THERMAL RADIATION CHARACTERISTICS

OF FLUIDIZED SOLIDS

by

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A thesis submitted in fulfilment of the requirement

for the degree of Doctor of Philosophy

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S U M M A R Y

Radiative heat transfer from an emulsion of high temperature fluidized solids is visualised, utilizing a continuum approach, as that from a volume emitter of voidage greater than zero. The initial considerations were of a simplified mathematical model which for an homogeneous constant property, emitting, absorbing and scattering medium may be described by the exact formulation for one dimensional, steady-state radiative transfer. From the analysis the dependancy of the emitted radiative flux levels upon the emissivity of the bed material emerged. Such observations were verified experimentally.

Following a study of the hydrodynamic behaviour of the bed surface, a model describing transient radiative transfer was evolved based upon the packet model with initial isothermal conditions. The small residence times of the surface eruptions, found experimentally, constrained the flux levels to be almost invariant with time.

The analysis was theoretically and experimentally extended to include the conductive heat transfer mode in consideration of surfaces immersed into a fluidized bed. Again, using the packet model the dominant conductive mode was apparent for early particle residence times. Radiation plays an increasingly significant role as the time history of the particle contact period proceeds. Contrary to previous suggestions, radiation contributes significantly to the overall energy transfer for the normal operating temperature range of fluidized bed heat exchangers, unless

sufficiently low particle or transfer surface emissivities are encountered.

Throughout the work a number of problems have been high-lighted where further effort is required.

ACKNOWLEDGEMENTS

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NOMENCLATURE

A   Surface area

Bi   Biot number

C   Velocity of light

C_p Specific heat capacity

C_D Drag coefficient

$D(\tau)$ Non-dimensional function defined by equation (B.11)

D   Diameter

d   Object distance measured from radiation pyrometer

$E_n(x)$ Exponential integral defined by equation (2.1)

erf(x)   Error function

$F_{1,2}$ Non-dimensional boundary flux

F_{k-j} View factor between surfaces k and j

f_o Bubble voidage

G   Irradiation

g   Acceleration due to gravity

h   Height of rise of ejected particle cloud, heat transfer coefficient

H   Bed height

$H_{\bar{\omega}}$ 'H'-function defined in [12]

I   Intensity of radiation

K   Coefficient, thermal conductivity

L   Thickness measured from bed surface, interparticle distance

m'   Defined by equation (1.3)

N(x)   Particle concentration distribution

n	Number of particles
N_λ	Steradiancy
Nu	Nusselt number
N_1	Conduction - Radiation number
$P(\theta)$	Phase function
P	Pressure
ppm	Parts per million
Q	Heat flow rate
q	Heat flux
Re	Reynolds number
T	Temperature, thermocouple
t	Absolute time
t'	Non-dimensional time
U	Velocity
V	Volume, velocity
W	Weight
$\bar{x}$	Particle diameters
x	Any position within the bed measured from the bed surface

Greek Symbols

α	Absorptivity
β	Angle
$\Delta t'$	Non-dimensional temperature increment
ΔP	Pressure difference
$\Delta \tau$	Optical depth increment
δ	Kronecker delta

ϵ	Emissivity, voidage
θ	Angle, non-dimensional temperature
λ	Wavelength
μ	$\cos \theta$, viscosity
ρ	Density, reflectivity
σ	Stefan Boltzmann constant
τ	Optical depth, transmissivity
τ_o	Optical thickness
ϕ	Azimuthal Angle
ϕ_s	Sphericity
ψ	Angle
ψ_1	Defined by equation (1.2)
ω	Solid angle
ω_o	Scattering albedo
Ω	Solid angle
ν	Refractive index

Subscripts

1	Boundary 1
2	Boundary 2
a	Absorption
b	Bed, black body
c	Conduction
d	Directional, detector
e	Effective, emulsion
f	Fluid, fluidized
g	Gas

h	Hemispherical
i	Isothermal, instantaneous
m	Minimum, mean
n	Non-isothermal
p	Particle
r	Radiation
s	Sample, scattering
T	Thermocouple
t	Target, total
W	Wall
x	Region
y	Region

Superscripts

(')	Refers to a non-dimensional quantity or dummy variable of integration
u	Upper triangular matrix
L	Lower triangular matrix

C H A P T E R   1

AN INTRODUCTION TO THERMAL RADIATION WITHIN BEDS OF
HIGH TEMPERATURE FLUIDIZED SOLIDS

1.1 Introduction

The analysis of radiative energy transport within and through
beds of fluidized solids, or gas-solid emulsions, has received only
limited attention during recent years. In the literature appears
a wealth of information concerning the two modes of heat transfer
in fluidized beds, namely conduction and convection, but for a
number of reasons the radiative mode has escaped an equal degree
of attention. With the advent of fluidized bed heat exchangers,
particularly where combustion of a gas/air mixture takes place
within the bed of fluidized solids, the temperatures encountered
are of a sufficient level to warrant the inclusion of the thermal
radiative component in the governing energy equation. The com-
bustion of a gas/air mixture within a bed of fluidized solids
takes place at temperatures much lower than the theoretical flame
temperature of such a mixture. For this to occur the heat losses,
which are predominantly radiative, from the upper, highly turbulent
free surface of the bed must be large. In fact, of order 50% of
the total heat capacity for a bed temperature of 1 000°C with a
theoretical flame temperature of 2 000°C.

Heat transfer in any porous media (emulsion) is by gas and
solid conduction as well as radiation. The mechanism of conduction
through the gas and solid are similar, in a sense that the heat
flux is proportional to thermal conductivity and local temperature
gradient. Radiation on the other hand is a long range process

and must be treated accordingly. Local inhomogeneities in the
emulsion affect the transmission of radiant energy. For example,
radiation traversing the emulsion may (a) pass through voids in
the porous bed, (b) be transmitted through the particles, (c) be
absorbed by the particles and subsequently re-emitted (Kirchoff's
Law) and (d) be scattered (re-orientated) by the particles. The
emulsion can be considered to be homogeneous and continuous.
This is justified if the gas voids and particles are essentially
in equilibrium, and if the particle spacing is sufficiently small
so that the temperature difference between adjacent particles is
small compared to the absolute temperature.

   Although the particular interest in this study is concerned
with shallow fluidized bed heat transfer, the initial model of
radiative transfer applies equally to beds of any depth.
Unfortunately, no strict definition of a shallow fluidized bed
exists and so, where necessary, a shallow bed is assumed to be
one in which bubble coalescence does not have sufficient time
to occur.

1.2 Review of Previous Studies

   The limited amount of thermal radiative studies in fluidi-
zed beds that have been conducted have shown varying degrees of
contradiction in a number of important fundamental areas. The
levels of bed temperature below which the contribution of thermal
radiation to the total energy exchange may be ignored is a first
example. Yoshida and Kunii[1] studied radiant heat transfer
from a bed of fluidized solids to two kinds of heat exchange pipe,

one of a high emissivity 0.8 and one of a low emissivity 0.17, the
bed walls having an emissivity of 0.8. The bed material was a
microspherical catalyst of size distribution 0.149-0.210 mm and
density 1 540 kg/m^3. The authors concluded from this experimental
study that the contribution due to radiation heat transfer was
not significant (approximately 5%) below bed temperatures of
1 000°C - 1 200°C. A fundamental assumption based on the work
of Zabrodsky[2] and used in [1], was that the degree of blackness
(effective emissivity) of the emulsion is equal to unity. In
contrast to the work of [1], Vedamurthy and Sastri[3] in a
mathematical study of radiation and conduction heat transfer to
the walls of a fluidized bed combustion concluded, for the case
of coal burning within the combustor, that radiation contributed
significantly (approximately 30%) to the overall heat transfer
down to bed temperatures of 800°C. In this case the assumption
was again that the emulsion effective emissivity was equal to
unity although no mathematical justification was given. A
number of further studies have been attempted but are concerned
primarily with radiative heat transfer to immersed surfaces
within a fluidized bed. These references are cited in studies
reported later in this work.

1.2.1 The fluidized bed effective emissivity

In the previous section the term bed effective emissivity
was introduced and requires a formal explanation. The term was
introduced by Zabrodsky[2] who recognised the fact that a parti-
culate bed may have an effective emissivity of a somewhat different

value from the emissivity of the actual particle material. The
term 'effective' is used to distinguish between the strict defini-
tion of emissivity of any isothermal surface emitter of voidage
zero and a volume emitter (emulsion of fluidized solids) of
voidage not equal to zero, with either isothermal or non-isothermal
particulate boundaries. In the analysis, Zabrodsky[2] approximated
the emulsion of fluidized solids by a dusty gas cloud. Using
the equation of energy attenuation (Beer's Law)

$$\frac{dI}{dx} = -K_a I \qquad (1.1)$$

an expression for the effective emissivity was derived for the
system. A complete description of this analysis appears in
Appendix A. It was subsequently shown in [2] for a typical
fluidized bed system that the effective emissivity ε_e was
approximately unity in all cases. This is not a surprising
conclusion as in addition to the high values of particle concen-
tration a fundamental assumption of Beer's Law is that all particles
are thermally black.

In contrast, the work of Rubstov and Syremyatnikov[4]
considered the problem of effective emissivity from an idealized
model of the geometry of particles in contact with a plane vertical
surface. A term

$$\psi_\lambda = \left[\frac{1}{m'}\right]^2 \qquad (1.2)$$

where $m' = (1 - \varepsilon_b)^{-\frac{1}{3}}$ $\qquad (1.3)$

and ε_b is the bed voidage was evolved as being the fraction of the surface overshadowed by the first row of particles. For n rows of particles the effective emissivity was derived as

$$\varepsilon_e = \sum_1^n \varepsilon_p \psi_K \quad \text{for } K = 1,\ldots.n \tag{1.4}$$

and ε_p = particle material emissivity. It was concluded that ε_e may differ significantly from unity depending largely upon the value of ε_p.

In a discussion of [4] by Zabrodsky[5] it was shown that the term $\psi = (1/m')^2$ was not strictly justified and, for spherical particles, must depend upon their packing arrangement, e.g. for loose cubic packing of spheres, the interparticle distance

$$L = 0.806 \, D_p (1 - \varepsilon_b)^{-\frac{1}{3}} \tag{1.5}$$

and

$$\psi_1 = \frac{\pi D_p^2}{4L^2} = 1.2\left[\frac{1}{m'}\right]^2 \tag{1.6}$$

Later experimental work by Pikashov et al[6] in which a narrow angle radiometer probe was placed in a fluidized bed with the thin quartz protecting glass situated in the horizontal plane, supports the results of [4] and concludes that the effective emissivity must deviate from unity. However, in a similar experimental analysis by Ilchenko et al[7], considerable cooling of the first row of particles in contact with the quartz protecting glass were observed. This would account for a reduction in the measured levels of effective emissivity.

Hence the 'state of the art' to date is confused being dependent on highly idealized systems and for this reason it is desirable to reconsider the problem of radiation heat transfer within an emulsion of fluidized solids using a physically and mathematically justifiable approach.

1.3 The Equation of Radiative Transfer

Considering Zabrodsky's[2] earlier hypothesis of a dust cloud it is obvious from equation (1.1) that the energy attenuation on traversing the emulsion is due to the absorption properties of the emulsion alone. Consideration of a more complete equation, i.e. the equation of radiative transfer, introduces other factors into the analysis.

The equation of radiative transfer in non-dimensional form may be written as

$$\mu \frac{dI}{d\tau} = -I(\tau) + \frac{\omega_o}{4\pi} \int_{4\pi} I(\tau, \theta') P(\theta, \theta') d\Omega + (1 - \omega_o) I_b(\tau) \quad (1.7)$$

A more complete description of the derivation of equation (1.7) and its adaption for use with the governing energy equation appears in Appendix B. Briefly the equation describes energy (radiation intensity I) attenuation and augmentation when traversing a differential element of emulsion. The first term on the right-hand side of equation (1.7) describes energy attenuation due to absorption and scattering of the incident beam. The second term describes energy augmentation due to scattering (reorientation) of energy in the direction of the incident beam. The third term

accounts for the emission of the particles within the emulsion.
Before proceeding to an analysis of equation (1.7), an explana-
tion of the parameters appearing in the equation are now given
in relation to a bed of fluidized solids.

1.4 Application of the Equation of Radiative Transfer to a Bed of Fluidized Solids - Description of Important Parameters

In a general sense any medium which absorbs, emits and
scatters radiation, whether a particle seeded flame, thermal
insulation material or a bed of fluidized solids may be equally
described by the equation of radiative transfer. It is convenient
here to consider the packet model of heat transfer described by
Figure 1.1 in the case of an emulsion of fluidized solids through
which radiative energy is transported. The packet model was
first suggested and developed by Mickley and Fairbanks[8] and
used to describe conduction heat transfer between an emulsion
and an immersed surface. A similar packet model is considered
in this study where all particles are regarded as having zero
relative motion, and heat is transferred only by the viewing (in
a radiative sense) of a surface by a packet of emulsion which is
constantly replenished with fresh particles. Obviously, conduc-
tion and convection heat transfer should be included into the
analysis; however, the initial consideration is of radiation
predominating.

1.4.1 Description of the important parameters encountered in the

radiative transport equation

(i) Scattering cross-section

The extent of scattering of an object (particle) is normally, particularly in the nuclear energy transport field, expressed in terms of a scattering cross-section. This is the apparent area that an object presents to an incident radiative beam in relation to the ability of the object to deflect (reorientate) radiative energy. This apparent area may be quite different from the physical cross-sectional area of the object as shown in Table 1.1 taken from Siegel and Howell[9], depending upon the ratio of the particle size to the radiative wavelength of the incident beam, $\pi D_p/\lambda$. The ratio of the scattering cross-section to the actual geometric projected area of the particle is known as the efficiency factor.

TABLE 1.1   (Reproduced from [9])

BODY	PHYSICAL CROSS-SECTION mm^2	CONDITIONS	TYPE OF SCATTERING	SCATTERING CROSS-SECTION
Particles of Diameter D_p	$\dfrac{\pi D_p^2}{4}$	$\lambda \gg D_p$ single scattering	Rayleigh	Proportional to V^2/λ^4
		$\lambda \simeq D_p$	Mie	Varies widely
		$\lambda \ll D_p$	Fraunhofer and Fresnel diffraction plus reflection	$\sim 2\left[\dfrac{\pi}{4} D_p^2\right]$

(ii) Extinction coefficient

With reference to Section 1.3, attenuation of radiative
energy is due to both scattering and absorption of an incident
beam. In the radiative transfer equation (1.7) it is convenient
to combine these two effects in terms of a single parameter, the
extinction coefficient. With reference to Table 1.1 for a bed of
fluidized particles, the ratio $\pi D_p \gg \lambda$ always applies and hence
the absorption coefficient (K_a) is simply defined in this study
as

$$K_a = \frac{N \pi D_p^2 \varepsilon_p}{4} \tag{1.8}$$

from Hottel and Sarofim[10] where ε_p is the particle material
emissivity and N is the particle concentration. The particle
concentration may be represented by a function of distance for
a known distribution, but this refinement produces unnecessary
complications at this stage of the work and so N is taken as a
constant value throughout the emulsion field. The scattering
coefficient K_s is similarly derived as the absorption coefficient
and defined as

$$K_s = 2 \left[\frac{N \pi D_p^2 \rho_r}{4} \right] \tag{1.9}$$

where ρ_r is the reflectivity of the particle material from [10].
The multiplying factor of 2 in equation (1.9) accounts for the
diffraction of energy around large particles where $\pi D_p \gg \lambda$ from [9].
The extinction coefficient is then simply $K_t = K_a + K_s$.

(iii) Phase function

A complete description of the phase function appears in [9].
Briefly, $P(\theta)$ is the intensity distribution, as a function of
circumferential angle θ, of scattered energy. A number of dif-
ferent functions of $P(\theta)$ are described in [9] for various shapes
and sizes of particles. In the case of fluidized particles with
$\pi D_p \gg \lambda$ the phase function of energy scattered over 4π steradians
is, for diffusely reflecting particles

$$P(\theta) \;=\; \frac{8}{3\pi}\,(\mathrm{Sin}\,\theta - \theta\,\mathrm{Cos}\,\theta) \qquad\qquad (1.10)$$

The inclusion of such a phase function to account for anisotropic
energy scattering, produces unnecessary complications in a solu-
tion of the radiative transfer equation. For this reason, all
energy scattering is assumed to be isotropic and then $P(\theta) = 1$.
A further justification of this fundamental assumption is shown
in the work of Love et al [11], for a generalised absorbing and
scattering medium, who concluded that anisotropic scattering
accounts for 150% change in the overall emitted flux levels at
$\tau_o = 0.1$, to 30% at $\tau_o = 2.0$.

However, the assumption of isotropic scattering is often
valid in an optically thick medium, the difference in flux levels
reducing as $\tau_o \to \infty$.

(iv) Scattering albedo

The scattering albedo describes the amount of energy
scattered to the total energy attenuated, i.e. $\omega_o = K_s/K_t$.

Some workers, Chandresekhar[12] and Kourganoff[13] use the value

of

$$\frac{K_a}{K_t} = (1 - \omega_o) ,$$

the degree of darkening, in an astrophysical context.

(v) Optical thickness, τ_o

An important dimensionless parameter in any radiation

absorbing, emitting and scattering medium is the optical thickness

of the medium. In this present study a fundamental assumption is

that all radiation is gray and independent of wavelength distri-

bution. Another assumption is that all radiative properties of

the fluidized emulsion are independent of temperature variation.

It therefore follows that the extinction coefficient K_t is also

independent of wavelength and temperature distribution. Hence

the characteristic physical dimension (L) of the emulsion may

be transformed into an optical dimension (photon penetration

depth) as $\tau_o = K_t L$. Physically, depending upon the constituents

of the emulsion, the optical thickness may vary between zero, for

a vacuum, and infinity for a solid object. In the case of a

fluidized bed the passage, through an element of emulsion, of a

beam of radiative energy is affected by other elements in close

proximity. Hence the regime of application of optical thickness

is for $\tau_o \gg 1$, i.e. the emulsion is regarded as being optically

thick. When considering the depth (x) of an emulsion of thickness

(L), the term optical depth $\tau = K_t x$ is used.

1.4.2. Considerations of a simplification of the equation of radiative transfer

(i) The alternate slab model

In the analysis of [3], the equation of radiative transfer was not considered, the radiative flux appearing in the energy equation in a simplified form of $q_r = \sigma T^4(x)$ with the assumption $\varepsilon_p = 1$ or, in nondimensional form,

$$q_r' = \theta^4(x) \tag{1.11}$$

From Appendix B the radiative flux equation is given for a black body ($\omega_o = 0$) emulsion, as

$$q_r'(\tau) = 2F_1 E_3(\tau) - 2F_2 E_3(\tau_o - \tau) + 2\int_0^\tau \theta^4(\tau') E_2(\tau - \tau') d\tau'$$

$$- 2\int_\tau^{\tau_o} \theta^4(\tau') E_2(\tau' - \tau) d\tau' \tag{1.12}$$

Equation (1.11) may be obtained from equation (1.12) by assuming the emulsion to be made up of finite slabs of solid material separated by interstices of negligible optical thickness, i.e. $\tau = \tau_o \to 0$. Hence equation (1.12) becomes

$$q_r' = 2F_1 E_3(0) - 2F_2 E_3(0)$$

and

$$q_r' = F_1 - F_2 \tag{1.13}$$

which is simply the radiative flux transfer between two black

parallel plates.

Again in [3] for use in a finite difference solution of the energy equation then (1.13) is recast in the form

$$q'_{r_i} = F_i - F_{i-1} \text{ for } i = 1, \ldots n \tag{1.14}$$

for the ith and (i-1)th layers of the emulsion, or

$$q'_{r_i} = \theta_i^4 - \theta_{i-1}^4 \tag{1.15}$$

However, when considering the physics of the emulsion, equation (1.15) neglects the contribution to the flux at the ith and (i-1)th layer of fluxes emitted by remote layers and passing through the intermediate interstices even though thermal radiative transfer is a long range (global) process. As (1.15) was obtained from assuming a negligible optical thickness between the layers it would seem reasonable to assume that the contribution of flux from adjacent and near adjacent layers is contributory. The simplification to obtain equation (1.15) is not consistent with the application of such an equation in a fluidized bed of $\varepsilon_b > 0$. The emissivity of such a model is equal to that of the material considered and from a consideration of Figure 2.1 such a result would underestimate the subsequent radiative fluxes for all cases except $\varepsilon_p = 1$.

In the work of Hill and Wilhelm[49] a similar slab model was considered in a study of combined radiative and conductive heat transfer in a quiescent gas-solid bed. However, for a

generalized bed material of particle emissivity, ε_p, the bed was considered to be made up of either infinite parallel planes, concentric cylinders of infinite length, or concentric spheres, each surface considered gray and partially transparent. The allowance for the transparent nature of the surfaces (slabs), thought of as the effective fraction of free area, is an imporvement over the model used by [3] accounting for the long range transfer of radiative energy. In addition, the effects of the reflectivity and absorbtivity of the surfaces were included. The resulting equations were derived by applying an energy balance between successive surfaces and summing across the field. The radiative flux equations were similar, from a heat transfer point of view to those of Hamaker[20] who considered a differential, rather than a finite slab approach to the problem. By way of interest the radiative flux equation obtained by [49] of

$$q'_r = \frac{\sigma(T_o^4 - T_{N+1}^4)}{\frac{1}{\alpha_o A_o} + \frac{1 + \rho_r - \tau_t}{1 - \rho_r + \tau_t} \sum_{i=1}^{N} \frac{1}{A_i} + \frac{\rho_{r\,N+1}}{\alpha_{N+1} A_{N+1}}} \tag{1.16}$$

simplifies to that of radiative transfer between two infinite parallel plates of temperatures T_o and T_1 respectively i.e. the second term in the denominator disappears. The work of [49] and [20] are in fact different approaches to the derivation and solution of the radiative transfer equation.

(ii) Optically thick approximation

A more realistic approximation of radiative transfer within

fluidized beds is the optically thick approximation. From Sparrow and Cess[26] it was shown that the radiative flux equation of Appendix B may be simplified in the case of $\tau_o >> 1$ of

$$q_r' = -\frac{4}{3}\frac{d\theta^4}{d\tau} \qquad (1.17)$$

in nondimensional form and implies that radiative fluxes at a plane are affected by its near neighbours only, being a function of the local temperature gradient. Unfortunately, this assumption breaks down in the vicinity of a boundary for reasons given in [26] and explained in Chapter 2 of this work.

(iii) The Zone Method

Developed by Hottel and Sarofim[10], this method consists of subdividing nonisothermal enclosures filled with a non-isothermal gas into areas and volumes that can be considered isothermal. An energy balance is then applied to each division leading to the solution of a set of simultaneous equations. The method is not mathematically elegant but forms a powerful practical tool having considerable advantage in multidimensional situations.

1.5 Concluding Remarks

From a consideration of the radiative transfer equation and its various components a number of approximate approaches to a solution have been considered. However, as the initial study forms a fundamental analysis of radiative transfer within and through emulsions of fluidized solids, an application of the exact

radiative transfer equation is desired. Where appropriate, comparisons are made with the approximate solutions. From the subsequent application of the radiative transfer equation it is convenient to model the fluidized bed as that of a packet of gas-solid emulsion with appropriate mean physical properties.

C H A P T E R   2

ANALYSIS OF THE RADIATIVE TRANSFER EQUATION

2.1 Introduction

Applications of radiant transport theory has received considerable attention over recent years, and appears in the literature of astrophysics [12] and [13], optics, Gumprecht *et al*[14], chemistry, Chu and Churchill[15] and nuclear energy, Davison [16]. One of the most thorough and clear presentations is given by Viskanta[17] who emphasized the heat transfer point of view. The work in [17] is particularly relevant to the present study of fluidized solid radiative transfer and for this reason [17] and subsequent studies by the same author are constantly cited throughout this text.

2.2 Development of the Mathematical Model

As a first step a simplified physical model of a fluidized emulsion is assumed along the lines of the packet model of [8]. A plane, gray layer of the emulsion packet is bounded by plane, parallel, gray, diffusely emitting - reflecting boundaries, according to the configuration of Figure B1 in Appendix B. In addition, a number of further simplifying assumptions are made and summarised as follows:

1. One-dimensional radiative heat transfer

2. Constant fluid and solid properties

3. Fluid and solid in local thermal equilibrium

4. Steady-state

5. Fluid and solid in dynamic equilibrium

6.   Constant absorption and scattering coefficients

7.   Isotropic scattering

8.   Emulsion, homogenous and isotropic and of constant voidage

With consideration of later experimental studies, the emulsion is further visualized as a homogeneous slab exchanging energy, by radiation only, with a remote heat transfer surface (detector).

2.3   Previous Solutions of the Radiative Transfer Equation

The equation of radiative transfer has been applied to a similar model as previously described and solved using a number of different techniques. Edwards and Bobco[18] in a generalized study of dispersion emission used a modified diffusion method of solution. This technique was based upon the method of moments used in an application of the equation of radiative transfer to nuclear transport problems. The particular application of [18] is in the field of radiation exchange from a seeded rocket engine exhaust plume to some remote heat transfer surface (e.g. engine base). An interesting and relevant feature of this analysis is the use of free boundary conditions which occur equally in the case of rocket engine exhaust plumes and beds of fluidized solids. The free boundary is applicable whenever an inward directed radiative flux on the emulsion boundary has a negligible influence on the temperature and radiative flux distribution within the emulsion. Further conclusions from [18] of a fundamental nature were the concept of radiative intensity, when non-dimensionalised

to the emulsion black body intensity, being equivalent to directional emissivity and that isotropic scattering does not lead to the classical Lambert diffuse emission at an emulsion boundary. Even in an emulsion of $\tau_o \to \infty$ the latter case is not met. This conclusion emphasizes the care which must be taken in interpreting experimental results when using a narrow angle (field of view) radiometer to view such an emulsion. Finally, the relation between an area formulation and a volume formulation of the problem was indicated to show that any scattering system may be solved as a field problem and the results used with an area configuration factor to obtain the irradiation at some remote surface. Alternatively, the irradiation may be obtained directly by application of the volume formulation as shown in [17] without introduction of a configuration factor concept.

In a study of radiation through a foggy (scattering) atmosphere, Schuster[19] introduced the now well-known two flux method of approximate solution of the radiative transfer equation. This method was extended for use in heat transfer systems by Hamaker[20] and consists of reducing the non-linear, integrodifferential equation to two simultaneous, linear differential equations, which in a simple plane parallel system are amenable to a closed form solution.

In a similar study area as [18], Morizumi and Carpenter[21] used a statistical probability approach to solve the radiative transfer equation. However, only the first term on the right-hand side of the equation was considered and similar to the equation used in [2]. The multiple scattering of radiative

energy was allowed for by the probability functions derived for an incident beam traversing a differential slab of emulsion. Comparisons of the analytical predictions with subsequent experimental results, using a narrow angle radiometer and a spectrometer, resulted in an average difference of 17% between the two.

2.4 Analysis

The equation of radiative transfer in conjunction with the energy equation in Appendix B is now amenable to a closed form solution and hence a numerical technique is employed. The solution is complicated by the fact that the exponential integral $E_1(|\tau - \tau'|)$ appearing in the energy equation is singular at the origin. This problem is overcome by approximating the temperature distribution $\theta^4(\tau)$ and the scattering function $D(\tau)$, with finite polynomial expansions, as suggested by Leung and Edwards[22] in a study of solar heat shields and solar heat collectors. This method was also used in a non-scattering system for the temperature distribution in a time dependant radiative heat transfer study by Viskanta and Bathla[23] and by Krishna Prasad and Hering[24] in a similar investigation.

After spacial discretization of the plane parallel layer of emulsion, bounded by gray, diffusely emitting, black boundaries, the problem is reduced to one of solving for unknown coefficients using a matrix method applied across the field.

2.4.1. Boundary conditions

As discussed previously, the free boundary of a fluidized

bed emulsion may be regarded for radiative heat transfer purposes
as one in which the incoming boundary radiative flux has negli-
gible effect on the temperature and flux distribution within
the emulsion. This being the case, the free boundary condition
used in this analysis is taken as equivalent to a black plane
wall at either zero absolute temperature with $F_1 = \theta_1^4 = 0$, or at
atmospheric temperature with θ_1^4 of order 0.002 for a bed tempera-
ture at 1 000°C. This boundary condition is obviously idealized
and suggests a temperature discontinuity at the free surface of
the emulsion. In reality, conduction and convection heat transfer
will occur in addition to radiation heat transfer to eliminate
the apparent discontinuity. The idealized boundary condition
used here is not physically justifiable but is used as a
mathematical expedience. A similar problem was observed by.
Deissler[25] in a study of radiative heat transfer to parallel
plates from an absorbing, emitting and scattering medium. The
case of an optically thick medium was taken, resulting in an ap-
proximation to the radiative flux equation of

$$q_r' = -\frac{4}{3}\frac{d\theta^4}{d\tau}$$

from [25] and [26]. Appropriate boundary conditions were evaluated
to account for the apparent discontinuity when conduction was
absent from the analysis. This technique is commonly referred to
as the radiation slip method.

The second boundary condition, deep within the emulsion, is
taken to be equivalent to a plane, black, diffusely radiating

surface at the bed temperature. This is in accordance with the
configuration of Figure B.1 of Appendix B, and Figure 1.1. The
assumption of such a black boundary was chosen to simulate the
radiative flux emitted from the volume of emulsion beyond a depth
of $\tau = 10$. Hence from Appendix B, $F_2 = \theta_2^4 = 1$. With the expo-
nential integral $E_3(\tau) = 0.0000038302$ for $\tau = 10$, the effects of
such an assumption are sensed only in the immediate vicinity of
this boundary.

With reference to Figure B.1 and Figure 1.1, solutions were
obtained for fluxes emitted in the negative x direction.

2.5 Discussion of Analytical Results

The method of solution of the energy equation is described
in Appendix D. The exponential integrals $E_n(\tau)$ encountered in the
solution of the radiative transfer equation from [9] are described
briefly as

for positive real arguments

$$E_n(\tau) = \int_0^1 \mu^{n-2} e^{-\tau/\mu} \, d\mu \qquad (2.1)$$

or

$$E_n(\tau) = \int_0^1 \mu^n \mu^{-2} e^{-\tau/\mu} \, d\mu \qquad (2.2)$$

and

$$E_n(\tau) = \frac{1}{\tau} \int_0^1 \mu^n \, d(e^{-\tau/\mu}) \qquad (2.3)$$

integrating by parts the recurrence relation is obtained as

$$n E_{n+1}(\tau) = e^{-\tau} - \tau E_n(\tau) \qquad n \geqslant 1 \qquad (2.4)$$

and

$$\int E_n(\tau) d\tau = -E_{n+1}(\tau) \quad n \geqslant 1 \tag{2.5}$$

From Appendix D the lowest case of exponential integral encountered is $E_2(\tau)$. Using the recurrence relation of equation (2.4) with values of $E_1(\tau)$ obtained from a computer algorithm, all higher cases of $E_n(\tau)$ may be obtained.

The solutions of Appendix D were tested for convergence using polynomial expansions for $\theta^4(\tau)$ and $D(\tau)$ of order 4 and 6, with a spacial step size reduced from 0.1 to 0.05 and to 0.02. For the convenience of studies, reported later, of a transient analysis of radiative transfer a spacial size less than 0.05 required a large amount of computer memory and exceeded the capacity of University of Aston ICL 1905E computer. Again, for a transient analysis computational time was also excessive. For these reasons the 1906A/CDC7600 installation at the University of Manchester Regional Computer Centre was utilized.

This technique, although not of a closed form as the analysis given in [18], is sufficiently flexible to account for possible non-isothermal conditions. It was not subject to the anomolies of [18], where in some cases ε_e exceeded unity.

From the theory developed in [18] and [21] the curves of effective emissivity against optical thickness are shown in Figure 2.1 for an isothermal slab, along with results obtained using the two flux method of [19] in Appendix C. The value of effective emissivity from each method reaches an asymtotic value

for large values of optical thickness. It is apparent from these curves that only a particulate material of emissivity equal to unity will produce an effective emulsion emissivity of unity at large emulsion optical thicknesses. It appears possible, in most cases, to have an effective emulsion emissivity of a higher value than the particulate material emissivity. This may be explained by consideration of the work reported in [26] on radiation from cavities. As a bundle of radiative energy traverses a cavity (or void in a fluidized bed of particles), the multiple reflections occurring within the cavity act to augment the emissive power of the cavity relative to that of a plane surface of identical temperature and material emissivity. In practice, the cavity effect is used in the design of black bodies. Because of the asymtotic nature of the curves in Figure 2.1 and for reasons which are later justified experimentally, the slab of emulsion is considered to be optically thick and an optical thickness of $\tau_o = 10$ is taken as being representative of the depth over which the changes of temperature and hence radiative flux may extend, but not exceed. The solutions of Appendix D were initially for an isothermal emulsion. However, as a non-isothermal system is more realistic, due allowance was made in the analysis to make possible the inclusion of a temperature distribution.

Results were obtained for a number of scattering albedo values ω_o. The curves are shown in Figure 2.2 along with curves derived from those of Figure 2.1.

Also shown on Figure 2.2 are exact values of ε_e (hemispherical) with $\tau_o = \infty$. These results were reported in [18] and derived from

the work of [12]. Briefly, the analysis is taken from a solution of the radiative transfer equation in the discipline of astrophysics [12] and adapted in [18] for use in a heat transfer context. Radiative intensities may be defined in terms of $H_{\bar{\omega}}$ functions which are tabulated in [12] for a range of ω_0 and θ values.

The value of ε_e (hemispherical) requires values of the first moments of the $H_{\bar{\omega}}$ functions again reported in [12]. The points of hemispherical emissivity taken from [18] and plotted in Figure 2.2 show a favourable comparison with the results obtained by the present analysis thus justifying its use.

Again, in Figure 2.2 the results obtained in the present analysis are greater than those from the work in [18] by up to 6.1%. As reported in [18], these results would underestimate the exact solution. Hence it is reasonable to assume that the analysis presented in this study is an improved approximation to the exact solution.

The curves of Figure 2.2 show conclusively the effect of particle material emissivity, through the scattering albedo ω_0 on the emulsion effective emissivity. This is in contrast with the findings in [2] who concluded that ε_e = unity in all cases but supports the postulation in [4] and [5]. The curves of Figure 2.3 demonstrate how the effective emissivity of absorbing, emitting and scattering media exceed the value of particle emissivity. As a comparison the effective emissivity of a diffusely emitting conical groove, from [26], is given emphasizing the effect of scattering.

2.6 Conclusions

The equation of radiative transfer is not amenable to closed form solution and a numerical technique, subject to initial simplifying assumptions, has been developed. The variation of effective emissivity, of an emulsion of fluidized solids, with change of scattering albedo is shown. The results compare favourably with the results of other authors and show the dependence of ε_e upon the type of particle material used. The limitation of the Zabrodsky model [2] originates in the neglect of the effects of energy scattering and energy emission of a particulate emulsion. The consequence is an overshadowing of the important features highlighted only by a solution of the complete equation of radiative transfer.

2.7 Suggestions of Areas for Further Study

The solution of the radiative transfer equation may be achieved by alternative methods such as the statistical Monte Carlo method developed for use in heat transfer systems by Howell and Perlmutter[27]. However, this technique requires considerable computer time and storage capacity and for this reason the classical differential approach is used in the present study. Another method of solution is a numerical technique based upon Gaussian quadrature and a matrix eigenvector application developed by Hsia[28] and Love[29] used in a study of steady-state radiative transfer with an-isotropic scattering. The work by Weston and Hauth[30] in a study of the transient cooling of an absorbing, emitting and isotropic scattering medium for the case of combined

radiation and conduction used a similar approach. Solutions
based on this approach would be useful in order to compare and
justify the technique used in the present study. In this work
of an emulsion of fluidized solids, a number of simplifying as-
sumptions were made and listed. The assumption of steady state
is one which may be readily relaxed and solutions obtained for
the more realistic situation of transient radiative transfer. In
theory, the analysis may also be extended to include conduction
heat transfer for the case of energy exchange to a surface
immersed within a fluidized bed. However, before these extensions
to the work are undertaken, it is desirable to conduct an experi-
mental programme to justify the results so far obtained.

C H A P T E R   3

DETERMINATION OF FLUIDIZATION AND RADIATIVE HEAT

TRANSFER PARAMETERS

3.1 Introduction

Before proceeding to an experimental programme of radiative
flux determination from emulsions of fluidized solids, it is
necessary to describe a few fundamental fluidizing parameters in
addition to those arising due to radiative heat transfer
considerations.

3.2 Minimum Fluidizing Velocity at High Bed Temperatures

A fundamental property of any bed of fluidized solids is
the minimum fluidizing velocity, U_{mf}. It is a basic parameter
used in mathematical models of fluidization and used extensively
in the two phase theory of fluidization of Davidson and Harrison[31].
Values of U_{mf} may be determined for any bed temperature using the
classical Ergun equation, Ergun[32] of

$$\frac{1.75}{\phi_s \epsilon_{mf}^3} \left[\frac{D_p U_{mf} \rho_g}{\mu} \right]^2 + \frac{150(1-\epsilon_{mf})}{\phi_s^2 \epsilon_{mf}^3} \left[\frac{D_p U_{mf} \rho_g}{\mu} \right] = \frac{D_p^3 \rho_g (\rho_p - \rho_g) g}{\mu^2} \quad (3.1)$$

resulting in the solution of a quadratic for U_{mf}. Equation (3.1)
for particle Reynolds numbers of less than 20 may be simplified to

$$U_{mf} = \frac{(\rho_p - \rho_g) g}{\mu} \frac{\phi_s^2 D_p^2}{150} \frac{\epsilon_{mf}^3}{(1-\epsilon_{mf})} \quad (3.2)$$

The two parameters, sphericity ϕ_s and voidage at minimum fluidizing
velocity ϵ_{mf} are not well tabulated for a comprehensive range of

particle types. To overcome this problem, a paper by Broughton[34] described a technique in a study of high temperature effects on U_{mf} which resulted in rewriting of equation (3.2) in the form

$$U_{mf} = \frac{(\rho_p - \rho_g) g \, D_p^2 \, \phi_s^2}{\mu_g \, f(\varepsilon_{mf})} \tag{3.3}$$

where

$$f(\varepsilon_{mf}) = \frac{150(1 - \varepsilon_{mf})}{\varepsilon_{mf}^3} = C \, \phi_s^2 \tag{3.4}$$

hence

$$U_{mf} = \frac{(\rho_p - \rho_g) g \, D_p^2}{C \, \mu_g} \tag{3.5}$$

[34] cited values of C used by other workers ranging from 1 233, Davidson and Harrison[35] to 1 650, Kunii and Levenspiel[36] and suggested in the absence of experimental data that a value of 1 440 be used producing an expected error range of up to 15%. Again [34] cites a number of references where equation (3.5) was used with acceptable results, although correlation was poor, with a value for C of 1420, in the work of Mii, Yoshida and Kunii[37] resulting in errors of up to 100% compared with experimental values of U_{mf}.

3.2.1. Experimental determination of U_{mf} at high bed temperatures

An experimental programme was set up in which bed pressure drops were measured at incremental gas flow velocities. Due to the low gas flow rates required in such experiments, it was not possible to use a conventional type of fluidized bed as combustion

becomes unstable. For this reason an 80 mm diameter x 300 mm high, mild steel fluidized bed reactor was designed and is shown in Figure 3.1. The incoming air was heated by a 2 kilo-watt 'cooker element' situated below the 2 in thick ceramic distributor plate. The position of the heating element, on heating the distributor plate, reduced considerably the thermal gradients which would normally occur in a fluidized bed just above a relatively cool distributor plate. Hence, the gas velocity remains constant throughout the total height of bed, an important criterion for U_{mf} measurements. Particles of a number of materials were sieved in a narrow size range. In order to achieve good fluidization, bed heights of up to 140 mm were used. If shallow beds are used in U_{mf} experiments, i.e. of less than approximately 100 mm in a bed diameter 80 mm, there is a possibility that the total weight of solids will not be fully supported by the fluidizing medium at or above minimum fluidizing conditions. This phenomenon is particularly noticeable at high bed temperatures, and demonstrated in the later experiments of Section 3.4 in which shallow beds of height 25 mm were used. The bed pressure drop levels were measured using an inclined liquid manometer from a pressure tapping just above the distributor plate and another tapping above the bed of solids. The experimental procedure was to lower the gas flow rate from an initially well fluidized bed while measuring the pressure drop at suitable intervals. This was then repeated for increasing gas velocities, again measuring pressure levels, up to a well fluidized condition. Repeated raising and lowering of the gas flow rates resulted in the typical curves shown in Figure 3.2

from which U_{mf} was determined in the normal way.

3.2.2. Discussion of results

For the materials used in these experiments, the curves of U_{mf} against bed temperatures are shown in Figure 3.3 (a, b & c). Unfortunately, it was not possible to raise the bed temperatures above 600°C using the electrical heating method. The normal bed temperature range of interest for radiation flux measurements is 800°C to 1 100°C. In order to compare the experimental results of U_{mf} with those obtained using both equation (3.1) and equation (3.5), the voidage at minimum fluidizing velocity was determined from

$$\varepsilon_{mf} = 1 - \frac{W_b}{A_b \rho_p H_{mf}} \qquad (3.6)$$

where H_{mf} was measured during the experiments, using a steel ruler immersed in the bed. This technique was rather crude as the true bed height was difficult to determine due to localized bubbling around the ruler as U_{mf} was reached. Over the range of bed temperatures used the value of H_{mf} changed by only 2 mm to 3 mm resulting in a small variation of the calculated ε_{mf} values. For the purpose of substitution in equation (3.1) and (3.5) a mean of ε_{mf} over the bed temperature range was taken. Values of sphericity $\emptyset_s$ were obtained using the paper of Rittenhouse[33] who tabulated diagrams of typical ranges of two-dimensional particle sphericity. Using a microscope and these tables, subjective estimates of sphericity were obtained. The results of ε_{mf} and

TABLE 3.1

MATERIAL: SILICA SAND; TEMPERATURE: 300 K; ρ_p: 2 630 kg/m³				
$\emptyset_s$	ε_{mf}	D_p mm	C	Re_{mf}
0.83	0.44	0.354	1431.4	2.086
0.85	0.424	0.55	1568.8	6.637
0.83	0.416	0.777	1766.3	14.86

MATERIAL: SILICON CARBIDE; TEMPERATURE: 300 K; ρ_p: 3 179 kg/m³				
$\emptyset_s$	ε_{mf}	D_p mm	C	Re_{mf}
0.75	0.47	0.354	1361.3	2.63
0.72	0.455	0.55	1674.1	7.51
0.75	0.45	0.777	1609.5	18.89

MATERIAL: ALUMINA; TEMPERATURE: 300 K; ρ_p: 3 850 kg/m³				
$\emptyset_s$	ε_{mf}	D_p mm	C	Re_{mf}
0.67	0.52	0.354	1140.7	3.74
0.69	0.501	0.55	1250.2	11.46
0.69	0.49	0.777	1365.8	25.1

MATERIAL: MAGNESIA; TEMPERATURE: 300 K; ρ_p: 3 820 kg/m³				
$\emptyset_s$	ε_{mf}	D_p mm	C	Re_{mf}
0.71	0.49	0.354	1289.9	3.45
0.71	0.482	0.55	1376.5	10.64
0.73	0.471	0.777	1425.1	24.76

sphericity are shown in Table 3.1 for the bed materials used along with the values of constant C and particle Reynolds number for each case. A wide range of values for C were obtained emphasizing the effect of particle sphericity and hence ε_{mf} on the resulting U_{mf} levels. Comparison of the sphericity and ε_{mf} results obtained here with the curves of Brown $et\ al$[41] shown in Figure 3.4 shows that all points were bounded by the curves of loose packing and normal packing. As these two quantities are subject to experimental error the favourable comparison with the expected curve of loose packing for the case of a fluidized bed places this error within acceptable limits.

The comparison of theoretical with experimental U_{mf} V_s temperature curves of Figure 3.3 (a, b and c) are reasonable in most cases showing least deviation for U_{mf} calculated from equation (3.5). The discrepancy with values calculated from equation (3.1) and equation (3.5) were most obvious at ambient temperature. This fact is reflected in the values of particle Reynolds numbers at ambient and high temperatures. In some cases, Re_{mf} was large and hence equation (3.5) was used at or approaching its limit of applicability. Although the greatest discrepancy between theoretical and experimental U_{mf} values occurred at ambient temperatures, at high bed temperatures of up to 600°C the error was acceptable and the trend is assumed to continue to bed temperatures of up to 1 100°C. The use of either equation (3.1) or equation (3.5) with a known value of C would give acceptable results at high bed temperatures in the absence of sphericity and ε_{mf} values. The use of C = 1 440 in equation (3.5) as suggested by [34] gives maximum difference at 600°C of 16% in the case of 0.354 mm diameter

alumina particles and of 4% for 0.777 mm diameter sand particles.

3.2.3 Conclusions

For all particle sizes and materials used in the later experimental studies of this work, U_{mf} was measured for bed temperatures up to 600°C. The correlations with equation (3.1) and equation (3.5) with a predetermined value for constant C in each case were reasonable. The greatest discrepancy occurred at ambient temperature but rapidly diminished as the bed temperature was increased. As stated in [34] the use of C = 1 440 in equation (3.5) is expected to give errors of up to 16% for high temperature beds. It is assumed that the results can be extrapolated to bed temperatures of up to 1 100°C and equation (3.1) or equation (3.5) used with confidence.

3.3 Determination of Particle Emissivity

As emphasized by the results of Chapter 2, Figure 2.2, the effective emissivity of an optically thick medium which absorbs and scatters radiation is strongly dependant upon particle emissivity. As described in Table 3.3 values of ε_p for the materials used in this study vary considerably. Such variations are understandable when the large number of parameters which effect emissivity measurements are considered. Some examples of these parameters are grain structure, transmissivity, wavelength dependency, trace additives and surface roughness. The problem has also been aggravated by the inadequate description given by many investigators of the materials studied. An additional difficulty arises due

to emission originating at considerable depths below the material surface, e.g. refractory oxides, wood and fibrous materials, where absorption and scattering of energy are significant parameters.

In order to have reasonable estimates of the particle emissivities used in this study it was necessary to produce test samples of the various materials and conduct an experimental programme of measurement.

3.3.1 Manufacture of test samples

The most suitable samples for emissivity measurements are solid slabs of equivalent material. Unfortunately, no such samples were available and so special test samples were produced from the readily available particulate material.

An initial consideration was given to the possible use of slabs made from compacted fine particulate material. However, all that is really achieved is a packed bed of solids which is essentially a volume emitter rather than the required surface emitter. Hence, a different method was considered where particles of each material in the size range 0.71 mm to 0.85 mm were embedded in a single layer on the surface of a sub-layer of suitable material. In order to reduce the contribution to the overall surface emitted flux of this sub-layer, a material of low emissivity was chosen, i.e. pure silver.

Using a firebrick mould which also acted as a support for the test sample, a small amount of silver was melted using a propane/air burner. Immersed in this melt was a Cr.AL thermocouple. A diagram of the sample configuration is shown in Figure 3.5. As the

silver became molten, a few particles were sprinkled onto the surface. The particles in direct contact with the surface became embedded and on subsequent cooling in air remained intact. All the loose particles then remaining were blown off using a compressed air jet. The resulting surface as described in Figure 3.5 revealed closely spaced particles interspaced by silver. The molten silver developed a small meniscus and with the lower density particles protruding from the silver the appearance of the surface was far from ideal. However, subsequent grinding of the surface completely removed the embedded particles and so was not continued. Although a source of error, for the purpose of analysis, such a sample surface was regarded as being flat. The size of each sample, as described by Figure 3.5 was chosen as ⅝ in diameter x ¼ in deep for the field of view of the radiation pyrometer subtended a circular area of 0.25 ins diameter at a target distance of 24 ins. Hence, the field of view was always filled by the sample.

3.3.2 Experimental procedure

The experimental configuration is described schematically in Figure 3.5. To reduce the losses of radiative flux from the system and to ensure energy interchange between pyrometer and sample surface only, the inner surface of the pyrometer extension tube was covered with aluminium foil. This highly reflecting surface, in reducing the temperature gradients at the test sample surface, also reduced the error incurred in assuming that the immersed thermocouple measured this surface temperature. In similar experiments conducted by Sully, Brandes and Waterhouse[72], coats

of powdered refractories of known thickness were applied to metal

strips. The surface temperature gradients were corrected by taking

emissivity measurements for a range of coating thicknesses and

extrapolating back to a zero thickness. However, absorption and

scattering of energy within and at the surface of these coatings

becomes significant and should be allowed for. In the work of

Pattison[73], solid cylindrical test samples were heated on the

outer surface by a propane/oxygen flame and measurements of

material emissivity obtained by viewing through this flame. In

the subsequent analysis due allowance was made for the effect of

flame emissivity. In this way the surface temperature was known

accurately, being obtained from a radiation pyrometer viewing a

0.25 in diameter x 1 in deep hole bored in the specimen acting as

a black body source.

Various measurement techniques have been cited in the litera-

ture, the most comprehensive of which appears in [43]. However,

a few simple methods were considered, Pirani[74], plotted the decay

of radiation with time using a total radiation receiver and extra-

polated back to zero time to obtain the sample emissivity. This

method being dependant upon the response time of the measuring

instrument was not considered. Michaud[75] used a similar technique

to [73]. This method of a flame directly heating the surface to

be measured was not desirable in this study. With the sample

temperature measured just below the surface of the silver sub-layer

the true value of surface temperature could not be determined with

any reasonable degree of certainty. Ideally, a furnace was required

to heat the sample. However, this was not readily available and

the test sample in this study was heated on its reverse side by a propane-air burner. Each sample was raised in temperature in steps up to a maximum of about 860°C. With the melting point of silver determined at approximately 935°C, it was not advisable to raise the sample temperature any further. For each steady value of sample temperature measured via a digital voltmeter, the pyrometer output was recorded. From the output fluctuations the error of the pyrometer output was estimated at ±5 K. Such experiments were repeated a number of times by constantly raising and lowering the sample temperature. A further sample of pure silver with no other material present was like-wise studied.

From the conservation of energy, particle emissivity ε_p may be simply obtained from the equation

$$q_{o,particle}A_p + q_{o,silver}A_s = q_{i,detector}A_{target} \tag{3.7a}$$

From a consideration of the system, neglecting radiative exchange with the pyrometer extension tube, then generally

$$\sum_{j=1}^{N} [\delta_{kj} - (1 - \varepsilon_k)F_{k-j}]q_{o,j} = \varepsilon_k \sigma T_k^4 \tag{3.7b}$$

with

$$\delta_{kj} = \begin{cases} 1 & - \text{ when } k = j \\ 0 & - \text{ when } k \neq j \end{cases}$$

where $q_{o,j}$ is the radiosity of surface j

$q_{i,j}$ is the irradiation of surface i

then

$$q_{o,p} - (1 - \varepsilon_p)F_{p-d}q_{o,d} \doteq \varepsilon_p \sigma T_p^4 \qquad (3.7c)$$

$$q_{o,s} - (1 - \varepsilon_s)F_{s-d}q_{o,d} = \varepsilon_s \sigma T_p^4 \qquad (3.7d)$$

and with

$$q_{o,d} = \sigma T_d^4 \Rightarrow negligible$$

Then equations (3.7c) and (3.7d) simplify to give with equation
(3.7a)

$$\varepsilon_p = \frac{T^4_{measured}}{T^4_{sample}} \frac{A_{target}}{A_{particle}} - \varepsilon_{silver}\frac{A_{silver}}{A_{particle}} \qquad (3.8)$$

The number of particles within the 0.25 inch diameter field of

view of the pyrometer were counted with the aid of a microscope.

Great care was exercised in ensuring correct alignment of the

sample and pyrometer. However, the exact position, relative to

the sample centre line of the area of view could not be precisely

determined. Hence the number of particles within an area of 0.25

in diameter were counted for this area centred at different posi-

tions over the sample surface. A mean value was then obtained.

From a few initial experiments it appeared that ε_p predicted

from the overall energy balance, gave values greater than unity,

particularly when A_p/A_t was small. Such an energy balance predicts

an inverse relationship between ε_p and the ratio A_p/A_t. In order

to investigate the validity of this relationship the initial experi-

ments were repeated for a number of A_p/A_t values. The experimental

technique eventually used was to initially measure the emissivity
of a sample of pure silver defined as

$$\varepsilon_{silver} = \frac{T^4_{pyrometer}}{T^4_{sample}}$$

from Appendix F. The sample was then remelted and a few particles
added to the molten surface. The emissivity of this sample was then
determined for a range of sample temperatures along with the ratio
A_p/A_t. This procedure was repeated for increasing values of A_p/A_t.

3.3.3 Discussion of Results

Typical experimental values of ε_{sample} against sample tem-
perature are given in Figure 3.6a for each particle material and
A_p/A_t value along with the curve of ε_{silver} for silver alone. As
expected for non metallic solids, the value of ε_{sample} decreased
with increase of sample temperature. This fact is well known and
reported in [9] and [10]. For silver, the curve predicts a similar
trend; however, as reported in [9] and [10] for metals, an
opposite trend was observed. The reason for this discrepancy may
be attributed to the formation of an oxide at the surface of the
silver sample, which would hence exhibit the trend reported for
a non-metallic material. At high sample temperatures, a levelling
out of ε_{sample} was observed for all the materials studied. Hence
it was a simple matter to extrapolate the results to higher tem-
peratures to the levels encountered in fluidization, i.e. up to
1 100°C.

The effects on ε_{sample} of varying the ratio A_p/A_t are shown

in Figure 3.6b. The curves were drawn, simply by inspection,
through the results of ε_{sample} and extrapolated to a value of A_p/A_t = unity. Hence the method of ε_p
determination was to take $\varepsilon_p = \varepsilon_{sample}$ at the extrapolated point
of $A_p/A_t = 1$, when the sample is totally covered by particulate
material. An explanation of this non-linearity effect lies in the
effects of the scattering of radiative energy by the essentially
particulate surface. The true surface area of the radiating
particles is much larger than the cross-sectional area assumed
in equation (3.7a) and tends towards a hemispherical surface.
Hence considerable particle to particle radiation takes place
resulting in a multiple scattering system.

This method is simple but approximate in that the temperature
of the particles are assumed to reach the measured silver sublayer
temperature. Realistically temperature gradients are in evidence
and it is therefore expected that the results underestimate the
true values of ε_p. An extension of the technique, although not
considered in this study, was described by [72] in which increasing
particle depths were used. This would be useful in estimating the
effects of surface temperature gradients.

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TABLE 3.2

Material	ε_p	Temperature Range K	Reference and Notes
Al$_2$O$_3$	0.55 - 0.45	1 000 - 13 000	[9]
Al$_2$O$_3$	0.35 - 0.53	1 473	[10] - Grain size 2 - 120 μm
Al$_2$O$_3$	0.42 - 0.25	750 - 1 000	[42]
Al$_2$O$_3$	0.41	1 200	[43]
Al$_2$O$_3$	0.45 - 0.25	1 273	[21]
MgO$_3$	0.5 - 0.45	1 000 - 13 000	[9]
MgO$_3$	0.3 - 0.48	1 473	[10] - Grain size 1 - 120 μm
MgO$_3$	0.3 - 0.2	1 000 - 2 200	[42]
MgO$_3$	0.2	1 200	[43] - Pressed and sintered measured in argon
MgO$_3$	0.45	1 200	[43]
SiO$_2$	0.62 - 0.46	1 173 - 1 573	[42] - Grain size 70 - 600 μm
SiO$_2$	0.75	1 300	[43] - Fused specimen 0.33 cm thick measured in vacuum 10^{-9} mm Hg
SiC	0.96 - 0.8	1 300	[43]
Fire clay	0.75	1 273	[42]

TABLE 3.3

Material	k_p kW/mK	ρ_p kg/m³	C_p kJ/kgK	ε_p Range	ω_o Range	ε_e Range
Silica Sand	0.00187	2360	0.8374	0.46 - 0.75	0.71 - 0.4	0.78 - 0.82
Silicon Carbide	0.018	3179	0.714	0.8 -	0.34 - 0.09	0.94 - 0.99
Alumina	0.0061	3850	0.714	0.25 - 0.55	0.86 - 0.63	0.62 - 0.82
Magnesium Oxide	0.007	3820	0.714	0.2 - 0.5	0.89 - 0.68	0.55 - 0.79
Fire Clay	0.00128	2600	1.172	0.75	0.4	0.91
Carbon	0.005	2300	0.952	~1.0	0	1.0

The predicted results of ε_p and corresponding values of ω_o along with ε_{ei} determined from Figure 2.3 are tabulated below in Table 3.4.

TABLE 3.4

	Silica Sand	Alumina	Magnesia	Silicon Carbide
ε_p	0.64	0.37	0.57	0.77
ω_o	0.529	0.773	0.601	0.374
ε_{ei}	0.87	0.71	0.84	0.93

3.3.4 Conclusions

Test samples from which particle emissivity was determined were produced from slabs of silver into the surface of which particles were embedded. Varying the number of particles per unit area the values of sample emissivity were observed to increase non-linearly with increase in A_p/A_t. The value of ε_p was then determined by extrapolation to $A_p/A_t = 1$. The method is essentially approximate and for the reasons described would tend to underestimate the true values of ε_p. However, the advantages of the technique lay in its simplicity of what is essentially a complex problem. It is expected that the results of ε_p so predicted enable values of fluidized emulsion effective emissivities to be predicted with greater confidence.

3.4 Determination of the Extinction Coefficient

In order to determine the extinction coefficient it was nec-
essary to know the value of particle concentration within the
emulsion. In the context of the assumptions so far postulated
for a fluidized emulsion, the constant voidage extends to the free
surface. As this free surface is assumed to be in dynamic equili-
brium, i.e. the gas and particles are motionless relative to one
another, and furthermore that at no time is the surface disturbed
by the passage of bubbles, then it is possible to derive an equation
of particle concentration which will be a time mean value. The
derivation of this particle concentration equation in terms of the
vertical pressure drop profile within the emulsion is shown in
Appendix E(a). The analysis is then extended to include the
radiative parameters of particle emissivity and reflectivity
resulting in the final equation for extinction coefficient K_t.
The unknown pressure profile existing within the emulsion, being
essentially a time-mean value, particularly in the vicinity of the
free surface, had to be determined experimentally. The resulting
pressure profile may then be approximated for purposes of analysis
by a least squares polynomial curve fitting procedure.

3.4.1 Experimental Determination of Bed Pressure Profile

The experimental configuration shown in Figure 3.7a of a
143 mm diameter x 100 mm high mild steel fluidized bed reactor,
was used with a supply of propane/air mixture. Bed pressure drop
profiles were determined, for a number of particle sizes, (sieved
to a narrow size range), materials, fluidizing velocity and bed

temperature, measured by an unshielded copper constantan thermo-
couple, immersed in the bed. As the characteristics of shallow
beds are of particular interest in this study only bed heights of
up to 25 mm, measured at atmospheric pressure, were studied.

To measure the pressure drop distribution, a 0.5 mm diameter
stainless steel pitot tube was traversed through the total depth
of the bed with pressure levels recorded at incremental heights.
In reality the pitot tube remained fixed while the bed was traversed
vertically. The pressure levels of such shallow beds are small and
difficult to measure quickly using a liquid manometer, with a long
response time of the liquid column and connecting tube. Hence a
'Kistler' differential pressure transducer, type 7251, capable of
resolution to 4 Pascals was used. The output from the transducer
was processed in a charge amplifier type 5001 and finally recorded
on an Ultra Violet Recorder (U-V-R). The differential transducer
is basically a dynamic pressure measuring instrument but may also
be used to measure quasi-static pressure levels, as required in
these experiments. To achieve this mode of operation the trans-
ducer was connected to the pressure probe via a three-way pneumatic
switch. The configuration is shown in Figure 3.7b. By means of
this switch the reference (zero) differential pressure is applied
to the transducer. Releasing this switch, and with the charge
amplifier in the 'Long' time constant mode, the required differential
pressure level was then applied to the transducer. A typical output
signal traced from the U.V.R. paper is shown in Figure 3.8. In
Figure 3.9 is shown the simple resistance matching network used to
match the high impedance transducer amplifier with the low impedance

U.V. recorder.

3.4.2. Discussion of Results

In the vicinity of the free bubbling surface, the experi-
mental points were difficult to obtain, a transducer signal to
noise ratio of unity was approached. Hence in this region the
pressure drop curves were extrapolated to zero at an 'observed'
mean bed height, i.e. the height of the bed taken when the pressure
probe became visible. This extrapolated distance was in most cases
of order 3 mm to 4 mm and constituted a small proportion of the
overall bed height. Examples of the experimental pressure profiles
are shown for a few of the cases studied in Figure 3.10 through
Figure 3.12. The curves resulted from a least squares fit of the
experimental points.

In most cases it was observed that the total bed pressure
drop measured at the distributor plate was not equal to the weight
of solids in the bed, i.e. $PA/W_b \neq 1$, even at high values of $U-U_{mf}$.
This suggests that the whole of the bed was not fully supported by
the fluidizing gas and may be explained from a consideration of what
occurs in the first few particle diameters just above the distri-
butor plate. Over the first few particle diameters of the bed large
temperature gradients occur. As a result, this portion of the bed
will not be fluidized constituting a measurable proportion of the
total height of bed, e.g. 20% for a 25 mm deep bed of particle size
0.50 mm and entrance region extending over 10 D_p. In very deep
beds this entrance region effect constitutes only a small percentage
of the total bed height and being difficult to measure could be

considered as an experimental error. Further to this explanation
the work of Hiby[45] who measured oscillation frequencies in the
region just above the distributor plate of beds up to 150 mm deep,
concludes that the first 10 D_p of the bed perform a periodic
oscillation coherent over the bed cross-section. This oscillation
of about the ten lowest layers within a deep bed continues and acts
as the origin for periodic gas bubble formation. Extension of the
work of [45] by Verloop and Heertjes[46] in shallow gas fluidized
beds, showed that the bed would oscillate homogeneously, i.e. the
particles moved in phase, only when the height of bed was smaller
than some critical height, e.g. for D_p = 0.55 mm, U/U_{mf} = 3,
ε_b = 0.52 and from [46] H_c = 79 mm.

Hence even though the bed is well fluidized with $1 < U/U_{mf} < 3$
for a height of less than 79 mm, a fully supported condition is not
reached. Although the studies of [45] and [46] were at ambient
temperature, extensions of the hypothesis to higher bed temperatures
appear to be valid.

From the pressure profiles so obtained, along with values
of the appropriate particle emissivity and reflectivity given in
Table 3.4, values of extinction coefficient K_t may be obtained
corresponding to values of optical depth τ = 10 shown on the
pressure distribution curves. The pressure curves were generally
of an approximately linear relationship up to optical depths of
τ = 10, and the errors due to non-linearities in deriving subse-
quent values of K_t from $K_t \sim 10/nD_p$ are small. The curves of K_t
against change of excess gas velocity $(U-U_{mf})$ are most appropriate
to this work and are shown in Figure 3.13 through Figure 3.21.

The curves were obtained from a least squares polynomial fit of the experimental points. No attempt was made to correlate the effect of bed temperature variation as only three values were studied. Values of K_t required at intermediate bed temperature levels were obtained by simple interpolation. As may be expected increasing bed temperatures and increasing gas velocities produce incremental bed expansions resulting in lower particle concentrations and hence lower values of K_t.

These experimental values of K_t for a fluidized emulsion have not, to the author's knowledge, been determined previously. However, studies on radiatively semi-transparent materials such as Dydydium glass by Melamed [38] in which the assumption of optical thickness, $\tau_o = \infty$ was made, show values of absorption coefficient K_a of a comparative level. Also from the analysis of Yokoburi[39] levels of K_t occurring in a particle seeded gas for carbon particles of size typically $D_p = 0.77$ mm gives $K_t = 1\ 500$ m^{-1}. These results compare favourably with values for K_t using particles, i.e. silicon carbide found in this work. As a contrast, values of K_a are given for non-seeded hydrogen at 100 atmospheres and 1 680 K of 10^{-5}m^{-1} from Williams et al[40]. This was a study of the wavelength dependency of K_a in sub-micrometre particle flames. For non-seeded CO_2, a value of K_a of approximately 10 m^{-1} at atmospheric pressure and 700 K was reported in [26]. In addition, [40] gave an approximate value for K_a of 1 000 m^{-1} for 1 μm diameter carbon at 2 200 K with a dispersion density (weight of particles per unit volume of emulsion) of 0.2 kg/m^3, which emphasizes how the radiative properties are improved when solid particles are added to a

hot gas.

3.4.3. Conclusions

Time average particle concentration levels may be estimated experimentally in the vicinity of a freely bubbling fluidized bed surface from a determination of the time mean static pressure distribution throughout the height of the bed. From such an analysis the important radiative parameter of extinction coefficient follows. The levels of K_t reported are comparable to those of similar systems in semi-transparent materials and particle seeded gases reported in the literature. The K_t values of a fluidized emulsion are as expected; some orders of magnitude greater than reported values of non seeded gases such as hydrogen and CO_2.

As $\tau = 10$ extends to appreciable physical depths within the bed, i.e. $x = 14.16$ mm for 0.354 mm diameter sand particles at 1 050°C, it is expected that radiative cooling will produce temperature gradients to similar physical depths.

3.5 Determination of the Time Average Temperature Gradient at the Surface of a Fluidized Bed

As stated in Chapter 1.4.1(v), all properties of the emulsion are assumed to be independant of temperature distribution. From the analysis of Chapter 2 an initial solution was of an isothermal slab of emulsion. However, a more realistic model is one in which thermal gradients exist at the bed surface due to radiative cooling of the particles. The existence of such surface

temperature gradients is a reasonable hypothesis if the heat transfer by radiation and convection to and from a single particle in a hot gas stream is considered.

For example, consider a single particle in a hot gas stream, initially at 1 323°K and suddenly exposed (radiatively) to an atmosphere at ambient temperature. Use of the lumped capacity technique is justified for a Biot number of less than 0.2. For a silicon carbide particle in air, the solid and fluid properties are:

$$D_p = 0.55 \text{ mm} \qquad\qquad \rho_f = 0.2715 \text{ kg/m}^3$$

$$k_p = 1.87 \text{ W/m K} \qquad\qquad \mu_f = 4.86 \times 10^{-5} \text{ Ns/m}$$

$$\rho_p = 3\ 179 \text{ kg/m}^3 \qquad\qquad k_f = 8.35 \times 10^{-2} \text{ W/m K}$$

$$c_p = 0.714 \text{ kJ/kg K} \qquad\qquad T_\infty = 300 \text{ K}$$

$$\varepsilon_p = 0.92 \qquad\qquad C_{pf} = 1.188 \text{ kJ/kg K}$$

$$T_0 = 1\ 323 \text{ K}$$

Using the Rantz-Marshall correlation for low Reynolds numbers:

$$h_c = 2 \frac{K_f}{D_p} = 0.3036 \text{ kW/m}^2 \text{ K} \tag{3.9}$$

$$h_r = \sigma \varepsilon_p (T_0^2 + T_\infty^2)(T_0 + T_\infty) = 0.1558 \text{ kW/m}^2 \text{ K} \tag{3.10}$$

$$\text{Bi} = \frac{(h_r + h_c) D_p}{2 k_p} = 0.068 \tag{3.11}$$

and

$$\frac{T(t) - T_\infty}{T_0 - T_\infty} = e^{-\left[\frac{h_r + h_c}{\rho_p \cdot C_p}\right] \frac{6}{D_p} t} \tag{3.12}$$

for t = 0.06 secs, a typical residence time for particles above

the bed.   Then

$$T(0.06) = 1196.07 \text{ K}$$

and

$$\frac{T(0.06)}{T_o} = 0.904 \tag{3.13}$$

In the analysis of a fluidized bed gas to particle heat transfer by [36], the heat transfer coefficient was determined for both steady and unsteady conditions from a simple heat balance equation, neglecting any radiative transfer. The important parameters in the resulting relationship were the temperatures of the inlet and exit gas and the temperature of the solids within the bed, assumed equal to the gas temperature throughout the bed. The temperature distribution throughout the bed implies significant losses from the bed surface by radiative heat transfer and the radiative term should be included in the overall heat balance equation. A paper by Juveland et al[44] attempted to measure this distribution for a number of fluidizing conditions in very shallow beds of heights up to 13.3 mm. A suction pyrometer was used to measure the gas temperature profile above the bed. Due to blockage of the pyrometer by entrained particles, temperatures less than ¼ in from the bed surface were not obtained. By back extrapolation from the resulting profiles the gas temperature at the bed surface was determined. In all cases the gas exit temperature was less than the bed temperature by up to 150°C. With these results and the assumption of [36] that the particles and gas at exit are in local thermal equilibrium, the resulting thermal radiation levels

emitted by the bed surface are accordingly reduced from the purely
isothermal case.

3.5.1. Experimental Determination of the Surface Temperature Gradients

The measurement of the temperature distribution at the highly
bubbling free surface of the bed is complicated by the large turn-
over rate of particles and all that may be hoped for is a time
mean estimation of the gradients. One method of observation would
be to view the bed in the horizontal plane with a narrow angle
radiation pyrometer and traverse the instrument through the total
height of bed. However, two major difficulties arise; that of
the unknown effective emissivity of the dense bed coupled with,
as the pyrometer views the diffuse cloud above the dense phase, an
unknown configuration factor, i.e. the field of view of the pyro-
meter may not be completely filled at all times. For reasons of
the above complications, this method was not pursued and a contact
measuring instrument was selected. Firstly, a suction pyrometer
was used and traversed through a 143 mm dia x 100 mm high mild
steel bed of bed depths 25 mm as in Section 3.3. To ensure suf-
ficiently low pyrometer suction rates compared with the total
throughput of propane-air mixture, a fraction not exceeding 10%,
a small entrance bore of 1.5 mm diameter was used. Unfortunately,
the fine hole tended to block, as in [44], so consistently that
results were only possible in the void region above the bed. For
this reason a shielded Cr.AL thermocouple probe was constructed
from twin bore alumina tubing and shown in Figure 3.22. The
shielded thermocouple probe measures essentially the temperature

of the gas flowing over the thermocouple bed. Gas exhaust holes were provided behind the thermocouple bead to induce such a flow. For use in the thermal balance equation, a thermocouple was inserted in the wall of the tube to monitor T_w.

Vertical traverses of the probe, both up and down, were achieved by actually traversing the bed, supported on a 'Griffin Labjack' maintaining the probe in a fixed position. A range of bed temperatures, excess gas velocities $(U-U_{mf})$, particle materials and particle sizes were used in the experiments to determine as far as possible the effects on the resulting temperature profiles of as many variables as practicable. The bed was traversed relative to the fixed probe well into the free board above the bed; the observed bed height determined at the point where the end of the probe emerged from the bed.

3.5.2. Analysis of the Shielded Thermocouple Probe

A simple heat balance on the probe of Figure 3.22 gives

$$h_c A_T (T_g - T_T) = h_r A_T (T_T - T_w) + \sigma A_T F_{\infty T} (T_T^4 - T_\infty^4)$$
$$+ \text{ conduction losses} \quad (3.14)$$

From [9] $F_{\infty T}$, the view factor between the thermocouple bead and the open end of the probe, is small and may be neglected. Then equation (3.14) becomes:

$$T_g = T_T + \frac{h_r}{h_c} (T_T - T_w) + C \quad (3.15)$$

From the Rantz-Marshall correlation for low Reynolds number flow:

$$h_c = \frac{2K_f}{D_T} \tag{3.16}$$

Assuming concentric cylinders:

$$h_r = \frac{\sigma(T_T^2 + T_w^2)(T_T + T_w)}{\frac{1}{\varepsilon_T} + \frac{A_T}{A_W}\left[\frac{1}{\varepsilon_W} - 1\right]} \tag{3.17}$$

which underestimates the exact value of h_r by less than 3%

with ε_T = 0.8 for an oxidised thermocouple head

ε_W = 0.5 for alumina

In equation (3.15) the conduction losses in the thermocouple
wires may be neglected. Assuming the wires to be fins protruding
3 mm from the base of the alumina tubing, the temperature correc-
tion was of order 10 K introducing an error of 1% in the tempera-
ture ratio Θ.

3.5.3. Discussion of Results

The measured temperature levels were non-dimensionalised to
the bed temperature plotted against probe position relative to the
observed surface of the bed. This vertical distance, x, was
transformed into an optical depth $\tau = K_t x$ and the experimental
results are shown in Figure 3.23 through Figure 3.25. Appropriate
values of K_t were obtained from the earlier reported results in
Chapter 3, linearly interpolating between the plotted curves where
necessary. It was desirable to check the repeatability of a few
of the experimental results. However, this was subject to the

ability of repeating the precise air and gas flow rates to achieve
the same bed temperature value measured at the same position
within the bed. It was not thought that the test rig or for that
matter, the method of gas temperature measurement justifies too
rigorous a check. Hence, experiments were repeated in the sense
that similar conditions were set up for each repeated test run
and a range of further results obtained.

The scatter of the experimental points shown in Figure 3.23
through Figure 3.25 were reasonable, of order ±5% for the bed
temperatures considered. However, it was observed that temperature
profiles obtained at bed temperatures below 850°C fell well out-
side the general range of experimental scatter resulting from
studies at higher bed temperatures. At bed temperatures below
850°C, incomplete combustion within the bed causes considerable
gas bubble explosions to occur at or near the bed free surface
resulting in high noise levels and higher than bed temperature
gas levels. Hence, studies for such unstable conditions resulting
from these low bed temperature levels were considered to give
unrepresentative results. For later analytical use it was con-
venient to obtain a mathematical relationship between θ and τ and
for this reason a least squares polynomial curve fitting procedure
was undertaken using a fourth degree polynomial in line with the
analysis of Chapter 2. The striking observation which was a
consistent feature of these experiments is the high curvature of
the temperature distribution curve deep within the bed, close to
the point of maximum temperature. Below this portion of the bed
the temperature decreases due to the cooling effect of the

distributor plate. At the other extremity of the bed close to
the free surface, the curvature is less severe with the gradient
reducing gradually into the free board above the bed surface.
The point of inflexion is an indication of the 'effective' surface
of the bed above which cooling of the gas by relatively cold
particles has no effect. The shape of the fitted polynomial ap-
proximations of Figure 3.23 through Figure 3.25 support this
observation.

A further observation was that particles of a high emissivity,
i.e. silicon carbide, tended to show a pronounced temperature
gradient whereas for particles of a low emissivity the slope of
the temperature curve was significantly reduced. This fact is
simply explained from a consideration of the particle radiative
heat transfer coefficient being dependant on the value of
particle emissivity from equation (3.8).

The degree of cooling of the bed surface is significant and
extends to large optical depths within the bed, e.g. in the case
of 0.777 mm silicon carbide the curve extends to $\tau = 6$, resulting
in an actual depth of $x = 16.63$ mm. For a shallow bed of total
depth 25 mm (measured at ambient temperature), the depth of
penetration of this time mean cooling curve represents 66.6% of
the total bed height. The assumption of an isothermal shallow bed
is far from reasonable.

In deep beds, although this percentage may become insignifi-
cant, the assumption of an isothermal bed still does not hold as
the important parameter is optical depth.   With the initial
assumption of an optical thickness of $\tau_0 = 10$, the cooling curves

are seen to extend in most cases almost to this depth and hence
flux levels emitted from the surface are reduced accordingly.
The fluxes originating at large optical depths deep within the
bed are reduced exponentially on passing through the emulsion
producing insignificant effects at the free surface - a point
discussed earlier in Chapter 2.

3.6   Conclusions

Using a somewhat crude method it has been possible to
measure the time averaged gas temperature gradients in the vici-
nity of the free surface of a fluidized bed. Although subject
to uncertainties, the nature of the results obtained produce
some interesting observations, particularly the depth within
the bed to which cooling of the gas and particles extends. At
this stage it is reasonable although not strictly true to assume
that particles and gas remain, on a time average basis, at the
same temperature. It is now feasible incorporating the results
of this and previous chapters to set up a programme of experiments
to measure the steady state radiative fluxes emitted by the
surface of a fluidized bed.

C H A P T E R   4

EXPERIMENTAL DETERMINATION OF THE STEADY STATE RADIATIVE

FLUXES EMITTED BY THE SURFACE OF A FLUIDIZED BED

4.1 Introduction

As the free surface of a fluidized bed is readily accessible it lends itself to the experimental determination of emitted radiative fluxes by simply viewing it with a suitable instrument. It has the major advantage over studies of heat transfer to immersed surfaces within the bed in that no disturbance to particle motion is imposed. The analysis of Chapter 2 may, in principle, be applied to a study of radiative heat transfer to a surface within the bed. In previous chapters essentially steady state or time average processes have been analysed and the assumption is continued in this study. Throughout the experimental study the radiative fluxes are assumed to emanate only from the particulate solids. The contribution of the air/gas (propane) mixture is taken to be negligible due to the relatively low emissivity of the combustion products. A short study of the NO_x levels produced during fluidized bed combustion of natural gas and air was carried out at the British Gas Corporation, Midlands Research Station. Using variable gas/air mixtures at bed temperatures from 900°C to 1 080°C, the maximum NO_x level observed was 9.7 ppm for a bed of 0.354 mm diameter sand particles at a temperature of 900°C, combustion taking place with 10% excess gas.

4.2 The Experimental Programme

4.2.1 Instrumentation

For the determination of the essentially fluctuating
radiative flux levels emitted from the bed surface it was desirable
to use an instrument with a fast response time. As will be des-
cribed in Chapter 5, the residence times of particles at the bed
surface ranged from approximately 0.02 seconds to 0.1 seconds.
Hence, an instrument response time considerably less than 0.02
seconds was required. Radiometers of the required response time
are available utilizing a variety of detectors, e.g. uncooled
thermistor bolometers or cooled Indium Antimonide detectors held
in an evacuated chamber. The output signal from such instruments
is essentially d.c. and in order to improve the amplification of
the signal a mechanical chopper, positioned in front of the
instrument viewing aperture, converts the radiative flux signal
to a.c. A full description of such instruments appears in Smith,
Jones and Chasmar[47]. In addition such instruments are capable
of wavelength dependant radiative flux investigations through the
use of narrow band filters.

Unfortunately, the cost of such instruments became prohibi-
tive and a radiation pyrometer manufactured by 'Land Pyrometers'
was purchased for the study. A schematic diagram of this instrument
in its operating position is shown in Figure 4.1b. The instrument
utilizes a thermopile detector sensitive by virtue of the arsenic
trisulphide protecting glass, to a wavelength range of 0.8 μm to
11 μm. Held in an evacuated chamber and with auxilliary electronics,
the detector has a response time of 0.06 seconds to 98% of input
signal. The distance of the lens from the thermopile was 6.325 in,
the focal length of the lens being 5.007 in. The instrument field

of view varied with distance from the target, i.e. from 1.4 ins
diameter at zero pyrometer to target distance, to 0.25 ins diameter
at a target distance of 24 ins. The narrow field of view of
1/100 radian was an advantageous requirement enabling a study to
be made of a small area of the bed surface at any one time. The
pyrometer is essentially a temperature measuring device over the
range of 300°C to 1 200°C, i.e. 0-5 volts signal output. The
output is normally non-linear but a linearising circuit was used
to simplify calibration. An emissivity control via an operation
amplifier is a standard addition on this instrument providing for
a range of target emissivity of between 0.5 and 1.0. Its normal
use is when the target emissivity is known, which is not the case
in a fluidized bed. On the contrary, it is this 'effective emis-
sivity' as described in Chapter 1 that is to be determined, being
equivalent to the non-dimensional radiative flux analysis of
Appendix B. Hence it is necessary to consider the operation of
a radiation pyrometer for use as an 'effective emissivity'
measuring device. Such an analysis is confined to Appendix F,
and simply defines $\varepsilon_e = T^4{}_{pyrometer}/T^4{}_{target}$ with $T_{target} = T_{bed}$
in this case. The instrument emissivity control was maintained
at a value of 1.

Because of the fluctuating nature of the input/output signal
the temperature levels from the pyrometer were monitored on an
Ultra Violet recorder using a galvonometer having a natural
frequency of 160 Hz. For matching of the high impedance pyrometer
with the low impedance U.V. recorder, the network of Figure 3.9
was again utilized. To improve the accuracy of this measuring

system the pyrometer output signal was amplified. To achieve
this the pyrometer was operated over small voltage ranges, i.e.
a 1.8 volt change giving full scale deflection of the galvonometer,
by offsetting the general voltage level via a series connected
battery and only allowing the variable signal through to the
U.V.R. In this way the variable signal was amplified without
recourse to a d.c. amplifier. The output U.V.R. trace was then
calibrated by simply imposing a known voltage to the pyrometer
input circuit. This was a technique provided for by the instrument
manufacturer along with calibration of the pyrometer against a
black body source.

4.2.2 Experimental Procedure

The experimental configuration was as shown in Figure 4.1a,b
With the bed surface exposed to atmosphere, large temperature
gradients occur at the free surface, described in Chapter 3.4
and as a consequence, these have to be minimised in order to
apply the isothermal analysis of Appendix B. To achieve a near
isothermal condition, the radiative losses from the bed surface
were reduced using a 1 inch thick ceramic plate placed above the
reactor. In the plate was cut a 15 mm wide slot extending over
half the plate width. The bed was then viewed by the pyrometer
through this slot. In this configuration there is a possibility
that the bed and top surface will provide a hohlraum condition,
i.e. the pyrometer would view a black body. However, a condition
of a hohlraum is that all surfaces of the enclosure remain iso-
thermal irrespective of surface emissivity, otherwise the second

law of thermodynamics would be contravened. To check this pos-
sibility the inside surface of the top ceramic plate was continu-
ously monitored, always remaining some $200°K-300°K$ below the
temperature of the bed. This bed temperature was measured by a
bare Cr AL thermocouple located deep in the bed at the position
of maximum temperature. A check on the isothermal nature of the
bed was carried out by comparing the radiation pyrometer output
signal with and without the top ceramic plate in position. Typical
U.V. recorder traces are shown in Figure 4.2 showing clearly, for
the case of an isothermal bed, the signal to noise ratio of order
unity. With this experimental configuration using a 143 mm dia x
100 mm deep mild steel reactor, a number of experiments were
carried out for variable fluidizing conditions and different
particle materials sieved in narrow size ranges of 0.31 - 0.42 mm,
0.5 - 0.6 mm and 0.71 - 0.82 mm. Geometric means of the particle
size ranges were then obtained. A photograph of the overall
experimental rig is given in Figure 4.1a.

4.3 Discussion of Results

An initial study of the horizontal variations to check for
any localised characteristics of emitted flux were determined by
traversing the bed in the horizontal plane across the field of view
of the pyrometer. The results indicated only significant varia-
tions near the wall of the reactor where cooler particles of
longer average residence times reside. The invariant nature of
the results are taken also to be an indication of even fluidization
over the bed surface. Following this, a further study was carried

out to determine the effect on emitted flux levels of a variation

in bed height. As the object was to study shallow beds, the flux

levels were measured for a number of heights up to 50 mm. The

results are given in Figure 4.3 in terms of the standard deviation

of temperature fluctuations obtained from the U.V. traces for

various bed heights and temperatures. The differences of standard

deviation did not present significant variations in mean tempera-

ture, and subsequently a bed height of H_{mf} = 25 mm (measured at

ambient temperature) was used throughout.

Using the experimental conditions previously described,

results of surface temperatures ($T_{pyrometer}$) were determined for

both an isothermal (ε_{ei}) and a non-isothermal (ε_{en}) (top ceramic

plate removed) bed. The mean surface temperatures along with the

standard deviations obtained from the recorded U.V.R. traces were

determined from up to 100 values of temperature taken at small

intervals over a 150 mm length of paper trace. With ε_{e} = $T^{4}_{pyrometer}/$

T^{4}_{bed}, results for the variable material and fluidizing parameters

are given in Figure 4.4 through Figure 4.11. The theoretical curves

for ε_{ei} and ε_{en} shown also in Figure 4.4 through Figure 4.11 are

derived using the analysis of Appendix B and Appendix D. These

theoretical predictions are dependant upon appropriate values of

particle emissivity ε_{p}. In the case of an isothermal bed, values

of ε_{ei} may be read directly from Figure 2.2 for an appropriate

range of albedo values. In the case of a non-isothermal bed,

values of ε_{en} were obtained from Appendix D after making due allowance

for the surface temperature gradients using the appropriate

curves reported in Figure 3.23 through Figure 3.25.

The measured mean surface temperatures obtained from the U.V.R. traces for an isothermal bed resulted in insignificant values of standard deviation. However, for a non-isothermal bed these values of standard deviation increased appreciably. A typical value of standard deviation equal to 14 K for the case of 0.55 mm sand particles at a bed temperature of 1 000°C and a $(U-U_{mf})$ value of 338.6 mm/s was recorded.

4.3.1 Experimental Observations

Referring to Figure 4.4 through Figure 4.11 it was observed that at low values of $(U-U_{mf})$, approximately 150 mm/s, the value of effective emissivity ε_e reduced particularly when the top plate above the bed was removed. This fact is most probably due to poor particle mixing and particle turn-over, both at the surface and deep within the bed. This results in particles residing far longer than normal at the bed surface. Being exposed to atmosphere, significant cooling by radiation ensues. Increasing the value of $(U-U_{mf})$ tended to improve the particle mixing and particle turn-over rate at the bed surface resulting in an initial increase and final levelling out of the effective emissivity values. Also, an increase in gas/air velocity would tend to expand the bed as a whole. An observation was that combustion within the bed improved whereas in the somewhat 'slumped' state of the bed at low gas/air velocities, combustion appeared unstable with an increase in the emitted noise level. At relatively high values of $(U-U_{mf}) \sim 450$ mm/s a tendency for the effective emissivity values to fall was observed. However, the trend was far from consistent and no further conclusions

may be drawn.

As expected, the effective emissivity or non-dimensional flux varied considerably from unity being strongly dependant on particle emissivity. Comparison of the experimental points with the theoretical predictions is complicated by the fact that at this stage theory does not allow for any variation of ε_e with change of excess gas velocity $(U-U_{mf})$.

For the case of sand and alumina particles the experimental points of Figure 4.4 through Figure 4.7 for high $U-U_{mf}$ values fall reasonably close to the predicted values for the isothermal bed. Theory tends to overpredict experiment by up to 6% for bed temperatures exceeding about 900°C but underpredict for less than 900°C. Similar trends were observed also for magnesia particles whereas for silicon carbide particles excellent agreement with theory was obtained.

With the bed surface open to atmosphere and due allowance made in the predictions for the surface temperature gradients present, using the curves of Figure 3.23 through Figure 3.25, comparison with experiment was generally poor. Such discrepancies are understandable considering the method by which the surface temperature gradients were obtained and the uncertainties associated with the resulting profiles. For an optically dense medium the surface emitted flux tends to follow the proportionality $q_r(o) \alpha \theta^4(o)$. Hence any small error in determining the surface temperature will be amplified by raising this surface temperature up to the power four. Such experiments confirm the inadequacy of the method used for the temperature gradient measurements described

in Chapter 3.

A comparison of the predictions for an isothermal bed with the essentially non-isothermal emulsion overpredicts by up to 50%.

As will be shown in Chapter 5, the response time of the radiation pyrometer is of the same order as the residence time of the surface ejected bubbles. Hence the results of ε_e which are strictly fluctuating may be significantly damped because of this time lag between physical occurrence and subsequent measurement. Ideally, a fast response instrument is required with a small field of view to follow the instantaneous evolution of radiative flux from single isolated bursting bubbles. However, the discrepancies between theory and experiment are still too large to be accounted for by the time lag of the measuring instrument. It is evident that considerable surface eruption interaction takes place with increased particle residence times at the bed surface resulting in significant surface temperature gradients.

Only three particle sizes were used although covering a reasonable size range. In all experiments no obvious effect on the results of varying particle size was observed.

With the reported discrepancies between prediction and experiment the results, at this stage, are useful to a thermal designer only in an empirical sense. For convenience the results may be reported by taking mean values at high $U-U_{mf}$ values. From such results particle emissivity may then be predicted for comparison with Table 3.4. Such mean values of ε_{ei} are reported in Table 4.1.

TABLE 4.1

	Silica Sand	Silicon Carbide	Alumina	Magnesia
ε_{ei}	0.82	0.92	0.67	0.76
ε_p	0.54	0.76	0.31	0.43

Experimentally, it was further observed that the effective emissivity changed significantly as the bed temperature was lowered. At bed temperatures less than 850°C the value of ε_e began to increase from the values observed at higher bed temperatures. For relatively high values of $(U-U_{mf})$ this fact was most prominent and reported in Figure 4.4 and Figure 4.8 for all particle materials at bed temperatures of 825°C. To check this observation further experiments were carried out using a high value of $(U-U_{mf}) = 340$ mm/s for both isothermal and non-isothermal bed conditions. Varying the bed temperature the previous observations were supported and the results recorded in Figure 4.12 and plotted as in [7] as a radiative flux (rather than effective emissivity) against bed temperature. The cross-hatched area encompasses the experimental results of [7] in which a similar deviation away from the absolute black body curve with increases in bed temperature was reported. The comparison is one in principle only and is not strictly valid as in the experiments of [7] a radiometer probe was immersed into the bed. In this case other factors should be considered, particularly the cooling of the particles in contact with the radiometer protecting glass which, as a consequence, would reduce the recorded

flux levels.

In a discussion of the work reported in [7] a recent paper, [5], quoted a table of particle emissivities and effective (measured) bed emissivities for the bed materials used in [7]. It was suggested in [5] that the radiometer quartz protection glass used in [7] was not cooled to the levels anticipated, i.e. 100°C-200°C, and in most cases would approach the temperature of the bulk of the bed. Hence it is fair to assume an almost isothermal emulsion radiating through a glass wall. Below in Table 4.2. are given the particle and effective emissivities taken from [5] via [7]. For comparison, values of ω_o were calculated from the reported values of ε_p and values of ε_e then predicted from Figure 2.3. A comparison between ε_e from this study and $\varepsilon_e^{[5]}$ from [5] are extremely close, shown by the percentage difference column of Table 4.2. Hence with an accurate knowledge of particle material emissivity and isothermal conditions prevailing, emitted radiative flux levels may be simply predicted with reasonable accuracy.

Theoretically, according to the analysis of Chapter 2, the curves of radiative flux against bed temperature remain parallel to the absolute curve for a black body. The trend away from this parallelism shown in the experimental points of Figure 4.12 being due to the fact that at bed temperatures below 850°C, combustion within the bed was not complete and occurred in the gas bubbles at or near the bed surface. This fact was noted in the surface temperature profile measurements of Chapter 3.3 resulting in higher local particle temperatures than were measured deep within the bed. This fact was further observed in the work

TABLE 4.2. Taken from [5] and [7]

BED MATERIAL	Dp mm	U/U_{mf}	Tb°C	ε_p	ε_e [5]	ω_o	ε_e	$\dfrac{\varepsilon_e - \varepsilon_e\,[5]}{\varepsilon_e\,[5]}$ %
River Sand	1 - 1.5	1.2 - 3.0	500 - 1 100	0.6	0.85	0.58	0.85	0
Chamotte	1 - 1.5	1.2 - 3.0	450 - 1 100	0.6	0.8	0.58	0.85	+6.25
Zirconium Dioxide	0.25 - 1	1.2 - 4.0	600 - 1 150	0.23	0.59	0.88	0.57	-3.38
Corundum (Alumina)	1.5 - 2	1.2 - 3.0	800 - 1 450	0.27	0.59	0.85	0.63	+6.78

of Sadilov and Baskakov[48] in an attempt to determine the temperature of the gas within the bubbles bursting at the bed surface. From the results of [48] this gas bubble temperature was up to 900 K higher for a bed of 0.32 mm diameter alumina particles at a gas velocity of 0.3 m/s than the measured bed temperature of 800°C. As the bed temperature was increased this temperature difference reduced rapidly to almost zero at a bed temperature of 1 100°C. For this reason only experimental results in beds above 900°C are regarded as being representative.

It is interesting at this point to compare the results of this study with those of Szekely and Fisher[71] who reported a study of bed to wall radiation heat transfer in which a fluidized bed of particles was heated by a radiant heater at 650 K through the glass walls of the bed. From strategically positioned thermocouples a heat balance was established over the whole system and heat transfer coefficients were determined. The theoretical analysis considered the radiative heat transfer to a single particle and after certain simplifying assumptions a simple relationship of

$$Q_r = F \sigma \pi D_p^2 (T_s^4 - T_p^4) \tag{4.1}$$

was reported, with

T_s = heater temperature

T_p = particle temperature

Included in F is the emissivity of the particulate material along with the view factor between a triangular array of particles and a plane surface. In all cases the predicted values of radiative

flux underestimated the experimental values of Q_r by a factor of about 1.5. As noted in [71] the simple analysis did not take account of reflection or reradiation from adjacent particles or for any additional heat absorbed by subsequent layers of particles. For this purpose the problem should be described by the exact integro-differential equations.

To this end, using the experimental conditions reported in [71], values of Q_r were predicted using the analysis of Appendix B and C; the calculated values are reported in Table 4.3 along with the predicted and experimental results from [71]. In this analysis with the bed heated through the transparent walls by an externally radiating source, the boundary conditions from [71] were $\theta_1^4 = 9.222$ with $T_1 = 650$ K and $T_b = 373$ K.

TABLE 4.3

	Silicon Carbide	Porous Alumina	Iron Shot	
ε_p	0.94	0.9	0.6 ⎫	
Predicted Q_r watts	∿95	∿90	∿110 ⎬ From [71]	
Experimental Q_r watts	∿160	∿140	∿135 ⎭	
ω_o	0.12	0.18	0.58 ⎫ Present	
Predicted Q_r watts	124.7	109.4	65.2 ⎬ Study	

where $Q_r = q_r \pi DL$

$D = 0.061$ m

$L = 0.067$ m

From Table 4.3 a significant increase in the predicted values
of Q_r are observed for Silicon Carbide and Alumina using the present
analysis. However, the discrepancy is still too large to be at-
tributed to experimental error. For the case of iron shot particles
the present analysis predicts results even smaller than those
predicted in [71]. Unfortunately, no details are given in [71]
of the source of particle emissivity determination. It is possible
in the case of iron shot that the value of ε_p is much larger than
0.6. Interestingly with ε_p = unity, theory predicts Q_r = 138.5 watts.

With the unacceptably large discrepancy between prediction
and experiment, it would be useful to repeat similar experiments
and qualify the results reported in [71].

4.3.2 Directional and Hemispherical Effective Emissivities

Both experimentally and theoretically according to the
analysis of Chapter 2, the reported values of emulsion effective
emissivity are, by definition, hemispherical. Functionally, the
directional emissivity of any emitter is defined as

$$\varepsilon_{ed} = \varepsilon_{ed}(\Theta, \tau_o, \omega_o)$$

In the analysis of Appendix B the radiative flux, and hence
effective emissivity, is assumed to be invariant with respect to
angle Θ, where Θ is measured from the normal to the emulsion surface
shown in Figure 4.1. Through subsequent integration, the effective
emissivity becomes

$$\varepsilon_e = \varepsilon_e(\tau_o, \omega_o) = \varepsilon_{eh}$$

In the work of [18] the directional effect of emissivity was shown to deviate from Lambert's Cosine Law, significantly at large values of angle Θ, particularly in media of a low optical thickness. This variation was shown to be less marked for optically dense media. In the work of [7] where a radiometer was immersed in the bed, Lambert's Law was assumed to hold but not strictly justified. To check the angular variation of effective emissivity in the case of a fluidized bed where an optical measuring instrument with a narrow field of view was used, an experimental programme was carried out.

The radiation pyrometer supported on a simple traversing mechanism was moved along a hemispherical path above the bed. Using a 143 mm diameter x 50 mm high mild steel reactor, the pyrometer viewed the bed surface to within 10° of the horizontal plane through a 15 mm wide x 20 mm high slot cut into the side of the bed. During operation a few particles were ejected through this slot but this did not produce any detremental effects. Using the same bed materials as in previous experiments, and for similar fluidizing conditions, results of effective emissivity were obtained with and without the top ceramic plate in position. The results are shown in Figure 4.13 through Figure 4.20. The deviation from Lambert's Cosine Law is small as suggested in [18] and confirms the earlier assumption of this work that hemispherical emissivity may be used throughout with the introduction of only a small error.

Strictly, for comparison with experiment, a two-dimensional analysis of the surface emitted fluxes should be conducted, a similar study of which was carried out in the work of Bobco[50].

However, in the interest of simplicity and to a first approxima-
tion, an analysis of the angular variation of emitted flux was
taken from [18]. This analysis was essentially a modified diffu-
sion approximation to the radiative transfer equation.

The comparison of experiment and prediction was expectedly
poor in some cases. The important trend away from Lambert's
Cosine Law was sustained, though not of a sufficiently significant
level to invalidate the assumption made previously of ε_{ed}
invariant with angle Θ.

4.4 Further Studies of a Bed Composed of a Binary Mixture

The previous experiments were idealized in that particles
of a very narrow size range were used. Industrial applications
of fluidized bed heat exchangers normally use particulate material
comprising a large size range. As an extreme of this condition,
the effect on emitted radiative flux levels in beds of binary
mixtures were studied. When using a binary mixture segregation of
the bed occurs. Segregation, according to the work of Cheung,
Nienow and Rowe[51] and of Gibilaro and Rowe[52], depends largely
upon the density ratio of the particles using and to a lesser
degree upon the diameter ratio of the particles. In this work
only the particle diameter ratio effects are considered as in heat
exchangers it is unlikely that particles of different type
(density) are used.

4.4.1 Experimental Observations

The experimental configuration and method of radiative flux

measurement has been described earlier in this Chapter and need

not be elaborated here. The results of two such experiments for

a non-isothermal bed using Silica Sand and Silicon Carbide are

reported in Figure 4.21 for a range of percentage fines content.

On start-up the bed segregated readily with combustion taking

place in the fluidized upper portion of the bed made up of fines

with the lower portion of large particles remaining unfluidized.

As the temperature of the lower portion rose above 750°C this also

became fluidized. The two segregated regions then became increas-

ingly mixed as the temperature of the lower portion increased

further. With combustion taking place deep within the lower

portion, at a temperature 950°C for sand particles and 935°C for

Silicon Carbide particles, a number of effective emissivity results

were observed for a range of $(U-U_{mf})$ values. The values of minimum

fluidizing velocity for the mixture were derived from the correla-

tion of [51] as $U_{mf\ mix} = (U_{mfL}/U_{mfs})^{a_L^2} U_{mfs}$, with subscript L and

S referring to Large and small particles respectively. The term

a_L is the fraction of large particles present, i.e. $a_L + a_s = 1$.

In a segregated bed the above correlation does not hold except

perhaps in the interface between the upper and lower portions.

For a low percentage fines content, mixing at the interface of the

upper and lower portions was significant although a major propor-

tion of the fines remained virtually suspended above the bed

surface.

As suspected, the measured results of effective emissivity

showed a marked drop of order 5% compared with the previous results

for a narrow particle size range. The suspended fines, having

extremely long residence times at the bed surface, were cooled significantly by radiation to atmosphere and also acted as an attenuater of the radiative fluxes emitted by the lower portion of large particles. Similar trends to those experienced on beds of a narrow particle size range were observed for increasing $(U-U_{mf})$ values, i.e. a sharp increase in effective emissivity with a final levelling off. However, as the percentage fines content was increased, different results were observed at low $(U-U_{mf})$ values. With more fines added a sufficient volume of small particles encouraged combustion to take place in the upper portion of the bed and at the interface. This was verified by traversing a thermo-couple through the total height of the bed and noting the point of maximum temperature. At low $(U-U_{mf})$ values with combustion taking place near to the bed surface, the lower portion became defluidized (similar to start-up conditions). Hence the upper portion acted as a single bed with a high turn-over rate of small particles thereby producing higher emitted radiative flux levels. Increasing $(U-U_{mf})$ encouraged a shift downwards of the combustion zone, this once more fluidizing the lower portion, improving the mixing of the two regimes near the bed surface. With small particles thus virtually suspended the radiative flux levels were reduced. These results were general for both silica sand and silcon carbide, although for silicon carbide the shift of the combustion zone to the lower portion occurred at a lower percentage fines content than for silica sand.

The experiments were not continued further. Nevertheless, they demonstrate that in an industrial design of a thermal heat

exchanger where large particle size ranges are to be used, the effect of segregation may have a significant effect upon the expected radiative flux levels emitted from the bed surface.

4.5   Application of the Analysis to Irradiation of a Remote Surface by a Fluidized Emulsion

The analysis so far has been concerned with the determination of the surface emergent radiative flux levels from a fluidized bed. Of interest to the thermal designer is the irradiation of a surface, remote from the bed, but nevertheless exchanging radiative energy with the bed. The solution of the radiative transfer equation has by nature transformed the problem of radiation from a volume emitter to one of transfer from an effective surface, i.e. with a corresponding value of effective emissivity. Hence irradiation of a remote surface is reduced to a geometrical problem when the appropriate emissivity and temperature of the remote surface are included in the solution of the radiative transfer equation. The analysis becomes essentially that of radiative transfer between opaque surfaces involving a determination of the relevant view factor.

4.6   Conclusions and Suggestions for Further Study

Radiative flux levels from the free surface of a fluidized bed have been obtained both theoretically and experimentally with the assumption of steady state heat transfer made throughout. An important parameter is the particle material emissivity and, due to the range of values reported in the literature, may be a source

of large discrepancy between predicted and experimental results.
However, values of particle emissivity reported in Chapter 3
produce a reasonable comparison between theoretical and experimental
values of emulsion effective emissivity for a closely controlled
study of an isothermal bed surface. The variations of particle
emissivity with the range of temperatures used in this work do not
appear to be large (from Chapter 3) and hence the assumption of
invariance in this study is not expected to be a major source of
error.

Further studies on beds radiating to atmosphere showed how
the complex, time-dependent, surface temperature gradients consid-
erably reduced the emitted fluxes compared with the controlled
isothermal case.

An attempt has been made to estimate these temperature
profiles using a shielded thermocouple probe reported in Chapter 3.
Unfortunately, such a method leads to large uncertainties as to the
accuracy of the resulting profiles which on substitution into the
theoretical analysis show considerable discrepancy with experiment
emphasizing the emitted flux dependance upon the type of remote
boundary present. With the emitted fluxes being proportional to
the surface temperature raised to the fourth power of any optically
dense medium, then the accurate determination of this surface
temperature is essential.

Further experiments studying the hemispherical variation of
the surface emitted flux show a deviation from Lambert's Cosine
Law. Although such variations occur over a small lower portion of
the hemisphere, they are not considered significant in this study

of an emulsion with $\tau_0 \gg 1$.

As industrial designers are concerned with beds of a large particle size range a few experiments were conducted to show the effect on surface emitted flux levels from binary mixtures. Although such experiments are somewhat ideal, they do show interesting trends which should be further studied for a variety of mixture types and fluidizing conditions. However, as many fundamental problem areas still remain, this line of approach was not continued.

Theoretically, the study has established that radiative flux determination from a volume emitter may be recast as an effective surface emitter, the transfer from which depends only upon the geometry (view factor) between the relevant surfaces and the thermal gradients present at the bed surface.

So far the analysis has been verified experimentally only under controlled laboratory conditions where the surface temperature gradients have been significantly reduced and isothermal conditions maintained throughout the bed. In order to predict the emitted flux levels for a variety of remote boundary conditions further study is required on the behaviour of the bubbling surface and its effect upon the emitted fluxes.

The assumption of a steady state process maintained throughout this study is an over-simplification of the real conditions, and a study of the fluctuating nature both thermally and hydrodynamically of single isolated erupting bubbles is desired to eliminate any surface interactions.

C H A P T E R   5

A STUDY OF THE BEHAVIOUR OF A FREELY BUBBLING FLUIDIZED
BED SURFACE

5.1 Introduction

Before proceeding to an unsteady analysis of radiative
transfer, a study of the behaviour of particles at the bed surface
is necessary. Any study of the surface is complicated by the a-
parent non-periodic nature of bubbling. However, according to the
hypothesis of [45], the observed periodic nature of bubble nuclea-
tion just above the distributor plate gives rise to similar pressure
fluctuations measured at this position. To a first approximation
it may be considered that these periodic fluctuations are due to
a subsequent periodic bursting of bubbles at the bed surface.
Following this initial premise, a simple model of bubble and
particle motion may be derived which enables representative, if
not exact, results to be obtained. From the point of view of
radiative heat transfer at the freely bubbling surface, such an
energy exchange is rendered unsteady not by the direct variation
of radiative intensity (as this term in the radiative transfer
equation is multiplied by the reciprocal of the velocity of light,
i.e.

$$\frac{1}{C} \frac{dI}{dt}),$$

but directly by the radiation cooling of the particles subject to
a residence time of exposure. Hence a first consideration in the
study of surface behaviour is to establish a residence time, or
residence time distribution of particles ejected by bursting bubbles.

It was shown earlier in Chapter 3 that on a time average
basis temperature gradients extended deep into the bed to optical
depths of order 10. Consideration of the temperature fluctuations
at the surface suggests two alternatives. First, of all a model
of an isothermal packet of emulsion brought to the surface at zero
time and exposed thereby producing a temperature distribution for
all times greater than zero may be assumed. Alternatively, as
there may be a significant time lapse between one bubble arriving
at the surface ejecting a packet of particles, and the next bubble
plus ejected packet, the subsequently exposed surface would cool
by radiation. A temperature distribution would be imposed at the
surface which in the model suggests that at zero time the emulsion
packet is not isothermal. Of course in a freely bubbling bed as
distinct from a single isolated bursting bubble, considerable bubble
interaction and lateral particle mixing takes place. Thus neither
of the two alternatives would strictly hold, although the second
one appears closer to reality.

One important assumption inherent in the two suggestions is
that particles ejected into the free board above the bed originate
from or near to the bed surface. From extensive studies of bubbles
in fluidized beds reported in [31], it appears that particles
ejected into the free board may well originate at the bed surface.
However, these may also be augmented by a proportion of particles
brought from deep within the bed, carried in the wake of the bubble.
Again the observations reported in [31] confirm that considerable
particle mixing takes place within the bubble wake as the bubble
passes through the bed. If this is the case, then particles ejected

in this manner into the free board would be considerably hotter

than particles ejected from the surface, given their momentum by

the diffusion of gas from the bubble as it reaches the surface.

It is a point of conjecture whether or not bubbles occur at all

in shallow fluidized beds which for the purposes of this study are

defined as beds in which bubbles may grow but do not have sufficient

time to coalesce. Again referring to the work of [45] and from

the observations of shallow bed pressure drops reported in

Chapter 3, it is quite feasible to imagine bubbles originating

at, or a few particle diameters below, the bed surface. This is

particularly so when the overall bed height approaches 10 D_p.

So in any study of the behaviour of a freely bubbling bed surface

the above postulations should be considered.

For the convenience of any initial study where a limited

amount of work has been previously reported, the simplest approach

is usually the most desirable. In this work the obvious starting

point was to study the behaviour of single isolated bursting

bubbles and build up a model to include further complexities if

initial observations allow. To this end with a consideration of

the previous postulations, a programme of experiments was conducted

in which single bubbles were injected into an incipiently fluidized

bed and the ensuing events recorded on film.

5.2 Particles Ejected by Single Isolated Bursting Bubbles

5.2.1 A Theoretical Model

An initial study of the appropriate equations of particle

motion at the bed surface was made in the light of the work on

particle ejection and entrainment by Zenz and Weil[55] and later by Do, Grace and Clift[56]. In particular, the work of [56] postulated a simple model based on three simplifying assumption and it is this analysis that is used in this study. Assumptions:

[1] Particles are ejected from the surface of the bed with an initial velocity V_i related to, but not necessarily the same as, the velocity of the bubble causing the particle ejection

[2] The subsequent particle motion is determined by a balance between gravity and drag forces, with

$$C_D = \frac{24}{R_e} (1 + 0.15 \, R_e^{0.687}) + \left[\frac{0.42}{1 + 4.25 \times 10^4 \, R_e^{-1.16}} \right] \quad (5.1)$$

[3] The gas in the free board has a uniform superficial velocity, U.

The equation of motion may be written from a consideration of assumption [2]; i.e.

$$\frac{dV}{dt} = \frac{-3}{4} \frac{C_D \, \rho_g \, V_r |V_r|}{\rho_p \, D_p} - \frac{(\rho_p - \rho_g) g}{\rho_p} \quad (5.2)$$

where V = dh/dt is positive in the upwards direction, and

V_r = V - U, and

h is the height of the particle measured from its ejected position, i.e. the dome of the erupting bubble.

Equation (5.2) may be solved numerically with boundary conditions of $V = V_i$ and h = 0 at t = 0.

The above theoretical model is similar to that of [55] but allows
for the more exact form of drag coefficient. [55] used the
standard relationship of

$$C_D = \frac{37.5}{R_e} \qquad (5.3)$$

A computer program using a Kutta-Merson numerical integration
procedure was written to solve equation (5.2) and checked with the
reported results of [56] using their initial boundary and flow
conditions. The predicted particle trajectory curves as a function
of time are described in [56]. These curves suggest observations
which may be expected from later experimental studies. It appears
that large particles are generally projected higher for the same
initial conditions and have longer residence times in the free
board than smaller particles, although very fine particles may
be immediately entrained and carried away if their terminal
settling velocity is exceeded. Further predictions by [56] suggest
that increasing the superficial gas velocity U increases the
height and residence time of any size of particle. Also, that if
particles return to the bed and are not entrained away, the maximum
height of rise is strongly dependant on the initial ejection
velocity V_i, with

$$\frac{h_{max}}{D_p} = \rho_p \frac{D_p V_i}{\mu \, 28.125} \left[1 - \frac{1}{V} \log_e (1 + V) \right] \qquad (5.4)$$

where $V = V_i / (V_t - U)$

   V_t = terminal settling velocity.

5.2.2 <u>Experimental Study</u>

In the work of [56] an experimental programme was conducted to verify the previously described mathematical model. A 'two dimensional' fluidized bed was used, constructed from plexiglass sheet 560 mm wide x 10 mm thick x 2 440 mm high. Although not given in the report, it would appear that a deep bed of glass beads was used for the study. As this present work is concerned with shallow beds, i.e. less than 100 mm, it was necessary to repeat such experiments and compare the results with the analytical predictions. A similar 'two-dimensional' fluidized bed was constructed from perspex sheet 600 mm wide x 10 mm thick x 600 mm deep. Sand particles of three sizes, i.e. 0.354 mm, 0.55 mm and 0.777 mm (geometric means) were used. The depth of particles within the bed was maintained at 65 mm, measured at minimum fluidization conditions, throughout the experiments. A schematic diagram is shown in Figure 5.1 describing the lighting arrangement similar to that employed in [56] for purposes of a photographic study of the bubble rise and subsequent bursting at the bed surface. Intense rear lighting of the bed enabled the two dimensional bubbles to be seen clearly. Above the bed a black matt screen (cardboard) was used with front illumination to allow sufficient contrast for particles ejected into the free board to be seen. The horizontal screen at the front of the bed was designed to allow illumination of ejected particles and prevent front illumination of the lower portion of the bed. The superficial gas velocity was set at $1 - 1.2 \ U_{mf}$ for the particles used, the incoming air bubbled through a humidifier to minimise any

electrostatic effects.

With a solenoid valve controlled by a rotating cam, a series of bubbles were ejected through a 1 mm diameter bore stainless steel tube, positioned at 10 mm above the distributor plate. The frequency of bubble generation was simply controlled by changing the speed of rotation of the cam. The bubble rise bursting at the bed surface, followed by particle ejection, was recorded for a number of experiments using an electrically driven 'Hycam' high speed camera operated at 200 frames per second. A pulse marker appeared on the film every 10 seconds as a check of frame speed. Sufficiently clear resolution of particles was obtained using a 75 mm telephoto lens. The velocity of bubbles and ejected particle trajectories were measured from a frame by frame analysis of the films.

Using the bubble nose as the point of reference, its mean rise velocity was obtained from a measurement of distance travelled divided by time of travel. The mean ejection velocity of particles at the surface of the bed was also obtained from the motion of the dome shaped surface disturbance as the bubble broke the surface. The determination of this velocity was made difficult by having to measure very short disturbance distances. However, use of the high speed camera and subsequent enlargement of each frame by an order 4.4 : 1 when viewed on a screen made it at least possible, although the measurements may be subject to a small error. The position with respect to time of the upper surface of the ejected cloud of particles was observed as the cloud rose and then returned to the bed surface.

5.3 Discussion of Results

As an example of the bubble rise and particle ejection, a sequence of photographs are shown in Figure 5.2. It was observed that on initial injection of a volume of gas into the bottom of the bed, an immediate disturbance was sensed at the bed surface shown in Figure 5.2a. Figure 5.2b and Figure 5.2c show clearly the rising bubble with its well defined wake. During examination of this bubble motion, the particles within the wake being interchanged with particles from the main emulsion could be clearly observed. Figure 5.2d shows the bubble approaching the bed surface. Clearly the bubble here reduces in size as gas from its interior diffuses rapidly through the small depth of emulsion of reduced resistance above it. Figure 5.2e and Figure 5.2f then describe the subsequent rise and fall of the ejected particulate cloud.

As observed in [56], the initial ejection velocity V_i of the particles exceeds the mean bubble rise velocity by typically 15%, whereas in this work it is between 15% and 25%. It appears that this extra velocity may be caused by the diffusion of gas from the bubble interior as the bubble breaks the surface supporting the evidence of Grace and Harrison[57].

Predicted and observed particle cloud trajectories are described in Figure 5.4 through Figure 5.6 with $V_i = 1.25\ U_b$ for the different cases studied. In all cases the comparisons are reasonable and justify the theoretical analysis in application to shallow fluidized beds for single isolated bubbles. Also shown in Figure 5.3 are the predicted cloud trajectories for high temperature conditions. The experimental results, although observed at ambient temperature,

compare favourably with the high temperature predicted curves.
Such comparisons are reasonable as the only variables in the pre-
dicted results are gas properties with bubble velocity taken from
ambient conditions. Although the gas viscosity increases by an
order of two from 20°C to 1 000°C, the gas density reduces by an
order of four.

In a freely bubbling shallow fluidized bed considerable
interaction between bubbles at the surface along with lateral
mixing of particles takes place. For this reason films were taken
of a freely bubbling system and typically shown in Figure 5.3.
The analysis of these films proved difficult and only a study of
the local variations occurring at the surface were possible. It
is highly unlikely that such local variations could be generalised
to occur across the whole of the bed surface. However, such
studies are interesting in that they show another extreme of
surface disturbance particularly when two bubbles coalesce near the
surface just before bubble eruption. As the shallow bed is con-
sidered for the purpose of this study to be one in which bubble
coalescence within the bed does not have sufficient time to take
place, only the occurrence of bubble coalescence at the surface
was considered. The important observations were that the initial
ejection velocity V_i is increased considerably by up to twice the
measured bubble velocity, i.e. $V_i = 2U_b$. This effect was also
reported by Botterill, George and Basford[57] in a study of coales-
cing three dimensional bubbles. As the occurrence of bubble
coalescence at the surface reduces as the bed height is reduced,
the study of such phenomena was not pursued but is reported as an

observation which should receive further attention. As the bed
heights used for purposes of radiative heat transfer were only up
to 30 mm, it seems reasonable to assume that the initial analysis
of single isolated bubbles may be applied to these beds.

Again from the point of view of radiation transfer an impor-
tant observation was the origin of the particles within the ejected
cloud. It appeared that for a single isolated bubble, the wake
fraction of the bubbles did not burst through the bed surface as
has been suggested by Basov *et al*[58] and Kehoe[59], but particles
within the cloud originated at the bed surface. However in a
freely bubbling bed where bubble coalescence near the bed surface
took place the wake, through increased momentum due to the accele-
ration of the now larger bubble, did break the surface and spout
to an appreciable height. Again this was an infrequent occurrence
and to a first approximation was not considered. According to this
observation it seems reasonable that at all times a temperature
gradient exists at the bed surface, confirming the experiments of
Chapter 3, and hence a purely isothermal assumption would over-
estimate the resulting radiative flux levels.

From the theoretical analysis the residence time of the
projected cloud of particles is simply determined from the maximum
time of cloud rise and fall.

5.3.1 Extension to a Three-Dimensional Bed

The studies have been concerned with an experimental study
of a two-dimensional bed. Such beds are convenient for experimental
study in that bubbles may be readily photographed from their

nucleation stage through to their bursting at the bed surface and subsequent collapse. For the more realistic case of a three-dimensional bubbling bed, Rowe, in the section on experimental properties of bubbles in [31] expressed doubt as to the great value of two-dimensional behaviour. Although this remains an area of uncertainty, it was assumed for the purpose of this study that the two-dimensional experimental verification of equation (5.2) applies to three-dimensional beds.

5.4 Conclusions

Using a simple analytical approach, the particle ejection heights and mean residence times may be predicted for single, isolated bursting bubbles. The analysis has been verified by experiment using a two-dimensional bed and it was assumed to represent the three-dimensional case. As beds of interest in this work are essentially shallow, the extra projected heights of particle clouds due to coalescence of bubbles at the bed surface are not considered. It appears that the majority of ejected particles constituting the particle cloud above the bed originate from the surface of the bed and hence it is reasonable to assume that large temperature gradients exist at all times within the cloud. It is expected that determination of the actual temperature gradient will be complicated by the lateral, unsteady mixing of particles within the cloud.

At elevated bed temperatures, using bubble velocities obtained at ambient conditions, the predicted cloud trajectories did not show any large variation over those predicted at room temperature.

However, in order to verify this fact it is desirable to determine bubble velocities in three-dimensional beds at elevated temperatures.

5.5   Determination of Bubble Velocity in a Three-Dimensional Bed

5.5.1   Introduction

Bubble velocities within three-dimensional beds are difficult to study and the work of Rowe and Everett[60],[61],[62] have resorted to an elegant method using an x-ray technique. Less elegant methods have been used by Werther[63] in which a miniature capacitance probe was immersed in beds of varying sizes. It was pointed out in [63] that the technique of photographing the bubbling surface of the bed and relating bubble eruption size to bubble velocity may lead to large errors, being restricted to studies at relatively low $(U-U_{mf})$ values. However, such a technique was used by Kassim[64] in a study of the effects in deep beds of various types of distributor plate design. Unfortunately, at high bed temperature no other method appears forthcoming and hence experiments were conducted using shallow beds, i.e. up to 70 mm height measured at ambient temperatures.

5.5.2   Experimental Procedure for the Determination of Bubble Eruption Diameters

Filming of the bed surface was carried out using a highly reflecting plate placed above the bed and inclined at an angle of 45° to the bed surface. A schematic diagram of the experimental arrangement is shown in Figure 5.7. A Bolex Camera operated at

64 frames per second was used with 75 mm lens. A previous study of the bubble eruption diameters occurring in shallow fluidized beds at ambient temperatures of heights less than 70 mm is reported in McGrath and Streatfield[65]. For comparison, initial experiments at ambient temperature used a square bed of dimensions 153 mm x 153 mm x 200 mm high, constructed from 4 mm thick perspex sheet. For a range of $(U-U_{mf})$ values using 0.15 mm sand particles, films were taken and bubble eruption diameters obtained from a frame by frame analysis. Only one particle size was used, the assumption being made from the two-phase theory of fluidization[35] that the governing parameters are the physical and fluidization properties of solids and gas.

The experiments were repeated at a bed temperature of 900°C using a 143 mm diameter x 150 mm high mild steel reactor with 0.354 mm sand particles. With combustion taking place within the bed the 'red glow' of emitted light did not give sufficient contrast to clearly define individual bubbles. However, by shining an angled photoflood lamp on to the bed surface, the sand particles appeared a similar colour to the studies at ambient temperature. This technique made it possible to obtain bubble eruption diameters at a number of $(U-U_{mf})$ values although at relatively high superficial gas velocities film analysis became impossible.

5.5.3 Film Analysis and Discussion of Results

The films were projected on to a screen for analysis, producing a 4.4 : 1 enlargement. Bubble eruption diameters D_{be} were measured across the horizontal diameter. An alternative, and

perhaps more accurate method, was that of [64] in which the maximum
and minimum diameter were measured and $D_{be} = \sqrt{D_{be\,max} \cdot D_{be\,min}}$.
In this study the diameter D_{be} of each bubble over the bed surface
was measured. In order to avoid measuring the same bubble at
different stages of its rise and fall, only every fourth frame was
analysed. The number of bubbles of a given size for each of over
50 frames were plotted on a bubble size distribution diagram. A
typical diagram is given in Figure 5.8 emphasizing the 'bell-shaped'
normal distribution curve. From such diagrams log-probability
curves were drawn to determine the mean bubble eruption diameters
at appropriate values of $(U-U_{mf})$. The probability co-ordinate was
determined from a cumulative summation of all bubble eruption
diameters. Such typical curves are reported in Figure 5.9 through
Figure 5.12 for ambient bed temperatures and Figure 5.13 through
Figure 5.16 for elevated bed temperatures. Mean bubble eruption
diameters are then obtained from these curves corresponding to the
50% probability line.

It was observed in the previous studies of [64] and [65] of
bubbles in two-dimensional beds that the bubble diameter was
related to its eruption diameter by $D_{be} = \frac{3}{2} D_b$, where all diameters
are now mean values. With this relationship, the bubble velocity
within a three-dimensional bed may be determined from the simple
equation of

$$U_b = U - U_{mf} + 0.711\sqrt{g\,D_b} \qquad (5.5)$$

derived from the two-phase theory of fluidization of [35]. From

the results of mean D_b so obtained a plot of D_b(mean) against

$H_{mf}.(U-U_{mf})$ for the range of experiments studied is described in

Figure 5.17. Curve ① was determined from experiments at ambient

temperatures whereas curve ② was obtained at elevated tempera-

tures of 900°C ±20°K using a least squares polynomial fit.

Interestingly, the work of Kato and Wen[65] for deep beds confirmed

the relationship of

$$D_b = 1.4 \, \rho_p \, D_p \, \frac{U}{U_{mf}} \, H \qquad\qquad (5.6)$$

This relationship compared with the experimental results observed

in this study for shallow beds at ambient temperatures tended to

underestimate D_b. In fact, comparison was improved to a maximum

difference of 5% by changing the constant in equation (5.6) to 2.8.

In the reported study of [65] bubble eruption diameters at ambient

temperatures were plotted against excess gas velocity $(U-U_{mf})$ for

a number of bed heights.

Comparison of the results of [65] with those of curve ①

and equation (5.6) show considerable discrepancy. It was observed

by [65] that appreciable bubbling occurred at minimum fluidizing

velocities with reported eruption diameters of up to 50 mm. It is

possible in such studies that due to distributor plate design local

fluidization occurred at low superficial gas velocities rather than

homogeneous fluidization. The possibility of local fluidization

occurring in the present study was reduced by using a high pressure

drop distributor plate. The occurrence of local fluidization in

[65] was further supported by the reported curves of a number of

surface eruptions against excess gas velocity. For a shallow bed
of height 13 mm, up to 45 surface eruptions were reported at mini-
mum fluidizing conditions, rising to a maximum of 80 eruptions at
$(U-U_{mf})$ = 120 mm/sec. Thereafter the number of eruptions fell
linearly with further increase of excess gas velocity. However,
as pointed out by [65] for particles less than 0.2 mm diameter the
bubble velocity exceeds the superficial gas velocities and a large
cloud occurs around such bubbles. Hence at the surface such bubbles
disperse their cloud and give too high a figure for the measured
eruption diameter. The method of eruption diameter determination
in [65] was to fluidize the bed with a 1 mm thick layer of fine
powdered alumina outlining the bursting bubble which was subsequently
measured with a ruler. At high excess gas velocities the gas flow
was shut off for bubble measurement. It is possible at such
velocities, particularly in the deep beds, that considerable bubble
coalescence occurred at the bed surface.

With such large discrepancies no comparisons were possible.

In the present study at elevated temperatures, no reasonable
correlation of the form of equation (5.6) was observed. A convenient
form of presentation was to obtain a polynomial plot, using a least
squares curve fitting technique of D_b against $H_{mf}(U-U_{mf})$. This
curve is only suitable for use within the range of H_{mf} and $U-U_{mf}$
values used in its determination, i.e. to values of $H_{mf}.(U-U_{mf})$
of 10×10^3 mm^2/s. However, from inspection the relationship
appears to approach a maximum and may be reasonably extrapolated
to values of $H_{mf}.(U-U_{mf})$ of 20×10^3 mm^2/s; thereafter large errors
are introduced.

The asymptotic nature of curve ② is reasonable as at elevated temperatures gas viscosity levels become appreciable and the seemingly linear relationship of curve ① at ambient temperature does not hold. Also the indicated finite size of bubble diameter when $H_{mf} = 0$ is reasonable as, at the distributor plate, the bubble size approaches the size of the distributor plate holes. As described in [64], equation (5.6) should be modified to

$$D_b = D_{bd} + 1.4 \rho_p D_p \frac{U}{U_{mf}} H \qquad (5.7)$$

where D_{bd} is related to the size of distributor plate holes. Curve ① of Figure 5.17 should also be modified in a similar way.

5.6 Determination of the Surface Bubble Residence Time in Three-Dimensional Beds

With the analysis derived in Section 5.2, the mean bubble residence times may be obtained with the boundary condition of initial ejection velocity V_i related to bubble velocity as

$$V_i = 1.25 U_b \qquad (5.8)$$

$$V_i = 1.25 \left[(U-U_{mf}) + 0.711\sqrt{g\,D_p} \right] \qquad (5.9)$$

Values of mean D_b are taken from the polynomial expression of $D_b = f(h_{mf}, U-U_{mf})$ described in Figure 5.17 and may be applied with $H_{mf}(U-U_{mf})$ not greater than 20×10^3 mm^2/s.

Figure 5.18 describes the predicted particle cloud trajectories

of a freely bubbling bed of 0.354 mm silica sand particles operating
at a $(U-U_{mf})$ value of 100 mm/s and a bed temperature of 1 000°C.
The influence of bed height on the resulting trajectory profiles
are as expected, i.e. with deeper beds D_b increases, V_i therefore
increases and hence h_{max} increases along with residence time t_r.
From the curves of Figure 5.18 the predicted particle cloud
residence times were obtained and are reported in Figure 5.19 with
residence time t_r defined as the time for the cloud to rise and
subsequently return to the bed surface. Hence for any solid
material and fluidizing conditions, the particle cloud residence
times may be predicted in a similar manner.

The foregoing analysis has ignored the residence time dis-
tribution of individual particles, assuming to a first approximation
that all particles reside for the same time. Such an extension of
the work is not justified while the uncertainties introduced by the
method of bubble velocity measurement remain.

5.7 Conclusions

Using an approximate technique a method of predicting mean
residence times of surface erupted particle clouds has been estab-
lished. Experimental mean bubble sizes are combined with the
solutions of the equations of motion of surface ejected particle
clouds. Being approximate in nature as the method is subject to
experimental limitations, all that may be expected at this stage is
a reasonable order of size of the mean cloud residence times.
There are many areas in which further work or even a completely
different approach are justified, particularly in bubble velocity

determination. The object of such a study is to enable an analy-
tical solution of unsteady radiative heat transfer to proceed. As
no previous studies are forthcoming, this chapter is regarded as a
first step which has of necessity been kept as simple as possible.
An unsteady solution although limited to varying degrees by the
above-mentioned considerations is nevertheless approached and
reported in the next chapters.

C H A P T E R   6

TRANSIENT THERMAL RADIATION FROM HIGH TEMPERATURE
FLUIDIZED SOLIDS

6.1 Introduction

Transient thermal radiation heat transfer in generalized media which absorb, emit and scatter energy has been a subject of considerable interest in recent years. Viskanta and Bathla[23] used the exact formulation to study the radiative heating and cooling of a plane layer of stagnant gas in which the scattering mode was neglected, i.e. $K_t = K_a$. Nemchinov[66] used the two flux approximation to study the transient cooling by radiation of a semi-infinite volume of gas. A number of other solutions, Onufriev[67], Adriankin[68], along with Viskanta and Lall[69] examined the problem for a spherical mass of gas. Recently, Prasad and Hering[24] used the exact formulation to study the transient heating of a plane layer of non-scattering, grey medium between parallel black plates.

Particular interest in this work is in the radiative energy transfer from the freely bubbling surface of a fluidized bed of solids to a remote heat transfer surface for a number of different bed materials. In this configuration radiative transfer is the predominant mode of energy exchange and hence in the subsequent analysis the conduction and convection heat transfer modes are neglected. As in the steady state approach of Chapter 2 the time dependant problem studied here is essentially one of energy transfer from an optically thick medium (emulsion) in which the scattering of radiative energy may be significant. The exact formulation given in Appendix B is employed and a technique developed in

Appendix C is used along with a finite difference approximation
of the energy equation.

6.2 Analysis

A plane parallel layer of a constant voidage absorbing,
emitting and scattering medium (emulsion) is confined between
uniform temperature, gray, diffusely emitting and reflecting boun-
daries shown in Figure B1 of Appendix B. From the analysis of
Appendix B, the one dimensional energy equation may be written as

$$\frac{\partial \theta}{\partial t'} = - \frac{\partial q_r'}{\partial \tau} \tag{6.1}$$

as defined in Appendix G, where the equation of radiative flux q_r'
is given by equation (B.10) of Appendix B. The model is of the
emulsion and boundaries initially at the same temperature (iso-
thermal) or with some known initial temperature distribution
imposed on the emulsion. At time zero the emulsion is subject to
a step change of temperature at one boundary (designated 1) in
Figure 1.1, the other boundary (designated 2) held at a constant
temperature throughout. Hence, fluxes are emitted in the negative
x direction with reference to Figure 1.1 and Figure B1. Equation
(6.1) is then subject to initial conditions of

$$t' = 0, \quad \theta(\tau,0) = \begin{bmatrix} 1 \\ \theta(\tau) \end{bmatrix} \tag{6.2}$$

and boundary conditions of

$$\tau = 0, \quad \theta(0,t') = \theta_1, \quad t' > 0 \tag{6.3}$$

$$\tau = \tau_o, \quad \theta(\tau_o, t') = .1, \quad t' > 0 \tag{6.4}$$

along with the radiative boundary conditions of

$$F_1 = \epsilon_1 \theta_1^4 + 2(1 - \epsilon_1) \left[F_2 E_3(\tau_o) + \int_0^{\tau_o} [(1 - \omega_o) \theta^4(\tau, t') \right.$$

$$\left. + \frac{\omega_o}{4} D(\tau', t')] E_2(\tau') d\tau' \right] \tag{6.5}$$

and

$$F_2 = \epsilon_2 \theta_2^4 + 2(1 - \epsilon_2) \left[F_1 E_3(\tau_o) + \int_0^{\tau_o} [(1 - \omega_o) \theta^4(\tau, t') \right.$$

$$\left. + \frac{\omega_o}{4} D(\tau' t')] E_2(\tau_o - \tau') d\tau' \right] \tag{6.6}$$

6.2.1 Numerical Method of Solution

The energy equation, being a partial, non-linear, integro-differential equation, is not amenable to a closed form solution in its present form. A numerical technique is used which has been previously described for the steady state solution in Appendix D so will not be repeated here. However, as the time dependant nature of the solution is now required, the extension of the numerical method to allow for this is described in Appendix G. Briefly, the solution is one of a finite difference approximation of the energy equation by computational means using the U.M.R.C.C. CDC7600/1906A installation. At each time interval a least squares polynomial curve fitting procedure is used to describe the temperature distribution. An independant check on this part of the program revealed agreement with its exact counterpart to the sixth significant figure

or less. The computer program developed for this analysis is reported in Appendix H. For the steady state solutions of Chapter 2, the same program was used but with the time dependant flux solution not included, i.e. simply the same as a solution at time zero.

6.3 Discussion of Results

An initial check of the computed results was made with the reported curves of the instantaneous time dependant radiative flux of [23]. Unfortunately, the reported analysis was only concerned with a zero scattering ($\omega_o = 0$) medium and as no other solutions for $\omega_o > 0$ are available to the author's knowledge, this single comparison will have to suffice. Figure 6.1 describes the comparison of the present solution with that of [23] for an initially iso-thermal emulsion, i.e. $\theta(\tau,0) = 1$ a maximum difference of order 1.4% was observed in the flux predictions. Further solutions for $0 \leqslant \omega_o \leqslant 1$ remained convergent and are assumed correct to a similar order of magnitude.

Surface radiative flux levels and temperature distributions were obtained for a number of scattering albedo values typical of fluidized solid materials and described in Figure 6.2 and Figure 6.3 for idealized boundary conditions. In all cases an optical thick-ness of $\tau_o = 10$ is assumed as being representative of the depth of the emulsion over which the variation of temperature and flux may extend. In Figure 6.2 an important observation is the reduced cooling rate of the emulsion as the scattering albedo is increased from zero. This is reasonable as the albedo is a function of

particle emissivity and with a reduced particle emissivity the
emulsion cannot emit or re-absorb energy at the same rate as a
black body. However for large times, solutions converge irrespec-
tive of albedo value. This reduced cooling rate is reflected in
the temperature distribution curves of Figure 6.3. Further solu-
tions are reported in Figure 6.4 and Figure 6.5 showing the effects
of varying the initial boundary temperature θ_1 and of the boundary
emissivity ε_1 on an initially isothermal emulsion $\theta(\tau,0) = 1$.
As may be expected an increase in the free boundary temperature θ_1
reduces the initial overall radiative flux exchange. The emulsion
then cools from this initial level but at a reduced rate. Similar
effects are observed as the free boundary emissivity ε_1 is reduced
from a value of unity. For low values of ε_1 energy in the vicinity
of the boundary is reflected back to the emulsion where it is re-
absorbed thus reducing the cooling rate. For values of ε_1 approach-
ing 0.5 with the reduced cooling rate, the change with time of
temperature distribution was exceedingly small and this set a limit
on the accuracy of the numerical solution. In fact, the solution
began to diverge for these low values of ε_1, the recommended range
of applications being $0.6 \leqslant \varepsilon_1 \leqslant 1$. The term appearing in the non-
dimensional time, i.e. $\rho \, C_p (1 - \varepsilon_b)/K_t \, \sigma \, T_b^{3}$, characterises the time
to radiate the entire energy of the volume of emulsion at a tem-
perature T_b. Solutions were obtained by this method up to non-
dimensional times of 0.1 after which actual computing time became
prohibitive. For completeness an approximate approach applicable
for long time solutions, i.e. for $t' \gg 1$ was employed, the dashed
lines of Figure 6.2 and Figure 6.3 drawn joining the two solutions.

Using the optically thick approximation, i.e. $\tau_0 \to \infty$ described

elsewhere [9] and [26], the energy equation becomes

$$\frac{\partial \theta}{\partial t'} = \frac{16}{3} \frac{\partial^2 \theta}{\partial \tau^2} \tag{6.7}$$

subject to boundary conditions of

$$\tau = 0, \quad \theta = \theta_1 + \left[\frac{1}{\varepsilon_1} - \frac{1}{2}\right] \frac{16}{3} \frac{\partial \theta}{\partial \tau}, \quad t' > 0 \tag{6.8}$$

$$\tau \to \infty, \quad \theta = 1, \quad t' > 0 \tag{6.9}$$

and an initial condition of

$$t' = 0, \quad \theta = 1, \quad \tau > 0 \tag{6.10}$$

An analytical solution may be obtained from Carslaw and Jaeger[70]
page 71, as

$$\theta(\tau,t') = \operatorname{erf} \frac{\tau}{8}\sqrt{\frac{3}{t'}} + \exp\left[\frac{3\tau}{16\left[\frac{1}{\varepsilon_1} - \frac{1}{2}\right]} + \frac{3}{16}\frac{t'}{\left[\frac{1}{\varepsilon_1} - \frac{1}{2}\right]^2}\right]$$

$$\operatorname{erfc}\left[\frac{t}{8}\sqrt{\frac{3}{t'}} + \frac{3}{4}\left[\frac{1}{\varepsilon_1} - \frac{1}{2}\right]\sqrt{\frac{t'}{3}}\right] \tag{6.11}$$

where

$$\theta(\tau,t') = \frac{T(\tau,t') - T_1}{T_b - T_1} \tag{6.12}$$

and

$$q_r'(0,t') = \frac{1}{4} \frac{\theta(0,t') \, (\theta_1^4 - 1)}{\left[\frac{1}{\varepsilon_1} - \frac{1}{2}\right]} \tag{6.13}$$

which may be readily solved. Equation (6.11) is only applicable
for a long time solution as the optically thick approximation does
not hold in the vicinity of any boundary (space or time). As
predicted by equation (6.11) all solutions converge after a suf-
ficient time lapse irrespective of scattering albedo or of initial
boundary conditions. As qualified later, such long time solutions
constitute what is essentially an academic exercise as only short
time solutions are applicable to the freely bubbling surface.

6.4 Comparison with Experimental Observations.

Experimental determination of the surface emitted radiative
fluxes have already been described in Chapter 4. The results, due
to the limitations of the radiation pyrometer, were essentially
time average measurements. In order to compare predictions from
the time solution with the experimental results it is necessary to
consider the residence time and residence time distribution of the
ejected particle clouds. A simplification appearing in the approach
of this problem described in Chapter 5 is to assume a uniform
residence time distribution of the ejected cloud. Hence on a time
average basis, the emitted radiative fluxes may be described by
the equation

$$q_{rm}(0,t_m) = \frac{1}{t_m} \int_0^{t_m} q_r(0,t)\,dt \tag{6.14}$$

where $q_r(0,t)$ is the instantaneous radiative flux. The results
of this approach are described in Figure 6.6 for the different bed
materials used in this study. Figure 6.6a describes the time
variation of instantaneous radiative flux with the uniform initial
temperature distribution shown in the inset, and Figure 6.6b
describes the time mean variation of the flux obtained by use of
equation (6.12). Such curves were plotted only to a maximum non-
dimensional time (t') of 0.1. From an initial consideration of
the particulate cloud residence times such a range of (t') appeared
to cover all possible cases. A striking feature of these curves
is the small variation of radiative flux with non-dimensional
time. These curves imply, depending on mean residence time values,
that the time mean radiative fluxes emitted by the bed surface
are essentially invariant with change of $(U-U_{mf})$. The curves of
Figure 6.7 and Figure 6.8 substantiate these predictions, being
essentially equivalent to the corresponding curves of Figure 4.5,
Figure 4.7, Figure 4.9 and Figure 4.11 found from the steady state
analysis. In order to obtain the curves of Figure 6.7 and
Figure 6.8 appropriate particle cloud residence times were obtained
from Figure 5.19 along with values of K_t reported in Figure 3.13
through Figure 3.21, H_{mf} in all cases being 25 mm (measured at
ambient temperatures).

Table 6.1 illustrates typical values of the appropriate
parameters for non-dimensional time determination for the case
of Silicon Carbide particles.

As may be observed from Table 6.1, although particle size
was not directly included in the analysis the effects appear in

TABLE 6.1

Silicon Carbide; ρ = 3 179 kg/m^3; C_p = 0.714 kJ/kg				
(a) U-U$_{mf}$ = 100 mm/s; H$_{mf}$ = 25 mm (at 20°C)				
	D$_p$ mm			T$_b$ °C
	0.354	0.55	0.777	
K$_t$	960	670	780	925
ε_b	0.55	0.46	0.41	
t$_m$	0.09	0.09	0.09	
t$'_m$	0.00825	0.00479	0.00511	
K$_t$	900	600	600	1 050
t$_m$	0.09	0.09	0.09	
t$'_m$	0.0104	0.00578	0.00529	
(b) U-U$_{mf}$ = 400 mm/s				
K$_t$	660	560	400	925
ε_b	0.71	0.59	0.51	
t$_m$	0.2	0.2	0.2	
t$'_m$	0.0196	0.0117	0.00701	
K$_t$	600	510	360	1 050
t$_m$	0.2	0.2	0.2	
t$'_m$	0.0239	0.0144	0.00849	

the non-dimensional time predictions through the value of extinction coefficient K_t and emulsion voidage ε_b. Experimentally, the effects of particle size appear in the value of minimum fluidizing velocity although from Figure 6.7 and Figure 6.8, no consistent characteristic was observed. An important variable affecting the value of t_m' is bed voidage. Using the relationship given in Leva $et\ al$[76] of

$$\frac{1 - \varepsilon_b}{\varepsilon_b^3} = \frac{U_{mf}}{U}\left[\frac{1 - \varepsilon_{mf}}{\varepsilon_{mf}^3}\right] \qquad (6.15)$$

the bed voidage may be written in terms of the cubic equation.

$$\varepsilon_b^3\,\alpha + \varepsilon_b - 1 = 0 \qquad (6.16)$$

where

$$\alpha = \left[\frac{1 - \varepsilon_{mf}}{\varepsilon_{mf}^3}\right]\left[1 - \frac{U - U_{mf}}{U}\right]$$

Now from the analysis of Chapter 3, equation (3.4)

$$\left[\frac{1 - \varepsilon_{mf}}{\varepsilon_{mf}^3}\right] = \frac{C\ \varnothing^2}{150} \qquad (6.17)$$

with C = 1 440 as validated by the high temperature correlations of U_{mf} determination reported in Figure 3(a,b,c). The appropriate values of ε_b reported in Table 6.1 are assumed invariant for the temperature ranges of interest. In all cases values of t_m' are 0.01 or less, thus the values of radiative flux predict an almost

invariant change with time. The effects of the initial temperature distribution using appropriate temperature profiles from Figure 3.23 through Figure 3.25 on the predicted radiative flux levels are described in Figure 6.9(a,b). No comparison is made with experimental results because as reported earlier in Chapter 4, considerable discrepancies were observed. This being the case, Figure 6.9(a,b) are presented for academic interest only. The initial temperature distribution reduces the cooling rate of the emulsion as evidenced by Figure 6.10 and Figure 6.11 compared with an equivalent emulsion subject to an initial uniform distribution. Such an effect is similar to the results encountered from an increase in the scattering albedo ω_o.

6.5   Conclusions and Suggestions for Further Study

Within the limitations of experimentally predicting particle cloud residence times, the essentially time dependant radiative fluxes emitted by an ejected isothermal emulsion are independant of excess gas velocity $(U-U_{mf})$. This invariance is also predicted experimentally although the response time of the measuring instrument (radiation pyrometer) may have limited these observations.

The transient analysis considers radiative transfer from a single bursting isothermal bubble and ejected particle cloud, with no interaction between adjacent bubbles at or near the bed surface. In reality considerable interaction does occur, particularly at high $(U-U_{mf})$ values, hence further studies are required on single bursting bubbles. However, even for considerably long residence times the change of radiative flux is not expected to be large.

The initial temperature distribution was assumed uniform. However, results indicate that depending upon the boundaries present, a temperature distribution will always be imposed upon the ejected cloud. With a high emissivity, low temperature boundary, the ejected cloud residence times and surface interaction of bubbles are sufficient to impose an initial significant temperature distribution. At the other extreme a boundary of low emissivity or high emissivity and high temperature the initial temperature variation is not large. Hence further analysis must be considered with this boundary dependant temperature distribution more adequately described.

An important observation is that when a heat transfer surface is brought into contact with the bed and energy exchange by conduction must be considered, radiative transfer plays an increasingly significant role as the residence time of the emulsion packet increases. However, for short residence times conduction heat transfer will be the predominant mode.

The model of transient radiative transfer developed in this chapter has certain shortcomings. Its verification by experimentation is far from complete and further improvement must be left until a suitable radiometer or alternative measuring instrument is available. Further uncertainties remain in the determination of extinction coefficient values. The values of K_t used in this transient model were obtained from a time averaged approximation, and further effort is required in determining more reliable values of K_t applicable in this transient model. The residence time of the ejected cloud of particles was obtained in a somewhat idealised

and simple way and requires further investigation, particularly
for the case when surface interaction of bubbles occur.

In essence, the attempt of such a transient study has, in
providing for further study, evolved a number of areas where
further knowledge is desired even though the model has of
necessity been kept as simple as possible.

CHAPTER 7

UNSTEADY COMBINED RADIATION AND CONDUCTION HEAT TRANSFER

TO SURFACES IMMERSED IN A FLUIDIZED BED

7.1 Introduction

During the history of fluidized bed heat transfer considerable attention has been focused on the mechanism controlling heat energy exchange between the bed and either the walls of the reactor or some immersed surface. Various workers[1] and Baskakov and Goldobin[83] immersed two surfaces of a high and low emissivity into a hot bed to determine the radiative heat transfer component. Overall heat transfer coefficients were obtained to an immersed spherical, copper calorimeter by Karchenko and Markhorin[84], although the size of the probe of 60 mm diameter was too large to apply the lumped capacity method of heat transfer with Biot numbers exceeding 0.2. Hence unaccounted for errors are expected in the reported values of heat transfer coefficient. As a contrast the work of Baskakov et al[85] used a radiometer probe immersed in the bed to measure the radiative fluxes and presented the variation of effective emissivity of the bed with change of temperature of the radiometer quartz protection glass. Such curves showed the influence of the cooling of the first row of particles in contact with the glass by conduction and radiation, reflected in a lowering of the measured radiative fluxes. These results may be compared with those of [6] where no such allowance for particle cooling was made, and further confirmed by the observations of [7].

In all the above analysis, the combined modes of conduction and radiation heat transfer are present. To date no reasonable

comparisons between existing theories and experiment are available.
The most comprehensive theoretical solution, without at present
any experimental data, was performed by [3]. Unfortunately in the
analysis no values of wall temperature was reported and hence a
comparison with the solution presented in this chapter is not
forthcoming.

At low bed temperatures where the predominant mode of heat
transfer is by conduction a number of studies have been performed,
a notable one of which is that of Kubie and Broughton[82] who
found excellent agreement with controlled residence time data
available in the literature. In essence the theoretical solutions
presented in [82] were based on the packet theory first established
in [8] which was modified to account for the local voidage and
property variations in the vicinity of a flat surface. An equation
relating bed voidage and non-dimensional distance was reported as

$$\varepsilon(\bar{x}) = 1 - 3(1 - \varepsilon_b)(\bar{x} - \tfrac{2}{3}\bar{x}^2) \text{ for } \bar{x} < 1$$

and

$$\varepsilon(\bar{x}) = \varepsilon_b \text{ for } \bar{x} > 1$$

where $\bar{x}$ = number of particle diameters.

Although not strictly correct in that conduction and radia-
tion must be solved simultaneously, the curves reported in Figure
7.1 are taken from the analysis of Chapter 6 for a purely radiating
system and from [82] for a purely conducting system. As the simul-
taneous solution of both modes of heat transfer are expected to
produce a mutual weakening of each component, the curves of

Figure 7.1 would tend to over predict the correct results. A

further assumption for the radiative curve of Figure 7.1 is that

no property variations occur which would again tend to over predict.

Also a constant temperature boundary condition was used in the

present analysis, whereas in [82] a constant flux boundary condi-

tion was considered. However, the interesting feature of such

curves is the obvious predominance of conduction for short emulsion

residence times. Radiation plays an increasingly significant role

as the time history of the emulsion packet proceeds. This fact was

pointed out by Lick[77] in an asymptotic approach for short and

long time solutions of the combined modes of heat transfer in a

plane absorbing slab. Also in evidence is the dependance of

radiative particle Nusselt number (h D_p/K_{be}) on particle size and

bed temperature. These results imply that the radiative heat

transfer coefficient is independant of particle size although an

indirect dependance appears in the non-dimensional time through

the extinction coefficient K_t. Such a result contrasts with the

pure conduction results where the heat transfer coefficient is

strongly dependant on particle size and reported elsewhere[83],[84]

and [85]. Hence with these different regimes of dependance a

general representation of conductive and radiative Nusselt numbers

is impractical and only solutions for specific conditions may be

reported.

In the absence of sufficient experimental data on radiation

and conduction heat transfer pertaining to the case of a fluidized

emulsion, a programme of experimentation was designed in which the

bed was viewed through the wall of a quartz glass reactor with a

radiation pyrometer. However, before such experiments are des-
cribed a mathematical model of the system is first evolved and
solutions compared with some existing data.

7.2 A Generalized Approach to Unsteady Combined
Radiation and Conduction Heat Transfer

7.2.1 Introduction

Simultaneous conductive and radiative transfer under
transient conditions, although a complex phenomenon, has become
a subject of considerable technical interest in recent years.
Initial solutions were subject to simplifications and approximations
to the non-linear, partial, integro-differential energy equation.
The two flux approximation was used in [66] and a linearisation,
combined with an exponential Kernel approximation to the radiative
flux equation was used in [77] for short and long asymptotic time
solutions. Heinisch and Viskanta[78] used a similarity solution
utilizing the optically thick approximation for radiative flux and
Hazzah and Beck[79] used a rigorous differential method to predict
energy transfer for the limiting cases of an optically thin and
thick medium. Recently the studies in [30] and Doornink and
Hering[80] utilised the exact formulation of the radiative
transfer equation. [80] assumed a non-scattering medium contained
between black parallel boundaries and utilized the function ap-
proximation technique essentially used in this study and described
in Appendix D. Scattering effects were included in the analysis
described in [30] where the study of a similar plane layer problem
also allowed for reflecting boundaries using for solution the

method of idempotents discussed by [29]. In all the previous

analyses a gray medium, homogeneous in nature and contained between

plane parallel boundaries was assumed. In this study a model of

similar geometry is assumed as being most relevant and of a simple

configuration for the study of heat transfer to the walls of a

fluidized bed reactor or to a probe immersed therein.

7.2.2 Analysis

The simultaneous transport of energy from a stationary

emulsion may be described in terms of the geometry of Figure B1

and Figure 1.1. The assumptions according to the emulsion are

again as described in the steady state analysis of Chapter 2. The

radiative flux equation is described in Appendix B, and for a

fluidized packet of emulsion in accordance with the analysis of

[82] for a variable property boundary layer, may be written in

the energy equation as

$$\frac{\partial}{\partial x}\left[K(x)\frac{\partial T}{\partial x}\right] - \frac{\partial q_r}{\partial x} = \rho(x)C_p(x)\frac{\partial T}{\partial t} \qquad (7.1)$$

The physical model described by equation (7.1) requires that the

radiative flux term q_r be written also in variable property form.

Such a term would result in unnecessary complications in solutions

at this stage and would not be consistent with the assumptions

initially written into the mathematical model. To reduce the com-

plexity of equation (7.1) in keeping with the non-variable property

radiative flux term, a mathematical expedience described by Wicke

and Fetting[81] may be utilised. The proposed model of [81] suggests

that heat is transferred through a gas film separating the emulsion
packet from the bed wall or immersed surface. The packet originates
from deep within the bed, resides at the wall for a specific time
and is then replaced by a fresh packet. In this analysis with such
a gas film attached to the heat transfer surface assuming mean
property values, equation (7.1) may be simplified to

$$\frac{\partial^2 \theta}{\partial \tau^2} - \frac{1}{4N_1} \frac{\partial q_r}{\partial \tau} = \frac{\partial \theta}{\partial t'} \qquad (7.2)$$

where in the two regimes value N_1 assumes a mean constant value,
i.e.

for the gas film:

$$N_1 = N_{1g} = \frac{K_{tg} K_g}{4 \sigma T_b^3} \qquad (7.3)$$

for the emulsion:

$$N_1 = N_{1e} = \frac{K_{tb} K_b}{4 \sigma T_b^3} \qquad (7.4)$$

The term N_1 is referred to in the literature as the conduction-
radiation parameter with $N_1 \to \infty$ for pure conduction and $N_1 \to 0$ for
pure radiation.

As described in [9] the parameter N_1 does not directly give
the relative values of conduction to emission as the ratio of these
values depends also upon both the temperature difference and tem-
perature level in addition to the value of N_1. Described in [80]
is the relative importance of the term N_1/τ_o^2. Radiative transfer

does not significantly effect the temperature distribution for $N_1/\tau_o^2 > 5.0$ and conduction predominates. As N_1/τ_o^2 or N_1 decreases, greater interaction of radiative and conductive transfer occurs. For $\tau_o > 1$, radiative transfer effects are dominant when $N_1 < 0.005$. Also pointed out in the work of [30] was the warning of interpreting t' as a measure of absolute time in comparing results for differing values of N_1, e.g. for a fixed optical thickness $\tau_o = 1$ increasing N_1 by 10^2 implies increasing thermal conductivity by a factor of 10^2. However, absolute time is inversely proportional to thermal conductivity, hence for $N_1 = 0.005$ and $t' = 0.001$, the corresponding comparison for $N_1 = 0.5$ should be at $t' = 0.1$. In this study the situation arose where non-dimensional time relating to the gas layer and the emulsion packet were not equivalent because of the difference in the corresponding values of N_1 (gas layer) and N_1 (emulsion packet). However, the two non-dimensional times of

$$t'_g = \frac{K_{tg}^2 K_g t}{(\rho C_p)_g} \tag{7.5}$$

and

$$t'_b = \frac{K_{tb}^2 K_b t}{(\rho C_p)_b} \tag{7.6}$$

may be related by equating through the absolute time t. Hence

$$t'_b = \frac{K_{tb}^2}{(\rho C_p)_b} \frac{(\rho C_p)_g t'_g}{K_{tg}^2 K_g} \tag{7.7}$$

In the analysis a weighted mean value of $(\rho C_p)_g$ was taken as

$$(\rho \, C_p)_{ge} = \frac{(\rho C_p)_g + (\rho C_p)_b}{2} \tag{7.8}$$

and

$$(\rho \, C_p)_b = (\rho \, C_p)_p \, (1 - \varepsilon_b) \tag{7.9}$$

In finite difference form equation (7.7) becomes

$$\Delta t'_b = \frac{K_{tb}^2 \, K_b \, (\rho \, C_p)_g}{(\rho \, C_p)_b \, K_{tg}^2 \, K_g} \, \Delta t'_g \tag{7.10}$$

It was convenient for computation of equation (7.2) to impose a value $\Delta t'_b$ in the finite difference approximation and hence the corresponding value of $\Delta t'_g$ was then determined. The value of $\Delta t'_b$ (the time increment) was set using a fixed spacial step size in equation (7.2) and convergence obtained with $\Delta \tau = 0.02$ and $\Delta t'_b = 0.0001$. From [82] the voidage variation of the emulsion packet takes place over one particle diameter. However, as the major property variation occurs over the first $0.5 \, D_p$ the depth of the gas layer in this study is taken equal to $0.5 \, D_p$. With the assumption of an optically thin gas layer which is transparent and offers no resistance to radiation transport from the bulk of the emulsion packet and bed wall, the extinction coefficient in the gas layer, K_{tg} is

$$K_{tg} = \frac{\tau_o}{0.5 \, D_p} \quad \text{with } \tau_o = 0.1 \tag{7.11}$$

The effective conductivity of the gas layer was taken as a mean between K_g and K_b, i.e.

$$K_{ge} = \frac{K_g + K_b}{2} \tag{7.12}$$

The thermal conductivity within the emulsion packet was determined from the analysis of Kunii and Smith[86] who derived a relationship for the effective conductivity of a porous medium in terms of the voidage and the solid and gas conductivities. Hence

$$\frac{K_{be}}{K_g} = \epsilon_b + \frac{\beta(1 - \epsilon_b)}{\emptyset_1 + \gamma\frac{K_g}{K_p}}$$

where constant $\beta = 1$ and $\gamma = \frac{2}{3}$ for $0.476 \leqslant \epsilon_b \leqslant 1.0$. The constant $\emptyset_1$ is reported graphically by [86] as a function of K_p/K_g. The curve of Figure 7.2 of K_{be}/K_g against ϵ_b was derived for the particulate material used in this study.

7.2.3 Initial and Boundary Conditions

In accordance with the simple model of an isothermal packet of emulsion arriving at the plane boundary at zero time, and heat transfer taking place with incremental time, the initial condition may be written

$$\theta(\tau,0) = \begin{Bmatrix} 1 \\ \theta(\tau) \end{Bmatrix}, \quad \tau \geqslant 0 \tag{7.13}$$

with boundary conditions of

$$\theta(0,t') = \theta_1, \quad t' \geqslant 0 \tag{7.14}$$

and

$$\theta(\tau_o, t') = 1.0, \quad t' > 0 \tag{7.15}$$

along with the radiative boundary conditions of

$$F_w = \varepsilon_w \theta_w^4 + (1 - \varepsilon_w)(F_w - q_r(o)) \tag{7.17}$$

and

$$F_2 = \varepsilon_2 \theta_2^4 + (1 - \varepsilon_2)(F_2 + q_r(\tau_o)) \tag{7.18}$$

As the gas layer is defined as an optically thin medium a simplification to equation (7.2) results in a modified energy equation for the optically thin limit of

$$\frac{\partial^2 \theta}{\partial \tau^2}\bigg|_{\tau = \tau_{og}} + \frac{(1 - \omega_o)}{2N_1 g}[F_w + F_e - 2\theta^4(\tau_{og}, t')] = \frac{\partial \theta}{\partial t'} \bigg|_{g}_{\tau' = \tau_{og}}$$

$$\tag{7.19}$$

For non-black boundaries F_w and F_e, the wall flux and emulsion flux respectively may be found from the equations relating incident, emitted and reflected radiative energy, hence

$$F_w = \frac{\varepsilon_w \theta_w^4 + (1 - \varepsilon_w) q_{re}'}{[1 - (1 - \varepsilon_w)(1 - \varepsilon_e)]} \tag{7.20}$$

and

$$F_e = \frac{q_{re}' + (1 - \varepsilon_e)\varepsilon_w \theta_w^4}{[1 - (1 - \varepsilon_e)(1 - \varepsilon_w)]} \tag{7.21}$$

with q_{re}' the non-dimensional radiation from the emulsion packet and ε_e the effective emissivity of the surface of the emulsion packet

designated $\varepsilon_e = q'_{re}$. As q'_{re} changes with time of exposure of the

packet, the value of ε_e is also time dependant. Using the optically

thin limit for the energy equation, (7.19) simplifies the starting

solution at each time interval of the finite difference approxi-

mation of equation (7.2). If equation (7.2) had not been modified

to account for the optically thin gas layer a difficulty arises in

determining with sufficient confidence the gradient $\partial q'_r / \partial \tau$ at

$\tau = \tau_{og}$.

Within the emulsion packet the energy equation (7.2) is

applied across the field at each time interval in explicit finite

difference form. A complete analysis of the finite difference

equations used to solve equation (7.2) and equation (7.19) appear

in Appendix I along with the computer programme written for this

purpose. The time solution of the radiative flux equation was

based upon the method used for the transient, pure radiative model

of Chapter 6 with appropriate modifications to include the conduc-

tive heat transfer mode.

7.2.4 A Qualification for the Use of a Mean Property Gas Boundary Layer

An initial study of the effects upon the predicted results

of varying the effective properties of the gas layer between a gas

value, e.g. $K_{ge} = K_g$ and a mean value, e.g.

$$K_{ge} = \frac{(K_g + K_b)}{2}$$

produced an insignificant change in the radiative flux distribution

with time but reduced the conductive flux level by a factor of almost 3. As the conductive values predicted, using a mean property gas layer, compared favourably with the results of [82], such a boundary layer approximation was used throughout.

7.3 Discussion of Predicted Results

In order to determine appropriate values of K_t for the emulsion packet in the vicinity of a flat surface, the simple analysis reported in Appendix E(b) was used in accordance with the boundary property variations of [82]. Curves of these values are plotted in Figure 7.3 for the particulate material studied. K_{tb} in the emulsion was then taken from the invariant portion of the curves from $D_p = 1$ onwards.

The variation of bed voidage ε_b with $U-U_{mf}$ was derived from the equation

$$\varepsilon_b^{\,3}\alpha + \varepsilon_b - 1 = 0$$

where

$$\alpha = \frac{(1 - \varepsilon_{mf})}{\varepsilon_{mf}^{\,3}} \left[1 - \frac{U - U_{mf}}{U} \right]$$

and

$$\left[\frac{1 - \varepsilon_{mf}}{\varepsilon_{mf}^{\,3}} \right] = \frac{C\phi_s^{\,2}}{150} \quad \text{with } C = 1\,440$$

discussed earlier in Chapter 3 and Chapter 6. Values of K_{tb} varying

with $U-U_{mf}$ were then determined and reported in Figure 7.4 through Figure 7.6.

Throughout the analysis an emulsion optical thickness of $\tau_0 = 10$ is assumed along with a black boundary condition at $\tau = \tau_0$ simulating the flux from deep within the bed, i.e. $F_2 = F_b = 1$. The instantaneous radiative and conductive heat transfer coefficients are defined as

$$h_r(t') = \frac{q_r(0,t')}{T_b - T_w} \tag{7.21}$$

and

$$h_c(t') = \frac{q_c(0,t')}{T_b - T_w}$$

with

$$q_c(0,t') = -K_{ge} \left. \frac{\partial \theta}{\partial \tau} \right|_{\tau = 0}$$

Using a uniform residence time distribution function $\psi(t') = 1$ of emulsion packets at the wall, time average heat trasnfer coefficients are defined as

$$h_{rm}(t'_m) = \frac{1}{t'_m} \int_0^{t'_m} h_r(t')\psi(t')dt' \tag{7.23}$$

and

$$h_{cm}(t'_m) = \frac{1}{t'_m} \int_0^{t'_m} h_c(t')\psi(t')dt' \tag{7.24}$$

In order to derive suitable values of the absolute emulsion packet

mean residence times, t_m, the expression of Gelperin and Einstein reported in [31] was used. With

$$t_m = \frac{H_{mf}^2}{0.711 H_f^2} \frac{(U - U_{mf})}{1.5 \ g} \left[\frac{H_{mf}}{H_f - H_{mf}} \right]^2 \qquad (7.25)$$

Using the relations

$$\frac{1 - \epsilon_b}{\epsilon_b^{\ 3}} = \frac{U_{mf}}{U} \left[\frac{1 - \epsilon_{mf}}{\epsilon_{mf}^{\ 3}} \right] \qquad (7.26)$$

$$H_f = H_{mf} \left[\frac{1 - \epsilon_{mf}}{1 - \epsilon_b} \right] \qquad (7.27)$$

equation (7.25) may be transformed to

$$t_m = \frac{U - U_{mf}}{10462.365} \left[\frac{1 - \epsilon_b}{1 - \epsilon_{mf}} \right]^2 \qquad (7.28)$$

with $(U - U_{mf})$ in mm/s.

An assumption in the analysis is that heat is transferred from the inner surface of the gas bubbles attached to the surface by radiation only. Hence to find the view factor between an assumed hemispherical bubble and the wall consider radiation between a gray heat source (inner bubble surface) and a gray heat sink (wall) completely enclosed in a radiating enclosure (surrounding emulsion). Then

$$\frac{1}{A_b \bar{F}_{bw}} = \frac{1}{A_b F_{bw}} + \frac{1}{A_b} \left[\frac{1}{\epsilon_e} - 1 \right] + \frac{1}{A_w} \left[\frac{1}{\epsilon_w} - 1 \right] \qquad (7.29)$$

with

$$A_b = \frac{\pi D_b^2}{2}, \qquad A_w = \frac{\pi D_b^2}{4}$$

and $F_{wb} = 1.0$, then $F_{bw} = \frac{1}{2}$. Then

$$\frac{1}{\bar{F}_{bw}} = \frac{1}{\varepsilon_e} + \frac{2}{\varepsilon_w} - 1 \qquad (7.30)$$

Hence

$$h_{re}(t_m') = h_{rm}(t_m')[1 - f_o] + h_{rm}(t_m') \frac{A_b}{A_w} \bar{F}_{bw} fo \qquad (7.31)$$

and consequently

$$h_{ce}(t_m') = h_{cm}(t_m')[1 - f_o] \qquad (7.32)$$

with

$$h_t(t_m') = h_r(t_m') + h_c(t_m') \qquad (7.33)$$

where

$$f_o = 1 - \left[\frac{1 - \varepsilon_b}{1 - \varepsilon_{mf}}\right] \qquad (7.34)$$

7.3.1 Comparison of Predicted with Reported Heat Transfer Data to an Immersed Probe

Unfortunately, very little experimental data exists in the literature which may be suitably compared with the present analysis. The work of [83], however, does provide for an initial and suitable

comparison. Using the reported physical properties for fireclay given in [83] of probe surface emissivity $\varepsilon_w = 0.8$, particle emissivity $\varepsilon_p = 0.75$ and for two particle sizes namely 0.35 mm and 0.63 mm, theoretically predicted curves of radiative and total heat transfer coefficient are reported in Figure 7.7(a,b) and compared with the experimental data from [83]. A list of appropriate values used in the analysis are given below in Table 7.1.

TABLE 7.1

Fireclay $\begin{pmatrix} \varepsilon_p = 0.75 & T_b = 1\ 123\ K \\ \omega_o = 0.4 & \varepsilon_w = 0.8 \end{pmatrix}$

			Units
D_p	0.35	0.63	mm
ε_b	0.72	0.55	-
K_{be}	0.0002	0.00027	kW/m K
ρ_p	2600	2600	kg/m^3
C_p	1.1724	1.1724	kJ/kg K
K_{tg}	114.3	95.7	m^{-1}
K_{tb}	1500	1400	m^{-1}
N_{1g}	0.0504	0.1747	-
N_{1b}	0.934	1.177	-
f_o	0.589	0.34	-

The probes used in [83] were spherical, one of a high and one of a low emissivity. The analysis presented here considers transfer between plane surfaces and may account for a certain level of error. The predicted radiative heat transfer coefficients of Figure 7.7a compare favourably with the data from [83], particularly for a particle size of 0.35 mm diameter. A further improvement between the results is observed if the contribution to radiative transfer from the inner surface of the bubble is neglected. However, such neglection is unreasonable and would lead to an even poorer comparison for the total heat transfer coefficient of Figure 7.7b than is already observed. The trend observed experimentally for total heat transfer to increase with reduced particle size is supported by the predicted results due to increased contribution from conduction with smaller particles.

With the varying opinions voiced in the literature on the significance of radiative transfer in fluidized beds it is interesting to note that in the predicted results of this study at a bed temperature of 1 123 K the percentage of radiative to total (conduction plus radiation) heat transfer coefficient varied from 12.7% at a probe wall temperature of 200°C to 28.3% at a wall temperature of 800°C. The experimental data from [83] gave 4.5% and 20% respectively. Hence, radiation appears to be a significant contributor to the total heat transfer even with a comparatively low temperature bed of 1 123 K (850°C).

It was reported in [3] and [7] that cooling within the emulsion extended up to 1.5 D_p after a suitable residence time. From the results of this study cooling was observed to extend up

to $\sim 4~D_p$ for 0.35 mm particles at a probe wall temperature of 200°C after a residence time of 0.138 seconds.

7.3.2 Comparison of the Instantaneous Nusselt-Fourier Relationship for a Coupled and Uncoupled Solution of Conduction and Radiation Heat Transfer to a Plane Surface

Reported in Figure 7.1 are the instantaneous conductive and radiative Nusselt number ($h~D_p/K_{be}$) variation with instantaneous Fourier number

$$\frac{K_{be}~t}{\rho_p C_p (1 - \epsilon_b) D_p^2}$$

The non-dimensional time, referring to the pure radiation and combined conduction and radiation cases, were transformed for convenience into equivalent Fourier numbers. With

$$F_o = \frac{K_{be}~t'}{D_p^2 K_{tb}~\sigma T_b^3}$$

for pure radiation and

$$F_o = \frac{t'}{K_{tb}^2 D_p^2}$$

for the combined mode. As previously stated these curves were obtained by uncoupling the conductive and radiative components of the energy equation and solving separately. Figure 7.8 in contrast to Figure 7.1 describes the results obtained from a simultaneous solution of the coupled radiation and conduction energy equation and compared with the curves taken from Figure 7.1. Unfortunately,

computational time became excessive and only coupled solutions to
a value of F_o = 1.0 were predicted. The interesting feature of
these curves is the small but noticeable reduction in the radiative
Nusselt numbers for the combined solution, emphasizing the mutual
weakening of each component when solved simultaneously. These
observations are further qualified by the temperature profiles of
Figure 7.12 which show considerably more cooling of the emulsion
layer adjacent to the boundary when conduction is present compared
with the case when conduction is neglected. Also in evidence are
the depths of penetration of the cooling curves, in both cases
exceeding the value of 1.5 D_p proposed by [3] and [7].

Referring to Figure 7.8 the conductive Nusselt number
appears to underestimate at early times the curve interpolated
from the predictions of [82] and overestimate for longer times.
However, such a comparison is not strictly valid as the analysis
of [82] considered a constant flux boundary condition whereas this
study considers a constant temperature boundary condition. Also
the curves obtained from [82] were interpolated for the materials
used in this study. Similar curves to Figure 7.8 may be drawn
for other materials and different boundary conditions. A few
such curves are described in Figure 7.9 and Figure 7.10 for sand
particles and varying boundary conditions. As expected, increasing
the wall temperature and decreasing the wall emissivity signifi-
cantly change the value of radiative Nusselt number although no
change in the conductive Nusselt number was observed. Figure 7.11
for silicon carbide particles shows a marked fall in both conduc-
tive and radiative Nusselt numbers compared with sand particles.

This may be attributed to the fact that silicon carbide has a
higher thermal conductivity than sand and becomes the influencing
term in the Nusselt number (h D_p/K_{be}) even though the heat transfer
coefficients (both radiative and conductive) have been signifi-
cantly increased.

Such curves of instantaneous Nusselt-Fourier number
variations are at this stage used only to predict and understand
the important trends of combined conduction and radiative heat
transfer to a plane boundary. In the predicted case of heat
transfer to a surface immersed in a fluidized bed, the emulsion
residence time, residence time distribution and bubble to wall
heat transfer must be considered. Hence for other materials and
boundary (wall) conditions the predicted results may be summarised
as follows:

(a) For bed materials of high particle emissivity, i.e. $\omega_o \to o$,
the contribution of radiation to the total heat transfer
is significant in fluidized beds operating at normal
temperatures, i.e. 800°C - 1 100°C when the irradiated
surface has an emissivity approaching unit. As $\omega_o \to 1$
the emitted flux levels are reduced accordingly.

(b) For low wall emissivities the contribution of radiation
decreases significantly, i.e. by an order of 50% for a
wall emissivity of 0.5.

(c) The radiative heat transfer coefficient is independent of
particle size although an increase of particle size is
reflected in an increase in the radiative Nusselt number
through the term (D_p/K_{be}), multiplying the heat transfer

coefficient h. The converse is true for conduction with
the heat transfer coefficient dependant and Nusselt number
independant of particle size.

(d) Conduction is independant of temperature level and only
dependant on temperature gradient. On the other hand,
radiation depends solely on the relative temperature levels
between the fluidized bed and containing walls or immersed
surface.

(e) Within the normal range of emulsion packet residence times
occurring in fluidized beds, the conductive mode of heat
transfer is predominant over the initial period of contact
with radiation becoming more prominant as time proceeds.
Furthermore, the simultaneous solution of combined con-
duction and radiation provides an insignificant weakening
of the radiative component compared with the solution of
the uncoupled energy equation. This reduces the complexity
of solving the energy equation and simplifies the
predictions of radiative transfer.

7.4 Experimental Determination of Heat Transfer to the
Wall of a Quartz Glass Reactor

To augment the previously reported results of heat transfer
to probes immersed in a fluidized bed where both conduction and
radiation are important, an experimental programme was designed in
which the radiation pyrometer described in Chapter 4 was positioned
in the horizontal plane viewing the bed through the glass wall of
the fluidized bed reactor. A schematic diagram of the experimental

configuration is described in Figure 7.13. The pyrometer exten-
sion tube was covered internally with aluminium foil to reduce
energy exchange with the bed and also directed cooling air to the
glass wall, the flow of which could be varied to maintain a con-
stant glass wall temperature. The temperature of the glass wall
was constantly monitored by two Cr.Al thermocouples embedded into
two small holes bored into the glass wall. These thermocouples
were strategically positioned above and below the pyrometer viewing
area of 0.25 in diameter. To reduce heat losses from the bed the
whole reactor was heavily lagged. Also to reduce radiative
transfer prior to the measuring section through the glass wall
to the insulation, a strip of aluminium foil was placed just below
the pyrometer viewing area. This technique enhanced the probability
of an isothermal packet being brought into the viewing area, an
assumption upon which the analysis is based, rather than one with
an imposed thermal gradient. In all experiments bed heights of
70 mm were used.

7.4.1 Experimental Procedure

Unfortunately it was not possible with the present system to
maintain low wall temperatures, i.e. of order 100°C - 200°C.
Hence, as an attenuation of the fluxes may be expected at high
glass wall temperatures along with a contribution from the glass
itself an initial study of the overall attenuation effects of the
quartz glass was performed. This was achieved by viewing the bed
free surface alternately through a sample of the quartz glass and
then with the glass sample removed. The attenuation was estimated

at a number of glass temperatures and found to reduce the true radiative flux by 14% ±3% over a temperature range of 600°C to 1 000°C.

Beds of silica sand, magnesia, alumina and silicon carbide particles with three different particle sizes of geometric means 0.354 mm, 0.55 mm and 0.777 mm were used. Initially the bed temperature was set at around 1 000°C. With the wall temperature then controlled to a fixed value by the air flow over the outer surface of the glass wall, pyrometer output was recorded for increasing values of $(U-U_{mf})$. Low values of $(U-U_{mf})$ < 100 mm/s were difficult to obtain as the bed tended to become poorly fluidized and combustion of the propane/air mixture became unstable. However, with care and speed reasonably repeatable results were obtained by quickly lowering the superficial air velocity to some fixed level and recording the pyrometer output before combustion became unstable and the bed temperature began to fall. The air velocity was then rapidly increased to establish stable conditions and a steady bed temperature.

7.4.2 Discussion of Results

As in this study a radiatively transparent wall was used, the boundary conditions in the analysis previously described in this chapter had to be modified. For the purpose of conduction heat transfer determination the wall temperature was taken as measured experimentally.

However, the radiative boundary condition simulating transfer to the radiation pyrometer was obtained by taking the temperature

and emissivity as ambient and unity respectively, e.g. F_1 = 0.00238 and with F_2 = 1 at T_b = 1 000°C. The results of these experiments along with the theoretically predicted values are described in Figure 7.14 through Figure 7.16 are typical time average radiative flux curves from which the predicted curves of Figure 7.17 through Figure 7.20 were obtained for a range of $(U-U_{mf})$ values.

For the case of silica sand and silicon carbide particles, comparison of experiment with the predicted results are reasonable although for the case of alumina and magnesia a significant deviation may be observed. The analysis predicts a definite dependancy of emitted radiative flux on particle size with q'_{rm} increasing for a decrease in particle size. As pointed out earlier, the opposite is true for conduction heat transfer. However, experimentally such trends were difficult to observe.

The general shape of the predicted curves were obtained experimentally with lower emitted fluxes at high emulsion packet residence times, i.e. low values of $(U-U_{mf})$. The fluxes increase rapidly at first as $(U-U_{mf})$ was steadily increased with a levelling out towards a maximum value. The fraction of bubbles present within the bed from the previously described analysis of f_o predicts a steady increase as $(U-U_{mf})$ and bed voidage increases. If this is the case then at increasingly shorter packet residence times the emitted flux should tend to decrease due to the predominance of radiation from the inner bubble surface over that from the emulsion. However, in reality this is not the case as shown in [36] the bubble fraction f_o attains a maximum at an

optimum value of $(U-U_{mf})$ after which it decreases. Hence, with this mechanism the emitted radiative fluxes should continue to increase as the proportion emitted by the bubble gives way to that emitted from the emulsion packet. Such effects were not studied experimentally and are only analytically predicted observations. At very high values of $(U-U_{mf})$ slugging of the bed would tend to occur along with a reduced bubble fraction and a different mechanism of particle motion takes place. In this case a reduced particle residence time at the wall may again become the dominant feature.

Without the advantage of controlled residence time and bubble fraction experiments, the predicted solutions are limited by the highly idealised equations which are used to determine t_m and f_o. From the pyrometer output trace a peak much greater than the general levels was observed at infrequent periods irrespective of $(U-U_{mf})$ values. It was apparent that such peaks were due to passing bubbles which had a 'hotter glow' than the surrounding emulsion suggesting that gas combustion was taking place at the bubble boundary thereby raising the particles in its near vicinity to levels above the general bed level. For this reason the applicability of transfer to the wall of the equation used to predict f_o becomes questionable. Its use may in fact be reasonable away from the wall or at a surface immersed within the bed, but not necessarily at the bed wall. Such observations are supported by the work of Donsi et al[87] who reported that solids flow tends to be vertically downwards at the bed wall and upwards in the core of the bed, and that a limitation on solids exchange

normal to the wall is related to a continuous curtain of particles
confining the bubble on the wall side. Such a curtain of particles
would shield the effects of direct bubble to wall radiative
transfer.

7.5 Conclusions and Suggestions for Further Study

The general analysis of combined conduction and radiative
heat transfer from an optically dense emulsion has been applied
to the case of a surface immersed within the bed compared with
the data obtained by other workers. The results are reasonable
as the size of probe used by some workers was too large to give
sufficiently small Biot numbers and was spherical with the analysis
developed in this study strictly applicable only to plane surfaces.

A purely analytical approach was undertaken initially to
predict the instantaneous values of conductive and radiative
Nusselt numbers comparing the results obtained from the coupled
energy equation with the uncoupled solutions. Insignificant
mutual weakening of either mode was observed for the coupled
solution.

The work has been extended experimentally to measure the
radiative fluxes in contact with the wall of a glass reactor. In
many ways this is similar to the studies of [6] where a radiometer
probe was immersed into a bed and the fluxes measured from the
particles in contact with quartz protecting glass. The advantage
of this study is the direct allowance given in the analysis to the
conductive transfer between glass and contacting particles,
although a disadvantage was the inability of the system to maintain
a sufficiently low glass wall temperature. In the analysis

of the previous study of [6] this was not carried out.

The experimental results, although far from comparing satis-
factorily with theory in some cases, show similar distinct trends.
Unfortunately, the analysis is still subject to areas of large
uncertainty, particularly the values of residence time of emulsion
packets at the wall and of bubble fraction determination. For
both these unknowns standard expressions were used to give repre-
sentative results. However, further studies are desirable on
single bubbles or bubble chains with the residence time and bubble
fraction controlled, in contrast to the freely bubbling bed studied
here. For this purpose and to facilitate a transient radiative
study, a radiometer with a much shorter response time is required
than the one used in the present study. As particle motion at the
bed wall and at the bed axis tends to be different, i.e. particles
move downward at the wall and upward at the centre of the bed,
then heat transfer results measured experimentally or postulated
analytically may be significantly different for the two regions.
Further effort is required in this direction with a radiometer
probe, utilizing a water-cooled quartz protecting glass placed at
both positions.

The theoretical analysis retains a physically unjustified
approximation; that of a gas film between the first row of
particles and the wall or immersed surface. Such an approximation
is used at this stage as a mathematical expedience. It was shown
in [82] that a physically justifiable variable property boundary
layer could be used to good effect for the case of conduction
heat transfer only. Further work is now necessary to solve the

energy equation including the radiative term utilizing such a

variable property boundary layer.

R E F E R E N C E S

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PHYSICAL SYSTEM OF PACKET MODEL
FOR RADIATIVE TRANSFER TO A
PLANE BOUNDARY

T_1, ε_1

$X = 0$
$\tau = 0$

$\mu = 0$

I

θ

$\mu = -1$ $\mu = 1$ $X = L$
$\tau = \tau_0$

$\mu = 0$

X, τ

$\mu = \cos \theta$
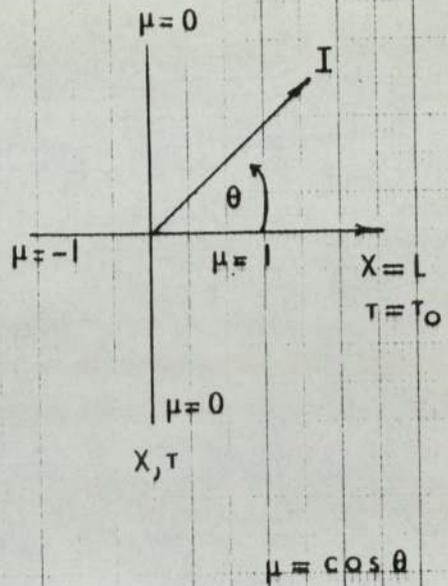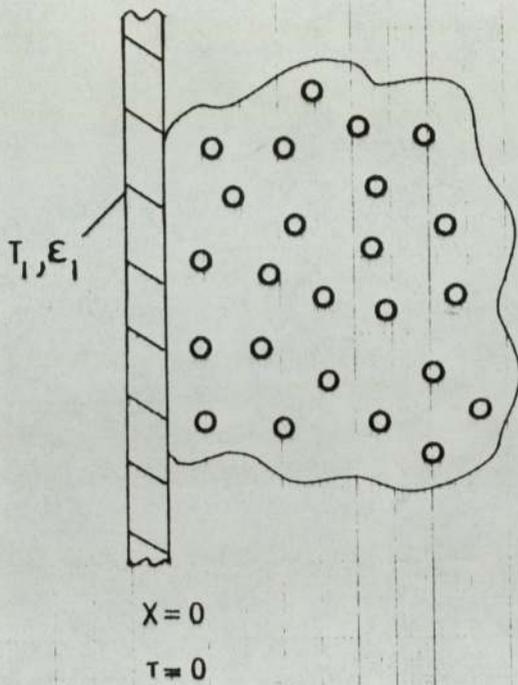
FIG 1.1

COMPARISON OF APPROXIMATE SOLUTIONS
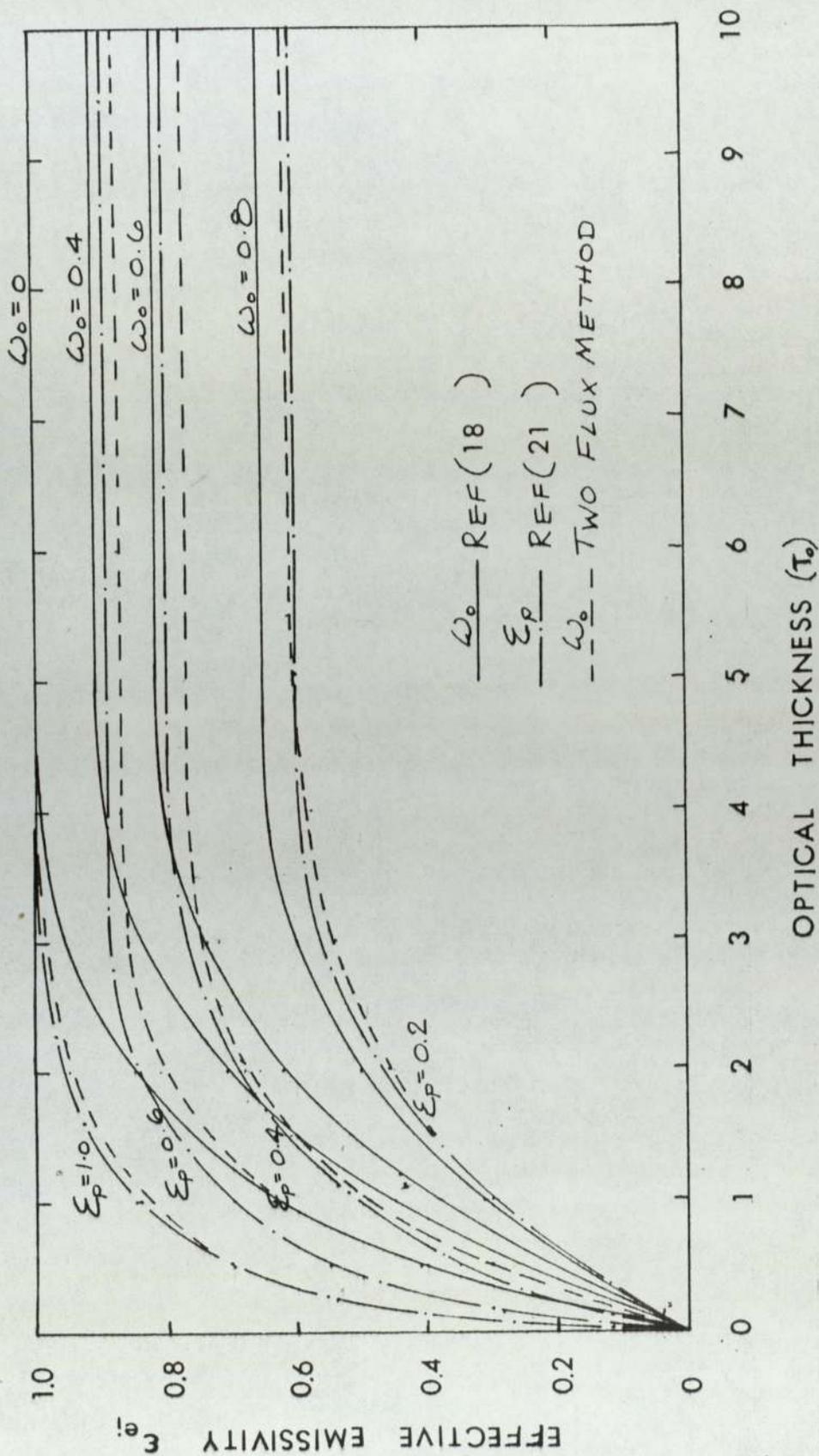
OF RADIATIVE TRANSFER EQUATION

FIG 2.1

COMPARISON OF APPROXIMATE SOLUTIONS
OF RADIATIVE TRANSFER EQUATION
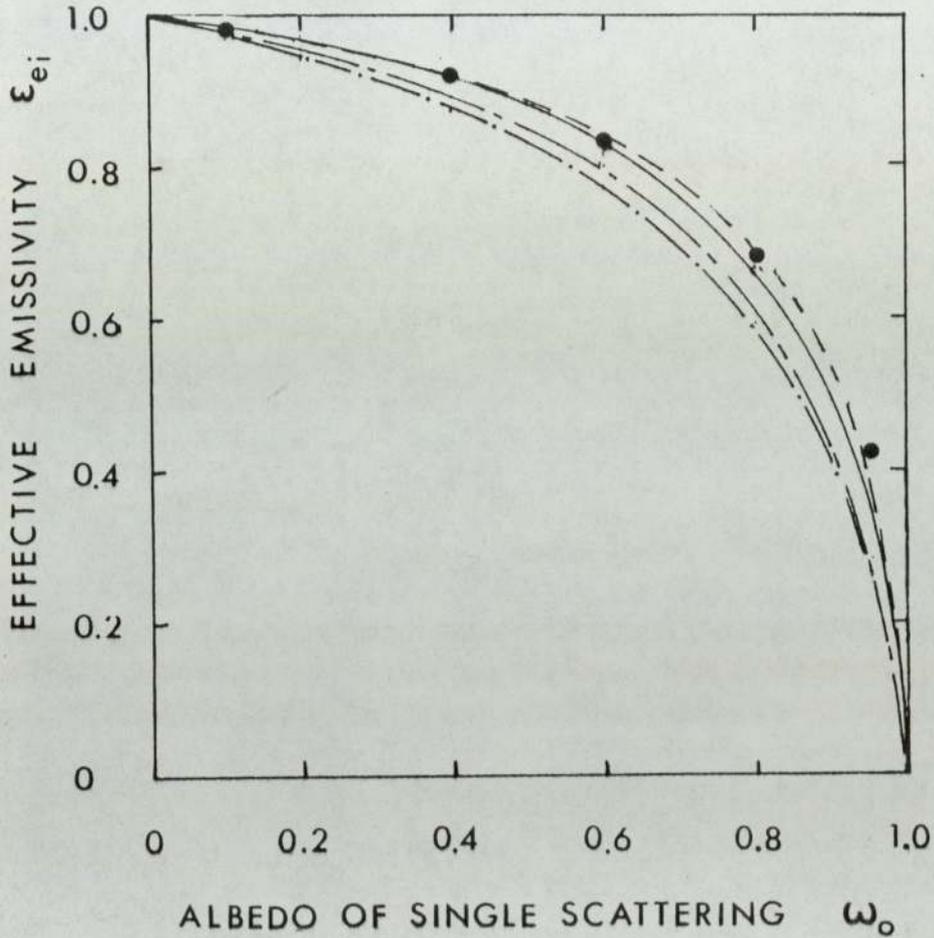
FIG. 2.2

EFFECT OF ALBEDO ON PARTICLE AND EFFECTIVE EMISSIVITY
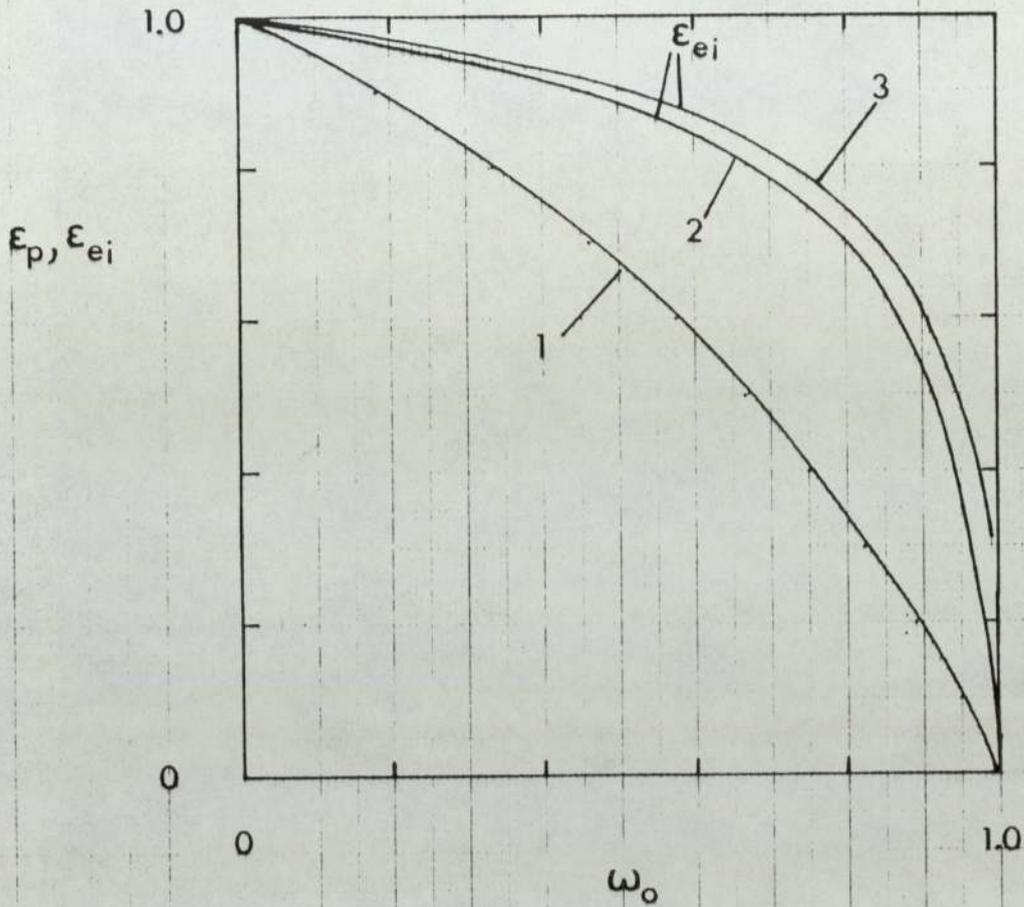
1, $\omega_o = \dfrac{1}{1 + \dfrac{\varepsilon_P}{2(1-\varepsilon_P)}}$

2, PRESENT WORK

3, DIFFUSE CONICAL GROOVE [26]

FIG 2.3

EXPERIMENTAL CONFIGURATION FOR U_{mf} DETERMINATION

80 mm dia

Cr-Al THERMOCOUPLE

EXHAUST

TO MANOMETER

THERMAL INSULATION

FLUIDISED BED

300 mm

TO MANOMETER

51mm THICK DISTRIBUTOR PLATE

TO POWER SUPPLY

AIR SUPPLY

2 KW HEATER ELEMENT
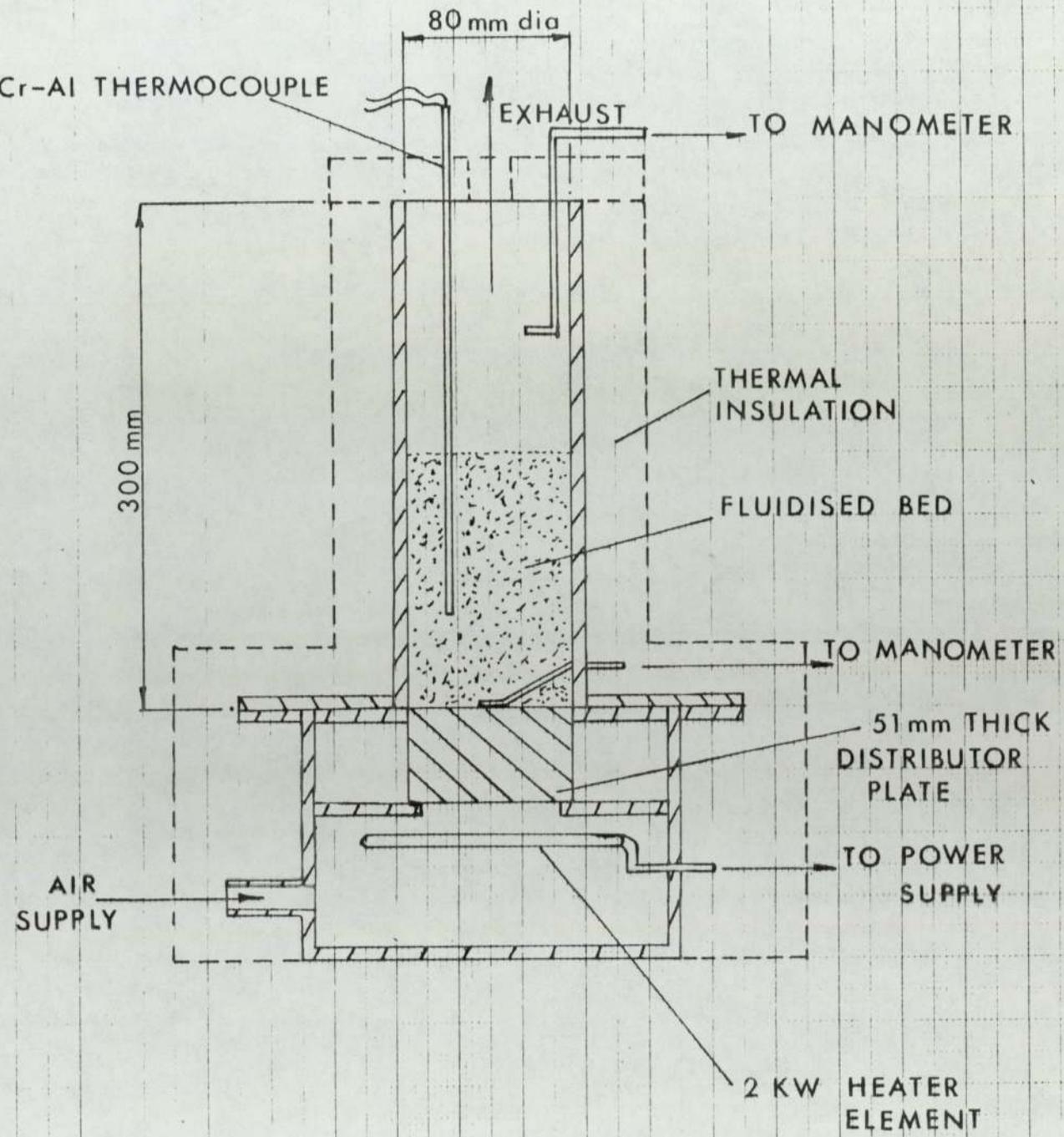
FIG 3.1

EFFECT OF GAS VELOCITY ON BED PRESSURE DROP

SILICA SAND

$D_p = 0.777$ mm

T_b °C

① 20

② 448

U_{mf}

SUPERFICIAL GAS VELOCITY U mm/sec

DIMENSIONLESS PRESSURE DROP $\dfrac{\Delta P_A}{W_b}$
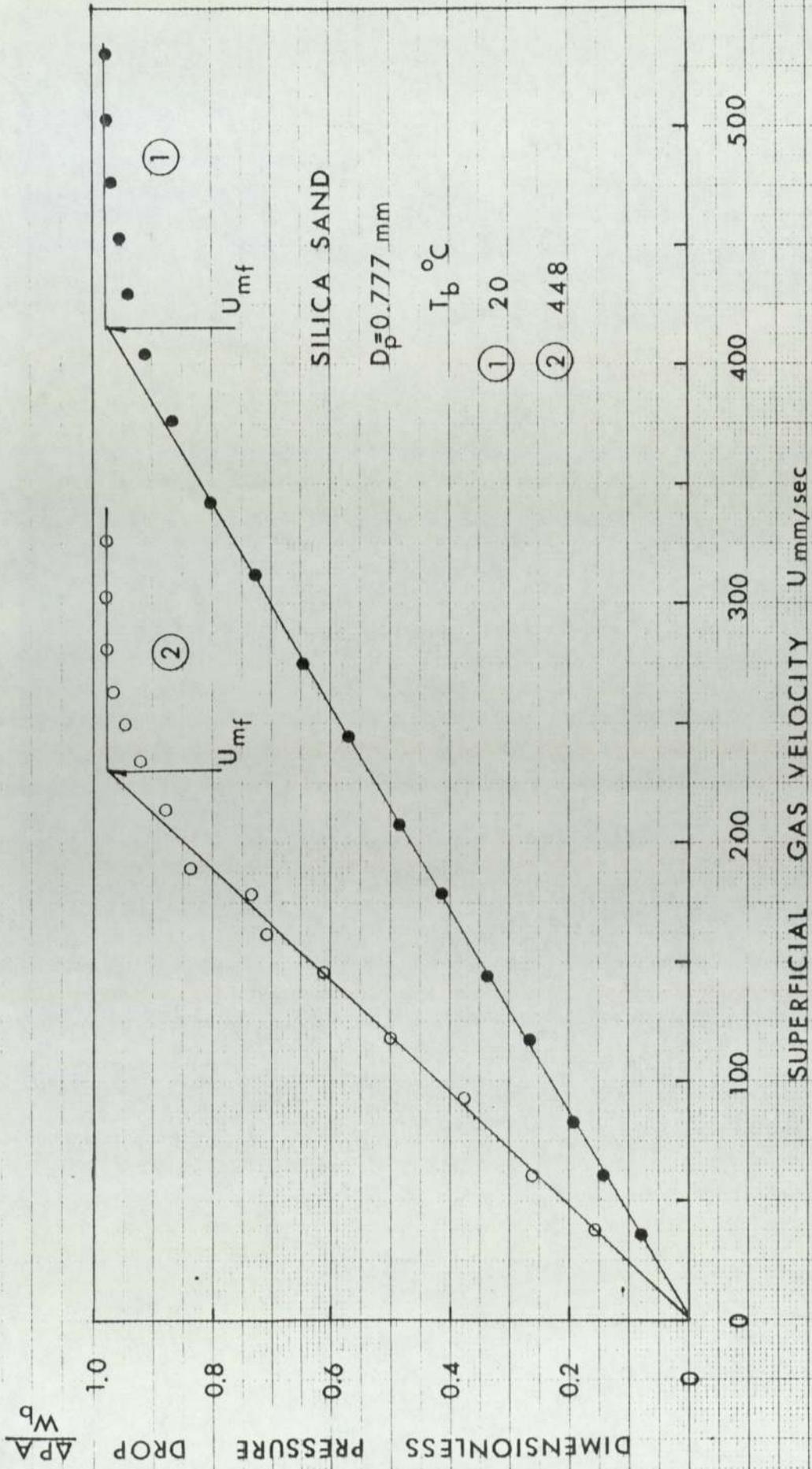
FIG 3.2

TEMPERATURE EFFECT ON MINIMUM FLUIDISING VELOCITY

D_p mm 0.354

MAT^L
• SILICA SAND
○ SILICON CARBIDE
■ ALUMINA
□ MAGNESIA
- - - EQN 3.1
—— EQN 3.5

BED TEMPERATURE °C

MINIMUM FLUIDISING VELOCITY mm/sec

300
200
100
0

100   200   300   400   500   600   700
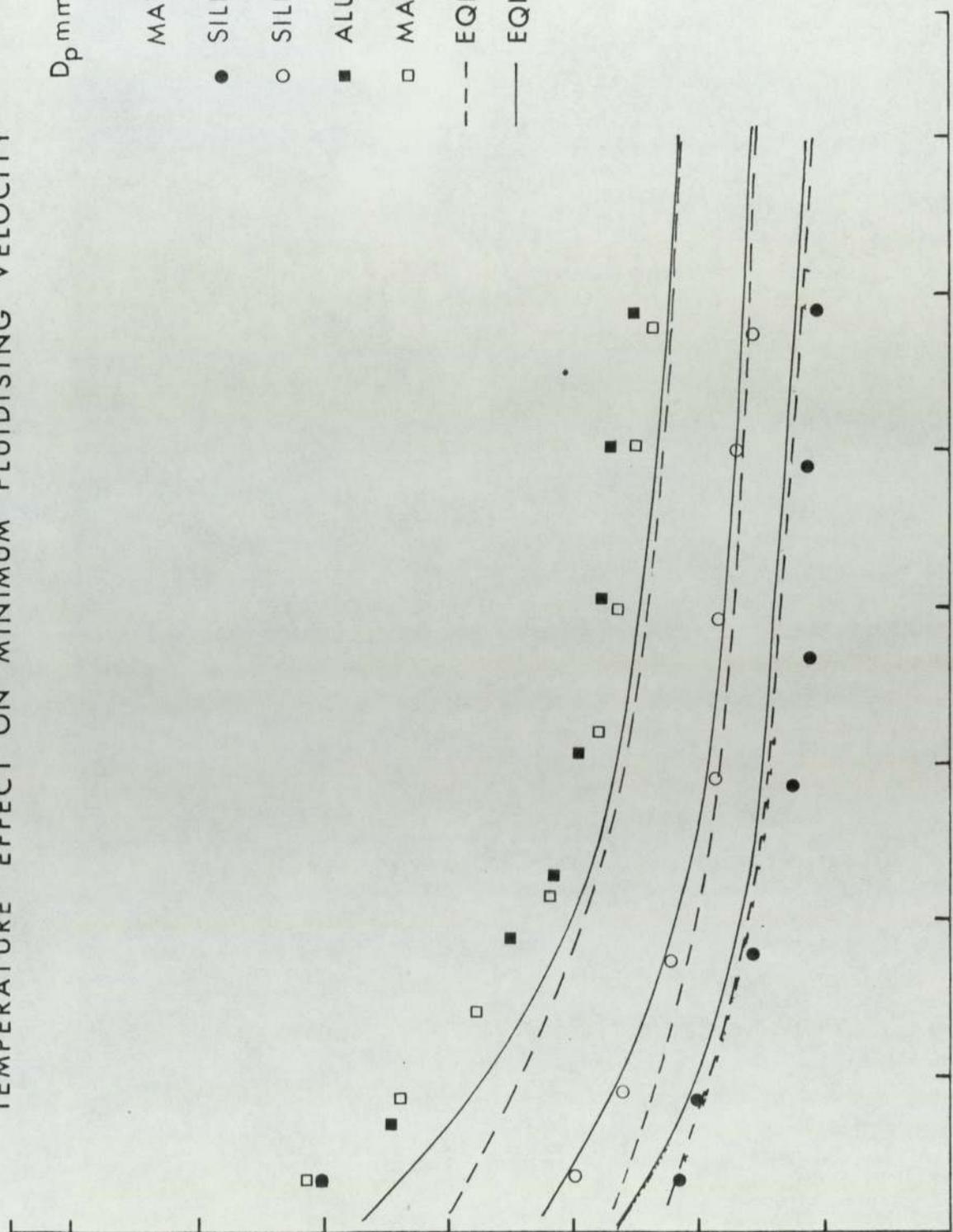
FIG 3.3a

TEMPERATURE EFFECT ON MINIMUM FLUIDISING VELOCITY

D_p mm 0.55

MAT^L

● SILICA SAND
○ SILICON CARBIDE
■ ALUMINA
□ MAGNESIA
--- EQN 3.1
— EQN 3.5

FIG 3.3b

BED TEMPERATURE °C

MINIMUM FLUIDISING VELOCITY mm/sec

TEMPERATURE EFFECT ON MINIMUM FLUIDISING VELOCITY

D_p mm 0.777

MAT^L

● SILICA SAND
○ SILICON CARBIDE
■ ALUMINA
□ MAGNESIA
--- EQN 3.1
— EQN 3.5

MINIMUM FLUIDISING VELOCITY mm/sec

BED TEMPERATURE °C
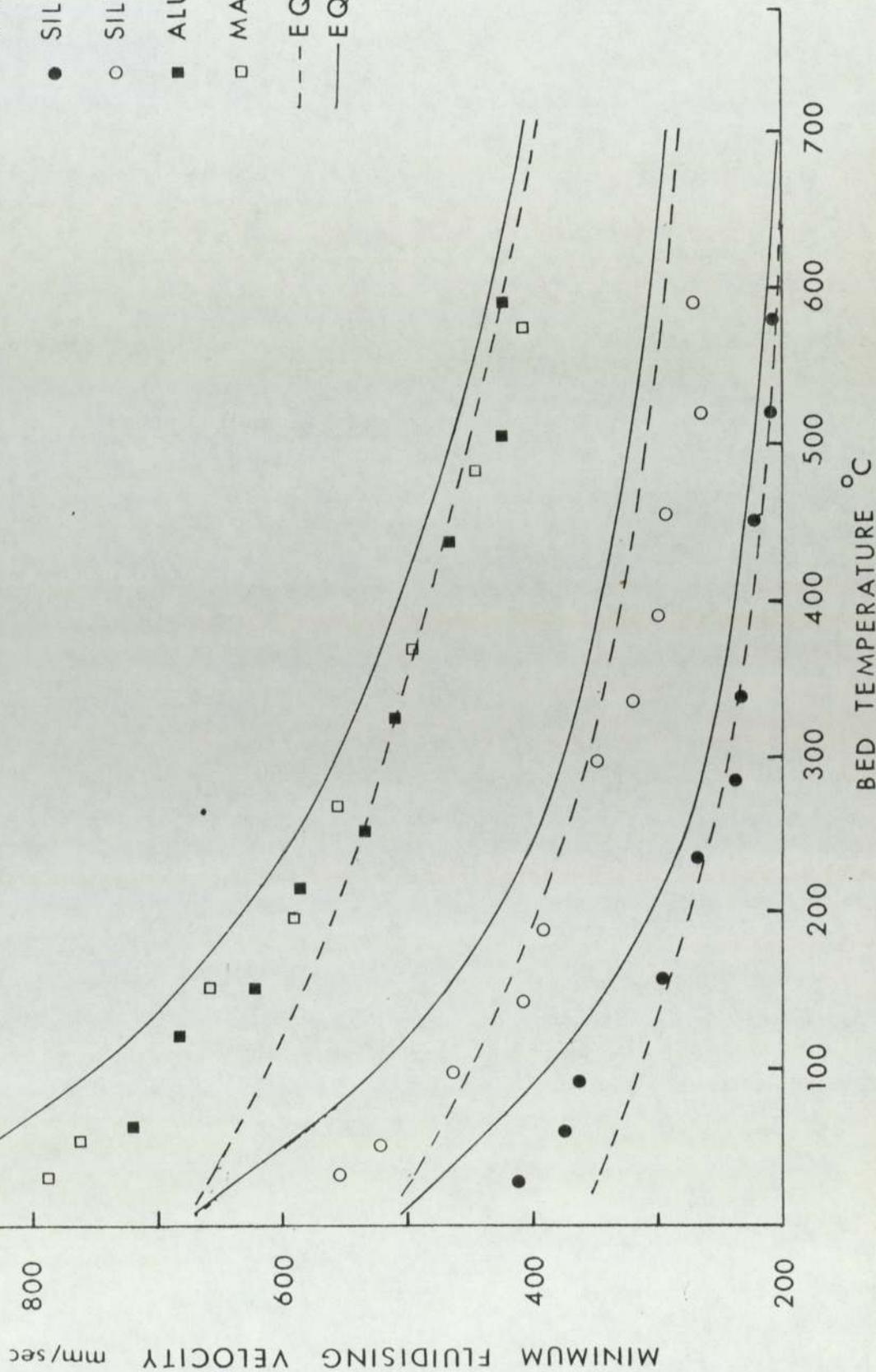
FIG 3.3c

PARTICLE SPHERICITY Vs MINIMUM VOIDAGE FROM [41]
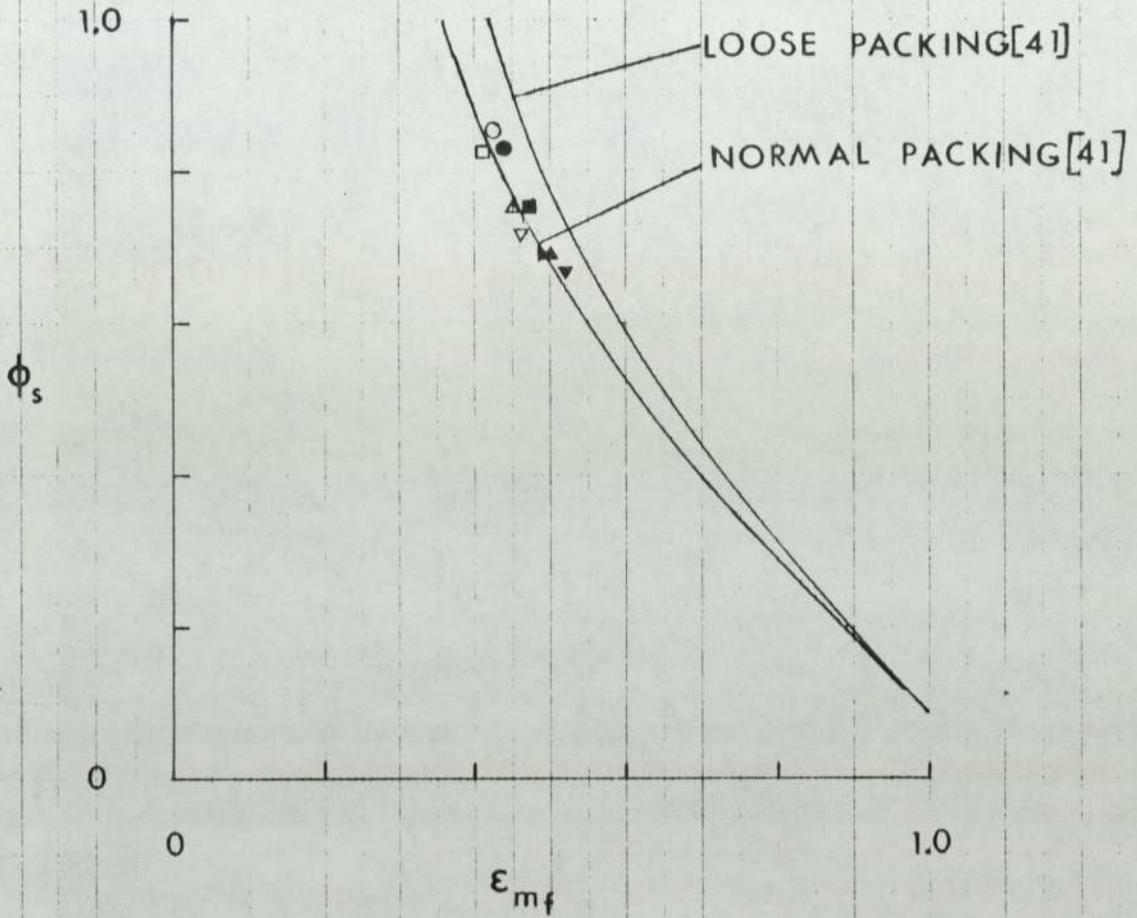
	MAT^L	D_p mm
●	SAND	0.354
○		0.55
□		0.777
▼	ALUMINA	0.354
▲		0.55
▶		0.777
■	SILICON CARBIDE	0.354
▽		0.55
△		0.777

FIG 3.4

CONFIGURATION FOR PARTICLE

EMISSIVITY MEASUREMENT

PYROMETER

WATER IN

WATER OUT

AIR IN

2 ft

EXTENSION
TUBE

TEST SAMPLE

BURNER

PROPANE-AIR
MIXTURE

ALUMINIUM
FOIL

PYROMETER
EXTENSION
TUBE

FIELD OF
VIEW

$\frac{5}{8}$" DIA X $\frac{1}{4}$" DEEP

FIREBRICK
MOULD

THERMOCOUPLE   SILVER
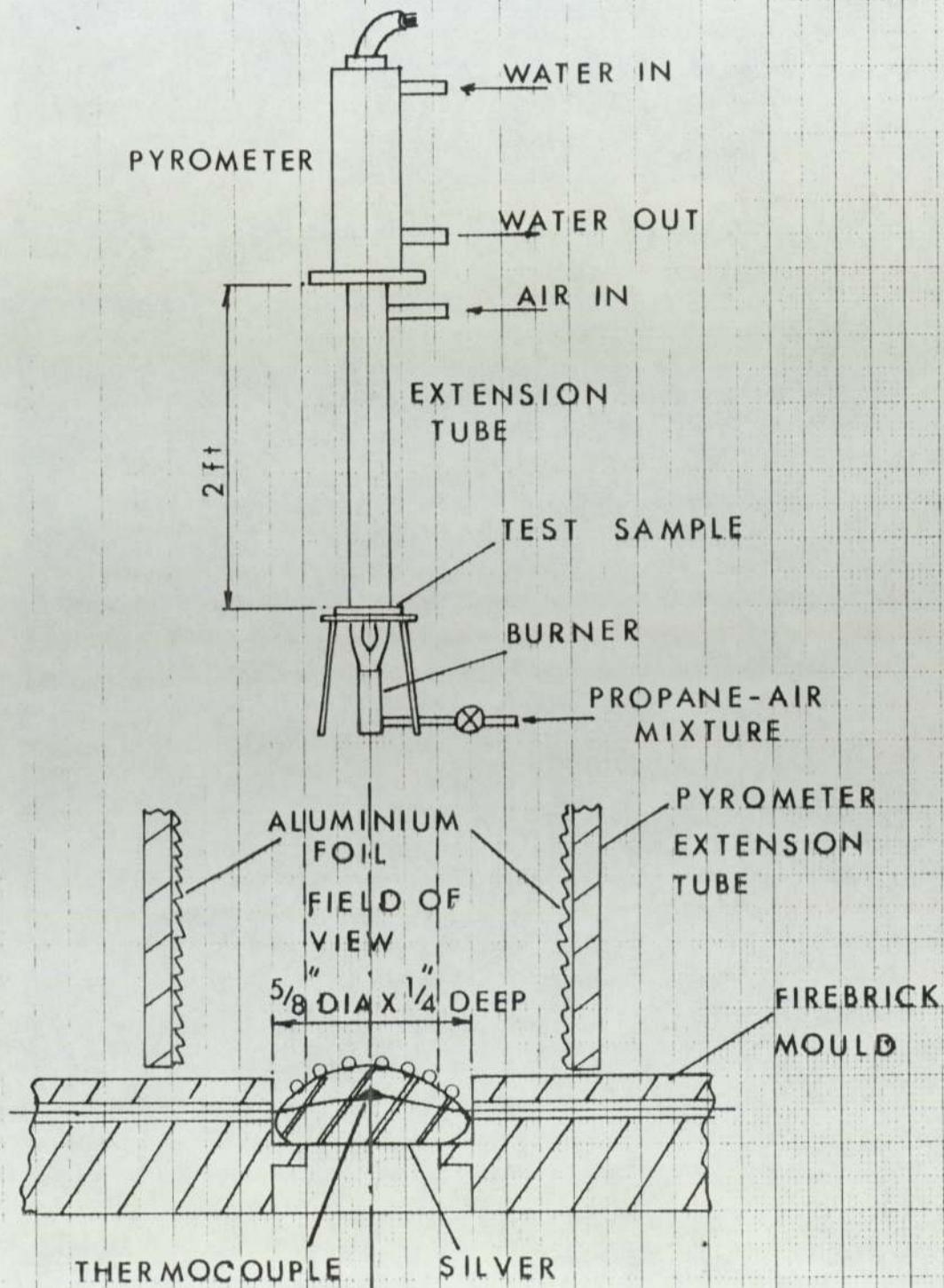
FIG 3.5

DETAIL OF TEST SAMPLE

VARIATION OF SAMPLE EMISSIVITY WITH TEMPERATURE
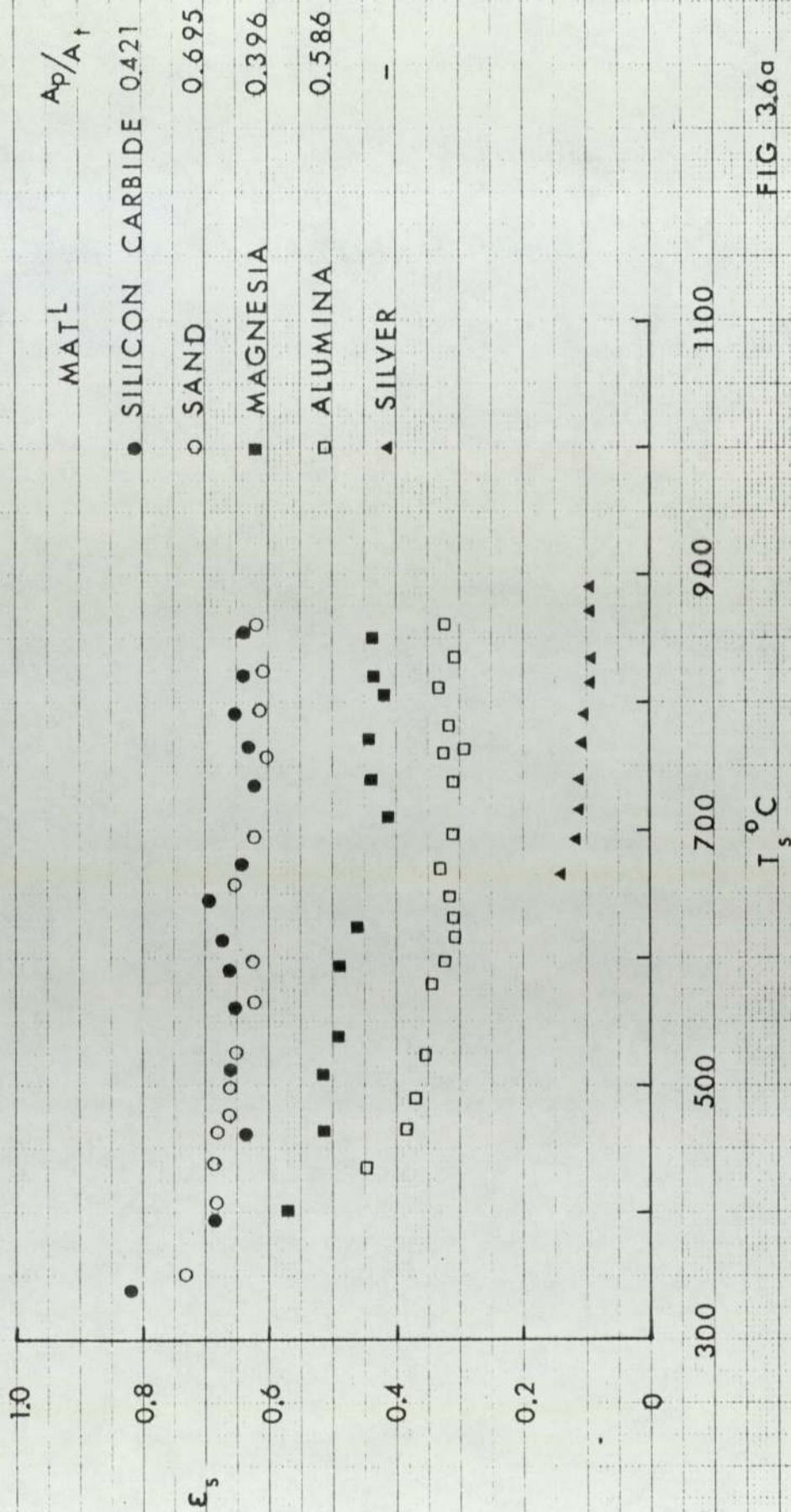
FIG 3.6a

VARIATION OF SAMPLE EMISSIVITY WITH PARTICLE AREA FRACTION
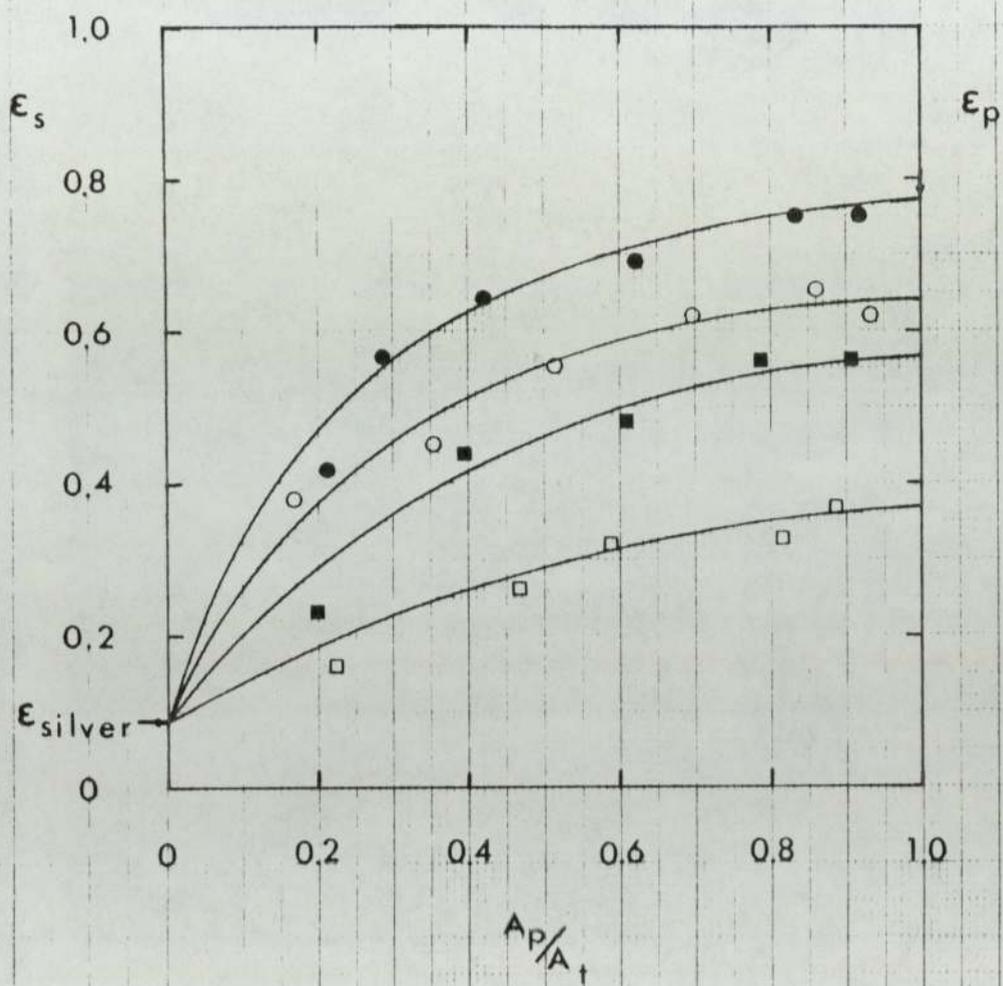
MAT^L	T_s $^\circ$C
o SILICA SAND	860
• SILICON CARBIDE	855
■ MAGNESIA	853
□ ALUMINA	863

FIG 3.6b

CONFIGURATION FOR MEASUREMENT OF BED PRESSURE DROP DISTRIBUTION

FIG 3.7a

DIFFERENTIAL PRESSURE TRANSDUCER

FIG 3.7b
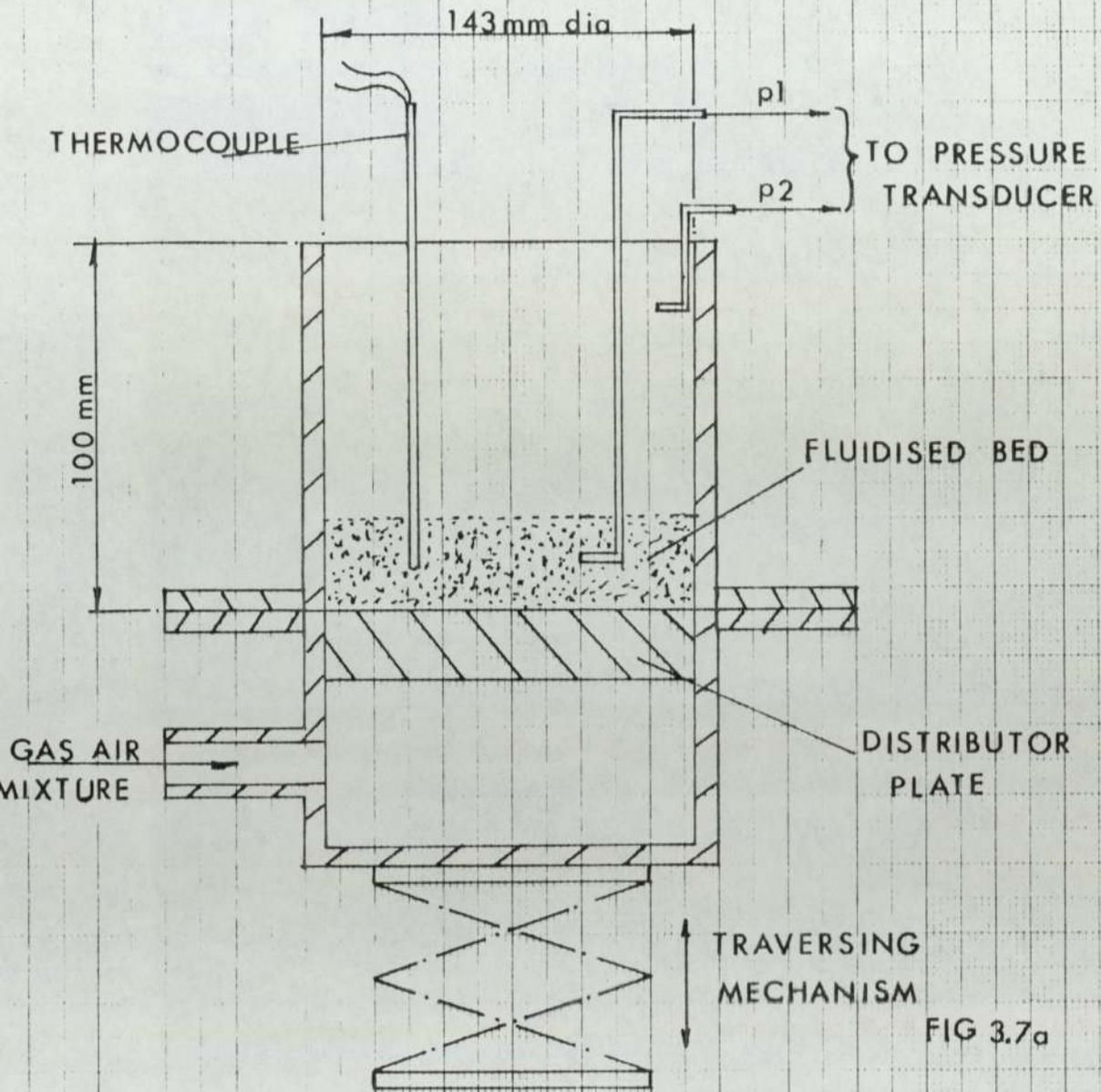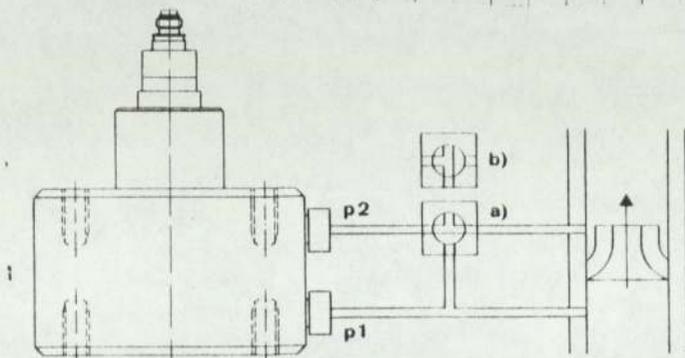
U-V-R TRACE OF TRANSDUCER OUTPUT AT
DIFFERENT PROBE POSITIONS IN A FLUIDISED BED
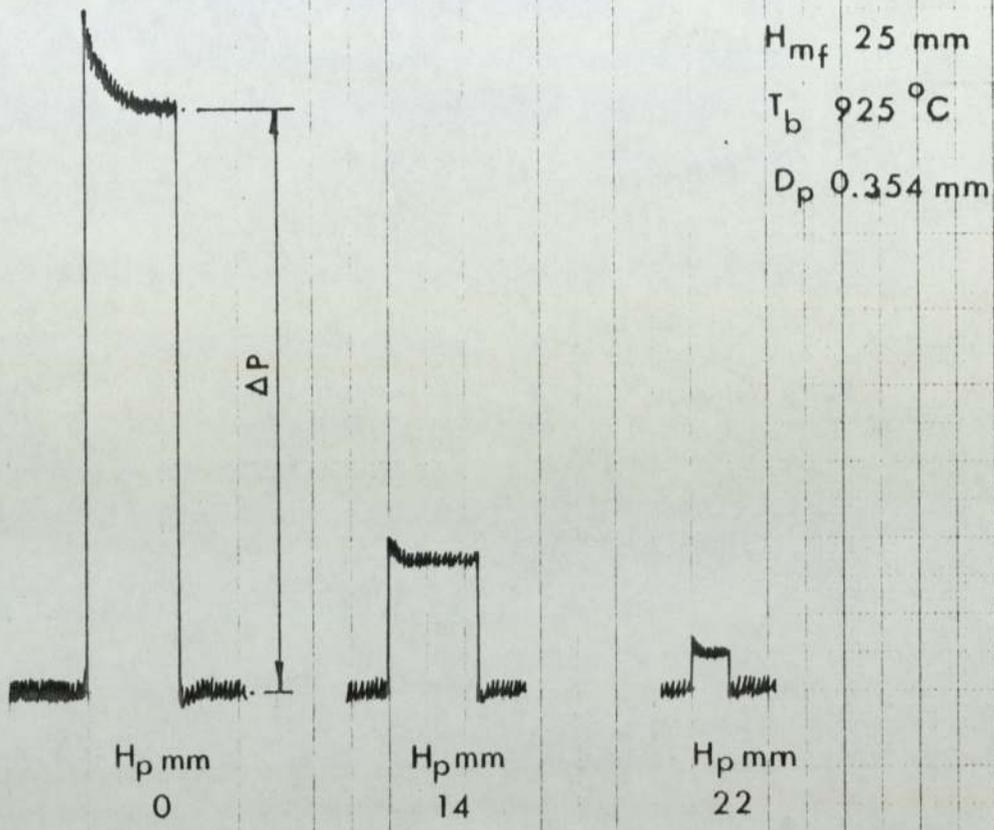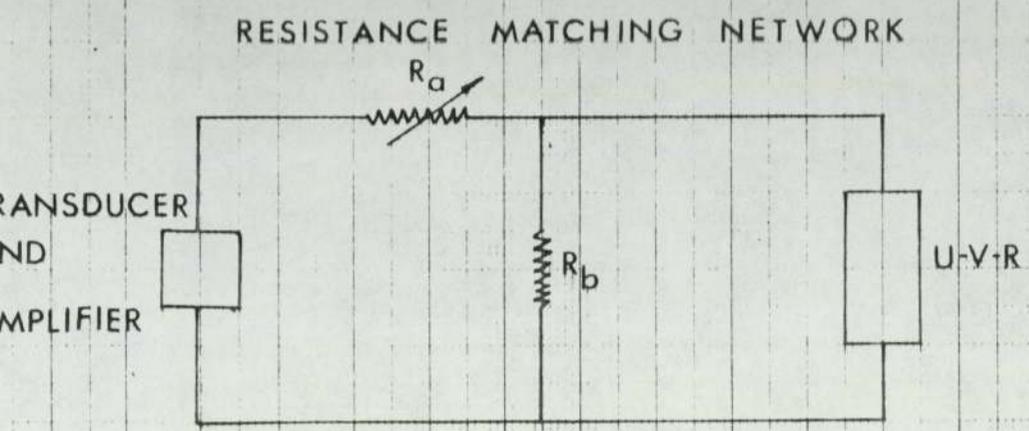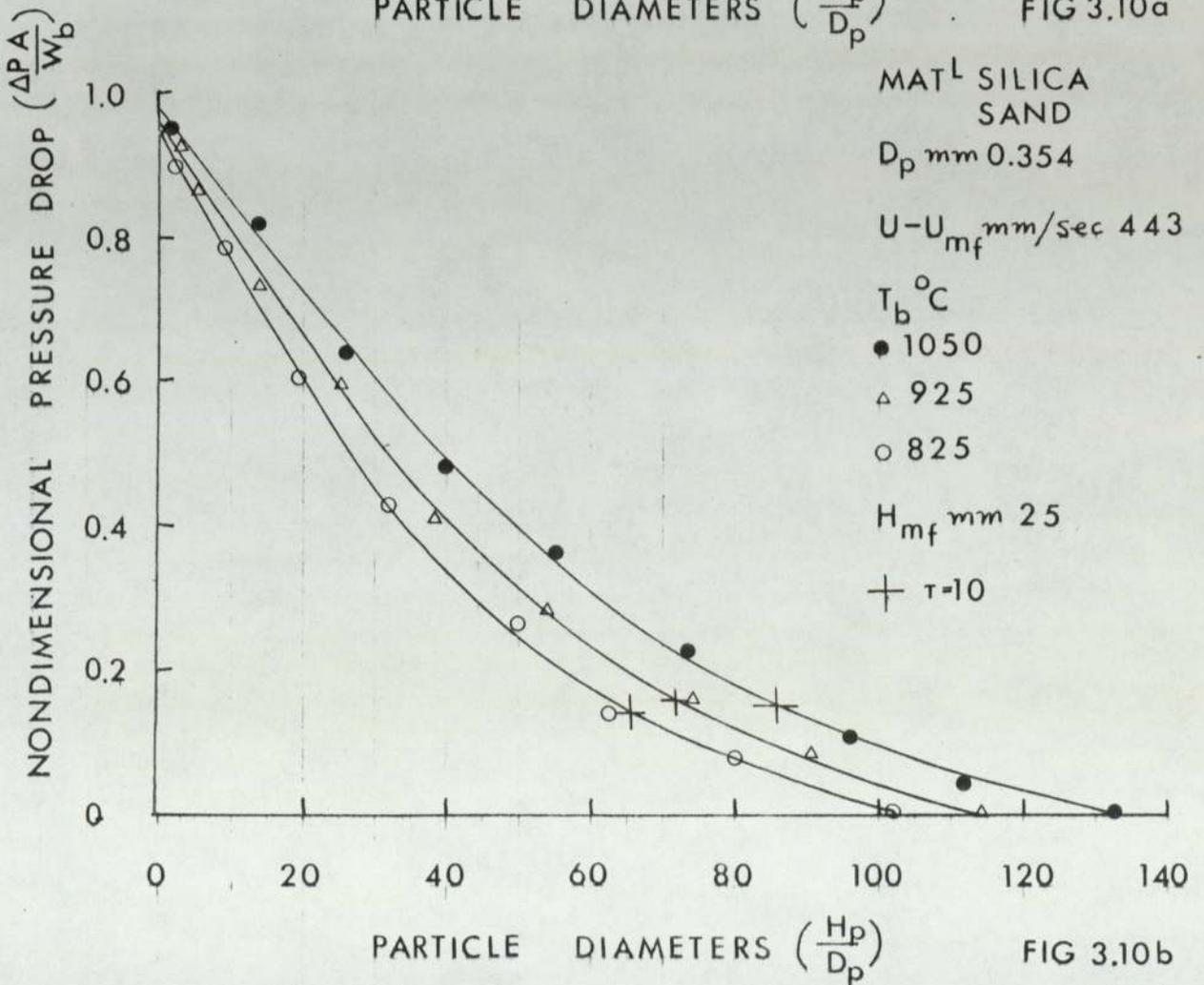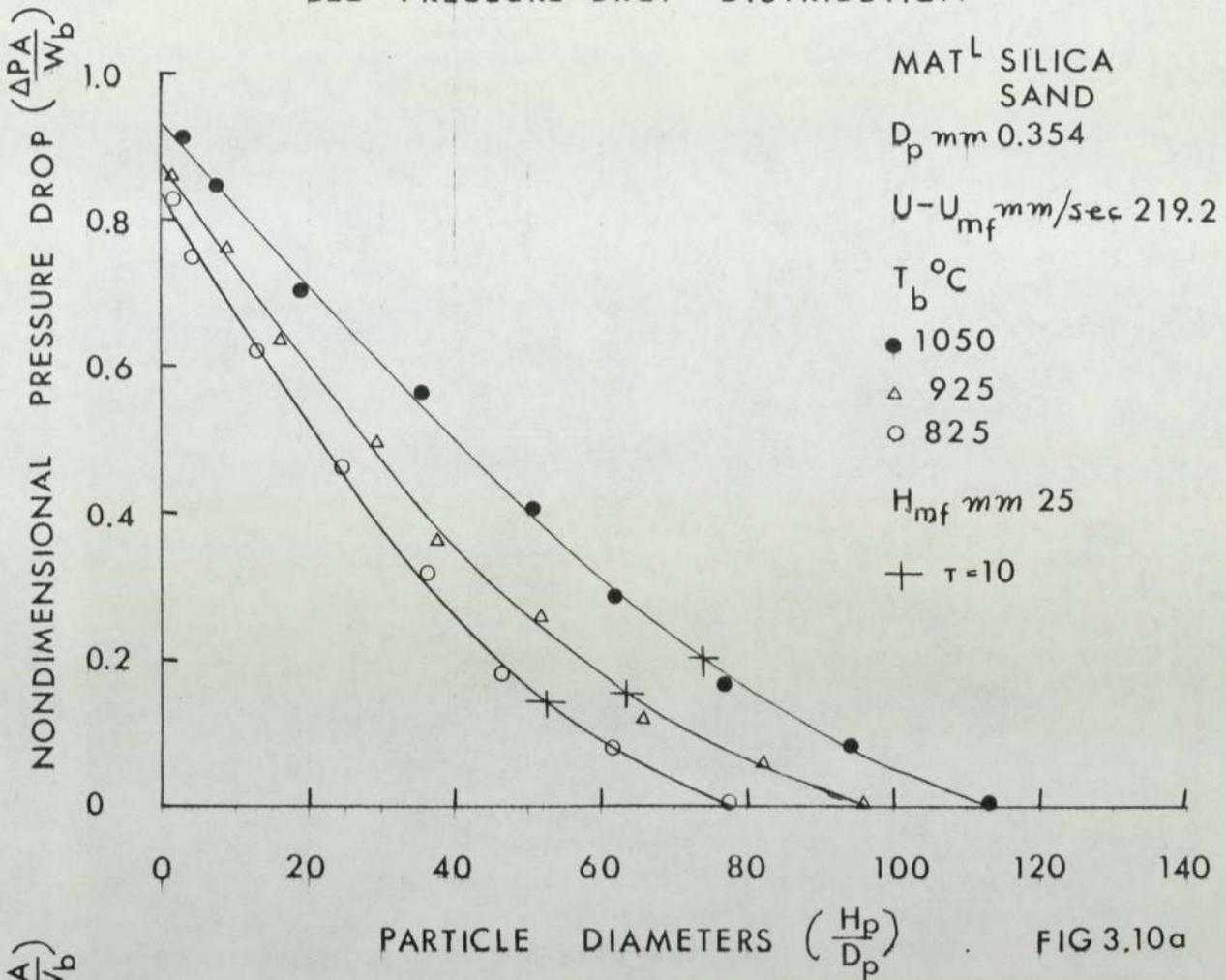
H_{mf} 25 mm

T_b 925 °C

D_p 0.354 mm

H_p mm
0

H_p mm
14

H_p mm
22

FIG 3.8

RESISTANCE MATCHING NETWORK

R_a

TRANSDUCER
AND
AMPLIFIER

R_b

U-V-R

FIG 3.9

BED PRESSURE DROP DISTRIBUTION

FIG 3.10a

FIG 3.10b

BED PRESSURE DROP DISTRIBUTION

MAT^L SILICON
CARBIDE
D_p mm 0.354

$U-U_{mf}$ mm/sec 219.2

T_b °C
● 1050
△ 925
○ 825

H_{mf} mm 25
+ τ=10

FIG 3.11a

MAT^L SILICON
CARBIDE
D_p mm 0.354

$U-U_{mf}$ mm/sec 440

T_b °C
● 1050
△ 925
○ 825

H_{mf} mm. 25
+ τ=10

FIG 3.11b

BED PRESSURE DROP DISTRIBUTION

FIG 3.12 a

FIG 3.12 b
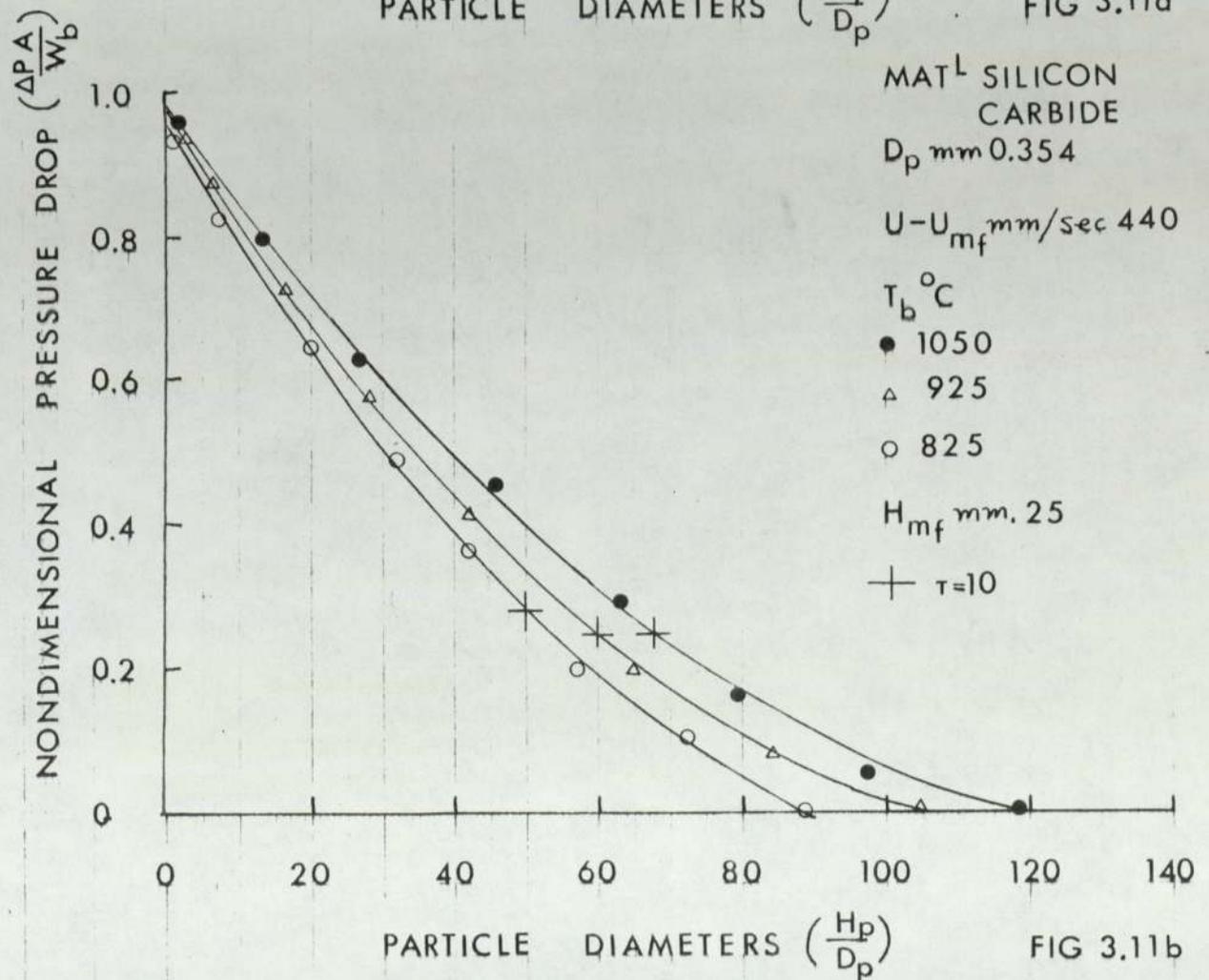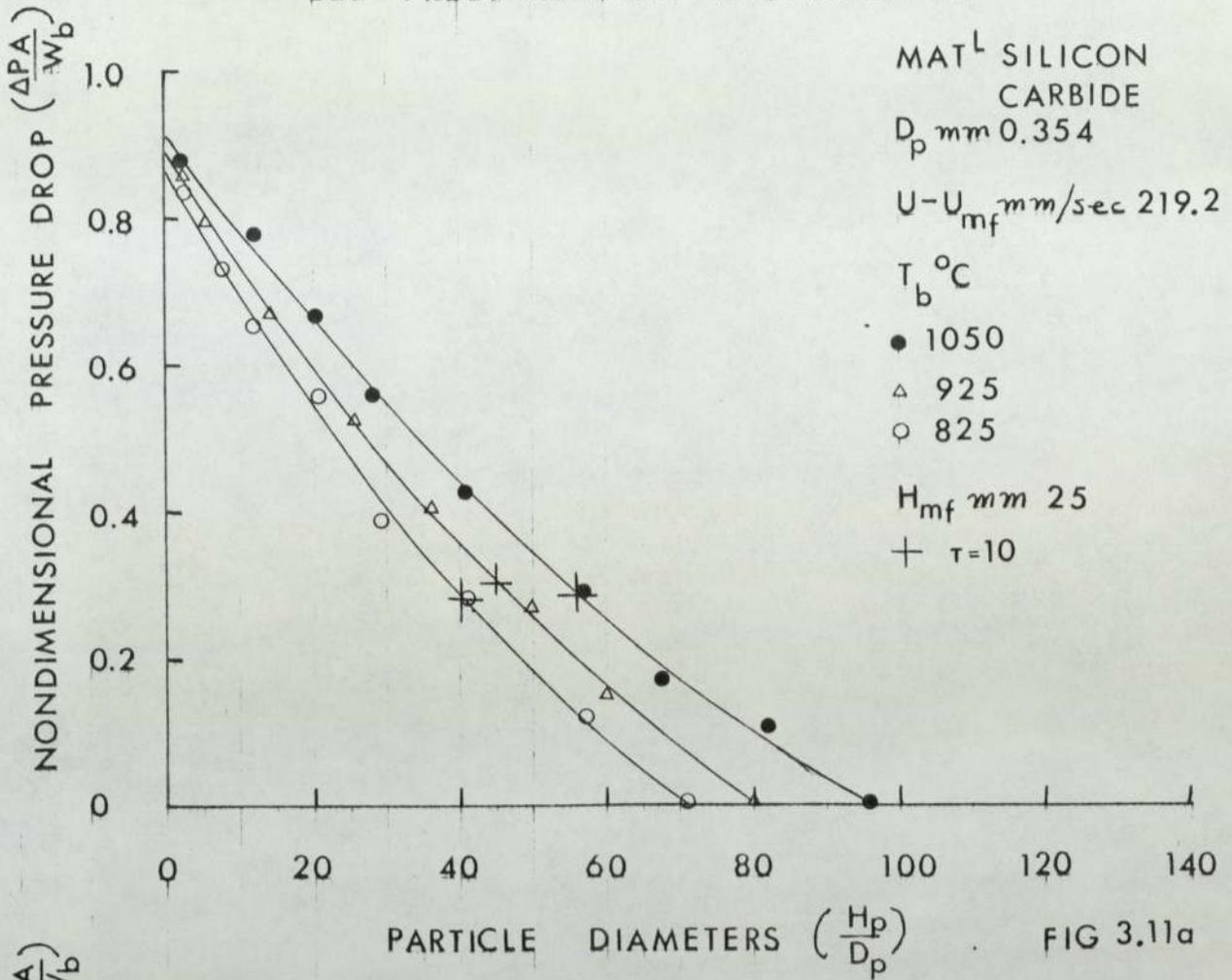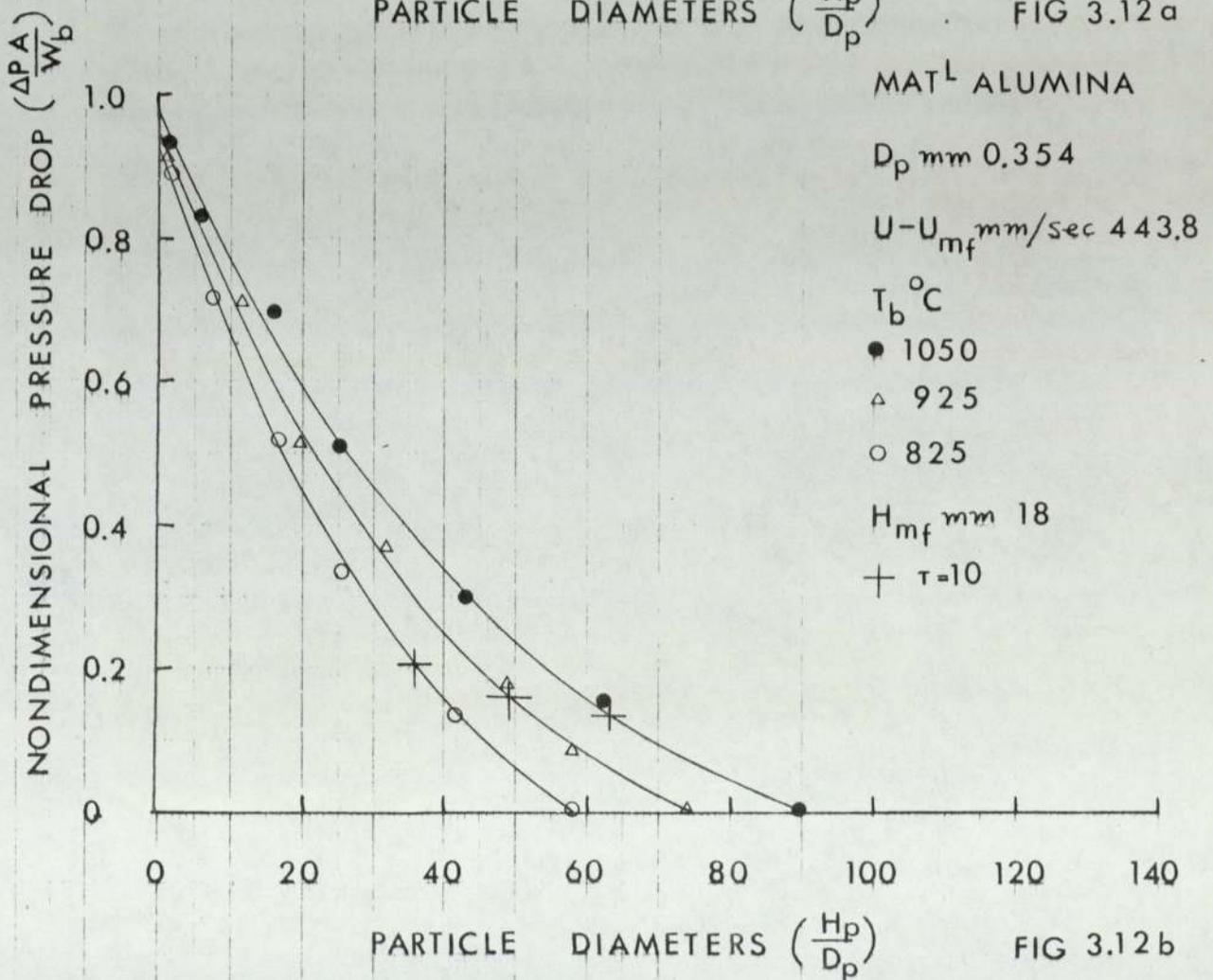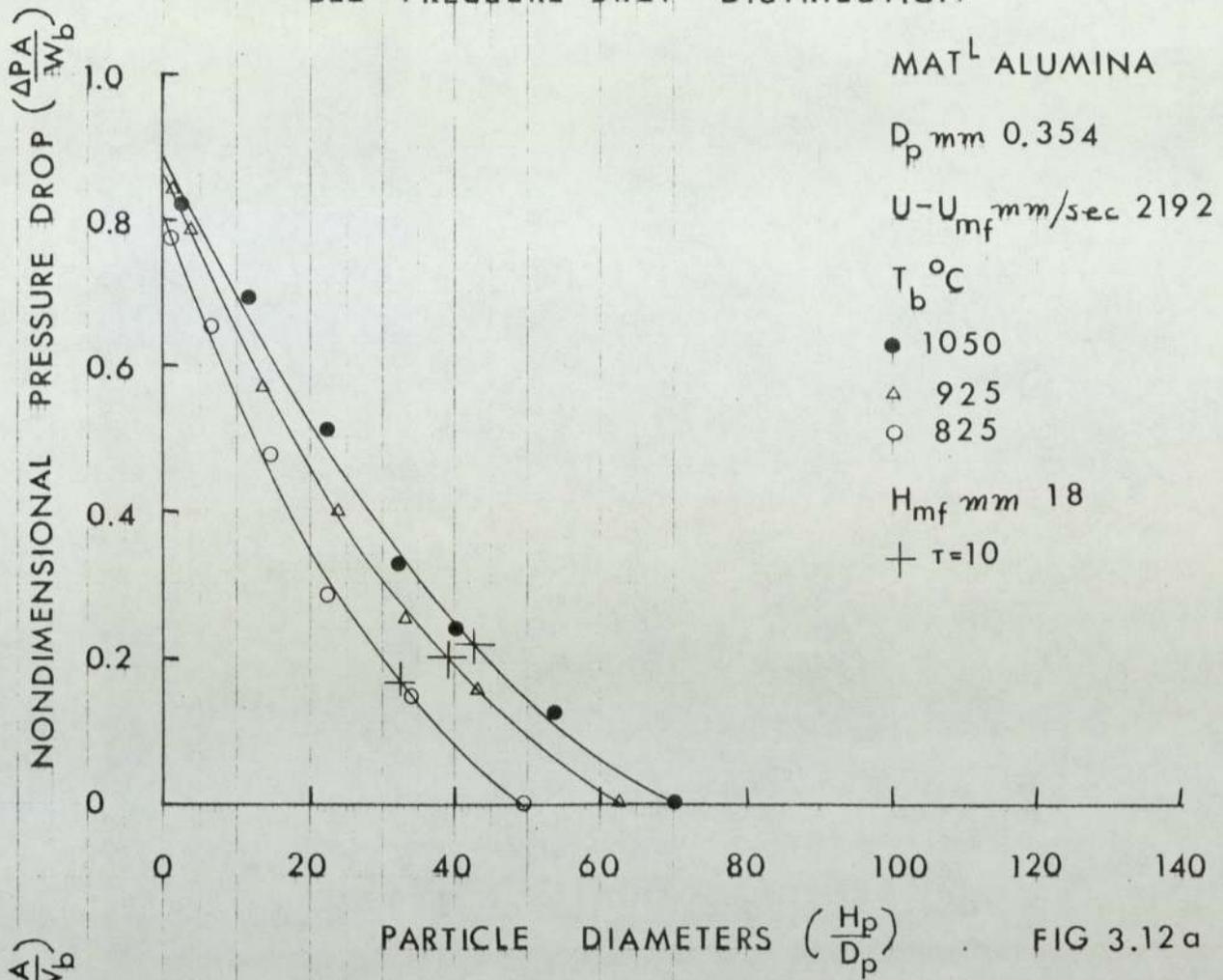
EFFECT OF EXCESS GAS VELOCITY AND BED TEMPERATURE

ON EXTINCTION COEFFICIENT

MAT^L SILICA SAND

D_p 0.354 mm

H_{mf} 25 mm

T_b °C	ε_p
● 825	0.64
○ 925	
▼ 1050	

EXTINCTION COEFFICIENT K_t mm^{-1}

$(U-U_{mf})$ mm/sec

1.4 1.2 1.0 0.8 0.6 0.4 0.2

0 100 200 300 400 500
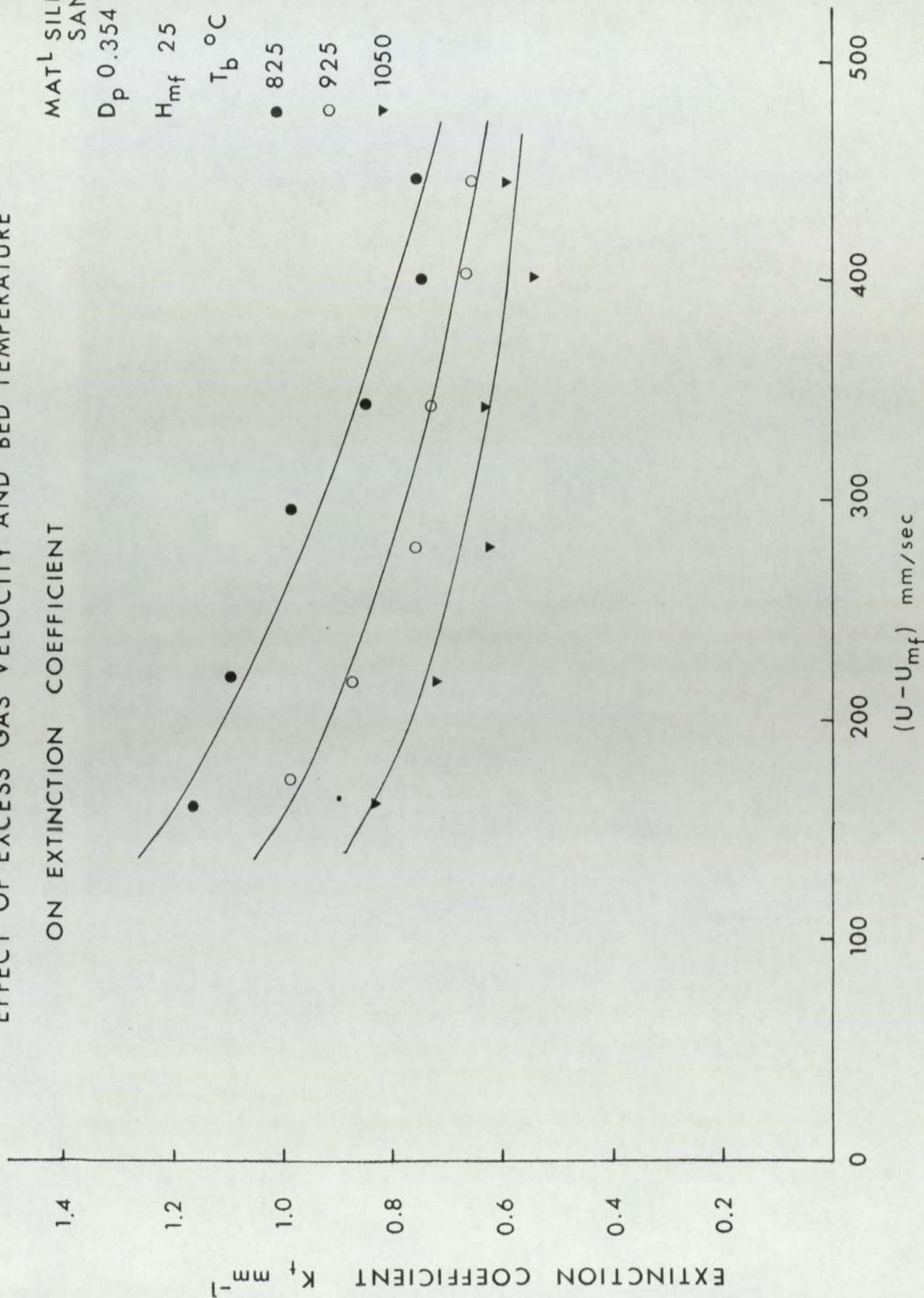
FIG 3.13

EFFECT OF EXCESS GAS VELOCITY AND BED TEMPERATURE

ON EXTINCTION COEFFICIENT

MAT^L SILICON
CARBIDE

D_p 0.354 mm

H_{mf} 25 mm
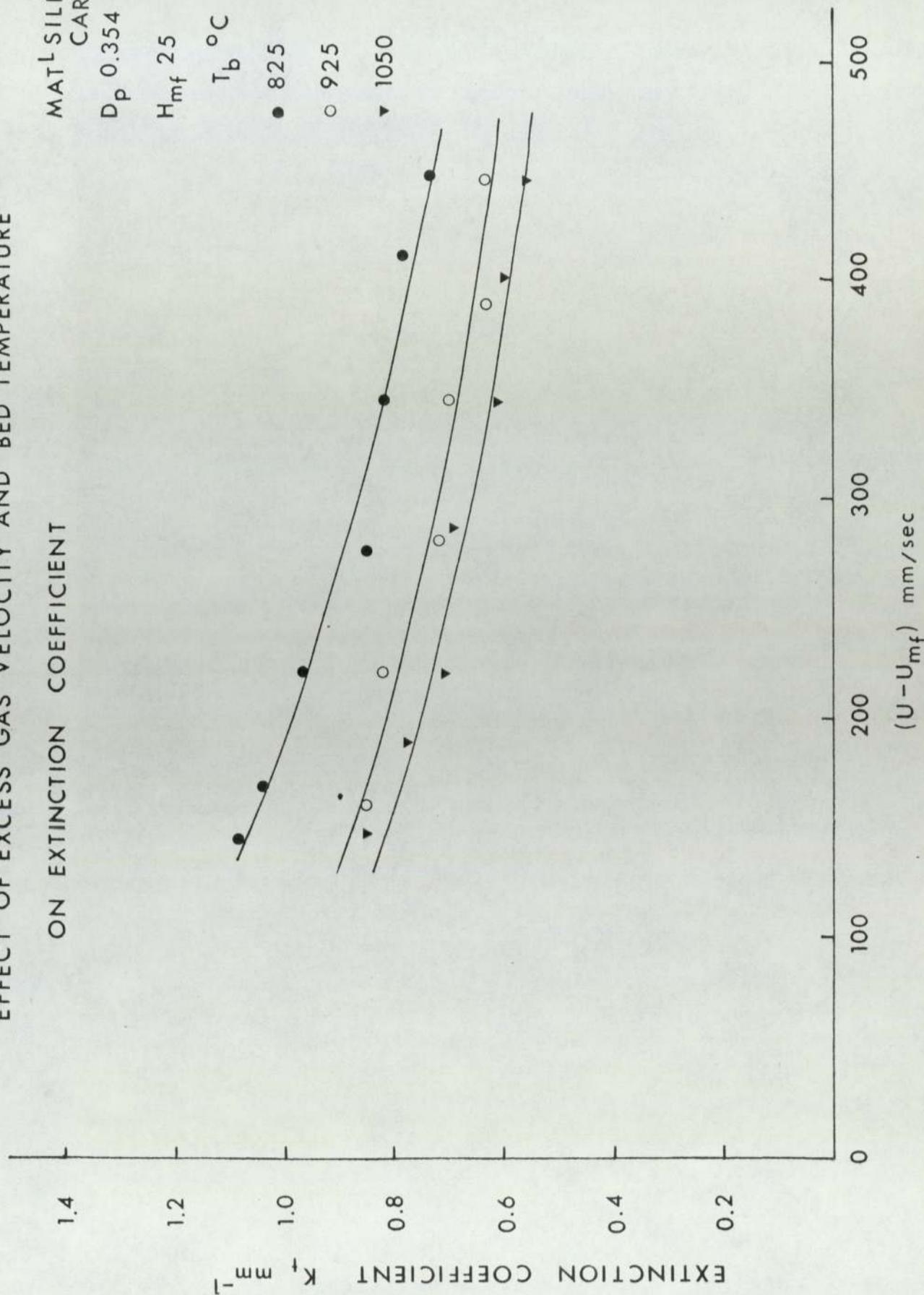
T_b °C	ε_p 0.77
● 825	
○ 925	
▼ 1050	

FIG 3.14

EXTINCTION COEFFICIENT K_t mm^{-1}

$(U-U_{mf})$ mm/sec

EFFECT OF EXCESS GAS VELOCITY AND BED TEMPERATURE

ON EXTINCTION COEFFICIENT

MAT^L ALUMINA

D_p 0.354 mm

H_{mf} 18 mm

T_b °C ε_p 0.37

● 825
○ 925
▼ 1050

$(U - U_{mf})$ mm/sec

EXTINCTION COEFFICIENT K_t mm^{-1}
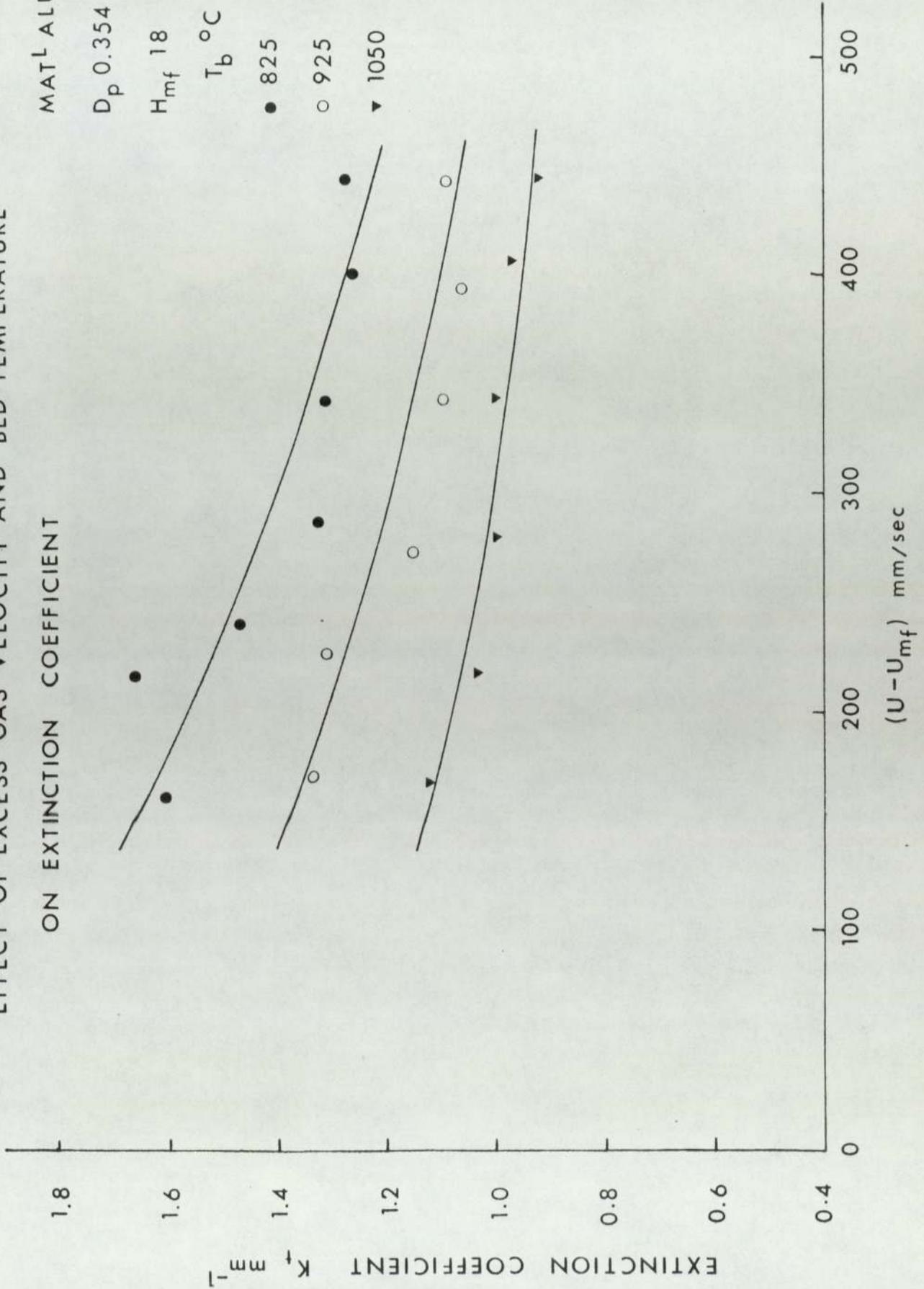
FIG 3.15

EFFECT OF EXCESS GAS VELOCITY AND BED TEMPERATURE

ON EXTINCTION COEFFICIENT

MAT^L SILICA SAND

D_p 0.55 mm

H_{mf} 25 mm

T_b °C ε_p 0.64

● 825

○ 925

▼ 1050

$(U-U_{mf})$ mm/sec

EXTINCTION COEFFICIENT K_t mm^{-1}
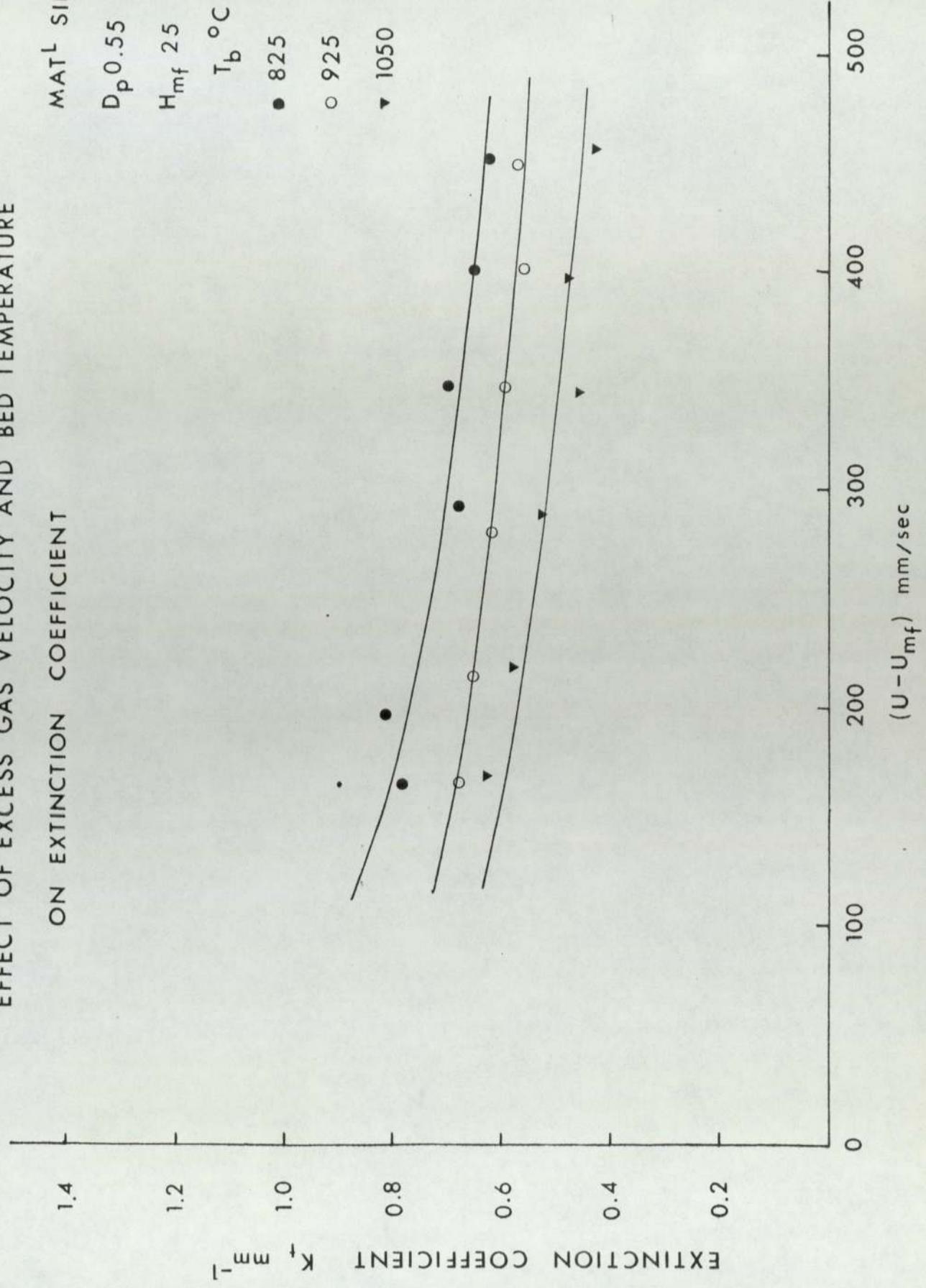
FIG 3.16

EFFECT OF EXCESS GAS VELOCITY AND BED TEMPERATURE

ON EXTINCTION COEFFICIENT

MAT^L SILICON
CARBIDE

D_p 0.55 mm

H_{mf} 25 mm

T_b °C ε_p
            0.77
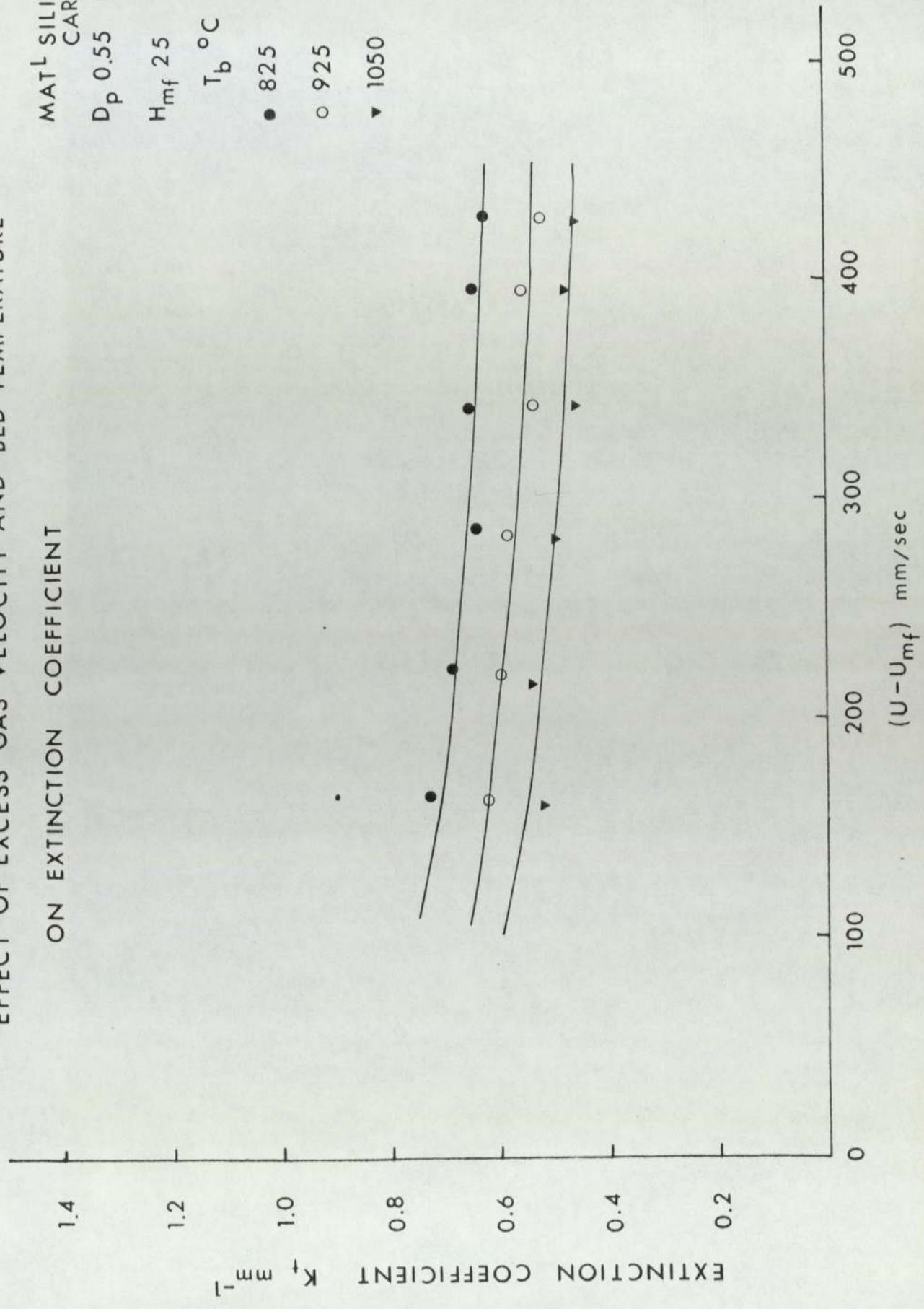
● 825

○ 925

▼ 1050

FIG 3.17

$(U-U_{mf})$ mm/sec

EXTINCTION COEFFICIENT K_t mm^{-1}

1.4

1.2

1.0

0.8

0.6

0.4

0.2

0   100   200   300   400   500

EFFECT OF EXCESS GAS VELOCITY AND BED TEMPERATURE

ON EXTINCTION COEFFICIENT

MAT^L ALUMINA

D_p 0.55 mm

H_{mf} 18 mm

	T_b °C	ε_p 0.37
•	825	
○	925	
▼	1050	
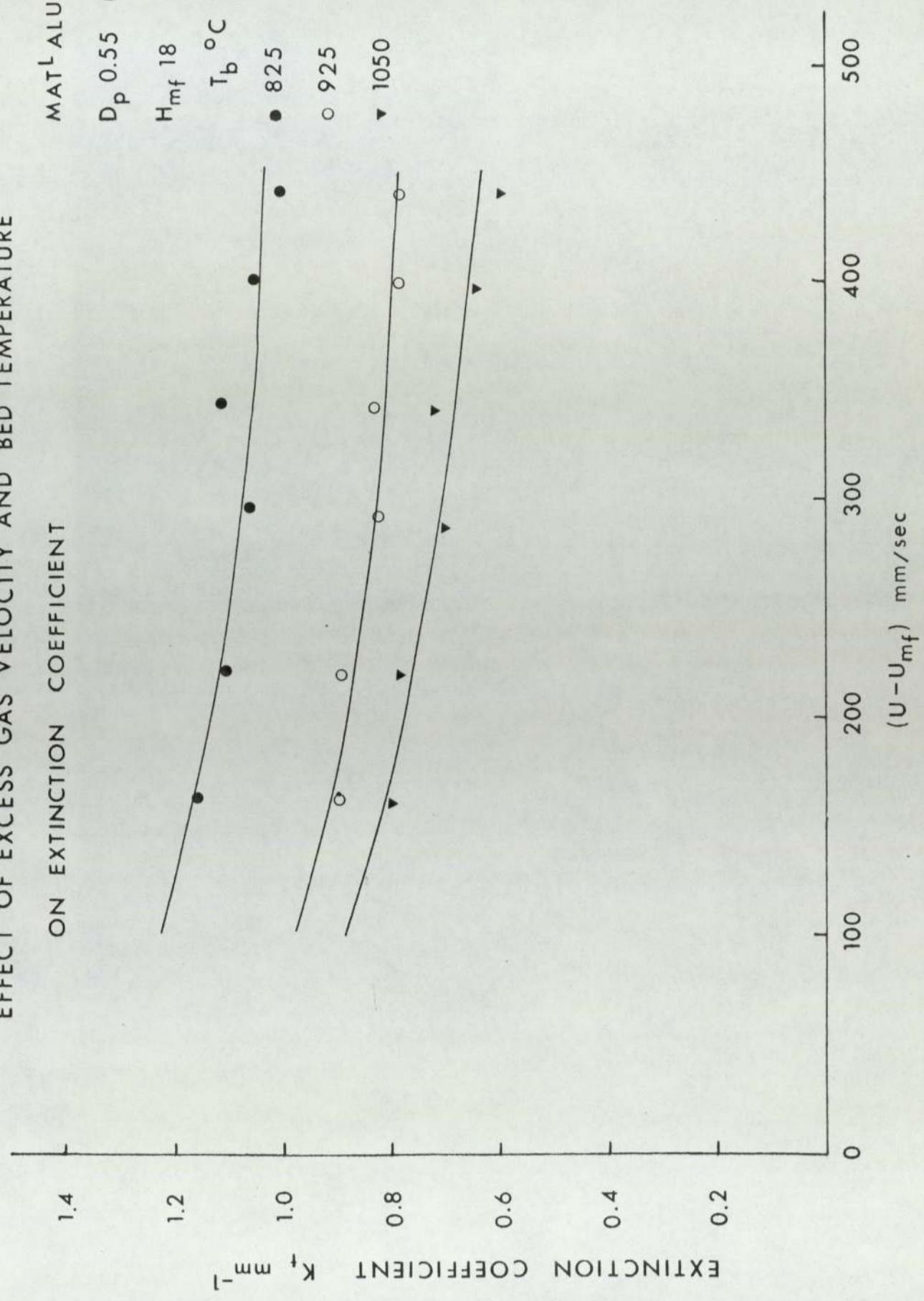
FIG 3.18

$(U-U_{mf})$ mm/sec

EXTINCTION COEFFICIENT K_t mm^{-1}

EFFECT OF EXCESS GAS VELOCITY AND BED TEMPERATURE

ON EXTINCTION COEFFICIENT

MAT^L SILICA
SAND

D_p 0.777 mm

H_{mf} 25 mm

T_b °C ε_p

● 825   0.64

○ 925

▼ 1050

$(U - U_{mf})$ mm/sec

EXTINCTION COEFFICIENT K_t mm^{-1}
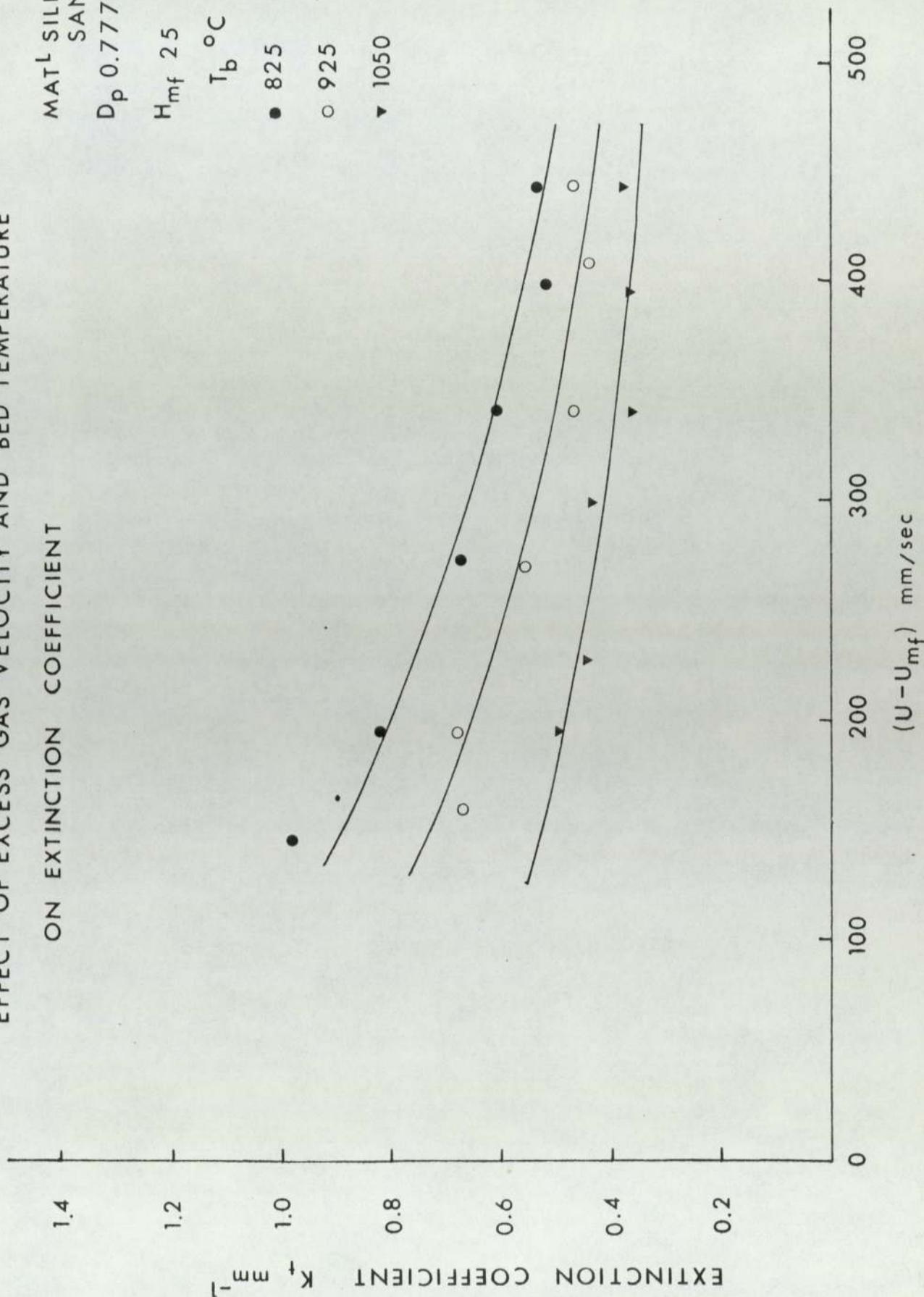
FIG 3.19

EFFECT OF EXCESS GAS VELOCITY AND BED TEMPERATURE

ON EXTINCTION COEFFICIENT

MAT^L SILICON
CARBIDE

D_p 0.777 mm

H_{mf} 25 mm

T_b °C	ε_p
	0.77
● 825	
○ 925	
▼ 1050	

EXTINCTION COEFFICIENT K_f mm^{-1}
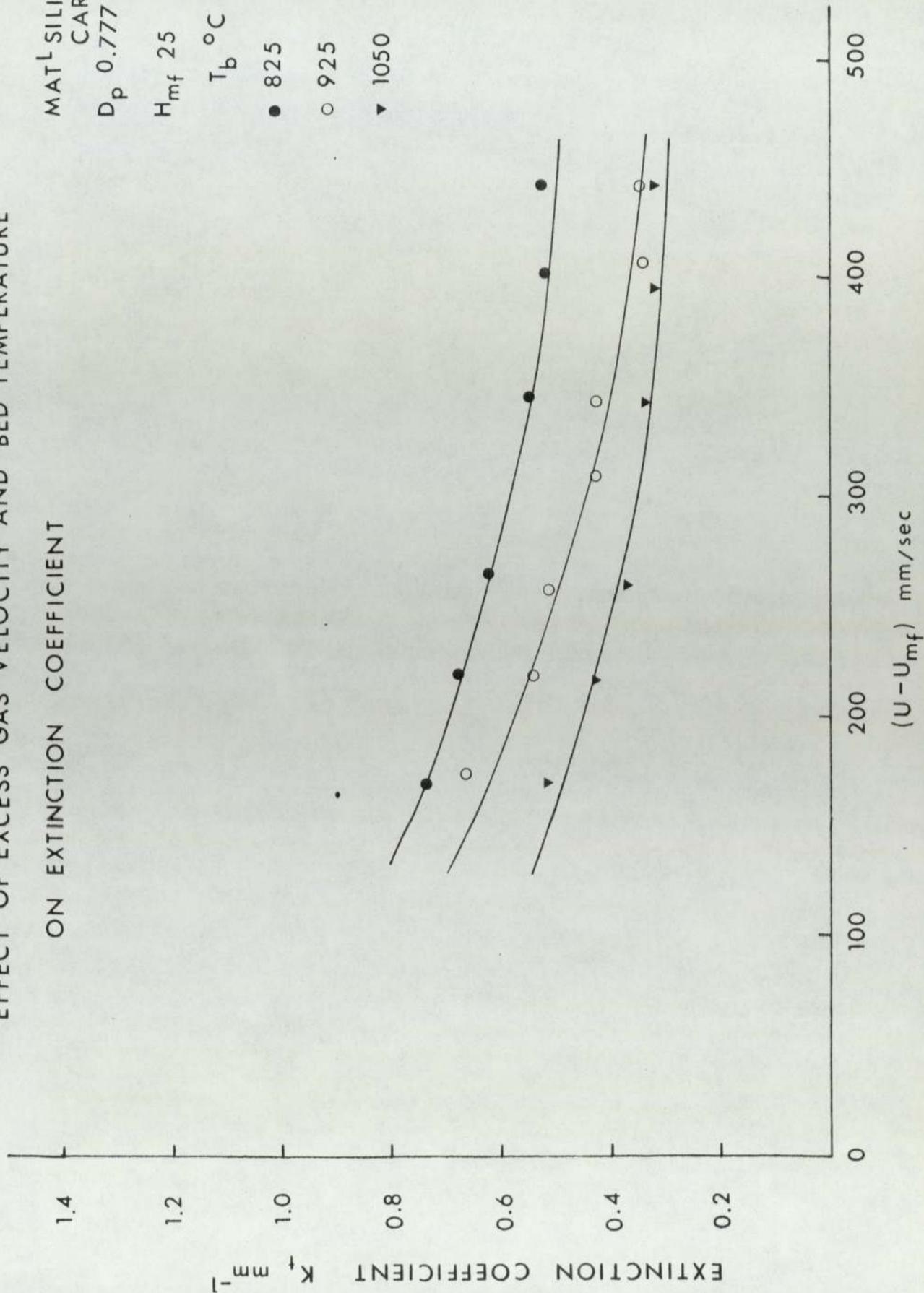
$(U-U_{mf})$ mm/sec

FIG 3.20

EFFECT OF EXCESS GAS VELOCITY AND BED TEMPERATURE

ON EXTINCTION COEFFICIENT

MAT^L ALUMINA

D_p 0.777 mm

H_{mf} 18 mm

T_b °C	ε_p 0.37
● 825	
○ 925	
▼ 1050	

EXTINCTION COEFFICIENT K_t mm^{-1}
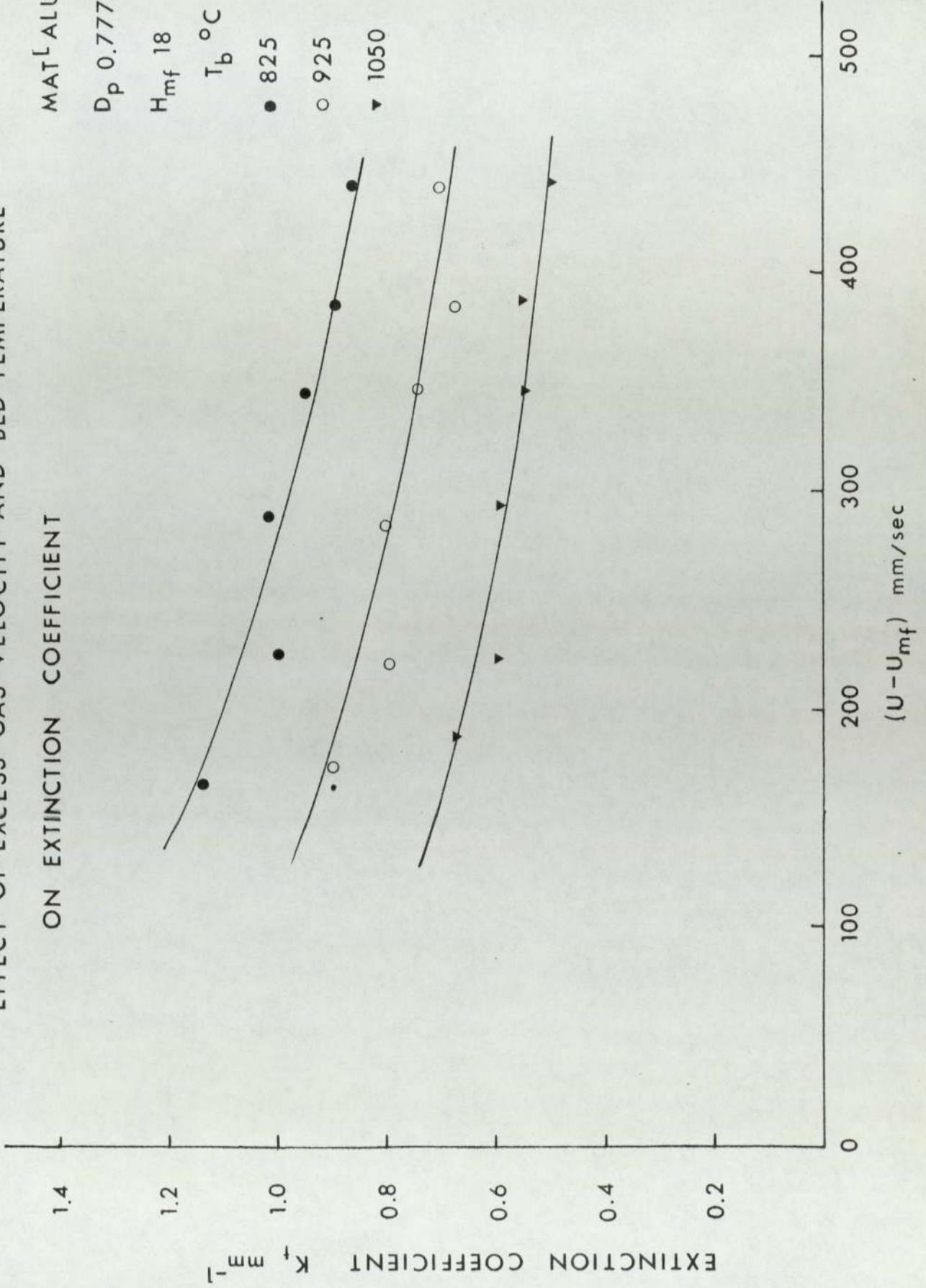
$(U - U_{mf})$ mm/sec

FIG 3.21

MEASUREMENT OF BED SURFACE TEMPERATURE GRADIENT

143 mm dia

THERMOCOUPLE

SHIELDED
THERMOCOUPLE
PROBE

100 mm

A

GAS - AIR
SUPPLY

5 mm   3mm

1 mm dia hole

4mm dia

Cr Al

Cr Al

ENLARGEMENT - A

FIG. 3.22

NONDIMENSIONAL TEMPERATURE θ

$$\theta = 0.9072 + 0.0072\,\tau + 0.00219\,\tau^2 - 0.000257\,\tau^3 + 0.00000574\,\tau^4$$

OPTICAL DEPTH τ
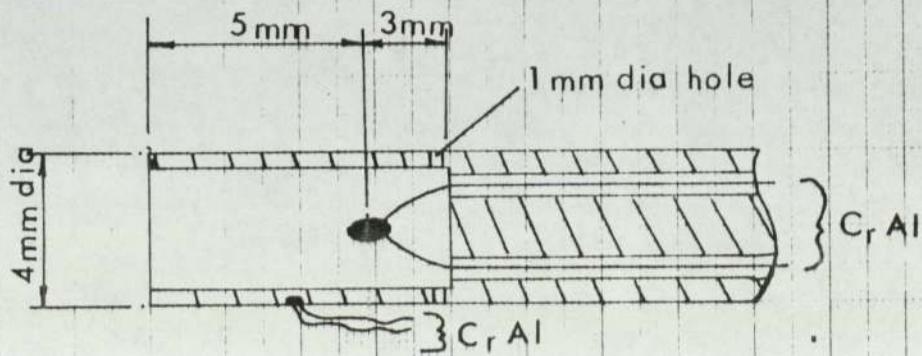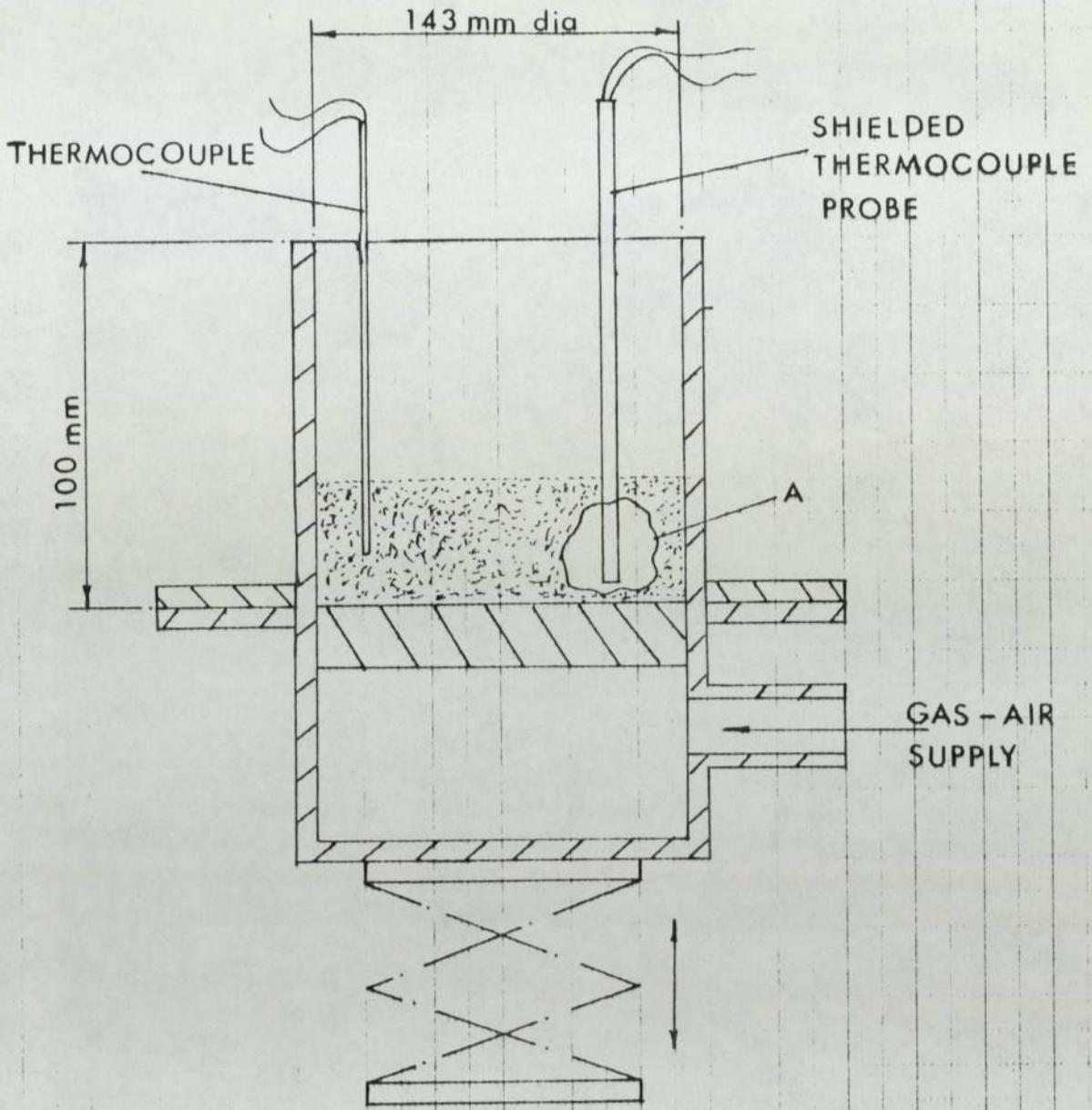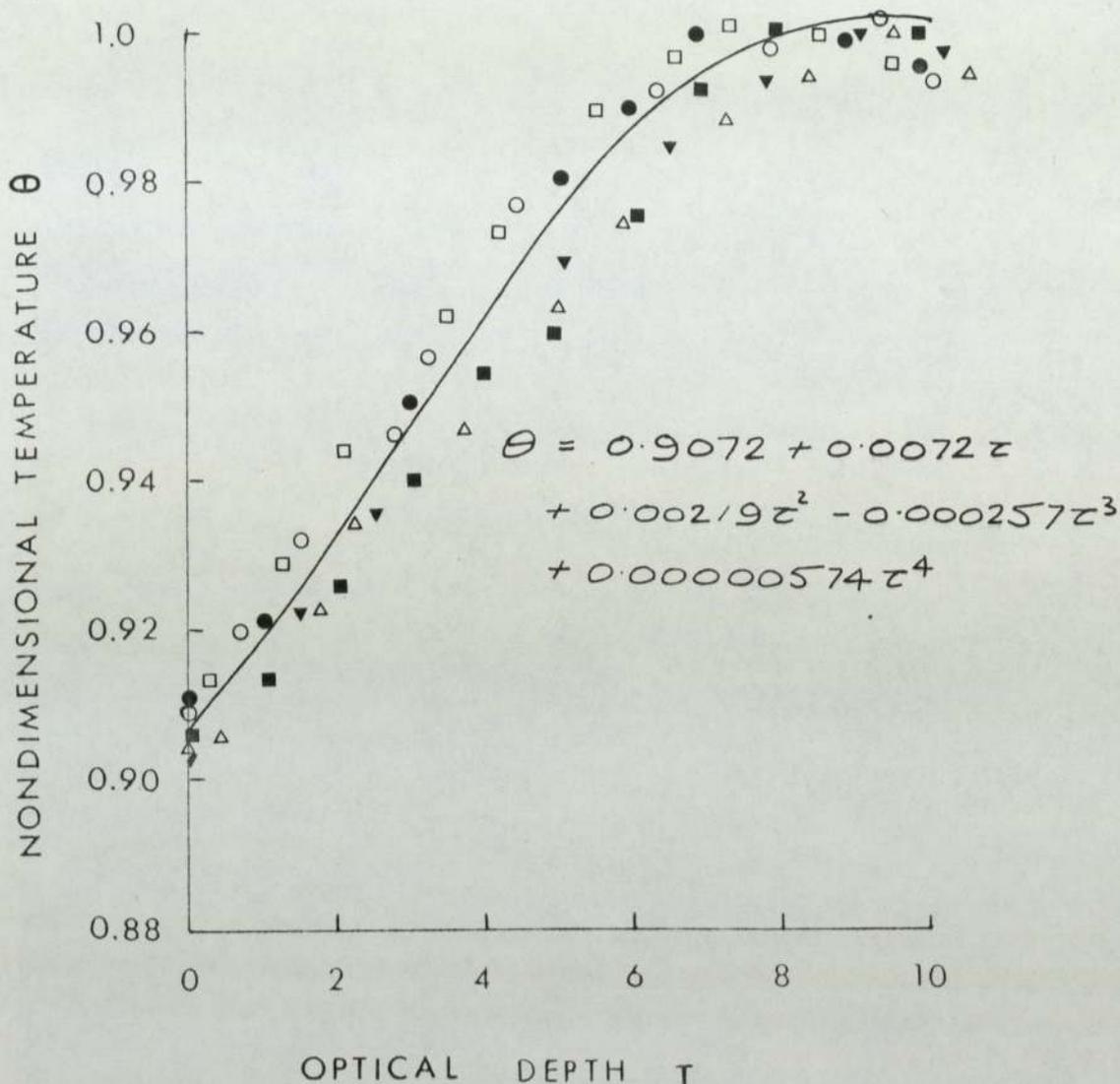
	$U - U_{mf}$ mm/sec	K_t mm^{-1}	$T_{bed} \pm 10$ °C	D_p mm	MAT^L	H_{mf} mm
●	439.0	0.479	1048	0.55	SILICON CARBIDE	25
■	220.0	0.535	1053	0.777		
○	440.0	0.612	868	0.354		
▼	221.0	0.654	874	0.55		
△	219.0	0.81	925	0.354		
□	443.0	0.372	928	0.777		

FIG 3.23

NONDIMENSIONAL TEMPERATURE θ (y-axis) values: 1.0, 0.98, 0.96, 0.94, 0.92, 0.90, 0.88

OPTICAL DEPTH τ (x-axis) values: 0, 2, 4, 6, 8, 10

$$\theta = 0.942 + 0.0061\tau + 0.000258\tau^2 - 0.00000304\tau^3 - 0.00000389\tau^4$$
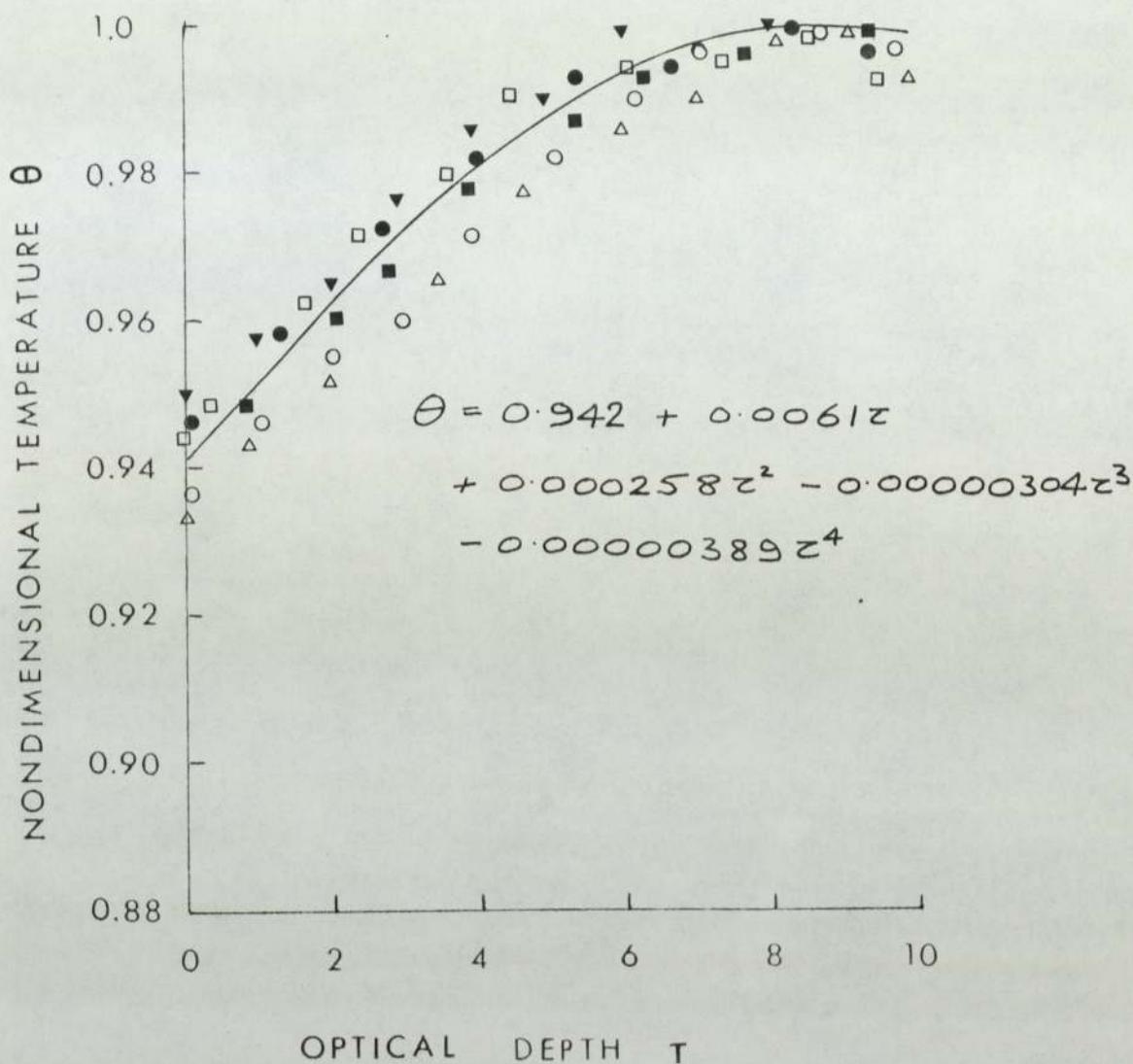
$U - U_{mf}$ mm/sec	K_t mm^{-1}	$T_{bed} \pm 10$ $^\circ$C	D_p mm	MAT^L	H_{mf} mm
● 440.0	0.5	1045	0.777	ALUMINA	25
■ 225.0	0.625	1053	0.55		
○ 442.0	0.81	865	0.354		
▼ 219.0	0.97	876	0.777		
△ 442.0	0.82	934	0.55		
□ 220.0	1.32	924	0.354		

FIG 3.24

NONDIMENSIONAL TEMPERATURE θ

OPTICAL DEPTH T

$$\theta = 0.9172 + 0.00848\tau + 0.00103\tau^2 - 0.000105\tau^3 - 0.0000000005\tau^4$$
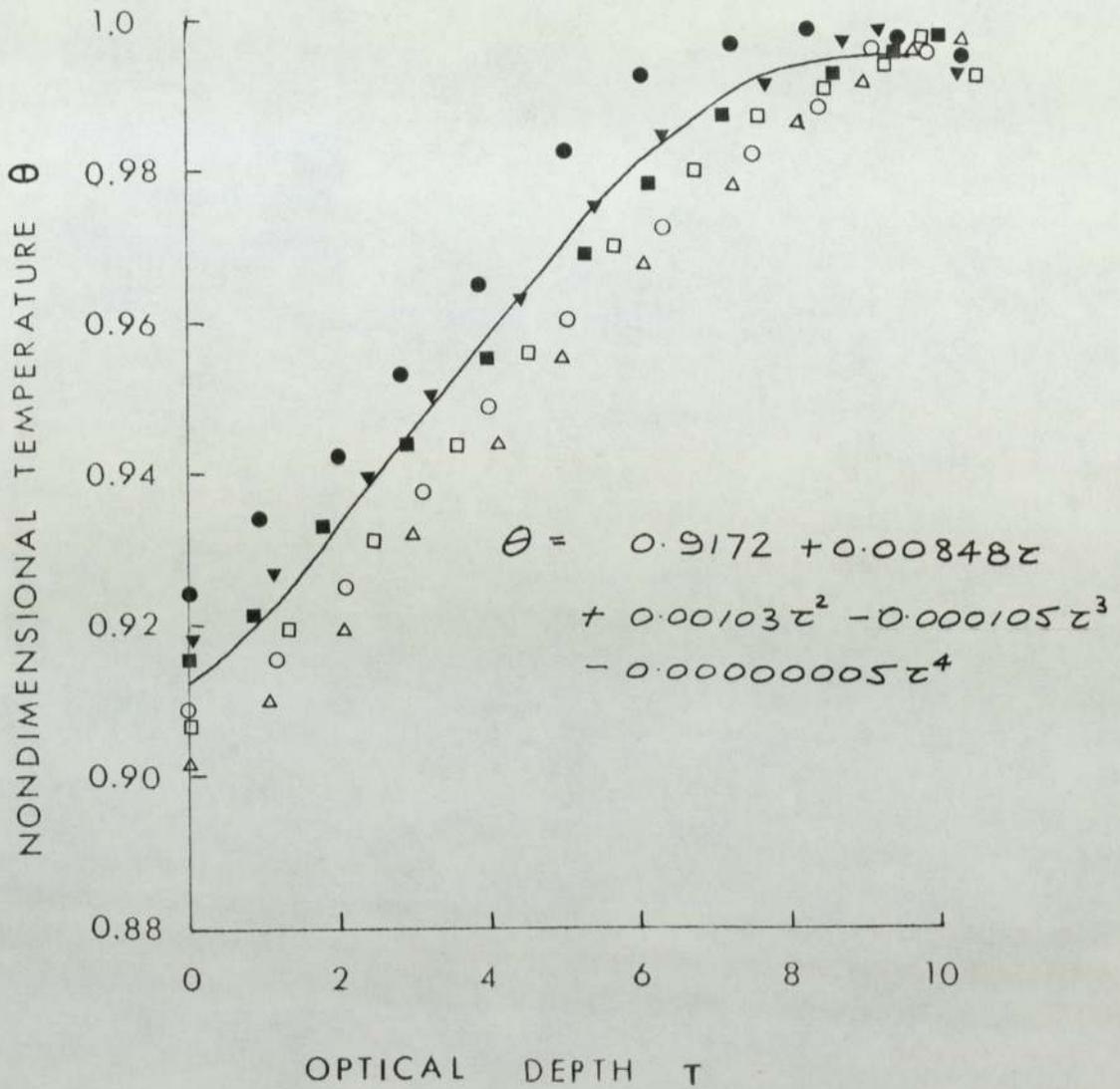
U−U$_{mf}$ mm/sec	K$_t$ mm^{-1}	T$_{bed}$ ±10 °C	D$_p$ mm	MAT^L	H$_{mf}$ mm
● 441.0	0.371	1052	0.777	SILICA SAND	25
■ 221.0	0.748	1050	0.354		
○ 440.0	0.643	875	0.55		
▼ 219.0	0.672	872	0.777		
△ 442.0	0.710	923	0.354		
□ 214.0	0.681	927	0.55		

FIG 3.25
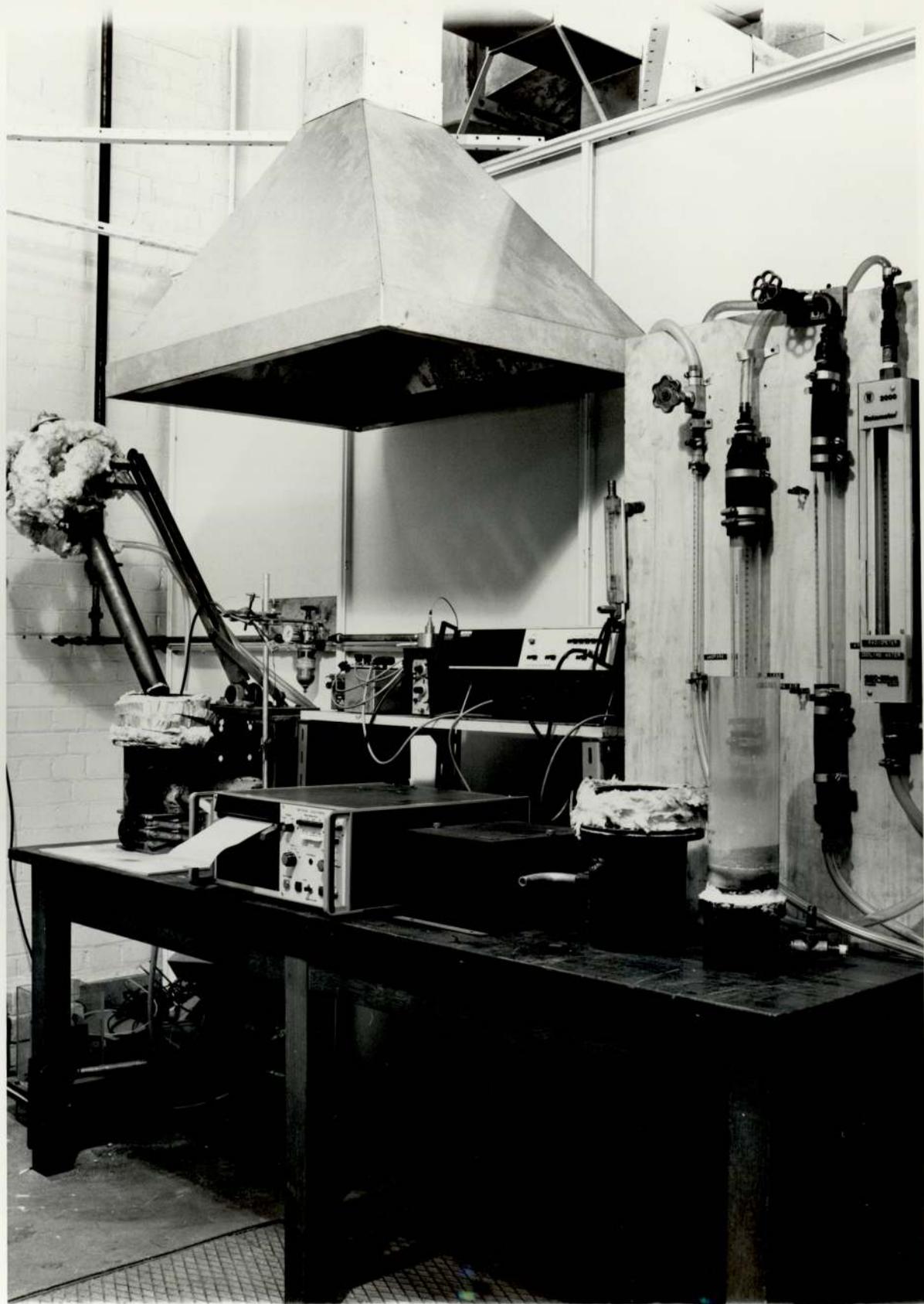
Photograph of the experimental arrangement   FIG 4.1a

CONFIGURATION FOR SURFACE EMITTED

FLUX DETERMINATION

SIGNAL

WATER IN

PYROMETER

WATER OUT

AIR IN

FIELD OF VIEW

AIR OUT

A

θ

C$_r$Al
THERMOCOUPLE

C$_r$Al
THERMOCOUPLE

GAS-AIR
MIXTURE

15 mm

VIEW-A
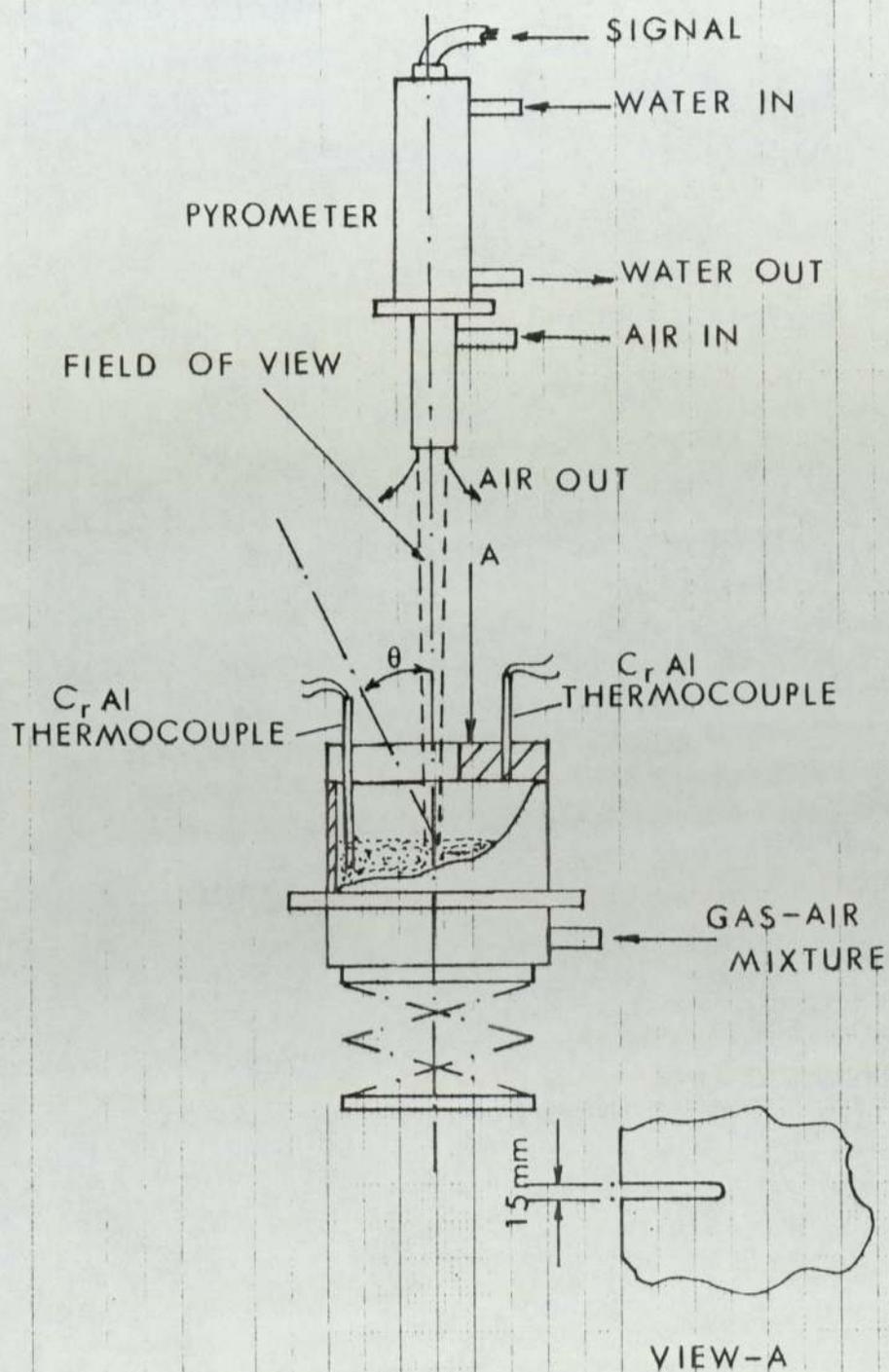
FIG 4.1b

TYPICAL PYROMETER OUTPUT TRACE FOR
AN ISOTHERMAL (A) AND NONISOTHERMAL (B) BED

(A)
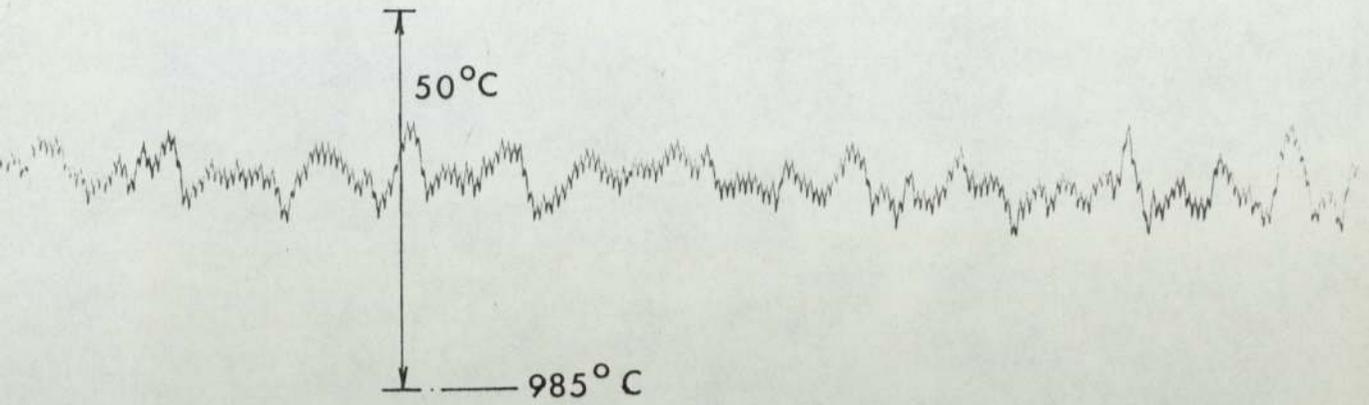
	$U - U_{mf}$ mm/sec	T_b °C	D_p mm	
(A)	443	1050	0.55	SAND
(B)	437	"	"	"
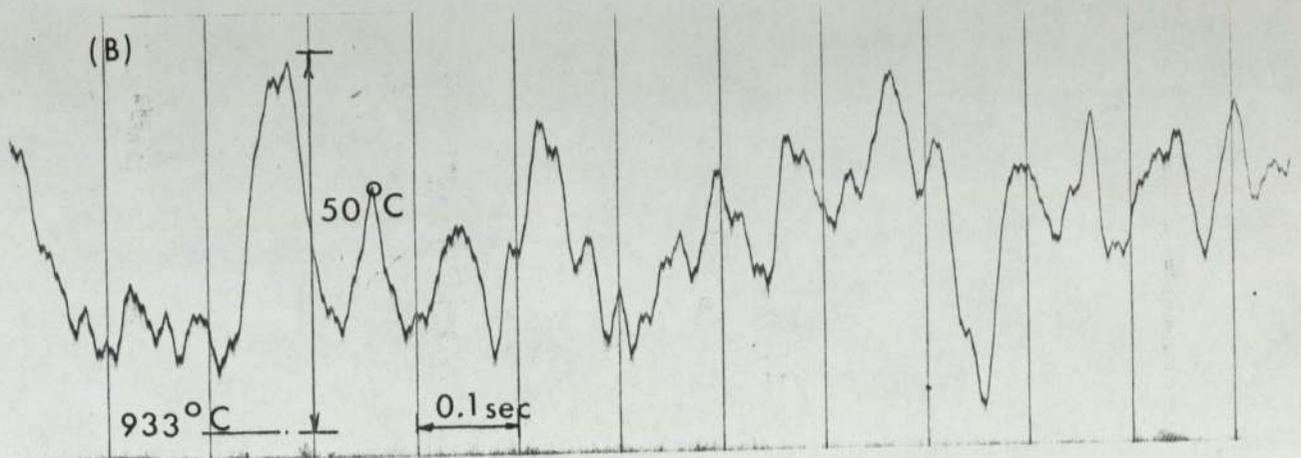
FIG 4.2

TYPICAL STANDARD DEVIATIONS
OF SURFACE TEMPERATURE FLUCTUATIONS

STANDARD DEVIATION °C

MEASURED (MEAN) SURFACE TEMPERATURE °C
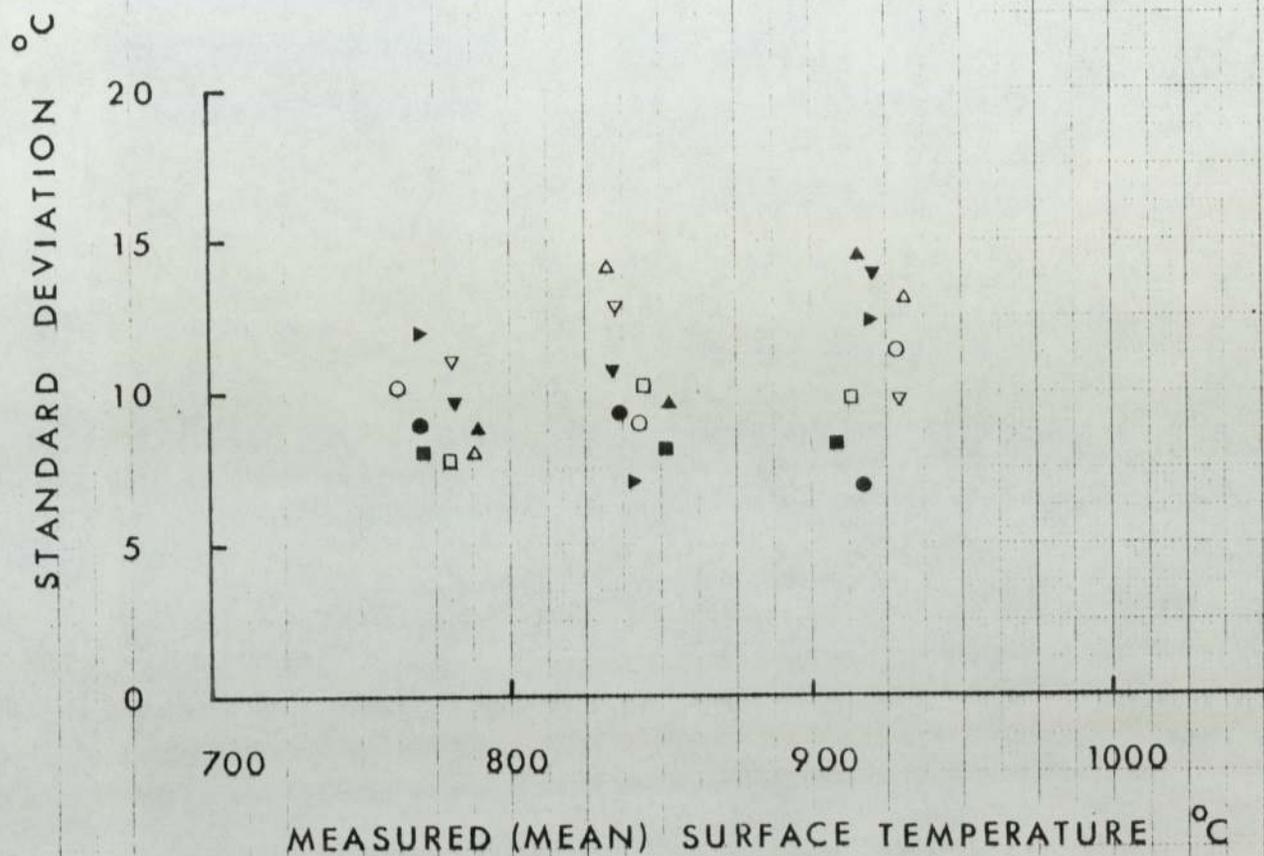
MAT^L SILICA SAND	D_P mm	H_{mf} mm	$U - U_{mf}$ mm/sec
●	0.55	15	214
○		33	—//—
□		50	—//—
■		15	338.6
▼		33	—//—
▽		50	—//—
▲		15	437.7
△		33	—//—
▶		50	—//—

FIG 4.3

EFFECT OF EXCESS GAS VELOCITY ON EFFECTIVE EMISSIVITY
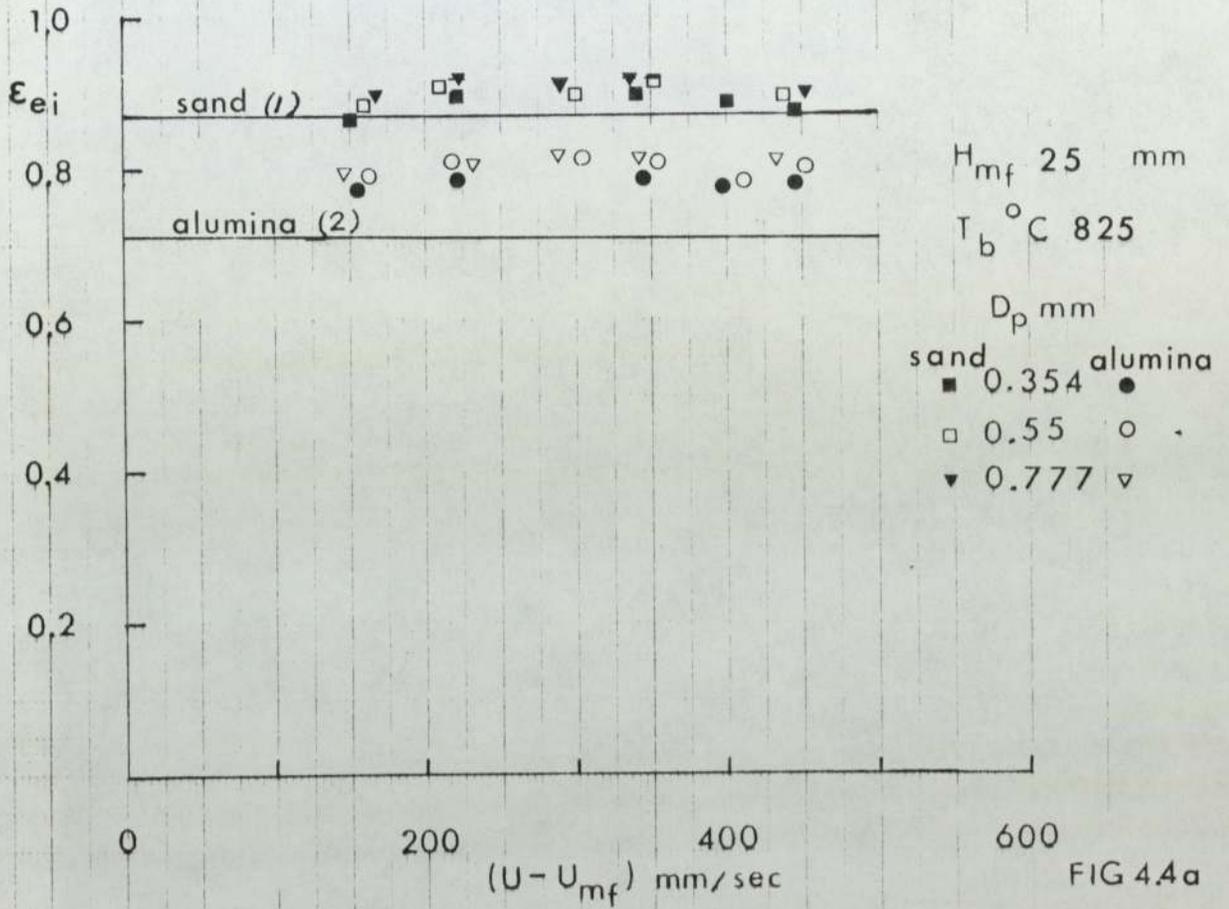
FIG 4.4a
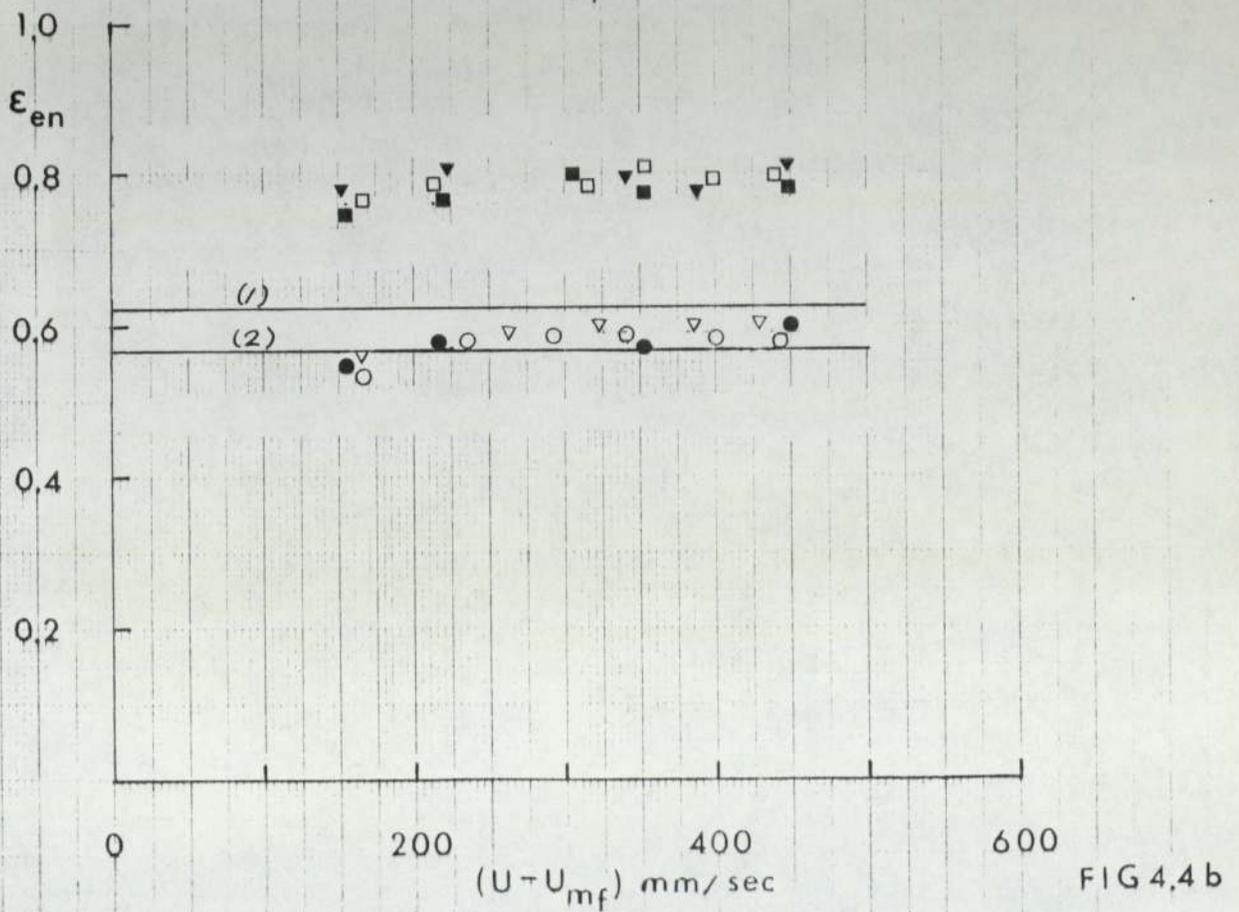
FIG 4.4b

EFFECT OF EXCESS GAS VELOCITY ON EFFECTIVE EMISSIVITY
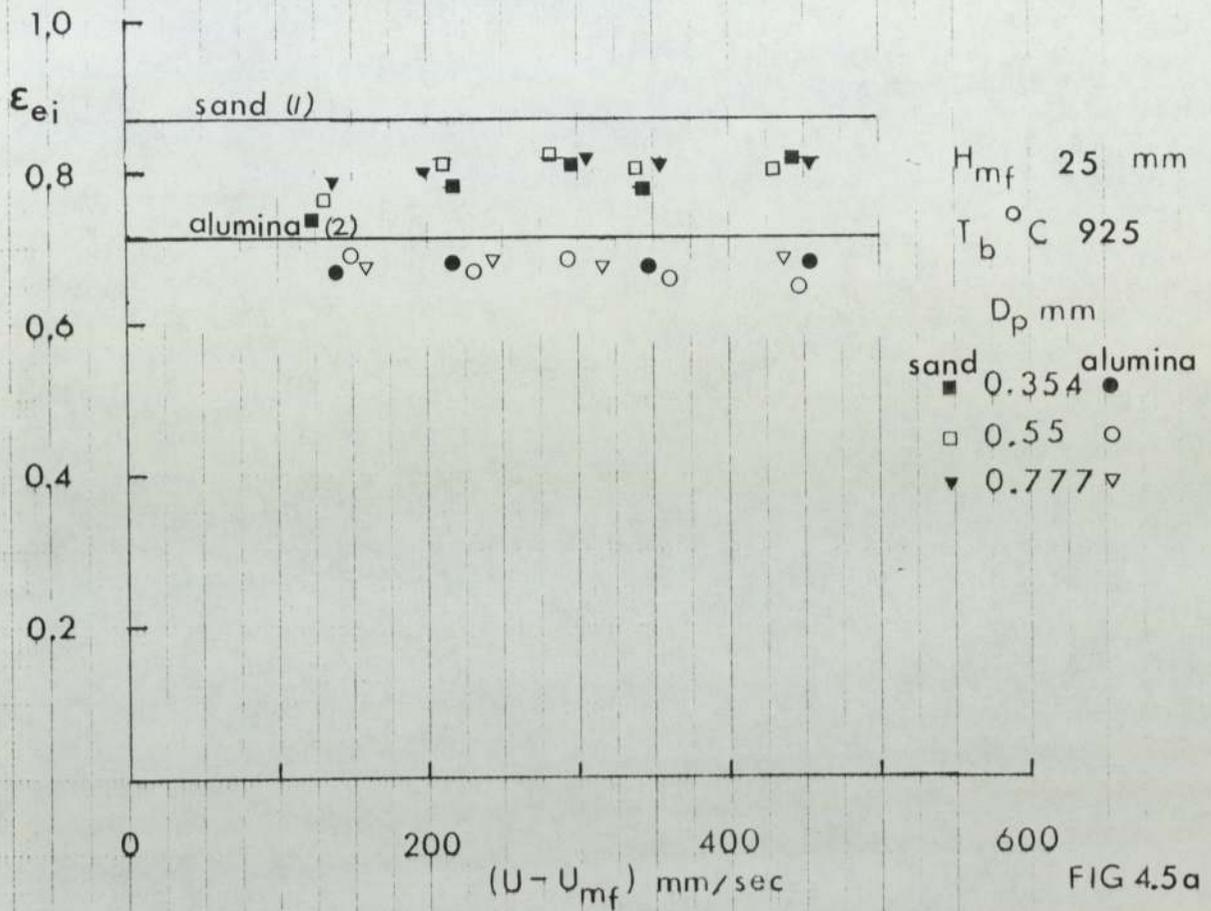
FIG 4.5a
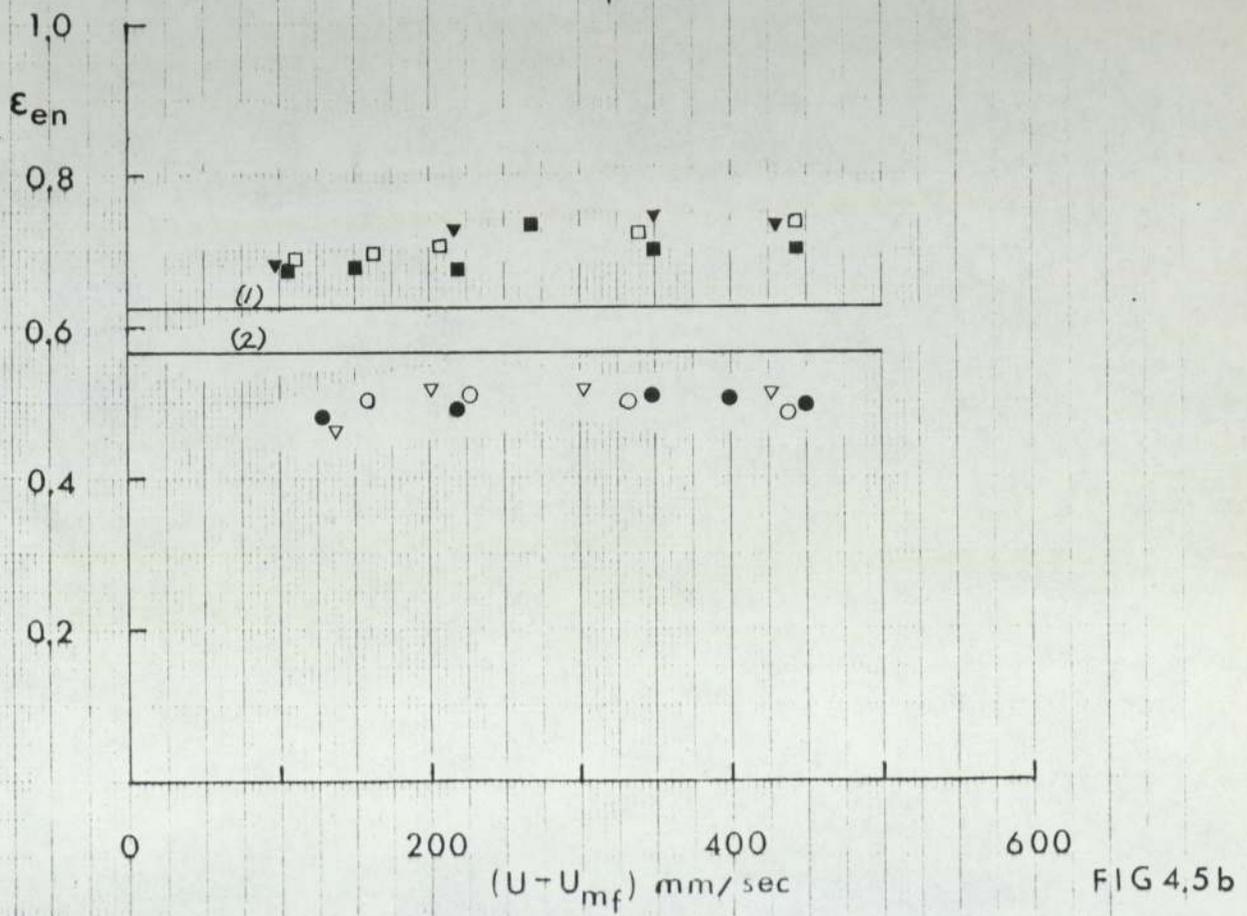
FIG 4.5b

EFFECT OF EXCESS GAS VELOCITY ON EFFECTIVE EMISSIVITY
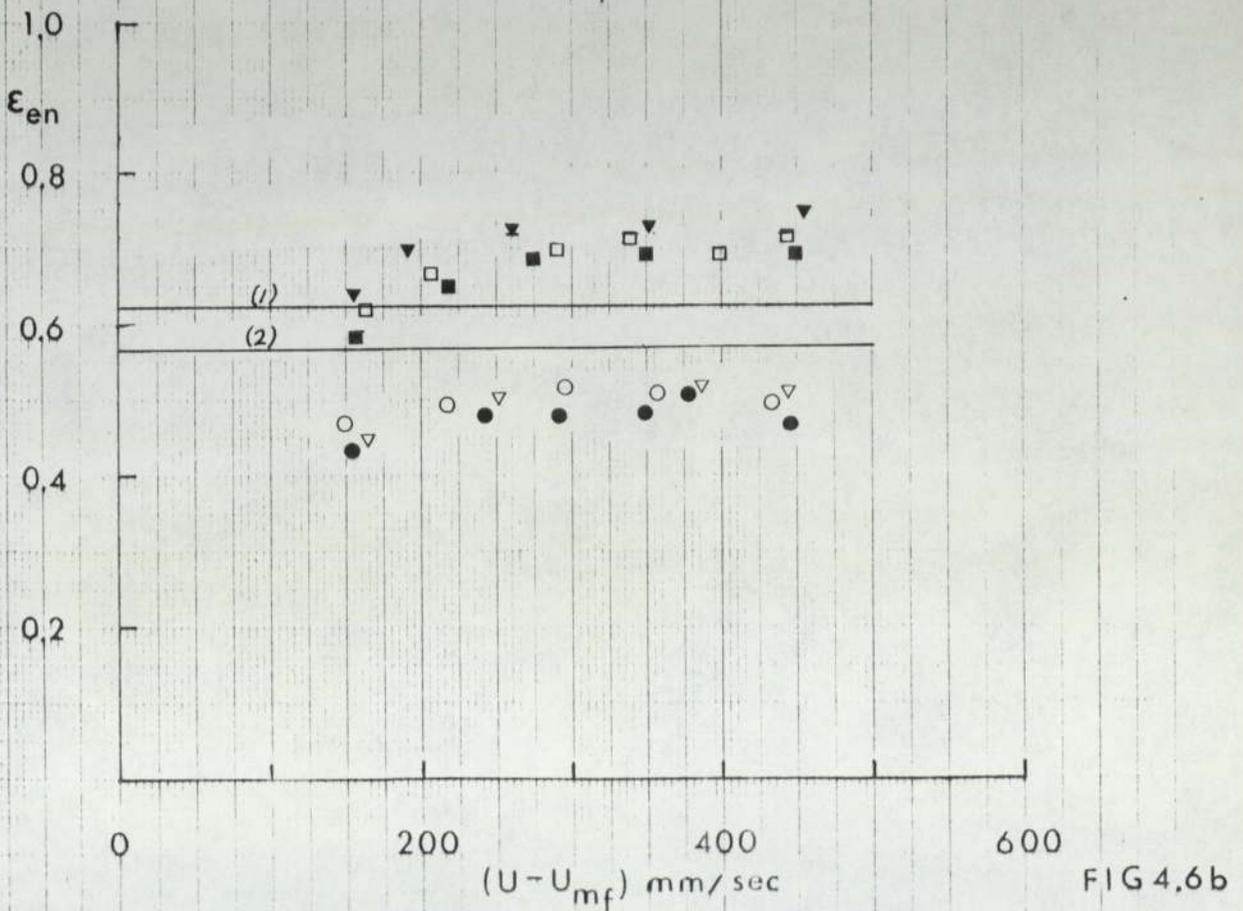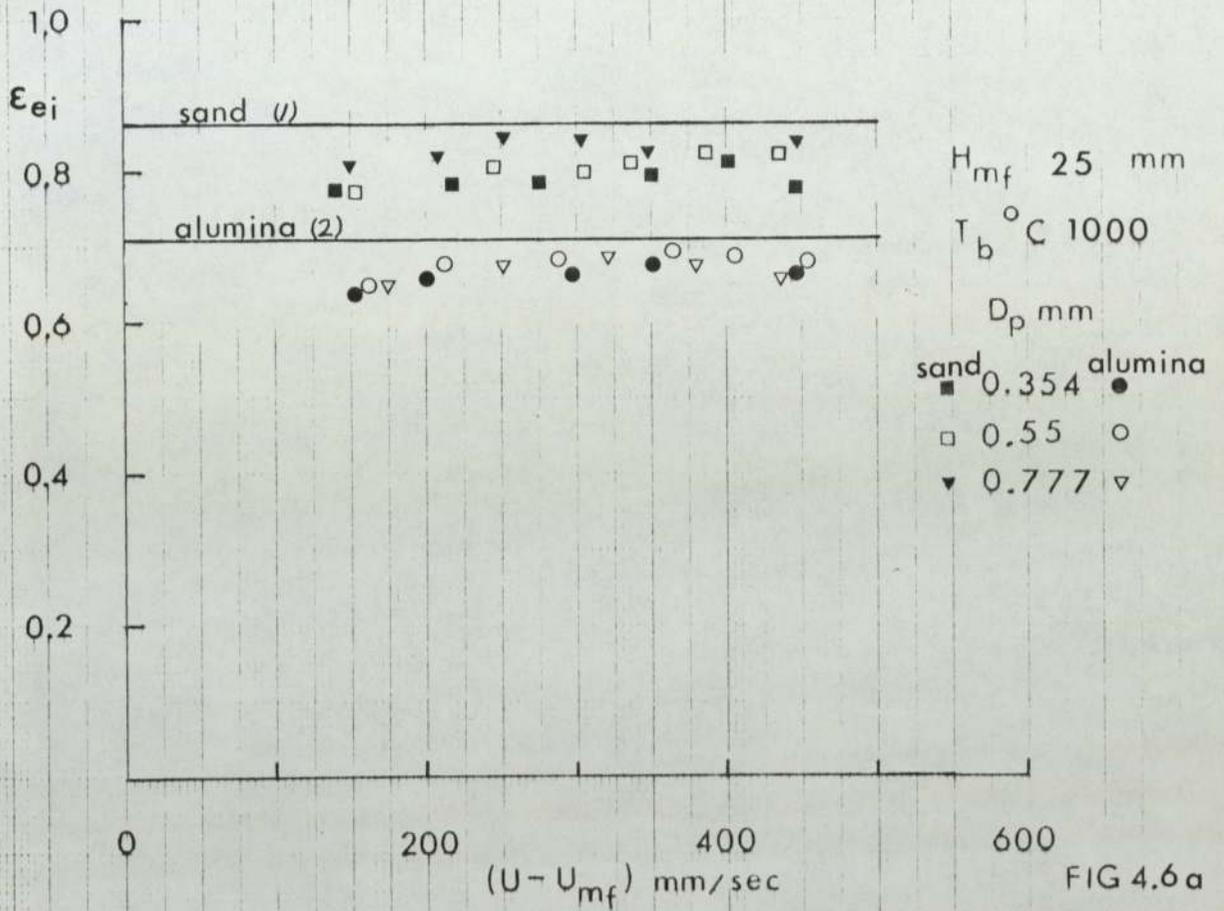
ε_{ei}

sand (1)

alumina (2)

H_{mf} 25 mm

T_b °C 1000

D_p mm

sand 0.354 alumina ●

□ 0.55 ○

▼ 0.777 ▽

$(U - U_{mf})$ mm/sec

FIG 4.6 a

ε_{en}

(1)

(2)

$(U - U_{mf})$ mm/sec

FIG 4.6 b

EFFECT OF EXCESS GAS VELOCITY ON EFFECTIVE EMISSIVITY

H_{mf} 25 mm

T_b °C 1050

D_p mm

sand 0.354 alumina
  ■ 0.354   ●
  □ 0.55   ○
  ▼ 0.777   ▽

FIG 4.7a

FIG 4.7b

EFFECT OF EXCESS GAS VELOCITY ON EFFECTIVE EMISSIVITY

H_{mf} 25 mm

T_b °C 825

D_p mm

silicon carbide	magnesia
■ 0.354	●
□ 0.55	○
▼ 0.777	▽

s-carbide (1)

magnesia (2)

$(U - U_{mf})$ mm/sec

FIG 4.8a

$(U - U_{mf})$ mm/sec

FIG 4.8b

EFFECT OF EXCESS GAS VELOCITY ON EFFECTIVE EMISSIVITY

s-carbide (1)

magnesia (2)

ε_{ei}

H_{mf} 25 mm

T_b °C 925

D_p mm

silicon carbide   magnesia

■ 0.354   ●

□ 0.55   ○

▼ 0.777   ▽

0   200   400   600

$(U - U_{mf})$ mm/sec   FIG 4.9a

ε_{en}

(2)

(1)

0   200   400   600

$(U - U_{mf})$ mm/sec   FIG 4.9b

EFFECT OF EXCESS GAS VELOCITY ON EFFECTIVE EMISSIVITY

FIG 4.10a

FIG 4.10b

EFFECT OF EXCESS GAS VELOCITY ON EFFECTIVE EMISSIVITY

s-carbide (1)

magnesia (2)

H_{mf} 25 mm

T_b °C 1050

D_p mm

silicon carbide   magnesia
- ■ 0.354 ●
- □ 0.55   ○
- ▼ 0.777 ▽

$(U - U_{mf})$ mm/sec

FIG 4,11a

(2)

(1)
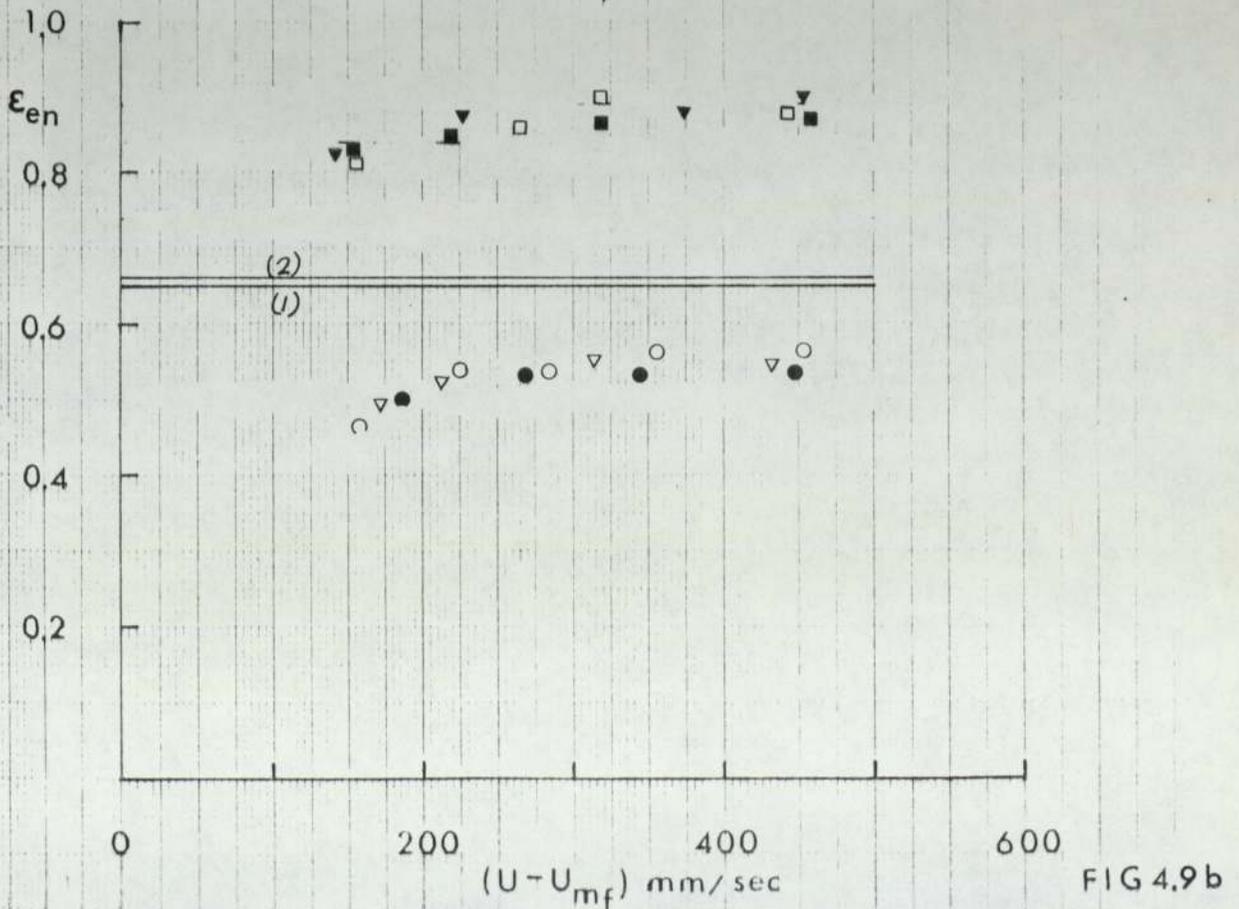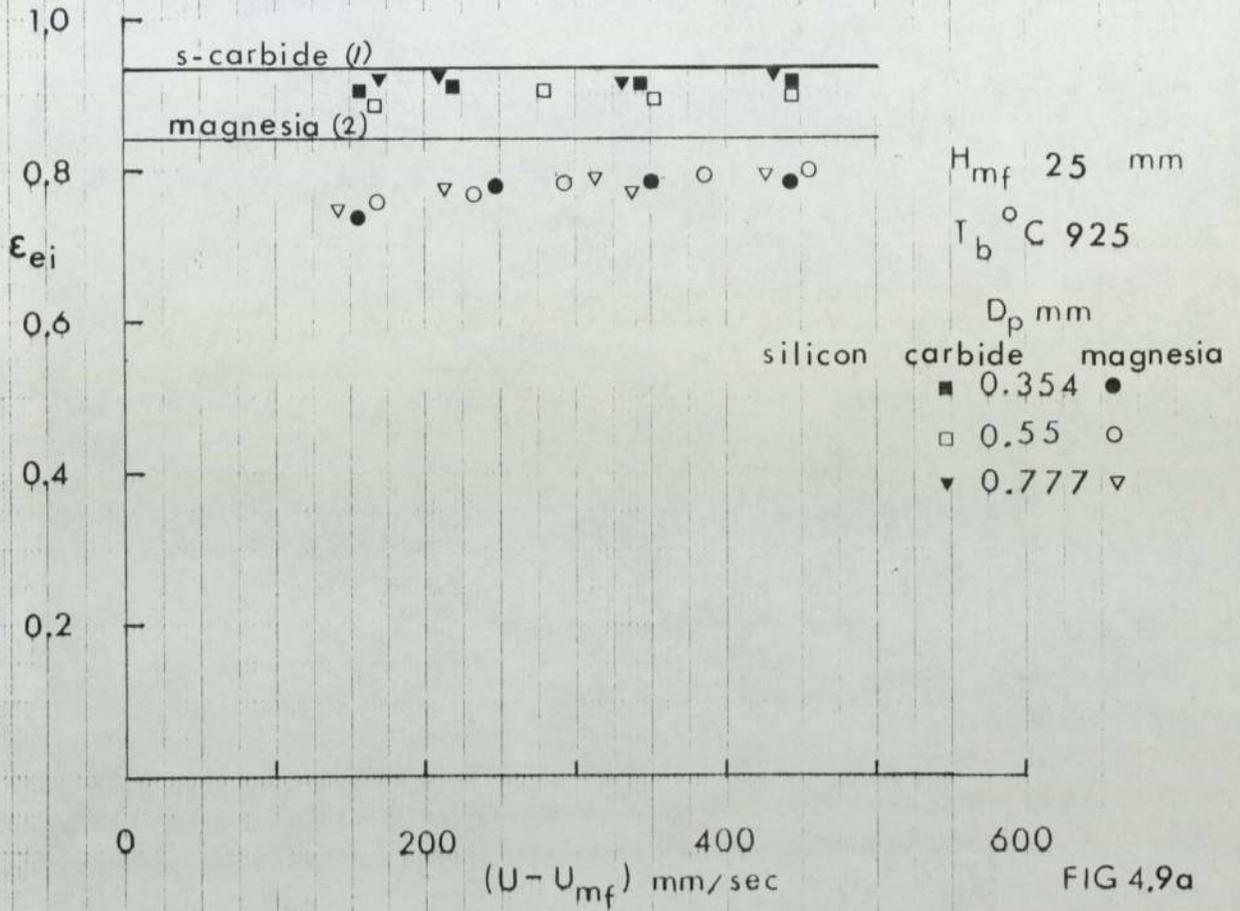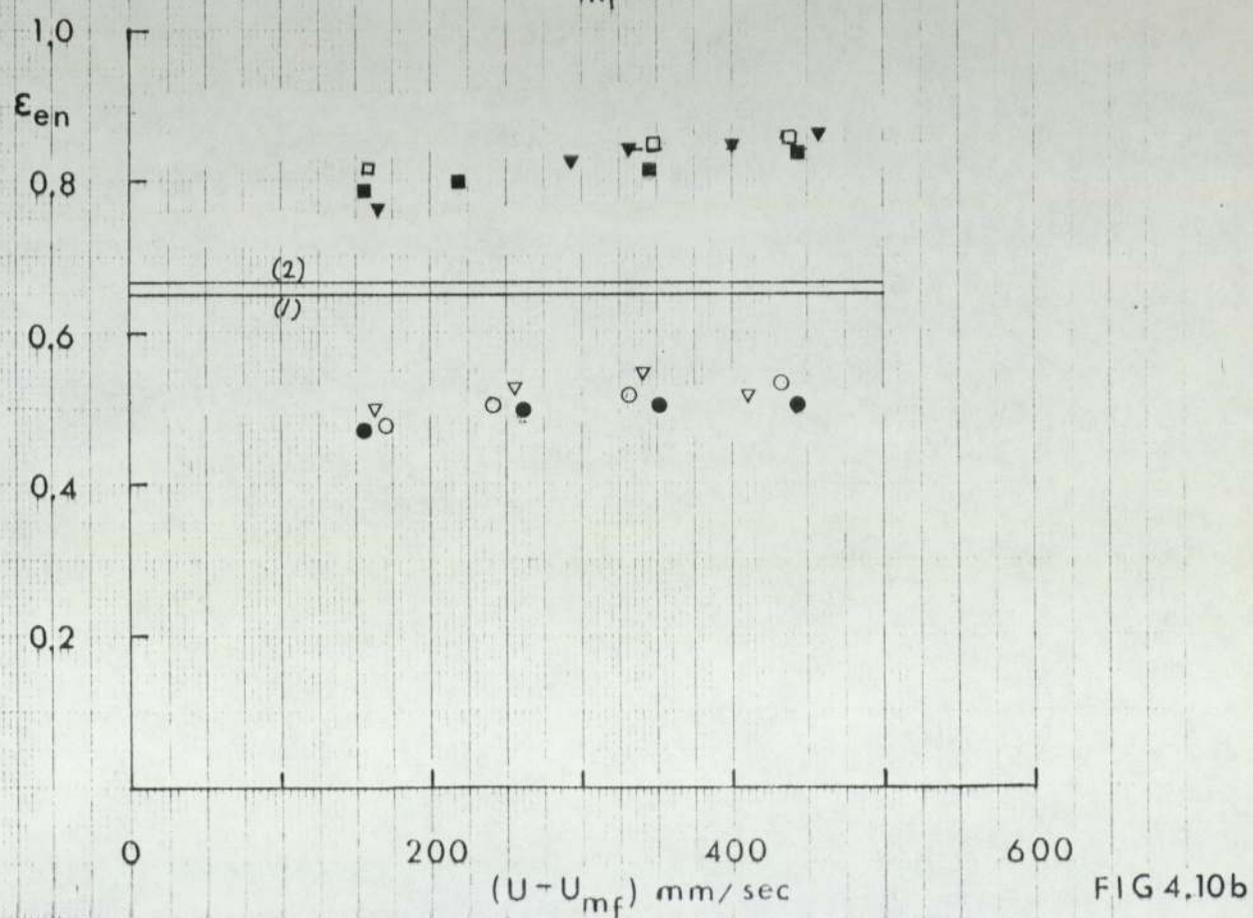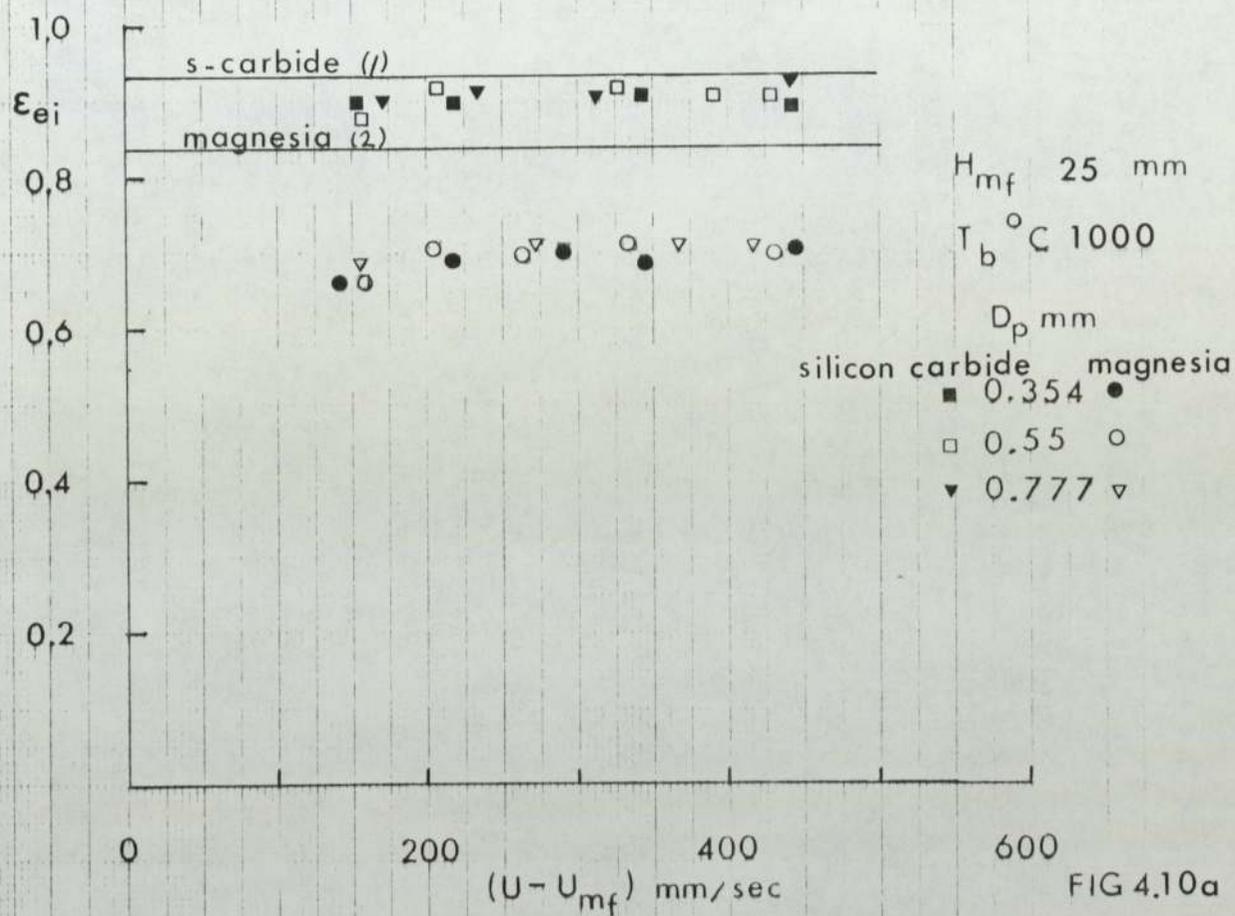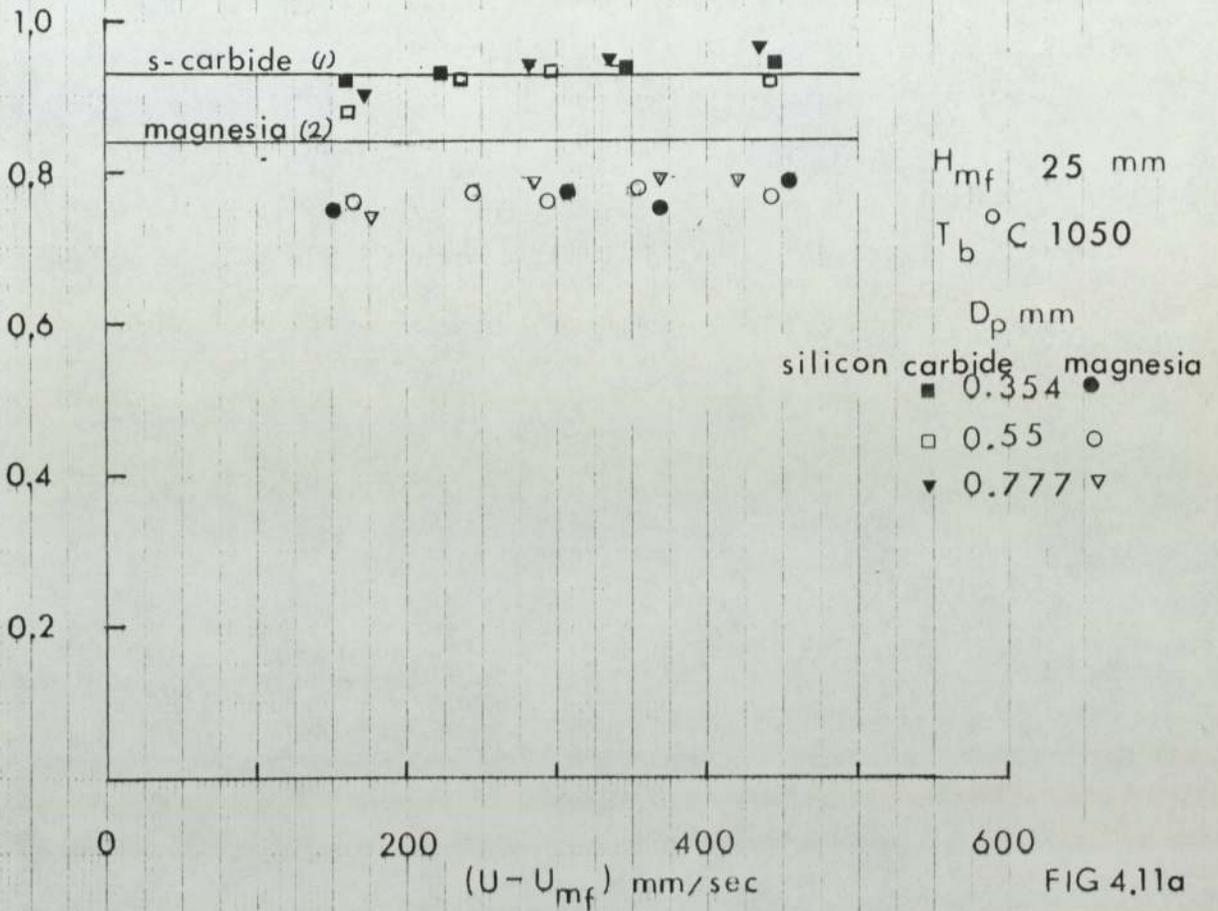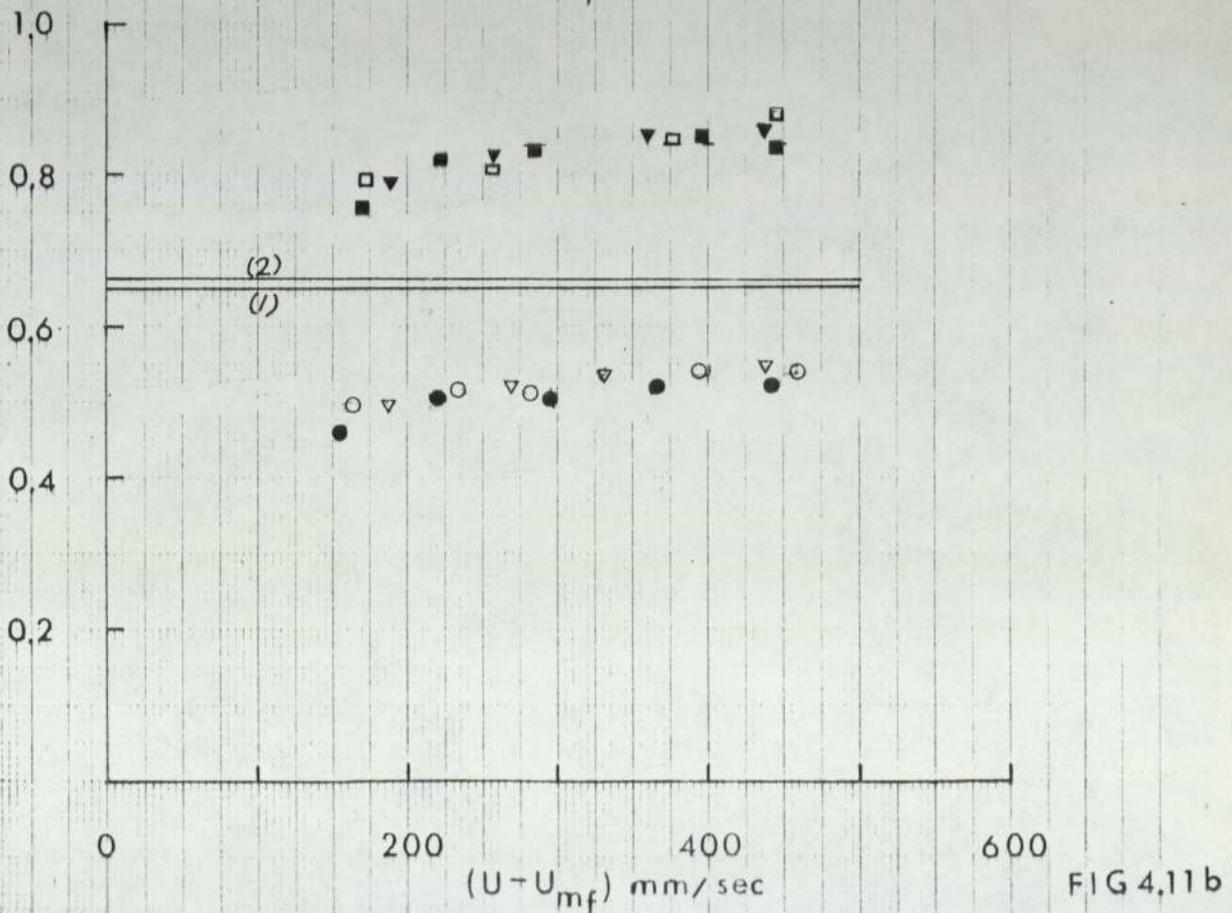
$(U - U_{mf})$ mm/sec

FIG 4,11b

EFFECT OF BED TEMPERATURE ON
SURFACE EMITTED RADIATIVE FLUX

RADIATIVE FLUX (WATTS $-M^{-2}$)

25×10^4

20

15

BLACK BODY

10

5

0

7   8   9   10   11   12 13×10^2

BED TEMPERATURE $^{\circ}C$

RANGE OF RESULTS
FROM [7]
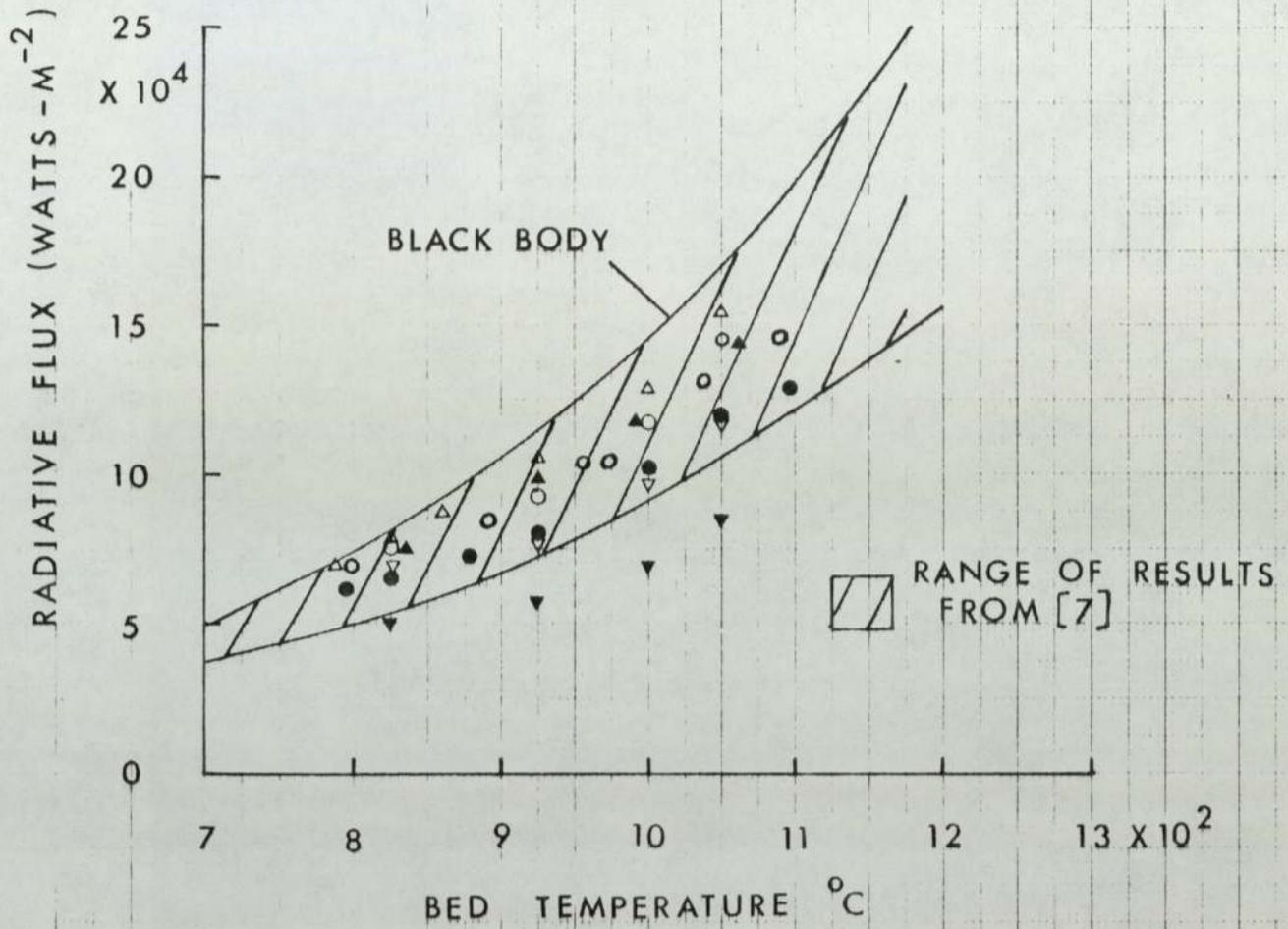
		MAT^L	D_P mm	$(U-U_{mf})$ mm/sec
a	⌀	SAND	0.55	340
b	●	— // —	— // —	— // —
a	▽	ALUMINA	0.354	344
b	▼	— // —	— // —	— // —
a	△	SILICON CARBIDE	0.777	334
b	▲	— // —	— // —	— // —

a ISOTHERMAL BED

b NONISOTHERMAL BED

FIG 4.12
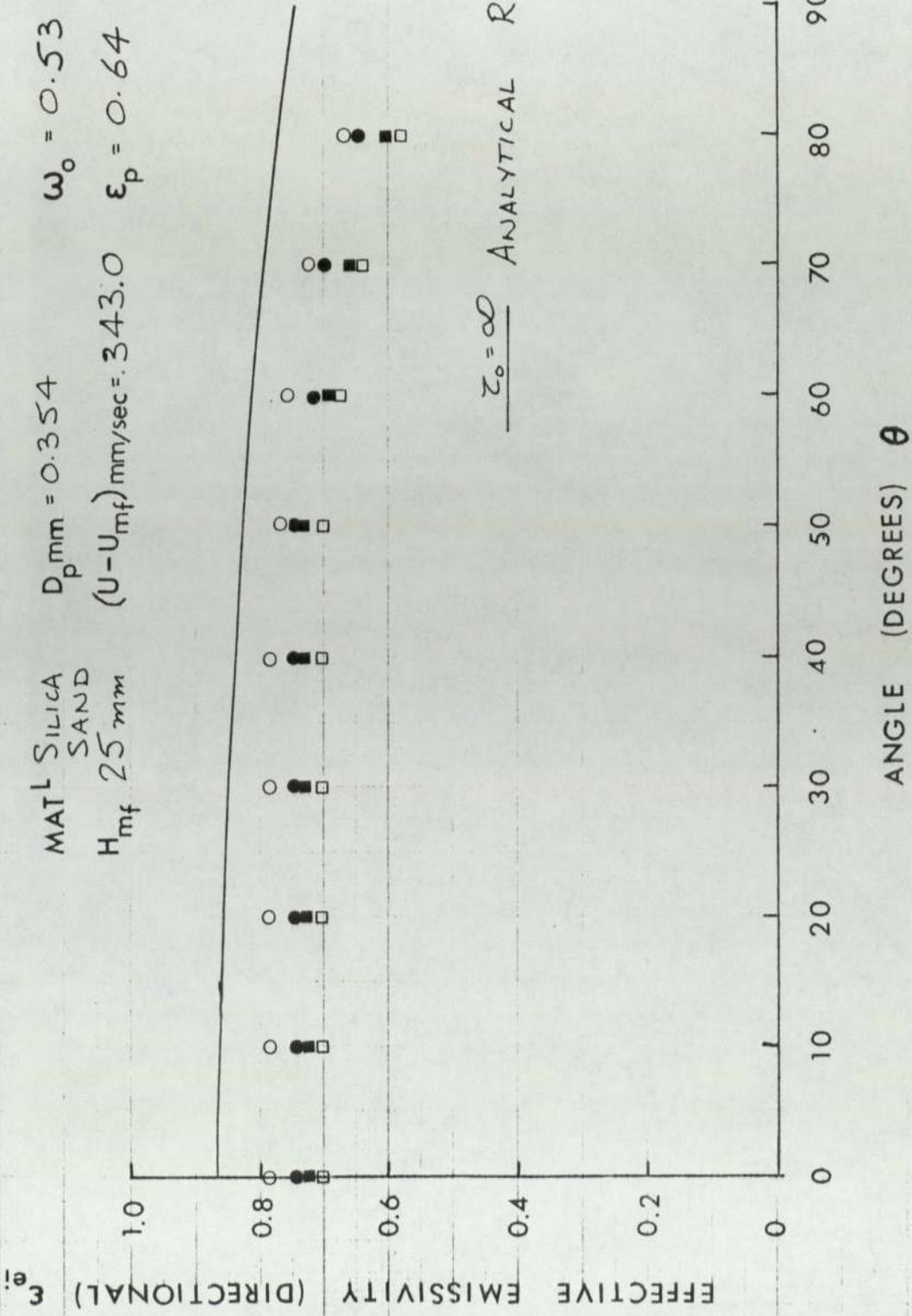
FIG 4.13

FIG 4.14

MAT^L SILICA SAND

	T_b °C	T_s °C
●	1050	647
□	1000	635
■	925	604
○	825	590

$\omega_o = 0.53$

$\epsilon_p = 0.64$

D_p mm 0.55

H_{mf} 25 mm

$(U-U_{mf})$ mm/sec 338.6

$Z_o = \infty$

_____ ANALYTICAL REF. (18)

ANGLE (DEGREES) θ

EFFECTIVE EMISSIVITY (DIRECTIONAL) ϵ_{ei}

FIG 4.15

FIG 4.16

FIG 4.17

MAT^L SILICA   D$_p$mm 0.354 ω_o = 0.53
        SAND
H$_{mf}$ 25 mm   (U–U$_{mf}$) mm/sec 343.0 ε_p = 0.64

T$_b$ °C		T$_s$ °C
●	1050	—
□	1000	—
■	925	—
○	825	—

EFFECTIVE EMISSIVITY (DIRECTIONAL) ε_{en}

1.0   0.8   0.6   0.4   0.2   0

0   10   20   30   40   50   60   70   80   90

ANGLE (DEGREES) θ

FIG 4.18

FIG 4.19

MAT^L SILICON CARBIDE D_p mm 0.354 $\omega_o = 0.37$

H_{mf} 25 mm $(U-U_{mf})$ mm/sec 340 $\varepsilon_p = 0.77$

T_b °C		T_s °C
●	1050	—
□	1000	—
■	925	—
○	825	—

1.0

0.8

0.6

0.4

0.2

0

EFFECTIVE EMISSIVITY (DIRECTIONAL) ε_{en}

0   10   20   30   40   50   60   70   80   90
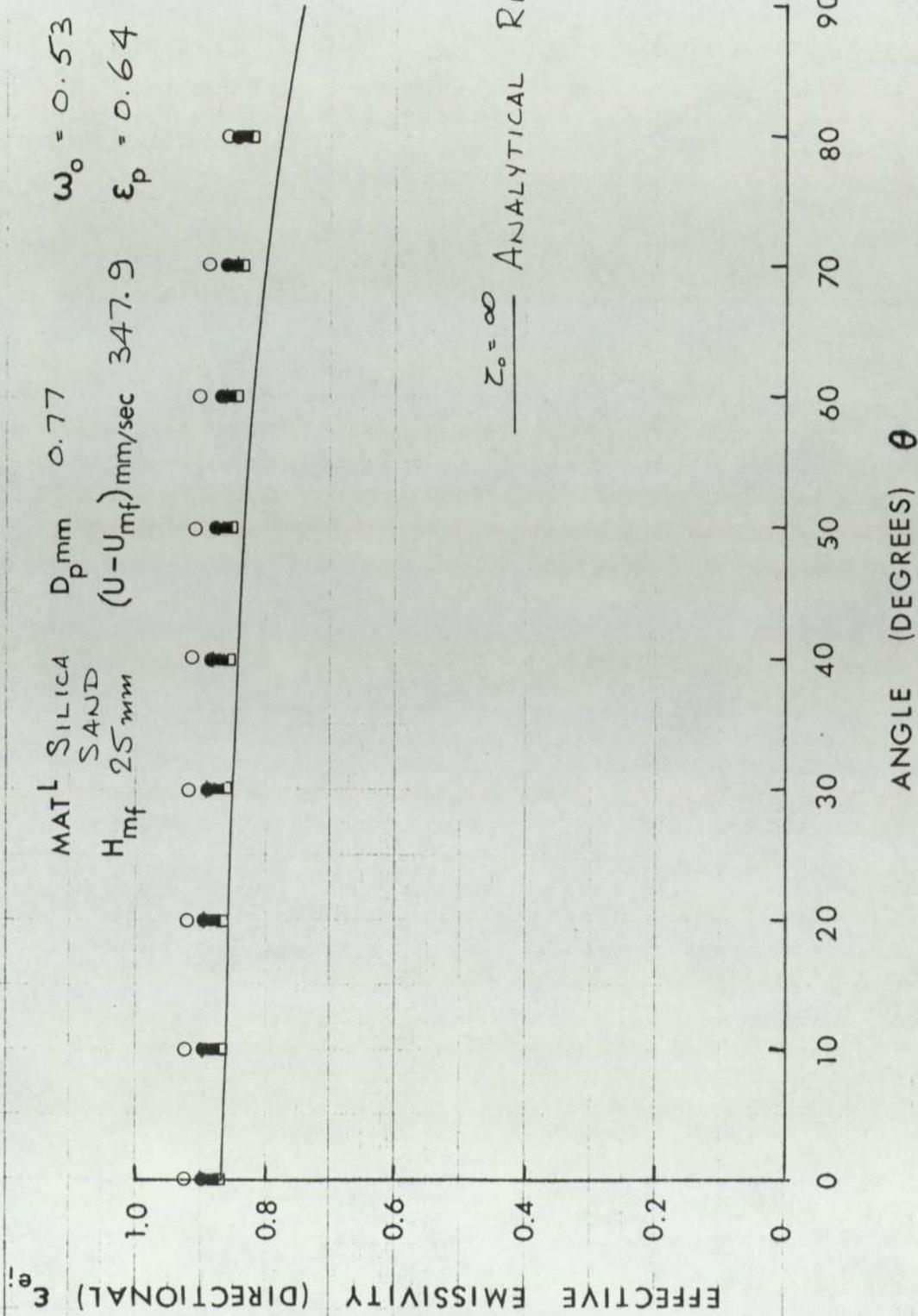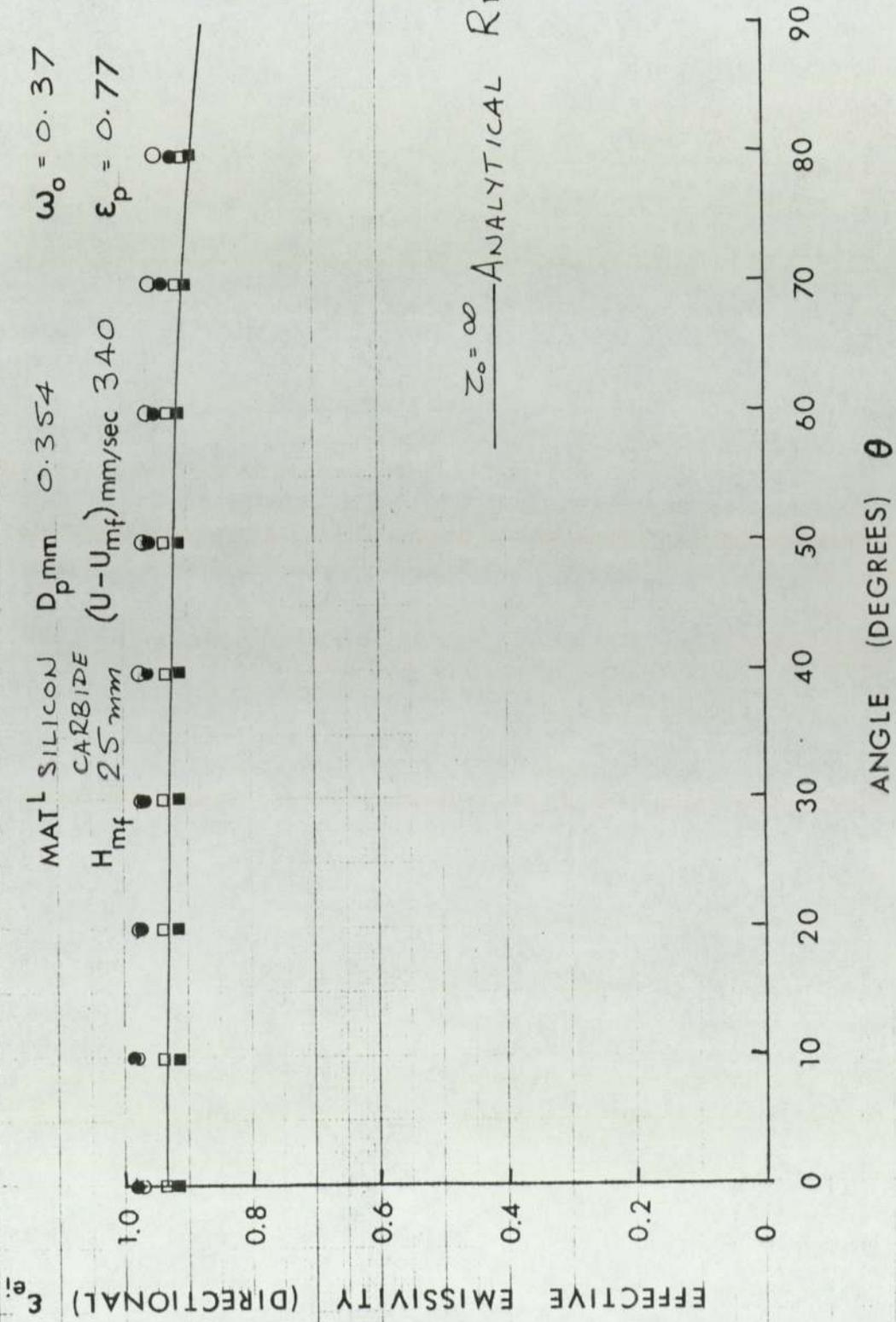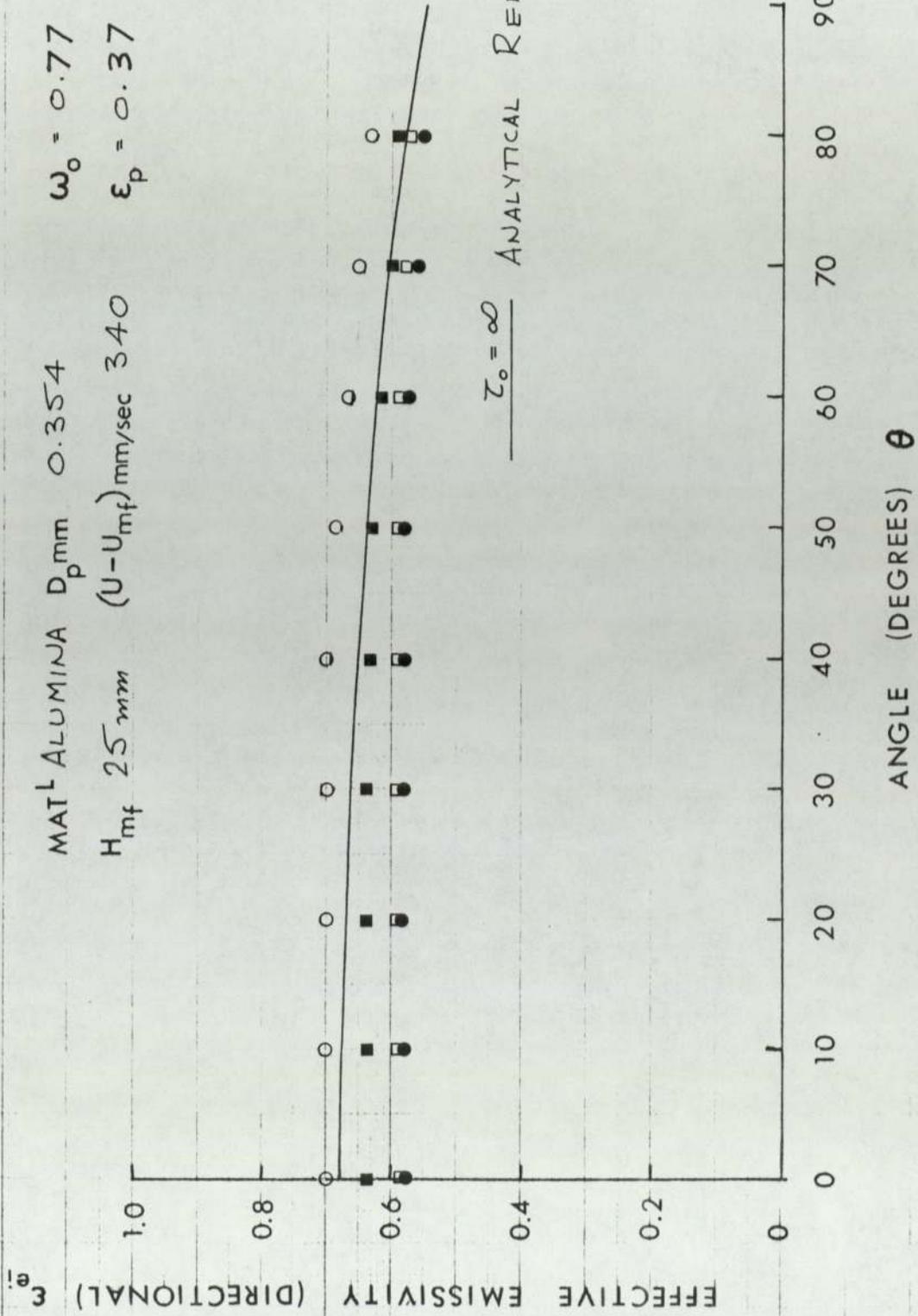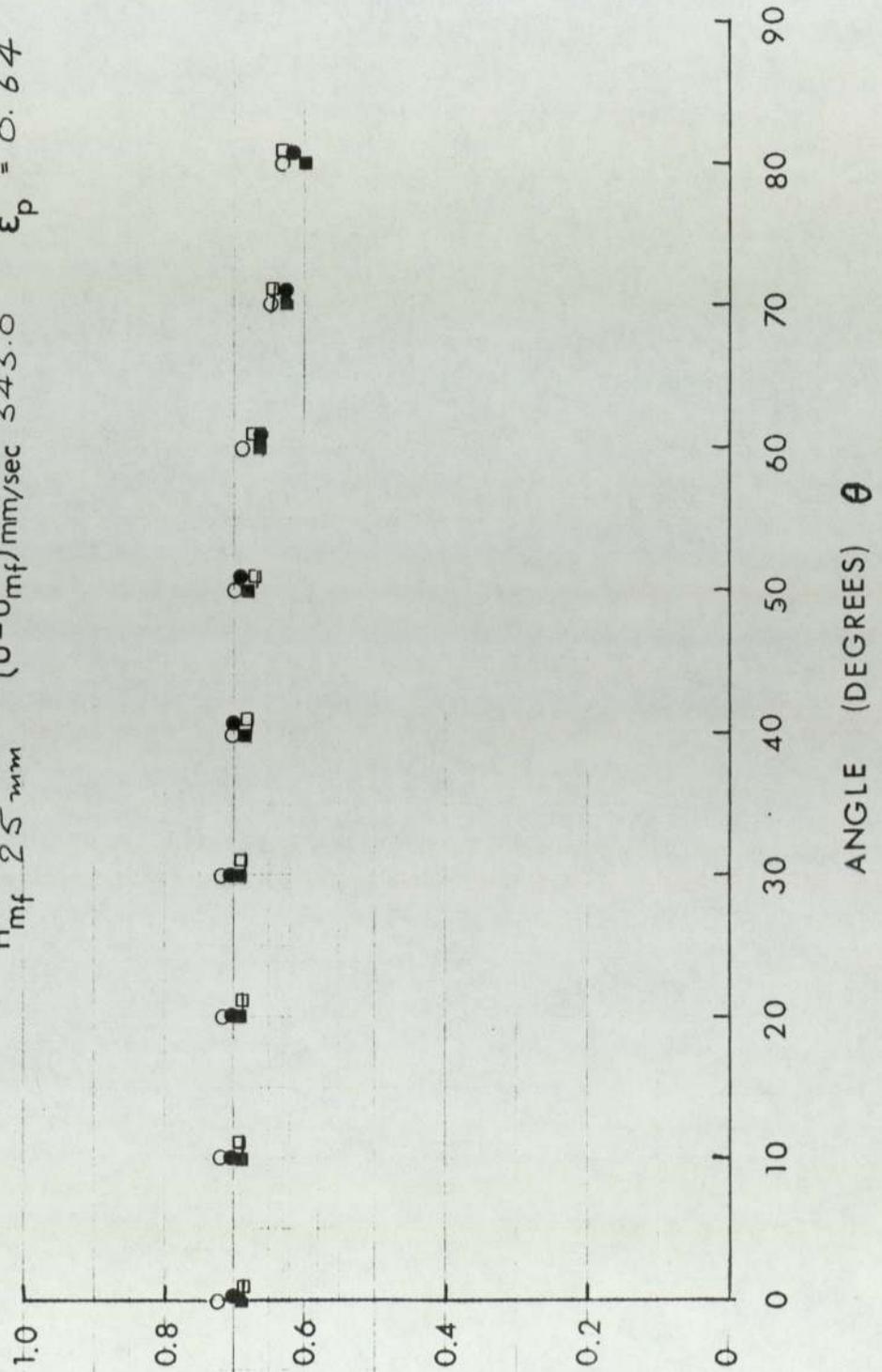
ANGLE (DEGREES) θ
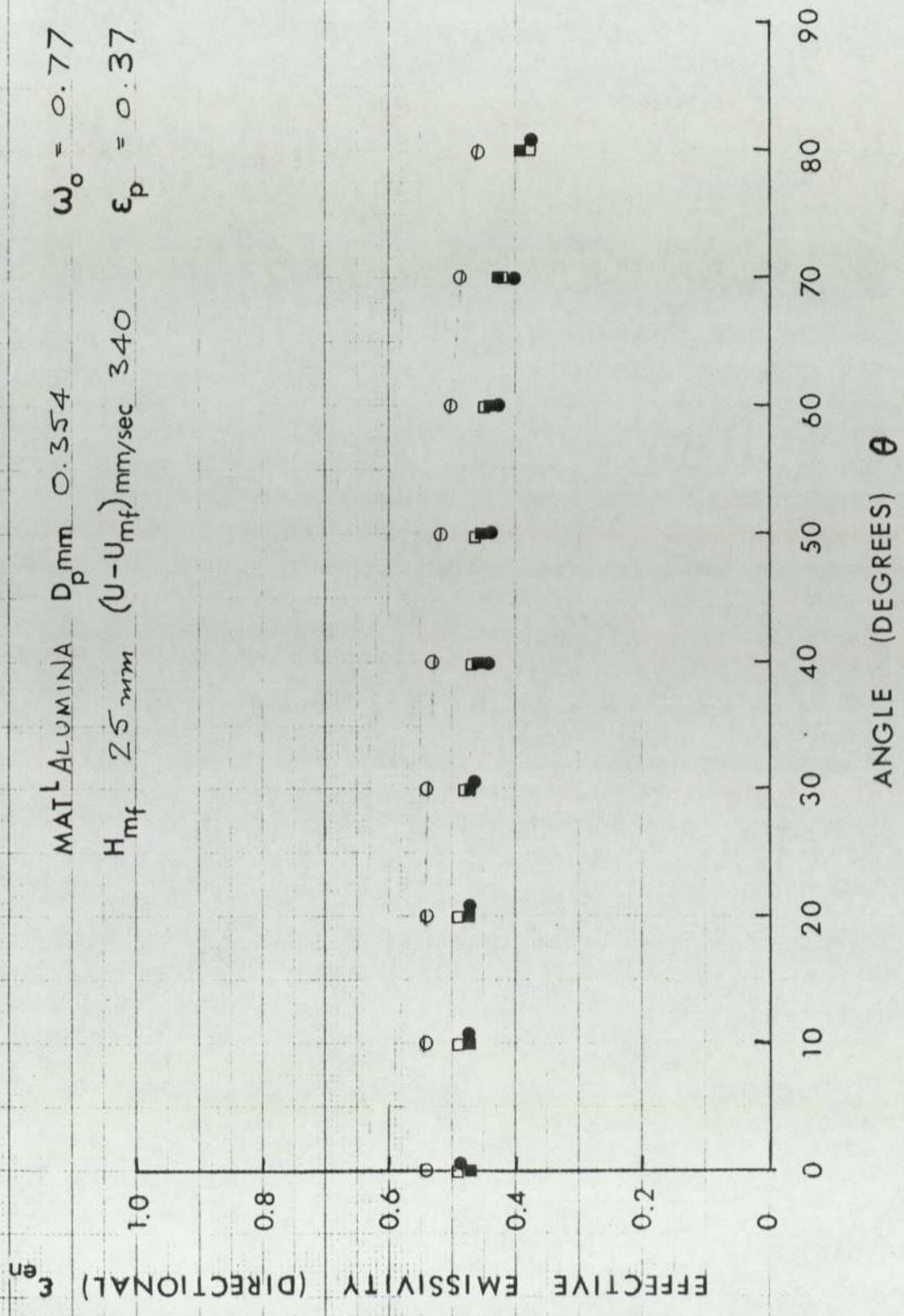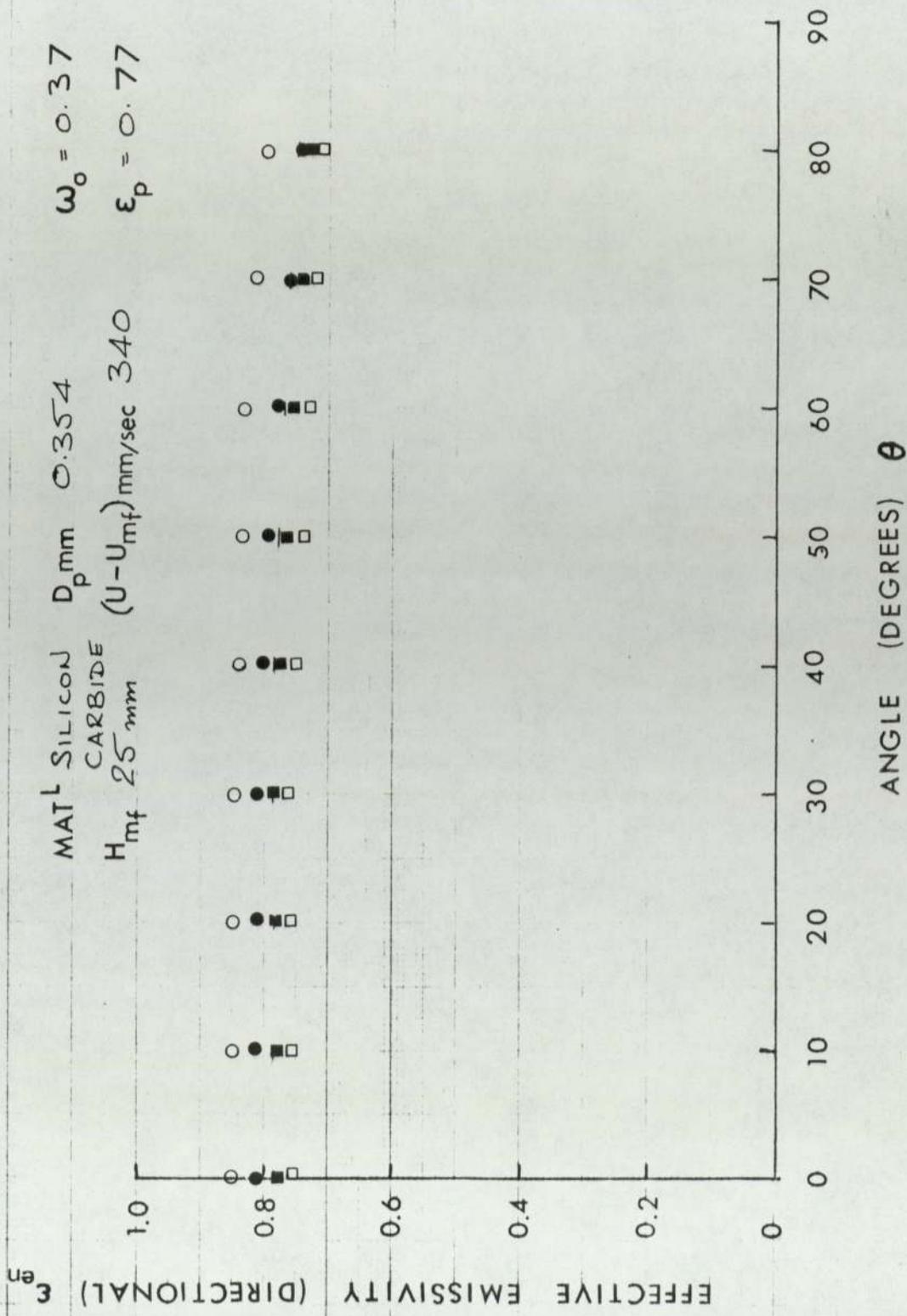
FIG 4.20

EFFECT OF EXCESS GAS VELOCITY ON
EFFECTIVE EMISSIVITY OF A BINARY
MIXTURE

MAT^L. SILICA SAND

H_{mf} 3.5 mm

T_b °C

950 ±10

D_p mm	%	%	%
0.275	5.1	10.3	22.8
	●	△	■
0.777	94.9	89.7	77.2
$U_{mf_{mix}}$ mm/s	158.3	133.4	89.5

ε_{en}

0   200   400   600

$(U-U_{mf})$ mm/sec   FIG 4.21a

MAT^L. SILICON CARBIDE

H_{mf} 4.2 mm

T_b °C

935 ±10

D_p mm	%	%	%
0.275	5.5	12.4	26.3
	●	△	■
0.777	94.5	87.6	73.7
$U_{mf_{mix}}$ mm/s	222.5	171.4	107.6

ε_{en}

0   200   400   600
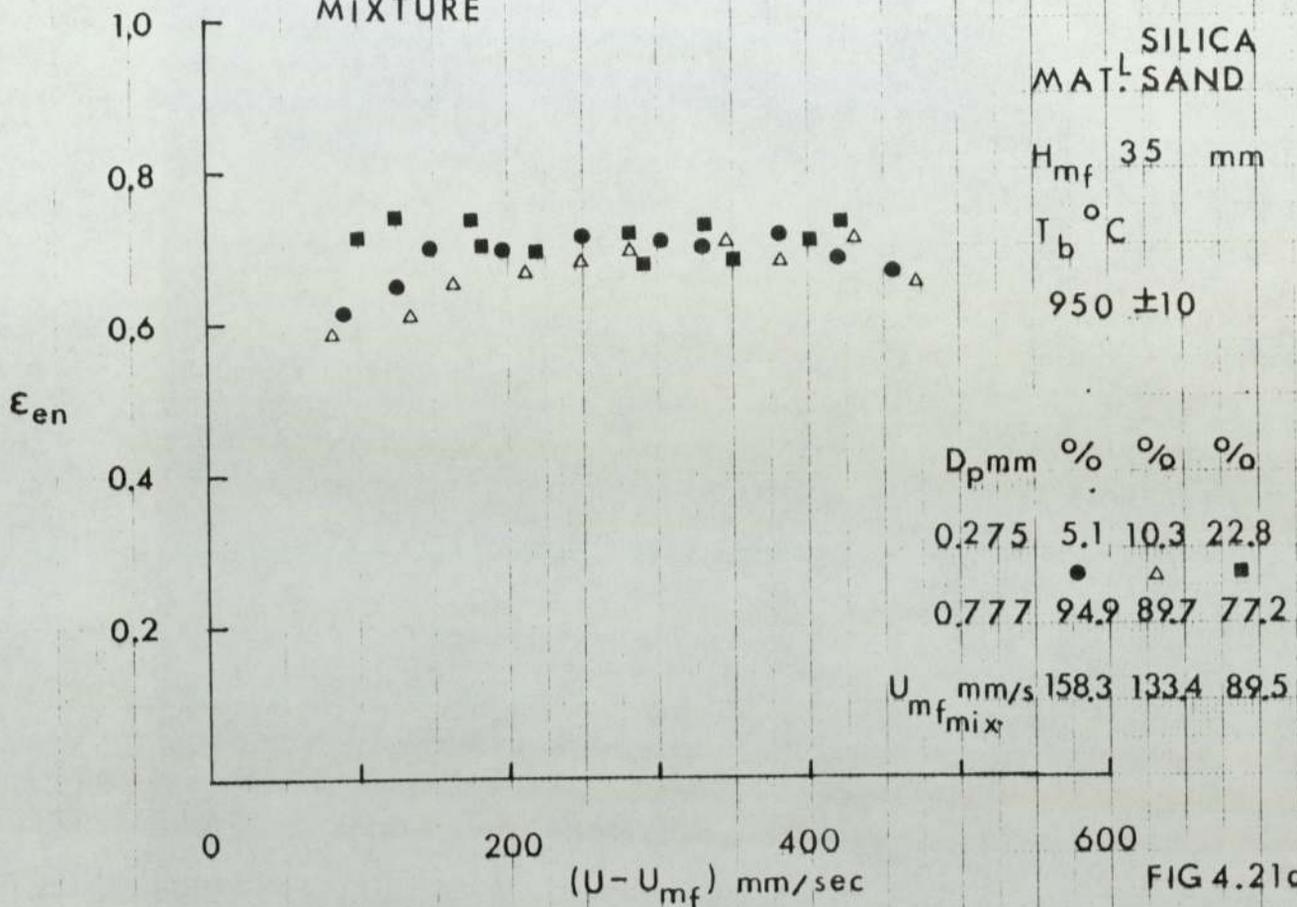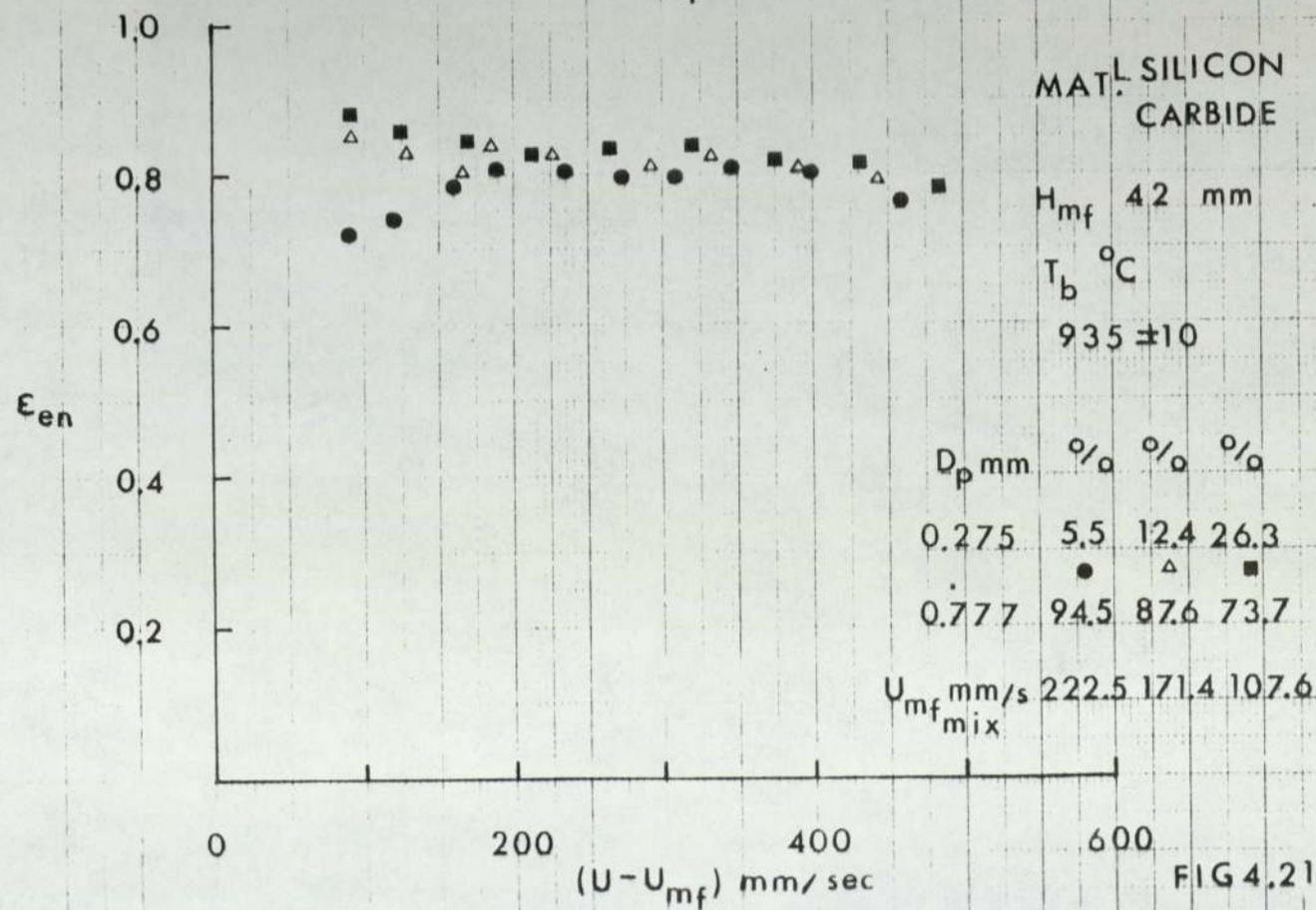
$(U-U_{mf})$ mm/sec   FIG 4.21b

EXPERIMENTAL CONFIGURATION FOR A STUDY OF '2-D' BUBBLES

PHOTOFLOOD

SCREEN

SCREEN

PHOTOFLOOD

BUBBLE INJECTION

HUMIDIFIED AIR SUPPLY
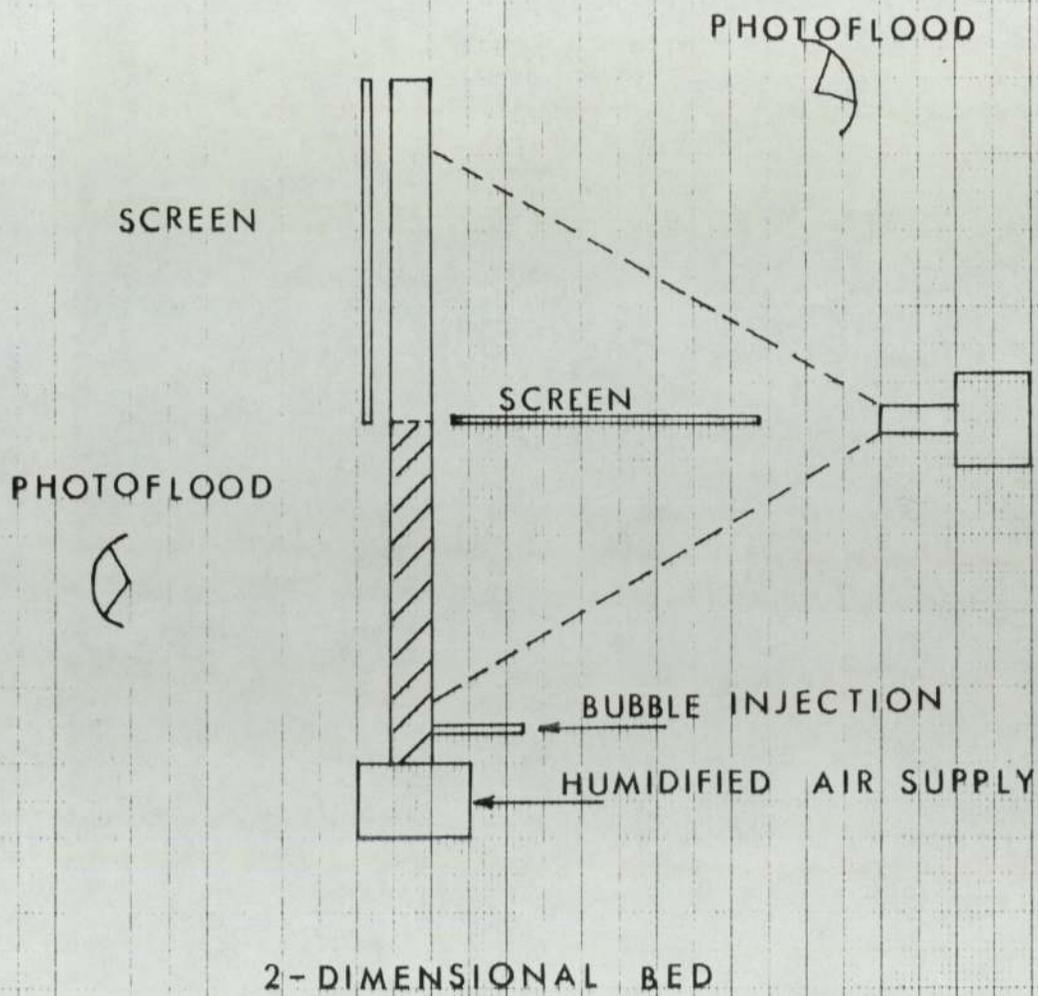
2-DIMENSIONAL BED

FIG 5.1
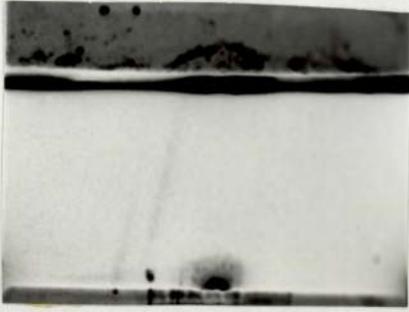
a)   0.05 sec
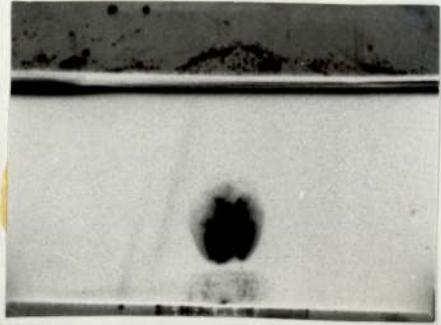
b)   0.15 sec
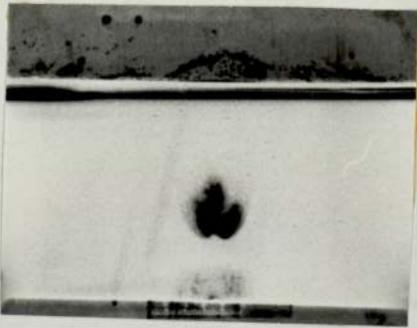
c)   0.2 sec
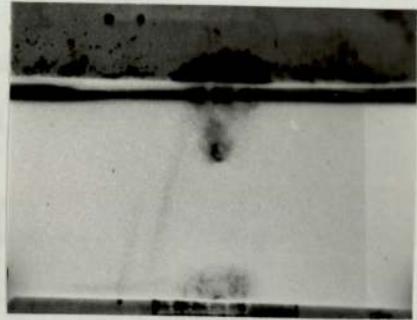
d)   0.3 sec
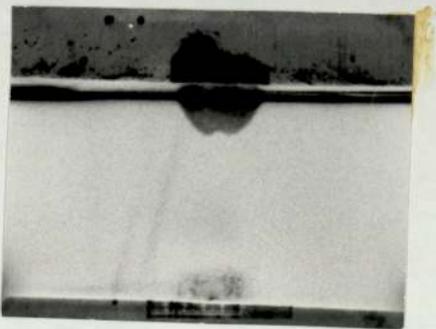
e)   0.35 sec
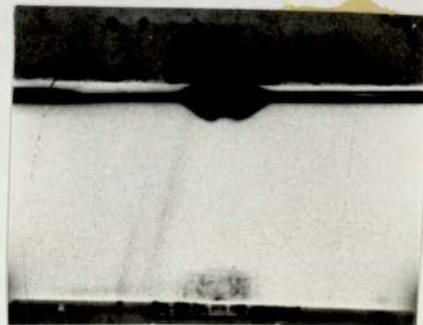
f)   0.39 sec
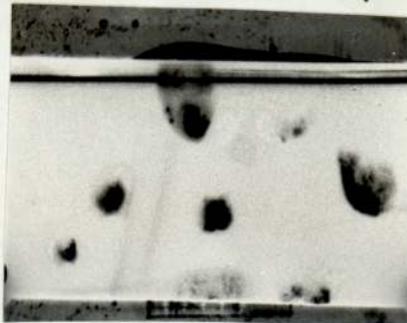
Freely bubbling bed

FIG 5.3

PREDICTED AND OBSERVED
PARTICLE CLOUD TRAJECTORIES
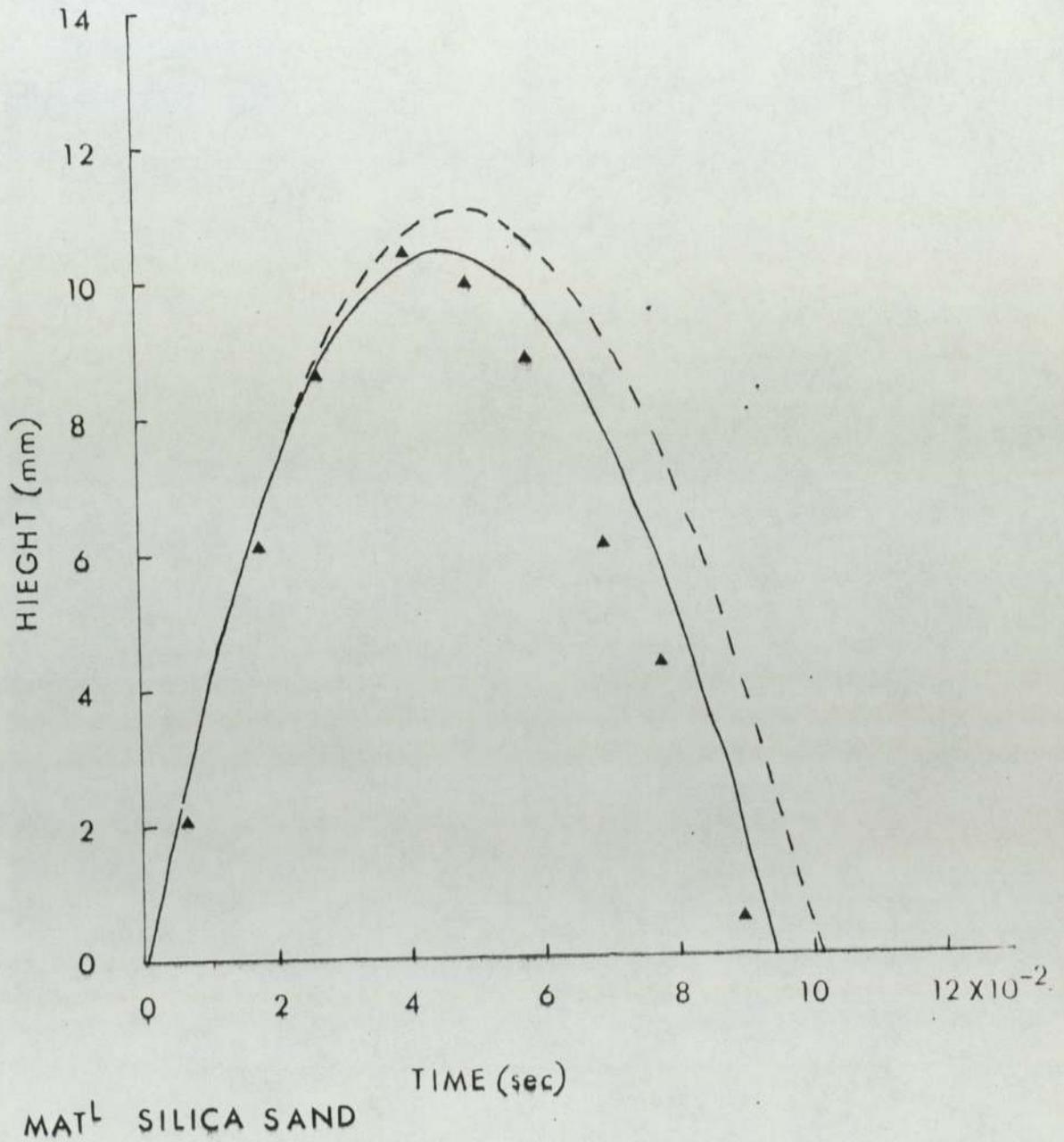
MATᴸ SILICA SAND

	U_b mm/sec	T_b °C	D_p mm
▲ ——	355	20	0.354
– – –	–//–	1000	–//–

FIG 5.4

PREDICTED AND OBSERVED
PARTICLE CLOUD TRAJECTORIES

MAT^L SILICA SAND

U$_b$ mm/sec	T$_b$ ^{o}C	D$_p$ mm
▲—336.9	20	0.777
— — — -//-	1000	-//-

FIG 5.5

PREDICTED AND OBSERVED
PARTICLE CLOUD TRAJECTORIES

MAT^L SILICA SAND

	U_b mm/sec	T_b ^{o}C	D_p mm
▲	323	20	0.55
–//–		1000	–//–

FIG 5.6

PHOTOGRAPHIC STUDY OF BED SURFACE

REFLECTING PLATE

45°
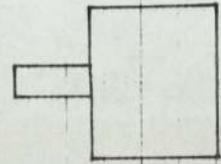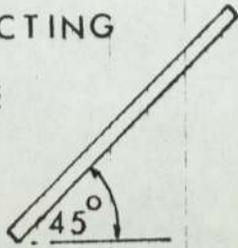
CAMERA
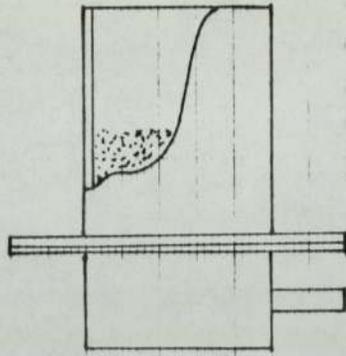
AIR SUPPLY

FLUIDISED BED

FIG 5.7

NUMBER OF BUBBLES/FRAME
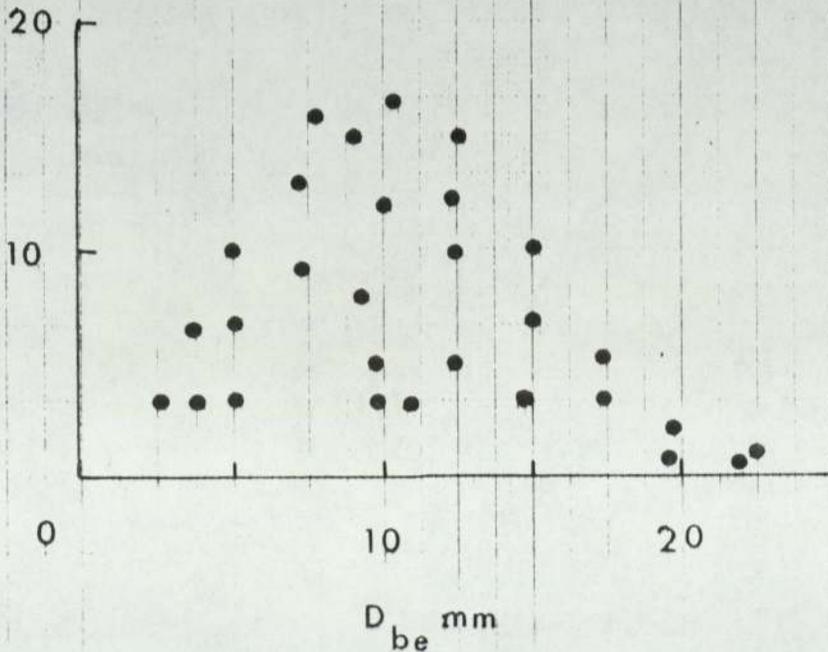
$U - U_{mf}$

40.76 mm/sec

$T_b \, °C$

20

FIG 5.8

D_{be} mm

FIG 5.9

MAT^L SAND

	U−U_{mf}\nmm/sec	T_b °C\n20	H_{mf}\nmm\n10	D_p\nmm\n0.15
▶	12.98			
▽	17.66			
■	23.73			
□	28.9			
○	40.76			

D_be\nmm

PROBABILITY

FIG 5.10

FIG 5.11

MAT^L SAND

U-U$_{mf}$ mm/sec	T$_b$ °C	H$_{mf}$ mm	D$_p$ mm
	20	50	0.15
► 12.98			
▽ 17.66			
■ 23.73			
□ 28.9			

D$_{be}$ mm

PROBABILITY

FIG 5.12

MAT^L SAND

U-U$_{mf}$ mm/sec	T$_b$°C	H$_{mf}$ mm	D$_p$ mm
	20	70	0.15
▶ 12.98			
▽ 17.66			
■ 23.73			
□ 28.9			

D$_{be}$ mm

PROBABILITY

FIG 5.13

FIG 5.14

FIG 5.15

MAT^L SAND $T_b^{\circ}C$ H_{mf} D_p

               mm   mm

             900   40   0.354

$U - U_{mf}$
mm/sec

▼   46.2

▽   91

■   168
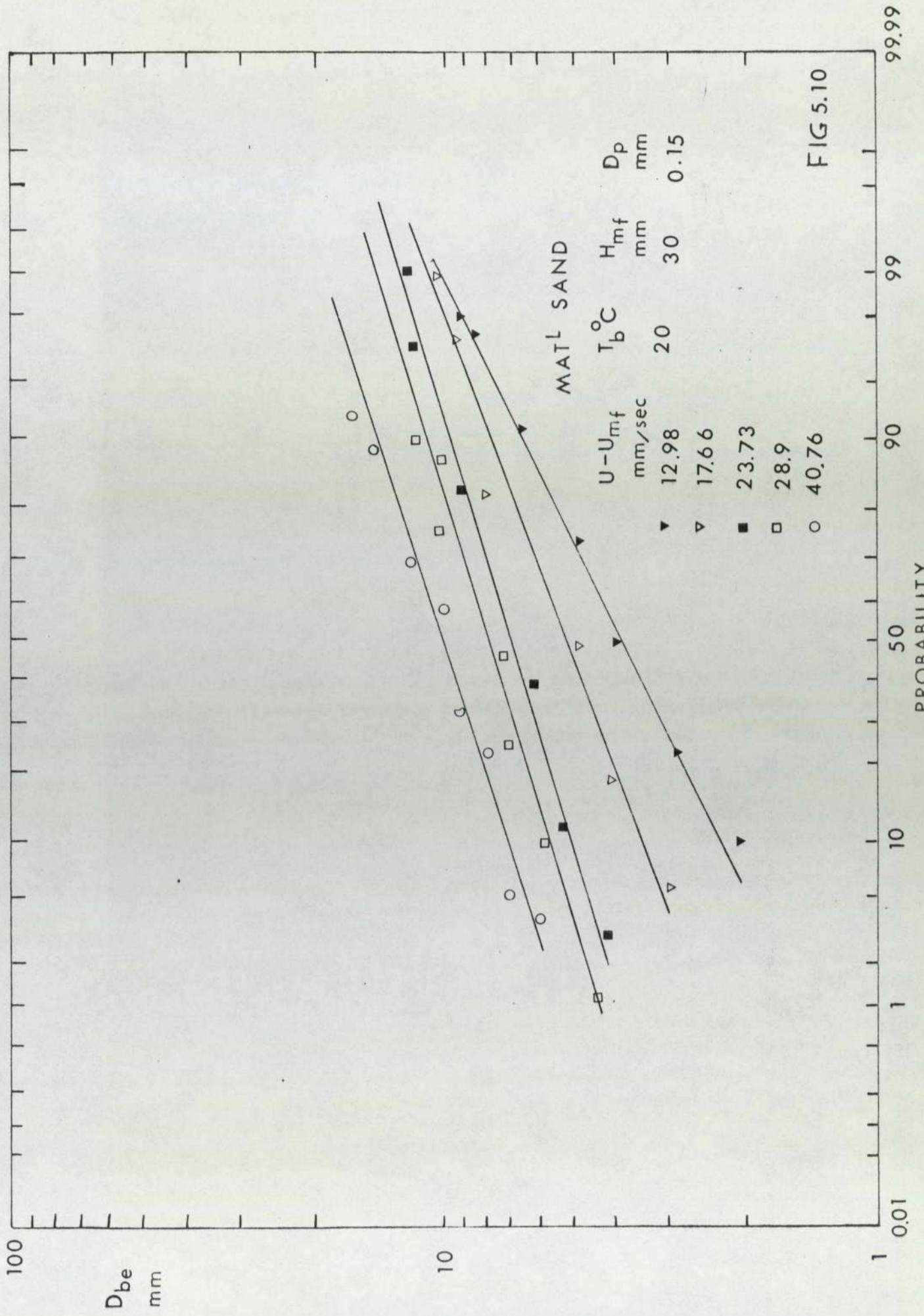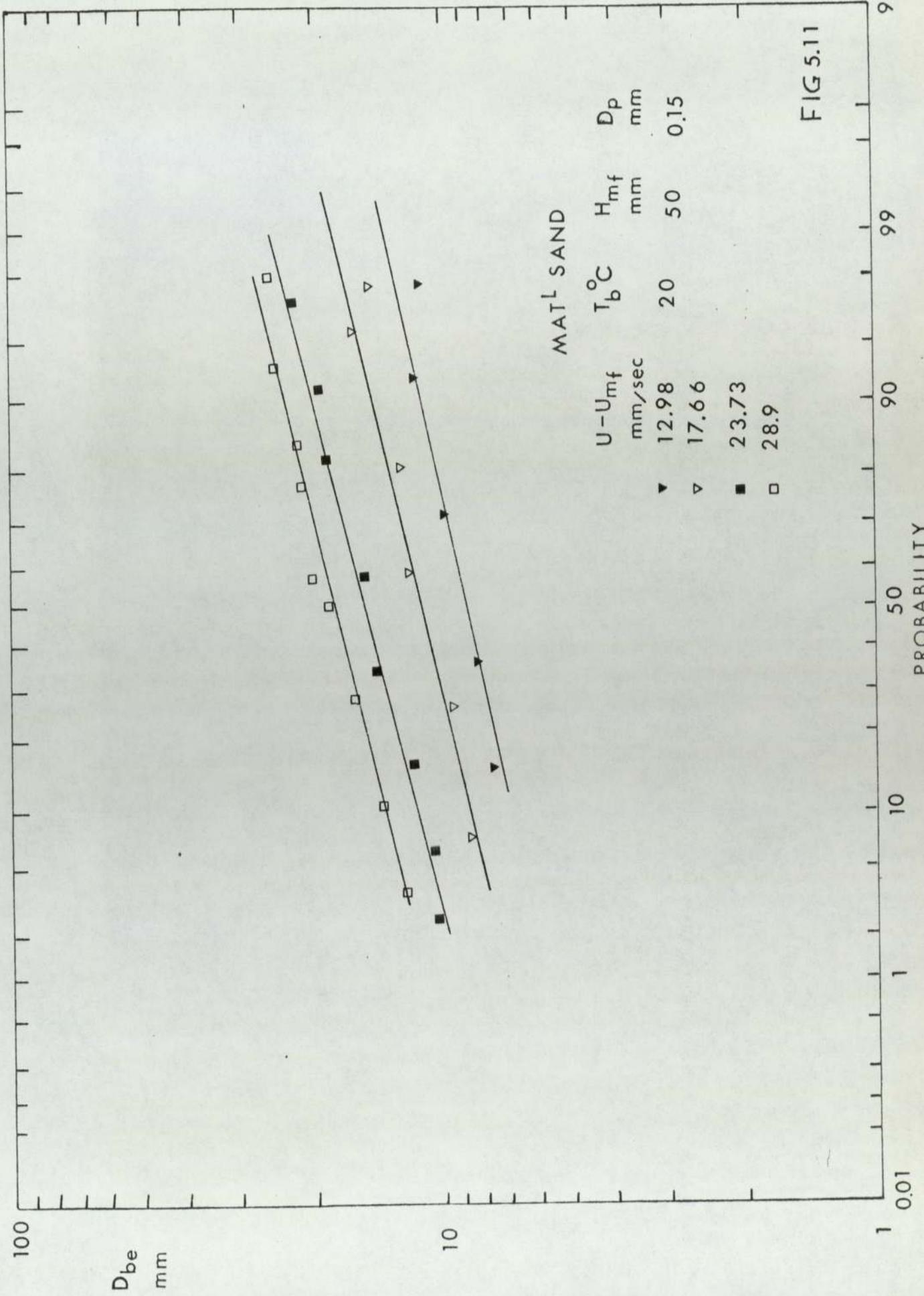
□   271

PROBABILITY

D_{be}
mm

100

10

0.01   1   10   50   90   99   99.99
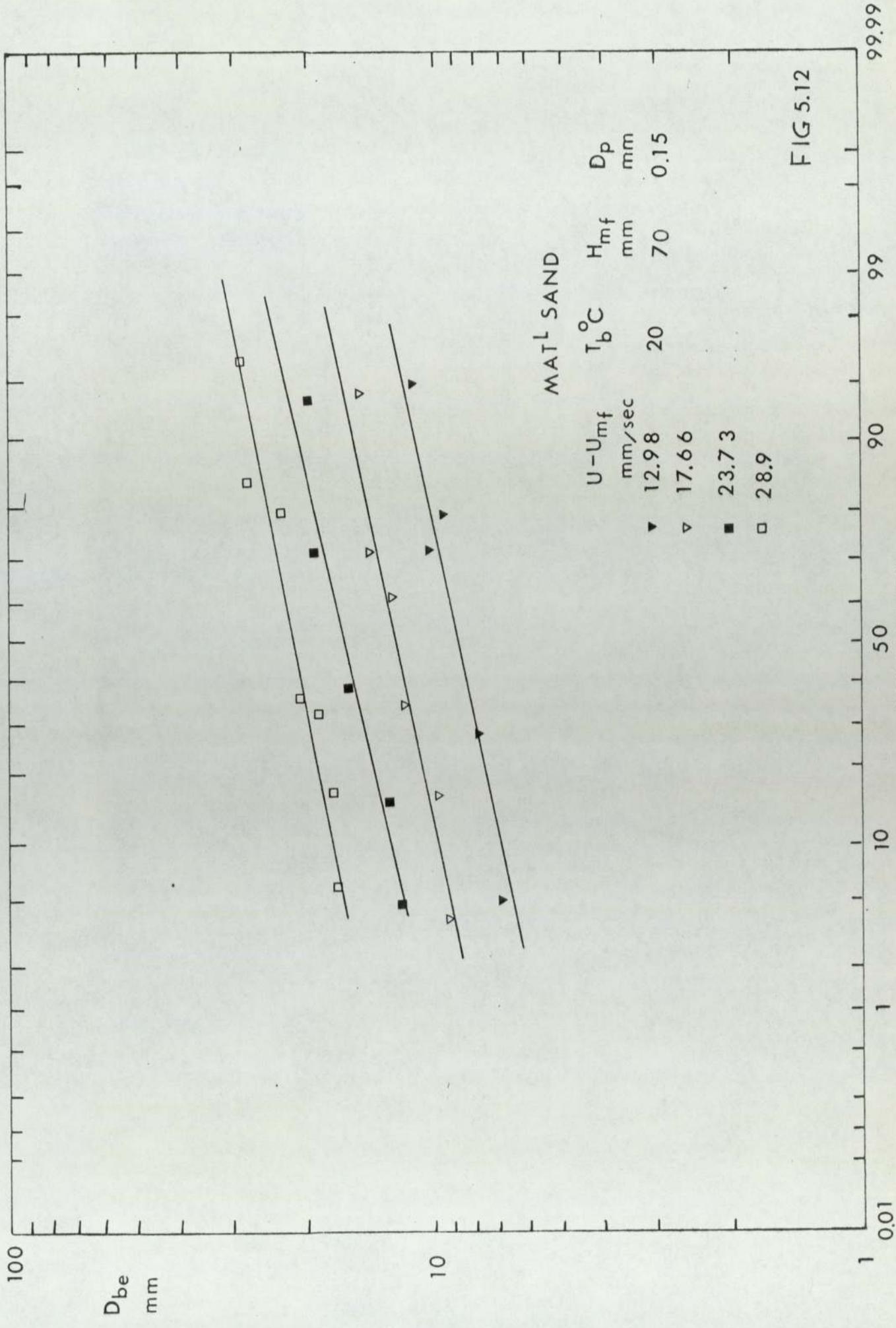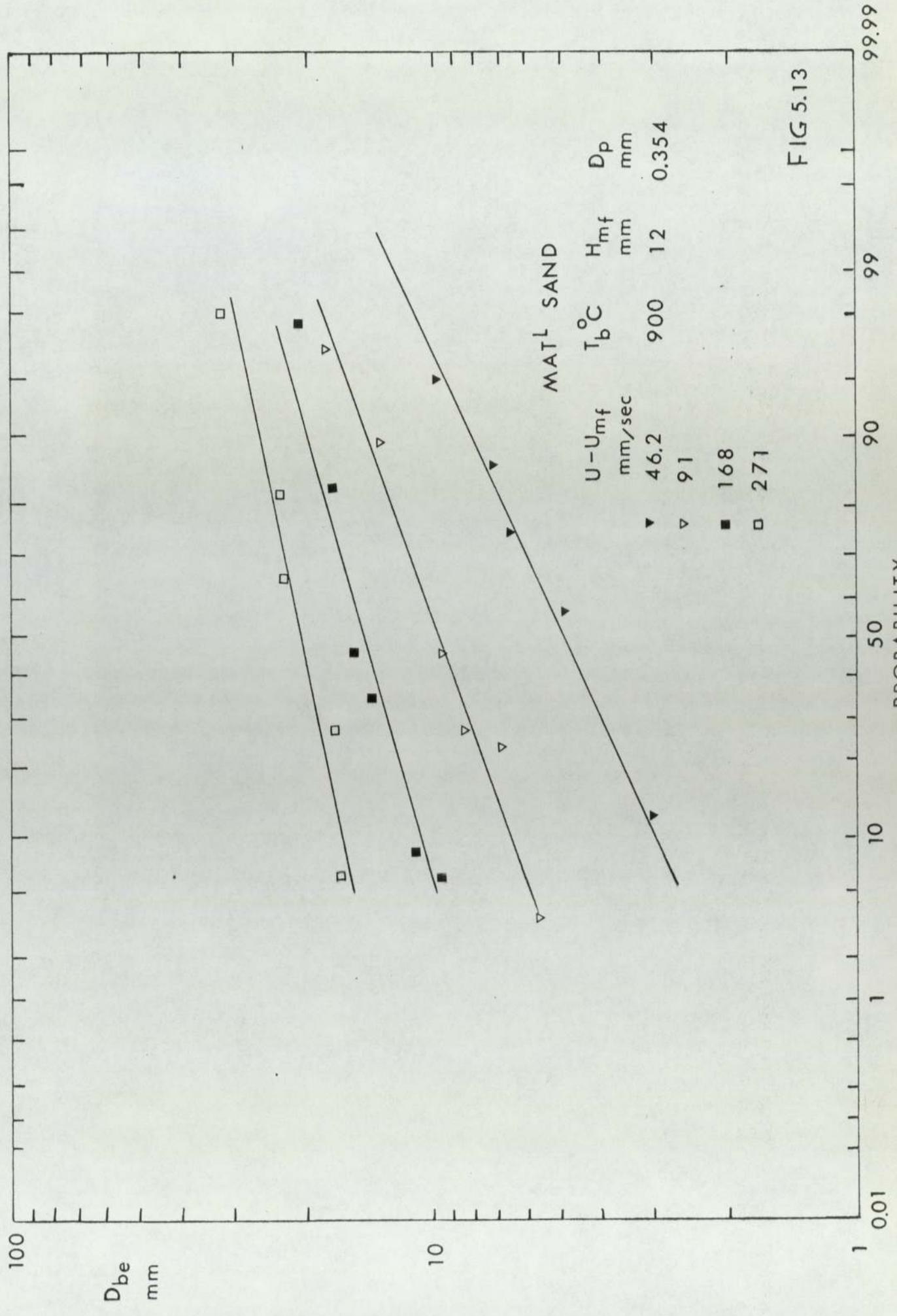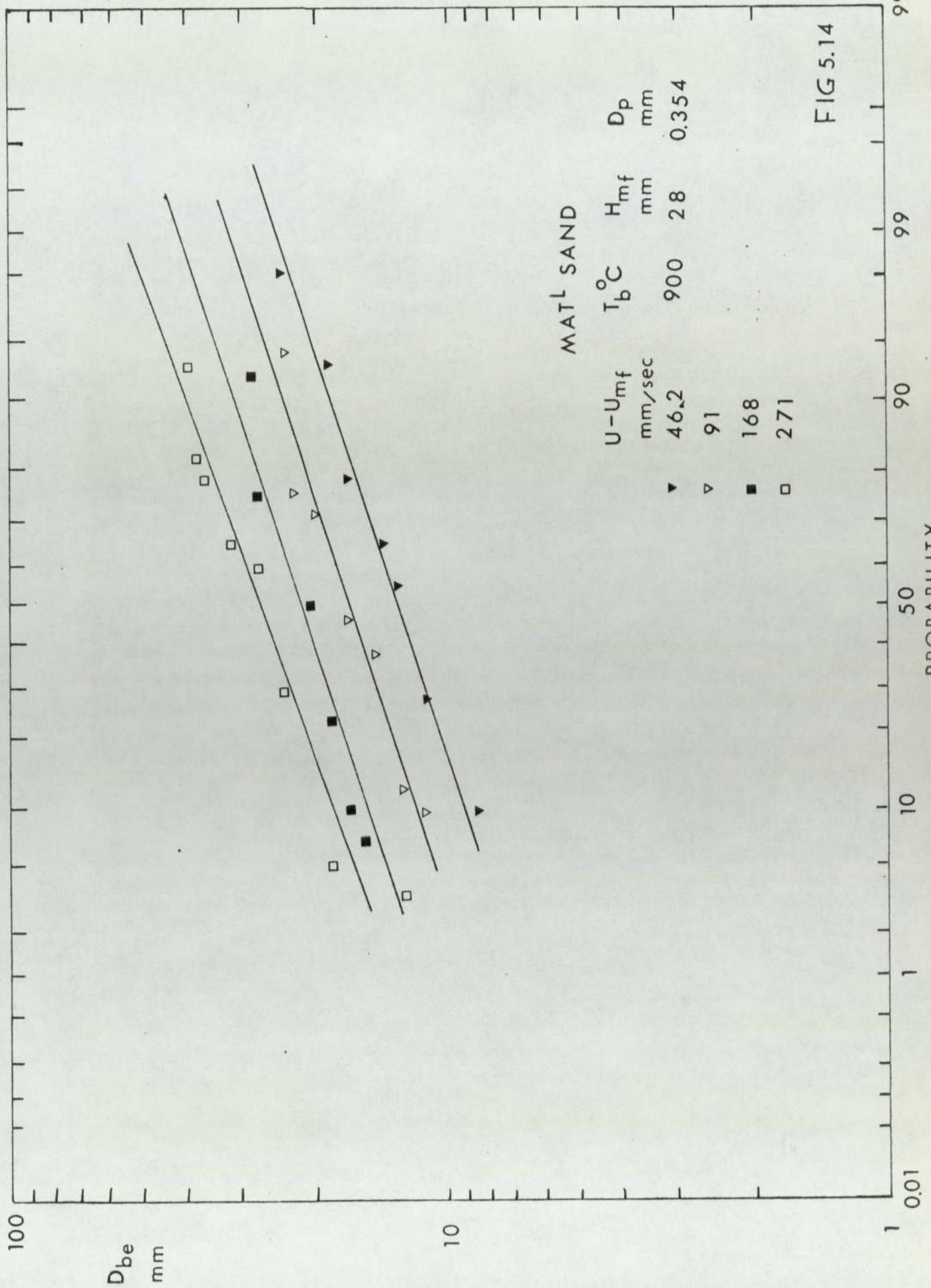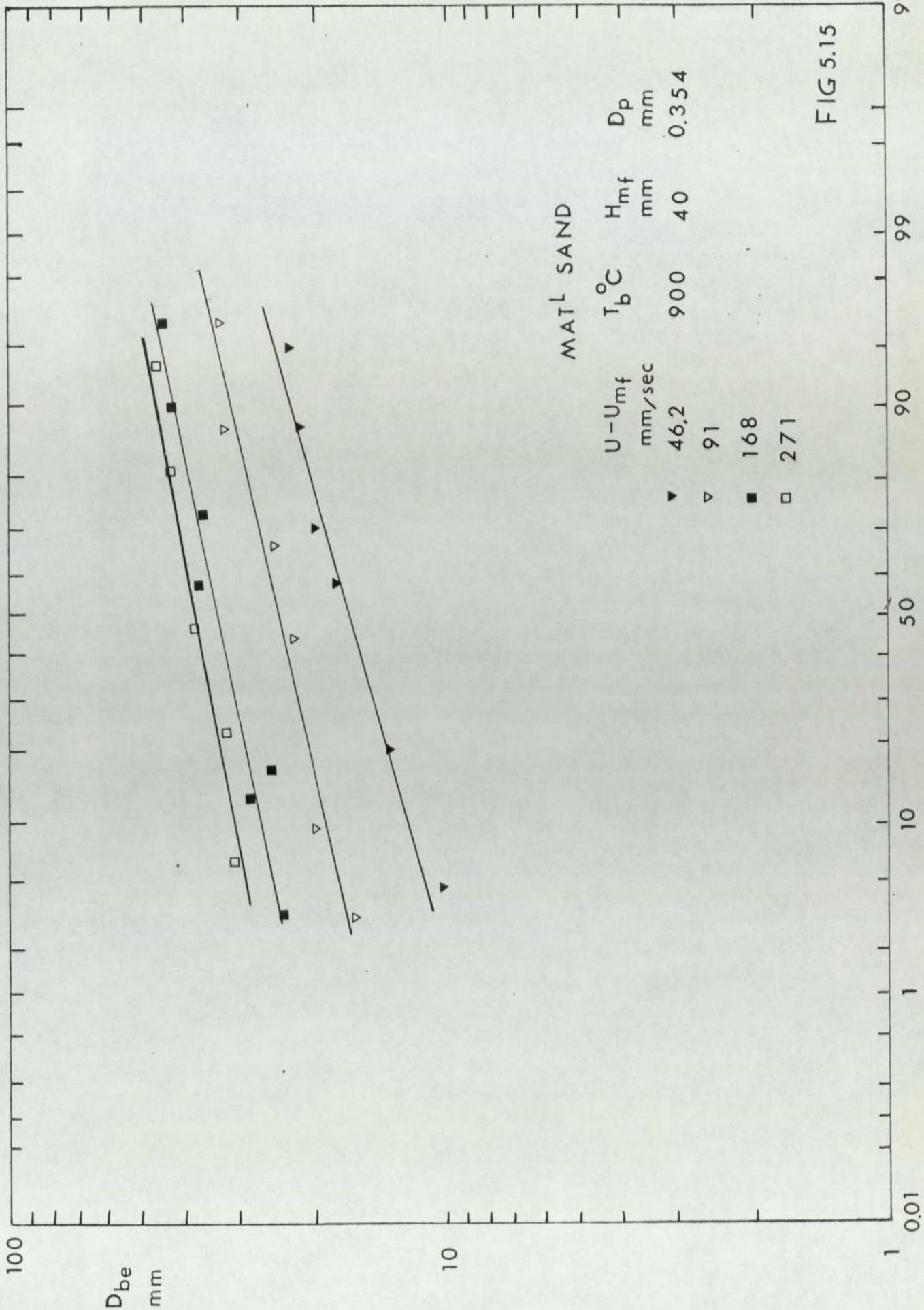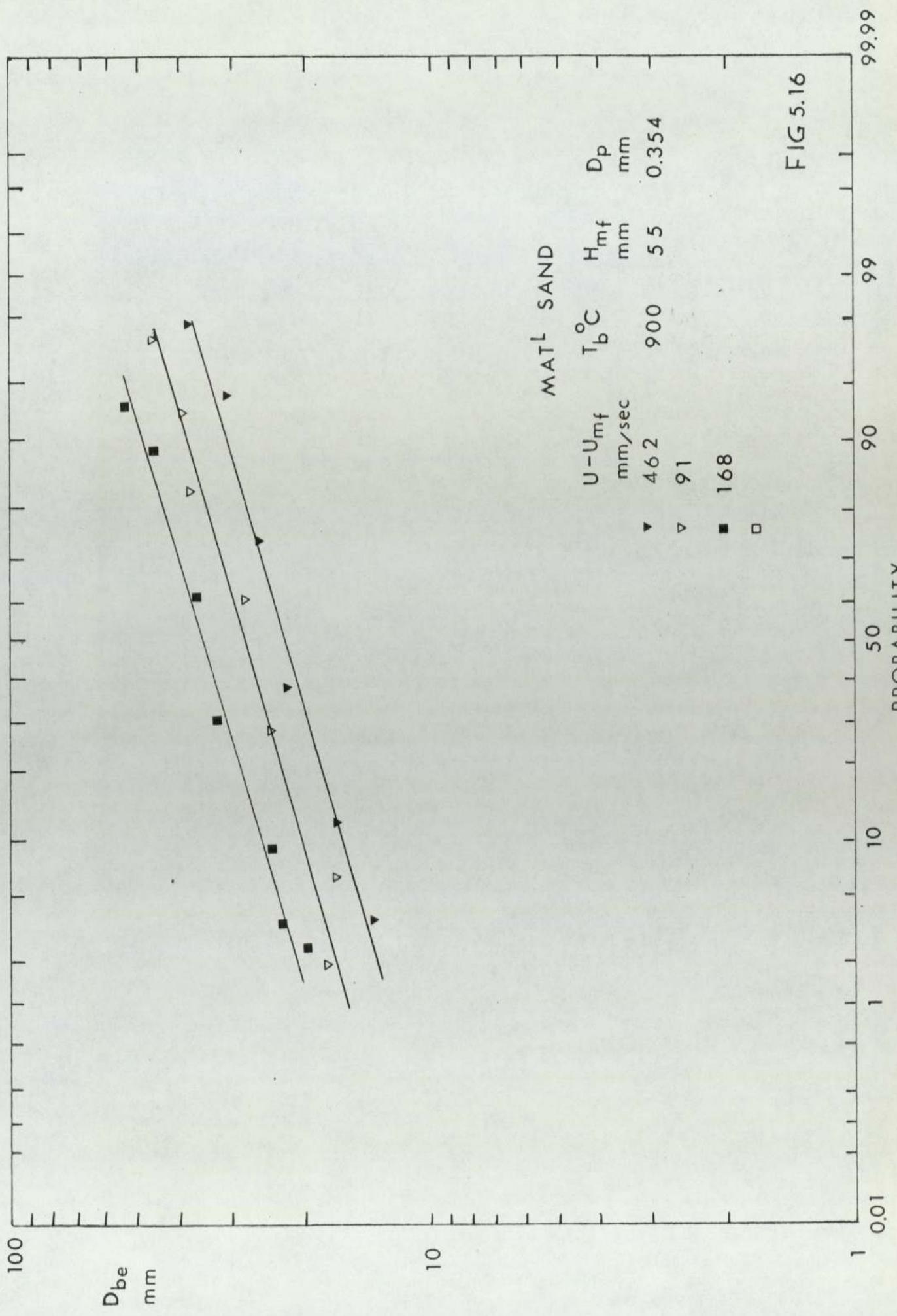
FIG 5.16

A CORRELATION OF D_b, $H_{mf}(U-U_{mf})$ AT AMBIENT AND ELEVATED BED TEMPERATURES

① $D_b = 7.5\ z$

② $D_b = 0.1795 + 7.077\,z - 1.031\,z^2 + 0.0714\,z^3 - 0.00166\,z^4$

$z = H_{mf}(U-U_{mf}) \times 10^{-3}$

$H_{mf}(U-U_{mf})$ mm²/sec

MEAN BUBBLE DIAMETER D_b(mm)

T_b °C 900

T_b °C 20
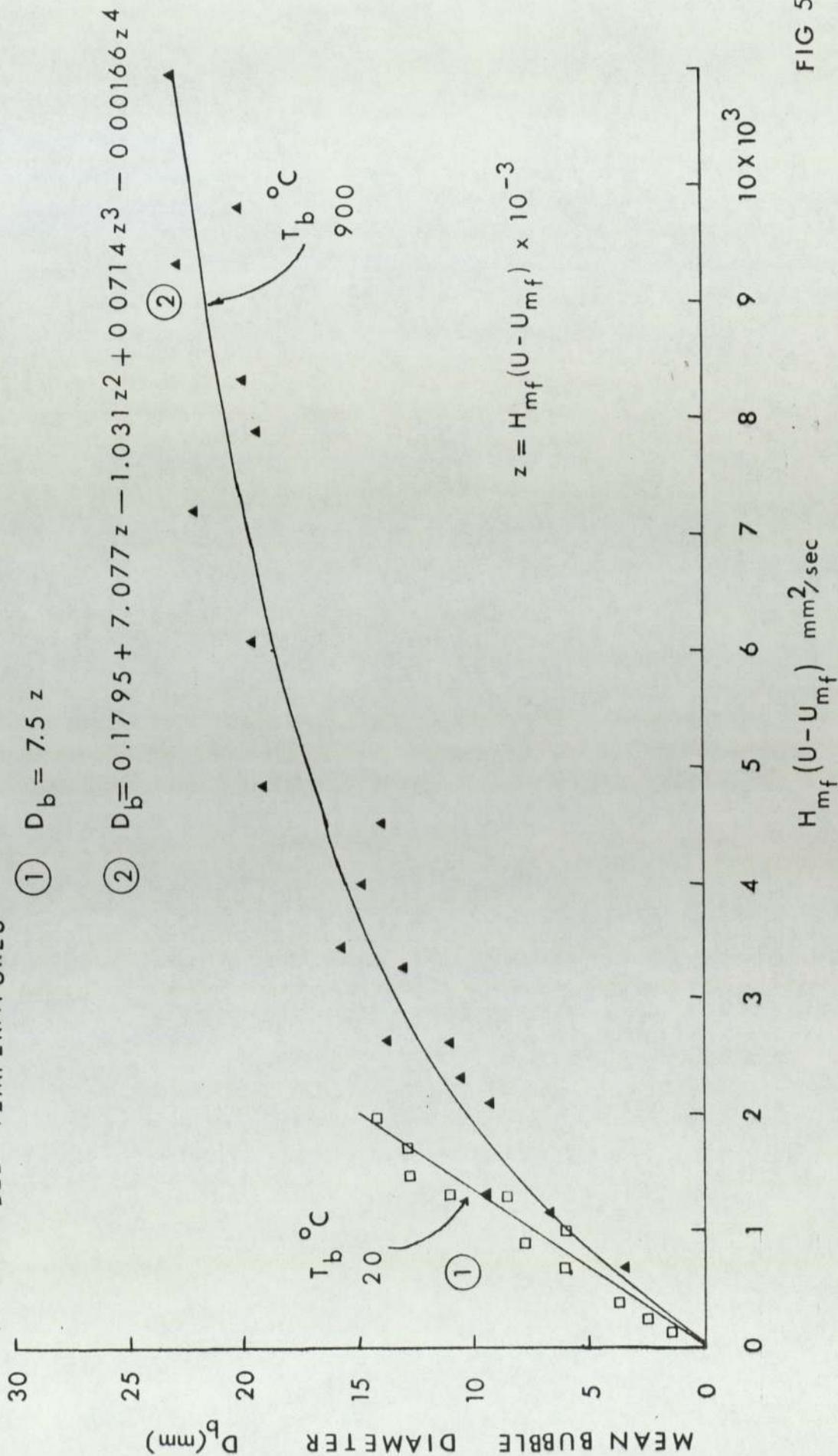
FIG 5.17

PREDICTED PARTICLE CLOUD TRAJECTORIES
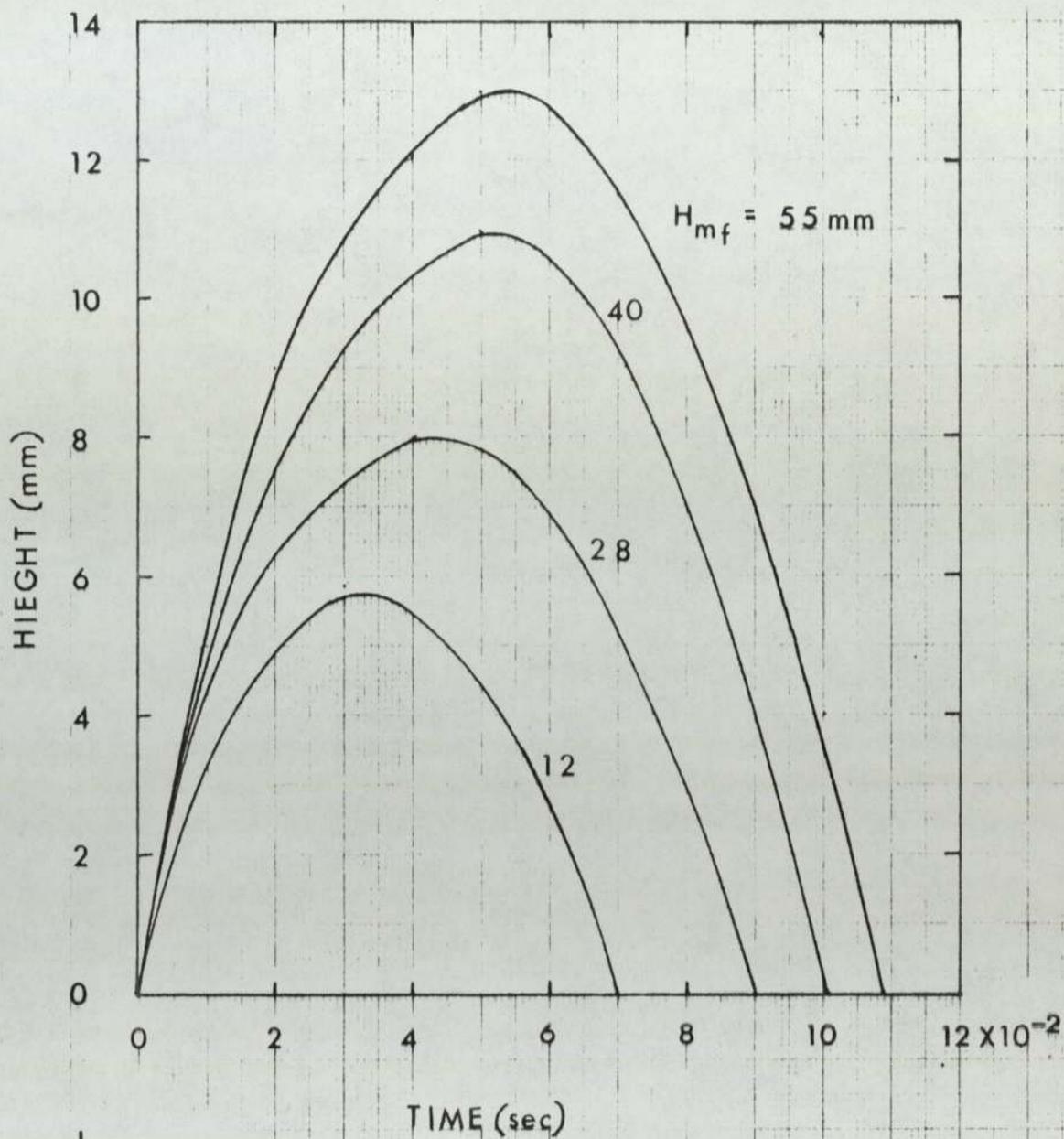
FIG 5.18

PREDICTED PARTICLE CLOUD RESIDENCE TIMES
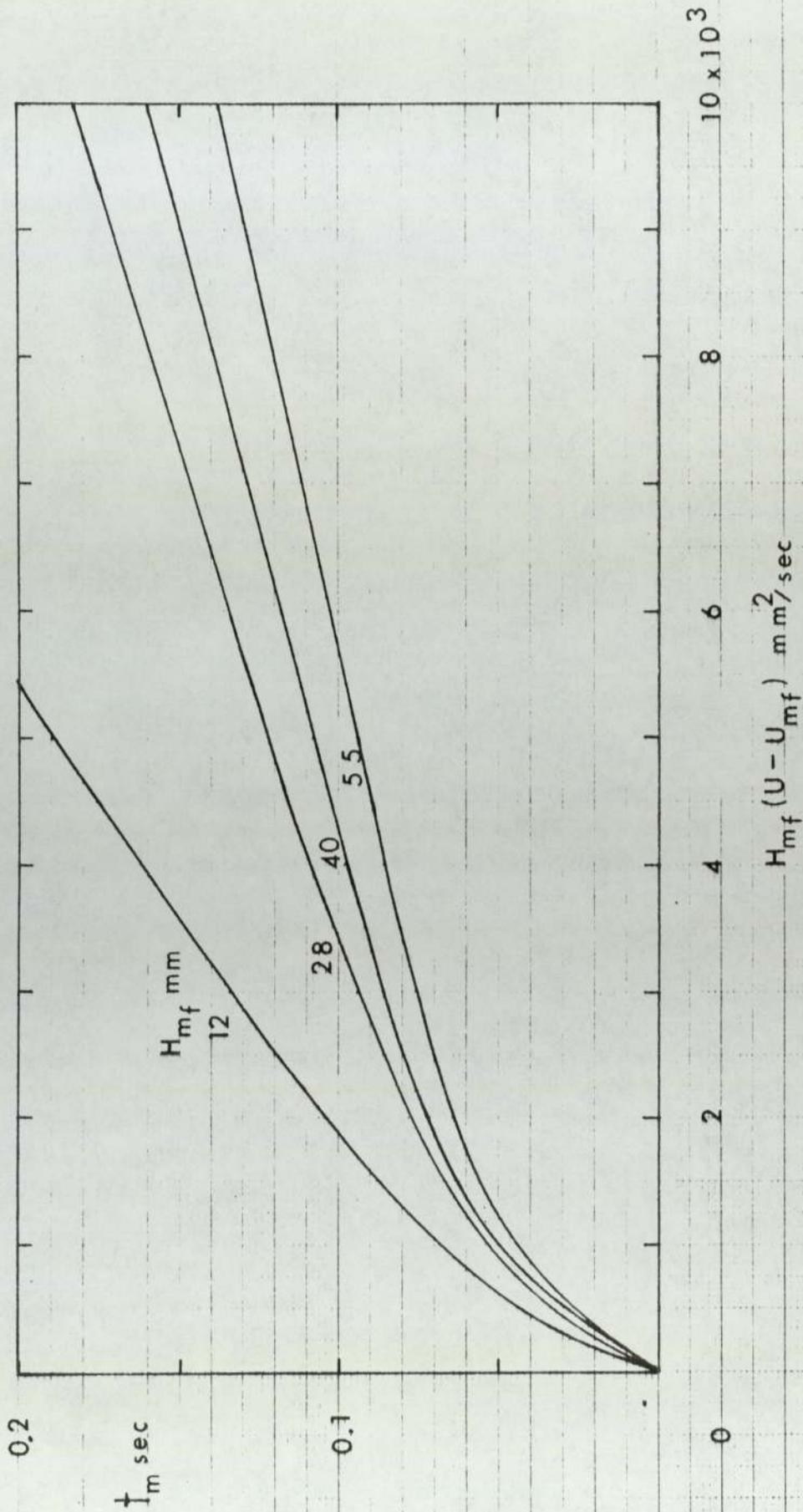
FIG.5.19

COMPARISON OF SOLUTIONS FOR TRANSIENT RADIATIVE TRANSFER
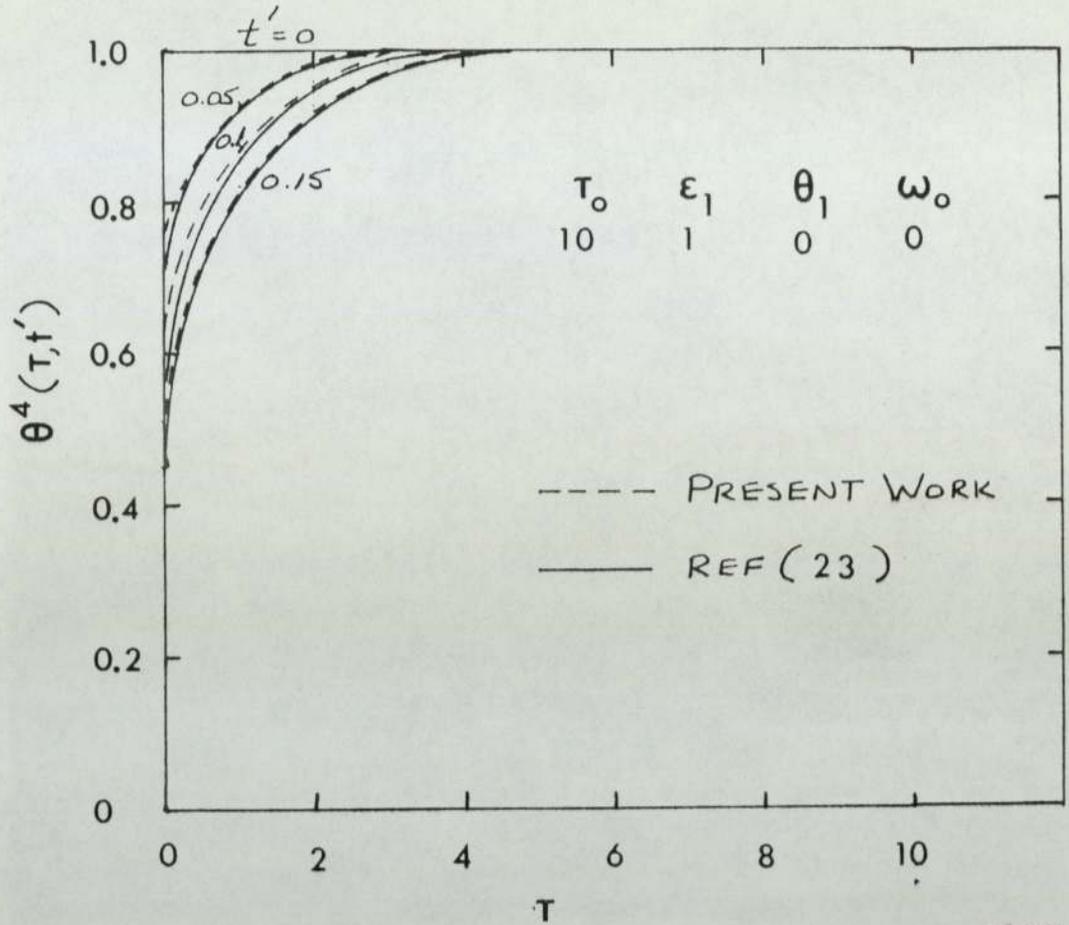
FIG. 6.1a
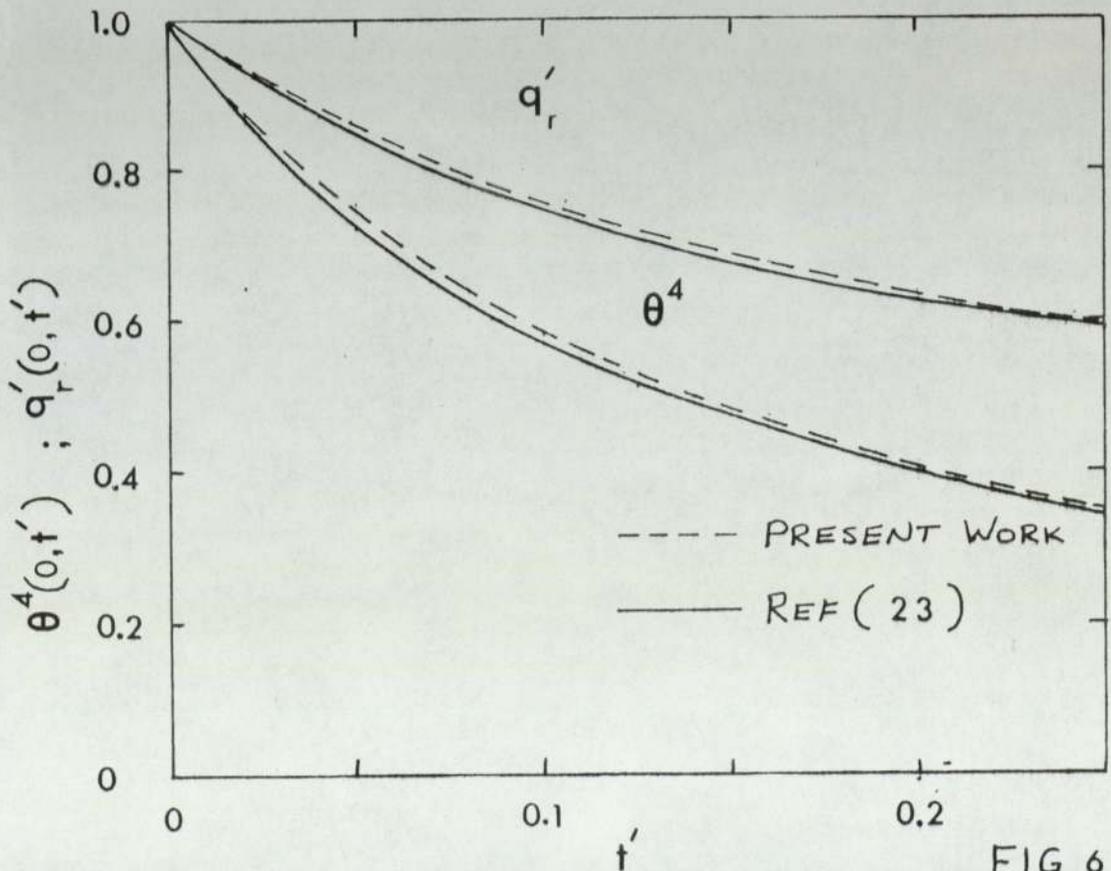
FIG. 6.1b

$q_r'(0,t')$

	ε_1	θ_1	ω_0
1	1.0	0	0.148
2	1.0	0	0.4
3	1.0	0	0.7
4	1.0	0	0.857
5	1.0	0	0.857

1.0

0.8

0.6

0.4

0.2

0

10^{-3} 10^{-2} 10^{-1} 1   10

$t' = \dfrac{\kappa_t \sigma T_b^3 t}{\rho_p c_p (1-\varepsilon_b)}$

FIG 6.2

TRANSIENT TEMPERATURE DISTRIBUTION

	T_0	ε_1	θ_1	ω_0	t'
1	10	1.0	0	0	0.1
2	10	1.0	0	0.148	0.1
3	10	1.0	0.	0.4	0.1
4	10	1.0	0	0.7	0.1
5	10	1.0	0	0.857	0.1

FIG. 6.3

	T_0	ε_1	θ_1	ω_0	t'
1	10	1.0	0.5	0	0.1
2	10	1.0	0.5	0.4	0.1
3	10	1.0	0.5	0.857	0.1
4	10	0.7	0.5	0	0.1
5	10	0.7	0.5	0.4	0.1
6	10	0.7	0.5	0.857	0.1

FIG 6.4

FIG 6.5

Legend:
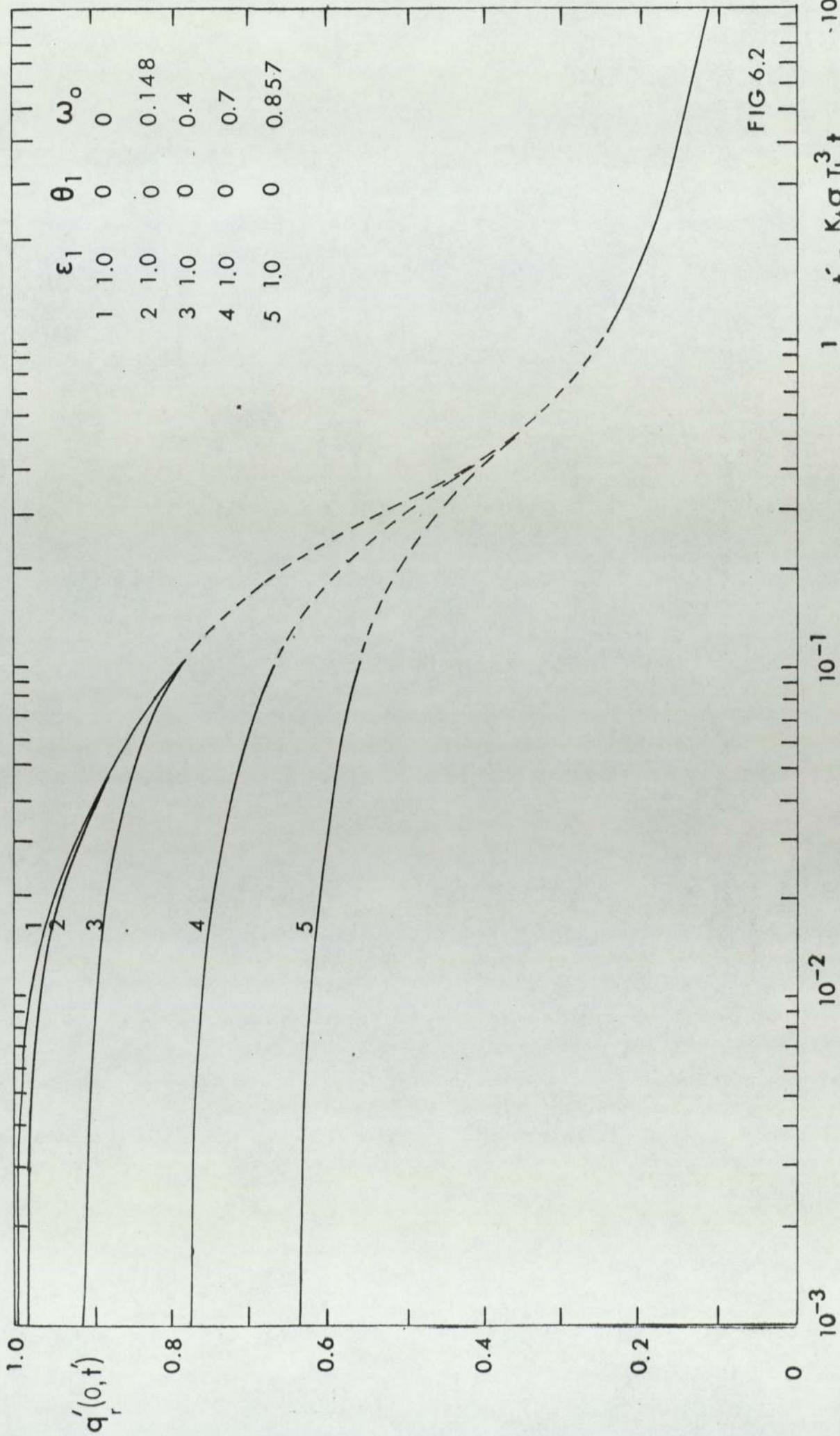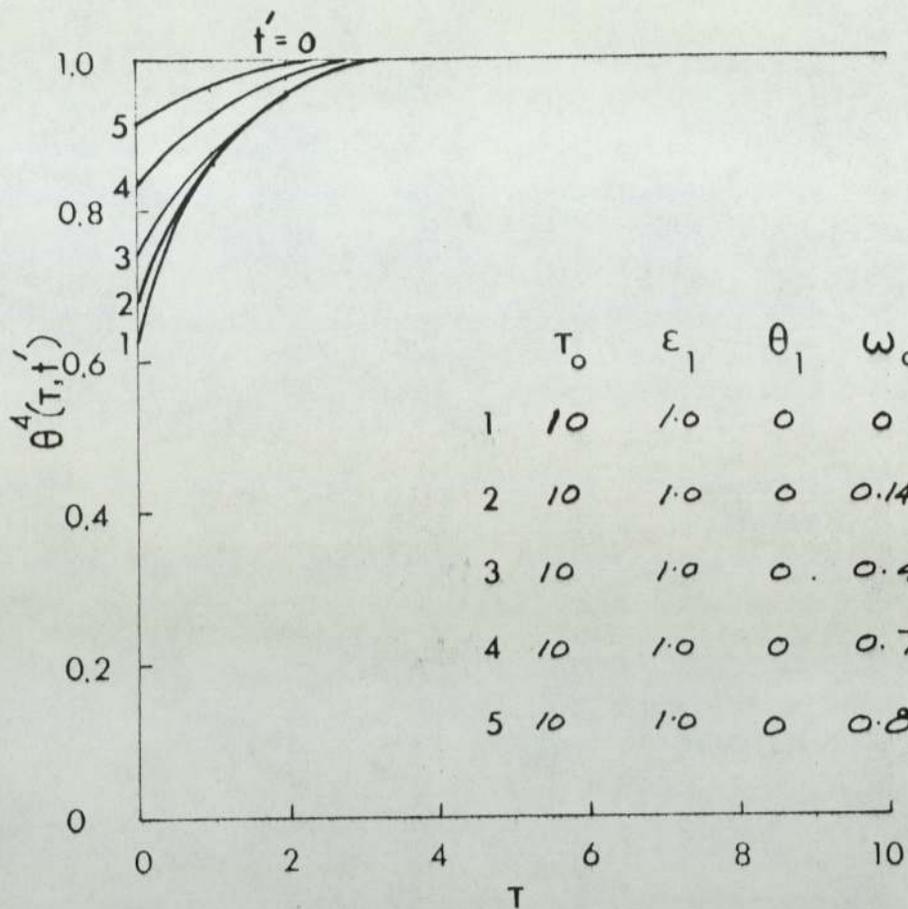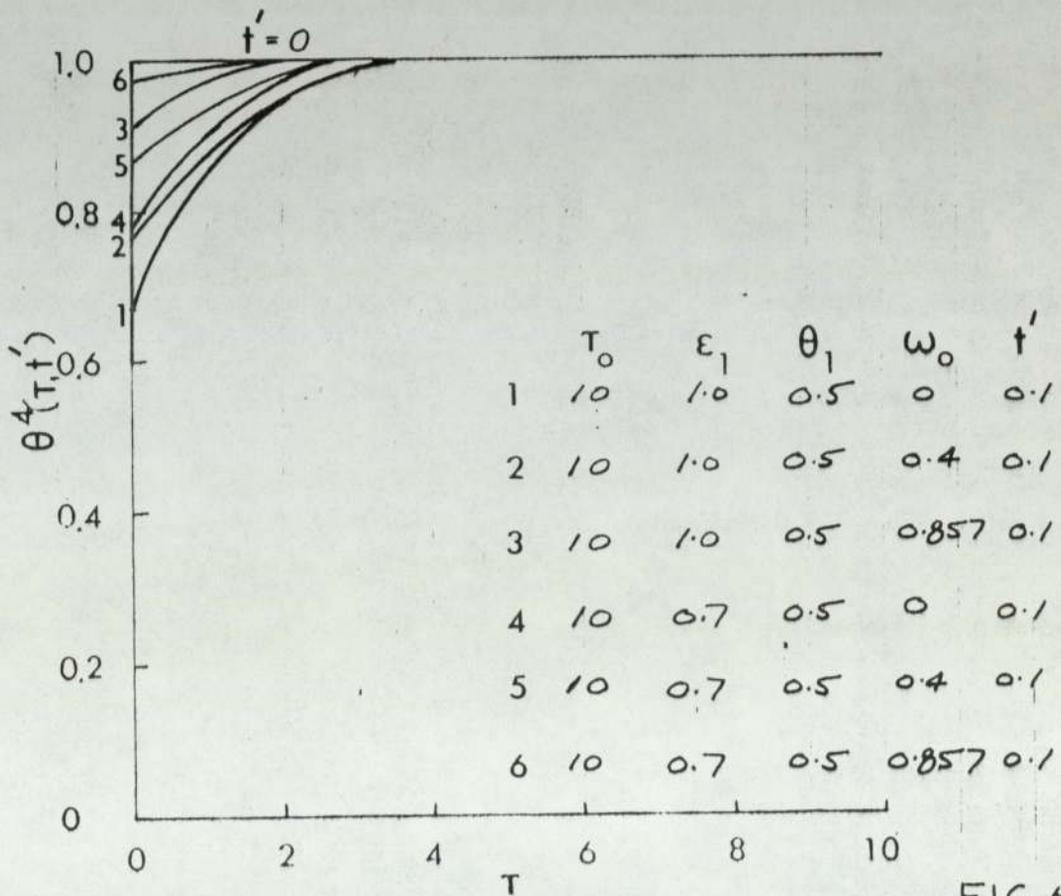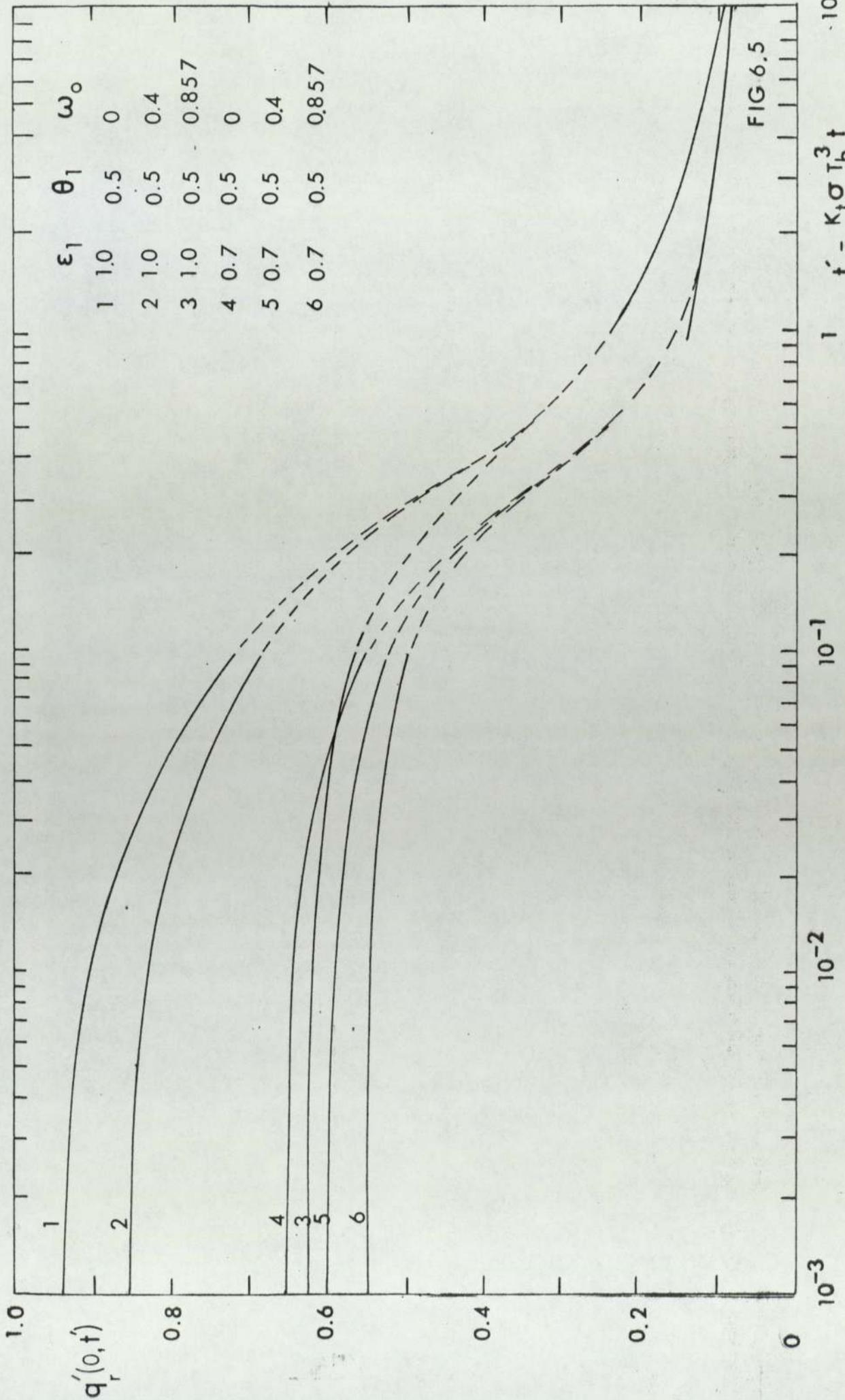
	ϵ_1	θ_1	ω_0
1	1.0	0.5	0
2	1.0	0.5	0.4
3	1.0	0.5	0.857
4	0.7	0.5	0
5	0.7	0.5	0.4
6	0.7	0.5	0.857

Vertical axis: $q_r'(0,t')$

Horizontal axis: $t' = \dfrac{K_t \sigma T_b^3 t}{\rho_p C_p (1-\epsilon_b)}$

TIME VARIATION OF RADIATIVE FLUX
EMITTED BY AN OPTICALLY DENSE MEDIUM

τ_0	ε_1	θ_1^4	ω_0	T_0
1	1,0	0.0028	0.374	10
2	1,0	$-\prime-$	0.529	
3	1,0	$-\prime-$	0.601	
4	1,0	$-\prime-$	0.773	

FIG 6.6a

FIG 6.6b
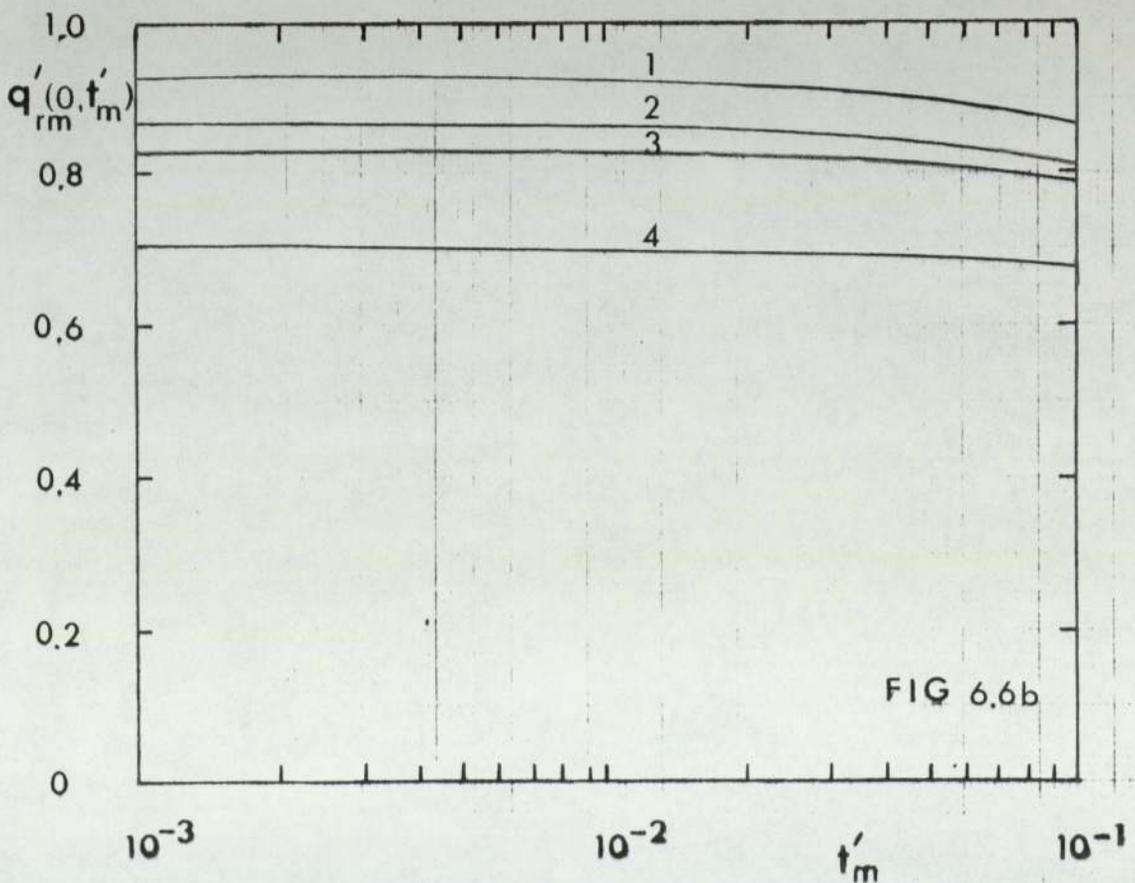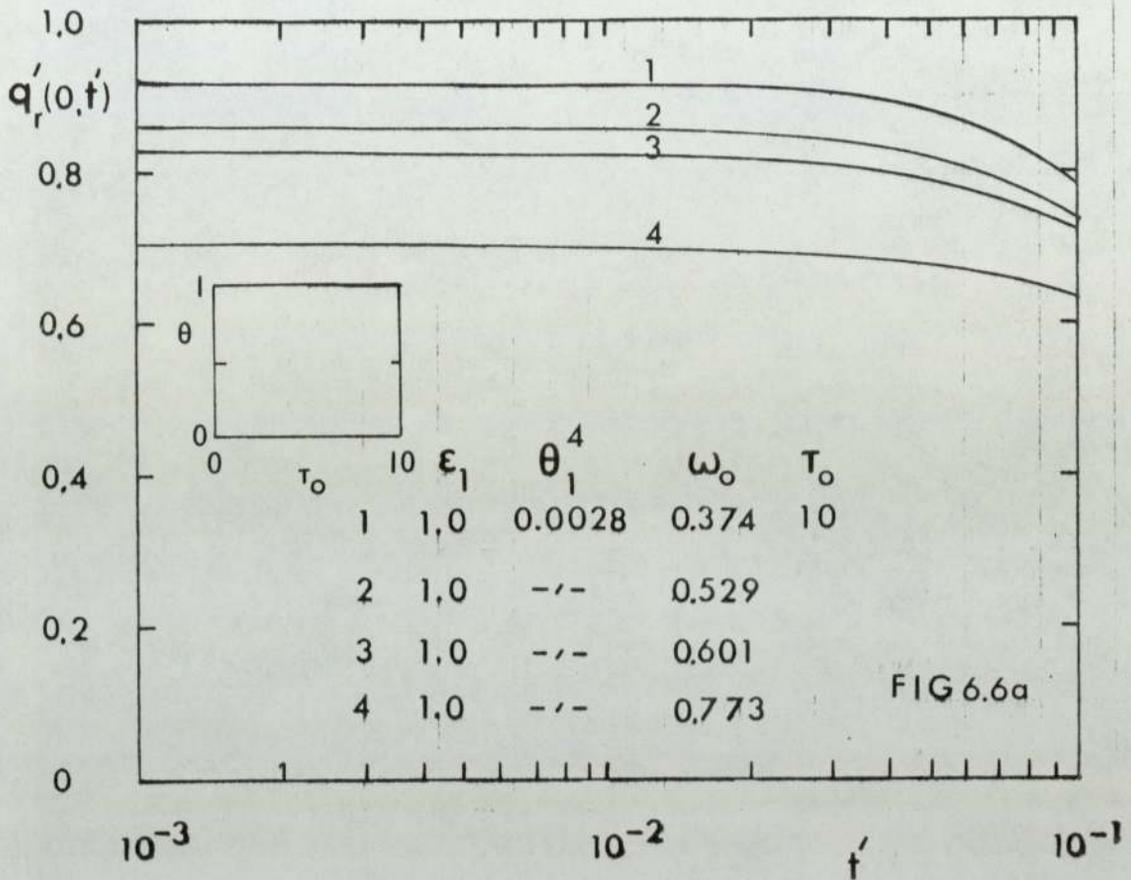
EFFECT OF EXCESS GAS VELOCITY ON
TIME MEAN RADIATIVE FLUX
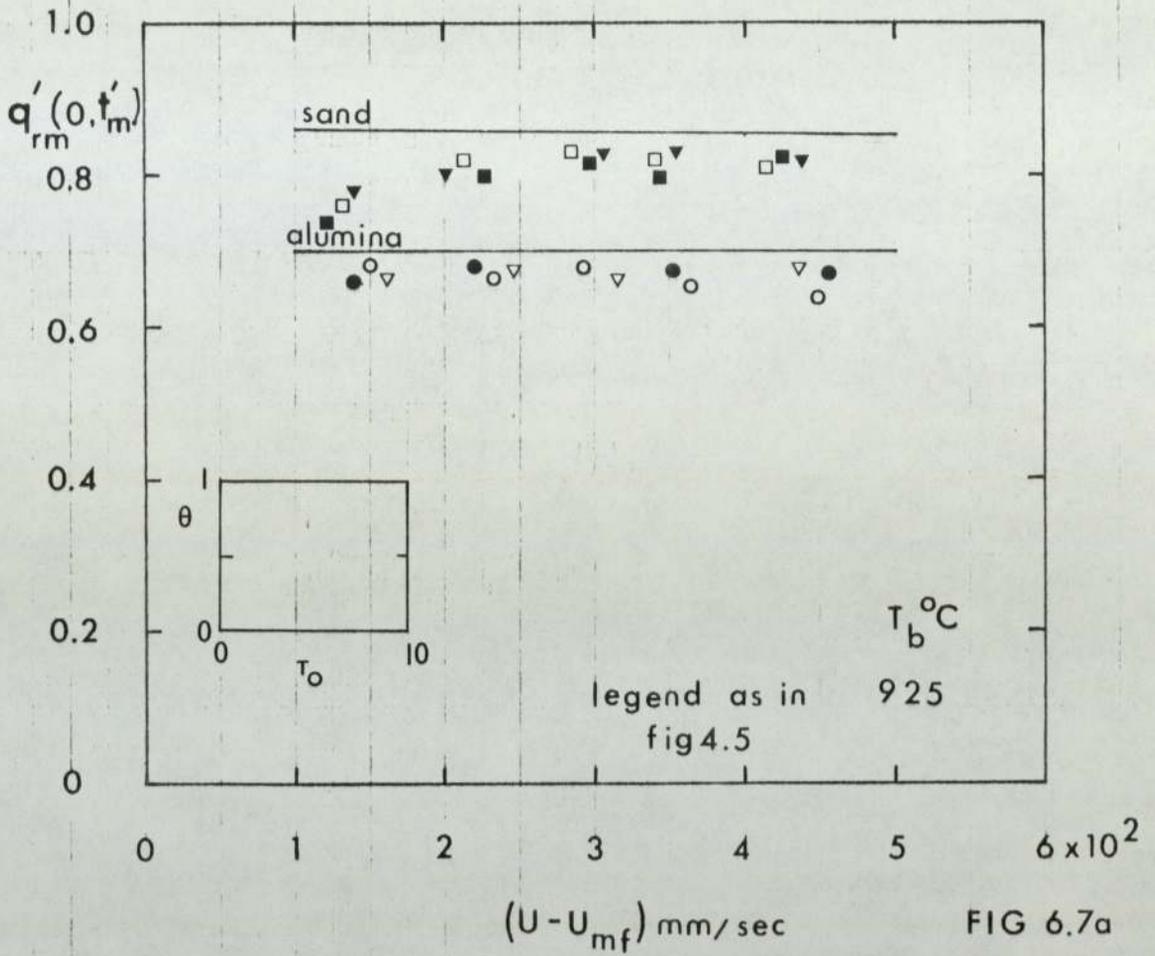
FIG 6.7a
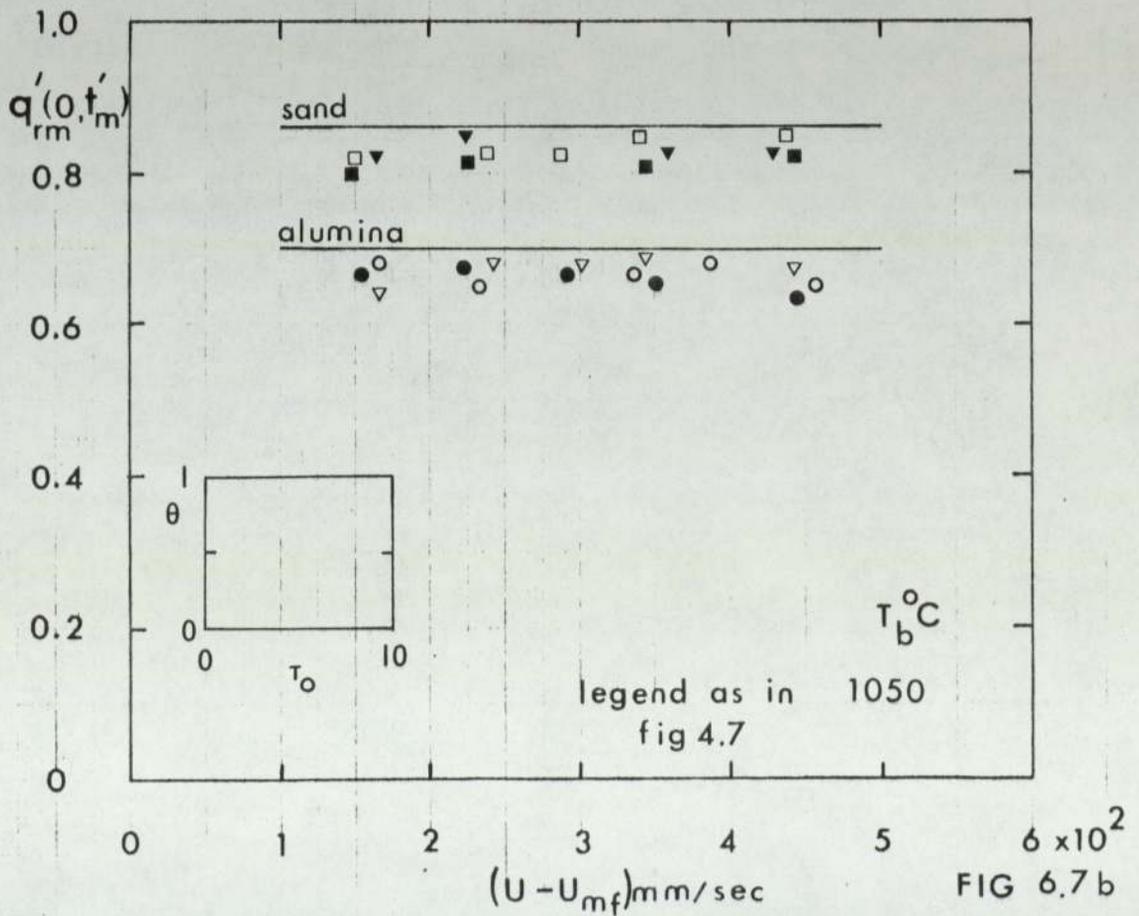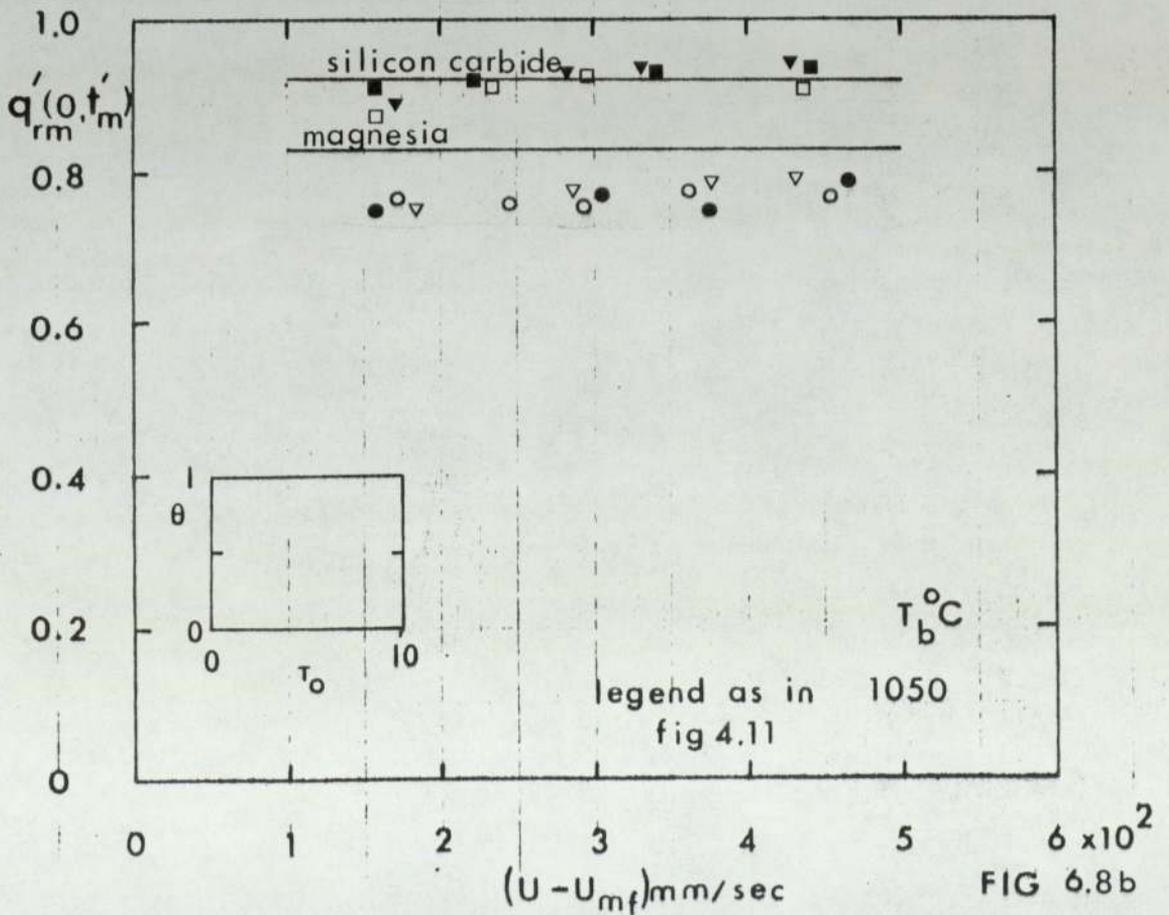
FIG 6.7 b

EFFECT OF EXCESS GAS VELOCITY ON
TIME MEAN RADIATIVE FLUX

$q'_{rm}(0,t'_m)$

silicon carbide

magnesia

$T_b \, ^\circ C$

legend as in
fig 4.9

925

$(U-U_{mf})$ mm/sec

FIG 6.8a

$q'_{rm}(0,t'_m)$

silicon carbide

magnesia

$T_b \, ^\circ C$

legend as in
fig 4.11

1050

$(U-U_{mf})$ mm/sec

FIG 6.8b

TIME VARIATION OF RADIATIVE FLUX
EMITTED BY AN OPTICALLY DENSE MEDIUM

	ε_1	θ_1^4	ω_0	T_0
1	1,0	0.0028	0.374	10
2	1,0	-,-	0.529	
3	1,0	-,-	0.601	
4	1,0	-,-	0.773	

FIG 6.9a

FIG 6.9b

TRANSIENT

TEMPERATURE DISTRIBUTION

$t' = 0$

	T_o	ε_1	θ_1	ω_o	t'
a	10	1.0	0.221	0.374	0.1
b	-//-	-//-	-//-	-//-	-//-

$\theta^4(\tau, t')$

θ, τ/τ_o

FIG. 6.10

$t' = 0$

	T_o	ε_1	θ_1	ω_o	t'
a	10	1.0	0.221	0.529	0.1
b	-//-	-//-	-//-	-//-	

$\theta^4(\tau, t')$
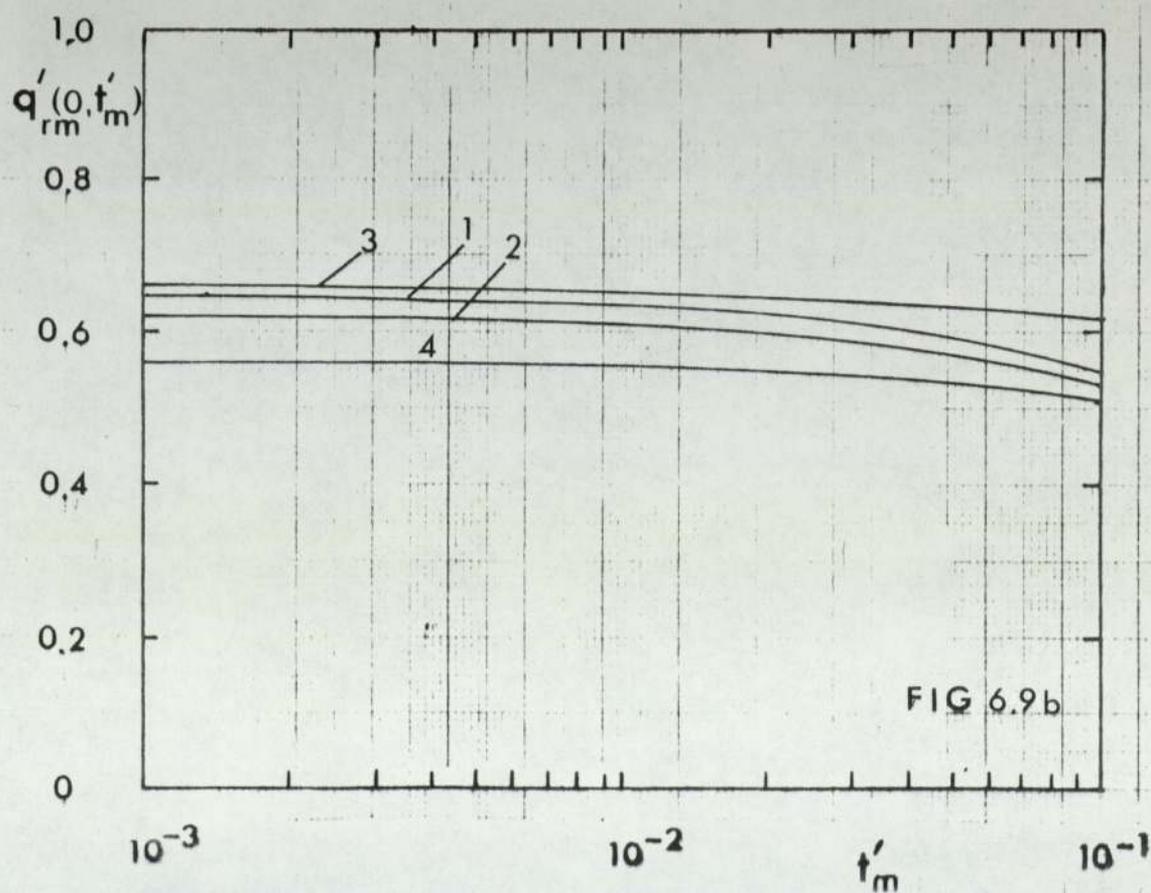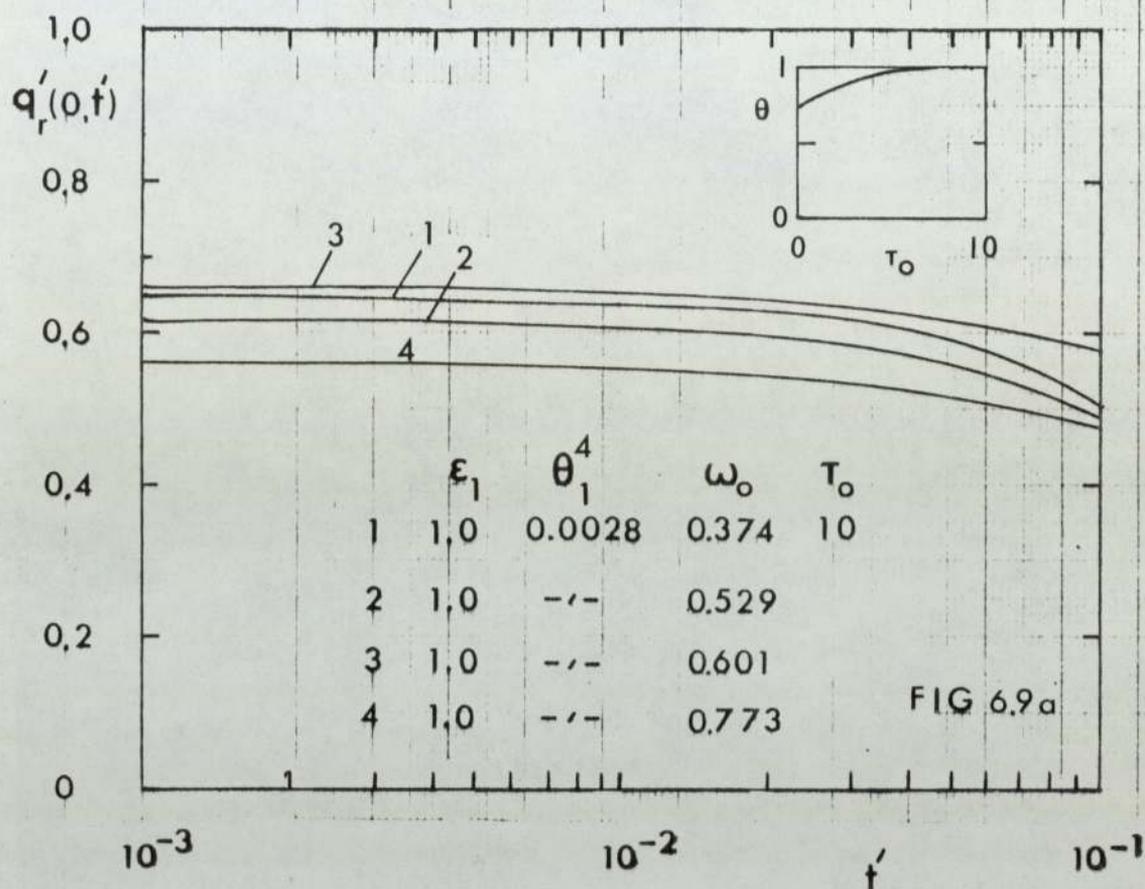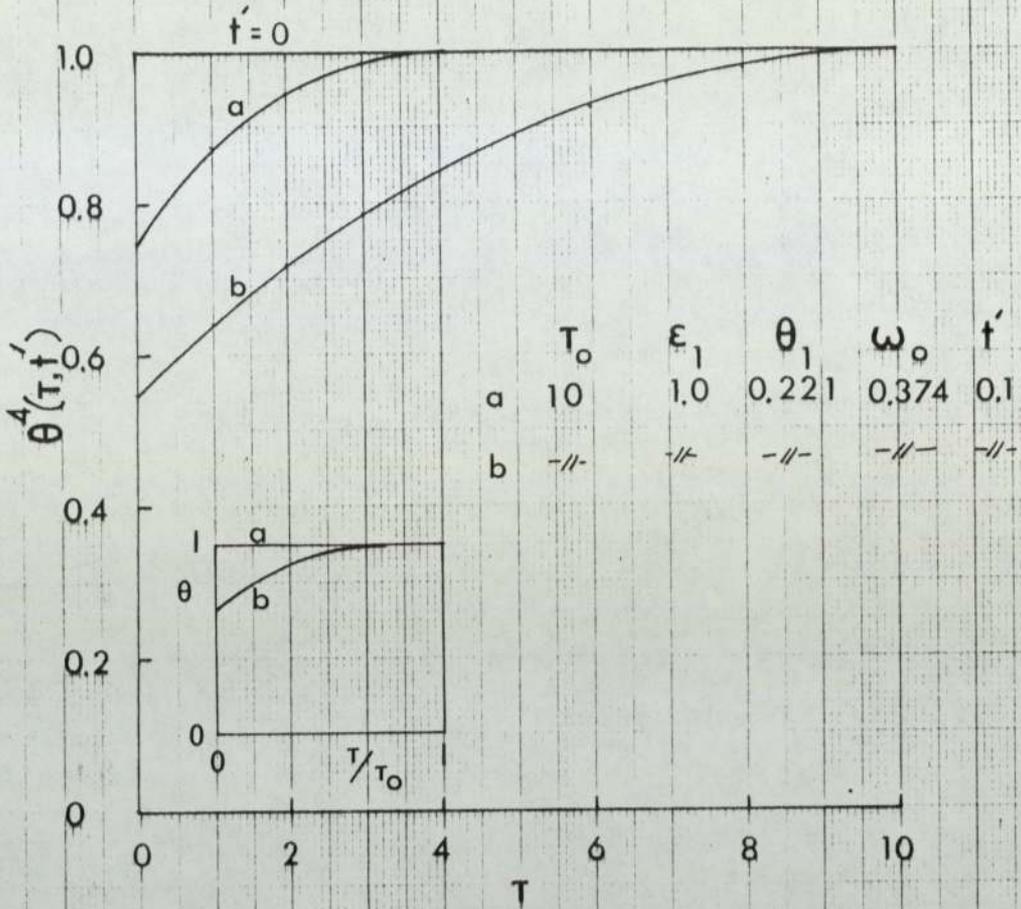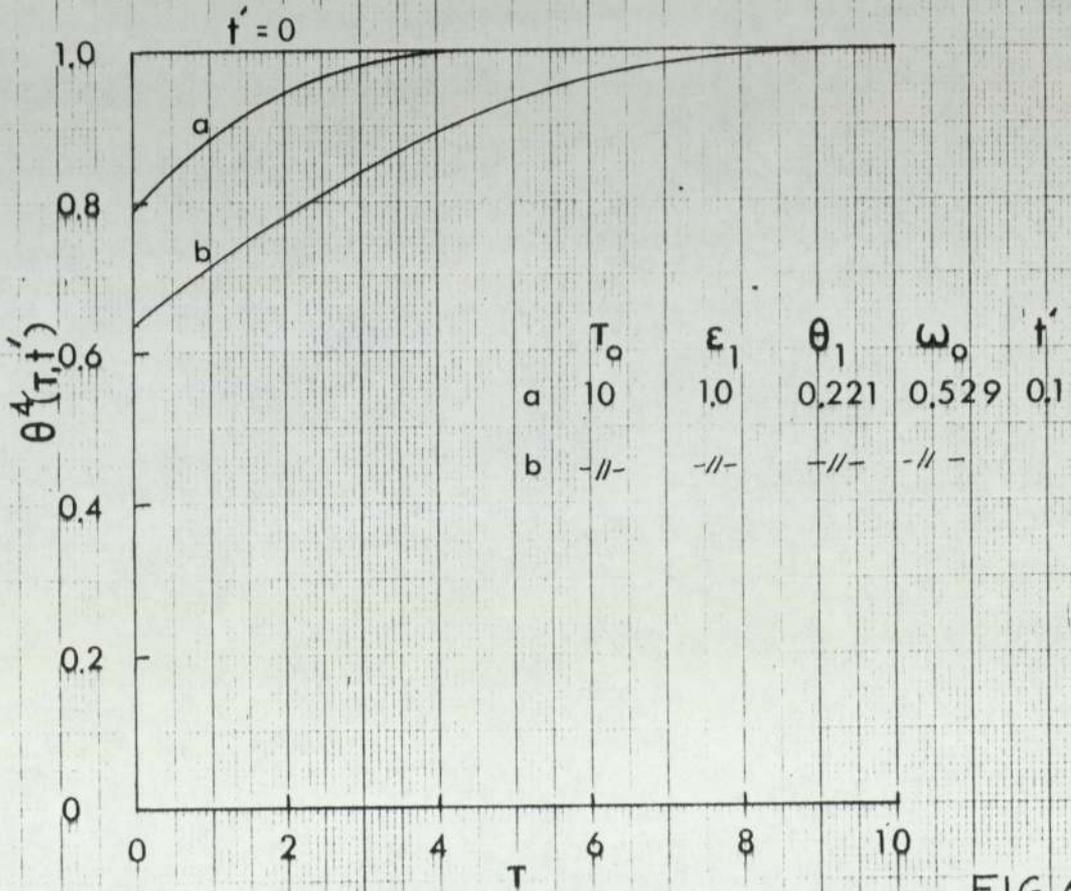
FIG 6.11

PREDICTED INSTANTANEOUS NUSSELT – FOURIER NUMBER RELATIONSHIP

SILICA SAND

$(N_{ui})_c$ from [82]

$(N_{ui})_r$ present work

	ε_w	θ_w	ω_o	$T_b\,^\circ K$	$D_p\,mm$	ε_b
a	1.0	0.5	0.529	1273	0.354	0.41
b	1.0	0.5	0.529	1273	0.777	0.41
c	1.0	0.5	0.529	1673	0.354	0.41

$(N_{ui})_{c,r}$

1.0

0.1

b
c

a

0.01   0.1   1.0   10

F_{oi}
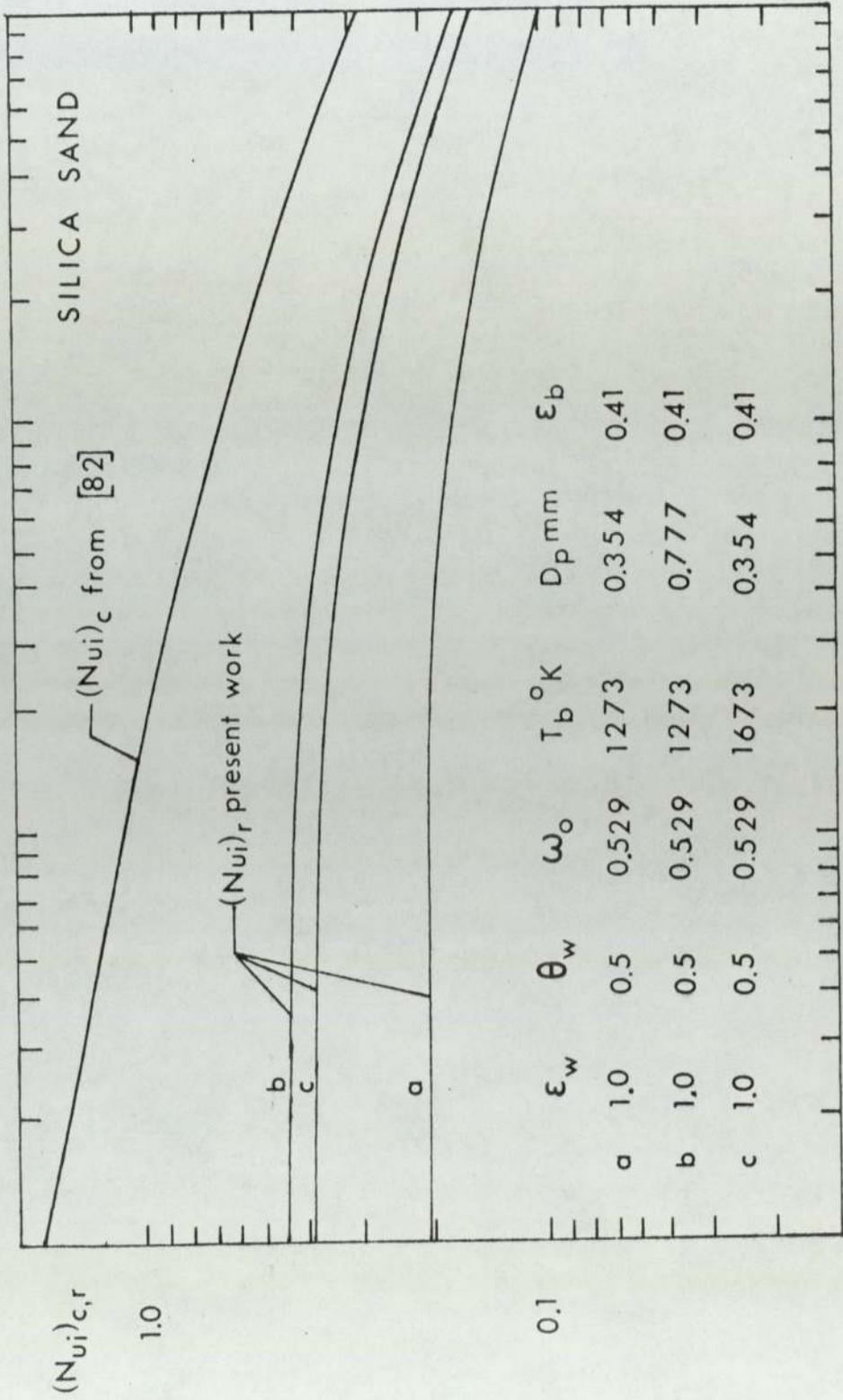
FIG 7.1

THERMAL CONDUCTIVITY OF A
POROUS MEDIUM from [86]
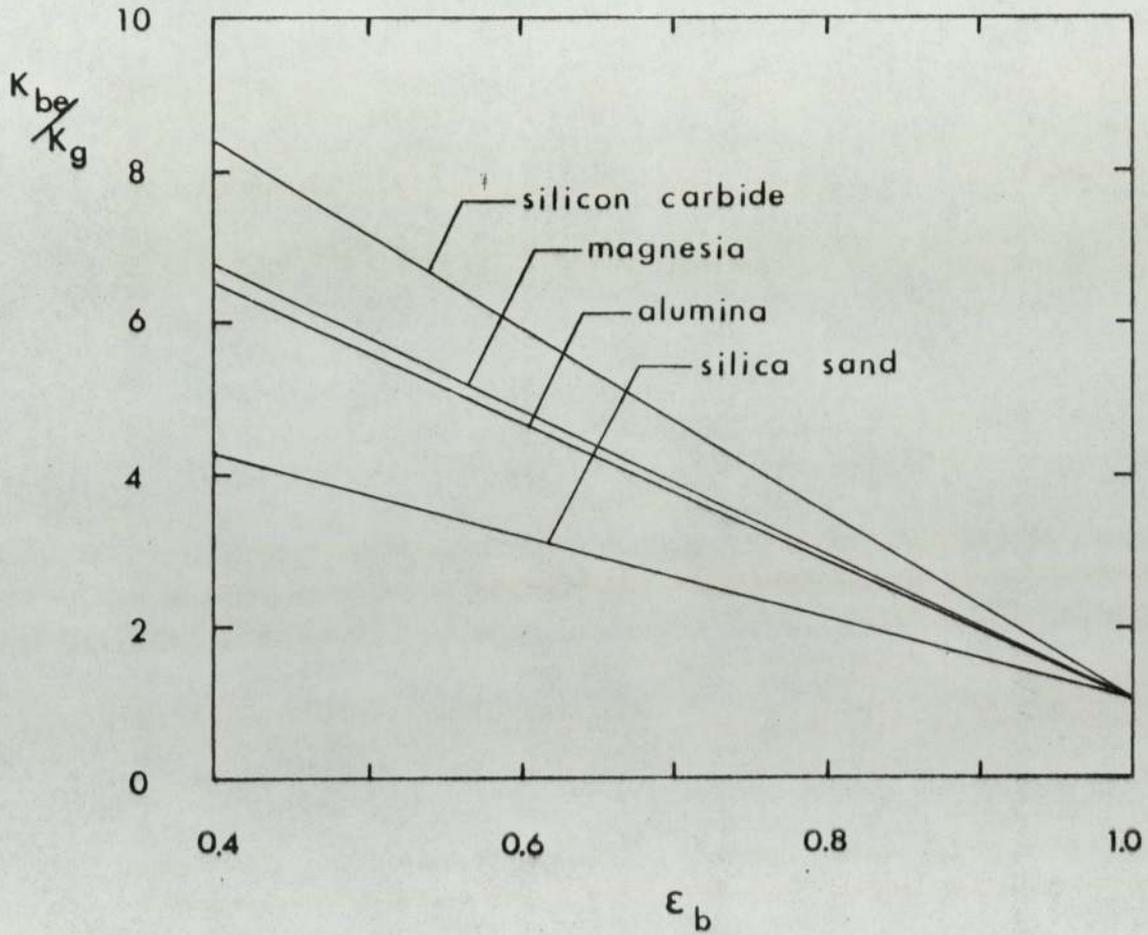
FIG 7.2

VARIATION OF EXTINCTION COEFFICIENT

AT A WALL
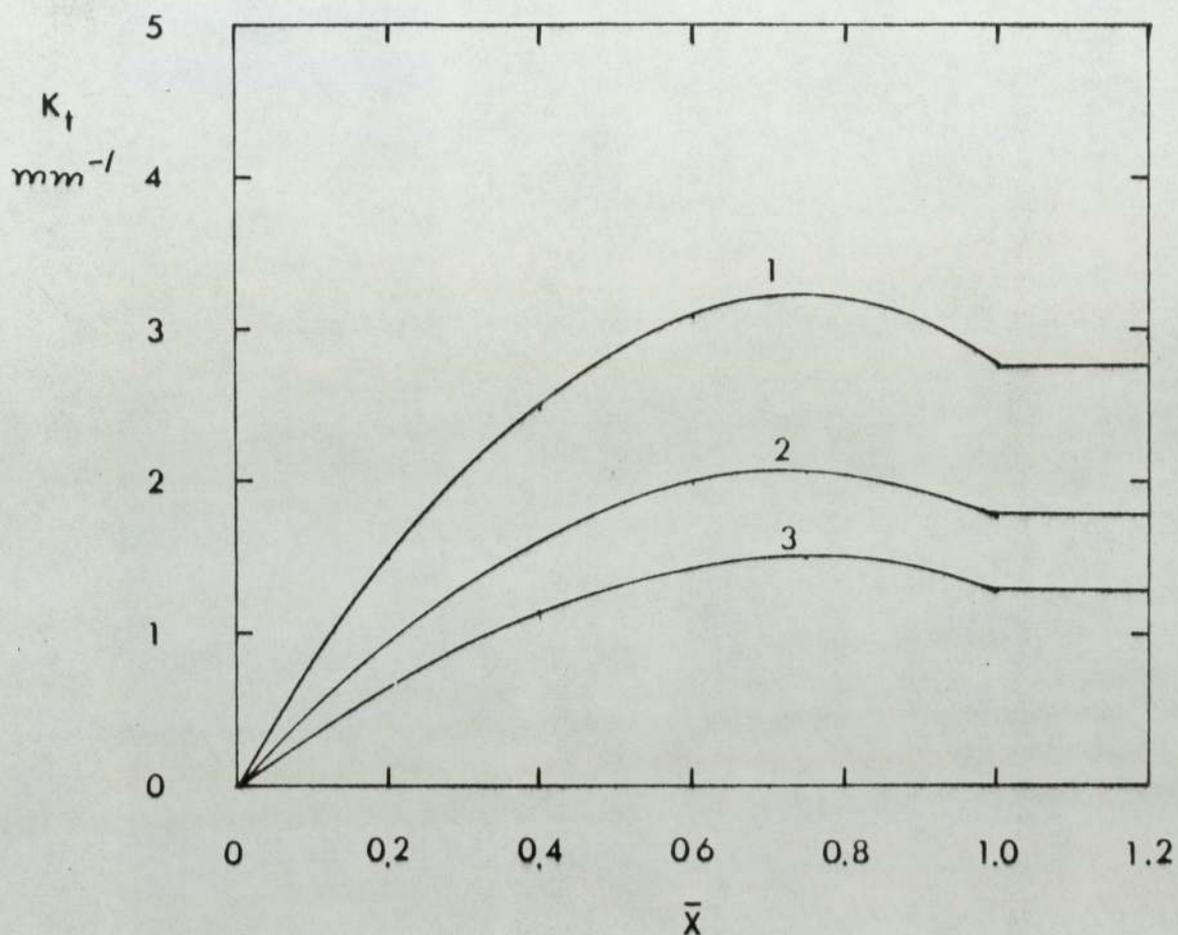
	MAT^L	D_p mm	ε_p	ε_b
1	ALUMINA	0.354	0.37	0.58
2	—//—	0.55	—//—	—//—
3	—//—	0.777	—//—	—//—

FIG 7.3

VARIATION OF EXTINCTION COEFFICIENT (IN EMULSION PACKET) WITH EXCESS GAS VELOCITY
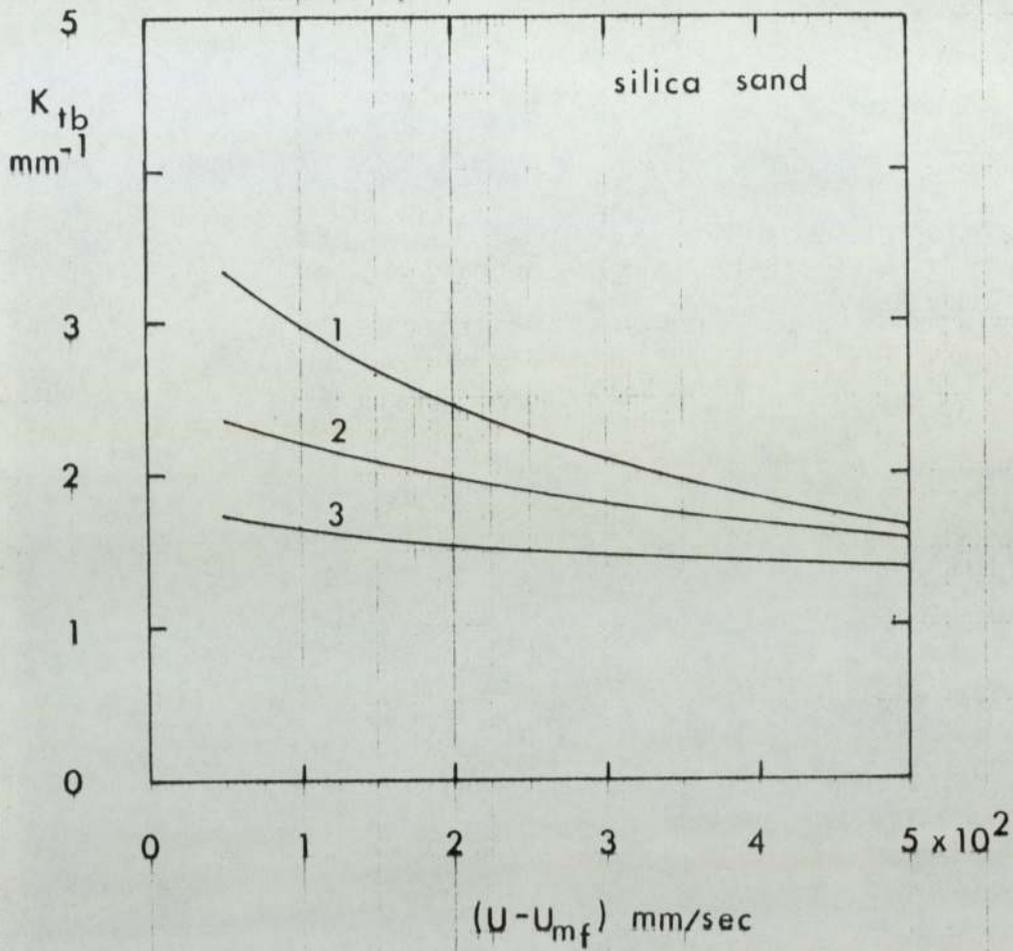
	ε_P	D_P mm
1	0.64	0.354
2	—//—	0.55
3	—//—	0.777

FIG 7.4

VARIATION OF EXTINCTION COEFFICIENT (IN EMULSION PACKET) WITH EXCESS GAS VELOCITY

ε_p and D_p mm
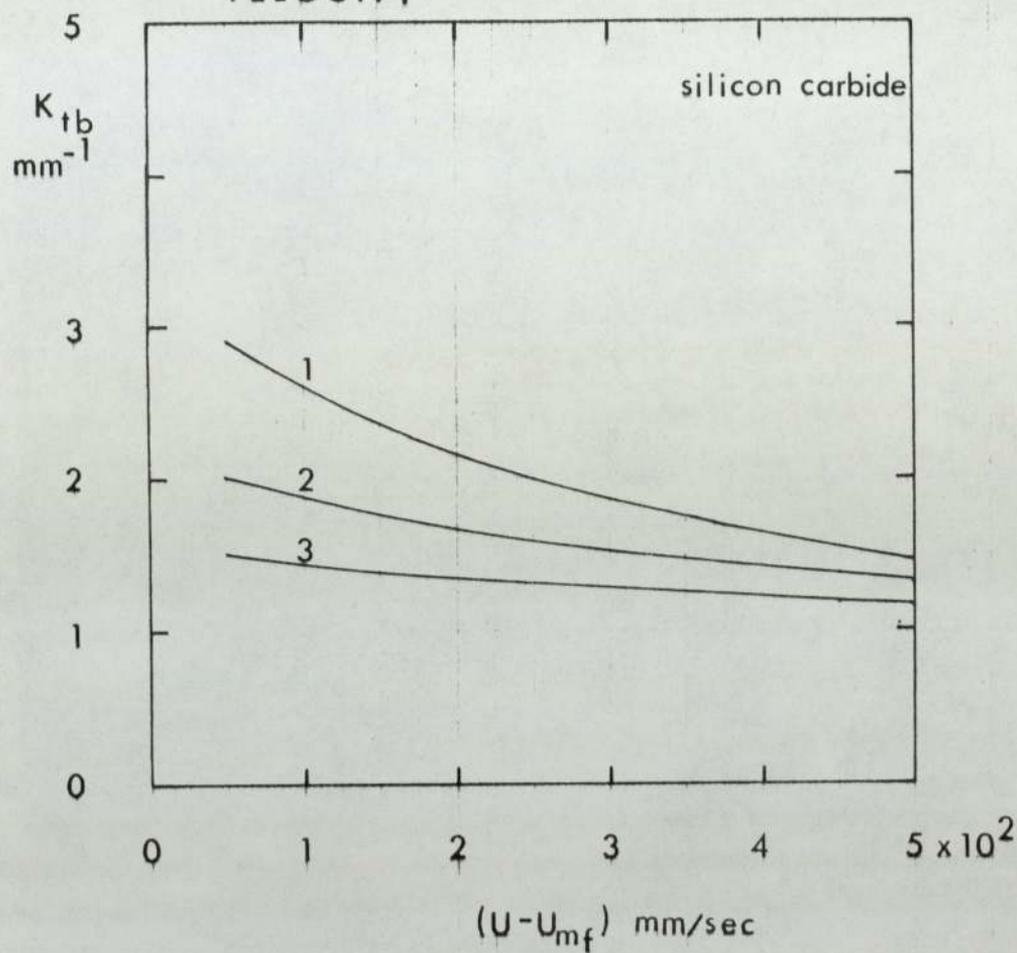
	ε_p	D_p mm
1	0.77	0.354
2	$-//-$	0.55
3	$-//-$	0.777

FIG 7.5

VARIATION OF EXTINCTION COEFFICIENT
(IN EMULSION PACKET) WITH EXCESS GAS
VELOCITY
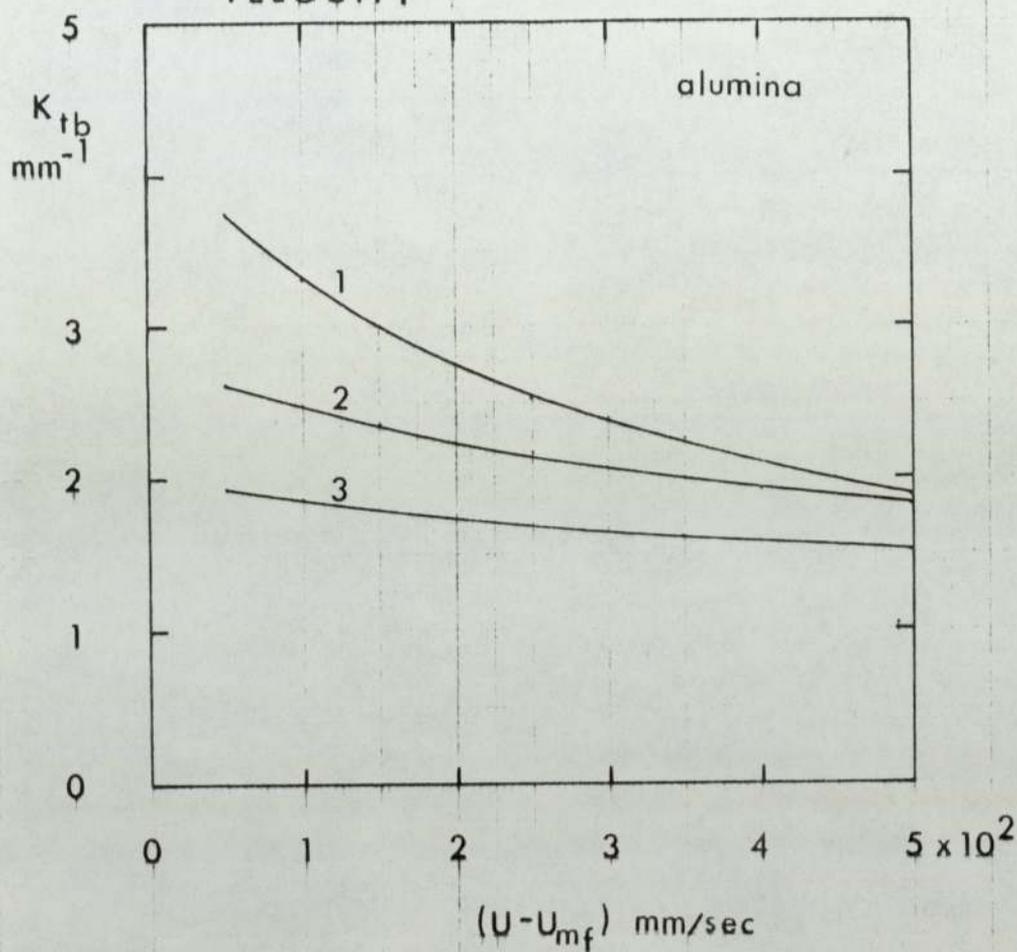
	ε_p	D_p mm
1	0.37	0.354
2	$-//-$	0.55
3	$-//-$	0.777

FIG 7.6

COMPARISON OF PREDICTED HEAT
TRANSFER COEFFICIENTS WITH EXPERIMENTAL
DATA FROM [83]

FIG 7.7a

FIG 7.7b

PREDICTED INSTANTANEOUS NUSSELT - FOURIER NUMBER RELATIONSHIP

SILICA SAND

uncoupled sol^n

--- coupled sol^n

$(N_{ui})_c$ from [82]

$(N_{ui})_c$ present work

$(N_{ui})_r$ present work

$(N_{ui})_{c,r}$

	ε_w	θ_w	ω_o	T_b °K	D_p mm	ε_b	N_{1g}	N_{1b}
a	1.0	0.5	0.529	1273	0.354	0.41	0.0496	2.413
b	1.0	0.5	0.529	1273	0.777	0.41	0.0226	1.1
c	1.0	0.5	0.529	1673	0.354	0.41	0.026	1.265

F_{oi}

FIG 7.8

PREDICTED INSTANTANEOUS NUSSELT – FOURIER NUMBER RELATIONSHIP

uncoupled sol^n
coupled sol^n

SILICA SAND

$(N_{ui})_c$ from [82]

$(N_{ui})_c$ present work

$(N_{ui})_r$ present work

$(N_{ui})_{c,r}$

	ϵ_w	θ_w	ω_o	$T_b \, ^\circ K$	$D_p \, mm$	ϵ_b	N_{1g}	N_{1b}
a	1.0	0.8	0.529	1273	0.354	0.41	0.0496	2.413
b	1.0	0.8	0.529	1273	0.777	0.41	0.0226	1.1
c	1.0	0.8	0.529	1673	0.354	0.41	0.026	1.265

0.01   0.1   1.0   10

F_{oi}

1.0   0.1   0.01

FIG 7.9

PREDICTED INSTANTANEOUS NUSSELT – FOURIER NUMBER RELATIONSHIP

—— uncoupled sol^n

--- coupled sol^n

SILICA SAND

$(N_{ui})_c$ from [82]

$(N_{ui})_c$ present work

$(N_{ui})_r$ present work

$(N_{ui})_{c,r}$

F_{oi}

	ε_w	θ_w	ω_o	$T_b\,^\circ K$	D_p mm	ε_b	N_{1g}	N_{1b}
a	0.7	0.5	0.529	1273	0.354	0.41	0.0496	2.413
b	0.7	0.5	0.529	1273	0.777	0.41	0.0226	1.1
c	0.7	0.5	0.529	1673	0.354	0.41	0.026	1.265

FIG 7.10

PREDICTED INSTANTANEOUS NUSSELT - FOURIER NUMBER RELATIONSHIP

SILICON CARBIDE

uncoupled sol^n ———
coupled sol^n - - - -

$(N_{ui})_c$ from [82]

$(N_{ui})_c$ present work

$(N_{ui})_r$ present work

	ε_w	θ_w	ω_o	T_b °K	D_p mm	ε_b	N_{1g}	N_{1b}
a	1.0	0.5	0.374	1273	0.354	0.41	0.089	4.31
b	1.0	0.5	0.374	1273	0.777	0.41	0.041	1.97
c	1.0	0.5	0.374	1673	0.354	0.41	0.0465	2.259

$(N_{ui})_{c,r}$

F_{oi}
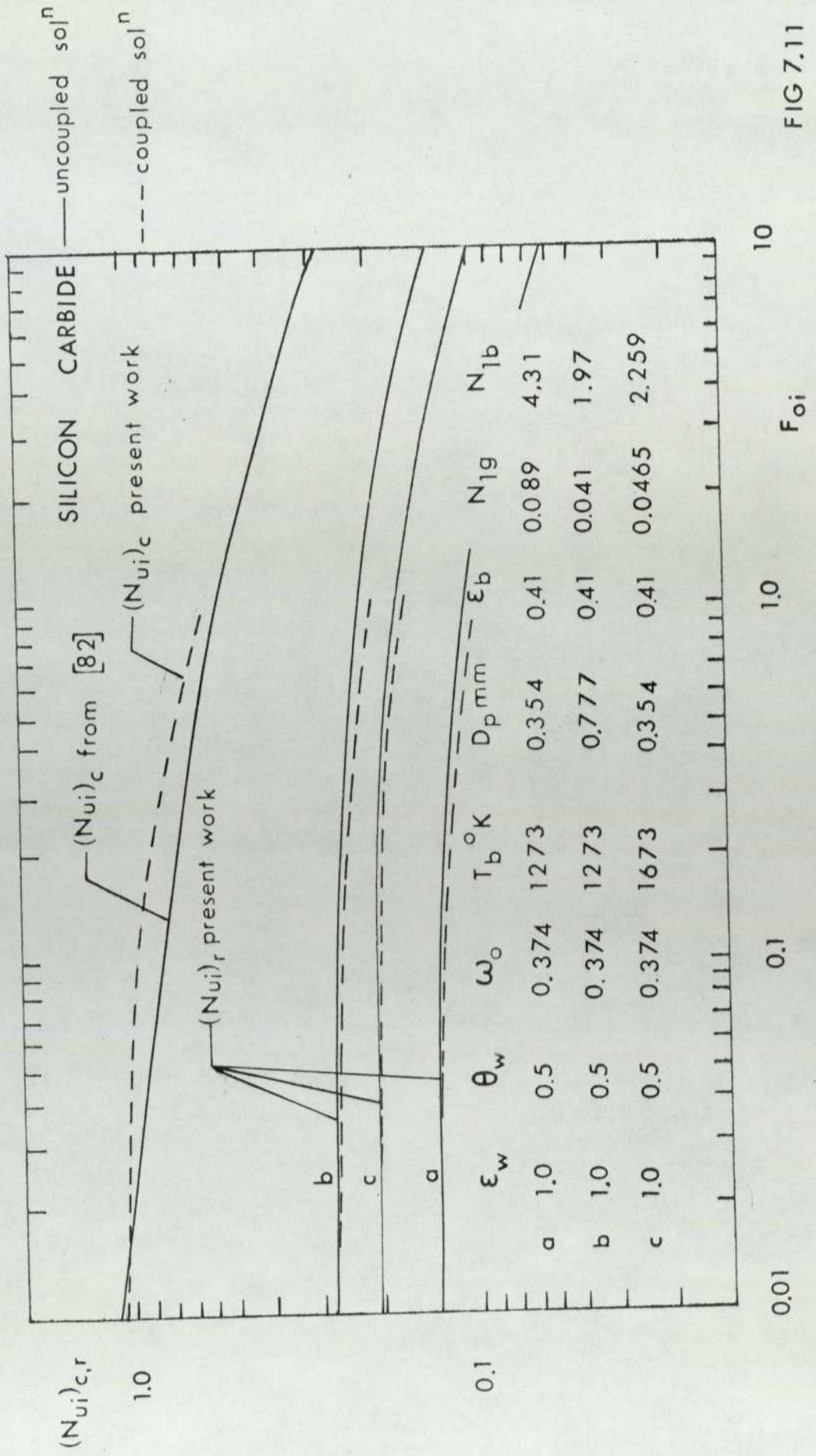
FIG 7.11

TRANSIENT
TEMPERATURE   DISTRIBUTION

$F_o = 0$

3.57 D_p

3.32 D_p

T_o	ε_w	θ_w	ω_o	F_o
10	1.0	0.5	0.529	0.345

———— radiation   only

— — — radiation plus conduction

$\theta^4(T, F_o)$

T
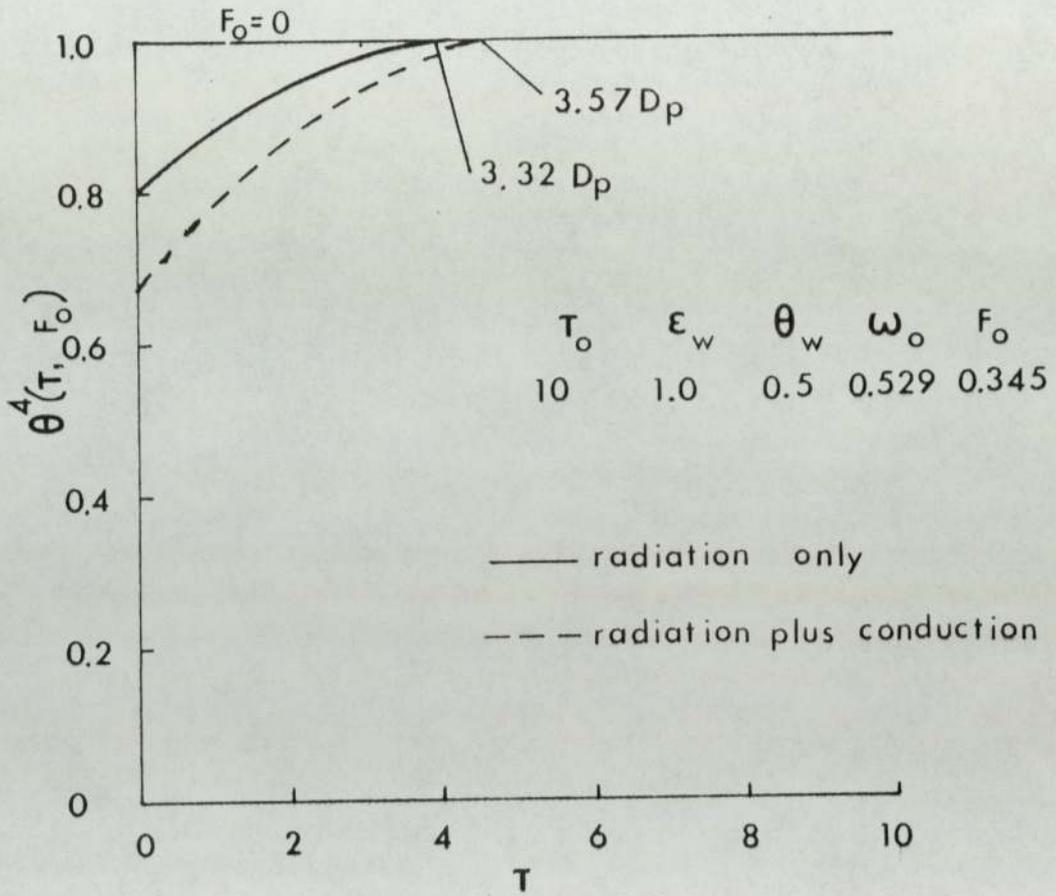
FIG. 7.12

EXPERIMENTAL CONFIGURATION FOR
BED TO WALL RADIATIVE FLUX
MEASUREMENTS

100mm DIA

QUARTZ GLASS
REACTOR

INSULATION

COOLING
AIR

PYROMETER

'X'

GAS-AIR
MIXTURE

ALUMINIUM
FOIL

GLASS
WALL

AIR
FLOW

THERMOCOUPLE

FIG 7.13

VIEW 'X'

ALUMINA

$q'_{rm}(0,t'_m)$

$T_b \, ^\circ K$ 1253

$D_p \, mm$ 0.55

	U−U$_{mf}$ mm/sec	ε_w	θ_w	ω_o	ε_p
		1.0	0.797	0.773	0.37
1	50				
2	100				
3	200				
4	300				
5	400				
6	500				

$$t'_m = \frac{K_t^2 \, K_b \, t_m}{\rho_p \, C_p (1 - \varepsilon_b)}$$

FIG 7.14

SILICA SAND

	ϵ_w	θ_w	ω_o	ϵ_p
	1.0	0.883	0.529	0.64

	$U-U_{mf}$ mm/sec
1	50
2	100
3	200
4	300
5	400
6	500

T_b °K
1251

D_p mm
0.55

$$t'_m = \frac{K_t^2 \, K_b \, t_m}{\rho_p \, c_p (1-\epsilon_b)}$$

$q'_{rm}(0,t'_m)$

FIG 7.15

FIG 7.16

SILICON CARBIDE

$$T_b \, ^{\circ}K \quad 1204$$

$$D_p \, mm \quad 0.55$$

$$t'_m = \frac{K_t^2 \, K_b \, t_m}{\rho_p \, c_p (1 - \varepsilon_b)}$$

$q'_{rm}(0, t'_m)$

		ε_w	θ_w	ω_o	ε_p
		1.0	0.692	0.374	0.77

	$U - U_{mf}$ mm/sec		
1	50		
2	100		
3	200		
4	300		
5	400		
6	500		

EFFECT OF EXCESS GAS VELOCITY ON
TIME MEAN RADIATIVE FLUX

silica sand

$q'_{rm}(0,t'_m)$

		D_p mm	θ_w	ε_w	ω_o	$T_b{}^\circ C$
a	▼	0.354	0.854	1.0	0.529	1257
b	■	0.55	0.883	-//-	-//-	1251
c	□	0.777	0.86	-//-	-//-	1261

$(U-U_{mf})$ mm/sec   FIG 7.17

alumina

$q'_{rm}(0,t'_m)$

		D_p mm	θ_w	ε_w	$\dot{\omega}_o$	$T_b{}^\circ C$
a	▼	0.354	0.81	1.0	0.773	1285
b	■	0.55	0.797	-//-	-//-	1253
c	□	0.777	0.795	-//-	-//-	1283

$(U-U_{mf})$ mm/sec   FIG 7.18

EFFECT OF EXCESS GAS VELOCITY ON TIME MEAN RADIATIVE FLUX

silicon carbide

D_p mm		θ_w	ε_w	ω_o	$T_b\ ^\circ C$
▼	0.354	0.72	1.0	0.374	1211
■	0.55	0.692	-//-	-//-	1204
□	0.777	0.883	-//-	-//-	1252

$(U - U_{mf})$ mm/sec

FIG 7.19

magnesia

D_p mm		θ_w	ε_w	ω_o	$T_b\ ^\circ C$
▼	0.354	0.8	1.0	0.601	1283
■	0.55	0.88	-//-	-//-	1223
□	0.777	0.87	-//-	-//-	1254

$(U - U_{mf})$ mm/sec

FIG 7.20

A P P E N D I C E S

APPENDIX A

THE EFFECTIVE EMISSIVITY OF A FLUIDIZED BED

From Zabrodsky[2] the fluidized bed may be approximated by a dusty gas stream, requiring a solution of the classical energy attenuation equation

$$\frac{dI}{dx} = -K_a I \tag{A.1}$$

integrating (A.1)

$$I = I_b e^{-K_a x} \quad \text{or} \quad \frac{I}{I_b} = e^{-K_a x} = \tau_t$$

Now $\tau_t + \alpha = 1$

Hence from Kirchoff's law

$$\alpha = \varepsilon_e = 1 - \tau_t = 1 - e^{-K_a x}$$

The constant K_a (absorption coefficient of gas layer) was determined in [2] as

$$K_a = \frac{1.5 (1 - \varepsilon_b)}{D_p} \tag{A.2}$$

Hence for a typical fluidized bed with values of $\varepsilon_b = 0.65$ and $D_p = 1.5$ mm, then

$$\varepsilon_e = 1 - 8.5 \times 10^{-5} \simeq 1$$

APPENDIX B

THE EQUATION OF RADIATIVE TRANSFER

Referring to Figure B.1 at the end of this Appendix, the equation of radiative transfer may be written for the one-dimensional case from [17] as:

$$\text{Cos}\theta \ \frac{dI}{dx} \ = \ -K_t I(x,\theta) \ + \ \frac{K_s}{4\pi} \int_{4\pi} I(x,\theta')P(\theta,\theta')d\Omega \ + \ K_a \ \nu^2 \ I_b(x)$$

$$(B.1)$$

where θ' and θ refer to the incident and scattered directions respectively using the transforms

$$K_t \ = \ K_a + K_s, \qquad \nu^2 \ = \ 1$$

$$\tau \ = \ \int_0^x K_t(x)dx, \qquad \tau_0 \ = \ \int_0^L K_t(x)dx$$

with K_t invariant with x

$$\omega_o \ = \ \frac{K_s}{K_t}, \qquad 1 - \omega_o \ = \ \frac{K_a}{K_t}, \qquad \mu \ = \ \text{Cos}\theta$$

Equation (B.1) becomes

$$\mu \ \frac{dI}{d\tau} \ = \ -I + \frac{\omega_o}{4} D(\tau) + (1 - \omega_o)I_b(\tau) \qquad (B.2)$$

where

$$D(\tau) \ = \ \int_{4\pi} I(\tau,\theta')P(\theta,\theta')d\Omega$$

For the one-dimensional case equation (B.2) may be written as two

simultaneous equations for the positive and negative τ and μ

direction as

$$\mu \frac{dI^+}{d\tau} + I^+ = (1 - \omega_o) I_b(\tau) + \frac{\omega_o}{4\pi} D(\tau) \tag{B.3}$$

and

$$\mu \frac{dI^-}{d\tau} + I^- = (1 - \omega_o) I_b(\tau) + \frac{\omega_o}{4\pi} D(\tau) \tag{B.4}$$

with boundary conditions of

$$I^+(\tau,\mu) = I^+(0,\mu), \qquad \tau = 0$$

$$I^-(\tau,\mu) = I^-(\tau_o,\mu), \qquad \tau = \tau_o$$

using an integrating factor $e^{\tau/\mu}$ then equations (B.3) and (B.4)

becomes

$$I^+(\tau,\mu) = I^+(0,\mu)e^{-\tau/\mu} + \int_o^\tau [(1-\omega_o)I_b(\tau') + \frac{\omega_o}{4\pi} D(\tau')]e^{-(\tau-\tau')/\mu} \frac{d\tau'}{\mu}$$

$$\tag{B.5}$$

and

$$I^-(\tau,\mu) = I^-(\tau_o,\mu)e^{(\tau_o-\tau)/\mu} - \int_\tau^{\tau_o} [(1-\omega_o)I_b(\tau') + \frac{\omega_o}{4\pi} D(\tau')]e^{-(\tau-\tau')/\mu} \frac{d\tau'}{\mu}$$

$$\tag{B.6}$$

where τ' is used as a dummy argument to distinguish from τ. The

Radiative Flux may be defined as

$$q_r(\tau) = 2\pi \int_o^1 I^+ \mu d\mu - 2\pi \int_o^{-1} I^- \mu d\mu \tag{B.7}$$

Then substitution of (B.5), (B.6) into (B.7)

$$q_r(\tau) = 2\pi \int_0^1 I^+(0,\mu)e^{-\tau/\mu}\mu\,d\mu - 2\pi \int_0^1 I^-(\tau_0,\mu)e^{-(\tau_0-\tau)/\mu}\mu\,d\mu$$

$$+ \; 2\pi \int_0^\tau [(1-\omega_0)I_b(\tau') + \frac{\omega_0}{4\pi}D(\tau')]E_2(\tau-\tau')d\tau'$$

$$- \; 2\pi \int_\tau^{\tau_0} [(1-\omega_0)I_b(\tau') + \frac{\omega_0}{4\pi}D(\tau')]E_2(\tau'-\tau)d\tau' \qquad (B.8)$$

and similarly

$$D(\tau) = 2\pi \int_0^1 I^+(\tau,\mu)d\mu - 2\pi \int_0^{-1} I^-(\tau,\mu)d\mu$$

Hence

$$D(\tau) = 2\pi \int_0^1 I^+(0,\mu)e^{-\tau/\mu}d\mu + 2\pi \int_0^1 I^-(\tau_0,\mu)e^{-(\tau_0-\tau)/\mu}d\mu$$

$$+ \; 2\pi \int_0^{\tau_0} [(1-\omega_0)I_b(\tau') + \frac{\omega_0}{4\pi}D(\tau')]E_1(|\tau-\tau'|)d\tau' . \qquad (B.9)$$

where $E_n(\tau)$ are exponential integrals.

Non-dimensionalizing with

$$q_r'(\tau) = \frac{q_r(\tau)}{\sigma T_b^{\,4}}, \qquad\qquad D'(\tau) = \frac{D(\tau)}{\sigma T_b^{\,4}}$$

$$\frac{T_1^{\,4}}{T_b^{\,4}} = \theta_1^{\,4}, \qquad\qquad \frac{T_2^{\,4}}{T_b^{\,4}} = \theta_2^{\,4}$$

$$I^+(0,\mu) = \frac{F_1}{\pi}, \qquad\qquad I^-(\tau_0,\mu) = \frac{F_2}{\pi}$$

$$2\pi \int_0^1 I^+(0,\mu)e^{-\tau/\mu}\mu\,d\mu = 2F_1 E_3(\tau)$$

$$2\pi \int_0^1 I^-(\tau_0,\mu)e^{-(\tau_0-\tau)/\mu}d\mu = 2F_2 E_3(\tau_0-\tau)$$

$$2\pi \int_0^1 I^+(0,\mu) e^{-\tau/\mu} d\mu = 2F_1 E_2(\tau)$$

$$2\pi \int_0^1 I^-(\tau_0,-\mu) e^{-(\tau_0-\tau)/\mu} d\mu = 2F_2 E_2(\tau_0-\tau)$$

$$\theta^4(\tau) = \frac{\pi I_b(\tau)}{\sigma T_b^4}$$

then equations (B.8) and (B.9) may be written

$$q_r'(\tau) = 2F_1 E_3(\tau) - 2F_2 E_3(\tau_0-\tau)$$

$$+ 2\int_0^\tau [(1-\omega_0)\theta^4(\tau') + \frac{\omega_0}{4} D'(\tau')] E_2(\tau-\tau') d\tau'$$

$$- 2\int_\tau^{\tau_0} [(1-\omega_0)\theta^4(\tau') + \frac{\omega_0}{4} D'(\tau')] E_2(\tau'-\tau) d\tau' \qquad (B.10)$$

and

$$D'(\tau) = 2F_1 E_2(\tau) + 2F_2 E_2(\tau_0-\tau)$$

$$+ 2\int_0^{\tau_0} [(1-\omega_0)\theta^4(\tau') + \frac{\omega_0}{4} D'(\tau')] E_1(|\tau-\tau'|) d\tau' \qquad (B.11)$$

Equations (B.10) and (B.11) are known as the 'radiative flux equations'.

In equation (B.10) the first two terms on the right-hand side of the equation are the contributions to the radiative flux $q_r'(\tau)$ from the two boundaries. The third and fourth terms are the contributions from the absorbing, emitting and scattering medium. For plane parallel boundaries appropriate radiative boundary conditions become from [26]

$$F_1 = \epsilon_1 \theta_1^4 + (1-\epsilon_1)(F_1 - q_r(0)) \qquad (B.12)$$

and

$$F_2 = \varepsilon_2 \theta_2{}^4 + (1 - \varepsilon_2)(F_2 + q_r(\tau_o)) \tag{B.13}$$

then at $\tau = 0$

$$F_1 = \varepsilon_1 \theta_1{}^4 + 2(1 - \varepsilon_1)\{F_2 E_3(\tau_o)$$

$$+ \int_0^{\tau_o} [(1-\omega_o)\theta^4(\tau') + \frac{\omega_o}{4} D'(\tau')] E_2(\tau') d\tau'\} \tag{B.14}$$

and at $\tau = \tau_o$

$$F_2 = \varepsilon_2 \theta_2{}^4 + 2(1 - \varepsilon_2)\{F_1 E_3(\tau_o)$$

$$+ \int_0^{\tau_o} [(1-\omega_o)\theta^4(\tau') + \frac{\omega_o}{4} D'(\tau')] E_2(\tau_o-\tau') d\tau'\}$$
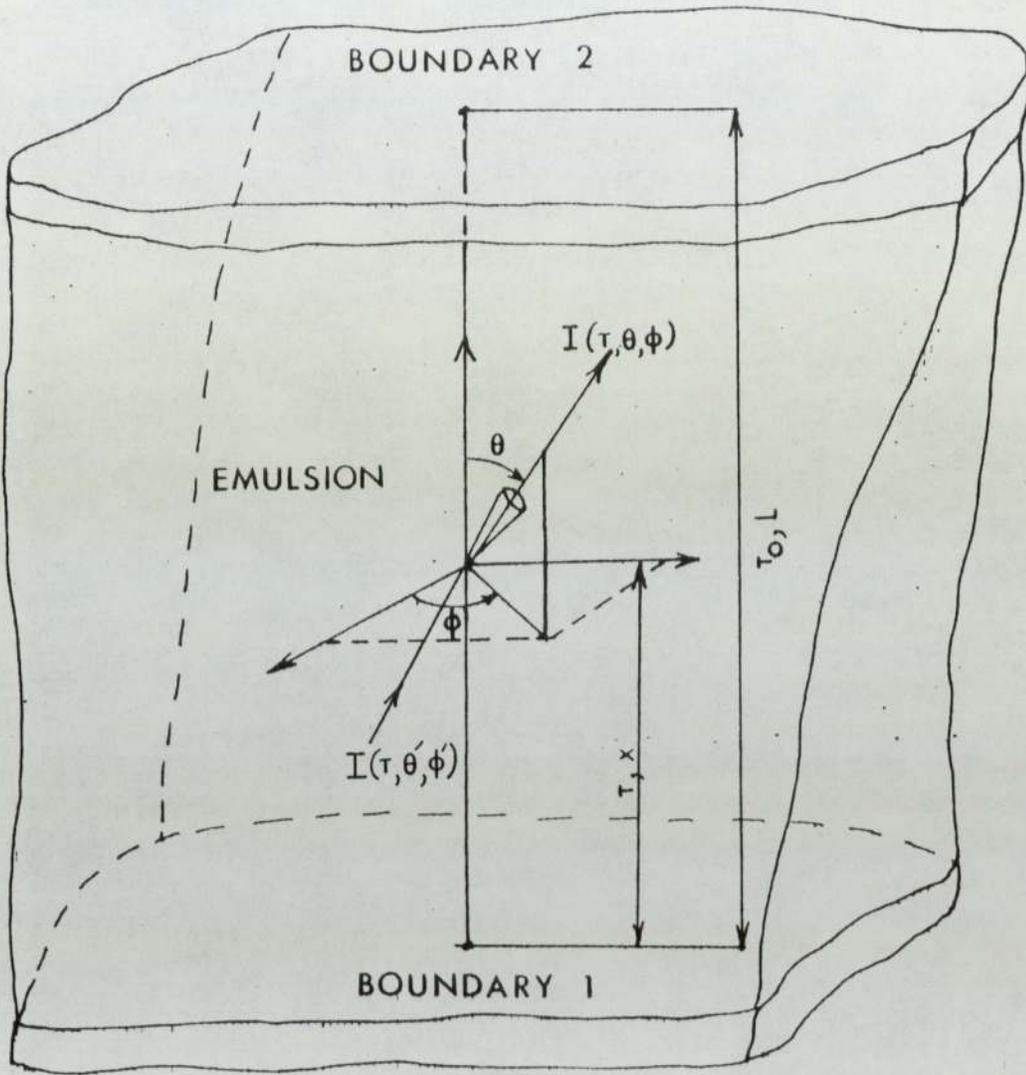
GEOMETRIC CONFIGURATION OF RADIATIVE TRANSFER
OF AN EMULSION BETWEEN PARALLEL BOUNDARIES

FIG. B1

APPENDIX C

THE TWO FLUX APPROXIMATION OF THE EQUATION OF RADIATIVE

TRANSFER APPLIED TO AN ISOTHERMAL MEDIUM

From Appendix B the equation of radiative transfer in non-dimensional form is

$$\mu \frac{dI}{d\tau} = -I + \frac{\omega_o}{4\pi} \int_{4\pi} IP(\theta')d\Omega + (1-\omega_o)I_b \qquad (C.1)$$

and may be transformed by assuming that radiation is scattered in the direction of the incident beam or at 180° to this direction. Hence from [10]

$$\frac{\omega_o}{4\pi} \int_{4\pi} IP(\theta')d\Omega = \omega_o[fI^+ + bI^-] \qquad (C.2)$$

For isotropic scattering $P(\theta') = 1$ and

$$f = \int_0^1 P(\theta')d\mu \qquad b = \frac{1}{2}\int_{-1}^0 P(\theta')d\mu$$

Hence equation (C.1) becomes

$$\mu \frac{dI^+}{d\tau} = (f\omega_o - 1)I^+_r + \omega_o bI^- + (1 - \omega_o)I_b \qquad (C.3)$$

and

$$\mu \frac{dI^-}{d\tau} = (1 - f\omega_o)I^- - \omega_o bI^+ - (1 - \omega_o)I_b \qquad (C.4)$$

The solution of (C.3) and (C.4) from [20] is:

$$I^+(\tau) = A(1 - m)e^{n\tau} + B(1 + m)e^{-n\tau} + I_b \qquad (C.5a)$$

$$I^-(\tau) = A(1 + m)e^{n\tau} + B(1 - m)e^{-n\tau} + I_b \qquad (C.5b)$$

for $f = b = \frac{1}{2}$, where

$$n = [a(a + 2s)]^{\frac{1}{2}} \qquad (C.6)$$

and

$$m = \frac{n}{a + 2s} \qquad (C.7)$$

where

$$a = (1 - f\omega_o)/\mu , \qquad (C.8)$$

$$s = \omega_o b/\mu \qquad (C.9)$$

subject to boundary conditions of

$$\tau = 0, \qquad I^+ = 0 \qquad (C.10)$$

$$\tau = \tau_o, \qquad I^- = 0 \qquad (C.11)$$

Hence

$$B = I_b \left[\frac{\dfrac{1}{(1 + m)e^{n\tau_o}} - \dfrac{1}{(1 - m)}}{\left[\dfrac{1 + m}{1 - m}\right] - \left[\dfrac{1 - m}{1 + m}\right]\dfrac{1}{e^{2n\tau_o}}} \right] \qquad (C.12)$$

and

$$A = \frac{-B(1 + m) - I_b}{(1 - m)} \qquad (C.13)$$

By assuming that I^+ and I^- are the total one dimensional hemispherical fluxes across the reference plane rather than intensities, then $\mu = 1$. According to [10] the optical depth for any one dimensional slab may be modified using $\tau = 2\tau$.

APPENDIX D

A SOLUTION OF THE RADIATIVE FLUX EQUATION FOR STEADY
STATE HEAT TRANSFER

The steady state energy equation may be written, for pure
radiation (neglecting conduction and convection), as

$$\frac{dq_r}{d\tau} = f(\tau) \neq 0$$

to remain consistent with the first differential of equation (B.10)
when an arbitrary temperature distribution is imposed. The Radiative
Flux equations (B.10) and (B.11) of Appendix B are:

$$q_r'(\tau) = 2F_1 E_3(\tau) - 2F_2 E_3(\tau_0 - \tau)$$

$$+ 2\int_0^\tau [(1 - \omega_0)\Theta^4(\tau') + \frac{\omega_0}{4} D'(\tau')]E_2(\tau - \tau')d\tau'$$

$$- 2\int_0^{\tau_0} [(1 - \omega_0)\Theta^4(\tau') + \frac{\omega_0}{4} D'(\tau')]E_2(\tau' - \tau)d\tau' \quad (D.1)$$

and

$$D'(\tau) = 2F_1 E_2(\tau) + 2F_2 E_2(\tau_0 - \tau)$$

$$+ 2\int_0^{\tau_0} [(1 - \omega_0)\Theta^4(\tau') + \frac{\omega_0}{4} D'(\tau)]E_1(|\tau - \tau'|)d\tau' \quad (D.2)$$

A solution of equations (D.1) and (D.2) may be obtained by firstly
considering equation (D.2). Approximating

$$\theta^4(\tau') = \sum_{i=1}^n C_i \tau'^{(i-1)} \quad (D.3)$$

and

$$D'(\tau') = \sum_{i=1}^{n} d_i \tau'^{(i-1)} \qquad (D.4)$$

With the coefficients C_i, i=1....n, initially known then substitution of equations (D.3) and (D.4) into equation (D.2) enables the integrals to be solved analytically term by term and the unknown coefficients d_i, i=1....n, to be obtained by satisfying equation (D.2) at 'n' locations.

This may be achieved by recasting the modified form of equation (D.2) as a matrix equation. Then equation (D.2) becomes

$$[V] \cdot [d] = [W] + [X] + [Y] + [Z] \cdot [d] \qquad (D.5)$$

Hence

$$[U] \cdot [d] = [P] \qquad (D.6)$$

where

$$[U] = [V] - [Z]$$

and

$$[P] = [W] + [X] + [Y]$$

$$\text{with} \quad [V] = \begin{bmatrix} \tau_1, & \tau_1^2, & \tau_1^3, & ., & ., & \tau_1^{(n-1)} \\ \tau_2, & \tau_2^2, & \tau_2^3, & ., & ., & \tau_2^{(n-1)} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \tau_n, & \tau_n^2, & \tau_n^3, & ., & ., & \tau_n^{(n-1)} \end{bmatrix}$$

$$[W] = \begin{bmatrix} 2F_1 E_2(\tau_1) \\ 2F_1 E_2(\tau_2) \\ \cdot \\ \cdot \\ \cdot \\ 2F_1 E_2(\tau_n) \end{bmatrix}$$

$$[X] = \begin{bmatrix} 2F_2 E_2(\tau_0 - \tau_1) \\ 2F_2 E_2(\tau_0 - \tau_2) \\ \cdot \\ \cdot \\ \cdot \\ 2F_2 E_2(\tau_0 - \tau_n) \end{bmatrix}$$

$$[Y] = \begin{bmatrix} I_1, & I_2\tau_1', & I_3\tau_1'^2 & \cdot & \cdot & \cdot & I_n\tau_1'^{(n-1)} \\ I_1, & I_2\tau_2', & I_3\tau_2' & \cdot & \cdot & \cdot & I_n\tau_2'^{(n-1)} \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ I_1, & I_2\tau_n', & I_3\tau_n' & \cdot & \cdot & \cdot & I_n\tau_n'^{(n-1)} \end{bmatrix} \cdot \begin{bmatrix} C_1 \\ C_2 \\ \cdot \\ \cdot \\ \cdot \\ C_n \end{bmatrix}$$

$$[Z] = \begin{bmatrix} J_1, & J_2\tau_1', & J_3\tau_1' & \cdot & \cdot & \cdot & J_n\tau'^{(n-1)} \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ J_1, & J_2\tau_n', & J_3\tau_n' & \cdot & \cdot & \cdot & J_n\tau_n'^{(n-1)} \end{bmatrix}$$

$$[d] = \begin{bmatrix} d_1 \\ d_2 \\ \cdot \\ \cdot \\ \cdot \\ d_n \end{bmatrix}$$

where I_i, $i=1....n$

and J_i, $i=1....n$

are obtained from a solution of the integrals in equation (D.2).

It remains to solve equation (D.6) for the unknowns d_i, $i=1....n$.

This is achieved by transforming $[U]$ into an upper and lower

matrix. Hence

$$[U] \cdot [d] = [P]$$

becomes

$$[U]^L \cdot [U]^U \cdot [d] = [P]$$

or

$$[U]^L \cdot [b] = [P] \qquad\qquad (D.7)$$

and

$$[U]^U \cdot [d] = [b] \qquad\qquad (D.8)$$

Hence [b] is solved from (D.7), substituted in (D.8) and solved for

[d].

With a solution for $D'(\tau)$ on substituting in equation (D.1),

$q_r'(\tau)$ may then be determined.

APPENDIX E

(a) <u>Determination of Extinction Coefficient from Bed</u>
<u>Pressure Drop Distribution</u>

Consider a unit volume of a gas/solid emulsion.

$$V_p \rho_p N = \bar{\rho} \qquad \text{bulk mass/unit volume} \qquad (E.1)$$

$$\therefore N = \frac{\bar{\rho}}{V_p \rho_p} \qquad \text{particles/unit volume} \qquad (E.2)$$

Across volume ΔV the pressure difference

$$\Delta P = \frac{g V_p \rho_p N \Delta V}{A} \qquad (E.3)$$

with $\Delta V = A \Delta x$; then

$$\frac{\Delta P}{\Delta x} = g V_p \rho_p N \qquad (E.4)$$

and

$$\underset{\Delta x \to o}{L_t} \frac{\Delta P}{\Delta x} = \frac{dP}{dx} = V_p \rho_p N g \qquad (E.5)$$

For a distribution of particles/unit volume $N(x)$, equation (5) may
be written

$$N(x) = \frac{1}{g V_p \rho_p} \frac{dP(x)}{dx} \qquad (E.6)$$

with

$$V_p = \frac{\pi}{6} D_p^{\,3}$$

$$N(x) = \frac{6}{\pi D_p^{3} \rho_p g} \frac{dP(x)}{dx} \qquad (E.7)$$

Extinction cross-section

$$K_t' = \frac{\pi D_p^{2}}{4} [\epsilon_p + 2(1 - \epsilon_p)]$$

Hence optical depth $\tau(x)$ is defined

$$\tau(x) = \int_0^x K_t' N(x) dx \qquad (E.8)$$

$\therefore$

$$\tau(x) = \frac{3}{2} D\frac{1}{\rho_p\rho_p g}[\epsilon_p + 2(1 - \epsilon_p)] \int_0^x \frac{dP}{dx}(x).dx \qquad (E.9)$$

As $P(x)$ is obtained experimentally. A least squares polynomial

fit may be used to obtain the form of

$$P(x) = \sum_{i=1}^{n} C_i x^{(i-1)} \qquad (E.10)$$

Then $\tau(x) = f_n(x)$ may be found. A value of $K_t(x)$ may then be

obtained from the relationship.

$$K_t(x) = K_t' N(x) = \frac{d\tau(x)}{dx} \qquad (E.11)$$

(b) <u>Determination of Extinction Coefficient at a Plane</u>

<u>Surface</u>

From (a) it was established that

$$N(x) = \frac{\bar{\rho}(x)}{V_p \rho_p} \qquad (E.12)$$

where

$$\bar{\rho}(x) = \rho_p(1 - \varepsilon_b(x)) \tag{E.13}$$

Hence

$$N(x) = \frac{(1 - \varepsilon_b(x))}{V_p} = \frac{6(1 - \varepsilon_b(x))}{\pi D_p^3} \tag{E.14}$$

with $\bar{x} = \frac{x}{D_p}$; then

$$N(\bar{x}) = \frac{6}{\pi D_p^3} (1 - \varepsilon_b(\bar{x})) \tag{E.15}$$

and

$$K_t' = \frac{\pi D_p^2}{4} [\varepsilon_p + 2(1 - \varepsilon_p)] \tag{E.16}$$

Hence

$$K_t(\bar{x}) = K_t' N(\bar{x}) \tag{E.17}$$

$$\bar{K}_t(\bar{x}) = \frac{K_t' 6}{\pi D_p^3} [1 - \varepsilon_b(\bar{x})] \tag{E.18}$$

From [82]:

$$\varepsilon_b(\bar{x}) = 1 - 3(1 - \varepsilon_b)(\bar{x} - \frac{2}{3}\bar{x}^2), \quad \bar{x} \leqslant 1 \tag{E.19}$$

and

$$\varepsilon_b(\bar{x}) = \varepsilon_b, \quad \bar{x} > 1 \tag{E.20}$$

APPENDIX F

OPERATION OF A RADIATION PYROMETER (RADIOMETER) SYSTEM
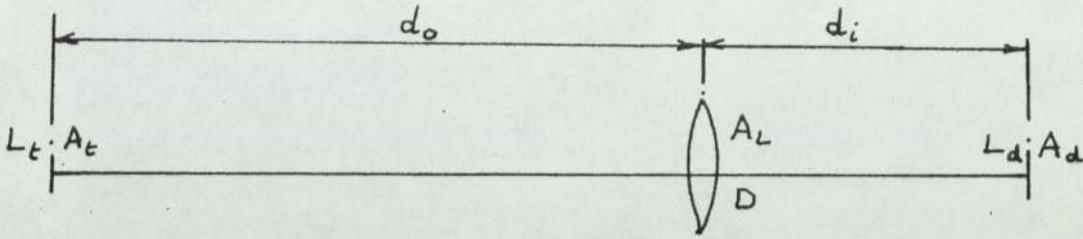
For a thin lens f = focal length.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad (F.1)$$

The focal length depends upon lens material and its curvature.
Target area which fills the detector A_d is

$$A_t = A_d \qquad (F.2)$$

From (F.1) and (F.2):

$$A_t = A_d \frac{(d_o - f)^2}{f^2} \qquad (F.3)$$

and for $d_o \gg f$

$$A_t \simeq A_d \frac{d_o^2}{f^2} \qquad (F.4)$$

For a detector diameter = L_d then

$$L_t \simeq L_d \frac{d_o}{f} \tag{F.5}$$

When the target dimensions are equal to or greater than the size
in equation (F.4) or (F.5), the target is resolved, therefore
requirement for target resolution is:

$$L_t \geqslant L_d \frac{d_o}{f} \tag{F.6}$$

Irradiance of the Detector

The irradiance G_L received at the radiometer lens decreases
with the square of the distance to the target. Hence total radia-
tion received is

$$A_L G_L = \frac{A_L A_t \varepsilon_t}{\pi d_o^2} \int_{\lambda_1}^{\lambda_2} N_\lambda \, d\lambda \quad \text{watts/steradian} \tag{F.7}$$

$$= \frac{D \, A_t \varepsilon_t}{4 \, d_o^2} \int_{\lambda_1}^{\lambda_2} N_\lambda \, d\lambda \qquad " \quad / \qquad " \tag{F.8}$$

with λ = wavelength

ε_t = target emissivity

N_λ = target radiance

and the radiation incident on the detector is

$$G_d = \frac{G_L A_L \tau_t}{A_d} = \frac{A_t \tau_t D^2 \varepsilon_t}{4 d_o^2 A_d} \int_{\lambda_1}^{\lambda_2} N_\lambda \, d\lambda \text{ watts/steradian } -m^2 \tag{F.9}$$

where τ_t is the transmission of the optical lens. The signal voltage from the detector is proportional to G_d. For a resolved target

$$\frac{A_t}{d_o^2 A_d} = \frac{1}{f^2}$$

therefore irradiance at the detector is

$$G_d = \frac{\tau_t D^2 \epsilon_t}{4f^2} \int_{\lambda_1}^{\lambda_2} N_\lambda d\lambda \quad \text{watts/steradian } -m^2 \qquad (F.10)$$

and

$$G_d = \frac{\tau_t D^2 \epsilon_t \pi}{4f^2} \int_{\lambda_1}^{\lambda_2} N_\lambda d\lambda \quad \text{watts/steradian } - m^2 \qquad (F.11)$$

Although signal strength V (mv) tends to decrease with the square of target distance, the area of the resolved target increases with square of target distance, resulting in an invariance of the signal with distance and size. Hence

$$V = \frac{K\tau_t D^2 \epsilon_t \pi}{4f^2} \int_{\lambda_1}^{\lambda_2} N_\lambda d\lambda \quad \text{(m volts)} \qquad (F.12)$$

Constant K includes the responsivity (volts/watt) of the detector and the gain of amplifier. Hence

$$V = K \cdot G_d \qquad (F.13)$$

and K may be found for black body calibration at relevant temperatures. For a wide spectral range

$$N(T_t^{\,4}) = \int_{\lambda_1}^{\lambda_2} N_\lambda d\lambda \qquad (F.14)$$

$$\therefore \quad V = \frac{K\tau_t D^2 \varepsilon_t \pi}{4f^2} N(T_t{}^4) \qquad\qquad (F.15)$$

From calibration with recorded temperature T_c

$$V = \frac{K\tau_t D^2 \varepsilon_d \pi}{4f^2} N(T_c{}^4) \qquad\qquad (F.16)$$

from (F.15) and (F.16) with $\varepsilon_d = 1$

$$\varepsilon_t = \frac{N(T_c{}^4)}{N(T_t{}^4)} \simeq \frac{T_c{}^4}{T_t{}^4} \qquad\qquad (F.17)$$

APPENDIX G

A SOLUTION OF THE RADIATIVE FLUX EQUATION FOR UNSTEADY

HEAT TRANSFER

The unsteady energy equation for an emulsion (packet) of fluidized particles in contact with a plane surface is:

$$\bar{\rho} \, \bar{C}_p \, \frac{\partial T}{\partial t} = - \frac{\partial q_r}{\partial x} \qquad (G.1)$$

where $\bar{\rho} \, \bar{C}_p = \rho_p C_p (1 - \varepsilon_b)$ and may be non-dimensionalised to

$$\frac{\partial \theta}{\partial t'} = - \frac{\partial q_r'}{\partial \tau} \qquad (G.2)$$

where

$$t' = \frac{K_t \, \sigma \, T_b^{\,3} t}{\rho_p C_p (1 - \varepsilon_b)}$$

subject to boundary conditions of

$$\tau = 0, \qquad \theta(0, t') = \theta_1, \qquad t' \geqslant 0 \qquad (G.3)$$

$$\tau = \tau_o, \qquad \theta(\tau_o, t') = 1, \qquad t' \geqslant 0 \qquad (G.4)$$

and initial conditions of

$$t' = 0, \qquad \theta(\tau, 0) = \begin{Bmatrix} 1 \\ \theta(\tau) \end{Bmatrix}, \qquad \tau \geqslant 0 \qquad (G.5)$$

A solution of equation (G.2) may be obtained using the analysis described in Appendix D. In this case the polynomial approximations become:

$$\theta^4(\tau,t') = \sum_{i=1}^{n} C_i(\tau,t')\tau^{(i-1)} \tag{G.6}$$

and

$$D(\tau,t') = \sum_{i=1}^{n} d_i(\tau,t')\tau^{(i-1)} \tag{G.7}$$

Referring to Appendix D, the matrix equation was solved for d_i, $i=1\ldots n$ and a solution for $q_r'(\tau,t')$ obtained initially at $t' = 0$ subject to appropriate boundary conditions. Equation (G.2) may then be written in finite difference form, i.e.

$$\left[\frac{\theta_n^{p+1} - \theta_n^{P}}{\Delta t'}\right] = -\left[\frac{q_{r\ n+1}' - q_{r\ n-1}'}{2\Delta\tau}\right]^{P} \tag{G.8}$$

where n refers to the nodal spacial position and p refers to the nodal position of instantaneous time. A temperature profile may then be predicted for the next time increment from equation (G.8). At any instant of time the coefficients $C_i(\tau,t')$ are obtained by a least squares method.

The matrix equation is then resolved, the flux equation determined and the procedure repeated for a new time increment. In order to start the solution at the boundary n = 1, for each new time increment, the approximation $q_{r\ n-1} \simeq q_{r\ n}$ was used and equation (G.8) becomes at $\tau = 0$, i.e. n = 1

$$\left[\frac{\theta_1^{p+1} - \theta_1^{P}}{\Delta t'}\right] = -\left[\frac{q_{r\ 2}' - q_{r\ 1}'}{\Delta\tau}\right]^{P} \tag{G.9}$$

APPENDIX H

A COMPUTER PROGRAM TO SOLVE THE STEADY STATE &

TRANSIENT RADIATIVE FLUX EQUATION

The following program solves the simultaneous equations (B.10) and (B.11) of Appendix B, initially for the steady state approximation. An extension is included to solve the transient energy equation (G.1) and, in finite difference form, equation (G.8) described in Appendix G. Simply the steady state solution of equations (B.10) and (B.11) with a described temperature distribution is given by the initial solution at time zero.

The important variables encountered in the program are described below.

(a) Input Variables

$FL1 = \theta_1^4$, the wall temperature to bed temperature ratio raised to the fourth power at boundary 1.

$FL2 = \theta_2^4$, the wall temperature to bed temperature ratio raised to the fourth power at boundary 2. In this analysis $\theta_2^4 = 1$.

WO   is the scattering albedo

EW   is the emissivity of boundary 1

EW2 is the emissivity of boundary 2

$C(I)$, $I = 1....m$ are the coefficients of the $(m-1)$ degree polynomial describing the initial fourth power temperature distribution, i.e.

$$\theta^4(\tau) = \sum_{i=1}^{m} C_i \tau^{(i-1)}$$

(b) <u>Output Variables</u>

$D(I)$, $I = 1....m$ are the coefficients of the $(m-1)$ degree
polynomial describing the incident radiative flux, i.e.

$$D(\tau) = \sum_{i=1}^{m} d_i \tau^{(i-1)}$$

$Q(I)$, $I = 1....n$ are the radiative flux levels at the corresponding
spacial position across the field. $Y1(I,J)$, $I = 1....n$, $J = m$
are the values of temperature across the field at position (I) and
time (J). ML is a counter.

(c) <u>Important Variables in the Main Body of the Program</u>

$Z(I)$, $I = 1....n$ designates spacial position across the field.

S13AAF   is a NAG subroutine used to determine the first

exponential integral, i.e. $E_1(x)$

DELX   is the spacial step size

DELT   is the temporal step size

LESQUT   is a least squares curve fitting subroutine

DEES   is a subroutine to determine the coefficients

$D(I)$, $I = 1....m$

GAUSS   is a subroutine called from subroutine LESQUT.

```
      PROGRAM LEWN(INPUT,OUTPUT,TAPE1=INPUT,TAPE2=OUTPUT)
      DIMENSION P(6,6),S1(8),U3(6,6),C(10)
      DIMENSION U4(6,6),U5(6,6),U6(6,6)
      DIMENSION QC(505),QT(505)
      DIMENSION D(8),Q(505),Y1(505,3),Y(505),X(10,505),YE(505)
      DIMENSION Z(505),ZZ(505),R(505),A(8,8),DD(8),U(505,6),S(505,6)
      COMMON Y1
      LEVEL 2,Y1
      REAL J1,J2,J3,J4,J5
      READ(1,*)FL1,FL2,W0,FW,EW2
      READ(1,*)(C(I),I=1,5)
      WRITE(2,22)FL1,FL2,W0,EW,EW2
   22 FORMAT(5E12.4)
      DELX=0.02
      DELT=0.0001
      NX=1
      BL1=FL1
      BL2=FL2
      N=501
      ML=0
      NN=N+1
      Z(1)=0.0
      DO 53 K=2,NN
   53 Z(K)=Z(K-1)+0.02
      DO 3 K=1,NN
      X(1,K)=1.0
      X(2,K)=Z(K)
      DO3 J=3,10
    3 X(J,K)=X(J-1,K)*Z(K)
      DO 56 I=1,NN
   56 Y(I)=C(1)+(C(2)*Z(I))+(C(3)*Z(I)*Z(I))+(C(4)*Z(I)*Z(I)*Z(I))
     1+(C(5)*(Z(I)**4))
   39 CONTINUE
      FL1=BL1
      FL2=BL2
      DO55 I=5,5
      L=N+1
      CALL LESQUT (I,10,L,X,Y,C,YE,R,RR)
   55 CONTINUE
      DO 57 I=1,NN
   57 Y(I)=C(1)+(C(2)*Z(I))+(C(3)*Z(I)*Z(I))+(C(4)*Z(I)*Z(I)*Z(I))
     1+(C(5)*(Z(I)**4))
      BD=Z(N)
      E1A=S13AAF(BD,IFAIL)
      E2A=(1.0/EXP(Z(N)))-(Z(N)*E1A)
      E3A=(1.0/2.0)*((1.0/EXP(Z(N)))-(Z(N)*E2A))
      E4A=(1.0/3.0)*((1.0/EXP(Z(N)))-(Z(N)*E3A))
      E5A=(1.0/4.0)*((1.0/EXP(Z(N)))-(Z(N)*E4A))
      E6A=(1.0/5.0)*((1.0/EXP(Z(N)))-(Z(N)*E5A))
      E7A=(1.0/6.0)*((1.0/EXP(Z(N)))-(Z(N)*E6A))
      H1=C(1)*(0.5-E3A)
      J1=       (0.5-E3A)
      H2=C(2)*(0.333-E4A-(Z(N)*E3A))
      J2=       (0.333-E4A-(Z(N)*E3A))
      H3=C(3)*(0.5-(2.0*E5A)-(2.0*Z(N)*E4A)-(Z(N)*Z(N)*E3A))
      J3=       (0.5-(2.0*E5A)-(2.0*Z(N)*E4A)-(Z(N)*Z(N)*E3A))
      H4=C(4)*(1.2-(6.0*E6A)-(6.0*Z(N)*E5A)-(3.0*Z(N)*Z(N)*E4A)
```

```
1-((Z(N)**5)*E5A))
   J4=        (1.2-(6.0*E6A)-(6.0*Z(N)*E5A)-(3.0*Z(N)*Z(N)*E4A)
1-((Z(N)**5)*E5A))
   H5=C(5)*(4.0-(24.0*E7A)-(24.0*Z(N)*E6A)-(12.0*Z(N)*Z(N)*E5A)
1-(4.0*(Z(N)**3)*F4A)-((Z(N)**4)*E3A))
   J5=        (4.0-(24.0*E7A)-(24.0*Z(N)*E6A)-(12.0*Z(N)*Z(N)*E5A)
1-(4.0*(Z(N)**3)*E4A)-((Z(N)**4)*E3A))
   W1=C(1)*(0.5-E3A)
   V1=        (0.5-E3A)
   W2=C(2)*((Z(N)*0.5)-0.333+E4A)
   V2=        ((Z(N)*0.5)-0.333+E4A)
   W3=C(3)*((Z(N)*Z(N)*0.5)-(2.0*Z(N)*0.333)+0.5-(2.0*E5A))
   V3=        ((Z(N)*Z(N)*0.5)-(2.0*Z(N)*0.333)+0.5-(2.0*E5A))
   W4=C(4)*(((Z(N)**3)*0.5)-(3.0*Z(N)*Z(N)*0.333)+(6.0*Z(N)*0.25)
1-(6.0*0.2)+(6.0*E6A))
   V4=        (((Z(N)**3)*0.5)-(3.0*Z(N)*Z(N)*0.333)+(6.0*Z(N)*0.25)
1-(6.0*0.2)+(6.0*E6A))
   W5=C(5)*(((Z(N)**4)*0.5)-(4.0*(Z(N)**3)*0.3333)+(12.0*Z(N)*Z(N
1)*0.25)-(24.0*Z(N)*0.2)+4.0-(24.0*E7A))
   V5=        (((Z(N)**4)*0.5)-(4.0*(Z(N)**3)*0.3333)+(12.0*Z(N)*Z(N
1)*0.25)-(24.0*Z(N)*0.2)+4.0-(24.0*E7A))
   FL1=(EW*BL1)+(2.0*(1.0-EW)*((BL2*EW2*E3A)+((1.0-W0)*(H1+H2+H3+H4
1+H5))))+(4.0*(1.0-EW)*(1.0-EW2)*(1.0-W0)*E3A*(W1+W2+W3+W4+W5))
   FL1=FL1/(1.0-(4.0*(1.0-EW)*(1.0-EW2)*E3A*E3A))
   FL2=(BL2*EW2)+(2.0*(1.0-EW2)*BL1*EW*E3A)+(2.0*(1.0-EW2)*(1.0-W0)
1*(W1+W2+W3+W4+W5))+(4.0*(1.0-EW2)*(1.0-EW)*(1.0-W0)*E3A*(H1+H2+
2H3+H4+H5))
   FL2=FL2/(1.0-(4.0*(1.0-EW)*(1.0-EW2)*E3A*E3A))
   XZ=(1.0-(4.0*(1.0-EW)*(1.0-EW2)*E3A*E3A))
   DO 58 I=1,5
   U3(I,1)=(1.0-EW)*W0*0.5*J1/XZ
   U3(I,2)=(1.0-EW)*W0*0.5*J2/XZ
   U3(I,3)=(1.0-EW)*W0*0.5*J3/XZ
   U3(I,4)=(1.0-EW)*W0*0.5*J4/XZ
   U3(I,5)=(1.0-EW)*W0*0.5*J5/XZ
58 CONTINUE
   DO 59 I=1,5
   U4(I,1)=E3A*(1.0-EW)*(1.0-EW2)*W0*V1/XZ
   U4(I,2)=E3A*(1.0-EW)*(1.0-EW2)*W0*V2/XZ
   U4(I,3)=E3A*(1.0-EW)*(1.0-EW2)*W0*V3/XZ
   U4(I,4)=E3A*(1.0-EW)*(1.0-EW2)*W0*V4/XZ
   U4(I,5)=E3A*(1.0-EW)*(1.0-EW2)*W0*V5/XZ
59 CONTINUE
   DO 60 I=1,5
   U5(I,1)=(1.0-EW2)*(1.0-EW)*W0*E3A*J1/XZ
   U5(I,2)=(1.0-EW2)*(1.0-EW)*W0*E3A*J2/XZ
   U5(I,3)=(1.0-EW2)*(1.0-EW)*W0*E3A*J3/XZ
   U5(I,4)=(1.0-EW2)*(1.0-EW)*W0*E3A*J4/XZ
   U5(I,5)=(1.0-EW2)*(1.0-EW)*W0*E3A*J5/XZ
60 CONTINUE
   DO 61 I=1,5
   U6(I,1)=0.5*(1.0-EW2)*W0*V1/XZ
   U6(I,2)=0.5*(1.0-EW2)*W0*V2/XZ
   U6(I,3)=0.5*(1.0-EW2)*W0*V3/XZ
   U6(I,4)=0.5*(1.0-EW2)*W0*V4/XZ
   U6(I,5)=0.5*(1.0-EW2)*W0*V5/XZ
61 CONTINUE
```

```
      DO40 I=1,N
      GG=Z(N)-Z(I)
      IF(Z(I).EQ.0.0) GO TO 560
      BE=Z(I)
      F1A=S13AAF(BE,IFAIL)
      E2A=(1.0/EXP(Z(I)))-(Z(I)*F1A)
      GO TO 561
  560 F2A=1.0
  561 CONTINUE
      IF(GG.EQ.0.0) GO TO 562
      F1B=S13AAF(GG,IFAIL)
      E2B=(1.0/EXP(GG))-(GG*F1B)
      GO TO 563
  562 E2B=1.0
  563 CONTINUE
   43 E3A=(1.0/2.0)*((1.0/EXP(Z(I)))-(Z(I)*F2A))
   62 F3B=(1.0/2.0)*((1.0/EXP(GG))-(GG*E2B))
      F4A=(1.0/3.0)*((1.0/EXP(Z(I)))-(Z(I)*E3A))
      E4B=(1.0/3.0)*((1.0/EXP(GG))-(GG*E3B))
      E5A=(1.0/4.0)*((1.0/EXP(Z(I)))-(Z(I)*E4A))
      F5B=(1.0/4.0)*((1.0/EXP(GG))-(GG*E4B))
      F6A=(1.0/5.0)*((1.0/EXP(Z(I)))-(Z(I)*E5A))
      E6B=(1.0/5.0)*((1.0/EXP(GG))-(GG*E5B))
      E7A=(1.0/6.0)*((1.0/EXP(Z(I)))-(Z(I)*F6A))
      F7B=(1.0/6.0)*((1.0/EXP(GG))-(GG*E6B))
      S(I,1)=(((2.0-F2A-E2B)*(2.0*(1.0-W0)))*C(1))+(2.0*FL1*F2A)+
     1(2.0*FL2*E2B)
      S(I,2)=(((((2.0*Z(I))+E3A-(Z(N)*F2B)-E3B)*(2.0*(1.0-W0)))*C(2))
      S(I,3)=(((((2.0*Z(I)*Z(I))+1.333-(2.0*E4A)-(Z(N)*Z(N)*E2B)
     1-(2.0*Z(N)*E3B)-(2.0*E4B))*(2.0*(1.0-W0)))*C(3))
      S(I,4)=(((((2.0*Z(I)*Z(I)*Z(I)))+(4.0*Z(I))+(6.0*E5A)-(6.0*E5B)-(
     1Z(N)*Z(N)*Z(N)*E2B)-(3.0*Z(N)*Z(N)*E3B)-(6.0*Z(N)*E4B))
     2*(2.0*(1.0-W0)))*C(4))
      S(I,5)=(((((2.0*(Z(I)**4))+(8.0*Z(I)*Z(I))+9.6-(24.0*E6A)-(4.0*
     1(Z(N)**3)*F3B) -((Z(N)**4)*E2B)-(12.0*Z(N)*Z(N)*E4B)-(24.0*
     2Z(N)*E5B) -(24.0*E6B))*(2.0*(1.0-W0)))*C(5))
      U(I,1)= (2.0-F2A-E2B)*W0/2.0
      U(I,2)= ((2.0*Z(I))+F3A-(Z(N)*E2B)-E3B)*W0/2.0
      U(I,3)= ((2.0*Z(I)*Z(I))+1.333-(2.0*E4A)-(Z(N)*Z(N)*E2B)
     1-(2.0*Z(N)*E3B)-(2.0*E4B)) *W0/2.0
      U(I,4)= ((2.0*Z(I)*Z(I)*Z(I))+(4.0*Z(I))+(6.0*F5A)-(6.0*E5B)-(
     1Z(N)*Z(N)*Z(N)*F2B)-(3.0*Z(N)*Z(N)*E3B)-(6.0*Z(N)*E4B)) *W0/2.0
      U(I,5)= ((2.0*(Z(I)**4))+(8.0*Z(I)*Z(I))+9.6-(24.0*E6A)-(4.0*
     1(Z(N)**3)*F3B) -((Z(N)**4)*E2B)-(12.0*Z(N)*Z(N)*E4B)-(24.0*
     2Z(N)*E5B) -(24.0*E6B)) *W0/2.0
   40 CONTINUE
      Z(1)=0.0
      DO 966 L=2,NN
  966 Z(L)=Z(L-1)+0.02
      DO 90 I=1,5
      DO 90 J=2,5
      P(I,1)=1.0
   90 P(1,J)=0.0
      DO 91 I=2,5
      DO 91 J=2,5
      II=(125*I)-124
      YW=Z(II)
```

```
      YYY=YW**(J-1)
   91 P(I,J)=YYY
      DO 92 I=2,5
      DO 92 J=1,5
   92 U(I,J)=U(((125*I)-124),J)
      DO 94 I=1,5
      DO 94 J=1,5
   94 A(I,J)=P(I,J)-U(I,J)-U3(I,J)-U4(I,J)-U5(I,J)-U6(I,J)
      DO 95 I=2,5
      DO 95 J=1,5
   95 S(I,J)=S(((125*I)-124),J)
      DO 98 I=1,5
   98 S1(I)=S(I,1)+S(I,2)+S(I,3)+S(I,4)+S(I,5)
      CALL DEES(S1,A,DD)
      DO 82 I=1,5
   82 D(I)=DD(I)
      FL1=FL1+(D(1)*U3(1,1))+(D(2)*U3(1,2))+(D(3)*U3(1,3))+(D(4)*
     1U3(1,4))+(D(5)*U3(1,5))+(D(1)*U4(1,1))+(D(2)*U4(1,2))+(D(3)*U4
     2(1,3))+(D(4)*U4(1,4))+(D(5)*U4(1,5))
      FL2=FL2+(D(1)*U5(1,1))+(D(2)*U5(1,2))+(D(3)*U5(1,3))+(D(4)*
     1U5(1,4))+(D(5)*U5(1,5))+(D(1)*U6(1,1))+(D(2)*U6(1,2))+(D(3)*U6
     2(1,3))+(D(4)*U6(1,4))+(D(5)*U6(1,5))
      DO 133 I=1,N
      GG=Z(N)-Z(I)
      IF(Z(I).EQ.0.0)GO TO 565
      BF=Z(I)
      E1A=S13AAF(BF,IFAIL)
      E2A=(1.0/EXP(Z(I)))-(Z(I)*E1A)
      GO TO 566
  565 E2A=1.0
  566 CONTINUE
      IF(GG.EQ.0.0) GO TO 567
      E1B=S13AAF(GG,IFAIL)
      E2B=(1.0/EXP(GG))-(GG*E1B)
      GO TO 568
  567 E2B=1.0
  568 CONTINUE
      E3A=(1.0/2.0)*((1.0/EXP(Z(I)))-(Z(I)*E2A))
      E3B=(1.0/2.0)*((1.0/EXP(GG))-(GG*E2B))
      E4A=(1.0/3.0)*((1.0/EXP(Z(I)))-(Z(I)*E3A))
      E4B=(1.0/3.0)*((1.0/EXP(GG))-(GG*E3B))
      E5A=(1.0/4.0)*((1.0/EXP(Z(I)))-(Z(I)*E4A))
      E5B=(1.0/4.0)*((1.0/EXP(GG))-(GG*E4B))
      E6A=(1.0/5.0)*((1.0/EXP(Z(I)))-(Z(I)*E5A))
      E6B=(1.0/5.0)*((1.0/EXP(GG))-(GG*E5B))
      E7A=(1.0/6.0)*((1.0/EXP(Z(I)))-(Z(I)*E6A))
      E7B=(1.0/6.0)*((1.0/EXP(GG))-(GG*E6B))
      F1=C(1)*(E3B-E3A)
      G1=D(1)*(E3B-E3A)
      F2=C(2)*(E4A-0.666+(Z(N)*E3B)+E4B)
      G2=D(2)*(E4A-0.666+(Z(N)*E3B)+E4B)
      F3=C(3)*((2.0*E5B)+(2.0*Z(N)*E4B)+(Z(N)*Z(N)*E3B)-(2.0*E5A)
     1-(1.333*Z(I)))
      G3=D(3)*((2.0*E5B)+(2.0*Z(N)*E4B)+(Z(N)*Z(N)*E3B)-(2.0*E5A)
     1-(1.333*Z(I)))
      F4=C(4)*((6.0*E6B)+(6.0*Z(N)*E5B)+(3.0*Z(N)*Z(N)*E4B)+((Z(N)
     1**3)*E3B)+(6.0*E6A)-2.4-(2.0*Z(I)*Z(I)))
```

```
      G4=D(4)*((6.0*F6B)+(6.0*Z(N)*F5B)+(3.0*Z(N)*Z(N)*E4B)+((Z(N)
     1**3)*E3B)+(6.0*E6A)-2.4-(2.0*Z(I)*Z(I)))
      F5=C(5)*((24.0*E7B)+(24.0*Z(N)*E6B)+(12.0*Z(N)*Z(N)*F5B)+
     1(4.0*(Z(N)**3)*E4B)+((Z(N)**4)*E3B)-(24.0*E7A)-(9.6*Z(I))-(2.66
     2*(Z(I)**3)))
      G5=D(5)*((24.0*E7B)+(24.0*Z(N)*E6B)+(12.0*Z(N)*Z(N)*E5B)+
     1(4.0*(Z(N)**3)*E4B)+((Z(N)**4)*E3B)-(24.0*E7A)-(9.6*Z(I))-(2.66
     2*(Z(I)**3)))
  133 Q(I)=(2.0*FL1*F3A)-(2.0*FL2*F3B)+(2.0*(1.0-W0)*(F1+F2+F3+F4+F5))
     1+((W0/2.0)*(G1+G2+G3+G4+G5))
      Q(NN)=Q(NN-2)
      MM=2
      DO 75 I=1,NN
      DO 75 J=MM,MM
   75 Y1(I,J-1)=SQRT(SQRT(Y(I)))
      DO 70 I=2,N
      DO 70 J=MM,MM
      Y1(1,J)=Y1(1,J-1)-((Q(2)-Q(1))*DELT/DELX)
   70 Y1(I,J)=Y1(I,J-1)-((DELT/DELX)*(Q(I+1)-Q(I-1))/2.0)
      Y1(NN,MM)=1.0
      DO71 J=MM,MM
      DO 71 I=1,NN
   71 Y(I)=Y1(I,J)**4
      IF(ML.EQ.0)GOTO 800
      NX=NX+1
      IF(NX.EQ.10)GO TO 800
      GOTO 801
  800 CONTINUE
      NX=0
      WRITE(2,4) (D(I),I=1,5)
    4 FORMAT(5E12.4)
      WRITE(2,608)(C(I),I=1,5)
  608 FORMAT(5F12.4)
      WRITE(2,501)ML
  501 FORMAT(I5)
      WRITE(2,110)(Q(I),I=1,N,50)
  110 FORMAT(8E12.4)
   74 WRITE(2,6)((Y1(I,J),J=MM,MM),I=1,N,50)
    6 FORMAT((4E20.8))
  801 CONTINUE
      DO 86 I=1,NN
   86 Y1(I,1)=Y1(I,2)
      IF(ML.EQ.1005)GOTO 20
      ML=ML+1
      GOTO 39
   20 STOP
      END
```

```
      SUBROUTINE GAUSS(M,AA,BB,X)
      DIMENSION A(20),AA(20),B(20,20),BB(20,M),X(M)
      DO 5 J=1,M
      X(J)=0
      A(J)=AA(J)
      DO 5 K=1,M
    5 B(J,K)=BB(J,K)
      IF(M.EQ.1)GOTO 20
      MY=M-1
      DO 10 L=1,MY
      LL=M+1-L
      MX=M-L
      DO 10 J=1,MX
      RB=B(J,LL)/B(LL,LL)
      A(J)=A(J)-A(LL)*RB
      DO 10 K=1,MX
   10 B(J,K)=B(J,K)-B(LL,K)*RB
   20 DO 40 K=1,M
      XB=0
      DO 30 J=1,K
   30 XB=XB+X(J)*B(K,J)
   40 X(K)=(A(K)-XB)/B(K,K)
      RETURN
      END
```

```
      SUBROUTINE LESQUT (M,MF,N,X,Y,C,YE,R,RR)
      DIMENSION A(20),B(20,20),X(MF,N),Y(N),C(MF),YE(N),R(N)
      DATA M1/20/
      IF(M.GT.M1)GOTO15
      M1=M
      DO 5 J=1,MF
      A(J)=0
      DO 5 K=1,MF
    5 B(J,K)=0
      DO 10 I=1,N
      DO 10 J=1,MF
      A(J)=A(J)+X(J,I)*Y(I)
      DO 10 K=1,MF
   10 B(J,K)=B(J,K)+X(J,I)*X(K,I)
   15 CALL GAUSS(M,A,B,C)
      RR=0
      DO 30 I=1,N
      YE(I)=0
      DO 20 J=1,M
   20 YE(I)=YE(I)+C(J)*X(J,I)
      R(I)=Y(I)-YE(I)
   30 RR=RR+R(I)*R(I)
      RETURN
      END
```

```
      SUBROUTINE DFFS(S1,A,DD)
      DIMENSION YY(8),DD(8),L(8,8),U1(8,8),S1(8),A(8,8)
      DO 100 I=1,5
      DO 100 J=1,5
100   L(I,J)=0.0
      DO 101 I=1,5
      DO 101 J=1,5
101   U1(I,J)=0.0
      L(1,1)=1.0
      L(2,2)=1.0
      L(3,3)=1.0
      L(4,4)=1.0
      L(5,5)=1.0
      DO 103 I=1,1
      DO 103 J=1,5
103   U1(I,J)=A(I,J)
      L(2,1)=A(2,1)/U1(1,1)
      DO 104 I=2,2
      DO 104 J=2,5
104   U1(I,J)=A(I,J)-(L(2,1)*U1(I-1,J))
      L(3,1)=A(3,1)/U1(1,1)
      L(3,2)=(A(3,2)-(L(3,1)*U1(1,2)))/U1(2,2)
      DO 105 I=3,3
      DO 105 J=3,5
105   U1(I,J)=A(I,J)-(L(3,1)*U1(I-2,J))-(L(3,2)*U1(I-1,J))
      L(4,1)=A(4,1)/U1(1,1)
      L(4,2)=(A(4,2)-(L(4,1)*U1(1,2)))/U1(2,2)
      L(4,3)=(A(4,3)-(L(4,1)*U1(1,3))-(L(4,2)*U1(2,3)))/U1(3,3)
      DO 106 I=4,4
      DO 106 J=4,5
106   U1(I,J)=A(I,J)-(L(4,1)*U1(I-3,J))-(L(4,2)*U1(I-2,J))-(L(4,3)*U1
     1(I-1,J))
      L(5,1)=A(5,1)/U1(1,1)
      L(5,2)=(A(5,2)-(L(5,1)*U1(1,2)))/U1(2,2)
      L(5,3)=(A(5,3)-(L(5,1)*U1(1,3))-(L(5,2)*U1(2,3)))/U1(3,3)
      L(5,4)=(A(5,4)-(L(5,1)*U1(1,4))-(L(5,2)*U1(2,4))-(L(5,3)*U1
     1(3,4)))/U1(4,4)
      DO 107 I=5,5
      DO 107 J=5,5
107   U1(I,J)=A(I,J)-(L(5,1)*U1(I-4,J))-(L(5,2)*U1(I-3,J))-(L(5,3)*
     1U1(I-2,J))-(L(5,4)*U1(I-1,J))
      YY(1)=S1(1)
      YY(2)=S1(2)-(L(2,1)*YY(1))
      YY(3)=S1(3)-(L(3,1)*YY(1))-(L(3,2)*YY(2))
      YY(4)=S1(4)-(L(4,1)*YY(1))-(L(4,2)*YY(2))-(L(4,3)*YY(3))
      YY(5)=S1(5)-(L(5,1)*YY(1))-(L(5,2)*YY(2))-(L(5,3)*YY(3))-(L
     1(5,4)*YY(4))
      DD(5)=YY(5)/U1(5,5)
      DD(4)=(YY(4)-(U1(4,5)*DD(5)))/U1(4,4)
      DD(3)=(YY(3)-(U1(3,5)*DD(5))-(U1(3,4)*DD(4)))/U1(3,3)
      DD(2)=(YY(2)-(U1(2,5)*DD(5))-(U1(2,4)*DD(4))-(U1(2,3)*DD(3)))
     1/U1(2,2)
      DD(1)=(YY(1)-(U1(1,5)*DD(5))-(U1(1,4)*DD(4))-(U1(1,3)*DD(3))
     1-(U1(1,2)*DD(2)))/U1(1,1)
      RETURN
      END
```

1

**** FINISH

APPENDIX I

UNSTEADY COMBINED CONDUCTION AND RADIATION HEAT TRANSFER

The energy equation for a one-dimensional plane layer of an absorbing, emitting and scattering medium in which conduction heat transfer is included may be simply described by Figure B.1 and written

$$\frac{\partial}{\partial x}\left[K(x)\ \frac{\partial T}{\partial x}\right] - \frac{\partial q_r(x)}{\partial x} = \rho(x)C_p(x)\ \frac{\partial T}{\partial t} \qquad (I.1)$$

The solution of equation (I.1) is complicated by the term $q_r(x)$ adding another degree of freedom to the integro-differential equation of radiative flux. Hence this study considers the equation (I.1) with constant properties. Equation (I.1) becomes

$$K_b\ \frac{\partial^2 T}{\partial x^2} - \frac{\partial q_r}{\partial x} = (\rho\ C_p)_p \frac{\partial T}{\partial t} \qquad (I.2)$$

using the transform $d\tau = K_t dx$ and the non-dimensional groups

$$\theta = \frac{T}{T_b}, \qquad\qquad N_1 = \frac{K_t\ K_b}{4\sigma T_b^{\ 3}}$$

$$t' = \frac{K_t^{\ 2}K_b\ t}{(\rho\ C_p)_p}$$

and for an emulsion $(\rho\ C_p)_p = \rho_p\ C_p\ (1 - \varepsilon_b)$ equation (I.2) becomes

$$\frac{\partial\theta^2}{\partial\tau^2} - \frac{1}{4N_1}\ \frac{\partial q_r'}{\partial\tau} = \frac{\partial\theta}{\partial t'} \qquad (I.3)$$

For two adjacent regions x and y then we have

$$\left.\frac{\partial^2\theta}{\partial\tau^2}\right|_x - \frac{1}{4N_1}\left.\frac{\partial q_r'}{\partial\tau}\right|_x = \left.\frac{\partial\theta}{\partial t'}\right|_x \qquad (I.4)$$

and

$$\left.\frac{\partial^2\theta}{\partial\tau^2}\right|_y - \frac{1}{4N_1}\left.\frac{\partial q_r'}{\partial\tau}\right|_y = \left.\frac{\partial\theta}{\partial t'}\right|_y \qquad (I.5)$$

which may be solved simultaneously for an equivalent absolute time t, through the relationship

$$t_x' = \frac{K_{tx}^2 K_{bx}}{(\rho C_p)_x} \cdot \frac{(\rho C_p)_y t_y'}{K_{ty}^2 K_{by}} \qquad (I.6)$$

In finite difference form equations (I.4) and (I.5) become

$$\left.\frac{(\theta_n^{p+1} - \theta_n^p)}{\Delta t'}\right|_{x,y} = \left.\frac{(\theta_{n+1} - 2\theta_n + \theta_{n-1})^p}{\Delta\tau^2}\right|_{x,y}$$

$$-\frac{1}{4N_1}\left.\frac{(q_{r\,n+1}' - q_{r\,n-1}')^p}{2\Delta\tau}\right|_{x,y} \qquad (I.7)$$

subject to initial and boundary conditions

$$t' = 0, \qquad \theta(\tau,0) = \begin{Bmatrix} 1 \\ \theta(\tau) \end{Bmatrix}, \qquad \tau \geqslant 0 \qquad (I.8)$$

$$\tau = 0, \qquad \theta(0,t') = \theta_1, \qquad t' \geqslant 0 \qquad (I.9)$$

$$\tau = \tau_o, \qquad \theta(\tau_o,t') = 1, \qquad t' \geqslant 0 \qquad (I.10)$$

The method of solution is essentially as described in Appendix G
and Appendix D. For an optically thin medium the first derivative
of equation (B.10) of Appendix B with respect to τ, becomes with
$\tau = \tau_o \ll 1$

$$- \frac{\partial q'_r}{\partial \tau} = 2(1 - \omega_o)[F_1 + F_2 - 2\theta^4(\tau,t')] \qquad (I.11)$$

where F_1 and F_2 are the non-dimensional boundary fluxes, i.e.
$F_1 = F_w$ and $F_2 = F_e$ (at the emulsion boundary). Hence equation
(I.3) becomes with equation (I.11)

$$\frac{\partial^2 \theta}{\partial \tau^2} + \frac{(1 - \omega_o)}{2N_1} [F_w + F_e - 2\theta^4(\tau,t') = \frac{\partial \theta}{\partial t'} \qquad (I.12)$$

and for the gas layer

$$\tau = \tau_{og} \ll 1, \qquad \theta^4(0,t') = \theta^4_w$$

and

$$\theta^4(\tau_{og},t') = \theta^4_e \quad \text{(the surface temperature of the emulsion)}$$

and (I.12) in finite difference form becomes with n = 1 (at the
emulsion boundary adjacent to the gas layer)

$$\frac{(\theta_1^{p+1} - \theta_1^p)}{\Delta t'} = \frac{(\theta_2 - 2\theta_1 + \theta_w)^p}{\Delta \tau^2} + \frac{(1 - \omega_o)}{2N_{1g}} (F_w + F_e - 2\theta_1^4)^p \qquad (I.13)$$

with the conductive flux

$$q_c^1 = -4N_1 \left. \frac{\partial \theta}{\partial \tau} \right|_{x,y}$$

and in finite difference form

$$q_c' = -4N_\downarrow \left.\frac{(\theta_{n+1} - \theta_{n-1})^p}{2 \, \Delta\tau}\right|_{g,b} \qquad (I.14)$$

APPENDIX J

A COMPUTER PROGRAM TO SOLVE THE TRANSIENT COMBINED

CONDUCTION AND RADIATION ENERGY EQUATION

The important variables are as described in Appendix H along with the following additions.

(a) Input Variables (SI units where appropriate)

RHOP is the density of the particulate material

CP is the specific heat capacity of the particulate material

RHOG is the density of the fluidizing gas

CPG is the specific heat capacity of the fluidizing gas

XB is the depth of the boundary gas film layer in particle
     diameters

KG is the thermal conductivity of the fluidizing gas

EB is the bed voidage

DP is the particle diameter

KT is the extinction coefficient of the emulsion

KB is the effective thermal conductivity of the emulsion

TB is the bed temperature

TW is the non-dimensional wall temperature

BB is an averaging constant, i.e. taken as 2.0

(b) Output Variables

QC(I) I = 1....n is the conductive flux across the field

QT(I) I = 1....n is the sum of the conductive and radiative
     fluxes across the field

HR is the radiative heat transfer coefficient

HC   is the conductive heat transfer coefficient

HT   = HR + HC

PERC = HR/HT

COND is the conductive flux kW/m^2

RAD is the radiative flux kW/m^2

FLUXT = (COND + RAD) kW/m^2

N1   is the conduction/radiation number associated with the
          emulsion

N2   is the conduction/radiation number association with
          the gas layer

KTG is the extinction coefficient of the gas layer

DELT1 is the non-dimensional time associated with N2

```
      PROGRAM TEWN(INPUT,OUTPUT,TAPE1=INPUT,-TAPE2=OUTPUT)
      DIMENSION P(6,6),S1(8),US(6,6),C(10)
      DIMENSION U4(6,6),U5(6,6),U6(6,6)
      DIMENSION QC(505),QT(505)
      DIMENSION D(8),Q(505),Y1(505,3),Y(505),X(10,505),YE(505)
      DIMENSION Z(505),ZZ(505),R(505),A(8,8),DD(8),U(505,6),S(505,6)
      COMMON Y1
      LEVEL 2,Y1
      REAL N2
      REAL N1,KT,KB
      REAL KG,KS
      REAL KTG
      REAL J1,J2,J3,J4,J5
      READ(1,*)FL1,FL2,WO,EW,EW2,RHOP,CP,RHOG
      READ(1,*)CPG,XB,KG,FB,DP,KT,KB,TB,TW
      READ(1,*)(C(I),I=1,5),BB
      WRITE(2,320)FL1,FL2,WO,EW,EW2,RHOP,CP,RHOG,BB
320   FORMAT(9E12.4)
      WRITE(2,325)CPG,XB,KG,FB,DP,KT,KB,TB,TW
325   FORMAT(9E12.4)
      SIGMA=56.7E-12
      N1=(KT*KB)/(4.0*SIGMA*(TB**3))
      DELX=0.02
      KTG=DELX/(DP*XB)
      KG=(KG+KB)/BB
      N2=(KTG*KG)/(4.0*SIGMA*(TB**3))
      DELT=(           (((RHOG*CPG)+(RHOP*CP*(1.0-EB)))/BB)*KT*KT*KB)/
     1(RHOP*CP*(1.0-EB)*KG*KTG*KTG)
      DELT1=0.0001/DELT
      DELT=0.0001
      WRITE(2,702)DELT1,DELT,KTG,N1,N2
702   FORMAT(5E12.4)
      IF(DELT1.GT.0.002)GOTO20
      CONTINUE
      NX=1
      BL1=FL1
      BL2=FL2
      N=501
      ML=0
      NN=N+1
      Z(1)=0.0
      DO 53 K=2,NN
53    Z(K)=Z(K-1)+0.02
      DO 3 K=1,NN
      X(1,K)=1.0
      X(2,K)=Z(K)
      DO3 J=3,10
3     X(J,K)=X(J-1,K)*Z(K)
      DO 56 I=1,NN
56    Y(I)=C(1)+(C(2)*Z(I))+(C(3)*Z(I)*Z(I))+(C(4)*Z(I)*Z(I)*Z(I))
     1+(C(5)*(Z(I)**4))
39    CONTINUE
      FL1=BL1
      FL2=BL2
      DO55 I=5,5
      L=N+1
      CALL LESQUT (1,10,L,X,Y,C,YE,R,RR)
```

```
   55 CONTINUE
      DO 57 I=1,NN
   57 Y(I)=C(1)+(C(2)*Z(I))+(C(3)*Z(I)*Z(I))+(C(4)*Z(I)*Z(I)*Z(I))
     1+(C(5)*(Z(I)**4))
      BD=Z(N)
      E1A=S13AAF(BD,IFAIL)
      E2A=(1.0/EXP(Z(N)))-(Z(N)*E1A)
      E3A=(1.0/2.0)*((1.0/EXP(Z(N)))-(Z(N)*E2A))
      E4A=(1.0/3.0)*((1.0/EXP(Z(N)))-(Z(N)*E3A))
      E5A=(1.0/4.0)*((1.0/EXP(Z(N)))-(Z(N)*E4A))
      E6A=(1.0/5.0)*((1.0/EXP(Z(N)))-(Z(N)*E5A))
      E7A=(1.0/6.0)*((1.0/EXP(Z(N)))-(Z(N)*E6A))
      H1=C(1)*(0.5-E3A)
      J1=      (0.5-E3A)
      H2=C(2)*(0.333-E4A-(Z(N)*E3A))
      J2=      (0.333-E4A-(Z(N)*E3A))
      H3=C(3)*(0.5-(2.0*E5A)-(2.0*Z(N)*E4A)-(Z(N)*Z(N)*E3A))
      J3=      (0.5-(2.0*E5A)-(2.0*Z(N)*E4A)-(Z(N)*Z(N)*E3A))
      H4=C(4)*(1.2-(6.0*E6A)-(6.0*Z(N)*E5A)-(3.0*Z(N)*Z(N)*E4A)
     1-((Z(N)**3)*E3A))
      J4=      (1.2-(6.0*E6A)-(6.0*Z(N)*E5A)-(3.0*Z(N)*Z(N)*E4A)
     1-((Z(N)**3)*E3A))
      H5=C(5)*(4.0-(24.0*E7A)-(24.0*Z(N)*E6A)-(12.0*Z(N)*Z(N)*E5A)
     1-(4.0*(Z(N)**3)*E4A)-((Z(N)**4)*E3A))
      J5=      (4.0-(24.0*E7A)-(24.0*Z(N)*E6A)-(12.0*Z(N)*Z(N)*E5A)
     1-(4.0*(Z(N)**3)*E4A)-((Z(N)**4)*E3A))
      W1=C(1)*(0.5-E3A)
      V1=      (0.5-E3A)
      W2=C(2)*((Z(N)*0.5)-0.333+E4A)
      V2=      ((Z(N)*0.5)-0.333+E4A)
      W3=C(3)*((Z(N)*Z(N)*0.5)-(2.0*Z(N)*0.333)+0.5-(2.0*E5A))
      V3=      ((Z(N)*Z(N)*0.5)-(2.0*Z(N)*0.333)+0.5-(2.0*E5A))
      W4=C(4)*(((Z(N)**3)*0.5)-(3.0*Z(N)*Z(N)*0.333)+(6.0*Z(N)*0.25)
     1-(6.0*0.2)+(6.0*E6A))
      V4=      (((Z(N)**3)*0.5)-(3.0*Z(N)*Z(N)*0.333)+(6.0*Z(N)*0.25)
     1-(6.0*0.2)+(6.0*E6A))
      W5=C(5)*(((Z(N)**4)*0.5)-(4.0*(Z(N)**3)*0.3333)+(12.0*Z(N)*Z(N
     1)*0.25)-(24.0*Z(N)*0.2)+4.0-(24.0*E7A))
      V5=      (((Z(N)**4)*0.5)-(4.0*(Z(N)**3)*0.3333)+(12.0*Z(N)*Z(N
     1)*0.25)-(24.0*Z(N)*0.2)+4.0-(24.0*E7A))
      FL1=(EW*BL1)+(2.0*(1.0-EW)*((BL2*EW2*E3A)+((1.0-WO)*(H1+H2+H3+H4
     1+H5))))+(4.0*(1.0-EW)*(1.0-EW2)*(1.0-WO)*E3A*(W1+W2+W3+W4+W5))
      FL1=FL1/(1.0-(4.0*(1.0-EW)*(1.0-EW2)*E3A*E3A))
      FL2=(BL2*EW2)+(2.0*(1.0-EW2)*BL1*EW*E3A)+(2.0*(1.0-EW2)*(1.0-WO)
     1*(W1+W2+W3+W4+W5))+(4.0*(1.0-EW2)*(1.0-EW)*(1.0-WO)*E3A*(H1+H2+
     2H3+H4+H5))
      FL2=FL2/(1.0-(4.0*(1.0-EW)*(1.0-EW2)*E3A*E3A))
      XZ=(1.0-(4.0*(1.0-EW)*(1.0-EW2)*E3A*E3A))
      DO 58 I=1,5
      U3(I,1)=(1.0-EW)*WO*0.5*J1/XZ
      U3(I,2)=(1.0-EW)*WO*0.5*J2/XZ
      U3(I,3)=(1.0-EW)*WO*0.5*J3/XZ
      U3(I,4)=(1.0-EW)*WO*0.5*J4/XZ
      U3(I,5)=(1.0-EW)*WO*0.5*J5/XZ
   58 CONTINUE
      DO 59 I=1,5
      U4(I,1)=E3A*(1.0-EW)*(1.0-EW2)*WO*V1/XZ
```

1

```
      U4(I,2)=F3A*(1.0-EW)*(1.0-EW2)*WO*V2/XZ -
      U4(I,3)=F3A*(1.0-EW)*(1.0-EW2)*WO*V3/XZ
      U4(I,4)=F3A*(1.0-EW)*(1.0-EW2)*WO*V4/XZ
      U4(I,5)=E3A*(1.0-EW)*(1.0-EW2)*WO*V5/XZ
   59 CONTINUE
      DO 60 I=1,5
      U5(I,1)=(1.0-EW2)*(1.0-EW)*WO*F3A*J1/XZ
      U5(I,2)=(1.0-EW2)*(1.0-EW)*WO*E3A*J2/XZ
      U5(I,3)=(1.0-EW2)*(1.0-EW)*WO*E3A*J3/XZ
      U5(I,4)=(1.0-EW2)*(1.0-EW)*WO*F3A*J4/XZ
      U5(I,5)=(1.0-EW2)*(1.0-EW)*WO*E3A*J5/XZ
   60 CONTINUE
      DO 61 I=1,5
      U6(I,1)=0.5*(1.0-EW2)*WO*V1/XZ
      U6(I,2)=0.5*(1.0-EW2)*WO*V2/XZ
      U6(I,3)=0.5*(1.0-EW2)*WO*V3/XZ
      U6(I,4)=0.5*(1.0-EW2)*WO*V4/XZ
      U6(I,5)=0.5*(1.0-EW2)*WO*V5/XZ
   61 CONTINUE
      DO40 I=1,N
      GG=Z(N)-Z(I)
      IF(Z(I).EQ.0.0) GO TO 560
      BE=Z(I)
      E1A=S13AAF(BE,IFAIL)
      E2A=(1.0/EXP(Z(I)))-(Z(I)*F1A)
      GO TO 561
  560 E2A=1.0
  561 CONTINUE
      IF(GG.EQ.0.0) GO TO 562
      E1B=S13AAF(GG,IFAIL)
      E2B=(1.0/EXP(GG))-(GG*E1B)
      GO TO 563
  562 E2B=1.0
  563 CONTINUE
   43 F3A=(1.0/2.0)*((1.0/FXP(Z(I)))-(Z(I)*F2A))
   62 E3B=(1.0/2.0)*((1.0/EXP(GG))-(GG*E2B))
      E4A=(1.0/3.0)*((1.0/FXP(Z(I)))-(Z(I)*E3A))
      E4B=(1.0/3.0)*((1.0/FXP(GG))-(GG*E3B))
      E5A=(1.0/4.0)*((1.0/FXP(Z(I)))-(Z(I)*E4A))
      E5B=(1.0/4.0)*((1.0/EXP(GG))-(GG*E4B))
      E6A=(1.0/5.0)*((1.0/EXP(Z(I)))-(Z(I)*E5A))
      E6B=(1.0/5.0)*((1.0/FXP(GG))-(GG*E5B))
      E7A=(1.0/6.0)*((1.0/FXP(Z(I)))-(Z(I)*E6A))
      E7B=(1.0/6.0)*((1.0/EXP(GG))-(GG*E6B))
      S(I,1)=(((2.0-E2A-E2B)*(2.0*(1.0-WO)))*C(1))+(2.0*FL1*E2A)+
     1(2.0*FL2*E2B)
      S(I,2)=(((((2.0*Z(I))+E3A-(Z(N)*E2B)-E3B)*(2.0*(1.0-WO)))*C(2))
      S(I,3)=(((((2.0*Z(I)*Z(I))+1.333-(2.0*E4A)-(Z(N)*Z(N)*E2B)
     1-(2.0*Z(N)*E3B)-(2.0*E4B))*(2.0*(1.0-WO)))*C(3))
      S(I,4)=(((((2.0*Z(I)*Z(I)*Z(I))+(4.0*Z(I))+(6.0*E5A)-(6.0*E5B)-(
     1Z(N)*Z(N)*Z(N)*E2B)-(3.0*Z(N)*Z(N)*E3B)-(6.0*Z(N)*E4B))
     2*(2.0*(1.0-WO)))*C(4))
      S(I,5)=(((((2.0*(Z(I)**4))+(8.0*Z(I)*Z(I))+9.6-(24.0*E6A)-(4.0*
     1(Z(N)**3)*E3B) -((Z(N)**4)*E2B)-(12.0*Z(N)*Z(N)*E4B)-(24.0*
     2Z(N)*E5B) -(24.0*E6B))*(2.0*(1.0-WO)))*C(5))
      U(I,1)= (2.0-E2A-E2B)*WO/2.0
      U(I,2)= ((2.0*Z(I))+E3A-(Z(N)*E2B)-E3B)*WO/2.0
```

```
      U(I,5)= ((2.0*Z(I)*Z(I))+1.333-(2.0*F4A)-(Z(N)*Z(N)*E2B)
     1-(2.0*Z(N)*E3B)-(2.0*F4B)) *W0/2.0
      U(I,4)= ((2.0*Z(I)*Z(I)*Z(I))+(4.0*Z(I))+(6.0*F5A)-(6.0*E5B)-(
     12(N)*Z(N)*Z(N)*E2B)-(3.0*Z(N)*Z(N)*E3B)-(6.0*Z(N)*E4B)) *W0/2.0
      U(I,5)= ((2.0*(Z(I)**4))+(8.0*Z(I)*Z(I))+9.6-(24.0*E6A)-(4.0*
     1(Z(N)**3)*F3B) -((Z(N)**4)*E2B)-(12.0*Z(N)*Z(N)*E4B)-(24.0*
     22(N)*F5B) -(24.0*E6B)) *W0/2.0
   40 CONTINUE
      Z(1)=0.0
      DO 966 L=2,NN
  966 Z(L)=Z(L-1)+0.02
      DO 90 I=1,5
      DO 90 J=2,5
      P(I,1)=1.0
   90 P(I,J)=0.0
      DO 91 I=2,5
      DO 91 J=2,5
      II=(125*I)-124
      YW=Z(II)
      YYY=YW**(J-1)
   91 P(I,J)=YYY
      DO 92 I=2,5
      DO 92 J=1,5
   92 U(I,J)=U(((125*I)-124),J)
      DO 94 I=1,5
      DO 94 J=1,5
   94 A(I,J)=P(I,J)-U(I,J)-U3(I,J)-U4(I,J)-U5(I,J)-U6(I,J)
      DO 95 I=2,5
      DO 95 J=1,5
   95 S(I,J)=S(((125*I)-124),J)
      DO 98 I=1,5
   98 S1(I)=S(I,1)+S(I,2)+S(I,3)+S(I,4)+S(I,5)
      CALL DEES(S1,A,DD)
      DO 82 I=1,5
   82 D(I)=DD(I)
      FL1=FL1+(D(1)*U3(1,1))+(D(2)*U3(1,2))+(D(3)*U3(1,3))+(D(4)*
     1U3(1,4))+(D(5)*U3(1,5))+(D(1)*U4(1,1))+(D(2)*U4(1,2))+(D(3)*U4
     2(1,3))+(D(4)*U4(1,4))+(D(5)*U4(1,5))
      FL2=FL2+(D(1)*U5(1,1))+(D(2)*U5(1,2))+(D(3)*U5(1,3))+(D(4)*
     1U5(1,4))+(D(5)*U5(1,5))+(D(1)*U6(1,1))+(D(2)*U6(1,2))+(D(3)*U6
     2(1,3))+(D(4)*U6(1,4))+(D(5)*U6(1,5))
      DO 133 I=1,N
      GG=Z(N)-Z(I)
      IF(Z(I).EQ.0.0)GO TO 565
      BF=Z(I)
      E1A=S13AAF(BF,IFAIL)
      E2A=(1.0/EXP(Z(I)))-(Z(I)*E1A)
      GO TO 566
  565 E2A=1.0
  566 CONTINUE
      IF(GG.EQ.0.0) GO TO 567
      E1B=S13AAF(GG,IFAIL)
      E2B=(1.0/EXP(GG))-(GG*E1B)
      GO TO 568
  567 E2B=1.0
  568 CONTINUE
      E3A=(1.0/2.0)*((1.0/EXP(Z(I)))-(Z(I)*E2A))
```

```
      E3B=(1.0/2.0)*((1.0/EXP(GG))-(GG*E2B))
      E4A=(1.0/3.0)*((1.0/EXP(Z(1)))-(Z(1)*E3A))
      F4B=(1.0/3.0)*((1.0/EXP(GG))-(GG*E3B))
      E5A=(1.0/4.0)*((1.0/EXP(Z(1)))-(Z(1)*F4A))
      E5B=(1.0/4.0)*((1.0/EXP(GG))-(GG*E4B))
      E6A=(1.0/5.0)*((1.0/EXP(Z(1)))-(Z(1)*E5A))
      E6B=(1.0/5.0)*((1.0/EXP(GG))-(GG*E5B))
      E7A=(1.0/6.0)*((1.0/EXP(Z(1)))-(Z(1)*E6A))
      F7B=(1.0/6.0)*((1.0/EXP(GG))-(GG*F6B))
      F1=C(1)*(E3B-E3A)
      G1=D(1)*(E3B-F3A)
      F2=C(2)*(E4A-0.666+(Z(N)*E3B)+E4B)
      G2=D(2)*(E4A-0.666+(Z(N)*F3B)+F4B)
      F3=C(3)*((2.0*E5B)+(2.0*Z(N)*E4B)+(Z(N)*Z(N)*E3B)-(2.0*E5A)
     1-(1.533*Z(1)))
      G3=D(3)*((2.0*E5B)+(2.0*Z(N)*E4B)+(Z(N)*Z(N)*F3B)-(2.0*E5A)
     1-(1.533*Z(1)))
      F4=C(4)*((6.0*E6B)+(6.0*Z(N)*E5B)+(3.0*Z(N)*Z(N)*E4B)+((Z(N)
     1**3)*E3B)+(6.0*E6A)-2.4-(2.0*Z(1)*Z(1)))
      G4=D(4)*((6.0*E6B)+(6.0*Z(N)*E5B)+(3.0*Z(N)*Z(N)*E4B)+((Z(N)
     1**3)*E3B)+(6.0*E6A)-2.4-(2.0*Z(1)*Z(1)))
      F5=C(5)*((24.0*E7B)+(24.0*Z(N)*E6B)+(12.0*Z(N)*Z(N)*E5B)+
     1(4.0*(Z(N)**3)*E4B)+((Z(N)**4)*E3B)-(24.0*E7A)-(9.6*Z(1))-(2.66
     2*(Z(1)**3)))
      G5=D(5)*((24.0*E7B)+(24.0*Z(N)*E6B)+(12.0*Z(N)*Z(N)*E5B)+
     1(4.0*(Z(N)**3)*E4B)+((Z(N)**4)*E3B)-(24.0*F7A)-(9.6*Z(1))-(2.66
     2*(Z(1)**3)))
  133 Q(I)=(2.0*FL1*E3A)-(2.0*FL2*E3B)+(2.0*(1.0-W0)*(F1+F2+F3+F4+F5))
     1+((W0/2.0)*(G1+G2+G3+G4+G5))
      Q(NN)=Q(NN-2)
      MM=2
      DO 75 I=1,NN
      DO 75 J=MM,MM
   75 Y1(I,J-1)=SQRT(SQRT(Y(I)))
      DO 70 I=2,N
      DO 70 J=MM,MM
      EWE=-Q(1)
      F1=(BL1+((1.0-EW)*(-Q(1))))/(1.0-((1.0-EW)*(1.0-EWF)))
      F2=((-Q(1))+((1.0-EWF)*BL1))/(1.0-((1.0-EW)*(1.0-EWE)))
      Y1(1,J)=Y1(1,J-1)+((DELT1/(DELX*DELX))*(Y1(2,J-1)-(2.0*Y1(1,J-1))
     1+TW))+((DELT1*(1.0-W0)/(2.0*N2))*(F1+F2-(2.0*Y1(1,J-1))))
   70 Y1(I,J)=Y1(I,J-1)+((DELT/(DELX*DELX))*(Y1(I+1,J-1)-(2.0*Y1(I,J-1
     1)))+Y1(I-1,J-1)))-((DELT/(4.0*N1*2.0*DELX))*(Q(I+1)-Q(I-1)))
      Y1(NN,MM)=1.0
      DO 306 J=MM,MM
      DO 306 I=2,N
      QC(1)=(4.0*N2)*(TW-Y1(1,J-1))/DELX
  306 QC(I)=(4.0*N1)*(Y1(I-1,J-1)-Y1(I+1,J-1))/(DELX*2.0)
      DO 310 I=1,N
  310 QT(I)=Q(I)+QC(I)
      HR=Q(1)*SIGMA*(TB**4)/((TW*TB)-TB)
      HC=QC(1)*SIGMA*(TB**4)/((TW*TB)-TB)
      HT=HC+HR
      PERC=HR/HT
      COND=QC(1)*SIGMA*(TB**4)
      RAD=Q(1)*SIGMA*(TB**4)
      FLUXT=COND+RAD
```

```
      DO71 J=MM,MM
      DO 71 I=1,NN
  71  Y(1)=Y1(I,J)**4
      IF(ML.EQ.0)GOTO 800
      NX=NX+1
      IF(NX.EQ.10)GO TO 800
      GOTO 801
 800  CONTINUE
      NX=0
      WRITE(2,4) (D(I),I=1,5)
   4  FORMAT(5E12.4)
      WRITE(2,608)(C(I),I=1,5)
 608  FORMAT(5E12.4)
      WRITE(2,501)ML
 501  FORMAT(I5)
      WRITE(2,110)(Q(I),I=1,N,50)
 110  FORMAT(8E12.4)
      WRITE(2,309)(QC(I),I=1,N,50)
 309  FORMAT((8E12.4))
      WRITE(2,311)(QT(I),I=1,N,50)
 311  FORMAT((8E12.4))
      WRITE(2,609)HR,HC,HT,PERC,COND,RAD,FLUXT
 609  FORMAT(7F12.4)
  74  WRITE(2,6)((Y1(I,J),J=MM,MM),I=1,N,50)
   6  FORMAT((4E20.8))
 801  CONTINUE
      DO 86 I=1,NN
  86  Y1(I,1)=Y1(I,2)
      IF(ML.EQ.1005)GOTO 20
      ML=ML+1
      GOTO 39
  20  STOP
      END
```

```fortran
      SUBROUTINE GAUSS(M,AA,BB,X)
      DIMENSION A(20),AA(20),B(20,20),BB(20,M),X(M)
      DO 5 J=1,M
      X(J)=0
      A(J)=AA(J)
      DO 5 K=1,M
    5 B(J,K)=BB(J,K)
      IF(M.EQ.1)GOTO 20
      MY=M-1
      DO 10 L=1,MY
      LL=M+1-L
      MX=M-L
      DO 10 J=1,MX
      RB=B(J,LL)/B(LL,LL)
      A(J)=A(J)-A(LL)*RB
      DO 10 K=1,MX
   10 B(J,K)=B(J,K)-B(LL,K)*RB
   20 DO 40 K=1,M
      XB=0
      DO 30 J=1,K
   30 XB=XB+X(J)*B(K,J)
   40 X(K)=(A(K)-XB)/B(K,K)
      RETURN
      END
```

```
      SUBROUTINE LESQUT (M,MF,N,X,Y,C,YE,R,RR)
      DIMENSION A(20),B(20,20),X(MF,N),Y(N),C(MF),YE(N),R(N)
      DATA M1/20/
      IF(M.GT.M1)GOTO15
      M1=M
      DO 5 J=1,MF
      A(J)=0
      DO 5 K=1,MF
    5 B(J,K)=0
      DO 10 I=1,N
      DO 10 J=1,MF
      A(J)=A(J)+X(J,I)*Y(I)
      DO 10 K=1,MF
   10 B(J,K)=B(J,K)+X(J,I)*X(K,I)
   15 CALL GAUSS(M,A,B,C)
      RR=0
      DO 30 I=1,N
      YE(I)=0
      DO 20 J=1,M
   20 YE(I)=YE(I)+C(J)*X(J,I)
      R(I)=Y(I)-YE(I)
   30 RR=RR+R(I)*R(I)
      RETURN
      END
```

```
      SUBROUTINE DEES(S1,A,DD)
      DIMENSION YY(8),DD(8),L(8,8),U1(8,8),S1(8),A(8,8)
      DO 100 I=1,5
      DO 100 J=1,5
100   L(I,J)=0.0
      DO 101 I=1,5
      DO 101 J=1,5
101   U1(I,J)=0.0
      L(1,1)=1.0
      L(2,2)=1.0
      L(3,3)=1.0
      L(4,4)=1.0
      L(5,5)=1.0
      DO 103 I=1,1
      DO 103 J=1,5
103   U1(I,J)=A(I,J)
      L(2,1)=A(2,1)/U1(1,1)
      DO 104 I=2,2
      DO 104 J=2,5
104   U1(I,J)=A(I,J)-(L(2,1)*U1(I-1,J))
      L(3,1)=A(3,1)/U1(1,1)
      L(3,2)=(A(3,2)-(L(3,1)*U1(1,2)))/U1(2,2)
      DO 105 I=3,3
      DO 105 J=3,5
105   U1(I,J)=A(I,J)-(L(3,1)*U1(I-2,J))-(L(3,2)*U1(I-1,J))
      L(4,1)=A(4,1)/U1(1,1)
      L(4,2)=(A(4,2)-(L(4,1)*U1(1,2)))/U1(2,2)
      L(4,3)=(A(4,3)-(L(4,1)*U1(1,3))-(L(4,2)*U1(2,3)))/U1(3,3)
      DO 106 I=4,4
      DO 106 J=4,5
106   U1(I,J)=A(I,J)-(L(4,1)*U1(I-3,J))-(L(4,2)*U1(I-2,J))-(L(4,3)*U1
     1(I-1,J))
      L(5,1)=A(5,1)/U1(1,1)
      L(5,2)=(A(5,2)-(L(5,1)*U1(1,2)))/U1(2,2)
      L(5,3)=(A(5,3)-(L(5,1)*U1(1,3))-(L(5,2)*U1(2,3)))/U1(3,3)
      L(5,4)=(A(5,4)-(L(5,1)*U1(1,4))-(L(5,2)*U1(2,4))-(L(5,3)*U1
     1(3,4)))/U1(4,4)
      DO 107 I=5,5
      DO 107 J=5,5
107   U1(I,J)=A(I,J)-(L(5,1)*U1(I-4,J))-(L(5,2)*U1(I-3,J))-(L(5,3)*
     1U1(I-2,J))-(L(5,4)*U1(I-1,J))
      YY(1)=S1(1)
      YY(2)=S1(2)-(L(2,1)*YY(1))
      YY(3)=S1(3)-(L(3,1)*YY(1))-(L(3,2)*YY(2))
      YY(4)=S1(4)-(L(4,1)*YY(1))-(L(4,2)*YY(2))-(L(4,3)*YY(3))
      YY(5)=S1(5)-(L(5,1)*YY(1))-(L(5,2)*YY(2))-(L(5,3)*YY(3))-(L
     1(5,4)*YY(4))
      DD(5)=YY(5)/U1(5,5)
      DD(4)=(YY(4)-(U1(4,5)*DD(5)))/U1(4,4)
      DD(3)=(YY(3)-(U1(3,5)*DD(5))-(U1(3,4)*DD(4)))/U1(3,3)
      DD(2)=(YY(2)-(U1(2,5)*DD(5))-(U1(2,4)*DD(4))-(U1(2,3)*DD(3)))
     1/U1(2,2)
      DD(1)=(YY(1)-(U1(1,5)*DD(5))-(U1(1,4)*DD(4))-(U1(1,3)*DD(3))
     1-(U1(1,2)*DD(2)))/U1(1,1)
      RETURN
      END
```

**** FINISH