THE COMPARISON OF THE PERFORMANCE OF SERVO MOTORS FEED-DRIVE SYSTEMS.

TO MY FAMILY

THE COMPARISON OF THE PERFORMANCE OF SERVO MOTORS FEED-DRIVE SYSTEMS.

BY

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SUMMARY

The rapid progress which has been made in the field of semi-conductor devices and also in the design of electric motors has considerably improved the performance of electric servo motor feed-drive systems. This allows a tighter specification for servo performance in such applications as machine tool feed-drives or robot actuation; such that electric drives are now much more common and are tending to supplant electro-hydraulic drives, except for high power levels. Therefore, there is now a demand to specify the capabilities of all types of servo motors in order to meet the tight performance requirements of modern control systems.

To achieve the above aim a general mathematical model is first established for all types of servo motors. The model is then modified for each type of servo motor. In order to predict the dynamic characteristics, a computer program package is developed to solve the mathematical model of the system. Provision is also made for investigating the non-linear characteristics of a system.

The performance of the following types of servo motors for different applications are studied:

1. D.C. motors with the following controllers:
   a) Thyristor controlled with current frequency of 50 Hz.
   b) Thyristor controlled with current frequency of 150 Hz.
   c) Pulse width modulated drive with current frequency of 2 kHz.
2. A.C. induction servo motors.
3. Stepping servo motors.
4. Electro-hydraulic servo motors.

The optimized performance capabilities of the servo motor systems are first predicted on the basis of the manufacturers' specifications. Compensation by a derivative feedback and the combination of a lead-lag network is then applied to further improve the predicted performance. This then allows a realistic comparison of the capabilities of electric and electro-hydraulic servo motors.

Key words: feed-drive systems; servo motors; velocity control; position control; machine tool; actuator.

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NOTATION

The following notations are used throughout the thesis. If they are only used in the local text then the notations will be redefined.

A	Circumferential area of the rotor.
a	Linear acceleration.
B	Magnetic flux density, and Bulk module of compressibility.
C	Torque developed on the flapper of electrohydraulic valve.
C_L	Damping coefficient (dynamic friction).
C_x, C_p	Rate of change of flow per unit displacement and pressure change of servo valve.
c_m	Voltage constant of electrical motors and displacement of hydraulic motor.
E,E'	Induced voltage of electrical motors (b.e.f.).
F	Force applied on the load in linear motion.
H	Magnetic force.
I	Current of the armature of electrical motors.
I_f	Current of the field of electrical motor.
I_m, I_L	Inertia of the rotor and load respectively.
K_1,K_3, K_4	Gain of position, velocity and current amplifier respectively.
K_2	Gain of the feedforward integrator.
K_a,K_m	Gain of velocity and acceleration feedback.
Kg	Gain of power amplifier.
K_v	Gain of electrohydraulic servo valve.
K_c	Gain of the current feedback.
K_t	Torque constant of motor.

K_s	Stiffness of the transmission shaft.
K'_t	Torque constant of stepping motor.
ℓ	Length of the rotor of the motor.
L	Inductance of the armature of the motor.
M	Mass of the load in linear motion.
m	Mark to space ratio of pulse width modulated signal.
N	Speed ratio of gear box.
N_1	Speed ratio of lead-screw.
n	Running velocity of A.C. motors.
n_1	Synchronous velocity of A.C. motors.
n	Number of phases energized in stepping motor.
p	Number of pole in electrical motors and pressure change in hydraulic motors.
p',p"	Pressure at the two ends of servo valve.
P_1,P_2	Pressure at the inlet and outlet of hydraulic motors respectively.
P_L	Power loss.
Q_m,q_m	Flow rate of hydraulic motors.
Q_L	Leakage flow.
R	Resistance of the armature of the motor.
R_1,R_2	Primary and secondary resistance of A.C. motors.
R_e	Effective resistance of A.C. motors.
R_m	Magnetizing resistance of A.C. motors.
r	Radius of the rotor of the motor.
s	Laplace transform operator.
S_ℓ	Slip ratio of A.C. motors.
S_1,S_2 etc.	Roots of the characteristic equation.
T	Effective torque from the load in linear motion.
T_L	External load torque.
T_m	Torque developed by the motor.

T_s	Torque on the shaft of the motor.
V	Output voltage from the power amplifier and volume of oil in hydraulic motor.
V_c, V_v, V_p	Output voltage from current, velocity and position amplifier respectively.
V_a	Voltage constant of firing circuits.
V_m, V_d	Peak supply voltage.
V_o	Volume of oil under pressure in hydraulic motor.
V_{RN}, V_{RS} etc.	Voltage potential difference on the various phases of A.C. motors.
x, y	Displacement of the spool of the servo valve.
x_1, x_2	Primary and secondary reactance of the A.C. motor.
x_m	Magnetizing reactance of A.C. motors.
x_e	Effective reactance of A.C. motors.
z	Number of conductor on the armature of the motor.
α	Angular acceleration and phase angle of electrical signal.
θ_i, θ_o	Input and output position of servo feed drive system.
θ_m	Position of the rotor of the motor.
θ	Position error of stepping motor.
$\tau_1 \; \tau_2 \; \tau_3$	Time constants of position and current amplifiers respectively.
τ_v	Time constant of servo valve.
ω_m, ω_o	Angular velocity of the motor and load respectively.
ω, ω_s	Voltage frequency of A.C. motors.
$\mu_o \mu_r$	The relative permeability of air and iron respectively.
ν	Tangential velocity of the rotor of the motor.
φ	Pole and air gap flux of D.C. and A.C. motors. Step angle of stepping motors.
φ_2	The rotor phase lag of A.C. motors.

$\Delta p, \Delta i$	Pressure and current change.
η_m	Efficiency of the servo motors.

NOTE ON THE LAYOUT OF THESIS

There are independent chapters for each type of servo motor. Explanatory graphs are incorporated into this work and they are generally facing their corresponding text for easy reference. The total results of the analyses for each type of servo motor are combined in a series of graphs and tables at the end of each chapter.

Apart from the main chapters, more in-depth study which is directly related to the work appears in Appendices A to H.

CHAPTER 1

INTRODUCTION

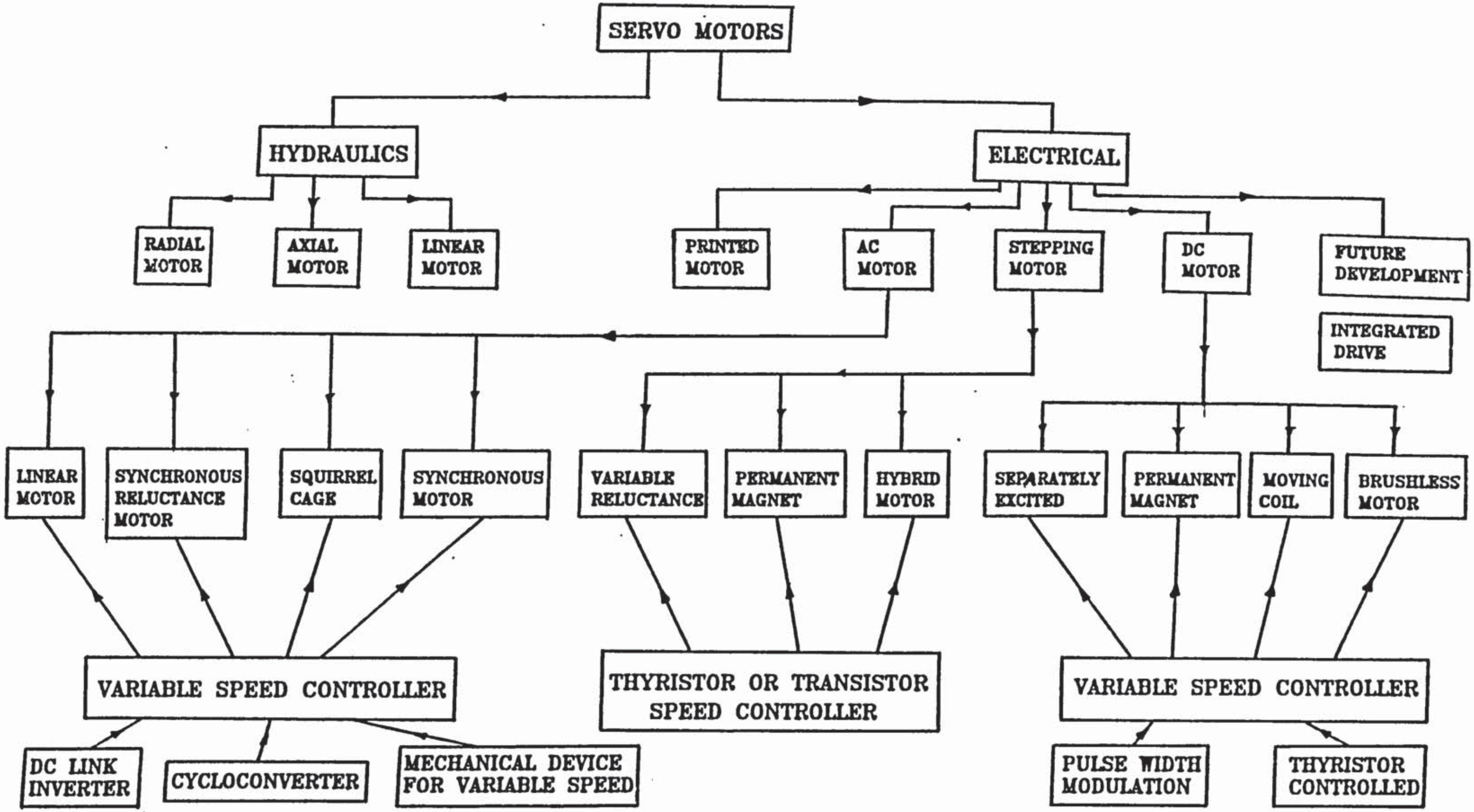


Fig. 1.1 Categorization of Servo Motors

1. INTRODUCTION

The rapid increase in the use of electronic circuits and computers, for the control of electric motors, has resulted in a demand for higher performance specifications in all types of precision servo controlled machine tools and robot motors.

The increases made in the development of high performance servo motor drive systems has complicated the choice of an appropriate servo motor for a particular application. This problem of choice is particularly prominent in machine tools and robot systems where the performance of the system is of the utmost importance.

The prime consideration would be to choose a servo motor to meet the dynamic and static requirements. Further considerations would be the capital cost, the reliability, the efficiency and the size of the complete servo motor drive.

High performance servo-motor drives may be categorized into hydraulic and electric motors, each of which is then made up of various types, as shown in Fig. 1.1, to meet specific requirements. The extent of the choice of servo motor drives is thus quite considerable. Static performance, of widely used industrial motors is readily available though little information is obtainable on newly developed motors.

Static characteristics of the available motors in the market can usually be obtained from the manufacturers' data. The dynamic performance of the servo motor drive is not easily available without modelling the system. In this work a simulation is made for various types of servo motors to obtain the dynamic performance of each type of motor for different applications.

Electro-hydraulic servo motors are widely used in high performance servo motor drive systems. Their capabilities are well known in industry. Usually electro-hydraulic servo valves are used to control these motors. These motors have been a desirable choice for many applications, where the power to size ratio were important, especially in heavy machinery such as mining and mobile equipment.

D.C. motors have usually been the first preference in small size, high performance applications due to their desirable torque-speed characteristic. The inherent problem with D.C. motors was the difficulty in controlling the voltage efficiently and economically at higher current rates.

Due to having a constant speed characteristic, A.C. motors have limited applications such as in machine tool systems but are the first choice for any constant speed applications.

Stepping motors have wide industrial applications but the practical difficulty of devising suitable electronic circuits for the switching of the supplied voltage has limited their use, in particular for the larger motors.

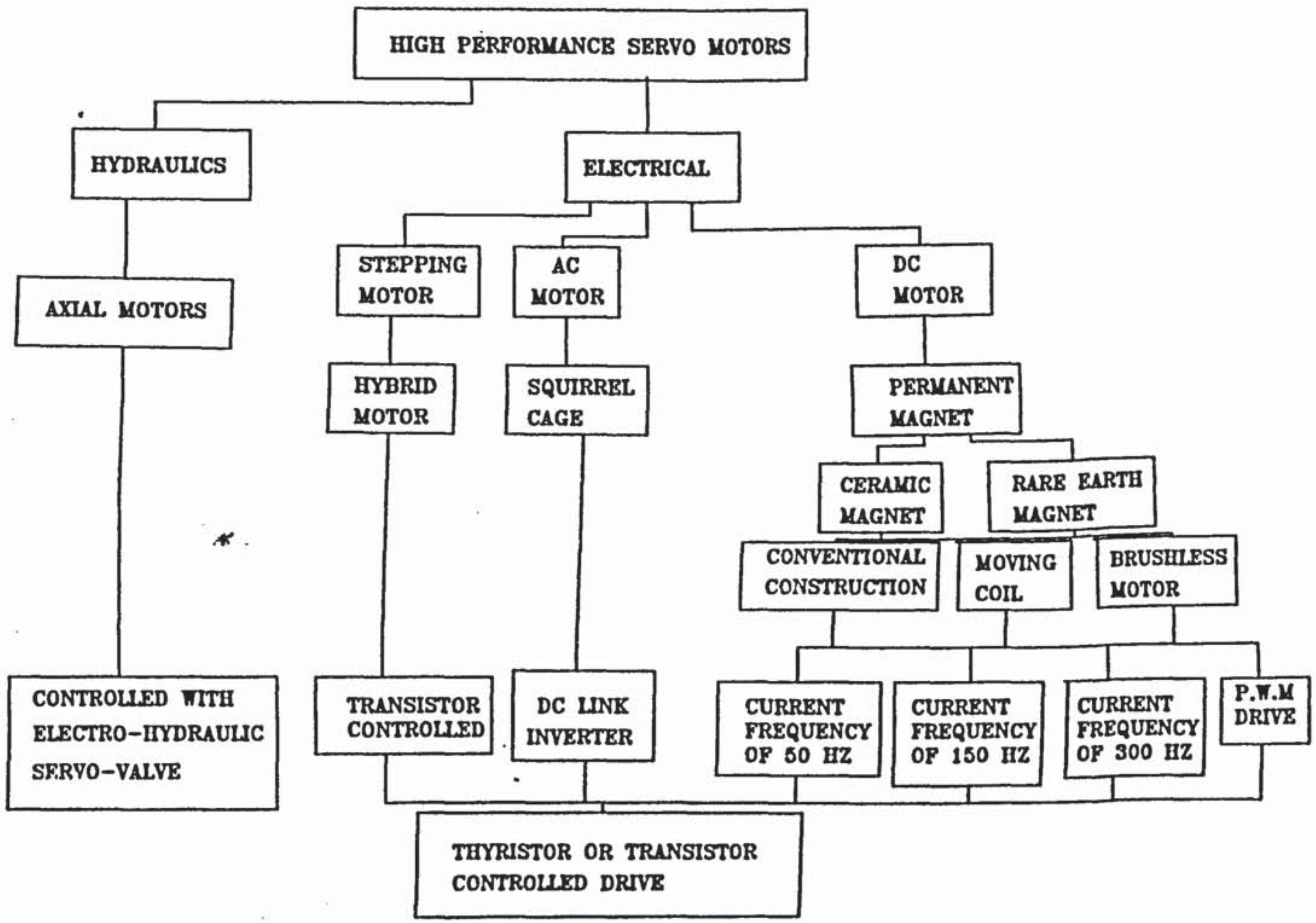


Fig. 1.2. Classification of servo motors for high performance applications.

Solid state power electronics has now opened up new prospects in motor control technology. After many years of development, thyristors and transistors are now used extensively in electrical motor applications. The thyristor is a semi-conductor device capable of controlling large currents. Since its invention in 1957 it has revolutionized the field of electric power control and conversion. Power transistor ratings have also increased dramatically. The increased performance and rapid reduction in the cost of electronic circuitry has resulted in many electrical servo motor drive system applications.

The available types of motors and controllers for general applications, as was shown in Fig. 1.1, may be reclassified for high performance applications as shown in Fig. 1.2.

In this work the performance and characteristics of the following servo motors with different controllers are simulated and established.

1. D.C. motors.
    a. Thyristor controlled D.C. motors.
    b. Pulse width modulated D.C. motors.
2. A.C. induction motors.
3. Stepping motors.
4. Electro-hydraulic motors.

The characteristics of rare-earth magnet D.C. motors and printed motors are also investigated.

Further work is then carried out to predict the best performance of each servo motor drive by means of compensation techniques. The

Safety Cover

Tacho

Servo Motor

Simulated Transmission Shaft

Load Inertia

Resolver

Strain Gauges for Torque Measurement

Control and Measuring Panel

Mechanical Torque Generator (Friction Torque)

Thyristor Controlled Power Amplifier

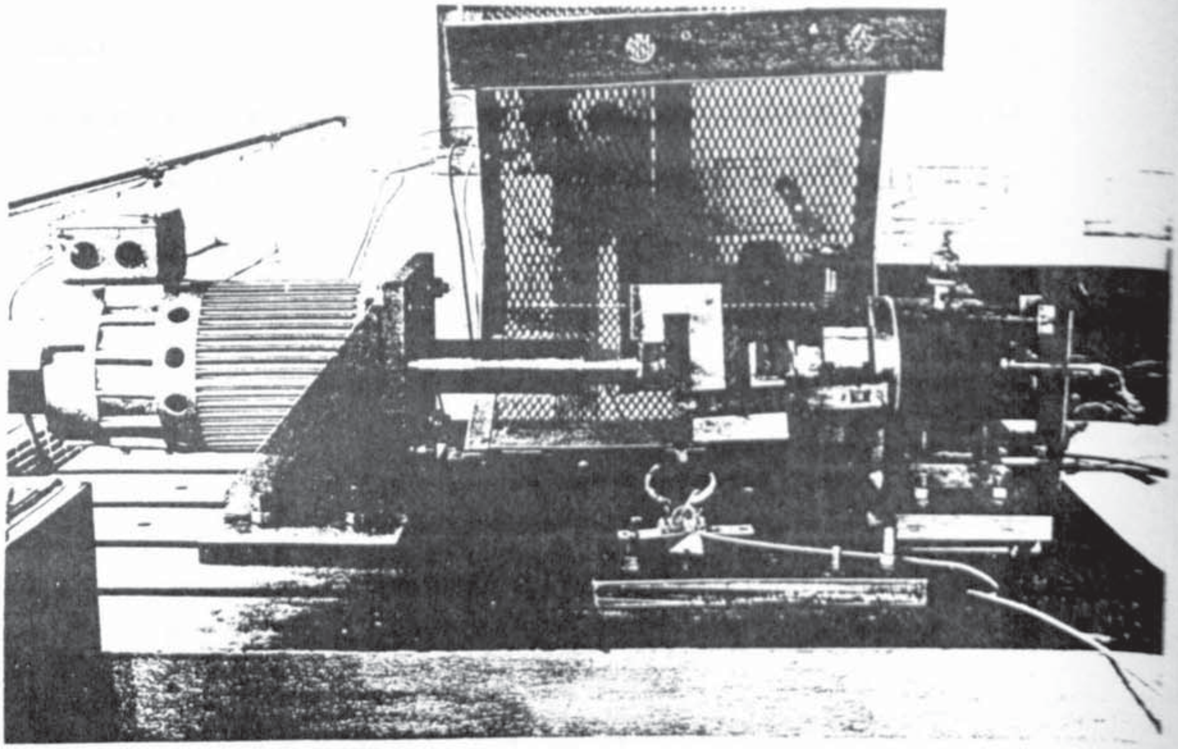
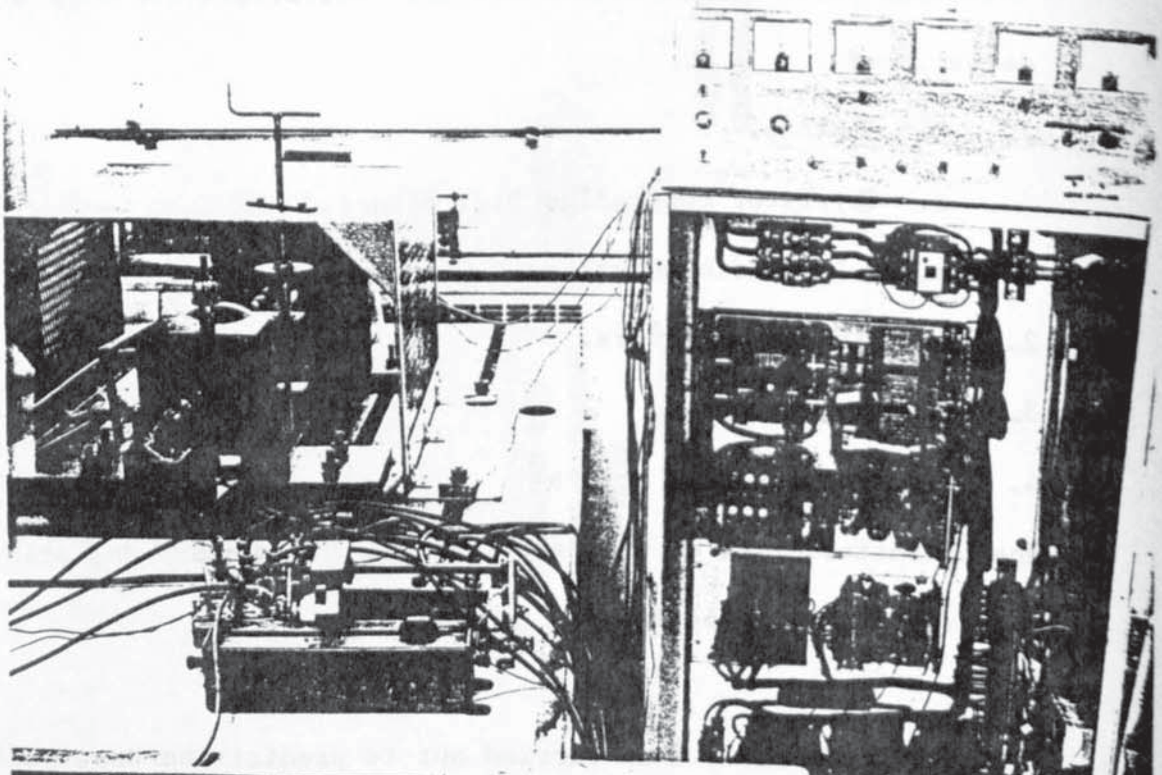


Fig. 1.3 Experimental Test Rig Set Up



most valuable compensation techniques for servo motor drive systems are lead-lag and derivative feedback.

In order to compare and confirm the theoretical performance predictions an experimental test rig was set up as shown in Fig. 1.3. The test rig was designed in such a way that the motor and drive could be easily replaced to test different servo motor drives. The inertial load and transmission stiffness were also varied to investigate the effect of different applications on the performance.

The physical parameters (size, weight, price, etc.) are then compared with the dynamic performance for each of the servo motor drives.

In Chapter 2, previous work on servo motor drive systems is briefly discussed.

In Chapter 3, a general mathematical model is developed to predict the behaviour of the system. This general model is then modified for each of the servo motor drives and analysed theoretically. To facilitate the theoretical analysis of the stability and accuracy of the servo motor drives,a computer program package is developed for use on a desk top computer. The program is in general form and can solve any linear or linearized control problem directly from the dynamic equations of the system. It is also capable of solving non-linear problems by varying the linearized coefficients as the state of the system changes. In Chapter 4, the characteristics and performance of D.C. servo motors with thyristor and pulse width modulated controlled drive are established and discussed. In Chapter 5, the general performance of thyristor or transistor controlled A.C. servo motor is analysed. In Chapter 6, the characteristics and performance of stepping servo motor drives are established and discussed. In Chapter 7, the characteristics and performance of electro-hydraulic servo motors are studied.

To achieve a higher performance from the standard servo motor drive systems the employment of lead-lag network in conjunction with derivative feedback is investigated for each of the above servo motor drives.

To meet higher performance requirements for more sophisticated servo motor systems, there has been a considerable amount of research to develop and improve new servo motors such as the rare earth magnet D.C. motor, brushless D.C. motors, printed motor and linear motors. The characteristics and performance of these servo motors are briefly studied in Chapters 4 and 8.

There are various other servo motor drives with limited applications such as 'hydraulic cylinder drive', 'pneumatic servo drive', 'constant speed A.C. motors with mechanical devices to achieve variable output speed, although these are beyond the scope of this thesis.

In Chapter 8, as a finale, the dynamic and static characteristics of all the above servo motor drive systems are compared.

CHAPTER 2

REVIEW OF PREVIOUS LITERATURE

2. PREVIOUS WORK

2.1 Electric D.C. Servo Motors

D.C. motors are well known in industry. The different types and their characteristics are amply described in many texts. A book by Hindmarsh [1] clearly explains the principle of operation, their applications and characteristics.

There are certain considerations that must be taken into account when selecting, designing, installing or maintaining electric motor control equipment. The principle purpose of a controller is fully presented by Lerich [2]. In most high performance systems, such as NC machine tools and aircraft actuators, the position of the shaft has to be controlled precisely, this can be done only by means of a servo amplifier. The basic operation of a servo amplifier is discussed by Tako [3].

D.C. servo motors in a broad power range can be designed with a wide variety of D.C. motors driven by versatile solid state controls. Jones [4] studied a simple way of matching a motor to its load using the dynamic characteristics of motors. He studied the applications of three main types of D.C. motors, i.e. (a) separately excited shunt motor; (b) permanent magnet motors; (c) moving coil motors. However, he neglected the study of the dynamic behaviour of the motor combined with the load and controller.

An attempt has been made by Szabados, Sinha and Dicenzo [5] to show how well a D.C. permanent magnet motors meets the requirements of the minimum-time position control problem. A second order mathematical model has been used and an approximate solution for the switching time has been obtained in an explicit form in this paper. It has further been shown that on-line estimation of three auxiliary parameters will permit the determination of the switching times even for the very difficult problem of load variation. The step drive of the D.C. servo motor is very useful for precise positioning control. In a paper by Matsui and Adachi [6], the step drive characteristic of the D.C. servo motor has been discussed. These kinds of drive are usually manufactured in small size only because of their low efficiency, their prohibitive costs and the complexity of their drive circuits.

Competition from various other servo motors has forced manufacturers of D.C. motors to design and develop new D.C. motors with increased performances in order to protect their share of the market in high performance servo motor drive system applications. The rare earth magnet, brushless and moving coil D.C. motors have been manufactured for some applications but they are still not in common use. Woodbury has discussed the basic control problems of brushless D.C. motors. He has shown that the motor typically has at least five independent parameters which must be specified properly to obtain the best performance. The gear ratio must be selected, and in addition, several parameters in the electronic commutation and control circuitry must be optimized. Woodbury has also derived the appropriate equations and a systematic method is outlined for arriving at an optimum configuration. To obtain fast, reliable motor starting and motor speed changes, the driver timing signals must, in general, be derived from some electronic rotor position sensing devices. A variety of such devices are outlined by Luzic [8], Sato and Semenov [9] and Sato [10].

A brushless D.C. servo motor described by Wise and Simons [11] consists of two basic components; 1) a brushless D.C. motor featuring a permanent magnet rotor with Hall effect devices for sensing rotor position, and 2) an electronic control package. One of the advantages of this kind of D.C. servo motor is the non-necessity of having a separate position sensing device.

Most of the above work has usually concentrated on D.C. motors of small sizes with specific drives designed for it. Nowadays due to the rapid reduction in the price of electronic circuitry especially thyristors, it has been possible to control large D.C. servo motors as well. The controller of D.C. servo motors are usually of two types: 1) Thyristor controlled D.C. servo motors, and 2) Pulse width modulated D.C. servo motors. It has been shown by Holmes [12] that a simple thyristor bridge controller with single phase A.C. supply can form the basis of a closed-loop control system for the speed control of a D.C. shunt motor. Holmes has also shown experimentally that closed-loop speed control is possible throughout the whole range of conduction. Two current limiting circuits are also proposed by Holmes, which tend to prevent an excessive loss of response speed. A general survey of various ways in which semiconductor controlled rectifiers may be applied to the control of electric motors and generators, and the operating characteristics thereby obtained, have been studied by King [13]. A brief account of the controlled rectifiers and the basic circuits in which they are used are also discussed in this paper.

Merret [14,15,16] has further studied the characteristics of thyristor speed control of D.C. shunt motors from a single phase supply. He has established the dynamic equations of thyristor and motor in all modes of operation. To avoid excessive currents during the transient operation some circuit arrangement is desirable to limit this current to an acceptable level. The nature of the problem of thyristor controlled D.C. motors and some practical circuits are described in his second paper. In his third paper, some instruction for the selection of a D.C. motor, to meet any specific set of requirements of torque and speeds, for single phase thyristor speed controlled D.C. motors are given.

Butler [17] discusses some practical design details for a controller which provides over 2 Kw output from 230 V single phase mains.

The optimization of control systems using thyristors is made difficult by the inherent non-linearity of power amplification by means of thyristors. A work by Nolf [18,19] studies the static and dynamic performance of the thyristor power amplifier when it is equipped with two types of easily used detector circuits. In part II of the paper the non-linearity of the thyristor amplifier resulting from the non-constant static and dynamic gains is further analysed to show the usefulness of the thyristor amplifier having a constant gain. Three solutions are proposed in his paper for linearizing the non-linearity of the thyristor.

The above survey shows the characteristics of different D.C. servo motors, and further provides information on the mathematical modelling of these servo motors. A considerable amount of research has been concentrated on individual servo motors and on improvement of electronic circuitry to increase the efficiency. However, the research papers by the above authors have neglected to give a useful and comprehensive comparison of the performance of the various types of D.C. servo motors.

2.2 Electrical A.C. Servo Motors

Many modern high performance systems require servo motor drives with a precise control of velocity and position, coupled with long-term stability. The D.C. servo motor has satisfied these requirements in many applications, but the mechanical commutator is often undesirable due to the regular maintenance required. A.C. motors such as the squirrel-cage induction motor and the synchronous reluctance motor both have a robust rotor construction which permits reliable and maintenance-free operation at high speeds. Now though, thyristor and transistor controlled A.C. motors have overcome the constant speed problem which had been inherent.

The characteristics and dynamic performances of various A.C. motors are well known, and many texts have described and illustrated their characteristics. A book by Laithwaite [20] explains the structure and the corresponding equivalent circuits of various A.C. motors. The theory of operation and the design of thyristor controlled variable speed A.C. motors with their consequent dynamic performances are described in detail by Murphy [21]. Presently solid state A.C. motor drives, using thyristors as switching devices, have been developed and marketed in Europe and the U.S.A. [22,23]. Falling thyristor prices and an appreciation of the performance possibilities has created a growing interest in solid state A.C. drives, and consequently the variable frequency drive in particular is becoming increasingly common in industry.

There are two basic types of static frequency converter, i) the D.C. link converter, and ii) cycloconverter. In the D.C. link converter the A.C. supply is first converted to pure D.C. The D.C. voltage is then fed to a static 'inverter', a device which converts D.C. power to A.C. power. The static inverter uses thyristors (or transistors) which are switched sequentially so that an alternating voltage wave form is delivered to the A.C. motor. Many suitable inverter circuits have been developed and are described amply in literature [24-29]. The second basic form of static frequency converter is the cycloconverter and this has also been used in variable speed A.C. drives [30-33]. In a cycloconverter the network frequency is converted directly to a lower output frequency without intermediate rectification.

A paper by Mea [34] investigates the performance and dynamic characteristics of two phase A.C. servo motors. By varying the amplitude of the voltage of one of the phases the slip ratio can be changed to achieve variable speed. These types of servo motors are usually in the smaller range because of their poor performance and efficiency. These motors are used in A.C. servo mechanisms and computers which require a rapid and accurate response characteristic. To achieve these characteristics, servo motors have small diameters and high resistance rotors. The high resistance is desirable in order to provide a more linear speed-torque characteristic for accurate control. The inherent damping of the motor decreases as the control-phase voltage is decreased. This paper by Mea also shows that the actual motor system performance is determined by the characteristics of the servo in the slow-speed regions. In these regions, the slope of the speed-torque curve is generally such that damping is not sufficient to maintain stability in a high gain servo loop. To establish a tighter servo loop, various damping means are used in this paper. Typical performance curves are also shown for some common servo motors.

Further work on position servo loops for synchronous and reluctance motors has been carried out by Otto [35]. He shows that it is economical to replace D.C. motors in servo systems with A.C. synchronous and induction motors, in spite of the generally more complex circuitry required by the A.C. principle. This paper mainly deals with synchronous motors that rely on shaft position reference to fit them into high performance position feedback systems. Otto also shows analytically that any synchronous motor, including the reluctance motor, can become a direct replacement for a converter drive D.C. motor if the phase voltages are the result of properly modulated shaft resolver output.

Most of the above papers and work have lain emphasis on particular A.C. motors for specific applications. They also tend to concentrate on circuit design to improve drive performance, although it is not of much use to model the complete servo motor drive systems. However, the papers help to further the understanding of the operation of the motor and to derive a mathematical model.

2.3 Electrical Stepping Motors

Much research has been done on improving the characteristics and performance of stepping motors. Most of the work has concentrated on increasing the efficiency and on the reduction of the cost of stepping motors in order to make them competitive with other types of motors. Due to recent advancements and the reduction in the cost of electronic components, these motors are now being used at an increased rate, especially in the larger size range.

A report by Sharpe [36] investigates the applications potential of these types of motors and the power ranges available related to standard frame size. In his report the general characteristics of stepping motors and their static performance is greatly emphasised. A manufacturers list is also provided.

The principle dynamic characteristics and the rotary displacement of the motor shaft with respect to time is fully discussed by Optiz [37]. In his paper the dynamic characteristics of three stepping motors at different step angles and inertial loads are compared. However, he neglects to study the effect of the stiffness of the coupling. In his paper, though a brief description of the use of the hydraulic torque amplifier is given. A non-dimensional analysis of the static characteristics of variable reluctance (VR) motors with large step angle is given by Harris, McIntosh and Taylor [38]. The speed of response of stepping motors is limited by the starting and stopping rate of steps. Some improvement and prediction of these stepping rates is studied by Lawrenson, Hughes and Acarnley [39].

One of the major problems of stepping motors is the low damping of the rotor, especially the VR type. An investigation of this problem using analogue damping techniques has been carried out by Goddijn [40]. The three types of stepping motors have advantages and disadvantages. The operation and design of permanent magnet stepping motors is very simple but because of their low efficiency, these motors are not very attractive to the buyer. The variable reluctance stepping motors are the most attractive because of their higher efficiency and lower inertias. The drawback is the low damping of the motor and the oscillation of the rotor about the desired position. The hybrid stepping motors, a combination of the PM and VR types improves the damping by increasing the inertia of the rotor. Kordik [41] has studied the basic difference between the hybrid and VR types of stepping motors. A mathematical model and an investigation of the stability of the PM stepping motor is given by Pollack [42]. The acceleration-time characteristic of stepping motors under open loop control condition is studied by Leenhouts and Singh [43]. In this paper a method to quantitatively evaluate the effectiveness of a drive structure and a pulse generation scheme is derived. Experimental results with a digital and analogue ramp controller is also presented and their corresponding performance is compared.

The dynamic performance of PM stepping motors is discussed by Hughes [44]. He has shown that the performance characteristics of these motors could be simplified to three parameters. The dynamic and static characteristics of the hybrid stepping motors with a mathematical model is fully studied by Pickup and Russell [45]. Although the results presented in this paper are appropriate to operation with a constant voltage drive circuit, the use of the model can be extended to study the effects on the dynamic characteristics of using dual-voltage supplies or chopped excitation,for instance. Kodrik [46] has studied different methods of introducing damping in VR stepping motors. His study is based on experimental results and provide some idea of the cause and the prevention of the oscillations.

Using velocity feedback to modulate phase currents, creating a resultant torque which opposes rotor motion, is suggested by Rakes [47] to be an effective way to reduce step motor transient response oscillation. Furthermore he has shown that this technique can be applied to 4 or 3 phase motors provided that two phase detent is used.

Stepping motors are widely used to drive x-y positioning mechanism in NC machinery, because their shaft can move in precise angular increments. However, the motor must be able to move the load quickly. This requires the ability to accelerate the screen and table to a very high speed in a short time and,as it tends to reach the desired position a deceleration to prevent overshoot,(just sufficient to attain the correct position). For this type of application it is shown by Claudio [48] that a smaller torque motor often works better than a motor in the larger size range. A step-by-step procedure for pinpointing the right size is also given by Claudio.

The inherent problem of low-torque availability from the smaller size range stepping motors, can be overcome by using Electrohydraulic stepping motors. A non-linear mathematical model is presented by Vilenius [49] for a modified commercially constructed electrohydraulic stepping motor. With the aid of this model he has examined the dynamic performance of the motor and the quality of the model. It can be deduced from the above survey that much research work has concentrated on individual motors whilst the effect of coupling on the stability and accuracy of the system has been neglected. However, by referring to the various references an overall picture of the dynamic and static characteristics of step motors can be grasped.

2.4 Electrohydraulic Servo Motors

Hydraulic motors of various types are of long standing and well established in industry. Rotary hydraulic motors are of many types; the single, double, or multi-vane type (vane type); a number of piston types; and some gear motors. The theory of operation of these motors is given by Morce[50]. The operation and performance of hydraulic motors in servo control systems depends on the control valve. Most high performance servo systems employ electrohydraulic valves to control the motors. The operation and characteristic of these valves is also discussed by Morce[50].

A mathematical analysis of valve controlled servo motors is analysed by Blackburn, Reethhaf and Shearer [51]. The speed-torque characteristic and the transient response of the system with the assumption that it is of second order is discussed in the above work. The design of electrohydraulic drives can be greatly aided by careful use of linearized design: a definite treatment of this topic is published by Merritt [52]. The low-speed behaviour of low-speed hydraulic motors has been extensively studied by Cessford [53]. Sung and Watanaba [54] have established a mathematical model for the control of load by hydraulic servo systems through elastic structure. They have shown that the use of a rotary motor,properly coupled with a speed reduction mechanism instead of a conventional rectilinear (Jack-type) drive permits a more flexible choice of parameters and results in a better overall system performance.

The effect of derivatives feedback as an active compensation of lightly damped electrohydraulic cylinder drives is studied by Bell and Pennington [55]. It also discusses the use of acceleration and pressure transducer signals to introduce damping into drive where the load mass is supported on anti-friction bearings; i.e. where the inherent mechanical damping is minimal. The analysis is based on the use of a linearized model. Although this analysis has been carried out for cylinder drives, the significance of this compensation technique is readily applicable to rotary feed drive systems.

An experimental study of the performances of an electrohydraulic servo drive has been carried out by Bell and Cowan [56]. The steady state and dynamic characteristics are examined in detail. Particular attention is paid to the threshold speed and the influence of velocity feedback. A comparison of a linear model constructed from manufacturers' data and measured performance through experimental means is also made. De Pennington, Mannetze and Bell [57] have shown that the modelling of the control valve is of prime importance in the design of electrohydraulic systems. They have further shown that a system with 'negative transient acceleration feedback' is an effective compensation to improve the performance of the electrohydraulic servo drive. The use of a digital computer is essential for this type of system design.

A wide range of electrohydraulic flow control valve designs have been employed and comprehensive reviews have been published by Shute and Turnbull [58] and Guillon [59]. Modern valves are now two stage devices, invariably employing feedback from the second stage to the first stage, which is controlled by an electric torque motor. The design of torque motors is studied by Cartwright [60]. Valves which offer the best dynamic and static performance are described by Bidlack [61]. The design and maintenance of hydraulic power supply for different purposes is examined by Sloper [62].

The above research works enable the overall characteristics of electrohydraulic servo motors and the accompanying problems of system modelling to be deduced. Little information is available on the actual accuracy and performances of complete servo motor drives which is of use to the designer.

2.5 Comparison of Different Servo Motor Drives

As was shown in the literature survey of different servo motors, most previous research emphasised the improving of individual servo motor drives. Much less work has been done to compare the performance of different servo motor drive systems, because of the complexity of the problem. Ball [63] has examined this problem, the limitations of different servo motors and the trends of future development.

Lawrenson [64] has compared the performances and characteristics of switched reluctance motor drive with other standard servo motors.

CHAPTER 3

MATHEMATICAL MODEL, COMPUTER SIMULATION, AND GENERAL PERFORMANCE OF SERVO MOTOR FEED-DRIVE SYSTEM

3. MATHEMATICAL MODEL AND COMPUTER SIMULATION OF THE SYSTEM

3.1 Introduction

In order to understand and control any feed drive system, one must obtain quantitative mathematical models of these systems. It is therefore necessary to analyse the relationship between the system variables and obtain a mathematical model. Since the systems under consideration are dynamic in nature, the descriptive equations are usually differential equations. Furthermore, if these equations can be linearized, then the Laplace transform can be utilised in order to simplify the method of solution.

Most research work mentioned previously concentrated on the development of a model of electronic circuits for a specific drive. For this reason it was either too complicated or failed to give an overall view of the performance of the system. The aim of this chapter is to analytically develop a general linearized mathematical model for servo motor feed drive systems, employing servo motors. This model may then be modified for any particular servo motor and application. The non-linear nature of individual servo motor drives and their influence on the performances of the system will be discussed for the corresponding servo motors in their respective chapters.

A high performance servo motor drive system usually consists of three sub-units:

i) the controller

ii) the servo motor

iii) the load mechanism.

A mathematical model will be derived for each of the sub-units by employing linear or linearized relations. Having established the dynamic equations of each sub-unit, an overall mathematical model is obtained both for velocity and position control. Effort has been made to develop a simple but general model to simulate and predict the behaviour of the system. The model is then modified and compared with the experimental test results for each servo motor drive as will be shown later.

To facilitate the analysis of the accuracy and the stability of the system a computer program package has been developed. This package is in a general form to handle any linear or non-linear control problem.

3.2 Performance Criteria in Servo Motor Drive Systems

In choosing a servo motor for a specific application, the static and dynamic characteristics of the complete system are the main considerations. The static characteristics, such as maximum torque, size and price are normally obtainable from manufacturers. The dynamic characteristics, however, are not easily obtainable without modelling the system. In any control system employing servo motors, two important dynamic characteristics must be considered:

a) The effect of a command signal, and

b) The effect of external disturbances such as a torque change.

These two dynamic characteristics should be taken into consideration for both velocity and position control.

The dynamic performances of the system can be defined by several parameters such as frequency band width, time constant and settling time, for a particular input variable. In order to investigate the complete dynamic behaviour, an impulse input response of the system would be an ideal test [76]. Because of the impracticality of this test it is not used frequently, particularly in servo motor drive systems. The most practical test is the step input response of the system. From the step input response of the system most of the performance parameters such as frequency response, settling time and time constant can be extracted. It is also easier for the designer to comprehend the step input response. The overshoot and the steady state error, which can be obtained from the test, are also of major importance in the design of servo motor drive systems. In machine tool and robotic applications, the following error of the system for a ramp input response is also important. By knowing the following error the user is able to compensate these errors when performing a defined path. The steady state and dynamic errors of the system due to the external torque are also important. It is therefore essential to obtain a step input torque response of the system. The speed of recovery can also be obtained from this test.

To investigate the stability and the effect of a particular parameter of the system on the performance,the root-locus technique is used.

The static and physical characteristics of servo motor drive systems may be defined as:

a) weight/power, weight of the motor to the rated power of the motor. It would be useful to compare the weight of the complete servo drive system. This is not so easy because different controllers have different weights. The inverse of this ratio indicates the density of the power.

b) Inertia/stand still torque, this defines the minimum time required to change the velocity by a unit value at the stand still torque.

c) Inertia/rated torque, this defines the minimum time required to change the velocity by a unit value at the rated torque.

d) Inertia x maximum velocity/stand still torque, this defines the minimum time required to reach the maximum velocity from rest at the stand still torque.

e) Inertia x maximum velocity/rated torque, this defines the minimum time required to reach the maximum velocity at the rated torque.

Usually, as the velocity of the motor increases, the maximum torque available decreases to a rated torque. The average values of minimum time required at rated and maximum torque will therefore give a more realistic prediction of the response time of the motor to reach the necessary velocity. The total inertia of rotor and load must also be considered to determine the prediction of the response time for the complete system. The above minimum response times are ideal values, i.e. the motor develops the rated or maximum torque instantaneously and stays at the limited values until the required velocity is reached, the torque then drops to zero to keep the velocity constant. However, this will not be true when the motor is operating with a controller. The dynamic behaviour of the complete system will limit the gain and consequently the response time. Compensation techniques are used to reach the above ideal optimum response times. Fig. 3.1 shows the ideal torque and velocity response of a motor. The ideal response is only achievable when the gain of the system is infinitely large and there is a perfect current or torque limiter to protect the motor from being overloaded.

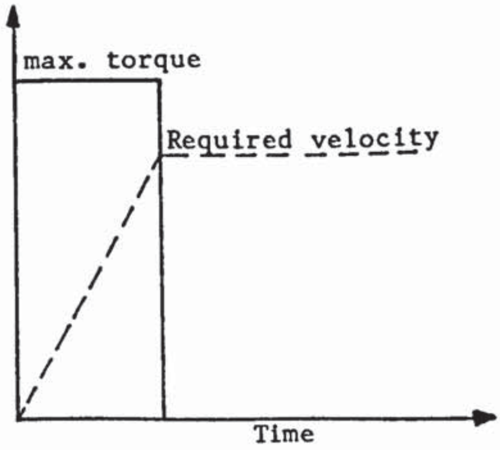


Fig. 3.1 Optimum velocity and torque response of an ideal motor. ----- velocity

—— torque

3.3 Mathematical Model of the System

3.3.1 Mathematical model of the controller

3.3.1.1 Introduction

The structure of a controller of a motor differs for the various types of motors and manufacturer's design. The controller of a high performance servo motor usually consists of: a) position amplifier, b) velocity amplifier, c) current amplifier (pressure amplifier in the case of hydraulic motors),and d) power amplifier. Most of the controllers of D.C. and A.C. servo motors have the above structure. Hydraulic servo motors may also have the same structure except that the servo valve is used as the power amplifier.

Stepping motors are basically open loop servo motors. The mathematical model of the controllers of stepping motors will be discussed in Chapter 6. In this section a mathematical model of a controller having the above structure is developed. The necessary modifications are made for each type of servo motor in their corresponding chapters.

3.3.1.2 Position amplifier.

There are basically two types of position amplifier and they may be categorized as either analogue or digital. In the analogue type a position transducer produces a voltage proportional to the difference between the shaft position and a reference position. In order to obtain continuous operation, the reference point changes as the shaft rotates. The command position voltage is then compared with the voltage of the position transducer. The error voltage is then amplified by the position amplifier to obtain an output voltage for the velocity amplifier. The transfer function of the position amplifier may be ideally written as:

$$\frac{V_p}{\theta_i - \theta_o} = K_1 \tag{3.1}$$

where

V_p is the output voltage of position amplifier (volts)

θ_i is the command position (rad)

θ_o is the actual position of the load or shaft (rad)

K_1 is the gain of the position amplifier (volts/rad)

Usually the position transducer contains some noise picked up due to the moving parts. In order to filter out this noise the position amplifier is designed as a first order lag network (Appendix A). This transfer function may be formulated as:

$$\frac{V_p}{\theta_i - \theta_o} = \frac{K_1}{1 + \tau_1 s} \tag{3.2}$$

where

τ_1 is the time constant of the network (sec.)

s is the Laplace transform operator.

A digital position amplifier makes the servo motor behave similar to the stepping motor. This kind of position amplifier is designed in such a way that servo motor systems can easily be interfaced to a mini-computer or microprocessor. The input signal is in pulse form and each pulse represents an increment of rotation. The input pulses are counted in a pulse counter. The number registered on this counter is then compared with the number registered from the position feedback counter. The result of this comparison is a pulse width modulated signal. This pulse width modulated signal is then transformed to an analogue voltage via a digital to analogue converter. This voltage is proportional to the position error and is fed to the position amplifier. A detailed description of this type of position amplifier is given in Appendix C. The transfer function of this type of position amplifier is similar to the analogue position amplifier.

3.3.1.3 Velocity amplifier.

The circuit diagram of a velocity amplifier with a feed forward integrator is shown in Appendix A. The voltage output from the position amplifier or the command velocity is compared with the velocity feedback. The error voltage is then amplified by the velocity amplifier and the output voltage is fed to the current amplifier. The transfer function of the velocity amplifer may be formulated as(see Appendix A):

$$\frac{V_v}{V_p - K_m \omega_m} = \frac{K_2 + K_3 s}{s} \tag{3.3}$$

where

V_v is the output voltage of velocity amplifier

ω_m is the angular velocity of the motor

K_m is the velocity feedback gain

K_2 and K_3 are the amplifier and feed forward integrator gain.

In the velocity amplifier, the velocity feedback is obtained from the motor because of its practicality. Usually a tacho generator is connected to the motor as a velocity transducer. When the motor is connected to the load via a flexible transmission shaft or coupling, then the performance of the system varies according to the location of the velocity transducer. This will be further investigated later.

In order to analyse possible ways of improving the performance of the system, the use of acceleration feedback is investigated. By introducing acceleration feedback in the velocity amplifier the transfer function may be re-written as:

$$\frac{V_v}{V_p - K_m\omega_m - K_a s\omega_m} = \frac{K_2 + K_3 s}{s} \tag{3.4}$$

where

K_a is the gain of the acceleration feedback.

In equation 3.4 the acceleration feedback is considered around the motor. In practice, it is easy to take the derivative of the velocity feedback as an acceleration feedback. This method must be employed with great care, because of the noise in the velocity transducer. If, however, the load is in linear motion then it is possible to use an acceleration transducer for acceleration feedback. The effect of acceleration feedback around the motor and from the load, on the performance of the system will be further discussed.

In order to filter out the noise of the velocity transducer, a first order lag network may be added to the velocity amplifier. This is necessary for the controller, where the output voltage is directly fed to the power amplifier. In this analysis and by assuming the existence of a current amplifier, a first order lag network will be added to the current amplifier. This filter will reduce noise from the velocity and current amplifiers.

3.3.1.4 Current amplifier.

The acceleration feedback is not easily obtainable, particularly when the load is in angular motion. To use the latter, an additional cost is inevitable. Another technique is to use current feedback as a semi-acceleration feedback. The current is usually proportional to the torque and the torque is proportional to the acceleration of the rotor of the motor. However, the drawback is that at steady state the current is no longer proportional to acceleration. The current shape of most of the electrical servo motors is in pulse form; the use of a low pass filter is therefore essential in order to obtain average values of current. The low pass filter on the feedback signal produces a phase lag which is undesirable and causes instability. The most effective way would be to have a first order lag network instead (Appendix A). For reasons of stability a lead network may also be added to the current network. The significant influence of the lead-lag network on the performance of the system will be discussed later.

The transfer function of the current amplifier then may be simplified as (Appendix A):

$$\frac{V_c}{V_v - K_c I} = \frac{K_4(1 + \tau_2 s)}{1 + \tau_3 s} \qquad (3.5)$$

where

K_c is the gain of the current feedback

V_c is the output voltage from the current amplifier

K_4 is the gain of the current amplifier

τ_2 and τ_3 are the time constants of the lead-lag network.

The significance of current feedback use compared with acceleration feedback use on the stability and accuracy of the system will be discussed later

Details of the electronic circuits which provide the above transfer functions for position, velocity and current amplifiers are given in Appendix A.

3.3.1.5 Power amplifier.

The transfer function of the power amplifier usually involves non-linear equations. By using perturbation techniques linearized relationships can be obtained. For electrical power amplifiers, the relationship between the input voltage, V_c, and output voltage V can be formulated as:

$$V = K_g V_c \tag{3.6}$$

where

K_g is the gain of the power amplifier.

The linearized relationships for hydraulic power amplifiers using a servo valve as a power amplifier may be formulated as:

$$q_m = c_x x - c_p p \tag{3.7}$$

where

q_m is the flow rate of the motor

x is the displacement of the spool of the servo valve

p is the back pressure of the motor

c_x is the flow change per unit displacement of servo valve

c_p is the flow change per unit change in pressure.

The dynamic equation of the electrohydraulic servo valves may be simplified as a first order system [61]:

$$x = \frac{K_v}{1 + \tau_v s} V_c \qquad (3.8)$$

where

K_v is the gain of servo valve

τ_v is the time constant of the servo valve.

The exact dynamic equations and the validity of the above linearized relation will be examined in more detail for each servo motor drive system later.

3.3.2 Mathematical model of servo motors

It is difficult to establish a unified mathematical model for all servo motors. The dynamic characteristics of D.C. and hydraulic motors are well established. It was shown by Szabados [65] that linear dynamic equations are satisfactory assumptions for most D.C. motors which have constant magnetic field flux. The dynamic equations of A.C. and stepping motors are very complicated, but for the sake of simplicity, linear equations similar to those of D.C. motors will be established here, though a more detailed discussion will follow later. Kirchoffs voltage equation applied to an analogous electrical motor circuit may be written as:

$$V = RI + LSI + c_m \omega_m \qquad (3.9)$$

A similar equation may be obtained for hydraulic motors through mass-continuity.

The torque available from the motor may be related to the

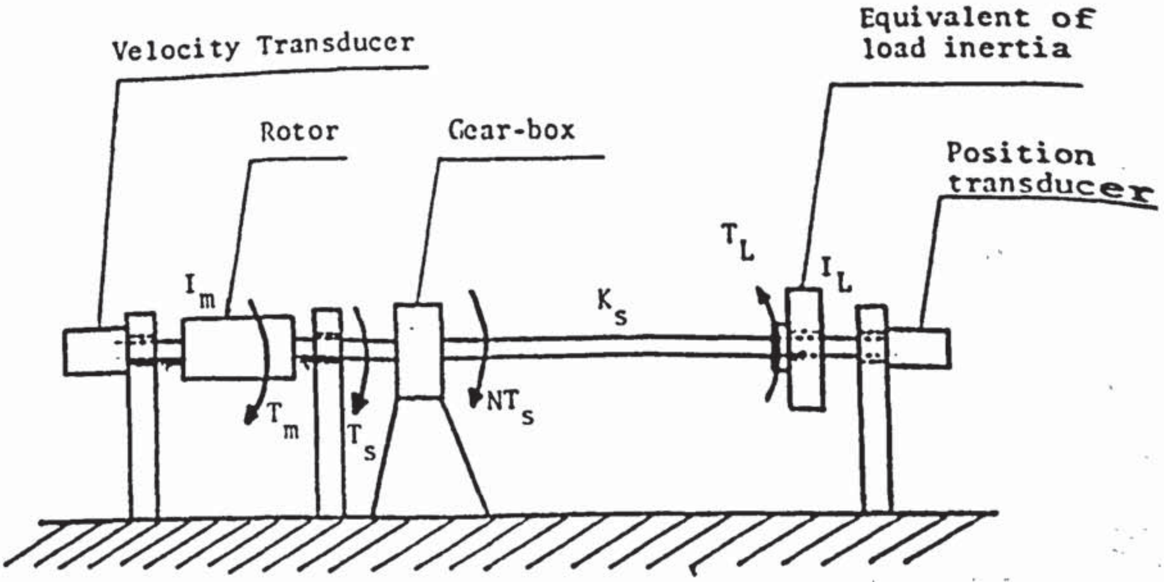


Fig. 3.2 : Schematic Diagram of the Mechanical Part of the System

current as:

$$T_m = K_t I \qquad (3.10)$$

The dynamic equation of the rotor with an inertia, I_m, and shaft torque, T_s can be formulated as:

$$T_m = I_m s\omega_m + T_s \qquad (3.11)$$

In equation 3.11 the dynamic friction is neglected and the effective friction is represented with the load.

3.3.3 Mathematical model of the loading mechanism

This part of the system as shown in Fig. 3.2 usually consists of the rotor of the motor, a gear box, a high stiffness power transmission mechanism and a working table or load inertia.

The torque developed in the shaft of the rotor, T_s, is given by equation 3.11. The relationship between torque output and shaft position may be formulated as:

$$T_s = \frac{K_s}{N}\left(\frac{\theta_m}{N} - \theta_o\right) \qquad (3.12)$$

where N is the gear reduction ratio of the gear box. For simplicity the inertia of the transmission shaft can be included with the inertial load.

The mass in linear motion can be represented as an inertia in angular motion. If the mass of the table is M and the pitch of the lead-screw is N_1, the equivalent inertia can be evaluated as:

$$F = Ma \qquad (3.13)$$

where F is the force applied to the table and a is the linear acceleration of the table, but:

$$a = \frac{N_1 \alpha}{2\pi} \tag{3.14}$$

and

$$T = \frac{N_1 F}{2\pi} \tag{3.15}$$

Substituting from 3.15 and 3.14 for F and a in 3.13 yields,

$$T = \frac{M N_1^2 \alpha}{4\pi^2} \tag{3.16}$$

or

$$T = I_L \alpha \tag{3.17}$$

Comparing 3.17 and 3.16 yields,

$$I_L = \frac{M N_1^2}{4\pi^2} \tag{3.18}$$

where I_L is the equivalent mass moment of inertia of the table in angular motion.

The dynamic equation of the load inertia (working table in the simulated form as shown in Fig. 3.2) neglecting the static friction may be written as:

$$NT_s - T_L = I_L s^2 \theta_o + C_L s \theta_o \tag{3.19}$$

where C_L is the effective equivalent dynamic friction of the system.

Having established the mathematical model of each part of the system the overall block diagram can be drawn as is shown in Fig. 3.3.

3.4 Computer Simulation of the Dynamic Analysis of Control Systems.

For the design of control systems, knowledge of the dynamic performance of the system is essential. In order to obtain the required performance, a designer needs to formulate the dynamic equations of the system, and to study: a) stability, b) transient response, and c) behaviour under steady state conditions. As the system becomes more complicated, the study and solution of its characteristic equations without the use of advanced aids may become impossible, especially so for non-linear systems. Because of these difficulties, a computer package which makes the analysis and design of control systems simpler was developed.

The program first evaluates the general transfer functions of the output variables with respect to any particular input variable of the linearized system in symbolic form. When all the necessary numerical values have been assembled, the program will then calculate the transfer function in numerical form. To investigate the stability, the roots of the characteristic equation are evaluated by use of a numerical technique. To investigate the effect of a change in any parameter on the system, the root-locus of the parameter can also be obtained by the program. Having obtained the roots of the characteristic equation, the transient response of the system for some standard inputs by the use of the inverse Laplace transform technique can be obtained. The Nyquist plot, frequency response and the phase angle response may also be obtained.

This program is also capable of handling a non-linear system. The non-linear equations that describe the system should be linearized about the steady state operating point, hence it is essential to find the steady state operating point. One of the subprograms of this package provides a numerical method of solving these non-linear equations. The dynamic characteristics of the system, then may be analysed by the small perturbations technique. This computer package allows the large signal response of the non-linear system to be studied by changing the non-linear coefficients at the end of each step. Hence the length of each step depends on the degree of non-linearity, and this piece-wise analytical solution allows larger step-lengths than a numerical integration routine.

The structure and theory of this package is given in detail in Appendix B.

3.5 General Performance and Influence of Parameters of a Linear Servo Motor Drive System.

3.5.1 Introduction

A servo motor drive system may be defined as a linear system if its dynamic characteristics can be described by the mathematical model obtained in the previous section. Servo motors have inherent limitations which tend to alter the dynamic characteristics of the system. These limitations and their effects on the performance characteristics will be discussed later. The parameters of position, velocity and current amplifiers may be optimized in order to obtain a stable system which conforms with the required performance. In this section the effect of each parameter on the dynamic performance of the system is investigated.

To facilitate analysis the root-locus technique is used. For the purpose of numerical analysis a 3 kW D.C. motor is first considered, but the results can be generalised for any type of servo motor drive system, if its dynamic characteristics can be described by the linear mathematical model already outlined. The investigation is first performed in velocity control and after establishing the influence of each of the parameters the effect of the position amplifier on the root-locus is finally investigated. The root-locus of the characteristic equation is symmetrical about the real axis. For this reason only the upper part of the root-locus is shown in this analysis. The root-locus of a particular parameter is only obtained over the range of interest and not the complete loci. The purpose is only to show the effect of variation of the parameter on the loci of the roots. When investigating the effect of a parameter on the system, other parameters are kept constant. The values of other parameters are chosen in such a way that firstly the system be underdamped and secondly, the initial points of loci of the particular root-locus are obtained from the root-locus of the previous graph. So, in this way, the effect of the parameter on the system can be seen more clearly.

3.5.2 Effect of velocity amplifier gain

Fig. 3.4 shows the root-locus of the system as the velocity amplifier gain is increased. There are two complex roots, one due to the natural frequency of loading mechanism (s_3) and the other due to the loop frequency (s_2). As the gain of the velocity amplifier is increased, the two roots move away from the real axis. The damping ratio of the loop frequency reduces and the damping ratio of load frequency increases initially and reduces again. For high performance systems, it is thus essential to use compensation techniques to improve the damping ratio.

3.5.3 Effect of feed forward integrator gain

Fig. 3.5 shows the root-locus as the gain of the feed-forward integrator is increased. An additional real root (s_1) can be extracted from this graph. This is due to the fact that the feed forward integrator increases the order of a system. As the gain of the integrator is increased the real root move away from the origin resulting in a faster integral action. The damping of the two complex roots is reduced and at the higher values it can become unstable. The feed forward integrator is usually used in servo motor drive systems in order to achieve zero steady state error. Up to a certain feed forward integrator gain value,the change in the damping of the other two roots is not significant. An excessive increase in this gain can cause the damping ratio of other roots to be decreased rapidly.

3.5.4 Effect of current feedback

Fig. 3.6 shows the root-locus as the gain of the current feedback is increased. An increase in the current feedback gain tends to reduce the frequency of the two complex roots. It also tends to increase the damping ratio of these two roots. The value of the real root increases as the gain of the current feedback is increased. An excessive increase in the current feedback gain results in a reduction of the damping ratio of the two complex roots. This is because the real root,due to the integrator action is becoming more dominant in the response. In order to prevent the reduction in damping ratio of the two complex roots, the integrator gain must be reduced. As can be seen from the graph, the current feedback may be used to introduce damping into the two complex roots.

3.5.5 Effect of acceleration feedback around the motor.

Fig. 3.7 shows the root-locus as the gain of acceleration feedback is increased. This acceleration feedback has not changed the value of the real root which is due to the feed forward integrator. The complex root (s_2) moves away from the vertical axis with an initial increase in its damping ratio. The damping ratio of the natural frequency of the load mechanism (s_3) is increased considerably. The acceleration feedback around the motor may be used to introduce damping into the natural frequency of the load mechanism.

3.5.6 Effect of acceleration feedback from the load.

Fig. 3.8 shows the root-locus as the gain of the acceleration feedback from the load is increased. This feedback increases the damping ratio of the loop frequency (s_2), but reduces the damping ratio of the natural frequency of the load (s_3) so that the system is unstable. By using a filtering device the effect of this feedback on the natural frequency of the load may be cancelled, and can be used as a compensation technique for the loop frequency of the system.

3.5.7 Effect of a first order lag network.

Transducer signals may contain noise. In order to filter out this noise a first order lag network is commonly used in servo motor drive systems. Fig. 3.9 shows the root-locus as the time constant of the lag network is increased. The first order lag network increases the order of the characteristic equation. An increase in the time constant of the lag network reduces the frequency and the damping ratio of the two complex roots (s_2,s_3) (which can become unstable). The two real roots tend to converge and result in complex roots with low frequency. It is hence deducible that a first order lag network is not desirable for servo motor drive systems. For filtering the noise and reducing the speed of response it is necessary to include a lag network into the system. When a first order lag network is used, other types of compensation techniques such as a lead network must be used to improve the undesirable effect.

3.5.8 Effect of a lag network in combination with current feedback.

In this analysis, the current feedback is kept constant and the root-locus, as the time constant of the lag network is increased, is obtained. Fig. 3.10 shows the root-locus as the time constant of the lag network is increased. The effect of a change in the time constant of the lag network in conjunction with current feedback is different from the previous cases. Firstly no significant change in the fundamental frequency and damping ratio (s_2) occurs. The frequency of the loading system gradually decreases as the time constant increases and results in a higher damping ratio (s_3). The two real roots, one due to the integrator and the other due to the time constant of the lag network converge and change into high frequency complex roots (s_1) with a reduction in its damping ratio.

3.5.9 Effect of the change of the time constant of lead network.

Fig. 3.11 shows the root-locus as the time constant of the lead network is increased. The effect of the lead network is almost opposite to that of the lag network. It introduces little damping into the fundamental frequency of the system and the load natural frequency (s_2,s_3) respectively. The complex roots s_3 move towards the real axis and eventually result in two real roots. One of these real roots moves away from the origin and the other moves towards the origin. It is deducible from this graph that by the use of the lead network the disadvantage of the lag network can be eliminated whilst the desirable filter effect (or reduction in speed of response) is preserved. When the time constant of the lead network becomes larger than that of the lag network, one of the roots tends to the origin resulting in a very slow response. This has a disadvantageous effect in many applications.

3.5.10 Effect of the gain of velocity amplifiers on a system in combination of lead-lag network and current feedback.

Fig. 3.12 shows the root-locus as the gain of the velocity amplifier is increased (with the feed forward integrator). The trend of the loci of the roots are similar to that of the system without compensation. Because of the two overdamped real roots, the gain can be increased much more than that of uncompensated system. At a certain value of the gain, the two real roots converge and separate into complex roots. Further increase in the gain results in higher frequency complex roots and lower damping ratio.

3.5.11 Effect of the gain of feed forward integrator on the compensated system.

Fig. 3.13 shows the root-locus as the gain of the integrator is increased. Initially an increase in the gain of the integrator tends to increase the damping ratio of the complex root (s_2) and reduces the damping ratio of the natural frequency of the load (s_3). Further increase in the gain causes the convergence of the two real roots into complex roots and also a decrease in the damping ratio. As can be seen from this graph, an excessive increase in the gain of the feed forward integrator may cause an unstable system.

3.5.12 Effect of acceleration feedback on the compensated system.

The effect of acceleration feedback is similar to that of the uncompensated system. Figs. 3.14 and 3.15 show the root-locus as the gain of the acceleration feedback is increased around the motor and load respectively. When the feedback gain around the motor is increased it tends to add to the damping ratio of both complex roots. Above a certain value of this gain the damping ratio of the loop frequency gradually reduces (s_2). The improvement in the damping of the natural frequency of the load is considerable (s_3). The acceleration feedback from the load improves the damping of the loop frequency (s_2) whilst the damping ratio of the load natural frequency decreases. These acceleration feedbacks have no effect on the two real roots. One of which is due to the feed forward integrator and the other is due to the lag network in the system.

3.5.13 Effect of acceleration feedback when the velocity feedback is from the load.

When the velocity feedback is obtained from the load instead of the motor, the damping of the loop frequency (s_2) increases but the damping ratio of the load natural frequency reduces considerably (as the starting points of the locus shown in Figs. 3.16 and 3.17). As can be seen from Figs. 3.15 and 3.16 the acceleration feedback around the motor improves both complex roots (s_2,s_3). The acceleration feedback from the load improves the loop frequency (s_2) but worsens the damping ratio of the load's natural frequency (s_3). Acceleration feedback in this case has no effect on the other two real roots (so_1, so_2).

3.5.14 Effect of gain and time constant of position amplifier on the system.

In this section the effect of the position amplifier on the roots of the characteristic equation are considered. Two types of characteristic equations, one overdamped and the other with a damping ratio of 0.7 of the fundamental root of the characteristic equation of the system in velocity control. Figs. 3.18 and 3.19 show the root-locus as the gain and the time constant of the position amplifier are increased, with a 0.7 damping ratio of loop frequency in velocity control. It can be seen from Fig. 3.18 that as the gain of the position amplifier is increased, the real root due to the position closed-loop (s_4) moves away from the origin. Initially the change on the other roots is not significant. Above a certain value of the gain, the damping ratio of the fundamental frequency of the system in velocity control reduces rapidly. The increase in the value of the real root, due to the position control loop, and the reduction in the damping ratio, of the frequency of the system in the velocity control, will limit the gain of the position amplifier and consequently the accuracy of the system. The increase in the time constant of the lag network results in a reduction of the damping ratio of the fundamental frequency of the system in velocity control. The existence of time lag network in the position amplifier is similar to that of the velocity control which is undesirable. Figs. 3.20 and 3.21 shows the root-locus as the gain and time constant of the position amplifier are increased with the system overdamped in velocity control. As can be seen from these graphs, the increase in the gain of the position amplifier reduces the damping ratio of the position control frequency with little change in other roots. The oscillation of the fundamental frequency in position control will limit the gain of the position amplifier. The increase in the time constant of the lag network results in a reduction of the damping ratio and the frequency of the position loop frequency (as can be seen in Fig. 3.21). It was noticed that the system with a damping ratio of 0.7 in velocity control can provide a more accurate and stable system in the position control mode. This is due to the further increase of the gain of the position amplifier with little reduction in the damping ratios of the other roots.

3.6 Conclusions.

In this chapter a linear mathematical model for most servo motor drive systems was obtained. The general performance of the system first in velocity control mode was established. The location of the roots of the characteristic equation generally determine the performance of the system. Without any compensation the location of roots depend on the parameters of the system such as inertia, resistance, inductance, natural frequency of the load mechanism. For a specific application the effects of using a feed forward integrator, a current feedback, a lead-lag network and acceleration feedback were investigated. The increase in the gain of the system generally increases the speed of response by moving the roots away from the origin of the complex plane. It also reduces the damping ratio of the roots. The addition of feed forward integrator increases the order of the characteristic equation. It was shown that above a certain value of the gain of the integrator further reduction in the damping ratio of the roots are caused.

Improvement on the damping ratio of the roots was achieved by using current or acceleration feedbacks. The acceleration feedback around the motor introduces damping in the high frequency root (due to the load mechanism). The acceleration feedback from the load increases the damping ratio of the loop frequency. Introduction of a lag network reduces the speed of response by moving the roots towards the origin of the complex plane. It also introduces two additional complex roots in the system. It is generally undesirable as the performance of the system is concerned. It will be shown later that due to the limitations of the controller of the servo motor, this lag network is used to reduce the speed of response of the system. It is also used as a low pass filter to reduce the noise of the transducers.

The lead network will introduce damping into the additional complex root produced by the lag network. Therefore, it is desirable to use a lead network whenever a lag network is used. It is also desirable to keep the time constant of lead network much lower than the time constant of the lag network.

The introduction of lead-lag network and current feedback as compensation allows the gain of the system to be increased much more than a system without compensation. The gain of the feed forward integrator must be kept below a certain value, otherwise it causes a rapid reduction of the damping ratio of the system. The effect of using acceleration feedback remains similar to that of an uncompensated system. It was shown when velocity feedback is obtained from the load instead of the motor, it increases the damping ratio of the loop frequency but it reduces the damping ratio of the natural frequency of the load mechanism. By use of acceleration feedback improvement on the damping ratio can be achieved.

It was shown that the performance of the system in the position control mode depends on its performance in the velocity control mode. A damping ratio of around 0.7 in velocity control allows a higher position gain to be used in position control. When the speed of response of the system in the position control mode approaches the speed of response of the system in the velocity control mode, the damping ratio of the fundamental frequency of the system reduces rapidly and results in an oscillatory or unstable response.

Having established the effect of various feedbacks and parameters of the system, the locations of the roots can be adjusted by varying the appropriate parameter or parameters. For each type of servo motor there are certain limitations on the speed of response of the system. By adjusting properly the parameters of the compensation networks, the limitations can be met and also the required performance is obtained. These limitations will be discussed for each type of servo motor drive system later.

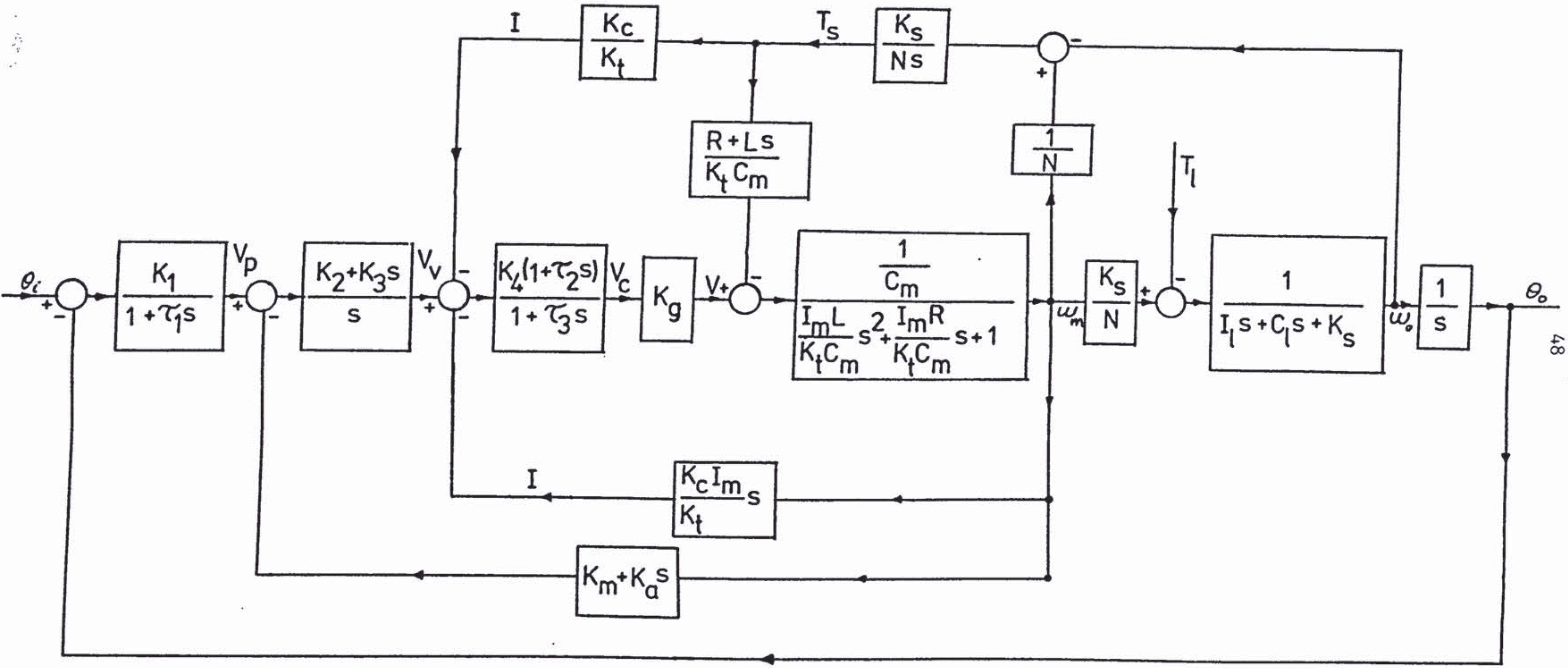


Fig. 3.3 Linear Mathematical Model of the System.

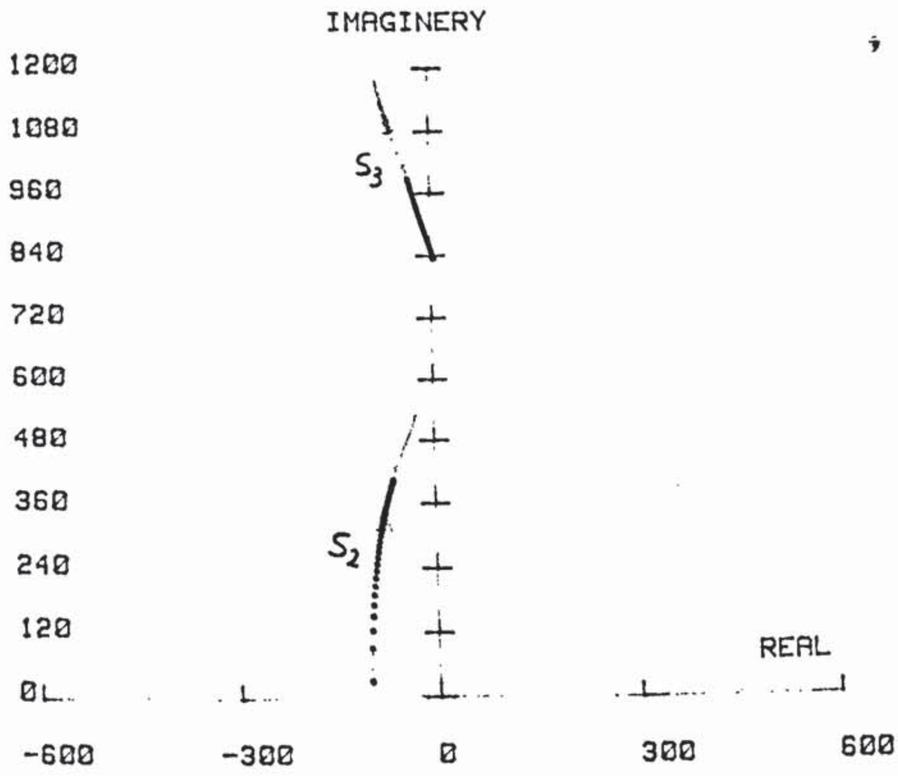


Fig. 3.4 Root-Locus as the Gain of the Velocity Amplifier is Increased.

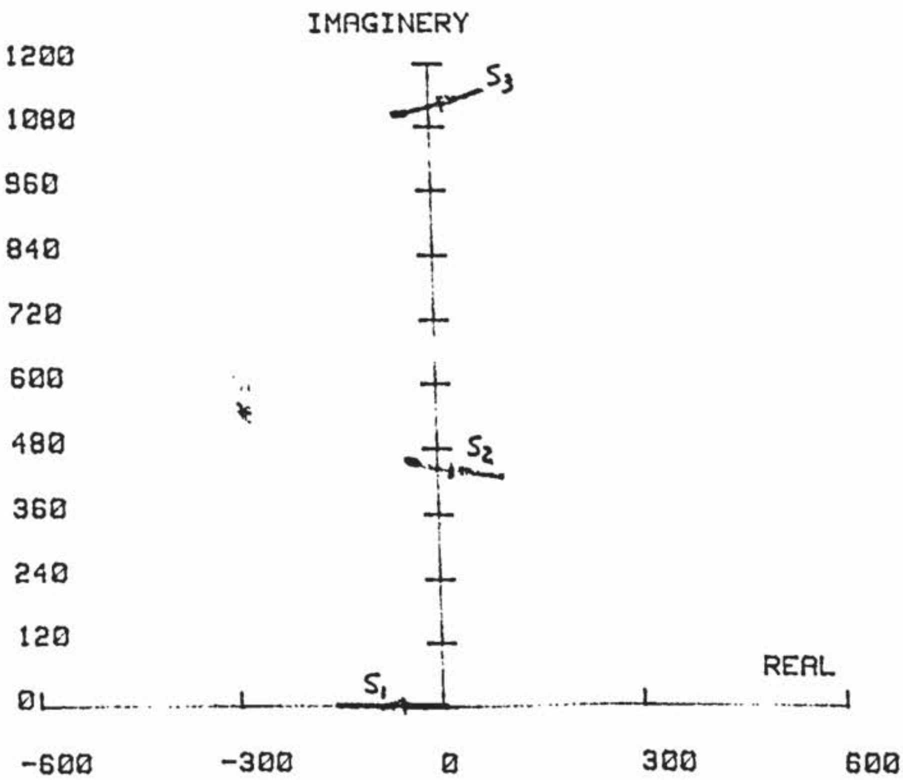


Fig. 3.5 Root-Locus as the Gain of Feed Forward Integrator is Increased.

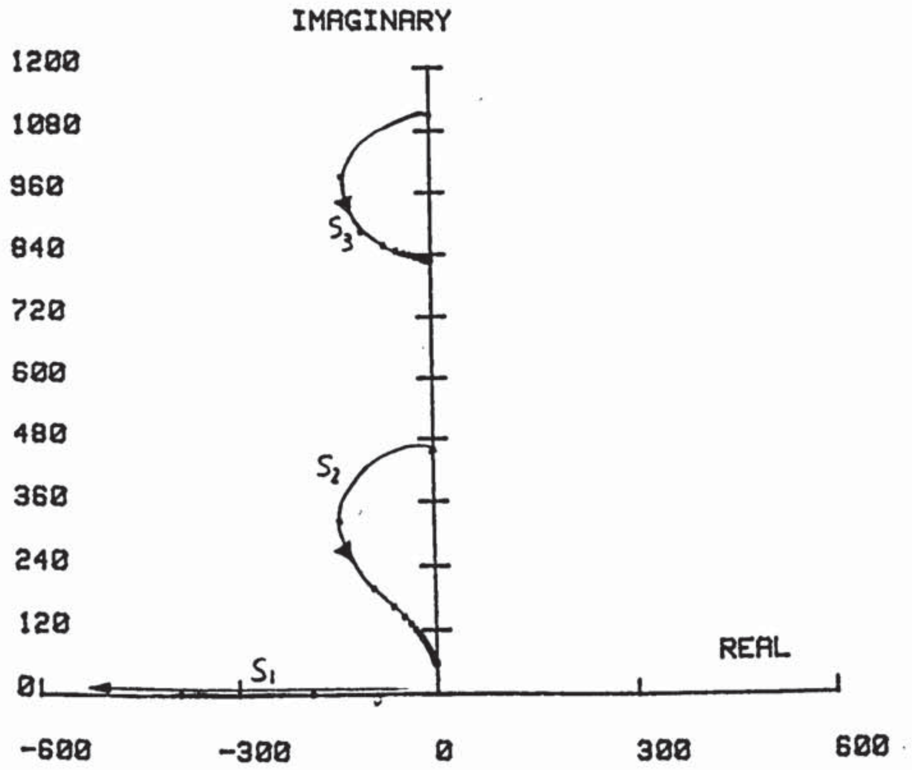


Fig. 3.6 Root-Locus as the Gain of the Current Feedback Gain is Increased.

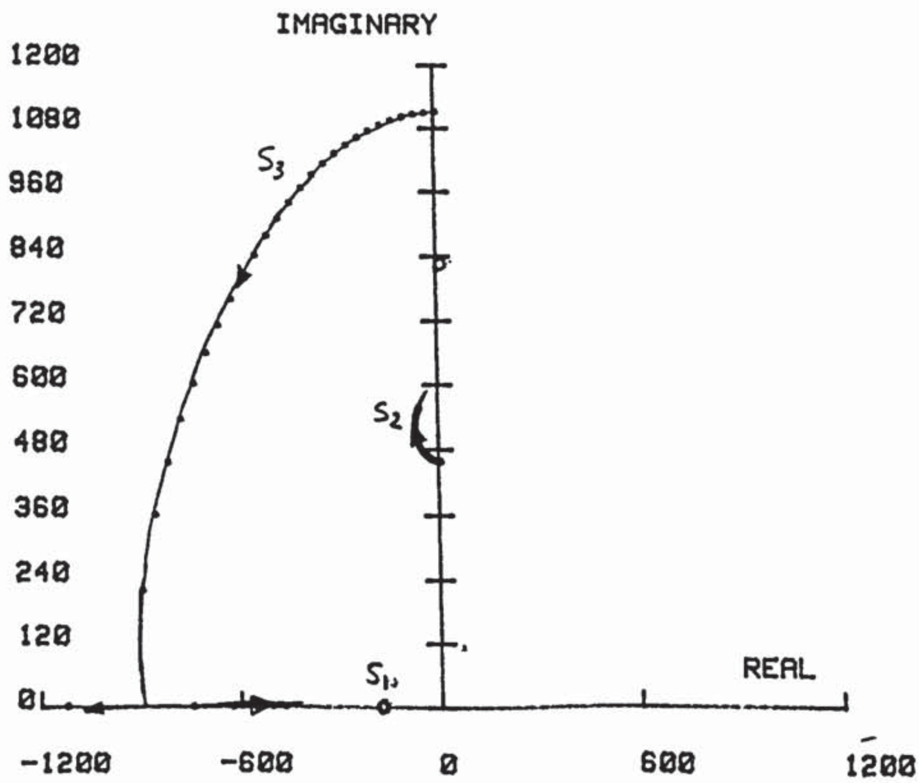


Fig. 3.7 Root-Locus as the Gain of the Acceleration Feedback Around the Motor is Increased.

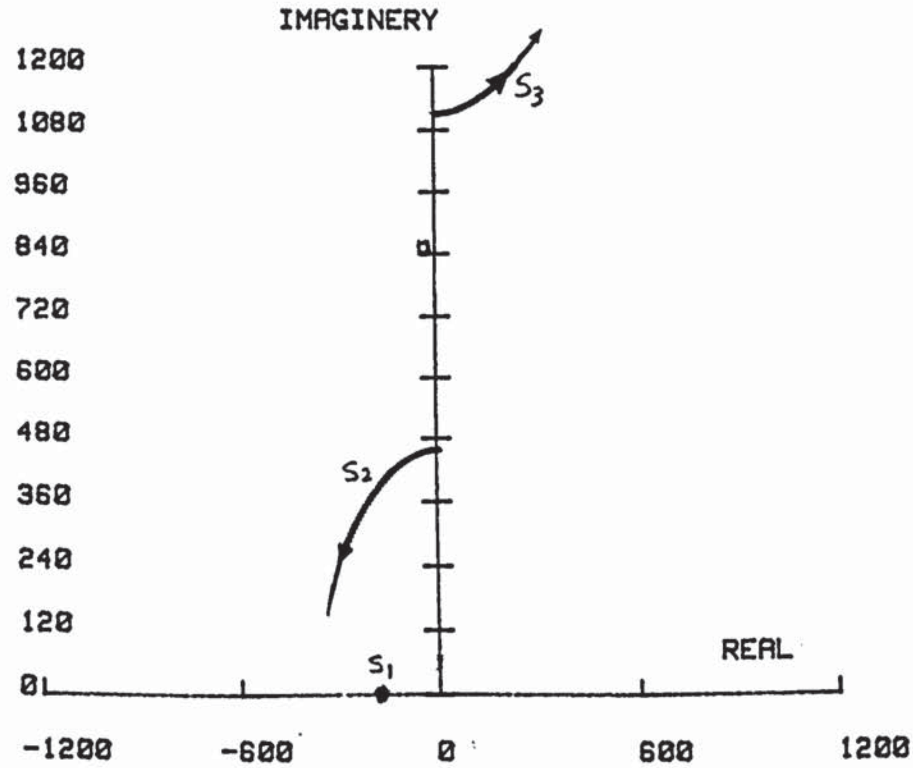


Fig. 3.8 Root-Locus as the Gain of the Acceleration Feedback From the Load is Increased.

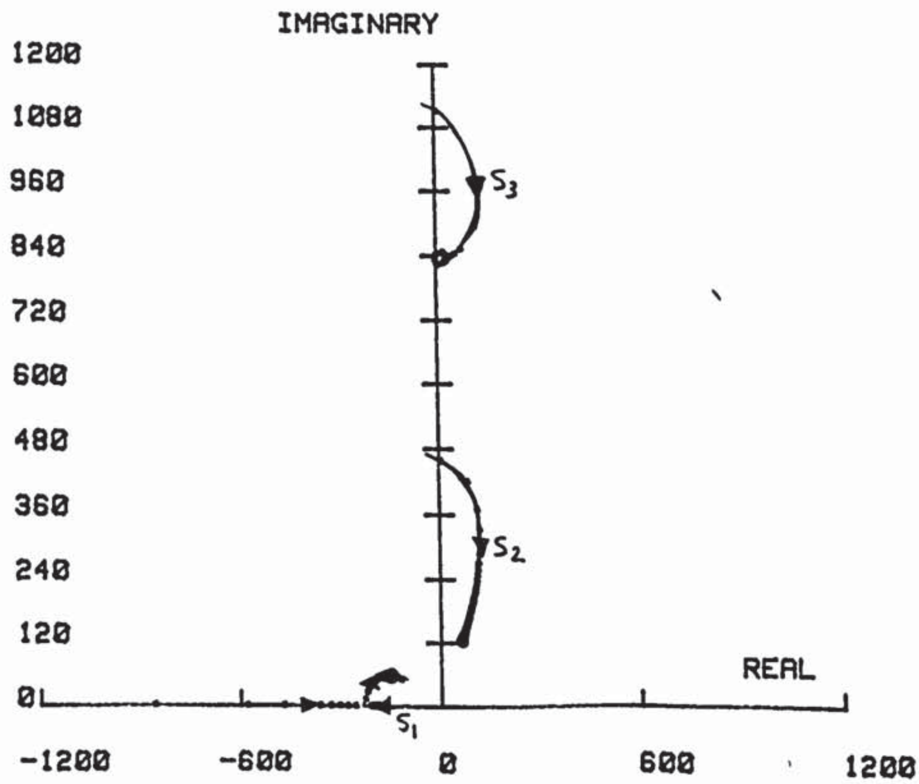


Fig. 3.9 Root-Locus as the Time Constant of the Lag Network is Increased.

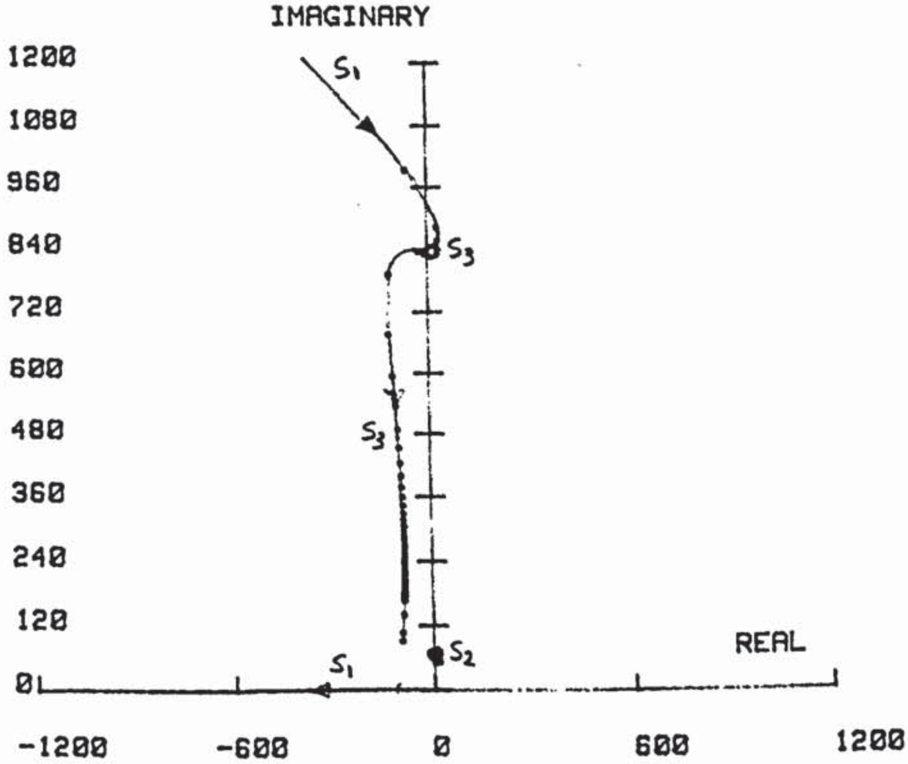


Fig. 3.10 Root-Locus as the Time Constant of the Lag Network is Increased. System with Current Feedback.

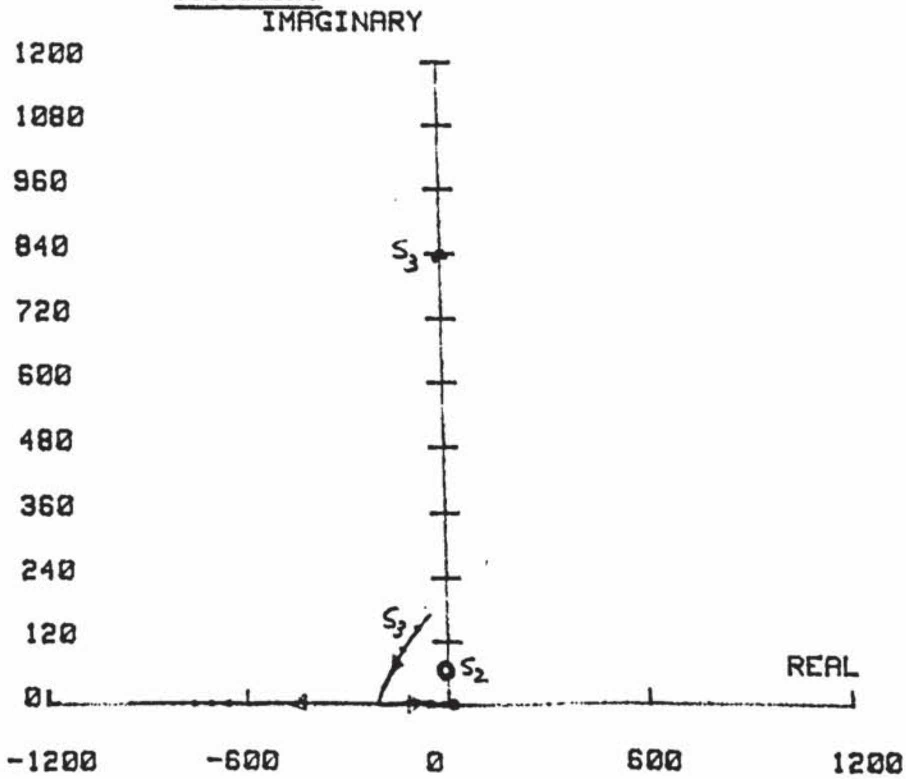


Fig. 3.11 Root-Locus as the Time Constant of the Lead-Network is Increased.

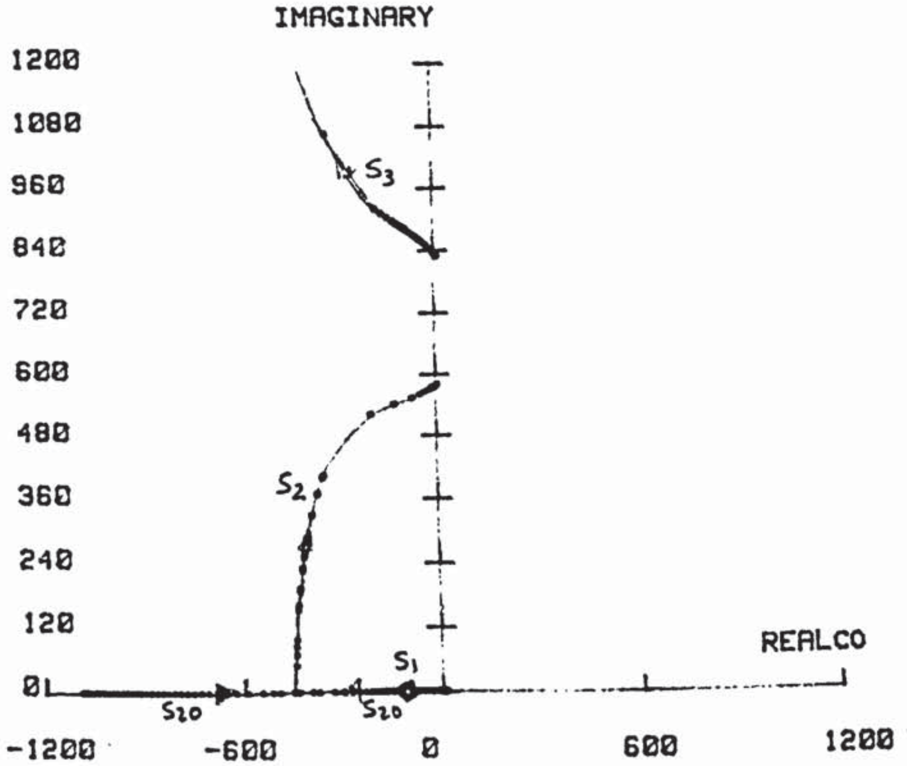


Fig. 3.12 Root-Locus as the Gain of Velocity Amplifier, on the System with Lead-Lag Network and Current Feedback is Increased.

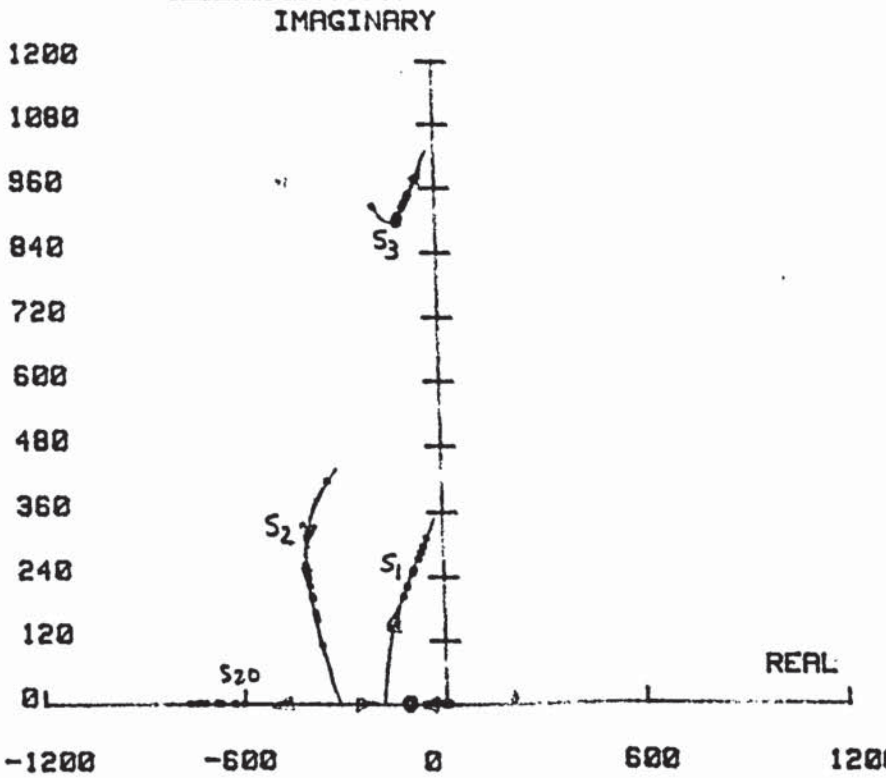


Fig. 3.13 Root-Locus as the Gain of Feed Forward Integrator, on the System with Lead-Lag Network and Current Feedback, is Increased.

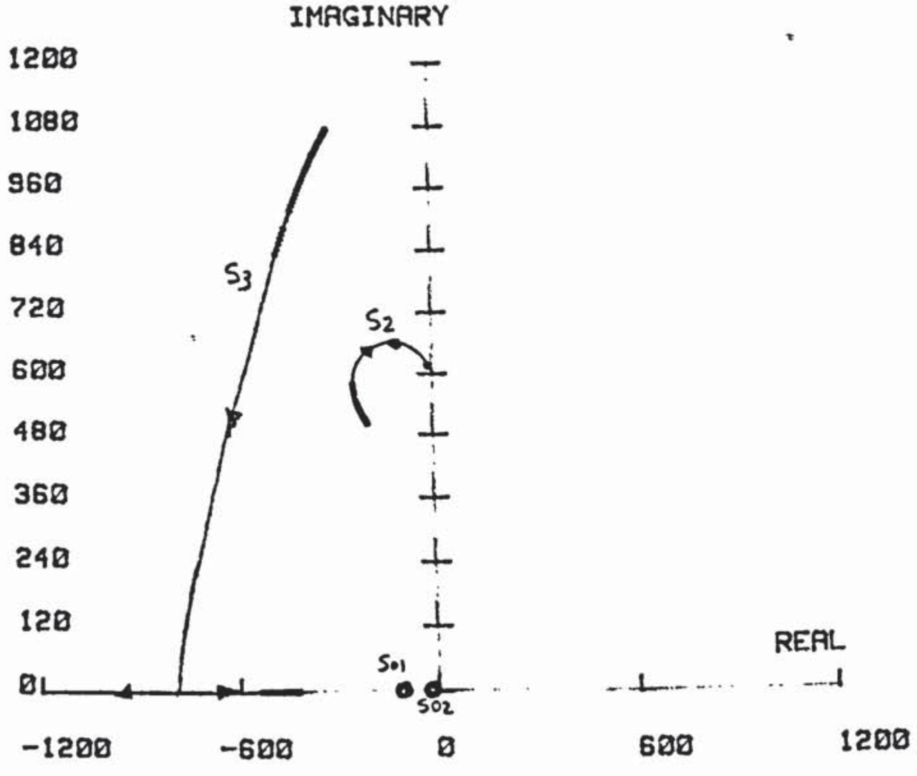


Fig. 3.14 Root-Locus as the Gain of Acceleration Feedback, Around the Motor on the Compensated System, is Increased.

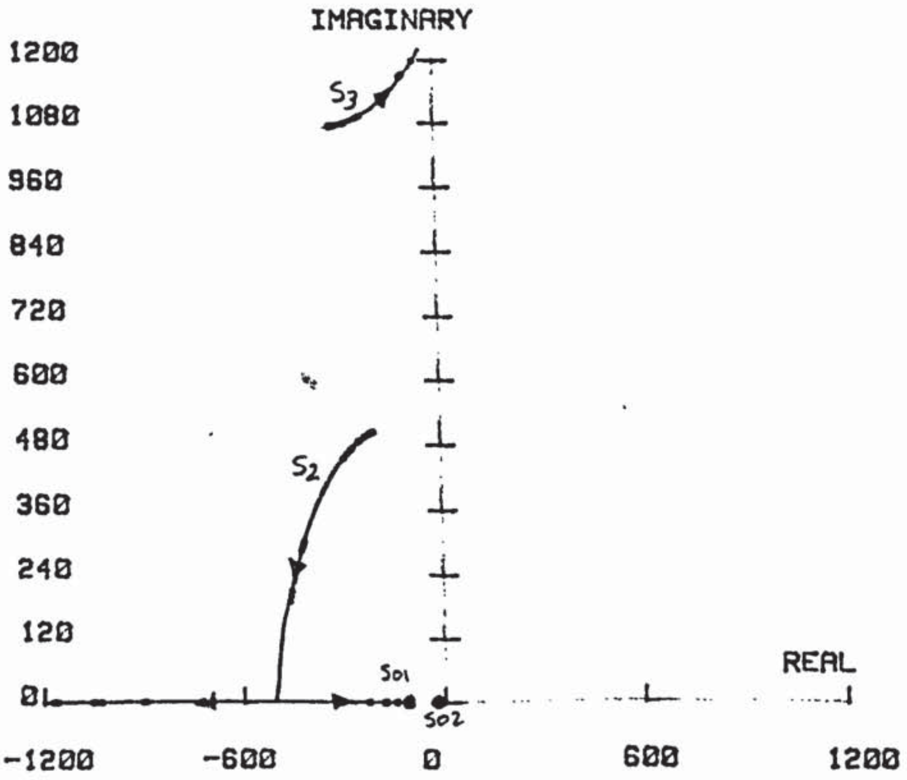


Fig. 3.15 Root-Locus as the Gain of the Acceleration Feedback, From the Load on the Compensated System, is Increased.

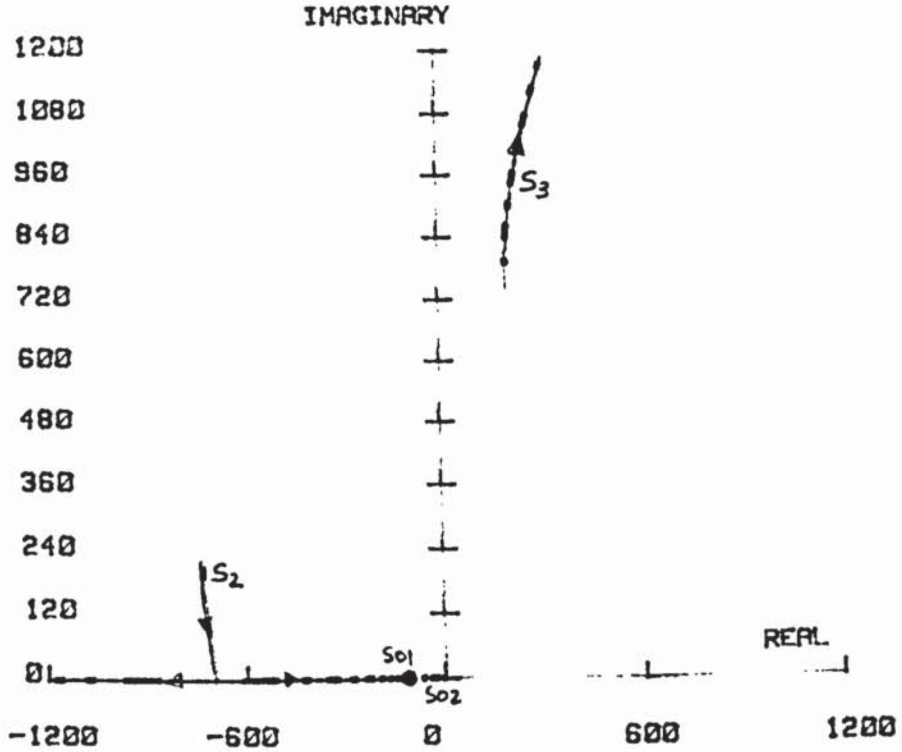


Fig. 3.16 Root-Locus as the Gain of the Acceleration Feedback From the Load is Increased. Velocity Feedback is Obtained From the Load.

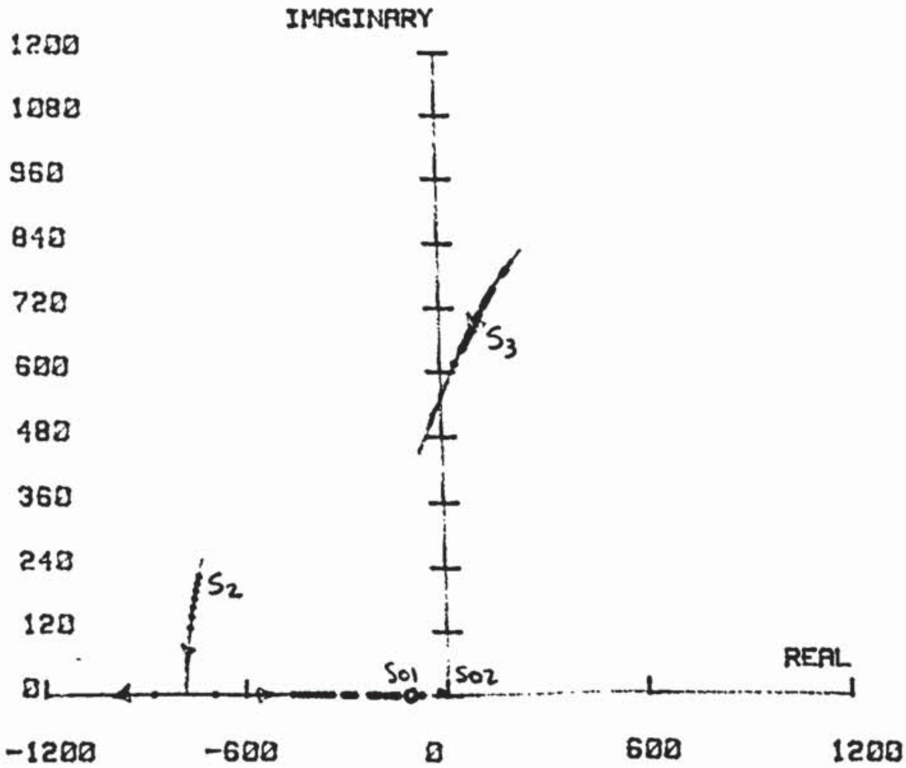


Fig. 3.17 Root-Locus as the Gain of the Acceleration Feedback Around the Motor is Increased. Velocity Feedback is Obtained From the Load.

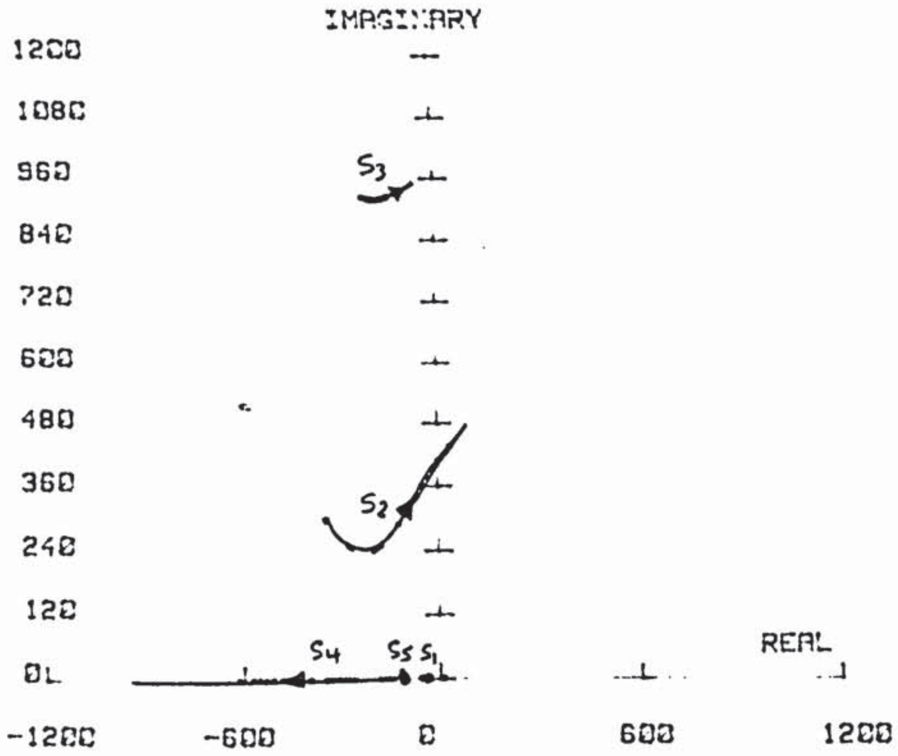


Fig. 3.18 Root-Locus as the Gain of Position Amplifier, on the System with 0.7 Damping Ratio in Velocity Control, is Increased.

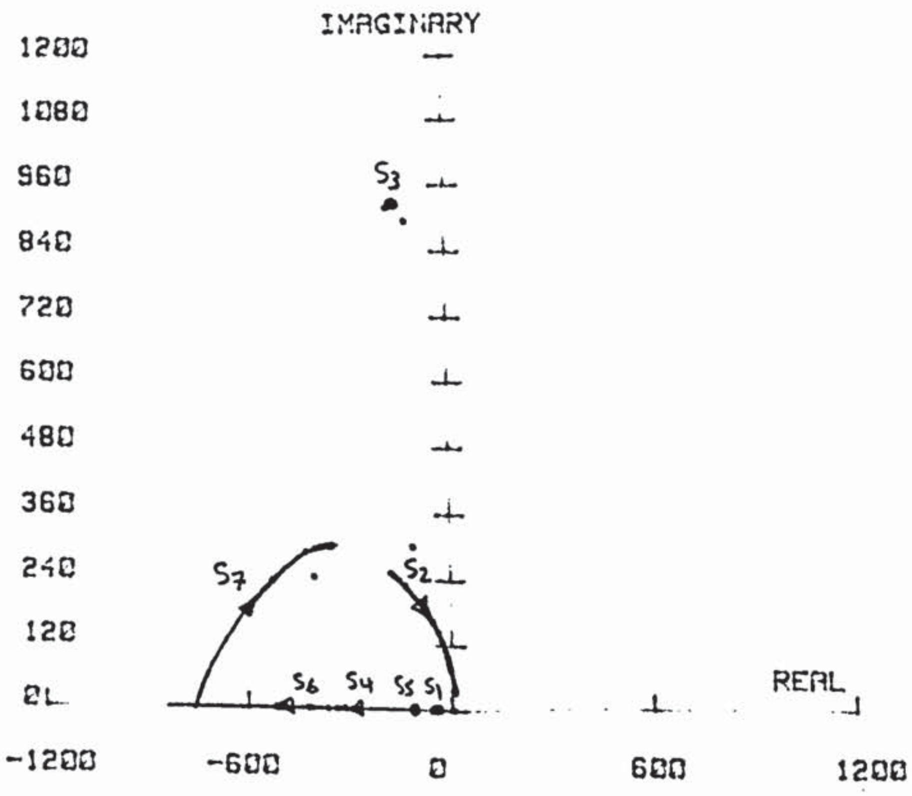


Fig. 3.19 Root-Locus as the Time Constant of the Position Amplifier is Increased.

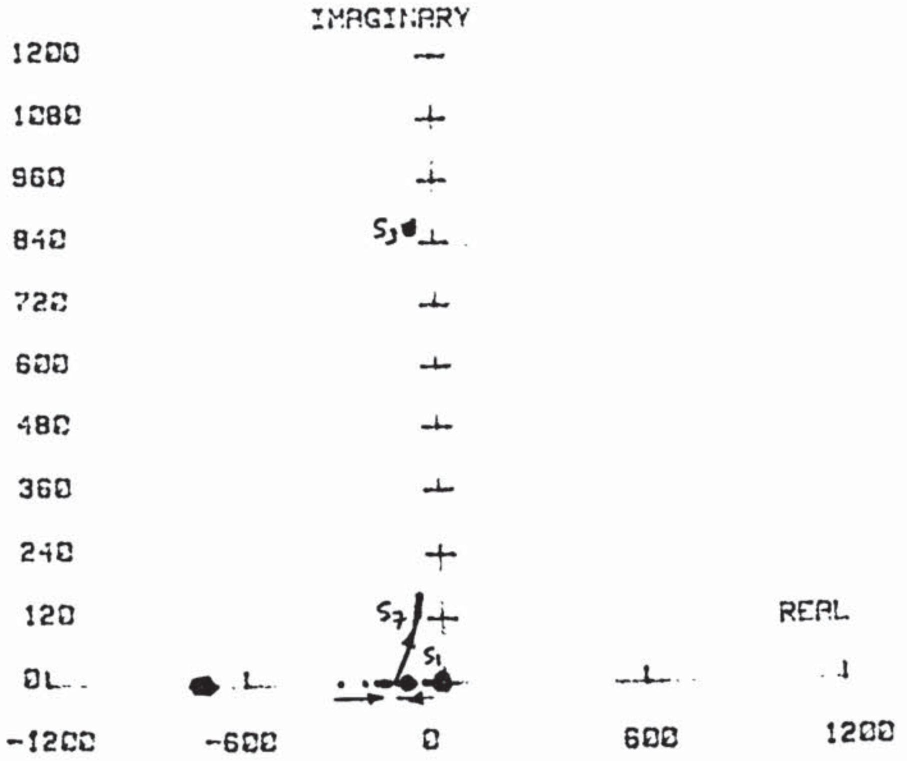


Fig. 3.20 Root-Locus as the Gain of Position Amplifier, on the System Overdamped in Velocity Control, is Increased.

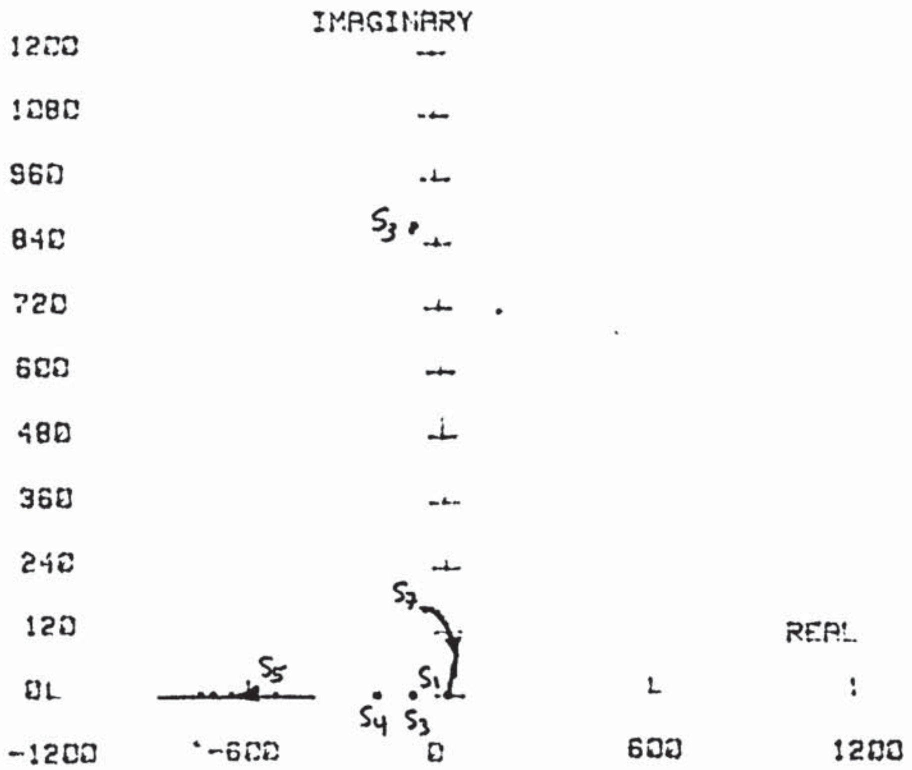


Fig. 3.21 Root-Locus as the Time Constant of the Position Amplifier is Increased.

CHAPTER 4

PERFORMANCE OF D.C. SERVO MOTORS

4. PERFORMANCE OF ELECTRICAL D.C. SERVO MOTORS

4.1 Introduction

The aim of this chapter is to study the performance and characteristics of electrical D.C. servo motors for velocity and position control systems. These motors have been used in industry for many years in different applications. The recent advancement in the automation technology has resulted in increased demand for high performance servo motors. The motors must be capable of having variable speed for velocity or position control. The controller of the motors plays a crucial part in the overall performance of the system.

During the past decade the advances made in power thyristors and transistors and their applications have caused a big change in the field of servo motor control. Originally, semi-controlled rectifiers (SCRs), as they were then called, were used only on a minor scale, but now the situation is completely different and they are the first choice for all control and power conversion apparatus. Thyristors have revolutionized the use of D.C. motors in industry, especially when a continuous variable speed is required. This transformation has been due to the dramatic improvement in thyristor capabilities. Because of the recent increase in the use of D.C. servo motors controlled with a thyristor power amplifier, there is a need to specify the performance of this kind of drive.

D.C. servo motors for velocity and position controlled applications are mainly of three types, i.e. a) separately excited, b) permanent magnet, and c) moving coil. The structure and theory of operation is discussed in this chapter.

The controllers of these motors are usually of two types and these are: a) thyristor controlled D.C. motors, and b) pulse width modulated D.C. motors. The characteristics of these controllers and their influence on the system's performance are investigated in this chapter.

4.2 Structure and Theory of Operation

4.2.1 Separately excited D.C. motors

Separately excited D.C. motors can be classified according to the arrangement of the field windings into: a) shunt, b) series and c) compound. In machine tools and robot applications the shunt motors are used. For this reason, only the shunt motor will be considered in this section. The field coils are wound to be connected in series with one another and the full line voltage is applied to the field circuit. In consequence, with a constant supply voltage, the field current is practically constant. The field current, being only required for magnetization, is a small fraction of the total full load current of the motor, i.e. field current plus armature current.

The armature is also connected to the supply and the current through it varies according to the load on the motor. The resistance of the armature is very low compared with the resistance of the field. In order to develop a mathematical model, a knowledge of the design procedure is required [67]. The relationship between the flux density and the magnetizing force is given by:

$$B = \mu_o \mu_r H \tag{4.1}$$

where

B is the flux density ($\omega b/m^2$)

μ_r is the relative permeability of iron

H is the magnetic force (ampere turns/metre)

μ_o is the permeability of air.

If the field contains N turns and a current I_f passes through it, the magnetizing force H is:

$$H = N I_f \qquad (4.2)$$

The linear velocity ν of the armature conductor, with angular velocity ω_m, may be written as:

$$\nu = r\omega_m \qquad (4.3)$$

where r is the radius of the armature. The magnetic flux density B can be re-written in terms of pole flux (φ) and area (A):

$$B = \frac{\varphi}{A} \qquad (4.4)$$

$$A = \frac{2\pi r \ell}{p} \qquad (4.5)$$

where p is the number of pole

ℓ is the length of the conductor.

The generated voltage, E' per conductor, can be calculated from the equation $E' = B\ell\nu$

$$\text{i.e. } E' = \frac{\varphi p \omega_m}{2\pi} \qquad (4.6)$$

If there are Z conductors moving through the field, their cumulative effect will be Z times the above voltage.

$$E = \frac{Z\varphi p \omega_m}{2\pi} \qquad (4.7)$$

or

$$E = [\frac{Z\varphi p}{2\pi}]\omega_m \qquad (4.8)$$

The quantity enclosed in brackets in 4.8 is a design constant and is commonly called the voltage constant, c_m, of the motor. It is an

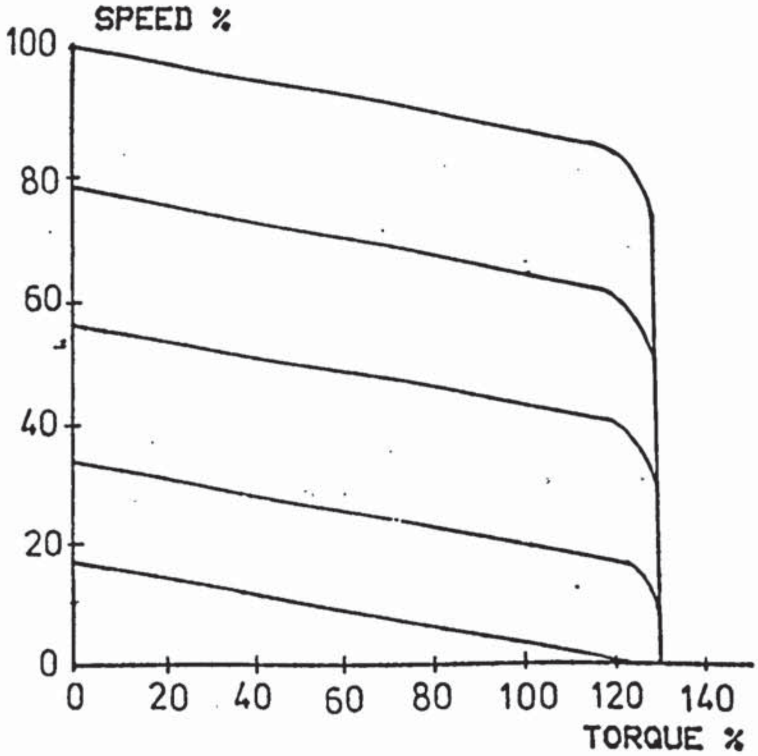


FIG 4.1.SPEED-TORQUE CHARACTERISTIC
OF A TYPICAL DC SHUNT MOTOR.

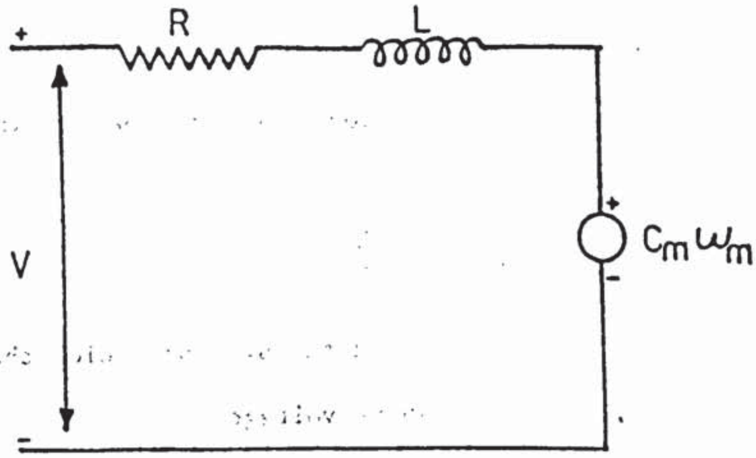


FIG 4.2.EQUIVALANT CIRCUIT DIAGRAM OF
OF DC SHUNT MOTOR.

important motor characteristic since it will determine the speed of the motor at a given value of applied voltage. The basic equation for torque is derived from the equation for force (F) of ($F = B\ell I$) in a manner similar to that used in obtaining the voltage equation. After the appropriate substitutions are made, the following expression for torque is obtained:

$$T_m = [\frac{Z\varphi p}{2\pi}] I \qquad (4.9)$$

where I is the current of the armature.
Neglecting the armature reaction, with a constant field current, the magnetic flux φ is constant. Equations 4.7 and 4.9 are linear in this case. A typical speed-torque characteristic is shown in Fig. 4.1. With the above assumption of constant flux and neglecting windage friction, eddy loss [67] the equivalent circuit diagram is shown in Fig. 4.2.

4.2.2 Permanent magnet D.C. servo motors

The field magnetic flux of these types of motors is generated by permanent magnet materials. Most conventional permanent magnet D.C. motors use ceramic magnets. Some new magnetic materials such as rare earth magnet have been produced recently, which improves the performance considerably. The characteristics and performances of rare earth magnet D.C. motors will be discussed later. In this section, only the characteristics of conventional permanent magnet D.C. motors will be discussed.

In these motors the stator magnetic flux remains essentially constant at all levels of armature current and, therefore, the speed-torque curve of the motor is linear over an extended range. The

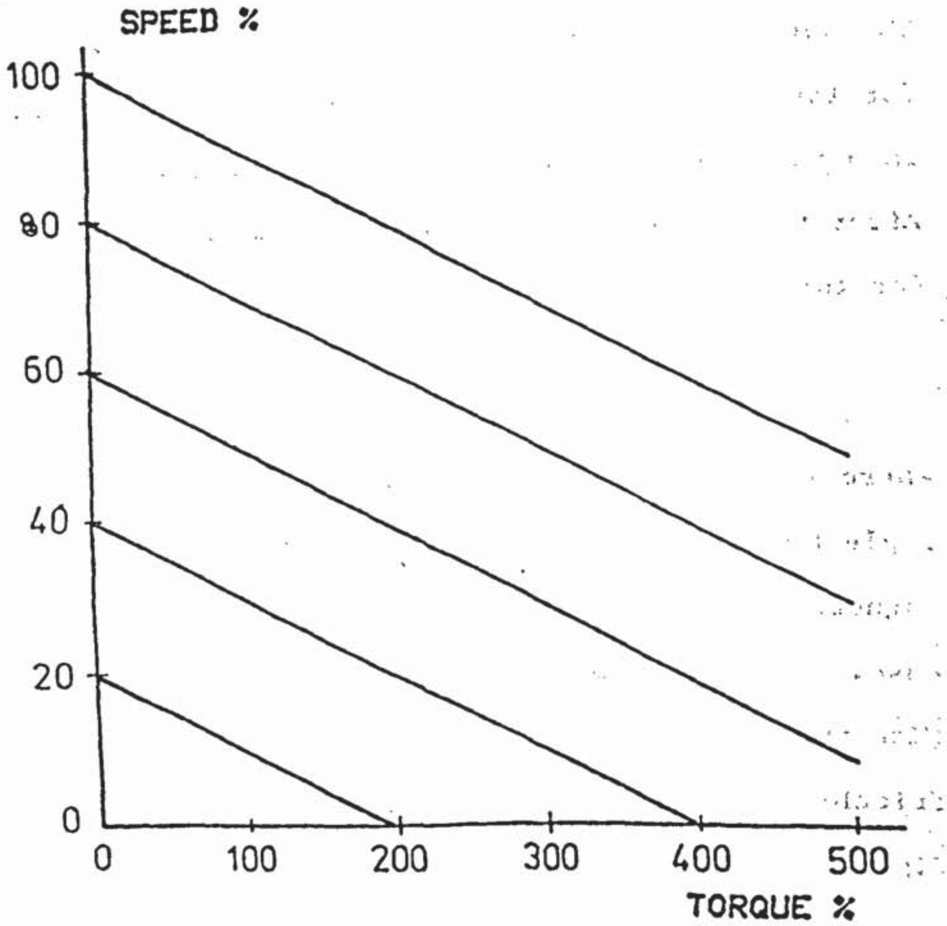


FIG 4.3.SPEED-TORQUE CHARACTERISTIC OF A TYPICAL PERMANENT MAGNET DC MOTOR.

stalled torque will tend to be higher than in shunt D.C. motors. The speed-torque characteristic of a typical permanent magnet D.C. motor is shown in Fig. 4.3. The advantages of these motors over separately excited shunt motors may be summarized as follows:

1. Linear speed-torque characteristics at all levels of armature current.
2. High stall torque, useful for high acceleration requirements.
3. There is no need for electric power to generate the field flux.
4. A smaller frame and lighter motor for a given output power.

4.2.3 Moving coil D.C. motors

In an effort to improve the performance of D.C. motors, designers have always sought to produce a motor with higher torque and lower inertia. The moving coil D.C. motor has satisfied this requirement. The conductor of the armature in place of an iron block are held in high strength plastic material. These motors are manufactured in two types: a) disc armature type (printed motor), and b) shell armature motor. The performance and advantages of these types of motors are discussed later.

4.3 Controllers of D.C. Servo Motors

4.3.1 Thyristor controlled D.C. servo motor

There are many techniques to regulate and control a D.C. servo motor. The use of controlled rectifiers to drive a D.C. motor is a long established technique. The use of thyristor devices has stimulated the development of a wide and varied range of power amplifier designs. In the absence of an ideal D.C. power amplifier, the use of an amplifier that chops a rectified alternating current wave form must guarantee that there can be continuity of armature current and that there can be a reverse supply to the armature.

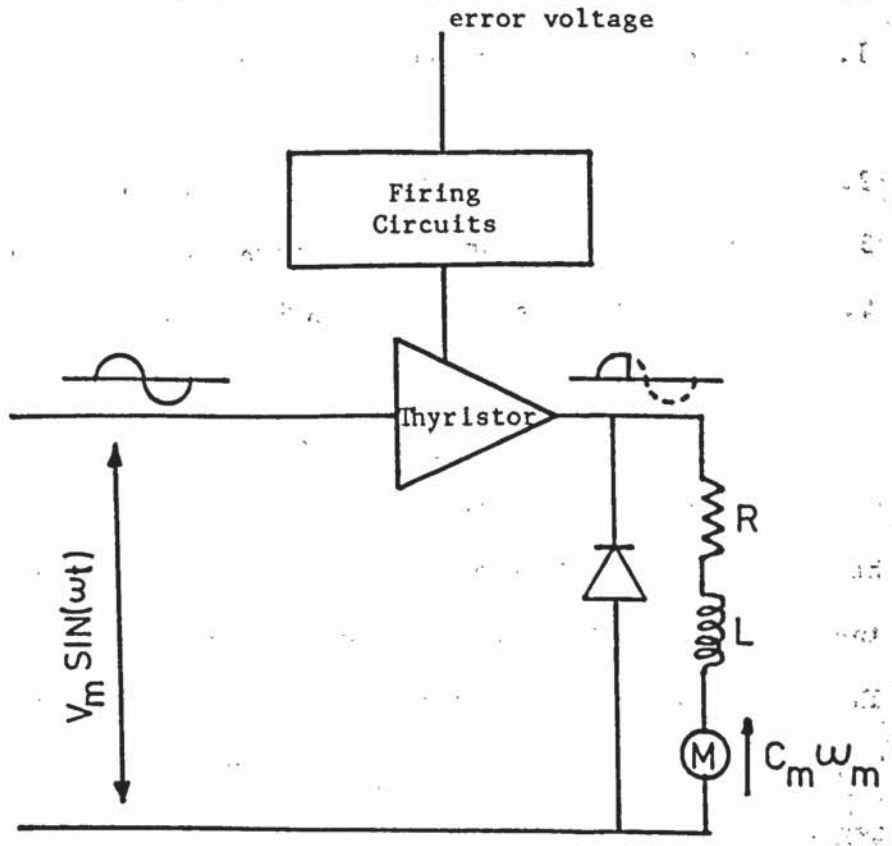


Fig. 4.4. Schematic representation of a thyristor controlled D.C. servo motor.

Power amplifier utilizing thyristors are mainly of three types: a) single phase half or full wave rectification, b) two phase full or half wave rectification and c) three phase half or full wave rectification. The performance of the D.C. servo motor drives vary according to the controller type.

The optimization of a control system using a thyristor power amplifier is difficult because of the inherent non-linearity of the power amplification. The mathematical model is complex because of the complexity of the firing circuits and the inherent non-linearity.

Fig. 4.4 shows the principal representation of a thyristor power amplifier with its input and its load. The assembly of one or of several thyristors and their control is comprised of two quite distinct parts. The first, of lower power, changes the input voltage into a train of impulses which serves to feed the triggering circuit of the thyristor. It consists of a detector circuit and an impulse generator. The second, which is the power part, may be represented schematically (Fig. 4.4) by a single thyristor fed by a single phase circuit held at amplitude V_m which is connected to the load.

There are two techniques of transforming a single voltage to obtain the firing angle. These two detector circuits are respectively the detector circuit to a level of constant charge and to a level of variable charge. These two detector circuits and the corresponding characteristics are given in detail in the [18,19] literature. In most control systems, the firing circuits to obtain a level of variable charge is used, because of minimum non-linearity in the power amplifier characteristic. The relationship between the firing angle,

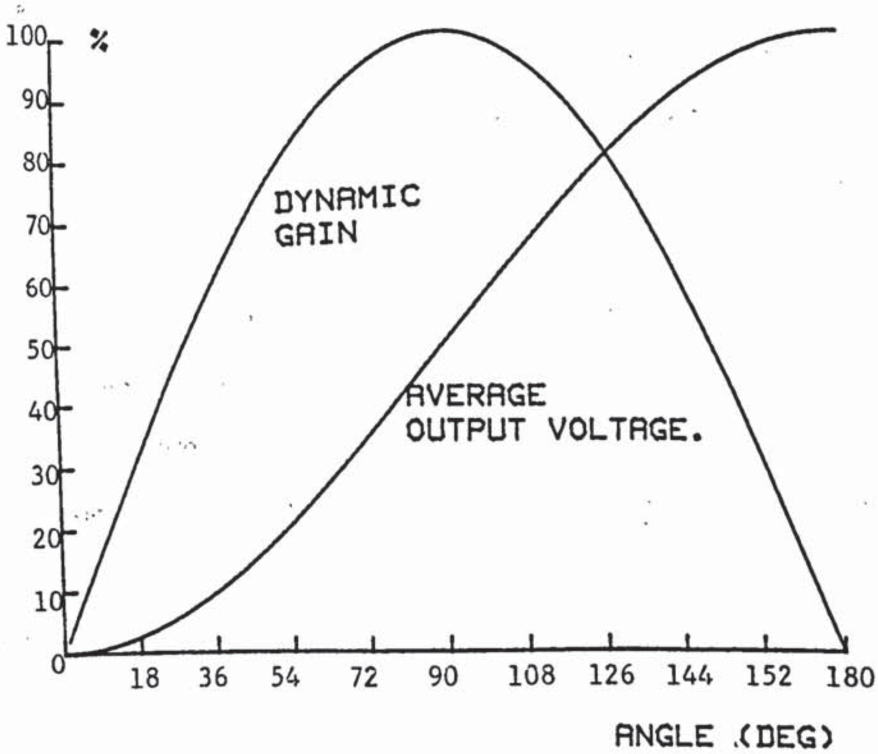


Fig. 4.5. The average output voltage and dynamic gain of a thyristor vs firing angle for single phase and resistive load.

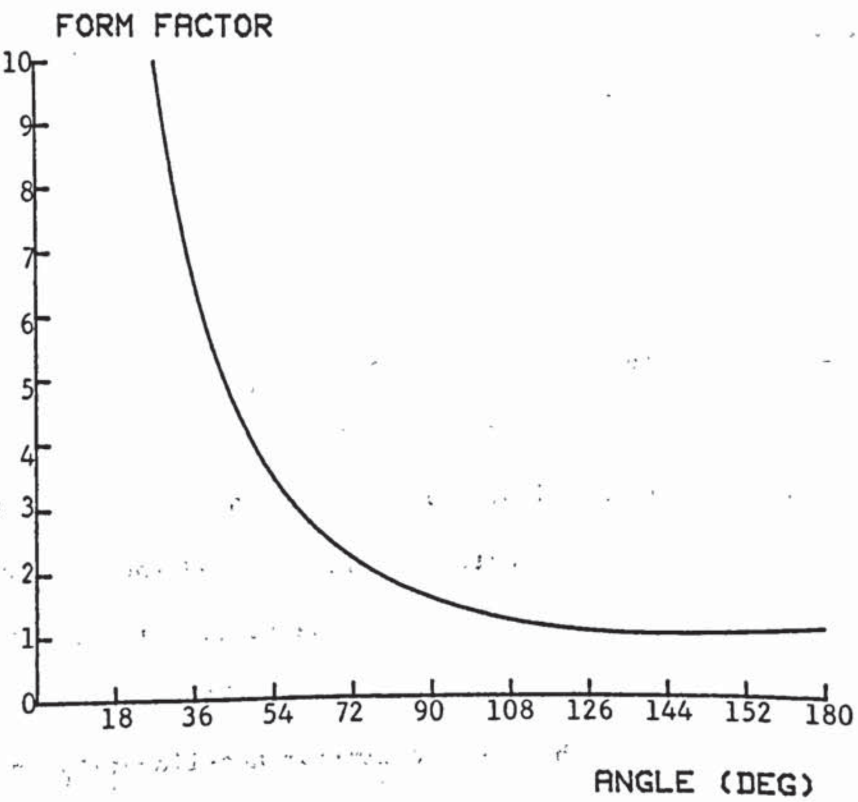


Fig. 4.6. Form factor of a thyristor controlled system with single phase and resistive load.

α, and the error voltage of V_c of the firing circuit can be written:

$$\alpha = \frac{\pi}{V_a} V_c \qquad (4.10)$$

where V_a is the base voltage of the firing circuit.

The average voltage V obtained from the thyristor may be used for the voltage equation of the motor provided that the inertia of the motor is high enough to prevent it from responding to the current waveform. With the assumption of only a resistive load the average voltage output from a thyristor supplied with single phase full wave rectification can be calculated as:

$$V = \frac{1}{\pi} \int_{0}^{\alpha} V_m \mathrm{Sin}\alpha \, d\alpha = \frac{V_m}{\pi} (1 - \cos\alpha) \qquad (4.11)$$

Substituting for α in equation 4.10 yields,

$$V = \frac{V_m}{\pi} (1 - \cos(\frac{\pi}{V_a} V_c)) \qquad (4.12)$$

It can be seen from equation 4.12 that the average voltage output from the thyristor, with respect to the input voltage, is non-linear. The dynamic gain of thyristor can be calculated as:

$$Kg = \frac{dV}{dV_c} = \frac{V_m}{V_a} \sin(\frac{\pi}{V_a} V_c) \qquad (4.13)$$

Fig. 4.5 shows the average output voltage and the dynamic gains of the thyristor with respect to the firing angle. Due to the existence of inductance in the motor, the characteristic equations of an average output voltage and the dynamic gain of the thyristor will change. This is not very significant for low inductance motors. A detailed mathematical calculation of this is given by Merrett [14].

The above calculation may be carried out for multi-phase power supply, these calculations are also given in detail by Merrett [14].

The existence of inductance and using multi-phase power supply reduces the non-linearity of the thyristor power amplifier.

In order to use the linear control theory, obtained in Chapter 3, to predict the system behaviour, equation 4.11 may be linearized about the steady state operating point, i.e.

$$V = Kg\ V_c \tag{4.14}$$

where

$$Kg = \frac{V_m}{V_a} \operatorname{Sin}\left(\frac{\pi}{V_a} V_c\right) \tag{4.15}$$

with the above linearization the mathematical model in Chapter 3 may be used to predict the behaviour of the system. Since the supply voltage is switched on at various firing angles, the power loss must be evaluated. The power losses in D.C. servo motors may be due to a host of factors, depending on the design of the motor; eddy current losses, hysteresis losses, armature commutation losses, viscous friction losses and armature resistance losses. The main loss to the first approximation is due to the armature resistance and this can be expressed by

$$P_L = RI^2_{rms} \tag{4.16}$$

In order to relate RMS current to average current in such cases, an expression relating the two has been established called the form factor (K):

$$K = \frac{I_{rms}}{I_{av}} \tag{4.17}$$

The I_{av} will determine the motor torque produced. The RMS and average current values depend on the firing angle. Fig. 4.6 shows the form factor of thyristor power amplifier. It shows that the switching characteristic introduces an additional resistive loss into the system. It is minimum when the average and RMS current are equal.

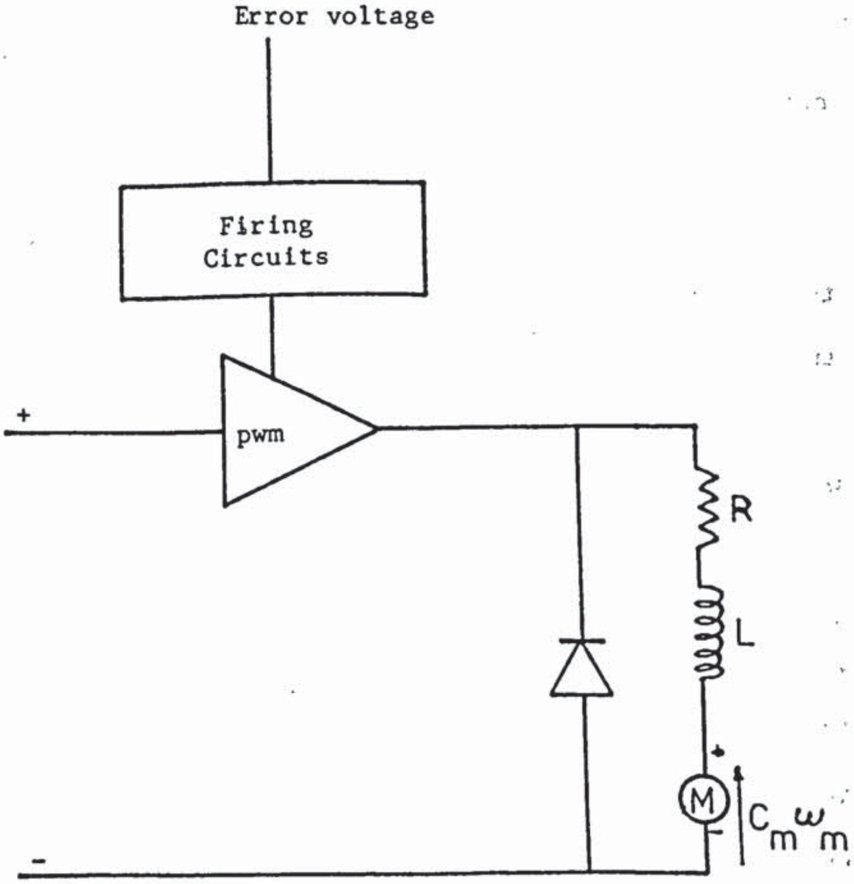


Fig. 4.7. Equivalent circuit of a D.C. motor in a pwm control system.

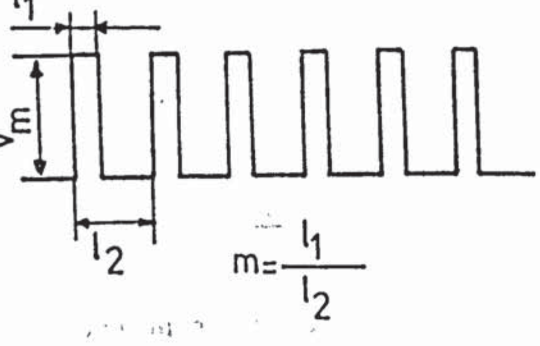


Fig. 4.8. Output voltage of a pwm system.

Substituting 4.17 into 4.16 for I_{rms} yields:

$$P_L = RK^2 I_{av}^2 \tag{4.18}$$

Hence, the form factor has a strong influence on motor heating and losses.

4.3.2 Pulse width modulated D.C. servo motors

As will be shown later, the performance of D.C. servo motors can be improved considerably by increasing the frequency of the supply voltage pulsation. In thyristor controlled systems the frequency of the supply voltage can be at maximum twice the main frequency. In order to achieve higher frequency pulses, the A.C. supply is rectified to D.C. The pulse width modulated (pwm) drive employs transistor or thyristors as switches to modulate the pulse width. Actually, the thyristor amplifier is pwm in spirit. Since pwm's are based on a continuously available D.C. power source, and since thyristor cannot "turn off" current conduction by themselves as transistors can, they must be provided with an additional "turn-off" circuit. The problems associated with such circuitry have limited the use of thyristors in pwm systems to high current, low switching rate applications, such as lift truck and vehicle traction controls.

Today, then, transistor-operated pwm switching amplifiers are used in most high performance, high power speed control systems and servo systems.

In examining voltage and current characteristics of a pwm system, first the ideal motor and its behaviour in a pwm system is considered. The motor equivalent circuit and supply modulated pulses are shown in Fig. 4.7.

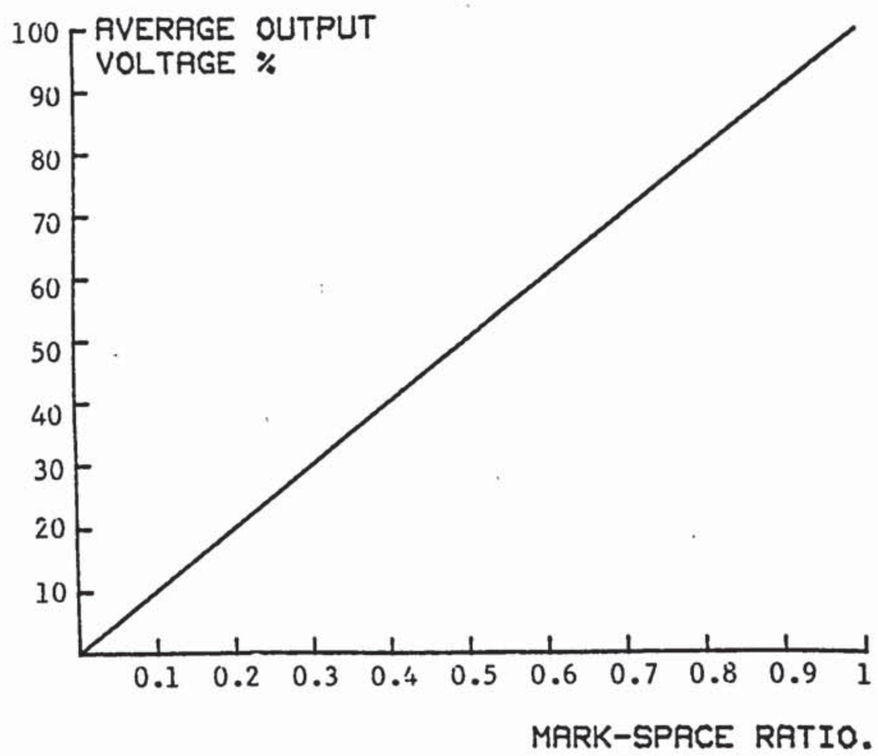


Fig. 4.9. Average output voltage of p.w.m. system with resistive load vs. mark-space ratio.

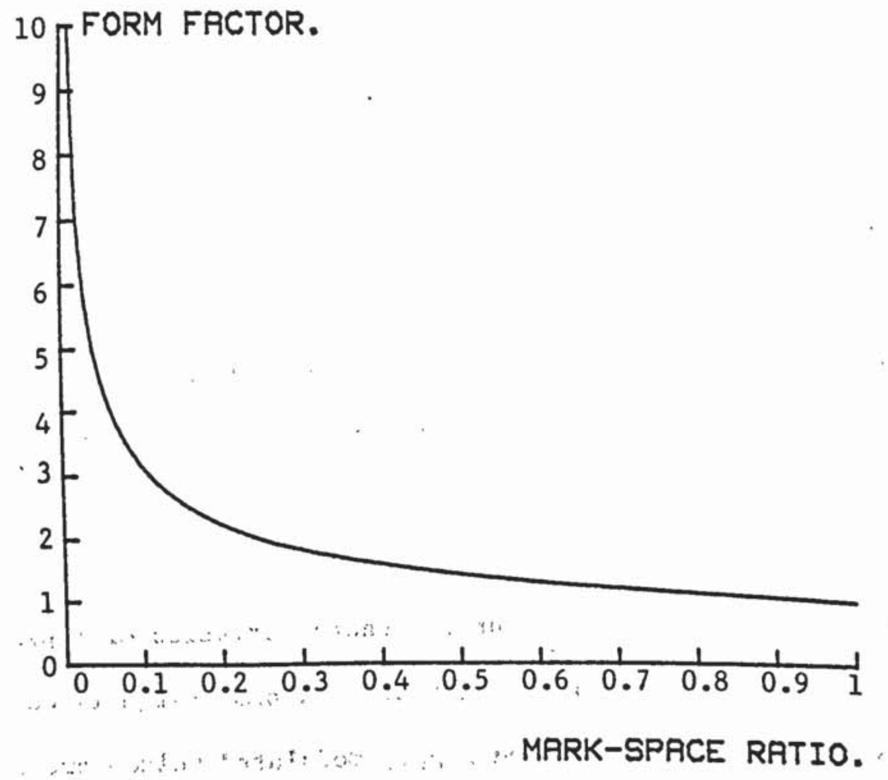


Fig. 4.10. Form factor of a p.w.m. system of resistive load vs. mark-space ratio.

Usually the output voltage of pwm systems is defined by the mark to space ratio (m) as shown in Fig. 4.8. The firing circuits can be designed in such a way that they produce a linear relationship between mark-space ratio and error voltage V_c:

$$m = K_f V_c \tag{4.19}$$

where K_f is the constant of the circuit.
The average output voltage can then be calculated as:

$$V = mV_m \tag{4.20}$$

Substituting 4.18 into 4.19 yields:

$$V = K_f V_m \ V_c \tag{4.21}$$

with dynamic gain Kg

$$Kg = K_f V_m \tag{4.22}$$

As can be seen the gain of the power amplifier of pwm systems is constant (Fig. 4.9). Therefore the mathematical model of Chapter 3 for the power amplifier may be used.

In order to calculate the resistive loss in the armature the exact shape of the current must be known. The wave form of the motor current during the switching mode is dependent not only on the switching rate, but on the motor speed, the total inductance, motor resistance and the current level in the last cycle.

A similar calculation made for thyristor controlled D.C. servo motors may be carried out to find the armature losses as 4.17. In the case of a pwm amplifier the form factor will depend on the pulse frequency, on the electrical time constant of the motor and on any associated series inductance. The average output voltage and form factor of a pwm system for resistive load are shown in Figs. 4.9 and

4.10 respectively. The inductance of the motor will reduce the form factor.

4.4 Performance of Thyristor Controlled Electrical D.C. Servo Motors

4.4.1 Introduction

In this section the performance of thyristor controlled D.C. servo motors is investigated. The mathematical model derived in Chapter 3 is employed with careful consideration of the non-linearity of thyristor power amplifier as already discussed. In order to substantiate the mathematical model, tests were carried out on specific D.C. motors and were compared with the theoretical prediction of their performances. The specifications of the motor and the controller under test are given in Appendix C. The controller was three phase half wave rectification and bidirection. The motor was of the permanent magnet type with a rated torque of 17.5 Nm and maximum speed of 1500 r.p.m. This servo motor system was supplied by Lucas Control Systems Ltd.

A servo motor drive system with good performance in velocity control mode will provide a better performance in position control. For this reason, emphasis is given mainly to velocity control, and, finally, the performance of the system in position control mode is analysed.

Having compared and confirmed the mathematical model results with the experimental results, the prediction of all types of thyristor controlled D.C. servo motors can be confidently derived.

4.4.2 Experimental and theoretical comparison

The numerical values of the parameter of the mathematical model, as derived in Chapter 3, are substituted in the computer program simulation. The transformation of the mathematical model acceptable for the program is given in Appendix D. Having entered all the necessary information into the computer program, the transfer functions of output speed and current with respect to an input command signal or external torque are derived as:

$$\frac{\omega_o}{V_p} = \frac{B_2 s^2 + B_1 s + B_0}{A_6 s^6 + A_5 s^5 + A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0} \tag{4.23}$$

$$\frac{I}{V_p} = \frac{C_5 s^5 + C_4 s^4 + C_3 s^3 + C_2 s^2 + C_1 s + C_0}{A_6 s^6 + A_5 s^5 + A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0} \tag{4.24}$$

$$\frac{\omega_o}{T_L} = \frac{D_5 s^5 + D_4 s^4 + D_3 s^3 + D_2 s^2 + D_1 s}{A_6 s^6 + A_5 s^5 + A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0} \tag{4.25}$$

$$\frac{I}{T_L} = \frac{E_5 s^5 + E_4 s^4 + E_3 s^3 + E_2 s^2 + E_1 s + E_0}{A_6 s^6 + A_5 s^5 + A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0} \tag{4.26}$$

The constants of the transfer functions are a function of the parameters of the system; these are given in Appendix D. Substituting numerical values for basic parameters the transfer function is evaluated in numerical form and consequently the roots of the characteristic equation and the required response obtained. The numerical values of the parameters of the D.C. servo motor under test and the source of these values are given below.

K_2 = 98 (volts/(volt/sec)

K_3 = 3.2 (volts/volt) — from circuit diagram of velocity amplifier

K_m = 0.095 (volts/(rad/sec) — direct test on the motor, volts/(rad/sec)

K_a = 0

$K_4 = 42$ (volts/volt) From the circuit diagram of the

$\tau_2 = 0.004$ (sec) current feedback loop, and were

$\tau_3 = 0.335$ (sec) confirmed experimentally.

$K_c = 0.08$ (volts/amp) current feedback gain, from test on the drive

$K_g = 13$ (volts/volt) thyristor gain, from test.

$R = 0.6$ (ohms)

$L = 0.0015$ (henry)

$c_m = 0.83$ (volts/(rad/sec)

Manufacturer's data

$K_t = 0.83$ (Nm/amp)

$I_m = 0.0216$ (Kg-m²)

$N = 1$

$K_s = 12000$ (N-m/rad) from the design of loading mechanism.

$I_L = 0.027$ (Kg-m²) load inertia from design.

$C_L = 0.1$ (Nm/(rad/sec)) damping coefficient, from test.

The experimental response of the velocity of the system was obtained by using the voltage from the velocity transducer (Tacho generator) and fast speed plotter (u.v. recorder). The response of the systems for a step input of voltage experimentally and theoretically is compared in Fig. 4.11. A good agreement exists between the results. The discrepancies are mainly due to static friction, which is neglected in the mathematical model, and the non-linearity of thyristor power amplifiers. The velocity response of the system for a step input of external torque is shown in Fig. 4.12. It can be seen that the speed drops initially as the torque is applied. Due to the integrator in the velocity amplifier, the speed recovers resulting in a zero steady state error. The amount of dynamic velocity drop of the motor depends on the overall gain of the system. The speed of recovery depends on the integrator gain in the velocity amplifier. An excessive increase in the integrator gain results in an unstable system. The velocity of the motor, however, tends to fluctuate with the current pulses to the motor. The amplitude of this oscillation depends on the inertia of the motor, the load, the stiffness of the coupling mechanism and the shape of the current. Figs. 4.13 and 4.14 show this fluctuation at different velocities and currents. As the velocity of the motor increases the current becomes smoother and results in smaller oscillations. An increase in current increases the amplitude of oscillation, due to the torque being proportional to the current of the motor. This oscillation of velocity was first thought of as coming from the noise induced in the velocity transducer. This was rejected by a mathematical analysis of the forced vibration of a rotor coupled to the load inertia. It was proved that there were genuine velocity fluctuations, which are more significant at low speeds due to the shape of the current (see Appendix E).

The frequency and phase angle response of the system for small amplitude oscillations is shown in Fig. 4.15. The experimental frequency and phase angle responses were obtained by using the fast Fourier transform technique on a Real Time Frequency Spectrum Analyser [72]. Similar results were also obtained by using a frequency generator and measuring the velocity amplitude and its phase angle. It can be seen from this graph that there is high frequency noise in the system. This noise could be due to a number of factors. Due to the nature of the current, any natural frequency of the loading mechanism could be excited, particularly the combination of a velocity transducer and the rotor of the motor. Some noise is also due to the velocity transducer, which was a tacho generator. The frequency of this noise usually depends on the velocity of the motor. However, some oscillation of a frequency equal to that of the current frequency also exists in the system.

The above responses were obtained where the motor was operating below the current limit. In order to investigate the action of the current limiter for large signals, tests were carried out for a large step input. The principle of the current limiter is based on the speed-torque characteristic of the motor. At rest the motor is capable of producing high torque, as the motor accelerating the available torque reduces. Therefore, the action of current limiter is to limit the surge of current when the motor is at rest, and slowly reduces this limit as the motor speeds up. Fig. 4.16 shows the response of the velocity of the motor for large signal input. As can be seen from this graph, the slope of the response is reduced as the velocity is increased. The theoretical response was obtained by reducing the input voltage to the velocity amplifier when the current rises above the limit. The response of the current corresponding to the velocity response of Fig. 4.16 is shown in Fig. 4.17. The experimental response of the current is also superimposed on this graph. Because of the nature of current pulses the upper, lower boundary and the average of the current response are shown on this graph. There is a good correlation between the theoretical and experimental response of the current. This shows that the assumption of an average current and voltage in the theoretical analysis is acceptable.

The performances of the system in the position control are investigated. The design of the position controller was based on the digital to analogue conversion. Each input pulse represents one increment of motion of the motor. This is shown in detail in Appendix C. A series of tests were carried out to investigate the position error. The input pulses were counted via a pulse counter and the actual position of the shaft was measured from the position transducer; (the phase difference of two sine waves), (see Appendix C). No measurable position error at steady state was noticed. Due to the nature of the position controller being practically a ramp input, it was not possible to obtain an experimental response of step input of position.

One important characteristic of a position control system is the following error in a ramp input of position. The transient response of the position of the motor in a ramp input is shown in Fig. 4.18. The experimental and theoretical comparison of the velocity of the motor in ramp input is shown in Fig. 4.19. The steady state error of the system, as a function of velocity of the motor, is shown in Fig. 4.20.

4.4.3 Possible improvement of the system using acceleration feedback.

In order to improve performances, it is essential to increase the gain of the system. There is a limit in thyristor controlled D.C. servo motors for the speed of response of the velocity of the motor. This is due to the fact that once a thyristor is fired, it will continue to conduct until the current is back to zero. The voltage supplied is in sine wave form therefore the conduction will stop as the current becomes zero. An attempt to increase the gain will result in a faster velocity response of the motor. The increase in the velocity response is achieved by a larger firing angle at transient condition. Further increase in the gain could cause the motor to overshoot at the first cycle of conduction and undershoot for the next cycle.

Another problem might occur due to the current limiter in the system. The current limiter will sense the large current and try to reduce the firing angle for the next cycle of voltage supplied. If the speed of response is high the current limiter will continue to increase and reduce the firing angle and could cause violent oscillation. It may also overload the motor at each cycle of conduction. The above two limitations can be summarized as:

a) An overshoot in the velocity response must not occur before completion of one cycle. This implies a limitation on the speed of velocity response of the motor.

b) The increase in the current of the motor must be below the current limit of the motor during one cycle of conduction. This also limits the speed of response of the motor.

The above limitations restrict improvement in the speed of response of the system. The optimum response depends on the current frequency and the load inertia of the system.

The effect of external torque, however, can be reduced by keeping the speed of response of the motor below the limit and increasing the gain. This can be achieved by compensation techniques. Most manufacturers tend to use current feedback compensation techniques. The current feedback will reduce the speed of response, hence the gain can be increased. Current feedback is not an efficient compensation technique due to the pulsation of the current. In order to obtain an average current, it is essential to have a low pass filter or a lag network, which is not desirable as far as the stability of the system is concerned, as has already been shown. Furthermore, at steady state,the current feedback will reduce the stiffness of the system for an external torque. Fig. 4.21 shows the response of the system for a step input of torque at two different current feedback gains. It can be seen from this graph that the improvement on the effect of external torque is not considerable.

The most efficient way of reducing the effect of torque is the use of acceleration feedback. The difficulty, however, is in installing an acceleration transducer. It is easier to have acceleration feedback, where the load is in linear motion. In order to substantiate the effect of acceleration feedback experimentally, the derivative of the velocity transducer signal was used. The circuit arrangement of derivation of the velocity signal is shown in Fig. 4.22. The voltage input to the velocity amplifier may be calculated as

$$V_2 = \frac{R_2(1 + R_1 cs)}{(R_1 + R_2)(1 + \frac{R_2 R_1}{R_1 + R_2} cs)} V_1 \qquad (4.27)$$

The lead network of 4.27 produces a signal which is the proportional plus the derivative of the velocity feedback. The lag network reduces the noise of the derivative signal. Fig. 4.23 shows the response of the system without an acceleration feedback where the gain of the velocity amplifier has been increased 5 times. Fig. 4.24 shows the improvement achieved by the use of acceleration feedback. The effect of external torque on the system is shown in Fig. 4.25. It shows that by use of acceleration feedback the improvement on the effect of external torque is more considerable. It can also be seen from these graphs that the acceleration feedback will increase the damping ratio of the natural frequency of the system.

By the use of acceleration feedback, the gain of the velocity amplifier can be increased. The increase of the gain, with the above procedure is limited only by the noise of the transducers and system. The main noise is due to current pulsation and this causes a considerable amount of noise in the velocity transducer.

4.4.4 Performances of thyristor controlled D.C. servo motors with the current frequency of 50 Hz.

4.4.4.1 Introduction

In this section performances of D.C. servo motors supplied by single phase half wave thyristor controlled rectification are investigated. This type of drive is usually used for low performance applications with bidirectional rotation. The performances of four different sizes of permanent magnet D.C. motors for different applications are considered. As long as the system is operating in the linear mode and below the current limit of the motor, the characteristics of all kinds and types of D.C. motors are similar. In this section the performances of permanent magnet D.C. motors are considered. Eventually, the different types of D.C. motors and their influence on the dynamic and static performance will be discussed and compared at the end of this chapter.

The influence of load inertia and load natural frequency on the performances of four different sized motors (1,3,5 and 10 kW) are investigated. The parameters of the velocity and current amplifiers are optimized in such a way that: a) they satisfy the limitations of the speed of response as mentioned before, and b) they achieve a 0.6 to 0.7 damping ratio of the system in the velocity control mode.

Having established the performance, then the effect of acceleration feedback on the performances of the system is further investigated. The static characteristics of the motors under consideration are given in Appendix E.

4.4.4.2 Effect of load inertia on the performances of the system.

The performance of the D.C. servo motor system is dependent on the total inertia of the load and rotor of the motor. In this section, by varying the stiffness of the transmission, the natural frequency of the loading system is kept constant. Three different load inertia are considered to obtain the trend of the performance of the system with respect to load inertia. These are between 0 to 3 $Kg\text{-}m^2$ and they are also presented as a percentage of rotor inertia for each motor. Zero inertia is where there is no load inertia or it is negligible. With the above inertia the parameters of the velocity and current amplifier are optimized to obtain the best performance and at the same time keeping the speed of the velocity below the limitation of the thyristor. Figs. 4.26, 4.27, 4.28 and 4.29 show the response of the system for a step input of velocity for different load inertias and for four different sizes of motor. The corresponding effects of a step input of external torque are shown in Figs. 4.32, 4.33, 4.34 and 4.33.

4.4.4.3 The effects of load natural frequency on performance.

The effect of load natural frequency on a 5 kW D.C. motor is considered here. The results, however, can be extended for any type and size of motor. The inertia of the load is chosen as half of the rotor inertia. Fig. 4.34 shows the response of the system for a step input of velocity for different load natural frequencies. It can be seen that at low load natural frequency the damping of the system is low and as the natural frequency is increased, the damping is also increased. The load natural frequency can be seen to cause high frequency oscillations on response. It was noticed that these oscillations only become significant at the low inertia side of the system. If the load inertia is higher than the inertia of the rotor then the oscillations only become significant at motor end. When the load natural frequency becomes smaller than the closed loop frequency, the system becomes unstable. To stabilize the system, the gain of the system must be reduced in order to reduce the speed of response.

4.4.4.4 The performance of the system in the position control mode.

Having established the characteristic performance in velocity control mode, the trend of performance in the position control mode follows that of the velocity control mode. The performance of the 5 kW D.C. servo motor in position control is only investigated here. Similar results, however, can be obtained for the other sizes of motor. The position amplifier gain is adjusted to obtain critically damping in the position closed loop fundamental frequency. Fig. 4.35 shows the response of the system for a unit step input of position for three different load inertias. As can be seen from this graph, the time response of the position control reduces as the inertia increases which is similar to the system in velocity control. The response of the system in position control is also slower than that of the system in velocity control. The effect of torque on the system in position control is shown in Fig. 4.36 for different load inertias. It can be seen that initially there is position error, but due to the integrator in the velocity control the error becomes zero. A similar response can also be obtained for other size motors. The following error of the system for a ramp input for the 5 kW D.C. servo motor with different loads is shown in Table 1.

4.4.5 Performance of thyristor controlled D.C. servo motors with current frequency of 150 Hz.

4.4.5.1 Introduction

These devices are usually used for high performance position and velocity controlled applications with bidirectional rotation. The 150 Hz current frequency is obtainable from a three phase A.C. voltage supply and half wave rectification. The direction of rotation is determined by the firing of thyristors in the positive or the negative cycle of the supply voltage.

As in the previous section, the performance of four different D.C. motors is investigated. The frequency of current being higher than that in the previous section, the current can rise faster, and consequently results in a better performance.

4.4.5.2 Effect of load inertia on the performance of the system.

Figs. 4.37, 4.38, 4.39 and 4.40 show the response of the system for a unit step input for the four motor sizes (1, 3, 5 and 10 kW) with different loads. Again, as the inertia of the load increases the response of the system becomes slower, and also the larger the motor the further the reduction of the speed of response. Comparing these results, with results obtained for the controller with a current frequency of 50 Hz, it can be seen that a faster response is obtained by using the controller with current frequency of 150 Hz. The effect of a step input of torque for the four different sizes, for different load inertias is shown in Figs. 4.41, 4.42, 4.43 and 4.44. It can be seen from these graphs that the velocity drop has reduced by using controllers with current frequency of 150 Hz.

4.4.5.3 Effect of load natural frequency on the system.

The effect of the load natural frequency on the 5 kW motor is considered. The results however can be extended for other size motors. Fig. 4.45 shows the step input response of the system for different load natural frequencies with load inertia of 50% of the rotor. The results are similar to the system with the controller of 50 Hz current frequency. The oscillations of the load natural frequency are more significant because of the higher gains of the system.

4.4.5.4 Performance of the system in position control.

Fig. 4.46 shows the performance of the 5 kW motor in the position control mode. It is evident that the speed of response is faster than the system with controller of 50 Hz current frequency. It can be seen that the total inertia (motor plus load) has considerable effect on the speed of response of the system. The speed of response is slower than in the velocity control mode.

The effect of a step input of torque on the position accuracy is shown in Fig. 4.47. It can be seen that, as the inertia is increased, so is the dynamic error in the system. The dynamic error is much less than the system using the controller with a 50 Hz current frequency. The following error with a ramp input for the 5 kW motor is shown in Table 2.

4.4.6 Effect of the non-linearity of power thyristors on the performance of the system.

It was shown that the output average voltage from the thyristor is non-linear and the degree of non-linearity is maximum when the load is purely resistive. The dynamic gain of the thyristor is zero at zero firing angle. This gain is increased as the firing angle is increased. It reaches its maximum value of a firing angle of $\frac{\pi}{2}$ and reduces to zero again at a firing angle of π. In order to investigate the effect of this non-linearity on the system, three different dynamic gains were considered for the 5 kW motor. The response of the system and the effect of external torque for these three different dynamic gains is shown in Figs. 4.48 and 4.49 respectively. As can be seen from these graphs at the low thyristor dynamic gain the system becomes less damped and oscillatory, but as the dynamic gain is increased, the damping increases. The stiffness of the system for an external torque is low at low dynamic gain and it is increased as the dynamic gain is increased. At very low dynamic gain the system becomes unstable. This reduction of damping and instability is due to the existence of the integrator in the system which becomes more effective at the lower system gain. When the torque is applied to the motor or the speed becomes higher, because of higher dynamic gains, the effect of non-linearity becomes less significant. In order to prevent the system becoming unstable at low dynamic gain, the designer must prevent the system operating at low firing angle or at a very high firing angle. This can be achieved by letting the thyristor be fired with the motor stationary in both directions, hence preventing it becoming unstable and also increasing the stiffness of the system for the external torque. The maximum firing angle must also be limited in order to prevent instability at high firing angles.

4.5 Pulse Width Modulated D.C. Servo Motors

4.5.1 Experimental and theoretical comparison of pulse width modulated D.C. servo motors

4.5.1.1 Introduction

It shown previously that as the frequency of the current supplied to the motor is increased a better performance can be obtained. With an A.C. supply a maximum frequency of 300 Hz can be achieved from a three phase power supply. In order to achieve higher current frequency a pulse width modulated drive (PWM) is used. The principle operation of this controller was shown previously. Furthermore, it was shown that the mathematical model obtained in Chapter 3 can also be used to predict the performance of the system. In order to substantiate the theoretical model a test similar to the thyristor controlled system was carried out. The frequency of the current of the PWM system can be varied according to the design of the controller. The system under test uses a current frequency of 2 kHz. The specification of motor and the PWM drive is given in Appendix C.

4.5.1.2 Experimental and Theoretical Comparison.

Two series of tests were carried out, a) step of velocity input and, b) step input of torque and the results are compared with the theoretical results obtained from the mathematical model. In these tests the system remains in a linear mode and none of the parameters saturate or reach a limiting value. Fig. 4.50 shows the experimental and theoretical results of a step input of velocity. Fig. 4.51 shows the effect of a step input of torque theoretically and experimentally. It can be seen from these graphs that there is a good agreement between the experimental and theoretical results. The small discrepancy is mainly due to the static friction.

For large step inputs the current limiter becomes active and limits the speed of response. A large step input test was carried out, the response of the velocity and current is shown in Fig. 4.52. It can be seen from this graph that when the current reaches the limit of the motor, the acceleration and consequently the speed of response of the velocity remains constant, until the current becomes smaller than the limit value, then the system will behave according to the mathematical model. There was good agreement between the theoretical and experimental results in position control mode, which is not included in this chapter. The theoretical analysis for position control will be shown later.

Due to the high frequency current supplied to the motor, the oscillation of the rotor becomes much smaller than for a thyristor controlled system. No measurable oscillation was noticed due to the frequency of the current, however, some noise existed due to the tacho generator and the induced noise, but these were very small.

4.5.2 The performance of a pulse width modulated system.

4.5.2.1 Introduction

The performance of a PWM servo motor system was analysed for both velocity and position control. Four sizes of motor similar to thyristor controlled servo motors were considered. The effect of load inertia and load natural frequency on the performance were investigated. The practical limitation of current rise, below the current limit during one cycle of voltage, is observed by limiting the gain of the system. Other parameters such as the time constants of the lead-lag network, the gain of the current feedback and the integrator gains were optimized in order to achieve a damping ratio of 0.6 to 0.7 for the velocity control mode. Having established the performance of the system the behaviour of the 5 kW motor was investigated for position control. Similar results may be expected for other size motors.

4.5.2.2 Effect of Load Inertia.

The effect of inertia on the performance of the PWM system was similar to that obtained for the thyristor controlled system. Figs. 4.53, 4.54, 4.55 and 4.56 show the response of the four motors with different load inertia but with the same load natural frequency. It can be seen from these graphs that the response of the PWM system is faster than the corresponding thyristor controlled one. This is due to the capability of a PWM system having a faster current rise whilst at the same time keeping it below the current limit of the motor. The effect of a step input of torque on the system is shown in Figs. 4.57, 4.58, 4.59 and 4.60. It can be seen from these graphs that the effect of torque is much smaller than for the corresponding thyristor controlled one.

4.5.2.3 Effect of Load Natural Frequency.

The effect of the load natural frequency on the 5 kW motor is shown in Fig. 4.61. It can be seen that at a low natural frequency the damping of the system reduced quite considerably. At higher load natural frequency, the damping of the system increases with some oscillation on the response due to the load natural frequency. The effect of load natural frequency on other size systems is similar but has different response time. The above response was obtained with a load inertia of 50% of the rotor inertia. The oscillation due to the load natural frequency is, however, more significant at the lower inertia end.

4.5.2.4 Performance of Pulse Width Modulated Systems for Position Control.

The trend of performance for position control follows those for velocity control, i.e. the system with better performance for velocity control also produces a better performance for position control. Fig. 4.62 shows the response of the 5 kW motor to a step input of position with different load inertia. It can be seen from this graph that the position amplifier gain was adjusted in such a way that there would be no position overshoot. The speed of response is also slower than the corresponding system for velocity control.

The effect of torque on the system is shown in Fig. 4.63. It can be seen that by increasing the load inertia the error is increased. This is because of the lower position amplifier gain, which is reduced for reasons of stability. The following error of the 5 kW motor with different load inertia is given in Table 3. It can be seen that, firstly, as the load inertia increases so does the following error, secondly, the following error of the PWM system is smaller than for the corresponding thyristor controlled systems.

4.6 The Effect of Acceleration Feedback on Performance

The gains of D.C. servo motor system is limited by the current rise to the motor during one cycle of supply voltage. This limits the speed of response of all kinds of D.C. servo motors and consequently the dynamic stiffness. In order to increase the stiffness, the gain of the system must be increased, without increasing the speed of response. Most designers of controllers use current feedback and lead-lag networks to keep the speed of response below the limit. It was shown earlier the improvement on the stiffness of the system for enternal torque was not considerable. Further, it was shown experimentally and theoretically that acceleration feedback will improve the stiffness considerably. The effect of acceleration feedback on all systems is similar. In this section the effect is shown for the 5 kW permanent magnet D.C. servo motor with a single phase thyristor control (current frequency of 50 Hz). The gain of the system is increased 10 times.

Acceleration feedback is used to improve damping and reduce the speed of response below the limit of current rise during one cycle. Fig. 4.64 shows the response of the system for a step input for different values of load inertia. Comparison of this graph with Fig. 4.28 shows that, firstly, the speed of response is constant, secondly, the acceleration feedback has introduced considerable damping in the system.

The effect of torque on this system is shown in Fig. 4.65 for different load inertia. Comparison of this graph with Fig. 4.33 shows that the effect of torque has been reduced one tenth of its former value. With this technique the increase in gain of the system is dependent only on the noise of the velocity transducer.

Acceleration feedback round the motor was used as this proved to be more effective. However, acceleration feedback from the load may be used if careful consideration of the load natural frequency is made to avoid the system becoming unstable.

4.7 Comparison of the Performance of the Different Types of D.C. Servo Motors.

4.7.1 Introduction

There are three predominant types of D.C. servo motors for machine tool and robotic applications,these are:

1) Separately excited D.C. servo motor.
2) Permanent magnet D.C. servo motor
   a) Ceramic permanent magnet type
   b) Alnico permanent magnet type
   c) Rare earth magnet type.
3) Moving coil D.C. servo motors
   a) Shell type rotor
   b) Disc type rotor (printed motor).
4) Brushless D.C. motors.

It was shown that the performance of a D.C. servo motor depends on its current capability and the speed of current rise. The speed of current rise depends on the type of controller used. In order to improve performance research is being carried out in three areas [73]. Firstly, to reduce the inertia of the rotor; i.e. moving coil types and brushless D.C. motor; secondly, to increase the current capability of the motor in transient conditions such as the permanent magnet type (rare earth magnet in particular); thirdly, to increase the frequency of the supply voltage to the motor, for instance, using a pwm drive with higher current frequency.

The dynamic performance of ceramic permanent magnet D.C. motors has been examined. It was shown that a motor with higher current capability and lower rotor inertia gives a better dynamic performance. Therefore, it can be said that from the mentioned types of motor, the types with lower values of inertia/max. torque ratio would provide a faster and more accurate response.

The separately excited D.C. servo motor, having a higher value of inertia/max. torque ratio and mass/power ratio has few applications in high performance systems. The brushless D.C. motors are still under improvement and although they have been used in some applications, but there is little data on their characteristics. This is due to the fact that manufacturers are reluctant to release data until they become more standardized.

The following design characteristics are considered for each type:

a) Mass/power ratio
b) Inertia/max. tcrque, where the motor is stationary
c) Inertia/rated torque
d) Inertia x max. velocity/max. torque
e) Inertia x max. velocity/rated torque.

The data is obtained from a limited number of manufacturers and could change for different designs.

4.7.2 Comparison of mass/power ratio.

Fig. 4.66 shows the mass/power ratio of different types of D.C. motors at different power rating. It can be seen that points are scattered across an average curve. This is due to the fact that designs of various manufacturers are made for different applications. Some motors with a high mass/power ratio are capable of producing high current for transient period. The mass/power ratios for a given power rating are similar. This is because the power available from a motor is limited by the heat generated due to the resistive loss and the temperature rise of motor. The mass/power ratios, can be seen in Fig. 4.66, to decrease as the power increases. Using forced ventilation will reduce this ratio but with the expenses of an additional mass of the blower.

There are indications that the brushless D.C. motors with rare earth magnet gives the lowest ratio and it follows with rare earth magnet printed motors, rare earth magnet D.C. motor and ceramic magnet D.C. motors have the highest ratio.

4.7.3 Comparison of inertia/torque.

This ratio is very important for predicting the dynamic performance of the motor. The lower the ratio the faster the response. The maximum torque in the case of D.C. motors is obtained when the motor is stationary and is only available for a short period of time for acceleration purposes. This ratio also indicates the response time for a unit change in velocity. Fig. 4.67 shows the inertia/max. torque ratio of different types with various power ratings.

The maximum torque available from a D.C. motor falls to the rated torque at full velocity. Fig. 4.68 shows, the inertia/rated torque ratio of different types, with various power ratings. This ratio indicates the response time of the motor for a unit change of velocity at the rated torque. However, the minimum response time of the motor varies between inertia/max. torque and the inertia/rated torque for a unit change in velocity.

These ratios increase as the load inertia increases. The total inertia of the rotor and the load must be considered for predicting the minimum response time. This will be discussed fully in Chapter 8 and Appendix H.

It can be seen from these graphs that printed circuit D.C. motors have the fastest response, and the rare earth magnet type follows. The ceramic type motors have higher response times compared with the other two types. By designing these motors, brushless further decrease in response time may be achieved. It can also be seen that the response time increases as the power of the motor increases. This is due to the fact that the increase in inertia required to obtain higher torques is more dominant.

4.7.4 Comparison of inertia x maximum velocity/torque.

This ratio determines the average time to go from zero velocity to the maximum velocity at the applied torque. Figs. 4.69 and 4.70 show this response time for maximum and rated torques. It can be seen from these graphs that the printed circuit motors have the fastest response followed by the rare earth magnet and ceramic magnet type motors. The response time increases as the power of the motor increases.

The effect of the maximum rated velocity on the response time is contradictory. An increase in the maximum rated velocity (which depends on the design specification of the motor) results in a smaller available torque but at a lower rotor inertia. However, an overall effect is a higher response time, i.e. a motor with a lower maximum rated velocity produces a faster response.

4.8 Brushless D.C. Motor

A brushless motor consists of a permanent magnet rotor, a wound stator, a rotor position assembly and a solid-state commutator. This combination allows the removal of the brushes and commutator and their well-known maintenance problems. The performance of brushless D.C. motors similar to other types depends significantly on the material used for the permanent magnet. In Figs. 4.66 to 4.70 the static characteristics of brushless motors, employing ordinary magnet, are compared with other types of D.C. motors. These motors are still under development and the data on their characteristics tend to differ considerably from different manufactures. The characteristics shown on these graphs are only obtained from some limited number of manufacturers. It can be seen from Fig. 4.66 that the (mass/power) ratio of these motors are similar to other types and there is no significant change of this ratio.

These motors, due to the removal of commutator and wiring on the rotor, possess a lower inertia than ceramic magnet type. This can be seen in Fig. 4.67 which compares the (Inertia/Maximum torque) ratio. The (inertia/rated torque) ratio further reveals this point as can be seen in Fig. 4.68.

Fig. 4.69 compares the (inertia x maximum velocity/maximum torque) of these motors with other types. It can be seen a faster static speed of response is obtained than the ceramic magnet type, but it is slower than rare earth magnet and printed type motors. This is due to the fact that the maximum available torque from ordinary magnet brushless D.C. motors is not as high as the rare earth magnet type.

Fig. 4.70 compares the (inertia x maximum velocity/rated torque) ratio of brushless motors with other types. It can be seen from this graph that the reduction of the inertia of brushless D.C. motors is more significant and results in the fastest speed of response at rated torque of the motor.

The above comparison is made on brushless D.C. motors with ordinary magnet material. Recently, new magnets such as rare earth magnets are incorporated with these motors and improves their performance considerably. This advance makes these motors the most appealing in high performance applications [66]. With the above comparison and the dynamic analyses of this chapter show that the optimum performance can be achieved with a D.C. servo motor feed-drive with the following specifications:

1. Making the motor brushless.
2. Designing the motor in printed circuit type (disk type rotor).
3. Using the rare earth magnet or other types of permanent magnet materials which gives the highest maximum torque.
4. Controlling the motor with pulse width modulated drive, with maximum pulse frequency.
5. Employing acceleration feedback with combination of a lead-lag network as compensation.

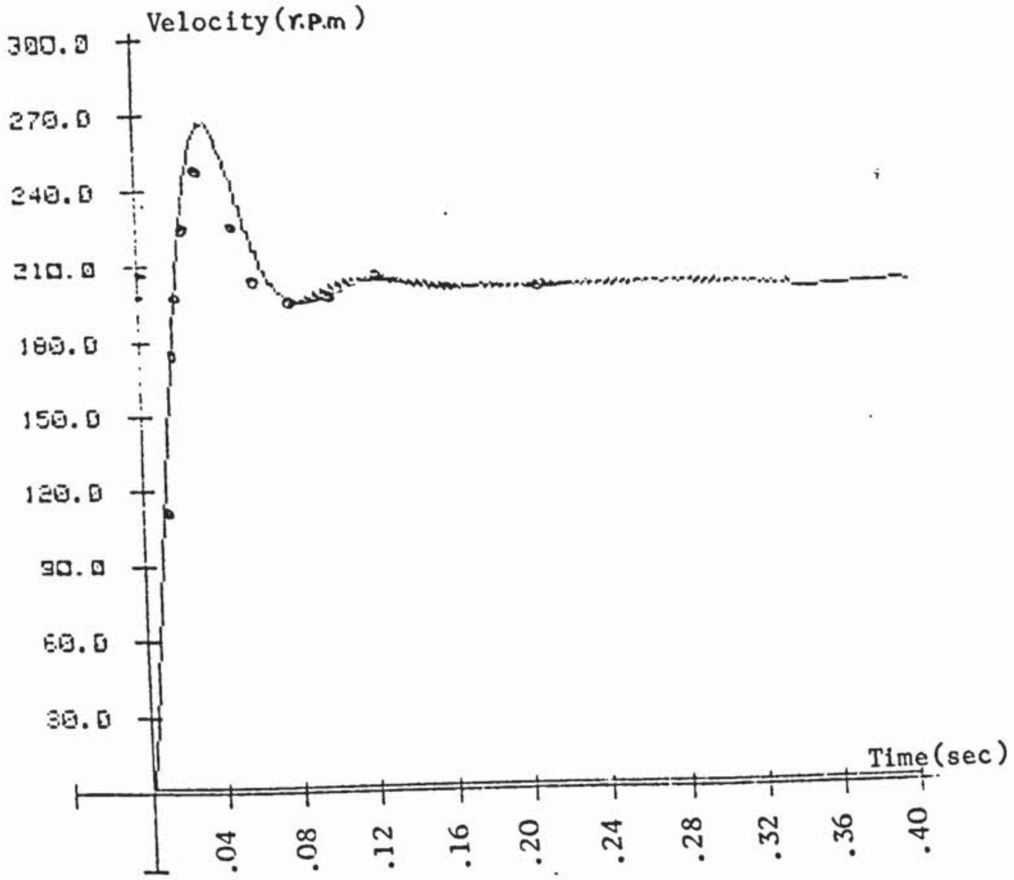


Fig. 4.11 Experimental and theoretical response of the system to a step input of velocity.

——— Theoretical

o o o Experimental

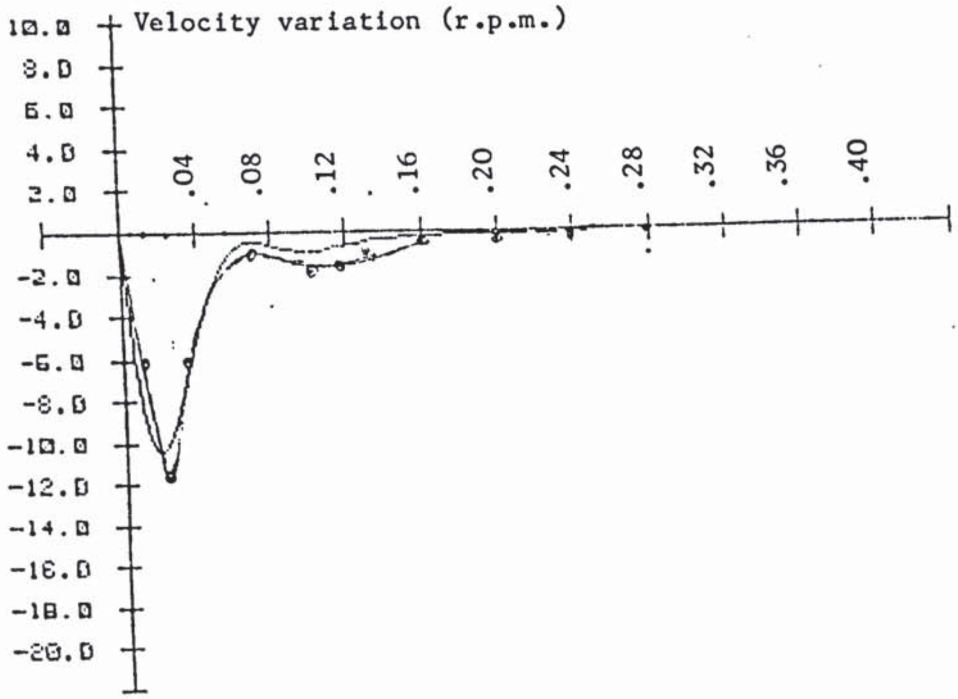


Fig. 4.12 Experimental and theoretical response of the system to a step input of torque of 10 N.m.

——— Theoretical

o o o Experimental

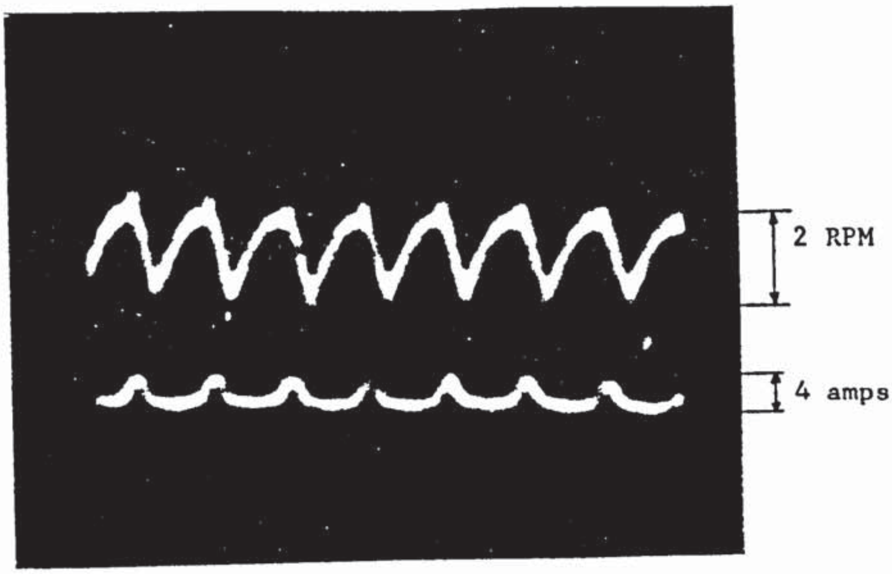


Fig. 4.13.a. Velocity fluctuation of the motor at no load and speed of 6 r.p.m. frequency of oscillation = 150 Hz.

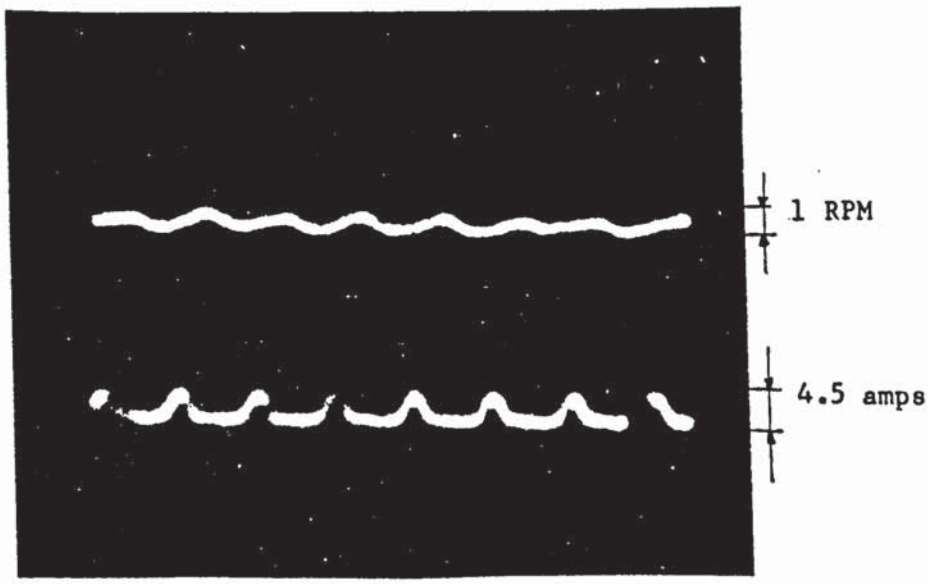


Fig. 4.13.b. Velocity fluctuation of the motor at no load and speed of 100 r.p.m. frequency of oscillation = 150 Hz.

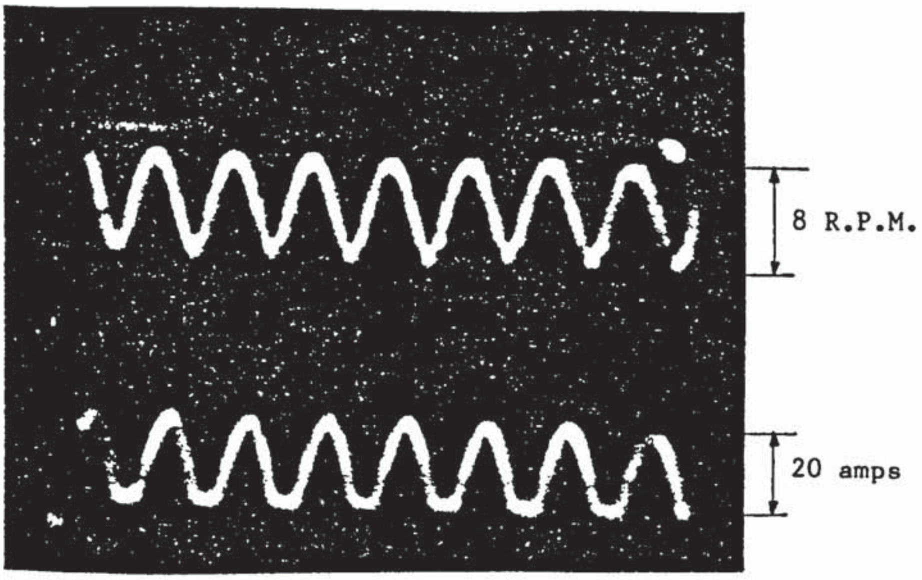


Fig. 4.14.a. Velocity fluctuation of the motor at current of 20 amps and speed of 6 r.p.m. Frequency of oscillation = 150 Hz.

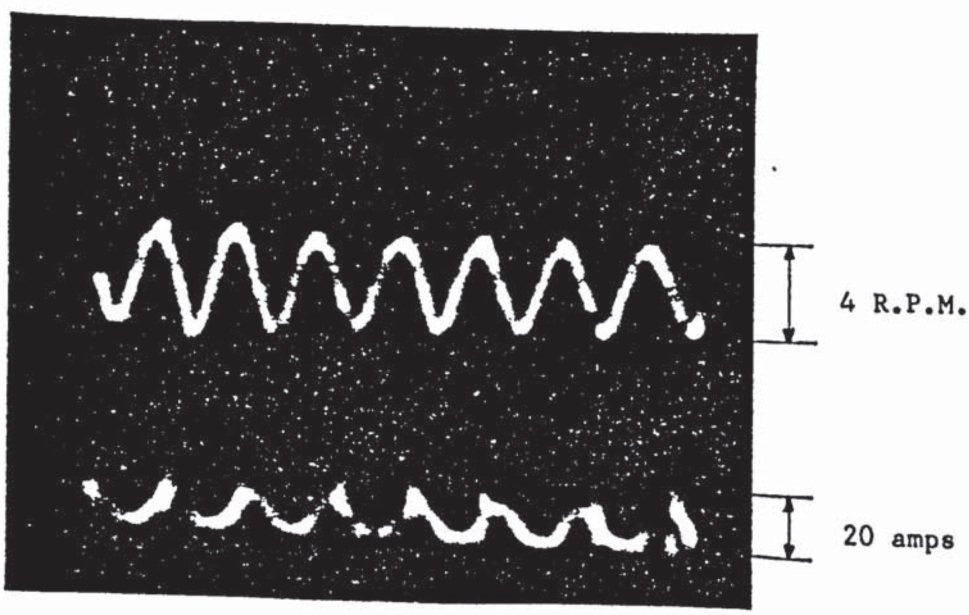


Fig. 4.14.b. Velocity fluctuation of the motor at current of 20 amps and speed of 100 r.p.m. Frequency of oscillation = 150 Hz.

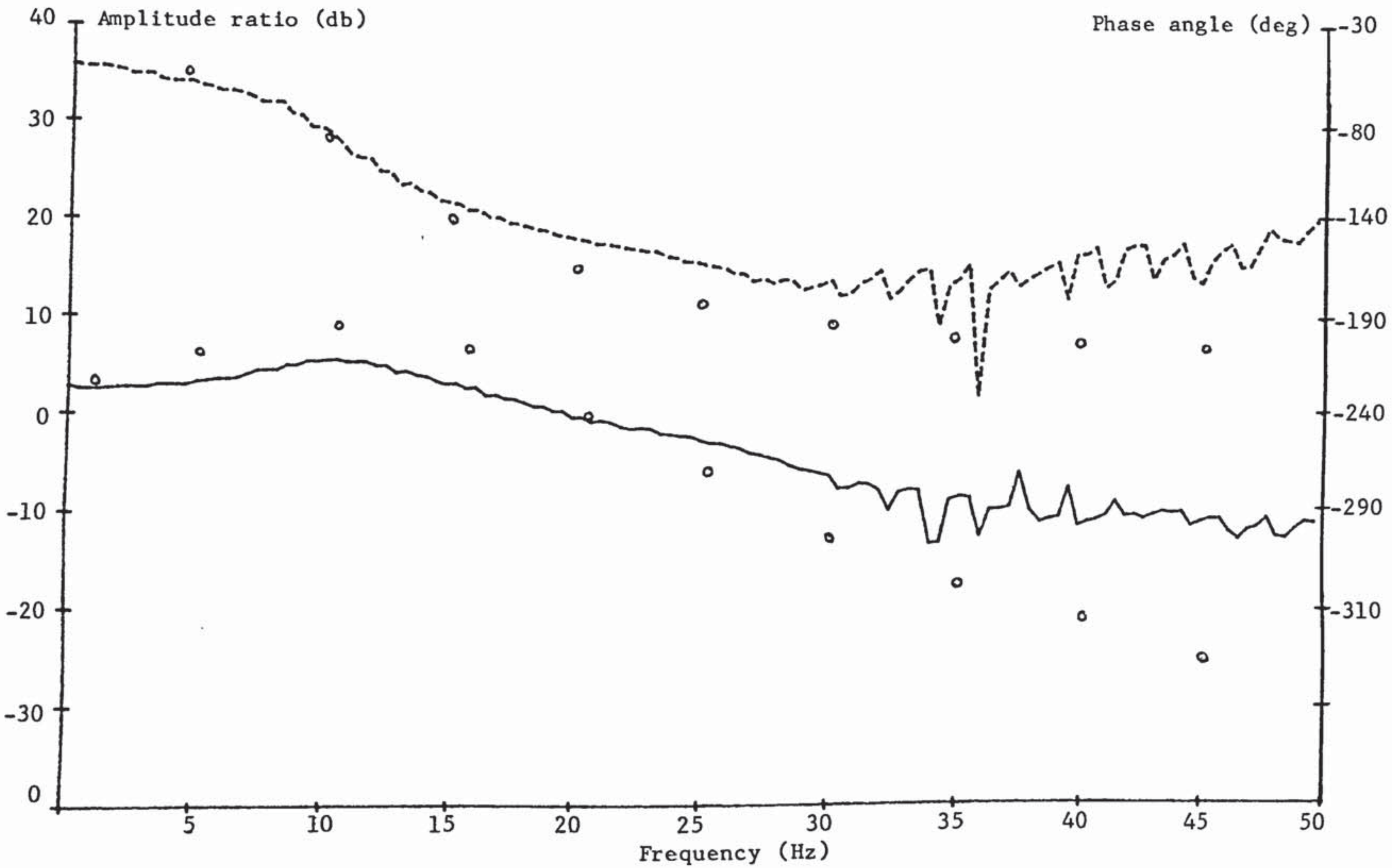


Fig. 4.15 Frequency and phase response of the system.

- - - - phase angle

——— Amplitude ratio

o o o o Theoretical

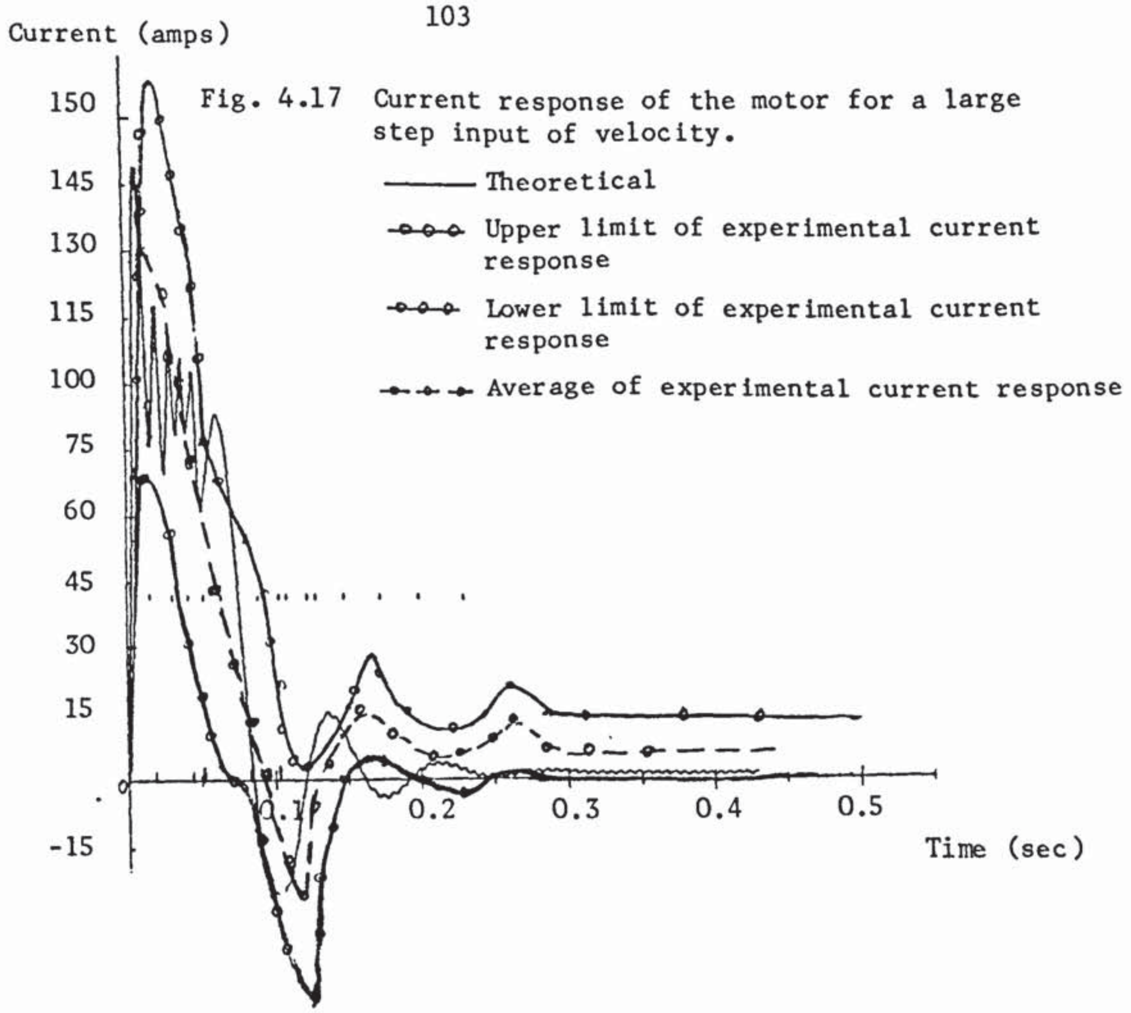


Fig. 4.17 Current response of the motor for a large step input of velocity.

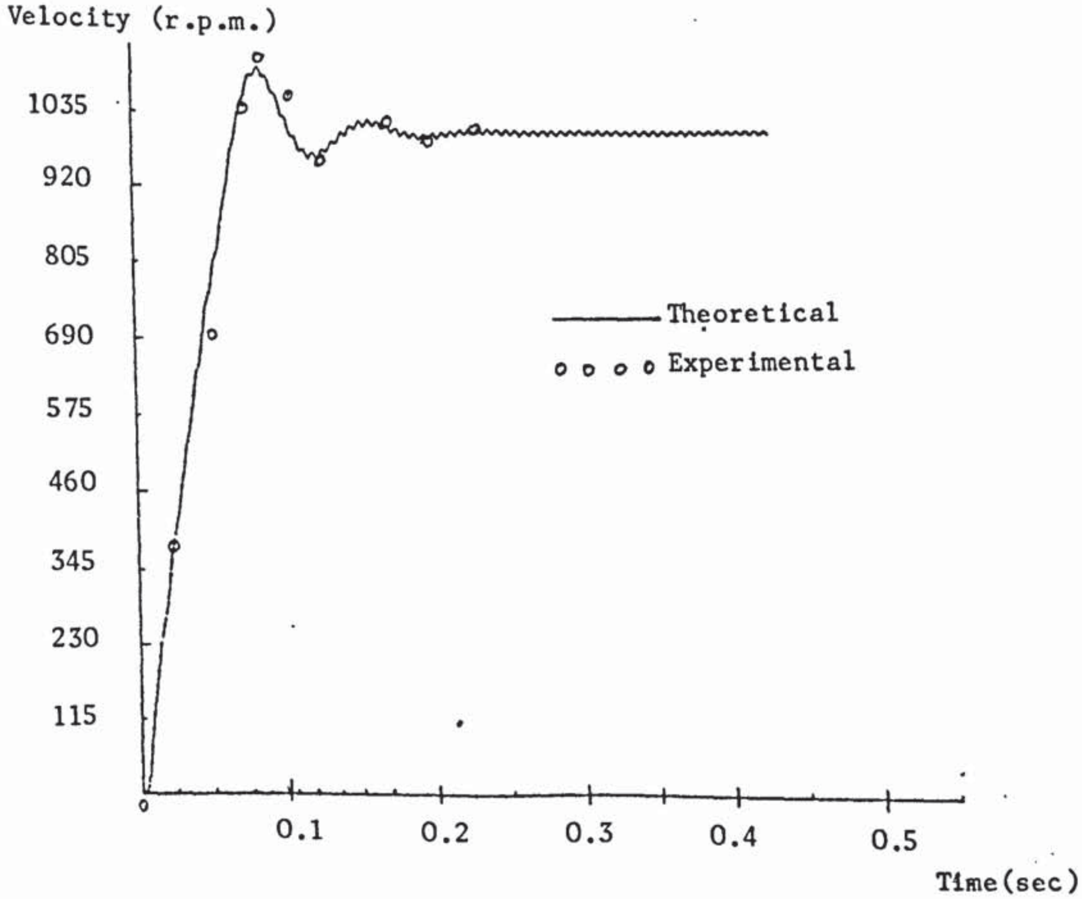


Fig. 4.16 Velocity response of the system for a large step input of velocity.

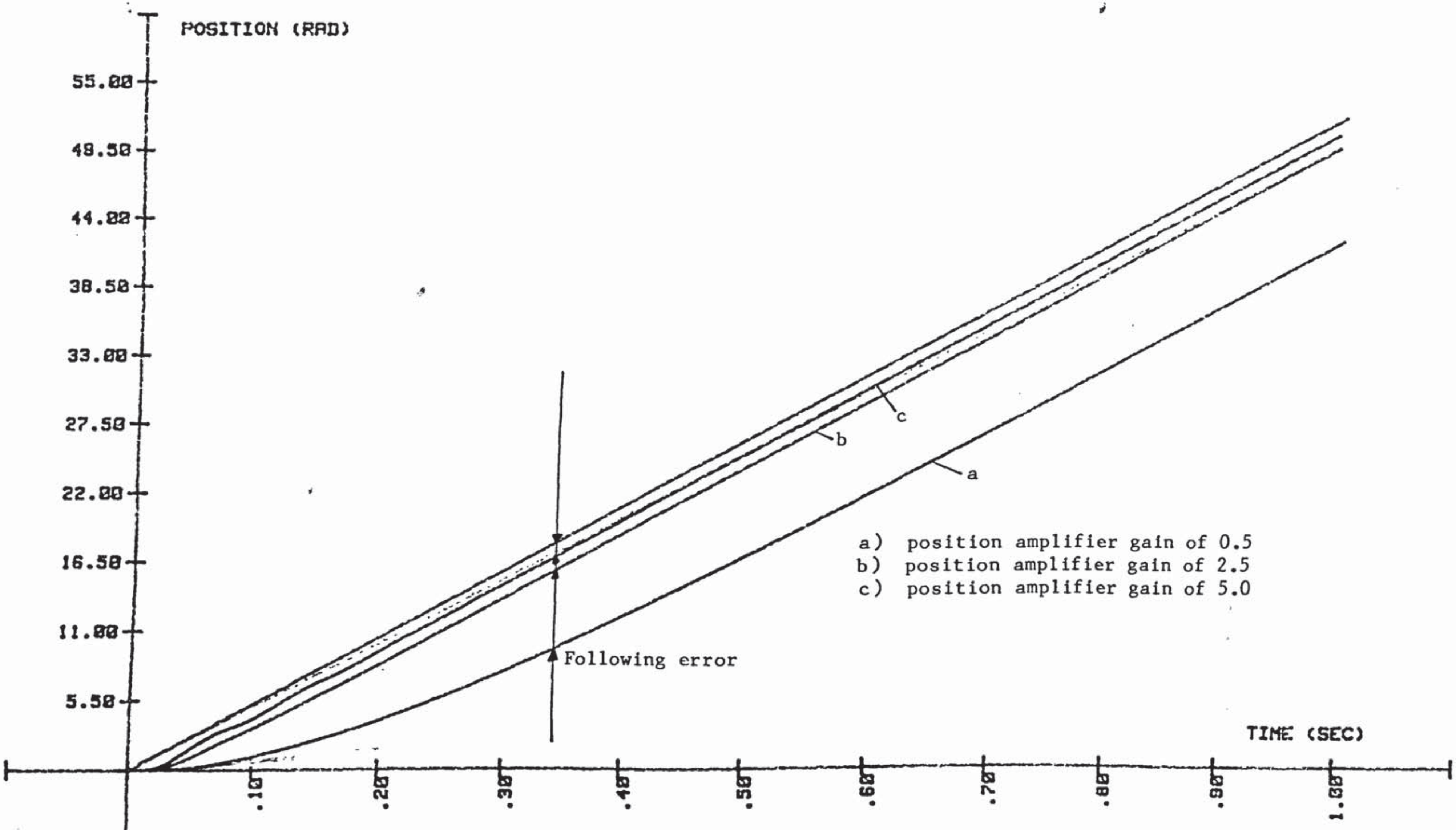


Fig. 4.18 The position response of the system to a ramp input showing the following error.

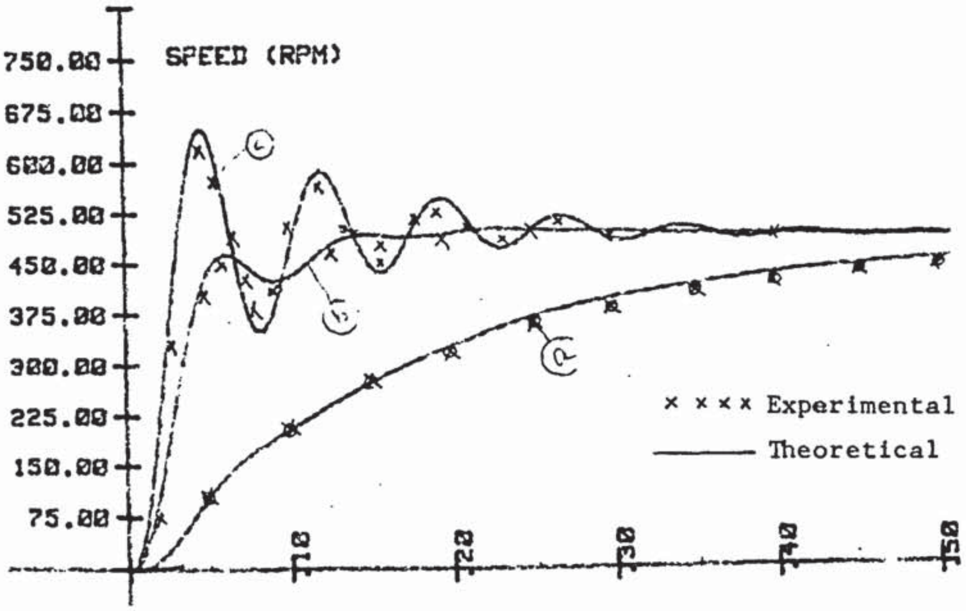


Fig. 4.19 Experimental and theoretical comparison of the velocity of the motor for a ramp input.

a) position amplifier gain of 0.5
b) position amplifier gain of 2.5
c) position amplifier gain of 5.0

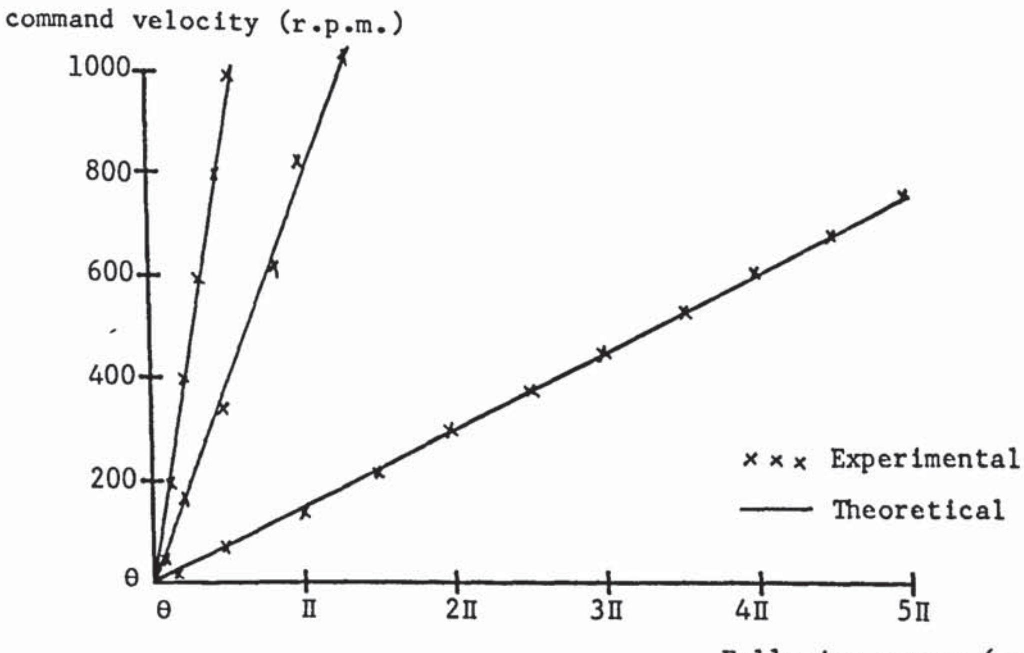


Fig. 4.20 Static characteristic of the system in position closed loop.

a) position amplifier gain of 0.5
b) position amplifier gain of 2.5
c) position amplifier gain of 5.0

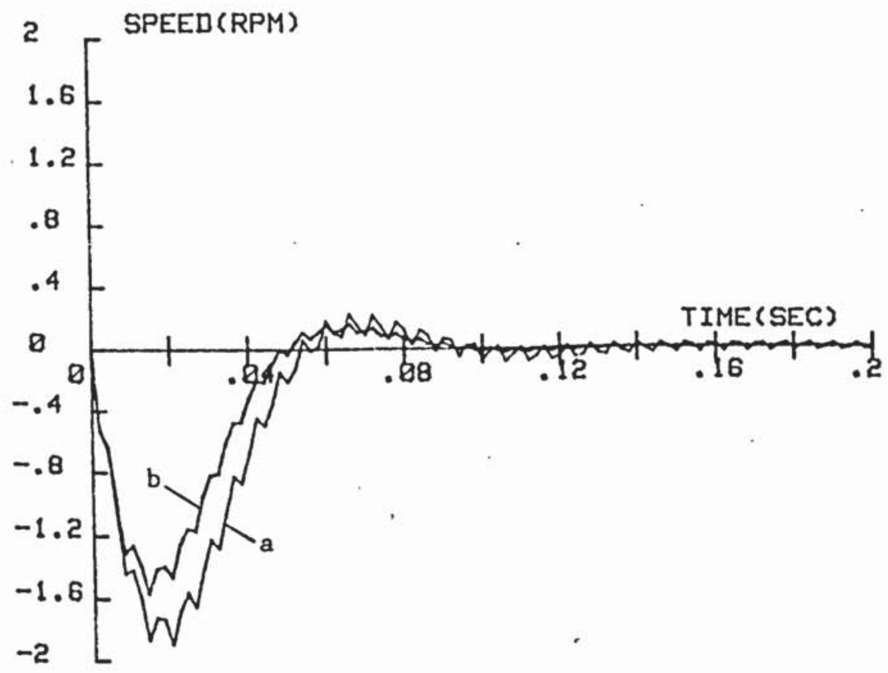


Fig. 4.21 Effect of current feedback for a unit step input of torque.

a) current feedback gain of 0.02
b) current feedback gain of 0.08

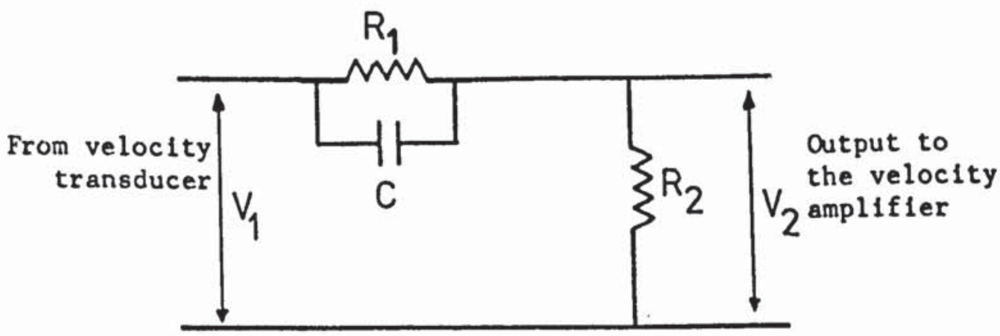


Fig. 4.22 Circuits diagram for obtaining proportional and derivative signal.

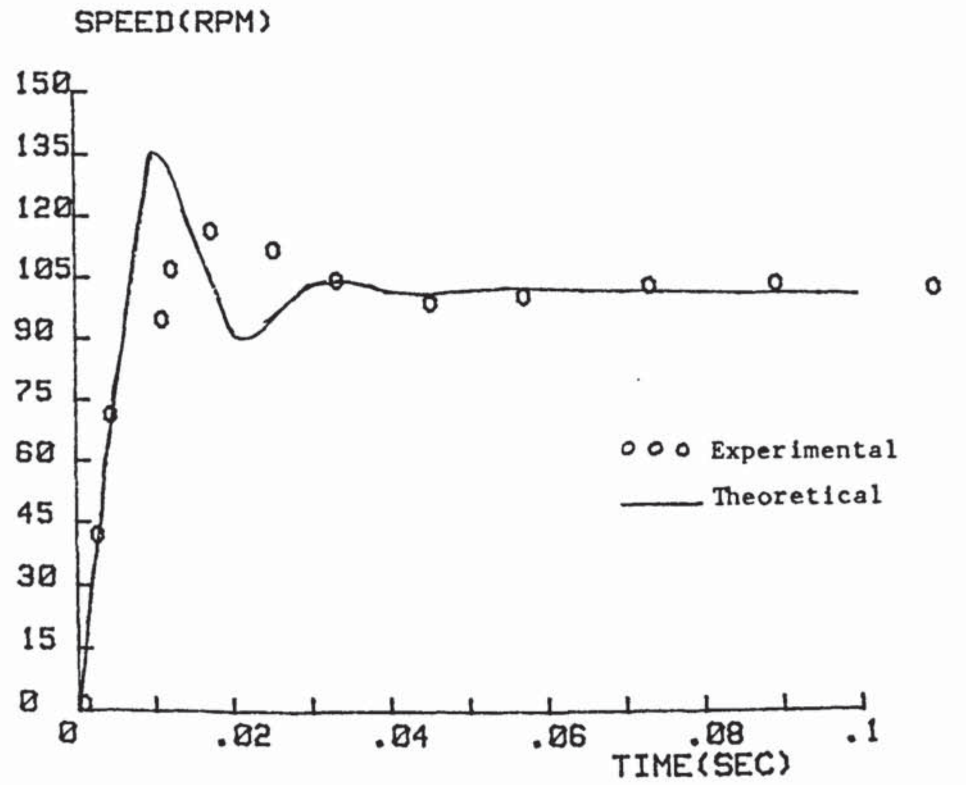


Fig. 4.24. Step input response, with acceleration feedback, and the velocity amplifier gain increased 5 times.

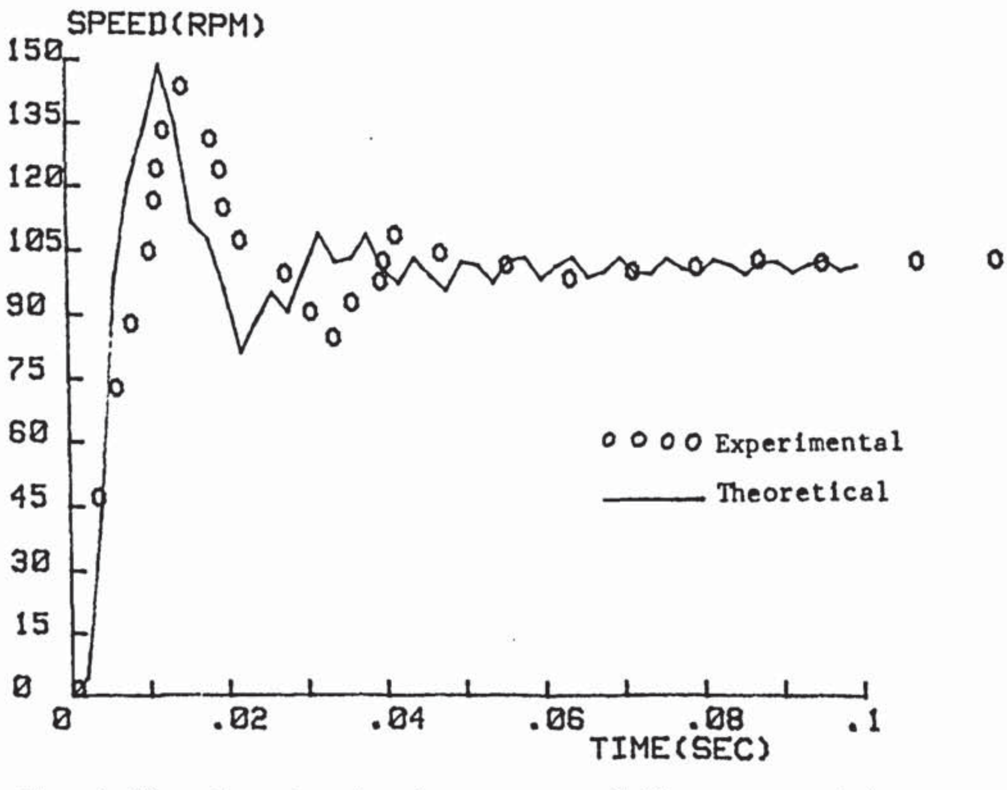


Fig. 4.23. The step input response of the system without acceleration feedback and with an increase of 5 times on the velocity amplifier.

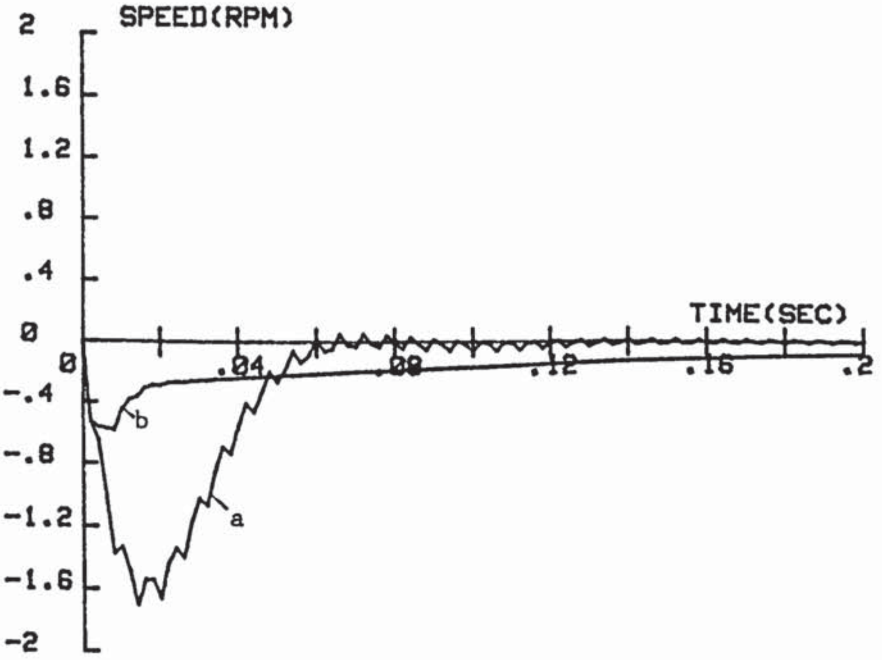


Fig. 4.25. Effect of a unit step input of external torque.

a) system without acceleration feedback
b) system with acceleration feedback and the gain of velocity amplifier increased 5 times.

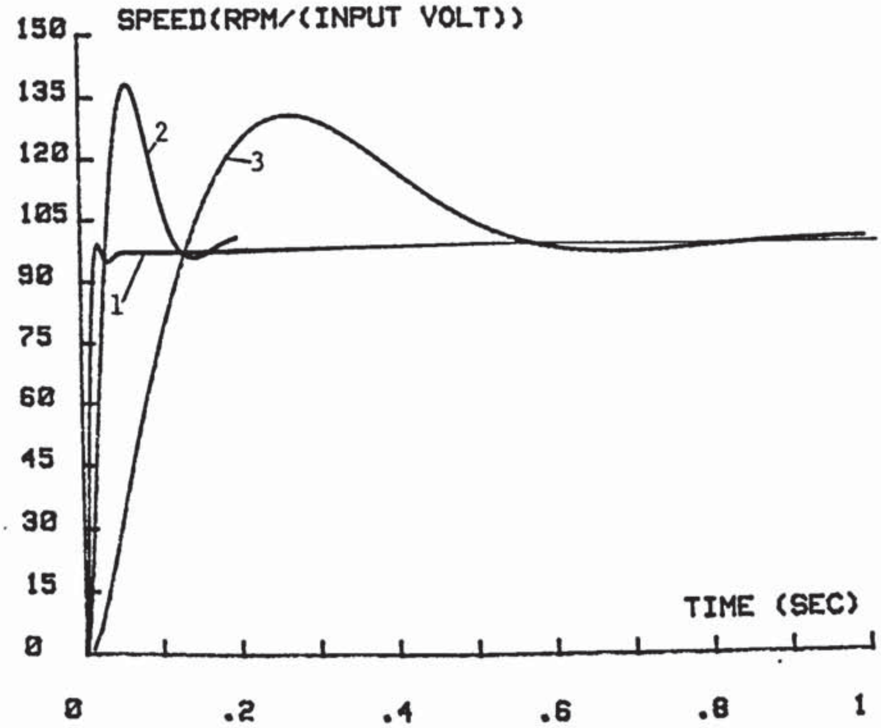


Fig. 4.26. Response to a step input of velocity for 1 kW D.C. servo motor. Thyristor controlled with current frequency of 50 Hz.

1) No load inertia.
2) Load inertia of 0.02 ($Kg.m^2$) equivalent to 200% of rotor inertia.
3) Load inertia of 0.1 ($kg.m^2$) equivalent to 1000% of rotor inertia.

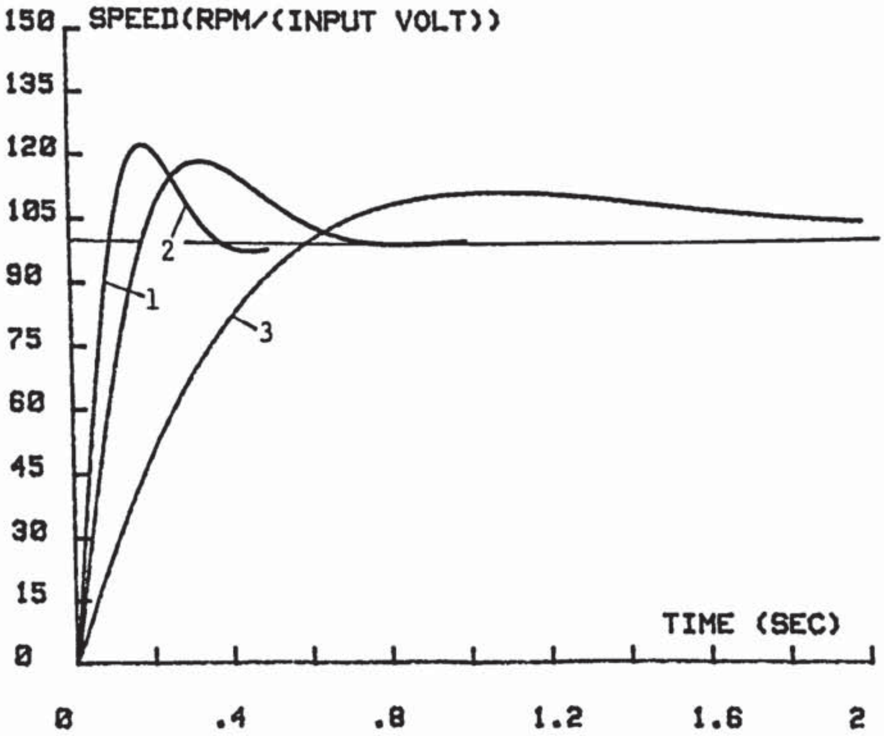


Fig. 4.27. Response to a step input of velocity for a 3 kW D.C. servo motor. Thyristor controlled with current frequency of 50 Hz.

1) No load inertia.
2) Load inertia of 0.02 equivalent to 100% of rotor inertia.
3) Load inertia of 0.1 equivalent to 500% of rotor inertia.

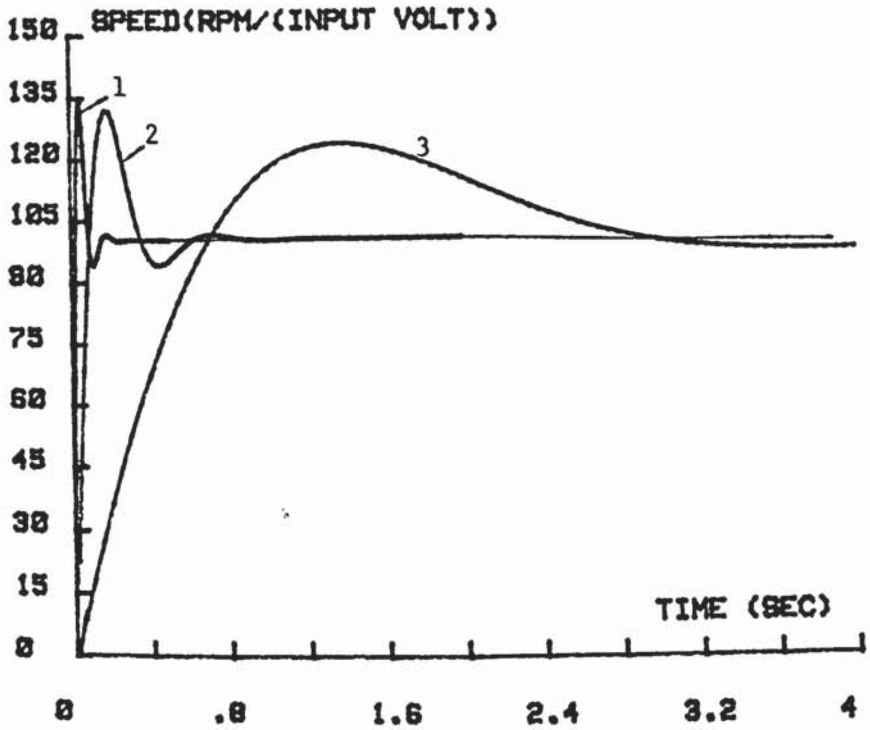


Fig. 4.28. Response to a step input of velocity for a 5 kW D.C. servo motor. Thyristor controlled with current frequency of 50 Hz.

1) No load inertia.
2) Load inertia of 0.044 kg.m² equivalent to 100% of rotor inertia.
3) Load inertia of 1 kg.m² equivalent to 250% of rotor inertia.

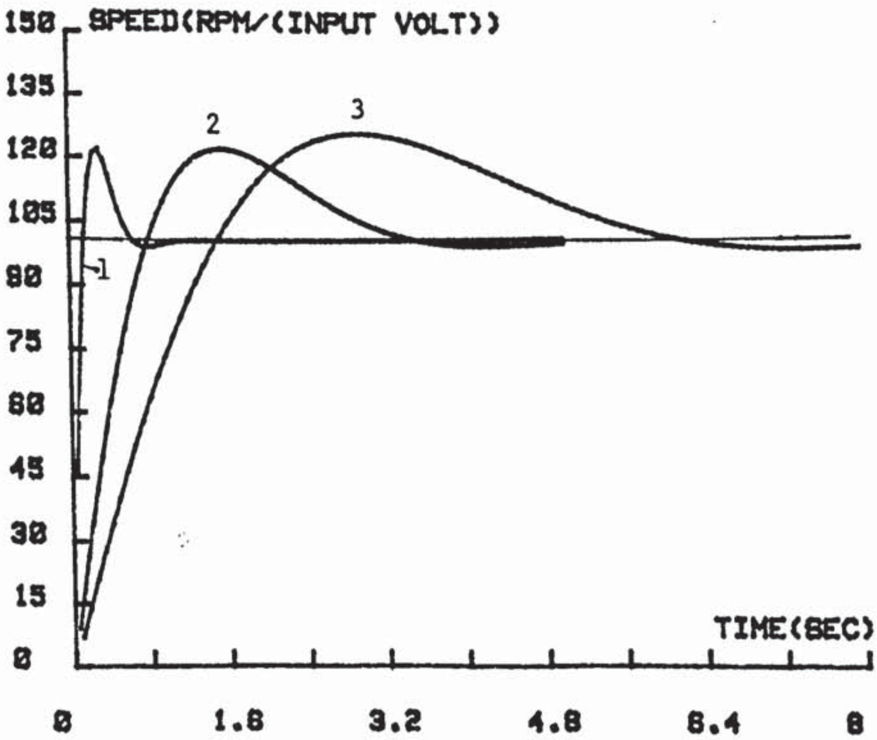


Fig. 4.29. Response to a step input of velocity for a 10 kW D.C. servo motor. Thyristor controlled with current frequency of 50 Hz.

1) No load inertia.
2) Load inertia of 1 kg.m² equivalent to 400% of rotor inertia.
3) Load inertia of 2.4 kg.m² equivalent to 1000% of rotor inertia.

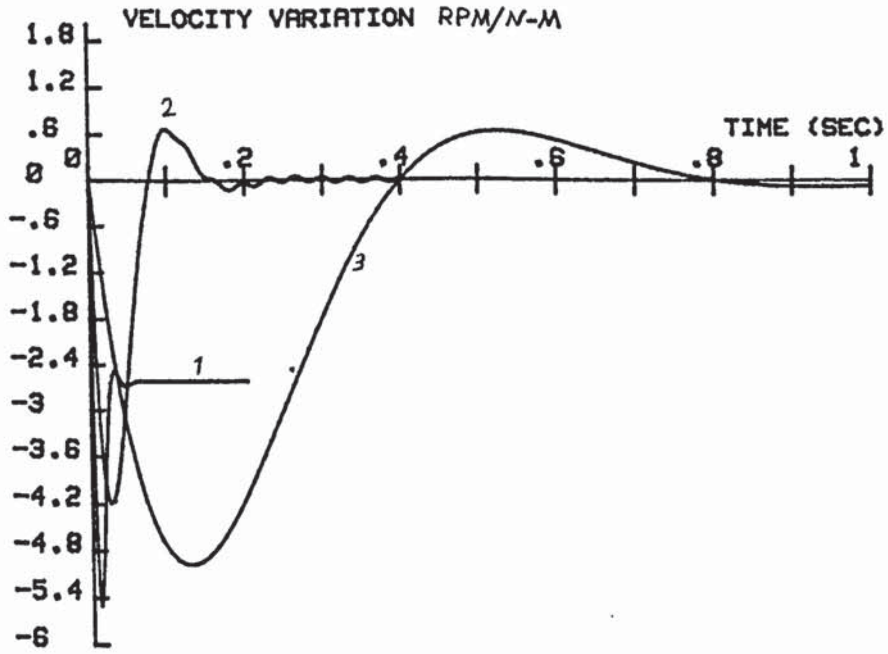


Fig. 4.30. Response to a unit step input of torque for a 1 kW D.C. servo motor. Thyristor controlled with current frequency of 50 Hz.

1) No load inertia and no feedforward integrator.
2) Load inertia of 0.02 $kg.m^2$ equivalent to 200% of rotor inertia.
3) Load inertia of 0.1 $kg.m^2$ equivalent to 1000% of rotor inertia.

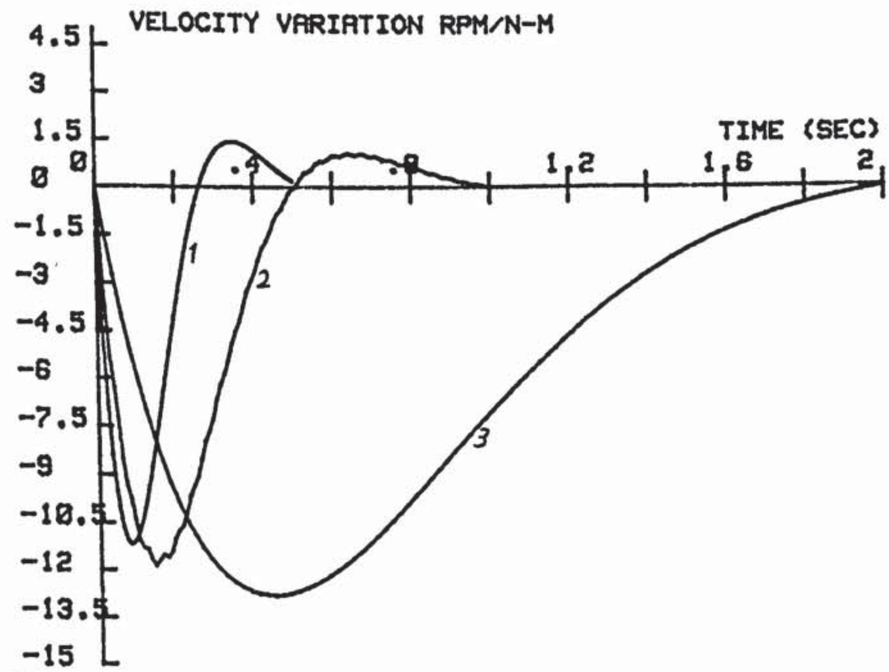


Fig. 4.31. Response to a unit step input of torque for a 3 kW D.C. servo motor. Thyristor controlled with current frequency of 50 Hz.

1) No load inertia.
2) Load inertia of 0.02 equivalent to 100% of rotor inertia.
3) Load inertia of 0.1 equivalent to 500% of rotor inertia.

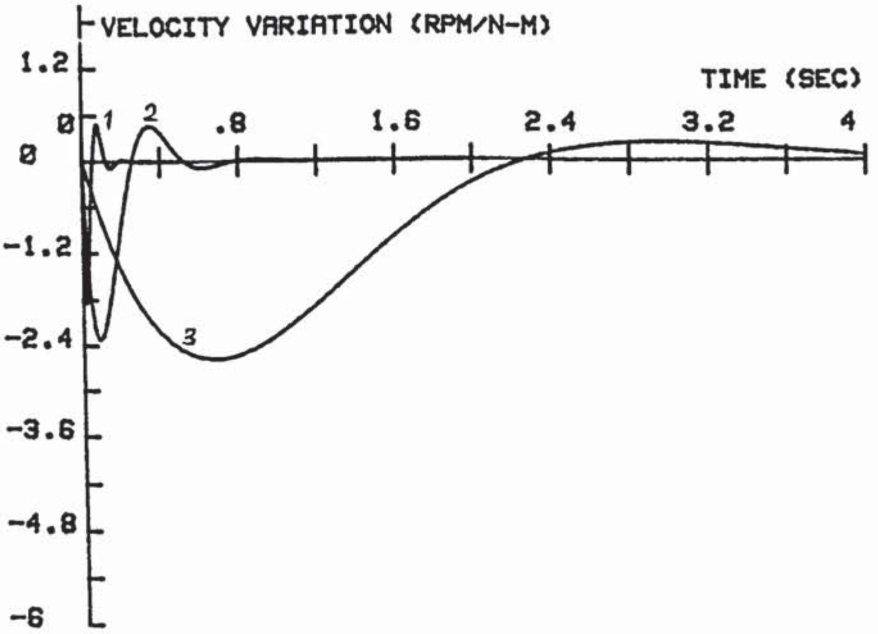


Fig. 4.32. Response to a unit step input of torque for a 5 kW D.C. servo motor. Thyristor controlled with current frequency of 50 Hz.

1) No load inertia.
2) Load inertia of 0.044 $kg.m^2$ equivalent to 100% of rotor inertia.
3) Load inertia of 1 $kg.m^2$ equivalent to 2300% of rotor inertia.

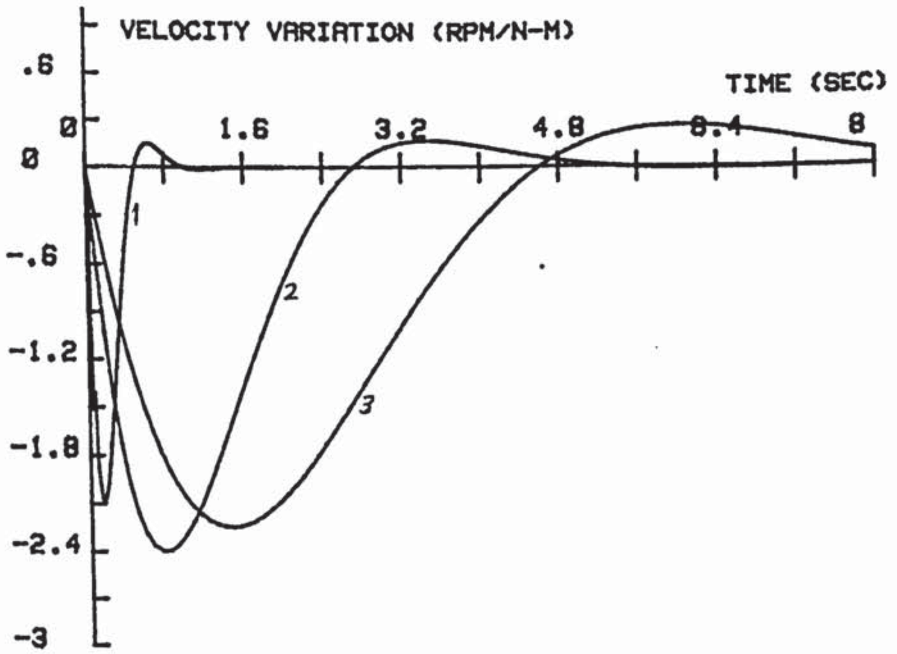


Fig. 4.33. Response to a unit step input of torque for a 10 kW D.C. servo motor. Thyristor controlled with current frequency of 50 Hz.

1) No load inertia.
2) Load inertia of 1 $kg.m^2$ equivalent to 400% of rotor inertia.
3) Load inertia of 2.4 $kg.m^2$ equivalent to 1000% of rotor inertia.

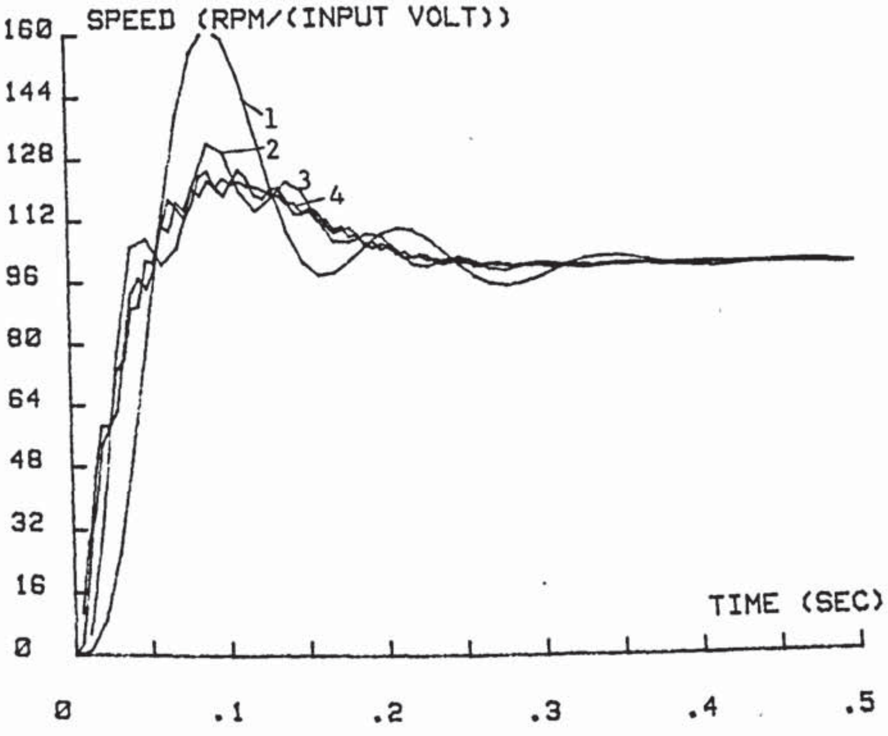


Fig. 4.34. Step input response of the system. Thyristor controlled with current frequency of 50 Hz.

1) Load natural frequency of 8 Hz.
2) Load natural frequency of 20 Hz.
3) Load natural frequency of 30 Hz.
4) Load natural frequency of 100 Hz.

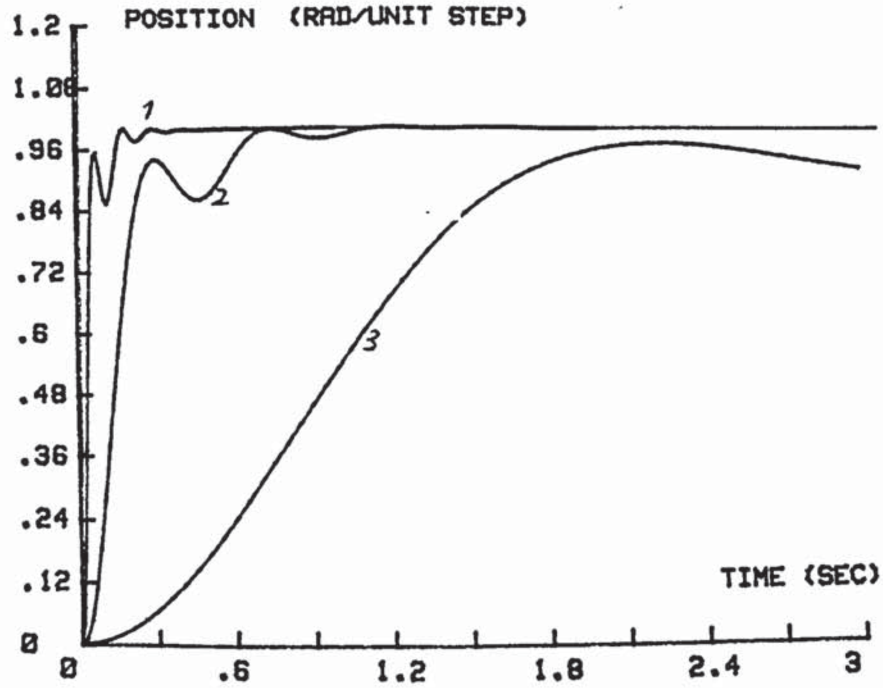


Fig. 4.35. Step input response in position control of a 5 kW D.C. servo motor. Thyristor controlled with current frequency of 50 Hz.

1) No load inertia.
2) Load inertia of 0.1 $kg.m^2$ equivalent to 230% of rotor inertia.
3) Load inertia of 1 $kg.m^2$ equivalent to 2300% of rotor inertia.

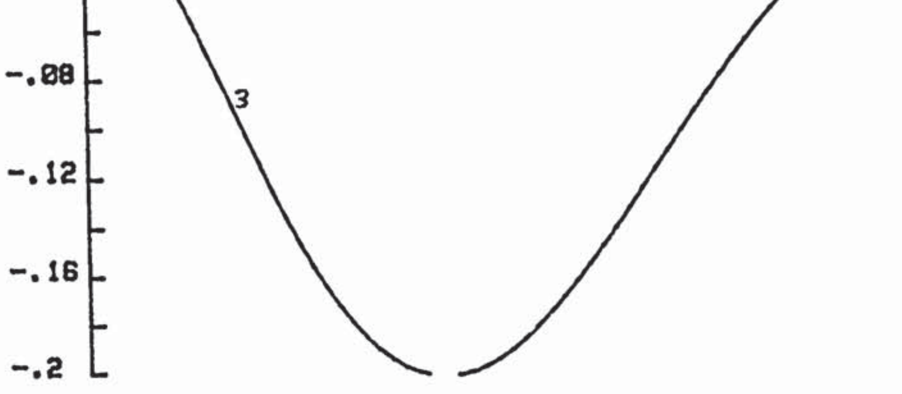


Fig. 4.36. Step torque response in position control of 5 kW D.C. servo motor. Thyristor controlled with current frequency of 50 Hz.

1) No load inertia.
2) Load inertia of 0.1 $kg.m^2$ equivalent to 230% of rotor inertia.
3) Load inertia of 1 $kg.m^2$ equivalent to 2300% of rotor inertia.

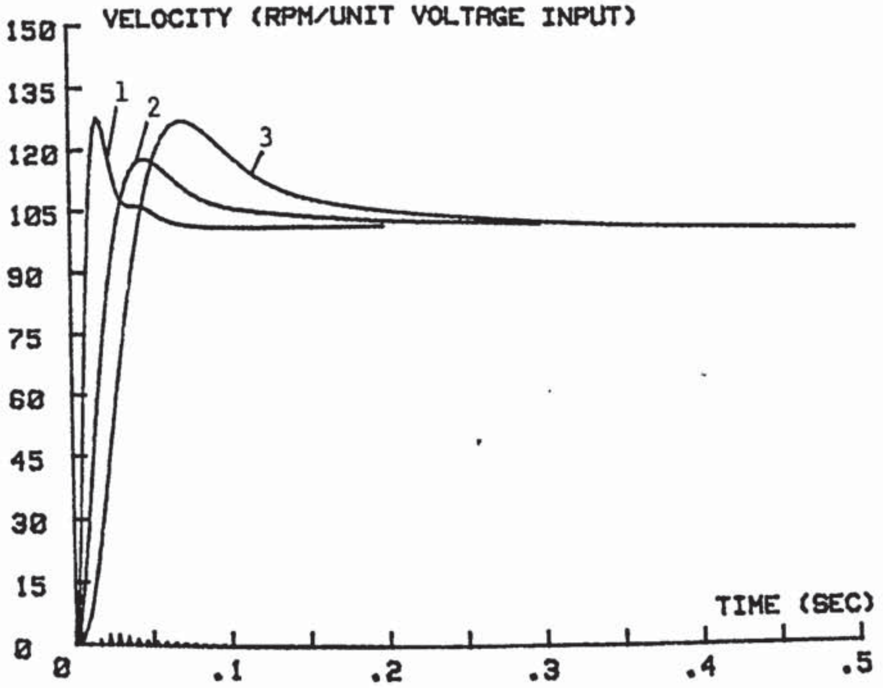


Fig. 4.37. Step input response of 1 kW D.C. servo motor. Thyristor controlled with current frequency of 150 Hz.

1) No load inertia.
2) Load inertia of 0.02 equivalent to 200% of rotor inertia.
3) Load inertia of 0.1 equivalent to 1000% of rotor inertia.

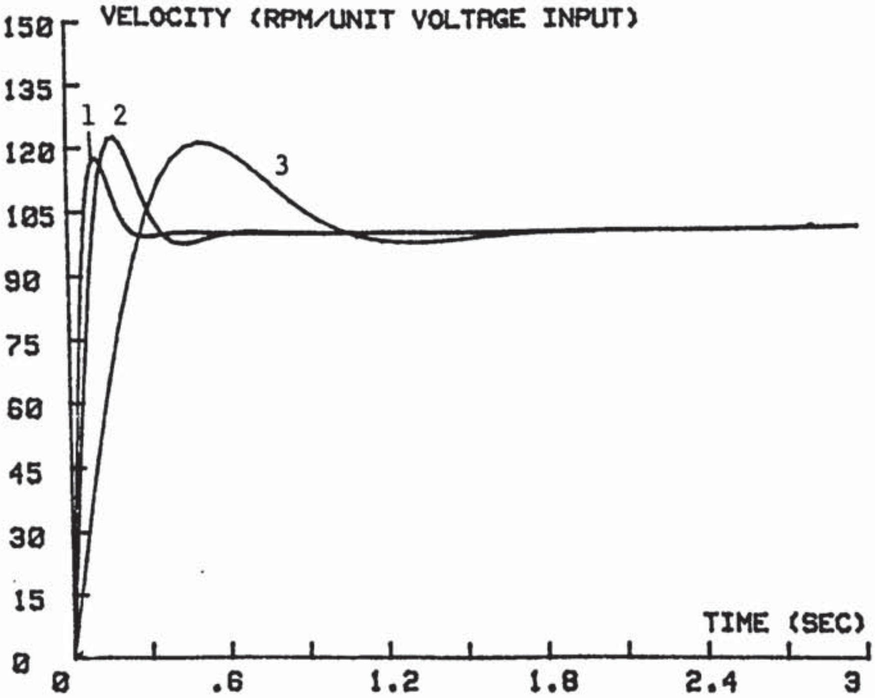


Fig. 4.38. Step input response of 3 kW D.C. servo motor. Thyristor controlled with current frequency of 150 Hz.

1) No load inertia.
2) Load inertia of 0.02 equivalent to 100% of rotor inertia.
3) Load inertia of 0.1 equivalent to 500% of rotor inertia.

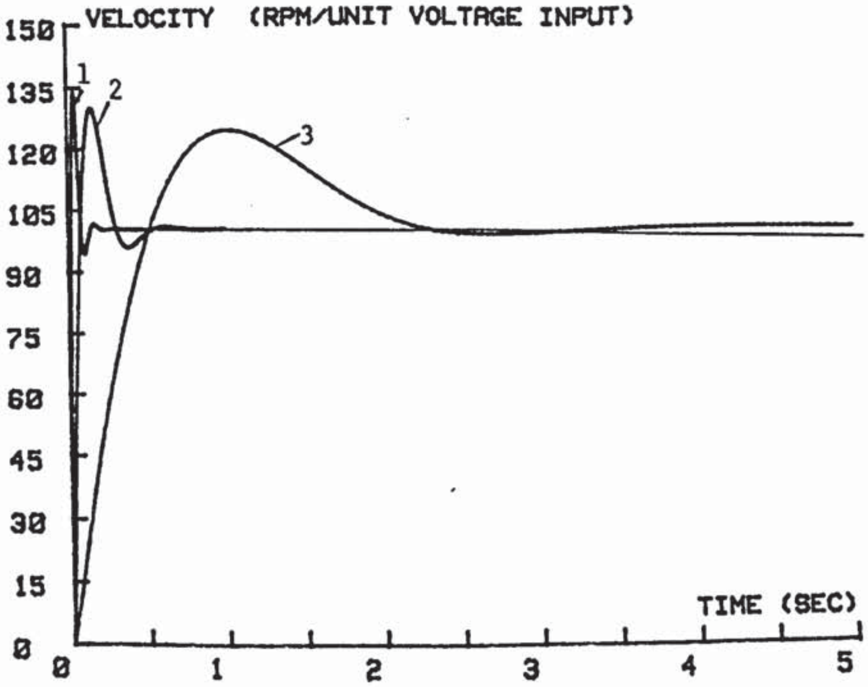


Fig. 4.39. Step input response of 5 kW D.C. servo motor. Thyristor controlled with current frequency of 150 Hz.

1) No load inertia.
2) Load inertia of 0.1 $kg.m^2$ equivalent to 230% of rotor inertia.
3) Load inertia of 1 $kg.m^2$ equivalent to 2300% of rotor inertia.

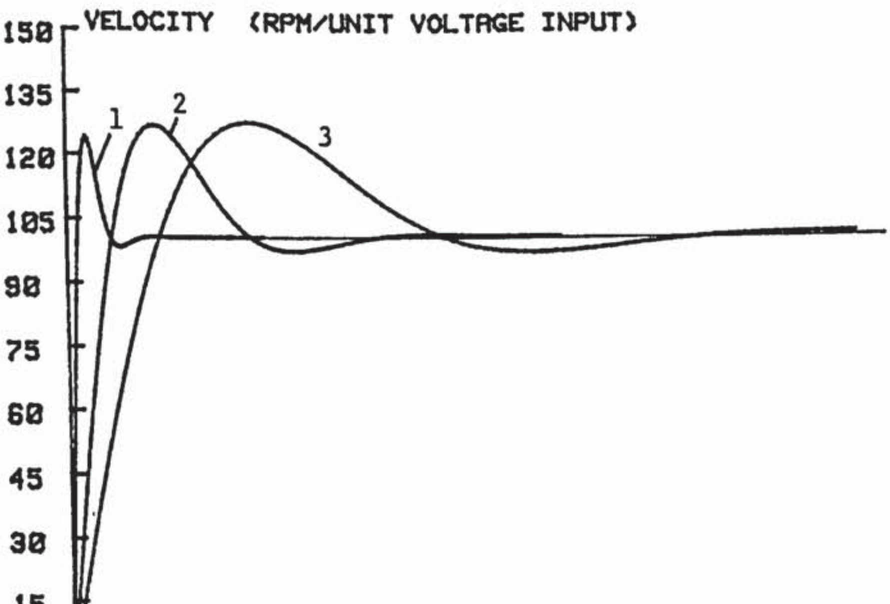


Fig. 4.40. Step input response of 10 kW D.C. servo motor. Thyristor controlled with current frequency of 150 Hz.

1) No load inertia.
2) Load inertia of 1 $kg.m^2$ (400% of rotor inertia).
3) Load inertia of 2.4 $kg.m^2$ (1000% of rotor inertia).

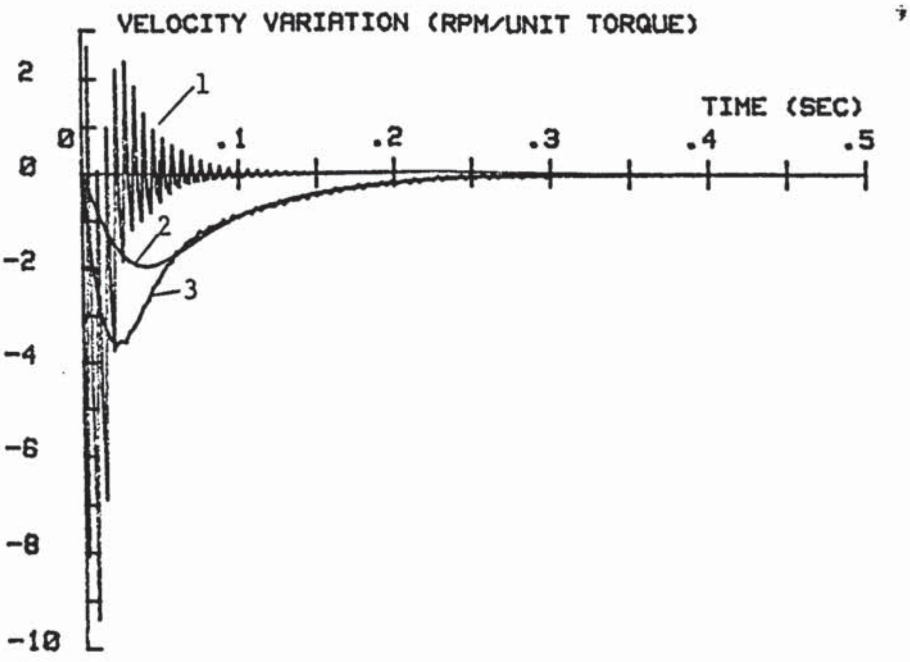


Fig. 4.41. Response to a unit step input of torque of a 1 kW D.C. servo motor. Thyristor controlled with current frequency of 150 Hz.

1) Very low load inertia.
2) Load inertia of 0.02 kg.m² (200% of rotor inertia).
3) Load inertia of 0.1 kg.m² (1000% of rotor inertia).

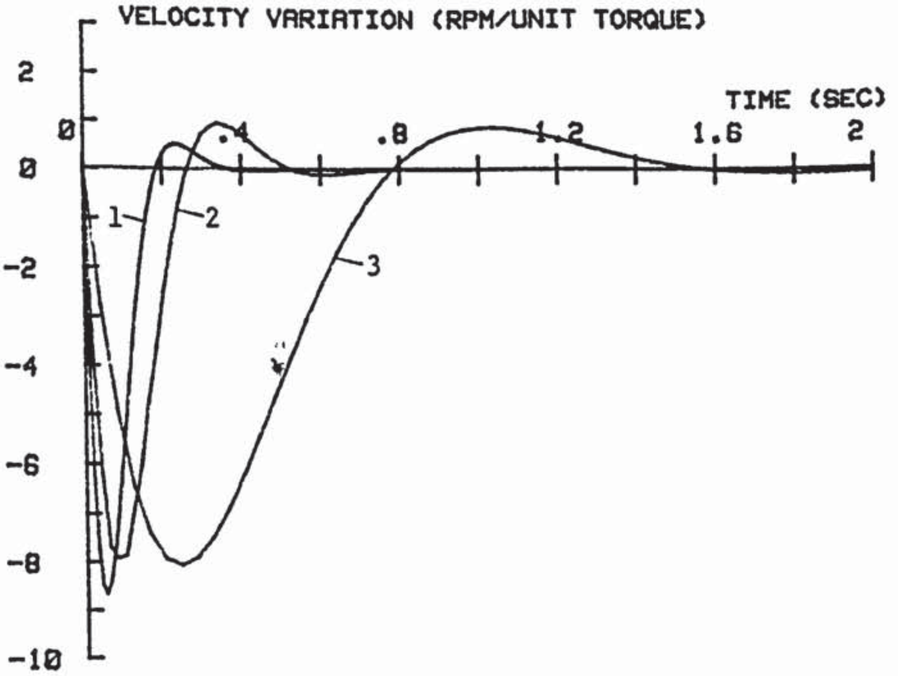


Fig. 4.42. Response to a unit step input of torque of a 3 kW D.C. servo motor. Thyristor controlled with current frequency of 150 Hz.

1) No load inertia.
2) Load inertia of 0.02 kg.m² (100% of rotor inertia).
3) Load inertia of 0.1 kg.m² (500% of rotor inertia).

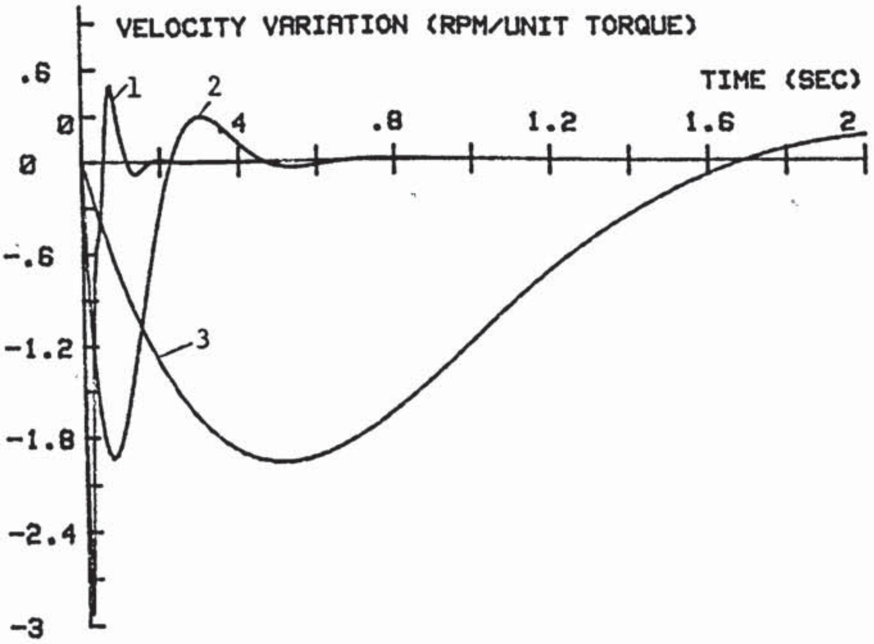


Fig. 4.43. Response to a unit step input of torque of a 5 kW D.C. servo motor. Thyristor controlled with current frequency of 150 Hz.

1) No load inertia.
2) Load inertia of 0.044 kg.m² (100% of rotor inertia).
3) Load inertia of 1 kg.m² (2300% of rotor inertia).

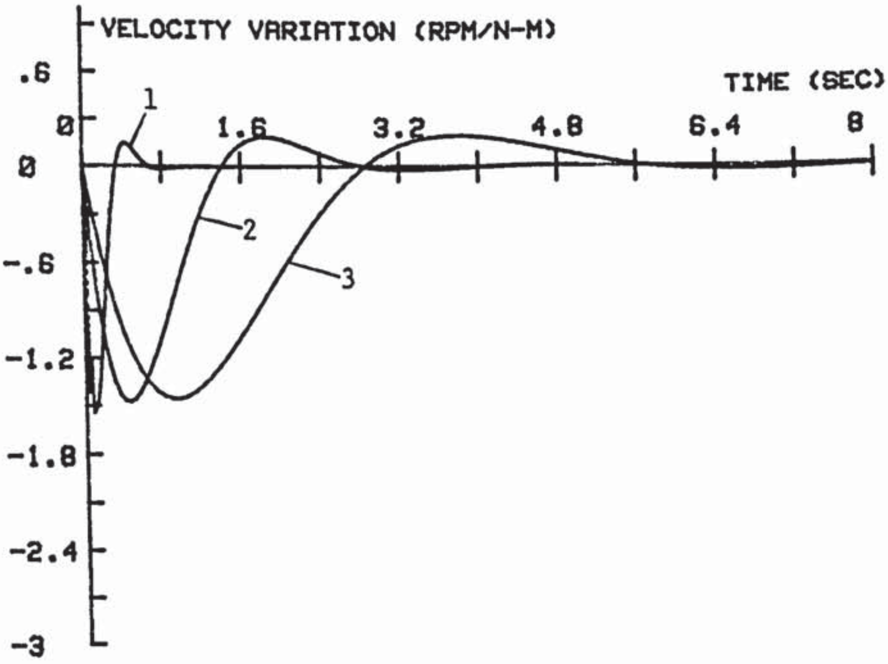


Fig. 4.44. Response to a unit step input of torque of a 10 kW servo motor. Thyristor controlled with current frequency of 150 Hz.

1) No load inertia.
2) Load inertia of 1 kg.m² (400% of rotor inertia).
3) Load inertia of 2.4 kg.m² (1000% of rotor inertia).

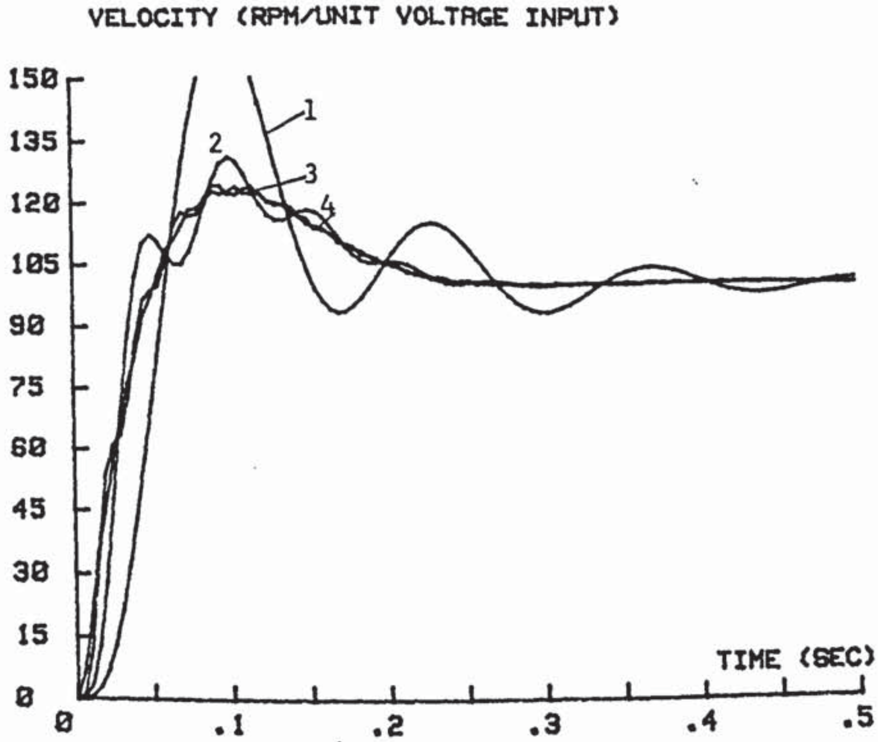


Fig. 4.45. Step input response of the system. Thyristor controlled with current frequency of 150 Hz.

1) Load natural frequency of 8 Hz.
2) Load natural frequency of 20 Hz.
3) Load natural frequency of 30 Hz.
4) Load natural frequency of 100 Hz.

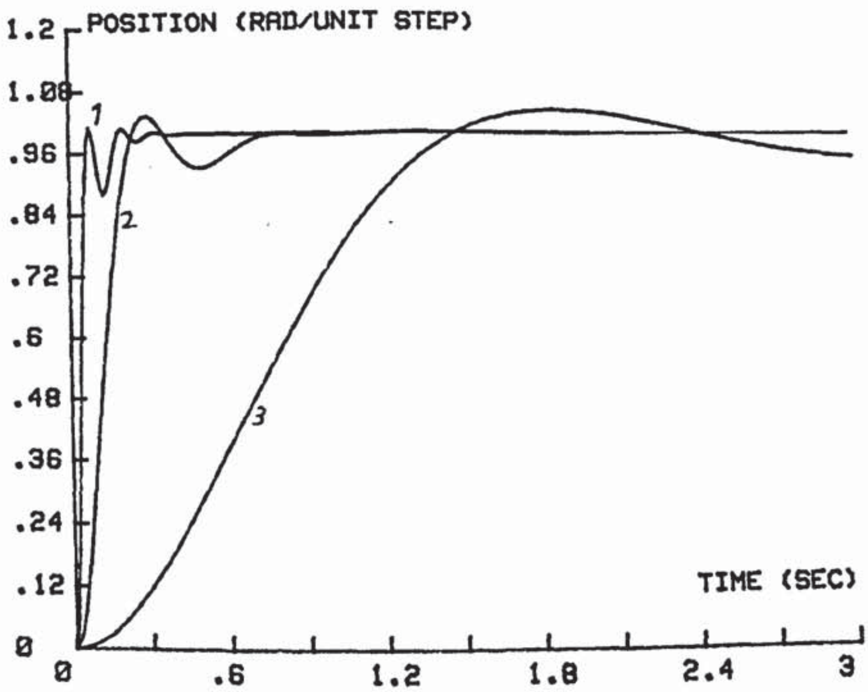


Fig. 4.46. Step input response in position control of a 5 kW D.C. servo motor. Thyristor controlled with current frequency of 150 Hz.

1) No load inertia.
2) Load inertia of 0.1 $kg.m^2$ equivalent to 230% of rotor inertia).
3) Load inertia of 1 $kg.m^2$ equivalent to 2300% of rotor inertia).

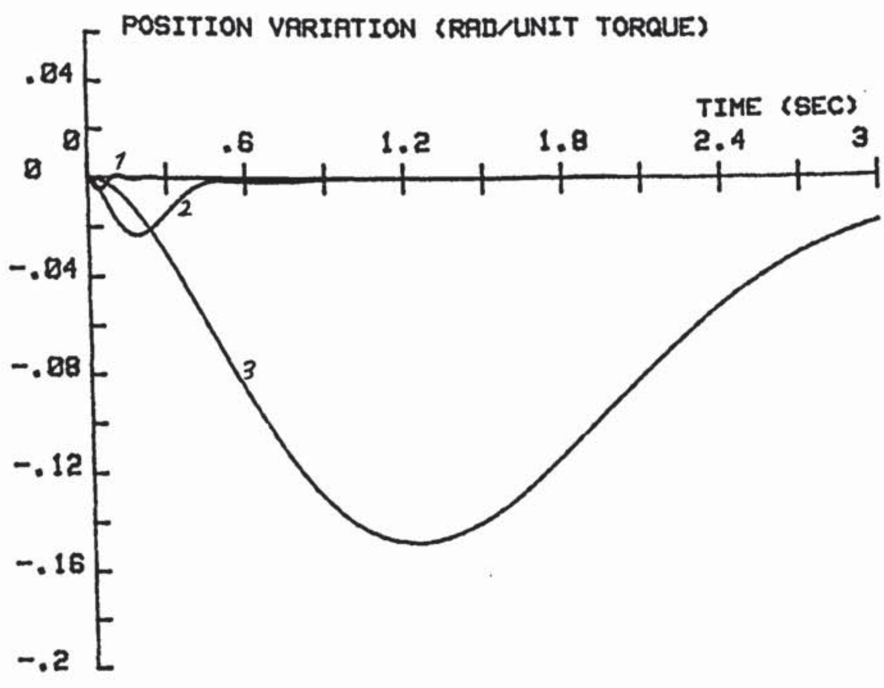


Fig. 4.47. Step input torque response in position control of a 5 kW D.C. servo motor. Thyristor controlled with current frequency of 150 Hz.

1) No load inertia.
2) Load inertia of 0.1 $kg.m^2$ equivalent of 230% of rotor inertia).
3) Load inertia of 1 $kg.m^2$ equivalent of 2300% of rotor inertia).

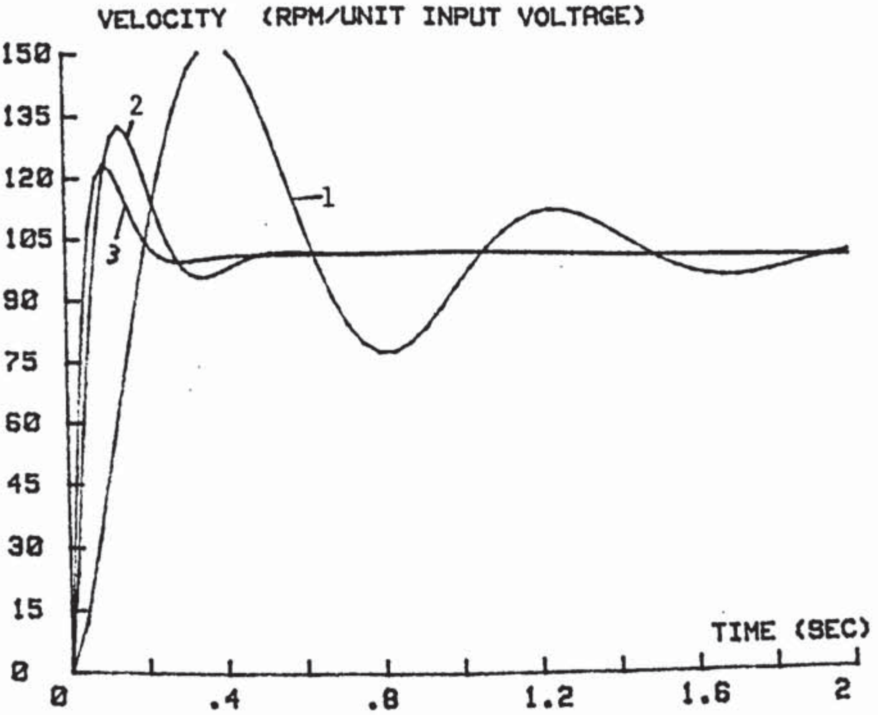


Fig. 4.48. Effect of non-linearity of a thyristor power amplifier.

1) Firing angle of 5% of maximum angle.
2) Firing angle of 25% of maximum angle.
3) Firing angle of 50% of maximum angle.

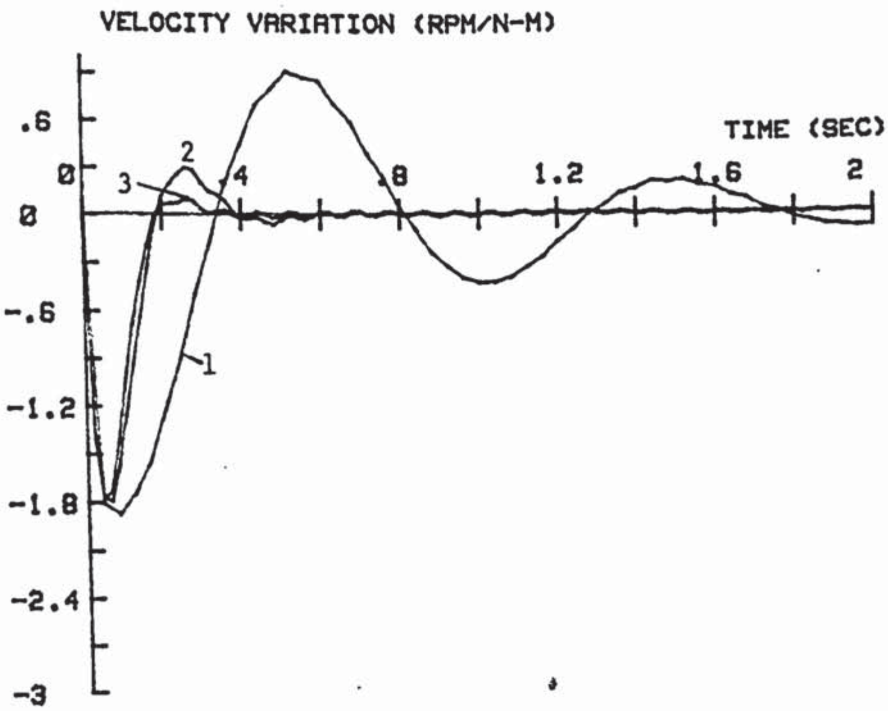


Fig. 4.49. Effect of non-linearity of a thyristor power amplifier on the external torque.

1) Firing angle of 5%.
2) Firing angle of 25%.
3) Firing angle of 50%.

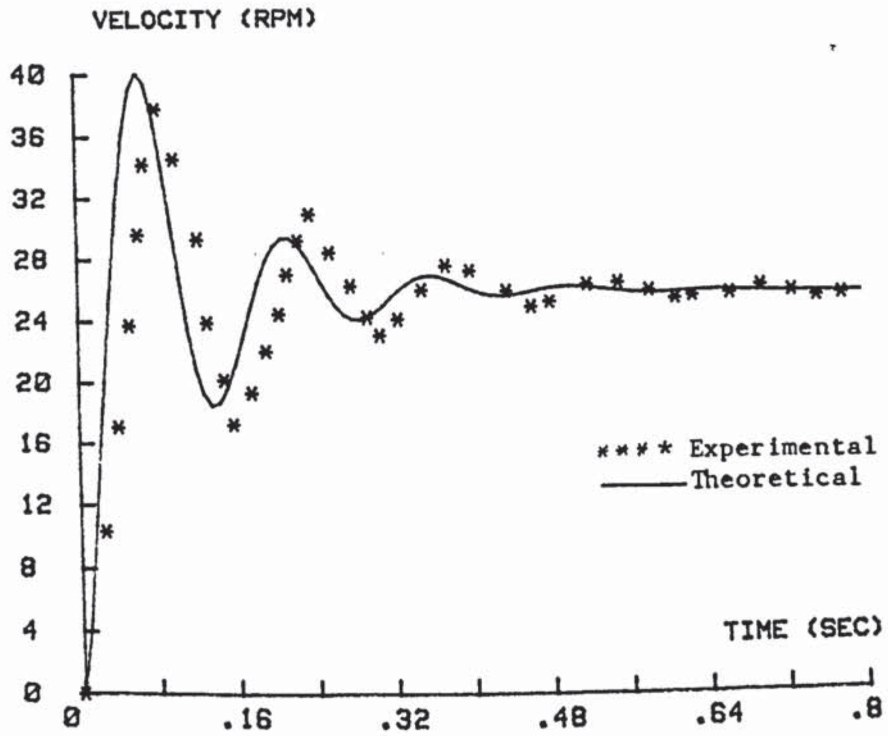


Fig. 4.50. Experimental and theoretical comparison of the step input velocity of a p.w.m. servo motor.

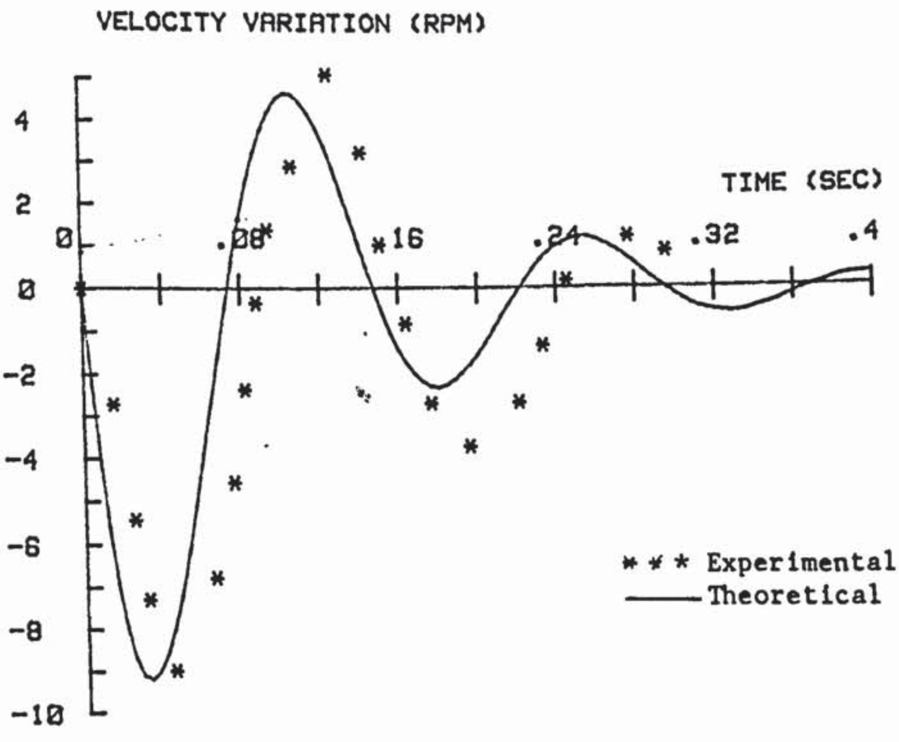


Fig. 4.51. Experimental and theoretical comparison of the response of a p.w.m. servo motor for step input of torque (4 N-M).

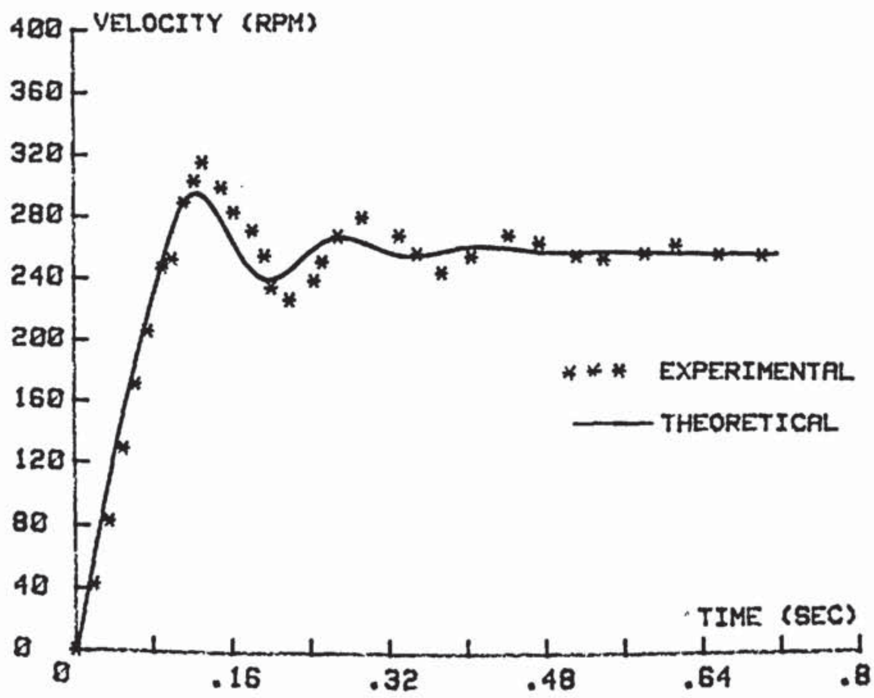


Fig. 4.52. Experimental and theoretical comparison of the response of the system for a large step input. Current is limited to 35 amps with rated current of 15 amps.

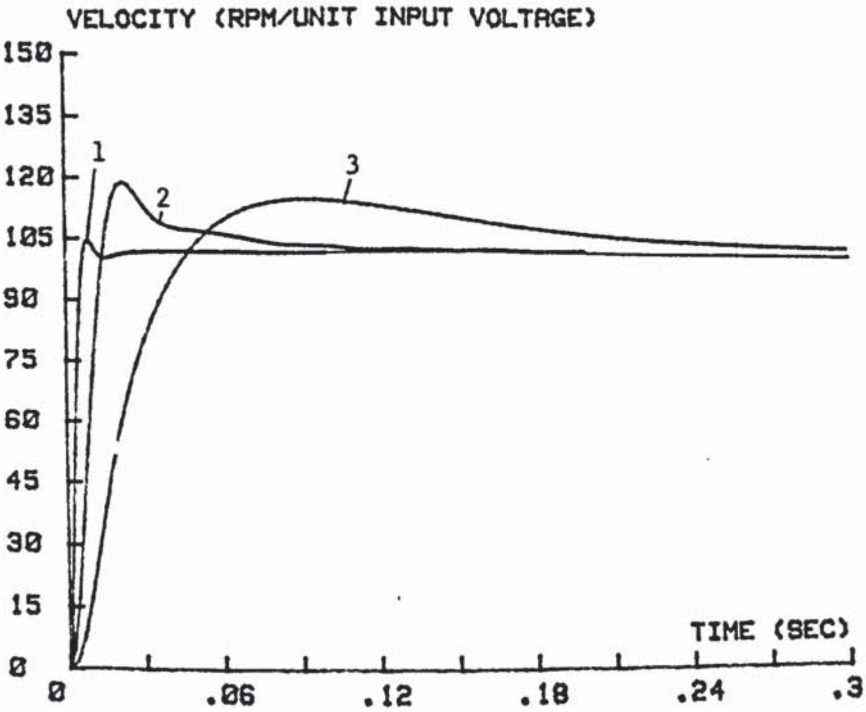


Fig. 4.53. Step input response of a 1 kW D.C. servo motor. p.w.m. drive with current frequency of 2 kHz.

1) No load inertia.
2) Load inertia of 0.02 kg.m² (200% of rotor inertia).
3) Load inertia of 0.1 kg.m² (1000% of rotor inertia).

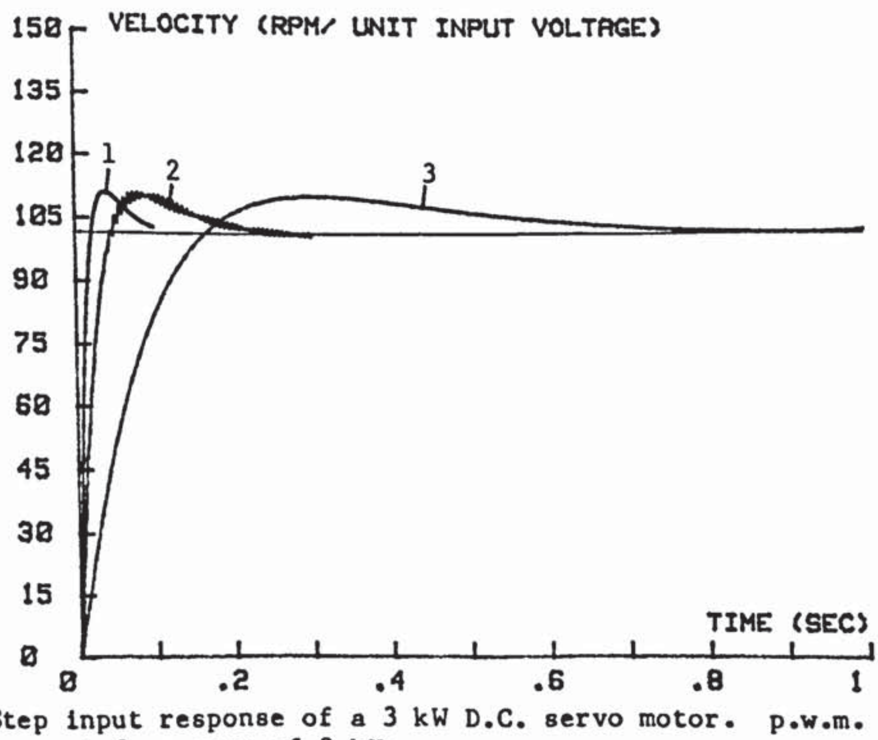


Fig. 4.54. Step input response of a 3 kW D.C. servo motor. p.w.m. drive with current frequency of 2 kHz.

1) No load inertia.
2) Load inertia of 0.02 kg.m² (100% of rotor inertia).
3) Load inertia of 0.1 kg.m² (500% of rotor inertia).

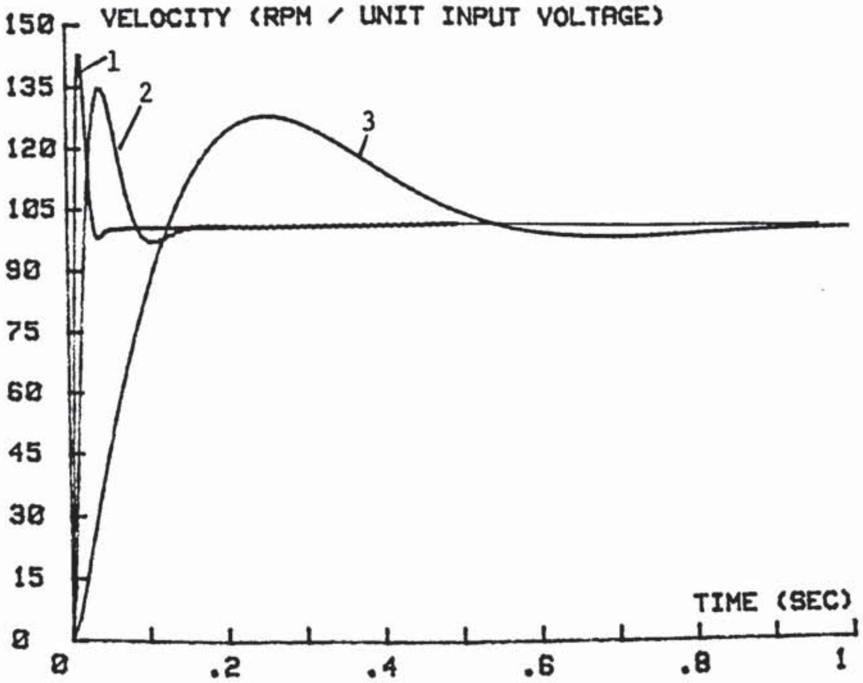


Fig. 4.55. Step input response of a 5 kW D.C. servo motor. p.w.m. drive with current frequency of 2 kHz.

1) No load inertia.
2) Load inertia of 0.1 kg.m² (230% of rotor inertia).
3) Load inertia of 1 kg.m² (2300% of rotor inertia).

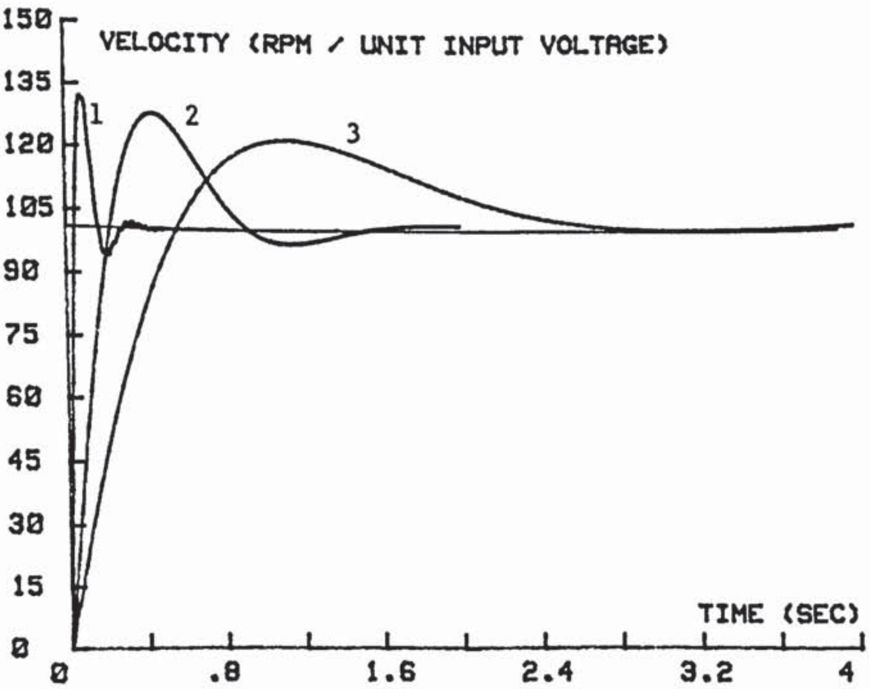


Fig. 4.56. Step input response of a 10 kW D.C. servo motor. p.w.m. drive with current frequency of 2 kHz.

1) No load inertia.
2) Load inertia of 1 kg.m² (400% of rotor inertia).
3) Load inertia of 2.4 kg.m² (1000% of rotor inertia).

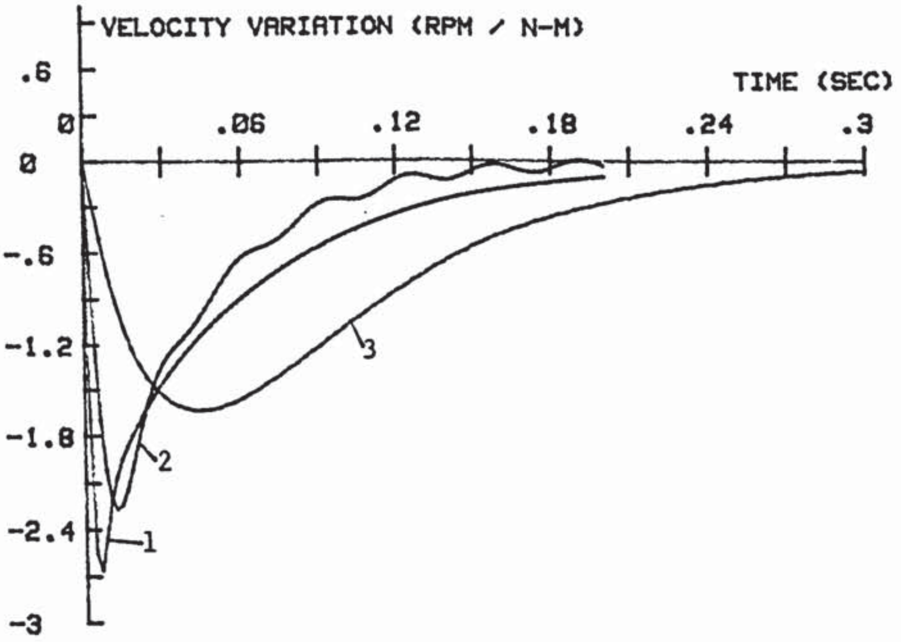


Fig. 4.57. Response of a 1 kW D.C. servo motor to a unit step input of torque. p.w.m. drive with current frequency of 2 kHz.

1) No load inertia.
2) Load inertia of 0.02 kg.m² (200% of rotor inertia).
3) Load inertia of 0.1 kg.m² (1000% of rotor inertia).

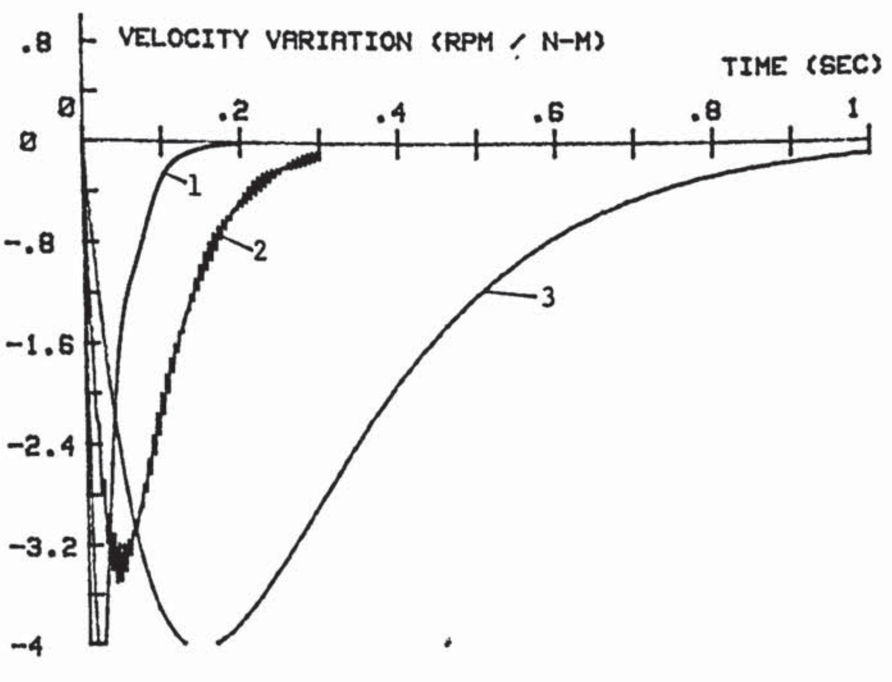


Fig. 4.58. Response of a 3 kW D.C. servo motor to a unit step input of torque. p.w.m. drive with current frequency of 2 kHz.

1) No load inertia.
2) Load inertia of 0.02 kg.m² (100% of rotor inertia).
3) Load inertia of 0.1 kg.m² (500% of rotor inertia).

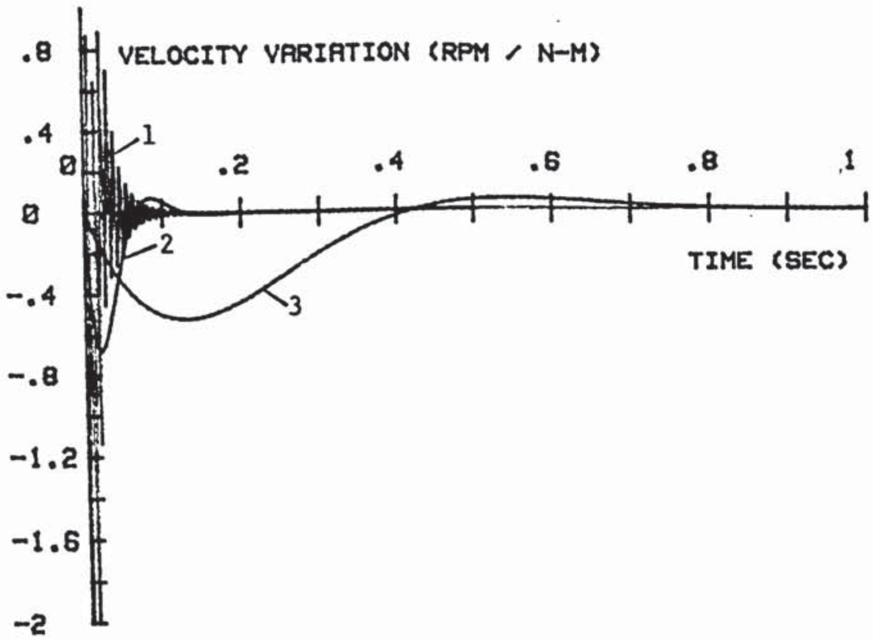


Fig. 4.59. Response of a 5 kW D.C. servo motor to a unit step input of torque. p.w.m. drive with current frequency of 2 kHz.

1) Very low inertia.
2) Load inertia of 0.044 kg.m² (100% of rotor inertia).
3) Load inertia of 1 kg.m² (2300% of rotor inertia).

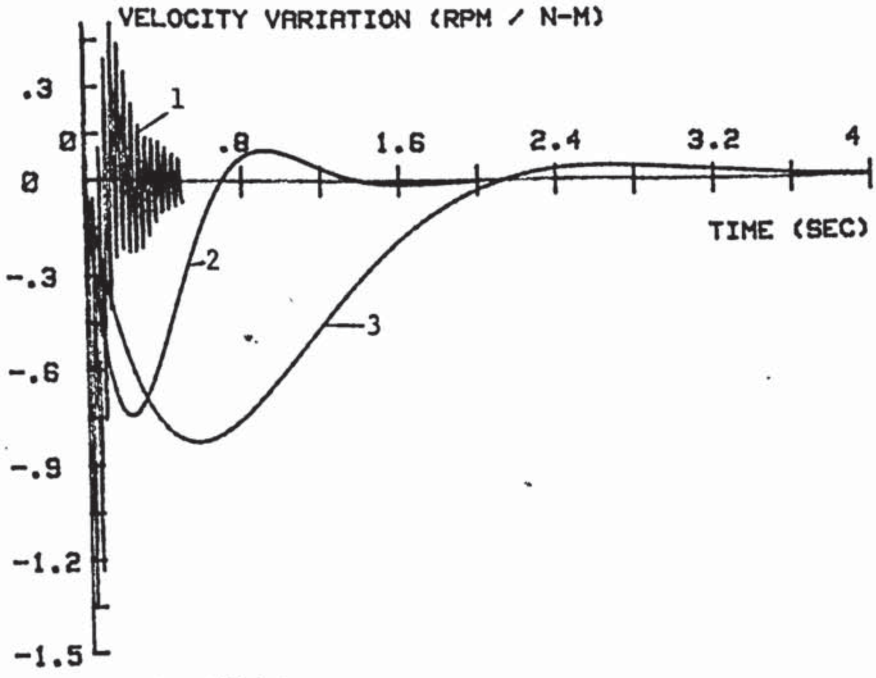


Fig. 4.60. Response of a 10 kW D.C. servo motor to a unit step input of torque. p.w.m. drive with current frequency of 2 kHz.

1) Very low load inertia.
2) Load inertia of 1 kg.m² (400% of rotor inertia).
3) Load inertia of 2.4 kg.m² (1000% of rotor inertia).

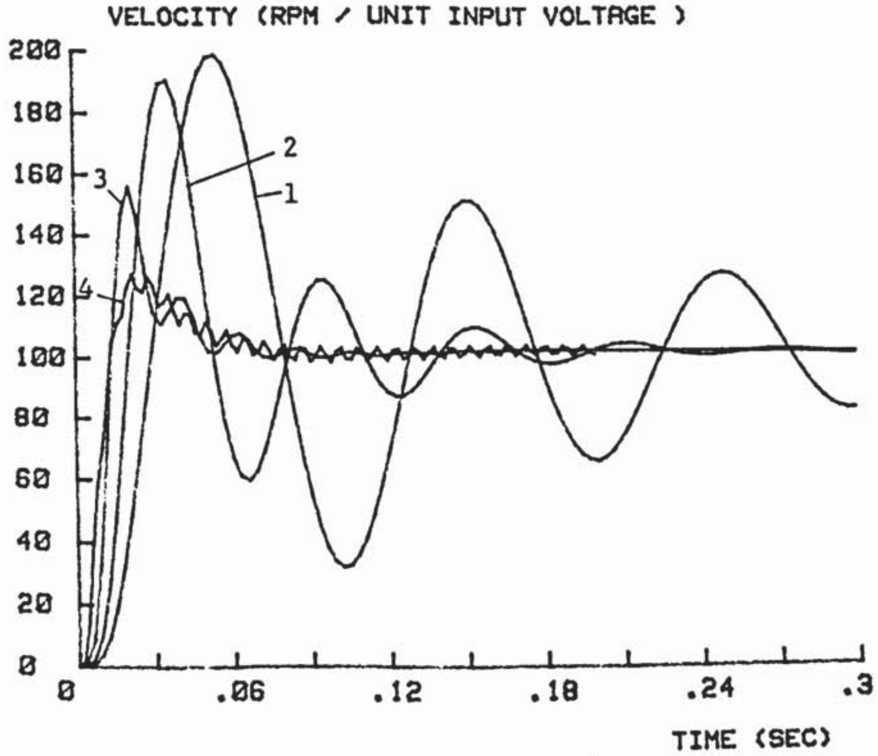


Fig. 4.61. Step input response of the system for different load natural frequency. p.w.m. drive with current frequency of 2 kHz.

1) Load natural frequency of 8 Hz.
2) Load natural frequency of 20 Hz.
3) Load natural frequency of 30 Hz.
4) Load natural frequency of 100 Hz.

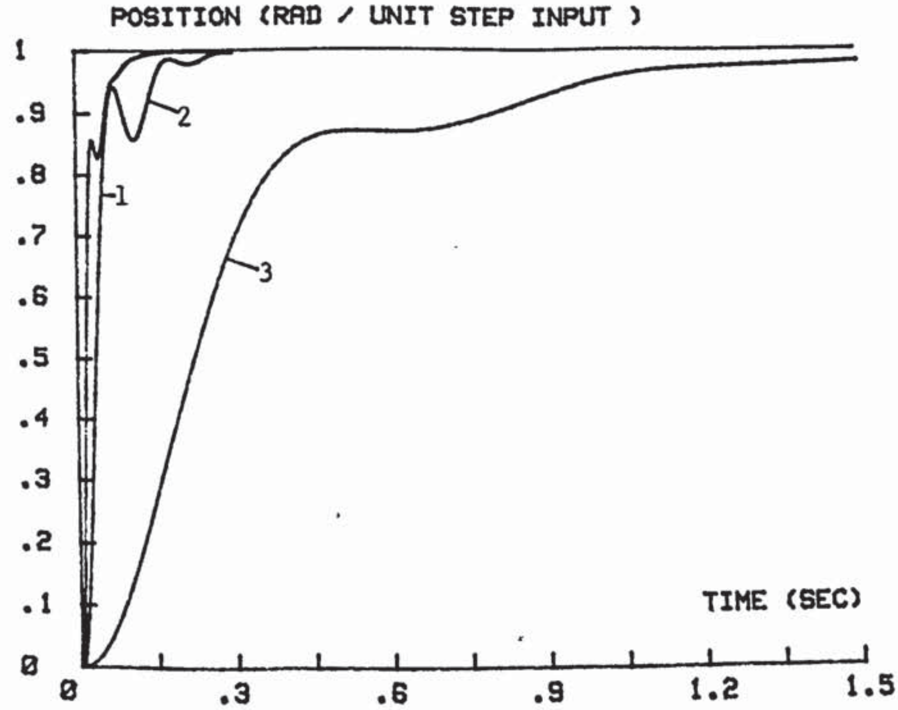


Fig. 4.62. Step input response in position control of a 5 kW D.C. servo motor. (p.w.m. drive with current frequency of 2 kHz.

1) No load inertia.
2) Load inertia of 0.1 kg.m² (230% of rotor inertia).
3) Load inertia of 1 kg.m² (2300% of rotor inertia).

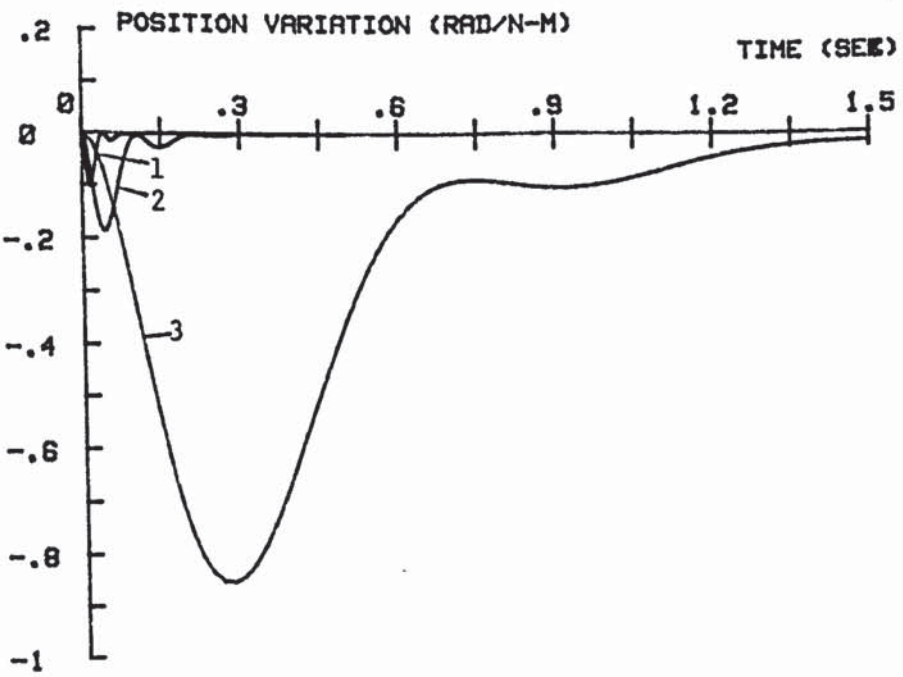


Fig. 4.63. Step input torque response in position control of 0.5 kW D.C. servo motor. (p.w.m. drive with current frequency of 2 kHz.

1) No load inertia.
2) Load inertia of 0.1 kg.m² (230% of rotor inertia).
3) Load inertia of 1 kg.m² (2300% of rotor inertia).

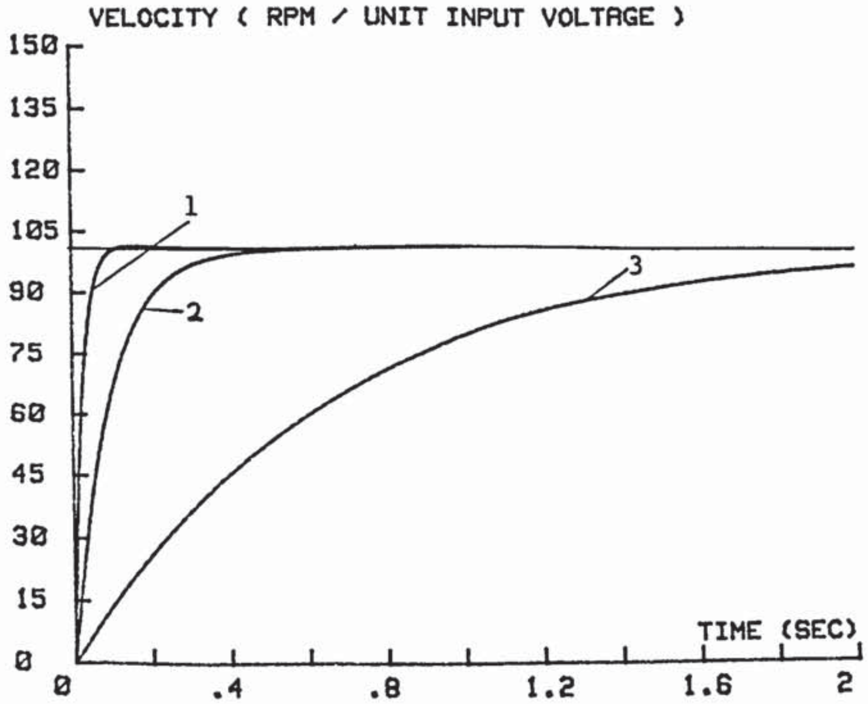


Fig. 4.64. Response of the 5 kW D.C. servo motor with acceleration feedback and a 10-fold increase in the gain. Thyristor controlled with current frequency of 50 Hz.

1) No load inertia.
2) Load inertia of 0.1 kg.m² (230% of rotor inertia).
3) Load inertia of 1 kg.m² (2300% of rotor inertia).

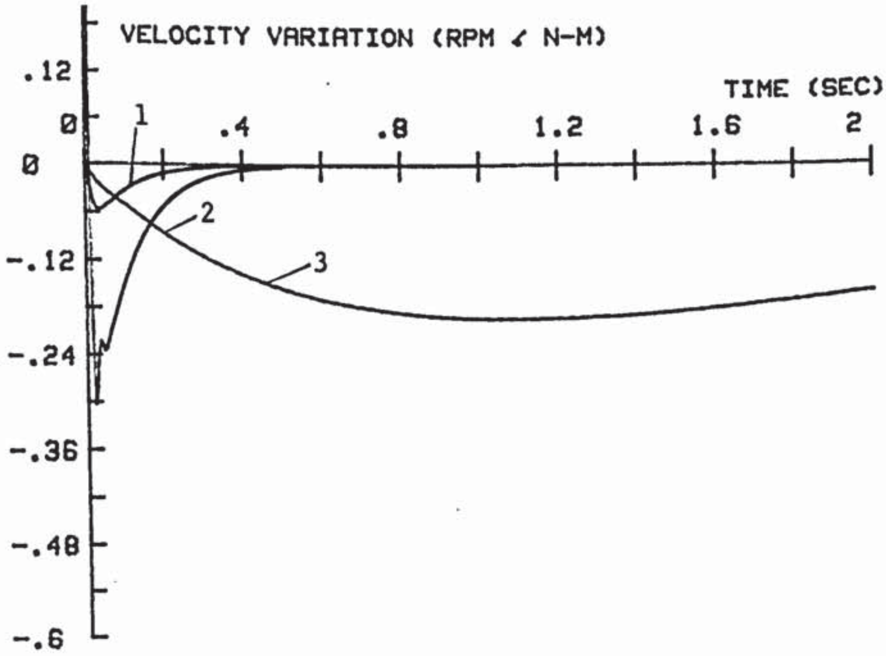


Fig. 4.65. Effect of external torque on the above system.

1) No load inertia.
2) Load inertia of 0.1 kg.m² (230% of rotor inertia).
3) Load inertia of 1 kg.m² (2300% of rotor inertia).

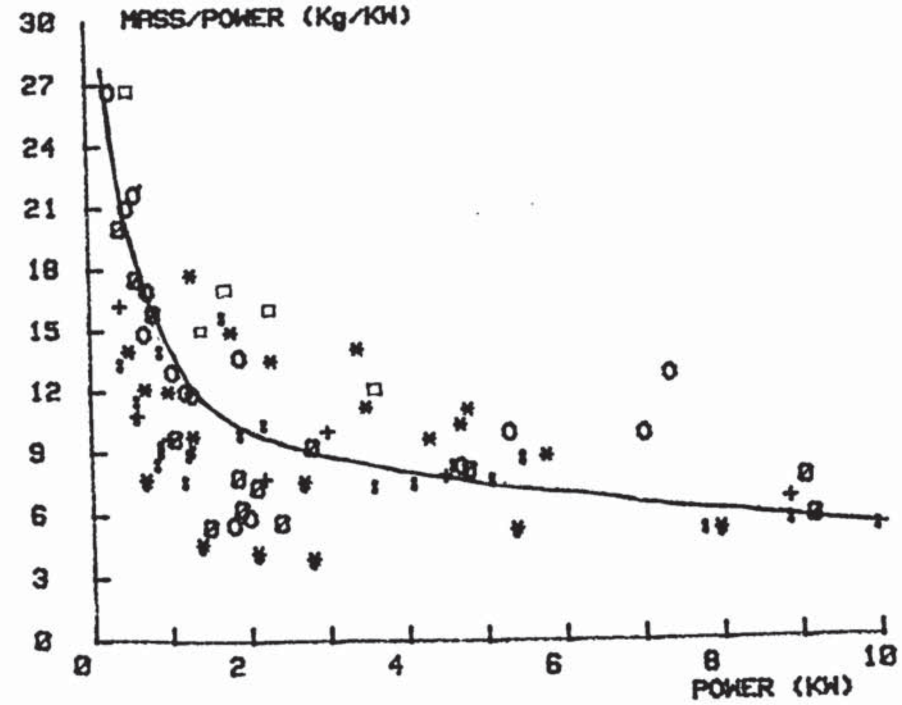


Fig. 4.66. Mass/power comparison of different types of D.C. motors.
* ceramic permanent magnet D.C. motors (less than 2000 r.p.m.)
: ceramic permanent magnet D.C. motors (higher than 1500 r.p.m.)
0 rare earth magnet D.C. motors (less than 1500 r.p.m.)
θ rare earth magnet D.C. motors (higher than 1500 r.p.m.)
+ disc type permanent magnet D.C. motor
□ brushless D.C. motor

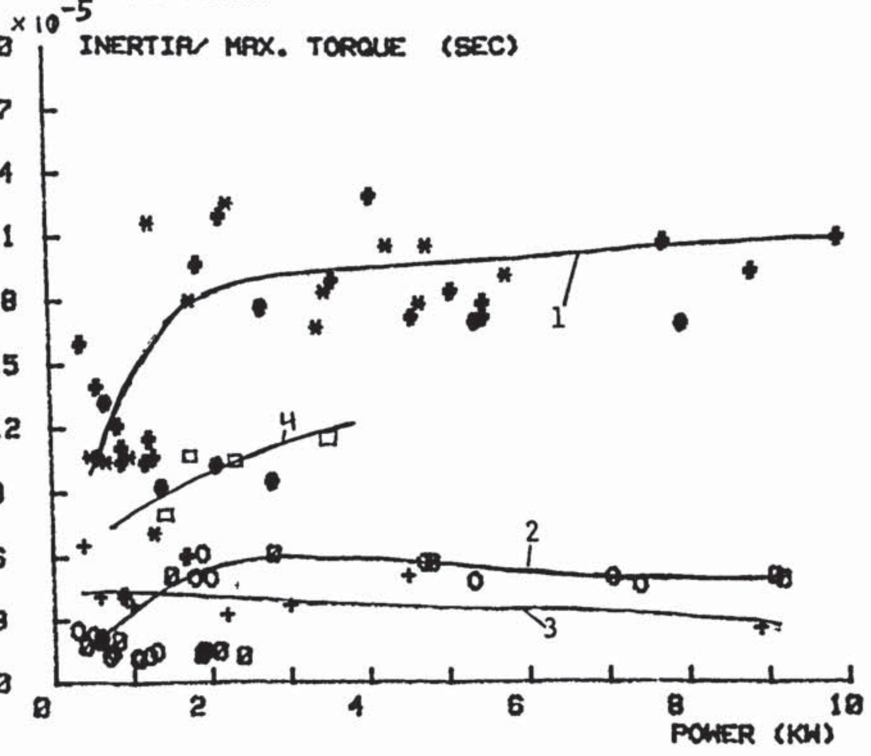


Fig. 4.67 Response time of different types of D.C. motors for a unit change in velocity at maximum torque.

1) Ceramic magnet
2) rare earth magnet
3) disc type rotor
4) brushless D.C. motor

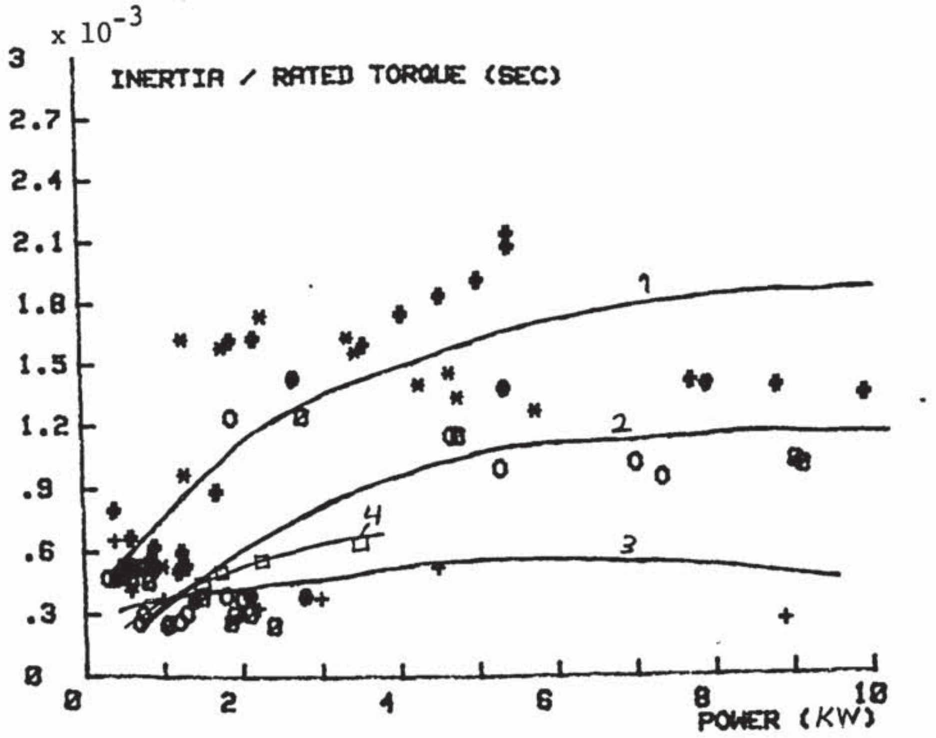


Fig. 4.68 Response time of different types of D.C. motors for a unit change in velocity at rated torque.

1) Ceramic magnet type
2) Rare earth magnet type
3) Disc type motor
4) Brushless type motor

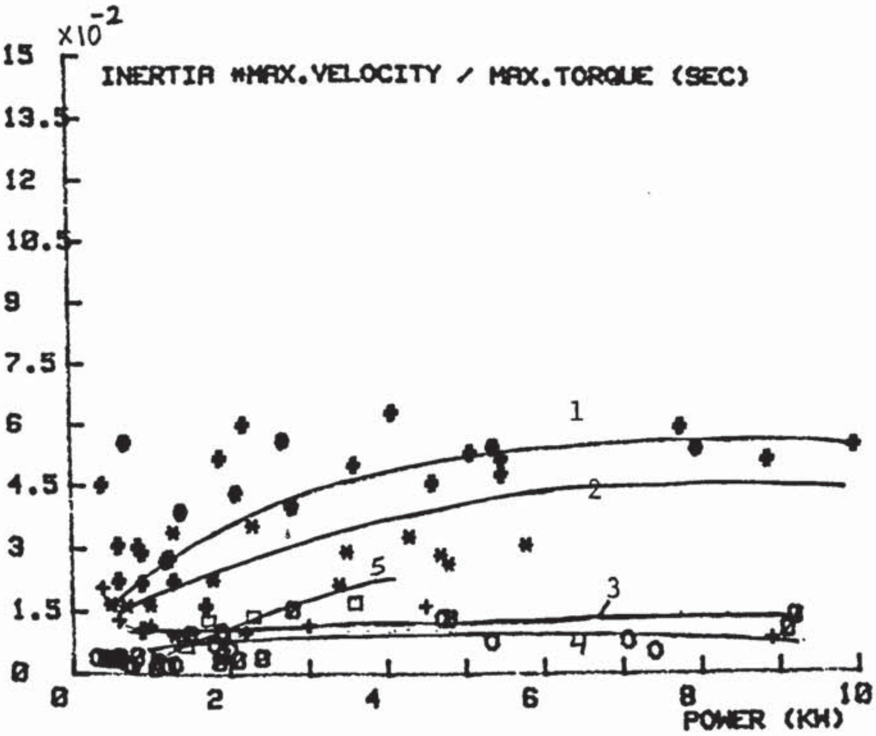


Fig. 4.69 Response time of different types of D.C. motors for maximum change in velocity at maximum torque.

1) Ceramic type motor (greater than 1500 r.p.m.)
2) Ceramic type motor (less, than 1500 r.p.m.)
3) Rare earth magnet type
4) Disc type motor
5) Brushless type motor

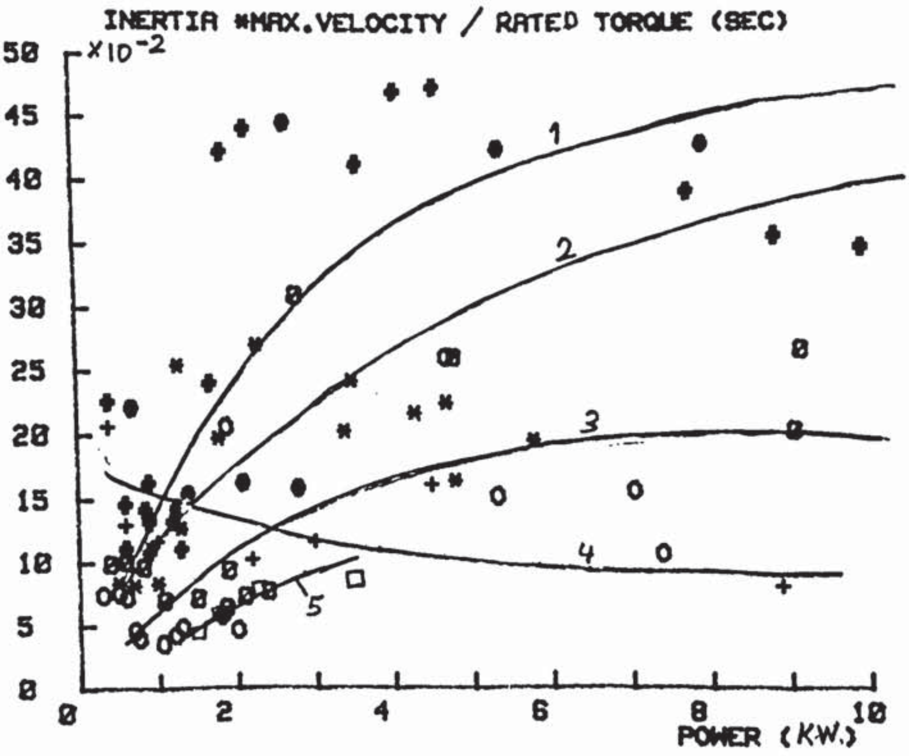


Fig. 4.70 Response time of different types of D.C. motors for maximum change in velocity at rated torque.

Table 1. The following error of the 5 kW D.C. servo motor (Thyristor controlled with current frequency of 50 Hz).

Load inertia $kg.m^2$	Following error Deg/r.p.m.
0	0.43
0.1	1.73
1	10.8

Table 2. The following error of the 5 kW D.C. servo motor (Thyristor controlled with current frequency of 150 Hz).

Load inertia $kg.m^2$	Following error Deg/r.p.m.
0	0.39
0.1	1.23
1	7.2

Table 3. The following error of the 5 kW D.C. servo motor (PWM drive with current frequency of 2 kHz).

Load inertia $kg.m^2$	Following error Deg/r.p.m.
0	0.14
0.1	0.28
1	1.9

CHAPTER 5

THE PERFORMANCE OF THE ELECTRICAL A.C. SERVO MOTOR DRIVE SYSTEM FOR VELOCITY AND POSITION CONTROL.

5. PERFORMANCE OF ELECTRICAL A.C. SERVO MOTORS

5.1 Introduction

Thyristor drives have been used extensively in industry. For normal industrial applications, a D.C. motor powered by a thyristor convertor is now a popular choice as a variable-speed drive. The performance of thyristor controlled and pulse width modulated D.C. servo motor drives were established in previous chapters. This chapter is concerned with solid-state variable speed drives using A.C. rather than D.C. motors. The static variable frequency A.C. drive uses a cage-rotor induction motor or synchronous reluctance motor powered by a static frequency convertor. This gives a versatile and robust variable speed machine which has an advantage over conventional variable-speed drives, of better reliability and reduced maintenance. The main objection to the static A.C. drive has been on economic grounds and its lower starting torque compared with D.C. servo motors. However, power semi-conductor prices are steadily decreasing as production volume grows and manufacturing techniques improve.

The replacement of the D.C. machine by an A.C. motor also has economic benefits. Increasing labour and material costs are weakening the position of the D.C. motor with its elaborate commutator construction. Because of these factors, and a growing awareness of the performance possibilities, a more widespread application of the solid-state A.C. drive is inevitable. In this chapter the basic characteristics of A.C. motors are discussed. The basic principle of the operation of static variable-frequency A.C. drives is investigated. The performance of an A.C. servo motor in velocity control, is first studied in open loop velocity control. The theoretical results are substantiated with experimental tests. The performance of four different sizes of A.C. servo motors in velocity closed-loop are established (1, 3, 5 and 10 kW). The performance of the 5 kW A.C. servo motor is further studied for position control. The use of current and acceleration feedback for improving the performance are analysed. Finally the static characteristics of different sizes of A.C. servo motor are given.

5.2 Characteristics of A.C. Servo Motors

5.2.1 Introduction

There are many different types of A.C. motors, as previously discussed in Chapter 2. Squirrel cage induction A.C. motors are the most economic and are also available in most standard sizes on the market. They have been attractive to the designer of servo motor drive systems. In this section only the characteristics of these types of motors are studied. Usually other types of A.C. motors have similar characteristics.

The induction A.C. motors may be subdivided into single and polyphase types. The single phase induction motors are commonly used at low power, usually less than 5 kW. They operate at low power-factor and are relatively inefficient when compared with polyphase motors. Though simplicity might be expected in view of the two-line supply, the analysis is quite complicated [32] and will not be given in detail here. The dynamic equations of polyphase induction motors will be analysed in this section, since they are the most attractive for high performance servo drive applications.

5.2.2 Mathematical model of A.C. servo motors

The induction motor has a symmetrical polyphase stator winding which is connected to a balanced polyphase A.C. supply. The wound-rotor machine also has a polyphase winding on the rotor, and the terminals are connected to slip-rings which are usually short-circuited. In a squirrel-cage motor, the individual rotor conductors are short-circuited by low-resistance end-rings. Thus, the rotor winding of the induction motor has no external connection, and receives its supply by induction from the stator. The stator currents establish a rotating air-gap m.m.f. wave, and this produces a flux wave of constant amplitude which rotates at uniform speed, moving through a double pole pitch in each cycle of the supply frequency. The speed of rotation of the field is therefore:

$$\omega_f = \frac{2\pi f}{p} \tag{5.1}$$

where,

ω_f is the speed of rotation of flux (rad/sec)

f is the supplied frequency (Hz)

p is the number of stator pole-pairs

ω_f is the synchronous velocity of the induction motor.

For normal operations, the supply voltage, and frequency are both constant. When the motor is at rest, the synchronously rotating field induces e.m.f.s at supply frequency in the stationary rotor. The rotor conductors are short circuited at each end and, if the circuit is purely resistive, the current and e.m.f. distributions are identical. The torque contribution of each rotor conductor is proportional to the product of the conductor current and the local flux density. This is the optimum condition for the production of electromagnetic torque. If the rotor circuit is purely inductive, the resultant torque is zero. In practice, the rotor power factor is always greater than zero, and hence a motor torque is developed which causes the machine to run-up from the standstill in the same direction as the air-gap field. If the motor is running at a speed n less than the synchronous speed n_1 the difference, (n_1-n), is called the slip speed of the machine and is usually expressed as a fraction of the synchronous speed to give the fractional slip s_ℓ as:

$$s_\ell = \frac{n_1-n}{n_1} \tag{5.2}$$

For normal loading conditions it can be fairly accurately assumed that the air-gap flux is constant [20]. The magnitude of the rotor e.m.f. is then proportional to the fractional slip s_ℓ. If the rotor runs in exact synchronism with the air-gap field, the induced e.m.f. is zero and the rotor develops zero torque. In practice, even on no-load, a small motor torque is required to overcome windage and friction, and the motor runs at a speed which is slightly less than the synchronous speed. When load is applied to the motor shaft, the speed drops further below synchronism and larger rotor currents are induced. When a rotor current flows, an additional component of stator current is induced. As in a transformer, this load component of stator current neutralizes the rotor m.m.f. and leaves the resultant air-gap flux practically unaltered, as previously assumed. With the above assumption the torque developed by the motor can be written as [20].

$$T_m = K\varphi I_2\cos\varphi_2 \tag{5.3}$$

where

K is a constant

I_2 is the r.m.s. current in a rotor conductor

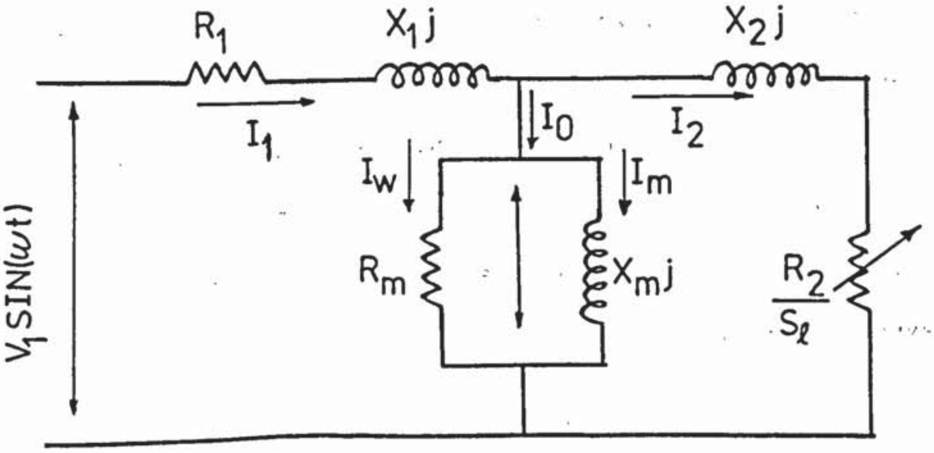


Fig. 5.1 Equivalent circuit diagram of an A.C. motor.

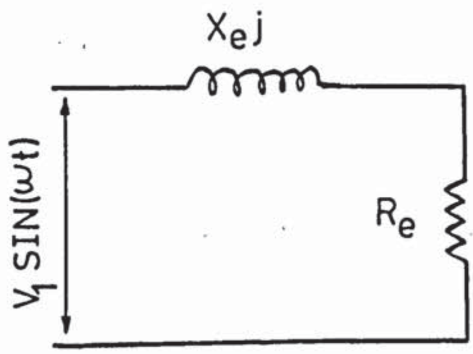


Fig. 5.2 Simplified circuit diagram of an A.C. motor. The values of inductance and resistance change as the state of the motor changes.

φ_2 is the rotor phase lag

$I_2\cos\varphi_2$ is the in-phase rotor current

φ is the air-gap flux.

In order to obtain the voltage equation of an A.C. induction motor the complete characteristics of stator and rotor must be known. To simplify the analysis, the equivalent circuit diagram [20] of an A.C. motor for one of the supply phases is shown in Fig. 5.1, where:

R_1 is the stator resistance (ohms)

$x_1 = L_1\omega$ is the stator reactance (ohms)

R_2 is the rotor resistance (ohms)

$x_2 = L_2\omega$ is the rotor reactance (ohms)

R_m is the magnetizing resistance (ohms)

$x_m = L_m\omega$ is the magnetizing reactance (ohms)

The R_2/s_ℓ resistance provide the equivalent back e.m.f. of the motor. The equivalent circuit may be simplified as shown in Fig. 5.2., where

$$Re = \frac{\dfrac{R_2/s_\ell}{(R_2/s_\ell)^2+(x_2)^2} + \dfrac{R_m}{R_m^2+(x_m)^2}}{\left(\dfrac{R_2/s_\ell}{(R_2/s_\ell)^2+(x_2)^2} + \dfrac{R_m}{R_m^2+(x_m)^2}\right)^2 + \left(\dfrac{x_2}{(R_2/s_\ell)^2+(x_2)^2} + \dfrac{x_m}{R_m^2+(x_m)^2}\right)^2} + R_1 \tag{5.4}$$

and

$$x_e = \frac{\dfrac{x_2}{(R_2/s_\ell)^2+x_2^2} + \dfrac{x_m}{R_m^2+x_m^2}}{\left(\dfrac{R_2/s_\ell}{(R_2/s_\ell)^2+x_2^2} + \dfrac{R_m}{R_m^2+x_m^2}\right)^2 + \left(\dfrac{x_2}{(R_2/s_\ell)^2+x_2^2} + \dfrac{x_m}{R_m^2+x_m^2}\right)^2} + x_1 \tag{5.5}$$

It can be seen from equation 5.4 and 5.5 that the effective resistance and inductance of the motor are, function of slip ratio s_ℓ (torque) and the supplied frequency ω. Figs. 5.3 and 5.4 shows these variations

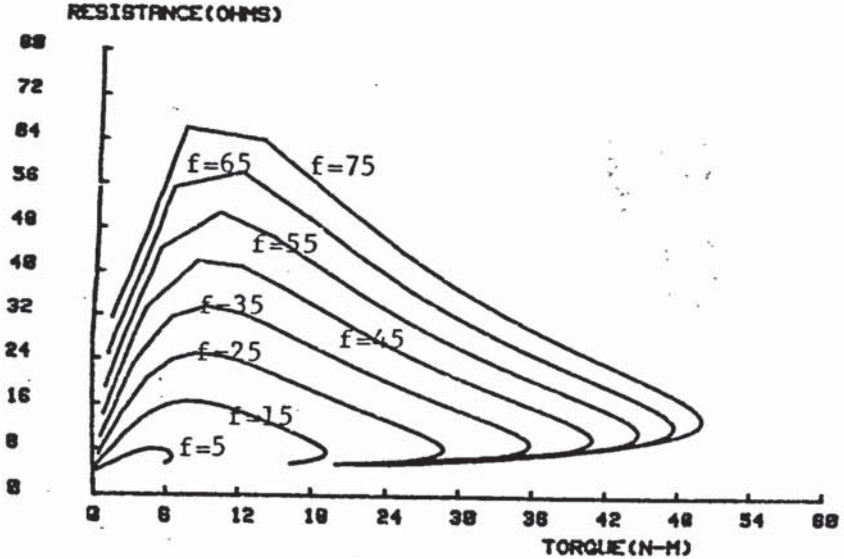


FIG 5.3.EQUIVALANT RESISTANCE OF THE A.C.MOTOR.

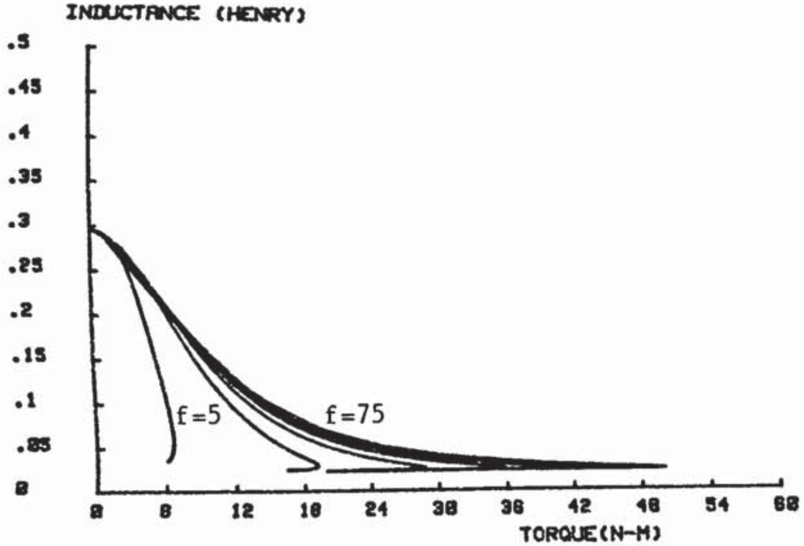


FIG 5.4.EQUIVALANT INDUCTANCE OF THE A.C.MOTOR.

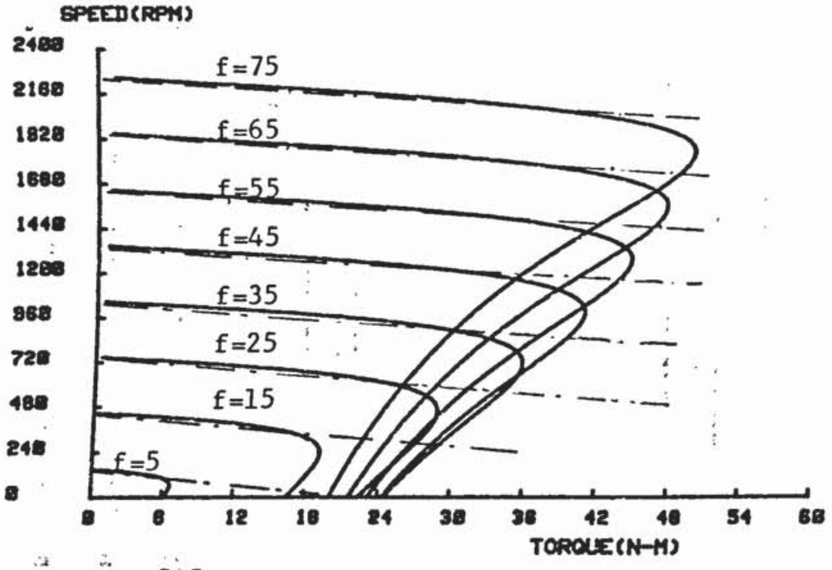


FIG 5.5 . SPEED-TORQUE CHARACTERISTIC OF THE A.C.MOTOR.

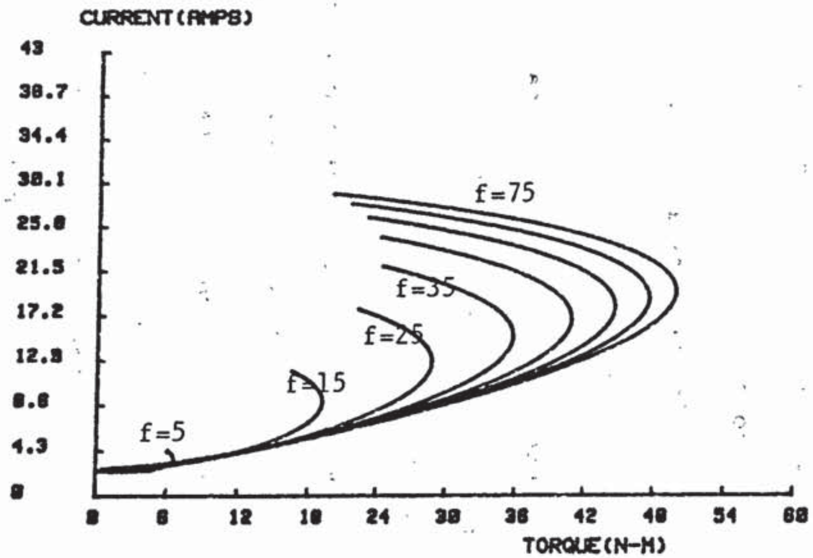


FIG 5.6 .CURRENT-TORQUE CHARACTERISTIC OF THE A.C.MOTOR.

as a function of torque and the supplied frequency for a 2.2 kW 3 phase induction A.C. motor. The effective inductance of the motor reduces as the torque is increased. The decrease in inductance is quicker at low supply frequency. The effective resistance initially increases as the torque increases and reaches a peak value. Further increase in torque results in a reduction in the effective resistance. The speed-torque characteristic of the A.C. motor is shown in Fig. 5.5. It can be seen that the speed-torque characteristic of the motor can be assumed linear over a wide range of speed and torque, as shown with a dotted line on this graph. The extent of the linearity of the speed-torque characteristic depends on the operating point of the motor. The speed drop may be assumed to be due to the RI voltage drop of the motor which is similar to that of D.C. motors. With the previous assumption, the voltage equation of the A.C. motor with constant air-gap flux may be written as:

$$V = RI + L\frac{dI}{dt} + c_m \omega_m \qquad (5.6)$$

In equation 5.6 the resistance R and inductance L are non-linear and vary according to the operating point of the motor as shown before. V is the supplied voltage to the motor with constant air gap flux. Equation 5.6 is valid only over small variations in the state of the A.C. motor. For stability and accuracy analysis the average values of R and L will be considered. The performance of the system at different operating points will be further studied at the end of this chapter. In order to establish a linear torque equation of the motor, the torque-current characteristic of the motor is obtained first. Fig. 5.6 shows the torque-current characteristic of a 2.2 kW A.C. motor at different supply frequencies. It can be seen from this graph that the current-torque characteristic is non-linear. A linear assumption is made here to establish the torque equation of the motor, with the aid of a small perturbation technique. With this assumption the torque equation can be written as:

$$T = K_t I \tag{5.7}$$

The constants K_t vary according to the state of the A.C. motor. In the transient condition, however, it covers the whole range of operation. The maximum values of K_t will be considered for numerical analysis. The effect of the variation of K_t (as the state of the motor changes) on the stability and accuracy of the system will be studied later.

With the above assumption and linearization techniques, the mathematical model obtained in Chapter 3 may be used for the prediction of the performance of A.C. motors.

5.3 Characteristics of a Static Frequency Convertor

5.3.1 Introduction

The idea of using a variable frequency supply to control the speed of A.C. motors is not new and rotating frequency convertors have been employed for many years. These are used principally in multi-motor mill drives and in special applications where a high operating frequency is chosen in order to permit the use of compact A.C. motors. Nowadays the rotating machine methods of variable-frequency A.C. power generations are being largely supplanted by static conversion methods.

The performance and reliability of a variable-speed A.C. system is improved if the rotating frequency convertor is replaced by static methods of variable-frequency power generation [30]. The renewed interest in static frequency conversion is due to the improved characteristics of the thyristor and transistors as compared with other switching devices. The thyristor is a more efficient switching device since the voltage drop in the on condition is only about 1 V [30].

There are two basic types of static frequency convertor, the D.C. link convertor and the cycloconvertor. In the D.C. link converter, the three phase A.C. supply is first rectified to D.C. in a standard rectifier and the resulting D.C. power is fed to a static inverter. An inverter is a device which converts D.C. power to A.C. power, and the static inverter uses thyristor (or transistors) which are switched sequentially so that an alternating voltage wave form is delivered to the A.C. motor. The output frequency is determined by the rate at which the inverter thyristors are triggered into conduction.

The second basic form of static frequency converter is the cycloconverter and this has also been used in variable-speed A.C. drives [33]. In a cycloconverter, the network frequency is converted directly to a lower output frequency without intermediate rectification. Since the output frequency is less than one third of the supply frequency, the drive is therefore suitable for low speed motor operation when the input is at normal mains frequency.

5.3.2 Characteristics of the static frequency converter.

A static converter which delivers variable-frequency power to a motor or transformer must also vary the terminal voltage as a function of frequency in order to maintain the proper magnetic conditions in

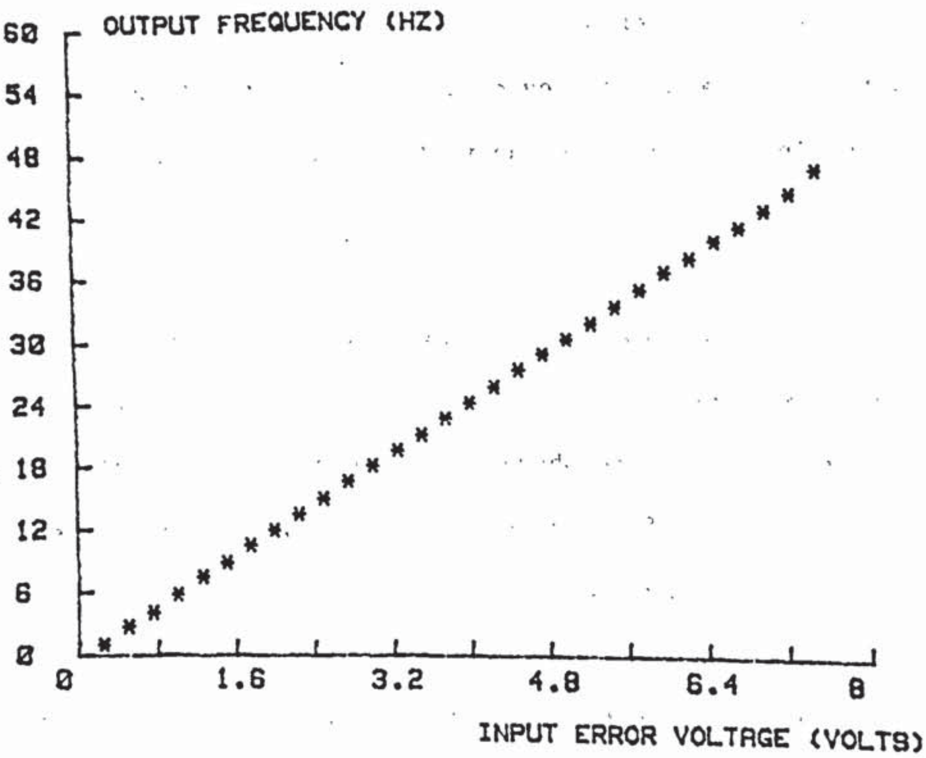


Fig. 5.7 The static characteristics of a three phase frequency converter.

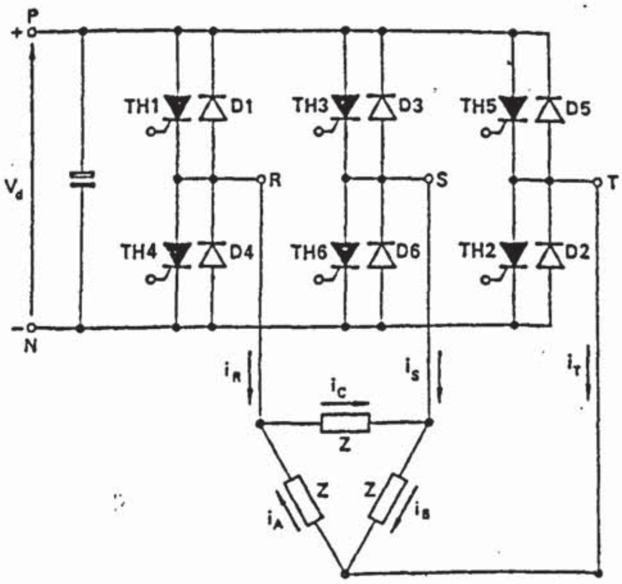


Fig. 5.8 The basic structure of a three phase bridge inverter circuit with a delta connected load.

the core. In practice, magnetic devices usually operate near saturation point in order to give maximum utilization of the core material. When the operating frequency is reduced, the applied voltage must be reduced proportionately or the saturation flux density is exceeded, resulting in excessive iron losses and a magnetizing current. Most of the static converters maintain constant air-gap flux. To achieve this the voltage/frequency ratios are held constant. This mode of operation is known as constant "volts/cycle". The above constant air-gap flux analysis assumes negligible winding resistance, whereas, in practice, at low frequencies the resistive potential difference becomes significant compared with the induced e.m.f. This causes a reduction in the air-gap flux and motor torque. In order to maintain the low-speed torque, the volts/Hz must be increased at frequencies below 20 Hz [25].

Ideally, the static frequency converter produces an output frequency of ω_s proportional to an input error voltage v_c with constant air-gap flux. The dynamic equation of a converter may than be written as:

$$\omega_s = K_g v_c \tag{5.8}$$

This assumption was confirmed by an experimental test on a 3 kW frequency converter, as shown in Fig. 5.7. With the above assumption the mathematical model obtained in Chapter 3 (by an analogy of output frequency (ω_s) with the average output voltage of thyristor controlled D.C. motor (v)) will be used to predict the behaviour of an A.C. servo motor driven by a static frequency converter.

5.3.3 The principle of operation of a frequency converter.

In this section the principle of operation of the D.C. link converter will be discussed. The principle of operation of the cycloconverter are given in references 30-33. There are many techniques to convert the D.C. voltage to variable frequency and variable voltage. Each has some advantages and disadvantages [24-29]. However, the basic operation of all techniques are the same. The static inverter must generate a three phase output of variable frequency and variable voltage. The simplest three phase inverter is shown in Fig. 5.8, in which the gating and commutating circuits have been omitted for clarity. The operation of the gating and commutating circuits and their types are given in references 24-29. The basic inverter operation is independent of the commutation method used.

In general, the output voltage wave form of a stepped-wave inverter consists of a series of steps, and each change in voltage level corresponds to a switching action in the inverter circuit. The number of steps on each cycle of output voltage depends on the design of the inverter. The output voltage, being composed of a number of steps, contains higher harmonic frequencies. Since harmonic effects may be undesirable, it is important to generate a wave form with a low harmonic content. By using more complex circuits with larger numbers of thyristors, the number of steps in the voltage waveform can be increased to give a closer approximation to the ideal sinusoidal waveshape.

The circuits of Fig. 5.8 give a six step voltage at each cycle. A three phase output is obtained by preserving a phase displacement

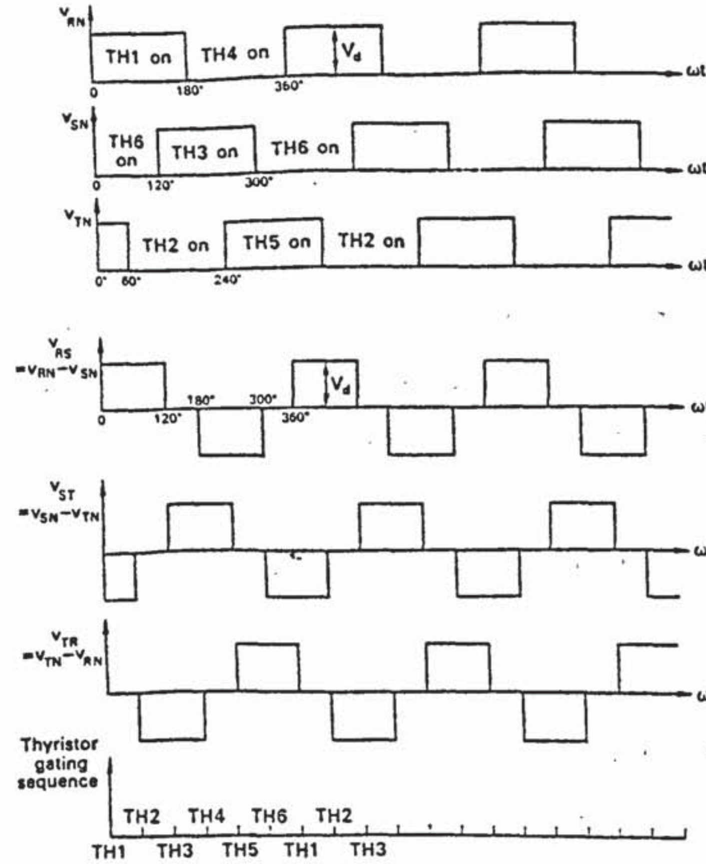


Fig. 5.9 Voltage waveforms and gating sequence for six step operations of the three-phase bridge inverter.

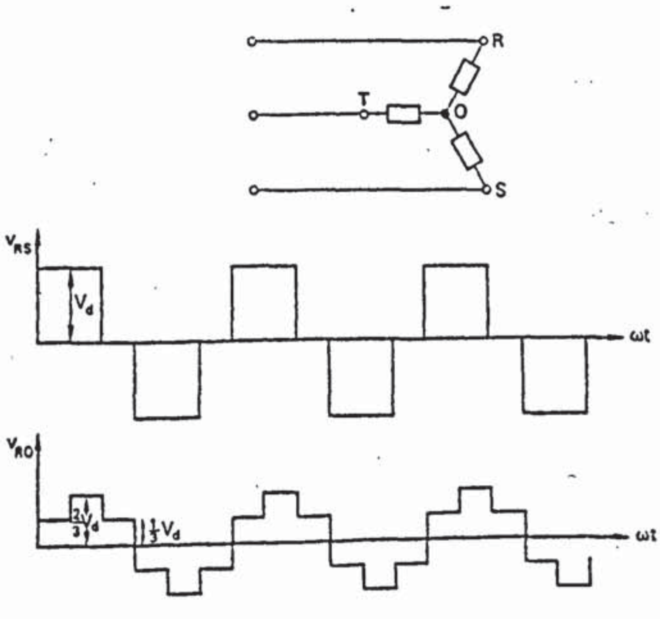


Fig. 5.10 Six step line-to-line and line to neutral voltages for a star connected load.

of 120° between the gating pulses delivered to each leg of the inverter. In Fig. 5.8 the thyristors are numbered in their correct firing sequence. This means that they must be gated at uniform intervals in the sequence TH1, TH2, TH3, TH4, TH5, TH6 in order to complete one cycle of output voltage. The output frequency of the inverter is determined by the gating frequency of thyristors. Fig. 5.9 shows the voltage waveforms and gating sequence for a six step operation of the three-phase bridge inverter, with reference to the negative line N. Taking the negative D.C. line as reference point N, and assuming instantaneous commutation, the terminal voltage V_{RN}, V_{SN} and V_{TN} have the square waveforms shown in Fig. 5.9. V_{RN} equals the supply voltage V_d when TH1 conducts, and is zero when TH4 conducts. The feedback diodes clamp the output voltage to the D.C. supply voltage and so maintains a flat-top square-wave output voltage on reactive loads.

The line to line voltages are obtained by subtraction. Therefore,

$$\begin{aligned} V_{RS} &= V_{RN} - V_{SN} \\ V_{ST} &= V_{SN} - V_{TN} \\ V_{TR} &= V_{TN} - V_{RN} \end{aligned} \tag{5.9}$$

Since the line voltage is obtained as the difference of the two square waves, it contains no third harmonic or multiples thereof, because these are zero-sequence harmonic and, therefore, are in phase in each of the three square wave voltages, V_{RN}, V_{SN} and V_{TN}.

The line voltage waveforms are also shown in Fig. 5.9, and are described as "stepped waves", or "quasi-square waves", the Fourier analysis gives the expression

$$V_{RS} = \frac{2\sqrt{3}}{\pi} V_d \ \{\sin\omega t - \frac{1}{5}\sin 5\omega t - \frac{1}{7}\sin 7\omega t + \frac{1}{11}\sin 11\omega t \ldots\} \qquad (5.10)$$

The r.m.s. value is 0.816 V_d and the fundamental component has an r.m.s. value of 0.78 V_d.

In the case of a star or wye-connected load, the line-to-neutral voltage has the waveform shown in Fig. 5.10 with steps of $V_d/3$ and $2V_d/3$. The harmonic content of the line and phase voltage is the same, and the difference in waveshape is due to a different phase relationship between the fundamental and harmonics. The complete expression for the phase voltage is

$$V_{RD} = \frac{3}{\pi} V_d \ \{\sin\omega t + \frac{1}{5}\sin\omega t + \frac{1}{7}\sin 7\omega t + \ldots\} \qquad (5.11)$$

These line and phase voltage waveforms are described as six-step waves since they are generated by an inverter with six uniformly spaced commutations per cycle. The lowest harmonic present is the fifth harmonic with an amplitude equal to 20 percent. of the fundamental. In practice, the six-step waveform has been used successfully in many adjustable speed A.C. drives. The elimination of the zero-sequence harmonic is advantageous as these can be seriously detrimental to motor performance. The actual voltage waveform may differ somewhat from the ideal waveshape due to commutation effects and internal voltage drops in the inverter circuit.

The current waveforms in the inverter due to the existence of inductance is different to the voltage waveforms.

5.4 The Performance of an A.C. Servo Motor

5.4.1 Experimental and theoretical comparisons of the performance of a 2.2 kW A.C. servo motor.

The A.C. motor under test was a standard three phase squirrel cage induction motor. The variable frequency inverter was a transistor 6-steps, three phase supply voltage [70]. The specifications of the motor and the inverter are given in reference [70]. The static speed-torque characteristic of the motor at different supply frequencies was tested, and the results were similar to the theoretical speed-torque characteristics of the motor as seen in Fig. 5.5. In this section the dynamic performance of the A.C. motor is compared with the theoretical results with the aid of the mathematical model obtained in Chapter 3. The non-linearity of the motor must be taken into account. The static variable frequency was designed to operate in open-loop velocity control. Hence, the comparison is given only for velocity control in open loop. The behaviour of the system in velocity and position closed-loop will be analysed theoretically for different sizes of A.C. servo motors. The average values of inductance will be considered. Since the system will operate over the whole range of the torque (in a transient condition) the assumption of average value of inductance is acceptable for dynamic analysis. The numerical values of the parameters of the mathematical model of A.C. servo motors under test and the source of these values are given below.

$K_2 = 0$	No feedforward integrator in the inverter
$K_3 = 1$	From a circuit diagram of the inverter
$K_a = 0$	No acceleration feedback
$K_4 = 1$	From a circuit diagram of the inverter
$\tau_2 = 0$	Adjustable time constant.
$\tau_3 = 0.4$ sec.	
$K_c = 0$	No current feedback in inverter.
$K_g = 6.5$	Overall gain of the inverter as was shown in Fig. 5.7.

$R = 4$	Starting point resistance, as was shown in Fig. 5.1.
$L = 0.15$	Average value of the inductance of the motor.
$C_m = 0.34$	Hz/(Rad/sec), from test on the motor.
$K_t = 3.33$	N.m/amp (from Fig. 5.6).
$I_m = 0.55 \times 10^{-3}$	From manufacturer's data
$N = 1$	No gear box.
$K_s = 100000$	
$I_L = 0.0216$	From the design of the loading mechanism.
$C_L = 0.1$	

Fig. 5.11 compares the experimental and theoretical results for a step input of velocity. As can be seen, there is a close degree of agreement between the two sets of results. The discrepancies arise because of the non-linearity of the dynamic characteristics of the A.C. motor, which was discussed earlier. The effect of a step input of external torque both experimentally and theoretically is compared in Fig. 5.12. When the step input of the torque is small and is within the range of the rated torque of the motor there is close agreement between the two results. The above tests confirm that the mathematical model modified for the A.C. servo motor can be used to predict the behaviour of the system for different sizes of A.C. servo motors both in closed or open-loop. For large step inputs, the current overload may be prevented by using a current limiter in the inverter. In most applications especially in the smaller size range of A.C. motors, this is not necessary, since the resistance of an A.C. motor is much higher than the corresponding D.C. motor. In the larger size range a current limiter may be used to slow down the current rise during transient conditions.

5.4.2 Performance of the different sizes of A.C. servo motors in velocity control.

5.4.2.1 Introduction.

The performances of four standard sizes [73] of A.C. servo motors (1, 3, 5, 10 kW) are investigated theoretically in this section. The analysis is based on the assumption that the system is in velocity closed loop. In addition current feedback and lead-lag network are available to compensate the system. The parameters of the compensation network are optimized to obtain the best performance of each of the systems. The characteristics of each of the A.C. servo motors are investigated at various load inertia with fixed load natural frequency. The effects of the natural frequency of the load at fixed load inertia is also studied. Having established the performance of each system in velocity control, the performance of the 5 kW A.C. motor is investigated in position control mode. The use of acceleration feedback as compensation is investigated for the 5 kW A.C. servo motor. Finally the effect of non-linearity of the A.C. motor is investigated for the 5 kW A.C. servo motor.

5.4.2.2 The Effect of Load Inertia on the Performance of the System.

The effect of load inertia on the performance of the A.C. servo motors is similar to that of D.C. servo motors. Figs. 5.13, 5.14, 5.15 and 5.16 show the performance characteristics of four different sizes of A.C. motors for a step input of velocity at various load inertia. For each case the parameters of the compensator are modified to achieve a damping ratio of approximately 0.6 of the fundamental frequency of the system. The compensators used are lead-lag network and current feedback. It was noticed that for A.C. motors a lead network with a combination of current feedback provides damping in the fundamental frequency of the system and consequently a higher value of gain can be used. It can be seen that for each size of A.C. motor the load inertia reduces the speed of response of the system. As the size of the motor increases, a further reduction in the speed of response occurs due to the increase in the inertia of the motor. The capability of the motor being overloaded for a short period of time has reduced the effect of load inertia on the dynamic performance as shown in the previous graphs. The effect of torque on the performance of the different size motors at a different load inertia for a unit step input of torque, is shown in Figs. 5.17, 5.18, 5.19 and 5.20. It can be seen that for a particular size of motor, as the load inertia is increased the dynamic velocity drop reduces, but the recovery time increases. As the size of the motor increases the velocity drop per unit torque reduces, effectively giving a constant velocity drop at rated torque (the rated torque increases as the sizes of the motor increase).

5.4.2.3 The Effects of Load Natural Frequency on the Performance.

Fig. 5.21 shows the performance of the 5 kW A.C. motor at various natural frequencies of load. At low load natural frequency the damping ratio of the fundamental frequency reduces with an increased damping in the load frequency. As the load natural frequency increases, the damping ratio of the loop frequency increases. The oscillation of the load natural frequency begins to appear on the response. With a further increase in the load natural frequency, the effect is only a small amplitude oscillation on the response. In this analysis the load inertia was chosen to 50% of rotor inertia, since the oscillation is more dominant at the lower inertia side of the load mechanism.

In this test all the other parameters of the system are kept constant. By changing the parameters of the amplifiers, however, the performance can be improved.

5.4.3 The Performance of the System in Position Control.

Having established the performance of the system in the velocity control mode, the trend of the characteristics of the system in position control mode follows its performance in velocity control mode. In this section the performance of only the 5 kW A.C. servo motor in position control is studied. Similar results may be obtained for other sizes of A.C. motors. The performance of the 5 kW A.C. motor is studied at different load inertia. The position amplifier gain is adjusted to prevent the system having an overshoot of position. Fig. 5.22 shows the response of the system for a step input of position. As can be seen from this graph, the speed of response reduces as the load inertia increases. The speed of response is also slower than the corresponding performance in velocity control mode. Fig. 5.23 shows the effect of external torque on the position accuracy. Initially, the position error increases as the torque is applied. The error is recovered eventually due to the feedforward integrator in the system. The value of dynamic position error depends on the gain of the position amplifier.

It can be seen from this graph that as the load inertia is increased so is the dynamic position error. This is due to the lower position amplifier gain at higher load inertia for stability purposes.

The speed of recovery is also reduced as the load inertia is increased. This is due to the lower loop frequency of the system at velocity control mode with higher load inertia. Further increase in the gain of the position amplifier results in causing the system to oscillate with the fundamental frequency of the system in velocity control mode. If the system is overdamped in velocity control, then the increase in the gain of position amplifier causes the system to oscillate at lower frequency in position control mode. In this case, in order to prevent the system having overshoot in position control, a much lower position amplifier gain is required. The following error of the system for a ramp input is shown in Table 5.1 at different load inertia. It can be seen that as the load inertia is increased so is the following error.

5.4.4 Performance of the System Using Acceleration Feedback

It was shown in the previous chapter that the acceleration feedback may be used as an active compensation to improve the performance of the system. In this section the effect of acceleration feedback on the performance of an A.C. servo motor is investigated. By using acceleration feedback, the damping of the system is increased and consequently the gain of the loop can be increased. The performance of the 5 kW motor is analysed here, and similar results can also be obtained for other sizes of servo motor. The gain of the loop is increased 10 times, thus by using acceleration feedback the performance is compensated. Fig. 5.24 shows the performance of the 5 kW A.C. servo motor for a step input of velocity at different load inertia. It can be seen that, the performance of this system is improved considerably with the aid of acceleration feedback as compensation. Fig. 5.25 shows the effect of torque on this system. The dynamic velocity variation of the system for external torque is reduced almost 10 times compared with the system that does not use acceleration feedback.

5.4.5 The Effect of Non-Linearity of an A.C. Motor on Performance.

As was discussed earlier, the dynamic equations of A.C. motors are non-linear. These non-linearities have some effect on the performance of the system. In the dynamic simulation, the non-linearities appear as variation in the equivalent resistance and inductance of the motor, as the state of operation of the A.C. motor changes. These non-linearities (variation in the equivalent resistance and inductance) were shown in Figs. 5.3 and 5.4. In this section the performance of the 5 kW A.C. motor at four different operation conditions are investigated. Figs. 5.26 and 5.27 show the performance of the A.C. motor for step input of velocity and torque at the following steady state conditions.

1. No load and low velocity. In this state the inductance of the motor is high and the resistance is the lowest value. These variations both result in a lower damping of the system. The reduction into the resistance increase the stiffness of the system for the external torque, but because of the low damping the dynamic velocity drop is higher than in other cases.
2. No load and high velocity. In this condition the inductance of the motor is still high but the value of the resistance is increased. This causes an increase in the damping ratio. It reduces the stiffness of the system for the external torque because of the increase in the resistance.
3. 100% of rated torque and low velocity. In this condition the resistance is increased but the inductance is reduced considerably.

The increase in the resistance causes more damping in the system, but the reduction in the inductance has an opposite effect.

4. 100% of the rated torque and high velocity. This causes a slight increase in inductance and resistance which both have a contradictory effect on the performance.

By the above analysis it is evident that the damping ratio of these kinds of servo motors is low at low velocity and low torque. However, this is not a drawback because the parameters of the compensation network can be adjusted to increase the damping in these conditions, which in turn, cause a higher damping ratio in other conditions. In the dynamic analysis by separating the variation of resistance due to the back e.m.f. as an internal feedback ($C_m \omega_m$) and taking the average values of the inductance, a good simulation of the system is given. By this technique the dynamic behaviour of the system can be predicted confidently.

The high frequency oscillation on the response of the system, for a step input of torque, is due to the natural frequency of the load. This is more dominant on the lower inertia side of the load mechanism.

5.5 The Static Performance of A.C. Servo Motors

5.5.1 Mass/Power Ratio

Fig. 5.28 shows the mass/power ratio of A.C. induction motors for three different maximum rated velocity capabilities. It can be seen from this graph that this ratio reduces as the power increases. The mass/power ratio reduces as the maximum velocity of the motor increases. This is mainly due to the fact that the amount of wiring of stator reduces for a higher velocity motor.

5.5.2 Inertia/Torque

As was discussed earlier, the ratio shows the minimum response time of the motor for a unit change in velocity at the applied torque. Figs. 5.29 and 5.30 show this ratio for the maximum and rated torque of an A.C. induction motor, at different power ratings and maximum velocity, respectively. The average values of these two ratios gives a more realistic minimum response time of the motor for a unit change of velocity. As can be seen from these two graphs, the ratio increases as the power of the motor increases. As the maximum velocity of the motor increases the ratio reduces slightly and results in a faster response of the motor for the unit change of velocity. This is due to the increase in the inertia of the motor at lower maximum velocity.

5.5.3 Comparison of the Inertia x Maximum Velocity/Torque Ratio of A.C. Motors

This ratio determines the minimum response of the motor for maximum velocity variations at the applied torque. Figs. 5.31 and 5.32 show the ratio of A.C. motors for the maximum and rated torque, for different power rating and maximum velocity, respectively. As can be seen from these graphs, this rate increases as the power rating of the motor increases, thereby resulting in a slower response. This ratio increases as the maximum velocity of the motor increases, which is the opposite effect as was shown in 5.5.2. At low velocity change the response of a motor with a higher maximum velocity is faster than the motor with a lower maximum velocity. For greater velocity change the speed of response is opposite to this.

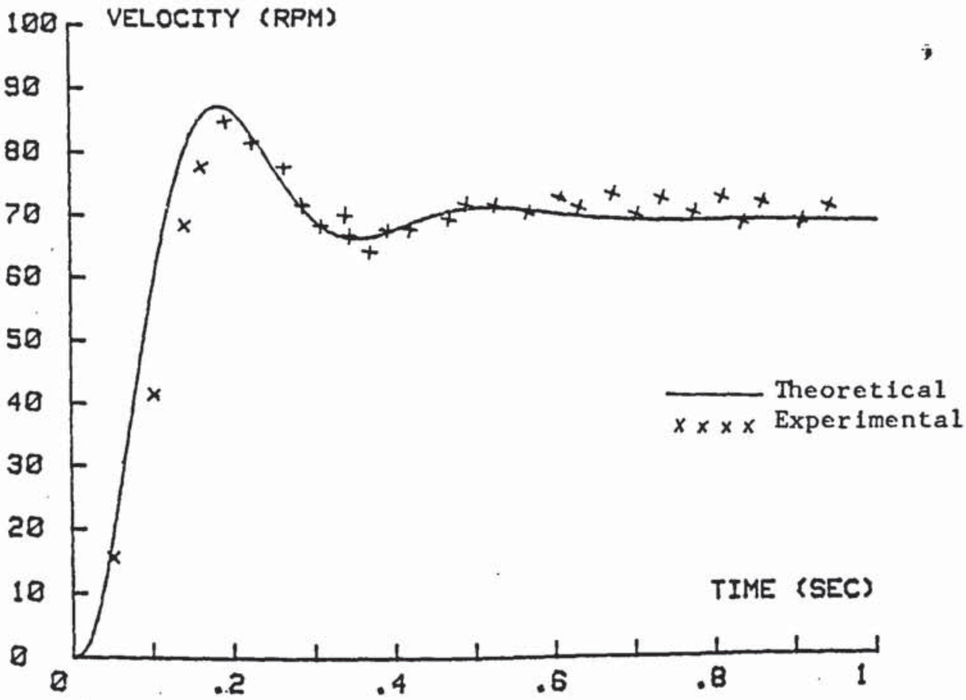


Fig. 5.11 Theoretical and experimental comparison of the A.C. servo motor drive system for a step input of velocity.

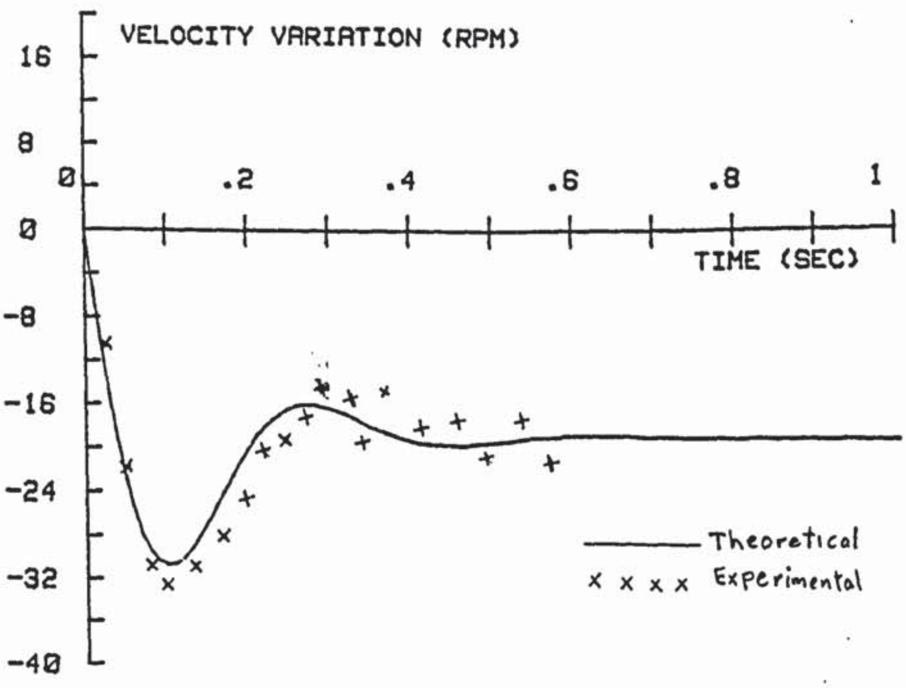


Fig. 5.12 Velocity variation of the A.C. servo motor for a unit step input of torque.

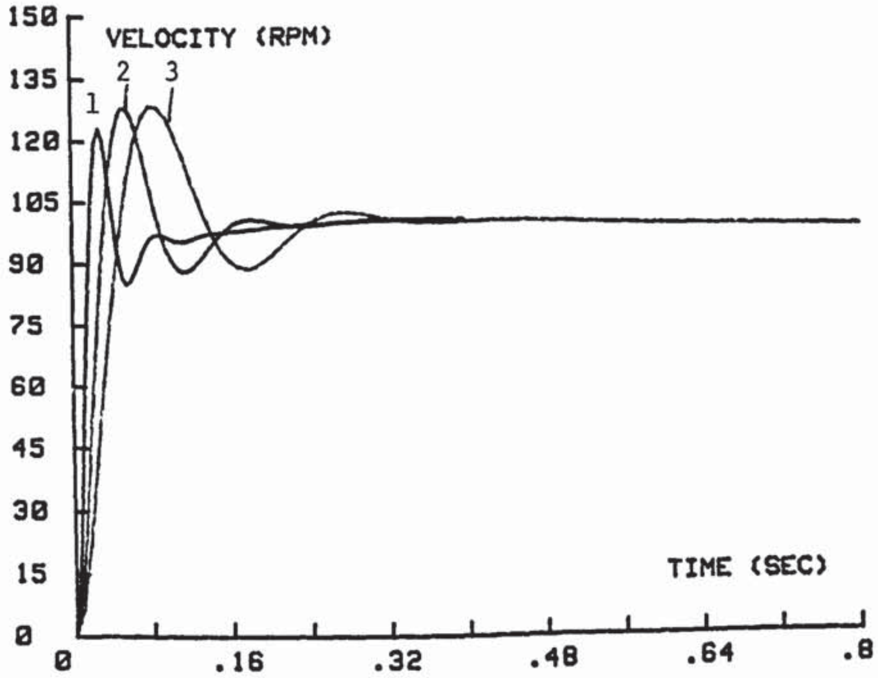


Fig. 5.13 Response of a 1 kW A.C. servo motor for a step input of velocity.

1. No load inertia
2. Load inertia of 0.007 $Kg.m^2$ = 200% of rotor
3. Load inertia of 0.017 $Kg.m^2$ = 500% of rotor

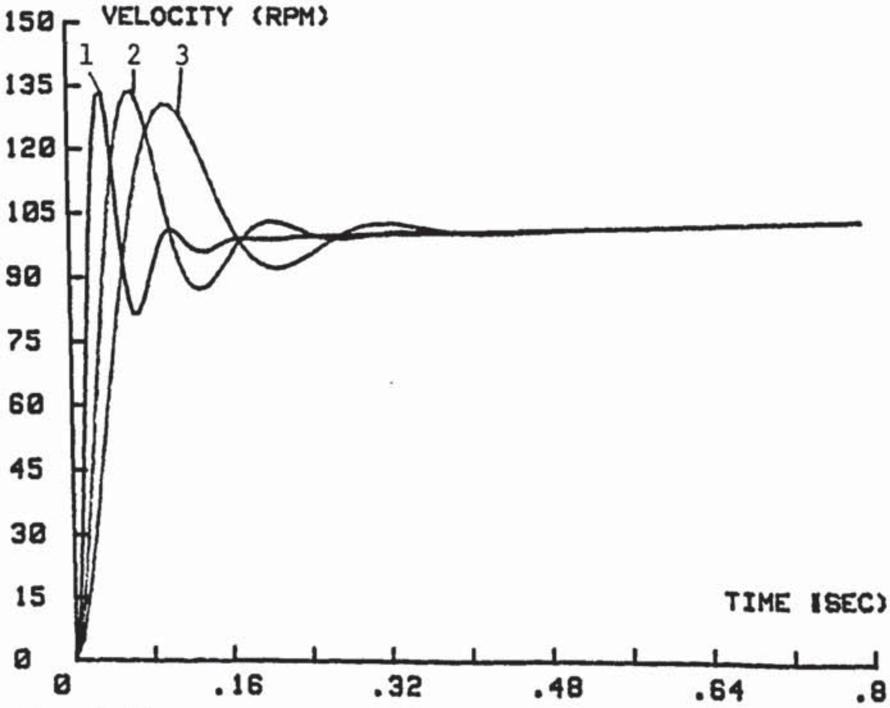


Fig. 5.14 Response of a 3 kW A.C. servo motor for a step input of velocity.

1. No load inertia
2. Load inertia of 0.022 $Kg.m^2$ = 200% of rotor
3. Load inertia of 0.055 $Kg.m^2$ = 500% of rotor

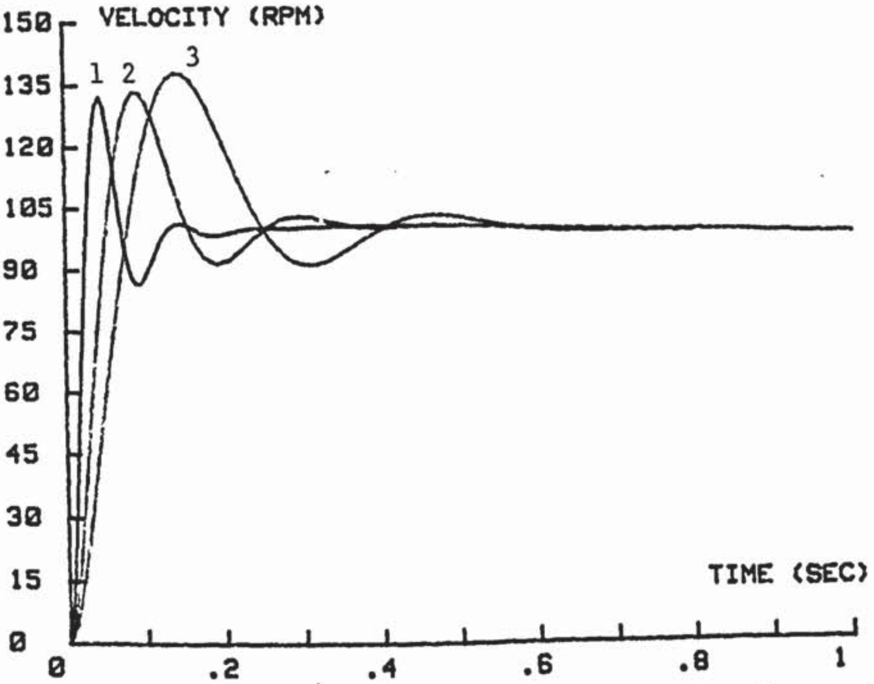


Fig. 5.15 Response of a 5 kW A.C. servo motor for a step input of velocity.

1. No load inertia
2. Load inertia of 0.06 $Kg.m^2$ = 200% of rotor
3. Load inertia of 0.15 $Kg.m^2$ = 500% of rotor

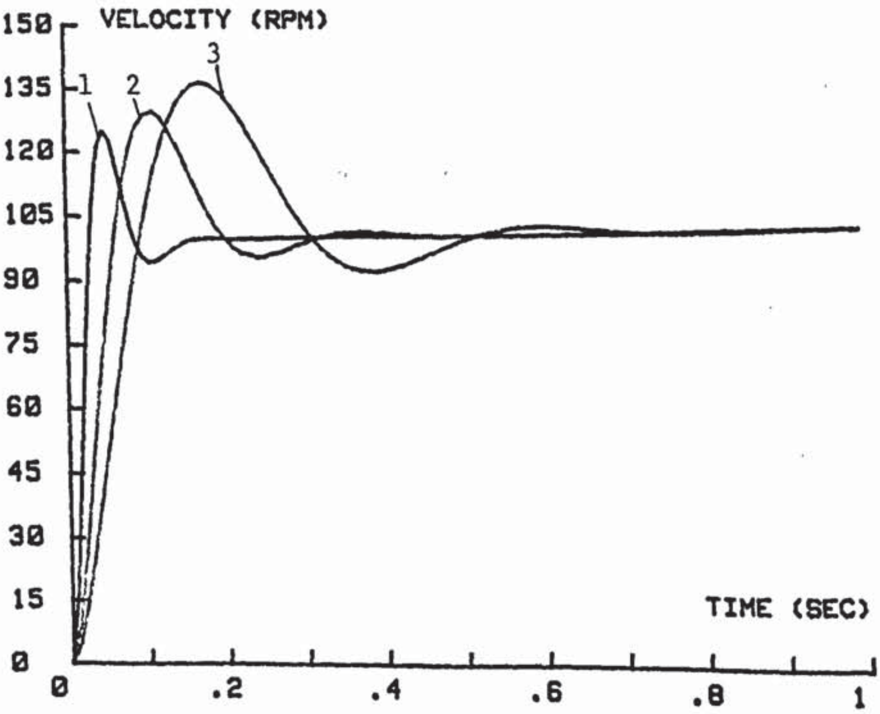


Fig. 5.16 Response of a 10 kW A.C. servo motor for a step input of velocity.

1. No load inertia
2. Load inertia of 0.14 $Kg.m^2$ = 200% of rotor
3. Load inertia of 0.35 $Kg.m^2$ = 500% of rotor

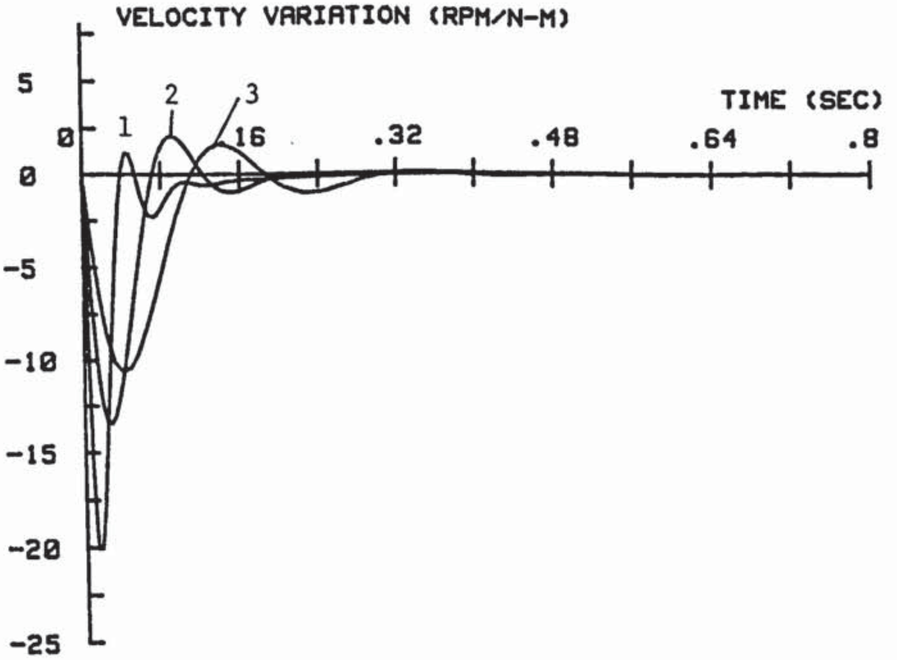


Fig. 5.17 Velocity variation of a 1 kW A.C. motor for a unit step input of torque.

1. No load inertia
2. Load inertia of 0.007 $Kg.m^2$ = 200% of rotor
3. Load inertia of 0.017 $Kg.m^2$ = 500% of rotor

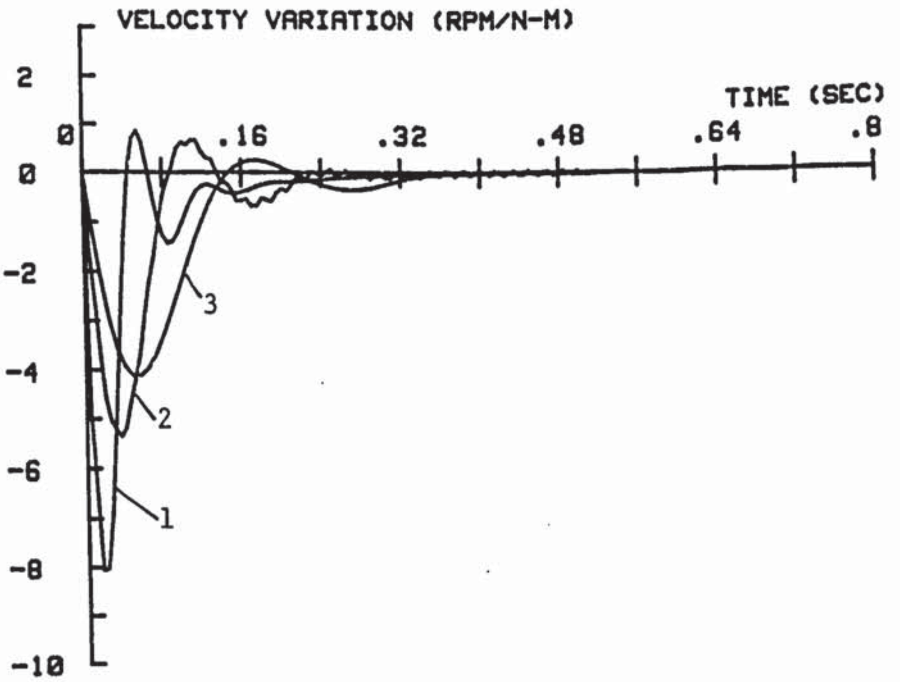


Fig. 5.18 Velocity variation of a 3 kW A.C. motor for a unit step input of torque.

1. No load inertia
2. Load inertia of 0.022 $Kg.m^2$ = 200% of rotor
3. Load inertia of 0.055 $Kg.m^2$ = 500% of rotor

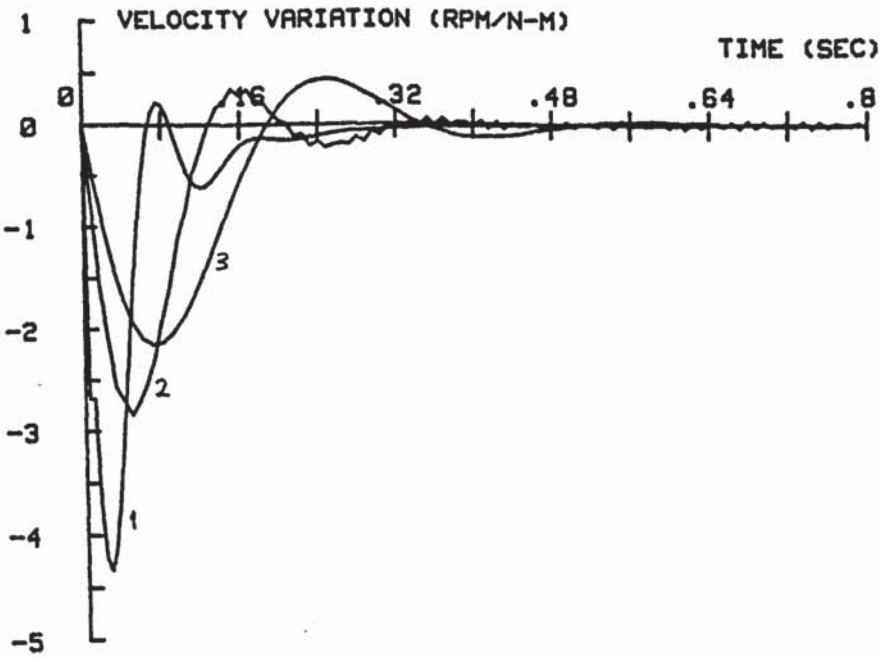


Fig. 5.19 Velocity variation of a 5 kW A.C. motor for a unit step input of torque.

1. No load inertia
2. Load inertia of 0.06 Kg.m² = 200% of rotor
3. Load inertia of 0.15 Kg.m² = 500% of rotor

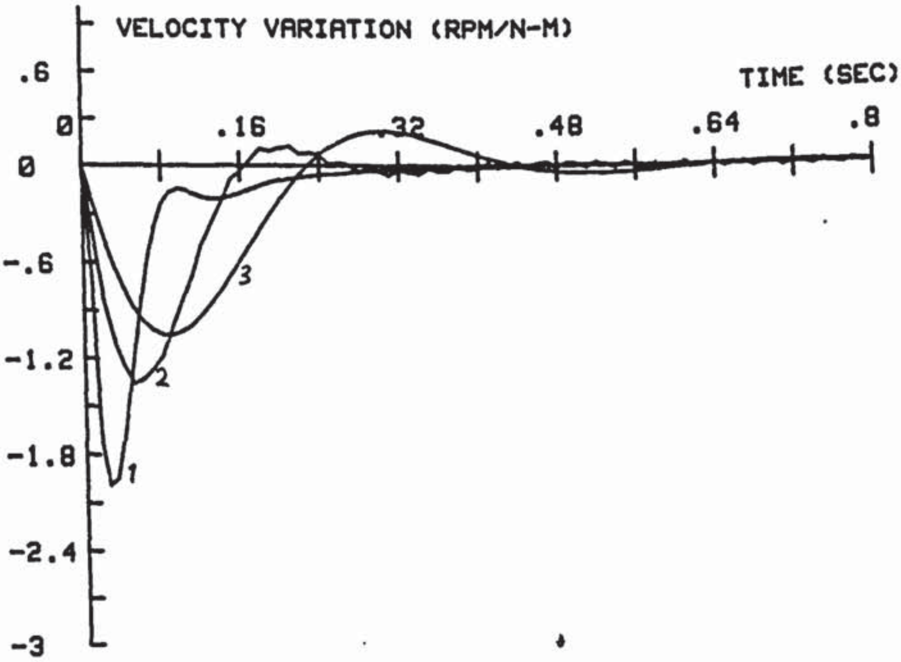


Fig. 5.20 Velocity variation of a 10 kW A.C. motor for a unit step input of torque.

1. No load inertia
2. Load inertia of 0.14 Kg.m² = 200% of rotor
3. Load inertia of 0.35 Kg.m² = 500% of rotor

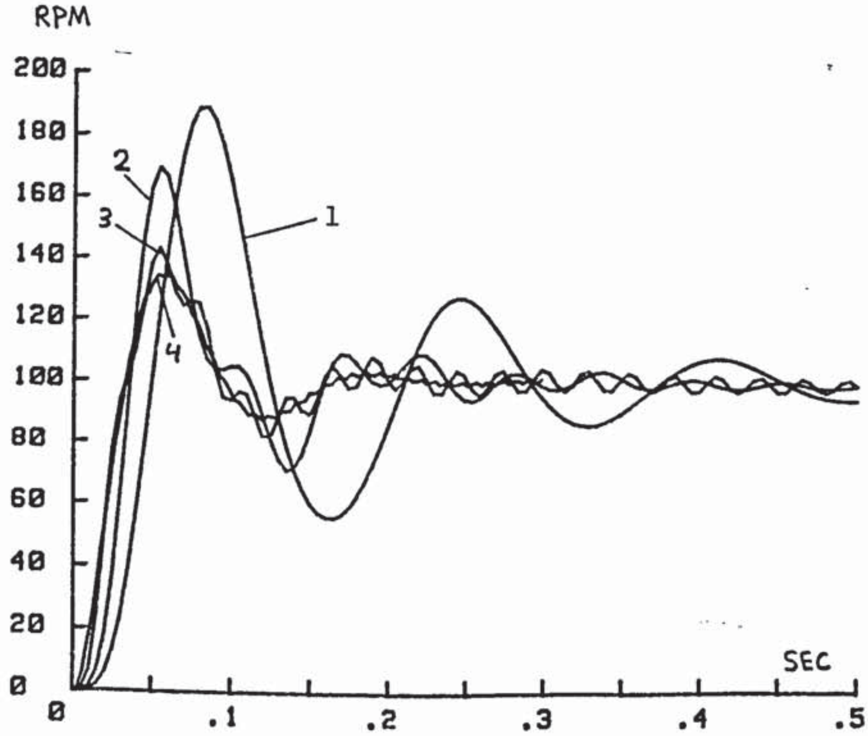


Fig. 5.21 The effect of load natural frequency on the performance of an A.C. servo motor system. Load inertia of 50% of rotor inertia.

1. Load natural frequency of 13 Hz
2. Load natural frequency of 18 Hz
3. Load natural frequency of 37 Hz
4. Load natural frequency of 113 Hz

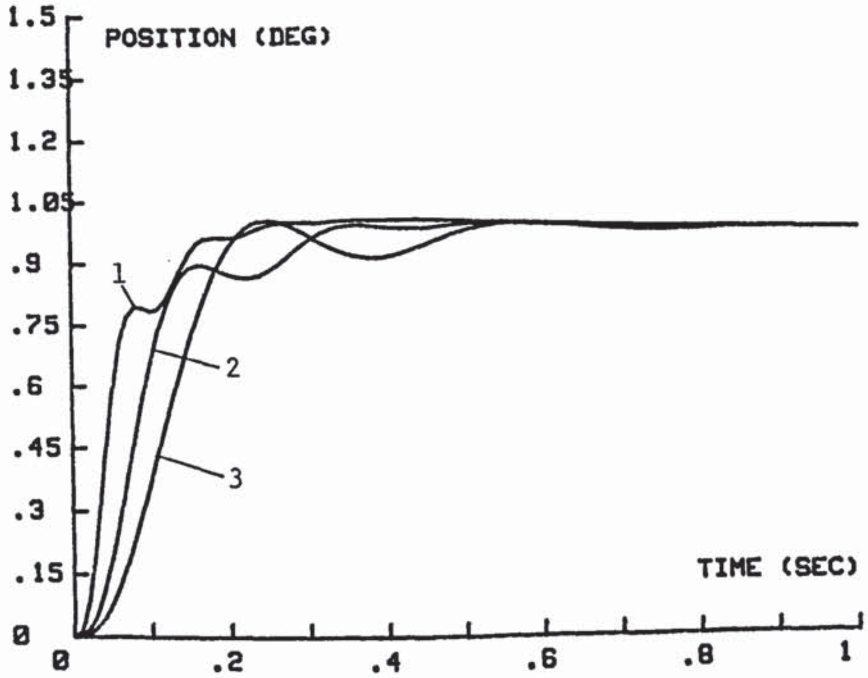


Fig. 5.22 The position response of a 5 kW A.C. motor for a unit step input of position.

1. No load inertia
2. Load inertia of 0.06 $Kg.m^2$ = 200% of rotor
3. Load inertia of 0.15 $Kg.m^2$ = 500% of rotor

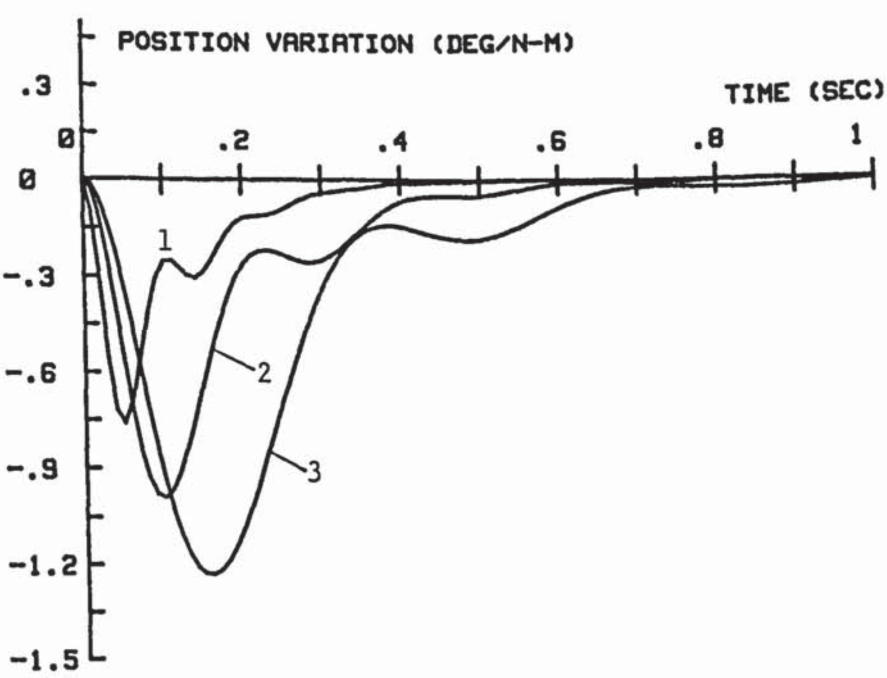


Fig. 5.23 Position variation of a 5 kW D.C. motor for a unit step input of torque.

1. No load inertia
2. Load inertia of 0.06 $Kg.m^2$ = 200% of rotor
3. Load inertia of 0.15 $Kg.m^2$ = 500% of rotor

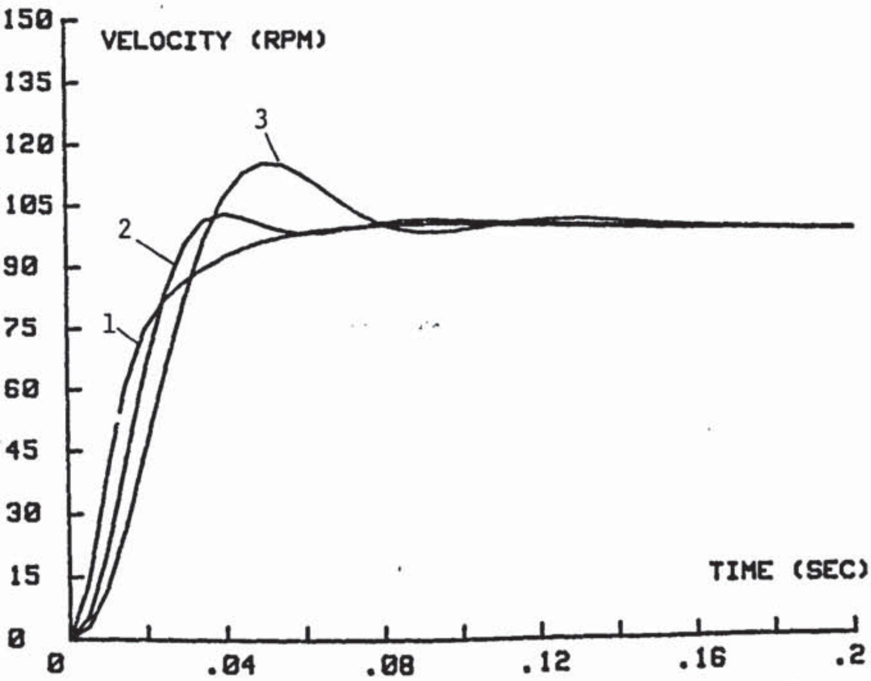


Fig. 5.24 Response of the 5 kW A.C. servo motor for a step input of velocity. The gain is increased 10 times and acceleration feedback is used to increase the damping.

1. No load inertia
2. Load inertia of 0.06 $Kg.m^2$ = 200% of rotor
3. Load inertia of 0.15 $Kg.m^2$ = 500% of rotor

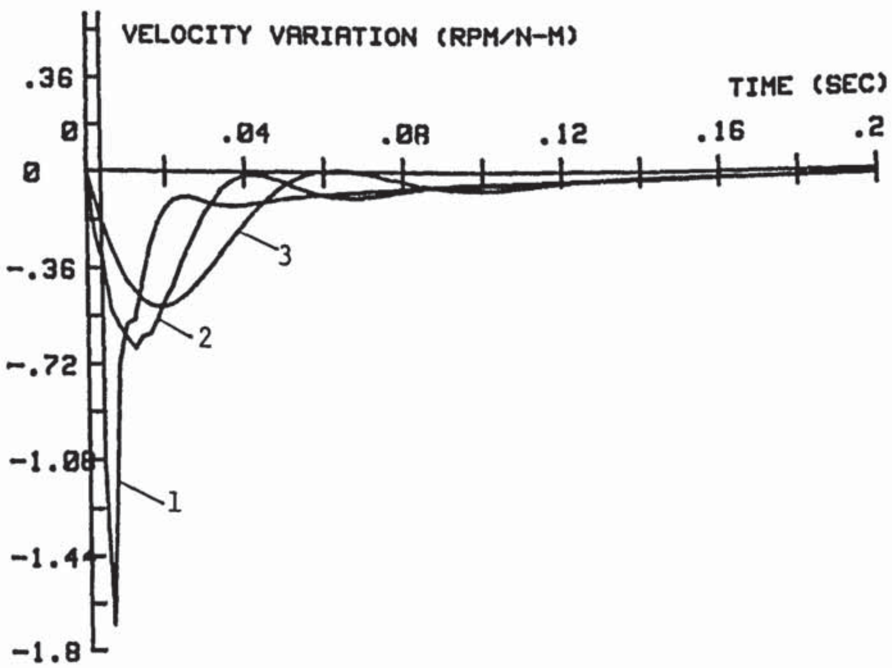


Fig. 5.25 Velocity variation of the 5 kW A.C. servo motor for a unit step input of torque.

1. No load inertia
2. Load inertia of 0.06 $Kg.m^2$ = 200% of rotor
3. Load inertia of 0.15 $Kg.m^2$ = 500% of rotor

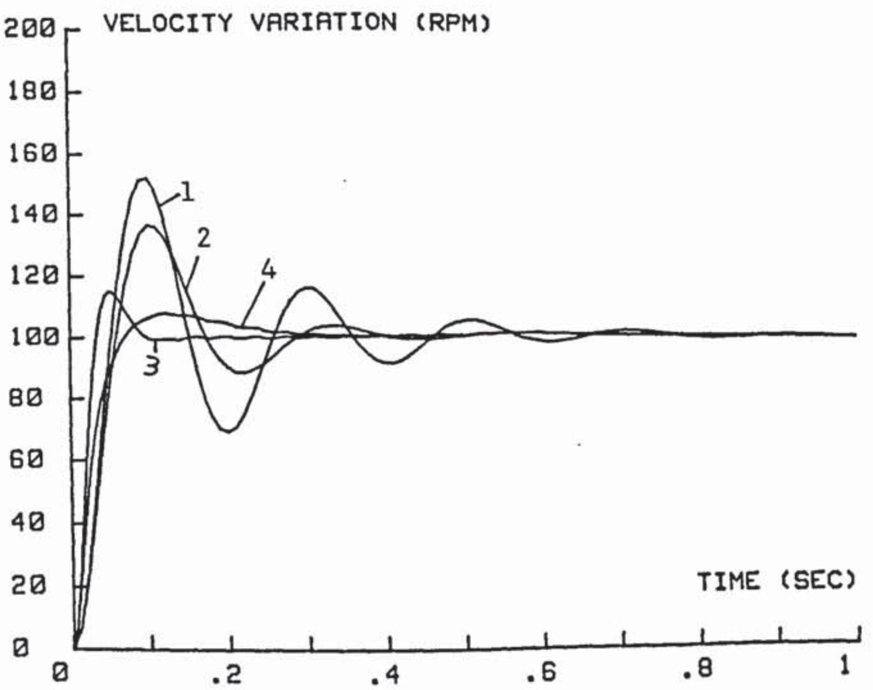


Fig. 5.26. Velocity response of the 5 kW A.C. servo motor at different steady state conditions.

1. No load low velocity
2. No load high velocity
3. 100% of rated torque low velocity
4. 100% of rated torque high velocity

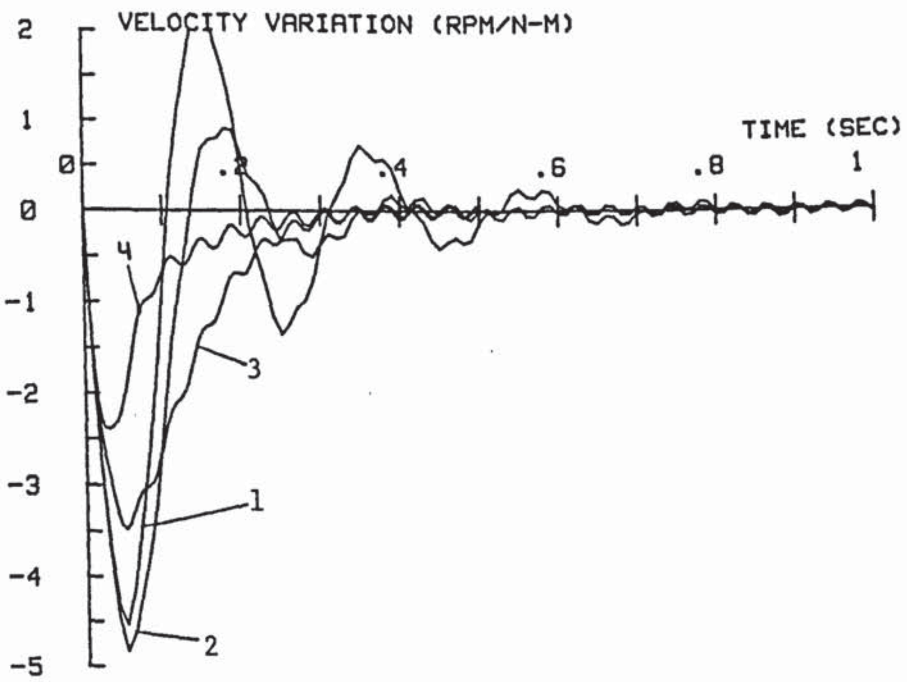


Fig. 5.27 Velocity variation of the 5 kW A.C. servo motor for a unit step input of torque at the different steady state condition.

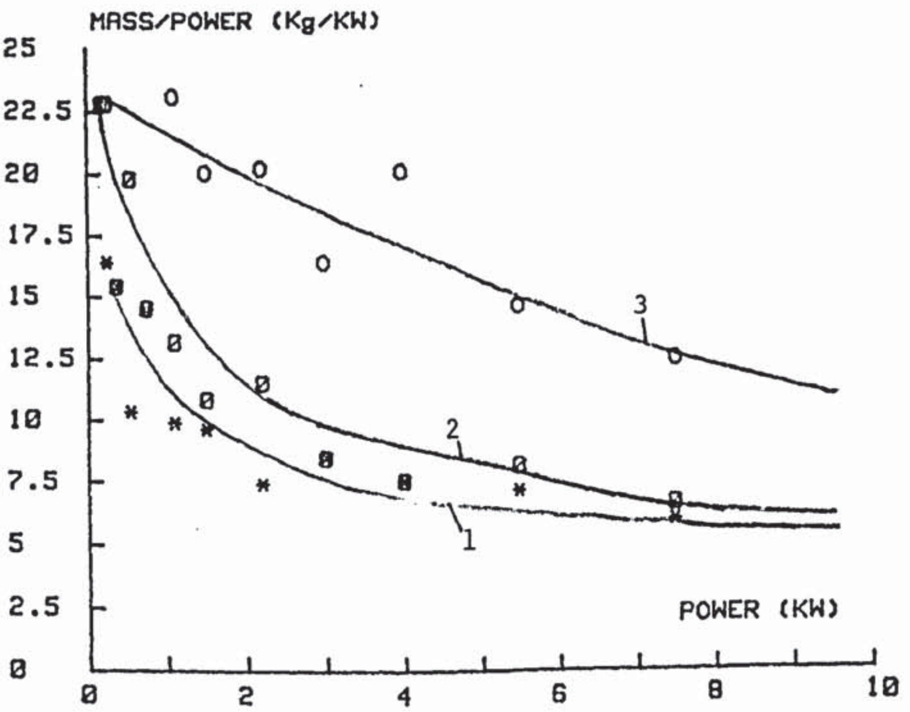


Fig. 5.28 Mass/power ratio of A.C. motors vs. power.

1. 2 pole induction A.C. motors with maximum velocity of 3000 r.p.m.
2. 4 pole induction A.C. motors, maximum velocity of 1500 r.p.m.
3. 8 pole induction A.C. motors, max. velocity of 750r.p.m.

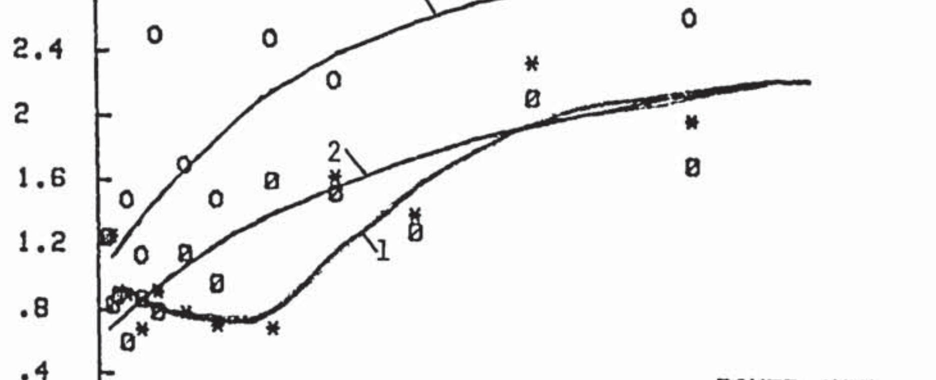


Fig. 5.29 Response time of A.C. motors vs. power.

1. 2 pole
2. 4 pole
3. 8 pole

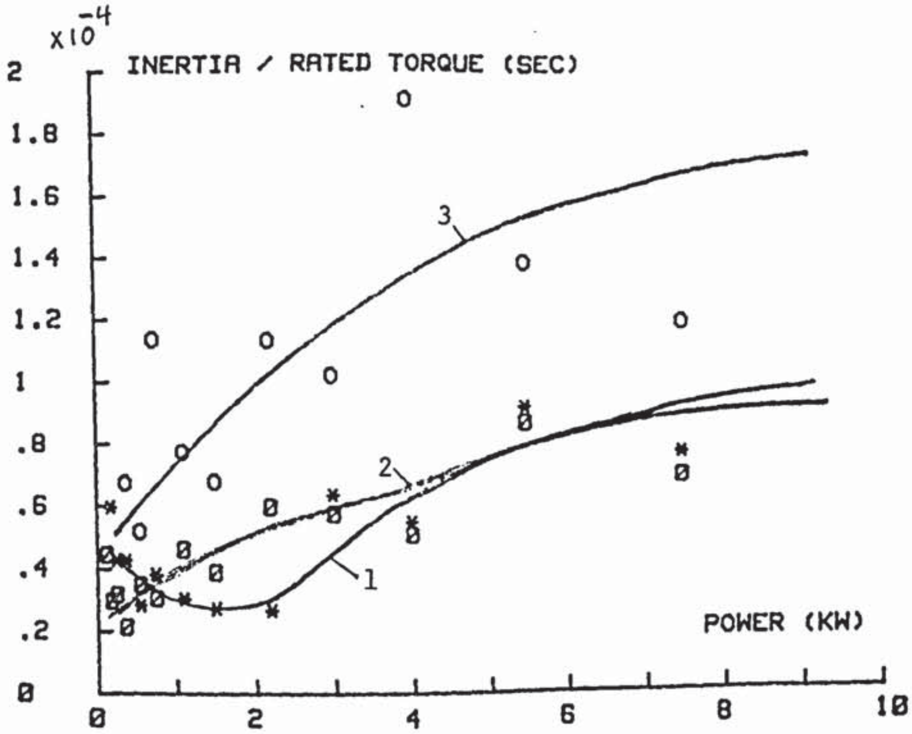


Fig. 5.30 Response time of an A.C. motor for a unit change of velocity at rated torque.

1. 2 pole
2. 4 pole
3. 8 pole

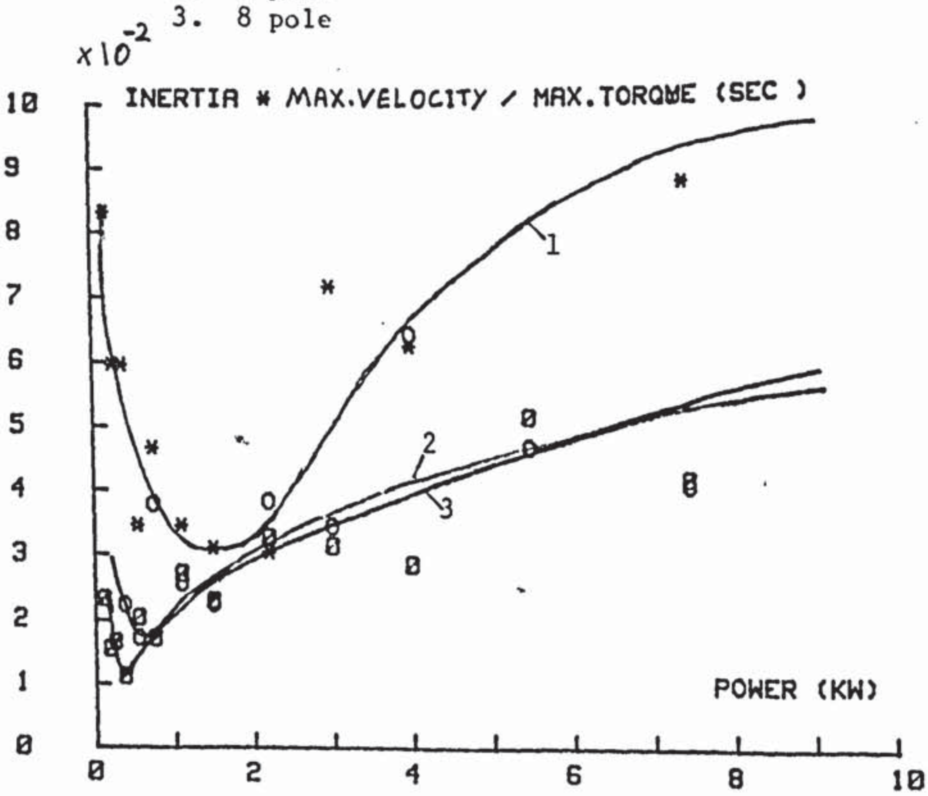


Fig. 5.31 Response time of A.C. motors for maximum change of velocity at maximum torque.

1. 2 pole
2. 4 pole
3. 8 pole

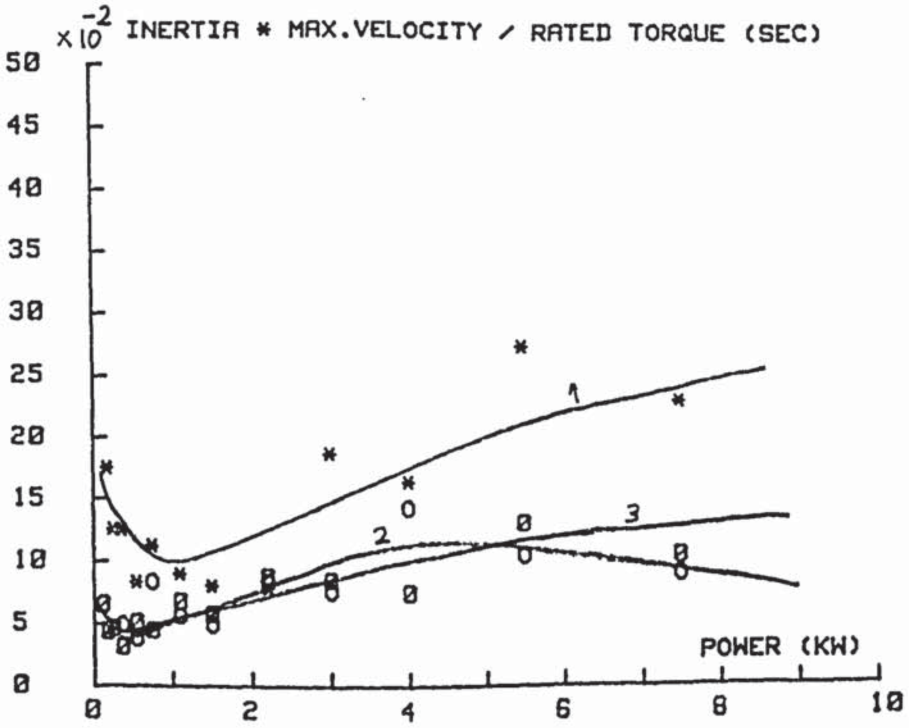


Fig. 5.32 Response time of A.C. motors for maximum change of velocity at rated torque.

1. 2 pole
2. 4 pole
3. 8 pole

Table 5.1 Following Error of 5 Kw A.C. Servo Motor

Load Inertia $kg.m^2$	Following Error Deg/rpm
0	0.38
0.06	0.63
0.15	0.8

CHAPTER 6

STEPPING SERVO MOTORS

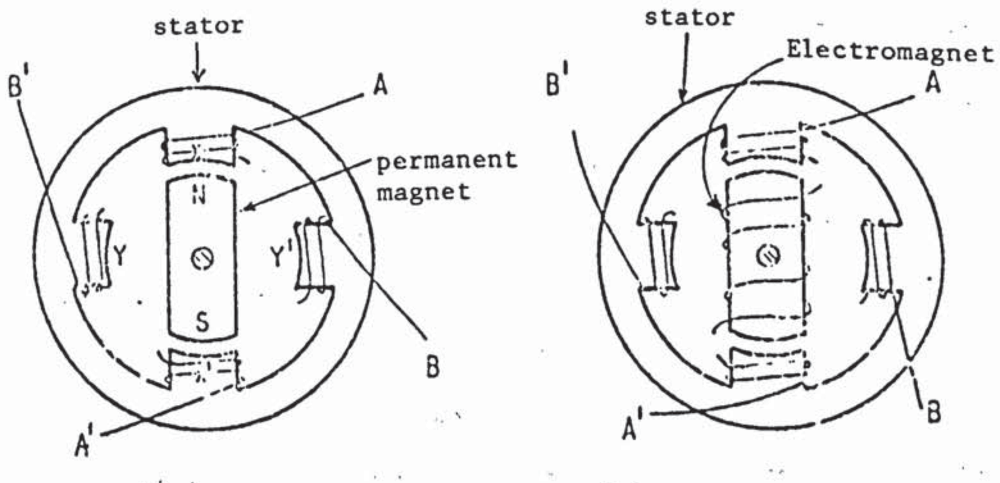
6. STEPPING SERVO MOTORS

6.1 Introduction

A stepping motor is defined as a rotary device whose output shaft moves in discrete steps when excited from a switched D.C. supply. Stepping motors are very practical devices for converting digital-pulse inputs into analogue shaft-output (or rotary) motion as required in more modern electric or electronic equipment. Each shaft revolution can be expressed in terms of a number of discrete identical steps or increments. Each step can be triggered by a single pulse. Many devices that provide incremental rotary motion can serve as stepping motors. These include true stepping motors, rotary solenoids, electromagnetic step clutches and linear rotary actuators. In this chapter only true stepping motors will be considered.

In the past, the drawbacks in the use of stepping motors have been related to their units which have been basically of the electro-mechanical or commutator type with the attendant problems of brushes and commutation. The advancement of transistor circuitry has eliminated these problems and has made provision for switching rates considerably higher than those previously possible. One other factor influencing the emergence of the stepping motor is the widespread adoption of digital computing devices whose output, in digital form, requires (in many cases) a digital to analogue converter at its output stage. This requirement is met by the stepping motors which is in itself a digital to analogue converter of the simplest form.

As the demand and applications for stepping motors has increased, so has the number of types and sizes made available to the market. Hence, there is a great demand to specify the performance of these motors for different applications. In this chapter, firstly the basic



(a) Permanent magnet rotor (b) D.C. energized rotor

Fig. 6.1 Active type stepping motor, two phase stator.

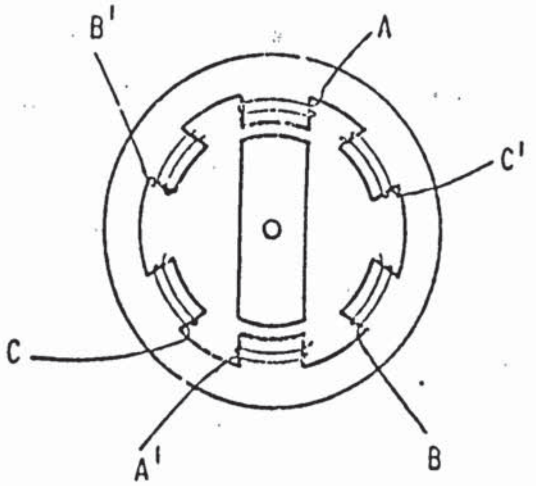


Fig. 6.2 A three phase reactive type stepping motor with salient pole type rotor.

characteristics of stepping motors are discussed. A brief description of the controller is also given. Finally, the performance of these motors are studied dynamically and statically.

6.2 Principle Operation of Stepping Motors

6.2.1 Introduction

The electromagnetic stepping motors are mainly three types; a) permanent magnet (PM), active rotor; b) variable reluctance (VR), reactive rotor; and c) hybrid type, a combination of PM and VR. The permanent magnet type stepping motor has either a permanent magnet rotor, or a rotor made of soft iron and is energized by D.C. voltage through slip-rings and brushes. The stator can be similar to that of a conventional 2-phase or 3-phase induction motor. Figs. 5.1(a) and 5.1(b) show the permanent magnet (PM) rotor and D.C. energized rotor types respectively. The need for slip-rings and brush gear makes the active-rotor version the less attractive of the two, although there are indications that this type of construction has more output torque compared to a PM type of a similar size.

The VR type of stepping motor uses a magnetically soft iron rotor which has teeth and slots similar to the rotor of an inductor alternator or salient-pole type rotor. The stator is similar to that of an active rotor stepping motor. A stepper motor with a salient pole type rotor is shown in Fig. 6.2

The hybrid type of motor uses a combination of permanent magnet and variable reluctance rotor structure and is similar to an inductor motor in construction. Fig. 6.3 shows a motor of this type. In this figure, the stator winding consists of eight coils, one around each

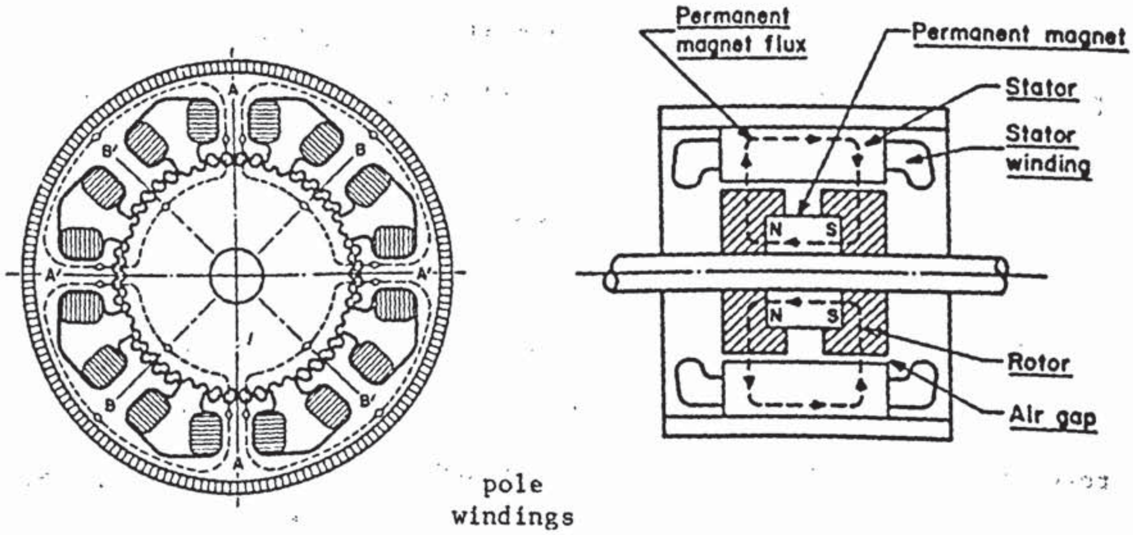


Fig. 6.3 Hybrid type of stepping motor.

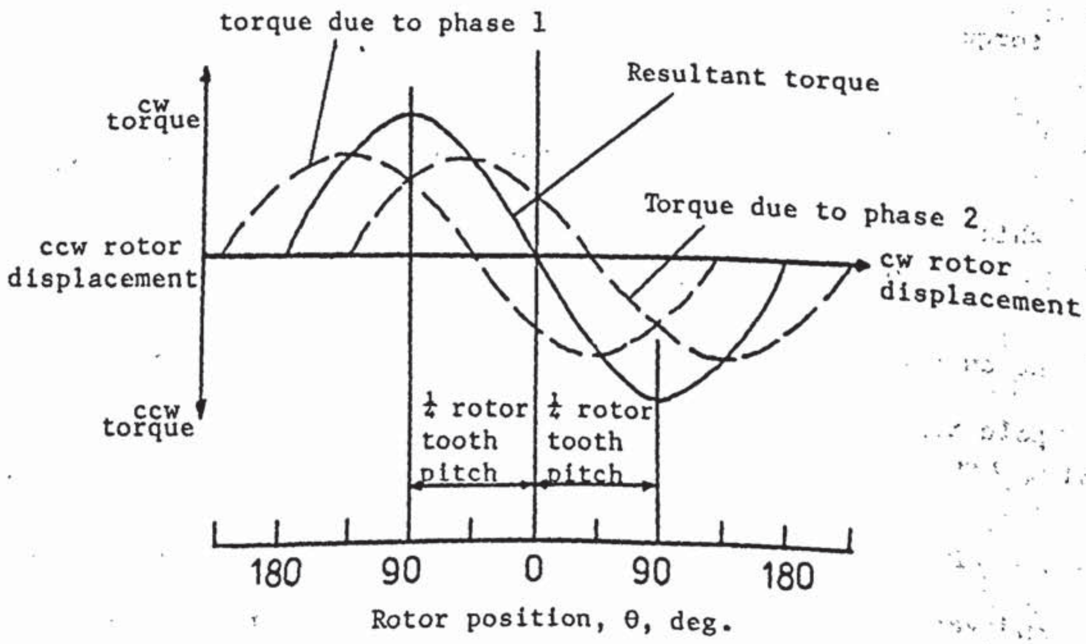


Fig. 6.4 Torque as a function of rotor position for generalized permanent magnet stepping motor - two phases energized.

tooth. These coils are connected as in a conventional 2-phase winding. Rotor punchings with teeth are mounted in two stacks on the shaft separated by a permanent magnet which is axially magnetized.

The torque developed by a step motor is a function of the rotor position and the winding currents. With small angle permanent magnet step motors the torque produced by each winding varies almost sinusoidally with position. The torque magnitude first increases linearly with current until saturation occurs, and then increases slower than linearly. In reluctance types of stepping motors an additional reluctance torque is also added to the exciting torque. As the torque equation of the hybrid type stepping motors is a combination of PM and VR types it will be discussed here. These types of motors also are the most commonly used because of their good performance and will be discussed later. Fig. 6.4 shows this torque as a function of rotor position in a generalised case for a two phase stator. Assuming the magnitude of the stator currents are equal, the developed torque (when both of the phases are energized) can be shown to be [71]:

$$T_m = n\, K_t'\, I \sin\theta \tag{6.1}$$

where

T_m = developed torque (N.m)

K_t' = motor torque constant, Nm/A

I = constant value of stator input current (energization current)

$= N_{RT}\varphi$

n = number of phases energized

N_{RT} = number of rotor teeth

φ = step angle, deg.

For a specific condition n, K_t' and I will be constant and equation 6.1 may be simplified as:

$$T_m = K'_{t_1} I \sin\theta \tag{6.2}$$

where K'_{t_1} = motor torque constant (N.m/amp)

I = stator current at each phase (amp)

θ = rotor position

As can be seen from equation 6.2, the torque equation of stepping motors is non-linear and the torque is a function of rotor position. Equation 6.2 may be linearized as

$$T_m = K_t I - K_\theta \theta \tag{6.3}$$

where

$$K_t = \left(\frac{dT_m}{dI}\right)_{\theta = \text{constant}} = K_t \sin\theta \quad \text{(N.m/amp)} \tag{6.4}$$

$$K_\theta = -\left(\frac{dT_m}{d\theta}\right)_{I=\text{constant}} = +K_t I \cos\theta \quad \text{(N.m/rad)} \tag{6.5}$$

Equation 6.3 shows that there is internal position feedback in stepping motors. The voltage equation of stepping motors can be simplified for each phase as:

$$V = RI + L\frac{dI}{dt} \tag{6.6}$$

where

V = voltage supplied to each phase.

R = resistance of each phase.

L = inductance of each phase.

I = current of each phase.

Equations 6.3 and 6.6 are the governing dynamic characteristics of the stepping motor. During dynamic operation, the motor windings are being continuously switched and effective torque utilization requires that pulses be timed properly. The static torque represents the

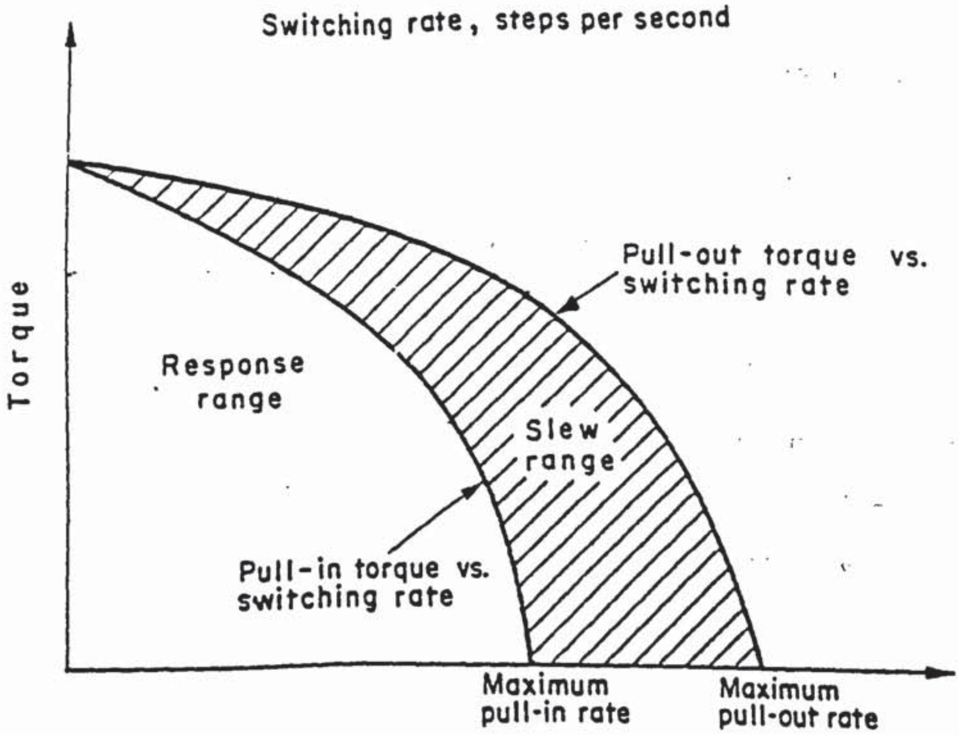


Fig. 6.5 Typical torque-speed characteristic of stepping motors.

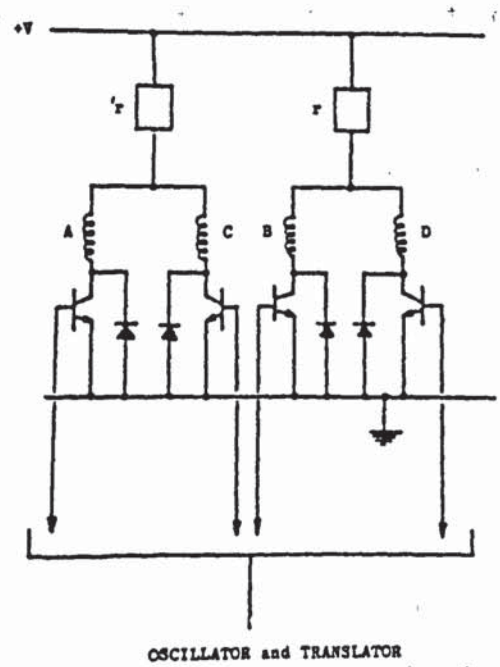


Fig. 6.6 A circuit diagram of the controller of a stepping motor. A, B, C and D are the stator winding (phases).

absolute maximum torque available instantaneously from a step motor. The dynamic torque-speed characteristic is indicative of the maximum steady-state torque available at a given speed with a particular motor-drive combination. Fig. 6.5 shows a typical speed-torque characteristic of stepping motors. The maximum torque available reduces as the speed increases. This is mainly due to the inductance of the stator and the magnetic drag. It is important to realise that the reduction in torque at different speeds is not due to the speed drop. The maximum error due to the applied torque is a maximum of one step angle. Torque above this point only causes the motor to lose steps and may halt the motor. A more complex mathematical model for the hybrid stepping motors are given by Pickup and Russell [45]. However, in their model the effect of load inertia and coupling between the two inertia is neglected. In order to obtain an overall performance of stepping motors the linearized model obtained in this section is used. With this modification the mathematical model obtained in Chapter 3 is used to predict the dynamic behaviour of the system.

6.3 Principle Operation of the Controller of the Stepping Motors

The purpose of stepping motor drive circuitry is, in addition to producing the usual motor excitation source,to monitor the successive input pulses applied to the circuit in order to control the sequence of energization of the motor stator windings; thus producing the required stepped angular motion of the rotor. Generally, drive circuitry consists of logically controlled power output stages; designed to accept information in the form of an external pulse train (of any shape) and convert it into separate, appropriately timed pulses which are amplified to drive three, four, five or more phases of a motor. The timing of these triggering pulses is arranged in various combinations or modes which control the sequence by which the windings are energized and the rate at which the rotor steps. Suitable logic circuits are used to feed these pulses in the sequence required, to the various phases and the rotor advances by a unit step per pulse. For a typical circuit, a bistable multivibrator in the form of a triggering flip flop would form the input to the drive, then, with the input pulses being applied and depending upon its initial state and sequence desired, its output would be fed to a number of switching flip flops with intermediate gating circuits used to carry out the necessary discrimination. It is the output from these flip flops which are then passed through amplifying stages to form drives to the windings.

The transient response of the stator current has a significant effect on the stepper motor response as the switching transients affect the maximum torque developed by the motor. The inductance and resistance of the stator coil place a restriction on the response time of the coil current. If the time between the application of each step command approaches this time, the current will not reach its expected value, and the maximum torque developed by the motor will be reduced. Therefore, the time constant, $^{L}/R$, can place an unacceptable constraint on the torque. In general, stepping motor drive circuits are designed to compensate for these effects. Fig. 6.6 shows a typical construction of stepping motor drives. Since stepping motors are basically open-loop control systems, the simple mathematical model of the drive obtained in Chapter 3 is used for the controller.

6.4 Dynamic Performance of the Stepping Servo Motor

6.4.1 Introduction

Stepping motors, due to their construction limitations, are usually in small power ratings. For this reason the performance of 1 and 2 kW stepping motors are analysed here. The performance prediction, however, can be extended to other ranges of stepping motors. The specification of these two motors are given in Appendix F. In this section the performance of stepping motors are investigated theoretically and the results may be compared with experimental work carried out by Pickup and Russell [45]. Since stepping motors are inherently position control devices, the dynamic performances are studied in position control. Finally, velocity control performances of stepping motors are studied. The effect of load inertia and load natural frequency are studied. The prediction of the performance of the motors in position closed-loop and the influence of compensation techniques are studied.

6.4.2 Effect of Load Inertia on the Performance of the Systems.

Figs. 6.7 and 6.8 show the response of a 1 and 2 kW stepping servo motor at different load inertia. As can be seen from these graphs, the damping of stepping motors are generally low and there is oscillation about the steady state points. The damping reduces as the load inertia increases. The response of the two sizes of motor are similar, but a faster response may be obtained from the smaller size stepping motor. The reduction in damping ratio due to load inertia is undesirable in most applications, therefore it is necessary to use compensation to reduce the oscillation. The use of compensation techniques for improving the performance will be discussed later.

The effect of step input of torque on the accuracy of the system is shown in Figs. 6.9 and 6.10. Two types of position error from these graphs can be seen; i.e. dynamic and static. The static position error is caused by the external torque, its value can be obtained from equation 6.2. The dynamic position error depends on the dynamic characteristics of the servo system, and it is larger than the static error. This position error depends on the total inertia and the excitation current. However the position error is limited to the step angle of the stepping motor. This will limit the maximum static and dynamic torque applied to the motor. Above this value it causes the motor to lose steps.

6.4.3 Effect of Load Natural Frequency

Load natural frequency has considerable effect on the performance of stepping motors. Fig. 6.11 shows the response of 2 kW stepping motors at different load natural frequency. It was noticed that the damping of the fundamental frequency of the system reduces as the load natural frequency reduces. At low load natural frequency, systems may become unstable. The increase in the load natural frequency, however, improves the damping of the loop frequency. Therefore, it is essential that the load natural frequency be higher than the loop frequency, unless some compensation technique is used to prevent the system from becoming unstable.

6.4.4 Effect of Acceleration Feedback on the System

It was noticed that the use of acceleration feedback around the motor will improve the damping of stepping motors especially at larger load inertia. Fig. 6.12 shows the response of the 2 kW stepping motor, with acceleration feedback as compensator. As can be seen the performances have improved considerably, although the speed of response has not been changed. The effect of torque on the compensated system is shown in Fig. 6.13. The effect of dynamic torque on the position error has been reduced considerably. Therefore, by the use of acceleration feedback as compensation, a larger dynamic torque can be applied to the motor.

6.4.5 Performance of Stepping Motors in Velocity Control Mode

It was discussed before that stepping motors are in principle position control systems. Each input pulse represents a one increment rotation. By applying a ramp input (a train of pulses) the motor may be operated in velocity control mode. Two important features are investigated in this section; i.e. low velocity and high velocity characteristics. Fig. 6.14 shows the position response of the 2 kW stepping motor for a small ramp input. The oscillation of the motor about each step is clearly seen on this graph. The corresponding velocity characteristic is shown in Fig. 6.15. It can be seen that the velocity is oscillatory. The average velocity is equal to the amplitude of ramp input. For greater velocity, the response is different. Fig. 6.16 shows the position response of the stepping motor for large amplitude of ramp input. Fig. 6.17 shows the corresponding velocity response. It can be seen from these graphs that the oscillation of the velocity reduces and the velocity almost stays at its steady state value. The oscillation of the stepping motor about its position is an undesirable feature and must be reduced by some damping technique. In practice the existence of permanent magnet in the rotor introduces an additional damping. It was shown in the previous section that the use of acceleration feedback introduces a considerable amount of damping into the system. Therefore the acceleration feedback may be used as an active damping compensation.

The maximum rate of input pulses is limited by the following error in the run-up condition. This following error is limited to the step angle of the motor. This is clearly seen on Fig. 6.14 and 6.16. The maximum input pulse rate at the start is usually given by the manufacturers. However, after the motor speeds up, the rate of input pulses can be increased. This rate depends on total inertia, friction and the inductance of motors. The external torque also reduces this rate. This is given in detail in the references [37, 39, 43].

6.5 Static Characteristics of Stepping Motors

6.5.1 Mass/Power Ratio

The mass/power ratio, which is an indication of the weight of the motor for hybrid types of stepping motors, is shown in Fig. 6.18. It can be seen from this graph that stepping motors are generally available in small power ratings. This ratio reduces as the power of the motor increases.

6.5.2 Inertia/Torque Ratio

Fig. 6.19 shows the inertia/max. torque as the function of power rating of the hybrid stepping motors. It can be seen that this ratio, which is the time of a unit change of velocity, increases as the power rating increases. The maximum torque is only obtainable when the motor is stationary. The torque reduces as the velocity of the motor increases as was shown in Fig. 6.5. Fig. 6.20 shows this ratio at

50% of maximum velocity. It can be seen that this ratio generally increases as the power increases. A comparison of these two graphs shows that the response time increases at a higher velocity due to the reduction in available torque.

6.5.3 Inertia x Max. Velocity/Torque

This ratio is the minimum response time of the motor to change its velocity from 0 to maximum velocity at the applied torque. This ratio at 50% of maximum torque is shown in Fig. 6.21. This ratio increases rapidly as the power rating of the motor increases (larger motors). This ratio at maximum torque (when the motor is stationary) is shown in Fig. 6.22. Similarly it can be seen from this graph that this ratio increases as the power rating of the motor (larger motors) increases. When the motor is commanded from one velocity to another, the available torque reduces. When calculating the response time,this fact must be considered. By taking the average torque at two operating points, a more accurate response time can be evaluated. An additional response time,due to the dynamic characteristic of the motor,must be added to the above response time which was discussed previously.

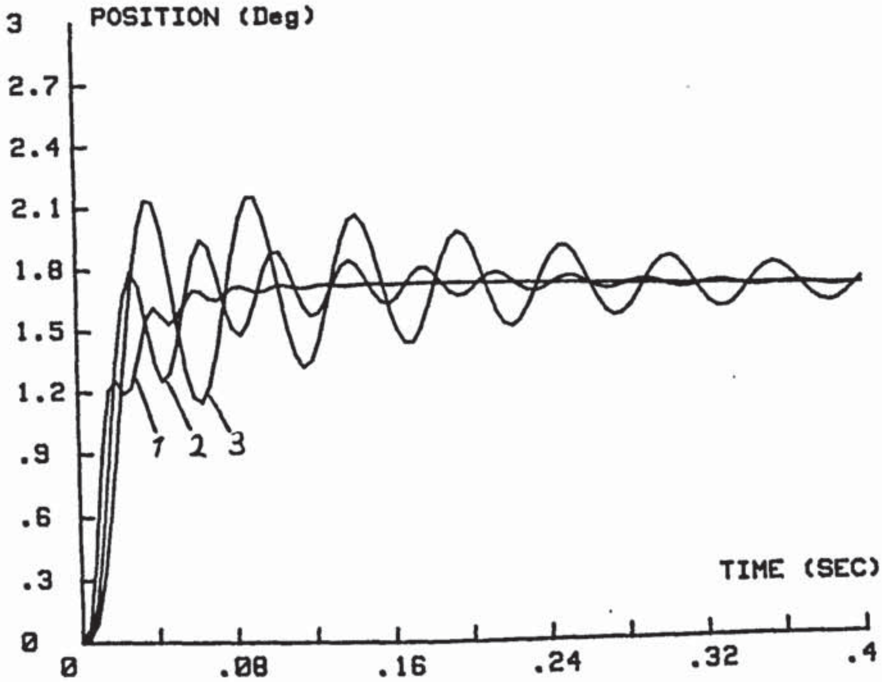


Fig. 6.7 Response of a 1 kW stepping motor for a single step displacement.

1. No load inertia
2. Load inertia of 0.012 $Kg.m^2$ = 200% of rotor
3. Load inertia of 0.03 $Kg.m^2$ = 500% of rotor

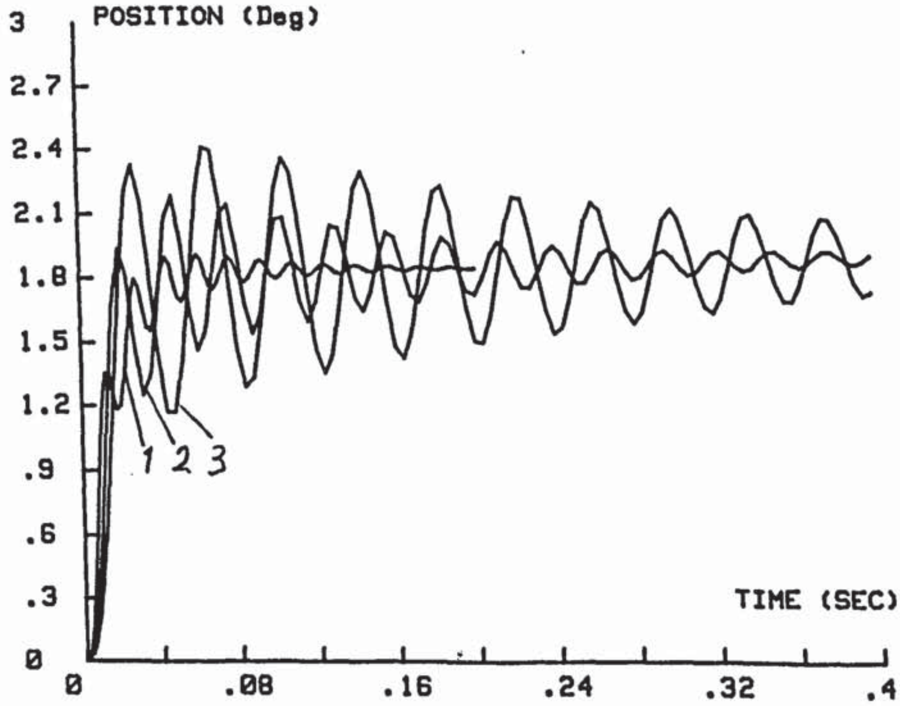


Fig. 6.8 Response of a 2 kW stepping motor for a single step displacement.

1. No load inertia
2. Load inertia of 0.018 $Kg.m^2$ = 200% of rotor
3. Load inertia of 0.045 $Kg.m^2$ = 500% of rotor

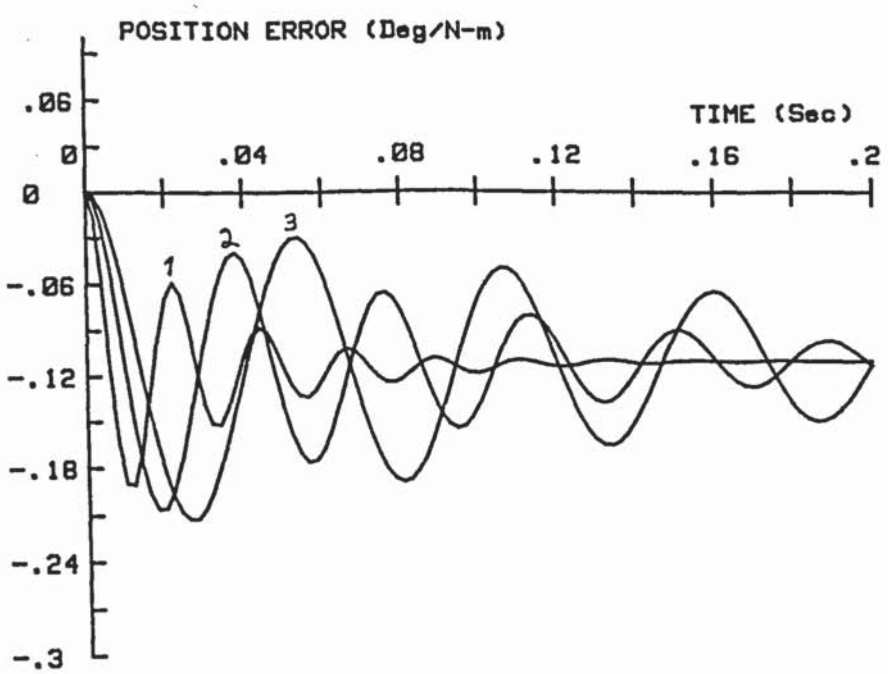


Fig. 6.9 Position variation of a 1 kW stepping motor for a unit step of torque.

1. No load inertia
2. Load inertia of 0.012 Kg.m² = 200% of rotor
3. Load inertia of 0.03 Kg.m² = 500% of rotor

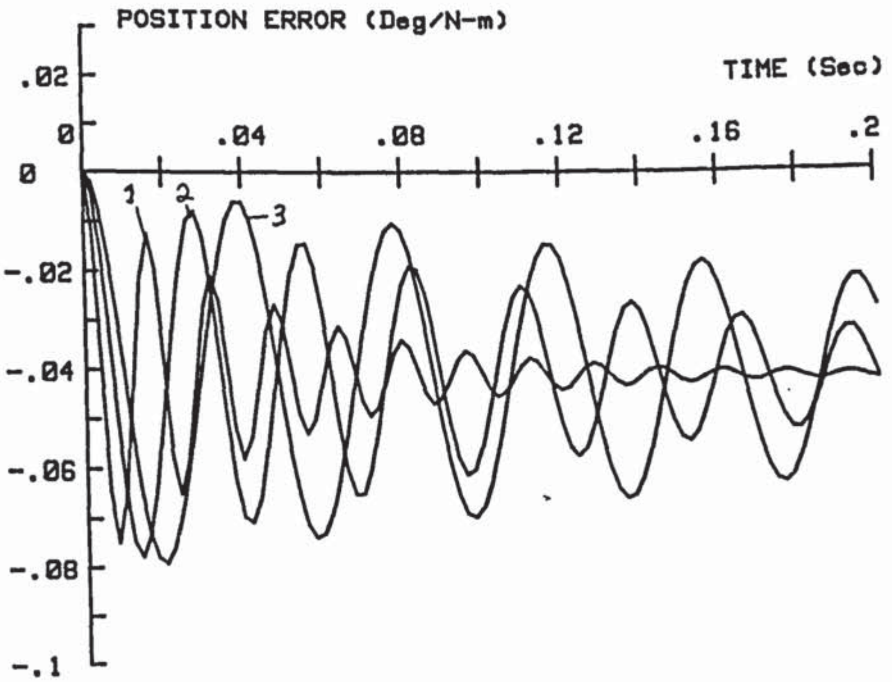


Fig. 6.10 Position variation of 2 kW stepping motor for a unit step input of torque.

1. No load inertia
2. Load inertia of 0.012 Kg.m² = 200% of rotor
3. Load inertia of 0.03 Kg.m² = 500% of rotor

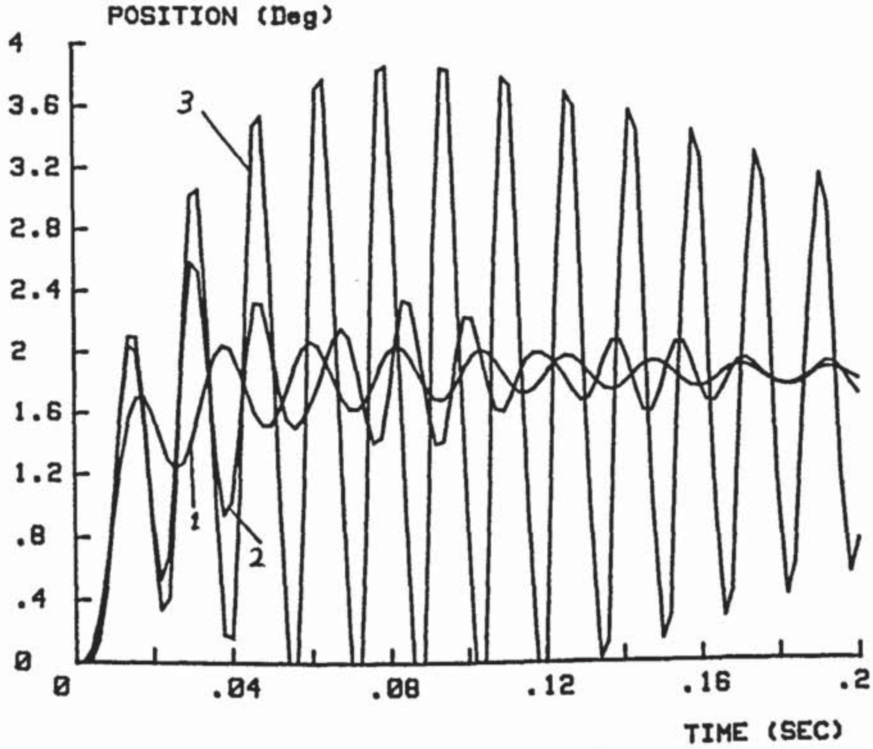


Fig. 6.11 Effect of load natural frequency on the 2 kW stepping motor.

1. Load natural frequency = 40 Hz
2. Load natural frequency = 23 Hz
3. Load natural frequency = 18 Hz

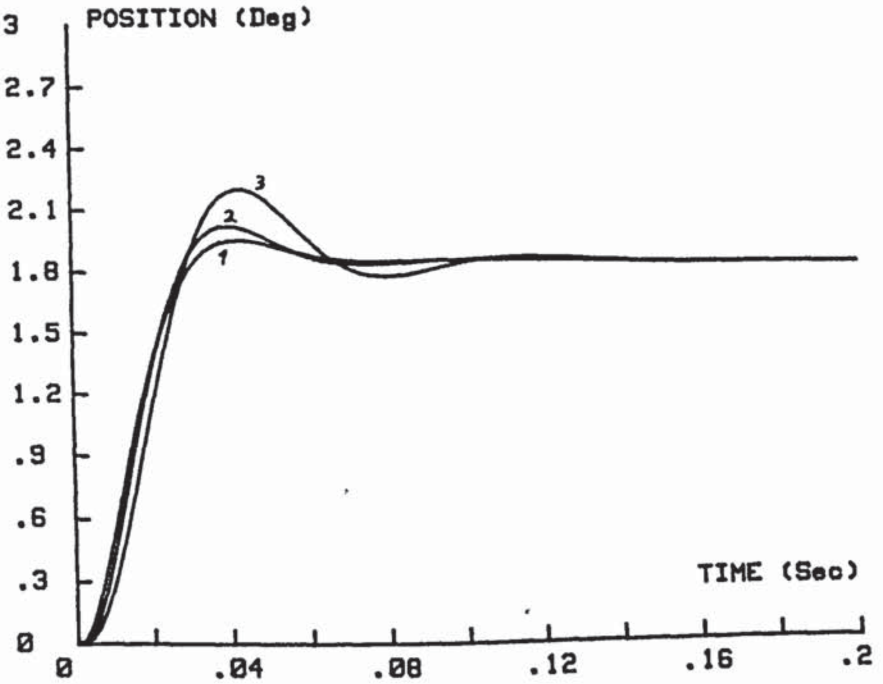


Fig. 6.12 Response of a 2 kW stepping motor for a single step displacement. Acceleration feedback is used to compensate the damping.

1. No load inertia
2. Load inertia of 0.018
3. Load inertia of 0.045

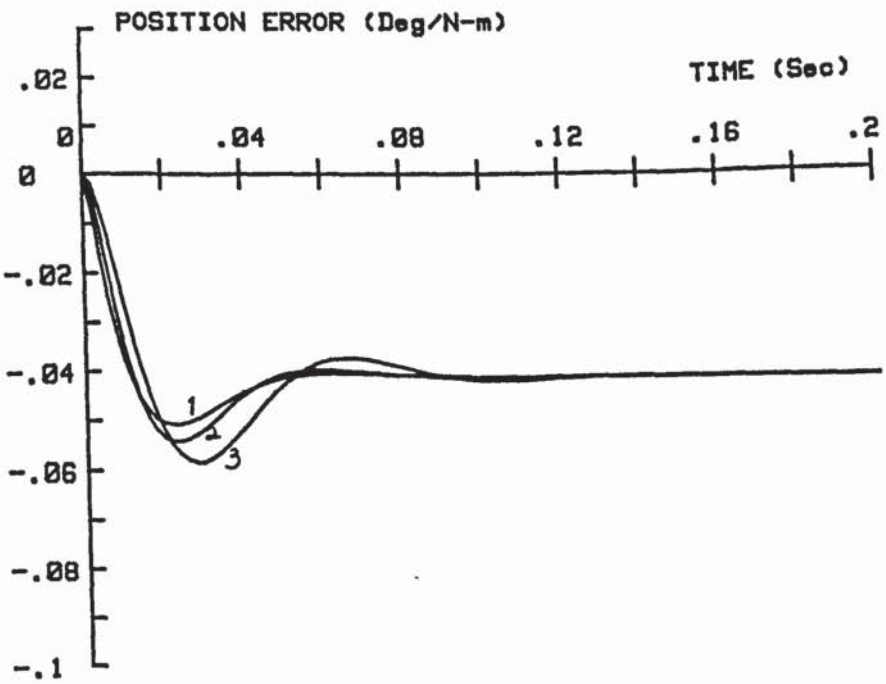


Fig. 6.13 Position variation of the 2 kW stepping motor for a unit step input of torque. Acceleration feedback is used to compensate the damping.

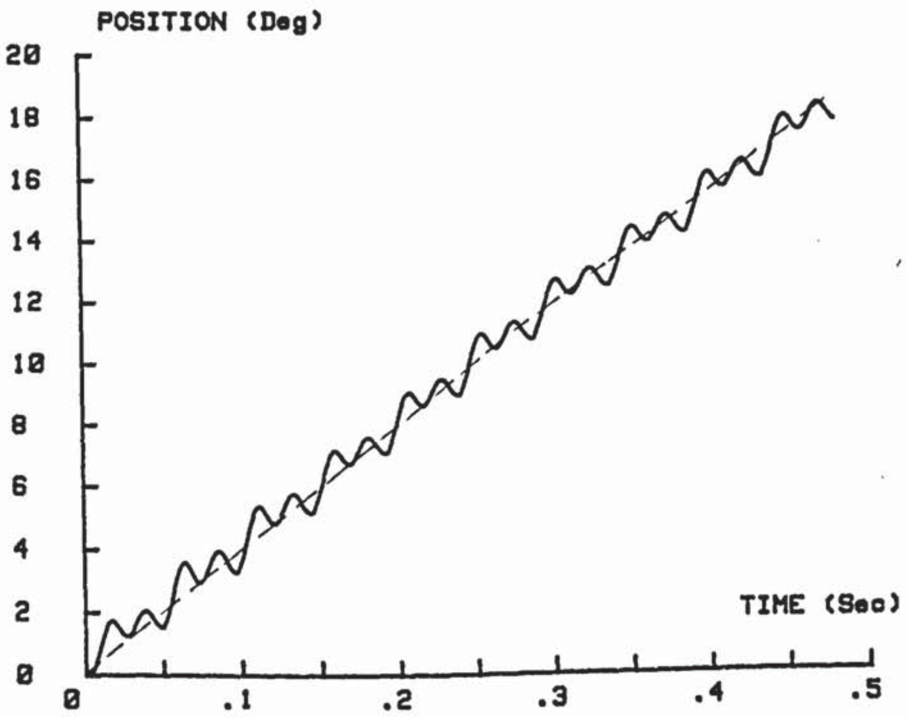


Fig. 6.14 Position response of the 2 kW stepping motor for a ramp input (load inertia = rotor inertia).

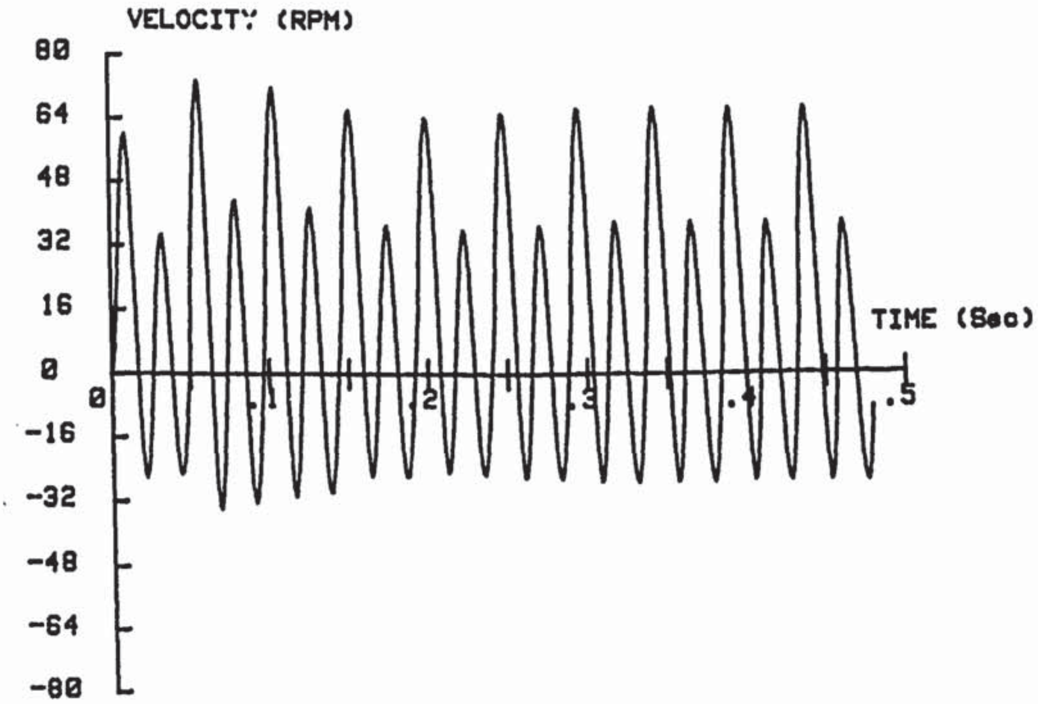


Fig. 6.15 Velocity response of the 2 kW stepping motor for a ramp input. Average velocity of 6 rpm (load inertia = rotor inertia).

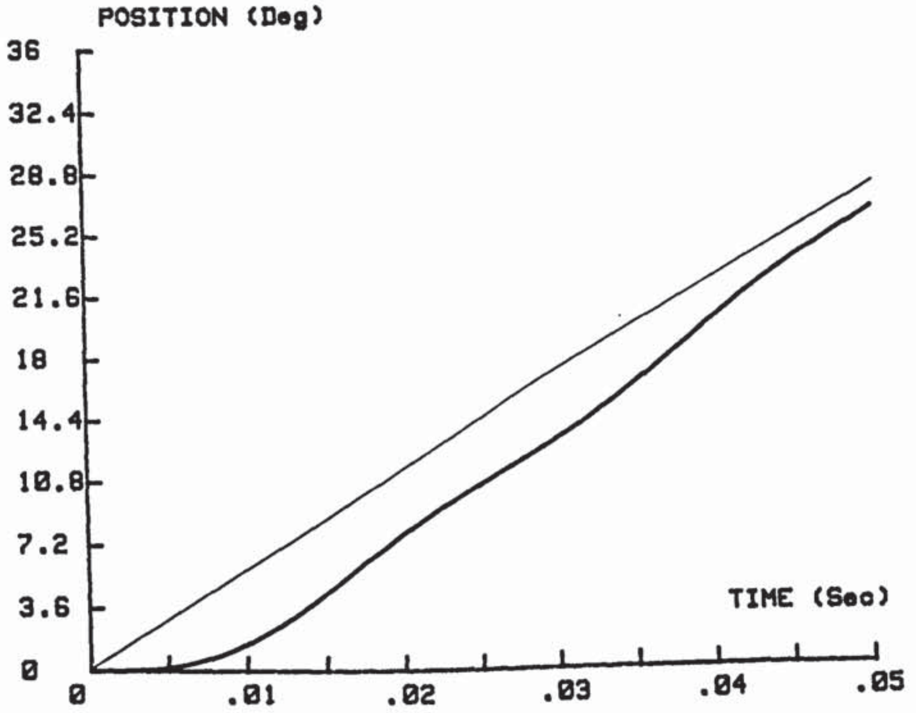


Fig. 6.16 Position response of the 2 kW stepping motor for a ramp input (load inertia = rotor inertia).

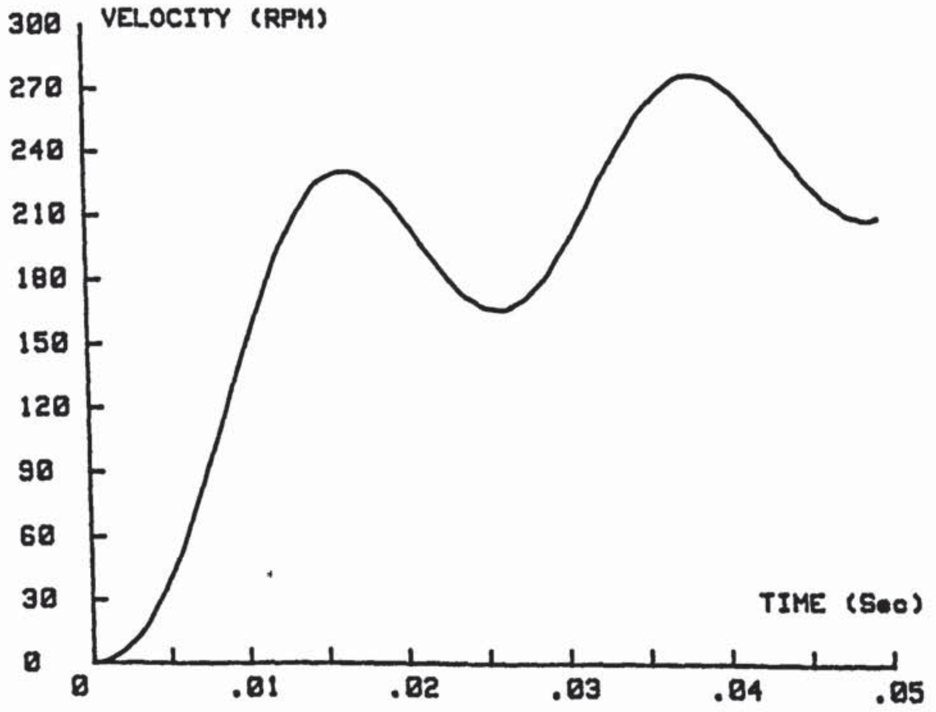


Fig. 6.17 Velocity response of the 2 kW stepping motor for a ramp input. Average velocity of 220 rpm (load inertia = rotor inertia).

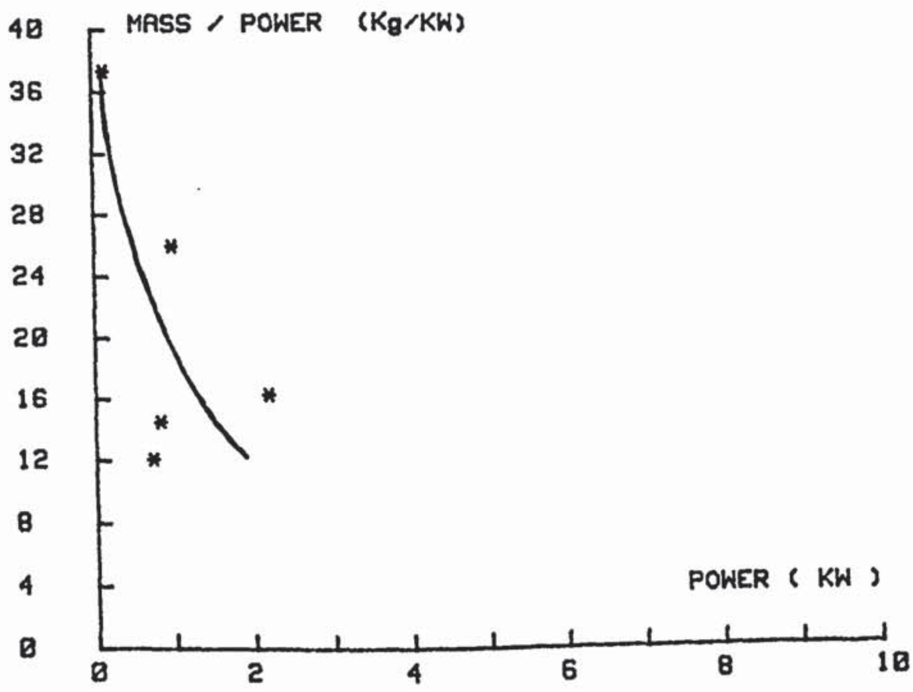


Fig. 6.18 Mass/power vs. power rating of stepping motor.

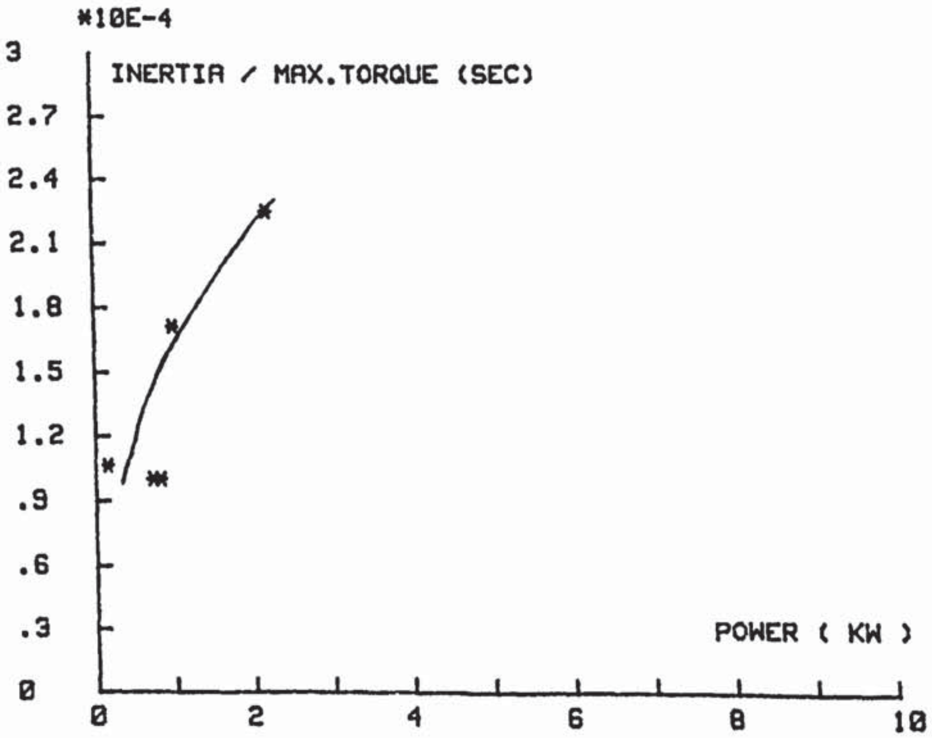


Fig. 6.19 Inertia/max. torque vs. power of stepping motors.

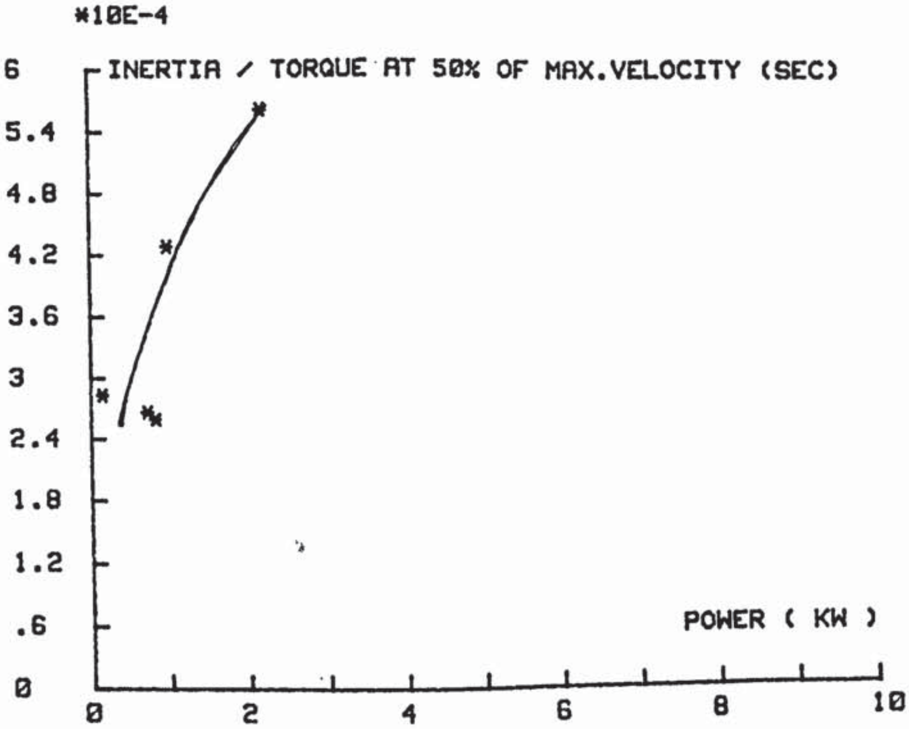


Fig. 6.20 Inertia/50% of rated torque vs. power rating of stepping motor.

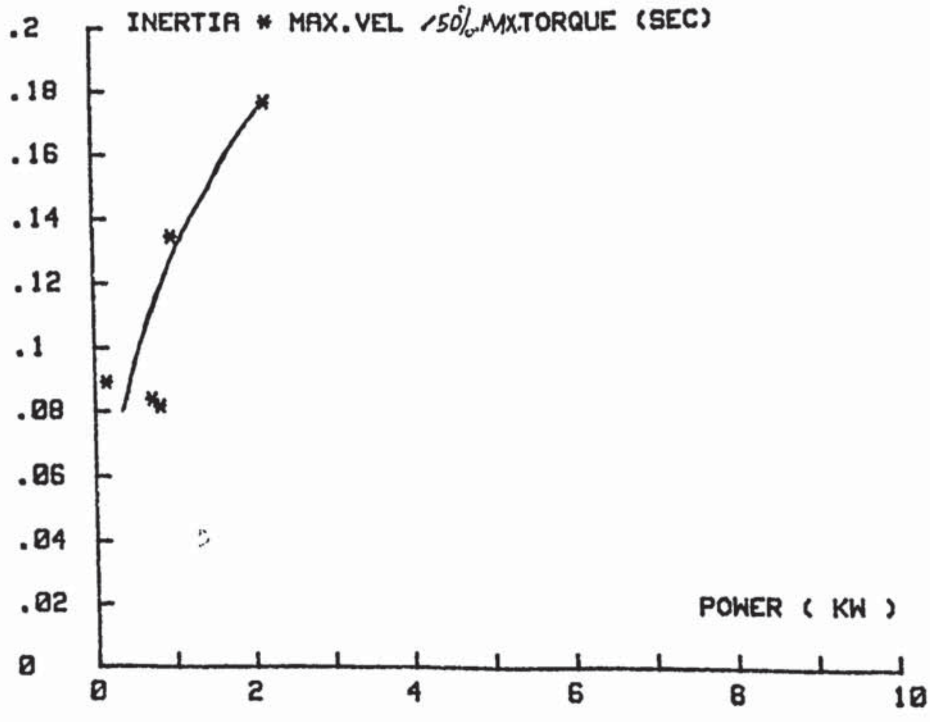


Fig. 6.21 Inertia x max. velocity/50% of max. torque vs. power rating of stepping motor.

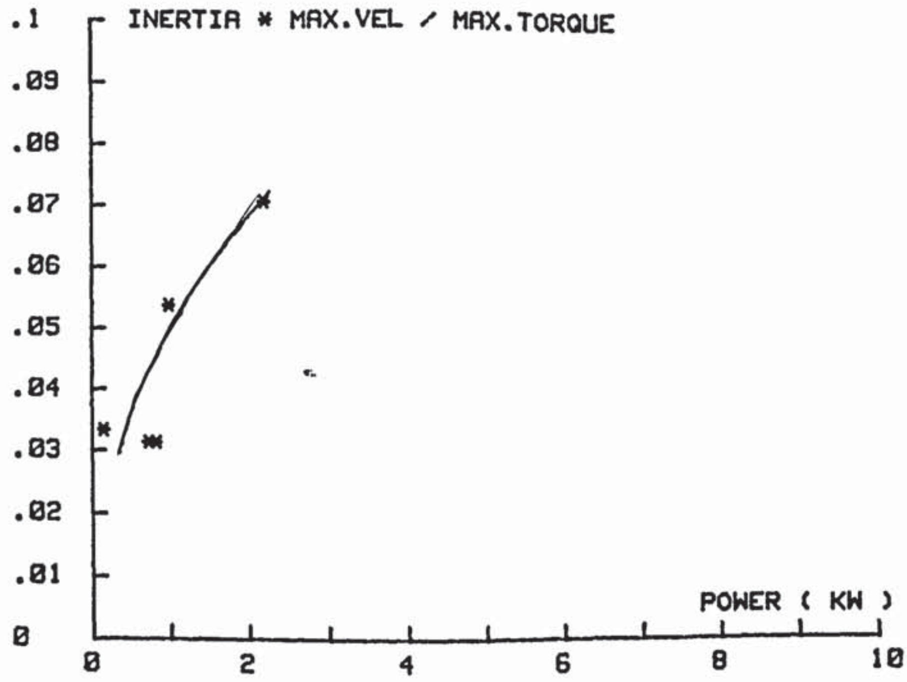


Fig. 6.22 Inertia x max. velocity/max. torque vs. power rating of stepping motor.

CHAPTER 7

ELECTROHYDRAULIC SERVO MOTORS

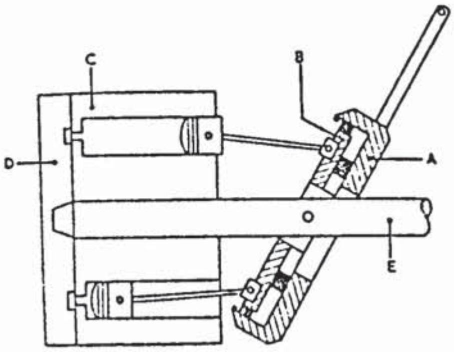


Fig. 7.1 Schematic diagram of an axial piston type motor.

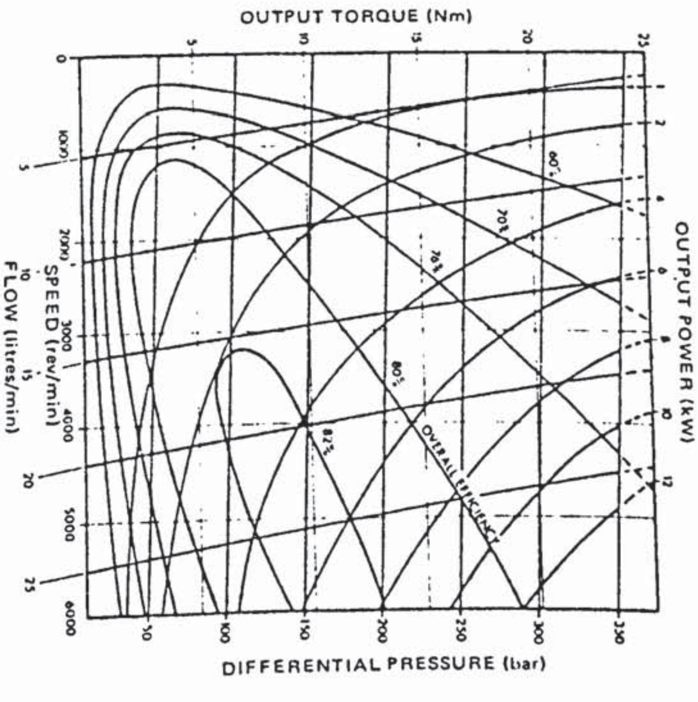


Fig. 7.2 A typical static characteristic of an axial piston type motor.

7. Electrohydraulic Servo Motors

7.1 Introduction

One major advantage fluid has over electrical power is that the former is not bound by the physical limitation of the material; for example, the saturation limit of steel limits the performance of the electro magnet, and available force per unit area from the hydraulic system is something like 10 times that of a saturated magnet. Therefore, hydraulic units are used whenever a large force for a small volume or a small weight is required. This applies particularly to heavy machinery such as mining machines and mobile equipment but is not so important as in machine tools where accuracy and stability are the prime requirements. In theoretical analysis, the hydraulic power units are more complicated than the electrical because of the degree of non-linearity in the mathematical model, which will be shown later. In this work the accuracy, stability and performance of a 10 kW electrohydraulic servo motor will be analysed. Although this analysis is done on a particular motor, similar results may be obtained for other power range of motors.

7.2 Principle Operation of Hydraulic Motor

As was discussed in the literature survey there are many types of hydraulic motors, i.e. radial and axial types. In high performance systems the axial type with constant ilow delivery is commonly used. This is due to its excellent accuracy and low mass/power ratio and performance. In this section the principle operation of this type of hydraulic motor is discussed. Fig. 7.1 shows a schematic diagram of an axial piston type motor and Fig. 7.2 shows its typical static performance characteristic. The angle of the swash plate may be fixed or variable. The angle defines the volume of oil delivered per one revolution. In most applications of machine tools, the angle of the swash plate is kept constant and the flow is controlled via an electrohydraulic servo valve. In the steady state the velocity of the motor will depend on the oil flow rate to it:

$$Q_m = C_m \omega_m \qquad (7.1)$$

where Q_m is the flow rate, C_m is the motor displacement and ω_m is the velocity of the motor. In order to complete the flow equation, the compressibility and leakage flow must also be added to equation 7.1. The leakage flow is proportional to the pressure difference between the inlet and outlet of the motor:

$$Q_L = \lambda_m \Delta p \qquad (7.2)$$

where λ_m is the motor leakage coefficient. When oil is discharged from a motor at high pressure, there will be some compression of oil volume due to the slight compressibility of the oil at high pressure. A volume of oil V_o at atmospheric pressure will be reduced to a volume V, at a gauge pressure p, according to the relation:

$$V = V_o(1 - \frac{p}{B}) \qquad (7.3)$$

where B is the Bulk modular of compressibility. The rate of change of volume with pressure is:

$$\frac{dV}{dp} = - \frac{V_o}{B} \qquad (7.4)$$

or

$$\frac{dV}{dt} = - \frac{V_o}{B} \frac{dp}{dt} \qquad (7.5)$$

Hence the total flow rate at high pressure may be written as:

$$Q_m = C_m \omega_m + \lambda_m \Delta p + \frac{V_o}{B} \frac{dp}{dt} \qquad (7.6)$$

By analogy of Q_m to voltage (V) and p to current I equation 7.6 is similar to the voltage equation of the electrical motors. Allowing

Fig. 7.3 Schematic diagram of electro-hydraulic servo valve.

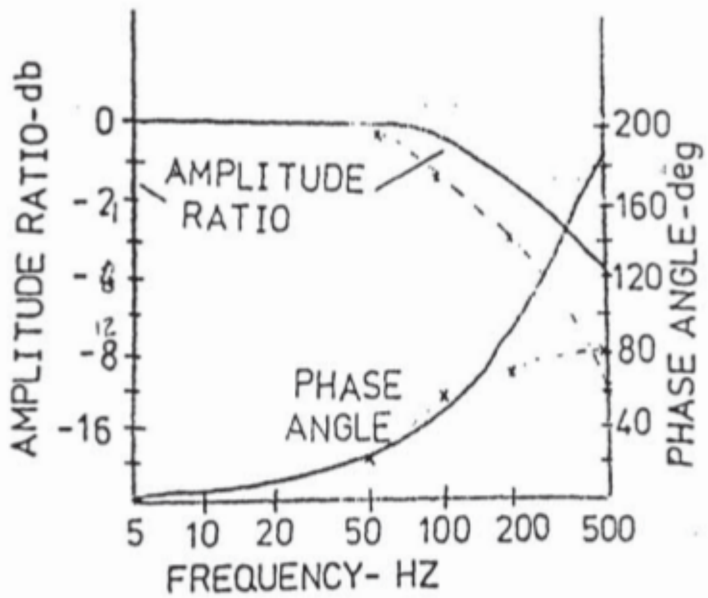


Fig. 7.4 Frequency response and phase angle of electro-hydraulic servo valve.

—— Exact response

-x-x-x- Response with assumption of first order system.

for mechanical losses between the rotor and the supplied power, the torque equation of the motor may be written as:

$$T_m = \eta_m C_m \Delta p \tag{7.7}$$

where η_m is the efficiency of the motor. Equation 7.7 is also similar to the torque equation of D.C. motors. The above analysis shows that the mathematical model obtained for electrical motors in Chapter 3 may be used for hydraulic motors. Although the above analysis is done on an hydraulic axial type motor, similar results may also be obtained for other types of hydraulic motors, which are beyond the scope of this work.

7.3 Controller of a Hydraulic Motor

The controller of a hydraulic motor is very similar to that of a D.C. motor except that the power amplifier is substituted by an electrohydraulic servo valve. The electrohydraulic servo valve is a component which, in response to an electric current input, provides a proportional output in the form of a hydraulic flow. Fig. 7.3 shows a typical one stage electrohydraulic servo valve. The types of servo valve now available commercially represent a wide variety of designs based on different principles [58]. A servo valve consists basically of a hydraulic valve which is controlled by an electric actuator, although many are more complex, having several hydraulic stages and various devices for internal feedback or stabilization. In this section the mode of operation of a fairly simple modern servo valve is considered. The input current, Δi, entering the servo valve is applied to a push-pull torque motor having a permanent magnet and two sets of windings, as shown at the top of the sectional elevation in Fig. 7.3 The torque developed (C) on the floating flapper plate is a function of Δi:

$$C = f(\Delta i) \tag{7.8}$$

Under the action of the torque C the flapper plate tends to close one of the variable orifices O_2 and to open the other: the result is a difference in the pressure p' and p". This is the principle of the operation of the double flapper and nozzle hydraulic potentiometer which constitutes the first hydraulic stage of this servo valve:

$$p' - p'' = f(C) \tag{7.9}$$

The valve spool is set in motion as a result of the action of the pressure difference (p' - p"). This movement compresses one of the end springs and extends the other. Movement ceases when the force from the springs balance the pressure force. Therefore, a certain spool position, y, corresponds to certain pressure difference (p' - p"):

$$y = f(p' - p'') \tag{7.10}$$

The input current, therefore, controls the position of the valve spool. In comparing equations 7.8, 7.9 and 7.10, the relation between spool position, y and the current Δi may be written as:

$$y = f(\Delta i) \tag{7.11}$$

The exact mathematical relationship between Δi and y is very complex and involves non-linear equations. It has been shown by Bell [57] that an approximation of a first order relation between y and Δi is sufficient in servo motor drive applications for predicting the performance of the servo valve. Therefore,

$$y = \frac{K_v}{1 + \tau_v S} \Delta i \tag{7.12}$$

where K_v and τ_v are the gain and the time constant of the valve. A typical frequency response of a servo valve is shown in Fig. 7.4. It can be seen from this graph that the assumption of equation 7.12 is valid, since the frequency band width of servo valves is much higher than the motor.

The flow equation through a servo valve with a spool opening y may be written as [53]:

$$Q = Ky\sqrt{(p_1 - p_2)} \tag{7.13}$$

where p_1 is the supply pressure and is constant. p_2 is the back pressure. The equation 7.13 is non-linear and may be linearized about the steady state:

$$q = C_y y - C_p p_2 \tag{7.14}$$

where

$$C_y = \left(\frac{\partial Q}{\partial y}\right)_{p_2 = \text{cons}} = K\sqrt{(p_1 - p_2)} \tag{7.15}$$

$$C_p = -\left(\frac{\partial Q}{\partial p_2}\right)_{y=\text{constant}} = \frac{+\,Ky}{2\sqrt{p_1 - p_2}} \tag{7.16}$$

The value of C_p and C_y depend on the operating point, i.e. p_2 and y. As can be seen from 7.15 and 7.16, at a constant spool opening y, C_y reduces as p_2 increases and C_p increases as p_2 increases. The effect of non-linearity of the flow equation and the influence of the variation of C_p and C_y on the performance will be discussed later.

7.4 The Dynamic Performance of the Electrohydraulic Servo Motor

7.4.1 Introduction

It has been shown experimentally by Cessford [53] that the linearized mathematical model obtained in the previous section is satisfactory for predicting the performance of electrohydraulic servo motors. In this section the performance of a hydraulic axial piston motor type pm60, from Lucas Fluid Power is investigated (Appendix F). The maximum power rating of this motor is up to a maximum of 12 kW. The characteristic of the electrohydraulic valve is shown in Fig. 7.5, with a time constant of 0.0016 sec. The performance of this motor with the modified mathematical model of Chapter 3 is studied for different applications in both velocity and position control modes. Furthermore, the optimized performance using compensation techniques such as acceleration feedback and lead-lag network is investigated. Finally, the effect of non-linearity of the flow equation, which results in a variation of C_p and C_y, on the system is studied.

7.4.2 The Effect of Load Inertia on the Performance of the System.

The performance of the system for a step input of velocity at different load inertia is shown in Fig. 7.5. As can be seen from this graph, the speed of response of the system reduces as the load inertia increases. The effect of torque on the system at different load inertia is shown in Fig. 7.6. The dynamic characteristic is the same as the input response. The dynamic velocity drop reduces as the inertia is increased. The speed of recovery reduces considerably as the load inertia is increased. Generally as the load inertia increases the performance of the system deteriorates without using compensation.

7.4.3 The Effect of Load Natural Frequency on the System.

The effect of load natural frequency is shown in Fig. 7.7. In contrast to the electrical motor, the reduction in the load natural frequency does not reduce the damping of the loop frequency but increases it. The oscillation of load natural frequency becomes more dominant in the response as seen in 7.7. Therefore, for designing the servo motor in the worse conditions, i.e. high stiffness, transmission must be considered which in turn will provide a good performance in normal conditions. At very low load natural frequency, however, the oscillation of the two inertia becomes dominant and is not desirable.

7.4.4 Performance of the System in a Position Control Mode.

As was shown in previous chapters, a system with a better perfor-mance in velocity control mode will give a better performance in position control. The performance of the system in position control for a unit step input of position is shown in Fig. 7.8. It can be seen that firstly the gain of the position amplifier is adjusted in such a way that there is no overshoot in position response. Secondly, as the result of the load inertia increase there is a slower response of position. The effect of torque on the position accuracy is shown in Fig. 7.9. The dynamic error increases as the load inertia is increased and also results in a slower speed of recovery. The following error of the system for a ramp input with different load inertia is given in Table 7.1. It can be seen that the following error increases as the load inertia is increased.

7.4.5 Improvement on the Performance Using Acceleration Feedback as Compensation.

In order to improve the performance of the system, the gain must be increased. The increase in the gain results in a reduction of the damping of the system. It was found that the acceleration feedback around the motor is an effective way of improving the performance of the system. The use of acceleration feedback on the hydraulic motors is investigated here. The study is only carried out in velocity control mode. The performance in position control, however, follows the performance in velocity control mode. The gain of the system is increased 10 times, then acceleration feedback gain is adjusted to increase the damping and to achieve 0.7 damping ratio in the fundamental frequency of the system. Fig. 7.10 shows the optimized response of the system for a step input of velocity.

The effect of torque is shown in Fig. 7.11. As can be seen from these two graphs, with the use of acceleration feedback, considerable improvement can be achieved on the performance. The increase of the gain in this manner is only limited by the existence of noise in the various transducers used.

7.4.6 The Effect of Non-Linearity of the Electrohydraulic Servo Valve.

As was shown in the previous section, the linearized coefficients of the flow equation varies as the state of the system changes. In this section,by keeping all the parameters of the various parts of the system constant, the effect of variation of C_p and C_y on the performance is investigated. It can be seen, that C_y is only a function of back pressure p_2 (applied torque). C_p is a function of back pressure, p_2 and, the displacement of the spool of the valve, y. It was found that at a constant loading the variation of C_p had little effect on the performance of the system. The damping of the system is at its lowest value where the motor is operating at low speed and no load conditions since C_y is maximum value and C_p is at its minimum value. As the speed increases C_p increases which introduces slight damping into the system, since it is an internal feedback (semi acceleration feedback). This effect is very little because the opening of the valve y is limited. When the load on the motor increases C_y reduces and C_p increases. The reduction in the value of C_y and the increase of C_p results in a lower loop gain. This introduces damping at the expense of lowering the speed of response. The increase in the value of C_p also increases the damping of the system. These are shown in Fig. 7.12. It was shown that the critical condition of stability is at slow speed no torque conditions.

In order to obtain a good performance it is essential that the system be optimized in this region.

7.5 Static Characteristics of the Hydraulic Motor

The hydraulic motors are capable of providing power of quite a wide range. The motor under investigation was capable of delivering up to 12 kW power. This, in a way is a disadvantage of the hydraulic motor, because at even a low power rating one has to choose the high power motor. Due to the wide range power rating of hydraulic motors the static characteristic of the 12 kW hydraulic motor is given in Table 7.2. These static characteristics will be compared with other types of electrical motor in Chapter 8.

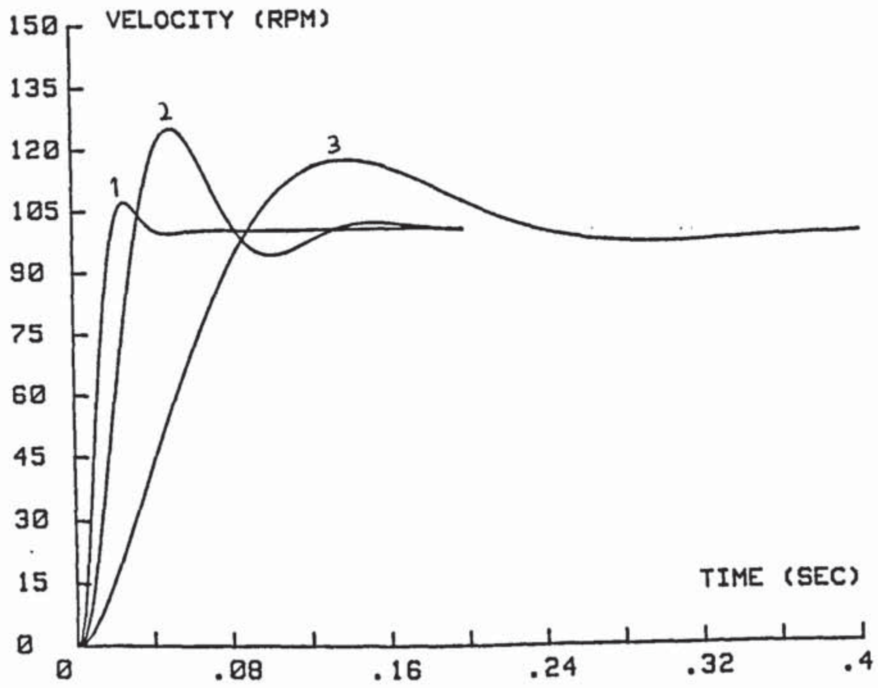


Fig. 7.5 Response of the hydraulic motor for a step input of velocity.

1. No load inertia
2. Load inertia of 0.0024 $Kg.m^2$ = 300% of rotor
3. Load inertia of 0.008 $Kg.m^2$ = 1000% of rotor

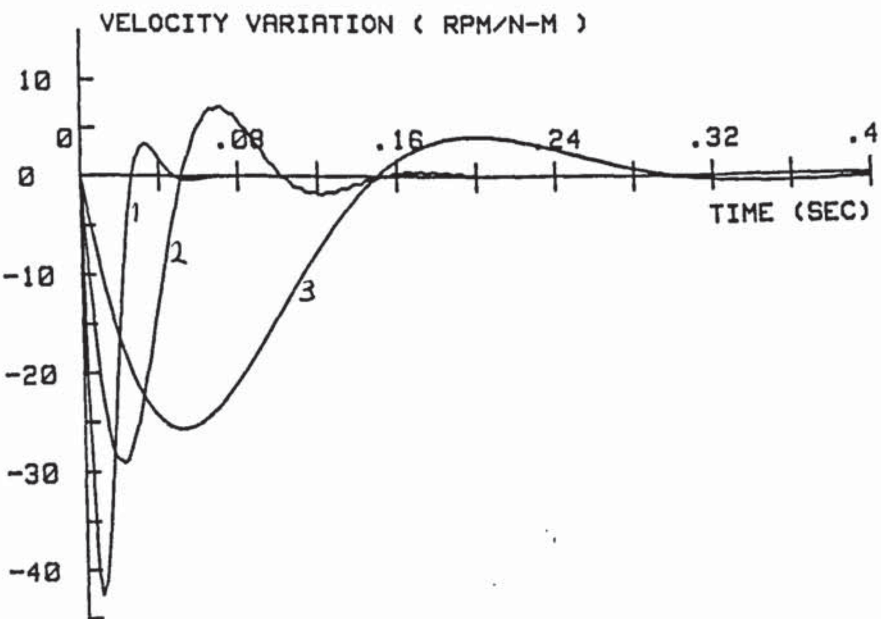


Fig. 7.6 Velocity variation of the hydraulic motor for a unit step input of torque.

1. No load inertia
2. Load inertia of 0.0024 $Kg.m^2$
3. Load inertia of 0.008 $Kg.m^2$

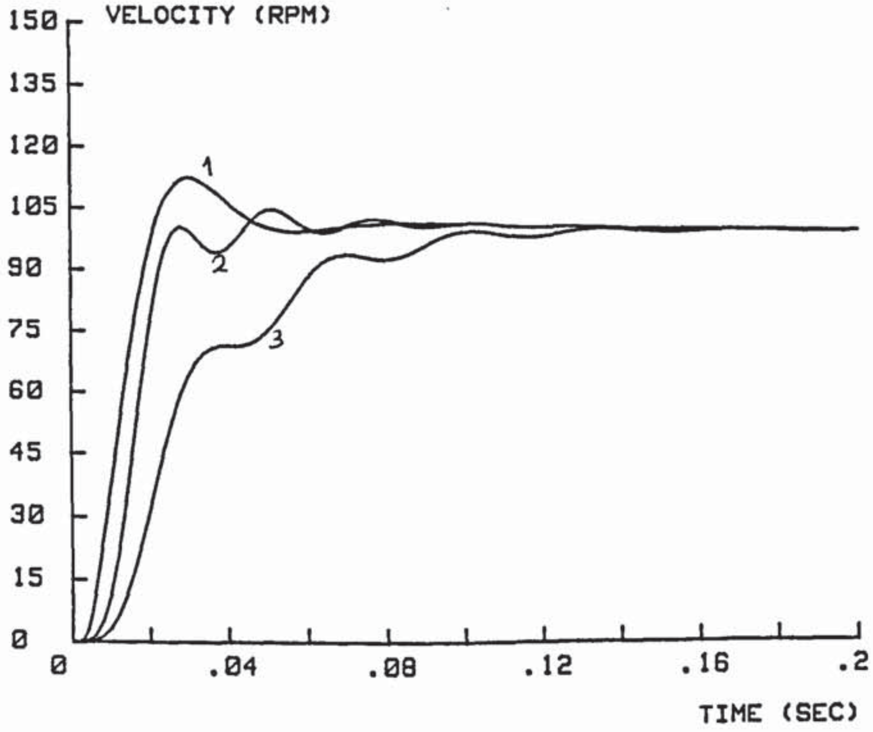


Fig. 7.7 Effect of load natural frequency on the system.

1. Natural frequency of 140 Hz
2. Natural frequency of 40 Hz
3. Natural frequency of 30 Hz

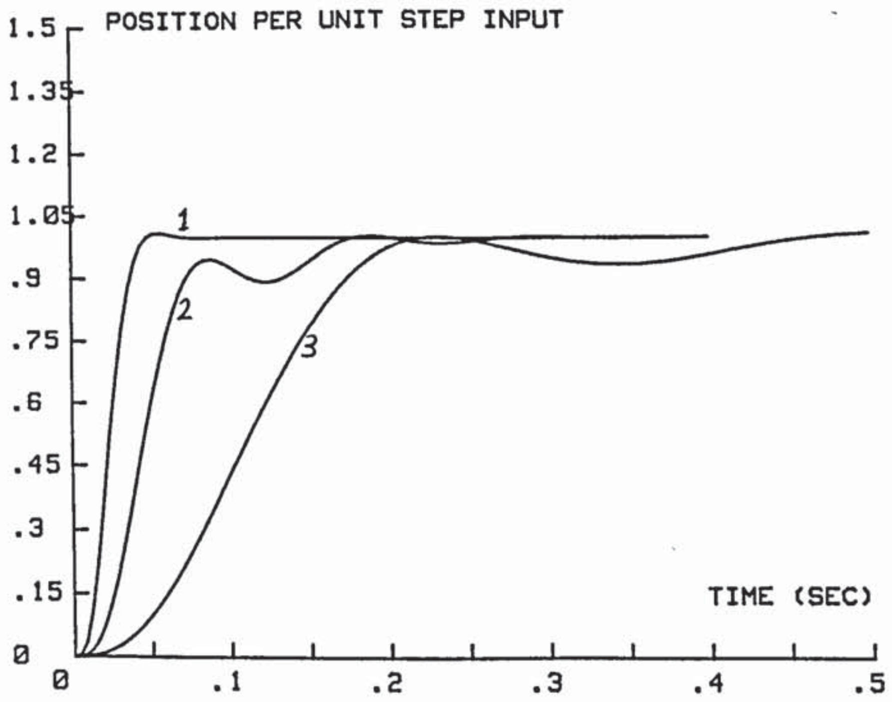


Fig. 7.8 Position response of the system for a unit step input.

1. No load inertia
2. Load inertia of 0.024 $Kg.m^2$
3. Load inertia of 0.008 $Kg.m^2$

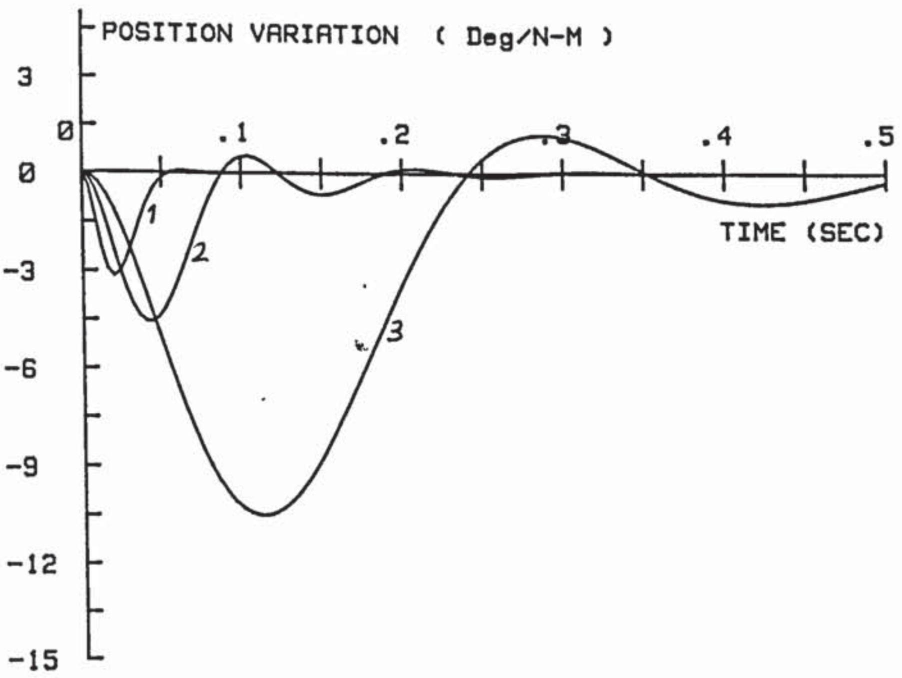


Fig. 7.9 Position variation of the system for a unit step input of torque.

1. No load inertia
2. Load inertia of 0.0024 $Kg.m^2$
3. Load inertia of 0.008 $Kg.m^2$

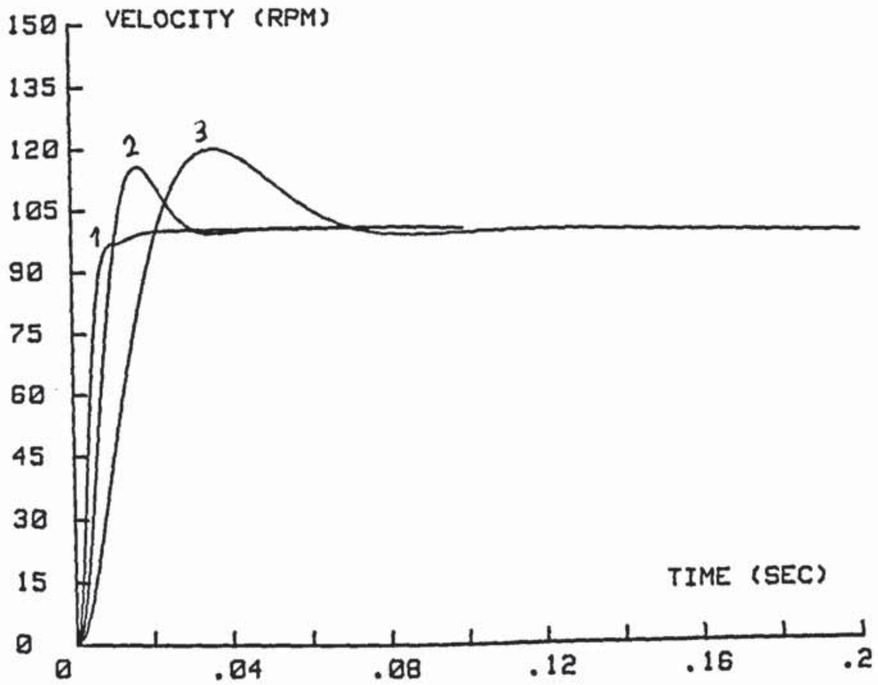


Fig. 7.10 Optimized response of the system for a step input of velocity.

1. No load inertia
2. Load inertia of 0.0024 $Kg.m^2$
3. Load inertia of 0.008 $Kg.m^2$

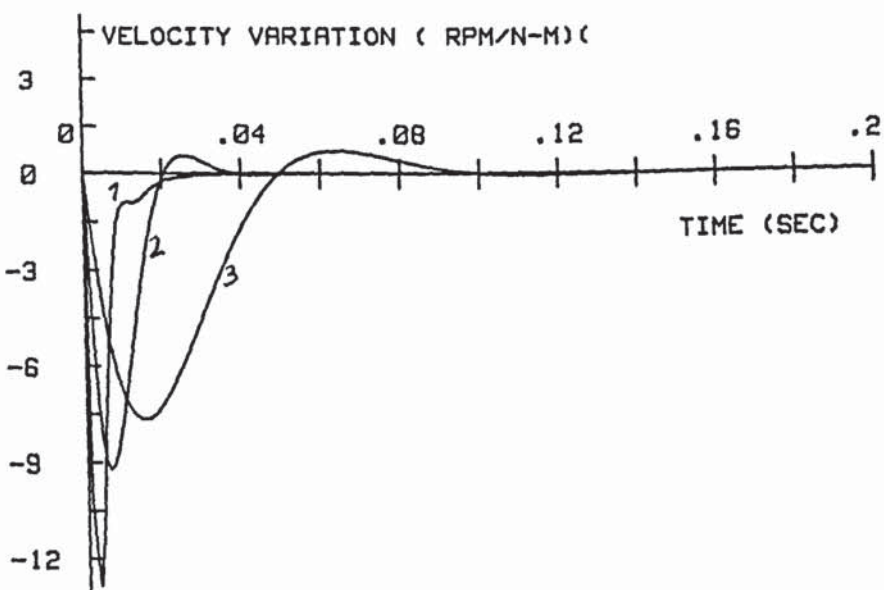


Fig. 7.11 Variation of the velocity for a unit step input of torque.

1. No load inertia
2. Load inertia of 0.0024 $Kg.m^2$
3. Load inertia of 0.008 $Kg.m^2$

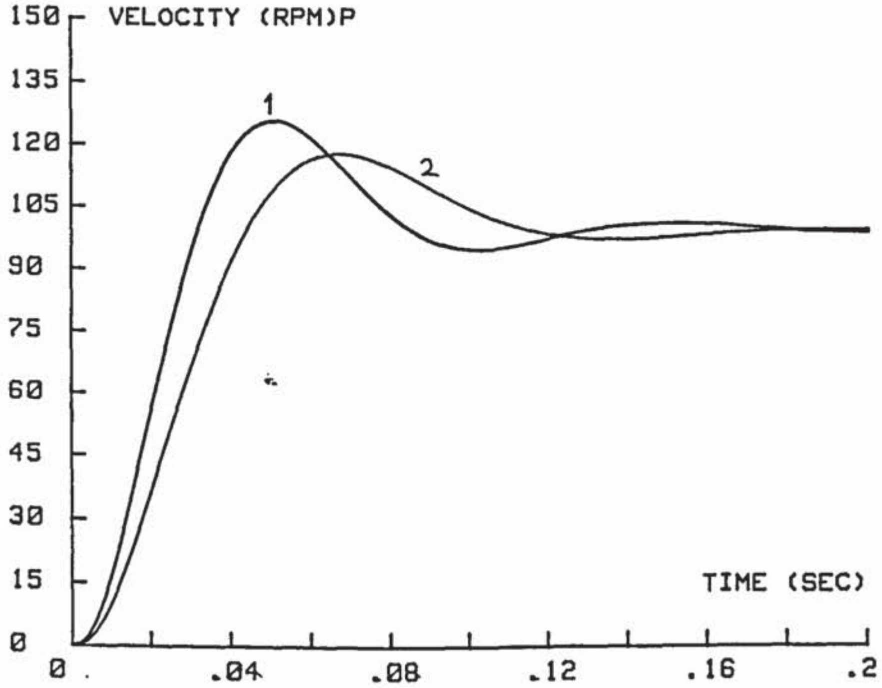


Fig. 7.12 Effect of non-linearity on the system.

1. Low speed, no load torque
2. High speed, 100% of rated torque

Table 7.1. Following error of electrohydraulic servo motor.

Load Inertia ($Kg.m^2$)	Following Error deg/(r.p.m.)
0	0.14
0.0024	0.3
0.008	0.7

Table 7.2. Static characteristics of hydraulic motors.

Parameter	Value	
Mass/power	0.3	(Kg/kW)
Inertia/max. torque	1.6×10^{-5}	(sec)
Inertia/rated torque	3.2×10^{-5}	(sec)
Inertia x max. velocity/max. torque	8.4×10^{-3}	(sec)
Inertia x max. velocity/rated torque	16.7×10^{-3}	(sec)

CHAPTER 8

COMPARISON OF THE DYNAMIC AND STATIC CHARACTERISTICS OF SERVO MOTORS.

8. A COMPARISON OF THE DYNAMIC AND STATIC CHARACTERISTICS OF SERVO MOTORS.

8.1 Introduction

In this chapter the dynamic performance of different types of servo motors are compared. A comparison of 1 HP servo motors for a specific application is published and presented in Appendix G. In the paper the static characteristics of up to 10 kW motors are also examined and therefore will not be included in this section.

This chapter is an extension of the published paper and will tackle the comparison in a different manner. All types of servo motors are compared at various load inertia within their respective power ranges of 1, 3, 5 and 10 kW. In this comparison it is assumed that the natural frequency of the power transmission mechanism is high and will not affect their performance. This was examined in depth for each type of motor in their respective chapters. In D.C. motor types a prediction of the performance of the rare earth magnet and brushless motors is also given.

It is desirable to use the static and dynamic characteristics established in this thesis as a means of choosing an appropriate servo motor for a specific application and a detailed procedure of this is given in Appendix H.

The performance of the following servo motor drives are compared in this chapter.

1. Ceramic magnet D.C. motors with the following controller.
   a) Thyristor controlled with current frequency of 50 Hz (they are shown on the enclosed graphs by 1a).

b) Thyristor controlled with current frequency of 150 Hz (they are shown on the enclosed graph by 1b).

c) pwm drive with current frequency of 2 kHz (they are shown on the enclosed graphs by 1c).

2. A.C. servo motors with D.C. link converter as controller (they are shown on the enclosed graphs by 2).
3. Stepping servo motors (they are shown on the enclosed graphs by 3).
4. Electrohydraulic servo motor (it is shown on the enclosed graphs by 4).
5. Prediction of the performance of rare earth magnet D.C. motors with pwm drive (they are shown on the enclosed graphs by broken lines -----).
6. Prediction of the performance of brushless D.C. motors with pwm drive (they are shown on the enclosed graphs by a dotted line -.-.-.-.).

8.2 Criteria of Comparison

For a realistic comparison, two important dynamic characteristics must be considered, i.e. 1) the speed of response to a command signal, and 2) the effect of external torque. The speed of response can be determined by the settling time as shown in this graph. The settling time is the time that the system will become fixed within some percentage of its steady state value (5% in this comparison). The settling time is used as a tool to compare the speed of response of the system. These settling times are obtained from the analyses previously made on each type of motor.

A typical response of the system for a step input of load torque is shown in Fig. 8.2. The shape of response is evidently similar to

Fig. 8.1. Two important errors can be seen, i.e. 1) steady state error, and 2) dynamic error. Most high performance system contains a feed forward integrator in the controller to achieve zero steady state error. The dynamic error of different types of servo motors will be compared here. The speed of recovery in some application is important. The speed of recovery is similar in value to the response time of the system and also depends on the gain of the feed forward integrator.

It was shown in Chapter 3 that a system with a better performance in velocity control mode will provide a better performance in position control mode. This was expanded further for each type of motor in Chapters 4, 5, 6 and 7. A damping ratio of around 0.7 in the fundamental frequency of velocity control mode will provide further improvement on the performance of position control mode. The parameters of controllers, i.e. velocity amplifier gain, the time constants of lead-lag network and the gain of current feedback are optimized in such a way that the best possible performance with damping ratio of 0.7 in velocity control mode is obtained.

For these reasons only the performance of the system in velocity control mode is considered.

8.3 Comparison of the Performance of D.C. Servo Motors

Figs. 8.3, 8.5, 8.7 and 8.9 show the settling time of four different sizes of motors versus load inertia. Upon inspection of these graphs the following conclusions for the speed of response of D.C. motors may be drawn.

1. The dynamic settling time for a specific motor and controller increases as the load inertia increases. The reduction in the speed of response is basically due to the reduction of the frequency

of the fundamental loop frequency. In addition the gain of the system must be further reduced for stability and the current limitation during one cycle of supplied voltage. The rate of increase in settling time depends on the maximum torque availability of the motor, i.e. the gain of the system can be further advanced for a motor with a higher maximum torque. Generally as the power of the motor increases, whilst keeping the rated velocity constant the maximum torque of the motor increases. Therefore it can be seen on the enclosed graphs for the 1, 5 and 10 kW motors that the slope of the settling time curve is reduced for a higher power rating motor. However, graph 8.5 shows that the slope of settling time of the 3 kW D.C. motors is increased when compared with the 1 kW motor. This is due to the fact that although the power has increased, the maximum torque available is the same as the 1 kW motor (Appendix F). The increase in the power is due to the higher rated torque and velocity of the motor.

2. At low load inertia a smaller motor gives a faster speed of response. At higher load inertia a larger motor gives faster speed of response. At a certain value of load inertia the curves of two sizes of motor intersect, which indicates that at this value the two motors possess the same dynamic settling time. Therefore a smaller size motor is more suitable below this point and vice versa above the intersection.
3. The settling time, for a specific motor and application, reduces as the current frequency of the controller increases. At higher frequency the controller allows faster current rise and consequently the gain of the system is further advanced. This was thoroughly examined in Chapter 4.
4. By use of some new magnet materials, such as the rare earth magnet, the performance of the motors further improved. This is shown in

the enclosed graphs by a broken line. The improvement achieved is due to the increase of the maximum torque capability of the motor which is almost double the ceramic magnet type. By making these motors brushless, further improvement is achieved because of the reduction of the rotor inertia as shown on the enclosed graphs. The settling time of brushless motors is obtained on the assumption that the maximum torque availability of the motor remains the same as the rare earth magnet type. Further data needs to be obtained from the manufacturer's to substantiate this assumption.

The dynamic error, due to a step input of load torque on the above types and power rating of motors versus load inertia, is shown in Figs. 8.4, 8.6, 8.8 and 8.10. The following conclusions may be drawn from the graphs.

1. The dynamic error of ceramic magnet type slightly increases as the load inertia increases. This is due to the reduction in gain of the system in order to observe the commutation current limit during one cycle of power supply. The dynamic error reduces however when using a controller with a higher current frequency. The error further reduces for rare earth magnet and brushless types of D.C. motors.
2. The error reduces as the size of the motor increases for various contradictory reasons. Firstly, the inertia of the motor and its maximum torque availability increases as the power of the motor increases. These two have an opposite effect on the gain of the system. The increase in inertia imposes a lower gain on the controller as mentioned before. The increase in maximum torque availability allows a higher gain to be used. The resultant gain is then dependent upon the more dominant effect of the two. It

was shown in Chapter 4, concerning the static characteristics of D.C. motors that the (inertia/maximum torque) ratio increases for larger motors. Therefore, the increase of inertia is slightly more dominant in most cases. This is particularly significant for 3 kW motors as shown in Fig. 8.6. For this 3 kW D.C. motor the maximum available torque is similar to the 1 kW motor, but the inertia has increased (Appendix F). This imposes a lower gain on the system of the 3 kW motor than is usually found in a 1 kW motor and this results in a higher dynamic error.

The resistance and inductance reduction of the larger motor is most important. This means that the motor is capable of developing higher and faster torque at a lower velocity drop than a smaller motor. It can be seen from the enclosed figures for 1, 5 and 10 kW motors that the reduction of resistance and inductance is more dominant and this results in a lower dynamic error.

8.4 A.C. Servo Motors

Figs. 8.3, 8.5, 8.7 and 8.9 show the settling time of 1, 3, 5 and 10 kW motors respectively. The resistance and inductance of A.C. motors are generally higher than the D.C. motors. For this reason, especially with the smaller sizes of motors, there is no need for a current limiter and also the current rise is not as large as D.C. motors. Therefore the restriction of gain due to the current rise during one cycle of supplied voltage is removed. The only restriction of the gain is then due to the stability of the system. Upon inspection of the figures of settling time, the following conclusions may be drawn:

1. The settling time for a specific motor increases as the load inertia increases. This is mainly due to the dynamic characteristics of

the system. In addition, the gain also has to be reduced to keep the system stable with adequate damping in the fundamental frequency of the system.

2. At low load inertia, the settling time increases for a larger motor. This is the exact opposite for high load inertia. As with D.C. motors, the settling time curves of each of the motors will intersect at a certain value of load inertia. Therefore, a larger size motor is more suitable above this point and vice versa below the intersection.,
3. With larger motors the slope of the settling time curve versus load inertia reduces more rapidly than its counterpart D.C. motor. This makes the A.C. motors more competitive with D.C. motors at higher power rating.

The dynamic error of these motors for a unit step input of load torque is also shown in Figs. 8.4, 8.6, 8.8 and 8.10 respectively. The following results can be obtained from these graphs.

1. The dynamic error reduces rapidly as the load inertia increases. Although the gain is reduced for reasons of stability, the reduction is not as much as D.C. motors, as was previously discussed. This will allow the A.C. motors to have a smaller dynamic error than D.C. motors at high load inertia. In contrast, the dynamic error at low load inertia is much higher when compared with D.C. motors. Firstly, this is because the resistance and inductance of these motors are high and this results in a large initial dynamic error. Secondly, the inertia of A.C. motors is smaller than the corresponding D.C. motors.
2. The dynamic error reduces as the power of the motor increases. This is predominantly due to the reduction of resistance and inductance of the larger motors. Lower resistance and inductance

results in a larger and faster development of the torque as opposed to less velocity drop.

8.5 Stepping Motors

The stepping motors are basically position control systems. Their characteristics in velocity control mode is slightly different. The input frequency determines the command velocity. The drawback with these kinds of motor is that they are not easily available at higher power ratings. The data available from a limited number of manufacturers was only up to 2 kW power rating. For this reason, the performance characteristics of a 1 kW stepping motor is compared here.

It can be seen from Fig. 8.4 (the curve is shown as 3) that the settling time of these types of motor increases rapidly as the load inertia increases. This is due to the high sensitivity of these motors to load inertia and the associate instability problem.

The settling time of these motors at low load inertia is also higher than the other motors. This is mainly due to the dynamic characteristics of these motors which are open-loop systems. There are indications that in operating these systems in closed-loop, the performance of the motors will improve. It was shown in Chapter 7 that by employing acceleration feedback, damping is introduced into the system and overcomes the well-known problem of instability.

The dynamic error of the stepping motor is limited to a maximum of one step angle. Therefore, the effect of torque on these systems is not shown here. There is, however, a dynamic velocity drop but it is not significant (due to a fast recovery).

8.6 Hydraulic Motors

One major drawback of hydraulic motors is that they are not easily available in a wide range of power rating, especially at the lower power range. The analyses were carried out on a 12 kW power range and this was the smallest standard size available in the markets. The performance of this motor is compared with other sizes of electrical motors up to 10 kW.

It can be seen from the graphs 8.3 to 8.10 (with curve no. 4) that the settling times of these motors increase rapidly as the load inertia increases. This is, in addition to the leakage and compressibility of oil, due to the back pressure. The back pressure reduces the flow rate. This phenomena does not exist in electrical motors, i.e. the increase in current will not reduce the supply voltage. For this reason, a lot of research is being done to develop a servo valve which is internally compensated. The pressure compensated servo valve is the most commonly used. By use of this type of valve the slope of the settling time may be reduced. The settling time of these motors is a lot smaller at low load inertia. This is mainly due to the fact that these motors have very low inertia as compared with electrical motors.

The dynamic error is high at low load inertia for two reasons. Firstly, the inertia of the motor is very small, even smaller than a 1 kW electrical motor. Secondly, some of the error is due to the back pressure as previously mentioned. The dynamic error, however, drops rapidly as the load inertia is increased. This is due to the fact that there is no other restriction on the gain of the system except its stability.

8.7 Comparison of Performance

8.7.1 As a Manufacturer's Design

From the analyses in this chapter, the following conclusions may be drawn:

1. The speed of response of hydraulic motors is higher than the corresponding electrical motors at low load inertia. The speed of response reduces dramatically as the load inertia increases. Above a certain value of load inertia the speed of response will be smaller than electrical motors. The brushless D.C. motors give the fastest speed of response compared with other electrical motors. Rare earth magnet D.C. motors give a good speed of response. Stepping motors give the lowest speed of response and reduce rapidly as the load inertia increases.

   At low power range A.C. motors have higher settling time than D.C. motors. The effect of load inertia on the performance is less than other types of motors. At higher power range A.C. motors become competitive with conventional D.C. motors. The effect of load inertia is further reduced.

   The settling time of electrical motors is obtained on the basis that the maximum torque was available during transient conditions. If the motor is operating continuously in acceleration and deceleration conditions, the settling time increases, since the heat generated limits the available torque. The increase in settling time depends on the ON and OFF period of the motor, which is usually given by the manufacturers.
2. The hydraulic motors give the highest dynamic error at low load inertia. This error reduces as the load inertia increases. The D.C. motors give the lower dynamic error. Stepping motors, being

basically position control systems, have the lowest dynamic error.

A.C. motors give a larger dynamic error than the D.C. motors. This error reduces rapidly as the load inertia increases and could become smaller than the D.C. motors.

3. Generally the dynamic speed of response of all types of systems reduces as the load inertia and the power rating of the motor increases. This implies that a larger motor does not necessarily give a faster dynamic response. A larger motor gives a faster response at the limit conditions where the motor provides the maximum torque for accelerating the load. (Appendix H).
4. As was discussed in this section, there are some advantages and some disadvantages for each type of motor. The final decision of choosing a motor depends on the priority of specific applications.

In addition to the static and dynamic characteristics, the capital cost and reliability are also of major importance. These two factors are very dependent on a variety of factors and are not easily obtainable. However, an approximate capital cost of 1 HP motors is compared in Appendix G. Due to the quality of design of different manufacturers and because of the capabilities of individual units, it was not possible to make a comprehensive cost comparison. Generally, the capital cost increases as the performance of the system increases. It can be said that, at the moment, brushless D.C. motors are more expensive than other types of electrical motors. Then follows the rare earth magnet, printed motors and ceramic magnet D.C. motors. The stepping motors seem to be cheaper than other D.C. motors. However, the A.C. motors are the least expensive but the hydraulic motors do have the cheapeat cost/power ratio, but are more expensive at low power range applications.

On the controller side, the D.C. link invertors are more costly than other types of controllers. The controller of stepping motors seems to be cheaper but its price depends a lot on the option included in the controller. The thyristor controlled drive follows the stepping motor. pwm drives are usually more expensive than the corresponding thyristor controlled ones.

8.7.2 A Comparison of Performance Using Compensation

As was previously shown, the use of acceleration feedback improves the performance of all types of motor at all power ranging. In Appendix G this is compared for each type of 1 HP motors. In this section, the comparison is made for 5 kW motors at different load inertia. Fig. 8.14 shows the settling time of different types of motors versus load inertia. The following results may be drawn from this graph.

1. The improvement on the dynamic speed of response of D.C. motors is not considerable. This is due to the limitation of the motor on the current rise during one cycle of power supply.
2. It improves the performance of A.C. motors considerably and makes them competitive with some conventional D.C. motors.
3. The speed of response of hydraulic motors has been further improved.

The effect of load torque on the compensated system is shown in Fig. 8.15. It can be seen that the dynamic error of all types of motors is improved considerably. The reduction in dynamic error is almost proportional to the increase in gain of the system.

It was found that by the use of acceleration feedback the gain of the system can be increased considerably. The increase in gain was only limited from the existence of noise in the transducer of the systems.

Therefore, it is essential to use high quality transducers in high performance systems.

Key to the Enclosed Graphs

The symbols used on the enclosed graphs refer to the following motors and controllers.

1a. Thyristor controlled ceramic magnet D.C. motor with current frequency of 50 Hz.

1b. Thyristor controlled ceramic magnet D.C. motor with current frequency of 150 Hz.

1c. Pulse width modulated ceramic magnet D.C. motor with current frequency of 2 kHz.

2. A.C. induction servo motor with D.C. link converter as controller.

3. Stepping servo motors.

4. Electrohydraulic servo motors.

------- rare earth magnet D.C. motors with pwm drive

-.-.-.- rare earth magnet brushless D.C. motors with pwm drive.

FIG 8.1. PERFORMANCE CRITERIA FOR A COMMAND SIGNAL.

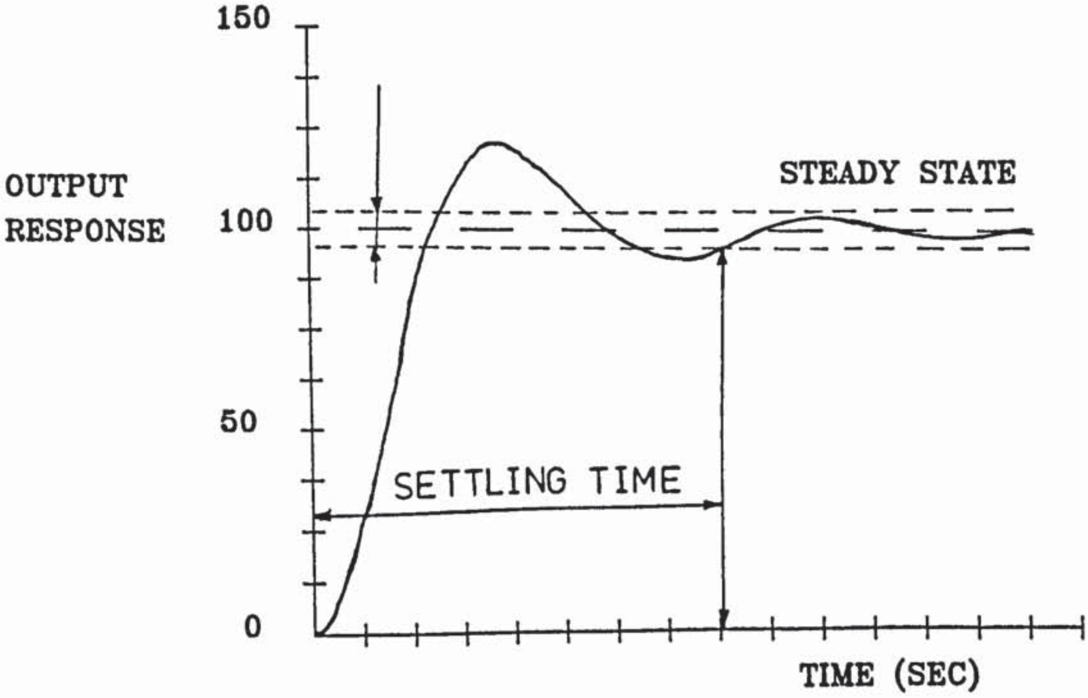
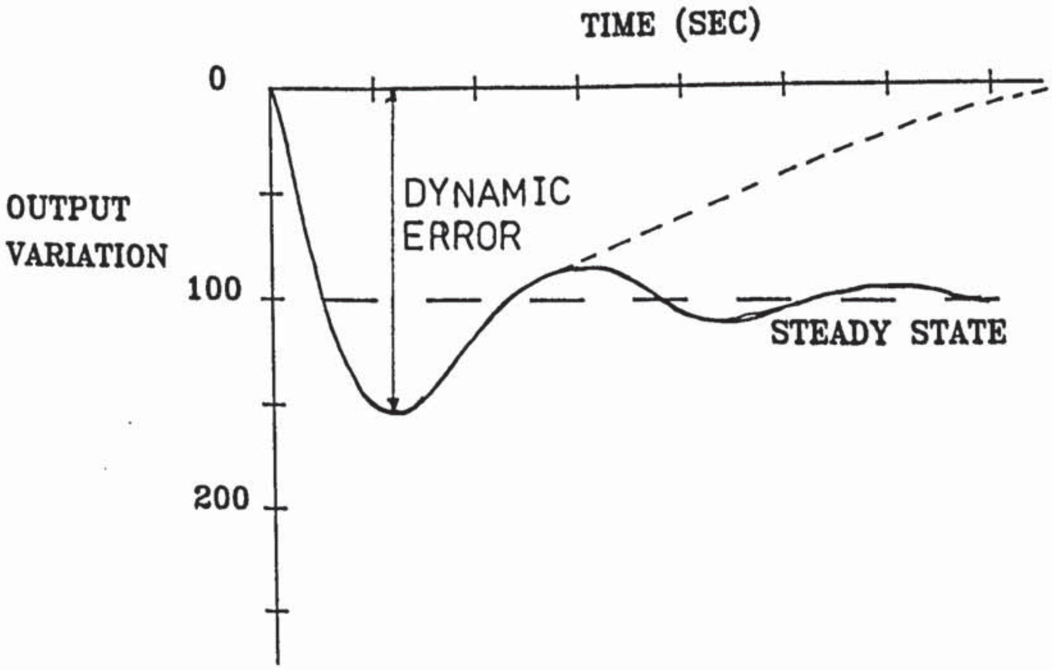


FIG 8.2. PERFORMANCE CRITERIA FOR EXTERNAL DISTURBANCES.



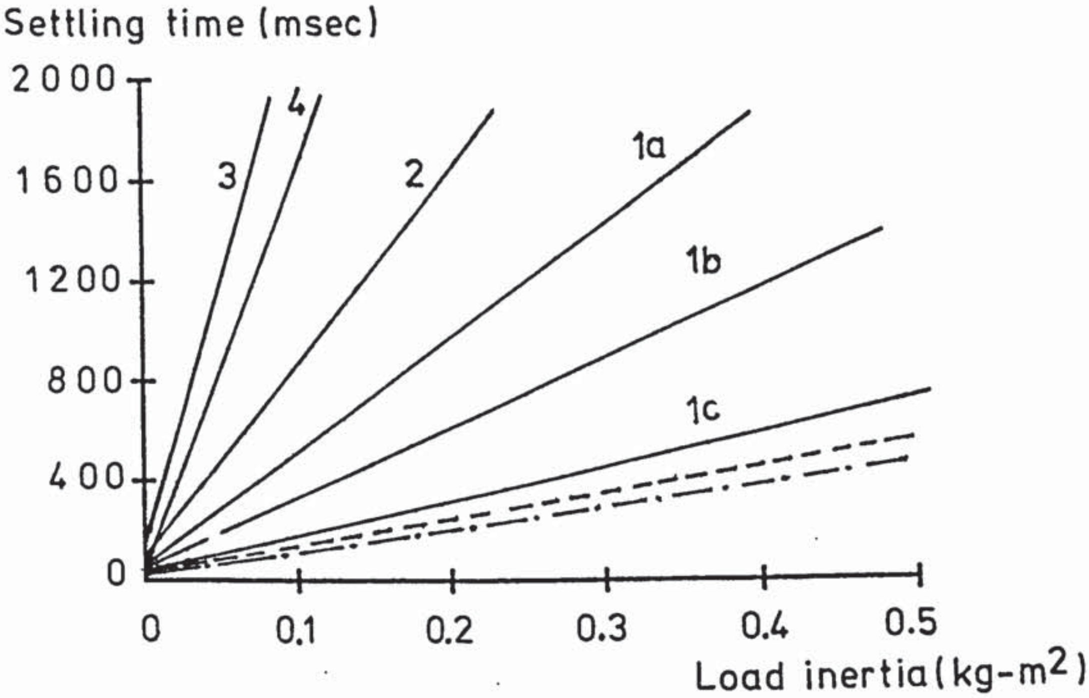


Fig. 8.3 Comparison of dynamic settling time of 1 kW servo motors.

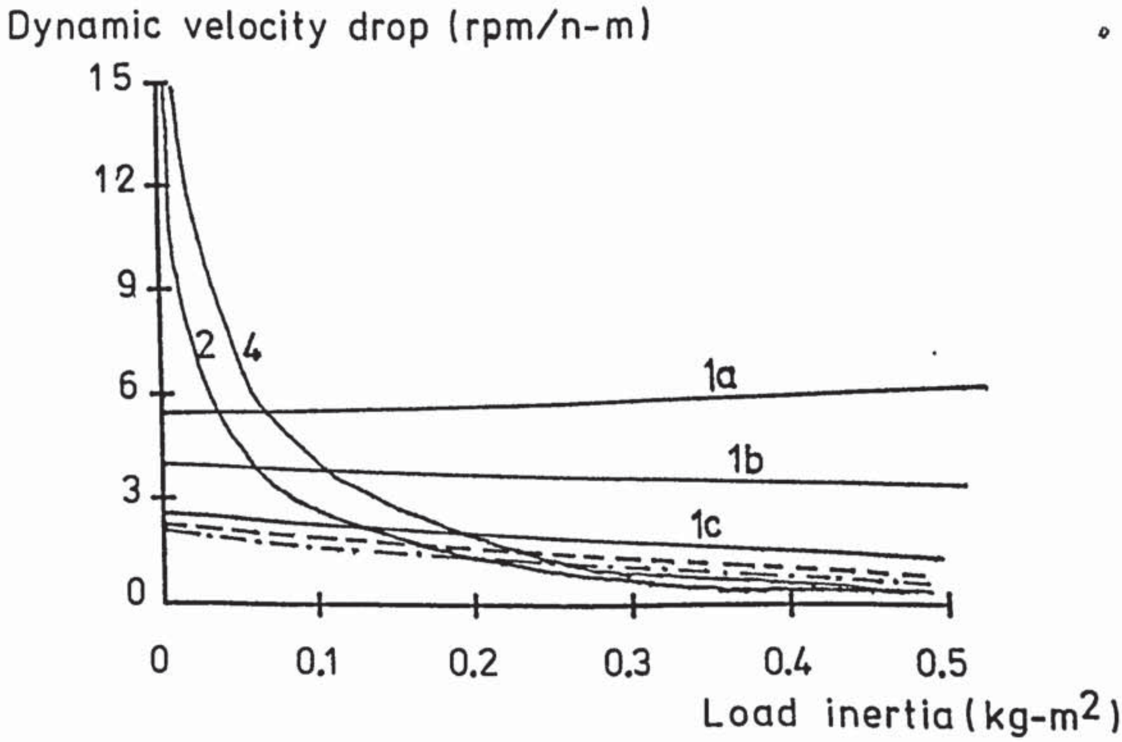


Fig. 8.4 Comparison of dynamic error of 1 kW servo motors for a unit step input of load torque.

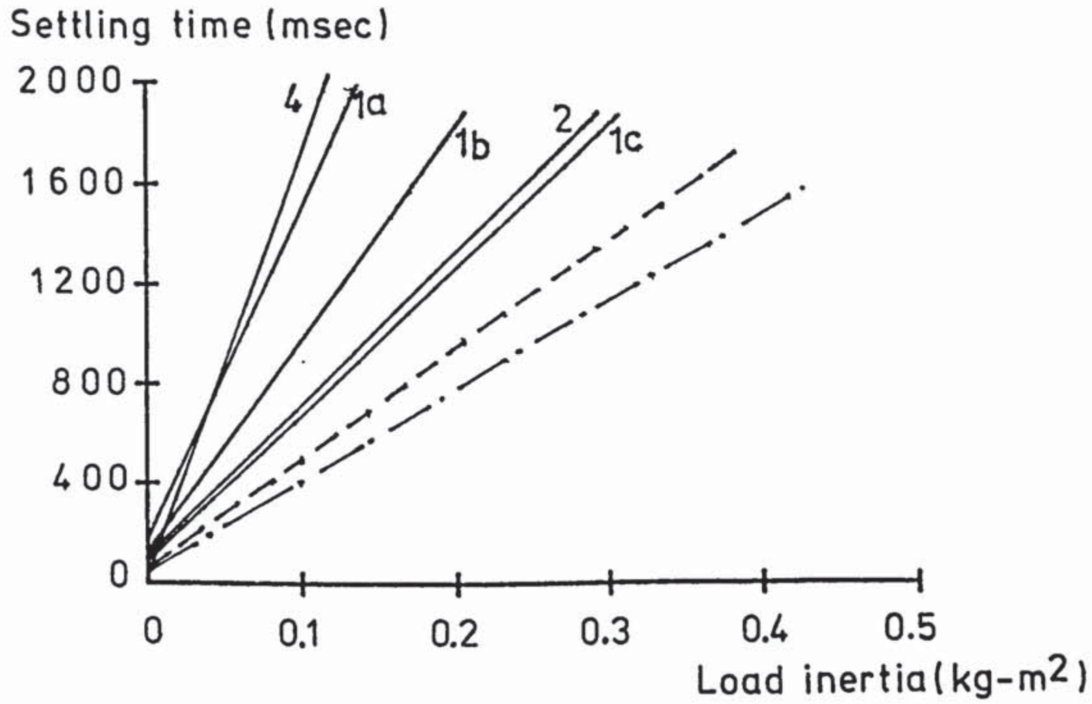


Fig. 8.5 Comparison of dynamic settling time of 3 kW servo motors.

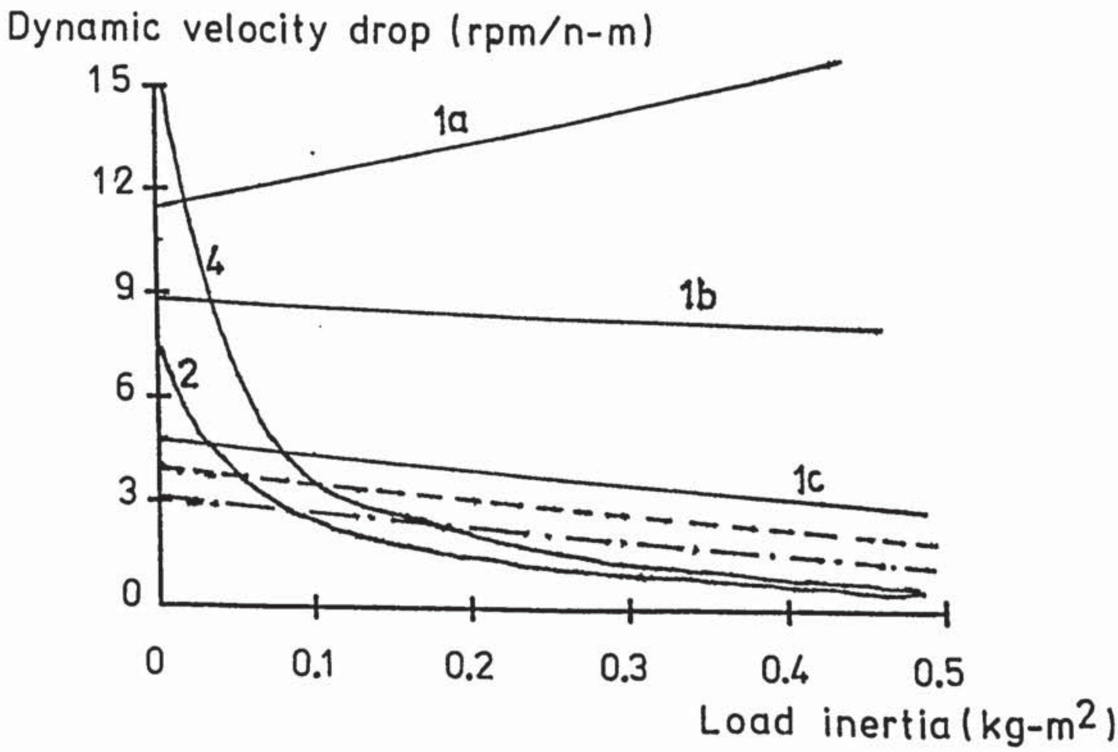


Fig. 8.6 Comparison of dynamic error of 3 kW servo motors for a unit step input of load torque.

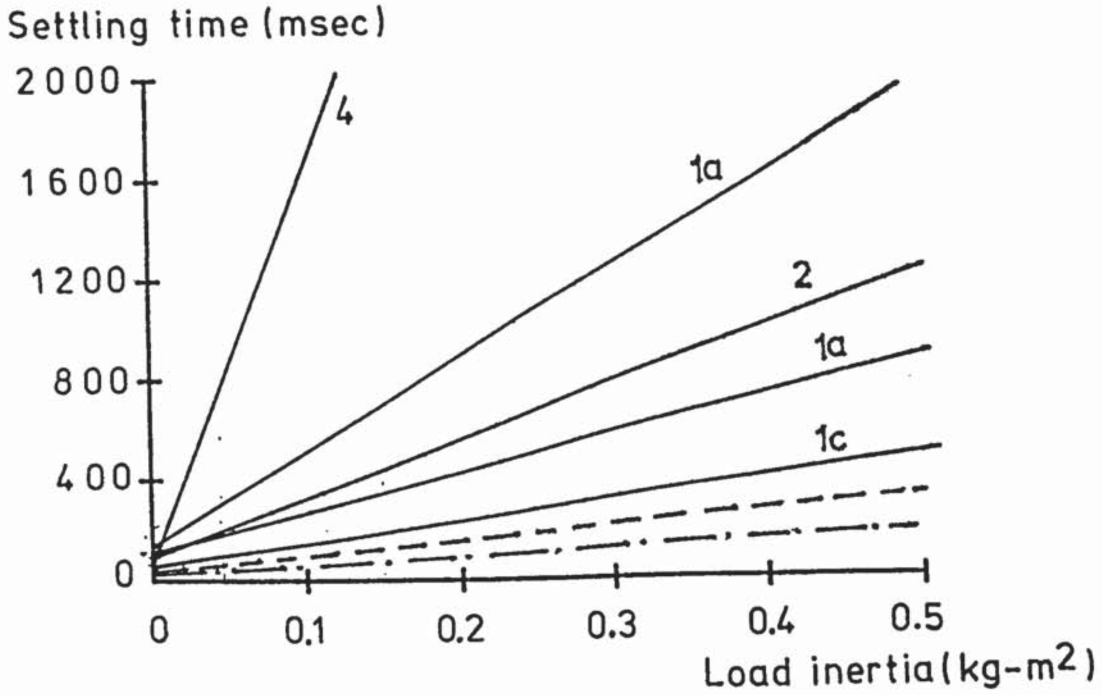


Fig. 8.7 Comparison of dynamic settling time of 5 kW servo motors.

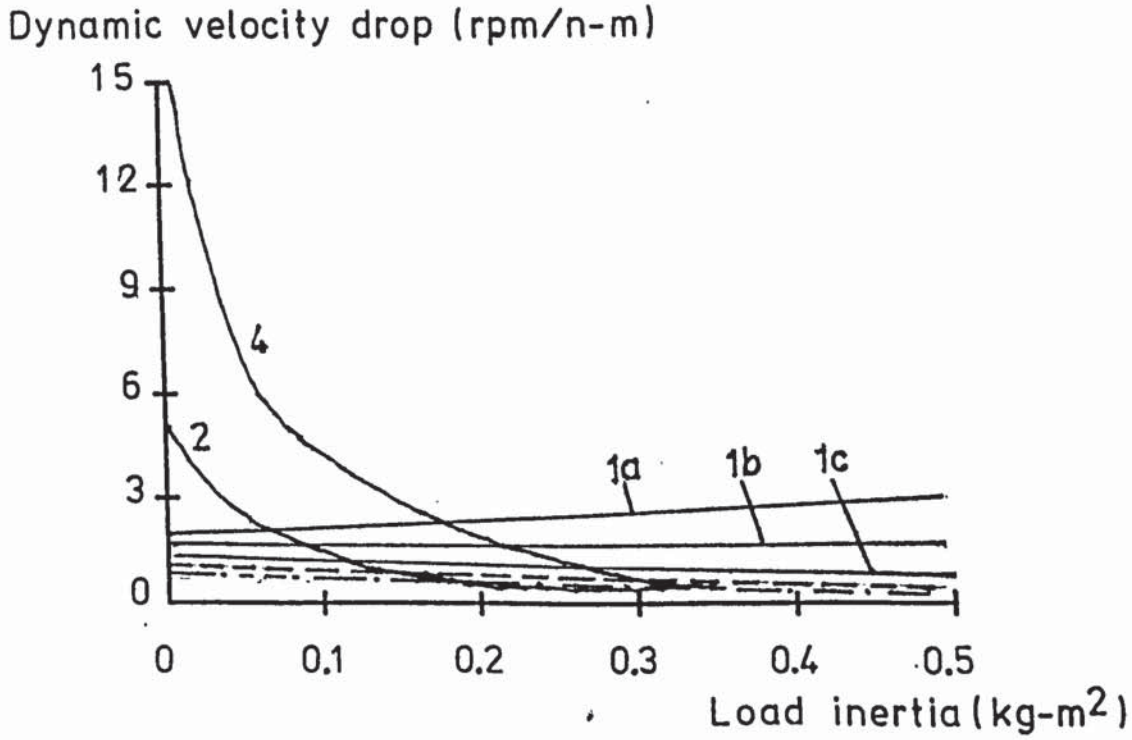


Fig. 8.8 Comparison of dynamic error of 5 kW servo motors for a unit step input of load torque.

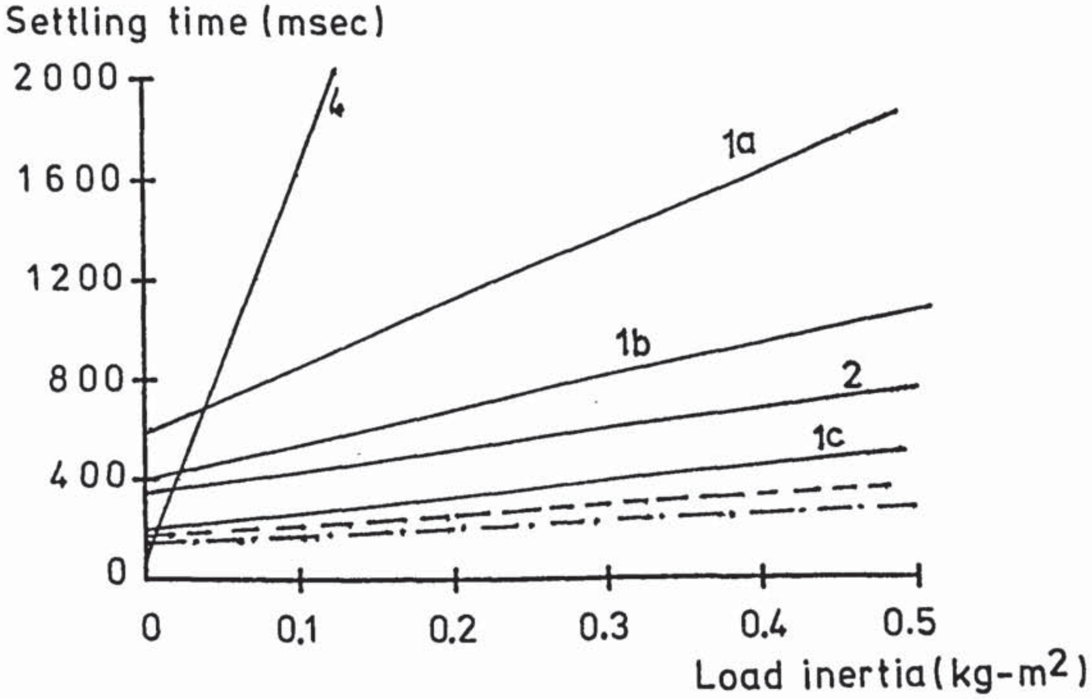


Fig. 8.9 Comparison of dynamic settling time of 10 kW servo motors.

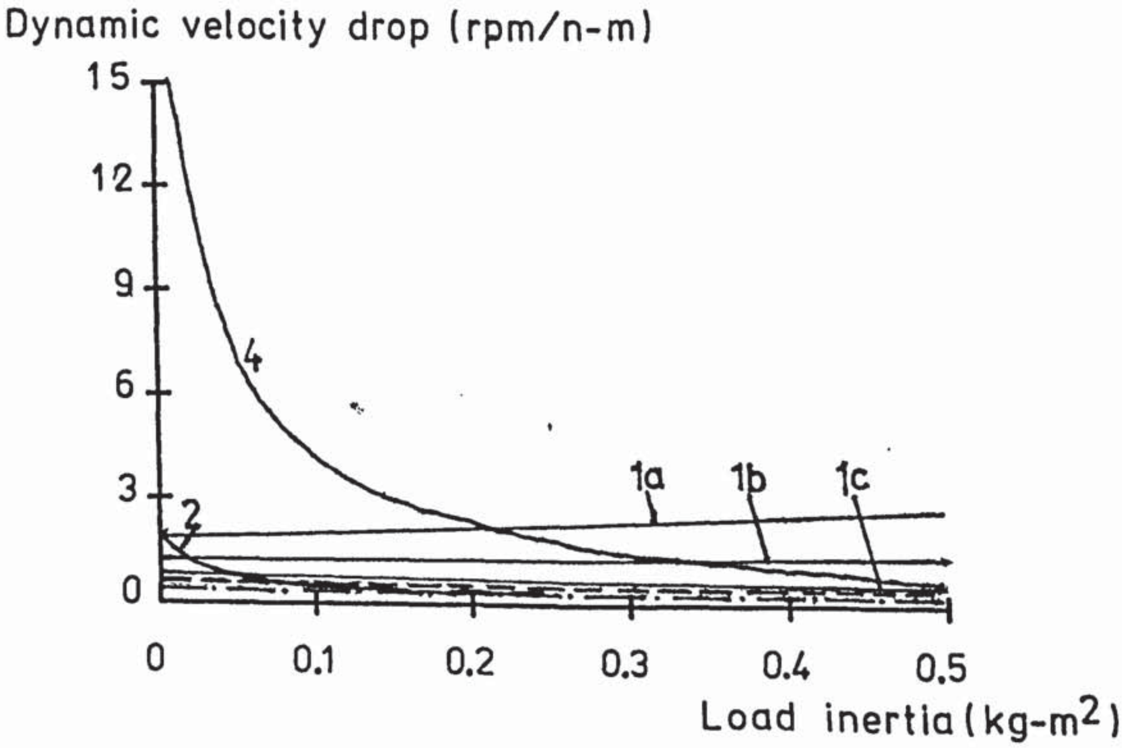


Fig. 8.10 Comparison of dynamic error of 10 kW servo motors for a unit step input of torque.

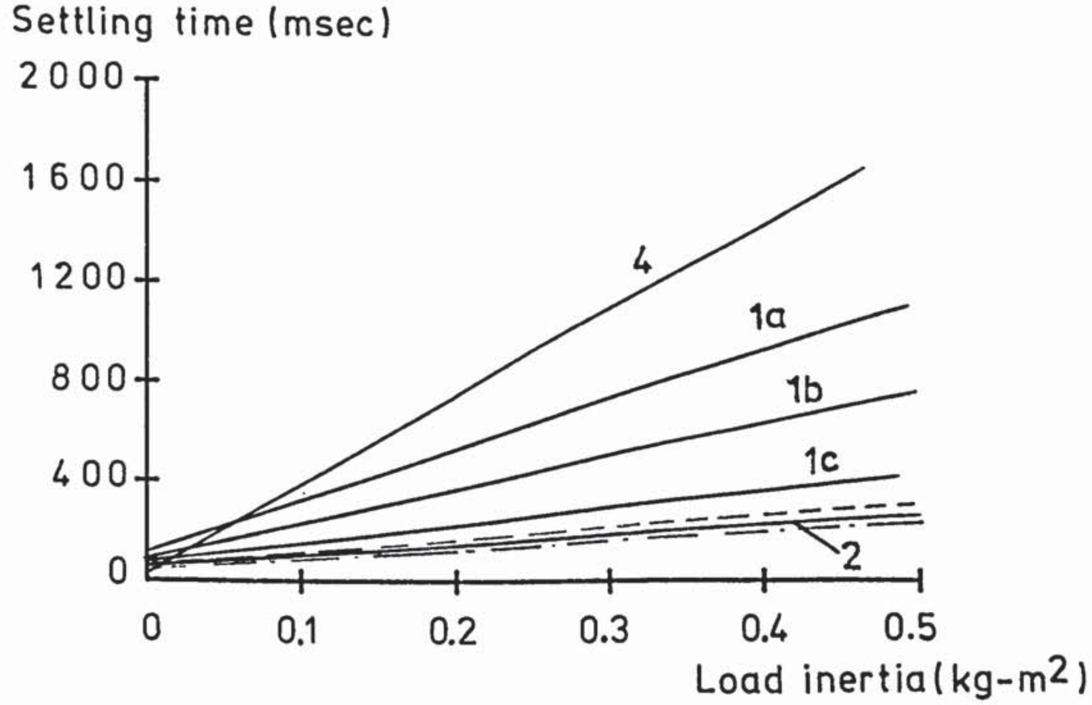


Fig. 8.11 Comparison of dynamic settling time of 5 kW servo motors. The gain of the system is increased 10 times, and acceleration feedback is used for compensation.

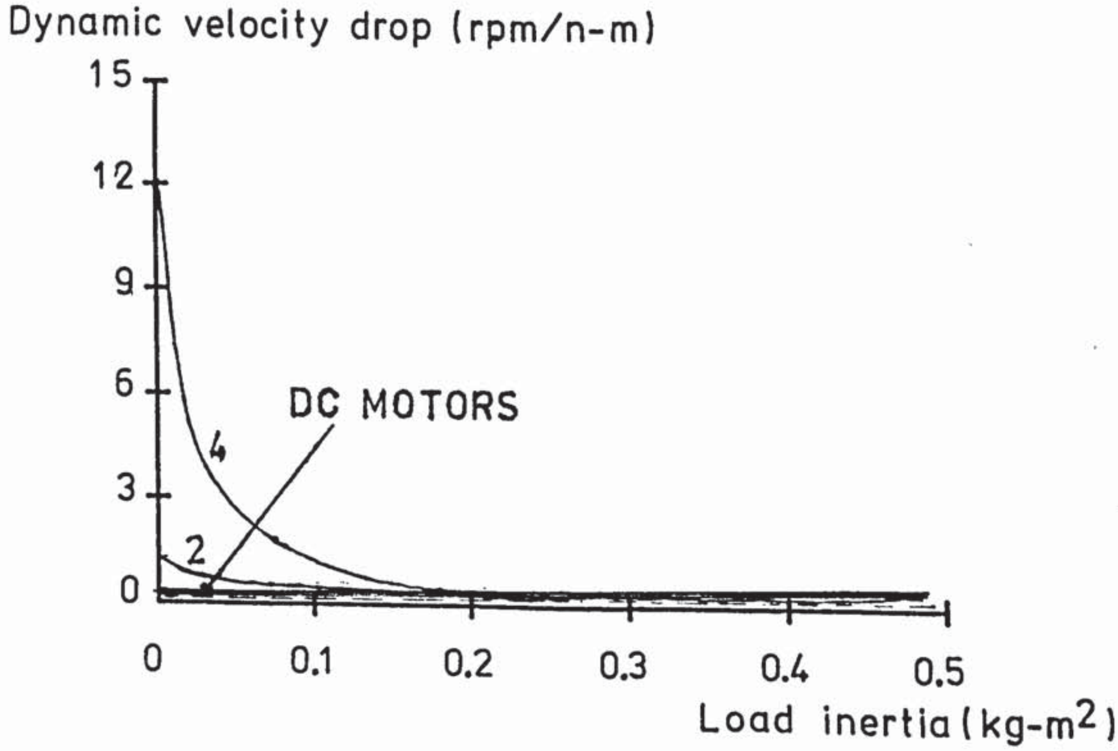


Fig. 8.12 Comparison of dynamic error of 5 kW servo motors for a unit step input of load torque. Gain of the system is increased 10 times and acceleration feedback is used for compensation.

CHAPTER 9

CONCLUSIONS AND SUGGESTIONS

9. CONCLUSIONS AND SUGGESTIONS

9.1 General Performance

In this work the performance and accuracy of servo motors were studied. For high performance applications the motors were classified as hydraulic and electrical. From the hydraulic motors the performance of the axial piston type, driven with an electrohydraulic valve was examined. Three types of electrical motors, i.e. a) D.C. motors, b) A.C. motors, and c) stepping motors were considered. The performance of ceramic magnet D.C. motors driven with thyristor controlled or pulse width modulated drive were established. Some characteristics of other D.C. motors such as rare earth magnet, printed circuit motors and brushless D.C. motors were also briefly examined. From A.C. types of motors the performance of a squirrel cage induction motor was studied. From stepping types of motors, the performance of the hybrid motor was studied.

A general linear mathematical model was established to study the performance of all types of motors. The model is in general form with some minor modifications and it may be applied to any type of motor or controller. The linear model was substantiated with a series of experimental tests on different types of servo motors. The analyses were then confidently carried out on motors of up to 10 kW.

In order to investigate the performance of the complex mathematical model, a computer program package was developed. The package is in a general form and can solve any linear control problem. The package is also capable of handling non-linear problems. This can be achieved by changing the coefficients of the linearized equation as the state of the system changes.

The package is capable of investigating, root-locus, frequency response and the transient response of the system. The method of solution is based on an exact solution of the differential equations. The only numerical technique employed is in calculating the roots of the characteristic equation. The package is capable of solving the transient response to some standard input functions and these are: 1) impulse input, step input, ramp input and acceleration input. However, this will not prevent the package from obtaining a response to an arbitrary input function, since any input function may be divided into a number of steps.

The package which employs an exact solution technique does not have the problem of non-convergency of numerical techniques. The length of step is based on the fundamental frequency of the system and this piece-wise analytical solution allows a longer step-length than a numerical integration routine.

In this analysis, some conclusions were drawn which are general for all types of motors, these are summarized below:

1. A servo motor with a good performance in velocity control mode will provide a better performance in position control mode. In the design of a servo motor therefore, it is essential firstly to optimize the system in velocity control mode and then the gain of position amplifier may be adjusted to obtain the best performance in position control mode.
2. A damping ratio of about 0.7 in the fundamental frequency of the system in velocity control mode will provide a higher accuracy in position control mode. The gain of velocity amplifier and the

time constants of lead-lag network can be adjusted to achieve this damping ratio.

3. The lag network in position and velocity amplifier is usually undesirable as far as the speed of response and accuracy is concerned. But their existence is usually essential in order to reduce the effect of noise from the transducers. The time constant of the lag network, therefore, must be minimized.
4. It was shown that the use of current feedback (pressure feedback in the case of the hydraulic motor) will provide some compensation and therefore, the gain of the system can be increased. However, because of the existence of this feedback at steady state, there was not a considerable improvement on the effect of external torque.
5. It was shown that the use of acceleration feedback and not the current feedback improves the speed of response for all types of motors. The effect of external torque also reduces considerably. The acceleration feedback introduces damping into the system and consequently the gain of the system can be increased considerably. The increase in the gain of the system with this technique only depends on the existence of the level of noise in the transducers. It was also shown that the acceleration feedback around the motor also introduces damping on the natural frequency of the loading mechanism.
6. It was noticed that the use of filter on the feedback had an undesirable effect on the stability of the system. Therefore, the transducers of high performance systems must be noise free. It was also noticed that the coupling of transducers to the shaft of the motor has a considerable effect on the systems performance.

Flexible coupling is not desirable, since the feedback signal may contain noise due to the oscillation of the rotor of the transducer. This problem is more significant in the case of D.C. motors, where the current (torque) supplied to the motor is in pulse form. This problem becomes much worse when the natural frequency of coupling is close to the current frequency of the motor.

7. Generally, the increase of the load inertia reduces the speed of response, both dynamically and statically. The increase of the load inertia slightly increases the dynamic error due to the external torque on D.C. servo motors. The dynamic error reduces as the load inertia increases for other types of motors.

8. As the size of the motor increases (higher power rating) the dynamic speed of response for a specific load increases. The static speed of response increases at the torque limit of the motor. Therefore the combination of load and rotor inertia have a contradictory effect on the overall speed of response of the system.

   To choose a servo motor on the basis of the speed of response is a trial and error procedure. This implies that it is not necessarily a larger motor that will provide a faster response. This is shown in detail in Appendix H.

   The dynamic error for a fixed external torque reduces for a larger motor but it seems that the dynamic error remains constant for the rated torque of all types of motors.

9. A low natural frequency of the loading mechanism will reduce the

damping of the system. This imposes a reduction on the gain of the system which results in a poorer performance. The effect is less significant on the hydraulic servo motor. This is mainly due to the inherent pressure feedback of hydraulic motors which reduces the flow rate.

9.2 D.C. Motors

It was found that the performance of D.C. motors was limited by the type of controller used. The main limitation was on thyristor controlled systems. The principle problem with thyristors is that once they are fired the conduction cannot be stopped without the current returning to zero. This imposes a major limitation on the gain of the system. An attempt to increase the gain could cause the motor to become overloaded during one cycle, and since the thyristor cannot be stopped during this cycle by the current limiter, the motor begins to oscillate violently. Therefore the performance of a D.C. motor driven by a thyristor depends to a great extent on the maximum current capability of the motor. For this reason, a controller with a higher voltage frequency and a motor with a higher current capability produces a better performance.

Another problem with D.C. motors is their oscillation with current frequency of the motor. This is more significant with low inertia motors. It also introduces a considerable amount of noise into the transducers, and could excite the rotor of the transducer at each interval of supplied current pulse. For this reason a concentrated effort has been made by manufacturers to produce controllers with higher current frequency such as pwm systems. The analyses were made on a pwm system with current frequency of 2 kHz. But now pwm systems with much higher current frequencies are available in the markets.

The following conclusions on the performance of D.C. motors may be drawn from the analyses.

1. The performance of D.C. motors depends on the current frequency of the controller. The performance improves as the supplied current frequency increases. pwm systems with higher current frequency give a better performance than the thyristor controlled drive.
2. Since the performance of D.C. motors also depends on their current capability during transient response, new magnetic materials produce higher current limitations on the motors. The rare earth magnet is one of these materials, which enables the motor to be capable of accepting almost 20 times its steady state current capability. Therefore motors with a rare earth magnet give a better performance.
3. It was shown that motors with lower rotor inertia give a better speed of response. The printed circuits and brushless D.C. motors are of this type. It can be said that brushless D.C. motors with a rare earth magnet give the best performance.
4. The non-linearity of the voltage equation of the thyristor was examined. It was shown that at low dynamic gain the system can become oscillatory due to the feed forward integrator (with low external torque stiffness). This problem was overcome by preventing the thyristor from being operated at very low dynamic

gain at the expense of efficiency.

5. The capital cost and reliability of the motors are also of major importance. As the capital cost is dependent on many parameters it was not possible to obtain an accurate price comparison. However, the capital cost is almost proportional to the performance. Therefore, it can be said that brushless D.C. motors with rare earth magnet are the most expensive and they are followed by the rare earth magnet, the printed circuits and the ceramic magnet. The reliability and problems of brush motors are well known, therefore motors without brushes are more reliable. The reliability of rare earth magnet motors have yet to be proved. It seems that because of the brittleness of the magnet, these types of motors are not as reliable as the ceramic magnet types.

9.3 A.C. Motors

The A.C. motors generally have a slow speed of response because of their high inductance and low maximum torque. The dynamic equations of A.C. motors are non-linear. It was shown that by linearizing the equations and by substituting an effective resistance and inductance a good prediction of the performance can be made. The effect of non-linearity on the performance was examined. It was found that a considerable amount of oscillation existed at low speed and under no-load conditions.

By the use of acceleration feedback the performance of A.C. motors was considerably improved and made them competitive with conventional D.C. motors.

The A.C. motors are the cheapest and the most reliable and therefore they are the most appealing. The applications of these motors will, no doubt, increase in the future. The only problem is that the controllers of these types of motors are more expensive than other types. The gradual reduction in the price of electronics shows a bright future for these motors. The only drawback is the maximum torque capability of these motors which is a lot less than the counterpart of D.C. motors.

9.4 Stepping Motors

The performance of the hybrid types were examined. These motors which are basically an open loop position control are very attractive in position control systems. They give excellent position accuracy within the operating limit.

These motors suffer greatly from high rotor inertia and low maximum torque especially the larger sizes. Another problem is that these motors have not been manufactured in a larger size simply because of their low efficiency. It was found that at the low natural frequency of the loading mechanism these motors become very oscillatory. At higher load inertia the performance degrades quite rapidly.

Stepping motors are cheaper than D.C. motors but are more expensive than A.C. motors. They are as reliable as A.C. motors. Their controllers are cheaper than the controllers of A.C. or D.C. motors. These motors also are not good for heavy duty cycles, since they loose steps easily. It was found that the use of acceleration feedback improves the problem of the low damping of these motors.

9.5 Hydraulic Motors

The performance of these motors was established. It was found that they produce a very fast dynamic response. The non-linearity of electrohydraulic valves causes no problem on these motors. Due to the inherent pressure feedback, these motors are more stable than electrical motors. However, this inherent feedback reduces the stiffness of the motor when the external torque is applied.

The low natural frequency of loading mechanism causes no problem on the stability of the system. The drawbacks of these motors are well known, such as the necessity of having a power pack, oil contamination and noise.

The major shortcoming of hydraulic motors, in the field of machine tools and robotic applications, is that they are not easily available in the smaller power range.

9.6 A Comparison

The servo motors may be compared with a number of criteria. The choice of a motor for a specific application depends on the priority of these criteria. These criteria may be: the speed of response, stiffness with external torque, the static and dynamic speed of response, capital cost, reliability and availability.

The hydraulic motors produce the fastest response dynamically and statically. The D.C. motors are second as far as the speed of response is concerned. The brushless type of D.C. motors, however, compete with hydraulic motors in this sense. The stepping and A.C. motors are the last choice of drive if the speed of response is the prime consideration. It was shown that by the use of acceleration feedback the A.C. motors become competitive with D.C. motors.

The stepping motors give the highest stiffness for the applied external torque. They are followed by D.C. and A.C. motors. Hydraulic motors give the lowest stiffness. The use of acceleration feedback increases the stiffness of all types of motors.

The capital cost of the complete drive unit must be compared with maintaining the similarity of all the parameters. These include power, the capability of controllers, their options on the possible feedback and degree of axis drive, the addition of velocity and position transducers. For these reasons it was not possible to compare the capital cost of all types of motors. There were indications that for low power ratings, the hydraulic servo motors are the most expensive. The pwm systems are the second most expensive servo motors. They are followed by thyristor controlled, A.C. and stepping motors.

At a higher power rating hydraulic motors become cheaper, because most of the component is similar to the lower power rating. The only major change comes from flow rate.

The A.C. and stepping motors are the most reliable types, because they are brushless. Hydraulic motors are very reliable apart from the contamination of oil and they require more maintenance care. The reliability of the brushes of D.C. motors and their maintenance care are well known. The development of brushless D.C. motors make them as competitive as A.C. motors.

The stepping motors are not readily available in larger sizes. The hydraulic motors are not easily available in smaller sizes. The D.C. and A.C. motors are available in almost all power ranges. The availability of rare earth and brushless D.C. motors have yet to be seen.

In some applications the size of the motors are also important. The hydraulic motors have the smallest size with the same power rating. They follow with A.C., stepping and D.C. motors. The brushless D.C. motors are capable of competing with hydraulic motors.

Finally it can be said that by the advances made in electronics and their gradual reduction in price and the overall performance achieved in the performance of electrical motors, they are now superceding the hydraulic motors in machine tools and robotic applications.

The possible future of hydraulic motors can be seen if the manufacturers of hydraulic motors develop more motors in smaller sizes and in a more readily available type. On this condition they may overcome the disadvantage of the use of oil as a source of power. In electrical types of motors the brushless D.C. and A.C. motors will hold most of the high performance applications. They tend to be an integrated part of the system, i.e. the rotor will be built in the moving part of the system.

9.7 Suggestions for Future Work

The following can be suggested for further work on servo motors:

1) An improved mathematical model may be used to predict the dynamic performance of the systems. This can be achieved by including

the non-linearity of the controller and motor. Static, windage friction may also be included in the mathematical model. This requires a very powerful numerical technique to handle the complex mathematical model.

2. The computer package may be modified to include the mathematical model of all types of motors. This will enable the user to input the parameter just for obtaining the performance of a specific application. Furthermore the data of different types and power ratings of motors, a performance criteria and a search routine may be included in the package in order that the best motor for a specific application may be obtained.
3. Further work may be carried out to study the performance of other types of motors. Although there are indications that these have little use in high performance applications.
4. More work can be carried out to compare the performance of the linear actuator with rotary motors.
5. Since brushless and rare earth magnet D.C. motors are now being developed, further work may be carried out to predict the optimum performance possibility of these motors.

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APPENDIX A

CIRCUITS DIAGRAM FOR PROPORTIONAL, INTEGRATOR AND LEAD-LAG NETWORK

A.1 Proportional Transfer Function

Fig. A.1 shows a circuits diagram for obtaining a proportional transfer function. The relationship between the output and error voltage can be obtained by writing the Kirchoff's voltage equation as:

$$\frac{v_o}{v_e} = \frac{R_2}{R_1} \qquad \text{A.1}$$

where R_2/R_1 is the gain of the amplifier.

A.2 Proportional Plus Integrator Transfer Function

Fig. A.2 shows a circuit diagram for obtaining a proportional plus a feed forward integrator transfer function. The relationship between output voltage (v_o) and the error voltage v_e may be obtained as:

$$\frac{v_o}{v_e} = \frac{R_2C_1s + 1}{R_1C_1s} = \frac{(R_2/R_1)s + (1/R_1C_1)}{s} \qquad \text{A.2}$$

where

R_2/R_1 is the gain of the amplifier

$1/R_1C_1$ is the gain of the integrator

A.3 Lag Network

Fig. A.3 shows a circuit diagram for obtaining a lag network. The transfer function of this amplifier can be written as:

$$\frac{v_o}{v_e} = \frac{R_2/R_1}{1 + R_2C_1s} \qquad \text{A.3}$$

where

R_2/R_1 is the gain of the amplifier

$R_2\ C_1$ is the time constant of the lag network.

A.4 Lead-Lag Network

Fig. A.4 shows a circuit diagram for obtaining a lead-lag network. By writing the Kirchoff's voltage equation the transfer function can be written as:

$$\frac{v_o}{v_e} = \frac{(R_2/R_1)(1 + R_1C_1s)}{1 + R_2C_2s} \qquad \text{A.4}$$

where

R_2/R_1 is the gain of the amplifier

R_1C_1 is the time constant of lead network

R_2C_2 is the time constant of lag network.

A.5 Application of These Transfer Functions

Generally the lag network with adjustable gain and time constant is used for the position and velocity amplifiers of a servo motor controller. The lag network is used to reduce the noise of the transducers.

The lead-lag network is used in the current amplifier of a controller. Usually the time constant of the lag network depends on the current frequency of the motor. This lag network is used to obtain an average value of the current. Therefore, this time constant is fairly large.

The lead network is usually used to compensate the electrical time constant and to improve the stability of the motor.

The feed forward integrator is used in the velocity amplifier in order to obtain zero steady state error. The gain of the integrator can be adjusted by the value of the capacitance C_1 which is independent of the amplifier.

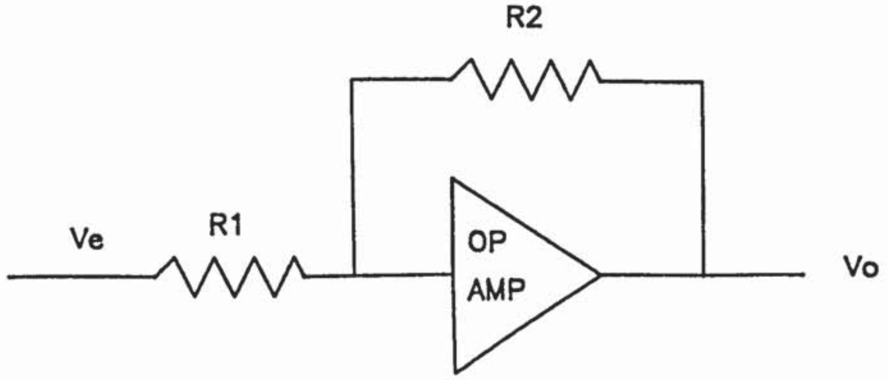


Fig A.1.Proportional amplifier.

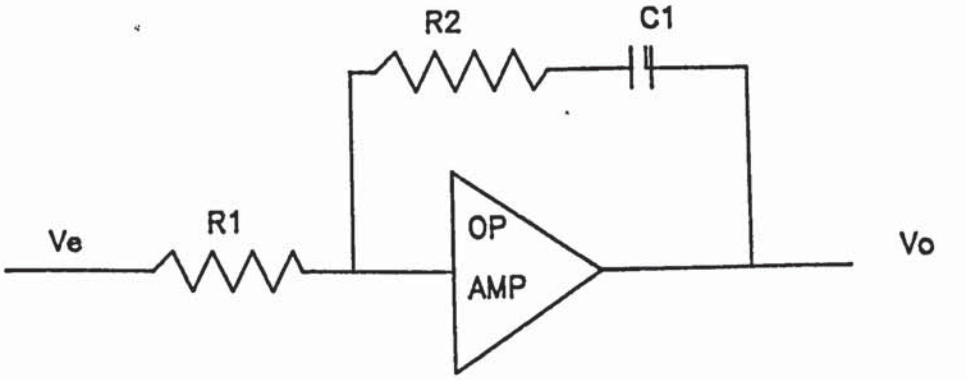


Fig A.2.Proportional plus integrator amplifier.

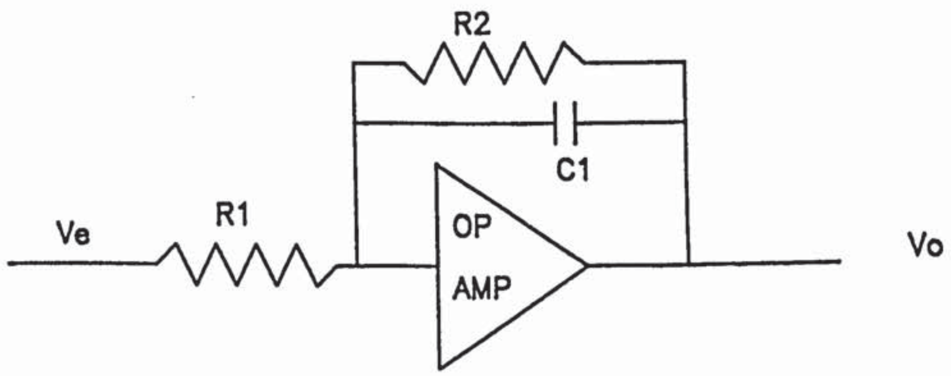


Fig A.3.Proportional and lag net-work.

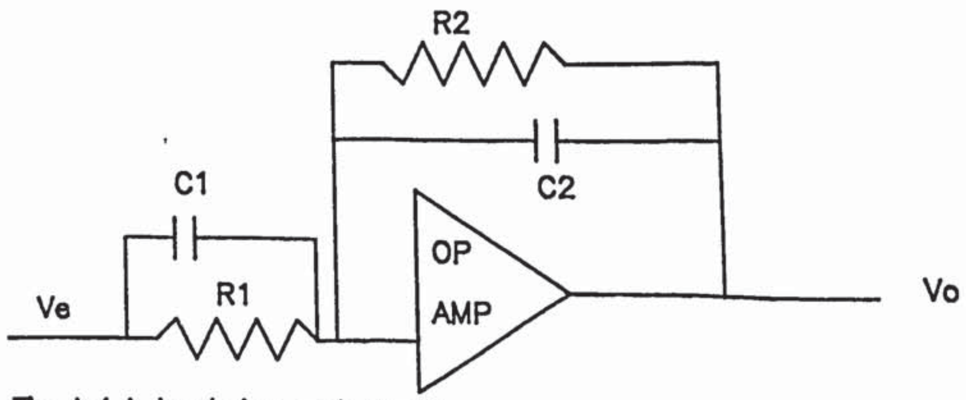


Fig A.4.A lead-lag net-work.

APPENDIX B

COMPUTER PROGRAME SIMULATION OF THE DYNAMIC DESIGN OF THE CONTROL SYSTEMS

B.1 - INTRODUCTION

For design of any control system a good knowledge of the dynamic performance of the system is essential. In order to obtain the required performance a designer needs to formulate the dynamic equations of the system, and to study the stability, transient responses and its behaviour under steady state conditions. As the system becomes more complicated, the study and solution of its characteristic equations by hand becomes more tedious and sometimes impossible, especially for a non-linear system. Because of these difficulties the aim of this work is to provide a computer package which makes the analysis and design of a system simpler.

A typical application of this package is the designing of the hydraulic control component commonly used in a modern control system. These are generally non-linear in the mathematical sense, but the non-linearities can be linearized by considering small perturbations about the steady state operating point. This technique of linearization enables the well known methods used for linear systems to be applied to non-linear systems, and this computer package allows the large-signal response of a non-linear system to be studied by changing where necessary, the non-linear coefficients for new values of variables at the end of each step. Therefore, the length of step is based on the degree of non-linearity, and the piece-wise analytical solution allows longer step-lengths than does a numerical integration routine.

This work presents a digital computer programme package to study the stability, transient response, frequency response and the steady state behaviour of a system using a desk-top computer. This method requires a set of linearized differential equations to be established using the state-space approach and the concept of small perturbations about the steady state conditions. Furthermore, the user of this package may also formulate a set of steady state equations if the steady state values are required.

The programme first evaluates the general transfer functions for any output variables with respect to any input variables of the linearized system in symbolic form. When all the necessary numerical values have been assembled, the programme will then ascertain the steady state operating point, if it is required, and the roots of the characteristic equation. Then the parameters can be changed in order to meet the desired performance and finally the transient response, frequency response or root-locus can be plotted for each output variable with respect to an input signal.

B.2.1 - METHOD AND GENERAL DESCRIPTION OF THE PROGRAM

A set of differential equations which describe a system can be written as,

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \ldots\ldots\ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} \ldots\ldots\ldots & a_{2n} \\ & \cdot\ \cdot\ \cdot & & \\ & \vdots & & \\ a_{n1} & a_{n2} & a_{n3} \ldots\ldots\ldots & a_{nn} \end{bmatrix} x \begin{bmatrix} Y_1 \\ Y_2 \\ \\ \cdot \\ \\ Y_n \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \ldots\ldots\ldots & b_{1m} \\ b_{21} & b_{22} \ldots\ldots\ldots & b_{2m} \\ \\ \\ \\ b_{n1} & b_{n2} \ldots\ldots\ldots & b_{nm} \end{bmatrix} x \begin{bmatrix} X_1 \\ X_2 \\ \\ \\ \\ X_m \end{bmatrix} \quad - 1$$

where,

$[a_{ij}]$ and $[b_{ij}]$ are the coefficient matrices of polynominals in operator D.n is the number of output variables, m is the number of input variables.

The transfer function of an output, y_i with respect to a certain input x_j, can be obtained by setting all the remaining inputs equal to zero. The result will be a system of linear equations;

$[A]\ \{y\} = \{B_J\}\ x_j\ (t)$ (2)

where $\{B_J\} = \begin{matrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{nj} \end{matrix}$

[A] and $\{B_j\}$ are the coefficients matrices of polynominal in D. If this system has no linearity dependent equations, then any output variable $y_i(t)$ can be found by applying Cramer's rule. Therefore,

$$y_i\ (t) = \frac{|A_i|\quad x_j(t)}{|A|} \qquad (3)$$

where,

$[A_i]$ = coefficient matrix of the system 2 with the ith column replaced by the coefficient column of the right hand side of the system 2.

The resulting transfer function is,

$$\frac{y_i\,(t)}{x_j(t)} = \frac{|A_i|}{|A|} \qquad (4)$$

Since the coefficient matrices [A] and [B] are polynominals in D, $[A_i]$ and [A] will be polynominals in D. It can also be seen that determinant $|A|$ remains the same for all variables y and x. Therefore, the determinant $|A|$ is the characteristic polynominal and $|A| = 0$ is the characteristic equation for the system 2.

In order to simplify the digital computer programme in evaluating the transfer function, this package requires the user to transform system 2 into a set of first order differential equations in the state space form. The transformation can easily be performed by introducing a set of new variables into system 2, in the following way.

When a certain output variable y has a k_{th} derivative as its highest derivative, then k new variables can be defined by;

$$\frac{dy}{dt} = y_{n+1}$$

$$\frac{d^2y}{dt^2} = y_{n+2} \qquad (5)$$

$$\vdots$$

$$\frac{d^ky}{dt^k} = y_{n+k}$$

The above process is carried out for all output and input variables, then a set of new variables will be added to the original one, i.e.

$$\frac{dx}{dt} = y_{n+q+1}$$

$$\frac{d^2x}{dt^2} = y_{n+q+2} \qquad (6)$$

$$\vdots$$

$$\frac{d^px}{dt^p} = y_{n+q+p}$$

q is the total of variables introduced for all the output variables.

p is the highest derivative of the x_j input variable.

Note that the derivative of an input variable is to be considered as an output variable in assembling the system matrix.

As this is performed for all variables y_i and x_j, then all higher order derivatives of y_i and x_j can be eliminated. A set of equations similar to the set of equations in (5) and (6) is then added to the system 1. Therefore,

$$a'_{11}\, y_1(t) + a'_{12}\, y_2(t) + \dots\dots + a'_{1n+q}\, y_{n+q}(t) =$$

$$b'_{11}\, x_1(t) + \dots\dots + b'_{1m+l}\, x_{m+l} \qquad (7)$$

$$a'_{n1}\, y_1(t) + a'_{n2}\, y_2(t) + \dots\dots + a'_{n\, n+q}\, y_{n+q}(t) =$$

$$b'_{n1}\, x_1(t) + \dots\dots\ b'_{nm+l}\, x_m(t)$$

l is the total of variables introduced for all the input variables.

The system of equations (7) is the same as in (1) except that new variables for the higher order derivatives are introduced. The coefficients a'_{ij} and b'_{ij} are constant. A set of equations which relates the higher derivatives to the lower one must be added to system 7 to complete the system 7.
The complete system equations using operator s instead of D can be written as;

$$a'_{11}\, y_1(s) + a'_{12}\, y_2(s) + \ldots\ldots\ldots + a'_{1p}\, y_p(s) = b'_{11}\, x_1(s) + \ldots . b'_{1m}\, x_m(s)$$

(8)

$$a'_{n1}\, y_1(s) + a'_{n2}\, y_2(s) + \ldots\ldots\ldots\ldots + a'_{np}\, y_p(s) = b'_{n1}\, x_1(s) + b'_{nn}\, x_m(s)$$

$$sy_1(s) - y_{n+1}(s) = 0$$

$$\vdots \qquad (9)$$

$$sy_n(s) - y_{n+q}(s) = 0$$

$$sx_1 - y_{n+q+1}(s) - 0$$

$$\vdots$$

$$sx_m - y_p(s) = 0$$

(8) is the original system of equations, with added variables and with constant coefficients. (9) are the complement equations with coefficients of either s, (-s) or -1, (+1).

Finally the new system equations can be written in matrix form as,

$$[A]\ \{Y\} = [B]\ \{X\} \qquad (10)$$

The coefficients matrices A an B are either a constant or S. Y and X are the output and input variables respectively.

This simplified form of system equations (10) is then processed by the Package. The transfer function of i th output variable with respect to j th input variable (i.e. $\frac{y_i}{x_j} = \frac{|A_i|}{|A|}$) is evaluated in symbolic form by Cramer's rule. The positions of the elements of [A] and [A_i] that constitute the coefficients of denominator and numerator of this transfer function are kept in a storage file which later in the programme numerical values will be substituted to proceed with the analysis.

B.2.2 STEADY STATE OPERATING POINTS

The non-linear equations that describe a system should be linearized about the steady state operating point, so that it is essential to find the steady state operating point. One of the subprogrammes in this Package provides a numerical method of solving these non-linear equations. The maximum size of this system of equations is limited to 15 because of the unsatisfactory convergence behaviour of a large system of simultaneous non-linear equations. This limitation will not affect a large system, because in any case some of the variables can be eliminated from the system in order to keep the above limitation.

B.3. GENERAL STRUCTURE OF THE PACKAGE

The general structure of the Package is shown in the flow-chart of fig.1 (page 262) and the branches are explained below.

Branch 1; this branch provides the facility of skipping the programme "CONTRO", which evaluates the transfer function in symbolic forms, in the case of a re-run.

Branch 2; if the steady state operating point can easily be obtained by the user or if it is not required a considerable amount of computer time can be saved by not calling the subprogramme "STYST", which ascertains the steady state conditions. This branch provides the facility of skipping this subprogramme if the steady state point is not required.

Branch 3; this branch is used to jump the subprogramme "ROOTS" if the roots of the characteristic equation are not required.

Branch 4; this branch is used to repeat the computations in the case when the user wants to change the basic parameters in order to obtain more satisfactory results.Therefore it provides the means to change the parameters easily.

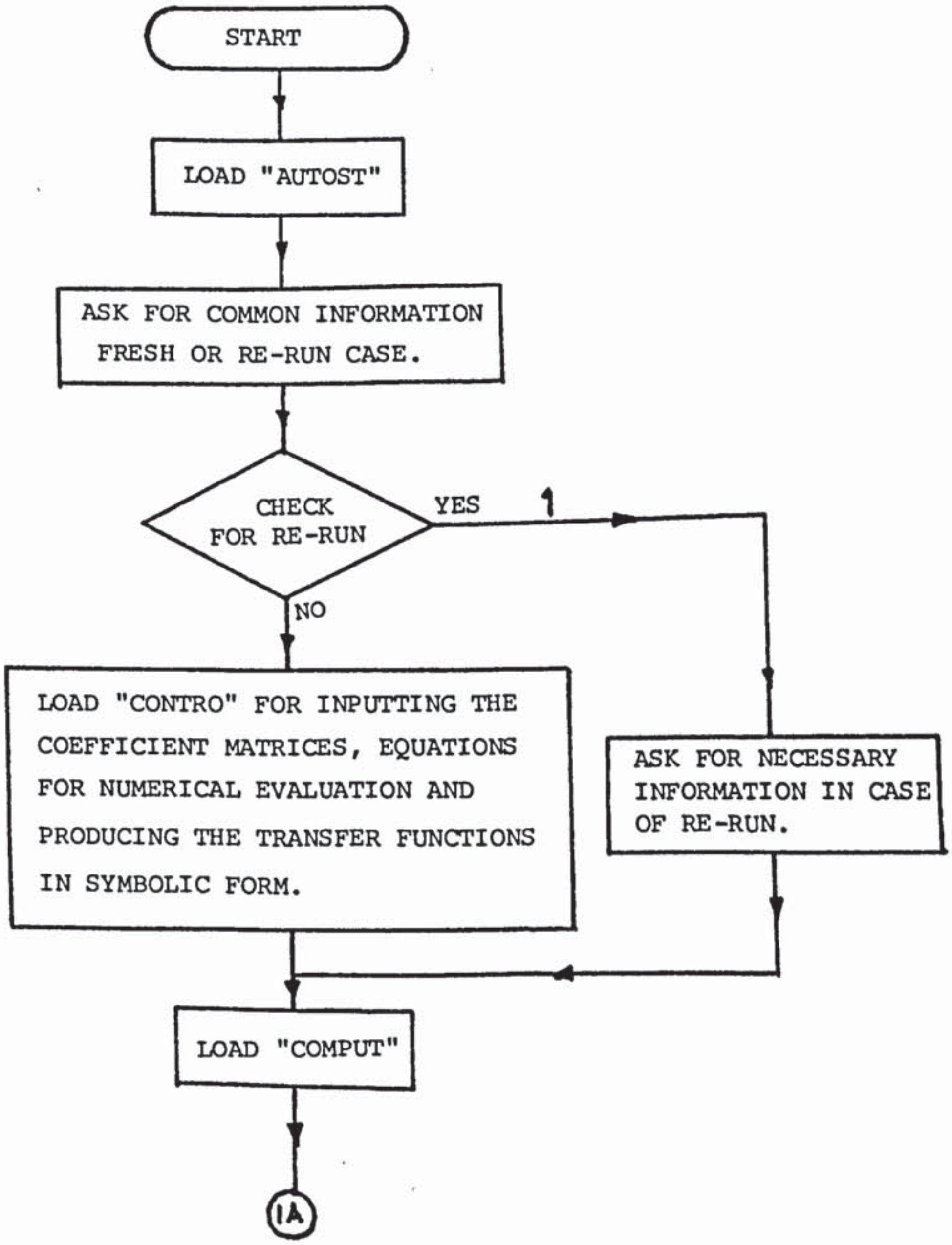
Branch 5; the branch calls the subprogramme "NYQI" and is used when the Nyquist plot, frequency response and the phase lag plot are required.

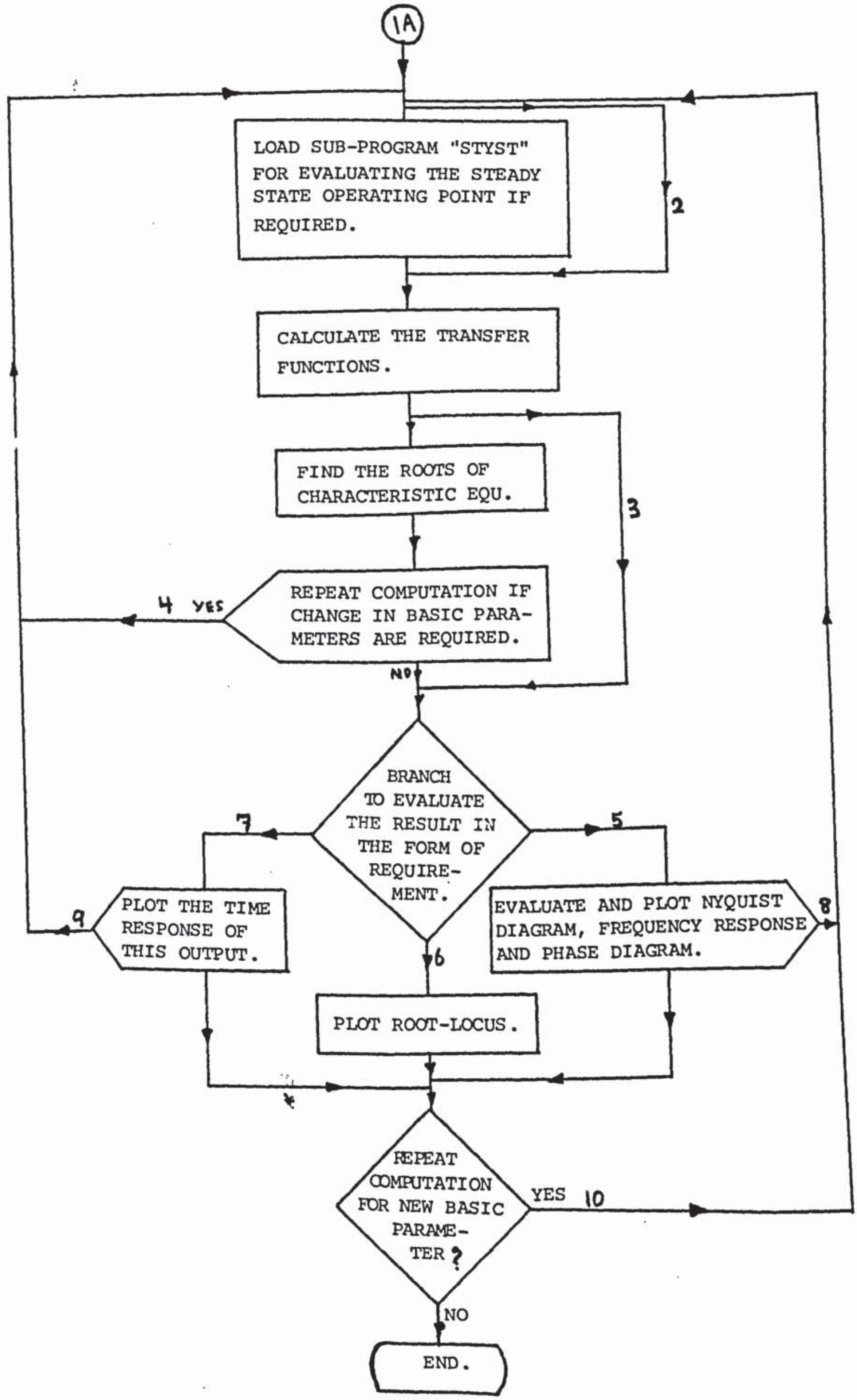
Branch 6; this branch calls the subprogramme "PLOT" in order to plot the root-locus of the system's characteristic equation.

Branch 7; this branch calls the subprogramme "RESPO" to evaluate and plot the time response of the output variable for a specific input function which is specified by the user.

Branches 8,9, these branches repeat the computation for the next output variables.
Branch 10, this branch provides the means to repeat the computation for new values of parameters or terminates the computation.

Fig 1: General flow chart of the package




1A
LOAD SUB-PROGRAM "STYST" FOR EVALUATING THE STEADY STATE OPERATING POINT IF REQUIRED.
2
CALCULATE THE TRANSFER FUNCTIONS.
FIND THE ROOTS OF CHARACTERISTIC EQU.
3
REPEAT COMPUTATION IF CHANGE IN BASIC PARAMETERS ARE REQUIRED.
4 YES
NO
BRANCH TO EVALUATE THE RESULT IN THE FORM OF REQUIREMENT.
7
5
PLOT THE TIME RESPONSE OF THIS OUTPUT.
9
EVALUATE AND PLOT NYQUIST DIAGRAM, FREQUENCY RESPONSE AND PHASE DIAGRAM.
8
6
PLOT ROOT-LOCUS.
REPEAT COMPUTATION FOR NEW BASIC PARAMETER?
YES 10
NO
END.

B.4. FUNCTION OF EACH PROGRAMME AND SUB-PROGRAMME

B.4.1 Programme "AUTOST"

This is the drive programme, it asks for the basic information, e.g.fresh/re-run, the kind of analysis required, etc. Then it calls the appropriate programme to carry out the study.

B.4.2 Programme "CONTRO"

This serves as an initialization programme, it requires the following data,

a) the equations that evaluate system's coefficient matrices. numerically.

b) numerical values of basic parameters

c) steady state equations if required

d) initial estimate for steady state operating point

e) the coefficients of the system matrices [A] and [B] in symbolic form.

Having entered the above data the programme proceeds with the evaluation of transfer functions for all the required output variables with respect to the required input variable (in symbolic form) according to Cramer's rule.

The determinant of a square matrix can be evaluated as,

$$\det|A| = \sum_{\underline{P}} \sigma_{\underline{P}}\, a_{p_1 1}\; a_{p_2 2}\; a_{p_3 3} \ldots\ldots a_{p_n n} \qquad (11)$$

The sum is taken over all n, permutation $\underline{P}$ of degree n, and $\sigma_{\underline{P}}$ is 1 or -1, depending on whether $\underline{P}$ is even or odd. A permutation of degree n is any arrangement of the first n itegers(i.e. it is a sequence of integers in which each integer between 1 and n appears exactly once). In other words, a permutation can be defined as an n - vector.

$$\underline{P} = \left\{p_1, p_2, p_3, \ldots\ldots p_n\right\}^t \qquad (12)$$

with $p_i \varepsilon \left\{1,2,3,\ldots\ldots n\right\}$ and $p_i \neq p_j$ for $i \neq j$

If an inversion is define as an instance a greater integer precedes a smaller one, then a permutation $\underline{P}$ is said to be even Or odd depending upon whether the number of inversions are even or odd respectively.

The amount of work involved in evaluating these determinants is not substantial because of the presence of a high percentage of zeros in the matrices. The resulting transfer functions are then stored. Specific numerical values can be substited in later stages.

B.4.3 Programme "COMPUT"

Unlike the programme "CONTRO", this programme "COMPUT" is used for numerical computations. This calls the subprogramme "STYST" to solve the set of steady state equations, if required. It calls the programme "ROOTS" to solve the characteristic equation. Then a root-locus can be obtained by calling the programme "PLOT". Nyquist plot, frequency response and phase plot may be obtained by calling the subprogramme "NYQI". Time response (transient response) can be obtained by calling the subprogramme "RESPO".

B.4.4 Sub-programme "ROOTS"

This segment is used to evaluate the zeros of the characteristic polynominal . It employs Bairstow's method which will evaluate couples roots of a polynominal, so that undamped natural frequencies and damping ratios of quadratic factors may be determined;it is self-initiating, using a quotient difference scheme to find the initial estimate.

B.4.5 Sub-programme "STYST"

This segment evaluates the steady state operating point. It uses a quasi-Newton method proposed by Brogdew to solve a system of non-linear equations which represents the steady state equations. This method is chosen because of its accuracy, its rate of convergence and its reliability[68].

B.4.6 Sub-programme "RESPO"

This subprogramme calculates and plots the transient response of each required output variables separately. The input variable is limited to an impulse, step, velocity or acceleration type. The method of finding the inverse laplace transform by partition is used in this subprogramme.

Let the transfer function of a particular output variable with respect to an input variable be,

$$\frac{x_o}{x_i} = \frac{f(D)}{P(D)} \qquad (13)$$

By applying the laplace transform and considering the initial condition in 13, gives.

$$x_o(s) = \frac{f(s)x(s)}{P(s)} + \frac{f_1(s)}{P(s)} \qquad (14)$$

substituting x(s) (the input function is known) in 14, gives

$$x_o(s) = \frac{f_2(s)}{P_1(s)} + \frac{f_1(s)}{P(s)} \qquad (15)$$

By knowing the zeros of polynominals $P_1(s)$ and $P(s)$ the inverse Laplace transform can be evaluated by partitions method.Assume that there are t zero roots in polynominal $P_1(s)$ and t_1 zero roots in polynominal $P(s)$ then 15 can be re-arranged as

$$x_o(s) = \frac{f_2(s)}{K_1 \prod_{i=1}^{n} (S-Z_i)} + \frac{f_1(s)}{K_2 \prod_{j=1}^{m} (S-Z_{1j})} = \frac{A_1}{S} + \frac{A_2}{S^2} + \ldots + \frac{A_t}{S^t} + \frac{A_{t+1}}{S-Z_{t+1}}$$

$$+ \frac{A_n}{S-Z_n} + \frac{A_{n+1}}{S} + \frac{A_{n+2}}{S^2} + \ldots \frac{A_{n+t_1}}{S^{t_1}} + \frac{A_{n+t_1+1}}{S-Z_{11}} + \frac{A_{n+t_1+2}}{S-Z_{12}} +$$

$$+ \frac{A_{n+m+t_1}}{S-Z_{1m}} \qquad (16)$$

The values of A_{i+t} which are the constants of non-zero roots are given by

$$A_{i+t} = \frac{f_2(Z_{i+t})}{\prod_{\substack{p=1 \\ p \neq i+t}}^{n} K(Z_{i+t} - Z_p)} \qquad (17)$$

and,

$$A_{n+t_1+j} = \frac{f_1(Z_{1j+t1})}{\prod_{\substack{p=1 \\ p \neq j+t_1}}^{m} K(Z_{1j+t1} - Z_{1p})}$$

Having calculated all the constants of the non-zero roots, then coefficients of zero roots are calculated by substituting arbitrary values for s, and solving a set of linear simultaneous equations for the rest of constant A_i.

When all the constant coefficients are obtained then the inverse of the Laplace transform can easily be obtained from 16, i.e.

$$x_o(t) = A_1 + A_2 t + \ldots\ldots \frac{A_p t^{p-1}}{(p-1)} + \sum_{i=p}^{q} A_i \, e^{Z_i t} \qquad (18)$$

This sub-programme is called to calculate the transient responses of the output variables, when the roots of the characteristic equation are evaluated.

The starting point of time, step of time and maximum values of time are arbitrary and must be entered by the user. Estimated maximum and minimum values of the output variable are required only for the purpose of plotting the response. More accurate values are known after the first run. There is an access to obtain a new response for different initial values.

At the end of the sub-programme the computer repeats the process of calculation for the second output variable.

B.4.7 Sub-programme "NYQI"

The transfer function of a particular output variable with respect to the error of a unity feedback system (Nyquist plot) or open-loop phase plot, can be written as,

$$\frac{x_o}{x_i} = \frac{f(s)}{p(s)} \qquad (19)$$

for a harmonic input 19 yields

$$\frac{x_o}{x_i} = \frac{f(j\omega)}{p(j\omega)} \qquad (20)$$

For different values of ω, $\frac{x_o}{x_i}$ can be calculated from 20. The result can be plotted in complex plane; i.e.

$$\left.\frac{x_o}{x_i}\right/_{\omega=\omega_o} = x + jy \qquad (21)$$

After calculating the ratio for a series of values of ω, where the number of points, and the lowest and highest values of ω are defined by the user, Nyquist plots, frequency response and phase angle plots can be plotted.

The above mentioned calculation is performed in sub-programme "NYQI".

The starting point of frequency, step of frequency, number of points, and interval of frequency in which the print-out is required, are some of the data which must be entered by the user.

This sub-programme is designed so that any change in the initial information can be altered to obtain new plots. At the end of the sub-programme the computer repeats this process for the next output variable (next transfer function).

B.4.8 Programme"Plot"

This sub-programme produces the resulting root-locus plot on the s-plane. Because of the occurance of conjugate pairs of complex roots, only the upper plane is produced. This can save space, time and memory in the computer.

B.5 - CONCLUSION

This computer package programme is an easy and efficient way of analysing the dynamic characteristics of a linear control system. The roots of the characteristic equation can easily be observed from the stability point of view. For the design of a system any parameter of the system can easily be changed and investigations made of the effect of it on the system.

The transient response is one of the most important characteristics of a system. For design purposes the transient response of a system for any disturbances should be well predicted. In some design cases the priority of the design is based on the transient response. For these reasons a sub-programme is written to evaluate the transient response for four standard input functions, impulse , step, velocity (ramp), and acceleration.

The root-locus of the characteristic equation with varying parameters, can also be analysed. The Nyquist plot and frequency response also can be obtained by calling the sub-programme "NYQI".

The non-linear system can be analysed by the small perturbations method i.e; the non-linear equations should be linearised about the steady state operating point.

The computer package has the following disadvantages;

1) To change the equations to a form acceptable to the computer takes time and needs skill.

2) For more complex systems the programme sometimes fails to evaluate the roots of the characteristic equation. This occurs mainly in systems whose characteristic equation is of a higher order than 6. It seems that this problem arises when there are two or more equal or close roots in the characteristic equation.

Further work might be done in order to simplify the input of the data. For example the programme may be re-designed to evaluate the state-space matrices form from the differential equation automatically.

Further work is being carried out to solve a non-linear system. The method which is employed at the moment is to first linearise the equations of non-linear form and then at each step of time the new values of the non-linear parameters be substituted for the previous values.

B.6 - OPERATION INSTRUCTION

1. Set up the systems governing differential equations in the matrix form containing only first order differential equations. The coefficients matrix of output and input variables must be named A and B respectively. Hence an element is denoted by A(I,j) or B(I,j) where I is the row subscript and j is the column subscript. These elements must be expressed as a function of,

   Input(1), Input(2), ... , z(1), z(2), ... , x(1), ...

   where z is the array that contains the numerical values of basic parameters describing the system; x is the array that contains the values of the steady state operating point and the output variables. Input is the array that contains the values of input variables.

2. Set up the steady state equations and re-arrange them in a set of equations to be zeroed. These equations must be expressed in the form of;

   F(I) = f(Input(1), Input(2), ... , Z(1), Z(2), ... x(1), ...)
   I = 1,2, ...

3. Switch the power on.

4. Insert the disk in the appropriate place

   4a. Type in LOAD "AUTOST: F8".

   4b. <RUN>.

5. Latch the PRINT ALL key.

6. When the question "SELECT THE MASS STORAGE UNIT" i.e. F8 (Disk), T15, T14, (Right and Left cassette) appears, type the appropriate unit in which the data should be stored.

7. If the storage unit is new, then by typing NO, to the question, "Have the data files been created?", the necessary data-files will be created in order to store the data.

8. By typing 1 the transient response of the system can be analysed and by typing 0 or 2 the Nyquist plot or root-locus can be investigated respectively.

9. Type in the number of output variables (X) to be analysed.

10. Type in the location of each output variable in the output variable vector.

11. Type in the location of the input variable. The programme only considers one input variable at a time and the rest of the input variables will be equated to zero automatically.

12. When the question "IS THIS A RE-RUN?" appears, type:

    (i) YES and press CONT if this is a re-run.

    (ii) NO and press CONT if this is a "fresh" application.

13. If it is a new problem, proceed to step 19.

14. Type n, the order of the coefficient matrix of the output variables, and press CONT.

15. Type m, the number of columns of the input variable coefficient matrix and press CONT.

16. Type in the number of basic parameters that describe the system and press CONT.

17. If the steady state solution is required type YES and press CONT.

18. Proceed to step (39).

19. Type the equation for evaluating a non-zero element of the system coefficient matrix i.e. A(1,1) = Z(1), A(1,2) = ... one by one.

20. Press CONT key.

21. Check that there is no mistake in the equation.

22. If the equation is entered correctly, press CONT and proceed to step (23); otherwise type NO and press CONT, then re-enter the corrected equation by repeating from step (19).

23. To enter another equation, press CONT and repeat from step (19).

24. To end inputting this set of equations, type NO and press CONT.

25. Type in the steady state equations (one by one) which had been re-arranged in the form of i.e.

```
F(1) = fn(Input(1), Input(2), ... Z(1), Z(2), ... X(1), ... )
F(2) = fn(   "         "          "      "          "
.
:
```

26. The procedure of entering the steady state equations is the same as for entering the elements of the coefficients matrices.

27. Type in the total number of basic parameters describing the system, the program will automatically call it an array Z.

28. Type in the value of Z(I), where Z(I) is the Ith basic parameter.

    Z(1) = 64, X(2) = 2.7 .... etc.

29. Type the total number of dependent variables that appear in the set of steady-state equations. The program will name these variables as an an array X.

30. Type in X(I) the initial estimate of Ith dependent variable in the set of steady state equations.

31. Type in the order of systems output variable coefficient matrix.

32. Type the total of non-zero elements in the matrix.

33. Type the row, column subscripts of a non-zero element, then the symbol of this element (this symbol should not contain more than 5 characters or the symbol s (s is reserved for Laplace transform operator).

34. Repeat step (33) until all the non-zero elements have been entered.

35. Repeat similar procedure (from step (31)) for matrix B, the input variable coefficient matrix.

36. To print out the arrays, type YES and press CONT; otherwise, type NO and press CONT.

37. To print out the polynominal, type YES and press CONT; otherwise, type NO and press CONT.

38. To find the steady state solution type YES and press CONT; otherwise type NO and press CONT. If the steady state solution is not required then proceed to step 40.

39. Type the numerical values of the Input variables for solving the steady state equations.

40. To read the array Z (basic parameters) from the data-file, type 2 and press CONT; otherwise type Ø and press CONT.

41. To read the array X (initial estimate of the steady state solution type 2 and press CONT; otherwise type Ø and press CONT.

42. If the steady state solutions are not required, then proceed to step (46).

43. If intermediate results of the steady state solution are required then type YES and press CONT; otherwise type NO and press CONT.

44. Type in the number of significant digits required for the steady state solutions. The number should not be zero.

45. Type the maximum number of iterations allowed for solving the steady-state equations.

46. To modify any basic parameters, type YES and press CONT. Then proceed to step (48).

47. If no modification is required, type NO and press CONT. Then proceed to step (78), step (51) or step (67) depending on the type of analysis you require i.e; Nyquist and frequency response or time response or root-locus.

48. Type the jth basic parameter, if it has to be modified.

49. Type the new value of this parameter.

50. Go to step (48) or type Ø to end modification.

Operation Instruction for Time Response

51. Type in YES and press CONT, if you require graphics; otherwise, type NO and press CONT. If the answer to question "DO YOU WANT GRAPHICS?" is NO then the response will be plotted but the axis will not be numbered for saving the time of computation.

52. Type in the initial values of output and input variables. D is the derivative operator. D↑Ø is the output or input and D↑1, D↑2 and D↑3 etc. are the first, second and third etc., derivatives of output or input variables.

53. Type in Ø,1,2,3 for choosing anyone of the four standard input functions. Ø for impulse input. 1 for step input. 2 for velocity (ramp) input and 3 for an acceleration input.

54. Type in the amplitude of the input function, if any one of the last three functions is chosen.

55. Type in the number of derivatives of the output variable you require.

56. Type in the starting point of time, with respect to this value the output evaluation will start.

57. Type in the step of time; the output value will be calculated at each step.

58. Type in the maximum value of time, where this value limits the evaluation.

59. Type in estimate values of maximum and minimum values of output variable, in order to scale the graphs.

60. Type in the interval of time in which the print-out is required.

61. Type in the magnification factor. This value only affects the plotting of the response i.e. the result will be multiplied by the magnification factor, then it will be plotted on the screen.

62. Adjust the position of the cursor by pressing the <⇆> or <↑↓> keys, then type in the title of each axes, etc.

63. When the calculation finishes, press CONT in order to get a hard copy of the response.

64. If any change on the calculation of response of this output is required, then type YES and press CONT; otherwise type NO and press CONT.

65. If at any stage of calculation of the time response, you wish to change on the initial conditions or input function, etc., press the PAUSE key and type CONT 1ØØØØ, then press the EXEC key. By doing this the calculation starts from the beginning of this sub-programme.

66. At the end of this sub-programme the process will be continued for the next required output variable.

Operation Instruction for Root-locus

67. To enlarge the graph, type 1 and press CONT, otherwise, type Ø and press CONT, then proceed to step (74).

68. Move the horizontal cursor line to the upper limit of the area to be enlarged by pressing the ↑ key.

69. Move the vertical cursor to the right-hand limit of this area by pressing the → key. Press CONT key.

70. Move the horizontal cursor line to the lower limit of this area by pressing the ↓ key.

71. Move the vertical cursor line to the left-hand limit of this area by pressing the ← key.

72. Press CONT.

73. Go to step (67).

74. Move the cursor to the desired position for labelling. Label the graph by typing in the letters.

75. After the graph has been labelled, press CONT to get the hard copy of this final result.

76. Remove the disk.

77. Switch the power OFF.

Operation Instruction for Nyquist plot and Frequency Response

78. Type in the starting point of frequency and press CONT key. The computation starts at this frequency.

79. Type in the number of points that you require.

80. Type in the increment of frequency that the calculation should proceed.

81. Type in the interval of frequency in which the print-out is required.

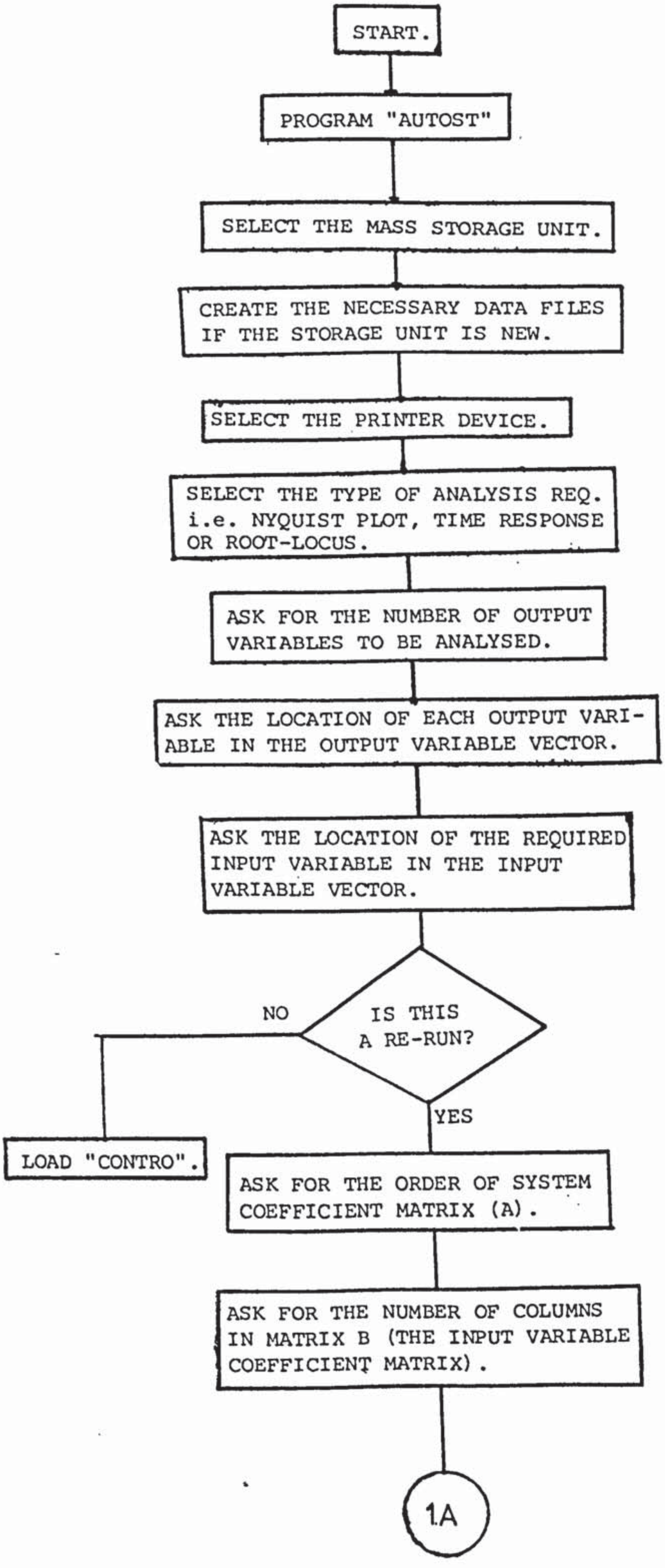
82. Move the cursor to label the graphs.

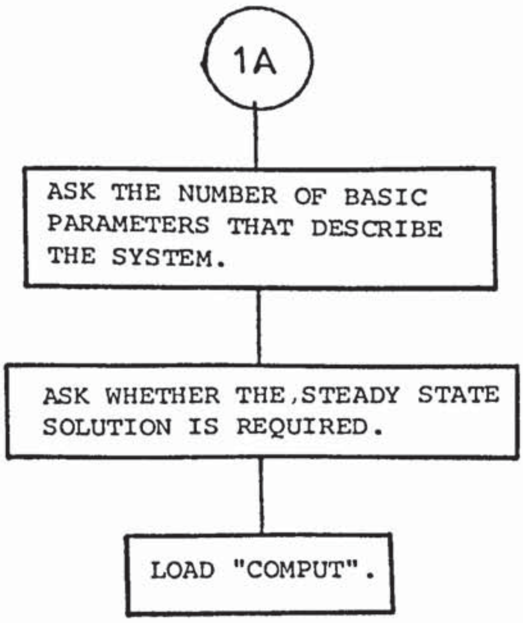
83. Type in YES and press CONT key if the frequency response and phase angle plot are required; otherwise type NO and press CONT key.

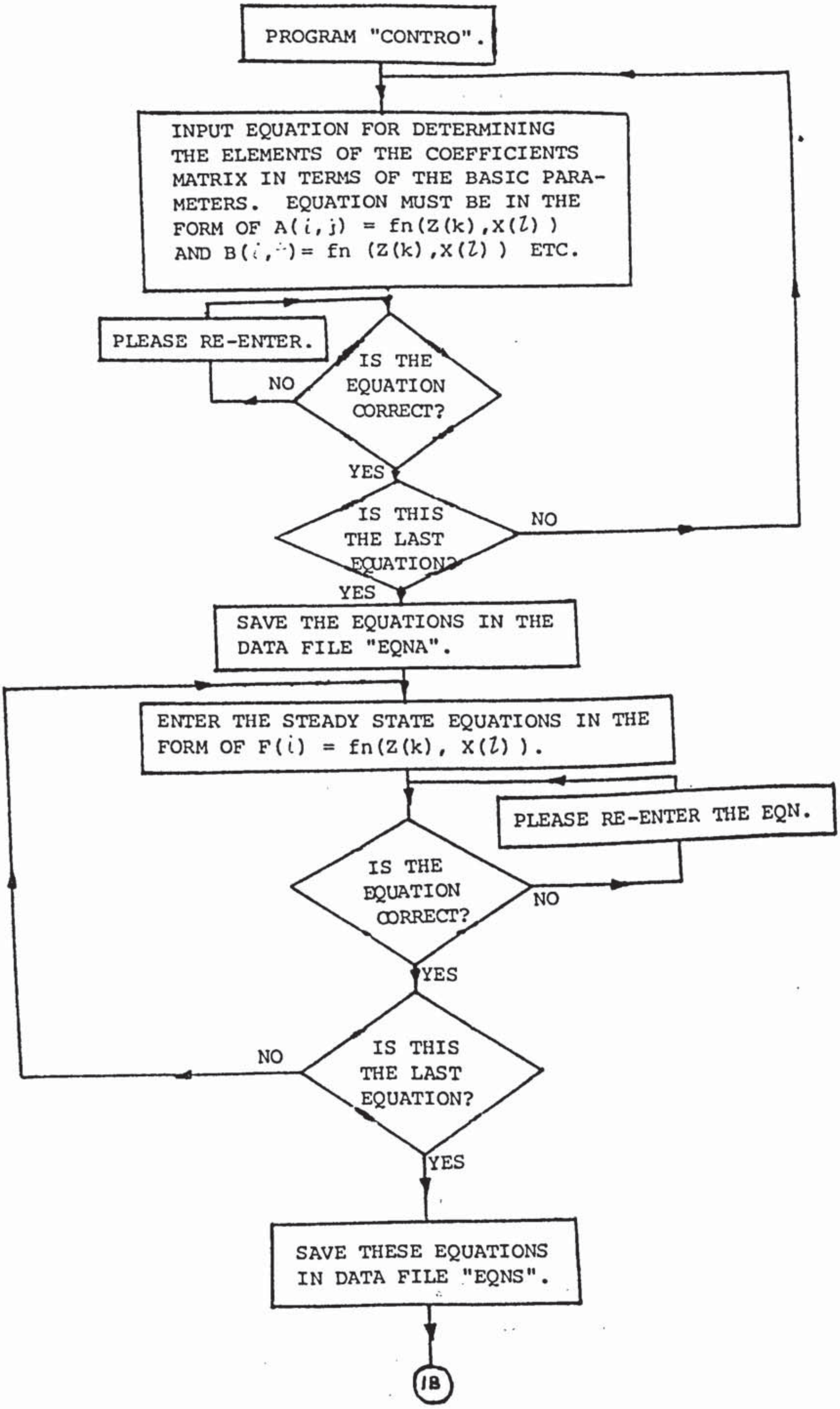
84. Type in YES and press CONT key if the phase angle plot is required; otherwise type NO and press CONT key.

85. Type in YES and press CONT key if any change in these plots are required; otherwise type in NO and press CONT key.

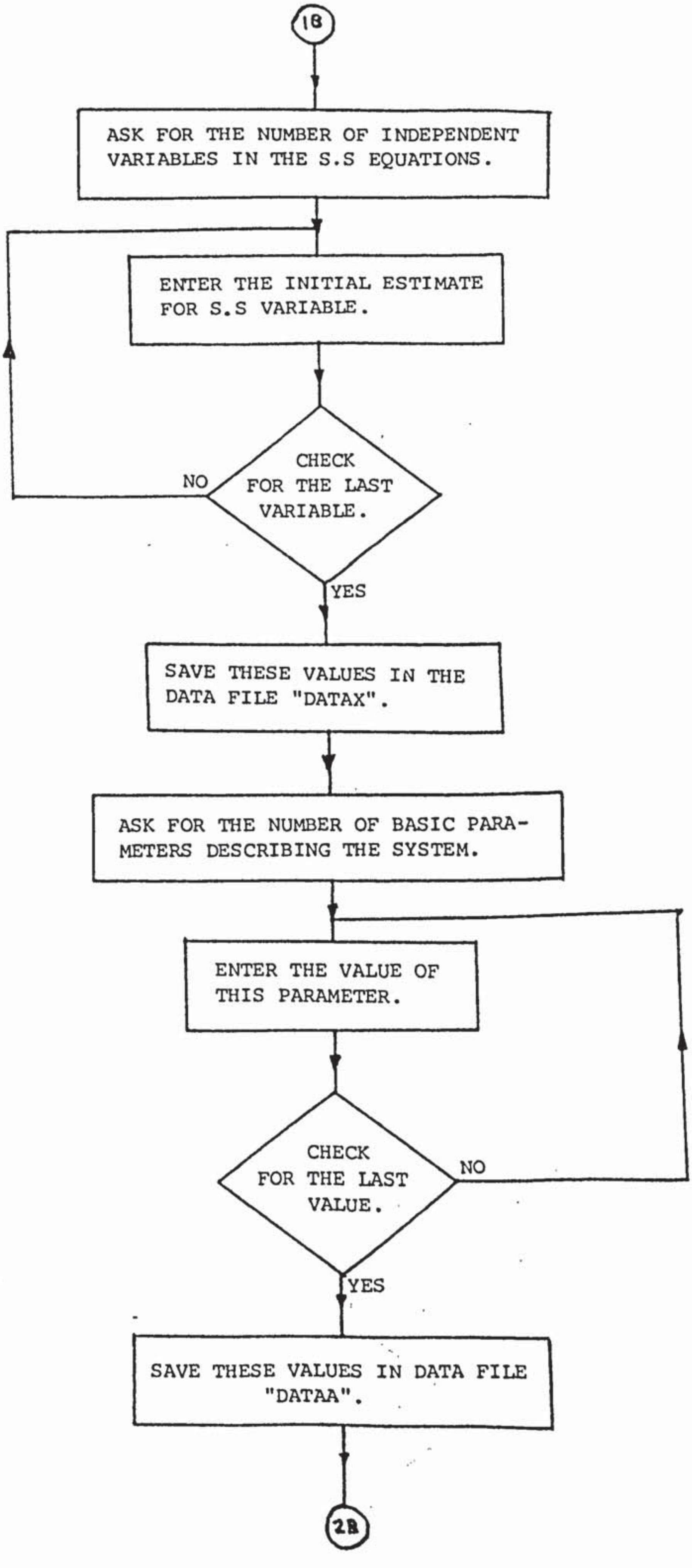
B.7. Flow charts of the Package.

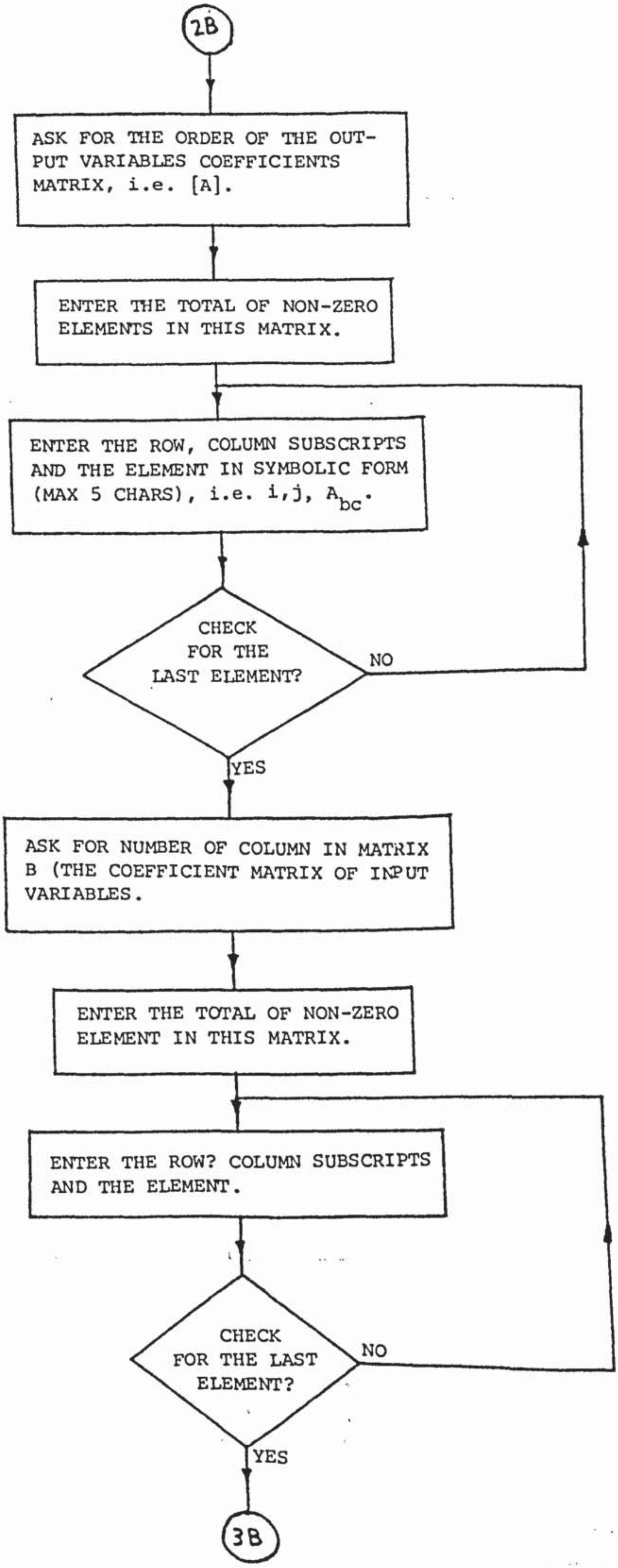

START.
PROGRAM "AUTOST"
SELECT THE MASS STORAGE UNIT.
CREATE THE NECESSARY DATA FILES IF THE STORAGE UNIT IS NEW.
SELECT THE PRINTER DEVICE.
SELECT THE TYPE OF ANALYSIS REQ. i.e. NYQUIST PLOT, TIME RESPONSE OR ROOT-LOCUS.
ASK FOR THE NUMBER OF OUTPUT VARIABLES TO BE ANALYSED.
ASK THE LOCATION OF EACH OUTPUT VARIABLE IN THE OUTPUT VARIABLE VECTOR.
ASK THE LOCATION OF THE REQUIRED INPUT VARIABLE IN THE INPUT VARIABLE VECTOR.
IS THIS A RE-RUN?
NO
YES
LOAD "CONTRO".
ASK FOR THE ORDER OF SYSTEM COEFFICIENT MATRIX (A).
ASK FOR THE NUMBER OF COLUMNS IN MATRIX B (THE INPUT VARIABLE COEFFICIENT MATRIX).
1.A

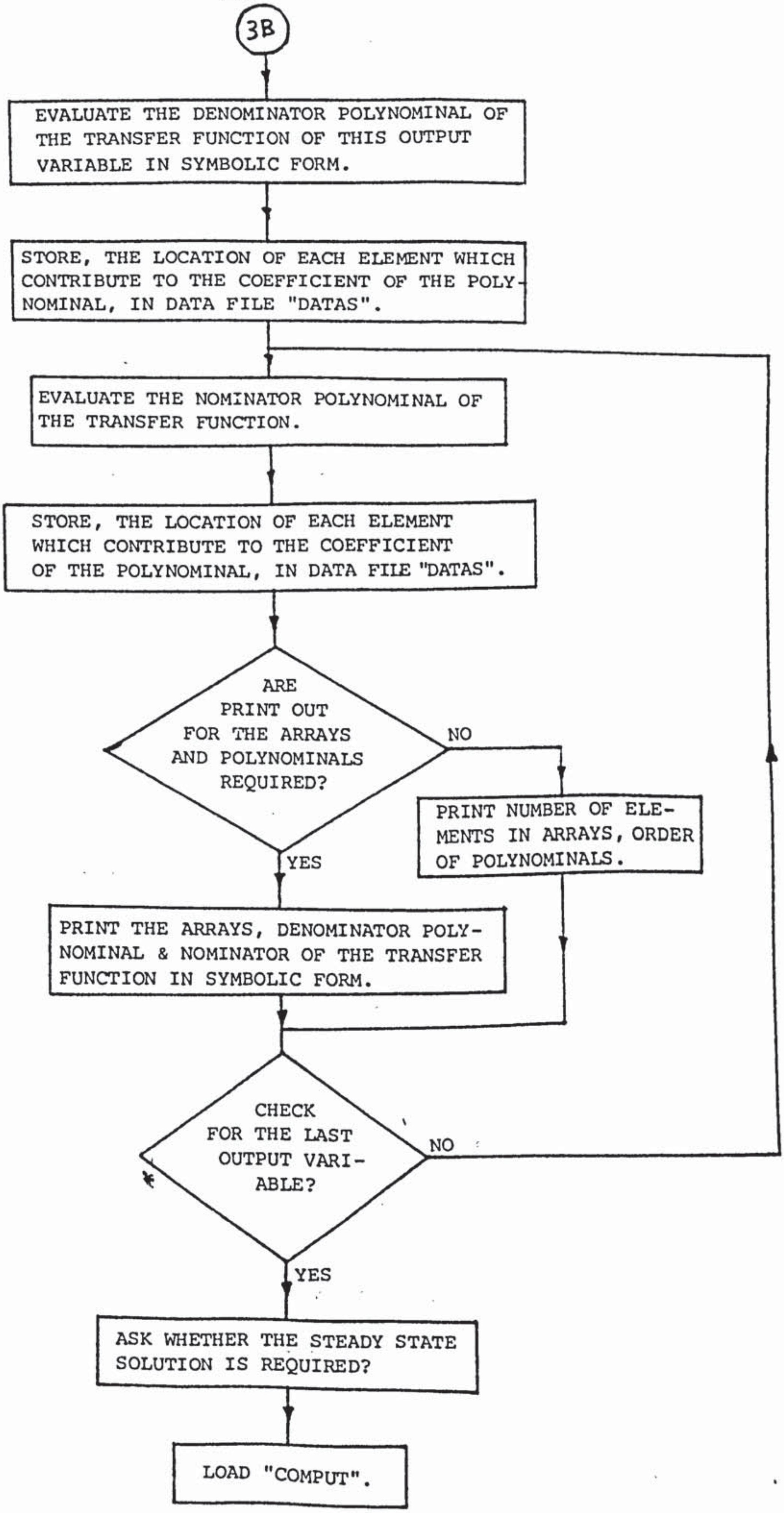

1A
ASK THE NUMBER OF BASIC PARAMETERS THAT DESCRIBE THE SYSTEM.
ASK WHETHER THE STEADY STATE SOLUTION IS REQUIRED.
LOAD "COMPUT".

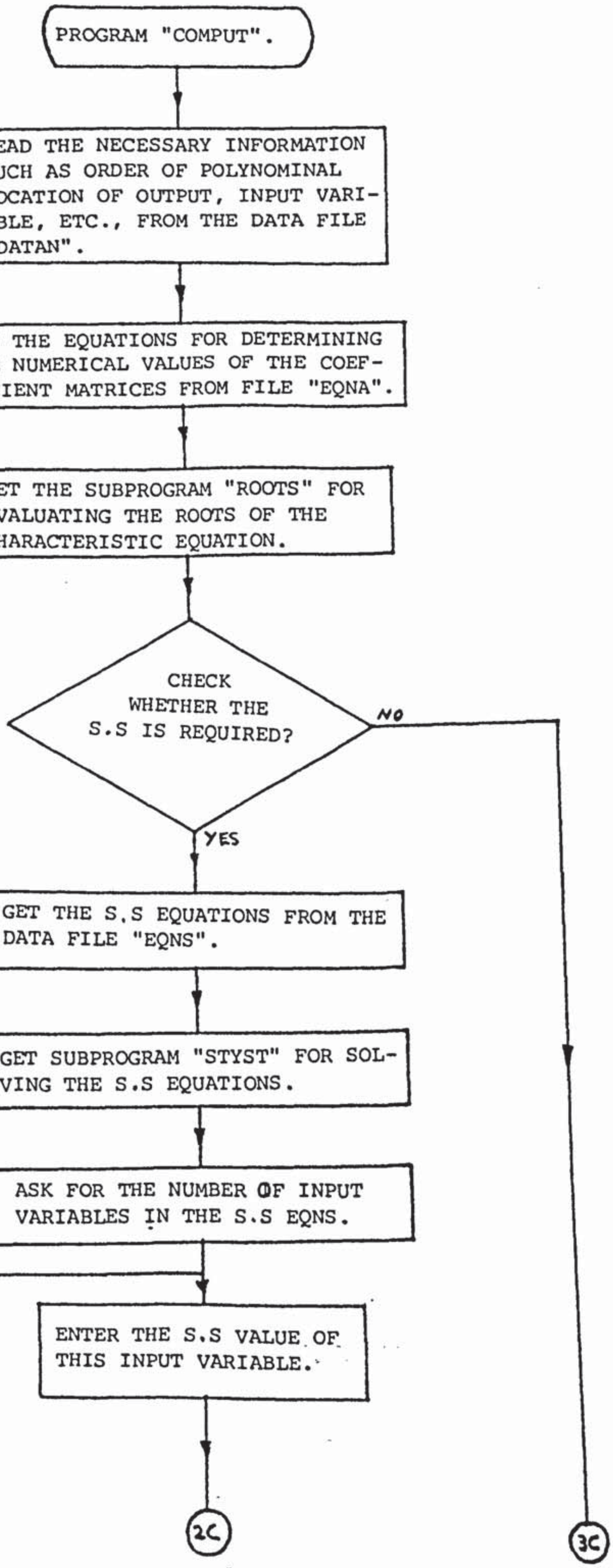


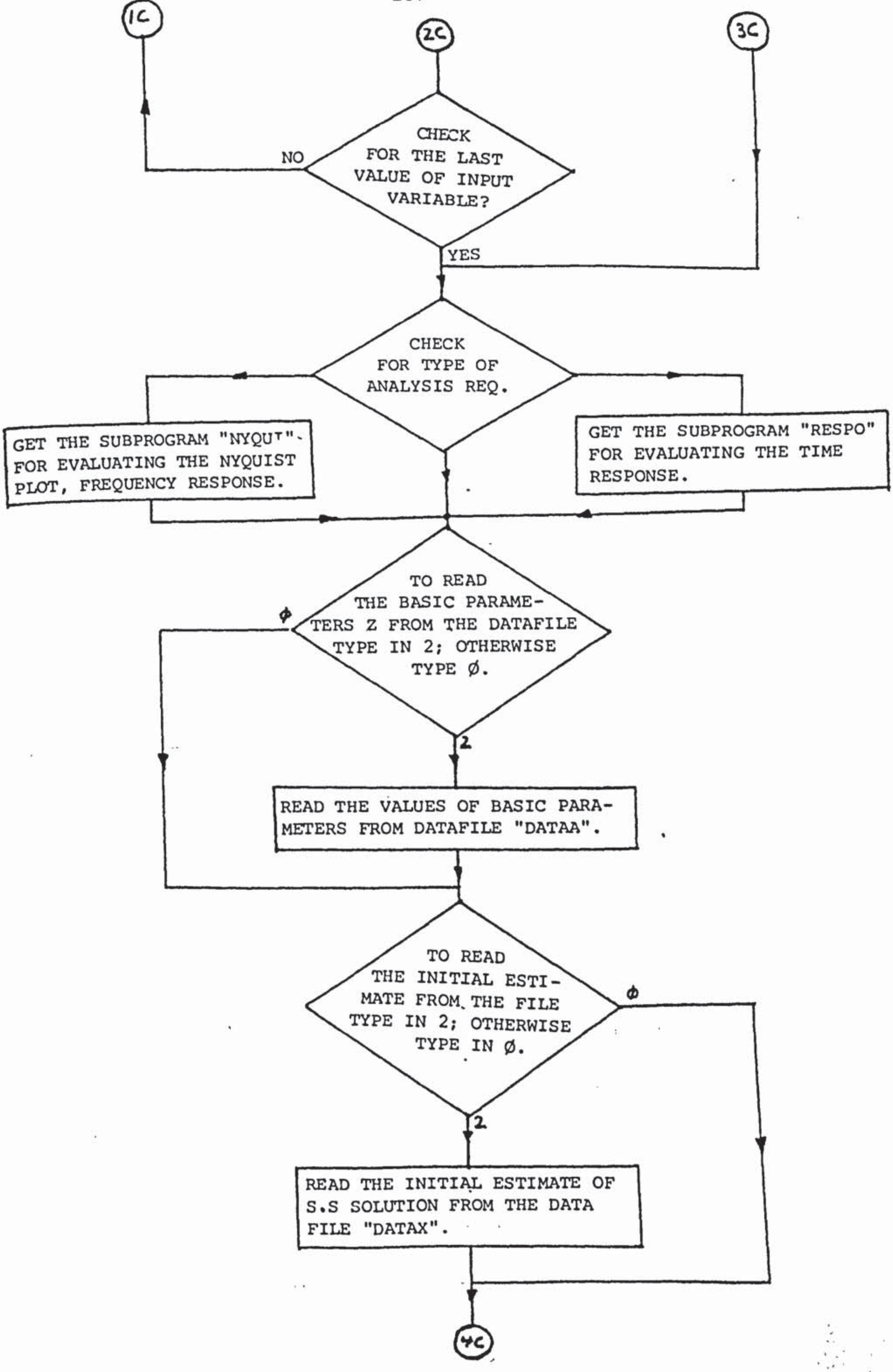
PROGRAM "CONTRO".
INPUT EQUATION FOR DETERMINING THE ELEMENTS OF THE COEFFICIENTS MATRIX IN TERMS OF THE BASIC PARAMETERS. EQUATION MUST BE IN THE FORM OF A(i,j) = fn(Z(k),X(l)) AND B(i,j) = fn (Z(k),X(l)) ETC.
PLEASE RE-ENTER.
NO
IS THE EQUATION CORRECT?
YES
IS THIS THE LAST EQUATION?
NO
YES
SAVE THE EQUATIONS IN THE DATA FILE "EQNA".
ENTER THE STEADY STATE EQUATIONS IN THE FORM OF F(i) = fn(Z(k), X(l)).
PLEASE RE-ENTER THE EQN.
IS THE EQUATION CORRECT?
NO
YES
NO
IS THIS THE LAST EQUATION?
YES
SAVE THESE EQUATIONS IN DATA FILE "EQNS".
1B

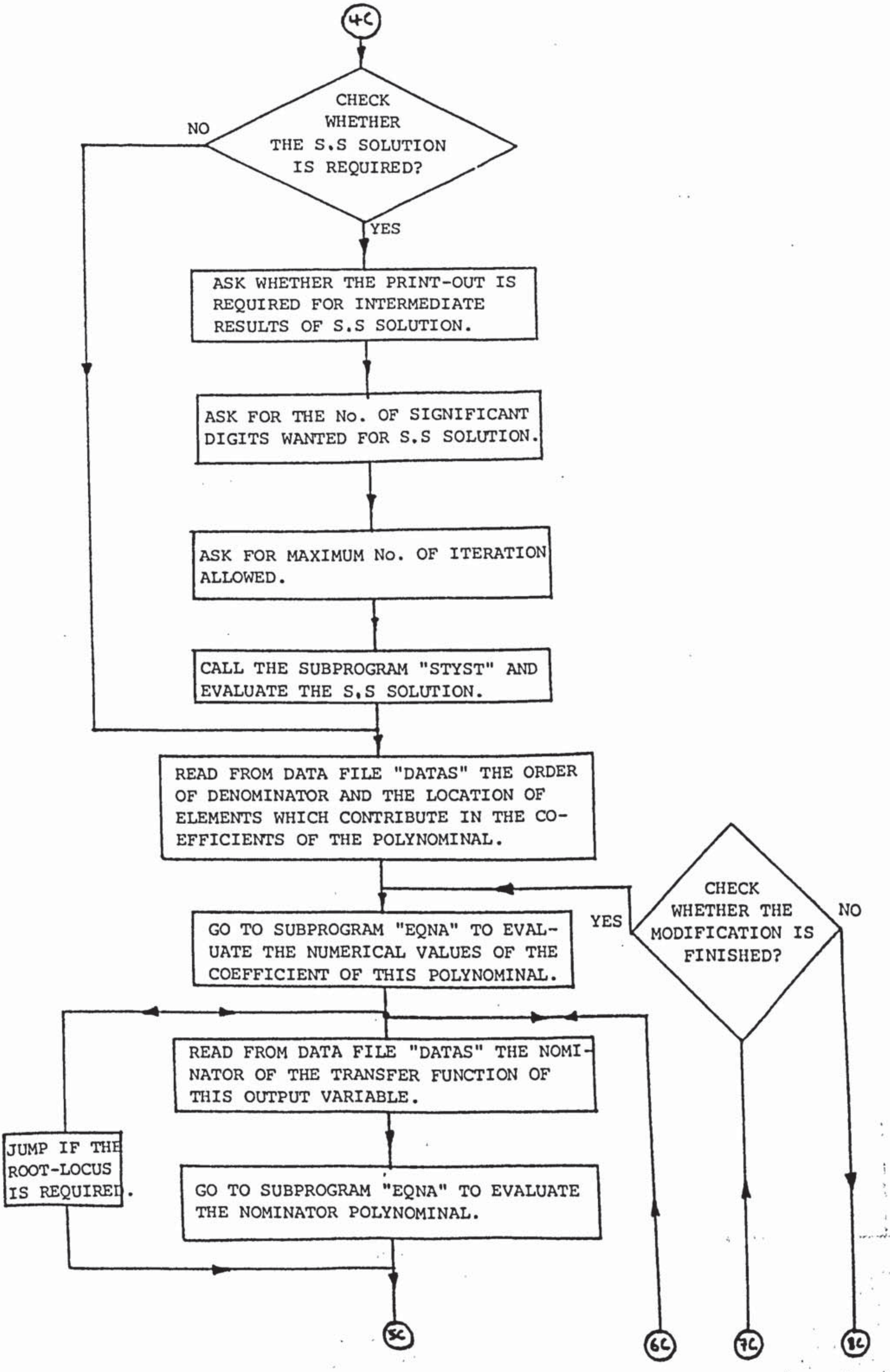

1B
ASK FOR THE NUMBER OF INDEPENDENT VARIABLES IN THE S.S EQUATIONS.
ENTER THE INITIAL ESTIMATE FOR S.S VARIABLE.
CHECK FOR THE LAST VARIABLE.
NO
YES
SAVE THESE VALUES IN THE DATA FILE "DATAX".
ASK FOR THE NUMBER OF BASIC PARAMETERS DESCRIBING THE SYSTEM.
ENTER THE VALUE OF THIS PARAMETER.
CHECK FOR THE LAST VALUE.
NO
YES
SAVE THESE VALUES IN DATA FILE "DATAA".
2B

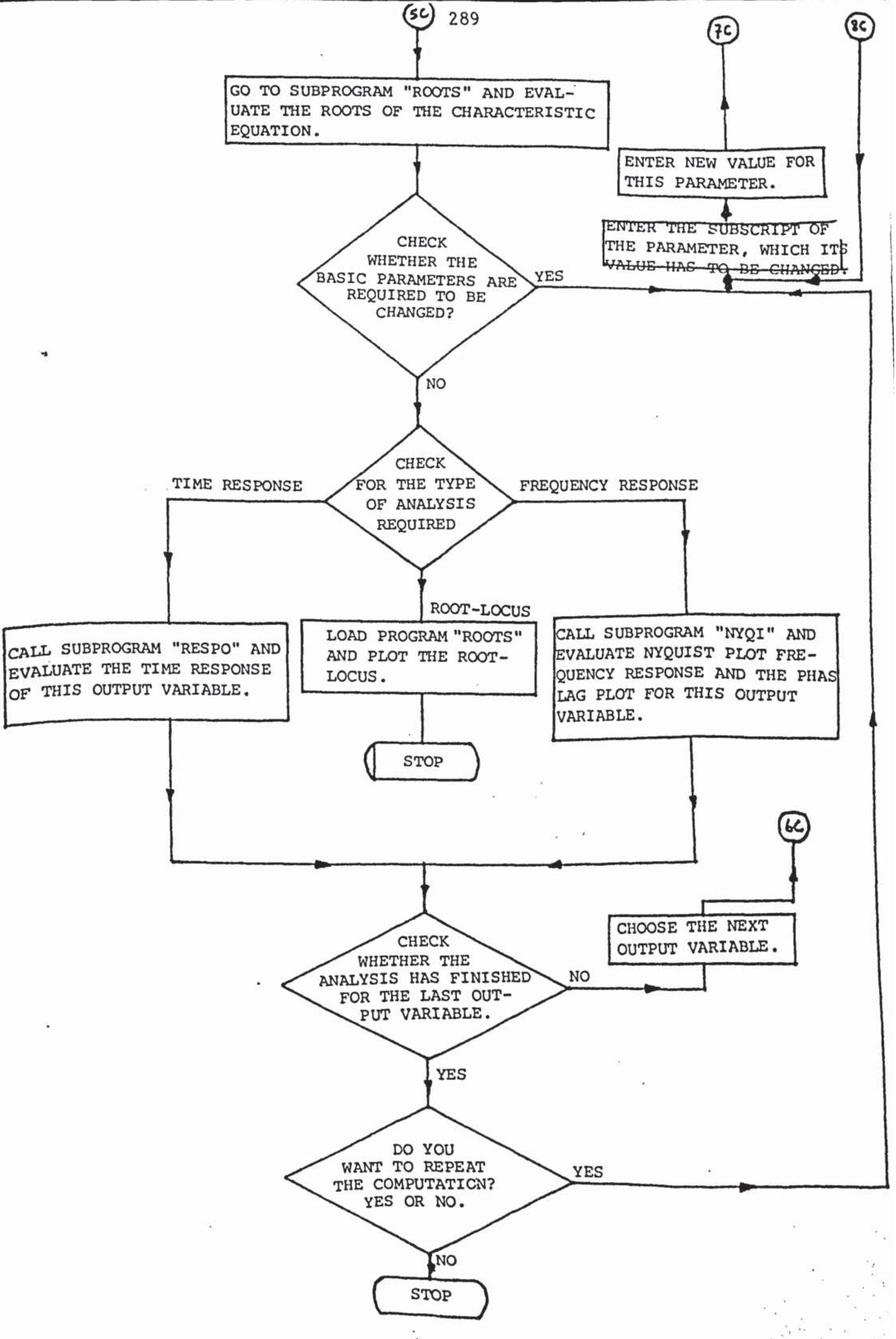

2B
ASK FOR THE ORDER OF THE OUT-PUT VARIABLES COEFFICIENTS MATRIX, i.e. [A].
ENTER THE TOTAL OF NON-ZERO ELEMENTS IN THIS MATRIX.
ENTER THE ROW, COLUMN SUBSCRIPTS AND THE ELEMENT IN SYMBOLIC FORM (MAX 5 CHARS), i.e. i,j, A_{bc}.
CHECK FOR THE LAST ELEMENT?
NO
YES
ASK FOR NUMBER OF COLUMN IN MATRIX B (THE COEFFICIENT MATRIX OF INPUT VARIABLES.
ENTER THE TOTAL OF NON-ZERO ELEMENT IN THIS MATRIX.
ENTER THE ROW? COLUMN SUBSCRIPTS AND THE ELEMENT.
CHECK FOR THE LAST ELEMENT?
NO
YES
3B

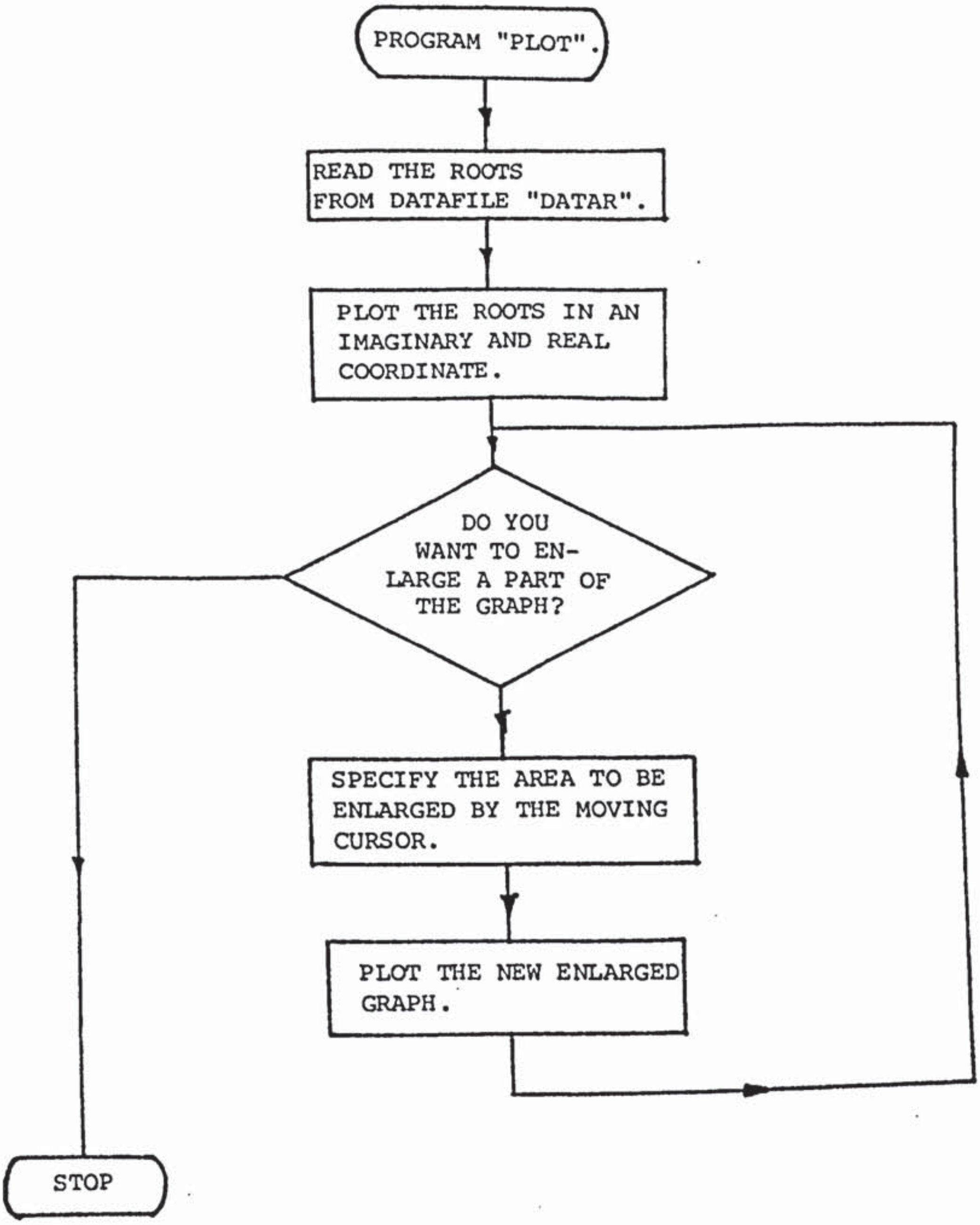

3B
EVALUATE THE DENOMINATOR POLYNOMINAL OF THE TRANSFER FUNCTION OF THIS OUTPUT VARIABLE IN SYMBOLIC FORM.
STORE, THE LOCATION OF EACH ELEMENT WHICH CONTRIBUTE TO THE COEFFICIENT OF THE POLYNOMINAL, IN DATA FILE "DATAS".
EVALUATE THE NOMINATOR POLYNOMINAL OF THE TRANSFER FUNCTION.
STORE, THE LOCATION OF EACH ELEMENT WHICH CONTRIBUTE TO THE COEFFICIENT OF THE POLYNOMINAL, IN DATA FILE "DATAS".
ARE PRINT OUT FOR THE ARRAYS AND POLYNOMINALS REQUIRED?
NO
PRINT NUMBER OF ELEMENTS IN ARRAYS, ORDER OF POLYNOMINALS.
YES
PRINT THE ARRAYS, DENOMINATOR POLYNOMINAL & NOMINATOR OF THE TRANSFER FUNCTION IN SYMBOLIC FORM.
CHECK FOR THE LAST OUTPUT VARIABLE?
NO
YES
ASK WHETHER THE STEADY STATE SOLUTION IS REQUIRED?
LOAD "COMPUT".

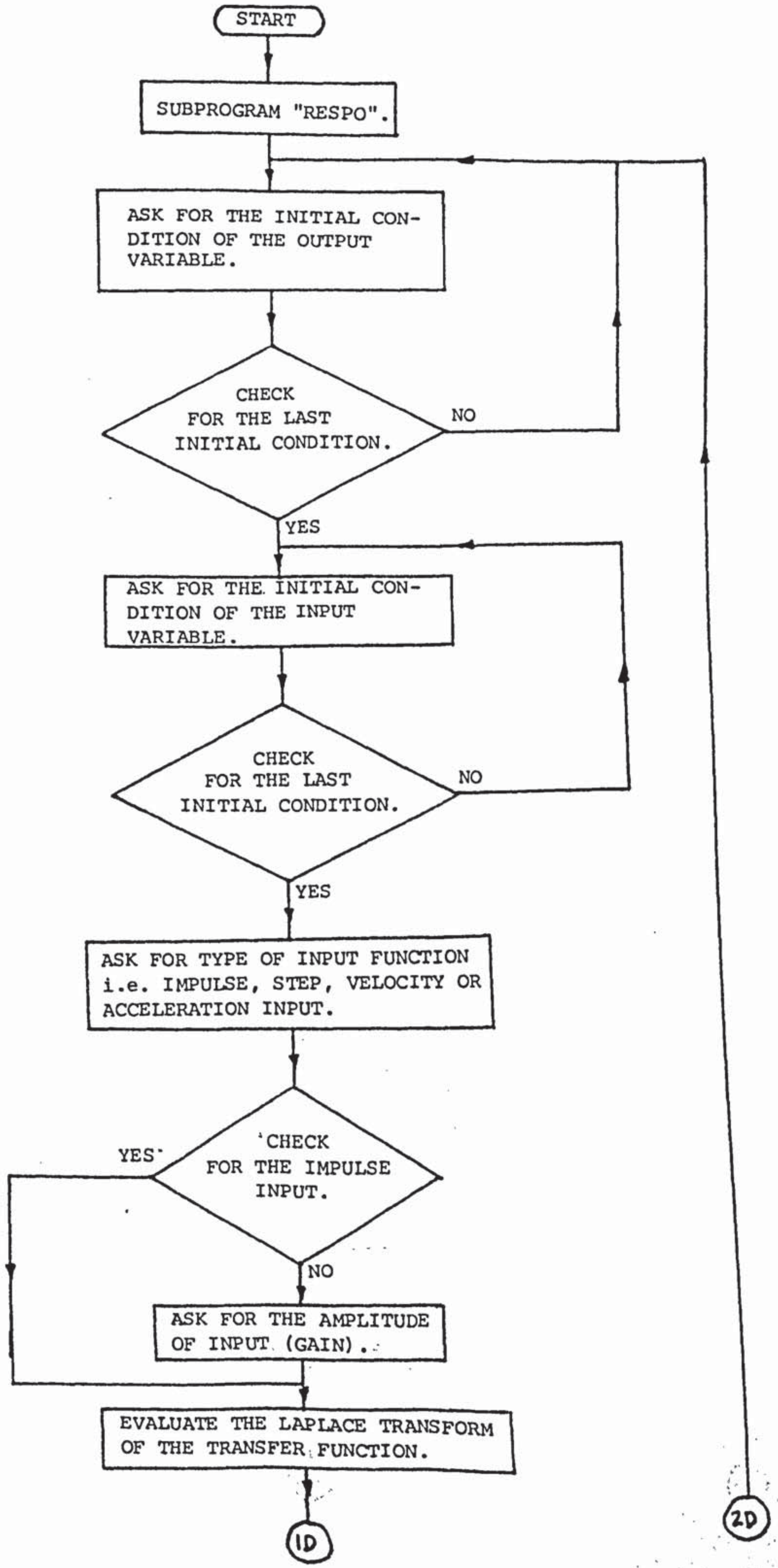

PROGRAM "COMPUT".
READ THE NECESSARY INFORMATION SUCH AS ORDER OF POLYNOMINAL LOCATION OF OUTPUT, INPUT VARIABLE, ETC., FROM THE DATA FILE "DATAN".
GET THE EQUATIONS FOR DETERMINING THE NUMERICAL VALUES OF THE COEFFICIENT MATRICES FROM FILE "EQNA".
GET THE SUBPROGRAM "ROOTS" FOR EVALUATING THE ROOTS OF THE CHARACTERISTIC EQUATION.
CHECK WHETHER THE S.S IS REQUIRED?
NO
YES
GET THE S.S EQUATIONS FROM THE DATA FILE "EQNS".
GET SUBPROGRAM "STYST" FOR SOLVING THE S.S EQUATIONS.
ASK FOR THE NUMBER OF INPUT VARIABLES IN THE S.S EQNS.
ENTER THE S.S VALUE OF THIS INPUT VARIABLE.
1C
2C
3C

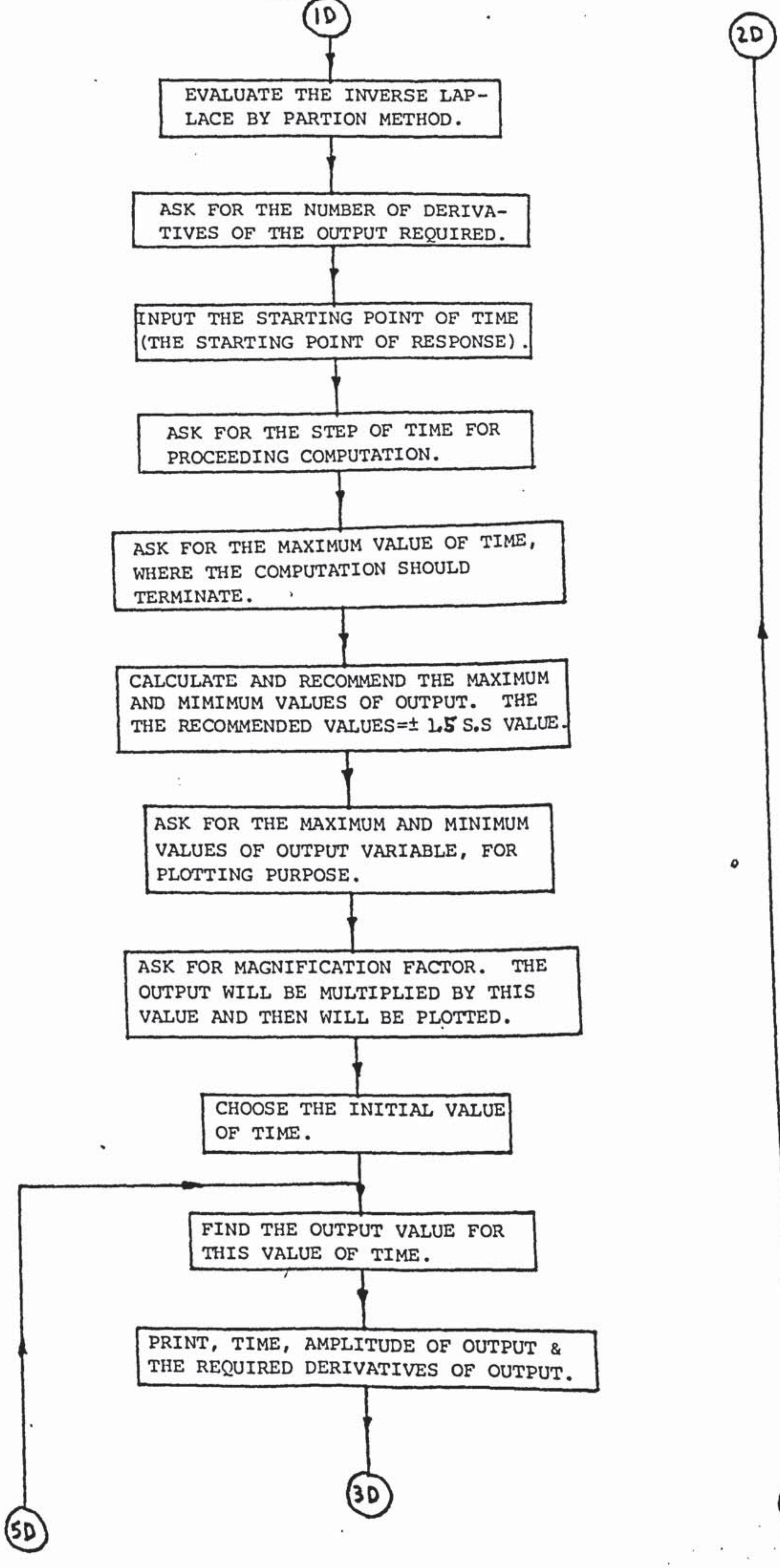

1C
2C
3C
CHECK FOR THE LAST VALUE OF INPUT VARIABLE?
NO
YES
CHECK FOR TYPE OF ANALYSIS REQ.
GET THE SUBPROGRAM "NYQUT" FOR EVALUATING THE NYQUIST PLOT, FREQUENCY RESPONSE.
GET THE SUBPROGRAM "RESPO" FOR EVALUATING THE TIME RESPONSE.
TO READ THE BASIC PARAMETERS Z FROM THE DATAFILE TYPE IN 2; OTHERWISE TYPE Ø.
Ø
2
READ THE VALUES OF BASIC PARAMETERS FROM DATAFILE "DATAA".
TO READ THE INITIAL ESTIMATE FROM THE FILE TYPE IN 2; OTHERWISE TYPE IN Ø.
Ø
2
READ THE INITIAL ESTIMATE OF S.S SOLUTION FROM THE DATA FILE "DATAX".
4C

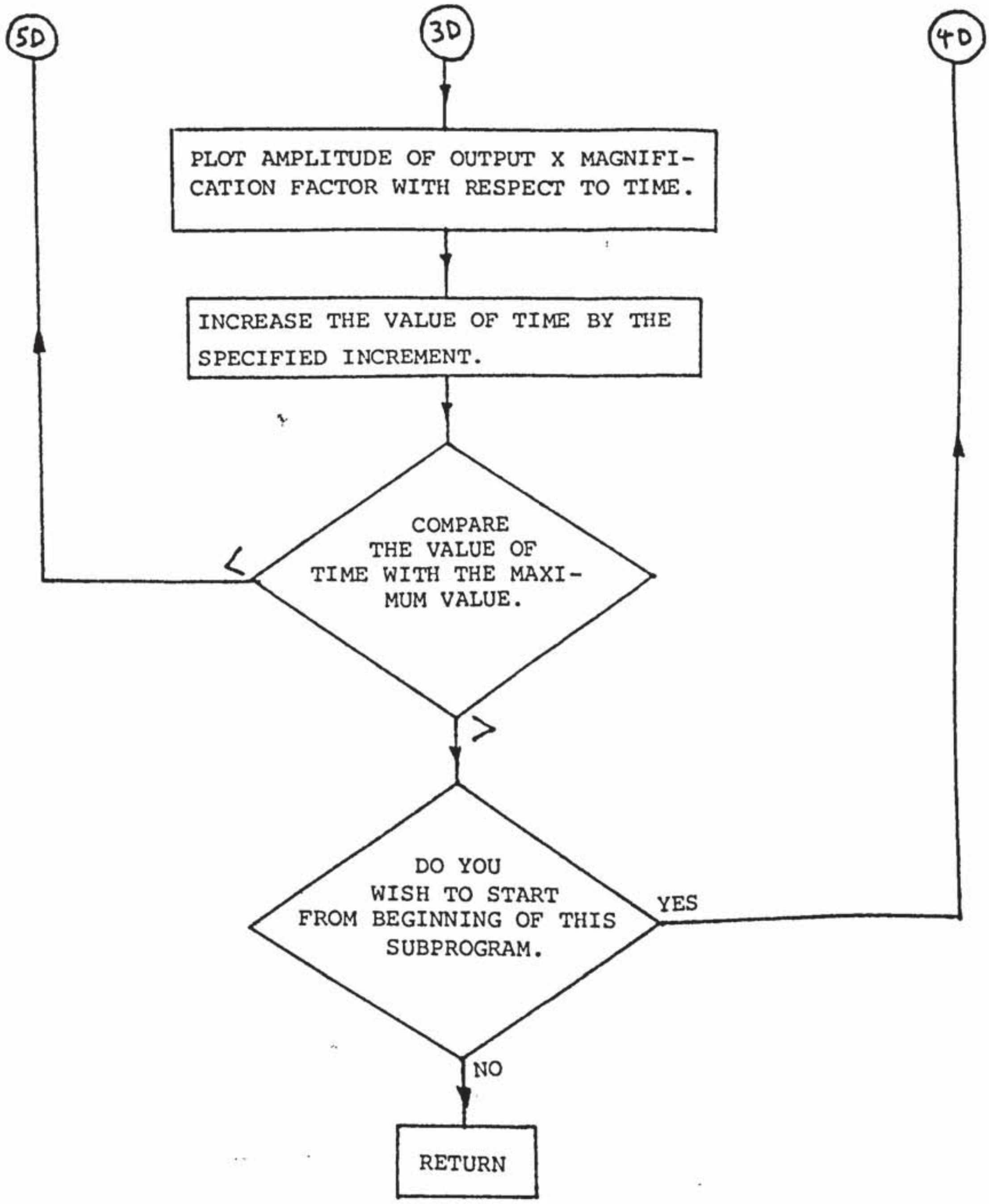

4C
CHECK WHETHER THE S.S SOLUTION IS REQUIRED?
NO
YES
ASK WHETHER THE PRINT-OUT IS REQUIRED FOR INTERMEDIATE RESULTS OF S.S SOLUTION.
ASK FOR THE No. OF SIGNIFICANT DIGITS WANTED FOR S.S SOLUTION.
ASK FOR MAXIMUM No. OF ITERATION ALLOWED.
CALL THE SUBPROGRAM "STYST" AND EVALUATE THE S.S SOLUTION.
READ FROM DATA FILE "DATAS" THE ORDER OF DENOMINATOR AND THE LOCATION OF ELEMENTS WHICH CONTRIBUTE IN THE CO-EFFICIENTS OF THE POLYNOMINAL.
GO TO SUBPROGRAM "EQNA" TO EVAL-UATE THE NUMERICAL VALUES OF THE COEFFICIENT OF THIS POLYNOMINAL.
YES
CHECK WHETHER THE MODIFICATION IS FINISHED?
NO
READ FROM DATA FILE "DATAS" THE NOMI-NATOR OF THE TRANSFER FUNCTION OF THIS OUTPUT VARIABLE.
JUMP IF THE ROOT-LOCUS IS REQUIRED.
GO TO SUBPROGRAM "EQNA" TO EVALUATE THE NOMINATOR POLYNOMINAL.
5C
6C
7C
8C

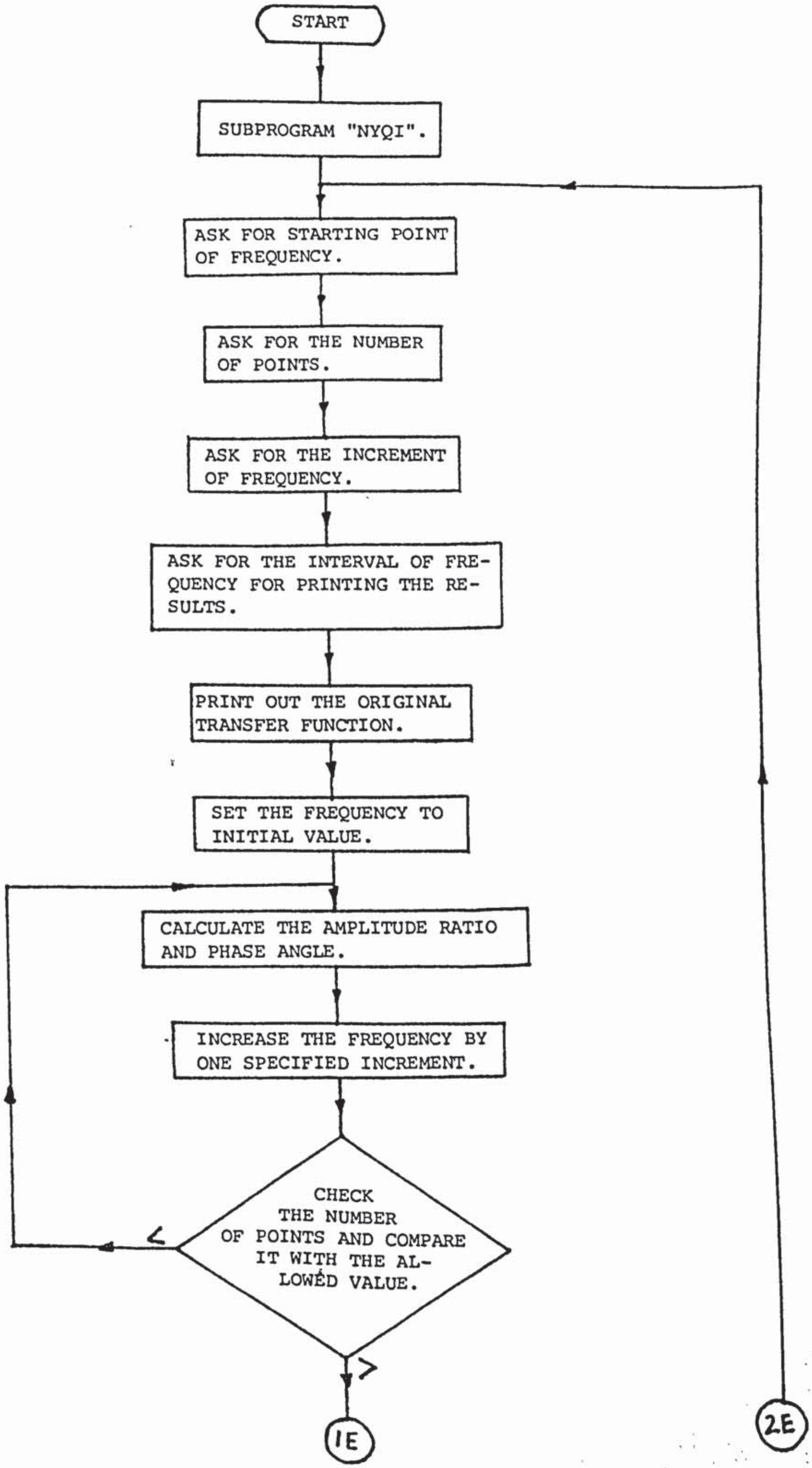

5C
7C
8C
GO TO SUBPROGRAM "ROOTS" AND EVALUATE THE ROOTS OF THE CHARACTERISTIC EQUATION.
ENTER NEW VALUE FOR THIS PARAMETER.
ENTER THE SUBSCRIPT OF THE PARAMETER, WHICH ITS VALUE HAS TO BE CHANGED.
CHECK WHETHER THE BASIC PARAMETERS ARE REQUIRED TO BE CHANGED?
YES
NO
CHECK FOR THE TYPE OF ANALYSIS REQUIRED
TIME RESPONSE
FREQUENCY RESPONSE
ROOT-LOCUS
CALL SUBPROGRAM "RESPO" AND EVALUATE THE TIME RESPONSE OF THIS OUTPUT VARIABLE.
LOAD PROGRAM "ROOTS" AND PLOT THE ROOT-LOCUS.
CALL SUBPROGRAM "NYQI" AND EVALUATE NYQUIST PLOT FREQUENCY RESPONSE AND THE PHAS LAG PLOT FOR THIS OUTPUT VARIABLE.
STOP
6C
CHOOSE THE NEXT OUTPUT VARIABLE.
CHECK WHETHER THE ANALYSIS HAS FINISHED FOR THE LAST OUTPUT VARIABLE.
NO
YES
DO YOU WANT TO REPEAT THE COMPUTATICN? YES OR NO.
YES
NO
STOP

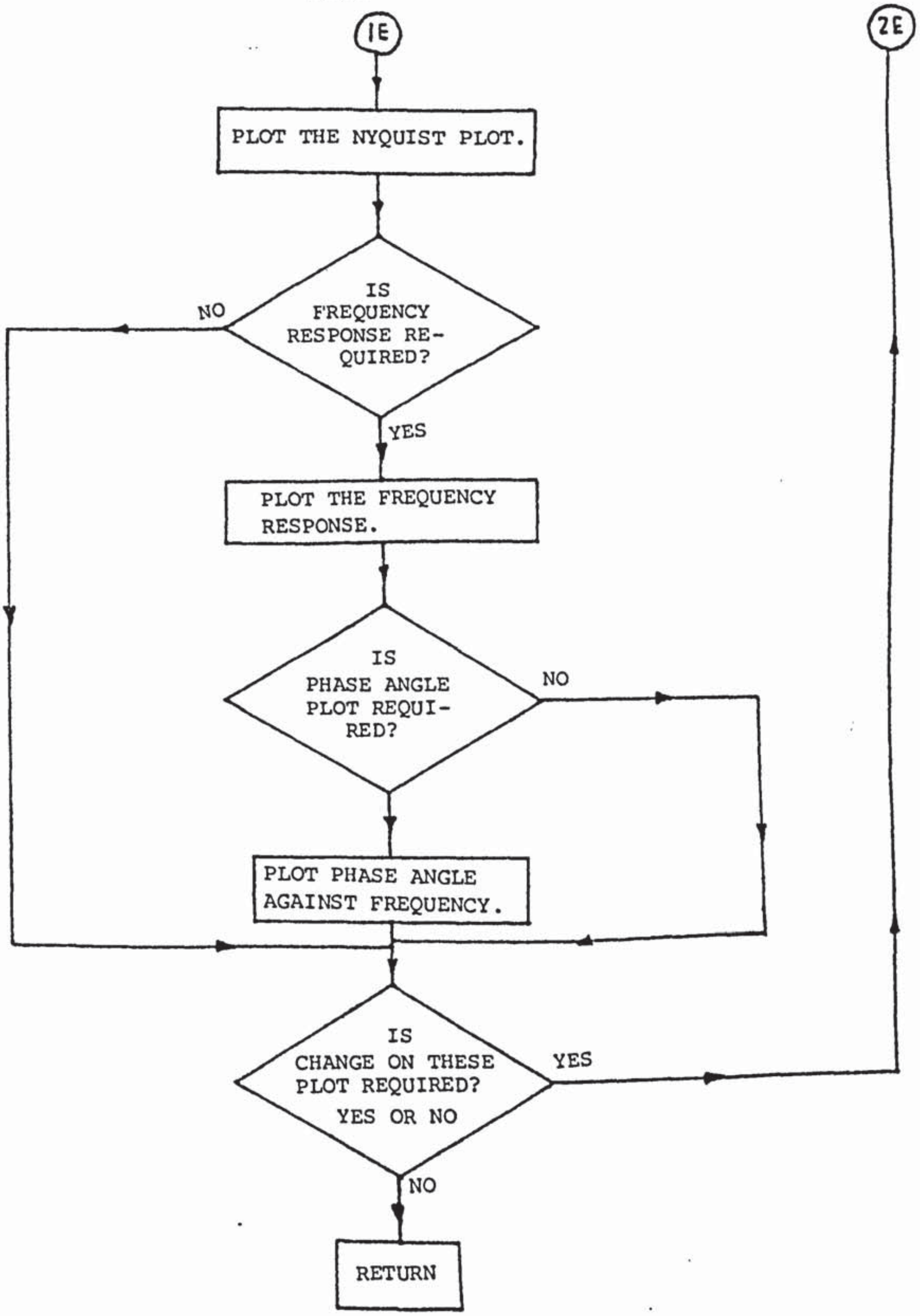

PROGRAM "PLOT".
READ THE ROOTS FROM DATAFILE "DATAR".
PLOT THE ROOTS IN AN IMAGINARY AND REAL COORDINATE.
DO YOU WANT TO EN-LARGE A PART OF THE GRAPH?
SPECIFY THE AREA TO BE ENLARGED BY THE MOVING CURSOR.
PLOT THE NEW ENLARGED GRAPH.
STOP


START
SUBPROGRAM "RESPO".
ASK FOR THE INITIAL CONDITION OF THE OUTPUT VARIABLE.
CHECK FOR THE LAST INITIAL CONDITION.
NO
YES
ASK FOR THE INITIAL CONDITION OF THE INPUT VARIABLE.
CHECK FOR THE LAST INITIAL CONDITION.
NO
YES
ASK FOR TYPE OF INPUT FUNCTION i.e. IMPULSE, STEP, VELOCITY OR ACCELERATION INPUT.
CHECK FOR THE IMPULSE INPUT.
YES
NO
ASK FOR THE AMPLITUDE OF INPUT (GAIN).
EVALUATE THE LAPLACE TRANSFORM OF THE TRANSFER FUNCTION.
1D
2D


1D
2D
EVALUATE THE INVERSE LAPLACE BY PARTION METHOD.
ASK FOR THE NUMBER OF DERIVATIVES OF THE OUTPUT REQUIRED.
INPUT THE STARTING POINT OF TIME (THE STARTING POINT OF RESPONSE).
ASK FOR THE STEP OF TIME FOR PROCEEDING COMPUTATION.
ASK FOR THE MAXIMUM VALUE OF TIME, WHERE THE COMPUTATION SHOULD TERMINATE.
CALCULATE AND RECOMMEND THE MAXIMUM AND MIMIMUM VALUES OF OUTPUT. THE THE RECOMMENDED VALUES=± 1.5 S.S VALUE.
ASK FOR THE MAXIMUM AND MINIMUM VALUES OF OUTPUT VARIABLE, FOR PLOTTING PURPOSE.
ASK FOR MAGNIFICATION FACTOR. THE OUTPUT WILL BE MULTIPLIED BY THIS VALUE AND THEN WILL BE PLOTTED.
CHOOSE THE INITIAL VALUE OF TIME.
FIND THE OUTPUT VALUE FOR THIS VALUE OF TIME.
PRINT, TIME, AMPLITUDE OF OUTPUT & THE REQUIRED DERIVATIVES OF OUTPUT.
3D
4D
5D


5D
3D
4D
PLOT AMPLITUDE OF OUTPUT X MAGNIFICATION FACTOR WITH RESPECT TO TIME.
INCREASE THE VALUE OF TIME BY THE SPECIFIED INCREMENT.
COMPARE THE VALUE OF TIME WITH THE MAXIMUM VALUE.
<
>
DO YOU WISH TO START FROM BEGINNING OF THIS SUBPROGRAM.
YES
NO
RETURN


START
SUBPROGRAM "NYQI".
ASK FOR STARTING POINT OF FREQUENCY.
ASK FOR THE NUMBER OF POINTS.
ASK FOR THE INCREMENT OF FREQUENCY.
ASK FOR THE INTERVAL OF FREQUENCY FOR PRINTING THE RESULTS.
PRINT OUT THE ORIGINAL TRANSFER FUNCTION.
SET THE FREQUENCY TO INITIAL VALUE.
CALCULATE THE AMPLITUDE RATIO AND PHASE ANGLE.
INCREASE THE FREQUENCY BY ONE SPECIFIED INCREMENT.
CHECK THE NUMBER OF POINTS AND COMPARE IT WITH THE ALLOWED VALUE.
<
>
1E
2E


1E
2E
PLOT THE NYQUIST PLOT.
IS FREQUENCY RESPONSE RE-QUIRED?
NO
YES
PLOT THE FREQUENCY RESPONSE.
IS PHASE ANGLE PLOT REQUI-RED?
NO
PLOT PHASE ANGLE AGAINST FREQUENCY.
IS CHANGE ON THESE PLOT REQUIRED? YES OR NO
YES
NO
RETURN

B.8. List of programmes and subprogrammes.

PROGRAM "AUTOST"

```
10   REM *** PROGRAM STORED IN "AUTOST:F8 " ***
20    LOAD KEY "SFK1:T15"
30    OPTION BASE 1
40    PRINTER IS 16
50    PRINT PAGE
60    PRINT "  ______________________________________________________________
______________________"
70    PRINT "  |
                    |"
80    PRINT "  | THIS PROGRAMME PACKAGE INVESTIGATES THE STABILITY
 AND TRANSIENT |"
90    PRINT "  |  RESPONSE OF A SYSTEM.
                    |"
100   PRINT "  |
                    |"
110   PRINT "  | THE ROOT-LOCUS, NYQUIST PLOT,FREQUENCY RESPONSE A
ND THE TIME        |"
120   PRINT "  | RESPONSE OF A SYSTEM CAN BE ANALYSED , DEPENDING
ON THE USER'S      |"
130   PRINT "  | CHOICE.
                    |"
140   PRINT "  |
                    |"
150   PRINT "  |______________________________________________________________
____________________|"
160   PRINT
170   PRINT
180   PRINT
190   PRINT
200   INPUT "DO YOU WANT INSTRUCTION ? YES/NO",H$
210   IF (H$="NO") OR (H$="no") THEN GOTO 1710
220   PRINT PAGE
230   PRINT " FIRST OF ALL YOU SHOULD WRITE THE GOVERNING DIFERENTIA
L '
240   PRINT " EQUATIONS OF THE SYSTEM."
250   PRINT
260   PRINT
270   PRINT " PROCEDURE OF RE-ARRANGING THE EQUATIONS IN A FORM ACCE
PTABLE TO "
280   PRINT " THE PROGRAMME IS SHOWN BY AN EXAMPLE AS FOLLOWS;"
290   PRINT
300   PRINT " LET'S ASSUME THAT A CONTROL SYSTEM CONSISTS OF THREE G
OVERNING"
310   PRINT " DIFERENTIAL EQUATIONS WITH THREE OUTPUT AND TWO INPUT
VARIABLES."
320   PRINT
330   PRINT " LET THE OUTPUT VARIABLES BE Y1,Y2,Y3 AND THE INPUT VAR
IABLES BE X1,X2 ."
340   PRINT
350   PRINT " FURTHERMORE,LET THE GOVERNING DIFERENTIAL EQUATIONS OF
 THE SYSTEM "
360   PRINT " BE AS SHOWN BELOW;"
370   PRINT
380   PRINT
390   PRINT "       PRESS  CONT  KEY TO PROCEED.   "
400   PRINT
410   PRINT
420   PRINT
430   PAUSE
440   PRINT " A3*A1*S^2(Y1)+A3*S(Y2)+A3*A6*S^2(Y2)+A4*(Y1)=A5*S(X1)+
A6*(X2)    (1)"
```

```
450   PRINT " A8+A7*S(Y1)+A8*S^2(Y2)=A9*(X1)
        (2)"
460   PRINT " A10+S^2(Y3)+A11*S(Y1)=A12*(X1)+A13*(X2)
        (3)"
470   PRINT
480   PRINT " WHERE;                                              "
490   PRINT "         S IS THE LAPLACE TRANSFORM OPERATOR ."
500   PRINT "         A1,A2,A3,...ETC ARE THE BASIC PARAMETERS THAT DE
SCRIBE THE"
510   PRINT "         SYSTEM."
520   PRINT
530   PRINT " NOW SOME NEW VARIABLES SHOULD BE INTRODUCED , THE NUMB
ER OF NEW "
540   PRINT " VARIABLES IS EQUAL TO THE TOTAL OF THE ORDER  OF THE M
AXIMUM DER-"
550   PRINT " IVATIVES OF INPUT AND OUTPUT VARIABLES."
560   PRINT " THEREFORE; FOR THR ABOVE SYSTEM, 7 NEW VARIABLES MUST
BE INTRODUCED "
570   PRINT " AS FOLLOWS;"
580   PRINT
590   PRINT
600   PRINT "          PRESS CONT KEY TO PROCEED        "
610   PAUSE
620   PRINT
630   PRINT
640   PRINT " Y4=S(Y1)       1"
650   PRINT " Y5=S(Y2)       2"
660   PRINT " Y6=S(Y3)       3"
670   PRINT " Y7=S^2(Y1)     4"
680   PRINT " Y8=S^2(Y2)     5"
690   PRINT " Y9=S^2(Y3)     6"
700   PRINT " Y10=S(X1)      7"
710   PRINT
720   PRINT
730   PRINT "ALSO , 7 NEW EQUATIONS MUST BE ADDED TO THE ORIGINAL EQ
UATIONS."
740   PRINT "i.e.;"
750   PRINT " S(Y1)=Y4       1"
760   PRINT " S(Y2)=Y5       2"
770   PRINT " ........"
780   PRINT "........."
790   PRINT " S(X1)=Y10      7"
800   PRINT
810   PRINT
820   PRINT "     PRESS CONT KEY TO PROCEED        "
830   PAUSE
840   PRINT
850   PRINT
860   PRINT "  THE ABOVE EQUATIONS IN MATRICES FORM WITH CONSTANT CO
EFFICIENTS"
870   PRINT "  (SYMBOLIC FORM) IS SHOWN BELOW;"
880   PRINT " _                                   _   _    _
          "
890   PRINT "| A   0   0   0   B   0   C   D   0   E  | |Y1 | | 0
E  |   |X1|"
900   PRINT "| 0   0   0   G   0   0   0   H   0   0  | |Y2 | | I
0  |   |X2|"
910   PRINT "| 0   0   0   J   0   0   0   0   K   0  | |Y3 | | L
M  |       "
920   PRINT "| S   0   0  -1   0   0   0   0   0   0  | |Y4 | | 0
0  |       "
```

```
930   PRINT "| 0   S   0   0  -1   0   0   0   0   0  | |Y5 | | 0
0 |       "
940   PRINT "| 0   0   S   0   0  -1   0   0   0   0  |*|Y6 |=| 0
0 |       "
950   PRINT "| 0   0   0   S   0   0  -1   0   0   0  | |Y7 | | 0
0 |       "
960   PRINT "| 0   0   0   0   S   0   0  -1   0   0  | |Y8 | | 0
0 |       "
970   PRINT "| 0   0   0   0   0   S   0   0  -1   0  | |Y9 | | 0
0 |       "
980   PRINT "|_0   0   0   0   0   0   0   0   0   1 _| |Y10|
 |_9   0 _|       "
990   PRINT
1000  PRINT " S,IS THE LAPLACE TRANSFORM OPERATOR AND SHOULD NOT BE
USED FOR CONSTANTS OF THE MATRICES."
1010  PRINT
1020  PRINT
1030  PRINT "   PRESS CONT KEY TO PROCEED    "
1040  PAUSE
1050  PRINT " THE BASIC PARAMETERS (A1,A2,..ETC) SHOUD BE WRITTEN IN
 ARRAY FORM"
1060  PRINT " ,NAMED AS Z;AS SHOWN BELOW:"
1070  PRINT
1080  PRINT " Z(1)=A1"
1090  PRINT " Z(2)=A2"
1100  PRINT " Z(3)=A3"
1110  PRINT " Z(4)=A4"
1120  PRINT " Z(5)=A5"
1130  PRINT " Z(6)=A6"
1140  PRINT " Z(7)=A7"
1150  PRINT " Z(8)=A8"
1160  PRINT " Z(9)=A9"
1170  PRINT " Z(10=A10"
1180  PRINT " Z(11)=A11'
1190  PRINT " Z(12)=A12'
1200  PRINT " Z(13)=A13'
1210  PRINT
1220  PRINT " LATER ON IN THE PROGRAMME THE NUMERICAL VALUES OF THES
E BASIC "
1230  PRINT " PARAMETERS ARE REQUIRED."
1240  PRINT "     PRESS CONT KEY TO PROCEED    "
1250  PAUSE
1260  PRINT
1270  PRINT " THE EQUATIONS FOR DETERMINING THE COEFFICIENTS MATRICE
S SHOULD BE"
1280  PRINT " WRITTEN AS;"
1290  PRINT
1300  PRINT
1310  PRINT " A(1,1)=Z(4)"
1320  PRINT " A(1,5)=Z(3)"
1330  PRINT " A(1,7)=Z(1)*Z(3)"
1340  PRINT " A(1,8)=Z(6)*Z(3)"
1350  PRINT " A(1,10)=-Z(5)"
1360  PRINT " A(2,4)=Z(8)*Z(7)"
1370  PRINT " A(2,8)=Z(8)"
1380  PRINT " A(3,4)=Z(11)"
1390  PRINT " A(3,9)=Z(10)"
1400  PRINT " B(1,2)=Z(6)"
1410  PRINT " B(2,1)=Z(9)"
1420  PRINT " B(3,1)=Z(12)"
```

```
1430  PRINT " B(3,2)=Z(13)"
1440  PRINT
1450  PRINT "   PRESS CONT KEY TO PROCEED    "
1460  PAUSE
1470  PRINT
1480  PRINT
1490  PRINT
1500  PRINT
1510  PRINT " THE STEADY STATE EQUATIONS CAN BE WRITTEN AS;"
1520  PRINT
1530  PRINT
1540  PRINT " F(1)=Z(4)*X(1)-Z(6)*Input(2)      ,ORIGINAL EQUATIONS
WITH S=0"
1550  PRINT
1560  PRINT " F(2)=Z(9)*Input(1)"
1570  PRINT
1580  PRINT " F(3)=Z(12)*Input(1)+Z(13)*Input(2)"
1590  PRINT
1600  PRINT " WHERE,"
1610  PRINT
1620  PRINT "  X  IS THE ARRAY OF STEADY STATE OUTPUT VALUES."
1630  PRINT
1640  PRINT " Input IS THE ARRAY OF INPUT VALUES."
1650  PRINT
1660  PRINT
1670  PRINT
1680  PRINT
1690  INPUT " DID YOU UNDERSTAND THE PROCEDURE ? YES/NO",Hg$
1700  IF (Hg$="NO") OR (Hg$="no") THEN GOTO 30
1710 INPUT "SELECT THE MASS STORAGE UNIT ie   F8,T14,T15  ?    ",D$
1720 IF D$="F8" THEN 1750
1730 IF D$="T15" THEN 1770
1740 IF D$="T14" THEN 1790
1750 D$=":F8"
1760  GOTO 1800
1770 D$=":T15"
1780  GOTO 1800
1790  D$=":T14"
1800 MASS STORAGE IS D$
1810 INPUT " HAVE THE DATA FILES BEEN CREATED ? YES/NO (NO WHEN THE
STORAGE UNIT IS NEW)  ",D$
1820  IF D$="YES" THEN 1930
1830  CREATE "DATAR",200
1840  CREATE "DATAR1",200
1850  CREATE "DATAN",20
1860  CREATE "DATAA",30
1870  CREATE "DATAS",200
1880  CREATE "DATAX",20
1890  CREATE "EQNA",30
1900  CREATE "EQNS",30
1910  CREATE "BASE1",30
1920  CREATE "BASE3",30
1930 INPUT "SELECT THE PRINTER DEVICE ie 0 FOR PAPER 16 FOR CRT .",K
oi
1940 PRINTER IS Koi
1950 COM INTEGER N,Z,U1,Kmod,B0,Kf2,Kno
1960  COM INTEGER Kon(25)
1970 INPUT "For NYQUIST PLOT,TIME RESPONSE,ROOT-LOCUS press 0,1 or 2
 respectively?",Kmod
1980  INPUT "How many output variables are required?",Kno
```

```
1990  REDIM Kon(Kno)
2000  FOR I=1 TO Kno
2010  DISP "Location of";I;" output variable in output vector ?";
2020  INPUT Kon(I)
2030  NEXT I
2040  INPUT "Location of input variable in input vector ?",Kf2
2050 INPUT "IS THIS A RE-RUN? YES/NO",A$
2060 IF A$="YES" THEN 2080
2070 LOAD "CONTRO:T15"
2080 INPUT "Order of system coefficient matrix?",N
2090 INPUT "Number of column in matrix B (Numbers of ROWS=Number of
ROWS in MATRIX A?",B0
2100 INPUT "Number of basic parameters that describe the system?",Z
2110 U1=0
2120 INPUT "Do you want to estimate the s.s. solution for a step inp
ut? YES/NO ",A$
2130 IF A$="YES" THEN 2150
2140 U1=1
2150 LOAD "COMPUT:T15"
2160 END
```

PROGRAM "COMPUT"

```
10   REM *** PROGRAM STORED IN "COMPUT:F8 " ***
20   REM
30   Locus$=""
40   REM *** DECLARATION FOR THE MAIN PROGRAM ***
50   OPTION BASE 1
60   COM INTEGER A0,Z9,U1,Kmod,B0,Kf2,Kno
70    COM INTEGER Kon(25)
80   COM INTEGER S(10000),S1(10000)
90    DIM S2(10000),Loc(10),St(10),Ste(10),Fin(10)
100  REAL Z(500),X(15),K(26),K1(26)
110   COM K11(25),Ko1(25)
120   COM Input(10)
130   REDIM K11(Kno+1),Ko1(Kno+1)
140   ASSIGN #2 TO "DATAN"
150   Kmnh=0
160   MAT READ #2;K11,Ko1
170   ASSIGN * TO #2
180  INTEGER N,Wo,No,Mo,V7
190   Mef=0
200   Kef=0
210  REM *** PREPARATION ***
220  REM *** THIS SEGMENT MUST BE PERFORMED AT THE BEGINNING***
230  REM *** OF THIS PROGRAM EVEN WHEN IT IS A RERUN ***
240   DIM Inial(10,10),Mnk(10)
250   REDIM Inial(Ko1(1),Kno)
260   MAT Inial=ZER
270   PRINT "THE ORDER OF SYSTEM'S OUTPUT VARIABLES COEFFICIENT MATR
IX IS:   ";A0;"X";A0
280   PRINT "THE ORDER OF SYSTEM'S INPUT VARIABLES COEFFICIENT MATRI
X IS:   ";A0;"X";B0
290   PRINT "NUMBER OF BASIC PARAMETERS IS:  ";Z9
300   PRINT " NUMBER OF OUTPUT VARIABLES REQUIRED IS:";Kno
320  IF A0=0 THEN 380
330  GET "EQNA",2770,340
340  GET "ROOTS:T15",4000,350
350   IF Kmod<>2 THEN 380
360  ASSIGN #5 TO "DATAR"
370  BUFFER #5
380  IF U1=1 THEN 520          !U1=1 means NO S.S. Soln. required
390  GET "STYST:T15",6100,400
400  GET "EQNS",8600,410
410   IF Kmod<>2 THEN 440
420  ASSIGN #5 TO "DATAR"
430  BUFFER #5
440  REM *** PREPARATION SEGMENT END ***
450  INPUT "NUMBER OF INPUTS IN THE STEADY STATE EQUATIONS",Mj
460  REDIM Input(Mj)
470   FOR I=1 TO Mj
480   DISP "THE";I;"th INPUT VALUE"
490   INPUT Input(I)
500   NEXT I
510   IF Kmnh=1 THEN 580
520 IF Kmod=1 THEN 540
530   GOTO 550
540   GET "RESPO:T15",10000,550
550   IF Kmod=0 THEN 570
560   GOTO 580
570   GET "NYQI:T15",10000,580
580  INPUT "TO READ Z FROM FILE, INPUT 2 OTHERWISE 0",V7
590  IF V7<1 THEN 630
```

```
600  ASSIGN #2 TO "DATAA"
610  MAT READ #2;Z(Z9)
620  ASSIGN * TO #2
630  INPUT "TO READ X FROM FILE, INPUT 2 OTHERWISE 0",V7
640  IF V7<1 THEN 680
650  ASSIGN #3 TO "DATAX"
660  READ #3;V7
670  MAT READ #3;X(V7)
680  IF U1=1 THEN 780
690  INPUT "Do you want to print out intermediate results for s.s. s
oln.?",A$
700  Wo=0
710  IF (A$="NO") OR (A$="no") THEN 730
720  Wo=1
730  INPUT "Give no. of signigicant digits wanted for s.s. soln.",No
740  INPUT "Give maximum no. of iterations allowed for s.s. soln.",M
o
750  REM *** SUBPROGRAM STYST IS CALLED FOR S.S. SOLN. ***
760  CALL Styst(Input(*),Z(*),X(*),V7,Wo,No,Mo,#3)
770   IF Mef=1 THEN 870
780  DIM A(25,25),B(25,25),K0(26),C(25)
790  REDIM A(A0,A0),B(A0,B0),C(A0)
800  REDIM S(K11(1)),S1(K11(2))
810   ASSIGN #2 TO "DATAS"
820   MAT READ #2;S
830   Mnh=1
840   O1=Ko1(1)
850   L1=K11(1)
860   REDIM K(O1+1)
870   GOSUB 2770
880   GOSUB 2370
890   IF Kmod<>2 THEN 910
900   ASSIGN * TO #2
910   GOSUB 1690
920   IF Kmod=2 THEN 1250
930   PRINT
940   PRINT "THE NUMERATOR POLYNOMINAL OF ";Kon(Mnh);"Th OUTPUT VARI
ABLE IS:"
950   REDIM S2(L1)
960   MAT S2=S
970   REDIM S(K11(Mnh+1))
980   MAT READ #2;S
990   Kno1=O1
1000  Kno2=L1
1010  REDIM K0(O1+1)
1020  MAT K0=K
1030  L1=K11(Mnh+1)
1040  O1=Ko1(Mnh+1)
1050  FOR I=1 TO A0
1060  C(I)=A(I,Kon(Mnh))
1070  A(I,Kon(Mnh))=B(I,Kf2)
1080  NEXT I
1090  REDIM K(O1+1)
1100  GOSUB 2370
1110  REDIM K1(Ko1(Mnh+1)+1)
1120  O1=Kno1
1130  L1=Kno2
1140  MAT K1=K
1150  REDIM K(O1+1)
1160  MAT K=K0
```

```
1170  REDIM S(L1)
1180  MAT S=S2
1190  FOR I=1 TO A0
1200  A(I,Kon(Mnh))=C(I)
1210  NEXT I
1220  Od=Ko1(Mnh+1)
1230  N=01
1240  IF Mnh>1 THEN 1290
1250  CALL Roots(K(*),N,Kmod,#5)
1260  IF Kmod=2 THEN 1870
1270  INPUT "Do you require the results for these values of paramete
r Y/N",A$
1280  IF (A$="N") OR (A$="NO") THEN 2130
1290  IF Kmod<>2 THEN 1340
1300  PRINT #2;END
1310  ASSIGN * TO #5
1320 REM *** REDUCING ORDER IF POSSIBLE ***
1330  LOAD "PLOT:T15"
1340  IF Kmod=0 THEN 1570
1350  IF Kmod=1 THEN 1360
1360  Or=Ko1(1)
1370  Mnk1=Ko1(1)
1380  IF K(Or+1)=0 THEN Or=Or-1
1390  IF K(Or+1)<>0 THEN 1470
1400  REDIM Mnk(Or+1)
1410  FOR I=1 TO Or+1
1420  Mnk(I)=K(I)
1430  NEXT I
1440  REDIM K(Or+1)
1450  MAT K=Mnk
1460  GOTO 1380
1470  Ko1(1)=Or
1480  CALL Respo(K(*),K1(*),Ko1(*),Mnh,Inial(*),Kef)
1490  IF Kef<=Kno THEN GOTO 1560
1500  REDIM K(Mnk1+1)
1510  MAT K=ZER
1520  FOR I=1 TO Ko1(1)+1
1530  K(I)=Mnk(I)
1540  NEXT I
1550  Ko1(1)=Mnk1
1560  GOTO 1580
1570  CALL Nyqi(K(*),K1(*),Ko1(*),Mnh)
1580 PRINT
1590  Mnh=Mnh+1
1600  IF Mnh<=Kno THEN 920
1610  ASSIGN * TO #2
1620  INPUT "Do you want to repeat computation YES/NO",M1$
1630  Kmnh=1
1640  Kef=Kef+1
1650  IF M1$="YES" THEN 1870
1660  END
1670  STOP
1680 REM *** REDUCING ORDER IF POSSIBLE ***
1690 N=01                                    !Start search from 01 o
rder term
1700 J9=N+1
1710 IF ABS(K(J9))>=1E-70 THEN 1750          !1E-70 as criterion
1720 N=N-1
1730 IF N<0 THEN 1760
1740 GOTO 1700
```

```
1750 IF N=01 THEN 1840
1760 FRINT
1770 FRINT "REDUCED SPECIFIC POLYNOMIAL COEFFICIENTS";
1780 FRINT TAB(45);"DEGREE IN S"
1790 FRINT
1800 FOR I=1 TO N+1
1810 FRINT USING 2630;K(I),I-1
1820 NEXT I
1830  O1=N
1840  RETURN
1850 CALL Roots(K(*),N,Kmod,#5)
1860  GOSUB 2660
1870 IF Locus$="YES" THEN GOTO 2100
1880 IF Locus$="NO" THEN GOTO 2050
1890 INPUT " ,DO YOU WANT TO FIND THE ROOT-LOCUS BY CHANGING THE PARA
METERS? MANUALLY  YES/NO",Locus$
1900  INPUT " NUMBER OF PARAMETERS TO BE VARIED",Numb
1910  FOR I=1 TO Numb
1920  PRINT "LOCATION OF ";I;" PARAMETERS IN ARRAY Z TO BE VARIED AT
 ";I;" LOOP?"
1921  PRINT
1930  INPUT Loc(I)
1940  INPUT " INPUT THE  STARTING, STEP OF VARIATION AND MAXIMUM VAL
UES OF THIS PARAMETER",St(I),Ste(I),Fin(I)
1950  NEXT I
1960  PRINT "THE ROOT-LOCUS IS PLOTTED BY VARIATION OF FIRST ;"
1961  PRINT
1970  FOR J=1 TO Numb
1980  PRINT "Z(";Loc(J);")  FROM ";St(J);" TO ";Fin(J);" BY STEP OF
";Ste(J)
1990  IF J=Numb THEN GOTO 2010
2000  PRINT " THEN BY INCREASING OF ONE STEP OF;"
2010  NEXT J
2020  Imn=0
2030      Imn=Imn+1
2031  IF Imn>Numb THEN GOTO 2240
2040      Jnc=0
2050  Z(Loc(Imn))=St(Imn)+Jnc*Ste(Imn)
2070 IF Jnc*Ste(Imn)+St(Imn)>Fin(Imn) THEN 2030
2075  Jnc=Jnc+1
2090  GOTO 2190
2100 INPUT "ANY MODIFICATION IN Z? YES/NO",B$
2110 IF B$="NO" THEN 2240
2120  GOTO 2230
2130 INPUT "GIVE SUBSCRIPT FOR THIS ELEMENT, 0 TO END MODIFICATION",
J1
2140 IF J1=0 THEN 2180
2150 INPUT "GIVE NEW VALUE FOR THIS ELEMENT",Z(J1)
2160 GOTO 2130
2170  Mef=1
2180 FRINT
2190 FRINT "NEW Z IS:"
2200 FOR I=1 TO Z9
2210 FRINT "Z[";I;"] =";Z(I)
2220 NEXT I
2230 ON U1+1 GOTO 760,770
2240 FRINT "DONE"
2250 FRINT #5;END
2260 ASSIGN * TO #5
2270  GOTO 2340
```

```
2280 READ #3,1;V7
2290 FOR J=1 TO V7
2300 PRINT #3;X(J)
2310 NEXT J
2320 PRINT #3;END
2330 ASSIGN * TO #3
2340 IF A0=0 THEN 2360
2350 LOAD "PLOT:T15"
2360 STOP
2370 REM *** SUBROUTINE TO CALCULATE POLYNOMIAL COEFFICIENTS ***
2380 MAT K=ZER(O1+1)
2390 FOR Hmno=0 TO O1
2400 O2=Hmno+1
2410 FOR J=1 TO L1-1
2420 T1=0
2430 IF S(J)<>Hmno THEN 2550
2440 T1=SGN(S(J+1))
2450 IF J=1 THEN 2540
2460 J1=J-1
2470 IF ABS(S(J1))<101 THEN 2540
2480 I2=INT(S(J1)/100)
2490 J2=S(J1)-100*I2
2500 T1=T1*A(I2,J2)
2510 IF J1<=1 THEN 2540
2520 J1=J1-1
2530 GOTO 2470
2540 K(O2)=K(O2)+T1
2550 NEXT J
2560 NEXT Hmno
2570 PRINT
2580 PRINT "COEFFICIENTS OF THE SPECIFIC POLYNOMIAL ";
2590 PRINT TAB(45);"DEGREE IN S"
2600 PRINT
2610 FOR I=1 TO O1+1
2620 PRINT USING 2630;K(I),I-1
2630 IMAGE 10X,MD.10DXE,23X,DD
2640 NEXT I
2650  RETURN
2660  IF Kmod=2 THEN 2510
2670  INPUT "Do you require the result for this values of parameters
 YES OR NO ",A$
2680  IF A$="NO" THEN 2510
2690  IF Kmod<>2 THEN 2730
2700  PRINT #5;END
2710  ASSIGN * TO #5
2720  LOAD "PLOT:F8 "
2730  GOSUB 9000
2740  Mnh=Mnh+1
2750  IF Mnh<=Kno THEN 920
2760  END
2770  STOP
```

PROGRAM "CONTRO"

PROGRAM: CONTRO   HP ENHANCED BASIC   Date: 1/11/83   Page 1

```
10   REM *** PROGRAM STORED IN "CONTRO:F8 "***
20   REM *** PROGRAM USED TO INPUT NECESSARY EQUATIONS ***
30   OPTION BASE 1
40   COM INTEGER N,Z9,U1,Kmod,Ko3,Kf2,Kno
50    COM INTEGER Kon(25)
60   COM INTEGER S(10000),S1(10000)
70   REM *** PREPARATION ***
80   DIM I$[80]
90   REM ** EQUATIONS FOR [A] AND B**
100  ASSIGN #1 TO "EQNA"
110  PRINT "EQUATIONS FOR DETERMINING A[I,J]'s ANDB(I,J)'s"
120  DISP "EQUATION FOR DETERMINING A[I,J]'s in the form A(I,J)=..an
d B(I,J)=.."
130  FOR I=2011 TO 2990
140  I$[1,5]=VAL$(I)
150  LINPUT "EQUATION FOR DETERMINING A[I,J]'s AND B(I,J)'S",I$[6]
160  DISP I$
170  BEEP
180  DISP "IF EQUATION IS CORRECT, PRESS <CONT> TO CONTINUE"
190  INPUT "IF EQUATION IS INCORRECT, Type NO   <CONT>",B$
200  IF B$="NO" THEN 220
210  GOTO 250
220  DISP "PLEASE RE-ENTER THE "
230  B$="YES"
240  GOTO 150
250  PRINT #1;I$
260  PRINT I$
270  DISP "To enter more equations, PRESS <CONT>"
280  BEEP
290  INPUT "To end inputting equations, Type NO   <CONT>",B$
300  IF B$="NO" THEN 320
310  NEXT I
320  PRINT #1;"2999 RETURN","3000 END",END
330  ASSIGN * TO #1
340  REM ** STEADY STATE EQUATIONS **
350  B$="YES"
360  ASSIGN #2 TO "EQNS"
370  PRINT "EQUATIONS FOR STEADY STATE OPERATION POINTS"
380  DISP "ENTER EQUATION IN FORM:  F(..)=..."
390  FOR I=8010 TO 8990 STEP 10
400  I$[1,5]=VAL$(I)
410  LINPUT "EQUATION FOR F-VECTOR",I$[6]
420  DISP I$[6]
430  BEEP
440  DISP "IF EQUATION IS CORRECT, PRESS <CONT> TO CONTINUE"
450  INPUT "IF EQUATION IS INCORRECT, Type NO   <CONT> ",B$
460  IF B$="NO" THEN 480
470  GOTO 510
480  B$="YES"
490  DISP "PLEASE RE-ENTER THE "
500  GOTO 300
510  PRINT #2;I$
520  PRINT I$
530  DISP "To enter more equations, PRESS <CONT>"
540  BEEP
550  INPUT "To end inputting equations, Type NO   <CONT>",B$
560  IF B$="NO" THEN 580
570  NEXT I
580  PRINT #2;"8999 RETURN","9000 SUBEND",END
590  ASSIGN * TO #2
```

```
600  REM ** INITIAL ESTIMATES FOR STEADY STATE SOLUTION **
610  ASSIGN #3 TO "DATAX"
620  INPUT "No. of output variables in the steady state eqns.",N
630  PRINT #3;N
640  IF N=0 THEN 700
650  FOR I=1 TO N
660  DISP "Initial estimate for X[";I;"] in the S.S. operating point
";
670  INPUT X
680  PRINT #3;X
690  NEXT I
700  PRINT #3;END
710  ASSIGN * TO #3
720  REM ** ENTER BASIC PARAMETERS **
730  ASSIGN #4 TO "DATAA"
740  INPUT "No. of basic parameters describing the system?",Z9
750  FOR I=1 TO Z9
760  DISP "Z[";I;"] =";
770  INPUT A
780  PRINT USING 790;I,A
790  IMAGE "Z[",DD,"] = ",MD.6DXE
800  PRINT #4;A
810  NEXT I
820  ASSIGN * TO #4
830  REM
840  REM *** THE FOLLOWING PART OF ***
850  REM *** THIS PROGRAM IS TO INPUT THE SYSTEM COEFFICIENT MATRIX
***
860  REM *** AND DETERMINE THE GENERAL CHARACTERISTIC POLYNOMIAL IN
***
870  REM *** SYMBOLIC FORM WITHOUT REFERENCE TO PARTICULAR NUMERICAL
*
880  REM *** VALUES OF THE COEFFICIENTS ***
890  INTEGER K(25)
900  DIM A$(25,25)[5],B$[5]
910  REM *** INPUT THE COEFFICIENTS OTHER THAN ZERO IN SYMBOLIC FORM
***
920  INPUT "Order of the system coefficient matrix? max. 25",N
930  IF N=0 THEN 3420                        !Skip if order is zero
940  INPUT "Total No. of non-zero elements?",N9
950  IF N9>0 THEN 980                        !Skip if all zero-elements
960  N=0
970  GOTO 3420
980  REDIM A$(N,N),K(N)                      !Redimension the working si
zes
990  ON ERROR GOTO 1030                      !Trap improper data entries
1000 FOR I1=1 TO N9                          !A FOR ... NEXT Loop is use
d
1010 INPUT "Row, column subscripts and the element (max. 5 chars.)",
I,J,A$(I,J)
1020 GOTO 1050
1030 DISP "You entered an unacceptable data, please RE-ENTER"
1040 GOTO 1010
1050 NEXT I1
1060 OFF ERROR                               !Cancel the trap
1070 REM *** PRINT THIS COEFFICIENT MATRIX ***
1080 PRINT
1090 PRINT "THE SYSTEM COEFFICIENT MATRIX IS :"
1100 PRINT
1110 FOR I=1 TO N
```

```
1120 FOR J=1 TO N
1130 IF A$(I,J)="" THEN 1160                    !Check for null element
1140 PRINT TAB((J-1)*6);A$(I,J);
1150 GOTO 1170
1160 PRINT TAB((J-1)*6);"0";                    !Print a 0 if null element
1170 NEXT J
1180 PRINT                                      !Cause a CR-LF action in pr
inter
1190 NEXT I
1200  INTEGER K1(25)
1210  DIM S$(25,25)[5],N$[5]
1220  INPUT "NUMBER OF COLUMNS IN MATRIX B",Ko3
1230  INPUT "TOTAL NUMBER OF NON-ZERO ELEMENTS IN B",Ko2
1240  REDIM S$(N,Ko3),K1(N)
1250  FOR I=1 TO Ko2
1260  INPUT "ROW,COLUMN AND ELEMENT ",Mn,Mb,S$(Mn,Mb)
1270  NEXT I
1280  PRINT
1290  PRINT "THE COLUMN B IS "
1300  FOR I=1 TO N
1310  FOR J=1 TO Ko3
1320  IF S$(I,J)="" THEN 1350
1330  PRINT TAB((J-1)*6);S$(I,J);
1340  GOTO 1360
1350  PRINT TAB((J-1)*6);"0";
1360  NEXT J
1370  PRINT
1380  NEXT I
1390  Mon=0
1400  PRINT
1410  DIM K11(25),Ko1(25)
1420  REDIM K11(Kno+1),Ko1(Kno+1)
1430  DIM M$(25)[5],G$(25)[5]
1440  REDIM G$(N)
1450  FOR Kt1=0 TO Kno
1460  IF Kt1=0 THEN 1510
1470  FOR Kt2=1 TO N
1480  G$(Kt2)=A$(Kt2,Kon(Kt1))
1490  A$(Kt2,Kon(Kt1))=S$(Kt2,Kf2)
1500  NEXT Kt2
1510  GOSUB 1710
1520  IF Kt1>0 THEN 1540
1530  ASSIGN #3 TO "DATAS"
1540  FOR S8=1 TO L1
1550  PRINT #3;S(S8)
1560  NEXT S8
1570  IF Kt1<Kno THEN 1600
1580  PRINT #3;END
1590  ASSIGN * TO #3
1600  K11(Kt1+1)=L1
1610  Ko1(Kt1+1)=O1
1620  IF Kt1=0 THEN 1660
1630  FOR I=1 TO N
1640  A$(I,Kon(Kt1))=G$(I)
1650  NEXT I
1660  NEXT Kt1
1670  ASSIGN #1 TO "DATAN"
1680  MAT PRINT #1;K11,Ko1
1690  ASSIGN * TO #1
1700  GOTO 3440
```

```
1710 REM *** FIND THE PATH ***
1720 C1=0                                   !C1 is the max. order of po
lynomial
1730 L1=1
1740 K1=0
1750 L=1
1760 K1=K1+1
1770 REM *** CHECK IF THE K1th ELEMENT IN THE FIRST COLUMN ***
1780 REM *** OF THIS SYSTEM MATRIX IS A "0"                 ***
1790 IF A$(K1,1)="" THEN 1810
1800 GOTO 1840
1810 REM *** CHECK IF THIS ELEMENT IS THE NON-EXISTENT (N+1)th ELEME
NT ***
1820 IF K1>=N THEN 2630                     !Branch to write an end-mar
k in S
1830 GOTO 1760                              !Continue the search
1840 REM *** NEW COLUMN UNDER INVESTIGATION IS COLUMN L2 ***
1850 L2=L+1                                 !Old column is L
1860 REM *** THE ROW SUBSCRIPT OF THE CONTRIBUTED ELEMENT FROM ***
1870 REM *** COLUMN L IS STORED IN THE Lth PLACE IN ARRAY K  ***
1880 K(L)=K1
1890 REM *** SEARCH FOR A CONTRIBUTED ELEMENT IN COLUMN L2 ***
1900 J=0
1910 J=J+1
1920 REM *** CHECK FOR NON-ZERO ELEMENT ***
1930 IF A$(J,L2)="" THEN 2030
1940 REM *** CHECK WHETHER THIS ROW ALREADY CONTRIBUTED AN ELEMENT *
**
1950 FOR I=1 TO L
1960 IF J=K(I) THEN 2030
1970 NEXT I
1980 REM *** CHECK WHETHER THE LAST COLUMN (COLUMN N) HAS BEEN REACH
ED ***
1990 IF L2>=N THEN 2120                     !The > sign is an insure po
licy
2000 L=L2
2010 K1=J
2020 GOTO 1850                              !Search for the next elemen
t
2030 IF J<N THEN 1910
2040 IF L>1 THEN 2070
2050 K1=K(1)
2060 GOTO 1820
2070 J=K1
2080 L2=L
2090 L=L-1
2100 K1=K(L)
2110 GOTO 2030
2120 K(L2)=J
2130 REM *** EVALUATE SIGN DUE TO INVERSIONS ***
2140 REM *** INITIALLY ASSIGN A +ve SIGN TO THE PRODUCT TERM ***
2150 N3=100                                 !Sign is represented by 100
 or -100
2160 REM *** START THE CHECK FROM THE 1st ELEMENT ***
2170 REM *** END THE CHECK AT THE (N-1)st ELEMENT ***
2180 FOR N4=1 TO N-1
2190 I=N4+1
2200 REM *** ELEMENTS TO BE COMPARED ARE FROM THE (N4+1)st ELEMENT T
O THE Nth **
2210 FOR J1=I TO N
```

```
2220 IF K(N4)<=K(J1) THEN 2250                 !No inversion if true
2230 REM *** IF INVERSION OCCURS THEN SIGN IS CHANGED ***
2240 N3=-N3
2250 NEXT J1
2260 NEXT N4                                   !Loops completed
2270 REM *** EVALUATE ORDER AND CORRECT SIGN OF PATH ***
2280 REM *** THEN STORE INFORMATION IN THE ARRAY S    ***
2290 O=0
2300 FOR I=1 TO N
2310 B$=A$(K(I),I)
2320 REM *** CHECK WHETHER THE CONTRIBUTED ELEMENT IS UNITY ***
2330 IF B$="1" THEN 2540                       !No correction is needed
2340 REM *** CHECK WHETHER THE CONTRIBUTED ELEMENT IS NEGATIVE UNITY
 ***
2350 IF B$="-1" THEN 2450                      !A change in sign is needed
2360 REM *** CHECK IF THIS ELEMENT IS A NEGATIVE LAPLACE DOMAIN VARI
ABLE ***
2370 IF (B$="-S") OR (B$="-s") THEN 2480
2380 REM *** CHECK IF THIS ELEMENT IS A POSITIVE LAPLACE DOMAIN VARI
ABLE ***
2390 IF (B$="S") OR (B$="s") THEN 2500
2400 REM *** ROW AND COLUMN SUBSCRIPTS ARE STORED IN S ***
2410 S(L1)=K(I)*100+I                          !Address of A(I,J) is store
d as 100I+J
2420 L1=L1+1
2430 GOTO 2540
2440 REM *** CHANGE IN SIGN DUE TO -1 ***
2450 N3=-N3
2460 GOTO 2540
2470 REM *** CHANGE IN SIGN DUE TO -S ***
2480 N3=-N3
2490 REM *** INCREASE IN ORDER DUE TO S OR -S ***
2500 O=O+1
2510 IF O<=O1 THEN 2540
2520 REM *** ASSIGN CURRENTLY MAXIMUM ORDER OF POLYNOMIAL TO O1 ***
2530 O1=O
2540 NEXT I                                    !End of searching in K
2550 REM *** STORE ORDER IN S ***
2560 S(L1)=O
2570 L1=L1+1
2580 REM *** STORE SIGN IN S ***
2590 S(L1)=N3
2600 L1=L1+1
2610 GOTO 2030                                 !Search for another path
2620 REM *** MARK AN END-MARK IN S ***
2630 S(L1)=9999                                !Max. No. representing add.
 is 2525
2640 REM *** PRINT OUT THE ARRAY S IF DESIRED ***
2650 INPUT "Do you want to print out the array S? YES/NO",B$
2660 IF (B$="NO") AND (Kt1=0) THEN 2900
2670  IF (B$="NO") AND (Kt1>0) THEN 2920
2680 PRINT
2690  IF Kt1>0 THEN 2740
2700  PRINT
2710  PRINT
2720 PRINT "THE MATRIX S STORED IS FOR THE CHARACTERISTIC POLYNOMINA
L :"
2730  GOTO 2760
2740  PRINT "THE MATRIX S STORED IS FOR THE NUMERATOR POLYNOMINAL OF
";Kt1;
```

```
2750  PRINT "OUTPUT VARIABLE)"
2760 PRINT
2770 P1=0
r control                                !P1 is the counter for pape
2780 FOR J=1 TO L1
2790 P1=P1+1
2800 IF P1<13 THEN 2830
2810 PRINT
rinter                                   !Causes a LF-CR action in p
2820 P1=1
2830 PRINT USING "#,X,MDDXDD";S(J)
2840 IF ABS(S(J))<>100 THEN 2870
2850 PRINT
2860 P1=1
2870 NEXT J
2880 PRINT
2890  IF Kt1>0 THEN 2920
2900 PRINT "THE MAXIMUM ORDER OF THE CHARACTERISTIC POLYNOMIAL IS: "
;O1
2910  GOTO 2940
2920  PRINT "THE MAXIMUM ORDER OF THE POLYNOMINAL(FOR NUMERATOR OF";
Kt1;
2930  PRINT "OUTPUT VARIABLE) IS:";O1
2940 PRINT "THE NUMBER OF ELEMENTS IN THE ARRAY S IS:";L1
2950 REM *** CHECK IF S IS A "ZERO" ARRAY ***
2960 IF S(1)=9999 THEN 3400              !Recall that end-mark is 99
99
2970 REM *** PRINT OUT THE GENERAL CHARACTERISTIC POLYNOMIAL IF DESI
RED ***
2980 INPUT "Do you want to print out the polynomial? YES/NO",B$
2990 IF B$="NO" THEN 3420
3000 PRINT
3010  IF Kt1>0 THEN 3040
3020 PRINT "THE GENERAL CHARACTERISTIC POLYNOMIAL (IN SYMBOLIC FORM)
 IS :"
3030  GOTO 3050
3040  PRINT "THE NUMERATOR POLYNOMINAL OF";Kt1;"OUTPUT VARIABLE  IN
SYMBOLIC FORM IS:";O1
3050 PRINT
3060 REM *** START SCANNING FROM THE 0th ORDER TERM ***
3070 O=0
3080 L5=0
3090 PRINT
3100 PRINT
3110 REM *** CHECK IF THE MAXIMUM ORDER HAS BEEN REACHED ***
3120 IF O>O1 THEN 3420
3130 PRINT USING 3230;O,O
3140 PRINT "THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :"
3150 L5=L5+1
3160 REM *** CHECK IF THE LAST ELEMENT HAS BEEN REACHED ****
3170 IF L5=L1 THEN 3210
3180 REM *** FIND THE ELEMENT IN S REPRESENTED THIS ORDER **
3190 IF S(L5)=O THEN 3240
3200 GOTO 3150
3210 O=O+1
3220 GOTO 3080
3230 IMAGE "ORDER = ",DD," (i.e. S^",DD," term)"
3240 L5=L5+1
3250 REM *** CHECK FOR THE SIGN OF THIS TERM ***
3260 PRINT
```

```
3270 IF S(L5)<0 THEN 3300
3280 PRINT "( 1)";
3290 GOTO 3310
3300 PRINT "(-1)";
3310 REM *** SEARCH AND PRINT THE CONTRIBUTING ELEMENT ***
3320 L6=L5-2
3330 IF L6=0 THEN 3150
3340 IF ABS(S(L6))<101 THEN 3150             !Sign of other term reache
c
3350 I0=INT(S(L6)/100)
3360 J0=S(L6)-100*I0
3370 PRINT " + (";A$(I0,J0);")";
3380 L6=L6-1
3390 GOTO 3330
3400 PRINT "ZERO POLYNOMIAL"                 !Stop if zero-polynomial
3410 STOP
3420 REM *** CHAIN TO "COMPUT:T15" ***
3430 RETURN
3440 INPUT "Do you want to find the s.s. operating pt.?YES/NO",B$
3450 L1=0
3460 IF B$="YES" THEN 3500
3470 L1=1
3480 IF (N=0) OR (N9=0) THEN 3500
3490 LOAD "COMPUT:T15"
3500 END
```

PROGRAM "PLOT"

```
10   REM *** PROGRAM STORED IN "PLOT:T15" ***
20   OPTION BASE 1
30   DIM X(3000),Y(3000)
40   PRINTER IS 0
50   PRINT TAB(5);"REAL PART";TAB(25);"IMAG PART"
60   ASSIGN #1 TO "DATAR"
70   ON END #1 GOTO 190
80   I=0
90   K1=K2=K3=0
100  I=I+1
110  READ #1;X(I),Y(I)
120  PRINT X(I),Y(I)
125  IF Y(I)<=K1 THEN 140
130  K1=Y(I)
140  IF X(I)<=K2 THEN 160
150  K2=X(I)
160  IF X(I)>=K3 THEN 100
170  K3=X(I)
180  GOTO 100
190  OFF END #1
200  S1=(K2-K3)/10
210  S2=12*S1
220  S3=1.2*K1
230  PLOTTER IS 13,"GRAPHICS"
240  GRAPHICS
250  FRAME
260  LOCATE 15,95,15,95
270  IF S3>S2 THEN 340
280  T1=K3-S1
290  T2=K2+S1
300  T3=-S1
310  T4=11*S1
320  T=S1
330  GOTO 400
340  T3=-.1*K1
350  T4=1.1*K1
360  T=.1*K1
370  S5=(K2+K3)/2
380  T1=S5-6*T
390  T2=S5+6*T
400  GOSUB 910
410  LDIR 0
420  LORG 5
430  FOR J=1 TO I-1
440  MOVE X(J),Y(J)
450  LABEL USING "K";"*'
460  NEXT J
470  DUMP GRAPHICS
480  POINTER 0,0,1
490  INPUT "TO ENLARGE GRAPH, INPUT 1 ELSE 0",C
500  REM
510  IF C=0 THEN 850
520  PRINT "MOVE CURSOR TO THE UPPER RIGHT CORNER OF AREA TO BE ENLA
RGED"
530  DIGITIZE B,D
540  PRINT "MOVE CURSOR TO THE LOWER LEFT CORNER OF AREA TO BE ENLAR
GED"
550  DIGITIZE A,C
560  GCLEAR
570  X=(B-A)/10
```

```
580  Y=(D-C)/10
590  IF X<Y THEN 660
600  T1=A-X
610  T2=B+X
620  T3=C-X
630  T4=T3+12*X
640  T=X
650  GOTO 710
660  T1=A-Y
670  T2=T1+12*Y
680  T3=C-Y
690  T4=D+Y
700  T=Y
710  GOSUB 910
720  LDIR 0
730  LORG 5
740  FOR J=1 TO I-1
750  IF (X(J)>A) AND (X(J)<B) AND (Y(J)>C) AND (Y(J)<D) THEN 770
760  GOTO 790
770  PLOT X(J),Y(J),-2
780  LABEL USING "K";"X"
790  NEXT J
800  REM
810  INPUT "TO ENLARGE GRAPH, INPUT 1 ELSE 0",G
820  IF G=0 THEN 880
830  POINTER B,D,1
840  GOTO 520
850  STOP
860  PRINT "SCALE USED IS :";.1*K1;" PER INTERVAL"
870  REM
880  LETTER
890  DUMP GRAPHICS
900  STOP
910  REM *** SUBROUTINE TO PLOT AXES AND LABEL THEM ***
920  SCALE T1,T2,T3,T4
930  T5=T6=0
940  IF T1*T2<0 THEN 960
950  T5=T2
960  IF T3*T4<0 THEN 980
970  T6=T3
980  AXES T,T,T5,T6
990  CSIZE 2
1000 LDIR 0
1010 LORG 2
1020 H1=T
1030 H2=T4
1040 Z=0
1050 IF T3>0 THEN Z=T6
1060 FOR S=Z TO H2 STEP H1
1070 IF S=0 THEN 1100
1080 MOVE T5,S
1090 LABEL USING "XM6D.DD";S
1100 NEXT S
1110 H2=T3
1120 H1=-H1
1130 IF H1<>T THEN 1040
1140 LDIR -PI/2
1150 LORG 2
1160 H1=T
1170 H2=T2
```

```
1180 Z=0
1190 IF T2<0 THEN Z=T5
1200 FOR S=Z TO H2 STEP H1
1210 IF (S=0) OR (S=T) THEN 1240
1220 MOVE S,T6
1230 LABEL USING "XXM6D.DD";S
1240 NEXT S
1250 H1=-H1
1260 H2=T1
1270 IF H1<>T THEN 1180
1280 RETURN
1290 END
```

SUBPROGRAM "STYST"

```
5010 SUB Styst(Input(*),Z(*),X(*),INTEGER N,W9,N1,M0,#1)
5020 REM *** PROGRAM STORED IN "STYST:T15" ***
5030 REM *** PROGRAM FOR ESTIMATING STEADY STATE OPERATING POINTS **
*
5040 OPTION BASE 1
5050 DIM H(15),F(15),P(15),E(15),Y(15),G(15),Q(15),M(15,15)
5060 REM
5070 IF N=0 THEN 6320
5080 IF N1<>0 THEN 5100
5090 N1=3
5100 REM *** DETERMINE MAX. NO. OF ITERATIONS ***
5110 IF M0<>0 THEN 5130
5120 M0=4*N+39
5130 R1=.4*10^(-N1)
5140 REDIM M(N,N),H(N),G(N),F(N),P(N),Q(N),E(N),Y(N)
5150 REM
5160 E1=0
5170 FOR I=1 TO N
5180 E1=E1+ABS(X(I))
5190 NEXT I
5200 E1=E1*R1*.5
5210 N2=INT(N/2+.55)
5220 S1=I1=N3=N4=0
5230 C1=1
5240 PRINT
5250 PRINT "Initial Estimate of the S.S. Solution Vector X :"
5260 PRINT
5270 FOR I=1 TO N
5280 PRINT USING 5290;I,X(I)
5290 IMAGE "X[",DD,"] = ",MD.6DXE
5300 NEXT I
5310 REM *** EVALUATE JACOBIAN MATRIX ***
5320 N5=1
5330 FOR I=1 TO N
5340 IF ABS(X(I))<1E-32 THEN 5370
5350 H(I)=.0001*X(I)
5360 GOTO 5380
5370 H(I)=1E-36
5380 NEXT I
5390 GOSUB 7000
5400 MAT G=F
5410 FOR I=1 TO N
5420 X(I)=X(I)+H(I)
5430 REM *** COMPUTE NEW VECTOR F ***
5440 GOSUB 7000
5450 REM
5460 REM *** X[I] RE-ESTABLISHED ***
5470 X(I)=X(I)-H(I)
5480 REM *** COMPUTE THE PARTIAL DERIVATIVES BY ONE-SIDED DIFFERENCI
NG ***
5490 FOR I2=1 TO N
5500 M(I2,I)=(F(I2)-G(I2))/H(I)
5510 NEXT I2
5520 NEXT I
5530 REM *** INVERT JACOBIAN MATRIX ***
5540 MAT M=INV(M)
5550 IF DET<>0 THEN 5690          !DET is the determinant of M
5560 REM *** NOW M IS SINGULAR BECAUSE DET = 0 ***
5570 REM
5580 S1=S1+1
```

```
5590 IF S1<5 THEN 5620     !CHECK No. of successive singularity
5600 PRINT "JACOBIAN MATRIX 5 TIMES SINGULAR"
5610 GOTO 6740    !STOP Execution for this set and START for another,
 if any
5620 PRINT "JACOBIAN MATRIX IS SINGULAR"
5630 FOR I=1 TO N
5640 IF X(I)<>0 THEN 5660
5650 X(I)=1E-35
5660 X(I)=1.03*X(I)                !Increase components of X by 3%
5670 NEXT I
5680 GOTO 5310                     !Branch to "Evaluate Jacobian Matr
ix"
5690 PRINT
5700 PRINT "JACOBIAN MATRIX EVALUATED AND INVERTED"
5710 PRINT
5720 MAT M=(-1)*M
5730 MAT F=G                       !Transfer data back to vector F
5740 REM *** COMPUTE VECTOR P ***
5750 MAT P=M*F
5760 REM *** CONSERVE F AND X ***
5770 MAT E=F
5780 MAT Y=X
5790 REM *** COMPUTE NEXT X ***
5800 FOR I=1 TO N
5810 X(I)=X(I)+P(I)
5820 NEXT I
5830 GOSUB 7000                    !Compute new values of F
5840 REM *** PRINT RESULTS ***
5850 IF W9=0 THEN 5930
5860 PRINT
5870 PRINT TAB(5);"X-VECTOR";TAB(23);"F EVALUATED"
5880 PRINT
5890 FOR I=1 TO N
5900 PRINT USING 5910;X(I),F(I)
5910 IMAGE 2(4X,MD.6DXE)
5920 NEXT I
5930 REM *** BRANCH TO JACOBIAN MATRIX ***
5940 I1=I1+1                       !Increase count for iterations by
 1
5950 K=0
5960 FOR I=1 TO N
5970 IF ABS(E(I))>=ABS(F(I)) THEN 5990
5980 K=K+1                         !New F[i] value larger than old
5990 NEXT I
6000 IF K<=N2 THEN 6080
6010 ON N5 GOTO 6020,6060
6020 N4=N4+1
6030 IF N4<5 THEN 5310             !Goto Evaluate Jacobian Matrix
6040 PRINT "NO FURTHER CONVERGENCE"
6050 GOTO 6740                     !Compute another set of S.S. pt.
6060 MAT X=Y                       !Transfer data back to X
6070 GOTO 5310                     !Goto Evaluate Jacobian Matrix
6080 REM *** TEST FOR CONVERGENCE ***
6090 FOR I=1 TO N
6100 IF X(I)=0 THEN 6120
6110 IF ABS((Y(I)-X(I))/X(I))>R1 THEN 6160
6120 NEXT I
6130 C1=C1+1
6140 IF C1<3 THEN 6400
6150 GOTO 6180
```

```
6160 C1=1
6170 COTO 6400
6180 FEM *** TEST FOR F = 0 ***
6190 FOR I=1 TO N
6200 IF ABS(F(I))>E1 THEN 6330
6210 NEXT I
6220 FRINT
6230 FRINT "STEADY STATE OPERATING PONIT SOLUTION VECTOR X"
6240 FRINT
6250 FOR J=1 TO N
6260 FRINT "X[";J;"] = ";X(J)
6270 NEXT J
6320 SUBEXIT
6330 N3=N3+1
6340 IF N3<15 THEN 6400
6350 FRINT
6360 FRINT "ITERATION HAS CONVERGED TO A POINT WHICH IS NOT"
6370 FRINT "A SOLUTION TO THE SYSTEM (I.E. STATIONARY POINT)"
6380 FRINT
6390 COTO 6740
6400 FEM *** CHECK NO. OF ITERATIONS ***
6410 IF I1<M0 THEN 6460
6420 FRINT
6430 FRINT "HAS NOT CONVERGED AFTER";M0;"ITERATIONS"
6440 FRINT
6450 COTO 6740
6460 FEM *** COMPUTE NEW ITERATION MATRIX ***
6470 N5=2                          !Set N5 to 2
6480 N4=0
6490 P2=0
6500 MAT G=F-E                     !Compute the vector Y
6510 FOR K=1 TO N
6520 H=0
6530 Q(K)=0
6540 FOR K1=1 TO N
6550 Q(K)=Q(K)+M(K1,K)*P(K1)
6560 H=M(K,K1)*G(K1)+H
6570 NEXT K1
6580 P2=P2+P(K)*H
6590 E(K)=H+P(K)
6600 IF ABS(E(K))>=1E30 THEN 6710
6610 NEXT K
6620 IF ABS(P2)<=1E-30 THEN 6710
6630 FOR K1=1 TO N
6640 FOR K2=1 TO N
6650 H=E(K1)*Q(K2)
6660 IF ABS(H)>=1E30 THEN 6710
6670 M(K1,K2)=M(K1,K2)-H/P2
6680 NEXT K2
6690 NEXT K1
6700 COTO 5740
6710 FRINT
6720 FRINT "OVERFLOW"
6730 COTO 5310                     !Branch to Evaluate Jacobian Matri
x
6740 STOP
7000 FEM *** SEGMENT TO CALCULATE F-VECTOR ***
```

SUB-PROGRAM "ROOTS"

PROGRAM: ROOTS   HP ENHANCED BASIC   Date: 1/11/83   Page 1

```
10   SUB Roots(K(*),INTEGER N,Kmod,#5)
20   REM *** PROGRAM STORED IN "ROOTS:T15" ***
30   OPTION BASE 1
40      ASSIGN #3 TO "BASE3"
50   N1=N                                   !Conserve N
60   REM *** PROGRAM STORED IN "ROOTS:T15" ***
70   REM *** FINDS THE ROOTS OF POLYNOMIAL ***
80   REM *** VERSION 2/AUGUST 1978 ***
90   REM N --- ORDER OF POLYNOMIAL
100  REM K --- COEFFICIENTS OF POLYNOMIAL
120  REM *** FOR COMPLEX PAIRED ROOT ONLY THE ONE WITH POSITIVE IMAG
INARY ***
140  REM *** IS STORED INTO FILE "DATAR:T15" ***
150  IF N<=0 THEN 1660
190  DIM A(26),B(26),X(26)
200  FOR I=1 TO N+1
210  A(I)=K(N+2-I)
220  B(I)=A(I)
230  NEXT I
240  PRINT
250  C1=0
260  D1=4
270  G=0
280  PRINT
290  PRINT
300  PRINT
310  PRINT "   The roots are:"
320  PRINT
330  PRINT TAB(14);"Real Part";TAB(32);"Imag Part";TAB(48);"Zeta(Dam
ping Ratio)"
340  PRINT
350  IF N<=2 THEN 1140
360  IF A(N+1)=0 THEN 1260
370  IF N/2-INT(N/2)=0 THEN 400
380  GOSUB 1380
390  GOTO 350
400  IF ABS(A(N-1))<1E-60 THEN 440
410  P=A(N)/A(N-1)
420  Q=A(N+1)/A(N-1)
430  GOTO 460
440  P=A(N)
450  Q=A(N+1)
460  FOR I=1 TO N+1
470  X(I)=A(I)
480  NEXT I
490  GOSUB 1330
500  FOR I=1 TO N-1
510  B(I)=X(I)
520  NEXT I
530  R=X(N)
540  S=A(N+1)-P*X(N)-Q*X(N-1)
550  GOSUB 1330
560  X(N)=P*X(N-1)-Q*X(N-2)
570  D=X(N-1)^2-X(N)*X(N-2)
580  IF ABS(D)>1E-60 THEN 610
590  PRINT "Solution unobtainable with this Program"
600  GOTO 1660
610  P1=P+(R*X(N-1)-S*X(N-2))/D
620  Q1=Q+(S*X(N-1)-R*X(N))/D
630  IF ABS(P)>1E-60 THEN 670
```

```
640  IF ABS(P1)>1E-60 THEN 670
650  IF ABS(Q)>1E-60 THEN 680
660  GOTO 690
670  IF ABS(P1/P-1)>.000001 THEN 690          !Relative error allowe
d 1E-6
680  IF ABS(Q1/Q-1)<.000001 THEN 830
690  P=P1
700  Q=Q1
710  C1=C1+1
720  IF C1=D1*100 THEN 740
730  GOTO 460
740  PRINT
750  PRINT "The solution did not converge after";C1;"iterations"
760  PRINT "To CONTINUE for 100more iterations TYPE 1 otherwise"
770  PRINT "TYPE 0"
780  INPUT K1
790  IF K1=1 THEN 810
800  GOTO 1660
810  D1=D1+1
820  GOTO 460
830  FOR I=2 TO N-1
840  A(I)=B(I)
850  NEXT I
860  N=N-2
870  D=P*P-4*Q
880  IF D<0 THEN 1020
890  D=SQR(D)
900  PRINT USING 940;(-P+D)/2,0,SGN(P-D)
910  PRINT USING 940;(-P-D)/2,0,SGN(P+D)
920   IF Kmod<>2 THEN 940
930  PRINT #5;(-P+D)/2,0,(-P-D)/2,0
940  IMAGE 4X,2(6X,MD.5DXE),6X,MD.5D
950   IF Kmod=2 THEN 980
960   PRINT #3;(-P+D)/2,0
970   PRINT #3;(-P-D)/2,0
980  C1=0
990  D1=4
1000 IF N-2>0 THEN 360
1010 GOTO 1140
1020 D=SQR(-D)
1030 Z1=P/2/SQR((P/2)^2+(D/2)^2)
1040 PRINT USING 940;-P/2,D/2,Z1
1050 PRINT USING 940;-P/2,-D/2,Z1
1060  IF Kmod<>2 THEN 1080
1070 PRINT #5;-P/2,D/2
1080 IF Kmod=2 THEN 1110
1090  PRINT #3;-P/2,D/2
1100  PRINT #3;-P/2,-D/2
1110 C1=0
1120 D1=4
1130 IF N-2>0 THEN 360
1140 IF N=1 THEN 1200
1150 IF N=0 THEN 1660
1160 P=B(2)/B(1)
1170 Q=B(3)/B(1)
1180 N=0
1190 GOTO 870
1200 PRINT USING 940;-B(2)/B(1),0,SGN(B(2)/B(1))
1210 IF Kmod<>2 THEN 1230
1220 PRINT #5;-B(2)/B(1),0
```

```
1230 IF Kmod=2 THEN 1250
1240  PRINT #3;-B(2)/B(1),0
1250 GOTO 1660
1260 PRINT USING 940;0,0,0
1270  IF Kmod<>2 THEN 1290
1280 PRINT #5;0,0
1290 IF Kmod=2 THEN 1310
1300  PRINT #3;0,0
1310 N=N-1
1320 GOTO 350
1330 X(2)=X(2)-P*X(1)
1340 FOR I=3 TO N
1350 X(I)=X(I)-P*X(I-1)-Q*X(I-2)
1360 NEXT I
1370 RETURN
1380 IF B(2)=0 THEN 1410
1390 X=-B(2)/B(1)
1400 GOTO 1420
1410 X=-B(N+1)/B(1)
1420 F=0
1430 F1=0
1440 FOR I=1 TO N+1
1450 J=N-I+2
1460 IF B(J)=0 THEN 1500
1470 F=B(J)*X^(I-1)+F
1480 IF I-1=0 THEN 1500
1490 F1=(I-1)*B(J)*X^(I-2)+F1
1500 NEXT I
1510 X1=X-F/F1
1520 IF ABS(X/X1-1)<.000001 THEN 1550
1530 X=X1
1540 GOTO 1420
1550 PRINT USING 940;X1,0,-SGN(X1)
1560  IF Kmod<>2 THEN 1580
1570 PRINT #5;X1,0
1580 IF Kmod=2 THEN 1600
1590  PRINT #3;X1,0
1600 N=N-1
1610 FOR I=2 TO N+1
1620 A(I)=B(I)+X1*A(I-1)
1630 B(I)=A(I)
1640 NEXT I
1650 RETURN
1660 N=H1
1670  ASSIGN * TO #3
1690 SUBEND
```

SUB-PROGRAM "NYQI"

```
10    SUB Nyqi(K(*),K1(*),Ko1(*),Mnh)
20    OPTION BASE 1
30    REM PROGRAM IS STORED IN NYQI
40    Or=Ko1(1)
50    Od=Ko1(Mnh+1)
60    INPUT "DO YOU WANT A NYQUIST OR A COMPLEX PHASE PLOT?, TYPE 1
FOR NYQUIST OR 0 FOR COMPLEX CLOSED LOOP",Kmj1
70    IF Kmj1=0 THEN 200
80    IF Od<Or THEN GOTO 170
90    DIM Kd(25)
100   REDIM Kd(Od+1)
110   FOR I=1 TO Or+1
120   Kd(I)=K(I)
130   NEXT I
140   REDIM K(Od+1)
150   MAT K=Kd
160   Or=Od
170   FOR I=1 TO Od+1
180   K(I)=K(I)-K1(I)
190   NEXT I
200   DIM P(25),Z(25),Z1(25)
210   INPUT "WHAT IS THE STARTING POINT OF FREQUENCY? ",W0
220 INPUT "NUMBER OF POINTS?( MAX 1000)",Np
230 INPUT "INCREMENT OF FREQUENCY?",Dw
240   INPUT "INTERVAL OF FREQUENCY FOR WHICH THE PRINT-OUT IS REQUIR
ED?",Int
250   REDIM Z(Or)
260   IF Nd=0 THEN 280
270   REDIM Z1(Nd)
280    Yj=0
290   Xj=0
300   Aj=0
310   Bj=0
320   PRINT
330   PRINT "The transfer function is :"
340   PRINT
350   I1=0
360   FOR I=1 TO Or+1
370   IF I>1 THEN 400
380   Con=K(1)
390   GOTO 410
400   Z(I-1)=K(I)
410   NEXT I
420   FOR I=1 TO Od+1
430   IF I>1 THEN 460
440   Con1=K1(1)
450   GOTO 470
460   Z1(I-1)=K1(I)
470   NEXT I
480   FOR I=Od+1 TO 1 STEP -1
490   PRINT K1(I);"S^";I-1;"+";
500   NEXT I
510   PRINT
520   PRINT "------------------------------------------------------------
------------------------"
530   PRINT
540   FOR I=Or+1 TO 1 STEP -1
550   PRINT K(I);"S^";I-1;"+";
560   NEXT I
570   PRINT
```

```
580  PRINT "Number of points=";Np,"Order of the denominator=";Or,"O
rder of the numerator=";Od
590  PRINT
600  PRINT
610 ASSIGN #3 TO "DATAR"
620  BUFFER #3
630  W=W0
640 FOR J=1 TO Np
650  P=2
660  FOR I=1 TO Or STEP 2
670  Yj=Yj+(-1)^P*Z(I)*W^I
680  P=P+1
690  NEXT I
700  P=1
710  IF Or=1 THEN 760
720  FOR I=2 TO Or STEP 2
730  Xj=Xj+(-1)^P*Z(I)*W^I
740  P=P+1
750  NEXT I
760  Xj=Xj+Con
770  IF Od=0 THEN 890
780  P=2
790  FOR I=1 TO Od STEP 2
800  Bj=Bj+(-1)^P*Z1(I)*W^I
810  P=P+1
820  NEXT I
830  P=1
840  IF Od=1 THEN 890
850  FOR I=2 TO Od STEP 2
860  Aj=Aj+(-1)^P*Z1(I)*W^I
870  P=P+1
880  NEXT I
890  Aj=Aj+Con1
900  Q=Xj
910 R=Yj
920  IF (Q=0) AND (R=0) THEN 940
930  GOTO 960
940  Q=.001
950  R=.001
960  Xj=(Aj*Q+Bj*R)/(Q^2+R^2)
970  Yj=(Bj*Q-Aj*R)/(Q^2+R^2)
980  PRINT #3;Xj,Yj
990  Xj=Yj=Aj=Bj=0
1000  W=J*Dw+W0
1010  DISP J
1020  NEXT J
1030  PRINT #3;END
1040  ASSIGN * TO #3
1050  I1=0
1060  PRINT
1070  PRINT "DATA FOR NYQUIST PLOT:"
1080  PRINT
1090 PRINT TAB(5);"REAL PART";TAB(25);"IMAG PART"
1100  IF I1=0 THEN 1130
1110  ASSIGN #3 TO "DATAR1"
1120  GOTO 1140
1130 ASSIGN #3 TO "DATAR"
1140  BUFFER #3
1150 ON END #3 GOTO 1390
1160 I=0
```

```
1170 K1=K2=K3=Ymin=0
1180 I=I+1
1190  IF I>Np THEN 1390
1200  IF I1=1 THEN 1250
1210  IF I1=2 THEN 1270
1220  IF I1=0 THEN 1230
1230  READ #3;Xj,Yj
1240  GOTO 1280
1250  READ #3;Xj,Yj,Wj
1260  GOTO 1280
1270  READ #3;Xj,Wj,Yj
1280  IF I1>0 THEN 1310
1290  IF INT(I/Int)<>I/Int THEN 1310
1300 PRINT Xj,Yj
1310  IF Yj<Ymin THEN Ymin=Yj
1320 IF Yj<=K1 THEN 1340
1330 K1=Yj
1340 IF Xj<=K2 THEN 1360
1350 K2=Xj
1360 IF Xj>=K3 THEN 1180
1370 K3=Xj
1380 GOTO 1180
1390 OFF END #3
1400  IF Ymin<-9999 THEN Ymin=-9000
1410  IF K3<-9999 THEN K3=-9000
1420 S1=(K2-K3)/10
1430 S2=12*S1
1440  S8=(K1-Ymin)/10
1450 S3=12*S8
1460 PLOTTER IS 13,"GRAPHICS"
1470 GRAPHICS
1480 FRAME
1490 LOCATE 20,90,20,90
1500 IF S3>S2 THEN 1510
1510 T1=K3-S1
1520 T2=K2+S1
1530 T3=Ymin-S8
1540 T4=K1+S8
1550 T=S1
1560 GOTO 1640
1570 S10=(K1+Ymin)/2
1580 T3=S10-6*S10
1590  T4=7*S10
1600 T=.1*K1
1610 S5=(K2+K3)/2
1620 T1=S5-6*T
1630 T2=S5+6*T
1640 GOSUB 1930
1650 LDIR 0
1660 LORG 5
1670  IF I1=0 THEN 1690
1680  IF (I1=1) OR (I1=2) THEN 1710
1690  ASSIGN #3 TO "DATAR"
1700  GOTO 1720
1710  ASSIGN #3 TO "DATAR1"
1720  BUFFER #3
1730 FOR J=1 TO I-1
1740  IF I1=1 THEN 1790
1750  IF I1=2 THEN 1810
1760  IF I1=0 THEN 1770
```

```
1770  READ #3;Xj,Yj
1780  GOTO 1820
1790  READ #3;Xj,Yj,Wj
1800  GOTO 1820
1810  READ #3;Xj,Wj,Yj
1820 PLOT Xj,Yj
1830 NEXT J
1840 ASSIGN * TO #3
1850  GRAPHICS
1860  WAIT 1000
1870 POINTER .5,.5,1
1880 LETTER
1890 DUMP GRAPHICS
1900  I1=I1+1
1910  EXIT GRAPHICS
1920  GOTO 2360
1930 REM *** SUBROUTINE TO PLOT AXES AND LABEL THEM ***
1940 SCALE T1,T2,T3,T4
1950 T5=T6=0
1960 IF T1*T2<0 THEN 1980
1970 T5=T2
1980 IF T3*T4<0 THEN 2000
1990 T6=T3
2000 AXES T,S8,T5,T6
2010 CSIZE 2
2020 LDIR 0
2030 LORG 2.2
2040 H1=T
2050 H2=T4
2060 Z=0
2070 IF T3>0 THEN Z=T6
2080  IF (I1=0) OR (I1=2) THEN S8=-S8
2090  IF (I1=0) OR (I1=2) THEN H2=T3
2100 FOR S=Z TO H2 STEP S8
2110 IF S=0 THEN 2140
2120 MOVE -T2/5,S
2130 LABEL USING "XM5D.DD";S
2140 NEXT S
2150 H2=T3
2160 H1=-H1
2170 IF H1<>T THEN 2060
2180 LDIR -PI/2
2190 LORG 2.2
2200 H1=T
2210 H2=T2
2220 Z=0
2230  IF I1=1 THEN H1=H1
2240  IF (I1=0) OR (I1=2) OR (I1=1) THEN T6=ABS(T3)/5
2250  IF I1=1 THEN T6=-T6
2260 IF T2<0 THEN Z=T5
2270 FOR S=Z TO H2 STEP H1
2280 IF (S=0) OR (S=T) THEN 2310
2290 MOVE S,T6
2300 LABEL USING "XXM4D.DD";S
2310 NEXT S
2320 H1=-H1
2330 H2=T1
2340 IF H1<>T THEN 2220
2350 RETURN
2360  IF I1>=3 THEN 2810
```

```
2370  IF I1>=2 THEN 2780
2380 INPUT "DO YOU WANT A PLOT OF THE  FREQUENCY RESPONSE AND PHASE
LAG?. YES OR NO",B$
2390  IF B$="NO" THEN 2810
2400  IF I1>=2 THEN 2780
2410  PRINT
2420  Ia=Ib=0
2430  PRINT "FREQUENCY  Rad/Sec","AMPLITUTE RATIO","PHASE LAG DEG"
2440  ASSIGN #4 TO "DATAR1"
2450  BUFFER #4
2460  ASSIGN #3 TO "DATAR"
2470  BUFFER #3
2480  FOR I=1 TO Np
2490  READ #3;Xj,Yj
2500  IF Kmj1=0 THEN GOTO 2550
2510          Sa=SQR(Yj^2+(Xj^2-Xj+Yj^2)^2)/((1+Xj)^2+Yj^2)
2520          IF (Xj=0) AND (Yj=0) THEN Xj=Yj=1E-60
2530 Pa=180/PI*ATN(Yj/(Xj+Xj^2+Yj^2))
2540  GOTO 2580
2550  Sa=SQR(Xj^2+Yj^2)
2560  IF Xj=0 THEN Xj=1E-60
2570  Pa=180/PI*ATN(Yj/Xj)
2580  IF (Pa<0) OR (Pa=0) THEN 2630
2590  IF (Pa>0) AND (Ib=1) THEN 2670
2600  Pa=-180+Pa
2610  Ia=1
2620  GOTO 2680
2630  IF Ia=0 THEN 2680
2640  Pa=-180+Pa
2650  Ib=1
2660  GOTO 2680
2670  Pa=-360+Pa
2680  Wj=W0+(I-1)*Dw
2690  IF INT(I/Int)<>I/Int THEN 2710
2700  PRINT Wj,Sa,Pa
2710  PRINT #4;Wj,Sa,Pa
2720  NEXT I
2730  PRINT #3;END
2740  ASSIGN * TO #3
2750  PRINT #4;"END"
2760  ASSIGN * TO #4
2770  GOTO 1100
2780  INPUT "DO YOU WANT A PLOT OF THE PHASE LAG? YES/NO",B$
2790  IF B$="NO" THEN 2810
2800  GOTO 1100
2810  INPUT " Is any change in this plot required yes or no",V$
2820  IF (V$="YES") OR (V$="yes") OR (V$="Yes") THEN 210
2830  FOR I=1 TO Od+1
2840  K(I)=K(I)+K1(I)
2850  NEXT I
2860  SUBEND
```

SUB-PROGRAM "RESPO"

```
10    SUB Respo(K(*),K1(*),Ko1(*),Mnh,Inial(*),Kef)
20    OPTION BASE 1
30    IF K(Ko1(1)+1)=0 THEN Or=Ko1(1)-1
40    IF K(Ko1(1)+1)<>0 THEN 120
50    Or=Ko1(1)
60    DIM Mnk(Or+1)
70    FOR I=1 TO Or+1
80    Mnk(I)=K(I)
90    NEXT I
100   REDIM K(Or+1),Inial(Or+1)
110   MAT K=Mnk
120   DIM Q(25),R(25),S(25),T(25),X(25),Y(25),U(25),V(25),P(20),Sa(6
,6),Oa(6),Oa1(6),Aa(6),Ka1(6,6),Iaa(6),Z(25),Z1(25),K2(25),Iniao(25)
,Iniai(25)
130   DIM U1(25),V1(25),Iaa1(6),Aa1(6),Div(10,2)
140   Od=Nd=Ko1(Mnh+1)
150   Or=Nc1=Ko1(1)
160   REDIM K2(Or),Iniao(Or)
170   IF Od=0 THEN 190
180   REDIM Iniai(Od)
190   INPUT "DO YOU WANT GRAPHICS ? YES/NO.",K1$
200   GOTO 210
210   PRINTER IS 16
220   PRINT PAGE
230   PRINT " IN THE FOLLOWING PART D IS THE DERIVATIVE OPERATOR,D^0
 INDICATES                    OUTPUT OR INPUT VARIABLE ,D^1 INDICATES
THE FIRST DERIVATIVE  "
240   PRINT "   OF OUTPUT OR INPUT VARIABLE.etc"
250   FOR I=1 TO Or
255   PRINT "What is the initial value of D^";I-1;"of this OUTPUT VA
RIABLE ?"
260   DISP "What is the initial value of D^";I-1;"of this OUTPUT VAR
IABLE ?"
270   INPUT Iniao(I)
280   NEXT I
290   GOTO 330
300   FOR I=1 TO Or
310   Iniao(I)=Inial(I,Mnh)
320   NEXT I
330   IF Od=0 THEN 400
340   FOR I=1 TO Od
345   PRINT "What is the initial value of D^";I-1;"of this INPUT VAR
IABLE? "
350   DISP "What is the initial value of D^";I-1;"of this INPUT VARI
ABLE? "
360   INPUT Iniai(I)
370   NEXT I
380   PRINT PAGE
390   PRINT
400   PRINT "WHAT IS THE INPUT FUNCTION ? "
410   PRINT
420   PRINT
430   PRINT
440   PRINT "  0)IMPULSE INPUT "
450   PRINT
460   PRINT "  1)STEP INPUT "
470   PRINT
480   PRINT "  2)VELOCITY INPUT (RAMP INPUT)"
490   PRINT
500   PRINT "  3)ACCELERATION INPUT "
```

```
510   PRINT
520   PRINT
530   PRINT
540   PRINT "FOR IMPULSE,STEP,RAMP,ACCELERATION TYPE 0,1,2 OR 3 RESP
ECTI ELY"
550   INPUT " TYPE IN THE REQUIRED FUNCTION",Fun
560   IF Fun<>0 THEN 590
570   IF Fun=0 THEN Inp=1
580   GOTO 620
590   INPUT "What is the amplitude of input ?",Inp
600   PRINT PAGE
610   PRINTER IS 16
620   REDIM Z(Nc1)
630   IF Nd=0 THEN 650
640   REDIM Z1(Nd)
650   FOR I=1 TO Or+1
660   IF I>1 THEN 690
670   Con=K(1)
680   GOTO 710
690   Z(I-1)=K(I)
700   Z(I-1)=Z(I-1)
710   NEXT I
720   Kmbhg=Z(Or)
730   Con=Con/Z(Or)
740   FOR I=1 TO Or
750   Z(I)=Z(I)/Z(Or)
760   NEXT I
770   FOR I=1 TO Od+1
780   IF I>1 THEN 810
790   Con1=K1(1)
800   GOTO 830
810   Z1(I-1)=K1(I)
820   Z1(I-1)=Z1(I-1)*Inp/Kmbhg
830   NEXT I
840   Con1=Con1*Inp/Kmbhg
850   MAT K2=ZER
860   FOR I=1 TO Or
870   FOR J=I+1 TO Or+1
880   K2(I)=K2(I)+K(J)*Iniao(J-I)
890   NEXT J
900   NEXT I
910   IF Od=0 THEN 970
911   IF Od>Or THEN GOTO 925
920   FOR I=1 TO Od
921   GOTO 930
925   FOR I=1 TO Or
930   FOR J=I+1 TO Od+1
940   K2(I)=K2(I)-K1(J)*Iniai(J-I)
950   NEXT J
960   NEXT I
970   Nc=Nc1
980   ASSIGN #3 TO "BASE3"
990   PRINT
1000  PRINT
1010  FOR I=1 TO Nc1
1020  READ #3;X(I),Y(I)
1030  IF (X(I)=0) AND (Y(I)=0) THEN X(I)=Y(I)=1E-20
1040  IF (X(I)=0) AND (Y(I)=0) THEN 1060
1050  GOTO 1070
1060  Nc=Nc-1
```

```
1070  NEXT I
1080  REDIM Q(Nc),R(Nc),X(Nc),Y(Nc),S(Nc),T(Nc),U(Nc),V(Nc)
1090  ASSIGN * TO #3
1100  IF Nc1-Nc+Fun=0 THEN 1130
1110  IF Nc1-Nc+Fun=0 THEN 1130
1120  REDIM Aa1(Nc1-Nc+Fun),Iaa1(Nc1-Nc+Fun)
1130  Lo1=Nc1-Nc+Fun
1140  GOSUB 1320
1150  Lo1=Nc1-Nc
1160  MAT U1=U
1170  MAT V1=V
1180  REDIM Z1(Or-1)
1190  FOR I=1 TO Or
1200  IF I>1 THEN 1230
1210  Con1=K2(I)/K(Or+1)
1220  GOTO 1240
1230  Z1(I-1)=K2(I)/K(Or+1)
1240  NEXT I
1250  Od=Nd=Or-1
1260  Fun1=Fun
1270  Fun=0
1280  MAT Aa1=Aa
1290  MAT Iaa1=Iaa
1300  GOSUB 1320
1310  GOTO 2090
1320  IF Lo1=0 THEN 1340
1330  REDIM Sa(Lo1,Lo1),Oa(Lo1),Oa1(Lo1),Aa(Lo1),Iaa(Lo1),Ka1(Lo1,Lo
1)
1340   FOR I=1 TO Nc
1350  FOR J=1 TO Nc
1360  IF J=I THEN 1550
1370  A1=-X(J)
1380  B1=-Y(J)
1390  A2=X(I)
1400  B2=Y(I)
1410  GOSUB 4570
1420  IF (I=1) AND (J=2) THEN 1450
1430  IF J=1 THEN 1450
1440  GOTO 1480
1450  A1=1
1460  B1=0
1470  GOTO 1500
1480  A1=A3
1490  B1=B3
1500  A2=Real
1510  B2=Imag
1520  GOSUB 4600
1530  A3=Real
1540  B3=Imag
1550  NEXT J
1560  Q(I)=Real
1570  R(I)=Imag
1580  IF Lo1=0 THEN 1680
1590  FOR K=1 TO Nc1-Nc+Fun
1600  A1=X(I)
1610  B1=Y(I)
1620  A2=Q(I)
1630  B2=R(I)
1640  GOSUB 4600
1650  Q(I)=Real
```

```
1660  R(I)=Imag
1670  NEXT K
1680  NEXT I
1690  MAT S=ZER
1700  MAT T=ZER
1710  IF Nd=0 THEN 1940
1720  FOR K=1 TO Nc
1730  FOR J=1 TO Nd
1740  FOR I=J TO Nd
1750  A1=X(K)
1760  B1=Y(K)
1770  IF I<>J THEN 1810
1780  A2=1
1790  B2=0
1800  GOTO 1830
1810  A2=Real
1820  B2=Imag
1830  GOSUB 4600
1840  NEXT I
1850  S(K)=S(K)+Real*Z1(Nd-J+1)
1860  T(K)=T(K)+Imag*Z1(Nd-J+1)
1870  NEXT J
1880  S(K)=S(K)+Con1
1890  NEXT K
1900  IF Fun<>3 THEN 1970
1910  MAT S=(2)*S
1920  MAT T=(2)*T
1930  GOTO 1970
1940  MAT S=CON
1950  MAT S=(Con1)*S
1960  MAT T=ZER
1970  FOR I=1 TO Nc
1980  A2=Q(I)
1990  B2=R(I)
2000  A1=S(I)
2010  B1=T(I)
2020  GOSUB 4630
2030  U(I)=Real
2040  V(I)=Imag
2050  NEXT I
2060  GOSUB 3530
2070  RETURN
2080  PRINT
2090  INPUT "HOW MANY DERIVATIVES OF THIS OUTPUT VARIABLE ARE REQUIR
ED MAX 10 ?",Nde
2100  INPUT "STARTING  POINT OF TIME ?",Tin
2110  INPUT "STEP OF TIME WHICH THE OUTPUT MUST BE EVALUATED ?",Delt
2120  INPUT "MAXIMUM VALUE OF TIME ?",Maxt
2130  FOR I=1 TO Or+1
2140  IF K(I)=0 THEN Ak1=0
2150  IF K(I)<>0 THEN Ak1=I
2160  IF Ak1<>0 THEN GOTO 2180
2170  NEXT I
2180  FOR I=1 TO Od+1
2190  IF K1(I)=0 THEN Ak11=0
2200  IF K1(I)<>0 THEN Ak11=I
2210  IF Ak11<>0 THEN GOTO 2230
2220  NEXT I
2230  Axs=K1(Ak11)/K(Ak1)
2240  PRINTER IS 16
```

```
2250  PRINT PAGE
2260  PRINT "FOR SCALING THE AXIS OF GRAPHS , YOU SHOUD ENTER ESTIMA
TED  MAXIMUM AND"
2270  PRINT
2280  PRINT "MINIMUM VALUES OF THE OUTPUT VARIABLE . THE RECOMMENDED
 VALUES ARE:"
2290  PRINT
2300  IF Axs<0 THEN GOTO 2350
2310  PRINT "MAXIMUM VALUE OF OUTPUT = ";1.5*Axs*Inp
2320  PRINT
2330  PRINT "MINIMUM VALUE OF OUTPUT = ";-1.5*Axs*Inp
2340  GOTO 2380
2350  PRINT "MAXIMUM VALUE OF OUTPUT = ";-1.5*Axs*Inp
2360  PRINT
2370  PRINT "MINIMUM VALUE OF OUTPUT = ";1.5*Axs*Inp
2380  INPUT " TYPE IN ESTIMATED MAXIMUM AND MINIMUM VALUES OF OUTPUT
 ?",Maxo,Mino
2390  INPUT "AT WHAT INTERVAL OF TIME THE PRINT OUT  IS REQUIRED ?",
In
2400  INPUT "MAGNIFICATION FACTOR ?",Mag
2410  PRINTER IS 0
2420  IF Nde=0 THEN 2440
2430  REDIM Div(Nde,2)
2440  PLOTTER IS 13,"GRAPHICS"
2450  GRAPHICS
2460  FRAME
2470  LOCATE 10,90,10,90
2480  IF Mino<0 THEN T=-Mino/10
2490  IF Mino>=0 THEN T=Maxo/10
2500  SCALE -Maxt/10,11*Maxt/10,Mino-T,Maxo+T
2510  CSIZE 2.4
2520  LDIR PI/2
2530  LORG 3
2540  AXES Maxt/10,T,0,0
2550  IF K1$="NO" THEN 2760
2560  FOR I=0 TO Maxt STEP Maxt/10
2570  IF I=0 THEN 2600
2580  MOVE I,-Maxo/3
2590  LABEL USING "XM 7D.DDD  ";I
2600  NEXT I
2610  LDIR 0
2620  LORG 2
2630  FOR I=Mino TO Maxo STEP Maxo/10
2640  IF I=0 THEN 2670
2650  MOVE -Maxt/3.5,I
2660  LABEL USING "XM 5D.DDD";I
2670  NEXT I
2680  MOVE -Maxt/10,Maxo+1.5*Maxo/10
2690  LABEL "VALUE OF OUTPUT VARIABLE"
2700  MOVE Maxt+.5*Maxt/10,-Maxo/8
2710  LABEL "TIME --- SEC"
2720  IF Fun1=0 THEN 2760
2730  FOR I=Tin TO Maxt STEP Maxt/100
2740  PLOT I,Inp*I^(Fun1-1)
2750  NEXT I
2760  Tim=Tin
2770  PRINT "VALUES OF TIME";"     ";"AMPLITUDE  OF  OUTPUT";"  ";"
       HIGHER DERIVATIVES"
2780  Z1=Z2=0
2790  FOR I=1 TO Nc
```

```
2800  IF Y(I)<>0 THEN 2860
2810  A1=U(I)*EXP(X(I)*Tim)
2820  B1=V(I)*EXP(X(I)*Tim)
2830  Z1=Z1+A1+U1(I)*EXP(X(I)*Tim
2840  Z2=Z2+B1+V1(I)*EXP(X(I)*Tim
2850  GOTO 3000
2860   A1=EXP(X(I)*Tim)*COS(Y(I)*Tim)
2870   B1=SIN(Y(I)*Tim)*EXP(X(I)*Tim)
2880   A2=U(I)
2890   B2=V(I)
2900   GOSUB 4600
2910   Z1=Z1+Real
2920   Z2=Z2+Imag
2930   A1=EXP(X(I)*Tim)*COS(Y(I)*Tim)
2940   B1=SIN(Y(I)*Tim)*EXP(X(I)*Tim)
2950   A2=U1(I)
2960   B2=V1(I)
2970   GOSUB 4600
2980   Z1=Z1+Real
2990   Z2=Z2+Imag
3000   NEXT I
3010    IF Nde=0 THEN 3520
3020    MAT Div=ZER
3030    FOR J=1 TO Nde
3040   IF (J=1) OR (J=2) OR (J=5) OR (J=6) OR (J=9) OR (J=10) THEN S
in=-1
3050   IF (J=3) OR (J=4) OR (J=7) OR (J=8) THEN Sin=1
3060   IF (J=1) OR (J=4) OR (J=5) OR (J=8) THEN Sin1=1
3070   IF (J=2) OR (J=3) OR (J=6) OR (J=7) THEN Sin1=-1
3080   FOR I=1 TO Nc
3090   IF Y(I)<>0 THEN 3130
3100   Div(J,1)=Div(J,1)+(U(I)+U1(I))*X(I)^J*EXP(X(I)*Tim)
3110   Div(J,2)=Div(J,2)+(V(I)+V1(I))*X(I)^J*EXP(X(I)*Tim)
3120   GOTO 3500
3130   A1=X(I)^J*EXP(X(I)*Tim)*COS(Y(I)*Tim)
3140   B1=X(I)^J*EXP(X(I)*Tim)*SIN(Y(I)*Tim)
3150   IF J/2=INT(J/2) THEN 3190
3160   A1=A1+Sin*Y(I)^J*SIN(Y(I)*Tim)*EXP(X(I)*Tim)
3170   B1=B1+Sin1*Y(I)^J*COS(Y(I)*Tim)*EXP(X(I)*Tim)
3180   GOTO 3210
3190   A1=A1+Sin*Y(I)^J*COS(Y(I)*Tim)*EXP(X(I)*Tim)
3200   B1=B1+Sin1*Y(I)^J*SIN(Y(I)*Tim)*EXP(X(I)*Tim)
3210   A2=U(I)+U1(I)
3220   B2=V(I)+V1(I)
3230   Sis=SIN(Y(I)*Tim)
3240   Exp=EXP(X(I)*Tim)
3250   Cos=COS(Y(I)*Tim)
3260   IF J=1 THEN 3470
3270   IF J=2 THEN 3320
3280   IF J=3 THEN 3350
3290   IF J=4 THEN 3380
3300   IF J=5 THEN 3410
3310   IF J=6 THEN 3440
3320   A1=A1-2*X(I)*Y(I)*Exp*Sis
3330   B1=B1+2*X(I)*Y(I)*Exp*Cos
3340   GOTO 3470
3350   A1=A1-3*X(I)*Y(I)*Exp*(X(I)*Sis+Y(I)*Cos)
3360   B1=B1+3*X(I)*Y(I)*Exp*(X(I)*Cos-Y(I)*Sis)
3370   GOTO 3470
3380   A1=A1+X(I)*Y(I)*Exp*(-4*X(I)^2*Sis-6*X(I)*Y(I)*Cos+4*Y(I)^2*S
is)
```

```
3390   B1=B1+X(I)*Y(I)*Exp*(4*X(I)^2*Cos-6*X(I)*Y(I)*Sis-4*Y(I)^2*Co
s
3400   GOTO 3470
3410   A1=A1+X(I)*Y(I)*Exp*(-5*X(I)^3*Sis-10*X(I)^2*Y(I)*Cos+10*X(I)
*Y(I)^2*Sis+5*Y(I)^3*Cos)
3420   B1=B1+X(I)*Y(I)*Exp*(5*X(I)^3*Cos-10*X(I)^2*Y(I)*Sis-10*X(I)*
Y(I)^2*Cos+5*Y(I)^3*Sis
3430   GOTO 3470
3440   A1=A1+X(I)*Y(I)*Exp*(-6*X(I)^4*Sis-15*X(I)^3*Y(I)*Cos+20*X(I)
^2*Y(I)^2*Sis+15*X(I)*Y(I)^3*Cos-6*Y(I)^4*Sis)
3450   B1=B1+X(I)*Y(I)*Exp*(6*X(I)^4*Cos-15*X(I)^3*Y(I)*Sis-20*X(I)^
2*Y(I)^2*Cos+15*X(I)*Y(I)^3*Sis+6*Y(I)^4*Cos)
3460   GOTO 3470
3470   GOSUB 4600
3480   Div(J,1)=Div(J,1)+Real
3490   Div(J,2)=Div(J,2)+Imag
3500   NEXT I
3510   NEXT J
3520   GOTO 3880
3530   IF Lo1=0 THEN 3870
3540   FOR K=1 TO Nc1-Nc+Fun
3550   IF (Fun=1) OR (Fun=0) THEN Pa=1
3560   IF Fun=2 THEN Pa=1
3570   IF Fun=3 THEN Pa=2
3580   Real1=Imag1=Oa=Oa1=0
3590   FOR I=1 TO Nc1
3600   Oa=Oa+Z(I)*K^I
3610   NEXT I
3620   Oa=Oa+Con
3630   Oa=Oa*K^Fun
3640   IF Nd=0 THEN 3680
3650   FOR M=1 TO Nd
3660   Oa1=Oa1+Z1(M)*K^M
3670   NEXT M
3680   Oa1=Oa1+Con1
3690   FOR N=1 TO Nc
3700   A1=U(N)
3710   B1=V(N)
3720   A2=K-X(N)
3730   B2=-Y(N)
3740   GOSUB 4630
3750   Real1=Real1+Real
3760   Imag1=Imag1+Imag
3770   NEXT N
3780   Oa(K)=(Oa1*Pa/Oa-Real1)*K^(Nc1-Nc+Fun)
3790   Oa1(K)=-Imag1*K^(Nc1-Nc+Fun)
3800   FOR S=1 TO Nc1-Nc+Fun
3810   Sa(K,S)=K^(Nc1-Nc+Fun-S)
3820   NEXT S
3830   NEXT K
3840   MAT Ka1=INV(Sa)
3850   MAT Aa=Ka1*Oa
3860   MAT Iaa=Ka1*Oa1
3870   RETURN
3880   IF Nc1-Nc+Fun1=0 THEN 3980
3890   FOR Qs=1 TO Nc1-Nc+Fun1
3900   IF Qs=1 THEN Pa1=1
3910   IF Qs=2 THEN Pa1=1
3920   IF Qs=3 THEN Pa1=2
3930   IF Qs=4 THEN Pa1=6
```

```
3940 IF Qs=5 THEN Pa1=24
3950 Z1=Z1+Aa1(Qs)*Tim^(Qs-1)/Pa1
3960 Z2=Z2+Iaa1(Qs)*Tim^(Qs-1)/Pa1
3970 NEXT Qs
3980 IF Lo1=0 THEN 4160
3990 FOR J=1 TO Lo1
4000 IF (J=1) OR (J=2) THEN Pa1=1
4010 IF J=3 THEN Pa1=2
4020 IF J=4 THEN Pa1=6
4030 IF J=5 THEN Pa1=24
4040 Z1=Z1+Aa(I)*Tim^(I-1)/Pa1
4050 Z2=Z2+Iaa(I)*Tim^(I-1)/Pa1
4060 NEXT J
4070 IF Nde=0 THEN 4250
4080 FOR J=1 TO Nde
4090 IF Nc1-Nc+Fun1-1-J<0 THEN 4160
4100 FOR Qs=1 TO Nc1-Nc+Fun1
4110 IF Qs-1-J<0 THEN 4140
4120 Div(J,1)=Div(J,1)+Aa1(Qs)*Tim^(Qs-1-J)
4130 Div(J,2)=Div(J,2)+Iaa1(Qs)*Tim^(Qs-1-J)
4140 NEXT Qs
4150 NEXT J
4160 IF Lo1<2 THEN 4250
4170 FOR J=1 TO Nde
4180 IF Lo1-1-J<0 THEN 4250
4190 FOR Qs=1 TO Lo1
4200 IF Qs-1-J<0 THEN 4230
4210 Div(J,1)=Div(J,1)+Aa(Qs)*Tim^(Qs-1-J)
4220 Div(J,2)=Div(J,2)+Iaa(Qs)*Tim^(Qs-1-J)
4230 NEXT Qs
4240 NEXT J
4250 IF Tim>0 THEN 4340
4260 MOVE 0,0
4270 GOTO 4340
4280 IF (Fun1=1) OR (Fun1=3) THEN 4300
4290 PLOT Tim,Tim
4300 IF (Fun1=2) OR (Fun1=1) THEN 4340
4310 LABEL ""
4320 PLOT Tim,Tim^2,1
4330 LABEL ""
4340 PLOT Tim,Z1*Mag,-1
4350 IF INT(Tim/In)<>Tim/In THEN 4390
4360 ON ERROR GOTO 4380
4370 PRINT Tim,Z1,Div(1,1),Div(2,1),Div(3,1),Div(4,1),Div(5,1)
4380 OFF ERROR
4390 Tim=Tim+Delt
4400 IF Tim<=Maxt THEN 2780
4410 IF K1$="NO" THEN 4450
4420 LETTER
4430 DUMP GRAPHICS
4440 PRINTER IS 16
4450 EXIT GRAPHICS
4460 INPUT "DO YOU WANT TO CHANGE THE INPUT FUNCTION ?  YES/NO",N1
$
4470 IF N1$="NO" THEN 4490
4480 GOTO 140
4490 ON ERROR GOTO 4530
4500 FOR I=1 TO Or-1! Or-1
4510 Inial(I+1,Mnh)=Div(I,1)
4520 NEXT I
```

```
4530   OFF ERROR
4540   Inial(1,Mnh)=Z1
4550   PRINTER IS 16
4560  GOTO 4660
4570  Real=A1+A2
4580  Imag=B1+B2
4590  RETURN
4600   Real=A1*A2-B1*B2
4610   Imag=A1*B2+B1*A2
4620   RETURN
4630   Real=(A1*A2+B1*B2)/(A2^2+B2^2)
4640   Imag=(B1*A2-A1*B2)/(A2^2+B2^2)
4650   RETURN
4660  SUBEND
```

PROGRAM "NONLIN"

INCLUDING THE NON-LINEAR

CHARACTERISTIC OF A SYSTEM

```
PROGRAM: NONLIN     HP ENHANCED BASIC   Date: 1/11/83          Page 1

10   REM *** PROGRAM STORED IN "COMPUT:T15" ***
20   REM
30   REM *** DECLARATION FOR THE MAIN PROGRAM ***
40   OPTION BASE 1
50   COM INTEGER A0,Z9,U1,Kmod,B0,Kf2,Kno
60    COM INTEGER Kon(25)
70   COM INTEGER S(10000),S1(10000)
80    DIM S2(10000)
90   REAL Z(500),X(15),K(26),K1(26)
100   COM Ki1(25),Ko1(25)
101   COM Input(10)
110   REDIM Ki1(Kno+1),Ko1(Kno+1)
120   ASSIGN #2 TO "DATAN"
121   Kmnh=0
130   MAT READ #2;Ki1,Ko1
140   ASSIGN * TO #2
150  INTEGER N,Wo,No,Mo,V7
160   Mef=0
161   Kef=0
162   Kpno=0
170  REM *** PREPARATION ***
180  REM *** THIS SEGMENT MUST BE PERFORMED AT THE BEGINNING***
190  REM *** OF THIS PROGRAM EVEV WHEN IT IS A RERUN ***
191   DIM Inial(10,10),Scal(10,10)
192   REDIM Inial(Ko1(1),Kno)
193   MAT Inial=ZER
194   DIM X1(10),Y1(10)
196 REDIM X1(Ko1(1)),Y1(Ko1(1))
200   PRINT A0,Z9,U1,Kmod,B0,Kf2,Kno
210  GOTO 240
220  GET "EQNA",2160,230
230  GET "ROOTS:T15",3600,240
240   IF Kmod<>2 THEN 270
250  ASSIGN #5 TO "DATAR"
260  BUFFER #5
270  IF U1=1 THEN 400          !U1=1 means NO S.S. Soln. required
280  GET "STYST:T15",6100,290
290  GET "EQNS",8600,300
300   IF Kmod<>2 THEN 330
310  ASSIGN #5 TO "DATAR"
320  BUFFER #5
330  REM *** PREPARATION SEGMENT END ***
340  INPUT "NUMBER OF INPUT IN THE STEADY STATE EQUATION",Mj
350  REDIM Input(Mj)
360   FOR I=1 TO Mj
370   DISP "THE";I;"th INPUT VALUE"
380   INPUT Input(I)
390   NEXT I
391   IF Kmnh=1 THEN 460
400 IF Kmod=1 THEN 430
410   GOTO 430
420   GET "RESPO:T15",10000,430
430   IF Kmod=0 THEN 450
440   GOTO 460
450   GET "NYQI:T15",10000,460
460  INPUT "TO READ Z FROM FILE, INPUT 2 OTHERWISE 0",V7
470  IF V7<1 THEN 510
480  ASSIGN #2 TO "DATAA"
490  MAT READ #2;Z(Z9)
500  ASSIGN * TO #2
```

```
510  INPUT "TO READ X FROM FILE, INPUT 2 OTHERWISE 0",V7
520  IF V7<1 THEN 560
530  ASSIGN #3 TO "DATAX"
540  READ #3;V7
550  MAT READ #3;X(V7)
560  IF U1=1 THEN 660
570  INPUT "Do you want to print out intermediate results for s.s. s
oln.?",A$
580  Wo=0
590  IF (A$="NO") OR (A$="no") THEN 610
600  Wo=1
610  INPUT "Give no. of signigicant digits wanted for s.s. soln.",No
620  INPUT "Give maximum no. of iterations allowed for s.s. soln.",M
o
630  REM *** SUBPROGRAM STYST IS CALLED FOR S.S. SOLN. ***
640  CALL Styst(Input(*),Z(*),X(*),V7,Wo,No,Mo,#3)
650   IF Mef=1 THEN 750
660  DIM A(25,25),B(25,25),K0(26),C(25)
670  REDIM A(A0,A0),B(A0,B0),C(A0)
680  REDIM S(Kl1(1)),S1(Kl1(2))
690   ASSIGN #2 TO "DATAS"
700   MAT READ #2;S
710   Mnh=1
720   O1=Ko1(1)
730   L1=Kl1(1)
740   REDIM K(O1+1)
750   GOSUB 2160
760   GOSUB 1760
770   IF Kmod<>2 THEN 790
780   ASSIGN * TO #2
790   GOSUB 1330
800   IF Kmod=2 THEN 1130
810   PRINT
820   PRINT "THE NOMINATOR POLYNOMINAL OF ";Kon(Mnh);"Th OUTPUT VARI
ABLE IS:"
830   REDIM S2(L1)
840   MAT S2=S
850   REDIM S(Kl1(Mnh+1))
860   MAT READ #2;S
870   Kno1=O1
880   Kno2=L1
890   REDIM K0(O1+1)
900   MAT K0=K
910   L1=Kl1(Mnh+1)
920   O1=Ko1(Mnh+1)
930   FOR I=1 TO A0
940   C(I)=A(I,Kon(Mnh))
950   A(I,Kon(Mnh))=B(I,Kf2)
960   NEXT I
970   REDIM K(O1+1)
980   GOSUB 1760
990   REDIM K1(Ko1(Mnh+1)+1)
1000  O1=Kno1
1010  L1=Kno2
1020  MAT K1=K
1030  REDIM K(O1+1)
1040  MAT K=K0
1050  REDIM S(L1)
1060  MAT S=S2
1070  FOR I=1 TO A0
```

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```
1380  A(I,Kon(Mnh))=C(I)
1390  NEXT I
1100  Od=Ko1(Mnh+1)
1110  N=01
1120  IF Mnh<1 THEN 1160
1130  CALL Roots(K(*),N,Kmod,#5)
1131  IF Kmod=2 THEN 1510
1140  GOTO 1160
1150  IF (A$="N") OR (A$="NO") THEN 1530
1160  IF Kmod<>2 THEN 1210
1170  PRINT #2;END
1180  ASSIGN * TO #5
1190 REM *** REDUCING ORDER IF POSSIBLE ***
1200  LOAD "PLOT:T15"
1210  IF Kmod=0 THEN 1250
1220  IF Kmod=1 THEN 1230
1230  CALL Respo(K(*),K1(*),Ko1(*),Mnh,Inial(*),Kef,X1(*),Y1(*),Scal
(*),Kno,Kno)
1240  GOTO 1260
1250  CALL Nyqi(K(*),K1(*),Ko1(*),Mnh)
1260 PRINT
1270  Mnh=Mnh+1
1280  IF Mnh<=Kno THEN 800
1290  ASSIGN * TO #2
1291  M1$="YES"
1292  Kmnh=1
1293  PRINTER IS 0
1294  Kef=Kef+1
1295  IF M1$="YES" THEN 1521
1300  END
1310  STOP
1320 REM *** REDUCING ORDER IF POSSIBLE ***
1330 N=01                                        !Start search from 01 o
rder term
1340 J9=N+1
1350 IF ABS(K(J9))>=1E-70 THEN 1390
1360 N=N-1                                       !1E-70 as criterion
1370 IF N<0 THEN 1400
1380 GOTO 1340
1390 IF N=01 THEN 1480
1400 PRINT
1410 PRINT "REDUCED SPECIFIC POLYNOMIAL COEFFICIENTS";
1420 PRINT TAB(45);"DEGREE IN S"
1430 PRINT
1440 FOR I=1 TO N+1
1450 PRINT USING 2020;K(I),I-1
1460 NEXT I
1480  RETURN
1490 CALL Roots(K(*),N,Kmod,#5)
1500  GOSUB 2050
1510 INPUT "ANY MODIFICATION IN Z? YES/NO",B$
1520 IF B$="NO" THEN 1640
1521 GOTO 1522
1522  Z(1)=70*.4*Inial(1,4)/SQR(2*(Inial(1,1)+2000))
1523  Z(2)=70*.005/SQR(ABS(2*(Inial(1,1)+2000))
1524  Z(3)=70*.005/SQR(ABS(2*(Inial(1,1)-Inial(1,2))))
1525  Z(4)=70*.25*Inial(1,5)/SQR(2*(Inial(1,2)-Inial(1,3))))
1526  Z(8)=70*.4*SQR(2*(Inial(1,3)+2000))
1527  Z(9)=70*.25*SQR(2*(Inial(1,1)+2000))
1528  PRINT "Z(1)=";Z(1),"Z(2)=";Z(2),"Z(3)=";Z(3),"Z(4)=";Z(4),"Z(8
)=";Z(8),"Z(9)=";Z(9)
```

Change of coefficients of linearized equation.

```
1529  GOTO 1628
1530 INPUT "GIVE SUBSCRIPT FOR THIS ELEMENT, 0 TO END MODIFICATION",
J1
1540 IF J1=0 THEN 1580
1550 INPUT "GIVE NEW VALUE FOR THIS ELEMENT",Z(J1)
1560 GOTO 1530
1570  Mef=1
1580 PRINT
1590 PRINT "NEW Z IS:"
1600 FOR I=1 TO Z9
1610 PRINT "Z[";I;"] =";Z(I)
1620 NEXT I
1621  GOTO 1629
1628 Kpno=Kpno+1
1629 PRINTER IS 16
1630 ON U1+1 GOTO 640,650
1640 PRINT "DONE"
1650 PRINT #5;END
1660 ASSIGN * TO #5
1661  GOTO 1730
1670 READ #3,1;V7
1680 FOR J=1 TO V7
1690 PRINT #3;X(J)
1700 NEXT J
1710 PRINT #3;END
1720 ASSIGN * TO #3
1730 IF A0=0 THEN 1750
1740 LOAD "PLOT:T15"
1750 STOP
1760 REM *** SUBROUTINE TO CALCULATE POLYNOMIAL COEFFICIENTS ***
1770 MAT K=ZER(O1+1)
1780 FOR Hmno=0 TO O1
1790 O2=Hmno+1
1800 FOR J=1 TO L1-1
1810 T1=0
1820 IF S(J)<>Hmno THEN 1940
1830 T1=SGN(S(J+1))
1840 IF J=1 THEN 1930
1850 J1=J-1
1860 IF ABS(S(J1))<101 THEN 1930
1870 I2=INT(S(J1)/100)
1880 J2=S(J1)-100*I2
1890 T1=T1*A(I2,J2)
1900 IF J1<=1 THEN 1930
1910 J1=J1-1
1920 GOTO 1860
1930 K(O2)=K(O2)+T1
1940 NEXT J
1950 NEXT Hmno
1960 PRINT
1970 PRINT "COEFFICIENTS OF THE SPECIFIC POLYNOMIAL ";
1980 PRINT TAB(45);"DEGREE IN S"
1990 PRINT
2000 FOR I=1 TO O1+1
2010 PRINT USING 2020;K(I),I-1
2020 IMAGE 10X,MD.10DXE,23X,DD
2030 NEXT I
2040  RETURN
2050  IF Kmod=2 THEN 1900
2060  INPUT "Do you require the result for this values of parameters
 YES OR NO ",A$
```

```
2070  IF A$="NO" THEN 1900
2080  IF Kmod<>2 THEN 2120
2090  PRINT #5;END
2100  ASSIGN * TO #5
2110  LOAD "PLOT:T15"
2120  GOSUB 9000
2130  Mnh=Mnh+1
2140  IF Mnh<=Kno THEN 800
2150  END                    Equation for numerical calculation of
2160 A(1,1)=Z(1)+Z(2)        the coefficients matrices.
2161 A(1,2)=-Z(2)
2162 A(1,4)=Z(8)
2163 A(1,6)=Z(5)/Z(7)
2164 A(1,8)=Z(10)
2165 A(2,1)=-Z(2)
2166 A(2,2)=Z(2)+Z(4)/(1+Z(4)/Z(3))
2167 A(2,5)=Z(9)/(1+Z(4)/Z(3))
2168 A(2,7)=Z(6)/Z(7)
2169 A(2,8)=-Z(10)
2170 A(2,9)=Z(11)/(1+Z(4)/Z(3))
2171 A(3,1)=-Z(10)
2172 A(3,2)=Z(10)
2173 A(3,4)=Z(15)
2174 A(3,8)=Z(14)
2175 A(3,10)=Z(12)
2176 A(4,2)=-Z(11)/(1+Z(4)/Z(3))
2177 A(4,5)=Z(17)+Z(9)*Z(11)/(Z(4)+Z(3))
2178 A(4,9)=Z(16)+Z(11)^2/(Z(4)+Z(3))
2179 A(4,11)=Z(13)
2180  A(5,2)=-Z(3)
2181  A(5,3)=Z(4)+Z(3)
2182  A(5,5)=Z(9)
2183  A(5,9)=Z(11)
3148 RETURN
3149 END
3600 SUB Roots(K(*),INTEGER N,Kmod,#5)
3610 REM *** PROGRAM STORED IN "ROOTS:T15" ***
3620 OPTION BASE 1
3630    ASSIGN #3 TO "BASE3"
3640 H1=N                                       !Conserve N
3650 REM *** PROGRAM STORED IN "ROOTS:T15" ***
3660 REM *** FINDS THE ROOTS OF POLYNOMIAL ***
3670 REM *** VERSION 2/AUGUST 1978 ***
3680 REM N --- ORDER OF POLYNOMIAL
3690 REM K --- COEFFICIENTS OF POLYNOMIAL
3700  DIM Kmno(N+1)
3710 REM *** FOR COMPLEX PAIRED ROOT ONLY THE ONE WITH POSITIVE IMAG
INARY ***
3720  MAT Kmno=K
3730 REM *** IS STORED INTO FILE "DATAR:T15" ***
3740  Mnbh=1/K(1)
3750 MAT K=(Mnbh)*K
3770 IF N<=0 THEN 5250
3780 DIM A(26),B(26),X(26)
3790 FOR I=1 TO N+1
3800 A(I)=K(N+2-I)
3810 B(I)=A(I)
3820 NEXT I
3830 PRINT
3840 C1=0
```

SUB-PROGRAM "RESPO2"

For limiting the current of the motor

at each required step of time

```
5020 F1=0
5030 FOR I=1 TO N+1
5040 J=N-I+2
5050 IF B(J)=0 THEN 5090
5060 F=B(J)*X^(I-1)+F
5070 IF I-1=0 THEN 5090
5080 F1=(I-1)*B(J)*X^(I-2)+F1
5090 NEXT I
5100 X1=X-F/F1
5110 IF ABS(X/X1-1)<.000001 THEN 5140
5120 X=X1
5130 GOTO 5010
5140 PRINT USING 4530;X1,0,-SGN(X1)
5150  IF Kmod<>2 THEN 5170
5160 PRINT #5;X1,0
5170 IF Kmod=2 THEN 5190
5180  PRINT #3;X1,0
5190 N=N-1
5200 FOR I=2 TO N+1
5210 A(I)=B(I)+X1*A(I-1)
5220 B(I)=A(I)
5230 NEXT I
5240 RETURN
5250 N=H1
5260  ASSIGN * TO #3
5280 SUBEND
10000 SUB Respo(K(*),K1(*),Ko1(*),Mnh,Inial(*),Kef,X(*),Y(*),Scal(*)
,Kpno,Inp,Inp1,INTEGER Kno,#8)
10010 OPTION BASE 1
10020 REM PROGRAM IS STORED IN RESPO
10030 DIM Q(25),R(25),S(25),T(25),U(25),V(25),P(20),Sa(6,6),Oa(6),Oa
1(6),Aa(6),Ka1(6,6),Iaa(6),Z(25),Z1(25),K2(25),Iniao(25),Iniai(25)
10040 DIM U1(25),V1(25),Iaa1(6),Aa1(6),Div(10,2)
10050 Od=Nd=Ko1(Mnh+1)
10060 Or=Nc1=Ko1(1)
10070 Kplo=0
10080 REDIM K2(Or),Iniao(Or)
10090 IF Od=0 THEN 10110
10100 REDIM Iniai(Od)
10110 K1$="NO"
10120 IF Kef<>0 THEN 10180
10130 FOR I=1 TO Or
10140 DISP "What is the initial value of D^";I-1;"(Xoutput(0))"
10150 INPUT Iniao(I)
10160 NEXT I
10170 GOTO 10210
10180 FOR I=1 TO Or
10190 Iniao(I)=Inial(I,Mnh)
10200 NEXT I
10210 IF Od=0 THEN 15910
10220 MAT Iniai=ZER
10230 IF Kpno=0 THEN 10330
10231 IF Mnh=1 THEN Inp1=Inp
10240 IF Mnh=2 THEN Iniai(1)=Inp1
10250 IF Mnh=1 THEN GOTO 10270
10260 GOTO 10310
10270 Iniai(1)=Inp1
10290 IF ABS(Inial(1,2))>=50 THEN Inp=0
10300 IF ABS(Inial(1,2))<50 THEN Inp=2.56
10310 DISP Iniai(1),Inp
```

Equation for limiting the current by reducing the input voltage at each step of time.

```
10320 GOTO 10340
10330 Inp=2.56
10340 REDIM Z(Nc1)
10350 IF Nd=0 THEN 10370
10360 REDIM Z1(Nd)
10370 FOR I=1 TO Or+1
10380 IF I>1 THEN 10410
10390 Con=K(1)
10400 GOTO 10430
10410 Z(I-1)=K(I)
10420 Z(I-1)=Z(I-1)
10430 NEXT I
10440 Kmbhg=Z(Or)
10450 Con=Con/Z(Or)
10460 FOR I=1 TO Or
10470 Z(I)=Z(I)/Z(Or)
10480 NEXT I
10490 FOR I=1 TO Od+1
10500 IF I>1 THEN 10530
10510 Con1=K1(1)
10520 GOTO 10550
10530 Z1(I-1)=K1(I)
10540 Z1(I-1)=Z1(I-1)*Inp/Kmbhg
10550 NEXT I
10560 Con1=Con1*Inp/Kmbhg
10570 MAT K2=ZER
10580 FOR I=1 TO Or
10590 FOR J=I+1 TO Or+1
10600 K2(I)=K2(I)+K(J)*Iniao(J-I)
10610 NEXT J
10620 NEXT I
10630 IF Od=0 THEN 10690
10640 FOR I=1 TO Od
10650 FOR J=I+1 TO Od+1
10660 K2(I)=K2(I)-K1(J)*Iniai(J-I)
10670 NEXT J
10680 NEXT I
10690 Nc=Nc1
10700 IF Mnh>1 THEN 10820
10710 ASSIGN #3 TO "BASE3"
10720 PRINT
10730 PRINT
10740 REDIM X(Nc1),Y(Nc1)
10750 FOR I=1 TO Nc1
10760 READ #3;X(I),Y(I)
10770 IF (X(I)=0) AND (Y(I)=0) THEN 10790
10780 GOTO 10810
10790 X(I)=1E-30
10800 Y(I)=1E-30
10810 NEXT I
10820 REDIM Q(Nc),R(Nc),X(Nc),Y(Nc),S(Nc),T(Nc),U(Nc),V(Nc)
10830 IF Mnh>1 THEN 10850
10840 ASSIGN * TO #3
10850 Fun=1
10860 GOTO 10890
10870 DISP "WHAT IS THE INPUT FUNCTION ,STEP,VELOCITY,ACCELARATION"
10880 INPUT " FOR STEP PRESS 1,FOR VELOCITY 2 AND FOR ACCELARATION P
RESS 3",Fun
10890 REDIM Aa1(Nc1-Nc+Fun),Iaa1(Nc1-Nc+Fun)
10900 Lo1=Nc1-Nc+Fun
```

```
10910 GOSUB 11090
10920 Lo1=Nc1-Nc
10930 MAT U1=U
10940 MAT V1=V
10950 REDIM Z1(Or-1)
10960 FOR I=1 TO Or
10970 IF I>1 THEN 11000
10980 Con1=K2(I)/K(Or+1)
10990 GOTO 11010
11000 Z1(I-1)=K2(I)/K(Or+1)
11010 NEXT I
11020 Od=Nd=Or-1
11030 Fun1=Fun
11040 Fun=0
11050 MAT Aa1=Aa
11060 MAT Iaa1=Iaa
11070 GOSUB 11090
11080 GOTO 11950
11090 IF Lo1=0 THEN 11110
11100 REDIM Sa(Lo1,Lo1),Oa(Lo1),Oa1(Lo1),Aa(Lo1),Iaa(Lo1),Ka1(Lo1,Lo
1)
11110  FOR I=1 TO Nc
11120 FOR J=1 TO Nc
11130 IF J=I THEN 11320
11140 A1=-X(J)
11150 B1=-Y(J)
11160 A2=X(I)
11170 B2=Y(I)
11180 GOSUB 14160
11190 IF (I=1) AND (J=2) THEN 11220
11200 IF J=1 THEN 11220
11210 GOTO 11250
11220 A1=1
11230 B1=0
11240 GOTO 11270
11250 A1=A3
11260 B1=B3
11270 A2=Real
11280 B2=Imag
11290 GOSUB 14190
11300 A3=Real
11310 B3=Imag
11320 NEXT J
11330 Q(I)=Real
11340 R(I)=Imag
11350 IF Lo1=0 THEN 11450
11360 FOR K=1 TO Nc1-Nc+Fun
11370 A1=X(I)
11380 B1=Y(I)
11390 A2=Q(I)
11400 B2=R(I)
11410 GOSUB 14190
11420 Q(I)=Real
11430 R(I)=Imag
11440 NEXT K
11450 NEXT I
11460 MAT S=ZER
11470 MAT T=ZER
11480 IF Nd=0 THEN 11710
11490 FOR K=1 TO Nc
```

```
11500 FOR J=1 TO Nd
11510 FOR I=J TO Nd
11520 A1=X(K)
11530 B1=Y(K)
11540 IF I<>J THEN 11580
11550 A2=1
11560 B2=0
11570 GOTO 11600
11580 A2=Real
11590 B2=Imag
11600 GOSUB 14190
11610 NEXT I
11620 S(K)=S(K)+Real*Z1(Nd-J+1)
11630 T(K)=T(K)+Imag*Z1(Nd-J+1)
11640 NEXT J
11650 S(K)=S(K)+Con1
11660 NEXT K
11670 IF Fun<>3 THEN 11740
11680 MAT S=(2)*S
11690 MAT T=(2)*T
11700 GOTO 17260
11710 MAT S=CON
11720 MAT S=(Con1)*S
11730 MAT T=ZER
11740 FOR I=1 TO Nc
11750 A2=Q(I)
11760 B2=R(I)
11770 A1=S(I)
11780 B1=T(I)
11790 GOSUB 14220
11800 U(I)=Real
11810 V(I)=Imag
11820 NEXT I
11830 GOSUB 13140
11840 RETURN
11850 PRINT
11860 Nde=6
11870 Tin=.0003
11880 Delt=.0003
11890 Maxt=.0003
11900 Maxo=110
11910 Mino=0
11920 In=.0003
11930 IF Kpno=0 THEN Tin=0
11940 GOTO 12020
11950 GOTO 11850
11960 INPUT "STARTING  POINT OF TIME ",Tin
11970 INPUT "STEP OF TIME WHICH THE MUST BE EVALUATED ",Delt
11980 INPUT "MAXIMUM VALUE OF TIME ",Maxt
11990 INPUT "ESTIMATED MAXIMUM AND MINIMUM VALUES OF OUTPUT ",Maxo,M
ino
12000 INPUT "AT WHAT INTERVAL OF TIME THE OUTPUT IS REQUIRED ",In
12010 INPUT "MAGNIFICATION FACTORE",Mag
12020 IF Nde=0 THEN 12060
12030 K1$="NO"
12040 REDIM Div(Nde,2)
12050 GOTO 12070
12060 PLOTTER IS 13,"GRAPHICS"
12070 IF K1$="NO" THEN 12310
12080 GRAPHICS
```

```
12090 FRAME
12100 LOCATE 10,90,10,90
12110 IF Mino<0 THEN T=-Mino/10
12120 IF Mino>=0 THEN T=Maxo/10
12130 SCALE -Maxt/10,11*Maxt/10,Mino-T,Maxo+T
12140 CSIZE 2.4
12150 LDIR PI/2
12160 LORG 3
12170 AXES Maxt/10,T,0,0
12180 IF K1$="NO" THEN 12310
12190 FOR I=0 TO Maxt STEP Maxt/10
12200 IF I=0 THEN 12230
12210 MOVE I,-Maxo/3
12220 LABEL USING "XM 6D.DDDDD";I
12230 NEXT I
12240 LDIR 0
12250 LORG 2
12260 FOR I=Mino TO Maxo STEP Maxo/10
12270 IF I=0 THEN 12300
12280 MOVE -Maxt/3.5,I
12290 LABEL USING "XM 5D.DDD";I
12300 NEXT I
12310 Tim=Tin
12311 IF Kpno=0 THEN 12340
12320 IF INT((Kpno+1)/10)=(Kpno+1)/10 THEN GOTO 12340
12330 GOTO 12360
12340 PRINTER IS 0
12350 GOTO 12370
12360 PRINTER IS 16
12370 IF (Mnh>1) OR (Kpno>0) THEN 12390
12380 PRINT "VALUES OF TIME";"     ";"AMPLITITUDE OF OUTPUT";"   ";"HI
GHER DERAVATIVE"
12390 Z1=Z2=0
12400 FOR I=1 TO Nc
12410 IF Y(I)<>0 THEN 12470
12420 A1=U(I)*EXP(X(I)*Tim)
12430 B1=V(I)*EXP(X(I)*Tim)
12440 Z1=Z1+A1+U1(I)*EXP(X(I)*Tim)
12450 Z2=Z2+B1+V1(I)*EXP(X(I)*Tim)
12460 GOTO 12610
12470  A1=EXP(X(I)*Tim)*COS(Y(I)*Tim)
12480  B1=SIN(Y(I)*Tim)*EXP(X(I)*Tim)
12490  A2=U(I)
12500  B2=V(I)
12510  GOSUB 14190
12520  Z1=Z1+Real
12530  Z2=Z2+Imag
12540  A1=EXP(X(I)*Tim)*COS(Y(I)*Tim)
12550  B1=SIN(Y(I)*Tim)*EXP(X(I)*Tim)
12560  A2=U1(I)
12570  B2=V1(I)
12580  GOSUB 14190
12590  Z1=Z1+Real
12600  Z2=Z2+Imag
12610  NEXT I
12620   IF Nde=0 THEN 13130
12630   MAT Div=ZER
12640   FOR J=1 TO Nde
12650  IF (J=1) OR (J=2) OR (J=5) OR (J=6) OR (J=9) OR (J=10) THEN S
in=-1
```

```
12660  IF (J=3) OR (J=4) OR (J=7) OR (J=8) THEN Sin=1
12670  IF (J=1) OR (J=4) OR (J=5) OR (J=8) THEN Sin1=1
12680  IF (J=2) OR (J=3) OR (J=6) OR (J=7) THEN Sin1=-1
12690  FOR I=1 TO Nc
12700  IF V(I)<>0 THEN 12740
12710  Div(J,1)=Div(J,1)+(U(I)-U1(I))*X(I)^J*EXP(X(I)*Tim)
12720  Div(J,2)=Div(J,2)+(V(I)+V1(I))*X(I)^J*EXP(X(I)*Tim)
12730  GOTO 13110
12740  A1=X(I)^J*EXP(X(I)*Tim)*COS(Y(I)*Tim)
12750  B1=X(I)^J*EXP(X(I)*Tim)*SIN(Y(I)*Tim)
12760  IF J/2=INT(J/2) THEN 12800
12770  A1=A1+Sin*Y(I)^J*SIN(Y(I)*Tim)*EXP(X(I)*Tim)
12780  B1=B1+Sin1*Y(I)^J*COS(Y(I)*Tim)*EXP(X(I)*Tim)
12790  GOTO 12820
12800  A1=A1+Sin*Y(I)^J*COS(Y(I)*Tim)*EXP(X(I)*Tim)
12810  B1=B1+Sin1*Y(I)^J*SIN(Y(I)*Tim)*EXP(X(I)*Tim)
12820  A2=U(I)+U1(I)
12830  B2=V(I)+V1(I)
12840  Sis=SIN(Y(I)*Tim)
12850  Exp=EXP(X(I)*Tim)
12860  Cos=COS(Y(I)*Tim)
12870  IF J=1 THEN 13080
12880  IF J=2 THEN 12930
12890  IF J=3 THEN 12960
12900  IF J=4 THEN 12990
12910  IF J=5 THEN 13020
12920  IF J=6 THEN 13050
12930  A1=A1-2*X(I)*Y(I)*Exp*Sis
12940  B1=B1+2*X(I)*Y(I)*Exp*Cos
12950  GOTO 13080
12960  A1=A1-3*X(I)*Y(I)*Exp*(X(I)*Sis+Y(I)*Cos)
12970  B1=B1+3*X(I)*Y(I)*Exp*(X(I)*Cos-Y(I)*Sis)
12980  GOTO 13080
12990  A1=A1+X(I)*Y(I)*Exp*(-4*X(I)^2*Sis-6*X(I)*Y(I)*Cos+4*Y(I)^2*S
is)
13000  B1=B1+X(I)*Y(I)*Exp*(4*X(I)^2*Cos-6*X(I)*Y(I)*Sis-4*Y(I)^2*Co
s)
13010  GOTO 13080
13020  A1=A1+X(I)*Y(I)*Exp*(-5*X(I)^3*Sis-10*X(I)^2*Y(I)*Cos+10*X(I)
*Y(I)^2*Sis+5*Y(I)^3*Cos)
13030  B1=B1+X(I)*Y(I)*Exp*(5*X(I)^3*Cos-10*X(I)^2*Y(I)*Sis-10*X(I)*
Y(I)^2*Cos+5*Y(I)^3*Sis)
13040  GOTO 13080
13050  A1=A1+X(I)*Y(I)*Exp*(-6*X(I)^4*Sis-15*X(I)^3*Y(I)*Cos+20*X(I)
^2*Y(I)^2*Sis+15*X(I)*Y(I)^3*Cos-6*Y(I)^4*Sis)
13060  B1=B1+X(I)*Y(I)*Exp*(6*X(I)^4*Cos-15*X(I)^3*Y(I)*Sis-20*X(I)^
2*Y(I)^2*Cos+15*X(I)*Y(I)^3*Sis+6*Y(I)^4*Cos)
13070  GOTO 13080
13080  GOSUB 14190
13090  Div(J,1)=Div(J,1)+Real
13100  Div(J,2)=Div(J,2)+Imag
13110  NEXT I
13120  NEXT J
13130  GOTO 13490
13140  IF Lo1=0 THEN 13480
13150  FOR K=1 TO Nc1-Nc+Fun
13160  IF (Fun=1) OR (Fun=0) THEN Pa=1
13170  IF Fun=2 THEN Pa=1
13180  IF Fun=3 THEN Pa=2
13190  Real1=Imag1=Oa=Oa1=0
```

```
13200 FOR I=1 TO Nc1
13210 Oa=Oa+Z(I)*K^I
13220 NEXT I
13230 Oa=Oa+Con
13240 Oa=Oa+K^Fun
13250 IF Nd=0 THEN 13290
13260 FOR M=1 TO Nd
13270 Oa1=Oa1+Z1(M)*K^M
13280 NEXT M
13290 Oa1=Oa1+Con1
13300 FOR N=1 TO Nc
13310 A1=U(N)
13320 B1=V(N)
13330 A2=K-X(N)
13340 B2=-Y(N)
13350 GOSUB 14220
13360 Real1=Real1+Real
13370 Imag1=Imag1+Imag
13380 NEXT N
13390 Oa(K)=(Oa1+Pa/Oa-Real1)*K^(Nc1-Nc+Fun)
13400 Oa1(K)=-Imag1*K^(Nc1-Nc+Fun)
13410 FOR S=1 TO Nc1-Nc+Fun
13420 Sa(K,S)=K^(Nc1-Nc+Fun-S)
13430 NEXT S
13440 NEXT K
13450 MAT Ka1=INV(Sa)
13460 MAT Aa=Ka1*Oa
13470 MAT Iaa=Ka1*Oa1
13480 RETURN
13490 FOR Qs=1 TO Nc1-Nc+Fun1
13500 IF Qs=1 THEN Pa1=1
13510 IF Qs=2 THEN Pa1=1
13520 IF Qs=3 THEN Pa1=2
13530 IF Qs=4 THEN Pa1=6
13540 IF Qs=5 THEN Pa1=24
13550 Z1=Z1+Aa1(Qs)*Tim^(Qs-1)/Pa1
13560 Z2=Z2+Iaa1(Qs)*Tim^(Qs-1)/Pa1
13570 NEXT Qs
13580 IF Lo1=0 THEN 13760
13590 FOR J=1 TO Lo1
13600 IF (J=1) OR (J=2) THEN Pa1=1
13610 IF J=3 THEN Pa1=2
13620 IF J=4 THEN Pa1=6
13630 IF J=5 THEN Pa1=24
13640 Z1=Z1+Aa(I)*Tim^(I-1)/Pa1
13650 Z2=Z2+Iaa(I)*Tim^(I-1)/Pa1
13660 NEXT J
13670 IF Nde=0 THEN 13860
13680 FOR J=1 TO Nde
13690 IF Nc1-Nc+Fun1-1-J<0 THEN 13760
13700 FOR Qs=1 TO Nc1-Nc+Fun1
13710 IF Qs-1-J<0 THEN 13740
13720 Div(J,1)=Div(J,1)+Aa1(Qs)*Tim^(Qs-1-J)
13730 Div(J,2)=Div(J,2)+Iaa1(Qs)*Tim^(Qs-1-J)
13740 NEXT Qs
13750 NEXT J
13760 IF Lo1<2 THEN 13860
13770 FOR J=1 TO Nde
13780 IF Lo1-1-J<0 THEN 13860,
13790 FOR Qs=1 TO Lo1
```

```
13800  IF Os-1-J<0 THEN 13830
13810  Div(J,1)=Div(J,1)+Aa(Os)*Tim^(Os-1-J)
13820  Div(J,2)=Div(J,2)+Iaa(Os)*Tim^(Os-1-J)
13830  NEXT Os
13840  NEXT J
13850  IF K1$="NO" THEN 13940
13860  IF K1$="NO" THEN 13940
13870  IF (Fun1=1) OR (Fun1=3) THEN 13890
13880  PLOT Tim,Tim
13890  IF (Fun1=2) OR (Fun1=1) THEN 13920
13900  LABEL ""
13910  PLOT Tim,Tim^2,1
13920  LABEL ""
13930  PLOT Tim,Z1*Mag,1
13940  IF INT(Tim/In)<>Tim/In THEN 13960
13950  ON ERROR GOTO 19152
13960  PRINT Tim*(Kpno+1),Z1,Div(1,1),Div(2,1)
13961  PRINT #8;Z1
13970  OFF ERROR
13980  Tim=Tim+Delt
13990  IF Tim<=Maxt THEN 12390
14000  IF K1$="NO" THEN 14060
14010  LETTER
14020  DUMP GRAPHICS
14030  PRINTER IS 16
14040  N1$="NO"
14050  GOTO 14090
14060  N1$="NO"
14070  GOTO 14090
14080  EXIT GRAPHICS
14090  IF N1$="YES" THEN 10050
14100  FOR I=1 TO Or-1
14110  Inial(I+1,Mnh)=Div(I,1)
14120  NEXT I
14130  Inial(1,Mnh)=Z1
14140  PRINTER IS 16
14150     GOTO 14710
14160 Real=A1+A2
14170 Imag=B1+B2
14180 RETURN
14190  Real=A1*A2-B1*B2
14200  Imag=A1*B2+B1*A2
14210  RETURN
14220  Real=(A1*A2+B1*B2)/(A2^2+B2^2)
14230  Imag=(B1*A2-A1*B2)/(A2^2+B2^2)
14240  RETURN
14250 BEEP
14260 IF (Kpno>0) OR (Mnh>1) OR (Kplo>0) THEN 14280
14270 PLOTTER IS 5,"INCREMENTAL",.05
14280 CSIZE 1
14290 IF Kno>3 THEN Plo=2
14300 IF Kno>6 THEN Plo=3
14310 IF Kno<3 THEN Plo=1
14320 IF (Kpno>0) OR (Mnh>1) OR (Kplo>0) THEN 14510
14330 LIMIT 0,300*Plo,0,900
14340 IF (Kpno>0) OR (Kplo>0) THEN 14510
14350 IF (Kplo>0) OR (Mnh>1) THEN 14510
14360 REDIM Scal(Kno,4)
14370 K=0
14380 FOR I=1 TO Kno
```

```
14390 IF I/6=INT(I/5) THEN K=K-1
14400 IF I>5 THEN 14430
14410 LOCATE 4+K*40,40+K*40,4+40*(I-1),40+(I-1)*40
14420 GOTO 14440
14430 LOCATE 4+K*40,40+40*K,4+(I-5)*40,40+(I-5)*40
14440 DISP "What the Max and Min values of X axis of";I;"th variable
s"
14450 INPUT Scal(I,2),Scal(I,1)
14460 DISP "What is the Max and Min values of";I;"th variables"
14470 INPUT Scal(I,4),Scal(I,3)
14480 SCALE Scal(I,1),Scal(I,2)*11/10,Scal(I,3),Scal(I,4)*11/10
14490 AXES Scal(I,2)/10,Scal(I,4)/10,0,0,2,2,.3
14500 NEXT I
14510 K=0
14520 IF Mnh/6=INT(Mnh/5) THEN K=K+1
14530 DISP Kplo
14540 IF Mnh>5 THEN 14570
14550 LOCATE 4+K*40,40+K*40,4+40*(Mnh-1),40+(Mnh-1)*40
14560 GOTO 14580
14570 LOCATE 4+K*40,40+40*K,4+(Mnh-5)*40,40+(Mnh-5)*40
14580 SCALE Scal(Mnh,1),Scal(Mnh,2)*11/10,Scal(Mnh,3),Scal(Mnh,4)*11
/10
14590 IF (Kpno>0) AND (Kplo=0) THEN Kmnj=Inial(1,Mnh)
14600 IF Kpno>0 THEN 14650
14610 IF Kplo>0 THEN 14630
14620 MOVE 0,Inial(1,Mnh)
14630 PLOT Tim,Z1,-1
14640 GOTO 14690
14650 IF Kplo>0 THEN 14670
14660 MOVE Maxt*Kpno,Inial(1,Mnh)
14670 PLOT Maxt*Kpno+Tim,Z1,-1
14680 Kmnj=Z1
14690 Kplo=Kplo+1
14700 RETURN
14710 PENUP
14720 SUBEND
```

APPENDIX C

SPECIFICATIONS AND CIRCUIT DIAGRAMS OF THE

THYRISTOR CONTROLLED AND THE PWM D.C. SERVO MOTOR

C.1 Specification of Controller

With the aid of LUCAS Control Systems Ltd., we were able to carry out an accurate testing of their servo motor. The complete servo drive Hyper-Loop for the feed drive of N.C. or C.N.C. machine tools consists of the following; a) three phase thyristor or pwm power amplifier; b) appropriate firing circuits, c) a low speed D.C. electric motor; d) a transformer, and e) choke coils.

The controller is of the panel-mounted type. On the panel (dimensions 485 x 610 mm, height of frame for two axes 245 mm, for three axes 270 mm and a recommended depth of casing 305 mm) there are all control circuits, a stabilized power source, a contactor for connecting the power circuits, a thermal protection relay, thyristor fuses and guard circuits. The transformer and the choke coils (two for each axis) are fitted separately.

A schematic circuit diagram of the power amplifier and controller are shown in Figs.C.1 and C.2 respectively. For velocity feedback, a tacho generator is built in the servo motor. The output voltage of this tacho generator is used as a velocity feedback.

The position transducer is the resolver type and its location is optional. A resolver is usually mounted at the end shaft of the motor. In our experiment we placed the position transducer at the load side, since the accuracy of the load is more important. A circuit diagram of the position amplifier is shown in Fig. C.3.

It can be seen from this graph that a 2 kHz sine and cosine wave are supplied to the resolver. The output from the resolver is a sine wave whose phase angle with one of the reference signals depends on the position of the rotor of the resolver. In this manner one mechanical revolution is transformed to 360° of an electrical phase angle.

The phase difference between input pulses and feedback pulses are measured by a discriminator. The output pulses of the discriminator are fed to a digital-analogue convertor. The output voltage is proportional to the width of the pulses. With this technique each input pulse represents one step rotation of the shaft of the motor.

For further details refer to the circuit diagrams of the controller of D.C. servo motor (Lucas Control System Ltd.). The specification of the D.C. motor under test is given in C.2. The low-speed motors Hyper-Loop are special multi-pole D.C. machines regulated by the armature voltage with excitation by ceramic permanent magnets. These motors are available in three versions:

1. TENV - totally enclosed non-ventilated
2. TEAO - totally enclosed, air over
3. BV - blower ventilated

The motor under test was of the first type. The purpose of these tests on the motors was to substantiate the mathematical model and to investigate their suitability for high performance applications.

The specifications of the pwm drive are similar to the thyristor controlled one. The only difference is in the type of power amplifier. It could employ either thyristors or transistors. The principle circuit diagrams of these systems are shown in Fig. 4.8. For further details refer to the manufacturers' circuit diagram.

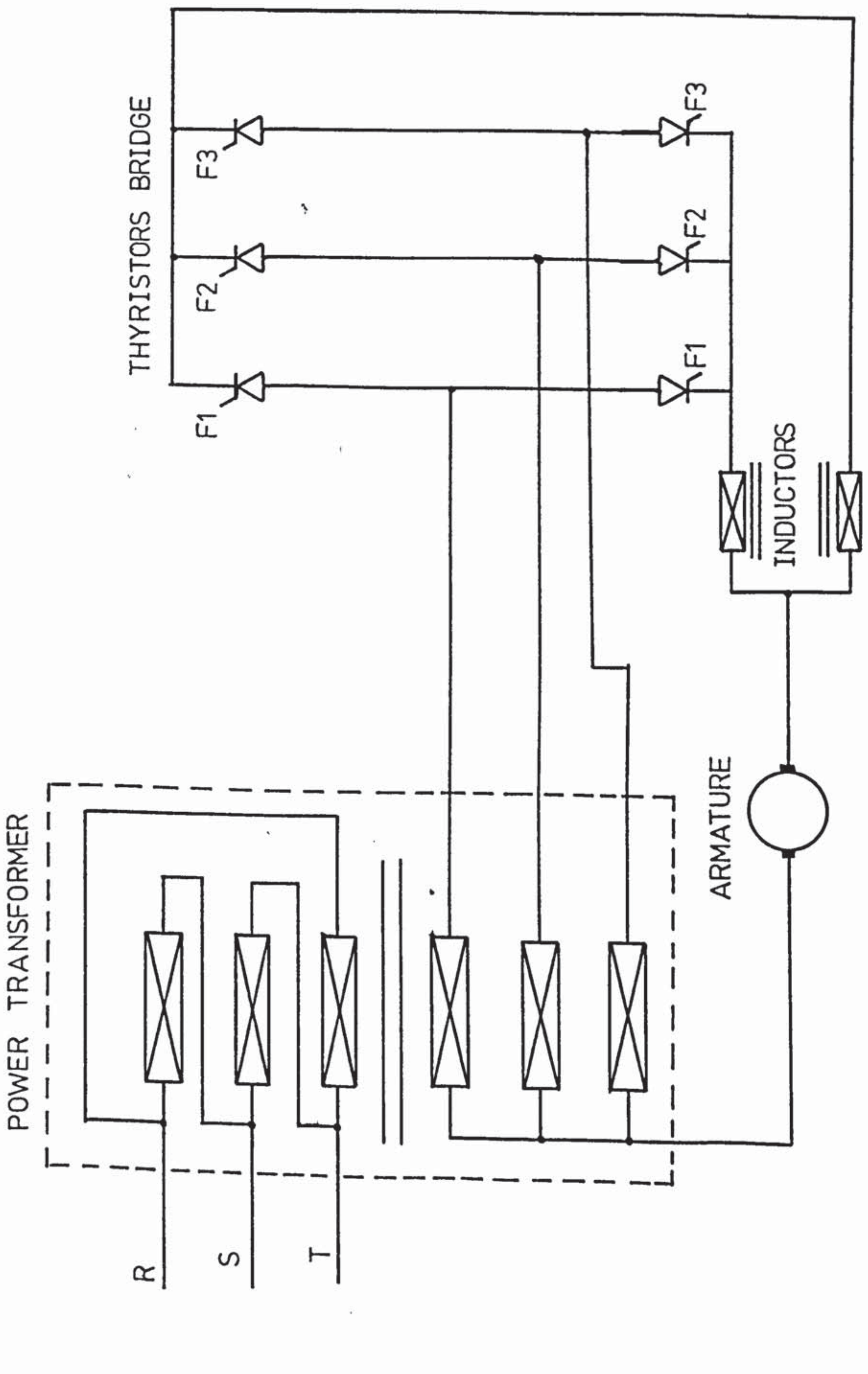


Fig C.1. Wiring diagram of thyristor power amplifier

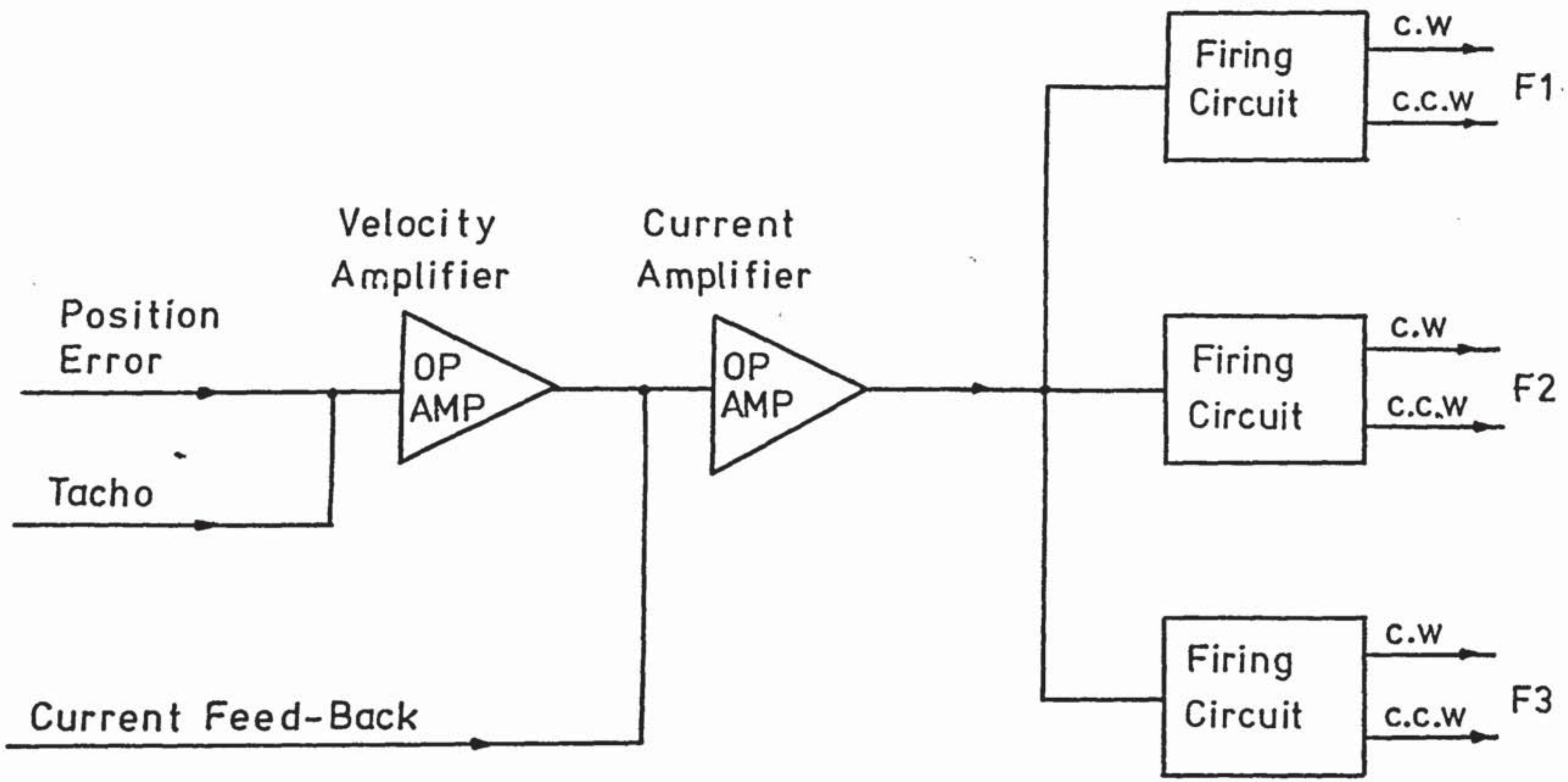


Fig C.2. Wiring diagram of firing circuit.

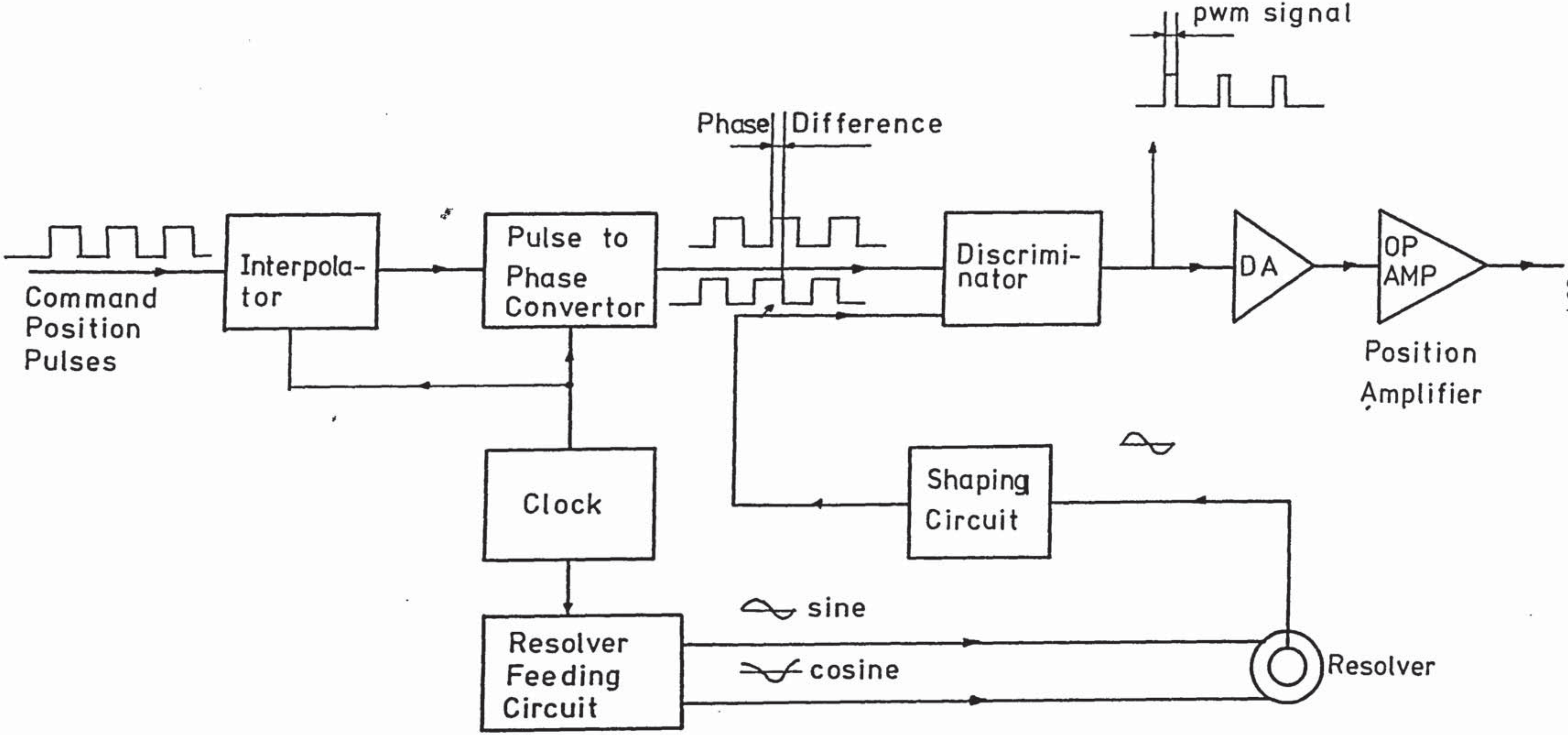


Fig C.3. Circuit diagram of digital position amplifier.

C.2 Specification of D.C. motor under Test

The motor under test with thyristor controlled drive had the following specifications:

Rated torque	17.5 (N.m)
Maximum limit torque	132 (N.m)
Electrical time constant	2.45 (msec)
Mechanical time constant	11.4 (msec)
Torque constant	0.83 (N.m/amp)
Voltage constant	0.83 (volts/(rad/sec))
Arm resistance less brushes	0.36 (ohms)
Arm inductance	0.88 (mhenry)
Maximum velocity	1420 (rpm)
Motor weight	34 Kg
Model	LB212/10 Lucas Control Systems.

The model under test with pwm drive had the following specifications:

Rated torque	6.8 (N.m)
Maximum limit torque	60 (N.m)
Electrical time constant	(2.8 (msec)
Mechanical time constant	10 (msec)
Torque constant	0.7 (N.m/amp)
Voltage constant	0.7 (volts/(rad/sec))
Arm resistance less brushes	2.1 (ohms)
Arm inductance	5.88 (mhenry)
Maximum velocity	1500 (rpm)
Motor weight	12.7 Kg
Model	407-1514 Lucas Control System.

APPENDIX D

TRANSFORMATION OF A MATHEMATICAL MODEL

COMPATIBLE TO THE COMPUTER PROGRAM PACKAGE

D. Transformation of Mathematical Model Compatible to the Computer Program Package

The dynamic equations derived in Chapter 3 can be summarized as:

$$\frac{V_p}{\theta_i - \theta_o} = \frac{K_1}{1 + \tau_1 s} \tag{1}$$

$$\frac{V_v}{V_p - K_m\omega_m} = \frac{K_2 + K_3 s}{s} \tag{2}$$

$$\frac{V_c}{V_v - K_c I} = \frac{K_4(1 + \tau_2 s)}{1 + \tau_3 s} \tag{3}$$

$$V = Kg''_c \tag{4}$$

$$V = RI + LSI + C_m\omega_m \tag{5}$$

$$T_m = K_t\, I \tag{6}$$

$$T_m = I_m s\omega_m + T_s$$

$$T_s = \frac{K_s}{N}\left(\frac{\theta_m}{N} - \theta_o\right) \tag{8}$$

$$NT_s - T_L = I_L s^2\theta_o + C_L s\theta_o \tag{9}$$

The above nine differential equations are the governing dynamic characteristic of a servo motor drive system. The above equations may be rearranged by separating the input and output variables as follows:

$$V_p + \tau_1 sV_p + K_1\theta_o = K_1\theta_i \tag{10}$$

$$sV_v + K_3K_m s\omega_m - K_3 sV_p + K_2K_m\omega_m - K_2V_p = 0 \tag{11}$$

$$\tau_3 sV_c + V_c - K_4V_v + K_4K_cI - K_4\tau_2 sV_v + K_4\tau_2 K_c sI = 0 \qquad (12)$$

$$V - KgV_c = 0 \qquad (13)$$

$$V - RI - LsI - C_m\omega_m = 0 \qquad (14)$$

$$T_m - K_T I = 0$$

$$T_m - I_m s\omega_m - T_s = 0 \qquad (15)$$

$$NT_s - \frac{K_s}{N}\theta_m + K_s\theta_o = 0 \qquad (16)$$

$$NT_s - I_L s^2\theta_o - C_L s\theta_o = T_L \qquad (17)$$

By introducing new variables for all higher derivative of the output variables, the complete mathematical model may be written in matrix form as:

$$AY = BX \qquad (18)$$

where, y and x are the vectors containing the output and input variables respectively.

A and B are the coefficients matrices.

Matrix equation (18) in the expanded form is shown in the next page.

By inputing the matrices A and B in symbolic form in the program, the transfer function of any output variable with respect to any input variable will be calculated in symbolic form. In order to proceed with numerical calculation a set of equations, as was shown in Appendix B, to determine the values of matrices A and B must be entered. A typical print out of the transfer functions of θ_o and I with respect to θ_i are shown in the following pages.

θ_o	V_p	ω_m	V_v	V_c	I	V	T_m	T_s	θ_m	$S\theta_o$	SV_p	$S\omega_m$	SV_v	SV_c	SI	$S^2\theta_o$	$S^2\omega_m$	$S^3\theta_o$
K_1	1										τ_1							
	$-K_2$	K_2K_m									$-K_3$	K_3K_m	1				B	C
			$-K_4$	1	K_4K_c								$K_4\tau_2$	τ_3	A			
				$-K_g$		1									$-L$			
		$-C_m$			$-R$	1												
					$-K_t$		1											
							1	-1				$-I_m$						
K_s								N	$\frac{-K_s}{N}$									
								N		$-C_L$						$-I_L$		
S										-1								
	S										-1							
		S										-1						
			S										-1					
				S										-1				
					S										-1			
										S						-1		
												S					-1	
																S		-1
		-1							S									

θ_i	T_L
K_1	
	1

where,
$A = K_4 \tau_2 K_c$

Transformation of mathematical model. Any symbols may be usd in matrices as shown in the following pages.

Sample Computer Print Out

THE SYSTEM COEFFICIENT MATRIX IS :

F	GH	M	IJ	0	HG	0	H	0	0	0	0	0	GF	KN	0	1	0	0
Ø	0	0	0	HJ	0	KJ	0	1	0	0	0	0	0	0	K	K	KJH	0
Ø	0	0	0	0	0	0	0	J	0	1	0	0	0	0	0	0	0	0
Ø	0	JK	0	JHJK	0	0	0	0	0	1	0	0	0	0	K	0	0	0
Ø	0	0	0	JKJK	0	0	0	0	0	0	1	0	0	0	0	0	0	0
Ø	0	0	0	0	0	0	0	0	0	0	1	-1	0	KKJ	0	0	0	0
NMN	0	KJ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	NM
J	0	0	0	0	0	0	0	0	0	0	0	M	K	0	0	0	0	0
S	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0
Ø	0	S	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0
Ø	0	0	0	S	0	0	0	0	0	0	0	0	0	0	-1	0	0	0
Ø	0	0	0	0	0	S	0	0	0	0	0	0	0	0	0	-1	0	0
Ø	0	0	0	0	0	0	0	S	0	0	0	0	0	0	0	0	-1	0
Ø	0	0	0	0	0	0	0	0	0	0	0	S	0	0	0	0	0	-1
Ø	-1	0	0	0	0	0	0	0	0	0	0	0	S	0	0	0	0	0
Ø	0	0	-1	0	0	0	S	0	0	0	0	0	0	0	0	0	0	0
Ø	0	0	0	0	-1	0	0	0	0	0	0	0	0	S	0	0	0	0
Ø	0	0	K	0	0	0	1	0	M	0	0	0	0	0	0	0	0	0
-1	0	0	0	0	0	0	0	0	S	0	0	0	0	0	0	0	0	0

THE COLUMN B IS

Ø	0
Ø	0
Ø	0
Ø	0
Ø	0
Ø	0
Ø	0
Ø	1
Ø	0
Ø	0
Ø	0
Ø	0
Ø	0
Ø	0
Ø	0
Ø	0
Ø	0
M	0
Ø	0

THE MATRIX S STORED IS FOR THE CHARACTERISTIC POLYNOMINAL :

```
1 01   7 02   5 03   2 04   3 05   8 08      1   -1 00
1 01   7 02   5 03   2 04   3 05   8 08  18 16       2  -1 00
1 01   7 02   5 03   3 05   8 08   2 12      2   -1 00
1 01   7 02   5 03   3 05   8 08   2 12  18 16       3  -1 00
7 01   1 02   5 03   2 04   3 05   8 08      1    1 00
7 01   1 02   5 03   2 04   3 05   8 08  18 16       2   1 00
7 01   1 02   5 03   3 05   8 08   2 12      2    1 00
7 01   1 02   5 03   3 05   8 08   2 12  18 16       3   1 00
7 01   4 02   5 03   8 08      2  -1 00
7 01   4 02   5 03   8 08  18 16      3  -1 00
7 01   4 02   5 03   8 08   2 13      3  -1 00
7 01   4 02   5 03   8 08   2 13  18 16      4  -1 00
7 01   2 03   3 05   8 08   6 10      3  -1 00
7 01   2 03   3 05   8 08   6 10  18 16      4  -1 00
7 01   4 03   8 08   6 10      3  -1 00
7 01   4 03   8 08   6 10  18 16      4  -1 00
7 01   4 03   8 08   6 10   2 13      4  -1 00
7 01   4 03   8 08   6 10   2 13  18 16      5  -1 00
7 01   5 03   2 04   3 05   8 08   1 10      2    1 00
7 01   5 03   2 04   3 05   8 08   1 10  18 16       3   1 00
7 01   5 03   2 04   3 05   8 08   1 17      3    1 00
7 01   5 03   2 04   3 05   8 08  18 16   1 17       4   1 00
7 01   5 03   3 05   8 08   1 10   2 12      3    1 00
7 01   5 03   3 05   8 08   1 10   2 12  18 16       4   1 00
7 01   5 03   3 05   8 08   2 12   1 17      4    1 00
7 01   5 03   3 05   8 08   2 12  18 16   1 17       5   1 00
7 01   8 08   6 10   4 11      4  -1 00
7 01   8 08   6 10   4 11  18 16      5  -1 00
7 01   3 05   8 08   6 10   2 11      4  -1 00
7 01   3 05   8 08   6 10   2 11  18 16      5  -1 00
7 01   8 08   6 10   4 11   2 13      5  -1 00
7 01   8 08   6 10   4 11   2 13  18 16      6  -1 00
8 01   1 02   5 03   2 04   3 05   7 14      2   -1 00
8 01   1 02   5 03   2 04   3 05   7 14  18 16       3  -1 00
8 01   1 02   5 03   3 05   2 12   7 14      3   -1 00
8 01   1 02   5 03   3 05   2 12   7 14  18 16       4  -1 00
8 01   4 02   5 03   7 14      3   1 00
8 01   4 02   5 03   7 14  18 16      4   1 00
8 01   4 02   5 03   2 13   7 14      4   1 00
8 01   4 02   5 03   2 13   7 14  18 16      5   1 00
8 01   7 02   2 03   3 05      2   1 00
8 01   7 02   2 03   3 05  18 16      3   1 00
8 01   7 02   4 03      2   1 00
8 01   7 02   4 03  18 16      3   1 00
8 01   7 02   4 03   2 13      3   1 00
8 01   7 02   4 03   2 13  18 16      4   1 00
8 01   7 02   4 11      3   1 00
8 01   7 02   4 11  18 16      4   1 00
8 01   7 02   3 05   2 11      3   1 00
8 01   7 02   3 05   2 11  18 16      4   1 00
8 01   7 02   4 11   2 13      4   1 00
8 01   7 02   4 11   2 13  18 16      5   1 00
8 01   2 03   3 05   6 10   7 14      4   1 00
8 01   2 03   3 05   6 10   7 14  18 16      5   1 00
8 01   4 03   6 10   7 14      4   1 00
8 01   4 03   6 10   7 14  18 16      5   1 00
8 01   4 03   6 10   2 13   7 14      5   1 00
8 01   4 03   6 10   2 13   7 14  18 16      6   1 00
8 01   5 03   2 04   3 05   1 10   7 14      3   -1 00
8 01   5 03   2 04   3 05   1 10   7 14  18 16       4  -1 00
8 01   5 03   2 04   3 05   7 14   1 17      4   -1 00
8 01   5 03   2 04   3 05   7 14  18 16   1 17       5  -1 00
```

```
 8 01    5 03    3 05    1 10    2 12    7 14       4   -1 00
 8 01    5 03    3 05    1 10    2 12    7 14   18 16       5   -1 00
 8 01    5 03    3 05    2 12    7 14    1 17       5   -1 00
 8 01    5 03    3 05    2 12    7 14   18 16    1 17       6   -1 00
 8 01    6 10    4 11    7 14       5    1 00
 8 01    6 10    4 11    7 14   18 16       6    1 00
 8 01    3 05    6 10    2 11    7 14       5    1 00
 8 01    3 05    6 10    2 11    7 14   18 16       6    1 00
 8 01    6 10    4 11    2 13    7 14       6    1 00
 8 01    6 10    4 11    2 13    7 14   18 16       7    1 00
 1 02    5 03    2 04    3 05    8 09    7 14       3   -1 00
 1 02    5 03    2 04    3 05    8 09    7 14   18 16       4   -1 00
 1 02    5 03    3 05    8 09    2 12    7 14       4   -1 00
 1 02    5 03    3 05    8 09    2 12    7 14   18 16       5   -1 00
 4 02    5 03    8 09    7 14       4    1 00
 4 02    5 03    8 09    7 14   18 16       5    1 00
 4 02    5 03    8 09    2 13    7 14       5    1 00
 4 02    5 03    8 09    2 13    7 14   18 16       6    1 00
 7 02    2 03    3 05    8 09       3    1 00
 7 02    2 03    3 05    8 09   18 16       4    1 00
 7 02    4 03    8 09       3    1 00
 7 02    4 03    8 09   18 16       4    1 00
 7 02    4 03    8 09    2 13       4    1 00
 7 02    4 03    8 09    2 13   18 16       5    1 00
 7 02    5 03    2 04    3 05    8 08    1 09       2   -1 00
 7 02    5 03    2 04    3 05    8 08    1 09   18 16       3   -1 00
 7 02    5 03    2 04    3 05    8 08    1 15       3   -1 00
 7 02    5 03    2 04    3 05    8 08    1 15   18 16       4   -1 00
 7 02    5 03    3 05    8 08    1 09    2 12       3   -1 00
 7 02    5 03    3 05    8 08    1 09    2 12   18 16       4   -1 00
 7 02    5 03    3 05    8 08    2 12    1 15       4   -1 00
 7 02    5 03    3 05    8 08    2 12    1 15   18 16       5   -1 00
 7 02    8 09    4 11       4    1 00
 7 02    8 09    4 11   18 16       5    1 00
 7 02    3 05    8 09    2 11       4    1 00
 7 02    3 05    8 09    2 11   18 16       5    1 00
 7 02    8 09    4 11    2 13       5    1 00
 7 02    8 09    4 11    2 13   18 16       6    1 00
 2 03    3 05    8 09    6 10    7 14       5    1 00
 2 03    3 05    8 09    6 10    7 14   18 16       6    1 00
 4 03    8 09    6 10    7 14       5    1 00
 4 03    8 09    6 10    7 14   18 16       6    1 00
 4 03    8 09    6 10    2 13    7 14       6    1 00
 4 03    8 09    6 10    2 13    7 14   18 16       7    1 00
 5 03    2 04    3 05    8 09    1 10    7 14       4   -1 00
 5 03    2 04    3 05    8 09    1 10    7 14   18 16       5   -1 00
 5 03    2 04    3 05    8 09    7 14    1 17       5   -1 00
 5 03    2 04    3 05    8 09    7 14   18 16    1 17       6   -1 00
 5 03    3 05    8 09    1 10    2 12    7 14       5   -1 00
 5 03    3 05    8 09    1 10    2 12    7 14   18 16       6   -1 00
 5 03    3 05    8 09    2 12    7 14    1 17       6   -1 00
 5 03    3 05    8 09    2 12    7 14   18 16    1 17       7   -1 00
 8 09    6 10    4 11    7 14       6    1 00
 8 09    6 10    4 11    7 14   18 16       7    1 00
 3 05    8 09    6 10    2 11    7 14       6    1 00
 3 05    8 09    6 10    2 11    7 14   18 16       7    1 00
 8 09    6 10    4 11    2 13    7 14       7    1 00
 8 09    6 10    4 11    2 13    7 14   18 16       8    1 00
 7 02    5 03    2 04    3 05    8 08    1 16   18 19       1    1 00
 7 02    5 03    2 04    3 05    8 08    1 18   18 19       0    1 00
 7 02    5 03    3 05    8 08    2 12    1 16   18 19       2    1 00
 7 02    5 03    3 05    8 08    2 12    1 18   18 19       1    1 00
99 99
THE MAXIMUM ORDER OF THE CHARACTERISTIC POLYNOMIAL IS:   8
THE NUMBER OF ELEMENTS IN THE ARRAY S IS: 935
```

```
THE GENERAL CHARACTERISTIC POLYNOMIAL (IN SYMBOLIC FORM) IS :

ORDER =  0 (i.e. S^ 0 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :

( 1) * (M) * (H) * (M) * (J) * (KJ) * (JKJK) * (KJ)

ORDER =  1 (i.e. S^ 1 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :

(-1) * (M) * (J) * (KJ) * (JKJK) * (KJ) * (F)
( 1) * (M) * (J) * (KJ) * (JKJK) * (M) * (NMN)
( 1) * (M) * (IJ) * (M) * (J) * (KJ) * (JKJK) * (KJ)
( 1) * (M) * (H) * (K) * (M) * (J) * (JKJK) * (KJ)

ORDER =  2 (i.e. S^ 2 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :

(-1) * (K) * (M) * (J) * (KJ) * (JKJK) * (KJ) * (F)
(-1) * (K) * (M) * (J) * (JKJK) * (KJ) * (F)
( 1) * (K) * (M) * (J) * (KJ) * (JKJK) * (M) * (NMN)
( 1) * (K) * (M) * (J) * (JKJK) * (M) * (NMN)
(-1) * (M) * (JKJK) * (JK) * (NMN)
( 1) * (KN) * (M) * (J) * (KJ) * (JKJK) * (NMN)
(-1) * (NN) * (J) * (KJ) * (JKJK) * (M) * (J)
( 1) * (J) * (HJ) * (KJ) * (J)
( 1) * (JHJK) * (KJ) * (J)
(-1) * (GF) * (M) * (J) * (KJ) * (JKJK) * (KJ)
( 1) * (M) * (IJ) * (K) * (M) * (J) * (JKJK) * (KJ)

ORDER =  3 (i.e. S^ 3 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :

(-1) * (K) * (K) * (M) * (J) * (JKJK) * (KJ) * (F)
( 1) * (K) * (K) * (M) * (J) * (JKJK) * (M) * (NMN)
(-1) * (K) * (M) * (JKJK) * (JK) * (NMN)
(-1) * (KJH) * (M) * (JKJK) * (JK) * (NMN)
(-1) * (KKJ) * (M) * (J) * (HJ) * (NMN)
(-1) * (KKJ) * (M) * (JHJK) * (NMN)
( 1) * (K) * (KN) * (M) * (J) * (KJ) * (JKJK) * (NMN)
( 1) * (HG) * (M) * (J) * (KJ) * (JKJK) * (NMN)
( 1) * (K) * (KN) * (M) * (J) * (JKJK) * (NMN)
(-1) * (K) * (NN) * (J) * (KJ) * (JKJK) * (M) * (J)
(-1) * (NN) * (K) * (J) * (JKJK) * (M) * (J)
( 1) * (NN) * (JKJK) * (JK) * (J)
( 1) * (K) * (J) * (HJ) * (KJ) * (J)
( 1) * (K) * (JHJK) * (KJ) * (J)
( 1) * (KJH) * (JHJK) * (KJ) * (J)
( 1) * (K) * (KJ) * (J)
( 1) * (K) * (J) * (KJ) * (J)
(-1) * (NN) * (KN) * (J) * (KJ) * (JKJK) * (J)
(-1) * (NN) * (K) * (J) * (KJ) * (JKJK) * (M)
( 1) * (K) * (J) * (HJ) * (KJ)
( 1) * (K) * (JHJK) * (KJ)
(-1) * (K) * (GF) * (M) * (J) * (KJ) * (JKJK) * (KJ)
(-1) * (GH) * (M) * (J) * (KJ) * (JKJK) * (KJ)
(-1) * (K) * (GF) * (M) * (J) * (JKJK) * (KJ)

ORDER =  4 (i.e. S^ 4 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :
```

```
(-1) * (K) * (KJH) * (M) * (JKJK) * (JK) * (NMN)
(-1) * (K) * (KKJ) * (M) * (J) * (HJ) * (NMN)
(-1) * (K) * (KKJ) * (M) * (JHJK) * (NMN)
(-1) * (KJH) * (KKJ) * (M) * (JHJK) * (NMN)
( 1) * (HG) * (K) * (M) * (J) * (KJ) * (JKJK) * (NMN)
( 1) * (K) * (K) * (KN) * (M) * (J) * (JKJK) * (NMN)
( 1) * (HG) * (K) * (M) * (J) * (JKJK) * (NMN)
(-1) * (K) * (KKJ) * (M) * (NMN)
(-1) * (K) * (KKJ) * (M) * (J) * (NMN)
(-1) * (K) * (NN) * (K) * (J) * (JKJK) * (M) * (J)
( 1) * (K) * (NN) * (JKJK) * (JK) * (J)
( 1) * (NN) * (KJH) * (JKJK) * (JK) * (J)
( 1) * (K) * (KJH) * (JHJK) * (KJ) * (J)
( 1) * (K) * (K) * (KJ) * (J)
( 1) * (K) * (K) * (J) * (KJ) * (J)
( 1) * (KJH) * (K) * (KJ) * (J)
( 1) * (NN) * (KKJ) * (J) * (HJ) * (J)
( 1) * (NN) * (KKJ) * (JHJK) * (J)
(-1) * (K) * (NN) * (KN) * (J) * (KJ) * (JKJK) * (J)
(-1) * (HG) * (NN) * (J) * (KJ) * (JKJK) * (J)
(-1) * (NN) * (K) * (KN) * (J) * (JKJK) * (J)
(-1) * (K) * (NN) * (K) * (J) * (KJ) * (JKJK) * (M)
(-1) * (NN) * (K) * (K) * (J) * (JKJK) * (M)
( 1) * (NN) * (K) * (JKJK) * (JK)
( 1) * (K) * (K) * (J) * (HJ) * (KJ)
( 1) * (K) * (K) * (JHJK) * (KJ)
( 1) * (KJH) * (K) * (JHJK) * (KJ)
(-1) * (K) * (GH) * (M) * (J) * (KJ) * (JKJK) * (KJ)
(-1) * (K) * (K) * (GF) * (M) * (J) * (JKJK) * (KJ)
(-1) * (GH) * (K) * (M) * (J) * (JKJK) * (KJ)
( 1) * (K) * (K) * (KJ)
( 1) * (K) * (K) * (J) * (KJ)
(-1) * (NN) * (KN) * (K) * (J) * (KJ) * (JKJK)

ORDER =  5 (i.e. S^ 5 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :

(-1) * (K) * (KJH) * (KKJ) * (M) * (JHJK) * (NMN)
( 1) * (HG) * (K) * (K) * (M) * (J) * (JKJK) * (NMN)
(-1) * (K) * (K) * (KKJ) * (M) * (NMN)
(-1) * (K) * (K) * (KKJ) * (M) * (J) * (NMN)
(-1) * (KJH) * (K) * (KKJ) * (M) * (NMN)
( 1) * (K) * (NN) * (KJH) * (JKJK) * (JK) * (J)
( 1) * (K) * (KJH) * (K) * (KJ) * (J)
( 1) * (K) * (NN) * (KKJ) * (J) * (HJ) * (J)
( 1) * (K) * (NN) * (KKJ) * (JHJK) * (J)
( 1) * (NN) * (KJH) * (KKJ) * (JHJK) * (J)
(-1) * (HG) * (K) * (NN) * (J) * (KJ) * (JKJK) * (J)
(-1) * (K) * (NN) * (K) * (KN) * (J) * (JKJK) * (J)
(-1) * (HG) * (NN) * (K) * (J) * (JKJK) * (J)
( 1) * (NN) * (K) * (KKJ) * (J)
( 1) * (NN) * (K) * (KKJ) * (J) * (J)
(-1) * (K) * (NN) * (K) * (K) * (J) * (JKJK) * (M)
( 1) * (K) * (NN) * (K) * (JKJK) * (JK)
( 1) * (NN) * (KJH) * (K) * (JKJK) * (JK)
( 1) * (K) * (KJH) * (K) * (JHJK) * (KJ)
(-1) * (K) * (GH) * (K) * (M) * (J) * (JKJK) * (KJ)
( 1) * (K) * (K) * (K) * (KJ)
( 1) * (K) * (K) * (K) * (J) * (KJ)
( 1) * (KJH) * (K) * (K) * (KJ)
( 1) * (NN) * (KKJ) * (K) * (J) * (HJ)
( 1) * (NN) * (KKJ) * (K) * (JHJK)
(-1) * (K) * (NN) * (KN) * (K) * (J) * (KJ) * (JKJK)
(-1) * (HG) * (NN) * (K) * (J) * (KJ) * (JKJK)
(-1) * (NN) * (K) * (KN) * (K) * (J) * (JKJK)

ORDER =  6 (i.e. S^ 6 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :
```

```
(-1) * (K) * (KJH) * (K) * (KKJ) * (M) * (NMN)
( 1) * (K) * (NN) * (KJH) * (KKJ) * (JHJK) * (J)
(-1) * (HG) * (K) * (NN) * (K) * (J) * (JKJK) * (J)
( 1) * (K) * (NN) * (K) * (KKJ) * (J)
( 1) * (K) * (NN) * (K) * (KKJ) * (J) * (J)
( 1) * (NN) * (KJH) * (K) * (KKJ) * (J)
( 1) * (K) * (NN) * (KJH) * (K) * (JKJK) * (JK)
( 1) * (K) * (KJH) * (K) * (K) * (KJ)
( 1) * (K) * (NN) * (KKJ) * (K) * (J) * (HJ)
( 1) * (K) * (NN) * (KKJ) * (K) * (JHJK)
( 1) * (NN) * (KJH) * (KKJ) * (K) * (JHJK)
(-1) * (HG) * (K) * (NN) * (K) * (J) * (KJ) * (JKJK)
(-1) * (K) * (NN) * (K) * (KN) * (K) * (J) * (JKJK)
(-1) * (HG) * (NN) * (K) * (K) * (J) * (JKJK)
( 1) * (NN) * (K) * (KKJ) * (K)
( 1) * (NN) * (K) * (KKJ) * (K) * (J)

ORDER =  7 (i.e. S^ 7 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :

( 1) * (K) * (NN) * (KJH) * (K) * (KKJ) * (J)
( 1) * (K) * (NN) * (KJH) * (KKJ) * (K) * (JHJK)
(-1) * (HG) * (K) * (NN) * (K) * (K) * (J) * (JKJK)
( 1) * (K) * (NN) * (K) * (KKJ) * (K)
( 1) * (K) * (NN) * (K) * (KKJ) * (K) * (J)
( 1) * (NN) * (KJH) * (K) * (KKJ) * (K)

ORDER =  8 (i.e. S^ 8 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :

( 1) * (K) * (NN) * (KJH) * (K) * (KKJ) * (K)
```

```
THE MATRIX S STORED IS FOR THE NUMERATOR POLYNOMINAL OF 1 OUTPUT VARIABLE)

   18 01    7 02    5 03    2 04    3 05    8 08    1 16      2    1 00
   18 01    7 02    5 03    2 04    3 05    8 08    1 18      1    1 00
   18 01    7 02    5 03    3 05    8 08    2 12    1 16      3    1 00
   18 01    7 02    5 03    3 05    8 08    2 12    1 18      2    1 00
   99 99
THE MAXIMUM ORDER OF THE POLYNOMINAL(FOR NUMERATOR OF 1 OUTPUT VARIABLE) IS: 3
THE NUMBER OF ELEMENTS IN THE ARRAY S IS: 37

THE NUMERATOR POLYNOMINAL OF 1 OUTPUT VARIABLE  IN SYMBOLIC FORM IS: 3


ORDER =  0 (i.e. S^ 0 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :


ORDER =  1 (i.e. S^ 1 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :

(  1) * (H) * (M) * (J) * (KJ) * (JKJK) * (KJ) * (M)

ORDER =  2 (i.e. S^ 2 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :

(  1) * (IJ) * (M) * (J) * (KJ) * (JKJK) * (KJ) * (M)
(  1) * (H) * (K) * (M) * (J) * (JKJK) * (KJ) * (M)

ORDER =  3 (i.e. S^ 3 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :

(  1) * (IJ) * (K) * (M) * (J) * (JKJK) * (KJ) * (M)
```

```
THE MATRIX S STORED IS FOR THE NUMERATOR POLYNOMINAL OF 2 OUTPUT VARIABLE)

   7 01   18 03   2 04   3 05   8 08   6 10   1 16      3  -1 00
   7 01   18 03   2 04   3 05   8 08   6 10   1 18      2  -1 00
   7 01   18 03   3 05   8 08   6 10   2 12   1 16      4  -1 00
   7 01   18 03   3 05   8 08   6 10   2 12   1 18      3  -1 00
   8 01    7 02  18 03   2 04   3 05   1 16      2   1 00
   8 01    7 02  18 03   2 04   3 05   1 18      1   1 00
   8 01    7 02  18 03   3 05   2 12   1 16      3   1 00
   8 01    7 02  18 03   3 05   2 12   1 18      2   1 00
   8 01   18 03   2 04   3 05   6 10   7 14   1 16      4   1 00
   8 01   18 03   2 04   3 05   6 10   7 14   1 18      3   1 00
   8 01   18 03   3 05   6 10   2 12   7 14   1 16      5   1 00
   8 01   18 03   3 05   6 10   2 12   7 14   1 18      4   1 00
   7 02   18 03   2 04   3 05   8 09   1 16      3   1 00
   7 02   18 03   2 04   3 05   8 09   1 18      2   1 00
   7 02   18 03   3 05   8 09   2 12   1 16      4   1 00
   7 02   18 03   3 05   8 09   2 12   1 18      3   1 00
  18 03    2 04   3 05   8 09   6 10   7 14   1 16      5   1 00
  18 03    2 04   3 05   8 09   6 10   7 14   1 18      4   1 00
  18 03    3 05   8 09   6 10   2 12   7 14   1 16      6   1 00
  18 03    3 05   8 09   6 10   2 12   7 14   1 18      5   1 00
  99 99
THE MAXIMUM ORDER OF THE POLYNOMINAL(FOR NUMERATOR OF 2 OUTPUT VARIABLE) IS:
THE NUMBER OF ELEMENTS IN THE ARRAY S IS: 173

THE NUMERATOR POLYNOMINAL OF 2 OUTPUT VARIABLE  IN SYMBOLIC FORM IS: 6


ORDER =  0 (i.e. S^ 0 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :


ORDER =  1 (i.e. S^ 1 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :

( 1) * (H) * (J) * (KJ) * (M) * (KJ) * (J)

ORDER =  2 (i.e. S^ 2 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :

(-1) * (H) * (KKJ) * (M) * (J) * (KJ) * (M) * (NMN)
( 1) * (IJ) * (J) * (KJ) * (M) * (KJ) * (J)
( 1) * (H) * (K) * (J) * (M) * (KJ) * (J)
( 1) * (H) * (K) * (J) * (KJ) * (M) * (KJ)

ORDER =  3 (i.e. S^ 3 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :

(-1) * (IJ) * (KKJ) * (M) * (J) * (KJ) * (M) * (NMN)
(-1) * (H) * (K) * (KKJ) * (M) * (J) * (M) * (NMN)
( 1) * (IJ) * (K) * (J) * (M) * (KJ) * (J)
( 1) * (H) * (NN) * (KKJ) * (J) * (KJ) * (M) * (J)
( 1) * (IJ) * (K) * (J) * (KJ) * (M) * (KJ)
( 1) * (H) * (K) * (K) * (J) * (M) * (KJ)

ORDER =  4 (i.e. S^ 4 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :

(-1) * (IJ) * (K) * (KKJ) * (M) * (J) * (M) * (NMN)
( 1) * (IJ) * (NN) * (KKJ) * (J) * (KJ) * (M) * (J)
( 1) * (H) * (NN) * (K) * (KKJ) * (J) * (M) * (J)
( 1) * (IJ) * (K) * (K) * (J) * (M) * (KJ)
( 1) * (H) * (NN) * (KKJ) * (K) * (J) * (KJ) * (M)

ORDER =  5 (i.e. S^ 5 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :
```

```
( 1) * (IJ) * (NN) * (K) * (KKJ) * (J) * (M) * (J)
( 1) * (IJ) * (NN) * (KKJ) * (K) * (J) * (KJ) * (M)
( 1) * (H) * (NN) * (K) * (KKJ) * (K) * (J) * (M)

ORDER =  6 (i.e. S^ 6 term)
THE COEFFICIENT OF THIS TERM (IN SYMBOLIC FORM) :

( 1) * (IJ) * (NN) * (K) * (KKJ) * (K) * (J) * (M)
```

Sample Numerical Print Out

```
NEW Z IS:
Z[ 1 ] = 0
Z[ 2 ] = 1
Z[ 3 ] = 0
Z[ 4 ] = 0
Z[ 5 ] = 1
Z[ 6 ] = 0
Z[ 7 ] = .335
Z[ 8 ] = 0
Z[ 9 ] = 1
Z[ 10 ] = .11
Z[ 11 ] = .0015
Z[ 12 ] = 0
Z[ 13 ] = 2.7
Z[ 14 ] = .009
Z[ 15 ] = 1
Z[ 16 ] = 10000
Z[ 17 ] = 17
Z[ 18 ] = .1
Z[ 19 ] = 0
Z[ 20 ] = 0
Z[ 21 ] = 0
Z[ 22 ] = 1
```

COEFFICIENTS OF THE SPECIFIC POLYNOMIAL	DEGREE IN S
0.0000000000 E+00	0
-1.1000000000 E+02	1
-1.8748250000 E+04	2
-6.5234540990 E+03	3
-8.5487089515 E+01	4
-5.8680022500 E-03	5
-7.6882500000 E-05	6

THE NUMERATOR POLYNOMINAL OF 1 Th OUTPUT VARIABLE IS:

COEFFICIENTS OF THE SPECIFIC POLYNOMIAL	DEGREE IN S
0.0000000000 E+00	0
-2.7000000000 E+04	1
0.0000000000 E+00	2

The roots are:

Real Part	Imag Part	Zeta(Damping Ratio)
0.00000 E+00	0.00000 E+00	0.00000
-7.33333 E+01	0.00000 E+00	1.00000
-5.87924 E-03	0.00000 E+00	1.00000
-2.98507 E+00	0.00000 E+00	1.00000
-1.55628 E-06	1.05437 E+03	.00000
-1.55628 E-06	-1.05437 E+03	.00000

APPENDIX E

FREQUENCY ANALYSIS OF THE MASS-SPRING MECHANISM OF THE LOAD SYSTEM

E. Theoretical Frequency Analysis of the Mass-Spring Mechanism of the Load System.

A schematic diagram of the simulated load mechanism is shown below.

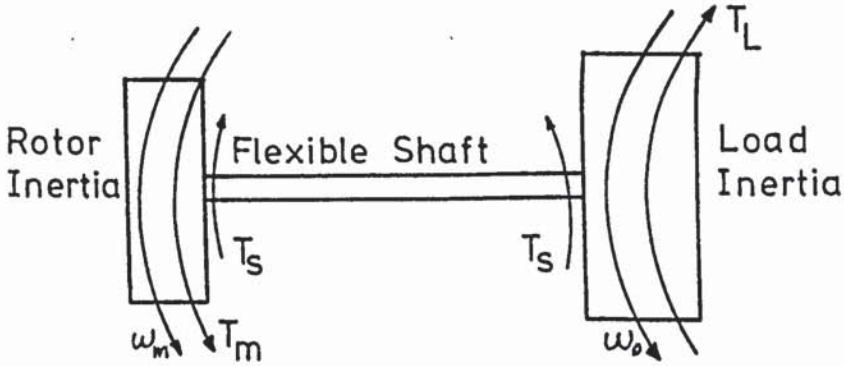


Fig. E.1 Simulation of the loading system.

The load mechanism can usually be simplified as an inertia of the rotor, I_m, a transmission shaft with stiffness K_s and the load inertia I_L.

To obtain the fundamental natural frequency the shaft may be assumed to be massless. However, for a more accurate evaluation of the natural frequency, its mass may be added to the load and rotor inertias. The dynamic equation of the rotor inertia can be written as:

$$T_m = I_m s\omega_m + T_s \tag{1}$$

where

$$T_s = K_s\left(\frac{\omega_m - \omega_o}{s}\right) \tag{2}$$

With an assumption of a dynamic friction of $C\,\omega_o$ the equation of the motion of the load inertia can be deducted as

$$T_s = I_L s\omega_o + C\,\omega_o + T_L \tag{3}$$

where T_L is the external torque.

As it was previously shown, the torque T_m is proportional to the current of the armature of the motor. Therefore,

$$T_m = K_\tau I \tag{4}$$

The transfer function of the output velocity with respect to the applied torque T_m and be obtained from equations 1 to 3 as:

$$\omega_m = \frac{(I_L s^2 + Cs + K_s)T_m - K_s T_L}{I_m I_L s^3 + I_m Cs^2 + (K_s I_m + K_s I_L)s + K_s C} \tag{5}$$

when there is no external torque T_L is equated to zero and this simplifies equation (5) to

$$\frac{\omega_m}{T_m} = \frac{(I_L s^2 + Cs + K_s)}{I_m I_L s^3 + I_m Cs^2 + (K_s I_m + K_s I_L)s + K_s C} \tag{6}$$

A similar relation for output velocity ω_o and the input torque T_m may also be obtained as

$$\frac{\omega_o}{T_m} = \frac{K_s}{I_m I_L s^3 + I_m Cs^2 + (K_s I_m + K_s I_L)s + K_s C} \tag{7}$$

In Chapter 4 it was shown that the current supplied to the motor was in pulse form. Therefore the torque developed by the motor is in pulse form (equation 4). The exact shapes of the pulses are dependent on many factors. However, to predict the response of the rotor and the load inertia to these torque pulsations a frequency analysis was carried out.

For a harmonic excitation with a frequency of ω the operator s may be substituted as $j\omega$ and equations 6 and 7 may be rearranged in terms of frequency ω. It can be shown that the undamped natural frequency can be evaluated as

$$\omega_n^2 = \frac{K_s(I_m + I_L)}{I_m I_L} = K_s\left(\frac{1}{I_m} + \frac{1}{I_L}\right)$$

It is evident from the above equations that the response of ω_m and ω_o are dependent on, a) excitation frequency, b) rotor and load inertia, and c) the stiffness of the shaft.

A typical frequency and phase angle response of the load inertia is shown in Fig. E.1 and Fig. E.2 respectively. It can be seen from this graph that the load will respond to the excitation force but the amplitude ratio reduces as the excitation frequency increases. The resonance frequency of the loading mechanism is also evident from this graph.

Table E.1 shows the amplitude ratio of the responses as a function of current frequency at various shaft stiffness and constant load inertia. Table E.2 shows these responses at various load inertia and constant shaft stiffness.

The measured oscillations of the motor to the current pulsation as shown in Chapter 4 were agreeable with these predictions.

When a thyristor controlled or a pwm servo motor drive system is used the following points must be considered.

1. The frequency of the current of the motor must not be close to the load natural frequencies or their harmonics.
2. Some oscillation will be transmitted and at low current frequency this will become significant. The major problem will be noticeable on the various transducers used.

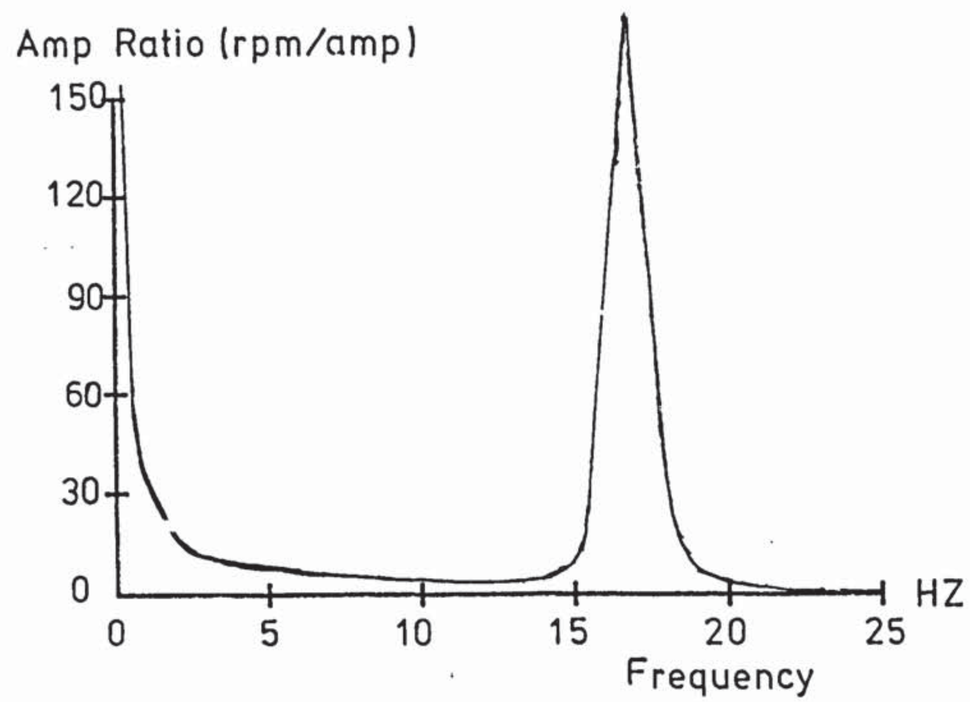


Fig E.1 . A typical frequency respnse of load structure.

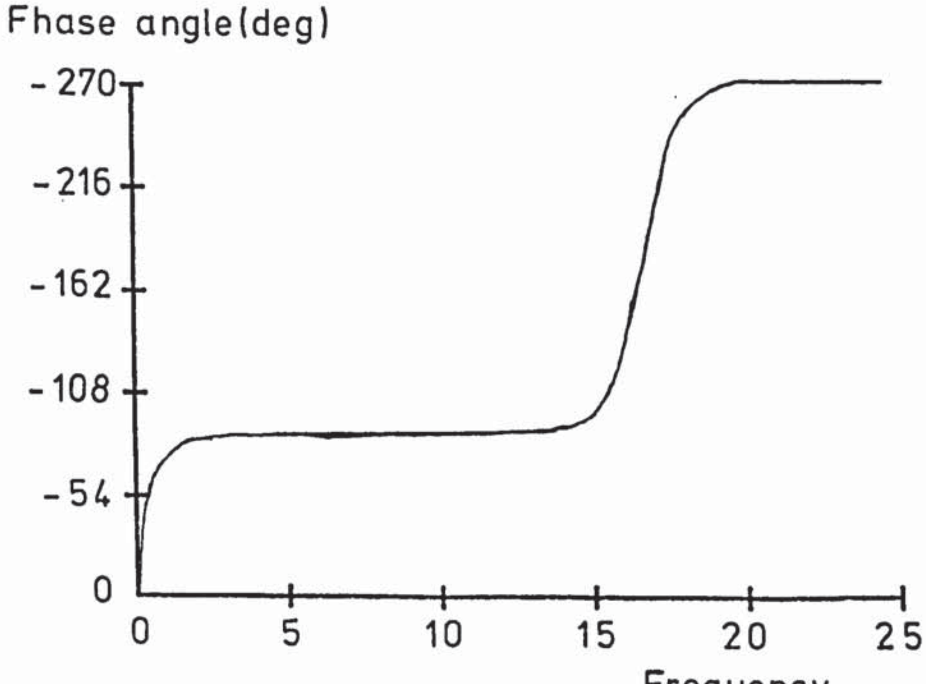


Fig E.2. A typical fhase angle of the load structure.

Current Frequency (Hz)	Load Nat. Freq. of 16 Hz. I_L=0.15, I_m=0.02 K_s = 1000		Load Nat. Freq. of 160 Hz. I_L=0.15, I_m=0.02 K_s = 100000		Load Nat. Freq. of 360 Hz. I_L=0.15, I_m=0.02 K_s = 500000	
	Amplitude Ratio (rpm/amp)					
	ω_o	ω_m	ω_o	ω_m	ω_o	ω_m
50	0.05	0.766	0.54	0.47	0.5	0.49
100	0.006	0.37	0.41	0.167	0.27	0.23
150	0.001	0.25	1.47	0.49	0.2	0.14
200	0.0008	0.185	0.21	0.29	0.18	0.095
250	0.0004	0.148	0.07	0.18	0.19	0.05
300	0.00023	0.12	0.032	0.14	0.29	0.019

Table E.1 Frequency response of load and rotor inertia at various natural frequency (by varying the shaft stiffness).

Current Frequency (Hz)	Load Nat. Freq. of 116 Hz. I_L=0.5, I_m=0.02 K_s = 100000		Load Nat. Freq. of 240 Hz. I_L=0.05, I_m=0.02 K_s = 100000		Load Nat. Freq. of 500 Hz. I_L=0.01, I_m=0.02 K_s = 100000	
	Amplitude Ratio (rpm/amp)					
	ω_o	ω_m	ω_o	ω_m	ω_o	ω_m
50	0.33	0.17	0.65	0.63	0.71	0.7
100	0.53	0.5	0.38	0.3	0.4	0.35
150	0.14	0.48	0.34	0.2	0.25	0.24
200	0.03	0.24	0.5	0.1	0.2	0.18
250	0.015	0.17	2.2	0.5	0.19	0.14
300	0.008	0.13	0.2	0.16	0.18	0.11

Table E.2 Frequency response of load and rotor inertia at various natural frequency (by varying the load inertia).

APPENDIX F

SPECIFICATION OF SERVO MOTORS

UNDER INVESTIGATION

F.1 D.C. Motors

Power Rating (kW)	Rotor Inertia ($Kg.m^2$)	Rated Velocity (rpm)	Maximum Torque Limit (N.m)	R (ohms)	L (henry)
1	0.0093	1260	113	0.58	0.0023
3	0.021	2500	113	0.12	0.0005
5	0.044	1200	230	0.16	0.00072
10	0.24	2500	800	0.025	0.00021

For further information about the specific characteristic of D.C. motors refer to catalogue of Hyper-Loop Lucas Control Systems Ltd., Torrington Avenue, Coventry CV4 9AJ.

F.2 A.C. Motors

Power Rating (kW)	Rotor Inertia ($Kg.m^2$)	Rated Velocity (rpm)	Maximum Torque Limit (N.m)
1	0.0034	1410	16
3	0.0115	1420	40
5	0.03	1440	80
10	0.072	1440	180

For further information about the specific characteristics of A.C. induction motors refer to catalogue of GEC Small Machine Ltd., Cakemore Road, Rowley Regis, Warley, West Midlands. B65 0QT

F.3 Stepping Motors

Power Rating (kW)	Rotor Inertia ($Kg.m^2$)	Rated Velocity (rpm)	Maximum Torque Limit (N.m)	R (ohms)	L (henry)
1	0.009	(3000max)	20	0.07	0.00125
2	0.009	(3000max)	40	0.11	0.0015

For further information for a specific stepping motor refer to the catalogue of GEC Small Machine Ltd., Cakemore Road, Rowley Regis, Warley, West Midlands. B65 0QT

F.4 Hydraulic Motor

Nominal displacement		4.55 (cm^3/rev)
Flow capacity		4.55 (litres/min)
Maximum velocity at full capacity	Peak	6000 (rpm)
	Continuous	4000
Maximum pressure	Peak	350 bar
	Continuous	280
Output torque (280 bar,3000 rpm)		17.5 N.m

For further information refer to catalogue of Commerical Shearing Inc., Industrial Hydraulic Division, Long Lane, Fazakerley, Liverpool L9 7BW.

APPENDIX G

PUBLISHED PAPER

APPENDIX H

THE CHOICE OF A SERVO MOTOR FOR A SPECIFIC APPLICATION

H.1 Introduction

It is problematic to choose an appropriate servo motor for a specific application. There are an increasing number of servo motors with different power ratings available and the designers are faced with the difficult choice of finding a suitable motor. Each motor has its advantages and disadvantages as already discussed.

There are many aspects to be taken into consideration when choosing the correct motor for a particular need. The major points are listed below

1) The motor must meet the requirements of:
    a) speed of response
    b) accuracy
    c) dynamic error due to an external disturbance
2) Capital cost
3) Reliability
4) Availability

The final decision of a motor, however, depends on the priority of the above points. In most high performance applications, the performance of the motor is the prime consideration, which is the concern of this chapter.

In this chapter, the results of this thesis will be put into graphs, so the designer can refer to them in order to readily obtain the settling time and dynamic error of the motor. He can repeat this calculation for all types of motors and make the final decision by comparing these results. The settling time and dynamic error of motors are given in Chapter 8 and will not be included in this chapter.

4.2 Procedure of Selection

Selection of a motor for a specific application is a trial and error procedure. The designer has to choose a power rating motor based on the power requirements of the load and then calculate its speed of response and dynamic error. He has to do the calculation for one of the types and then by referring to the graphs of the comparison, a prediction of the speed of response of the other types can be obtained. If the motor does not satisfy the requirements, the calculation must be repeated for a higher or lower power rating motor. It must be noted that a higher power rating motor sometimes reduces the speed of response, if the inertia of the rotor is more dominant than the load inertia (see Chapter 8 and Appendix G).

The overall settling time of the system consists of three parts and these are:

1) Dynamic settling time
2) Static settling time due to the load inertia
3) Static settling time due to the inertia of the rotor of the motor

and these may be written as:

$$T_a = T_d + T_s + T_s' \qquad (H.1)$$

where

T_a is the overall settling time

T_d is the dynamic settling time

T_s, T_s' are the static settling time of load and rotor inertia respectively.

The dynamic settling time of all types of motors were studied in detail in this thesis and they are summarized in Chapter 8.

The static settling time of the motors are usually obtainable from the manufacturers' data. The settling times due to a unit of load inertia and a unit velocity change are given in Figs. H.1 to H.6 for ceramic magnet, printed circuit, rare earth magnet, A.C., stepping and hydraulic motors respectively. Let the settling time given in the graphs at the power rating P be τ_s. If the load inertia is I_ℓ and the velocity change is ω then the static settling time can be obtained as:

$$T_s = I_\ell \times \omega \times \tau_s + T_s' \tag{H.2}$$

The settling time, T_s', due to the rotor of the motor can be obtained by referring to Fig. H.7. If the settling time from this graph be τ_s' at the power rating of P then T_s' can be calculated as:

$$T_s' = \omega \times \tau_s' \tag{H.3}$$

Substituting equation H.3 into H.2 yields

$$T_s = \omega(I_\ell \tau_s + \tau_s') \tag{H.4}$$

Therefore the overall settling time can be obtained as:

$$T_a = \omega(I_\ell \tau_s + \tau_s') + T_d \tag{H.5}$$

where

τ_s and τ_s' are obtained from Figs. H.1 to H.7.

T_d is obtained from the graphs in Chapter 8.

The settling time, T_a, provides a means of investigating whether a particular motor meets the requirements of speed of response.

The corresponding dynamic error due to an external torque can also be obtained from graphs in Chapter 8.

As can be seen from Figs. H.1 to H.6, the setting time due to the load inertia reduces as the power rating of the motor increases. The settling time due to the inertia of the rotor increases as the power rating increases as shown in Fig. H.7. The optimum value is obtained by trial and error procedure.

The overall settling time obtained from equation H.5 is approximate and neglects the friction and external torque. If there is friction or external torque during the transient response an additional settling time must be added to the equation H.5. If η% of the motor torque is used for external torque or friction, then the overall settling time can be obtained as:

$$T_a = (1 - \eta)[\omega(I_\ell \tau_s + \tau_s') + T_d] \tag{H.6}$$

H.3 A Worked Example

Let us assume that a designer wants to choose a motor to drive a load inertia of 0.1 Kg.m² from 0 speed to 1200 r.p.m. in one second.

Firstly the approximate power requirement can be calculated as:

$$T = I\alpha = 0.1\ (120) = 12\ \text{N.m}$$

$$\therefore \quad P = T\omega = 12 \times 120 = 1440\ \text{Watt} = 1.44\ \text{kW}$$

where

α is the average angular acceleration.

In order to select the type of the motor which provides the required speed of response, firstly the calculation of equation H.5 will be carried out for stepping motors. The calculation is first carried out for a 1 kW stepping motor. From Figs. H.5 and H.7 the setting time τ_s and τ_s' for a 1 kW motor can be obtained as

$$\tau_s = 0.0375 \text{ (sec)} \quad \text{and} \quad \tau_s' = 0.24 \times 10^{-3} \text{ (sec)}$$

Substituting these values in equation H.5 yields,

$$T_a = 40\pi(0.1 \times 0.0375 \times 0.24 \times 10^{-3}) + T_d = 0.48 + T_d \qquad \text{(H.7)}$$

It can be seen from equation H.7 that a 1 kW stepping motor will provide the required speed of response statically. The value of T_d can be obtained from Fig. 8.3 of Chapter 8 as

$$T_d \cong 2.2 \text{(sec)} \qquad \text{(H.8)}$$

Substituting (H.8) in to H.7 yields,

$$T_a \cong 2.68 \text{ (sec)} \qquad \text{(H.9)}$$

Therefore, by considering the dynamic and static settling time, a 1 kW stepping motor will not provide the necessary speed of response. It can be shown that even a larger stepping motor cannot provide the necessary speed of response.

Let us try the calculation for a 1 kW A.C. motor. τ_s and τ_s' can be obtained from Figs. H.4 and H.7 as:

$$\tau_s = 0.064 \text{ (sec)} \quad \text{and} \quad \tau_s' = 0.25 \times 10^{-3} \text{(sec)}$$

Therefore:

$$T_a = 0.798 + T_d \qquad \text{(H.10)}$$

T_d for a 1 kW A.C. motor from Fig. 8.1 can be obtained as:

$$T_d = 0.82 \text{ (sec)}$$

therefore,

$$T_a = 1.618 \text{ (sec)}$$

Therefore a 1 kW A.C. motor will not provide the required speed of response. If the calculation be repeated for a 3 kW A.C. motor we will find that

$$T_a = 1.26 \text{ (sec)}$$

Therefore a 3 kW A.C. motor will not provide the necessary speed of response although the overall settling time has reduced considerably compared with a 1 kW motor. By repeating the calculation for a 5 kW A.C. motor it can be found that

$$T_a = 0.65 \text{ (sec)}$$

Therefore a 5 kW A.C. motor will provide the required speed of response.

The calculations may be repeated for other types of servo motors. It can be found that the following servo motors also will provide the required speed of response.

1. A 1 kW thyristor controlled ceramic magnet D.C. motor with $T_a = 0.83$ sec.
2. A 0.5 kW pulse width modulated ceramic magnet D.C. servo motor with $T_a \cong 0.8$ sec.
3. Other types of D.C. motors, such as rare earth magnet, printed circuits and brushless, also meet the requirements at a smaller power range.

The hydraulic motor due to having large dynamic settling time at the specified load inertia will not provide the required performance, although the motor is capable of producing up to 12 kW power. By using compensation such as acceleration feedback, this motor is also capable of meeting the requirements as shown in Chapter 8.

Having obtained those motors, which satisfy the requirements, then the capital cost, size (power range) and reliability also must be considered. As was shown, a 5 kW A.C. motor will be suitable for the aboe requirements. The same requirements can be met by a much smaller D.C. motor.

For some applications may only one of the type of the available motors meet the requirements. In this case the designer has no choice but to use this motor.

Having made the calculation of the settling time, the dynamic error of the motor which is an indication of accuracy, can be obtained from the graphs of Chapter 8. Although the dynamic settling time and error of only 1,3,5 and 10 kW motors are given in Chapter 8, a good prediction of the performance of other sizes of motors can be obtained by comparing these graphs.

After a preliminary selection of the motor is made, then a more accurate performance prediction can be made by the use of the computer program package and the use of the mathematical model of Chapter 3.

If an external torque or friction exist during the transient conditions, the above calculation of settling time can be made by the use of equation H.6.

An approximate dimension of different types of motors are enclosed in Figs. H.8 to H.14 for reference purposes.

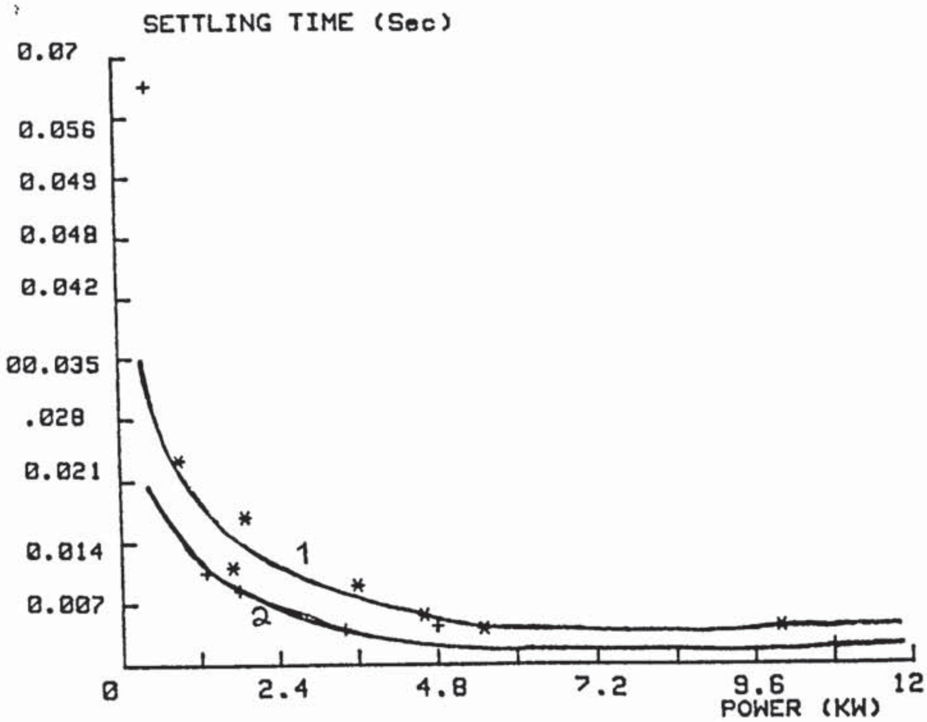


Fig H.1. Settling time of ceramic magnet d.c. motors for a unit change of velocity at a unit load inertia vs power rating.
1. Maximum velocity of 2500 R.P.M.
2. Maximum velocity of 1200 R.P.M.

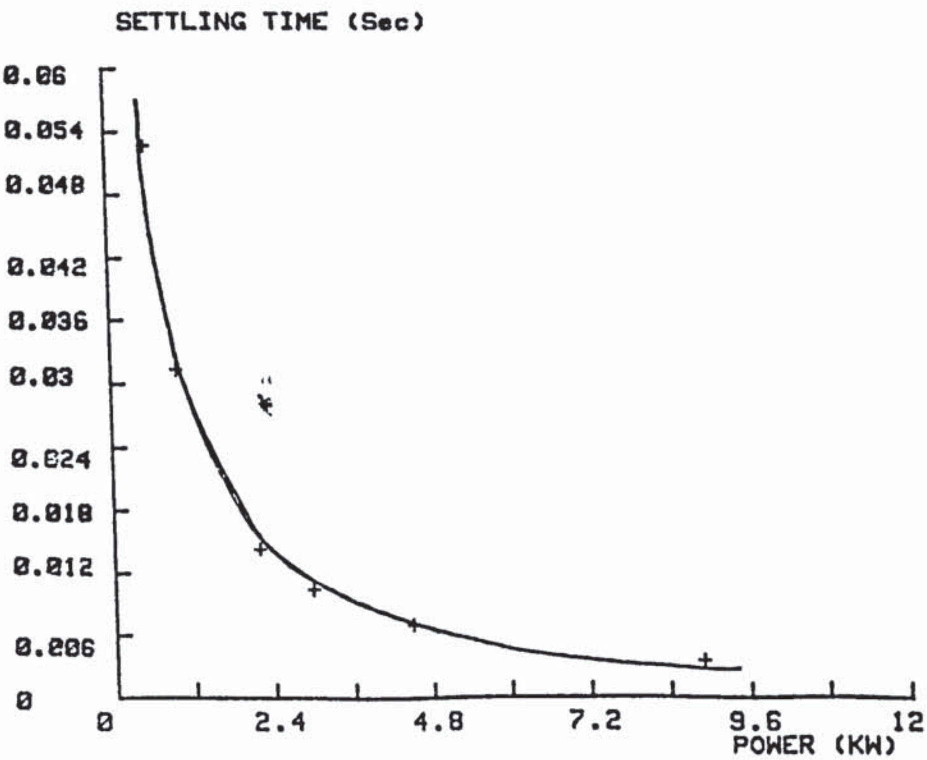


Fig H.2. Settling time of printed d.c. motors for unit change of velocity at a unit load inertia vs power.

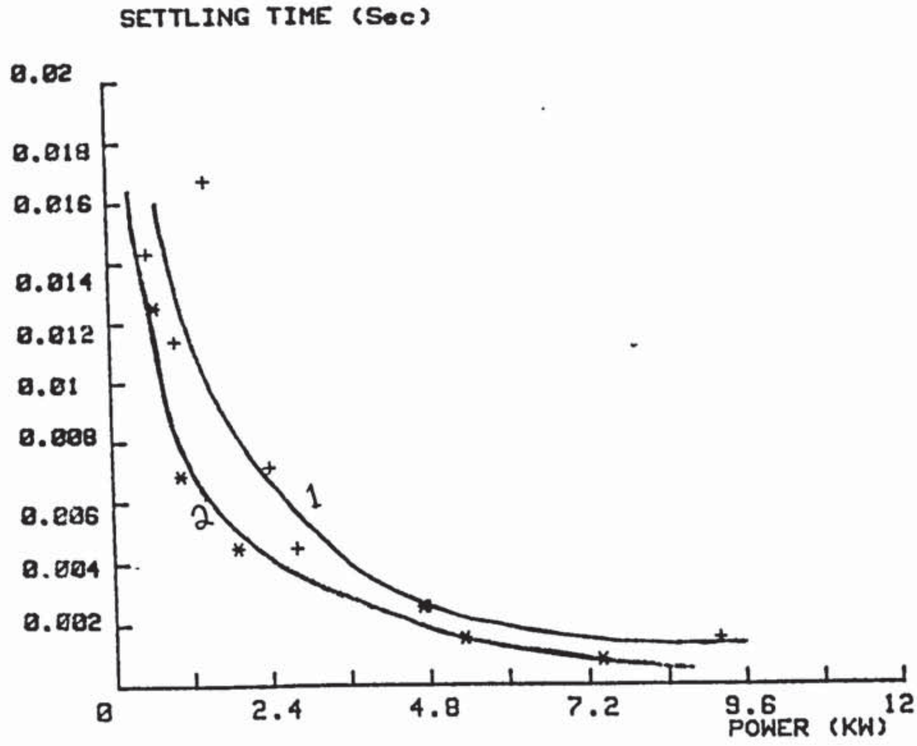


Fig H.3. Settling time of rare earth magnet d.c. motors for a unit change of velocity at a unit value of load inertia vs power rating.
1. Maximum velocity of 2500 R.P.M.
2. Maximum velocity of 1200 R.P.M.

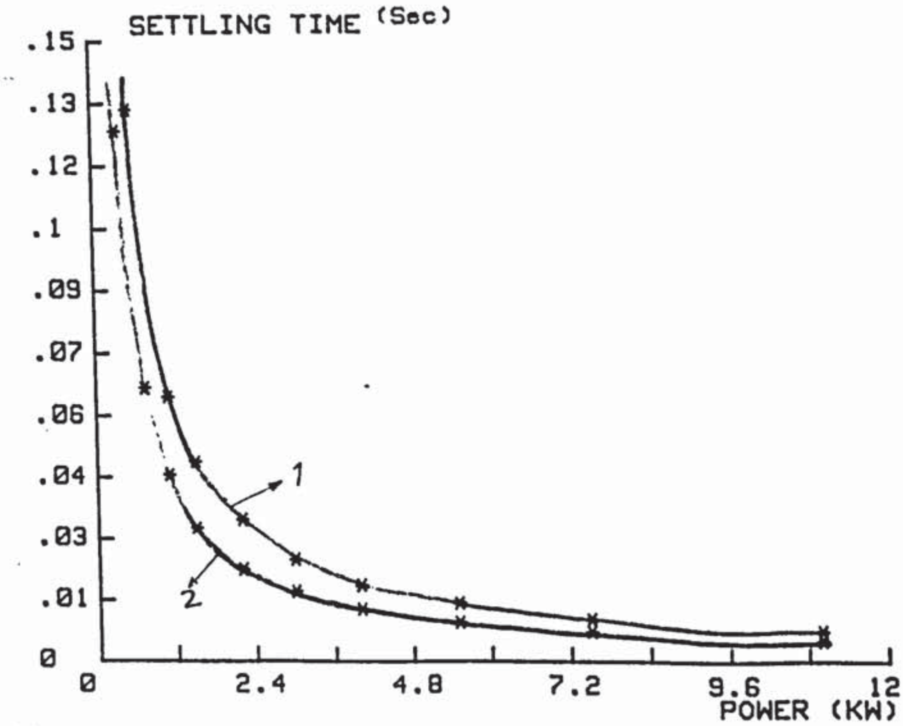


Fig H.4. Settling time of a.c. motors for unit change of velocity at a unit load inertia VS power.
1. Maximum rated velocity of 1500 R.P.M.
2. Maximum rated velocity of 950 R.P.M.

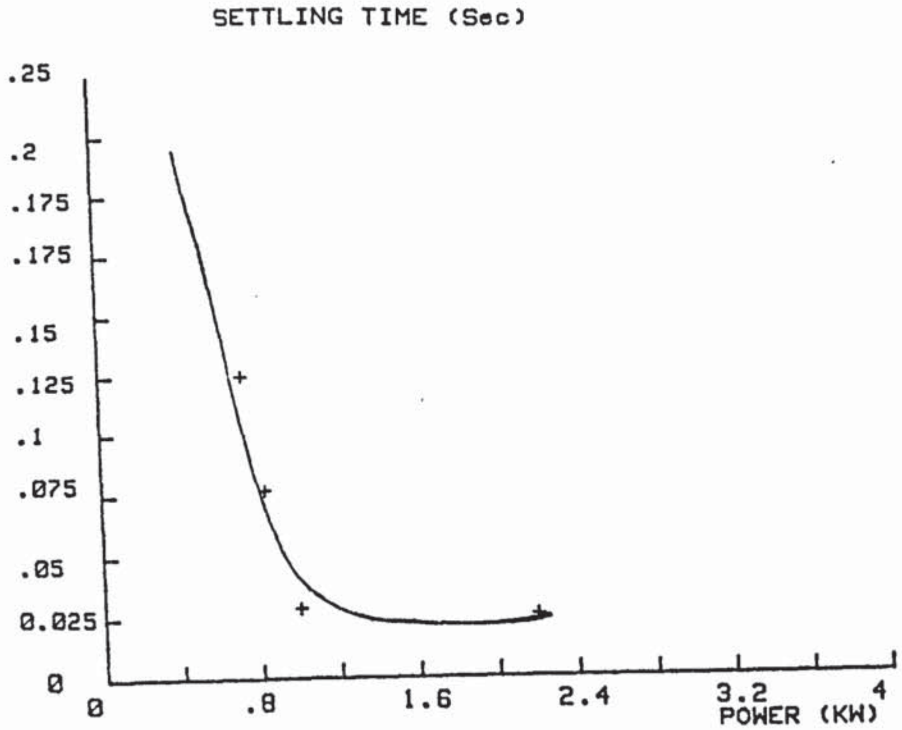


Fig H.5. Settling time of stepping motor for a unit change of velocity at a unit load inertia vs power.

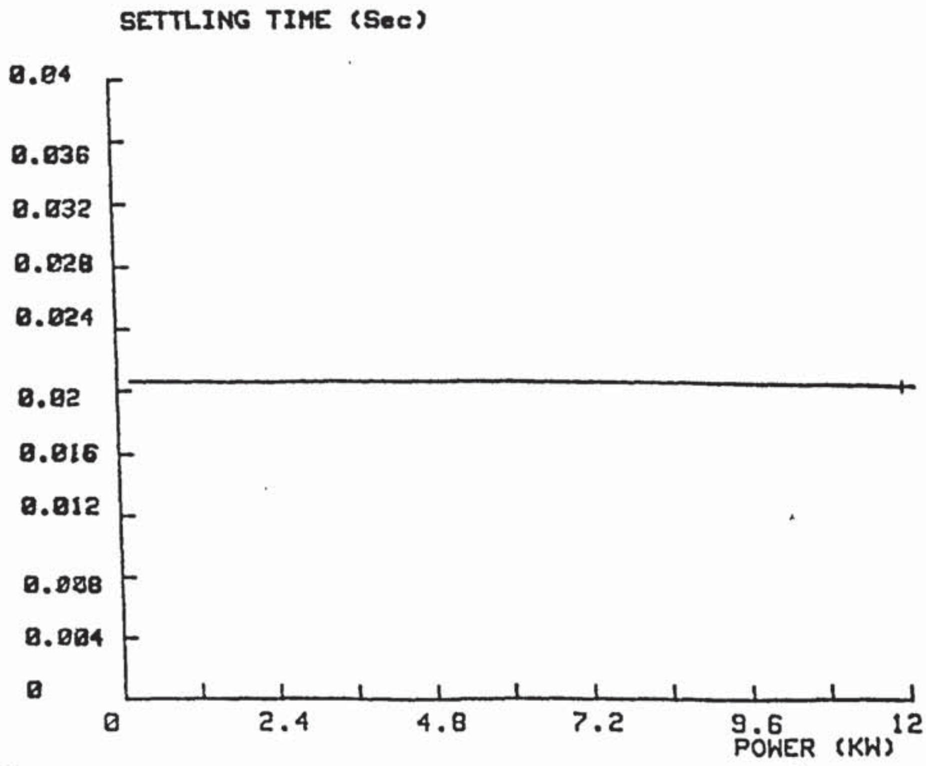
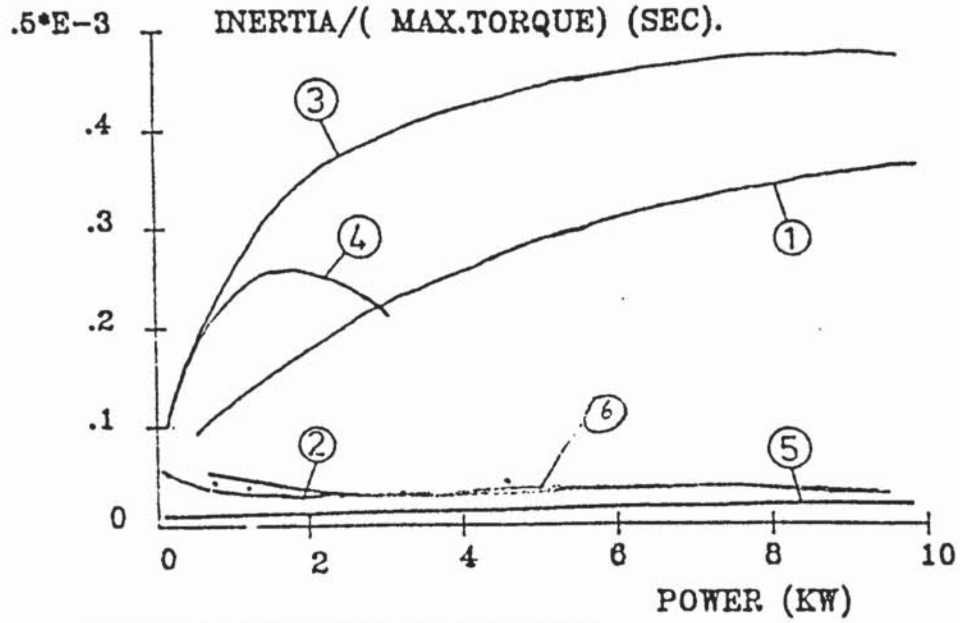


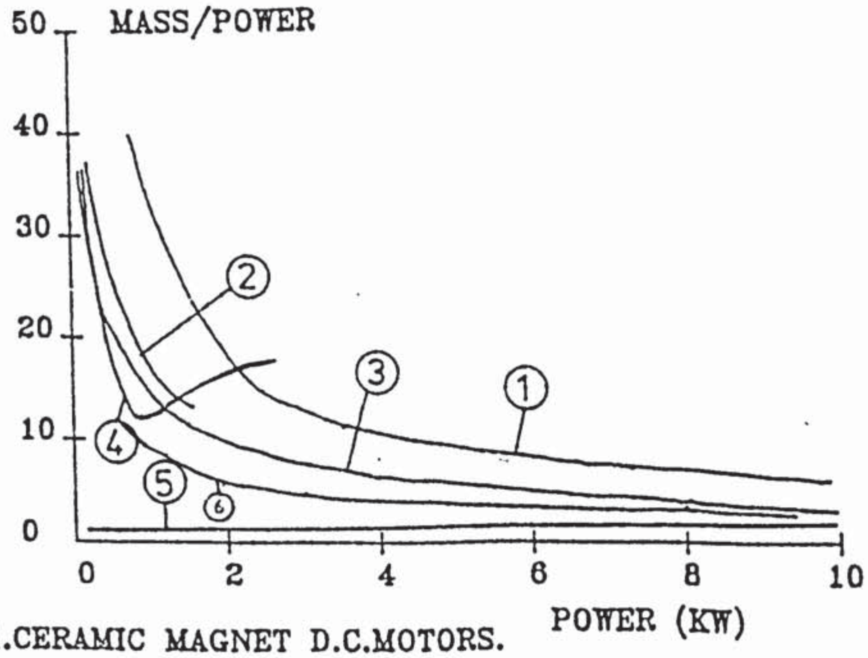
Fig H.6. Settling time of hydraulic motor for a unit change of velocity at a unit load inertia vs power (single motor with a maximum power rating of 12 kW).

FIG H.7. COMPARISON OF MINIMUM SETTLING TIME FOR A UNIT CHANGE OF VELOCITY .



1.CERAMIC MAGNET D.C.MOTORS.
2.RARE EARTH MAGNET D.C.MOTORS.
3.A.C.MOTORS.
4.STEPPING MOTORS.
5.HYDRAULIC MOTORS.
6.PRINTED D.C MOTORS.

FIG H.8 . COMPARISON OF MASS/POWER RATIO.



1.CERAMIC MAGNET D.C.MOTORS.
2.RARE EARTH MAGNET D.C.MOTORS.
3.A.C MOTORS.
4.STEPPING MOTORS.
5.HYDRAULIC MOTORS.
6.PRINTED D.C MOTORS.

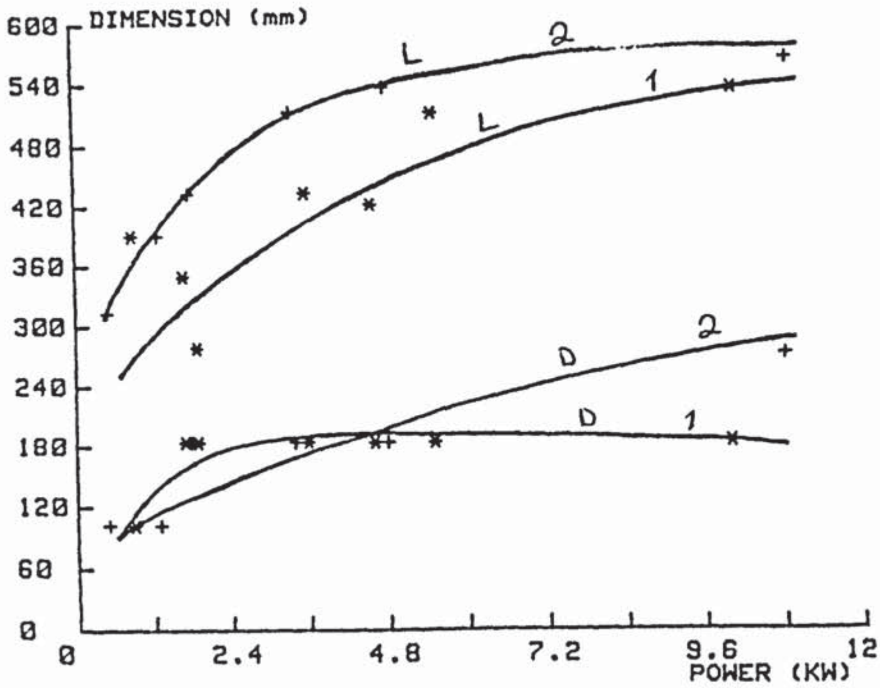


Fig H.9. Dimensions of ceramic magnet d.c. motors at different power rating.
1. Maximum velocity of 2500 R.P.M.
2. Maximum velocity of 1200 R.P.M.
L Length of motor.
D Diameter of motor.

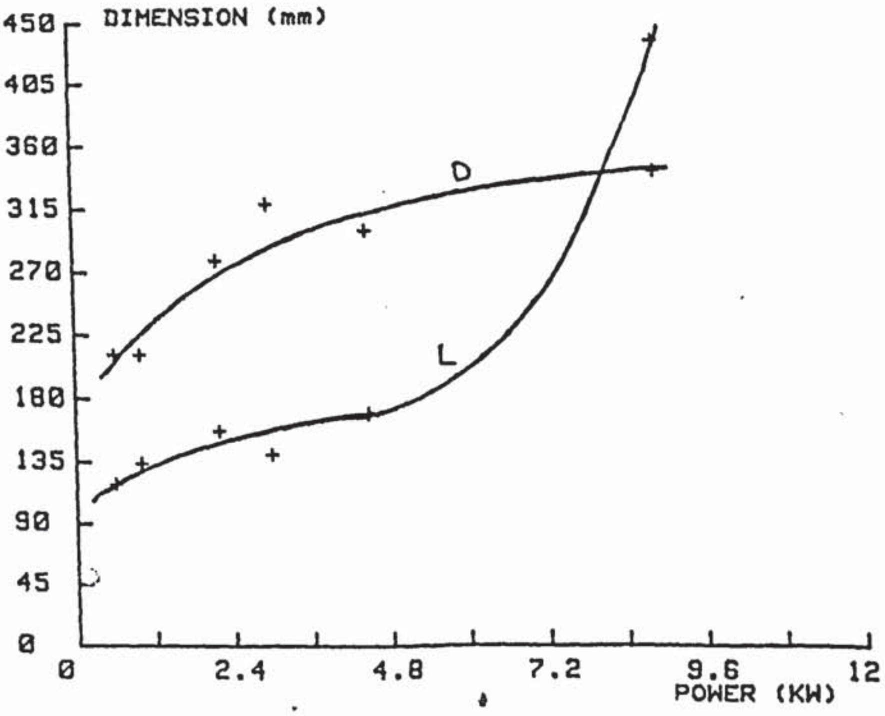


Fig H.10. Dimensions of printed d.c. motors at different power rating.
L Length of motor.
D Diameter of motor.

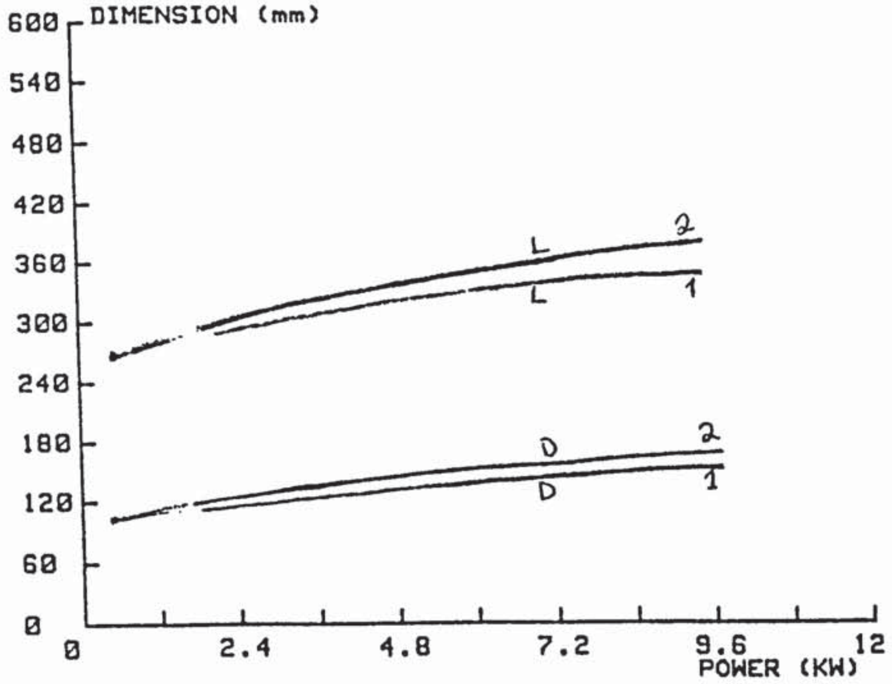


Fig H.11. Dimensions of rare earth magnet d.c. motors at different power rating.

1. Maximum velocity of 2500 R.P.M.
2. Maximum velocity of 1200 R.P.M.

L Length of the motor.

D Diameter of the motor.

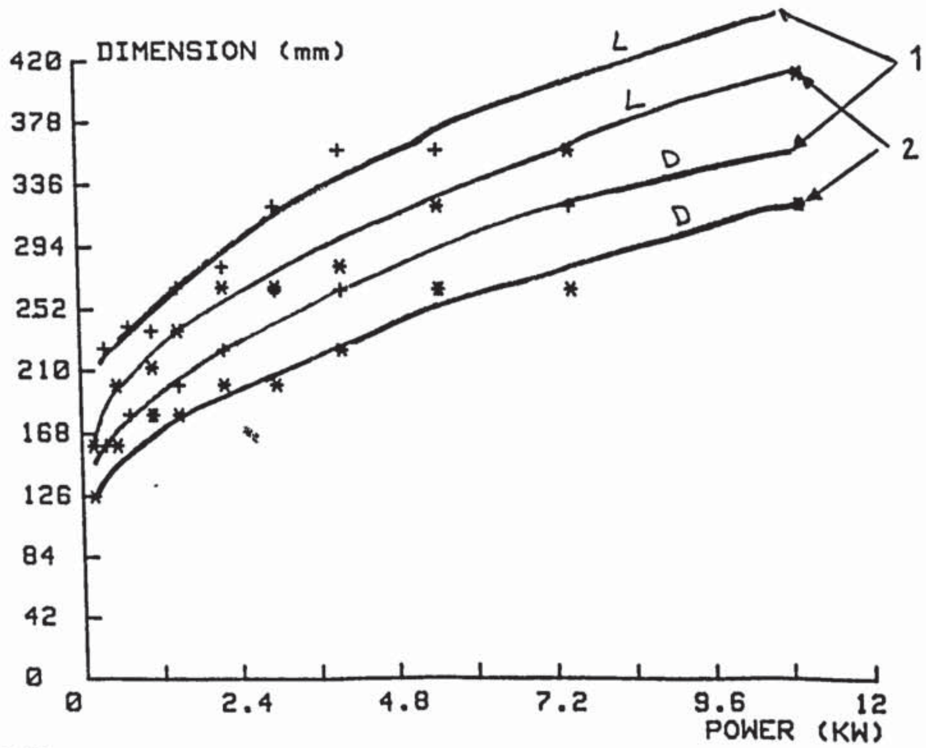


Fig H.12. Dimensions of a.c. motors at different power rating.

L Length of motor

D Diameter of motor

1. Maximum rated velocity of 1500 R.P.M
2. Maximum rated velocity of 950 R.P.M.

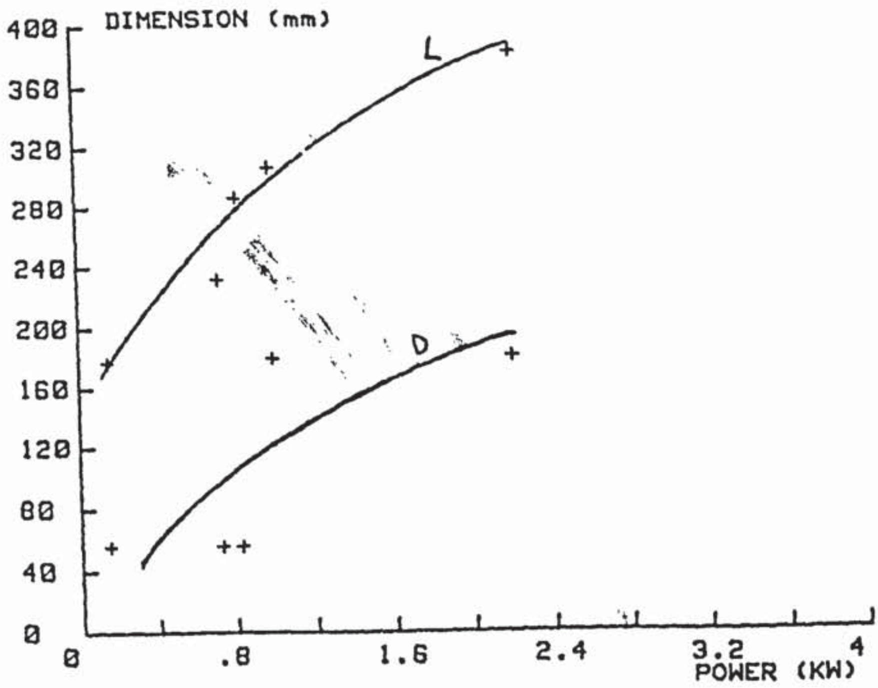


Fig H.13. Dimensions of stepping motors at different power rating.
L Length of motor
D Diameter of motor

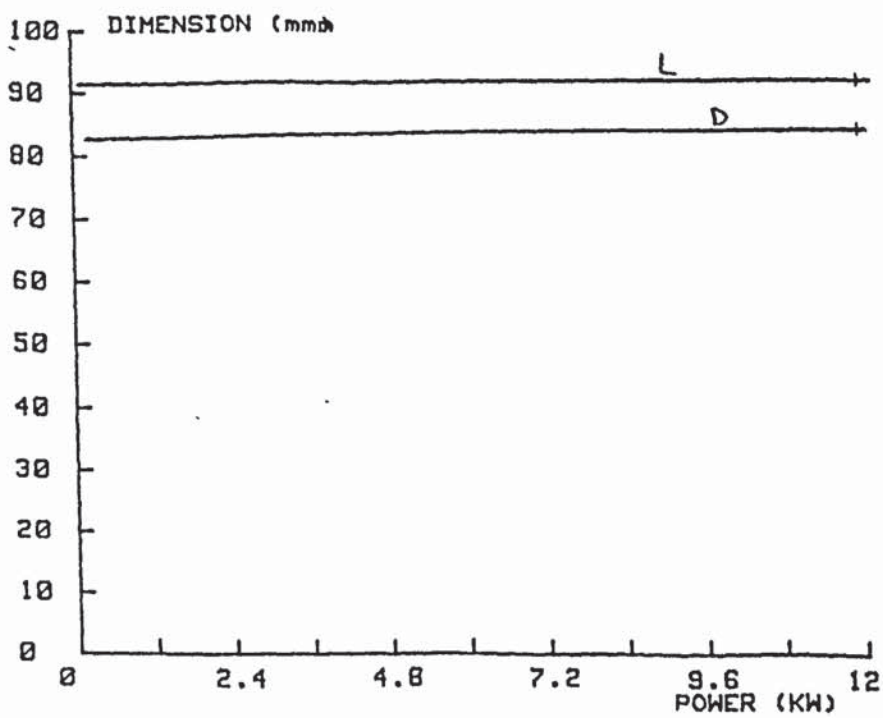


Fig H.14. Dimension of hydraulic motor at different power rating (single motor with a maximum power rating of 12 kW).
L Length of motor.
D Diameter of motor.