# SOME ANALYTICAL ASPECTS OF SAMPLING PLANS

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## SUMMARY

This thesis reports on the results of the analyses of certain aspects of sampling inspection plans. The investigation has been confined to attributes (as distinct from variables) plans and in this respect the analyses have been concerned with two main aspects of single and double plans. These are:-

(i) the Average Outgoing Quality Limit (AOQL) of the plan.

(ii) the Average Sample Number (ASN) of the plan.

In the former connection the investigation has been concerned with the evaluation of the AOQL analytically and the determination of the fraction defective of the incoming material to give the AOQL. The analyses have been applied to both single and double sampling plans.

In the latter connection the investigation has been concerned with the evaluation of the maximum ASN analytically and the determination of the fraction defective of the incoming material to give the maximum value of ASN. The analyses have been confined only to double sampling plans because in the case of single sampling the ASN is constant and is equal to n, the sample size.

These analytical treatments have led to the development of the following two theorems:-

1. For any single sampling plan where the acceptance number is c, the probability of acceptance of material of incoming quality whose value is such

as to give the maximum value of the outgoing quality is equal to (c + 1) times the probability of occurence of (c + 1) defective items in the sample.

i.e. $$Pa = \frac{(c + 1)e^{-np} n^{c+1} p^{c+1}}{(c + 1)!}$$

2. In any double sampling plan where the acceptance numbers on the first sample and combined samples are respectively $c_1$ and $c_2$, the value of the fraction defective of the incoming material p, to give the maximum value of the ASN is given by:-

$$p = \frac{\sqrt[(c_2 - c_1)]{\frac{c_2!}{c_1!}}}{n_1}$$

where $n_1$ is the size of the first sample.

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# DECLARATION

No part of the work described in this thesis has been submitted in support of an application for another degree or qualification of this or any other University or Institute of learning.

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## PART I

## INTRODUCTION

As the complexity of equipment is increasing, the achievement and maintenance of quality and reliability are becoming very real problems. Today the emphasis is on what has become to be called "total quality control". This involves:-

(i) defining quality standards,

(ii) appraising the conformance to the standards,

(iii) taking action whenever the standards are ~~exceeded,~~ not attained.

(iv) planning for improvement in the standards.

The techniques of quality control are basic to any manufacturing process and can be used in industries ranging from engineering to food, clothing, paper and textiles, although the methods of approach are somewhat different depending upon the type of manufacture. In mass production quality control is centered on the product, but in job-lot manufacture it is centered on the process.

Statistics are used in a total quality control programme where and when they may be considered to be appropriately applied, but are only one part of the overall pattern. The four statistical tools used are:-

(i) frequency distributions,

(ii) control charts,

(iii) sampling plans,

(iv) special methods.

A study of published texts on quality control has indicated that gaps exist in the treatment of certain aspects of sampling plans, particularly from an analytical point of view and the object of the investigation was to make good some of these deficiencies. The aspects in question are:-

(i) the Average Outgoing Quality Limit (AOQL) of single sampling plans,

(ii) The Average Outgoing Quality Limit (AOQL) of double sampling plans,

(iii) The Average Sample Number (ASN) of double sampling plans.

Alongside the analytical treatments a further object of the investigation was to devise methods for accurately determining the fraction defective of the incoming material which will result in the maximum value of the Average Outgoing Quality (ie. the AOQL) and that which will result in the maximum value of the ASN. It is particularly important from an economic point of view that these values be known, as the AOQ and the ASN will reveal the costs of the sampling plans. There is insufficient evidence from the results of the investigation, however, to show whether or not there is any correlation between the value of the fraction defective of the incoming material to give the maximum value of the AOQ and that to give the maximum value of the ASN.

The analytical treatments have led to the development of the two theorems stated in the summary. Arising from the first of these, a table has been prepared from which the AOQL of a plan may be readily determined, or, conversely, the table may be used to design a plan to assure a given AOQL. A similar table has been prepared in respect of certain combinations of $c_1$ and $c_2$ for double sampling plans where $c_1$ is the acceptance number on the first sample and $c_2$ the acceptance number on combined samples.

In the following section on some fundamental concepts associated with sampling plans, the natures of the above aspects are fully explained.

## PART II

## SOME FUNDAMENTAL CONCEPTS ASSOCIATED WITH SAMPLING PLANS

### 1. Quality and Quality Control

#### 1. (i) Definitions

The quality of a given characteristic is acceptable if the characteristic meets specification requirements.

Quality control of which sampling inspection forms only a part may be defined as:-

"An effective system for co-ordinating the quality maintenance and quality improvement efforts of various groups in an organisation so as to enable production at the most economic levels which allow for full customer satisfaction".

#### 1. (ii) Economic Aspects

Economic aspects of quality and quality control are illustrated in Figures 1 and 2.

Figure 1 shows the relationship between quality of design and cost. Absolute perfection can be achieved only at infinite cost and in practice an optimum economic level is sought after. This optimum level will occur at a point where the tangents to the two curves are parallel. Above this point, although the quality of design will be higher, a large rise (a) in the cost of achieving the increase in the quality of design will occur with very little rise (b) in the value. Below this point there will be a reduction in cost (c) but with a considerable drop in value (d).

Figure 2 shows the relationship between quality of conformance (that is the degree to which a characteristic meets the specification requirements) and cost. One hundred per cent conformance would only result in a prohibitive cost of quality control although the cost of rejects would be zero. At the other end of the scale where there is no degree of conformance, the cost of rejects would be prohibitive but there would be no cost of quality control. The theorectical cost of wages and materials would remain constant whether the characteristic is within the specification requirements or not. The total cost is the algebraic sum of the three curves. The optimum degree of conformance is where the total cost is a minimum. It will be seen from the totallcost curve that this cost can be the same for characteristics exceeding or falling short of the optimum. Whereas on the one hand the costs of rejects is lower, this is offset by an increased cost in quality control, on the other hand the reduction in the cost of quality control is offset by an increase in the cost of rejects.

## 2. Sampling Inspection

### 2. (i) Purposes of sampling inspection

Inspection is an essential part of the quality control function. One hundred per cent inspection of the product may be carried out or inspection may be on a systematic sampling basis. The former, apart from being uneconomic is not necessarily one hundred per cent efficient and unless one hundred per cent inspection is particularly stipulated - as it is for some types of product - most inspection today is done on a sampling basis.

Sampling inspection may be used in two ways:-

(i) for control of the process,

(ii) for acceptance of the product.

The most effective way of controlling the process is by the use of control charts. In conjunction with these, special process capability studies may be carried out. A process capability study involves techniques for appraising all the sources and nature of variability such as within-piece, piece-to-piece, time-to-time, and in the case of multi-stream processes such as multi-spindle machine tools and multi-cavity diecasting machines, stream-to-stream. However, such studies are outside the scope of the present work.

As far as acceptance of the product is concerned, once it has been established that a process is in control and that the level of control is satisfactory the control chart may then be adopted as standard for the process and used for acceptance of the product.

Sampling inspection is used not only for the acceptance of material at the source of production, but for the acceptance of incoming material from an outside supplier. In this connection the term "acceptance sampling" is generally used .

## 2 (ii) Distinction between attributes and variables inspection.

Whether inspection is one hundred per cent or on a sampling basis, it can be either by attributes or by variables. In attributes inspection, inspection merely involves testing to see whether or not a characteristic meets specification requirements without reference to degree and may be by the use of a "GO" - "NOT GO" gauge or be purely visual. In variables inspection, actual measurements are carried out on the characteristics. There exist both control charts and sampling inspection plans to cover each of these categories and although reference has been made to variables inspection in this thesis, it is essentially with attributes plans that the work is concerned.

## 2. (iii) Acceptance sampling by attributes

There are four main types of attributes sampling plans:-

Single,

Double,

Multiple,

Sequential.

The schematic arrangement of each type is shown in Figures 3, 4, 5 and 6 respectively.

In single sampling a lot is either accepted or rejected as the result of the inspection of a single sample. Thus the probability of rejection is one minus the probability of acceptance. The plan is designated by a sample size n and an acceptance number c.

In double sampling a lot may be accepted or rejected on the first sample, but unlike single sampling if it is not accepted on the first sample - as distinct from being outrightly rejected - a second sample is taken. Unless a decision is reached on the first sample, it is reached on the second sample.

Multiple sampling is a logical extension of double sampling and reference to Figure 5 will show that the taking of samples could continue until the lot is exhausted without a decision being reached. However, most multiple sampling plans are designed to force a decision after a certain number of inconclusive samples have been taken.

In the first three schemes, samples are of finite size but in sequential sampling the items are taken from the lot one at a time and the number of defective items against the total number of items inspected is plotted on a chart. Two decision lines are placed on the chart to mark the acceptance and rejection regions. As soon as a point falls below the lower line the lot is accepted without any further inspection. As soon as a point falls above the upper line the lot is rejected. As long as the points fall between the two lines, inspection is continued. As with multiple sampling this could continue until the lot is exhausted but in practice a decision is reached after a certain specified number of items has been inspected.

2 (iv) Comparative advantages of single, double and multiple sampling.

The comparative advantages of single, double and multiple sampling are illustrated in Figure 7. Sequential sampling is not included in the table as it is basically different from the other three schemes. Consider each of the listed aspects in turn.

Protection:- By this is meant the protection afforded to the consumer against the acceptance of a bad lot by virtue of an optimistic sample; also the protection afforded to the producer against the rejection of a good lot by virtue of a pessimistic sample. In general, all three schemes can be so designed that lots of specified quality shall have the same chance of acceptance (or rejection). In other words, the operating characteristic curves can be almost alike.

The total inspection cost:- In single sampling for the same protection the sample size is always higher than the size of the first sample in double sampling and that, in turn, higher than the size of the first sample in multiple sampling. Material whose incoming fraction defective approaches zero will nearly always be accepted on the first sample and thus the total inspection cost, on the basis that the cost is evaluated as so much per piece inspected, will be greatest in single sampling, less in double and least in multiple. By simple reasoning, material of very poor quality will nearly always be rejected on the first sample thus rendering unnecessary the taking of subsequent samples.

Variability of inspection load:- In single sampling a lot is accepted or rejected as the result of the inspection of a single sample irrespective of the value of the fraction defective of the incoming material. Thus the inspection load is constant.

In double and multiple sampling a lot will be accepted on the first sample if the fraction defective of the incoming material is equal to zero. As the quality of the incoming material deteriorates so will become greater the necessity for taking subsequent samples, so that the inspection load will vary with the quality of the incoming material.

Accurate estimation of lot quality:- Small samples tend to be optimistic, that is, they tend to reflect a rather better quality than is actually present. The greater the size of the sample the more representative is it of the lot from which it is drawn. As stated above, the sample size for single sampling is higher than that of the first sample for double and multiple sampling and thus single sampling gives the best estimation of lot quality.

Amount of record keeping:- In single sampling it is necessary to enter only the result of the inspection of a single sample. In the other two schemes, the results of the inspection of more than one sample must be entered. Thus, the amount of record keeping is least in single, more in double and most in multiple sampling.

Psychological:- This aspect is from the point of view of the producer. In single sampling a lot submitted for inspection has only one chance of acceptance. In other schemes, however, if a lot is not accepted on the first sample it has a chance of being accepted on a subsequent sample. Any advantage is only illusionary, however, as each of the three types of plan may be designed to give the same protection.

## 3 Fundamental Properties of Sampling Plans

### 3 (i) The operating characteristic of a sampling plan.

Associated with any sampling plan is the operating characteristic or O.C. curve. The curve shows the relationship between the fraction or per cent defective of the incoming material and its probability of acceptance. The ideal shape of the curve is shown in Figure 8. Incoming material up to some specified Acceptable Quality Level would have a 1.0 probability of acceptance and anything in excess of this would have a zero probability of acceptance.

No sampling plans, however, are so discriminating. There is always an element of risk in sampling and although the shape of the operating characteristic of some plans may approach the ideal shape very closely, a more practical shape of the curve is shown in Figure 9.

There are certain "key" points on the curve:-

$p_1$ is known as the acceptable quality level and is used in conjunction with the Producer's Risk, $\alpha$ . This is the probability that a lot of material of incoming AQL (Acceptable Quality Level) will be rejected. The symbol $p_2$ is known as the Lot Tolerance Fraction Defective or Lot Tolerance Per cent Defective (LTPD) and is used in conjunction with the Consumer's Risk, $\beta$ . This is the probability that alot of material of LTPD quality will be accepted. ($\alpha$ and $\beta$ are akin to the terms Type I error and Type II error used in statistics which represent respectively the probability of rejection of a true hypothesis and the probability of acceptance of a false hypothesis).

3 (ii) The concept of average outgoing quality (AOQ).

The evaluation of Average Outgoing Quality (AOQ) is based on the premise that rejected lots are one hundred per cent inspected, defective items being either repaired or replaced with effective items. Thus, if the incoming material is of very high quality, that is, having a fraction defective approaching zero, then the outgoing material will also be of very high quality because nearly all the lots will be accepted. On the other hand, if the incoming material is of very low quality, the outgoing material will again be of very high quality, because nearly all the lots will be rejected.

Suppose incoming material whose fraction defective is p has a probability of acceptance of Pa, then out of 1000 lots, say, 1000 Pa lots will be accepted and go out at p fraction defective. The remaining (1 - Pa) 1000 lots will be rejected and as the defective items are either repaired or replaced with effective items then these lots will go out at zero fraction defective. Thus the Average Outgoing Quality is (1000 Pa x p + (1 - Pa)1000 x 0 )/1000 = p x Pa; that is the Average Outgoing Quality is equal to the product of the fraction defective of the incoming material and its probability of acceptance.

As the value of p increases so will the value of the A.O.Q. until a critical value of p is reached when the A.O.Q. reaches a maximum value. Increasing p still further will cause the A.O.Q. to fall. The maximum value of the A.O.Q. is referred to as the A.O.Q.L. or Average Outgoing Quality Limit.

For any single sampling plan, the following assumptions are made:-

1. The lot size N is constant.
2. One hundred per cent inspection finds all the defective items.
3. The defective items are replaced with effective items.
4. N is large in comparison with the sample size, n.

An exact determination of the A.O.Q. is given by the formula $p.Pa\left(1 - \frac{n}{N}\right)$.

As N is generally large in comparison with n, for all practical purposes the A.O.Q. is evaluated as $p \times Pa$.

To illustrate the application of the above consider the single sampling plan n = 100, c = 1.

Table I indicates the evaluation of the probability of acceptance and the A.O.Q. for arbitrarily chosen values of p and Figures 10 and 11 show respectively the O.C. and the A.O.Q. curves for this plan.

As the A.O.Q.L is the maximum value of the A.O.Q. which itself is the product of p and Pa, the A.O.Q.L. is the maximum area under the O.C. curve.

Table I seems to indicate that the A.O.Q.L. is about 0.008370 but it is not evident from the table what the actual value of the A.O.Q.L. is and Figure 11 indicates that it is of the order of 0.0084.

A more exact determination using the theorem developed hereafter gives the value as 0.00839742.

The differences are small and in the particular example quoted are insignificant from a practical point of view. However, not always would there be such close agreement and it is always better to evaluate the A.O.Q.L. as accurately as possible in the first instance (which the above method does not allow) and to round it off afterwards if necessary.

The same procedure may be applied to the evaluation of the A.O.Q. and hence the A.O.Q.L. of double sampling plans. Consider the double sampling plan

$$n_1 = 40, \; c_1 = 0; \; n_2 = 60, \; c_2 = 3.$$

As the computations are more involved than those for single sampling it will be sufficient to take one point on the O.C. and A.O.Q. curves to illustrate the method.

The probability of acceptance on combined samples (i.e. the total probability of acceptance, Pa) is the sum of the probabilities of acceptance on the first and second samples.

$Pa_1$ = probability of no defective items

$Pa_2$ = probability of 1 defective item in first sample with 2 or less defective items in second sample.

+probability of 2 defective items in first sample with 1 or less defective items in second sample.

+probability of 3 defective items in first sample with no defective items in second sample.

If the incoming material is 0.03 fraction defective, $n_1 p = 40 \times 0.3 = 1.2$

and $n_2 p = 60 \times 0.3 = 1.8$

From tables of the cumulative Poisson distribution (appendix 17)

$Pa_1 = 0.301$

and $Pa_2 = (0.663 - 0.301)\ (0.731)$

$+ (0.897 - 0.663)\ (0.463)$

$+ (0.966 - 0.897)\ (0.165)$

$= 0.384$

Hence $Pa = 0.301 + 0.384$

$= 0.685$

This will be a point on the O.C. curve of the plan at $p = 0.03$.

The A.O.Q. at $p = 0.03$ is $p \times Pa$

$= 0.03 \times 0.685$

$= 0.02055$

## 3 (iii) The average sample number curve

In single sampling incoming material is either accepted or rejected as the result of the inspection of a single sample and the Average Sample Number will therefore be constant irrespective of the value of the fraction defective of the incoming material. Even though the rejection number may be reached before the inspection of the sample has been completed, the sample is nevertheless fully inspected for the inspection records.

In double sampling although a decision can be reached as the result of the inspection of the first sample, material not accepted on the first sample - but not rejected - will have a further chance of being accepted on a second sample. If the incoming material has a fraction defective of zero then all the lots will be accepted on the first sample and no second sample will be taken. As the quality of the incoming material deteriorates then so will become greater the necessity for taking more second samples until in the limit material 100% defective will be rejected on the first sample although this limit will not be reached in practice. Thus for incoming material of zero fraction defective the Average Sample Number will be equal to $n_1$ and for material of very poor quality it will approach $n_1$. Between the two, there is a value of p which will give a maximum value of the ASN.

The determination of the ASN will depend upon whether:-

Method (i) the second sample is completely inspected,

Method (ii) inspection of the second sample is curtailed or truncated when the number of defective items in the combined samples is found to exceed the acceptance number, $c_2$.

Method (ii) has found some favour as less inspection is involved, but this is not true generally for extremes of quality where material would be accepted or rejected on the first sample accordingly.

The writer favours method (i) for the following reasons:-

1. A better estimate of lot quality is obtained on account of the greater number of items inspected.

2. Currently published sampling inspection tables such as the American MIL-STD 105D (Ref.1) and the British equivalent DEF -131A (Ref.2) assume no curtailment of the second sample. These standards are widely used today throughout both countries, particularly by Government Departments, and for this reason the section on ASN in this thesis is based on the assumption that the second sample is completely inspected. In any case truncated inspection is used only when a lot is rejected and quality improvement efforts should result in greater numbers of lots being accepted.

When the second sample is completely inspected, the ASN is given by:-

$$ASN = n_1 + P_2 n_2$$

Where $P_2$ is the probability of taking a second sample.

Again consider the above double sampling plan with the incoming material at 0.025 fraction defective. The expected number of defective items in the first sample $m_1 = n_1 p = 40 \times 0.025 = 1.0$ The probability of acceptance on the first sample is the probability of no defective items and from tables of the cumulative Poisson distribution equal to 0.368.

The probability of rejection on the first sample is the probability of four or more defective items in the sample and is equal to 1 minus the probability of three or less. From the same tables, this is equal to 1 - 0.981 = 0·019

Therefore the probability of a decision on the first sample is equal to 0.368 + 0·019 = 0·387 and the probability of taking a second sample is 1 - 0·387 = 0·613

Hence the ASN = 40 + 0·613 x 60 = 76·78

(This figure is higher than that obtained when truncated inspection is used as will be seen by reference to Appendix 19).

For the above plan, values of the ASN for a range of values of p have been calculated on the basis of complete inspection of the second sample. The computation for this is shown in Table II with the corresponding graph in Figure 12.

3 (iv) The average total inspection (A.T.I.) curve

On the basis that rejected lots are one hundred per cent inspected, the Average Total Inspection will depend upon the sampling plan itself, the number of items in the lot, and the fraction defective of the incoming material.

In the case of single sampling a sample

of n items is always inspected. If Pa is the probability of acceptance then the probability of inspecting the residue is 1 - Pa and if N is the number of items in the lot, the number of items in the residue N - n. Hence the average number of items inspected in the residue is (1 - Pa) (N - n) and the Average Total Inspection is given by

$$A.T.I. = n + (1 - Pa)(N - n)$$

Evaluations of the A.T.I. for three different single sampling plans for a given value of p are shown in Table III.

The greater the value of p the greater the A.T.I. so this section has not been treated analytically. Nevertheless, Figure 13 shows the A.T.I. curve for the single sampling plan n = 100, c = 1 and a lot size N = 1000, where c is the acceptance number.

In double sampling, the Average Total Inspection is given by.

$$A.T.I. = n_1 Pa_1 + (n_1 + n_2)Pa_2 + N(1 - Pa)$$
$$= n_1 + n_2(1 - Pa_1) + \left[N - (n_1 + n_2)\right](1 - Pa)$$

Again, the greater the value of p the greater the A.T.I. as will be seen from Figure 13A in respect of the double sampling plan $n_1 = 100$, $c_1 = 0$; $n_2 = 200$, $c_2 = 1$ and a lot size of 1300.

Thus, as in the case of single sampling, this topic has not been treated analytically.

The O.C. and A.O.Q. curves are concerned with the protection provided by the plan.

The A.S.N. and A.T.I. curves reveal the costs of the plan.

## 4 Some notes on the design of sampling plans

This thesis is not concerned essentially with the design of sampling plans, but it would not be out of place to make some reference to this and to show how sampling plans may be designed so that incoming material of a given quality shall have some specified probability of acceptance.

If only one point on the O.C. curve is specified, then in theory an infinite number of plans may be designed whose O.C. curves all pass through this point. If two points are specified then the plan will be completely defined as the curve follows the Poisson distribution.

Suppose that in single sampling it is required that incoming material 4% defective shall have one in ten chance of being accepted. As sample sizes and acceptance numbers must be integers it may not be possible to design a plan to meet this **requirement** completely but the plan would be sufficiently close to meet the requirement from a practical point of view.

The procedure may follow one of two paths:-

(i) decide on a sample size and determine the acceptance number,

(ii) decide on an acceptance number and determine the sample size.

The latter is by far the more widely adopted method as acceptance numbers are generally small in comparison with the sample size, starting at zero upwards.

Suppose it is desired to make the acceptance number zero, then from tables of the cumulative Poisson distribution (Appendix 16 (iii)) when c = 0 and Pa = 0.1, np = 2.31 (by interpolation). Thus $n = \frac{2.31}{p} = \frac{2.31}{0.04} = 58$

The plan is therefore n = 58, c = 0

Similarly, at the same value of Pa = 0.1

when c = 1, np = 3.9 and n = 98

when c = 2, np = 5.33 and n = 133

The operating characteristic curves of these plans will all pass through the point $p = 0.04$, Pa = 0.10, but will not pass through any other common point (Figure 13B) On the basis that rejected batches are 100% inspected, each of these plans will involve a different amount of Average Total Inspection. Table III shows the computation of the A.T.I. when the incoming material is 0.5% fraction defective and the lot size N is 500.

It will be seen that the plan n = 98, c = 1 will give the minimum A.T.I. This is not necessarily true for other values of N however. Two other plans whose O.C. curves pass through the point Pa = 0.1, p = 0.04 are n = 168, c = 3 and n = 200, c = 4. When N = 1000, the A.T.I. values for the five plans are respectively 294, 176, 157, 177 and 203. Thus, in the case of N = 1000, the plan n = 133, c = 2 will give the minimum A.T.I.

Plans shown in published sampling tables are those which will result in the minimum A.T.I. for a given combination of lot size and incoming material fraction (or per cent) defective. To minimise the number of plans involved, the plans are based on ranges of lot sizes and ranges of fraction defective rather than on individual values.

Consider the plan whose O.C. curve passes through the points Pa = 0.1, p = 0.07; Pa = 0.9, P = 0.01

Only one plan will meet - or very nearly meet - this requirement. The procedure is to take arbitrary values of c and from Poisson tables find np at Pa = 0.100 and at Pa = 0.900, find their ratio and take as the acceptance number the particular values of c for which the ratios of the np s is 7, or very nearly 7, (because $\frac{np_2}{np_1} = \frac{p_2}{p_1} = \frac{0.07}{0.01} = 7$)

Table IV shows the method of determining the required value of c.

As the value of c is increased, the ratio decreases and it will be seen that 7.345 is the nearest to 7 when c = 1.

When np = 3.893, n = 3.893/0.07 = 55.6

and when np = 0.530, n = 0.530/0.01 = 53.0

Because the ratio of the nps is not exactly 7 (if it were so the values of n would be the same) it will be sufficiently accurate for all practical purposes to take the mean value of n as 54.

The plan is therefore n = 54, c = 1

To illustrate how closely this plan meets the requirements the probabilities of acceptance may be calculated for p = 0.01 and p = 0.07

When $p = 0.01$, $np = 0.54$ and $Pa = 0.897$

When $p = 0.07$, $np = 3.78$ and $Pa = 0.109$

The same basic principles can be extended to the design of double, multiple and sequential plans. All types of plan may be so designed so that their O.C. curves are very nearly the same.

## PART III

## ANALYTICAL TREATMENTS

### 5. The A.O.Q.L. of Single Sampling Plans

#### 5 (i) Literature Survey

From a literature survey no evidence of an analytical approach has been found. Most authors who deal with the A.O.Q.L. of a single sampling plan choose arbitrary values of p, find p x Pa for each value of p and note where the maximum value of p x Pa occurs.

In this connection Burr (Ref. 3) takes as an example the single sampling plan n = 150, c = 4 and concludes that the A.O.Q.L. is about 0.017.

Duncan (Ref. 4), following this method, takes as an example the single sampling plan n = 100, c = 2.

Both these authors plot the graph of p x Pa and estimate where the maximum value occurs.

Feigenbaum (Refs. 5 & 6), on the other hand, sketches the A.O.Q. curve for the plan n = 60, c = 0 but shows no computations, merely stating that the A.O.Q.L. occurs when the incoming material is 1.7% defective and is equal to 0.68% (The writer does not agree with this figure as application of his first theorem shows it to be 0.61%).

Similarly Freeman, Friedman, Mosteller and Wallis (Ref. 7) sketch the A.O.Q. curve for n = 225, c = 14 but do not show the computations, merely stating that the A.O.Q.L. is 4.2%

Grant (Ref. 8) takes n = 75, c = 1 and finds that the maximum A.O.Q. is 1.12% when p = 2.2%

Hill (Ref. 9) considers the plan n = 25, c = 1 and using the standard method of computation arrives at the conclusion that the A.O.Q.L. is 3.32 when p = 0.06

Huitson and Keen (Ref. 10) consider the plan n = 20, c = 1 and show how the A.O.Q. curve can be constructed, adding that the A.O.Q.L. can be read off at about 4.4%

Juran's book (Ref. 11) contains a chapter on "Statistical methods in the quality function" by J. W. Enell. As with the others, no analytical treatment is attempted. Computations of p x Pa are made for the plan n = 78, c = 1 and the A.O.Q.L. given as 0.01100.

Smith (Ref. 12) takes as an example the plan n = 180, c = 2, and merely quotes the A.O.Q.L. as 0.63%.

## 5 (ii) Background to the present investigation

Having examined the treatment by others of this aspect of sampling, the writer concluded that an analytical treatment would be more desirable as it would result in a determination of the value of the fraction defective of the incoming material to give the A.O.Q.L. to as high a degree of accuracy as one wished.

The initial investigation (Ref. 13) which led to the development of the first of the two theorems stated earlier consisted in taking values of 0, 1, 2, 3 etc. for c, the acceptance number, in turn and determining in each case the value of m (i.e. np) as follows. (As stated in appendix 16 (iii) the Poisson distribution is by far the most widely used probability distribution in sampling inspection and is applied here).

When the acceptance number c = 0 the probability of acceptance of the lot is given by

$$Pa = e^{-np}$$ i.e the first term of the Poisson distribution.

The A.O.Q., Q, is given by Q = p.Pa

$$\text{Thus } Q = p.e^{-np}$$

$$\text{and } \frac{dQ}{dp} = e^{-np} - npe^{-np}$$

For maximum Q, $\frac{dQ}{dp} = 0$

As $e^{-np} \neq 0$ and as np = m

$$1 - m = 0 \quad \text{or } \underline{m = 1}$$

When c = 1,

$$Pa = e^{-np}(1 + np)$$

$$\text{and } Q = e^{-np}(p + np^2)$$

$$\frac{dQ}{dp} = e^{-np}(1 + 2np) - e^{-np}(np + n^2p^2)$$

Again as $e^{-np} \neq 0$ and as np = m

$$1 + 2m - m - m^2 = 0$$

$$m^2 - m - 1 = 0$$

$$\text{or } m = \frac{+1 \pm \sqrt{1^2 + 4}}{2}$$

As m cannot be negative this gives $\underline{m = 1.618}$

When c = 2

$$Pa = e^{-np}(1 + np + \tfrac{1}{2}n^2p^2)$$

$$Q = e^{-np}(p + np^2 + \tfrac{1}{2}n^2p^3)$$

$$\frac{dQ}{dp} = e^{-np}(1 + 2np + \tfrac{3}{2}n^2p^2) - e^{-np}(np + n^2p^2 + \tfrac{1}{2}n^3p^3)$$

For maximum Q, $\frac{dQ}{dp} = 0$ and as $e^{-np} \neq 0$ and np = m

$$\underline{1 + m + \tfrac{1}{2}m^2 = \tfrac{1}{2}m^3}$$

When c = 3

$$Pa = e^{-np}(1 + np + \tfrac{1}{2}n^2p^2 + 1/6n^3p^3)$$

$$Q = e^{-np}(p + np^2 + \tfrac{1}{2}n^2p^3 + 1/6n^3p^4)$$

$$\frac{dQ}{dp} = e^{-np}(1 + 2np + 3/2n^2p^2 + 2/3n^3p^3)$$

$$-e^{-np}(np + n^2p^2 + \tfrac{1}{2}n^3p^3 + 1/6n^4p^4)$$

For maximum Q, $\frac{dQ}{dp} = 0$ and as $e^{-np} \neq 0$ and np = m

$$\underline{1 + m + \tfrac{1}{2}m^2 + 1/6m^3 = 1/6m^4}$$

When c = 4

$$Pa = e^{-np}(1 + np + \tfrac{1}{2}n^2p^2 + 1/6n^3p^3 + 1/24n^4p^4)$$

$$Q = e^{-np}(p + np^2 + \tfrac{1}{2}n^2p^3 + 1/6n^3p^4 + 1/24n^4p^5)$$

$$\frac{dQ}{dp} = e^{-np}(1 + 2np + 3/2n^2p^2 + 2/3n^3p^3 + 5/24n^4p^4)$$

$$- e^{-np}(np + n^2p^2 + \tfrac{1}{2}n^3p^3 + 1/6n^4p^4 + 1/24n^5p^5)$$

For maximum Q, $\frac{dQ}{dp} = 0$ and as $e^{-n_1p} \neq 0$ and np = m

$$\underline{1 + m + \tfrac{1}{2}m^2 + 1/6m^3 + 1/24m^4 = 1/24m^5}$$

Equations for values of c in excess of c = 4 were not developed as it became apparent that these followed a general pattern and the equations were first examined by tabulating them "en bloc":-

When c = 0 $\quad 1 = m$

When c = 1 $\quad 1 + m = m^2$

When c = 2 $\quad 1 + m + \frac{m^2}{2!} = \frac{m^3}{2!}$

When c = 3 $\quad 1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} = \frac{m^4}{3!}$

When c = 4 $\quad 1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \frac{m^4}{4!} = \frac{m^5}{4!}$

By the inspection of the above, the equation was written down for c = 5 :-

$$1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \frac{m^4}{4!} + \frac{m^5}{5!} = \frac{m^6}{5!}$$

and for c = 6

$$1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \frac{m^4}{4!} + \frac{m^5}{5!} + \frac{m^6}{6!} = \frac{m^7}{6!}$$

Both sides of the equation were multiplied by $e^{-m}$ giving:-

$$e^{-m}\left(1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \text{------------} + \frac{m^{c-1}}{(c-1)!} + \frac{m^c}{c!}\right) = e^{-m}\frac{m^{c+1}}{c!}$$

$$P_a = \frac{(c+1)e^{-m}m^{c+1}}{(c+1)!}$$

The L.H. side of the equation gives the probability of acceptance of material of lot quality p, that will result in the AOQL and the R.H. side of the equation (c+1) times the probability of exactly (c+1) defective items in the sample.

With the aid of tables of the cumulative Poisson distribution, values of m were determined for a range of values from c = 0 to c = 10 and these are shown in Table V.

When m was plotted against c it was found that with the exception of the cases where c = 0 and c = 1, the points lay on a reasonable straight line and the linear regression equation connecting m and c from c = 2 to c = 10 was determined as

$$m = 0.725c + 0.761$$

The graph of this equation is shown in Figure 14.

Values of m were determined to only two places of decimals as n is large in comparison with p.

Later, the writer determined for each value of c from c = 0 to c = 24 a factor K for determining the AOQL of a given single sampling the results of which were published in the Journal of the Institute of Quality Assurance (Ref.14)

Reference will again be made to this factor in the section on the present investigation.

5 (iii) The present investigation

In the present investigation the range of values of c was extended up to c = 30. It was considered that this was a sufficiently high value of c, although DEF-131A (Ref.2) and MIL-STD-105D (Ref.1) gives values of c up to c = 44, values of this order are associated with high AQL values and the occasions when such values would be used are few. Rather than take individual values of c as was done in the original investigation, the theorem was developed by taking the general case on its own.

In a single sampling plan where n is the sample size and c the acceptance number, the probability of acceptance of a lot of material whose incoming fraction defective is equal to p is given by

$$Pa = e^{-np}\left(1 + np + \frac{n^2p^2}{2!} + \text{———} + \frac{n^{c-1}p^{c-1}}{(c-1)!} + \frac{n^c p^c}{c!}\right)$$

If Q denotes the Average Outgoing Quality, then Q = p.Pa

$$= e^{-np}\left(p + np^2 + \frac{n^2p^3}{2!} + \text{———} + \frac{n^{c-1}p^c}{(c-1)!} + \frac{n^c p^{c+1}}{c!}\right)$$

Differentiating with respect to p we get:-

$$\frac{dQ}{dp} = e^{-np}\left(1 + 2np + 3\frac{n^2p^2}{2!} + \text{———} + \frac{c.n^{c-1}p^{c-1}}{(c-1)!} + \frac{(c+1)n^c p^c}{c!}\right)$$

$$- e^{-np}\left(np + n^2p^2 + \frac{n^3p^3}{2!} + \text{———} + \frac{n^c p^c}{(c-1)!} + \frac{n^{c+1}p^{c+1}}{c!}\right)$$

For maximum Q, $\frac{dQ}{dp} = 0$, from which

$$e^{-np}\left(1 + np + \frac{n^2p^2}{2!} + \text{———} + \frac{n^{c-1}p^{c-1}}{(c-1)!} + \frac{n^c p^c}{c!}\right)$$

$$= e^{-np} \frac{n^{c+1}p^{c+1}}{c!}$$

Multiply the numerator and denominator of the R.H. fraction by (c + 1). The L.H. side of the equation then becomes equal to

$$(c + 1)\, e^{-np} \frac{n^{c+1}p^{c+1}}{c!\,(c+1)}$$

$$= (c + 1)\, e^{-np} \frac{n^{c+1}p^{c+1}}{(c + 1)!}$$

The left hand side of the equation gives the probability of c or less defective items and the right hand side (c + 1) times the probability of exactly (c + 1) defective items.

In conjunction with the Computer Centre of the University, a programme was developed and values of m and Pa were determined. Details of the programme are given in Appendix 14 and the values of m and Pa together with their product in Table VI

Although in the original investigation (Ref.13) values of c were taken up to c = 10 and a linear relationship established between m and c, the present investigation has shown that this relationship is only approximate and is not true for higher values of c as is shown by the curvature of the graph in Figure 15.

As c is always a whole number and because of this the distribution is discrete as distinct from continuous, no attempt has been made to establish an equation connecting m and c for values of c in excess of c = 10. It is therefore recommended that the tabulated values of me be used for all corresponding values of c.

In the original investigation graphs of the fraction defective of the incoming material, p, to give the AOQL were plotted against the sample size n, for various values of c. It is considered that no useful purpose would be served by reproducing them all here. However, that for c = 20 is shown in Figure 16.

It would have been practically impossible to have produced graphs for all values of c and at the time of the original investigation it was considered sufficient just to show graphs for a few values of c, viz c = 0, c = 5 and c = 10.

Their object was to determine the value of p direct from the graph rather than by dividing m by the sample size, and for intermediate values of c values of p could be determined approximately by interpolation.

In the present example when n = 110, say, the value of p can be read from the L.H. scale of the graph as 0.145.

From table VI, at c = 20, m = 15.9206 and hence p = 15.9206/110 = 0.14473.

Also in the original investigation a graph of the probability of acceptance of incoming material to give the AOQL was plotted against the acceptance number c for values of c up to c = 10. This has been extended for values of c up to c = 30 and is shown in Figure 17.

Reference has already been made in the section on the background to the present investigation to a factor K developed by the writer.

As the AOQL is the product of p and Pa and as $p = \frac{m}{n}$, the AOQL may be written as $\frac{mPa}{n}$.

The factor K is the product of m and Pa and values of K for values of c ranging from c = 0 to c = 30, are shown in Table VI.

The table may be used in two ways :-

1. To determine the AOQL of any given sampling plan,
2. To design a sampling plan to assure a given AOQL.

In the former connection, suppose it is required to determine the AOQL of the single sampling plan n = 120, c = 3. Reference to Table IV shows that at c = 3, K = 1.9424. Hence the AOQL of the plan is 1.9424/120 = 0.01619 or approximately 1.62%

In the latter connection, suppose it is required to design a single sampling plan to assure an AOQL of 3% and that it is decided that the acceptance number be 4. The table shows that at this value of c, K = 2.5435. Hence the sample size n = 2.5435/0.03 = 85.

The plan is therefore:-

n = 85, c = 4.

The relationship between K and c is represented graphically in Figure 18. As was the case of the relationship between m and c, the graph exhibits a marked curvature but again no attempt was made to determine a mathematical relationship because of the discrete nature of the distribution. In any case it would not necessarily follow that any such mathematical relationship would hold good for values of c in excess of 30. Such values could be determined using the method for c = 0 to c = 30.

## 5 (iv) Discussion of results.

The results of the foregoing investigation have, in general, been discussed in context as the nature of the investigation and the method of its treatment was such that it was considered preferable to do this rather than to treat the discussion as a separate issue.

It has been pointed out in the literature survey that those authors who have dealt with the topic have merely used graphical methods, taking arbitrary values of p, in order to determine that value which will result in the AOQL.

This method will result only in an approximate value for p and, aside from other considerations, can be very lengthy whereas the writer's theorem and the developments arising therefrom may be used to evaluate the value of p to as great a degree of accuracy as is required.

The authors named may consider that their method is sufficiently accurate for practical purposes and indeed this could well be so in some applications as p is small in comparison with n.

However, occasions might arise when it becomes necessary to evaluate p to a greater degree of accuracy and it was with this in mind that the investigation was carried out.

## 6. The AOQL of Double Sampling Plans

### 6 (i) Literature survey

Authors generally have made reference to published sampling tables based on AOQL values with little or none to the computation of the AOQL value of a double sampling plan.

Burr (Ref.3) does not specifically deal with the AOQL of double sampling plans but refers to published sampling tables based on AOQL values.

Duncan (Ref.4) merely states that the AOQ is given by the product of p and Pa and sketches the AOQ curve for the plan $n_1 = 50$, $c_1 = 2$; $n_2 = 100$, $c_2 = 6$ but does not attempt to show the computations involved.

Feigenbaum (Ref. 5 & 6) as Burr, does not deal with this but again refers to published sampling tables based on AOQL values.

Freeman, Friedman, Mosteller and Wallis (Ref.7) also do not cover this aspect but make reference to tables based on AOQL values.

Grant (Ref.8) makes no reference to the AOQL of double sampling plans.

Huitson and Keen (Ref.10) refer only to published sampling tables based on AOQL.

Again the only reference to the AOQL of double sampling plans is in respect of published sampling tables in the chapter on "Statistical methods in the quality function" by J. W. Enell in the book by Juran (Ref.11).

## 6 (ii) Background to the present investigation

As was the case of single sampling the writer considered that the topic was one which would lend itself well to analytical treatments. These have recently been carried out and publication of the method of treatment and results is pending (Ref.15).

The computation by analytical methods of the AOQL of double sampling plans and hence the value of p to give the AOQL is more complicated than in single sampling. The following relates to double sampling plans where $c_1 = 0$ and $c_2$ ranges from $c_2 = 1$ to $c_2 = 6$.

## 6 (iii) The present investigation

In a double sampling plan where $c_1$ is the acceptance number on the first sample and $c_2$ the acceptance number on combined samples, then the probability acceptance number on combined samples, then the probability of acceptance on the first sample is the probability of $c_1$ or less defective items. The probability of acceptance on the second sample is the probability of $c_1 + 1$ defective items in the first sample with $c_2 - (c_1 + 1)$ or less defective items in the second sample plus the probability of $c_1 + 2$ defective items in the first sample with $c_2 - (c_1 + 2)$ or less defective items in the second sample plus ------- etc. ------ the probability of $c_2$ defective items in the first sample with no defective items in the second sample. The probability of acceptance on combined samples is the sum of the probabilities of acceptance on the first and second samples.

Thus $Pa = Pa_1 + Pa_2$

The Average Outgoing Quality (AOQ) is equal to p times Pa where p is the fraction defective of the incoming material.

The Average Outgoing Quality Limit (AOQL) is the maximum value of the AOQ.

When $c_1 = 0$ and $c_2 = 1$

$$Pa_1 = e^{-n_1 p} \text{ and } Pa_2 = e^{-n_1 p}\, n_1 p e^{-n_2 p}$$

$$Pa = Pa_1 + Pa_2 = e^{-n_1 p} + n_1 p e^{-(n_1 + n_2)p}$$

$$Q = AOQ = p.Pa = pe^{-n_1 p} + n_1 p^2 e^{-(n_1 + n_2)p}$$

$$\frac{dQ}{dp} = e^{-n_1 p} - n_1 pe^{-n_1 p} + 2n_1 pe^{-(n_1 + n_2)p} - (n_1^2 + n_1 n_2)p^2 e^{-(n_1 + n_2)p}$$

For maximum Q, $\frac{dQ}{dp} = 0$ and as $e^{-n_1 p} \neq 0$

$$1 - n_1 p + 2n_1 pe^{-n_2 p} - (n_1^2 p^2 + n_1 n_2 p^2)e^{-n_2 p} = 0$$

Putting $n_1 p = m_1$ and $n_2 p = m_2$ we get

$$1 - m_1 + 2m_1 e^{-m_2} - (m_1^2 + m_1 m_2)e^{-m_2} = 0$$

$$1 - m_1 = (m_1^2 + m_1 m_2 - 2m_1)e^{-m_2}$$

i.e. $$m_2 = \log_e \frac{m_1^2 + m_1 m_2 - 2m_1}{1 - m_1}$$

If $n_2 = kn_1$, $m_2 = km_1$

then $$km_1 = \log_e \frac{m_1^2 + km_1^2 - 2m_1}{1 - m_1}$$

i.e. $$m_1 = \frac{1}{k} \log_e \frac{(k+1)m_1^2 - 2m_1}{1 - m_1} = \frac{1}{k} \log_e \frac{2m_1 - (k+1)m_1^2}{m_1 - 1}$$

---

It must be pointed out that for $k = 1$ and the corresponding value of $m_1 = 1$, although both the numerator and the denominator of the expression

$\frac{1}{k} \log_e \frac{2m_1 - (k+1)m_1^2}{m_1 - 1}$ are equal to zero, and so indeterminate, the expression is nevertheless not meaningless because for $k = 1$:-

$$(2m_1 - 2m_1^2)e^{-m_1} = m_1 - 1$$

the solution of which is satisfied by $m_1 = 1$.

That this is so can be shown by considering a practical example. For a lot size of from 151 to 280 items and an AQL of 1.5%, DEF131A (Ref.2) quotes the following double sampling plan (normal inspection).

$n_1 = 20,\ c_1 = 0;\ n_2 = 20,\ c_2 = 1$

Now $Pa = Pa_1 + Pa_2 = e^{-n_1 p} + e^{-n_1 p} n_1 p e^{-n_2 p}$

and $Q = p.Pa = p.e^{-n_1 p} + e^{-(n_1+n_2)p} n_1 p^2$

As $n_1 = n_2 = 20$, i.e. $k = 1$, we may rewrite the expression for the Average Outgoing Quality as

$$\underline{Q = p.e.^{-20p} + 20p^2 e^{-40p}}$$

The computation of the AOQL is shown in Table VII and the AOQ curve in Figure 19.

It will be seen that the AOQL whose value is 0.02516 occurs when $p = 0.05$.

Thus $m_1 = n_1 p = 20 \times 0.05 = 1.0$

When $c_1 = 0$ and $c_2 = 2$

$$Pa_1 = e^{-n_1 p} \text{ and } Pa_2 = e^{-n_1 p} n_1 p \times e^{-n_2 p}(1 + n_2 p)$$

$$+ e^{-n_1 p} \frac{n_1^2 p^2}{2!} \times e^{-n_2 p}$$

$$= e^{-(n_1+n_2)p}(n_1 p + n_1 n_2 p^2) + e^{-(n_1+n_2)p} \frac{n_1^2 p^2}{2!}$$

$$= e^{-(n_1+n_2)p}(n_1 p + n_1 n_2 p^2 + \tfrac{1}{2} n_1^2 p^2)$$

Probability of acceptance on combined samples

$$Pa = Pa_1 + Pa_2$$

$$= e^{-n_1 p} + e^{-(n_1+n_2)p}(n_1 p + n_1 n_2 p^2 + \tfrac{1}{2} n_1^2 p^2)$$

Average Outgoing Quality (AOQ) = $Q = p.Pa$

$$= e^{-n_1 p} p + e^{-(n_1+n_2)p}(n_1 p^2 + n_1 n_2 p^3 + \tfrac{1}{2} n_1^2 p^3)$$

Differentiating with respect to p, we get:-

$$\frac{dQ}{dp} = e^{-n_1 p} - e^{-n_1 p} n_1 p$$

$$+ e^{-(n_1+n_2)p}(2n_1 p + 3n_1 n_2 p^2 + 3/2 n_1^2 p^2)$$

$$- e^{-(n_1+n_2)p}(n_1^2 p^2 + n_1^2 n_2 p^3 + \tfrac{1}{2} n_1^3 p^3 + n_1 n_2 p^2 + n_1 n_2^2 p^3 + \tfrac{1}{2} n_1^2 n_2 p^3)$$

For maximum Q, $\frac{dQ}{dp} = 0$

As $e^{-n_1 p} \neq 0$ and putting $n_1 p = m_1$ and $n_2 p = m_2$ we get:-

$$1 - m_1 + e^{-m_2}(2m_1 + 3m_1 m_2 + 3/2 m_1^2 - m_1^2 - m_1^2 m_2 - \tfrac{1}{2} m_1^3 - m_1 m_2 - m_1 m_2^2 - \tfrac{1}{2} m_1^2 m_2)$$

$$= 0$$

If $n_2 = kn_1$ i.e. $m_2 = km_1$ then

$$1 - m_1 + e^{-km_1}(2m_1 + 3km_1^2 + 1\tfrac{1}{2}m_1^2 - m_1^2 - km_1^3 - \tfrac{1}{2}m_1^3 - km_1^2 - k^2 m_1^3 - \tfrac{1}{2}km_1^3)$$

$$= 0$$

$$1 - m_1 + e^{-km_1}\left[2m_1 + (3k + 1\tfrac{1}{2} - 1 - k)m^2 - (k + \tfrac{1}{2} + k^2 + \tfrac{1}{2}k)m_1^3\right] = 0$$

$$1 - m_1 + e^{-km_1}\left[2m_1 + (2k + \tfrac{1}{2})m_1^2 - (k^2 + 1\tfrac{1}{2}k + \tfrac{1}{2})m_1^3\right] = 0$$

$$(m_1 - 1)e^{km_1} = 2m_1 + (2k + \tfrac{1}{2})m_1^2 - (k^2 + 1\tfrac{1}{2}k + \tfrac{1}{2})m_1^3$$

$$m_1 = \frac{1}{k}\log_e \frac{2m_1 + (2k + \tfrac{1}{2})m_1^2 - (k^2 + 1\tfrac{1}{2}k + \tfrac{1}{2})m_1^3}{m_1 - 1}$$

---

When $c_1 = 0$ and $c_2 = 3$

$$Pa = e^{-n_1 p} + e^{-(n_1+n_2)p}(n_1 p + n_1 n_2 p^2 + \tfrac{1}{2} n_1 n_2^2 p^3 + \tfrac{1}{2} n_1^2 p^2 + \tfrac{1}{2} n_1^2 n_2 p^3 + 1/6 n_1^3 p^3)$$

$$Q = pPa = e^{-n_1 p} p + e^{-(n_1+n_2)p}(n_1 p^2 + n_1 n_2 p^3 + \tfrac{1}{2} n_1 n_2^2 p^4 + \tfrac{1}{2} n_1^2 p^3$$

$$+ \tfrac{1}{2} n_1^2 n_2 p^4 + 1/6 n_1^3 p^4)$$

$$\frac{dQ}{dp} = e^{-n_1 p} - e^{-n_1 p} n_1 p + e^{-(n_1+n_2)p}\Big[2n_1 p + 3n_1 n_2 p^2 + 2n_1 n_2^2 p^3$$

$$+3/2 n_1^2 p^2 + 2n_1^2 n_2 p^3 + 2/3 n_1^3 p^3 - n_1^2 p^2 - n_1^2 n_2 p^3 - \tfrac{1}{2} n_1^2 n_2^2 p^4$$

$$-\tfrac{1}{2} n_1^3 p^3 - \tfrac{1}{2} n_1^3 n_2 p^4 - 1/6 n_1^4 p^4 - n_1 n_2 p^2 - n_1 n_2^2 p^3 - \tfrac{1}{2} n_1 n_2^3 p^4$$

$$-\tfrac{1}{2} n_1^2 n_2 p^3 - \tfrac{1}{2} n_1^2 n_2^2 p^4 - 1/6 n_1^3 n_2 p^4\Big]$$

For maximum Q, $\frac{dQ}{dp} = 0$

$e^{-n_1 p} \neq 0$. Put $n_1 p = m_1$ and $n_2 p = m_2$

If $n_2 = km_1$, then $m_2 = km_1$

Thus:-

$$1 - m_1 + e^{-km_1}\Big[2m_1 + 2m_1(km_1) + m_1(km_1)^2 - \tfrac{1}{2} m_1 (km_1)^3 + \tfrac{1}{2} m_1^2$$

$$+\tfrac{1}{2} m_1^2 (km_1) - m_1^2 (km_1)^2 + 1/6 m_1^3 - 2/3 m_1^3 (km_1) - 1/6 m_1\Big] = 0$$

$$1 - m_1 + e^{-km_1}\Big[2m_1 + 2Km_1^2 + k^2 m_1^3 - \tfrac{1}{2} k^3 m_1^4 + \tfrac{1}{2} m_1^2 + \tfrac{1}{2} km_1^3 - k^2 m_1^4$$

$$+1/6 m_1^3 - 2/3 km_1^4 - 1/6 m_1^4\Big] = 0$$

$$1 - m_1 + e^{-km_1}\Big[2m_1 + (2k+\tfrac{1}{2})m_1^2 + (k^2 + \tfrac{1}{2}k + 1/6)m_1^3 - (\tfrac{1}{2}k^3 + k^2 + 2/3k + 1/6)m_1^4\Big]$$

$$= 0$$

$$m_1 = \frac{1}{k} \log_e \frac{2m_1 + (2k+\tfrac{1}{2})m_1^2 + (k^2 + \tfrac{1}{2}k + 1/6)m_1^3 - (\tfrac{1}{2}k^3 + k^2 + 2/3k + 1/6)m_1^4}{m_1 - 1}$$

When $c_1 = 0$ and $c_2 = 4$

$$Pa = e^{-n_1 p} + e^{-n_1 p} n_1 p e^{-n_2 p}(1+n_2 p+\tfrac{1}{2}n_2^2 p^2+1/6 n_2^3 p^2)$$

$$+e^{-n_1 p}\tfrac{1}{2}n_1^2 p^2 e^{-n_2 p}(1+n_2 p+\tfrac{1}{2}n_2^2 p^2)$$

$$+e^{-n_1 p}1/6 n_1^3 p^3 e^{-n_2 p}(1+n_2 p)$$

$$+e^{-n_1 p}1/24 n_1^4 p^4 e^{-n_2 p}$$

$$= e^{-n_1 p}+e^{-(n_1+n_2)p}(n_1 p+n_1 n_2 p^2+\tfrac{1}{2}n_1 n_2^2 p^3+1/6 n_1 n_2^3 p^4+\tfrac{1}{2}n_1^2 p^2$$

$$+\tfrac{1}{2}n_1^2 n_2 p^3+\tfrac{1}{4}n_1^2 n_2^2 p^4+1/6 n_1^3 p^3+1/6 n_1^3 n_2 p^4+1/24 n_1^4 p^4)$$

$$Q = p.Pa = e^{-n_1 p} p = e^{-(n_1+n_2)p}(n_1 p^2+n_1 n_2 p^3+\tfrac{1}{2}n_1 n_2^2 p^4+1/6 n_1 n_2^3 p^5$$

$$+\tfrac{1}{2}n_1^2 p^3+\tfrac{1}{2}n_1^2 n_2 p^4+\tfrac{1}{4}n_1^2 n_2^2 p^5+1/6 n_1^3 p^4+1/6 n_1^3 n_2 p^5+1/24 n_1^4 p^5)$$

$$\frac{dQ}{dp} = e^{-n_1 p}-e^{-n_1 p}n_1 p+e^{-(n_1+n_2)p}(2n_1 p+3n_1 n_2 p^2+2n_1 n_2^2 p^2$$

$$+5/6 n_1 n_2^3 p^4+3/2 n_1^2 p^2+2n_1^2 n_2 p^3+5/4 n_1^2 n_2^2 p^4+2/3 n_1^3 p^3+5/6 n_1^3 n_2 p^4+$$

$$5/24 n_1^4 p^4)$$

$$-e^{-(n_1+n_2)p}(n_1^2 p^2+n_1^2 n_2 p^3+\tfrac{1}{2}n_1^2 n_2^2 p^4+1/6 n_1^2 n_2^3 p^5+\tfrac{1}{2}n_1^3 p^3+\tfrac{1}{2}n_1^3 n_2 p^4$$

$$+\tfrac{1}{4}n_1^3 n_2^2 p^5$$

$$+1/6 n_1^4 p^4+1/6 n_1^4 n_2 p^5+1/24 n_1^5 p^5+n_1 n_2 p^2+n_1 n_2^2 p^2+\tfrac{1}{2}n_1 n_2^3 p^4+1/6 n_1 n_2^4 p^5$$

$$+\tfrac{1}{2}n_1^2 n_2 p^3+\tfrac{1}{2}n_1^2 n_2^2 p^4+\tfrac{1}{4}n_1^2 n_2^3 p^5+1/6 n_1^3 n_2 p^4+1/6 n_1^3 n_2^2 p^5+1/24 n_1^4 n_2 p)$$

For maximum Q, $\frac{dQ}{dp} = 0$

$e^{-n_1 p} \neq 0$. Put $n_1 p = m_1$ and $n_2 p = m_2$

If $n_2 = kn_1$, then $m_2 = km_1$

Thus:-

$$1-m_1+e^{-km_1}(2m_1+3km_1^2+2k^2m_1^3+5/6k^3m_1^4+3/2m_1^2+2km_1^3+5/4k^2m_1^4+2/3m_1^3$$

$$+5/6km_1^4+5/24m_1^4-m_1^2-km_1^3-\tfrac{1}{2}k^2m_1^4-1/6k^3m_1^5-\tfrac{1}{2}m_1^3-\tfrac{1}{2}km_1^4-\tfrac{1}{4}k^2m_1^5$$

$$-1/6m_1^4-1/6km_1^5-1/24m_1^5-km_1^2-k^2m_1^3-\tfrac{1}{2}k^3m_1^4-1/6k^4m_1^5$$

$$-\tfrac{1}{2}km_1^3-\tfrac{1}{2}k^2m_1^4-\tfrac{1}{4}k^3m_1^5-1/6km_1^4-1/6k^2m_1^5-1/24km_1^5) = 0$$

$$m_1 = \frac{1}{k}\log_e\left(\left[2m_1+(2k+\tfrac{1}{2})m_1^2+(k^2+\tfrac{1}{2}k+1/6)m_1^3 + (1/3k^3+\tfrac{1}{4}k^2+1/6k+1/24)m_1^4-(1/6k^4+1/6k^3+2/3k^2+5/24k+1/24)m_1^5\right]\Big/ m_1-1\right)$$

When $c_1 = 0$ and $c_2 = 5$

$$Pa = e^{-n_1 p} + e^{-n_1 p} n_1 p e^{-n_2 p}(1 + n_2 p + \tfrac{1}{2} n_2^2 p^2 + 1/6 n_2^3 p^3 + 1/24 n_2^4 p^4)$$

$$+ e^{-n_1 p} \tfrac{1}{2} n_1^2 p^2 e^{-n_2 p}(1 + n_2 p + \tfrac{1}{2} n_2^2 p^2 + 1/6 n_2^3 p^3)$$

$$+ e^{-n_1 p} 1/6 n_1^3 p^3 e^{-n_2 p}(1 + n_2 p + \tfrac{1}{2} n_2^2 p^2)$$

$$+ e^{-n_1 p} 1/24 n_1^4 p^4 e^{-n_2 p}(1 + n_2 p)$$

$$+ e^{-n_1 p} 1/120 n_1^5 p^5 e^{-n_2 p}$$

$$Q = p.Pa = e^{-n_1 p} p + e^{-(n_1 + n_2)p}(n_1 p^2 + n_1 n_2 p^3 + \tfrac{1}{2} n_1 n_2^2 p^4 + 1/6 n_1 n_2^2 p^5$$

$$+ 1/24 n_1 n_2^4 p^6 + \tfrac{1}{2} n_1^2 p^3 + \tfrac{1}{2} n_1^2 n_2 p^4 + \tfrac{1}{4} n_1^2 n_2^2 p^5 + 1/12 n_1^2 n_2^3 p^6$$

$$+ 1/6 n_1^3 p^4 + 1/6 n_1^3 n_2 p^5 + 1/12 n_1^3 n_2^2 P^6 + 1/24 n_1^4 p^5 + 1/24 n_1^4 n_2 p^6$$

$$+ 1/120 n_1^5 p^6)$$

$$\frac{dQ}{dp} = e^{-m_1} - e^{-m_1} m_1 + e^{-(m_1 + m_2)}(2m_1 + 3m_1 m_2 + 2m_1 m_2^2 + 5/6 m_1 m_2^3$$

$$+ \tfrac{1}{4} m_1 m_2^4$$

$$+ 3/2 m_1^2 + 2m_1^2 m_2 + 5/4 m_1^2 m_2^2 + \tfrac{1}{2} m_1^2 m_2^3 + 2/3 m_1^3 + 5/6 m_1^3 m_2 + \tfrac{1}{2} m_1^3 m_2^2$$

$$+ 5/24 m_1^4 + \tfrac{1}{4} m_1^4 m_2 + 1/20 m_1^5)$$

$$- e^{-(m_1 + m_2)}(m_1^2 + m_1^2 m_2 + \tfrac{1}{2} m_1^2 m_2^2 + 1/6 m_1^2 m_2^3 + 1/24 m_1^2 m_2^4 + \tfrac{1}{2} m_1^3 + \tfrac{1}{2} m_1^3 m_2$$

$$+ \tfrac{1}{4} m_1^3 m_2^2 + 1/12 m_1^3 m_2^3 + 1/6 m_1^4 + 1/6 m_1^4 m_2 + 1/12 m_1^4 m_2^2 + 1/24 m_1^5 + 1/24 m_1^5 m_2$$

$$+ 1/120 m_1^6 + m_1 m_2 + m_1 m_2^2 + \tfrac{1}{2} m_1 m_2^3 + 1/6 m_1 m_2^4 + 1/24 m_1 m_2^5 + \tfrac{1}{2} m_1^2 m_2 + \tfrac{1}{2} m_1^2 m_2^2$$

$$+ \tfrac{1}{4} m_1^2 m_2^3 + \tfrac{1}{12} m_1^2 m_2^4 + \tfrac{1}{6} m_1^3 m_2 + 1/6 m_1^3 m_2^2 + \tfrac{1}{12} m_1^3 m_2^3 + \tfrac{1}{24} m_1^4 m_2$$

$$+ 1/24 m_1^4 m_2 + 1/120 m_1^5 m_2)$$

For maximum Q, $\frac{dQ}{dp} = 0$

$e^{-m_1} \neq 0$. If $n_2 = kn_1$ then $m_2 = km_1$ Thus:-

$$1-m_1+e^{-km_1}\Big(2m_1+3km_1^{2}+2k^{2}m_1^{3}+5/6k^{3}m_1^{4}+\tfrac{1}{4}k^{4}m_1^{5}+3/2m_1^{2}+2km_1^{3}$$

$$+5/4k^{2}m_1^{4}+\tfrac{1}{2}k^{3}m_1^{5}+2/3m_1^{3}+5/6km_1^{4}+\tfrac{1}{2}k^{2}m_1^{5}+5/24m_1^{4}+\tfrac{1}{4}km_1^{5}+1/20m_1^{5}$$

$$-m_1^{2}-km_1^{3}-\tfrac{1}{2}k^{2}m_1^{4}-1/6k^{3}m_1^{5}-1/24k^{4}m_1^{6}-\tfrac{1}{2}m_1^{3}-\tfrac{1}{2}km_1^{4}-\tfrac{1}{4}k^{2}m_1^{5}$$

$$-1/12k^{3}m_1^{6}-1/6m_1^{4}-1/6km_1^{5}-1/12k^{2}m_1^{6}-1/24m_1^{5}-1/24km_1^{6}-1/120_1^{5}-km_1^{2}$$

$$-k^{2}m_1^{3}-\tfrac{1}{2}k^{3}m_1^{4}-1/6k^{4}m_1^{5}-1/24k^{5}m_1^{6}-\tfrac{1}{2}km_1^{3}-\tfrac{1}{2}k^{2}m_1^{4}-\tfrac{1}{4}k^{3}m_1^{5}$$

$$-1/12k^{4}m_1^{6}-1/6km_1^{4}-1/6k^{2}m_1^{5}-1/12k^{3}m_1^{6}-1/24km_1^{5}-1/24k^{2}m_1^{6}-1/120km_1^{6}\Big)$$

$$= 0$$

$$m_1 = \frac{1}{k}\log_e\left(\frac{\left[2m_1+(2k+\tfrac{1}{2})m_1^{2}+(k^{2}+\tfrac{1}{2}k+1/6)m_1^{3}+(1/3k^{3}+\tfrac{1}{4}k^{2}+1/6k+1/24)m_1^{4}+(1/12k^{4}+1/12k^{3}+1/12k^{2}+1/24k+1/120)m_1^{5}-(1/24k^{5}+1/8k^{4}+1/6k^{3}+1/8k^{2}+1/120k+1/120)m_1^{6}\right]}{m_1-1}\right)$$

When $c_1=0$ and $c_2=6$

$$Pa = e^{-n_1p}+e^{-n_1p}n_1pe^{-n_2p}(1+n_2p+\tfrac{1}{2}n_2^2p^2+1/6n_2^3p^3+1/24n_2^4p^4+1/120n_2^5p^5)$$

$$+e^{-n_1p}\tfrac{1}{2}n_1^2p^2e^{-n_2p}(1+n_2p+\tfrac{1}{2}n_2^2p^2+1/6n_2^3p^3+1/24n_2^4p^4)$$

$$+e^{-n_1p}1/6n_1^3p^3e^{-n_2p}(1+n_2p+\tfrac{1}{2}n_2^2p^2+1/6n_2^3p^3)$$

$$+e^{-n_1p}1/24n_14p^4e^{-n_2p}(1+n_2p+\tfrac{1}{2}n_2^2p^2)$$

$$+e^{-n_1p}1/120n_1^5p^5e^{-n_2p}(1+n_2p)$$

$$+e^{-n_1p}1/720n_1^6p^6e^{-n_2p}$$

$$= e^{-n_1p}+e^{-(n_1+n_2)p}(n_1p+n_1n_2p^2+\tfrac{1}{2}n_1n_2^2p^3+1/6n_1n_2^3p^4+1/24n_1n_2^4p^5$$

$$+1/120n_1n_2^5p^6+\tfrac{1}{2}n_1^2p^2+\tfrac{1}{2}n_1^2n_2p^3+\tfrac{1}{4}n_1^2n_2^2p^4+1/12n_1^2n_2^3p^5+1/48n_1^2n_2^4p^6$$

$$+1/6n_1^3p^3+1/6n_1^3n_2p^4+1/12n_1^3n_2^2p^5+1/36n_1^3n_2^3p^6+1/24n_1^4p^4$$

$$+1/24n_1^4n_2p^5$$

$$+1/48n_1^4n_2^2p^6+1/120n_1^5p^5+1/120n_1^5n_2p^6+1/720n_1^6P^6)$$

$$Q = p.Pa = e^{-n_1p}p+e^{-(n_1n_2)p}(n_1p^2+n_1m_2p^3+\tfrac{1}{2}n_1n2^2p^4+1/6n_1n_2^3p^5$$

$$+1/24n_1n_2^4p^6+1/120n_1n_2^5p^7+\tfrac{1}{2}n_1^2p^3+\tfrac{1}{2}n_1^2n_2p^4+\tfrac{1}{4}n_1^2n_2^2p^5$$

$$+1/12n_1^2n_2^3p^6+1/48n_1^2n_2^4p^7+1/6n_1^3p^4+1/6n_1^3n_2p^5+1/12n_1^3n_2^2p^6$$

$$+1/36n_1^3n_2^3p^7+1/24n_1^4p^5+1/24n_1^4n_2p^6+1/48n_1^4n_2^2p^7+1/120n_1^5p^6$$

$$+1/120n_1^5n_2p^7+1/720n_1^6p^7)$$

$$\frac{dQ}{dp} = e^{-m_1}-e^{-m_1}m_1+e^{-(m_1+m_2)}(2m_1+3m_1m_2+2m_1m_2^2+5/6m_1m_2^3)$$

$$+\tfrac{1}{4}m_1m_2^4+7/120m_1m_2^5+3/2m_1^2+2m_1^2m_2+5/4m_1^2m_2^2+\tfrac{1}{2}m_1^2m_2^3$$

$$+7/48m_1^2m_2^4+2/3m_1^3+5/6m_1^3m_2+\tfrac{1}{2}m_1^3m_2^2+7/36m_1^3+5/24m_1^4$$

$$+\tfrac{1}{4}m_1^4m_2+7/48m_1^4m_2^2+1/20m_1^5+7/120m_1^5m_2+7/720m_1^6-m_1^2-m_1^2m_2$$

$$-\tfrac{1}{2}m_1^2m_2^2-1/6m_1^2m_2^3-1/24m_1^2m_2^4-1/120m_1^2m_2^5-1/3m_1^3-\tfrac{1}{2}m_1^3m_2$$

$$-\tfrac{1}{4}m_1^3m_2^2-1/12m_1^3m_2^3-1/48m_1^3m_2^4-1/6m_1^4-1/6m_1^4m_2-1/12m_1^4m_2^2$$

$$-1/36m_1^4m_2^3$$

$$-1/24m_1^5-1/24m_1^5m_2-1/48m_1^5m_2^2-1/120m_1^6-1/120m_1^6m_2-1/720m_1^7$$

$$-m_1m_2$$

$$-m_1m_2^2-\tfrac{1}{2}m_1m_2^3-1/6m_1m_2^4-1/24m_1m_2^5-1/120m_1m_2^6-\tfrac{1}{2}m_1^2m_2-\tfrac{1}{2}m_1^2m_2^2$$

$$-\tfrac{1}{4}m_1^2m_2^3-1/12m_1^2m_2^4-1/48m_1^2m_2^5-1/6m_1^3m_2-1/6m_1^3m_2^2-1/12m_1^3m_2^3$$

$$-1/36m_1^3m_2^4-1/24m_1^4m_2-1/24m_1^4m_2^2-1/48m_1^4m_2^3-1/120m_1^5m_2$$

$$-1/120m_1^5m_2^2-1/720m_1^6m_2)$$

For maximum Q, $\frac{dQ}{dp} = 0$

If $n_2 = kn_1$ i.e. $m_2 = km_1$ and as $e^{-m_1} \neq 0$

$$1-m_1+e^{-km_1}(2m_1+2km_1^2+k^2m_1^3+1/3k^3m_1^3+1/12k^4m_1^5+1/60k^5m_1^6-1/20k^6m_1^7$$

$$+\tfrac{1}{2}m_1^2+\tfrac{1}{2}km_1^3+\tfrac{1}{4}k^2m_1^3+1/12k^3m_1^5+1/48k^4m_1^6-7/240k^5m_1^7+1/6m_1^3+1/6km_1^4$$

$$+1/12k^2m_1^5+1/36k^3m_1^6-7/144k^4m_1^7+1/24m_1^4+1/24km_1^5+1/48k^2m_1^6$$

$$-7/144k^3m_1^7$$

$$+1/120m_1^5+11/720km_1^6-1/45k^2m_1^7+1/20m_1^6-7/720km_1^7-1/720m_1^7) = 0$$

$$m_1 = \frac{i}{k}\log_e\left\{\left[2m_1+(2k+\tfrac{1}{2})m_1^{2}+(k^{2}+\tfrac{1}{2}k+1/6)m_1^{3}+(1/3k^{3}+\tfrac{1}{4}k^{2}+1/6k+1/24)m_1^{4}+(1/12k^{4}+1/12k^{3}+1/12k^{2}+1/12k+1/120)m_1^{5}+(1/60k^{5}+1/48k^{4}+1/36k^{3}+1/48k^{2}+1/120k+1/20)m_1^{6}-(1/120k^{6}+7/240k^{5}+7/144k^{4}+7/144k^{3}+1/40k^{2}+7/720k+1/720)m_1^{7}\right]\Big/ m_1-1\right\}$$

Table VIII shows values of k with corresponding values of $m_1$. The values of k were decided upon by an examination of the Dodge-Romig Double Sampling Tables (Ref.16). Graphs of k against $m_1$ are shown in Figures 20 to 25.

Using regression methods, the best fitting curves have been determined giving the following results

$c_1 = 0; \ c_2 = 1$

$$m_1 = 1.6069-1.4771k+1.716k^2-1.3574k^3+0.67392k^4-0.18317k^5+0.020576k^6 \qquad (1)$$

$c_1 = 0; \ c_2 = 2$

$$m_1 = 2.1004-1.4627k+0.72702k^2-0.14815k^3 \qquad (2)$$

$c_1 = 0; \ c_2 = 3$

$$m_1 = 2.5700-1.5462k+0.57125k^2-0.084028k^3 \qquad (3)$$

$c_1 = 0; \ c_2 = 4$

$$m_1 = 2.2638-0.64807k+0.08500k^2 \qquad (4)$$

$c_1 = 0; \ c_2 = 5$

$$m_1 = 2.6742-0.77603k+0.091429k^2 \qquad (5)$$

$c_1 = 0; \ c_2 = 6$

$$m_1 = 2.8710-0.73717k+0.074286k^2 \qquad (6)$$

Although second degree polynomials give a better type of fit than first degree (linear) polynomials in the cases for $c_2 = 4$, $c_2 = 5$ and $c_2 = 6$, linear regression equations have also been determined for these cases giving the following results.

$c_1 = 0,\ c_2 = 4$

$$m_1 = 1.909038 - 0.2996k$$

$c_1 = 0,\ c_2 = 5$

$$m_1 = 2.0580 - 0.30060k$$

$c_1 = 0,\ c_2 = 6$

$$m_1 = 2.1586 - 0.27660k$$

As $r = -0.999$ with 95% confidence but limit of $-1.000$ to $-0.992$ when $c_2 = 4$, $-0.999$ with 95% confidence limits of $-1.000/-0.990$ when $c_2 = 5$ and again $-0.999$ with 95% confidence limits of $-1.000/-0.992$ when $c_2 = 6$, linear equations may be sufficiently accurate from a practical point of view for these values of $c_2$.

In DEF - 131A (Ref. 2) and MIL -STD 105D, (Ref. 1) doubling sampling plans have the same sample size for both first and second samples, i.e. $k = 1$.

When $c_1 = 0$, the equations for the probability of acceptance of material to give the AOQL when $c_1 = 1$, $c_2 = 2$, $c_2 = 3$, $c_2 = 4$, $c_2 = 5$ and $c_2 = 6$, become respectively:-

$$Pa = e^{-m_1} + e^{-2m_1}m_1 \qquad (7)$$

$$Pa = e^{-m_1} + e^{-2m_1}(m_1 + 1\tfrac{1}{2}m_1^2) \qquad (8)$$

$$Pa = e^{-m_1} + e^{-2m_1}(m_1 + 1\tfrac{1}{2}m_1^2 + 1\ 1/6m_1^3) \qquad (9)$$

$$Pa = e^{-m_1} + e^{-2m_1}(m_1 + 1\tfrac{1}{2}m_1^2 + 1\ 1/6m_1^3 + 5/8m_1^4) \qquad (10)$$

$$Pa = e^{-m_1} + e^{-2m_1}(m_1 + 1\tfrac{1}{2}m_1^2 + 1\ 1/6m_1^3 + 5/8m_1^4 + 31/120m_1^5) \qquad (11)$$

$$Pa = e^{-m_1} + e^{-2m_1}(m_1 + 1\tfrac{1}{2}m_1^2 + 1\ 1/6m_1^3 + 5/8m_1^4 + 31/120m_1^5 + 7/80m_1^6) \qquad (12)$$

In Philips Double Sampling Tables (Ref. 17) and Columbia Double Sampling Tables (Ref. 18), $n_2 = 2n_1$, i.e. $k = 2$.

In these conditions the above equations may be rewritten.

$$Pa = e^{-m_1} + e^{-3m_1}m_1 \qquad (13)$$

$$Pa = e^{-m_1} + e^{-3m_1}(m_1 + 2\tfrac{1}{2}m_1^2) \qquad (14)$$

$$Pa = e^{-m_1} + e^{-3m_1}(m_1 + 2\tfrac{1}{2}m_1^2 + 3\ 1/6m_1^3) \qquad (15)$$

$$Pa = e^{-m_1} + e^{-3m_1}(m_1 + 2\tfrac{1}{2}m_1^2 + 3\ 1/6m_1^3 + 2\ 17/24m_1^4 \qquad (16)$$

$$Pa = e^{-m_1} + e^{-3m_1}(m_1 + 2\tfrac{1}{2}m_1^2 + 3\ 1/6m_1^3 + 2\ 17/24m_1^4 + 1\ 91/120m_1^5) \qquad (17)$$

$$Pa = e^{-m_1} + e^{-3m_1}(m_1 + 2\tfrac{1}{2}m_1^2 + 3\ 1/6m_1^3 + 2\ 17/24m_1^4 + 1\ 91/120m_1^5 + 133/144m_1^6) \qquad (18)$$

Where necessary extension of the graphs connecting $m_1$ and $c_2$ to include k = 1 and k = 2 indicated that the curves were still smooth ones and hence the equations (1) to (6) of the graphs were used to determine the values of $m_1$ for k = 1 and k = 2.

These values were substituted in equations (7) to (18) to determine Pa. Values of $K = m_1 Pa$ are shown in Table IX for k = 1 and k = 2.

In the same way that Table VI was used to determine the AOQL of a single sampling plan or to design a single sampling plan to assure a given AOQL, Table IX may likewise be used in respect of double sampling plans.

In the former connection, if a double sampling plan is specified as

$$n_1 = 100,\ c_1 = 0;\ n_2 = 100,\ c_2 = 3.$$

from the table, at $c_2 = 3$ and k = 1, K = 0.99297.

Hence the AOQL of the plan is 0.099297/100

= 0.0099 or approximately 1%

In the latter connection suppose it is desired to assure an AOQL of 2% and that $c_2 = 2$, also that $n_2$ shall be twice $n_1$.

From the table at $c_2 = 2$ and k = 2

K = 0.54542: hence $n_1$ = 0.54542/0.02 = 27.

The plan is therefore:-

$$n_1 = 27,\ c_1 = 0;\ n_2 = 54,\ c_2 = 2.$$

Graphs of K against $c_2$ for k = 1 and k = 2 [Figures

26 and 27] indicate a slight curvature. Second degree polynomials have been determined giving the following results:-

k = 1

$$K = 0.18491 + 0.27824c_2 + 0.00062c_2^2$$

k = 2

$$K = 0.208956 + 0.173893c_2 + 0.00061c_2^2$$

As the coefficients of $c_2^2$ are small in each case, and in the foregoing $c_2^2$ has a maximum value of 36, separate linear regression equations were determined giving

k = 1

$$K = 0.185909 + 0.27824c_2$$

k = 2

$$K = 0.209879 + 0.173893c_2$$

As the range of values of $c_2$ from $c_2 = 1$ to $c_2 = 6$ represents a practical range in conjunction with $c_1 = 0$ no attempt was made to evaluate a prediction equation for $m_1$ in respect of values of $c_2$ in excess of $c_2 = 6$, although in the case of k = 1, the relationship between $m_1$ and $c_2$ is almost linear.

## 6 (iv) Discussion of results.

As was the case in single sampling the

results have loaned themselves better to discussion in context and the comments in "Discussion of results" in single sampling are similarly applicable to double sampling.

The analysis could be extended to include other combinations of $c_1$ and $c_2$. However, the computation of $m_1$ for the general case is outside the scope of the present thesis but the complexity can be seen by considering its derivation.

The probability of acceptance on the first sample is the probability of $c_1$ or less defective items and is given by

$$Pa_1 = e^{-n_1 p}\left(1+n_1 p+\frac{n_1 p^2}{2!} + \frac{n_1{}^3 p^3}{3!} + \text{------} + \frac{n_1^{(c_1-1)} p^{(c_1-1)}}{(c_1-1)!} + \frac{n_1{}^{c_1} p^{c_1}}{c_1!}\right)$$

The probability of acceptance on the second sample is the probability of exactly $(c_1+1)$ defective items in the first sample with $(c_2-c_1-1)$ or less defective items in the second sample plus the probability of exactly $(c_1+2)$ defective items in the first sample with $(c_2-c_1-2)$ or less defective items in the second sample

plus etc.,

plus the probability of exactly $(c_2-1)$ defective items in the first sample with 1 or less defective items in the second sample, plus the probability of exactly $c_2$ defective items in the first sample with no defective items in the second sample and is given by:-

$$Pa_2 = e^{-n_1 p} \frac{n_1^{(c_1+1)} p^{(c_1+1)}}{(c_1+1)!} e^{-n_2 p}\left(1+n_2 p+\frac{n_2^2 p^2}{2!}+\frac{n_2^3 p^3}{3!} + \text{------}\right.$$

$$\left.\text{------} + \frac{n_2^{(c_2-c_1-2)} p^{(c_2-c_1-2)}}{(c_2-c_1-2)!} + \frac{n_2^{(c_2-c_1-1)} p^{(c_2-c_1-1)}}{(c_2-c_1-1)!}\right)$$

$$+e^{-n_1 p} \frac{n_1^{(c_1+2)} p^{(c_1+2)}}{(c_1+2)!} e^{-n_2 p}\left(1+n_2 p+\frac{n_2^2 p^2}{2!}+\frac{n_2^3 p^3}{3!} + \text{------}\right.$$

$$\left.\text{------} + \frac{n_2^{(c_2-c_1-3)} p^{(c_2-c_1-3)}}{(c_2-c_1-3)!} + \frac{n_2^{(c_2-c_1-2)} p^{(c_2-c_1-2)}}{(c_2-c_1-2)!}\right)$$

+ etc.,

$$+e^{-n_1 p} \frac{n_1^{(c_2-1)} p^{(c_2-1)}}{(c_2-1)!} e^{-n_2 p}(1+n_2 p)$$

$$+e^{-n_1 p} \frac{n_1^{c_2} p^{c_2}}{c_2!} e^{-n_2 p}$$

The probability of acceptance on combined samples $Pa = Pa_1 + Pa_2$ and as the Average Outgoing Quality, Q, is given by $Q = pPa$, Q is equal to:=

$$e^{-n_1 p}\left(p+n_1 p^2+\frac{n_1^2 p^3}{2!} + \frac{n_1^3 p^4}{3!} + \text{---} + \frac{n_1^{(c_2-1)} p^{c_1}}{(c_1-1)!} + \frac{n_1^{c_1} p^{(c_1+1)}}{c_1!}\right)$$

$$+e^{-(n_1+n_2)p}\left(\frac{n_1^{(c_1+1)} p^{(c_1+2)}}{(c_1+1)!} + \frac{n_1^{(c_1+1)} n_2 p^{(c_1+3)}}{(c_1+1)!}\right.$$

$$+ \frac{n_1^{(c_1+1)} n_2^2 p^{(c_1+4)}}{(c_1+1)!\ 2!}$$

$$+\frac{n_1^{(c_1+1)} n_2^{3} p^{(c_1+5)}}{(c_1+1)!\,3!} + \cdots + \frac{n_1^{(c_1+1)} n_2^{(c_2-c_1-2)} p^{c_2}}{(c_1+1)!\,(c_2-c_1-2)!}$$

$$+\left.\frac{n_1^{(c_1+1)} n_2^{(c_2-c_1-1)} p^{(c_2+1)}}{(c_1+1)!\,(c_2-c_1-1)}\right)$$

$$+e^{-(n_1+n_2)p}\left(\frac{n_1^{(c_1+2)} p^{(c_1+3)}}{(c_1+2)!} + \frac{n_1^{(c_1+2)} n_2 p^{(c_1+4)}}{(c_1+2)!} + \frac{n_1^{(c_1+2)} n_2^{2} p^{(c_1+5)}}{(c_1+2)\,2!}\right.$$

$$+\frac{n_1^{(c_1+2)} n_2^{3} p^{(c_1+6)}}{(c_1+2)!\,3!} + \cdots + \frac{n_1^{(c_1+1)} n_2^{(c_2-c_1-3)} p^{c_2}}{(c_1+2)!\,(c_2-c_1-3)!}$$

$$+\left.\frac{n_1^{(c_1+2)} n_2^{(c_2-c_1-2)} p^{(c_2+1)}}{(c_1+2)!\,(c_2-c_1-2)!}\right)$$

+ etc.,

$$+e^{-(n_1+n_2)p}\left(\frac{n_1^{(c_2-1)} p^{c_2}}{(c_2-1)!} + \frac{n_1^{(c_2-1)} n_2 p^{(c_2+1)}}{(c_2-1)!}\right)$$

$$+e^{-(n_1+n_2)p}\left(\frac{n_1^{c_2} p^{(c_2+1)}}{c_2!}\right)$$

If the expression for Q is differentiated with respect to p and equated to zero, $n_1p$ put equal to $m_1$ $n_2p$ put equal to $m_2$ and this in turn equal to $km_1$, as $e^{-n_1p}$ and $e^{-n_2p}$ are not equal to zero, an expression may be found for $m_1$, again in the form

$$m_1 = \frac{1}{k} \log_e(\text{some function of } m_1)$$

In order to determine values of $m_1$ for $c_1 = 0$ and the range of values of $c_2$ from $c_2 = 1$ to $c_2 = 6$, very good initial approximations were first made by trial and error.

From a practical point of view this method could not be applied to the general case and since nothing is then known about the location of the roots of the equation, i.e. no initial approximations are known, and as the nature of the equation is very complex convergence may well tend to be slow, and in fact convergence may not be achieved at all. Additional work will have to be carried out to see if convergence does or does not occur.

## 7 The ASN of Double Sampling Plans.

### 7 (i) Literature survey

Again, a literature survey has shown no evidence of an analytical approach to this topic.

Burr (Ref. 3) takes a particular double sampling plan, viz $n_1 = 100$, $c_1 = 2$; $n_2 = 200$, $c_2 = 5$, chooses arbitrary values of p, evaluates the ASN arithmetically and plots the ASN against p.

Duncan (Ref. 4), as Burr, takes only a numerical example, viz, $n_1 = 50$, $c_1 = 2$: $n_2 = 100$, $c_2 = 6$ and evaluates the ASN from arbitrary values of p.

Feigenbaum (Refs. 5 & 6) makes no reference to the ASN of double sampling plans.

Freeman, Friedman, Mosteller and Wallis (Ref. 7) make no reference to this aspect.

Grant (Ref. 8) merely makes relative comparisons of the amounts of inspection involved under single, double and multiple plans.

Hill (Ref. 9) only states a formula for determining the ASN, viz:-

ASN = first sample size + (probability that a second sample is needed) x (second sample size).

Huitson and Keen (Ref. 10) make no mention of this.

Juran's book (Ref.11) contains a chapter on "Statistical methods in the Quality Function" by J. W. Enell who does not deal specifically with this but shows graphically comparisons of the average number of items inspected in single and double sampling.

## 7 (ii) Background to the present investigation

From both practical and economic points of view it is important to know whether some given value of the fraction defective of the incoming material will result in the maximum value of the ASN, and in particular, the actual value of the fraction defective of the incoming material which will result in the maximum value of the ASN. It was with this in mind that the writer decided to treat the subject analytically as to choose arbitrary values of p in order to determine the maximum value of the ASN could be very tedious and time consuming, and would give only an approximate value of p to give the maximum ASN.

The treatment in the initial investigation (Ref. 19) which led to the development of second of the two theorems stated earlier was as follows:-

In a double sampling plan, the probability of acceptance on the first sample is given by

$$Pa_1 = e^{-n_1 p}\left(1+n_1 p+\frac{n_1^2 p^2}{2!}+\text{--------}+\frac{n_1^{c_1-1} p^{c_1-1}}{(c_1-1)!}+\frac{n_1^{c_1} p^{c_1}}{c_1!}\right)$$

The probability of a rejection on the first sample is given by

$$Pr_1 = 1-e^{-n_1 p}\left(1+n_1 p+\frac{n_1^2 p^2}{2!}+\text{--------}+\frac{n_1^{c_1-1} p^{c_1-1}}{(c_1-1)!}+\frac{n_1^{c_1} p^{c_1}}{c_1!}\right)$$

$$+\frac{n_1^{c_1+1} p^{c_1+1}}{(c_1+1)!}+\text{--------}+\frac{n_1^{c_2-1} p^{c_2-1}}{(c_2-1)!}+\frac{n_1^{c_2} p^{c_2}}{c_2!})$$

The probability of a decision on the first sample is given by $P_1 = Pa_1 + Pr_1$.

$$= 1-e^{-n_1 p}\left(\frac{n_1^{c_1+1} p^{c_1+1}}{(c_1+1)!}+\frac{n_1^{c_1+2} p^{c_1+2}}{(c_1+2)!}+\text{------}+\frac{n_1^{c_2-1} p^{c_2-1}}{(c_2-1)!}+\frac{n_1^{c_2} p^{c_2}}{c_2!}\right)$$

The probability of taking a second sample $P_2$ is equal to $1 - P_1$.

Hence:-

$$P_2 = e^{-n_1 p}\left\{\frac{n_1^{c_1+1} p^{c_1+1}}{(c_1+1)!} + \frac{n_1^{c_1+2} p^{c_1+2}}{(c_1+2)!} + \cdots\cdots + \frac{n_1^{c_2-1} p^{c_2-1}}{(c_2-1)!} + \frac{n_1^{c_2} p^{c_2}}{c_2!}\right\}$$

Differentiating with respect to p gives:-

$$\frac{dP_2}{dp} = e^{-n_1 p}\left\{\frac{n_1^{c_1+1} p^{c_1}}{c_1!} + \frac{n_1^{c_1+2} p^{c_1+1}}{(c_1+1)!} + \cdots\cdots + \frac{n_1^{c_2-1} p^{c_2-2}}{(c_2-2)!} + \frac{n_1^{c_2} p^{c_2-1}}{(c_2-1)!}\right\}$$

$$- e^{-n_1 p}\left\{\frac{n_1^{c_1+2} p^{c_1+1}}{(c_1+1)!} + \frac{n_1^{c_1+3} p^{c_1+2}}{(c_1+2)!} + \cdots\cdots + \frac{n_1^{c_2} p^{c_2-1}}{(c_2-1)!} + \frac{n_1^{c_2+1} p^{c_2}}{c_2!}\right\}$$

For the maximum value of $P_2$, $\frac{dP_2}{dp} = 0$, and as $e^{-n_1 p} \neq 0$, then

$$\frac{n_1^{c_1+1} p^{c_1}}{c_1!} = \frac{n_1^{c_2+1} p^{c_2}}{c_2!}$$

$$n_1^{c_2-c_1} p^{c_2-c_1} = \frac{c_2!}{c_1!}$$

$$n_1 p = \sqrt[(c_2-c_1)]{\frac{c_2!}{c_1!}} = m_1$$

---

It was found that if $c_1$ was retained constant and $c_2$ varied, the relationship between $m_1$ and $c_2$ was almost linear for a practical range of values of $c_2$ and of the form $m_1 = ac_2 + b$. When $c_1$ was zero and $c_2$ was within the range from $c_2 = 1$ to $c_2 = 7$, the equation was

$$m_1 = 0.396c_2 + 0.619$$

The graph of this equation is shown in Figure 28A which also shows the plot of the values of $m_1$. The closeness to linearity was examined by evaluating the coefficient of correlation r, where -

$$r = \frac{\sum c_2 m_1}{\sqrt{\sum (c_2)^2 \sum (m_1)^2}}$$ giving a value of 0.9928.

Linear regression equations for other values of $c_1$ up to $c_1 = 10$ with ranges of $c_2$ commencing at $c_2 = c_1 + 1$ had been determined and in all cases the coefficient of correlation approximated to unity. For example, when $c_1$ was equal to 2, for a range of values of $c_2$ from $c_2 = 3$ to $c_2 = 15$. the equation connecting $m_1$ and $c_2$ was found to be

$$m_1 = 0.424c_2 + 1.8$$

with a coefficient of correlation r = 0.993.

It was further found that the relationship between b and $c_1$ approximated very closely to linearity and could be represented by the regression equation

$$b = 0.559c_1 + 0.600,$$

and with the range of values of $c_1$ from $c_1 = 0$ to $c_1 = 10$ the coefficient of correlation was 0.9965.

Thus the general equation $m_1 = ac_2 + b$ could be rewritten as:=

$$m_1 = ac_2 + 0.559c_1 + 0.600$$

It was suggested that the value of "a" depended upon the values of $c_1$ and $c_2$ but was of the order of 0.43.

It was concluded that although the equation

$$m_1 = {}^{(c_2-c_1)}\sqrt{\frac{c_2!}{c_1!}}$$

gave an exact value of $m_1$ it mattered little whether one used this equation or the one immediately preceding it as $n_1$ was large in comparison with p and it was the value of p in which one was ultimately interested (i.e. $p = \frac{m_1}{n_1}$).

## 7 (iii) The present investigation

In the present investigation, values of $m_1$ were determined to a greater degree of accuracy with the aid of a computer programme developed in conjunction with the Computer Centre of the University, the range of values of $c_1$ being extended up to $c_1 = 30$. The values are shown in Tables X-0 to X-30 and details of the computer programme are given in appendix 15.

It was apparent that the relationship between $m_1$ and $c_2$ was again almost linear whatever the value of $c_1$. Hence, linear regression equations together with the corresponding coefficients of correlation were determined, giving the following results.

$c_1 = 0$

$m_1 = 0.3960c_2 + 0.619484$ $\qquad r = 1.000$

$\underline{c_1 = 1}$

$m_1 = 0.4183c_2 + 1.198058 \qquad r = 1.000$

$\underline{c_1 = 2}$

$m_1 = 0.4239c_2 + 1.792605 \qquad r = 1.000$

$\underline{c_1 = 3}$

$m_1 = 0.4267c_2 + 2.388179 \qquad r = 1.000$

$\underline{c_1 = 4}$

$m_1 = 0.4324c_2 + 2.939176 \qquad r = 1.000$

$\underline{c_1 = 5}$

$m_1 = 0.4354c_2 + 3.502442 \qquad r = 1.000$

$\underline{c_1 = 6}$

$m_1 = 0.4353c_2 + 4.097955 \qquad r = 1.000$

$\underline{c_1 = 7}$

$m_1 = 0.4307c_2 + 4.773094 \qquad r = 1.000$

$\underline{c_1 = 8}$

$m_1 = 0.4326c_2 + 5.338236 \qquad r = 1.000$

$\underline{c_1 = 9}$

$m_1 = 0.4351c_2 + 5.885883 \qquad r = 1.000$

$c_1 = 10$

$m_1 = 0.4350c_2 + 6.482102$  $r = 1.000$

$c_1 = 11$

$m_1 = 0.4561c_2 + 6.619657$  $r = 1.000$

$c_1 = 12$

$m_1 = 0.4583c_2 + 7.132006$  $r = 1.000$

$c_1 = 13$

$m_1 = 0.4602c_2 + 7.643155$  $r = 1.000$

$c_1 = 14$

$m_1 = 0.4620c_2 + 8.153259$  $r = 1.000$

$c_1 = 15$

$m_1 = 0.4637c_2 + 8.662520$  $r = 1.000$

$c_1 = 16$

$m_1 = 0.4651c_2 + 9.171022$  $r = 1.000$

$c_1 = 17$

$m_1 = 0.4665c_2 + 9.678748$  $r = 1.000$

$c_1 = 18$

$m_1 = 0.4678c_2 + 10.18603$  $r = 1.000$

$\underline{c_1 = 19}$

$m_1 = 0.4689c_2 + 10.69270$ $\quad r = 1.000$

$\underline{c_1 = 20}$

$m_1 = 0.4700c_2 + 11.19887$ $\quad r = 1.000$

$\underline{c_1 = 21}$

$m_1 = 0.4710c_2 + 11.70469$ $\quad r = 1.000$

$\underline{c_1 = 22}$

$m_1 = 0.4720c_2 + 12.21016$ $\quad r = 1.000$

$\underline{c_1 = 23}$

$m_1 = 0.4729c_2 + 12.71528$ $\quad r = 1.000$

$\underline{c_1 = 24}$

$m_1 = 0.4737c_2 + 13.21995$ $\quad r = 1.000$

$\underline{c_1 = 25}$

$m_1 = 0.4745c_2 + 13.72444$ $\quad r = 1.000$

$\underline{c_1 = 26}$

$m_1 = 0.4752c_2 + 14.22872$ $\quad r = 1.000$

$\underline{c_1 = 27}$

$m_1 = 0.4759c_2 + 14.73262$ $\quad r = 1.000$

$c_1 = 28$

$$m_1 = 0.4766c_2 + 15.23648 \qquad r = 1.000$$

$c_1 = 29$

$$m_1 = 0.4772c_2 + 15.74009 \qquad r = 1.000$$

$c_1 = 30$

$$m_1 = 0.4778c_2 + 16.24345 \qquad r = 1.000$$

The value of the coefficient of correlation, r, has been determined to the third place of decimals in each case, but to test further the closeness to linearity of the equation:-

$$m_1 = (c_2 - c_1)\sqrt{\frac{c_2!}{c_1!}}$$

95% confidence limits for r have been determined giving the following range of values.

| | |
|---|---|
| $c_1 = 0$ | 1.000/1.000 |
| $c_1 = 1$ | 0.999/1.000 |
| $c_1 = 2$ | 0.999/1.000 |
| $c_1 = 3$ | 0.999/1.000 |
| $c_1 = 4$ | 0.999/1.000 |
| $c_1 = 5$ | 1.000/1.000 |
| $c_1 = 6$ | 1.000/1.000 |
| $c_1 = 7$ | 1.000/1.000 |
| $c_1 = 8$ | 1.000/1.000 |

| | |
|---|---|
| $c_1 = 9$ | 1.000/1.000 |
| $c_1 = 10$ | 1.000/1.000 |
| $c_1 = 11$ | 1.000/1.000 |
| $c_1 = 12$ | 1.000/1.000 |
| $c_1 = 13$ | 1.000/1.000 |
| $c_1 = 14$ | 1.000/1.000 |
| $c_1 = 15$ | 1.000/1.000 |
| $c_1 = 16$ | 1.000/1.000 |
| $c_1 = 17$ | 1.000/1.000 |
| $c_1 = 18$ | 1.000/1.000 |
| $c_1 = 19$ | 1.000/1.000 |
| $c_1 = 20$ | 1.000/1.000 |
| $c_1 = 21$ | 1.000/1.000 |
| $c_1 = 22$ | 1.000/1.000 |
| $c_1 = 23$ | 1.000/1.000 |
| $c_1 = 24$ | 1.000/1.000 |
| $c_1 = 25$ | 1.000/1.000 |
| $c_1 = 26$ | 1.000/1.000 |
| $c_1 = 27$ | 1.000/1.000 |
| $c_1 = 28$ | 1.000/1.000 |
| $c_1 = 29$ | 1.000/1.000 |
| $c_1 = 30$ | 1.000/1.000 |

Graphs of the above linear regression equations are shown in Figure 28B. Differences in the slopes are not very marked and in fact in the earlier investigation it was suggested that this could be taken as 0.43 for all practical purposes. However, the present investigation has shown that this could no longer hold throughout the whole range of values of $c_1$.

Values of "a" and "b" for each value of $c_1$ together with their 95% confidence limits have been separately tabulated and these are shown in Table XI and Table XII respectively.

The relationship between $c_1$ and "a" and $c_1$ and "b" are shown graphically in Figure 29 and Figure 30 respectively.

It will be seen that in each graph there is a marked change in the nature of the relationship after $c_1 = 10$.

As far as the relationship between "a" and $c_1$ is concerned, because of the irregularity of the graph from $c_1 = 0$ to $c_1 = 10$, no attempt has been made to establish a relationship between these two variables for this range of values of $c_1$.

For the range $c_1 = 11$ to $c_1 = 30$ the relationship is much more clearly defined. Both first(linear) and second degree polynomial equations have been determined giving respecively the following results:-

$$a = 0.446803 + 0.001096c_1$$

$$a = 0.44681 + 0.001096c_1 - 1.5 \times 10^{-8} c_1^2$$

As was found in the earlier investigation there is a marked linear relationship between b and $c_1$ although its nature changes after $c_1 = 10$.

Linear regression equations connecting b and $c_1$ for values of $c_1$ from $c_1 = 0$ to $c_1 = 10$ and for values of $c_1$ from $c_1 = 11$ to $c_1 = 30$ have therefore been determined together with their respective coefficients of correlation.

These are as follows:-

$c_1 = 0$ to $c_1 = 10$

$b = 0.5875c_1 + 0.60927$

$r = 1.000$

95% confidence limits for r:- 1.000/1.000

$c_1 = 11$ to $c_1 = 30$

$b = 0.5062578c_1 + 1.066407$

$r = 1.000$

95% confidence limits for r:- 1.000/1.000

As far as the relationship between "a" and $c_1$ is concerned there is very little practical difference between the value of "a" using a first degree polynomial and that using a second degree polynomial. On plotting the linear equation it was found that all the points lay between the 95% confidence limits (Figure 31).

Hence the linear equation $0.446803 + 0.001096c_1$ may be used for the determination of "a" within the range of values of $c_1$ from $c_1 = 11$ to $c_1 = 30$.

The original equation.

$$m_1 = ac_2 + b$$

now becomes

$$\underline{m_1 = ac_2 + 0.5875c_1 + 0.60927}$$

for a range of values of $c_1$ from $c_1 = 0$ to $c_1 = 10$, the appropriate value of "a" being taken from Table XI and

$$m_1 = 1.0664 + 0.5063c_1 + 0.0011c_1c_2 + 0.4469c_2$$

for a range of values of $c_1$ from $c_1 = 11$ to $c_1 = 30$.

7 (iv) Discussion of results

Because of the nature of the investigation the results have been discussed in context as was the case of the AOQL of single and double sampling plans.

The formula:-

$$p = \frac{\sqrt[(c_2-c_1)]{\dfrac{c_2!}{c_1!}}}{n_1}$$

will give an exact value for p:-

The equations expressing $m_1$ in terms of $c_1$ and $c_2$ were developed as an alternative means for the determination of $m_1$ (and hence p). It was intended that this alternative method be used for values of $c_1$ and $c_2$ considered. It may nevertheless, with discrimination, be used to determine $m_1$ for values of $c_1$ and $c_2$ outside these ranges. For example when $c_1 = 40$ and $c_2 = 60$, the writer's second theorem gives $m_1$ as 50.77 and the alternative equation $m_1$ as 50.17.

The choice of method will be influenced by the degree of accuracy to which one wants to determine $m_1$. It must be pointed out that p is small in comparison with $n_1$ and when $m_1$ is divided by $n_1$ the difference in the value of p in the two cases be insignificant. Where accuracy is required it is recommended that the theorem be used in the determination of $m_1$ for all combinations of $c_1$ and $c_2$.

Somewhat out of context but from a purely mathematical point of view it is interesting to note that despite its non-linear appearence an equation of the form

$$Y = (x - u)\sqrt{\frac{x!}{u!}}$$

(u is a constant and $x > u$, x and u being integers) gives a relationship between y and x which is almost linear.

## 8 Conclusions and Summary

### 8 (i) Conclusions

The AOQL of single sampling plans:-

1. The probability of acceptance, Pa, of material of incoming quality such as will result in the AOQL is given by

$$Pa = \frac{(c+1)e^{-np}n^{c+1}p^{c+1}}{(c+1)!}$$

2. From $c = 2$ to $c = 10$ the relationship between $m$ and $c$ is almost linear and has been evaluated as $m = 725c + 0.761$ with a coefficient of correlation of 0.9984.

3. The equation can not (as was previously thought to be the case) be used as a prediction equation for $m$ for all values of $c$ because as the values of $c$ is increased curvature becomes quite marked.

4. For values of $c$ up to $c = 30$, corresponding values of $m$ may be read from the table.

5. The value of $c$ at 30 represents a practical value. However, for values of $c$ in excess of 30, corresponding values of $m$ may be determined by making use of the theorem. This will give an exact value of $m$.

6. The value of $m$ to give the AOQL is independent of the individual values of $n$ and $p$.

7. The value of $m$ to give the AOQL depends only on the acceptance number $c$.

8. The theorem may be used to determine the AOQL of any given single sampling plan.

9. The theorem may be used to design a single sampling plan to assure a given AOQL.

The AOQL of double sampling plans:-

10. The value of $m_1$ to give the AOQL is again independent of the individual values of $n_1$ and $p$.

11. The value of $m_1$ to give the AOQL is independent of the individual values of $n_1$ and $n_2$ but does depend on their ratio and on the acceptance numbers on the first and combined samples.

12. The value of $m_1$ can be expressed by an equation of the form

$$m_1 = \frac{1}{k} \log_e \left\{ \frac{\text{some function of } m_1}{\text{some other function of } m_1} \right\}$$

Although the investigation has been essentially confined to $c_1 = 0$ with $c_2$ ranging from $c_2 = 1$ to $c_2 = 6$, further exploration in the field has shown that:-

(i) The numerator consists of $c_2$ positive term and one negative term i.e. the total number of terms in the numerator is one more than the acceptance number on combined samples.

(ii) For a given value of $c_2$ the numerator has $c_2 - 1$ terms in common with that for $c_2 - 1$.

(iii) For a given value of $c_2$, the denominator is constant irrespective of the value of $c_2$ and the number of terms in the denominator is equal to $c_1 + 2$.

(iv) The denominator consists of one positive term and $c_1 + 1$ negative terms. It may be shown that:-

when $c_1 = 1$, the denominator is $m_1^2 - m_1 - 1$,

when $c_1 = 2$, the denominator is $\frac{1}{2}m_1^3 - \frac{1}{2}m_1^2 - m_1 - 1$

when $c_1 = 3$, the denominator is $1/6m_1^4 - 1/6m_1^3 - \frac{1}{2}m_1^2 - m_1 - 1$

when $c_1 == 4$, the denominator is $1/24m_1^5 - 1/24m_1^4 - 1/6m_1^3 - \frac{1}{2}m_1^2 - m_1 - 1$

The denominators also consist of common terms.

All the statements under 12. appear to hold good for any combination of $c_1$ and $c_2$.

13. The equations expressing $m_1$ in terms of k, i.e. $n_2/n_1$ may be used to predict $m_1$ for all practical values of k.

14. Arising from 13. the equations may then be used to determine the AOQL of any double sampling plan within the limits of $c_1$ and $c_2$ under consideration or conversly to design a double sampling plan to assure a given AOQL.

The ASN of double sampling plans:-

15. The value of $m_1$ to give the maximum value of the ASN may be expressed by the equation:-

$$m_1 = (c_2 - c_1)\sqrt{\frac{c_2!}{c_1!}}$$

It is thus independent of the value of $n_2$, the size of the second sample and gives an exact value of $m_1$.

16. For a given value of $c_1$, the relationship between $m_1$ and $c_2$ is almost linear (r = 1.000 for the values of $c_1$ and $c_2$ under consideration) and from a practical point of view can be expressed in the alternative form,

$$m_1 = ac_2 + b$$

17. The relationship between "a" and $c_1$ depends on the range of values of $c_1$.

(i) For $c_1 = 0$ to $c_1 = 10$, the relationship between "a" and $c_1$ is an irregular one and no attempt has been made to establish a relationship.

(ii) For $c_1 = 11$ to $c_1 = 30$ the relationship is more clearly defined. It is not a strict linear relationship but the graph of a linear equation falls between the 95% confidence limits for "a" (Figure 31). This relationship may be expressed as

$$a = 0.446803 + 0.001096c_1$$

18. The relationship between b and $c_1$ again depends on the range of values of $c_1$ and, as is the case of "a" there is a definite "break" at $c_1 = 10$.

(i) For $c_1 = 0$ to $c_1 = 10$, the relationship is linear and has been determined as:-

$$b = 0.5875c_1 + 0.60927$$

with a coefficient of correlation of 1.000 (95% confidence limits for r are 1.000/1.000)

(ii) For $c_1 = 11$ to $c_1 = 30$, the relationship is linear and has been determined as:-

$$b = 0.5062578c_1 + 1.066407$$

with a coefficient of correlation of 1.000 (95% confidence limits for r are 1.000/1.000).

19. Arising from 17. and 18. the original equation

$m_1 = ac_2 + b$ now becomes:-

$m_1 = ac_2 + 0.5875c_1 + 0.60927$ for $c_1 = 0$ to $c_1 = 10$

and $m_1 = 1.0664 + 0.5063c_1 + 0.0011c_1c_2 + 0.4469c_2$ for

$c_1 = 11$ to $c_1 = 30$.

The value of "a" is obtained from Table IX.

8 (ii) Summary

As indicated elsewhere in this thesis the authors of standard works on quality control have all taken arbitrary values ofp in order to determine the AOQL and the maximum ASN of a plan. The method is very time consuming and can give only an approximate value for p. It might be sufficiently accurate for some practical purposes but occasions could arise when it becomes necessary to determine p to a greater degree of accuracy than that obtained using the above method.

In the case of the AOQL of single sampling plans it is a straightforward matter if use is made of the writer's first theorem as this will give an exact value for p.

In the case of the AOQL of double sampling plans the computation of the value of p is less straightforward. Use can be made of the prediction equations for $m_1$ ——— and hence p ——— developed by the writer. They all follow the same general pattern and will give a value of p more accurate than that obtained using the method involving choosing arbitrary values of p and evaluating the AOQ for each value of p chosen.

In the case of the ASN of double sampling plans the value of p may be accurately determined by making use of the writer's second theorem. The equations on page have been put forward as an alternative, but not quite so accurate, method for the determination of p, but certainly more accurate than the method used by the authors stated.

## 9 Future Work

As already suggested in the section on "The AOQL of Double Sampling Plans", the analysis could be extended to cover all other combinations of $c_1$ and $c_2$. It would then be possible to prepare a table of factors for determining the AOQL of <u>any</u> double sampling plan.

However, with regard to another topic one tool of quality control already referred to is control charts. As with sampling inspection schemes these fall into two categories - attributes and variables. In the former category are charts for fraction defective, number defective and number of defects. There are a greater number of charts in the second category such as charts for individual values, mid-range, median, standard deviation etc., but the two most widely used are average and range charts.

Most of the authors referred to in this thesis treat the subject of their construction and use in controlling the process and in acceptance of the product very thoroughly and have correctly stated that in the case of variables charts the sample size is generally much less than that for attribute charts, but on the other hand, any such saving in the number of items inspected could be more than offset by the greater amount of time spent in taking actual measurements.

However, comparisons of their relative effectiveness are seldom made.

In order for a chart to be effective in detecting change it is vital that the change be detected as soon as possible after it has taken place. The conventional type of control chart is not as efficient as it should be in this respect and it is possible for some defective parts to be produced before the change is finally detected.

A chart which has proved much favour during the last decade particularly in the process industries or where the product is in a continuous stream such as in textile or confectionary manufacture is the cummulative sum chart or cussum chart as it is popularly called.

It is beyond the scope of the present work to discuss in detail the principle of construction and use of the cusum chart but although it was originally designed as a variables chart it is the writer's opinion that it might well act as an alternative to, or even a replacement of, number defective and number of defects charts.

Preliminary tests by the writer using a simulation technique, details of which are given in Reference 20, have so far indicated that a change in the process average is detected sooner with a cusum chart than with a number defective or number of defects chart. However, the tests are in their preliminary stages and although the results so far are very encouraging, much work remains to be done before any conclusive evidence can be obtained.

## 10. Tables

| p | np | Pa | AOQ = p.Pa |
|---|---|---|---|
| 0.0002 | 0.02 | 1.000 | 0.000200 |
| 0.0015 | 0.15 | 0.990 | 0.001405 |
| 0.0050 | 0.50 | 0.910 | 0.004550 |
| 0.0070 | 0.70 | 0.844 | 0.005908 |
| 0.0080 | 0.80 | 0.809 | 0.006472 |
| 0.010 | 1.00 | 0.736 | 0.007360 |
| 0.015 | 1.50 | 0.558 | 0.008370 |
| 0.020 | 2.00 | 0.406 | 0.008120 |
| 0.026 | 2.60 | 0.267 | 0.006942 |
| 0.030 | 3.00 | 0.199 | 0.005970 |
| 0.036 | 3.60 | 0.126 | 0.004536 |
| 0.040 | 4.00 | 0.092 | 0.003680 |
| 0.046 | 4.60 | 0.056 | 0.002576 |
| 0.050 | 5.00 | 0.040 | 0.002000 |
| 0.060 | 6.00 | 0.017 | 0.001020 |
| 0.070 | 7.00 | 0.007 | 0.000490 |
| 0.080 | 8.00 | 0.003 | 0.000240 |
| 0.090 | 9.00 | 0.001 | 0.000090 |

TABLE I    Evaluation of O.C. and AOQ curves for the single sampling plan m = 100, c = 1.

| p | np | $Pa_1$ | $Pr_1$ | $Pd_1 = Pa_1 + Pr_1$ | $P_2 = 1 - Pd_1$ |
|---|---|---|---|---|---|
| 0.0025 | 0.10 | 0.905 | 0.000 | 0.905 | 0.095 |
| 0.005 | 0.20 | 0.819 | 0.000 | 0.819 | 0.181 |
| 0.010 | 0.40 | 0.670 | 0.001 | 0.671 | 0.329 |
| 0.015 | 0.60 | 0.549 | 0.003 | 0.553 | 0.447 |
| 0.020 | 0.80 | 0.449 | 0.009 | 0.458 | 0.542 |
| 0.025 | 1.00 | 0.368 | 0.019 | 0.387 | 0.613 |
| 0.030 | 1.20 | 0.301 | 0.034 | 0.335 | 0.665 |
| 0.035 | 1.40 | 0.247 | 0.054 | 0.301 | 0.699 |
| 0.040 | 1.60 | 0.202 | 0.079 | 0.281 | 0.719 |
| 0.045 | 1.80 | 0.165 | 0.109 | 0.274 | 0.726 |
| 0.050 | 2.00 | 0.135 | 0.143 | 0.278 | 0.722 |
| 0.060 | 2.40 | 0.091 | 0.221 | 0.312 | 0.688 |
| 0.070 | 2.80 | 0.061 | 0.308 | 0.369 | 0.631 |
| 0.080 | 3.20 | 0.041 | 0.397 | 0.438 | 0.562 |
| 0.090 | 3.60 | 0.027 | 0.485 | 0.512 | 0.488 |
| 0.100 | 4.00 | 0.018 | 0.567 | 0.585 | 0.415 |
| 0.125 | 5.00 | 0.007 | 0.735 | 0.742 | 0.258 |
| 0.150 | 6.00 | 0.002 | 0.849 | 0.851 | 0.149 |
| 0.175 | 7.00 | 0.001 | 0.918 | 0.919 | 0.081 |
| 0.200 | 8.00 | 0.000 | 0.958 | 0.958 | 0.042 |
| 0.225 | 9.00 | 0.000 | 0.979 | 0.979 | 0.021 |
| 0.250 | 10.00 | 0.000 | 0.990 | 0.990 | 0.010 |

TABLE II Evaluation of the ASN for the Double Sampling Plan $n_1 = 40$, $c_1 = 0$; $n_2 = 60$, $c_2 = 3$, with complete inspection of the second sample

| $P_2(n_2)$ | ASN = $n_1+P_2(n_2)$ |
|---|---|
| 5.70 | 47.70 |
| 10.86 | 50.86 |
| 19.74 | 59.74 |
| 26.82 | 66.82 |
| 35.52 | 72.52 |
| 36.78 | 76.78 |
| 39.90 | 79.90 |
| 41.94 | 81.94 |
| 43.14 | 83.14 |
| 43.56 | 83.56 |
| 43.32 | 83.32 |
| 41.28 | 81.28 |
| 37.86 | 77.86 |
| 33.72 | 73.72 |
| 29.28 | 69.28 |
| 24.90 | 64.90 |
| 15.48 | 55.48 |
| 8.94 | 48.94 |
| 4.86 | 44.86 |
| 2.52 | 42.52 |
| 1.26 | 41.26 |
| 0.60 | 40.60 |

TABLE II - Continued from page 88.

| PLAN | np | Pa | 1-Pa | N-n | (1-Pa) x (N-n) | n + (1-Pa) x (N-n) |
|---|---|---|---|---|---|---|
| n = 58<br>c = 0 | 0.29 | 0.749 | 0.241 | 442 | 111 | 169 |
| n = 98<br>c = 1 | 0.49 | 0.913 | 0.087 | 402 | 35 | 133 |
| n = 133<br>c = 2 | 0.66 | 0.971 | 0.029 | 367 | 11 | 144 |

Table III Computation of ATI for the three single sampling plans n = 58, c = 0; n = 98, c = 1; n = 133, c = 2. When p = 0.5% and N = 500.

| c | np | | RATIO |
|---|---|---|---|
| | Pa = 0.1 | Pa = 0.9 | |
| 0 | 2.310 | 0.106 | 21.792 |
| 1 | 3.893 | 0.530 | 7.345 |
| 2 | 5.329 | 1.100 | 4.845 |

Table IV Computation for value of c for a single sampling plan whose O.C. curve passes through the two points Pa = 0.1, p = 0.07; Pa = 0.9, p = 0.01.

| c | m = np |
|---|---|
| 0 | 1.000 |
| 1 | 1.618 |
| 2 | 2.27 |
| 3 | 2.95 |
| 4 | 3.64 |
| 5 | 4.35 |
| 6 | 5.06 |
| 7 | 5.80 |
| 8 | 6.55 |
| 9 | 7.30 |
| 10 | 8.06 |

Table V Values of m to give AOQL in Single Sampling. (Original Investigation).

| c | m | Pa | K = mPa |
|---|---|---|---|
| 0 | 1.0000 | 0.3679 | 0.3679 |
| 1 | 1.6180 | 0.5191 | 0.8400 |
| 2 | 2.2695 | 0.6041 | 1.3711 |
| 3 | 2.9452 | 0.6595 | 1.9424 |
| 4 | 3.6395 | 0.6989 | 2.5435 |
| 5 | 4.3490 | 0.7285 | 3.1682 |
| 6 | 5.0712 | 0.7517 | 3.8120 |
| 7 | 5.8041 | 0.7705 | 4.4719 |
| 8 | 6.5464 | 0.7860 | 5.1457 |
| 9 | 7.2970 | 0.7992 | 5.8314 |
| 10 | 8.0549 | 0.8104 | 6.5277 |
| 11 | 8.8194 | 0.8202 | 7.2334 |
| 12 | 9.5900 | 0.8287 | 7.9476 |
| 13 | 10.3660 | 0.8363 | 8.6695 |
| 14 | 11.1471 | 0.8431 | 9.3984 |
| 15 | 11.9328 | 0.8492 | 10.1338 |
| 16 | 12.7228 | 0.8548 | 10.8751 |
| 17 | 13.5169 | 0.8598 | 11.6219 |
| 18 | 14.3147 | 0.8644 | 12.3738 |
| 19 | 15.1560 | 0.8687 | 12.1305 |
| 20 | 15.9206 | 0.8726 | 13.8918 |
| 21 | 16.7284 | 0.8762 | 14.6571 |
| 22 | 17.5391 | 0.8796 | 15.4265 |
| 23 | 18.3526 | 0.8827 | 16.1996 |
| 24 | 19.1687 | 0.8856 | 16.9763 |
| 25 | 19.9874 | 0.8884 | 17.7563 |
| 26 | 20.8485 | 0.8910 | 18.5396 |
| 27 | 21.6318 | 0.8934 | 19.3258 |
| 28 | 22.4574 | 0.8957 | 20.1150 |
| 29 | 23.2851 | 0.8979 | 20.9070 |
| 30 | 24.1148 | 0.8999 | 21.7016 |

Table VI

Values of $m_1$Pa and mPa to give AOQL in Single Sampling (Present Investigation).

| (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|
| $p$ | $e^{-20p}$ | $pe^{-20p}$ | $e^{-40p}$ | $p^2$ | $20p^2$ |
| 0.01 | 0.8187 | 0.008187 | 0.6703 | 0.0001 | 0.002 |
| 0.02 | 0.6703 | 0.013406 | 0.4493 | 0.0004 | 0.008 |
| 0.03 | 0.5488 | 0.016464 | 0.3012 | 0.0009 | 0.018 |
| 0.04 | 0.4493 | 0.017972 | 0.2019 | 0.0016 | 0.032 |
| 0.05 | 0.3679 | 0.018395 | 0.1353 | 0.0025 | 0.050 |
| 0.06 | 0.3012 | 0.018072 | 0.0907 | 0.0036 | 0.072 |
| 0.07 | 0.2466 | 0.017262 | 0.0608 | 0.0049 | 0.098 |
| 0.08 | 0.2019 | 0.016152 | 0.0408 | 0.0064 | 0.128 |
| 0.09 | 0.1653 | 0.014877 | 0.0273 | 0.0081 | 0.162 |
| 0.10 | 0.1353 | 0.013530 | 0.0183 | 0.0100 | 0.200 |
| 0.12 | 0.0907 | 0.010884 | 0.00823 | 0.0144 | 0.288 |
| 0.14 | 0.0608 | 0.008512 | 0.00370 | 0.0196 | 0.392 |
| 0.16 | 0.0408 | 0.006528 | 0.00166 | 010256 | 0.512 |
| 0.18 | 0.0273 | 0.004914 | 0.000747 | 0.0324 | 0.648 |
| 0.20 | 0.0183 | 0.003660 | 0.000335 | 0.0400 | 0.800 |

Table VII Computation of AOQ for the Double Sampling Plan $n_1 = 20$, $c_1 = 0$; $n_2 = 20$, $c_2 = 1$.

| (7) | (8) |
|---|---|
| (4)(6) | AOQ = (3)+(7) |
| 0.0013406 | 0.0095276 |
| 0.0035944 | 0.0170004 |
| 0.0054216 | 0.0218856 |
| 0.0064608 | 0.0244328 |
| 0.0067650 | 0.0251600 |
| 0.0065304 | 0.0246024 |
| 0.0059584 | 0.0232204 |
| 0.0052224 | 0.0213744 |
| 0.0044226 | 0.0192996 |
| 0.0036600 | 0.0171900 |
| 0.0023704 | 0.0132542 |
| 0.00145040 | 0.0099624 |
| 0.00084992 | 0.0073779 |
| 0.000484056 | 0.0053981 |
| 0.000268000 | 0.0039280 |

Table VII - Continued from page 93.

| VALUE OF $k = \frac{n_2}{n_1}$ | VALUE OF $m_1$ WHEN $c_1 = 0$ | | | |
|---|---|---|---|---|
| | $c_2 = 1$ | $c_2 = 2$ | $c_2 = 3$ | $c_2 = 4$ |
| 0.2 | 1.3704 | | | |
| 0.3 | 1.2865 | | | |
| 0.5 | 1.1646 | | | |
| 0.6 | 1.1195 | | | |
| 0.8 | 1.0499 | 1.3197 | | |
| 0.9 | | 1.2648 | | |
| 1.0 | 1.0000 | 1.2166 | 1.5111 | |
| 1.1 | 0.9804 | 1.1740 | | |
| 1.2 | | 1.1360 | 1.3916 | |
| 1.3 | | 1.1021 | | |
| 1.4 | | | 1.2946 | |
| 1.5 | 0.9268 | | | |
| 1.6 | | | 1.2144 | |
| 1.8 | | | 1.1473 | 1.3727 |
| 1.9 | | | | 1.3392 |
| 2.0 | 0.8969 | | 1.0904 | 1.3076 |
| 2.1 | | | | 1.2777 |
| 2.2 | | | | 1.2495 |
| 2.3 | | | | 1.2228 |

Table VIII Values of $m_1$ to give AOQL in Double Sampling for various values of $n_2/n_1$ and $c_2$ ($c_1 = 0$).

| | $c_2 = 1$ | | | |
|---|---|---|---|---|
| 2.4 | | | | |
| 2.5 | 0.8923 | | | |
| 2.6 | | | | |
| 2.7 | | | | |
| 2.8 | | | | |
| 2.9 | | | | |
| 3.0 | | | | |
| 3.1 | | | | |
| 3.2 | | | | |
| 3.3 | | | | |

Table VIII - Continued from Page 95.

| | $c_2 = 5$ | $c_2 = 6$ |
|---|---|---|
| 2.4 | 1.3384 | |
| 2.5 | 1.3055 | |
| 2.6 | 1.2746 | |
| 2.7 | 1.2455 | |
| 2.8 | 1.2181 | |
| 2.9 | | 1.3580 |
| 3.0 | | 1.3280 |
| 3.1 | | 1.2997 |
| 3.2 | | 1.2728 |
| 3.3 | | 1.2473 |

| $c_2$ | k = 1 | | k = 2 | |
|---|---|---|---|---|
| | Pa | $K = m_1 Pa$ | Pa | $K = m_1 Pa$ |
| 1 | 0.50322 | 0.50322 | 0.46851 | 0.42021 |
| 2 | 0.59792 | 0.72743 | 0.58655 | 0.54542 |
| 3 | 0.65712 | 0.99297 | 0.64616 | 0.70457 |
| 4 | 0.74946 | 1.27460 | 0.67768 | 0.88613 |
| 5 | 0.79066 | 1.57310 | 0.71629 | 1.06573 |
| 6 | 0.85465 | 1.88717 | 0.75386 | 1.28896 |

Table IX Values of Pa and $m_1$ Pa to give AOQL in Double Sampling for $n_2/n_1 = 1$ and $n_2/n_1 = 2$ ($c_1 = 0$).

$c_1 = 0$

| $c_2$ | $m_1$ |
|---|---|
| 1 | 1.00000 |
| 2 | 1.41421 |
| 3 | 1.81712 |
| 4 | 2.21336 |
| 5 | 2.60517 |
| 6 | 2.99380 |
| 7 | 3.38002 |

Table X - 0

$c_1 = 1$

| $c_2$ | $m_1$ |
|---|---|
| 2 | 2.00000 |
| 3 | 2.44949 |
| 4 | 2.88450 |
| 5 | 3.30975 |
| 6 | 3.72792 |
| 7 | 4.14068 |
| 8 | 4.54916 |
| 9 | 4.95416 |
| 10 | 5.35627 |

Table X - 1

$c_1 = 2$

| $c_2$ | $m_1$ |
|---|---|
| 3 | 3.00000 |
| 4 | 3.46410 |
| 5 | 3.91487 |
| 6 | 4.35588 |
| 7 | 4.78939 |
| 8 | 5.21693 |
| 9 | 5.63959 |
| 10 | 6.05817 |
| 11 | 6.47329 |
| 12 | 6.88542 |
| 13 | 7.29496 |
| 14 | 7.70220 |
| 15 | 8.10741 |

Table X - 2

$c_1 = 3$

| $c_2$ | $m_1$ |
|---|---|
| 4 | 4.00000 |
| 5 | 4.47214 |
| 6 | 4.93242 |
| 7 | 5.38356 |
| 8 | 5.82739 |
| 9 | 6.26521 |
| 10 | 6.69799 |
| 11 | 7.12649 |
| 12 | 7.55128 |
| 13 | 7.97284 |
| 14 | 8.39154 |
| 15 | 8.80770 |
| 16 | 9.22158 |
| 17 | 9.63340 |
| 18 | 10.04337 |
| 19 | 10.45163 |
| 20 | 10.85833 |

Table X - 3

$c_1 = 4$

| $c_2$ | $m_1$ |
|---|---|
| 5 | 5.00000 |
| 6 | 5.47723 |
| 7 | 5.94392 |
| 8 | 6.40217 |
| 9 | 6.85347 |
| 10 | 7.29892 |
| 11 | 7.73928 |
| 12 | 8.17553 |
| 13 | 8.60789 |
| 14 | 9.03690 |
| 15 | 9.46294 |
| 16 | 9.88631 |
| 17 | 10.30725 |
| 18 | 10.72600 |
| 19 | 11.14275 |
| 20 | 11.55765 |
| 21 | 11.97086 |
| 22 | 12.38251 |

Table X - 4

$c_1 = 5$

| $c_2$ | $m_1$ |
|---|---|
| 6 | 6.00000 |
| 7 | 6.48074 |
| 8 | 6.95205 |
| 9 | 7.41559 |
| 10 | 7.87257 |
| 11 | 8.32394 |
| 12 | 8.77046 |
| 13 | 9.21271 |
| 14 | 9.65119 |
| 15 | 10.08630 |
| 16 | 10.51838 |
| 17 | 10.94773 |
| 18 | 11.37458 |
| 19 | 11.79916 |
| 20 | 12.22164 |
| 21 | 12.64220 |
| 22 | 13.06098 |
| 23 | 13.47809 |
| 24 | 13.89367 |
| 25 | 14.30781 |

Table X - 5

$c_1 = 6$

| $c_2$ | $m_1$ |
|---|---|
| 7 | 7.00000 |
| 8 | 7.48331 |
| 9 | 7.95811 |
| 10 | 8.42573 |
| 11 | 8.88719 |
| 12 | 9.34331 |
| 13 | 9.79473 |
| 14 | 10.24199 |
| 15 | 10.68554 |
| 16 | 11.12573 |
| 17 | 11.56290 |
| 18 | 11.99731 |
| 19 | 12.42920 |
| 20 | 12.85877 |
| 21 | 13.28620 |
| 22 | 13.71165 |
| 23 | 14.13525 |
| 24 | 14.55715 |
| 25 | 14.97744 |
| 26 | 15.39623 |
| 27 | 15.81361 |
| 28 | 16.22966 |
| 29 | 16.64446 |
| 30 | 17.05809 |

Table X - 6

$c_1 = 7$

| $c_2$ | $m_1$ |
|---|---|
| 8 | 8.00000 |
| 9 | 8.48528 |
| 10 | 8.96281 |
| 11 | 9.43368 |
| 12 | 9.89877 |
| 13 | 10.35877 |
| 14 | 10.81426 |
| 15 | 11.26571 |
| 16 | 11.71353 |
| 17 | 12.15805 |
| 18 | 12.59956 |
| 19 | 13.03833 |
| 20 | 13.47457 |
| 21 | 13.90848 |
| 22 | 14.34022 |
| 23 | 14.76995 |
| 24 | 15.19781 |
| 25 | 15.62391 |
| 26 | 16.04837 |
| 27 | 16.47129 |
| 28 | 16.89276 |
| 29 | 17.31285 |
| 30 | 17.73165 |
| 31 | 18.14922 |
| 32 | 18.56563 |
| 33 | 18.98093 |
| 34 | 19.39518 |
| 35 | 19.80843 |
| 36 | 20.22072 |
| 37 | 20.63211 |
| 38 | 21.04261 |
| 39 | 21.45229 |
| 40 | 21.86116 |

Table X - 7

$c_1 = 8$

| $c_2$ | $m_1$ |
|---|---|
| 9 | 9.00000 |
| 10 | 9.48683 |
| 11 | 9.96655 |
| 12 | 10.44009 |
| 13 | 10.90817 |
| 14 | 11.37142 |
| 15 | 11.83033 |
| 16 | 12.28535 |
| 17 | 12.73682 |
| 18 | 13.18506 |
| 19 | 13.63034 |
| 20 | 14.07290 |
| 21 | 14.51295 |
| 22 | 14.95066 |
| 23 | 15.38621 |
| 24 | 15.81973 |
| 25 | 16.25136 |
| 26 | 16.68122 |
| 27 | 17.10940 |
| 28 | 17.53602 |
| 29 | 17.96115 |
| 30 | 18.38488 |
| 31 | 18.80729 |
| 32 | 19.22843 |
| 33 | 19.64837 |
| 34 | 20.06718 |
| 35 | 20.48490 |
| 36 | 20.90158 |
| 37 | 21.21727 |
| 38 | 21.73202 |
| 39 | 22.14585 |
| 40 | 22.55882 |
| 41 | 22.97095 |
| 42 | 23.38229 |
| 43 | 23.79285 |

Table X - 8

$c_1 = 9$

| $c_2$ | $m_1$ |
|---|---|
| 10 | 10.00000 |
| 11 | 10.48809 |
| 12 | 10.96961 |
| 13 | 11.44536 |
| 14 | 11.91596 |
| 15 | 12.38196 |
| 16 | 12.84381 |
| 17 | 13.30188 |
| 18 | 13.75652 |
| 19 | 14.20801 |
| 20 | 14.65659 |
| 21 | 15.10248 |
| 22 | 15.54589 |
| 23 | 15.98698 |
| 24 | 16.42591 |
| 25 | 16.86282 |
| 26 | 17.29783 |
| 27 | 17.73105 |
| 28 | 18.16259 |

| $c_2$ | $m_1$ |
|---|---|
| 29 | 18.59254 |
| 30 | 19.02099 |
| 31 | 19.44802 |
| 32 | 19.87370 |
| 33 | 20.29809 |
| 34 | 20.72126 |
| 35 | 21.14326 |
| 36 | 21.56415 |
| 37 | 21.98937 |
| 38 | 22.40278 |
| 39 | 22.82062 |
| 40 | 23.23752 |
| 41 | 23.65353 |
| 42 | 24.06866 |
| 43 | 24.48298 |
| 44 | 24.89649 |
| 45 | 25.30924 |

Table X - 9

$c_1 = 10$

| $c_2$ | $m_1$ | $c_2$ | $m_1$ |
|---|---|---|---|
| 11 | 11.00000 | 31 | 20.07388 |
| 12 | 11.48913 | 32 | 20.50391 |
| 13 | 11.97216 | 33 | 20.93258 |
| 14 | 12.44977 | 34 | 21.35994 |
| 15 | 12.92252 | 35 | 21.78606 |
| 16 | 13.39089 | 36 | 22.21100 |
| 17 | 13.85527 | 37 | 22.63481 |
| 18 | 14.31602 | 38 | 23.05753 |
| 19 | 14.77343 | 39 | 23.47921 |
| 20 | 15.22776 | 40 | 23.89990 |
| 21 | 15.67926 | 41 | 24.31963 |
| 22 | 16.12812 | 42 | 24.73844 |
| 23 | 16.57452 | 43 | 25.15637 |
| 24 | 17.01863 | 44 | 25.57345 |
| 25 | 17.46059 | 45 | 25.98971 |
| 26 | 17.90053 | 46 | 26.40518 |
| 27 | 18.33859 | 47 | 26.81988 |
| 28 | 18.77485 | 48 | 27.23385 |
| 29 | 19.20943 | 49 | 27.64711 |
| 30 | 19.64241 | 50 | 28.05969 |

Table X - 10

$c_1 = 11$

| $c_2$ | $m_1$ |
|---|---|
| 12 | 12.00000 |
| 13 | 12.49000 |
| 14 | 12.97430 |
| 15 | 13.45350 |
| 16 | 13.92810 |
| 17 | 14.39860 |
| 18 | 14.86250 |
| 19 | 15.32820 |
| 20 | 15.78810 |
| 21 | 16.24500 |
| 22 | 16.69910 |
| 23 | 17.15060 |
| 24 | 17.59960 |
| 25 | 18.04650 |
| 26 | 18.49120 |
| 27 | 18.93390 |
| 28 | 19.37470 |
| 29 | 19.81370 |
| 30 | 20.25110 |
| 31 | 20.68680 |

Table X - 11

$c_1 = 12$

| $c_2$ | $m_1$ |
|---|---|
| 13 | 13.00000 |
| 14 | 13.49070 |
| 15 | 13.97610 |
| 16 | 14.45670 |
| 17 | 14.93300 |
| 18 | 15.40520 |
| 19 | 15.87380 |
| 20 | 16.33890 |
| 21 | 16.80100 |
| 22 | 17.26010 |
| 23 | 17.71650 |
| 24 | 18.17040 |
| 25 | 18.62190 |
| 26 | 19.07120 |
| 27 | 19.51830 |
| 28 | 19.96350 |
| 29 | 20.40690 |
| 30 | 20.84840 |
| 31 | 21.28830 |
| 32 | 21.72660 |

Table X - 12

$\underline{c_1 = 13}$

| $c_2$ | $m_1$ |
|---|---|
| 14 | 14.00000 |
| 15 | 14.49140 |
| 16 | 14.97770 |
| 17 | 15.45960 |
| 18 | 15.93720 |
| 19 | 16.41100 |
| 20 | 16.88130 |
| 21 | 17.34830 |
| 22 | 17.81230 |
| 23 | 18.27350 |
| 24 | 18.73200 |
| 25 | 19.18800 |
| 26 | 19.64170 |
| 27 | 20.09326 |
| 28 | 20.54270 |
| 29 | 20.99020 |
| 30 | 21.43580 |
| 31 | 21.87970 |
| 32 | 22.32190 |
| 33 | 22.76250 |

Table X - 13

$\underline{c_1 = 14}$

| $c_2$ | $m_1$ |
|---|---|
| 15 | 15.00000 |
| 16 | 15.49190 |
| 17 | 15.97910 |
| 18 | 16.46200 |
| 19 | 16.94090 |
| 20 | 17.41620 |
| 21 | 17.88800 |
| 22 | 18.35670 |
| 23 | 18.82250 |
| 24 | 19.28540 |
| 25 | 19.74590 |
| 26 | 20.20390 |
| 27 | 20.65960 |
| 28 | 21.11310 |
| 29 | 21.56460 |
| 30 | 22.01420 |
| 31 | 22.46200 |
| 32 | 22.90800 |
| 33 | 23.35230 |
| 34 | 23.79510 |

Table X - 14

$c_1 = 15$

| $c_2$ | $m_1$ |
|---|---|
| 16 | 16.00000 |
| 17 | 16.49240 |
| 18 | 16.98040 |
| 19 | 17.46420 |
| 20 | 17.94420 |
| 21 | 18.42080 |
| 22 | 18.89400 |
| 23 | 19.36420 |
| 24 | 19.83150 |
| 25 | 20.29620 |
| 26 | 20.75830 |
| 27 | 21.21810 |
| 28 | 21.67570 |
| 29 | 22.13110 |
| 30 | 22.58450 |
| 31 | 23.03600 |
| 32 | 23.48570 |
| 33 | 23.93370 |
| 34 | 24.38010 |
| 35 | 24.82490 |

Table X - 15

$c_1 = 16$

| $c_2$ | $m_1$ |
|---|---|
| 17 | 17.00000 |
| 18 | 17.49290 |
| 19 | 17.98150 |
| 20 | 18.46610 |
| 21 | 18.94720 |
| 22 | 19.42490 |
| 23 | 19.89940 |
| 24 | 20.37090 |
| 25 | 20.83970 |
| 26 | 21.30590 |
| 27 | 21.76960 |
| 28 | 22.23110 |
| 29 | 22.69030 |
| 30 | 23.14740 |
| 31 | 23.60260 |
| 32 | 24.05590 |
| 33 | 24.50740 |
| 34 | 24.95730 |
| 35 | 25.40550 |
| 36 | 25.85210 |

Table X - 16

$c_1 = 17$

| $c_2$ | $m_1$ |
|---|---|
| 18 | 18.00000 |
| 19 | 18.49320 |
| 20 | 18.98240 |
| 21 | 19.46790 |
| 22 | 19.94980 |
| 23 | 20.42860 |
| 24 | 20.90420 |
| 25 | 21.37700 |
| 26 | 21.84710 |
| 27 | 22.31470 |
| 28 | 22.77990 |
| 29 | 23.24280 |
| 30 | 23.70360 |
| 31 | 24.16240 |
| 32 | 24.61920 |
| 33 | 25.07410 |
| 34 | 25.52730 |
| 35 | 25.97890 |
| 36 | 26.42880 |
| 37 | 26.87720 |

Table X - 17

$c_1 = 18$

| $c_2$ | $m_1$ |
|---|---|
| 19 | 19.00000 |
| 20 | 19.49360 |
| 21 | 19.98330 |
| 22 | 20.46950 |
| 23 | 20.95230 |
| 24 | 21.43190 |
| 25 | 21.90860 |
| 26 | 22.38260 |
| 27 | 22.85390 |
| 28 | 23.32280 |
| 29 | 23.78930 |
| 30 | 24.25360 |
| 31 | 24.71580 |
| 32 | 25.17610 |
| 33 | 25.63440 |
| 34 | 26.09090 |
| 35 | 26.54570 |
| 36 | 26.99880 |
| 37 | 27.45030 |
| 38 | 27.90030 |

Table X - 18

$c_1 = 19$

| $c_2$ | $m_1$ |
|---|---|
| 20 | 20.00000 |
| 21 | 20.49390 |
| 22 | 20.98410 |
| 23 | 21.47090 |
| 24 | 21.95440 |
| 25 | 22.43500 |
| 26 | 22.91260 |
| 27 | 23.38760 |
| 28 | 23.86010 |
| 29 | 24.33010 |
| 30 | 24.79790 |
| 31 | 25.26350 |
| 32 | 25.72710 |
| 33 | 26.18870 |
| 34 | 26.64840 |
| 35 | 27.10640 |
| 36 | 27.56260 |
| 37 | 28.01720 |
| 38 | 28.47020 |
| 39 | 28.92170 |

Table X - 19

$c_1 = 20$

| $c_2$ | $m_1$ |
|---|---|
| 21 | 21.00000 |
| 22 | 21.49420 |
| 23 | 21.98480 |
| 24 | 22.47220 |
| 25 | 22.95640 |
| 26 | 23.43770 |
| 27 | 23.91630 |
| 28 | 24.39230 |
| 29 | 24.86580 |
| 30 | 25.33690 |
| 31 | 25.80580 |
| 32 | 26.27270 |
| 33 | 26.73750 |
| 34 | 27.20030 |
| 35 | 27.66140 |
| 36 | 28.12070 |
| 37 | 28.57830 |
| 38 | 29.03430 |
| 39 | 29.48870 |
| 40 | 29.94170 |

Table X - 20

$c_1 = 23$

| $c_2$ | $m_1$ |
|---|---|
| 24 | 24.00000 |
| 25 | 24.49490 |
| 26 | 24.98670 |
| 27 | 25.47550 |
| 28 | 25.96150 |
| 29 | 26.44480 |
| 30 | 26.92570 |
| 31 | 27.40410 |
| 32 | 27.88030 |
| 33 | 28.35430 |
| 34 | 28.83630 |
| 35 | 29.29620 |
| 36 | 29.76430 |
| 37 | 30.23050 |
| 38 | 30.69510 |
| 39 | 31.15790 |
| 40 | 31.61910 |
| 41 | 32.07880 |
| 42 | 32.53700 |
| 43 | 32.99380 |

Table X - 23

$c_1 = 24$

| $c_2$ | $m_1$ |
|---|---|
| 25 | 25.00000 |
| 26 | 25.49510 |
| 27 | 25.98720 |
| 28 | 26.47640 |
| 29 | 26.96290 |
| 30 | 27.44680 |
| 31 | 27.92830 |
| 32 | 28.40750 |
| 33 | 28.88450 |
| 34 | 29.35930 |
| 35 | 29.83210 |
| 36 | 30.30300 |
| 37 | 30.77200 |
| 38 | 31.23930 |
| 39 | 31.70480 |
| 40 | 32.16870 |
| 41 | 32.63100 |
| 42 | 33.09180 |
| 43 | 33.55120 |
| 44 | 34.00910 |

Table X - 24

$c_1 = 21$

| $c_2$ | $m_1$ |
|---|---|
| 22 | 22.00000 |
| 23 | 22.49440 |
| 24 | 22.98550 |
| 25 | 23.47340 |
| 26 | 23.95820 |
| 27 | 24.44030 |
| 28 | 24.91970 |
| 29 | 25.39650 |
| 30 | 25.87100 |
| 31 | 26.34320 |
| 32 | 26.81320 |
| 33 | 27.28110 |
| 34 | 27.74710 |
| 35 | 28.21110 |
| 36 | 28.67340 |
| 37 | 29.13400 |
| 38 | 29.59290 |
| 39 | 30.65020 |
| 40 | 30.50590 |
| 41 | 30.96020 |

Table X - 21

$c_1 = 22$

| $c_2$ | $m_1$ |
|---|---|
| 23 | 23.00000 |
| 24 | 23.49470 |
| 25 | 23.98610 |
| 26 | 24.47450 |
| 27 | 24.95990 |
| 28 | 25.44270 |
| 29 | 25.92280 |
| 30 | 26.40050 |
| 31 | 26.87580 |
| 32 | 27.34890 |
| 33 | 27.81990 |
| 34 | 28.28890 |
| 35 | 28.75600 |
| 36 | 29.22120 |
| 37 | 29.68460 |
| 38 | 30.14630 |
| 39 | 30.60640 |
| 40 | 31.06500 |
| 41 | 31.52200 |
| 42 | 31.97760 |

Table X - 22

$c_1 = 25$

| $c_2$ | $m_1$ |
|---|---|
| 26 | 26.00000 |
| 27 | 26.49530 |
| 28 | 26.98760 |
| 29 | 27.47730 |
| 30 | 27.96420 |
| 31 | 28.44870 |
| 32 | 28.93080 |
| 33 | 29.41070 |
| 34 | 29.88840 |
| 35 | 30.36400 |
| 36 | 30.83760 |
| 37 | 31.30940 |
| 38 | 31.77930 |
| 39 | 32.24750 |
| 40 | 32.71400 |
| 41 | 33.17890 |
| 42 | 33.64220 |
| 43 | 34.10400 |
| 44 | 34.56440 |
| 45 | 35.02340 |

Table X - 25

$c_1 = 26$

| $c_2$ | $m_1$ |
|---|---|
| 27 | 27.00000 |
| 28 | 27.49550 |
| 29 | 27.98810 |
| 30 | 28.47810 |
| 31 | 28.96550 |
| 32 | 29.45050 |
| 33 | 29.93310 |
| 34 | 30.41360 |
| 35 | 30.89200 |
| 36 | 31.36830 |
| 37 | 31.84280 |
| 38 | 32.31530 |
| 39 | 32.78610 |
| 40 | 33.25520 |
| 41 | 33.72260 |
| 42 | 34.18840 |
| 43 | 34.65270 |
| 44 | 35.11550 |
| 45 | 35.57690 |
| 46 | 36.03690 |

Table X - 26

$c_1 = 27$

| $c_2$ | $m_1$ |
|---|---|
| 28 | 28.00000 |
| 29 | 28.49560 |
| 30 | 28.98850 |
| 31 | 29.47880 |
| 32 | 29.96660 |
| 33 | 30.45210 |
| 34 | 30.93530 |
| 35 | 31.41640 |
| 36 | 31.89540 |
| 37 | 32.37240 |
| 38 | 32.84760 |
| 39 | 33.32090 |
| 40 | 33.79250 |
| 41 | 34.26240 |
| 42 | 24.73070 |
| 43 | 35.19740 |
| 44 | 35.66260 |
| 45 | 36.12630 |
| 46 | 36.58870 |
| 47 | 37.04970 |

Table X - 27

$c_1 = 28$

| $c_2$ | $m_1$ |
|---|---|
| 29 | 29.00000 |
| 30 | 29.49580 |
| 31 | 29.98890 |
| 32 | 30.47950 |
| 33 | 30.96770 |
| 34 | 31.45360 |
| 35 | 31.93730 |
| 36 | 32.41900 |
| 37 | 32.89860 |
| 38 | 33.37630 |
| 39 | 33.85210 |
| 40 | 34.32620 |
| 41 | 34.79850 |
| 42 | 35.26920 |
| 43 | 35.73830 |
| 44 | 36.20580 |
| 45 | 36.67190 |
| 46 | 37.13660 |
| 47 | 37.59980 |
| 48 | 38.06170 |

Table X - 28

$c_1 = 29$

| $c_2$ | $m_1$ |
|---|---|
| 30 | 30.00000 |
| 31 | 30.49590 |
| 32 | 30.98920 |
| 33 | 31.48010 |
| 34 | 31.96870 |
| 35 | 32.45510 |
| 36 | 32.93930 |
| 37 | 33.42140 |
| 38 | 33.90160 |
| 39 | 34.37990 |
| 40 | 34.85640 |
| 41 | 35.33110 |
| 42 | 35.80420 |
| 43 | 36.27560 |
| 44 | 36.74550 |
| 45 | 37.21380 |
| 46 | 37.68070 |
| 47 | 38.14620 |
| 48 | 38.61030 |
| 49 | 39.07310 |

Table X - 29

$c_1 = 30$

| $c_2$ | $m_1$ |
|---|---|
| 31 | 31.00000 |
| 32 | 31.49600 |
| 33 | 31.98960 |
| 34 | 32.48080 |
| 35 | 32.96970 |
| 36 | 33.45640 |
| 37 | 33.94100 |
| 38 | 34.42370 |
| 39 | 34.90440 |
| 40 | 35.38330 |
| 41 | 35.86040 |
| 42 | 36.33580 |
| 43 | 36.80950 |
| 44 | 37.28170 |
| 45 | 37.75230 |
| 46 | 38.22140 |
| 47 | 38.68910 |
| 48 | 39.15540 |
| 49 | 39.62030 |
| 50 | 40.08400 |

Table X - 30

$$m_1 = ac_2 + b$$

| $c_1$ | a | b |
|---|---|---|
| 0 | 0.3960 | 0.619484 |
| 1 | 0.4183 | 1.198058 |
| 2 | 0.4239 | 1.792605 |
| 3 | 0.4267 | 2.388179 |
| 4 | 0.4324 | 2.939176 |
| 5 | 0.4354 | 3.502442 |
| 6 | 0.4353 | 4.097955 |
| 7 | 0.4307 | 4.773094 |
| 8 | 0.4326 | 5.338236 |
| 9 | 0.4351 | 5.885883 |
| 10 | 0.4350 | 6.482102 |
| 11 | 0.4561 | 6.619657 |
| 12 | 0.4583 | 7.132006 |
| 13 | 0.4602 | 7.643155 |
| 14 | 0.4620 | 8.153259 |
| 15 | 0.4637 | 8.662520 |
| 16 | 0.4651 | 9,171022 |
| 17 | 0.4665 | 9.678748 |
| 18 | 0.4678 | 10.18603 |
| 19 | 0.4689 | 10.69270 |
| 20 | 0.4700 | 11.19887 |
| 21 | 0.4710 | 11.70469 |
| 22 | 0.4720 | 12.21016 |
| 23 | 0.4729 | 12.71528 |
| 24 | 0.4737 | 13.21995 |
| 25 | 0.4745 | 13.72444 |
| 26 | 0.4752 | 14.22872 |
| 27 | 0.4759 | 15.23262 |
| 28 | 0.4766 | 15.23648 |
| 29 | 0.4772 | 15.74009 |
| 30 | 0.4778 | 16.24345 |

Table XI Values of "a" and "b" in the equation $m_1 = ac_2 + b$ to give maximum ASN in Double Sampling.

| $c_1$ | Lower Confidence Limit For "a" | Upper Confidence Limit For "a" | Lower Confidence Limit For "b" | Upper Confidence Limit For "b" |
|---|---|---|---|---|
| 0 | 0.3906 | 0.4013 | 0.595604 | 0.643364 |
| 1 | 0.4119 | 0.4248 | 1.155960 | 1.240156 |
| 2 | 0.4186 | 0.4293 | 1.740601 | 1.844609 |
| 3 | 0.4220 | 0.4313 | 2.328192 | 2.448166 |
| 4 | 0.4279 | 0.4369 | 2.873153 | 3.004198 |
| 5 | 0.4311 | 0.4396 | 3.432587 | 3.572296 |
| 6 | 0.4314 | 0.4391 | 4.022343 | 4.173567 |
| 7 | 0.4274 | 0.4339 | 4.689003 | 4.857184 |
| 8 | 0.4295 | 0.4358 | 5.250330 | 5.426142 |
| 9 | 0.4320 | 0.4382 | 5.795033 | 5.976734 |
| 10 | 0.4321 | 014379 | 4.386694 | 6.577505 |
| 11 | 0.4525 | 0.4598 | 6.538450 | 5.700863 |
| 12 | 0.4547 | 0.4618 | 7.049662 | 7.214350 |
| 13 | 0.4568 | 0.4637 | 7.559843 | 7.726467 |
| 14 | 0.4587 | 0.4654 | 8.068995 | 8.237524 |
| 15 | 0.4604 | 0.4669 | 8.577457 | 8.747582 |
| 16 | 0.4620 | 0.4683 | 9.085202 | 9.256841 |
| 17 | 0.4634 | 0.4696 | 9.592222 | 9.765274 |
| 18 | 0.4648 | 0.4708 | 10.09883 | 10.27323 |
| 19 | 0.4660 | 0.4719 | 10.60489 | 10.78051 |
| 20 | 0.4672 | 0.4729 | 11.11051 | 11.28724 |
| 21 | 0.4683 | 0.4738 | 11.61577 | 11.79361 |

Table XII to be continued.

| | | | | |
|---|---|---|---|---|
| 22 | 0.4693 | 0.4747 | 12.12079 | 12.29952 |
| 23 | 0.4702 | 0.4755 | 12.62542 | 12.80515 |
| 24 | 0.4711 | 0.4763 | 13.12970 | 13.31020 |
| 25 | 0.4720 | 0.4770 | 13.63372 | 13.81516 |
| 26 | 0.4727 | 0.4777 | 14.13765 | 14.31980 |
| 27 | 0.4735 | 0.4783 | 14.64117 | 14.82407 |
| 28 | 0.4742 | 0.4789 | 15.14468 | 15.32829 |
| 29 | 0.4749 | 0.4795 | 15.64791 | 15.83228 |
| 30 | 0.4755 | 0.4800 | 15.15101 | 16.33589 |

Table XII Upper and Lower 95% Confidence Limits for "a" and "b" in the Equation $m_1 = ac_2 + b$.

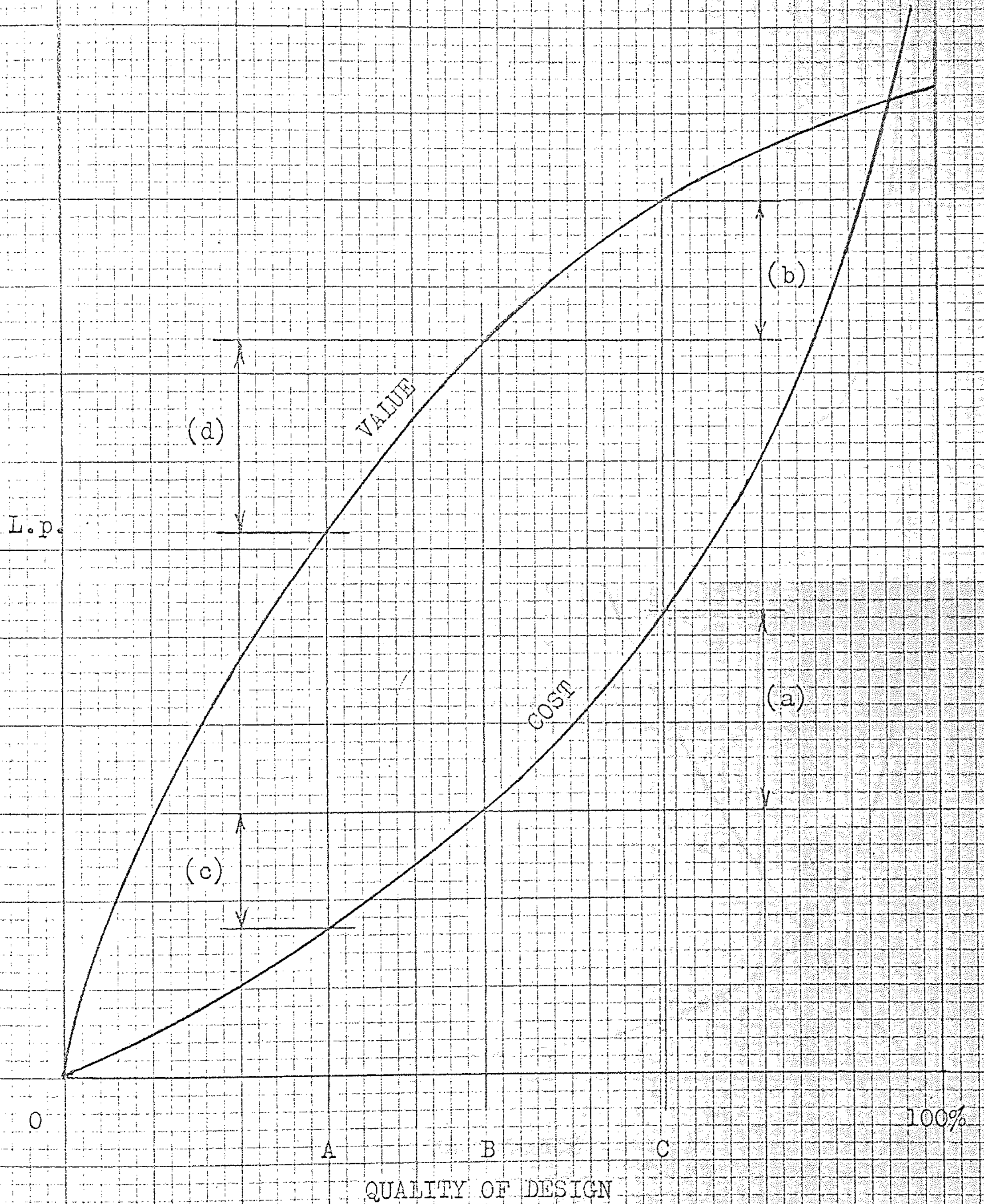



FIG.1 ECONOMICS OF QUALITY OF DESIGN

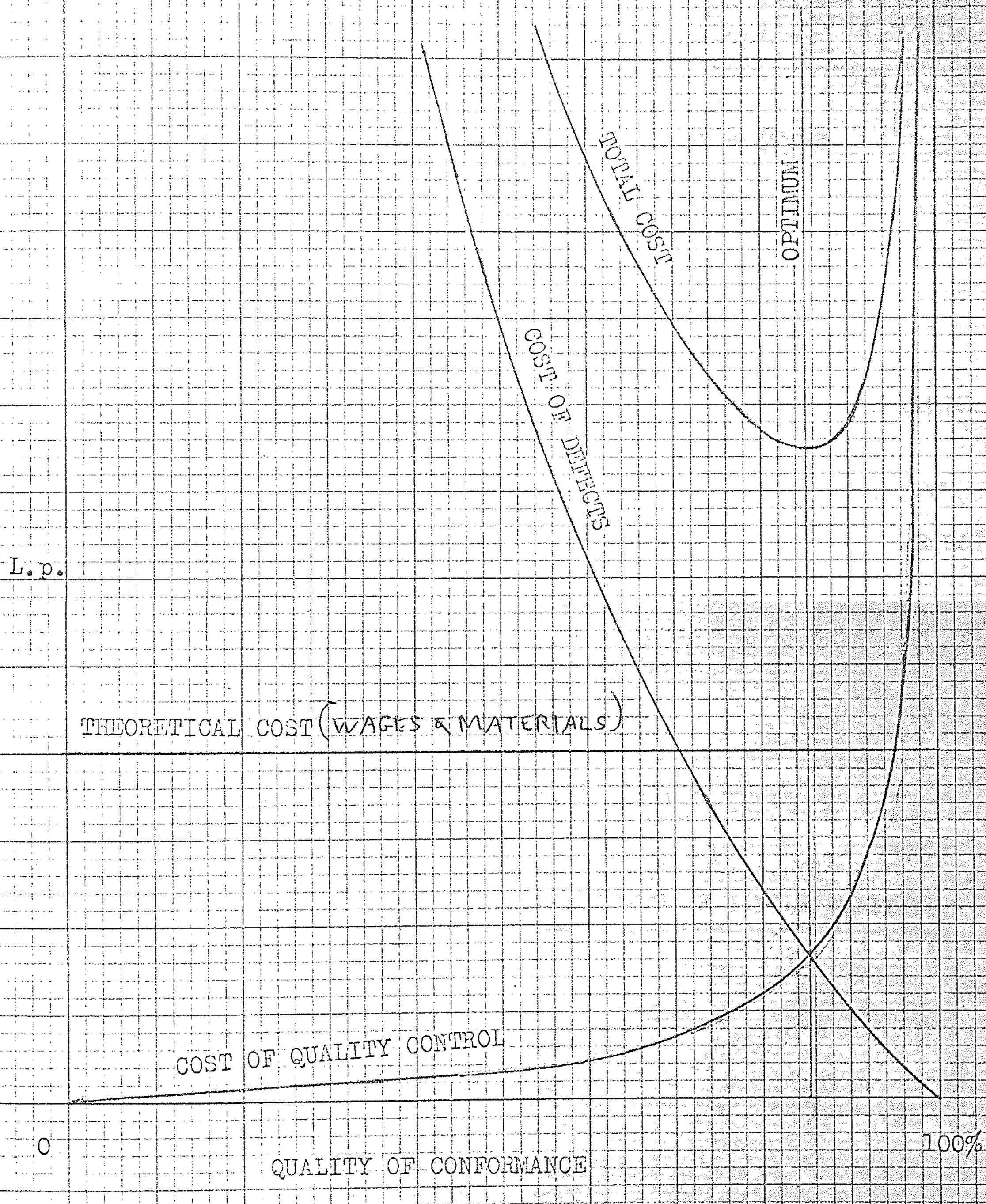



FIG.2 ECONOMICS OF QUALITY OF CONFORMANCE

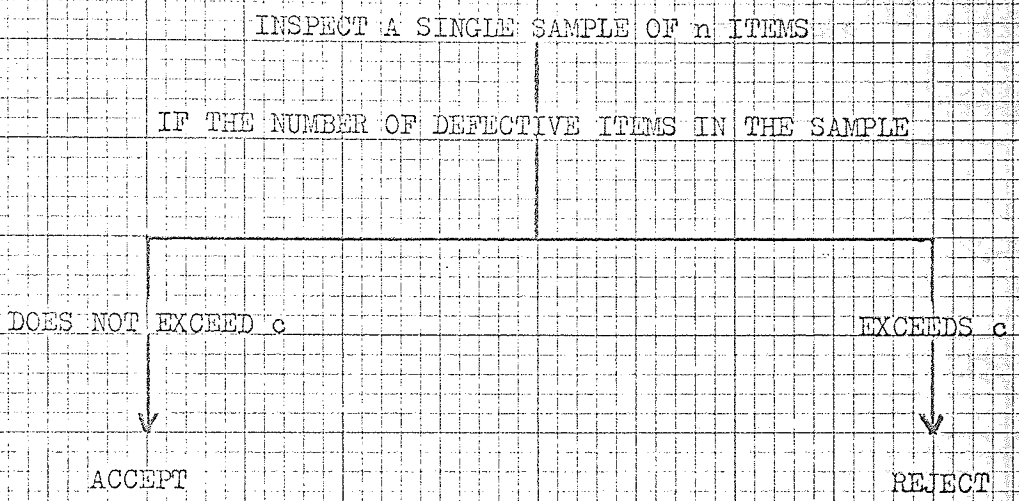



FIG.3 SCHEMATIC ARRANGEMENT OF SINGLE SAMPLING

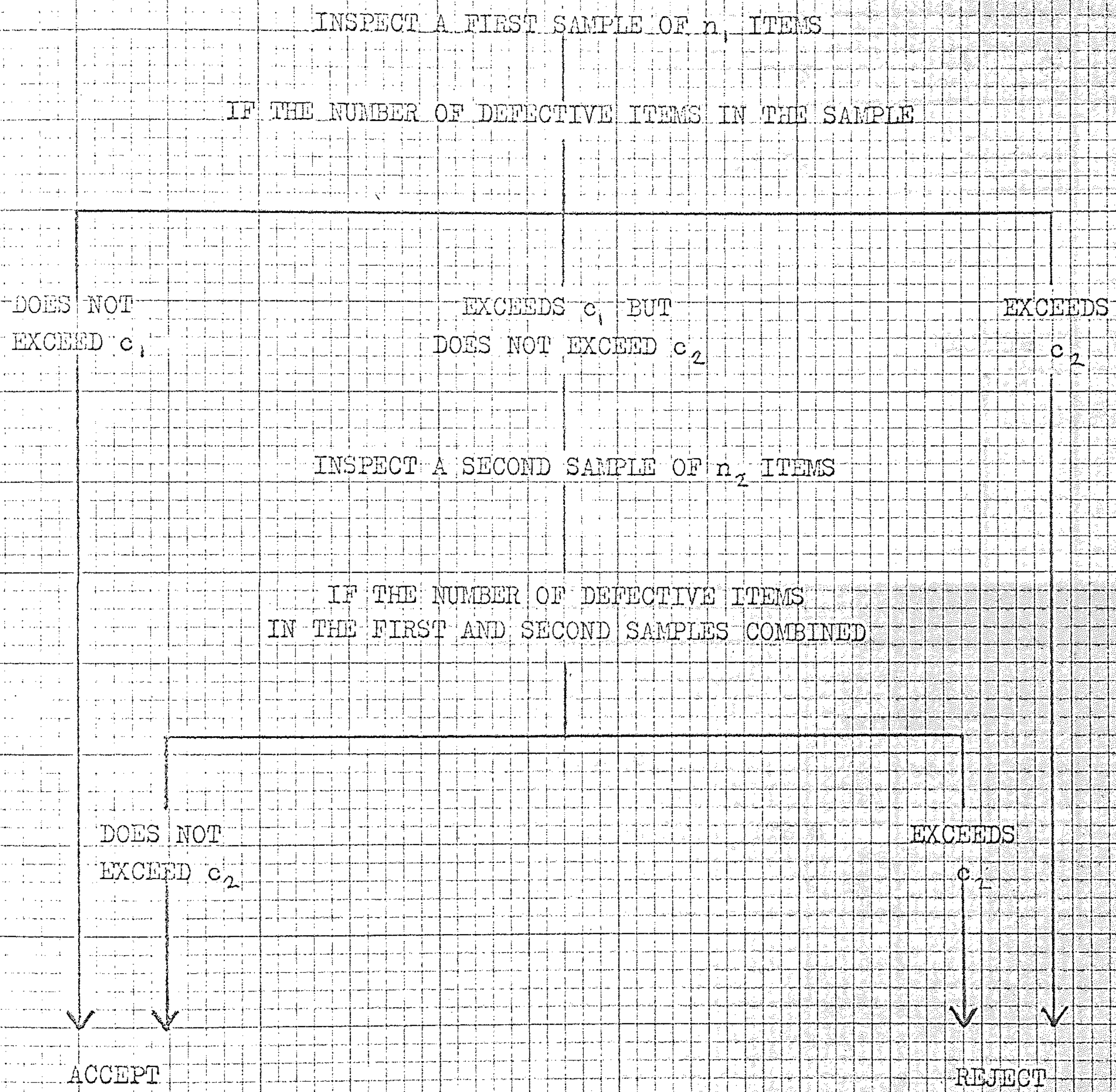



FIG.4 SCHEMATIC ARRANGEMENT OF DOUBLE SAMPLING

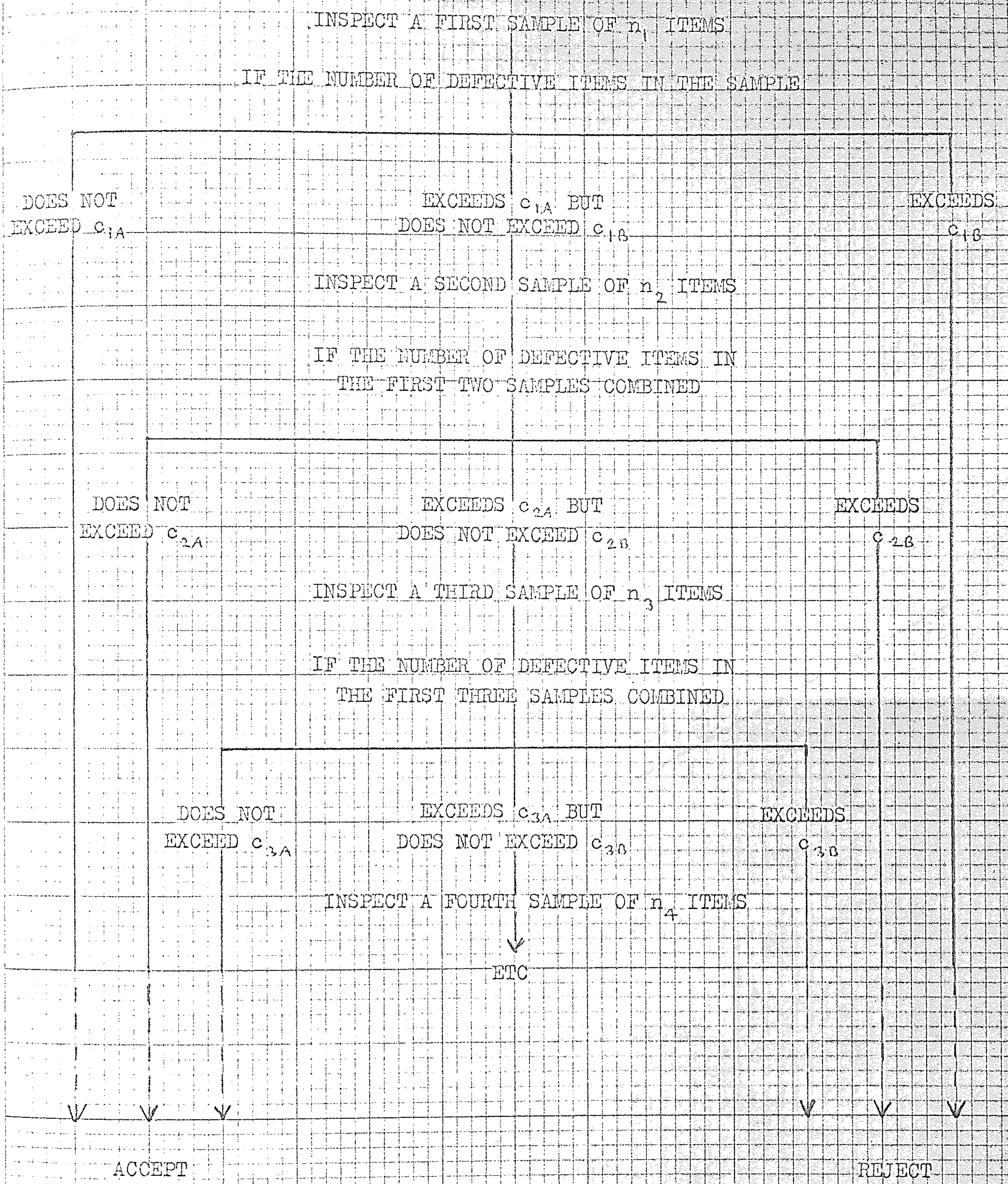



FIG.5 SCHEMATIC ARRANGEMENT OF MULTIPLE SAMPLING

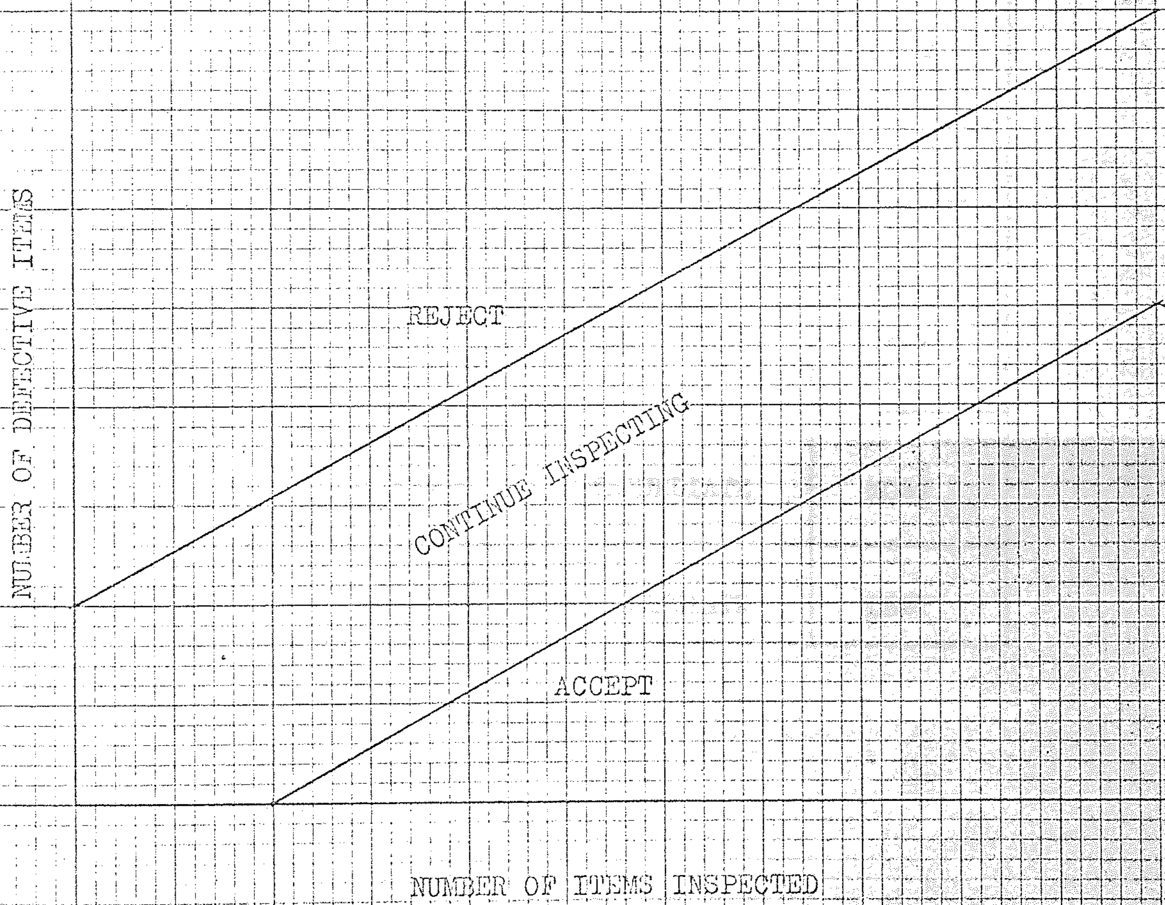



FIG. 6 SCHEMATIC ARRANGEMENT OF SEQUENTIAL SAMPLING

| | SINGLE | DOUBLE | MULTIPLE |
|---|---|---|---|
| PROTECTION | APPROXIMATELY EQUAL | | |
| TOTAL INSPECTION COST | MOST EXPENSIVE | INTERMEDIATE | LEAST |
| VARIABILITY OF INSPECTION LOAD | CONSTANT | VARIABLE | VARIABLE |
| ACCURATE ESTIMATION OF LOT QUALITY | BEST | INTERMEDIATE | WORST |
| AMOUNT OF RECORD KEEPING | LEAST | INTERMEDIATE | MOST |
| PSYCHOLOGICAL | WORST | INTERMEDIATE | BEST |

FIG.7 COMPARATIVE ADVANTAGES OF SINGLE, DOUBLE AND MULTIPLE SAMPLING

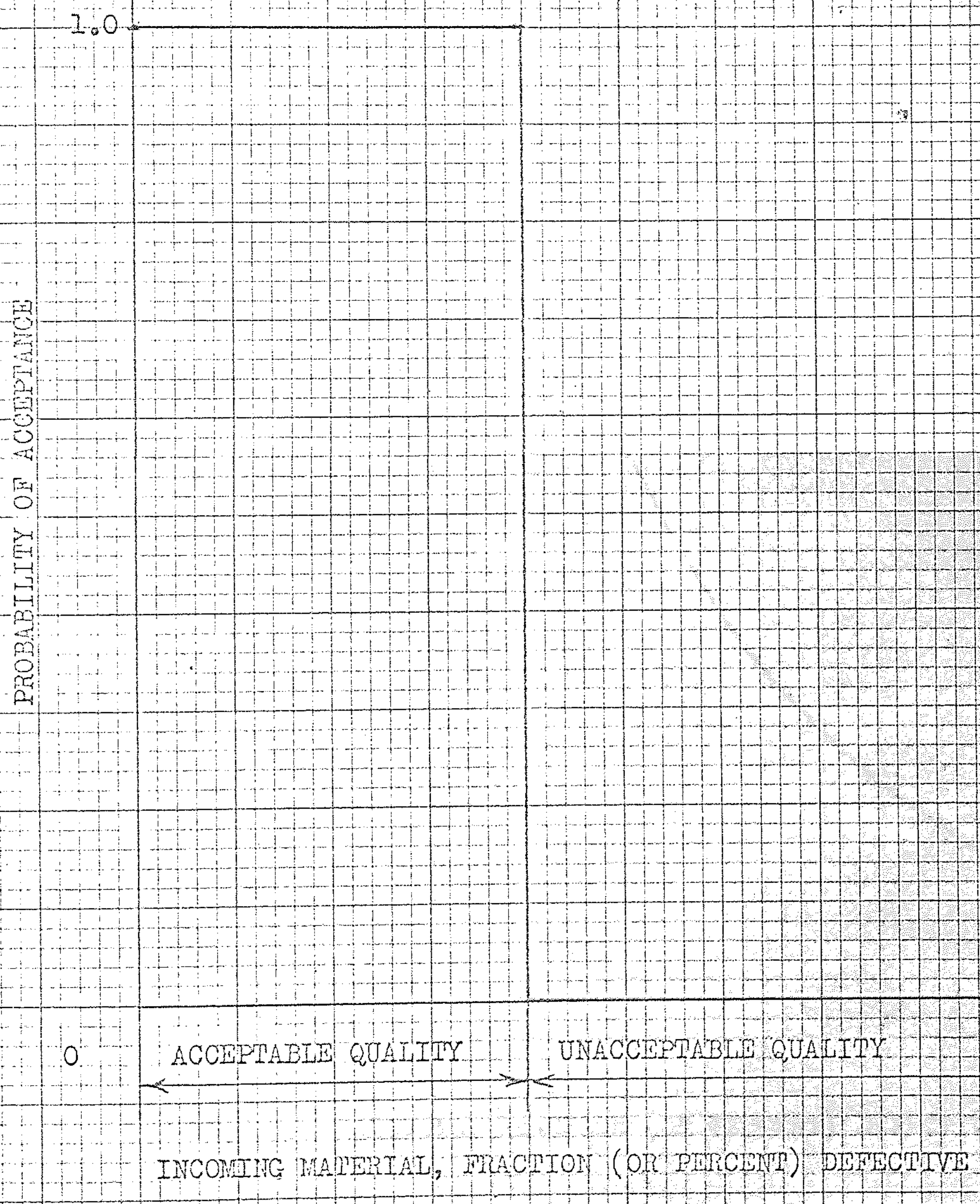



FIG.8 IDEAL SHAPE OF OPERATING CHARACTERISTIC CURVE

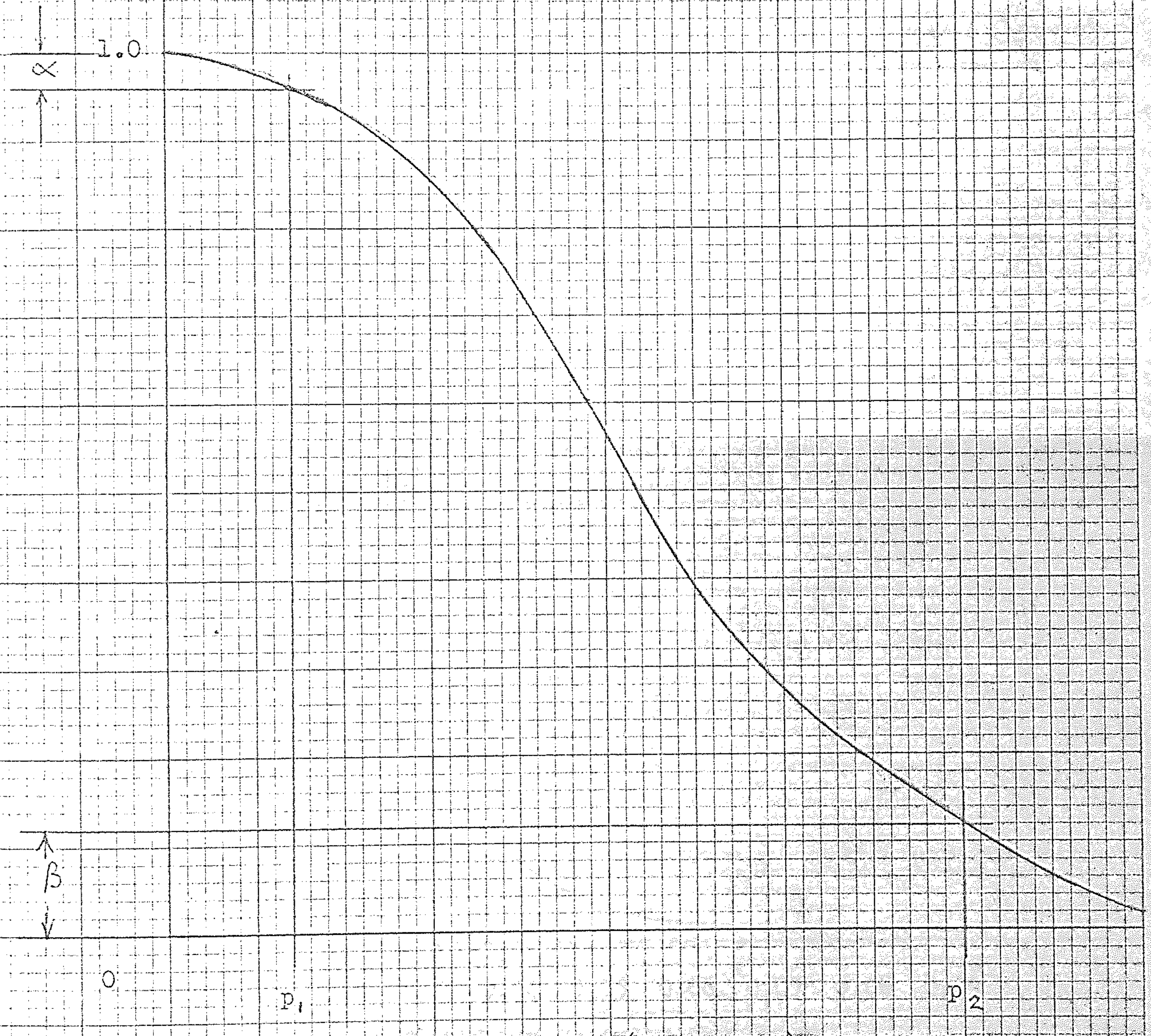



FIG.9 PRACTICAL SHAPE OF OPERATING CHARACTERISTIC CURVE

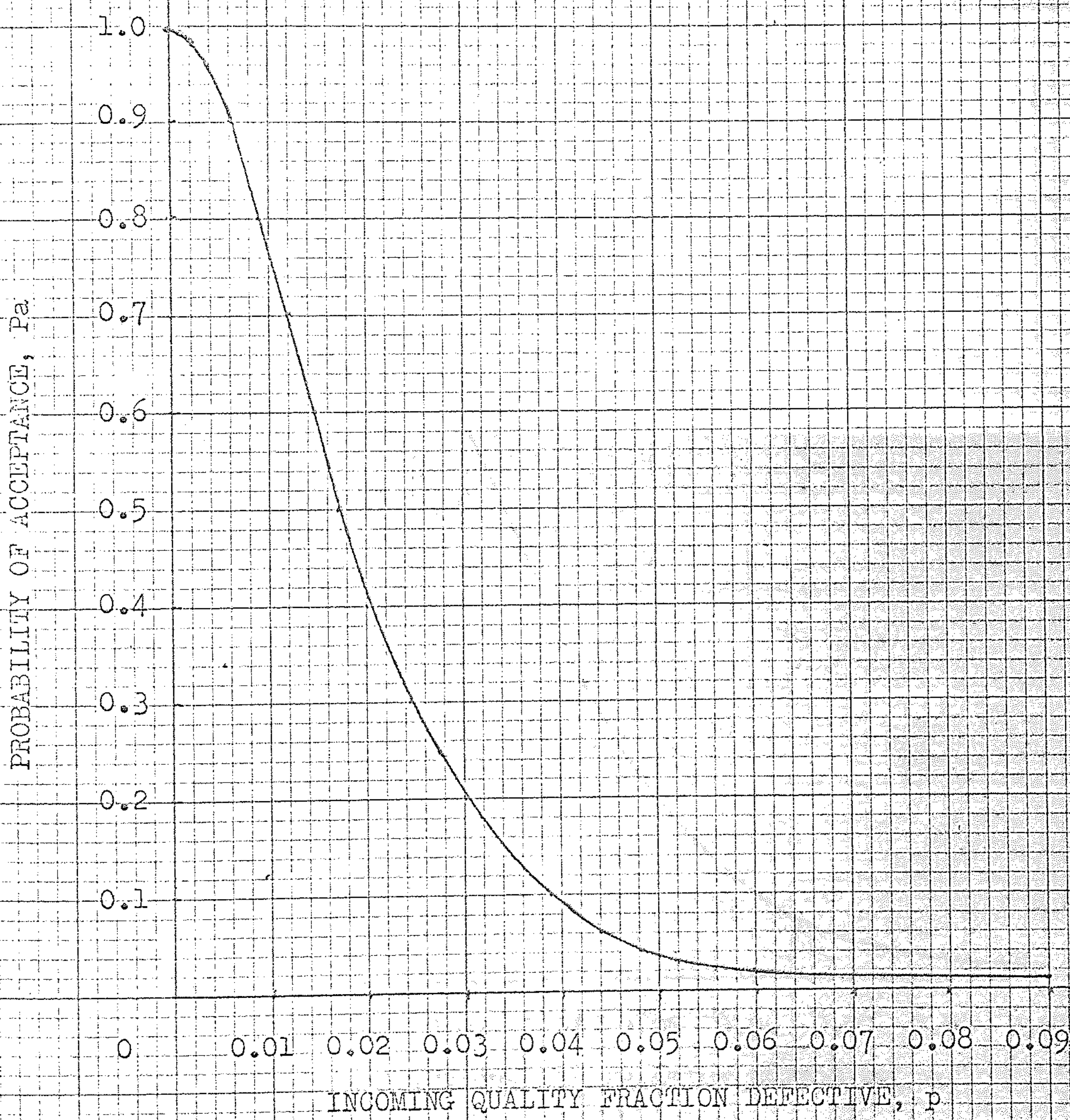



FIG.10 OPERATING CHARACTERISTIC CURVE FOR THE SINGLE SAMPLING PLAN, n= 100, c= 1

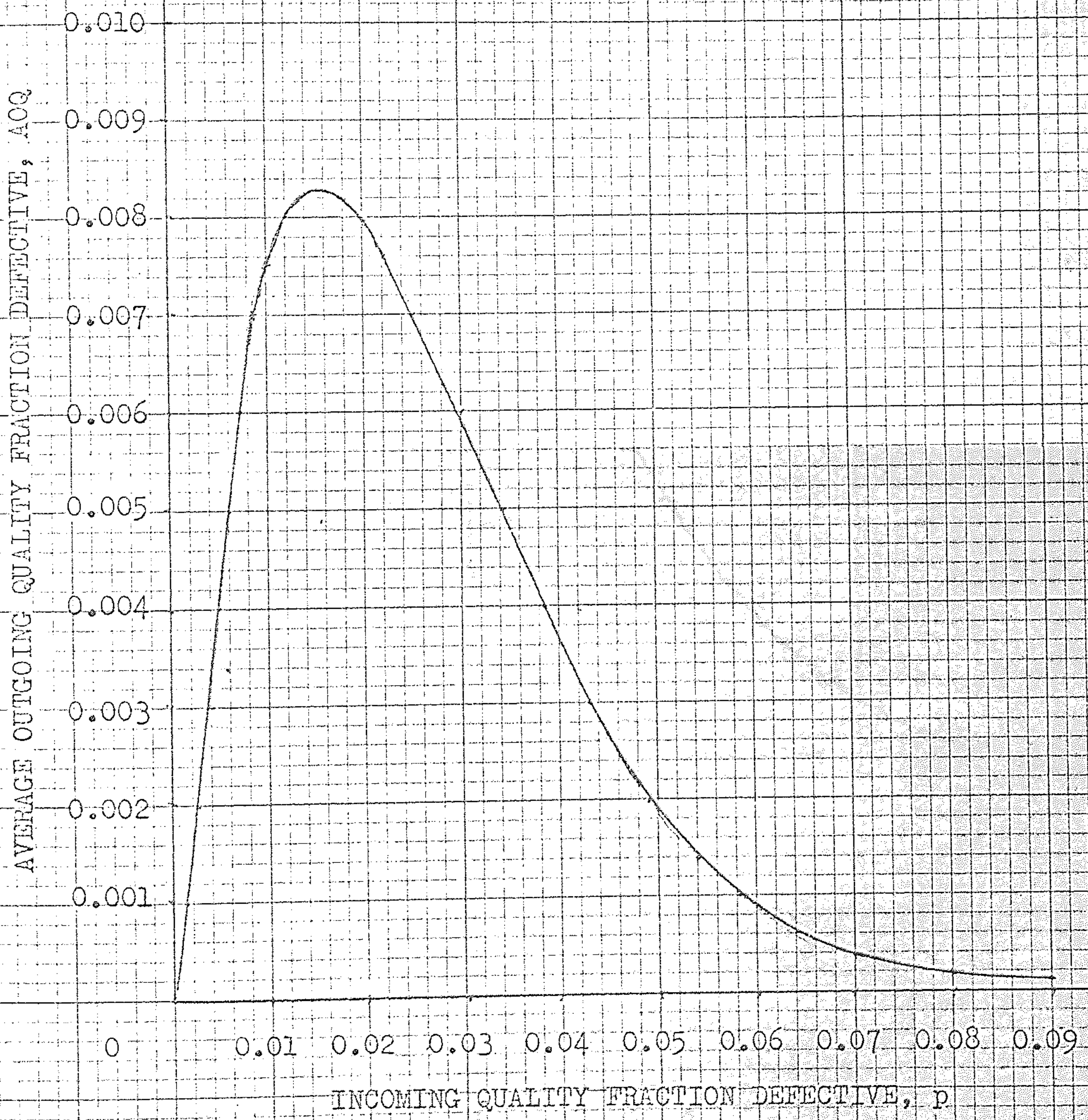



FIG. 11 AVERAGE OUTGOING QUALITY CURVE FOR THE SINGLE SAMPLING PLAN, n = 100, c = 1

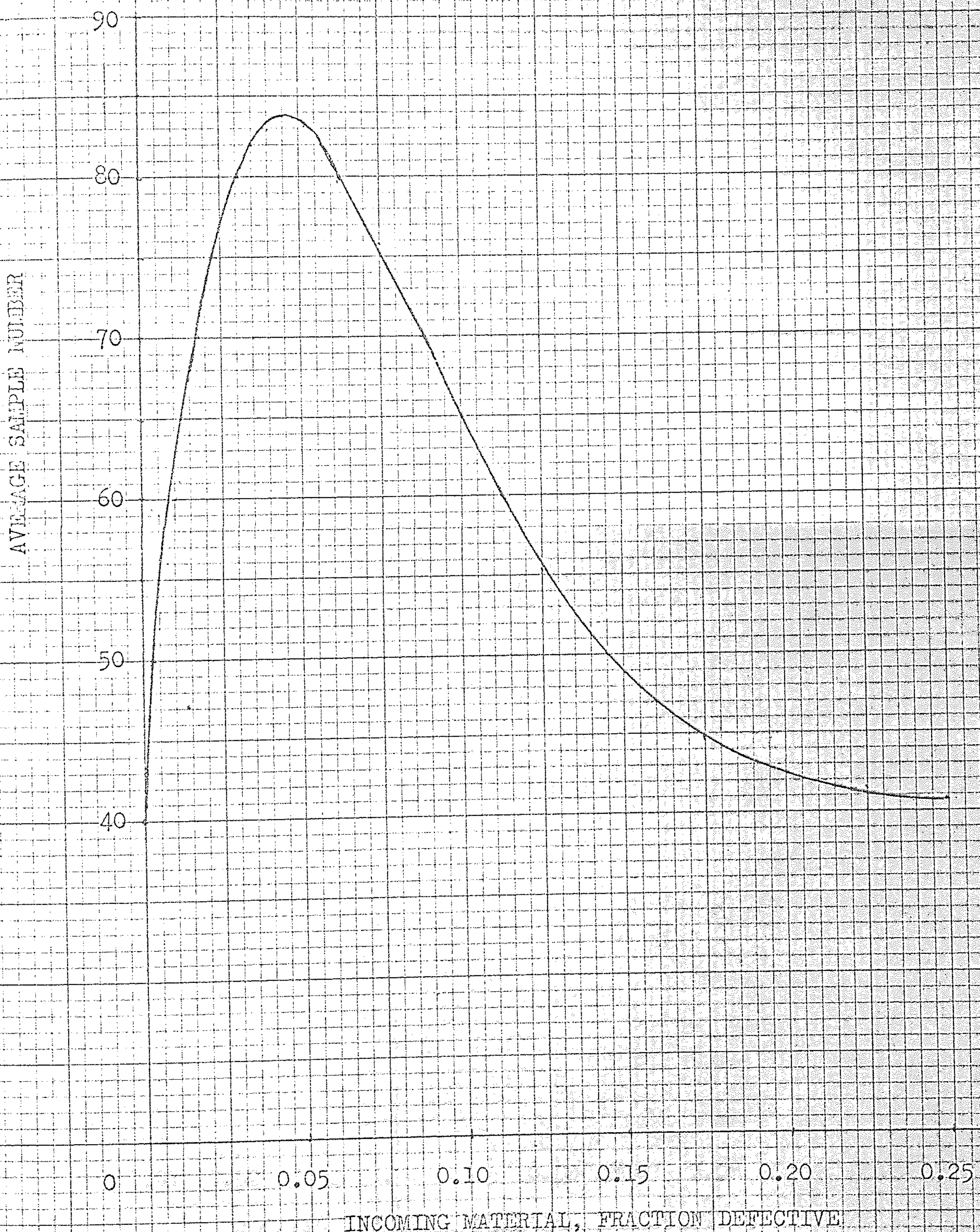



FIG.12 A.S.N. CURVE FOR THE DOUBLE SAMPLING PLAN

$n_1 = 40$, $c_1 = 0$; $n_2 = 60$, $c_2 = 3$

(COMPLETE INSPECTION OF SECOND SAMPLE)

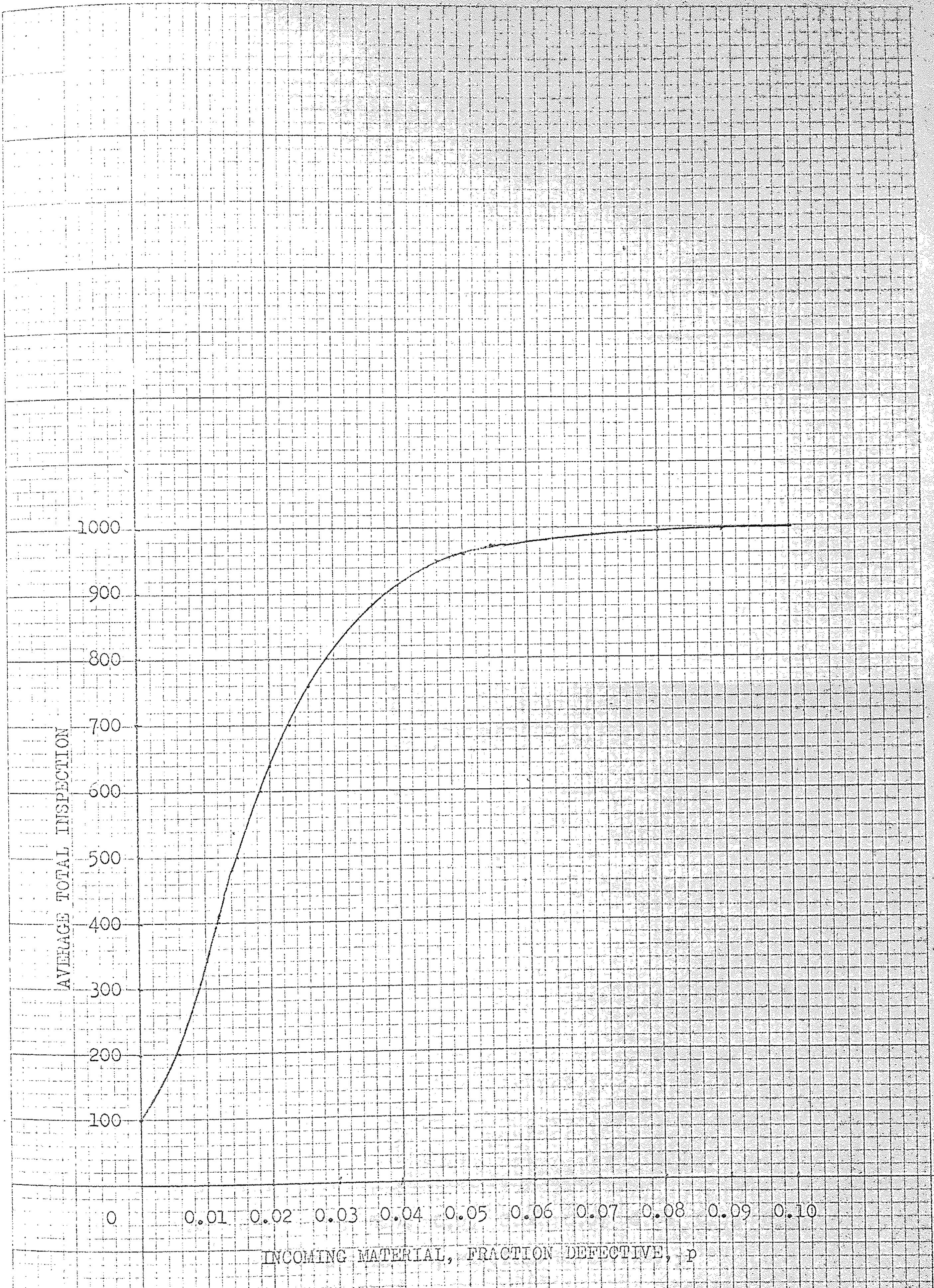



FIG.13 A.T.I. CURVE FOR THE SINGLE SAMPLING PLAN, n = 100, c = 1

N = 1000

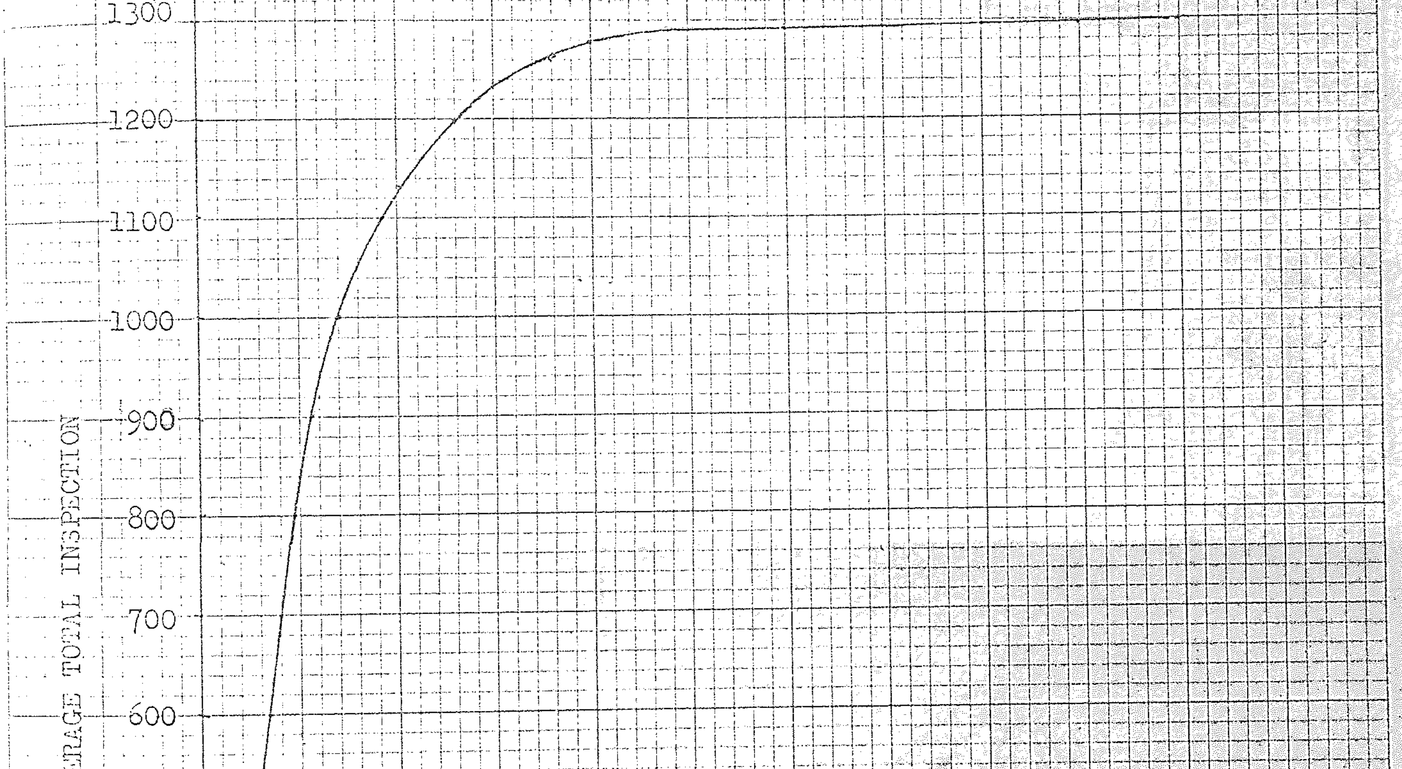



FIG. 13A A.T.I. CURVE FOR THE DOUBLE SAMPLING PLAN, $n_1 = 100$, $c_1 = 0$; $n_2 = 200$, $c_2 = 1$, $N = 1300$

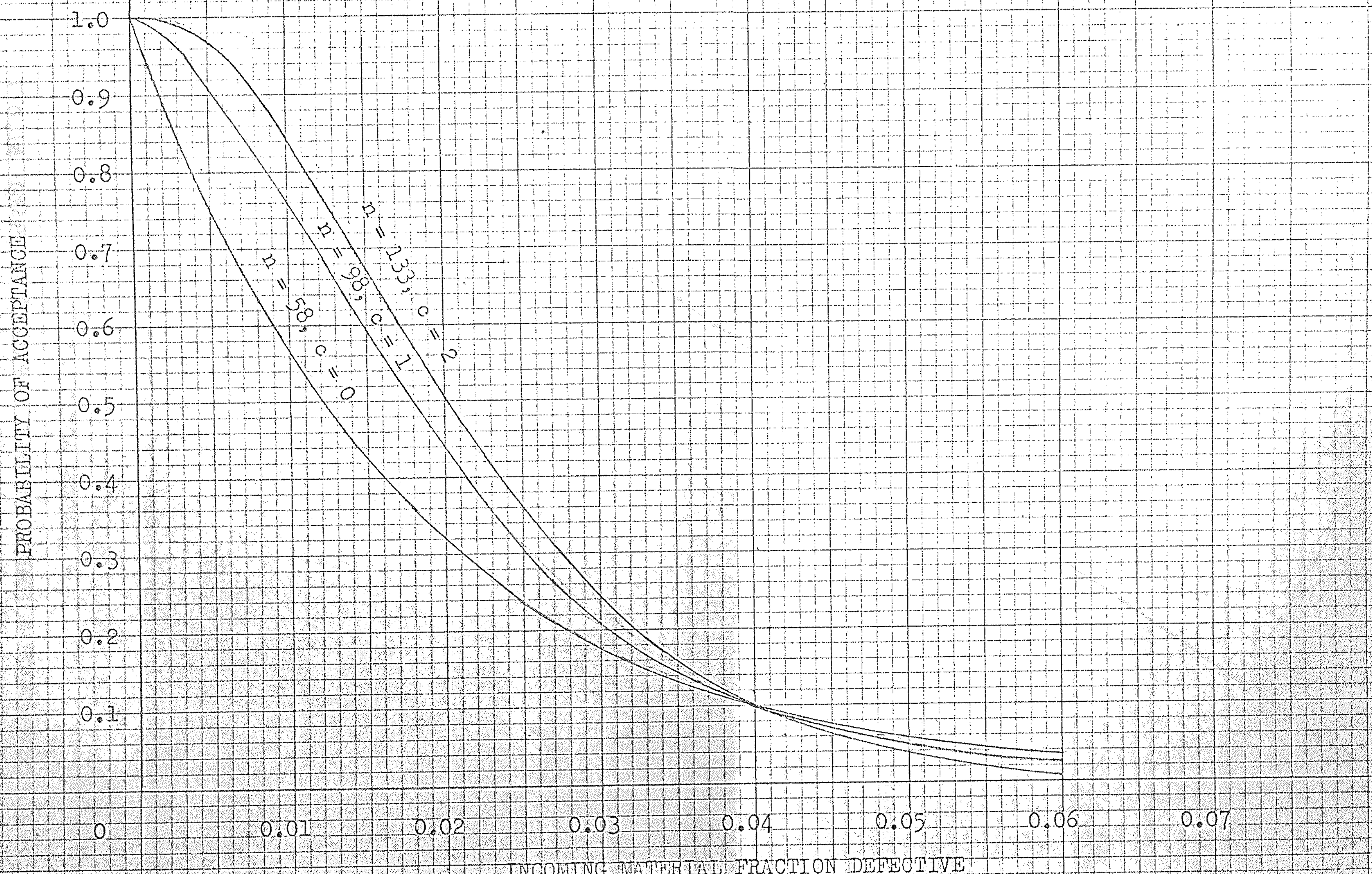



FIG. 13B O.C. CURVES FOR THE SINGLE SAMPLING PLANS n = 58, c = 0; n = 98, c = 1; n = 133, c = 2

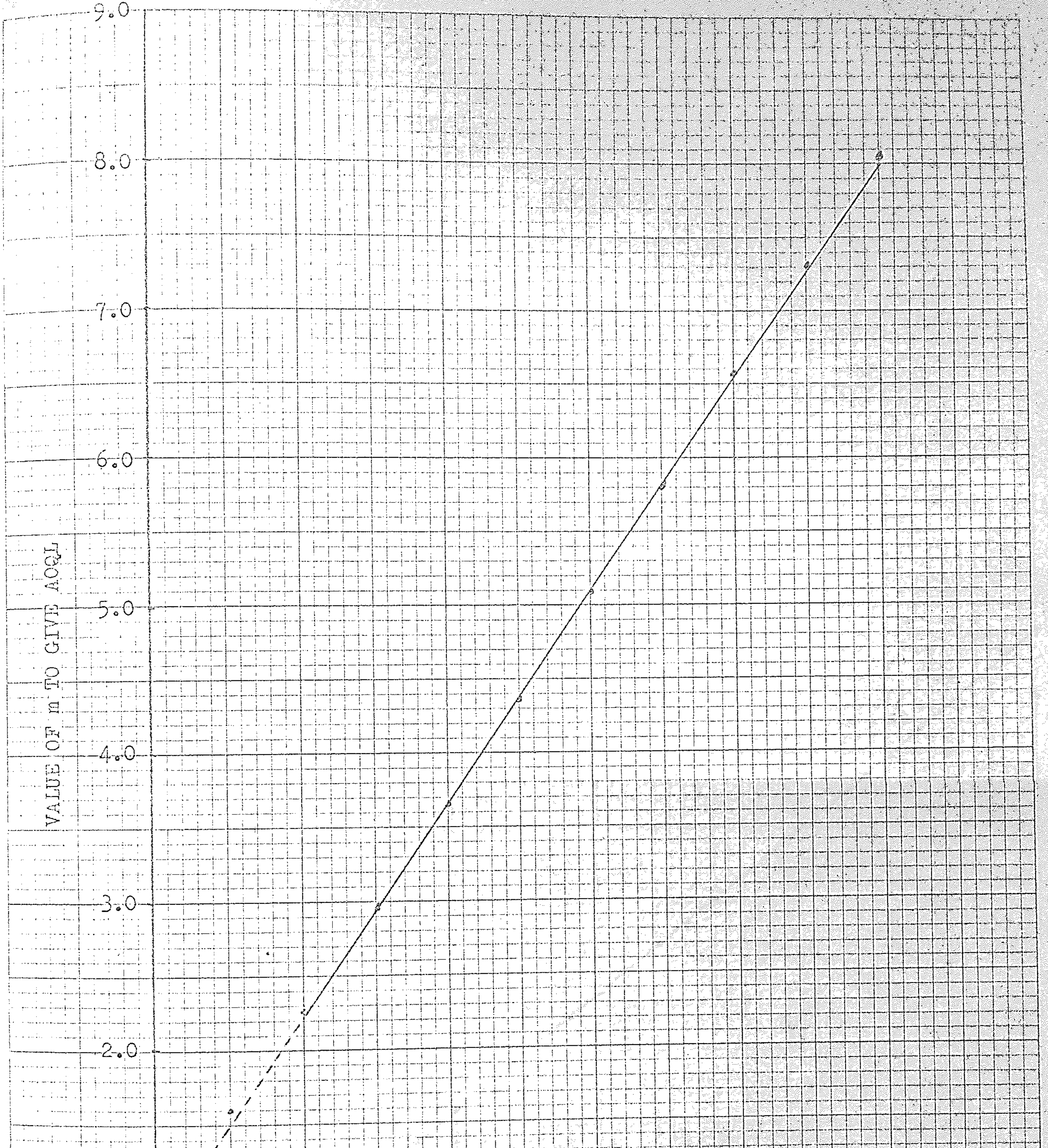



FIG. 14 GRAPH OF LINEAR REGRESSION EQUATION CONNECTING n & c

(c = 2 TO c = 10)

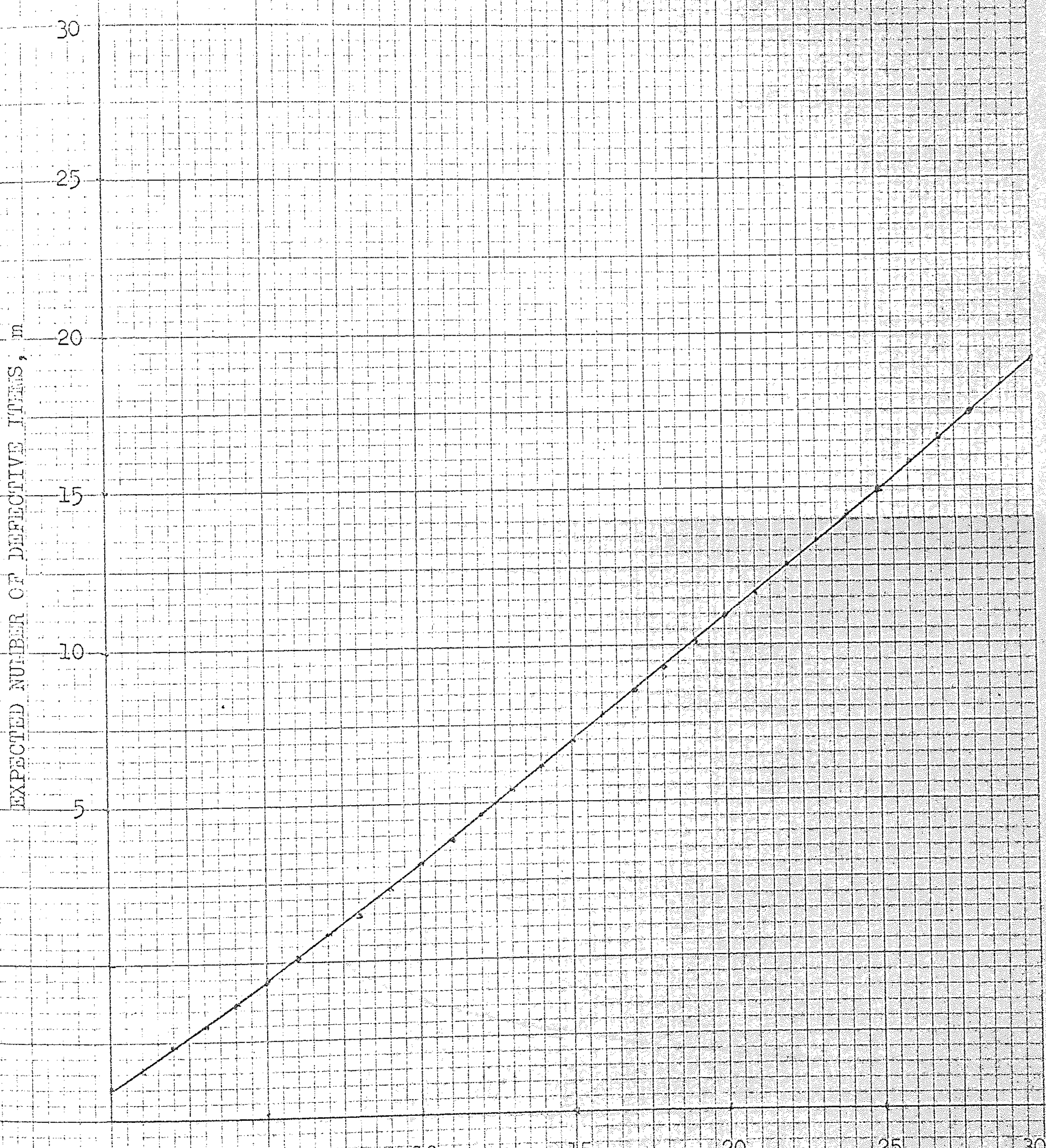



FIG. 15 GRAPH OF RELATIONSHIP BETWEEN m AND c

(c = 0 TO c = 30)

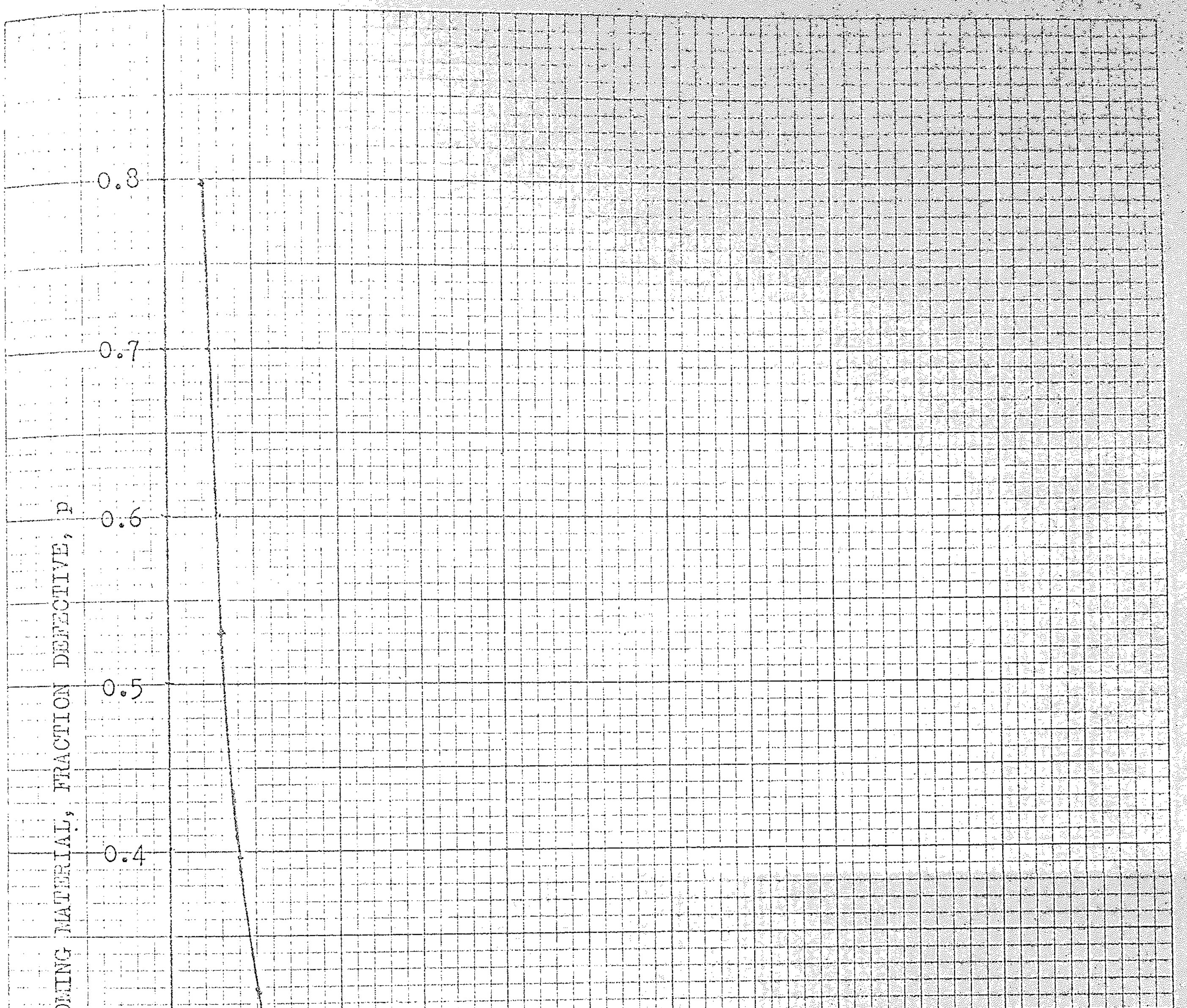



FIG. 16 RELATIONSHIP BETWEEN FRACTION DEFECTIVE OF INCOMING MATERIAL TO GIVE AOQL AND SAMPLE SIZE n, FOR c = 20

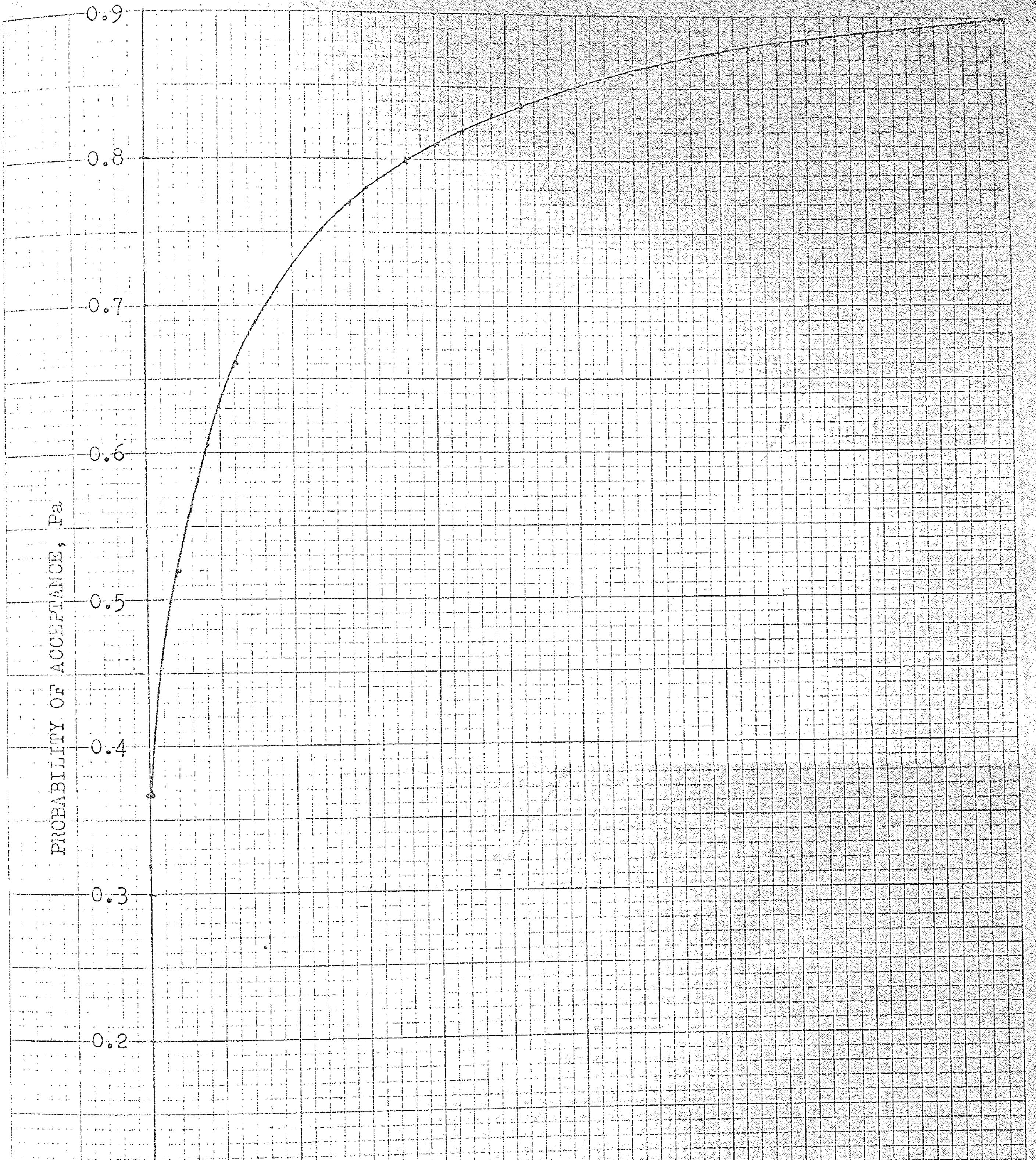



FIG. 17 RELATIONSHIP BETWEEN PROBABILITY OF ACCEPTANCE OF MATERIAL TO GIVE AOQL AND THE ACCEPTANCE NUMBER

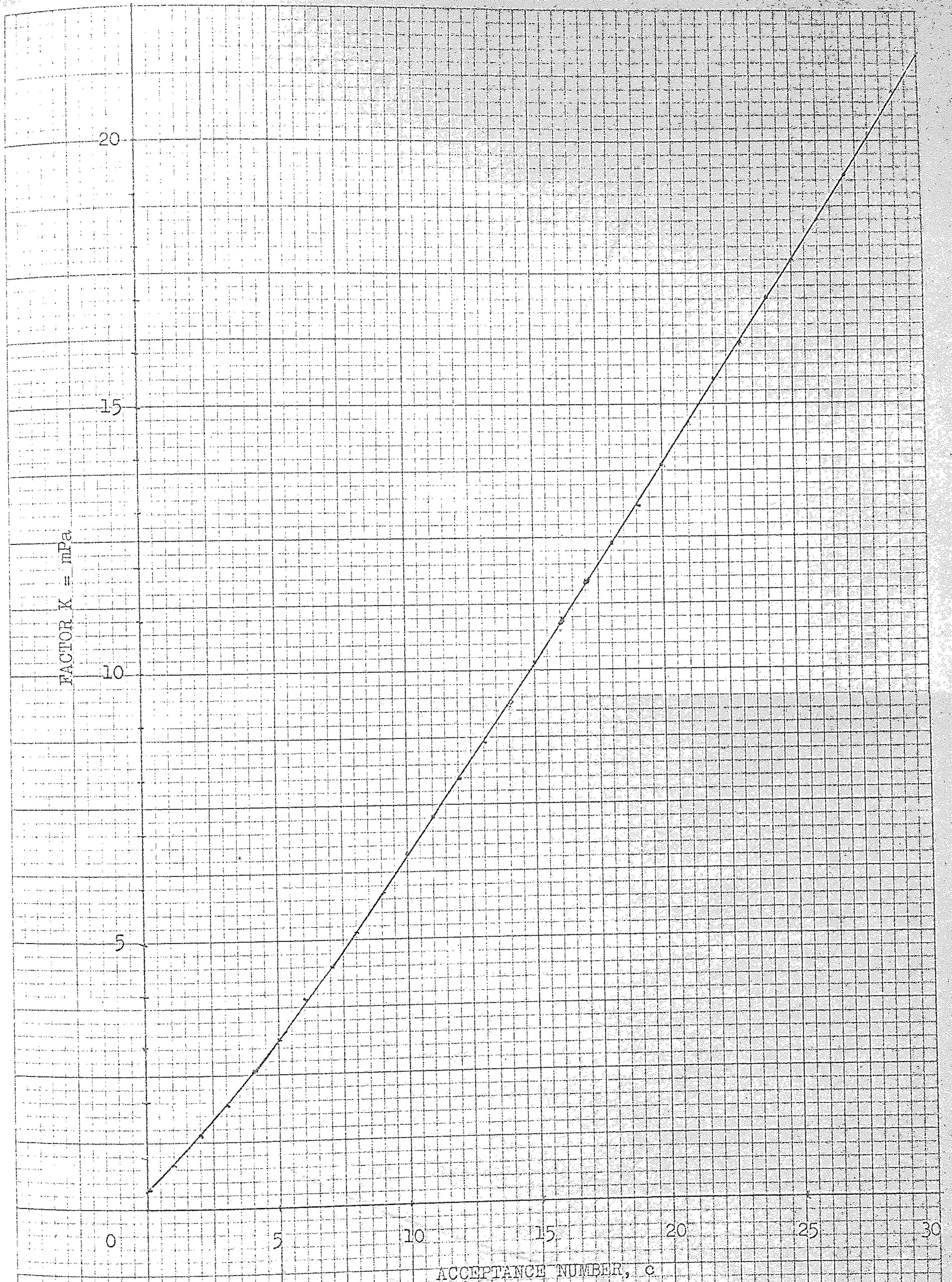



FIG.18 RELATIONSHIP BETWEEN mPa AND c

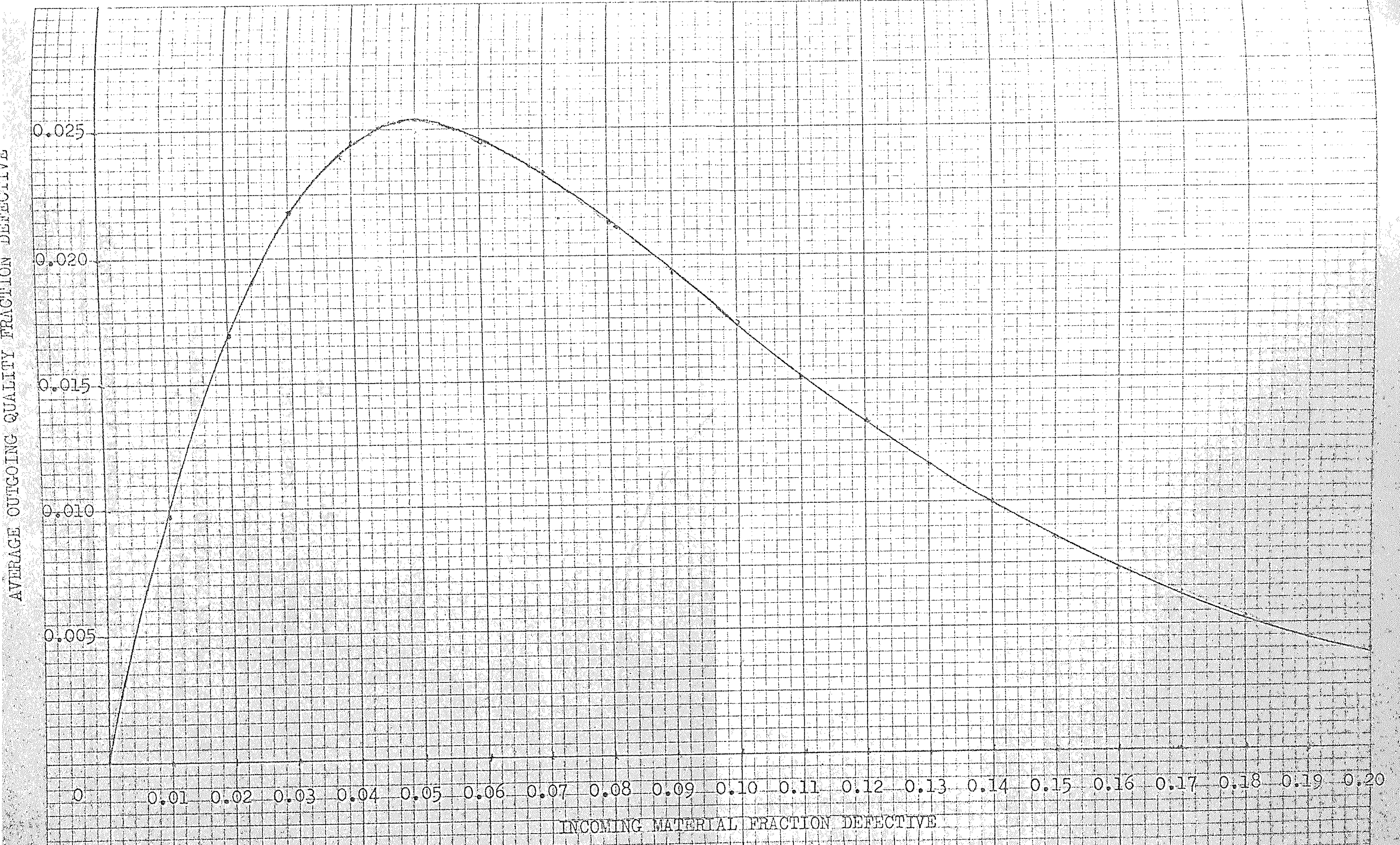
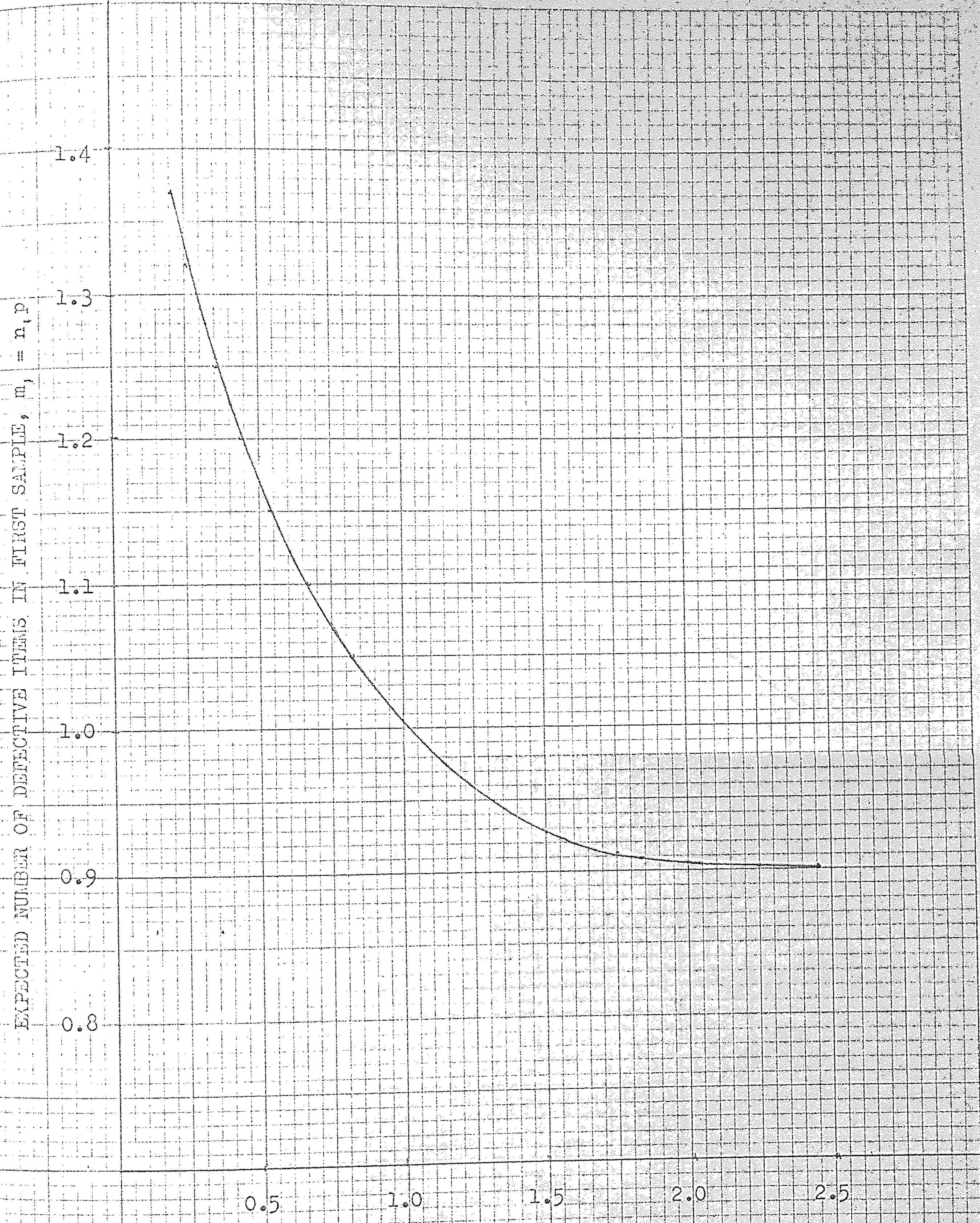



FIG. 19 AVERAGE OUTGOING QUALITY CURVE FOR THE DOUBLE SAMPLING PLAN $n_1 = 20$, $c_1 = 0$; $n_2 = 20$, $c_2 = 1$




ACCEPTANCE NUMBER ON FIRST SAMPLE, $c_1 = 0$

ACCEPTANCE NUMBER ON COMBINED SAMPLES, $c_2 = 1$

FIG. 20 RELATIONSHIP BETWEEN $m_1$ AND $n_2/n_1$ FOR $c_1 = 0$ AND $c_2 = 1$

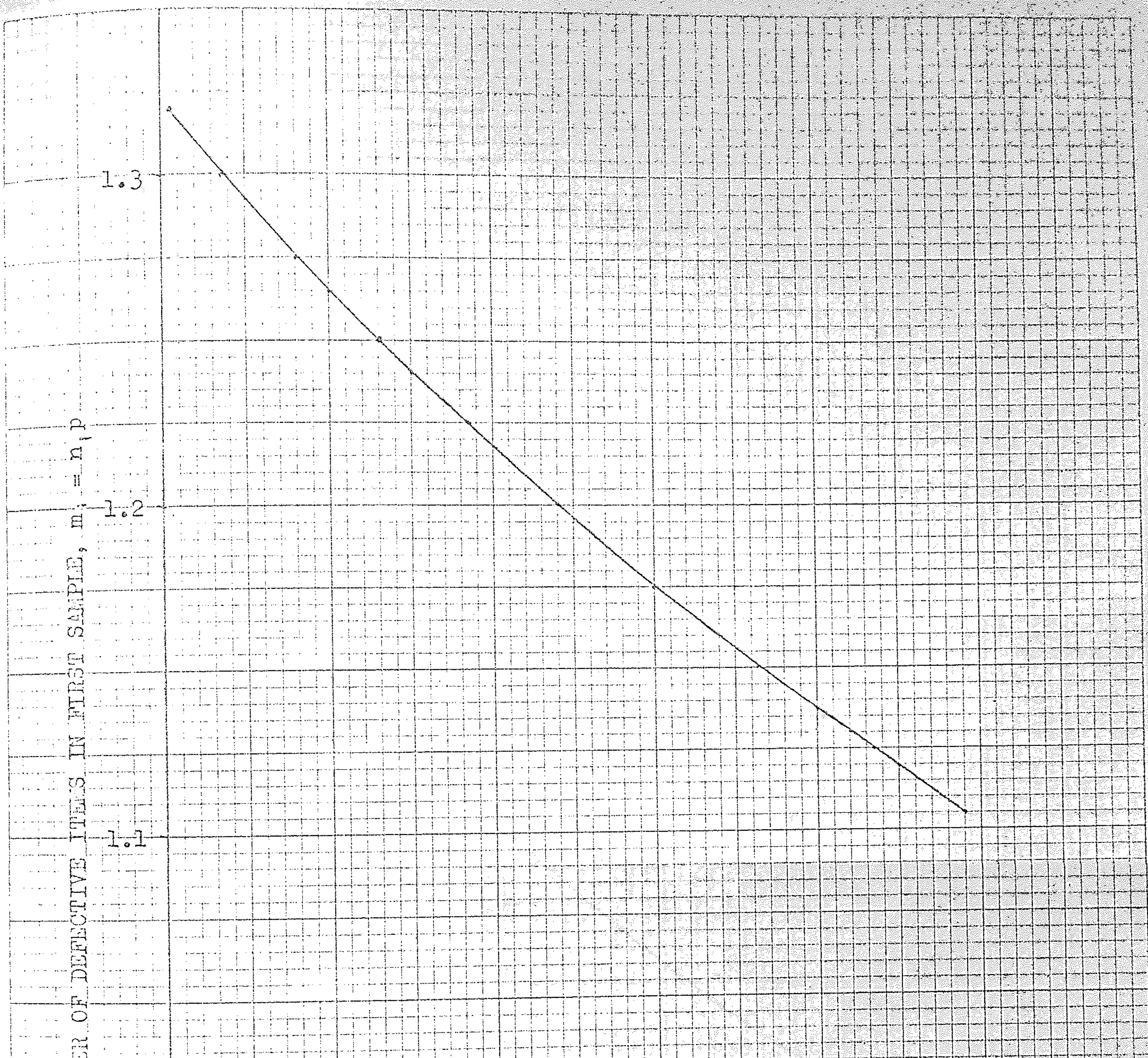
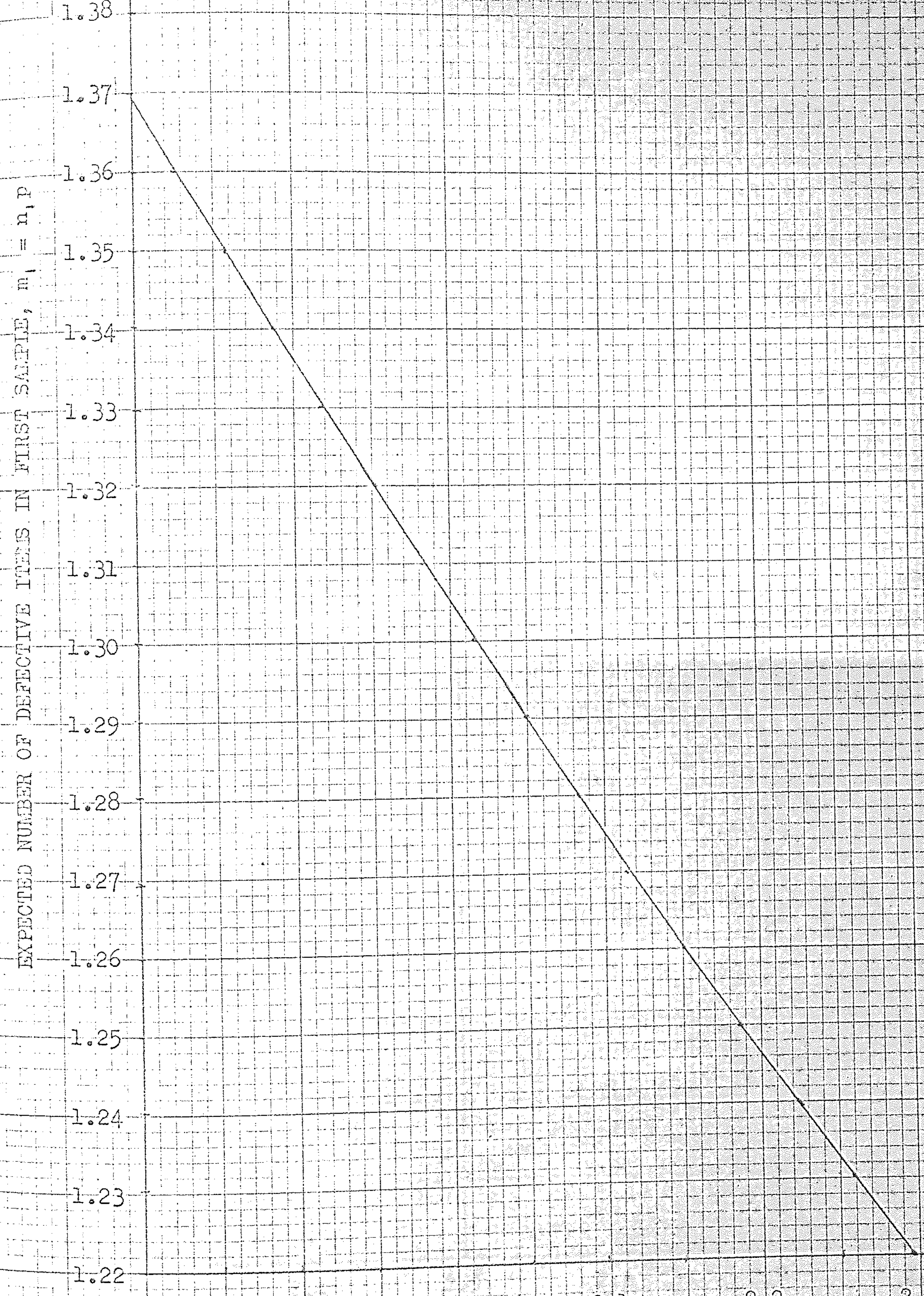



FIG. 21 RELATIONSHIP BETWEEN $m_1$ AND $n_2/n_1$ FOR $c_1 = 0$ AND $c_2 = 2$

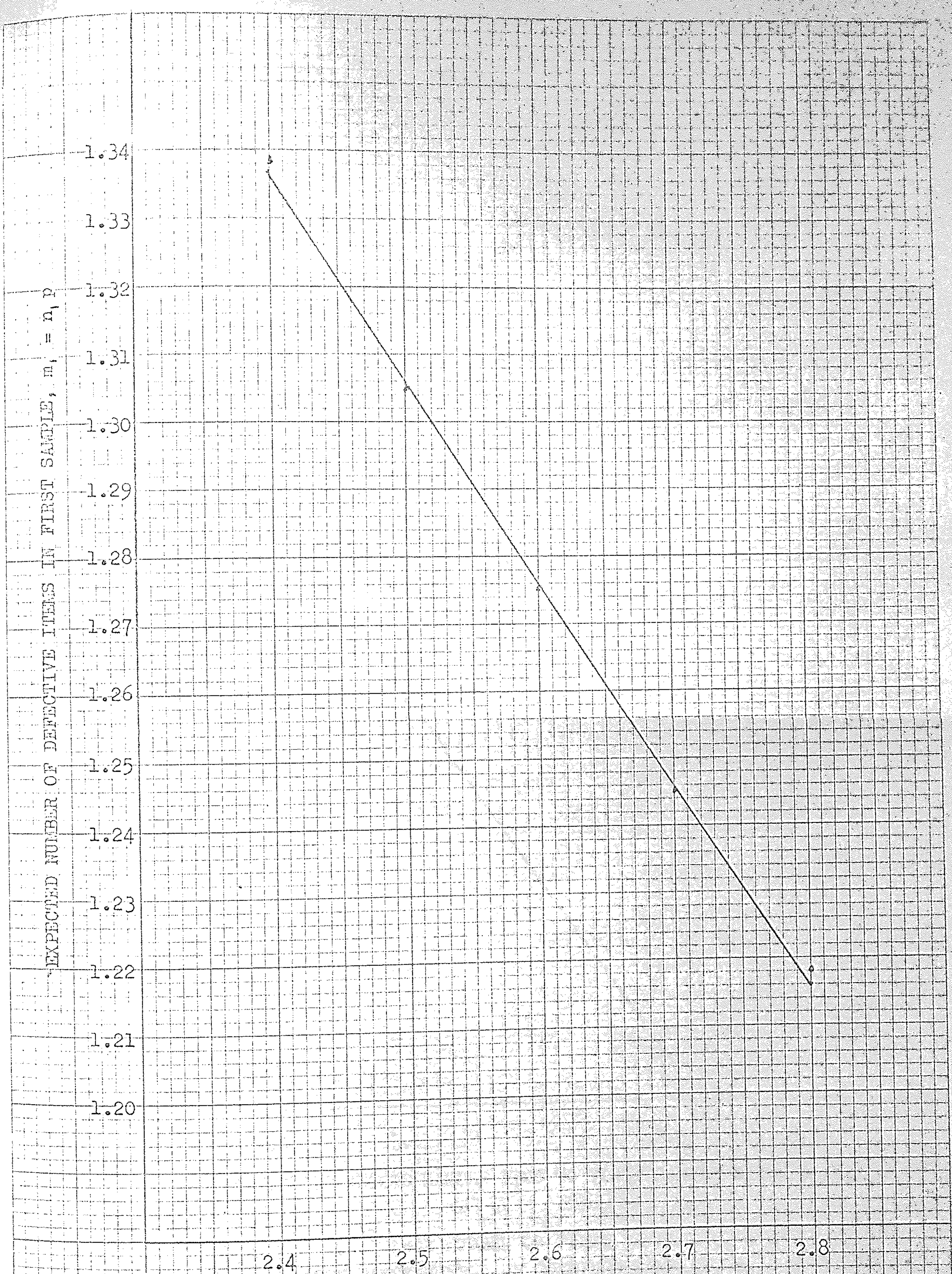

FIG. 22 RELATIONSHIP BETWEEN $m_1$ AND $n_2/n_1$ FOR $c_1 = 0$ AND $c_2 = 3$




ACCEPTANCE NUMBER ON FIRST SAMPLE, $c_1 = 0$

ACCEPTANCE NUMBER ON COMBINED SAMPLES, $c_2 = 4$

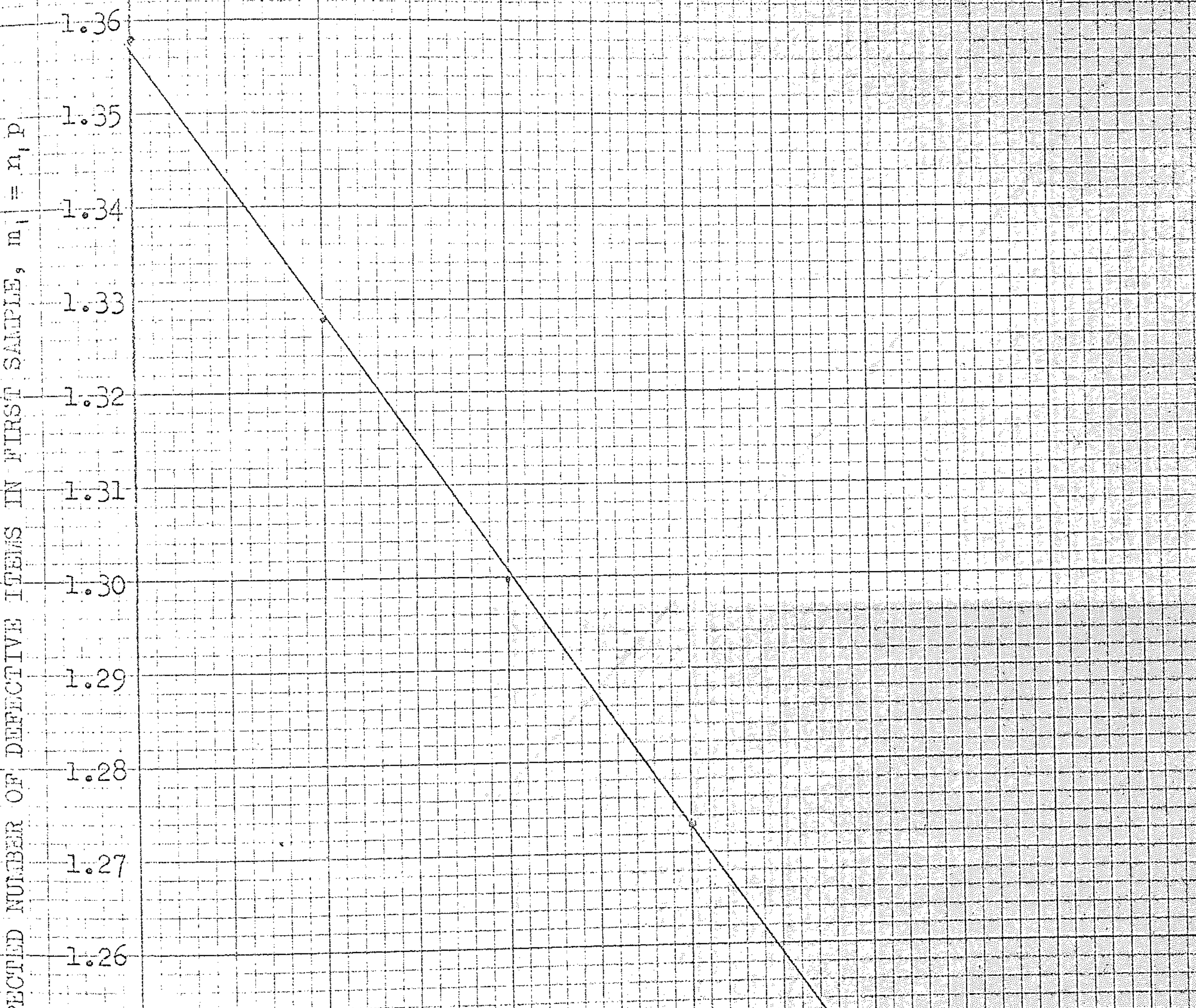
FIG. 23 RELATIONSHIP BETWEEN $m_1$ AND $n_2/n_1$ FOR $c_1 = 0$ AND $c_2 = 4$




ACCEPTANCE NUMBER ON FIRST SAMPLE, $c_1 = 0$

ACCEPTANCE NUMBER ON COMBINED SAMPLES, $c_2 = 5$

FIG. 24 RELATIONSHIP BETWEEN $m_1$ AND $n_2/n_1$ FOR $c_1 = 0$ AND $c_2 = 5$




ACCEPTANCE NUMBER ON FIRST SAMPLE, $c_1 = 0$
ACCEPTANCE NUMBER ON COMBINED SAMPLES, $c_2 = 6$

FIG. 25 RELATIONSHIP BETWEEN $m_1$ AND $n_2/n_1$ FOR $c_1 = 0$ AND $c_2 = 6$

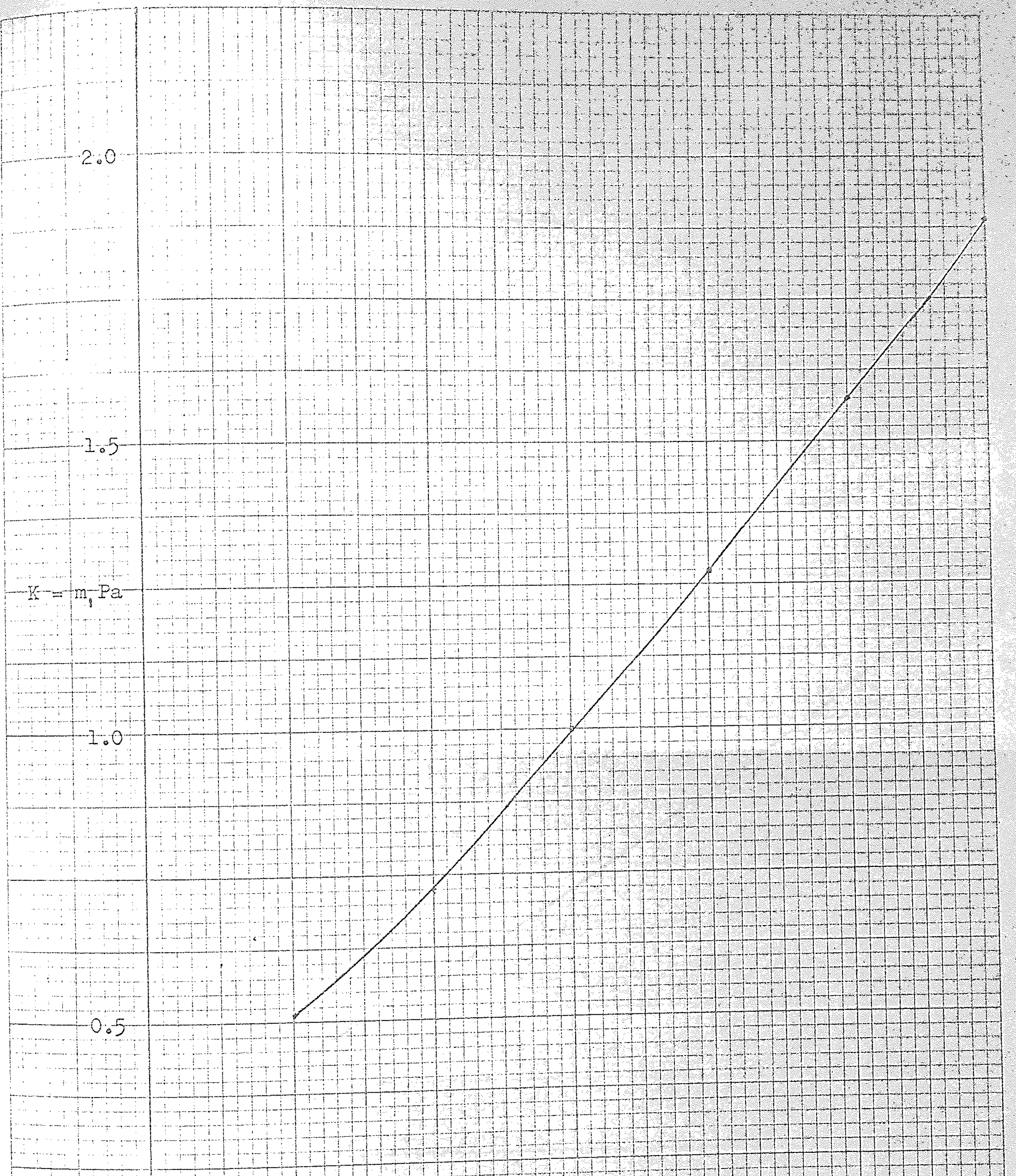



FIG. 26 RELATIONSHIP BETWEEN $m_1 Pa$ AND $c_2$ FOR $c_1 = 0$ AND $n_2/n_1 = 1$

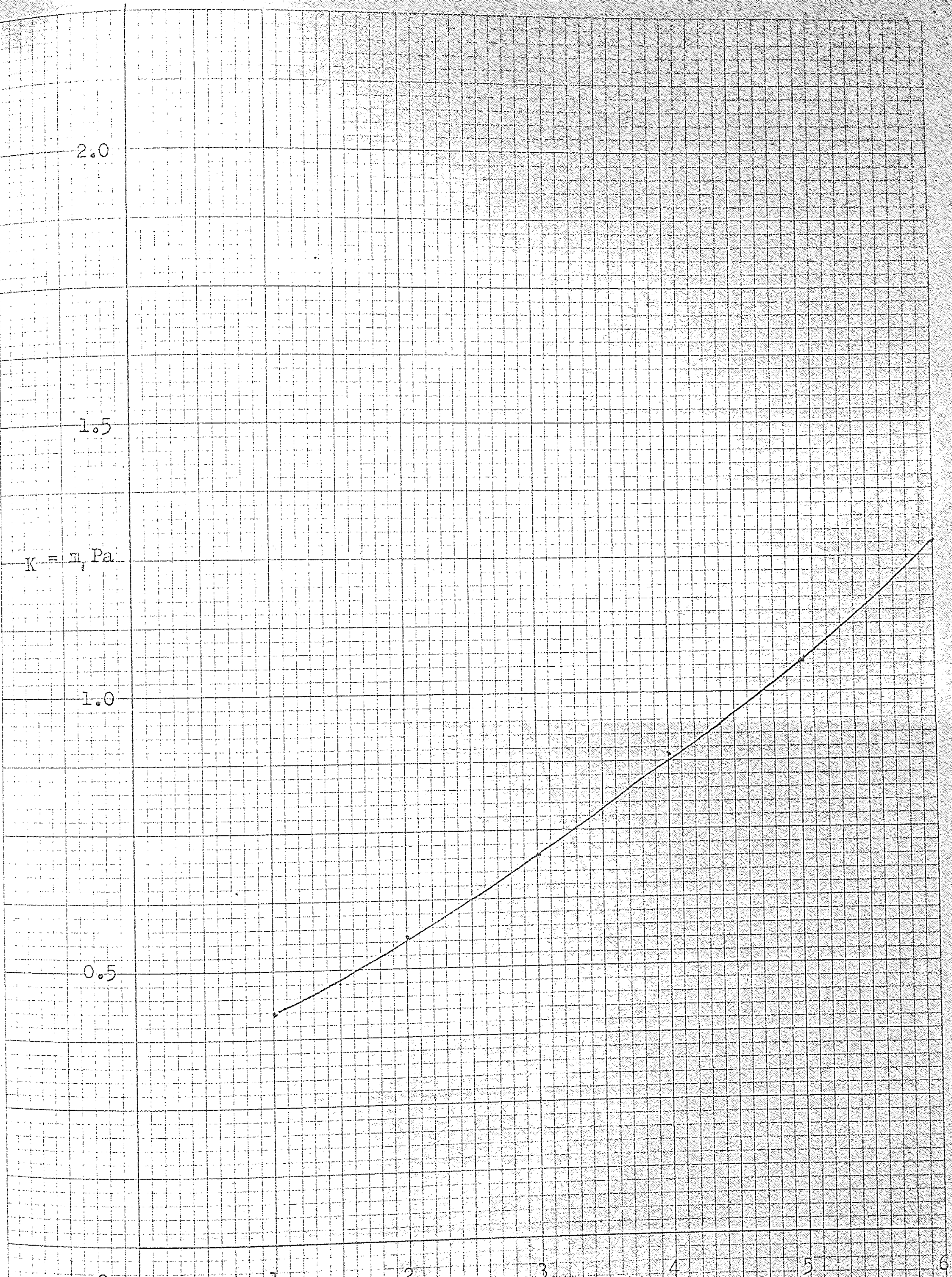



FIG. 27 RELATIONSHIP BETWEEN $m_1 Pa$ AND $c_2$ FOR $c_1 = 0$ AND $n_2/n_1 = 2$


FIG. 28A RELATIONSHIP BETWEEN $m_1$ TO GIVE MAXIMUM A.S.N. AND $c_2$ ($c_1 = 0$, $c_2 = 1$ to $c_2 = 7$)

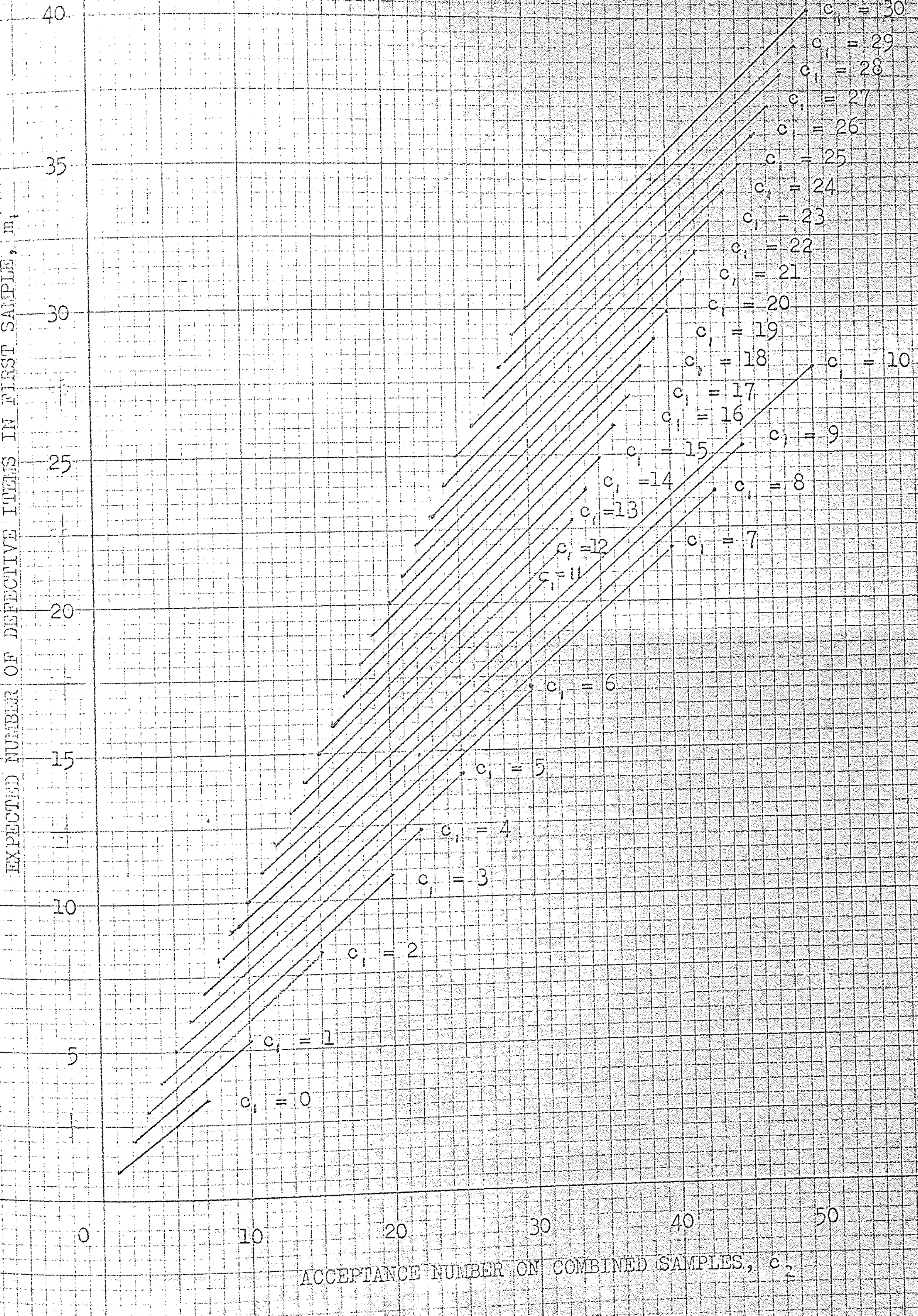



FIG. 28B RELATIONSHIP BETWEEN $m_1$ TO GIVE MAXIMUM A.S.N. AND $c_2$

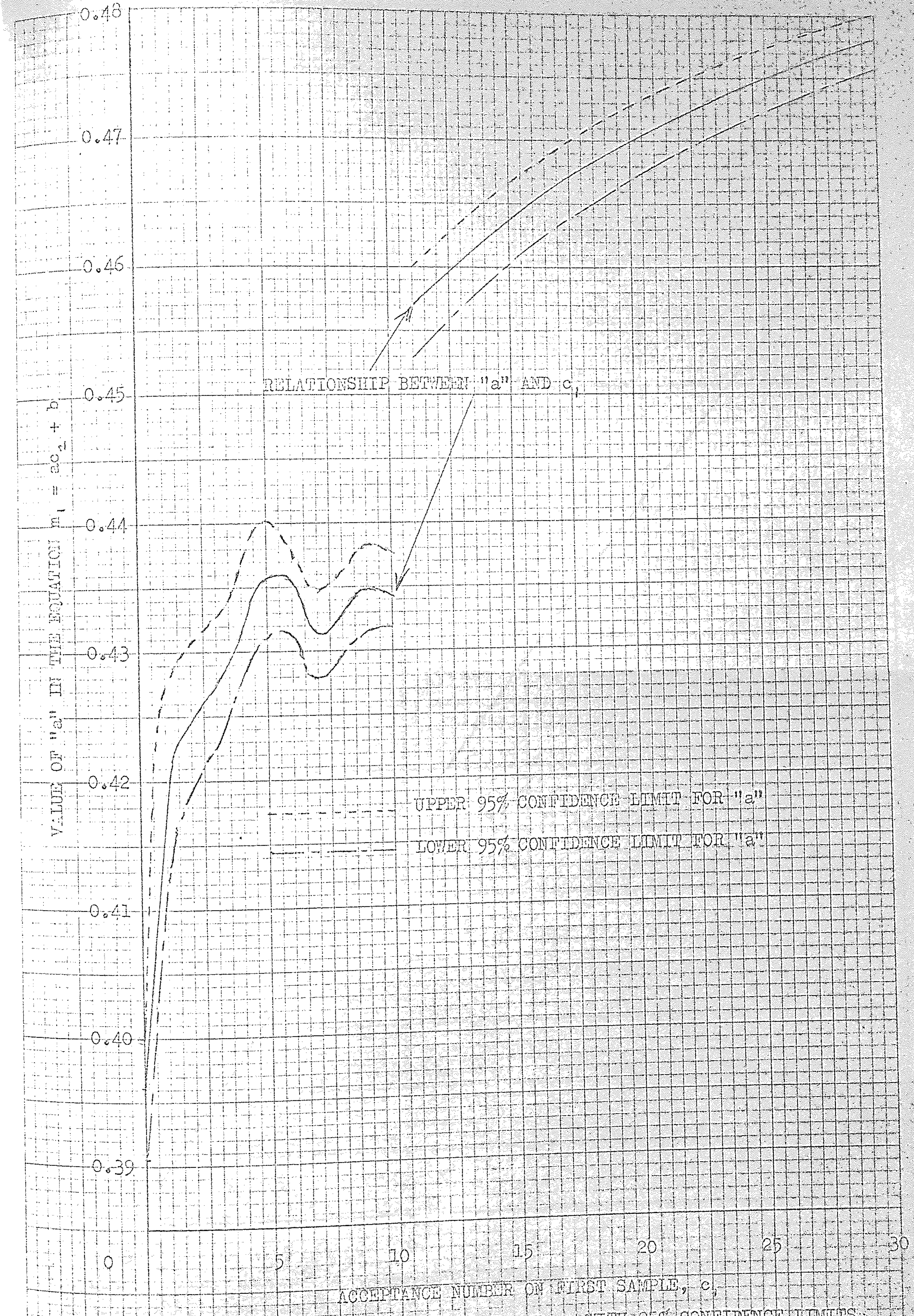



FIG. 29 RELATIONSHIP BETWEEN "a" AND $c_1$ WITH 95% CONFIDENCE LIMITS

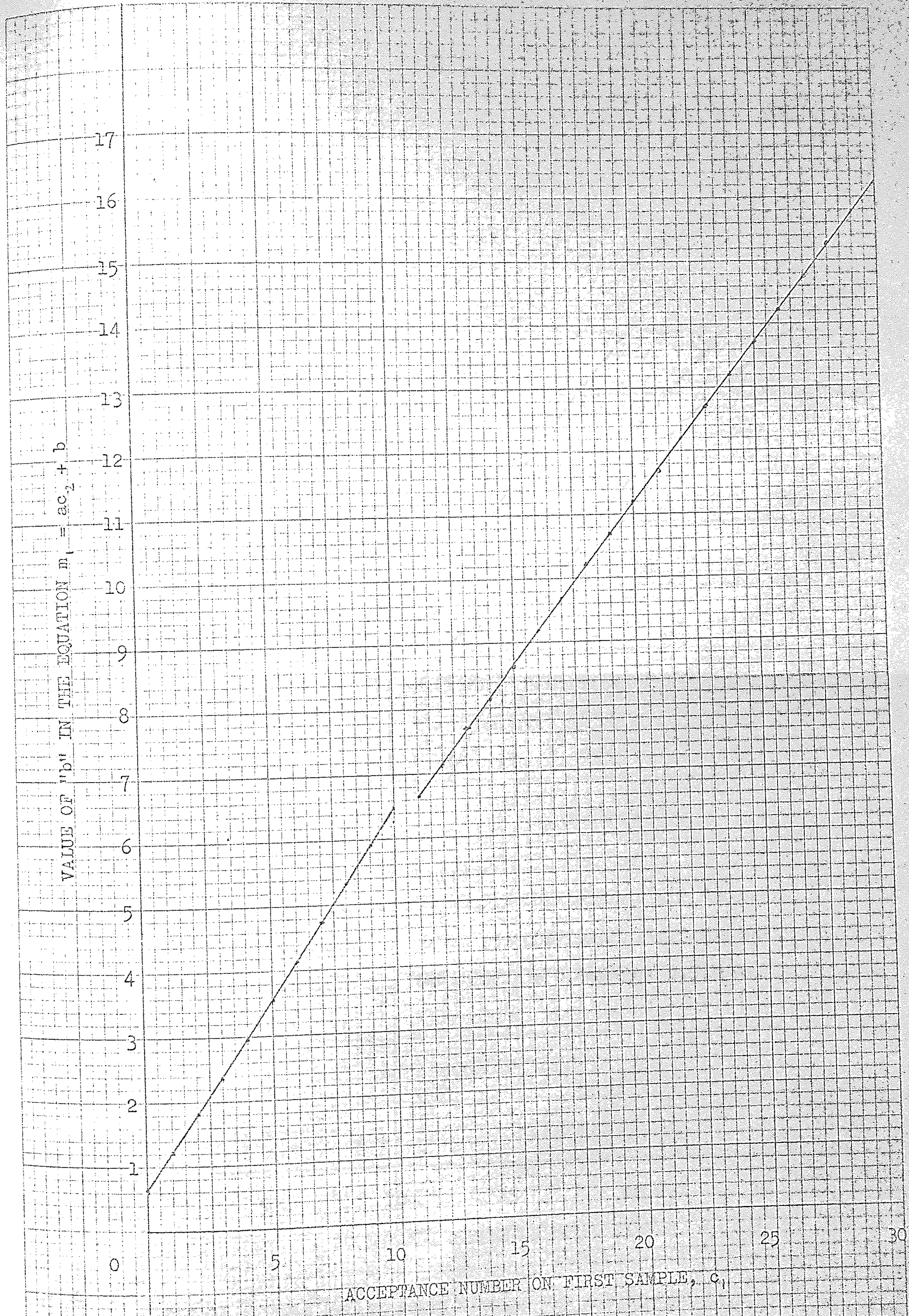



FIG. 30 RELATIONSHIP BETWEEN "b" AND $c_1$

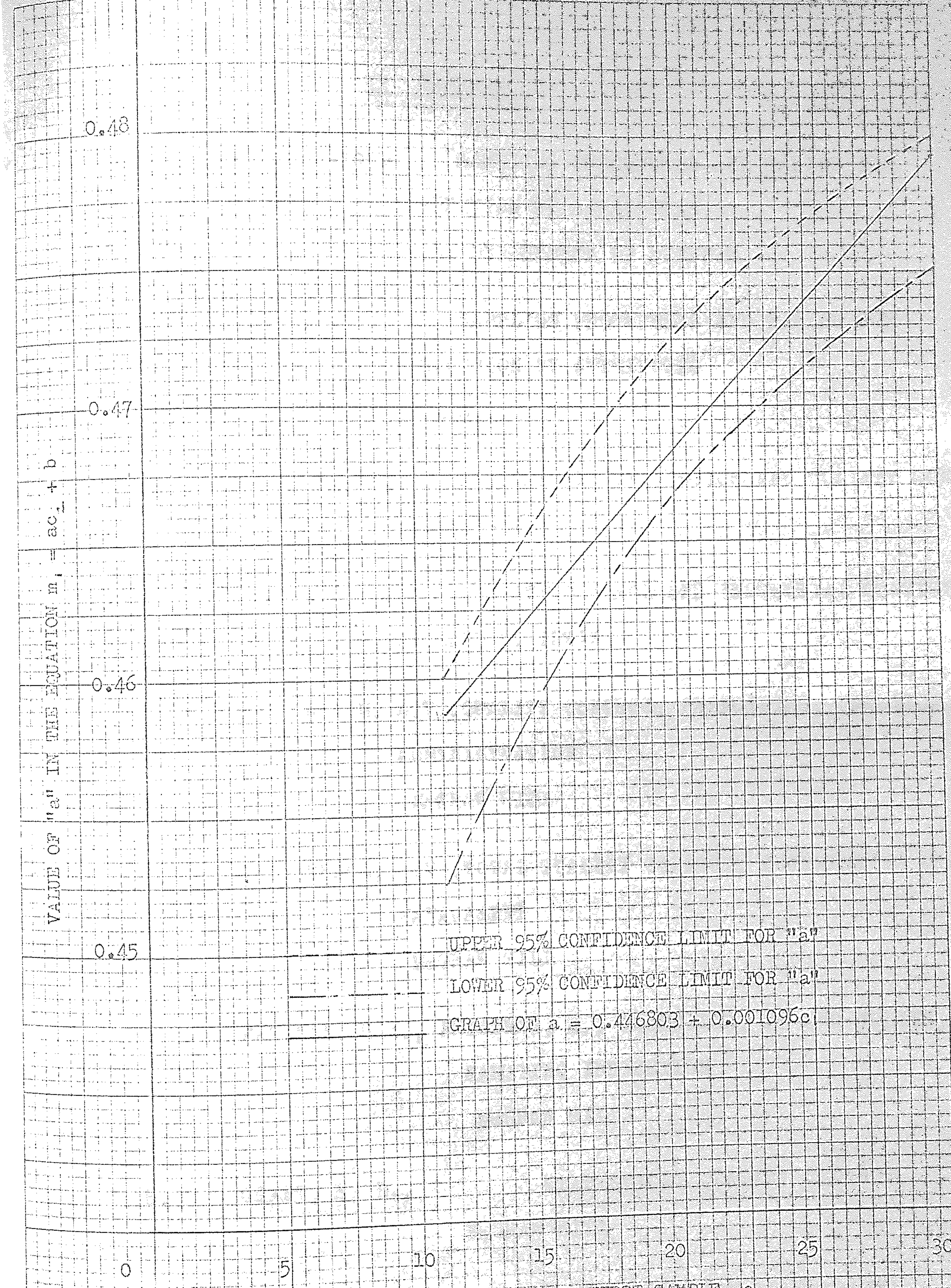



FIG. 31 LINEAR RELATIONSHIP BETWEEN "a" AND $c_1$

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## 13 LIST OF SYMBOLS

| | | |
|---|---|---|
| $\alpha$ | = | Producer's Risk |
| $\beta$ | = | Consumer's Risk |
| AOQ | = | AVERAGE OUTGOING QUALITY |
| AOQL | = | AVERAGE OUTGOING QUALITY LIMIT |
| AQL | = | ACCEPTABLE QUALITY LEVEL |
| ASN | = | AVERAGE SAMPLE NUMFER |
| ATI | = | AVERAGE TOTAL INSPECTION |
| $c$ | = | Acceptance Number in Single Sampling |
| $c_1$ | = | Acceptance Number on First Sample in Double Sampling |
| $c_2$ | = | Acceptance Number on Combined Samples in Double Sampling |
| K | = | A factor for determining the AOQL |
| $k$ | = | Ratio of second sample size to first sample size in double sampling |
| LTPD | = | LOT TOLERANCE PERCENT DEFECTIVE |
| $m$ | = | Expected number of defective items in sample in single sampling |
| $m_1$ | = | Expected number of defective items in first sample in double sampling |
| $m_2$ | = | Expected number of defective items in second sample in double sampling |
| N | = | Lot or batch size |
| $n$ | = | Size of sample in single sampling |
| $n_1$ | = | Size of first sample in double sampling |
| $n_2$ | = | Size of second sample in double sampling |
| O.C. | = | OPERATING CHARACTERISTIC |
| $p$ | = | Fraction defective of incoming material |
| $p_1$ | = | Fraction defective of incoming material in association with the Producer's Risk $\alpha$ |

| | | |
|---|---|---|
| $p_2$ | = | Fraction defective of incoming material in association with the Consumer's Risk $\beta$ |
| Pa | = | Total probability of acceptance of a lot or batch of material |
| $Pa_1$ | = | Probability of acceptance on first sample in double sampling |
| $Pa_2$ | = | Probability of acceptance on second sample in double sampling |
| $Pr_1$ | = | Probability of rejection on first sample in double sampling |
| $P_2$ | = | Probability of taking a second sample in double sampling |
| Q | = | An alternative symbol for AOQ |
| r | = | Coefficient of correlation |

## PART IV

## APPENDICES

### 14 Computer Programme (AOQL of Single Sampling Plans)

```
STATEMENT
NO.

0     'BEGIN''REAL'PA,M,MBAR;
2     'INTEGER'C;
3     'REAL''PROCEDURE'F(M,C,PA);
4     'REAL'M,PA; 'INTEGER'C;
6     'BEGIN''REAL'SUM,TERM; 'INTEGER'R;  TERM:=SUM:=1;
10    'FOR'R:=1'STEP'1'UNTIL'C+1'DO'
10    'BEGIN'
11    TERM:=TERM*M/('IF'R<C+1'THEN'R'ELSE'-1):
12    SUM:=SUM+TERM;
13    'END':
14    F:=EXP(-M)*SUM:  PA:=  TERM*EXP(-M):
16    'END'F:
17    'REAL''PROCEDURE'FDASH(M,C):
18    'REAL'M: 'INTEGER'C:
20    'BEGIN''INTEGER'I: 'REAL'CFAC:
23    CFAC:=1:
24    'FOR'I:=1'STEP'1'UNTIL'C'DO'CFAC:=I*CFAC:
25    FDASH:=M↑C/CFAC*EXP(-M)*(M-C-2):
26    'END'FDASH:    C:=0:
28    NEXT
28    M:=READ:
29    'FOR'MBAR:=M-F(M,C,PA)/FDASH(M,C)'WHILE'
29    ABS(MBAR-M)>0.000005'DO'BEGIN'M=MBAR:OUTPUT(FDASH(M,C)):'END':
33    WRITETEXT('('VALUE%OF%C')'):SPACE (4):OUTPUT(C):
36    WRITETEXT('('ROOT%M')'):SPACE  (2):OUTPUT(M):
39    WRITETEXT('('VALUE%PA')'):SPACE(2):OUTPUT(PA+F(M,C,PA)):
42    'IF'C<30'THEN''BEGIN'  C:=C+1:'GOTO'NEXT:'END':
46    'END':
```

# 15 Computer Programme (ASN of Double Sampling Plans)

```
STATEMENT
NO.

0       'TRACE' 2
0       'BEGIN'
1       'INTEGER' I,J,C1,C2A,C2B:
2       'REAL' M1:
3       'ARRAY' FACT(0:50):
4       FACT(0):=1:
5       'FOR' I:=1'STEP' 1 'UNTIL' 50 'DO' FACT(I):=FACT(I-1)*I:
6       PAPERTHROW:
7       L1:
7       C1:=READ:
8       'IF' C1>99'THEN'  'GOTO' FIN:
9       WRITETEXT('('C1=')'):
10      PRINT(C1,2,0):
11      NEWLINE(2):
12      C2A:=READ:
13      C2B:=READ:
14      J:=FACT(C1):
15      WRITETEXT('(''('2S3)'C2'('6S')'M1'('C')'')'):
16      'FOR' I:=C2A  'STEP'  1  'UNTIL'  C2B  'DO'
16      'BEGIN'
17      M1:=EXP( LN(FACT(I)/J)/(I-C1)):
18      PRINT(I,2,0):
19      SPACE(3):
20      PRINT(M1,1,5):
21      NEWLINE(1):
22      'END':
23      'GOTO' L1:
24      FIN:
24      'END' OF PROGRAM:
        ****
```

## 16 Basic Probability distributions used in Quality Control

This thesis is not concerned fundamentally with probability theory although the application of probability theory is an essential part of quality control. It is therefore considered not out of place to discuss some of the basic probability distributions.

The four most widely used distributions are

The hypergeometric distribution

The binomial distribution

The Poisson distribution

The normal distribution

### 16 (i) The hypergeometric distribution

The number of ways of arranging n things all together is n! and the number of ways (combinations) of drawing r from n! is $\frac{n!}{r!\,(n-r)!}$

If a lot of 1000 items is 1% defective then the number of defective items in the lot is 10 (alternatively, the number of defective items in the lot may be given, and not the fraction defective, but this makes no difference to the computations involved).

If the sample size is 100, the number of ways os drawing 100 from 1000 is $\frac{1000!}{100!\;900!}$

If the acceptance number c = 2, then the probability of acceptance of the lot is the probability of 2 or less defective items in the sample and is equal to the sum of the probabilities of 0, 1, and 2 defective items in the sample.

The probability of acceptance is therefore equal to $\dfrac{990!\ 100!\ 900!}{100!\ 890!\ 1000!}$

$+ \dfrac{990!\ 10!\ 100!\ 900!}{99!\ 891!\ 1!\ 9!\ 1000!}$

$+ \dfrac{990!\ 10!.100!\ 900!}{98!\ 892!\ 2!\ 8!\ 1000!}$

These, when evaluated, will give exact probabilities but, in view of the nature of the calculations involved, in practice approximations are used which will give values which are sufficiently close for all practical purposes.

In general, the smaller value of the fraction defective p and the greater the value of the sample size n the smaller is the error.

16 (ii) The binomial distribution

If a random sample of n items is drawn from a lot whose fraction defective is p then the terms of expansion of $(q + p)^n$ give respectively the probabilities of 0, 1, 2, 3 etc. defective items in the sample, where $q = 1 - p$.

In the example quoted above $n = 100$ and $p = 0.01$ so that the expansion of $(0.99 + 0.01)^{100}$ would give the probabilities.

Fortunately, calculations are much less involved as tables of the binomial distribution are published from which it is possible to read off direct the probabilities involved.

16 (iii) The Poisson Distribution

If x is the average or expected number of occurences of an event then the terms of the expansion of $e^{-x}xe^{+x}$

give respectively the probability of 0, 1, 2, 3 etc. occurences of the event.

In a sample of n items drawn from a lot which is p fraction defective, the expected number of defective items in the sample, m = np.

Thus the terms of the expansion of

$e^{-np}(1 + np + \frac{n^2p^2}{2!} + \text{-----------}+)$ give respectively the probabilities of 0, 1, 2, e etc. defective items in the sample.

Cumulative tables of the Poisson distribution (Appendix 17) are available from which the probability of acceptance may be read off direct. In the example quoted above, np = 100 c 0.01 = 1.0 and by referring to these tables it will be seen that the probability of acceptance (i.e. the probability of 2 or less) is 0.920.

The Poisson distribution assumes constant probability from draw to draw and ignores the changing probabilities resulting from the the depletion of the lot by the drawing of samples.

However, in sampling inspection the Poisson distribution is by far the most widely used and in this thesis the Poisson distribution is used throughout.

## 16 (iv) The normal distribution

The results of the measurements from many industrial processes follow a normal distribution. Used as a probability distribution, the total area of the normal curve is unity, and in this connection the area under the curve between the value of a variable x = 0 to x = x gives the probability of this value being less than x. Unlike other distributions referred to (except in the case of the binomial distribution when p = q = $\frac{1}{2}$) it is a symmetrical distribution terminating at $\pm$ . For most practical purposes, however, it may be assumed to terminate at just over $\pm$ 3 standard deviations from the mean. However, no further reference will be made to the distribution in this thesis.

## 17 CUMULATIVE POISSON DISTRIBUTION

Probability of c or less events

| m \ c | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| .02 | .980 | 1.000 | | | | |
| .04 | .961 | .999 | 1.000 | | | |
| .06 | .942 | .998 | 1.000 | | | |
| .08 | .923 | .997 | 1.000 | | | |
| .10 | .905 | .995 | 1.000 | | | |
| .15 | .861 | .990 | .999 | 1.000 | | |
| .20 | .819 | .982 | .999 | 1.000 | | |
| .25 | .779 | .974 | .998 | 1.000 | | |
| .30 | .741 | .963 | .996 | 1.000 | | |
| .35 | .705 | .951 | .994 | 1.000 | | |
| .40 | .670 | .938 | .992 | .999 | 1.000 | |
| .45 | .638 | .925 | .989 | .999 | 1.000 | |
| .50 | .607 | .910 | .986 | .998 | 1.000 | |
| .55 | .557 | .894 | .982 | .998 | 1.000 | |
| .60 | .549 | .878 | .977 | .997 | 1.000 | |
| .65 | .522 | .861 | .972 | .996 | .999 | 1.000 |
| .70 | .497 | .844 | .966 | .994 | .999 | 1.000 |
| .75 | .472 | .827 | .959 | .993 | .999 | 1.000 |
| .80 | .449 | .809 | .953 | .991 | .999 | 1.000 |
| .85 | .427 | .791 | .945 | .989 | .998 | 1.000 |
| .90 | .407 | .772 | .937 | .987 | .998 | 1.000 |
| .95 | .387 | .754 | .929 | .984 | .997 | 1.000 |
| 1.00 | .368 | .736 | .920 | .981 | .996 | .999 |
| 1.1 | .333 | .699 | .900 | .974 | .995 | .999 |
| 1.2 | .301 | .663 | .897 | .966 | .992 | .998 |
| 1.3 | .273 | .627 | .857 | .957 | .989 | .988 |
| 1.4 | .247 | .592 | .833 | .946 | .986 | .997 |
| 1.5 | .223 | .558 | .809 | .934 | .981 | .996 |
| 1.6 | .202 | .525 | .783 | .921 | .976 | .994 |
| 1.7 | .183 | .493 | .757 | .907 | .970 | .992 |
| 1.8 | .165 | .463 | .731 | .891 | .964 | .990 |
| 1.9 | .150 | .434 | .704 | .875 | .956 | .987 |
| 2.0 | .135 | .406 | .677 | .857 | .947 | .983 |
| 2.2 | .111 | .355 | .623 | .819 | .928 | .975 |
| 2.4 | .091 | .308 | .570 | .779 | .904 | .964 |
| 2.6 | .074 | .267 | .518 | .736 | .877 | .951 |
| 2.8 | .061 | .231 | .469 | .692 | .848 | .935 |

| m \ c | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| 1.00 | 1.000 | | | | |
| 1.1 | 1.000 | | | | |
| 1.2 | 1.000 | | | | |
| 1.3 | 1.000 | | | | |
| 1.4 | .999 | 1.000 | | | |
| 1.5 | .999 | 1.000 | | | |
| 1.6 | .999 | 1.000 | | | |
| 1.7 | .998 | 1.000 | | | |
| 1.8 | .997 | .999 | 1.000 | | |
| 1.9 | .997 | .999 | 1.000 | | |
| 2.0 | .995 | .999 | 1.000 | | |
| 2.2 | .993 | .998 | 1.000 | | |
| 2.4 | .988 | .997 | .999 | 1.000 | |
| 2.6 | .983 | .995 | .999 | 1.000 | |
| 2.8 | .976 | .992 | .998 | .999 | |
| 3.0 | .966 | .996 | .996 | .999 | 1.000 |

Ctd. from page 162

| m \ c | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 3.2 | .041 | .171 | .380 | .603 | .781 | .895 |
| 3.4 | .033 | .147 | .340 | .558 | .744 | .871 |
| 3.6 | .027 | .126 | .303 | .515 | .706 | .844 |
| 3.8 | .022 | .107 | .269 | .473 | .668 | .816 |
| 4.0 | .018 | .092 | .238 | .433 | .629 | .785 |
| 4.2 | .015 | .078 | .210 | .395 | .590 | .753 |
| 4.4 | .012 | .066 | .185 | .359 | .551 | .720 |
| 4.6 | .010 | .056 | .163 | .326 | .513 | .686 |
| 4.8 | .008 | .048 | .143 | .294 | .476 | .651 |
| 5.0 | .007 | .040 | .125 | .265 | .440 | .616 |
| 5.2 | .006 | .034 | .109 | .238 | .406 | .581 |
| 5.4 | .005 | .029 | .095 | .213 | .373 | .546 |
| 5.6 | .004 | .024 | .082 | .191 | .342 | .512 |
| 5.8 | .003 | .021 | .072 | .170 | .313 | .478 |
| 6.0 | .002 | .017 | .062 | .151 | .285 | .446 |
| 6.2 | .002 | .015 | .054 | .134 | .259 | .414 |
| 6.4 | .002 | .012 | .046 | .119 | .235 | .384 |
| 6.6 | .001 | .010 | .040 | .105 | .213 | .355 |
| 6.8 | .001 | .009 | .034 | .093 | .192 | .327 |
| 7.0 | .001 | .007 | .030 | .082 | .173 | .301 |

| | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|
| 3.2 | | | | | | |
| 3.4 | 1.000 | | | | | |
| 3.6 | 1.000 | | | | | |
| 3.8 | .999 | 1.000 | | | | |
| 4.0 | .999 | 1.000 | | | | |
| 4.2 | .999 | 1.000 | | | | |
| 4.4 | .998 | .999 | 1.000 | | | |
| 4.6 | .997 | .999 | 1.000 | | | |
| 4.8 | .996 | .999 | 1.000 | | | |
| 5.0 | .995 | .998 | .999 | 1.000 | | |
| 5.2 | .993 | .997 | .999 | 1.000 | | |
| 5.4 | .990 | .996 | .999 | 1.000 | | |
| 5.6 | .988 | .995 | .998 | .999 | 1.000 | |
| 5.8 | .984 | .993 | .997 | .999 | 1.000 | |
| 6.0 | .980 | .991 | .996 | .999 | .999 | 1.000 |
| 6.2 | .975 | .989 | .995 | .998 | .999 | 1.000 |
| 6.4 | .969 | .986 | .994 | .997 | .999 | 1.000 |
| 6.6 | .963 | .982 | .992 | .997 | .999 | .999 |
| 6.8 | .955 | .978 | .990 | .996 | .998 | .999 |
| 7.0 | .947 | .973 | .987 | .994 | .998 | .999 |

| m \ c | 6 | 7 | 8 | 9 | 10 | 17 |
|---|---|---|---|---|---|---|
| 3.2 | .955 | .983 | .994 | .998 | 1.000 | |
| 3.4 | .942 | .977 | .992 | .997 | .999 | |
| 3.6 | .927 | .969 | .988 | .996 | .999 | |
| 3.8 | .909 | .960 | .984 | .994 | .998 | |
| 4.0 | .889 | .949 | .979 | .992 | .997 | |
| 4.2 | .867 | .936 | .972 | .989 | .996 | |
| 4.4 | .844 | .921 | .964 | .985 | .994 | |
| 4.6 | .818 | .905 | .955 | .980 | .992 | |
| 4.8 | .791 | .887 | .944 | .975 | .990 | |
| 5.0 | .762 | .867 | .932 | .968 | .986 | |
| 5.2 | .732 | .845 | .918 | .960 | .982 | |
| 5.4 | .702 | .822 | .903 | .951 | .977 | |
| 5.6 | .670 | .797 | .886 | .941 | .972 | |
| 5.8 | .638 | .771 | .867 | .929 | .965 | |
| 6.0 | .606 | .744 | .847 | .916 | .957 | |
| 6.2 | .574 | .716 | .826 | .902 | .949 | |
| 6.4 | .542 | .687 | .803 | .886 | .939 | |
| 6.6 | .511 | .658 | .780 | .869 | .927 | 1.000 |
| 6.8 | .480 | .628 | .755 | .850 | .915 | 1.000 |
| 7.0 | .450 | .599 | .729 | .830 | .901 | 1.000 |

| m \ c | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 7.2 | .001 | .006 | .025 | .072 | .156 | .276 |
| 7.4 | .001 | .005 | .022 | .063 | .140 | .253 |
| 7.6 | .001 | .004 | .019 | .055 | .125 | .231 |
| 7.8 | .000 | .004 | .016 | .048 | .112 | .210 |
| 8.0 | .000 | .003 | .014 | .042 | .100 | .191 |
| 8.5 | .000 | .002 | .009 | .030 | .074 | .150 |
| 9.0 | .000 | .001 | .006 | .021 | .055 | .116 |
| 9.5 | .000 | .001 | .004 | .015 | .040 | .089 |
| 10.0 | .000 | .000 | .003 | .010 | .029 | .067 |
| 11. | .000 | .000 | .001 | .005 | .015 | .038 |
| 12 | .000 | .000 | .001 | .002 | .008 | .020 |
| 13 | .000 | .000 | .000 | .001 | .004 | .011 |
| 14 | .000 | .000 | .000 | .000 | .002 | .006 |
| 15 | .000 | .000 | .000 | .000 | .001 | .003 |

| | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|
| 7.2 | .937 | .967 | .984 | .993 | .997 | .999 |
| 7.4 | .926 | .961 | .980 | .991 | .996 | .998 |
| 7.6 | .915 | .954 | .976 | .989 | .995 | .998 |
| 7.8 | .902 | .945 | .971 | .986 | .993 | .997 |
| 8.0 | .888 | .936 | .966 | .983 | .992 | .996 |
| 8.5 | .849 | .909 | .949 | .973 | .986 | .993 |
| 9.0 | .803 | .876 | .926 | .959 | .978 | .989 |
| 9.5 | .752 | .836 | .898 | .940 | .967 | .982 |
| 10.0 | .697 | .792 | .864 | .917 | .951 | .973 |
| 11 | .579 | .689 | .781 | .854 | .907 | .944 |
| 12 | .462 | .576 | .682 | .772 | .844 | .899 |
| 13 | .353 | .463 | .573 | .675 | .764 | .835 |
| 14 | .260 | .358 | .464 | .570 | .669 | .756 |
| 15 | .185 | .268 | .363 | .466 | .568 | .664 |

| | 22 | 23 | 24 | 25 | 26 | 27 |
|---|---|---|---|---|---|---|
| 10.0 | 1.000 | | | | | |
| 11 | .999 | 1.000 | | | | |
| 12 | .997 | 1.999 | .999 | 1.000 | | |
| 13 | .992 | .996 | .998 | .999 | 1.000 | |
| 14 | .983 | .991 | .995 | .997 | .999 | .999 |
| 15 | .967 | .981 | .989 | .994 | .997 | .998 |

| m \ c | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| 7.2 | .420 | .569 | .703 | .810 | .887 |
| 7.4 | .392 | .539 | .676 | .788 | .871 |
| 7.6 | .365 | .510 | .648 | .765 | .854 |
| 7.8 | .338 | .481 | .620 | .741 | .835 |
| 8.0 | .313 | .453 | .593 | .717 | .816 |
| 8.5 | .256 | .386 | .523 | .653 | .763 |
| 9.0 | .207 | .324 | .456 | .587 | .706 |
| 9.5 | .165 | .269 | .392 | .522 | .645 |
| 10.0 | .130 | .220 | .333 | .458 | .583 |
| 11 | .079 | .143 | .232 | .341 | .460 |
| 12 | .046 | .090 | .155 | .242 | .347 |
| 13 | .026 | .054 | .100 | .166 | .252 |
| 14 | .014 | .032 | .062 | .109 | .176 |
| 15 | .008 | .018 | .037 | .070 | .118 |

| m \ c | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|
| 7.2 | .999 | 1.000 | | | |
| 7.4 | .999 | 1.000 | | | |
| 7.6 | .999 | 1.000 | | | |
| 7.8 | .999 | 1.000 | | | |
| 8.0 | .998 | .999 | 1.000 | | |
| 8.5 | .997 | .999 | .999 | 1.000 | |
| 9.0 | .995 | .998 | .999 | 1.000 | |
| 9.5 | .991 | .996 | .998 | .999 | 1.000 |
| 10.0 | .986 | .993 | .997 | .998 | .999 |
| 11 | .968 | .982 | .991 | .995 | .998 |
| 12 | .937 | .963 | .979 | .988 | .994 |
| 13 | .890 | .930 | .957 | .975 | .986 |
| 14 | .827 | .883 | .923 | .952 | .971 |
| 15 | .749 | .819 | .875 | .917 | .947 |

| m \ c | 28 | 29 | 30 |
|---|---|---|---|
| 14 | 1.000 | | |
| 15 | .999 | 1.000 | |

## 18 Published Sampling Tables

### 18 (i) Dodge-Romig Sampling Tables

Dodge and Romig (Ref. 16) provide four different sets of tables:-

Single sampling lot tolerance

Double sampling lot tolerance

Single sampling AOQL

Double sampling AOQL

The first and second sets of tables are classified according to lot tolerance per cent defective at a constant consumer's risk of 0.10.

The third and fourth sets are classified according to the average outgoing quality limit which they assure.

Lot tolerance plans emphasize a constant low consumer's risk with varying AOQLs, that is, they are designed to give considerable assurance that individual lots of poor material will seldom be accepted. The AOQL plans emphasize the limit on poor quality in the long run, but do not offer constant assurance that individual lots of low quality will not get through. They are appropriate only when rejected lots are 100 per cent inspected. The relative importance of these two objectives will guide the choice of plan.

### 18 (ii) Columbia Sampling Tables

These were originally prepared by the Statistical Research Group at Columbia University following a request by the U. S. Navy and were later incorporated in a peace time manual "Sampling Inspection".

Five inspection levels are available and the producer's risk is 5% for each level. Single double, multiple and sequential plans are available. In the case of double sampling $n_2 = 2n_1$ and in multiple sampling for a given plan the sample sizes are all equal.

18 (iii) MIL-STD 105 D Sampling Procedures and Tables for inspection by attributes.

These tables are published by the U. S. Department of Defense. The plans provide for three general and four special inspection levels.

Single, double and multiple plans are available.

Like the Columbia tables, for a given multiple plan the sample sizes are all the same, but in double sampling plans $n_2 = n_1$.

18 (iv) DEF-131-A Sampling Procedures and Tables for inspection by attributes.

These tables are published by the Ministry of Defence and, with the exception of a section of sequential sampling which these tables carry are identical with the american MIL-STD-105D.

Associated with DEF-131-A is DEF-7-A which is a guide to the use of the tables.

The appropriate American, British and Canadian authorities have collaborated in the preparation of the tables, the Canadian equivalent being CA-C-115.

18 (v) BS 6001: 1972 Specification for Sampling Procedures and Tables for Inspection by attributes.

BS 6000: 1972 Guide to the use of BS 6001

These tables, published by the British Standards Institute are identical with DEF-131-A.

18 (vi) BS 9000: 1967 Specification for general requirements for Electronic Parts of Assessed Quality.

BS 9001: 1967 Specification for sampling procedures and tables for inspection by attributes for Electronic Parts of Assessed Quality.

These tables are similar to the preceding ones except that they are directed specifically to electronic parts.

18 (vii) Other published sampling tables.

Many companies have devised their own sampling inspection tables.

Phillips Electrical Limited have devised single, double and multiple sampling tables based on what the company terms the "point of control" which is that percentage defective of the incoming material which stands equal chances of acceptance and rejection. The point of control ranges from ¼% to 10%. In single and double sampling the Company specifies that the samples should be taken from at least five different parts of the batch and in double sampling $n_2 = 2n_1$.

The company's multiple sampling scheme is as follows:-

Take 5 samples of N parts.

1) Pass batch if each sample contains less than p rejects.

2) Reject batch if any sample contains more than p rejects or more than one sample contains p rejects.

3) Take a second 5 samples of N parts if just one sample contains as many as p rejects and then:-

a) Pass batch if each of the second 5 samples contains less than p rejects,

b) Reject batch if any of the second 5 samples contains p or more rejects.

Tabulated values of N and P are related to the point of control and to the range of batch sizes.

Companies within the Volvo group use tables based on MIL-STD-105D, and although this calls for single, double and multiple sampling with normal, tightened or reduced inspection, Volvo uses only double sampling with normal and reduced inspection. The group has produced an admirable booklet entitled "Volvo Inspection".

## 19 B.S.I. Publications

B.S. 600: 1935 APPLICATION OF STATISTICAL METHODS TO INDUSTRIAL STANDARDIZATION AND QUALITY CONTROL

B.S. 1313:1947 FRACTION DEFECTIVE CHARTS FOR QUALITY CONTROL

B.S.2564:1955 DRAFTING SPECIFICATIONS BASED ON LIMITING THE NUMBER OF DEFECTIVE ITEMS PERMITTED IN SMALL SAMPLES.

B.S.4778:1971 GLOSSARY OF GENERAL TERMS USED IN QUALITY ASSURANCE

B.S.4891:;972 A GUIDE TO QUALITY ASSURANCE

B.S.6000:1972 A GUIDE TO THE USE OF B.S.6001

B.S.6001:1972 SPECIFICATION FOR SAMPLING PROCEDURES AND TABLES FOR INSPECTION BY ATTRIBUTES

B.S.9000:1967 SPECIFICATION FOR GENERAL REQUIREMENTS FOR ELECTRONIC PARTS OF ASSESSED QUALITY

B.S.9001:1967 SPECIFICATION FOR SAMPLING PROCEDURES AND TABLES FOR INSPECTION BY ATTRIBUTES FOR ELECTRONIC PARTS OF ASSESSED QUALITY.

## 20 The A.S.N. of double sampling plans with curtailed inspection of the second sample.

The A.S.N. with curtailed inspection of the second sample is given by the equation:-

$$ASN = n_1 + \sum_{s = c_1 + 1}^{c_2} Pn_1{:}\ s \left[ n_2 P'' n_2{:}c_2-s + \frac{c_2 - s+1}{p} P' n_2+1{:}c_2-s+2 \right]$$

Where $Pn_1:s$ = Probability of exactly s defectives out of $n_1$

$P''n_2:c_2-s$ = Probability of $c_2-s$ or less defectives out of $n_2$

$P'n_2+1:c_2-s+2$ = Probability of $c_2-s+2$ or more defectives out of $n_2+1$.

s is a summation variable.

To illustrate the application of the above equation consider the double sampling plan $n_1 = 40$, $c_1 = 0$; $n_2 = 60$, $c_2 = 3$. The computation of the ordinate of the ASN curve at $p = 0.025$ is shown in the table that follows.

| (1) $s$ | (2) $c_2 - s$ | (3) $Pn_1 : s$ | (4) $P'' n_2 : c_2 - s$ |
|---|---|---|---|
| $c_1 + 1 = 1$ | 2 | 0.368 | 0.809 |
| $c_1 + 2 = 2$ | 1 | 0.184 | 0.588 |
| $c_1 + 3 = 3 = c_2$ | 0 | 0.061 | 0.223 |
| total | | | |

| (5) $P' n_2 + 1 : c_2 - s + 2$ | (6) $\frac{c_2 - s + 1}{p}$ | (7) $n_2$ (4) + (5)(6) | (8) (3)(7) |
|---|---|---|---|
| 0.069 | 120 | 56.82 | 20.91 |
| 0.197 | 80 | 51.04 | 9.39 |
| 0.450 | 40 | 31.38 | 1.91 |
| | | | 32.21 |

$$ASN = 40 + 32.21 = 72.21$$

[The ASN with complete inspection of the second sample is 76.18]

# A USEFUL TABLE FOR DETERMINING THE AOQL

# OF ANY GIVEN SINGLE SAMPLING PLAN

J D MORRISON

"THE QUALITY ENGINEER", Vol 36, No 2

Aston University

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DEMONSTRATION AIDS FOR USE IN THE

TEACHING OF QUALITY CONTROL

J D MORRISON

"BULLETIN OF MECAHNICAL ENGINEERING" No 11

THE AOQL OF SINGLE SAMPLING PLANS

J D MORRISON

"THE QUALITY ENGINEER", Vol 31, No 5

THE FRACTION DEFECTIVE OF INCOMING

MATERIAL TO GIVE THE MAXIMUM ASN

IN DOUBLE SAMPLING

J D MORRISON

"THE QUALITY ENGINEER", Vol 34, No 5