MACPROP

A Revised Program for the Refinement
of Satellite Orbital Parameters.

Marion Swinerd

Master of Philosophy

The University of Aston in Birmingham

September 1982

THE UNIVERSITY OF ASTON IN BIRMINGHAM

Marion Swinerd

M. Phil 1982

MACPROP

A Revised Program for the Refinement of Satellite Orbital Parameters

Summary

Computer programs for the refinement of the orbital parameters of an artificial Earth satellite have been in use since 1957. One of the programs in current use is PROP. This program takes an initial estimate of the orbital parameters of an Earth satellite, together with observations of the satellite, and refines the parameters by a least squares differential correction process. The current version of PROP, known as PROP6, has been investigated and a new version, MACPROP, written. A description of the refinement process is presented, together with a detailed investigation of each segment of the program.

Changes to the program have been made to remove some programming errors. Additional facilities have been written into the program so that the user has greater control over some of the physical quantities used by the program. Further changes have been made to make PROP easier to use. Each change has been described and details of the use of the additional facilities are given.

MACPROP has been tested, using some of the new facilities, and the results are compared with those obtained by using PROP6. MACPROP is found to be slightly smaller and much faster than PROP6. The performance of PROP is improved by using MACPROP, particularly if some of the physical quantities used within PROP are varied. The orbital parameters obtained from MACPROP differ slightly from those obtained from PROP6 and may be accepted by the user with greater confidence, in the light of this investigation.

Key words: Satellite, Orbit, Program

List of Contents | Page numbers

List of Contents (continued)   Page Numbers

List of Contents (continued) Page Numbers

List of Tables

Page numbers

List of Figures | Page Numbers

1. Introduction

In an idealised situation a satellite can be envisaged orbiting an isolated, spherical, atmosphere-free Earth. In this circumstance, according to classical mechanics, the satellite would move in an unchanging elliptical orbit. However, in reality, the satellite is perturbed by a number of agencies. The orbit is influenced, principally, by the gravitational effect of the non-spherical Earth, the Moon and the Sun, and by aerodynamic forces due to the atmosphere. In some cases the pressure of solar radiation may be significant.

A computer program for the refinement of orbital parameters has been in use in the United Kingdom since 1957. The user of such a program takes an initial estimate of the orbital elements of an Earth satellite, together with observations of the satellite, and uses these in combination to refine the elements by an iterative least squares differential-correction procedure. Thus the researcher may obtain a series of orbits of a particular satellite over a period of time, and this series will specify the time evolution of the orbital elements of the satellite. The temporal behaviour of the elements can be related mathematically to the perturbing forces through Lagrange's planetary equations (Roy 1978), where, in terms of the usual Keplerian elements (see Section 3),

$$\frac{da}{dt} = \frac{2}{na}\frac{\partial U}{\partial M} ,$$

$$\frac{de}{dt} = \frac{1}{na^2e}\left\{(1-e^2)\frac{\partial U}{\partial M} - (1-e^2)^{\frac{1}{2}}\frac{\partial U}{\partial \omega}\right\} ,$$

$$\frac{di}{dt} = \frac{\operatorname{cosec} i}{na^2(1-e^2)^{\frac{1}{2}}}\left\{\cos i\,\frac{\partial U}{\partial \omega} - \frac{\partial U}{\partial \Omega}\right\} ,$$

$$\frac{d\Omega}{dt} = \frac{\operatorname{cosec} i}{na^2(1-e^2)^{\frac{1}{2}}}\frac{\partial U}{\partial i} ,$$

$$\frac{d\omega}{dt} = \frac{1}{na^2}\left\{ \frac{(1-e^2)^{\frac{1}{2}}}{e} \frac{\partial U}{\partial e} - \frac{\cot i}{(1-e^2)^{\frac{1}{2}}} \frac{\partial U}{\partial i} \right\},$$

$$\frac{dM}{dt} = n - \frac{1}{na^2}\left\{ \frac{(1-e^2)}{e} \frac{\partial U}{\partial e} + 2a \frac{\partial U}{\partial a} \right\},$$

U being the potential due to the perturbing forces.

Using the link between observation and theory, the researcher is able to determine the properties of the disturbing sources. Information about the state of the upper atmosphere at the satellite's altitude, for example densities and zonal winds, may be obtained. Similarly, the shape of the geoid may be deduced.

In 1962 the method of orbit determination was to use a collection of programs written for a Pegasus Mk I computer by Merson, Tayler and Gooding (Merson 1962). Later a program for the refinement of orbital parameters (PROP) was written in Fortran, suitable for running on an ICL 1900 series computer. Several versions of the program have been written and the latest, PROP6 (Gooding 1974), was released in 1974. The program has been written and maintained by a team at the Royal Aircraft Establishment (RAE), Farnborough, and was designed primarily to be suitable for running on the RAE's ICL 1907 computer. A User Guide (Gooding, Tayler 1968) for the PROP3 version was issued in 1968.

PROP6 is used currently as a principal research tool in the Earth Satellite Research Unit (ESRU) of the University of Aston in Birmingham. In order that the program may be more fully understood, and so that modifications to the program could be made, an independent investigation of PROP6 has been carried out at ESRU and several changes made to the program. The findings of the investigation and details of the changes are presented here. The thesis is presented in a form such that it may be used directly as a program document by anyone contemplating further modifications to the program.

In some instances the theoretical background of the PROP subroutines is not available in previous documentation of the program. In these cases the theory has been re-developed and presented here. Other subroutines have an elementary theoretical background which, nevertheless, has been included for completeness.

2. The Program

The program PROP has evolved over a number of years, from a simple program written in 1957, through a more sophisticated version that was run on a Pegasus machine until 1968, to a program written in Fortran II for running on an Atlas machine. The latter was the first program to be called PROP. The current version, PROP6, is written in ICL Extended Fortran (ICL 1971) with two of the segments written in semi-compiled code. This makes PROP6 suitable for running only on ICL 1900 series computers. In the new version of PROP the two semi-compiled segments have been removed but the version of Fortran used is ICL Extended Fortran. In this way the program has been made more transportable between main frame computers. Minor modifications may be necessary to overcome differences between versions of Fortran.

The program has been divided into a master segment, approximately 38 subroutine segments and two function segments. Each segment has been considered separately in Section 4.

The segments have been arranged in an overlay structure to ensure that the maximum core size required is less than 32k (1k = 1024 words of storage). The overlay structure divides the available core store into three "areas". One area is termed the "permanent" area. Segments assigned to this area are resident in core throughout a PROP job, together with segments added to PROP by the consolidator, for example circular functions. The other two areas are known as "overlay area 1" and "overlay area 2". At any one time during a PROP job an overlay area may contain only one of the units assigned to it, the other units being held in backing store. When a segment which has been assigned to an overlay unit is called, the system checks whether that unit is already in core store. If it is not it will be copied in to the appropriate area from the backing store and entered in the normal way. The assignment of segments to units and areas is illustrated in Figure 2.1.

Figure 2.1 The Overlay Structure

Permanent area:

PROP (master segment)

DATA (data block segment)

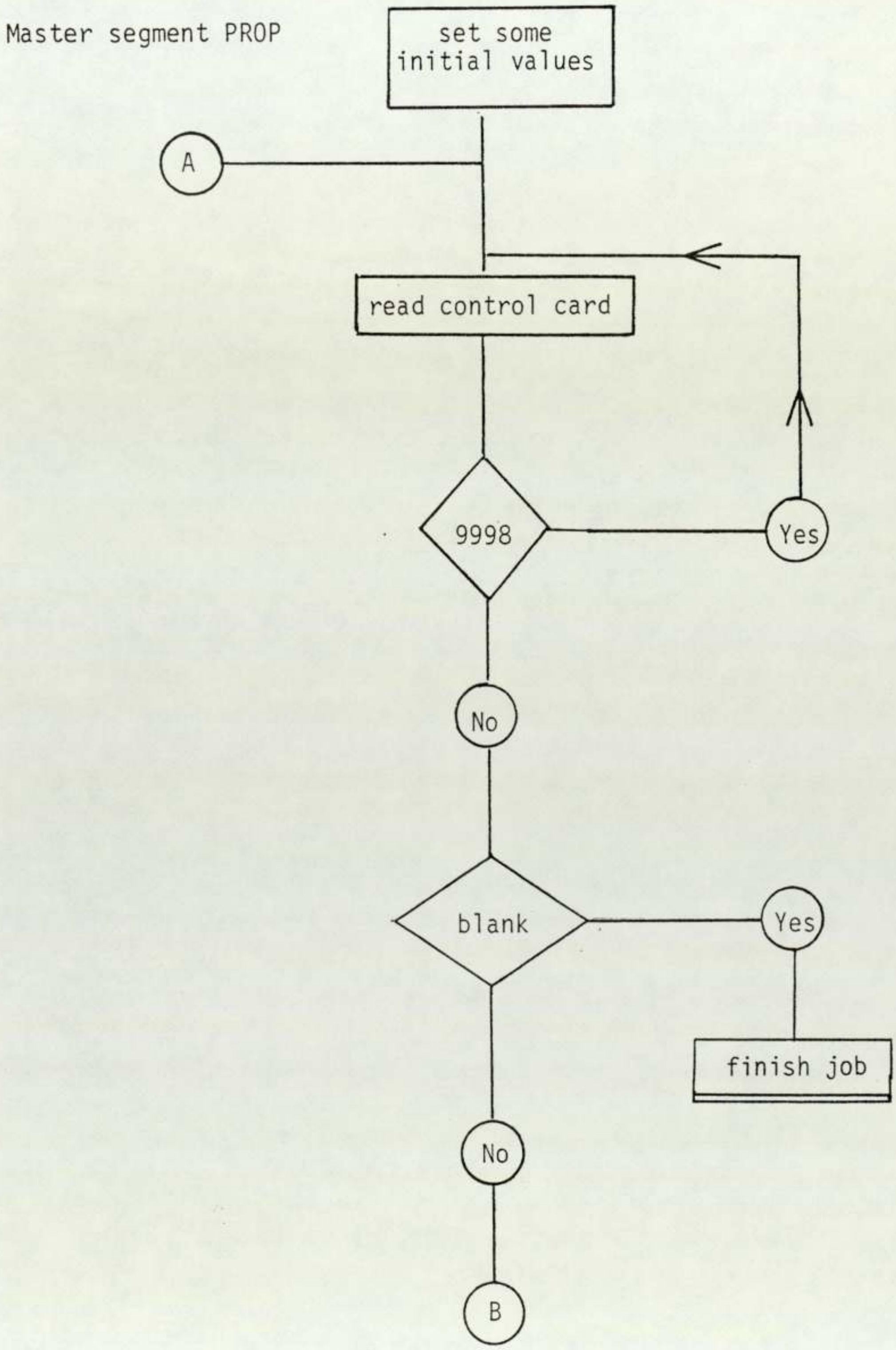
EAFKEP, MOVELS, SIDANG, TRINV — small segments

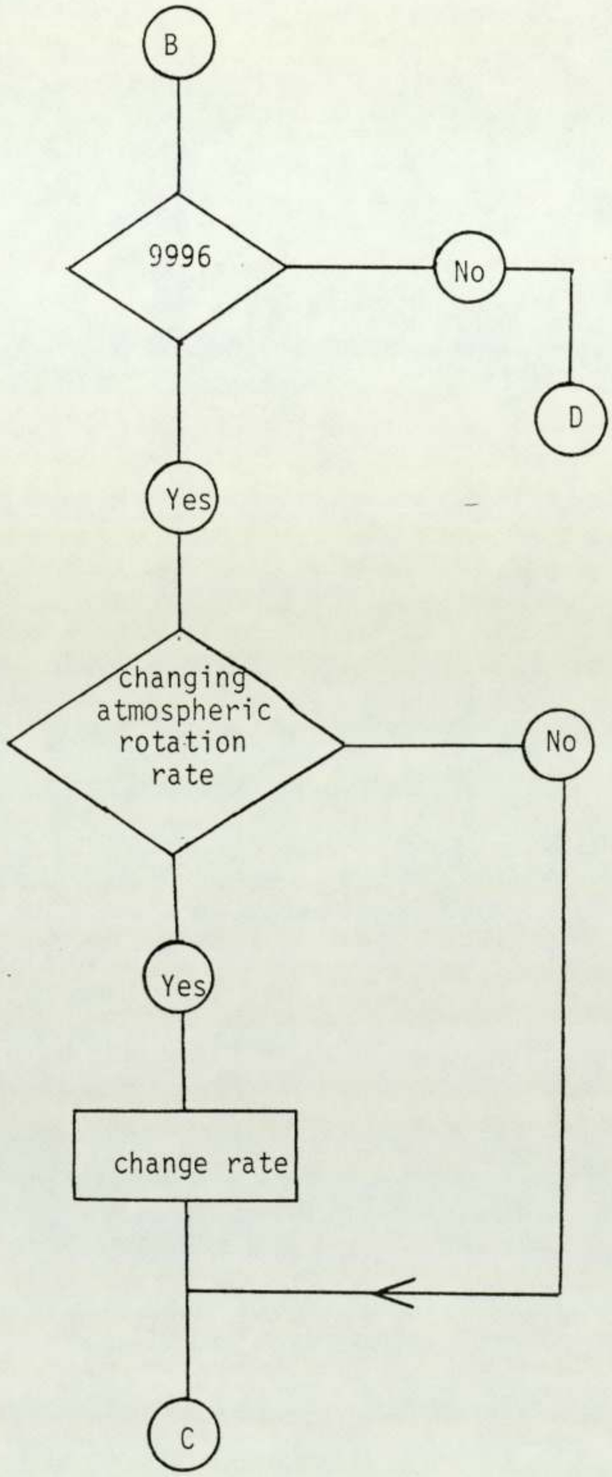
CONVRT, SATXYZ, SHOPER, SINXOX, XLONG — derivation of satellite position

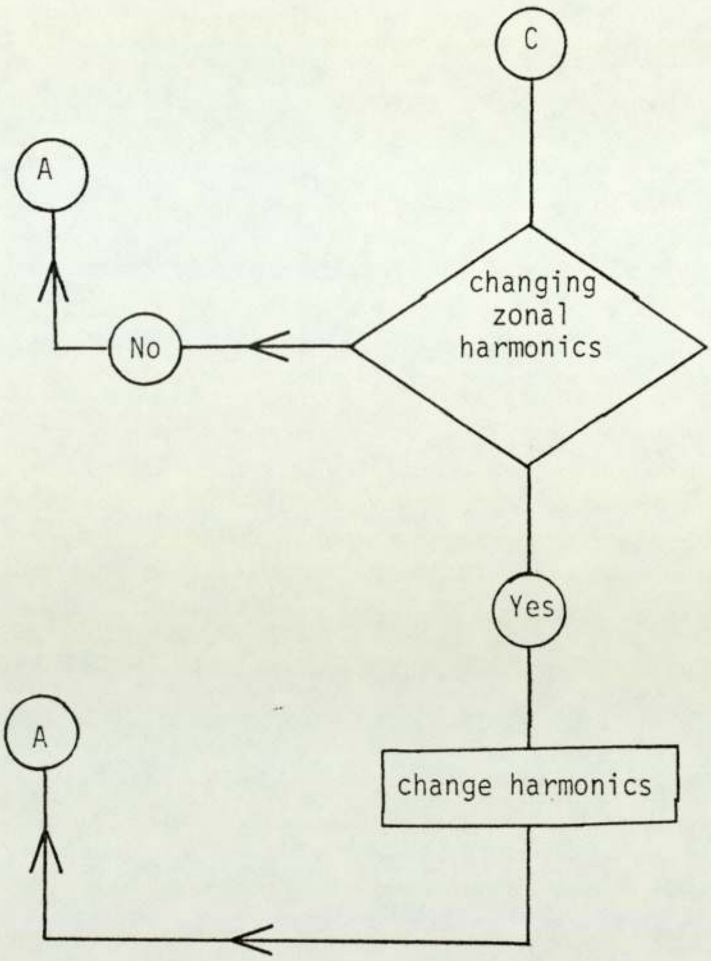
Area 1:	Area 2:
Unit 1 (1,1) CARSFE, DHMS, MJDATE, OBSIN, PRDATE — reading of observations SDCELS	Unit 1 (2,1) ECENTR, EPARAS, PARSHL — derivation of satellite position derivatives
Unit 2 (1,2) DIFCOR — the differential correction process	Unit 2 (2,2) ALTELS, CONVER, ELPRIN, MA01A, NEWELS, PRELON, REJECT, SDPRIN — subordinate to DIFCOR
Unit 3 (1,3) COTOEL, ELREAD, NUPOLY, PVREAD, SATELS — obtaining initial elements GEOCOR, GEOCRN, SENSIN — reading satellite observer locations	
Unit 4 (1,4) PRENUT, PROCES, REFCOR — correction of observations	

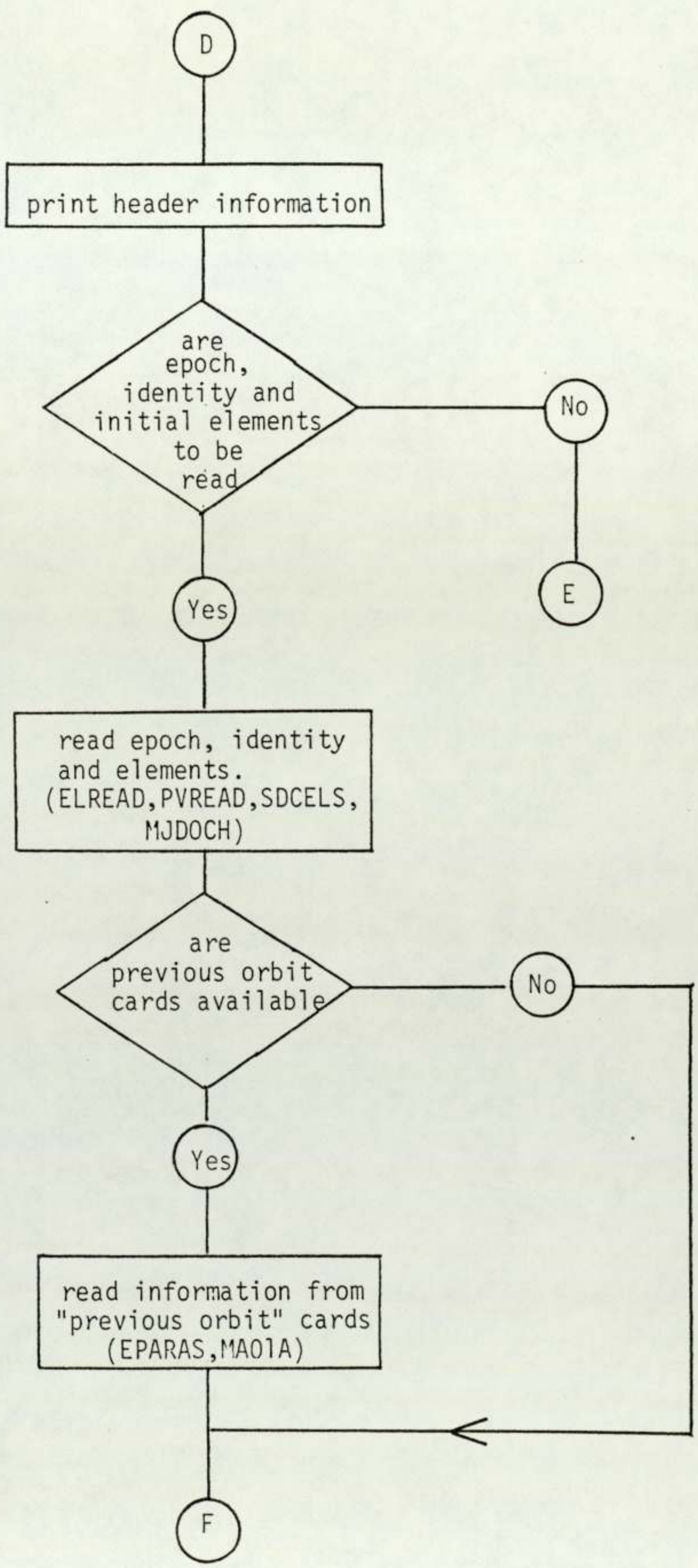
The flow of the program is illustrated in Figure 2.2

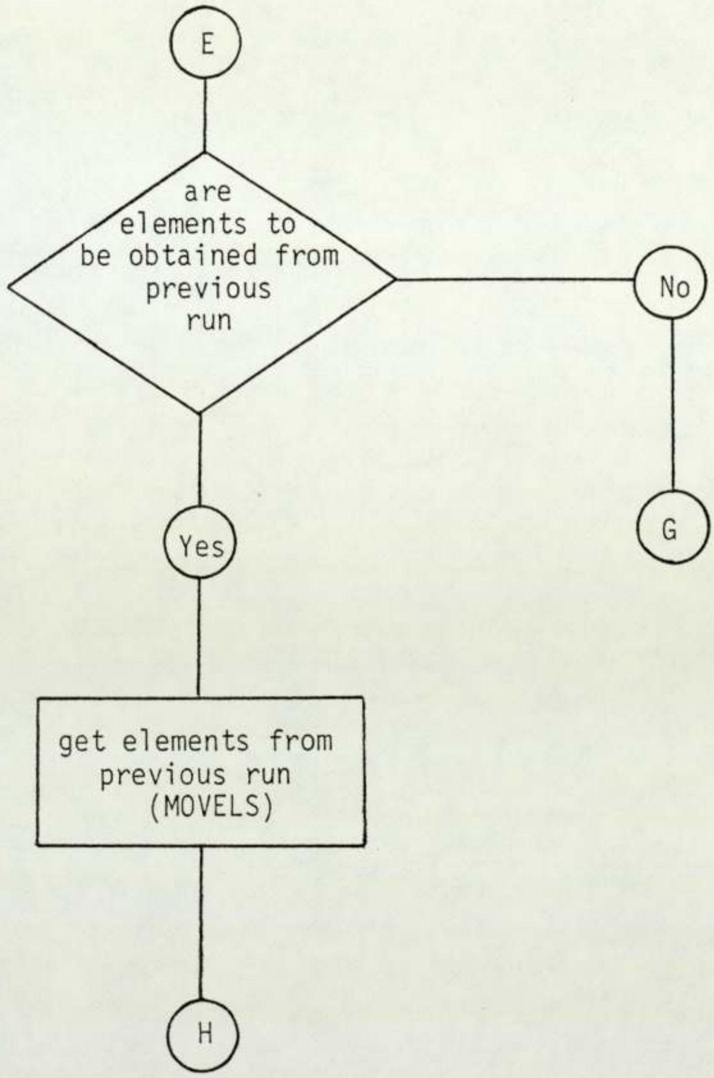
Figure 2.2 The Program Flow

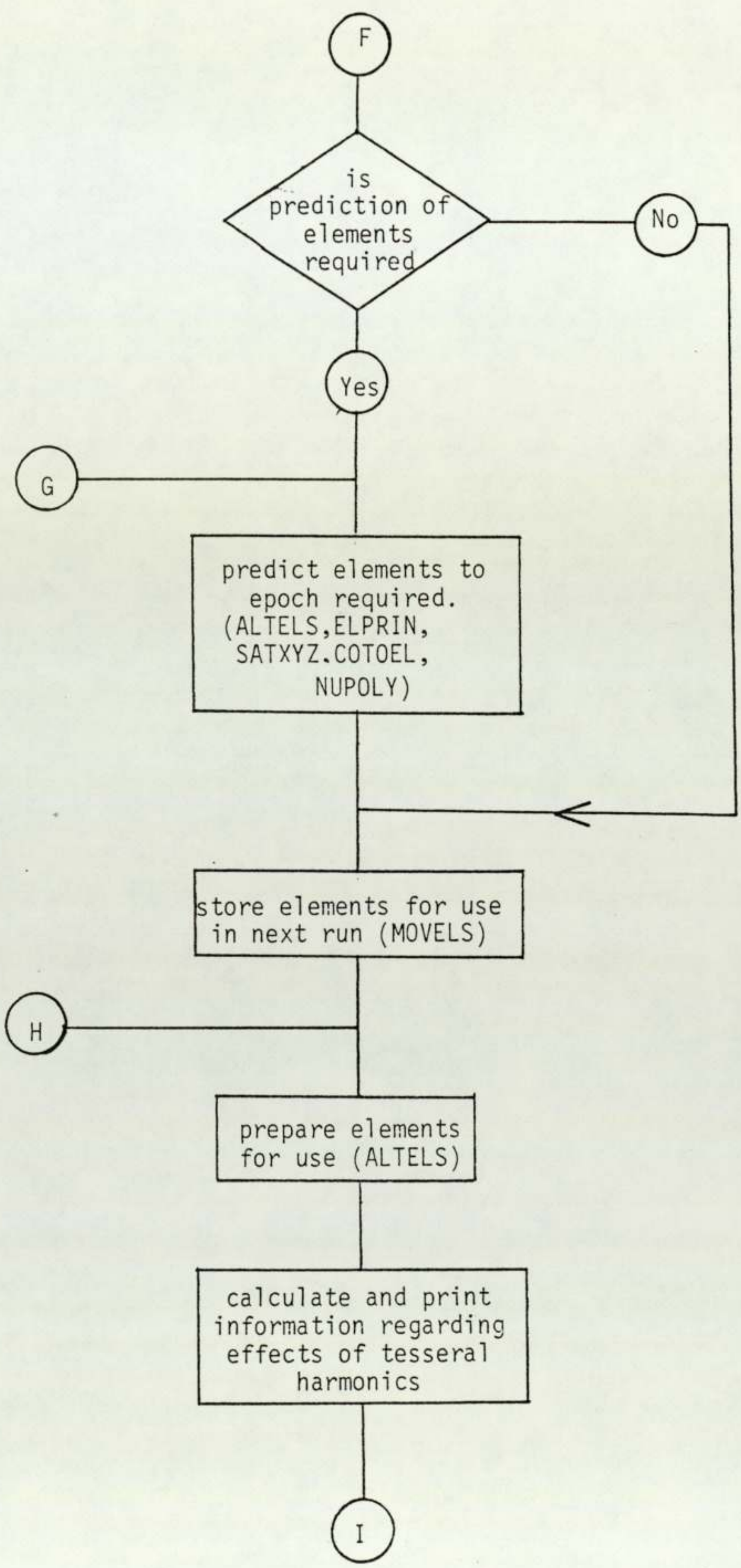


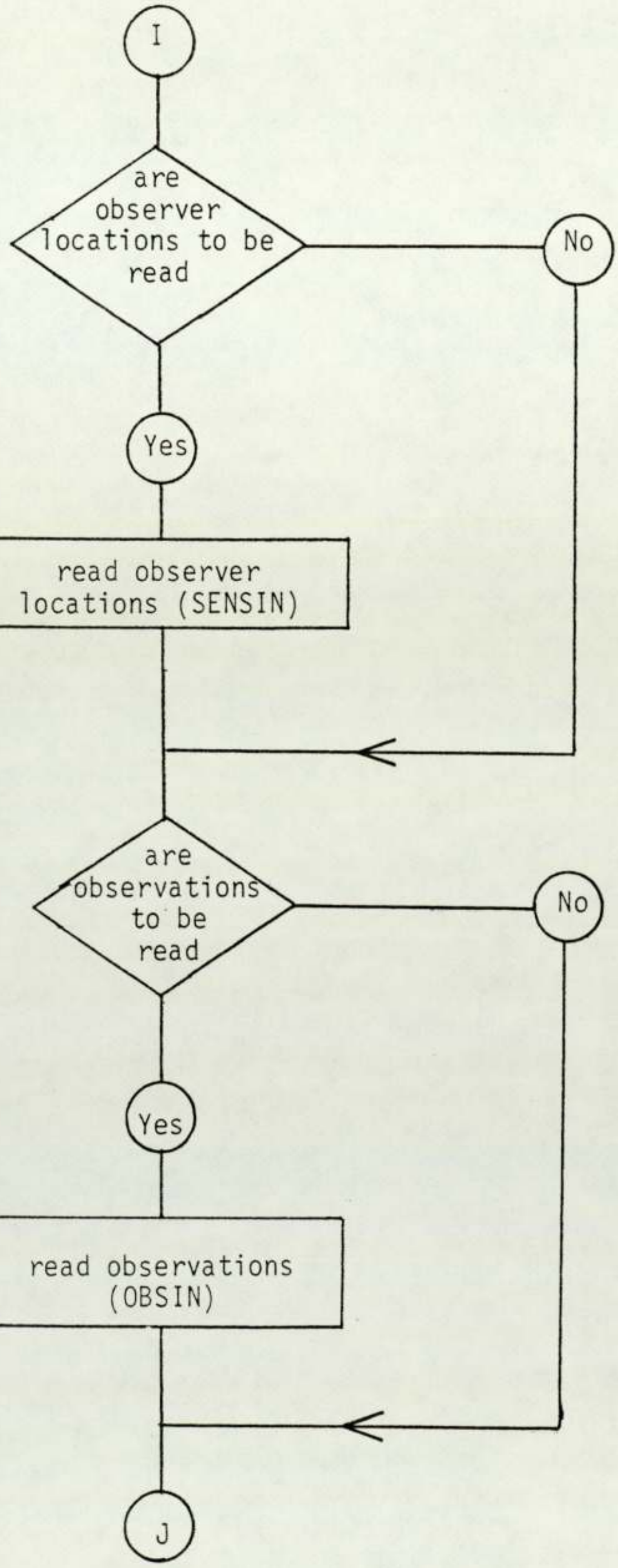

B
9996
No
D
Yes
changing atmospheric rotation rate
No
Yes
change rate
C

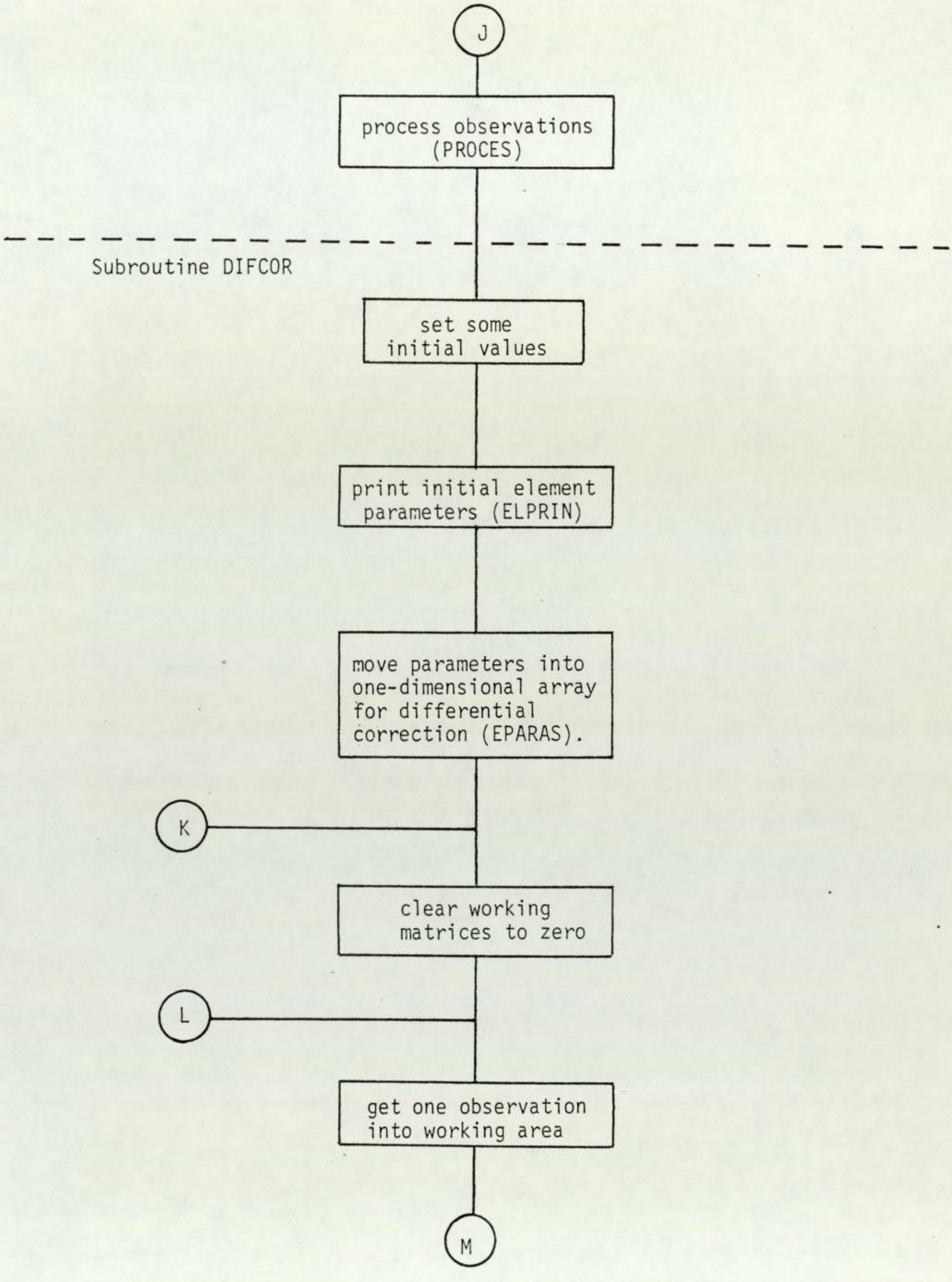

C
A
changing
zonal
harmonics
No
Yes
A
change harmonics

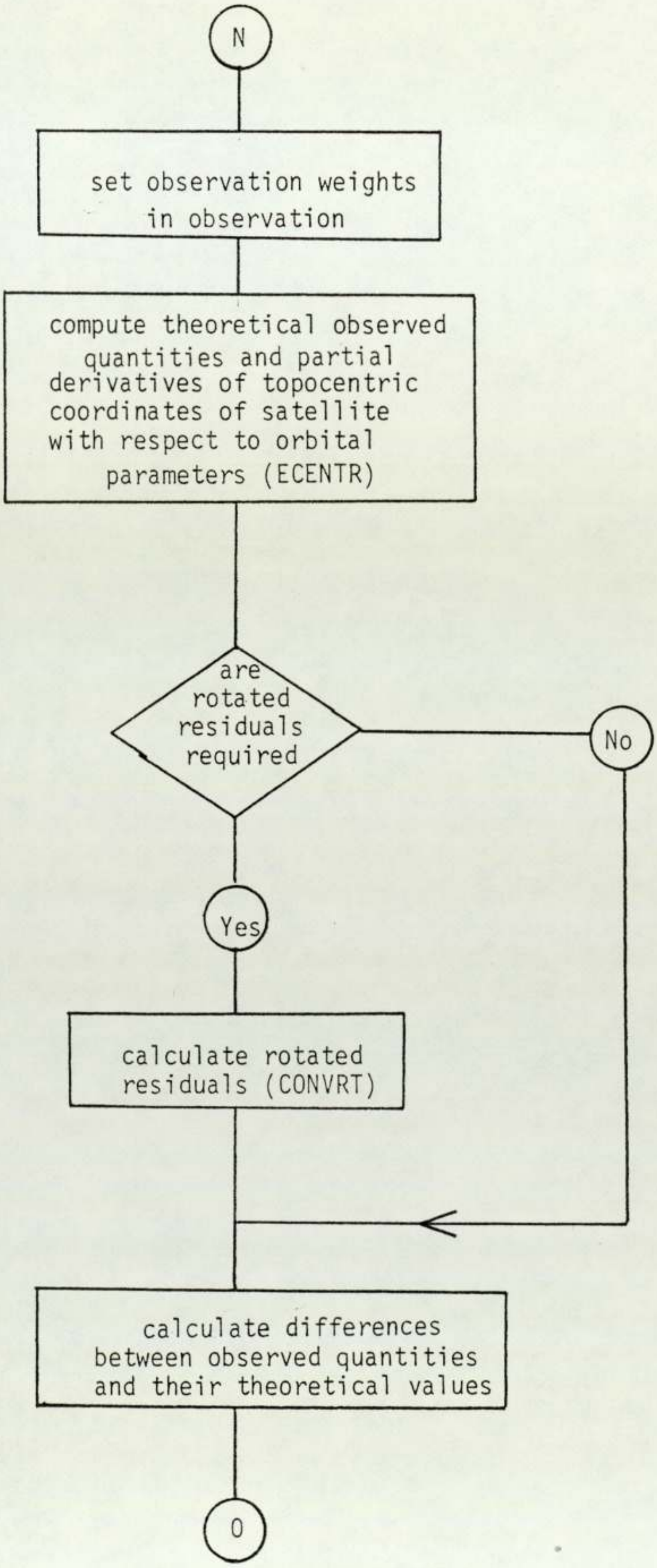

D
print header information
are epoch, identity and initial elements to be read
No
E
Yes
read epoch, identity and elements. (ELREAD,PVREAD,SDCELS, MJDOCH)
are previous orbit cards available
No
Yes
read information from "previous orbit" cards (EPARAS,MA01A)
F

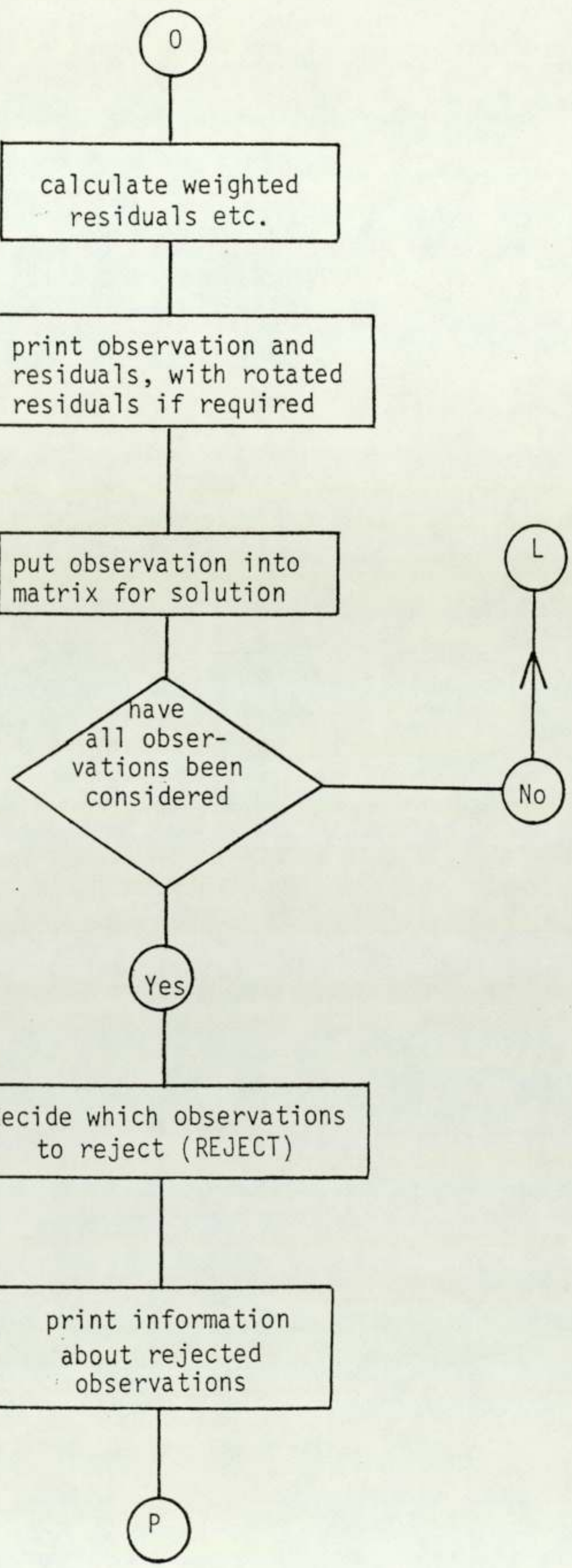

E
are elements to be obtained from previous run
No
G
Yes
get elements from previous run (MOVELS)
H

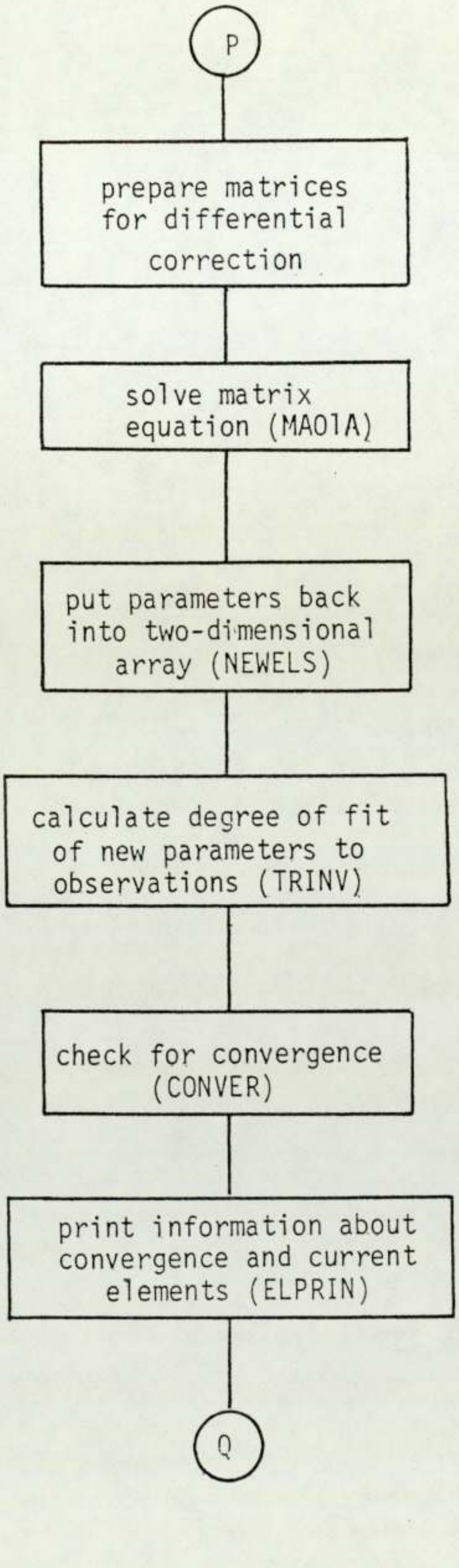

F
is prediction of elements required
No
Yes
G
predict elements to epoch required. (ALTELS,ELPRIN, SATXYZ.COTOEL, NUPOLY)
store elements for use in next run (MOVELS)
H
prepare elements for use (ALTELS)
calculate and print information regarding effects of tesseral harmonics
I

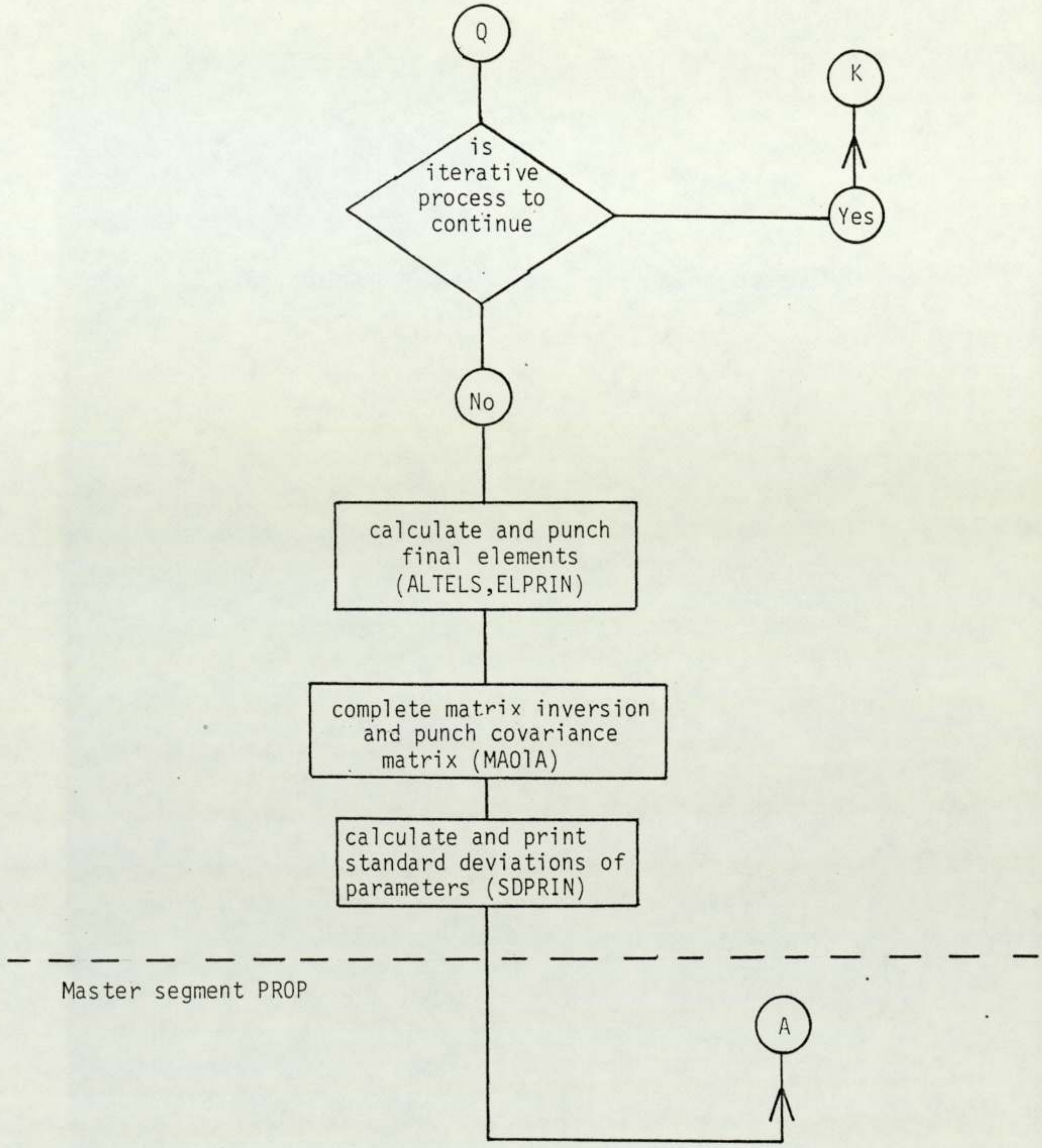

I
are observer locations to be read
No
Yes
read observer locations (SENSIN)
are observations to be read
No
Yes
read observations (OBSIN)
J


J
process observations (PROCES)
Subroutine DIFCOR
set some initial values
print initial element parameters (ELPRIN)
move parameters into one-dimensional array for differential correction (EPARAS).
K
clear working matrices to zero
L
get one observation into working area
M


N
set observation weights in observation
compute theoretical observed quantities and partial derivatives of topocentric coordinates of satellite with respect to orbital parameters (ECENTR)
are rotated residuals required
No
Yes
calculate rotated residuals (CONVRT)
calculate differences between observed quantities and their theoretical values
O


O
calculate weighted residuals etc.
print observation and residuals, with rotated residuals if required
put observation into matrix for solution
have all obser-vations been considered
No
L
Yes
decide which observations to reject (REJECT)
print information about rejected observations
P


P
prepare matrices for differential correction
solve matrix equation (MA01A)
put parameters back into two-dimensional array (NEWELS)
calculate degree of fit of new parameters to observations (TRINV)
check for convergence (CONVER)
print information about convergence and current elements (ELPRIN)
Q


Q
K
is iterative process to continue
Yes
No
calculate and punch final elements (ALTELS,ELPRIN)
complete matrix inversion and punch covariance matrix (MA01A)
calculate and print standard deviations of parameters (SDPRIN)
Master segment PROP
A

3. Generation of the Orbit

The orbital model used in PROP is that given by Merson (1966). In this model there are five basic elements, denoted by e, i, Ω, ω and M. With the exception of M, the mean anomaly, these are illustrated in Figures 3.1 and 3.2. The sixth element is chosen to be the mean motion, n, which is related to the mean anomaly M by

$$n = \frac{dM}{dt}$$

(where t denotes time), and to the semi-major axis of the elliptical orbit, a, by Kepler's third law.

In Figures 3.1 and 3.2, O is the gravitational centre of the Earth, about which the satellite follows an elliptical path with O as one focus. The centre of the circumscribed circle is at C. The point of closest approach of the satellite to the Earth is at A, the perigee point. The line O♈ points toward ♈, the first point of Aries, which is a 'fixed' point in the sky. N, the point at which the projection of the orbit crosses the plane of the equator with the satellite travelling north, is called the ascending node. The distance a is the radius of the circumscribed circle and is equal to the semi-major axis of the orbit.

The five basic elements are then given by:

e, the eccentricity of the ellipse,

i, the inclination of the orbital plane to the equatorial plane,

Ω, the right ascension of the ascending node, (the angle ♈ON, measured positive eastwards),

ω, the argument of perigee, (the angle NOA'), and

M, the mean anomaly when the satellite is at S, given by Kepler's equation

$$M = E - e \sin E,$$

where E is the eccentric anomaly.

Figure 3.1 Plan View of the Elliptical Orbit.

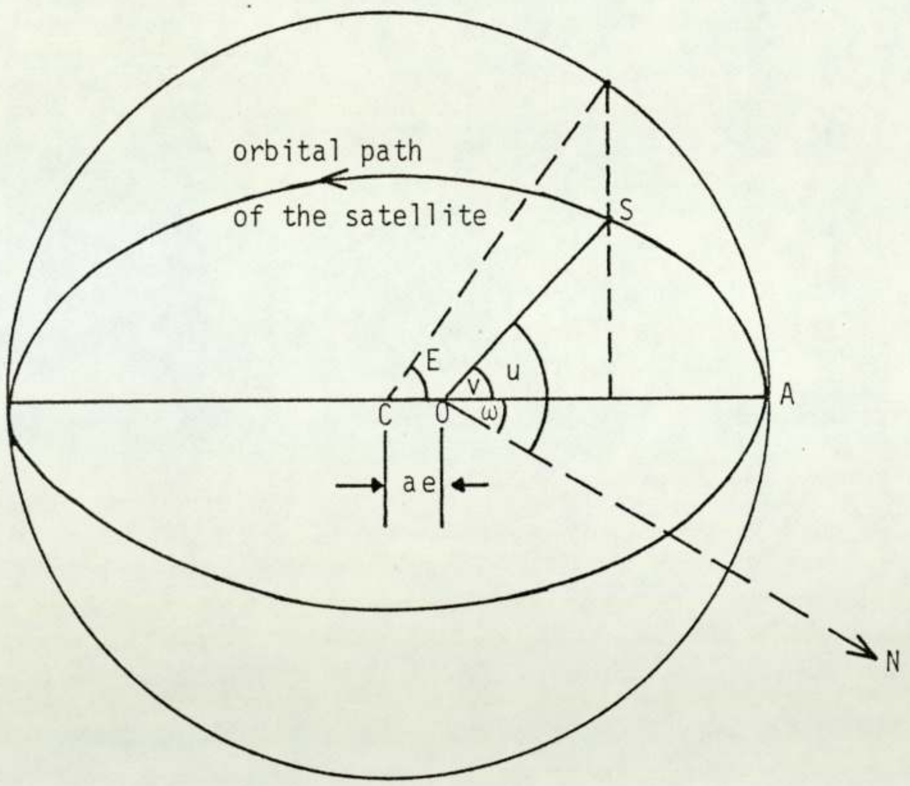
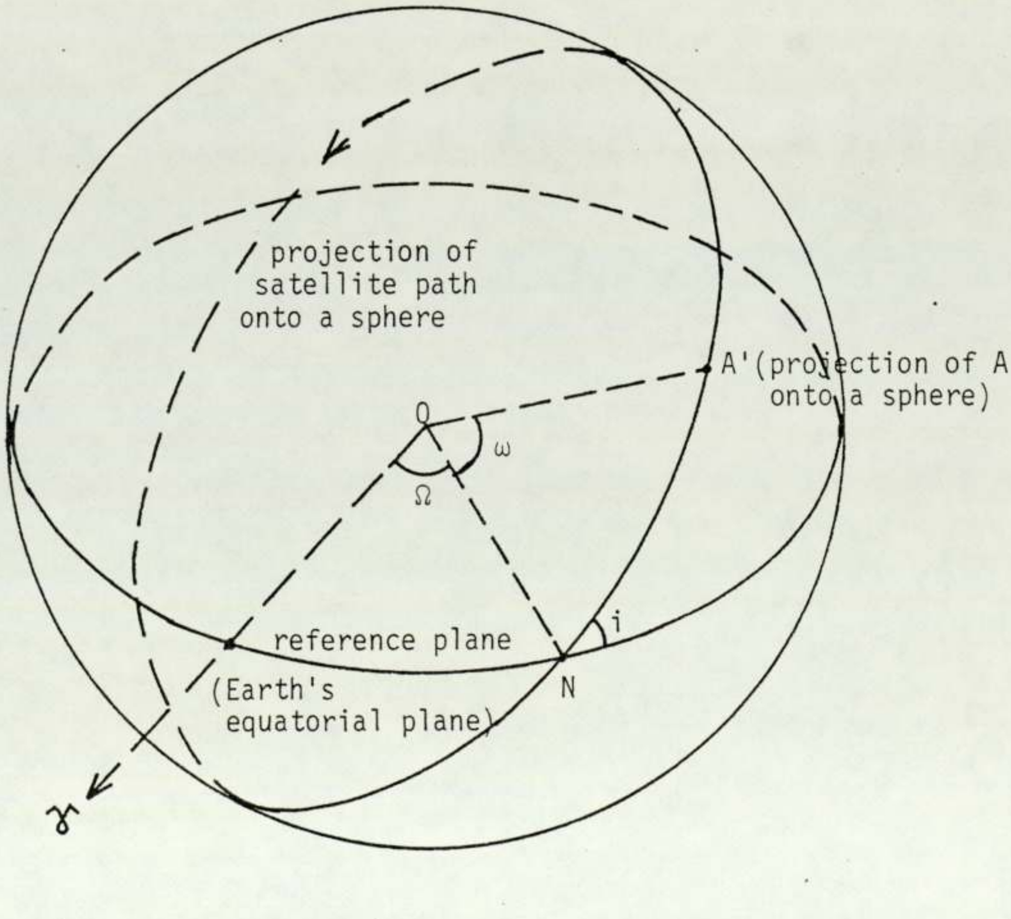


Figure 3.2 The Relationship of the Orbital Plane to the Reference Plane.



The other two angles shown in Figure 3.1 are v, the true anomaly, and u, the argument of latitude.

The semi-major axis, a, and the mean motion, n, are related by Kepler's third law, so that if the Earth is assumed to be perfectly spherical

$$a = \left(\frac{\mu}{n^2} \right)^{1/3} ,$$

where μ (= 398602 km^3/sec^2) is the Earth's gravitational constant.

The elements used within PROP are mean elements, which are time averages over one revolution of the osculating elements. For practical purposes these may be regarded as osculating elements from which the first-order short-periodic perturbations have been removed.

In PROP the basic five elements are approximated by polynomials in time. To these a perturbing factor is added to give the mean values at time t, thus

$$\left. \begin{aligned} e &= e_0 + \sum_{j=1}^{5} e_j t^j + de , \\ i &= i_0 + \sum_{j=1}^{5} i_j t^j + di , \\ \Omega &= \Omega_0 + \sum_{j=1}^{5} \Omega_j t^j + d\Omega , \\ \omega &= \omega_0 + \sum_{j=1}^{5} \omega_j t^j + d\omega , \\ M &= M_0 + \sum_{j=1}^{5} M_j t^j + dM , \end{aligned} \right\} \qquad (3.1)$$

where e_0, i_0, Ω_0, ω_0 and M_0 are mean values at the epoch (zero time) and e, i, Ω, ω and M are mean values at time t. The perturbation factors given by de, di, $d\Omega$, $d\omega$ and dM are all defined to be zero at the epoch. The mean motion, n, becomes

$$n = \frac{dM}{dt} = \sum_j jM_j t^{j-1} \quad . \qquad (3.2)$$

We will refer to a general orbital element (any one of e, i, Ω, ω, M) as ε so equations (3.1) may be written

$$\varepsilon = \varepsilon_0 + \sum_{j=1}^{5} \varepsilon_j t^j + d\varepsilon , \quad (3.3)$$

the ε_j, j = 0,1,...,5 being referred to as orbital coefficients.

To obtain a refined set of orbital coefficients at a particular epoch, an initial estimate of the orbital coefficients is required by PROP, normally in the form

$$e_0, e_1, e_2, e_3, e_4, e_5$$

$$i_0, i_1, i_2, i_3, i_4, i_5$$

$$\Omega_0, \Omega_1, \Omega_2, \Omega_3', \Omega_4, \Omega_5$$

$$\omega_0, \omega_1, \omega_2, \omega_3, \omega_4, \omega_5$$

$$M_0, M_1, M_2, M_3, M_4, M_5$$

with values of $k_e, k_i, k_\Omega, k_\omega, k_M$ relating to the degrees of the polynomials to be used to represent the elements (degree of polynomial representing element $\varepsilon = k_\varepsilon - 1$). Also input are values of m_ε, the number of coefficients of element ε to be refined, where $m_\varepsilon \leqslant k_\varepsilon$. These are the orbital parameters. The coefficients that are not to be refined are held fixed. The input of a typical run of PROP will be such that

$$k_e = k_i = k_\Omega = k_\omega = 2 ,$$

$$k_M = 3 ,$$

$$m_e = m_i = m_\Omega = m_\omega = 1 ,$$

$$m_M = 3 ,$$

and e_1, i_1, Ω_1 and ω_1 input as 0.0. The values given for e_0, i_0, Ω_0, ω_0, M_0, M_1 and M_2 will be refined but e_1, Ω_1 and ω_1 will not. However, the values printed for e_1, i_1, Ω_1 and ω_1 may not be zero as the input values are "exclusive" elements (elements excluding the effects of perturbations) but values output to the line printer include the effects of perturbations.

Refinement of the orbital parameters is done by comparing the input estimated orbit with a set of observations of the satellite made over a short period of time (typically a few days) centred about the epoch.

Each observation consists of a number of observed quantities η_{0_k}. There may be any number, one to six, of observed quantities in an observation, together with the date and time when the observation was made, and the observer location. PROP calculates a theoretical value, η_{T_k}, of each of the observed quantities from the initial estimate of the orbital coefficients using Equations (3.1) and (3.2). The difference between the observed and theoretical quantities is given by

$$\Delta\eta_k = \eta_{0_k} - \eta_{T_k} ,$$

but the observed quantities are functions of the orbital parameters. Hence

$$\Delta\eta_k = \sum_{j=1}^{NUMPAR} \Delta\varepsilon_j \frac{\partial\eta_k}{\partial\varepsilon_j} , \qquad (3.4)$$

where

$$NUMPAR = \sum_{\varepsilon} m_\varepsilon ,$$

and ε_j are the orbital parameters to be refined. The $\Delta\varepsilon_j$ represent changes to be made to ε_j to improve the initial estimate of the orbit.

Each observed quantity has a standard deviation σ_k assigned to it. From Equation (3.4) we obtain

$$\left(\frac{1}{\sigma_k}\right)^2 \frac{\partial\eta_k}{\partial\varepsilon_c} \Delta\eta_k = \left(\frac{1}{\sigma_k}\right)^2 \frac{\partial\eta_k}{\partial\varepsilon_c} \sum_{j=1}^{NUMPAR} \Delta\varepsilon_j \frac{\partial\eta_k}{\partial\varepsilon_j}$$

$$c = 1,\ldots,NUMPAR$$

and hence, transposing left to right,

$$\sum_{k=1}^{ktot} \left(\frac{1}{\sigma_k} \right)^2 \frac{\partial \eta_k}{\partial \varepsilon_c} \sum_{j=1}^{NUMPAR} \Delta\varepsilon_j \frac{\partial \eta_k}{\partial \varepsilon_j} = \sum_{k=1}^{ktot} \left(\frac{1}{\sigma_k} \right)^2 \frac{\partial \eta_k}{\partial \varepsilon_c} \Delta\eta_k , \qquad (3.5)$$

$$c = 1,\ldots,NUMPAR$$

where ktot is the total number of observed quantities in all the observations. Equations (3.5) may be represented by a matrix equation

$$\mathcal{M}^T \mathcal{M} \underset{\sim}{x} = \mathcal{M}^T \underset{\sim}{y} \qquad (3.6)$$

where the superscript T denotes transposition and $\mathcal{M}$ is the matrix

$$\begin{bmatrix} \frac{1}{\sigma_1}\frac{\partial \eta_1}{\partial \varepsilon_1} & \frac{1}{\sigma_1}\frac{\partial \eta_1}{\partial \varepsilon_2} & \cdots & \cdots & \cdots & \cdots & \frac{1}{\sigma_1}\frac{\partial \eta_1}{\partial \varepsilon_{NUMPAR}} \\ \frac{1}{\sigma_2}\frac{\partial \eta_2}{\partial \varepsilon_1} & \frac{1}{\sigma_2}\frac{\partial \eta_2}{\partial \varepsilon_2} & \cdots & \cdots & \cdots & \cdots & \frac{1}{\sigma_2}\frac{\partial \eta_2}{\partial \varepsilon_{NUMPAR}} \\ \vdots & \vdots & & & & & \vdots \\ \frac{1}{\sigma_{ktot}}\frac{\partial \eta_{ktot}}{\partial \varepsilon_1} & \frac{1}{\sigma_{ktot}}\frac{\partial \eta_{ktot}}{\partial \varepsilon_2} & \cdots & \cdots & \cdots & & \frac{1}{\sigma_{ktot}}\frac{\partial \eta_{ktot}}{\partial \varepsilon_{NUMPAR}} \end{bmatrix} ,$$

$\underset{\sim}{x}$ is the vector

$$\begin{bmatrix} \Delta\varepsilon_1 \\ \Delta\varepsilon_2 \\ \vdots \\ \Delta\varepsilon_{NUMPAR} \end{bmatrix}$$

and $\underset{\sim}{y}$ is the vector

$$\begin{bmatrix} \Delta\eta_1/\sigma_1 \\ \Delta\eta_2/\sigma_2 \\ \vdots \\ \Delta\eta_{ktot}/\sigma_{ktot} \end{bmatrix} .$$

Equation (3.6) may be solved for the vector $\underset{\sim}{x}$. The estimate of the orbit may then be improved by addition of the $\Delta\varepsilon_j$ to the appropriate ε_j, and the process repeated until convergence is achieved. The definition of convergence will be discussed later (subsection 4.3, CONVER).

4. The Segments.

This section contains a detailed description of the segments of the program PROP. Each subsection describes one segment. The subsections are arranged in alphabetical order for ease of reference. Consequently, on several occasions a segment has been referred to before it has been discussed. The appropriate subsection number is given, therefore, whenever a segment is referred to in another subsection.

Where the RAE have issued a specification for the segment, the reader is referred to that specification for details, although some of the information given there has been repeated here.

For each segment, a brief description of the action of the segment is given, followed by the input and output parameters and the use made of COMMON areas of store. This is followed by a more detailed account of the action of the segment, in which all the changes that have been made during the current investigation are indicated.

Technical details, including the type of segment (master, subroutine or function), the size (both in number of lines of coding and number of compiled words), the position of the segment in the overlay structure and how the segment inter-relates with the other segments in PROP are given in Table 4.1.

The description of the use of COMMON areas of store is normally restricted to a specification of which items in COMMON are input, output and used by the segment. The normal significance of the contents of each item in COMMON is given in Appendix I.

All variables, items in COMMON and subroutine parameters are of their implicit type unless otherwise stated. That is, variables and arrays with names beginning with one of the letters I to N inclusive are of type "integer", and others are of type "real".

Table 4.1 Technical Details of the Segments

Subsection	Segment name	Type	Size in lines	Size in words (optimised)	Overlay unit	Called by segments:	Calls segments:
4.1	ALTELS	subroutine	81	415 (438)	2,2	DIFCOR (4.7) PROP (4.28)	PRELON (4.25)
4.2	CARSFE	subroutine	15	107 (106)	1,1	OBSIN (4.22)	
4.3	CONVER	subroutine	32	98 (96)	2,2	DIFCOR (4.7)	
4.4	CONVRT	subroutine	53	297 (297)		OBSIN (4.22) PROCES (4.27)	TRINV (4.40)
4.5	COTOEL	subroutine	30	271 (269)	1,3	PROP (4.28) SATELS (4.32)	TRINV (4.40)
4.6	DHMS	subroutine	15	75 (74)	1,1	OBSIN (4.22)	
4.7	DIFCOR	subroutine	573	3133 (3156)	1,2	PROP (4.28)	ALTELS (4.1) CONVER (4.3) ECENTR (4.9) ELPRIN (4.10) EPARAS (4.12) MA01A (4.16) NEWELS (4.20) REJECT (4.31) SDPRIN (4.35) TRINV (4.40)
4.8	EAFKEP	function	13	63 (59)		SATXYZ (4.33)	
4.9	ECENTR	subroutine	311	1639 (1655)	2,1	DIFCOR (4.7)	PARSHL (4.23) SATXYZ (4.33) SIDANG (4.38) TRINV (4.40)

Table 4.1 (continued) Technical Details of the Segments

Subsection	Segment name	Type	Size in lines	Size in words (optimised)	Overlay unit	Called by segments:	Calls segments:
4.10	ELPRIN	subroutine	56	332 (352)	2,2	DIFCOR (4.7) PROP (4.28)	
4.11	ELREAD	subroutine	25	128 (136)	1,3	PROP (4.28)	
4.12	EPARAS	subroutine	17	73 (76)	2,1	DIFCOR (4.7) PROP (4.28)	
4.13	ERROR	semi-compiled					
4.14	GEOCOR	subroutine	42	229 (224)	1,3	SENSIN (4.36)	SIDANG (4.38) TRINV (4.40)
4.15	GEOCRN	subroutine	76	326 (313)	1,3	SENSIN (4.36)	TRINV (4.40)
4.16	MA01A	subroutine	114	613 (717)	2,2	DIFCOR (4.7) PROP (4.28)	
4.17	MILTIM	semi-compiled					
4.18	MJDATE	subroutine	33	182 (173)	1,1	OBSIN (4.22) PRDATE (4.24) PROP (4.28) SDCELS (4.34)	
4.19	MOVELS	subroutine	8	41 (48)		PROP (4.28)	
4.20	NEWELS	subroutine	17	87 (98)	2,2	DIFCOR (4.7)	

Table 4.1 (continued) Technical Details of the Segments

Subsection	Segment name	Type	Size in lines	Size in words (optimised)	Overlay unit	Called by segments:	Calls segments:
4.21	NUPOLY	subroutine	15	67 (67)	1,3	PROP (4.28)	
4.22	OBSIN	subroutine	824	3935 (3928)	1,1	PROP (4.28)	CARSFE (4.2) CONVRT (4.4) DHMS (4.6) MJDATE (4.18) PRDATE (4.24) SIDANG (4.38)
4.23	PARSHL	subroutine	100	607 (643)	2,1	ECENTR (4.9)	
4.24	PRDATE	subroutine	14	43 (42)	1,1	OBSIN (4.22) PROP (4.28)	MJDATE (4.18)
4.25	PRELON	subroutine	202	958 (923)	2,2	ALTELS (4.1)	SIDANG (4.38)
4.26	PRENUT	subroutine	77	386 (377)	1,4	PROCES (4.27)	TRINV (4.40)
4.27	PROCES	subroutine	84	499 (475)	1,4	PROP (4.28)	CONVRT (4.4) PRENUT (4.26) REFCOR (4.30) SATXYZ (4.33) SIDANG (4.38)
4.28	PROP	master	287	1246 (1261)			ALTELS (4.1) COTOEL (4.5) DIFCOR (4.7) ELPRIN (4.10) ELREAD (4.11) EPARAS (4.12) MA01A (4.16) MJDATE (4.18)

Table 4.1 (continued) Technical Details of the Segments

Subsection	Segment name	Type	Size in lines	Size in words (optimised)	Overlay unit	Called by segments:	Calls segments:
							MOVELS (4.19) NUPOLY (4.21) OBSIN (4.22) PRDATE (4.24) PROCES (4.27) SATXYZ (4.33)
4.29	PVREAD	subroutine	25	114 (120)	1,3	PROP (4.28)	SATELS (4.32)
4.30	REFCOR	subroutine	23	170 (169)	1,4	PROCES (4.27)	
4.31	REJECT	subroutine	38	208 (206)	2,2	DIFCOR (4.7)	
4.32	SATELS	subroutine	43	163 (172)	1,3	PVREAD (4.29)	COTOEL (4.5) SATXYZ (4.33)
4.33	SATXYZ	subroutine	127	705 (694)		ECENTR (4.9) PROCES (4.27) PROP (4.28) SATELS (4.32)	EAFKEP (4.8) SHOPER (4.37) XLONG (4.41)
4.34	SDCELS	subroutine	133	894 (852)	1,1	PROP (4.28)	MJDATE (4.18)
4.35	SDPRIN	subroutine	29	143 (155)	2,2	DIFCOR (4.7)	
4.36	SENSIN	subroutine	87	511 (509)	1,3	PROP (4.28)	GEOCOR (4.14) GEOCRN (4.15)
4.37	SHOPER	subroutine	36	292 (286)		SATXYZ (4.33)	

Table 4.1 (continued) Technical Details of the Segments

Subsection	Segment name	Type	Size in lines	Size in words (optimised)	Overlay unit	Called by segments:	Calls segments:
4.38	SIDANG	function	11	34 (34)		ECENTR (4.9) GEOCOR (4.14) OBSIN (4.22) PRELON (4.25) PROCES (4.27)	
4.39	SINXOX	subroutine	18	81 (80)		XLONG (4.41)	
4.40	TRINV	subroutine	8	56 (56)		CONVRT (4.4) COTOEL (4.5) DIFCOR (4.7) ECENTR (4.9) GEOCOR (4.14) GEOCRN (4.15) PRENUT (4.26)	
4.41	XLONG	subroutine	88	529 (505)		SATXYZ (4.33)	SINXOX (4.39)

In order to appreciate the following subsections to their full extent, it is recommended that the contents of Figure 2.2 be studied in detail.

4.1 ALTELS (Gooding 1969a)

This segment alters the form of the orbital elements of the satellite, which are held in a standard array.

There is one parameter:

IND: input indicator of the type of conversion of elements required (see table 4.1.1).

The use of COMMON is:

/ORBIT/

NOMIAL (6) is input.

ELEMT (6x6) is input and output.

JJ and RESDEL (4) are equivalenced to MJD,X,Y,Z,XDOT.

They are output only if IND = 0. JJ = number of NOMIAL (1), (2), (3) and (4) which are non-zero. The contents of RESDEL are described later in this subsection.

/PRECON/

ZONSEC (4) and DRASEC (4) are input or output, depending upon the value of IND. If they are output they are set during the call of PRELON.

The action taken by ALTELS is governed by the setting of IND, as shown in Table 4.1.1. The terms "exclusive" and "inclusive" elements have been defined by Gooding and Tayler (1968) and "hybrid" elements are discussed by Gooding (1969a).

Table 4.1.1 Action of ALTELS According to the Setting of IND.

IND	Action
<0	convert exclusive to inclusive elements
0	convert hybrid to inclusive elements
>0	convert inclusive to exclusive elements

In a normal run of PROP the first call of ALTELS (by segment PROP (4.28)) is with IND = -1, to convert the exclusive elements input to PROP by the analyst to inclusive elements suitable for differential correction by the segment DIFCOR (4.7).

DIFCOR then makes a series of calls of ALTELS with IND = 0, one at the end of each iteration. These convert the hybrid elements which are the outcome of the differential correction process to inclusive elements suitable for printing and, if necessary, for a further iteration. With IND = 0 ALTELS also calculates the quantities RESDEL and JJ. RESDEL is derived from the hybrid input ELEMT(j,2), j = 1,..,4; ZONSEC and DRASEC. ZONSEC and DRASEC will have been assigned the values of the secular perturbations due to zonal harmonics and drag within the previous call of ALTELS. RESDEL is then given by

RESDEL(j) = ELEMT(j,2) - ZONSEC(j) - OLDN(1)×DRASEC(j) j = 1,..,4.

The RESDEL are converted from units of radians and seconds to degrees and days. OLDN(1) is set to ELEMT(6,2) at the end of the previous call of ALTELS. RESDEL is printed out by DIFCOR preceded by the message: IMPORTANT - ANY OF THE FIRST FOUR SUB-1 ELEMENTS PRESENT HAS ITS FULL VALUE, THE MAIN SECULAR COMPONENT OF WHICH HAS BEEN COMPUTED INTERNALLY. THE EXTERNAL COMPONENTS, AND ONLY THESE APPEAR IN PUNCHED CARD FORMAT, ARE, RESPECTIVELY,.

The final call of ALTELS is made by DIFCOR with IND = +1 to convert the inclusive elements from the final iteration to exclusive elements suitable for punching on the card output.

If prediction of elements from an earlier epoch is required to obtain initial elements for the PROP run then two preliminary calls of ALTELS are made. The first call is made with IND = -1 to obtain elements in a form suitable for prediction. The second call is made with IND = +1 to convert the predicted elements to exclusive form suitable for starting a PROP run.

4.2 CARSFE

This segment converts the given Cartesian coordinates of a point to spherical polar coordinates.

There are six parameters:

X, Y, Z	input Cartesian coordinates of a point (x,y,z),
R, THETA, PHI	output spherical polar coordinates of a point (ρ,θ,ϕ).

ρ is in the same units as x, y and z. θ and ϕ are both output in radians.

This subroutine is new to PROP, having been derived from the program formerly used to pre-process observations supplied by the US Naval Research Laboratory. Observations from this source may now be read directly in to PROP without pre-processing. The output parameters are defined by:

$$\rho = (x^2 + y^2 + z^2)^{\frac{1}{2}} ,$$

$$\theta = \tan^{-1}\left[z/(x^2 + y^2)^{\frac{1}{2}}\right]$$

and $\phi = \tan^{-1}(y/x)$

as shown in Figure 4.2.1.

In the special case where $x^2 + y^2 < 10^{-40}$ the output parameters are defined by:

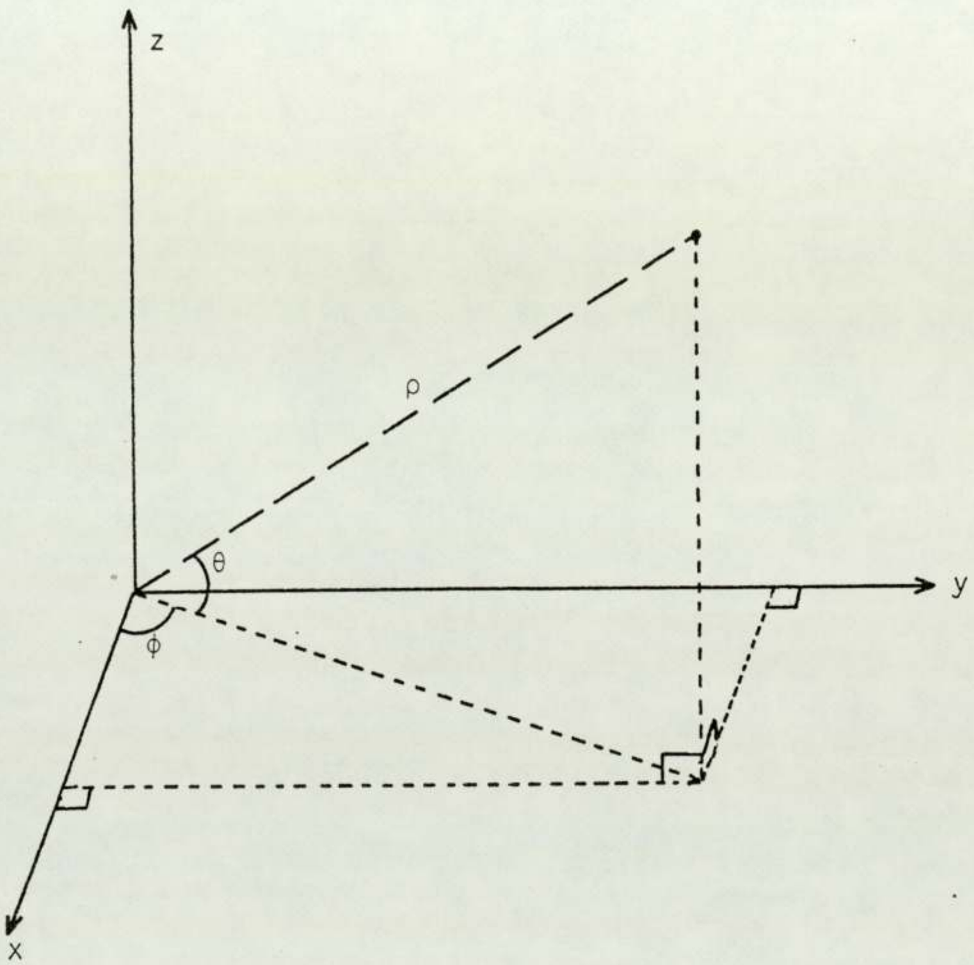
$$\rho = |z|,$$

$$\theta = \tan^{-1}\left[z/(x^2 + y^2)^{\frac{1}{2}}\right] \approx \pi/2$$

and $\phi = x^2 + y^2$.

This is different from the output of the original CARSFE in the NRL-pre-processor where, in the case of $x^2 + y^2 < 10^{-40}$, θ is defined to be zero.

Figure 4.2.1 Conversion of (x,y,z) to (ρ,θ,ϕ) by CARSFE.



4.3 CONVER (Gooding, no date available).

This segment tests for convergence of the differential correction process.

There are 8 parameters:

EPSLON: input measure of fit of the observations to the estimated orbit resulting from the previous iteration, $\mathcal{E}_{j-1}$.

EPSNEW: input measure of fit of the observations to the current estimated orbit, $\mathcal{E}_j$.

NOWREJ: input number of observations newly rejected by the current iteration.

MODE: input parameter restricting when convergence can be achieved (see Table 4.3.1).

ITNUM: input number of current iteration.

MAXITN: input maximum number of iterations allowed.

IVERGE: input indicator of whether the previous one or two iterations are converging or not;
output indicator of whether the previous and current iterations are converging or not.

NVER: output indicator of the state of the convergence.

The subroutine specification issued by the RAE (Gooding, no date available) is for an earlier version of CONVER than the one now available. The output parameters of CONVER and the definition of "convergent" and "divergent" are given in Table 4.3.1, where

$$\mathcal{E} = \frac{\mathcal{E}_{j-1} - \mathcal{E}_j}{\mathcal{E}_{j-1}} \ .$$

Table 4.3.1 Output Parameters of CONVER.

Input Conditions	Output		Description of Condition
	NVER	IVERGE	
$-10^{-3} \leqslant \mathcal{E} \leqslant 10^{-2}$ and NOWREJ = MODE = 0	1	0	converged
$\mathcal{E} < -10^{-3}$ and IVERGE = 2	-1	2	diverged
ITNUM $\geqslant$ MAXITN and $\mathcal{E} > 10^{-2}$ or $\mathcal{E} < -10^{-3}$ and IVERGE < 2 or MODE > 0 or NOWREJ > 0	-1	0	maximum number of iterations exceeded
ITNUM < MAXITN, $\mathcal{E} < -10^{-3}$ and IVERGE < 2	0	IVERGE (entry) +1	continuing
ITNUM < MAXITN and $\mathcal{E} > 10^{-2}$	0	0	continuing
ITNUM < MAXITN, $-10^{-3} \leqslant \mathcal{E} \leqslant 10^{-2}$ and MODE > 0 or NOWREJ > 0	0	0	continuing

The process must diverge on three successive iterations, or MAXITN must be reached, for the differential correction process to fail to converge.

4.4 CONVRT (Tayler, 1969a)

This segment performs topocentric conversions between azimuth/elevation, right ascension/declination and direction cosines.

There are seven parameters:

N: input indicator of conversion required (see table 4.4.1);

X1, X2: input quantities for conversion;

BETA: input geographic latitude of the topocentre, in radians, β;

LAMBDA: input geographic longitude + modified sidereal angle of the topocentre at the time of the observation (see 4.38, SIDANG), in radians, λ;

Y1, Y2: output converted quantities.

We will denote

Azimuth = Az

Elevation = El

Right ascension = α, Declination = δ — relative to the true equator and equinox of 1950.0

East-west direction cosine = ℓ

North-south direction cosine = m.

Az, El, α and δ (shown in Figure 4.4.2) are all input and output in radians. The conversions performed by CONVRT are shown in Table 4.4.1.

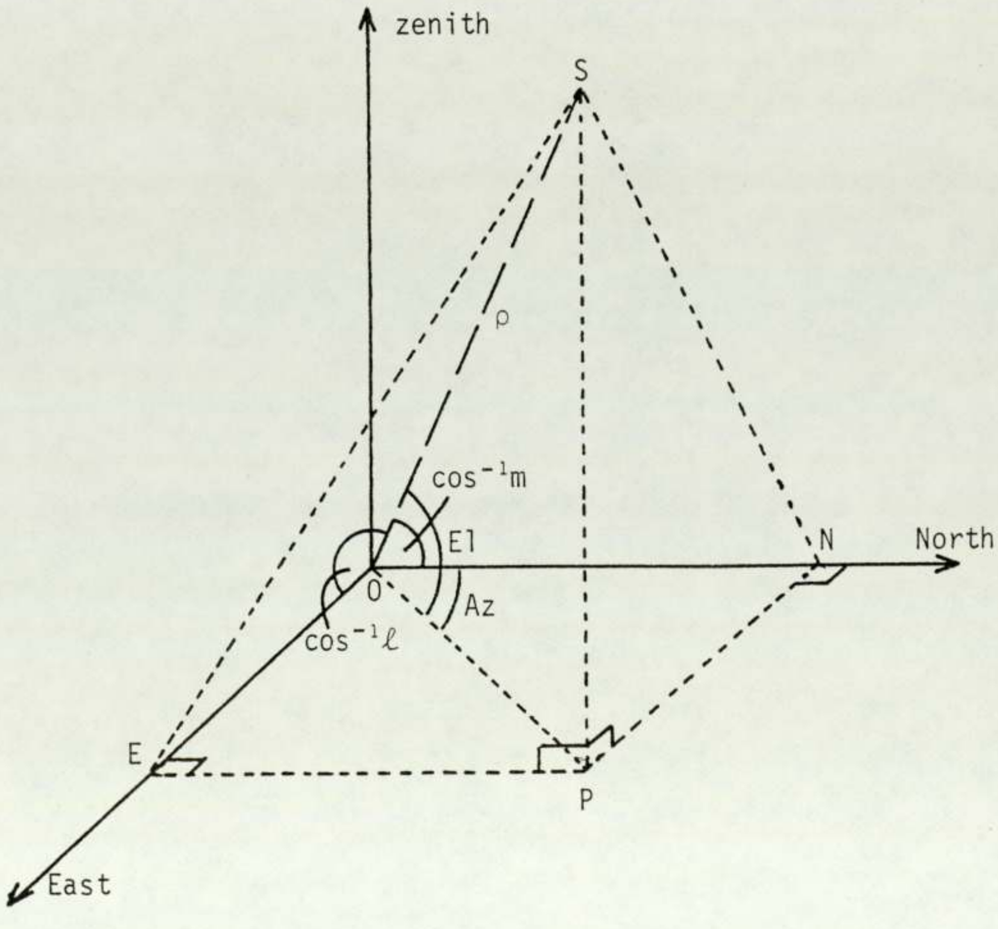
In practice, the program PROP never converts observed quantities to direction cosines so CONVRT is never called with N = 2 or N = 4. The derivations of the equations governing the conversions performed by CONVRT are not readily available in the literature so they have been re-derived here.

Table 4.4.1 Conversions Performed by CONVRT.

N	X1	X2	Y1	Y2
1	Az	El	α	δ
2	Az	El	ℓ	m
3	α	δ	Az	El
4	α	δ	ℓ	m
5	ℓ	m	Az	El
6	ℓ	m	α	δ

To convert between ℓ, m and Az, El consider a satellite at S observed by an observer at O (Figure 4.4.1)

Figure 4.4.1 Relationship between Azimuth, Elevation and Direction Cosines.



Let P be the sub-satellite point in the North-East plane and ρ the observer-satellite distance. By consideration of the right angled triangle SPE, the distance SE is given by

$$(SE)^2 = (SP)^2 + (PE)^2,$$

and using the cosine rule in the triangle SOE we obtain

$$(SE)^2 = \rho^2 + (OE)^2 - 2\rho(OE)\ell.$$

Hence

$$\ell = \frac{\rho^2 + (OE)^2 - (SP)^2 - (EP)^2}{2\rho(OE)}$$

$$= \frac{\rho^2 + \rho^2\cos^2 El \sin^2 Az - \rho^2 \sin^2 El - \rho^2 \cos^2 El \cos^2 Az}{2\rho^2 \cos El \sin Az}$$

$$= \cos El \sin Az.$$

Similarly

$$m = \cos El \cos Az.$$

Hence conversions between azimuth/elevation and direction cosines may be obtained.

To obtain conversions between azimuth/elevation and right ascension/declination the observer's latitude and longitude (relative to ♈, the first point of Aries) have to be considered. Again let the observer be at O and the satellite at S (Figure 4.4.2). We may then consider the spherical triangle APB (Figure 4.4.3), where PZB, BA, ASP, SZ and BS are all parts of great circles.

Figure 4.4.2 Relationship between Azimuth, Elevation and Right Ascension, Declination.

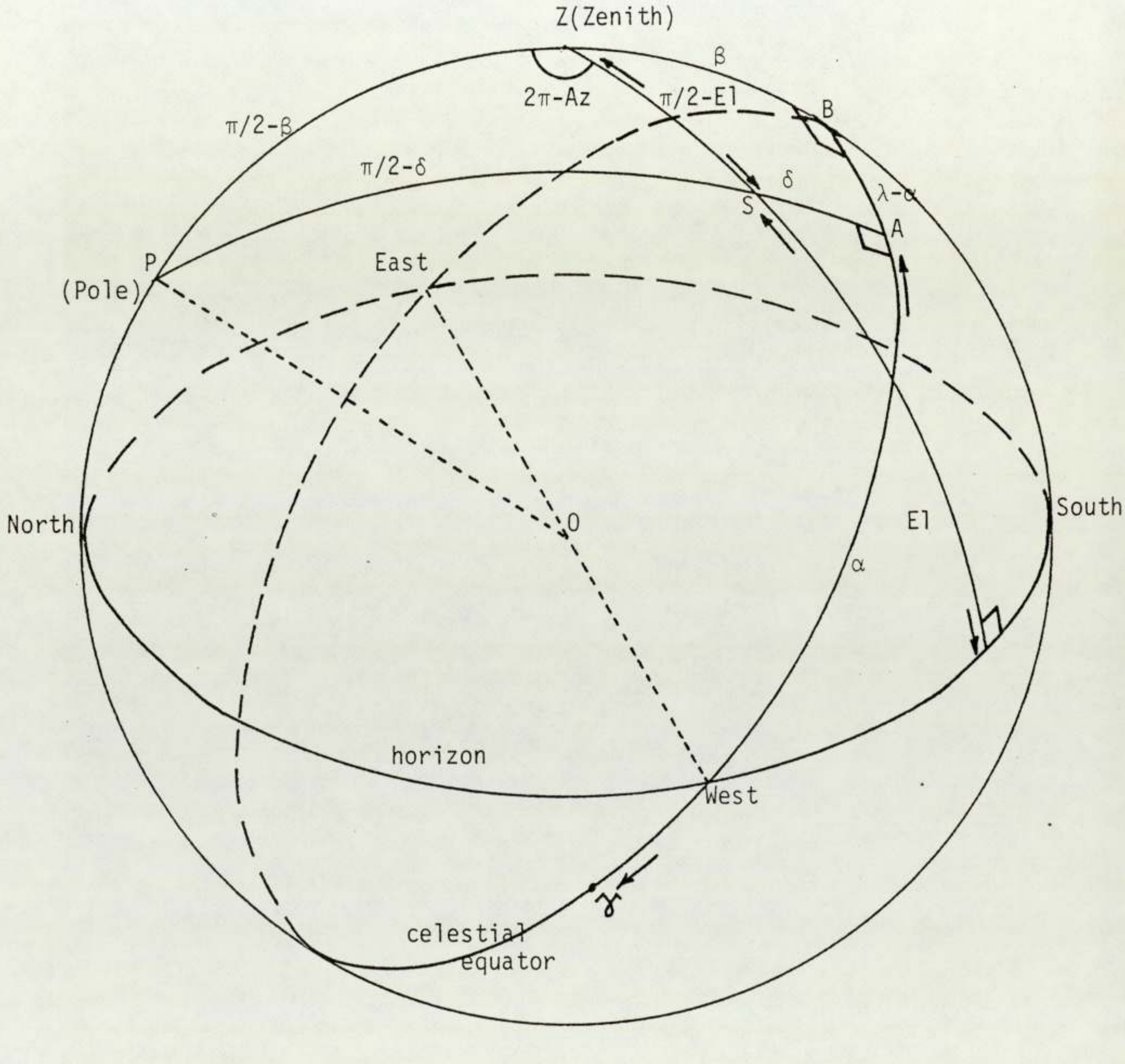
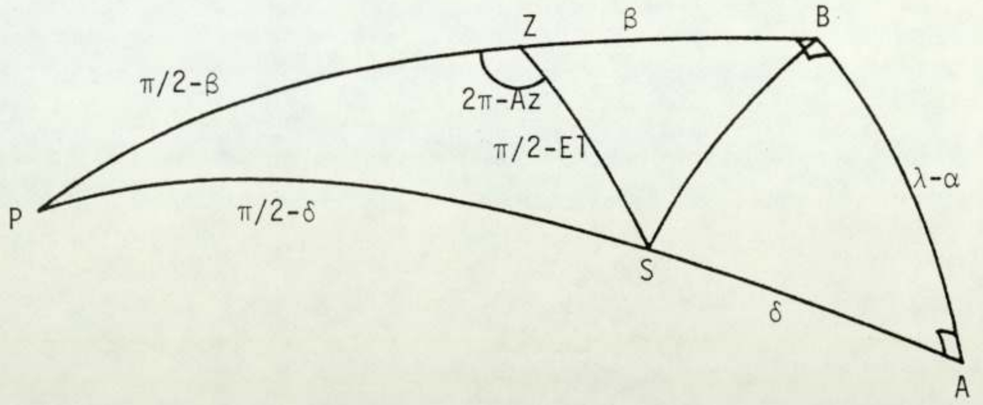


Figure 4.4.3. Spherical Triangles for Consideration of the Relationship between Azimuth, Elevation, Right Ascension and Declination.



Applying the cosine rule to the spherical triangle SPZ we obtain

$$\cos(\pi/2 - \delta) = \cos(\pi/2 - \beta)\cos(\pi/2 - El) + \sin(\pi/2 - \beta)\sin(\pi/2 - El)\cos(2\pi - Az)$$

and hence

$$\sin\delta = \sin\beta \sin El + \cos\beta \cos El \cos Az. \qquad (4.4.1)$$

Applying the sine rule to the spherical triangle APB we obtain

$$\frac{\sin(S\hat{P}Z)}{\sin(\lambda-\alpha)} = \frac{\sin(\pi/2 - \beta + \beta)}{\sin \pi/2} = 1,$$

so that

$$S\hat{P}Z = \lambda - \alpha.$$

Similarly, application of the sine rule to triangle SPZ gives

$$\frac{\sin(\lambda - \alpha)}{\sin(\pi/2 - El)} = \frac{\sin(2\pi - Az)}{\sin(\pi/2 - \delta)} ,$$

which becomes

$$\sin(\lambda - \alpha)\cos\delta = -\cos El \sin Az. \qquad (4.4.2)$$

From triangle SPZ, we obtain

$$\cos(\pi/2 - El) = \cos(\pi/2 - \beta)\cos(\pi/2 - \delta) + \sin(\pi/2 - \beta)\sin(\pi/2 - \delta)\cos(\lambda - \alpha),$$

and hence

$$\sin El = \sin\beta \sin\delta + \cos\beta \cos\delta \cos(\lambda - \alpha).$$

From triangle SZB we obtain

$$\cos(BS) = \cos(\pi/2 - El)\cos\beta + \sin(\pi/2 - El)\sin\beta\cos(\pi - Az)$$

$$= \sin El \cos\beta - \cos El \sin\beta \cos Az.$$

Also, from triangle ASB,

$$\cos(BS) = \cos\delta\cos(\lambda - \alpha) + \sin\delta\sin(\lambda - \alpha)\cos\pi/2$$

so that

$$\cos\delta\cos(\lambda - \alpha) = \sin El \cos\beta - \cos El \sin\beta\cos Az.$$

Expanding this, we find that

$$\cos\delta\cos\lambda\cos\alpha + \cos\delta\sin\lambda\sin\alpha = \sin El\cos\beta - \cos El\sin\beta\cos Az. \tag{4.4.3}$$

Now, from Equation (4.4.2) we obtain

$$\cos\delta\sin\lambda\cos\alpha - \cos\delta\cos\lambda\sin\alpha = -\cos El\sin Az. \tag{4.4.4}$$

Multiplying Equation (4.4.3) by $\cos\lambda$ and Equation (4.4.4) by $\sin\lambda$ and adding, we find that

$$\cos\delta\cos\alpha(\cos^2\lambda + \sin^2\lambda) + \cos\delta\sin\lambda\cos\lambda\sin\alpha - \cos\delta\cos\lambda\sin\lambda\sin\alpha$$

$$= \cos\lambda\sin El\cos\beta - \cos\lambda\cos El\sin\beta\cos Az - \cos El\sin Az\sin\lambda.$$

Hence

$$\cos\delta\cos\alpha = -\sin\beta\cos\lambda\cos El\cos Az - \sin\lambda\cos El\sin Az + \cos\beta\cos\lambda\sin El. \tag{4.4.5}$$

Similarly, multiplying Equation (4.4.3) by $\sin\lambda$ and Equation (4.4.4) by $\cos\lambda$ and subtracting we obtain

$$\cos\delta\sin\alpha = \sin\lambda\sin El\cos\beta - \sin\lambda\cos El\sin\beta\cos Az + \cos\lambda\cos El\sin Az. \tag{4.4.6}$$

Equations (4.4.1), (4.4.5) and (4.4.6) may be combined to give the matrix equation used by CONVRT for conversions between azimuth/elevation and right ascension/declination; that is

$$\begin{bmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{bmatrix} = \begin{bmatrix} -\sin\beta\cos\lambda & -\sin\lambda & \cos\beta\cos\lambda \\ -\sin\beta\sin\lambda & \cos\lambda & \cos\beta\sin\lambda \\ \cos\beta & 0 & \sin\beta \end{bmatrix} \begin{bmatrix} \cos El\cos Az \\ \cos El\sin Az \\ \sin El \end{bmatrix} .$$

When conversions between direction cosines and right ascension/declination are required the azimuth and elevation are found as intermediate quantities.

4.5 COTOEL

This segment converts the velocity and position of a satellite, in a geocentric Cartesian coordinate system, to osculating orbital elements.

There are fourteen parameters:

X Y Z	input geocentric Cartesian position coordinates of the satellite, x,y,z;
XDOT YDOT ZDOT	input geocentric velocity of the satellite, $\dot{x},\dot{y},\dot{z}$;
EMU:	input Earth's gravitational constant, μ;
A:	output semi-major axis of elliptical satellite orbit, a;
E ORBINC RANODE ARGPER EM EN	output osculating elements of the satellite's orbit, e, i, Ω, ω, M and n. (See Section 3, Generation of the Orbit).

The parameters of COTOEL are in a consistent system of units, with angles expressed in radians. In PROP x, y, z and a are given in kilometres; $\dot{x}$, $\dot{y}$, $\dot{z}$ in km/sec; μ in km^3/sec^2 and n in radians/sec.

There are four calls of COTOEL within the program PROP. The call from segment PROP (4.28) places the osculating elements into an array ELMAIN. The call is made only if prediction of initial elements from another epoch is required. The other three calls of COTOEL will be discussed in subsection 4.32 which describes the segment SATELS.

The derivations of the equations governing the conversions performed by COTOEL are not readily available in the literature, so they have been rederived here. Undisturbed motion is assumed throughout COTOEL.

We will express the geocentric radial distance of the satellite as

$$r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$$

and its velocity as

$$V = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{\frac{1}{2}}.$$

The semi-latus rectum, p, is defined by

$$p = a(1 - e^2).$$

Then Kepler's first law implies that

$$r = \frac{p}{1 + e \cos v} \tag{4.5.1}$$

where v is the true anomaly, and the second law gives

$$r^2\dot{v} = h \tag{4.5.2}$$

where h is the angular momentum. Newton's law of gravitation gives

$$h^2 = \mu p \tag{4.5.3}$$

and we have also

$$V^2 = \dot{r}^2 + (r\dot{v})^2 \tag{4.5.4}$$

Differentiating Equation (4.5.1) with respect to time t gives

$$\dot{r} = \frac{(1 + e \cos v)\dot{p} - p\frac{d}{dt}(1 + e \cos v)}{(1 + e \cos v)^2} .$$

Now, for undisturbed motion

$$\dot{p} = \dot{e} = 0$$

so

$$\dot{r} = \frac{pe\dot{v} \sin v}{(1 + e \cos v)^2} = \frac{he \sin v}{p} \tag{4.5.5}$$

by Equations (4.5.1) and (4.5.2). Substituting from Equations (4.5.1), (4.5.2) and (4.5.5) into Equation (4.5.4) gives

$$V^2 = \left(\frac{he \sin v}{p} \right)^2 + \left(\frac{h(1 + e \cos v)}{p} \right)^2$$

from which we find

$$a = \left(\frac{2}{r} - \frac{V^2}{\mu} \right)^{-1} \qquad (4.5.6)$$

This is used in COTOEL to calculate a.

Now, let ϕ be the angle between the radius vector $\underset{\sim}{r}$ and the velocity vector $\underset{\sim}{V}$, that is,

$$\phi = \sin^{-1} \left(\frac{r\dot{v}}{V} \right) .$$

Then

$$\underset{\sim}{r} . \underset{\sim}{V} = r.V \cos \phi = x\dot{x} + y\dot{y} + z\dot{z}.$$

Now

$$(1 - \cos^2\phi)^{\frac{1}{2}} = \sin \phi = \frac{r\dot{v}}{V} = \frac{r^2 . \dot{v}}{r.V} = \frac{h}{rV} = \frac{(\mu p)^{\frac{1}{2}}}{r.V}$$

$$= \frac{(\mu a(1 - e^2))^{\frac{1}{2}}}{rV}$$

and hence

$$e = \left[1 - \frac{r^2 V^2}{\mu a} (1 - \cos^2 \phi) \right]^{\frac{1}{2}} .$$

That is

$$e = \left[\left(\frac{rV \cos \phi}{\sqrt{\mu a}} \right)^2 + \left(\frac{rV^2}{\mu} - 1 \right)^2 \right]^{\frac{1}{2}} , \qquad (4.5.7)$$

which is used in COTOEL to calculate e.

Now we define a vector $\underset{\sim}{B}$ by

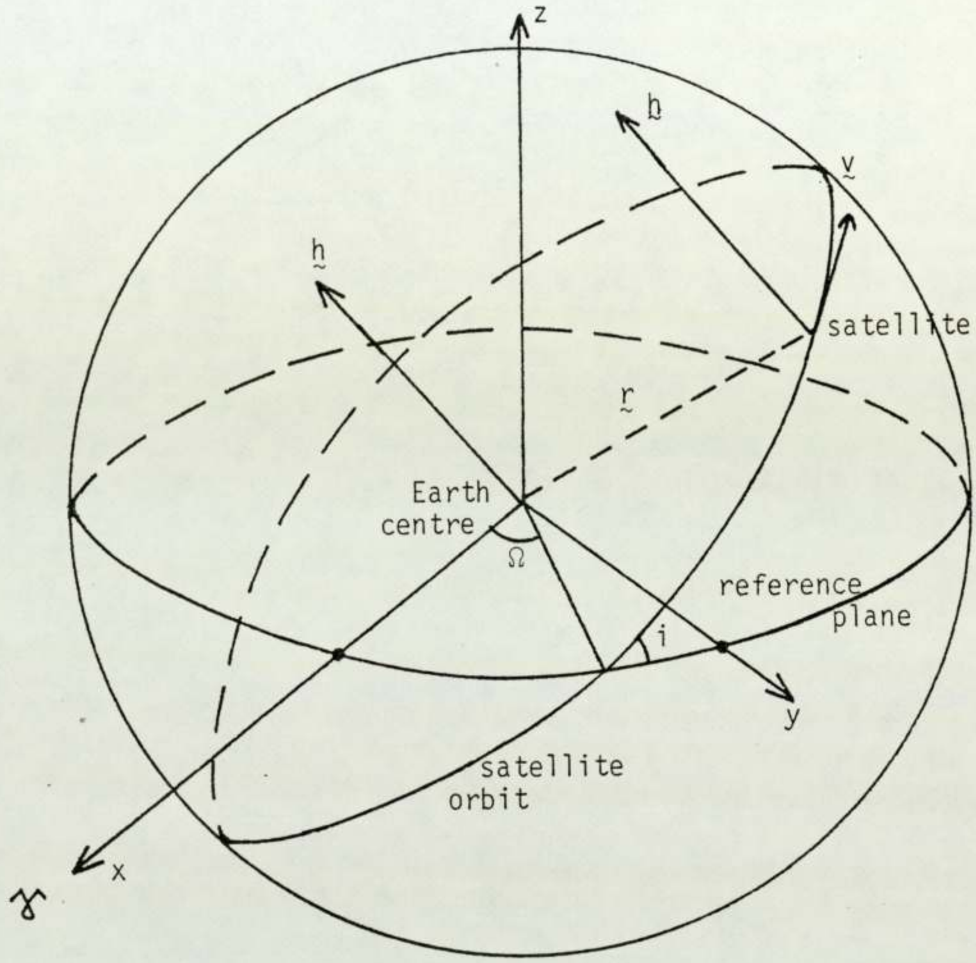
$$\underset{\sim}{B} = \underset{\sim}{r} \times \underset{\sim}{V} .$$

Then $\underset{\sim}{B}$ has components

$$B_x = y\dot{z} - z\dot{y}, \; B_y = z\dot{x} - x\dot{z}, \; B_z = x\dot{y} - y\dot{x}$$

and is perpendicular to the plane of the orbit. However, $|\underset{\sim}{B}|$ = h so we will put $\underset{\sim}{B} \equiv \underset{\sim}{h}$, $B_x \equiv h_x$, $B_y \equiv h_y$, $B_z \equiv h_z$, (see Figure 4.5.1).

Figure 4.5.1. The Vector $\underset{\sim}{h}$



It may be seen that

$h_z = h \cos i$, $h_x = \pm h \sin i \sin \Omega$, $h_y = \mp h \sin i \cos \Omega$,

so that

$$\tan \Omega = \frac{\sin \Omega}{\cos \Omega} = - \left(\frac{h_x}{h_y} \right) .$$

That is

$$\Omega = \tan^{-1}\left(\frac{y\dot{z} - z\dot{y}}{z\dot{x} - x\dot{z}}\right) \tag{4.5.8}$$

and

$$i = \tan^{-1}\left(\frac{\sin i}{\cos i}\right)$$

$$= \tan^{-1}\left[\frac{(h^2\sin^2 i\,\sin^2\Omega + h^2\sin^2 i\,\cos^2\Omega)^{\frac{1}{2}}}{h\cos i}\right]$$

$$= \tan^{-1}\left[\frac{(h_x{}^2 + h_y{}^2)^{\frac{1}{2}}}{h_z}\right]$$

$$= \tan^{-1}\left[\frac{\left[(y\dot{z} - z\dot{y})^2 + (z\dot{x} - x\dot{z})^2\right]^{\frac{1}{2}}}{x\dot{y} - y\dot{x}}\right] . \tag{4.5.9}$$

These are the expressions used in COTOEL to give i and Ω.

The expression used in COTOEL to give n is

$$n = \mu^{\frac{1}{2}}\, a^{-3/2} \tag{4.5.10}$$

which is Kepler's third law.

To obtain an expression for ω, the argument of perigee, we first consider a vector $\underset{\sim}{e}$ where

$$\underset{\sim}{e} = \left(\frac{V^2}{\mu} - \frac{1}{r}\right)\underset{\sim}{r} - \frac{rV\cos\phi}{\mu}\,\underset{\sim}{V} , \tag{4.5.11}$$

which is in the plane of the satellite orbit.

Then

$$|\underset{\sim}{e}|^2 = \left(\frac{V^2}{\mu} - \frac{1}{r}\right)^2 r^2 + \left(\frac{rV\cos\phi}{\mu}\right)^2 V^2$$

$$- 2rV\left(\frac{V^2}{\mu} - \frac{1}{r}\right)\left(\frac{rV\cos\phi}{\mu}\right)\cos\phi$$

$$= \left(\frac{rV^2}{\mu} - 1\right)^2 + \frac{(rV\cos\phi)^2}{\mu a}$$

$$= e^2.$$

We define

$$\left.\begin{aligned} \underset{\sim}{e} &= e_r \underset{\sim}{r} + e_V \underset{\sim}{V} \\ \text{where } e_r &= \frac{V^2}{\mu} - \frac{1}{r} , \\ \text{and } \quad e_V &= \frac{-rV\cos\phi}{\mu} . \end{aligned}\right\} \tag{4.5.12}$$

Further, we define the angle θ_{er} by

$$\underset{\sim}{e}.\underset{\sim}{r} = e\,r\cos\theta_{er} , \tag{4.5.13}$$

that is, θ_{er} is the angle between $\underset{\sim}{e}$ and $\underset{\sim}{r}$.

However, from Equation (4.5.12)

$$\underset{\sim}{e}.\underset{\sim}{r} = e_r\, \underset{\sim}{r}.\underset{\sim}{r} + e_V\, \underset{\sim}{V}.\underset{\sim}{r} \tag{4.5.14}$$

so, combining Equations (4.5.13) and (4.5.14), we obtain

$$r\cos\theta_{er} = \left(\frac{V^2}{\mu} - \frac{1}{r}\right)\frac{r^2}{e} - \frac{(rV\cos\phi)^2}{\mu e}$$

$$= \frac{V^2r^2}{\mu e}(1 - \cos^2\phi) - \frac{r}{e}$$

$$= \frac{V^2r^2}{\mu e}\sin^2\phi - \frac{a}{e}(1 - e\cos E)$$

$$= \frac{V^2r^2}{\mu e}\ \frac{a^2(1 - e^2)}{r(2a - r)} - \frac{a}{e}(1 - e\cos E)$$

(by equation (4.75) of Roy (1978))

$$= a\cos E - \frac{a}{e} + \frac{a^2(1 - e^2)}{(2a - r)}\frac{r}{e}\left(\frac{2}{r} - \frac{1}{a}\right)$$

$$= a\cos E - \frac{a}{e} + \frac{a^2(1 - e^2)}{(2a - r)}\frac{2}{e} - \frac{a(1 - e^2)}{(2a - r)}\frac{r}{e}$$

$$= a\cos E - \frac{a(2a - r) + 2a^2(1 - e^2) - ar(1 - e^2)}{e(2a - r)}$$

$$= a(\cos E - e)$$

$$= r\cos v.$$

Hence we may deduce that

$$\theta_{er} \equiv v,$$

that is, $\underset{\sim}{e}$ is in the direction of the argument of perigee. Now, $\underset{\sim}{h} = \underset{\sim}{r} \times \underset{\sim}{V}$ is perpendicular to the plane of the orbit so that $\underset{\sim}{h} \times \underset{\sim}{e}$ must be in the plane of the orbit, in the direction of the semi-minor axis of the ellipse. If we define a unit vector $\hat{\underset{\sim}{\ell}}$ along the line of the ascending node, so that

$$\hat{\underset{\sim}{\ell}} = \hat{\underset{\sim}{i}} \cos\Omega + \hat{\underset{\sim}{j}} \sin\Omega \ ,$$

then

$$\hat{\underset{\sim}{\ell}}.\underset{\sim}{e} = e \cos\omega \qquad (4.5.15)$$

and

$$\hat{\underset{\sim}{\ell}}.(\underset{\sim}{h} \times \underset{\sim}{e}) = h\, e \cos(\pi/2 + \omega)$$

$$= -h\, e \sin\omega . \qquad (4.5.16)$$

Here $\hat{\underset{\sim}{i}}$ is a unit vector toward ♈ and $\hat{\underset{\sim}{j}}$ is a unit vector perpendicular to $\hat{\underset{\sim}{i}}$, in the reference plane.

From Equations (4.5.15) and (4.5.16) we obtain

$$\tan\omega = \frac{-\ \hat{\underset{\sim}{\ell}}.(\underset{\sim}{h} \times \underset{\sim}{e})/h\, e}{\hat{\underset{\sim}{\ell}}.\underset{\sim}{e}/e} \qquad (4.5.17)$$

The term $\cos\omega$ may be expressed as

$$\cos\omega = \frac{1}{e}\left[(\hat{\underset{\sim}{i}} \cos\Omega + \hat{\underset{\sim}{j}} \sin\Omega).(e_r \underset{\sim}{r} + e_V \underset{\sim}{V})\right]. \qquad (4.5.18)$$

Now

$$e_r = \frac{V^2}{\mu} - \frac{1}{r} = \frac{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}{\mu} - \frac{1}{r} = \frac{1}{r} - \frac{1}{a}$$

$$= \frac{1}{a(1 - e\cos E)} - \frac{1}{a} = \frac{e\cos E}{r} \qquad (4.5.19)$$

and Equation (4.46) of Roy (1978) gives

$$\cos\phi = \frac{-e\sin E}{(1 - e^2\cos^2 E)^{\frac{1}{2}}} .$$

We note also that

$$1 - e^2\cos^2 E = (1 - e\cos E)(2 - (1 - e\cos E))$$

$$= \frac{r}{a}\left(2 - \frac{r}{a}\right)$$

$$= \frac{r^2}{a}\left(\frac{2}{r} - \frac{1}{a}\right)$$

$$= \frac{r^2 V^2}{a\mu},$$

so we obtain

$$e_V = \frac{-rV\cos\phi}{\mu} = \pm\left[\frac{-rV}{\mu}\ \frac{(-e\sin E(\mu a)^{\frac{1}{2}})}{rV}\right]$$

$$= \pm\left(\frac{a}{\mu}\right)^{\frac{1}{2}} e\sin E.$$

We take

$$e_V = -\left(\frac{a}{\mu}\right)^{\frac{1}{2}} e\sin E \tag{4.5.20}$$

from geometrical considerations.

Substituting from Equations (4.5.19) and (4.5.20) into Equation (4.5.18) we obtain

$$\cos\omega = \frac{1}{e}\left(\hat{\underset{\sim}{i}}\cos\Omega + \hat{\underset{\sim}{j}}\sin\Omega\right)\left(\frac{e\cos E}{r}\underset{\sim}{r} - \left(\frac{a}{\mu}\right)^{\frac{1}{2}} e\sin E\,\underset{\sim}{V}\right)$$

$$= \left(\frac{x}{r}\cos E - \dot{x}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\sin E\right)\cos\Omega$$

$$+ \left(\frac{y}{r}\cos E - \dot{y}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\sin E\right)\sin\Omega. \tag{4.5.21}$$

To find an expression for $\sin\omega$ we note that

$$\underset{\sim}{e} = \left(\frac{V^2}{\mu} - \frac{1}{r}\right)\underset{\sim}{r} - \frac{r\dot{r}}{\mu}\underset{\sim}{V}$$

where

$$\dot{r} = V\cos\phi,$$

and hence

$$\underset{\sim}{h} \times \underset{\sim}{e} = \left(\frac{V^2}{\mu} - \frac{1}{r}\right) \underset{\sim}{h} \times \underset{\sim}{r} - \frac{r\dot{r}}{\mu}(\underset{\sim}{h} \times \underset{\sim}{V}).$$

However

$$\underset{\sim}{h} = \underset{\sim}{r} \times \underset{\sim}{V},$$

so we obtain

$$\underset{\sim}{h} \times \underset{\sim}{e} = \left(\frac{V^2}{\mu} - \frac{1}{r}\right)\left[(\underset{\sim}{r} \times \underset{\sim}{V}) \times \underset{\sim}{r}\right] - \frac{r\dot{r}}{\mu}\left[(\underset{\sim}{r} \times \underset{\sim}{V}) \times \underset{\sim}{V}\right]$$

$$= \left(\frac{V^2}{\mu} - \frac{1}{r}\right)\left(r^2\underset{\sim}{V} - r\dot{r}\underset{\sim}{r}\right) - \frac{r\dot{r}}{\mu}\left(r\dot{r}\underset{\sim}{V} - V^2\underset{\sim}{r}\right).$$

This is arrived at by expanding the triple vector products and noting that

$\underset{\sim}{r}.\underset{\sim}{V} = r\dot{r}$; $\underset{\sim}{V}.\underset{\sim}{V} = V^2$ and $\underset{\sim}{r}.\underset{\sim}{r} = r^2$.

Thus

$$\underset{\sim}{h} \times \underset{\sim}{e} = \left(\frac{V^2r^2}{\mu} - r - \frac{(r\dot{r})^2}{\mu}\right)\underset{\sim}{V} + \left(\frac{-V^2r\dot{r}}{\mu} + \dot{r} + \frac{V^2r\dot{r}}{\mu}\right)\underset{\sim}{r}$$

$$= \left[\frac{(V^2 - \dot{r}^2)r^2}{\mu} - r\right]\underset{\sim}{V} + \dot{r}\underset{\sim}{r} . \qquad (4.5.22)$$

We know that

$$\dot{r} = V\cos\phi = \frac{a\,e\,\sin E}{r}\left(\frac{\mu}{a}\right)^{\frac{1}{2}}$$

and

$$h^2 = \mu p = \mu a(1 - e^2).$$

That is

$$\frac{h^2}{\mu} - r = a(1 - e^2) - a(1 - e\cos E)$$

$$= ae(\cos E - e),$$

so Equation (4.5.22) may be re-expressed as

$$\underset{\sim}{h} \times \underset{\sim}{e} = ae(\cos E - e)\underset{\sim}{V} + \frac{ae\sin E}{r}\left(\frac{\mu}{a}\right)^{\frac{1}{2}}\underset{\sim}{r} . \qquad (4.5.23)$$

Substituting Equation (4.5.23) into Equation (4.5.16) we obtain

$$\sin\omega = \frac{-1}{he}\left[\left(\hat{\underset{\sim}{i}}\cos\Omega + \hat{\underset{\sim}{j}}\sin\Omega\right)\cdot\left(ae(\cos E - e)\underset{\sim}{V} + \frac{ae\sin E}{r}\left(\frac{\mu}{a}\right)^{\frac{1}{2}}\underset{\sim}{r}\right)\right]$$

$$= \frac{-1}{h}\left\{\left[a(\cos E - e)\dot{x} + \frac{x}{r}\sin E(\mu a)^{\frac{1}{2}}\right]\cos\Omega\right.$$

$$\left. + \left[a(\cos E - e)\dot{y} + \frac{y}{r}\sin E(\mu a)^{\frac{1}{2}}\right]\sin\Omega\right\} \qquad (4.5.24)$$

where $h = (\mu a)^{\frac{1}{2}}(1 - e^2)^{\frac{1}{2}}$

and $\tan\Omega = \dfrac{y\dot{z} - z\dot{y}}{x\dot{z} - z\dot{x}}$.

From Equations (4.5.21) and (4.5.24), which are given by Merson (1977), we may obtain, by the manipulation given in Appendix II,

$$\omega = \tan^{-1}\left\{\frac{\dfrac{z}{r}\,e\cos E - \dfrac{\dot{z}e\sin E}{(\mu/a)^{\frac{1}{2}}}}{\left[\dfrac{z}{r}\,e\sin E + \dfrac{(e\cos E - e^2)\dot{z}}{(\mu/a)^{\frac{1}{2}}}\right]\Big/(1 - e^2)^{\frac{1}{2}}}\right\} \qquad (4.5.25)$$

which is the required expression as used in COTOEL for ω.

4.6 <u>DHMS</u> (Odell, 1969)

This segment converts a time expressed in days and decimals of a day, to days, hours, minutes, seconds and decimals of a second.

There are five parameters:

DAY: input time, in days, to be converted,

IDAY: output nearest integer less than DAY,

IHR: output non-negative integer number of hours relative to IDAY,

IMIN: output non-negative integer number of minutes relative to IHR,

SEC: output non-negative real number of seconds relative to IMIN.

It should be noted that a negative value of DAY gives a negative output in IDAY and non-negative output in IHR, IMIN and SEC. For example, if

DAY = -1.25

then

IDAY = -2,

IHR = 18,

IMIN = 0 and

SEC = 0.0.

The facility in DHMS to convert an angle in degrees and decimals of a degree to degrees, minutes, seconds and decimals of a second is not used by the program PROP.

4.7 DIFCOR

This segment applies the differential correction process to the initial estimates of the orbital parameters to obtain an improved estimate, by an iterative "least squares" method.

There are three parameters:

DATA: input (21 × MAXIM) array of observation data,

MAXIM: input maximum number of observations,

FAILED: output, logical indicator of success or failure of the convergence of the iterative process.

The PROP6 version of DIFCOR has two further parameters, CENTRE and MA01, which are input, subroutine names. These were always set to ECENTR and MA01A respectively in PROP6. The transfer of these subroutine names as parameters has been removed from the new version of PROP so that both the segments ECENTR(4.9) and MA01A(4.16) may be legally placed in overlay units.

The use of COMMON is:

/CNTROL/

IP, IPUNCH, IITT, ILINES, RIDENT, MAXITN, MINOBS, JELTYP and NOTHER are all input.

/ORBIT/

EMU, ERAD, MJDOCH, NOMIAL(6), MODEL(5) and EJ(40) are all input although EJ(1) and EJ(3) to EJ(40) are not used by DIFCOR.

XDOT, YDOT, ZDOT are all set during each call of ECENTR.

ELEMT(6 × 6) is input with initial values and output with final values.

MJD,X, Y, Z, XDOT are equivalenced to JJ and RESDEL (j), j = 1,..,4 which are input to DIFCOR by the segment ALTELS(4.1) when it is called by DIFCOR with parameter 0. This occurs at the end of each iteration, when the value placed in XDOT by ECENTR is no longer of use.

COMMON//

JTYPE, JRATES, JDCS and OBSNEX(23) are set by DIFCOR to specify each observation in turn, to ECENTR.

COSDEC, CALC(6) and PARDER(6 × 20) are set by ECENTR for each observation in turn.

NUMOBS, NUMPAR, REJLEV(3), OEMTM(20 × 20), OPARAS(20), NOODOF and OSUMSQ are input.

EPSLON is output.

Blank COMMON

INDRES is input.

XX, YY, ZZ, RANGE, RA and SINDEC are set by ECENTR for each observation in turn.

DEC is set by DIFCOR for each observation in turn.

The segment DIFCOR is at the heart of the program PROP. DIFCOR refines the initial orbital parameters by the least squares method described in Section 3. The process minimises

$$\sum_{k=1}^{ktot} R_k^{\,2}$$

where ktot is the total number of observed quantities and, using the notation of Section 3,

$$R_k = \sum_{j=1}^{NUMPAR} \frac{\partial\eta_k}{\partial\varepsilon_j} \Delta\varepsilon_j - \Delta\eta_k \ .$$

Six aspects of DIFCOR are discussed further in subsections 4.7.1 to 4.7.6. These are the mode of iteration, transformation of parameters, covariance and correlation, incorporation of a previously determined orbit, printing of rotated residuals and optional suppression of some of the line printer output.

A minor modification has been introduced into the segment DIFCOR such that the printing of the final orbital elements is preceded by the printing of the date of the epoch, and the identity of the satellite, under investigation.

4.7.1 Mode of Iteration

The orbital parameters refined by DIFCOR are governed by the contents of the array MODEL. When DIFCOR is called by PROP the array MODEL holds integers derived from the control parameter MMMMM input by the user. However if the variable MODE is non-zero on entry, then MODEL will be copied into an array MN(5) and MODEL will be temporarily overwritten.

When

MODE $\geqslant$ 2

MODEL is set to

0, 0, 0, 0, minimum (2,MN(5))

so the parameters refined in mode 2 are M_0 or M_0 and M_1.

When

MODE = 1

MODEL is set to

minimum (1,MN(1)), 0, 0, 0, minimum (3,MN(5))

so the parameters refined in mode 1 are contained in the set e_0, M_0, M_1, M_2. When MODE is zero then MODEL is set as it was input.

The mode of iteration is decremented by unity after each iteration during which no new observations were rejected, until MODE is zero.

4.7.2 Transformation of Parameters

The user may specify that transformations of parameters are to be made. The transformations have been discussed by Gooding (1978). This will normally be necessary if the orbit to be analysed is nearly circular or nearly equatorial, in which cases some of the normal orbital elements become ill-defined. The user specifies that transformations are required by setting the tens and units digits of the control variable JELTYP non-zero, as described by Gooding and Tayler (1968). The transformations specified will only be made when the mode is zero and the element coefficients to be transformed are orbital parameters to be refined. This will be assumed to be the case for the remainder of subsection 4.7.2.

It is possible, but it would be unusual, for the two digits of JELTYP to be set unequal to each other. In general, if the orbit is nearly circular JELTYP should be set to 11, if it is nearly equatorial JELTYP should be set to 22 and if it is both, JELTYP should be set to 33.

Within DIFCOR the first (tens) digit of JELTYP is transferred to JEL(1) and the second (units) digit to JEL(2). Logical variables are then set according to JEL(1), such that

LGC1 = TRUE if JEL(1) = 2 or 3, implying an equatorial orbit,

LGC2 = TRUE if JEL(1) = 1 or 3, implying a circular orbit.

LGC1 and LGC2 are then used to control the transformations necessary to prevent ill-conditioned matrices of partial derivatives from being used.

If the logical variables indicate that the orbit is equatorial but not circular, then partial derivatives

$$\left.\frac{\partial \eta_I}{\partial \Omega_0}\right|_{\omega_0} \quad \text{and} \quad \left.\frac{\partial \eta_I}{\partial \omega_0}\right|_{\Omega_0}$$

which are calculated during the call of the segment ECENTR, are approximately equal and therefore they are replaced by

$$\left.\frac{\partial \eta_I}{\partial \Omega_0}\right|_{\omega_0 + \Omega_0 \cos i_0} \qquad \text{and} \qquad \left.\frac{\partial \eta_I}{\partial(\omega_0 + \Omega_0 \cos i_0)}\right|_{\Omega_0} .$$

The notation $\left.\frac{\partial \eta_I}{\partial \varepsilon_j}\right|_{\varepsilon_k}$ denotes the partial derivative of η_I with respect to ε_j, keeping ε_k constant, where ε_j, ε_k need not be the standard orbital parameters.

Similarly, if the orbit is circular but not equatorial, partial derivatives

$$\left.\frac{\partial\eta_I}{\partial\omega_0}\right]_{M_0} \quad \text{and} \quad \left.\frac{\partial\eta_I}{\partial M_0}\right]_{\omega_0}$$

are approximately equal and therefore they are replaced by

$$\left.\frac{\partial\eta_I}{\partial\omega_0}\right]_{\omega_0+M_0} \quad \text{and} \quad \left.\frac{\partial\eta_I}{\partial(\omega_0+M_0)}\right]_{\omega_0} .$$

Expressions for these partial derivatives may be derived as follows, remembering that the η_I are functions of the orbital parameters, that is

$$\eta_I \equiv \eta_I(e_0,\ e_1,\ —,\ e_5,\ i_0,—,\ \Omega_0,—,\ \omega_0,\ —,\ M_0,\ —).$$

Let $\{\varepsilon\}$ be the set of orbital parameters, apart from Ω_0 and ω_0 and let

$\cos i_0 = \kappa$, a constant.

Define

$x \equiv \Omega_0$, $y \equiv \omega_0 + \kappa\Omega_0$,

hence $\Omega_0 = x$, $\omega_0 = y - \kappa x$.

Then

$$\left.\frac{\partial\eta_I}{\partial\Omega_0}\right]_{\omega_0+\kappa\Omega_0,\{\varepsilon\}} \equiv \left.\frac{\partial\eta_I}{\partial x}\right]_{y,\{\varepsilon\}}$$

$$= \frac{\partial\eta_I}{\partial\Omega_0}\frac{\partial\Omega_0}{\partial x} + \frac{\partial\eta_I}{\partial\omega_0}\frac{\partial\omega_0}{\partial x}$$

$$= \frac{\partial\eta_I}{\partial\Omega_0} - \kappa\frac{\partial\eta_I}{\partial\omega_0} ,$$

and

$$\left.\frac{\partial\eta_I}{\partial(\omega_0+\kappa\Omega_0)}\right]_{\Omega_0} \equiv \left.\frac{\partial\eta_I}{\partial y}\right]_x$$

$$= \frac{\partial \eta_I}{\partial \Omega_0} \frac{\partial \Omega_0}{\partial y} + \frac{\partial \eta_I}{\partial \omega_0} \frac{\partial \omega_0}{\partial y}$$

$$= \frac{\partial \eta_I}{\partial \omega_0} .$$

Similarly, if we now define

$$x \equiv \omega_0 \ , \quad y \equiv M_0 + \omega_0$$

and let $\{\varepsilon\}$ be the set of orbital parameters apart from M_0 and ω_0 then

$$\omega_0 = x \ , \quad M_0 = y - x.$$

We obtain

$$\left.\frac{\partial \eta_I}{\partial \omega_0}\right|_{M_0+\omega_0,\{\varepsilon\}} \equiv \left.\frac{\partial \eta_I}{\partial x}\right|_{y,\{\varepsilon\}}$$

$$= \frac{\partial \eta_I}{\partial \omega_0} \frac{\partial \omega_0}{\partial x} + \frac{\partial \eta_I}{\partial M_0} \frac{\partial M_0}{\partial x}$$

$$= \frac{\partial \eta_I}{\partial \omega_0} - \frac{\partial \eta_I}{\partial M_0}$$

and

$$\left.\frac{\partial \eta_I}{\partial (M_0+\omega_0)}\right|_{\omega_0,\{\varepsilon\}} \equiv \left.\frac{\partial \eta_I}{\partial y}\right|_{x,\{\varepsilon\}}$$

$$= \frac{\partial \eta_I}{\partial \omega_0} \frac{\partial \omega_0}{\partial y} + \frac{\partial \eta_I}{\partial M_0} \frac{\partial M_0}{\partial y}$$

$$= \frac{\partial \eta_I}{\partial M_0} .$$

If the orbit is both nearly equatorial and nearly circular then the parameters Ω_0, ω_0, M_0 are transformed to Ω_0, $\omega_0 + \Omega_0 \cos i_0$, $M_0 + \omega_0 + \Omega_0 \cos i_0$ by a combination of the processes described above.

With the partial derivatives being transformed before they are incorporated into $\mathcal{M}^T\mathcal{M}$ and $\mathcal{M}^T \underset{\sim}{y}$ the resultant parameters from the iteration have to be transformed back at the end of each iteration.

LGC1 and LGC2 also influence the processes described under subsection 4.7.3.

The contents of JEL(2) are used only at the end of each iteration. The increments to the orbital parameters, $\Delta\varepsilon_j$, are added to the previous estimates, ε_j' to give new estimates of the orbital parameters, e_0, i_0, Ω_0, ω_0, M_0, ... If JEL(2) = 1, indicating a nearly circular, non-equatorial, orbit, the convergence may be improved by temporarily transforming from e_0, ω_0 and M_0 to $e_0 \cos\omega_0$, $e_0 \sin\omega_0$ and $\omega_0 + M_0$. If JEL(2) = 2, indicating a nearly equatorial, non-circular, orbit, then convergence may be improved by temporarily transforming from i_0 and Ω_0 to $\sin i_0 \cos\Omega_0$ and $\sin i_0 \sin\Omega_0$. If JEL(2) = 3, indicating an orbit which is both nearly circular and nearly equatorial, the appropriate transformation is from e_0, i_0, Ω_0, ω_0 and M_0 to $\sin i_0 \cos\Omega_0$, $\sin i_0 \sin\Omega_0$ $e_0\cos(\omega_0 + \kappa\Omega_0)$, $e_0\sin(\omega_0 + \kappa\Omega_0)$ and $\omega_0 + \kappa\Omega_0 + M_0$. As previously, $\kappa = \cos i_0$ is assumed to be a constant.

The transformations are made as follows. When JEL(2) = 1, $\Delta\omega$ is given by

$$\Delta\omega = \omega_0 - \omega_0'$$

where prime indicates the value resulting from the previous iteration. Thence

$$e_0' \cos\omega_0' + \Delta(e_0' \cos\omega_0') = e_0 \cos\omega_0' - \Delta\omega\, e_0' \sin\omega_0'$$

and

$$e_0' \sin\omega_0' + \Delta(e_0' \sin\omega_0') = e_0 \sin\omega_0' + \Delta\omega\, e_0' \cos\omega_0' .$$

These terms are the new

$$e_{0_{new}} \cos \underset{\sim}{\omega}_{0_{new}} \quad \text{and} \quad e_{0_{new}} \sin \omega_{0_{new}}$$

respectively, from which the new $e_{0_{new}}$ and $\omega_{0_{new}}$ are derived. If M_0 is also a parameter then the new $M_{0_{new}}$ is given by

$$M_{0_{new}} = M_0 + \omega_0' + \Delta\omega - \omega_{0_{new}} .$$

When JEL(2) = 2, Δi and $\Delta\Omega$ are given by

$$\Delta i = i_0 - i_0' , \quad \Delta\Omega = \Omega_0 - \Omega_0' .$$

Thence

$$\sin i_0' \cos \Omega_0' + \Delta(\sin i_0' \cos \Omega_0') = \sin i_0' \cos\Omega_0' + \Delta i \cos i_0' \cos\Omega_0' - \Delta\Omega \sin i_0' \sin\Omega_0'$$

and

$$\sin i_0' \sin\Omega_0' + \Delta(\sin i_0' \sin\Omega_0') = \sin i_0' \sin\Omega_0' + \Delta i \cos i_0' \sin\Omega_0' + \Delta\Omega \sin i_0' \cos\Omega_0'.$$

These terms are the new

$$\sin i_{0_{new}} \cos\Omega_{0_{new}} \quad \text{and} \quad \sin i_{0_{new}} \sin\Omega_{0_{new}}$$

respectively, from which the new $i_{0_{new}}$ and $\Omega_{0_{new}}$ are derived. If ω_0 is also a parameter then the new $\omega_{0_{new}}$ is given by

$$\omega_{0_{new}} = \omega_0 + \cos i_{0_{new}} \left\{[\Omega_0] - \Omega_{0_{new}}\right\}$$

where $[\Omega_0]$ is Ω_0 normalised to be in the interval $-\pi$ to π.

When JEL(2) = 3 the above transformations are applied in combination.

4.7.3 The Covariance Matrix and Correlation Coefficients

At the end of the iterative process in DIFCOR the covariances of the orbital parameters are calculated and then the correlation coefficients. The covariance matrix is output to the card punch channel and the correlation coefficients to the main line printer output channel. If a transformation of parameters has been made to overcome ill-conditioning of the partial derivative matrices (LGC1 or LGC2 set TRUE), then this transformation affects the contents of the covariance matrix. However, the transformation is reversed before the correlation coefficients are calculated.

To obtain the covariance matrix we use the notation of Section 3. Let $\hat{\underset{\sim}{x}}$ be the vector of increments to the estimated orbital parameters that would give the true set of orbital parameters. Let $\underset{\sim}{x}$ be the estimate of $\hat{\underset{\sim}{x}}$, obtained from the final iteration by DIFCOR. Similarly, let $\hat{\underset{\sim}{y}}$ and $\underset{\sim}{y}$ be the true and estimated vectors of $\Delta\eta_k/\sigma_k$. Following Gooding (1970b), if Ex[ ] denotes a statistical expectation, the covariance matrix of the orbital parameters is given by

$$\begin{aligned}\mathrm{Cov}(\underset{\sim}{x}) &= \mathrm{Ex}\left[(\underset{\sim}{x} - \hat{\underset{\sim}{x}})(\underset{\sim}{x} - \hat{\underset{\sim}{x}})^T\right] \\ &= (\mathcal{M}^T\mathcal{M})^{-1}\mathcal{M}^T\mathrm{Ex}\left[(\underset{\sim}{y} - \hat{\underset{\sim}{y}})(\underset{\sim}{y} - \hat{\underset{\sim}{y}})^T\right]\mathcal{M}(\mathcal{M}^T\mathcal{M})^{-1}\end{aligned}$$

from Equation (3.5). If we assume that the weightings, σ_k, of the observed quantities, η_k, are reasonable then

$$\mathrm{Ex}\left[(\underset{\sim}{y} - \hat{\underset{\sim}{y}})(\underset{\sim}{y} - \hat{\underset{\sim}{y}})^T\right] = \mathcal{E}^2 I$$

where I is the unit matrix and $\mathcal{E}$ the convergence parameter (see sub-section 4.3, CONVER).

Hence

$$\mathrm{Cov}(x) = (\mathcal{M}^T\mathcal{M})^{-1}\,\mathcal{E}^2. \tag{4.7.1}$$

The covariance of two of the orbital parameters, ε_j, ε_k, is given by

$$\mathrm{Cov}(\varepsilon_j,\varepsilon_k) = \mathrm{Cov}(x)_{jk} = \mathcal{E}^2\left[(\mathcal{M}^T\mathcal{M})^{-1}\right]_{jk}$$

and correlation coefficients may then be obtained by

$$\mathrm{Correlation}\ (\varepsilon_j,\varepsilon_k) = \frac{\mathrm{Cov}(\varepsilon_j,\varepsilon_k)}{\left\{\mathrm{Cov}(\varepsilon_j,\varepsilon_j)\mathrm{Cov}(\varepsilon_k,\varepsilon_k)\right\}^{\frac{1}{2}}}\ .$$

4.7.4 The Previous Orbit Facility

When the "previous orbit" facility is being used, the master segment PROP (see subsection 4.28) passes to DIFCOR the array

$$\text{OEMTM} = \left[\frac{\text{Cov}(x)}{\mathcal{E}^2} \right]^{-1}$$

where Cov(x) is the covariance matrix from the previous orbit and $\mathcal{E}$ is the final convergence parameter from the previous orbit. We may see, by Equation (4.7.1), that OEMTM is identical to the final $\mathcal{M}^T\mathcal{M}$ from the previous orbit.

The previous orbit is incorporated at each mode zero iteration. The segment REJECT (see subsection 4.31) accumulates accepted observation data in EMTM(j,k,1), EMTY(j,1) and SUMSQ(1),(j,k=1,..,NUMPAR). The previous orbit is then added by

$$\text{EMTM(j,k,1)} = \text{EMTM(j,k,1)} + \text{OEMTM(j,k)} \qquad \text{j,k=1,..,NUMPAR}$$

$$\text{EMTY(j,1)} = \text{EMTY(j,1)} + \sum_{k=1}^{\text{NUMPAR}} \text{OEMTM(j,k)} \times (\text{OPARAS(k)} - \text{PARAS(k)}) \qquad \text{j=1,..,NUMPAR}$$

and

$$\text{SUMSQ(1)} = \text{SUMSQ(1)} + \text{OSUMSQ(1)} + \sum_{j=1}^{\text{NUMPAR}} \sum_{k=1}^{\text{NUMPAR}} \text{OEMTM(j,k)} + (\text{OPARAS(j)} - \text{PARAS(j)}).$$

In this way the number of degrees of freedom is increased,

NUMDOF = NUMDOF + NUMPAR + NOODOF,

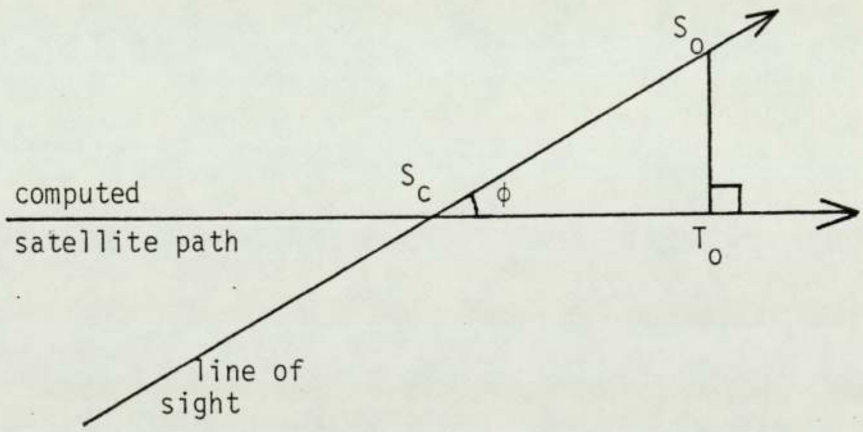
where NOODOF is the final number of degrees of freedom of the previous orbit.

4.7.5 Printing of Rotated Residuals

The segment DIFCOR will calculate and print rotated residuals when the input variable INDRES is non-zero and observations of type 1, 2, 3 or 7 are being considered. The quantities printed, and their relevance, are described by Gooding (1974). The quantities are derived here.

Consider first, observations of type 1, that is, observations of range only. In Figure 4.7.1 we show the satellite's computed position S_c and observed position S_o.

Figure 4.7.1. Rotated Residuals, Type 1 Observations.



It is assumed the line of sight is the same for both positions. The angle between the line of sight and the satellite path is ϕ. T_o is the foot of the perpendicular from S_o to the satellite path. The rotated residual is the time taken (in seconds) for the satellite to travel from S_c to T_o, at the calculated velocity of the satellite. By calling the segment ECENTR (see subsection 4.9), DIFCOR has available the computed topocentric satellite position vector

$$\underset{\sim}{\rho}_c = (x_c, y_c, z_c)$$

and velocity vector

$$\underset{\sim}{V} = (\dot{x}, \dot{y}, \dot{z}).$$

The computed right ascension and declination are also available and hence the observed position vector

$$\underset{\sim}{\rho}_o = (x_o, y_o, z_o)$$

may be computed. We define a vector $\underset{\sim}{\xi} = \overrightarrow{S_o S_c}$ by

$$\underset{\sim}{\xi} = (\underset{\sim}{\rho}_o - \underset{\sim}{\rho}_c) = (x_o - x_c,\ y_o - y_c,\ z_o - z_c)$$

$$= (\xi_1,\ \xi_2,\ \xi_3).$$

The scalar product of $\underset{\sim}{\xi}$ and $\underset{\sim}{V}$ gives

$$\underset{\sim}{\xi}.\underset{\sim}{V} = \xi V \cos\phi = \xi_1 \dot{x} + \xi_2 \dot{y} + \xi_3 \dot{z}$$

where $V = |\underset{\sim}{V}|$ and $\xi = |\underset{\sim}{\xi}|$. The distance $S_c T_o$ is given by $\xi \cos\phi$ and hence the residual required is

$$\frac{\xi \cos\phi}{V} = \frac{\underset{\sim}{\xi}.\underset{\sim}{V}}{V^2} = (\xi_1 \dot{x} + \xi_2 \dot{y} + \xi_3 \dot{z})/V^2.$$

Now consider observations of type 2. That is, observations of right ascension and declination which may have been derived from observations of azimuth and elevation. It is assumed that the observed range equals the calculated range. That is, referring to Figure 4.7.2, with the observer at O

$$OS_o = OS_c.$$

The line MN is perpendicular to both the line of sight and the calculated satellite path. The point Q_o is at a point such that $S_o NMQ_o$ forms a rectangle. The point T_o is the foot of the perpendicular from Q_o to $S_c M$. The residuals printed are:

δu = angle subtended at O by $S_c Q_o$,

δw = angle subtended at O by S_oQ_o,

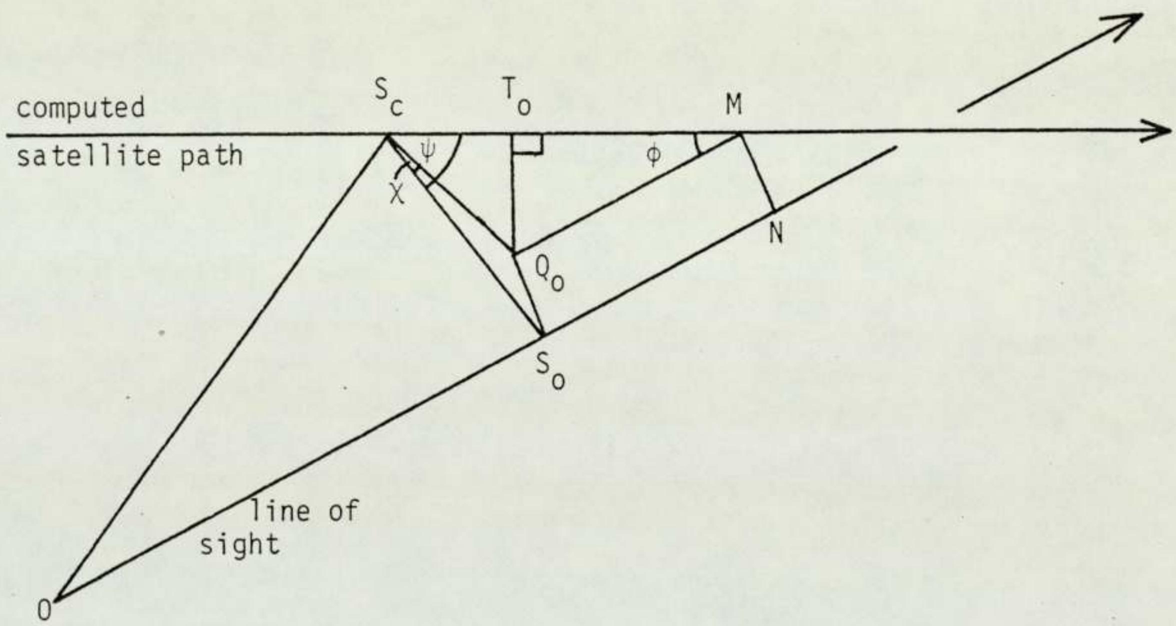
t_{ST} = time taken for the satellite to travel from S_c to T_o,

t_{SQ} = time taken for the satellite to travel from S_c to Q_o, and

t_{SM} = time taken for the satellite to travel from S_c to M.

The angles δu and δw are printed in degrees but will be derived in radians here. The times t_{ST} , t_{SQ} and t_{SM} are printed in seconds and it is assumed that the satellite travels at the calculated speed.

Figure 4.7.2 Rotated Residuals, Types 2 and 3 Observations



To obtain these residuals we define vectors $\underset{\sim}{V}$ along S_cT_oM and $\underset{\sim}{\xi}$ along S_cS_o as we did when considering type 1 observations. We define a unit vector along the line of sight by

$$\underset{\sim}{\zeta} = (x_o/\rho,\ y_o/\rho,\ z_o/\rho) = (\zeta_1,\ \zeta_2,\ \zeta_3)$$

where ρ is the RANGE calculated by ECENTR and x_0, y_0, z_0 are derived from ρ and the observed right ascension and declination. The vector product of $\underset{\sim}{\zeta}$ and $\underset{\sim}{V}$ gives another vector

$$\underset{\sim}{\nu} = \underset{\sim}{\zeta} \times \underset{\sim}{V}/V \sin\phi = (\nu_1, \nu_2, \nu_3).$$

$\underset{\sim}{\nu}$ is a unit vector, since $|\underset{\sim}{\zeta} \times \underset{\sim}{V}|$ is $V \sin\phi$, perpendicular to both $\underset{\sim}{\zeta}$ and $\underset{\sim}{V}$. Therefore $\underset{\sim}{\nu}$ is in the direction MN. Forming the vector product of $\underset{\sim}{\nu}$ and $\underset{\sim}{\zeta}$ defines another unit vector

$$\underset{\sim}{\tau} = \underset{\sim}{\nu} \times \underset{\sim}{\zeta}/|\underset{\sim}{\nu} \times \underset{\sim}{\zeta}| = (\tau_1, \tau_2, \tau_3).$$

If we assume that the angle S_cQ_oM is a right angle, then the vector $\underset{\sim}{\tau}$ is in the direction S_cQ_o. This assumption is implicit in the explanation of rotated residuals given by Gooding (1974). Provided that the observed and calculated directions are within a few minutes of arc the assumption is a reasonable one.

If we take the angle χ to be as shown in Figure 4.7.2. then we obtain

$$S_cQ_o = \xi \cos\chi$$

$$= \underset{\sim}{\tau}.\underset{\sim}{\xi} = (\tau_1\xi_1 + \tau_2\xi_2 + \tau_3\xi_3)$$

and

$$S_oQ_o = \xi \cos(\pi/2 - \chi)$$

$$= \underset{\sim}{\nu}.\underset{\sim}{\xi} = (\nu_1\xi_1 + \nu_2\xi_2 + \nu_3\xi_3).$$

Hence the residuals δu and δw are given by

$$\delta u = (\underset{\sim}{\tau}.\underset{\sim}{\xi})/\rho \text{ radians}$$

and

$$\delta w = (\underset{\sim}{\nu}.\underset{\sim}{\xi})/\rho \text{ radians.}$$

Further, if we take ψ to be as shown in Figure 4.7.2., we obtain

$$\underset{\sim}{V}.\underset{\sim}{\xi} = V\xi \cos\psi = (\dot{x}\xi_1 + \dot{y}\xi_2 + \dot{z}\xi_3)$$

and hence

$$t_{ST} = (\underset{\sim}{V}.\underset{\sim}{\xi})/V^2 \text{ seconds.}$$

The final two residuals are given by

$$t_{SQ} = \rho\,\delta u/V \text{ seconds}$$

where δu is expressed in radians, and

$$t_{SM} = t_{SQ}/\sin\chi \text{ seconds}$$

again assuming that the angle S_cQ_oM is a right angle.

Observations of type 3 are observations of direction cosines, which may be converted into observations of right ascension and declination. The geometrical situation is then the same as for type 2 observations. However, angular residuals δu and δw are not printed. Time residuals t_{ST}, t_{SQ} and t_{SM} are printed.

Observations of type 7 are observations either of range, right ascension and declination or of range, azimuth and elevation. If azimuth and elevation were observed they will have been converted already to right ascension and declination.

The geometry of the situation, shown in Figure 4.7.3., is very similar to the geometry of type 2 observations. However, the range OS_o is observed as ρ_o which is unequal, in general, to the calculated range OS_c of ρ_c. The residuals printed are

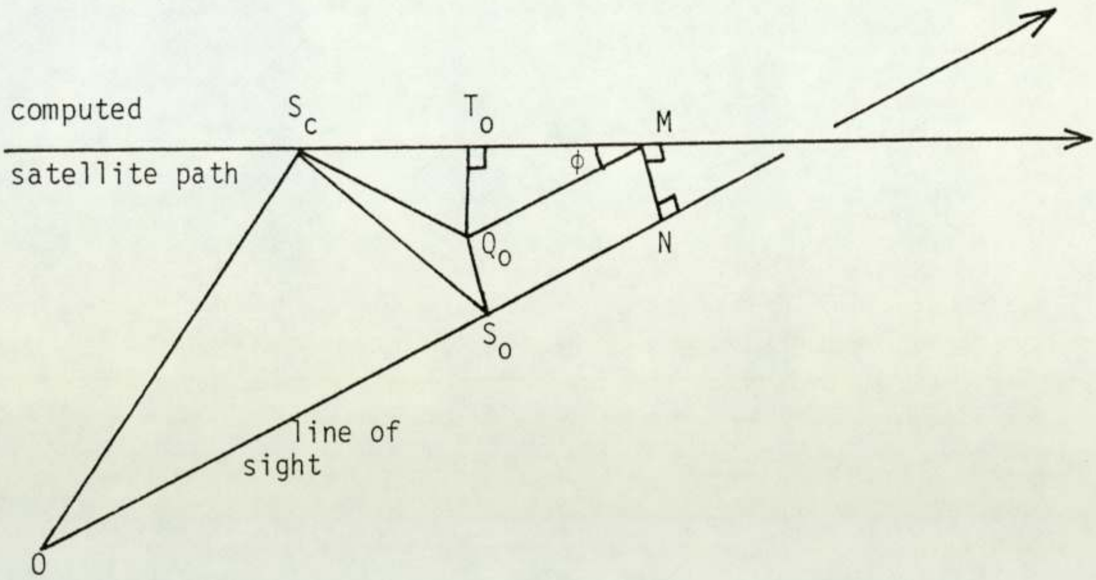
δu = angle subtended at O by S_cQ_o,

δw = angle subtended at O by S_oQ_o,

and

t_{ST} = time taken for the satellite to travel from S_c to T_o.

Figure 4.7.3 Rotated Residuals, Type 7 Observations



4.7.6 Optional Suppression of Printing

A change has been introduced throughout the segment DIFCOR to facilitate the optional suppression of the printing of information generated within each iteration performed by DIFCOR. This change has been made to work when the new PROP is run on an ICL 1900 series computer under GEORGE3 or GEORGE4 operating systems. Further changes may be necessary if the new PROP is to run on some other machine or under some other operating system.

A new item in COMMON/CNTROL/ of IITT has been introduced. All line printer output, from the writing of

FIRST ITERATION

until the test for convergence shows the process to be complete, is written to channel number IITT. Final printing, including the final set of orbital elements, is then written to the main line printer channel, number IP.

To facilitate the directing of output, which may or may not be the final output, the logical variable MSLGC has been introduced, together with the variable ITCH. MSLGC is set FALSE only when the final iteration has been made. ITCH is set such that

ITCH = IITT if MSLGC is TRUE,

ITCH = IP   if MSLGC is FALSE.

Output which may be final is printed on channel ITCH.

A further change has been made so that slightly different formats are used to print two digit iteration numbers. For example, the seventh iteration is headed

FURTHER$_{\nabla}$ITERATION$_{\nabla}$-$_{\nabla}$NUMBER$_{\nabla\nabla}$7

but the tenth iteration is headed

FURTHER$_{\nabla}$ITERATION$_{\nabla}$-$_{\nabla}$NUMBER$_{\nabla\nabla}$10

When the number of the final iteration is known this is then printed on output channel 12, using the format:

19H$_{\nabla}$FINAL$_{\nabla}$ITERATION$_{\nabla}$-$_{\nabla}$,I3

and the write statement is followed by a

PAUSE

statement. Output channel number 12 is a new output channel assigned, in the steering segment, to TYO, that is the monitoring file. Hence the number of the final iteration may be detected under the operating system and, if required, only information relevant to the final iteration is printed from channel IITT. A macro (file containing job control instructions) has been written to aid the user with this new facility. A listing of the macro, with explanatory notes, is given in Appendix IV.

4.8 EAFKEP

This function segment returns a value of the eccentric anomaly, estimated from the input values of the mean anomaly and the eccentricity.

There are two parameters

EM: input mean anomaly M (radians),

E: input eccentricity e.

The function EAFKEP places a value of the eccentric anomaly E_c into EAFKEP by solving Kepler's equation

$$E_c = M + e \sin E_c \qquad (4.8.1)$$

by an iterative method. The number of iterations is fixed by the value of the eccentricity e. If e is greater than 0.03 then five iterations are used. Otherwise only three are performed.

Equation (4.8.1) implies that

$$E_{c_t} = M + e \sin E_{c_t} \qquad (4.8.2)$$

where E_{c_t} is the true eccentric anomaly.

However

$$E_{c_t} = E_{c_a} + \delta E_c \qquad (4.8.3)$$

where E_{c_a} is an approximation to E_{c_t}. Hence, substituting Equation (4.8.3) into the right hand side of Equation (4.8.2) we obtain

$$E_{c_t} = M + e \sin (E_{c_a} + \delta E_c)$$

$$= M + e(\sin E_{c_a} \cos \delta E_c + \sin \delta E_c \cos E_{c_a}). \qquad (4.8.4)$$

If δE_c is small then

$$\cos \delta E_c \simeq 1, \quad \sin \delta E_c \simeq \delta E_c.$$

Using these two approximations in Equation (4.8.4) and substituting the result into the left-hand side of Equation (4.8.3) we find that

$$E_{c_a} + \delta E_c = M + e \sin E_{c_a} + e \delta E_c \cos E_{c_a}$$

and hence

$$\delta E_c = \frac{M - E_{c_a} + e \sin E_{c_a}}{1 - e \cos E_{c_a}} \quad . \tag{4.8.5}$$

An initial estimate E_{c_1} of E_{c_t} may be taken as

$$E_{c_1} = M$$

and further estimates obtained from Equation (4.8.5) using

$$E_{c_n} = E_{c_{n-1}} + \frac{\left(M - E_{c_{n-1}}\right) + e \sin E_{c_{n-1}}}{1 - e \cos E_{c_{n-1}}} \quad .$$

4.9 ECENTR (Gooding, 1968e)

This segment takes the current estimate of the orbital elements, together with the location of an observer and the date and time of an observation, and computes a theoretical observation. The segment also computes partial derivatives of the observed quantities with respect to the orbital parameters.

The following discussion is concerned with the specification of a theoretical observation. That is, all "observed" quantities are calculated from the current estimate of the satellite's position, based on the orbital model.

There is one parameter:

IND input, indicating whether the observer coordinates have already been rotated so that they are relative to Υ (IND = 0), or not (IND = 1). IND is always zero in the program PROP, as the station coordinates are rotated by a previous call to the segment PROCES(4.27).

The use of COMMON is

/ORBIT/

MJD and TIME output, having been set prior to the call of SATXYZ.

X, Y, Z, XDOT, YDOT and ZDOT, output having been set during the call of SATXYZ.

DERIV(6 × 20) output having been set during the call of PARSHL.

COMMON//

JTYPE, JRATES, JDCS, OBSNEX(23) and NUMPAR are input.

COSDEC, CALC(6) and PARDER(6 × 20) are output.

blank COMMON

XX, YY, ZZ, RANGE, RA, SINDEC and DEC are output.

/WORKIN/

The first 223 real locations of /WORKIN/ are used as ROT(3 × 3), TOPDER(6×20), DCROT(2×2), DCDER(4×20), ROTDOT(3×3) and RCSDEC(3). These arrays hold intermediate quantities during the execution of ECENTR. However, they are not used until after the calls of segments SATXYZ and PARSHL, both of which use /WORKIN/ for other purposes. The array RCSDEC(3) is new, and does not appear in the PROP6 version of ECENTR.

On entry to ECENTR, MJD and TIME are copied from OBSNEX(3) and (4) into COMMON/ORBIT/. The segment SATXYZ is then called with parameter zero. This gives the satellites geocentric position and velocity according to the current estimate of the orbit, and including all perturbations, in X, Y, Z, XDOT, YDOT, ZDOT of COMMON/ORBIT/. The segment PARSHL is then called with parameter JRATES. This fills the array DERIV with partial derivatives of X, Y, and Z with respect to the orbital parameters and, if JRATES is non-zero, partial derivatives of XDOT, YDOT and ZDOT with respect to the orbital parameters. Topocentric satellite coordinates and velocities, x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$ may then be found from

$$x = X - XSTA,$$

$$y = Y - YSTA,$$

$$z = Z - ZSTA,$$

where XSTA, YSTA, ZSTA are the observer coordinates in OBSNEX, (XSTA toward ♈), and

$$\dot{x} = XDOT + YSTA \times \omega_E ,$$

$$\dot{y} = YDOT - XSTA \times \omega_E ,$$

$$\dot{z} = ZDOT,$$

where ω_E is the rotation rate of the Earth.

If the observation type, JTYPE, is 16, that is x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$ observed, then all the quantities required are available. The output arrays are set such that

CALC = x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$

and

PARDER(I1,I2) = DERIV(I1,I2); I1 = 1,..,6; I2 = 1,..,NUMPAR.

Otherwise, observed quantities and their derivatives must be calculated from x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$ and DERIV.

The range ρ, right ascension α, and declination δ of the satellite are calculated from

$$\left.\begin{aligned} \rho &= (x^2 + y^2 + z^2)^{\frac{1}{2}} \\ \alpha &= \tan^{-1}(y/x) \\ \delta &= \tan^{-1}(z/(x^2+y^2)^{\frac{1}{2}}) . \end{aligned}\right\} \qquad (4.9.1)$$

The array ROT is then filled, such that

$$\text{ROT} = \begin{bmatrix} \frac{\partial\rho}{\partial x} & \frac{\partial\rho}{\partial y} & \frac{\partial\rho}{\partial z} \\ \cos\delta\frac{\partial\alpha}{\partial x} & \cos\delta\frac{\partial\alpha}{\partial y} & \cos\delta\frac{\partial\alpha}{\partial z} \\ \frac{\partial\delta}{\partial x} & \frac{\partial\delta}{\partial y} & \frac{\partial\delta}{\partial z} \end{bmatrix} \qquad (4.9.2)$$

and the top three rows of TOPDER are filled such that

$$\left.\begin{aligned} &\text{TOPDER}(1,j) = \frac{\partial\rho}{\partial\varepsilon_j} \;,\quad \text{TOPDER}(2,j) = \cos\delta\,\frac{\partial\alpha}{\partial\varepsilon_j} \;, \\ &\text{TOPDER}(3,j) = \frac{\partial\delta}{\partial\varepsilon_j} \;,\quad \text{where } j = 1,..,\text{NUMPAR and} \end{aligned}\right\} \qquad (4.9.3)$$

the ε_j are the orbital parameters.

The term $\frac{\partial\alpha}{\partial z}$ is zero, by Equation (4.9.1), but will be left in the expressions derived for consistency.

If both JRATES and JDCS are zero then enough information is now available to fill the arrays CALC and PARDER with the required quantities. If JRATES is non-zero then rates of change of some quantities were observed. In this case further basic quantities are calculated as

$$\left.\begin{aligned}\dot{\rho} &= \dot{x}\frac{\partial\rho}{\partial x} + \dot{y}\frac{\partial\rho}{\partial y} + \dot{z}\frac{\partial\rho}{\partial z} ,\\ \dot{\alpha}\cos\delta &= \dot{x}\cos\delta\frac{\partial\alpha}{\partial x} + \dot{y}\cos\delta\frac{\partial\alpha}{\partial y} + \dot{z}\cos\delta\frac{\partial\alpha}{\partial z} ,\\ &\text{and}\\ \dot{\delta} &= \dot{x}\frac{\partial\delta}{\partial x} + \dot{y}\frac{\partial\delta}{\partial y} + \dot{z}\frac{\partial\delta}{\partial z} .\end{aligned}\right\} \qquad (4.9.4)$$

The array ROTDOT is then filled such that

$$\text{ROTDOT} = \begin{bmatrix} \frac{d}{dt}\left(\frac{\partial\rho}{\partial x}\right) & \frac{d}{dt}\left(\frac{\partial\rho}{\partial y}\right) & \frac{d}{dt}\left(\frac{\partial\rho}{\partial z}\right) \\ \cos\delta\frac{d}{dt}\left(\cos\delta\frac{\partial\alpha}{\partial x}\right) & \cos\delta\frac{d}{dt}\left(\cos\delta\frac{\partial\alpha}{\partial y}\right) & \cos\delta\frac{d}{dt}\left(\cos\delta\frac{\partial\alpha}{\partial z}\right) \\ \cos\delta\frac{d}{dt}\left(\frac{\partial\delta}{\partial x}\right) & \cos\delta\frac{d}{dt}\left(\frac{\partial\delta}{\partial y}\right) & \cos\delta\frac{d}{dt}\left(\frac{\partial\delta}{\partial z}\right) \end{bmatrix},$$

(4.9.5)

and the bottom three rows of TOPDER are filled such that

$$\left.\begin{aligned} \text{TOPDER}(4,j) &= \frac{\partial\dot{\rho}}{\partial\varepsilon_j} , \quad \text{TOPDER}(5,j) = \cos^2\delta\,\frac{\partial\dot{\alpha}}{\partial\varepsilon_j} ,\\ \text{TOPDER}(6,j) &= \cos\delta\frac{\partial\dot{\delta}}{\partial\varepsilon_j} , \quad j = 1,\_\_\_\_,\text{NUMPAR} .\end{aligned}\right\} \qquad (4.9.6)$$

If JDCS is zero then enough information is available to fill the arrays CALC and PARDER with the required quantities. If, however, JDCS is non-zero, then either direction cosines, or their rates of change, were observed. In either case the direction cosines, ℓ, m, are calculated from

$$\left.\begin{aligned} \ell &= \cos\delta\sin(\alpha-\lambda) \\ m &= -\sin\beta\cos\delta\cos(\alpha-\lambda) + \cos\beta\sin\delta \end{aligned}\right\} \qquad (4.9.7)$$

where λ is the longitude of the observer (relative to ♈) and β the latitude, obtained from the array OBSNEX.

The array DCROT is then filled, such that

$$\text{DCROT} = \begin{bmatrix} \dfrac{1}{\cos\delta} \dfrac{\partial \ell}{\partial \alpha} & \dfrac{\partial \ell}{\partial \delta} \\ & \\ \dfrac{1}{\cos\delta} \dfrac{\partial m}{\partial \alpha} & \dfrac{\partial m}{\partial \delta} \end{bmatrix}, \tag{4.9.8}$$

and the top half of DCDER is filled such that

$$\text{DCDER}(1,j) = \frac{\partial \ell}{\partial \varepsilon_j}, \quad \text{DCDER}(2,J) = \frac{\partial m}{\partial \varepsilon_j}, \quad j = 1, \_\_\_\_, \text{NUMPAR}. \tag{4.9.9}$$

If the direction cosines were observed but not their rates of change, this is indicated by JDCS being non-zero and JRATES being zero. In this case enough information is available to fill the arrays CALC and PARDER with the required quantities. If, however, JRATES is also non-zero, then the rates of change of ℓ and m, $\dot{\ell}$ and $\dot{m}$, are derived as

$$\left.\begin{aligned} \dot{\ell} &= -\dot{\delta}\sin\delta\sin(\alpha-\lambda) + (\dot{\alpha}-\omega_E)\cos\delta\cos(\alpha-\lambda), \\ \dot{m} &= (\dot{\alpha}-\omega_E)(\sin\beta\cos\delta\sin(\alpha-\lambda)) + \dot{\delta}(\sin\beta\sin\delta\cos(\alpha-\lambda) \\ &\qquad + \cos\beta\cos\delta), \end{aligned}\right\} \tag{4.9.10}$$

where ω_E is the Earth's rotation rate, and then the bottom half of DCDER is filled such that

$$\text{DCDER}(3,j) = \frac{\partial \dot{\ell}}{\partial \varepsilon_j}, \quad \text{DCDER}(4,j) = \frac{\partial \dot{m}}{\partial \varepsilon_j}, \quad j = 1, \_\_\_\_, \text{NUMPAR}. \tag{4.9.11}$$

When all the required observed quantities, and their partial derivatives, have been calculated, JTYPE is examined to ascertain which quantities are to be stored in CALC and PARDER. These quantities, shown in Table 4.9.1, are copied from the calculated values of the observed quantities, TOPDER and DCDER as appropriate.

Some discrepancies have been found between what should be stored in the arrays and what is actually stored by ECENTR. These discrepancies have been corrected in the new version of ECENTR, and the changes are indicated here. As a preliminary, we note that the Equations (4.9.1) may be rearranged to give

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} \rho \cos\delta \cos\alpha \\ \rho \cos\delta \sin\alpha \\ \rho \sin\delta \end{bmatrix}, \tag{4.9.12}$$

and that, if $\eta = \rho$, α or δ, then

$$\frac{\partial \dot{\eta}}{\partial \dot{x}} = \frac{\partial \eta}{\partial x} \quad \text{and} \quad \frac{d}{dt}\left(\frac{\partial \eta}{\partial x}\right) = \frac{\partial \dot{\eta}}{\partial x} , \tag{4.9.13}$$

and similarly for the derivatives involving y and z.

The first discrepancy arises in the calculation of $\cos^2\delta \frac{\partial \dot{\alpha}}{\partial \varepsilon_j}$, to be put into TOPDER(5,j), j=1,____,NUMPAR. The PROP6 version of ECENTR makes use of the fact that

$$\cos^2\delta \frac{\partial \dot{\alpha}}{\partial \varepsilon_j} = \cos\delta \frac{\partial}{\partial \varepsilon_j} (\dot{\alpha}\cos\delta) + \dot{\alpha} \sin\delta \cos\delta \frac{\partial \delta}{\partial \varepsilon_j} ,$$

but uses, for $\cos\delta \frac{\partial}{\partial \varepsilon_j} (\dot{\alpha} \cos\delta)$, the expression

$$\text{ROTDOT}(2,1) \frac{\partial x}{\partial \varepsilon_j} + \text{ROTDOT}(2,2) \frac{\partial y}{\partial \varepsilon_j} + \text{ROTDOT}(2,3) \frac{\partial z}{\partial \varepsilon_j}$$

$$+ \text{ROT}(2,1) \frac{\partial \dot{x}}{\partial \varepsilon_j} + \text{ROT}(2,2) \frac{\partial \dot{y}}{\partial \varepsilon_j} + \text{ROT}(2,3) \frac{\partial \dot{z}}{\partial \varepsilon_j} . \tag{4.9.14}$$

Table 4.9.1 Contents of the Arrays CALC and PARDER

JTYP	CALC						PARDER					
	(1)	(2)	(3)	(4)	(5)	(6)	(1,j)	(2,j)	(3,j)	(4,j)	(5,j)	(6,j)
1	ρ						$\frac{\partial\rho}{\partial\varepsilon_j}$					
2	α	δ					$\cos\delta\frac{\partial\alpha}{\partial\varepsilon_j}$	$\frac{\partial\delta}{\partial\varepsilon_j}$				
3	ℓ	m					$\frac{\partial\ell}{\partial\varepsilon_j}$	$\frac{\partial m}{\partial\varepsilon_j}$				
4	$\dot{\rho}$						$\frac{\partial\dot{\rho}}{\partial\varepsilon_j}$					
5	$\dot{\alpha}\cos\delta$	$\dot{\delta}$					$\cos^2\delta\frac{\partial\dot{\alpha}}{\partial\varepsilon_j}$	$\cos\delta\frac{\partial\dot{\delta}}{\partial\varepsilon_j}$				
6	$\dot{\ell}$	$\dot{m}$					$\frac{\partial\dot{\ell}}{\partial\varepsilon_j}$	$\frac{\partial\dot{m}}{\partial\varepsilon_j}$				
7	ρ	α	δ				$\frac{\partial\rho}{\partial\varepsilon_j}$	$\cos\delta\frac{\partial\alpha}{\partial\varepsilon_j}$	$\frac{\partial\delta}{\partial\varepsilon_j}$			
8	ρ	ℓ	m				$\frac{\partial\rho}{\partial\varepsilon_j}$	$\frac{\partial\ell}{\partial\varepsilon_j}$	$\frac{\partial m}{\partial\varepsilon_j}$			
9	$\dot{\rho}$	$\dot{\alpha}\cos\delta$	$\dot{\delta}$				$\frac{\partial\dot{\rho}}{\partial\varepsilon_j}$	$\cos^2\delta\frac{\partial\dot{\alpha}}{\partial\varepsilon_j}$	$\cos\delta\frac{\partial\dot{\delta}}{\partial\varepsilon_j}$			

Table 4.9.1 (cont'd) Contents of the Arrays CALC and PARDER

JTYPE	CALC						PARDER					
	(1)	(2)	(3)	(4)	(5)	(6)	(1,j)	(2,j)	(3,j)	(4,j)	(5,j)	(6,j)
10	$\dot{\rho}$	$\dot{\ell}$	$\dot{m}$				$\frac{\partial\dot{\rho}}{\partial\varepsilon_j}$	$\frac{\partial\dot{\ell}}{\partial\varepsilon_j}$	$\frac{\partial\dot{m}}{\partial\varepsilon_j}$			
11	ρ	$\dot{\rho}$					$\frac{\partial\rho}{\partial\varepsilon_j}$	$\frac{\partial\dot{\rho}}{\partial\varepsilon_j}$				
12	α	δ	$\dot{\alpha}\cos\delta$	$\dot{\delta}$			$\cos\delta\,\frac{\partial\alpha}{\partial\varepsilon_j}$	$\frac{\partial\delta}{\partial\varepsilon_j}$	$\cos^2\delta\,\frac{\partial\dot{\alpha}}{\partial\varepsilon_j}$	$\cos\delta\,\frac{\partial\dot{\delta}}{\partial\varepsilon_j}$		
13	ℓ	m	$\dot{\ell}$	$\dot{m}$			$\frac{\partial\ell}{\partial\varepsilon_j}$	$\frac{\partial m}{\partial\varepsilon_j}$	$\frac{\partial\dot{\ell}}{\partial\varepsilon_j}$	$\frac{\partial\dot{m}}{\partial\varepsilon_j}$		
14	ρ	α	δ	$\dot{\rho}$	$\dot{\alpha}\cos\delta$	$\dot{\delta}$	$\frac{\partial\rho}{\partial\varepsilon_j}$	$\cos\delta\,\frac{\partial\alpha}{\partial\varepsilon_j}$	$\frac{\partial\delta}{\partial\varepsilon_j}$	$\frac{\partial\dot{\rho}}{\partial\varepsilon_j}$	$\cos^2\delta\,\frac{\partial\dot{\alpha}}{\partial\varepsilon_j}$	$\cos\delta\,\frac{\partial\dot{\delta}}{\partial\varepsilon_j}$
15	ρ	ℓ	m	$\dot{\rho}$	$\dot{\ell}$	$\dot{m}$	$\frac{\partial\rho}{\partial\varepsilon_j}$	$\frac{\partial\ell}{\partial\varepsilon_j}$	$\frac{\partial m}{\partial\varepsilon_j}$	$\frac{\partial\dot{\rho}}{\partial\varepsilon_j}$	$\frac{\partial\dot{\ell}}{\partial\varepsilon_j}$	$\frac{\partial\dot{m}}{\partial\varepsilon_j}$
16	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	$\frac{\partial x}{\partial\varepsilon_j}$	$\frac{\partial y}{\partial\varepsilon_j}$	$\frac{\partial z}{\partial\varepsilon_j}$	$\frac{\partial\dot{x}}{\partial\varepsilon_j}$	$\frac{\partial\dot{y}}{\partial\varepsilon_j}$	$\frac{\partial\dot{z}}{\partial\varepsilon_j}$

The values of $\partial x/\partial \varepsilon_j$, $\partial \dot{x}/\partial \varepsilon_j$ etc. are taken from the array DERIV.

Now

$$\text{ROT}(2,1) = \cos\delta \frac{\partial \alpha}{\partial x} \;,\quad \text{ROT}(2,2) = \cos\delta \frac{\partial \alpha}{\partial y} \quad \text{and ROT}(2,3) = \cos\delta \frac{\partial \alpha}{\partial z}$$

by Equation (4.9.2), and

$$\text{ROTDOT}(2,1) = \cos\delta \frac{d}{dt}\left(\cos\delta \frac{\partial \alpha}{\partial x}\right) ,$$

$$\text{ROTDOT}(2,2) = \cos\delta \frac{d}{dt}\left(\cos\delta \frac{\partial \alpha}{\partial y}\right) ,$$

$$\text{ROTDOT}(2,3) = \cos\delta \frac{d}{dt}\left(\cos\delta \frac{\partial \alpha}{\partial z}\right) ,$$

by Equation (4.9.5). Hence the Expression (4.9.14) may be written

$$\cos\delta \left\{ \frac{d}{dt}\left(\cos\delta \frac{\partial \alpha}{\partial x}\right)\frac{\partial x}{\partial \varepsilon_j} + \frac{d}{dt}\left(\cos\delta \frac{\partial \alpha}{\partial y}\right)\frac{\partial y}{\partial \varepsilon_j} + \frac{d}{dt}\left(\cos\delta \frac{\partial \alpha}{\partial z}\right)\frac{\partial z}{\partial \varepsilon_j} \right.$$

$$\left. + \frac{\partial \alpha}{\partial x}\frac{\partial \dot{x}}{\partial \varepsilon_j} + \frac{\partial \alpha}{\partial y}\frac{\partial \dot{y}}{\partial \varepsilon_j} + \frac{\partial \alpha}{\partial z}\frac{\partial \dot{z}}{\partial \varepsilon_j} \right\} ,$$

which, using Equation (4.9.13), may be rearranged to give

$$\cos\delta \left\{ \cos\delta \left[\frac{\partial \dot{\alpha}}{\partial x}\frac{\partial x}{\partial \varepsilon_j} + \frac{\partial \dot{\alpha}}{\partial y}\frac{\partial y}{\partial \varepsilon_j} + \frac{\partial \dot{\alpha}}{\partial z}\frac{\partial z}{\partial \varepsilon_j} \right] + \frac{\partial \dot{\alpha}}{\partial \dot{x}}\frac{\partial \dot{x}}{\partial \varepsilon_j} + \frac{\partial \dot{\alpha}}{\partial \dot{y}}\frac{\partial \dot{y}}{\partial \varepsilon_j} + \frac{\partial \dot{\alpha}}{\partial \dot{z}}\frac{\partial \dot{z}}{\partial \varepsilon_j} \right.$$

$$\left. - \dot{\delta}\sin\delta \left[\frac{\partial \alpha}{\partial x}\frac{\partial x}{\partial \varepsilon_j} + \frac{\partial \alpha}{\partial y}\frac{\partial y}{\partial \varepsilon_j} + \frac{\partial \alpha}{\partial z}\frac{\partial z}{\partial \varepsilon_j} \right] \right\}$$

which does not equal

$$\cos^2\delta \frac{\partial \dot{\alpha}}{\partial \varepsilon_j} .$$

Therefore this part of the segment ECENTR has been changed. We write

$$\cos^2\delta \frac{\partial\dot{\alpha}}{\partial\varepsilon_j} = \cos^2\delta\left[\frac{\partial\dot{\alpha}}{\partial x}\frac{\partial x}{\partial\varepsilon_j} + \frac{\partial\dot{\alpha}}{\partial y}\frac{\partial y}{\partial\varepsilon_j} + \frac{\partial\dot{\alpha}}{\partial z}\frac{\partial z}{\partial\varepsilon_j} + \frac{\partial\dot{\alpha}}{\partial\dot{x}}\frac{\partial\dot{x}}{\partial\varepsilon_j} + \frac{\partial\dot{\alpha}}{\partial\dot{y}}\frac{\partial\dot{y}}{\partial\varepsilon_j} + \frac{\partial\dot{\alpha}}{\partial\dot{z}}\frac{\partial\dot{z}}{\partial\varepsilon_j}\right]$$

which, by Equation (4.9.13), becomes

$$\cos^2\delta \frac{\partial\dot{\alpha}}{\partial\varepsilon_j} = \cos^2\delta\left[\frac{d}{dt}\left(\frac{\partial\alpha}{\partial x}\right)\frac{\partial x}{\partial\varepsilon_j} + \frac{d}{dt}\left(\frac{\partial\alpha}{\partial y}\right)\frac{\partial y}{\partial\varepsilon_j} + \frac{d}{dt}\left(\frac{\partial\alpha}{\partial z}\right)\frac{\partial z}{\partial\varepsilon_j}\right]$$

$$+ \cos\delta\left[\cos\delta\frac{\partial\alpha}{\partial x}\frac{\partial\dot{x}}{\partial\varepsilon_j} + \cos\delta\frac{\partial\alpha}{\partial y}\frac{\partial\dot{y}}{\partial\varepsilon_j} + \cos\delta\frac{\partial\alpha}{\partial z}\frac{\partial\dot{z}}{\partial\varepsilon_j}\right].$$

(4.9.15)

However

$$\cos^2\delta\,\frac{d}{dt}\left(\frac{\partial\alpha}{\partial x}\right) = \cos\delta\,\frac{d}{dt}\left(\cos\delta\,\frac{\partial\alpha}{\partial x}\right) + \dot{\delta}\,\frac{\partial\alpha}{\partial x}\cos\delta\,\sin\delta$$

$$= \text{ROTDOT}(2,1) + \dot{\delta}\,\sin\delta\,\text{ROT}(2,1)$$

and similarly for the derivatives involving y and z.

Hence

$$\cos^2\delta \frac{\partial\dot{\alpha}}{\partial\varepsilon_j} = \text{ROTDOT}(2,1)\frac{\partial x}{\partial\varepsilon_j} + \text{ROTDOT}(2,2)\frac{\partial y}{\partial\varepsilon_j} + \text{ROTDOT}(2,3)\frac{\partial z}{\partial\varepsilon_j}$$

$$+ \cos\delta\left[\text{ROT}(2,1)\frac{\partial\dot{x}}{\partial\varepsilon_j} + \text{ROT}(2,2)\frac{\partial\dot{y}}{\partial\varepsilon_j} + \text{ROT}(2,3)\frac{\partial\dot{z}}{\partial\varepsilon_j}\right]$$

$$+ \dot{\delta}\,\sin\delta\,\text{TOPDER}(2,j)\,. \qquad (4.9.16)$$

$\cos^2\delta \frac{\partial\dot{\alpha}}{\partial\varepsilon_j}$, j=1,..,NUMPAR are computed from the expression given in Equation (4.9.16).

The second discrepancy arises in the calculation of $\cos\delta \frac{\partial\dot{\delta}}{\partial\varepsilon_j}$, to be put in to TOPDER(6,j), j=1,..,NUMPAR. The PROP6 version of ECENTR sets

$$\mathrm{TOPDER}(6,j) = \mathrm{ROTDOT}(3,1)\frac{\partial x}{\partial\varepsilon_j} + \mathrm{ROTDOT}(3,2)\frac{\partial y}{\partial\varepsilon_j} + \mathrm{ROTDOT}(3,3)\frac{\partial z}{\partial\varepsilon_j}$$

$$+ \mathrm{ROT}(3,1)\frac{\partial\dot{x}}{\partial\varepsilon_j} + \mathrm{ROT}(3,2)\frac{\partial\dot{y}}{\partial\varepsilon_j} + \mathrm{ROT}(3,3)\frac{\partial\dot{z}}{\partial\varepsilon_j} ,$$

which, by Equations (4.9.2), (4.9.5) and (4.9.14), is equivalent to

$$\mathrm{TOPDER}(6,j) = \cos\delta\left[\frac{\partial\dot{\delta}}{\partial x}\frac{\partial x}{\partial\varepsilon_j} + \frac{\partial\dot{\delta}}{\partial y}\frac{\partial y}{\partial\varepsilon_j} + \frac{\partial\dot{\delta}}{\partial z}\frac{\partial z}{\partial\varepsilon_j}\right]$$

$$+ \left[\frac{\partial\dot{\delta}}{\partial\dot{x}}\frac{\partial\dot{x}}{\partial\varepsilon_j} + \frac{\partial\dot{\delta}}{\partial\dot{y}}\frac{\partial\dot{y}}{\partial\varepsilon_j} + \frac{\partial\dot{\delta}}{\partial\dot{z}}\frac{\partial\dot{z}}{\partial\varepsilon_j}\right]$$

$$\neq \cos\delta\frac{\partial\dot{\delta}}{\partial\varepsilon_j} .$$

The new version of ECENTR contains a modification which corrects this discrepancy.

The changes to TOPDER(5,j) and (6,j) were made by introducing the array RCSDEC, such that

RCSDEC(1) = 1, RCSDEC(2) = RCSDEC(3) = $\cos\delta$

and changing one other line of code such that $\dot{\delta}\sin\delta$ TOPDER(2,j) is added to TOPDER(5,j), rather than $\dot{\alpha}\sin\delta\cos\delta$ TOPDER(3,j) .

One further change was made to ECENTR. In the calculation of $\dot{\ell}$ and $\dot{m}$ the rotation rate of the Earth was represented by the variable EARTHW. This variable was undefined, and has been replaced by EOMEGA which is set, in the DATA statement, to the rotation rate of the Earth.

These changes may not be fully tested since observations of $\dot{\alpha}$, $\dot{\delta}$, $\dot{\ell}$ or $\dot{m}$ are not yet available for use in the program PROP.

4.10 ELPRIN

This segment outputs a standard array of elements to either or both of two output channels.

There are nine parameters:

NOMIAL: input 6 element array giving the degrees of the polynomials held in the array ELEMT,

ELEMT: input (6 × 6) array containing the coefficients of the polynomials representing the six orbital elements,

EMU: input Earth's gravitational constant, μ,

ERAD: input Earth's mean equatorial radius, R km,

EJ2: input second zonal harmonic of the Earth's gravitational field, J_2,

MJDOCH: input date of the elements to be output, in Modified Julian Date form (see subsection 4.18),

JPUNCH: input control parameter,

IP: input channel number for line printer output,

IPUNCH: input channel number for card punch output.

The only output from ELPRIN is to either, or both, of the output channels IP and IPUNCH. There are no output parameters and COMMON is not used. The direction of the output is governed by the parameter JPUNCH, as shown in Table 4.10.1.

Table 4.10.1 Direction of Output from ELPRIN

JPUNCH	Output Channel(s)
< 0	IPUNCH
$= 0$	IP and IPUNCH
> 0	IP

The output to channel IP is in a form suitable for printing on a line-printer and the output to channel IPUNCH is suitable for cardpunch output. The condition

JPUNCH = 0

is never used within the program PROP.

The coefficients, held in ELEMT, are converted from units of radians and seconds to degrees and days before being output.

When the output is being directed to channel IP (the line printer output), the coefficients are preceded by a line stating that the "derived semi-major axis" is a where a is calculated from

$$a = \left(\frac{\mu}{n_0^2}\right)^{1/3} - \frac{J_2}{2} R^2 \left(\frac{\mu}{n_0^2}\right)^{-1/3} \left(1 - \frac{3}{2} \sin^2 i_0\right) \left(1 - e_0^2\right)^{-3/2} \quad .$$

Here

$$\left.\begin{aligned} e_0 &= \mathrm{ELEMT}(1,1) \\ i_0 &= \mathrm{ELEMT}(2,1) \\ n_0 &= \mathrm{ELEMT}(6,1) \end{aligned}\right\} \quad \text{mean elements at the epoch.}$$

J_2 is the Earth's second zonal harmonic and R is the mean equatorial radius of the Earth, as given by Gooding and Tayler (1968).

A change has been introduced into the new version of the program PROP such that each line of coefficients output to channel IP is preceded by some identification of the element for which coefficients are being output, thus

ECCENTRICITY

INCLINATION

R.A. OF NODE

ARG OF PERIGEE

MEAN ANOMALY

The letters are placed where, formerly, spaces were output.

When the output is being directed to channel IPUNCH (the card punch output), the coefficients on each card are preceded by two numbers, separated by a solidus, /. The two numbers are MJDOCH and the card number (1, 2, 3, 4 or 5).

In both cases only the first five sets of coefficients held in ELEMT are output. The contents of ELEMT(6,j) are only used in the calculation of the semi-major axis, a.

The call of ELPRIN by segment PROP(4.28) is made to print original elements on the line printer if they are being used as a basis for the prediction of initial elements at some other epoch.

There are three calls of ELPRIN by the segment DIFCOR(4.7). The first of these is to print the initial elements on the main line printer output. The second is with IP set to ITCH to print the elements at the end of each iteration, on the line printer. The value of ITCH is set before calling ELPRIN, by testing whether the final iteration has just been performed. If the elements are from the final iteration, ITCH is set so that they are printed on the main line printer output. Otherwise they are printed on the "intermediate" line printer output. This redirecting of intermediate output is new to this version of PROP and is part of a time and paper saving facility, see subsection 4.7.6.

The third call of ELPRIN by DIFCOR is to punch final elements on to the card punch output.

4.11 ELREAD (Gooding, 1968a)

This segment reads orbital elements from the input channel, scales them appropriately, and stores them in an array.

There are four parameters:

ELEMT: output (6 × 6) array of coefficients of the polynomials representing the orbital elements,

MJDOCH: input Modified Julian Date (see subsection 4.18) for which elements are required,

IR: input channel number from which elements are to be read,

LOGCHK: output logical indicator of whether elements read are acceptable.

The only call of ELREAD in the program PROP occurs if the control parameter NEWSAT is 0 or 1, in which case initial elements are read from the card reader into the array ELEMT. The format of the cards has been described by Gooding and Tayler (1968). The contents of ELEMT are read in units of degrees and days but are converted, by ELREAD, to units of radians and seconds as appropriate. The contents of ELEMT(6,j) are derived from ELEMT(5,j+1), j=1,..,5.

The check within ELREAD that all elements are for a date of MJDOCH and that the cards are in the correct order is optional, and it is bypassed if the date and card number on the cards are zero. If the test is not bypassed and fails, the parameter LOGCHK is set FALSE, otherwise LOGCHK is returned as TRUE.

4.12 EPARAS (Gooding, 1968b)

This segment copies orbital parameters from a two-dimensional array to a one-dimensional array.

There are four parameters:

ELEMT: input (6×6) array holding the orbital parameters,

MODEL: input 5 element array holding the number of orbital parameters to be copied from each of the first five rows of ELEMT,

PARAS: output 20 element array to hold the orbital parameters,

NUMPAR: output number of orbital parameters copied.

The segment PROP(4.28) only calls EPARAS if "previous orbit" cards are read. It causes the initial elements to be copied from ELEMT to OPARAS. The call of EPARAS by DIFCOR(4.7) is made at the start of each iteration to copy orbital parameters from ELEMT to PARAS ready for differential correction.

4.13 ERROR

This semi-compiled segment has been removed from the new version of PROP. Its function was to enable program execution to continue after an execution error. This was introduced originally so that runs in a multi-run job were not jeopardised by an execution error occurring in a previous, independent run. The segment is only available in the semi-compiled version that is not compatible with the Fortran compiler available at Aston. Since it is not customary to run jobs of independent multi-runs, no penalty is suffered by its removal.

4.14 GEOCOR (Tayler, Gooding, 1969a)

This segment converts the geocentric Cartesian coordinates of a sensor (satellite observer) to a geographic latitude and longitude of the sensor and the height of the sensor above a reference spheroid.

There are eight parameters:

MJD: input modified Julian date (see subsection 4.18), or zero,

TIME: input time as a decimal of a day relative to MJD, or zero,

X, Y, Z: input geocentric Cartesian coordinates of the sensor, x, y, z, in kilometres,

SLAT: output latitude of the sensor, β, in radians,

SLONG: output longitude of the sensor, in radians,

HEIGHT: output height of the sensor above the reference spheroid, ht, in kilometres.

The only call of GEOCOR within the program PROP is from SENSIN(4.36), the segment which controls the reading of sensor cards. Sensor cards are often referred to as "observer cards". The call is made with MJD and TIME both set to zero and the function SIDANG(4.38) is not used. Therefore, although the subroutine was written to deal with both sensor and satellite positions, we will take x, y, z to be the position of an observer relative to the centre of the Earth. x is measured toward the Greenwich meridian, in the equatorial plane, y is perpendicular to x and also in the equatorial plane and z is such that x, y, z form a right-handed Cartesian system.

It is assumed that the shape of the Earth is as given by "Assumption C" of Fischer (1960). In this model the Earth's surface is assumed to be the shape that would be generated by rotating an ellipse about the minor axis. The Fischer spheroid used within PROP was derived from gravimetric geoid heights and an imposed flattening of 1/298.3.

The resulting value of the equatorial radius is 6378.166 km and the polar radius is given by

6378.166 - 6378.166/298.3 ≃ 6356.784 km.

The point x,y,z may be above or below the surface of the spheroid. The height returned by GEOCOR is the perpendicular distance between the spheroid and the point x,y,z.

The subordinate segment TRINV(4.40) is called to calculate the longitude given by

$$\text{SLONG} = \tan^{-1}(y/x)$$

and a distance w given by

$$w^2 = x^2 + y^2.$$

β and ht are then found by considering coordinates in the meridional plane, that is, a vertical plane through the line of longitude and the Earth's centre.

Consider a sensor S, outside the Fischer spheroid, and consider the angle β in terms of the eccentric angle E (Figure 4.14.1). The curve P_1P_2 is a section of an ellipse of semi-major axis A, semi-minor axis B where

A = 6378.166 km and B = 6356.784 km.

The sensor S has coordinates (w,z) and the sub-sensor point P has coordinates (w_0,z_0). However, P is on an ellipse so we may write

$$\frac{w_0^2}{A^2} + \frac{z_0^2}{B^2} = 1$$

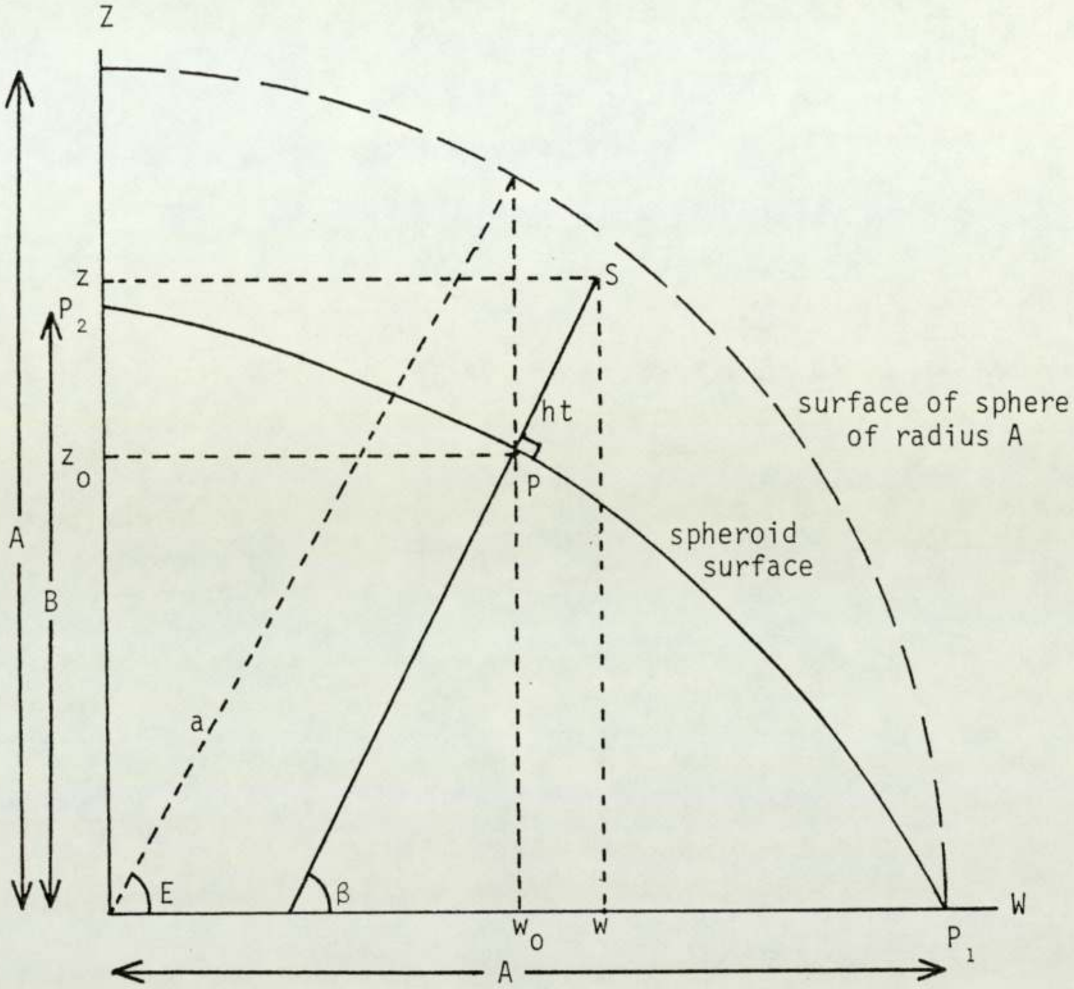
which may be rearranged to give

$$z_0 = B\left(1 - \frac{w_0^2}{A^2}\right)^{\frac{1}{2}} .$$

Differentiating this we obtain

$$\frac{dz_0}{dw_0} = -\frac{B^2 w_0}{A^2 z_0} .$$

Figure 4.14.1 A Sensor Location Relative to the Fischer Spheroid



The slope of the straight line through (w_0,z_0), perpendicular to the ellipse is given by

$$\tan \beta = -\frac{dw_0}{dz_0} = \frac{A^2z_0}{B^2w_0} \qquad (4.14.1)$$

and hence the equation of the line may be written

$$Z = \frac{A^2z_0}{B^2w_0} W + K$$

for some constant K. Since the line goes through the point (w_0,z_0) the above expression gives

$$K = z_o\left(1 - \frac{A^2}{B^2}\right)$$

and hence the equation of the line is given by

$$Z = \frac{A^2 z_o}{B^2 w_o} W + z_o\left(1 - \frac{A^2}{B^2}\right). \tag{4.14.2}$$

Now

$$w_o = A\cos E \text{ and } z_o = B\sin E$$

so Equation (4.14.2) may be written

$$Z = \frac{A\sin E}{B\cos E} W + B\sin E(1 - A^2/B^2).$$

This may be rearranged to give

$$BZ\cos E - AW\sin E = \sin E\cos E\,(B^2 - A^2).$$

We now note that the line also goes through S at (w,z) so, if we write

$$\Theta = \cos E, \quad \Phi = \sin E$$

we obtain two equations in Θ and Φ; viz,

$$\Theta^2 + \Phi^2 = 1 \tag{4.14.3}$$

and

$$Bz\Theta - Aw\Phi = \Theta\Phi(B^2 - A^2) \tag{4.14.4}$$

where A, B, z and w are all known. The two simultaneous equations (4.14.3) and (4.14.4) for Θ and Φ are solved iteratively in the subroutine GEOCOR using the method given by Tayler and Gooding (1969a). The development of the method is indicated by Tayler (1962) and is given here.

To obtain a first approximation, Θ_1, Φ_1, of the solution to Equations (4.14.3) and (4.14.4), we assume that

$$A^2 = B^2$$

and hence Equation (4.14.4) becomes

$$Bz\Theta_1 = Aw\Phi_1 \ . \qquad (4.14.5)$$

This gives

$$\Theta_1 = \frac{Aw}{Bz}\Phi_1 \ , \qquad \Phi_1 = \frac{Bz}{Aw}\Theta_1 \ .$$

Substituting these values for Θ_1, Φ_1 into Equation (4.14.3) we obtain

$$\Theta_1^2 = 1 - \Phi_1^2 = 1 - \left(\frac{Bz}{Aw}\right)^2 \Theta_1^2$$

$$\Phi_1^2 = 1 - \Theta_1^2 = 1 - \left(\frac{Aw}{Bz}\right)^2 \Phi_1^2$$

and hence

$$\Theta_1^2 = \frac{1}{1 + \left(\frac{Bz}{Aw}\right)^2} = \frac{(Aw)^2}{(Aw)^2 + (Bz)^2} \ ,$$

$$\Phi_1^2 = \frac{(Bz)^2}{(Aw)^2 + (Bz)^2} \ .$$

We take

$$\Theta_1 = \frac{Aw}{d} \ , \quad \Phi_1 = \frac{Bz}{d} \ ,$$

where, from geometrical considerations, d is given by (4.14.6)

$$d = + \left[(Aw)^2 + (Bz)^2\right]^{\frac{1}{2}} \ .$$

Let an improved approximation of Θ, Φ be given by

$$\Theta_2 = \Theta_1 + \delta\Theta_1 \ , \quad \Phi_2 = \Phi_1 + \delta\Phi_1 \ .$$

Substituting Θ_2, Φ_2 into Equations (4.14.3) and (4.14.4) we obtain

$$\left.\begin{aligned} &(\Theta_1 + \delta\Theta_1)^2 + (\Phi_1 + \delta\Phi_1)^2 = 1 \\ &Bz(\Theta_1 + \delta\Theta_1) - Aw(\Phi_1 + \delta\Phi_1) = (B^2 - A^2)\Theta_1\Phi_1 \ . \end{aligned}\right\} \qquad (4.14.7)$$

However, Equation (4.14.3) holds for Θ_1, Φ_1. Using Equations (4.14.5) and Equations (4.14.7) we have

$$\Theta_1 \, \delta\Theta_1 + \Phi_1 \, \delta\Phi_1 = -\frac{1}{2}(\delta\,\Theta_1^2 + \delta\,\Phi_1^2) \tag{4.14.8}$$

and

$$Bz\delta\Theta_1 - Aw\delta\Phi_1 = (B^2 - A^2)\Theta_1\Phi_1. \tag{4.14.9}$$

Multiplying Equation (4.14.8) by Aw and Equation (4.14.9) by Φ_1, and adding the resulting two equations we obtain

$$(Aw\Theta_1 + Bz\Phi_1)\delta\Theta_1 = \Phi_1(B^2 - A^2)\Theta_1\Phi_1 - \frac{Aw}{2}(\delta\Theta_1^2 + \delta\Phi_1^2) . \tag{4.14.10}$$

Similarly, multiplying Equation (4.14.8) by Bz and Equation (4.14.9) by Θ_1, and subtracting one result from the other, we have

$$(Aw\Theta_1 + Bz\Phi_1)\delta\Phi_1 = -\Theta_1(B^2 - A^2)\Theta_1\Phi_1 - \frac{Bz}{2}(\delta\Theta_1^2 + \delta\Phi_1^2). \tag{4.14.11}$$

By the use of Equations (4.14.6), Equations (4.14.10) and (4.14.11) reduce to

$$\left.\begin{aligned} \delta\Theta_1 &= C_1\Theta_1\Phi_1 - \tfrac{1}{2}\Theta_1(\delta\Theta_1^2 + \delta\Phi_1^2) \\ \delta\Phi_1 &= -C_2\Theta_1\Phi_1 - \tfrac{1}{2}\Phi_1(\delta\Theta_1^2 + \delta\Phi_1^2), \end{aligned}\right\} \tag{4.14.12}$$

where

$$C_1 = \frac{\Phi_1(B^2 - A^2)}{d} \quad , \qquad C_2 = \frac{\Theta_1(B^2 - A^2)}{d} \quad .$$

Further approximations of Θ, Φ are then obtained from

$$\Theta_{j+1} = \Theta_1 + \delta\Theta_j$$

$$\Phi_{j+1} = \Phi_1 + \delta\Phi_j$$

where

$$\delta\Theta_j = C_1\Theta_j\Phi_j - \frac{1}{2}\Theta_1\left[\delta\Theta_{j-1}^{\;2} + \delta\Phi_{j-1}^{\;2}\right],$$

$$\delta\Phi_j = -C_2\Theta_j\Phi_j - \frac{1}{2}\Phi_1\left[\delta\Theta_{j-1}^{\;2} + \delta\Phi_{j-1}^{\;2}\right].$$

The process is deemed to have converged when

$$|\delta\Theta_j - \delta\Theta_{j-1}| + |\delta\Phi_j - \delta\Phi_{j-1}| \leq 10^{-9} .$$

After obtaining approximate solutions Θ, Φ for $\cos E$ and $\sin E$ the observer latitude, β, is obtained from

$$\tan \beta = \frac{A\Phi}{B\Theta}$$

and the height of the observer above the geoid is given by

$$ht = (w - A\Theta) \cos \beta + (z - B\Phi) \sin \beta.$$

4.15 GEOCRN

This segment converts the geocentric Cartesian coordinates of a sensor to a geographic latitude and longitude of the sensor, and the height of the sensor above a reference spheroid.

There are six parameters:

X, Y, Z: input geocentric Cartesian coordinates of the sensor, x, y, z, in kilometres,

SLAT: output latitude of the sensor, β, in radians,

SLONG: output longitude of the sensor, in radians,

HEIGHT: output height of the sensor above the reference spheroid, ht, in kilometres.

GEOCRN is a new segment written for the new version of PROP, to be used as an alternative to GEOCOR(4.14). Control over which segment is used is left in the hands of the user, as described in subsection 4.36, SENSIN. Whenever GEOCOR is called within PROP the first two parameters, MJD and TIME, are always zero. Therefore it was possible to write GEOCRN without these two parameters, and without using the segment SIDANG(4.38).

The method used in GEOCOR to obtain the output parameters appears to be a method unique to that problem. It was not a familiar technique. Tayler (1962) mentions that an alternative method, based on polar coordinates, was examined but discarded as it used "considerably more computer time." However, another technique, based upon the familiar Newton's method of finding the roots of a polynomial has been examined and employed in this new segment GEOCRN.

The starting point of the technique employed in GEOCRN is Equation (4.14.2). Substitution of the coordinates of the sensor, (w,z), into this equation gives

$$z_o = \frac{zB^2w_o}{A^2w + B^2w_o - A^2w_o} \quad . \tag{4.15.1}$$

However, the sub-sensor point (w_o, z_o) is on an ellipse, so that

$$\frac{w_o^2}{A^2} + \frac{z_o^2}{B^2} = 1 \ .$$

Substituting for z_o from Equation (4.15.1) we obtain

$$\frac{w_o^2}{A^2} + \frac{(zB^2w_o)^2}{B^2(A^2w + B^2w_o - A^2w_o)^2} = 1 \tag{4.15.2}$$

which may be rearranged to give a quartic in w_o, viz.,

$$w_o^4(A^2 - B^2)^2 - 2A^2w(A^2 - B^2)w_o^3 + A^2(A^2w^2 + B^2z^2 - (A^2 - B^2)^2)w_o^2$$

$$+ 2A^4w(A^2 - B^2)w_o - A^6w^2 = 0 \tag{4.15.3}$$

As in the segment GEOCOR, a Fischer spheroid shape is assumed for the Earth's surface, so, to obtain a first approximation to w_o, we may say that

$$A^2 = B^2.$$

Consequently, the quartic Equation (4.15.3) reduces to the quadratic equation

$$(A^2w^2 + A^2z^2)w_o^2 - A^4w^2 = 0$$

giving

$$w_o = \pm \frac{wA}{(w^2 + z^2)^{\frac{1}{2}}} \quad .$$

We require the root w_o nearest to w, so we choose the positive sign thus

$$\frac{w_o}{w} = + \frac{A}{(w^2 + z^2)^{\frac{1}{2}}} \quad .$$

If we express Equation (4.15.3) in terms of $\Theta = w_0/w$ we obtain

$$(A^2 - B^2)^2\Theta^4 - 2A^2(A^2 - B^2)\Theta^3 + \frac{A_2}{w^2}(A^2w^2 + B^2z^2 - (A^2 - B^2)^2)\Theta^2$$
$$+ \frac{2A^4}{w^2}(A^2 - B^2)\Theta - \frac{A^6}{w^2} = 0 = f(\Theta). \qquad (4.15.4)$$

Equation (4.15.4) may be solved for Θ by Newton's method, i.e. by putting

$$\Theta_{j+1} = \Theta_j - \frac{f(\Theta_j)}{f'(\Theta_j)}$$

where primes indicate differentiation with respect to Θ and Θ_j, Θ_{j+1} are successive approximations to Θ. We use

$$\Theta_0 = \frac{A}{+(w^2 + z^2)^{\frac{1}{2}}}$$

as our initial estimate of w_0/w.

The process is deemed to have converged when

$$\left| \frac{f(\Theta_j)}{f'(\Theta_j)} \right| \leqslant 10^{-9} .$$

From the value of Θ found, w_0 may be determined from

$$w_0 = w\Theta.$$

Finally z_0 may be obtained from

$$\frac{w_0^2}{A^2} + \frac{z_0^2}{B^2} = 1 ,$$

care being taken that z_0 has the same sign as z.

If w is small, less than 50 km, then w_0/w may become indeterminate. In this case we consider the similar quartic in $z_0/z = \Phi$, viz.,

$$(A^2 - B^2)^2\Phi^4 + 2B^2(A^2 - B^2)\Phi^3 + \frac{B^2}{z^2}(A^2w^2 + B^2z^2 - (A^2 - B^2)^2)\Phi^2$$
$$- 2\frac{B^4}{z^2}(A^2 - B^2)\Phi - \frac{B^6}{z^2} = 0 = g(\Phi).$$

A first approximation to the required root of this quartic is

$$\Phi_0 = \frac{B}{+(w^2 + z^2)^{\frac{1}{2}}}$$

and further approximations to the root are given by

$$\Phi_{j+1} = \Phi_j - \frac{g(\Phi_j)}{g'(\Phi_j)} .$$

In this case primes indicate differentiation with respect to Φ. The process is deemed to have converged when

$$\left| \frac{g(\Phi_j)}{g'(\Phi_j)} \right| \leqslant 10^{-9} .$$

w_0 is obtained from

$$\frac{w_0^2}{A^2} + \frac{z_0^2}{B^2} = 1 ,$$

care being taken that w_0 has the same sign as w.

Having obtained values of w_0 and z_0, the latitude, β, is given by

$$\tan \beta = \frac{A^2 z_0}{B^2 w_0}$$

and the height, ht, by

$$ht = (w - w_0) \cos \beta + (z - z_0) \sin \beta.$$

Tests have been carried out to compare the results of GEOCOR and GEOCRN. No significant difference was found in the values stored in the array of station coordinates by SENSIN, whether GEOCOR or GEOCRN was used. Neither was there any significant difference in execution time between the two segments.

The conditions of using Newton's method, that the first estimate is reasonably good and that the function is continuous, are met by the functions under consideration in GEOCRN.

4.16 MA01A (York, Hopper, 1968)

This segment either solves a set of algebraic linear simultaneous equations, or inverts a matrix, or does both of these operations, depending upon the values of the input parameters.

There are seven parameters:

A: input and output, square, M × M matrix;

B: input and output, M × N matrix;

M: input dimension of A and B,

N: input second dimension of B, N is also used as a control parameter,

M1: input indicator of operation required,

IA: input maximum value of M,

IB: input maximum value of N.

All calls of MA01A in the program PROP are with IA = 20, IB = 1 and M = NUMPAR (the number of orbital parameters to be refined).

If M1 = 0 and N = 1 then the set of linear equations represented by $\underset{\sim}{Ax} = \underset{\sim}{B}$ are solved for $\underset{\sim}{x}$. (N = 1 implies that the matrix B may be regarded as a vector $\underset{\sim}{B}$). The solution ($\underset{\sim}{x}$) of the equation is placed in $\underset{\sim}{B}$ for output. The content of A on output is a partially inverted form of the input matrix A.

If M1 > 0 and N = 1 then the same action is taken as if M1 = 0, N = 1 and the inversion of A is completed.

If M1 < 0 then it is assumed that a previous call of MA01A has been made with M1 = 0, N = 1. The completion of the inversion of the matrix A is performed. In this case $\underset{\sim}{B}$ is unaltered by MA01A.

If M1 > 0 and N ⩽ 0 then the matrix A is inverted while $\underset{\sim}{B}$ is unchanged.

The call of MA01A made by segment PROP(4.28) is with parameters:

OEMTM,OPARAS,NUMPAR,0,1,20,1.

In this case OPARAS is ignored and the NUMPAR × NUMPAR matrix OEMTM is inverted. This call is only made if the control parameter NOTHER is non-zero. In this case there has been a previous run at the epoch of interest. The covariance matrix from that run has been read into OEMTM and then the contents of OEMTM divided by OEPSLN. OEPSLON is the final value of the convergence parameter $\mathcal{E}$ of the previous run.

The first call of MA01A by the segment DIFCOR(4.7) is with parameters

BOTTOM,TOP,NUMPAR,1,JELT,20,1.

JELT is obtained from the control parameters JELTYP as

JELT = JELTYP/100,

the calculation taking place in integer arithmetic. Therefore JELT is usually zero. BOTTOM is the NUMPAR × NUMPAR matrix of the top layer of the NUMPAR × NUMPAR × 3 array EMTM and TOP is a NUMPAR × 1 vector from the NUMPAR × 3 matrix EMTY. As the call of MA01A is immediately preceded by a call of the subroutine REJECT(4.31), BOTTOM and TOP hold information relevant to all accepted observed quantities. Thus MA01A returns a partially inverted matrix in BOTTOM and the solution, $\underset{\sim}{x}$, to the equations represented by BOTTOM $\underset{\sim}{x}$ = $\underset{\sim}{\text{TOP}}$ in $\underset{\sim}{\text{TOP}}$.

If JELT is strictly positive then the matrix BOTTOM will have been completely inverted.

The final call of MA01A is made by DIFCOR with parameters

BOTTOM,TOP,NUMPAR,1,-1,20,1.

This call is made when the differential correction of orbital parameters has been completed and a completely inverted BOTTOM is required. The call is only made if JELT is zero as, if JELT is non-zero, BOTTOM will have been inverted already.

A change has been made to MA01A to recalculate one of the internal variables, MM = M - 1, at one point in the subroutine. The change was made for technical reasons.

In order that MA01A could be placed in an overlay unit rather than in the "permanent" area of core the EXTERNAL statement referring to MA01A has been removed from the segment PROP and the calls of DIFCOR and MA01A have been adjusted accordingly.

4.17 MILTIM

This semi-compiled segment has been removed from the program PROP. Its function was to ascertain how much mill-time had been used by the program. It was introduced during development work on PROP and is only available in a semi-compiled version which is not compatible with the ICL Fortran compiler available at Aston. MILTIM is called by PROP6 at various stages during the program run if the "thousands" digit of the control parameter JELTYP is non-zero. The amount of mill-time used is then printed on the line printer. No penalty will be suffered by PROP users due to the removal of this facility.

4.18 MJDATE

This segment converts a date input in one format into another format.

There are six parameters:

IND: input indicator of the conversion required,

MJD, IDATE, IYY, IMM, IDD: expressions of the date in various formats. Each may be input or output, depending upon the value of IND.

The format conversion made by the segment MJDATE is controlled by the value of the input parameter IND, as shown in Table 4.18.1.

Table 4.18.1 Conversion of Date Formats by MJDATE

IND	input	output
1	MJD	IYY,IMM,IDD,IDATE
2	IDATE	MJD
3	IYY,IMM,IDD	MJD
4	IDATE	IYY,IMM,IDD

MJD is the Modified Julian Date, which is defined relative to the Julian day number, JD, by

$$MJD = JD - 2400000.5.$$

The origin of JD is defined to be Greenwich mean noon of 1 January 4713 BC and the day number increases by one each day. In this way dates in the twentieth and twenty-first centuries AD have JD's greater than 2,400,000. The MJD, used in satellite work, is therefore more manageable and the day starts, more conveniently, at midnight rather than noon.

For example, 18 November 1960 started at a Modified Julian Date of 37256.0. A table of MJD's is given by Tayler (1967).

IDATE is the civil date in the format yymmdd, where

dd is the day number in the month,

mm is the month number in the year,

yy is the year number in the twentieth century.

For example, for the date 18 November 1960, IDATE is 601118.

IYY, IMM and IDD are the three parts of the civil date yy, mm and dd respectively, so for the date 18 November 1960, IYY = 60, IMM = 11 and IDD = 18.

The subroutine MJDATE only works for dates in the twentieth century. The program PROP never calls MJDATE with IND = 4.

4.19 MOVELS

This segment copies the contents of a 6×6 real array into another 6×6 array.

There are two parameters:

SOURCE: input 6×6 array,

DESTIN: output 6×6 array.

This subroutine is a simple, new subroutine for copying the entire contents of SOURCE into DESTIN. It was written so that the initial element coefficients of each PROP run could be stored in an array STELS immediately before being differentially corrected. If the next run in the same PROP job then has a control parameter NEWSAT = 8 then the initial element coefficients for the PROP run can be obtained by copying from STELS by MOVELS, (see subsection 4.28, PROP).

4.20 NEWELS (Gooding 1968c)

This segment copies orbital parameters from a one-dimensional array to a two-dimensional array.

There are three parameters:

ELEMT: output, 6×6 array of orbital parameters,

MODEL: input, 5 element array giving the number of parameters to be copied into each row of ELEMT,

PARAS: input, 20 element array of orbital parameters.

The subroutine NEWELS may be regarded as the reciprocal to the subroutine EPARAS(4.12). PARAS is copied into ELEMT such that, if the contents of MODEL are m_e, m_i, m_Ω, m_ω, m_M then

PARAS(1),...,PARAS(m_e) are copied into ELEMT(1,1),...,ELEMT(1,m_e);
PARAS(m_e+1),...,PARAS(m_e+m_i) are copied into ELEMT(2,1),...,ELEMT(2,m_i)
etc.

ELEMT(6,j) are derived from ELEMT(5,j+1) by

ELEMT(6,j) = j×ELEMT(5,j+1)

for j=1,...,m_M-1.

Other coefficients in ELEMT are unchanged by NEWELS.

NEWELS is called at the end of each iteration in DIFCOR(4.7) to move the current estimate of the orbital parameters from PARAS to ELEMT so that they may be altered, by ALTELS(4.1), from hybrid to inclusive elements, and printed.

NEWELS is also called after completion of the iterative process to move the standard deviations of orbital parameters from PARAS to STDEVS(6×6).

4.21 NUPOLY

This segment changes the coefficients of a polynomial representing an orbital element so that the polynomial gives the value of the element when the time input is relative to a new epoch.

There are four parameters:

A: input and output, array of coefficients of the polynomial, a_j, a'_j ,

M: input maximum number of coefficients held in A,

N: input (number of coefficients held in A) - 1,

XBAR: input displacement of the origin required, $\overline{x}$.

NUPOLY is called by the segment PROP when initial elements are to be predicted from those supplied which are for MJDINCS days before the epoch required. NUPOLY is called six times, in a loop with I=1,..,6, with

M = 6, N = NOMIAL(I), $\overline{x}$ = MJDINCS$\times$86400, (86400 is the number of seconds in a day), and a_j = ELEMT(I,j), the supplied orbital element coefficients.

For example, when I = 1, and NOMIAL(1) = 3 say, then the eccentricity is given by

$$e = a_1 t^0 + a_2 t^1 + a_3 t^2 + a_4 t^3$$

where t denotes time, in seconds. To obtain a polynomial which will give the eccentricity at some time t relative to an epoch $\overline{x}$ seconds later, a_1, a_2, a_3 and a_4 are replace by a'_1, a'_2, a'_3 and a'_4 where

$$a'_1 = a_1 + a_2\overline{x} + a_3\overline{x}^2 + a_4\overline{x}^3,$$

$$a'_2 = a_2 + 2a_3\overline{x} + 3a_4\overline{x}^2,$$

$$a'_3 = a_3 + 3a_4\overline{x},$$

$$a'_4 = a_4.$$

4.22 OBSIN

This segment controls the reading of observation cards and the storing and printing of the information derived from them.

There are six parameters:

DATA: output, (21 × MAXIM) array of information derived from the observation cards,

MAXIM: input, maximum number of observations allowed,

STASHN: input, (8 × MAXSTA) array of information about the observer locations, pre-set by the segment SENSIN(4.36),

MAXSTA: input, maximum number of observers,

NUMSTA: input, number of observers.

OBSFAI: output,logical indicator of success or failure of OBSIN to find legal observation control cards (LOOXEE cards). This is a new parameter.

The use of COMMON is:

/CNTROL/

IR, IP, ILINES, RIDENT, ITIMEC and IOBSNS input.

/ORBIT/

EMU, ERAD, MJDOCH and ELEMT(6 × 6) input.

COMMON//

SIGMA(6) input.

NUMOBS output.

There are five DATA statements at the beginning of OBSIN. These statements pre-set the contents of some variables. The variables are:

PI = 3.14159265359 ($= \pi$),

CKM = 1.609343 (factor to convert miles to kilometres),

MSIGN2 = 1H- (a negative sign),

THETA(6) = 0.149873, 0.054806, -0.074689, -0.099002, -0.169646,
-0.188909
(see subsection 4.22.9),

MONICA(36) = 3HCOL, 3HEG6, 3HBPO, 3HIN6, 3HFTM, 3HYR6, 3HGFO, 3HRK6,
3HJOB, 3HUR6, 3HLIM, 3HAP6, 3HMOJ, 3HAV6, 3HNEW, 3HFL6, 3HROS,
3HRAN, 3HMAD, 3HGAR, 3HCAR, 3HVON, 3HULA, 3HSK6, 3HMAD, 3HGA6,
3HORO, 3HRA6
(see subsection 4.22.3),

TIMCOD(9) = 0.0001, 0.001, 0.003, 0.01, 0.03, 0.1, 0.3, 1.0, 3.0
(see subsection 4.22.1)

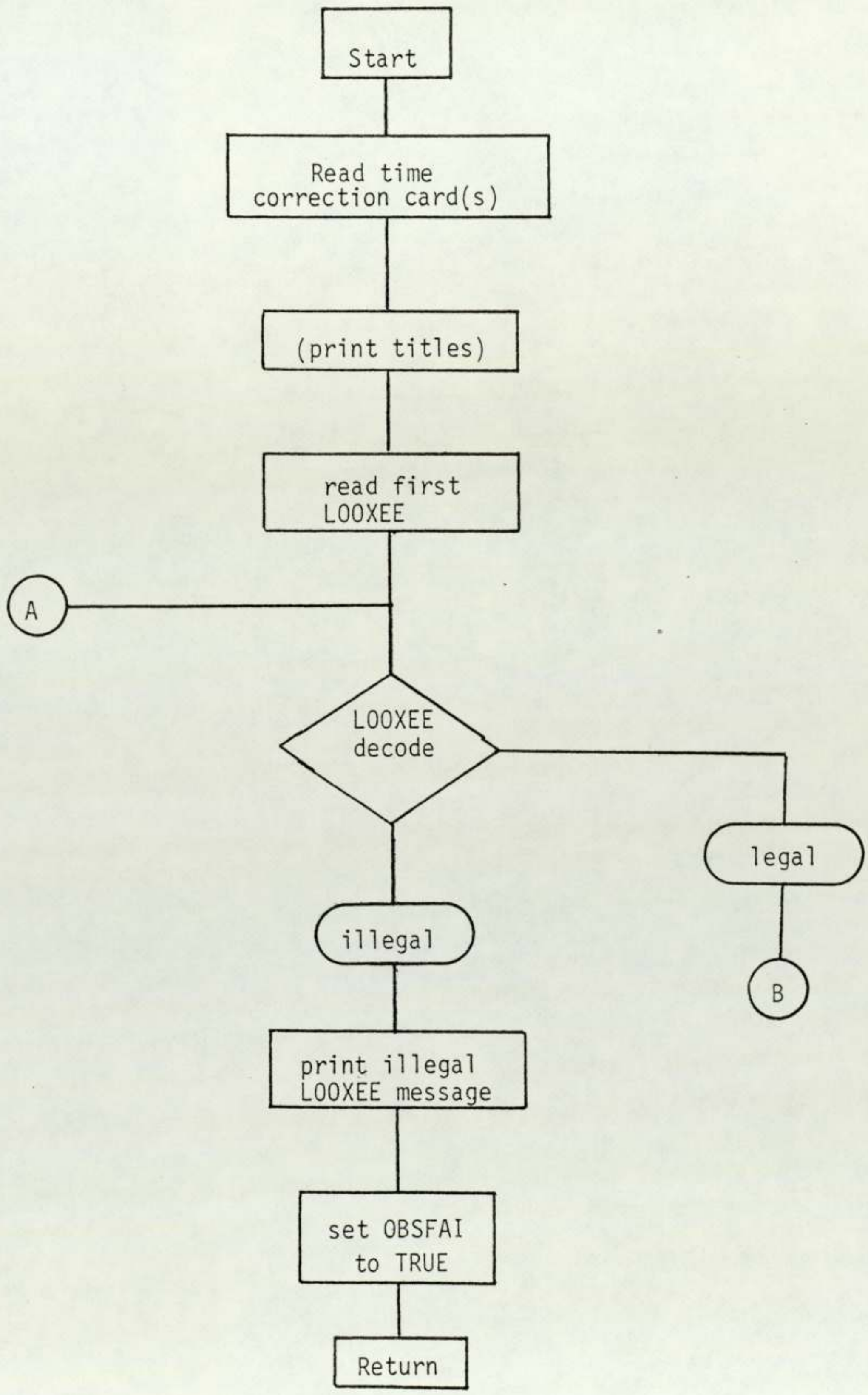
ANGCOD(49) = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,
18, 19, 20, 21, 23, 25, 27, 31, 35, 41, 49, 60, 72, 90, 114, 144,
186, 237, 306, 399, 516, 681, 900, 1170, 1500, 1950, 2580, 3300,
4500, 5760, 5356, 10800
(see subsection 4.22.1),

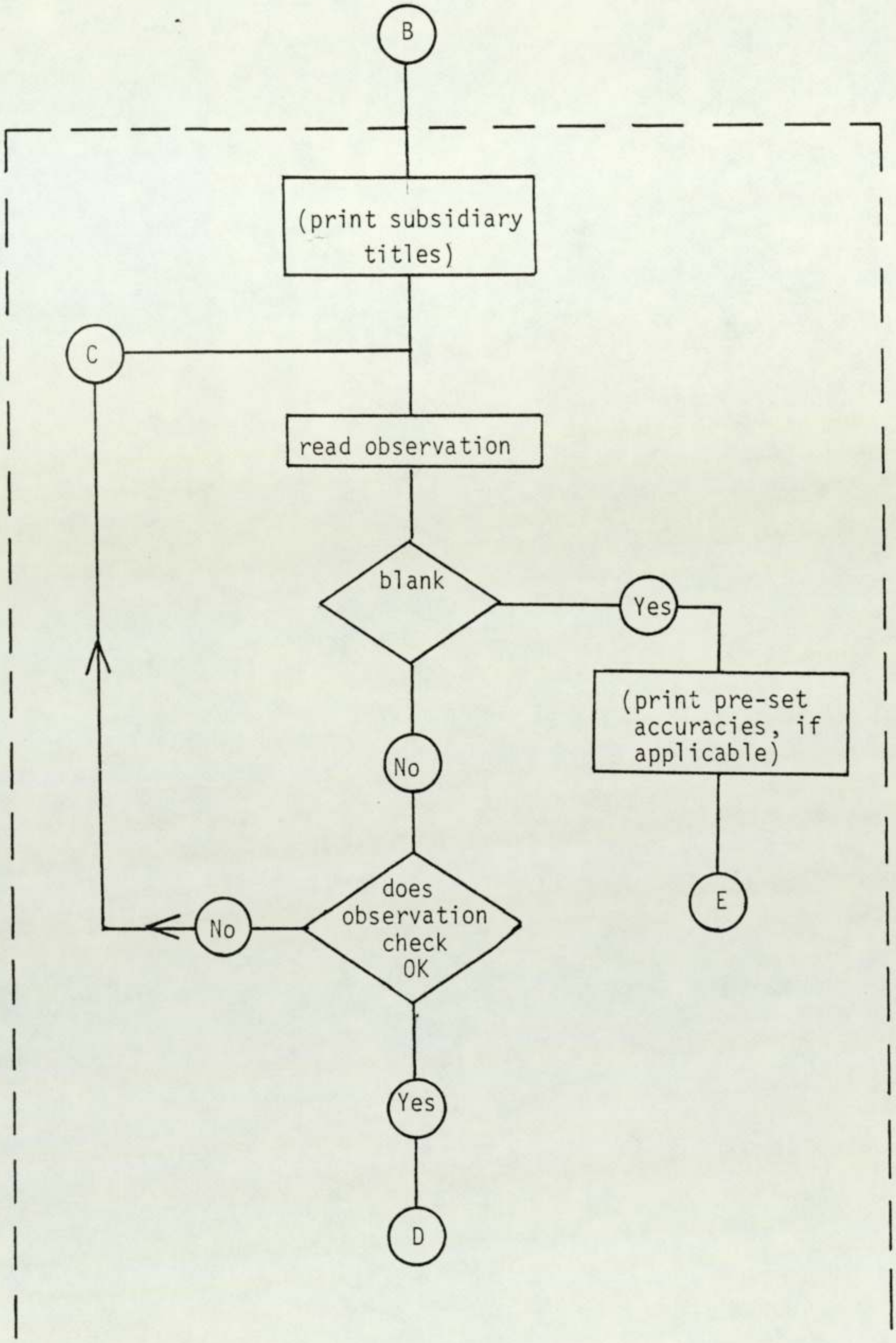
NESRO(26) = 2, 0, 3, 0, 10, 6, 19, 11, 0, 0, 0, 8, 1, 4, 12, 5, 7, 9,
16, 0, 18, 0, 14, 0, 0, 13
(see subsection 4.22.5).

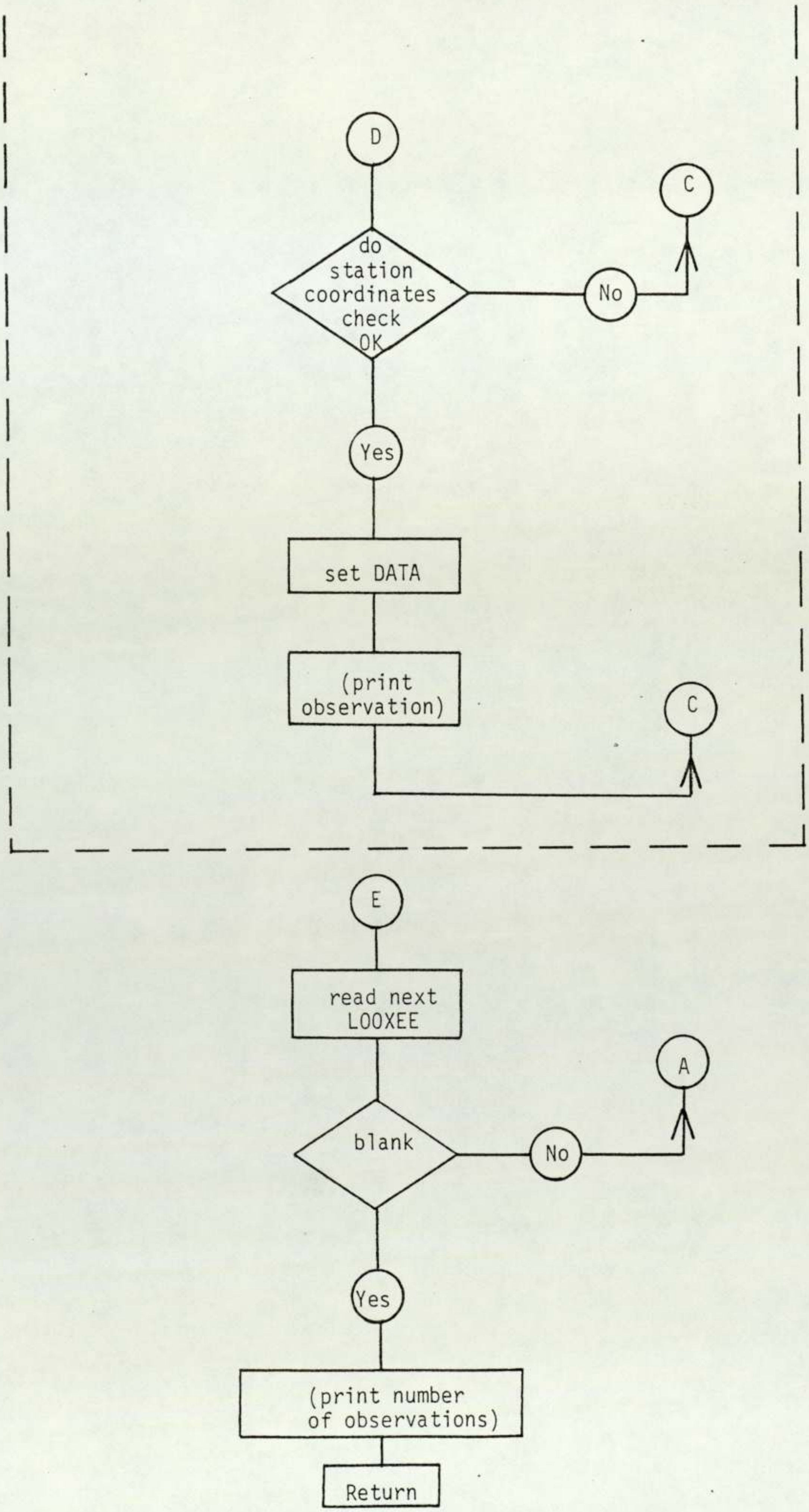
The segment OBSIN may be used to read in observations in a variety of formats. Observations in the same format are grouped together in the data deck: each group is preceded by a card indicating the format of the next group of observations. This card is referred to as the "LOOXEE" card as the number on the card is read in to the variable LOOXEE. Each group of observations is terminated by a blank card and the complete set of observations is terminated by a further blank card.

The complete set of observations may be preceded by one, two or three time correction cards. The presence of these cards is indicated by the input variable ITIMEC.

Figure 4.22.1 Outline Flow of OBSIN




B
(print subsidiary titles)
C
read observation
blank
Yes
(print pre-set accuracies, if applicable)
E
No
does observation check OK
No
Yes
D


D
do station coordinates check OK
No
C
Yes
set DATA
(print observation)
C
E
read next LOOXEE
A
blank
No
Yes
(print number of observations)
Return

Printing of observation information may or may not be required. This is indicated by the input variable IOBSNS. If IOBSNS is non-zero then printing is required. An outline flowchart of OBSIN is given in Figure 4.22.1. In this figure optional printing is indicated by brackets.

The part of the flowchart enclosed in a dashed-line box represents different parts of code depending upon which format of observations is being read in. The control of execution between blocks of code is governed mostly by assigned GOTO statements.

A systematic change has been introduced to OBSIN to overcome a problem that might otherwise arise in 1984. The largest integer that may be held on ICL 1900 series computer is 8388607. Satellites launched after 1984.0 will have identity numbers greater than 8400000. Therefore the identity of the satellite under investigation is held in the real variable RIDENT. Whenever a seven digit satellite identity is read from an observation card it is read, with the format F7.0, into the real variable RJDENT. The previous version of OBSIN read the integer variable JDENT with the format I7 to be compared with the integer IDENT. The test of identity is now

IF(ANINT(RIDENT).EQ.ANINT(RJDENT)).

The binary version of PROP6 does not check the satellite identity when the number is too large to be held in an integer.

The main function of OBSIN is to fill the array DATA with data derived from the observations read in. One column of DATA is used for each observation read in.

The contents of DATA(k,j) k=1,..,21, j=1,..,NUMOBS are

DATA(1,j) = indicator of the type of observation,

DATA(2,j) = indicator of corrections to be applied to the observation. These corrections will be applied when the segment PROCES(4.27) is called by PROP(4.28) after the return from OBSIN, DATA(3,j) = date of observation, in Modified Julian Date (MJD) form,

DATA(4,j) = time of observation, relative to DATA(3,j), in decimals of a day, after corrections have been applied,

DATA(5,j), DATA(6,j), DATA(7,j) = x_s, y_s, z_s, geocentric Cartesian coordinates of the observing station, in km, x_s toward the Greenwich meridian, in the equatorial plane,

DATA(8,j) = sin(latitude)
DATA(9,j) = cos(latitude)
DATA(10,j) = sin(longitude)
DATA(11,j) = cos(longitude)
— geographic coordinates of the observing station, longitude measured relative to the Greenwich meridian, positive Eastwards,

DATA(12,j) to DATA(17,j) = observed quantities,

DATA(18,j) to DATA(21,j) = accuracies of observed quantities.

The observed quantities stored are combinations of range (ρ km), right ascension (α radians), declination (δ radians), direction cosines (ℓ,m) and their rates of change ($\dot{\rho}$, $\dot{\alpha}$, $\dot{\delta}$, $\dot{\ell}$, $\dot{m}$), or the Cartesian quantities x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$. If the observed quantities are azimuth and elevation (or their rates of change), then these are converted to α,δ (or $\dot{\alpha},\dot{\delta}$) before they are stored. The correspondence between observation type, observed quantities and their accuracies, and where the information is stored in the array DATA, is given in Table 4.22.1.

Observations currently available are of types 1, 2, 3, 4 and 7. Parts of OBSIN to deal with other types of observation have not been written. However, the information given here should make the writing of these parts relatively easy if and when such observations become available.

The contents of DATA(2,j) are given by Table 4.22.2 for data types 2 and 7 (α,δ or azimuth,elevation reported). If the observation is of type 1, 3 or 4 (ρ and/or ℓm reported) then DATA(2,j) = 1.

Table 4.22.1 Location of Observed Quantities in DATA

DATA(1,j) (TYPE)	DATA (12,j)	(13,j)	(14,j)	(15,j)	(16,j)	(17,j)	(18,j)	(19,j)	(20,j)	(21,j)
1	ρ (km)						σ_ρ (km)			
2	α (radians)	δ (radians)					$\sigma_{\alpha\delta}$ (radians)			
3	ℓ	m					$\sigma_{\ell m}$			
4	$\dot{\rho}$ (km/sec)						$\sigma_{\dot{\rho}}$ (km/sec)			
5	$\dot{\alpha}$ (rads/sec)	$\dot{\delta}$ (rads/sec)					$\sigma_{\dot{\alpha}\dot{\delta}}$ (rads/sec)			
6	$\dot{\ell}$ (sec^{-1})	$\dot{m}$ (sec^{-1})					$\sigma_{\dot{\ell}\dot{m}}$ (sec^{-1})			
7	ρ	α	δ				σ_ρ	$\sigma_{\alpha\delta}$		
8	ρ	ℓ	m				σ_ρ	$\sigma_{\ell m}$		
9	$\dot{\rho}$	$\dot{\alpha}$	$\dot{\delta}$				$\sigma_{\dot{\rho}}$	$\sigma_{\dot{\alpha}\dot{\delta}}$		

Table 4.22.1 (continued) Location of Observed Quantities in DATA

DATA(1,j) (TYPE)	DATA (12,j)	(13,j)	(14,j)	(15,j)	(16,j)	(17,j)	(18,j)	(19,j)	(20,j)	(21,j)
10	$\dot\rho$	$\dot\ell$	$\dot m$				$\sigma_{\dot\rho}$	$\sigma_{\dot\ell\dot m}$		
11	ρ	$\dot\rho$					σ_ρ	$\sigma_{\dot\rho}$		
12	α	δ	$\dot\alpha$	$\dot\delta$			$\sigma_{\alpha\delta}$	$\sigma_{\dot\alpha\dot\delta}$		
13	ℓ	m	$\dot\ell$	$\dot m$			$\sigma_{\ell m}$	$\sigma_{\dot\ell\dot m}$		
14	ρ	α	δ	$\dot\rho$	$\dot\alpha$	$\dot\delta$	σ_ρ	$\sigma_{\alpha\delta}$	$\sigma_{\dot\rho}$	$\sigma_{\dot\alpha\dot\delta}$
15	ρ	ℓ	m	$\dot\rho$	$\dot\ell$	$\dot m$	σ_ρ	$\sigma_{\ell m}$	$\sigma_{\dot\rho}$	$\sigma_{\dot\ell\dot m}$
16	x (km)	y (km)	z (km)	$\dot x$ (km/sec)	$\dot y$ (km/sec)	$\dot z$ (km/sec)	σ_{xyz} (km)	$\sigma_{\dot x\dot y\dot z}$ (km/sec)		

σ_ρ , $\sigma_{\alpha\delta}$ etc. represent accuracies of the observed ρ, $\alpha\delta$ etc.

Table 4.22.2 Contents of DATA(2,j)

Observation	DATA(2,j)	
	Tens digit	Units digit
Azimuth and Elevation		
uncorrected for refraction	1	0
fully corrected for refraction	2	0
α, δ		
corrected for astronomical refraction but not parallactic refraction (see subsection 4.30, REFCOR)	0	
fully corrected for refraction	3	
referred to the equinox and equator:		
true of date		0
of 1855.0		1
of 1875.0		2
of 1900.0		3
of 1950.0		4
mean of date		5

The first part of OBSIN reads time correction cards, if ITIMEC is non-zero. The format of these cards has been described by Gooding and Tayler (1968). Time corrections read in are added to the observed time. However, any atomic time (A1) correction is not used by OBSIN. If such a correction is read in then a warning that it is not used will be printed out by the new version of OBSIN. If WWV(UTC) time correction card, or cards, are read in the correction is not applied to observations read in under a LOOXEE of 111111 or 222222. The new version of OBSIN prints out a warning to this effect if such cards are read in.

OBSIN has also been modified in this section so that the segment MJDATE has distinct input and output parameters. The previous version was giving failures at this point when compiled by the Fortran compiler available at Aston University.

Most of the formats that may be read in by OBSIN have been described by Gooding and Tayler (1968). An outline of one other has been given by Gooding (1974). Where the format has not been fully described previously in PROP documentation it has been given in the following subsections. The PROP user should consult Gooding and Tayler (1968) for the others. There are now nine legal values of LOOXEE; 111111, 222222, 333333, 444444, 444441, 555555, 666666, 777777 and 888888. These will be considered now, in turn.

4.22.1 LOOXEE = 111111

This section of code checks initially that NUMOBS $\leqslant$ MAXIM and RJDENT = RIDENT. The station search consists of checking that the coordinates of the observing station, identified by number on the observation card, are in the array STASHN. If they are not found the observation is rejected. Otherwise the coordinates are copied from STASHN to DATA.

If the observed quantities were Azimuth and Elevation these are converted to Right ascension and Declination. The accuracy of the observation, $\sigma_{\alpha\delta}$, stored in DATA(18,j), is the following function of the two variables, NASD and NTSD, read from the observation card,

v_1 = 1/20 if NTSD = 0

= TIMCOD(NTSD) if NTSD ≠ 0,

v_2 = SIGMA(4) if NASD = 0

= ANGCOD(NASD)*0.4848136812*10⁻⁵ if NASD ≠ 0.

(The factor 0.4848136812*10⁻⁵ gives a conversion from seconds of arc to radians. It should be noted that NASD is not the accuracy of the observation in seconds of arc, when NASD > 21),

$$e_0 = \text{ELEMT}(1,1),$$

$$n_0 = \text{ELEMT}(6,1),$$

$$a^* = (\mu/n_0^2)^{1/3} \;,$$

$$\dot{\theta} = n_0/\left[2^{\frac{1}{2}}(1 + e_0^2/2 - R/a^*)\right],$$

then

$$\sigma_{\alpha\delta} = \left[v_2^2 + \frac{1}{2}\,(\dot{\theta} v_1)^2\right]^{\frac{1}{2}}. \qquad (4.22.1)$$

4.22.2 LOOXEE = 222222

Checks on NUMOBS, RJDENT and the observation station number NOSTAT, are made as described in subsection 4.22.1. If the quantities observed were direction cosines then a check is made that both quantities are present.

The observational accuracy, stored in DATA(18,j), is a function of which quantity was observed. The accuracy of observed range is SIGMA(2), range rate has an observational accuracy of SIGMA(3) and direction cosines are assigned an accuracy of SIGMA(1).

When the last observation under this LOOXEE card has been read the values of SIGMA(1), SIGMA(2) and SIGMA(3) are printed out, if observations are being printed.

4.22.3 LOOXEE = 333333

Initially NUMOBS is checked against MAXIM. No check of the satellite identity may be made as it is not on the observation cards. The observing station identity is read into a two element array NAMEST, using the format 2A3. The two elements of NAMEST are compared with the contents of the array MONICA for some positive integer NOSTAT $\leqslant$ 18, such that

NAMEST(1).EQ.MONICA(2*NOSTAT-1).AND.NAMEST(2).EQ.MONICA(2*NOSTAT).

The value of NOSTAT for which the above condition holds is used as the station number for which a search is made through the array STASHN, as described in subsection 4.22.1.

The accuracy stored in DATA(18,j) is SIGMA(2) if range was observed, and SIGMA(3) if range rate was observed. These values of SIGMA are printed out after the last observation under this LOOXEE card, if observations are being printed.

4.22.4 LOOXEE = 444444

Observations under this looxee are read in from pairs of cards. The first of a pair will be rejected if it does not contain an east-west direction cosine, ℓ, or if the next card also contains an east-west direction cosine. In the latter case the second card is assumed to be the first of a pair. If the second card of a pair does not contain a north-south direction cosine, m, or if $m^2 + \ell^2 \geqslant 1$, or if the times of the observations of ℓ and m differ by more than 0.005 seconds then the pair of cards is rejected. Having obtained a pair of valid direction cosines, NUMOBS is checked against MAXIM and then the station name

NAMEST checked as described in subsection 4.22.3. The time of the observation is taken to be the average of the times read from the two cards, corrected by WWV if applicable.

The observational accuracy, stored in DATA(18,j) is SIGMA(1), and this is printed after the last observation under this LOOXEE card, if observations are being printed.

4.22.5 LOOXEE = 444441

This format was introduced, but not described in detail, by Gooding (1974). In introducing the format, Gooding refers to the DCOP program of Walter, Wales and Pallaschkle (1968), but DCOP accepts a variety of formats, only one of which is acceptable to OBSIN under this looxee. The format is given in Table 4.22.3.

The satellite identity is checked against RIDENT. This is a new check, omitted from the previous version of OBSIN. The number of observations is checked against MAXIM.

The station number is translated by

NOSTAT = NESRO(NOSTAT)

before searching through the array STASHN for the station coordinates, as described in subsection 4.22.1.

If no WWV time correction has been read in then the correction on the observation card, CORR, is subtracted from the observed time. However, if WWV time corrections have been read in these will be used instead. A change has been incorporated into OBSIN to prevent the time correction on the observation card from being ignored, whether or not WWV time correction cards had been read in.

The observational accuracy stored in DATA(18,j) is SIGMA(1) and this value is printed after the last observation under this LOOXEE card has been printed, if observations are being printed.

Table 4.22.3 The Format of Observations under LOOXEE = 444441

Column Numbers	Format	Variable	Description
1-7	F7.0	RJDENT	Satellite identity.
8-12	5X	-	-
13-17	I5	NOSTAT	Observing station number
18-23	I6	MDAY	Observation date, either in MJD form (right justified) or as year, month, day, (yymmdd)
24-25	I2	MHR	Observation time, in hours, minutes, seconds. A decimal point is assumed between columns 29 and 30.
26-27	I2	MIN	
28-33	F6.4	SEC	
34-42	F9.8	DCL	East-west direction cosine, ℓ. A decimal point is assumed between columns 34 and 35.
43	1X	-	-
44-52	F9.8	DCM	North-south direction cosine, m. A decimal point is assumed between columns 44 and 45.
53-64	12X	-	-
65-71	F7.4	CORR	Time correction, in seconds. A decimal point is assumed between columns 67 and 68.

4.22.6 LOOXEE = 555555

A check is made that RJDENT = RIDENT and that NUMOBS does not exceed MAXIM. The observing station coordinates are copied from the array STASHN to the array DATA as described in subsection 4.22.1. If the observed quantities were Azimuth and Elevation then these are converted to Right Ascension and Declination before being stored in DATA. The observational accuracy stored in DATA(18,j) is SIGMA(4). The value of SIGMA(4) is converted from radians to seconds of arc and printed out after the last observation under this LOOXEE, if observations are being printed.

4.22.7 LOOXEE = 666666

After preliminary checking that NUMOBS does not exceed MAXIM and that RJDENT equals RIDENT, the station coordinates are sought in STASHN, as described in subsection 4.22.1. If the observed quantities were Azimuth and Elevation then these are converted to Right Ascension and Declination before being stored in the array DATA.

If range is an observed quantity then the range accuracy given on the observation card, RSD, is stored in DATA(18,j), unless RSD is zero. In this case the range accuracy is taken to be SIGMA(2). If angles were observed but the angular accuracy read from the card, ASD, is zero, then the accuracy of the observed angles and time is assumed to be SIGMA(4). This value is stored in DATA(18,j) unless range was observed also, in which case SIGMA(4) is stored in DATA(19,j). If the value of ASD read is non-zero then the accuracy stored in DATA(18,j) or DATA(19,j) is given by Equation (4.22.1), with

v_1 = TSD, timing accuracy read from the observation card, which may be zero,

and

v_2 = ASD, converted to radians.

4.22.8 LOOXEE = 777777

The format of observations under this LOOXEE is given in Table 4.22.4.

Table 4.22.4 The Format of Observations under LOOXEE = 777777

Column Numbers	Format	Variable	Description
1-6	2A3	NAMEST	Observing station name
7-8	2X	-	-
9-14	3I2	MY MM MD	Observation date, in year, month, day form (yymmdd), or in MJD form (right justified in the six columns)
15-16 17-18 19-26	I2 I2 F8.6	MHR MIN SEC	Observation time in hours, minutes, seconds. A decimal point is assumed between columns 20 and 21
27-30	4X	-	-
31-45	E15.0	RANGE	Range, in feet
46-60	E15.0	RAAZ	Azimuth, in degrees
61-75	E15.0	DECEL	Elevation, in degrees
			Azimuth and elevation should be fully corrected for refraction.

No preliminary check of satellite identity is possible as the identity number is not given on the observation cards in this format. However, NUMOBS is checked against MAXIM. The station coordinates are found by the method described in subsection 4.22.3.

If range is non-zero then a range accuracy of SIGMA(5) is stored in DATA(18,j). If angles were observed their accuracy is taken to be SIGMA(6) converted from degrees to radians. If range and angles were observed then SIGMA(6) is stored in DATA(19,j). If only angles were observed the value is stored in DATA(18,j).

4.22.9 LOOXEE = 888888

This format has been introduced to OBSIN so that observations provided by the United States Naval Research Laboratory (NRL) may be read in directly by OBSIN. With all previous versions of OBSIN, the NRL observations had to be pre-processed by a separate program to adjust them to the format suitable for reading in under a LOOXEE of 666666. This pre-processing will now be done internally to PROP. It does not significantly increase the amount of time taken by a typical PROP job. The part of the observation pack containing NRL observations should be constructed as shown in Figure 4.22.2. There are normally two NRL sub-LOOXEE cards and groups of observations but there may be more, or less. The format of the cards is given in Table 4.22.5.

When reading in NRL observations, OBSIN checks that NUMOBS does not exceed MAXIM. If the satellite identity RJDENT on an NRL sub-LOOXEE card does not equal RIDENT then the subsequent group of NRL observations is ignored.

Figure 4.22.2 Construction of LOOXEE = 888888 Part of an Observation Pack.

888888 (LOOXEE card)

NRL sub-LOOXEE card

} Group of NRL observations

blank card (end of sub-LOOXEE)

NRL sub-LOOXEE card

} Group of NRL observations

blank card (end of sub-LOOXEE)

blank card (end of all NRL observations)

Table 4.22.5 The Format of Cards under LOOXEE = 888888

Column Numbers	Format	Variable	Description
NRL sub-looxee card:			
1-3	3X	-	-
4-10	F7.0	RJDENT	Satellite identity.
11-20	F10.0	ASD	Angular accuracy (degrees).
21-25	F5.3	RSD	Range accuracy (km). A decimal point is assumed between columns 22 and 23.
26-30	F5.0	RMNEL	Minimum acceptable elevation (degrees).
NRL observation cards:			
1-4	4X	-	-
5-10	3I2	MY MM MD	Date of observation in year, month, day form (yymmdd), or in MJD form (right justified in the six columns).
11	1X	-	-
12-13	I2	MHR	Time of observation, in hours, minutes, seconds. A decimal point is assumed between columns 18 and 19.
14-15	I2	MIN	
16-20	F5.2	SEC	
21-27	7X	-	-
28	I1	NTYPE	Station type.
29-30	I2	NOSTAT	Station number.
31-40	F10.2	EWANGL	Three observed quantities. Decimal points are assumed between columns 38 and 39, 48 and 49, 58 and 59. (NSANGL is type REAL)
41-50	F10.2	NSANGL	
51-60	F10.2	RANGE	

The angular accuracy, ASD, is converted to radians and assigned to each of the angular observations in the subsequent group of observations. If the range accuracy, RSD, read in is zero then a range accuracy of 1 kilometre is assigned to each observation of range in the subsequent group of observations.

On each observation card the type and number of the observing station are given by NTYPE and NOSTAT. If NTYPE is zero then the observation is ignored.

If NTYPE equals one, then the station number, NOSTAT, must be one of 1, 2, 3, 4, 5, 6, 28. In this case the input variables EWANGL and NSANGL may be interpreted as illustrated in Figure 4.22.3. (NSANGL has been declared a variable of type REAL).

The angle A' is related to the azimuth, Az, of the satellite by:

$$\left.\begin{array}{l} Az = A' - THETA(NOSTAT),\ (1 \leqslant NOSTAT \leqslant 6), \\ Az = A',\ (otherwise). \end{array}\right\} \qquad (4.22.2)$$

We may see that

$$\tan A' = OP_2/P_1P_2$$

and

$$\tan E\ell = P_2P_3/[(OP_2)^2 + (P_1P_2)^2]^{\frac{1}{2}}$$

where $E\ell$ is the satellite's elevation.

However

$$\sin(EWANGL) = OP_2/OP_3 = OP_2/[(OP_2)^2 + (P_2P_3)^2]^{\frac{1}{2}},$$

$$\cos(EWANGL) = P_2P_3/OP_3 = P_2P_3/[(OP_2)^2 + (P_2P_3)^2]^{\frac{1}{2}},$$

$$\sin(NSANGL) = P_1P_2/OS = P_1P_2/[(OP_2)^2 + (P_2P_1)^2 + (P_1S)^2]^{\frac{1}{2}},$$

and

$$\cos(NSANGL) = OP_3/OS = [(OP_2)^2 + (P_2P_3)^2]^{\frac{1}{2}}/[(OP_2)^2 + (P_1P_2)^2 + (P_1S)^2]^{\frac{1}{2}}.$$

Therefore

$$\tan A' = (OP_2/OP_3)(OP_3/OS)/(P_1P_2/OS)$$

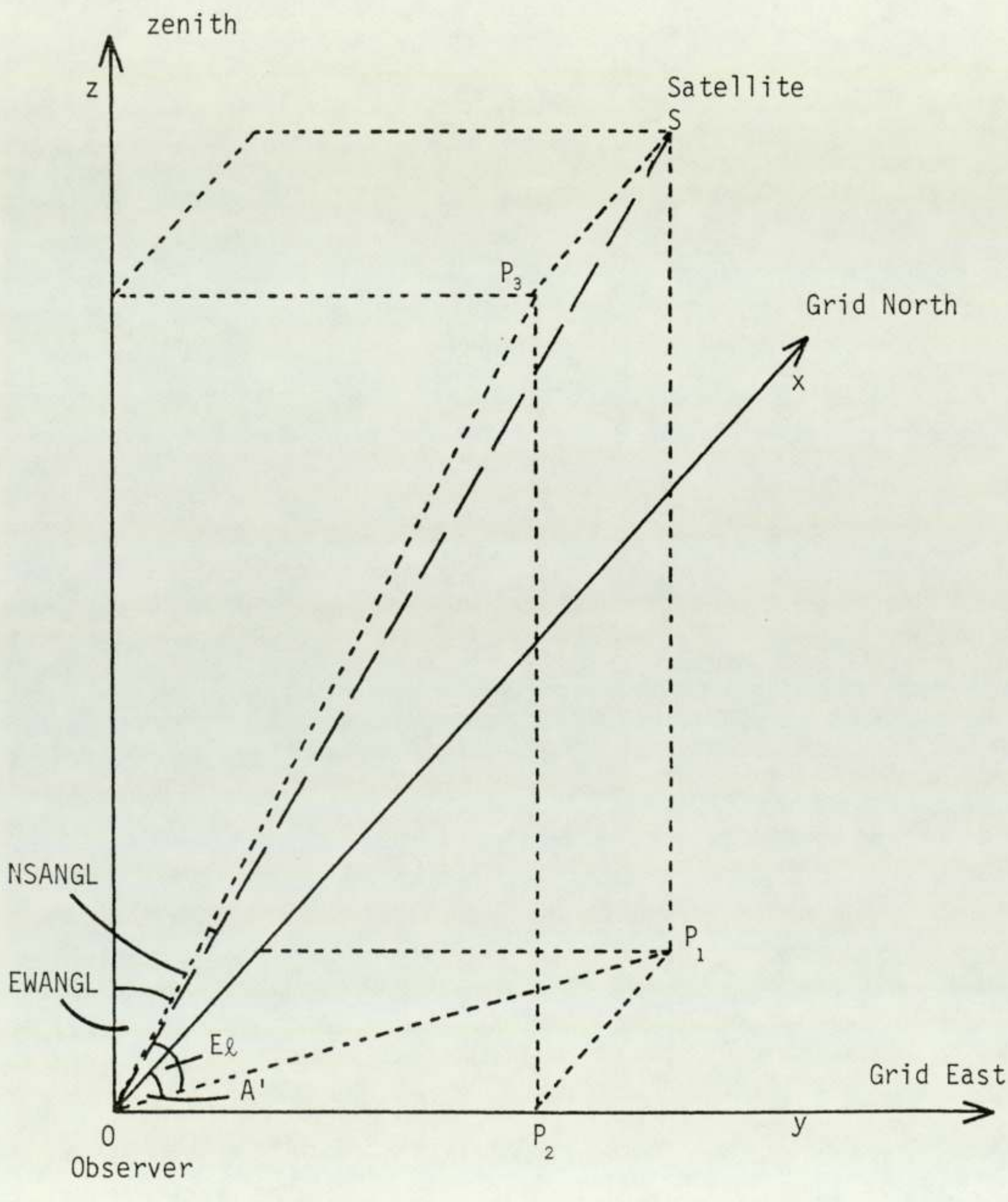
$$= \sin(\mathrm{EWANGL})\cos(\mathrm{NSANGL})/\sin(\mathrm{NSANGL}) \,, \qquad (4.22.3)$$

and

$$\tan E\ell = (P_2P_3/OP_3)(OP_3/OS)/\left[(OP_2/OS)^2 + (P_1P_2/OS)^2\right]^{\frac{1}{2}}$$
$$= (P_2P_3/OP_3)(OP_3/OS)/\left[(P_1P_2/OS)^2 + \{(OP_2/OP_3)(OP_3/OS)\}^2\right]^{\frac{1}{2}}$$

$$= \frac{\cos(\mathrm{EWANGL})\cos(\mathrm{NSANGL})}{\left[\sin^2(\mathrm{NSANGL}) + \sin^2(\mathrm{EWANGL})\cos^2(\mathrm{NSANGL})\right]^{\frac{1}{2}}} \,. \qquad (4.22.4)$$

Figure 4.22.3 Interpretation of EWANGL and NSANGL



Equations (4.22.2), (4.22.3) and (4.22.4) are used in OBSIN to derive the equivalent observed Azimuth and Elevation from EWANGL, NSANGL and NOSTAT.

The calculated value of the elevation, $E\ell$, is compared with RMNEL and if

$$RMNEL \geqslant E\ell, \; RMNEL \neq 0,$$

then the observation is rejected as being too close to the horizon for accuracy. The observed range, RANGE, is converted from miles to kilometres by the factor CKM before being stored.

If NTYPE is greater than one then the station number is set to 29, regardless of the value of NOSTAT on the observation card. In this case the three observed quantities, EWANGL, NSANGL and RANGE are interpreted as topocentric Cartesian coordinates of the satellite, z, y, x respectively, in miles. The stored range of the satellite is thus

$$\text{range} = \left[(EWANGL)^2 + (NSANGL)^2 + (RANGE)^2\right]^{\frac{1}{2}} \times CKM.$$

The Azimuth and Elevation are given by

$$\text{Azimuth} = \tan^{-1}(NSANGL/RANGE),$$

$$\text{Elevation} = \tan^{-1}\left[EWANGL/(RANGE^2 + NSANGL^2)^{\frac{1}{2}}\right].$$

Elevation is not compared with RMNEL when the station type is greater than one.

The calculations under a LOOXEE card of 888888 which involve the $\tan^{-1}$ function are performed by a call to the segment CARSFE. This segment uses the ICL extended Fortran function ATAN2, which allows $\tan^{-1}(x/0)$, giving an angle of $\pi/2$.

The observing station coordinates are found by the method described in subsection 4.22.1.

4.22.10 Illegal LOOXEE card

If a LOOXEE card has a number on it which is not recognised by the program as a legal LOOXEE then the output parameter OBSFAI is set to TRUE and a return executed so that execution may continue with the next independent run in the PROP job. This is a change from the PROP6 version of OBSIN which forced an execution error in this circumstance. Execution then continued with the next independent run because the segment ERROR existed but this segment has been removed (see subsection 4.13, ERROR).

4.23 PARSHL (Tayler, Gooding,1970).

This segment computes partial derivatives of the geocentric Cartesian coordinates of the satellite with respect to the set of orbital parameters.

The subroutine has one, input, parameter:

JRATES: indicator of whether derivatives of position coordinates only (JRATES = 0) or position and velocity coordinates (JRATES = 1) are required.

The use of COMMON IS:

/ORBIT/

L, NOMIAL(6), ELEMT(6 × 6), X≡x, Y≡y, Z≡z, XDOT≡$\dot{x}$, YDOT≡$\dot{y}$, ZDOT≡$\dot{z}$, QPD(30) and MODEL(5) are all input.

DERIV(6 × 20) is output.

The subroutine PARSHL is constructed in three parts. The first part sets up preliminary quantities for the calculation of the derivatives of spatial coordinates with respect to orbital parameters. The third part sets up the equivalent preliminary quantities for the calculation of the derivatives of velocity components with respect to orbital parameters. The central part calculates the required derivatives. If JRATES is zero then the third part is not executed and the central part is executed once only. If velocity derivatives are required the third part is executed and then the central part is executed for a second time.

There is a test at the end of the central section designed to execute a RETURN on completion of the section if either velocity derivatives are not required or the section has been executed twice already. This test has had to be modified. It used to read:

```
6 IF (JRATES.EQ.0.OR.M1.EQ.4)RETURN
```

but the label 6 is the end of a "DO loop". Hence a warning was generated by the compiler. The execution obtained by the above statement is equivalent to

```
  IF(JRATES.EQ.0.OR.M1.EQ.4)RETURN
6 CONTINUE
```

which is not what is intended. The code has been changed to

```
6 CONTINUE
  IF(JRATES.EQ.0.OR.M1.EQ.4)RETURN
```

as required. This makes only a small change to the output of PARSHL. In what follows we assume, for simplicity, that velocity derivatives are required.

The preliminary quantities calculated in PARSHL are the partial derivatives of the satellite coordinates, and velocity components, with respect to the orbital elements. These quantities are set in the (6×6) matrix AE. Using the notation of Tayler and Gooding (1970), we denote:

$$X \equiv (x,y,z)^T \equiv (X_1,X_2,X_3)^T$$

$$\dot{X} \equiv (\dot{x},\dot{y},\dot{z})^T \equiv (\dot{X}_1,\dot{X}_2,\dot{X}_3)^T$$

$$A \equiv (A_1,A_2,A_3)^T \equiv (\mathrm{QPD}(16),\mathrm{QPD}(17),\mathrm{QPD}(18))^T$$

$$B \equiv (B_1,B_2,B_3)^T \equiv (\mathrm{QPD}(19),\mathrm{QPD}(20),\mathrm{QPD}(21))^T$$

$$F \equiv (F_1,F_2,F_3)^T \equiv (\mathrm{QPD}(10),\mathrm{QPD}(11),\mathrm{QPD}(12))^T$$

$$G \equiv (G_1,G_2,G_3)^T \equiv (\mathrm{QPD}(13),\mathrm{QPD}(14),\mathrm{QPD}(15))^T$$

and

$$W \equiv (W_1,W_2,W_3)^T \equiv (-y,x,0)^T.$$

The contents of the array AE are then given by:

$$AE(k,1) = \frac{\partial X}{\partial e} = -a\cos v\, A + \frac{r(1+p/r)}{(1-e^2)} \sin v B,$$

$$AE(k,2) = \frac{\partial X}{\partial i} = rF,$$

$$AE(k,3) = \frac{\partial X}{\partial \Omega} = W,$$

$$AE(k,4) = \frac{\partial X}{\partial \omega} = rB,$$

$$AE(k,5) = \frac{\partial X}{\partial M} = \frac{a\, e \sin v}{(1-e^2)^{\frac{1}{2}}} A + \frac{r(p/r)^2}{(1-e^2)^{3/2}} B,$$

$$AE(k,6) = \frac{\partial X}{\partial n} = -(2/3n)X,$$

$$AE(k+3,1) = \frac{\partial \dot{X}}{\partial e} = \frac{(\mu/p)^{\frac{1}{2}}}{(1-e^2)} \left[- (p/r)\sin v\, A + (e+\cos v)B\right],$$

$$AE(k+3,2) = \frac{\partial \dot{X}}{\partial i} = \dot{r}F + \frac{(\mu p)^{\frac{1}{2}}}{r} G,$$

$$AE(k+3,3) = \frac{\partial \dot{X}}{\partial \Omega} = \dot{W}$$

$$AE(k+3,4) = \frac{\partial \dot{X}}{\partial \omega} = \dot{r}B - \frac{(\mu p)^{\frac{1}{2}}}{r} A,$$

$$AE(k+3,5) = \frac{\partial \dot{X}}{\partial M} = -\frac{(\mu p)^{\frac{1}{2}}/r}{(1-e^2)^{3/2}} \left(\frac{p}{r}\right) A,$$

and

$$AE(k+3,6) = \frac{\partial \dot{X}}{\partial n} = (1/3n)\dot{X}$$

for $k = 1, 2, 3$.

The expression for $\frac{\partial \dot{X}}{\partial i}$ is not exactly as given by Tayler and Gooding (1970). However, the expression given here matches that used in the program and, as with all the other expressions used in the calculation of AE, has been verified.

To obtain derivatives with respect to orbital parameters, ε_j, from the matrix AE, we consider the element ε expressed as a polynomial

$$\varepsilon = \varepsilon_0 + \varepsilon_1 t + \varepsilon_2 t^2 + \ldots + \varepsilon_{\mathrm{NOMIAL}(\varepsilon)-1}\ t^{\mathrm{NOMIAL}(\varepsilon)-1} \quad ,$$

from which we obtain

$$\frac{\partial\varepsilon}{\partial\varepsilon_j} = t^j \quad , \; j = 0,..,\mathrm{NOMIAL}(\varepsilon)-1.$$

(Time from the epoch, t, is equivalent to QPD(9)).

Hence, for $\varepsilon = e, i, \Omega, \omega$ we have

$$\frac{\partial\zeta}{\partial\varepsilon_j} = \frac{\partial\zeta}{\partial\varepsilon}\frac{\partial\varepsilon}{\partial\varepsilon_j} = t^j \frac{\partial\zeta}{\partial\varepsilon}$$

where $\zeta = X, \dot{X}$

$j = 0,..,\mathrm{MODEL}(\varepsilon)-1.$

Since $n = \sum_j j\, M_j\, t^{j-1}$ we treat derivatives with respect to M_j slightly differently, viz.,

$$\frac{\partial\zeta}{\partial M_j} = t^j \frac{\partial\zeta}{\partial M} + j^{t^{j-1}} \frac{\partial\zeta}{\partial n} \quad .$$

The derivatives obtained thus are stored in the (6×20) array DERIV.

Two types of corrections are applied to the contents of DERIV, as discussed by Gooding (1974). The first type, termed "J_2 corrections", arise from the first order secular perturbations due to zonal harmonics. Using his notation, Merson (1966) gives these perturbations in the form

$$\dot{\Omega}_{\mathrm{sec}} = -\sum_{\ell=2}^{L} J_\ell\, \gamma_\ell^0\, \alpha_\ell^0\, B_\ell^0\, A_\ell^1$$

and

$$\dot{\omega}_{sec} = -\dot{\Omega}_{sec}\cos i_0 + \sum_{\ell=2}^{L} J_\ell\,\gamma_\ell^0\,A_\ell^0(\ell B_\ell^0 + \beta_\ell^0\,B_\ell^1).$$

(Note that Merson's A_ℓ^0 and B_ℓ^0 etc. are not related to our A and B). Similar perturbations to other elements are zero. The dominant, J_2, term of $\dot{\Omega}_{sec}$ is

$$\begin{aligned}\Omega_{(1)} &= -J_2\,\gamma_2^0\,\alpha_2^0\,B_2^0\,A_2^1\\ &= -\frac{3}{2}J_2(R/p_0)^2 n_0\cos i_0,\end{aligned}$$

and the dominant term of $\dot{\omega}_{sec}$ is

$$\begin{aligned}\omega_{(1)} &= \frac{3}{2}J_2(R/p_0)^2\,n_0\cos i_0 + J_2\gamma_2^0\,A_2^0(2B_2^0 + \beta_2^0 B_2^1)\\ &= \frac{3}{4}J_2(R/p_0)^2\,n_0(4-5\sin^2 i_0)\end{aligned}$$

where $p_0 = a_0(1-e_0^2)$

and $n_0 \equiv M_1$.

(The expression for $\Omega_{(1)}$ is given the wrong sign in Tayler and Gooding (1970)).

Therefore $\Omega_{(1)}$ and $\omega_{(1)}$ are functions of e_0, i_0 and M_1. Derivatives with respect to these parameters must be modified. Thus

$\dfrac{\partial\zeta}{\partial e_0}$ is increased by $\dfrac{\partial\zeta}{\partial\Omega_1}\dfrac{\partial\Omega_{(1)}}{\partial e_0} + \dfrac{\partial\zeta}{\partial\omega_1}\dfrac{\partial\omega_{(1)}}{\partial e_0}$,

$\dfrac{\partial\zeta}{\partial i_0}$ is increased by $\dfrac{\partial\zeta}{\partial\Omega_1}\dfrac{\partial\Omega_{(1)}}{\partial i_0} + \dfrac{\partial\zeta}{\partial\omega_1}\dfrac{\partial\omega_{(1)}}{\partial i_0}$,

and

$\dfrac{\partial\zeta}{\partial M_1}$ is increased by $\dfrac{\partial\zeta}{\partial\Omega_1}\dfrac{\partial\Omega_{(1)}}{\partial n_0} + \dfrac{\partial\zeta}{\partial\omega_1}\dfrac{\partial\omega_{(1)}}{\partial n_0}$.

In each case, $\dfrac{\partial\zeta}{\partial\Omega_1}$ and $\dfrac{\partial\zeta}{\partial\omega_1}$ are given by

$t\frac{\partial\zeta}{\partial\Omega}$ and $t\frac{\partial\zeta}{\partial\omega}$ respectively.

The second type of corrections are termed "M-corrections". They arise because the derivatives have been calculated with respect to the inclusive mean anomaly $M_{j_{inc}}$, whereas they are required with respect to the exclusive $M_{j_{exc}}$. Tayler and Gooding (1970) give

$$M_{j_{inc}} = M_{j_{exc}} - \Omega_j \cos i_0 - \omega_j \quad , \quad j > 1.$$

In DERIV we have, using the same notation as in subsection 4.7 DIFCOR, and letting $\{\varepsilon\}$ be the set of orbital parameters apart from M_j, Ω_j and ω_j

$$\left.\frac{\partial\zeta}{\partial M_{j_{inc}}}\right|_{\Omega_j,\omega_j,\{\varepsilon\}} , \quad \left.\frac{\partial\zeta}{\partial\Omega_j}\right|_{M_{j_{inc}},\omega_j,\{\varepsilon\}} \text{ and } \left.\frac{\partial\zeta}{\partial\omega_j}\right|_{M_{j_{inc}},\Omega_j,\{\varepsilon\}} ,$$

whereas we require

$$\left.\frac{\partial\zeta}{\partial M_{j_{exc}}}\right|_{\Omega_j,\omega_j,\{\varepsilon\}} , \quad \left.\frac{\partial\zeta}{\partial\Omega_j}\right|_{M_{j_{exc}},\omega_j,\{\varepsilon\}} \text{ and } \left.\frac{\partial\zeta}{\partial\omega_j}\right|_{M_{j_{exc}},\Omega_j,\{\varepsilon\}} .$$

As in subsection 4.7, DIFCOR, we may alter the stored derivatives to those required since:

$$\left.\frac{\partial\zeta}{\partial M_{j_{exc}}}\right|_{\Omega_j,\omega_j,\{\varepsilon\}} = \left.\frac{\partial\zeta}{\partial M_{j_{inc}}}\right|_{\Omega_j,\omega_j,\{\varepsilon\}} \frac{\partial M_{j_{inc}}}{\partial M_{j_{exc}}} + \left.\frac{\partial\zeta}{\partial\Omega_j}\right|_{M_{j_{inc}},\omega_j,\{\varepsilon\}} \frac{\partial\Omega_j}{\partial M_{j_{exc}}}$$

$$+ \left.\frac{\partial\zeta}{\partial\omega_j}\right|_{M_{j_{inc}},\Omega_j,\{\varepsilon\}} \frac{\partial\omega_j}{\partial M_{j_{exc}}}$$

$$= \left.\frac{\partial\zeta}{\partial M_{j_{inc}}}\right|_{\Omega_j,\omega_j,\{\varepsilon\}} ,$$

$$\left.\frac{\partial\zeta}{\partial\Omega_j}\right|_{M_{j_{exc}},\omega_j,\{\varepsilon\}} = \left.\frac{\partial\zeta}{\partial M_{j_{inc}}}\right|_{\Omega_j,\omega_j,\{\varepsilon\}} \frac{\partial M_{j_{inc}}}{\partial\Omega_j} + \left.\frac{\partial\zeta}{\partial\Omega_j}\right|_{M_{j_{inc}},\omega_j,\{\varepsilon\}} \frac{\partial\Omega_j}{\partial\Omega_j}$$

$$+ \left.\frac{\partial\zeta}{\partial\omega_j}\right|_{M_{j_{inc}},\Omega_j,\{\varepsilon\}} \frac{\partial\omega_j}{\partial\Omega_j}$$

$$= \left.\frac{\partial\zeta}{\partial M_{j_{inc}}}\right|_{\Omega_j,\omega_j,\{\varepsilon\}} (-\cos i_0) + \left.\frac{\partial\zeta}{\partial\Omega_j}\right|_{M_{j_{inc}},\omega_j,\{\varepsilon\}}$$

and

$$\left.\frac{\partial\zeta}{\partial\omega_j}\right|_{M_{j_{exc}},\Omega_j,\{\varepsilon\}} = \left.\frac{\partial\zeta}{\partial M_{j_{inc}}}\right|_{\Omega_j,\omega_j,\{\varepsilon\}} \frac{\partial M_{j_{inc}}}{\partial\omega_j} + \left.\frac{\partial\zeta}{\partial\Omega_j}\right|_{M_{j_{inc}},\omega_j,\{\varepsilon\}} \frac{\partial\Omega_j}{\partial\omega_j}$$

$$+ \left.\frac{\partial\zeta}{\partial\omega_j}\right|_{M_{j_{inc}},\Omega_j,\{\varepsilon\}} \frac{\partial\omega_j}{\partial\omega_j}$$

$$= \left.\frac{\partial\zeta}{\partial M_{j_{inc}}}\right|_{\Omega_j,\omega_j,\{\varepsilon\}} (-1) + \left.\frac{\partial\zeta}{\partial\omega_j}\right|_{M_{j_{inc}},\Omega_j,\{\varepsilon\}}$$

where $\frac{\partial\zeta}{\partial M_{j_{inc}}} = t^j \frac{\partial\zeta}{\partial M}$.

M-corrections are applied for $j \geqslant 1$.

If the correction to the program code at label 6 had not been made, then J_2 and M corrections would not be completed during the final execution of the second part of the segment. The terms involving $\frac{\partial\zeta}{\partial\omega_1}$ in J_2 corrections would have been omitted and the M correction to $\frac{\partial\zeta}{\partial\omega_j}$ would not have been made.

A further change has been made to the segment PARSHL as M-type corrections were being calculated wrongly. The factor t^j involved in the calculation of $\frac{\partial\zeta}{\partial M_{j_{inc}}}$ was wrongly programmed such that $t^j = t^1$ always. This change only affects PROP runs where Ω_2 and ω_2 are orbital parameters.

4.24 PRDATE (Tayler, Odell, 1969)

This segment prints a date on the line printer output.

There are two parameters:

MJD: input Modified Julian Day number of the date to be printed (see subsection 4.18),

IP: input channel number of line printer upon which the output is required.

There are no output parameters to PRDATE and COMMON areas of store are not used. The only output is on the specified line printer. The date is converted from MJD to civil form by MJDATE and then printed, in columns 16 to 26, in the form

19yy mmm dd.

19yy is the year, mmm are the first three letters of the month and dd is the day number in the month.

PRDATE is called by the segment PROP(4.28) to print the epoch for which elements are being derived, and by segment OBSIN(4.22) to print the date of each observation.

4.25 PRELON (Merson, Gooding, Tayler, no date available)

This segment computes time independent quantities that are required for the calculation of secular and long periodic perturbations of the satellite orbit.

There is one parameter:

IND: input indicator of which perturbations are to be considered.

IND = 0 → drag, zonal harmonics and tesseral harmonics,

IND = 1 → zonal harmonics and tesseral harmonics.

In the program PROP, IND is always zero.

The use of COMMON is:

/ORBIT/

EMU ≡ μ, ERAD ≡ R, EJ22 ≡ $J_{2,2}$, ELAM22 ≡ $\lambda_{2,2}$, L, DENSCH ≡ H, MJDOCH, ELEMT(6×6) and EJ(40) ≡ $J_1, J_2, \ldots, J_{40}$ are input.

QPD(25) to QPD(30) inclusive, are output.

/PRECON/

ZONSEC(4), DRASEC(4), TDRAG(2), F ≡ f, QSQ ≡ q^2, EX, P4 ≡ p_4, P5 ≡ p_5, P10 ≡ p_{10}, P11 ≡ p_{11}, P13 ≡ p_{13}, P14 ≡ p_{14}, P15 ≡ p_{15}, P16 ≡ p_{16}, P17 ≡ p_{17}, TW ≡ 3W, STL ≡ $\sin 2\ell_{2,2}$, CTL ≡ $\cos 2\ell_{2,2}$, TTHMO ≡ $2\dot{\ell}_{2,2}$, Z4F ≡ $z_4 f$ and ABCD (38×8) are output.

ATMROT ≡ Λ is input.

The segment PRELON bases its calculations upon the current estimate of the orbit, which is contained in the array ELEMT. Specifically, the following equivalences are used:

e_0 ≡ ELEMT(1,1), i_0 ≡ ELEMT(2,1), Ω_0 ≡ ELEMT(3,1)

ω_0 ≡ ELEMT(4,1), n_0 ≡ ELEMT(6,1) and $\dot{n}$ ≡ ELEMT(6,2).

The semi-major axis, a_0, is calculated using the definition given by Gooding and Tayler (1968),

$$a_0 = (\mu/n_0^2)^{1/3} - \frac{J_2}{2} R^2 (1 - e_0^2)^{-3/2} (1 - 3f/2)(\mu/n_0^2)^{-1/3} .$$

Many of the output variables in /PRECON/ are available directly since

$q^2 = 1 - e_0^2$,

$f = \sin^2 i_0$,

$EX = e_0 \sin i_0$,

$p_4 = \sin i_0$,

$p_5 = \cos i_0$,

$p_{10} = e_0 \cos i_0$,

$p_{11} = (21/16) J_2^2 (R/p_0)^4 \; n_0 e_0^2 \sin i_0 \cos i_0 (1 - 15f/7)$,

$p_{13} = 3J_2 (R/p_0)^2 \; n_0 e_0 (4 - 5f)$,

$p_{14} = (15/2) J_2 (R/p_0)^2 \; n_0 e_0 f \cos i_0$,

$p_{15} = \sin 2(\omega_0 - \pi/2) = -2 \sin\omega_0 \cos\omega_0$,

$p_{16} = \cos 2(\omega_0 - \pi/2) = 1 - 2\cos^2\omega_0$

and

$p_{17} = (21/16) J_2^2 (R/p_0)^4 \; n_0 e_0 \sin i_0 (1 - 15f/14)$.

The quantities $z_4 f$, $\cos 2\ell_{2,2}$, $\sin 2\ell_{2,2}$, 3W and $2\dot{\ell}_{2,2}$ were introduced in the PROP5 version of PROP (Gooding, Odell, 1973) so that perturbations due to tesseral harmonics could be given a more satisfactory representation than formerly. The quantities are given by

$z_4 f = n_0 J_2 (R/p_0)^2 f$,

$3W = 3J_{2,2} \, n_0 (R/p_0)^2$,

$\ell_{2,2} = \hat{\theta} + \lambda_{2,2} - \Omega_0$

where $\hat{\theta}$ is the modified sidereal angle at MJDOCH (see subsection 4.38, SIDANG). Hence the angle $\ell_{2,2}$ is the angle between the axis of the bulge in the equator represented by $J_{2,2}$ and Υ, at the epoch under investigation.

$$\dot{\ell}_{2,2} = \omega_E - \dot{\Omega}_{sec}$$

where ω_E is EOMEGA, a new variable introduced, and defined, by a new DATA statement setting ω_E equal to the rotation rate of the Earth, in radians.sec^{-1}. $\dot{\Omega}_{sec}$ is the secular rate of change of Ω. Hence $\dot{\ell}_{2,2}$ is the rate of change of $\ell_{2,2}$, in radians.sec^{-1}.

The quantities QPD(25) to QPD(30) inclusive were introduced when the segment PARSHL (subsection 4.23) was amended to make "J_2 corrections" to partial derivatives. The quantities required are the partial derivatives of $\Omega_{(1)}$ and $\omega_{(1)}$ with respect to e_0, i_0 and n_0 where

$$\Omega_{(1)} \doteq -\frac{3}{2} J_2(R/p_0)^2 n_0 \cos i_0$$

and

$$\omega_{(1)} = \frac{3}{4} J_2(R/p_0)^2 n_0(4 - 5 \sin^2 i_0).$$

These quantities are not mentioned in the PRELON segment specification by Merson, Gooding and Tayler. They are set such that

$$\text{QPD(25)} = \frac{\partial\Omega_{(1)}}{\partial e_0} = -6J_2(R/p_0)^2 n_0 e_0 \cos i_0(1 - e_0^2)^{-1} ,$$

$$\text{QPD(26)} = \frac{\partial\Omega_{(1)}}{\partial i_0} = (3/2)J_2(R/p_0)^2 n_0 \sin i_0,$$

$$\text{QPD(27)} = \frac{\partial\Omega_{(1)}}{\partial n_0} = -(7/2)J_2(R/p_0)^2 \cos i_0,$$

$$\text{QPD(28)} = \frac{\partial\omega_{(1)}}{\partial e_0} = 3J_2(R/p_0)^2 n_0 e_0(4 - 5f)(1 - e_0^2)^{-1},$$

$$\text{QPD(29)} = \frac{\partial\omega_{(1)}}{\partial i_0} = -(15/2)J_2(R/p_0)^2 n_0 \sin i_0 \cos i_0$$

and

$$QPD(30) = \frac{\partial \omega_{(1)}}{\partial n_0} = (7/12)J_2(R/p_0)^2(12 - 15f).$$

If we take a_k, b_k, c_k and d_k as defined by Merson (1966), the second stage of PRELON fills the first four columns of the array ABCD with quantities

$$ABCD(k,1) = \overline{a}_k ,$$
$$ABCD(k,2) = \overline{b}_k ,$$
$$ABCD(k,3) = \overline{c}_k \text{ and}$$
$$ABCD(k,4) = \overline{d}_k \qquad k = 1,\ldots,L-2$$

where

$$\overline{\psi}_k = \psi_k/(e_0 \sin i_0)^k,$$

in which ψ is any one of a, b, c and d.

Quantities $\overline{b}_0$, $\overline{c}_0$ and $\overline{d}_0$, also defined by Merson (1966), are calculated in the second stage of PRELON.

The third stage of PRELON modifies the contents of ABCD such that the eight quantities in the kth row of ABCD are:

$$a_k^{\,c},\ b_k^{\,c},\ c_k^{\,c},\ d_k^{\,c},\ a_k^{\,s},\ b_k^{\,s},\ c_k^{\,s} \text{ and } d_k^{\,s}$$

given by

$$\psi_k^{\,c} = (\psi_k/e_0 \sin i_0) \cos k(\omega_0 - \pi/2),$$

$$\psi_k^{\,s} = (\psi_k/e_0 \sin i_0) \sin k(\omega_0 - \pi/2)$$

in which ψ is any one of a, b, c and d.
In this way no division by $e_0 \sin i_0$ is made so potential singularities are avoided.

Secular perturbations, $\dot{e}_{sec}$, $\dot{i}_{sec}$, $\dot{\Omega}_{sec}$ and $\dot{\omega}_{sec}$ are calculated such that

$$\text{ZONSEC}(1) = \dot{e}_{sec} = 0,$$

$$\text{ZONSEC}(2) = \dot{i}_{sec} = 0,$$

$$\text{ZONSEC}(3) = \dot{\Omega}_{sec} = -\frac{\overline{c}_0}{2} - \frac{3}{8} J_2^2 (R/p_0)^4 \, n_0 \cos i_0 \left[9 + e^2 - 12q - f\left(10 - \frac{5e^2}{4} - 18q\right)\right]$$

and

$$\text{ZONSEC}(4) = \dot{\omega}_{sec} = \frac{1}{2}(\overline{b}_0 + \overline{d}_0 + \cos i_0 \overline{c}_0) + \frac{1}{4} J_2^2 (R/p_0)^4 \cos i_0 \left\{(12 - 15f)\left[3\left(1 + \frac{e^2}{4} - q\right) - f\left(\frac{43}{16} - \frac{e^2}{32} - \frac{9q}{2}\right)\right] - \frac{15}{4} e^2 \cos^4 i_0\right\} .$$

These are equivalent to the expressions given by Merson (1966) for secular changes in Ω and ω when J_2^2 terms are taken into account. The secular changes in e and i are time dependent functions of J_2^2 and are calculated when required by the segment XLONG (subsection 4.41).

The final section of PRELON calculates perturbations due to drag and it is not executed if the input parameter IND is non-zero. Three changes have been made to this section of code in the new version of PROP.

The first change is that the rotation rate of the Earth, ω_E, is taken from the new DATA statement, rather than being written out specifically each time it is used. This makes the execution of the section easier to follow.

The second change involves the rotation rate of the upper atmosphere. Merson, Gooding and Tayler state that this is assumed to be 8.75×10^{-5} radians.sec^{-1}. However, the program used a more precise value of $8.750547574 \times 10^{-5}$ radians.sec^{-1}, which is $1.2\omega_E$. The original choice of the factor $\Lambda = 1.2$ was probably influenced by King-Hele (1969), who concludes an average value of Λ to be 1.2. Other, later, work (King-Hele, Walker, 1977) might indicate a more appropriate choice of Λ to the user, for the orbit and epoch under investigation. The assumed rotation rate of the upper atmosphere has been changed to $\Lambda\omega_E$ radians.sec^{-1}, where Λ has a value selected by the user (see PROP, subsection 4.28).

The third change in the section concerns the programming of the expression used for DRASEC(2). As given by Merson, Gooding and Tayler, and also by Gooding (1969b), intermediate variables are calculated in the form

$$z = a_0 e_0 / H,$$

$$y_1 = \frac{240z + 153z^2 + 68z^3 + 24z^4}{480 + 306z + 196z^2 + 80z^3 + 24z^4} \quad ,$$

$$y_2 = \frac{30z^2 + 16z^3 + 12z^4}{240 + 153z + 98z^2 + 40z^3 + 12z^4} \quad .$$

From these the segment derives variables

$$F^{\frac{1}{2}} = 1 - (\Lambda\omega_E/n_0)(1 - e_0)^{3/2} \cos i_0/(1 + e_0)^{\frac{1}{2}} \ ,$$

$$(1 - e_0^2)k_1/k_2 = (1 - e_0)y_1 + \frac{e_0}{2}\left(\frac{1 - e_0}{1 + e_0}\right)(1 + y_2 + 2y_1 - 4y_1^2),$$

$$k_3/k_4 = \frac{(1 - e_0)^3}{(1 + e_0)} y_2 + e_0 \left(\frac{1 - e_0}{1 + e_0}\right)^2 (2 + 3e_0)(2y_2 - y_1 - y_1 y_2) \ ,$$

and

$$k_4/k_2 = \frac{(1 - e_0)^3}{(1 + e_0)} + 2e_0(2 + e_0)\left(\frac{1 - e_0}{1 + e_0}\right)^2 (1 - y_1).$$

DRASEC and TDRAG are then set such that

$$DRASEC(1) = -(2/3n_0)(1 - e_0^2)k_1/k_2,$$

$$DRASEC(2) = -(\Lambda\omega_E \sin i_0/6n_0^2F^{\frac{1}{2}}q)k_4/k_2,$$

$$DRASEC(3) = DRASEC(4) = 0,$$

$$TDRAG = (\dot{n}/n_0)(\Lambda\omega_E/6n_0F^{\frac{1}{2}}q)k_3/k_2$$

and

$$TDRAG(2) = TDRAG(1) \times \sin i_0.$$

Examination of these expressions shows a discrepancy between these and the expressions given by Merson, Gooding and Tayler and by Gooding. The expression for k_4/k_2 should be

$$k_4/k_2 = \frac{(1-e_0)^3}{(1+e_0)} + 2e_0(2+e_0)\frac{(1-e_0^2)}{(1+e_0)^2}(1-y_1)$$

$$= \frac{(1-e_0)^3}{(1+e_0)} + 2e_0(2+e_0)\left(\frac{1-e_0}{1+e_0}\right)(1-y_1).$$

Consequently the program has been amended to remove one factor of $(1-e_0)/(1+e_0)$ from the second term of k_4/k_2. This change will have a small impact upon the secular rate of change of the orbital inclination due to drag. The effect of the change is less for less eccentric orbits than for highly eccentric orbits. It is likely that changing the value of Λ will have more impact upon the orbit obtained, and this has been investigated in Section 5.

4.26 PRENUT (Tayler, Gooding, 1969b)

This segment makes corrections in the observed right ascension, α, and declination, δ, for precession and nutation. The result is to give α and δ relative to the reference system used within PROP. The equinox of this system is the projection of the equinox of 1950.0 onto the equator of date.

There are five parameters:

ALPHA: input and output right ascension, α, in radians,

DELTA: input and output declination, δ, in radians,

MJD: input date of the observation of α,δ, in modified Julian date form (see subsection 4.18),

TIME: input time of the observation of α,δ as a decimal of a day,

INEQEQ: input indicator of the date of the equator and equinox to which the input α,δ refer.

Corrections are applied to α,δ by PRENUT in three stages. The first stage corrects for the smooth, long periodic motion of the pole of the equator about the pole of the ecliptic. This motion has a period of approximately 26,000 years and is called precession. The amount of the correction is a function of the number of days between the epoch of the star catalogue (indicated by INEQEQ) and 1950.0, and the time between 1950.0 and the time of the observation.

The second stage corrects α,δ for the short periodic, irregular, motion of the true pole around the mean pole. This motion has an amplitude of approximately 9 seconds of arc, a period of approximately 18.6 years, and is called nutation. The amount of the correction is a function of the time between 1950.0 and the time of the observation.

The third stage corrects α for the rotation between the true equinox and the pseudo-equinox used by PROP (the projection of the mean equinox of 1950.0 on the equator of date).

The expressions used to give each stage of the corrections are given by Gooding and Tayler (1968). A more complete description of the effects of precession and nutation is given in Section 2 of the Explanatory Supplement to the Astronomical Ephemeris (H.M. Nautical Almanac Office, 1961).

4.27 PROCES (Tayler, Gooding, 1969c)

This segment corrects an array of satellite observations for Earth rotation, light time, refraction, precession and nutation.

There are three parameters:

DATA: input and output, (21 × MAXIM) array of observations.

MAXIM: input maximum number of observations,

IND: input indicator of whether all corrections are required.

The use of COMMON is:

/CNTROL/

IOBSNS is input

/ORBIT/

MJD, TIME, X, Y and Z are overwritten as each observation is considered in turn.

COMMON//

NUMOBS is input.

After processing each observation, PROCES sets DATA(2,) for that observation to unity. If IOBSNS = 2 this is the only processing done by PROCES.

On entry to PROCES the observing station Cartesian coordinates for each observation are stored in DATA(5,), DATA(6,) and DATA(7,). DATA(5,) is measured along an axis in the equatorial plane and toward the Greenwich meridian. Similarly, the sine and cosine of the observing station are stored in DATA(10,) and DATA(11,). Longitude is measured relative to the Greenwich meridian, positive Eastwards. To replace the values with values relative to the first point of Aries, ♈ , the function SIDANG(4.38) is used to give the modified sidereal angle, $\hat{\theta}$. The parameters to SIDANG are the date and time of the observation. The "rotated" values are then given by

$$X_{rotated} = \text{DATA}(5,\ ) \times \cos\hat{\theta} - \text{DATA}(6,\ ) \times \sin\hat{\theta},$$

$$Y_{rotated} = \text{DATA}(5,\ ) \times \sin\hat{\theta} + \text{DATA}(6,\ ) \times \cos\hat{\theta}$$

$$\sin(\text{longitude,rotated}) = \text{DATA}(10,\ ) \times \cos\hat{\theta} + \text{DATA}(11,\ ) \times \sin\hat{\theta}$$

$$\cos(\text{longitude,rotated}) = -\text{DATA}(10,\ ) \times \sin\hat{\theta} + \text{DATA}(11,\ ) \times \cos\hat{\theta}.$$

If IND is non-zero these values are stored in DATA(5,), DATA(6,), DATA(10,) and DATA(11,) respectively, DATA(2,) is set to unity and the next observation is considered. Otherwise light time, precession, nutation and refraction are taken into account. This is always the case in the program PROP.

The Cartesian coordinates of the satellite, x,y,z at the time of the observation, are calculated by the subroutine SATXYZ(4.33). The range, ρ, of the satellite from the observer may then be calculated from

$$\rho = \left\{(x - X_{rotated})^2 + (y - Y_{rotated})^2 + (z - \text{DATA}(7,\ ))^2\right\}^{\frac{1}{2}} .$$

If DATA(1,) = 1, 7, 8, 11, 14 or 15 then range is an observed quantity and the variable ρ is overwritten with the observed range, from DATA(12,). Light time correction is applied to the time of the observation by replacing the contents of DATA(4,) with

$$\text{DATA}(4,\ ) - \frac{\rho}{2.592 \times 10^{10}} ,$$

(2.592×10^{10} is the velocity of light in km/day).

An approximate light time correction is made to the observer position to give the observer position at the time the light left the satellite. If the longitude of the observer when the light is received is λ then the longitude when the light is transmitted is $\lambda - \delta\lambda$ where

$$\delta\lambda = 2\pi \times \text{ light travel time in days}$$

$$= \frac{2\pi\rho}{2.592 \times 10^{10}}$$

$\delta\lambda$ is small, and hence

$$\left.\begin{aligned}\sin(\lambda-\delta\lambda) &\simeq \sin\lambda - \delta\lambda\cos\lambda,\\ \cos(\lambda-\delta\lambda) &\simeq \cos\lambda + \delta\lambda\sin\lambda.\end{aligned}\right\} \quad (4.27.1)$$

Similarly the x and y coordinates of the observer are changed during light travel time to x_{light}, y_{light} where

$$\left.\begin{aligned}x_{light} &= X_{rotated} + \delta\lambda\ Y_{rotated}\ ,\\ y_{light} &= Y_{rotated} - \delta\lambda\ X_{rotated}\ .\end{aligned}\right\} \quad (4.27.2)$$

In the subroutine PROCES 2π is approximated by 6.3, hence

$$\delta\lambda \simeq \Delta\lambda = \frac{6.3\rho}{2.592\times10^{10}}\ .$$

The subroutine is written in such a way that the corrected values stored are:

$$\left.\begin{aligned}\mathrm{DATA}(5,\ ) &= X_{rotated} + \Delta\lambda\ Y_{rotated}\ ,\\ \mathrm{DATA}(6,\ ) &= Y_{rotated} - \Delta\lambda\left[X_{rotated} + \Delta\lambda\ Y_{rotated}\right],\\ \mathrm{DATA}(10,\ ) &= \sin\beta - \Delta\lambda\cos\lambda\ ,\\ \mathrm{DATA}(11,\ ) &= \cos\beta + \Delta\lambda(\sin\lambda - \Delta\lambda\cos\lambda).\end{aligned}\right\} \quad (4.27.3)$$

The differences between Equations (4.27.1), (4.27.2) and Equations (4.27.3) are of the order of $(\Delta\lambda)^2\ Y_{rotated}$ and $(\Delta\beta)^2\cos\lambda$ which are very small. Hence the error involved is negligible.

Further corrections to the observations, for precession, nutation and refraction, are required only if DATA(1,) = 2, 7, 12 or 14. That is, the observed quantities include either right ascension and declination or azimuth and elevation. The corrections are applied by calling the subroutines PRENUT(4.26) and REFCOR(4.30) as appropriate. The actual corrections applied are controlled by the contents of DATA(2,).

After all corrections have been applied DATA(2,) is set to unity before processing the next observation.

4.28 PROP

The segment PROP is the master segment of the program PROP. It is the first segment to be entered after the program has been loaded and calls the other segments of the program as they are required.

Since PROP is the master segment there are no parameters and it is not very meaningful to describe items in COMMON as input or output. Some items are preset, by the BLOCK DATA, to initial values as given in Appendix I. The following items in COMMON are referred to directly by the segment.

All of COMMON/CNTROL/ except IITT and ILINES, and all of COMMON /ORBIT/ except QPD(30) and DERIV(6 × 20) are referenced directly. In COMMON/PRECON/ only TW, STL, CTL, TTHMO and ATMROT are used. In COMMON// SIGMA(6), OEMTM(20 × 20), OPARAS(20), NOODOF, OSUMSQ, REJLEV(3), EPSLON and NUMPAR are all referenced, and in blank COMMON just INDRES is used.

The segment PROP controls the setting of initial values of some of the variables. These may be read from the user input "control card" or may be set to default values. The satellite and epoch under investigation are set by PROP, under user control. Initial elements are derived in various ways by the segment PROP. The method used is indicated by the user via the control parameter NEWSAT. PROP controls the input of observer coordinates and observations and then calls the differential correction process. The segment also controls the running of multiple runs within one job.

There have been a number of changes made to PROP, mostly to increase the users' control and the program flexibility.

The statement

EXTERNAL ECENTR,MA01A

has been removed. The segments ECENTR and MA01A are now referred to directly by the segment DIFCOR, rather than being parameters of DIFCOR. This has allowed ECENTR and MA01A to be placed legally in overlay units.

The statement

REAL MILTIM

has been removed, along with all references to the segment MILTIM, as described in subsection 4.17.

The statements

```
INTEGER EMERGE,EMERG1,EMERG2,EMERG3
DATA EMERG1,EMERG2,EMERG3/4H9998,4H////,4H/// /
```

have been removed, together with all references to these integers and the segment ERROR (see subsection 4.13). They were used in the off-lining of input, as described by Gooding (1974). They ensured also that runs in a multi-run job were not jeopardised by the failure of a previous, independent, run in the same job. These facilities have been removed. A physical card reader is not used during a PROP run. Instead a file-store file previously created via a card reader and, possibly, a VDU, is used. Hence off-lining of input is no longer necessary. The removal of off-lining gives an advantage if PROP fails when only some of the input data has been read, since the number of cards read by a job is output on the monitor file. The insurance against previous independent runs failing had to be removed, as described in subsection 4.13 ERROR. It is, therefore, no longer necessary to terminate input packs of data with

///

////

9998 cards may still be included to separate independent runs in a multi-run job. However, a fatal execution error causes a job to terminate.

A new legal value of NEWSAT, given on the control card by the user, has been introduced. If NEWSAT equals 8 then the initial elements for the PROP run are taken as identical to the initial elements for the previous run in the same job. These elements will have been stored in the new array STELS(6 × 6) by calling the segment MOVELS in the previous run (see subsection 4.19), and are restored to the array ELEMT(6 × 6) by calling MOVELS. If NEWSAT equals 8 then the epoch and satellite identity are also taken to be identical with the previous run, hence no epoch/identity card and no element cards should follow NEWSAT = 8 in the data pack. NEWSAT = 8 should not be used in the first run of a job.

The option was introduced because users wished to repeat an orbit determination, using a slightly different set of orbital parameters. This may now be done in a multi-run job. The data pack is set up with the control card, epoch/identity card, initial elements, previously determined orbit cards (if applicable), observer location cards and observation cards followed by two 'zero', or blank, cards. This will now be followed by a second control card, set such that:

KKKKK,MMMMM define the alternative model to be used,

MODE may be changed from that on the first control card, but this is unlikely,

NEWSAT will be set equal to 8,

the value of MJDINC is ignored,

MAXITN, MINOBS and JELTYP may be changed from those on the first control card, but this is unlikely,

NOTHER may be set to zero. If it is set to non-zero then it must be set to the same value as it had on the previous control card. "Previously determined orbit" cards should not be after this control card in either case but a non-zero NOTHER will cause the previous "previously determined orbit" to be incorporated into this run.

ITIMEC will be ignored,

ISENSR and IOBSNS will be set to 2,

L, EJ22, DENSCH and EPSLON may be different from those on the previous control card.

This second control card will be followed either by a zero, or blank, card or by a further control card.

When NEWSAT equals 5 or 7, "SDC" elements are input by PROP calling the segment SDCELS (see subsection 4.34). A change has been introduced such that the satellite identity that appears on the element cards is returned as an integer parameter by SDCELS. This satellite identity is printed out by the segment PROP. It will probably not be identical to the satellite identity on the epoch/ identity card and it is printed for the users' information.

The satellite identity on the epoch/identity card is held in the real variable RIDENT. This change was introduced because satellites launched in or after 1984 will have identity numbers greater than the largest integer that may be held in an ICL 1900 series computer. See subsection 4.22, OBSIN.

A small change has been made to make values of MAXITN greater than 99 illegal. If more than 99 iterations are requested, as a maximum, a warning message is printed, and the job will skip to the next '9998' pre-control card and start the next independent run in the job. If there are no subsequent 9998 cards the job will fail with the message

FILE *CRO EXHAUSTED

on the monitoring file. This has been done to aid the optional suppression of printing, described in subsection 4.7.6.

A new, optional, control card, called the "Aston Control Card" (ACC) has been introduced. An ACC may precede any ordinary control card in a PROP run. The ACC is identified by having

9996

punched in the first four columns, and may hold two further pieces of information.

In columns 66 to 70 of the ACC may be punched, in a format of F5.0, an alternative value of ATMROT. This is the atmospheric rotation rate. It is assumed, when calculating the drag on the satellite, that the atmosphere is rotating around the Earth at a rate of ATMROT times the rotation rate of the Earth, see subsection 4.25, PRELON. A default value of ATMROT of 1.2 is used in any run for which an ACC is not used to alter its value. If an ACC is present with columns 66 to 70 either left blank or zero, then the default value is assumed. The value of ATMROT used is printed at the beginning of each PROP run.

The ACC may also be used to change the zonal harmonics (J's) used in PROP. Once changed in a PROP job by an ACC, the new set of J's is used for all runs in the job, unless replaced using another ACC. Modifications to the J's used in any run may be made, as described by Gooding (1974), by altering the value of L on the control card of a run, or by setting L negative and temporarily replacing up to five of the J's for the run. To change the set of J's by an ACC,columns 6 to 10 of the ACC should contain a positive integer in the format I5. This integer, JMAX, must not exceed 40. If it does, then the job will be stopped, with a warning message. After the ACC there should then follow cards containing the JMAX new J's, each multiplied by a factor of 10^6. They should be arranged, five values per card, in the format 5F10.3. If JMAX is not a multiple of five then the last "new J" card will contain less than five values. The "new J's" should be given, starting from J_1 (usually very small) in ascending order, with no gaps in the sequence. The J's to be used are printed at the beginning of each run.

It should be noted that PROP calls the subroutines DATE and TIME. These subroutines are available via the compiler on the ICL 1900 series machines. The program may need modifying if it is to be run on some other machine. They each have one parameter, output set to the date and time at which the subroutines are called, respectively. The output parameters are in a format suitable for printing with an A8 format.

4.29 PVREAD

This segment reads position and velocity components of a satellite, prints them and converts them into orbital elements.

There are three parameters:

IR: input channel number from which the satellite position and velocity are to be read,

IP: input channel number to which the satellite position and velocity are to be sent for printing,

LOGCHK: output logical indicator of success, or failure, of reading.

The use of COMMON is:

/ORBIT/

MJDOCH is input,

X, Y, Z, XDOT, YDOT and ZDOT are output as read by PVREAD.

ELEMT(6 × 6) is output during the call of SATELS.

The output parameter LOGCHK is set to TRUE initially. The internal variable MCHECK is read from channel IR with X, Y, Z, XDOT, YDOT and ZDOT using the format:

I5,3X,6F12.0.

The position and velocity are then printed out on channel IP.

If MCHECK is non-zero but not equal to MJDOCH then the output parameter is set to FALSE.

The position and velocity are converted to mean elements e_0, i_0, Ω_0, ω_0, M_0 and M_1 in ELEMT by the segment SATELS(4.32). Steps are taken to ensure that X, Y, Z, XDOT, YDOT and ZDOT in COMMON/ORBIT/ are re-instated to the values read from channel IR, after returning from SATELS.

PVREAD is called by PROP(4.28) only if the control parameter NEWSAT = 4.

4.30 REFCOR (Tayler, 1969c)

This segment corrects observations for the effects of refraction.

There are five parameters:

N: input indicator of the corrections required,

E: input observed elevation (E radians),

R: input geocentric range of the satellite (r km),

RS: input radial distance of the observer from the Earth's centre (R_s km),

ECORR: output corrected elevation (E_{corr} radians).

The indicator, N, may take any one of four different values:

N = 1 → correct optical observation for total refraction,

N = 2 → correct optical observation for parallactic refraction,

N = 3 → correct radio observation for total refraction,

N = 4 → correct radio observation for parallactic refraction.

The cases N = 3 and N = 4 are never used within the current version of PROP, all radio observations being fully corrected for refraction before being read in by the segment OBSIN (see subsection 4.22). However, REFCOR has been written to correct uncorrected radio observations in case OBSIN is modified to read such observations at some future date.

The case N = 1 is used when elevation is an observed, uncorrected quantity. The case N = 2 is used when an observation of right ascension and declination has been made and corrected for astronomical refraction. Such an observation will have been converted to an equivalent azimuth and elevation by PROP before REFCOR is called.

The method of correcting observations is given by Tayler (1969c), and by Gooding and Tayler (1968). It is also given by Merson (1962), who refers to Brown (1957). Tayler (1969c), Merson (1962) and Brown (1957) all refer to Garfinkel (1944), which has been "adapted to automatic computers", Garfinkel (1967), but requires a knowledge of atmospheric conditions at the time the observation was made.

The correction is made by first calculating

$$\chi = \tan^{-1}\left[\frac{\cos E}{8.4\sin E}\right]$$

and

$$\Delta E = 10^{-3}\left[d\tan^7\chi + c\tan^5\chi + b\tan^3\chi + a\tan\chi\right]$$

where a, b, c and d are given in Table 4.30.1.

Table 4.30.1 Refraction coefficients.

	a	b	c	d
Total refraction	4.63731	3.18832	0.86834	1.20386
Parallactic refraction	4.63688	2.73481	0.84958	0.03983

To correct for total refraction

$E' = E - \Delta E,$

$\gamma = \cos^{-1}((R_S/r)1.000276454\cos E)$

and $\theta = \gamma - E'$

are calculated. To correct for parallactic refraction E', γ and θ are calculated as

$E' = E + \Delta E$

$\gamma = \cos^{-1}((R_S/r)1.000276454\cos E')$

and $\theta = \gamma - E.$

In both cases, E_{corr} is given by

$$E_{corr} = \tan^{-1}(\cot\theta - (R_S/r)\operatorname{cosec}\theta).$$

If the observation is made by radio signal E_{corr} must then be modified on the basis of

$$(E_{corr} - E)_{radio} = 5(E_{corr} - E)_{optical} \quad \text{if } N = 3,$$

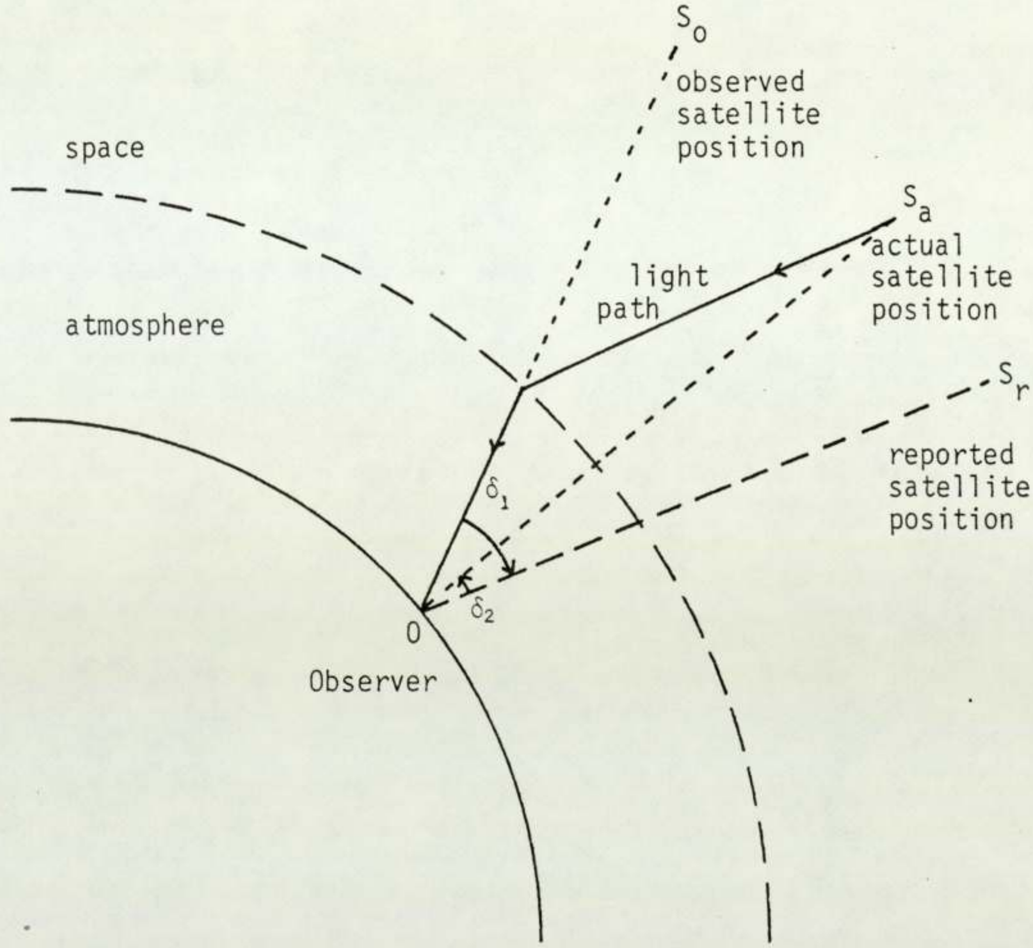
and

$$(E_{corr} - E')_{radio} = 5(E_{corr} - E')_{optical} \quad \text{if } N = 4.$$

The factor 5 is assumed on the basis of a result given by Gaposchkin (1964) and is thought to be a function of the frequency used. It may have to be modified in the light of experience.

Parallactic refraction, or refractive parallax, is described by McCrosky and Posen (1968). A simplified view of the situation is shown in Figure 4.30.1, where the atmosphere is assumed to have a discrete boundary. When the satellite is at S_a the observer at O observes the satellite to be at S_o, the light from the satellite having been refracted by the atmosphere. However, to record the satellite's position in terms of right ascension and declination, the observer notes its position relative to the background of stars. The positions of the stars are found from a star atlas, corrected for astronomical refraction, δ_1. The satellite's position is reported to be S_r. However, the stars are infinitely far away compared with the range of the satellite and the parallax, δ_2, is corrected for by REFCOR when IND = 2 or 4.

Figure 4.30.1 Parallactic Refraction



4.31 REJECT (Gooding, 1968d)

This segment decides which levels of observations are to be accepted and which rejected. It collects all those levels to be accepted into level 1.

There are nine parameters:

LEVEL: Input 4 element array of number of observations assigned to each of the four levels.

NUMPAR: Input number of orbital parameters.

LEVREJ: Input maximum number of observations which may be rejected.

JLEV: Input and output 4 element array of number of observed quantities assigned to each of the four levels. On output JLEV(1) holds the number of accepted observed quantities.

SUMSQ: Input and output 3 element array of the sums of the squares of the weighted residuals of the observed quantities in the top three levels. On output SUMSQ(1) holds the sum of the squares of the weighted residuals of all the accepted observed quantities.

EMTM: Input and output $(20 \times 20 \times 3)$ array of three (20×20) matrices $\mathcal{M}^T\mathcal{M}$, at the top three levels. On output EMTM(, , 1) holds $\mathcal{M}^T\mathcal{M}$ of all the accepted observations.

EMTY: Input and output (20×3) array of three 20 element vectors $\mathcal{M}^T\underset{\sim}{y}$ at the top three levels. On output MTY(,1) holds $\mathcal{M}^T\underset{\sim}{y}$ of all the accepted observations.

MAXEPT: Output maximum level now accepted.

NOWREJ: Output number of observations rejected.

REJECT accepts and rejects quantities which have previously been assigned to four different levels. It does this on the basis of the contents of the array LEVEL. Each value in LEVEL is compared with zero and LEVREJ to ascertain whether that level is to be accepted or rejected.

No more than one non-empty level may be rejected. Level 4 is always rejected. If LEVEL(I) > LEVREJ, I ≠ 4, then level I is not rejected. It is not possible to reject level 2 and accept level 3.

The effect of REJECT is summarised in Table 4.31.1

Table 4.31.1 The Effect of REJECT

Input			Output		
LEVEL(4)	LEVEL(3)	LEVEL(2)	MAXEPT	NOWREJ	levels added into level 1
$\ell_4 \neq 0$	$\ell_3 \neq 0$	anything	3	ℓ_4	2, 3
$\ell_4 \neq 0$	$\ell_3 = 0$	$\ell_2 \neq 0$	2	ℓ_4	2
$\ell_4 \neq 0$	$\ell_3 = 0$	$\ell_2 = 0$	1	ℓ_4	none
$\ell_4 = 0$	$\ell_3 \geqslant$ LEVREJ	$\ell_2 \neq 0$	3	0	2, 3
$\ell_4 = 0$	$\ell_3 \geqslant$ LEVREJ	$\ell_2 = 0$	3	0	3
$\ell_4 = 0$	$0 < \ell_3 <$ LEVREJ	$\ell_2 \neq 0$	2	ℓ_3	2
$\ell_4 = 0$	$0 < \ell_3 <$ LEVREJ	$\ell_2 = 0$	1	ℓ_3	none
$\ell_4 = 0$	$\ell_3 = 0$	$\ell_2 \geqslant$ LEVREJ	2	0	2
$\ell_4 = 0$	$\ell_3 = 0$	$\ell_2 <$ LEVREJ	1	ℓ_2	none

At the end of the subroutine REJECT the parameter NOWREJ has been set to LEVEL(I) where I is the number of the non-empty level rejected, or zero if none were rejected. MAXEPT is set to the maximum accepted level. All accepted levels are "added in" to level 1.

For example, if levels 3 and 4 are rejected but level 2 is accepted, then the output arrays are adjusted by

JLEV(1) = JLEV(1) + JLEV(2),

SUMSQ(1) = SUMSQ(1) + SUMSQ(2),

EMTM(I1,I2,1) = EMTM(I1,I2,1) + EMTM(I1,I2,2)

I1,I2 = 1,..,NUMPAR,

EMTY(I1,1) = EMTY(I1,1) + EMTY(I1,2)

I1 = 1,..,NUMPAR

MAXEPT is set to 2 and NOWREJ to LEVEL(3) + LEVEL(4).

This occurs if either

LEVEL(4) ≠ 0, LEVEL(3) = 0, LEVEL(2) ≠ 0,

or LEVEL(4) = 0, 0 < LEVEL(3) < LEVREJ, LEVEL(2) ≠ 0,

or LEVEL(4) = 0, LEVEL(3) = 0, LEVEL(2) ⩾ LEVREJ.

4.32 SATELS (Gooding 1974, Appendix A)

This segment computes six mean orbital elements of the satellite from six osculating Cartesian components of position and velocity.

There are no parameters. The use of COMMON is:

/ORBIT/

EMU $\equiv \mu$, MJDOCH, X $\equiv$ x, Y $\equiv$ y, Z $\equiv$ z, XDOT $\equiv \dot{x}$, YDOT $\equiv \dot{y}$, ZDOT $\equiv \dot{z}$ and EJ(2) $\equiv J_2$ are input.

ELEMT(6×6) is output, together with MJD set equal to MJDOCH and TIME set to zero.

The segment SATELS is called when the initial estimate of the satellite orbit is in the form of Cartesian position and velocity components, $x, y, z, \dot{x}, \dot{y}, \dot{z}$, which have been read by the segment PVREAD (see subsection 4.29). We denote the components in general by w_k, k = 1,—,6. The w_k read by PVREAD are osculating components $w_{k_{osc}}$. The required contents of ELEMT are

ELEMT(1,1) = e_0, ELEMT(2,1) = i_0, ELEMT(3,1) = Ω_0, ELEMT(4,1) = ω_0,
ELEMT(5,1) = M_0 and ELEMT(6,1) = ELEMT(5,2) = n_0

with the rest of ELEMT set to zero, where e_0, i_0, Ω_0, ω_0, M_0 and n_0 are mean elements. In general we denote the set of elements by ε_j, j = 1,—,6, with $\varepsilon_{j_{mean}}$ denoting the mean value of ε_j and $\varepsilon_{j_{osc}}$ denoting the osculating value of ε_j. The $\varepsilon_{j_{osc}}$ which may be derived from the input $w_{k_{osc}}$ are related to the $\varepsilon_{j_{mean}}$ by

$$\varepsilon_{j_{osc}} = \varepsilon_{j_{mean}} + \delta\varepsilon_{j_{mean}}$$

where $\delta\varepsilon_{j_{mean}}$ is the short periodic perturbation of ε_j. The calculation of $\delta\varepsilon_{j_{mean}}$ should be based upon the mean elements $\varepsilon_{j_{mean}}$. The $\delta\varepsilon_{j_{mean}}$

are, therefore, not directly available.

To overcome this, the $\varepsilon_{j_{osc}}$ are obtained from the $w_{k_{osc}}$ by calling the segment COTOEL (see subsection 4.5). The segment SATXYZ (see subsection 4.33) is then called in a reverse manner. That is, with parameter -1 and EJ(2) temporarily set to $-J_2$. In this way a set of approximate Cartesian components, $w_{k_{app}}$, based upon elements

$$\varepsilon_{j_{osc}} - \delta\varepsilon_{j_{osc}} \qquad j = 1,—,6$$

is obtained.

The segment COTOEL is then called to convert these $w_{k_{app}}$ to the elements

$$\varepsilon_{j_{app}} = \varepsilon_{j_{osc}} - \delta\varepsilon_{j_{osc}} \quad .$$

EJ(2) is reset to J_2 and SATXYZ called again with parameter -1 to obtain another set of components, $w_{k_{near}}$. This time the coordinates are related to

$$\varepsilon_{j_{app}} + \delta\varepsilon_{j_{app}} \qquad j = 1,—,6 \; .$$

Mean Cartesian components $w_{k_{mean}}$ are then calculated from

$$w_{k_{mean}} = w_{k_{osc}} + w_{k_{app}} - w_{k_{near}}$$

and thence, by a further call of the segment COTOEL, the elements related to $w_{k_{mean}}$ are found. These elements are output in the array ELEMT.

4.33 SATXYZ (Tayler, Gooding, no date available)

This segment computes geocentric Cartesian coordinates of the position and velocity of a satellite at a given date and time from a set of orbital elements.

There is one parameter only:

IND: input indicator of which perturbations are to be taken into account in the calculations.

IND = -1 → short periodic perturbations,

IND = 0 → all perturbations,

IND = 1 → long periodic and luni-solar perturbations,

IND = 2 → long periodic perturbations.

The option to include luni-solar perturbations is nominal at present as the segment to calculate them, XLUSOL, does not exist.

The use of COMMON is

/ORBIT/

EMU ≡ μ, ERAD ≡ R, MJDOCH, NOMIAL(6), ELEMT(6 × 6), MJD and TIME are input.

X ≡ y, Y ≡ y, Z ≡ z, XDOT ≡ $\dot{x}$, YDOT ≡ $\dot{y}$, ZDOT ≡ $\dot{z}$ and QPD(1) to QPD(24) inclusive are output.

/WORKIN/

The first 24 locations of /WORKIN/ are used as workspace during the execution of SATXYZ.

The segment SATXYZ is called by four different segments of the program PROP. The segment PROP(4.28) calls SATXYZ, with parameter IND = 1 and J_2 set, temporarily, to zero, when the prediction of initial elements from one epoch to another is required. The segment PROCES(4.27) calls SATXYZ, with parameter IND = 0, so that the time taken for light to travel from the satellite to the observer may be estimated. The segment SATELS(4.36) calls SATXYZ, with parameter IND = -1, several times during the conversion of osculating Cartesian coordinates to mean elements. In the above three cases the output quantities QPD(1) to QPD(24) are calculated, but not required by the calling segment. The fourth segment to call SATXYZ is ECENTR(4.9), with parameter IND = 0. In this case the output of QPD(1) to QPD(24) are used by the calling segment, as well as the x, y, z, $\dot{x}$, $\dot{y}$ and $\dot{z}$ calculated.

The required quantities in QPD are:

QPD(1) = $\bar{a}$, QPD(2) = $\bar{e}$, QPD(3) = $\bar{n}$, QPD(4) = $\cos\bar{v}$,
QPD(5) = $\sin\bar{v}$, QPD(6) = $\overline{p/r}$, QPD(7) = $\dot{r}$, QPD(8) = $(1-\bar{e}^2)^{\frac{1}{2}}$,
QPD(9) = t, QPD(22) = r, QPD(23) = $(\mu p)^{\frac{1}{2}}/r$, QPD(24) = $(\mu/p)^{\frac{1}{2}}$

where barred quantities are related to unperturbed quantities and unbarred quantities are related to osculating quantities. Here t is the time, in seconds, between the epoch MJDOCH and the observation time MJD.TIME.

The required contents of QPD(10) to QPD(21) are the three components of each of four vectors, where:

QPD(10), (11), (12) = $\bar{F}_1$, $\bar{F}_2$, $\bar{F}_3$;
QPD(13), (14), (15) = $\bar{G}_1$, $\bar{G}_2$, $\bar{G}_3$;
QPD(16), (17), (18) = A_1, A_2, A_3 and
QPD(19), (20), (21) = B_1, B_2, B_3.

These vectors are given by:

$$\bar{F} = \begin{bmatrix} \sin\bar{u} \ \sin\bar{\Omega}\sin i \\ -\sin\bar{u} \ \cos\bar{\Omega}\sin i \\ \sin\bar{u} \ \cos i \end{bmatrix}, \qquad \bar{G} = \begin{bmatrix} \cos\bar{u} \ \sin\bar{\Omega}\sin i \\ -\cos\bar{u} \ \cos\bar{\Omega}\sin i \\ \cos\bar{u} \ \cos i \end{bmatrix},$$

$$A = \begin{bmatrix} x/r \\ y/r \\ z/r \end{bmatrix} = \begin{bmatrix} \cos u\cos\Omega - \cos i\sin u\sin\Omega \\ \cos u\sin\Omega + \cos i\sin u\cos\Omega \\ \sin u\sin i \end{bmatrix}$$

and $B = \begin{bmatrix} -\sin u \cos\Omega - \cos i \cos u \sin\Omega \\ -\sin u \sin\Omega + \cos i \cos u \cos\Omega \\ \cos u \sin i \end{bmatrix}$,

where, again, barred quantities are related to unperturbed elements and unbarred quantities relate to perturbed, osculating quantities. We note that

$$B = \frac{\partial A}{\partial u}$$

and, if $\overline{A}$ and $\overline{B}$ denote A and B calculated using $\overline{u}$ and $\overline{\Omega}$ instead of u and Ω, then

$$\overline{F} = \frac{\partial\overline{A}}{\partial i} \quad \text{and} \quad \overline{G} = \frac{\partial\overline{F}}{\partial\overline{u}} = \frac{\partial^2\overline{A}}{\partial i \partial\overline{u}} .$$

We will refer to items in the array ELEMT by the convention implied by

$$\text{ELEMT} = \begin{bmatrix} e_0 & e_1 & e_2 & e_3 & e_4 & e_5 \\ i_0 & i_1 & i_2 & i_3 & i_4 & i_5 \\ \Omega_0 & \Omega_1 & \Omega_2 & \Omega_3 & \Omega_4 & \Omega_5 \\ \omega_0 & \omega_1 & \omega_2 & \omega_3 & \omega_4 & \omega_5 \\ M_0 & M_1 & M_2 & M_3 & M_4 & M_5 \\ n_0 & n_1 & n_2 & n_3 & n_4 & n_5 \end{bmatrix} .$$

It would be most unusual for all 36 items in ELEMT to be non-zero. In general the number of non-zero items in the Ith row of ELEMT is given by NOMIAL(I) + 1. The unperturbed elements at time t are then given by:

$$\overline{e} = e_0 + e_1 t + \text{———} + e_{\text{NOMIAL(1)}} t^{\text{NOMIAL(1)}} ,$$

$$\overline{i} = i_0 + i_1 t + \text{———} + i_{\text{NOMIAL(2)}} t^{\text{NOMIAL(2)}} ,$$

$$\overline{\Omega} = \Omega_0 + \Omega_1 t + \text{———} + \Omega_{NOMIAL(3)} t^{NOMIAL(3)} ,$$

$$\overline{\omega} = \omega_0 + \omega_1 t + \text{———} + \omega_{NOMIAL(4)} t^{NOMIAL(4)} ,$$

$$\overline{M} = M_0 + M_1 t + \text{———} + M_{NOMIAL(5)} t^{NOMIAL(5)} ,$$

$$\overline{n} = n_0 + n_1 t + \text{———} + n_{NOMIAL(6)} t^{NOMIAL(6)} .$$

The value of $\overline{M}$ is corrected so that it lies in the range

$$-\pi < \overline{M} \leqslant \pi.$$

Further unperturbed quantities may then be calculated, viz.,

$f = \sin^2 \overline{i}$,

$\overline{a} = (\mu/\overline{n}^2)^{1/3} - \frac{1}{2} J_2 R^2 (\mu/\overline{n}^2)^{-1/3} (1 - 3f/2)(1 - \overline{e}^2)^{-3/2}$,

$\overline{p} = \overline{a}(1 - \overline{e}^2)$

and $\overline{q} = (1 - \overline{e}^2)^{1/2}$.

A value of $\overline{E}$, the eccentric anomaly, is found from Kepler's equation by using the function EAFKEP with parameters $\overline{M}$ and $\overline{e}$. The true anomaly, $\overline{v}$, is then given by

$$\overline{v} = 2 \tan^{-1} \left[\left(\frac{1 + \overline{e}}{1 - \overline{e}} \right)^{\frac{1}{2}} \tan \frac{\overline{E}}{2} \right]. \qquad (4.33.1)$$

The expression (4.33.1) is derived here.

For any angle θ

$$\tan^2 \frac{\theta}{2} = \frac{\sin^2 \theta/2}{\cos^2 \theta/2} = \frac{\frac{1}{2}(1 - \cos\theta)}{\frac{1}{2}(1 + \cos\theta)} . \qquad (4.33.2)$$

The true anomaly, $\overline{v}$, may be related to the eccentric anomaly, $\overline{E}$, via the range $\overline{r}$ by

$$\frac{\overline{a}(1 - \overline{e}^2)}{1 + \overline{e}\cos\overline{v}} = \overline{r} = \overline{a}(1 - e\cos\overline{E})$$

from which we obtain

$$\cos\overline{v} = \frac{\cos\overline{E} - \overline{e}}{1 - \overline{e}\cos\overline{E}} \quad . \qquad (4.33.3)$$

Substituting Equation (4.33.3) into Equation (4.33.2) with $\theta \equiv \overline{v}$, we obtain

$$\tan^2\frac{\overline{v}}{2} = \frac{1 - \left(\dfrac{\cos\overline{E} - e}{1 - \overline{e}\cos\overline{E}}\right)}{1 + \left(\dfrac{\cos\overline{E} - e}{1 - \overline{e}\cos\overline{E}}\right)}$$

which may be rearranged to give

$$\tan^2\frac{\overline{v}}{2} = \left(\frac{1 - \cos\overline{E}}{1 + \cos\overline{E}}\right)\left(\frac{1 + \overline{e}}{1 - \overline{e}}\right)$$

$$= \tan^2\frac{\overline{E}}{2}\left(\frac{1 + \overline{e}}{1 - \overline{e}}\right)$$

by Equation (4.33.2), and hence the required result, Equation (4.33.1).

The final two unperturbed quantities calculated are

$$\overline{p}/\overline{r} = 1 + \overline{e}\cos\overline{v}$$

and

$$\overline{u} = \overline{v} + \overline{\omega} \ .$$

The quantity $(\mu/\overline{p})^{\frac{1}{2}}$ is not calculated in the new version of SATXYZ since it is no longer required (see subsection 4.41, XLONG).

The segments SHOPER (subsection 4.37) and XLONG are called with barred quantities as their parameters to obtain

$di_s, \sin\bar{i}d\Omega_s, du_s + \cos\bar{i}d\Omega_s, dp_s/2\bar{p}, dr_s/\bar{p}, d\dot{r}_s$

and

$di_\ell, \sin\bar{i}d\Omega_\ell, du_\ell + \cos\bar{i}d\Omega_\ell, da_\ell, de_\ell, \bar{e}dv_\ell$.

The suffix s indicates short periodic perturbations calculated by the segment SHOPER, and the suffix ℓ indicates long periodic perturbations calculated by the segment XLONG. Some of these perturbations are combined to give

$di = di_s + di_\ell$,

$\sin\bar{i}\,d\Omega = \sin\bar{i}\,d\Omega_s + \sin\bar{i}\,d\Omega_\ell$

and

$du + \cos\bar{i}\,d\Omega = du_s + \cos\bar{i}\,d\Omega_s + du_\ell + \cos\bar{i}\,d\Omega_\ell$.

The perturbed inclination, i, is given by

$i = \bar{i} + di$.

Hence

$$\sin i = \sin(\bar{i} + di) = \sin\bar{i}\cos di + \sin di \cos\bar{i}$$
$$\simeq \sin\bar{i} + di\cos\bar{i}$$

and

$$\cos i = \cos(\bar{i} + di) = \cos\bar{i}\cos di - \sin\bar{i}\sin di$$
$$\simeq \cos\bar{i} - di\sin\bar{i}.$$

The vectors $\bar{F}$ and $\bar{G}$ are then calculated and stored in QPD(10), (11), (12) and QPD(13), (14), (15) respectively. The vectors $\bar{A}$ and $\bar{B}$ are calculated and stored in the workspace /WORKIN/. From $\bar{A}$, $\bar{B}$, $\bar{F}$ and $\bar{G}$ the vectors A and B may be calculated, since

$$A = \bar{A} + d\bar{A}$$
$$= \bar{A} + \frac{\partial\bar{A}}{\partial\bar{u}}du + \frac{\partial\bar{A}}{\partial\bar{\Omega}}d\Omega$$
$$= \bar{A} + (du + \cos\bar{i}\,d\Omega)\bar{B} - (\sin\bar{i}\,d\Omega)\bar{G}$$

and

$$B = \overline{B} + d\overline{B}$$

$$= \overline{B} + \frac{\partial\overline{B}}{\partial\overline{u}}\, du + \frac{\partial\overline{B}}{\partial\overline{\Omega}}\, d\Omega$$

$$= \overline{B} - (du + \cos\overline{i}\, d\Omega)\overline{A} + (\sin\overline{i}\, d\Omega)\overline{F}.$$

The vectors A and B are stored in QPD(16), (17), (18) and QPD(19), (20), (21) in a normalised form, such that, if

$$A \equiv (A_1, A_2, A_3),\ B \equiv (B_1, B_2, B_3)$$

then

$$(A_1^2 + A_2^2 + A_3^2)^{\frac{1}{2}} = 1,$$

$$(B_1^2 + B_2^2 + B_3^2)^{\frac{1}{2}} = 1 \text{ and}$$

$$A_1B_1 + A_2B_2 + A_3B_3 = 0.$$

In this way A and B represent unit, mutually perpendicular, vectors. This normalisation has been discussed by Gooding (1974).

The quantities

QPD(7) = $\dot{r}$, QPD(22) = r, QPD(23) = $(\mu p)^{\frac{1}{2}}/r$ and QPD(24) = $(\mu/p)^{\frac{1}{2}}$ must still be calculated, together with x, y, z, $\dot{x}$, $\dot{y}$ and $\dot{z}$. To obtain these, some preliminary quantities are required. These are:

$$e'\cos v' = (\overline{e} + de_\ell)\cos(\overline{v} + dv_\ell)$$

$$= (\overline{e} + de_\ell)(\cos\overline{v}\cos dv_\ell - \sin\overline{v}\sin dv_\ell)$$

$$\simeq (\overline{e} + de_\ell)(\cos\overline{v} - dv_\ell\sin\overline{v})$$

$$\simeq (\overline{e} + de_\ell)\cos\overline{v} - \overline{e}dv_\ell\sin\overline{v}$$

and

$$e'\sin v' = (\overline{e} + de_\ell)\sin(\overline{v} + dv_\ell)$$

$$= (\overline{e} + de_\ell)(\sin\overline{v}\cos dv_\ell + \cos\overline{v}\sin dv_\ell)$$

$$\simeq (\overline{e} + de_\ell)(\sin\overline{v} + dv_\ell\cos\overline{v})$$

$$\simeq (\overline{e} + de_\ell)\sin\overline{v} + \overline{e}dv_\ell\cos\overline{v}$$

from which we obtain

$$p' = (\overline{a} + da_{\ell})\left[1 - (e'\cos v')^2 - (e'\sin v')^2\right]$$

$$= a'(1 - e'^2).$$

QPD(22) is then given by

$$QPD(22) = \frac{p'}{1 + e'\cos v'} + \frac{p'}{\overline{p}}\, dr_s = r.$$

QPD(24) is given by

$$QPD(24) = (\mu/p')^{\frac{1}{2}}$$

and QPD(23) is given by

$$QPD(23) = \left(\frac{\mu}{p'}\right)^{\frac{1}{2}} p'\left(1 + \frac{dp_s}{2\overline{p}}\right) \Big/ r$$

$$\simeq \frac{(\mu p')^{\frac{1}{2}}}{r}\left(1 + \frac{dp_s}{\overline{p}}\right)^{\frac{1}{2}}$$

$$= \frac{\mu^{\frac{1}{2}}}{r}\left(p' + \frac{p'}{\overline{p}}\, dp_s\right)^{\frac{1}{2}}$$

$$\simeq (\mu p)^{\frac{1}{2}}/r.$$

The final item in QPD is given by

$$QPD(7) = \left(\frac{\mu}{p'}\right)^{\frac{1}{2}} e'\sin v' + d\dot{r}_s$$

$$= \dot{r}.$$

The output variables x, y, z are then given by

$$x = rA_1,\ y = rA_2,\ z = rA_3$$

since

$$A = \begin{bmatrix} x/r \\ y/r \\ z/r \end{bmatrix}.$$

Finally the variables $\dot{x}$, $\dot{y}$, $\dot{z}$ are given by

$$\dot{x} = \dot{r}A_1 + [(\mu p)^{\frac{1}{2}}/r]B_1 ,$$

$$\dot{y} = \dot{r}A_2 + [(\mu p)^{\frac{1}{2}}/r]B_2 ,$$

$$\dot{z} = \dot{r}A_3 + [(\mu p)^{\frac{1}{2}}/r]B_3 .$$

4.34 SDCELS (Cook, Clarke, Gooding, 1972)

This segment reads orbital elements in the format issued by the Space Defense Center (Colorado, U.S.A.), converts them to "PROP type" elements and stores them in the standard PROP array.

There are eight parameters:

ID: output satellite identification number for which the elements have been read by SDCELS. This is a change from PROP6.

IR: input channel number from which the elements are to be read.

MJDOCH: input Modified Julian Date (see subsection 4.18) for which elements are required.

NOCRD1: Input control parameter,

NCARD: input control parameter,

L: input logical control parameter,

ELEMT: output (6×6) array of elements,

LOGCHK: output logical indicator of success or failure of reading elements.

SDCELS is called by PROP(4.28) only if the control parameter NEWSAT is 5 or 7. If NEWSAT = 5 then the parameters to SDCELS are:

JDENT,IR,MJDOCH,0,3,.TRUE.,ELEMT,LOGCHK.

If NEWSAT = 7 then the parameters are:

JDENT,IR,MJDOCH,2,0,.TRUE.,ELEMT,LOGCHK.

In both cases the input parameter MJDOCH is the Modified Julian Date of the epoch under investigation. The option of inputting MJDOCH as zero and it being output as the date of the elements read is unused by the program PROP. Similarly the parameter L is input as TRUE, indicating that exclusive elements are required. If L = FALSE then inclusive elements would be output but this option is unused by the program PROP.

When NEWSAT = 5, three SDC element cards are read as follows.

The first card holds the variables:

IC2,NOSAT2,MJD,TIME,M_0,Ω_0,ω_0,e_0,i_0

to be read using the format:

I1,2(1X,I5),F9.8,3(1X,F8.4),1X,F8.7,1X,F8.4.

The second card holds the variables:

IC3,NOSAT3,M_1,M_2,Ω_1,ω_1,e_1,i_1

to be read using the format:

I1,1X,I5,1X,F11.8,1X,F11.9,2(1X,F8.5),2(1X,E8.5).

The third card holds the variables:

IC4,NOSAT4,M_3,M_4,Ω_2,ω_2,e_2,i_2

to be read using the format:

I1,1X,I5,2(1X,E11.8),3(1X,E8.5),1X,F8.0.

If the exponent of e_2 has two digits then this variable will have been read wrongly. This is corrected by:

$$\text{If } |e_2| \leqslant 0.99999 \times 10^{-10} \text{ then } e_2 = e_2 \times 10.$$

When NEWSAT = 7, two SDC element cards are read, as follows.

The first card holds variables:

IC1,NOSAT1,MYR,MD,TIME,M_2,M_3

to be read using the format:

I1,1X,I5,11X,I2,I3,F9.8,1X,F10.8,1X,E8.5.

The second card holds the variables:

IC2,NOSAT2,i_0,Ω_0,e_0,ω_0,M_0,M_1

to be read using the format:

I1,1X,I5,2(1X,F8.4),1X,F7.7,2(1X,F8.4),1X,F11.8.

In this case the date of the elements has been read in the form of a year number, MYR, and a day number within that year, MD. These are converted to a Modified Julian Date, MJD, by calling the subroutine MJDATE with parameters:

3,MJD,I,MYR,1,MD.

In both cases tests are carried out on the card numbers ICk. If ICk is non-zero and ICk ≠ k then the output parameter LOGCHK is set to FALSE and the subroutine returns to the calling segment (PROP). The output parameter ID is set equal to the first non-zero NOSATk read in. If a subsequent NOSATk is non-zero and not equal to ID then LOGCHK is set to FALSE and the subroutine returns to the calling segment.

If the checks on card numbers and satellite identity are passed then the elements are converted, by the algorithm given by Cook, Clarke and Gooding (1972), to PROP type exclusive elements. These elements are stored in ELEMT before returning to the calling segment with LOGCHK set TRUE. The differences between SDC elements and PROP type elements are discussed by Gooding (1970a).

Changes have been incorporated into SDCELS to overcome a problem arising from the satellite identification number. In the PROP6 version of SDCELS the satellite identity was input as the first parameter, ID. This was compared with the satellite identity given on the SDC element cards. As the ID input was normally the seven digit satellite identification found on observation cards the five digit satellite identification on the SDC cards had to be set to zero when preparing the input. This meant that the check on satellite identity was bypassed. In the new version of SDCELS, ID is an output parameter. On returning to the calling segment, PROP, the five digit satellite identity found on the SDC element cards may be printed out for the user to check manually.

The segment SDCELS has been moved from overlay unit (1,3) to overlay unit (1,1). This was to enable the segment MJDATE(4.18) to be called. MJDATE is in overlay unit (1,1) and it may not be called by a segment in an overlay unit (1,k), k ≠ 1.

4.35 SDPRIN

This segment prints out values of standard deviations of orbital elements.

There are four parameters:

NOMIAL: input 6 element array governing how many numbers are to be printed,

ELEMT: input (6×6) array of numbers to be printed,

SMAXSD: input standard deviation of the derived semi-major axis,

IP: input channel number to which the output is to be sent for printing.

There are no output parameters to SDPRIN and it does not use COMMON. The only output of SDPRIN is that printed on channel IP.

The subroutine is analogous to the subroutine ELPRIN(4.10). The standard deviation of the derived semi-major axis, SMAXSD, is printed. This is followed by the printing of the standard deviations of the coefficients of the polynomials, which have been output by a call of the subroutine ELPRIN. The standard deviations are converted from units of radians and seconds to degrees and days before printing.

The amount of printing is controlled by NOMIAL. If NOMIAL(I) = j-1 then ELEMT(I,1),....,ELEMT(I,j) are printed, I=1,..,5.

The only call of SDPRIN in the program PROP is with parameters:

NOMIAL,STDEVS,SMAXSD,IP

where STDEVS is a 6×6 array holding the standard deviations.

One change has been made to SDPRIN, viz., the redundant label, 1, has been removed.

4.36 SENSIN

This segment reads sensor (observer) locations into an array.

There are three parameters:

STASHN: output (8 × MAXSTA) array of sensor location information,

MAXSTA: input maximum number of sensors permitted,

NUMSTA: output number of sensor cards read.

The use of COMMON is:

/CNTROL/

IR, IP and ISENSR are input.

SENSIN reads a maximum of MAXSTA sensor location records (observer cards) from channel IR and, if ISENSR is non-zero, prints the information on channel IP. Information about each sensor location is stored in the array STASHN. This information is:

STASHN(1,I) = identification number of the sensor,

STASHN(2,I) = x, STASHN(3,I) = y, STASHN(4,I) = z: geocentric Cartesian coordinates of the sensor, x measured in the equatorial plane, toward the Greenwich meridian,

STASHN(5,I) = sin(latitude), STASHN(6,I) = cos(latitude), STASHN(7,I) = sin(longitude), STASHN(8,I) = cos(longitude): geographic latitude and longitude of the sensor,

I=1,...,NUMSTA.

The sensor information may be presented in any one of four forms, as described by Gooding and Tayler (1968). The form of the information is indicated by the first digit on the sensor card, which is read into the variable MARKER.

The sensor name is read into the variable NAMEST and printed but it is not stored in STASHN. Similarly the sensor's height above a reference spheroid is read, or derived, and printed, but not stored.

If MARKER = 3 then the height is defined to be zero. The latitude and longitude read from the sensor card are converted from degrees to radians and their sines and cosines found. All the information required for printing and storing is then available.

If MARKER = 2 or 4 then a subroutine is called to calculate the geographic coordinates from the Cartesian coordinates given on the sensor card. The subroutine called if MARKER = 2 is GEOCOR(4.14). If MARKER = 4 then GEOCRN(4.15) is called, MARKER = 4 being a new MARKER and GEOCRN a new subroutine. The subroutine called is the only difference between the MARKERs.

If MARKER = 0 or 1 then the sensors' Cartesian coordinates are calculated from the geographic coordinates within the segment SENSIN. In these calculations the Earth is assumed to be ellipsoidal in shape. If MARKER = 0 then the semi-major axis, A, of the generating ellipse is assumed to be 6378.166 km, and the semi-minor axis, B, to be 6356.784 km. This is the Fischer spheroid, a more complete description of which is given in subsection 4.14, GEOCOR. If MARKER = 1 then the values of A and B are read from the sensor card.

In either case let β,λ be, respectively, the geographical latitude and longitude of the sensor and ht its height above the spheroid. β,λ and ht are all read from the sensor card. Assume that the sensor is at S and the centre of the Earth at C. We consider a vertical section through C and S. (Figure 4.36.1).

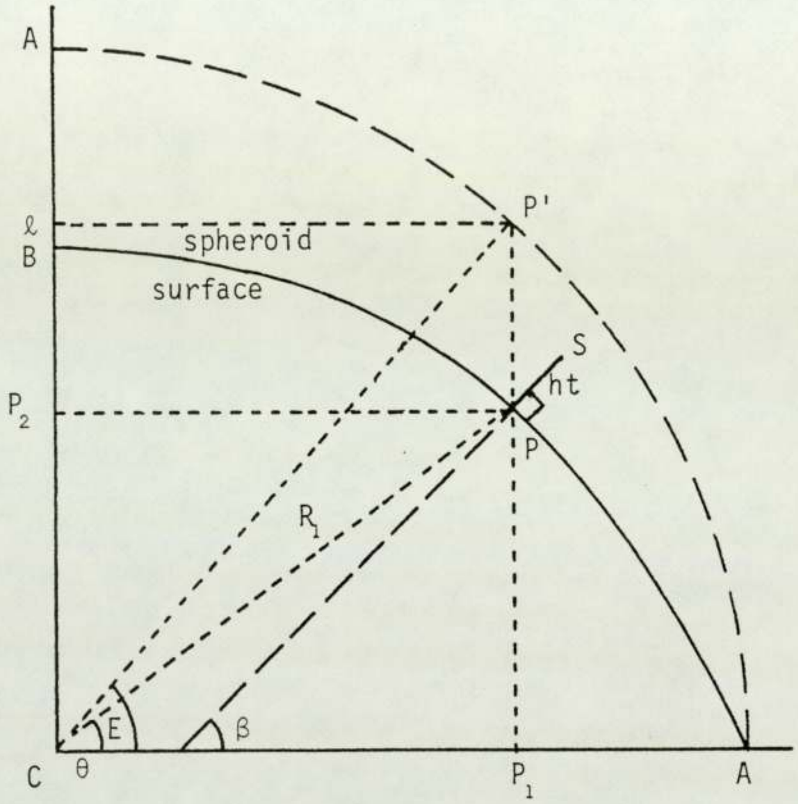
The curve AB, through the point P, is defined to be part of an ellipse. If we consider the coordinates (p_1,p_2) of the sub-sensor point P we obtain

$$\frac{p_1^2}{A^2} + \frac{p_2^2}{B^2} = 1 \qquad (4.36.1)$$

and hence the slope of the ellipse, at P, is given by

$$\frac{dp_2}{dp_1} = -\frac{Bp_1}{A^2}\left(1 - \frac{p_1^2}{A^2}\right)^{-\frac{1}{2}} .$$

Figure 4.36.1 A Sensor Position Relative to a Reference Spheroid.



The slope of the perpendicular to the ellipse is then given by

$$-\frac{dp_1}{dp_2} = \frac{A^2p_2}{B^2p_1} = \tan\beta.$$

If we consider the point P', at coordinates (p_1, ℓ) on the circle, centre C, radius A, then

$$\ell^2 = A^2 - p_1^2 .$$

Substituting for p_1^2 from Equation (4.36.1) we find that

$$\ell^2 = A^2\left[1 - \left(1 - \frac{p_2^2}{B^2}\right)\right]$$

and hence

$$\ell = \frac{Ap_2}{B} ,$$

The eccentric angle, E, is given by

$$\tan E = \frac{\ell}{p_1}$$

and we obtain

$$\frac{A}{B} \tan E = \frac{A\ell}{Bp_1} = \frac{A^2 p_2}{B^2 p_1} = \tan \beta.$$

We have also

$$p_1 = A \cos E , \quad p_2 = B \sin E$$

and

$$\tan \theta = \frac{p_2}{p_1} = \frac{B \sin E}{A \cos E} = \frac{B}{A} \tan E .$$

Hence we find that the radial distance of P is given by

$$R_1 = p_1^2 + p_2^2 = A^2\cos^2 E + B^2 \sin^2 E$$

where E is given by

$$\tan E = \frac{B}{A} \tan \beta.$$

Cartesian coordinate, x,y,z, of S may then be determined from

$$x = (R_1 \cos \theta + ht \cos \beta)\cos \lambda,$$

$$y = (R_1 \cos \theta + ht \cos \beta)\sin \lambda,$$

$$z = R_1 \sin \beta + ht \sin \beta.$$

The subroutine SENSIN has been changed to avoid potential problems arising from a user having more than MAXSTA sensor records in a PROP run. (MAXSTA has been set to 50 in PROP). If more than MAXSTA sensor locations are presented then NUMSTA is set to MAXSTA, and a warning message of the form

MORE THAN k STATION CARDS PRESENT, THE REST HAVE BEEN IGNORED

where k = MAXSTA, is printed out. The remaining sensor cards are ignored. Without this change SENSIN could have unpredictable effects on the rest of the program if MAXSTA was exceeded.

4.37 SHOPER (Tayler, 1969e)

This segment calculates short periodic perturbations in the satellite orbit, due to the second zonal harmonic, J_2.

There are fourteen parameters:

ECC: input orbital eccentricity, e,

SI: input sin(i)

CI: input cos(i)

(SI, CI: where i is the orbital inclination,)

OMEGA: input argument of perigee of the orbit, ω radians,

V: input true anomaly of the satellite, v radians,

SV: input sin(v)

CV: input cos(v)

U: input argument of latitude of the satellite, u radians,

FMANOM: input mean anomaly of the satellite, M_f radians, adjusted so that $|v - M_f| < \pi$,

F: input $f = \sin^2(i)$

P: input semi-latus rectum of the orbit, $p = a(1 - e^2)$, (km),

POVERR: input p/r, where r is the radial distance, in km, of the satellite from the Earth's centre,

Q: input $q = (1 - e^2)^{\frac{1}{2}}$,

TSP: output 6 element array of short periodic perturbations.

The use of COMMON is:

/ORBIT/

EMU $\equiv \mu$, ERAD $\equiv$ R and EJ(40) are input to SHOPER. EJ(2) $\equiv J_2$ is used but the rest of the array EJ is unused.

The array TSP is filled by SHOPER such that

$$TSP(1) = \frac{1}{2} KC \sin i \cos i,$$

$$TSP(2) = -K\left(v - M_f + e \sin v - \frac{1}{2} S\right) \sin i \cos i,$$

$$\mathrm{TSP}(3) = Kh\left[v - M_f + e\sin v + \frac{1}{3} ge\sin v(g + \cos v)\right] + \frac{1}{3} Kf\left[\frac{1}{4}\sin 2u + e\sin(v + 2\omega)\right],$$

$$\mathrm{TSP}(4) = \frac{1}{2} K(hq + fC),$$

$$\mathrm{TSP}(5) = \frac{1}{3} K\left[\frac{1}{2} f\cos 2u - h(1 + g\cos v - qr/p)\right],$$

and

$$\mathrm{TSP}(6) = \frac{1}{3} (\mu/p)^{\frac{1}{2}} K \left\{h\sin v\left[-\frac{1}{2} qe + g(p/r)^2\right] - f(p/r)^2 \sin 2u\right\},$$

where

$$K = (3/2)J_2 (R/p)^2,$$

$$C = \cos 2u + e\left[\cos(v + 2\omega) + \frac{1}{3}\cos(3v + 2\omega)\right],$$

$$S = \sin 2u + e\left[\sin(v + 2\omega) + \frac{1}{3}\sin(3v + 2\omega)\right],$$

$$h = 1 - \frac{3}{2} f,$$

and

$$g = e/(1 + q).$$

The argument of latitude, u, is given by

$$u = v + \omega$$

and hence

$$v + 2\omega = 2u - v,\quad 3v + 2\omega = 2u + v.$$

Therefore the contents of TSP are set to di_s, $d\Omega_s \sin i$, $du_s + d\Omega_s \cos i$, $dp_s/2p$, dr_s/p and $d\dot{r}_s$ respectively, where these are the short periodic perturbations given by Merson (1966) and Gooding (1969b). The expressions given for the short periodic perturbations were developed from the expressions given by Kozai (1959) for the short periodic perturbations of the basic elements.

4.38 SIDANG (Tayler, 1969b)

This function segment calculates the modified sidereal angle, in radians, at a given date and time.

There are two input parameters:

MJD:	Modified Julian Date (see subsection 4.18)	for which the modified sidereal angle is required.
TIME:	time as a fraction of a day	

The modified sidereal angle SIDANG is the angle between the Greenwich meridian and the first point of Aries, Υ, at a time of MJD.TIME.

SIDANG is calculated first in revolutions as:

$$SIDANG = a_1 + a_2 \text{ float}(MJD - a_3) + a_4 TIME$$

where

$a_1 = 0.277987616$

= number of revolutions of the Greenwich meridian from Υ at an epoch of 1950.0,

$a_2 = 0.00273781191$

= (number of revolutions made, per day, by the Greenwich meridian, relative to Υ) -1,

$a_3 = 33282$

= Modified Julian Date of 1950.0,

$a_4 = 1 + a_2$.

SIDANG is then modified to give an angle, in radians, such that

$$0 \leqslant SIDANG < 2\pi.$$

4.39 SINXOX (Tayler, 1969d)

This segment evaluates three trigonometrical functions of a given angle.

There are four parameters:

X: input angle in radians, x,

F1X: output first required function of x, $f_1(x)$,

F2X: output second required function of x, $f_2(x)$,

F3X: output third required function of x, $f_3(x)$.

The three required functions are:

$$f_1(x) = \frac{\sin x}{x}, \quad f_2(x) = \frac{1 - \cos x}{x^2}, \quad f_3(x) = \frac{x - \sin x}{x^3}$$

If $|x|$ is greater than 0.1 (radians) then the standard SIN and COS functions are used to derive $f_1(x)$ and $f_2(x)$. $f_3(x)$ is then obtained from

$$f_3(x) = [1 - f_1(x)]/x^2 .$$

Otherwise series expansions are used to obtain $f_2(x)$ and $f_3(x)$, the series being truncated at x^6. Thus

$$f_2(x) = \left[\left(\frac{-x^2}{8!} + \frac{1}{6!}\right) x^2 - \frac{1}{4!}\right] x^2 + \frac{1}{2!} \quad ,$$

$$f_3(x) = \left[\left(\frac{-x^2}{9!} + \frac{1}{7!}\right) x^2 - \frac{1}{5!}\right] x^2 + \frac{1}{3!}$$

and also

$$f_1(x) = 1 - x^2 f_3(x).$$

The subroutine has been changed so that $\frac{1}{n!}$, n=1,..,9 are stated to 11 significant digits, rather than 11 decimal places, as this is the maximum working accuracy of the ICL 1904S unless "double precision" is used.

4.40 TRINV

This segment converts two-dimensional Cartesian coordinates to polar coordinates.

There are four parameters:

Y: input Cartesian coordinate, y,

X: input Cartesian coordinate, x,

R: output polar coordinate, r,

TH: output polar coordinate, θ.

The output parameters are given by

$$r = (x^2 + y^2)^{\frac{1}{2}} \ ,$$

$$\theta = \tan^{-1}(y/x),$$

except in the special case where $r = 0$. In this case θ is defined to be zero.

4.41 XLONG (Merson, Tayler, Gooding, 1969)

This segment calculates long-periodic perturbations in the satellite orbit.

There are five parameters:

T: input time t, in seconds, at which perturbations are required, relative to the epoch of the satellite's orbital elements,

POVERR: input p/r where $p = a(1 - e^2)$ is the semi-latus rectum of the orbit and r is the geocentric radial distance of the satellite,

SV: input sin(v) } where v is the satellite's true anomaly,

CV: input cos(v) }

TLP: output 6 element array of long periodic perturbations, di_ℓ, $d\Omega_\ell \sin i$, $du_\ell + d\Omega_\ell \cos i$, da_ℓ, de_ℓ and edv_ℓ.

There are two changes in this list of parameters to that given by Merson, Tayler and Gooding (1969). In 1969 the output array TLP held perturbations di_ℓ, $d\Omega_\ell \sin i$, $du_\ell + d\Omega_\ell \cos i$, $dp_\ell/2p$, dr_ℓ/p and $d\dot{r}_\ell$. The change to the current array was made before the PROP6 version of PROP was issued to remove an anomaly that is discussed by Gooding (1974).

The second change is the removal of the parameter SRMOP = $(\mu/p)^{\frac{1}{2}}$ in the new version of XLONG, with corresponding changes in the calling segment SATXYZ(4.33) as this parameter is not used by XLONG.

The use of COMMON is:

/ORBIT/

L and ELEMT(6×6) are input

/PRECON/

ZONSEC(4), TDRAG(2), F ≡ f, QSQ ≡ q^2, P4 ≡ p_4, P5 ≡ p_5, P10 ≡ p_{10}, P11 ≡ p_{11}, P13 ≡ p_{13}, P14 ≡ p_{14}, P15 ≡ p_{15}, P16 ≡ p_{16}, P17 ≡ p_{17}, TW ≡ 3W, STL ≡ $\sin 2\ell_{2,2}$, CTL ≡ $\cos 2\ell_{2,2}$, TTHMO, Z4F ≡ $z_4 f$ and ABCD(38×8) are all input to XLONG having been set in a prior call of the segment PRELON(4.25).

The segment XLONG may be divided into four parts. The first part calculates some preliminary quantities. The next three parts calculate the contributions to the long periodic perturbations by drag, zonal harmonics and tesseral harmonics respectively. We will first give the expressions for these perturbations as they appear in the segment XLONG, and then show that these expressions are equivalent to those given by Merson (1966) and Gooding (1969b and 1974).

The expressions used by XLONG are

$$\left.\begin{aligned}
\mathrm{TLP}(1) &= di_\ell = di_{drag} + di_J + di_{2,2}\ , \\
\mathrm{TLP}(2) &= d\Omega_\ell \sin i = d\Omega_{drag} \sin i + d\Omega_J \sin i + d\Omega_{2,2} \sin i\ , \\
\mathrm{TLP}(3) &= du_\ell + d\Omega_\ell \cos i = du_{drag} + d\Omega_{drag} \cos i + du_J \\
&\qquad + d\Omega_J \cos i + du_{2,2} + d\Omega_{2,2} \cos i, \\
\mathrm{TLP}(4) &= da_\ell = da_{drag} + da_J + da_{2,2}\ , \\
\mathrm{TLP}(5) &= de_\ell = de_{drag} + de_J + de_{2,2} \\
\mathrm{TLP}(6) &= edv_\ell = edv_{drag} + edv_J + edv_{2,2}\ .
\end{aligned}\right\} \quad (4.41.1)$$

The preliminary quantities calculated by XLONG are

$$\begin{aligned}
L_1 &= \sum_{k=1}^{L-2} f_1(k\dot{\omega}_S t)(a_k{}^C + d_k{}^C - b_k{}^C) \\
&\qquad - (k\dot{\omega}_S t) f_2(k\dot{\omega}_S t)(a_k{}^S + d_k{}^S - b_k{}^S), \\
L_2 &= \sum_{k=1}^{L-2} f_1(k\dot{\omega}_S t)(b_k{}^C + d_k{}^C) - (k\dot{\omega}_S t) f_2(k\dot{\omega}_S t)(b_k{}^S + d_k{}^S), \\
L_3 &= \sum_{k=1}^{L-2} k\left[f_1(k\dot{\omega}_S t) a_k{}^C - (k\dot{\omega}_S t) f_2(k\dot{\omega}_S t) a_k{}^S\right], \\
L_4 &= \sum_{k=1}^{L-2} f_1(k\dot{\omega}_S t) c_k{}^C - (k\dot{\omega}_S t) f_2(k\dot{\omega}_S t) c_k{}^S, \\
L_5 &= \sum_{k=1}^{L-2} k\left[(k\dot{\omega}_S t) f_3(k\dot{\omega}_S t) a_k{}^C + f_2(k\dot{\omega}_S t) a_k{}^S\right]
\end{aligned}$$

$$+ p_{17}\left[p_{16}\, 2\dot{\omega}_S t f_3(2\dot{\omega}_S t) + p_{15}\; f_2(2\dot{\omega}_S t)\right],$$

$$L_6 = \sum_{k=1}^{L-2} k\left[(k\dot{\omega}_S t) f_2(k\dot{\omega}_S t) a_k{}^C + f_1(k\dot{\omega}_S t) a_k{}^S\right]$$

$$+ p_{17}\left[p_{16}\, 2\dot{\omega}_S t f_2(2\dot{\omega}_S t) + p_{15}\, f_1(2\dot{\omega}_S t)\right],$$

$$L_7 = p_{16}\, 2\dot{\omega}_S t f_2(2\dot{\omega}_S t) + p_{15}\, f_1(2\dot{\omega}_S t),$$

$$L_8 = p_{16}\, f_1(2\dot{\omega}_S t) - p_{15}\, 2\dot{\omega}_S t f_2(2\dot{\omega}_S t),$$

$$L_9 = p_{16}\, 2\dot{\omega}_S t f_3(2\dot{\omega}_S t) + p_{15}\, f_2(2\dot{\omega}_S t),$$

and

$$L_{10} = t\sum_{k=1}^{L-2} k\left[(k\dot{\omega}_S t) f_2(k\dot{\omega}_S t) a_k{}^C + f_1(k\dot{\omega}_S t) a_k{}^S\right]$$

$$+ t\, p_{17}\left[p_{16}\, 2\dot{\omega}_S t f_2(2\dot{\omega}_S t) + p_{15}\, f_1(2\dot{\omega}_S t)\right]. \qquad (4.41.2)$$

where

$f_1(x) = (\sin x)/x$, $f_2(x) = (1 - \cos x)/x^2$, and $f_3(x) = (x - \sin x)/x^3$.

The latter functions are all calculated by calling the segment SINXOX(4.39).

The following intermediate quantities are also calculated by XLONG,

$$\left.\begin{aligned} \mathrm{DEOQSQ} &= p_5 L_{10}, \\ \mathrm{EDMOQC} &= -tp_5\left[(L_3 + e_0^3 L_1)/q^2 + p_{17} L_8\right], \end{aligned}\right\} \qquad (4.41.3)$$

$$\left.\begin{aligned} C &= t\left[\cos 2\ell_{2,2}\, f_1(2\dot{\ell}_{2,2}\, t) - 2\dot{\ell}_{2,2}\, t \sin 2\ell_{2,2}\, f_2(2\dot{\ell}_{2,2}\, t)\right], \\ S &= t\left[\sin 2\ell_{2,2}\, f_1(2\dot{\ell}_{2,2}\, t) + 2\dot{\ell}_{2,2}\, t \cos 2\ell_{2,2}\, f_2(2\dot{\ell}_{2,2}\, t)\right], \\ SI &= t^2\left[\sin 2\ell_{2,2}\, f_2(2\dot{\ell}_{2,2}\, t) + 2\dot{\ell}_{2,2}\, t \cos 2\ell_{2,2}\, f_3(2\dot{\ell}_{2,2}\, t)\right]. \end{aligned}\right\} \qquad (4.41.4)$$

The constituent parts of Equations (4.41.1) are then given by:

$$\left.\begin{aligned}
di_{drag} &= tT_2L_8\,, \\
d\Omega_{drag}\sin i &= tp_5L_7T_1, \\
du_{drag} + d\Omega_{drag}\cos i &= tL_7(1-f)^{\frac{1}{2}}T_1, \\
da_{drag} = de_{drag} &= edv_{drag} = 0.0,
\end{aligned}\right\} \quad (4.41.5)$$

$$\left.\begin{aligned}
di_J &= -p_{10}L_{10}\,, \\
d\Omega_J\sin i &= t(p_{10}L_3 - feL_4 + p_{11}L_8 - tp_{14}L_5), \\
du_J + d\Omega_J\cos i &= tp_5\Big\{-L_3\Big[e + (p/r+1)\cos v\Big]/q^2 \\
&\quad -e\Big[(p/r)^2L_1/q^2 - L_2\Big] \\
&\quad + (p/r+1)\sin v\,L_6 \\
&\quad + p_{17}L_8\Big[(5/2)e - (p/r+1)\cos v\Big] \\
&\quad + tp_{13}L_5\Big\}\,, \\
de_J &= \mathrm{DEOQSQ}\times q^2, \\
edv_J &= e\sin v(p/r+1)\times\mathrm{DEOQSQ} + (p/r)^2\times\mathrm{EDMOQC}\,, \\
da_J &= 0.0,
\end{aligned}\right\} \quad (4.41.6)$$

$$\left.\begin{aligned}
di_{2,2} &= -3Wp_5S, \\
d\Omega_{2,2}\sin i &= 3Wp_4p_5C - (9/2)W\,SI\,p_5z_4f, \\
du_{2,2} + d\Omega_{2,2}\cos i &= (9/2)WfC\left[1+\left(\frac{p}{rq}\right)^2\right] + 18Wz_4fp_4SI, \\
edv_{2,2} &= \frac{9}{2}WfC(p/r)^2e/q^2, \quad \text{and} \\
da_{2,2} &= de_{2,2} = 0.
\end{aligned}\right\} \quad (4.41.7)$$

Perturbations due to tesseral harmonics, $di_{2,2}$ etc., were first introduced to PROP in this form at the PROP5 stage (Gooding, Odell,1973).

Before showing that Equations (4.41.5), (4.41.6) and (4.41.7) are equivalent to the expressions given by Merson (1966) and Gooding (1969b and 1974) for long periodic perturbations, it is convenient to prove a result given in Appendix A12 of Merson (1966), viz.,

$$S_k(\dot{\omega}) = \int_0^t \sin k(\omega - \pi/2)dt$$

$$= t\left[k\dot{\omega}tf_2(k\dot{\omega}t)\cos k(\omega_0 - \pi/2) + f_1(k\dot{\omega}t)\sin k(\omega_0 - \pi/2)\right], \quad (4.41.8)$$

$$C_k(\dot{\omega}) = \int_0^t \cos k(\omega - \pi/2)dt$$

$$= t\left[f_1(k\dot{\omega}t)\cos k(\omega_0 - \pi/2) - k\dot{\omega}tf_2(k\dot{\omega}t)\sin k(\omega_0 - \pi/2)\right], \quad (4.41.9)$$

and

$$\int_0^t S_k(\dot{\omega})dt = t^2\left[k\dot{\omega}tf_3(k\dot{\omega}t)\cos k(\omega_0 - \pi/2)\right.$$

$$\left. + f_2(k\dot{\omega}t)\sin k(\omega_0 - \pi/2)\right], \quad (4.41.10)$$

where $\omega_0 = \omega$ at time $t = 0$, and $\dot{\omega} = d\omega/dt$, is constant.

To demonstrate these results, we change the variable in the integrals of Equations (4.41.8) and (4.41.9) to θ, such that

$$\theta = k(\omega - \pi/2).$$

Consider, first, the integral in Equation (4.41.8). This may be written

$$\frac{1}{k\dot{\omega}} \int_{k(\omega_0 - \pi/2)}^{k(\omega_0 + \dot{\omega}t - \pi/2)} \sin\theta \, d\theta$$

$$= \frac{t}{k\dot{\omega}t}\left[\cos k(\omega_0 - \pi/2) - \cos k(\omega_0 + \dot{\omega}t - \pi/2)\right] \quad (4.41.11)$$

$$= \frac{t}{k\dot{\omega}t}\left[\cos k(\omega_0 - \pi/2) - \cos k\dot{\omega}t \cos k(\omega_0 - \pi/2) + \sin k\dot{\omega}t \sin k(\omega_0 - \pi/2)\right]$$

$$= t\left[\frac{(1 - \cos k\dot{\omega}t)}{k\dot{\omega}t} \cos k(\omega_0 - \pi/2) + \frac{\sin k\dot{\omega}t}{k\dot{\omega}t} \sin k(\omega_0 - \pi/2)\right]$$

$$= t\left[k\dot{\omega}t f_2(k\dot{\omega}t) \cos k(\omega_0 - \pi/2) + f_1(k\dot{\omega}t) \sin k(\omega_0 - \pi/2)\right]$$

which is the right hand side of Equation (4.41.10), as required.

Similarly the integral in Equation (4.41.9) may be written

$$\frac{1}{k\dot{\omega}} \int_{k(\omega_0 - \pi/2)}^{k(\omega_0 + \dot{\omega}t - \pi/2)} \cos\theta \, d\theta$$

$$= \frac{t}{k\dot{\omega}t}\left[\sin k(\omega_0 + \dot{\omega}t - \pi/2) - \sin k(\omega_0 - \pi/2)\right]$$

$$= \frac{t}{k\dot{\omega}t}\left[\sin k(\omega_0 - \pi/2) \cos k\dot{\omega}t + \cos k(\omega_0 - \pi/2) \sin k\dot{\omega}t - \sin k(\omega_0 - \pi/2)\right]$$

$$= t\left[\frac{\sin k\dot{\omega}t}{k\dot{\omega}t} \cos k(\omega_0 - \pi/2) - \frac{(1 - \cos k\dot{\omega}t)}{k\dot{\omega}t} \sin k(\omega_0 - \pi/2)\right]$$

$$= t\left[f_1(k\dot{\omega}t) \cos k(\omega_0 - \pi/2) - k\dot{\omega}t f_2(k\dot{\omega}t) \sin k(\omega_0 - \pi/2)\right]$$

which is the right hand side of Equation (4.41.9), as required.

To verify Equation (4.41.10), we substitute the right hand side of Equation (4.41.11) into the left hand side of Equation (4.41.10) and obtain

$$\int_0^t S_k(\dot{\omega}) dt = \int_0^t \frac{1}{k\dot{\omega}}\left[\cos k(\omega_0 - \pi/2) - \cos k(\omega_0 + \dot{\omega}t - \pi/2)\right] dt$$

$$= \frac{1}{k\dot{\omega}}\left[t \cos k(\omega_0 - \pi/2)\right]_0^t - \frac{1}{k\dot{\omega}}\left[\frac{1}{k\dot{\omega}} \sin k(\omega_0 + \dot{\omega}t - \pi/2)\right]_0^t$$

$$= \frac{t^2}{k\dot{\omega}t} \cos k(\omega_0 - \pi/2) - \frac{t^2}{(k\dot{\omega}t)^2} \left[\sin k(\omega_0 + \dot{\omega}t - \pi/2) - \sin k(\omega_0 - \pi/2) \right]$$

$$= t^2 \left\{ \frac{\cos k(\omega_0 - \pi/2)}{k\dot{\omega}t} - \frac{1}{(k\dot{\omega}t)^2} \left[\sin k(\omega_0 - \pi/2) \cos k\dot{\omega}t \right.\right.$$

$$\left.\left. + \cos k(\omega_0 - \pi/2) \sin k\dot{\omega}t - \sin k(\omega_0 - \pi/2) \right] \right\}$$

$$= t^2 \left[\frac{(k\dot{\omega}t - \sin k\dot{\omega}t)}{(k\dot{\omega}t)^2} \cos k(\omega_0 - \pi/2) + \frac{(1 - \cos k\dot{\omega}t)}{(k\dot{\omega}t)^2} \sin k(\omega_0 - \pi/2) \right]$$

$$= t^2 \left[k\dot{\omega}t f_3(k\dot{\omega}t) \cos k(\omega_0 - \pi/2) + f_2(k\dot{\omega}t) \sin k(\omega_0 - \pi/2) \right]$$

which is the right hand side of Equation (4.41.10), as required.

We now use the results given in Equations (4.41.8), (4.41.9) and (4.41.10), together with the definitions of $a_k{}^c$ etc, given in Appendix I, to re-express L_j, j=1,..,10 as given in Equations (4.41.2).

$$L_1 = \sum_{k=1}^{L-2} \frac{(a_k + d_k - b_k)}{e_0 \sin i_0} \left[f_1(k\dot{\omega}_S t) \cos k(\omega_0 - \pi/2) - k\dot{\omega}_S t f_2(k\dot{\omega}_S t) \sin k(\omega_0 - \pi/2) \right]$$

$$= \frac{1}{t} \sum_{k=1}^{L-2} \frac{(a_k + d_k - b_k)}{e_0 \sin i_0} C_k(\dot{\omega}_S). \qquad (4.41.12)$$

$$L_2 = \sum_{k=1}^{L-2} \frac{(b_k + d_k)}{e_0 \sin i_0} \left[f_1(k\dot{\omega}_S t) \cos k(\omega_0 - \pi/2) - k\dot{\omega}_S t f_2(k\dot{\omega}_S t) \sin k(\omega_0 - \pi/2) \right]$$

$$= \frac{1}{t} \sum_{k=1}^{L-2} \frac{(b_k + d_k)}{e_0 \sin i_0} C_k(\dot{\omega}_S). \qquad (4.41.13)$$

$$L_3 = \sum_{k=1}^{L-2} \frac{k a_k}{e_0 \sin i_0} \left[f_1(k\dot{\omega}_S t) \cos k(\omega_0 - \pi/2) - k\dot{\omega}_S t f_2(k\dot{\omega}_S t) \sin k(\omega_0 - \pi/2) \right]$$

$$= \frac{1}{t} \sum_{k=1}^{L-2} a_k' \, C_k(\dot{\omega}_S), \qquad (4.41.14)$$

using the definition of a_k' given by Appendix A6 of Merson (1966).

$$L_4 = \sum_{k=1}^{L-2} \frac{c_k}{e_0 \sin i_0} \Big[f_1(k\dot{\omega}_S t) \cos k(\omega_0 - \pi/2)$$

$$- k\dot{\omega}_S t f_2(k\dot{\omega}_S t) \sin k(\omega_0 - \pi/2) \Big]$$

$$= \frac{1}{t} \sum_{k=1}^{L-2} \frac{c_k}{e_0 \sin i_0} C_k(\dot{\omega}_S) \, . \qquad (4.41.15)$$

$$L_5 = \sum_{k=1}^{L-2} \frac{ka_k}{e_0 \sin i_0} \Big[k\dot{\omega}_S t f_3(k\dot{\omega}_S t) \cos k(\omega_0 - \pi/2)$$

$$+ f_2(k\dot{\omega}_S t \sin k(\omega_0 - \pi/2) \Big]$$

$$+ p_{17} \Big[\cos 2(\omega_0 - \pi/2) 2\dot{\omega}_S t f_3(2\dot{\omega}_S t) + \sin 2(\omega_0 - \pi/2) f_2(2\dot{\omega}_S t) \Big]$$

$$= \frac{1}{t^2} \sum_{k=1}^{L-2} a_k' \int_0^t S_k(\dot{\omega}_S) dt + \frac{p_{17}}{t^2} \int_0^t S_2(\dot{\omega}_S) dt. \qquad (4.41.16)$$

$$L_6 = \sum_{k=1}^{L-2} \frac{ka_k}{e_0 \sin i_0} \Big[k\dot{\omega}_S t f_2(k\dot{\omega}_S t) \cos k(\omega_0 - \pi/2)$$

$$+ f_1(k\dot{\omega}_S t) \sin k(\omega_0 - \pi/2) \Big]$$

$$+ p_{17} \Big[\cos 2(\omega_0 - \pi/2) 2\dot{\omega}_S t f_2(2\dot{\omega}_S t) + \sin 2(\omega_0 - \pi/2) f_1(2\dot{\omega}_S t) \Big]$$

$$= \frac{1}{t} \sum_{k=1}^{L-2} a_k' \, S_k(\dot{\omega}_S) + \frac{p_{17}}{t} S_2(\dot{\omega}_S). \qquad (4.41.17)$$

$$L_7 = \cos 2(\omega_0 - \pi/2) 2\dot{\omega}_S t f_2(2\dot{\omega}_S t) + \sin 2(\omega_0 - \pi/2) f_1(2\dot{\omega}_S t)$$

$$= \frac{1}{t} S_2(\dot{\omega}_S). \qquad (4.41.18)$$

$$L_8 = \cos 2(\omega_0 - \pi/2) f_1(2\dot{\omega}_S t) - \sin 2(\omega_0 - \pi/2) 2\dot{\omega}_S t f_2(2\dot{\omega}_S t)$$

$$= \frac{1}{t} C_2(\dot{\omega}_S). \qquad (4.41.19)$$

$$L_9 = \cos 2(\omega_0 - \pi/2) 2\dot{\omega}_S t f_3(2\dot{\omega}_S t) + \sin 2(\omega_0 - \pi/2) f_2(2\dot{\omega}_S t)$$

$$= \frac{1}{t^2} \int_0^t S_2(\dot{\omega}_S) dt. \qquad (4.41.20)$$

and

$$L_{10} = t \sum_{k=1}^{L-2} \frac{k a_k}{e_0 \sin i_0} \Big[k\dot{\omega}_S t f_2(k\dot{\omega}_S t) \cos k(\omega_0 - \pi/2)$$

$$+ f_1(k\dot{\omega}_S t) \sin k(\omega_0 - \pi/2) \Big]$$

$$+ t p_{17} \Big[\cos 2(\omega_0 - \pi/2) 2\dot{\omega}_S t f_2(2\dot{\omega}_S t) + \sin 2(\omega_0 - \pi/2) f_1(2\dot{\omega}_S t) \Big]$$

$$= \sum_{k=1}^{L-2} a_k' \, S_k(\dot{\omega}_S) + p_{17} S_2(\dot{\omega}_S). \qquad (4.14.21)$$

Similarly, the expressions given for C, S and SI in Equations (4.41.4) may be shown to be equivalent to

$$\left. \begin{aligned} C &= \int_0^t \cos(2\ell_{2,2}) dt, \\ S &= \int_0^t \sin(2\ell_{2,2}) dt, \\ \text{and} \quad SI &= \int_0^t S \, dt = \int_0^t \int_0^t \sin(2\ell_{2,2}) dt \, dt . \end{aligned} \right\} \qquad (4.41.22)$$

We may now establish that the expressions given in Equations (4.41.3) for DEOQSQ and EDMOQC are equivalent to those given by Merson (1966), for de_J/q^2 and edM_J/q^3 respectively.

Equation (4.41.3) gives

$$\text{DEOQSQ} = p_5 L_{10}$$

which, by Equation (4.41.21), becomes

$$\text{DEOQSQ} = \sin i \sum_{k=1}^{L-2} a_k' S_k(\dot{\omega}_S) + \sin i \; p_{17} S_2(\dot{\omega}_S)$$

$$= \sin i \sum_{k=1}^{L-2} a_k' S_k(\dot{\omega}_S) + (21/16) J_2^2 (R/p)^4 n_0 e_0 f(1 - 15f/14) S_2(\dot{\omega}_S)$$

$$= de_J/q^2 \qquad (4.41.23)$$

as given in Equation (A42) of Merson (1966).

Similarly,

$$\text{EDMOQC} = -\, t p_5 \left[(L_3 + e_0^2 L_1)/q^2 + p_{17} L_8 \right]$$

$$= -t \sin i \left[\frac{1}{q^2 t} \sum_{k=1}^{L-2} a_k' C_k(\dot{\omega}_S) + \frac{e_0^2}{q^2 t} \sum_{k=1}^{L-2} \frac{(a_k + d_k - b_k)}{e_0 \sin i_0} C_k(\dot{\omega}_S) \right.$$

$$\left. + \frac{p_{17}}{t} \; C_2(\dot{\omega}_S) \right]$$

$$= \frac{e}{q^2} \sum_{k=1}^{L-2} (b_k - a_k - d_k) C_k(\dot{\omega}_S) - \frac{\sin i}{q^2} \sum_{k=1}^{L-2} a_k' C_k(\dot{\omega}_S)$$

$$- (21/16) J_2^2 (R/p)^4 n_0 e_0 f(1 - 15f/14) C_2(\dot{\omega}_S)$$

$$= e\, dM_J/q^3 \qquad (4.41.24)$$

as given in Equation (A41) of Merson (1966).

In the light of the results given in Equations (4.41.12) to (4.41.24) we re-examine the constituent parts of the long periodic perturbations, given by Equations (4.41.5), (4.41.6) and (4.41.7).

Equations (4.41.5) become

$$di_{drag} = t \, T_2 L_8$$

$$= T_2 C_2(\dot{\omega}_S), \qquad (4.41.25)$$

$$d\Omega_{drag}\sin i = tp_5 L_7 T_1$$
$$= T_1 \sin i\; S_2(\dot{\omega}_S), \qquad (4.41.26)$$

$$du_{drag} + d\Omega_{drag}\cos i = tL_7(1-f)^{\frac{1}{2}}\, T_1$$
$$= T_1 \cos i\; S_2(\dot{\omega}_S), \qquad (4.41.27)$$

and

$$da_{drag} = de_{drag} = edv_{drag} = 0.0. \qquad (4.41.28)$$

Equations (4.41.25), (4.41.26), (4.41.27) and (4.41.28) comply with the expressions given in Appendix B7 of Gooding (1969b), for long periodic perturbations due to drag.

Equations (4.41.6) become

$$di_J = -p_{10} L_{10}$$
$$= -\, e\cos i\left[\sum_{k=1}^{L-2} a_k{}'\; S_k(\dot{\omega}_S) + p_{17}\, S_2(\dot{\omega}_S)\right]$$
$$= -\, e\cos i\left[\sum_{k=1}^{L-2} a_k{}'\; S_k(\dot{\omega}_S)\right.$$
$$\left. + (21/16)J_2^2(R/p)^4 n_0 e_0 \sin i_0 (1 - 15f/14) S_2(\dot{\omega}_S)\right], \qquad (4.41.29)$$

$$d\Omega_J \sin i = t(p_{10} L_3 - fe_0 L_4 + p_{11} L_8 - tp_{14} L_5)$$
$$= e_0 \cos i_0 \sum_{k=1}^{L-2} a_k{}'\; C_k(\dot{\omega}_S) - f e_0 \sum \frac{c_k}{e_0 \sin i_0}\; C_k(\dot{\omega}_S)$$
$$+ p_{11}\, C_2(\dot{\omega}_S) - p_{14}\left[\sum_{k=1}^{L-2} a_k{}'\int_0^t S_k(\dot{\omega}_S)dt + p_{17}\int_0^t S_2(\dot{\omega}_S)dt\right]$$
$$= \sum_{k=1}^{L-2}(e_0 \cos i_0 a_k{}' - \sin i\; c_k)C_k(\dot{\omega}_S)$$
$$+ (21/16)J_2^2(R/p)^4 n_0 e_0^2 \sin i_0 \cos i_0 (1 - 15f/7) C_2(\dot{\omega}_S)$$
$$- (15/2)J_2(R/p)^2 n_0 e_0 f \cos i_0 \left[\sum_{k=1}^{L-2} a_k{}'\int_0^t S_k(\dot{\omega}_S)dt\right.$$

$$+ (21/16)J_2^2(R/p)^4 n_0 e_0 \sin i_0 (1 - 15f/14)\int_0^t S_2(\dot{\omega}_S)dt\Big], \qquad (4.41.30)$$

and

$$du_J + d\Omega_J \cos i = tp_5\Big\{-L_3\Big[e + (p/r+1)\cos v\Big]/q^2$$

$$- e\Big[(p/r)^2 L_1/q^2 - L_2\Big] + (p/r+1)\sin v\, L_6 + tp_{13} L_5$$

$$+ p_{17} L_8\Big[(5/2)e - (p/r+1)\cos v\Big]\Big\}$$

$$= t\sin i\Big\{ -\frac{1}{t}\sum_{k=1}^{L-2} a_k' C_k(\dot{\omega}_S)\Big[e + (p/r+1)\cos v\Big]/q^2$$

$$- e(p/r)^2(1/q^2)(1/t)\sum_{k=1}^{L-2} \frac{(a_k + d_k - b_k)}{e_0 \sin i_0}\, C_k(\dot{\omega}_S)$$

$$+ (e/t)\sum_{k=1}^{L-2} \frac{(b_k + d_k)}{e_0 \sin i_0}\, C_k(\dot{\omega}_S)$$

$$+ (p/r+1)\sin v\Big[\frac{1}{t}\sum_{k=1}^{L-2} a_k' S_k(\dot{\omega}_S) + \frac{p_{17}}{t} S_2(\dot{\omega}_S)\Big]$$

$$+ p_{13}\Big[\frac{1}{t}\sum_{k=1}^{L-2} a_k'\int_0^t S_k(\dot{\omega}_S)dt + \frac{p_{17}}{t}\int_0^t S_2(\dot{\omega}_S)dt\Big]$$

$$+ p_{17}\Big[(5/2)e - (p/r+1)\cos v\Big]\frac{1}{t}\, C_2(\dot{\omega}_S)\Big\}$$

$$= \Big[e + (p/r+1)\cos v\Big]\frac{\sin i_0}{q^2}\cdot\sum_{k=1}^{L-2} a_k' C_k(\dot{\omega}_S)$$

$$+ \frac{ap}{r^2}\sum_{k=1}^{L-2}(b_k - a_k - d_k)\, C_k(\dot{\omega}_S) + \sum_{k=1}^{L-2}(b_k + d_k)\, C_k(\dot{\omega}_S)$$

$$+ (1 + p/r)\sin v \sin i\sum_{k=1}^{L-2} a_k' S_k(\dot{\omega}_S)$$

$$+ (21/16)J_2^2(R/p)^4 n_0 e_0 f(1 - 15f/14)\left[\left[\frac{5e}{2} - (1 + p/r)\cos v\right] C_2(\dot{\omega}_S) + (1 + p/r)\sin v \; S_2(\dot{\omega}_S)\right]$$

$$+ 3J_2(R/p)^2 n_0 e_0 (4 - 5f)\sin i_0 \left[\sum_{k=1}^{L-2} a_k' \int_0^t S_k(\dot{\omega}_S)dt\right.$$

$$\left.+ (21/16)J_2^2(R/p)^4 n_0 e_0 \sin i_0 (1 - 15f/14)\int_0^t S_2(\dot{\omega}_S)dt\right]. \qquad (4.41.31)$$

The expressions given in Equations (4.41.29), (4.41.30) and (4.41.31) comply with those given in Appendix A11 of Merson (1966) as Equations (A38), (A39) and (A40).

The expression given for da_J in Equation (4.41.6) is correct since, as stated in Section 5.3 of Gooding (1974), we "re-introduce da as a basic perturbation, its long periodic value being set to zero."

The expression for de_J comes from Equation (4.41.23) since

$$de_J = DEOQSQ \times q^2 = \frac{de}{q^2} \times q^2 = de.$$

The expression given for edv_J becomes, using Equations (4.41.23) and (4.41.24),

$$edv_J = e\sin v(1 + p/r)\frac{de_J}{q^2} + \left(\frac{p}{r}\right)^2 \frac{edM_J}{q^3}. \qquad (4.41.32)$$

To demonstrate that edv_J may be expressed as this combination of de_J and dM_J, we write v as

$$\left.\begin{aligned} v &= 2\tan^{-1}\phi \\ \text{where} \quad \phi &= \left(\frac{1 + e}{1 - e}\right)^{\frac{1}{2}} \tan\frac{E}{2}. \end{aligned}\right\} \qquad (4.41.33)$$

Hence

$$dv = 2\left(\frac{d}{d\phi}\tan^{-1}\phi\right)d\phi\ . \tag{4.41.34}$$

However,

$$\frac{d}{d\phi}(\tan^{-1}\phi) = (1+\phi^2)^{-1} = \left(1+\tan^2\frac{v}{2}\right)^{-1} = \left(\sec^2\frac{v}{2}\right)^{-1}, \tag{4.41.35}$$

and

$$d\phi = \frac{\partial\phi}{\partial e}\,de + \frac{\partial\phi}{\partial E}\,dE. \tag{4.41.36}$$

The eccentric anomaly, E, is related to the mean anomaly, M, by

$$M = E - e\sin E,$$

from which we obtain

$$dE(1 - e\cos E) = dM + de\sin E$$

and hence

$$dE = \frac{a}{r}\,dM + \frac{a\sin E}{r}\,de. \tag{4.41.37}$$

Substituting Equation (4.41.37) into Equation (4.41.36), we have

$$d\phi = \frac{\partial\phi}{\partial e}\,de + \frac{\partial\phi}{\partial E}\left(\frac{a}{r}\,dM + \frac{a\sin E}{r}\,de\right)$$

$$= \left(\frac{\partial\phi}{\partial e} + \frac{\partial\phi}{\partial E}\,\frac{a\sin E}{r}\right)de + \frac{\partial\phi}{\partial E}\,\frac{a}{r}\,dM\ . \tag{4.41.38}$$

From Equation (4.41.33) we obtain

$$\frac{\partial\phi}{\partial E} = \frac{1}{2}\left(\frac{1+e}{1-e}\right)^{\frac{1}{2}}\sec^2\frac{E}{2} \tag{4.41.39}$$

and

$$\frac{\partial\phi}{\partial e} = \left(\frac{1-e}{1+e}\right)^{\frac{1}{2}}\frac{\tan\frac{E}{2}}{(1-e)^2}\ . \tag{4.41.40}$$

Substituting Equations (4.41.35), (4.41.38) and (4.41.40) into Equation (4.41.34) we find that

$$dv = 2\left(\sec^2\frac{v}{2}\right)^{-1}\left[\frac{\tan\frac{E}{2}}{(1-e)^2}\left(\frac{1-e}{1+e}\right)^{\frac{1}{2}} de\right.$$

$$\left. + \frac{a\sin E}{r}\frac{\partial\phi}{\partial E} de + \frac{a}{r}\frac{\partial\Phi}{\partial E} dM\right]. \tag{4.41.41}$$

If we consider the coefficient of dM,

$$K_1 = \frac{2}{\sec^2\frac{v}{2}}\quad\frac{a}{r}\frac{\partial\phi}{\partial E},$$

then, by Equation (4.41.39)

$$K_1 = \frac{2}{\sec^2\frac{v}{2}}\quad\frac{a}{r}\frac{1}{2}\left(\frac{1+e}{1-e}\right)^{\frac{1}{2}}\sec^2\frac{E}{2},$$

which may be rearranged, using Equations (4.41.33), to give

$$K_1 = \frac{a}{r}\frac{1}{\sec^2\frac{v}{2}}\left(\frac{1+e}{1-e}\right)^{\frac{1}{2}}\left[1+\tan^2\frac{v}{2}\left(\frac{1-e}{1+e}\right)\right]$$

$$= \frac{a}{rq}(1+e\cos v) = \frac{p^2}{r^2q^3}. \tag{4.41.42}$$

Similarly, the coefficient of de may be written as

$$K_2 = \frac{2}{\sec^2\frac{v}{2}}\quad\frac{\tan\frac{E}{2}}{(1-e)^2}\left(\frac{1-e}{1+e}\right)^{\frac{1}{2}} + \frac{p^2\sin E}{r^2q^3},$$

which may also be rearranged, using Equations (4.41.33), to give

$$K_2 = \frac{2}{\sec^2\frac{v}{2}}\left(\frac{1-e}{1+e}\right)^{\frac{1}{2}}\frac{\tan\frac{v}{2}}{(1-e)^2}\left(\frac{1-e}{1+e}\right)^{\frac{1}{2}} + \frac{p^2\sin E}{r^2q^3}$$

$$= \frac{\sin v}{q^2} + \frac{p^2 \sin E}{r^2 q^3}$$

$$= \frac{\sin v}{q^2} (1 + p/r). \qquad (4.41.43)$$

Substituting Equations (4.41.42) and (4.41.43) into Equation (4.41.41) we obtain

$$dv_J = \frac{\sin v}{q^3} (1 + p/r)\ de_J + \frac{p^2}{r^2 q^3}\ dM_J$$

and hence Equation (4.41.31) for edv_J.

Finally, the expressions given for long periodic perturbations caused by tesseral harmonics, given in Equations (4.41.7), become

$$di_{2,2} = -3Wp_5S$$
$$= -3J_{2,2}\, n_0 (R/p)^2 \sin i_0 \int_0^t \sin(2\ell_{2,2})dt, \qquad (4.41.44)$$

$$d\Omega_{2,2} \sin i = 3Wp_4p_5C - \frac{9W}{2} SI\ p_5\ z_4 f$$
$$= 3J_{2,2}\, n_0 (R/p)^2 \sin i_0 \cos i_0 \int_0^t \cos(2\ell_{2,2})dt$$
$$-(9/2)J_{2,2}\, n_0 (R/p)^2 \sin i_0 n_0 J_2 (R/p)^2 f \int_0^t\int_0^t \sin(2\ell_{2,2})dt\,dt, \qquad (4.41.45)$$

$$du_{2,2} + d\Omega_{2,2} \cos i = (9/2)WfC\left[1 + (p/rq)^2\right] + 18W\ z_4 f\, p_4\, SI$$
$$= (9/2)J_{2,2}\, n_0 (R/p)^2 f\left(1 + \frac{pa}{r^2}\right)\int_0^t \cos(2\ell_{2,2})\ dt$$
$$+ 18J_{2,2}\, n_0 (R/p)^2 n_0 J_2 (R/p)^2 f \cos i_0 \int_0^t\int_0^t \sin(2\ell_{2,2})dt\,dt, \qquad (4.41.46)$$

$$edv_{2,2} = (9/2)WfC \left(\frac{p}{r}\right)^2 \frac{e}{q^2}$$

$$= (9/2)J_{2,2}\, n_0 (R/p)^2 f \left(\frac{p}{r}\right)^2 \frac{e}{q^2} \int_0^t \cos(2\ell_{2,2})dt, \qquad (4.41.47)$$

and

$$de_{2,2} = da_{2,2} = 0.0.$$

The expression (4.41.44) for $di_{2,2}$ is exactly that given in Section 3 of Gooding (1974), for the $J_{2,2}$ perturbation in i.

The expression (4.41.45) may be written

$$d\Omega_{2,2} \sin i = 3J_{2,2}\, n_0 (R/p)^2 \sin i_0 \cos i_0 \int_0^t \cos(2\ell_{2,2})dt$$

$$- \frac{3}{2} J_2 n_0 (R/p)^2 f \int_0^t 3J_{2,2}\, n_0 (R/p)^2 \sin i_0 \left[\int_0^t \sin(2\ell_{2,2})dt\right] dt$$

$$= 3J_{2,2}\, n_0 (R/p)^2 \sin i_0 \cos i_0 \int_0^t \cos(2\ell_{2,2})dt$$

$$+ \frac{3}{2} J_2 n_0 (R/p)^2 f \int_0^t (di_{2,2})dt,$$

which is also given in Section 3 of Gooding (1974), when the terms that were neglected at the PROP5 stage, indicated by Gooding by primes, are included.

The expression (4.41.46) may be written

$$du_{2,2} + d\Omega_{2,2} \cos i = (9/2)J_{2,2}\, n_0 (R/p)^2 f \left(1 + \frac{pa}{r^2}\right) \int_0^t \cos(2\ell_{2,2})dt$$

$$+ 6J_2 n_0 (R/p)^2 \sin i_0 \cos i_0 \int_0^t 3 \sin i_0 J_{2,2}\, n_0 (R/p)^2 \left[\int_0^t \sin(2\ell_{2,2})dt\right] dt$$

$$= (9/2)J_{2,2}\, n_0 (R/p)^2 f \left(1 + \frac{pa}{r^2}\right) \int_0^t \cos(2\ell_{2,2})dt$$

$$- 6J_2 n_0 (R/p)^2 \sin i_0 \cos i_0 \int_0^t (di_{2,2})dt,$$

also given in Section 3 of Gooding (1974).

The perturbation $da_{2,2}$ is zero by definition and $dp_{2,2}/(2p)$ is given by Gooding (1974) to be zero. Since

$$p = a(1 - e^2),$$

it follows that

$$de_{2,2} = 0.$$

Analogous to Equation (4.41.32), we have

$$edv_{2,2} = \frac{e \sin v}{q^2}(1 + p/r)de_{2,2} + \frac{p^2 e}{r^2 q^3} dM_{2,2} .$$

However, since $de_{2,2}$ is zero, we may write

$$edv_{2,2} = \frac{p^2 e}{r^2 q^3} dM_{2,2} . \qquad (4.41.48)$$

We have also

$$dM_{2,2} = \frac{q^3}{e \sin v} \frac{dr_{2,2}}{p}$$

(shown in Appendix III).

Substituting this into Equation (4.41.48) we obtain

$$edv_{2,2} = \frac{p^2 e}{r^2 q^3} \frac{q^3}{e \sin v} \frac{dr_{2,2}}{p}$$

$$= \left(\frac{p}{r}\right)^2 \frac{1}{\sin v} \frac{dr_{2,2}}{p} .$$

Hence, using the expression given in Section 3 of Gooding (1974) for $dr_{2,2}/p$, we have

$$edv_{2,2} = \left(\frac{p}{r}\right)^2 \frac{1}{\sin v} (9/2)f \frac{e}{q^2} \sin v \, J_{2,2} n_0 (R/p)^2 \int_0^t \cos(2\ell_{2,2})dt$$

$$= (9/2)J_{2,2} n_0 (R/p)^2 f(p/r)^2 \frac{e}{q^2} \int_0 \cos(2\ell_{2,2})dt$$

as given in Equation (4.41.47).

5. Test Runs of the New PROP

A number of test runs of the new PROP have been made so that a comparison between this and PROP6 may be undertaken. The new PROP is referred to as MACPROP, although strictly that is the name of the macro for running the new PROP (see Appendix IV). MACPROP was then run using some of the new options that are not available in PROP6. In this way the effect of changing the assumed atmospheric rotation rate, Λ, and the values of the zonal harmonics, the J's, could be investigated.

The satellite chosen for these test runs was Explorer 24 (1964-76A) in the five months prior to its decay in October 1968. This satellite had been analysed, using PROP6, and a determination of zonal wind speeds made, by Swinerd (1981). The satellite was chosen for the present work because it was a balloon satellite. The orbit of such a satellite is greatly influenced by zonal winds, especially near to decay, and estimates of these wind speeds were available from the published analysis.

The original orbit determination was made at 18 epochs and the same 18 epochs have been used here. The initial control cards, initial elements, station location data and observations used were identical to those used by Swinerd. When multi-run MACPROP jobs were required further control cards were inserted, after the observations. These cards indicated, via the control parameters NEWSAT, IOBSNS and ISENSR, that the initial elements, station data and observations were identical to those of the previous run in the job. "Aston Control Cards" (ACC's), see subsection 4.28, PROP, were used to change the values of Λ and the J's used.

5.1 Comparison between PROP6, MACPROP and OPTimised MACPROP.

The first comparisons were made by running three separate jobs for each epoch. The first job used PROP6, to repeat the results obtained by Swinerd. The second job used MACPROP, using the binary version created using the default compiler. The third job used MACPROP with the parameter OPT. This had the effect of using the binary version created using the optimising compiler. This version of PROP will be referred to as MACPROP(OPT).

There was no difference between the results obtained using MACPROP and those obtained from MACPROP(OPT). The optimised version saved approximately 13% of the computer time taken and used 195 extra words of core store.

This check on the execution of MACPROP(OPT) has been carried out since there are indications that there are errors in the optimising compiler. The test runs carried out demonstrate that, under the conditions used, MACPROP(OPT) executes correctly. However, if control parameters of NEWSAT or NOTHER are non-zero, or if JELTYP is neither 00 nor 11, then the MACPROP user should perform some similar parallel test runs before relying upon MACPROP(OPT). Further, these tests read all the observations under a "LOOXEE" format of 666666. Other formats should be treated with similar caution initially if MACPROP(OPT) is used.

The output from PROP6 and MACPROP was compared. MACPROP ran approximately one third faster than PROP6 and used 320 words less core storage. All runs, except epoch 11 (MJD 40063) converged in the same number of iterations, with the same observation rejections and with a maximum change of $\mathcal{E}$ (the convergence parameter) of 0.001. Epoch 11 took one less iteration with MACPROP, when $\mathcal{E}$ was 0.869 rather than 0.788. PROP6 rejected observation number 25, indicating that its weighted residual was between $3\mathcal{E}$ and $4\mathcal{E}$ (denoted by *), and observation number 28, indicating that its weighted residual was between $4\mathcal{E}$ and $10\mathcal{E}$ (denoted by **). MACPROP only rejected observation number 28, with one *.

The changes in orbital parameters obtained by using MACPROP rather than PROP6 were usually less than 8% of the standard deviation of that parameter. The exception to this was, again, epoch 11. At this epoch the parameters a, e_0, M_1 and M_3 were altered by approximately one standard deviation each. The parameter M_2 was altered by approximately half a standard deviation and the parameters i_0, Ω_0, ω_0 and M_0 were altered by less than 12.5% of a standard deviation each. M_4 and M_5 were not parameters at that epoch.

Overall, the changes obtained by using MACPROP, rather than PROP6, are so small that all PROP6 runs may be regarded as valid. The change to MACPROP does, however, improve the performance of PROP in some cases.

5.2 Effect of Changing the Values of the Zonal Harmonics used in MACPROP.

MACPROP and MACPROP(OPT) will be regarded as synonymous in this and further subsections of Section 5.

MACPROP may be run using a completely different set of zonal harmonics (J's) to those stored in the binary version of the program. To investigate the effect of changing the J's MACPROP was run, for all 18 epochs used by Swinerd (1981), with the default even harmonics and the set of 9 odd J coefficients given by King-Hele, Brookes and Cook (1981). The resultant output was compared with that obtained by running MACPROP with the default J's.

The changes obtained were slight. One convergence parameter increased by 0.001 to 0.983 when the new set of J's were used. There was virtually no change in the orbital parameters obtained. The use of the new J's caused the standard deviations of M_3, M_4 and M_5 to decrease slightly at the later epochs, near to satellite decay.

5.3 Effect of Assuming Different Atmospheric Rotation Rates

Swinerd (1981) gives various values of the atmospheric rotation rate, Λ, which may be associated with the various epochs investigated.

A value Λ_{av} has been obtained as an average rotation rate, over the whole period under analysis. It was obtained by a least squares method. Two values of $\Lambda_{\frac{1}{2}}$ were obtained when the period under investigation was, arbitrarily, split at MJD 40086 and a least squares fit of Λ obtained for each portion of the period under investigation.

Five values of Λ_{eye} were obtained by fitting averaged rotation rate curves to subsets of the data.

Finally, values of Λ_{fit} could be derived from the dependence of Λ on local solar time given by Swinerd (1981). Eighteen values of Λ_{fit} were obtained, one associated with the local solar time at each epoch under investigation.

The values of Λ found from Swinerd (1981) are given in Table 5.3.1.

PROP6 has within it, an assumed atmospheric rotation rate of $\Lambda = 1.2$ revolutions per day, and this is the default value of Λ used in MACPROP. The rotation of the upper atmosphere produces a force, normal to the orbital plane, on the satellite. Therefore the value of Λ affects the orbital inclination, i_0, and its rate of change, i_1. MACPROP has been run, using an ACC to change the assumed Λ for each epoch, so that the extent of the effect may be investigated. The values of Λ used were those given in Table 5.3.1. In addition a value of $\Lambda = 1.0$ was used for each epoch. The default set of zonal harmonics was used in each case.

All runs converged in the same number of iterations and with the same observation rejections as MACPROP with the default Λ. The convergence parameter $\mathcal{E}$ varied by a maximum of 0.003.

The changes in orbital coefficients achieved by varying Λ were small compared with the standard deviations of the parameters. However, as expected, the value of i_0 changed in over 80% of the runs. The value of i_1, which was not a parameter, changed slightly in all but three of the 91 runs. The values of i_0 and i_1 obtained at each epoch were linearly dependent upon Λ.

Table 5.3.1. Values of Λ from Swinerd (1981)

Nomenclature	Λ	Period of applicability (MJD)	Corresponding Epoch numbers
Λ_{av}	1.06	39994 - 40142	1,—,18
$\Lambda_{\frac{1}{2},1}$	0.98	39994 - 40086	1,—,14
$\Lambda_{\frac{1}{2},2}$	1.14	40086 - 40142	14,—,18
$\Lambda_{eye,1}$	1.1	39994 - 40022	1,—,5
$\Lambda_{eye,2}$	0.9	40029 - 40057	6,—,10
$\Lambda_{eye,3}$	1.1	40063 - 40106	11,—,16
$\Lambda_{eye,4}$	0.8	40107 - 40126	none
$\Lambda_{eye,5}$	1.3	40127 - 40142	17,18
$\Lambda_{fit,1}$	1.192	39994	1
$\Lambda_{fit,2}$	1.244	40001	2
$\Lambda_{fit,3}$	1.085	40008	3
$\Lambda_{fit,4}$	0.974	40016	4
$\Lambda_{fit,5}$	0.910	40022	5
$\Lambda_{fit,6}$	0.851	40029	6
$\Lambda_{fit,7}$	0.815	40035	7
$\Lambda_{fit,8}$	0.795	40041	8
$\Lambda_{fit,9}$	0.806	40050	9
$\Lambda_{fit,10}$	0.909	40057	10
$\Lambda_{fit,11}$	1.312	40063	11
$\Lambda_{fit,12}$	1.313	40068	12
$\Lambda_{fit,13}$	1.231	40077	13
$\Lambda_{fit,14}$	1.138	40086	14
$\Lambda_{fit,15}$	1.083	40091	15
$\Lambda_{fit,16}$	0.915	40106	16
$\Lambda_{fit,17}$	1.252	40127	17
$\Lambda_{fit,18}$	1.329	40142	18

Changing the assumed value of Λ in MACPROP changed the standard deviations of the orbital parameters. Decreases of up to 0.5% in standard deviation were obtained in the tests carried out.

5.4 The Effect of Combining New J's and an Estimated Λ

Two runs were made at each epoch with the new set of J's, input via an ACC, together with a change in Λ. The rotation rates chosen for these tests were $\Lambda = 1.0$ and $\Lambda = \Lambda_{fit}$. Comparisons were made between the output obtained and that obtained when the default J's and Λ were used.

No difference in the number of iterations and observation rejections was found. The convergence parameter $\mathcal{E}$ varied by a maximum of 0.003.

The effect of using the new J's and altering the value of Λ was most apparent at epoch 18, just prior to the satellite's decay.

When Λ was set at 1.0 the standard deviations of the orbital parameters at this epoch increased. When Λ was set to 1.329, which was Λ_{fit} for this epoch, the standard deviations decreased. This indicates that the larger value of Λ is a more appropriate estimate of the atmospheric rotation rate for that epoch than the default value of $\Lambda = 1.2$, at the height of this particular satellite.

6. Conclusions

The computer program PROP is a principal tool in Earth satellite research. The original PROP available to researchers at Aston University was a binary version of PROP6, that had been compiled elsewhere on a slightly different computer to that available at Aston. Some of the documentation of PPOP6 was out of date and incomplete. Changes to PROP could not be made easily since the source code was not available at Aston. Without complete documentation some lack of confidence in the program was experienced by users, who felt that anomalous results may be due either to errors in the program or to their lack of understanding of the program.

To overcome these problems a new source of PROP has been created at Aston University. This new program is different to PROP6 in several aspects so it is referred to as MACPROP to distinguish it from PROP6.

PROP has been investigated, in detail, segment by segment. An overall description of the action of the program has been given, followed by an upgraded description of each segment. Reference has been made to the previous documentation where that is possible. Much of the theoretical background of the segments, missing from that documentation, is derived here.

During these investigations several, minor, programming errors have been found, and corrected. In addition, a number of new facilities have been written into MACPROP.

A major source of satellite observations, for use in PROP, is the U.S. Naval Research Laboratory (NRL). These observations are supplied in a format that is not acceptable to PROP6 and the observations must be preprocessed by another program. An additional observation format has been written into MACPROP so that NRL observations may be read directly in the format provided.

The values of the zonal harmonics of the Earth's gravitational field (J's) are held, in binary code, in PROP. The PROP6 user may overwrite a maximum of five of these values with alternatives which will be in effect for one PROP run. In addition to this feature the user of the new program, MACPROP, may also define a completely new set of up to forty harmonics. These will be in effect for the remainder of the PROP job after they are read in, which may be several PROP runs.

The orbit of an Earth satellite is affected by the rotation of the atmosphere through which it passes. In PROP6 this atmospheric rotation rate is set at 1.2 revolutions per day. The MACPROP user may specify an alternative value if it is required.

A PROP user may wish to make two, very similar, runs in one PROP job. If the only difference is information on the control card, then the MACPROP user may use a new value of one of the control parameters. Thus it is not necessary to input the epoch/identity and initial elements to the program again. The PROP6 user does not have this facility. The situation arises, typically, when the user is unsure how many coefficients in the polynomial representing mean anomaly should be treated as orbital parameters.

A PROP user may give the position of a satellite observer in geocentric Cartesian coordinates. These are converted to latitude, longitude and height above a reference spheroid, within PROP. The method of conversion used in PROP6 is an iterative method specific to PROP. The method has been investigated and is described. However, MACPROP gives the user the option of employing a more familiar technique, Newton's method of finding the roots of a polynomial, in performing the required conversion.

PROP generates a large amount of line printer output. Approximately half of this information is printed at the intermediate steps of the iterative process of refining orbital parameters. When using MACPROP this output may, optionally, be suppressed.

For technical reasons the binary version of PROP previously available does not check the satellite identity on observation cards when it is used to refine the orbital parameters of a satellite launched in, or after, 1984. Modifications have been incorporated into MACPROP so that the checks are carried out for all satellites.

All these changes, together with some, more cosmetic, changes, have been described in the appropriate sections. The description includes instructions for the user to employ the various options available.

MACPROP has been tested and its output compared with that obtained from PROP6. Some, minor, changes to the orbital parameters were obtained, due to the correction of the programming errors. However, all PROP6 runs may be regarded as valid. Further improvements to the orbits obtained by MACPROP can be made by using an alternative set of J's and an appropriate value of the atmospheric rotation rate.

MACPROP is considerably faster than PROP6 and uses slightly less core storage. A faster version of MACPROP, created using an optimising compiler, is also available.

The program PROP has been checked, independently of its original authors, and several errors removed. A detailed description of how each part of the program works is now available. Therefore the user may regard orbits obtained from MACPROP with greater confidence.

Appendix I. COMMON Areas of Store.

The definition and use of COMMON areas of store has been given by ICL (1971). Several COMMON blocks are used within PROP and a description of their usual contents is given here. Some items in COMMON have their initial values set by a segment BLOCK DATA.

COMMON block /ORBIT/ contents:

EMU: Earth's gravitational constant μ. Preset by BLOCK DATA to 398602.0 km^3/sec^2.

ERAD: Mean equatorial radius of the Earth, R. Preset by BLOCK DATA to 6378.163 km.

EJ22: Tesseral harmonic of the geoid, $J_{2,2}$. Preset by BLOCK DATA to 1.8×10^{-6}.

ELAM22: Angle between the Greenwich meridian and the equatorial bulge of $J_{2,2}$, measured positive eastwards, $\lambda_{2,2}$. Preset by BLOCK DATA to -18.0°.

L: Number of Earth's zonal harmonics to be considered. Preset by BLOCK DATA to 20.

DENSCH: Density scale height, H. Preset by BLOCK DATA to 25.0 km.

MJDOCH: Modified Julian Date of the epoch.

NOMIAL: 6 element array of degrees of the six polynomials used to represent the six orbital elements.

ELEMT: (6×6) array of coefficients of the six polynomials representing the six orbital elements.

MJD: Modified Julian Date.

TIME: Time, decimals of a day relative to MJD.

X, Y, Z: Geocentric Cartesian coordinates of satellite's position.

XDOT YDOT ZDOT	Geocentric Cartesian coordinates of satellite's velocity.

QPD: 30 element array of quantities related to partial derivatives of satellite position and velocity, such that:

QPD(1) = a, QPD(2) = e, QPD(3) = n,

QPD(4) = $\cos v$, QPD(5) = $\sin v$, QPD(6) = p/r,

QPD(7) = $\dot{r}$, QPD(8) = $(1 - e^2)^{\frac{1}{2}}$, QPD(9) = t,

QPD(10) to QPD(12) = $\overline{F}$
QPD(13) to QPD(15) = $\overline{G}$
QPD(16) to QPD(18) = A
QPD(19) to QPD(21) = B
(see subsection 4.33, SATXYZ)

QPD(22) = r, QPD(23) = $(\mu p)^{\frac{1}{2}}/r$, QPD(24) = $(\mu/p)^{\frac{1}{2}}$,

QPD(25) = $\frac{\partial\Omega_{(1)}}{\partial e_0}$, QPD(26) = $\frac{\partial\Omega_{(1)}}{\partial i_0}$, QPD(27) = $\frac{\partial\Omega_{(1)}}{\partial n_0}$

QPD(28) = $\frac{\partial\omega_{(1)}}{\partial e_0}$, QPD(29) = $\frac{\partial\omega_{(1)}}{\partial i_0}$, QPD(30) = $\frac{\partial\omega_{(1)}}{\partial n_0}$

(see subsection 4.25, PRELON)

MODEL: 5 element array of number of coefficients in the five basic element polynomials to be treated as parameters.

DERIV: (6×20) array of partial derivatives of satellite position and velocity with respect to the orbital parameters, $\frac{\partial x}{\partial \varepsilon_j}$, $\frac{\partial y}{\partial \varepsilon_j}$, $\frac{\partial z}{\partial \varepsilon_j}$, $\frac{\partial \dot{x}}{\partial \varepsilon_j}$, $\frac{\partial \dot{y}}{\partial \varepsilon_j}$, $\frac{\partial \dot{z}}{\partial \varepsilon_j}$.

EJ: 40 element array of the Earth's zonal harmonics, J_n. The first 21 are preset by BLOCK DATA to

10^{-18}, 1082.637×10^{-6}, -2.531×10^{-6}, -1.619×10^{-6},

-0.246×10^{-6}, 0.558×10^{-6}, -0.326×10^{-6}, -0.209×10^{-6},

-0.094×10^{-6}, -0.233×10^{-6}, 0.159×10^{-6}, -0.188×10^{-6},

-0.131×10^{-6}, 0.085×10^{-6}, -0.026×10^{-6}, 0.048×10^{-6},

-0.258×10^{-6}, -0.137×10^{-6}, 10^{-18}, -0.087×10^{-6},

20×10^{-18} .

The COMMON block /PRECON/ contents are given by:

ZONSEC: 4 element array of secular changes in e, i. Ω and ω due to zonal harmonics.

DRASEC: 4 element array of secular changes in e, i, Ω and ω due to atmospheric drag.

TDRAG: 2 element array of quantities related to the effect of drag on the satellite orbit.

F: $\sin^2 i_0$ $(=f)$

QSQ: $1 - e_0^2$ $(=q^2)$

EX: $e_0 \sin i_0$

P4: $\cos i_0$

P5: $\sin i_0$

P10: $e_0 \cos i_0$

P11: $(21/16)J_2^2(R/p)^4 n_0 e_0^2 \sin i_0 \cos i_0 (1 - 15f/7)$

where $R \equiv$ ERAD and $p \equiv a(1 - e_0^2)$.

P13: $3J_2(R/p)^2 n_0 e_0 (4 - 5f)$

P14: $(15/2)J_2(R/p)^2 n_0 e_0 f \cos i_0$

P15: $\sin 2(\omega_0 - \pi/2)$

P16: $\cos 2(\omega_0 - \pi/2)$

P17: $(21/16)J_2^2(R/p)^4 n_0 e_0 \sin i_0 (1 - 15f/14)$

TW: $3J_{2,2} n_0 (R/p)^2$

STL: $\sin 2\ell_{2,2}$

CTL: $\cos 2\ell_{2,2}$

where $\ell_{2,2}$ = (modified sidereal angle at MJDOCH) $+ \lambda_{2,2} - \Omega_0$.

TTHMO: $2\dot{\ell}_{2,2}$

Z4F: $f\, n_0 J_2 (R/p)^2$

ABCD: (38 × 8) array containing the main output from PRELON. The 8 quantities in the k^{th} row of ABCD may be denoted a_k^c, b_k^c, c_k^c, d_k^c, a_k^s, b_k^s, c_k^s and d_k^s, given by

$$\psi_k^c = (\psi_k/e_0 \sin i_0) \cos k(\omega_0 - \pi/2),$$

$$\psi_k^s = (\psi_k/e_0 \sin i_0) \sin k(\omega_0 - \pi/2)$$

in which ψ is any one of a, b, c or d and a_k, b_k, c_k and d_k are defined by Merson (1966).

ATMROT: Factor, Λ, giving the rotation rate of the upper atmosphere when multiplied by the rotation rate of the Earth. Preset to 1.2 at the beginning of each run of PROP. This is a new variable, not present in PROP6.

All of the COMMON block /PRECON/, except ATMROT, is set by the segment PRELON, but F is preset to 0.0 by BLOCK DATA.

The COMMON block /CNTROL/ contents are:

IR: Channel number from which input to PROP is read. Preset to 1 by BLOCK DATA.

IP: Channel number to which the main lineprinter output from PROP is sent. Preset to 2 by BLOCK DATA.

IPUNCH: Channel number to which the cardpunch output from PROP is sent. Preset to 3 by BLOCK DATA.

IITT: Channel number to which the intermediate lineprinter output from PROP is sent. Preset to 4 by BLOCK DATA. This is a new channel which is not used in this way in PROP6.

ILINES: Number of lines that may be printed on one page of lineprinter output. Preset to 60 by BLOCK DATA. (Preset to 66 in PROP6).

RIDENT: Identity number of the satellite. This is a real variable, unlike PROP6 which uses the integer variable IDENT.

KKKKK: Control information to be placed in NOMIAL.

MMMMM: Control information to be placed in MODEL.

MODE: Mode of iteration.

NEWSAT: Control information defining where the initial estimate of the orbital elements is to be obtained.

MJDINC: Number of days over which prediction of initial elements from those given is required.

MAXITN: Maximum number of iterations allowed.

MINOBS: Minimum number of accepted observations allowed.

JELTYP: Parameter governing transformation of certain elements to prevent ill-conditioned matrices and to aid convergence.

NOTHER: Indicator of whether "previous-orbit" cards are included in the data deck.

ITIMEC: Indicator of what "time-correction" cards are included in the data deck.

ISENSR: Indicator of the extent to which sensor card data should be read and printed.

IOBSNS: Indicator of the extent to which observation card data should be read and printed.

The blank COMMON block is normally defined in two sections. The first section, sometimes denoted by

COMMON//

contains, for one observation under consideration:

JTYPE: Denotes the type of observation.

JRATES: Indicates whether rates of change of some quantities were observed.

JDCS: Indicates whether direction cosines were observed.

OBSNEX: 23 element array containing all the data relevant to the observation.

COSDEC: $\cos \delta_T$, where δ_T is the theoretical declination, according to the current estimate of the orbit.

CALC: 6 element array containing the theoretical observed quantities η_{T_k} .

PARDER: (6×20) array containing partial derivatives $\frac{\partial \eta_{T_k}}{\partial \varepsilon_j}$.

The first section of blank COMMON also contains:

SIGMA: 6 element array of standard deviations of some observed quantities. Preset by PROP at the beginning of a PROP job to 0.00029, 0.15, 0.0005, 0.0017, 0.1 and 0.0333 respectively.

NUMOBS: Number of observations read in to PROP run.

OEMTM: Covariance matrix from a previous run.

OPARAS: Initial orbital parameters if "previous run" cards are read in.

NOODOF: Number of degrees of freedom of a "previous run".

OSUMSQ: Sum of the squares of weighted residuals of a "previous run".

REJLEV: 3 element array of rejection levels, preset by PROP at the beginning of a PROP job to 3.0, 4.0 and 10.0 respectively.

EPSLON: Current measure of convergence, $\mathcal{E}$.

NUMPAR: Number of orbital parameters.

The second section of blank COMMON is always denoted by

COMMON

and it contains:

INDRES: Indicator of whether rotated residuals are required.

and, for one observation under consideration, the theoretical values, according to the current estimate of the orbit, of:

XX, YY, ZZ: Topocentric Cartesian coordinates of the satellite,

RANGE: Topocentric range of the satellite,

RA: Right ascension of the satellite,

SINDEC: $\sin \delta_T$,

DEC: δ_T .

The COMMON block /WORKIN/ is not used to pass values between segments but only as a workspace area. The use of this is described in the subsections relevant to those segments. In each case the dummy array REST, present in PROP6, has been removed from the end of the COMMON block, as it is unused.

Appendix II   Manipulation of the expression for ω

In subsection 4.5 we obtained

$$\sin\omega = -\frac{1}{h}\left\{\left[a(\cos E - e)\dot{x} + \frac{x}{r}\sin E(\mu a)^{\frac{1}{2}}\right]\cos\Omega + \left[a(\cos E - e)\dot{y} + \frac{y}{r}\sin E(\mu a)^{\frac{1}{2}}\right]\sin\Omega\right\} \qquad (4.5.24)$$

and

$$\cos\omega = \left\{\left[\frac{x}{r}\cos E - \dot{x}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\sin E\right]\cos\Omega + \left[\frac{y}{r}\cos E - \dot{y}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\sin E\right]\sin\Omega\right\} \qquad (4.5.21)$$

where

$$h^2 = \mu a(1 - e^2),$$

$$\tan\Omega = \frac{y\dot{z} - z\dot{y}}{x\dot{z} - z\dot{x}}$$

and $r^2 = x^2 + y^2 + z^2$.

If we now let

$$W = \tan\omega = \frac{\sin\omega}{\cos\omega}$$

then

$$W = \frac{1}{h}\left\{-\left[\frac{x}{r}(\mu a)^{\frac{1}{2}}\sin E + \dot{x}a(\cos E - e)\right]\cos\Omega - \left[\frac{y}{r}(\mu a)^{\frac{1}{2}}\sin E + \dot{y}a(\cos E - e)\right]\sin\Omega\right\} \times$$

$$\left\{\left[\frac{x}{r}\cos E - \dot{x}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\sin E\right]\cos\Omega + \left[\frac{y}{r}\cos E - \dot{y}\left(\frac{a}{u}\right)^{\frac{1}{2}}\sin E\right]\sin\Omega\right\}^{-1}$$

$$= -\left\{\left[\frac{x}{r}\ \sin E + \dot{x}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}(\cos E - e)\right]\cos\Omega\right.$$

$$+\left.\left[\frac{y}{r}\ \sin E + \dot{y}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}(\cos E - e)\right]\sin\Omega\right\} \times$$

$$(1 - e^2)^{-\frac{1}{2}}\left\{\left[\frac{x}{r}\cos E - \dot{x}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\sin E\right]\cos\Omega\right.$$

$$+\left.\left[\frac{y}{r}\cos E - \dot{y}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\sin E\right]\sin\Omega\right\}^{-1} \qquad \text{(A.II.1)}$$

Now

$$x = r.\cos v = a(\cos E - e),$$

$$\frac{y}{b} = \sin E, \text{ where } b = a(1 - e^2)^{\frac{1}{2}},$$

$$\text{and } \cos E = \frac{a - r}{ae}\ .$$

Hence Equation (A.II.1) may be written

$$W = -\left\{\left[\frac{x}{r}\frac{y}{b} - \frac{x\dot{x}}{a}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\right](x\dot{z} - z\dot{x})\right.$$

$$+\left.\left[\frac{y^2}{rb} + \frac{\dot{y}x}{a}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\right](y\dot{z} - z\dot{y})\right\} \times$$

$$\left\{\left[\frac{x}{r}\left(\frac{a - r}{ae}\right) - \frac{\dot{x}y}{b}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\right](x\dot{z} - z\dot{x})\right.$$

$$+\left.\left[\frac{y}{r}\left(\frac{a - r}{ae}\right) - \frac{y\dot{y}}{b}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\right](y\dot{z} - z\dot{y})\right\}^{-1}(1 - e^2)^{-\frac{1}{2}}$$

$$= \frac{N_1}{D_1} \text{ say,}$$

where

$$N_1 = \frac{-y}{rb}\left\{x(x\dot{z} - z\dot{x}) + y(y\dot{z} - z\dot{y})\right\} - \frac{x}{a}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\left\{\dot{x}(x\dot{z} - z\dot{x}) + \dot{y}(y\dot{z} - z\dot{y})\right\}$$

$$= \frac{-y}{rb}\left\{\dot{z}r^2 - zr\dot{r}\right\} - \frac{x}{a}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\left\{\dot{z}r\dot{r} - zV^2\right\} , \qquad \text{(A.II.3)}$$

and

$$D_1 = (1-e^2)^{\frac{1}{2}}\left\{\left(\frac{a-r}{rae}\right)\left[x(x\dot{z} - z\dot{x}) + y(y\dot{z} - z\dot{y})\right]\right.$$

$$\left.\frac{-y}{b}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\left[\dot{x}(x\dot{z} - z\dot{x}) + \dot{y}(y\dot{z} - z\dot{y})\right]\right\}$$

$$= (1-e^2)^{\frac{1}{2}}\left[\left(\frac{a-r}{rae}\right)(\dot{z}r^2 - zr\dot{r}) - \frac{y}{b}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}(\dot{z}r\dot{r} - zV^2)\right], \qquad \text{(A.II.4)}$$

where

$$V^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2 .$$

Now

$$x = a(\cos E - e) = a\left[\left(\frac{a-r}{ae}\right) - e\right] = \frac{a(1-e^2) - r}{e}$$

and

$$r = a(1 - e\cos E)$$

so

$$\dot{r} = ae\dot{E}\sin E .$$

That is

$$\sin E = \frac{\dot{r}}{ae\dot{E}} .$$

We have also

$$M = E - e\sin E$$

so

$$n = \dot{E} - \dot{E}e\sin E .$$

That is

$$\dot{E} = \frac{n}{1 - e\cos E} = \frac{na}{r} .$$

Hence we may write

$$\frac{y}{b} = \sin E = \frac{r\dot{r}}{a^2 en} .$$

Therefore Equations (A.II.3) and (A.II.4) may be expressed as

$$N_1 = \frac{-\dot{r}}{a^2 en}(\dot{z}r^2 - zr\dot{r}) - \frac{a(1-e^2)-r}{ae}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}(\dot{z}r\dot{r} - zV^2)$$

$$= \frac{N_2}{ae} \text{ say,}$$

and

$$D_1 = (1-e^2)^{\frac{1}{2}}\left[\left(\frac{a-r}{rae}\right)(\dot{z}r^2 - zr\dot{r}) - \frac{r\dot{r}}{a^2 en}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}(\dot{z}r\dot{r} - zV^2)\right]$$

$$= \frac{D_2}{ae} \text{ say.}$$

Thus

$$W = \frac{N_1}{D_1} = \frac{N_2/ae}{D_2/ae} = \frac{N_2}{D_2}$$

where

$$N_2 = -\dot{r}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}(\dot{z}r^2 - zr\dot{r}) - \left[a(1-e^2) - r\right]\left(\frac{a}{\mu}\right)^{\frac{1}{2}}(\dot{z}r\dot{r} - zV^2) \quad \text{(A.II.5)}$$

$$\left(\text{since } na = \left(\frac{\mu}{a}\right)^{\frac{1}{2}}\right),$$

and

$$D_2 = (1-e^2)^{\frac{1}{2}}\left[\left(\frac{a-r}{r}\right)(\dot{z}r^2 - zr\dot{r}) - r\dot{r}\left(\frac{a}{\mu}\right)(\dot{z}r\dot{r} - zV^2)\right]. \quad \text{(A.II.6)}$$

This gives

$$W = \frac{\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\left\{zr\dot{r}^2 - a(1-e^2)(\dot{z}r\dot{r} - zV^2) - zrV^2\right\}}{(1-e^2)^{\frac{1}{2}}\left\{\left(\frac{a-r}{r}\right)(\dot{z}r^2 - zr\dot{r}) - r\dot{r}\left(\frac{a}{\mu}\right)(\dot{z}r\dot{r} - zV^2)\right\}} . \quad \text{(A.II.7)}$$

Now,

$$h^2 = r^2V^2 - r^2\dot{r}^2$$

and

$$h^2 = \mu a(1 - e^2)$$

so we may write

$$V^2 = \frac{\mu a(1 - e^2) + (r\dot{r})^2}{r^2} \qquad \text{(A.II.8)}$$

and

$$(r\dot{r})^2 = r^2V^2 - \mu a(1 - e^2). \qquad \text{(A.II.9)}$$

Also we have

$$V^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right),$$

and so Equation (A.II.5) may be rearranged to give

$$N_2 = \left(\frac{a}{\mu}\right)^{\frac{1}{2}}\left\{zr\dot{r}^2 - a(1 - e^2)(\dot{z}r\dot{r} - zV^2) - \frac{zr}{r^2}\left[\mu a(1 - e^2) + (r\dot{r})^2\right]\right\}$$

$$= \left(\frac{a}{\mu}\right)^{\frac{1}{2}}\left\{-a(1 - e^2)(\dot{z}r\dot{r} - zV^2) - \mu a(1 - e^2)\,\frac{z}{r}\right\}$$

$$= \left(\frac{a}{\mu}\right)^{\frac{1}{2}}\left\{\mu a(1 - e^2)\left[\frac{zV^2}{\mu} - \frac{\dot{z}r\dot{r}}{\mu} - \frac{z}{r}\right]\right\}. \qquad \text{(A.II.10)}$$

Using Equation (A.II.10) in Equation (A.II.7) we obtain

$$W = \frac{a(\mu a)^{\frac{1}{2}}(1 - e^2)\left(\frac{zV^2}{\mu} - \frac{\dot{z}r\dot{r}}{\mu} - \frac{z}{r}\right)}{(1 - e^2)^{\frac{1}{2}}\left\{\left(\frac{a - r}{r}\right)(\dot{z}r^2 - zr\dot{r}) + \left(\frac{a}{\mu}\right)\left[-\dot{z}(r\dot{r})^2 + zr\dot{r}V^2\right]\right\}}$$

$$= \frac{(1 - e^2)^{\frac{1}{2}}\left(\frac{zV^2}{\mu} - \frac{\dot{z}r\dot{r}}{\mu} - \frac{z}{r}\right)}{\frac{a - r}{ra(\mu a)^{\frac{1}{2}}}\left[\dot{z}r^2 - zr\dot{r}\right] + \frac{1}{\mu(\mu a)^{\frac{1}{2}}}\left[-\dot{z}r^2\dot{r}^2 + zr\dot{r}V^2\right]} \qquad \text{(A.II.11)}$$

$$= \frac{N_3}{D_3} \quad \text{say,}$$

where

$$N_3 = (1 - e^2)^{\frac{1}{2}}\left[\frac{z}{r}\left(\frac{rV^2}{\mu} - 1\right) - \frac{\dot{z}r\dot{r}}{\mu}\right] \qquad (A.II.12)$$

and

$$D_3 = \frac{z}{(\mu a)^{\frac{1}{2}}}\left[-\frac{r\dot{r}(a-r)}{ra} + \frac{r\dot{r}V^2}{\mu}\right] + \frac{\dot{z}}{(\mu a)^{\frac{1}{2}}}\left[\frac{r^2(a-r)}{ra} - \frac{r^2\dot{r}^2}{\mu}\right]$$

$$= \frac{z}{a(\mu a)^{\frac{1}{2}}}\left[-\left(\frac{a-r}{r}\right)r\dot{r} + \left(\frac{a}{\mu}\right)r\dot{r}V^2\right] + \frac{\dot{z}}{a(\mu a)^{\frac{1}{2}}}\left[\left(\frac{a-r}{r}\right)r^2 - \left(\frac{a}{\mu}\right)r^2\dot{r}^2\right]$$

$$= \frac{z\dot{r}}{(\mu a)^{\frac{1}{2}}}\left[-\left(\frac{a-r}{r}\right)\frac{r}{a} + \frac{rV^2}{\mu}\right] + \dot{z}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\left(\frac{\mu}{a}\right)^{\frac{1}{2}}\frac{1}{a(\mu a)^{\frac{1}{2}}}\left[\frac{ar^2}{r} - \frac{r^3}{r} - \left(\frac{a}{\mu}\right)r^2\dot{r}^2\right]$$

$$= \frac{z\dot{r}}{(\mu a)^{\frac{1}{2}}}\left[\frac{-ra + r^2}{ra} + \frac{rV^2}{\mu}\right] + \dot{z}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\left[\frac{ar}{a^2} - \frac{r^2}{a^2} - \frac{ar^2\dot{r}^2}{a^2\mu}\right]$$

$$= \frac{z\dot{r}}{(\mu a)^{\frac{1}{2}}}\left[-1 + \frac{r}{a} + \frac{rV^2}{\mu}\right] + \dot{z}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\left[\frac{r}{a} - \frac{r}{a}\cdot\frac{r}{a} - \frac{(r\dot{r})^2}{\mu a}\right].$$

Expressions (A.II.8) and (A.II.9) then give

$$D_3 = \frac{z\dot{r}}{(\mu a)^{\frac{1}{2}}}\left[-1 + \frac{r}{a} + \frac{r\mu}{\mu}\left(\frac{2}{r} - \frac{1}{a}\right)\right]$$

$$+ \dot{z}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\left[\left(2 - \frac{rV^2}{\mu}\right) - \frac{r}{a}\left(2 - \frac{rV^2}{\mu}\right) - \frac{(r^2V^2 - \mu a(1-e^2))}{\mu a}\right]$$

$$= \frac{z\dot{r}}{(\mu a)^{\frac{1}{2}}} + \dot{z}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\left[2 - \frac{rV^2}{\mu} - 2\left(2 - \frac{rV^2}{\mu}\right) + (1 - e^2)\right]$$

$$= \frac{z\dot{r}}{(\mu a)^{\frac{1}{2}}} + \dot{z}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\left[\frac{rV^2}{\mu} - 1 - e^2\right]. \qquad (A.II.13)$$

Substituting Equations (A.II.12) and (A.II.13) into Equation (A.II.11) we have

$$W = \frac{(1-e^2)^{\frac{1}{2}}\left[\frac{z}{r}\left(\frac{rV^2}{\mu}-1\right)-\frac{\dot{z}r\dot{r}}{\mu}\right]}{\frac{z\dot{r}}{(\mu a)^{\frac{1}{2}}}+\dot{z}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\left[\frac{rV^2}{\mu}-1-e^2\right]}$$

$$= \frac{(1-e^2)^{\frac{1}{2}}\left[\frac{z}{r}\left(\frac{rV^2}{\mu}-1\right)-\frac{\dot{z}r\dot{r}}{(\mu a)^{\frac{1}{2}}}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\right]}{\frac{zr\dot{r}}{r(\mu a)^{\frac{1}{2}}}+\dot{z}\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\left[\frac{rV^2}{\mu}-1-e^2\right]}$$

$$= \frac{(1-e^2)^{\frac{1}{2}}\left[\frac{z}{r}\ e\cos E-\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\dot{z}\,e\sin E\right]}{\frac{z}{r}\ e\sin E+\left(\frac{a}{\mu}\right)^{\frac{1}{2}}\dot{z}\,e\,(\cos E-e)}$$

$$= \frac{\left[\frac{z}{r}\ e\cos E-\frac{\dot{z}\,e\sin E}{(\mu/a)^{\frac{1}{2}}}\right]}{\left[\frac{z}{r}\ e\sin E+\frac{(e\cos E-e^2)\dot{z}}{(\mu/a)^{\frac{1}{2}}}\right]\Big/(1-e^2)^{\frac{1}{2}}}$$

which is the required expression (4.5.25) used by COTOEL for tan ω.

Appendix III The Relationship between the Long Periodic Perturbations due to Tesseral Harmonics, in Mean Anomaly and Radius.

We wish to express the perturbation in mean anomaly, M, due to tesseral harmonics ($dM_{2,2}$) as a function of the perturbation in radial distance, r, due to tesseral harmonics ($dr_{2,2}$). We note that the corresponding perturbations in semi-major axis and eccentricity ($da_{2,2}$ and $de_{2,2}$) are both zero.

We have

$$M = E - e \sin E \tag{A.III.1}$$

where E is the eccentric anomaly, and

$$r = a(1 - e \cos E). \tag{A.III.2}$$

From Equation (A.III.2) we obtain

$$\cos E = \frac{a - r}{ae}$$

and hence

$$\sin E = \left[1 - \left(\frac{a-r}{ae}\right)^2\right]^{\frac{1}{2}} . \tag{A.III.3}$$

Substituting from Equation (A.III.3) into Equation (A.III.1), we obtain M in terms of a, e and r

$$M = \sin^{-1}\left\{\left[1 - \left(\frac{a-r}{ae}\right)^2\right]^{\frac{1}{2}}\right\} - e\left[1 - \left(\frac{a-r}{ae}\right)^2\right]^{\frac{1}{2}} .$$

Hence

$$\frac{dM_{2,2}}{dr_{2,2}} = \frac{d}{dr_{2,2}}\left\{ \sin^{-1}\left[1 - \left(\frac{a-r}{ae}\right)^2\right]^{\frac{1}{2}} - e\left[1 - \left(\frac{a-r}{ae}\right)^2\right]^{\frac{1}{2}}\right\} \tag{A.III.4}$$

We write

$$\theta = \left[1 - \left(\frac{a-r}{ae}\right)^2\right]^{\frac{1}{2}} . \tag{A.III.5}$$

Then Equation (A.III.4) becomes

$$\frac{dM_{2,2}}{dr_{2,2}} = \frac{d}{d\theta}\left[\sin^{-1}\theta - e\theta\right]\frac{d\theta}{dr_{2,2}} \quad . \qquad \text{(A.III.6)}$$

Now

$$\frac{d}{d\theta}\sin^{-1}\theta = (1-\theta^2)^{-\frac{1}{2}}$$

$$= \left[1 - 1 + \left(\frac{a-r}{ae}\right)^2\right]^{-\frac{1}{2}}$$

by Equation (A.III.5)

$$= \frac{ae}{a-r} \quad , \qquad \text{(A.III.7)}$$

and

$$\frac{d}{d\theta}e\theta = e \ , \qquad \text{(A.III.8)}$$

since $de_{2,2} = 0$ and $da_{2,2} = 0$.

Similarly we have

$$\frac{d\theta}{dr_{2,2}} = \frac{1}{2}\left[1 - \left(\frac{a-r}{ae}\right)^2\right]^{-\frac{1}{2}} \frac{d}{dr_{2,2}}\left[1 - \left(\frac{a-r}{ae}\right)^2\right]$$

$$= \frac{1}{2}\left[1 - \left(\frac{a-r}{ae}\right)^2\right]^{-\frac{1}{2}}(-2)\left(\frac{a-r}{ae}\right)\frac{d}{dr_{2,2}}\left(\frac{a-r}{ae}\right]$$

$$\frac{\left(\frac{a-r}{ae}\right)\left(\frac{-1}{ae}\right)\left(\frac{-2}{2}\right)}{\left[1 - \left(\frac{a-r}{ae}\right)^2\right]^{\frac{1}{2}}} \quad . \qquad \text{(A.III.9)}$$

Substituting from Equations (A.III.7), (A.III.8) and (A.III.9) into Equation (A.III.6) we obtain

$$\frac{dM_{2,2}}{dr_{2,2}} = \frac{\left[\left(\frac{ae}{a-r}\right) - e\right]\left(\frac{a-r}{ae}\right)\left(\frac{1}{ae}\right)}{\left[1 - \left(\frac{a-r}{ae}\right)^2\right]^{\frac{1}{2}}} ,$$

which, using Equation (A.III.3), becomes

$$\frac{dM_{2,2}}{dr_{2,2}} = \frac{\left[\left(\frac{ae}{a-r}\right) - e\right]\left[\frac{a-r}{(ae)^2}\right]}{\sin E}$$

$$= \left(\frac{1}{ae} - \frac{a-r}{a^2 e}\right) \Big/ \sin E$$

$$= (r/a)/(a\, e \sin E)$$

$$= \frac{(1-e^2)}{ae(1+e\cos v)\sin E} . \qquad \text{(A.III.10)}$$

If we express $\sin E$ in terms of $E/2$, this then becomes

$$\frac{dM_{2,2}}{dr_{2,2}} = \frac{(1-e^2)}{ae(1+e\cos v)2\sin(E/2)\cos(E/2)}$$

which may be rearranged, using Equation (4.41.33), to obtain

$$\frac{dM_{2,2}}{dr_{2,2}} = \frac{(1-e^2)^{\frac{1}{2}}}{ae\sin v}$$

and hence

$$\frac{dM_{2,2}}{dr_{2,2}} = \frac{q\,q^2}{ae\sin v\, q^2}$$

$$= \frac{q^3}{e\sin v\, a(1-e^2)}$$

$$= \frac{q^3}{e \sin v . p} .$$

From this we obtain

$$dM_{2,2} = \frac{q^3}{e \sin v} \frac{dr_{2,2}}{p} .$$

Appendix IV The Macro

A new macro, MACPROP, for running the new version of PROP, has been written. A listing of the macro is given in Figure A.IV.1. Users control the running of their PROP jobs by parameters passed to the macro. Possible parameters are:

PROGxxxxxxx: This parameter is mandatory. It passes to the macro the name of the file in which the input data for the PROP job is stored. The name of this file should be of the form

xxxxxxxDATA

where xxxxxxx is any string of up to seven letters and numbers, beginning with a letter. This parameter also defines the names of the output files, normally xxxxxxxLIST, xxxxxxxCARD and xxxxxxxITT.

OPT: This parameter is optional. If it is used then the binary version of PROP loaded will be that compiled using the optimising compiler, XFEV. If it is not used then the version of PROP loaded will be that compiled using the normal compiler, XFIV. Execution of the program should be faster if OPT is used. However, there are some indications of errors in this version of the compiler. If OPT is used then the output files are called xxxxxxxOLIST, xxxxxxxOCARD and xxxxxxxOITT. The input file should still be called xxxxxxxDATA.

MACRO: This parameter is optional. It causes the macro to execute an EXIT at the end of execution, rather than an ENDJOB. In this way the user may set up a macro-macro file holding several calls of the macro MACPROP. A job may then be run using the macro-macro file for job control.

In this way several data files may be run in the same job, sequentially.

SUPPRESS: This parameter is optional. It causes much of the line printer information generated by a PROP job to be suppressed, as described in subsection 4.7.6 DIFCOR. If SUPPRESS is used and a job fails with a fatal execution error then the files containing the suppressed information are kept on the user's filestore.

KEEP: This parameter is optional. It may be used in conjunction with the parameter SUPPRESS. When used it causes the files xxxxxxxITT (or xxxxxxxOITT) to be kept on the user's filestore.

Both binary versions of PROP are stored in the user number :SWS9120. Traps have been removed so that any user whose user number begins with :SW may execute them. The macro MACPROP is also stored in user number :SWS9120 and traps have been removed so that the group of :SW users may copy it into their own user number for use, by issuing

COPY :SWS9120.MACPROP,MACPROP

from their own user number. The input data file should be in the user's own user number and all output files will be created there.

Figure A.IV.1 MACPROP

```
      WHENEVER COMMAND ERROR, GO TO 50
      IF ABSENT (PROG),GO TO 50
      SETPARAM X,(%(PROG))
      IF NOT EXISTS (%XDATA),GO TO 50
      IF PRESENT (OPT),(LOAD :SWS9120.PROPOPT) -
            ELSE (LOAD :SWS9120.PROPBIN)
      ASSIGN *CR0,%XDATA
      LISTFILE %XDATA,*LP,NUMBER
      IF PRESENT (OPT), SETPARAM X, (%(PROG)O)
      ASSIGN *LP0,%XOUT (LIMIT 10000)
      ASSIGN *LP1,%XITT (LIMIT 10000)
      LISTFILE %XOUT,*LP
      ASSIGN *CP0,%XCARD
      LISTFILE %XCARD,*LP
      ERASE %XOUT
      ERASE %XCARD
      ENTER 0
10    IF DELETED, GO TO 20
      IF HALTED (EE),GO TO 50
      IF FAILED,GO TO 50
      IF DISPLAY(FINAL),GO TO 60
      DISPLAY 0,RUN ERROR
      IF PRESENT(MACRO),EXIT
      ENDJOB
20    IF PRESENT(SUPPRESS),GO TO 30
      LISTFILE %XITT,*LP
30    IF PRESENT(KEEP),GO TO 50
40    ERASE %XITT
      IF EXISTS (%XITT),GO TO 40
50    IF PRESENT(MACRO),EXIT
      ENDJOB
60    IF ABSENT(SUPPRESS),GO TO 70
      SETPARAM W,DISPLAY(21,23)
      LISTFILE %XITT,*LP,-
      FROMC/FURTHER ITERATION - NUMBER  %W/
      ASSIGN *LP1,%XITT(+1) (LIMIT 10000)
70    RESUME
      GO TO 10
\
\  NEW PROP MACRO
\
\  MACRO TO RUN PROP ORBITAL ANALYSIS PROGRAM
\  IN THE BINARY VERSION COMPILED AND CONSOLIDATED AT
\  THE UNIVERSITY OF ASTON IN BIRMINGHAM.
\
```

continued...

```
\   USE RJ MOP---,MACPROP,PARAM(PROG<RUNAME>)
\   IF ALL AVAILABLE OUTPUT IS REQUIRED.
\   USE RJ MOP---,MACPROP,PARAM(PROG<RUNAME>,SUPPRESS)
\   IF SUPPRESSION OF ITERATION INFORMATION IS REQUIRED.
\   USE RJ MOP---,MACPROP,PARAM(PROG<RUNAME>,SUPPRESS,KEEP)
\   IF SUPPRESSION OF ITERATION INFORMATION IS REQUIRED,
\   BUT YOU WISH TO KEEP THE SUPPRESSED
\   INFORMATION ON YOUR FILESTORE.
\
\   THE PARAMETER OPT MAY BE INCLUDED TO RUN THE VERSION
\   OF PROP COMPILED WITH THE OPTIMISING COMPILER.
\   THE PARAMETER MACRO MAY BE INCLUDED TO CAUSE THIS MACRO
\   TO EXIT AT THE END, RATHER THAN ENDJOB.
\
\   <RUNAME> MUST BE OF THE FORM XXXXXXX
\   WHERE THE DATAFILE NAME IS OF THE FORM XXXXXXXDATA.
\   THE LINE PRINTER LISTINGS ARE LABELLED XXXXXXXOUT AND
\   XXXXXXITT AND THE CARDFILE IS LABELLED XXXXXXXCARD.
\   IF THE OPTIMISED VERSION OF PROP IS BEING USED
\   THEN THE OUTPUT FILES WILL BE LABELLED XXXXXXXOOUT,
\   XXXXXXXOITT AND XXXXXXXOCARD.
\
```

References

BROWN, D. C., 1957 R.C.A. Data Reduction Tech. Report 39
A Treatment of Analytical Photogrammetry (with Emphasis on Ballistic Camera Applications).

COOK, G. E., CLARKE, R., GOODING, R. H., 1972
RAE Space Department Subroutine Specification SDCELS (Mark 2 version).

FISCHER, I., 1960 J. of Geophys. Res., 64 (7) 2067-76
An Astrogeodetic World Datum from Geoidal Heights based on the Flattening f = 1/298.3.

GAPOSCHKIN, E. M., 1964 SAO Special Report No. 161
Differential Orbit Improvement (DOI-3)

GARFINKEL, G., 1944 Astron. J. 50 (8) 169-179
An Investigation in the Theory of Astronomical Refraction.

GARFINKEL, G., 1967 Astron. J. 72 (2) 235-254
Astronomical Refraction in a Polytropic Atmosphere

GOODING, R. H., no date available
RAE Space Department Subroutine Specification CONVER.

GOODING, R. H., 1968a
RAE Space Department Subroutine Specification ELREAD.

GOODING, R. H., 1968b
RAE Space Department Subroutine Specification EPARAS.

GOODING, R. H., 1968c
RAE Space Department Subroutine Specification NEWELS.

GOODING, R. H., 1968d
RAE Space Department Subroutine Specification REJECT.

GOODING, R. H., 1968e
RAE Space Department Subroutine Specification ECENTR.

GOODING, R. H., 1969a
RAE Space Department Subroutine Specification ALTELS (Mark 2 version).

GOODING, R. H., 1969b RAE Tech. Report 69104, London, HMSO
A PREP Users' Manual.

GOODING, R. H., 1970a RAE Tech. Memo Space 138, London, HMSO
Some Comments on SDC Orbital Elements, with Special Reference to the Satellite Ariel 3.

GOODING, R. H., 1970b RAE Tech. Memo Space 156, London, HMSO
Orbit Determination at the Royal Aircraft Establishment.

GOODING, R. H., 1974 RAE Tech. Report 74164, London, HMSO
The Evolution of the PROP6 Orbit Determination Program, and Related Topics.

GOODING, R. H., 1978 RAE Tech. Memo Space 265, London, HMSO
Orbital Elements: a Lecture.

GOODING, R. H., ODELL, A. W., 1973 RAE Tech. Report 73088, London, HMSO,
The Look-angle Prediction Programs PLAP, ESPLAP and 6130 PLAP.

GOODING, R. H., Tayler, R. J., 1968. RAE Tech. Report 68299, London, HMSO.
A PROP3 Users' Manual.

H.M. NAUTICAL ALMANAC OFFICE, 1961 London, HMSO.
Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac.

ICL, 1971 London, ICL
Extended Fortran - ICL Students Edition

KING-HELE, D. G., 1969 RAE Tech. Report 69121, London, HMSO.
Average Rotational Speed of the Upper Atmosphere from Changes in Satellite Orbits.

KING-HELE, D. G., BROOKES, C. J., COOK, G.E., 1981
Geophys. J. R. Astr. Soc., 64 3-30
Odd Zonal Harmonics in the Geopotential, from Analysis of 28 Satellite Orbits.

KING-HELE, D. G., WALKER, D. M. C., 1977 Planet. Space Sci. 25 313-336
Upper-Atmosphere Zonal Winds: Variation with Height and Local Time.

KOZAI, Y., 1959 Astron. J. 64 (1274) 367-377
The Motion of a Close Earth Satellite.

McCROSKY, R. E., POSEN, A., 1968 SAO Special Report 273
Special Data Reduction Procedures for Prairie Network Meteor Photographs.

MERSON, R. H., 1962 RAE Tech. Note Space 16, London, HMSO.
A Pegasus Computer Programme for the Improvement of the Orbital Parameters of an Earth-Satellite.

MERSON, R. H., 1966 RAE Tech. Report 66255, London, HMSO
The Dynamic Model of PROP, a Computer Program for the Refinement of the Orbital Parameters of an Earth Satellite.

MERSON, R. H., 1977 RAE Tech. Report 77083, London, HMSO
Satellite Orbit and Attitude Theory and its Use with Skynet 2.

MERSON, R. H., GOODING, R. H., TAYLER, R. J., no date available.
RAE Space Department Subroutine Specification PRELON.

MERSON, R. H., TAYLER, R. J., GOODING, R. H., 1969
RAE Space Department Subroutine Specification XLONG.

ODELL, A. W., 1969
RAE Space Department Subroutine Specification DHMS.

ROY, A. E., 1978 Adam Hilger Ltd., Bristol
Orbital Motion.

SWINERD, G. G., 1981 Planet. Space Sci., 29, 349-358
Determination of Zonal Wind Speeds at Heights Near 530 km from the Orbit of Explorer 24 (1964-76A).

TAYLER, R. J., 1962 RAE Tech. Note Space 23, London, HMSO
A Pegasus Computer Programme for the Preparation of Ephemerides of an Earth-Satellite.

TAYLER, R. J., 1967 RAE Tech. Memo Space 101, London, HMSO
A Table of Modified Julian Day Numbers for the Years 1955 to 1999.

TAYLER, R. J., 1969a
RAE Space Department Subroutine Specification CONVRT.

TAYLER, R. J., 1969b
RAE Space Department Function Specification SIDANG.

TAYLER, R. J., 1969c
RAE Space Department Subroutine Specification REFCOR.

TAYLER, R. J., 1969d
RAE Space Department Subroutine Specification SINXOX.

TAYLER, R. J., 1969e
RAE Space Department Subroutine Specification SHOPER.

TAYLER, R. J., GOODING, R. H., no date available
RAE Space Department Subroutine Specification SATXYZ.

TAYLER, R. J., GOODING, R. H., 1969a
RAE Space Department Subroutine Specification GEOCOR.

TAYLER, R. J., GOODING, R. H., 1969b
RAE Space Department Subroutine Specification PRENUT.

TAYLER, R. J., GOODING, R. H., 1969c
RAE Space Department Subroutine Specification PROCES.

TAYLER, R. J., GOODING, R. H., 1970
RAE Space Department Subroutine Specification PARSHL.

TAYLER, R. J., ODELL, A. W., 1969
RAE Space Department Subroutine Specification PRDATE.

WALTER, H. G., WALES, I. M., PALLASCHKLE, S., 1968 ESRO SR-8
The Differential Correction of Close-Earth Satellite Orbits. Part II, The Differential Correction Orbit Programme (DCOP).

YORK, E., HOPPER, M., 1968
RAE Space Department Subroutine Specification MA01A (modified by R. H. Gooding).