MATERIALS, ENGINEERING, AND THE ECONOMY: AN INPUT-OUTPUT STUDY OF TECHNICAL DECISIONS IN THE UNITED KINGDOM

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SUMMARY

The object of this work is to make a quantitative estimate of the effects on the UK economy of some technical changes in the use of materials in the manufacture of engineering and construction output.

This is achieved by an analysis based on the 1968 input-output tables for the UK economy. The estimates include the effects on the output of every industry, on the UK imports bill, on total labour and capital stock requirements, and on the prices of UK industrial products.

The technical changes considered include a substitution of plastic for steel in motor car bodies, a reduction of the material content of final engineering and construction products, and a reduction in the steel waste arising in engineering industries. A comparison is made of the energy used in engineering and construction industries directly, and the energy used indirectly by way of materials. The effects of some technical changes in motor car manufacturing are compared with the effects of some non-technical changes in motor car use. Finally, the relation between national economic variable and engineering materials are analysed in detail, and the contribution changes in material use by the engineering and construction industries can make to national economic objectives are estimated and compared for each material.

The work shows that technical changes in materials use in UK engineering and construction industries may achieve considerable national savings of labour, capital stock, imports, energy, etc. But for savings of significance to the whole economy, technical changes need to be widespread over all materials and industries, and should lead to material savings rather than materials substitution.

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I must also record my thanks at having been invited to spend a number of very pleasantly instructive evenings at meeting of the Midlands Materials Engineering Club which helped me to become acquainted with a subject quite strange to me.

But my greatest debt is to my supervisor Professor H. J. Pick. He has been my teacher, and has opened a new world to me.

Thanks also to my wife Johanna for her encouragement during the past three years.

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NOTE

This work is an interdisciplinary study in materials engineering and economics. The emphasis is on the relation between the two and does not make deep incursions into either engineering or economics.

In writing this report I have attempted to make it readable to a general and not a specialist readership.

The results of the analyses are recorded in a substantial number of rather large tables. To ease the burden for the reader the most important features of the tables are highlighted in the text and in diagrams.

CHAPTER ONE

INTRODUCTION

(1-1) OVERALL OBJECTIVE

The overall objective of this work is to make a quantitative estimate of the effects on the UK economy of a number of technical changes in the use of materials in the manufacture of engineering and construction output. This is done to contribute to understanding the relation between engineering technology and the economy. It is hoped that such understanding may eventually lead to the development of technical and manufacturing changes which are of national benefit but for which there may be no impetus within the present divisions of manufacturing enterprises.

(1-2) THE PROBLEM

(1-2.1) Origin: Materials in the Firm

The work has its origin in studies of materials in engineering firms.

In most firms materials cost is high — generally considerably higher than labour. In the US Pick(1) found that material costs in 1967 accounted for 25 - 75% of the cost of manufacturing industries while labour costs were in the 10 - 30% range, and for the UK the proportions are of the same order. A cost analysis of some 130 industries by Rawicz-Szczerbo(2) in the

UK revealed that in over 80% of them material costs were greater than labour.

Typically the production cost breakdown for a firm might be as shown in Table (1-1) from which it will be seen that a 10% reduction in material costs could increase profits by 45%. For comparison, a 10% reduction in labour costs would increase profits by only 15%.

The relative size of profits and materials costs for firms can be further assessed from a survey of 1778 US manufacturing companies in 1955 which revealed that on the average a 2% reduction in material costs could generate profits equivalent to those of a 17% increase in sales. One company, General Electric Co. (USA), estimated that a 2% reduction in materials costs would yield \$20m additional profits after taxes.[(3)]

In addition to directly being a large part of the production costs of engineering firms, materials to some extent also determine many other costs. Materials acquisition, materials stocking and storage, and materials waste disposal are obviously materials determined costs; but so are a part of capital, energy and labour costs. This is so because the manufacturing properties of the materials determine the type of capital equipment, factory floor space, machining rates, etc. And in turn the machining rates determine the number of machines, energy usage, man-hours of labour, and so forth (see Pick [(4)]).

The above clearly indicates that there is a potentially large source of company profits in attempts to reduce or control materials expenditure. This may have been overlooked by many firms. In one particular UK firm Bahiri found what is probably not so rare:

> "It seems that very little attention was paid to improving the materials utilization of existing materials, and no one could give us even an approximate idea about the yield of the principal materials used by the company. While they had never taken any measurement of waste or scrap of production materials, they had gone so far as changing the toilet towels to affect savings." ....(Bahiri[5], p. 107)

There are ample case studies to illustrate that more efficient materials management – from purchasing research and materials stocking to machining and quality controlling – has led to substantial savings for firms (see for example Rawicz-Szczerbo[2], Bahiri[5], Zenz[6]).

The object of the present work is not to add to these studies of benefits to individual firms, but to draw on them and to progress to a study of possible benefits which might accrue to the national economy as a whole if there were changes in the use of materials by individual firms.

(1-2.2) Problem: Materials in the Economy

In the context of the national economy as a whole, materials are part of a whole range of natural and economic resources. The manufacturing sector of a modern industrialised economy can be represented as a highly complex network of interdependent engineering processes which progressively convert natural resources to final products.

Figure (1.1) is a simplified[1] picture of such a conversion network, and this is used here to illustrate the discussion. Natural resources are inputs to the system. Within the network these resources flow from process to process and from industry to industry until eventually they emerge fully converted and assembled into final products. Manufacturing resources such as capital stock and manpower are used to operate the network.

Individual conversion sequences can be identified as for chemicals in Figure (1.2), and for some materials as in Figure (1.3) where each diagram indicates the increase in value of the material in response to work done as it progresses through the conversion network[2]. But it must be recalled that the sequences of Figures (1.2) and (1.3), and the conversion network of Figure (1.1), are highly simplified summaries of manufacturing as it really is. The true complexity cf the conversion network can be better gauged by contemplating the manufacture of the 100 or so components of only one product — the motor car body illustrated in Figure (1.4)(a). Each single component of the motor body is the end product of a number of engineering processes, as illustrated for example in Figure (1.4)(b). And further, the steel sheet of Figure (1.4)(b) is derived from many further upstream and branching processes.

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1. Obviously the network is in fact far more complex than illustrated here, and involves considerable feedback (e.g. scrap metal). The complexity can be gauged for example by recalling that energy is an input into almost all of the vast number of engineering processes. It should also be remembered that in a modern economy many resources are imported and not extracted from nature within the one economy.

2. The aluminium sequence also indicates the progressive reduction in weight which occurs for all materials in the conversion network partly because they need to be extracted from the large volume of matter in which they are found in nature, and partly because engineering processes are wasteful of materials.

In the context of an entire resources conversion network a product at any point is seen to be derived from a large number of natural resources — the total resource use often being far greater than is readily apparent. Finished steel, for example, obviously draws directly on resources of iron ore. Less obvious but equally essential are other resources used indirectly, such as coal and oil: coal to make the coke and generate the electricity used in steel making; oil to transport the iron ore, coke and other materials to the steelworks, and also to transport the coal to the coke and electricity works, etc. Similarly the total manpower and capital stock required to manufacture steel is not simply that employed directly by the steelworks: it includes the manpower and capital stock used to make and supply all the essential raw materials, energy, transport, etc., required by the steel industry. In Figure (1.5)(a), for example, final product F is derived from 5 natural resources N; manpower and capital equipment is required for 14 engineering processes x on route to the manufacture of this one final product.

Similarly each resource may be used in the manufacture of a wide range of products, and may be machined in many engineering processes, as illustrated in the example of Figure (1.5)(b).

It follows from this description of the interdependence of engineering processes, resources, and final products, that a change in the efficiency with which materials are used by individual firms will do far more than simply increase the profitability of that single firm as discussed in the previous section. Such a change by one firm will have implications for a whole range of natural resources and for the manpower and capital equipment employed in a vast array of upstream engineering processes, as is illustrated, for example, in Figure (1.6).

In this work such a total systems framework is used to make a quantitative analysis of the implications on a wide range of the UK's economic resources of changes in the use of structure materials (i.e. excluding textiles) in the manufacture of engineering and construction commodities.

(1-2.3) Reasons for Study

The study is undertaken in the hope that it contributes to understanding more fully the relationship between engineering design and the national economy, and to understanding more fully the alternative resource costs of alternative engineering design.

Such understanding may lead to the assessment of the suitability of alternative engineering designs, not only in the context of the individual firm, but also in the context of the national economy, with national objectives in mind. In turn this could lead to encouragement being given to the development of engineering designs which

(i) increase the efficiency of the conversion network as a whole so that resources may be

(a) conserved,

or (b) redeployed for additional or alternative production,

(ii) impede the disruptive effects on the economy of potential restrictions in supply or abrupt price changes in some imported resources.

The development of such engineering designs may be considered as a contribution to the management of the economy and to public welfare.

(1-2.4) The Potential for Change

It is not proposed in this work to analyse how the technical changes considered can actually be made to take place, nor to discuss the means of financing the change, of restructuring industry, of ensuring a favourable political climate, etc. These are research projects in themselves.

Here it is simply assumed, what is well known to the profession, that engineering is highly flexible in the combination of material, labour, capital, energy, etc., required to manufacture a commodity which conforms to some specified objectives.[3] Given appropriate specifications and stimuli, it is believed that engineering design can respond accordingly.

And there appears to be substantial room for change in industrial practice. Galloway, Director of PERA, puts it thus:

> "Recent discussions with production development experts from most industrial countries, including USA, Germany, Japan, Russia, Sweden, etc., reveal growing concern that so many manufacturing techniques which are technologically possible and economically desirable are still not extensively utilised in manufacturing industries. The progressive factories where technological advances in manufacturing techniques are profitably applied without undue delay are in the minority ..... In some instances the application of techniques which are of long standing and well proven is extremely low (5% to 10% of potential). It is not unlikely that .... application generally throughout industry is considerably less than 50%. As manufactured output comprises 70% of British total productive output, this means that over 35% of our total output is inefficiently produced." .... (Galloway[13], p. 323)

3. The engineering journals are full of appropriate examples. See for example Pick (10 - 12).

Bahiri, in a study of a large number of UK engineering firms, analysed machine utilization, machining speeds, etc., and relation of machine capacity (size, performance) to use. He estimated that

> "..... only approximately 12% of industry is machining at anything approaching economic cutting conditions."
>
> ...... (Bahiri [5], p. 118)

The study also revealed what appears to be substantial inefficiency in materials and labour utilization.

If indeed there is considerable inefficiency in UK industry, and engineering design is sufficiently flexible, then it follows that technical changes could release part of the vast resources used to manufacture and process materials in the UK; and these may be conserved or redeployed more productively.

(1-3) METHODS OF ANALYSIS

(1-3.1) Process Analysis and Industry Analysis

Estimation of the effects on the economy of changes in the use of materials is a problem in interdependent process analysis. Such analysis is not new for single industries (see for example Markowitz[14], Manne[15], and Chenery and Clark[16], esp. ch. 4). But such individual process analysis does not appear practical for an entire economy because of the vast number of processes.[4]

4. Chenery and Clark(18), p. 128, 129, are of the opinion that process analysis may be theoretically undesirable, as well as empirically infeasible, in economic analysis because it may be difficult to identify decision-making and operating units.

In this work the interdependent process analysis is approximated by an interdependent industry analysis using the standard econometric technique of input-output analysis, based on readily available UK interindustry transactions matrices.[17]

Input-output analysis is well established, and it is not proposed in this work to repeat or add to the vast technical literature on the subject (see UN bibliographical series[18]).[5] Description of input-output techniques here is limited to the following: in chapter 2 is a brief illustration of how input-output tables provide an approximate quantitative description of the engineering processes of Figure (1.1); in chapter 3 alternative input-output formulations are compared for suitability for the present work; and in chapter 8 some simple calculating formulae on the input-output matrices are presented (and tested in chapter 9).

(1-3.2) Input-Output Analysis and Technology

There is a substantial literature on input-output analysis and technical or technological change. Indeed the the UN bibliographical series[18] includes such a category. But most of the works listed appear to fall into one of two categories which are not directly relevant to the present work.

Firstly, much of the published work appears to be directed towards updating input-output technical coefficients so that predictive input-output models can make allowances for technological change. A number of mathematical techniques have been developed for this purpose, the most

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5. Readers unfamiliar with input-output techniques are referred to two excellent publications: a simple review by Miernyk (19), and the very detailed guide by UN (20).

well known perhaps being the "rAs" method. A comprehensive critical survey of these methods can be found in Lecomber[21] and Omar[22]. Generally the users of these techniques have not attempted to identify the reason for the changes, and as such their work is of little help in the present work.

Secondly, and of more interest, there are considerable historical studies to assess the change in input-output coefficients in the past. One approach in this field has been to use the final demand in one year, say 1958, with the technical coefficients in another year, say 1947, to estimate what the industrial structure would have been in 1958 under the technology of 1947. By comparison with the observed industrial structure of 1958, the effects of technological change from 1947 to 1958 can be assessed.

Such an approach has been used for example by Strout[23] and Reardon[24] to assess the effect of technological change on energy use in the USA. Both of these works used three input-output tables (1939, 1947, 1958) to estimate the effects of technological change over two time periods.

A second study of particular interest which has used this technique is by Carter[25, 26]. On materials this study showed that in the USA between 1947 and 1958 the total value of materials declined as a percentage of direct plus indirect inputs; this decline was largely attributable to decline in the most important materials; there was noted diversification of materials used with less important materials increasing their relative percentage shares, but while the new materials made inroads on older ones they did not replace them completely in any industry.

These historical studies are of interest in that they report some of the effects of technological changes, but they fail to identify the precise engineering changes which led to these effects. Obviously the effects are the results of innumerable engineering and other changes.

(1-3.3) The Static Input-Output Model

In contrast to such multi-year studies, the approach adopted here is to select one single year, 1968, and assess what the effects on the UK economy might have been had there been alternative modes of production or engineering designs in the UK engineering and construction industries. This is done by comparing the industrial structure actually observed in 1968 with that structure consistent with the observed final demand and the alternative technical coefficients. The effects observed thus are attributable to the specific change under consideration.

The year 1968 was selected because this is the year of the latest available input-output table for the UK.[6]

There are two major limitations to be emphasized about the approach and the model used.

Firstly, the model used here is static and refers to only one year. It does not describe the economy in any transition period from the decision to incorporate the technical change until the year 1968; nor does it illustrate how to finance the technical change or what the effects of any alternative capital investment programme in the intervening years might be, etc.

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6. At the commencement of this work only the 1968 provisional table was available. See NOTE on page 97 .

Secondly, the model is of the type which has come to be known as an "open" input-output model. It is not a complete model of the economy and does not relate all economic variables to ensure balance. It does not relate, for example, imports with exports to ensure balance in foreign transactions, employment with consumer demand, output and capital stock with capital investment, etc., to ensure consistency. In this analysis all final demand is taken constant at the 1968 level.

This assumption of constancy of final demand includes fixed capital formation, and as such has important implications for the interpretation of the results reported in this work. It can, for example, be argued that a saving of material in the economy may induce a reduction in current new capital requirements of materials producing industries; hence less materials are required to produce capital equipment, which further reduces the capital requirements of materials producing industries, and thus further the materials, ... , and so forth. Such progressive rounds via capital formation, which add to the effects of the technical changes, are not included in this analysis. Thus, for example, the energy content of materials reported in chapter 6 and 9 do not include the energy content of the capital equipment used in the manufacture of the materials.

Such inclusion would require a more sophisticated model than the current account static input-output model used here.[7] It is only the current use of resources which are included here.

7. At the time of writing data on capital flows do not appear to be readily available in sufficient detail for such a model using the full input-output classifications of industries.

(1-4) OUTLINE OF REPORT

Chapter 2 is an introduction to input-output analysis. Firstly it illustrates the relation between the interdependent engineering processes of Figure (1.1) and input-output analysis of the entire economy. Secondly elementary input-output techniques are used to make an overall assessment of the role of materials, capital stock, manpower, and imports in the manufacture of engineering and construction output.

In chapters 3 to 7 the object is to estimate the effects on the economy of specified changes in material use.

Chapter 3 (mathematical) presents three alternative formulations for the use of the UK input-output tables, and compares the formulations for suitability for the present work.

Chapter 4 tests the formulations of chapter 3 for ease of use. The application is to estimate the economic effects of a change towards producing passenger motor car bodies from plastic instead of steel.

Chapter 5 is a study of the economic effects of an increase in the productivity of materials used by the engineering and construction industries: firstly by reduction of the materials content of commodities; secondly by reduction of the process steel scrap arising in the manufacture of the commodities; and thirdly by a combination of the two.

Chapter 6 analyses the energy used by manufacturing industries. It is shown that more energy is used to make the materials purchased by these industries than is used to convert the materials into final products.

Chapter 7 compares the economic effects of possible changes in the technology of motor car production with the effects of possible non-technical changes in the use of cars.

In chapters 8 to 9 the object is to reverse the analysis of chapters 3 to 7, and to estimate the engineering design changes in the use of materials necessary to achieve specified national objectives.

Chapter 8 (mathematical) presents simple computation formulae which may be used to relate technical change and the economy, without the need for computer matrix inversion.

Chapter 9 uses the formulae developed in chapter 8 and tests them for ease of use, and tests the accuracy of linear approximations. The application is as follows: for each engineering material an analysis and comparison is made of the resources used to manufacture the material, the material component of prices, and the material component of regional employment; hence and estimate and comparison is made of the saving of each material, or the substitution of material, necessary by the UK manufacturing industries so that specified national objectives can be contributed to.

Chapter 10 presents some overall conclusions and suggestions for further work.

CHAPTER TWO

INTRODUCTORY INPUT-OUTPUT ANALYSIS

The object of this chapter is firstly to briefly illustrate the applicability of input-output analysis as a technique to assess the effect of changes in the use of materials in the processes of the resources conversion network, and secondly to make a preliminary overall assessment of the potential magnitude of some of the effects on the UK economy.

(2-1) FROM RESOURCES CONVERSION NETWORK TO INPUT-OUTPUT ANALYSIS

(2-1.1) The Input-Output Table

Figure (1.1) illustrates the conversion of natural resources into final products through a network of interdependent engineering processes.

In theory, such a network can be translated into the form of a matrix as illustrated in Figure (2-1), in which every entry records the flow of material, energy, etc., from one process to another. Most of the elements of such a matrix will in fact be zero because the output of every engineering process in the economy does not flow into every other engineering process. In such a table one of the conversion sequences from Figure (1.1) from resources into final products proceeds about the table as illustrated for one example in Figure (2.1), which might refer to the steel sequence of Figure (1.3)(a). An increase in final demand for one product or a change in design favouring one material over another creates a disturbance throughout the matrix exactly as was demonstrated in Figure (1.6).

However, in practice, because of the almost limitless number of engineering processes, and because of the lack of data on individual processes, it is not possible to tabulate such a matrix. To overcome these difficulties all the engineering processes which fall within the operation of what is identified in national accounts as an "industry" can be aggregated, as illustrated in Figure (2.2) for three "industries". In such an aggregated conversion network materials flow from "industry" to "industry": resource N, for example, is extracted in industry I_1 (along with other natural resources), refined in industry I_2, and manufactured into components and assembled in industry I_3 into final product F.

The aggregated industry-to-industry conversion network can be tabulated in the form of the matrix illustrated in Figure (2.1) with "process" refering to "industry". The matrix now is of a far more manageable size, and, above all, data on the flows between industries are readily available in the national current accounts of most countries.

Such a table is an "input-output" table: the columns are the inputs of every industry, the rows the outputs. Input-output tables form part of the current accounts of most countries. The size of the table depends on the degree of process aggregation, or, equivalently, the definition of "industry". The tables for Mali (1958) for example identify only 8 industries; the tables for the USA (1967) identify 484 industries.[20] (Clearly the smaller the level of process aggregation and the larger the table, the better for the present work.)

In the UK the 1968 provisional[1] tables identify 70 industries. These are compiled as two separate matrices as shown in Figure (2.3). One matrix is the "make" matrix which records the commodities made by each industry; the other is the "absorption" matrix which records the commodities and imports, etc., bought by each industry and by final buyers. A conversion sequence of the type illustrated in Figure (1.3) might proceed through the the two tables as illustrated in Figure (2.3): the material is imported by industry I_i which turns it into commodity C_n bought by industry I_k; industry I_k turns it into commodity C_ℓ which is bought by industry I_j to make commodity C_m bought by final buyers.

(2-1.2) Use of Tables in Analysis

The previous section illustrated the description of a resources conversion network as an input-output accounting table. In the present section the use of the table in estimation and prediction is briefly illustrated.

Definitions:

n the number of industries in the table

$D_{i,j}$ the i,j^{th} element of the input-output accounting table, i.e. the sale from industry i to industry j

t_i the total output of industry i

f_i the total final demand for the output of industry i

$$d_{i,j} = \frac{D_{i,j}}{t_j}$$

1. Provisional tables, updated from the 1963 tables, were used in the early sections of this work. The full 1968 tables published during the course of this work identify 90 industries. See NOTE on page 97 .

D, t, f   the matrices of elements $d_{i,j}$, t_i, f_i, respectively

For each industry i = 1, 2, ..., n

$$\begin{pmatrix}\text{Total}\\ \text{output}\end{pmatrix} = \begin{pmatrix}\text{Total sales}\\ \text{to industries}\end{pmatrix} + \begin{pmatrix}\text{Sales to}\\ \text{final buyers}\end{pmatrix}$$

Therefore

$$t_i = \sum_j D_{i,j} + f_i$$

$$= \sum_j \frac{D_{i,j}}{t_j} t_j + f_i$$

$$= \sum_j d_{i,j} t_j + f_i$$

which in matrix terms is

(2.1) $t = Dt + f$

This equation is an accounting identity on the input-output table which follows directly from the definitions of the variables; it expresses an internal balance in the accounting table. However, the link between the input-output table and equation (2.1) may be severed to a greater or lesser degree, and by making a number of assumptions equation (2.1) ceases to be an accounting identity and can be used for prediction and estimation.

In many input-output models t, f, and D are all incorporated as variables, and the published input-output tables perhaps provide only base data for D (as in forecasting).

In this work the object is to estimate what the effects on UK industrial output (and hence resource use) might have been if in 1968 there had been changes in the use of materials by the manufacturing industries. To this end equation (2.1) can be used with the 1968 tables as follows:

(a) final demand f is taken as constant at the level recorded in the input-output tables (except in chapter 7),

(b) it is assumed that all coefficients of matrix D are constant[2] with respect to t at the value obtainable from the input-output tables, except for the manufacturing industries in which the coefficients are constant with respect to t at values which change in response to technical changes,

(c) t is solved for in equation (2.1) from the known f and specified D.

The method clearly is very simple and elegant, requiring only that the changes in D be specified, and then t is given by

$$t = (I - D)^{-1}f$$

But the very simplicity of the method is a major problem in its use. The level of process aggregation is such that it is difficult to identify individual inputs to individual processes, and considerable within industry back-up data and analyses are necessary to estimate the effects on the economy of specific design changes. The alternative to such detailed within industry analysis is to accept the approximation of "process" by "industry", accept broad definitions of commodities — including materials —, and be far less specific in design changes.

In the following chapters both approaches are adopted at different times.

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2. Thus for all i,j if t_j increases by Δt_j then $D_{i,j}$ increases by $\Delta D_{i,j} = \frac{D_{i,j}}{t_j} \Delta t_j$ so as to ensure constant $\frac{D_{i,j}}{t_j}$.

(2-2) APPLICATION: A PRELIMINARY OVERALL ASSESSMENT

The previous sections have briefly illustrated the relation between input-output tables and resources conversion networks, and how these tables can be used to estimate the economic effects of changes in the use of materials by the manufacturing industries. In the following sections elementary input-output techniques are used with the object of making a brief overall assessment of the total use of materials, and of capital stock, manpower, and imports via materials by UK engineering and construction industries. Input-output Table D is used as the basis for the assessment.

(2-2.1) The Direct Matrices

The 70 industries of the input-output tables were identified as "materials" industries, "engineering" (including construction) industries, or "other" industries. The designations are indicated on Table (2.1) which tabulates for each of the 70 industries the total inputs from all UK materials, engineering, and "other" industries, and imports in these categories.

The motor vehicle industry, for example, purchased £429.1m of the output of UK materials industries, £991.7m from engineering industries, £243.6m from "other" industries, as well as directly importing £24.8m, £80.7m and £3.1m of materials, engineering, and "other" commodities.[3] This input was used by the motor vehicle industry to produce a gross output of £2636.7m. That is, $\frac{429.1}{2636.7} \times 100 = 16.3\%$ of the total input was direct UK produced materials, $\frac{24.8}{2636.7} \times 100 = 0.9\%$ was directly imported

3. Figures throughout the text are not rounded. This is done to identify with the tables, and should not be taken to imply corresponding accuracy.

materials, and the total direct material input in motor vehicle production was 17.2% of all input costs.

Table (2.2) tabulates the percent of direct material input costs for all engineering industries. The percentage varies from 6.7% for the electricity and telecommunications industry, to 49.5% for cans and metal boxes. For all but four of the industries the percentage direct material input cost is less than 25% of gross output (= total input costs).

It will be noticed that these percentage material inputs are substantially different from the 45% reported "typical" by Rawicz-Szczerbo[(2)], the 30 - 70% reported by Pick[(1)], and similar percentages reported by Bahiri[(5)], and Zenz[(6)] (see section (1-2.1)). The major reason appears to be that the above writers have used data on materials costs derived from the (financial) accounts of firms, and these accounts may include substantial quantities of highly processed materials.

Pick, for example, obtains from the UK Census of Production[(27)] a direct material input cost of 70% in the motor vehicle industry (from Census Table 4). But "materials" in the Census include completed motor bodies, gear boxes, dynamos and magnetos, locks, etc. (Census Table 10). In the present work these are not considered as materials, but as part of the 50% of bought-out components of the average UK motor vehicle factory (Rhys[(28)], ch. 3). Table (2.3) records the total purchases of materials as distinct from components by the motor vehicle industry from Census Table 10, and it is seen that materials are 16% of gross output — which is of the same order[4] as the 17% reported in Table (2.2).

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4. Differences are due to the exclusion of rubber (tyres, etc.) from Table (2.3). the inclusion of textiles and leather, and the lack of data on "unspecified materials" in Table (2.3).

The data of Table (2.1) is further aggregated into a 3 x 3 inter-industry matrix in Table (2.4), in which gross output is measured free from the duplication of intra-industry transactions. The table also records the primary input costs.[5]

Table (2.4) shows that the total UK materials output in 1968 was £5647.7m, of which £3555.6m was sold directly to engineering industries, £855.3m to "other" industries, and £1236.8m to final buyers (including export). To produce this £5647.7m gross output the materials industries imported £873.7m of materials for further processing, a total of £154.7m of other imports, employed £7936m of capital stock for one year, and a total of 1098.1 thousand man-years of labour.

As well as directly purchasing £3555.6m of materials from domestic suppliers, the engineering industries directly imported £428.9m of materials. Their other imports summed to £657m, and they employed £14329m of capital stock and a total of 5301.2 thousand workers.

Table (2.5) tabulates the direct inputs to each industry in coefficient form, and shows that the engineering industries direct material inputs were 23.0% from domestic suppliers and 2.8% from foreign suppliers, for a total of 25.8%. Each £1 of materials and engineering output directly required the use of £1.4052 and £0.9272 of capital stock, and 0.1945×10^{-3} and 0.3430×10^{-3} man-years of labour, respectively.[6]

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5. The primary inputs recorded in the UK input-output tables include "Sales by Final Buyers". "Taxes less Subsidies", and "Gross Profits and Other Trading Income". These are not of interest in the present work, and are included in Tables (2.4) to (2.8) only for completion to balance the inputs and outputs. In subsequent chapters primary inputs include purchase or use of imports, capital stock, labour ("operatives" and "others", both given in numbers and earnings, and "total income from employment"), and total value added. Details of data sources are given in the NOTE on page 97.

6. Total direct materials input in the engineering industries is 19.5% when gross output includes intra-industry transactions, and is an average for all UK engineering firms.

(2-2.2) The Direct Plus Indirect Matrices

Table (2.5) tabulates the direct input requirements per £1 of gross output of materials, engineering, and "other" industries. The total requirements however include the indirect.

Table (2.6) tabulates the total requirements to produce each £1 of materials, engineering, and "other" final demand.[7] To satisfy each £1 of final demand for UK engineering commodities a total output of £0.2457 is necessary in the materials industries, £1.0387 in the engineering industries, and £0.2128 in "other" industries; a total import of £0.0714 of materials and £0.0728 of all other imports is required in the UK, and £1.8272 of total capital stock and 0.4871×10^{-3} man-years of labour.[8]

The data of Table (2.6) are analysed here in three ways to illustrate the role of materials in the economy.

Consider firstly the final destination of materials, which is obtained by multiplying the vector of final demand in each final category (consumer, public authorities, capital formation, stocks, and exports, each aggregated into materials, engineering, and "other") by the interindustry total coefficient matrix of Table (2.6). The results for materials are tabulated in Table (2.7), and show that the UK materials output was used more to

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7. After Heesterman(29), ch. 3, if $\begin{bmatrix} D \\ \hdashline U \end{bmatrix}$ is the partitioned matrix of Table (2.5) then Table (2.6) is $\begin{bmatrix} (I - D)^{-1} \\ \hdashline U(I - D)^{-1} \end{bmatrix}$

8. Total domestic and total imported materials cannot be simply added to obtain total materials requirement, as this entails some duplication. Table (2.6) records total requirements per £1 of final demand. To obtain total requirements per £1 of gross output Table (2.6) can be divided by the diagonal elements, as done in the work of Lin (30). This is only a slight adjustment.

produce export goods while for imported materials the reverse was the case. Altogether, 10% of total materials were used to manufacture the goods purchased by public authorities or went into stocks; the remaining 90% were fairly evenly divided between the production of consumer, export and capital goods, as illustrated in Figure (2.4).

Secondly consider value added. From the interindustry coefficient matrix of total requirements in Table (2.6) value added can be traced back to source. The engineering industries, for example, used £0.2457 of UK produced materials per £1 final demand. But each £1 of materials directly required £0.2891 of labour (TIFE) and £0.1224 of profits, etc. (GP&OTI) (Table (2.5)). Hence each £1 of final engineering output indirectly is based on £0.2457 x 0.2891 = £0.0710 of labour and £0.2457 x 0. 1224 = £0.0301 of profits, etc. in the materials industries. Similarly the other coefficients of the total interindustry requirements when multiplied by the labour and profits, etc. elements of Table (2.5) lead to the results of Table (2.8).

Table (2.8) shows that of the total value of engineering output, only 55.3% has been added in the engineering industries; 10% has been added in the UK materials industries, 15.4% in "other" industries, and 14.4% is imported. Some 17.3% of the value of engineering output is value incorporated in the material, and the domestic materials industry is responsible for 10.1% of this. The UK materials industries are seen to add 42.7% to the value of their own output; 23.5% of the value is imported, of which 17.1% lies in the imported materials.

Finally consider the total direct plus indirect use of resources by the UK materials industries, and by the engineering industries via materials.

The total use of resources can be estimated by multiplying the coefficients of Table (2.6) by gross output, and this gives the results of Table

(2.9). The production of £5647.7m of UK material output indirectly required £5647.7 x (1.0370 - 1) = £208.8m of materials for its own production, as well as a total requirement of £5647.7 x 0.1713 = £967.5m of imported materials, and £357.5 of all other imports, £13017.1m of capital stock, a total of 2035.8 thousand man-years of labour, as well as other inputs.

These are the total requirements to produce £5647.7m of UK material output, and thus the engineering industries used $\frac{3555.6}{5647.7}$ of the above inputs indirectly via direct purchases of UK produced materials. Table (2.9) shows that via direct purchases of UK produced materials, the engineering industries indirectly used a further £131.6m of UK produced materials, £609.1m of imported materials, £225.1m of all other imports, £8195.3m of capital stock, a total of 1281.5 thousand man-years of labour, as well as other inputs. In addition they imported £428.9m of materials directly.

To put these figures into perspective in the UK economy, they are 19.0% of the £5462m imports by UK industriy[9] in 1968 (11.3% of £9171m UK total imports), 9.3% of the £88.1 x 10^9 UK industrial capital stock (5.5% of £149.8 x 10^9 UK total), and 7.6% of the 16.9m UK industrial labour force (5.0% of 25.6m UK total work force). The total value added in the UK materials industries in manufacturing the materials used directly by the engineering industries was 8.4% of total £30101m value added by UK industry and 6.9% of all £36810m value added in the UK in 1968.

(2-3) CONCLUSIONS

The first part of this chapter has demonstrated how input-output accounting tables can be used as approximations to a resources conversion

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9. Industries included in the input-output classifications.

network, and how they can be used to estimate the effects on the UK economy of changes in the use of materials by the UK manufacturing industries. It was shown that the method is simple but requires considerable back-up data and analysis.

In the second part of this chapter elementary input-output analysis was used to make an overall assessment of the use of materials, and capital stock, labour and imports via materials, by the engineering and construction industries.

It has been demonstrated that for most engineering and construction industries some 10 - 25% of the direct input costs are material costs, that 10% of the value of all engineering and construction commodities is incorporated in the UK produced materials used directly and indirectly, and 7% in imported materials. The value added by the engineering and construction industries to their output is more than 5 times that added by the UK materials industries. In terms of the conversion network or sequences of the type illustrated in Figures (1.1) to (1.3), it means that far more work is done in cutting, forming, shaping, and assemblying the materials in the downstream stages of the sequences than in the upstream stages of materials extraction and refining.

Further it has been demonstrated that the use of £8200m of capital stock for one year, 1280 thousand man-years of manpower, and £1040m of imports were required in 1968 to make the materials (or directly import them) used by the engineering and construction industries. In the context of the entire economy this represents 6% of total UK 1968 capital stock, 5% of total labour, and 11% of total imports (and similar magnitudes are demonstrated in chapter 6 for energy resources).

It can be concluded from this analysis, which it must be remembered does not included materials determined non-material inputs (see section (1-2.1)), that any change in the use of materials by the engineering industries which release a part of these resources for redeployment may be of significant benefit to the UK. But it is also clear that one cannot expect too much. It is unlikely that such changes by themselves could lead to dramatic short term or even medium term advantage to the UK economy as a whole.

CHAPTER THREE

ALTERNATIVE INPUT-OUTPUT FORMULATIONS

(3-1) INTRODUCTION

The object of this chapter is to present and compare three alternative formulations for the use of the UK input-output tables to estimate the economic effects of technical changes in the use of materials by engineering and construction industries. For the present work it is necessary to study in detail the elements of the tables, and their interaction. The formulations are compared with this requirement in mind.

Alternative formulations arise because of the availability of a number of different input-output tables for the UK. The tables are initially tabulated in commodity x industry form as illustrated in Figure (3.1). Table A (Make) records the commodities produced by each industry, and Table B (Absorption) records the commodities purchased by each industry. Table C is the third data table which records the commodities directly imported by each industry. Subsequent tables are derived from these data tables. Tables A and B are transformed into industry x industry form in Table D, and into commodity x commodity form in Table K (of the 1968 tables).[1] The transformation from Tables A and B into symmetric Tables D and K is achieved by what have come to be known as "industry technology", "commodity technology", and "hybrid technology"

1. The commodity x commodity tables was not included in the 1968 provisional tables used in the early stages of this work.

assumptions (see Stone[31], UN[20], Gigantes[32], Armstrong[33], and the preamble to the 1968 input-output tables[17]).

Many input-output analyses are based on symmetric Tables D and K. It is shown in this chapter that there are alternative formulations based directly on the basic data Tables A and B. The comparison shows that there may be substantial differences between the formulations with respect to ease of use and results obtained. The chapter also confirms[2] the analysis of Armstrong[33] that the transformation of Tables A and B into symmetric Table D is not achieved by technology assumptions but by inter-industry purchasing assumptions: it follows that in an analysis based on Table D it may be difficult to separate changes in technology from changes in purchasing assumptions.

NOTE: A formulation based on the commodity x commodity table was not considered for this work for two reasons. Firstly, the table was not included in the 1968 provisional tables. Secondly, a criterion for choice of formulation for this work is ready compatability with other sources of input data which generally are available for industries rather than commodities[3]; the writer did not have access to the resources necessary for substantial data conversion[4].

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2. The present writer independently arrived at the same conclusions as Armstrong. It is interesting to note that while the CSO refers to Armstrong's paper his conclusions are ignored in the preamble to the 1968 input-output tables.

3. For example: Census of Production(27), Employment and Productivity Gazette(34), National Income and Expenditure(35), UK Energy Statistics (36), primary inputs in input-output tables, etc.

4. The labours involved in data transformation is apparent, for example, from Barna(37), Stone(31), and Evans and Hoffenberg(38).

(3-2) FORMULATIONS

(3-2.1) Definitions

Matrix Order	Matrix	Definition
n x n	Flow A	the matrix of total sales by each industry of each commodity (make matrix)
n x n	Flow B	the matrix of total purchases by each industry of each commodity (absorption matrix)
n x n	Flow D	the matrix of total purchase by each industry of the products of each industry (interindustry flow matrix)
s x n	Flow U	the matrix of total purchase by each industry of primary inputs
n x m	Flow W_C, Flow W_I	the matrix showing for each category of final demand the total purchase of commodities, industrial products, respectively
n x 1	f_C, f_I	the matrix of total final demand for each commodity, industrial product, respectively
n x 1	z	the matrix of total output of each commodity
n x 1	t	the matrix of total output (= input) of each industry
s x 1	g	the matrix of total demand for primary inputs by all industries
n x 1	p_C, p_I	the matrix of price indices for commodities, industries, respectively

Matrix Order	Matrix	Definition
	A^*, A, B, D, U	the coefficient matrices of the corresponding flow matrices (defined in equation (3.1))
	$a_{i,j}$, $b_{i,j}$, $d_{i,j}$	the i,j elements of the coefficient matrices
	$\hat{x}$	a diagonal matrix with vector x on the diagonal
n x 1	θ	the matrix whose j^{th} element is the sum of industry j primary inputs in coefficient form (see equation (3.26))
	D^+, D^*, Flow W_I^+, Flow W_I^*, f_I^+, f_I^*	matrices corresponding to D, Flow W_I, and f_I obtained by the PMPA and POPA, respectively.(see equations (3.12) to (3.19))

The relationships between the coefficient matrices and the flow matrices are

(3.1)(a) $A = (\hat{z})^{-1}(\text{Flow } A)$ (d) $D = (\text{Flow } D)(\hat{t})^{-1}$

(b) $B = (\text{Flow } B)(\hat{t})^{-1}$ (e) $A^* = (\text{Flow } A)(\hat{t})^{-1}$

(c) $U = (\text{Flow } U)(\hat{t})^{-1}$

and form (3.1)(a) and (3.1)(e) we make the identity

(3.2) $$A^*(\hat{t}) = (\hat{z})A$$

(3-2.2) Formulations and Assumptions

Three alternative formulations of these matrices may be used to estimate total output of industries and commodities, and total use of primary inputs. Each formulation is based on different assumptions.

FORMULATION I

Basing our analysis on the interindustry table the most usual input-output formulation

$$(3.3) \qquad t = (I - D)^{-1} f_I$$

is obtained. This equation, as given in the static case, is only an accounting identity which follows from the definition of D above. When this equation is used for prediction of total output, however, it ceases to be an identity and becomes an estimator based on the following assumption about D:

Constant Industry Input Assumption (CIIA)

"Every industry purchases its industrial inputs in a constant ratio."

The total use of primary inputs by all industries is given by

$$(3.4) \qquad g = Ut$$

which becomes

$$(3.5) \qquad g = U(I - D)^{-1} f_I$$

when (3.3) is used to substitute for t.

FORMULATION II

Basing our analysis on the make and the absorption matrices we may form the simultaneous equations

(3.6)(a) $t = A'z$ (b) $z = Bt + f_C$

and solve them to obtain

(3.7)(a) $z = (I - BA')^{-1} f_C$

(b) $t = (I - A'B)^{-1} A'f_C = A'(I - BA')^{-1} f_C$

which as predictors of total output are based on the following two assumptions about A and B, respectively:

Constant Market Share Assumption (CMSA)

"Every industry maintains a constant share of each commodity market in which it participates."

Constant Commodity Input Assumption (CCIA)

"Every industry purchases its commodity inputs in a constant ratio."

The total use of primary inputs by all industries is given by

(3.8) $g = UA'(I - BA')^{-1}f_C$

when (3.7)(b) is substituted in (3.4).

FORMULATION III

From the make and absorption matrices we may also form

(3.9) $z = A^*t$

which together with (3.3)(b) solves to give

(3.10)(a) $t = (A^* - B)^{-1}f_C$ (b) $z = A^*(A^* - B)^{-1}f_C$

These predictors of total output of industries and commodities are based on the CCIA about B as above, and also on an assumption about A*;

<u>Constant Commodity Output Assumption</u> (CCOA)

"Every industry produces its commodity output in a constant ratio."

The total use of primary inputs in this formulation is given by

$$(3.11) \qquad g = U(A^* - B)^{-1} f_C$$

when (3.10) is substituted in (3.4).

Each of the above three formulations may be used to estimate the output of each industry for given final demand and specified coefficient matrices D, A and B, or A* and B in the case of formulations I, II, and III, respectively. In subsequent sections the three formulations are compared.

(3-3) COMPARISON: IDENTIFICATION OF PHYSICAL FLOWS

To use the above formulations to assess the implications of technical changes it is necessary to identify the physical substance recorded in the input-output tables

This identification may be easier for the transactions of Tables A and B than for Table D partly because these tables are more readily compatible with other sources of data (particularly the Census of Production from which Tables A and B are largely derived) and partly because the transactions in Tables A and B refer to specific commodities purchased by each industry while in Table D the transactions refer to total purchases by one industry from another industry. Because each industry produces a whole range of commodities, the entries in the inter-industry matrix may refer to a number of commodities. Each industry of course produces one very dominant commodity (average index of specialization is 91.8%) but there is considerable variation, particularly in the manufacturing industries.

This difficulty of identifying the substance represented be each element in Table D may be gauged for example from the synthetic resins and plastic industry. Matrix D has 53 non-zero elements in the purchases by this industry (column D17) while matrix B has only 46 (column B17). The difference clearly is due to the purchase of off-diagonal elements from some industries. The average difference ($|B17 - D17|$) is 0.34 and the standard deviation is 1.15. For larger industries this difference is accentuated (e.g. $|B40 - D40|$ has $\bar{x} = 1.23$, $s = 2.36$). For specific items one may note that the synthetic resins and plastic industry purchases only £0.1M of pharmaceutical chemicals commodities (B(15,17) = 0.1) and yet from the latter industry the former purchases £3.5M of commodities (D(15,17) = 3.5). Similarly B(18,17) = 109.6 while D(18,17) = 100.6. Clearly these differences are due to off- diagonal purchases: in the first case the synthetic resins and plastics industry purchased from the pharmaceutical chemicals industry the latter's principal product plus off-diagonal commodities (over 97% of the purchases were off-diagonal commodities); in the second case the purchases from the other chemicals and allied industries industry fell short of the synthetic resins and plastic industry's purchases of the former's principal commodities, the difference being made up by off-diagonal purchases from other industries.

To establish how these differences are made up — in the first case what off-diagonal commodities are purchased from the relevant industry, and in the second case from what industries the relevant off- diagonal elements are purchased[5] — it is necessary to examine in detail the transformation used by the CSO in deriving Table D from Tables A and B. This may be a major task.

It is concluded that it may be easier to identify the physical substance represented by the transactions of Tables A and B than by the transactions of Table D. Consequently, from this point of view, the 2-matrix formulations II and III are favoured over the 1-matrix formulation I.

(3-4) COMPARISON: PURCHASING ASSUMPTIONS

The differences between the two 2-matrix formulations and the 1-matrix formulation are in part due to different interindustry purchasing assumptions and corresponding relationships.

The effect of these different interindustry purchasing assumptions can be illustrated by considering an increase in demand (final or industrial) for a given commodity x. If commodity x is purchased from industry A then the direct resource effect of the change is dependent on the resources employed directly in industry A, and the indirect resource effect is dependent on the resources used by the industries which supply A — which in turn depends on industry A's purchasing preferences and also those of its upstream suppliers, If, alternatively, commodity x is purchased from industry B then the resource effect depends on the resources employed in industry B and its suppliers, and hence on the purchasing prefences of industry B and its suppliers. Consequently

5. In addition one cannot be sure that the above minimum of the two sets of figures relates to principal commodity purchases. For example, one cannot be sure that industry 17's demand for commodity 15 will be purchased from industry 15; D(15,17) = 3.5 may refer largely to off-diagonal purchases. Similarly industry 17's purchase of commodity 18 may be greater than £9M of off-diagonal purchases from industries other than industry 18.

the resource effects of the same increase in demand may be different, depending on purchasing assumptions throughout the economy.[6]

Clearly for the present work such differences may be important, and it is proposed in this section to analyse in some detail the differences between the formulations which arise because of different purchasing assumptions.

(3-4.1) Interindustry Transactions Table

Input-output Table D is an accounting table of interindustry transactions. There is no assumption in formulation I as to what is produced, sold or purchased by each industry. Hence equation (3.3) expresses a relationship, not about techniques of production, but about interindustry transactions.

This relationship is such that any increase in the demand for any one of the different commodities produced by an industry generates identical transactions and hence identical resource effects; and yet an increase in demand for the same commodity from different producing industries generates different interindustry transactions and resource effects.

Further, changes in the coefficients of matrix D do not necessarily reflect changes in technology — they may simply reflect changes in the industry from which the same commodity is purchased. And yet such changes may have significant consequences on resource use as estimated by formulation I.

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6. Of course, if the technology to produce a given commodity in all industries which produce that commodity were identical (for all commodities), then differences in purchasing assumptions would be irrelevant.

To use formulation I for technical change studies it is necessary either to use data on interindustry purchasing patterns, or to ignore the distinction between commodities and industries and assume:

Identical Industry Product Assumption (IIPA)

"For all industries, all products produced by the same industry are identical."

The first of these appears beyond the scope of the present work, and the second would have to be adopted. Clearly, however the assumption appears unrealistic, and for detailed studies of technical changes the use of formulation I appears to be disadvantageous in this respect.

(3-4.2) Two Alternative Purchasing Assumptions

For comparative purposes consider the following two alternative assumptions which may be used to translate Tables A and B into an interindustry table:

Proportional Market Purchase Assumption (PMPA)

"Each commodity is purchased from every producer of that commodity in proportion to each industry's market share of that commodity."

Proportional Output Purchase Assumption (POPA)

"Every purchaser purchases from each industry all commoditites that the industry makes, and in proportion to the output mix of the industry."

Firstly, applying the PMPA to the industries to obtain the interindustry matrix D^+, and its elements d^+, we have

$$\text{Flow } d^+_{i,j} = \text{purchase by industry j of the output of industry i}$$

$$= \sum_k \begin{pmatrix}\text{industry j's purchase} \\ \text{of commodity k}\end{pmatrix} \begin{pmatrix}\text{proportion of commodity k} \\ \text{produced by industry i}\end{pmatrix}$$

$$= \sum_k \left(\text{Flow } b_{k,j}\right) \left(\frac{\text{Flow } a_{k,i}}{z_k}\right)$$

$$= \sum_k \left(\text{Flow } a_{k,i}\right) \left(\frac{1}{z_k}\right)\left(\text{Flow } b_{k,j}\right)$$

Consequently

$$(3.12) \qquad \text{Flow } D^+ = (\text{Flow } A)'(\hat{z})^{-1}(\text{Flow } B) = A'B\,\hat{t}$$

and hence

$$(3.13) \qquad D^+ = A'B$$

Similarly in the final demand categories

$$(3.14)(a) \quad \text{Flow } W^+_I = (\text{Flow } A)'(\hat{z})^{-1}(\text{Flow } W_C) = A'(\text{Flow } W_C)$$

$$(b) \quad f^+_I = A'f_C$$

where the superscript + indicates that the relevant matrix is obtained mathematically via the PMPA and not from the input-output tables. Substituting (3.13) and (3.14) into (3.7) we obtain

$$(3.15) \qquad t = (I - D^+)^{-1} f^+_I$$

Secondly, applying the POPA to the demand by the industries for commodities to obtain the interindustry matrix D* and its elements d*, we have

$$\text{Flow } b_{i,j} = \text{purchase by industry j of commodity i}$$

$$= \sum_k \begin{pmatrix}\text{industry j's purchase} \\ \text{of industry k's output}\end{pmatrix} \begin{pmatrix}\text{proportion of industry k's} \\ \text{output which is commodity i}\end{pmatrix}$$

$$= \sum_k \left(\text{Flow } d^*_{k,j}\right)\left(\frac{\text{Flow } a_{i,k}}{t_k}\right)$$

$$= \sum_k \left(\text{Flow } a_{i,k}\right)\left(\frac{1}{t_k}\right)\left(\text{Flow } d^*_{k,j}\right)$$

Consequently

(3.16) $\quad \text{Flow } B = (\text{Flow } A)(\hat{t})^{-1}(\text{Flow } D^*) = (A^*)(D^*)(\hat{t})$

and hence

(3.17) $\quad D^* = (A^*)^{-1}B$

Similarly in the final demand categories

(3.18)(a) $\quad \text{Flow } W^*_I = (\text{Flow } A)^{-1}(\hat{z})(\text{Flow } W_C) = (A^*)^{-1}(\text{Flow } W_C)$

(b) $\quad f^*_I = (A^*)^{-1}f_C$

where the superscript * indicates that the relevant matrices are obtained mathematically via the POPA and not from the input-output tables.

From (3.10)(a) we obtain a rearrangement as follows

$$t = (A^* - B)^{-1}f_C$$

$$= \{ A^*(I - (A^*)^{-1}B) \}^{-1}f_C$$

$$= \{I - (A^*)^{-1}B\}^{-1}(A^*)^{-1}f_C$$

Now substituting from (3.16) and (3.18) we obtain

(3.19) $\quad t = (I - D^*)^{-1}f^*_I$

Clearly (3.15) and (3.19) are of the same form as (3.3) and the output of each of the industries in all formulations will be identical iff $A'B = D = (A^*)^{-1}B$ and $A'f_C = f_I = (A^*)^{-1}f_C$. In the static case when all three formulations are merely identities, total output calculated from given final demand and coefficient matrices will clearly be the same under each method. This means that if D^+ and D^* are not exactly equal to D, then f_I^+ and f_I^* will also not be exactly equal to f_I but will exactly compensate for the difference of D^+ and D^* respectively, so that the same total output is given by (3.3), (3.15), and (3.19).

For the 1968 provisional tables a quick check shows that f_I^+ is closer to f_I than f_I^* is. For f_I^+ the deviation from f_I is more than 1% in only 8 cases out of the 70 with a maximum deviation of 5.7% and an average absolute deviation of 3.3. For f_I^* the deviation is more than 1% in 57 cases out of the 70 with a maximum of 89% and an average absolute deviation of 20.9.

This does not however imply that the PMPA is a "better" or more "realistic" purchasing assumption than the POPA. It only shows that the PMPA can be used to approximate the CSO's derived interindustry transactions, while the POPA leads to a very different interindustry transactions matrix.[7] The evaluation of the POPA and the PMPA on the basis of the above comparison thus is not very meanigful. The purchasing assumptions ought to be evaluated on the basis of which is the more realistic, and clearly the POPA appears implausible. Purchases

7. This is not altogether surprising because the PMPA to a large extent formed the basis of the CSO's derivation of Table D from Tables A and B (see introduction to the 1963 tables, CSO(17), p. 6).

from an industry do not realistically consist of the whole range of commodities produced by the industry. A model which assumes this would have to allow for negative[8] purchases from some industries by some buyers as they off-load the unwanted commodities they were forced to buy along with the commodities they did want.

But because the POPA appears more implausible than the PMPA it does not follow that formulation III is less valid than formulation II. While the rearrangement of (3.7)(b) into (3.15) and (3.10)(a) into (3.19) can be interpreted in the light of the PMPA and the POPA respectively, this does not in itself imply that these alternative market disciplines are embedded in formulations II and III respectively.

Indeed, in formulations II and III the total output of each industry (and commodity) is independent of any purchasing assumptions. An increase in final demand for any one commodity results in changes in the total output of each industry so that in each commodity market total supply equals total demand. There is no assumption, explicit or implicit, as to which industry purchases what from where, even though in formulation II it is assumed that each industry maintains a constant share in each commodity market.

(3-4.3) <u>Variations in Purchasing Assumptions</u>

Consider the range of possible interindustry relationships more closely with the following theorems.

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8. See section (3-4.5) below for expanded discussion.

Define locally for any single commodity market:

$\gamma_{i,j}$ the interindustry sale from industry i to industry j ($\gamma_{i,i}$ are the intraindustry sales)

ϕ_i the final demand purchase from industry i

ζ the total final demand for the commodity

A_i the total sales of industry i

B_i the total purchases of industry i

z the total output of that commodity, i.e. $\sum_i A_i = z$

THEOREM (3.1)

"A necessary and sufficient condition for a purchasing assumption in a single commodity market to be feasible is that it provides a solution to the system

$$\left.\begin{aligned} \phi_i + \sum_j \gamma_{i,j} &= A_i \\ \sum_j \gamma_{j,i} &= B_i \end{aligned}\right\} \quad \text{for } i = 1, \ldots\ldots, n$$

and $\phi_i, \gamma_{i,j} \geq 0$ for i,j = 1, ..., n."

Proof It is clearly necessary that the purchasing assumption provides a non-negative solution to the following equations

Sales Equation for Industry i

sales to final buyer + total interindustry sales = total sales

$$\phi_i + \sum_j \gamma_{i,j} = A_i \quad (3.20)$$

for i = 1,........, n.

Purchase Equation for Industry I

total interindustry purchases = total purchases

(3.21) $$\sum_{j} \gamma_{j,i} = B_i \quad \text{for } i = 1,\ldots\ldots,n$$

To prove sufficiency it is required to show that final buyers are also satisfied by a purchasing assumption which provides a solution to equations (3.20) and (3.21).

Purchase Equation for Final Buyers

(3.22) $$\sum_{i} \phi_i = \zeta$$

Equations (3.20), (3.21) and (3.22) are 2n + 1 equations, but these are not all independent. Summing (3.20) and (3.21) for each industry we have, respectively,

(3.23) $$\sum_{i}' \phi_i + \sum_{i}\sum_{j} \gamma_{i,j} = \sum_{i}' A_i$$

(3.24) $$\sum_{i}\sum_{j} \gamma_{j,i} = \sum_{i} B_i$$

However $\sum_{i}\sum_{j}' \gamma_{i,j} = \sum_{i}'\sum_{j} \gamma_{j,i}$ and thus with (3.23) and (3.24) we obtain

$$\sum_{i} \phi_i + \sum_{i}' B_i = \sum_{i}' A_i$$

$$\sum_{i} \phi_i = \sum_{i}' A_i - \sum_{i} B_i$$

$$\doteq z - \sum_{i}' B_i$$

$$= \zeta$$

since for every industry the residual from interindustry sales must go to final buyers.

Thus equation (3.22) is dependent on (3.20) and (3.21), and we are left with 2n independent equations in $n^2 + n$ variables (i.e. $n^2 - n$ degrees of freedom), and sufficiency has been proven.

THEOREM (3.2)

"The PMPA is a feasible purchasing assumption."

Proof To prove this theorem we need to show that equations (3.20) and (3.21) are solved by purchases in accordance with the PMPA. Under a market shares assumption ϕ_i and $\gamma_{i,j}$ are given by:

$$\left.\begin{aligned} \phi_i &= \frac{A_i}{z}\,\zeta \\ \gamma_{i,j} &= \frac{A_i}{z}\,B_j \\ \phi_i,\ \gamma_{i,j} &\geq 0 \end{aligned}\right\} \quad \text{for } i,j = 1,\ldots\ldots,n$$

In equation (3.20)

$$\begin{aligned} \frac{A_i}{z}\zeta + \sum_j \frac{A_i}{z} B_j &= \frac{A_i}{z}\zeta + \frac{A_i}{z}\sum_j B_j \\ &= \frac{A_i}{z}\left(\zeta + \sum_j B_j\right) \\ &= \frac{A_i}{z}\, z \qquad \text{as before} \\ &= A_i \qquad \text{as required} \end{aligned}$$

In equation (3.21) the above expression for $\gamma_{i,j}$ may be expressed with subscripts interchanged and summed, thus:

$$\sum_j \gamma_{j,i} = \sum_j \frac{A_j}{z} Bi$$

$$= \frac{B_i}{z} \sum_j A_j$$

$$= \frac{B_i}{z} z \quad \text{as before}$$

$$= B_i \quad \text{as required.}$$

The implication of the above two theorems is that there is a whole range of possible interindustry purchasing assumptions in any single commodity market, subject to the condition that equations (3.20) and (3.21) be satisfied. Clearly the "hybrid technology" assumptions used by the CSO define one set of purchasing assumptions: the PMPA is an alternative. There are many possibilities for finding solutions to (3.20) and (3.21), one algorithmic approach is given in Appendix A1 .

(3-4.4) <u>Implicit Commodity Assumption</u>

The previous sections emphasized that in formulations II and III total output is estimated so that total demand and supply are equal in every commodity market; there are no specified interindustry relationships. It follows that there must be two strong conditions implicit in these formulations. These conditions could be overcome by specifying purchasing assumptions, as in formulation I.

Firstly, it is necessary that all commodities in the same commodity market are identical, irrespective of the industry of production: a final buyer must find them all equivalent; an industrial buyer must find them all equally suitable as an input into his production process. If this were not so, and industries produced variations of commodities in the same classification, then in formulation III a change in the share of the total output produced by any one industry may alter the composition of the commodities in that classification, while in formulation II an increase in demand by a final buyer purchasing only one variation cannot be satisfied. Clearly this condition is not often met because commodity classifications are of necessity broad, and non-principal commodities produced by industries are usually only a part of the broad class of commodities. 'Motor vehicles and tractor' commodities produced by the 'other electrical goods' industry, for example, is clearly electrical motor equipment and not passenger cars, tractors, buses, lorries, caravans, etc., as produced by the 'motor vehicles and tractors' industry.

Secondly, it is necessary that all commodities in the same commodity market are equally available to all purchasers: they must actually be able to be sold to purchasers and used as inputs in the case of industrial purchasers. The mathematics of formulations II and III will ensure only that an increase in demand for every commodity due to some change (technical or final demand) is met by an increase in supply. This supply of the commodity may however be produced by industries which in practice cannot actually make it available to satisfy the demand, perhaps because of physical immobility of the commodity. Electricity generated within an establishment may not, for example, be available for use by final buyers or other establishments.

These two conditions are summarized as the following assumption which is implicit in formulations II and III:

Identical Commodity and Availability Assumption (ICAA)

"For all commodity markets, all commodities in the same market are identical and equally available to all purchasers."

(3-4.5) The Inverse Matrices

Consider the differences between the usual Leontief inverse matrix $(I - D)^{-1}$ of formulation I and the inverse[9] matrices $A'(I - BA')^{-1}$ and $(I - BA')^{-1}$ of formulation II, and $(A^* - B)^{-1}$ and $A^*(A^* - B)^{-1}$ of formulation III.

The j^{th} column of $(I - D)^{-1}$ is the total output of every industry necessary to produce one unit of final demand for the output of industry j. The j^{th} columns of both[10] $A'(I - BA')^{-1}$ and $(A^* - B)^{-1}$ are the total output of every industry necessary to produce one unit of final demand for commodity j, irrespective of which industry supplies final buyers. The j^{th} columns of both[10] $(I - BA')^{-1}$ and $A^*(A^* - B)^{-1}$ are the total output of every commodity necessary to produce one unit of final demand for commodity j, irrespective of which industry supplies final buyers.

The difference between the matrices is emphasized by the fact that $(I - D)^{-1}$, $A'(I - BA')^{-1}$ and $(I - BA')^{-1}$ are strictly positive, while $(A^* - B)^{-1}$ and $A^*(A^* - B)^{-1}$ both have some negative elements[11].

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9. $A'(I - BA')^{-1} = \{ (A')^{-1} - B \}^{-1}$ and $A^*(A^* - B)^{-1} = \{I - B(A^*)^{-1}\}^{-1}$ and thus both may be termed 'inverse'. Note that BA' and $B(A^*)^{-1}$ are commodity x commodity coefficient matrices obtained under 'industry technology' and 'commodity technology' assumptions, respectively (see Armstrong(33), and Stone(31)).

10. Under different assumptions, and hence the columns are different.

11. For the provisional tables $(A^* - B)^{-1}$ has considerably more negative elements than $A^*(A^* - B)^{-1}$.

Consequently increased final demand for one industry's output in formulation I or commodity in formulation II will increase the output of every industry and commodity. In formulation III however increased final demand for one commodity will increase the output of most industries and commodities, but may decrease the output of others. Most of these negative elements are small in absolute terms, although with the 1968 provisional tables element 38, 37 of $(A^* - B)^{-1}$ is -0.08: a £1000 increase in final buyer demand for 'cans and metal boxes' commodities (classification 38) with all other final demand constant thus decreases the output of the 'other metal goods' industry (classification 38) by £80. Similarly element 35,28 is -0.04: a £1000 incraese in final demand for 'office machinery' (classification 28) with all other final demand constant thus decreases the output of 'electronics and telecommunications' commodities (classification 35) by £40.

The explanation of this negative effect with respect to the output of industries is quite simple. Consider the first order effects: if final buyers purchase an increased amount of only one commodity then the suppliers of this commodity will need to increase their output; the output of their other (secondary) products will also be increased proportionally; as there has been no increase in final demand for these other commodities the major producers of these, who do not necessarily produce the first commodity, will need to reduce production to maintain constant availabilities for final buyers. The total picture is of course not as straight forward as this because of higher order effects, but the explanation of the negative elements in matrix $(A^* - B)^{-1}$ is clear.

Similarly the negative elements of $A^*(A^* - B)^{-1}$ can be explained: an increase in Δz_i in final demand for commodity i will increase the output of industries $j \in J$ by t_j, and decrease the output of industries $k \in K$ by $|\Delta t_k|$; the use of commodity m will increase in industries $k \in K$ by $b_{m,k}|\Delta t_k|$; in balance the total use of commodity m may increase or decrease. I.E. $\Delta z_m \gtrless 0$ if $\sum_{j\varepsilon J} b_{m,j}\Delta tj \gtrless \sum_{k\varepsilon K} b_{m,k}|\Delta t_k|$, respectively.

Because of these negative elements the coefficients of $(A^* - B)^{-1}$ and $A^*(A^* - B)^{-1}$ cannot be interpreted as total — direct plus indirect — production input coefficients per unit of final demand in the same way as the coefficients of $(I - D)^{-1}$ are often interpreted. The elements of the j^{th} column of $(A^* - B)^{-1}$ and $A^*(A^* - B)^{-1}$ can be interpreted only as the marginal readjustment from equilibrium required in the output of industries and commodities, respectively, so that total supply equals total demand in each commodity market when a change in one unit of final demand for commodity j occurs.

Similarly the coefficient matrices of formulation II must be interpreted with care, although in this case the non-negativity of the elements does not mitigate against the interpretation of the coefficients as total production input coefficients.

(3-4.6) Conclusions on Purchasing Assumptions

This section has emphasized that formulation I is a formulation about interindustry transactions, dependent on the CSO's interindustry purchasing assumptions. Changes in coefficients could be the result of changed interindustry purchasing assumptions and not only technical change. Use of this formulation for the present purpose requires the assumption that all products in each industry are identical — ignoring that industries produce a range of commodities. This unrealistic assumption appears a disadvantage of formulation I.

Formulations II and III are free from purchasing assumptions. They require only that the total demand in any commodity market equals total supply. Coefficient changes reflect only technical changes. But this freedom from interindustry purchasing assumptions implies that all commodities in each classification are identical and equally available to all potential customers, irrespective of industry of production. This unrealistic implicit assumption appears to be a disadvantage of formulations II and III.

Further this section has illustrated the differences between the inverse matrices of the formulations. Care needs to be taken to interpret the matrices in light of the assumptions on which the formulations are based. In formulation III the inverse coefficient matrices can certainly not be interpreted as total production input coefficients.

(3-5) COMPARISON: TECHNOLOGICAL INPUT-OUTPUT BALANCE

Perhaps the most important difference between the three formulations is in how a balance between input, technology, and output is achieved.

Consider firstly formulation III. In formulation III the columns of matrices A* and B are specified, as illustrated in Figure (3.2). That is, a commodity input mix and a commodity output mix is specified for each industry in light of the technology which converts the inputs into outputs. Each industry's share of each commodity market is mathematically dependent on final demand, and on the coefficient matrices. A change in technology in this formulation is reflected by either altered inputs, or altered outputs, or both.

The fact that the outputs may be easily specified by a user of formulation III is particularly advantageous as technological change may well alter the sales of an industry as well as its purchases. Firstly, for example, the by-products and waste products of the old and the new processes may be quite different: the changing of motor body manufacture from steel to plastics clearly implies that the waste products will be of plastic materials rather than scrap steel. Secondly, the original secondary products (which may be linked to the principal product by a common material, process or market) may be very different from the principal product after technological change, and may be discontinued in favour of new secondary products. For example, a manufacturer whose primary product was ceramic bathtubs may find ceramic tiles a profitable secondary product, but if he changed to production of glass bathtubs then the production and processing of ceramics only for tiles may no longer be profitable and the production of tiles out of glass may not be feasible either, and yet he may be able to manufacture glass jewellery as a profitable new secondary product.

In formulation II the rows of matrix A and the columns of matrix B are specified as illustrated in Figure (3.2). The relative commodity outputs of each industry (column of A) are mathematically determined. In this formulation there is no guarantee that for each industry a given output commodity mix (a column of A) is achievable by a known conversion technology from the specified inputs (a column of B). For example, a change in demand for commodity i will increase industry j's output of commodity i so as to maintain industry j's market share. But industry j's output of other commodities will not increase (proportionally), the result being that the inputs and outputs may not be technologically balanced. Use of this formulation for predicting the results of technical changes thus is difficult as the inputs and the proposed technology in one industry must be reconciled with specific market shares, which are dependent on all industries.

This formulation is particularly unrealistic if the off-diagonal sales by an industry are by-products of the principal engineering process, or if they are secondary products sharing a common material or technology with the principal product. The output of these off-diagonal products should be related to the output of the principal product of the industry, irrespective of the changing market share. The above manufacturer who made glass jewellery out of the excess material from his glass bathtubs would not seek to increase his glass jewellery output so as to maintain his share of that market should demand rise. Instead, his glass jewellery ouput is more likely to be governed by the demand for glass bathtubs.

In formulation I the columns of matrix D are specified. A change in technology which alters the output of an industry as well as the inputs would be diffucult to reconcile with the interindustry relationship embedded in the matrix. To estimate the altered total output of every industry it is necessary to identify and reallocate the purchases by other industries from the industry in which the composition of output changed. That is, an examination of the customers' purchasing assumptions (and possibly production processes) may be required to anticipate new interindustry relationships. In this formulation the coefficients in matrix D must be altered to reflect both changes in technology and changes in market disciplines.

It is concluded that formulation III is the simplest formulation in which inputs and outputs can be changed to ensure technological balance.

(3-6) PRICE RELATIONSHIPS

Technical changes may result in price changes (see Figures (5.2) and (5.3) below) and for completion the above formulations are extended accordingly.

Price changes may be expressed in terms of commodity prices or industry prices.

In the usual industry x industry formulation the standard input-output price equation is

$$(3.25) \qquad p_I = (I - D')^{-1} \theta$$

where θ is the matrix with j^{th} element

$$(3.26) \qquad \theta_j = \sum_{i=1}^{s} u_{i,j}$$

In commodity x industry formulations prices may be expressed in terms of the output of industries and in terms of commodities, and the expression will depend on the functional relationship between commodity prices and industry prices.

Firstly, using matrix A as weights to relate commodity and industry prices we have

$$p_{C_i} = \sum_j \begin{pmatrix}\text{proportion of commodity } i \\ \text{produced by industry } j\end{pmatrix} \times \begin{pmatrix}\text{price of output} \\ \text{of industry } j\end{pmatrix}$$

$$= \sum_j \left(\frac{\text{Flow } a_{i,j}}{z_i}\right)\left(p_{I_j}\right)$$

$$= \left(\frac{1}{z_i}\right) \sum_j \left(\text{Flow } a_{i,j}\right)\left(p_{I_j}\right)$$

Hence from (3.1)(a)

(3.27) $$p_C = (\hat{z})^{-1}(\text{Flow } A)(p_I) = A\, p_I$$

From the usual input-ouput price consideration we have

(3.28) $$p_I = B'p_C + \theta$$

which corresponds to (3.25). Equations (3.27) and (3.28) solve to give

(3.29)(a) $$p_I = (I - B'A)^{-1}\theta$$ (b) $$p_C = A(I - B'A)^{-1}\theta$$

From (3.13) we may substitute for B'A in (3.29)(a) to give

(3.30) $$p_I = \{ I - (D^+)' \}^{-1}\theta$$

Consequently (3.29)(a) is of the same form as (3.25), but with an interindustry matrix obtained via the PMPA.

Alternatively, using A* as weights we have

$$p_{I_i} = \text{price of the products of industry } i$$

$$= \sum_j \begin{pmatrix}\text{proportion of industry } i\text{'s} \\ \text{output which is commodity } j\end{pmatrix} \left(\text{price of commodity } j\right)$$

$$= \sum_j \left(\frac{\text{Flow } a_{j,i}}{t_i}\right)\left(p_{C_j}\right)$$

$$= \left(\frac{1}{t_i}\right)\sum_j \left(\text{Flow } a_{j,i}\right)\left(p_{C_j}\right)$$

Hence from (3.1)(e)

(3.31) $$p_I = (\hat{t})^{-1}(\text{Flow } A)'p_C = (A^*)'p_C$$

which with (3.28) solves to give

(3.32)(a) $p_I = \{ (A^* - B)^{-1}A^* \}'\theta$ (b) $p_C = \{ (A^* - B)^{-1} \}'\theta$

Equation (3.32)(a) may be rearranged as

$$p_I = \{ I - (A^*)^{-1}B \}^{-1}\theta$$

into which (3.17) may be substituted to give

(3.33) $p_I = \{ I - (D^*)' \}^{-1}\theta$

Consequently equation (3.32)(a) is of the same form as (3.25) but with an interindustry matrix obtained via the POPA.

It should be pointed out that in this second functional relatioship between commodity prices and industry prices an increase or decrease in some primary input costs may lead to some price decreases or increases, respectively. This is so because the coefficient matrices of (3.32) have some negative elements (see section (3-4.5) above).

It must also be emphasized that in the case of commodity x industry formulations the method of estimating changed total output due to technical changes and the method of estimating price changes are independent. That is, price equations (3.29) or (3.32) can be used with either formulation II or formulation III.

To ensure a consistent approach, in this work prices shall be expressed in terms of industry prices only. In formulation I price changes will be estimated from (3.25). In formulations II and III price changes will be estimated from (3.30) but with D^+ obtained via the PMPA (which was shown in section (3-4) not only to be feasible but also an approximation to the CSO's interindustry assumptions) after the technical change represented by alterations in A and B, or A* and B. For formulation III it thus is necessary to first estimate total commodity and industry output, secondly to estimate the new market shares matrix via identity (3.2), and thirdly to estimate D^+ via (3.13), before using equation (3.30) for price estimation.

(3-7) SUMMARY AND CONCLUSIONS

This chapter presented three formulations which estimate total output of industries, total primary input requirements, and price levels, from given technical coefficient matrices and given final demand. One formulation was based on the interindustry Table D while the other formulations were based on the basic data Tables A and B of the UK input-output tables. The formulations were compared for suitability for the present purpose of studying in detail the elements of the matrices, and their interaction, and the effects of changes.

The three formulations, the assumptions, and the equations used (in order) for estimating the effects of technical changes are summarized overleaf.

SUMMARY OF FORMULATIONS

FORMULATION I	FORMULATION II	FORMULATION III
Data: industry classification only: TABLE D	Data: commodity and industry classification: TABLES A, B	Data: commodity and industry classification: TABLES A, B

(A) ESTIMATE COEFFICIENT MATRICES.

Equations: (3.1)(d), (c)	Equations: (3.1)(a), (b), (c)	Equations: (3.1)(b), (e), (c)

(B) CHANGE COEFFICIENTS IN MATRICES TO REFLECT TECHNICAL CHANGE.

(C) ESTIMATE TOTAL OUTPUT OF INDUSTRIES AND USE OF PRIMARY INPUTS.

Assumes: CIIA, IIPA	Assumes: CCIA, CMSA, ICAA	Assumes: CCIA, CCOA, ICAA
Equations: (3.3), (3.5)	Equations: (3.7)(a), (b), (3.8)	Equations: (3.10)(a), (b), (3.11)

(D) ESTIMATE PRICE LEVELS

Based on inter-industry matrix	Generates inter-industry matrix via PMPA	Generates interindustry matrix via PMPA
Equations: (3.26), (3.25)	Equations: (3.26), (3.2), (3.13), (3.30)	Equations: (3.26), (3.2), (3.13), (3.30)

It has been shown in this chapter that for the present purpose the two 2-matrix formulations II and III have advantages over the 1-matrix formulation I in that they make for easier identification of the physical substance represented by the elements of the matrices; they do not rely on fixed interindustry relationships; they do not necessitate the unrealistic assumption that each industry produces only one product and not a range of commodities; and they allow both inputs and outputs of each industry to be specified. A disadvantage of formulations II and III, however, is that they require that all commodities in the same class to be identical and freely available to all potential customers. Of the two 2-matrix formulations, formulation III has advantages over formulation II in that it is relatively easy to ensure that outputs and inputs conform to the conversion technology in each industry.

It is concluded that there may be substantial differences between the formulations with respect to both ease of use and results obtained. This conclusion shall be tested in the following chapter.

CHAPTER FOUR

THE PLASTIC MOTOR VEHICLE BODY

The previous chapter presented and compared the assumptions of three alternative formulations to use the UK input-output tables to estimate the economic effects of a technical change in UK industries. In this chapter these formulations are used to estimated the effects on the UK economy of a wide scale change to producing passenger motor bodies from plastic material rather than iron and steel.

The chapter is in two parts. In Part A the object is to compare the formulations for ease of use and for differences in results; the analysis is based on data which was readily available although in parts not very reliable. In Part B the object is to estimate the effects of the technical change; the results are based on more reliable data which became available at a later stage in the work.

PART A — COMPARISON OF FORMULATIONS

(4A-1) DISAGGREGATING THE INPUT-OUTPUT TABLES

The 70 order input-output tables were disaggregated as illustrated in Figure (4.1).

Industry 17, "synthetic resins, plastic materials and synthetic rubber", was disaggregated to isolate that plastic material which may be used for motor vehicle body production, so that only those industries which produced the relevant type of plastic material, or had inputs into the production of that material, would have increased output.

ABS was chosen as a material suitable for the manufacture of a plastic motor body (see Butler et al[9]). As ABS constitutes 1.4% of the output of industry 17 (Census of Production, Report 34, Table 5) the disaggregation of row and column 17 of Tables A, B, and D of the input-output tables could reasonably be done on such a proportionate basis if no other information were available. However, expert knowledge of the plastic industry was used to override this 1.4% proportion as often as possible.[1] Classification 17(a) was taken as ABS while 17(b) was taken as the remaining part of classification 17.

Similarly the passenger motor body section of classification 40, "motor vehicles and tractors", needs to be isolated. This classification includes commercial goods vehicles, road tractors, buses, battery-electric vehicles, trailers and caravans, as well as passenger cars and taxis. But as Table (4.1) shows, passenger vehicles constitute 70% of the completed vehicle output in value terms, and an approximate first step in the disaggregation of the input-output tables can be based on this proportion.

Having separated the passenger cars and taxis from the rest, the industry was pictured as shown in Figure (4.2) which was used as a guide to disaggregate classification 40 into the following four classifications:

- 40(a) Passenger motor body in white
- 40(b) Remaining parts of motor vehicle
- 40(c) Assembly of 40(a) with 40(b)
- 40(d) Other non-passenger car sections of 40

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1. I am indebted to Mr. Whalley of the Rubber and Plastics Research Association for his valuable co-operation.

Throughout the disaggregation the data was interpreted in the light of the average 1300 cc motor car produced in the UK (see Table (4.1)), and was controlled by four interrelated considerations.

(i) Proportions

The census data revealed that 56% of the total output is completed motor vehicles, and that 70% of these are passenger cars. Similarly other proportions were built up with the help of expert knowledge of the motor vehicle industry[2], as shown in Table (4.2). From these crude proportions Table (4.3) was constructed which shows the proportions in which classification 40 of the input-output tables could be disaggregated.

(ii) Expertise

The proportions of Table (4.3) were useful starting points, but in fact expertise was able to override these simple proportions in most classifications to lead to quite different proportions in the disaggregation of individual elements. The assembly process, 40(c), for example purchases no raw materials such as aluminium (indutsry 20) but only finished components such as assembled motor bodies (industry 40(a)), automotive parts (industry 40(b)), as well as construction and mechanical handling equipment (industry 27), electricity (industry 64), etc.

Consider for example the following two disaggregations:

(a) Flow B(19,40) – the motor industry's purchase of iron and steel.

The data supplied for inputs to the production of a 1300 cc motor vehicle included iron and steel use as tabulated in Table (4.4). The

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2. An expert employed in the motor vehicle industry was privately consulted. He critically commented on the disaggregation and supplied some valuable data used here.

proportions of iron and steel, by value going into body panel etc. for the average car thus is 38.8%. If passenger cars are 70% of the market then b(19,40) is disaggregated as shown in Table (4.5). This disaggregation tends to be corroberated by the data of census Table 10 and Iron and Steel Annual Statistics[39] which record steel purchases by the motor vehicle industry by thickness of sheet, and hence its use can be anticipated.

(b) Flow B(40,19) – the iron and steel industry's purchase of motor vehicle commodities.

In the input-output tables scrap arising in an industry is recorded as part of the principal product of the industry, and element B(40,19) represents in part a sale of process scrap to the iron and steel industry. If the scrap iron and steel sales as recorded in census Table 5 are upgraded to reflect sales from the whole industry then £9.6m of B(40,19) = 15.7 can be expected to be due to scrap sales.

Table (4.6) tabulates the disaggregation of Flow B(40,19) partly based on the proportions of Table (4.3) and the help of an expert.

(iii) Coefficients

Independently the disaggregation was also approached from the back door. A crude set of technical coefficients for the motor body industry was built up to reflect what the disaggregation should eventually reveal in coefficient form.

Some guidance in estimating these coefficients was given by Butler, et al[9] (who give the distribution of labour, material and tooling costs for sports car production at a rate of 500 per week), Census Table 4, and the figures supplied for the 1300 cc motor vehicle as shown in Table (4.7). Table (4.8) tabulates the distribution of input costs used as a guide, and agreed to by the expert privately consulted.

(iv) Balance

The input-output tables also impose considerable restrictions on the disaggregation. The row totals and column totals for Tables A, B, and D must all be consistent, and this requirement together with the above three interrelated controls very often revealed inconsistencies in judgement. Lack of balance initially was quite marked, indicating insufficient understanding of what the figures of the input-output classifications referred to, or that the translation of data based on engineering knowledge of production of one type of vehicle into economic figures relating to the industry as a whole was in error. Often the correct questions had not been asked.

These four interrelated considerations jointly controlled the disaggregation. The final disaggregation did not conform with each control individually – they were often in conflict with each other. The compromises resulted, for example, in B(19,40) becoming 55.0, 93.1, 0, 74.0 and the entries of Table (4.8) actually resulting in 37,26,13,2½, 10, 10 percent respectively.

It should be noted that the greatest effort was made only in isolating the motor body industry. The purpose of disaggregation into four separate industries was only to force more questions to be asked about the structure of the motor industry so that the motor body industry could be isolated with the maximum of confidence in accuracy: individually classifications 40(b), (c), and (d) were not necessarily identified as accurately, but in toto they do reflect the non-body sector of the motor industry, and in the computations these will be aggregated into one non-body part of the motor industry (thus leaving a 72 order matrix).

The first column of Table (4.9) shows the most important part of the disaggregation – column B(40a), the inputs into the motor body industry (altogether the disaggregation consists of 36 columns and rows which cannot all be shown here).

(4A-2) TECHNICAL COEFFICIENTS FOR THE PLASTIC MOTOR BODY

The object of this section is to find some plausible figures for the technical coefficients for plastic motor body production which may be used to obtain a first estimate of the changes in the economy and to test the three formulations presented in the previous chapter. The coefficients presented here are suggested from a study of some of the relevant literature of plastic use in engineering, and have been agreed as plausible by the experts consulted.

The literature of plastic application in engineering and plastic as a competition for other materials is, of course, very extensive. (References 9, 40 - 50 were found instructive and interesting by the present writer.) In automobile production plastics have, in one form or another, steadily made their presence felt. The literature describes a great variety of components manufactured from plastic, as well as numerous experimental and special purpose vehicles with all-plastic bodies. It is, however, largely true that plastics have not fulfilled the promises many "experts" had believed them to hold in the automotive industry. Only a very small proportion of the world's cars have plastic bodies. Sheet steel has remained a most durable material. The plastic component content of cars has risen steadily to near 40lb/unit in some instances. "This steady increase has amounted to evolution and not revolution." (Butler, et al[9] p. 7).

Broadly, the following considerations guided the building of technical coefficients for the plastic motor car body:

LABOUR

One advantage of using plastic rather than metal is that complex shapes can be moulded far more readily. The metal car body is assembled from over 100 components which are welded together, as illustrated in Figure (1.4). Some 70% of the labour costs are incurred in making the several thousand spot welds needed.[9] With plastic, however, 70 to 90% of the vehicle could be in the form of only three components[9,48] which are then joined by adhesives, Clearly substantial reductions in labour costs may be realised.

MATERIAL

The higher cost of plastic material clearly act against the plastic motor body. To obtain precise figures on material costs is clearly not possible until a reasonable attempt were made to design the vehicle. Fundamental design questions would need to be resolved: is there to be a metal frame, which provides the strength, on which to hang panels of plastic material so that the plastic acts merely as a shield, or is the plastic required to take stresses as well? The amount of plastic required would clearly be different if strength characteristics or volume characteristics were called for. Nevertheless, it appears reasonable to suppose that some 60% of the cost of the plastic motor body would be due to the main plastic material, while adhesives would amount to a further 5%[9]. Iron and steel usage would drop from 26% to near 2%. There will be far less waste material and paint requirement for plastic bodies[46].

TOOLING AND CAPITAL

Another advantage of plastic bodies is that the tooling required is likely to be considerably less. As indicated above, large parts of the plastic body can be moulded in one stage, while with metal many different components would have to be individually formed, and each single metal component may require several stampings[9]. Other overheads such as floor space, etc., will also be reduced. Frequent model changes would be easier and less costly. Certainly the £3½ - £4 M tooling costs for a 1500 cc motor body and the 2 - 3 year lead time could be much reduced [48]. Butler, et al[9] suggest that capital costs for plastic motor body production might be some 10 - 25% of the steel body counterpart.

ENERGY

Shaping plastic requires less power, but does require some increase in operating temperatures. Overall the energy requirements are expected to be less with plastic.

TRANSPORT

Transportation costs are expected to be somewhat reduced for the plastic car body because of a decreased weight in both the material input and in the finished body.

Table (4.9) presents the plausible coefficients obtained for the inputs to the plastic car body industry in the third column as opposed to the coefficients for the steel body in column two. The figures are clearly very tentative, and are to be used to test the formulations and provide a rough guide of the likely overall effects. There was a considerable difficulty experienced in obtaining even these rough estimates of likely coefficients. No change has been made in the coefficients for which the likely changes were completely unknown. This column vector of input coefficients was substituted for the steel body coefficient inputs in the formulations of the previous chapter.

(4A-3) RESULTS

The results of the calculations are tabulated in Table (4.10). The computations were carried out with the 72 order matrices (the non-body part of the motor industry aggregated into one category) for each of the three formulations, and also with the full 74 order matrices and the 71 order matrices (ABS not separated from the plastic materials industry, motor body separated from the motor industry) for formulation III.

It is apparent that the results using each of the alternative methods are basically similar.

Consider firstly the differences between the formulations applied to the 72 order matrices.

Overall the results are not greatly different from one formulation to another for most industries, but the output of ABS is noticeably different in the case of formulation II. Under each of the three methods the output of ABS rises from £6.3m to £128.6m, £115.9m and £134.7m under formulation I, II, and III respectively – despite the fact that the motor body industry purchases £122.4m of ABS in each case. Under formulation II the results are much as expected. The plastic industry maintains a constant percentage of the total market and a substantial part of the increased ABS required by the car body industry is supplied by other industries which produce ABS as a secondary product. Industry 61 (other manufacturing) for example originally produced only £0.2m worth of ABS out of a total output of £1226.9m, while after the change this industry produces £3.8m of ABS out of a total output of £1227.7m. This corresponds to a nineteenfold increase in the proportion of ABS output to total output, and other industries had similar increases in this proportion. (The demand for their principal product remained roughly constant while ABS demand increased nineteenfold.) The ABS industry itself in this case produces only £114.1m ABS, a constant 90% of the market.

Under formulation III the increased demand of ABS has passed entirely on to that industry. The other industries which also produce some small amounts of ABS do not produce significantly more since their total output, to which their ABS production is linked, does not rise significantly. The market share of these industries has reduced to approximately one nineteenth of their original values while the ABS industry now produces over 99% of that commodity.

In formulation I the results are not very different from formulation III, except perhaps for the ABS industry. This might be taken as implying that the difficulties of identifying the substance represented by the elements of matrix D were reasonably well overcome. It would be more reasonable, however, to suggest that the differences were slight because the changes proposed did not effect many industries significantly, and further, that the differences in the assumptions upon which the formulations are based and differences in the elements of matrices B and D are not such as to lead to substantially different results. This conclusion is supported by Armstrong(33) who found that differences in assumptions used to translate make and absorption matrices into symmetric matrices does not lead to substantially different elements in the symmetric matrices: hence formulations based on different matrices may not, for the present purposes, lead to substantially different results.

Consider now the results obtained by the use of formulation III with matrices of different order. Again there is little difference in the results.

In the case of the 74 order matrices model the similarity of the results with the 72 order model suggests that the disaggregation of the motor industry was reasonably accurate for each of the four new classifications. As appealing as this view might be, it is probably more correct to suggest that because changes have not been postulated in the three non-body industries, the aggregation or disaggregation of these does not affect the result greatly.

In the case of the 71 order model there is slightly more variation due to differences between the inputs used for the ABS industry and the full plastics industry. But it needs to be pointed out that the disaggregation of the plastics industry proved very difficult because of a lack of sufficient access to expertise in that field. Also, the ABS industry is such as small part of the total plastic materials industry that its elements may be swamped by the inherent errors of the input-output tables. For these reasons it is inappropriate to emphasize the differences between the 71 order model and the 72 and 74 order models.

(4A-4) CONCLUSIONS

The object of Part A of this chapter has been to compare the formulations developed in chapter 3 for differences in results and for ease of use in studies of the economic effects of technical changes in the manufacturing industries.

Above all, the present example has demonstrated that there are substantial difficulties in identifying the substance represented by the elements of the input-output tables, and difficulties in specifying the alternative technology in the input-output framework. These difficulties appear to far outweigh any differences between the formulations with respect to ease of use: these difficulties are liable to lead to errors far greater than any differences between the formulations.

For convenience, in this work formulation III has been used in Part B of this chapter and in chapter 5, while formulation I has been used in subsequent chapters (for reasons given in NOTE on page 97).

In Part B of this chapter 71 order matrices are used: the passenger motor body is disaggregated from the total motor industry. The disaggregation of the plastic materials industry was abandoned because of insufficient ready access to expert knowledge of that industry.

PART B — ESTIMATE OF EFFECTS OF TECHNICAL CHANGE

(4B-5) A REAPPRAISAL OF DATA[3]

The problem which arose repeatedly in attempting to obtain technical coefficients for plastic motor body production was to specify the production rate. Different production rates mean different techniques and plastic type to be used. Production of 100 units per week could

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3. I am indebted to Professor B. B. Hundy, Professor of Automobile Manufacture, Cranfield Institute of Technology, for his valuable guidance and assistance in reappraising the data of Part A, and making further data available. Part B was completed several months after Part A.

most economically be achieved with hand layout techniques using fibre glass, while at rates above 200 per week this appears no longer to be economical (although Gurney[49] suggests that rates of 1000 per week may be economical). In reviewing the disaggregation of Part A of this chapter with variable production rates in mind, some inconsistencies were found and the data had to be amended.

In 1968 the UK motor vehicle industry produced at least 42 different motor vehicles (some of the low volume production is collected as "other" in the data [50]) with production rates varying from less than one to nearly 6,000 per week. As shown in Table (4.11), at the high rate only 4 vehicle types account for over 50% of total production, while at the lower end 26 vehicle types account for only 12.4%.

With the help of an expert and data made available by him the production costs of Table (4.11) were estimated for average cars in each of the production rates. The estimates for plastic car bodies are based on urethane. The high production rates refer to small cars (Mini, Viva, Cortina, BMC 1100/1300), the midrange cars include some large cars (jaguar, Zephyr, Corsair etc.) while the low production rate cars are a mixture of small sports cars (e.g. Lotus, Rapier, Sprite) and specialist cars (e.g. E-type, RR, Jensen, Reliant etc.). The surprisingly high total figure for the low production rate cars is largely due to the average being pushed up by the specialist, expensive vehicles.

Figure (4.3) is a plot of the costs of plastic and steel motor body production with production rate, and this is used here to illustrate the differences between the costs and the underlying ideas. Figure (4.2) refers, of course, to the production cost economies of scale of only one car. It is not a diagram appropriate to the UK car production as a whole, as quite different cars are produced at the various rates. A plot of UK car production — rate vs cost — would have substantial discontinuities.

The precise shape of the curves will of course vary with the type and size of car body under consideration, as well as the type of plastic and technology used. The curves would no doubt be discontinuous at a number of points because the technique of production would alter, as well as the material used, as the production rate increases and other techniques become economically optimal. Nevertheless, the relationship between the cost of bodies under different materials appears to be roughly as illustrated.

At low rates the plastic body is cheaper to produce because of very low capital costs offsetting the higher material cost per unit. A set of moulds cost about £125,000 which can produce up to 350 bodies per week with 2 shifts and normal setting times. The tools for steel body production are at least 8 times this amount, and at such low production rates they are not used economically. As the production rate increases the cost of tools for both materials increases, but far more rapidly for plastic forming tools. For steel body production tools to operate at higher production rates are somewhat more expensive, but other tools can simply be used more intensively to operate closer to their optimum rate. Plastic motor body forming tools are limited in the intensity to which they can be used by setting times.[4] Increases in production rates

4. See Gurney (49), and: "However, forming cycles are much slower for plastic than for steel. The main body of the Formacar take 2½ min.; similar steel-body parts can be stamped out in less than 30 sec." (West (47) p. 78)

thus imply purchases of tools to operate at higher rates, which are considerably more expensive, as well as duplicating sets of moulds which have low economic usage rates. As production rates rise the widely held view that plastic components are less capital intensive than metal components appears to become invalid. At high production rates plastic car bodies are more expensive not only because of the higher price of plastic material, but also because of higher capita costs which causes the "plastics" curve of Figure (4.2) to rise rapidly.

It needs to be pointed out that the data for plastic motor bodies presented in Table (4.11) are still subject to the same doubt which was raised in Part A of this chapter – particularly for the higher production rates. The costing of plastic motor bodies was found to be very difficult by the expert consulted. For large production rates the price of the plastic car body is substantially higher than for the steel car body, and costing in this area of the market had consequently not been attempted previously by the expert consulted. At the low rates production of plastic car bodies – particularly sports car bodies – has been in progress for a long time. The data for this end of the scale were thus far more readily available.

The data of Table (4.11) when summed contradicted some of the figures which were used in the disaggregation in Part A, and it was found some errors had been made. In particular, the gross output of the motor body industry had been overestimated, and consequently the purchase of iron and steel, labour, capital, etc., by the motor body industry had also been overestimated. The adjusted figures for the disaggregation of column B(40) is shown in Table (4.12) for motor body inputs, as well as the figures for steel and plastic motor bodies in coefficient form. The last figures are an expansion of the data of Table (4.11) to encompass all commodity purchases, but totalled for the whole range of UK motor vehicles. That is, the figures for plastic and steel body production as recorded in Table (4.11) were expanded for each production rate range into 70 commodity purchases, and then these were summed over all production rate to obtain figures for the whole industry as shown in Table (4.12).

It needs to be pointed out that the plastic car body, for the industry as a whole, is some 13% more expensive[5] than the steel car body. For the high production rate the figure is 28%. Column 4 of Table (4.12) gives the technical coefficients for the plastic car body relative to its own higher price. To obtain figures relative to the old price the coefficients need to be multiplied by 1.13. The cost of plastic, for example, relative to the old price thus becomes 0.6667, and these adjusted coefficients are the ones used for the computation of new industrial output.

(4B-6) RESULTS AND CONCLUSIONS

Table (4.13) shows the percentage change in gross output of each industry after the change to plastic has taken place. These results are largely similar to those obtained in Part A, Table (4.10), above.

The results indicate that there are no substantial overall changes in the UK economy due to this technical change.

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5. Using data from Table (4.3) this 13% increase in the body cost leads to a 3.3% increase in final motor vehicle cost, and a 1.6% in the total industry output — which does not appear significant for the economy as a whole.

The reduction in the iron and steel industry's gross output is only 2.6% (£57.4m), and hence the suppliers of the iron and steel industry do not have any substantial loss of demand for their products, the exception being the coke industry. The plastic industry does have a substantial increase in its output, 26.6% (£120.8m), but this industry in 1968 did not make significant demands on most UK industries. Only for the chemical industry is there a significant, 1.7% (£38.8m), induced increased output.

For the energy industries the gross output of coal is reduced by 0.1% (£0.8m), coke 1.1% (£2.6m), and gas by 0.1% (£0.6m), while oil output rises by 0.2% (£1.9m) and electricity by 0.2% (£3.2m).

For the materials industries other than iron and steel and plastic, the technical change has induced reductions in the gross output of paint, aluminium, other non-ferrous metals, and increases in mining and quarrying, pottery and glass, timber, and paper and board output.

Table (4.14) tabulates the resultant changes in the primary inputs. The technical change indicates an increased imports requirement of £18m (0.3% of total industrial imports), £250m (0.3%) in total capital stock, and a saving of 25 thousand (0.2%) man-years of total labour.

But while the overall changes do not appear to be substantial there are considerable differences in how the totals are made up.

The motor vehicle industry's total loss of jobs is 36 th. – 35 th. of which are in the body workshop. The original workforce of 44 th. in the motor body industry is reduced to only 9 th., and these will need to be skilled in plastic forming and joining, rather than in metal forming, cutting and welding. In the steel industry the loss of jobs is

11 th., the major portion being in plants manufacturing steel sheets and wide continuous coil.[6] These losses are partly offset by increased employment of 15 th. in the plastic materials industry, 5.5 th. in the chemical industry, and a net gain of 7.3 th. in other industries.

Similarly the composition of the imports to the UK and the type of capital stock alters, as does the pattern of interindustry flow of goods about the UK economy.

It is concluded that a substitution of plastic for steel in the manufacture of all UK produced passenger motor vehicle bodies would significantly alter only the output of the plastic material, chemical, iron and steel, and coke industries. These changed outputs would not spread to other industries: in no other industries would a substantial change in gross output be indirectly induced, nor would the total primary input requirements for all UK industries alter significantly. Howevery, the composition of the totals — the pattern of interindustry transactions and industrial use of primary inputs, particularly labour — may alter significantly.

6. Using data from chapter 5 on the composition of iron and steel output, the 2.6% reduction in output reflects a reduction of approximately 24% in the output of steel sheets and continuous wide coil.

CHAPTER FIVE

CHANGES IN MATERIALS CONVERSION EFFICIENCY

(5-1) INTRODUCTION

The object of this chapter is to estimate the effects on the UK economy of changes in the efficiency with which materials are converted into final products. Two changes are considered: firstly, a uniform 10% reduction in the direct material content of final products; secondly, a uniform 50% reduction in the direct iron and steel scrap arising in each engineering industry.[1]

Define locally (see Figure (5.1)):

m_t	the total material input
m_s	the material which ends up as scrap
m_p	the material which ends up in the product
P	the value of the commodity produced

Under conditions of constant prices we shall consider producing the same P with:

Section (5-2) a 10% decrease in m_t while $\frac{m_s}{m_t}$ remains constant

Section (5-3) a 50% decrease in $\frac{m_s}{m_t}$ while m_p remains constant

1. Data is not readily available in sufficient detail on non-ferrous waste to allow this to be included in the study.

Section (5-3) the combination of the above two parts

The technical means of achieving these changes is not specified here in detail, but it is assumed that no other industrial input is used in an altered amount.[2] The material saving could however be accompanied by primary input savings, or indeed they could be achieved because of a substitution of capital costs (perhaps more expensive less wasteful machines) or labour costs (perhaps more expensive skilled labour) for material cost. These options are illustrated in Figure (5.2).

In this work the maximum employment increases, wage increases, or price reductions achievable by allocating the material cost savings to the options are highlighted. These three options can be considered as three vertices of a tetrahedron as illustrated in Figure (5.3). The fourth vertex represents the use of the material cost savings for payments to the capital account or to profits. Any point within the tetrahedron is a compromise between the four option vertices, and shows that the material saving was achieved by substituting more labour, paying higher wages and salaries, and (possibly) using more capital, and resulted in price reductions and (possibly) higher profits.

(5-2) REDUCTION IN MATERIAL CONTENT OF COMMODITIES

(5-2.1) Reasons for Reduction

The reduction of the material content of a commodity which satisfies a given need reflects more a change in design than process. The impetus for design changes may originate from:

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2. That is for example, energy and transport costs do not alter when less materials are used.

(a) Taste Factors: Consumer taste change in favour of

(i) slim-line commodities

(ii) plainer commodities; cars without fins, etc.

(b) Respecification of Need

(i) by consumer; the consumer may find that he does not need a large car, boat, TV, etc.

(ii) by engineer; correct study of an engineering component may reveal that it has been overspecified in terms of strength, etc., requirements.

(c) Design

(i) component; study of specifications may reveal possible design alterations, such as a thin veneer over a strong frame giving similar strength characteristics with less material than a solid component.

(ii) assembly; a different design of an assembly may achieve the same objective more economically with respect to materials.

(d) Technology

(i) in materials; new materials with greater strength.

(ii) in design; eg. rotary engine, electronics, etc.

(iii) in process; a new process may be able to use materials more efficiently, e.g. stamping thinner, more efficient castings, etc.

(5-2.2) Results

The industries for this study were classified as:[3]

MATERIALS: 2, 4, 13, 17, 19 - 21, 52 - 54, 56, 60

ENGINEERING: 22 - 42, 55, 56, 61, 62

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3. From the Census industry 2 is seen to be only ½ materials, and industry 56 is 1/3 materials, 2/3 engineering. Allowances were made in the analysis for these partial classifications: see Appendix A2.

and in matrix B of formulation III (chapter 3) the inputs from these materials industries the the engineering industries was decreased by 10%, as was the direct import of material in row 1 of matrix U (using data from Table C). The results are tabulated in Table (5.1).

Column (1) clearly shows shows that there are substantial savings in gross output for each of the materials industries. But there are also substantial savings in other industrial outputs as well. In the case of energy £11.5m of coal, £8.6m of oil, £11.1m of coke, £4.1m of gas, and £12.9m of electricity is saved.

In the case of the primary inputs the change has induced a saving of £130m of imports, £980m of capital stock, and 114 th. man-years of labour. The imports savings are largely direct material imports to the materials and engineering industries – other types of imports are not greatly affected. Capital stock and labour savings, however, are more spread and only 2/3 falls in the materials industries – fully one third of the total savings have been indirectly induced in the engineering and "other" industries.

The material cost saving sums to £368.5m, and this could be used to finance the technical change. The major part of this cost saving is in the construction, other metal goods, and motor vehicle industries.

(a) If the material saving is achieved without substitution then this cost saving is available for profits, or can be passed on to

(i) consumer: Column (4) shows the price reductions possible for each commodity. Overall the final price reduction for all UK produced goods is 1.0% (Table (5.1)(c)).

(ii) labour: Column (5) shows the average rise in TIFE in the engineering industries. While labour suffers a loss of 114 th. jobs, its total income rises by £368.5m - £154.4m = £214m because of the rises in the engineering industries.

(b) If the material saving is achieved by substitution then the cost saving can be used to pay for new capital stock, or

(i) higher paid labour: The average wages will rise as in (a)(ii) above.

(ii) more labour at constant wages: The cost saving can finance 291.4 th. man-years of labour at constant TIFE. Column (6) shows that most of this labour will be in the construction, other metal goods, and motor vehicle industries. Total employment in engineering industries will rise by 291.4 - 10.2 = 281.2 th. jobs and fall by 69.6 th. and 34.3 th. in the materials and "other" industries respectively, with a net gain of 77.3 th. jobs and a rise of £368.5m - £154.4m = £214.1m in total income (TIFE).

(5-3) REDUCTION IN IRON AND STEEL WASTE

Materials are wasted in many engineering processes. As much as half of the material purchased may be degraded into scrap in processes such as machining, forging, and stamping (see Tables (5.2), (5.3), and (5.4), and similar results reported by Bahiri[5] and Singh[53]).

In the context of the resources conversion network of Figure (1.1) this waste implies that all the resources which were used upstream to produce the wasted material, are also wasted. A part of the UK's energy, capital stock, labour, imports, etc., was employed to manufacture materials which are wasted in downstream engineering processes. And most of this resource waste cannot be averted by materials recycling, because recycling entails reprocessing and re-employment of many of the resources necessary for materials manufacturing [4] (see Becker and Pick[(59)]).

In this section it is proposed to assess the effects on the UK's manufacturing resources of a 50% reduction in the engineering industries.

It is emphasized that this is in fact only a small part of the total scrap arising in the UK. As well as "process scrap" arising in the engineering industries, "circulating scrap" arises in the iron and steel industry, and "capital scrap" arises when capital goods are discarded at the end of their useful working life [5] (nomenclature after Boughey[(60)]). Figure(5.4) illustrates the flow of scrap in these three categories about the UK, and this model was used as a guide in collecting data.

4.And also the use of considerable resources to collect and process scrap before returning it to the material manufacturers (see references 54 - 58).

5. It is of interest to note that scrap arising within each of these categories may vary with non-technical factors. For circulating scrap the proportion varies with product: "For example, in the case of sections, the proportion is about 19%, increasing to 32% for plates, 38% for tubes and about 47% for forgings. Changes in the pattern of steel produced, therefore, materially affect the overall average own arising ratio." (Boughey (60), p. 16). Similarly each engineering industry has a different proportion of scrap arising, and changes in the consumer pattern of demand may alter process scrap arising. Capital scrap arising may depend on investment decisions over a long period of time.

(5-3.1) Reasons for Reduction

(a) Technical Changes

A change in process used may do much to reduce waste, as illustrated in the excellent report by the Institute of Production Engineers[(52)]; the introduction of electrostatic painting dramatically reduced paint waste (and labour costs) as shown by the data from one company in Table (5.5); the wider use of powder metal forming techniques may reduce metal waste in the future, etc. [6]

(b) Design Changes

Modern designs emphasizing clean, straight lines may be machined without the waste of more ornate designs.

(c) Operational Changes

(i) by engineers ensuring that casts are not unduely large so as to require excessive machining, that sections of metal sheet are suitable for machining into specified shapes, etc.

(ii) by workmen in handling material input and output to avoid damage [7], in careful machining to reduce the number of rejects and spoils [8], etc.

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6. Within the iron and steel industry continuous casting has reduced circulating scrap since it wastes only 6% as against 20% for conventional methods (Boughey (60)).

7. "Small pits produced by stamping atmospheric pollution into the sheet (metal), coupled with scared lines produced by indifferent handling, may reduce the strength of sheet by more than 10% where the two faults coincide on both sides of the sheet." (Lloyd-Lucas (61), p. 162)

8. Boredom and tiredness resulting in poor workmanship and spoils was relieved by a management approach in one company, as reported by Lloyd-Lucas. He comments: "It is not uncommon for scrap and rework to be as much as 10%, sometimes as high as 50%, of the end cost of a component. Reference to the balance sheets, even of successful companies, will show that the actual profit on turnover is small. It may be as low as 2%, rarely as high as 10%, Yet scrap and rework costs are blithely accepted at 2%, 5% or 10% as being inevitable, or at least acceptable." (Lloyd-Lucas (61), p. 161)

(5-3.2) Methods and Data

A reduction in the proportion of metal wasted results in a decreased scrap metal sale, and a decreased purchase of new metal.

Using m_t, m_s, and m_p as defined at the outset, with superscripts (1) or (2) on m_t, m_s to denote the quantity of metal before and after the change, respectively, the a δ% change in the proportion [9] of metal wasted, $\frac{m_s}{m_t}$, leads to

(5.1) $$m_t^{(2)} = \frac{m_t^{(1)}}{1 \div \frac{\delta}{100} \frac{m_s^{(1)}}{m_p}}$$

(5-2) $$m_s^{(2)} = m_t^{(2)} - m_p$$

as the purchases of new metal and scrap arising after the change.

In physical terms clearly

$$\begin{pmatrix}\text{Quantity of}\\ \text{scrap reduced}\end{pmatrix} = \begin{pmatrix}\text{Quantity of}\\ \text{new metal saved}\end{pmatrix}$$

Therefore

$$m_s^{(1)} - m_s^{(2)} = m_t^{(1)} - m_t^{(2)}$$

9. A reduction in the proportion of material waste is considered rather than a reduction in the absolute amount because the assumptions of formulation III are expressed as proportions. A reduction in the absolute amount of scrap arising could (theoretically) be technically inconsistent with output and with new metal purchases.

In value terms if π_n and π_s are the price per ton of new metal and scrap respectively, then

$$(5.3) \qquad \begin{pmatrix} \text{Reduction in value} \\ \text{of scrap sold} \end{pmatrix} = \pi_s (m_s^{(1)} - m_s^{(2)})$$

$$(5.4) \qquad \begin{pmatrix} \text{Reduction in value} \\ \text{of new metal purchased} \end{pmatrix} = \pi_n (m_t^{(1)} - m_t^{(2)})$$

$$(5.5) \qquad \text{Cost saving} = (\pi_n - \pi_s)(m_t^{(1)} - m_t^{(2)})$$

Use of these equations to estimate the effects of waste reduction requires data on the tons of scrap arising in each industry, the value of this scrap, and the value of the new metal saved when the scrap arising is reduced. These data were largely obtained from the Census of Production reports. However, often metal transactions by each industry were not recorded in physical quantities in the Census. In the case of the motor vehicle industry, for example, the scrap sold is recorded in value terms only, and as this is 37% of the total value of process scrap sold by the UK engineering industries the omission represents a serious data gap. Such data gaps were filled by using the known prices for metal transactions from similar industries. Supplementary sources of data were used[39, 62], and guidance from experts was obtained.[10]

The data compiled on total scrap transactions is tabulated in Table (5.6) and illustrated in the context of the ferrous metal flow diagram of Figure (5.5). The data on new metal and scrap transactions by each input-output engineering industry is tabulated in Table (5.7) from which it is seen that the proportion of material waste varied from 7.4% (insu-

10. I am indebted to Mr. D. Keeling, Chief Scrap Buyer, British Steel Corporation, for his assistance.

lated wires and cables) to 53.5% (other vehicles) with an average of 18.5%. The loss in value per ton of metal turned into scrap varied from 73% (engineers' small tools) to 95% (industrial engines) with an average of 88%. The motor vehicle industry which accounted for 37% of all process scrap turned 29.4% of its iron and steel purchases into scrap, losing 89% of its value.

New metal is classified in Table (5.7) by each of the 12 categories used in the Census, but only one figure for the total scrap arising in each industry is known. It is not known from which of the metal types purchased by each industry the scrap originates. This presents the analysis with a major difficulty because the price of new metal bought varies substantially from pig iron at an average of £22/ton to steel castings at an average of £200/ton. Clearly there will be substantial differences in the value of new metal saved, equation (5.4), and in the cost saving, equation (5.5), if the scrap in an industry was obtained largely from the lower priced new metal, or if it was obtained from a higher priced new metal.

To overcome this problem an attempt was made to estimate the scrap arising from each of the twelve groups of metal in each industry by regression techniques. The total scrap arising in each industry is known; the new metal purchased by each industry is also known. It is not unlikely that the industry variation in the proportion of new metal wasted arises in part due to the variation in the proportions in which industries purchase the twelve new metal types, as well as a general variation in the efficiency of metal conversion between industries.

Using m_s and m_t as defined in section (5-1) and

m_j the metal of type j purchased, $j = 1,....,12$

α_j the proportion of metal of type j wasted

i a suffix for indistry $i = 1, ..., 20$

γ_i an efficiency factor of material conversion for industry i

we have $\alpha_j\gamma_i$ as the proportion of metal of type j wasted in industry i. The α_j can be estimated by a multilinear regression model

$$\left(\frac{m_s}{m_t}\right)_i = \sum_j \alpha_j \left(\frac{m_j}{m_t}\right)_j$$

and γ_i from

$$\gamma_i = \frac{\text{observed scrap in industry i}}{\text{regression estimated scrap in industry i}}$$

$$= \left(\frac{m_s}{m_t}\right)_i \Big/ \widehat{\left(\frac{m_s}{m_t}\right)}_i$$

It was hoped that this model, and a corresponding one in terms of prices, would give an estimate of the proportion of scrap originating in each industry from each metal type in physical and monetary terms. However, the analysis was unsuccessful. Table (5.8) shows the results obtained. The regression model could explain only 65% of the total variance, and the prediction of scrap for each industry was in error by an average of 28%. But the main problem was that the α_j did not conform to $0 \leq \alpha_j \leq 1$, so that the regression coefficients could not validly be interpreted as proportions of metal waste.

Several variations of the data were used in the model; leaving out unusual industries, combining industries (e.g. electrical, mechanical), combining metal types, using the full 40 industries of the Census reports for which the data was available, etc. But in each case the results were similar to those reported in Table (5.8). A test was also made of the correlation between the proportion of total metal wasted , $\frac{m_s}{m_t}$, with the proportion of metal type j purchased, $\frac{m_j}{m_t}$, for each j (except tinplate, j = 8, which was purchased by only 9 of the 20 industries). The results are illustrated in Figure (5.6). For none of the 11 metal types is there any significant correlation: for forgings 22%. steel castings 24%, and for "other" metal 46% of the total variance could be attributed to a least squares regression line, but for all other metal types almost none of the total variance could be attributed to such regression.

The conclusion to be drawn is that there is considerable variation between industries in the efficiency of material utilization. This variation dominates and frustrates attempts to estimate average proportions of scrap arising by metal type over all industries. (Perhaps this is not unexpected when one considers the varying nature of the industries and processes, and the ranges of material utilization reported in Tables (5.2), (5.3), and (5.4).) We are thus unable to reduce the proportion of scrap arising of each individual metal type, or to use the price of that new metal type to reduce the industry's purchase of new metal.

The procedure adopted here is to reduce the proportion of scrap arising for the industry as a whole, and to reduce the purchase of new metal by using the average price of metal purchased by the industry from the domestic and foreign suppliers so as to maintain a constant ratio between the two sources.

(5-3.3) Direct and Indirect Scrap

Before discussing the results of the reduction of process scrap arising in the engineering industries, it is of interest to estimate the total scrap arising in the production of final products. Table (5.7) records the direct scrap arising, but there is also substantial indirect scrap arising in other engineering industries, and circulating in the iron and steel industry.

The indirect process scrap arising in other engineering industries was estimated by summing over all engineering industries the product of total purchases (direct plus indirect coefficient) from those industries and the process scrap generated in that industry as a proportion of gross output, and subsequently subtracting the direct scrap arising from this total. Indirect scrap circulating in the iron and steel industry is total (direct plus indirect coefficient) purchase from the iron and steel industry multiplied by the value of scrap circulating per unit output of iron and steel $\left(\frac{132.4}{2206.3} = 0.06 \right)$.

The results are tabulated in coefficient form in Table (5.9). In all cases the direct scrap arising is less than the indirect.

Each £m of motor vehicle output, for example, generated £3800 of scrap directly in the motor vehicle industry, plus £1200 indirectly in other engineering industries, plus £12100 circulating scrap in the iron and steel industry. The £2637m gross output of the motor vehicle industry in 1968 thus generated £10m (1.1 th. tons) of scrap directly in the motor vehicle industry, £3m (0.3 th. tons) indirectly in other engineering industries, and £32m (24.4 th. tons) circulating scrap in the iron and steel industry. In toto, the manufacture of the motor vehicle industry's gross output generated almost 50% of all 1968 process scrap and almost 25% of all circulating scrap.

(5-3.4) Results

The data of Table (5.7) was used with formulae (5.1) to (5.5) to estimate the effects of a δ = 50% reduction in the direct scrap arising in each engineering industry (see Appendix A2 for details of the program used).

Industrial output and primary input savings are tabulated in Table (5.10). Savings of domestic iron and steel amount to £172m, and induced savings of other engineering materials include £1.2m of aluminium, £8.0m of other non-ferrous metals, £0.9m of plastic, and £1.9m of rubber. Energy savings are £7.4m of coal, £3.8m of oil, £8.7m of coke, £2.3m of gas, and £4.8m of electricity. A total of £30m of imports are saved, £460m of capital stock, and 62.5 th. man-years of labour. Most of the import savings are due to direct savings of imported iron and steel by the scrap generating industries and imports of iron ore by the iron and steel industry, but 1/3 of the saving is indirectly induced in "other" industries. Almost ½ of capital stock and labour savings have been indirectly induced in the "other" and the scrap generating industries.

The cost saving is tabulated in Table (5.11). The material cost saving sums to £126.1m for domestic iron and steel and £8.5m for imported iron and steel, the total being £134.6m. Against this the scrap generating industries lose £16.1m because of decreased scrap sales, and the net cost saving is £118.5m which could be used to finance the waste reduction. Almost 40% of the cost saving is in the motor vehicle industry.

Table (5.12) shows some of the effects of allocating the cost saving to the options of Figures (5.2) and (5.3).

(a) If the waste reduction is achieved without substitution then the cost saving is available for profits, or can be passed on to

(i) consumers: Column (1) shows the price reductions possible for each commodity. Overall the final price reduction for UK produced goods is 0.4% (Table (5.12) (b)).

(ii) labour: Column (2) shows the average rise in TIFE in the scrap generating industries. While labour suffers a loss of 62.5 th. jobs, its total income rises by £118.5m - £76.0m = £42.5m because of the rises in the scrap generating industries.

(b) If the material saving is achieved by substitution then the cost saving can be used to pay for new capital stock, or

(i) higher paid labour: The average wages and salaries will rise as in (a)(ii) above.

(ii) more labour at constant wages: The cost saving will finance 98.9 th. man-years of labour at constant TIFE. Column (3) shows that most of this labour will be in the motor vehicle and other metal goods industries. Total employment in the scrap generating industries will rise by 98.9 - 13.1 = 85.8 th. jobs and fall by 32.3 th. and 17.1 th. in the iron and steel, and "other" industries, respectively, with net gain of 46.4 th. jobs and a rise of £118.5m - £76.0m = £42.5m in total income (TIFE).

(5-4) REDUCTION IN MATERIAL CONTENT AND IRON AND STEEL WASTE

The results when both changes are combined are tabulated in Table (5.13). The results are approximately the sum of the results for each separate change.[11]

Savings include material savings of £333m of UK produced iron and steel, £25.3m of aluminium, £71.8m of other non-ferrous metals, £20.9m of plastic, £23.6m of rubber, £19.0m of timber, £17.0m of pottery and glass, £18.7m of mining and quarry products, £11.3m of paint, £55.8m of building materials, etc.

Energy savings are £18.1m of coal, £12.4m of oil, £18.9m of coke, £5.9m of gas, and £17.5m of electricity. Total imports savings are £157m, £1430m of capital stock is saved, and 173.4 th. man-years of labour: £20.3m of the imports savings, £54.4m of the capital stock, and 62.5 th. of the labour savings are in the iron and steel industry.

11. The savings for the combination of the changes are less than the sum of the savings from each individual change because the "interaction" of the changes. The additivity and interaction of changes is discussed in chapters 8 and 9.

The cost saving sums to £487m, and this could be used to finance the changes. The major part of this cost saving is in the construction, other metal goods, and motor vehicle industries.

(a) If the material saving and waste reduction is achieved without substitution then the cost saving is available for profit, or it can be passed on to

(i) consumers: Column (4) shows the price reductions possible for each commodity. Overall, (Table (5.13) (c)) final price reductions for UK produced goods fall by 0.8% for consumers (a saving of £132m), 1.2% for public authorities (£40m), 2.2% for capital purchasers (£154m), 1.7% for foreign buyers (£135m), and 1.3% for all final goods (£497m). The loss of revenue from export is £31m greater than the import savings, and, in the absence of economic policy changes, the technical changes could have adverse effects on the Balance of Trade.

(ii) labour: Column (5) shows the average rise in TIFE in the industries making the changes. While labour suffers a loss of 173.4 th. jobs, its total income rises by £487.0m - £222.9m = £264.1m because of the rises in the industries making the changes.

(b) If the materials saving and waste reduction is achieved by substitution then the cost saving can be used to pay for new capital stock, or

(i) higher paid labour: The average wages will rise as in (a)(ii) above.

(ii) <u>more labour at constant wages</u>: The cost saving will finance 390.7 th. man-years of labour at constant TIFE. Column (6) shows that most of this labour will be in the construction, other metal goods, and motor vehicles industries. Total employment in the scrap generating industries will rise by 276.6 - 21.1 = 255.5 th. jobs, in other engineering industries by 114.1 - 0.5 = 113.6 th. jobs. Employment will fall by 62.5 th. in the iron and steel industry, 38.2 th. in other materials industries, and by 51.1 th. in "other" industries. The net gain in employment is 217.3 th. jobs, and total income (TIFE) rises by £487.0m - £222.9m = £264.1m.

(5-5) CONCLUSIONS

The object of this chapter has been to estimate the effects on the UK economy of changes in the efficiency with which materials are converted into final products.

It has been demonstrated that design changes which reduce the material content of final engineering and construction output may result in substantial savings of a wide range of manufacturing resources. Similarly changes which reduce the ferrous metal waste arising in the engineering industries will also save a wide range of the resources which directly and indirectly were used to manufacture the wasted metal. A 50% reduction of iron and steel waste by itself releases almost as much of some of the resources as a 10% reduction of the total material content of final products: clearly a reduction in the waste of all other materials in engineering industries, and also in the materials manufacturing industries, will add to the results reported here.

Further, it must be recalled that the savings estimated here are exclusive of savings of materials determined non-material inputs (see section (1-2.1)). A reduction in material waste and content of output in a factory may also result in saving part of the energy, capital stock, labour, imports, etc., directly and indirectly used in materials acquisition, storage, machining, waste disposal, etc., in that factory. Such additional savings may be considerable: with a lathe, for example, the machining time — and hence the electriciy, labour and capital stock requirements — is directly proportional to the amount of waste material to be removed; a reduction in waste here could lead to significant non-material savings directly in that process. And a similar situation exists in blast furnaces, rolling mills, and a wide range of other manufacturing processes. Such savings will further add to the results reported here.

It can be concluded that widespread changes which reduce the material content of manufactured goods and reduce material waste could release substantial resources for redeployment for additional or alternative production.

NOTE

PUBLICATION OF 1968 INPUT-OUTPUT TABLES

This work was commenced in October 1972 using the 70 order provisional input-output tables supplied by the CSO. In late December 1973 the full 90 order 1968 tables were published, and these were processed and ready for use on the Aston University computer in March 1974. Subsequent work was based on the full 1968 tables.

At the same time a review was made of the methods and data used, and the following changes were made.

Method

(a) While for the above work the full 70 order matrices were used in the computations (although the resultant changes for industries such as food, textiles, etc., are not reported here), it was decided to aggregate the 90 order tables to 60 as shown in Appendix A3. This was done for computing convenience.

(b) The previous chapters were based on formulation III of chapter 3. The work of subsequent chapters is based on formulation I. The experience gained so far illustrated that the results of the alternative formulations are not significantly different; errors of data are likely to be far more significant than the differences between the formulations. Hence the 1-matrix formulation I has been adopted because it is considerably more convenient with respect to computing than the 2-matrix formulations. Gross output is measured free from duplication in subsequent chapters.

Data

The initial computations included primary input as recorded in the input-output tables. These were considered to be unsuitable from an engineering point of view, and were replaced by additional data on labour and capital stock. Details of data sources and methods are given in Appendix A3. The analyses of the previous chapters were repeated with the new data to obtain the effects on primary inputs reported above.

CHAPTER SIX

THE ENERGY CONTENT OF MATERIALS USED IN ENGINEERING AND CONSTRUCTION[1]

The object of this chapter is to demonstrate that a large part of the total energy used in the manufacture of UK engineering and construction output is used indirectly via materials. In fact, the energy content of materials used by these industries is shown to be considerably greater than the energy used directly in the conversion of the materials into final goods. A comparison is made of the effects on the economy's total energy requirements of material saving and direct energy saving in the engineering and construction industries.

Energy materials such as creosote and pitch, which account for less than 3% of energy, have been ignored here. No distinction is made between the use of fuels as sources of energy and as process materials (e.g. coke in steel making, oil in plastic and chemicals). Only the energy content of UK produced materials is included; hence energy savings are savings of UK produced or directly imported energy (not by foreign suppliers of materials) and material savings do not apply to materials directly imported by engineering and construction industries.

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1. Some of the results of this chapter were reported in Pick and Becker (63).

(6-1) DIRECT ENERGY CONSUMPTION

The direct energy consumption in the UK may be obtained for each consumer from Table D of the input-output tables, and Table C for imports of oil.

However, the tables are expressed as the value of energy purchased. Consequently differences in the price charged to different consumers will bias the comsumption of energy in favour of those consumers charged highest prices. Secondly, the table records the total purchases by consumers from the energy industries. This total transaction may include some non-energy purchases such as electric, gas, etc., installations and repairs.

However, adjustments can be made for these two factors by comparing Table D with the energy consumption recorded in Tables 9 to 12 of UK Energy Statistics. 2 (36) In particular the substantial differences between domestic consumer and industrial prices can be allowed for. Table (6.1) records the total energy used in original units for domestic, public authorities, and industrial users obtained from the Energy Statistics. Table (6.2) tabulates industrial energy prices, heat equivalent units, and value-to-therm conversion factors.

From the data of Tables (6.1) and (6.2) the distribution of energy consumption tabulated in Table (6.3) is obtained by

(a) fixing energy used by domestic consumers and public authorities from Table (6.1),

(b) using the relative industrial distribution of energy purchases

2. The Energy Statistics classifications are not sufficiently detailed for input-output applications: nor are they readily compatible with those of the input-output tables.

as obtained from the input-output tables to conform to the respective totals of Table (6.1),

(c) adding to the total of Table (6.1) the energy transfers within the energy producing sector, and also adding (or subtracting) stocks and exports, as recorded in the Energy Statistics.

The distribution of Table (6.3) is given in percentage terms, and multiplication of these percentages by either original units or heat equivalent units shown at the bottom of the table gives the energy consumption by each consumer. For example, total coal use by the materials industries in 1968 was 22.6 x 0.544 = 12.3 M tons = 5741 x 0.544 = 3123 M therms. Multiplying the percentage by the total heat equivalent units allows subsequent addition accross the table to give the total distribution of direct energy comsumption recorded in the right hand side of Table (6.3).

It is seen that the materials producing industries directly consume far more coke, gas coal and oil than the engineering and construction industries directly consume. In the case of electricity the consumption is of a similar order. For energy of all types in heat equivalent units the materials producers directly consumed more than twice as much as the engineering and construction industries directly consumed. This is clearly illustrated in Figure (6.1): the iron and steel industry alone directly consumed as much energy of all types as all the engineering and construction industries consumed directly.

(6-2) TOTAL ENERGY CONSUMPTION

The fact that the materials producing industries consume a large part of the total energy used in the UK, coupled with the fact that the engineering and construction industries use much of this material, suggests that the indirect consumption of energy by the engineering and construction industries via materials may be high.

This suggestion can be assessed from the input-output tables for each industry, but here only the total engineering and construction industries, and some subgroups of these, are considered.

Table D was aggregated into four small tables:

(a) an 11 order table with 5 energy industries, one material, 4 engineering and construction (as in Table (6.3)), and one "other" industry comprising all remaining industries,

(b) an 8 order table with the 4 engineering and construction industries further aggregated (from the 11 order table),

(c) a 7 order table with the 5 energy industries aggregated into 1 (from the 11 order table),

(d) a 4 order table with industries of total energy, total materials, total engineering and construction, and "other" industries.

The 4 order matrix and its inverse is tabulated in Table (6.4), which is used here to illustrate the analysis. Gross output is taken free from duplication in all cases. Imports of refined and crude oil are included in the analysis, also free from duplication.[3]

Directly, from Table (6.4), the engineering and construction industries purchased a total of £280.8m + £24.5m = £305.3m of energy to produce £15220.7m of output.

Directly plus indirectly, however, the engineering and construction industries purchased £0.0452 of energy per £1 of gross output from UK suppliers plus £0.0037 of oil imports. Thus the production of £15220.7m of engineering and construction gross output required a total of 15220.7 x (0.0452 + 0.0037) = £744.3m of total UK produced energy plus imported oil. The total energy use by the engineering and construction industries thus is nearly 2½ times as much as the direct energy use.

Much of this indirect is via materials. These industries directly purchased £3409.5m of materials produced in the UK. Each £1 of materials production required a total of £0.0997 of domestic energy plus £0.0062 of oil imports. Hence the engineering and construction industries purchased 340.9 x (0.0997 + 0.0062) = £361.8m of energy via direct purchases of materials, 20% more than was directly purchased (£305.3m).

The total direct plus indirect expenditure on energy by the materials, the engineering and construction industries, and final buyers is illustrated in Figure (6.2). The illustration also includes the energy flows in heat equivalent units. These however are only approximate (see footnotes). They may be in error because it is not clear which value-to-

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3. All category 5 of the input-output imports Table C was taken as refined oil, and category 3 in the case of imports to the UK oil (5), chemicals (6), and plastic materials (8) industries was taken as crude. In the 11 and 8 order matrices oil imports to UK oil industry was deleted, while in the 7 and 4 order matrices oil imports to the total energy industry was deleted — thus eliminating duplication.

therm conversion factors are appropriate for the indirect flows in this highly aggregated analysis. From Table (6.2) the cost per therm is seen to vary substantially for alteranative forms of energy. It is suggested, however, that the difference between the energy purchased directly and indirectly via materials by the engineering and construction industries is even greater in heat equivalent units than in monetary units. This may be expected because the engineering and construction industries purchase comparatively more of their direct energy as relatively high cost electricity, while the materials industries purchase comparatively more of the lower cost energy (e.g. coal and coke in steelmaking).

In an analogous way the total purchase of energy by the engineering and construction industries was separated into direct, indirect via materials, and indirect not elsewhere specified (nes.) for the 7, 8, and 11 order matrices. The results are tabulated in Table (6.5) (the results of the above example being in the lower right hand corner) and illustrated in Figure (6.3).

It is clearly evident that a major part of the total use of energy by the engineering and construction industries is indirectly used via materials. Altogether, these industries use 8.3 times more coal via materials than they use directly, 10.3 times more coke, 1% more oil, 53% more gas, but only 80% as much electricity. Overall, 20% more energy is used via materials than is used directly. It is particularly the construction industry which uses more total energy via materials than it does directly, twice as much in fact. The "other manufacturing" industries use only 13% more energy via materials than directly, the "general engineering 1% more, while the "transport equipment" industry uses only 93% as much via materials as directly.

(6-3) RESULTS OF CONSERVATION: CONCLUSIONS

The implication of the above description of energy flows in the economy is that materials savings by the engineering and construction industries may contribute more towards national energy conservation than will energy directly used by these industries.

This is confirmed by comparing the effects of a 10% saving of UK produced materials with a 10% saving of energy purchased by the engineering and construction industries in the full 60 order matrix. Table (6.6) shows the resultant change in the output of every industry, and Table (6.7) shows the changed output of energy converted into original units using the data of Tables (6.2) and (6.3).

Substantially more coke, gas and coal is saved via the 10% materials directed savings than via 10% energy directed savings; in the case of oil the savings are of a similar magnitude; but in the case of electricity the savings are greater via direct electricity savings than via materials savings.

In addition to saving more energy, the materials directed saving will also save more other industrial output and primary inputs, as is shown in Table (6.6): 8.8 times more imports, 7.7 times more capital stock, and 14.9 times more total labour may be saved via materials directed policies than via energy directed ones. The cost saving is 11.5 times greater under the former policy, thus indicating that 11.5 times more money can be used to finance the technical change or benefit the economy (depending on the options of Figures (5.2) and (5.3)) via materials saving than via energy saving.

It is concluded that material savings by the engineering and construction industries may indirectly contribute more to energy conservation than will direct energy savings; and further, the former may provide the economy with substantially greater non-energy savings as well.

CHAPTER SEVEN

THE MOTOR CAR AND ITS SUBSTITUTES

The object of this chapter is to estimate and compare the effects on the UK economy of three possible changes in the technology of motor car production and two possible non-technical changes in the use of cars in the UK. Both long term and short term effects are considered. The motor vehicle industry plays a major role in the economy[28,64-66] and it may be expected that changes relating to it could contribute much to resource savings.

(7-1) MODELS

The basic models of postulated changes are:

Changes in Production Technology

A Improvement in manufacturing technology at constant design, i.e. using less of all inputs to produce the same motor vehicle. A uniform 10% reduction of all inputs is postulated.

B Reduction in weight at constant manufacturing technology, i.e. using less materials to produce a lighter motor vehicle. A uniform 10% reduction of all material inputs is postulated.

C Increase in service life of cars. A 30% increase in service life of all cars leading to a 30% decrease in annual demand for new cars and parts is postulated.[1]

Changes in Use

D Decrease in driving speed. A uniform 30% dcrease in driving speed leading to a 30% fuel saving and a 30% increase in service life of cars is postulated.

E Part substitution of private by public transport. A 20% substitution in passenger traffic leading to 20% fuel savings by private motorists and a 20% decrease in annual demand for cars is postulated. 70% of the savings are spent on public transport — rail and road — in the existing pattern of 33% on rail and 67% on buses.

For each of the above basic models a short term and a long term model is considered. The short term model corresponds to a situation in which the change has been newly introduced — at the beginning of the current year — and thus affects only new cars. The long term model corresponds to a situation in which the change was well established and affects the entire stock of cars. (In model A there is no difference between the long term and the short term case.)

In addition, a number of alternative assumptions are postulated for the above basic models. The assumptions are listed in the diagrammatic representations of the models on the following pages.

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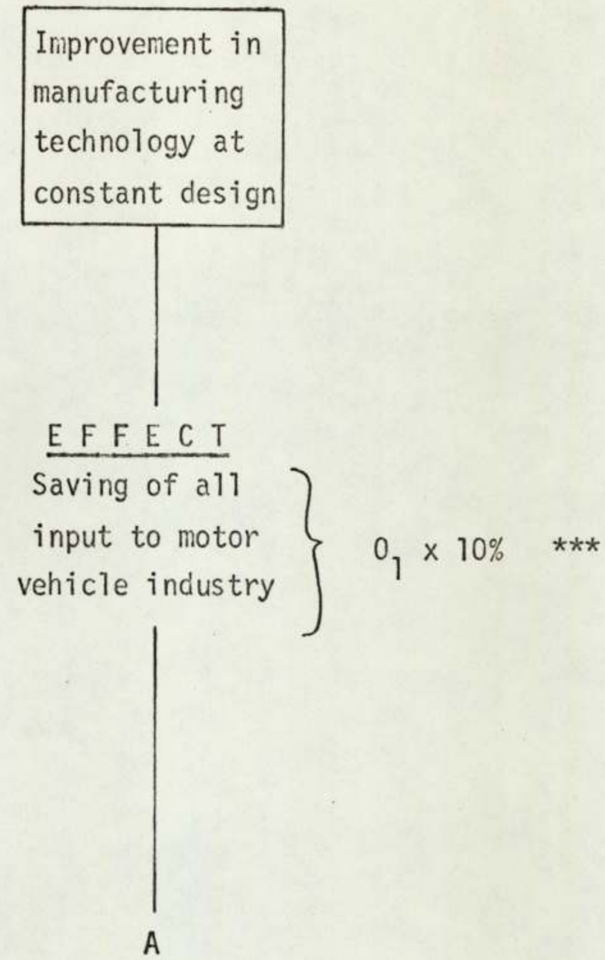
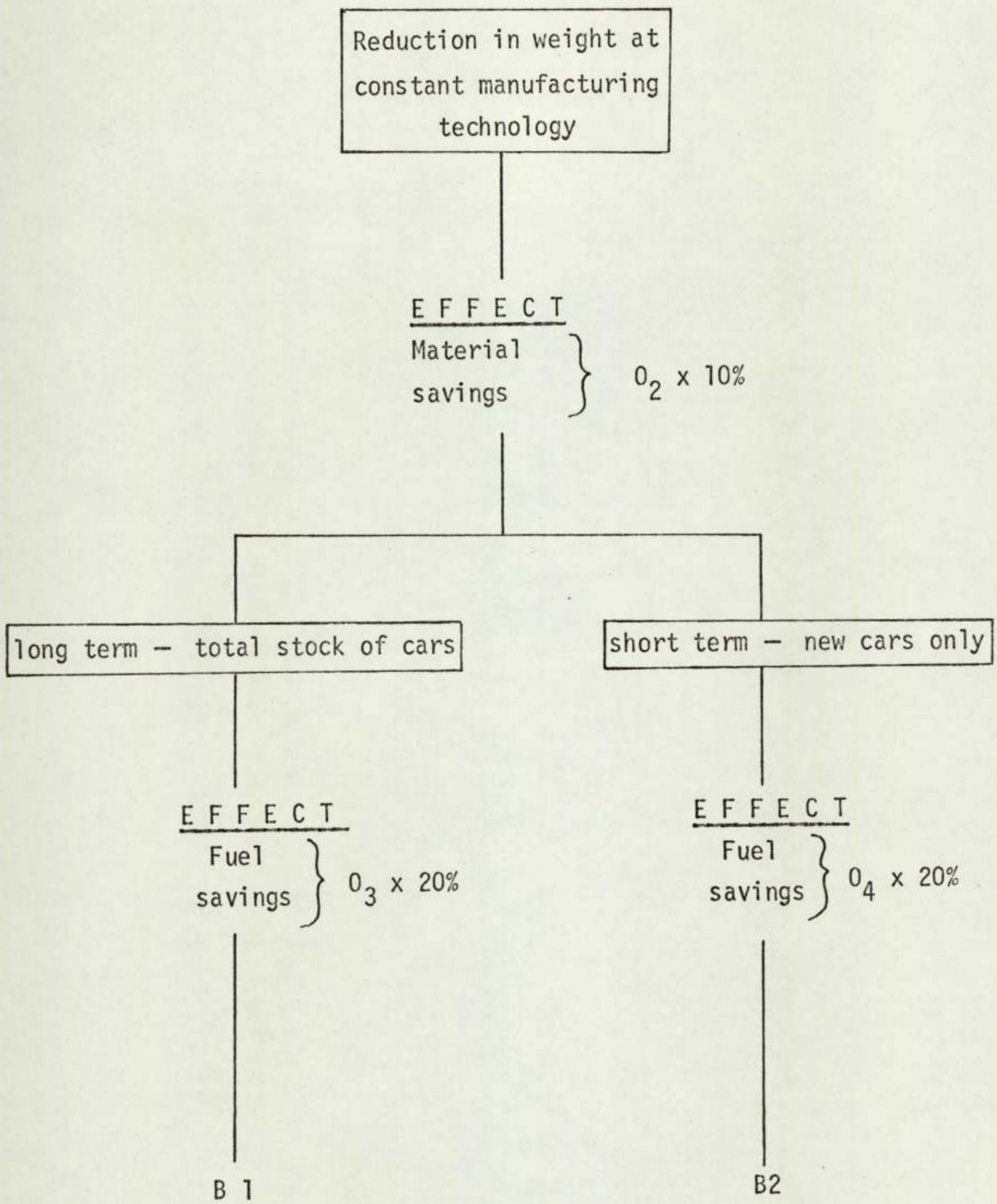
1. Inspection by A. B. Svensk Bilproving (67) in Sweden showed that the life of the average BMC car was 9.8 years while some cars such as Volvo managed 13.6 years which is nearly 40% more. The average life was 11.8 years — still 20% above BMC cars.

M O D E L   A

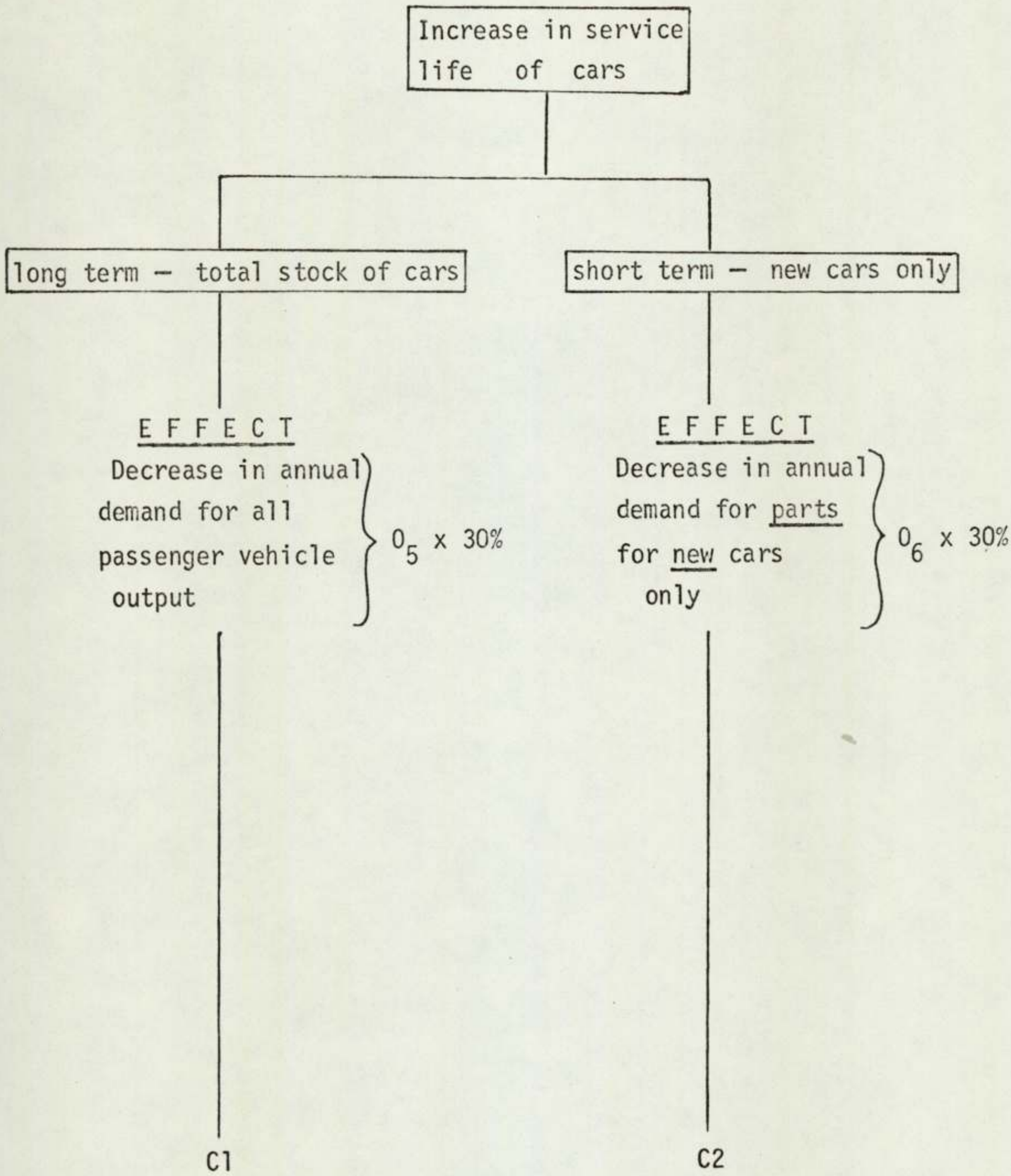


*** NOTE O_1 refers to matrix operation 1 (input reduction) listed in Table (7.1), while 10% indicates that the operation is conducted at the 10% level (10% reduction).

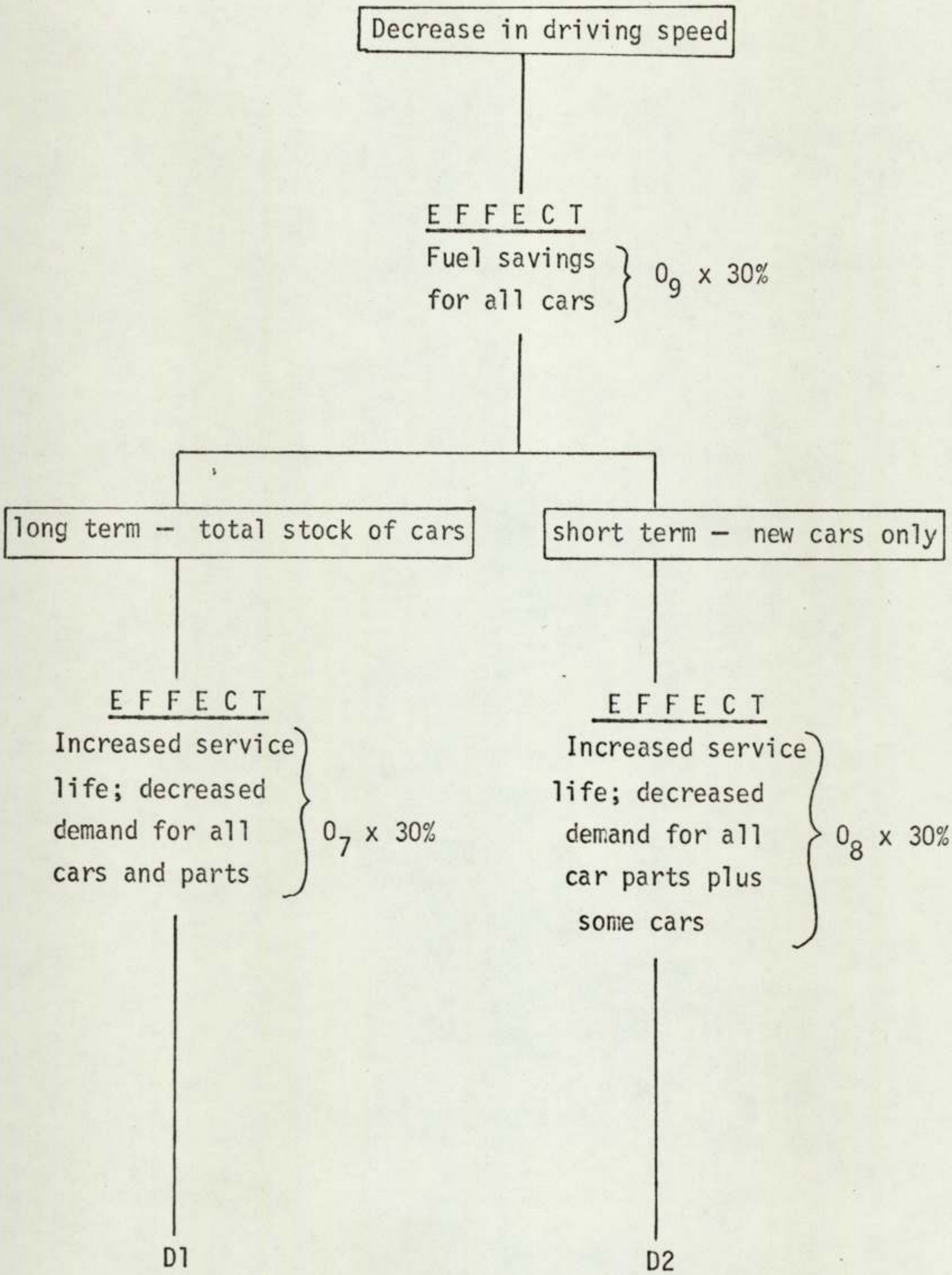
MODEL B


Reduction in weight at constant manufacturing technology
EFFECT
Material savings
$0_2 \times 10\%$
long term – total stock of cars
short term – new cars only
EFFECT
Fuel savings
$0_3 \times 20\%$
EFFECT
Fuel savings
$0_4 \times 20\%$
B 1
B2

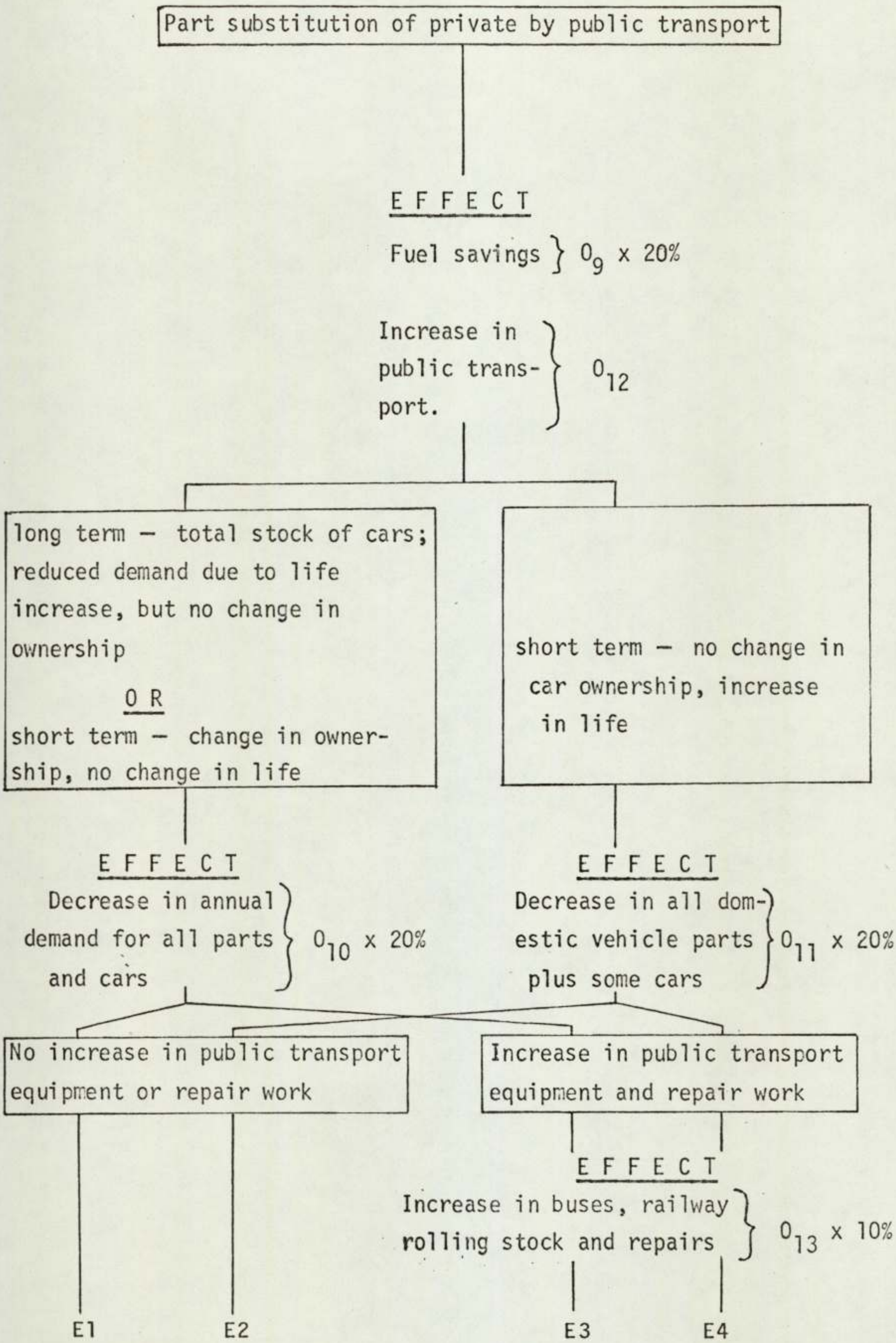
MODEL C


Increase in service life of cars
long term – total stock of cars
short term – new cars only
EFFECT
Decrease in annual demand for all passenger vehicle output
0_5 x 30%
EFFECT
Decrease in annual demand for parts for new cars only
0_6 x 30%
C1
C2

MODEL D


Decrease in driving speed
EFFECT
Fuel savings for all cars
$0_9 \times 30\%$
long term – total stock of cars
short term – new cars only
EFFECT
Increased service life; decreased demand for all cars and parts
$0_7 \times 30\%$
EFFECT
Increased service life; decreased demand for all car parts plus some cars
$0_8 \times 30\%$
D1
D2

MODEL E


Part substitution of private by public transport
EFFECT
Fuel savings } 0_9 x 20%
Increase in public transport. } 0_{12}
long term – total stock of cars; reduced demand due to life increase, but no change in ownership
OR
short term – change in ownership, no change in life
short term – no change in car ownership, increase in life
EFFECT
Decrease in annual demand for all parts and cars } 0_{10} x 20%
EFFECT
Decrease in all domestic vehicle parts plus some cars } 0_{11} x 20%
No increase in public transport equipment or repair work
Increase in public transport equipment and repair work
EFFECT
Increase in buses, railway rolling stock and repairs } 0_{13} x 10%
E1
E2
E3
E4

In addition to the above models a summation model, S, is also considered for both the long and the short term cases:

$$S_1 = A + B_1 + C_1 + D_1 + E_1 \quad \text{(long term)}$$

$$S_2 = A + B_2 + C_2 + D_2 + E_2 \quad \text{(short term)}$$

The matrix operations for these models were conducted in the order of the models as summed.

(7-2) COMPUTATIONS

The operations tabulated in Table (7.1) were performed on matrices D and F_I stored in the computer in the combinations and levels indicated for each model on the previous pages, and formulation I of chapter 3 was used to estimate the effects of the changes on the economy. Before presenting the results consider an alternative approximate means of estimating the effects. This alternative is used here to point out which of the changed tabulated in Table (7.1) are significant, and which are not significant in each model.

Essentially all the above models are combinations of nine different identifiable basic operations, with each operation incorporated in the models at some level. These nine operations are listed in Table (7.2). Table (7.3) is the estimation[2] of the effects of the operation, each individually, on the primary inputs. For operation (6), for example,

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2. The operations listed in Table (7.2) are not directly compatible with the operations of Table (7.1) because the former does not include changes within the body of matrix D for operations (3) to (9), while the latter does. Hence the use of the operations of Table (7.2) will be approximate. However, the approximation is expected to be good (see chapters 8 and 9).

if the final demand for road transport were zero then column (6) of Table (7.3) lists the decrease in the use of primary inputs in the economy [3].

Define locally:

Q_j as the full effect on the economy of operation j

M_i as the effect on the economy of model i

k_j as a constant giving the level of Q_j in model i

then

$$M_i = \sum_{j=1}^{9} k_j Q_j$$

and for the above models we have, for example,

$$A = k_1 Q_1$$

$$B_1 = k_2 Q_2 + k_3 Q_3 + k_8 Q_8 + k_9 Q_9$$

The full effects tabulated in Table (7.3) thus are the models M_i with $k_i = 1$ and $k_j = 0$ for $j \neq i$.

The usefulness of this system lies in its ability to easily estimate the alterations in the effects, M_i, of technical changes when the assumptions alter the coefficients k_j. In particular it is possible to see how the effects build up and to isolate the specific hypotheses and operations which are responsible for the major portion of the changes on the economy.

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3. Stated another way, to produce the total final demand for £411.2m of road transport it is necessary to employ £16.8m of imports, £662.1m of capital stock, 155.5 thousand operatives, 53.3 thousand other employees, etc.

Table (7.4) lists the coefficients k_j obtained for the models postulated above, and Table (7.5) lists the minimum level of a coefficient k_j so that the primary input is altered by at least £1.0m (1.0 th. man-years in the case of labour) by operation j. For example, Table (7.5) shows that k_3 must be greater than 445×10^{-5} before imports are changed by more than £1.0m by operation (3) of Table (7.2). Table (7.4) shows that in all models the coefficients k_3 are greater than 445×10^{-5}. Hence operation (3) will have a discernible effect on imports for each model. We conclude that the fuel savings (indirectly) induced by the changes in all models discernibly affect imports to the UK.

From Table (7.4) we observe that operation (8) – changes in distributive services – is not of significance for most primary inputs in models B2, C2, D1, and D2. Operation (9) is of no significance in model B2, and of marginal significance in model B1. Similarly operation (3) is of marginal significance in model B2. For these models these operations could have been omitted.

But Table (7.4) shows that for most models all of the operations considered contribute something to the overall results reported below, even if this contribution is small.

(7-3) RESULTS

The operations tabulated in Table (7.1) were applied at the levels indicated, and the results of the computer analysis are tabulated in Table (7.6).

(7-3.1) Results for UK Industrial Output

Overall the results indicate that a saving of all inputs used to manufacture motor vehicles is necessary to achieve significant savings in the output of UK industries. The effect on UK industrial output of a reduction in the materials requirement of the motor vehicle industry, of a reduction of petrol requirements for driving cars, of a reduction in public transport demand, are relatively small.

The saving of motor vehicle inputs can either be a direct saving by improvement in manufacturing technology (model A), or an indirect saving by reducing demand for motor vehicles as a side effect of changes such as increasing the service life of cars (model C), decreasing the driving speed (model D) and hence the service life of cars, or substituting private transport by public (model E).

In the case of model C2 technical changes which increase the service life of cars will not substantially alter demand for cars in the short term because the changes only apply to new cars, and hence there will be few savings in the short term. But in the long term, when the entire stock of cars is of the long life type, the demand for cars will be substantially altered and some significant changes are possible. Because the technical changes in this model have been assumed to alter the service life of the car by 30%, a (approximately) 30% reduction in the inputs to motor car production is induced: hence the results of model C1 are approximately three times those of model A.

A decrease in driving speed (model D) will achieve savings of oil, but the UK oil supplying industry does not use substantial amounts of the output of the other UK industries, and hence the oil saving itself will not save substantial amounts of other UK industrial output. However, the decrease in driving speed has a side effect of increasing the service life of cars and decreasing the maintenance requirements, thus reducing motor vehicle demand and leading to wide spread savings via the motor industry. In contrast to model C, many of these savings are realized in the short term because the entire stock of cars benefits immediately. However, the extension of service life to the older cars is not substantial as these have already been subjected to the wear of normal (higher) driving speeds. In the long term when the entire stock of cars has always been driven at lower speeds the reduced demand for motor vehicle output will be greater, and so will the consequent savings to the economy. But the long term savings via the decrease in service life induced by speed reduction are not as large as the long term savings resulting from manufacturing changes in the motor vehicle industry which increase the service life by an equivalent amount. This is so because the latter reduced demand for all cars manufactured in the UK, including export cars, while the former decreases only domestic demand for the UK car industry (as well as domestic demand for imported cars and parts).

Similarly for model E, the substitution of private by public transport leads to savings in industrial output via the reduction of motor vehicle demand: neither the savings of UK industrial output via fuel savings, nor the increased public transport demand are as significant as the savings via the UK motor vehicle industry.

A comparison of the savings of energy and engineering materials is illustrated in Figure (7.1).

In the short term it is in models A and D2 – improvements in manufacturing technology and decrease in driving speeds – that the largest savings of energy (except oil), and engineering materials are achieved. In model B2 material savings of a similar magnitude are achieved (as may be expected); relatively high coke (and coal) savings are also induced via steel and cast iron savings. In the short term relatively small energy and material savings are achieved by either substituting public transport for private (in fact coal, electricity, and rubber requirements are increased), or by technical changes which increase the service life of cars.

However, in the long term it is the change to the manufacture of cars with increased service lives which saves far more energy, (except oil) and engineering materials than any of the other changes postulated here.

(7-3.2) Results for Primary Inputs

For primary inputs the savings via fuel savings, and the increases via public transport increased demand, are significant (as is confirmed by a comparison of Tables (7.4) and (7.5)) because these industries and the distributive services industry which distributes fuel use substantial amounts of capital stock, labour, and, in the case of the oil industry, substantial imports. Figure (7.2) illustrates the changes on the primary inputs.

In the short term model D2 stands out as having greater effect on primary inputs, particularly imports, than any other single model. In model A capital stock and labour savings are comparable to those of model D2, but for the other models the savings are small in comparison, and in E2 and E4 there are considerable increases in capital stock and labour requirements.

In the long term it is the technical changes which increase the cars' service lives which save more capital stock and labour than any other change, although the decrease in driving speed will save more imports (of oil). Under model E, the substitution of public for private transport, the results indicate that despite the direct and indirect savings by the motor vehicle industry, the economy has a net increase in capital stock (£1500m) and labour (94 th. man-years) requirements.

However, these increases must be interpreted with considerable caution for two reasons.

Firstly,it has been assumed that the transfer from private to public transport is on the basis of cost: 70% of the money saved by the consumer by using his car 20% less is spent on public transport. This is rather arbitrary – one could equally have equated milage savings, in which case the relative price of public and transport needs to be considered. Clearly many options are possible depending on consumer behavior and these will influence the amount by which public transport increases.

Secondly, in this case the linearity assumption of input-output analysis is clearly unrealistic. Consider, for example, capital stock. From Table (7.6) the railway gross output rises by 22.6% and road transport by 19.3%. By the input-output constant input assumption the corresponding increased direct capital stock requirements by these industries are also 22.6% and 19.3%, which is £1600m and £310m respecively. But a large part of the £7100m of capital stock of the railways is lines, stations, tunnels, bridges, etc., which clearly can be used more intensively and do not need to be increased strictly in proportion to increased traffic. However, even if the estimated increased direct capital stock requirements were reduced to 10% of the above (£160m) for the railways, and to a perhaps more plausible £150m for the road transport industry, the net effect on the economy would still not be a saving of capital stock.

(7-4) CONCLUSIONS

The object of this exercise has been to estimate the effects on the UK economy of some changes in the technology of motor car production, and to compare these effects with those resulting from some non-technical changes in the use of cars in the UK.

It has been shown that in the short term a decrease in driving speed will achieve far larger imports, capital stock, manpower, and oil savings than will any of the postulated changes in motor car production technology. Decreased driving speeds will also achieve comparable energy (other than oil) and material savings to those achievable by improvements in motor car manufacturing technology or reduction in motor car weight. The saving of resources by decreased driving speeds is achieved not because of direct petrol savings but indirectly because of the resultant increased service life of the car.

In the long term however, far greater savings (except for oil and car imports) are achievable by technical changes within the motor car industry resulting in the manufacture of cars with equivalently increased service lives. This change will save £1400m of capital stock, £75m of imports, as well as other manufacturing resources.

But by far the greatest savings are achievable by a summation of all the changes considered here. In the long term it is estimated that a combination of the five changes postulated here would have saved £14m of coal in 1968, £9m of coke, £110m of oil, £9m of gas, £23m of electricity, £44m of cast iron, £102m of steel, £23m of aluminium, £26m of other non-ferrous metals, £12m of plastic material, £35m of rubber, £210m of imports, 310 thousand man-years of labour, and £1140m of capital stock, as well as a wide range of other UK industrial resources.

CHAPTER EIGHT

SIMPLIFYING FORMULAE AND APPROXIMATIONS

(8-1) INTRODUCTION

In the previous chapters estimates were made of the effects on national manufacturing resources of specified design changes in the manufacturing industries. These estimates were derived from the formulations of chapter 3. In each case a matrix inversion on the computer was necessary.

In this chapter alternative formulae are developed which relate national variables directly with the parameters defining design criteria for design changes in the manufacturing industries. These formulae serve two major purposes.

Firstly they can be used to estimate the effects of specified design changes from the data of standard input-output tables without the need to invert a matrix on the computer. The formulae are such that a desk calculator may be used.

Secondly they can be used in reverse to obtain parameters defining criteria for design changes which will achieve specified national objectives.

The chapter contains simple examples of the use of these formulae. In chapter 9 they will be extensively applied to test for ease of use, and to assess the errors of linear approximations.

NOTE: Readers may prefer to first read the simple overall view of the mathematics in this and the following chapter given in Appendix A4.

(8-2) SINGLE DOMESTIC INPUT CHANGE

This section presents two theorems which may be used for studies about technical changes resulting in the changed use of one domestically produced input to industries, and the changed use of any or all primary inputs by industries. The technical changes may take place in all, or any combination of, industries.

For the present purposes the technical changes refer to decreased or increased use of materials by manufacturing industries. As illustrated in Figure (8.1), the row of input coefficients relating to the material involved in the technical change, row p, alters by ε_m for the manufacturing industries m in which the technical change takes place; elsewhere $\varepsilon_j = 0$. The primary input change will refer to that part of the imports row relating to direct import of the same material by the manufacturing industries. Thus in row 1 of the primary input coefficient matrix we have a change of $\nu_{i,m}$ for the manufacturing industries m and $\nu_{i,j} = 0$ elsewhere.

(8-2.1) Theorems

Definitions (In addition to definitions of section (3-2.1))

$t_i, g_i, p_i, \theta_i, u_{i,j}, d_{i,j}, f_i$

the elements of matrices t, g, p_I, θ, U, D, f_I respectively

$\delta_{i,j}$ the elements of $(I - D)^{-1}$, i.e. the total input coefficient from industry i to industry j

$\gamma_{i,j}$ the elements of $U(I - D)^{-1}$, i.e. the total primary input coefficient of primary input i to industry j

m a subscript which may take any value corresponding to the manufacting industries in which the technical change occurs (columns of D)

$\sum_m$ a summation only over those industries m involved in the technical change

p a subscript corresponding to the material used in a changed amount

ε_m an increase in $d_{p,m}$

$\nu_{i,m}$ an increase in $u_{i,m}$

Δ a forward difference operator signifying an increase in a variable

(n) a superscript to a variable which has taken a new value e.g $t^{(n)} = t + \Delta t$

For convenience define

$$\sigma_i = \sum_m \varepsilon_m \delta_{m,i}$$

From (3.3), (3.4) and (3.25)

(8.1)(a) $t_i = \sum_j \delta_{i,j} f_j$ (b) $g_i = \sum_j u_{i,j} t_j$ (c) $p_i = \sum_j \delta_{j,i} \theta_j$

respectively, while from the above definition

(8.2) $$\gamma_{i,j} = \sum_k u_{i,k} \delta_{k,j}$$

THEORM (8.1)

If elements m of any one row p of the interindustry coefficient matrix D is increased by an amount ε_m such that for all m $d^{(n)}_{p,m} = d_{p,m} + \varepsilon_m$ subject to $\varepsilon_m \geq -d_{p,m}$, then

$$(8.3) \qquad \delta^{(n)}_{i,j} = \delta_{i,j} + \frac{1}{1 - \sigma_p} \delta_{i,p}\sigma_j \quad \text{for } i,j = 1,\ldots\ldots,n$$

PROOF

Define ξ such that $D^{(n)} = D + \xi$. That is, ξ is an n x n matrix with elements ε_j in the p^{th} row and 0 elsewhere ($\varepsilon_j = 0$ if $j \neq m$).

Consider the matrix X such that

$$(8.4) \qquad (I - D^{(n)})^{-1} = (I - D)^{-1}X$$

That is

$$\begin{aligned} I &= (I - D^{(n)})(I - D)^{-1}X \\ &= \left(I - \{D + \xi\}\right)(I - D)^{-1}X \\ &= \left(\{I - D\} - \xi\right)(I - D)^{-1}X \\ &= \left(\{I - D\}\{I - D\}^{-1} - \xi\{I - D\}^{-1}\right)X \\ &= \left(I - \xi\{I - D\}^{-1}\right)X \end{aligned}$$

Hence

$$X = \left(I - \xi\{I - D\}^{-1}\right)^{-1}$$

Thus in (8.4)

$$\begin{aligned} (I - D^{(n)})^{-1} &= (I - D)^{-1}\left(I - \xi\{I - D\}^{-1}\right)^{-1} \\ &= (I - D)^{-1}(I - \Gamma)^{-1} \end{aligned}$$

where $\Gamma = \xi(I - D)^{-1}$. If $\Gamma^N \to 0$ as $N \to \infty$ then

$$\begin{aligned} (I - D^{(n)})^{-1} &= (I - D)^{-1}(I + \Gamma + \Gamma^2 + \Gamma^3 + \Gamma^4 + \ldots\ldots\ldots) \\ &= (I - D)^{-1}(I + \Lambda) \\ &= (I - D)^{-1} + (I - D)^{-1}\Lambda \end{aligned}$$

Therefore

(8.5) $\quad (I - D^{(n)})^{-1} = (I - D)^{-1} + \Phi$

where $\Lambda = \Gamma + \Gamma^2 + \Gamma^3 + \Gamma^4 + \ldots\ldots$, and $\Phi = (I - D)^{-1}\Lambda$

Let $\psi^k_{i,j}$, $\lambda_{i,j}$, $\phi_{i,j}$ be the elements of Γ^k, Λ, Φ respectively and consider these in turn. From the definition of ξ we have for all j

$$\psi_{i,j} = \begin{cases} \sum_m \varepsilon_m \delta_{m,j} & i = p \\ 0 & i \neq p \end{cases}$$

By definition $\sigma_j = \sum_m \varepsilon_m \delta_{m,j}$ and thus

$$\psi_{i,j} = \begin{cases} \sigma_j & i = p \\ 0 & i \neq p \end{cases}$$

$$\psi^2_{i,j} = \begin{cases} \sigma_p \sigma_j & i = p \\ 0 & i \neq p \end{cases}$$

$$\psi^3_{i,j} = \begin{cases} \sigma_p^2 \sigma_j & i = p \\ 0 & i \neq p \end{cases}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\psi^k_{i,j} = \begin{cases} \sigma_p^{k-1} \sigma_j & i = p \\ 0 & i \neq P \end{cases}$$

Hence for all j

$$\lambda_{i,j} = \begin{cases} \sum_{k=1}^{\infty} \sigma_p^{k-1} \sigma_j & i = p \\ 0 & i \neq p \end{cases}$$

The geometric progression to ∞ may be summed to give

$$\lambda_{i,j} = \begin{cases} \dfrac{\sigma_j}{1 - \sigma_p} & i = p \\ 0 & i \neq p \end{cases}$$

if $\sigma_p < 1$, and thus

$$\phi_{i,j} = \frac{1}{1 - \sigma_p} \delta_{i,p}\sigma_j \quad \text{for all } i,j$$

and hence in (8.5)

$$\delta^{(n)}_{i,j,} = \delta_{i,j} + \frac{1}{1 - \sigma_p} \delta_{i,p} \sigma_j$$

as required.

Corollary (8.1.1)

Under the conditions of Theorem (8.1)

(8.6) $$\gamma^{(n)}_{i,j} = \gamma_{i,j} + \frac{1}{1 - \sigma_p} \delta_{i,p} \sigma_j$$

for $i = 1,\ldots\ldots,s,$ and $j = 1,\ldots\ldots\ldots,n$

Proof

From (8.2)

$$\begin{aligned} \gamma^{(n)}_{i,j} &= \sum_k' u_{i,k} \delta^{(n)}_{k,j} \\ &= \sum_k' u_{i,k} \left(\delta_{k,j} + \frac{1}{1 - \sigma_p} \delta_{k,p} \sigma_j\right) \\ &= \sum_k' u_{i,k} \delta_{k,j} + \frac{1}{1 - \sigma_p}\left(\sum_k' u_{i,k}\delta_{k,p}\right) \sigma_j \end{aligned}$$

$$= \gamma_{i,j} + \frac{1}{1-\sigma_p} \gamma_{i,p} \sigma_j$$

as required.

Corollary (8.1.2)

Under the conditions of Theorem (8.1) iff $j = p$

$$(8.7) \qquad \frac{\delta^{(n)}_{i,j}}{\delta^{(n)}_{k,j}} = \frac{\delta_{i,j}}{\delta_{k,j}} \qquad \text{for } i,\ k = 1,\ldots\ldots,n$$

and

$$(8.8) \qquad \frac{\gamma^{(n)}_{i,j}}{\delta^{(n)}_{k,j}} = \frac{\gamma_{i,j}}{\delta_{k,j}} \qquad \text{for } i = 1,\ldots\ldots,n,\ k = 1,\ldots\ldots,s$$

Proof

From (8.3) for $i = 1,\ldots\ldots,n$

$$\delta^{(n)}_{i,j} = \delta_{i,j} + \frac{1}{1-\sigma_p} \delta_{i,p} \sigma_j$$

$$= \frac{\delta_{i,j} - \delta_{i,j}\sigma_p + \delta_{i,p}\sigma_j}{1-\sigma_p}$$

$$= \frac{\delta_{i,j}}{1-\sigma_p} \qquad \text{iff } j = p.$$

Hence for $i,\ k = 1,\ldots\ldots,n$

$$\frac{\delta^{(n)}_{i,j}}{\delta^{(n)}_{k,j}} = \frac{\delta_{i,j}}{1-\sigma_p} \Big/ \frac{\delta_{k,j}}{1-\sigma_p}$$

$$= \frac{\delta_{i,j}}{\delta_{k,j}} \qquad \text{iff } j = p$$

as required for equation (8.7).

The proof of equation (8.8) uses (8.3) and (8.6) in a similar way.

THEOREM (8.2)

If

(a) elements m of any one row p of the interindustry coefficient matrix D is increased by an amount ε_m such that for all m

$$d^{(n)}_{p,m} = d_{p,m} + \varepsilon_m, \text{ subject to } \varepsilon_m \geq -d_{p,m}$$

(b) elements m of the i^{th} row of the primary input x industry matrix U is increased by an amount $\nu_{i,m}$ such that for all i,j

$$u^{(n)}_{i,m} = u_{i,m} + \nu_{i,m}, \text{ subject to } \nu_{i,m} \geq -u_{i,m},$$

then for i = 1,....,n

$$(8.9) \quad \Delta t_i = \frac{\sum_m \varepsilon_m t_m}{1 - \sigma_p} \delta_{i,p} = \Delta t_p \left(\frac{\delta_{i,p}}{\delta_{p,p}} \right)$$

and for i = 1,....,s

$$(8.10) \quad \Delta g_i = \sum_m \nu_{i,m} t_m + \frac{\sum_m \varepsilon_m t_m}{1 - \sigma_p} \left(\gamma_{i,p} + \sum_m \nu_{i,m} \delta_{m,p} \right)$$

$$= \sum_m \nu_{i,m} t_m + \frac{\Delta t_p}{\delta_{p,p}} \left(\gamma_{i,p} + \sum_m \nu_{i,m} \delta_{m,p} \right)$$

Furthermore, if every element of the total coefficient primary input cost matrix θ is increased by an amout $\Delta\theta_j$ such that for all j $\theta^{(n)}_j = \theta_j + \Delta\theta_j$ then for all i = 1,....,n

$$(8.11) \quad \Delta p_i = \sum_j \delta_{j,i} \Delta\theta_j + \frac{\sigma_i}{1 - \sigma_p} \left(1 + \sum_j \delta_{j,p} \Delta\theta_j \right)$$

(N.B. In equation (8.11) the summation over j may also be taken as a restricted summation, as $\sum_m$ is, if $\Delta\theta_j = 0$ for some j. Equation (8.11) may be used to estimate the industry price changes due to primary input price changes other than, or in addition to, primary input changes. I.E. j may take values additional to, or equivalent to, m. If $j \equiv m$ and $\Delta\theta_m = -\varepsilon_m$ for all m then it follows that $\Delta p_i = 0$ for all i. Hence any technical change which does not change total input costs does not alter prices - as may be expected.)

PROOF of (8.9)

For i = 1, ..., n

$$\Delta t_i = t_i^{(n)} - t_i$$

$$= \sum_j \delta_{i,j}^{(n)} f_j - \sum_j \delta_{i,j} f_j \qquad \text{from (8.1)(a)}$$

$$= \sum_j \left(\delta_{i,j}^{(n)} - \delta_{i,j}\right) f_j$$

$$= \sum_j \frac{1}{1-\sigma_p} \delta_{i,p} \sigma_j f_j \qquad \text{from (8.3)}$$

$$= \frac{1}{1-\sigma_p} \delta_{i,p} \sum_j \sigma_j f_j$$

$$= \frac{1}{1-\sigma_p} \delta_{i,p} \sum_j \left\{\sum_m \varepsilon_m \delta_{m,j}\right\} f_j \qquad \text{by definition}$$

$$= \frac{1}{1-\sigma_p} \delta_{i,p} \sum_m \left\{\varepsilon_m \left(\sum_j \delta_{m,j} f_j\right)\right\}$$

Therefore

$$(8.12) \quad \Delta t_i = \frac{1}{1-\sigma_p} \delta_{i,p} \sum_m \varepsilon_m t_m \qquad \text{from (8.1)(a)}$$

as required for (8.9).

Furthermore, rearranging (8.12)

$$\Delta t_i = \frac{\sum_m' \varepsilon_m t_m}{1 - \sigma_p} \cdot \delta_{p,p} \cdot \left(\frac{\delta_{i,p}}{\delta_{p,p}} \right)$$

$$= \Delta t_p \left(\frac{\delta_{i,p}}{\delta_{p,p}} \right) \quad \text{from (8.12) for } i = p$$

as required for (8.9).

PROOF of (8.10)

For $i = 1, \ldots\ldots, s$

$$\Delta g_i = g_i^{(n)} - g_i$$

$$= \sum_j u_{i,j}^{(n)} t_j^{(n)} - \sum_j u_{i,j} t_j \quad \text{from (8.1)(b)}$$

$$= \sum_j \left(u_{i,j} + \nu_{i,j} \right) \left(t_j + \frac{\sum_m' \varepsilon_m t_m}{1 - \sigma_p} \delta_{j,p} \right) - \sum_j u_{i,j} t_j$$

from (8.9)

Since $\nu_{i,j} = 0$ for $j \neq m$

$$\Delta g_i = \sum_m \nu_{i,m} t_m + \frac{\sum_m' \varepsilon_m t_m}{1 - \sigma_p} \left(\sum_j u_{i,j} \delta_{j,p} + \sum_m \nu_{i,m} \delta_{m,p} \right)$$

Therefore

$$(8.13) \quad \Delta g_i = \sum_m \nu_{i,m} t_m + \frac{\sum_m' \varepsilon_m t_m}{1 - \sigma_p} \left(\gamma_{i,p} + \sum_m \nu_{i,m} \delta_{m,p} \right)$$

from (8.2)

as required for (8.10).

Furthermore, rewriting (8.13) as

$$\Delta g_i = \sum_m \nu_{i,m}\, t_m + \frac{\sum_m \varepsilon_m t_m}{1 - \sigma_p} \cdot \delta_{p,p} \cdot \left(\frac{1}{\delta_{p,p}}\right)\left(\gamma_{i,p} + \sum_m \nu_{i,m}\, \delta_{j,p}\right)$$

$$= \sum_m \nu_{i,m}\, t_m + \frac{\Delta t_p}{\delta_{p,p}} \left(\gamma_{i,p} + \sum_m \nu_{i,m}\, \delta_{m,p}\right)$$

as required for (8.10).

PROOF of (8.11)

For i = 1,........,n

$$\Delta p_i = p_i^{(n)} - p_i$$

$$= \sum_j \delta_{j,i}^{(n)}\, \theta_j^{(n)} - \sum_j \delta_{j,i}\, \theta_j \text{ from (8.1)(c)}$$

$$= \sum_j \left(\delta_{j,i} + \frac{1}{1 - \sigma_p}\, \delta_{j,p}\, \sigma_i\right)\left(\theta_j + \Delta\theta_j\right) - \sum_j \delta_{j,i}\, \theta_j$$

from (8.3)

$$= \sum_j \delta_{j,i}\, \Delta\theta_j + \frac{\sigma_i}{1 - \sigma_p}\left(\sum_j \delta_{j,p}\, \theta_j + \sum_j \delta_{j,p}\, \Delta\theta_j\right)$$

$$= \sum_j \delta_{j,i}\, \Delta\theta_j + \frac{\sigma_i}{1 - \sigma_p}\left(1 + \sum_j \delta_{j,p}\, \Delta\theta_j\right)$$

as required for (8.11) since $\sum_j \delta_{j,p}\, \theta_j = p_p = 1$ initially.

(8-2.2) Application: Ease of Use

The above formulae are based on simple summations, and not matrix inversion. Consequently the effects of a technical change may be estimated directly from the standard input-output tables using an ordinary desk calculator. Use of a computer may not be necessary.

Theorem (8.1) allows the new elements of the inverse matrix, after a technical change, to be directly estimated without inversion; and similarly corollary (8.1.1) estimates the new elements of the total primary input coefficient matrix. (Clearly, however, these formulae would only be used if a small number of elements of the new matrices were required.)

Corollary (8.1.2) demonstrates an important feature of the matrices. The technical change alters all the elements of $(I - D)^{-1}$ and $U(I - D)^{-1}$; however, in the p^{th} column the elements are altered by a constant ratio $\frac{1}{1 - \sigma_p}$. Hence the relative inputs into the materials producing industries, $\frac{\delta_{i,p}}{\delta_{p,p}}$ and $\frac{\gamma_{i,p}}{\delta_{p,p}}$, do not alter, but for all other industries the relative inputs do alter.

Theorem (8.2) provides easy estimates of the changed total output of every industry, the changed total use of primary inputs by industry, and price changes resulting from the technical change. The ease of use of these formulae can be gauged from the following example.

<u>EXAMPLE (8.1)</u>

Consider the effects of a reduction in the use of domestic steel by the motor vehicle and construction industries.

<u>Data</u>: p = 10 (steel), m = 31 (motor vehicles), 52 (construction).

$d_{10,31} = 0.09572$, $d_{10,52} = 0.02463$, $t_{31} = 1852.2$, $t_{52} = 4954.3$,

$\delta_{31,10} = 0.00921$, $\delta_{52,10} = 0.00883$, $\delta_{10,10} = 1.02222$,

$\delta_{31,31} = 1.00367$, $\delta_{31,52} = 0.00478$, $\delta_{1,10} = 0.05433$, $\gamma_{2,10} = 4.06023$

Variables: unknown .... Δt_i, Δg_i, Δp_i

known ...... $\varepsilon_{31} = -0.015$ i.e. 15.7% decrease in $d_{10,31}$

$\varepsilon_{52} = -0.005$ i.e 20.3% decrease in $d_{10,52}$

$\gamma_{i,m} = 0$ for all i,m

Summations: $\sigma_p = \sum_m \varepsilon_m \delta_{m,p} = -0.00019$, $\sum'_m \varepsilon_m t_m = -52.6$,

$$\sum'_m \nu_{i,m} t_m = 0, \quad \sum'_m \nu_{i,m} \delta_{m,p} = 0, \quad \sum'_j \delta_{j,i} \Delta\theta_j = 0$$

Changed total output of industries

From (8.9)

$$\Delta t_p = \frac{\sum'_m \varepsilon_m t_m}{1 - \sigma_p} \delta_{p,p} = -53.8$$

Thus the technical change has resulted in a saving of £53.8m of steel gross output.

For all other industries $\Delta t_i = \Delta t_p \left(\frac{\delta_{i,p}}{\delta_{p,p}}\right) = -53.8 \times \frac{\delta_{i,10}}{1.02222}$

E.G. i = 1 (coal), $\Delta t_1 = -2.9$. The total coal saving is £2.9m of gross output.

Changed total use of primary input

From (8.10)

$$\Delta g_i = \Delta t_p \left(\frac{\gamma_{i,p}}{\delta_{p,p}}\right) = -53.8 \times \frac{\gamma_{i,10}}{1.02222}$$

E.G. i = 2 (capital stock), $\Delta g_i = -213.7$. The total capital stock saving is £213.7m.

Price changes

In (8.11) $\Delta\theta_j = 0$, hence

$$p_i = \frac{\sigma_i}{1 - \sigma_p} = \frac{-(0.015)\delta_{i,31} - (0.005)\,\delta_{i,52}}{1.00208}$$

Therefore $\Delta p_i = -0.01497\delta_{i,31} - 0.00499\delta_{i,52}$

E.G. $i = 31$ (motor vehicles), $\Delta p_i = -0.015$. The price reduction in motor vehicle output is 1.5%.

From the above example it is evident that in estimating Δt_i and Δg_i the main computational effort is in calculating the summations σ_p, $\sum'_m \nu_{i,m}\, t_m$, $\sum'_m \nu_{i,m}\, \delta_{m,p}$, $\sum'_m \varepsilon_m\, t_m$. If a technical change takes place in a large number of industries then the computational effort in calculating these summations increases. However, once the summations have been calculated and inserted into (8.9) and (8.10) then the changed output of every industry and the changed use of primary inputs is obtained from the same equation. For estimating Δp_i, however, a different summation, σ_i, is required for each industry i. Hence considerable computations may be necessary if the change takes place in a large number of industries and many price changes are required.

(8.2.3) Application: Design Criteria and National Objectives

The previous section illustrated the use of Theorem (8.2) in estimating the effects of a technical change. The equations may also be used in reverse to estimate the changed use of a material by manufacturing industries necessary to achieve a specified national objective. Consider the following example:

EXAMPLE (8.2)

What change in the use of domestically produced steel by the motor vehicle and construction industries is necessary to reduce the total use of capital stock in the UK by at least £200m?

Data: As in example (8.1)

Variables: unknown ..... $\varepsilon_{31}, \varepsilon_{52}$

known .... $\nu_{i,m} = 0$ for all i,m

$\Delta g_2 \leq -200$ (capital stock)

Summations: $\sigma_p = \sum_m \varepsilon_m \delta_{m,p} = 0.00921\varepsilon_{31} + 0.00883\varepsilon_{52}$

$$\sum_m \varepsilon_m t_m = 1852.2\varepsilon_{31} + 4954.3\varepsilon_{52}$$

$$\sum_m \nu_{i,m} t_m = 0, \quad \sum_m \nu_{i,m} \delta_{m,p} = 0$$

Solution:

From (8.10)

$$\Delta g_2 = \frac{\sum_m \varepsilon_m t_m}{1 - \sigma_p} \gamma_{2,10}$$

Therefore

$$-200 \geqslant \frac{1852.2\varepsilon_{31} + 4954.3\varepsilon_{52}}{1 - 0.00921\varepsilon_{31} - 0.00883\varepsilon_{52}} \times 4.06023$$

That is, any $\varepsilon_{31}, \varepsilon_{52}$ satisfying

(8.14) $\quad -37.59\varepsilon_{31} - 100.57\varepsilon_{52} \geq 1$

represents the changed use of steel by the motor vehicle and construction industries, respectively, which will reduce total UK capital stock requirements by at least £200 M. If, for example, no change occurs in the motor vehicle industry then $\varepsilon_{31} = 0$, $\varepsilon_{52} \leq -0.010$, and hence a $\frac{|\varepsilon_{52}|}{d_{10,52}} \times 100 \geq 40.6\%$ reduction in the use of steel by the construction industry will achieve the objective. If $\varepsilon_{52} = 0$ the $\varepsilon_{31} \leq -0.027$, and hence a 28.2% or greater reduction in the motor vehicle industry alone will achieve the objective. If the change is to take place uniformly in both industries so that $\varepsilon_j = d_{p,j}\, k$ is substituted in equation (8.14) (8.14), where 100k% is the change in steel use by each industry, then (8.14) becomes

$$-(37.59)(0.09572)k - (100.57)(0.02463)k \geq 1$$

Hence a $|100k| \geq 16.5\%$ reduction in the use of steel by both the motor vehicle and the construction industry will reduce the total UK capital stock requirements by at least £200 M.

It should be pointed out that in the above example it is not only the capital stock requirements which are reduced. The material saving also leads to saving of other primary inputs, as well as the output of all other industries. But the specified objective has been satisfied. It has been demonstrated that Theorem (8.2) may be used to obtain parameters of material use by manufacturing industries. These parameters define design criteria for manufacturing commodities so that national objectives are satisfied.

(8-2.4) Application: Identification of Major Components and Approximations

One important reason for using the equations of Theorem (8.2), rather than a computer, to estimate the effect of changes in the use of materials is that the user becomes familiar with the data. He can identify the industries and the components of the formulae which make large contributions to the overall results.

One consequence is that it may be possible to obtain approximation formulae. For example, equation (8.11) can be written as

$$\Delta p_i = \sum_j{}' \delta_{j,i} \Delta\theta_j + \frac{\sigma_i}{1 - \sigma_p} + \frac{\sigma_i}{1 - \sigma_p} \sum_j{}' \delta_{j,p} \Delta\theta_j$$

I.E. (Total price change) = (Price change due to changed primary input use) + (Price change due to changed domestic material use) + (Price change due to interaction of changed domestic material use with changed primary input use)

The interaction term is the product of two summations. If each summation is small then the interaction term is relatively small, and could be omitted. Similarly, if σ_p is substantially less than 1 (as was the case in example (8.1)) then the denominator $1 - \sigma_p$ may be omitted from the equations of Theorem (8.2).

If the interaction term and σ_p in fact are small then we have the following:

LINEAR APPROXIMATIONS

(8.15) $\Delta t_i \simeq \left(\sum_m{}' \varepsilon_m t_m\right) \delta_{i,p}$ for $i = 1, \ldots, n$

(8.16) $\Delta g_i \simeq \sum_m{}' \nu_{i,m} t_m + \left(\sum_m{}' \varepsilon_m t_m\right) \gamma_{i,p}$ for $i = 1,\ldots,s$

(8.17) $\Delta p_i \simeq \sum_j{}' \delta_{j,i} \Delta\theta_j + \sigma_i$ for $i = 1,\ldots,n$

(8-3) MULTIPLE INPUT CHANGES

The theorems in the previous sections of this chapter are applicable to changes in the elements of only one row of the interindustry matrix. D. Technical changes may however alter the elements of several rows. A substitution of plastic for steel in the manufacturing industries, for example, will be reflected by changes in these two rows, as well as a reduction of the inputs for the steel based technology and an increase in the inputs for the plastic based technology.

The formulae to estimate the effects of such multiple changes may be derived from the corresponding formulae of Theorems (8.1) and (8.2) by combination of the expressions for each individual change. From (8.3) and (8.9), for example, the r^{th} change is related to the $(r-1)^{th}$ change by

$$\delta_{i,j}^{(r)} = \delta_{i,j}^{(r-1)} + \frac{1}{1-\sigma_{p_r}^{(r-1)}} \delta_{i,p_r}^{(r-1)} \sigma_j^{(r-1)}$$

$$\Delta t_i^{(r)} = \frac{\sum_j \varepsilon_j^{(r)} t_j^{(r)}}{1-\sigma_{p_r}^{(r-1)}} \delta_{i,p_r}^{(r-1)}$$

By backsubstitution expressions for the total effect of all changes in terms of the original data can be obtained. However the formulae become very complex. For example, under two changes, ε_j in row p and λ_j in row q we obtain

$$\delta_{i,j}^{(n)} = \delta_{i,j} + \frac{(\sigma_j + \mu_j\sigma_q - \mu_q\sigma_j)\delta_{i,p} + (\mu_j + \mu_p\sigma_j - \mu_j\sigma_p)\,\delta_{i,q}}{1 - \sigma_p - \mu_q - \mu_p\sigma_q + \mu_q\sigma_p}$$

$$\Delta t_i = \frac{(T_\varepsilon - T_\varepsilon\mu_q + T_\lambda\sigma_q)\delta_{i,p} + (T_\lambda - T_\lambda\sigma_p + T_\varepsilon\mu_p)\delta_{i,q}}{1 - \sigma_p - \mu_q - \mu_p\sigma_q - \mu_q\sigma_p}$$

where $\mu_i = \sum_j \lambda_j \, \delta_{j,i}$, $T_\lambda = \sum_j \lambda_j \, t_j$, $T_\varepsilon = \sum_j \varepsilon_j \, t_j$

and all other symbols as previously defined in section (812.1).

A simpler approach is to use the approximation formulae (8.15), (8.16), and (8.17).

Definitions (in addition to definitions of section (8-2.1))

ℓ a subscript which may take any value corresponding to the inputs which are involved in the technical change by industries m (rows of D), including p

$\sum'_{\ell}$ a summation only over those inputs ℓ involved in the technical change

$\Pi_{\ell,m}$ a proportion signifying the increase in input ℓ per unit increase in p in industry m. I.E. $\Delta d_{\ell,m} = \Pi_{\ell,m}\varepsilon_m$, NB $\Pi_{p,m} = 1$

Π_{ℓ} a proportion signifying the increase in input ℓ per unit increase in p in all industries. I.E. $\Pi_{\ell} = \Pi_{\ell,m}$ if Π_{ℓ} is constant (common technology) for all m.

$\Pi^{(\theta)}_{i,m}$, $\Pi^{(\theta)}_{i}$
as $\Pi_{\ell,m}$, Π_{ℓ} but relevant to primary input i

k a constant signifying the proportion increase in the use of material p by industry m, i.e. $k \geq -1$

APPROXIMATION FORMULAE

If in each manufacturing industry m, an increase of ε_m in the use of material p and $\Pi_{\ell,m}\varepsilon_m$ in related inputs from industry ℓ (i.e. $\Delta d_{\ell,m} = \Pi_{\ell,m}\varepsilon_m$ and $\Pi_{p,m} = 1$), and $\Pi^{(\theta)}_{i,m}\,\varepsilon_m$ in related primary input i (i.e. $\Delta u_{i,m} = \nu_{i,m} = \Pi^{(\theta)}_{i,m}\varepsilon_m$) then

(8.18) $$\Delta t_i \simeq \sum'_{\ell} \sum'_{m} \Pi_{\ell,m}\, \varepsilon_m\, t_m\, \delta_{i,\ell} \quad \text{for } i = 1,\ldots,n$$

(8.19) $$\Delta g_i \simeq \sum_m \Pi^{(\theta)}_{i,m}\, \varepsilon_m\, t_m + \sum_\ell \sum_m \Pi_{\ell,m}\, \varepsilon_m\, t_m\, \gamma_{i,\ell}$$
for $i = 1,\ldots\ldots,s$

(8.20) $$\Delta p_i \simeq \sum_j \delta_{j,i}\, \Delta\theta_j + \sum_\ell \sum_m \Pi_{\ell,m}\, \varepsilon_m\, \delta_{i,m}$$
for $i = 1,\ldots\ldots,n$

If the technical change is uniform and common to all industries (i.e. $\varepsilon_m = kd_{p,m}$, $\Delta d_{\ell,m} = k\Pi_\ell d_{\ell,m}$, $\Delta u_{i,m} = k\Pi^{(\theta)}_i d_{\ell,m}$) then

(8.21) $$\Delta t_i \simeq k \sum_m d_{p,m}\, t_m \sum_\ell \Pi_\ell \delta_{i,\ell}$$

(8.22) $$\Delta g_i \simeq k \sum_m d_{p,m}\, t_m \left(\Pi^{(\theta)}_i + \sum_\ell \Pi_\ell\, \gamma_{i,\ell} \right)$$

(8.23) $$\Delta p_i \simeq \sum_j \delta_{j,i}\, \Delta\theta_j + k \sum_m d_{p,m}\, \delta_{i,m} \sum_\ell \Pi_\ell$$

EXAMPLE (8.3)

What uniform replacement of domestic steel by plastic in the motor vehicle and construction industries will save at least £200 M of capital stock if in both industries 1 unit of steel can be replaced by 2 units of plastic, and related savings are 1/20 units of bolts & screws, 1/10 units of electricity, 5 units of capital stock, 8/10 units of operative labour, but 1/10 units of additional chemical adhesives are required?

Equation: (8.22) with $i = 2$ (capital stock)

Data: $p = 10$ (steel); $m = 31$ (motor vehicles), 52 (construction);
$\ell = 6$ (chemicals), 8 (plastic), 10 (steel), 36 (bolts), 54 (electricity).

Data (continued):

$t_{31} = 1852.2$, $t_{52} = 4954.3$, $d_{10,31} = 0.09572$, $d_{10,52} = 0.02463$,

$\gamma_{2,6} = 3.13128$, $\gamma_{2,10} = 4.06023$, $\gamma_{2,54} = 7.27922$, $\gamma_{2,8} = 3.30634$,

$\gamma_{2,36} = 2.18881$.

Variable: unknown ..... k

known .... $\Delta g_2 \leq -200$, $\Pi_6 = -0.1$, $\Pi_8 = -2.0$, $\Pi_{10} = 1.0$,

$\Pi_{36} = 0.05$, $\Pi_{54} = 0.1$, $\Pi_2^{(\theta)} = 5.0$

NB. $\Pi_3^{(\theta)} = 0.8$ (labour) not required

Summations; $\sum_m d_{10,m}\, t_m = 299.3$, $\sum_\ell \Pi_\ell\, \gamma_{2,\ell} = -2.02821$

Solution:

From (8.22)

$$\Delta g_2 \simeq k \sum_m d_{10,m}\, t_m \left(\Pi_2^{(\theta)} + \sum_\ell \Pi_\ell\, \gamma_{2,\ell} \right)$$

Therefore

$$-200 \geq k\,(299.3)(5.0 - 2.02821)$$

and hence $k \leq -0.225$. Thus at least 22.5% of the steel used by the motor vehicle and construction industries must be replaced by plastic to save at least £200 M of capital stock.

(8-4) CONCLUSIONS

This chapter has developed formulae which relate national manufacturing resources with parameters defining criteria for design changes in the manufacturing industries — the design changes being such that materials and related inputs required by the manufacturing industries are altered. The national resource utilization is changed because of the consequent different total requirement for manufacturing resources used in producing the material and related inputs.

It was shown that if the technical change results in only one domestically produced material being used in an altered amount then the formulae are simple. However, if several domestic materials and related inputs are used in changed amounts then the formulae become complex. In this chapter linear approximations are given for multiple change cases. These approximation formulae are linear combinations of the resources used to manufacture the materials and related inputs involved in the technical change: that is, the formulae are linear combinations of columns of $(I - D)^{-1}$ and $U(I - D)^{-1}$.

Simple examples have been given in this chapter on the use of the formulae. In the next chapter the formulae are applied to relate design parameters in the manufacturing industries with resources used to produce engineering materials. The application will assess the formulae for ease of use, and estimate the errors involved in making simple linear assumptions.

CHAPTER NINE

ECONOMIC PROPERTIES OF ENGINEERING MATERIALS

(9-1) INTRODUCTION

In chapters 3 to 7 some specified changes in the use of materials by manufacturing industries were postulated, and the effects on the economy estimated.

In this chapter such an approach is abandoned in favour of simply estimating and comparing some economic properties of engineering materials in the UK, and illustrating the significance of these economic properties to engineering oriented attempts at saving resources, etc., and at overcoming the effects of disruptions to the economy. The technical changes in each industry are not specified; they may be any changes which lead to a uniform k% decrease, increase or substitution in materials use in all manufacturing industries. The economic properties included are the resource intensity of materials, the materials component of prices, and the materials component of regional employment. While all resources are included, energy capital stock, labour and imports are highlighted and used in illustrations.

The analysis is based on the formulae of chapter 8. The chapter includes an assessment of the significance of the interaction terms in the formulae, and of the errors involved in assuming linearity.

ENGINEERING MATERIALS (parameter p)

Material	Industry number	Abbreviation
cast iron	9	c
steel	10	s
aluminium	11	a
other non-ferrous metals	12	n
plastic	8	p
rubber	49	r

MANUFACTURING INDUSTRIES (parameter m)

Industry numbers 13 to 39, 44, 50 to 52

(9-2) MATERIALS AND RESOURCES

(9-2.1) Comparative Resource Intensities

The total resource intensities of the engineering materials are tabulated in Table (9.1) for energy, capital stock, manpower, and imports. These are the direct plus indirect inputs per £ of material output in the UK, $\frac{\delta_{i,p}}{\delta_{p,p}}$ and $\frac{\gamma_{i,p}}{\delta_{p,p}}$.

These resource intensities are compared in Figure (9.1). In the UK £1 of cast iron for example directly and indirectly required £0.06343 of coal in its manufacture in 1968, and as such, in value terms, cast iron is more coal intensive than any of the other engineering materials. Steel, for example, is only 84% as coal intensive. Figure (9.1) shows that of the UK manufactured engineering materials[1] cast iron is most

1. Construction materials are compared in Appendix A5.

coal, coke, and labour intensive; plastic is most oil intensive; steel is most gas, electricity and capital stock intensive; and the non-ferrous group of metals are most imports intensive.

(9-2.2) Use of Resources via Materials by Manufacturing Industries

The resource intensities can be used in equation (8.9) to estimate the total resource use by the manufacturing industries via each material.

In equation (8.9) substitute $\varepsilon_m = -kd_{p,m}$, so that a uniform 100k% decrese in material p used by all industries m can be estimated by

$$(9.1) \qquad \Delta t_p = - \frac{\left(\sum_m d_{p,m} t_m\right) \delta_{p,p} \, k}{1 + \left(\sum_m d_{p,m} \delta_{m,p}\right) k}$$

The summations are over 31 manufacturing industries; the elements are tabulated in the published input-ouput tables. The summations and the diagonal elements $\delta_{p,p}$ are tabulated in Table (9.2). In the case of cast iron, for example, equation (9.1) becomes

$$\Delta t_9 = - \frac{211.2 \times 1.00438 \, k}{1 + 0.00187 \, k}$$

and if k = 1 then $\Delta t_9 = -211.7$. Thus, the manufacturing industries directly and indirectly purchase £211.7m of cast iron, 68.2% of UK 1968 gross output.

Having established the direct plus indirect material purchase, the total use of UK industrial inputs via materials by these manufacturing industries is given by the second part of equation (8.9), and primary inputs by the second part of equation (8.10), which are directly calculable from the total requirements matrices. In the case of cast iron, for example, these equations are

$$\Delta t_i = -211.7\left(\frac{\delta_{i,9}}{\delta_{9,9}}\right)$$

$$\Delta g_i = -211.7\left(\frac{\gamma_{i,9}}{\delta_{9,9}}\right)$$

Using the data of Table (9.1) we find that the total coal and labour used to make the cast iron by the manufacturing industries is 211.7 x 0.0634 = £13.4m and 211.7 x 0.5238 = 110.9 th., respectively.

Table (9.3) tabulates the results for each input as a percentage of gross output or UK industrial use of primary input. The imports row of the primary inputs table includes direct imports of material by the manufacturing industries. It is clear from Table (9.3) that more industrial and primary inputs are used by the manufacturing industries by way of steel than by way of any other individual engineering material.

The results are illustrated in Figure (9,2) for energy, imports, capital stock, and labour. It is clear that despite the considerable variation in resource intensities of the engineering materials illustrated in Figure (9.1), the fact that steel is the dominant material purchased by the manufacturing industries implies that at least four times more energy, capital stock and labour is used via steel by the manufacturing industries than via any other engineering material. The imports bill via the other non-ferrous group of metals is roughly equivalent to that via steel.

(9-2.3) Effect of Design Changes

The above has compared the resource intensity of the materials and the total manufacturing industries' use of resources via each material. It is now proposed to consider how many of these resources can be saved or released for redeployment for additional or alternative production.

Savings of UK industrial output are given by

$$\Delta t_i = - \frac{\left(\sum_m d_{p,m} t_m\right) \delta_{i,p}\, k}{1 + \left(\sum_m d_{p,m} \delta_{m,p}\right) k}$$

when $\varepsilon_m = -kd_{p,m}$ is substituted in equation (8.9). The above expression is a non-linear function of k, the proportion of material saved in each manufacturing industry. Consider, however, the linear approximation of equation (8.15) which becomes

$$\Delta t_i^{(approx)} = - \left(\sum_m d_{p,m} t_m\right) \delta_{i,p}\, k$$

The percentage error of this approximation is

$$\frac{\Delta t_i^{(approx)} - \Delta t_i}{\Delta t_i} \times 100 = 100 \left(\sum_m d_{p,m} \delta_{m,p}\right) k$$

The summations are tabulated in Table (9.2) and are seen to be small. The errors in estimating resource[2] savings by making linear assumptions are 1.7k% in the case of steel, 1.1k% for other non-ferrous metals, 0.2k% for each of cast iron, aluminium and plastic, and 0.1k% for

2. For primary inputs the errors can be estimated analogously, and are identical.

rubber. Figure (9.3) is a plot of both Δt_p and $\Delta t_p^{(approx)}$ for each material. Only in the case of steel and other non-ferrous metals do the two graphs discernibly separate, and this only occurs at very high values of k. Clearly the linear approximation is satisfactory for proportions of materials which design and process changes might feasibly save.

It is concluded that a reduction in the proportion of a material used by the engineering industries will release a corresponding proportion of the resources used in the manufacture of that material. Thus from Table (9.3) we see, for example, that a 100k% reduction in the use of steel by the engineering industries will save 7.2k% of coal output, 6.0k% of stone, slate, etc. output, 17.0k% of other mining and quarrying, 28.1k% of coke,....., and so forth.

To enable easy comparison Figure (9.4) is a diagramatic representation of the savings of energy, imports, capital stock, and labour resulting from a 100k% saving of each UK produced material by the engineering industries for feasible values of 100k (i.e. $\leq$ 20%). An 18% direct saving in cast iron for example results in a total saving of £38m of cast iron, £2.4m of coal, £2.4m of coke, £1.2m of oil, £1.5m of electricity, £4.1m of imports, £108.8m of capital stock, and 20.0 th. man-years of labour.

Figure (9.4) allows easy comparison of the material savings necessary to achieve specified objectives. For example, a £10m reduction in imports cannot be achieved by feasible reductions in the use of UK produced[3] cast iron, plastics, or rubber. It can however be achieved by a 5% reduction in UK produced steel, a 14% reduction in UK produced

3. Clearly some of the saving could be achieved by reduction of direct imports of materials without the associated side effects.

aluminium, or a 6% reduction in UK produced non-ferrous metals. But it must be noted that the other effects associated with these various solutions to the same problem are vastly different. In the case of steel for example this reduction in imports is accompanied by a £200m saving of capital equipment and 23 th. man-years of total labour: in the case of aluminium the associated changes include a saving of £60m of capital stock and 8 th. man-years of total labour, while in the case of non-ferrous metals capital stock savings are £30m and total labour savings are 5 th. man-years.

From Figure (9.4) it can be observed that feasible savings of one material by itself will not release for redeployment significant proportions of energy or primary inputs, and that significant constraints on the economy such as oil or coal shortages cannot be overcome by design changes which reduce the use of one material only.

(9-3) MATERIALS AND PRICES

In Figures (5.2) and (5.3) it was illustrated that if materials saving occurs without substitution of other production factors and the cost savings are passed on to customers, then commodity prices could fall. In this section it is proposed to compare the price reductions achievable via savings of each material by the manufacturing industries, and also to compare such reductions with price rises induced by increases in coal mining income and increases in oil import prices.

(9-3.1) Materials Component of Prices

The total materials component of prices is the total — direct plus indirect — cost of materials purchased per £1 of output. It follows that if material savings occur in all industries then prices may fall in proportion to the total material component of prices. But in this chapter technical changes are considered only in the manufacturing industries, and hence the only part of the indirect relevant is that part bought indirectly from a manufacturing industry which initially directly purchased the material.

The price reduction achievable by savings of domestic materials are given from equation (8.11), and by substituting $\varepsilon_m = -d_{p,m}k$ and $\Delta\theta_j = \Delta\theta_m k$ for $j = m$, $\Delta\theta_j = 0$ for $j \neq m$ we have

$$(9.2)\quad \Delta p_i = -\frac{\left(\sum_m d_{p,m}\delta_{m,i}\right)k}{1 + \left(\sum_m d_{p,m}\delta_{m,p}\right)k} - \left(\sum_m \delta_{m,i}\,\Delta\theta_m\right)k + \frac{\left(\sum_m d_{p,m}\delta_{m,i}\right)(k)^2}{1 + \left(\sum_m d_{p,m}\delta_{m,p}\right)k} \times \left(\sum_m \delta_{m,i}\,\Delta\theta_m\right)$$

where $\Delta\theta_m$ is the change in primary input due to a technical change which saves 100k% of directly imported materials. That is

$$\begin{pmatrix}\text{Total}\\ \text{price}\\ \text{reduction}\end{pmatrix} = \begin{pmatrix}\text{Price reduction due}\\ \text{to changed use of}\\ \text{domestic material}\end{pmatrix} + \begin{pmatrix}\text{Price reduction due}\\ \text{to changed use of}\\ \text{imported material}\end{pmatrix} + \begin{pmatrix}\text{Price change due to interaction}\\ \text{of domestic material reduction}\\ \text{with imported material reduction}\end{pmatrix}$$

Consider firstly the price reductions due to saving of domestic material. If $\Delta\theta_m = 0$ the equation (9.2) becomes

$$\Delta p_i = - \frac{\left(\sum_m d_{p,m}\delta_{m,i}\right) k}{1 + \left(\sum_m d_{p,m}\delta_{m,p}\right) k}$$

The summations $\sum_m d_{p,m}\delta_{m,i}$ are tabulated for each material p and industry i in Table (9.4), and these may be used to calculate the price reductions. But consider the linear approximation of equation (8.17) which by substituting $\Delta\theta_m = 0$ and $\varepsilon_m = -d_{p,m}k$ becomes

$$\Delta p_i^{(approx)} = - \left(\sum_m d_{p,m}\delta_{m,i}\right) k$$

The error of this linear approximation is $100 \left(\sum_m d_{p,m}\delta_{m,p}\right) k\%$ which was already noted to be very small. Hence Table (9.4) is also a good approximation to the price changes resulting from a 100k% saving of UK produced materials by each manufacturing industry. For example, a 100k% saving of domestic steel could reduce the price of motor vehicles by 12.1k%, areospace equipment by 5.0k%, etc.

Consider secondly the price reduction resulting from a decreased use of directly imported materials. This is given from equation (9.2), by substituting $\varepsilon_m = 0$, as

$$\Delta p_i = - \left(\sum_m \delta_{m,i} \, \Delta\theta_m\right) k$$

These summations are tabulated in Table (9.5) for each industry i and material p. The price reductions by way of reduction of imported material are clearly small.

Reduction of both imported and domestic material may lead to the price reductions tabulated in Table (9.6). The comparison of Table (9.6) with Tables (9.4) and (9.5) clearly indicates that the interaction term of equation (9.2) is relatively small, as may be expected.[4]

Table (9.6) shows clearly that in most industries the steel component of prices is far greater than the component of any other single engineering material. For total final output a 100k% saving of material may lead to a 0.6k% price reduction in the case of cast iron, 3.4k% for steel, 0.5k% for aluminium, 1.4k% for other non-ferrous metals, 0.7k% for plastic, and 0.5k% for rubber. For consumer products price reductions are least, while for capital and export goods they are greater.

In Figure (9.5) the material price components are compared for some selected products and for final buyers. While for the products selected there is considerable variation in which material is the largest component of prices, for final buyers steel is dominant: possible price reductions via the other non-ferrous group of metals are only 45% those of steel, and all other materials are less than 25% those of steel.

(9-3.2) Comparison With Inflationary Pressures

In this section it is proposed to compare the above potential price reductions resulting from material savings with two arbitrary sources

4. The interaction term is the product of two terms which are substantially less than unity. For small, feasible, k the interaction term will be less significant still.

of price increases:[5]

(i) an increase in coal mining income (TIFE)

(ii) an increase in the price of all oil imported by UK industries

Define locally:

$\Delta\theta_j^*$ primary input in industry j associated with inflationary problem (e.g. for (i) above $\Delta\theta_1^* =$ TIFE in coal mining industry and $\Delta\theta_j^* = 0$, $j \neq 1$)

$\Delta\theta_m^t$ import of material by industry m

k^* constant giving proportion rise in inflationary problem

k^t proportion of material saved by all manufacturing industries

Hence from (8.11)

$$(9.3)\quad \Delta p_i = \left(\sum_j \delta_{j,i}\Delta\theta_j^*\right)k^* - \left(\sum_m \delta_{m,i}\Delta\theta_m^t\right)k^t - \frac{\left(\sum_m d_{p,m}\delta_{m,i}\right)k^t}{1 + k^t\left(\sum_m d_{p,m}\delta_{m,p}\right)}$$

$$+ \left(\frac{\sum_m d_{p,m}\delta_{m,i}}{1 + k^t\left(\sum_m d_{p,m}\delta_{m,p}\right)}\right)\left(\sum_m \delta_{m,p}\,\Delta\theta_m^t\right)\left(k^t\right)^2$$

$$- \left(\frac{\sum_m d_{p,m}\delta_{m,i}}{1 + k^t\left(\sum_m d_{p,m}\delta_{m,p}\right)}\right)\left(\sum_j \delta_{j,p}\,\Delta\theta_j^*\right) k^t\, k^*$$

5. See footnotes to Table (9.7) for details of what is included in the calculations of the effects of the inflationary pressure.

That is,

$$\begin{pmatrix}\text{Total}\\ \text{price}\\ \text{change}\end{pmatrix} = \begin{pmatrix}\text{Price change due}\\ \text{to inflationary}\\ \text{problem}\end{pmatrix} + \begin{pmatrix}\text{Price change due}\\ \text{to imported}\\ \text{material saving}\end{pmatrix} + \begin{pmatrix}\text{Price change due}\\ \text{to domestic}\\ \text{material saving}\end{pmatrix}$$

$$+ \begin{pmatrix}\text{Price change due to}\\ \text{interaction of}\\ \text{imported and domestic}\\ \text{material saving}\end{pmatrix} + \begin{pmatrix}\text{Price change due to}\\ \text{interaction of inflat-}\\ \text{ionary problem and}\\ \text{domestic material saving}\end{pmatrix}$$

The data for these formulations are recorded in the following tables:

$\sum'_{m} d_{p,m}\ \delta_{m,i}$ Table (9.4)

$\sum'_{m} \delta_{m,p}\ \Delta\theta^{t}_{m}$ Table (9.5)

$\sum'_{j} \delta_{j,i}\ \Delta\theta^{*}_{j}$ Table (9.7)

The effect of an increase in coal mining income and oil import price rises can be estimated from equation (9.3) by setting $k^t = 0$, hence

$$\Delta p_i = \left(\sum_j \delta_{j,i}\ \Delta\theta^*_j\right) k^*$$

Thus Table (9.7) records the effect of a 100k*% rise in these primary inputs.

A 100k*% increase in coal mining income results, for example, in a 64k*% increase in coal prices, and hence an increase of 39k*% in coke, 13k*% in electricity, 4k*% in cast iron, 3k*% in steel, k*% in motor vehicles, etc., and 1.3k*% in total final products.

Similarly a 100k*% increase in the price of oil imports causes a 74k*% rise in the price of oil and oil products refined in the UK, 7k*% in chemicals, 5k*% in plastics, 4k*% in steel, 2.7k*% in electricity, 1.8k*% in motor vehicles, etc., and 2.3k*% in total final output of UK industrial products.

To estimate what these rises would have been in the face of price reductions via material savings, each term of equation (9.3) was calculated for each combination of industry, material, and inflationary source (i.e. 5 terms for each of 60 x 6 x 2 = 720 combinations).

EXAMPLE (9.1)

Consider for example steel savings (p = 10), and coal mining income rise.

i = 1 coal prices

$$\Delta p_1 = 0.63656k^* - 0.00106k^t - \frac{0.01322k^t}{1 + 0.01656k^t} + \frac{0.01322}{1 + 0.01656k^t} \times 0.00118\left(k^t\right)^2 - \frac{0.01322}{1 + 0.01656k^t} \times 0.03415\ k^t\ k^*$$

If $100k^t = 100k^* = 1\text{x}100\%$ then

$$\Delta p_1 = 63.7 - 0.1 - 1.3 + 0 - 0 = 62.2\%$$

i = 38 cans and metal boxes prices

$$\Delta p_{38} = 0.01884k^* - 0.03289k^t - \frac{0.48391k^t}{1 + 0.01656k^t} + \frac{0.48391}{1 + 0.01656k^t} \times 0.00118\left(k^t\right)^2 - \frac{0.48391}{1 + 0.01656k^t} \times 0.03415\ k^t\ k^*$$

If $100k^t = 100k^* = 1\text{x}100\%$ then

$$\Delta p_i = 1.9 - 3.3 - 47.6 + 0.1 - 1.6 = -50.5\%$$

The analysis showed that in all cases the interaction terms were small in comparison to the sum of the first three term, and furthermore, as was shown above, that the third term could be approximated by the numerator so that an approximate price change is

$$\Delta p_i^{(approx)} = \left(\sum_j \delta_{j,i} \Delta\theta_j^*\right) k^* - \left(\sum_m' \delta_{m,i} \Delta\theta_m^t\right) k^t - \left(\sum_m d_{p,m} \delta_{m,i}\right) k^t$$

(For Example (9.1) above $\Delta p_1^{(approx)} = 62.2\%$, $\Delta p_{38}^{(approx)} = -49.8\%$ which clearly are very good approximations.) The average error of the approximation was found to be less than 0.1% for $k^* = k^t = 1$, and clearly for feasible k^*, k^t the approximation is good.

As $\Delta p_i^{(approx)}$ is simply the price change resulting from the inflationary source plus the change due to material saving, the joint effect is simply

$$k^t \times \text{(Table (9.6))} + k^* \times \text{(Table (9.7))}$$

These results can be compared as follows. If $\Delta p_i^{(approx)} = 0$ then

$$\frac{k^*}{k^t} = \frac{\sum_m' \delta_{m,i} \Delta\theta_m^t + \sum_m' d_{p,m} \delta_{m,i}}{\sum_j \delta_{j,i} \Delta\theta_j^*}$$

The ratio k^*/k^t represents the proportion of the inflation which can be overcome by a policy of materials saving. A 100k*% reduction in material use by all engineering industries will nullify the price rise in product i caused by a

$$100 \left(\frac{\sum_m' \delta_{m,i} \Delta\theta_m^t + \sum_m d_{p,m} \delta_{m,i}}{\sum_j' \delta_{j,i} \Delta\theta_j^*}\right) k^t \ \%$$

rise in coal mining income or oil import prices..

The ratio $^{k^*}/_{k^t}$ is tabulated in Table (9.8) for each industry, material, and inflationary source. In Table (9.8)(i) for example a $100k^t$% saving in steel by all manufacturing industries is able to nullify the rise in coal prices resulting from a $2k^t$% rise in coal mining income, while for stone, etc. extraction the effects of a $151k^t$% rise in mining income can be nullified, for other mining and quarrying $50k^t$%, for coke $4k^t$%, etc. For most of the engineering commodities the price rises caused by what may be considered feasible increases in coal mining income can be contained by feasible savings in steel. In the case of motor vehicles for example a 10% saving in steel is able to nullify a 127% rise in coal mining income. For total final products a $100k^t$% saving in cast iron, steel, aluminium, other non-ferrous metals, plastic, and rubber is able to overcome a $46k^t$%, $262k^t$%, $38k^t$%, $108k^t$%, $54k^t$% and $38k^t$%, respectively, rise in coal mining income.

It is concluded from Table (9.8) that a reduction in material requirements – particularly steel – by the manufacturing industries could overcome the price increases in total final output induced by a considerable increase in coal mining income and oil import prices. The relative commoditiy prices may however be substantially changed: prices could rise in non-engineering products and fall in engineering products, the consequent possibly being price increases in total consumer goods and price reductions in capital and export goods.

(9-4) MATERIALS AND REGIONAL EMPLOYMENT

In section (9-2) it was illustrated that there is substantial difference in the labour intensity of the engineering materials, and in the total labour required to manufacture the materials used by the UK engineering and construction industries. In this section it is proposed to extend the analysis to include direct plus indirect employment in materials manufacturing in each of the 11 standard geographical regions of the UK.

(9-4.1) Data and Methods

In the absence of readily available regional input-output tables [6] the analysis here is based on an extension of the constant input assumption (of labour) to employment in each region:

Constant Regional Distribution of Employment Assumption (CRDEA)

"In every industry each unit of output is produced by employment which has the same regional distribution as the total employment in that industry."

Clearly this is a highly simplifying assumption which may be invalid for some purchases, such as electricity (especially with respect to Northern Ireland). The results presented here need to be interpreted in the light of this assumption.

Data on the regional distribution of employment in the input-output industries is tabulated in Table (9.9) (sources given in footnote), and this is put into the context of total regional employment in Table (9.10). Table (9.9) is used to estimate the direct plus indirect regional employment in industries as follows.

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6. The theory of regional input-output analysis is highly advanced, see UN bibliography (18).

Define locally:

R 11 x 60 matrix of distribution of employment in each region and industry (Table (9.9)).

$\hat{e}$ a diagonal matrix of total employment in each industry (operatives plus others from primary input matrix)

From these definitions the total employment in each region is $R(\hat{e})$, the direct coefficient is $R(\hat{e})(\hat{t})^{-1}$, and the direct plus indirect employment in each region and industry per £1 of final demand is $R(\hat{e})(\hat{t})^{-1}(I - D)^{-1}$. (This equation is analogous to the total requirments equation (3.5) for all primary inputs.)

(9-4.2) Results

Table (9.11) tabulates the direct, indirect, and total regional distribution of employment in materials manufacturing. It is seen that while the direct is often concentrated in a few regions, the indirect is more in line with the regional distribution of total UK employment tabulated in Table (9.10). The result is that direct plus indirect employment in materials manufacturing is spread more evenly over the UK.[7] This is illustrated in the case of steel in Figure (9.6): while in East Anglia and Northern Ireland there is no direct employment in steel manufacturing, these regions do participate indirectly.

7. To some extent the "evening-out" of employment is due to the assumptions made about the distribution of regional employment for those industries not covered by the census reports. See footnote (2), Table (9.9).

However, despite this spread of total employment in materials manufacturing over the UK, there is still considerable regional specialization. Figure (9.7) compares the direct plus indirect employment in materials manufacturing as a proportion of total employment in each region. It is shown that employment in cast iron and steel production is intensive in an arc running from Wales to the East Midlands and then to Scotland; employment in aluminium and other non-ferrous metals production is intensive in Wales and the West Midlands; employment in plastic and rubber production is intensive in the western half of the UK.

The above results refer to total employment in materials manufacturing. But not all of these materials are used by the UK engineering and construction industries in which design changes are postulated here. The direct plus indirect regional employment in the production of each material for the UK engineering and construction industries is tabulated in Table (9.12), and compared in Figure (9.8). In the West Midlands, for example, 26.9 th. man-years of labour were used to produce the cast iron used by UK engineering and construction industries in 1968, 76.7 th. in steel, 18.2 th. in aluminium, 26.5 th. in other non-ferrous metals, 4.4 th. in plastic and13.0th. in rubber; and thus a 100k% saving of each of these materials by all engineering and construction industries in 1968 could have released for redeployment (subject to the options of Figures (5.2) and (5.3)) a proportionate quantity of manpower in the West Midlands, as well as in every other region of the UK.

Figure (9.8) shows that despite the considerable variation of regional employment with material, in all regions far more manpower is used to manufacture the steel used by UK engineering and construction industries than any other single material.

(9-5) A 10% SAVING OF ALL ENGINEERING MATERIALS

The above sections have demonstrated and compared the effects of savings in each engineering material by the manufacturing industries. In the present section it is proposed to briefly illustrate the effects of simultaneously saving all engineering materials.

Table (9.13) tabulates the results of a 10% saving in all engineering materials. This percentage was selected as being highly feasible from an engineering point of view, and the effect of other percentage savings may be obtained from this table on a pro-rata basis. The results were obtained via formulation I of chapter 3 and not from the formulae of chapter 8 because these become highly complex for multiple material changes (see section (8-4)). But it will be noticed that the results of Table (9.13) are almost identical to the sum of 10% of the results reported for each separate material in Tables (9.3), (9.6) and (9.12). Clearly the interaction of the savings are small, and the linear approximations (8.18) to (8.23) (with $\Pi_{\ell,m} = \Pi_{\ell} = -1$) are good.

Table (9.13) shows that there are substantial savings in the output of many industries, including the energy industries. Primary input savings include £81m of imports, £675m of capital stock, and 85 th. man-years of labour. These labour savings are most intensive in the materials producing regions of Wales, the Midlands, Northern, and Yorkshire and Humberside.

The cost saving sums to £240m and this could be used to finance the technical change. The largest part of this cost saving is in the motor vehicle industry.

(a) If the material saving is achieved without substitution then this cost saving is available for profits, or can be passed on to

(i) consumers; Column (4) shows the price reductions possible for each commodity. Overall the final price reduction for UK produced goods is 0.6%, a saving of £240m to purchasers (Table (9.13)(c)).

(ii) labour: Column (5) shows the average rise in TIFE in the manufacturing industries. While labour suffers a loss of 85 th. jobs, its total income rises by £240.7m - £104.0m = £136.7M.

(b) If the material saving is achieved by substitution then the cost saving can be used to pay for new capital stock, or

(i) higher paid labour: The average income will rise as in (i)(a) above.

(ii) more labour at constant income: The cost saving will finance 206.3 th. jobs at constant TIFE. Column (6) shows that a large part of this will be in the motor vehicle and in the other metal goods industries. This extra employment will be most intensive in the West Midlands and North West. In all regions except Wales the loss of direct and indirect materials production jobs is overcome by the increased labour demended in the manufacturing industries (Table (9.13)(d)). TIFE rises as in (a)(ii) above.

(9-6) MATERIALS SUBSTITUTION

In all of the previous sections design changes which reduce the material content of manufactured commodities were considered. In the present section design changes resulting in material substitution are considered. Steel was selected as the material to be substituted for because of the dominance of this material as shown above. Plastic and aluminium were selected as substituting materials because the physical properties of these materials enable them to be, and are increasingly being, used as substitutes in a wide range of applications where steel has been the traditional material.

The analysis is based on approximation formulae (8.21) and (8.22). In each of the above sections it was shown that linear approximations were good. It may be expected that these equations will also be sufficiently accurate for the present section.

(9-6.1) Substitution of Steel by Aluminium

Consider firstly a substitution by only one material, aluminium.

In equations (8.21), (8.22) put $\Pi_i^{(\theta)} = 0$, $\Pi_\ell = 0$ for $\ell \neq 10,11$, write s and a for suffixes $\ell = 10, 11$ (steel aluminium) respectively, substitute $\Pi_s = -1$ to indicate that steel is to be substituted for, and $\sum_m d_{s,m}t_m = 1095.6$ as the total steel purchased by the manufacturing industries (from Table (9.2)). Thus equations (8.21) and (8.22) become

$$(9.4) \qquad \Delta t_i = 1095.6k(\delta_{i,a} \Pi_a - \delta_{i,s})$$

$$(9.5) \qquad \Delta g_i = 1095.6k\ (\gamma_{i,a}\,\Pi_a - \gamma_{i,s})$$

for all industrial and primary inputs, respectively. Data for equation (9.4) are obtainable directly from input-output Table E; these are tabulated in Table (9.14) with data for equation (9.5).

Clearly from these equations resources will only be saved if $\Pi_a < \frac{\delta_{i,s}}{\delta_{i,a}}, \frac{\gamma_{i,s}}{\gamma_{i,a}}$. That is, whether more or less of a resource is required depends on the relative amount of aluminium required for substitution, and on the relative total use of resources in the manufacture of each material. Steel manufacturing, for example, requires 5¼ times as much coal per £1 of output than aluminium does. If £1 of steel in the manufacturing industries is replaced by less than £5¼ of aluminium then coal is saved: if more than £5¼ of aluminium is required then coal output must increase before the substitution can occur.

The critical values of the substitution cost ratio $\Pi_a^* = \frac{\delta_{i,s}}{\delta_{i,a}}, \frac{\gamma_{i,s}}{\gamma_{i,a}}$ are tabulated in Table (9.14). With the obvious important exception of imports, the use of each manufacturing resource will be lower unless substantially more than £1 of aluminium is required to replace £1 of steel. Similarly total use of labour will be lower, but there will be substantial regional differences.[8]

The considerable variation of Δt_i, Δg_i with Π_a and k are illustrated in Figure (9.9) for the materials, energy, capital stock, imports,

8. Table (9.14) indicates similar results if plastic is used as the substitute for steel.

total UK labour, and labour in the West Midlands. The diagrams show, for example, that at a 100k = 15% substitution rate and a substitution cost ratio of Π_a = 1.5, steel requirements fall by £165m, aluminium increases by £250m. Other savings include £7m of coal, £9m of coke, £2m of oil, £4m of gas, and £50m of capital stock: increased requirements include £70m of total labour (in the West Midlands labour requirements rise by 15 th.); electricity requirements remain unchanged.

The regional implications of the substitution of steel by aluminium (and also plastic) are illustrated in Figure (9.10). Such substitution would, for example, result in employment decreases in Wales if $\Pi_a = 1$ but increases if $\Pi_a = 2$ (substitution by plastic would decrease employment in Wales even if $\Pi_p = 2$), while in the Northern region employment would decrease even if $\Pi_a = 2$ (substitution by plastic would increase employment in Northern if Π_p is slightly greater than 1).

The illustration of equations (9.5) and (9.6) in Figure (9.9) makes for easy comparison of the effects of changes in engineering designs. Alternatively it is also possible to illustrate equations (9.5) and (9.6) in the form of resource isoquants, as in Figure (9.11) for imports and capital stock: each curve passes through combinations of Π_a and k for which increases or decreases in total resource requirements are identical. Such diagrams are particularly useful as a means of finding design criteria under which specified national resource savings can be satisfied. Figure (9.11) illustrates for example that substitution of aluminium for steel in the manufacturing industries in 1968 would have saved £300m of capital stock only if k and Π_a lie in the region below the -300 capital stock isoquant. If a further requirement is for example that imports must not rise the the 0 imports isoquant becomes an additional constraint which reduces the feasible region for Π_a and k, and hence for engineering design, further.

(9-6.2) Substitution of Steel by Aluminium and Plastic

If plastic as well as aluminium can be used as substitutes for steel in the manufacturing industries the the extension of equations (9.4) and (9.5) are

(9.6) $$\Delta t_i = 1095.6k \, (\delta_{i,a} \Pi_a + \delta_{i,p} \Pi_p - \delta_{i,s})$$

(9.7) $$\Delta g_i = 1095.6k \, (r_{i,a} \Pi_a + r_{i,p} \Pi_p - r_{i,s})$$

where suffix p refers to plastic. The data of Table (9.14) can be used in these equations to estimate the resource implications of a change from steel to plastic and aluminium materials for known k, δ_p, δ_a, r_p and r_a.

The resource isoquants of equations (9.6) and (9.7) are an extension of the diagrams of Figure (9.10) into three dimensions, and these may be used to find bounds on k, Π_a, and Π_p which achieve specified national objectives. As an arbitrary example suppose that total UK manpower requirements are to be reduced by at least 20 thousand, of which no more than 5 thousand is to be in the West Midlands, and oil consumption must not rise. The constraints on the material substitution parameters are given by

(i) $$1095.6k \, (0.34016\Pi_a + 0.31335\Pi_p - 0.44436) \leq -20$$

(ii) $$1095.6k \, (0.10713\Pi_a + 0.03081\Pi_p - 0.07099) \geq -5$$

(iii) $$1095.6k \, (0.01739\Pi_a + 0.04782\Pi_p - 0.04023) \leq 0$$

Any k, Π_a and Π_p satisfying these constraints define design criteria for the substitution. A cross section of the 3 dimensional feasible region corresponding to a 100k = 15% material substitution is illustrated in Figure (9.12) from which it is seen that there is considerable flexibility in the parameters, and hence in the design criteria, which will achieve the specified national objective. At point X, for example, the objective is achieved by substituting 15% of steel used by the manufacturing industries by aluminium costing 40% plus plastic costing 50% as much as the steel being replaced. This change will increase national requirements for aluminium by £65.1m, plastic by £82.3m, and imports by £16.8m, while saving £166.2m of steel, £7.0m of coal, £9.3m of coke, £1.5m of oil, £5.3m of gas, £2.9m of electricity, £230m of capital stock, and 24.9 thousand man-years of total labour including 2.1 thousand in the West Midlands.

(9-6.3) General Materials Substitution Models

General materials substitution models to include several materials are simply extensions to equations (9.4) to (9.7) above, and these may be used to estimate the effects on national resource utilization, and to estimate the feasible region for parameters defining criteria to achieve national objectives.

But it must be emphasized that as well as materials, such general models should include materials determined non-material inputs to engineering processes (see section (1-2.1)). In this chapter only the resources used to manufacture the materials were included in the models, but clearly the resources required to use the materials in the engineering industries also play a part, and indeed are part of the total economic properties of the materials.

It was indicated in chapter 1 that some of the energy, capital, labour, etc., used in the engineering processes are dependent on the materials used. In chapter 4 the change from steel to plastic in the manufacture of motor bodies, for example, entailed also a change in energy because plastic has a lower melting point and is easier formed than steel; in labour because a plastic body does not require all the manpower needed to weld and join the many parts of a steel body; in capital because plastic can be moulded into complex shapes easier than steel; in chemicals and bolts, screws, nuts, etc., because plastic components are joined by adhesives and not screws and bolts; ......, and so forth.

Consideration of such materials determined inputs to engineering processes clearly needs to be included in materials oriented attempts at economic planning. But to do so is no easy task. It requires a detailed study and understanding of a wide range of materials, of designs, and of engineering processes throughout industry. This is beyond the scope of the present work.[9]

9. A brief attempt was made to try to find a simple regression relationship between materials purchased by the engineering industries and the use of energy, capital stock, labour, etc. The approach was similar to the regression analysis of scrap metal in chapter 5, and the results were also similar: there is far too great variation between industries to obtain any simple relationship between materials and other inputs.

(9-7) SUMMARY AND CONCLUSIONS

There have been two major objectives in this chapter. The first of these objectives has been to assess the significance of the interaction terms in the formulae of chapter 8 and the errors involved in assuming linearity. It has been shown that for all of the changes considered here the interaction terms of the formulae and the errors of the linear approximations were insignificant. It is concluded that the effect of a technical change of the type considered here can be estimated directly from the published inverse input-output matrix E. Matrix inversion is unnecessary.[10]

Secondly the objective of this chapter has been to illustrate and compare the different economic properties of engineering materials, to illustrate some implications of these economic properties, and how these differences can be exploited to make some contributions to national economic objectives by design changes in the manufacturing industries.

It has been shown that the materials differ considerable in their resource use: in value terms cast iron is the most coal, coke, and labour intensive; plastic is most oil intensive; and the non-ferrous group of metals are the most imports intensive. Similarly it has been shown that there are considerable differences between the regional intensity of employment in materials manufacturing.

But steel is the dominant material used by the manufacturing industries, and hence, despite the considerable differences in the

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10. This has also been found for US input-output tables by Nimitkiat-klai (71), a postgraduate student supervised by the present writer.

resource intensities of the materials, at least four times more energy, capital stock and manpower is used by the manufacturing industries via steel than via any other single material. In the case of imports, the bill for the non-ferrous group of metals is roughly equal to the imports bill via steel. Similarly the steel component of the average price of UK final output was seen to be far greater than the price component of any other single material.

It folows that changes in the use of steel by the UK manufacturing industries will contribute far more to achieving national objectives such as resource conservation and inflation reduction than will changes in the use of any other single engineering material. A straightforward 10% saving of UK produced steel by the manufacturing industries, for example, will save £5.9m of coal, £6.4m of coke, £4.3m of oil, £4.2m of gas, £5.1m of electricity, £30m of imports, £440m of capital stock, and 48 thousand man-years of total UK labour (which would be most intensive in Wales): the change could have overcome the average price rise in all final products resulting from a 15% rise in oil import price to the UK in 1968, or a 26% rise in coal mining income.

The dominance of steel in respect to resource conservation can be illustrated by noting that when a 10% saving of all engineering materials was considered in section (9-5), the resource savings — except for imports — were only some 30 to 70% higher than those reported above for steel alone.

The final section of the chapter illustrated how estimates can be made of the effect on the economy's resources of design induced material substitution in the manufacture of final products. Steel was used as an example of the material to be substituted for by aluminium and plastic materials. The effects were shown to depend on the relative total resource intensities of the alternative materials, and on the relative amount of substituting material required. It was further illustrated how substitution parameters — and hence design criteria — can be estimated so that specified national resource objectives can be contributed to by changes in the use of materials in the manufacture of engineering and construction output.

CHAPTER TEN

C O N C L U S I O N S

(10-1) RECAPITULATION: OVERALL OBJECTIVES AND METHODS

The overall objective of this work has been to make a quantitative estimate of the effects on the UK economy of technical changes in the use of materials by the engineering and construction industries. This is a problem of analysing a system of interdependent engineering processes which progressively convert natural resources into final products. In this work an input-output analysis of UK industries has been used as an approximation to the process analysis. The analysis was based on current account interindustry transactions tables for one year only, and as such only the current effects of technical changes are included in the analysis. Possible capital investment changes directly or indirectly associated with the technical changes have not been included, and these may add to the effects reported in this work.

The technical changes considered included a substitution of steel by plastic in motor bodies, a reduction of the material content of final products, and a reduction in the steel waste arising in engineering industries. A comparison was made of the energy used in engineering and construction industries directly, and the energy used indirectly by way of materials. The effects of some technical changes in car manufacturing were compared with the effects of some non-technical changes in car use.

Some economic properties of engineering materials, and the contribution changes in material use by engineering industries can make to national objectives were compared for each engineering material.

(10-2) CONCLUSIONS

The estimates and conclusions for each individual study were presented in detail in each chapter. In this section it is not proposed to repeat these, but only to present some overall conclusions.

(10-2.1) On Methods

Input-output analysis is clearly a powerful and simple technique to estimate the effects on the economy of technical changes in engineering processes. But the ease of use of the national tables and the estimates obtained from them are very dependent on the level of process aggregation used in their construction (i.e. table size).

The high level of aggregation in the UK tables is such that much of the detailed effects on the economy are lost. A change of £1m of steel requirements, for example, may have far more significant local effects if it refers only to sheet steel than if it refers to all the output of the iron and steel industry – and similarly for other materials. Such valuable detailed interrelatedness is lost in the analysis.

But perhaps more significant for this work is that the high level of aggregation of the tables makes it difficult to identify the inputs and outputs of individual processes and the physical substance represented by the elements of the tables. Such identification is essential to estimate the economic effects of specific technical changes. The experience of this work is that likely errors of identifying input-output data and specifying alternative technical input coefficients may far overshadow any differences between alternative input-output formulations, as presented here in chapter 3.

It is concluded that from an input-output model based largely on the UK national tables:

(a) Many detailed economic effects are not readily revealed.

(b) Estimation of the economic effects of very specific engineering design changes may be difficult.

(c) Broad economic effects of more generally specified technical changes in materials use, or any other input, can be estimated.

To this end it has been demonstrated (chapters 8, 9) that simple linear combinations of the columns of the standard total requirements coefficient matrices are good and useful approximations if the technical changes refer to a limited number of input rows. Such linear combinations make for easy estimation of the technical changes which could achieve specified national objectives.

(10-2.2) <u>On Technical Changes in Materials Use</u>

The direct plus indirect use of resources in materials manufacturing in the UK are large. It has been shown that production and import of all materials used to manufacture final engineering and construction output absorbed £100m (12% of total UK) of the UK's coal output, £70m (30%) of coke, £115m (10%) of oil, £50m (9%) of gas, £110m (7%) of electricity, 1280 thousand man-years (5%) of labour, £1040m (11%) of imports, the use of £8200m (6%) of capital stock for one year, as well as a wide range of other resources. Changes in the use of materials by the UK engineering and construction industries thus may conserve, or release for alternative or additional production, a part of these resources, as well as a part of the materials determined non-material inputs used in the conversion of the materials into final products.

The analyses of the changes considered in this work have shown that indeed considerable resources could be released for redeployment. But in the context of the entire economy none of the changes, as estimated here, appear likely by themselves to lead to major short term benefit to the UK economy.

In particular, it can be concluded from these analyses that to achieve a wide range of resource savings which may be significant to the economy as a whole the following are required:

(a) Technical changes need to be widespread over industries.

> Individual changes within one industry may lead to benefits for that industry but are unlikely to result in significant resource savings for the economy as a whole. This is so even for an industry as large as the motor car industry (see chapters 4, 7).

(b) Technical changes need to be widespread over materials.

> Individual changes resulting in the changed use of only one material may lead to significant savings of that material but are unlikely to result in substantial savings

of other resources. Steel is by far the dominant material used in the UK engineering and construction industries and more resources are used in its manufacture than any other material, but even so changes in the use of steel alone will not lead to significant savings for the economy as a whole (see chapters 5, 9).

(c) Technical changes need to result in material saving rather than material substitution.

Material substitution will of course substantially alter the use of the materials directly involved, but to induce significant savings of other resources to the economy as a whole it is necessary that resources used in the manufacture of the saved material are not absorbed to increase manufacture of substituting materials.

But this is not to say that material substitution cannot be of some benefit to the economy. It has been shown that the materials differ substantially in their resource use (and other economic properties), and hence there may be scope for exploiting some of these differences to obtain savings for some resources (see chapters 4, 9). Further, if there are substantial differences between the resources used to processs the materials in the engineering and construction industries then these differences could also be exploited for resource savings (see chapter 4).

(10-3) THE WIDER CONTEXT

This work has considered technical changes relating to direct materials use in engineering and construction industries. Obviously there are many other technical and manufacturing changes which could also make contributions to national objectives.

All such possible changes may be important variables for economic planning. However, in a highly complex and interrelated economy the effects of such changes are often difficult to predict, and it may certainly not be clear what changes are likely to contribute most to given national objectives.

Consider, for example, the effects of alternative actions on overall energy requirements.

It has been shown in this work that materials are strongly linked with energy. It follows that any action taken by the manufacturing industries to contribute to national energy conservation cannot be seen in isolation from the effect on materials. If such action saves direct energy at the expense of decreased material utilization then the net effect on the economy may not be a contribution to energy conservation.

And there are further extensions. One third of all materials produced in the UK are used to make capital equipment (chapter 2). Hence if direct energy conservation by the manufacturing industries results in increased capital requirements then such direct energy savings could be outweighed by the indirect energy required to make and convert the materials used in capital equipment manufacture, particularly in the short term. Perhaps the manufacturing industries' contribution to national energy saving may lie more in increasing materials and capital productivity than in direct energy saving? Perhaps even materials saving itself may be better achieved by increases in capital productivity?

Clearly, direct changes in the use of materials in the manufacture of final products, as considered in this work, are only one part of many widespread interdependent technical and manufacturing changes which could contribute to increasing the efficiency of the whole manufacturing system. At a time when the management of the economy appears to be increasingly difficult, and when economic and manufacturing constraints threaten to rapidly change for the worse, increasing the efficiency of the manufacturing system appears essential.

To this end, one first step clearly is to increase understanding of the relation between engineering and the economy. Such understanding may lead to appropriate timely encouragement, perhaps originating from Government bodies[1], being given to the development of engineering designs and processes which maximize benefits to the economy as a whole.

And the contribution of changes in materials use may be very welcome.

1. At the time of writing NEDO was commencing a study of materials in the UK economy. The supervisor for the present work, Professor H. J. Pick, has been largely responsible for instigation of that study.

(10-4) FURTHER WORK

Given that changes in materials use is only a small part of many interdependent technical and manufacturing changes, there are clearly numerous possibilities for studying changes in related fields — including engineering, political, sociological, legal, economic, etc., studies to find appropriate means of bringing about technical changes which are of national benefit[2].

Leaving these aside, development of work along the lines of materials as considered here appears to lie in two obvious and major areas.

Firstly there is the extension into a dynamic model which includes the multiplying effects of changes on capital investment[3]. Such a model would allow estimation of the long term effects of changes, and as such may lead to a different assessment of the benefits of technical changes. Further, it may allow estimation of future materials requirements, and the optimum response over time to potential future supply problems.

The second major area of work lies in data collection. The input-output data as gathered by the CSO is of limited use for work of this type because of the very high degree of procees aggregation. The UK tables identify, for example, only four metal classifications — "iron castings, etc.", "other iron and steel", "aluminium and aluminium

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2. Indeed, what is "national benefit", and how does one relate national benefit with the interests of one industry? An interesting discussion on this point is H. W. Broude: "Steel Decisions and the National Economy." (72), which is set in an input-output framework.

3. The theory of dynamic input-output models is well developed, see UN bibliography (18). Cambridge (31,73) and Harvard (74) universities have undertaken substantial work in this field.

alloys", and "other non-ferrous metals". Important manufacturing metals like copper (and brass), tin, lead, chromium, zinc, nickel, etc., are not singly identified. For other materials the lack of detail is similar, and as such it is difficult to estimate the affects of changes in these from the standard national tables.

And not only is a greater horizontal disaggregation required, so is a greater vertical disaggregation. For effective studies of this type considerably greater detail is required on raw materials and processes in materials manufacturing (as for the sequences of Figure (1.3)), and also on processes and materials within engineering industries.

A major piece of work thus is to gather data for an input-output table which approximates more closely a process table and which is more useful for materials technology studies. From such a table a more reliable estimate can be made of

(a) the effect on the economy of shortages or price rises of individual materials.

(b) the effect on individual materials of changes in

(i) industrial demand as occurs due to design or process change in materials manufacturing, engineering, construction, or other industries.

(ii) final demand by consumers or foreign buyers, or as occurs from changes in government expenditure, capital investment programmes, etc.

With such a table in a dynamic model there are clearly possibilities for some really useful and exciting work.

APPENDIX A1

AN ALGORITHM TO TRANSLATE MAKE AND ABSORPTION MATRICES INTO INTERINDUSTRY MATRICES

There are clearly many solutions to Theorem (3.1). One algorithmic approach to find solutions based on a priority of final buyer, industry (subscript order) purchase is:

Step A: Choose for each i = 1,......,n-1

$$\mathrm{Max}\left(\zeta - \sum_{k=1}^{i-1} \phi_k - \sum_{k=i+1}^{n} A_k , 0\right) \leq \phi_i \leq \mathrm{Min}\left(\zeta - \sum_{k=1}^{i-1} \phi_k , A_i\right)$$

Step B: For each j = 1,....,n-1

(a) choose for each i = 1,.....,n-1

$$\mathrm{Max}\left\{B_j - \sum_{k=1}^{i-1} \gamma_{k,j} - \sum_{k=i+1}^{n} \left[A_k - {}_k - \sum_{m=1}^{j-1} \gamma_{k,m}\right], 0\right\}$$

$$\leq \gamma_{i,j} \leq \mathrm{Min}\left\{B_j - \sum_{k=1}^{i-1} \gamma_{k,j} , A_i - \phi_i - \sum_{k=1}^{j-1} \gamma_{i,k}\right\}$$

(b) evaluate $\gamma_{n,j} = B_j - \sum_{i=1}^{n-1} \gamma_{i,j}$

Step D: Evaluate for each i = 1,........,n

$$\gamma_{i,n} = A_i - \phi_i - \sum_{j=1}^{n-1} \gamma_{i,j}$$

The inequalities on the right-hand side above arise because a buyer neither purchases more from a source currently under consideration than he still needs, not can he purchase more than is available from that source. The left-hand side inequalities arise because firstly a purchase must be non-negative and secondly because the following must hold:

$$\begin{pmatrix}\text{Remaining availabilities}\\ \text{from sources still}\\ \text{to be considered}\end{pmatrix} \geqslant \begin{pmatrix}\text{Purchases still}\\ \text{to be made}\end{pmatrix}$$

$$= \begin{pmatrix}\text{Total purchases}\\ \text{from all sources}\end{pmatrix} - \begin{pmatrix}\text{purchases from}\\ \text{sources already}\\ \text{considered}\end{pmatrix} - \begin{pmatrix}\text{Purchases from sources}\\ \text{currently under consideration}\end{pmatrix}$$

Thus:

$$\begin{pmatrix}\text{Purchases from}\\ \text{source currently}\\ \text{under consideration}\end{pmatrix} \geqslant \begin{pmatrix}\text{Total purchases}\\ \text{from all sources}\end{pmatrix} - \begin{pmatrix}\text{Purchases from sources}\\ \text{already considered}\end{pmatrix} - \begin{pmatrix}\text{Remaining availabilities}\\ \text{from sources still}\\ \text{to be considered}\end{pmatrix}$$

The final buyer evaluation and selection is straight forward, but for the intermediate purchases it may be helpful to keep an account as shown in Figure (A1.1) where the $\Upsilon_{i,j}$ are subtracted simultaneously from the two appropriate accounts in the order indicated by the double arrow first and then the single arrow.

<u>EXAMPLE (A1.1)</u> Consider the following example:

$$\text{Flow A} = \begin{pmatrix}1.2901 & 0.0465\\ 0.2151 & 0.4656\end{pmatrix} \overset{z}{\begin{pmatrix}1.3366\\ 0.6807\end{pmatrix}} \qquad \text{Flow B} = \begin{pmatrix}0.1505 & 0.1862\\ 0.4945 & 0.1862\end{pmatrix} \overset{f_c}{\begin{pmatrix}1\\ 0\end{pmatrix}} \overset{z}{\begin{pmatrix}1.3366\\ 0.6807\end{pmatrix}}$$

$$t = \begin{pmatrix}1.5052 & 0.5121\end{pmatrix} \qquad \text{Flow U} = \begin{pmatrix}0.8602 & 0.1397\end{pmatrix}$$

$$t = \begin{pmatrix}1.5052 & 0.5121\end{pmatrix}$$

Consider commodity 1. The equations of Theorem (3.1) become:

$$\phi_1 + \Upsilon_{1,1} + \Upsilon_{1,2} = 1.2901 \qquad \Upsilon_{1,1} + \Upsilon_{2,1} = 0.1505$$

$$\phi_2 + \Upsilon_{2,1} + \Upsilon_{2,2} = 0.0465 \qquad \Upsilon_{1,2} + \Upsilon_{2,2} = 0.1862$$

and the above solution algorithm becomes

<u>Step A</u>: Choose $0.9535 \leq \phi_1 \leq 1$

<u>Step B</u>: Evaluate $\phi_2 = 1 - \phi_1$

Step C: (a) Choose $0.1040 + \phi_2 \leq \Upsilon_{1,1} \leq 0.1505$

(b) Evaluate $\Upsilon_{2,1} = 0.1505 - \Upsilon_{1,1}$

Step D: Evaluate $\Upsilon_{1,2} = 1.2901 - \phi_1 = \Upsilon_{1,1}$

$\Upsilon_{2,2} = 0.0465 - \phi_2 - \Upsilon_{2,1}$

Consider the following alternative solutions:

(i) Suppose the final buyer prefers to buy commodity 1 from industry 2 so that his purchases from industry 1 are chosen to be a minimum.

Step A: Select $\phi_1 = 0.9535$

Step B: $\phi_2 = 0.0465$

Step C: (a) $0.1505 \leq \Upsilon_{1,1} \leq 0.1505$ i.e. $\Upsilon_{1,1} = 0.1505$

(b) $\Upsilon_{2,1} = 0$

Step D: $\Upsilon_{1,2} = 0.1861$

$\Upsilon_{2,2} = 0$

(ii) Suppose the final buyer prefers to buy commodity 1 from industry 1 so that his purchases from this industry are chosen to be a maximum.

Step A: Select $\phi_1 = 1$

Step B: $\phi_2 = 0$

Step C: (a) $0.1040 \leq \Upsilon_{1,1} \leq 0.1505$

At this stage industry 1 may choose to purchase its commodity 1 from itself so the $\Upsilon_{1,1}$ is chosen at a maxium, i.e. $\Upsilon_{1,1} = 0.1505$

thus (b) $\Upsilon_{2,1} = 0$

Step D: $\Upsilon_{1,2} = 0.2901$, $\Upsilon_{2,2} = 0.0465$

which has implied a maximum of intraindustry sales and a minima of interindustry sales.

Alternatively industry 1 may elect to purchase its commodity 1 from industry 2 so that $\Upsilon_{1,1}$ is chosen at a minimum, i.e. $\Upsilon_{1,1} = 0.1040$

thus (b) $\Upsilon_{2,1} = 0.0465$

<u>Step D</u>; $\gamma_{1,2} = 0.1862$, $\gamma_{2,2} = 0$

which has implied a minima of intraindustry sales and a maxima of interindustry sales.

(iii) Suppose that all sales were according to the PMPA. The market share of each industry in commodity 1 is 0.9652 and 0.0348 for industry 1 and industry 2 respectively. Thus under PMPA $\phi_1 = (0.9652)(1) = 0.9652$, $\phi_2 = (0.0348)(1) = 0.0348$, $\gamma_{1,1} = (0.9652)(0.1505) = 0.1453$, $\gamma_{1,1} = (0.9652)(0.1862) = 0.1797$, $\gamma_{2,1} = (0.0348)(0.1505) = 0.0052$, $\gamma_{2,2} = (0.0348)(0.1862) = 0.0065$, which does not contradict the above equations.

APPENDIX A2

COMPUTATIONS FOR CHAPTER 5

Section (5-2)

The 10% saving in material input into the engineering and construction industries was achieved by subtraction of 10% from the material input coefficients in matrix B of formulation III, subject to the following :

(i) Industry 56, timber and miscellaneous wood manufactures, being both materials producing and materials using, and each component being identified in the ratio 1:2 (producer:user), the material inputs into this industry were taken as inputs into the user part and thus also reduced by 10%, except for forestry inputs which were considered inputs to the materials part – the sawmills – and thus not decreased. Purchases of other industries of the output of industry 56 were reduced by 1/3 of 10%. Purchases of industry 56 from itself were considered as sales of cut timber to the carpentry section of the industry, and thus also reduced by 10%.

(ii) Industry 2, forestry and fishing, is half materials producing (forestry plus sawmills) and half 'Other' industry. The purchases of the materials using industries of this industry were identified as timber and thus reduced by 10%.

Section (5-3)

The reductions in the proportion of material waste was achieved by the following steps on the computer for all the scrap generating industries:

For J = 22 to 42, J $\neq$ 32

(a) read π_n, π_s, $m_t^{(1)}$, $m_s^{(1)}$

(b) Calculate $m_t^{(2)} = m_t^{(1)} / (1 + \frac{1}{2}(m_s^{(1)}/m_t^{(1)} - m_s^{(1)}))$

(c) Reduce B(19,J) by $\pi_n(m_t^{(1)} - m_t^{(2)})/T(J)$

This reduces the purchase of new metal by each industry.

(d) Reduce A*(J,J) by $\pi_s(m_t^{(1)} - m_t^{(2)})/T(J)$

This reduces the sale of scrap by the scrap generating industry.

(e) Colümns J of matrix A*, and the 19th column of matrix B were adjusted to sum to 1 (B plus U) by dividing each element by the sum of the column after the other coefficient adjustments.

In matrix B the method of readjusting maintains other inputs in a constant ratio to each other (e.g. labour to profits), but not to the total. The whole column should perhaps have been evaluated independently, not according to some mathematical formula, but according to where the deficiency in scrap availabilities is made up. An alternative perhaps would have been to have placed the reductions in scrap purchases into increased ore purchases, but clearly this requires a detailed study of the Iron and Steel industry, and is beyond the scope of this work. The method of adjustment gives added weight to the primary inputs, which is why these did not fall in the iron and steel industry by the same percentage as the output of the whole industry.

Section (5-4)

The combined results were computed on the basis of section (5-3) first, then section (5-2). If the order had been reversed then there might have been some small differences in the results.

APPENDIX A3

PRIMARY INPUT DATA

(A3-1) Aggregation of Industries

The industries of the 90 order 1968 input-output tables were aggregated into 60, as shown in Table (A3.1), for the analyses of chapters 6 to 9.

(A3-2) Capital Stock

It was considered essential to include in the analyses the effects on capital stock.

Such data is not available for the input-output classifications in the UK official statistics. To estimate these is a substantial task (see Redfern[75], Barna[76], Dean[77], Feinstein[78]), and appears beyond the resources available for this work. What has been done here is to obtain a crude estimate of capital stock based on the figures of Table 64, National Income and Expenditure[35] (and some further data made available by the CSO on request) and a comparison with US data.

Table (A3.2) records the capital stock at 1970 prices for broad classes of input-output industries. These can be disaggregated to give capital stock for each of the 90 input-output industries by using "Gross Profits and Other Trading Income" as weights. For example, the "Iron and Steel" industry has an estimated £4.0 x 10^9 capital stock in 1968 (1970 prices) and a total GP&OTI of £100.4 M. Hence the "Iron castings" industry has a capital stock of £ $\frac{21.0}{100.4} \times 4.0 \times 10^9$ = £837 M and the "other iron and steel" industry £ $\frac{79.4}{100.4} \times 4.0 \times 10^9$ = £3163 M at 1970 prices. Corresponding disaggregation of the data of Table (A3.2) for each of the 90 input-output industries and subsequent re-aggregation to 60 industries leads to the capital stock figures of column (1). Table (A3.3).

The use of GP&OTI as weight for the disaggregation is suggested because these are the links between production accounts and capital accounts, via income and expenditure accounts (CSO[79] p. 61-64). Thus it may be (loosely) expected that an industry with a high capital stock could have a high GP&OTI.

As a control, the capital stock obtained by the above method was compared with US capital stock data. Table (A3.3) tabulates capital stock and labour costs for comparable US industries aggregated from the Annual Survey of Manufactures [80], 1968. Capital stock, C, and labour cost, L, in the US and UK can be related for every industry by

$$\left(\frac{C}{L}\right)_{UK} = k\left(\frac{C}{L}\right)_{US}$$

where k is a constant. Column (4) of Table (A3.3) tabulates $\left(\frac{C}{L}\right)_{US} \cdot L_{UK}$ and by comparison with the UK capital stock figures of column (1) we obtain the values of k in column (5). For most industries k lies around 2.6, but it varies considerably from 0.4 for the aluminium industry to 13.8 for wheeled tractors.

If we assume that for all industries

$$C_{UK} \simeq 2.6 \left(\frac{C}{L}\right)_{US} \cdot L_{UK}$$

then we can use this relationship to modify the figures of column (1) in the cases where k was substantially different from 2.6. The adjustments are shown in column (6): they are consistent with the totals for the groups of industries shown in Table (A3.2).

Finally, the capital stock figures were converted to 1968 prices by multiplication by a deflator of 0.9, suggested from Table 61, National Income and Expenditure [35] $(\frac{4397}{4864} = 0.9)$.

Clearly the methods used are very crude, and the results reported in this work need to be interpreted in this light. Capital stock figures are given in £M to enable a small change in the economy to record an effect on capital stock. Obviously, however, they are not accurate to this level.

(A3-3) Labour

It was also considered essential to include in the analyses results for labour in greater detail than given in the input-output tables primary input matrix.

The data used is tabulated in Table (A3.4). For industries 1 to 55 the data was obtained from the Census reports (Table 1). For industries 56 to 60 the estimates of employment and wages etc., are based on a weighted average for industries 1 to 55, the weights being "Total Income From Employment."

APPENDIX A4

A NOTE ON THE MATEMATICAL CONTENT OF CHAPTERS 8 AND 9

Consider a final demand change for any one product x. The changed resource requirement (output of each industry and use of primary input) is given by the product of the direct plus indirect resources which produced unit x and the changed final demand for product x. That is, one simply multiplies the changed demand for x by column x of matrices $(I - D)^{-1}$ and $U(I - D)^{-1}$ of the standard tables. This is so for any product x or for any combination of products.

If this were also the case for industrial demand then analysis of the effects of technical change could be much simplified. But each element of D plays a part in each element of $(I - D)^{-1}$, so that multiplication of the changed industrial demand for product x by the fixed elements of $(I - D)^{-1}$ and $U(I - D)^{-1}$ of the standard tables is not theoretically correct.

What is shown in chapter 8 is that when only one row of D alters, then certainly all elements of $(I - D)^{-1}$ and $U(I - D)^{-1}$ alter, but the elements of column x (and only column x) alter by a constant ratio (corollary (8.1.2)). Hence the ratios of column x of the standard matrices $(I - D)^{-1}$ and $U(I - D)^{-1}$ can be used to estimate the effects of changed industrial use of product x (as in Theorem (8.2)).

Further, the results of chapter 9 show that, while a change in D alters all elements of $(I - D)^{-1}$ and $U(I - D)^{-1}$, the alterations are not significant for the changes in D considered here. Hence the analysis can be based on simple linear combinations of the columns of $(I - D)^{-1}$ and $U(I - D)^{-1}$ of the standard input-output tables, and a change in industrial demand of the simple type considered in chapter 9 is approximately equivalent to a corresponding change in final demand.

That is, the estimators

$$\Delta t = \{ (I - D^{(n)})^{-1} - (I - D)^{-1} \}f$$

$$\Delta g = U\{ (I - D^{(n)})^{-1} - (I - D)^{-1} \}f$$

can be approximated by

$$\Delta t = (I - D)^{-1}(D^{(n)} - D)(\hat{t})$$

$$\Delta g = U(I - D)^{-1}(D^{(n)} - D)(\hat{t})$$

APPENDIX A5

BUILDING MATERIALS AND THE CONSTRUCTION INDUSTRY

For comparison with Figures (9.1) and (9.2) Figures (A5.1) and (A5.2) illustrate the comparative resouce use in the manufacture of building materials, and hence the construction industries use of resouces via these materials .

MATERIAL	INDUSTRY NUMBER	ABBREVIATION
Stone, sand, etc.	2	e
Bricks	40	b
Glass & pottery	41	g
Cement	42	m
Other building materials	43	o
Timber	45	t
Paint	7	i
Plastic products*	50	d

*Previously plastic products were considered as a materials using industry. It makes products from plastic materials (industry 8) which could have been made from other materials. Here it is considered as a materials producing industry as its omission would seriously understate the use of plastic by the construction industry, particulary since imports of plastic materials by the UK are via the plastic products industry.