VIBRATION CHARACTERISTICS

OF

FABRICATED SPACE FRAME

by

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In memory

of

my Father

Chief I. O. Pessu

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SUMMARY

This thesis describes an investigation to determine the natural frequencies and modal shapes of fabricated space frames.

Both theoretical and experimental approaches have been used in the investigations. Finite Element method have been extensively used in the theoretical analysis.

Variations between the experimental and theoretical results have been traced to errors in the assumed joint boundary conditions. Working on simpler frame structures, a method of joint representation has been established to take full account of the actual joint boundary conditions. The joint representation adopted is especially suited for finite element analysis and very easily applied to any frame analysis. All forms of joint conditions can easily be taken account of by this type of boundary condition representations.

Given a suitable size of computer, the vibration analysis of any space frame can be carried out readily with the computer programme developed. The method of matrices reduction is recommended to keep computer core requirement to a minimum.

BOUNDARY VIBRATION CHARACTERISTICS SPACE FRAMES

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CONTENTS

NOTATION

a	Shape Function constant
A	Cross sectional area
e	(subscript) element
E	Young's Modulus of Elasticity
F	Local or beam force
G	Rigidity Modulus
I	Second moment of area
[I]	Identity matrix
k, K	Stiffness coefficient
ℓ	Length of a beam
[L]	Lower triangular matrix
m, M	Mass
q	System or Frame displacement
Q	System or Frame force
[R]	Transformation matrix
t	Time in sec.
T	Kinetic energy
u	Local or beam displacement
U	Strain energy
ω	Natural frequency
x,y,z	Distance from the origin
x-,y-,z-	Axes
X-.Y-,Z	Axes
[Z]	Zero matrix

NOTATION continued...

α, β, γ	Transformation angles
ρ	Mass density
θ	Angle of twist
ϕ	Phase angle
[ ]	Matrix
{ }	Column vector
$[\ ]^t$	Transpose of a matrix
$[\ ]^{-1}$	Inverse of a matrix
D.O.F.	Degrees of freedom.

CHAPTER ONE

CHAPTER ONE

INTRODUCTION

1.1 Preamble

A structure can be classified as one-dimensional, two-dimensional or three-dimensional according to the character of its components. To a first approximation, a beam is usually analysed as a line structure by representing it by its centroidal axis. Hence, a beam can be regarded as a one-dimensional structure even if the line is curved. A shell or plate is represented by its middle surface for the purpose of analysis. A small element of it (shell or plate) will therefore be capable of extension in two dimensions. In reality, most bodies are three-dimensional, but only a few of them are analysed as such.

A structure whose components are all one-dimensional is usually called a framed structure. This is the type of structure dealt with in this work. Such a frame as a whole usually exhibits three-dimensional extension. A space frame, in particular, has components which span three-dimensional space. Hence it extends in three dimensions.

Structural analysis spans a broad spectrum of investigations. Some of the main areas of investigation include: stress distribution, displacement distribution, structural stability, thermo-elasticity, plasticity, creep, stress concentrations, fatigue and crack propagation, vibration frequencies and normal modes of vibration. As the title of this thesis suggests, the ~~suppose~~ purpose of this work is an investigation of the vibration frequencies and normal modes of vibration of space frames.

Recent advances in science and technology have required greater accuracy and speed of analysis. This requirement has also reflected itself in structural technology. Greater accuracy and speed will guarantee that more factors can be taken into account at design stages.

The old practice of ignoring vibration considerations during the design stages has many disadvantages. The vibration engineer who has to solve the vibration hazard of completed structures is left with very little option. Usually, the main choice available is the application of one type of vibration isolation or another. But in modern structures, such as long span bridges, tall buildings etc., vibration isolation is even more difficult to apply.

In recent years, the presence of giant oil rigs has attracted some attention. Much effort is being put into detailed analysis and checks on the conditions of the rigs. Owing to the severe wave battering they receive, oil rigs are prone to fatigue failure, stress concentrations occur at the joints giving rise to additional danger. Accurate means of determining these stresses are always being sought (ref.10). Huge sums of money are spent each year for diving checks on the several feet of submerged parts of the oil rigs.

An accurate knowledge of the vibration characteristics of space frames investigated in this work, would be applicable for checks on oil rigs in particular and other structures in general.

1.2 Scope

The work described in this thesis is an attempt to study frame vibration with simple frames. A space frame was initially chosen for the study. Although it was expected to be a simple space frame, the results obtained showed that an accurate analysis of it presented many difficulties.

Theoretical and experimental vibration analysis of the space produced some differences in the results. Attempts have been made to trace the sources of error resulting in the differences noticed. The main reason for the error has been traced to the incorrect prediction and representation of the joint boundary conditions of the space frame.

In order to overcome this error, attention was paid to one- and two-dimensional static and dynamic analysis of small component members of the original framed structure. These analyses were to simulate accurately the actual effects of the various joints.

A further eight chapters describe the work done in this project. Chapter 2 describes the design of the space frame required for this work. Chapter 3 describes the analytical method used in the theoretical work. Finite Element methods have been used extensively. Chapter 4 contains the various computer programmes written for the theoretical analysis, and Chapter 5 describes the experimental methods used in verifying the theoretical results.

Chapter 6 contains the theoretical and experimental vibration analysis of the space frame as well as some discussion of the results. This leads to Chapter 7 where the difference in results of the previous chapter are analysed and solutions sought by means of smaller one- and two-dimensional frames. Chapter 8 contains a further discussion of the results obtained and their applications. The conclusions are in the final chapter - Chapter 9.

CHAPTER TWO

CHAPTER TWO

FABRICATED SPACE FRAME DESIGN

2.1 Design Objectives

Any structure in general has a primary function of supporting and transferring externally applied loads to the reaction points. For a civil engineering structure, the reaction points refer to the points of the structure which are attached to a rigid foundation.

Structural design usually involves the analysis of known structural configurations which are subjected to known distributions of static or dynamic loads, displacements and temperatures. It is really a structural synthesis which should lead to the most efficient design (or optimum design) for the specified load and temperature environment. Thus, most structural design may proceed in steps of (i) a prior very rough appraisal of the statics of the structures to obtain some measure of the way the loads are carried, (ii) guessing or estimating the sizes of the members or components, (iii) analysis of the structure i.e. attempting to solve for the internal forces, or stresses and the resulting deformations under the action of the prescribed loading conditions, (iv) checking the strength of the elements of the structure and (v) successively modifying the original guesses for optimum use of the materials and load carrying capacity of the structure.

In the case of the design of the framed structure required for this work, most of the above steps in structural design are not necessary. There is no prescribed load that the structure is expected to carry. However, some factors have to be borne in mind in the choice of the structure used for the vibration investigation.

First of all, the structure has to be linear. Hence, its material as well as the joints have to behave linearly.

Secondly, for ease of construction and low cost of the frame, its members should be made from standard sized locally available materials. Also the joints should be of corresponding standard materials.

Thirdly, it is envisaged that the frame will be dismantled and rebuilt adding more components when necessary. Hence fixed joints are not recommended. Welded joints in particular should be avoided.

2.2 Design

Fig. 2.2.1 shows the Fabricated Space Frame used in this work. It is essentially a cube shaped frame standing on four vertical leg members, where it is clamped to a rigid bed.

All the beam members are made of standard 3/4 inch square tubes - fig. 2.2.2. Its material is mild steel with Young's Modulus, E, 207 kN/mm^2 and Modulud of Rigidity, G, 80 kN/mm^2.

The top four corners of the space frame (fig. 2.2.1 joints 1, 2, 3 and 4) are each made from the 3 points joint shown in fig.2.2.3(a). While the other four corners (joints 5, 6, 7 and 8) are each made from the 4 points joint shown in fig. 2.2.3(b). Both these joints are standard joints which take the chosen 3/4 inch square section tubes of fig. 2.2.2. They are orthogonal and therefore they do not take the cross-members directly.

These joints also contain some plastic padding in order to produce a tight fitting with the square tube beams. This in turn leads to some non linear behaviour of the frame at these joints. To overcome this effect, the joints were further strengthened with small plates shown in fig. 2.2.4. These plates were bolted on all faces of the corner joints.

In addition to strengthening the joints, the plates became the points of attachment of the cross-member beam to the corner joints. Again, the cross-members are bolted to the plates.

The feet of the frame are brazed to plates as shown in fig. 2.2.5. This forms the means of clamping the frame to the rigid bed.

Treating the fabricated space frame (fig. 2.2.1) as a rigid jointed frame, the number of joints (j) is 12; the number of members (m) is 22; and the total number of reactive elements (forces and couples) at the support (S) is 24. Then the degree of indeterminancy (r) of the space frame is given by (ref.5).

$$r = 6m - 6j + S$$
$$= 132 - 72 + 24$$

i.e. $r = 84$ which is greater than zero.

Thus the space frame is rigid and not a mechanism.

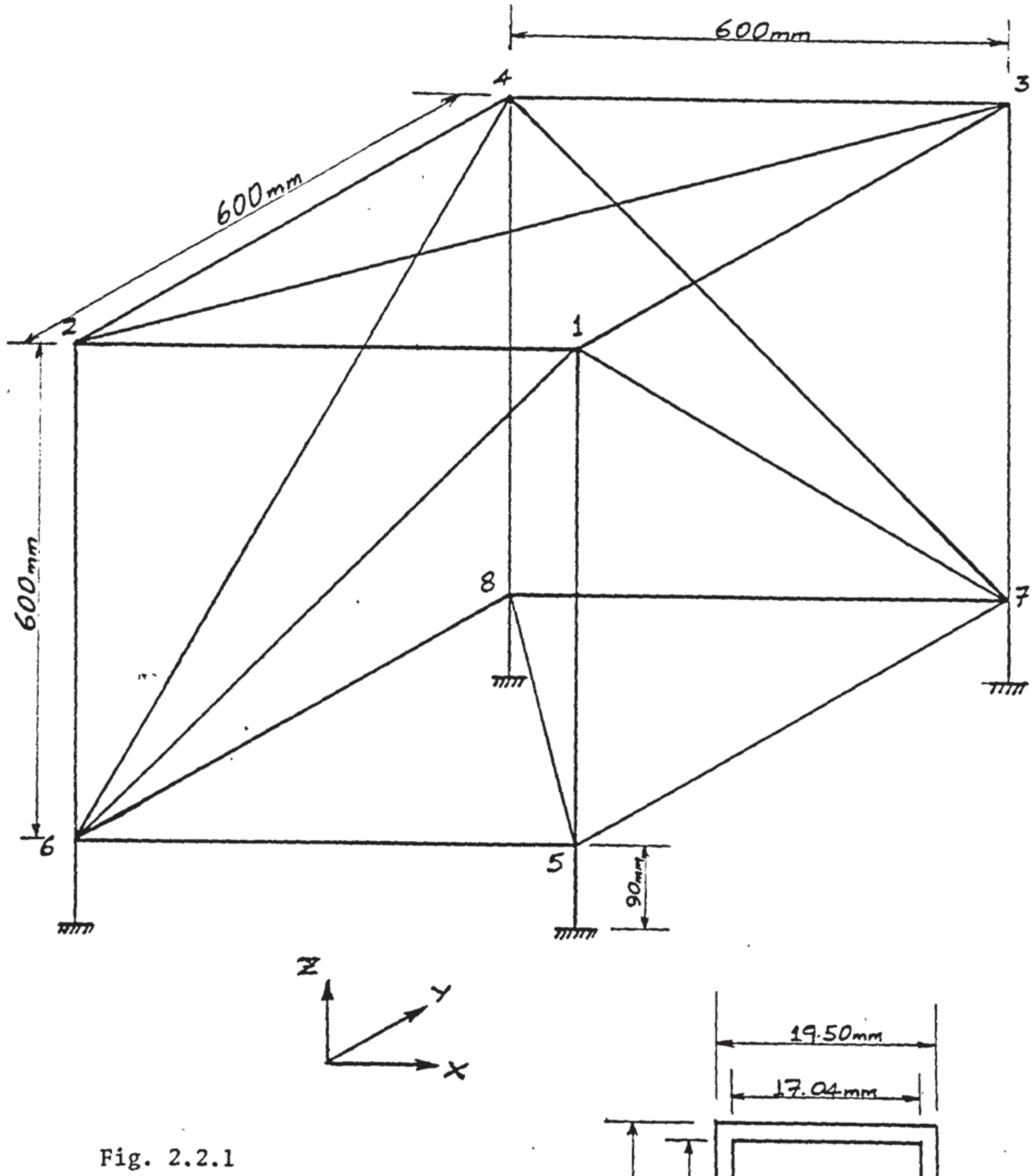


Fig. 2.2.1

Fabricated Space Frame

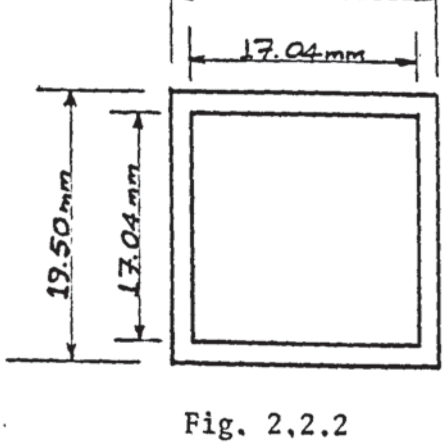


Fig. 2.2.2

Cross-section of the beam member

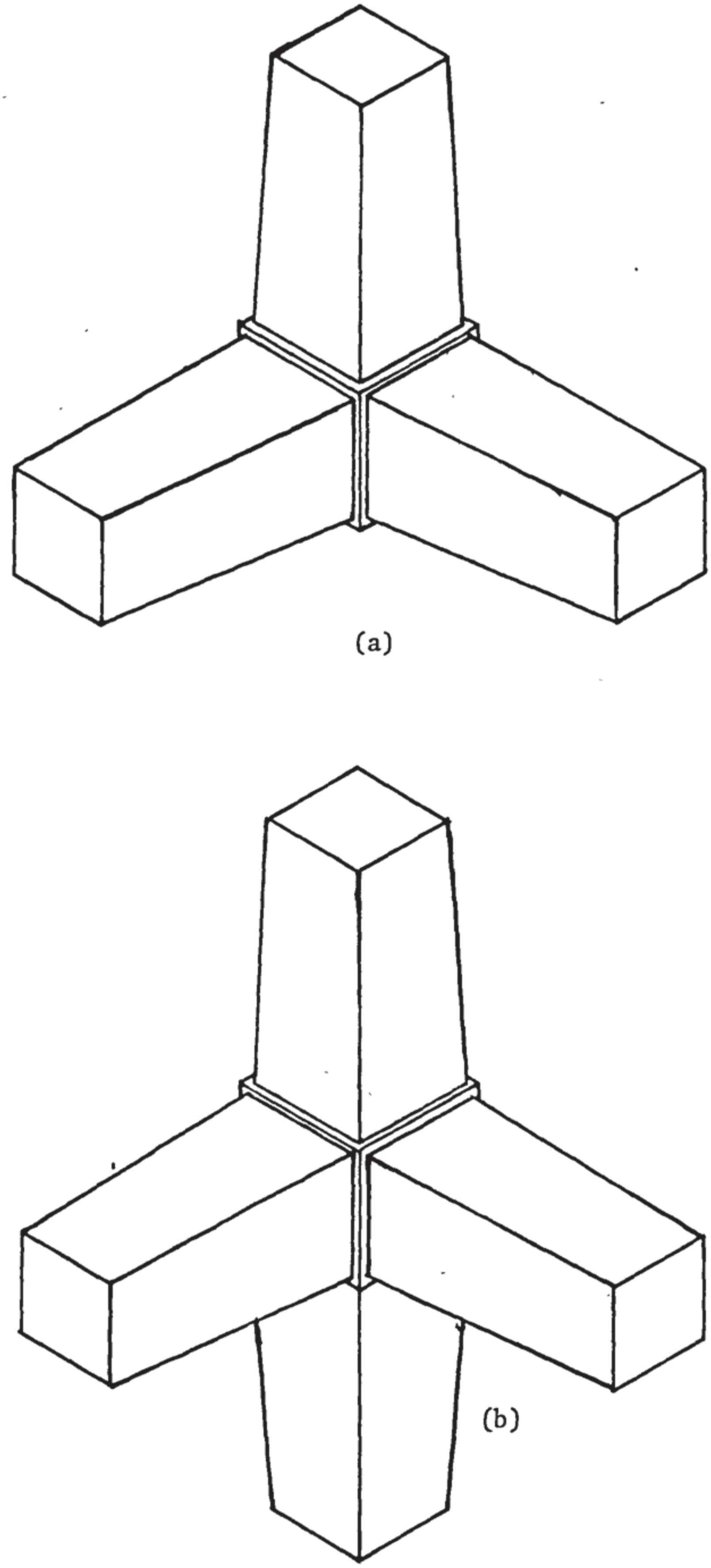
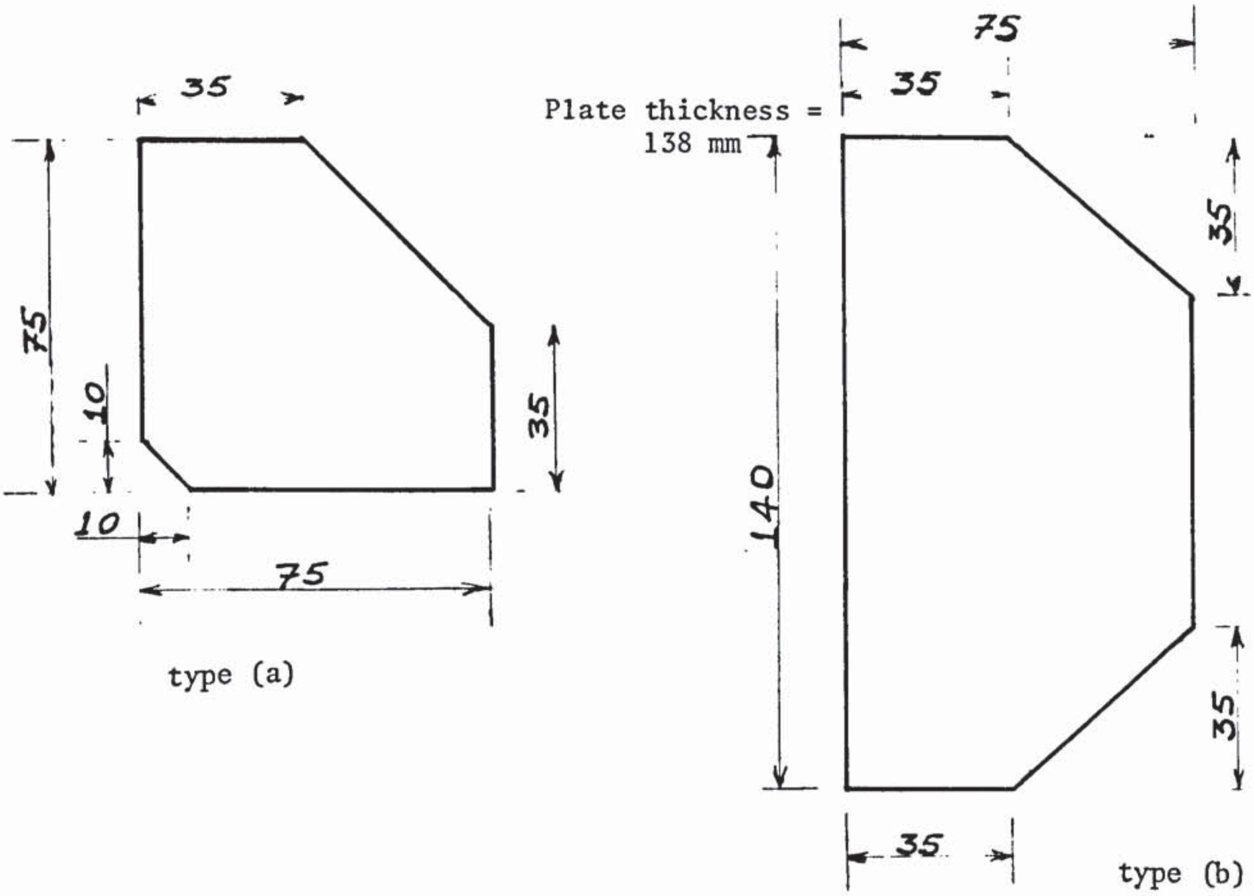


Fig. 2.2.3 <u>Corner joints of space frame</u>

Note: all dimensions are in mm



type (a)

type (b)

Fig. 2.2.4

<u>Corner plates</u>

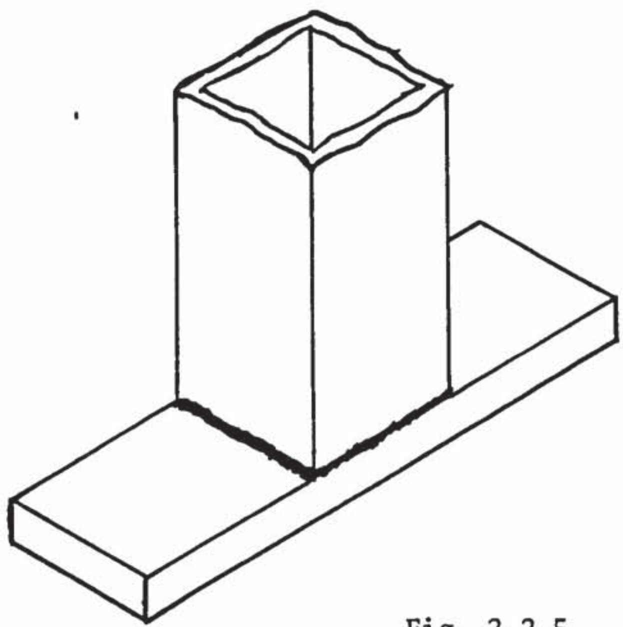


Fig. 2.2.5

<u>Leg Member</u>

CHAPTER THREE

CHAPTER THREE

FINITE ELEMENT METHOD APPLIED TO SPACE FRAME VIBRATION

3.1 Introduction

Two methods may be said to exist for the analysis of structures. They include analytical and numerical methods.

The difficulties and limitations associated with analytical methods are well known and cannot be over-emphasised. Closed-form solutions are possible only in special cases, and approximate solutions may be arrived at for some simple structures. But, generally, analytical methods cannot be applied to complex structures.

Numerical methods are the most practical methods for the solution of complex structural analysis. This has been re-enforced by the advent of the digital computer.

Numerical methods of structural analysis can be further divided into two types namely (i) numerical solution of the differential equations and (ii) matrix methods based on discrete-element idealization.

(i) Numerical solution of the differential equations is based on the mathematical approximations of the differential equations. The process is achieved either by finite-difference techniques or by direct numerical integration. Again, practical limitations exist in the application of this method. Hence, it is mainly restricted to the analysis of simple structures.

The numerical solution of the differential equations usually ends up with equations which can be cast into matrix notations. But the method is still not classified as a matrix method since the original formulationsdo not entail matrix connotations.

(ii) In the matrix method of structural analysis, matrix algebra is used throughout all the stages of development of the analysis. First of all, the structure is idealized into an assemblage of discrete structural elements. The assumed displacements are then combined into a matrix equation satisfying the boundary conditions at the joints of these elements.

Matrix based methods of structural analysis are very suitable for automation and programming for digital computers. The analysis is based on very simple steps of numerical work. This method is therefore suitable for the analysis of complex structure once there is an access to a suitably sized digital computer.

Matrix method of analysis has been found to be very suitable for the analysis of this work.

The Finite Element method of structural analysis falls into this category of matrix method of numerical analysis. In frame structural analysis in particular, the Finite Element method is much preferred to its other numerical method counterparts (such as the finite Difference method which is an older method) because of its versatility and flexibility of usage.

The application of the Finite Element method to frame vibration involves imagining the frame to be actually broken up into a number of beam 'elements' of 'finite' lengths. This concept has given rise to its name. As already discussed in Chapter One, frame structures contain one-dimensional components (beams). Generally a structure would be imagined to be actually broken up into a number of 'elements' of 'finite' dimensions. A structure of n(= 1, 2, 3) dimensions of space will have to be broken up into a system of n-dimensional finite elements.

The frame is subdivided into finite elements connected by nodes as shown in fig. 3.1.1. These finite elements may have equal or unequal lengths. The versatility of the Finite Element method means that variations in the element lengths can be easily taken into account without any additional difficulty.

The next step in this method of analysis is the determination of the "element stiffness and mass matrices" of the individual elements representing the frame. These are then assembled to form the "overall stiffness and mass matrices" for the entire "discretized" frame by requiring that the continuity of displacements and equilibrium of forces prevail at all nodes in the finite element model of the body. This will lead to the equation of vibrating motion in matrix form

$$[M]\{\ddot{q}\} + [K]\{q\} = \{Q\} \qquad 3.1.1$$

where [M] = overall mass matrix of the frame

[K] = overall stiffness matrix of the frame

{Q} = column matrix of exciting force

{q} = displacement column matrix

and $\{\ddot{q}\}$ = acceleration column matrix

In free vibration {Q} = 0 and {q} is a harmonic function of time. Then

$$\{q\} = \{u\}\sin(\omega t + \phi) \qquad 3.1.2a$$

and

$$\{\ddot{q}\} = -\{u\}\omega^2\sin(\omega t + \phi) \qquad 3.1.2b$$

Substituting equations 3.1.2 in equation 3.1.1 yields:

$$[K]\{u\} = \omega^2[M]\{u\} \qquad 3.1.3$$

Equation 3.1.3 represents an eigenvalue problem. The solution of this eigenvalue problem will yield the eigenvalues $\omega_1{}^2$, $\omega_2{}^2$, $\omega_3{}^2$,..... hence ω_1, ω_2, ω_3,..... which corresponds to the natural frequencies of vibration of the discretized frame whilst the corresponding $\{u\}_1$, $\{u\}_2$, $\{u\}_3$,..... are its natural modes of vibration.

In summary, therefore, the finite element solution of the Free vibration of a given frame structure requires the execution of the following operations in this order:

(i) Discretization or subdivision of the frame into a system of finite elements.

(ii) Derivation of the "element stiffness and mass matrices" for each individual element representing the framed structure.

(iii) Assembly of the "overall stiffness and mass matrices" of the frame.

(iv) Solution of eigenvalue problem (equation 3.1.3)

(v) Where necessary, as is most often the case, the eigenvectors are plotted to get a feel of the modal shape of free vibration of the frame.

3.2 Consistent mass and stiffness matrices of beam element

It has already been shown that the discretization of a framed structure should produce beam finite elements. Hence the next step in its vibration analysis is derivation of the consistent mass and stiffness matrices of the beam element.

Consider a beam element shown in fig. 3.2.0. Its extremities are identified by the letters A and B. These represent its points of connection to the nodes of the finite element discretization of the frame.

The beam element is in three-dimensional space and its orthogonal axes x_e, y_e and z_e are chosen such that the x_e-axis lie on the beam neutral axis.

If the beam element is subjected to a set of arbitrary external forces, then it will give rise to six internal reactive forces at each extremity of the beam. These will have their associated displacements. Forces here denote both forces and moments; and displacements include linear and angular displacements.

As shown in fig.3.2.0, these forces include:

(i) Axial forces F_1 and F_7

(ii) Shearing forces F_2, F_3, F_8, and F_9

(iii) Twisting moments F_4 and F_{10}

and (iv) Bending moments F_5, F_6, F_{11}, and F_{12}

The corresponding displacements are:

(i) Axial displacements u_1 and u_7

(ii) Transverse displacements u_2, u_3, u_8, and u_9

(iii) Twisting angles u_4, and u_{10}

and (iv) Bending angles u_5, u_6, u_{11}, and u_{12}

The positive directions of these displacements correspond to the positive directions of the corresponding forces as shown in fig. 3.2.0.

The consistent mass and stiffness matrices of the beam element is of order 12 x 12. In this case, since the element axes are chosen to coincide with the principal axes of the beam cross section, it is now possible to construct the 12 x 12 matrices from sets of 2 x 2 and 4 x 4 sunmatrices. From the engineering theories of beam bending and torsion, it is obvious that the axial forces F_1 and F_7 are functions of their corresponding displacements u_1 and u_7 only; the same is true also for the twisting moments (torques) F_4 and F_{10} in relation to their twisting angles u_4 and u_{10}.

For arbitrarily chosen bending planes, the bending moments and shearing forces in the $x_e y_e$ plane would depend on their corresponding displacements as well as on the displacements corresponding to the forces in the $x_e y_e$ plane. But in this case, the choice of axis have been such that the $x_e y_e$ and $x_e z_e$ planes coincide with the principal axes of the cross section. Hence the bending and shearing in these planes can be considered to be independent of each other.

All forces acting on the beam elements can then be separated into four groups and considered independently of each other. With suitable choice of corresponding displacement patterns within these groups expressions can be obtained for the kinetic energy (T_e) and strain energy (U_e) of the beam in terms of the displacements. The consistent mass and stiffness matrix terms will then be derived from these energy expressions.

3.2.1 Axial Vibration in x_e axis

Fig. 3.2.1 shows the beam element under consideration. The beam is undergoing an axial deformation or vibration. Elementary mechanics of materials show that the state of strain varies linearly within the beam element. Here, vibration is involved, hence the displacement is a function of time (t) also.

Thus a suitable displacement function is of the form

$$u(x,t) = a_0 + a_1 x = [1 \quad x] \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} \qquad 3.2.1$$

Applying the element boundary conditions of $u(o,t) = u_1$ and $u(\ell,t) = u_7$, we have

$$\begin{Bmatrix} u_1 \\ u_7 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & \ell \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} \qquad 3.2.2$$

or

$$\{u\} = [H]\{a\} \qquad 3.2.3$$

Solving for $\{a\}$ in equation 3.2.2 we have

$$\begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & \ell \end{bmatrix}^{-1} \begin{Bmatrix} u_1 \\ u_7 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/\ell & 1/\ell \end{bmatrix} \begin{Bmatrix} u_1 \\ u_7 \end{Bmatrix}$$

Substituting into equation 3.2.1,

$$u(x;t) = [1 \quad x] \begin{bmatrix} 1 & 0 \\ -1/\ell & 1/\ell \end{bmatrix} \begin{Bmatrix} u_1 \\ u_7 \end{Bmatrix}$$

so that $$u(x,t) = [(1 - x/\ell) \quad x/\ell] \begin{Bmatrix} u_1 \\ u_7 \end{Bmatrix} \qquad 3.2.4$$

or $$u(x,t) = [N_1(x) \quad N_2(x)] \begin{Bmatrix} u_1 \\ u_7 \end{Bmatrix} \qquad 3.2.5$$

Let A be the cross sectional area of the beam. Then the strain energy of the beam in axial direction is given by

$$U = \frac{1}{2} \int_0^\ell EA\left(\frac{\partial u(x,t)}{\partial x}\right)^2 dx \qquad 3.2.6$$

Substituting equation 3.2.4 in 3.2.6, we have

$$U = \frac{1}{2} \int_0^\ell \left([u_1 \; u_7] \begin{Bmatrix} -1/\ell \\ -1/\ell \end{Bmatrix} EA[-1/\ell \quad 1/\ell] \begin{Bmatrix} u_1 \\ u_7 \end{Bmatrix}\right) dx \qquad 3.2.7$$

On integration this reduces to the form

$$U = \frac{1}{2} [u_1 \quad u_7] \begin{bmatrix} \frac{EA}{\ell} & -\frac{EA}{\ell} \\ -\frac{EA}{\ell} & \frac{EA}{\ell} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_7 \end{Bmatrix} \qquad 3.2.8$$

This is of the form

$$U = \frac{1}{2}\{u\}^t[K]\{u\} \qquad 3.2.9$$

Thus comparison of equations 3.2.8 and 3.2.9 shows that

$$[K] = \begin{bmatrix} K_{1,1} & K_{1,7} \\ & \\ K_{7,1} & K_{7,7} \end{bmatrix} = \frac{EA}{\ell} \begin{bmatrix} 1 & -1 \\ & \\ -1 & 1 \end{bmatrix} \qquad 3.2.10$$

which is the axial stiffness matrix of the beam element.

Similarly, the kinetic energy of the beam in axial motion is given by

$$T = \frac{1}{2} \int_0^{\ell} \rho A \left(\frac{\partial u(x,t)}{\partial t}\right)^2 dx \qquad 3.2.11$$

where ρ is the mass density of the beam.

Substituting from equation 3.2.4 into 3.2.11 and noting that $\{u_1 \quad u_7\}^t$ is in fact a function of time t, integrating and simplifying gives

$$T = \frac{1}{2} \left\{ \frac{\partial u_1}{\partial t} \quad \frac{\partial u_7}{\partial t} \right\}^t \begin{bmatrix} \frac{\rho A \ell}{3} & \frac{\rho A \ell}{6} \\ & \\ \frac{\rho A \ell}{6} & \frac{\rho A \ell}{3} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_1}{\partial t} \\ \\ \frac{\partial u_7}{\partial t} \end{Bmatrix} \qquad 3.2.12$$

This is again of the form

$$T = \frac{1}{2} \left\{\frac{\partial u}{\partial t}\right\}^t [m] \left\{\frac{\partial u}{\partial t}\right\} \qquad 3.2.13$$

Thus, from equations 3.2.12 and 3.2.12, the axial mass matrix of the beam element is given by

$$[m] = \begin{bmatrix} m_{1,1} & m_{1,7} \\ m_{7,1} & m_{7,7} \end{bmatrix} = \rho A \ell \begin{bmatrix} \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix} \qquad 3.2.14$$

3.2.2 Twisting about the x_e-axis

The beam element under torsional vibration is as shown in fig. 3.2.2. As with the axial case, the angle of twist varies linearly along the beam in the form

$$\theta(x,t) = a_{10} + a_{11}x$$

$$\text{i.e.} \quad \theta(x,t) = [1 \quad x]\begin{Bmatrix} a_{10} \\ a_{11} \end{Bmatrix} \qquad 3.2.15$$

The appropriate boundary conditions are

$$\theta(o,t)=u_4 \quad \text{and} \quad u'(\ell,t) = u_{10}$$

Hence from equation 3.2.15 we have

$$\begin{Bmatrix} u_4 \\ u_{10} \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & \ell \end{bmatrix}\begin{Bmatrix} a_{10} \\ a_{11} \end{Bmatrix} \qquad 3.2.16$$

$$\text{or} \quad \{u\} = [H]\{a\} \qquad 3,2.17$$

From equation 3.2.16, we have

$$\begin{Bmatrix} a_{10} \\ a_{11} \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & \ell \end{bmatrix}^{-1}\begin{Bmatrix} u_4 \\ u_{10} \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/\ell & 1/\ell \end{bmatrix}\begin{Bmatrix} u_4 \\ u_{10} \end{Bmatrix}$$

Substituting for {a} into equation 3.2.15, we have

$$\theta(x,t) = [1 \quad x]\begin{bmatrix} 1 & 0 \\ -1/\ell & -1/\ell \end{bmatrix}\begin{Bmatrix} u_4 \\ u_{10} \end{Bmatrix}$$

Thus

$$\theta(x,t) = [(1 - \tfrac{x}{\ell}) \quad \tfrac{x}{\ell}]\begin{Bmatrix} u_4 \\ u_{10} \end{Bmatrix} \qquad 3.2.18$$

Let I_x be the polar second moment of area of the beam cross-section about the x_e axis.

Then the torsional strain energy of the beam is given by

$$U = \frac{1}{2}\int_{o}^{\ell} GI_x \left(\frac{\partial\theta(x,t)}{\partial x}\right)^2 dx \qquad 3.2.19$$

Substituting equation 3.2.18 into 3.2.19, integrating and simplifying yields

$$U = \frac{1}{2}[u_4 \quad u_{10}] \begin{bmatrix} \frac{GI_x}{\ell} & -\frac{GI_x}{\ell} \\ -\frac{GI_x}{\ell} & \frac{GI_x}{\ell} \end{bmatrix} \begin{Bmatrix} u_4 \\ u_{10} \end{Bmatrix} \qquad 3.2.20$$

which is of the form

$$U = \frac{1}{2}\{u\}^t[K]\{u\} \qquad 3.2.21$$

Hence the torsional stiffness matrix of the beam element is given by

$$[K] = \begin{bmatrix} K_{4,4} & K_{4,10} \\ K_{10,4} & K_{10,10} \end{bmatrix} = \frac{GI_x}{\ell} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad 3.2.22$$

And the torsional kinetic energy of the beam element is given by

$$T = \frac{1}{2}\int_{o}^{\ell} \rho I_x \left(\frac{\partial\theta(x,t)}{\partial t}\right)^2 dx \qquad 3.2.23$$

Substituting equation 3.2.18 into 3.2.23, integrating and simplifying we have

$$T = \frac{1}{2}\left[\frac{\partial u_4}{\partial t} \quad \frac{\partial u_{10}}{\partial t}\right] \begin{bmatrix} \frac{\rho I_x \ell}{3} & \frac{\rho I_x \ell}{6} \\ \frac{\rho I_x \ell}{6} & \frac{\rho I_x \ell}{3} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_4}{\partial t} \\ \frac{\partial u_{10}}{\partial t} \end{Bmatrix} \qquad 3.2.24$$

which is of the form

$$T = \frac{1}{2}\left\{\frac{\partial u}{\partial t}\right\}^t [m] \left\{\frac{\partial u}{\partial t}\right\} \qquad 3.2.25$$

Thus the torsional mass matrix of the beam element is given by

$$[m] = \begin{bmatrix} m_{4,4} & m_{4,10} \\ m_{10,4} & m_{10,10} \end{bmatrix} = \rho A \ell \begin{bmatrix} \frac{I_x}{3A} & \frac{I_x}{6A} \\ \frac{I_x}{6A} & \frac{I_x}{3A} \end{bmatrix} \qquad 3.2.26$$

3.2.3 Shearing and bending in the $x_e y_e$ plane

The beam under consideration is shown in fig. 3.2.3. Engineering theory of bar bending indicates that the deformation is characterised by the deflection curve taken up by the centre ~~time~~ line of the bar.

The element has four degrees of freedom. Hence a suitable displacement model is of the form

$$u(x,t) = a_{20} + a_{21}x + a_{22}x^2 + a_{23}x^3$$

or

$$u(x,t) = [1 \quad x \quad x^2 \quad x^3] \begin{Bmatrix} a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \end{Bmatrix} \qquad 3.2.27$$

The boundary conditions are

$$u(o,t) = u_2, \qquad \frac{\partial u}{\partial x}(o,t) = u_6$$

$$u(\ell,t) = u_8, \quad \text{and} \quad \frac{\partial u}{\partial x}(\ell,t) = u_{12}$$

Substituting the above boundary conditions into equation 3.2.27 we have:

$$\begin{Bmatrix} u_2 \\ u_6 \\ u_8 \\ u_{12} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & \ell & \ell^2 & \ell^3 \\ 0 & 1 & 2\ell & 3\ell^2 \end{bmatrix} \begin{Bmatrix} a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \end{Bmatrix} \qquad 3.2.28$$

i.e. $\{u\} = [H]\{a\}$

and $\{a\} = [H]^{-1}\{u\}$ 3.2.29

Now

$$[H]^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & \ell & \ell^2 & \ell^3 \\ 0 & 1 & 2\ell & 3\ell^2 \end{bmatrix}$$

i.e.

$$[H]^{-1} = \begin{bmatrix} 1 & 0 & -\frac{3}{\ell^2} & \frac{2}{\ell^3} \\ 0 & 1 & -\frac{2}{\ell} & \frac{1}{\ell^2} \\ 0 & 0 & \frac{3}{\ell^2} & -\frac{2}{\ell^3} \\ 0 & 0 & -\frac{1}{\ell} & \frac{1}{\ell^2} \end{bmatrix} \qquad 3.2.30$$

Substituting equations 3.2.29 and 3.2.20 in equation 3.2.27, we have

$$u(x,t) = [1 \quad x \quad x^2 \quad x^3] \begin{bmatrix} 1 & 0 & -\frac{3}{\ell^2} & \frac{2}{\ell^3} \\ 0 & 1 & -\frac{2}{\ell} & \frac{1}{\ell^2} \\ 0 & 0 & \frac{3}{\ell^2} & \frac{2}{\ell^3} \\ 0 & 0 & -\frac{1}{\ell} & \frac{1}{\ell^2} \end{bmatrix} \begin{Bmatrix} u_2 \\ u_6 \\ u_8 \\ u_{12} \end{Bmatrix} \qquad 3.2.31$$

Let I_z be the second moment of area of the cross section about the z_e-axis.

Then, neglecting the effects of shear deformation, the strain energy of the beam element under the action of the shearing forces and bending moments in the $x_e y_e$ plane is given by

$$U = \frac{1}{2} \int_0^{\ell} EI_z \left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)^2 dx \qquad 3.2.32$$

Substituting equation 3.2.31 into equation 3.2.32 and simplifying we have

$$U = \frac{1}{2} \begin{Bmatrix} u_2 \\ u_6 \\ u_8 \\ u_{12} \end{Bmatrix}^t \begin{bmatrix} \frac{12EI_z}{\ell^3} & & \text{symmetric} & \\ \frac{6EI_z}{\ell^2} & \frac{4EI_z}{\ell} & & \\ -\frac{12EI_z}{\ell^3} & -\frac{6EI_z}{\ell^2} & \frac{12EI_z}{\ell^3} & \\ \frac{6EI_z}{\ell^2} & \frac{2EI_z}{\ell} & -\frac{6EI_z}{\ell^2} & \frac{4EI_z}{\ell} \end{bmatrix} \begin{Bmatrix} u_2 \\ u_6 \\ u_8 \\ u_{12} \end{Bmatrix} \qquad 3.2.33$$

Thus the beam stiffness matrix in flexure in the $x_e y_e$ plane is given by

$$[K] = \begin{bmatrix} k_{2,2} & & \text{symmetric} & \\ k_{2,6} & k_{6,6} & & \\ k_{2,8} & k_{6,8} & k_{8,8} & \\ k_{2,12} & k_{6,12} & k_{8,12} & k_{12,12} \end{bmatrix} = \frac{EI_z}{\ell^3} \begin{bmatrix} 12 & 6\ell & -12 & 6\ell \\ 6\ell & 4\ell^2 & -6\ell & 2\ell^2 \\ -12 & -6\ell & 12 & -6\ell \\ 6\ell & 2\ell^2 & -6\ell & 4\ell^2 \end{bmatrix}$$

3.2.34

Also, the kinetic energy of the beam element in $x_e y_e$ plane due to shearing forces and bending moments is given by

$$T = \frac{1}{2}\int_0^{\ell} \rho A \left(\frac{\partial u(x,t)}{\partial t}\right)^2 dx \qquad 3.2.35$$

Substituting from equation 3.2.31 into equation 3.2.35 and simplifying we have

$$T = \begin{Bmatrix} \frac{\partial u_2}{\partial t} \\ \frac{\partial u_6}{\partial t} \\ \frac{\partial u_8}{\partial t} \\ \frac{\partial u_{12}}{\partial t} \end{Bmatrix}^t \begin{bmatrix} \frac{13\rho A\ell}{35} & & \text{symmetric} & \\ \frac{11\rho A\ell^2}{210} & \frac{\rho A\ell^3}{105} & & \\ \frac{9\rho A\ell}{70} & \frac{13\rho A\ell^2}{420} & \frac{13\rho A\ell}{35} & \\ -\frac{13\rho A\ell^2}{420} & -\frac{\rho A\ell^3}{140} & \frac{11\rho A\ell^2}{210} & \frac{\rho A\ell^3}{105} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_2}{\partial t} \\ \frac{\partial u_6}{\partial t} \\ \frac{\partial u_8}{\partial t} \\ \frac{\partial u_{12}}{\partial t} \end{Bmatrix}$$

Thus the beam mass matrix in flexure in the $x_e y_e$ plane (neglecting the effects of shear deformation) is given by

$$[m] = \begin{bmatrix} m_{2,2} & & \text{symmetric} & \\ m_{2,6} & m_{6,6} & & \\ m_{2,8} & m_{6,8} & m_{8,8} & \\ m_{2,12} & m_{6,12} & m_{8,12} & m_{12,12} \end{bmatrix} = \rho A\ell \begin{bmatrix} \frac{13}{35} & \text{symmetric} & & \\ \frac{11\ell}{210} & \frac{\ell^2}{105} & & \\ \frac{9}{70} & \frac{13\ell}{420} & \frac{13}{35} & \\ -\frac{13}{420} & -\frac{\ell^2}{140} & -\frac{11\ell}{210} & \frac{\ell^2}{105} \end{bmatrix}$$

3.2.36

3.2.4 Shearing and bending in the $x_e z_e$ plane

Fig. 3.2.4 shows the beam under consideration. Positive directions of forces and displacements are as illustrated.

Four degrees of freedom is envisaged and a suitable displacement model is of the form

$$u(x,t) = a_{30} + a_{31}x + a_{32}x^2 + a_{33}x^3$$

or

$$u(x,t) = [1 \quad x \quad x^2 \quad x^3] \begin{Bmatrix} a_{30} \\ a_{31} \\ a_{32} \\ a_{33} \end{Bmatrix} \qquad 3.2.37$$

The geometric boundary conditions as illustrated in fig.3.2.4 are as follows

$$\begin{aligned} u(o,t) &= u_3, \quad \frac{\partial\partial u}{\partial x}(o,t) = -u_5 \\ u(\ell,t) &= u_9, \quad \frac{\partial u}{\partial x}(\ell,t) = -u_{11} \end{aligned} \qquad 3.2.38$$

Hence, equations 3.2.38 and 3.2.37 give

$$\begin{Bmatrix} u_3 \\ -u_5 \\ u_9 \\ -u_{11} \end{Bmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & \ell & \ell^2 & \ell^3 \\ 0 & 1 & 2\ell & 3\ell^3 \end{vmatrix} \begin{Bmatrix} a_{30} \\ a_{31} \\ a_{31} \\ a_{33} \end{Bmatrix} \qquad 3.2.39$$

Notice the sign of the displacements u_5 and u_{11} into equations 3.2.38 and 3.2.39. A comparison of figs. 3.2.4 and 3.3.3 shows that the positive direction of the bending moments F_5 and $F_{11}$is opposite to that of F_6 and F_{12}. The directions of the positive bending moments in the $x_e z_e$ planes are different. This has been taken into account in the above equations.

Thus, from equation 3.2.39, we have

$$\begin{Bmatrix} a_{30} \\ a_{31} \\ a_{32} \\ a_{33} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & \ell & \ell^2 & \ell^3 \\ 0 & 1 & 2\ell & 3\ell^3 \end{bmatrix}^{-1} \begin{Bmatrix} u_3 \\ -u_5 \\ u_9 \\ -u_{11} \end{Bmatrix}$$

and

$$\begin{Bmatrix} a_{30} \\ a_{31} \\ a_{32} \\ a_{33} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -\frac{3}{\ell^2} & \frac{2}{\ell^3} \\ 0 & 1 & -\frac{2}{\ell} & \frac{1}{\ell^2} \\ 0 & 0 & \frac{3}{\ell^2} & \frac{2}{\ell^3} \\ 0 & 0 & -\frac{1}{\ell} & \frac{1}{\ell^2} \end{bmatrix} \begin{Bmatrix} u_3 \\ -u_5 \\ u_9 \\ -u_{11} \end{Bmatrix} \qquad 3.2.40$$

Substituting equation 3.2.40 into equation 3.2.37 we have

$$u(x,t) = [1 \quad x \quad x^2 \quad x^3] \begin{bmatrix} 1 & 0 & -\frac{3}{\ell^2} & \frac{2}{\ell^3} \\ 0 & -1 & \frac{2}{\ell} & -\frac{1}{\ell^2} \\ 0 & 0 & \frac{3}{\ell^2} & \frac{2}{\ell^3} \\ 0 & 0 & \frac{1}{\ell} & -\frac{1}{\ell^2} \end{bmatrix} \begin{Bmatrix} u_3 \\ u_5 \\ u_9 \\ u_{11} \end{Bmatrix}$$

3.2.41

Let I_y be the second moment of area of cross section about the y_e-axis.

Then, neglecting the effects of shear deformation, the strain energy of the beam element under the action of shearing forces and bending moments in the $x_e z_e$ plane is given by

$$U = \frac{1}{2} \int_0^{\ell} EI_y \left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)^2 dx \qquad 3.2.42$$

Substituting equation 3.2.41 into equation 3.2.42 integrating and simplifying, we have

$$U = \frac{1}{2} \begin{Bmatrix} u_3 \\ u_5 \\ u_9 \\ u_{11} \end{Bmatrix} \begin{bmatrix} \frac{12EI_y}{\ell} & & & \\ -\frac{6EI_y}{\ell^2} & \frac{4EI_y}{\ell} & & \\ -\frac{12EI_y}{\ell^3} & \frac{6EI_y}{\ell} & \frac{12EI_y}{\ell} & \\ -\frac{6EI_y}{\ell^2} & \frac{2EI_y}{\ell} & \frac{6EI_y}{\ell^2} & \frac{4EI_y}{\ell} \end{bmatrix} \begin{Bmatrix} u_3 \\ u_5 \\ u_9 \\ u_{11} \end{Bmatrix} \qquad 3.2.43$$

From equation 3.2.43, the flexural beam stiffness matrix in the plane is given by

$$[K] = \begin{bmatrix} k_{3,3} & & \text{symmetric} & \\ k_{3,5} & k_{5,5} & & \\ k_{3,9} & k_{5,9} & k_{9,9} & \\ k_{3,11} & k_{5,11} & k_{9,11} & k_{11,11} \end{bmatrix}$$

$$
= \begin{bmatrix}
\frac{12EI_y}{\ell} & & \text{symmetric} & \\
-\frac{6EI_y}{\ell^2} & \frac{4EI_y}{\ell} & & \\
-\frac{12EI_y}{\ell^3} & \frac{6EI_y}{\ell^2} & \frac{12EI_y}{\ell} & \\
-\frac{6EI_y}{\ell^2} & \frac{2EI_y}{\ell} & \frac{6EI_y}{\ell^2} & \frac{4EI_y}{\ell}
\end{bmatrix} \qquad 3.2.44
$$

Similarly, the expression for the kinetic energy of the beam is given by equation 3.2.35. Substitution of equation 3.2.41 into 3.2.25 gives

$$
T = \begin{Bmatrix} \frac{\partial u_3}{\partial t} \\ \frac{\partial u_5}{\partial t} \\ \frac{\partial u_9}{\partial t} \\ \frac{\partial u_{11}}{\partial t} \end{Bmatrix}^t
\begin{bmatrix}
\frac{13\rho A\ell}{35} & & & \\
-\frac{11\rho A\ell^2}{210} & \frac{\rho A\ell^3}{105} & & \\
\frac{9\rho A\ell}{70} & -\frac{13\rho A\ell^2}{420} & \frac{13\rho A\ell}{35} & \\
\frac{13\rho A\ell^2}{420} & -\frac{\rho A\ell^3}{140} & \frac{11\rho A\ell^2}{210} & \frac{\rho A\ell^3}{105}
\end{bmatrix}
\begin{Bmatrix} \frac{\partial u_3}{\partial t} \\ \frac{\partial u_5}{\partial t} \\ \frac{\partial u_9}{\partial t} \\ \frac{\partial u_{11}}{\partial t} \end{Bmatrix}
$$

3.2.45

And from equation 3.2.45, the flexural beam mass matrix in the $x_e y_e$ plane is given by

$$
[m] = \begin{bmatrix}
m_{3,3} & & \text{symmetric} & \\
m_{3,5} & m_{5,5} & & \\
m_{3,9} & m_{5,9} & m_{9,9} & \\
m_{3,11} & m_{5,11} & m_{9,11} & m_{11,11}
\end{bmatrix} .
$$

$$= \rho A\ell \begin{bmatrix} \frac{13}{35} & & & \\ -\frac{11}{210} & \frac{\ell^2}{105} & & \\ \frac{9}{70} & -\frac{13\ell}{420} & \frac{13}{35} & \\ \frac{13\ell}{420} & -\frac{\ell^2}{140} & \frac{11\ell}{210} & \frac{\ell^2}{105} \end{bmatrix} \qquad 3.2.46$$

3.2.5 Beam matrices in assembled form

From the above analysis and results, the 12 x 12 consistent mass and stiffness matrices of the beam element can be obtained.

Assembling equations 3.2.10, 3.2.22, 3.2.34 and 3.2.44, we have the complete stiffness matrix $[K_e]$ of the beam element given by equation 3.2.47.

Similarly, assembling equations 3.2.14, 3.2.26, 3.2.36 and 3.2.46, we have the complete mass matrix $[M_e]$ of the beam element given by equation 3.2.48

$$[K_e] = \begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12 \end{array}
\begin{bmatrix}
\frac{EA}{\ell} & & & & & & & & & & & \\
0 & \frac{12EI_z}{\ell^3} & & & & & & & & & & \\
0 & 0 & \frac{12EI_y}{\ell^3} & & & & & & \text{symmetric} & & & \\
0 & 0 & 0 & \frac{GI_x}{\ell} & & & & & & & & \\
0 & 0 & -\frac{6EI_y}{\ell^2} & 0 & \frac{4EI_y}{\ell} & & & & & & & \\
0 & \frac{6EI_z}{\ell^2} & 0 & 0 & 0 & \frac{4EI_z}{\ell} & & & & & & \\
-\frac{EA}{\ell} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{\ell} & & & & & \\
0 & -\frac{12EI_z}{\ell^3} & 0 & 0 & 0 & -\frac{6EI_z}{\ell^2} & 0 & \frac{12EI_z}{\ell^3} & & & & \\
0 & 0 & -\frac{12EI_y}{\ell^3} & 0 & \frac{6EI_y}{\ell^2} & 0 & 0 & 0 & \frac{12EI_y}{\ell^3} & & & \\
0 & 0 & 0 & -\frac{GI_x}{\ell} & 0 & 0 & 0 & 0 & 0 & \frac{GI_x}{\ell} & & \\
0 & 0 & -\frac{6EI_y}{\ell^2} & 0 & \frac{2EI_y}{\ell} & 0 & 0 & 0 & \frac{6EI_y}{\ell^2} & 0 & \frac{4EI_y}{\ell} & \\
0 & \frac{6EI_z}{\ell^2} & 0 & 0 & 0 & \frac{2EI_z}{\ell} & 0 & -\frac{6EI_z}{\ell^2} & 0 & 0 & 0 & \frac{4EI_z}{\ell}
\end{bmatrix}
\qquad 3.2.47$$

$$\begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array}$$

$$[M_e]=\rho A \ell \begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12 \end{array} \left[\begin{array}{cccccccccccc}
\frac{1}{3} & & & & & & & & & & & \\
0 & \frac{13}{35} & & & & & & & \text{symmetric} & & & \\
0 & 0 & \frac{13}{35} & & & & & & & & & \\
0 & 0 & 0 & \frac{I_x}{3A} & & & & & & & & \\
0 & 0 & -\frac{11\ell}{210} & 0 & \frac{\ell^2}{105} & & & & & & & \\
0 & \frac{11\ell}{210} & 0 & 0 & 0 & \frac{\ell^2}{105} & & & & & & \\
\frac{1}{6} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & & & & & \\
0 & \frac{9}{70} & 0 & 0 & 0 & \frac{13\ell}{420} & 0 & \frac{13}{35} & & & & \\
0 & 0 & \frac{9}{70} & 0 & -\frac{13\ell}{420} & 0 & 0 & 0 & \frac{13}{35} & & & \\
0 & 0 & 0 & \frac{I_x}{3A} & 0 & 0 & 0 & 0 & 0 & \frac{I_x}{3A} & & \\
0 & 0 & \frac{13\ell}{420} & 0 & -\frac{\ell^2}{140} & 0 & 0 & 0 & \frac{11\ell}{210} & 0 & \frac{\ell^2}{105} & \\
0 & -\frac{13\ell}{420} & 0 & 0 & 0 & -\frac{\ell^2}{140} & 0 & -\frac{11\ell}{210} & 0 & 0 & 0 & \frac{\ell^2}{105}
\end{array}\right] \quad 3.2.48$$

$$\begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array}$$

3.3 Beam properties in frame coordinate system

The mass and stiffness matrices obtained in section 3.2.5 consist of 12 x 12 dimensional arrays. These have been derived with respect to a convenient set of orthogonal axes x_e, y_e, z_e such that the x_e lies on the beam neutral axis while the $x_e y_e$ and $x_e z_e$ plane coincide with the principal axis of the beam cross-section. The choice of this coordinate system has led to a simplified derivation and results in equations 3.2.47 and 3.2.48.

The set of axes x_e, y_e, z_e are therefore localised axis or beam element axes. The beam element considered is one of many beam elements in the finite element discretization of the space frame in question. Each element will generally have a different set of axes such that these axes will not coincide with each other.

It is therefore necessary to define a global or frame set of coordinate system to which each beam element properties will have to be transformed. Fig. 3.3.0 shows a typical beam element in three-dimensional space. The x_e, y_e, z_e axes define the beam element coordinate system as explained in section 3.2. While the X, Y, Z axis define the global or frame coordinate system. A transformation matrix should exist which relates the beam element properties (displacements, forces, stiffness, mass etc.) in the element coordinate system x_e, y_e, z_e to their frame coordinate counterparts.

3.3.1 Plane axes transformation

It may be helpful to look at the transformation of the beam element properties from its local system to the frame system in a plane frame situation. Here, the process is much easier to derive and understand.

Fig. 3.3.1 shows a beam element connecting two joints A and B in a plane finite element discretization. The x_e- and y_e-axes are the element coordinate axes, while the X- and Y-axes are the frame coordinate axes. Angle α is the angle of rotation from X-axis to x_e-axis, the position direction being the anti-clockwise rotatoon shown in the figure.

The forces (forces and moments) acting at the two joints A and B are as shown in the figure in both the element coordinate system and the frame coordinate system.

The following are the equilibrium of forces equations at the the two joints.

At joint A.

$$FX_A + Fx_{eA}\cos\alpha - Fy_{eA}\sin\alpha = 0$$
$$FY_A + Fx_{eA}\sin\alpha + Fy_{eA}\cos\alpha = 0$$
$$M_A + M_{eA} = 0$$

At joint B.

$$FX_B + Fx_{eB}\cos\alpha - Fy_{eB}\sin\alpha = 0$$
$$FY_B + Fx_{eB}\sin\alpha + Fy_{eB}\cos\alpha = 0$$
$$M_B + M_{eB} = 0$$

In matrix notation, we have

At joint A

$$\begin{Bmatrix} FX_A \\ FY_A \\ M_A \end{Bmatrix} = \begin{bmatrix} -\cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & -\cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} Fx_{eA} \\ Fy_{eA} \\ M_{eA} \end{Bmatrix} \qquad 3.3.1$$

At joint B

$$\begin{Bmatrix} FX_B \\ FY_B \\ M_B \end{Bmatrix} = \begin{bmatrix} -\cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & -\cos\alpha & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{Bmatrix} Fx_{eB} \\ Fy_{eB} \\ M_{eB} \end{Bmatrix} \qquad 3.3.2$$

Thus the transformation for a plane beam element is of the form

$$\begin{Bmatrix} FX_A \\ FY_A \\ M_A \\ FX_B \\ FY_B \\ M_B \end{Bmatrix} = \begin{bmatrix} & [R\ M] & & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ 0 & 0 & 0 & & & \\ 0 & 0 & 0 & & [R\ M] & \\ 0 & 0 & 0 & & & \end{bmatrix} \begin{Bmatrix} Fx_{eA} \\ Fy_{eA} \\ M_{eA} \\ Fx_{eB} \\ Fy_{eB} \\ M_{eB} \end{Bmatrix} \qquad 3.3.3$$

The transformation matrix is therefore a 6 x 6 matrix. The same matrix will transform the displacement, stiffness and mass matrices.

It should be noted that a plane beam element, as analysed above, has six degrees of freedom. This in turn requires 6 x 6 transformation matrix.

A space beam element has twelve degrees of freedom. Thus a 12 x 12 transformation matrix is expected for effective transformation of the element properties to the space frame coordinate system.

It will be seen that transformation matrix can be built from 3 x 3 submatrices of the form [R M] above.

In the case of a space frame, there will be three rotations of axes rather than one rotation in the plane frame just analysed. The transformation of forces (excluding moments) only will be considered first. Moments transformation will then be deduced from the results. The three types of rotations are considered in turn below.

3.3.2 Rotation about the Z-axis

In fig. 3.3.2 the frame-axes system is represented by (X, Y, Z) at the joint A. A second axes system represented by (x_1, y_1, z_1) is also present at the same joint A, and is different from the frame-axes system only because of a clockwise rotation of α about the positive direction of the Z-axis.

Two sets of equilibrating forces (FX, FY, FZ) and (Fx_1, Fy_1, Fz_1) act in the frame axes system (X, Y, Z) and $x_1, y_1, z_1)$ respectively as shown in fig. 3.3.2(b). Equilibrium equations for the two sets of forces are as follows:

$$FX + Fx_1\cos\alpha - Fy_1\sin\alpha = 0$$

$$FY + Fx_1\sin\alpha - Fy_1\cos\alpha = 0$$

$$FZ + Fz_1 = 0$$

In matrix notations, we have

$$\begin{Bmatrix} FX \\ FY \\ FZ \end{Bmatrix} = \begin{bmatrix} -\cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & -\cos\alpha & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{Bmatrix} Fx \\ Fy \\ Fz \end{Bmatrix} \qquad 3.3.4$$

or $$\{F\} = [R\alpha]\{F_1\} \qquad 3.3.5$$

3.3.3 Rotation about the y_1-axis

In fig. 3.3.3, the axes system (x_1, y_1, z_1) defined in section 3.3.2 is shown in addition to a new set of axes system represented by (x_2, y_2, z_2). This new set of axes system differs from the previous one only because of a clockwise rotation of β about the positive direction of the y_1-axis.

The sets of forces (Fx_1, Fy_1, Fz_1) and (Fx_2, Fy_2, Fz_2) are equilibrating forces acting at the same joint A in the old (x_1, y_1, z_1) and new (x_2, y_2, z_2) set of axes systems respectively.

Equilibrium equations for these two sets of forces are as follows:

$$Fx_1 + Fx_2\cos\beta + Fz_2\sin\beta = 0$$

$$Fy_1 + Fy_2 = 0$$

$$Fz_1 - Fx_2\sin\beta + Fz_2\cos\beta = 0$$

and in matrix notation

$$\begin{Bmatrix} Fx_1 \\ Fy_1 \\ Fz_1 \end{Bmatrix} = \begin{bmatrix} -\cos\beta & 0 & -\sin\beta \\ 0 & -1 & 0 \\ \sin\beta & 0 & -\cos\beta \end{bmatrix} \begin{Bmatrix} Fx_2 \\ Fy_2 \\ Fz_2 \end{Bmatrix} \qquad 3.3.6$$

or $$\{F_1\} = [R\,\beta]\{F_2\} \qquad 3.3.7$$

3.3.4 Rotation about the x_2-axis

Here (fig. 3.3.4) the set of axes system (x_2, y_2, z_2) is as defined in the last subsection 3.3.3. The set of axes (x_e, y_e, z_e) represents the beam element coordinate system as previously defined. The latter axes defers from the (x_2, y_2, z_2) system only because of a positive (clockwise) rotation of γ about the positive direction of the x_2 axis.

Forces (Fx_e, Fy_e, Fz_e) are the final set of forces acting on the beam element in the beam element coordinate system. These forces, therefore, equilibrate the former set of forces (Fx_2, Fy_2, Fz_2) as illustrated in fig. 3.3.4(b).

Again, the equilibrium equations for these two sets of forces are as follows:

$$\begin{aligned} Fx_2 + Fx_e &= 0 \\ Fy_2 + Fy_e\cos\gamma - Fz_e\sin\gamma &= 0 \\ Fz_2 + Fy_e\sin\gamma + Fz_e\cos\gamma &= 0 \end{aligned}$$

and in matrix notation, we have

$$\begin{Bmatrix} Fx_2 \\ Fy_2 \\ Fz_2 \end{Bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & -\cos\gamma \end{bmatrix} \begin{Bmatrix} Fx_e \\ Fy_e \\ Fz_e \end{Bmatrix} \qquad 3.3.8$$

or $$\{F_2\} = [R\gamma]\{F_e\} \qquad 3.3.9$$

3.3.5 Overall transformation matrix

The results of the three step rotation as discussed in subsections 3.3.2, 3.3.3 and 3.3.4 above have the final effect of rotation of the frame coordinate system (X, Y, Z) into the beam element coordinate system (x_e, y_e, z_e) for the joint A.

Thus from equations 3.3.5, 3.3.7 and 3.3.9 we have

$$\{F\} = [R\ \alpha]\ [R\ \beta]\ [R\ \gamma]\ \{Fe\} \qquad 3.3.10$$

or $$\{F\} = [R\ M]\{Fe\} \qquad 3.3.11$$

where $[R\ M] = [R\ \alpha]\ [R\ \beta]\ [R\ \gamma]$

or

$$[R\ M] = \begin{bmatrix} -\cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & -\cos\alpha & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -\cos\beta & 0 & -\sin\beta \\ 0 & -1 & 0 \\ \sin\beta & 0 & -\cos\beta \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

3.3.12

So far, only sets of three orthogonal forces (excluding moment) have been considered. But generally for a space frame, and the beam element under investigation, there is the set of three orthogonal moment also acting on the beam element. These moments will act about the orthogonal axes.

With the strict adherence made so far to the convention that a positive moment is a clockwise moment when viewed along its axis, the operations described above for transformation of forces will also be applicable to moments transformation. Thus equations 3.3.10 and 3.3.11 also hold true for moments.

Let Q_1, Q_2, Q_3, Q_4, Q_5, Q_6 be the forces (and moments) acting on the beam element at the joint A in the frame coordinate system.

And Q_7, Q_8, Q_9, Q_{10}, Q_{11}, Q_{12} be the forces (and moments) acting on the beam element at the other joint B in the frame coordinate system.

Also, let Fe_1, Fe_2,.....Fe_6 and Fe_7, Fe_8,.....Fe_{12} be the other set of forces (and moments) acting on the beam element at the joint A and B respectively in the beam element coordinate system.

Then the 12 equilibrium equations relating actions on the element in the frame coordinate system and the beam element coordinate system can be seen to be similar to the results obtained above. In fact, the transformation in matrix terms is given by:

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{12} \end{Bmatrix} = \begin{bmatrix} [R\ M] & [Z] & [Z] & [Z] \\ [Z] & [R\ M] & [Z] & [Z] \\ [Z] & [Z] & [R\ M] & [Z] \\ [Z] & [Z] & [Z] & [R\ M] \end{bmatrix} \begin{Bmatrix} Fe_1 \\ Fe_2 \\ \vdots \\ Fe_{12} \end{Bmatrix} \qquad 3.3.13$$

$$\text{where} \qquad [Z] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad 3.3.14$$

and from equation 3.3.12

$$[R\,M] = \begin{bmatrix} -\cos\alpha.\cos\beta, & \sin\alpha.\cos\gamma - \cos\alpha.\sin\beta.\sin\gamma, & -\sin\alpha.\sin\gamma - \cos\alpha.\sin\beta.\cos\gamma \\ -\sin\alpha.\cos\beta, & -\cos\alpha.\cos\gamma - \sin\alpha.\sin\beta.\sin\gamma, & \cos\alpha.\sin\gamma - \sin\alpha.\sin\beta.\cos\gamma \\ \sin\beta, & -\cos\beta.\sin\gamma, & -\cos\beta.\cos\gamma \end{bmatrix}_{(3\times 3)} \qquad 3.3.15$$

Equation 3.3.13 is of the form

$$\{Q\} = [R]\{F_e\} \qquad 3.3.16$$

$$[R] = \begin{bmatrix} [R\,M] & [Z] & [Z] & [Z] \\ [Z] & [R\,M] & [Z] & [Z] \\ [Z] & [Z] & [R\,M] & [Z] \\ [Z] & [Z] & [Z] & [R\,M] \end{bmatrix} \qquad 3.3.17$$

All force transformations discussed so far also hold exactly for displacements. Thus, if the corresponding displacements of the beam element in frame coordinate system are denoted by $q_1, q_2, \ldots.. q_{12}$, then equation 3.3.16 can be written for the displacement as follows

$$\{q\} = [R]\{u\} \qquad 3.3.18$$

where

$\{u\}^t = [u_1\ u_2\ u_3, \ldots.. u_{12}]$, as defined in section 3.2

represents the displacements of the beam elemented in the beam element coordinate system.

Now, the strain energy of the beam element is given by

$$U = \{u\}^t[k_e]\{u\} \qquad 3.3.19$$

It is worth noting that the transformation matrix [R] is an orthogonal one. Thus, its inverse is equal to its transpose. Hence, from equation 3.3.18, we have

$$\{u\} = [R]^{-1}\{q\}$$

$$= [R]^{t}\{q\} \qquad 3.3.20$$

And substituting equation 3.3.20 into equation 3.3.19 we have, that the strain energy of the beam element is given by

$$U = \{q\}^{t}[R][k_e][R]^{t}\{q\} \qquad 3.3.21$$

Equation 3.3.21 is of the form

$$U = \{q\}^{t}[k]\{q\} \qquad 3.3.22$$

which is an expression of the strain energy of the beam element in terms of the displacements q_i (i = 1,...12) in the frame coordinate system. The matrix [k] is a 12 x 12 matrix and it is the stiffness matrix of the beam element in the frame coordinate system. From equations 3.3.21 and 3.3.22, it can be deduced that

$$[k] = [R][k_e][R]^{t}$$

where the matrix $[k_e]$ is given by equation 3.2.47.

Similarly the kinetic energy of the beam element is given by

$$T = \{\frac{\partial u}{\partial t}\}^{t}[m_e]\{\frac{\partial u}{\partial t}\} \qquad 3.3.24$$

Substituting equation 3.3.20 into equation 3.3.24, we have, that the kinetic energy of the beam element is given by

$$T = \{\frac{\partial q}{\partial t}\}^{t}[R][m_e][R]^{t}\{\frac{\partial q}{\partial t}\} \qquad 3.3.25$$

Again, this is of the form

$$T = \{\frac{\partial q}{\partial t}\}^t [m]\{\frac{\partial q}{\partial t}\} \qquad 3.3.26$$

which is an expression of the kinetic energy of the beam element in terms of the displacements $'q_i$ (i = 1,2,....12) in the force coordinate system.

Comparing equations 3.3.25 with 3.3.26 we have

$$[m] = [R][m_e][R]^t \qquad 3.3.27$$

where the matrix $[m_e]$ is given by equation 3.2.48.

The matrix [m] is a 12 x 12 matrix which represents the mass matrix of the beam element in the frame coordinate system.

3.4 Assembly of system mass and stiffness matrices

The mass and stiffness matrices obtained after the coordinate transformation, express the beam element properties in terms of the system (or global) coordinate system. These need to be assembled into the overall matrices for the frame. Thus the contribution of the beam element in question to the frame mass and stiffness matrices are to be identified and added accordingly. The code number method is utilised here.

The transformed beam element matrices are each 12 x 12 matrices. The first 6 rows or columns of these matrices are related to the frame coordinates at the end A of the beam, which the other 6 (7 - 12) rows or columns are related to frame coordinates at the other end B of the beam element. Thus, the matrices are such that the rows and columns 1,2 and 3 relate to the translated displacement components in the X-, Y-, and Z-directions of the frame axis system at the end A. The rows and columns 4, 5 and 6 relate to the rotational displacement components about the X-, Y-, and Z-axes of the frame axis system at the end A. Similarly, the rows and columns 7, 8 and 9 relate to the translational displacement components in the X-, Y- and Z-directions of the frame axis system at the end B. And the rows and columns 10, 11 and 12 relate to rotational displacement components about the X-, Y-, and X-axes of the frame axes system at the end B. The idea of the code number method is to assign to each of these 12 beam element matrix rows and columns, a number which represents the corresponding frame coordinate at those points.

Each of the 12 beam element coordinates should have a corresponding coordinate in the frame coordinate system. Any element coordinate which does not contribute to the frame coordinate system is assigned a zero code number. All other element coordinates are given code numbers equal to the value of the coordinate in the frame coordinate system. Thus the code number at any point is a positive (including zero) integer not greater than the total number of degrees of freedom of the discretized frame structure. It is worth noting that the inclusion of the zero code number makes it possible to analyse 1-dimensional and plane frame structures from the general 3-dimensional beam finite element discretization model.

As an example, suppose a beam element connects two points A and B of a frame structure as shown in fig. 3.4.1. The frame coordinates are as shown at the two ends. After the appropriate transformation, a 12 x 12 matrix is obtained for both the mass and stiffness matrices of the beam element. The code numbers for the addition of the beam element contribution to the frame mass and stiffness can easily be written down as follows:

local coordinate	1	2	3	4	5	6	7	8	9	10	11	12
frame coordinate	18	8	20	0	0	7	0	4	2	0	1	3

The 12 integer values representing the frame coordinates constitute the code numbers for the proper assembly of the beam element contribution to the frame mass and stiffness matrices. Note that repeated code numbers (except zeros) for the same beam element is meaningless and wrong.

In the above example, the frame coordinate 18 corresponds to the transformed beam coordinate 1. Thus the assembly routine is such that all beam matrix elements on the rows and columns 1 are to be added to the appropriate frame matrix elements on the rows and columns 18. A progressive addition is done in this manner from the rows and columns 1 to 12 of the beam matrix elements marching with the given code numbers. Zero code numbers imply no beam matrix element contribution at that point and hence no addition.

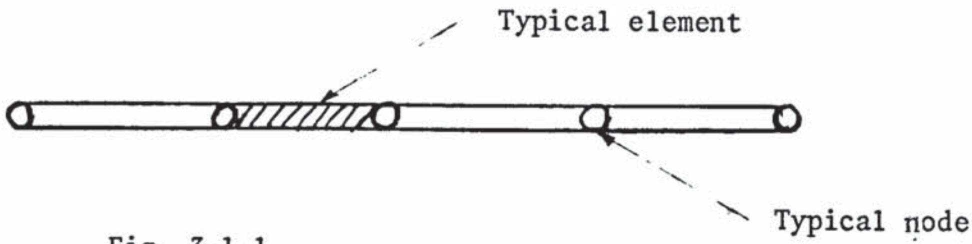


Fig. 3.1.1

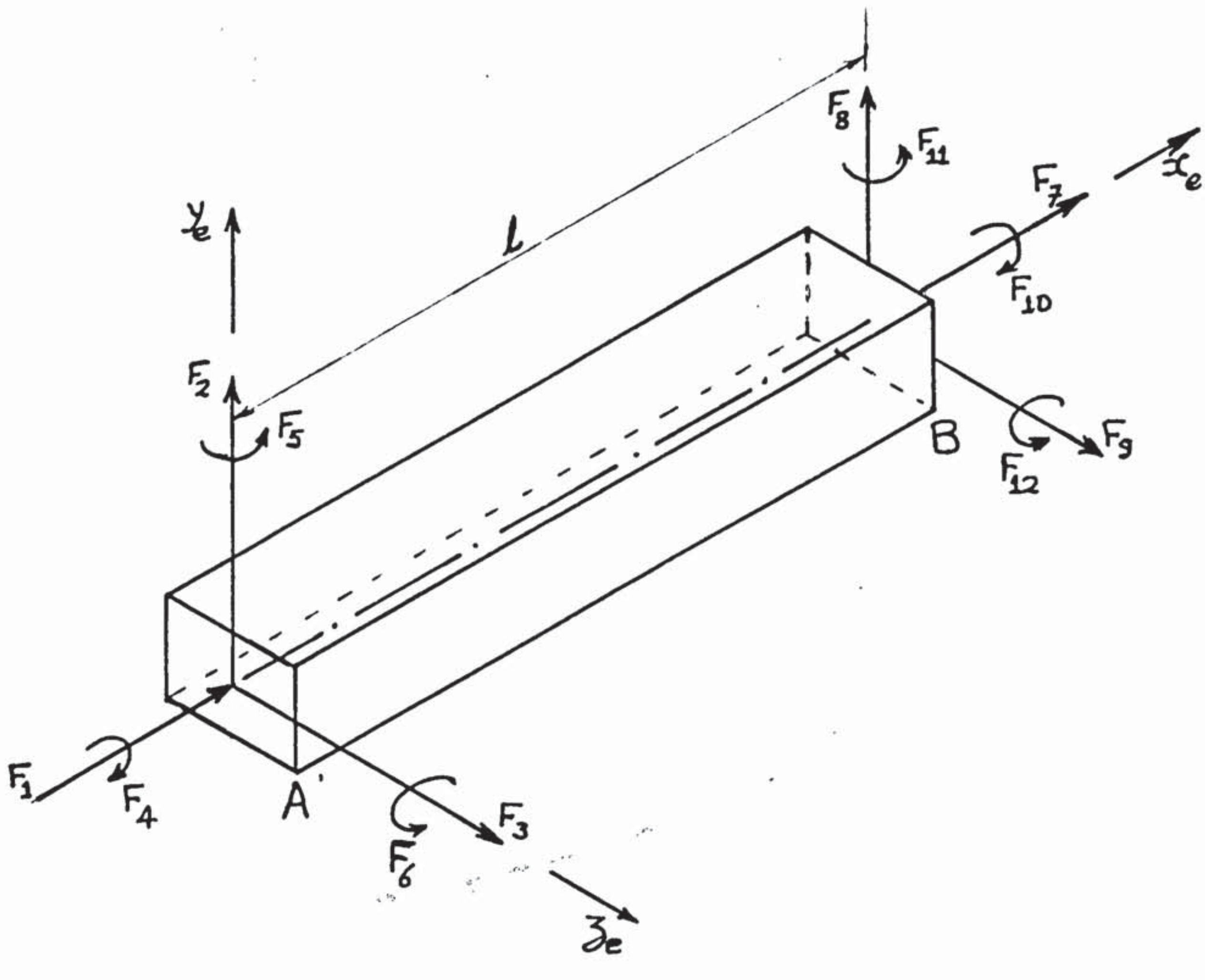


Fig. 3.2.0

Beam Element

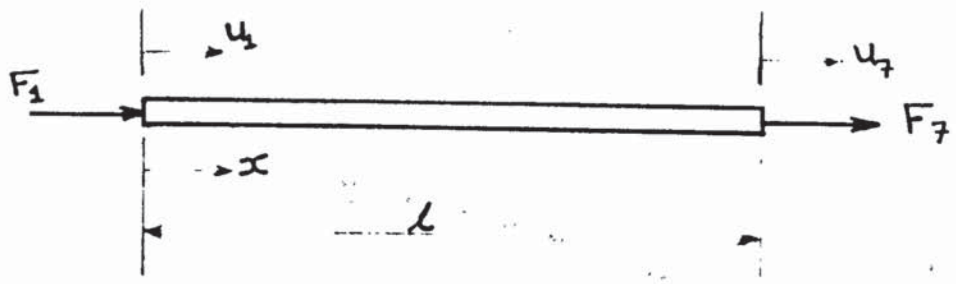


Fig. 3.2.1

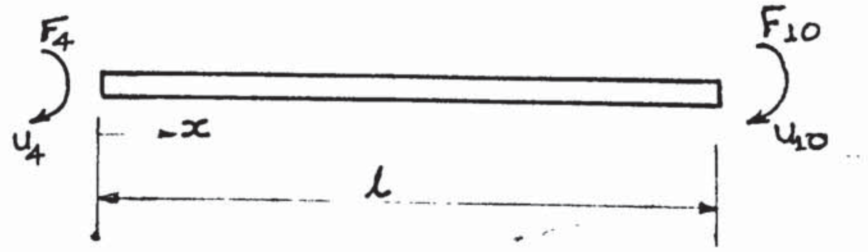


Fig. 3.2.2

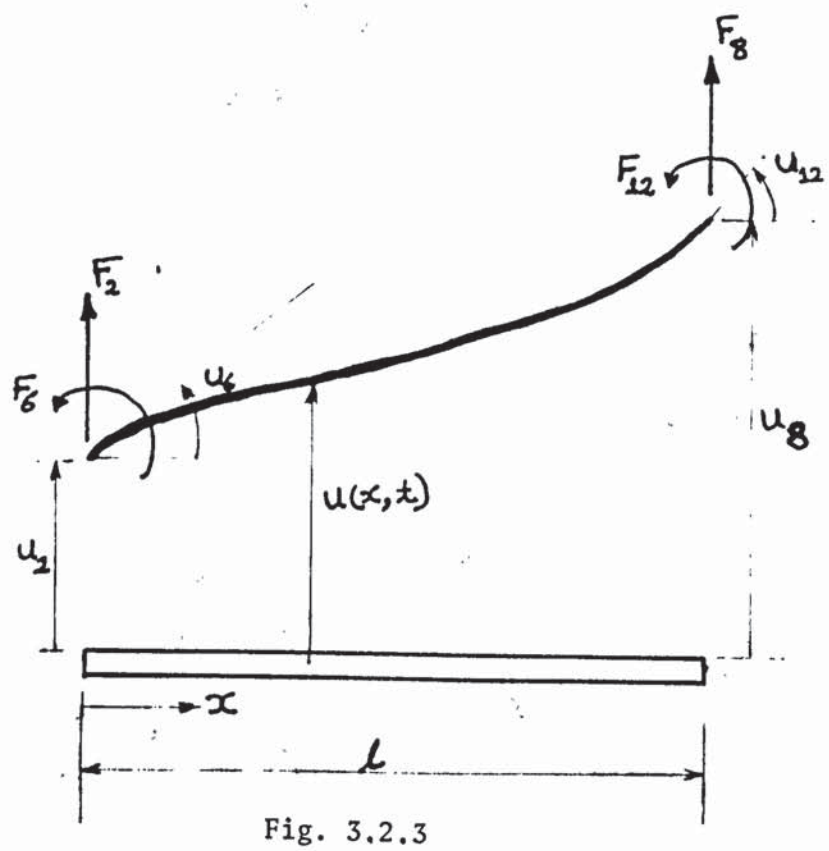


Fig. 3.2.3

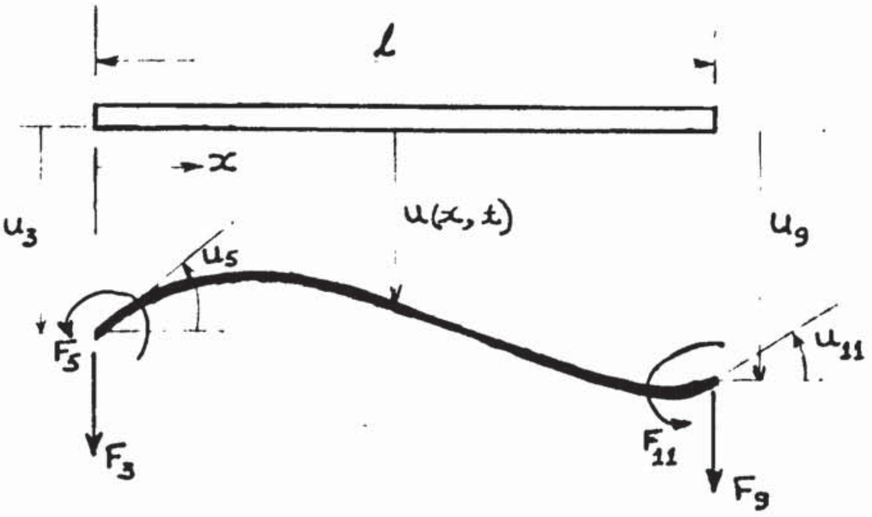


Fig. 3.2.4

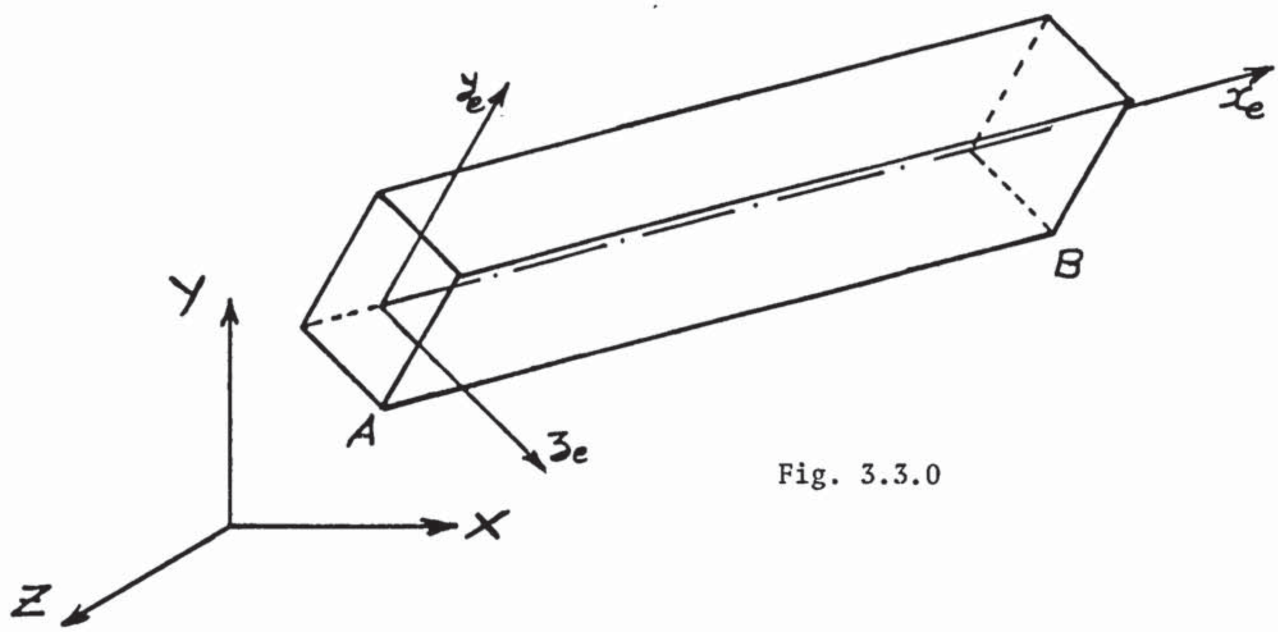


Fig. 3.3.0

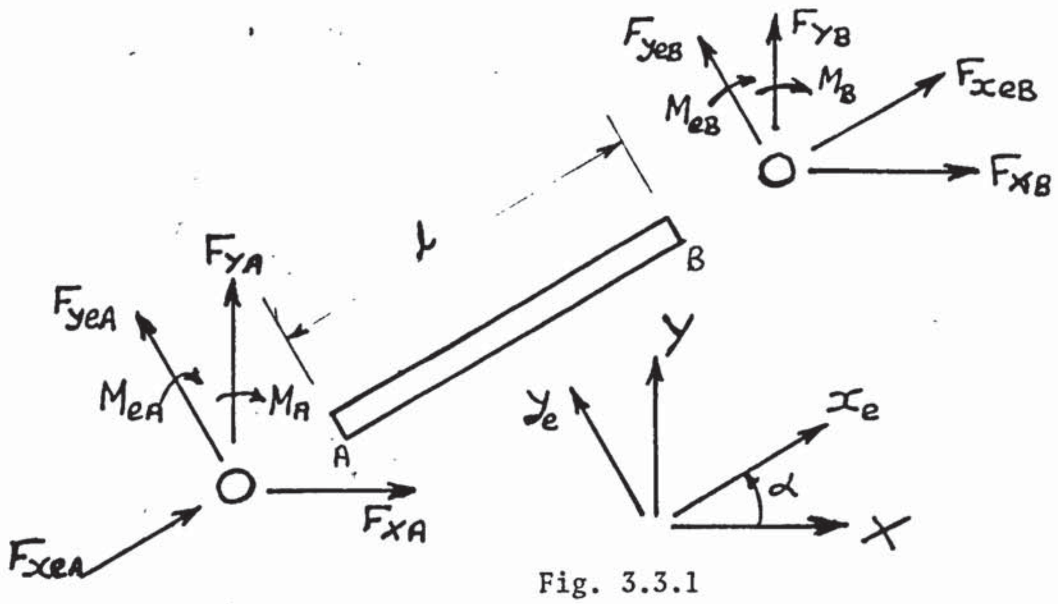
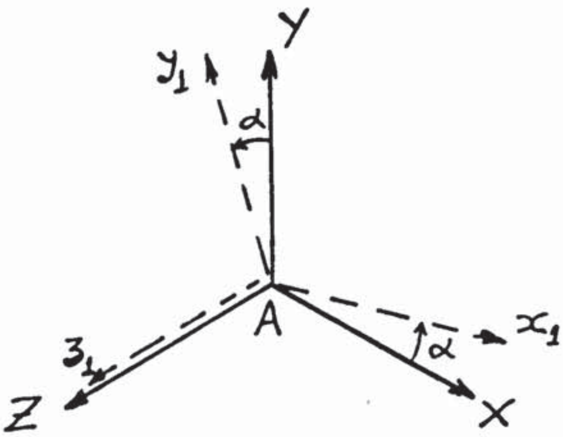
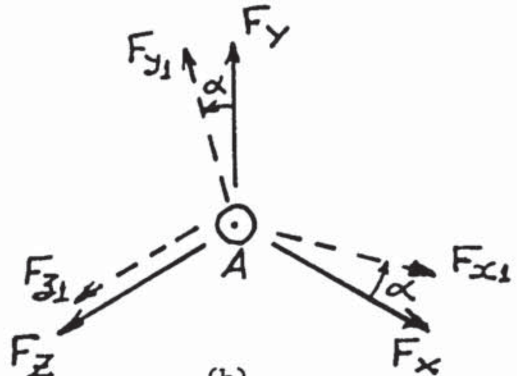


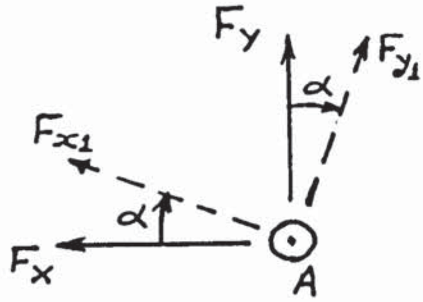
Fig. 3.3.1



(a) Rotation α about Z-axis



(b)

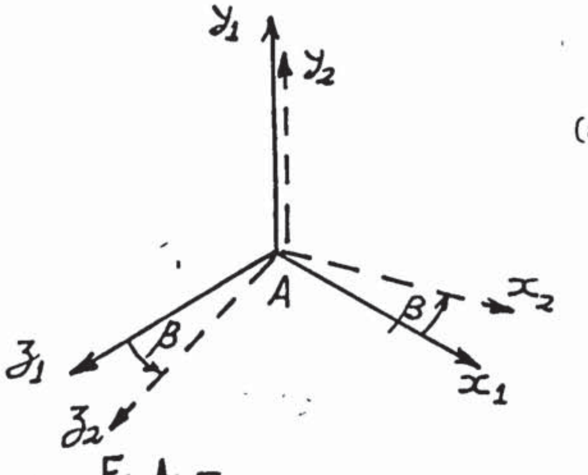


(c)

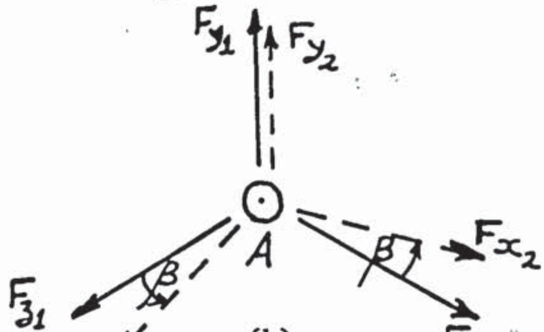
Forces in equilibrium

View along + z direction

Fig. 3.3.2

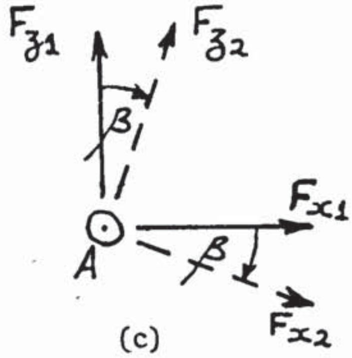


(a) Rotation β about y_1-axis



(b)

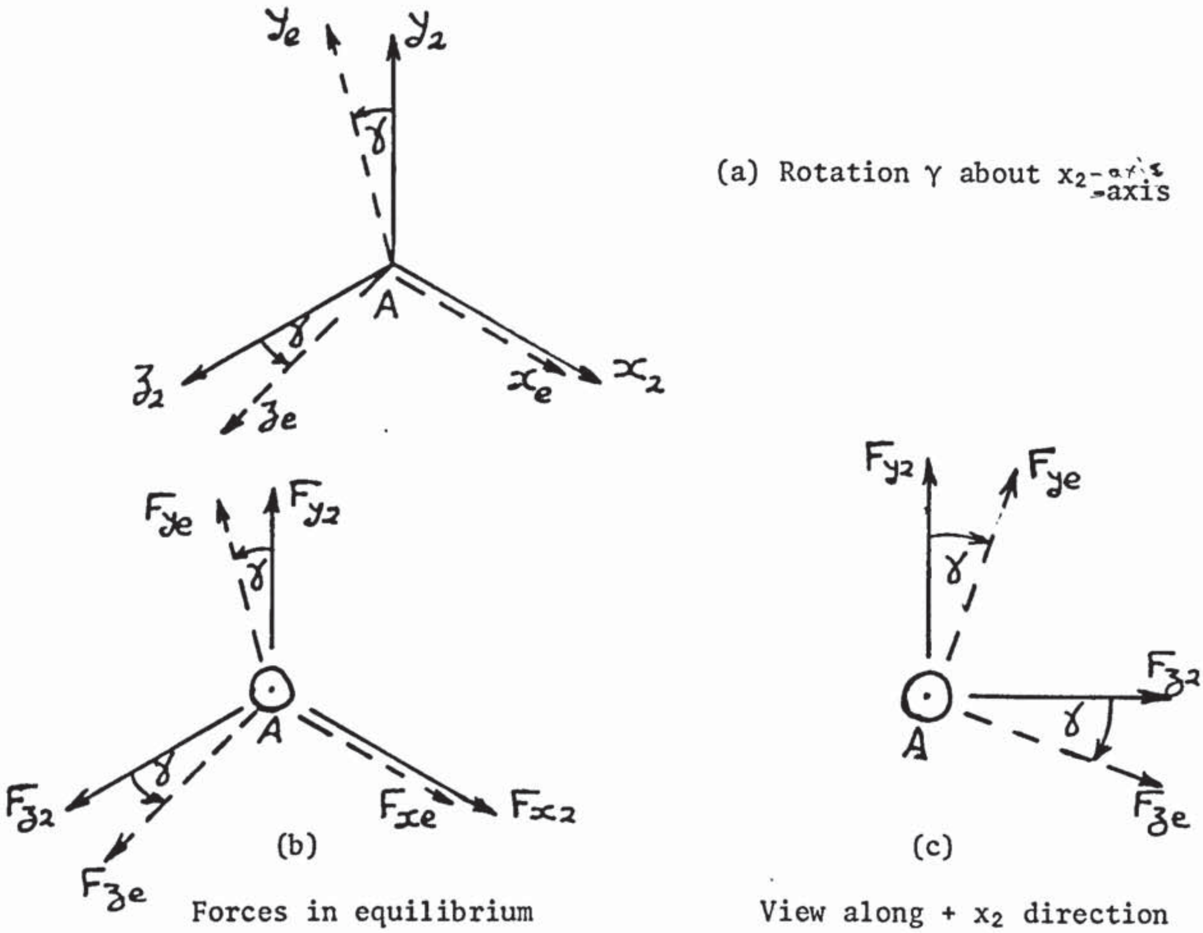
Forces in equilibrium



(c)

View along + y_1 direction

Fig. 3.3.3.



(a) Rotation γ about x_2-axis

(b) Forces in equilibrium

(c) View along + x_2 direction

Fig. 3.3.4

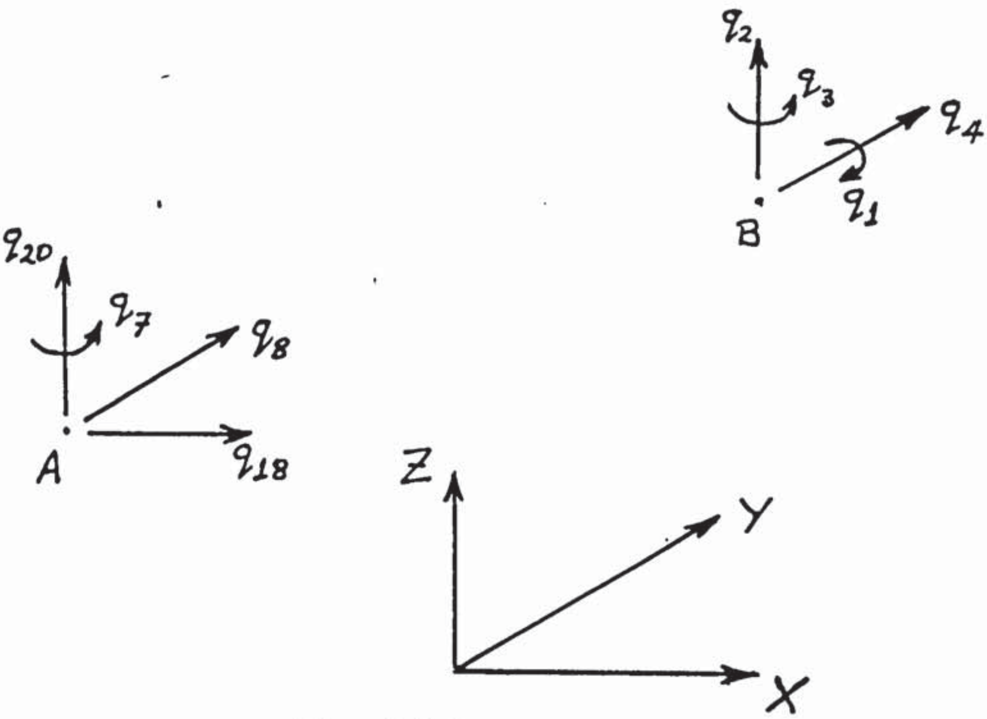


Fig. 3.4.1

CHAPTER FOUR

CHAPTER FOUR

DEVELOPMENT OF COMPUTER PROGRAMMES

4.1 Introduction

The digital computer is a very useful modern machine which lends itself to the perform prodigious feats of arithmetic calculations in small fractions of a second. To this end, it has become an indispensable tool to the scientist in general and the engineer in particular.

A proper use of the computer lies in ones ability to translate a problem into simple repeated steps of operations in a form which lends itself to the mode of working of the computer. In order to perform a particular job, the computer must be fed with the set of numbers to operate upon (data) and the set of operations required (programme).

In this work, as always, the theories and processes of solution of our problems have to be well represented in the form of computer programmes and data for the correct results to be obtained. In the previous chapter the theories for obtaining the mass and stiffness matrices of a beam element have been developed. The eventual goal of obtaining a computational dynamic analysis of a space frame structure can then be achieved through the sensible use of the digital computer.

Three main computer programmes have been developed in this work. The first of these programmes is called MASSTIFPROP. As the name tends to suggest, it produces the overall mass and stiffness matrices (properties) of a framed structure when supplied with the suitable properties of the constituent beam elements. The results of the last chapter are widely used in this programme.

Although the aim of this work is the dynamic analysis of structures, it has been found that static consideration becomes useful in the process. Recalling equation 3.1.1 (section 3.1) the equation of vibrating motion of a discretized body in matrix form is

$$[M]\{\ddot{q}\} + [K]\{q\} = \{Q\}$$

In statics, the acceleration term $\{\ddot{q}\}$ equals a zero vector $\{0\}$. Thus the static equilibrium equation is of the form

$$[K]\{q\} = \{Q\} \qquad 4.1.1$$

This represents a set linear algebraic equations from which the displacements $\{q\}$ can be solved for any given force vector $\{Q\}$. Here, a programme called STATICPROB has been developed to solve these systems of linear algebraic equations.

Thirdly, the programme NAGFEIGNVAL solves the eigenvalue problem of the form given by equation 3.1.3 (section 3.1) namely

$$[K]\{u\} = \omega^2[M]\{u\}$$

The results from this programme include the natural frequencies (eigenvalues) and normal modes (eigenvectors) of the system under consideration. The eigenvectors could be plotted to obtain a pictorial view of the modes of free vibration of the structure.

It is to be remarked that the discretization of any structure by finite elements usually lead to large order overall mass and stiffness matrices. Most modern digital computers are capable of working with large order matrices. But even so, the largest computers available have limited sizes hence they have limits to the size of matrices that they can input into them.

Space frames are particularly prone to producing very large order matrices in finite element discretizations. To reduce the effective core requirement of the computer programme, it is useful to reduce the number and size of the arrays declared in the programme to the bearest minimum.

One useful property of the mass and stiffness matrices of a real structure is symmetry. Thus, no data is lost by storing only the matrices as only upper or lower triangular matrices. The corresponding strict lower or upper triangular matrices then form useful core spaces for other data in the programme.

Furthermore, the overall mass and stiffness matrices of a structure are generally sparse matrices. But through intelligent numbering of the nodes (and nodal displacement coordinates) the actual matrices can be made to become band matrices with as little band width as possible (refs. 1 and 5). For efficient use of the computer, advantage should be taken of the banded nature of the matrices.

In writing the computer programmes for this work, the factors discussed above have been put into advantageous use wherever possible. But in some cases, it has not been practicable to actually save computer core by using these matrix properties. Hence, inevitable need for large core storage still exists. This in turn has placed a very high limitation on the number of degrees of freedom of structures which could be analysed on the available computer.

All the computer programmes described in this chapter are written in the Extended FORTRAN language. They have been developed on an ICL 19045 computer which is in operation at the University of Aston Computer Centre. The computer is controlled by the GEORGE 3 operating system.

The computer system carries a large library of standard routines which can be called for the specific data manipulations. A particular library on this computer is the NAG FORTRAN LIBRARY (NAG here means the NOTTINGHAM ALGORITHMIC GROUP). Among the many routines with this NAG Fortran Library are a large variety of them designed for matrix manipulations.

Some of these routines have been used in two of the computer programmes written in this work namely STATICPROB and NAGFEIGNVAL

4.2 Mass and Stiffness matrices programme

The computer programme called into play here is named MASSTIFPROP. This is a general programme which assembles the mass and stiffness matrices of a space frame structure. Strictly speaking, the frame structure does not have to be a 3-dimensional one. Plain frames as well as one-dimensional beams are also analysed by this programme.

MASSTIFPROP assumes that the structure is made up of similar uniform beam elements (all of the same cross section and elastic properties) with a total of 12 degree-offfreedom per beam element as discussed in Chapter 3. The 12 x 12 matrices of equations 3.2.47 and 3.2.48 (section 3.2.5), for the beam finite element stiffness and mass matrices respectively, are used for the analysis. If required, only a light modification of the programme is needed to adapt it for frame structures with uniform beam elements of different elastic constants and cross sections.

The beam finite element properties are transformed from the element (local) coordinate system into the frame (global) coordinate system. Through the use of the code method, the contribution of the beam finite element is added to the appropriate coordinates in frame coordinate system.

The fact that both stiffness and mass matrices are symmetric have been put into consideration in the assembled matrices for the frame structure. Both matrices have been stored as upper and lower triangular matrices in a rectangular array. Thus computer core storage has been minimized.

The programme also incorporates the addition of concentrated mass and stiffness properties to any coordinate of the frame. Thus joint masses (and other masses) can be added to the appropriate elements of the mass matrix, while the effect of spring stiffness can be added to the corresponding elements of the frame stiffness matrix.

4.2.1 Programme MASSTIFPROP

A flow chart of MASSTIFPROP is shown in fig. 4.2.1. Appendix A1 contains a compiled listing of the programme.

The method of modular programming has been used here and the flow chart is actually a modular flow chart. Modular programming implies a formal use of segmentation of a computer programme. In a Fortran programme, this involves the use of a minimum amount of the programme logic in the MASTER or main segment. The master segment, therefore, consists mainly of data areas, loops and call statements. The call statements refer to subroutine segments which carry out the details of the problem working.

As shown in the flow chart, the first segment in the programme is the Programme Description Segment. Channel numbers are here associated with specific unit numbers of the computer peripherals. This allows the possibility of reading the input data from various files (large input data are envisaged) and also writing the output mass and stiffness matrices onto computer files for use by latter programmes.

Array variables and other variables are next declared in a common statement, since these variables are used in the called subroutine segments. The most important variable here is the array ASSEMMATRIX. This is a rectangular array whose dimensions should at least consist of one plus the "number-of-degrees-of-freedom" of the frame rows and "number-of-degrees-oflfreedom" (NDOF) columns. The assembled mass and stiffness matrices are then held in this array as lower and upper triangular matrices respectively. Thus the stiffness matrix is held as an upper triangular matrix of the array ASSEMMATRIX starting with the element ASSEMMATRIX (1,1) through to the element ASSEMMATRIX(NDOF,NDOF). And the mass matrix is held as a lower triangular matrix of the array ASSEMMATRIX starting with the element ASSEMMATRIX(2,1) through to the element ASSEMMATRIX (NDOF+1,NDOF).

The number of problems to be analysed is read in to create a loop all through the remainder of the programme. This enables any number of systems of framed structure to be analysed in one run of the programme.

The channel number of the data input and output files are also read from the input card. From now on, all data read statements refer to the stated input channel number.

Other data elements are then read. These include the number of degrees of freedom of the frame structure, the number of beam finite elements of the discretized frame, the Young's and Rigidity modulii of the component beam elements of frame, the mass per unit length of the beam, the second moments of area of the beam element cross-section about its x-, y-, and z-axis, and the area of cross-section of the beam element. These data are also printed out on paper for checking purposes.

The subroutine segment ZEROASSEM is called to initialize the array ASSEMMATRIX. ZEROASSEM set all elements of the ASSEMMATRIX array equal to zero ready for subsequent adding of the contribution from each beam finite element to the frame mass and stiffness matrices. Thus subroutine also sets to zero two other arrays needed in a latter part of the programme.

The element code read should have values of 0, 1 or 2. A value of 0 implies that the next beam finite element has both length and orientation to the frame coordinate system different from the last element considered. Thus the programme continues with the next statement. A value of 1 implies that the next beam finite element has the same length as the previous element but at a different orientation. Thus the programme should skip the next statement which is a call statement. A value of 2 implies that the next beam finite element has the same length and orientation as the last element considered. Hence, the programme can skip the next two call statements.

Subroutine ELEMENT reads the length of the beam finite element and calculates the element mass (EMASS) and stiffness (ESTIF) matrices as 12 x 12 arrays from the equations 3.2.48 and 3.2.47 respectively of subsection 3.2.5.

Subroutine DIRECTN reads the coordinate transformation angles α, β and γ (section 3.3) and transforms the element mass and stiffness matrices from the element coordinate system to the frame coordinate system using equations 3.3.27 and 3.3.23. The subroutine calls another subroutine TRANSFORM, which produces the transformation matrix [R M] using equation 3.3.15.

Subroutine ASSEMBLE assembles (or added) the contribution of the beam finite element properties to the system mass and stiffness matrices. This is where the code number method is utilised.

The beam finite element length and transformation angles α, β, γ are also printed on paper as a checking procedure. If all finite elements of the discretized frame have not been covered, programme control goes back to read the element code for the next beam finite element.

Subroutine POINTPROP takes care of the presence of point or concentrated mass and stiffness properties in the nodes of the frame structure. It reads the row and column coordinates as well as the corresponding value of the mass and/or stiffness and adds the values to the appropriate elements of the assembled matrices.

Subroutine STOREMATX writes the rectangular array ASSEMMATRIX into a specified file for that purpose. This is the file from which future progtammes (STATICPROB and NAGFEIGNVAL) will read the mass and stiffness matrices of the structure. It can also be listed on paper for checking purposes. But this should be avoided as much as possible except in test cases where the mass and stiffness matrices are of very small order of magnitude. In large structures where the number of degrees of freedom runs into a hundred or more, large amounts of paper will be required to list the matrices.

At this point in the programme the loop for the number of problems to be analysed is completed. This multiple problem loop is useful where it is required to analyse the same frame structure with varying degrees of finite element refinement. It is therefore possible to make an immediate comparison of results of the analysis of a structure with increasing amount of refinement on the finite element discretization, and to see if the results justify the increased work in solving the structure with higher degrees of freedom.

4.2.2 Programme MASSTIFPROP-Input Data

A total of up to 12 input data card types go to produce a complete combination of input data for successful execution of the programme MASSTIFPROP. All, but the first card type, could appear more than once, in any one job.

The required input data are summarised below.

Card Type 1

Columns 1 - 5 Integer I5 - Number of problems

Card Type 2.

Columns 1 - 5 Integer I5 - Input file channel number

= 1, 8, 9, 10, 11 or 12.

Columns 6- 10 Integer I5 - Output file channel number

= 3, 3, 4, 6 or 7.

Card Type 3.

Columns 1 - 5 Integer I5 - Number of degrees of freedom of the structure.

Columns 6 - 10 Integer I5 - Number of beam finite elements in the structure.

Card Type 4.

Columns 1 - 10 Real F10.2 - Young's Modulus of elasticity of the beam element material N/mm^2.

Columns 11 - 20 Real F10.2 - Modulus of Rigidity of the beam element material N/mm^2

Columns 21 - 30 Real F10.7 - Mass per unit length of beam element Kg/mm.

Cart Type 5.

Columns 1 - 10 Real F10.2 - Second polar moment of area of the beam cross section mm^4

Columns 11 - 20 Real F10.2 - Second moment of area of the beam cross section about the y-axis mm^4

Columns 21 - 30 Real F10.2 - Second moment of area of the beam cross section about the z-axis mm^4

Columns 31 - 40 Real F10.2 - Area of cross section of the beam element mm^2

Card Type 6.

Columns 1 - 3 Integer I3 - Element code = 0, 1 or 2.

Card Type 7.

Columns 1 - 10 Real F10.2 - Length of beam finite element mm.

Card Type 8.

Columns 1 - 10 Real F10.2 - Rotation angle α degrees

Columns 11 - 20 Real F10.2 - Rotation angle β degrees

Columns 21 - 30 Real F10.2 - Rotation angle γ degrees

Card Type 9.

Columns	1- 5	Integer I5	-	Code number	for	finite	element	coordinate	1.
"	6-10	Integer I5	-	Code number	for	finite	element	coordinate	2.
"	11-15	"	-	"	"	"	"	"	3.
"	16-20	"	-	"	"	"	"	"	4.
"	21-25	"	-	"	"	"	"	"	5.
"	26-30	"	-	"	"	"	"	"	6.
"	31-35	"	-	"	"	"	"	"	7.
"	36-40	"	-	"	"	"	"	"	8.
"	41-45	"	-	"	"	"	"	"	9.
"	46-50	"	-	"	"	"	"	"	10.
"	51-55	"	-	"	"	"	"	"	11.
"	56-60	"	-	"	"	"	"	"	12.

Card Type 10.

Columns 1 - 5 Integer I5 - Number of point masses

Columns 6 - 10 " I5 - " " " stiffnesses

Card Type 11.

Columns 1 - 5 Integer I5 - Row number of point mass.

" 6 - 10 " I5 - Column number of point mass.

" 11 - 20 Real F10.4 - Value of point mass Kg.

Card Type 12.

Columns 1 - 5 Integer I5 - Row number of point stiffness

" 6 - 10 " I5 - Column number of point stiffness.

" 11 - 20 Real E10.3 - Value of point stiffness N-mm.

4.2.3 Input Data Example

It will be useful to give an illustration of the input data required for the analysis of a frame structure. Fig. 4.2.4(a) shows a simple space frame structure which is to be analysed with the aid of programme MASSTIFPROP.

All beam members are of the square cross section as shown in fig. 4.2.4(b). The masses (m_1 and m_2) at the two joints represent concentrated masses. A linear spring k_1 is also present and it represents an axis connection of the beam member to a fixed base.

The materials of the frame beam components are the same. And it is to be assumed that all free joints of the frame each have six degrees of freedom - three linear and three rotational. The two feet fixings each have zero degrees of freedom, while the spring fixed end has four degrees of freedom - one linear and 3 rotational.

Fig. (c) and (d) of fig. 4.2.4 show two finite element discretizations of the frame structure. Fig. 4.2.4(c) represents a 4 element (3 nodes) discretization as a first approximation analysis. Fig. 4.2.4 (d) represents an 8 element (7 nodes) discretization as a second approximation analysis of the frame structure. Hence the former is a 16 D.O.F. system and the latter is a 40 D.O.F. system.

Below are the essential figures for obtaining the required data for the analysis of the structure:

Young's Modulus of beam material E	$= 207\ kN/mm^2$
Rigidity " " " " G	$= 80\ kN/mm^2$
Mass per unit length of beam m_ℓ	$= 3.08 \times 10^{-3} kg/mm$
Polar second moment of area of beam cross section I_x	$= 26600\ mm^4$
Second moment of area of beam cross section about the y_e-axis I_y	$= 13300\ mm^4$
Second moment of area of beam cross section about the z_e-axis I_z	$= 13300\ mm^4$
Area of cross section of the beam A	$= 400\ mm^2$
Mass m_1	$= 1.25$ kg
Mass m_2	$= 0.75$ kg
Spring constant k_1	$= 7 \times 10^6\ N/mm^2$

The input data required for the analysis of the above two finite element discretization models of the frame structure is as shown on the following two pages of computer card data layout sheets - Table 4.2.1.

Note that it has been assumed that the mode of input of these data to the computer programme MASSTIFPROP is via the card reader and that the resulting mass and stiffness for the 16 D.O.F. and 40 D.O.F. systems are required to be stored on files assigned to channel numbers 3 and 4 respectively.

Table 4 · 2 · 1

```
    2
    1    3
   16    4
207000.00 80000.00 0.0030800
 26600.00 13300.00  13300.00      400.00
 0
   800.00
    90.00     0.00      0.00
    0    0    0    0    0    0    1    2    3    4    5    6
  2
    0    0    0    0    0    0    7    8    9   10   11   12
 0
   750.00
     0.00     0.00      0.00
    1    2    3    4    5    6    7    8    9   10   11   12
 1
     0.00    90.00      0.00
    7    8    9   10   11   12    0    0   13   14   15   16
    6    1
    1    1   1.2500
    2    2   1.2500
    3    3   1.2500
    7    7   0.7500
    8    8   0.7500
    9    9   0.7500
   13   13 7.000E+06

    1    4
   40    8
207000.00 80000.00 0.0030800
 26600.00 13300.00  13300.00      400.00
```

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```
    0
    400.00
     90.00     0.00      0.00
    0    0    0    0    0    0    1    2    3    4    5    6
  2
    1    2    3    4    5    6    7    8    9   10   11   12
  2
    0    0    0    0    0    0   19   20   21   22   23   24
  2
   19   20   21   22   23   24   25   26   27   28   29   30
  0
    375.00
      0.00     0.00      0.00
    7    8    9   10   11   12   13   14   15   16   17   18
  2
   13   14   15   16   17   18   25   26   27   28   29   30
  1
      0.00    90.00      0.00
   25   26   27   28   29   30   31   32   33   34   35   36
  2
   31   32   33   34   35   36    0    0   37   38   39   40
    6    1
    7    7    1.2500
    8    8    1.2500
    9    9    1.2500
   25   25    0.7500
   26   26    0.7500
   27   27    0.7500
   37   37 7.000E+06
****
```

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4.3 Static analysis programme

The idea of the static analysis of the frame structure arose at some stage in the work when it was becoming more and more difficult to understand the reason(s) for the unsatisfactory result of the dynamic analysis and experimentations. The practical means of the verification of the dynamic results should involve testing the accuracy of the stiffness matrix of the structure as obtained by computer programme MASSTIFPROP. This point will be discussed in detail in Chapter Six. But for the moment, it should be sufficient to note that the problem required to be solved is a static one.

In the static analysis of a structure, the problem involves the solution of a system of linear equations whose terms include the stiffness coefficients, structural displacements and the applied force(s). In matric terms, the system of linear equations is represented by equation 4.1.1 (section 4.1) namely

$$[K]\{q\} = \{Q\}$$

where [K] is the stiffness matrix of the structure.

{q} is the column matrix representing the nodal displacement of the structure

and {Q} is the column matrix representing the nodal force vector acting on the structure.

Knowing the applied force vector {Q}, it is usually required to solve for the resulting displacement vector {q}. Here, the computer programme STATICPROB has been developed to help solve this problem of statics.

4.3.1 Programme STATICPROB

A full compiled listing of this programme is shown in Appendix A.2. With the aid of the NAG Fortran Library, the computer programme STATICPROB, is much simpler. What the programme needs is the assembled stiffness matrix of the structure as output by the programme MASSTIFPROP plus the non zero elements of the force vector {Q} representing the magnitude of the applied force(s) acting on the structure and specified in the frame (or global) system of coordinates.

The NAG-Fortran Library routine FO4ASF is called to solve for the displacements. In its documentation, it is described that FO4ASF calculates the accurate solution of a set of real symmetric positive definite linear equations with a single right hand side (of the form [K]{q} = {Q}, by Cholesky's decomposition method.

Given the set of linear equations, [K]{q}= {Q}, the routine FO4ASF uses Cholesky's method to decompose [K] into triangular matrices such that

$$[K] = [L][L]^T$$

where [L] is lower triangular.

An approximation to {q} is found by forward and backward substitution The residual vector

{r} = {Q} - [A]{q} is then calculated and a correction, {d} to {q} is found by the solution of $[L][L]^T\{d\} = \{r\}$. The vector {q} is then replaced by ({x} + {d}) and the process repeated until full machine accuracy is obtained.

The programme STATICPROB prints out the two vectors - force {Q} and displacement {q} - as two separate vectors along side each other.

Here also, the programme can be used to solve more than one set of matrix equations. It is possible to solve for the displacement vectors using the same stiffness matrix [K] but different combinations of the force vector {Q}. The programme has its own Programme Description Segment to allow the use of multiple files as in programme MASSTIFPROP. Thus several problems with different stiffness matrices [K] can be solved in one run of the programme.

4.3.2 <u>Programme STATICPROB-Input Data</u>

The input data cards for this programme are relatively simple and few. They include.

Card Type 1

Columns 6 - 10 Integer I5 - Number of problems.

Card Type 2.

Columns 6 - 10 Integer I5 - Problem Indicator = 0 or 1.

Columns 11 - 15 Integer I5 - Input file channel number = 3, 4, 5, 6, 7 or 8.

Card Type 3.

Columns 6 - 10 Integer I5 - Number of non-zero elements in the force vector.

Card Type 4.

Columns 1 - 5 Integer I5 - Coordinate of force element

Columns 6 - 15 Real F10.2 - Magnitude of force in Newtons.

In card type 2, the value of 0 for the problem indicator implies that the stiffness matrix is the same as for the last problem solved. A value of 1 for the problem indicator implies that a new stiffness matrix is to be read and used for the computation. Card type 4 is to be provided for each non-zero element of the force vector specified in card type 3.

4.4 Eigenvalue Programme

The determination of the vibration characteristics of any structure should involve the solution of an eigenvalue problem of the form given by equation 3.1.3 (section 3.1) namely

$$[K]\{u\} = \omega^2[M]\{u\} \quad \text{or} \quad [M]\{u\} = \lambda[K]\{u\}$$

The solution of this equation will yield the natural frequencies ω of the structure and the corresponding modal shape (or eigenvector) represented by $\{u\}$. In this work a computer programme has been developed to solve such eigenvalue problems namely Programme NAGFEIGNVAL.

4.4.1 Programme NAGFEIGNVAL

A full compiled listing of this computer programme is shown in Appendix A.3.

Once again, the NAG Fortran Library have been used extensively in this programme. The major data required by programme NAGFEIGNVAL comes from the file produced by programme MASSTIFPROP and containing the mass and stiffness matrices of the structure in question. The programme has a built in set of four options of the solution required for any particular eigenvalue problem. These options include solving for (i) some (less than 25%) of the eigenvalues (frequencies) only, (ii) all the eigenvalues (iii) some (less than 25%) of both eigenvalues and eigenvectors only and (iv) all eigenvalues and eigenvectors. By so doing, the programme caters for small problems (where all the eigenvalues and eigenvectors could easily be computed) as well as large problems (where it would be undesirable and wasteful to compute all parameters.

Eight main subroutines from the NAG Fortran Library are used in this programme for the solution of the eigenvalue problem. They include F01AEF, F01AGF, F02BFF, F02ADF, F02BEF, F01AHF, F01AFF, and F02AEF. Other NAG Fortran Library routines called by this problem include X01AAF, X02AAF and X02ADF, where X01AAF fetches the mathematical constant π and X02AAF and X02ADF fetches other machine constants (computer dependent constants) needed by the above NAG Fortran Library routines.

F01AEF reduces the eigenproblem $[M]\{u\} = [K]\{u\}$ to the standard symmetric eigenproblem $[P]\{Z\} = \lambda\{Z\}$. [K] being a real symmetric positive definite matrix it can be factorised using Cholesky's method so that $[K] = [L][L]^T$. Therefore, $[M]\{u\} = \lambda[K]\{u\}$ implies

$$[L]^{-1}[M]([L]^T)^{-1}([L^T]\{u\}) = \lambda([L]^T\{u\}$$

which is the standard eigenproblem

$$[P]\{Z\} = \lambda\{Z\}$$

where $[P] = [L]^{-1}[M]([L]^T)^{-1}$

and $\{Z\} \quad [L]^T\{u\}$

Routine F01AGF gives the Householder reduction of a real symmetric matrix to the tridiagonal form for use in F02BFF and F02BEF.

F02BFF calculates selected eigenvalues of a real symmetric tri-diagonal matrix, where, if the eigenvalues are numbered in ascending order, the numbers of the first and last eigenvalues required are given.

Routine F02ADF calculates all the eigenvalues of the eigenvalue problem using Householder reduction and the QL algorithm.

Routine F02BEF calculates selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix, where the selected eigenvalues lie between two given values.

Routine F01AHF derives the eigenvectors of real symmetric matrix from the eigenvectors of the tridiagonal form where the tridiagonal matrix was produced by F01AGF.

Routine F01AFF derives the eigenvectors {u} of the problem [M]{u} = [K]{u} from the corresponding eigenvectors {Z} = [L]{u} of the derived standard symmetric eigenproblem. The eigenvector {u} is normalized such that

$$\{u\}^T[K]\{u\} = 1$$

Routine F02AEF calculates all the eigenvalues and eigenvectors of the eigenvalue problem using Householder reduction and the QL algorithm. The eigenvectors are also normalised such that

$$\{u\}^T[K]\{u\} = 1$$

Further details of the mathematical processes and theories used in the above routines can be obtained from ref.21.

The programme NAGFEIGNVAL also has the facility for multiple problems i.e. solving more than one eigenvalue problem in one computer run. Its Programme Description Segment allows the assignment of multiple files as input and output files. Where desired and when both eigenvalues and eigenvectors are computed, the results can also be stored on computer files for possible future use.

4.4.2 Programme NAGFEIGNVAL-Input Data

Apart from the input file containing the mass and stiffness matrices from programme MASSTIFPROP, this programme requires very little input data. Only a maximum of 4 different card types constitutes the set of data cards required by this programme. They include:

Card Type 1.

Columns 6 - 10 Integer I5 - Number of problems.

Card Type 2.

Columns 6 - 10 Integer I5 - Input file channel number = 3,4,5,6 or 7.

Columns 11 - 15 Integer I5 - Output file channel number = 0, 8, 9, 10, 11 or 12.

Columns 16 - 20 Integer I5 - Problem indicator = 1, 2, 3 or 4.

Card Type 3.

Columns 6 - 10 Integer I5 - Mode number of the lowest natural frequency required.

Columns 11 - 15 Integer I5 - Mode number of the highest natural Frequency required.

Card Type 4.

Columns 6 - 10 Integer I5 - Number of eigenvalues in the required range.

Columns 11 - 20 Real E10.3 - Lower bound of the required eigenvalue λ_{min}

Columns 21 - 30 Real E10.3 - Upper bound of the required eigenvalue λ_{max}

In card type 2, the value of zero for the output file channel number implies that no output file is written. The values of 1, 2, 3 and 4 for the problem indicator corresponding to the respective four problem options stated earlier. Thus a value of 1 implies that some eigenvalues only are required etc.

Card type 3 only applies if problem indicator equals 1. Card type 4 only applied when problem indicator equals 3 - i.e. some eigenvalues and corresponding eigenvectors are required.

4.5 Programme Tests

The greatest problem with computing is the high possibility of obtaining the wrong results from an otherwise well written computer programme. Possible sources of error range from the 'serious' cases of errors in the fundamental theories used in formulating the problem to the 'trivial' cases of errors in punching of the programme and/or data. It is, therefore, absolutely essential for all computer programmes to undergo very rigorous and probing tests before being put into use.

All three computer programmes described earlier in this chapter have undergone such extensive tests and the writer has been very satisfied with the results. It will not be useful to describe and discuss all the tests here. But two tests stand out here and a presentation of these gives some idea of what to expect in using these programmes. These tests include the analysis of a

(a) Simply supported beam,

(b) Portal frame.

4.5.1 Simply Supported Beam

The simply supported beam is as shown in fig. 4.5.1(a). It has been analysed using the finite element discretizations of 1-element, 2-elements, 4-elements and 8-elements systems producing 2-, 4-, 8-, and 16- D.O.F. systems respectively. The 8 finite elements (16 D.O.F.) discretization is shown in fig. 4.5.1.(b).

The beam properties are:

$$E = 207 \text{ kN/mm}^2$$

$$G = 80 \text{ kN/mm}^2$$

$$I_x = 10046 \text{ mm}^4$$

$$I_y = 5023 \text{ mm}^4$$

$$I_z = 5023 \text{ mm}^4$$

$$\rho A = m_\ell = 0.00065 \text{ Kg/mm}$$

$$A = 90 \text{ mm}^2$$

Static displacements of the beam under the action of a central load Q have been computed by programme STATICPROB. Fig. 4.5.2(a) shows a sketch of this static deflection curve. The deflection curve represents a half-sine wave as is to be expected from a knowledge of elementary beam theory. The results with fewer beam element discretizations produce the same pattern of curve, only with fewer coordinate values.

Table 4.5.1 shows the computed natural frequencies of the beam with 1-element, 2-elements, 4-elements and 8-elements discretized systems. Also the last column of this table includes the first 8 exact natural frequencies of the beam. The exact n^{th} natural frequencies of the beam is given by (Chapter 5 of ref.16)

$$\omega_n = (n\pi)^2 \sqrt{\frac{EI_z}{m_\ell \ell^4}}$$

where ℓ is the length of the beam.

As can be seen from the table, the computed natural frequencies of the beam improves as the finite element model is refined. This is in keeping with known theoretical prediction, with the frequencies reducing and approaching the exact values. The reliability of the finite element method used here is well illustrated by the fact that the first 5 computed natural frequencies are within 1% of the exact value. In fact, the first 2 frequencies are exact and the error in the third is only 0.13%. The table illustrates the degree of finite element refinement required to obtain reasonable accuracy in the computed frequencies. For example, no improvement is obtained in the first natural frequency between the 4-elements and 8-element systems. The 2-elements system gives a very good value for the first natural frequency.

Table 4.5.1

Simply Supported Beam Vibration

Mode	Computed Natural Frequencies in Hz				Exact natural frequencies in Hz
	1 element 2 D.O.F.	2-elements 4 D.O.F.	3-elements 8 D.O.F.	8-elements 16 D.O.F.	
1	69.72	63.08	62.84	62.84	62.82
2	319.54	278.91	252.30	251.40	251.28
3		701.12	575.80	566.12	565.38
4		1278.17	1115.68	1008.88	1005.12
5			1773.48	1584.36	1570.50
6			2804.41	2301.32	2261.52
7			4201.36	3171.27	3078.18
8			5112.68	4462.73	4020.48
9				5552.50	
10				7089.29	

The first 3 modes of the beam vibration have been plotted in fig.4.5.2. These represent 1/2, - 1 , and 1.1/2 - sine waves as was expected.

4.5.2 Portal Frame

The Portal Frame is shown in fig. 4.5.3(a). Finite element analysis has been carried out on the portal frame as 3-element (6 D.O.F.), 6-element (15 D.O.F.) and 12-element (33 D.O.F.) systems with each node having 3 D.O.F. (2 linear displacements and 1 rotational displacement). The case of the 12 finite element (33 D.O.F.) discretization of the portal frame is shown in fig. 4.5.3.(b).

The properties of the component beams of the portal frame are the same as for the simply supported beam.

Static displacements of the portal frame under the action of a horizontal force at its corner have been computed by programme STATICPROB as well as the static displacements under the action of a vertical central load. Fig. 4.5.4(a) and fig. 4.5.5(a) show the corresponding static deflection shapes for these two cases.

Table 4.5.2 gives a direct comparison of the computed natural frequencies of the portal frame under the various degrees of refinement of the finite element discretization model.

These are plotted in figs. 4.5.4(b), 4.5.5(b) and 4.5.6 representing the first, second and third modal shapes of vibration of the portal frame respectively.

As with the case of the beam the computed frequencies here show a reducing trend with the improved idealization of the finite element model. With this in mind it seems reasonable to assume (Table 4.5.2) that the first 4 natural frequencies with the 33 D.O.F. system should be very reliable. In particular, the first two frequencies should be very accurate.

The first two modal shapes plotted in figs.4.5.4(b) and 4.5.5(b) correspond to the static deflection shapes shown in figs. 4.5.4(a) and 4.5.5(a) respectively. All three modal shapes plotted are in accordance with expected shapes as obtained by other methods of frequency analysis of a portal frame (ref.17).

Table 4.5.2

Plane Frame Vibration

Mode	Computed Natural Frequencies in Hz		
	3-Elements 6 D.O.F.	6-Elements 15 D.O.F.	12-Elements 33 D.O.F.
1	20.43	20.40	20.40
2	96.17	80.93	80.45
3	207.96	132.80	131.42
4	1097.77	144.38	142.26
5	1178.10	325.52	287.80
6	2004.64	426.09	353.15
7		533.19	408.37
8		832.98	622.75
9		1116.65	743.42
10		1276.38	772.80
11		1303.91	1111.93
12		2377.91	1268.70
13		4078.30	1328.91
14		4229.74	1342.73
15		5149.06	1544.27
16			1955.12

Flow Chart - MASSTIFPROP

Fig. 4.2.1

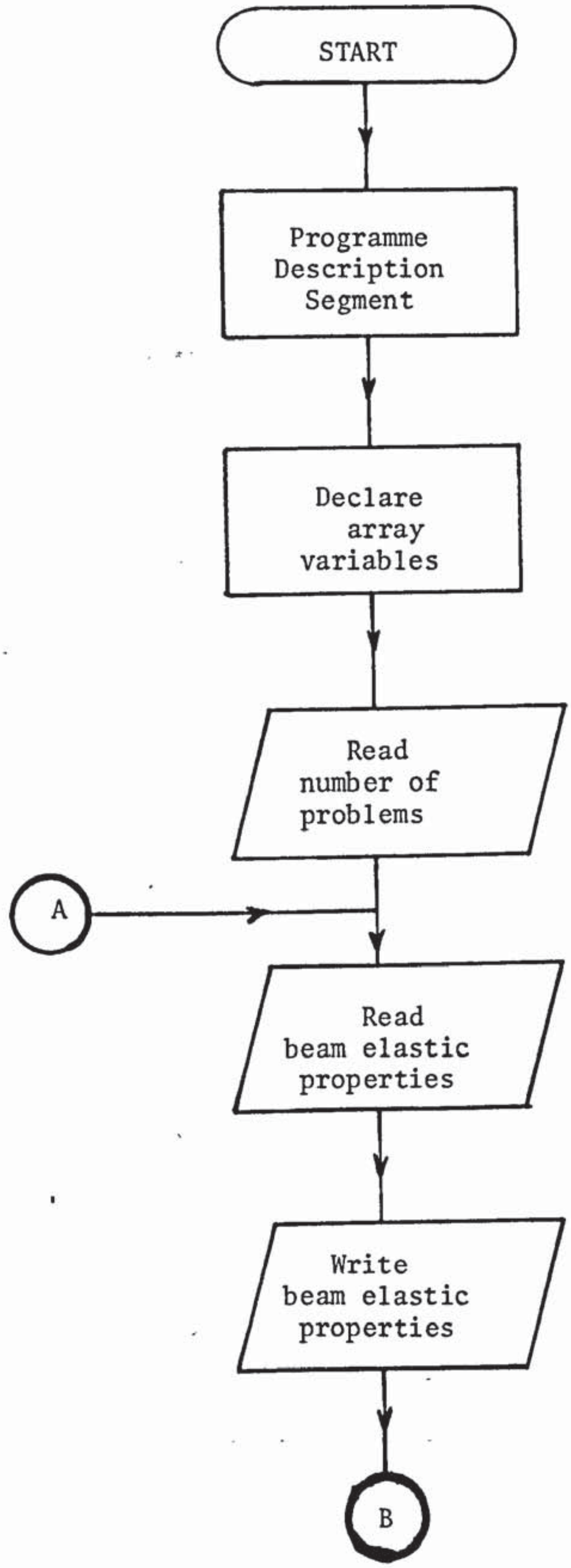


Fig. 4.5.1 continued

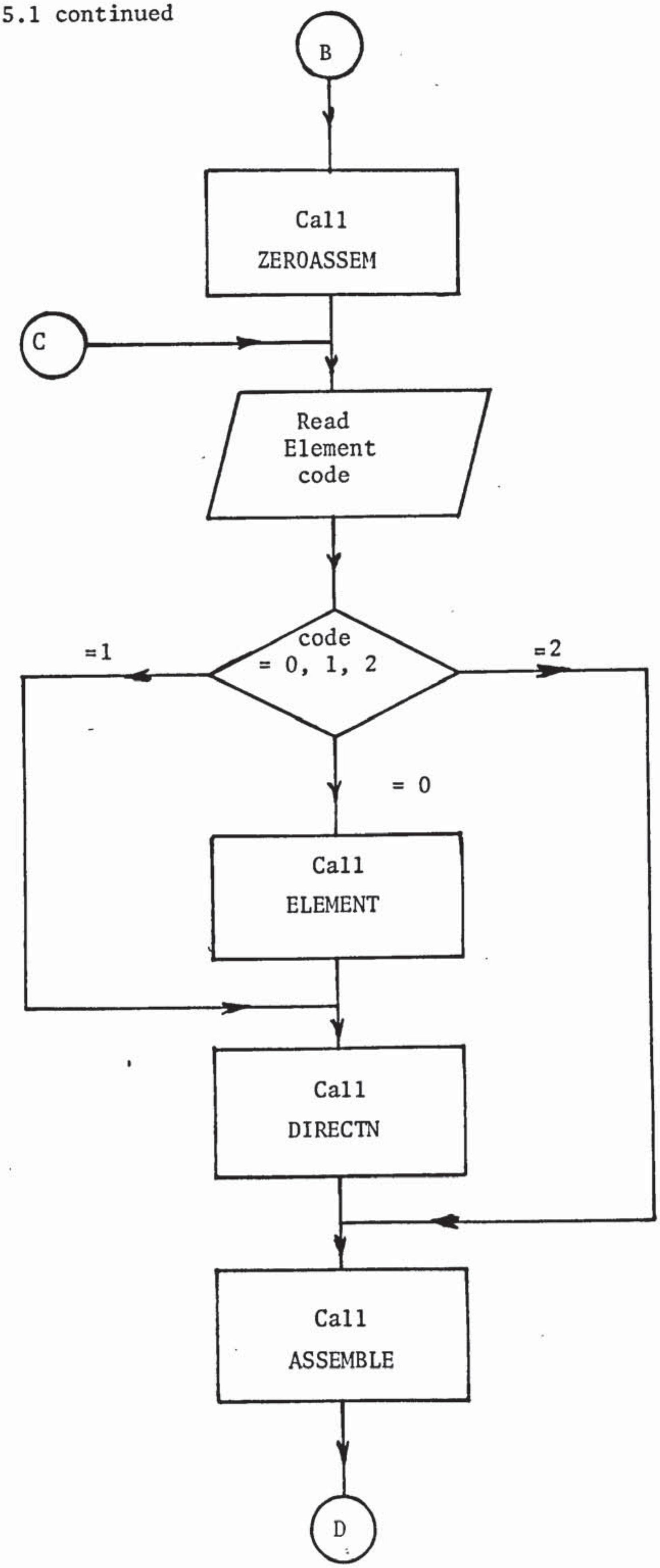
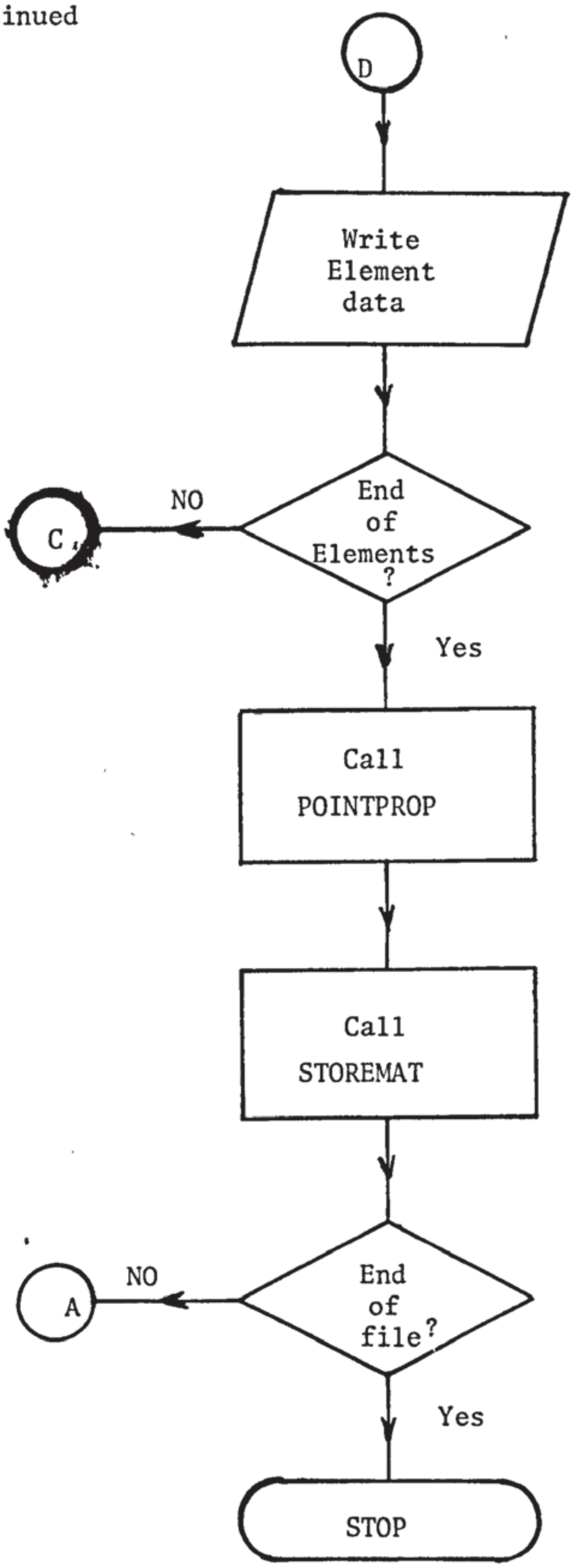


Fig. 4.2.1 continued



All dimensions are in mm

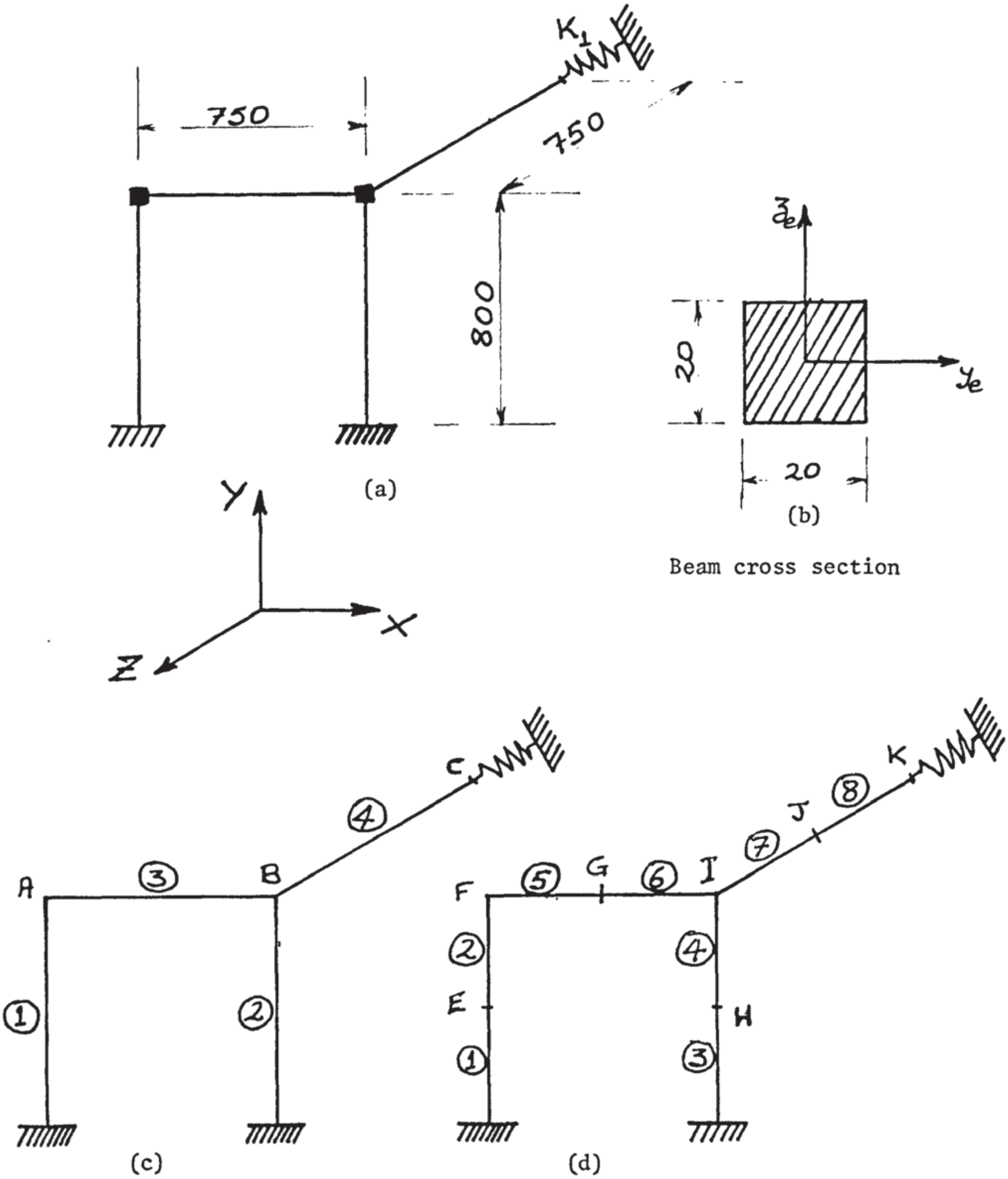
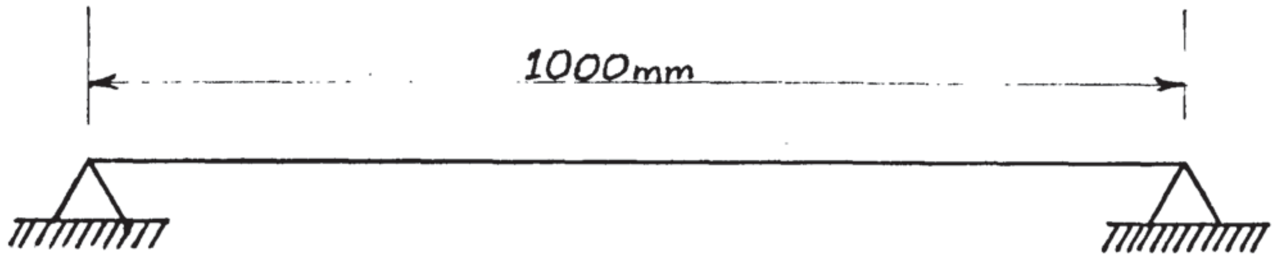
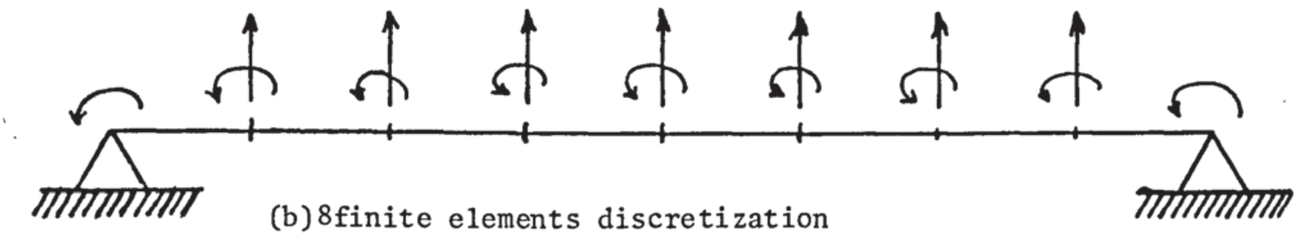


Fig. 4.2.4

Frame Example

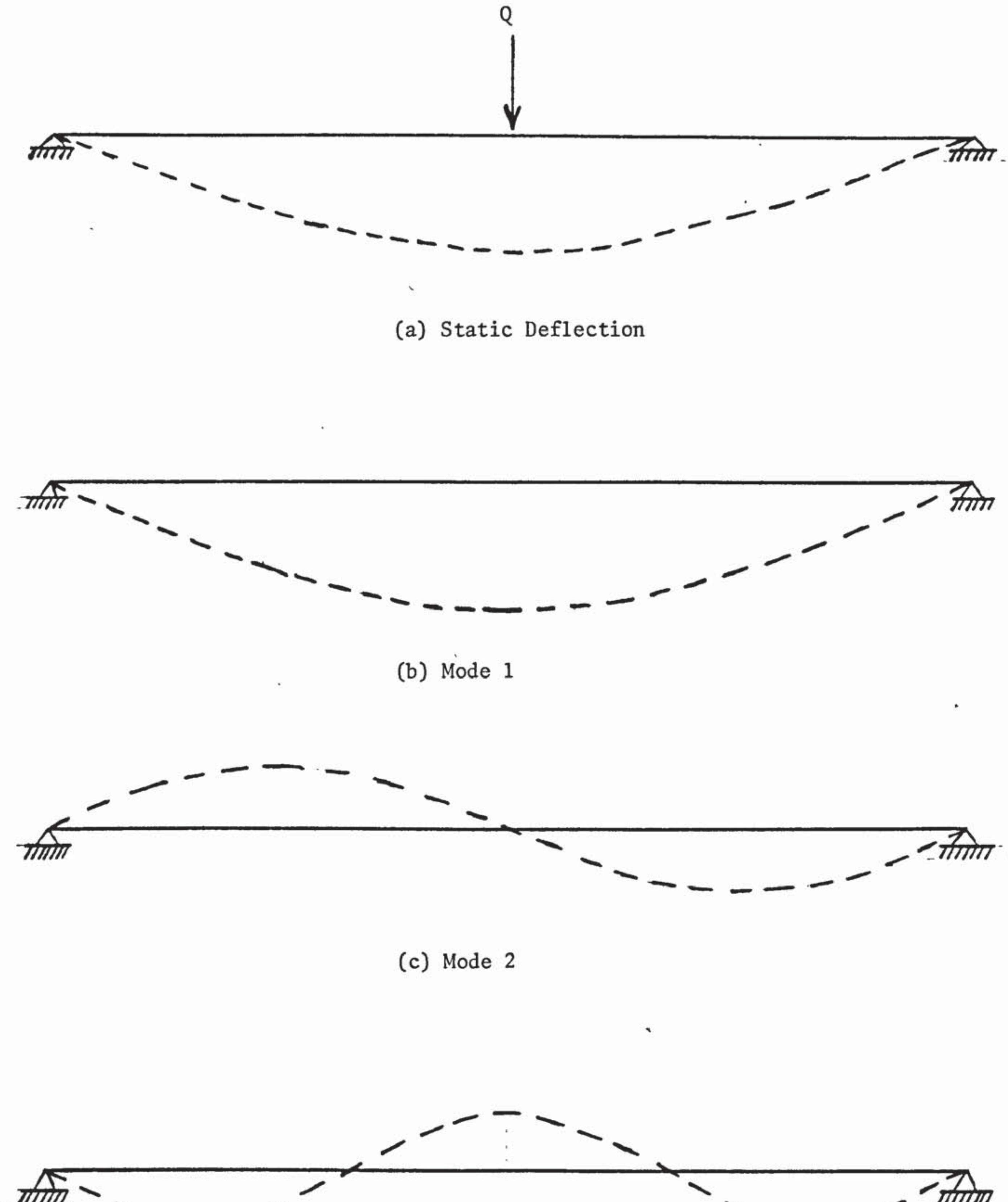


(a) simply supported beam



(b)8finite elements discretization

Fig. 4.5.1



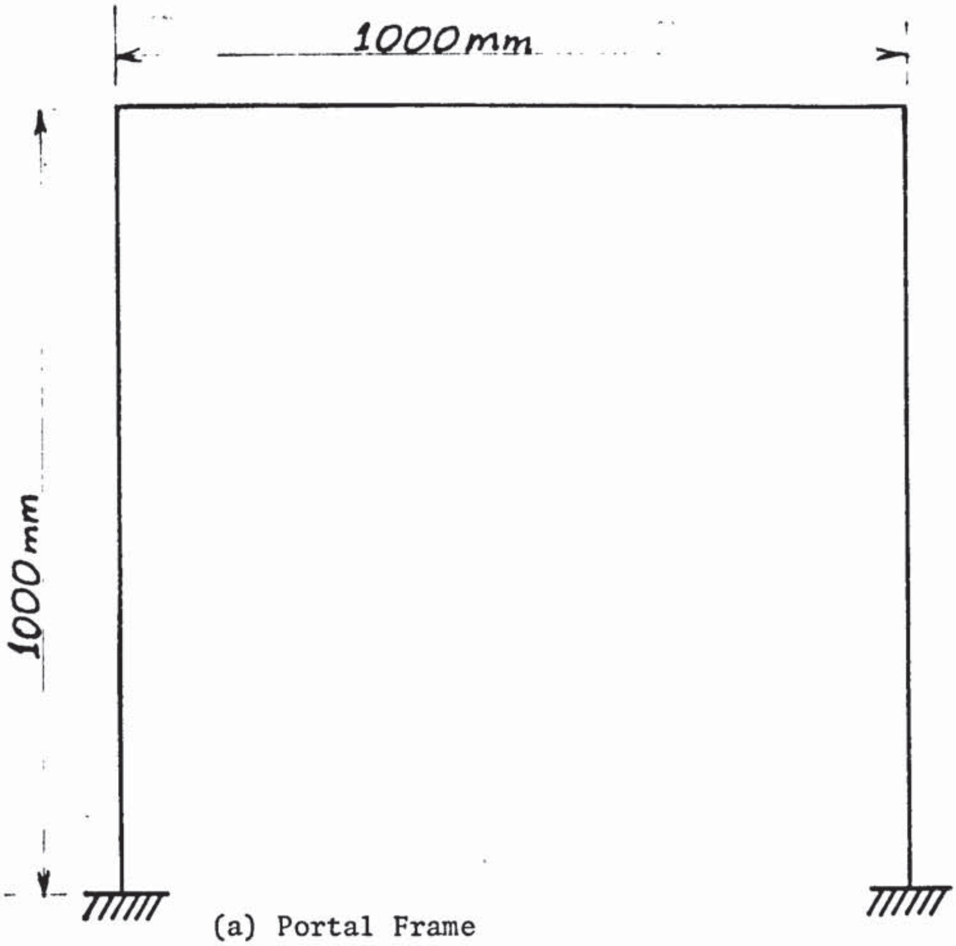
(a) Static Deflection

(b) Mode 1

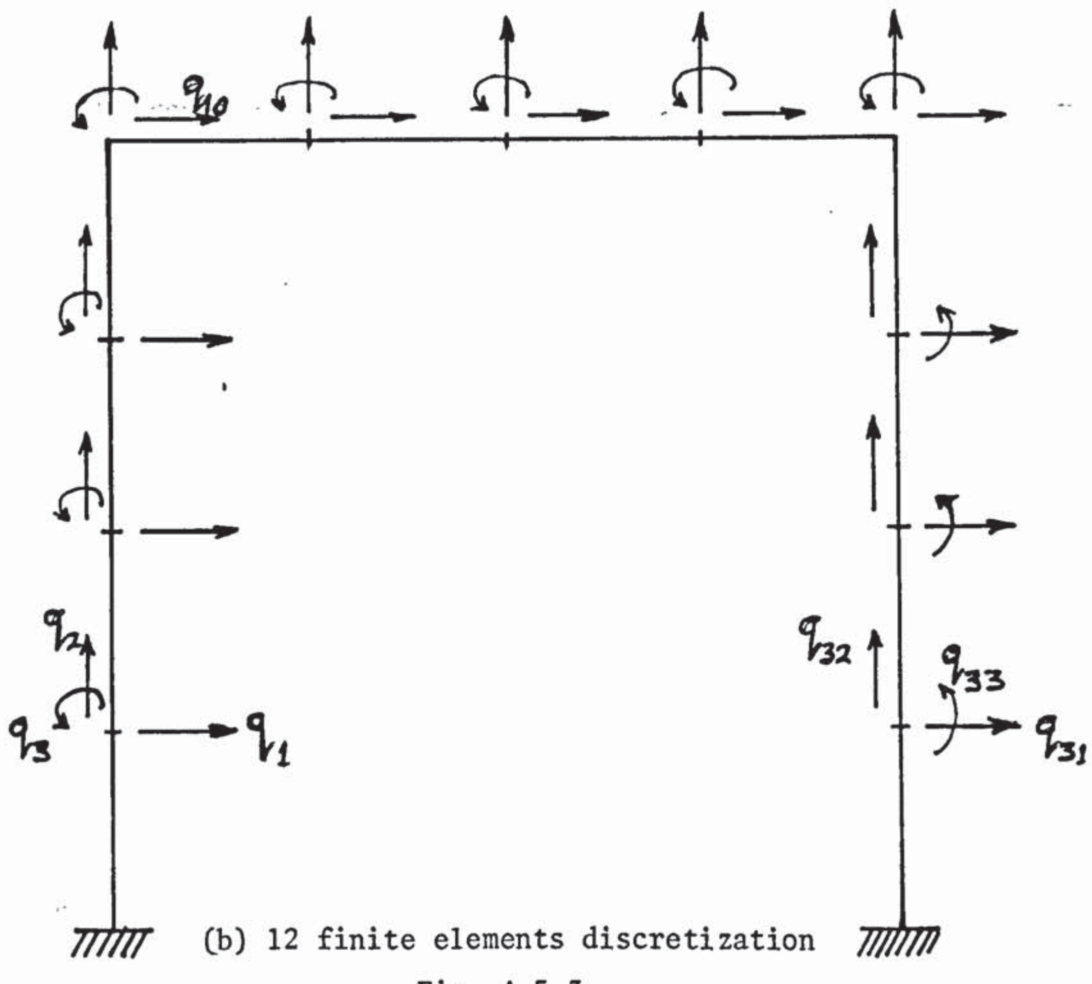
(c) Mode 2

(d) Mode 3

Fig. 4.5.2 Simply Supported Beam analysis

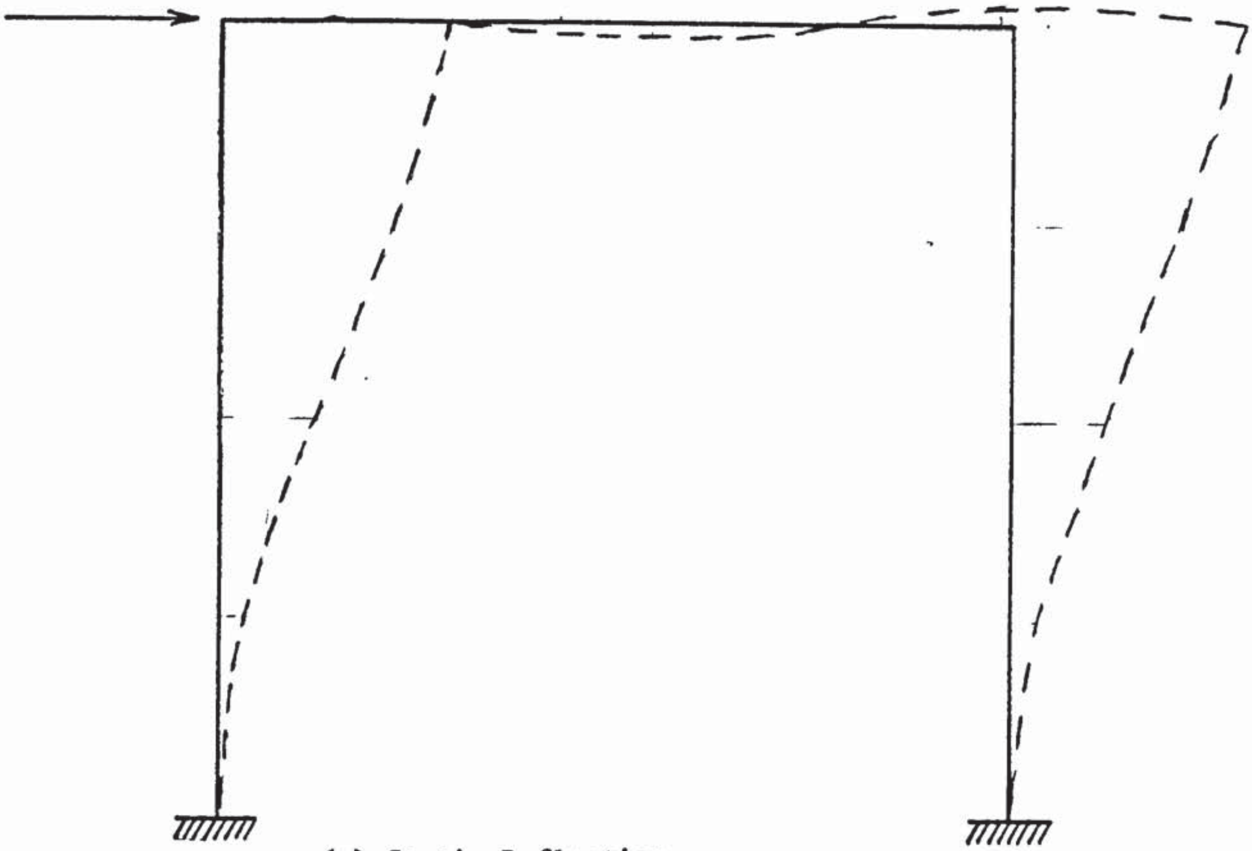


(a) Portal Frame

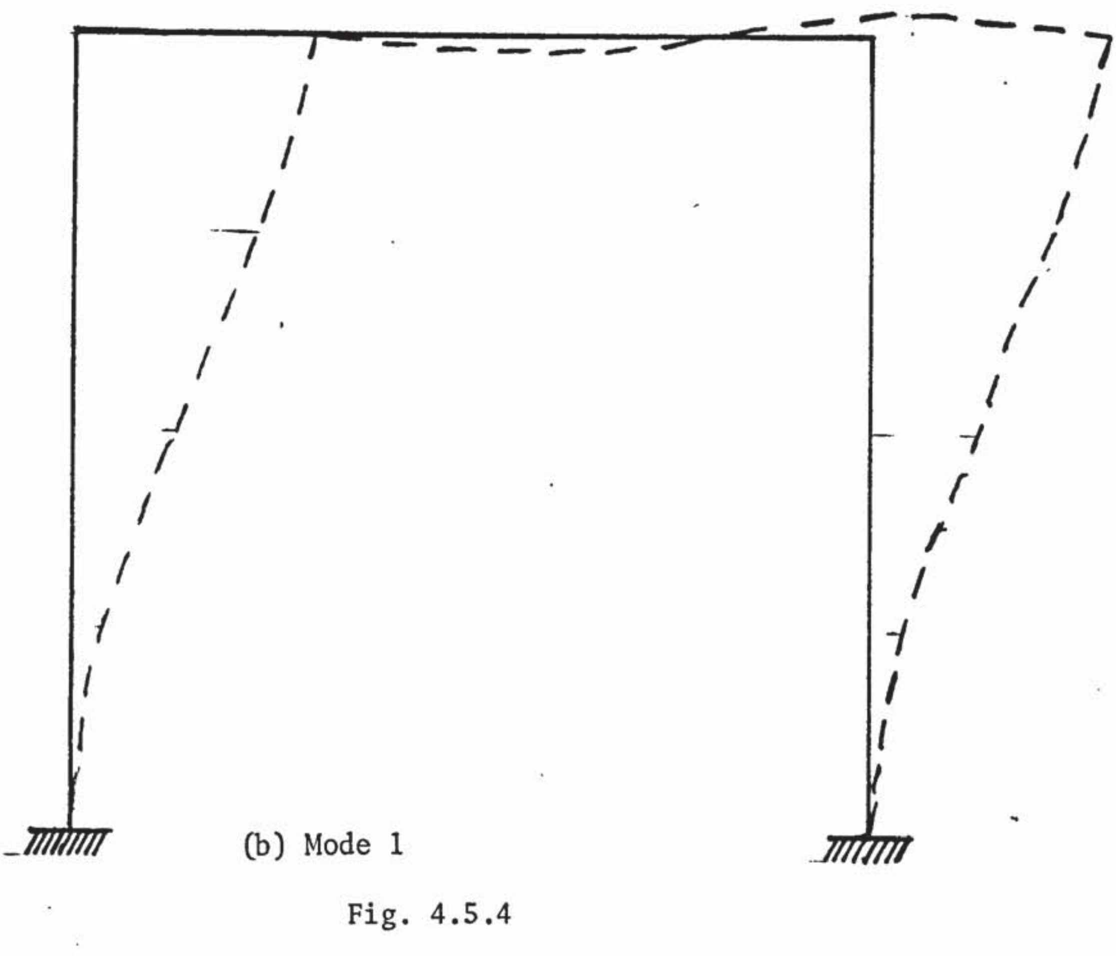


(b) 12 finite elements discretization

Fig. 4.5.3

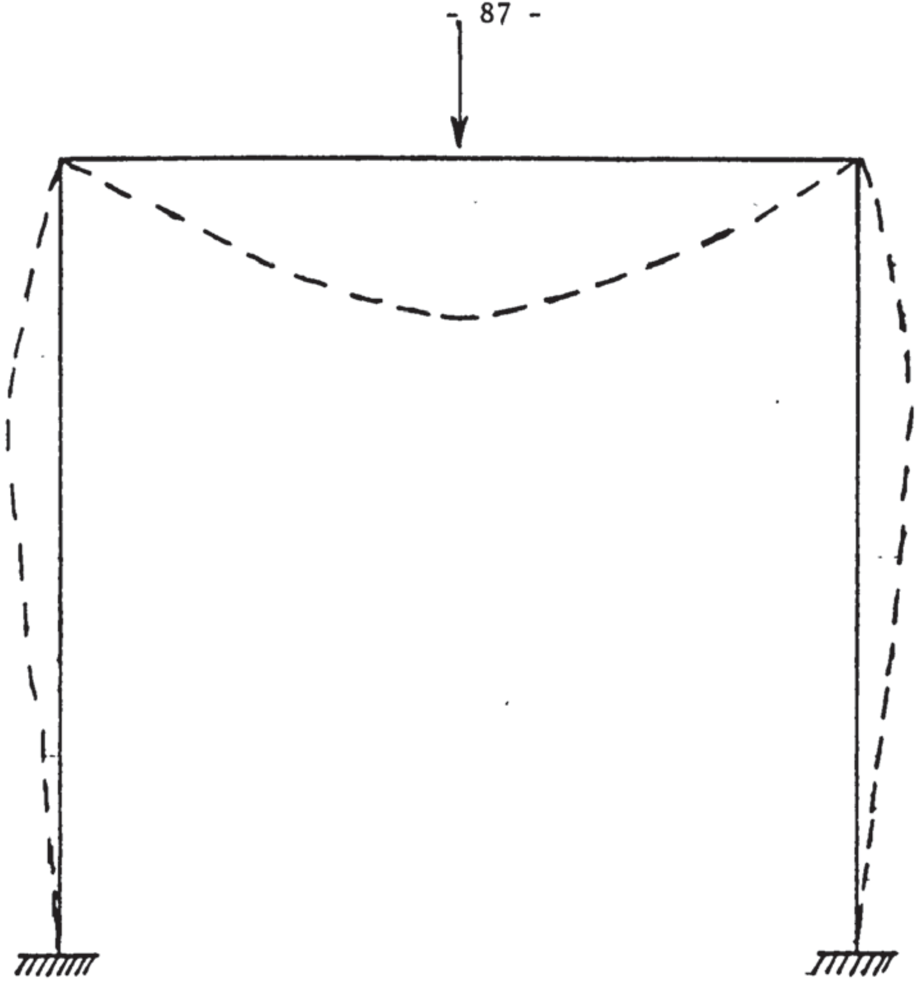


(a) Static Deflection

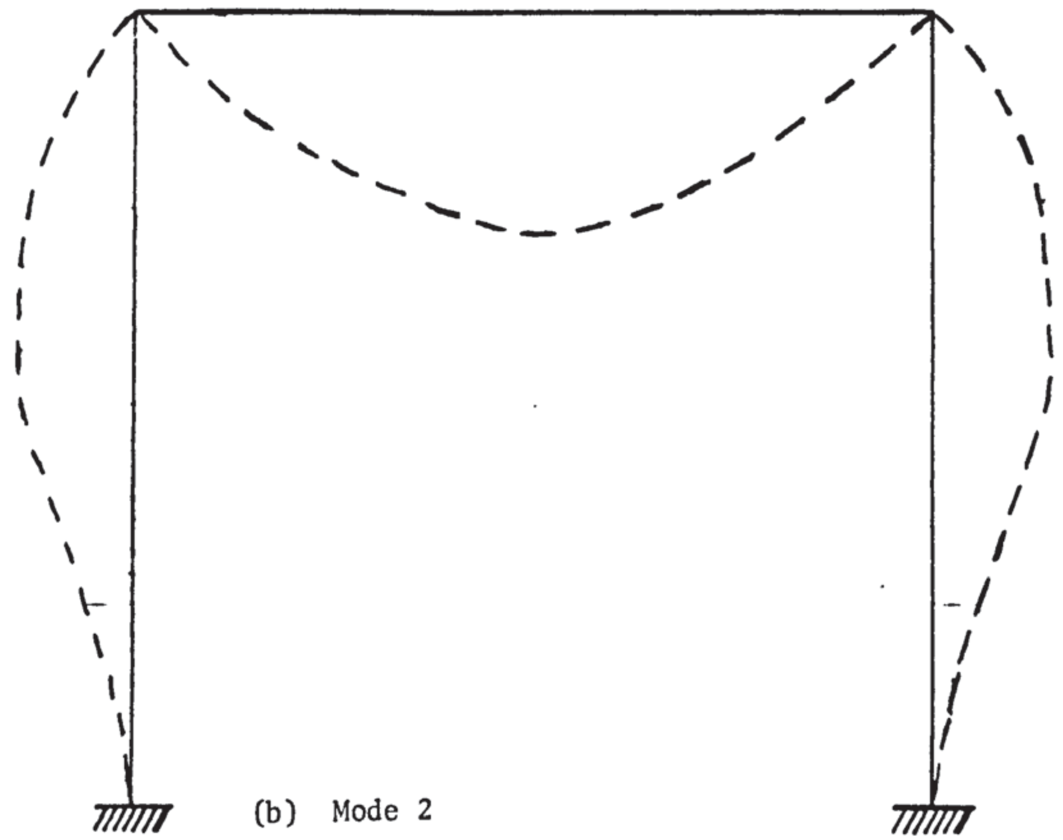


(b) Mode 1

Fig. 4.5.4

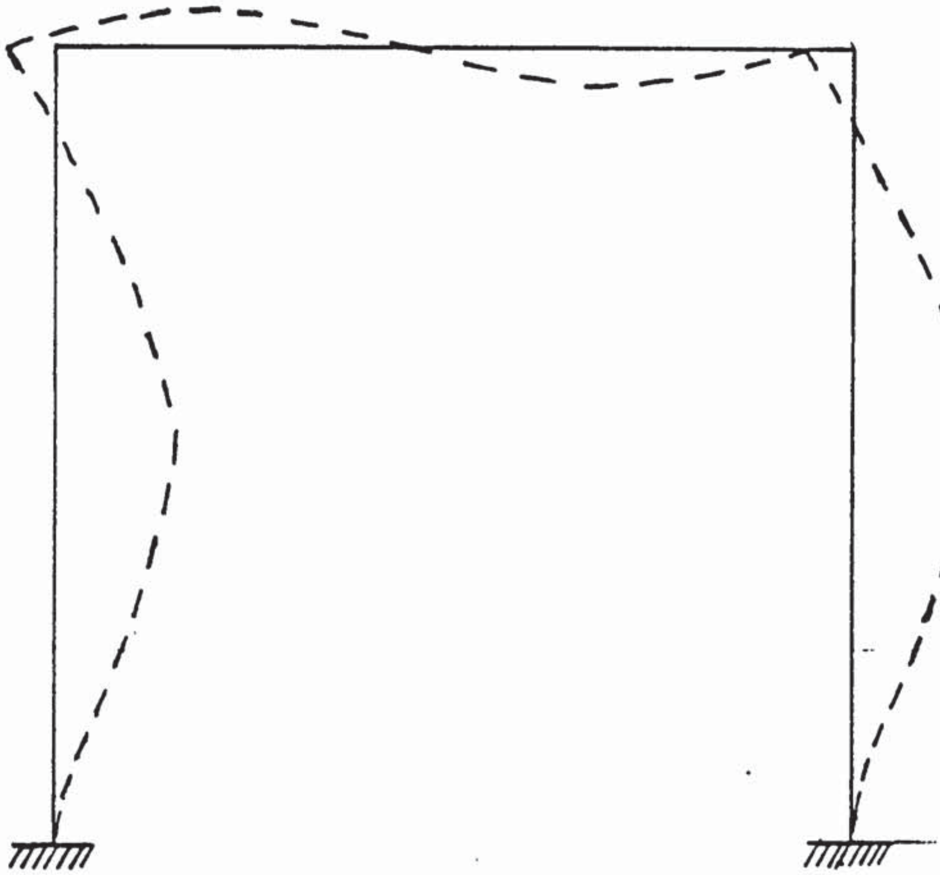


(a) Static Deflection



(b) Mode 2

Fig. 4.5.5



Mode 3

Fig. 4.5.6

CHAPTER FIVE

CHAPTER FIVE

EXPERIMENTAL TECHNIQUE

5.1 Introduction

Both static and dynamic experimentations have been involved in this work. But the main one is the dynamic experimentation where vibration characteristics are measured.

The relevant static experiments are concerned with the measurement and verification of the structural stiffness matrices used in the analytical work. The static experiments have here been restricted to simple measurements in force-displacement experiments. All that is required include a suitable arrangement for applying the load at a point of the structure and some dial gauges for measuring the corresponding displacements at various points of the structure. Average values of the force per unit displacement at the various points can then be calculated and the equivalent stiffness coefficient can be calculated.

The dynamic experimentation is a more involved set up. Basically, what is required is a means of setting the structure under forced vibration and measuring the response at various points of the structure. When the forcing frequency coincides with a natural frequency of vibration

The sweep oscillator is the basic input instrument while the accelerometer is the basic output instrument.

The sweep oscillator is basically a wave signal generator. For the purpose of this work on vibration analysis, the sweep oscillator is used to generate sine wave signals at any required frequency. The frequency is either set at a fixed value or varied (sweeped) automatically or manually between any set frequency limits.

The signal from the sweep oscillator is a weak one, and the power amplifier amplifies it to a reasonable level required by the vibrator. The vibrator then applies the amplified signal to the structure in the form of an oscillatory force to set the structure under forced vibration.

The accelerometer picks up the response of the structure at any point on the structure for eventual measurement. It should be noted that the accelerometer is an acceleration measuring transducer, not a displacement transducer as the vibration response is supposed to measure. However, it is known that the amplitude of vibration is proportional to the pick value of the corresponding acceleration at that point, and acceleration transducers are more convenient than displacement transducers. Moreover, the modal shape of vibration required represents the relative shape of oscillation of the structure and not the absolute magnitude of its amplitude of vibration. Thus the use of the accelerometer meets all the requirements of the results expected from the experiment.

of the structure, resonance occurs. By passing the measuring transducer round various points of the structure, the pick responses at these points can be obtained to produce the modal shape of vibration of the structure at that natural frequency.

For a complicated structure (a space frame is a complicated one), the measurement of the resonant frequencies and the corresponding modal shapes is not an easy job. An arbitrary choice of the point of excitation could lead to difficulties in producing the resonance at certain natural frequencies of the structure. For example, exciting the structure at a nodal point (point of zero amplitude of vibration) of a certain natural frequency would result in missing out the resonance at that natural frequency. In particular, it is very useful to excite at a point of maximum amplitude of vibration of a given modal natural frequency. Hence, it is customary to change the point of excitation of a structure to obtain the different modes of vibration.

The beam members of a frame structure are actually a continua. As such, measurement of its amplitudes of vibration have to be made at many closed points in order to produce a true form of the modal shape. In the theoretical analysis, both linear and angular displacements (or amplitudes) can be calculated. But experimental measurements are restricted to linear displacements only. This is an added difficulty in the experimentation.

5.2 Vibration Instrumentation

A block diagram of the vibration instrumentation used in this work is shown in fig. 5.2.1. Other components may be included in the actual instrumentation set up, but the basic ones are shown in the figure.

Again, the response signal picked up by the accelerometer is amplified by the charger amplifier. Owing to some degree of non-linearity in the structure and/or other external interference, the response signal obtained could be anything but a pure sine wave. The Dynamic Analyzer receives this impure sine signal and essentially acts as an inherently frequency-tuned bandpass filter. The concept of its operation is as illustrated in fig. 5.2.2. The impure sine signal represents the signal input to the analyzer while the original signal from the sweep oscillator is fed in as the tuning frequency input. Output from the analyzer is represented by the filtered output signal which is a pure sine signal at the tuning frequency.

The valve voltmeter measures the R.M.S. value of the signal whose values for various points on the structure provides the modal shape of vibration of the structure at a particular frequency. The XY recorder can also be used to make a graphical plot of the response over a frequency range in the form of response (on Y-axis) vs frequency (on X-axis). The sweep oscillator provides the frequency input to the plotter on a D.C. scale proportional to log frequency or linear frequency. Points of relatively high response on the plot constitute possible resonance (or natural) frequencies of vibration of the structure.

The C.R.O. (Cathode Ray Oscilloscope) gives a more immediate view of the response. In addition, a comparison of the response signal and the original signal from the oscillator gives an instant relative phase shift of the response signals on the C.R.O. In fact, the in-phase and out-of-phase positions of the two signals have been used to designate positive and negative signs respectively to the response of the structure at any particular point.

A phase meter could be used for the above purpose, but this is considered to be too sophisticated and un-necessary for this simple case of in- and out-phase measurement.

A frequency counter is usually connected to the instrumentation to give a more reliable reading of the frequency of the signal generated by the oscillator. As a check, the frequency counter is also used to know the actual frequency of the output signal from the analyzer. Both frequencies should read the same.

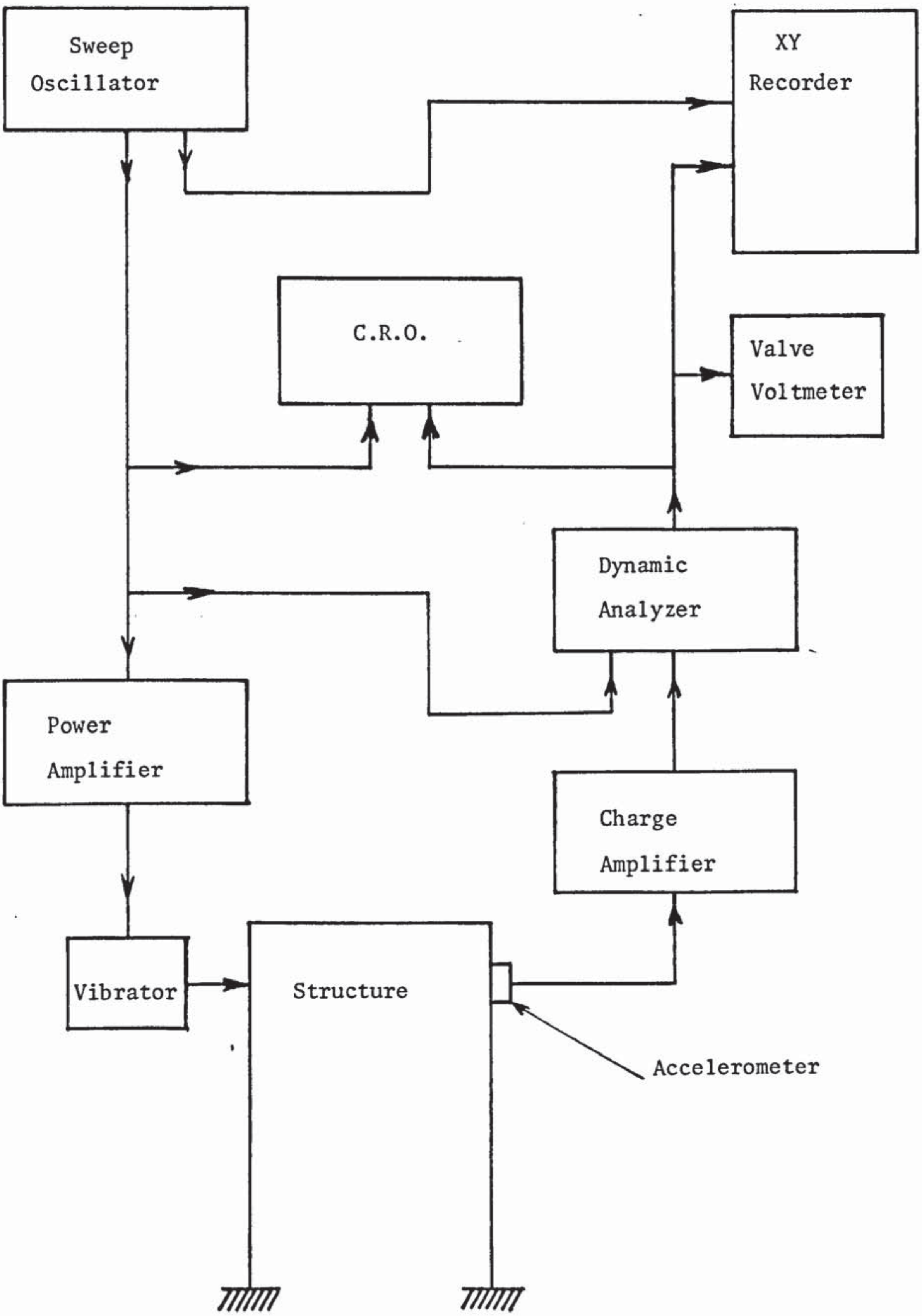


Fig. 5.2.1 Block diagram of Vibration Instrumentation

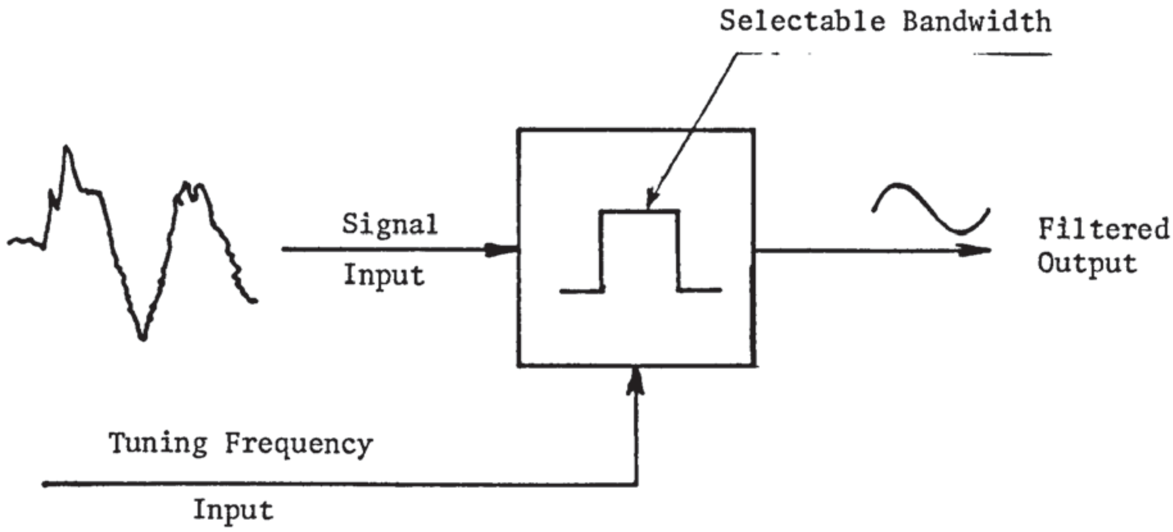


Fig. 5.2.2 Operation of Dynamic Analyser

CHAPTER SIX

CHAPTER SIX

ANALYSIS OF FABRICATED SPACE FRAME

6.1 Introduction

The fabricated space frame (fig.2.2.1, Chapter 2) is here analysed. Both theoretical and experimental vibration analysis are presented here.

The space frame is discretized into the 30 nodes system shown in fig.6.1.1. This involves using the original 8 joints of fig.2.2.1, the 4 feet joints and the mid points of each of the long member beams which makes up the space frame. No midpoint nodes have been included for the short feet members. The node numbering and the coordinate numbering which follows it, have been done to reduce to a minimum the eventual band width (refs.3,5 etc.) of the system mass and stiffness matrices. This should have in turn reduced the actual computer storage space required in the analysis. But unfortunately, efficient computer routines were not available to solve the vibration problem obtained whilst making use of the banded nature of the matrices.

Theoretical and experimental natural frequencies and modal shapes have been obtained. Plots of the modal shapes for the first three natural frequencies have been made for both theoretical and experimental results for comparison. In the discussion which follows, the theoretical and experimental results are discussed in detail. Probable reasons for errors are analysed and possible solutions are suggested.

6.2 Theoretical Vibration Analysis

The discretized form of the fabricated space frame shown in fig.6.1.1 has been used in this section. It is assumed here that all eight corner joints (nodes 6, 7, 8, 9, 18, 19, 20 and 21) are each capable of 6 D.O.F. - 3 translational motion in the X-Y, and Z-axes and 3 rotational motion about the X-, Y-, and Z-axes in the YZ, XZ and XY planes respectively. This assumption implies that these corner joints remain orthogonal at all times.

The midpoint nodes should naturally have 6 D.O.F. each. This will guarantee continuity at these nodes since the beams are originally continued at these points.

The feet nodes (27, 28, 29 and 30) are assumed to be perfectly clamped. Thus both translational and rotational displacements at these nodes are zeroes.

The above type of discretization implies that the structure becomes a 40 elements, 26 nodes finite element system. With 6 D.O.F. at each node, the structure is to be analysed as a 156 D.O.F.system.

In fact, the first possible discretization of the space frame should be as defined above but without the mid point nodes. Thus the structure becomes an 8 nodes (22 elements) finite element system having a total of 48 D.O.F. (see fig.2.2.1 in Chapter 2).

Any further discretization will involve more nodes hence more D.O.F. This will then make the matrices too large for the available computer. Thus the theoretical analysis of the fabricated space frame has been carried out with the 40 elements, 156 D.O.F. system and the 22 elements, 48 D.O.F. system analysis for comparison and check.

The essential data required for the analysis include:

Modulus of Elasticity of beam material, E = 207 kN/mm^2

Modulus of Rigidity of beam material, G = 80 kN/mm^2

Area of corss-section of beam, A = 90 mm^2

Mass per unit length of beam, $\rho A = 6.5 \times 10^{-4}$ Kg/mm

Second moment of area about X-axis, I_x = 10046 mm^4

Second moment of area about Y-axis, I_y = 5023 mm^4

Second moment of area about Z-axis, I_z = 5023 mm^4

Equivalent concentrated mass at joints 6,7,8 and 9 = 0.269 kg

Equivalent concentrated mass at joints 18,19,20 and 21 = 0.284 kg

And after making allowances for the joint length,

Length of the beam such as 6-8 = 560 mm

Length of cross member beam = 770 mm

Length of feet beam = 91 mm

Table 6.2.1 gives the computed first 10 natural frequencies of the fabricated space frame for both the 48 D.O.F. and 156 D.O.F. systems. It is to be expected, the computed frequencies are lower for the 156 D.O.F. system than for the 48 D.O.F. system. The former gives a better

TABLE 6.2.1

Computed Natural Frequencies for the Space Frame

MODE	Computed Natural Frequencies Hz	
	48 D.O.F.	156 D.O.F.
1	165.38	146.81
2	172.96	151.51
3	215.25	174.82
4	224.08	175.52
5	226.36	180.39
6	252.98	183.86
7	306.21	199.89
8	307.01	201.70
9	308.62	221.07
10	327.62	221.71

representation of the structure than the latter. With further increases in the number of elements (hence number of D.O.F.), the computed frequencies should, theoretically, tend to the lower limiting values equal to the actual natural frequencies. But even with the 156 D.O.F. system, the frequencies should be close enough to the true. values.

The corresponding computed modal shapes (or eigenvectors) for the first 10 modes are included in Appendix B as Appendix B1.

6.3 Experimental Frequencies and Modes

In the experimental vibration work on the fabricated space frame, the basic node system used in the theoretical finite element analysis has been followed.

The space frame is vibrated at a convenient point and the response is measured at other mode points 1 - 26 shown in fig. 6.2.1. Unlike the theoretical computations, only a maximum of 3 linear displacement coordinates could be measured for each of the 26 finite element nodes. The rotational displacements are left out of the modal measurements. However, apart from these measured displacements, it was sometimes useful to measure the displacements on a beam between the stated nodes in order to access the actual shape of vibration along the beam. This is because, in some cases, the response at the stated 26 nodes on the structure did not yield enough information required.

Appendix B2 shows the experimentally measured natural frequencies and corresponding modal responses for the first three modes. No direct comparison should be made between the listed absolute values of the responses across the modes. The responses are only to be compared with each other from point to point on the same mode. This is because the absolute values of these responses depended on the magnitude and point of application of the exciting force. In the first place, the magnitude of the exciting force was kept constant during the response measurement only for individual mode experimentations. Secondly, the point of excitation was not fixed for the three mode measurements. To obtain the best results, a structure should be excited at a point of possible maximum vibration response for the particular natural frequency intended to be measured.

6.4 Theoretical and Experimental Mode Plots

The theoretical and experimental natural frequencies as obtained in sections 6.2 and 6.3 do not agree. But their corresponding modal shapes should be plotted for comparison.

The fabricated space frame under consideration is a complicated one and plotting the modal shapes in 3-dimensional space is even more complicated. To ease the task, plane plots of the theoretical vibrating shapes of the structure were first made. The effects of the rotational displacements are better seen in these 2-dimensional plots. The complete 3-dimensional plots are built more readily from the plane plots. The 3-dimensional plots are difficult to interpret, but in combination with the 2-dimensional plots, the 3-dimensional mode of vibration is better visualised.

Six plane plots are required to obtain a complete picture of the space frame mode of vibration. Hence 6 plane plots have been made for each of the modes plots of vibration. Appemdix B contains the plane plots for the first three theoretical modes of vibration of the fabricated space frame. Figs. C1.1-6 (Appendix C1) are the plane plots for mode 1; figs. C2.1-6 (Appendix C2) are the plane plots for mode 2; and figs. C3.1-6 (Appendix C3) are the plane plots for mode 3.

Plots of the theoretical modes of vibration of the space frame in 3-dimensions are shown in fig. 6.4.1(a) for mode 1; fig.6.4.2(a) for mode 2; and fig. 6.4.3(a) for mode 3. The corresponding plots of the experimental modes are shown in fig.6.4.1(b) for mode 1; fig.6.4.2(b) for mode 2; and fig. 6.4.3(b) for mode 3.

In all three cases, the theoretical and experimental modal shapes are similar. The third mode of vibration represents the first symmetric mode of vibration of the space frame (ref.17). Here, there is relatively no horizontal motion of the tops of the stanchions and the centres of the top member beams remain horizontal. Each of the component beams (except the short feet beams) vibrates as if it were an isolated hinged-hinged beam vibrating at its first mode. The first and second modal shapes represent anti-symmetric modes of vibration. In each case, there is a sideways sway of the fabricated space frame.

6.5 Discussion of Results

The results of the theoretical and experimental vibration analysis of the fabricated space frame is satisfactory only in relation to the modal shapes obtained. The theoretical analysis used in this work has produced a very good forecast of the true modal shapes of vibration for the first three modes analysed. However, the corresponding natural frequencies have not been so close.

TABLE 6.5.1

MODE	Natural Frequencies (Hz)	
	Computed	Experimental
1	146.81	96.00
2	151.51	104.00
3	174.82	114.00

Table 6.5.1 shows the first three computed and experimental natural frequencies of the fabricated space frame. The computed frequencies are an average of about 50% higher than the experimental values. This is not good enough, and it is necessary to trace the possible source(s) of error in the theoretical approach.

Of all the parameters used in the analysis of the space, the most likely source of error is in the derived mass and stiffness matrices, from which the natural frequencies and modal shapes were computed. Errors of up to 50% higher computed natural frequencies should suggest a possible 125% over-estimation of the frame stiffness matrix or 125% under-estimation of the frame mass matrix or some combination of factions of these. Moreover, the close agreement in the corresponding modal shapes further raises the suspicion of such percentage estimation. But this line of approach has proved unreasonable upon investigation. Very extensive checks on the programmes and data have not shown any trace of such factional possibilities. In addition, tests on other structures of known solution have produced the correction solutions, with the same programmes.

Several other lines of investigation have also been followed. Among these lines is one of reconsideration of the boundary conditions at the joints of the frame. The initial step in this direction only involved a reconsideration of the conditions at the brazed feet of the frame. It was supposed that the joint at the feet could be capable of rotation about the three axis, and theoretical analysis was carried out on such a fabricated space frame.

This assumption implies an additional 12 D.O.F. to the systems as analysed in section 6.2. Thus, referring to fig. 6.1.1; the 48 D.O.F. and 156 D.O.F. systems now become 60 D.O.F. and 168 D.O.F. systems respectively. Table 6.5.2 shows the computed natural frequencies

TABLE 6.5.2

MODE	Computed Natural Frequencies (Hz)			
	48 D.O.F.	60 D.O.F.	156 D.O.F.	168 D.O.F.
1	165.38	102.09	146.81	98.80
2	172.96	105.65	151.51	101.91
3	215.25	127.06	174.82	122.84
4	224.08	207.17	175.52	167.03
5	226.36	224.31	180.39	174.68
6	252.98	248.40	183.86	183.08
7	306.21	258.34	199.89	185.77
8	307.01	297.29	201.70	201.66
9	308.62	307.43	221.07	217.72
10	327.62	307.43	221.71	221.35

for the new systems plus the previous results (Table 6.2.1) for the 48 D.O.F. and 156 D.O.F. systems for comparison

The computed natural frequencies with the released feet joint seemed to give reasonable values to within 8% of the experimental values. But it can be seen that there is some degree of inconsistency in the values.

The assumption involving a complete release of the feet joint as to be capable of rotational displacement is by far a wild guess of the true conditions of the space frame. It is most unlikely that the bulk of the error in the originally assumed joint boundary conditions lies in the rotational rigidity of the feet joint. As such, the sharp closure of the gap between the experimental frequencies and these computed frequencies should give an indication that the feet rotational release does not represent the real situation.

The corresponding modal shapes for the 168 D.O.F. system were also computed, and it may seem worth while to plot these modal shapes for comparison with the other modal shapes (experimental and the computed 156 D.O.F. system). But bearing in mind the time and effort it takes to plot a 3-dimensional modal shape of vibration, this was avoided as very little usefulness was expected from the results.

Of the mass and stiffness matrices of the space frame the more practical one to be checked in isolation is the stiffness matrix. This can be done by considering the statics of the fabricated space frame. Load-deflection experiments can be performed on the space frame. Also, the corresponding deflection of the frame can be calculated for a given load from the stiffness matrix and the static equation of equation 4.1.1.

$$[K]\{q\} = \{Q\} \qquad 6.5.1$$

In fact, this gave rise to writing the static analysis programme of section 4.3.

The stiffness matrices for both the 156 D.O.F. and 168 D.O.F. systems have been used in computing the displacements for applied load at convenient points. Experimental displacements at the same points were measured to obtain the average deflections of the frame structure for a fixed point load. Two such point load analyses were carried out. Table 6.5.3 shows the experimental and theoretical displacements at some points (coordinates) for a load of -200.0 N applied at the point 4X (coordinate 19) of the fabricated space frame. Table 6.5.4 shows the other set of values for a load of -200 N applied at the point 16X (coordinate 91) of the fabricated space frame.

TABLE 6.5.3

Applied Force = -200 N at point 4X

Coordinates		Frame Displacements (mm)		
		Experimental	Theoretical	
			156 D.O.F.	168 D.O.F.
1Y	2	0.087	0.069	0.067
3X	13	-0.022	-0.005	-0.031
4X	19	-0.400	-0.308	-0.335
5Y	26	-0.132	-0.057	-0.060
10X	55	-0.054	-0.019	-0.059
10Y	56	-0.009	-0.002	-0.007
12Y	68	0.090	0.014	0.012
13X	73	-0.029	-0.007	-0.042
15Y	86	0.049	-0.052	-0.058
16X	91	-0.107	-0.074	-0.113

The two tables show that there are very great differences between the experimental and theoretical displacements. The differences are very complex and no definite pattern of variation can be traced. The 168 D.O.F. system has not produced a closer displacement value to the experimental ones. Earlier it was seen that the 168 D.O.F. produced natural frequencies which seem to be closer to the experimental values.

TABLE 6.5.4

Applied Force = -200 N at point 16X

Coordinates		Frame Displacements (mm)		
		Experimental	Theoretical	
			156 D.O.F.	168 D.O.F.
1Y	2	0.091	0.014	0.012
3X	13	-0.036	-0.007	-0.042
4X	19	-0.077	-0.074	-0.113
5Y	26	0.063	-0.052	-0.058
10X	55	-0.077	-0.027	-0.082
10Y	56	0.013	-0.001	-0.010
12Y	68	0.117	0.037	0.047
13X	73	-0.137	-0.018	-0.065
15Y	86	0.060	-0.045	-0.056
16X	91	-1.760	-0.617	-0.685

If these closer frequencies were genuine, then one would have expected the displacement pattern to be closer also. This is a further confirmation of the earlier inference that the released feet system does not represent the real system.

Also, the general state of the displacement pattern in the two tables is that the theoretical displacements are lower than the experimental values. This implies that the theoretical stiffness matrix makes the space frame stiffer than it really is. This, in turn, should lead to higher computed natural frequencies as already obtained.

However, the complexity of the space frame means that very little can be expected in terms of improvement of the theoretical stiffness matrix from such static deflection tests on the space frame. The prime source of error in the theoretical stiffness matrix should be as regards the actual boundary conditions at the joints of the fabricated space frame. These joints include not only the brazed feet joints, but the other joints at the 8 corner points of the frame.

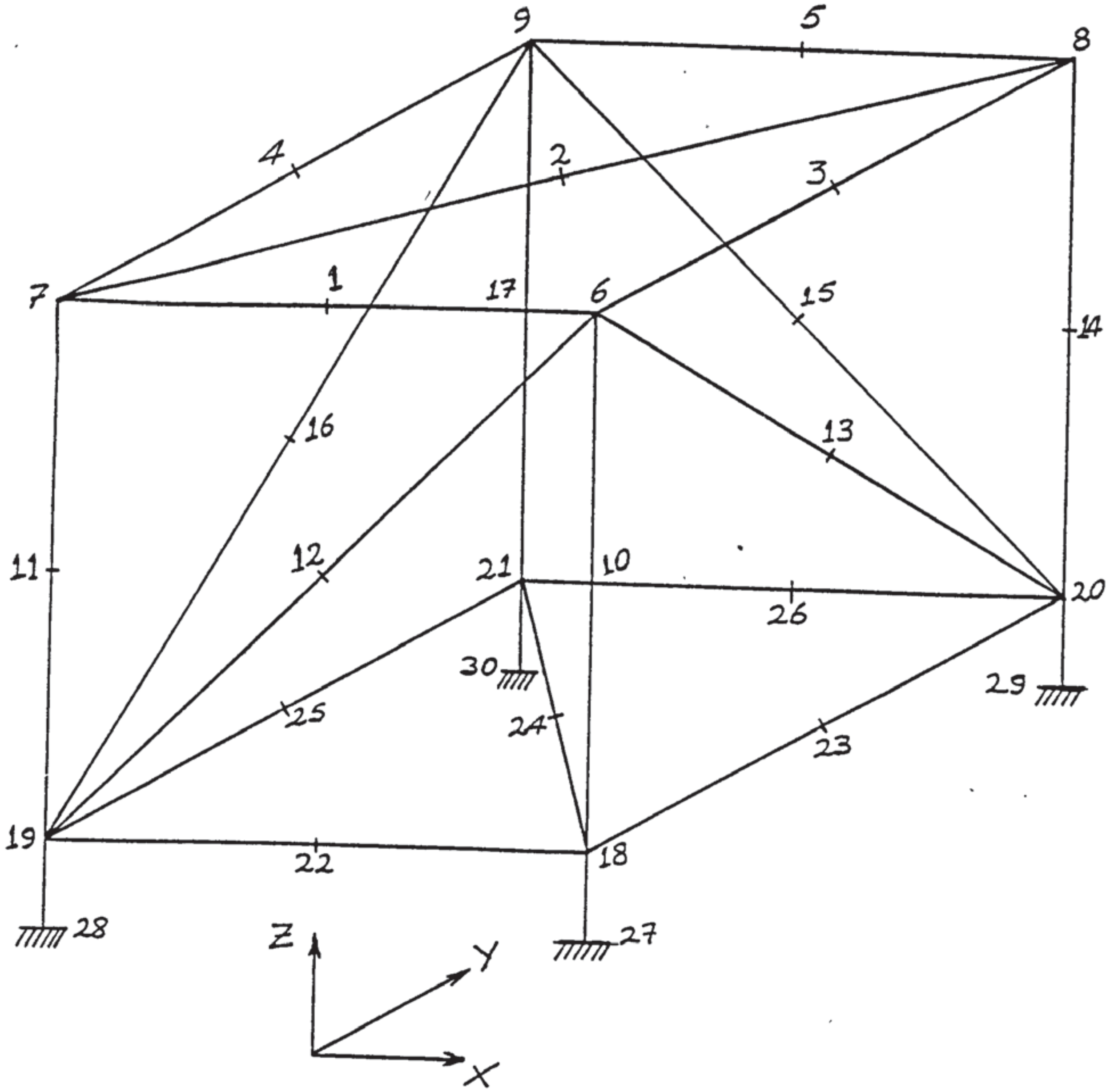


Fig. 6.1.1 30 Nodes discretized Space Frame

THEORETICAL: MODE 1

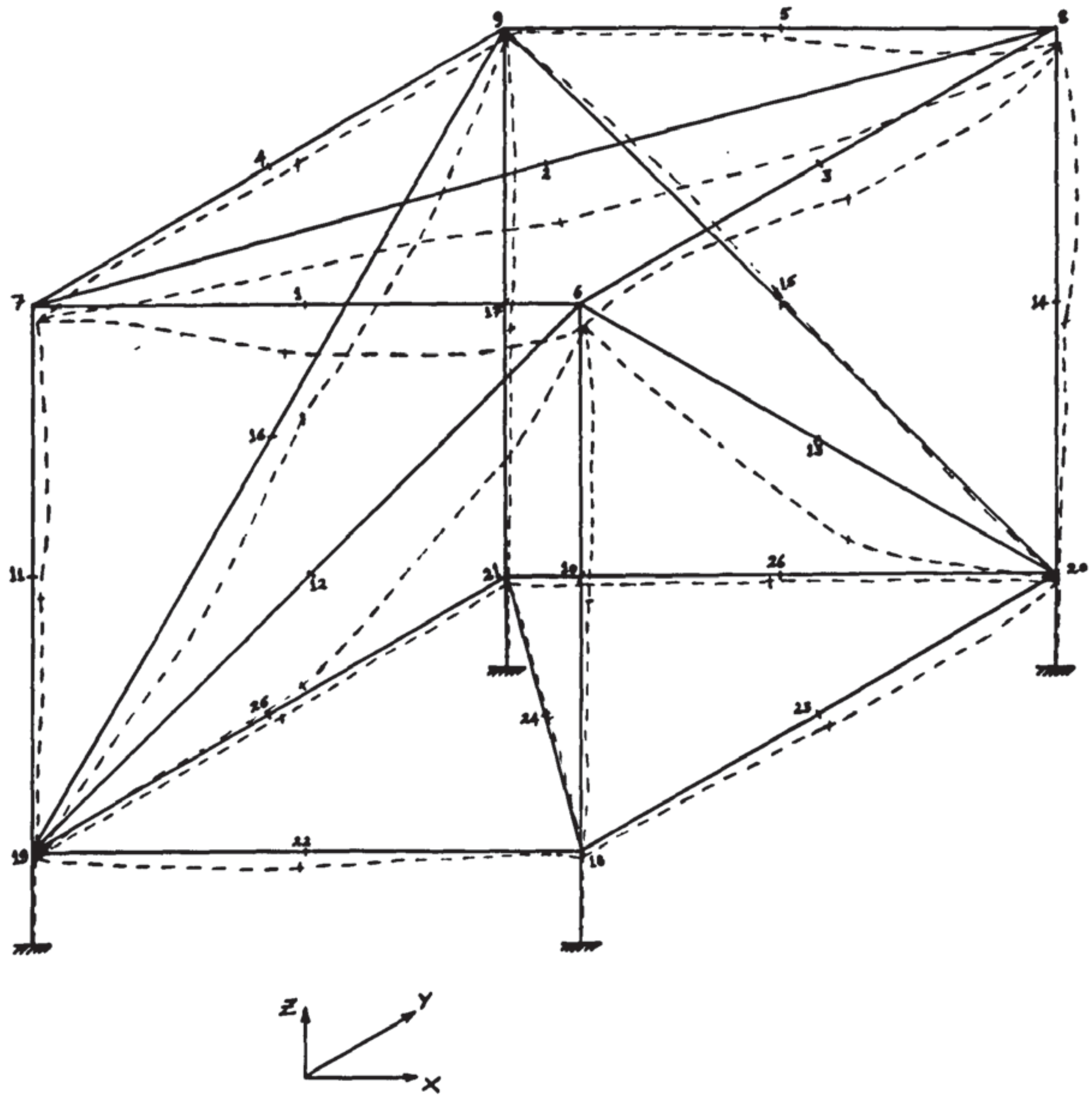
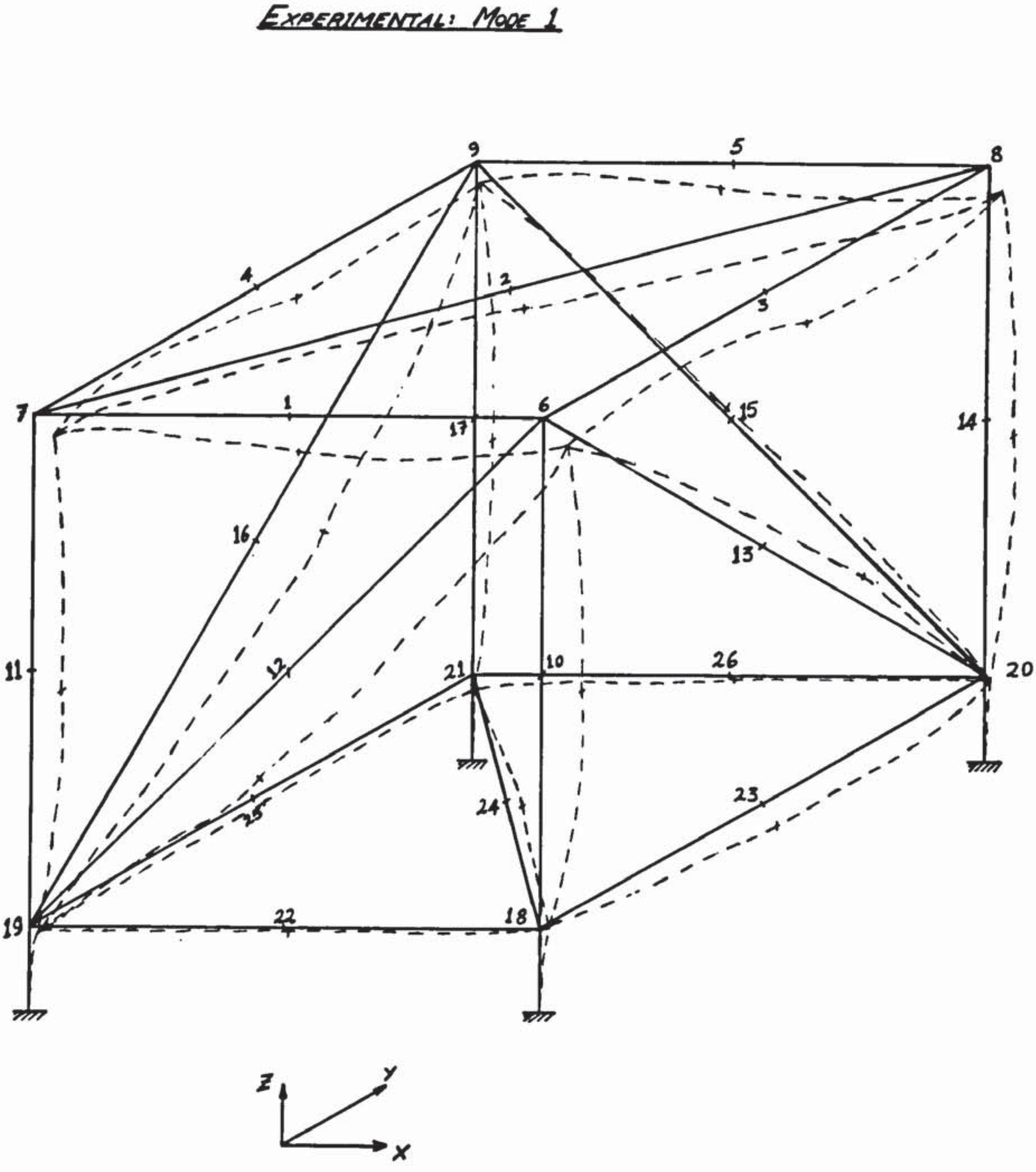


Fig. 6.4.1(a)


EXPERIMENTAL: MODE 1
9
5
8
4
2
3
7
1
6
17
15
14
16
13
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10
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19
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Z
Y
X

Fig. 6.4.1(b)

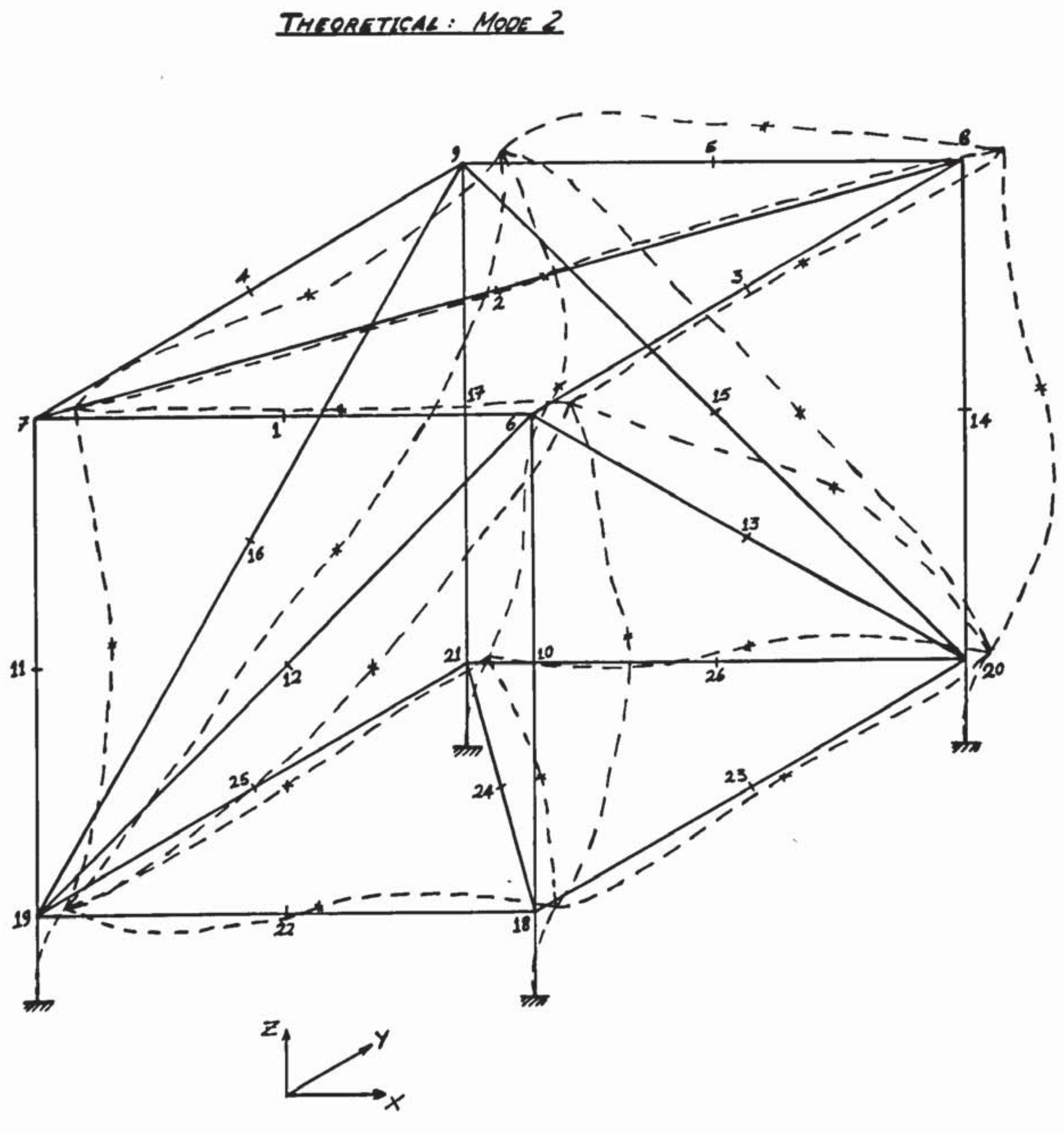
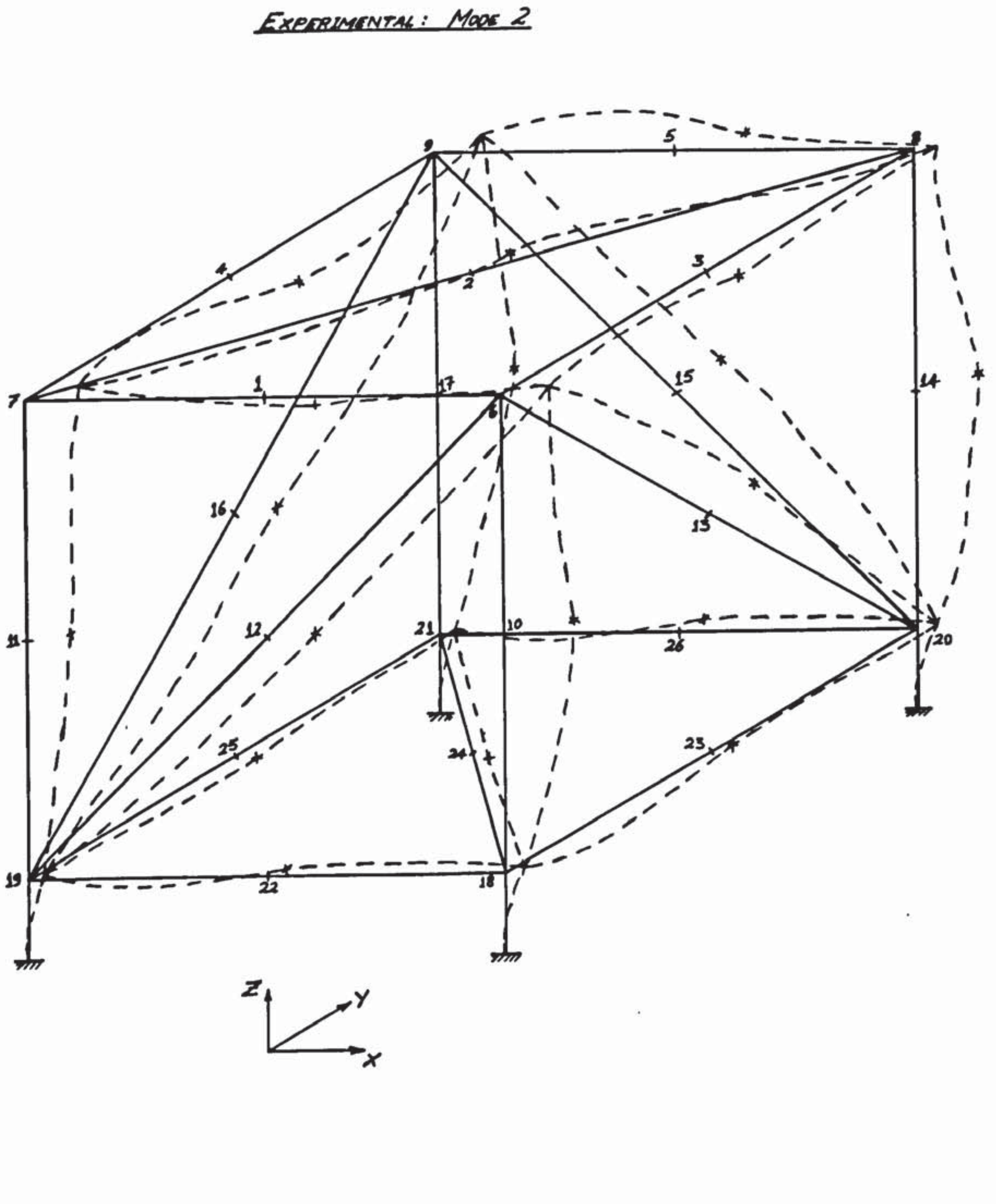

THEORETICAL: MODE 2
9
6
8
4
2
3
57
7
1
6
15
14
16
13
11
12
21
10
26
20
25
24
23
19
22
18
Z
Y
X

Fig. 6.4.2(a)


EXPERIMENTAL: MODE 2
1
2
3
4
5
6
7
8
9
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11
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17
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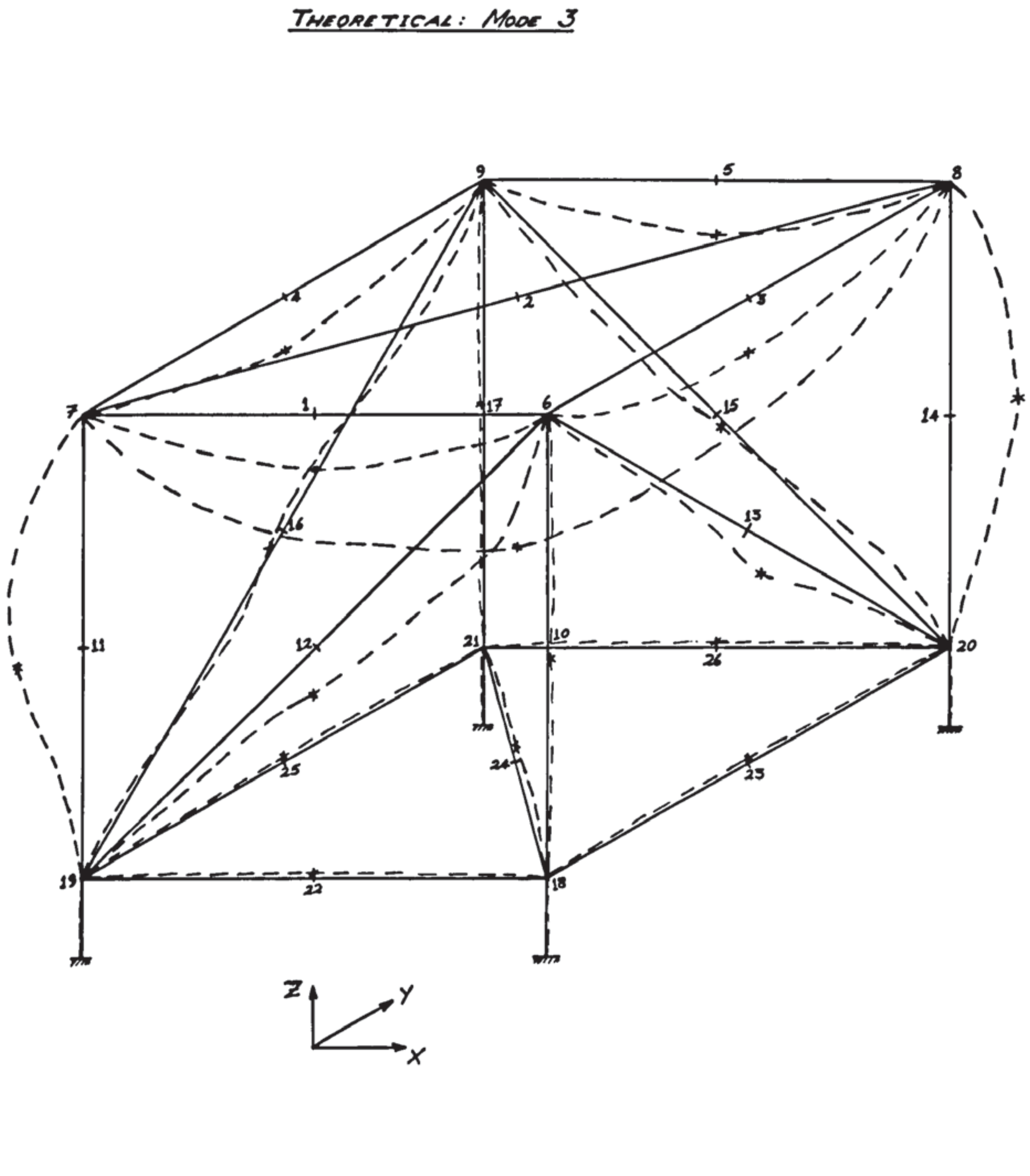
21
22
23
24
25
26
Z
Y
X

Fig. 6.4.2(b)


THEORETICAL: MODE 3
9
5
8
4
2
3
7
1
17
6
15
14
16
13
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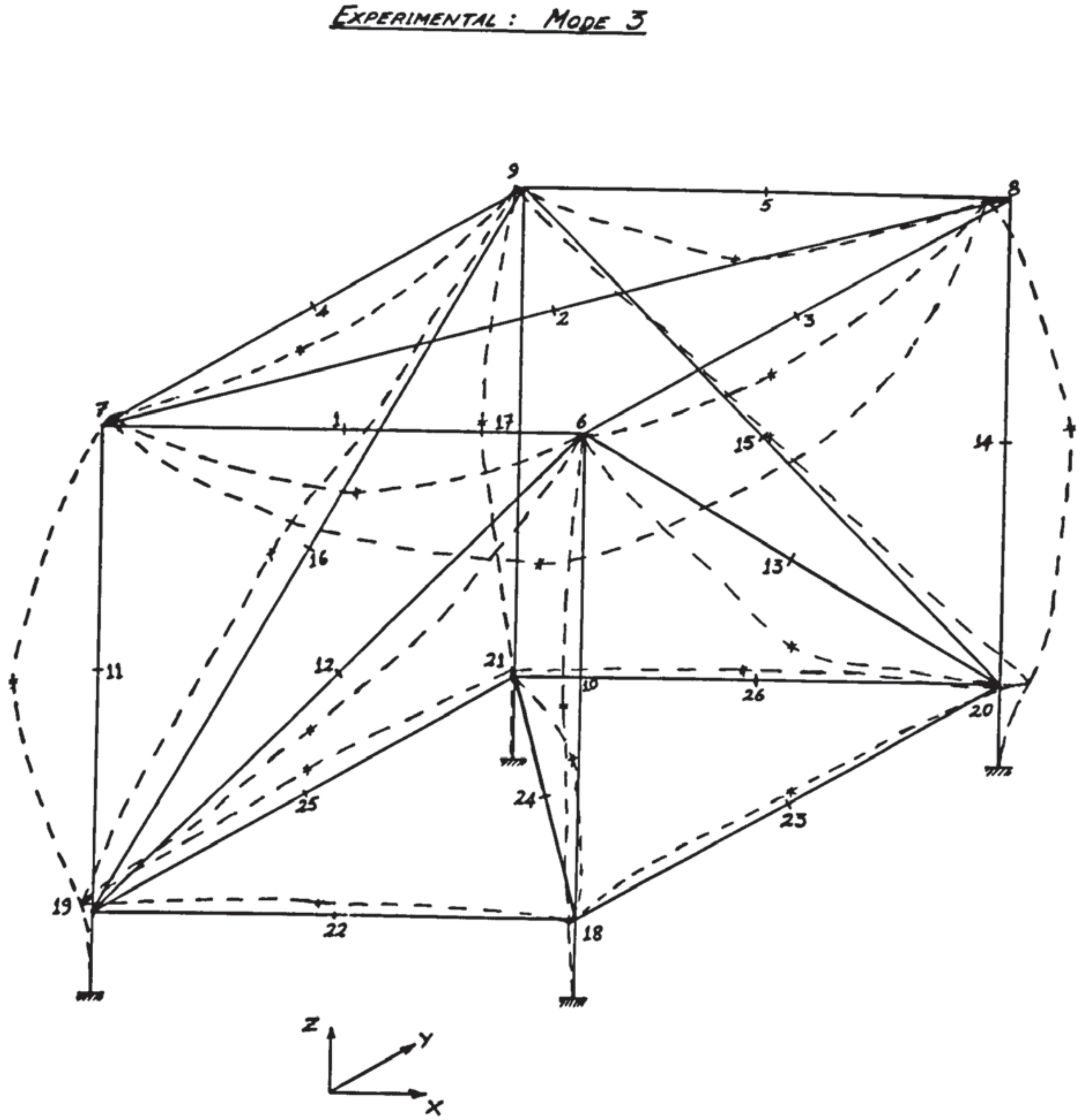
21
10
26
20
25
24
23
19
22
18
Z
Y
X

Fig. 6.4.3(a)


EXPERIMENTAL : MODE 3
9
8
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4
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Z
Y
X

Fig. 6.4.3(b)

CHAPTER SEVEN

CHAPTER SEVEN

INVESTIGATION OF BOUNDARY CONDITIONS

7.1 Introduction

The results of the experimental and theoretical vibration work on the space frame discussed in the last chapter have made to necessary to reconsider the assumed boundary conditions at the joints of the frame. This chapter describes the various steps taken to access the true boundary conditions at the joints.

The mass and stiffness matrices of a beam element (equations 3.2.48 and 3.2.47) derived in Chapter 3, represent the beam properties for a general free-free finite beam element. From these general matrices, the particular properties for any combination of beam element boundary conditions can be obtained. In elementary beam theory, the various types of boundary conditions include free, guided, hinged, and clamped end conditions.

These types of end conditions represent idealizations of the real boundary conditions for theoretical convenience. The greatest practical problems usually involve marching real systems such that the end conditions coincide exactly with these known ideal end conditions. An alternative approach is to find suitable theoretical end conditions which describe the real end conditions exactly. This alternative approach has been used here. Some possible methods of such approaches are illustrated as follows.

The clamped end of a beam represents a joint of zero rotation and transverse displacement. And the hinged end of a beam represents a joint of zero bending moment and transverse displacement. Now, a single span beam may be meant to be a clamped-clamped beam, but its behaviour could imply that the end conditions lie somewhere between a clamped-clamped beam and a hinged-hinged beam. Possibly, one first reaction to the situation is to calculate the fraction of the non-zero rotation and bending moment to the ideal value with assumed hinged and clamped end condition. A multiplication of the corresponding rows and columns of the beam properties matrices by this fraction would then be expected to yield the matrices required to describe the real end conditions exactly. Unfortunately, this method does not give any reasonable similation of the real system. Even on the simplest case of a simple span beam, the static deflection curve obtained from such theoretical analysis does not compare with the real system.

A second method of representing the real end conditions is given in ref.6, and is illustrated in fig. 7.1.1. A beam is shown in fig.7.1.1(a) hinged at the end B but with a non-ideal support at end A. This method of representation involves imagining the beam to be extended past the support as shown in fig.7.1.1(b). The elastic stiffness of this hypothetic extension A'A will determine the degree of end fixity at the support A. The length (ℓ_c) of this hypothetical equivalent "connection span" becomes an important parameter in the analysis. It is convenient, with its method, to assume the same value of flexural rigidity for the actual beam and the hypothetical equivalent span, thus variations in the degree of fixity will be fully expressed in terms of varying span length, ℓ_c.

This method looks simple enough for a simple problem but difficulties do arise when larger structures are being considered. For instance, in 2- and 3-dimensional frame structures, the degree of end fixity at a joint and hence the length (ℓ_c) of the hypothetical equivalent "connection span" will be different in the various directions (X-, Y-, Z-axes). Therefore, computation of the stiffness of the equivalent system will become more complicated and undesirable. Moreover, illustration of such a system on a drawing becomes impossible because of varying 'length' of the beam element in various axes directions.

A third method of representation seems to avoid the above difficulties, and gives a logically more satisfying picture of the real system. Here, it is imagined that an appropriate spring is included at the non-ideal joint to account for stiffness at that joint, as illustrated in fig.7.1.2. Fig. 7.1.2(a) shows a propped cantilever beam AB with the end A as a non-ideal clamping. Fig. 7.1.2(b) shows the equivalent system with a spring of stiffness K_c at the end A. The stiffness of the spring will be such that it accounts for the resisting bending moment and the rotation at the joint A of the real beam system.

Any type of joint fixity can be represented in this way. The number of degrees of freedom at the joint does not give additional difficulties in the representation and the logic of computation. The method is suited for the analysis of any frame structure by the finite element method. The actual boundary conditions at the joint are represented completely by the addition of "concentrated stiffnesses" to the appropriate elements of the assembled stiffness matrix of the structure. This method of representation of the actual joint boundary conditions have been used in this work.

The problem of assessing the boundary conditions at the joint is mainly a static one. As such the estimation of the stiffness K_c at the joint can be done from surely static considerations. Dynamic analysis can also be used later to confirm the results obtained.

The various types of joint conditions in the space frame are considered individually (or in small groups) in simple beam and plane frame configurations. Static tests are carried out on these simple members to obtain load-deflection relationships for the simple frame systems. The simple frame systems are then analysed using energy methods (ref.5) or elementary theory of beams to obtain expressions for the fictitious spring constants (K_c) of the springs in the equivalent system, in terms of the applied load and corresponding deflections.

The values calculated for the spring constants (K_c) at the joint are then fed into the programme MASSTIFPROP to obtain the mass and stiffness matrices of the simple frame system under consideration. The stiffness matrix obtained is tested on the programme STATICPROB to compute a static deflection shape of the simple frame structure. Computed and experimental static deflection shapes should be equivalent.

The simple frame structures are further analysed dynamically both experimentally and theoretically. Results of the natural frequencies and modal shapes are indications of the accuracy and reliability of the method of joint representation used.

Basically, three types of joints are present in the fabricated space frame of fig.2.2.1. These joints are simulated and analysed in the remaining part of this chapter to obtain the true boundary conditions.

7.2 Single Leg Member

Here, the brazed feet member is considered to determine its true boundary condition. This feet joint is supposed to behave like a clamped joint but this has not been the case. Fig. 7.2.1 shows the beam member used to investigate the feet boundary condition.

The single leg member, which is in the form of a cantilever, is first analysed statically. Load deflection shape of the member is obtained experimentally for both X- and Y-directions as shown in fig. 7.2.1. Fig.7.2.2 shows the point of application of the force and the other points 1, 2, 3 and 4 where the corresponding beam deflections are measured.

The deflections were plotted (graphs not shown) for all points to obtain the equivalent deflections of the beam for a load of 20 N, as shown in Table 7.2.1. Also, this table contains computed values for the deflection when the beam is assumed to be an ideal cantilever. Additional points (or nodes) 1a, 2a, 3a and 4a (fig. 7.2.2.) have been used for this computation. Fig. 7.2.3 shows a graphical comparison of the deflection shape for the experimental and theoretical results.

It can be seen from the graphs of fig.7.2.3 that the brazed feet are not acting like a clamped joint in both X- and Y-directions. Also, the boundary conditions are different in both these directions. The beam is analysed to develop a suitable theoretical model to the real beam boundary conditions, as follows.

Fig. 7.2.4 shows a model simulation of the beam. The condition at the brazed joint is represented by a spring (of stiffness k) which resists the bending moment at that point. By the elementary theory of beam bending the bending moment (M) at a point (x) along the beam is given by

$$EI \frac{d^2y}{dx^2} = M = -Wx \qquad 7.2.1$$

Integrating once, we have

$$EI \frac{dy}{dx} = -\frac{W}{2}x^2 + C_1 \qquad 7.2.2.$$

Boundary condition, at $x = \ell$, $\frac{dy}{dx} = \theta$

Substituting this in equation 7.2.2, we have

$$C_1 = EI\theta + \frac{W}{2}\ell^2$$

Therefore, equation 7.2.2 becomes

$$EI \frac{dy}{dx} = -\frac{W}{2}x^2 + \frac{W}{2}\ell^2 + EI\theta_1 \qquad 7.2.3$$

Integrating again, we have

$$EIy = -\frac{W}{6}x^3 + \frac{W}{2}\ell^2 x + EI\theta x + C_2 \qquad 7.2.4$$

TABLE 7.2.1

20 N Load at Point 4

Point	Deflections in mm		
	Experimental		Computed
	X-direction	Y-direction	
1a			0.049
1	0.50	0.25	0.189
2a			0.406
2	1.30	0.85	0.687
3a			1.020
3	2.35	1.67	1.392
4a			1.789
4	3.45	2.65	2.199

Boundary conditions,

at $x = 0$, $y = -\delta$

and at $x = \ell$, $y = 0$

Substituting these boundary conditions, into equation 7.2.4 we have

$$-EI\delta = C_2$$

and

$$0 = \frac{W}{3}\ell^3 + EI\theta\ell + C_2$$

Thus

$$\delta = \frac{W}{3EI}\ell^3 + \ell\theta \qquad 7.2.5$$

Now, taking moments about the point B, we have

$$K\theta = W\ell$$

or $$\theta \quad \frac{W\ell}{K} \qquad 7.2.6$$

Substituting equation 7.2.6 into 7.2.5, we have

$$\delta = \frac{W}{3EI}\ell^3 + \frac{W\ell^2}{K} \qquad 7.2.7$$

or $$\frac{\delta}{W} = \frac{\ell^3}{3EI} + \frac{\ell^2}{K} \qquad 7.2.8$$

Now, for the beam under consideration,

$\ell = 700$ mm, $E = 207{,}000$ N/mm^2

$I = 5043$ mm^4

In the X-direction,

$W = 20$ N and $\delta_x = 3.45$ mm

Substituting the above values into equation 7.2.8, and solving for the stiffness K_x in the X-direction, we have that

$$K_x = \underline{7.835 \times 10^6 \text{ N-mm}}$$

In the Y-direction,

$W = 20$ N and $\delta_y = 2.65$ mm.

Similarly

$$K_y = \underline{2.174 \times 10^7 \text{ N-mm}}$$

These values of stiffness constants were used in computing the beam elastic properties matrices by the computer programme MASSTIFPROP. The static deflection shapes obtained in both cases (X- and Y-directions) correspond to those obtained experimentally (fig. 7.2.3).

Having obtained static agreement, the beam was subjected to dynamic tests in the X-direction. The first three natural frequencies and modal shapes were obtained experimentally. The natural frequencies and modal shapes were also computed, with the beam discretized into 16 elements (33 D.O.F.).

TABLE 7.2.2

Mode	Natural Frequencies Hz	
	Computed	Experimental
1	34.31	33.80
2	237.39	224.60
3	696.82	645.00

The first three natural frequencies of the beam (computed and experimental) are shown in Table 7.2.2. These show an agreement to within less than 10% of each other. The modal shapes are the same for both computed and experimental results. The static deflection curve and the first three modal shapes are shown in fig. 7.2.5. It can be seen that the first modal shape corresponds to the static deflection curve.

7.3 Bent Cantilever Structure

The bent cantilever (fig. 7.3.1) considered here represents the single leg member just analysed plus an additional horizontal member fixed to its free end. The aim here is to obtain the joint boundary condition at the point B. The model of the bent cantilever is as shown in fig. 7.3.2.

Static deflection tests were carried out on the structure. Final values were obtained for the average deflections δ_1 and δ_2 at the points B and C for an applied load (W) of 20 N at the free end C. The values for the joint stiffnesses K_1 and K_2 at joints A and B respectively are then obtained by theoretical analysis of the structure as follows.

Referring to fig. 7.3.3, the bent cantilever structure can be analysed as two beam members and the elementary beam theory applied to each.

For member (2):

Taking point C as the origin, the bending moment of the beam is given by

$$EI \frac{d^2y}{dx^2} = M = -Wx \qquad 7.3.1$$

i.e. $$EI \frac{dy}{dx} = -\frac{W}{2}x^2 + C_1$$

At $x = \ell_2$, $\frac{dy}{dx} = \theta_3$

$$C_1 = \frac{W}{2}\ell_2{}^2 + EI\theta_3$$

And

$$EI\,\frac{dy}{dx} = -\,\frac{W}{2}\,x^2 + \frac{W}{2}\,\ell_2{}^2 + EI\theta_3$$

Integrating again, we have

$$EIy = -\,\frac{W}{6}\,x^3 + \frac{W}{2}\,\ell_2{}^2 x + EI\theta_3 x + C_2 \qquad 7.3.2$$

At $x = \ell_2 \quad y = 0$

and at $x = 0, \quad y = -\delta_2$

Substituting these boundary conditions into equation 7.3.2, we have

$$\delta_2 = \frac{W\ell_2{}^3}{3EI} + \ell_2\theta_3 \qquad 7.3.3$$

Taking moments about point B for member (2), we have

$$K_2(\theta_3 - \theta_2) = W\ell_2$$

or $$\theta_3 = \frac{W\ell_2}{K_2} + \theta_2 \qquad 7.3.4$$

Substituting equation 7.3.4 into equation 7.3.3 we have

$$\delta_2 = \frac{W\ell_2{}^3}{3EI} + \frac{W\ell_2{}^2}{K_3} + \ell_2\theta_2 \qquad 7.3.5$$

For member (1):

Taking point B as the origin, the bending moment of the beam is given by

$$EI\,\frac{d^2y}{dx^2} = M = -W\ell_2 \qquad 7.3.6$$

Thus $$EI\,\frac{dy}{dx} = -W\ell_2 x + C_2$$

At $x = 0, \quad \frac{dy}{dx} = \theta_2$

$$C_2 = EI\theta_2$$

And $$EI\frac{dy}{dx} = -W\ell_2 x + EI\theta_2 \qquad 7.3.7$$

Integrating

$$EIy = -\frac{1}{2}W\ell_2 x^2 + EI\theta_2 x + C_3 \qquad 7.3.8$$

At $x = 0, \quad y = -\delta_1$

and at $x = \ell_1, \quad y = 0$

Substituting these boundary conditions into equation 7.3.8, we obtain

$$\theta_2 = \frac{\delta_1}{\ell_1} + \frac{\ell_1\ell_2}{2EI} \qquad 7.3.9$$

Also at $x = \ell_1, \frac{dy}{dx} = \theta_1$, and from equation 7.3.7 we have

$$EI\theta_1 = -W\ell_1\ell_2 + EI\theta_2 \qquad 7.3.10$$

Now,

$$K_1\theta_1 = -K_2(\theta_3 - \theta_1) = W\ell_2 \qquad 7.3.11$$

From equations 7.3.9, 7.3.10 and 7.3.11, we have

$$\delta_1 = \frac{W\ell_1{}^2\ell_2}{2EI} \quad \frac{W\ell_1\ell_2}{K_1} \qquad 7.3.12$$

And from equations 7.3.5 and 7.3.9, we have

$$\delta_2 = \left(\frac{\ell_2}{3} + \frac{\ell_1}{2} + \frac{EI}{K_2}\right)\frac{W\ell_2{}^2}{EI} + \frac{\ell_2}{\ell_1}\delta_1 \qquad 7.3.13$$

Now $E = 207{,}000\ N/mm^2$, $I = 5023\ mm^4$

$\ell_1 = 700$ mm , $\ell_2 = 600$ mm

$W = 20$ N

$\delta_1 = 4$ mm , and $\delta_2 = 8.25$ mm

Substituting these values in equation 7.3.12 and 7.3.13, we have the following values:

$$K_1 = \underline{7.165 \times 10^6 \text{ N-mm}}$$

and $$K_2 = \underline{7.109 \times 10^6 \text{ N-mm}}$$

The bent cantilever was discretized into an 8 element 22 D.O.F.) system as shown in fig. 7.3.4. Using the above values for the stiffness constants (K_1 and K_2), the system is analysed to obtain the static deflection shape. The values obtained are in agreement with the experiment deflections.

Table 7.3.1 shows the experimental and computed values for the first three natural frequencies of the bent cantilever. These values are within 5% of each other. The computed and experimental modal shapes also correspond with each other. These modal shapes as well as the static deflection shape are plotted in fig. 7.3.5. And as expected, the modal shape for the first natural frequency takes the form of the static deflection configuration.

TABLE 7.3.1

Mode	Natural Frequencies Hz	
	Computed	Experimental
1	13.3	13.7
2	37.9	38.4
3	193.8	185.1

7.4 Portal Frame

The purpose of this section is to further verify the results of the previous section. The equivalent joint stiffness constants (K_1 and K_2) obtained in section 7.3 are here tested on a portal frame where a pair each of the two types of joints are present. Thus, no experimental static tests are carried out in this section.

Fig. 7.4.1 shows the model portal frame as a 12 elements (29 D.O.F) discretized system. Static deflection shapes were computed for two cases of the portal frame being acted upon by a horizontal sideways force at coordinate 8 and a central vertical force at coordinate 15.

Experimental and computed natural frequencies and modal shapes were also obtained. Table 7.4.1 shows the first three natural frequencies of the portal frame for both experimental and computed cases. These values are within 10% of each other.

The experimental and computed modal shapes are also very similar. The sideways sway static deflection shape and the first three modal shapes are shown in fig. 7.4.2. The first modal shape is similar to the sideways sway static deflection shape while the second modal shape is similar to the static deflection shape under the action of a central vertical force (not shown in the figure).

TABLE 7.4.1

Mode	Natural Frequencies Hz	
	Computed	Experimental
1	27.88	28.5
2	165.06	157.1
3	206.62	212.6

The agreement obtained above between the computed and experimental vibration characteristics of the real portal frame are reasonable. Thus, the values obtained for the joints stiffnesses, as well is the method of representation of the real system in the computer programmes, are reliable.

7.5 Cross Member

In this section, the aim is to determine the actual boundary conditions for the cross member joints of the space frame structure. As shown in fig. 7.5.1, the cross member beam is bolted at both ends to the stiffening plates of the space frame joints. With reference to the axes notation of fig. 7.5.1, the essential boundary parameters investigated here are the equivalent joint bending stiffnesses K_y and K_z in the Y- and Z-axis respectively.

Static deflection tests were carried out on the beam while loaded at point 4 and deflections measured at points 1 to 7 and A1 and B1 as indicated in fig. 7.5.2. From the measurements taken, the average deflections at these points have been calculated for 100 N loading and shown in Table 7.5.1. Also in the same table, are computed static deflections for the beam assuming clamped-clamped and hinged-hinged end conditions. Plots of these deflection shapes (not shown) indicate that the real end conditions of the cross-member beam are neither of a clamped or hinged nature.

Fig. 7.5.3 shows the model system which represents the real cross-member beam under flexure in either the Y- or Z-directions of fig.7.5.1. Here, the beam is assumed to be symmetric, thus the equivalent joint stiffnesses (K) at both end joints are equal.

TABLE 7.5.1

Points	Deflections in-mm			
	Experimental		Theoretical	
	Y-direction	Z-direction	Clamped ends	Hinged ends
A1	0.025	0.135		
1	0.067	0.280	0.036	0.336
2	0.175	0.530	0.114	0.629
3	0.280	0.750	0.193	0.836
4	0.340	0.760	0.229	0.915
5	0.290	0.690	0.193	0.836
6	0.190	0.510	0.114	0.629
7	0.080	0.270	0.036	0.336
B1	0.046	0.135		

Now from the elementary theory of beams, the bending moment (M) is given by

$$M = EI\,\frac{d^2y}{dx^2} = \frac{W}{2}x + K\theta - W[x - \frac{\ell}{2}] \qquad 7.5.1$$

where the term in square brackets [ ] equals zero when its value is negative.

Integrating equation 7.5.1, we have

$$EI\,\frac{dy}{dx} = \frac{W}{4}x^2 + K\theta x - \frac{W}{2}[x - \frac{\ell}{2}]^2 + D_1 \qquad 7.5.2$$

and

$$EIy = \frac{W}{12}x^3 + \frac{1}{2}K\theta x^2 - \frac{W}{6}[x - \frac{\ell}{2}]^3 + D_1x + D_2 \qquad 7.5.3$$

Now, at x = 0, y = 0, and from equation 7.5.3 we have:

$$D_2 = 0$$

and at x = 0, $\frac{dy}{dx} = \theta$, and equation 7.5.2 gives

$$D_1 = EI\theta$$

Equation 7.5.3 becomes

$$EIy = \frac{W}{12}x^3 + \frac{1}{2}K\ x^2 - \frac{W}{6}[x - \frac{\ell}{2}]^3 + EI\theta x \qquad 7.5.4$$

Also, at $x = \frac{\ell}{2}$, $y = \delta$, and equation 7.5.4 gives

$$EI\delta = \frac{W\ell^3}{96} + \frac{1}{2}\ell\theta(\frac{1}{4}K\ell + EI) \qquad 7.5.5$$

and at $x = \frac{\ell}{2}$, $\frac{dy}{dx} = 0$ and equation 7.5.2 gives

$$\frac{1}{2}\theta(K\ell + 2EI) = -\frac{W\ell^2}{16} \qquad 7.5.6$$

Eliminating θ from equations 7.5.5 and 7.5.6 we have

$$\delta = \frac{W\ell^3}{96EI}[1 - \frac{3}{2}.\frac{K\ell + 4EI}{K\ell + 2EI}] \qquad 7.5.7$$

Note that substituting K = 0 and K = ∞ in the above equation 7.5.7, we would obtain the standard beam theory central deflection for a hinged-hinged beam and a clamped-clamped beam respectively.

Now $E = 207{,}000\ N/mm^2$, $I = 5023\ mm^4$

$\ell = 770\ mm$, $W = 100\ N$

and $\delta_y = 0.340\ mm$, $\delta_z = 0.760\ mm$

Substituting these values into equation 7.5.7, gives the following results

$$K_y = \underline{1.394 \times 10^7\ N\text{-}mm}$$

and $$K_z = \underline{7.865 \times 10^5\ N\text{-}mm}$$

Computations of the static deflections of the cross-member beam using the above values of stiffness K_y and K_z in the 8 elements system (fig. 7.5.2) have both produced similar results as the experimental values in Table 7.5.1. Natural frequencies and modal shapes for both Y- and Z-directions were computed using the about joint stiffnesses.

Experimental vibration tests have been carried out to verify the computed results in the z-direction. Table 7.5.2 gives a comparison of the computed and experimental first three natural frequencies of vibration. From the table the first natural frequency is very accurate indeed, the third is within 2% of each other while the second has the most error of about 10%. However, generally, the correlation between the computed and experimental natural frequencies are also very close. Fig. 7.5.4 gives the static deflection and the first three modal shapes. As expected the first mode is similar to the static deflection shape.

TABLE 7.5.2

Mode	Natural Frequencies Hz	
	Computed	Experimental
1	117.27	117.60
2	435.96	484.70
3	967.08	984.20

7.6 Torsion Experiments

Basically, three types of joints are present in the space frame analysed in Chapter 6. These joints have been investigated so far only in terms of their resistance to bending. Thus, the equivalent joint stiffnesses obtained so far in this chapter refer to bending joint stiffnesses. In this section, the aim is to obtain the real joint stiffnesses in relation to torsional forces.

Owing to difficulties in investigating the vibration of non-circular section bars in torsion, the investigations in this section are very much restricted. In fact, only static experiments have been carried out. Theoretical models can be obtained for the joints to describe the real boundary conditions exactly.

The three types of joints of interest here include (a) the brazed feet of section 7.2 (fig. 7.2.1); (b) the corner joint B of fig.7.3.1 and (c) the bolted joint of the cross-member beam shown in fig. 7.5.1.

In all three types of joints, the theoretical model representing the torsional boundary conditions are similar . Fig. 7.6.1 gives such a model.

Applying elementary theory of beams to the beam under torsion, the torsional equation is given approximately by

$$T = -GI_x \frac{d\theta}{dx} \qquad 7.6.1$$

and

$$Tx = -GI_x\theta + D_o \qquad 7.6.2$$

Now at $x = \ell$, $\theta = \theta_B$

and at $x = 0$, $\theta = \theta_A$

Thus equation 7.6.2 becomes

$$T = GI_x(\theta_A - \theta_B) \qquad 7.6.3$$

Also

$$K_T\theta_B = T$$

or $$\theta_B = T/K_T \qquad 7.6.4$$

Equations 7.6.3 and 7.6.4 give

$$K_T = \frac{T}{\theta_A - \frac{T\ell}{GI_x}} \qquad 7.6.5$$

a) For the brazed feet, the following data were obtained

$G = 80{,}000\ N/mm^2$; $I_x = 10046\ mm^4$

$\ell = 695\ mm$; $T = 600\ N\text{-}mm$

and $\theta_A = \frac{0.250}{40} = 0.00625$

Substituting these in equation 7.6.5 gives

$$K_T = \underline{1.047 \times 10^5 \text{ N-mm}}$$

b) For the corner joint B, the following are the relevant data:

$G = 80{,}000\ \text{N/mm}^2$; $I_x = 10046\ \text{mm}^4$

$\ell = 590\ \text{mm}$; $T = 600\ \text{N-mm}$

and $$\theta_A = \frac{0.364}{40} = 0.0091 \text{ rad}$$

Equation 7.6.5 gives

$$K_T = \underline{6.929 \times 10^4 \text{ N-mm}}$$

c) For the bolted joints of the cross-member beam, the configuration is as shown in fig. 7.6.2. Equation 7.6.5 holds with the notation in the figure.

Here, the relevant data includes:

$G = 80{,}000\ \text{N/mm}^2$; $I_x = 10046\ \text{mm}^4$

$\ell = 385\ \text{mm}$; $T = 300\ \text{N-mm}$

and

$$\theta_A = \frac{0.450}{40} = 0.01125 \text{ rad}$$

and from equation 7.6.5, we get

$$K_T = \underline{2.701 \times 10^4 \text{ N-mm}}$$

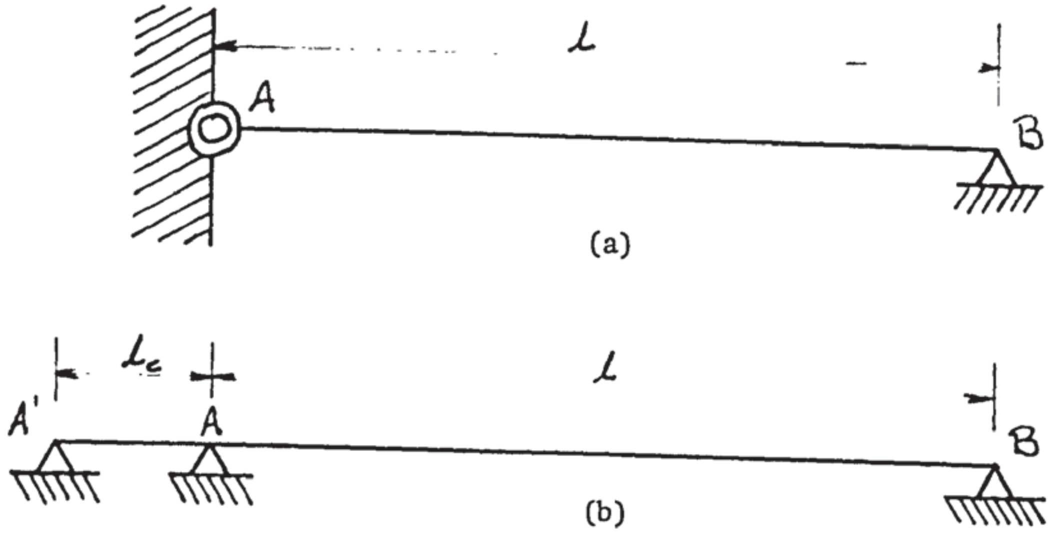


Fig. 7.1.1

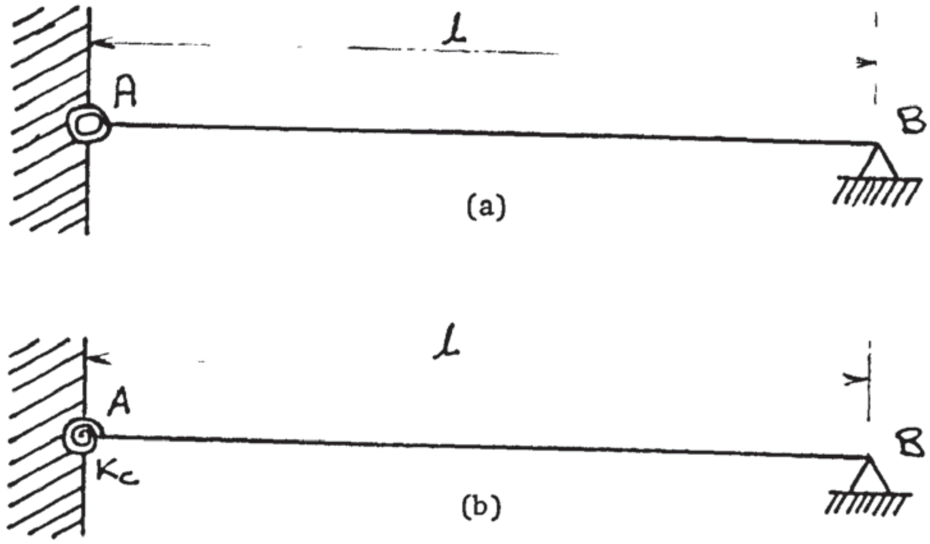


Fig. 7.1.2

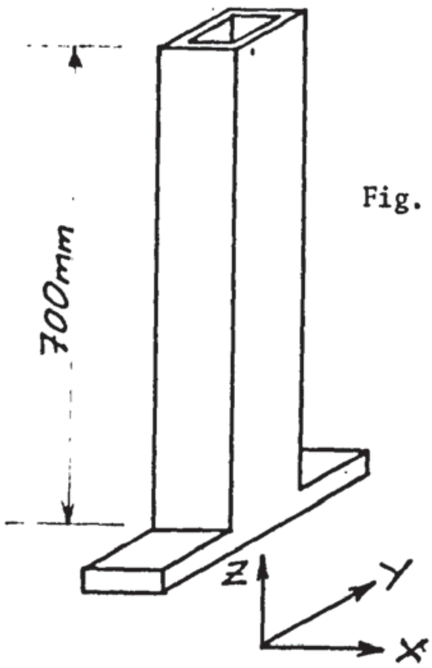


Fig. 7.2.1

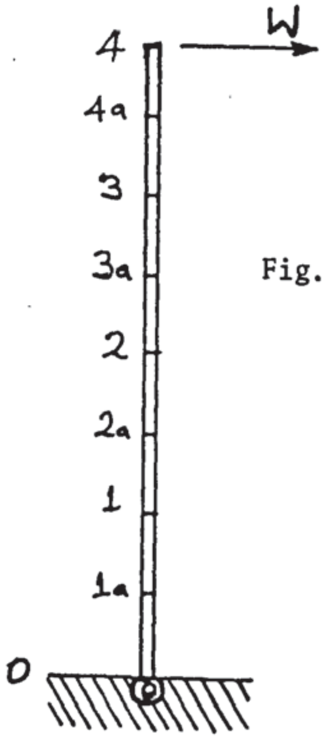
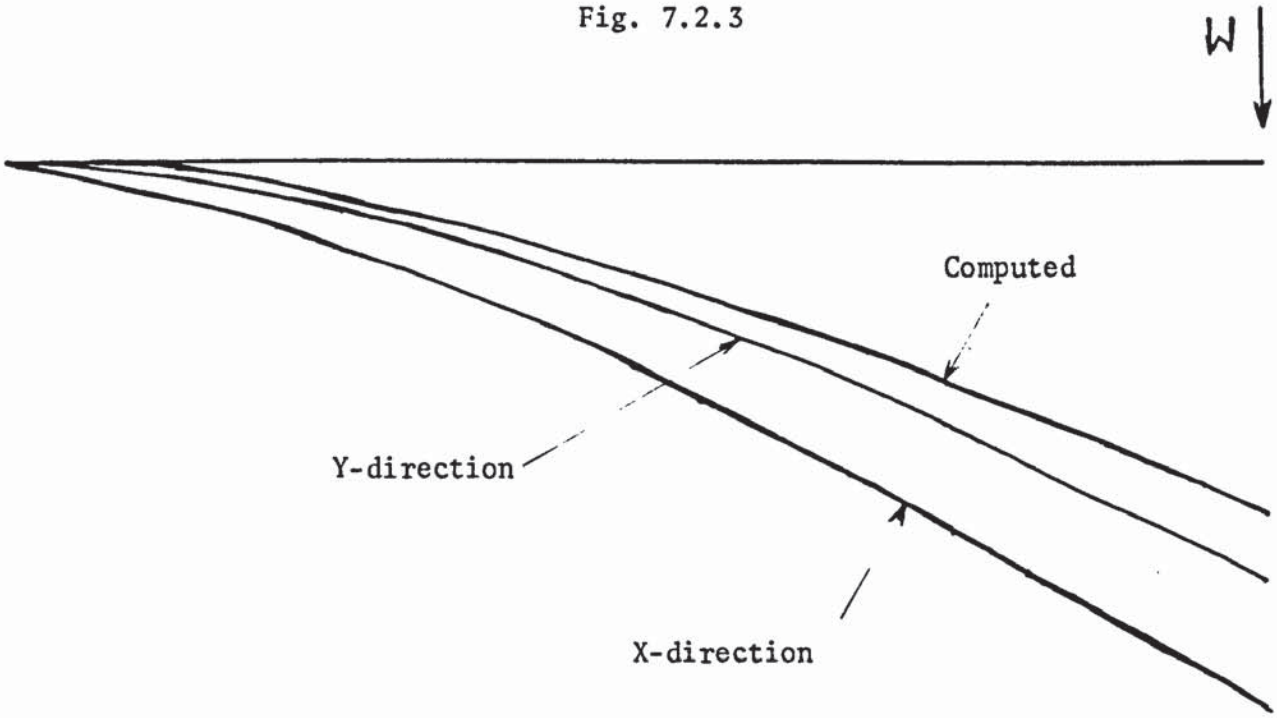


Fig. 7.2.2

Deflection Shape

Fig. 7.2.3



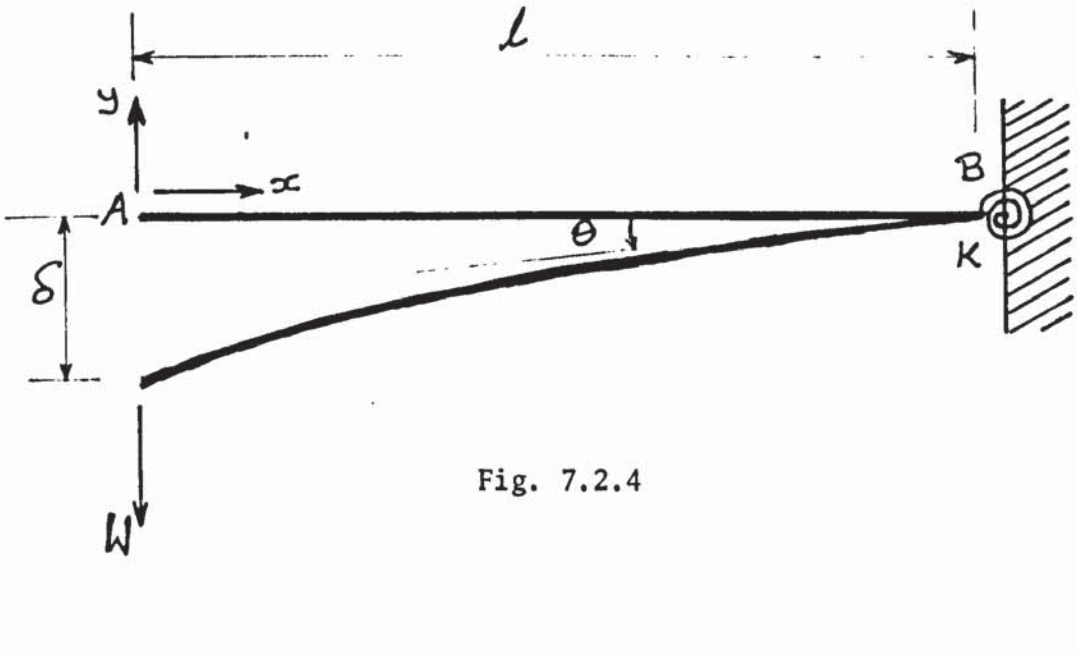


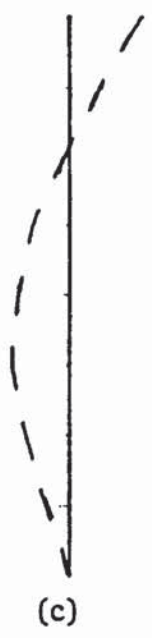
Fig. 7.2.4

(a)

Static Deflection

(b)

Mode 1



(c)

Mode 2

(d)

Mode 3

Fig. 7.2.5

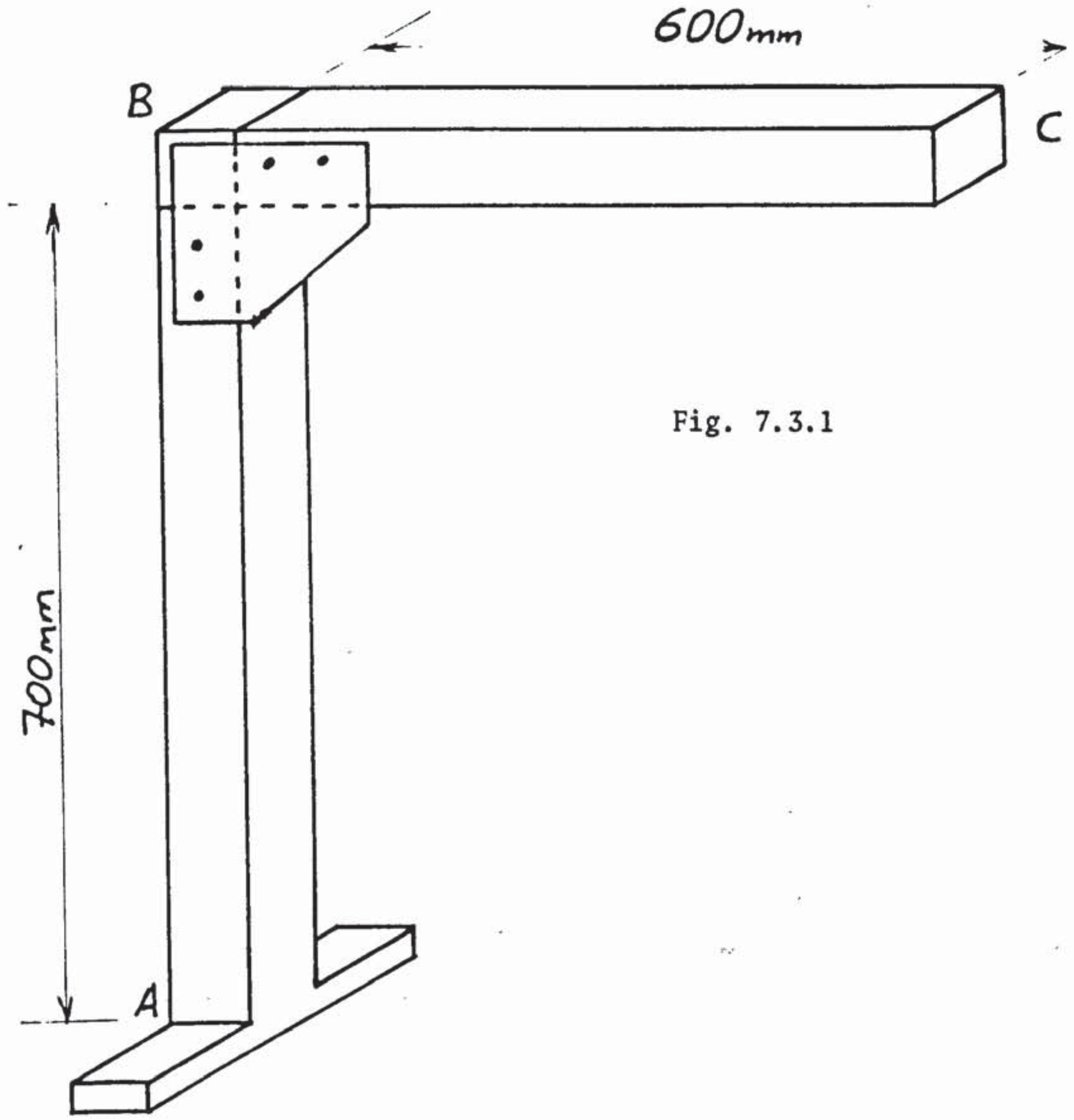


Fig. 7.3.1

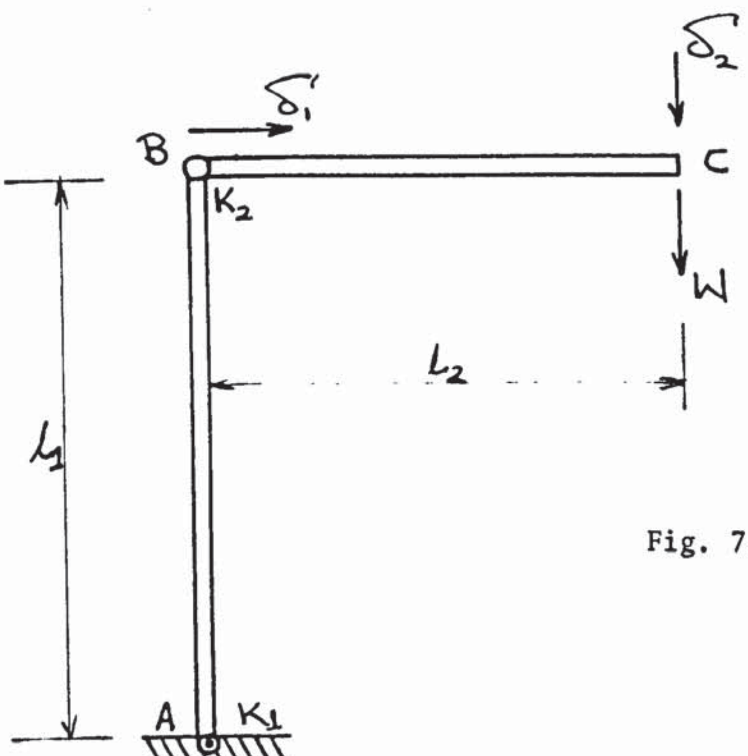


Fig. 7.3.2

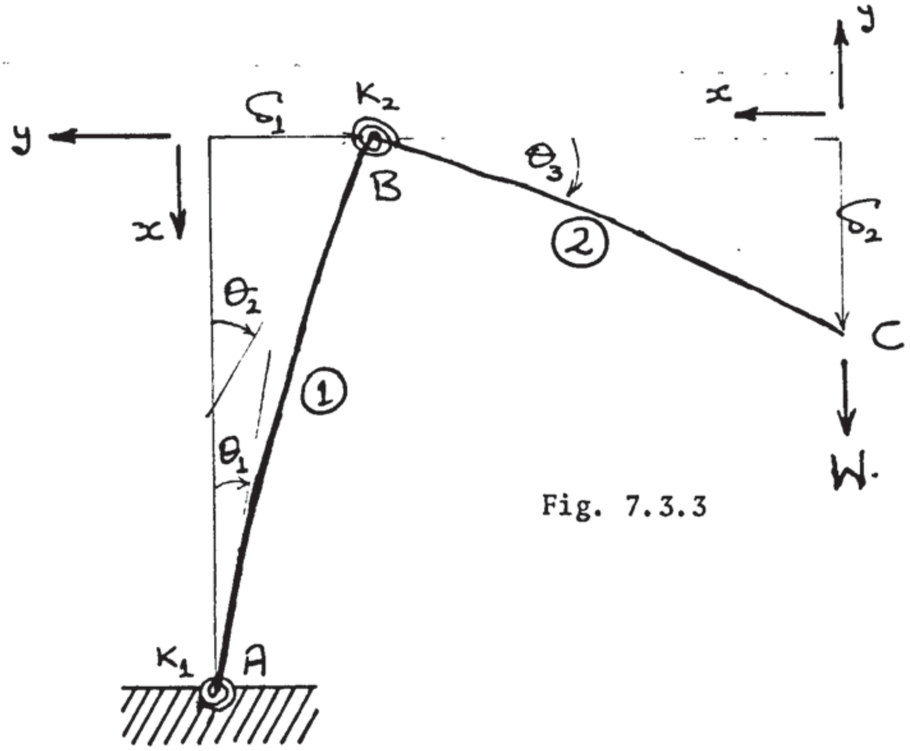


Fig. 7.3.3

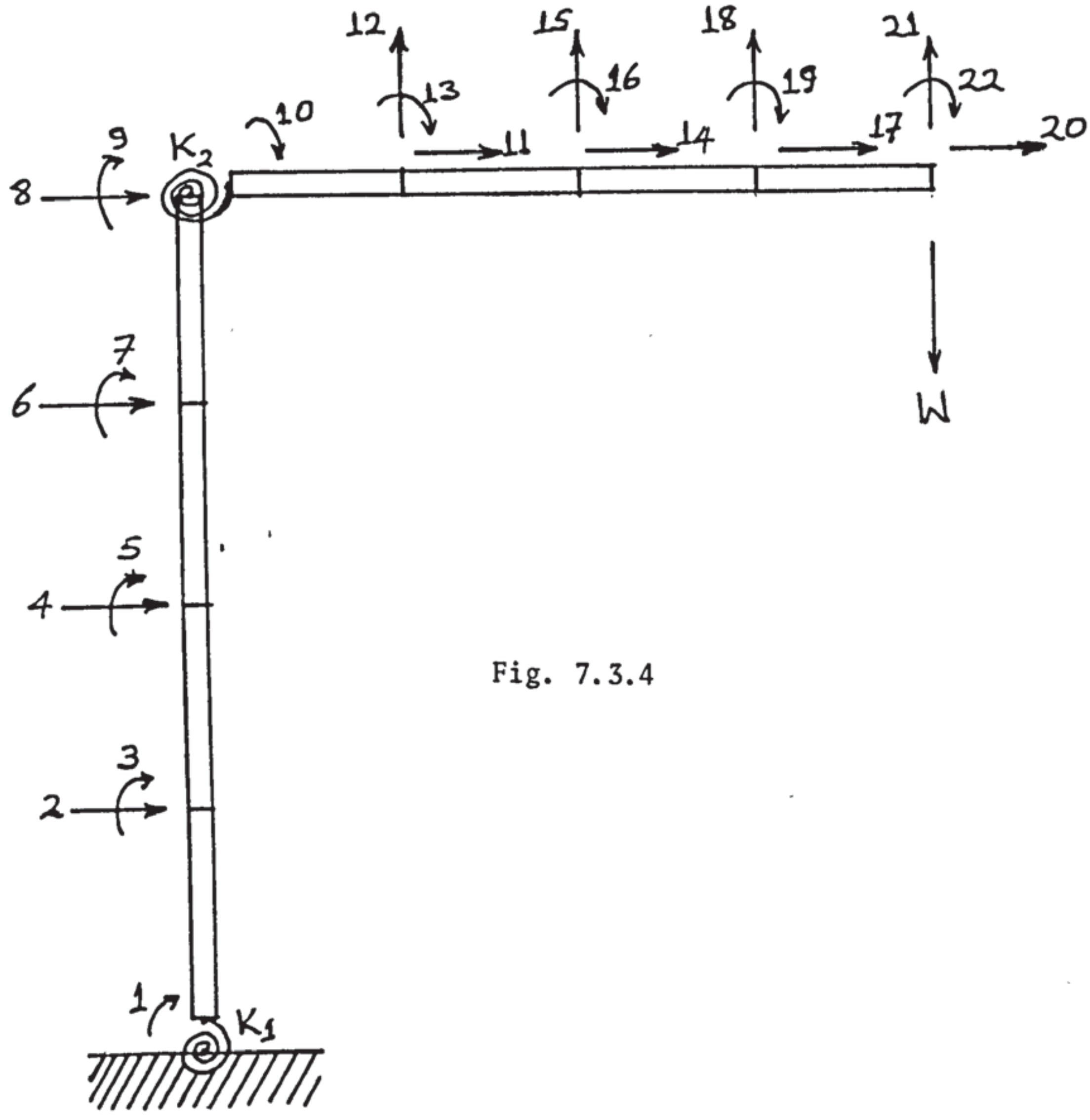


Fig. 7.3.4

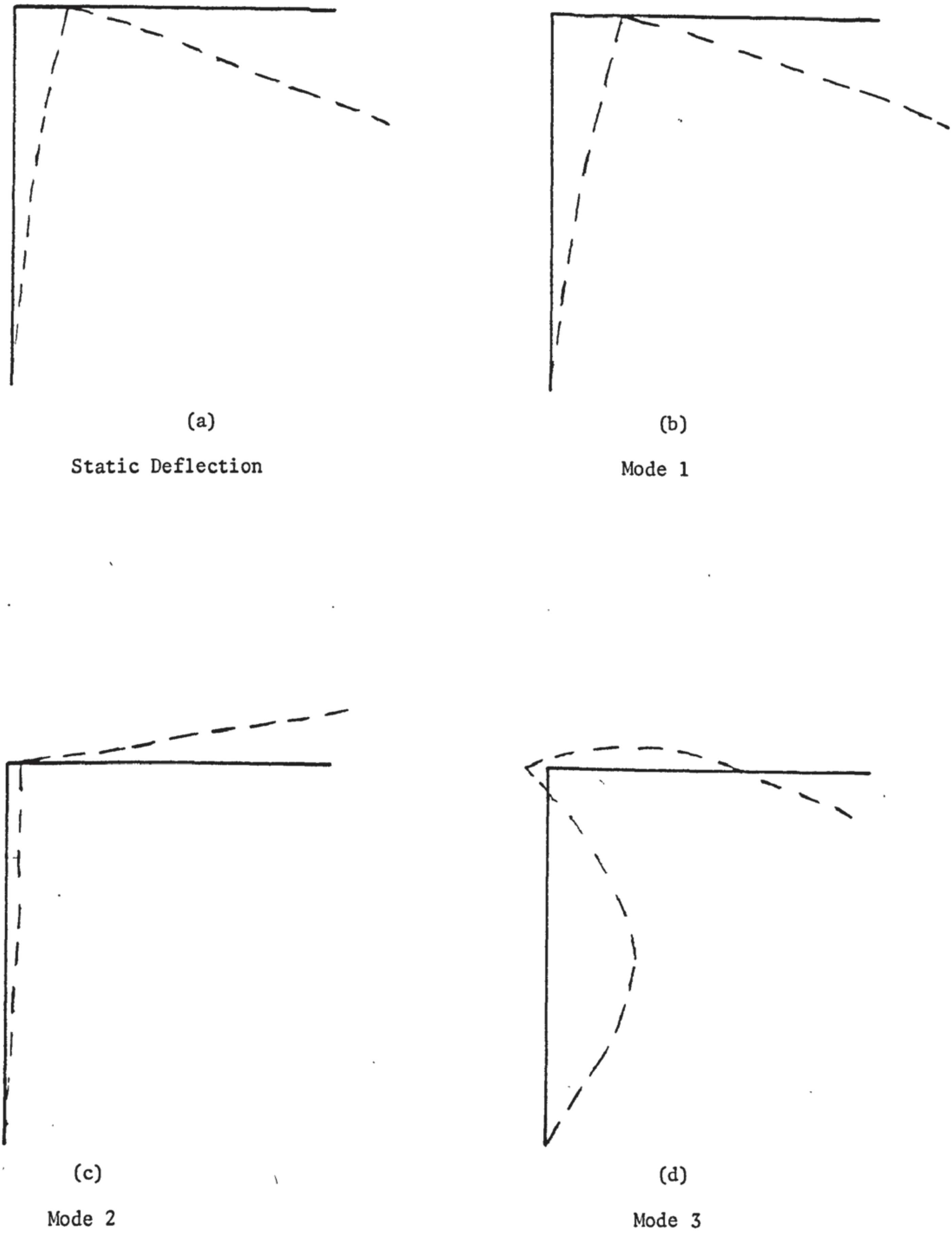


Fig. 7.3.5 <u>Bent Cantilever Shapes</u>

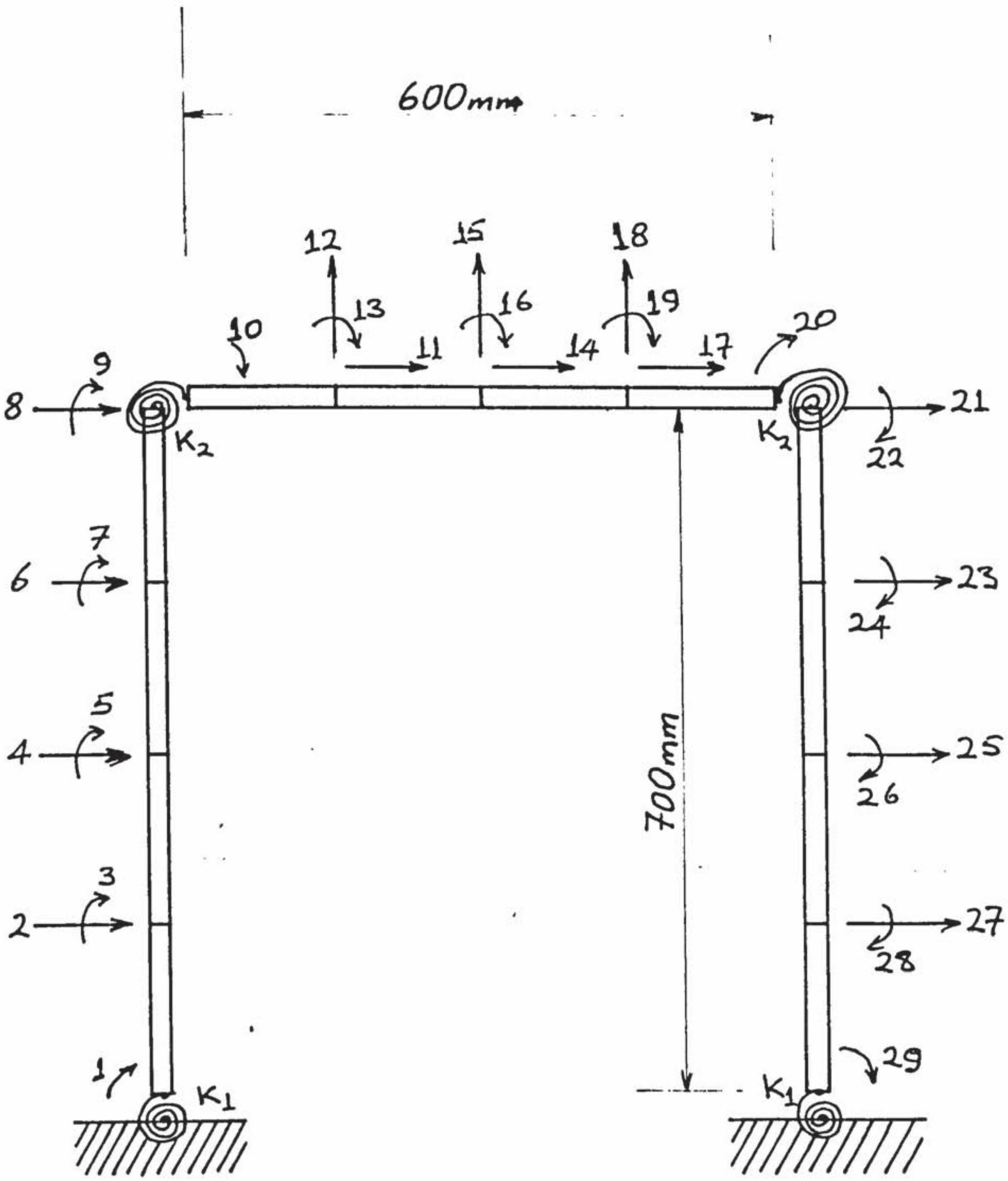


Fig. 7.4.1 Portal Frame

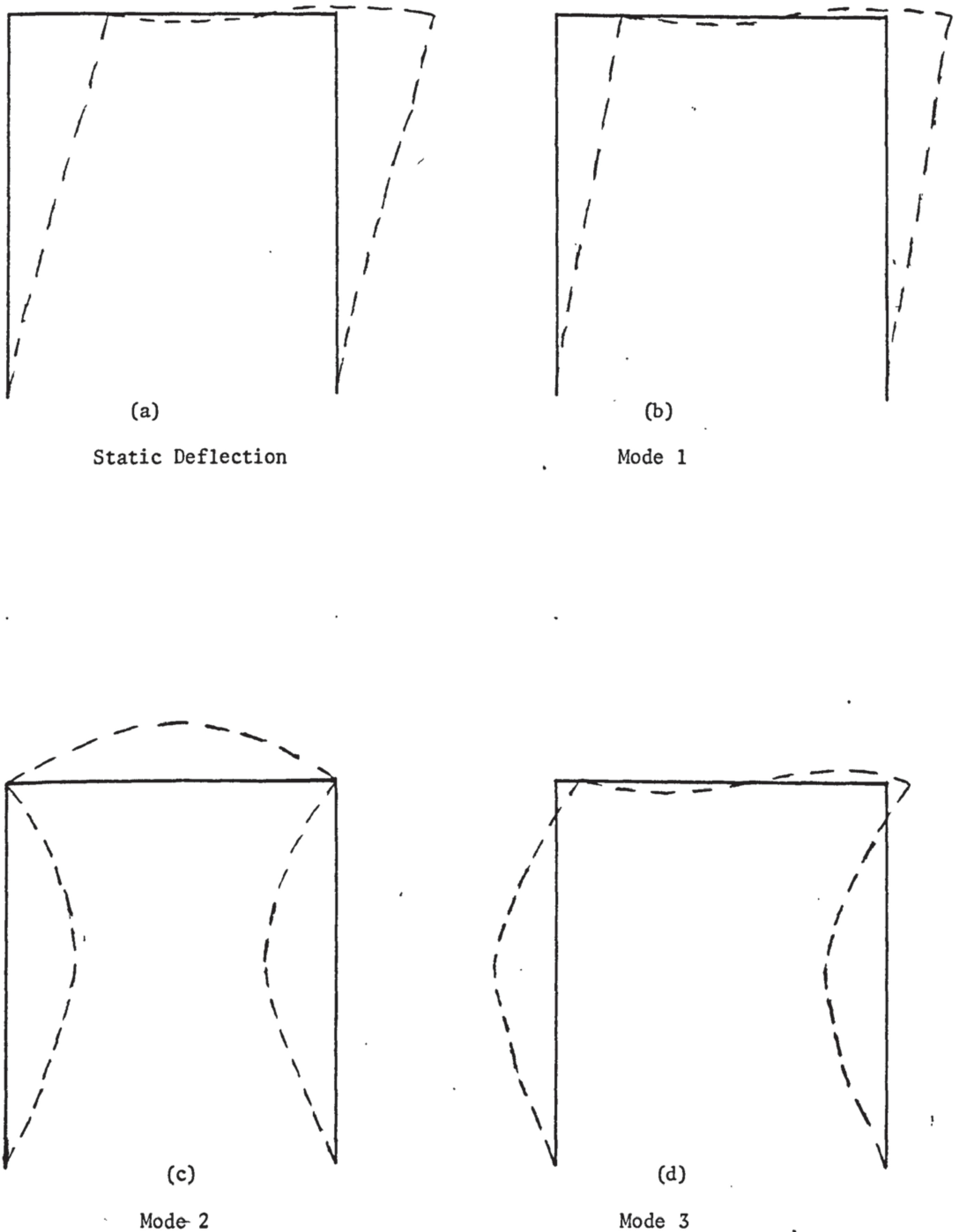


Fig. 7.4.2 <u>Portal Frame</u>

Fig. 7.5.1 Cross Member beam

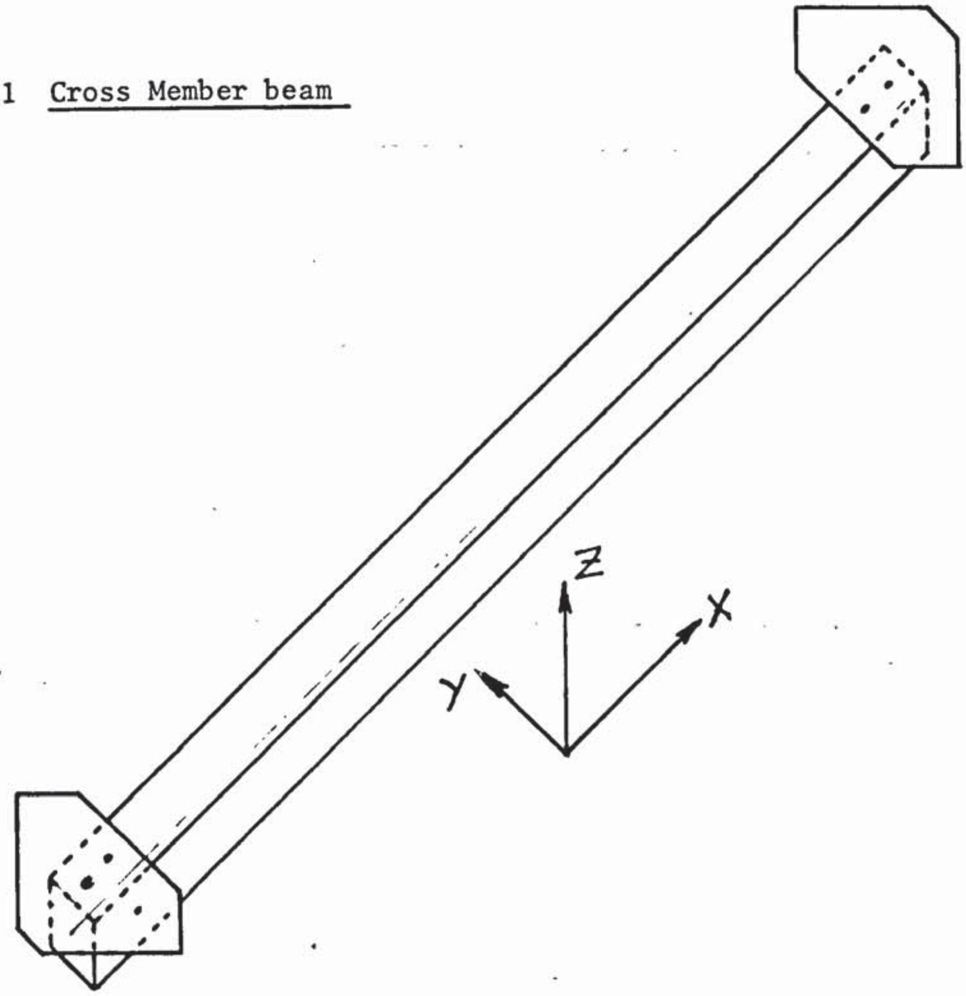


Fig. 7.5.2

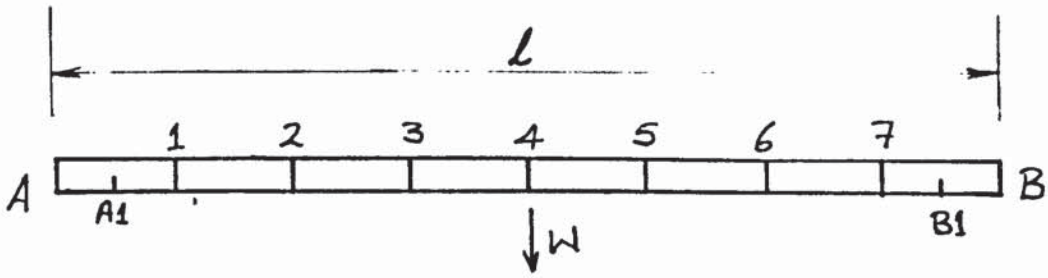
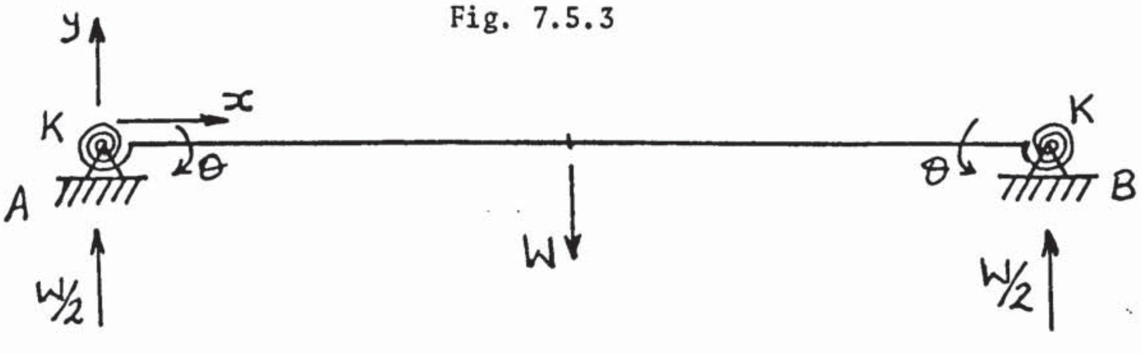


Fig. 7.5.3



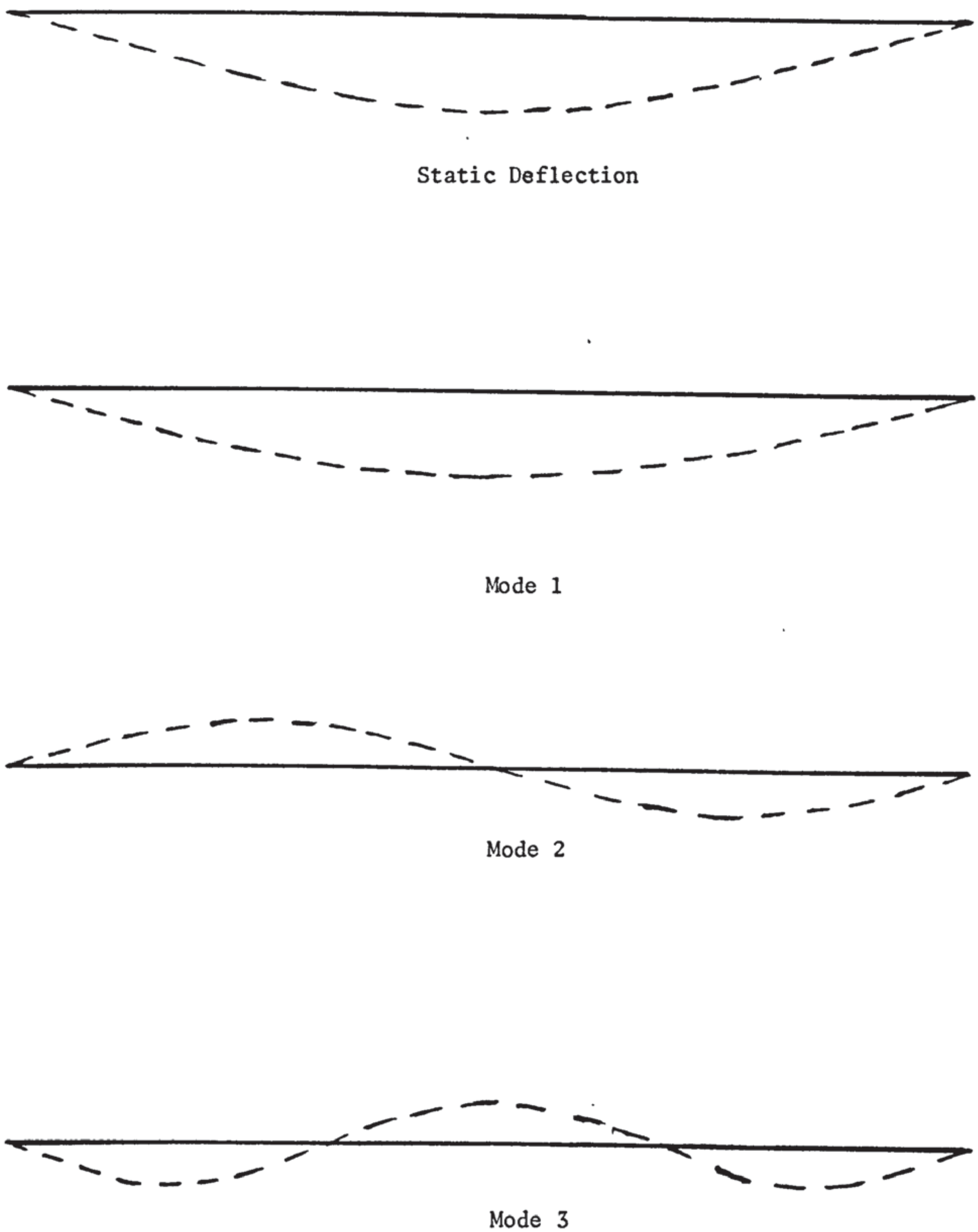


Fig. 7.5.4 Cross-member beam shapes

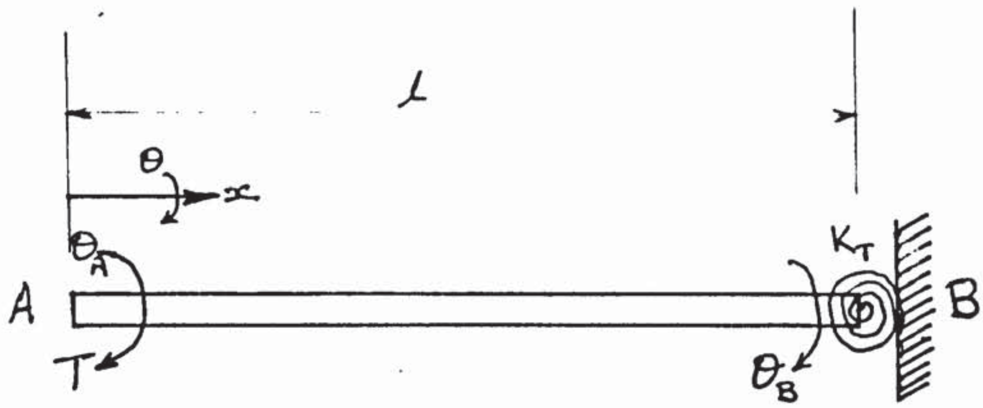


Fig. 7.6.1 Torsional beam model

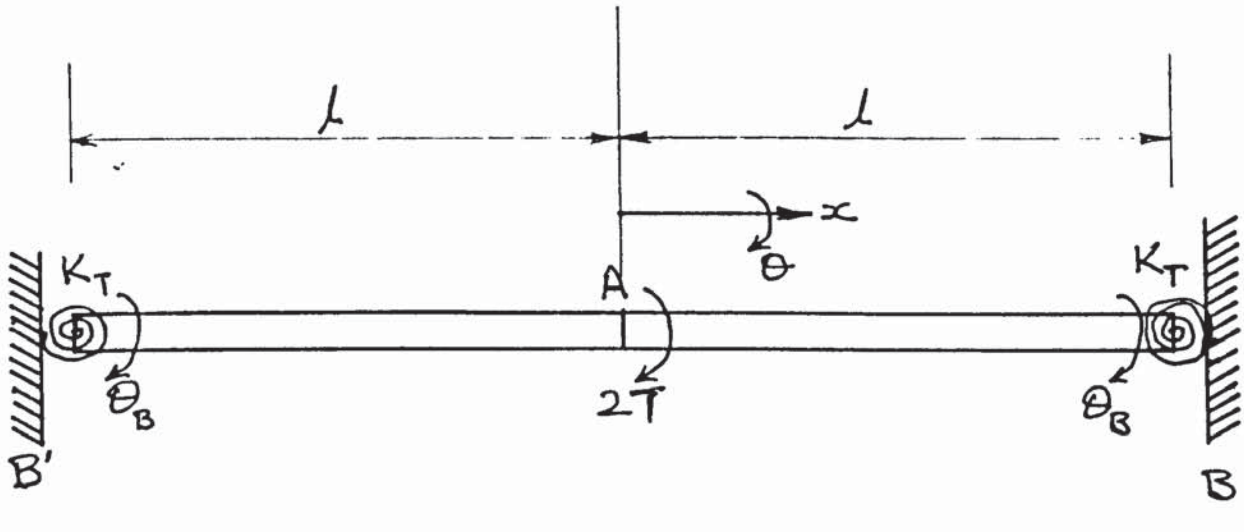


Fig. 7.6.2

CHAPTER EIGHT

CHAPTER EIGHT

DISCUSSIONS

8.1 Introduction

The investigations of the joint boundary conditions have yielded some positive results which are basic in the accurate analysis of frame structures. Having taken due account of the true joint boundary conditions, the computed static deflections, natural frequencies and modal shapes have all been equivalent to their experimental counterparts.

The true joint boundary conditions were not taken into account in the theoretical analyses of the space frame in Chapter 6. Having obtained the relevant joint stiffnesses, they should now be applied to the fabricated space frame.

8.2 Application of joint boundary conditions to Fabricated Space Frames

The three types of joints present in the fabricated space frame have been analysed in Chapter 7. These joints boundary conditions should be applied to the space frame of fig. 6.1.1 to produce a system which gives a closer representation of the space frame. Such application would in turn increase the number of degrees of freedom of the space frame during analysis.

Application of these boundary conditions will not affect the translational displacement coordinates of the space frame. Thus at each of the 26 finite element nodes of fig. 6.1.1, there will still exist 3 displacement coordinates. But the rotational coordinate will defer at all the real joints and remain the same at each of the beam midpoint finite element joints.

The brazed feet joints will now have 3 rotational coordinates each. These will defer from the type considered in the case of 168 D.O.F. system, because the rotational release will only be partial, not total as previously analysed. The partial release implies that the joint behaves in a form somewhere between a hinged and a clamped joint. The release factor is then accounted for by the fictitious spring as shown in Chapter 7.

Also,additional displacement coordinates can be deduced at each of the 8 corner joints. In fact, at these joints, each orthogonal beam members will contribute 3 rotational displacement coordinates.

Thus the 40 elements, finite element discretization of the Fabricated space frame (fig. 6.1.1) will yield a total of 228 D.O.F. system. Of these, 78 will be due to translational displacements, and 150 due to rotational displacements. The estimated computer core to run the eigenvalue problem programme NAGFEIGNVAL for such a 228 D.O.F. system is about 240 K. But the ICL 19045 computer currently available at the University Computer Centre only provide a maximum core size of

100K - less than half the requirement. The alternative computer at the Regional Computer Centre in Manchester only provides a maximum core size of 200K, which is still less then that required.

A practical way of avoiding the need for so much computer core is the reduction of the working number of degrees of freedom of the system (refs.1, 9, 15 etc.). This is achieved through some manipulation of the mass and stiffness matrices to obtain reduced matrices for the dynamic analysis. This usually involves distinguishing between the translational and rotational displacements of the structure as discussed in the next section.

8.3 Eliminating rotational displacements

The declared coordinates of the space frame include translational and rotational displacements. The experimental modal shape measurements involved only the translational displacement components. Generally, in dynamic analysis of structures, not all the static displacements are considered. For example, in the conventional dynamic analysis of aircraft wing structures only the deflections normal to the wing midplane are retained. By the same reasoning, it is useful, in this analysis of space frames, to retain only the translational displacements of the frame and eliminate the rotational displacements.

The first step in this elimination process is the partitioning of the stiffness matrix [K] and the displacement vector {q} of the structure in the form:

$$[K] = \begin{bmatrix} [K]_{t,t} & [K]_{t,r} \\ [K]_{r,t} & [K]_{r,r} \end{bmatrix} \qquad 8.3.1$$

and

$$\{q\} = \begin{Bmatrix} \{q\}_t \\ \{q\}_r \end{Bmatrix} \qquad 8.3.2$$

The vector $\{q\}_t$ refers to all the translational displacements which are to be retained as the degrees of freedom of the structure for the analysis. The vector $\{q\}_r$ refers to all the rotational displacements which are to be eliminated for the dynamic analysis. The stiffness matrix as partitioned in equation 8.3.1 is such that it is compatible with the partitional displacement vector.

The static equilibrium equation is given by equation 4.1.1 as

$$[K]\{q\} = \{Q\} \qquad 8.3.3$$

and in its partitioned form, it is given by

$$\begin{bmatrix} [K]_{t,t} & [K]_{t,r} \\ [K]_{r,t} & [K]_{r,r} \end{bmatrix} \begin{Bmatrix} \{q\}_t \\ \{q\}_r \end{Bmatrix} = \begin{Bmatrix} \{Q\}_t \\ \{Q\}_r \end{Bmatrix} \qquad 8.3.4$$

Assuming that the external forces $\{Q\}_r$ corresponding to the rotational displacements are equal to zero, we have from equation 8.3.4:

$$q_r = -[K]^{-1}_{r,r} [K]_{r,t} \{q\}_t \qquad 8.3.5$$

provided $[K]_{r,r}$ is not singular.

Substituting equation 8.3.5 into equation 8.3.4, we have

$$\{Q\}_t = ([K]_{t,t} - [K]_{t,r} [K]^{-1}_{r,r} [K]_{r,t}).\{q\}_t \qquad 8.3.6$$

or $$\{Q\}_t = [K]_c\{q\}_t \qquad 8.3.7$$

where

$$[K]_c = [K]_{t,t} - [K]_{t,r} [K]^{-1}_{r,r} [K]_{r,t} \qquad 8.3.8$$

$[K]_c$ represents the condensed stiffness matrix of the structure.

Let $[M]_c$ be the corresponding condensed mass matrix of the system. Let virtual displacements $\{\delta q\}$ be applied to the structure.

Then it follows from the ~~the~~ equivalence of the virtual work of the two equivalent mass representations of the continuous system that

$$\{\delta q\}^T_t(-[M]_c\{\ddot{q}\}_t = \{\delta q\}^T(-[M]\{\ddot{q}\})$$

or

$$\{\delta q\}^T_t[M]_c\{\ddot{q}\}_t = [\{\delta q\}^T_t\{\delta q\}^T_r][M] \begin{Bmatrix}\{\ddot{q}\}_t \\ \{\ddot{q}\}_r\end{Bmatrix} \qquad 8.3.9$$

Substituting equation 8.3.5 into equation 8.3.9 we have

$$\{\delta q\}^T_q[M]_c\{\ddot{q}\}_t = \{\delta q\}^T_t[A]^T_c[M][A]_c\{\ddot{q}\}_t \qquad 8.3.10$$

where

$$[A]_c = \begin{bmatrix} [I] \\ -[K]^{-1}_{r,r} [K]_{r,A} \end{bmatrix} \qquad 8.3.11$$

and [I] is an identity matrix.

Thus from equation 8.3.10, the condensed mass matrix of the structure is given by

$$[M]_c = [A]_c^T[M][A]_c \qquad 8.3.12$$

Equations 8.3.12 and 8.3.8 give the condensed mass and stiffness matrices respectively for any structure. When these equations are applied to the 40 elements finite element discretized space frame, two 78 x 78 matrices will be obtained for the dynamic analysis of the structure. This will represent a considerable reduction in computer core required for the dynamic analysis.

However, before arriving at these condensed matrices, large sized matrices are still to be manipulated. These may yet require more computer core than is currently obtainable. But there are ways to avoid the complete use of these large matrices during their manipulation.

Unfortunately, time has also run out for this work. Hence, it has not been possible to continue with the modifications required to arrive at the theoretical dynamic analysis required.

8.4 Further Observations

Some interesting observations can be made between the results of the preliminary investigations and the analysis of the fabricated space frame. In both cases, the experimental and theoretical modal shapes are similar.

In the case of the simple structures investigations, both the frequencies and modal shapes agree reasonably. Here, the real boundary conditions at the joints have been taken into account in the computations. Hence, the agreement between the computed and experimental vibration characteristics are to be expected.

The modal shapes of vibration obtained for these simple structures are very similar to the standard shapes where ideal joint boundary conditions are assumed. (Some of these standard modal shapes can be seen in ref.17). Thus, assumption of ideal boundary conditions at the joints should lead to reasonable theoretical modal shapes approximating the modal shapes for the real simple structure. The corresponding natural frequencies show high errors.

A similar pattern of behaviour has been shown by the fabricated space frame. The computed modal shapes have been based on assumed ideal joint boundary conditions. These have agreed reasonably well with the experimental modal shapes for the first three modes. But the corresponding computed natural frequencies show errors of up to 50%.

It is therefore expected that the application of the real joint boundary conditions to the fabricated space frame should lead to better agreements between the experimental and theoretical natural frequencies. The corresponding improvement in the overall pattern of the theoretical modal shapes should be minimal.

Natural frequencies have also been computed for the simple structures with assumed ideal joint boundary conditions. These were compared with the experimental values. The percentage errors in these natural frequencies have ranged from 10% to 35% in a somewhat random manner. But the percentage errors in the case of the fabricated space frame (as stated in section 6.5) is about 50%.

With the real joint boundary conditions, the percentage error in the computed first three natural frequencies is less than 10%. There is no direct interpolation between these percentage errors in the simple structures which can be applied to the fabricated space frame. The joint arrangement in the space frame is such that the commulative effects of the assumption of ideal joint boundary conditions cannot be deduced from the results of simple structures. But it is at least expected that application of the real boundary conditions should lead to very reasonable reductions in the error margin obtained.

CHAPTER NINE

CHAPTER NINE

CONCLUSION AND SUGGESTIONS

9.1 Conclusion

The vibration characteristics of fabricated space frame have been found both theoretically and experimentally. The modal shapes obtained theoretically and experimentally are very close. But the corresponding natural frequencies of vibration show that the theoretical values are as uch as 50% higher.

Investigations have shown that the errors in the natural frequencies are not as a result of a direct 125% over-estimation of the theoretical stiffness matrix. Rather, it is a result of the joint boundary conditions not being the ideal ones originally assumed in the computations.

Static experiments, theoretical computations and experimental vibration tests on simpler 1- and 2-dimensional frames have shown that due account should be taken of the actual joint conditions. Having done this, the experimental and theoretical natural frequencies and modal shapes have assumed much closer values for the simpler frame structures.

The method of representation of real joint conditions by the appropriate spring stiffnesses have been shown to be very effective.

It provides a practical and diagrammatic representation of the joint. In finite element methods, such representation is easily incorporated in the analysis.

For the simple frames, the modal shapes obtained are similar to those obtained when the joints are assumed to be ideal. This gives an indication that similar results should be obtained for the fabricated space frame. The theoretical modal shapes of the fabricated space frame (with ideal joints assumptions) are close to the experimental ones. Therefore, the theoretical modal shapes should be correct.

With an adequate size of computer the data obtained for the real joint boundary conditions can be fed into the MASSTIFPROP programme developed to obtain the true mass and stiffness matrices of the space frame. This, in turn, should yield the true vibration characteristics of the space frame. Alternatively, the suggested method (Chapter 8) of reducing the size of the matrices would yield the required results.

This work demonstrates the need and means of obtaining a more accurate assessment of the true joint conditions in vibration analysis of structures. This is not usually applied by the structural engineer at the design stages. A factor of safety is usually applied to account for the difference in the theoretical and real deflections of the structure. Such practice is unsatisfactory in vibration work. A true account of the joint boundary conditions, as shown in this work, should be preferred.

9.2 Suggestions for further work

It has been seen that there is an increase in the number of degrees of freedom of a frame when the real joint boundary conditions are taken into account in the finite element analysis. In the case of the fabricated space frame discussed in this work, the resulting problem has become too big for the available computer. It is possible to reduce the problem, for the same structure, to the capacity of the computer.

One method of reducing the problem has been discussed in Section 8.3 as the elimination of the rotational displacements from the matrix eigenvalue problem. The condensed mass and stiffness matrices obtained would be more manageable on the computer.

Another computational investigation relates to the effective use of the properties of the mass and stiffness matrices. The common mathematical methods do not take full advantage of the symmetric and banded nature of the mass and stiffness matrices in the solution of the eigenvalue problem. In static problems where such matrix properties are more fully used, problems of far more degrees of freedom can be solved in computers.

Having obtained better results for the space frame with the real joint boundary conditions, it will be useful to investigate the transient vibration response of the space frame. Optimum shock pulse should be investigated which will result in an optimum transient vibration response of the frame. Pessu (ref.33) suggests that the optimum shock pulse be a half sine wave whose duration is related to the natural frequencies of the structure.

Such optimum excitation analysis could be extended to situations where transient vibration response of structures is a parameter for assessment of the structural condition.

APPENDIX A

COMPUTER PROGRAMMES

BEST COPY

AVAILABLE

Variable print quality

APPENDIX A1

Compiled listing of Programme MASSTIFPROP

```
#####    #          ##          HHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHH
#    #   #          #           HHHH                   HHHHHHHHHHHH
#    #   #          #           HHHH                   HHHHHHHHHHHH
#    #   # #        #           HHHH                   HHHHHHHHHHHH    :EMP8041.COM
#    #   # #        #           HHHH                   HHHHHHHHHHHH
#    #   #####      #           HHHH                   HHHHHHHHHHHH
#####        #     #####        HHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHH

OF :EMP8041,COMPMASSTIF(1/B1B0)     PRODUCED ON  1JUN78 AT 13.12.43

T ASTON    IN ':EMP8041,COMPMASSTIF' ON  1JUN78 AT 13.12.45 USING  U14

   :EMP8041,COMPMASSTIF(/B1B0)

:EMP8041,COMPMASSTIF, 1JUN78 13.05.02 TYPE: BACK  10 : STREAM A : PRIORITY 1 : E
 JOB :EMP8041,COMPMASSTIF
JOB IS NOW FULLY STARTED
 COMPMASSTIF
 UAFORTRAN PROG MASSTIFPROP,OWNPD,NORUN
 MACROS,UAFORTRAN(205/)
        TA AB,CH,JL

: STATUS REPORT NUMBER 3  DATE 25 FEB  77

OMPILATION BY #XFIV MK 5B    DATE  01/06/78  TIME  13/09/35

                LIST(LP)
                PROGRAM (FXXX)
                INPUT  1 = CR0
                OUTPUT 2 = LP0
                OUTPUT 3 = TP0
                OUTPUT 4 = TP1
                OUTPUT 5 = TP2
                OUTPUT 6 = TP3
                OUTPUT 7 = TP4
                INPUT  8 = TR0
                INPUT  9 = TR1
                INPUT 10 = TR2
                INPUT 11 = TR3
                INPUT 12 = TR4
                COMPRESS INTEGER AND LOGICAL
                EXTENDED DATA
                TRACE 2
                END

                TRACE 1
                MASTER MASSTIFFNESS
          C
                COMMON/BLOCK0/ASSEMMATRIX(81,80)/
               1        BLOCK1/AMASS(12,12),ASTIF(12,12)/
               2        BLOCK2/WMASS(3,3),WSTIF(3,3),WKMAS(3,3),WKSTF(3,3)/
               3        BLOCK3/EMASS(12,12),ESTIF(12,12),ICOORD(12)/
               4        BLOCK4/TRNSM(3,3),TRNST(3,3)/
               5        BLOCK5/EMOD,GMOD,UNTMASS,XI,YI,ZI,AREA,ELENT/
```

```
     6              BLOCK6/NDOF,INP,IOUT,IPUNCH/
     7              BLOCK7/J,J0,J1,J2,K,K0,K1,K2,ND/
     8              BLOCK8/ALFA,BETA,GAMA
C
C
C
      INPC    =  1
      IOUT    =  2
C
C
C
C  THIS IS A GENERAL PROGRAM WHICH ASSEMBLES THE MASS AND STIFFNESS
C     MATRICES OF A SPACE FRAME STRUCTURE. IT ASSUMES THAT THE STRUCTURE
C     CONSISTS OF UNIFORM BEAM ELEMENTS (OF ANY CROSS SECTION) WITH A
C     TOTAL OF 12 D.O.F. FOR EACH BEAM. BY APPLYING THE B.CS. TO EACH
C     BEAM, THE CONTRIBUTION OF EACH BEAM IS ADDED TO THE ASSEMBLED MASS
C     AND STIFFNESS MATRICES. THESE PROPERTIES ARE ASSUMED TO BE
C     SYMMETRIC AND HENCE, ARE STORED IN A RECTANGULAR ARRAY ASSEMMATRIX
C     THE STIFFNESS MATRIX OCCUPIES THE UPPER TRIANGULAR MATRIX START-
C     ING WITH ELEMENT (1,1) THROUGH TO ELEMENT (NDOF,NDOF). WHILE THE
C     MASS MATRIX IS STORED AS A LOWER TRIANGULAR MATRIX STARTING WITH
C     ELEMENT (2,1) THROUGH TO ELEMENT (NDOF + 1,NDOF).
C        THERE IS ROOM ALSO FOR ADDING CONSENTRATED MASS AND
C     STIFFNESS PROPERTIES TO THE APPROPRIATE ELEMENTS OF THE
C     MATRICES.
C
C
C
      READ (INPC,10)          N
      DO 110 M = 1,N
C
      READ (INPC,10)          INP,IPUNCH
C
      READ(INP,10) NDOF,NOELEM
      READ(INP,20) EMOD,GMOD,UNTMASS
      READ(INP,30) XI,YI,ZI,AREA
C
C
   10 FORMAT(2I5)
   20 FORMAT(2F10.2,F10.7)
   30 FORMAT(4F10.2)
   40 FORMAT(I3)
   50 FORMAT(1H1//6X,28HNUMBER OF DEGREES OF FREEDOM,11X,'=',I9//6X,18HN
     1UMBER OF ELEMENTS,21X,'=',I9//6X,19HYOUNGS   MODULUS  E,20X,'=',F1
     22.2,17H  NEWTONS / SQ MM//6X,19HRIGIDITY MODULUS  G,20X,'=',F12.2,
     317H  NEWTONS / SQ MM//6X,20HMASS PER UNIT LENGTH,19X,'=',F12.7,10H
     4  KG / MM //6X,34HSECOND MOMENT OF AREA ABOUT X-AXIS,5X,'=',F12.2,
     511H   (MM ** 4)//34X,6HY-AXIS,5X,'=',F12.2,11H   (MM ** 4)//34X,6HZ-
     6AXIS,5X,'=',F12.2,11H   (MM ** 4)//6X,28HELEMENT CROSS-SECTIONAL AR
     7EA,11X,'=',F12.2,12H  SQ MM      ///5X,12H     ELEMENT,10X,6HLENGTH
     8,10X,10HANGLE ALFA,8X,10HANGLE BETA,8X,10HANGLE GAMA/11X,6HNUMBER,
     912X,'MM',14X,7HDEGREES,11X,7HDEGREES,11X,7HDEGREES/)
C
      WRITE(IOUT,50) NDOF,NOELEM,EMOD,GMOD,UNTMASS,XI,YI,ZI,AREA
C
      CALL ZEROASSEM
C
      DO 100 I = 1,NOELEM
      READ(INP,40) ICODE
      IF(ICODE - 1)  60,70,80
C
   60 CALL ELEMENT
```

```
   70 CALL DIRECTN
   80 CALL ASSEMBLE
      WRITE(IOUT,90) I,ELENT,ALFA,BETA,GAMA
   90 FORMAT(9X,I5,9X,F10.2,3(8X,F10.2))
  100 CONTINUE
C
      CALL POINTPROP
C
      ND  =  7
      CALL STOREMATX
C
  110 CONTINUE
C
      STOP
      END

SEGMENT, LENGTH    151, NAME   MASSTIFFNESS

C
C
C
C
      SUBROUTINE ZEROASSEM
C
      COMMON/BLOCK0/ASSEMMATRIX(81,80)/
     1       BLOCK3/EMASS(12,12),ESTIF(12,12),ICOORD(12)/
     2       BLOCK6/NDOF,INP,IOUT,IPUNCH/
     3       BLOCK7/J,J0,J1,J2,K,K0,K1,K2,ND
C
      K0    =    NDOF   +   1
      DO 400 K1 = 1,K0
      DO 400 K2 = 1,NDOF
      ASSEMMATRIX(K1,K2)    =    0.0
  400 CONTINUE
C
      DO 450 K1 = 1,12
      DO 450 K2 = 1,12
      EMASS(K1,K2)     =     0.0
      ESTIF(K1,K2)     =     0.0
  450 CONTINUE
C
      RETURN
      END

SEGMENT, LENGTH     72, NAME   ZEROASSEM

C
C
C
      SUBROUTINE ELEMENT
C
      COMMON/BLOCK3/EMASS(12,12),ESTIF(12,12),ICOORD(12)/
     1       BLOCK5/EMOD,GMOD,UNTMASS,XI,YI,ZI,AREA,ELENT/
     *       BLOCK6/NDOF,INP,IOUT,IPUNCH/
     2       BLOCK7/J,J0,J1,J2,K,K0,K1,K2,ND
C
      READ (INP,500)       ELENT
  500 FORMAT (F10.2)
```

```
C
      C1    =   UNTMASS * ELENT
      C2    =   C1  *  ELENT
      C3    =   C2  *  ELENT
C
      EMASS(1,1)     ,  EMASS(7,7)    =   C1 / 3.0
      EMASS(2,2)     ,  EMASS(8,8)    =   C1 * 13.0 / 35.0
      EMASS(3,3)     ,  EMASS(9,9)    =   EMASS(2,2)
      EMASS(4,4)     ,  EMASS(10,10)  =   C1 * XI / (AREA * 3.0)
      EMASS(5,5)     ,  EMASS(11,11)  =   C3 / 105.0
      EMASS(6,6)     ,  EMASS(12,12)  =   EMASS(5,5)
      EMASS(1,7)     ,  EMASS(7,1)    =   C1 / 6.0
      EMASS(2,6)     ,  EMASS(6,2)    =   C2 * 11.0 / 210.0
      EMASS(2,8)     ,  EMASS(8,2)    =   C1 * 9.0 / 70.0
      EMASS(2,12)    ,  EMASS(12,2)   = - C2 * 13.0 / 420.0
      EMASS(3,5)     ,  EMASS(5,3)    = - EMASS(2,6)
      EMASS(3,9)     ,  EMASS(9,3)    =   EMASS(2,8)
      EMASS(3,11)    ,  EMASS(11,3)   = - EMASS(2,12)
      EMASS(4,10)    ,  EMASS(10,4)   =   C1 * XI / (AREA * 6.0)
      EMASS(5,9)     ,  EMASS(9,5)    =   EMASS(2,12)
      EMASS(5,11)    ,  EMASS(11,5)   = - C3 / 140.0
      EMASS(6,8)     ,  EMASS(8,6)    =   EMASS(3,11)
      EMASS(6,12)    ,  EMASS(12,6)   =   EMASS(5,11)
      EMASS(8,12)    ,  EMASS(12,8)   =   EMASS(3,5)
      EMASS(9,11)    ,  EMASS(11,9)   =   EMASS(2,6)
C
C
C
      ESTIF(1,1)     ,  ESTIF(7,7)    =   EMOD * AREA / ELENT
      ESTIF(2,2)     ,  ESTIF(8,8)    =   EMOD * ZI * 12.0 / ELENT ** 3
      ESTIF(3,3)     ,  ESTIF(9,9)    =   EMOD * YI * 12.0 / ELENT ** 3
      ESTIF(4,4)     ,  ESTIF(10,10)  =   GMOD * XI / ELENT
      ESTIF(5,5)     ,  ESTIF(11,11)  =   EMOD * YI * 4.0 / ELENT
      ESTIF(6,6)     ,  ESTIF(12,12)  =   EMOD * ZI * 4.0 / ELENT
      ESTIF(1,7)     ,  ESTIF(7,1)    = - ESTIF(1,1)
      ESTIF(2,6)     ,  ESTIF(6,2)    =   EMOD * ZI * 6.0 / ELENT ** 2
      ESTIF(2,8)     ,  ESTIF(8,2)    = - ESTIF(2,2)
      ESTIF(2,12)    ,  ESTIF(12,2)   =   ESTIF(2,6)
      ESTIF(3,5)     ,  ESTIF(5,3)    = - EMOD * YI * 6.0 / ELENT ** 2
      ESTIF(3,9)     ,  ESTIF(9,3)    = - ESTIF(3,3)
      ESTIF(3,11)    ,  ESTIF(11,3)   =   ESTIF(3,5)
      ESTIF(4,10)    ,  ESTIF(10,4)   = - ESTIF(4,4)
      ESTIF(5,9)     ,  ESTIF(9,5)    = - ESTIF(3,5)
      ESTIF(5,11)    ,  ESTIF(11,5)   =   EMOD * YI * 2.0 / ELENT
      ESTIF(6,8)     ,  ESTIF(8,6)    = - ESTIF(2,6)
      ESTIF(6,12)    ,  ESTIF(12,6)   =   EMOD * ZI * 2.0 / ELENT
      ESTIF(8,12)    ,  ESTIF(12,8)   =   ESTIF(6,8)
      ESTIF(9,11)    ,  ESTIF(11,9)   =   ESTIF(5,9)
C
C
      RETURN
      END

MENT, LENGTH  358, NAME  ELEMENT

C
C
C
      SUBROUTINE TRANSFORM
C
```

```
      COMMON/BLOCK4/TRNSM(3,3),TRNST(3,3)/
     1       BLOCK7/J,J0,J1,J2,K,K0,K1,K2,ND/
     2       BLOCK8/ALFA,BETA,GAMA
C
      PAI      =  3.1415927 / 180.0
      SALFA    =  SIN(ALFA * PAI)
      SBETA    =  SIN(BETA * PAI)
      SGAMA    =  SIN(GAMA * PAI)
      CALFA    =  COS(ALFA * PAI)
      CBETA    =  COS(BETA * PAI)
      CGAMA    =  COS(GAMA * PAI)
C
      TRNSM(1,1)  =  -CALFA * CBETA
      TRNSM(1,2)  =   SALFA * CGAMA - CALFA * SBETA * SGAMA
      TRNSM(1,3)  =  -SALFA * SGAMA - CALFA * SBETA * CGAMA
      TRNSM(2,1)  =  -SALFA * CBETA
      TRNSM(2,2)  =  -CALFA * CGAMA - SALFA * SBETA * SGAMA
      TRNSM(2,3)  =   CALFA * SGAMA - SALFA * SBETA * CGAMA
      TRNSM(3,1)  =   SBETA
      TRNSM(3,2)  =  -CBETA * SGAMA
      TRNSM(3,3)  =  -CBETA * CGAMA
C
C
      DO 590 K1 = 1,3
      DO 590 K2 = 1,3
      TRNST(K1,K2)  =  TRNSM(K2,K1)
  590 CONTINUE
C
      RETURN
      END

SEGMENT, LENGTH   149, NAME  TRANSFORM

C
C
C
      SUBROUTINE DIRECTN
C
      COMMON/BLOCK1/AMASS(12,12),ASTIF(12,12)/
     1       BLOCK2/WMASS(3,3),WSTIF(3,3),WKMAS(3,3),WKSTF(3,3)/
     2       BLOCK3/EMASS(12,12),ESTIF(12,12),ICOORD(12)/
     3       BLOCK4/TRNSM(3,3),TRNST(3,3)/
     4       BLOCK6/NDOF,INP,IOUT,IPUNCH/
     5       BLOCK7/J,J0,J1,J2,K,K0,K1,K2,ND/
     6       BLOCK8/ALFA,BETA,GAMA
C
      READ (INP,600)   ALFA,BETA,GAMA
  600 FORMAT (3F10.2)
C
      IF (ALFA.NE.0.0)      GO TO 620
      IF (BETA.NE.0.0)      GO TO 620
      IF (GAMA.NE.0.0)      GO TO 620
C
      DO 610 K1 = 1,12
      DO 610 K2 = 1,12
      AMASS(K1,K2)    =    EMASS(K1,K2)
      ASTIF(K1,K2)    =    ESTIF(K1,K2)
  610 CONTINUE
      RETURN
C
```

```
  620 CALL TRANSFORM
C
      DO 680 J  =  1,4
      J0      =     J * 3 - 3
      DO 680 K  =  1,4
      K0      =     K * 3 - 3
      DO 630 J1 =  1,3
      DO 630 K1 =  1,3
      WKMAS(J1,K1)      =     EMASS(J0+J1,K0+K1)
      WKSTF(J1,K1)      =     ESTIF(J0+J1,K0+K1)
  630 CONTINUE
C
      DO 650 J1  =  1,3
      DO 650 K1  =  1,3
      WRKMS      =     0.0
      WRKSF      =     0.0
      DO 640 JK  =  1,3
      WRKMS      =     WRKMS  +  TRNSM(J1,JK)  *  WKMAS(JK,K1)
      WRKSF      =     WRKSF  +  TRNSM(J1,JK)  *  WKSTF(JK,K1)
  640 CONTINUE
C
      WMASS(J1,K1)     =     WRKMS
      WSTIF(J1,K1)     =     WRKSF
  650 CONTINUE
C
      DO 670 J1  =  1,3
      DO 670 K1  =  1,3
      WRKMS      =     0.0
      WRKSF      =     0.0
      DO 660 JK  =  1,3
      WRKMS      =     WRKMS  +  WMASS(J1,JK)  *  TRNST(JK,K1)
      WRKSF      =     WRKSF  +  WSTIF(J1,JK)  *  TRNST(JK,K1)
  660 CONTINUE
C
      AMASS(J0+J1,K0+K1)     =     WRKMS
      ASTIF(J0+J1,K0+K1)     =     WRKSF
  670 CONTINUE
C
  680 CONTINUE
C
      RETURN
      END

NT, LENGTH    357, NAME  DIRECTN

C
C
C
      SUBROUTINE ASSEMBLE
C
      COMMON/BLOCK0/ASSEMMATRIX(81,80)/
     1         BLOCK1/AMASS(12,12),ASTIF(12,12)/
     2         BLOCK3/EMASS(12,12),ESTIF(12,12),ICOORD(12)/
     3         BLOCK6/NDOF,INP,IOUT,IPUNCH/
     4         BLOCK7/J,J0,J1,J2,K,K0,K1,K2,ND
C
      READ(INP,710)  (ICOORD(J),J  =  1,12)
  710 FORMAT(12I5)
C
      DO 730 J  =  1,12
```

```
      IF (ICOORD(J).LT.0)              GO TO 715
      IF (ICOORD(J).LE.NDOF)           GO TO 730
  715 WRITE (IOUT,720)
  720 FORMAT (//5X,23HINVALID COORDINATE DATA)
C
      RETURN
C
  730 CONTINUE
C
      DO 750 J = 1,12
      IF (ICOORD(J).EQ.0)              GO TO 750
      J1       =     ICOORD(J)
      DO 740 K = 1,12
      IF (ICOORD(K).EQ.0)              GO TO 740
      K1       =     ICOORD(K)
      IF (J1.GT.K1)                    GO TO 740
      ASSEMMATRIX(J1,K1)    =  ASSEMMATRIX(J1,K1)     +  ASTIF(J,K)
      ASSEMMATRIX(K1+1,J1)  =  ASSEMMATRIX(K1+1,J1)   +  AMASS(J,K)
  740 CONTINUE
C
  750 CONTINUE
C
      RETURN
      END

GMENT, LENGTH   152, NAME  ASSEMBLE

C
C
C
      SUBROUTINE POINTPROP
C
      COMMON/BLOCK0/ASSEMMATRIX(81,80)/
     1         BLOCK6/NDOF,INP,IOUT,IPUNCH/
     2         BLOCK7/J,J0,J1,J2,K,K0,K1,K2,ND
C
      READ (INP,760)                NP,INDOF
  760 FORMAT (2I5)
C
      WRITE (IOUT,770)          NP
  770 FORMAT (///6X,22HNUMBER OF POINT MASSES,17X,1H=,I9/)
C
      IF (NP.LE.0)                  GO TO 830
C
  780 FORMAT (2I5,F10.4)
      DO 820 J = 1,NP
      READ (INP,780)                J1,K1,PMASS
      IF (J1.LT.1)              GO TO 790
      IF (J1.GT.NDOF)           GO TO 790
      IF (K1.LT.1)              GO TO 790
      IF (K1.LE.NDOF)           GO TO 810
C
  790 WRITE (IOUT,800)              J,J1,K1
  800 FORMAT (/5X,10HINVALID    ,I5,21HTH MASS COORDINATE   (,I4,2H ,,I4,2
     *H ))
      GO TO 820
C
  810 J2    =    J1 + 1
      IF (J1.GE.K1)                 GO TO 815
      J2    =    K1 + 1
```

```
      K1    =    J1
  815 ASSEMMATRIX(J2,K1)     =     ASSEMMATRIX(J2,K1)  +  PMASS
  820 CONTINUE
C
  830 WRITE (IOUT,840)             INDOF
  840 FORMAT (///6X,27HNUMBER OF POINT STIFFNESSES,12X,1H=,I9/)
C
      IF (INDOF.GT.0)              GO TO 850
C
      RETURN
C
  850 DO 880 J = 1,INDOF
      READ (INP,855)               J1,K1,PSTIF
  855 FORMAT (2I5,E10.3)
      IF (J1.LT.1)                 GO TO 860
      IF (J1.GT.NDOF)              GO TO 860
      IF (K1.LT.1)                 GO TO 860
      IF (K1.LE.NDOF)              GO TO 870
C
  860 WRITE (IOUT,865)             J,J1,K1
  865 FORMAT (/5X,10HINVALID    ,I5,26HTH STIFFNESS COORDINATE  (,I4,2H ,
     *,I4,2H ))
      GO TO 880
C
  870 IF (J1.LE.K1)                GO TO 875
      J2    =    J1
      J1    =    K1
      K1    =    J2
  875 ASSEMMATRIX(J1,K1)     =     ASSEMMATRIX(J1,K1)  +  PSTIF
  880 CONTINUE
C
      RETURN
      END

MENT, LENGTH   220, NAME  POINTPROP

C
C
C
      SUBROUTINE STOREMATX
C
      COMMON/BLOCK0/ASSEMMATRIX(81,80)/
     1      BLOCK6/NDOF,INP,IOUT,IOUTFL/
     2      BLOCK7/J,J0,J1,J2,K,K0,K1,K2,ND
C
      IF (IOUTFL.GT.0)             GO TO 900
C
      RETURN
C
C
C
  900 WRITE (IOUTFL,920)           NDOF
C
      JK   =   NDOF + 1
      DO 910 J = 1,JK
C
      WRITE (IOUTFL,930)           (ASSEMMATRIX(J,K),K = 1,NDOF)
  910 CONTINUE
C
  920 FORMAT (5X,2I5)
```

```
      930 FORMAT (5X,5F15.4)
    C
            RETURN
            END

MENT, LENGTH   627 NAME  STOREMATX

    C
    C
            FINISH

PILATION -  NO ERRORS

 FILE  1    54 BUCKETS USED

ED BY XPCK 12H   DATE   01/06/78   TIME 13/11/55

SHORTLIST

IN ED (FORTSEMICOMP)

LIB ED (SUBGROUPSRF4.SUBROUTINES)

WORK ED (FORTWORKFILE)

***

XXX
DATA (22AM)
ROGRAM (DBM)
      20480

UAFORTRAN; NORMAL EXIT
 ****
0.24 DELETED;CLOCKED 0.16
0.24 FINISHED ; 1 LISTFILES
JOBTIME USED 24 ; MAXIMUM CORE USED 32576
JOB UNITS 52

      NUMBER OF PAGES    9
```

APPENDIX A2

Compiled listing of Programme STATICPROB

```
#### #  #3  0000000000000000000000000000000000000000000000000
  #  #   #  0000                000000000000
  #  #   #  0000                000000000000
  # # #  #  0000                000000000000      :EMP8041.COM
  # # #  #  0000                000000000000
  # #### #  0000                000000000000
####  #  ####  00000000000000000000000000000000000000000000000

F :EMP8041,COMPSTATICS(1/81B0)    PRODUCED ON  1JUN78 AT 13.21.26

 ASTON   IN ':EMP8041,COMPSTATICS' ON  1JUN78 AT 13.21.27 USING  U14

  :EMP8041,COMPSTATICS(/81B0)

MP8041,COMPSTATICS, 1JUN78 13.13.40 TYPE: BACK  IO : STREAM A : PRIORITY 1 : F
JOB :EMP8041,COMPSTATICS
OB IS NOW FULLY STARTED
 COMPSTATICS
UAFORTRAN PROG STATICPROB,OWNPD,LIB NAGF,NORUN
MACROS,UAFORTRAN(205/)
    TA AB,CM,JL

 STATUS REPORT NUMBER 3  DATE 25 FEB  77

MPILATION BY #XFIV MK 3H    DATE  01/06/78  TIME  13/18/31

          LIST(LP)
          PROGRAM (FXXX)
          INPUT   1 = CR0
          INPUT   3 = TR0
          INPUT   4 = TR1
          INPUT   5 = TR2
          INPUT   6 = TR3
          INPUT   7 = TR4
          INPUT   8 = TR5
          OUTPUT  2 = LP0
          COMPRESS INTEGER AND LOGICAL
          EXTENDED DATA
          TRACE 2
          END

          TRACE 1
C
          MASTER  STATICS
C
C
          DIMENSION    STIFF(80,80),FORCE(80),XVECT(80),WK1(80),WK2(80)
C
C
C
          ISTIF  =  80
          IFAIL  =  0
C
          IMPT   =  1
```

```
      IOUT   =  2
C
C
      READ (INPT,10)            NUMBER
   10 FORMAT (5X,2I5)
C
      DO 500 NUMB = 1,NUMBER
C
      READ (INPT,10)            IND,INPD
      IF (IND.EQ.1)             GO TO 20
C
      IF (IFAIL.EQ.0)           GO TO 50
      GO TO 80
C
C
   20 IFAIL  =  0
      READ (INPD,10)            NDOF
      DO 40 I = 1,NDOF
C
      READ (INPD,30)            (STIFF(I,J),J = 1,NDOF)
   30 FORMAT (5X,5F15.4)
C
   40 CONTINUE
      READ (INPD,30)            (WK1(J),J = 1,NDOF)
C
   50 READ (INPT,10)            NFORC
C
      DO 55 I = 1,NDOF
      FORCE (I)  =  0.0
   55 CONTINUE
C
      DO 70 I = 1,NFORC
      READ (INPT,60)            JFORC,WFORC
   60 FORMAT (I5,F10.2)
      FORCE (JFORC)  =  WFORC
   70 CONTINUE
C
C
C
      CALL F04ASF (STIFF,ISTIF,FORCE,NDOF,XVECT,WK1,WK2,IFAIL)
C
C
C
      IF (IFAIL.EQ.0)           GO TO 120
   80 IF (IFAIL.EQ.2)           GO TO 100
C
      WRITE (IOUT,90)           NUMB
   90 FORMAT (2H1 //5X,41HSTIFFNESS MATRIX IS NOT POSITIVE DEFINITE,10X,
     *11HDATA SET    ,I5/)
C
      GO TO 500
C
  100 WRITE (IOUT,110)          NUMB
  110 FORMAT (2H1 //5X,35HSTIFFNESS MATRIX IS ILL-CONDITIONED,10X,11HDAT
     *A SET     ,I5/)
C
      GO TO 500
C
  120 WRITE (IOUT,130)          NDOF,NFORC
  130 FORMAT (2H1 ///25X,31HNUMBER OF DEGREES OF FREEDOM     =,I5//25X,31HN
     *UMBER OF APPLIED FORCES          =,I5/)
C
```

```
      LCT   =   40
      NPG   =   NDOF / LCT
      NPO   =   NPG * LCT
      IF (NPO.GE.NDOF)          GO TO 140
      NPG   =   NPG + 1
C
C
C
  140 DO 200 I = 1,NPG
C
      IC2   =   I * LCT
      IC1   =   IC2 - LCT + 1
      IF (IC2.LE.NDOF)          GO TO 150
      IC2   =   NDOF
C
  150 WRITE (IOUT,160)          I,NPG
  160 FORMAT (70X,4HPAGE,I3,4H  OF,I3//20X,10HCOORDINATE,10X,5HFORCE,10X
     *,12HDISPLACEMENT/37X,11HN OR N - MM,8X,9HMM OR RAD/)
C
      DO 180 J = IC1,IC2
C
      WRITE (IOUT,170)          J,FORCE(J),XVECT(J)
  170 FORMAT (22X,I5,10X,F10.2,9X,E11.4)
C
  180 CONTINUE
      WRITE (IOUT,190)
  190 FORMAT (2H1 ////////)
C
C
  200 CONTINUE
C
C
C
  500 CONTINUE
      STOP
      END

MENT, LENGTH   248, NAME  STATICS

      FINISH

PILATION -  NO ERRORS

UBFILE  :   11 BUCKETS USED

ED BY XPCK 12H   DATE   01/06/78   TIME 13/19/54

HORTLIST

 ED (FORTSEMICOMP)

B ED (SUBGROUPNAGF.SUBROUTINES)

B ED (SUBGROUPSRF4.SUBROUTINES)

RK ED (FORTWORKFILE)

*
```

APPENDIX A3

Compiled listing of Programme NAGFEIGNVAL

```
####    #        ##      LLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLL
  #    #        #        LLLL                      LLLLLLLLLLL
  #    #        #        LLLL                      LLLLLLLLLLL
  #    # #      #        LLLL                      LLLLLLLLLLL       :EMP8041,COM
  #    # #      #        LLLL                      LLLLLLLLLLL
  #    ####     #        LLLL                      LLLLLLLLLLL
####    #       #####    LLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLL

F :EMP8041,COMPEIGN(1/B180) . PRODUCED ON 1JUN78 AT 13.34.17

 ASTON    IN ':EMP8041,COMPEIGN' ON 1JUN78 AT 13.34.18 USING U14

  :EMP8041,COMPEIGN(/B180)

MP8041,COMPEIGN, 1JUN78 13.29.30 TYPE: BACK IO : STREAM A : PRIORITY 1 : ENTE
 JOB :EMP8041,COMPEIGN
JOB IS NOW FULLY STARTED
 COMPEIGN
 UAFORTRAN PROG NAGFEIGNVAL,OWNPD,LIB NAGF,NORUN
 MACROS,UAFORTRAN(205/)
      TA AB,CH,JL

 STATUS REPORT NUMBER 3  DATE 25 FEB  77

OMPILATION BY XFIV MK 3B    DATE  01/06/78  TIME  13/30/03

          LIST(LP)
          PROGRAM (FXXX)
          INPUT    1 = CR0
          INPUT    3 = TR0
          INPUT    4 = TR1
          INPUT    5 = TR2
          INPUT    6 = TR3
          INPUT    7 = TR4
          OUTPUT   2 = LP0
          OUTPUT   8 = TP0
          OUTPUT   9 = TP1
          OUTPUT  10 = TP2
          OUTPUT  11 = TP3
          OUTPUT  12 = TP4
          COMPRESS INTEGER AND LOGICAL
          EXTENDED DATA
          TRACE 2
          END


          TRACE 1
          MASTER NAGEIGNPROB
     C
          DIMENSION      AMASS(80,80),STIFF(80,80),
         1               ZVEC(80,80),ZPAT(80,20),WK(80,7),
         2               DL(80),D(80),E(80),E2(80),
         3               ROOT(80),WU(80),ICOUNT(20)
     C
          EQUIVALENCE    (AMASS(1,1),ZVEC(1,1))
```

```
C
      LOGICAL        LOG(80)
C
      INPT   =  1
      IOUT   =  2
C
      READ (INPT,10)          NUMBER
      IA     =  80
      TOL    =  X02ADF (IN)
      RELF   =  X02AAF (IN)
      PICT   =  X01AAF (IN)
      EPS1   =  1.00E-10
C
      DO 500 NUMB = 1,NUMBER
      READ (INPT,10)          IPTD,IUTD,IND
      READ (IPTD,10)          NDOF
   10 FORMAT (5X,3I5)
   15 FORMAT (5X,I5,2F10.3)
   20 FORMAT (5X,5F15.4)
      READ (IPTD,20)          (DL(J),J = 1,NDOF)
      DO 30 J = 1,NDOF
      STIFF(1,J)      =   DL(J) * 1000.0
   30 CONTINUE
C
      NDF   =  NDOF - 1
      DO 60 I = 1,NDF
      READ (IPTD,20)          (DL(J),J = 1,NDOF)
      DO 40 J = 1,I
      AMASS(J,I)   =   DL(J)
   40 CONTINUE
      IO     =  I + 1
      DO 50 J = IO,NDOF
      STIFF(IO,J)   =   DL(J) * 1000.0
   50 CONTINUE
   60 CONTINUE
C
      READ(IPTD,20)           (DL(J),J = 1,NDOF)
      DO 65 J = 1,NDOF
      AMASS(J,NDOF) =     DL(J)
   65 CONTINUE
C
      IFAIL  =  0
      IF (IND.EQ.2)           GO TO 80
      IF (IND.EQ.4)           GO TO 100
C
      CALL F01AEF (NDOF,AMASS,IA,STIFF,IA,DL,IFAIL)
      CALL F01AGF (NDOF,TOL,AMASS,IA,D,E,E2)
      IF (IND.EQ.3)           GO TO 90
C
C  SOME EIGNVALUES ( < 25 X ) ONLY ARE REQUIRED
C
      READ (INPT,10)          NM1,NM2
      M1    =  NDOF - NM2 + 1
      M2    =  NDOF - NM1 + 1
      MM12  =  M2 - M1 + 1
      CALL F02BFF (D,E,E2,NDOF,M1,M2,MM12,EPS1,RELF,EPS2,IZ,ROOT,WU)
      NC    =  M2 - M1 + 1
      DO 70 J = 1,NC
      D(J)  =  ROOT(J)
   70 CONTINUE
      GO TO 110
C
```

```
C  ALL EIGENVALUES ARE REQUIRED
C
   80 CALL FU2ADF (AMASS,IA,STIFF,IA,NDOF,D,WU,IFAIL)
      M1   =  1
      NM1  =  1
      NC   =  NDOF
      GO TO 110
C
C  SOME EIGNVALUES AND EIGNVECTORS ( < 25 % ) ARE REQUIRED
C
   90 READ (INPT,15)              N,ALB,UB
      IFAIL   =  1
      CALL FU2BEF (NDOF,D,ALB,UB,RELF,EPS1,E,E2,N,NC,ROOT,ZPAT,IA,
     *              ICOUNT,WK,LOG,IFAIL)
C
      IF (IFAIL.EQ.0)              GO TO 9980
      IF (IFAIL.EQ.2)              GO TO 9950
      WRITE (IOUT,9910)
 9910 FORMAT (2H1 //)
      WRITE (IOUT,9920)            N
 9920 FORMAT (15X,36HESTIMATED EIGENVALUES IN RANGE  N   =,I5/)
      WRITE (IOUT,9930)            NC
 9930 FORMAT (15X,36HNUMBER OF EIGENVALUES IN RANGE NC   =,I5/)
      WRITE (IOUT,9940)            NC
 9940 FORMAT (15X,20HRERUN PROGRAMME WITH,12X,4HC  =,I5/)
      GO TO 500
C
 9950 WRITE (IOUT,9910)
      WRITE (IOUT,9960)
 9960 FORMAT (15X,40HEIGENVECTOR ACCURACY AFTER 5 ITERATIONS /)
      IF (N.EQ.NC)                 GO TO 9980
      WRITE (IOUT,9920)            N
      WRITE (IOUT,9930)            NC
      GO TO 500
C
 9980 NM1   =  1
      NM2   =  NM1 + NC - 1
      CALL FU1AHF (NDOF,NM1,NM2,AMASS,IA,E,ZPAT,IA)
      CALL FU1AFF (NDOF,NM1,NM2,STIFF,IA,DL,ZPAT,IA)
C
      DO 95 J = 1,NC
      D(J)   =   ROOT(J)
C
      DO 95 K = 1,NDOF
      ZVEC(K,J)   =  ZPAT(K,J)
   95 CONTINUE
      GO TO 110
C
C  ALL EIGENVALUES AND EIGENVECTORS ARE REQUIRED
C
  100 CALL FU2AEF (AMASS,IA,STIFF,IA,NDOF,D,ZVEC,IA,DL,E,IFAIL)
      M1   =  1
      NM1  =  1
      NC   =  NDOF
C
C  PRINT FREQUENCIES
C
  110 DO 120 I  =  1,NC
      INN       =  NC - I + 1
      WKAREA    =  SQRT (1 / (ABS (D(INN))))
      WU(I)     =  SIGN (WKAREA,D(INN))
      ROOT(I)   =  WU(I) / (2.0 * PICT)
```

```
  120 CONTINUE
      WRITE (IOUT,130)          NDOF,NC
  130 FORMAT (2H1 //15X,35HNUMBER OF DEGREES OF FREEDOM      = ,I5//15X,3
     15HNUMBER OF EIGENVALUES COMPUTED    = ,I5///20X,20HCOMPUTED FREQUEN
     2CIES//20X,10HRAD / SEC ,18X,4H HZ /)
      J0   =  50
      I0   =  NM1 - 1
      I1   =  1
      I2   =  J0
      IF (NC.GT.J0)              GO TO 140
      I2   =  NC
  140 DO 160 I = I1,I2
      IK   =  I0 + I
      WRITE (IOUT,150)           IK,WU(I),ROOT(I)
  150 FORMAT (7X,I5,3X,F15.2,8X,F15.2)
  160 CONTINUE
C
      IF (NC.LE.I2)              GO TO 180
      I1   =  I2 + 1
      I2   =  I2 + J0
      WRITE (IOUT,170)
  170 FORMAT (2H1 /////20X,10HRAD / SEC ,18X,4H HZ /)
      IF (I2.LE.NC)              GO TO 140
      I2   =  NC
      GO TO 140
C
  180 IF (IND.LT.3)              GO TO 500
C
  200 I1   =  1
      I2   =  NC
      J0   =  58
      N0   =  NDOF / 5
      N1   =  N0 * 5
      IF (N1.GE.NDOF)            GO TO 210
      N0   =  N0 + 1
  210 N0   =  N0 + 5
      LN   =  N0
      WRITE (IOUT,220)
  220 FORMAT (2H1 /)
      DO 240 I = I1,I2
      INN  =  NDOF - I + 1
      WRITE (IOUT,230)           I,ROOT(I),(ZVEC(J,INN),J = 1,NDOF)
  230 FORMAT (//9X,5H MODE,I5,2X,20HNATURAL FREQUENCY   =,F15.2,3H HZ/26X
     *,13H MODAL SHAPE //30(5X,5E13.4/))
      LN   =  LN + N0
      IF (LN.LE.J0)              GO TO 240
      LN   =  N0
      WRITE (IOUT,220)
  240 CONTINUE
C
      IF (IUTD.LE.0)             GO TO 500
C
  250 N1   =  I2 - I1 + 1
      WRITE (IUTD,260)           NDOF,I1,N1
      WRITE (IUTD,270)           (ROOT(I),I = I1,I2)
  260 FORMAT (5X,3I5)
  270 FORMAT (5X,7F10.2)
      DO 290 I = I1,I2
      INN  =  NDOF - I + 1
      WRITE (IUTD,280)           I,(ZVEC(J,INN),J = 1,NDOF)
  280 FORMAT (2X,I5,40(T9,5E13.5/))
  290 CONTINUE
```

```
      WRITE (IOUT,300)          I1,I2
  300 FORMAT (2H1 /5X,4HTHE ,I5,5H  TO ,I5,56H  FREQUENCIES AND MODAL SH
     *APES HAVE BEEN WRITTEN ON FILE/)
C
  500 CONTINUE
      STOP
      END

MENT, LENGTH   719, NAME  NAGEIGNPROB

      FINISH

PILATION -  NO ERRORS

JBFILE  1     20 BUCKETS USED

ED BY XPCK 12H    DATE    01/06/78   TIME 13/33/05

IORTLIST

I ED (FORTSEMICOMP)

IB ED (SUBGROUPNAGF.SUBROUTINES)

IB ED (SUBGROUPSRF4.SUBROUTINES)

IRK ED (FORTWORKFILE)
**

X
TA (22AM)
GRAM (DBM)
FORTRAN: THE PROGRAM NEEDS MORE CORE. YOUR MZ IS PROBABLY TOO SMALL
***
14 FINISHED : 1 LISTFILES
BTIME USED 14 ; MAXIMUM CORE USED 40960
B UNITS 30

LLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLL
                                LLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLL
    NUMBER OF PAGES     5       LLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLL
                                LLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLL
LLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLL
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APPENDIX B

FABRICATED SPACE FRAME FREQUENCIES

and

MODAL SHAPES DATA

APPENDIX B1

Computed modal shape for the 156 D.O.F.

Fabricated Space Frame

MODE 1 NATURAL FREQUENCY = 146,81 HZ

MODAL SHAPE

0,1572E-03	-0,2943E-03	-0,1108E-03	-0.6296E-06	0,4254E-06
0,3824E-06	0,6063E-03	-0,6062E-03	0.1556E-10	-0,6822E-07
-0,6821E-07	0,7038E-14	0,2943E-03	-0.1572E-03	-0,1108E-03
0,4254E-06	-0,6297E-06	-0,3824E-06	0.2943E-03	-0,1572E-03
0,1108E-03	0,4254E-06	-0,6297E-06	0.3824E-06	0,1572E-03
-0,2943E-03	0,1108E-03	-0,6296E-06	0.4254E-06	-0,3824E-06
0,1539E-03	-0,1539E-03	-0,3837E-04	-0.1188E-05	-0,1188E-05
-0,6601E-10	0,1601E-03	-0,1601E-03	0.1844E-09	-0,6782E-07
-0,6783E-07	-0,1433E-05	0,1601E-03	-0.1601E-03	-0,1825E-09
-0,6782E-07	-0,6783E-07	0,1433E-05	0.1539E-03	-0,1539E-03
0,3837E-04	-0,1188E-05	-0,1188E-05	0.6665E-10	0,2816E-03
-0,2816E-03	-0,2210E-04	0,3501E-06	0.3501E-06	-0,4702E-10
0,2333E-03	-0,2332E-03	0,2437E-09	-0.4572E-07	-0,4574E-07
-0,1070E-05	0,4320E-03	-0,5472E-03	-0.3370E-03	-0,1514E-06
0,1810E-06	-0,2072E-06	0,5473E-03	-0.4319E-03	-0,3370E-03
0,1810E-06	-0,1513E-06	0,2071E-06	0.2333E-03	-0,2332E-03
-0,2385E-09	-0,4372E-07	-0,4574E-07	0.1070E-05	0,4320E-03
-0,5472E-03	0,3370E-03	-0,1514E-06	0.1810E-06	0,2072E-06
0,5473E-03	-0,4319E-03	0,3370E-03	0.1810E-06	-0,1513E-06
-0,2071E-06	0,2816E-03	-0,2816E-03	0.2210E-04	0,3501E-06
0,3501E-06	0,4706E-10	0,6356E-04	-0.6355E-04	-0,5773E-05
0,7871E-06	0,7372E-06	-0,2716E-10	0.7394E-04	-0,7393E-04
0,2968E-09	0,1176E-05	0,1176E-05	-0.7009E-06	0,7394E-04
-0,7393E-04	-0,2997E-09	0,1176E-05	0.1176E-05	0,7009E-06
0,6356E-04	-0,6355E-04	0,5773E-05	0.7871E-06	0,7872E-06
0,2715E-10	0,6383E-04	-0,1347E-03	-0.3409E-04	0,9843E-06
-0,4845E-06	0,2071E-06	0,1347E-03	-0.6882E-04	-0,3409E-04
-0,4844E-06	0,9344E-06	-0,2071E-06	0.6371E-04	-0,6369E-04
-0,2123E-11	-0,4045E-06	-0,4045E-06	0.8619E-14	0,1347E-03
-0,6862E-04	0,3409E-04	-0,4844E-06	0.9844E-06	0,2071E-06
0,6883E-04	-0,1347E-03	0,3409E-04	0.9843E-06	-0,4845E-06
-0,2071E-06				

MODE 2 NATURAL FREQUENCY = 151,51 HZ

MODAL SHAPE

0,1263E-03	0.2441E-03	-0,9167E-04	0.7244E-06	0,3733E-06
0,2956E-06	0,1293E-03	0,1294E-03	0,1184E-11	-0,8152E-07
0,8153E-07	-0,1825E-13	0,2441E-03	0,1264E-03	0,9168E-04
-0,3733E-06	-0,7243E-06	0,2956E-06	0,2441E-03	0,1264E-03
-0,9168E-04	-0,3733E-06	-0,7243E-06	-0,2956E-06	0,1263E-03
0,2441E-03	0,9167E-04	0,7244E-06	0.3733E-06	-0,2956E-06
0,1234E-03	0,1234E-03	0,1844E-08	0.1302E-05	-0,1302E-05
-0,1219E-05	0,1209E-03	0,1290E-03	0,1709E-05	0,1419E-06
-0,1419E-06	0,8712E-10	0,1290E-03	0.1290E-03	-0,1709E-05
0,1419E-06	-0,1419E-06	-0,8709E-10	0.1234E-03	0,1234E-03
-0,1845E-08	0,1302E-05	-0,1302E-05	0.1219E-05	0,2972E-03
0,2972E-03	0.1068E-08	-0,7184E-06	0.2183E-06	-0,8667E-06
0,2427E-03	0,2427E-03	0,2810E-05	0.1825E-06	-0,1825E-06
0,6632E-10	0,4589E-03	0,6468E-03	-0,3536E-03	-0,5379E-06
0,5308E-07	-0,1015E-06	0,6468E-03	0,4589E-03	0,3536E-03
-0,5310E-07	0,5380E-06	-0,1015E-06	0,2427E-03	0,2427E-03
-0,2810E-05	0,1825E-06	-0,1825E-06	-0,6625E-10	0,4589E-03
0,6468E-03	0,3536E-03	-0,5379E-06	0,5308E-07	0,1015E-06
0,6468E-03	0,4589E-03	-0,3536E-03	-0,5310E-07	0,5380E-06
0,1015E-06	0,2972E-03	0,2972E-03	-0,1060E-08	-0,2184E-06
0,2183E-06	0,8667E-06	0,7283E-04	0,7283E-04	0,2792E-09
-0,9910E-06	0,9909E-06	-0,5087E-06	0,8285E-04	0,8286E-04
0,3905E-05	-0,1354E-05	0,1354E-05	0,4506E-10	0,8285E-04
0,8286E-04	-0,3905E-05	-0,1354E-05	0,1354E-05	-0,4503E-10
0,7283E-04	0,7283E-04	-0,2823E-09	-0,9910E-06	0,9909E-06
0,5087E-06	0,7794E-04	0,1311E-03	-0,2674E-04	-0,1176E-05
-0,5878E-06	0,1022E-06	0,1311E-03	0,7795E-04	0,2674E-04
0,5873E-06	0,1176E-05	0,1023E-06	0,2541E-03	0,2541E-03
-0,4536E-11	-0,9970E-06	0,9970E-06	-0,7470E-14	0,1311E-03
0,7795E-04	-0,2674E-04	0,5878E-06	0.1176E-05	-0,1023E-06
0,7794E-04	0,1311E-03	0,2674E-04	-0,1176E-05	-0,5878E-06
-0,1022E-06				

MODE 3 NATURAL FREQUENCY = 174,82 HZ

MODAL SHAPE

-0,2062E-05	-0,9605E-06	-0,3450E-03	-0,1997E-05	-0,3222E-06
-0,1141E-07	-0,1818E-10	0,1715E-10	-0,1602E-02	-0,6569E-15
-0,3702E-14	0,7763E-13	0,9608E-06	-0,2061E-05	-0,3450E-03
-0,3222E-06	-0,1997E-05	0,1142E-07	-0,9608E-06	0,2061E-05
-0,3450E-03	0,3222E-06	0,1997E-05	0,1142E-07	-0,2062E-05
0,9605E-06	-0,3450E-03	0,1997E-05	0,3222E-06	-0,1141E-07
0,2842E-05	-0,2842E-05	-0,7683E-05	-0,1386E-05	-0,1386E-05
-0,1028E-11	0,1274E-05	0,1274E-05	-0,1219E-04	-0,2591E-05
0,2591E-05	0,1082E-11	-0,1274E-05	-0,1274E-05	-0,1219E-04
0,2591E-05	-0,2591E-05	0,9637E-12	-0,2842E-05	0,2842E-05
-0,7683E-05	0,1336E-05	0,1386E-05	-0,1048E-11	0,1316E-03
-0,1316E-03	-0,4337E-05	0,3214E-06	0,3214E-06	-0,7127E-12
-0,2328E-03	-0,2328E-03	-0,7065E-05	0,6074E-06	-0,6074E-06
0,1434E-11	0,1366E-03	-0,2445E-03	-0,1920E-03	-0,1402E-06
0,4490E-06	-0,4717E-06	0,2445E-03	-0,1866E-03	-0,1920E-03
0,4490E-06	-0,1402E-06	0,4717E-06	0,2328E-03	0,2328E-03
-0,7065E-05	-0,6074E-06	0,6074E-06	0,1294E-11	-0,1866E-03
0,2445E-03	-0,1920E-03	0,1402E-06	-0,4490E-06	-0,4717E-06
-0,2445E-03	0,1866E-03	-0,1920E-03	-0,4490E-06	0,1402E-06
0,4717E-06	-0,1316E-03	0,1316E-03	-0,4337E-05	-0,3214E-06
-0,3214E-06	-0,7848E-12	0,3434E-06	-0,3433E-06	-0,9773E-06
0,1625E-06	0,1625E-06	-0,4326E-12	-0,3874E-05	-0,3874E-05
-0,1918E-05	0,1721E-06	-0,1721E-06	0,1692E-11	0,3874E-05
0,3874E-05	-0,1918E-05	-0,1721E-06	0,1721E-06	0,1695E-11
-0,3434E-06	0,3433E-06	-0,9773E-06	-0,1625E-06	-0,1625E-06
-0,4740E-12	-0,1768E-05	-0,2584E-05	0,2623E-04	0,1680E-06
-0,1559E-09	0,9792E-08	0,2585E-05	0,1768E-05	0,2623E-04
-0,1551E-09	0,1680E-06	-0,9792E-08	-0,4055E-11	-0,6725E-11
0,9624E-04	-0,1399E-13	-0,4537E-14	0,5797E-12	-0,2585E-05
-0,1768E-05	0,2623E-04	0,1550E-09	-0,1680E-06	-0,9792E-08
0,1768E-05	0,2584E-05	0,2623E-04	-0,1680E-06	0,1558E-09
0,9792E-08				

MODE 4 NATURAL FREQUENCY = 175,52 HZ

MODAL SHAPE

0,2568E-04	0,2787E-03	-0,1622E-03	0.9719E-06	0,2072E-06
0,3644E-06	0,4598E-11	-0,5279E-11	0.1705E-08	-0,1345E-13
-0,2627E-13	-0,5200E-06	-0,2787E-03	0.2568E-04	0,1622E-03
-0,2072E-06	-0,9719E-06	0,3644E-06	-0.2787E-03	-0,2568E-04
0,1622E-03	0,2072E-06	0,9719E-06	0.3644E-06	-0,2568E-04
-0,2787E-03	-0,1622E-03	-0,9719E-06	-0.2072E-06	0,3644E-06
0,2598E-04	0,2598E-04	-0,2877E-11	0.1371E-05	-0,1371E-05
-0,2092E-05	0,2523E-04	-0,2528E-04	0.1367E-10	0,5646E-06
0,5646E-06	0,1228E-05	-0,2528E-04	0.2528E-04	0,1348E-10
-0,5646E-06	-0,5646E-06	0,1228E-05	-0.2598E-04	-0,2598E-04
-0,3321E-11	-0,1371E-05	0,1371E-05	-0.2092E-05	0,1618E-03
0,1618E-03	0,1532E-11	-0,3184E-06	0.3184E-06	-0,1379E-05
-0,2792E-04	0,2792E-04	0,7142E-11	-0.9541E-07	-0,9541E-07
0,1489E-05	0,2368E-03	0,8196E-03	-0.2189E-03	-0,4022E-07
0,4104E-06	0,5401E-06	0,8196E-03	0.2368E-03	0,2189E-03
-0,4104E-06	0,4022E-07	0,5401E-06	0.2792E-04	-0,2792E-04
0,9387E-11	0,9541E-07	0,9541E-07	0.1489E-05	-0,2368E-03
-0,8198E-03	-0,2189E-03	0,4022E-07	-0.4104E-06	0,5401E-06
-0,8198E-03	-0,2368E-03	0,2189E-03	0.4104E-06	-0,4022E-07
0,5401E-06	-0,1618E-03	-0,1618E-03	-0.5188E-11	0,3184E-06
-0,3184E-06	-0,1379E-05	0,1249E-04	0.1249E-04	-0,7734E-12
-0,2777E-06	0,2777E-06	-0,6529E-06	0.9600E-05	-0,9600E-05
0,2697E-11	-0,2409E-07	-0,2409E-07	0.1738E-05	-0,9600E-05
0,9600E-05	0,3163E-11	0,2409E-07	0.2409E-07	0,1738E-05
-0,1249E-04	-0,1249E-04	-0,3267E-12	0.2777E-06	-0,2777E-06
-0,6529E-06	0,1106E-04	0,2021E-03	0.2529E-04	-0,1516E-06
-0,6519E-07	-0,2175E-06	0,2021E-03	0.1106E-04	-0,2529E-04
0,6519E-07	0,1516E-06	-0,2175E-06	0.4319E-11	0,4207E-11
-0,7258E-10	0,5183E-15	0,2831E-14	0.4381E-06	-0,2021E-03
-0,1106E-04	-0,2529E-04	-0,6519E-07	-0.1516E-06	-0,2175E-06
-0,1106E-04	-0,2021E-03	0,2529E-04	0.1516E-06	0,6519E-07
-0,2175E-06				

MODE 5 NATURAL FREQUENCY = 180,39 HZ

MODAL SHAPE

0,1000E-03	-0,2070E-03	-0,9981E-04	0.5917E-06	0,3103E-06
0,2083E-06	0,3491E-11	-0,4196E-11	0.1344E-08	0,5295E-14
0,8187E-14	0,9596E-06	-0,2079E-03	0.1000E-03	0,9981E-04
-0,3103E-06	-0,5917E-06	0,2083E-06	0.2079E-03	-0,1000E-03
0,9981E-04	0,3103E-06	-0,5917E-06	0.2083E-06	-0,1000E-03
0,2079E-03	-0,9981E-04	-0,5917E-06	-0.3103E-06	0,2083E-06
0,9720E-04	0,9720E-04	-0,3710E-10	0.1192E-05	-0,1192E-05
0,1881E-05	0,1025E-03	-0,1025E-03	0.1305E-10	-0,1361E-07
-0,1361E-07	-0,5347E-06	-0,1025E-03	0.1025E-03	0,1393E-10
0,1361E-07	0,1361E-07	-0,5342E-06	-0.9720E-04	-0,9720E-04
-0,3750E-10	-0,1192E-05	0,1192E-05	0.1881E-05	0,2630E-03
0,2630E-03	-0,1747E-10	-0,2143E-06	0.2143E-06	0,1119E-05
0,2145E-03	-0,2145E-03	0,1205E-10	-0.2292E-06	-0,2292E-06
-0,3591E-06	0,5512E-03	-0,3118E-03	-0.4671E-03	0,6784E-06
0,2585E-07	0,1410E-05	-0,3118E-03	0.5512E-03	0,4671E-03
-0,2585E-07	-0,6784E-06	0,1410E-05	-0.2145E-03	0,2145E-03
0,7835E-11	0,2292E-06	0,2292E-06	-0.3591E-06	-0,5512E-03
0,3118E-03	-0,4671E-03	-0,6784E-06	-0.2585E-07	0,1410E-05
0,3118E-03	-0,5512E-03	0,4671E-03	0.2585E-07	0,6784E-06
0,1410E-05	-0,2630E-03	-0,2630E-03	-0.2525E-10	0,2143E-06
-0,2143E-06	0,1119E-05	0,6041E-04	0.6041E-04	-0,6524E-11
-0,7565E-06	0,7565E-06	0,3478E-06	0.7054E-04	-0,7054E-04
0,4888E-11	0,1249E-05	0,1249E-05	-0.1808E-06	-0,7054E-04
0,7054E-04	0,7833E-11	-0,1249E-05	-0.1249E-05	-0,1808E-06
-0,6041E-04	-0,6041E-04	-0,4166E-11	0,7565E-06	-0,7565E-06
0,3478E-06	0,6559E-04	-0,5108E-04	-0.4175E-04	0,2475E-06
-0,5164E-06	0,3210E-06	-0,5108E-04	0,6559E-04	0,4175E-04
0,5164E-06	-0,2475E-06	0,3210E-06	-0.1327E-11	-0,2343E-12
0,9224E-10	-0,3065E-14	0,1725E-13	0,1861E-06	0,5108E-04
-0,6559E-04	0,4175E-04	-0,5164E-06	0.2475E-06	0,3210E-06
-0,6559E-04	0,5108E-04	-0,4175E-04	-0.2475E-06	0,5164E-06
0,3210E-06				

MODE 6 NATURAL FREQUENCY = 183,86 HZ

MODAL SHAPE

0,1538E-04	-0.2700E-03	0,8400E-04	0.4794E-06	-0,2161E-06
0,7729E-06	0.1062E-02	-0,1062E-02	-0.3229E-10	-0,5445E-07
-0,5445E-07	-0.9170E-14	-0,2700E-03	-0.1538E-04	0,8400E-04
-0,2161E-06	0.4794E-06	-0,7729E-06	0.2700E-03	-0,1538E-04
-0,8400E-04	-0.2161E-06	-0,4794E-06	0.7729E-06	0,1538E-04
-0,2700E-03	-0.8400E-04	0,4794E-06	-0.2161E-06	-0,7729E-06
0,1244E-04	-0.1244E-04	-0,1072E-04	0.1008E-05	0,1008E-05
0,2062E-11	0.1326E-04	-0,1826E-04	0.1784E-10	-0,5396E-07
-0,5396E-07	-0.2936E-05	0,1826E-04	-0.1826E-04	-0,1630E-10
-0,5396E-07	-0.5396E-07	0,2936E-05	0.1244E-04	-0,1244E-04
0,1072E-04	0.1008E-05	0,1008E-05	-0.2086E-11	-0,1042E-03
0,1042E-03	-0.6060E-05	-0,1349E-06	-0.1349E-06	0,1590E-11
-0,2846E-04	0.2846E-04	0,1387E-10	0.2116E-06	0,2116E-06
-0,1261E-05	-0.2424E-03	0,3210E-03	0.2363E-03	0,2748E-06
-0,1057E-06	0.2373E-06	-0,3210E-03	0.2424E-03	0,2363E-03
-0,1057E-06	0.2748E-06	-0,2373E-06	-0.2846E-04	0,2846E-04
-0,1424E-10	-0.2116E-06	-0,2116E-06	0.1261E-05	-0,2424E-03
0,3210E-03	-0.2363E-03	0,2748E-06	-0.1057E-06	-0,2373E-06
-0,3210E-03	0.2424E-03	-0,2363E-03	-0.1057E-06	0,2748E-06
0,2373E-06	-0.1042E-03	0,1042E-03	0.6060E-05	-0,1349E-06
-0,1349E-06	-0.1629E-11	-0,1361E-04	0.1361E-04	-0,1383E-05
-0,2035E-06	-0.2035E-06	0,1127E-11	-0.1399E-04	0,1399E-04
0,1123E-10	-0.4185E-06	-0,4185E-06	0.4253E-06	-0,1399E-04
0,1399E-04	-0.1170E-10	-0,4185E-06	-0.4185E-06	-0,4253E-06
-0,1361E-04	-0.1361E-04	-0,1383E-05	-0.2035E-06	-0,2035E-06
-0,1131E-11	-0.1383E-04	0,5364E-04	0.1749E-04	-0,3125E-06
0,1642E-06	-0.1107E-06	-0,5364E-04	0.1383E-04	0,1749E-04
0,1642E-06	-0.3125E-06	0,1107E-06	-0.1366E-04	0,1366E-04
0,5012E-11	-0.1177E-06	-0,1177E-06	0.7678E-16	-0,5364E-04
0,1383E-04	-0.1749E-04	0,1642E-06	-0.3125E-06	-0,1107E-06
-0,1383E-04	0.5364E-04	-0,1749E-04	-0.3125E-06	0,1642E-06
0,1107E-06				

MODE 7 NATURAL FREQUENCY = 199.89 HZ

MODAL SHAPE

0,5716E-06	-0.5772E-05	0.4316E-04	-0.1764E-06	-0.8881E-07
-0,1868E-07	0.5126E-06	0.5127E-06	-0.6455E-10	-0.3915E-07
0,3915E-07	0.6216E-14	-0.5772E-05	0.5716E-06	-0.4316E-04
0,8881E-07	0.1764E-06	-0.1868E-07	-0.5772E-05	0.5716E-06
0,4316E-04	0.8881E-07	0.1764E-06	0.1868E-07	0.5716E-06
-0,5772E-05	-0.4316E-04	-0.1764E-06	-0.8881E-07	0.1868E-07
0,6301E-06	0.6301E-06	-0.1740E-11	-0.4155E-06	0.4155E-06
0,7337E-07	0.5105E-06	0.5105E-06	0.7606E-06	0.6461E-07
-0,6461E-07	0.1501E-12	0.5105E-06	0.5105E-06	-0.7606E-06
0,6461E-07	-0.6461E-07	-0.1866E-12	0.6301E-06	0.6301E-06
0,1395E-11	-0.4155E-06	0.4155E-06	-0.7337E-07	-0.2210E-04
-0,2210E-04	-0.1721E-11	0.1564E-06	-0.1564E-06	-0.1194E-05
0,3831E-06	0.3831E-06	0.3538E-06	-0.3065E-07	0.3065E-07
0,1077E-12	-0.1117E-03	-0.1250E-03	0.1143E-03	0.3159E-09
-0,9666E-07	-0.1236E-06	-0.1250E-03	-0.1117E-03	-0.1143E-03
0,9666E-07	-0.3158E-09	-0.1236E-06	0.3831E-06	0.3831E-06
-0,3538E-06	-0.3065E-07	0.3065E-07	-0.1396E-12	-0.1117E-03
-0,1250E-03	-0.1143E-03	0.3159E-09	-0.9666E-07	0.1236E-06
-0,1250E-03	-0.1117E-03	0.1143E-03	0.9666E-07	-0.3158E-09
0,1236E-06	-0.2210E-04	-0.2210E-04	0.6754E-13	0.1564E-06
-0,1564E-06	0.1194E-05	0.9489E-05	0.9489E-05	-0.4280E-12
-0,9215E-07	0.9215E-07	-0.2449E-05	0.4635E-05	0.4635E-05
-0,5449E-07	0.9826E-07	-0.9826E-07	0.6914E-13	0.4635E-05
0,4635E-05	0.5449E-07	0.9826E-07	-0.9826E-07	-0.8610E-13
0,9489E-05	0.9489E-05	0.5676E-14	-0.9215E-07	0.9215E-07
0,2449E-05	0.7078E-05	0.2268E-03	0.1688E-04	0.3071E-08
0,1430E-08	0.6481E-06	0.2268E-03	0.7078E-05	-0.1688E-04
-0,1430E-08	-0.3071E-08	0.6481E-06	0.1160E-02	0.1160E-02
-0,4442E-10	-0.9313E-07	0.9313E-07	-0.9374E-14	0.2268E-03
0,7078E-05	0.1688E-04	-0.1430E-08	-0.3071E-08	-0.6481E-06
0,7078E-05	0.2268E-03	-0.1688E-04	0.3071E-08	0.1430E-08
-0,6481E-06				

MODE 3 NATURAL FREQUENCY = 201,70 HZ

MODAL SHAPE

0,8608E-05	-0.4476E-05	-0,6906E-04	-0.3805E-06	0,5714E-06
-0,4819E-07	0.9968E-13	-0,7638E-12	0.6773E-03	-0,4174E-14
-0,5932E-15	-0.2446E-12	0,4477E-05	-0.8608E-05	-0,6906E-04
0,5714E-06	-0.3805E-06	0,4819E-07	-0.4477E-05	0,8608E-05
-0,6906E-04	-0.5714E-06	0,3805E-06	0.4819E-07	-0,8608E-05
0,4476E-05	-0.6906E-04	0,3805E-06	-0.5714E-06	-0,4819E-07
-0,1199E-04	-0.1199E-04	-0,7623E-05	-0.1464E-05	-0,1464E-05
-0,1130E-11	0.5192E-05	0,5192E-05	0.3508E-06	0,7072E-06
-0,7072E-06	0.7312E-12	-0,5192E-05	-0.5192E-05	0,3508E-06
-0,7072E-06	0.7072E-06	0,7343E-12	-0.1199E-04	0,1199E-04
-0,7623E-05	0.1464E-05	0,1464E-05	-0.1189E-11	0,1688E-03
-0,1688E-03	-0.1451E-05	0,3186E-06	0.3186E-06	-0,7194E-12
0,1247E-03	0.1247E-03	-0,4396E-06	-0.1607E-07	0,1607E-07
0,4034E-12	0.5151E-03	-0,2148E-03	-0.5038E-03	-0,1788E-06
0,2599E-06	-0.8363E-06	0,2148E-03	-0.5151E-03	-0,5038E-03
0,2599E-06	-0.1788E-06	0,8362E-06	-0.1247E-03	-0,1247E-03
-0,4396E-06	0.1607E-07	-0,1607E-07	0.4279E-12	-0,5151E-03
0,2148E-03	-0.5038E-03	0,1788E-06	-0.2599E-06	-0,8363E-06
-0,2148E-03	0.5151E-03	-0,5038E-03	-0.2599E-06	0,1788E-06
0,8362E-06	-0.1688E-03	0,1688E-03	-0.1451E-05	-0,3186E-06
-0,3186E-06	-0.5911E-12	-0,7669E-05	-0.7669E-05	-0,2724E-06
0,2819E-06	0.2819E-06	-0,3315E-12	0.1452E-04	0,1452E-04
-0,1228E-05	-0.5444E-06	-0,5444E-06	0.7560E-13	-0,1452E-04
-0,1452E-04	-0.1228E-05	0,5444E-06	-0.5444E-06	0,1120E-12
-0,7669E-05	0.7669E-05	-0,2724E-06	-0.2819E-06	-0,2819E-06
0,5302E-13	0.1112E-04	0,4514E-05	-0.2442E-04	-0,1320E-06
-0,2169E-06	-0.6224E-07	-0,4514E-05	-0.1112E-04	-0,2442E-04
-0,2169E-06	-0.1320E-06	0,6224E-07	0.9577E-10	0,9531E-10
0,2684E-03	-0.1087E-13	-0,8740E-14	-0.1475E-13	0,4514E-05
0,1112E-04	-0.2442E-04	0,2169E-06	0.1320E-06	0,6224E-07
-0,1112E-04	-0.4514E-05	-0,2442E-04	0.1320E-06	0,2169E-06
-0,6224E-07				

MODE [illegible] NATURAL FREQUENCY = 221,07 HZ

MODAL SHAPE

0,2820E-06	-0,2689E-07	-0,7475E-05	-0,2264E-07	-0,1816E-08
-0,1592E-08	-0,1507E-11	-0,3627E-11	-0,6754E-04	0,1280E-13
-0,2262E-13	0,9266E-14	0,2884E-07	-0,2820E-06	-0,7475E-05
-0,1816E-08	-0,2264E-07	0,1592E-08	-0,2890E-07	0,2820E-06
-0,7475E-05	0,1816E-08	-0,2264E-07	0,1592E-08	-0,2820E-06
0,2883E-07	-0,7475E-05	0,2264E-07	0,1316E-08	-0,1592E-08
0,3018E-06	-0,3018E-06	-0,2862E-05	-0,1301E-07	-0,1301E-07
0,2583E-12	0,2606E-06	0,2606E-06	-0,1754E-05	-0,3197E-07
0,3197E-07	0,3760E-13	-0,2606E-06	-0,2606E-06	-0,1754E-05
0,3197E-07	-0,3197E-07	0,4201E-13	-0,3018E-06	0,3018E-06
-0,2862E-05	0,1301E-07	0,1301E-07	-0,3312E-12	-0,8583E-04
0,8583E-04	-0,2524E-05	0,2518E-06	0,2518E-06	0,1508E-12
0,2613E-04	0,2613E-04	-0,1426E-05	0,8869E-07	-0,8869E-07
0,1851E-13	0,1432E-03	0,1258E-03	-0,1446E-03	-0,3139E-07
-0,8193E-07	-0,1270E-06	-0,1258E-03	-0,1432E-03	-0,1446E-03
-0,8193E-07	-0,3139E-07	0,1270E-06	-0,2613E-04	-0,2613E-04
-0,1426E-05	-0,8869E-07	-0,8869E-07	0,3029E-13	-0,1432E-03
-0,1258E-03	-0,1446E-03	0,3139E-07	0,8193E-07	-0,1270E-06
0,1258E-03	0,1432E-03	-0,1446E-03	0,8193E-07	0,3139E-07
0,1270E-06	0,8583E-04	-0,8583E-04	-0,2524E-05	-0,2518E-06
-0,2518E-06	-0,1393E-12	-0,6525E-05	0,6525E-05	-0,2173E-05
-0,8768E-06	-0,8768E-06	0,1186E-13	0,9799E-06	0,9799E-06
-0,1091E-05	-0,2998E-06	0,2996E-06	-0,1166E-13	-0,9799E-06
-0,9799E-06	-0,1091E-05	0,2996E-06	-0,2998E-06	0,2894E-13
0,6525E-05	-0,6525E-05	-0,2175E-05	0,8768E-06	0,8768E-06
-0,1527E-13	-0,2780E-05	0,5256E-05	-0,1133E-03	-0,5923E-06
0,1538E-06	0,1570E-07	-0,5256E-05	0,2780E-05	-0,1133E-03
0,1538E-06	-0,5923E-06	-0,1570E-07	-0,7416E-11	-0,8157E-11
-0,1525E-02	-0,2059E-13	0,1500E-13	0,7560E-14	0,5256E-05
-0,2780E-05	-0,1133E-03	-0,1538E-06	0,5923E-06	-0,1570E-07
0,2780E-05	-0,5256E-05	-0,1133E-03	0,5923E-06	-0,1538E-06
0,1570E-07				

MODE 10 NATURAL FREQUENCY = 221,71 HZ

MODAL SHAPE

0,1648E-04	0,2093E-03	-0,1042E-03	-0.3516E-06	-0,1682E-06
0,5060E-06	0,1303E-04	0,1809E-04	-0.6410E-11	-0,1240E-06
0,1240E-06	0,1535E-14	-0,2093E-03	-0.1648E-04	-0,1042E-03
0,1682E-06	0,3517E-06	0,5060E-06	0.2093E-03	0,1648E-04
0,1042E-03	-0,1682E-06	-0,3517E-06	-0.5060E-06	0,1648E-04
0,2093E-03	-0,1042E-03	-0,3516E-06	-0.1682E-06	-0,5060E-06
0,1489E-04	0,1489E-04	-0,1398E-09	-0.8816E-06	0,8816E-06
-0,1970E-05	0,1800E-04	0,1800E-04	0.5060E-05	0,1831E-06
-0,1831E-06	0,6060E-11	0,1800E-04	-0.1300E-04	-0,5060E-05
0,1831E-06	-0,1831E-06	-0,6090E-11	0.1489E-04	0,1489E-04
0,1391E-09	-0,8816E-06	0,8816E-06	0.1970E-05	-0,8508E-04
-0,8508E-04	-0,3231E-10	0,1519E-06	-0.1519E-06	-0,9666E-06
0,1813E-04	0,1813E-04	0,3982E-05	-0.1317E-06	0,1317E-06
0,5140E-11	-0,4183E-03	0,4808E-03	0.4271E-03	-0,8938E-06
-0,1916E-06	-0,4887E-06	0,4808E-03	-0.4185E-03	-0,4271E-03
0,1916E-06	0,8938E-06	-0,4888E-06	0.1813E-04	0,1813E-04
-0,3982E-05	-0,1317E-06	0,1317E-06	-0.5171E-11	-0,4183E-03
0,4808E-03	-0,4271E-03	-0,8938E-06	-0.1916E-06	0,4887E-06
0,4808E-03	-0,4183E-03	0,4271E-03	0.1916E-06	0,8938E-06
0,4888E-06	-0,8508E-04	-0,8508E-04	0.7738E-10	0,1519E-06
-0,1519E-06	0,9666E-06	-0,2864E-05	-0.2864E-05	-0,2012E-10
0,1080E-06	-0,1080E-06	0,5034E-07	-0.3145E-06	-0,3146E-06
0,2883E-05	0,1226E-06	-0,1226E-06	0.4161E-11	-0,3145E-06
-0,3146E-06	-0,2883E-05	0,1226E-06	-0.1226E-06	-0,4170E-11
-0,2864E-05	-0,2864E-05	0,1947E-10	0.1080E-06	-0,1080E-06
-0,5034E-07	-0,1394E-05	-0,6988E-05	0.3406E-05	0,1161E-06
0,6843E-07	-0,2037E-07	-0,6988E-05	-0.1594E-05	-0,3406E-05
-0,6844E-07	-0,1161E-06	-0,2037E-07	-0.6555E-04	-0,6555E-04
-0,2220E-09	0,1094E-06	-0,1094E-06	-0.8867E-14	-0,6988E-05
-0,1594E-05	0,3406E-05	-0,6844E-07	-0.1161E-06	0,2037E-07
-0,1594E-05	-0,6988E-05	-0,3406E-05	0.1161E-06	0,6843E-07
0,2037E-07				

APPENDIX B2

Experimental Natural Frequencies and Modal Shapes for the 26 nodes

Fabricated Space Frame

Coordinates			Response at resonance frequencies		
			Mode 1 96 Hz	Mode 2 104 Hz	Mode 3 114 Hz
1	X	1	1.73	1.98	0.19
	Y	2	-1.68	1.30	0.20
	Z	3	-0.24	-1.25	-2.05
2	X	7	2.40	0.97	0.21
	Y	8	-2.40	1.60	-0.56
	Z	9	0.51	0.48	-7.50
3	X	13	2.20	1.20	-0.44
	Y	14	-1.18	0.88	-0.32
	Z	15	-0.34	-0.81	-1.60
4	X	19	2.40	2.60	-0.62
	Y	20	-1.41	1.98	0.21
	Z	21	0.38	-1.40	-1.40
5	X	25	1.40	1.45	-0.46
	Y	26	-2.00	3.50	-0.49
	Z	27	0.24	-0.80	-1.90
6	X	31	1.63	2.10	0.21
	Y	32	-1.05	1.05	-0.34
	Z	33	-0.25	-0.30	0.11
7	X	37	1.83	1.85	0.17
	Y	38	-1.43	1.85	0.20
	Z	39	0.10	-0.28	0.23

Coordinates			96 Hz	104 Hz	114 Hz
8	X	43	1.50	0.79	-0.43
	Y	44	-1.30	0.70	-0.29
	Z	45	0.10	-0.24	0.13
9	X	49	1.30	1.05	-0.48
	Y	50	-1.40	2.10	0.22
	Z	51	0.08	0.09	-0.10
10	X	55	2.00	3.00	1.00
	Y	56	1.00	1.57	-1.80
	Z	57	0.03	-0.11	-0.20
11	X	61	1.90	1.55	-1.70
	Y	62	-1.22	1.15	-1.10
	Z	63	0.09	-0.21	0.20
12	X	67	-0.61	1.60	1.15
	Y	68	-2.75	1.60	-2.40
	Z	69	-0.61	-0.53	-0.55
13	X	73	4.02	-1.13	2.00
	Y	74	-0.70	1.74	-2.18
	Z	75	-0.70	0.88	-1.57
14	X	79	1.60	2.10	1.10
	Y	80	-0.96	1.95	1.00
	Z	81	0.10	-0.06	-0.02
15	X	85	2.40	3.50	-1.93
	Y	86	-3.40	-0.77	2.40
	Z	87	1.60	2.70	-1.32
16	X	91	3.65	1.20	-2.00
	Y	92	-0.52	1.64	0.93
	Z	93	0.52	-0.48	-0.62
17	X	97	1.70	2.10	-1.66
	Y	98	-1.34	3.20	0.44
	Z	99	-0.07	0.09	-0.10

APPENDIX C

2-DIMENSIONAL PLOTS OF SPACE FRAME COMPUTED MODAL SHAPES

APPENDIX C1

Mode 1

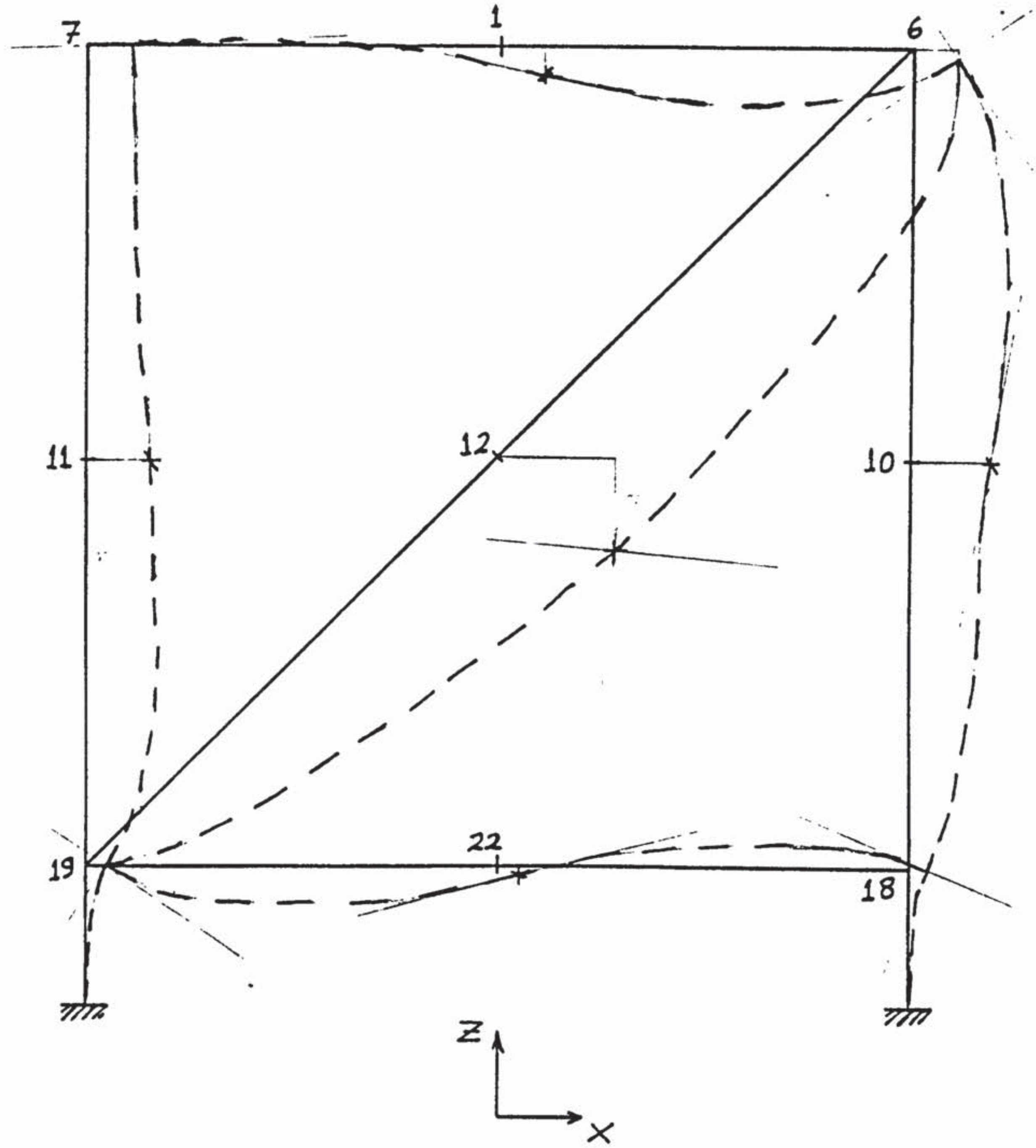


Fig. C.1.1

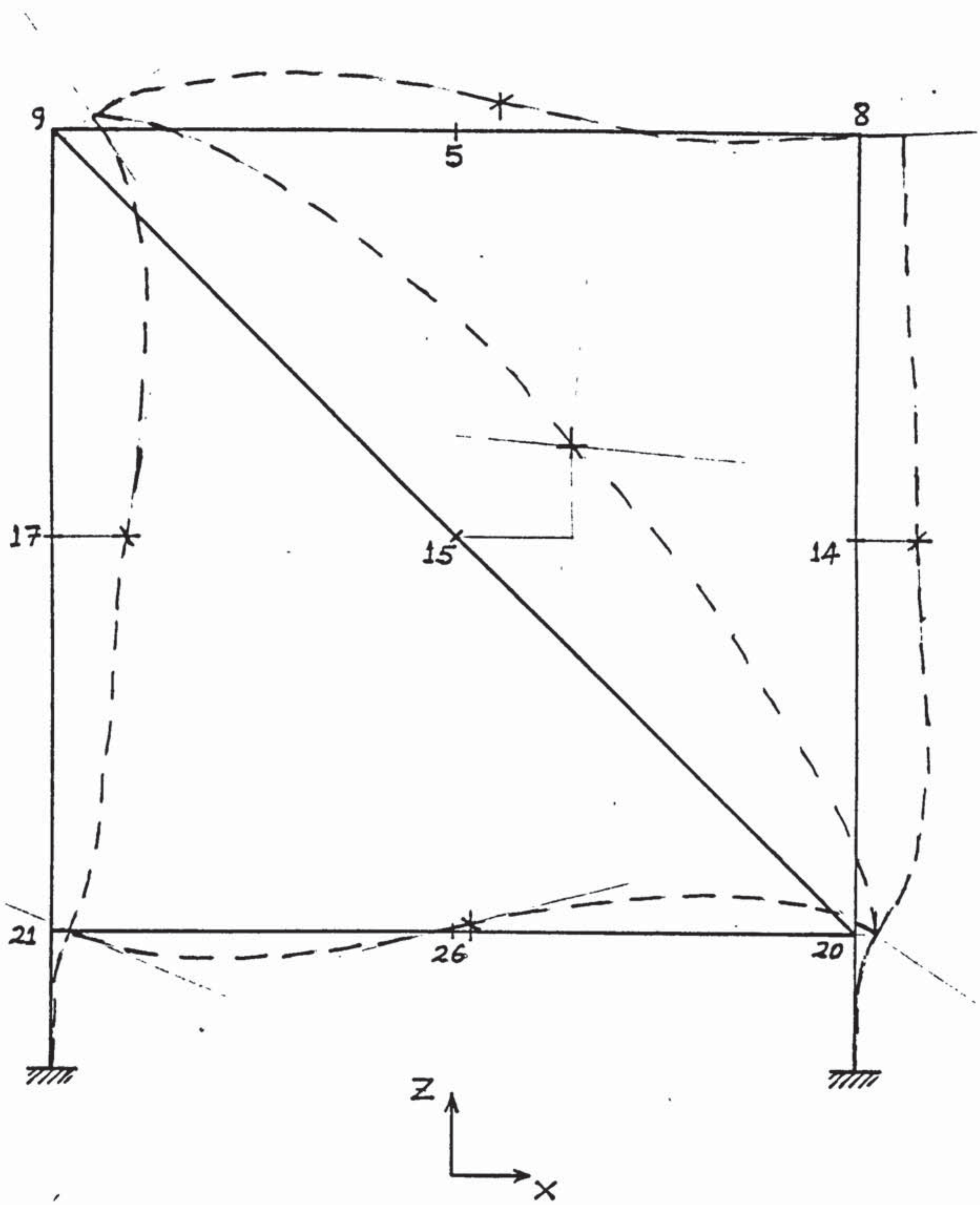


Fig. C.1.2

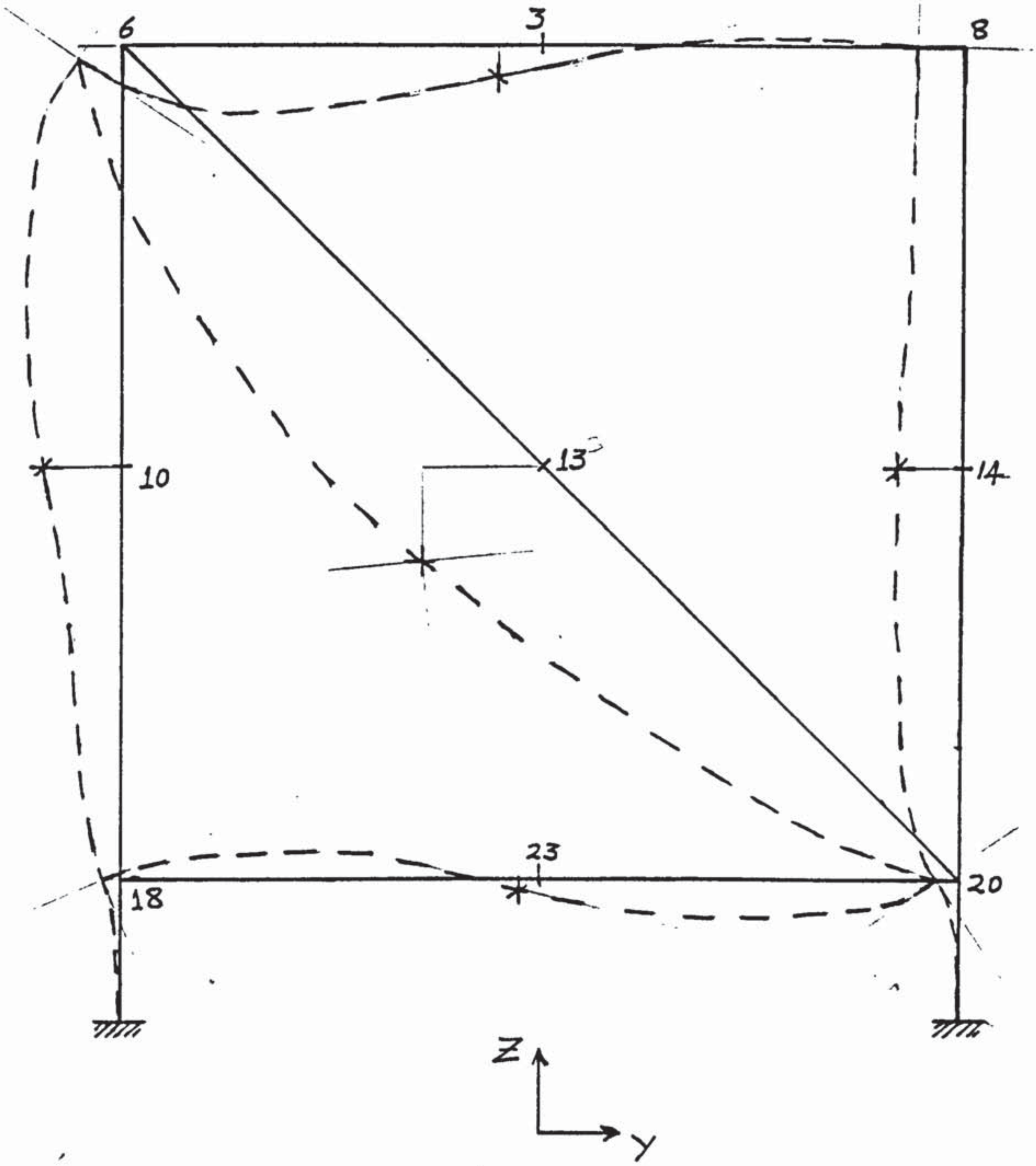


Fig. C.1.3

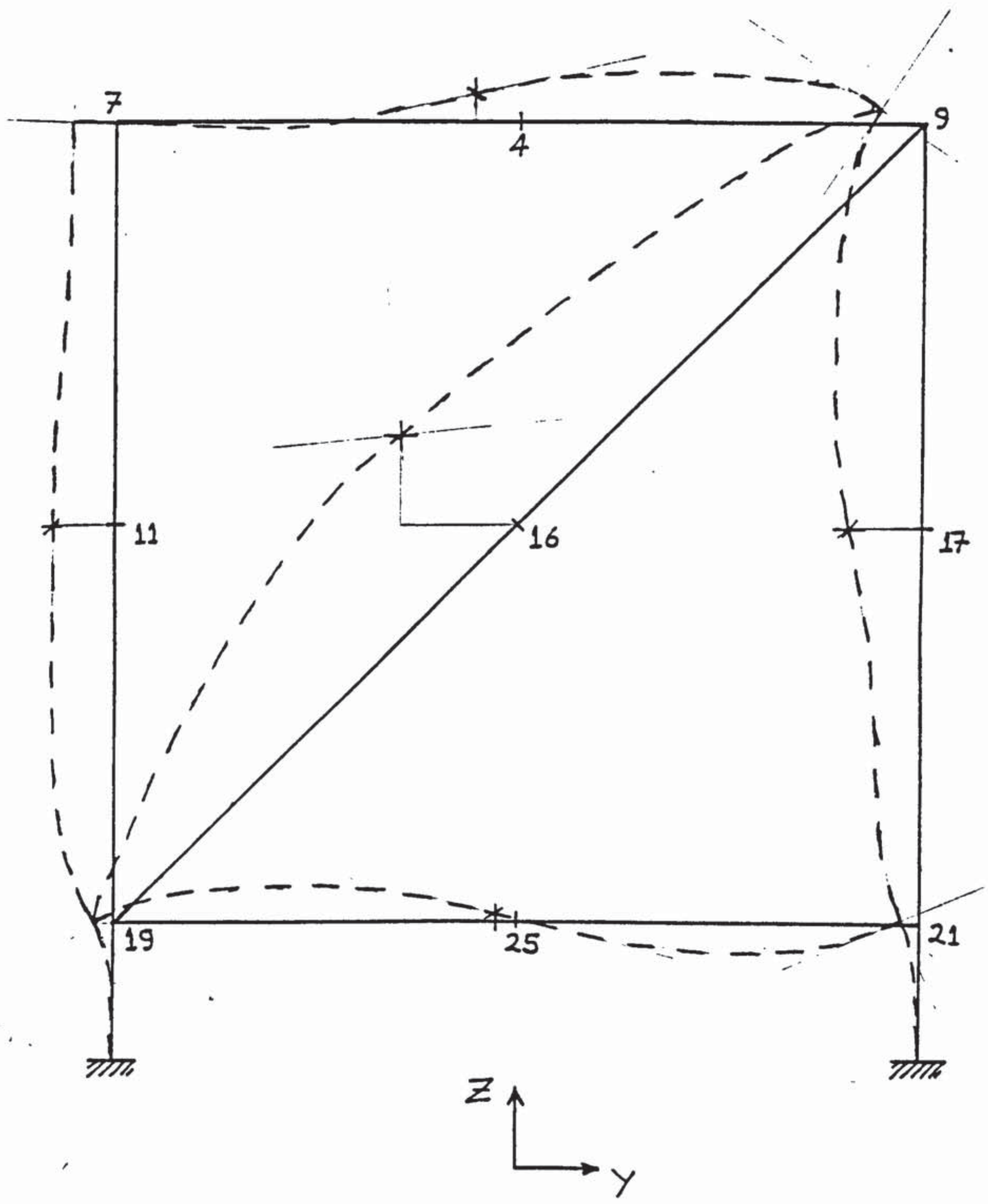


Fig. C.1.4

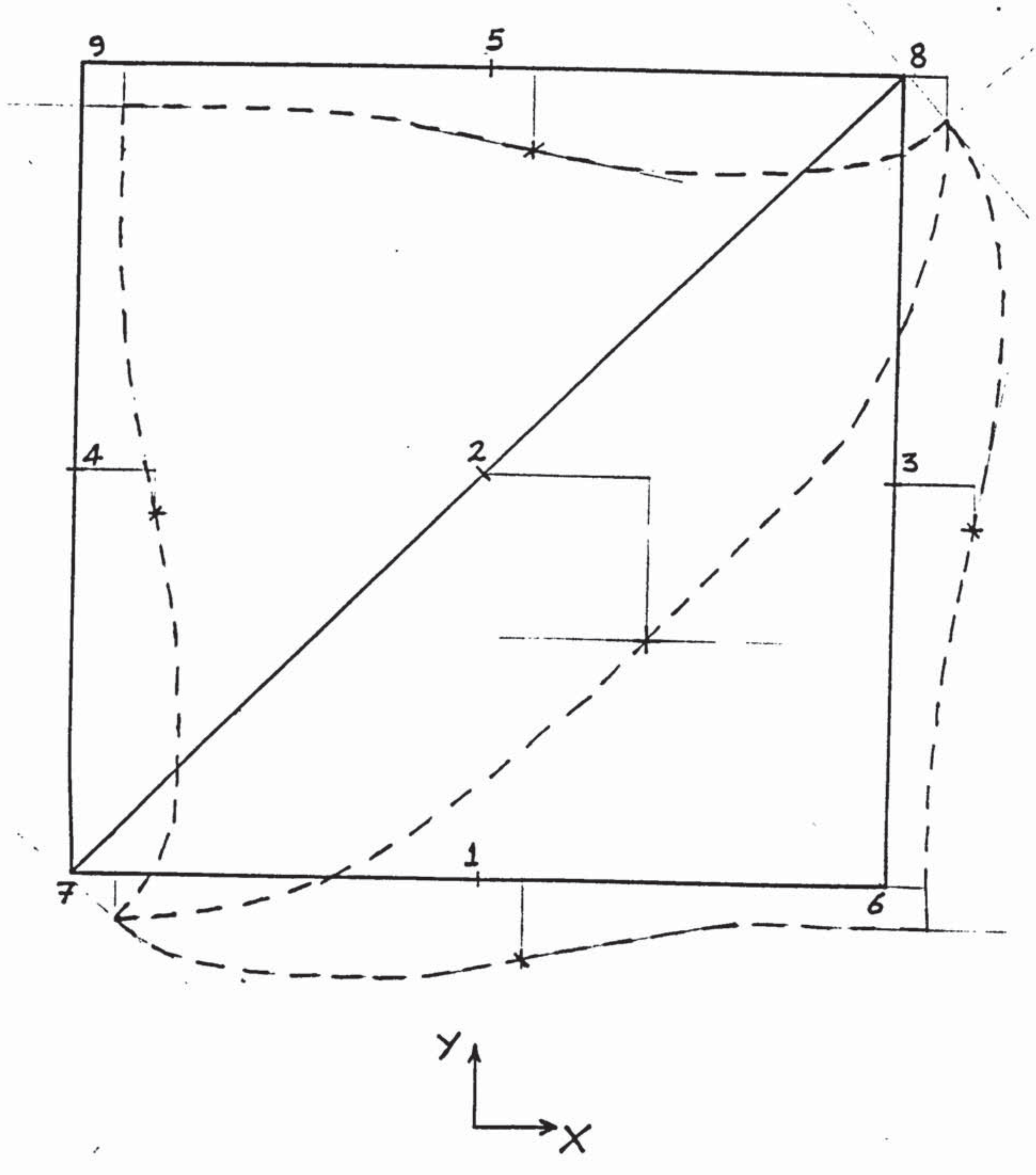


Fig. C.1.5

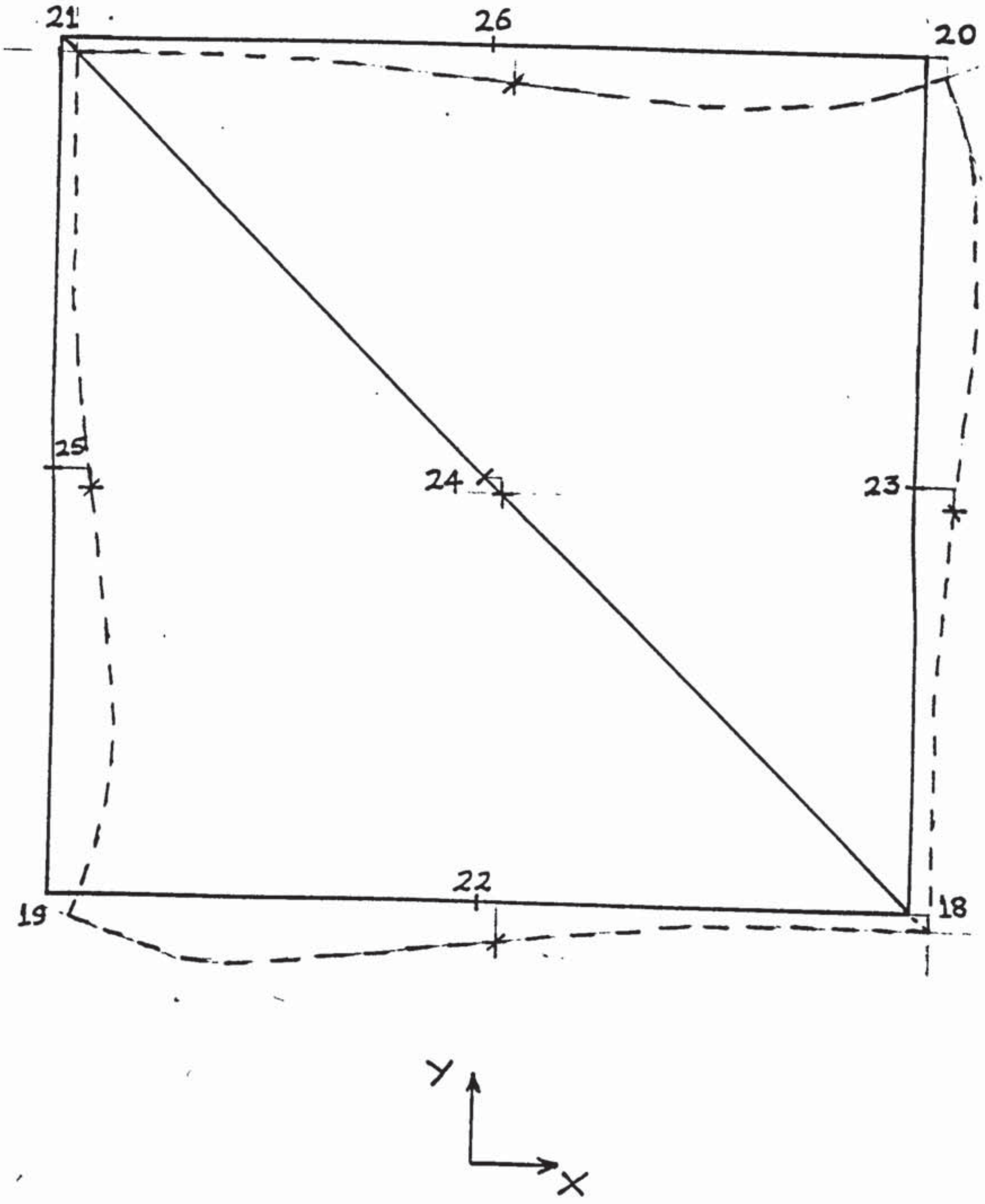


Fig. C.1.6

APPENDIX C2

Mode 2

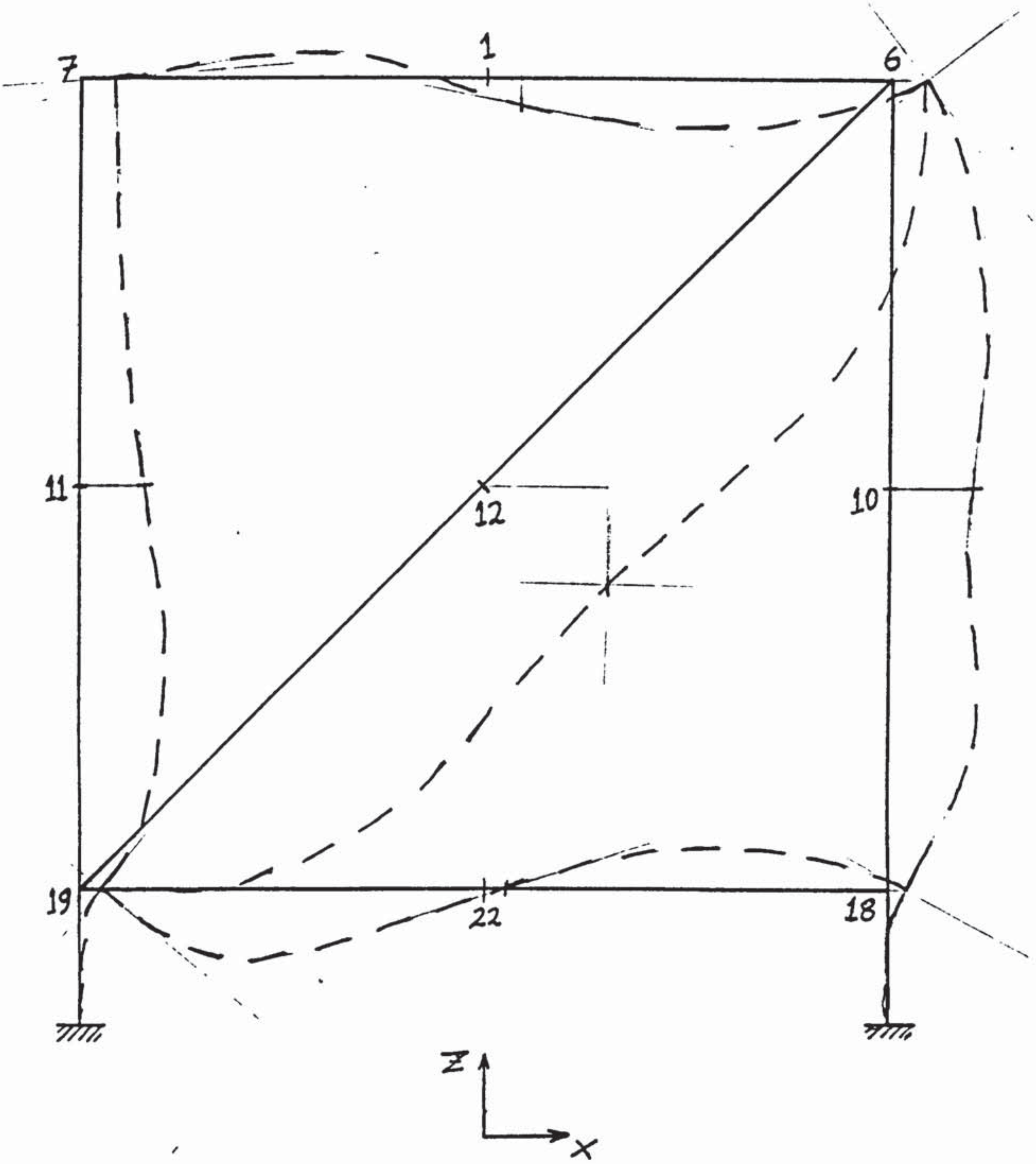


Fig. C.2.1

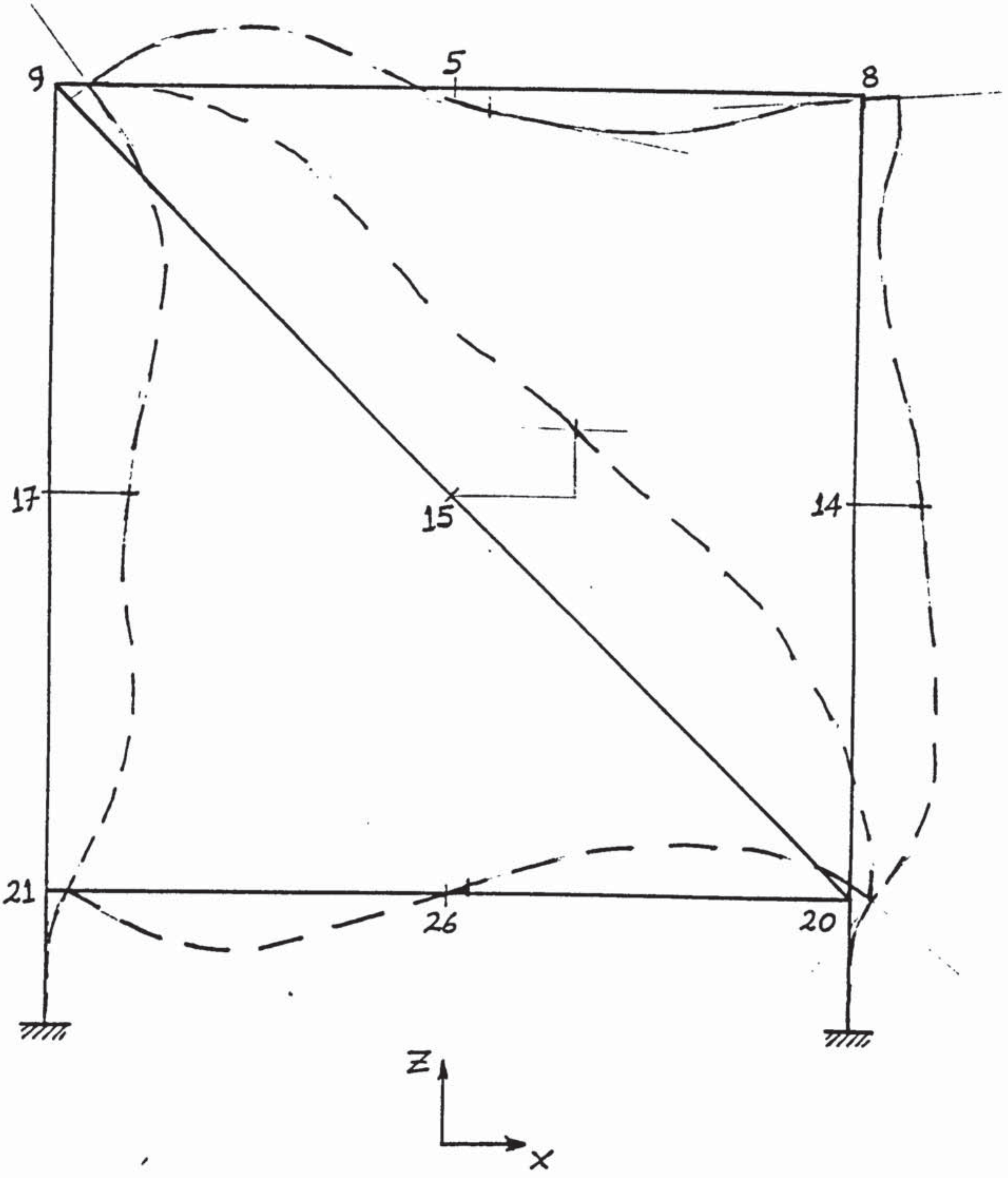


Fig. C.2.2

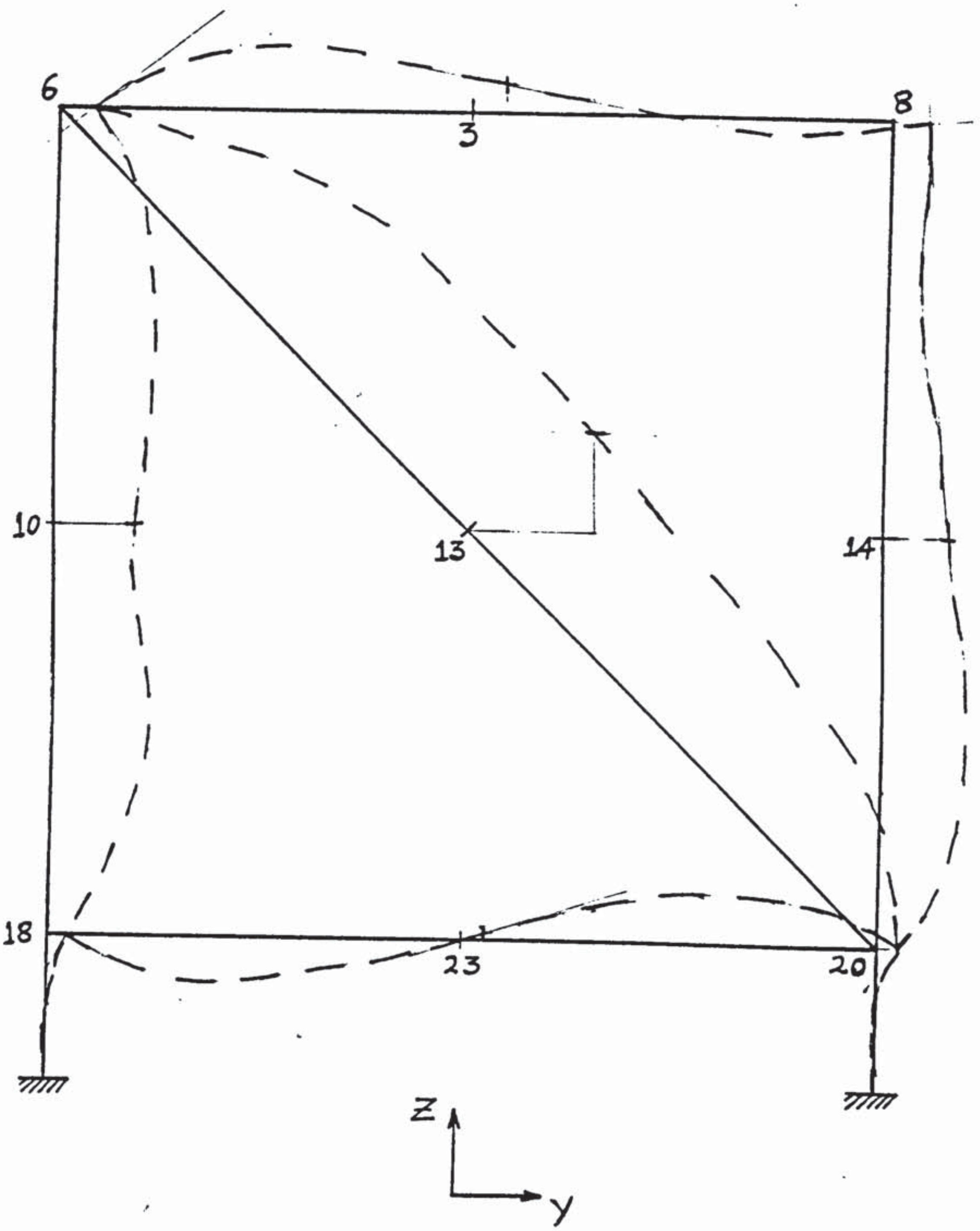


Fig. C.2.3

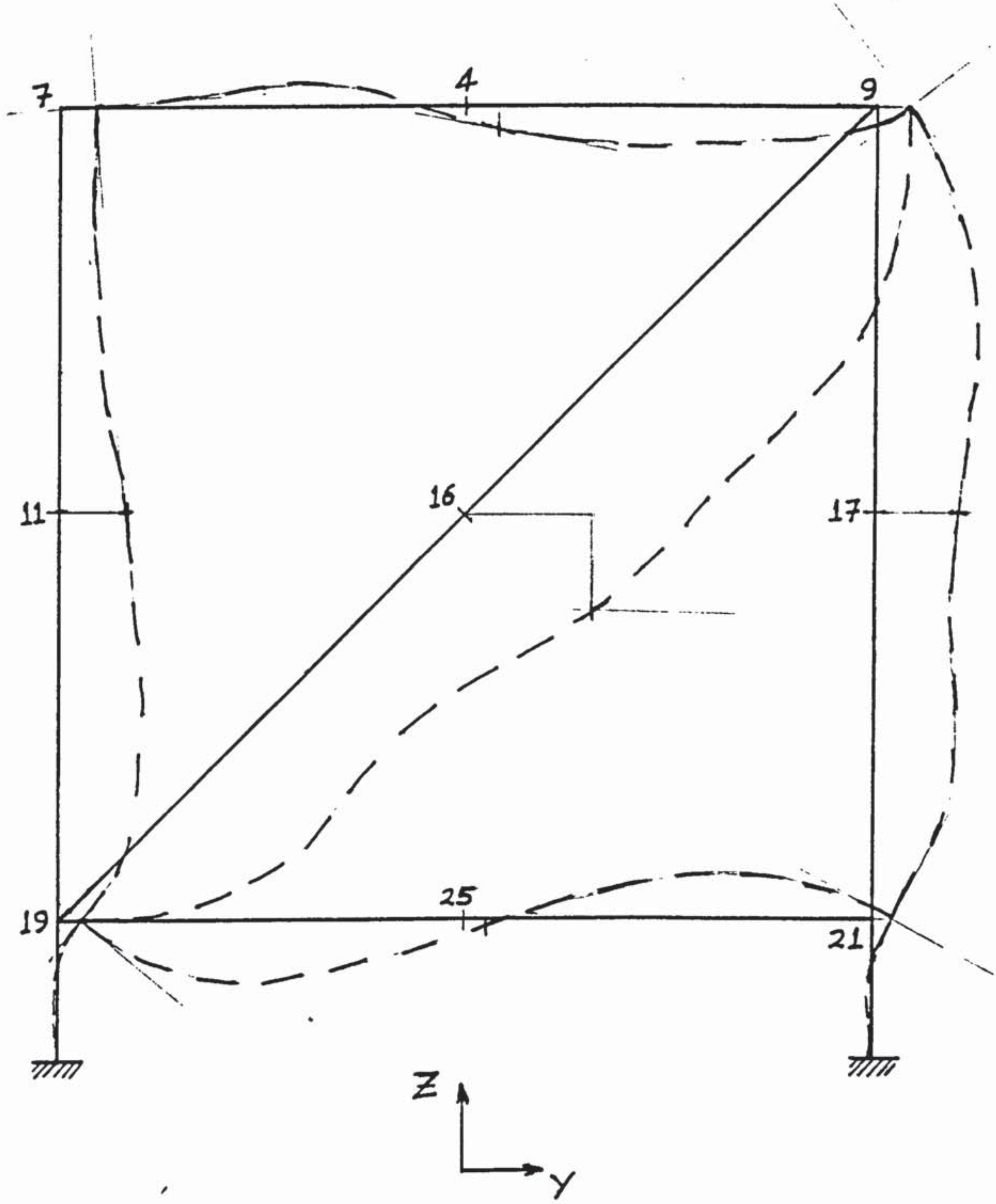


Fig. C.2.4

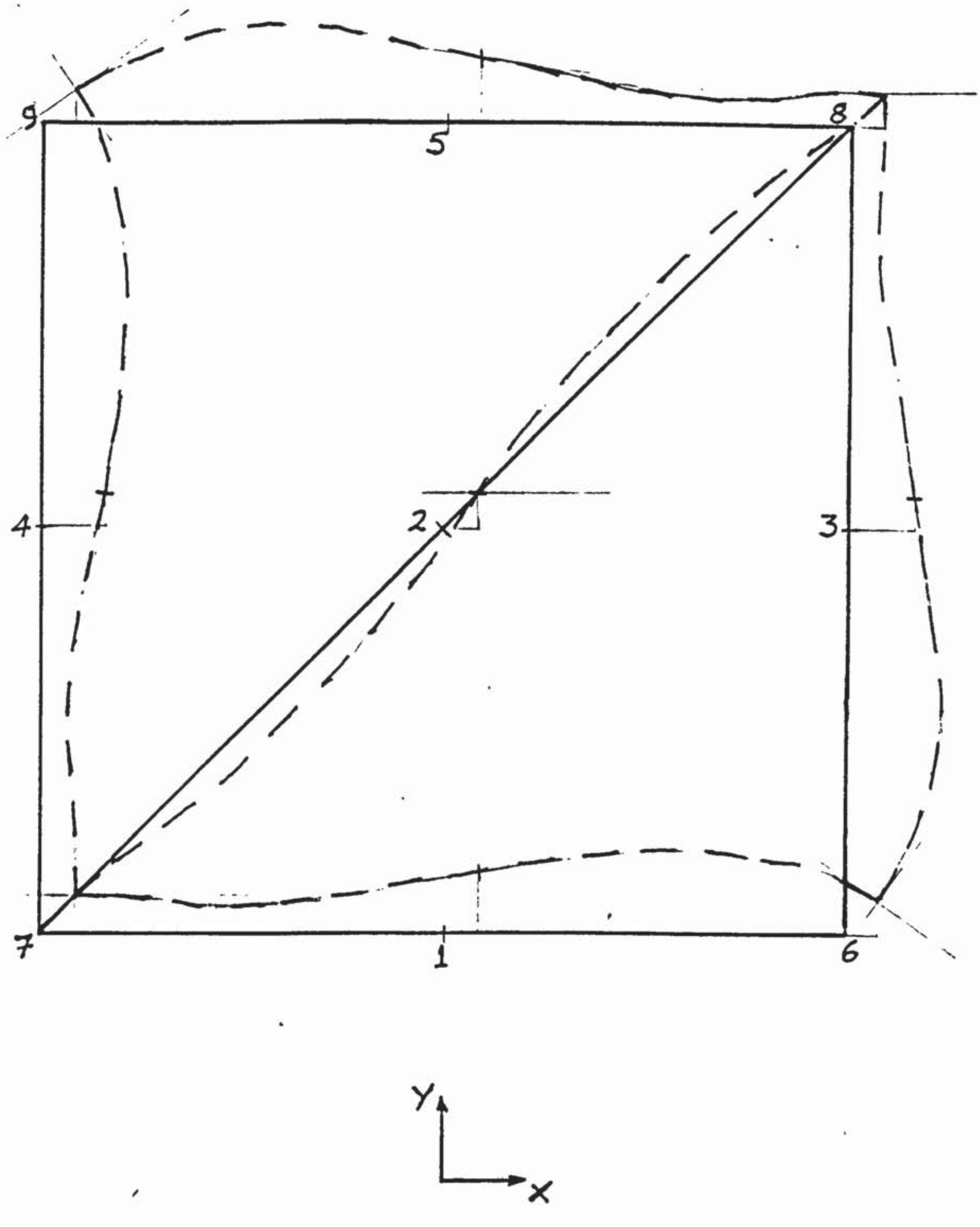


Fig. C.2.5

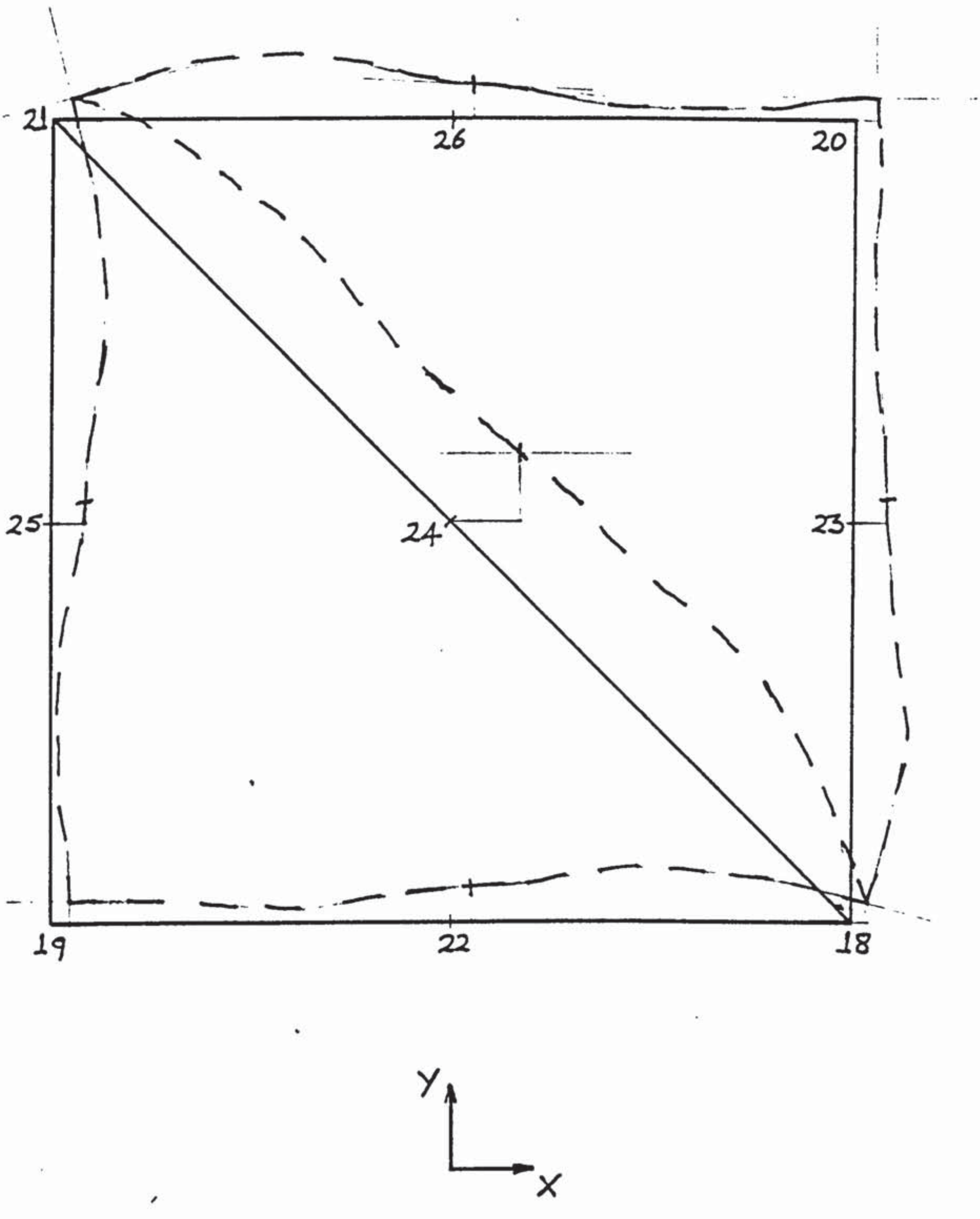


Fig. C.2.6

APPENDIX C3

Mode 3

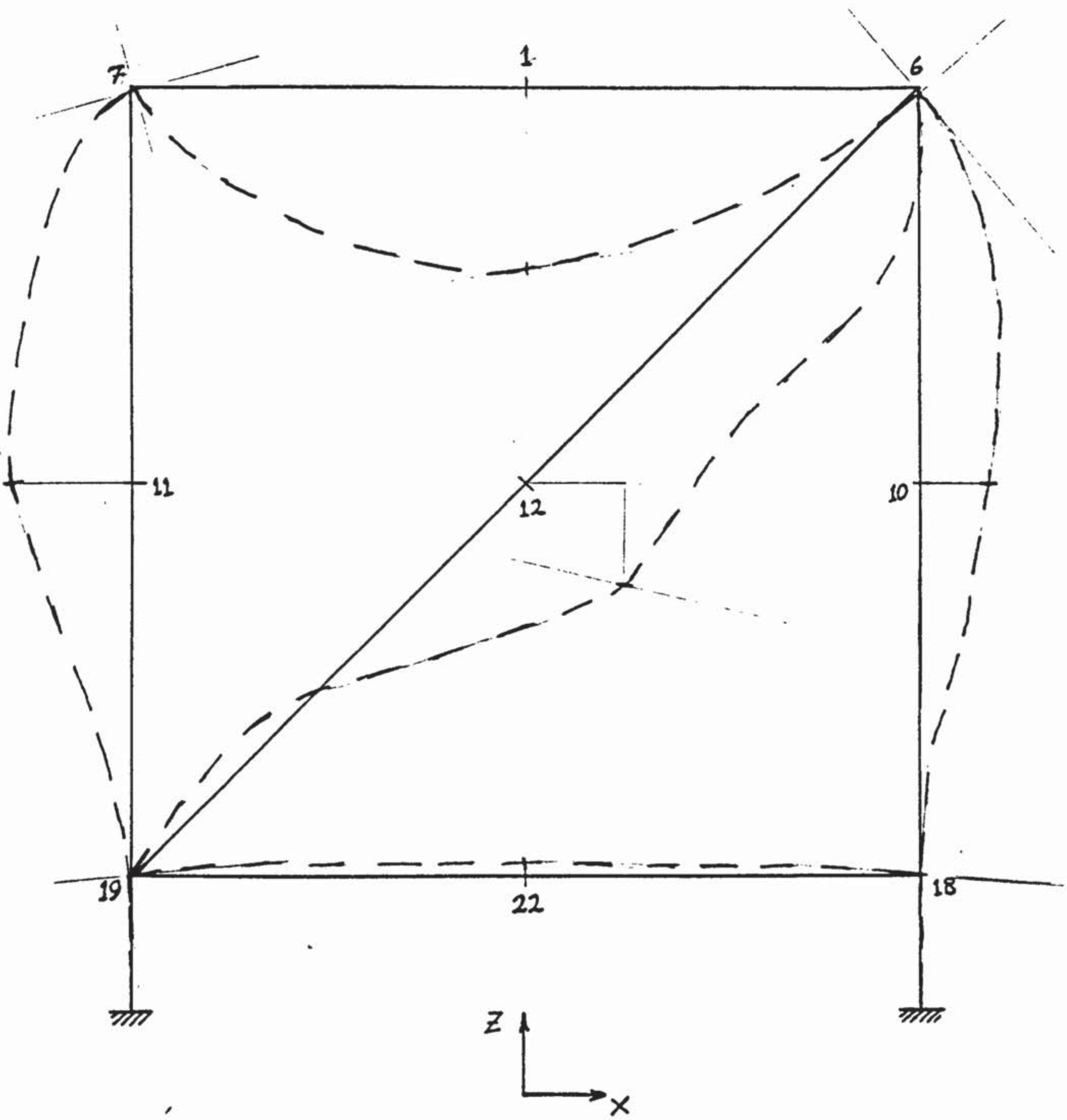


Fig. C.3.1

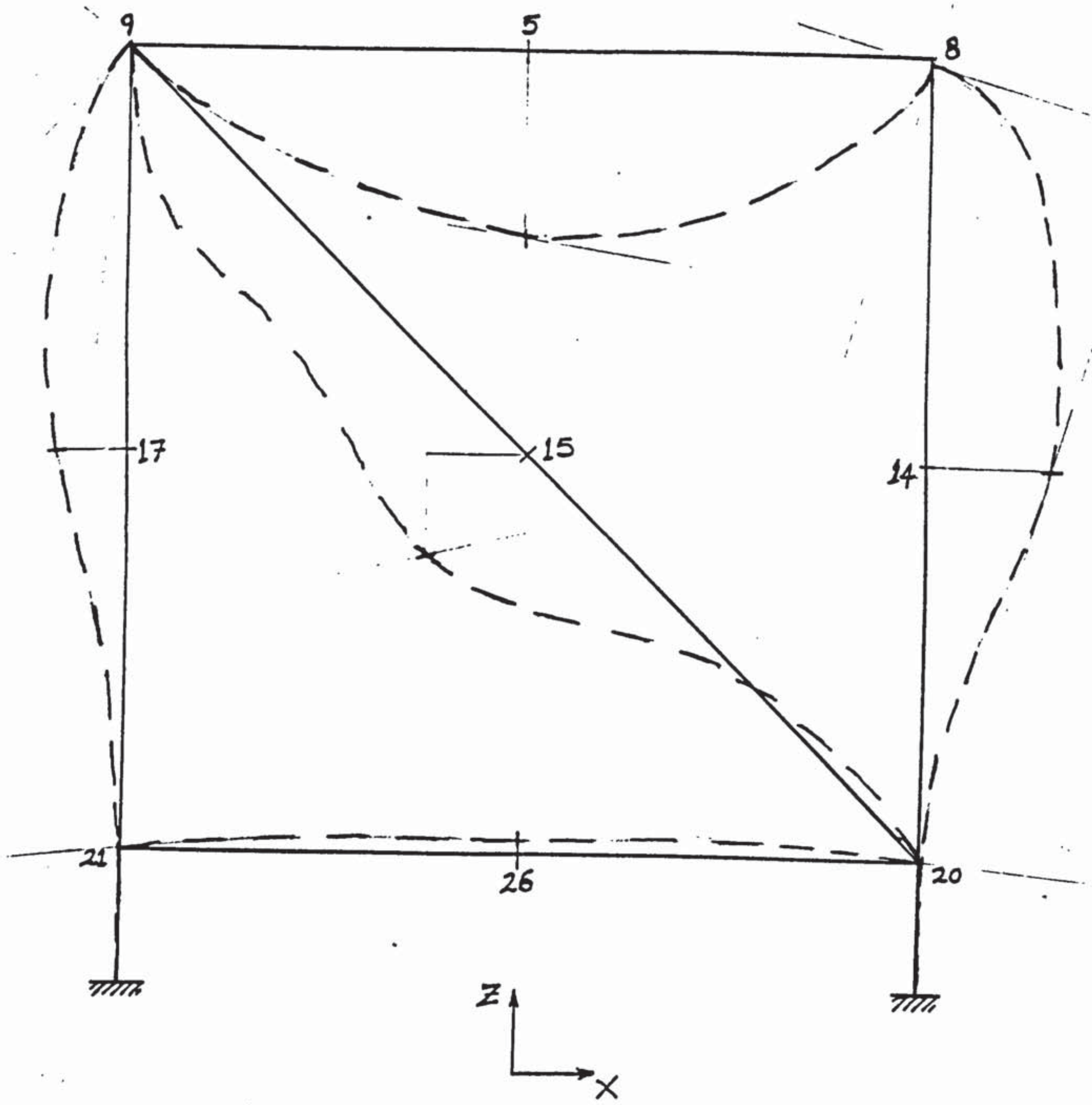


Fig. C.3.2

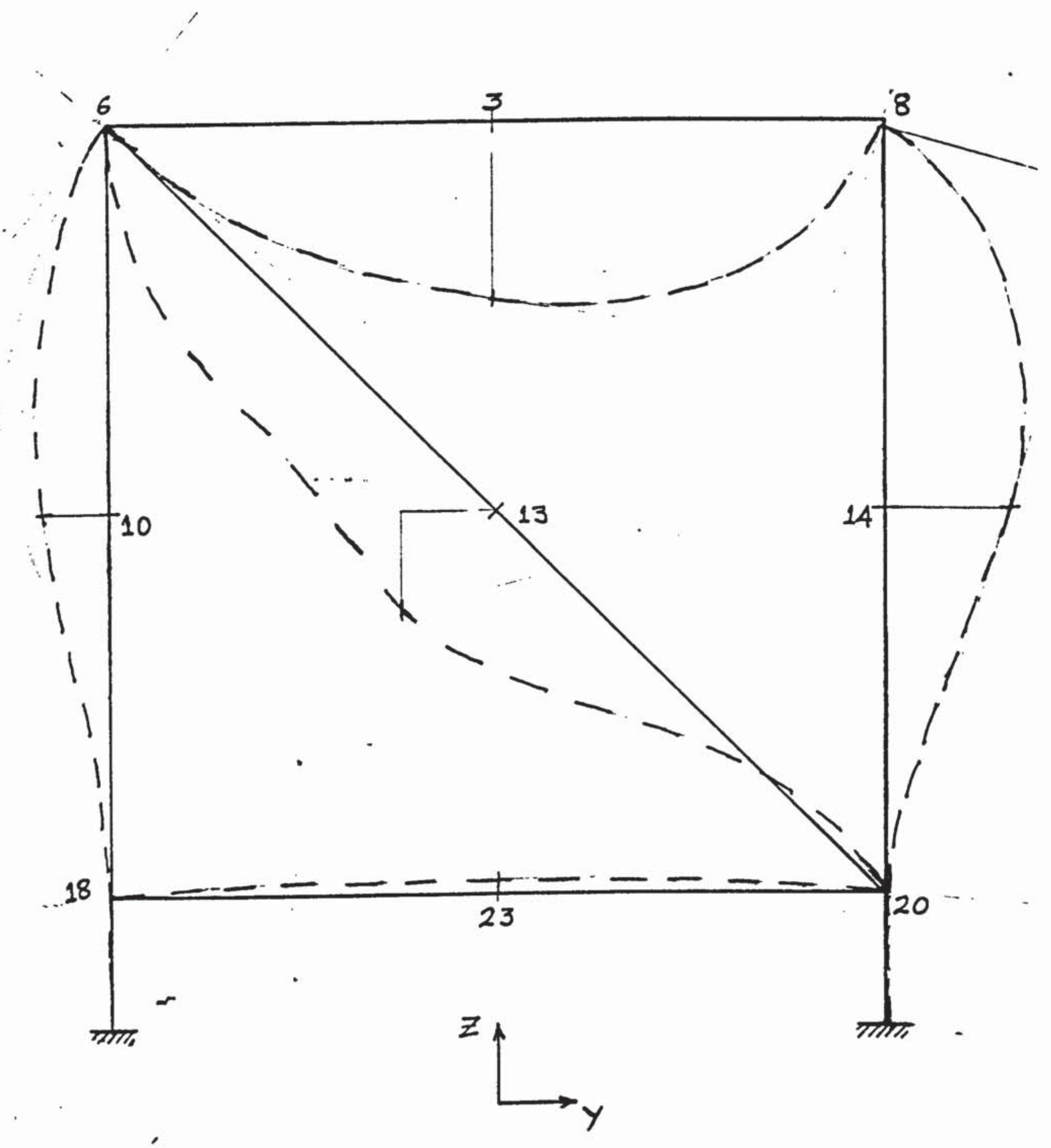


Fig. C.3.3

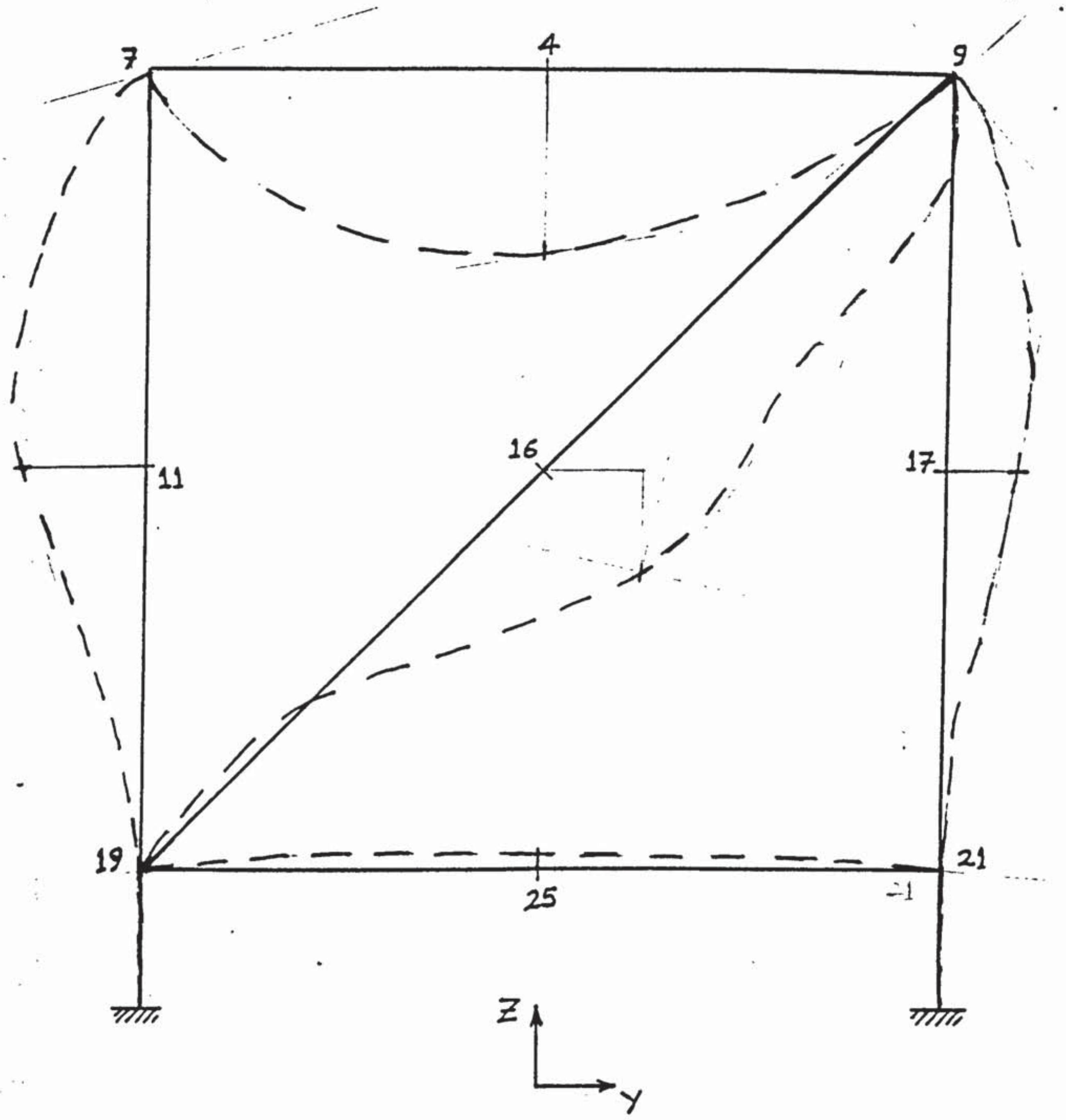


Fig. C.3.4

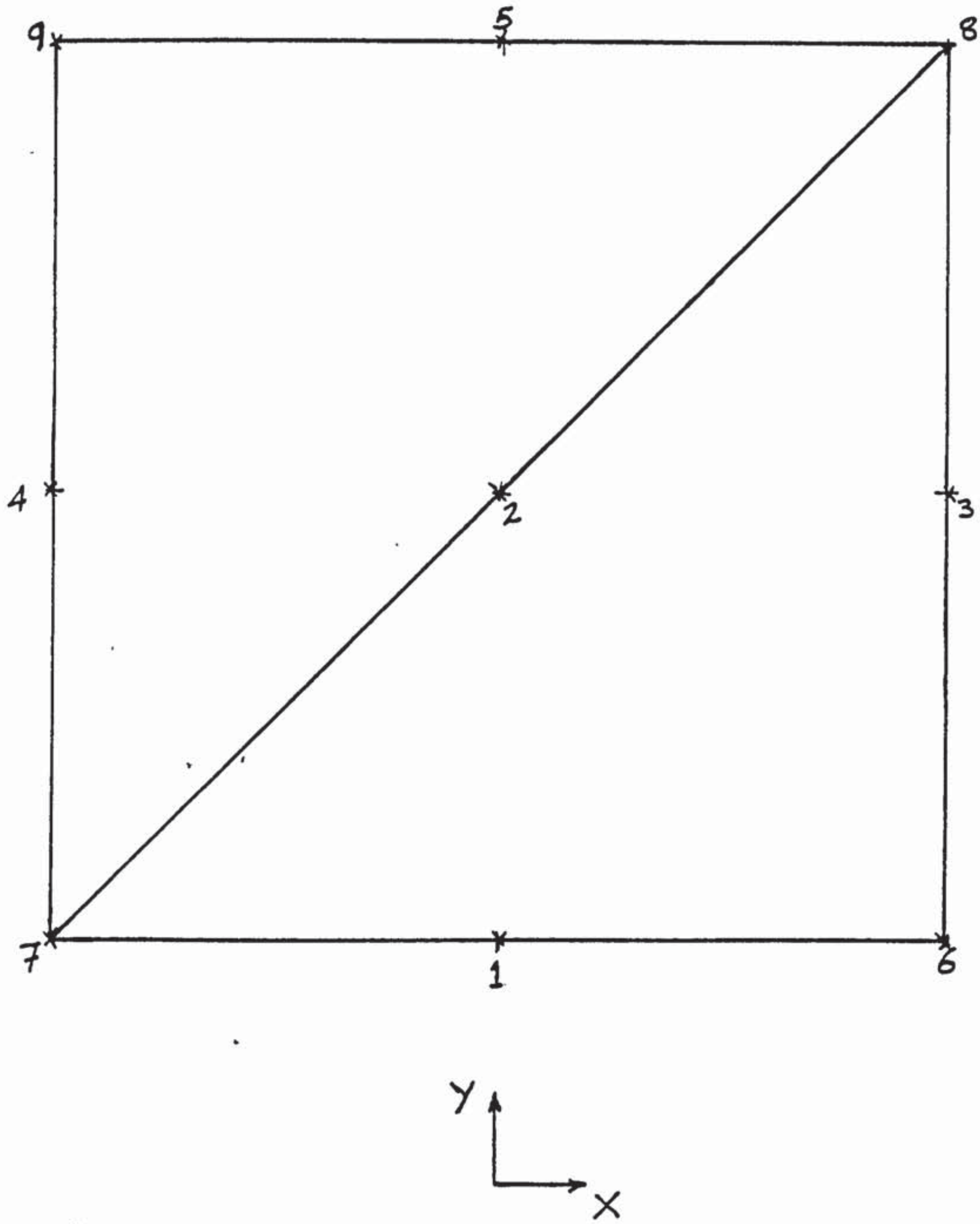


Fig. C.3.5

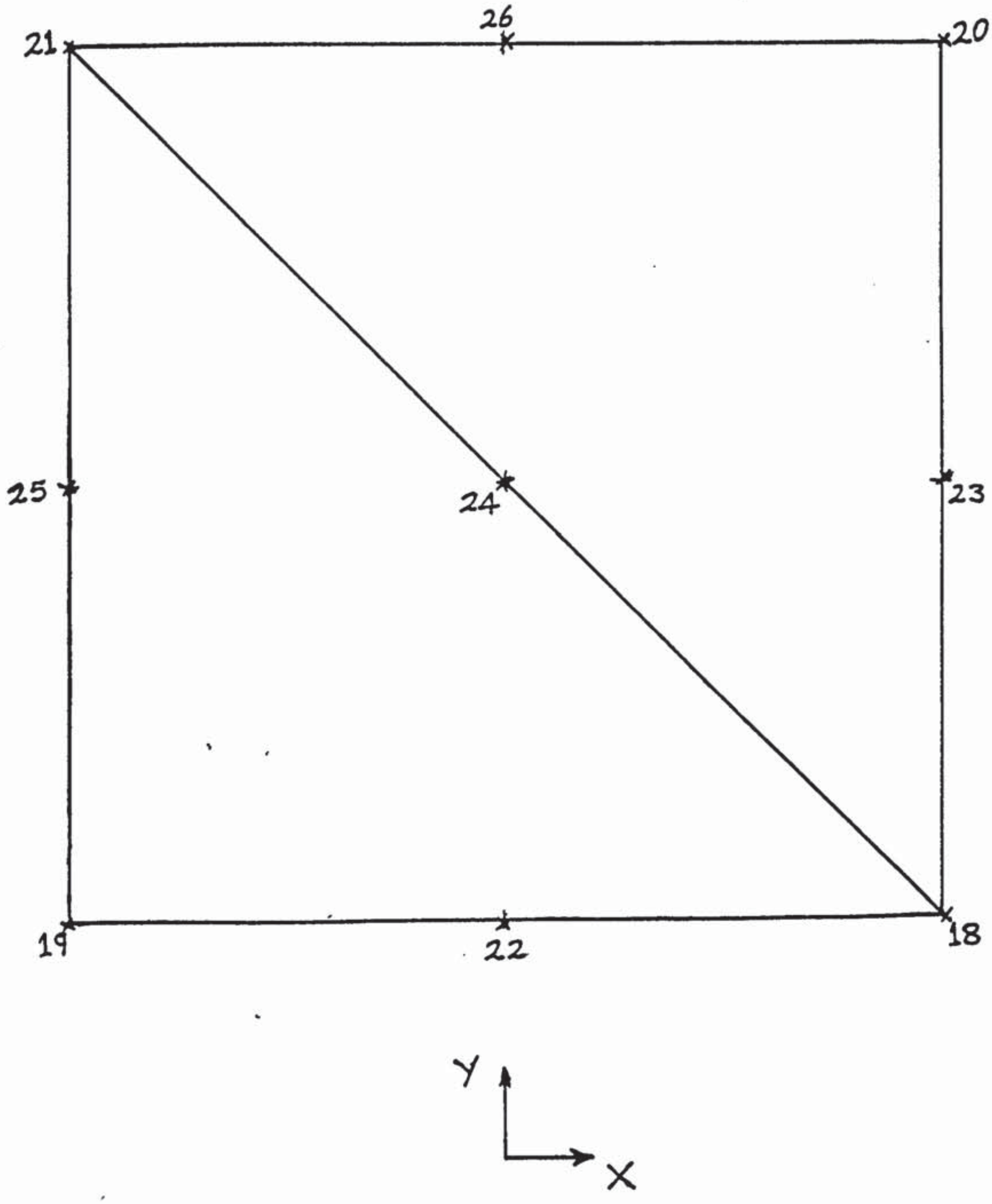


Fig. C.3.6

REFERENCES

1. J.S.Przemieniecki, "Theory of Matrix Structural Analysis", McGraw-Hill Inc., 1968.

2. Y.C.Gung, "Foundations of Solid Mechanics", Prentice-Hall Inc.,1965.

3. B.Nath, "Fundamentals of Finite Elements for Engineers", The Athlone Press of the University of London, 1974.

4. K.I.Majid, "Optimum Design of Structures", Newnes-Butterworths,1974.

5. T.H.Richards, "Energy Methods in Stress Analysis", Ellis Horwood Ltd.-Publisher, 1977.

6. Leonard Kelman Stevens, "Design of Frames with Deformation the Prime Criterion", Journal of the Strutural Division, Proceedings of the A.S.C.E., Vol.103, 1962, pages 55 - 73.

7. Kurt Eisemann, Lin Woo, and Saul Namyet, "Space Frane Analysis by Matrices and Computer", Journal of the Structural Division, Proceedings of the A.S.C.E., Vol.103, 1962, pages 245 - 277.

8. Daniel J. Gorman, "Free Vibration Analysis of Beams and Shafts", John Wiley & Sons Inc., 1975.

9. Moshe F. Rubinstein, "Matrix Computer Analysis of Structures", Prentice-Hall, Inc. 1966.

10. The Financial Times, London, Wednesday 5th December 1975, page 12.

11. J.M.Prentis, "Dynamics of Mechanical Systems", Longman Group Ltd.1970.

12. H.B.Harrison, "Computer Methods in Structural Analysis", Prentice-Hall Inc., 1973.

13. William T. Thomson, "Vibration Theory and Applications", Prentice-Hall Inc., 1965.

14. G.H.Ryder, "Strength of Materials", The Macmillan Press Ltd., 1969.

15. V.Kolovsek, "Dynamics in Engineering Structures", Butterworth & Co. (Publishers) Ltd., London, 1973.

16. W.C.Hurty and M.F.Rubinstein, "Dynamics of Structures", Prentice-Hall Inc., 1964.

17. R.E.D.Bishop and D.C.Johnson, "The Mechanics of Vibration", Campridge University Press,. 1960.

18. M.S.Gregory, "Linear Framed Structures", Longmans Green & Co.Ltd.1966.

19. G.L.Rogers, "Dynamics of Framed Structures", John Wiley & Sons Inc.1959.

20. A.S.Hall & R.W.Woodhead, "Frame Analysis", John Wiley & Sons,Inc.1961.

21. Wilkinson,J.H. & Reinsch C, "Handbook for Automatic Computation, Vol.II, Linear Algebra ", Springer-Verlag, 1971.

22. Jamad Joseph Azar, "Matrix Structural Analysis", Pergamon Press, 1972.

23. Kardestuncer, Hayretfin, "Elementary Matrix Analysis of Structures", McGraw-Hill, 1974.

24. D.H.Norrie and G.de Vries, "The Finite Element Method", Academic Press Inc., 1973.

25. T.H.Richards and Y.T.Leung, "An Accurate Method in Structural Vibration Analysis", Journal of Sound and Vibration, Vol.55, Number 3, 8th Dec. 1977, pages 363 - 376.

26. Lewis C.Schwidt, Peter R.Morgan and Leonard K. Stevens, "Effects of Imperfection on Space Frame Stiffness", Journal of the Structural Division, Proceedings of the A.S.C.E.,Vol.33,1977, pages 197 - 210.

27. Henry R. Grooms and James Rowe, "Substructuring and Conditioning", Journal of the Structural Div; Proceedings of the A.S.C.E. vol.103, 1977, pages 507 - 514.

28. Kristiansen, V.R. Soebel, W., and Hamilton, J.F., "An Investigation of Scaling laws for Vibrating Beams and Plates with Special Attention to the Effects of Shear and Rotatory Inertia", Journal of Sound and Vibration, vol.20, 1972, pages 113-122.

29. Hawson, W.P., and Williams, F.W. "Natural Frequencies of frames with axially loaded Timoshenko members", Journal of Sound and Vibration, vol.26, 1973, pages 503 - 515

30. MacBain, J.C., and Genin, J, "Natural Frequencies of a beam considering support characteristics", Journal of Sound and Vibration, vol.27,1973, pages 197 - 206.

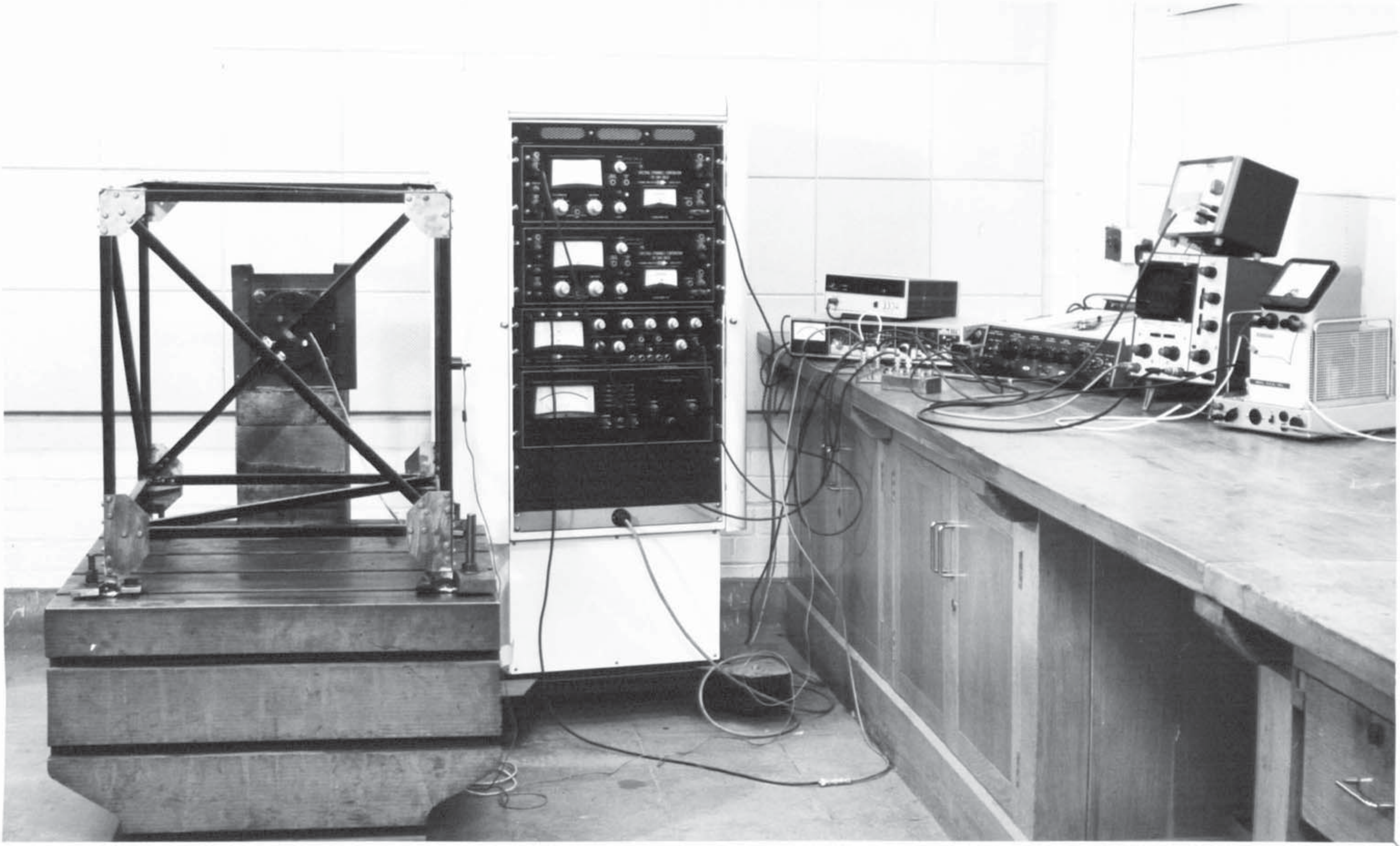
31. Gordon H.Holze and Arthur P.Bores, "Free Vibration Analysis using substructuring", Journal of the Structural Div, Proceedings of the A.S.C.E., vol.101, 1975, pages 2627 - 2639.

32. Joachim S. Rodrigues, "Node Numbering Optimization in Structural Analysis", Journal of the Structural Div, Proceedings of the A.S.C.E. vol.101, 1975, pages 361 - 376.

33. Pessu, A.M.O., "Optimization of the Transient Response of a Multi-Degree of Freedom System", M.Sc. Thesis, 1975, Mechanical Engineering Department, The University of Aston in Birmingham.

34. Guyan, R.J., "Reduction of Stiffness and Mass Matrices", J.Am.Inst. Aeron.Astron., vol.3, 1965, page 380.

35. Paul H. Wirstching, A.M.Asee and Peter H.Prasthofer., "Preliminary Dynamic Assessment of Deepwater Platforms", Journal of the Structural Division, Proceedings of the A.S.C.E., vol.102, 1976, pages 1447-1462.



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