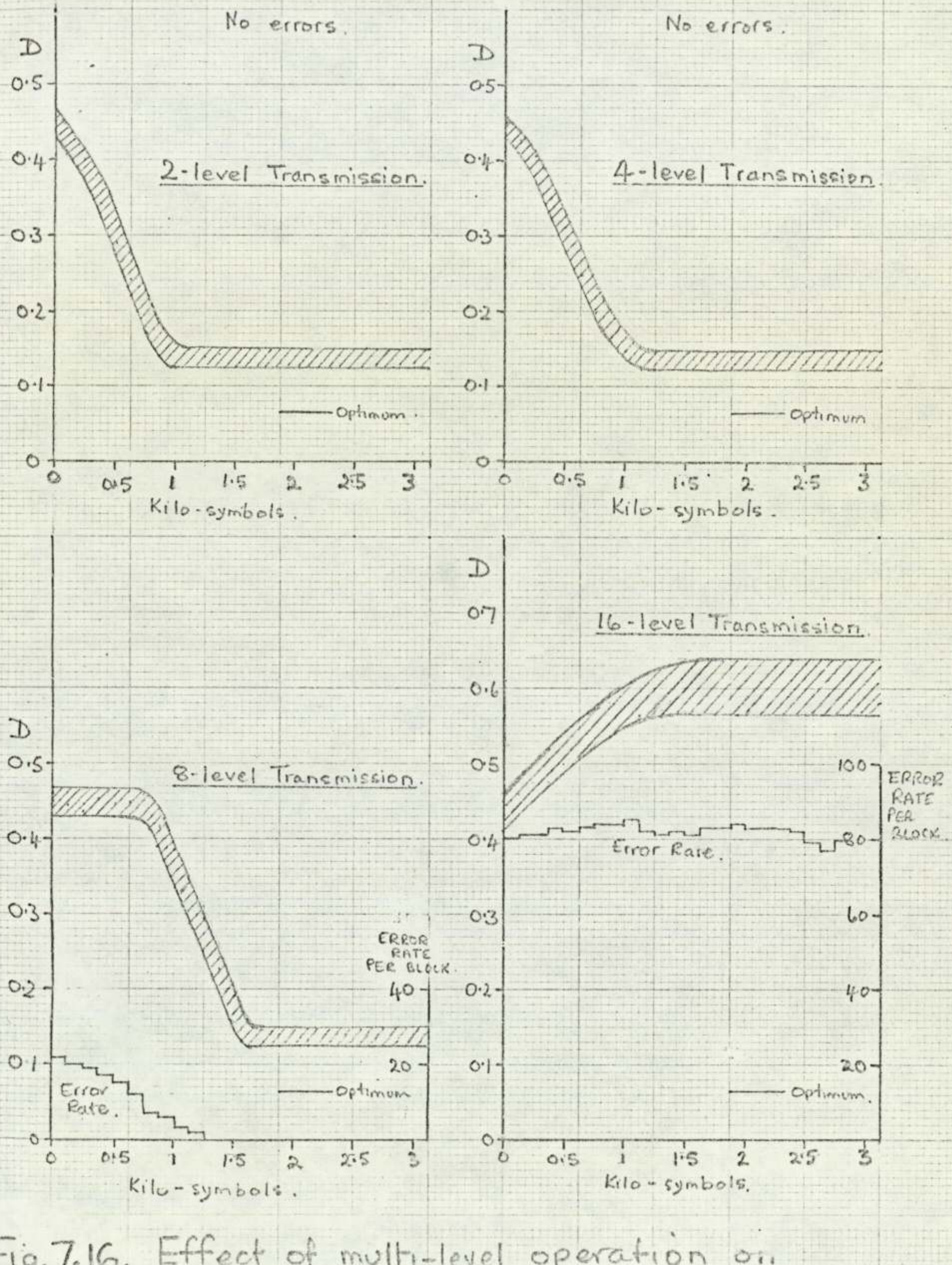


Fig. 7.16. Effect of multi-level operation on convergence of adaptive strategy.

four, eight and sixteen levels of amplitude modulation. In the case under consideration, the binary eye was open before equalisation and the four-level eye was not sufficiently closed to cause errors when a 127-bit p.r.b.s. was used as the data stream. Under the no-error condition it can be seen that the rate of convergence is independent of the number of operating levels. At eight-level operation the system initially produced a small but significant bit error rate (approximately 1 in 18). Since the encoding method ensures that the majority of errors will occur as only one bit error per symbol, the symbol error rate is thus of the order of 1 in 6. Consequently there are significantly more correct than incorrect symbols received. The effect on the convergence of the strategy of an initial error rate of this order is to delay any significant reduction in the intersymbol interference D until first of all the error rate has been significantly reduced. A reduction of the error rate occurs immediately transmission begins. Once the error rate has been reduced, the rate of convergence is the same as that for two and four-level operation.

At sixteen-level operation the system produced a bit error rate of approximately 1 in 6. This means that about half the symbols will be in error. This, in turn, means that the error signal is as often incorrect as it is correct and thus the tap increment decisions will be random in nature. The outcome of this is that the error rate is maintained and the equaliser does not converge at all.

It is evident that, in order for the equaliser coefficient settings to converge, it is necessary for the "eye" pattern at the output of the equaliser to be sufficiently near to the "open" condition to ensure that a greater proportion of the symbols will be decoded correctly than will be decoded incorrectly.

It is clear, therefore, that it is impossible to guarantee convergence for any system where the initial distortion $D_0 > 1$. Two measures, however, can be taken to ensure convergence where $D_0 < 1$. Firstly, the equaliser should be set initially to a condition where it is not increasing the intersymbol interference above that of the unequalised channel. The obvious choice to ensure this is to set all the coefficients to zero except the reference tap coefficient which is set to unity. Under these conditions the equaliser does not operate at all on the channel impulse response. Secondly, it will be necessary to commence equalisation with the number of modulation levels reduced to "open the eye". In general a short period of binary transmission of random data before transmission of actual information is all that is necessary.

b) Effect of Multi-level Operation on Convergence of Optimisation Strategy Applied to the Recursive Equaliser. Fig.7.17 shows the rate of convergence of the optimisation strategy applied to the same line characteristic and using the same size equaliser, but this time the equaliser is connected in the recursive configuration. It will be seen that there is no significant difference in the rate of convergence or in the error-rate as a result of the change in configuration.

Fig.7.18 shows the comparison between the convergence rates for the basic and the recursive configuration for a different line characteristic and a different size of equaliser. There is again no significant difference between the two convergence rates. It should also be noted that the rate of convergence for the two lines is identical

Rate of Convergence and Block Error Rates - W2H

Recursive Equaliser Configuration. p=6, q=12.
Averaging = 32 samples excess, Block size = 127 Symbols.
Coefficient Increment = 0·01.

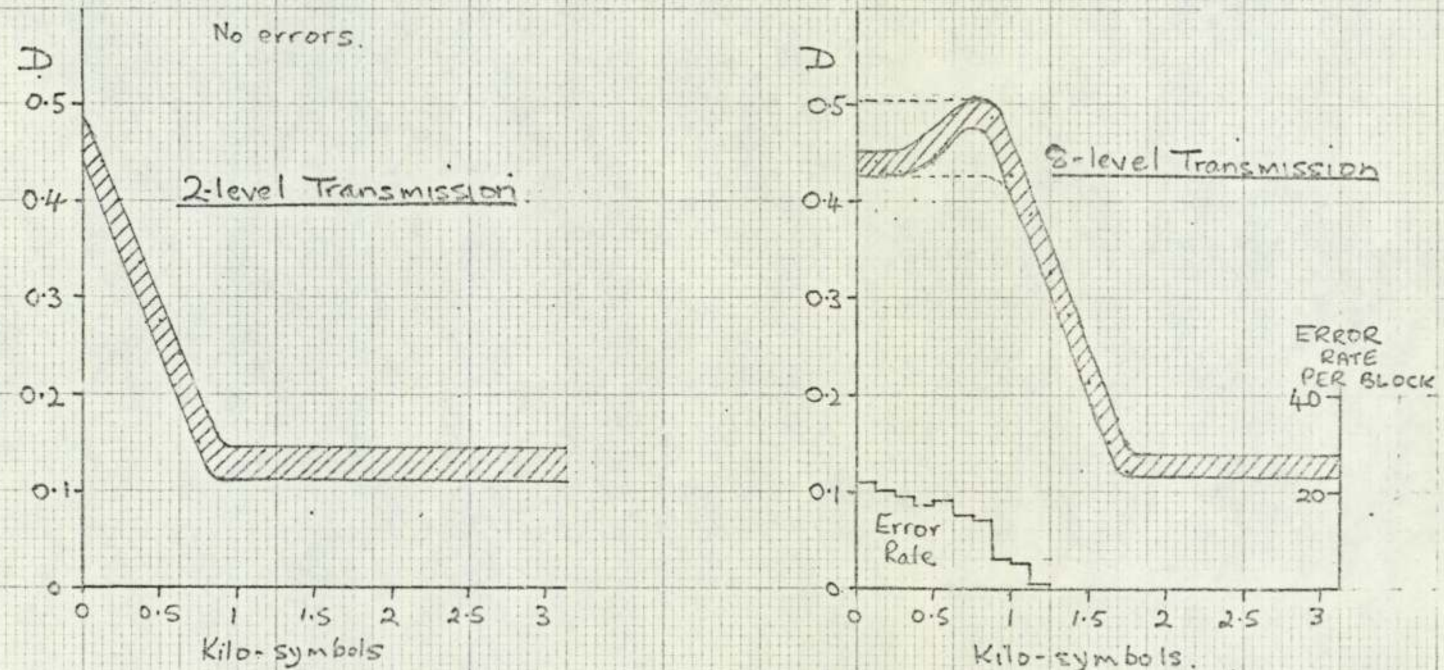


Fig. 7.17 Effect of multi-level operation on convergence of adaptive strategy applied to the recursive equaliser.

Rate of Convergence and Block Error Rates – L5A.

4-level Transmission, p = 6, q = 6. Block Size = 127 Symbols.
Coefficient Increment = 0·01, Averaging = 32 Samples excess.

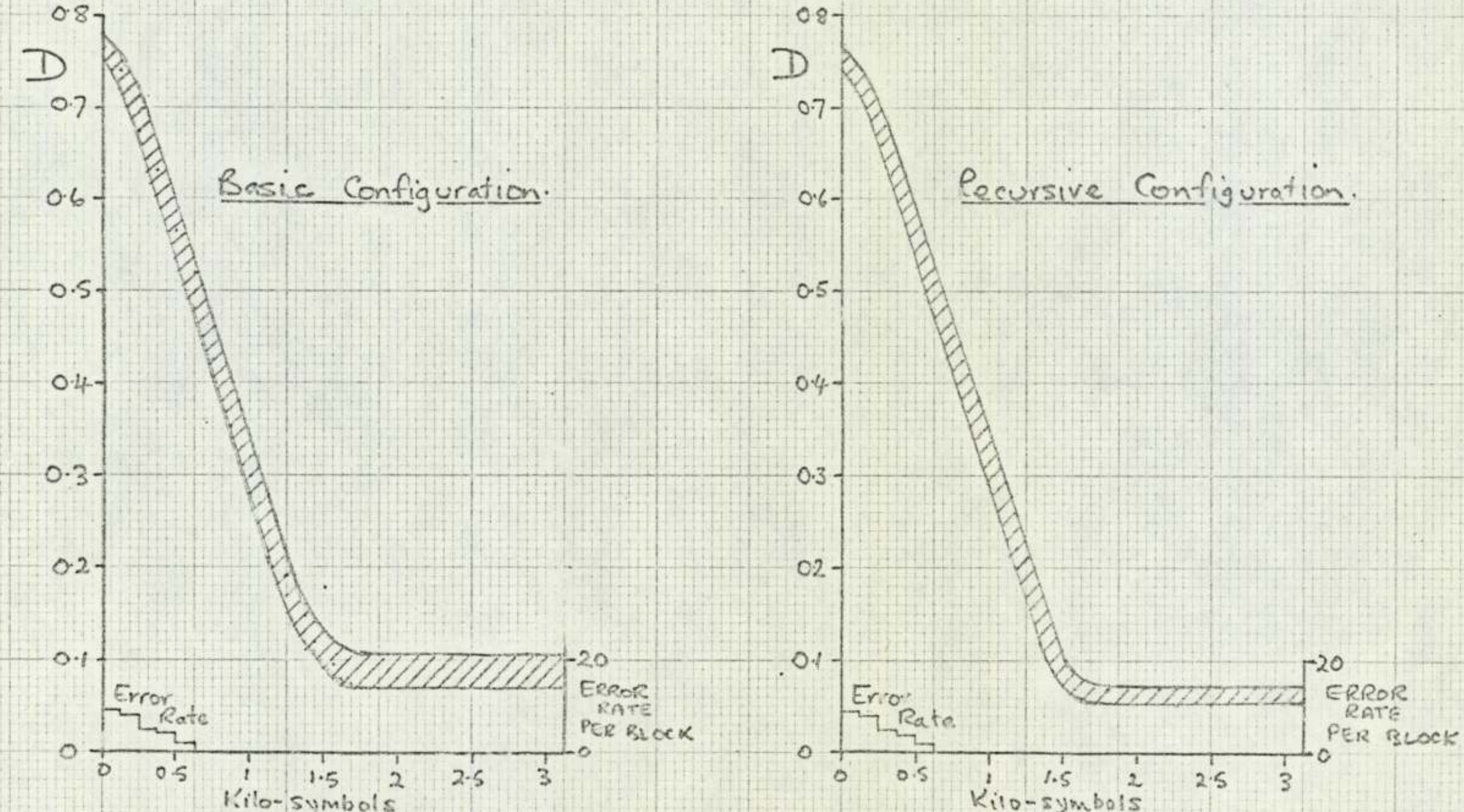


Fig 7.13 Comparison of convergence rates for normal and recursive adaptive equalisers

over the linear portion of the convergence curve, that is, the part of the curve after the error rate has become insignificant and until the final equalised condition is achieved.

c) Effect of Size of Coefficient Increment and Averaging on Rate of Convergence. The size of the coefficient increment and the amount of averaging provided on the coefficient updating signal are important parameters because, together, they determine the rate of convergence of the equaliser to its optimum setting. In order to attain the fastest possible rate of convergence, it is necessary to use the largest possible coefficient step size. In section 5.5 it was shown that there is a maximum increment step size, associated with a given channel response, with which it is possible to achieve a given maximum acceptable value of intersymbol interference D. This maximum step size is, in turn, associated with an optimum number and disposition of delay elements in the transversal filter.

When using the maximum increment step size, averaging on the updating signal is necessary to restrict the random variation of the coefficient setting to one increment step in each direction only. This averaging allows coefficient updating only after a significant error indication has been obtained. Because the equaliser has memory in the coefficient updating logic, it is necessary, once the coeffiecients have been updated, to allow the latest error information to propagate through the logic before a second updating is effected. This fixes the lower limit on the amount of averaging at at least L decisions. If this is not done, the multiple updatings will have the effect of increasing the mean error at each coefficient to greater than $\frac{1}{2}\Delta$ (See section 5.5). As the number of erroneous decisions due to system noise is generally comparatively small, their effect on the amount of averaging required can normally be neglected.

Since no improvement can be achieved over restricting the mean error at each coefficient to $\frac{1}{2}\Delta$, the provision of more averaging than is necessary to do this simply increases the rate of convergence with no reduction in the value of intersymbol interference attained. Fig.7.19 shows the effect of increasing the averaging, maintaining the other parameters constant. In the case illustrated, where L = 13, it is seen that a significant reduction in random variation in the equalised intersymbol interference is achieved by increasing the averaging from 8 samples excess to 16 samples excess. Increasing the averaging further to 32 samples excess has little effect, however, on the random veriation. It is clear, therefore, that the optimum averaging has already been reached by 16 samples excess and that any further averaging simply incurs a penalty in convergence rate.

However, for a given convergence rate, it is necessary to optimise the averaging and the increment size together. This is because, although decreasing the averaging increases the random variation, it is possible to associate with this a decrease in increment step size which would decrease the random variation. The theoretical predictions of coefficient behaviour, based on idealised assuptions, are given in section 7.5 and are illustrated in fig.7.12. The results obtained by computer simulation are given in fig.7.20 and are seen to be in accord with the previously developed theory. If the averager excess value is increased to a value significantly greater than the number of equaliser coefficients, there is an associated increase in the random variation of the intersymbol interference. This is evident from fig.7.12 and from a comparison of fig.7.19(c) with the results given in fig.7.20.

R1K Recursive Equaliser Configuration, p=6, q=6, 2-level Transmission, Block size = 127 Symbols.

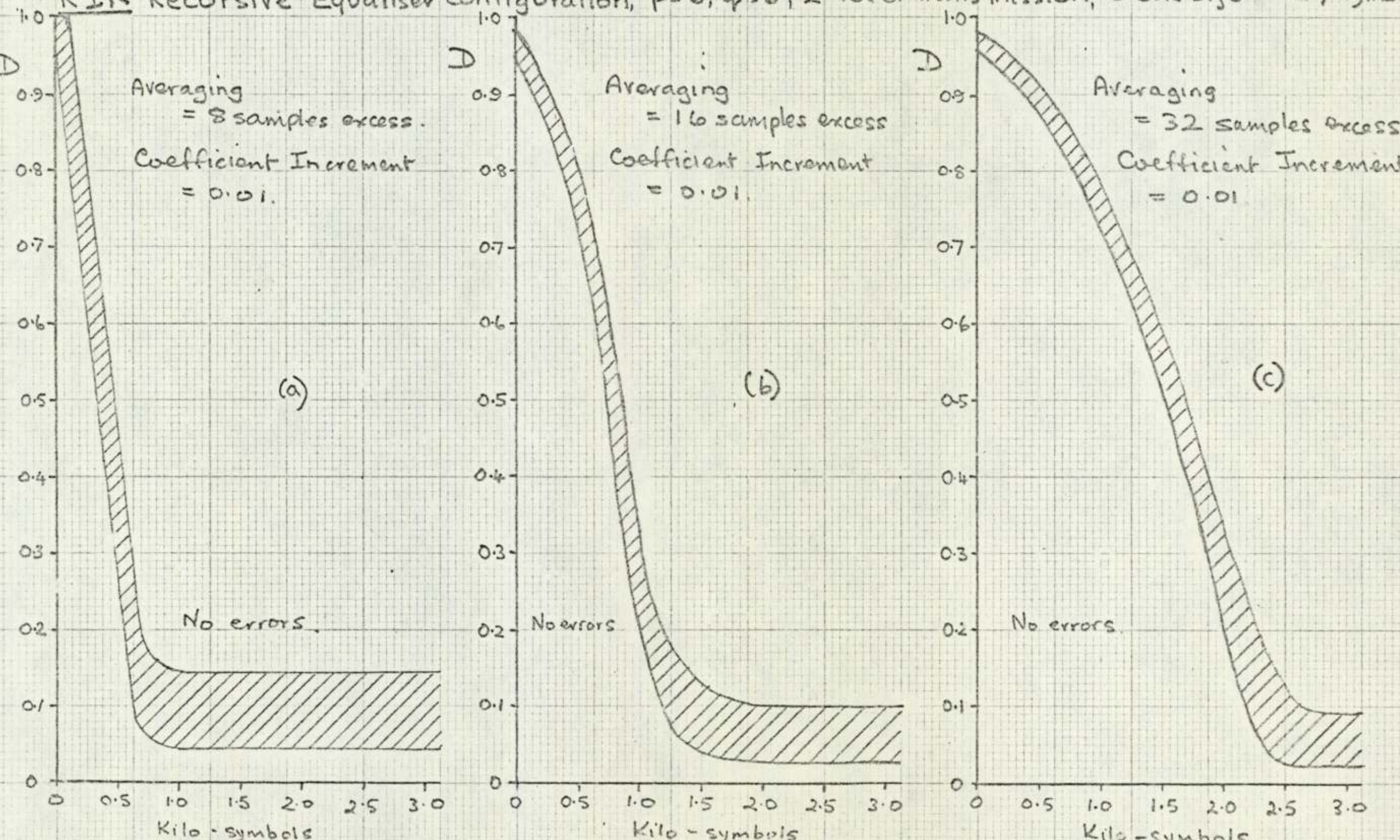


Fig 7.19. Effect of averaging on rate of convergence and random variation of adaptive equaliser

R1K Recursive Equaliser Configuration p=6, q=6 2-level Transmission Block size = 127 Symbols.

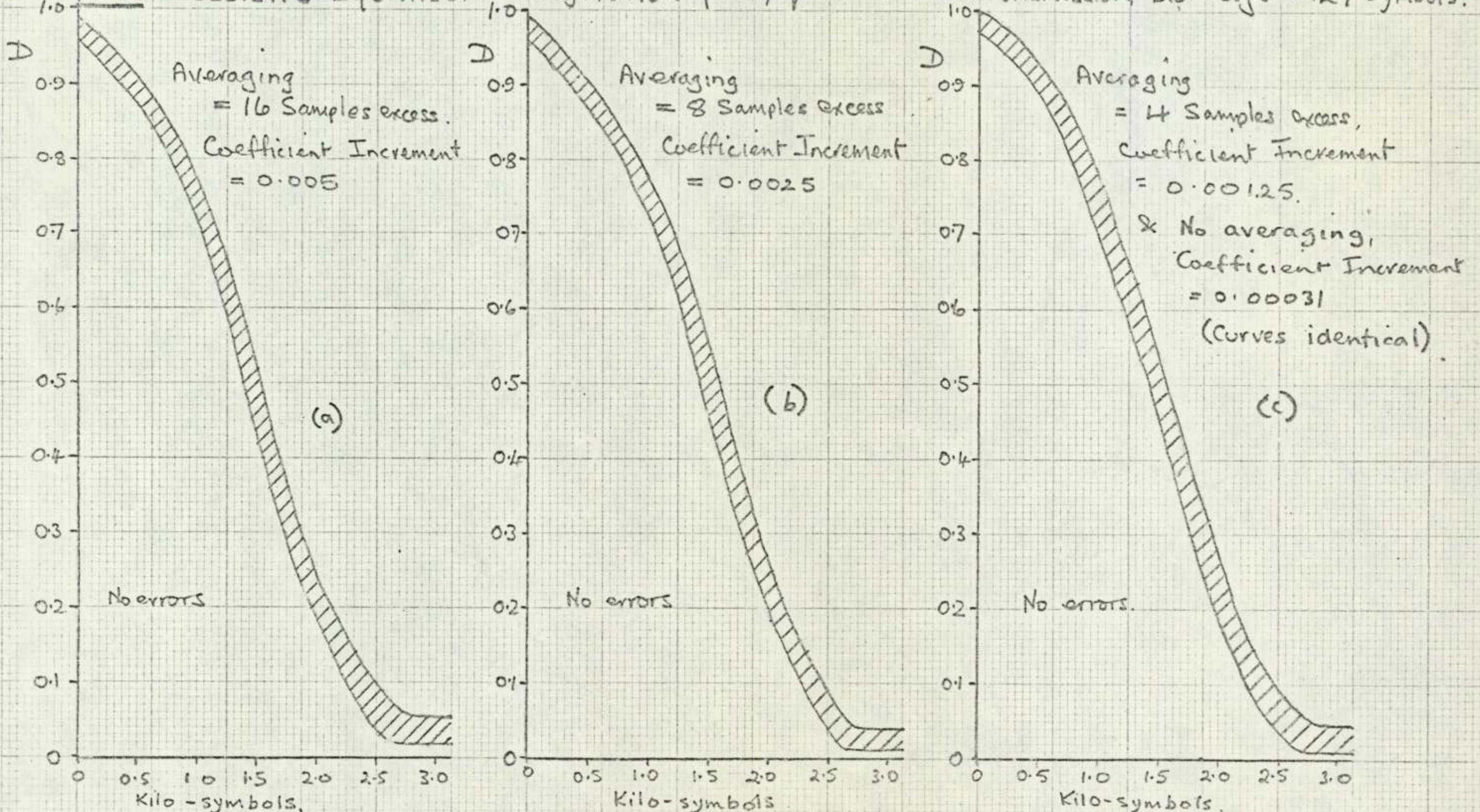


Fig 7.20. Effect of increment size versus averaging on random variation of adaptive equaliser. (convergence rate constant).

It is clear from these results that, assuming the number of erroneous decisions due to system noise is small, any advantage gained by the use of averaging is completely offset by the necessity to use larger increments to maintain the convergence rate constant.

If no averaging is used, the systematic variation in the coefficient setting will be L steps in each direction from the optimum value. The mean error in the coefficient setting is thus $\frac{\Delta L}{2}$ and the total mean deterioration in intersymbol interference (D_q) is thus given by:

$$D_q = \frac{\Delta L^2}{2}$$

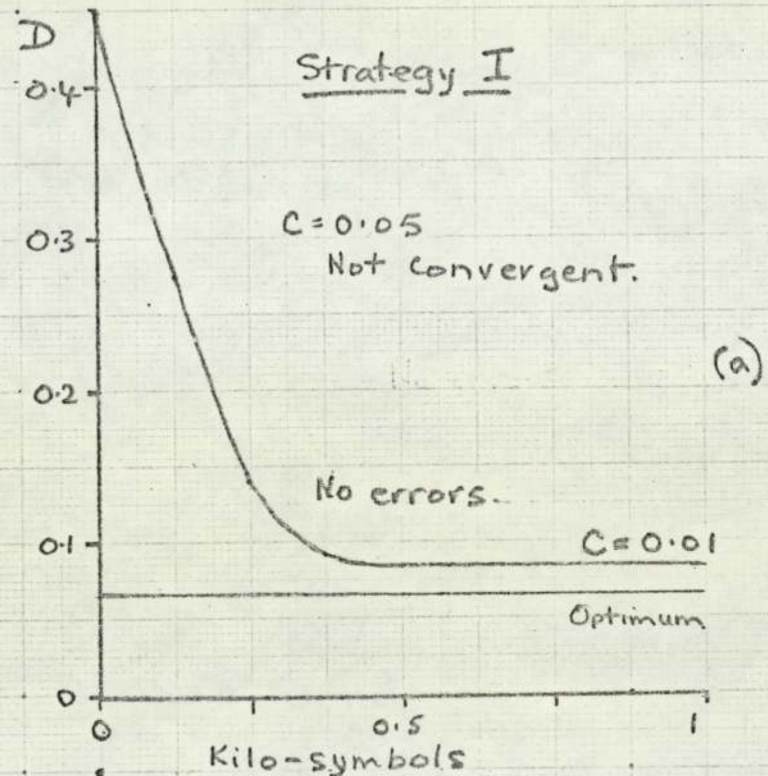
For the equaliser considered in fig.7.20, this gives a value of $D_q = 0.0262$. Since the same equaliser with ideal coefficient values would be capable of reducing the intersymbol interference D to a negligible value (See fi.5.2(a)), the mean value of the random variation would be expected to be equal to the mean deterioration D_q. That this is in fact the case is seen from fig.7.20(c).

7.9. Analysis of Tests using Computer Simulation, Phase 2.

a) Comparison of Strategies I and II using weighted coefficient increment steps. Phase 2 of the program of computer simulation tests commenced with a comparison of the performance of strategies I and II (as descibed in section 7.2) using weighted coefficient increment steps. The results of these tests are given in fig.7.21. The values of the convergence coefficient C are given on the graphs. The coefficient scale assumes that Y(t) and Z(t), and hence E(t), are normalised so that their amplitudes are unity when they correspond at the slicer with the level representing the maximum transmitted symbol amplitude.

For strategy II, the delay in the error signal is simply

W2H - Basic Equaliser Configuration, $\rho = 6$, $q = 12$
2 level transmission



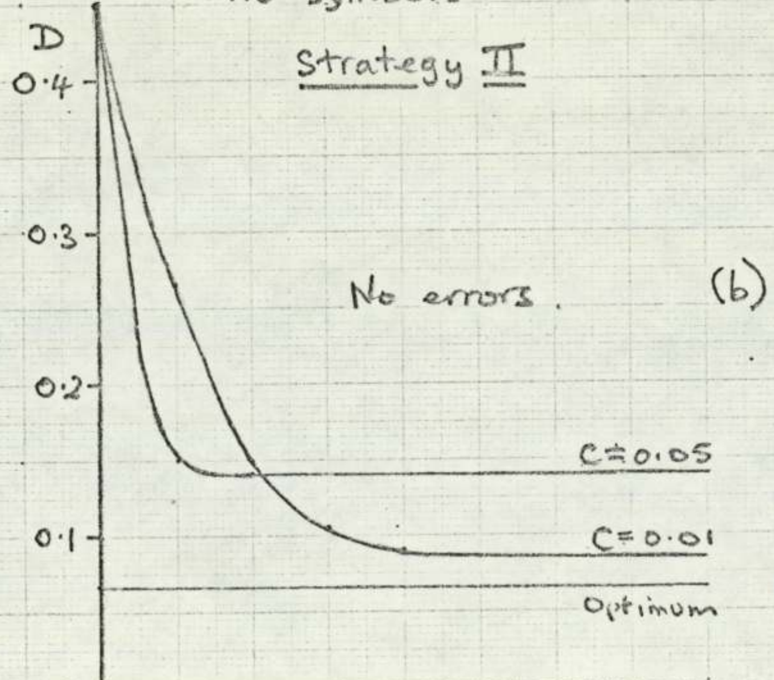


Fig 7.21 Comparison of Strategies I & II
Weighted increment steps.

one symbol period and the value C corresponds directly with α in the conditional expression 7.16. For strategy I, however, the delay in the error signal is equal to p symbol periods and the effect of an update is thus not seen for this time duration. The correspondence between C and α is thus $\alpha = C \times p$, since p updates will occur before any change becomes effective on the error signal information. The maximum value of any component of the vector $\underline{W}$ will be $\frac{A}{A-1}$ if the condition that the 'eye pattern' is open is satisfied. The maximum vector amplitude is thus

$$|\underline{W}| = \sqrt{L \times \left(\frac{A}{A-1}\right)^2}$$

Since $\alpha \leqslant \frac{2}{|\underline{W}|^2}$,

$$\therefore \quad \alpha \leqslant \frac{2}{L \times \left(\frac{A}{A-1}\right)^2}$$

In the example tested, we have $A = 2$, $L = 19$, and $p = 6$,

$$\therefore \ \alpha_{max} = 0.105$$

Thus C_{max} for strategy I = 0.0175

and for strategy II = 0.105

It will be seen from fig.7.21 that strategy I does not converge with a value of C = 0.05, whereas satisfactory convergence was obtained using strategy II. This is in accord with the prediction. With C = 0.01, there is no significant difference in the performance of the two strategies. Since much larger values of C can be used with strategy II, it is possible to obtain much faster convergence rates than are possible with strategy I. All further tests were therefore carried out using strategy II.

b) <u>Effect of Multilevel Operation on Convergence of Strategy II.</u>

Fig.7.22 shows the rate of convergence of strategy II, using weighted coefficient increment steps, and the block error rate, for a given channel response and equaliser operating with four

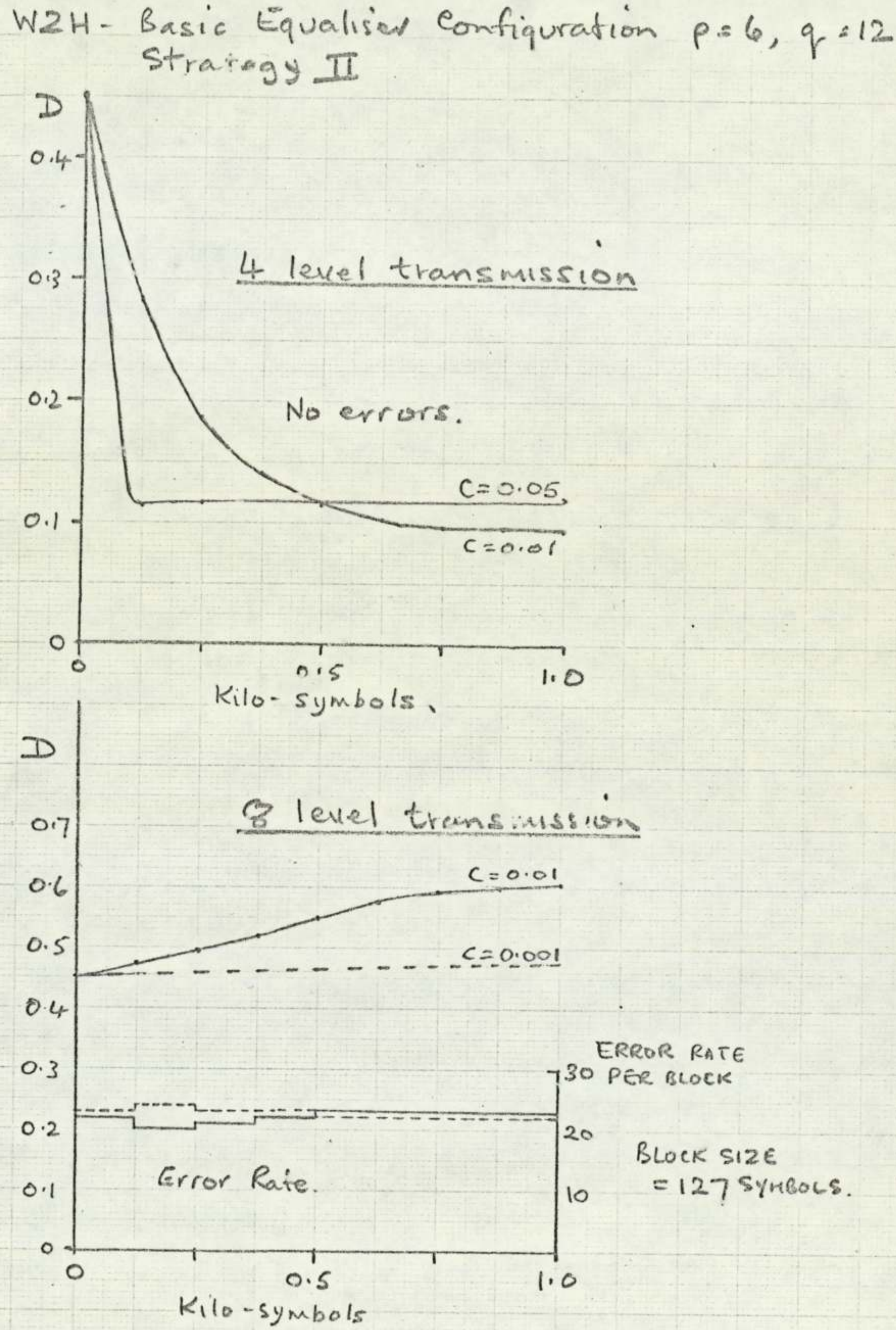


Fig. 7.22. Convergence of Strategy II (weighted increments), Multilevel Transmission.

and eight levels of amplitude modulation. The rate of convergence for the same equaliser and channel response operating with two levels of amplitude modulation are given in fig.7.21(b). In the case under consideration, the binary eye was open before equalisation and the four-level eye was not sufficiently closed to cause errors when a 127 bit p.r.b.s. was used as the data stream. Under the no-error condition it can be seen that the rate of convergence is independent of the number of operating levels.

At eight-level operation the system produced a significant error rate and the strategy failed to converge. Even by reducing the convergence coefficient from 0.01 to 0.001, it was still not possible to achieve convergence with eight-level operation. The results obtained for the same channel response and equaliser using fixed increments (given in fig.7.16.) show that it was possible to obtain convergence with eight-level operation and fixed increment steps, although the convergence time was somewhat indeterminate. The small advantage obtainable by using fixed increments is far outweighed, however, by the improvement obtained in the final value of intersymbol interference obtained after convergence has been completed. This improvement is discussed in more detail later.

c) Convergence of Strategy II applied to the Recursive Equaliser.

Fig.7.23 shows the rate of convergence obtained using the equaliser connected in the recursive configuration. The graph labelled W2H is for the same line characteristic and the same size equaliser as used for the tests in sections a and b. It will be seen that there is no significant difference in the rate of convergence or the error-rate as a result of the change of configuration. The graph labelled R1K is for a different line characteristic and a different size of equaliser, but using the same convergence coefficient C.

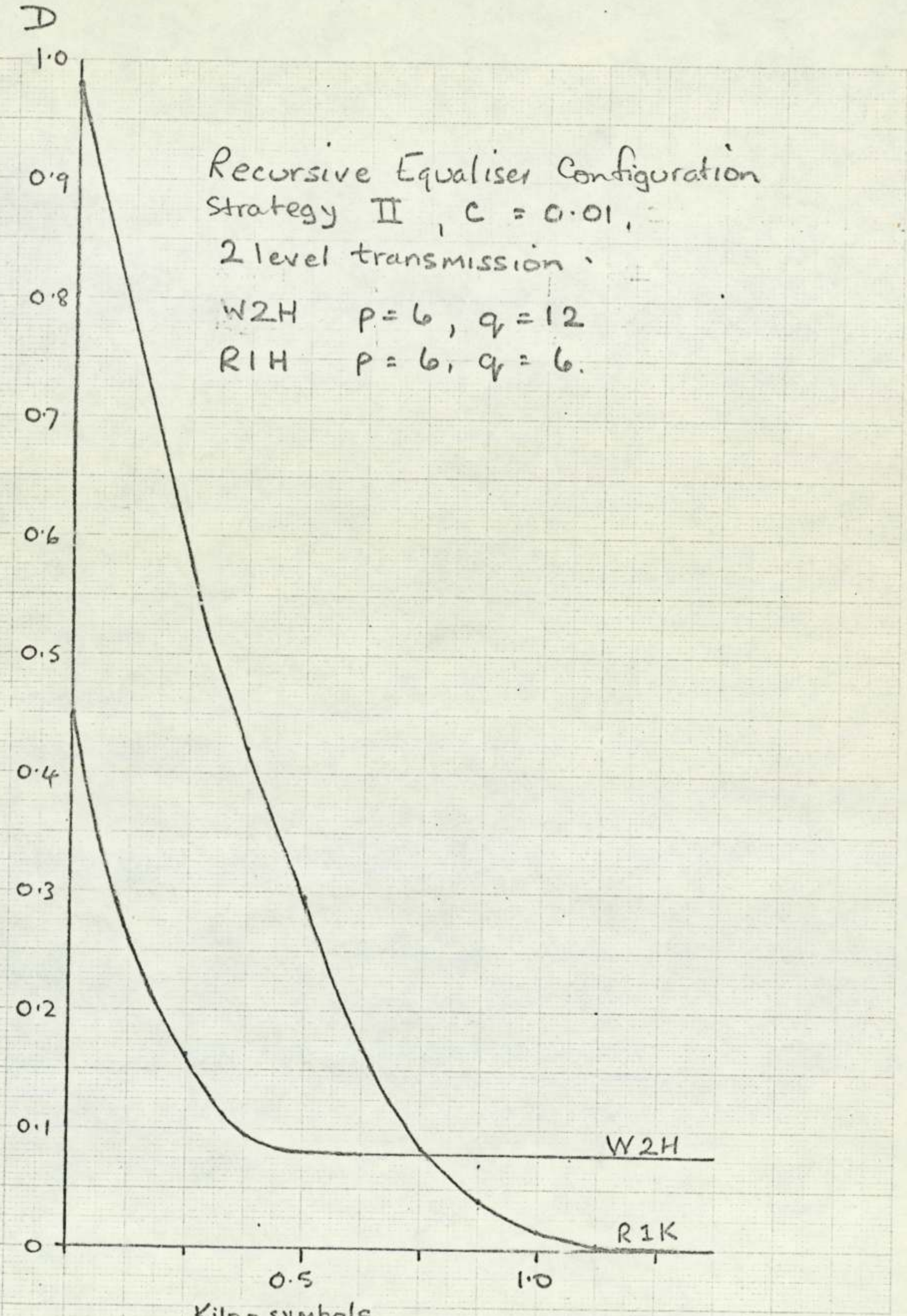
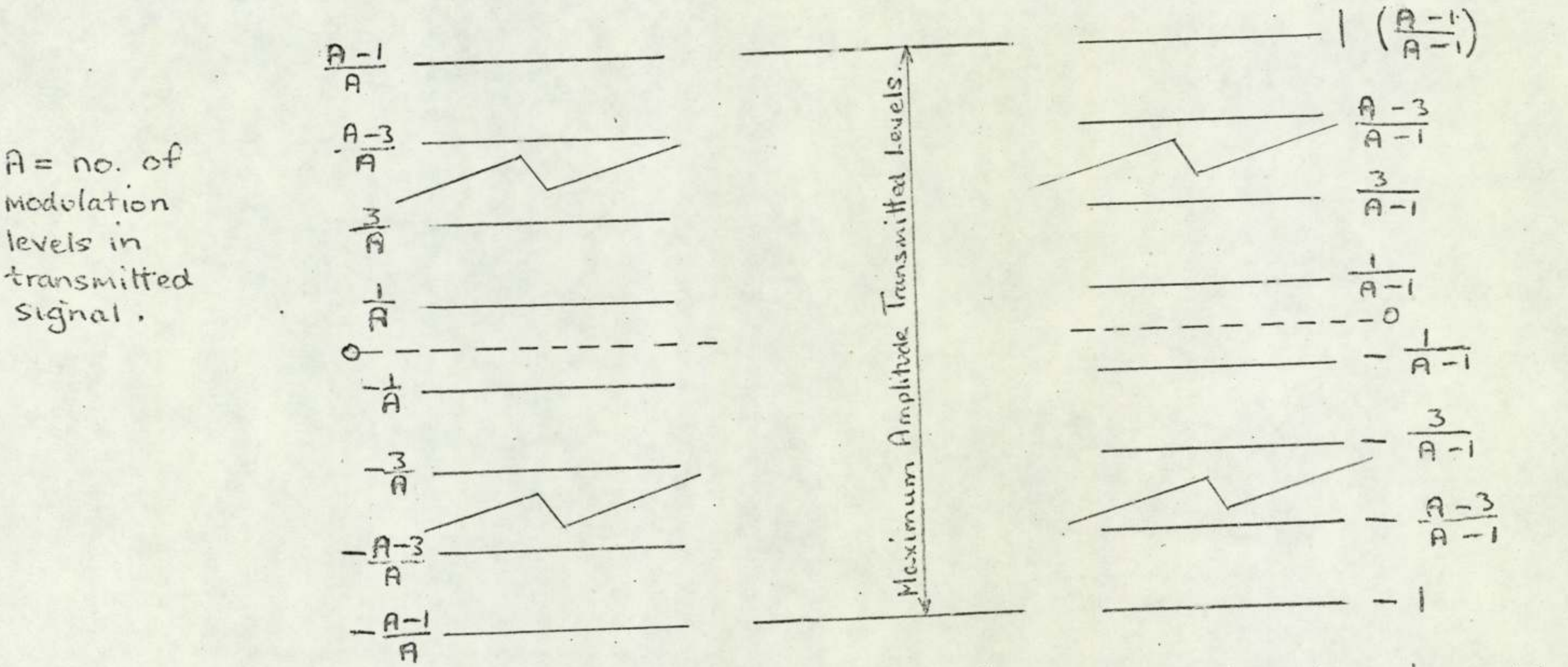


Fig 7.23. Convergence of Strategy II (weighted increments, Recursive Configuration

7.10. Rate of Convergence of Adaptive Strategies Using Weighted Increments.

From the previous sections it will be seen that the rate of convergence of the adaptive equaliser strategies using weighted coefficient increment steps is a function of the transmitted symbol rate, the number of levels of amplitude modulation and the convergence coefficient C. We may, however, normalise Y(t) and Z(t), and hence E(t), so that their amplitudes are $\frac{1}{2}$ when they correspond at the slicer to the mean of the magnitude of the transmitted symbol amplitudes (See fig.7.24(a)). In order to maintain the same rate of convergence as when the normalisation is as described in section 7.9(a), (See fig.7.24(b)), the value of the convergence coefficient C' must be made equal to $C\left(\frac{A}{A-1}\right)^2$. In this case it is found that the rate of convergence, for a given value of C', is independent of the number of levels of amplitude modulation in the transmitted signal.

It is evident from the computer simulation results detailed in the previous sections, that the decrease in intersymbol interference D is an exponential decay towards the fully-equalised value of intersymbol interference D_e. The rate of convergence is not significantly affected by the size of the equaliser (i.e. number of variable coefficients) or by the equaliser configuration. Fig.7.25 shows a normalised rate of convergence curve obtained from a comparison of the various simulation tests carried out. The deviations from this curve are, in practice, extremely small. The horizontal axis of the graph is given by y = C'.N, where N is the number of transmitted symbols. The time-lapse t is given by t = N/S, where S is the transmission rate in symbols/s. The time constant τ of the graph is found empirically to be given by



a) Mean of magnitude of transmitted level $= \frac{1}{2}$.

b) Magnitude of maximum transmitted level $= 1$.

Fig 7.24. Definition of Normalised Voltage Scale.

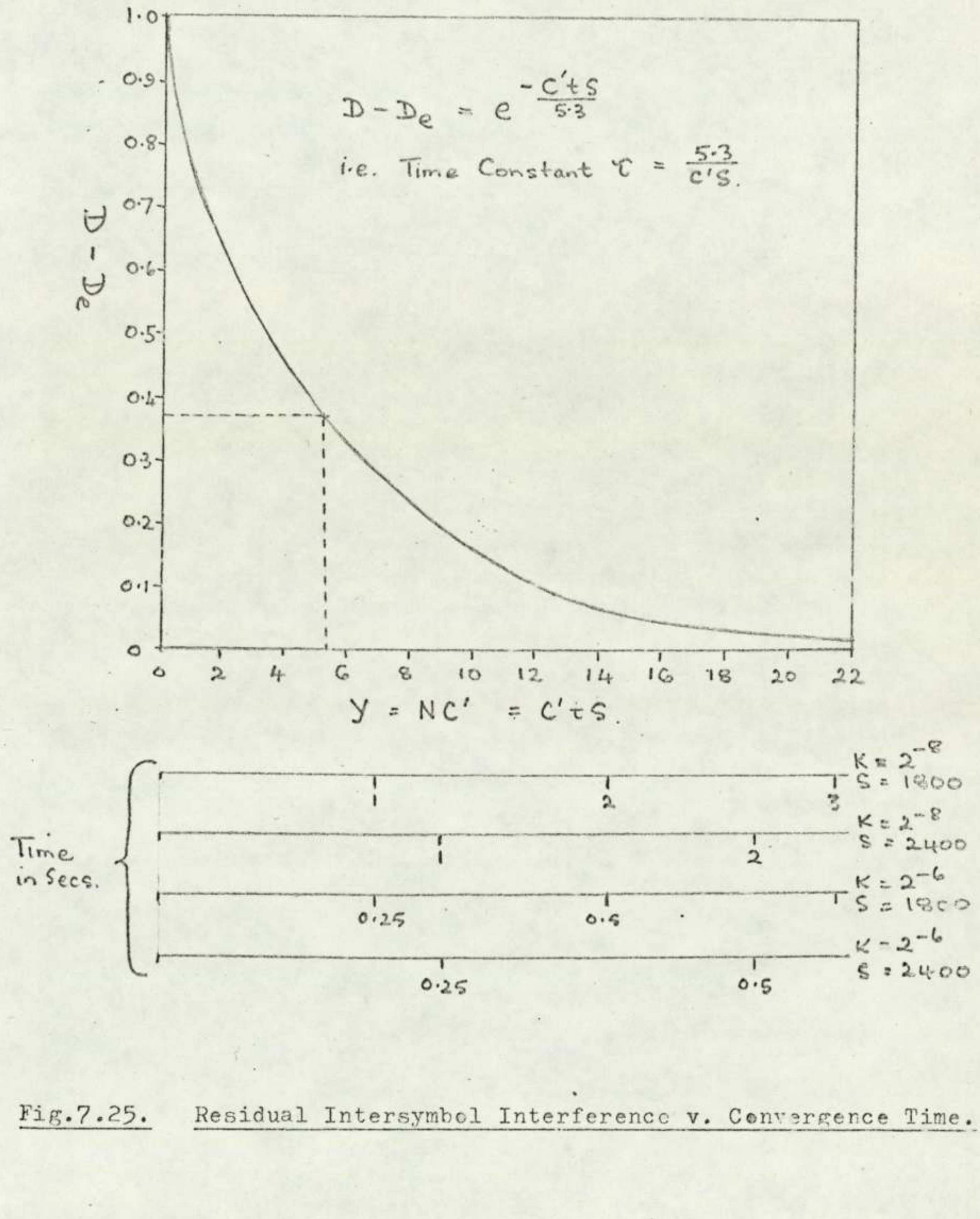


Fig.7.25. Residual Intersymbol Interference v. Convergence Time.

$$\tau = \frac{5.3}{C'S}.$$

The time scales for $C' = 2^{-6}$ (0.0156) and 2^{-8} (0.0039) and for S = 1800 and 2400 symbols/s. are indicated below the graph in fig.7.25.

7.11. Effect of System Noise on Weighted Increment Equaliser Strategies.

So far in the computer simulation we have assumed the system noise to be negligible and it would appear that the equaliser may be operated at the maximum convergence rate commensurate with system stability. In the presence of noise, however, there is a penalty in error-rate associated with any increase in the rate of equaliser convergence. The degradation in performance associated with a faster convergence rate arises from the increased coefficient increments made as a result of error signals produced by the system noise. It will be shown that this degradation can best be considered in terms of an equivalent noise penalty which degrades the error-rate performance of the modem.

Assume that the equaliser is in the fully equalised condition and let the mean signal power = P_s. If the system signal-to-noise ratio (S.N.R.) = x db., then the mean noise power $P_n = P_s 10^{-x/10}$. Assuming the noise is gaussian, then the mean noise voltage magnitude $E_n = 1.253\sqrt{P_s}\,10^{-x/20}$. Normalising the voltage measurements so that the multilevel signal is represented by the values shown in fig.7.24(a), we have

$$P_s = \frac{2}{A^3}\sum_{n=1}^{A/2}(2n-1)^2$$

and the mean signal voltage magnitude $E_s = \frac{1}{2}$.

The mean error signal voltage magnitude due to noise = $C'.E_n.E_s$

where C' is the convergence coefficient of the equaliser,

$$= 1.253\,\frac{C'}{2}\sqrt{P_s}\,10^{-x/20}.$$

Since each equaliser coefficient is incremented by this amount, the mean distortion voltage magnitude at the input to the equaliser summing network will be given by

$$1.253 \frac{C'L}{2} \sqrt{P_s}\, 10^{-x/20}$$

where L is the number of equaliser coefficients.

Hence the RMS distortion voltage

$$= \frac{C'L}{2} \sqrt{P_s}\, 10^{-x/20}$$

and the mean distortion power

$$P_d = \frac{C'^2L^2}{4} \cdot P_s\, 10^{-x/10}$$

Now the effective system S.N.R.

$$= \frac{P_s}{P_n + P_d}$$

$$= \frac{P_s}{10^{-x/10} P_s \left(1 + \frac{C'^2L^2}{4}\right)}$$

$$= 10 \log_{10} x\left(1 + \frac{C'^2L^2}{4}\right)$$

$$= 10 \log_{10} x + 10 \log_{10}\left(1 + \frac{C'^2L^2}{4}\right) \text{ db.}$$

Thus the degradation in performance is given by the equivalent noise penalty

$$10 \log_{10} \left(1 + \frac{C'^2L^2}{4}\right) \text{ db.} *$$

Graphs of noise penalty against C' for equalisers of length 18, 33 and 45 taps are plotted in fig.7.26. Values of points indicated on the graph are given in table 7.1.

The effect of the noise penalty on the system error-rate may be deduced from the error-rate versus signal-to-noise ratio curves given in fig.7.27. These curves are from Bennett and Davey[7.7].

*The contributing noise signals have been assumed to be uncorrelated, giving a 'worst-case' degradation.

TABLE 7.1.

C'	Degradation db.		
	L = 18	L = 33	L = 45
2^{-4}	1.367	3.160	4.742
2^{-5}	0.330	1.024	1.746
2^{-6}	0.086	0.282	0.508
2^{-7}	0.022	0.073	0.133
2^{-8}	0.004	0.017	0.035
2^{-10}	0	0	0

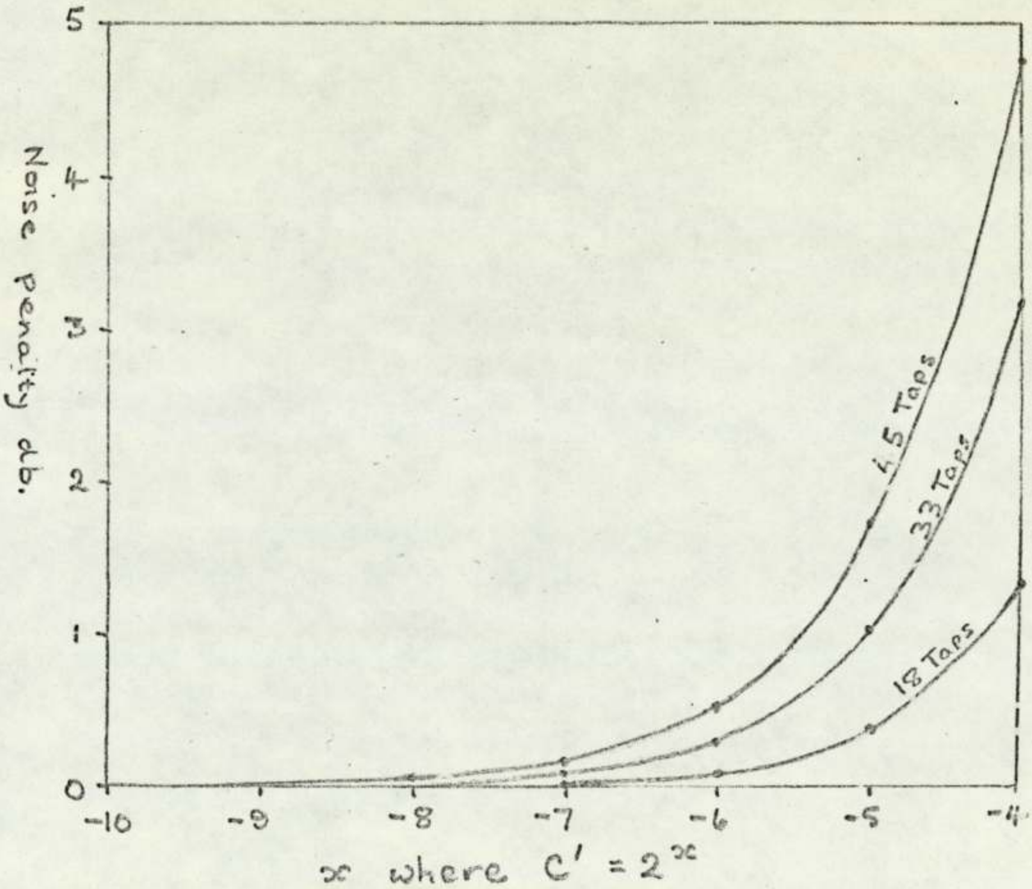


Fig.7.26. Noise Penalty v. Convergence Coefficient C'.

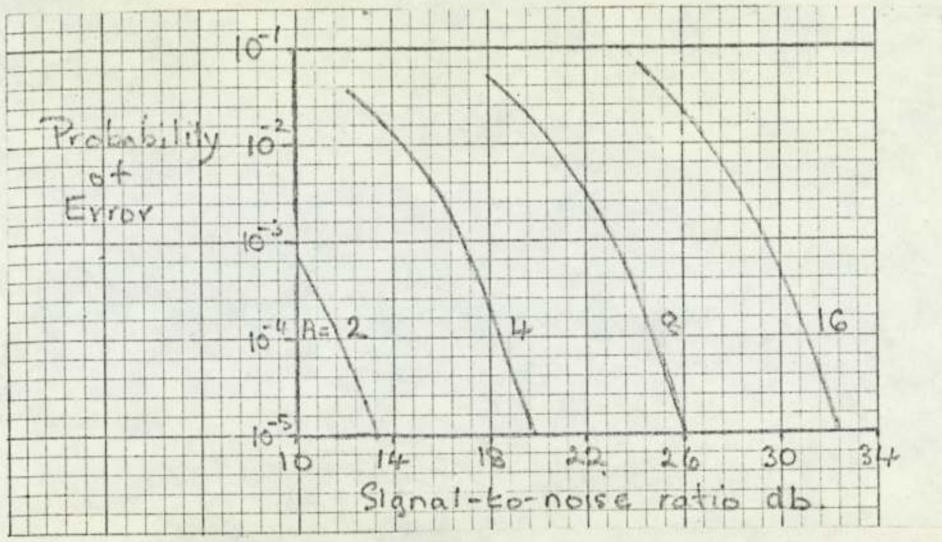


Fig.7.27. Symbol Error-Rate for A-ary Signalling.

7.12. Comparison of Fixed and Weighted Increment Strategies.

A comparison between the rate of convergence and degree of equalisation that can be attained using Strategy I with fixed coefficient increments (Lucky's algorithm[7.6]) and Strategy II with weighted coefficient increments is given in fig.7.28. These results[7.8] are those obtained by computer simulation of a 13-coefficient recursive equaliser with 6 feed-back coefficients, used in conjunction with equivalent base-band response R1K. It will be seen that, whereas the initial convergence rates are similar, the degree of equalisation finally achieved is far superior in the case of the weighted increment strategy. Also, the random variation in the fully-equalised state due to the quantisation of the increment steo size is entirely eliminated. The computer simulation does assume noise-free conditions and some variation will normally occur as a result of noise. The convergence coefficient for the weighted increment strategy has been chosen, however, so that only under extremely adverse conditions will the effect on this strategy be as great as that on the fixed increment algorithm.

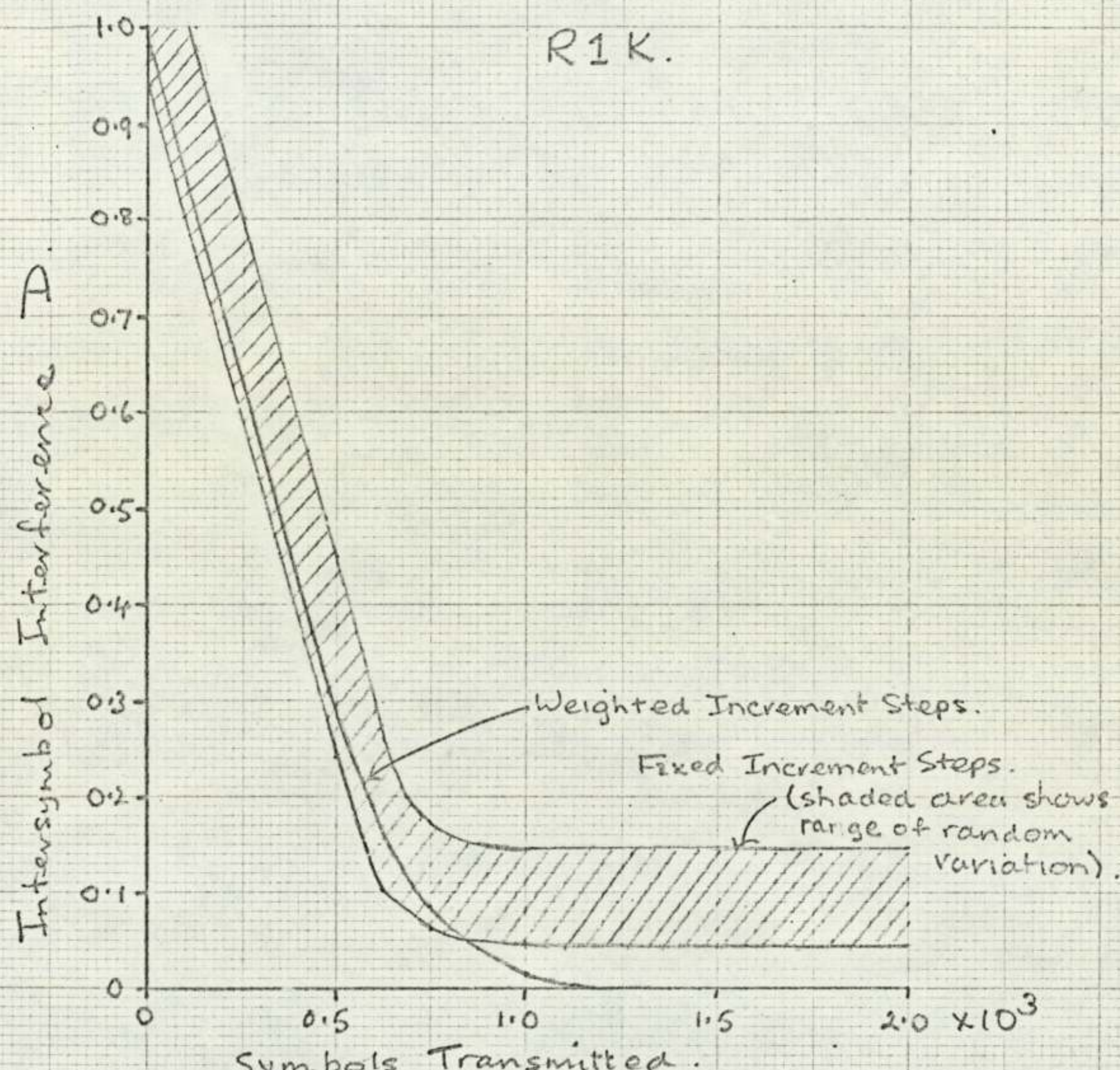


Fig 7.28. Comparison of Rate of Convergence of Strategies with Fixed and Weighted Increments.

8. DUAL MODE OPERATION.

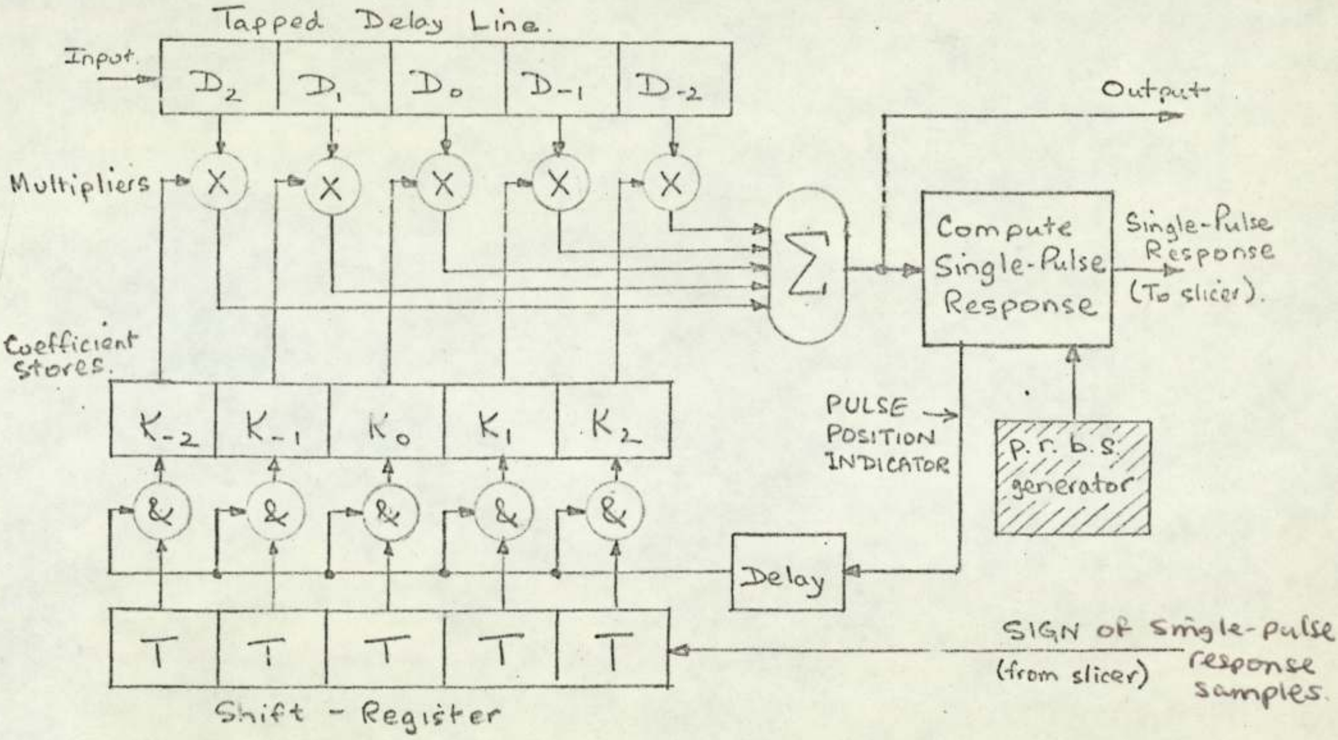
8.1. Introduction.

In the previous sections we have seen that the automatic transversal equaliser is capable of operation in either of two modes. In the automatic pre-set mode a test pattern is transmitted through the data channel which allows the channel single-pulse response to be computed from the received signal. From this response it is then possible to set the adjustable equaliser coefficients either directly or by means of an iterative strategy before the transmission of actual data is commenced. In the adaptive mode no test pattern is required as the coefficient values are determined from the error signals obtained from the received data.

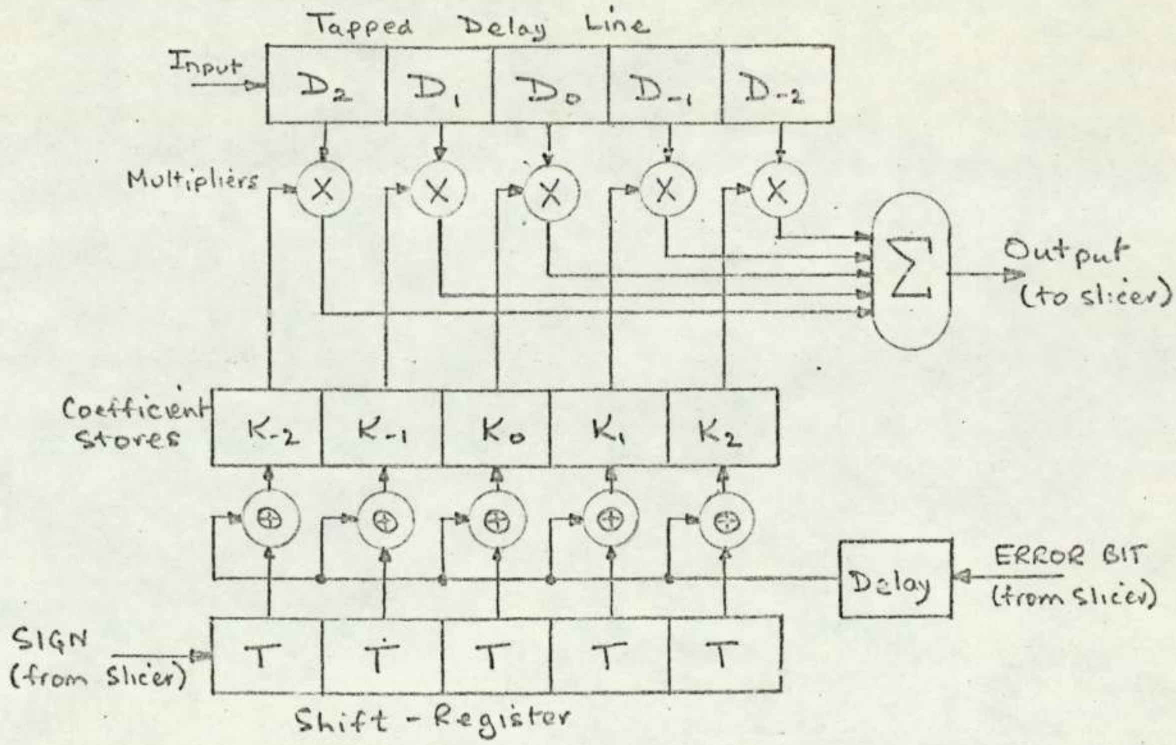
The main advantage of the adaptive mode of operation is that the coefficient values are continually varied so as to follow and compensate for changes in channel characteristic as they occur. The initial setting-up time is generally considerably longer, however, than is required with the automatic pre-set mode of operation. Thus there are circumstances where a hybrid system is highly desirable which commences operation in the automatic pre-set mode and then continues operation in the adaptive mode once satisafactory equalisation has been attained. Dual-mode operation can be especially advantageous where economic considerations dictate that a fixed coefficient increment strategy must be used for the adaptive mode of operation.

8.2. General Principles of Dual-Mode Operation.

The schematic diagrams for the automatic pre-set strategy, decscribed in section 6.2, and the adaptive strategy, described in section 7.2, are shown in fig.8.1.



a) Automatic Pre-Set Equaliser.



b) Adaptive Equaliser.

Fig.8.1. Schematic Diagrams of Automatic Equaliser Strategies.

A comparison of the schematic diagrams shows that there are two main differences in the logic requirements for the two modes of operation. Firstly, the shift register holding the sign digits is fed from opposite directions relative to the coefficient stores. In order to change from the automatic pre-set to the adaptive mode it is therefore necessary to be able to carry out a reversal of shift-register connection. This requires either the facility to reverse the direction of shift or to reverse the order of output connections. The latter option is preferable since, in this case, the register always contains meaningful information. With the former option, there is a short period whilst the register empties and refills when it contains incorrect information. This is not so disadvantageous as it appears, however, since other logic will also, of necessity, contain nonsense information for a few sample epochs after switching. In any case, the duration of these errors is such that they will normally be eliminated in the averaging provided on the coefficient stores.

Secondly, the coefficient incrementing logic consists of 'AND' gates in fig.8.1(a), whereas in fig.8.1(b) it consists of 'exclusive OR' functions which correlate the sign of the error bits with the sign of the equaliser output signal. For dual-mode operation, common use can be made of the logic elements by a simple rearrangement of the logic functions as described in the following section.

A change in the input and output connections to the delay and shift-register is also necessary. In the particular embodiment described in the next section, the delay is obtained by means of a short shift-register. This shift-register is designated shift-register B to distinguish it from the main shift-register, which is designated shift-register A.

8.3. A Practical Implementation.

We now consider a practical method of implementing the dual-mode requirements described in the previous section[8.1]. For the purposes of this description a 10-coefficient equaliser embodiment has been assumed. This method is, however, applicable to any size of equaliser. A schematic diagram of the switching logic and shift-registers is given in fig.8.2. Details of the shift-registers are given in fig.8.3.

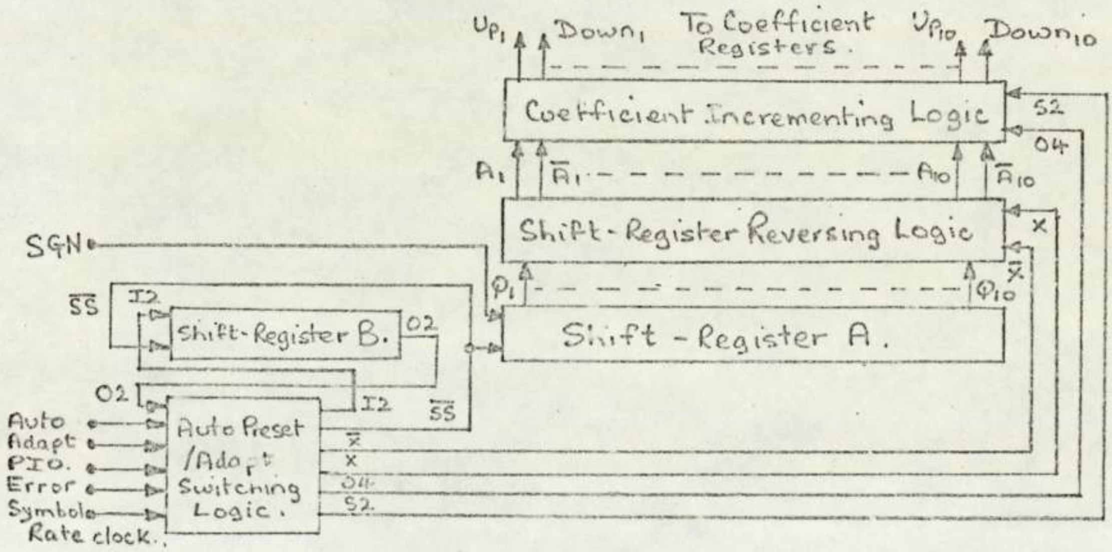


Fig.8.2. Shift-Register and Switching Logic.

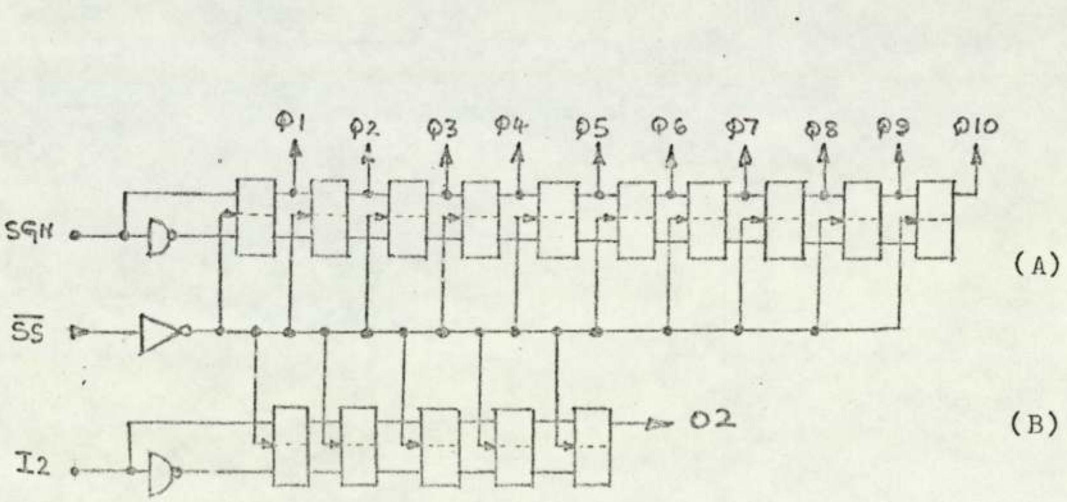
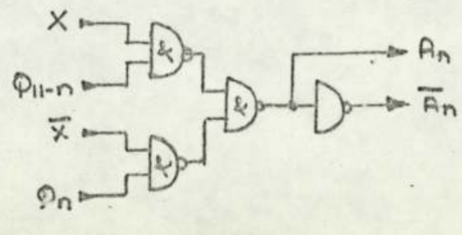


Fig.8.3. Shift-Registers A and B.

It will be noted that the input to shift-register A is the sign of the output symbol from the equaliser (SGN) in both modes of operation. The direction of operation of the shift-register is changed by re-orientation of the output connections using the logic arrangement shown in fig.8.4.



10 Circuits n = 1 to 10.

Fig.8.4. Shift-Register Reversing Logic.

The signal X defines the mode of operation and is 1(+ve) in the adaptive mode and 0(E) in the automatic pre-set mode. Either one of the two input gates is thus opened, depending on the mode of operation. The output from these two gates are "ORed" before connection to the coefficient updating logic.

Fig.8.5 shows the input and output switching logic for shift-register B.

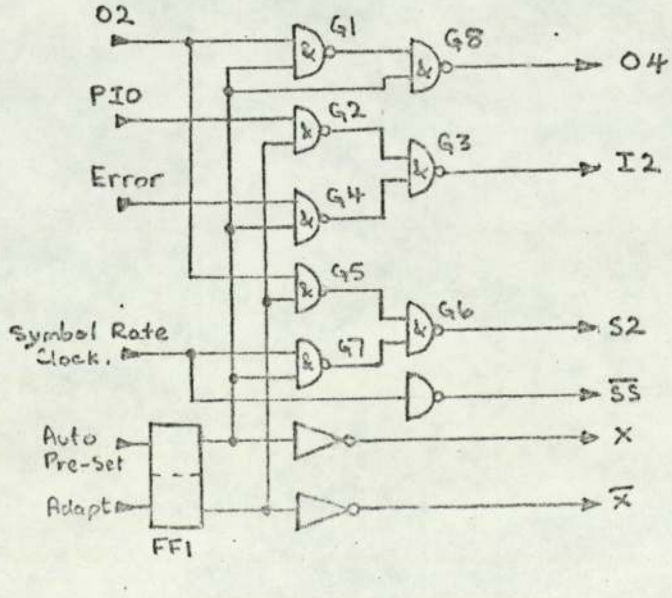


Fig.8.5. Auto Pre-Set/Adapt. Switching Logic.

The bistable multivibrator FF1 defines the mode of operation of the equaliser. Gates G2 to G4 connect either the output from the pulse-position indicator (PIO) or the error bit from the slicer to the input of shift-register B. In the adaptive mode, the output from shift-register B appears as O4 in fig.8.5. In the automatic pre-set mode, the signal O4 takes the condition O(E). The switching of signal O4 is obtained by means of gates G1 and G8. The increment command signal S2 is the shift-register B output in the automatic pre-set mode and the symbol-rate clock in the adaptive mode. This is obtained by means of gates G5 to G7.

The coefficient increment logic is shown in fig.8.6.

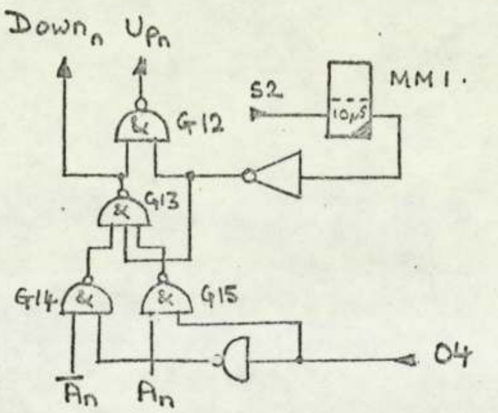


Fig.8.6. Coefficient Incrementing Logic.

Monostable multivibrator MM1 reshapes and retimes the increment command S2. In the automatic pre-set mode, O4 = O(E) and the gates G12 to G15 form an AND function between S2 and A_n. This is equivalent to the AND gate functions in fig.8.1(a). In the adaptive mode, the gates G12 to G15 becomes an exclusive-OR function between A_n and O4 in a manner equivalent to the exclusive-OR functions of fig.8.1(b). Tap incrementation is carried out on each symbol, since the signal S2 is derived from the symbol-rate clock from the modem.

8.4. Choice of Training Pattern for Automatic Pre-Set Mode.

In order to simplify the mode switching operation, it is desirable that the training pattern transmission used in the automatic pre-set mode should also be acceptable to an equaliser operating in the adaptive mode. It is then only necessary to ensure that the training pattern transmission lasts longer than the period of time allowed for automatic pre-set operation, instead of having to arrange simultaneous switching of pattern transmission at the send station and mode switching at the receive station. The pseudo-random binary sequence (p.r.b.s.) training pattern described in section 6.2(c) is suitable for this purpose.

9. DIGITAL IMPLEMENTATION.

9.1. Introduction.

In this chapter we consider methods of digital implementation which enable an adaptive equaliser to be realised using large-scale integration (L.S.I.) techniques. The requirement for an adaptive equaliser for modems designed to transmit high-speed data over the switched-telephone network was discussed in chapter 7. The use of L.S.I. makes the more complex strategies described in that chapter become economically viable. With these strategies it is possible to achieve faster convergence and more precise equalisation than is generally possible using the strategies normally employed with more conventional methods of implementation.

This work has led to the development of a single L.S.I. chip design which can be used to implement both basic "feed-forward" and recursive equalisers with various dispositions of the coefficients about the reference tap. The original concept was a 'chip per coefficient' philosophy, but it was found possible to provide three coefficients on a single chip, with an attendant saving in operational complexity.

9.2. Choice of Adaptive Strategy.

The strategy chosen for the digital implementation is the alternative strategy (Strategy II) described in section 7.2 and characterised by equation 7.7. This strategy was shown to be suitable for use both with the conventional feed-forward and the recursive transversal equaliser. The strategy minimises the mean-square error between the equalised and an ideal channel response and the equaliser coefficients are incremented in proportion to the magnitude of the estimate of the error in the coefficient setting. A schematic diagram for the strategy is given in fig.9.1.

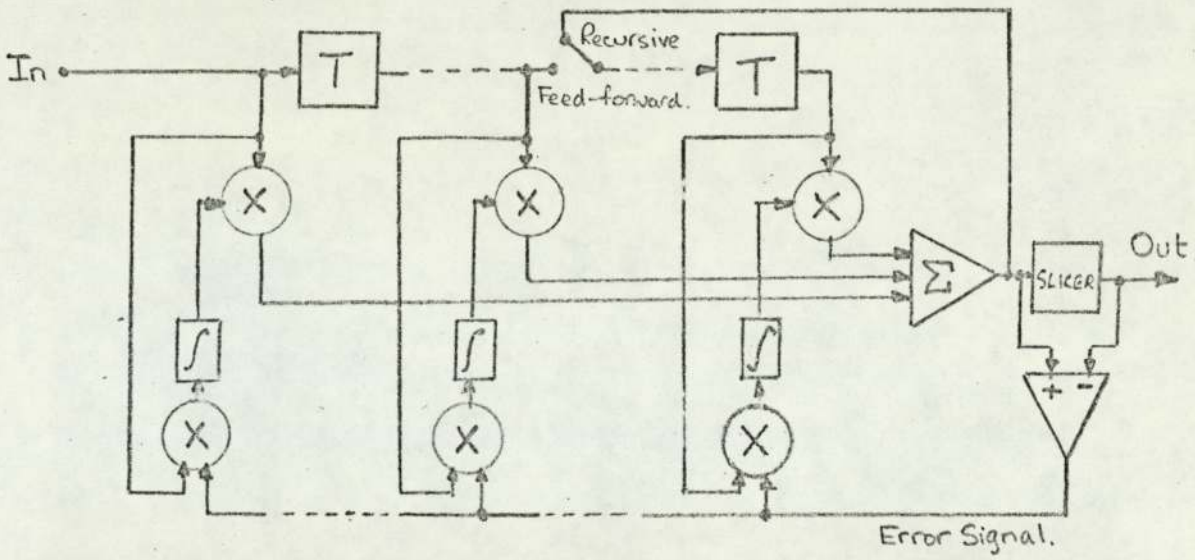


Fig.9.1. Basic System Schematic Diagram of Adaptive Equaliser.

9.3. Digital Implementation - General.

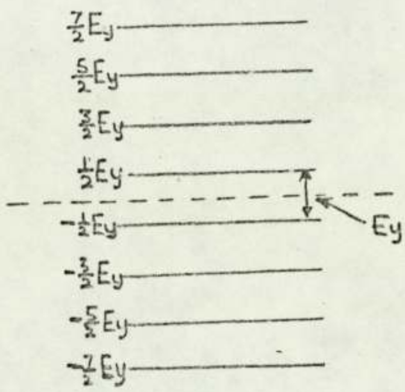
In a digital implementation it is necessary to quantise the sampled input signal as described in section 5.10. This enables the samples to be manipulated in digital number format. An analogue-to-digital converter is needed at the delay-line input and the delay-line then becomes a series of shift-register stages. The coefficients must also be stored in digital number form. Digital multiplication is used and the coefficient integrators become digital accumulators.

Two problems arise from the use of digital techniques, namely the accuracy to which it is necessary to represent the quantities involved in the computations and the most efficient method of performing tne arithmetic operations. We therefore consider these two problems in greater detail.

9.4. Accuracy Requirements for Quantised Representation of Line Signal and Coefficient Values.

The complexity of the digital logic requirements is extremely dependent on the accuracy with which it is necessary to represent the line signal and the coefficient values in their quantised form. In this section we consider the effects of quantisation on both these parameters.

a) <u>Line Signal.</u> The quantised line signal can be resolved into two components, the actual line signal and an additive noise component. The two components together form the input signal to the digital equaliser. The equivalent noise power of the additive noise component may be determined as follows. Consider a multilevel signal as illustrated in fig.9.2, where A = number of transmitted levels (alphabet) and E_y = the symbol interval.



<u>Fig.9.2.</u> <u>Multilevel Signal.</u>

Assuming random data, the mean transmitted signal power P_s is given by:

$$P_s = \frac{2}{A} \sum_{n=1}^{A/2} \left(\frac{E_y}{2}\right)^2 (2n - 1)^2$$

If x = number of binary bits used to represent the amplitude quantisation and y = number of binary bits used to represent one transmitted symbol ($\log_2 A$), then the quantisation interval is

$$E_x = \frac{E_y}{2^{(x - y)}}$$

But the mean equivalent noise power due to quantisation P_q can be shown[9.1] to be given by:

$$P_q = \frac{E_x^2}{12}$$

$$\therefore P_q = \frac{E_y^2}{12 \times 2^{2(x - y)}}$$

Hence the mean signal to equivalent-noise-power ratio is

$$\frac{P_s}{P_q} = \frac{6 \times 2^{2(x - y)}}{A} \sum_{n=1}^{A/2} (2n - 1)^2$$

This is plotted against x for various values of y in fig.9.3.

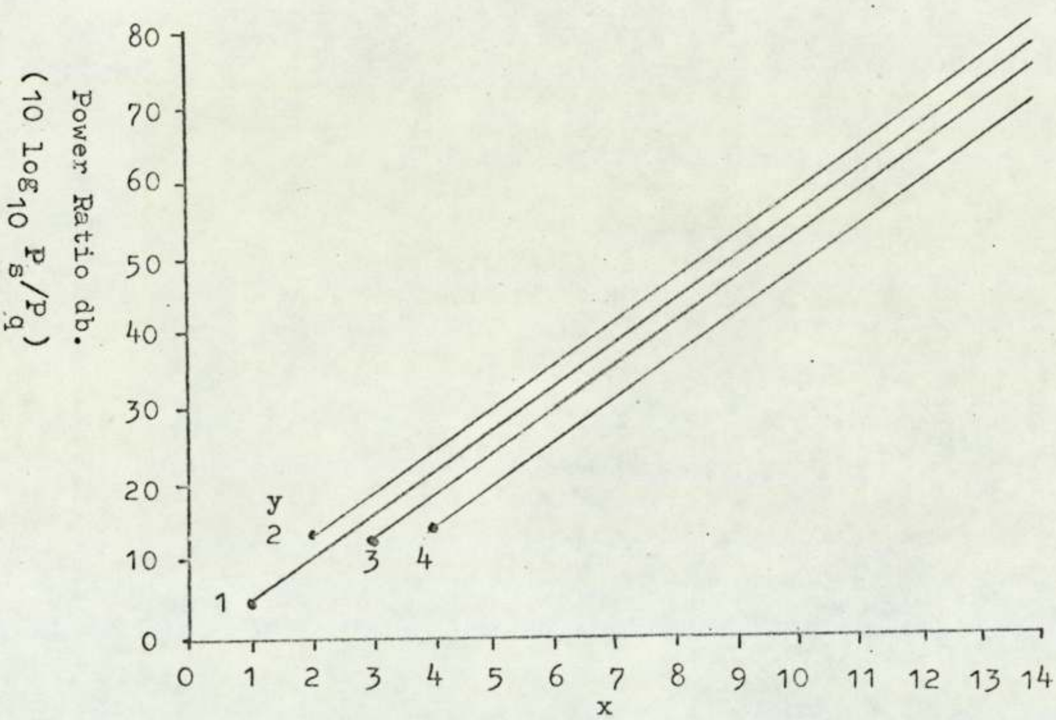


Fig.9.3. Mean Signal to Equivalent-Noise-Power against x and y.

Since the quantisation noise is uncorrelated with the existing noise in the system, we may assume that their resultant effect is additive. We shall use this assumption in order to determine the number of quantisation levels needed to ensure that the existing signal/noise ratio does not deteriorate more than a given amount. Calculations have been made for existing signal/noise ratios of 40db. and 30db. for deteriorations of not exceeding 0.5db. and 0.1db.. The results of these calculations are given in the following table:

Original s/n ratio	Deterioration	Equivalent additive s/n ratio
40db.	0.5db.	49.1db.
40db.	0.1db.	56.4db.
30db.	0.5db.	39.1db.
30db.	0.1db.	46.4db.

We can now determine from fig.9.3 the values of x required to ensure that the equivalent additive noise power does not exceed these values. The values of x so determined are given in the following table:

y(bits/symbol)	1	2	3	4
40 - 0.5db.	9	9	9	10
40 - 0.1db.	10	10	10	11
30 - 0.5db.	7	7	8	8
30 - 0.1db.	8	8	9	10

From the table it is clear that most of the requirements are satisfied by the use of 10 bits for quantisation of the line signal

b) <u>Coefficient Values.</u> The total mean deterioration in intersymbol interference D_q arising from the quantisation of the coefficient values is given by

$$D_q = \tfrac{1}{2}\Delta L,$$

where Δ is the quantisation step size and L is the number of taps in the delay-line. Since the coefficient must be capable of variation within the range -1 to +1,

$$\Delta = \frac{2}{2^x}$$

$$\text{Thus} \qquad D_q = 2^{-x}L.$$

Fig.9.4 shows Dq plotted against x for various values of L.

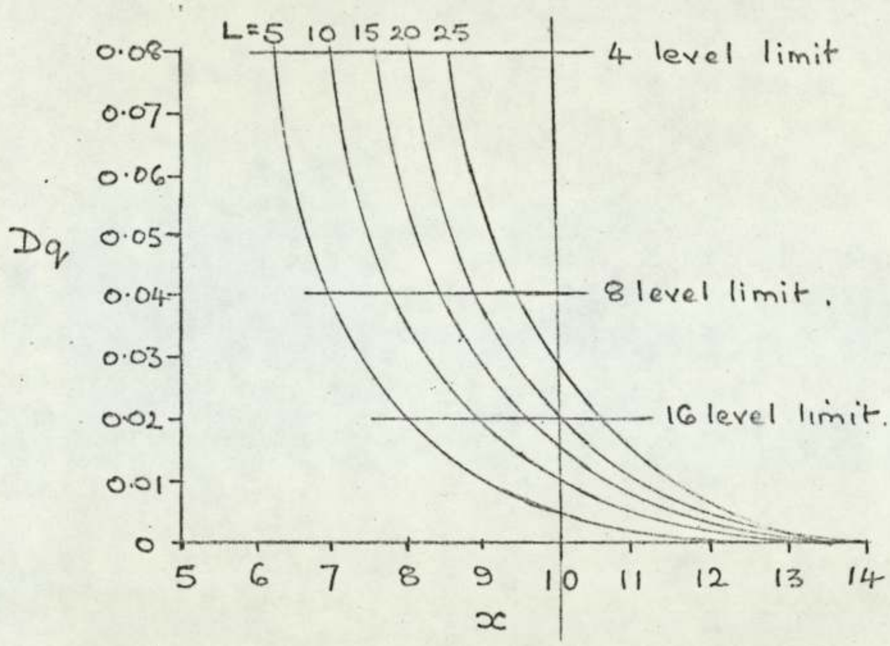


Fig.9.4. D_q Versus x for Various Values of L.

For 16-level transmission, D_q should not exceed about 0.02, for 8-level about 0.04 and for 4-level about 0.08. Since 16-level transmission would normally only be considered under very favourable conditions, fig.9.4 indicates that the use of 10 bits for quantisation of the coefficients is again a reasonable choice.

9.5. Coefficient Update Computation.

The time initially required to set the equaliser up and the rate at which it will follow changes in channel characteristic are dependent on the constant of integration of the coefficient integrators. Although short setting-up times are highly desirable, there is, unfortunately, a penalty in error-rate performance associated with any increase in the rate of equaliser convergence. This degradation of performance was analysed in section 7.11. It is evident from fig.7.26 and table 7.1 that $C' = 2^{-6}$ is generally the largest value of convergence coefficient that can be used without a serious degradation in modem performance. A problem therefore arises in the coefficient update calculation in that the product of the line signal, error signal and convergence coefficient must be at least 2^{-10} in order to update the least significant bit of the coefficient value. A cursory examination of the magnitude of these signals is enough to indicate that with 8-level operation only occasional updates will occur and at 16 levels the probability of an update is almost zero. In order to cope with this, it is necessary to provide additional least-significant bits to the coefficient value, although these need not be used in the actual 'transversal filter' part of the calculation. Three extra bits are found to be satisfactory for most purposes, making a requirement for 13 bits in all for the coefficient value store.

9.6. Number Representation.

A reduction in the amount of logic required to perform the arithmetic operations has been achieved by the use of negative radix binary arithmetic[9.2]. In this system of number representation the radix is -2, so that alternate bits represent positive and negative numbers respectively. (i.e. +64,-32,+16,-8,+4,-2,+1.) The advantages of this system of number representation are that there is no "end around carry" and that the addition and multiplication operations are performed direct, without regard to the polarity of the numbers involved. Both of these properties give a reduction in arithmetic logic over that required for conventional binary number methods of representation.

Another feature of the negative radix binary number representation is that the range of numerical values that can be represented by a given number of bits is not symmetrical about the zero value. In fact, the positive and negative ranges are in the ratio of approximately two-to-one, with twice as many positive representations as negative representations for an odd number of bits and vice-versa for an even number of bits. Since the total range of values taken by the coefficients is +2 to -1, this feature can be used to advantage in this application.

9.7. L.S.I. Implementation.

To make the most effective use of L.S.I. it is necessary to partition the logic into identical blocks. A suitable block is formed by one equaliser stage and its associated updating logic. A schematic diagram of the logic partitioning is given in fig.9.5.

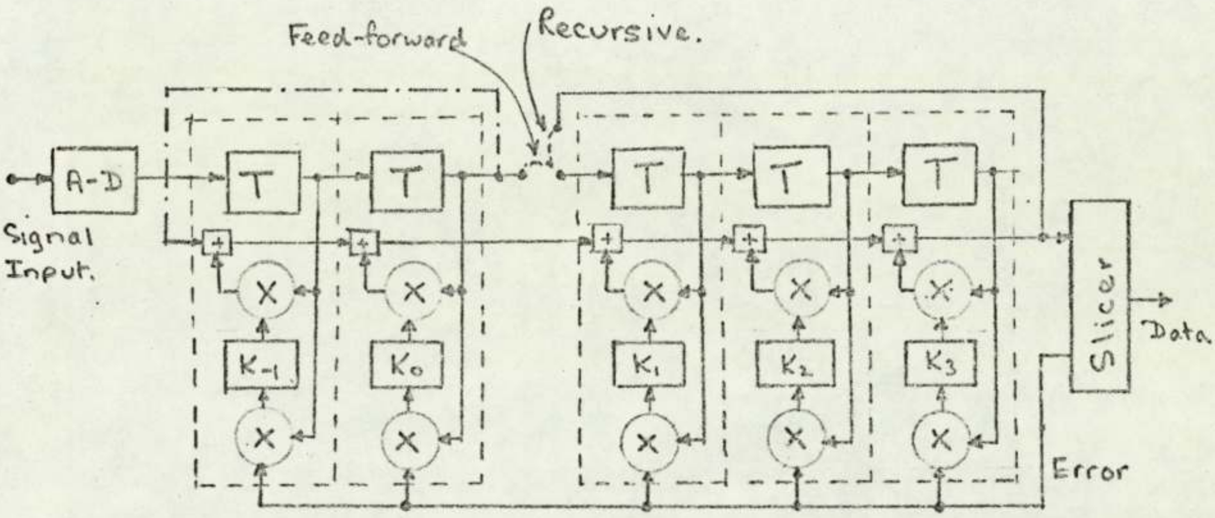


Fig.9.5. Schematic Diagram of Digital Adaptive Equaliser.

It will be seen that the blocks may be interconnected in either the feed-forward or the recursive configuration. Where negative radix binary representation is not used, it is possible to make the reference coefficient logic identical to all the other coefficients by adding in an additional signal so that varying the coefficient value about zero gives a signal whose magnitude varies about unity. This can be achieved simply by adding the signal at the reference coefficient tapping-point in the delay-line into the unused adder associated with the first stage as shown by the broken line in the diagram.

The most efficient use of logic is achieved by serial processing, providing the logic can be operated fast enough to achieve real-time processing. About 600 serial logical operations are necessary to perform the arithmetic for each received symbol. For a voice-band modem operating at 2400 symbols/s, a logic clock-rate of 1.5MHz. is therefore required. This is within the performance limits of M.O.S. large-scale integration [9.3].

A simplified schematic logic diagram for a single-tap stage is given in fig.9.6.

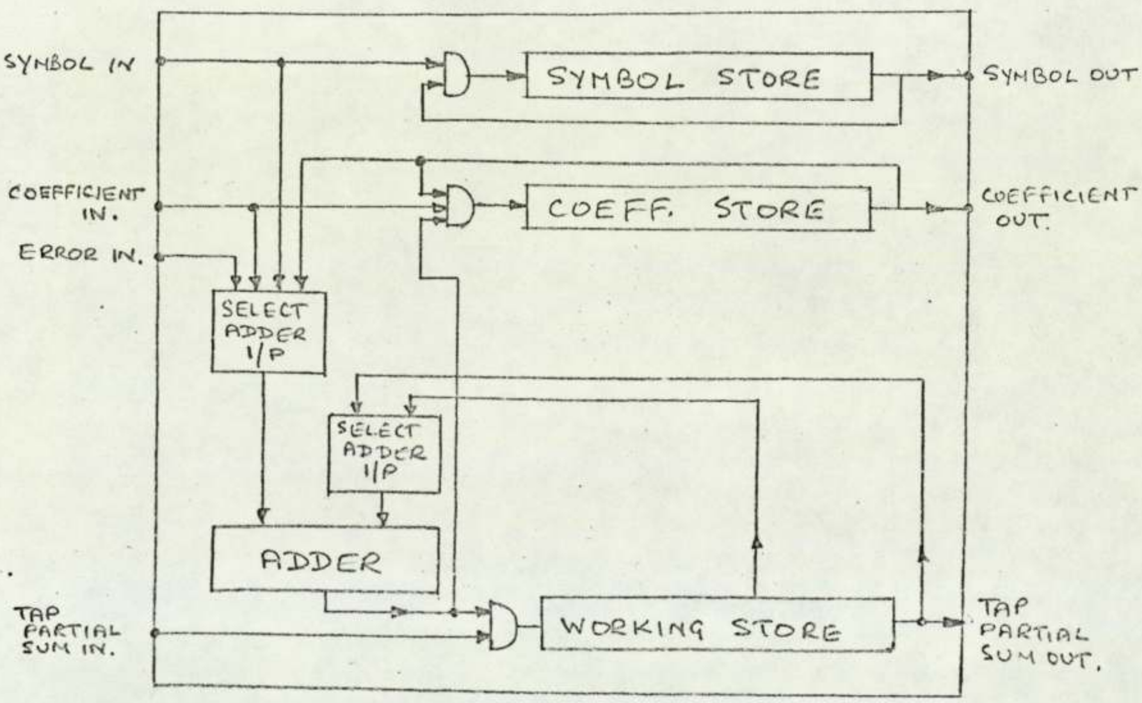


Fig.9.6. Equaliser Tap Schematic Diagram.

The timing and control signals are omitted from the diagram for the sake of clarity. Each tap stage consists of shift-registers for the unequalised data symbol store. The working store is used to store the partial products during the multiplication operation and the partial sum during the final summation of the tap multiplier outputs. An adder performs all the arithmetical addition and multiplication operations in conjunction with some logical functions controlling the adder inputs. The facility to read out and replace the coefficient values has been provided. This facility is useful in data systems where a central station automatically interrogates remote terminals on a routine basis, because it enables the central station equaliser to be set up without having to re-equalise from initial conditions each time the called station is changed.

The original concept was to construct the equaliser on a 'tap-per-chip' basis, but it was, in fact, found most economical to provide three taps per L.S.I. chip. This also results in some saving in logical operations per received symbol because it becomes possible to sum the three tap outputs together simultaneously on each chip, reducing the number of final additions between the chips by a factor of three. The interconnection of the tap stages for a single chip is given in fig.9.7.

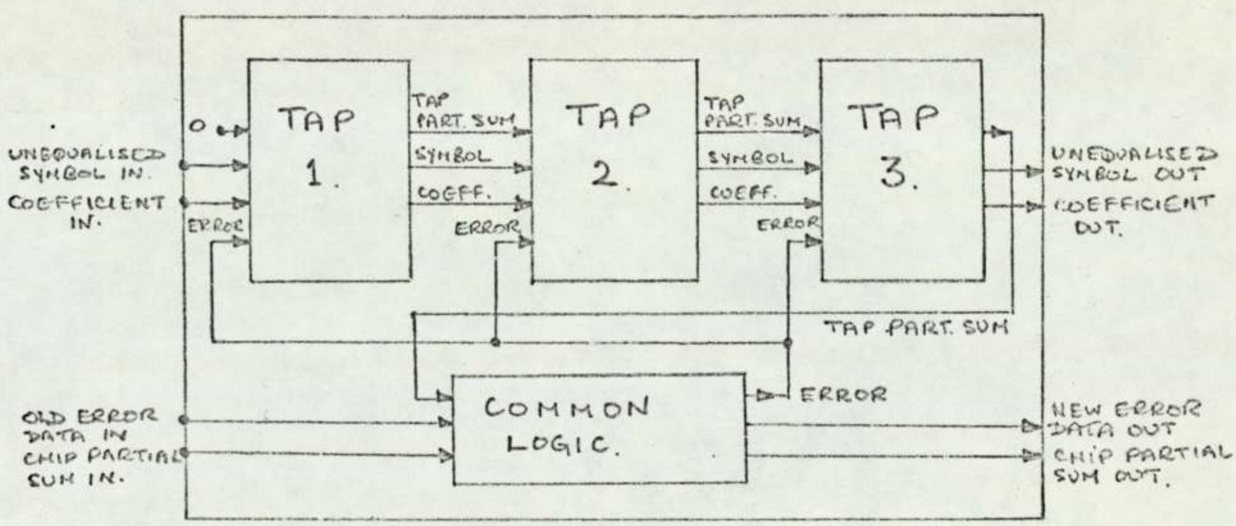


Fig.9.7. Equaliser Chip Block Diagram.

A schematic diagram of the common logic is given in fig.9.8. This common logic forms the chip partial sum by adding the tap-3 partial sum to the partial sum from the previous chip. The new error data computation is carried out only in the final chip. The computation facility is provided on all chips simply to maintain the chips identical. The interconnection of the chips is shown diagramatically in fig.9.9.

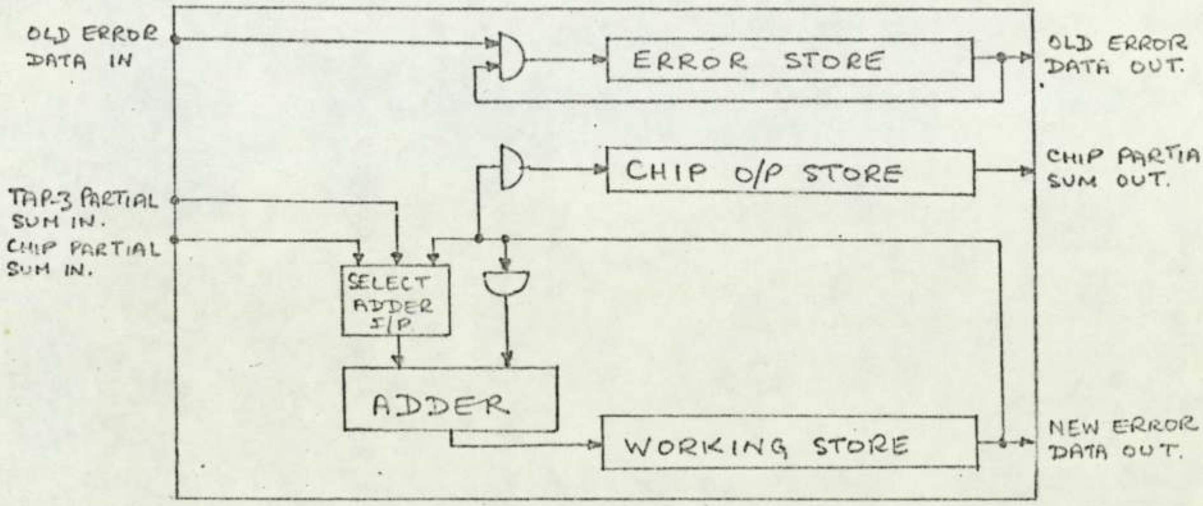
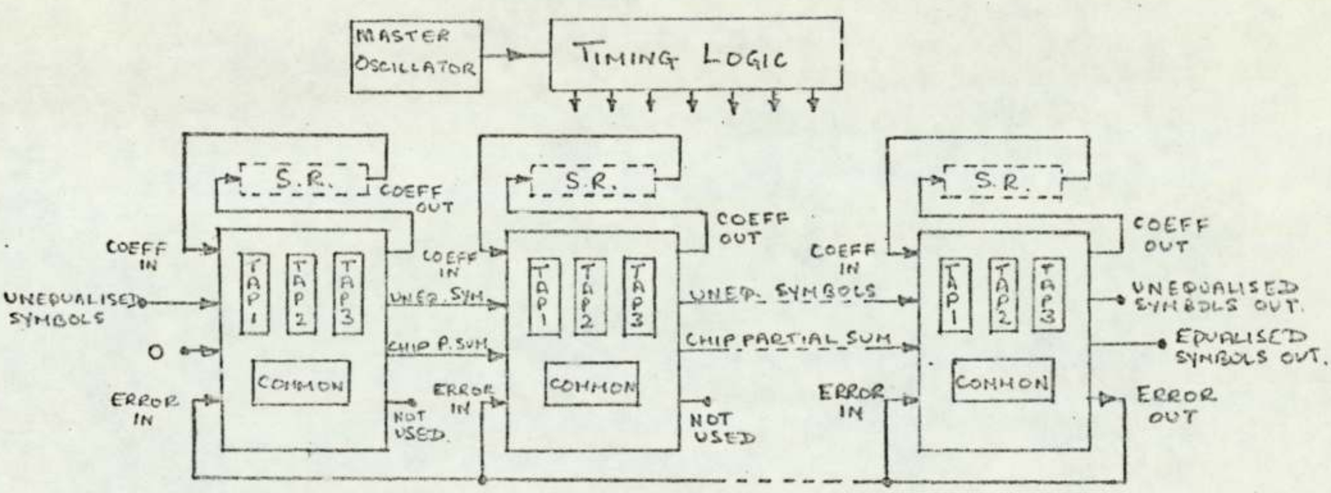


Fig.9.8. Common Logic Schematic Diagram.



S.R. = Shift Register, these are optional for interrogation system applications.

Fig.9.9. Equaliser System Schematic Diagram.

From the system design described in this section, a detailed logic design, suitable for M.O.S. L.S.I. was carried out.

9.8. L.S.I. Logic Design.

The detailed design work described in this section was carried out by the M.O.S. design team at Standard Telecommunication Laboratories Ltd., Harlow. A detailed schematic diagram of the chip logic is given in fig.9.10.

It will be seen that 14 bits are used for the coefficient store shift-registers. The additional bit, over the 13 bits shown to be necessary in section 9.5, is required because of the asymmetrical nature of the negative radix number representation. This gives only two-thirds the range of number representations in one direction from zero, compared with that obtained using conventional binary representation for a given number of register bits. The weighting factors of the shift-register bits are given in fig.9.11.

SYMBOL, ERROR, XPARK.

14	13	12	11	10	9	8	7	6	5	4	3	2	1
-2^{1}	-2^{0}	-2^{-1}	-2^{-2}	-2^{-3}	-2^{-4}	-2^{-5}	-2^{-6}	-2^{-7}	-2^{-8}	-2^{-9}	-2^{-10}	-2^{-11}	-2^{-12}

COEFF

28	27	26	25	24	23	22	21	20	19	18	17	16	15
-2^{1}	-2^{0}	-2^{-1}	-2^{-2}	-2^{-3}	-2^{-4}	-2^{-5}	-2^{-6}	-2^{-7}	-2^{-8}	-2^{-9}	-2^{-10}	-2^{-11}	-2^{-12}

COEFF

14	13	12	11	10	9
-2^{-13}	-2^{-14}	-2^{-15}	-2^{-16}	-2^{-17}	-2^{-18}

PARTSUM

27	26	25	24	23	22	21	20	19	18	17	16	15	14
-2^{1}	-2^{0}	-2^{-1}	-2^{-2}	-2^{-3}	-2^{-4}	-2^{-5}	-2^{-6}	-2^{-7}	-2^{-8}	-2^{-9}	-2^{-10}	-2^{-11}	-2^{-12}

DELAY

8	7	6	5	4	3	2	1
—	—	—	—	—	—	—	—

WORKSTORE

13	12	11	10	9	8	7	6	5	4	3	2	1
-2^{0}	-2^{-1}	-2^{-2}	-2^{-3}	-2^{-4}	-2^{-5}	-2^{-6}	-2^{-7}	-2^{-8}	-2^{-9}	-2^{-10}	-2^{-11}	-2^{-12}

Fig.9.11. Shift-Register Weighting Factors.

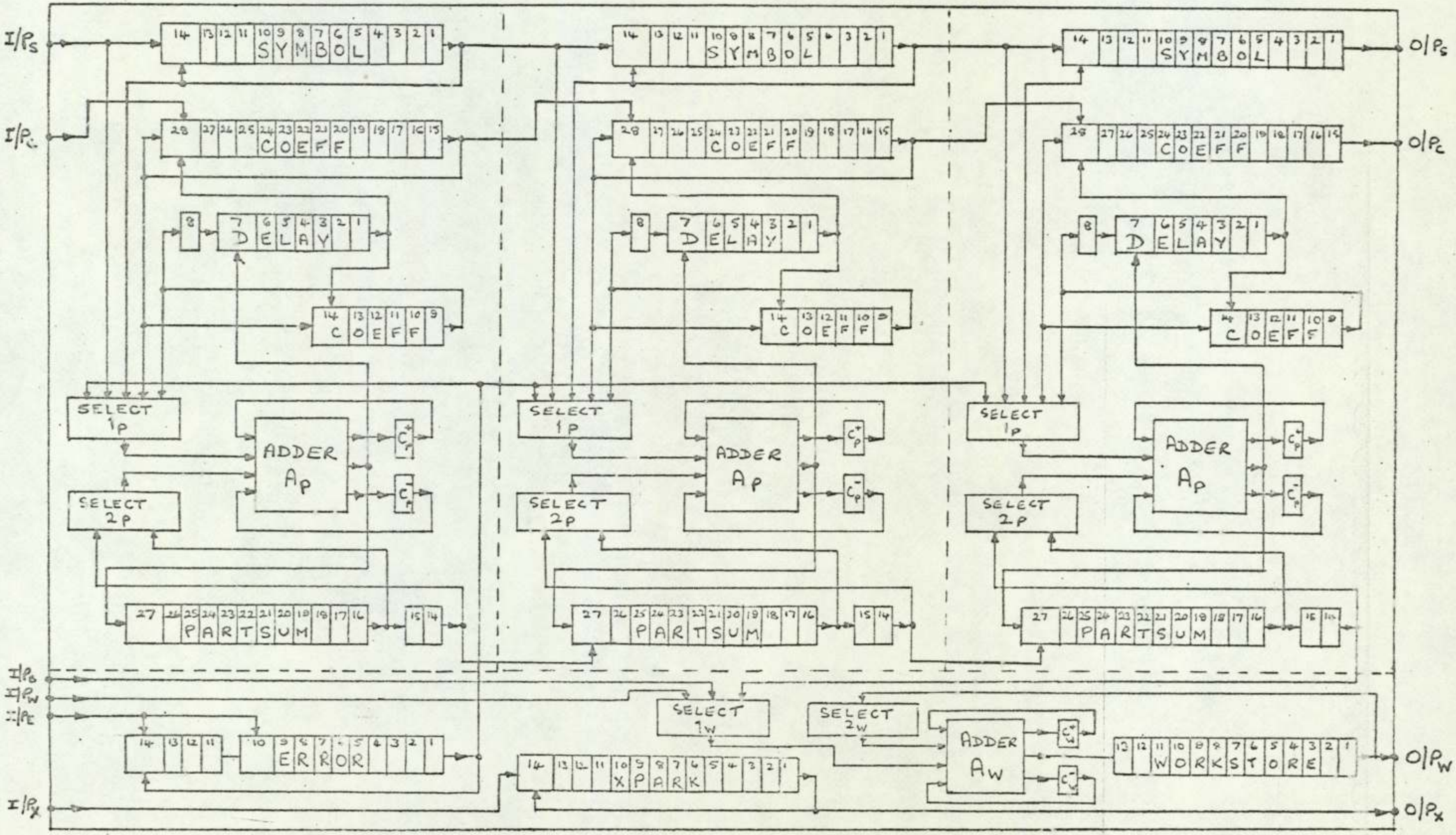


Fig. 9.10 Equaliser M.O.S. Chip Schematic Diagram.

The chip operation is controlled by clock-pulses and switching signals as illustrated in fig.9.12. The master oscillator (M.O.) operates at 3 MHz and is divided by two to give two-phase clock-pulses ${}_1\emptyset_1$ and ${}_2\emptyset_1$ at 1.5 MHz. These clock-pulses are used to circulate the shift-register contents. Certain operations are required once each time a single number representation has been circulated, that is, at every 14th clock-pulse. Secondary clock-pulses are therefore derived by dividing ${}_1\emptyset_1$ and ${}_2\emptyset_1$ by 14, giving ${}_1\emptyset_{14}$ and ${}_2\emptyset_{14}$, as shown.

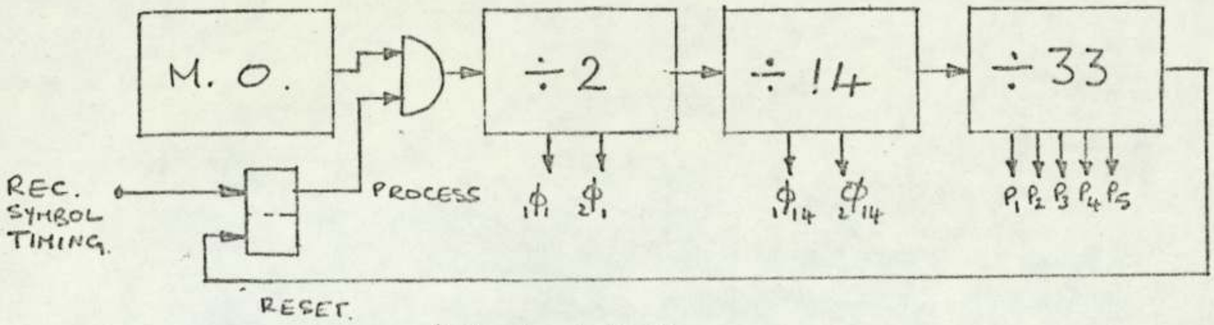
The computation is carried out in five phases. The duration of each phase and the operations carried out in each period are shown in table 9.1. The signals P1 to P5 control the logic switching for these operations and are derived from ${}_1\emptyset_{14}$, as shown in fig.9.12. At the end of period P5, the computation is terminated and the clock-pulses inhibited until the modem indicates that another received symbol is available to the equaliser for processing.

If the equaliser is operated in the recursive configuration, a sixth period P6, of 14 clock-pulses duration, is required to transfer the recursive-feedback symbol from the final WORKSTORE register into the final chip XPARK register. The flow of information between the chips can be seen from the chip interconnection diagram given in fig.9.13.

An analysis of the logical operations performed by each chip is given in table 9.2. Details of the logic functions and clock-pulse requirements necessary to perform these operations are given in table 9.3.

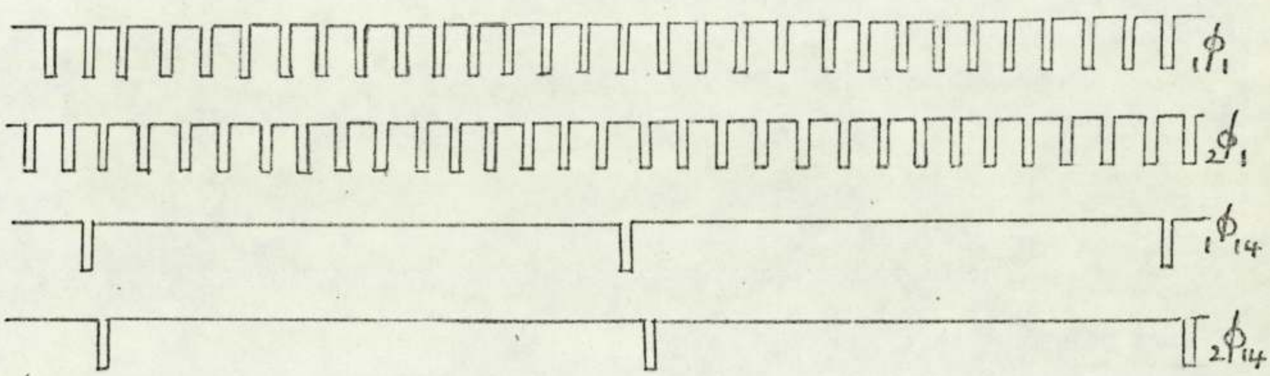
9.9. Feasibility Model.

A feasibility model was constructed from the design described in the previous section using T.T.L. logic. The model consisted of a nine-tap equaliser, the equivalent of



a) Control Logic.

b) Clock-Pulses.



c) Switching Signals.

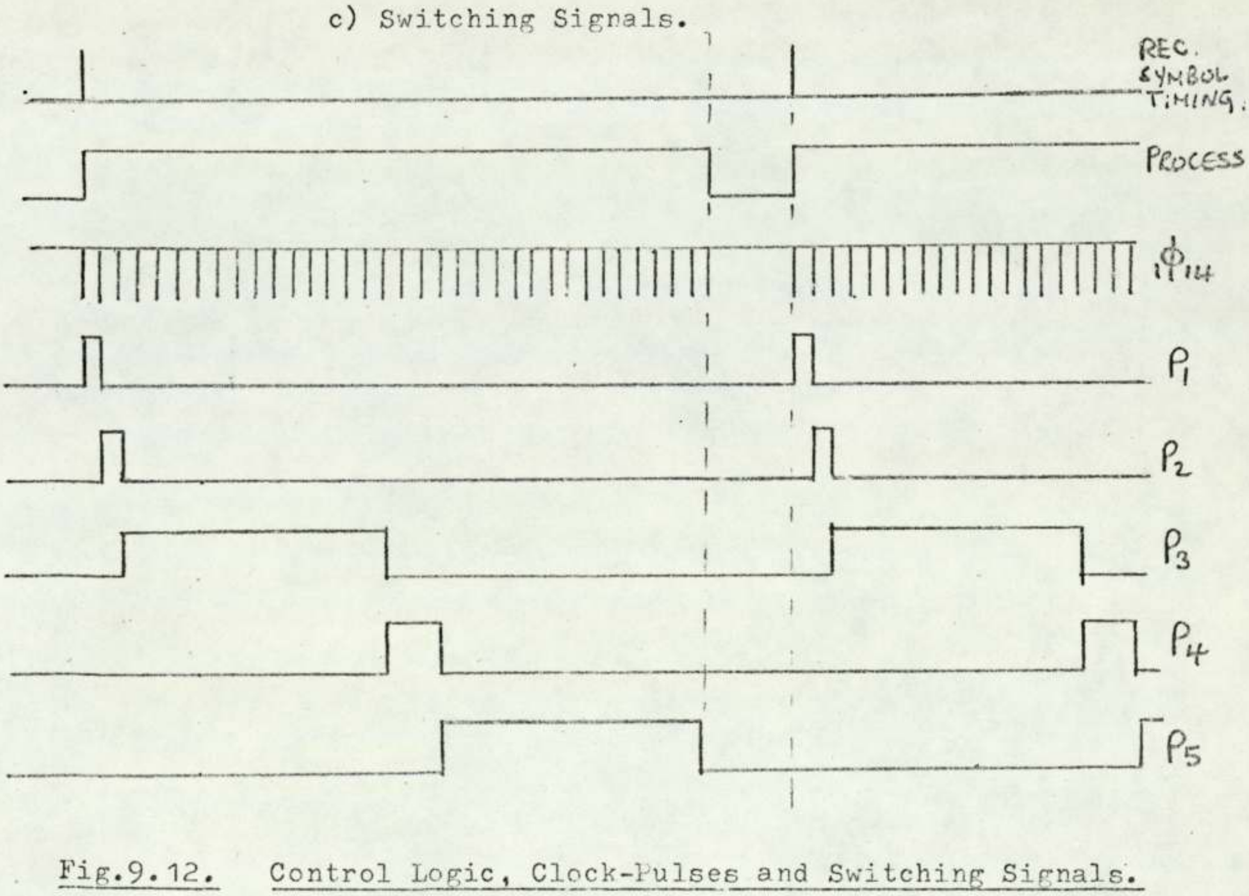


Fig.9.12. Control Logic, Clock-Pulses and Switching Signals.

Phase P1. 14 Clock-pulses.	a) Read in/out SYMBOL - Data symbols. b) Read in/out XPARK (Except on final chip) - Data symbols. c) Read out WORKSTORE (Final chip only) - Equalised data symbols to modem. d) Add coefficient update in PARTSUM to COEFF. e) Subtract estimated correct equalised symbol from actual equalised symbol (Final chip only) - error into WORKSTORE.
Phase P2. 14 Clock-pulses.	a) Read in ERROR from final chip WORKSTORE. b) Read out WORKSTORE (Final chip only). c) Add coefficient update in PARTSUM to COEFF.
Phase P3. 196 Clock-pulses.	a) Multiply SYMBOL by COEFF, forming equalised symbol partial sums in tap PARTSUMs.
Phase P4. 42 Clock-pulses.	a) Sum tap PARTSUMs into chip WORKSTORE. b) Read in/out COEFF (When required).
Phase P5. 196 Clock-pulses.	a) Sum chip WORKSTOREs into final chip WORKSTORE. b) Multiply ERROR by SYMBOL to form coefficient update in PARTSUM.

Table 9.1. Chip Computation Operations.

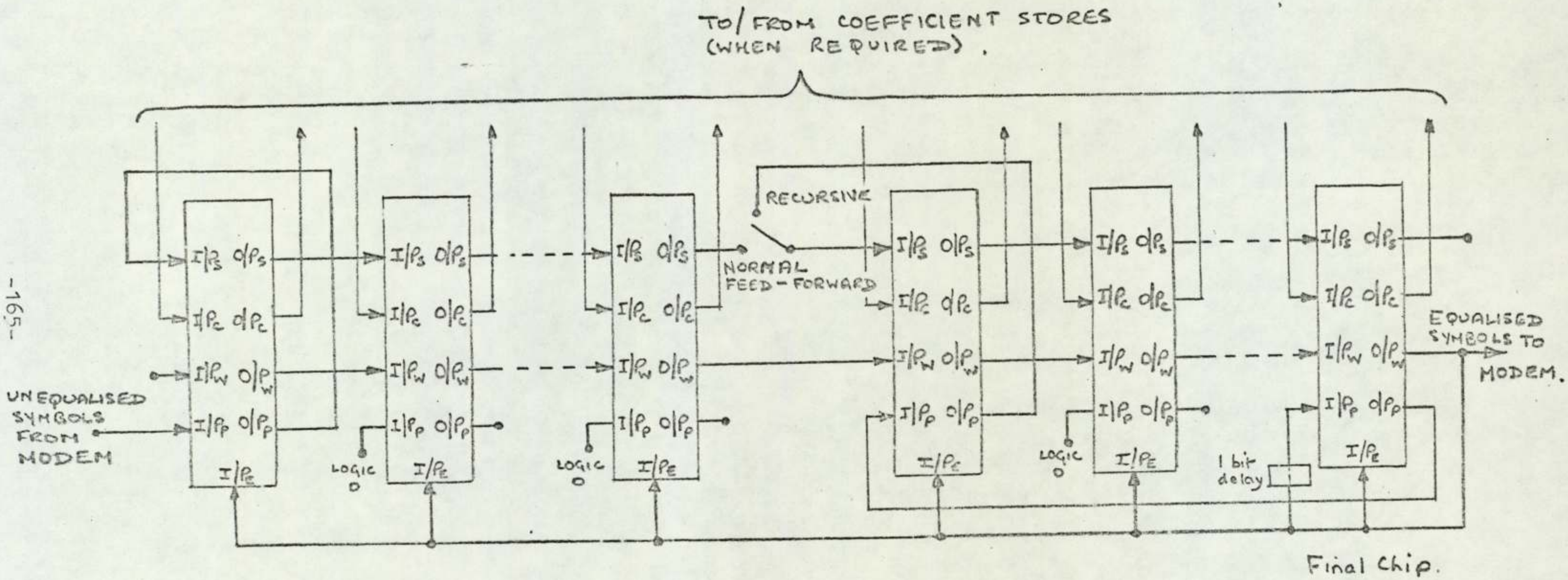


Fig 9. 13. Chip Interconnection Schematic Diagram.

Register.	Phase.				
	P1 14 Bits.	P2 14 Bits.	P3 196 Bits.	P4 42 Bits.	P5 196 Bits.
SYMBOL $S_{14} - S_1$	Shift. $S_{14} = I/P_s$	Shift. $S_{14} = S_1$	Shift/14. $S_{14} = S_1$	Shift. $S_{14} = S_1$	Shift/14. $S_{14} = S_1$
COEFF $C_{28} - C_{15}$	Shift. $C_{28} = D_1$	Shift. $C_{28} = D_1$	Shift. $C_{28} = C_{15}$	Shift. $C_{28} = C_{15}$ *	Shift. $C_{28} = C_{15}$
COEFF $C_{14} - C_9$	Shift. $C_{14} = C_{15}$	Shift. $C_{14} = C_{15}$	Shift. $C_{14} = D_1$	Shift. $C_{14} = D_1$ *	Shift. $C_{14} = D_1$
DELAY D_8	Shift. $D_8 = C_9$	Shift. $D_8 = C_9$	Shift. $D_8 = C_9$	Shift. $D_8 = C_9$	Shift. $D_8 = C_9$
DELAY $D_7 - D_1$	Shift. $D_7 = C_9 + P_{14}$	Shift.	Shift. $D_7 = D_8$	Shift. $D_7 = D_8$	Shift. $D_7 = D_8$
PARTSUM $P_{27} - P_{16}$	Shift. $P_{27} = 0$	Shift. $P_{27} = 0$	Shift. $P_{27} = C_{15}.I/P_s + P_{16}$	Shift. $P_{27} = I/P_s$	Shift. $P_{27} = S_1.E_1 + P_{16}$
PARTSUM $P_{15} - P_{14}$	Shift. $P_{15} = P_{16}$	Shift. $P_{15} = P_{16}$	Shift. $P_{15} = P_{16}$	Shift. $P_{15} = P_{16}$	Shift/14. $P_{15} = P_{16}$
ERROR $E_{14} - E_{11}$	Shift. $E_{14} = E_1$	Shift. $E_{14} = I/P_E$	Shift. $E_{14} = E_1$	Shift. $E_{14} = E_1$	Shift. $E_{14} = E_1$
ERROR $E_{10} - E_1$	Shift. $E_{10} = E_{11}$	Shift. $E_{10} = E_{11}$	Shift. $E_{10} = E_{11}$	Shift. $E_{10} = E_{11}$	Shift. $E_{10} = E_{11}$
WORKSTORE $W_{13} - W_1$	Shift. $W_{13} = I/P_B - W_1$ **	Shift. $W_{13} = 0$	Shift. $W_{13} = W_1$	Shift. $W_{13} = P_{14} + W_1$	Shift. $W_{13} = I/P_W + W_1$
XPARK $X_{14} - X_1$	Shift. $X_{14} = I/P_X$	Shift. $X_{14} = X_1$	Shift/14. $X_{14} = X_1$	Shift. $X_{14} = X_1$	Shift/14. $X_{14} = X_1$

* When coefficients are required to be exchanged $C_{28} = I/P_C$ & $C_{14} = 0$.

** Final chip only, other chips $W_{13} = 0$.

Table 9.2. Analysis of Chip Logical Operations.

$S_{14} = I/P_s.P1 + S_1.\overline{P1}$

$C_{28} = D_1.(P1 + P2) + C_{15}.\overline{(P1 + P2)}$ ($C_{28} = I/P_E$ for exchange COEFF)

$C_{14} = C_{15}.(P1 + P2) + D_1.\overline{(P1 + P2)}$ ($C_{14} = 0$ for exchange COEFF)

$D_8 = C_9$

$D_7 = A_p.(P1 + P2) + D_8.\overline{(P1 + P2)}$

$P_{27} = I/P_s.P4 + A_p.(P3 + P5)$

$P_{15} = P_{16}$

$1_p = C_9.(P1 + P2) + S_1.E_1.P5 + C_{15}.I/P_s.P3$

$2_p = P_{14}.(P1 + P2) + P_{16}.(P3 + P5)$

$A_p = 1_p + 2_p$ (Full serial addition, with carries.)

$E_{14} = I/P_E.P2 + E_1.\overline{P2}$

$E_{10} = E_{11}$ (Slower adaptation rates can be obtained if I/P_E is connected to E_{11} during phase P2.)

$W_{13} = A_w$

$X_{14} = I/P_X.P1 + X_1.\overline{P1}$

$1_w = I/P_B.P1 + P_{14}.P4 + I/P_W.P5$

$2_w = W_1.\overline{P2}$

$A_w = 1_w \pm 2_w$ (Full serial addition and subtraction, with carries and borrows, subtract during phase P1.)

a) Logic functions.

All units require clock pulses $_1\emptyset_1$ and $_2\emptyset_1$.

The following units also require $_1\emptyset_{14}$ and $_2\emptyset_{14}$:

$S_{14} - S_1$, $P_{15} - P_{14}$, $X_{14} - X_1$.

b) Clock requirements.

Table 9.3. Chip Logic Functions and Clock Requirements.

three M.O.S. equaliser chips.

Extensive tests were carried out using this model. Various equivalent base-band characteristics were simulated by means of a transversal filter, which was used to introduce distortion into a data stream consisting of a 511-bit pseudo-random binary sequence. The filter consisted of 40 taps, each capable of adjustment about a value of ± 1, normalised to the reference coefficient output. However, since only 9 stages are provided in the equaliser, the dispersion introduced by the filter was limited to nine symbol epochs. It was found that values of distortion D much greater than unity were required before the equliser failed to converge. The rates of convergence obtained were in accordance with those predicted by the theory given earlier in this thesis.

The equaliser functioned equally satisfactorily in both the normal and recursive configurations. Problems of instability, which were anticipated in the recursive mode, were not encuontered. Only by forcing the equaliser into an unstable condition, by specific selection of the characteristic to be equalised, was any instability obtained.

Once equalisation had been achieved, the equaliser adapted to changes of characteristic as rapidly as it was possible to vary the characteristic being equalised by altering the settings of the potentiometers on the line simulation filter. Although rate of adaptation was difficult to measure, it was clear that the rates obtained were similar to those expected from the predictions.

Following the successful testing of the feasibility model, work has proceeded towards the provision of masks and tooling for full-scale production of L.S.I. chips for a digital adaptive equaliser according to this design.

10. CONCLUSIONS.

10.1. General.

The earlier chapters of this work were devoted to a study of methods and strategies available for the equalisation of channels for high-speed data transmission. From this study it is evident that there is no optimum system, but rather that the system must be selected in the light of the circumstances in which it is to be operated.

The choice between the automatic pre-set mode and the adaptive mode depends on the variations in channel characteristics that are likely to be encountered. Much faster setting-up can be achieved using the pre-set mode, but such an equaliser is incapable of dealing with any changes in characteristic which occur after the initial setting-up period. A compromise is possible, whereby, at the expense of some additional hardware, operation can be commenced in the automatic preset mode and then continues its operation adaptively.

Another disadvantage of the adaptive equaliser is that its satisfactory operation depends on at least the major proportion of the data symbols being received correctly. If this condition is not satisfied, then the error signal, which is derived from the received signal, will be meaningless. Because of this, convergence cannot be guaranteed if the received data "eye-pattern" is initially completely closed. In practice, the line characteristic has to be extremely poor before the point of no-convergence is normally reached.

The choice of equaliser configuration depends entirely on the channel characteristics likely to be encountered. Where the channel single-pulse response has little dispersion, the conventional feed-forward transversal equaliser is satisfactory.

Such conditions normally apply to systems operating over leased "data-quality" telephone lines. Where the dispersion is considerable, and especially where the dispersed samples consist of substantial echoes, the recursive equaliser comes into its own. Doubts arising from the possibility of obtaining unstable networks have not been substantiated by either the computer simulation or the practical tests carried out. The recursive equaliser is particularly suitable for systems operating over h.f. radio channels, where multipath effects give rise to substantial echo signals. Echoes also frequently occur on switched-telephone network connections, often with delays of several milliseconds. These,again, can be most effectively dealt with using the recursive equaliser.

A variety of adaptation strategies exist for the adaptive equaliser. The most effective strategy is that which minimises the mean-square error in the received signal (MSE), with coefficient increments in proportion to the error signal magnitude. Unfortunately, the complexity of the increment hardware makes the use of proportional increments uneconomic for equalisers implemented using discrete component circuits. The use of M.O.S. L.S.I., however, makes the use of proportional increments economically viable.

The improved convergence rates that can be obtained by using proportional increments enables the adaptive equaliser to be used in applications which would otherwise have necessitated the use of automatic pre-set initialisation. It is still necessary to use a pre-set strategy in cases where the data "eye-pattern" is initially completely closed, since any data errors will give rise to false coefficient error indications. Provided the binary "eye" remains open, however, it is possible to set up multilevel systems using the adaptive strategy by operating initially with binary data only. Once the distortion has been reduced to the point where it is possible to support multilevel data, it is only necessary to change the receive modem slicer levels for multilevel operation and to signal the send modem that multilevel data may now be transmitted.

In practice it was found that it was possible to select modem parameters such that it was almost always possible to transmit binary data through both the leased "data-quality" and the switched-telephone networks with error-rates that did not interfere with the initial convergence of the adaptive equaliser. Work therefore proceeded to provide a design for a general-purpose M.O.S. L.S.I. equaliser chip based on the MSE adaptive equaliser, having proportional coefficient increments. This design has been presented in detail and commercial manufacture is now planned.

10.2. Further Work.

Two further aspects of this work are currently being pursued:

a) Implementation using Standard Logic Blocks.

Work is continuing at the University of Aston[10.1] on the design of a similar equaliser, but using standard logic blocks. This design is intended for situations where the production quantities are small and the provision of M.O.S. masks and tools is therefore uneconomical. Use is made of high-speed T.T.L. so that the arithmetic operations can be multiplexed between the stages. General-purpose multistage shift-register packages are being used to store the signal samples and coefficient values.

b) Use of Adaptive Equalisers with Partial Response Data Systems.

The adaptive equaliser simulation programs are being modified to enable simulation tests to be carried out on the use of the adaptive equaliser in conjunction with partial response data systems[10.2]. Preliminary tests have been carried out at S.T.L., Harlow, for an adaptively equalised class IV partial response system.* The results of these initial tests confirm the feasibility of such an arrangement, but the system performance was somewhat disappointing in comparison to that obtained with the conventional data systems as described in this work. A full programme of tests is necessary, however, before sufficient results are available to make a conclusive system comparison.

* Each symbol in the class 4 partial-response system is transmitted in the form 1,0,-1.

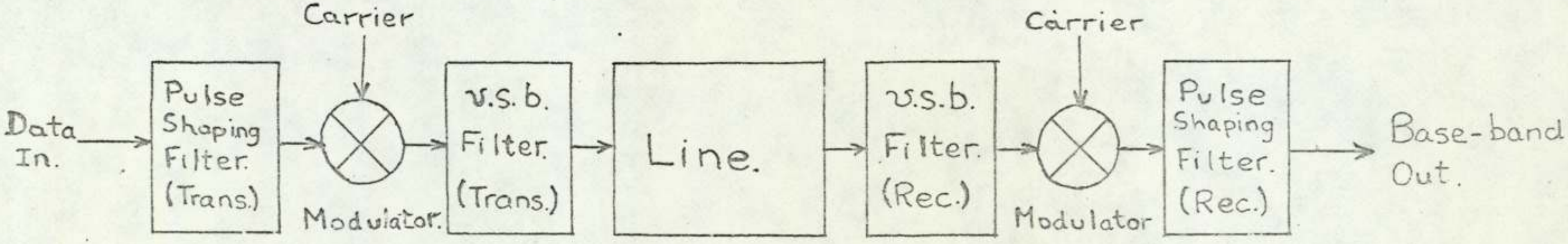
APPENDIX 1

CHARACTERISTICS OF TYPICAL TRANSMISSION CHANNELS USED FOR TEST PURPOSES.

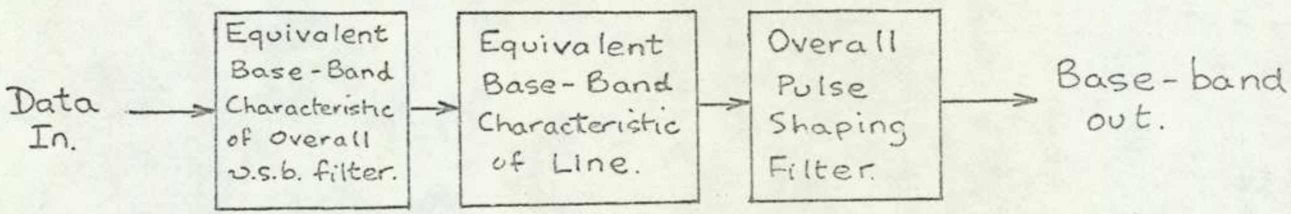
This appendix gives details of the characteristics of typical transmission channels used throughout this work in conjunction with the various computer simulation test programs.

The equivalent base-band channel characteristics of various vestigial-side-band (v.s.b.) modem configurations are computed from typical line characteristics using the computer program described in I.T.T. Technical Report No. STL 1029 [A.1]. The block diagram of a v.s.b. modem system is given in fig.A.1(a). This reduces to the equivalent system configuration illustrated in fig.A.1(b), which forms the basis of the computer program. The characteristics of the pulse-shaping filter, the line and the v.s.b. filter are read into the computer as program data, together with the modulated carrier frequency and data symbol transmission rate. The program determines the composite response of the line and the v.s.b. filter and then computes the equivalent base-band response between the input to the modulator and the output of the demodulator, assuming that both these functions are performed by linear multipliers. The program enables the phase of the demodulating carrier to be varied, but for these computations it is assumed to have been accurately determined. The response so obtained is then combined with the pulse shaping filter response to give the overall equivalent base-band characteristic.

Unless otherwise stated, the overall pulse-shaping filter response has been assumed to have a raised-cosine roll-off low-pass amplitude characteristic with a linear phase



a) v.s.b. Modem System Configuration.



b) Equivalent Base-Band Configuration.

Fig A.1. Equivalent Base-Band Channel of v.s.b Modem System.

characteristic and the overall v.s.b. filter response has been assumed to have a linear roll-off with a linear phase characteristic. The amplitude characteristics of these two filters are illustrated in fig.A.2.

The line characteristics used in the computations are shown in figs.A.3 to A.5. The characteristics designated W1 and W2 in fig.A.3 are estimated worst-case characteristics for special-quality data circuits to CCITT recommendation M102. The M102 limits are also shown in the diagram. The line characteristics L1 to L5, given in figs.A4 and A.5, are measured characteristics of actual leased connections supplied by the G.P.O. The compositions of these connections were as follows:

Line 1. R/CB2694 + BM-L1023, looped at Birmingham.

Line 2. R/CB2694 + L-XF1000, looped at Fenny Stratford.

Line 3. R/CB2694 + BM-L1020, looped at Birmingham.

Line 4. R/CB2694 + BM-L1201, looped at Birmingham.

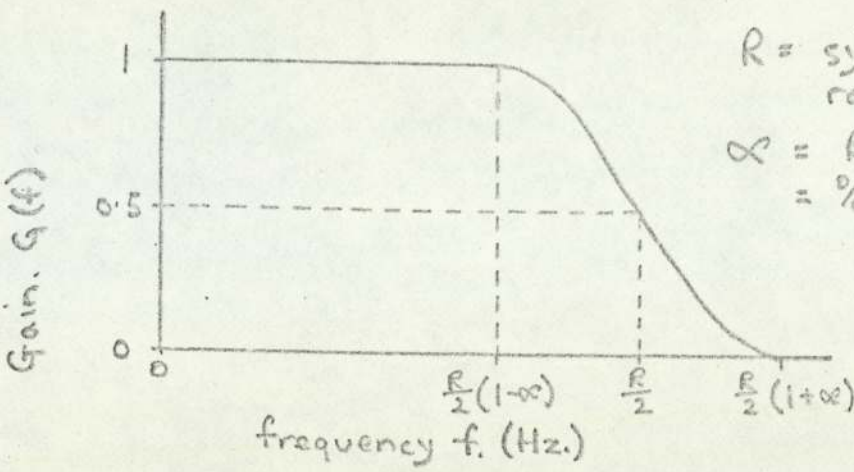
Line 5. R/CB2694, looped at Faraday House, London. Group delay equalisers removed.

R/CB2694 comprised 24.5 miles of 20/88/1.136 loaded audio cable from S.T.L., Harlow to Faraday House, London. Amplifiers and 25 mile group-delay equalisers were provided in both directions of transmission.

BM-L1023 comprised two tandem carrier channels from Faraday House to Birmingham via Bristol. Total distance 219 miles.

L-XF1000 comprised 8.5 miles of 20/88/1.136 loaded audio cable, plus 38.5 miles of 40/88/1.136 loaded audio cable from Faraday House to Fenny Stratford. One amplifier was provided for each direction of transmission.

BM-L1020 comprised one carrier channel on microwave from Faraday House to Birmingham. Total distance 131 miles.

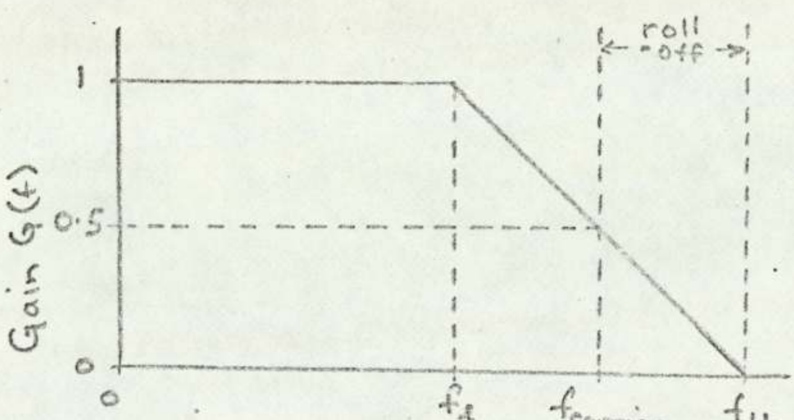


R = symbol transmission rate. (bauds).

α = Roll-off factor = % roll-off ÷ 100

$$G(f) = 1, \qquad 0 \le f \le (1-\alpha)\frac{R}{2}$$
$$G(f) = \frac{1}{2} - \frac{1}{2}\sin\left(\frac{2f-R}{\alpha R}\right)\pi, \qquad (1-\alpha)\frac{R}{2} \le f \le (1+\alpha)\frac{R}{2}$$
$$G(f) = 0, \qquad (1+\alpha)\frac{R}{2} \le f \le \infty$$

a) Pulse shaping filter. (Raised Cosine)



$$\text{roll-off} = \frac{f_u - f_l}{2}.$$
$$f_{carrier} = \frac{f_u + f_l}{2}.$$

$$G(f) = 1, \qquad 0 \le f \le f_l$$
$$G(f) = 1 - (f - f_l)/(f_u - f_l), \qquad f_l \le f \le f_u$$
$$G(f) = 0, \qquad f_u \le f \le \infty$$

b) v.s.b. filter.

Fig. A2. Assumed Amplitude Characteristics of v.s.b. and Pulse Shaping Filters.

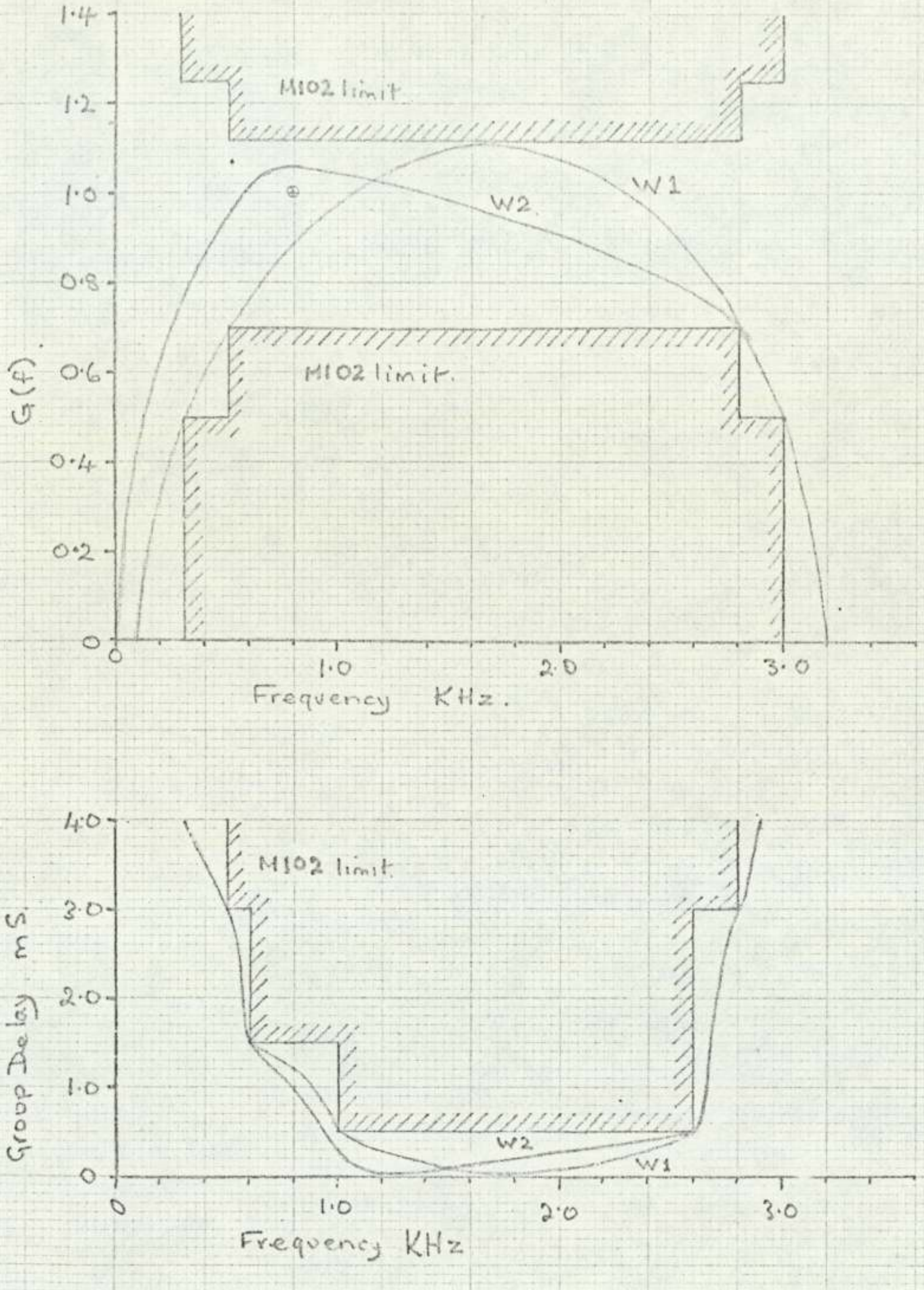


Fig. A3 Estimated Worst-Case Characteristics, for Special Quality Data Circuits to C.C.I.T.T. recommendation M102.

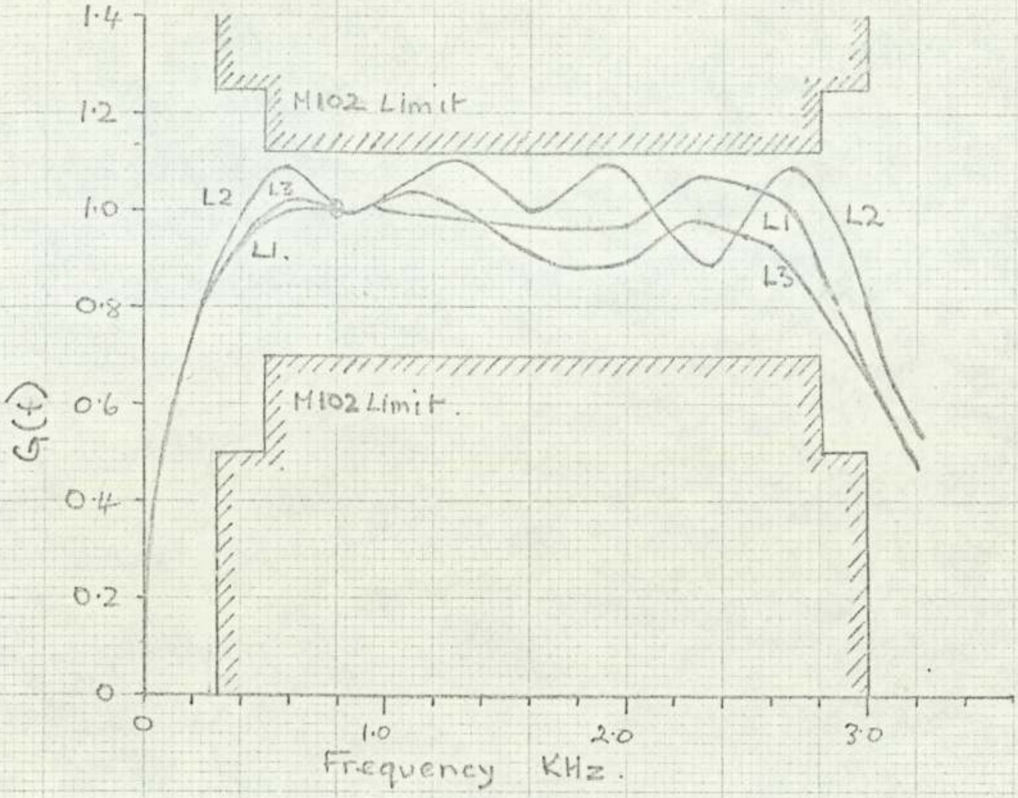


Fig A4 Actual Characteristics of Special Quality Data Circuits to C.C.I.T.T Recommendation M102 (Lines L1 - L3).

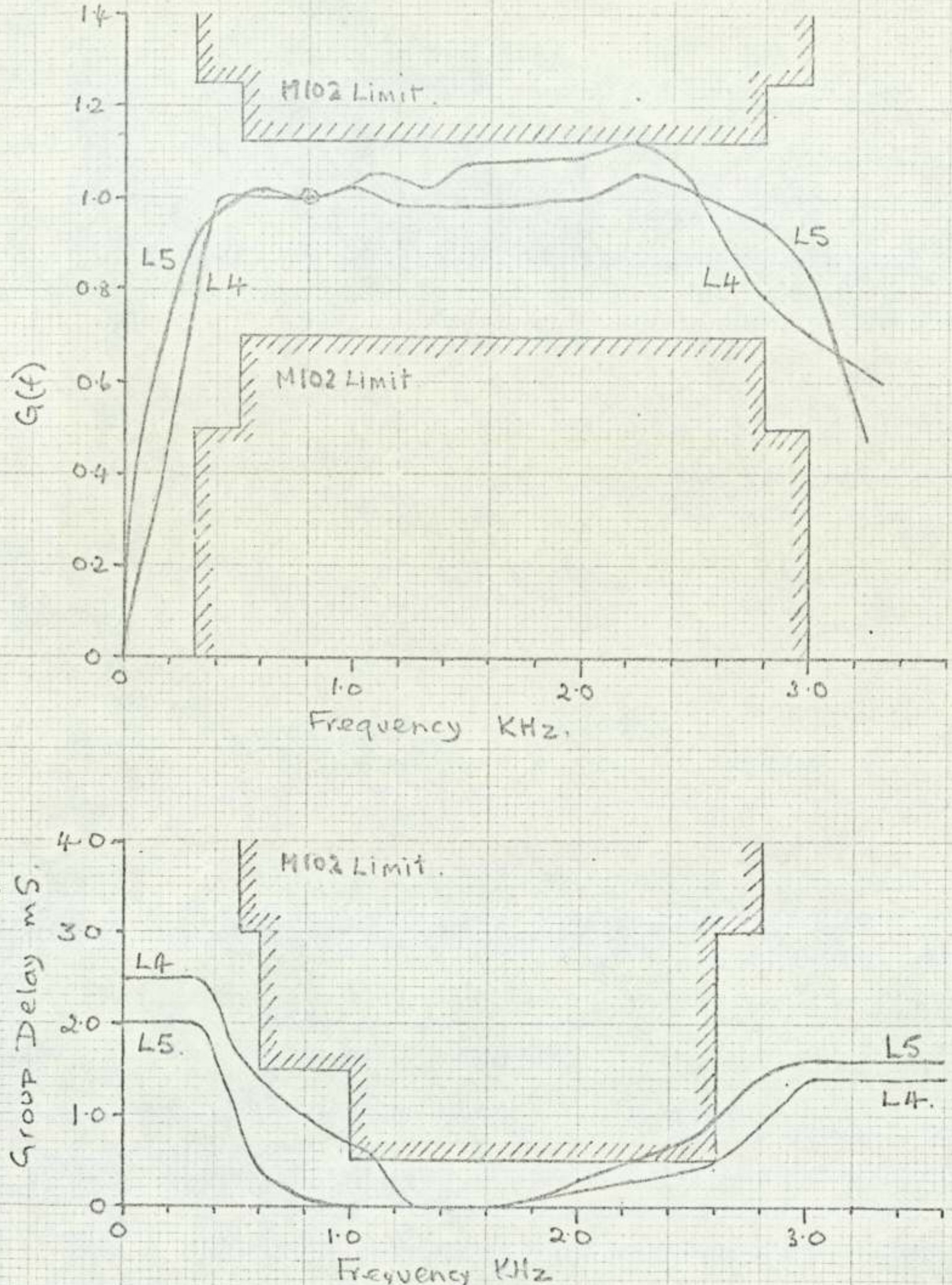


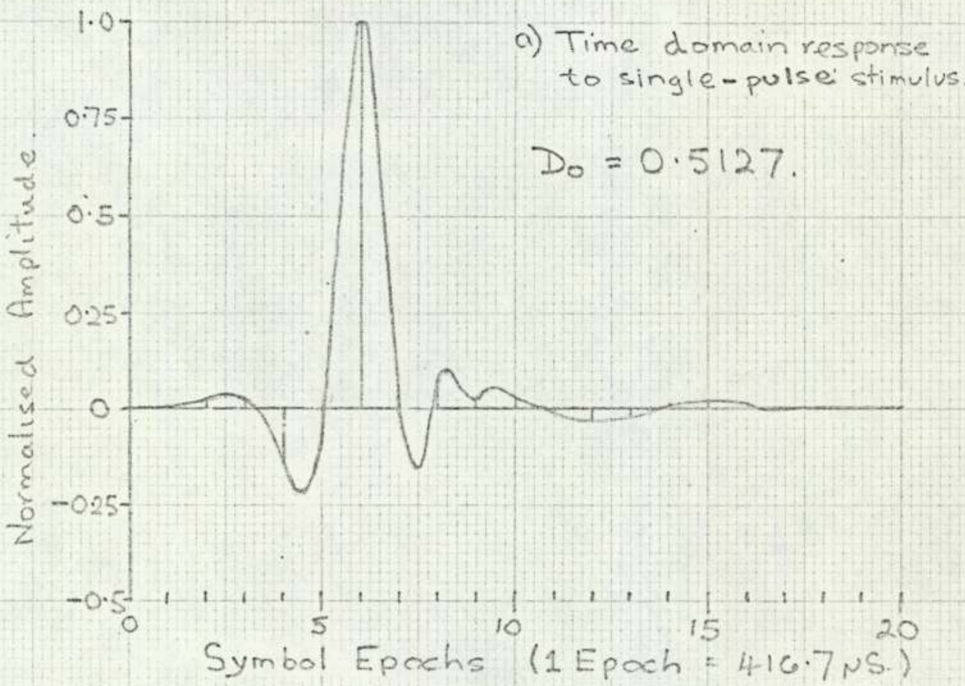
Fig A.5. Actual Characteristics of Special Quality Data Circuits to C.C.I.T.T recommendation M102. (Lines L4 & L5)

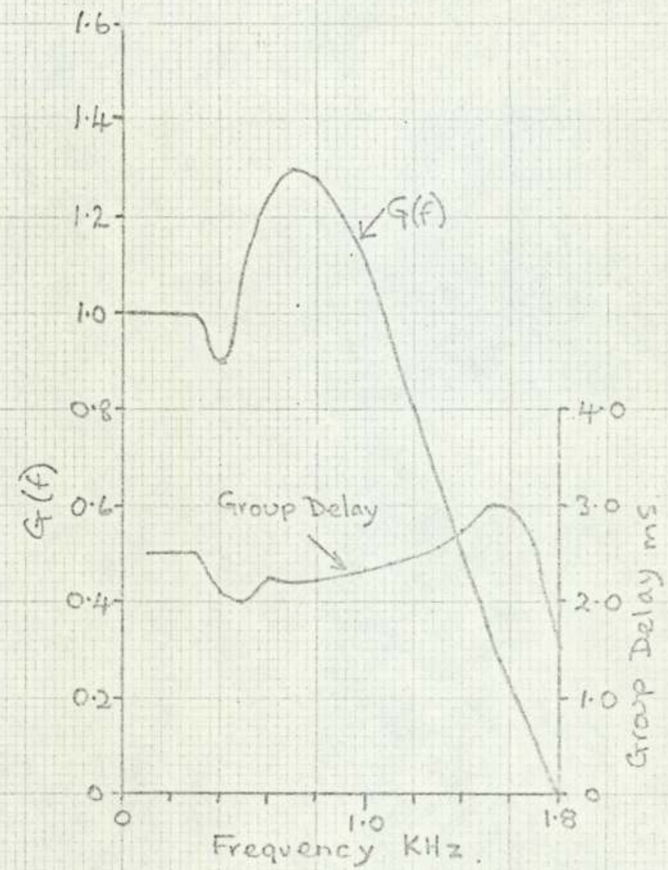
BM-L1021 comprised one carrier channel on coaxial line system from Faraday House to Birmingham. Total distance 123 miles.

The estimated characteristics were used in the initial experiments because actual characteristics did not become available until later in the work. Subsequent work shows that the estimates were, in fact, reasonable representations of M102 quality lines such as are likely to be encountered in practice.

The equivalent base-band characteristics given in figs.A.6 to A.16 were computed for various modem configurations associated with each line characteristic. Details of the line and modem parameters are given with each characteristic, as well as an appropriate designatory code. The first two characters of the designatory code indicate the line characteristic used and the third character the modem parameters as indicated in the following table.

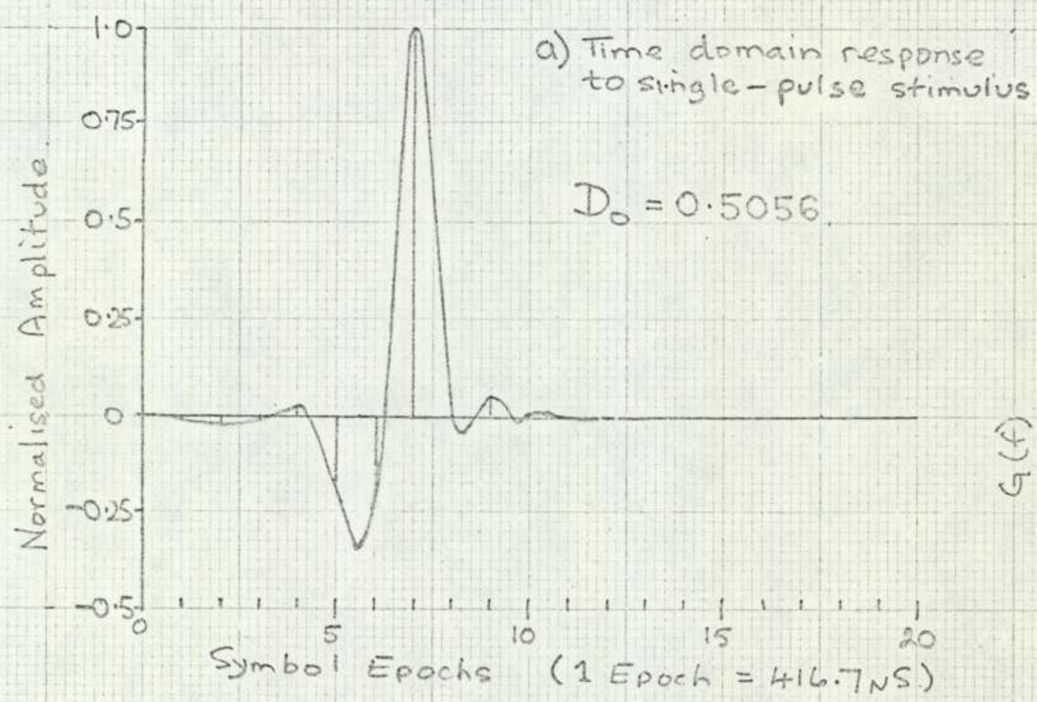
Parameter Designatory Code.	Symbol Rate. Symbols/s.	Carrier Frequency. Hz.	Shaping Filter Roll-Off.%	Vsb.Filter Roll-Off. Hz.
A	2400	2400	50	600
B	2400	2200	25	200
C	3200	2600	25	300
D	3200	2800	25	200
E	3600	2700	50	300
F	3600	2550	25	300
G	2400	2600	50	300
H	2400	2600	25	300
J	As A, but filter characteristics are actual measured values.			
K	2400	Estimated effect of echoes on base-band characteristic.		

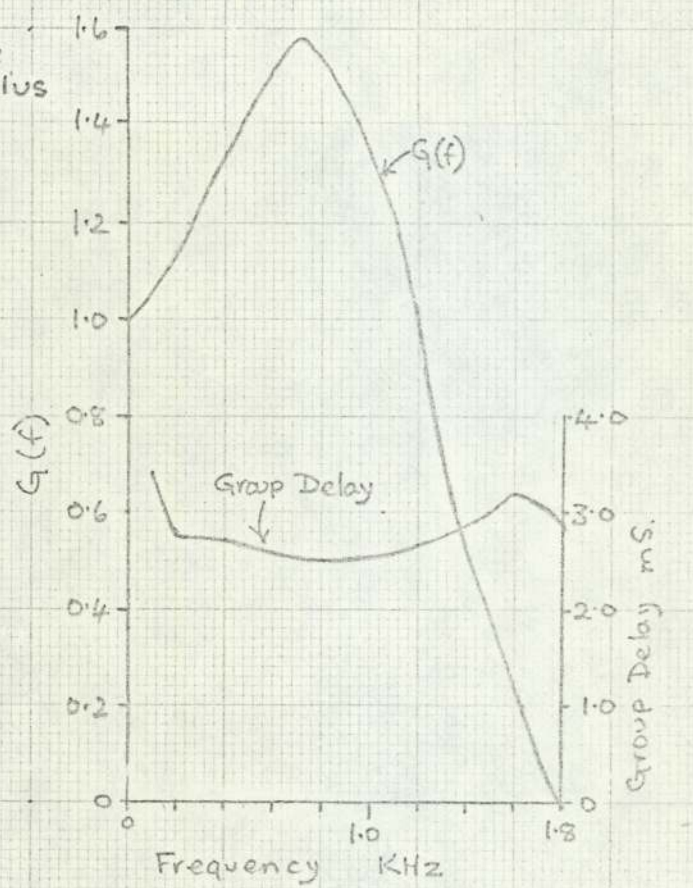




Line:	W1
Symbol Rate:	2,400 Bauds.
V.S.B Carrier:	2,400 Hz.
V.S.B Filter:	600 Hz roll-off.
Shaping Filter:	50% raised cosine roll-off.

Fig A.6.
Equivalent Base-Band Characteristic W1A.

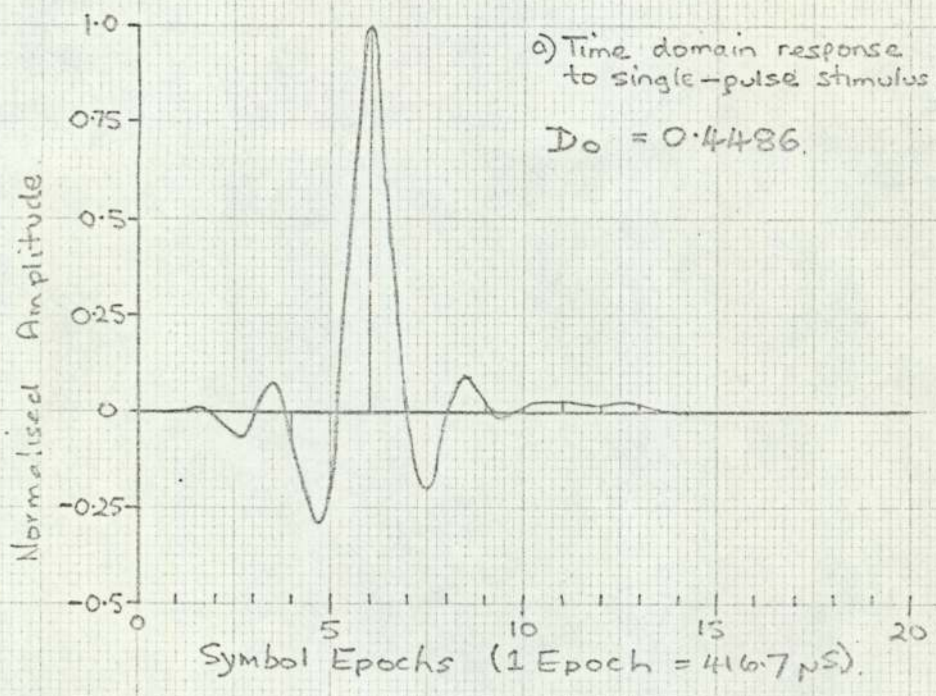


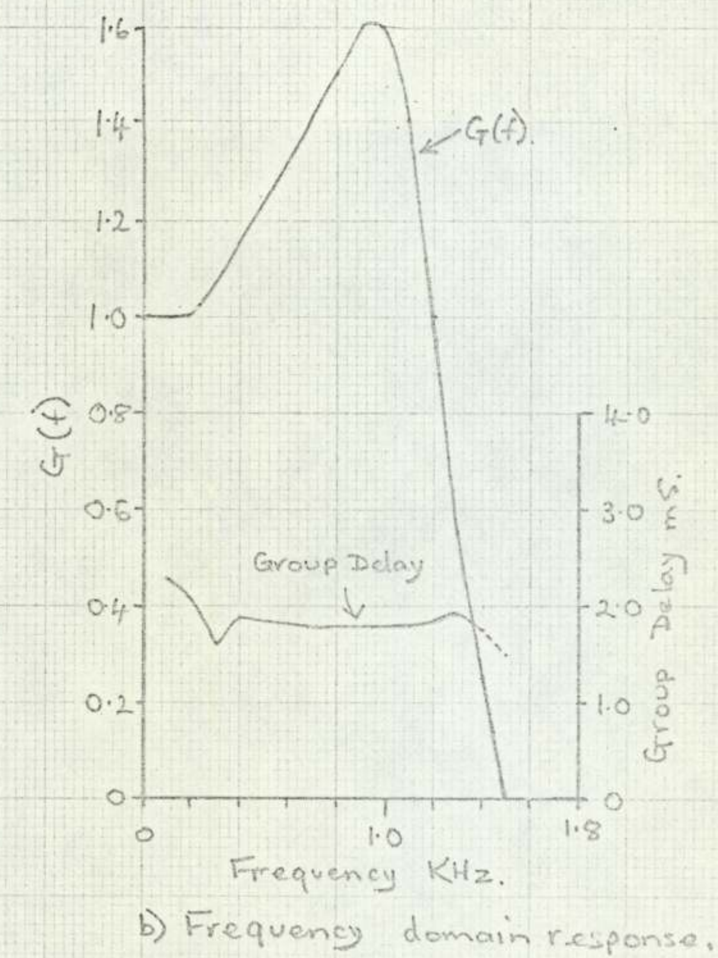


b) Frequency domain response.

Line	W1
Symbol Rate:	2,400 Bauds.
V.S.B. Carrier:	2,600 Hz.
V.S.B. Filter:	300Hz. roll-off.
Shaping Filter:	50% raised cosine roll-off

Fig. A.7.
Equivalent Base-Band Characteristic W1 G.

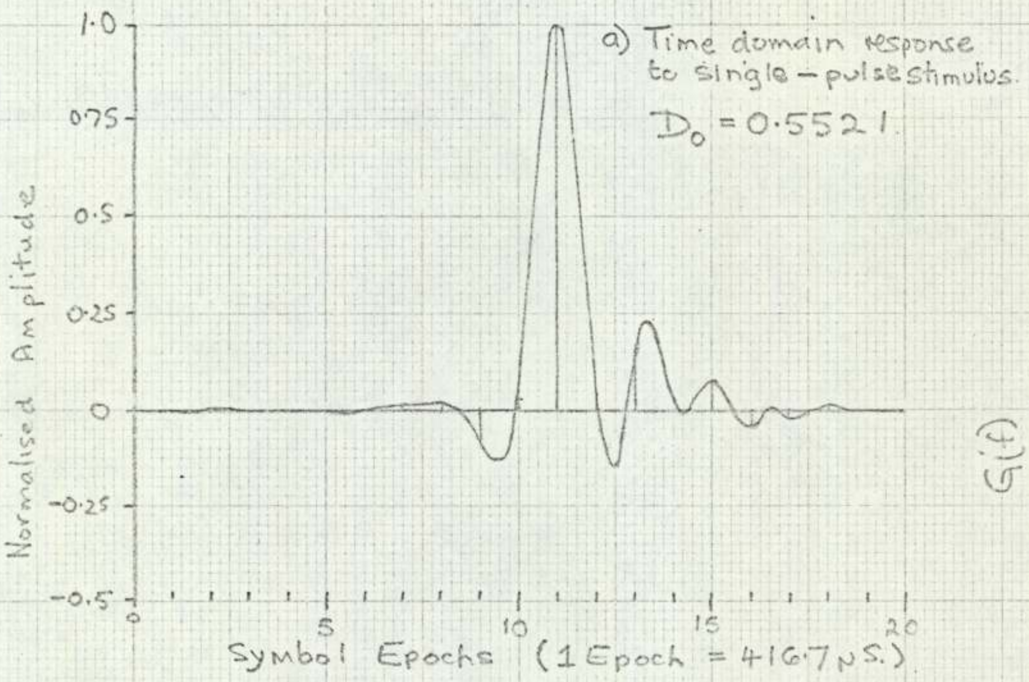


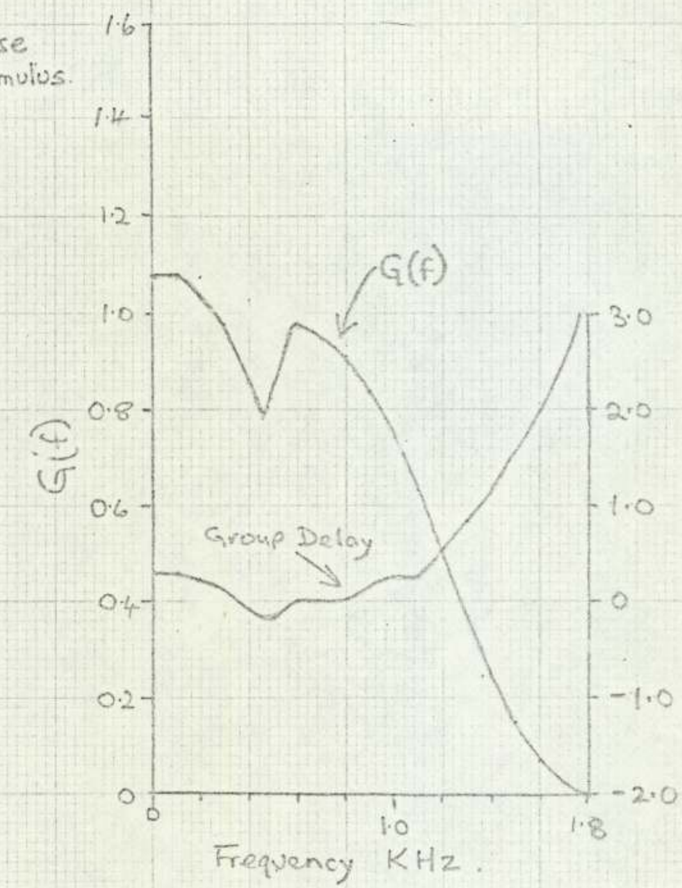


Line:	W 2
Symbol Rate:	2,400 Bauds.
V.S.B. Carrier:	2,600 Hz.
V.S.B. Filter:	300 Hz roll-off.
Shaping Filter:	25% raised cosine rolloff

Fig. A.8.

Equivalent Base-Band Characteristic W2H.





b) Frequency domain response

Line:	L1
Symbol Rate:	2400 Bauds.
V.S.B Carrier:	2400 Hz.
VSB Filter:	600 Hz roll-off
Shaping Filter:	50% Raised-Cosine roll-off.

Fig A.9.
Equivalent Base-Band Characteristic L1A.

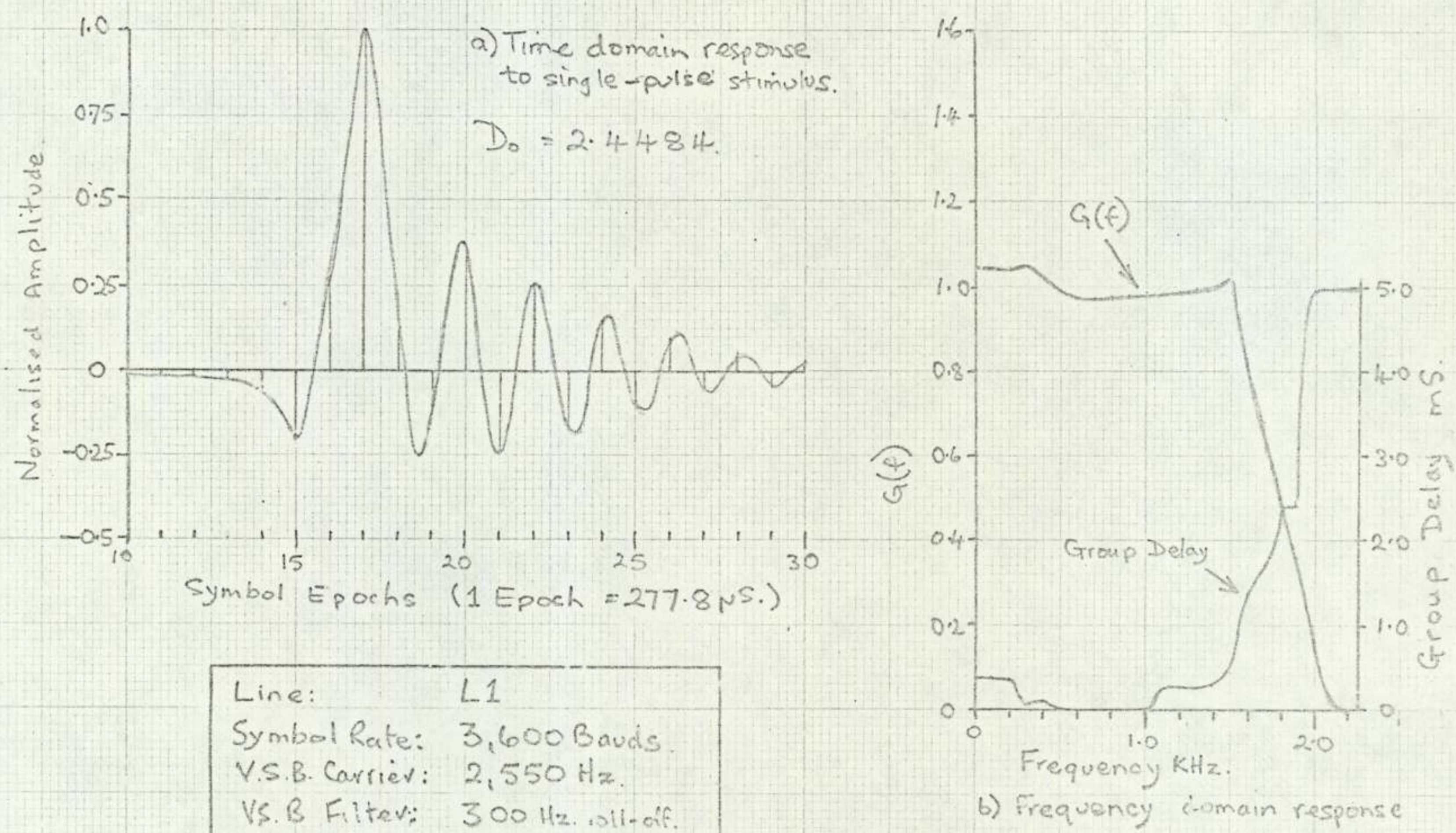
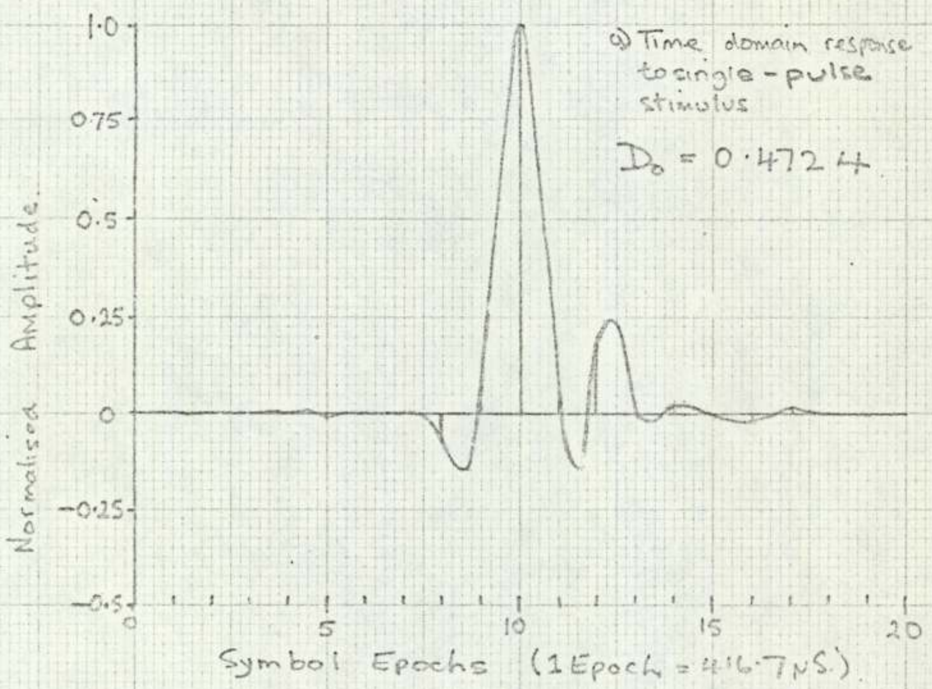
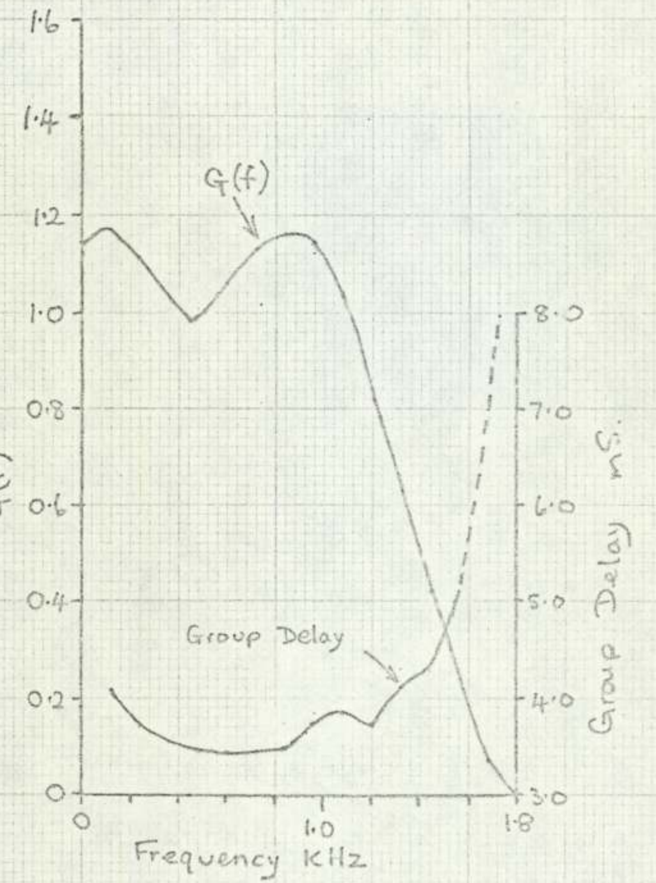


Fig. A.10.

Equivalent Base-Band Characteristic L1F.





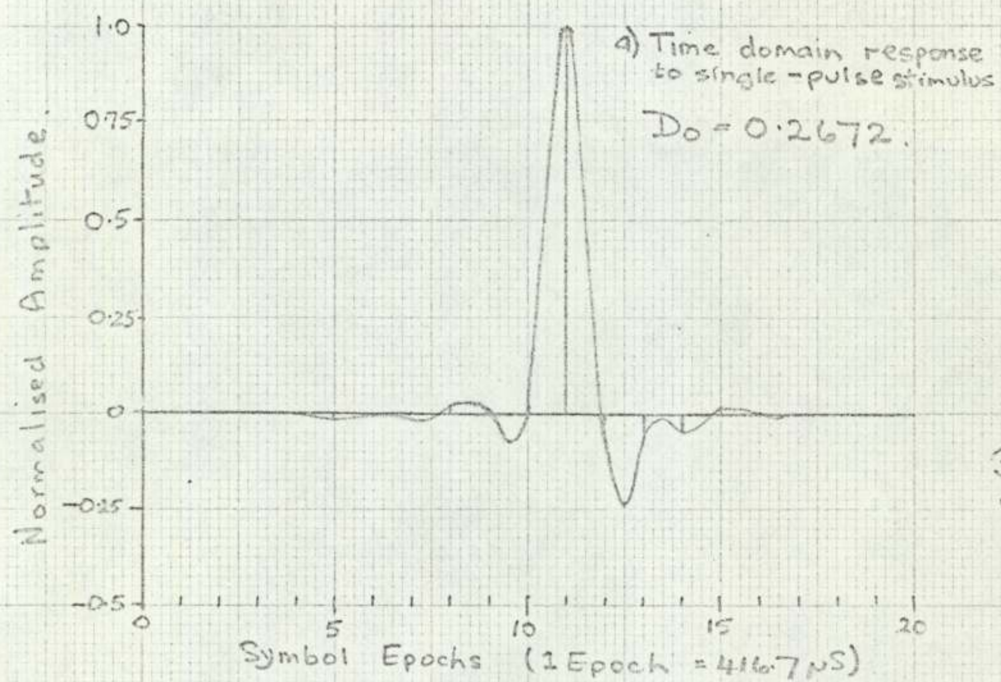
b) Frequency domain response.

Line:	L 1
Symbol Rate:	2400 Bauds.
VSB Carrier:	2400 Hz.
VSB Filter:	600 Hz roll-off
Shaping Filter:	50% Raised Cosine roll-off

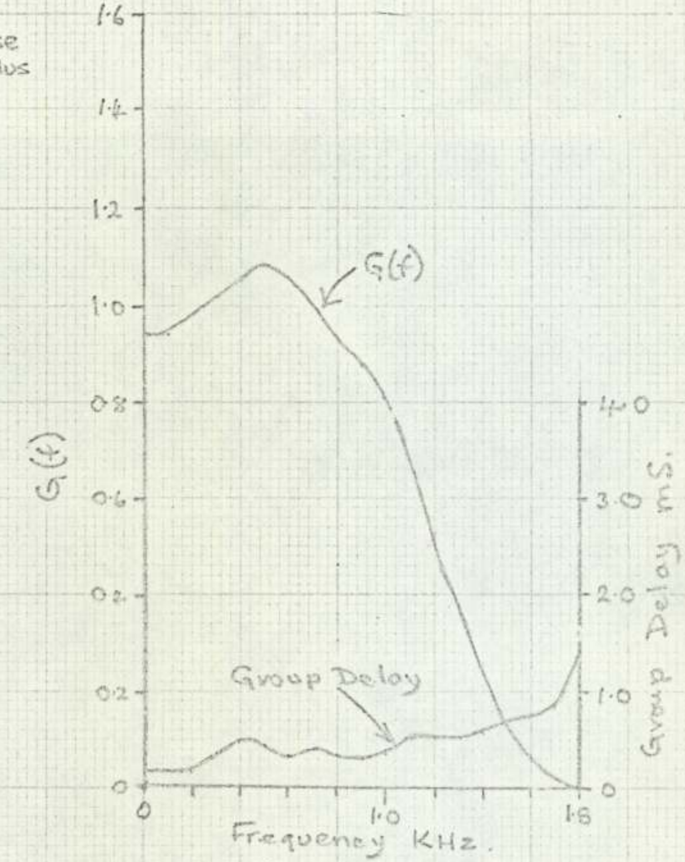
N.B. Filter Characteristics are actual measured values (c.f. with L1A)

Fig A.11.

Equivalent Base-Band Characteristic L1J.

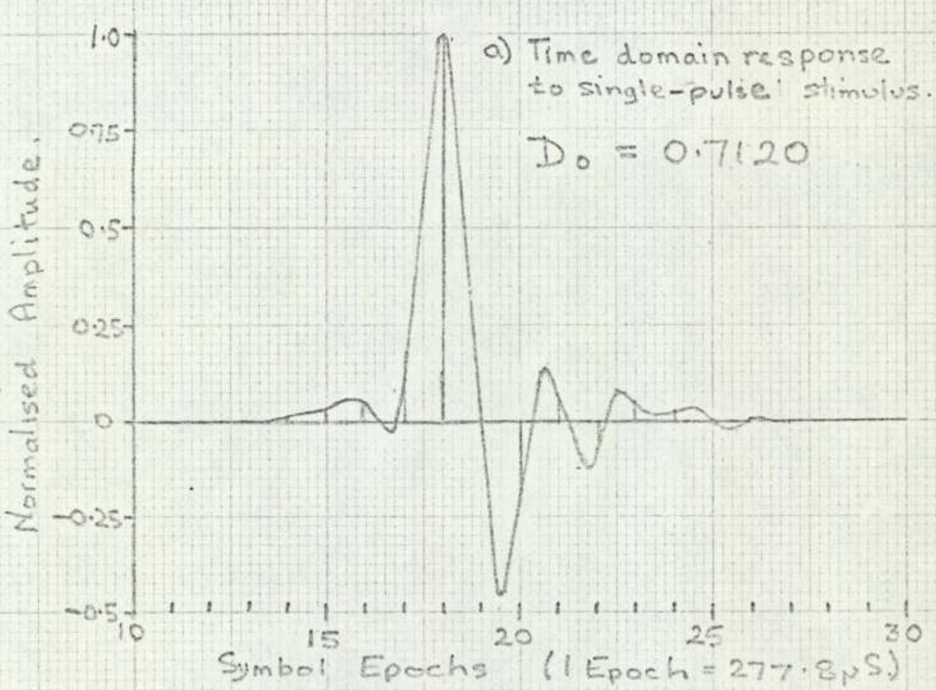


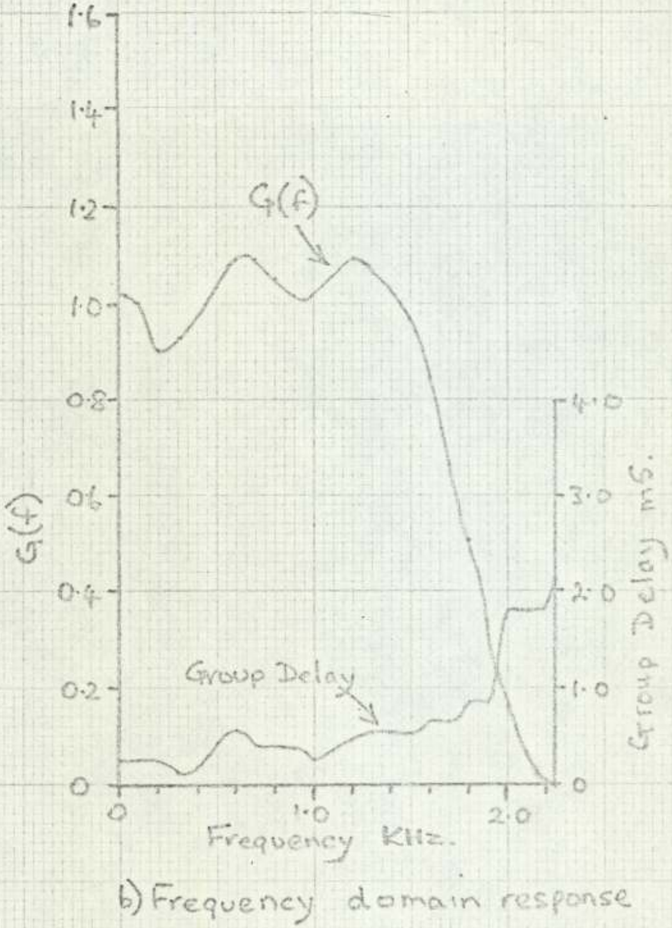
Line:	L2
Symbol Rate:	2400 Bauds
VSB Carrier:	2400 Hz.
VSB Filter:	600 Hz. roll-off
Shaping Filter:	50% raised cosine roll-off



b) Frequency domain response.

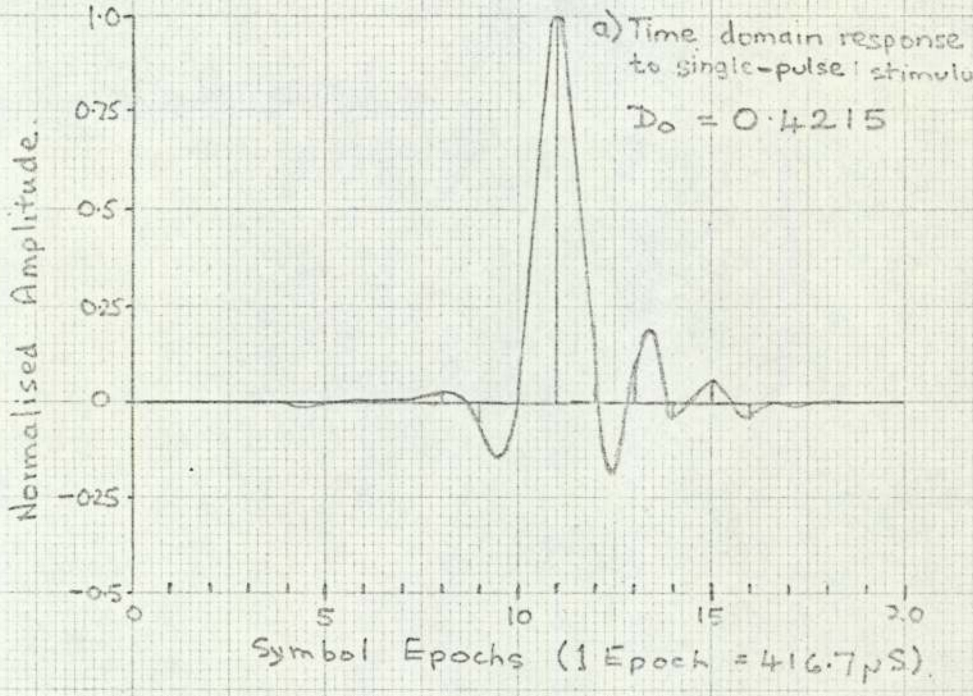
Fig A.12.
Equivalent Base-Band Characteristic L2A.

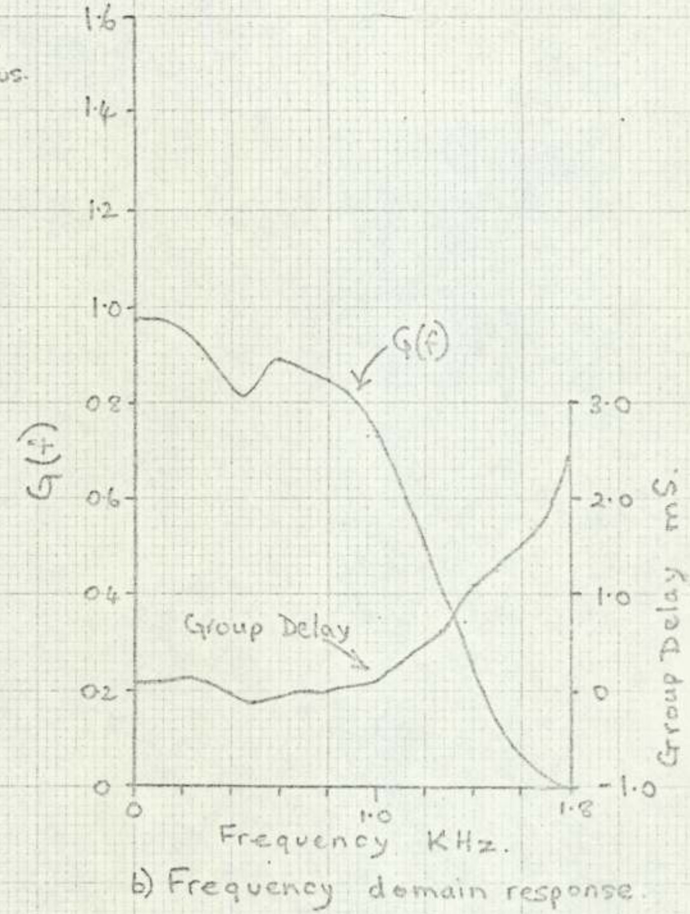




Line:	L2.
Symbol Rate:	3600 Bauds.
VSB Carrier:	2550 Hz.
VSB Filter:	300 Hz. roll-off
Shaping Filter:	25% raised cosine roll-off

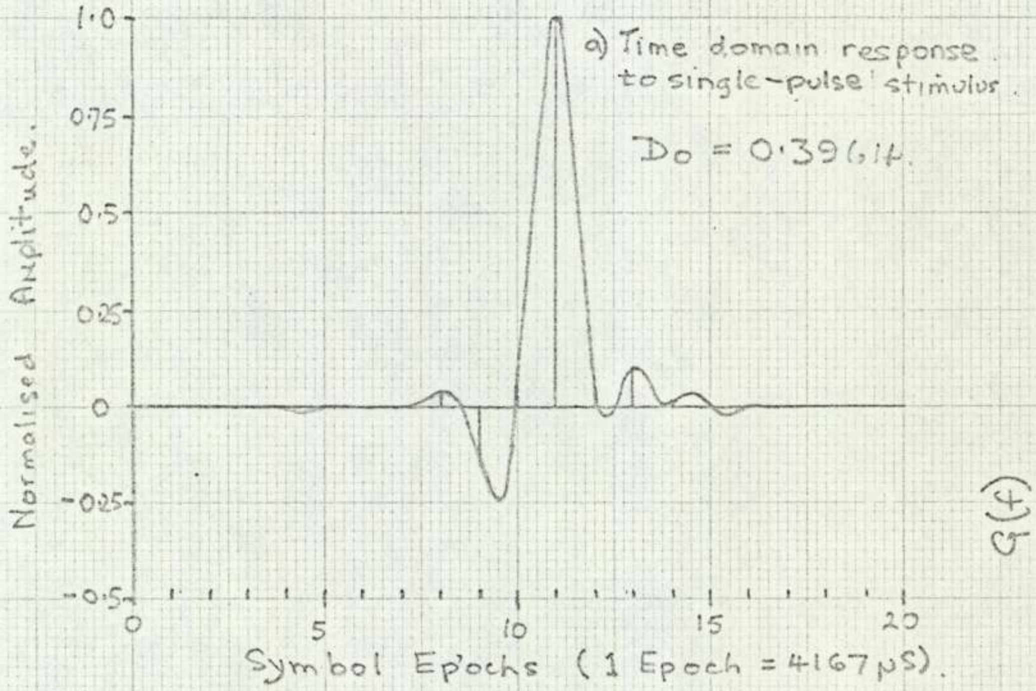
Fig. A13.
Equivalent Base-Band Characteristic L2F.

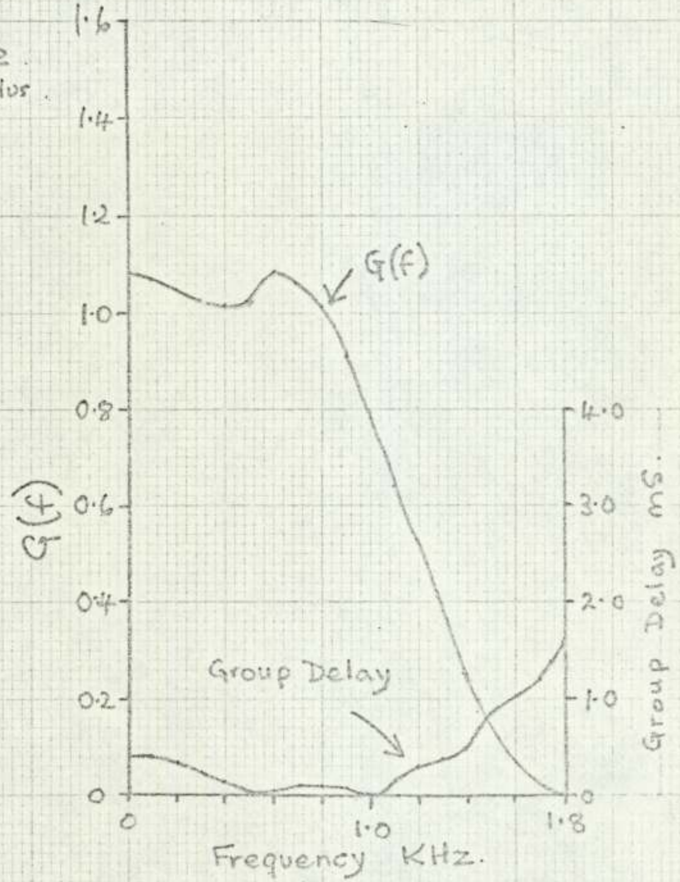




Line:	L3
Symbol Rate:	2400 Bauds.
V.S.B. Carrier:	2400 Hz.
V.S.B Filter:	600Hz roll-off
Shaping Filter:	50% raised cosine roll-off

Fig. A.14.
Equivalent Base-Band Characteristic L3A.





b) Frequency domain response.

Line:	L4.
Symbol Rate:	2400 Bauds
VSB Carrier:	2400 Hz
VSB Filter:	600Hz. roll-off.
Shaping Filter:	50% raised cosine roll-off

Fig. A.15.

Equivalent Base-Band Characteristic L4A.

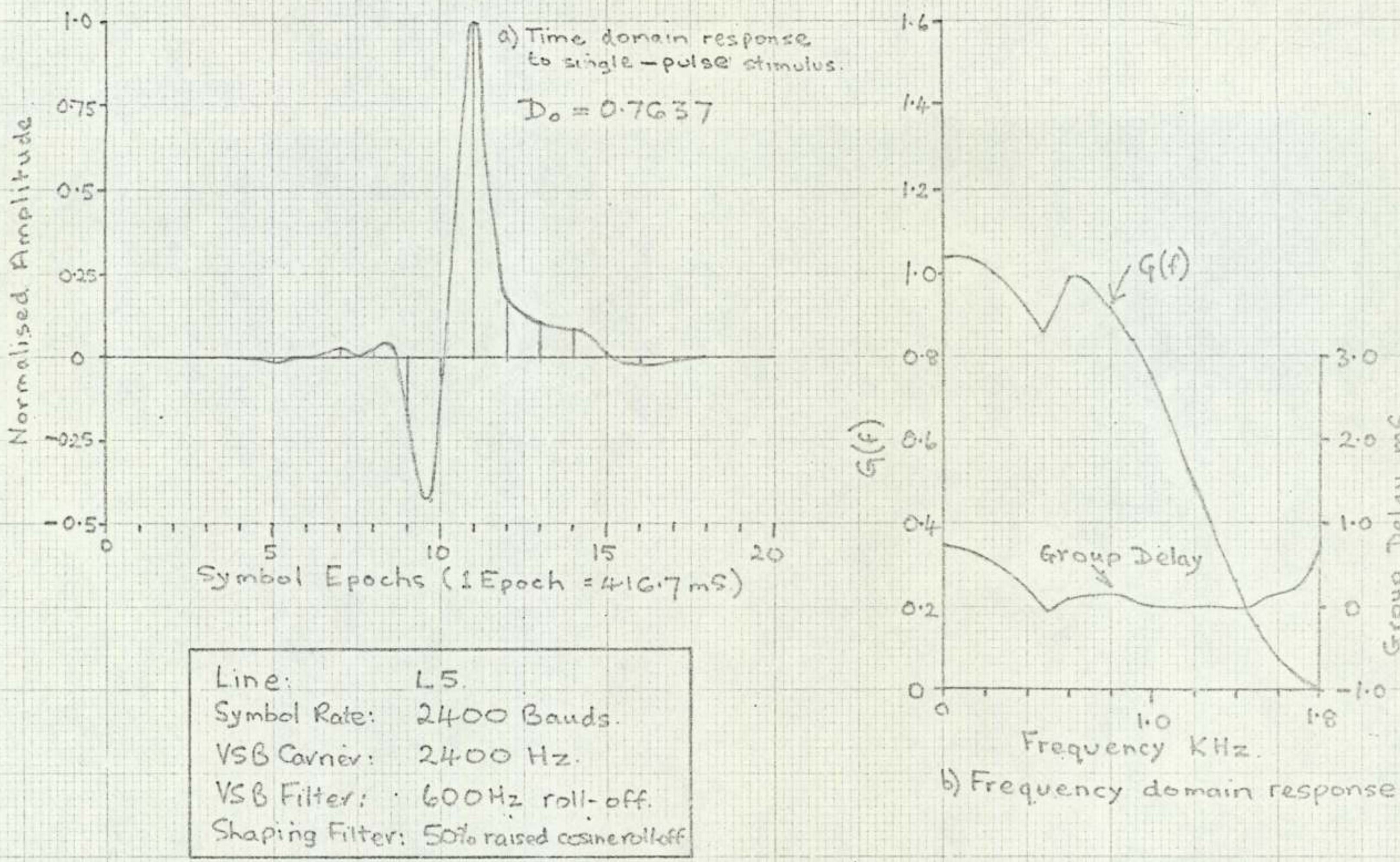
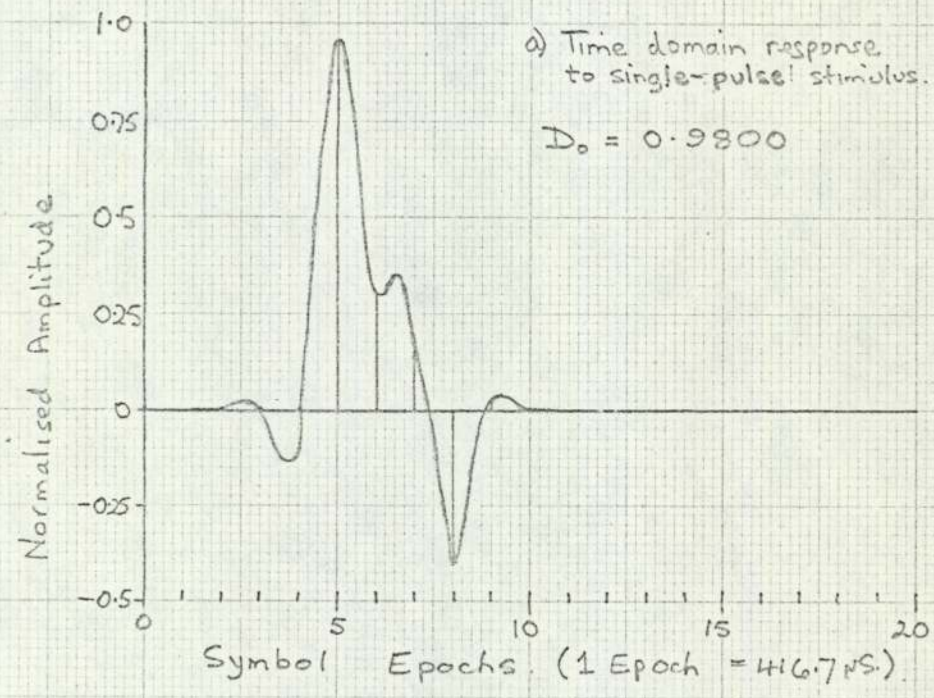


Fig A.16.

Equivalent Base-Band Characteristic L5A.

The equivalent base-band characteristic R1K, given in fig.A.17, is an estimated characteristic associated with a channel, such as an h.f. radio voice channel, which has significant echoes in the channel single-pulse response. The characteristics X1 and X2, given in figs. A18 and A19, are time-domain responses which were arbitarily chosen to have unequalised intersymbol interferences D_0 greater than unity. The computed equivalent frequency characteristics for these responses are also given.

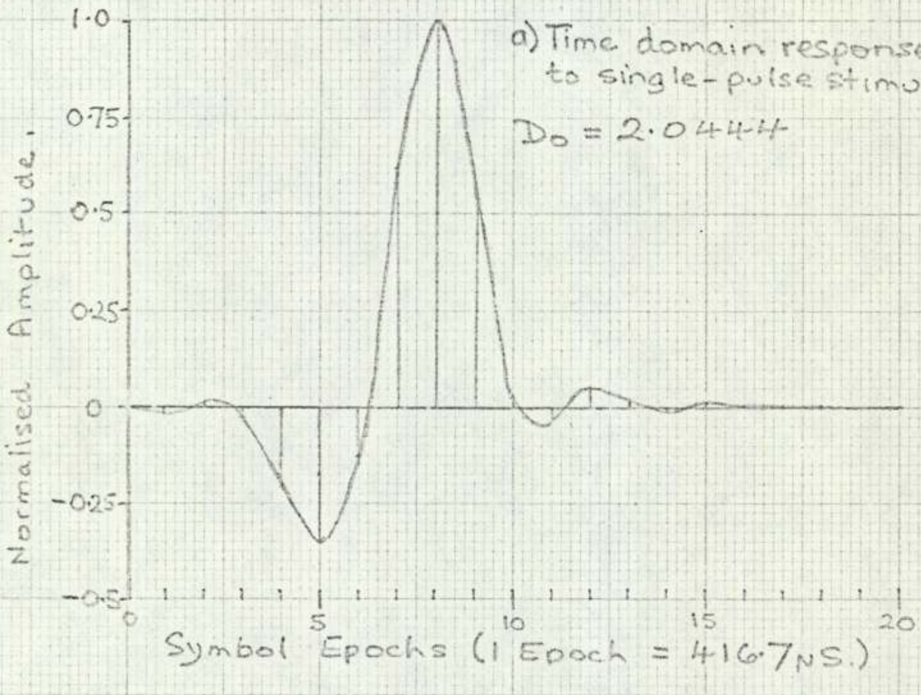
Some equivalent base-band channels are referred to in table 2.1 and in the computations detailed in section 5.3 for which full equivalent base-band characteristics have not been computed. Details of these characteristics do not therefore appear in this appendix.



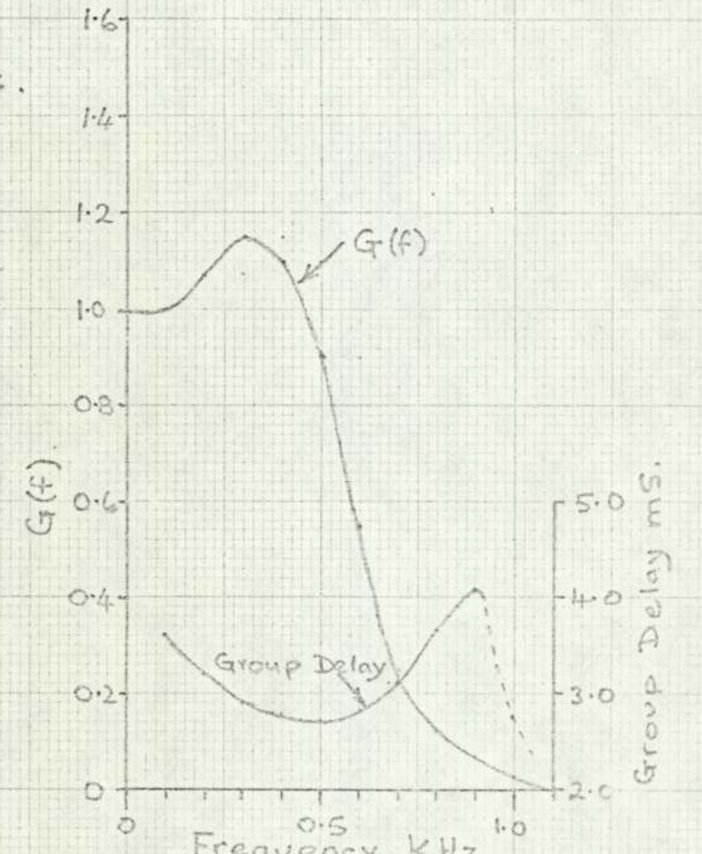
Estimated effect of echoes as might be encountered on h.f. or tropposcatter radio channels. Symbol Rate = 2400 Hz.

Fig. A 17.

Equivalent Base-Band Characteristic R1K.

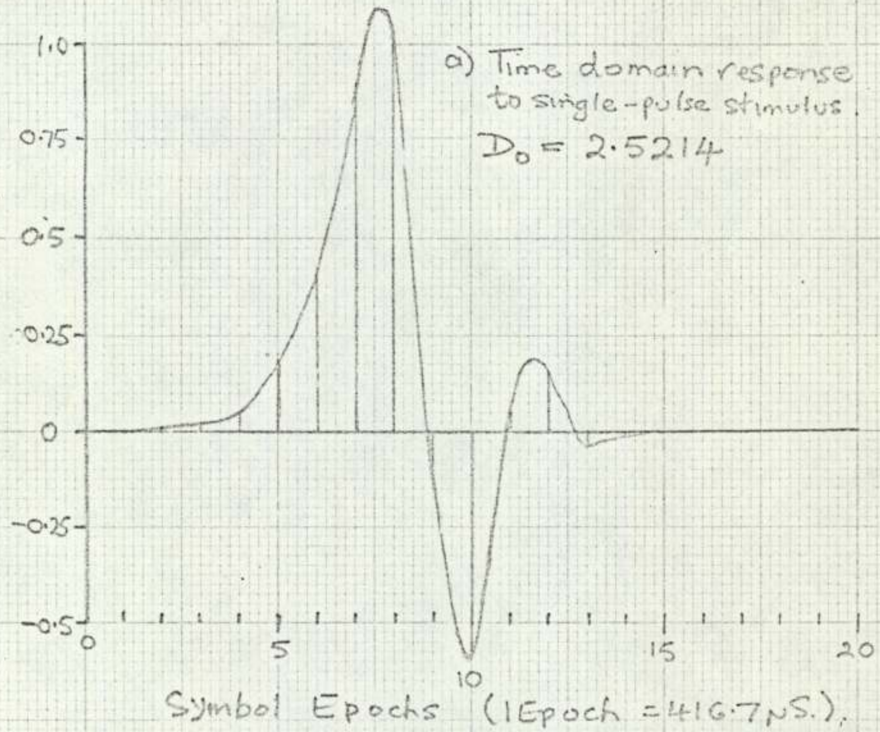


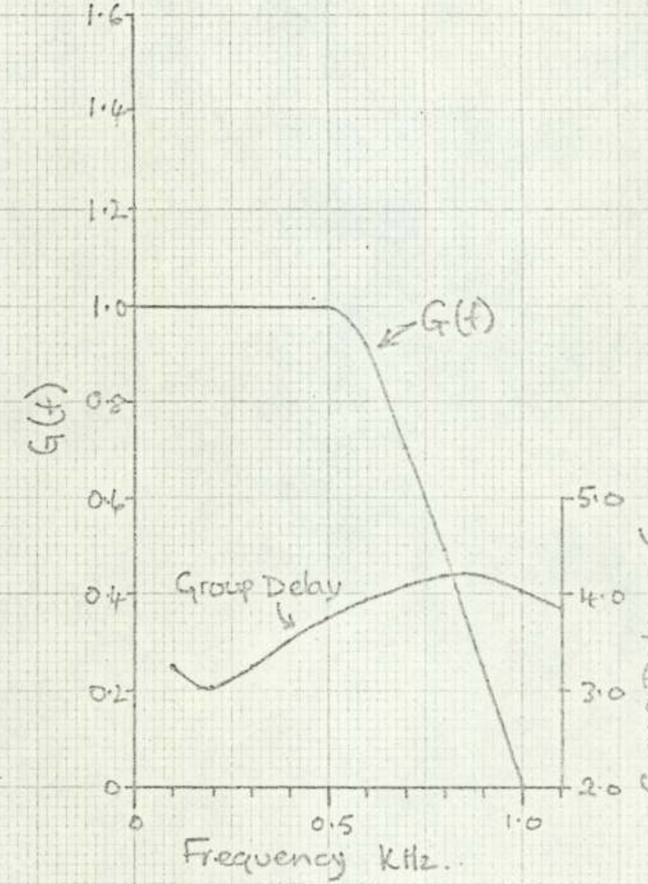
Arbitarily selected time-domain response to have $D_o > 1.0$.



b) Frequency domain response.

Fig. A.18.
Equivalent Base-Band Characteristic X1.





b) Frequency domain response.

Arbitarily Selected
time-domain response
to have $D_0 > 1.0$.

Fig. A.19.

Equivalent Base-Band Characteristic X2.

APPENDIX 2

LISTING OF COMPUTER PROGRAM TO COMPUTE D_{min}.

This program computes the minimum valves of intersymbol interference (D_{min}) that can be obtained for a given channel response, using various numbers of equaliser delay modules, and with various dispositions of these modules about the reference tap position. The program operates by solving the set of simultaneous equations (3.5) to obtain the optimum values of the multiplier coefficient settings. From these coefficient values the residual intersymbol interference (D_{min}) remaining after equalisation is then computed. The method of operation limits the use of the program to channels having an unequalised intersymbol interference $Do < 1$.

The program is written in FORTRAN IV language and the program card deck is compatible with the IBM/360 operating system.

```
      DIMENSION FT(1500),GT(180),COEFF(1600),VECTOR(40)
      READ (1,10) MTAP,NTAP,NSYM,NPK,NMAXI
   10 FORMAT (5I5)
      READ (1,12) NS
   12 FORMAT(I5)
      N=2*NS
      READ (1,11)(FT(I),I=1,N)
   11 FORMAT (8F10.5)
      NGT=NSYM+1
      DO 80 IN=1,5
      LIM=(4+IN)*2
      DO90 IL=1,LIM
      MT=IL
      NT=LIM-IL
      WRITE (3,1010) MT,NT
 1010 FORMAT ('1MT=',I5,'  NT=',I5)
      NCONS=MT+NT+1
      DO 110 I=1,40
  110 VECTOR(I)=0.0
      NC=MT+1
      VECTOR(NC)=1.0
      DO 111 I=1,180
  111 GT(I)=0.0
      DO 120 I=1,NSYM
      JA=NCONS+I
      JB=20*I-20+NMAXI*2
  120 GT(JA)=FT(JB)
      DO 20 J=1,NCONS
      DO 30 I=1,NCONS
      K=I-J+NPK+NCONS
      IA=I+(J-1)*NCONS
   30 COEFF(IA)  = GT(K)
   20 CONTINUE
      CALL SIMQ(COEFF,VECTOR,NCONS,KS)
      CALL XUOFF
      WRITE (3,1001) KS
 1001 FORMAT ('0KS=',I5)
      WRITE (3,1002)
 1002 FORMAT ('0TAP COEFFICIENTS')
      DO 60 I=1,NCONS
      WRITE (3,1003) VECTOR(I)
 1003 FORMAT (F10.4)
   60 CONTINUE
      SUM=0.0
      NYT=NGT+NCONS
      DO 21 I=1,NYT
      YT=0
      DO 31 J=1,NCONS
      K= NCONS-J+1
      L=I+J-1
   31 YT=YT+VECTOR(K)*GT(L)
   21 SUM=SUM+ABS(YT)
      WRITE (3,1004) SUM
 1004 FORMAT ('0SUM=',F10.4)
   90 CONTINUE
   80 CONTINUE
      STOP
      END
```

APPENDIX 3

LISTING OF AUTOMATIC PRE-SET EQUALISER SIMULATION PROGRAM.

This program simulates the operation of the automatic pre-set transversal equaliser. Details of the method of operation are given in section 6.5.

The program is written in FORTRAN IV language and the program card deck is compatible with the IBM/360 operating system.

```
C     EQUALISER SIMULATION PROGRAM
C
      DIMENSION FT(4001),C(22),SGN(22),SGNYT(201),NDELTA(9),STEP(2)
      READ (1,2000) MODEL,NRUNS,NDELTA,NSTEP,STEP
 2000 FORMAT (12I5,2F10.5)
C     MODEL 1 IS LUCKY1,2 IS LUCKY2,3 IS TURNER
      READ(1,1001) MTAP,NTAP,NSYM,NPK,NPTS,NREF,NFORM
 1001 FORMAT (7I5)
C     MTAP IS NUMBER OF TAPS BEFORE REF,NTAP IS NUMBER AFTER.
C     NSYM IS NUMBER OF COMPLETE SYMBOLS INPUT MAX NSYM IS39,NPK IS
C     NUMBER OF SYMBOL WITH PEAK,   NPTS IS THE NUMBER OF POINTS PER
C     SYMBOL AND MUST BE A FACTOR OF 100. NREF IS REFERENCE SAMPLE.
      DO 10 I=1,4001
   10 FT(I)=0.0
      NINC=100/NPTS
      MQ=(NPTS-NREF+1)*NINC+1
      LIM=(100*NSYM)+MQ
      IF(NFORM-1) 950,950,951
  950 READ (1,1000)(FT(I),I=MQ,LIM,NINC)
 1000 FORMAT (8F10.5)
      GO TO 952
  951 READ(1,1100)(FT(I),I=MQ,LIM,NINC)
 1100 FORMAT(10X,F10.5,10X,F10.5,10X,F10.5,10X,F10.5)
  952 LIMX=LIM-NINC
      DO 11 I=MQ,LIMX,NINC
      K=I+100/NPTS
      DIF=(FT(I)-FT(K))/NINC
      JL=NINC-1
      DO 12 J=1,JL
      L=I+J
   12 FT(L)=FT(I)-DIF*J
   11 CONTINUE
      DO 242 IDELTA=1,NRUNS
      NDEL=NDELTA(IDELTA)
      DO 240 ICH=1,NSTEP
      WRITE (7,2001) MODEL,MTAP,NTAP
 2001 FORMAT ('OMODEL ',I2,5X,'MTAP=',I2,5X,'NTAP=',I2)
      WRITE (7,2002) NDEL,STEP(ICH)
 2002 FORMAT('ONDEL=',I4,5X,'TAP INCREMENT=',F10.5)
      NCONS = MTAP+NTAP+1
      DO 35 I=1,NCONS
   35 C(I)=0.0
      C(MTAP+1)=1.0
      N=0
      KOUNT=0
      IF(NCONS-11) 41,41,42
   41 WRITE (7,1010) (I,I=1,NCONS)
 1010 FORMAT ('1   N',9X,'SUM',3X,11(4X,'C(',I2,')'))
      GO TO 40
   42 WRITE (7,1010) (I,I=1,11)
      WRITE (7,1020) (I,I=12,NCONS)
 1020 FORMAT (' ',19X,11(4X,'C(',I2,')'))
      SUMA = 0
      SUMB = 0
```

```
      SUMC = 0
   40 SUM=0
      DO 114 L=1,100
      KA=0
      YT=0
      DO 110 I=1,NCONS
      K=100*(NPK+MTAP+1+L-I)-(I-MTAP-1)*NDEL+1
      IF (K) 120,120,121
  120 FTX=0
      GO TO 110
  121 IF(K-4001) 123,123,120
  123 FTX=FT(K)
  110 YT=YT+C(I)*FTX
      IF (YT) 111,112,111
  112 KA=KA+1
      IF (KA-3) 114,113,113
  111 KA=0
      IF(MODEL-2) 901,902,902
  901 IF(L-NTAP) 130,130,114
  130 JB=MTAP+L+1
      SGN(JB)=SIGN(1.0,YT)
      GO TO 114
  902 JB=101+L
      SGNYT(JB)=SIGN(1.0,YT)
  114 SUM=SUM+ABS(YT)
  113 DO 115 J=1,100
      KA=0
      YT=0
      DO 116 I=1,NCONS
      K=100*(NPK+MTAP+1-J-I)-(I-MTAP-1)*NDEL+1
      IF (K) 122,122,124
  122 FTX=0
      GO TO 116
  124 IF(K-4001) 125,125,122
  125 FTX=FT(K)
  116 YT=YT+C(I)*FTX
      IF (YT) 117,118,117
  118 KA=KA+1
      IF (KA-3) 115,119,119
  117 KA=0
      IF(MODEL-2) 903,904,904
  903 IF(J-MTAP) 131,131,115
  131 JC=MTAP-J+1
      SGN(JC)=SIGN(1.0,YT)
      GO TO 115
  904 JC=101-J
      SGNYT(JC)=SIGN(1.0,YT)
  115 SUM=SUM+ABS(YT)
      IF(N-200) 119,70,70
   70 WRITE(7,71)
   71 FORMAT ('0 N EXCEEDS 200')
      GO TO 240
  119 N=N+1
      IF (NCONS-11) 141,141,142
  141 WRITE (7,1011) N,SUM,(C(I),I=1,NCONS)
```

```
 1011 FORMAT (I5,F15.5,11F9.4)
      GO TO 143
  142 WRITE (7,1011) N,SUM,(C(I),I=1,11)
      WRITE (7,1012) (C(I),I=12,NCONS)
 1012 FORMAT(20X,11F9.4)
  143 C(MTAP+1)=1.0
      DO 200 I=1,MTAP
      IF(MODEL-2) 907,906,906
  906 SIGMA=0.0
      DO 526 J=JC,JB
      IF (J-101)527,526,527
  527 IF(MODEL-2) 908,909,910
  909 JD=100*(NPK+MTAP-I+J-100)+1
      GO TO 912
  910 JD=100*(NPK+MTAP-I+J-100)+1-(I-MTAP-1)*NDEL
  912 IF (JD) 513,513,512
  513 FTD=0
      GO TO 515
  512 IF(JD-4001) 514,514,513
  514 FTD=FT(JD)
  515 JE=100*(NPK-101+J)+1
      IF (JE) 518,518,517
  518 FTE=0
      GO TO 520
  517 IF(JE-4001) 519,519,518
  519 FTE=FT(JE)
  520 IF (MODEL-2) 908,913,914
  913 JF=100*(NPK+MTAP+1-I)+1
      GO TO 915
  914 JF=100*(NPK+MTAP+1-I)+1-(I-MTAP-1)*NDEL
  915 IF (JF) 523,523,522
  523 FTF=0
      GO TO 525
  522 IF(JF-4001) 524,524,523
  524 FTF=FT(JF)
  525 SIGMA=SIGMA+(FTD-FTE*FTF)*SGNYT(J)
  526 CONTINUE
      SGN(I)=SIGN(1.0,SIGMA)
  907 C(I)=C(I)-STEP(ICH)*SGN(I)
  300 K=100*(NPK+MTAP+1-I)-(I-MTAP-1)*NDEL+1
      IF (K) 322,322,324
  322 FTX=0
      GO TO 200
  324 IF(K-4001) 325,325,322
  325 FTX=FT(K)
  200 C(MTAP+1)=C(MTAP+1)-C(I)*FTX
      MA=2+MTAP
      DO 203 I=MA,NCONS
      IF(MODEL-2) 917,916,916
  916 SIGMA=0.0
      DO 546 J=JC,JB
      IF (J-101)547,546,547
  547 IF(MODEL-2) 908,919,920
  919 JD=100*(NPK+MTAP-I+J-100)+1
      GO TO 922
```

```
  920 JD=100*(NPK+MTAP-I+J-100)+1-(I-MTAP-1)*NDEL
  922 IF (JD) 533,533,532
  533 FTD=0
      GO TO 535
  532 IF(JD-4001) 534,534,533
  534 FTD=FT(JD)
  535 JE=100*(NPK-101+J)+1
      IF (JE) 538,538,537
  538 FTE=0
      GO TO 540
  537 IF(JE-4001) 539,539,538
  539 FTE=FT(JE)
  540 IF (MODEL-2) 908,923,924
  923 JF=100*(NPK+MTAP+1-I)+1
      GO TO 925
  924 JF=100*(NPK+MTAP+1-I)+1-(I-MTAP-1)*NDEL
  925 IF (JF) 543,543,542
  543 FTF=0
      GO TO 545
  542 IF(JF-4001) 544,544,543
  544 FTF=FT(JF)
  545 SIGMA=SIGMA+(FTD-FTE*FTF)*SGNYT(J)
  546 CONTINUE
      SGN(I)=SIGN(1.0,SIGMA)
  917 C(I)=C(I)-STEP(ICH)*SGN(I)
  303 K=100*(NPK+MTAP+1-I)-(I-MTAP-1)*NDEL+1
      IF (K) 422,422,424
  422 FTX=0
      GO TO 203
  424 IF(K-3000) 425,425,422
  425 FTX=FT(K)
  203 C(MTAP+1)=C(MTAP+1)-C(I)*FTX
      SUMC = SUMB
      SUMB = SUMA
      SUMA = SUM
      IF(ABS(SUMC - SUM) - .001) 230,230,40
  230 KOUNT=KOUNT+1
      IF (KOUNT-5) 40,40,240
  240 WRITE (3,1013)
 1013 FORMAT ('0 KOUNT COMPLETED')
  242 CONTINUE
      WRITE (3,1014)
 1014 FORMAT ('0 PROBLEM COMPLETED')
      GO TO 911
  908 WRITE(3,2003)
 2003 FORMAT ('0 PROGRAM EXECUTION ERROR')
  911 STOP
      END
```

APPENDIX 4

LISTING OF ADAPTIVE EQUALISER SIMULATION PROGRAM.

This program simulates the operation of the adaptive transversal equaliser. Details of the method of operation are given in section 7.6.

The program is written in FORTRAN IV language and the program card deck is compatible with the IBM/360 operating system.

```
             C       ADAPTIVE EQUALISER SIMULATION PROGRAM                              RBAE   1
             C                                                                          RBAE
 0001                DIMENSION FT(800),VT(40),STEP(4),C(40),XT(40),KT(80),Y(100),ZT(40)RBAE   2
                    *,ISRA(40),ISRB(40),NAV(40),M(4),LA(4)                              RBAE   3
 0002                READ (1,2000) NCONF,MT,NT,NSTRAT,LTHR,NFB,NAVE,NSTEP,STEP          RBAE   4
 0003          2000 FORMAT (8I5,4F10.5)                                                 RBAE   5
 0004                READ(1,1001) MTAP,NTAP,NSYM,NPK,NPTS,NREF,NFORM,NBPS               RBAE  11
 0005          1001 FORMAT (8I5)                                                        RBAE  12
 0006                NLEV=2**NBPS                                                       RBAE 121
 0007                NSL=NLEV-1                                                         RBAE 122
 0008                DO 240 I=1,4                                                       RBAE 123
 0009           240 M(I)=1                                                              RBAE 124
 0010                NCONS=MT+NT+1                                                      R21E   6
 0011                NA=MT+1                                                            RBAE   7
 0012                NB=NA+NSYM                                                         RBAE   8
 0013                NC=MT+2                                                            RBAE  90
 0014                ND=NB+1                                                            RBAE  10
 0015                NE=NPK+MT                                                          RBAE 101
 0016                NG=NE*NBPS                                                         RBAEA101
 0017                NF=MAXO(NB,LTHR,NG)                                                RBAE 102
 0018                DO 10 I=1,400                                                      RBAE  13
 0019            10 FT(I)=0.0                                                           RBAE  14
 0020                LIM=NPTS*NSYM                                                      RBAE  15
 0021                IF(NFORM-1) 950,950,951                                            RBAE  16
 0022           950 READ (1,1000)(FT(I),I=1,LIM)                                        RBAE  17
 0023          1000 FORMAT (8F10.5)                                                     RBAE  18
 0024                GO TO 952                                                          RBAE  19
 0025           951 READ (1,1100)(FT(I),I=1,LIM)                                        RBAE  20
 0026          1100 FORMAT (10X,F10.5,10X,F10.5,10X,F10.5,10X,F10.5)                    RB1E  21
 0027           952 DO 11 I=1,NSYM                                                      RBAE  22
 0028                NX=(I-1)*NPTS+NREF                                                 RBAE  23
 0029            11 VT(I)=FT(NX)                                                        RBAE  24
 0030                VN=VT(NPK)                                                         RBAE  25
 0031                DO 12 I=1,NSYM                                                     RBAE  26
 0032            12 VT(I)=VT(I)/VN                                                      RBAE  27
 0033                DO 5 ISTEP=1,NSTEP                                                 RBAE  28
 0034                KEND=0                                                             RBAE  29
 0035                KTOT=0                                                             RBAE  30
 0036                N=0                                                                RBAE  31
 0037                NERR=0                                                             RBAE  32
```

```
      SUMA=0.0                                                          RBAE 33
      SUMB=0.0                                                          RBAE 34
      SUMC=0.0                                                          RBAE 35
      LSEQ=2**LTHR-1                                                    RBAE 36
      DO 50 I=1,NF                                                      RBAE 37
   50 KT(I)=1                                                           RBAE 38
      TOT=0.0                                                           RBAE 39
      DO 51 I=1,NSYM                                                    RBAE 40
      LZ=NSYM+1-I                                                       RBAE 41
      TOT=TOT+VT(LZ)                                                    RBAE 42
   51 XT(I)=TOT                                                         RBAE 43
      OUT=TOT                                                           RBAE 44
      SOUT=SIGN(1.2,OUT)                                                RBAE 45
      IOUT=IFIX(SOUT)                                                   RBAE 46
      IF(OUT-NSL) 56,56,65                                              RBAE 47
   56 IF(OUT+NSL) 64,57,57                                              RBAE 48
   57 JOUT=OUT+NLEV                                                     RBAE 49
      KOUT=MOD(JOUT,2)                                                  RBAE 50
      IF(KOUT) 64,64,65                                                 RBAE 51
   64 ISER=-1                                                           RBAE 54
      GO TO 63                                                          RBAE 55
   65 ISER=1                                                            RBAE 56
   63 DO 62 I=1,NCONS                                                   RBAE 57
      ISRA(I)=IOUT                                                      RBAE 58
      ISRB(I)=ISER                                                      RBAE 59
      ZT(I)=TOT                                                         RBAE 60
      C(I)=0.0                                                          RBAE 61
   62 NAV(I)=0                                                          RBAE 62
      C(NA)=1.0                                                         RBAE 63
      IF(NCONS-10) 53,53,54                                             RBAE 64
   53 WRITE(7,1010) (I,I=1,NCONS)                                       RBAE 65
 1010 FORMAT('1   N',4X,'SUM',5X,'NORM',4X,'ERRORS',10(4X'C(',I2,')'))  RBAE 66
      GO TO 55                                                          RBAE 67
   54 WRITE(7,1010) (I,I=1,10)                                          RBAE 68
      WRITE(7,1020) (I,I=11,NCONS)                                      RBAE 69
 1020 FORMAT(' '30X,10(4X,'C(',I2,')'))                                 RBAE 70
   55 DO 67 I=1,100                                                     RBAE 71
   67 Y(I)=0.0                                                          RBAE 72
      IF(NCONF-2) 20,21,21                                              RBAE 73
   20 SUM=0.0                                                           RBAE 74
```

```
          NIT=NCONS+NSYM-1                                RBAE 75
          DO 114 L=1,NIT                                  RBAE 76
          YT=0.0                                          RBAE 77
          DO 110 I=1,NCONS                                RBAE 78
          K=L+1-I                                         RBAE 79
          IF(K) 120,120,121                               RBAE 80
      120 VTX=0.0                                         RBAE 81
          GO TO 110                                       RBAE 82
      121 IF(K-NSYM) 123,123,120                          RBAE 83
      123 VTX=VT(K)                                       RBAE 84
      110 YT=YT+C(I)*VTX                                  RBAE 85
          IF(L-MT-NPK) 124,125,124                        RBAE 86
      125 DIVD=YT                                         RBAE 87
          GO TO 114                                       RBAE 88
      124 SUM=SUM+ABS(YT)                                 RBAE 89
      114 Y(L)=YT                                         RBAE 90
          DIST=SUM/DIVD                                   RBAE 91
          GO TO 30                                        RBAE 92
       21 SUM=0.0                                         RBAE 93
          IF(MT) 81,81,82                                 RBAE 935
       82 DO 115 L=1,MT                                   RBAE 94
          YT=0.0                                          RBAE 95
          DO 111 I=1,NCONS                                RBAE 96
          K=L+1-I                                         RBAE 97
          IF (K) 126,126,127                              RBAE 98
      126 VTX=0.0                                         RBAE 99
          GO TO 111                                       RBAE100
      127 IF (K-NSYM) 128,128,126                         RBAE101
      128 VTX=VT(K)                                       RBAE102
      111 YT=YT+C(I)*VTX                                  RBAE103
          SUM=SUM+ABS(YT)                                 RBAE104
      115 Y(L)=YT                                         RBAE105
       81 DO 116 L=NA,NB                                  RBAE106
          YT=0.0                                          RBAE107
          DO 112 I=1,NA                                   RBAE108
          K=L+1-I                                         RBAE109
          IF (K)129,129,130                               RBAE110
      129 VTX=0.0                                         RBAE111
          GO TO 112                                       RBAE112
      130 IF(K-NSYM) 131,131,129                          RBAE113
```

```
  131 VTX=VT(K)                                           RBAE114
  112 YT=YT+C(I)*VTX                                      RBAE115
      DO 113 I=NC,NCONS                                   RBAE116
      J =L+1-I+MT                                         RBAE117
      IF(J) 117,117,118                                   RBAE118
  117 YTX=0.0                                             RBAE119
      GO TO 113                                           RBAE120
  118 YTX=Y(J)                                            RBAE121
  113 YT=YT+C(I)*YTX                                      RBAE122
      Y(L)=YT                                             RBAE123
      IF (L-MT-NPK) 137,138,137                           RBAE124
  138 DIVD=YT                                             RBAE125
      GO TO 116                                           RBAE126
  137 SUM=SUM+ABS(YT)                                     RBAE127
  116 CONTINUE                                            RBAE128
      KOUNT=0                                             RBAE1295
      DO 119 L=ND,100                                     RBAE129
      YT=0.0                                              RBAE130
      DO 132 I=NC,NCONS                                   RBAE131
      J=L+1-I+MT                                          RBAE132
      IF (J) 133,133,134                                  RBAE133
  133 YTX=0.0                                             RBAE134
      GO TO 132                                           RBAE135
  134 YTX=Y(J)                                            RBAE136
  132 YT=YT+C(I)*YTX                                      RBAE137
      Y(L)=YT                                             RBAE138
      IF(ABS(YT)-.0001) 135,135,119                       RBAE139
  135 KOUNT=KOUNT+1                                       R21E140
      IF(KOUNT-NT) 119,136,136                            RBAE141
  119 SUM=SUM+ABS(YT)                                     RBAE142
      WRITE(7,1002)                                       RBAE143
 1002 FORMAT(' IMPULSE RESPONSE NOT CONVERGENT',/)        RBAE144
      NTERM=1                                             RBAE1445
  136 DIST=SUM/DIVD                                       RBAE145
   30 IF(NCONS-10) 31,31,32                               RBAE146
   31 WRITE (7,1011) N,DIST,DIVD,NERR,(C(I),I=1,NCONS)    RBAE147
 1011 FORMAT('0',I5,2F9.5,I6,' ',10F9.4)                  RBAE148
      GO TO 33                                            RBAE149
   32 WRITE (7,1011) N,DIST,DIVD,NERR,(C(I),I=1,10)       RBAE150
      WRITE (7,1012) (C(I),I=11,NCONS)                    RBAE151
```

```
 1012 FORMAT(31X,10F9.4)                                         RBAF152
   33 NERR=0                                                     RBAF153
      IF(N.GT.6600) GO TO 24                                     RBAF1536
   80 SUMC=SUMB                                                  RBAF154
      SUMB=SUMA                                                  RBAF155
      SUMA=DIST                                                  RBAF156
      IF(ABS(SUMC-DIST)-.001) 23,23,22                           RBAF157
   23 KEND=KEND+1                                                RBAF158
      IF(KEND-5) 22,22,24                                        RBAF159
   22 DO 58 J=1,NBPS                                             RBAF1591
      DO 25 I=2,NF                                               RBAF160
      LX=NF+2-I                                                  RBAF161
      LY=NF+1-I                                                  RBAF162
   25 KT(LX)=KT(LY)                                              RBAF163
      KT(1)=0-KT(LTHR)*KT(NFB)                                   RBAF164
   58 M(J)=KT(1)                                                 RBAF164A
      IF(M(4)) 200,200,201                                       RBAF164B
  200 IF(M(3)) 202,202,203                                       RBAF164C
  201 IF(M(3)) 204,204,205                                       RBAF164D
  202 IF(M(2)) 206,206,207                                       RBAF164E
  203 IF(M(2)) 208,208,209                                       RBAF164F
  204 IF(M(2)) 210,210,211                                       RBAF164G
  205 IF(M(2)) 212,212,213                                       RBAF164H
  206 GCM=11                                                     RBAF164I
      GO TO 214                                                  RBAF164J
  207 GCM=9                                                      RBAF164K
      GO TO 214                                                  RBAF164L
  208 GCM=13                                                     RBAF164M
      GO TO 214                                                  RBAF164N
  209 GCM=15                                                     RBAF164O
      GO TO 214                                                  RBAF164P
  210 GCM=5                                                      RBAF164Q
      GO TO 214                                                  RBAF164R
  211 GCM=7                                                      RBAF164S
      GO TO 214                                                  RBAF164T
  212 GCM=3                                                      RBAF164U
      GO TO 214                                                  RBAF164V
  213 GCM=1                                                      RBAF164W
      MGC=IFIX(GCM)
  214 GC=ISIGN(MGC,M(1))                                         RBAF164X
```

```
      DO 26 I=2,NSYM                                                    RBAE165
      LP=NSYM+2-I                                                       RBAE166
      LQ=NSYM+1-I                                                       RBAE167
   26 XT(LP)=XT(LQ)                                                     RBAE168
      XT(1)=0                                                           RBAE169
      DO 29 I=1,NSYM                                                    RBAE170
      LR=NSYM+1-I                                                       RBAE171
   29 XT(I)=XT(I)+VT(LR)*GC                                             RBAE172
      IF(NCONF-2) 40,41,41                                              RBAE173
   40 DO 18 I=2,NCONS                                                   RBAE174
      L=NCONS+2-I                                                       RBAE175
      J=NCONS+1-I                                                       RBAE176
   18 ZT(L)=ZT(J)                                                       RBAE177
      ZT(1)=XT(NSYM)                                                    RBAE178
      GO TO 42                                                          RBAE179
   41 DO 17 I=2,NT                                                      RBAE181
      L=NCONS+2-I                                                       RBAE182
      J=NCONS+1-I                                                       RBAE183
   17 ZT(L)=ZT(J)                                                       RBAE184
      ZT(NC)=OUT                                                        RBAE185
      DO 16 I=2,NA                                                      RBAE186
      L=NA+2-I                                                          RBAE187
      J=NA+1-I                                                          RBAE188
   16 ZT(L)=ZT(J)                                                       RBAE189
      ZT(1)=XT(NSYM)                                                    RBAE190
   42 OUT=0.0                                                           RBAE191
      DO 15 I=1,NCONS                                                   RBAE192
   15 OUT=OUT+ZT(I)*C(I)                                                RBAE193
      SOUT=SIGN(1.2,OUT)                                                RBAE194
      IOUT=IFIX(SOUT)                                                   RBAE195
      IF(OUT-NSL) 43,43,49                                              RBAE196
   43 IF(OUT+NSL) 48,44,44                                              RBAE197
   44 JOUT=OUT+NLEV                                                     RBAE198
      KOUT=MOD(JOUT,2)                                                  RBAE199
      IF(KOUT) 48,48,49                                                 RBAE200
   48 ISER=-1                                                           RBAE203
      GO TO 47                                                          IBA52 4
   49 ISER=1                                                            RBAE205
   47 IF(NSTRAT-2) 60,61,61                                             RBAE206
   60 DO 13 I=2,NCONS                                                   RBAE207
```

```
           LX=NCONS+2-I                                             RBAE208
           LY=NCONS+1-I                                             RBAE209
        13 ISRA(LX)=ISRA(LY)                                        RBAE210
           ISRA(1)=IOUT                                             RBAE211
           DO 14 I=2,NA                                             RBAE212
           LX=NA+2-I                                                RBAE213
           LY=NA+1-I                                                RBAE214
        14 ISRB(LX)=ISRB(LY)                                        RBAE215
           ISRB(1)=ISER                                             RBAE216
           DO 52 I=1,NCONS                                          RBAE217
           NAV(I)=NAV(I)-ISRA(I)*ISRB(NA)                           RBAE218
           IF(IABS(NAV(I))-NAVE) 52,66,66                           RBAE219
        66 ARGA=STEP(ISTEP)                                         RBAE220
           ARGB=FLOAT(NAV(I))                                       RBAE221
           C(I)=C(I)+SIGN(ARGA,ARGB)                                RBAE222
           NAV(I)=0                                                 RBAE2225
        52 CONTINUE                                                 RBAE223
           GO TO 19                                                 RBAE224
        61 WRITE (7,2001)                                           RBAE225
      2001 FORMAT ('   THIS STRATEGY NOT YET AVAILABLE')             RBAE226
        19 IF(OUT)220,220,221                                       RBAE226A
       220 LA(1)=-1                                                 RBAE226B
           GO TO 222                                                RBAE226C
       221 LA(1)=1                                                  RBAE226D
       222 GD=ABS(OUT)-8.0                                          RBAE226E
           IF(GD) 223,223,224                                       RBAE226F
       223 LA(4)=1                                                  RBAE226G
           GO TO 225                                                RBAE226H
       224 LA(4)=-1                                                 RBAE226I
       225 GC=ABS(GD)-4.0                                           RBAE226J
           IF(GC) 226,226,227                                       RBAE226K
       226 LA(3)=-1                                                 RBAE226L
           GO TO 228                                                RBAE226M
       227 LA(3)=1                                                  RBAE226N
       228 GB=ABS(GC)-2.0                                           RBAE226O
           IF(GB) 229,229,230                                       RBAE226P
       229 LA(2)=-1                                                 RBAE226Q
           GO TO 231                                                RBAE226R
       230 LA(2)=1                                                  RBAE226S
       231 DO 232 I=1,NBPS                                          RBAE226T
```

```
      IA=NG+1-I                                              RBAE226U
      IF(LA(I)-KT(IA))27,28,27                               RBAE227
   27 NERR=NERR+1                                            RBAE228
   28 N=N+1                                                  RBAE229
  232 KTOT=KTOT+1                                            RBAE230
      LBLK=LSEQ*NBPS                                         RBAE2305
      IF(KTOT-LBLK) 22,34,34                                 RBAE231
   34 KTOT=0                                                 RBAE232
      GO TO 55                                               RBAE233
   24 WRITE (7,1016)                                         RBAE234
 1016 FORMAT('  TABLE OF IMPULSE RESPONSE SAMPLES.')         RBAE235
      WRITE (7,1015)Y                                        RBAE236
 1015 FORMAT(/,10F10.5)                                      RBAE237
      WRITE(3,1013)                                          RBAE238
 1013 FORMAT ('0 PROBLEM COMPLETED')                         RBAE239
    5 CONTINUE                                               RBAE240
      WRITE (3,1014)                                         RBAE241
 1014 FORMAT ('0 PROGRAM COMPLETED')                         RBAE242
      STOP                                                   RBAE243
      END                                                    RBAE244
```

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