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Mathematical Modelling and Optimal Multivariable Control
of Chemical Processes

A Thesis submitted for the degree of
Doctor of Philosophy
by

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1984

VOLUME (1)

بسم الله الرحمن الرحيم

DEDICATIONS

To the soul of my father who introduced me to the field of engineering since I remember. To my mother who is patiently waiting for me at home in Sudan.

To my lovable wife Salma, my daughter Reem and my two sons Omer and Nawari.

To my sisters and brothers Mey, Sohier, Zahawi, Nawari, Fadwa, Susan, and Nasreen.

To all those who guided me throughout my educational upbringing.

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Mathematical Modelling and Optimal Multivariable Control
of Chemical Processes

Summary:

This work reports the development of a mathematical model and distributed, multivariable computer-control for a pilot plant double-effect climbing-film evaporator.

A distributed-parameter model of the plant has been developed and the time-domain model transformed into the Laplace domain. The model has been further transformed into an integral domain conforming to an algebraic ring of polynomials, to eliminate the transcendental terms which arise in the Laplace domain due to the distributed nature of the plant model. This has made possible the application of linear control theories to a set of linear-partial differential equations.

The models obtained have well tracked the experimental results of the plant.

A distributed-computer network has been interfaced with the plant to implement digital controllers in a hierarchical structure.

A modern multivariable Wiener-Hopf controller has been applied to the plant model. The application has revealed a limitation condition that the plant matrix should be positive-definite along the infinite frequency axis.

A new multivariable control theory has emerged from this study, which avoids the above limitation. The controller has the structure of the modern Wiener-Hopf controller, but with a unique feature enabling a designer to specify the closed-loop poles in advance and to shape the sensitivity matrix as required. In this way, the method treats directly the interaction problems found in the chemical processes with good tracking and regulation performances. Though the ability of the analytical design methods to determine once and for all whether a given set of specifications can be met is one of its chief advantages over the conventional trial-and-error design procedures. However, one disadvantage that offsets to some degree the enormous advantages is the relatively complicated algebra that must be employed in working out all but the simplest problem.

Mathematical algorithms and computer software have been developed to treat some of the mathematical operations defined over the integral domain, such as matrix fraction description, spectral factorization, the Bezout identity, and the general manipulation of polynomial matrices. Hence, the design problems of Wiener-Hopf type of controllers and other similar algebraic design methods can be easily solved.

Key words: distributed-parameter, ring of operators, Wiener-Hopf controllers, algebraic controllers, polynomial matrices algorithms,

VOLUME (1)

CHAPTER (1)

INTRODUCTION

| | | |
|-----|------------------------|----|
| 1.1 | INTRODUCTION | 1 |
| 1.2 | MATHEMATICAL MODELLING | 5 |
| 1.3 | CONTROL SYSTEMS | 10 |
| 1.4 | THE CHEMICAL PLANT | 19 |

CHAPTER (2)

LITERATURE REVIEW

| | | |
|-----|---------------------------------------|----|
| 2.1 | MATHEMATICAL MODELLING | 21 |
| 2.2 | PROCESS CONTROL | 24 |
| 2.3 | ECONOMIC VIABILITY OF PROCESS CONTROL | 39 |
| 2.4 | SYNOPSIS | 41 |

CHAPTER 3

EXPERIMENTAL SET-UP

| | | |
|-------|---|----|
| 3.1 | INTRODUCTION | 45 |
| 3.2 | THE DOUBLE EFFECT EVAPORATOR | 45 |
| 3.3 | PROCESS DESCRIPTION | 49 |
| 3.4 | PROCESS INSTRUMENTATION | 54 |
| 3.5 | PNEUMATIC VALVES | 56 |
| 3.6 | COMPUTER DATA-NETWORK | 60 |
| 3.6.1 | Introduction | 60 |
| 3.6.2 | The Honeywell Minicomputer System | 62 |
| 3.6.3 | Honeywell Peripheral Equipment | 62 |
| 3.6.4 | The Honeywell Analogue Digital I/O system (HADIO S) | 63 |

| | | |
|-----|---|----|
| 3.7 | THE MOTOROLA M6800 MICROPROCESSOR SYSTEM. | 64 |
| | 3.7.2 Peripheral Devices. | 66 |
| 3.8 | REMOTE SIGNAL CONDITIONING. | 67 |
| 3.9 | CONCLUSION | 69 |

CHAPTER (4)

MATHEMATICAL MODELLING

| | | |
|-----|--|-----|
| 4.1 | SCOPE: | 68 |
| 4.2 | THE PROCESS PHYSICS: | 69 |
| | 4.2.1 Introduction: | 69 |
| | 4.2.2 Non Equilibrium State: | 72 |
| 4.3 | LIQUID PHASE REGION | 76 |
| 4.4 | THE TWO PHASE REGION: | 81 |
| 4.5 | STEAM CHEST: | 87 |
| 4.6 | MODELLING OF THE PREHEATER AND THE SECOND EFFECT : | 98 |
| | 4.6.1 The Preheater : | 98 |
| | 4.6.2 The Second Effect: | 100 |
| 4.7 | SUMMARY: | 101 |

CHAPTER (5)

MODERN WIENER-HOPF MULTIVARIABLE CONTROLLER

| | | |
|-----|--|-----|
| 5.1 | INTRODUCTION: | 102 |
| 5.2 | GENERAL SPECIFICATIONS | 107 |
| 5.3 | WIENER-HOPF CONTROLLER | 108 |
| | 5.3.1 Summary of the Philosophy of Modern Wiener-Hopf Design method: | 109 |
| | 5.3.2 Modern Optimal Wiener-Hopf Controller: | 115 |
| 5.4 | COMMENTS | 131 |

CHAPTER (6)

MULTIVARIABLE CLOSED-LOOP POLES ALLOCATION AND MINIMAL SENSITIVITY : A Synthesis Approach

| | | |
|-----|---|-----|
| 6.1 | INTRODUCTION | 134 |
| | 6.1.1 Abstract : | 134 |
| 6.2 | SYNTHESIS OF THE CONTROLLER | 136 |
| | 6.2.1 Analysis | 138 |
| 6.3 | SOME RELATIONS IN THE SISO CASE | 144 |
| 6.4 | SENSITIVITY ANALYSIS | 146 |
| | 6.4.1 Ideal Case : $S = 0$ | 149 |
| | 6.4.2 Minimal Solution : $S \rightarrow 0$ | 150 |
| 6.5 | OPTIONS AVAILABLE TO THE DESIGNER ON SISO CASE: | 153 |
| 6.6 | ALTERNATIVE SOLUTION FOR MATRIX K | 155 |
| | 6.6.1 Option (1) | 155 |
| | 6.6.2 Option (2) | 158 |
| | 6.6.3 Option (3) | 159 |
| 6.8 | SUMMARY | 161 |

CHAPTER (7)

COMPUTER-AIDED CONTROL SYSTEM DESIGN and SYNTHESIS PACKAGE

| | | |
|-----|--|-----|
| 7.1 | INTRODUCTION | 163 |
| 7.2 | APA NOTATIONS : | 166 |
| | 7.2.1. Data Structure | 166 |
| 7.3 | SPECTRAL FACTORIZATION OF POLYNOMIAL MATRICES | 167 |
| 7.4 | ELEMENTARY OPERATIONS ON POLYNOMIAL MATRICES | 172 |
| | 7.4.1 Fields and Rings | 173 |
| | 7.4.2 Greatest Common Divisors and Coprime Matrices | 176 |
| | 7.4.3 Relatively Prime Matrices (coprime matrices) | 177 |
| | 7.4.4 Unimodular Matrices | 177 |

| | | |
|-------|---|-----|
| 7.4.5 | Computer Algorithm | 184 |
| 7.5 | INVERSION OF POLYNOMIAL MATRICES | 186 |
| 7.6 | PARTIAL FRACTION | 192 |
| 7.6.1 | Algorithm Theory | 192 |
| 7.6.2 | Infinite Series for Functions of a complex Variable: | 194 |
| 7.6.3 | Residue Theorem : | 195 |
| 7.6.4 | Computer Algorithm | 197 |
| 7.7 | MULTIPLICATION OF POLYNOMIAL MATRICES | 198 |
| 7.8 | MULTIPLICATION OF A POLYNOMIAL MATRIX by ITS TRANSPOSE INVERTED INDETERMINATE | 200 |
| 7.9 | SUMMATION OF RATIONAL POLYNOMIAL MATRICES | 201 |
| 7.10 | SOLUTION OF THE BEZOUT IDENTITY | 203 |
| 7.11 | TIME RESPONSE OF TRANSFER FUNCTIONS | 204 |
| 7.12 | EXAMPLES | 207 |
| | THE LIBRARY COMPENDIUM | 211 |
| | EPILOGUE | 249 |

CHAPTER (8)

REAL-TIME IMPLEMENTATION OF CONTROLLERS

| | | |
|-------|---|-----|
| 8.1 | INTRODUCTION | 250 |
| 8.2 | THE s-DOMAIN MODEL: | 255 |
| 8.2.1 | Rings of Delay Operators and Conformal Mapping | 259 |
| 8.2.2 | Zero-Order Hold and Sampler | 263 |
| 8.3 | MODEL TRANSFORMATION | 272 |
| 8.4 | WIENER-HOPF CONTROLLER ALGORITHM | 284 |
| 8.4.1 | Optimal Wiener-Hopf Controller | 285 |
| 8.4.2 | Initial Simplification for the Programs | 291 |
| 8.5 | THE ON-LINE COMPUTATIONAL FACILITIES | 302 |
| 8.5.1 | System Hardware Organisation | 303 |

| | | |
|---------|---|-----|
| 8.5.2 | The Communication Protocol | 305 |
| 8.5.3 | Communication Software | 306 |
| 8.5.3.a | HADIOS Executive Revision 03 | 306 |
| 8.5.3.b | The 6800 Executive | 308 |
| 8.5.4 | On Line Graphics | 310 |
| | The H316 Tektronix Graphic Package: | |
| 8.5.5 | Summary of the communication the software | 311 |
| 8.6 | THE COMPUTERS HIERARCHICAL STRUCTURE AND CONTROL TASK DISTRIBUTION | 314 |
| 8.6.1 | Control Task Distribution | 319 |
| 8.7 | THE DIGITAL COMPENSATORS: Real-Time Programs | 323 |
| 8.7.1 | Honeywell 316H Digital Compensator | 324 |
| 8.7.2 | Motorola M6800 Microcomputer Compensator | 324 |
| 8.8 | GENERAL REMARKS | 326 |

CHAPTER (9)

RESULTS ANALYSIS AND DISCUSSION

| | | |
|---------|--|-----|
| 9.1 | MATHEMATICAL MODELLING | 328 |
| 9.1.1 | Data logging | 328 |
| 9.1.2 | Results from the Model | 331 |
| 9.1.2.1 | The Heat Transfer Coefficient | 341 |
| 9.1.2.2 | Modelling of The Parameters Variation | 343 |
| 9.1.3 | The Interaction of The Plant Variables | 348 |
| 9.1.4 | The Equivalent Tube | 350 |
| 9.1.5 | Comparison of The Steady-State Results | 354 |
| 9.1.6 | Vapour Pressure Surges | 362 |
| 9.1.7 | Measurements Aspect | 373 |
| 9.2 | THE MODEL TRANSFORMATION | 374 |
| 9.2.1 | Markov Parameters | 375 |
| 9.2.2 | Pade` Approximation | 379 |
| 9.3 | THE MODERN WIENER-HOPF CONTROLLER | 381 |
| 9.3.1 | Left-Right Factorization | 382 |
| 9.3.1.1 | The Effect of Instruments Poles | 390 |
| 9.3.2 | The Solution of The Bezout Identity | 391 |
| 9.3.2.1 | The Question of Minimum Polynomial Degree | 396 |
| 9.3.3 | The Spectral Factorization | 399 |

| | | |
|-----|--|-----|
| 9.4 | APA: | |
| 9.5 | THE DIGITAL CONTROL AND THE HIERARCHICAL STRUCTURE | 409 |
| | 9.5.1 The Distributive Feature of The Digital Control | 410 |
| | 9.5.2 The Control Task Distribution Model | 420 |
| | 9.5.3 Process Cybernetics | 423 |

CHAPTER (10)

CONCLUSIONS AND FUTURE WORK

| | | |
|------|---|-----|
| 10.1 | MATHEMATICAL MODELLING | 426 |
| | 10.1.1 Towards The Future | 429 |
| 10.2 | SYSTEM SOFTWARE | 429 |
| 10.3 | MODEL TRANSFORMATION | 431 |
| 10.4 | WIENER-HOPF CONTROLLER | 432 |
| 10.5 | CLOSED LOOP POLES ALLOCATION and MINIMAL SENSITIVITY (CLOPAMS) | 434 |
| | 10.5.1 Conclusion | 434 |
| | 10.5.2 Future Work | 435 |
| 10.6 | THE HIERARCHICAL STRUCTURE | 436 |
| | 10.6.1 Future Trends | 437 |

VOLUME (2)

APPENDIX A

| | | |
|-----|--|----|
| A.1 | INTRODUCTION | 1 |
| A.2 | THE TWO PHASE VELOCITY | 16 |
| A.2 | LIQUID PHASE | 4 |
| A.3 | THE TWO PHASE REGION | 11 |
| A.5 | TUBES WALL | 19 |
| A.6 | STEAM CHEST | 21 |
| | MODELLING OF THE PREHEATER AND SECOND EFFECT | 27 |
| | THE SIGNAL-FLOW GRAPH | 33 |

APPENDIX B

| | |
|-------------------------------------|----|
| PARAMETER EVALUATION FOR SIMULATION | 38 |
| PADE` APPROXIMATION | 39 |

APPENDIX C

| | |
|-----------------------------|----|
| THERMODYNAMICS CORRELATIONS | 43 |
|-----------------------------|----|

APPENDIX D

| | |
|-----------------------------|----|
| FLOW THROUGH ORIFICE-PLATES | 44 |
|-----------------------------|----|

APPENDIX E

| | |
|--------------------------|----|
| PROGRAMS AND DATA FILES: | 47 |
| DYSIM | |
| PLOT1 | |
| ST5112 | |
| STEADY | |
| PARAM2 | |
| RS4212 | |
| RS5212 | |

APPENDIX F

| | |
|---------------------|----|
| THE M6800 EXECUTIVE | 60 |
|---------------------|----|

| | |
|---|----|
| ERROR HANDLING IN M6800 EXECUTIVE | 63 |
| HADIOS SUBROUTINES | 65 |
| ERROR HANDLING AND SENSE SWITCH USAGE IN H316 | 70 |

APPENDIX G

| | |
|------------------|----|
| SUBROUTINE GRAPH | 72 |
|------------------|----|

APPENDIX H

| | |
|-------------------------------------|-----|
| PROGRAMS AND DATA FILES TO FIND THE | 76 |
| WIENER-HOPF CONTROLLERS | |
| THE LIBRARY APA | 130 |

APPENDIX I

| | |
|------------------------------------|-----|
| THE ON-LINE UNIVERSAL COMPENSATORS | 179 |
| REFERENCES | 185 |
| BIBLIOGRAPHY OF RELEVANT INTEREST | 200 |

List of Figures

| | | |
|-------------|--|-----|
| Fig. [3.1] | Double Effect Evaporator: Process Description | 50 |
| Fig. [3.2] | Double Effect Evaporator: Process instrumentation | 55 |
| Fig. [3.3] | Arrangement of The Pneumatic valves | 59 |
| Fig. [3.4] | Organization of the Data Flow | 61 |
| Fig. [4.1] | The Flow Pattern along a Climbing Film Evaporator | 70 |
| Fig. [4.2] | The Two Main Flow regions | 75 |
| Fig. [4.3] | Section of The Boiling region | 75 |
| Fig. [4.4] | The Signal-Flow Graph | 93 |
| Fig. [5.1] | Filter $W(s)$ to Minimize the Error $e(t)$ | 112 |
| Fig. [5.2] | Multivariable Feedback Controller | 116 |
| Fig. [6.1] | Multivariable Feedback Control Structure | 137 |
| Fig. [7.1] | Controller-Realization of Transfer Functions | 206 |
| Fig. [8.1]a | A Sampler | 264 |
| Fig. [8.1]b | Zero-Order Hold Operation | 264 |
| Fig. [8.2] | The Decomposition of The Stairsteps Waveform into many pulses | 266 |
| Fig. [8.3] | Train of Pulses | 266 |
| Fig. [8.4] | The Behaviour of the Model in The s-Domain and z-Domain, (steam perturbations) | 279 |
| Fig. [8.5] | The Behaviour of the Model in The s-Domain and z-Domain, (feed flow rate perturbations) | 280 |
| Fig. [8.6] | The Behaviour of the Model in The s-Domain | 281 |

| | | |
|---------------|---|-----|
| | and z-Domain, (feed flow rate perturbations) | |
| Fig. [8.7] | Flow-Chart showing the steps in Synthesizing Wiener-Hopf Controller | 289 |
| Fig. [8.8] | Control Task Distribution | 316 |
| Fig. [8.9] | Hierarchical Structure of Compensators | 318 |
| Fig. [8.10] | Switching Time Forms for the Solenoid Valves | 322 |
| Fig. [9.1.1]a | Response of the Evaporator Outlet Temperature To step increase in steamflow rate | 334 |
| Fig. [9.1.1]b | Response of the Evaporator Product to step increase in steam flow rate | 335 |
| Fig. [9.1.2]a | Response of the Evaporator Outlet Temperature to step increase in Feed Flow rate | 336 |
| Fig. [9.1.2]b | Response of the Evaporator Product to step increase in Feed Flow rate | 337 |
| Fig. [9.1.3]a | Response of the Evaporator Outlet Temperature to step decrease in Feed Flow rate | 338 |
| Fig. [9.1.3]b | Response of the Evaporator Product to step decrease in Feed Flow rate | 339 |
| Fig. [9.1.4] | A Model for Plant Variables interactions | 344 |
| Fig. [9.1.5] | The Cyclone and the Orifice-Plate | 366 |
| Fig. [9.1.6] | Orifice-Plate Section | 368 |
| Fig. [9.5.1] | Star Topology | 414 |
| Fig. [9.5.2] | Ring Topology | 414 |
| Fig. [9.5.3] | Bus Topology | 414 |
| Fig. [9.5.4] | A Model of Hierarchical Control Structure | 422 |

Tables

| | | |
|----------------|---|-----|
| Table [3.1] | Physical Dimensions of the Double Effect Evaporator | 46 |
| Table [8.1] | Transfer function of steam with boiling temperature | 282 |
| Table [8.2] | Transfer function of feed flow rate with boiling temperature | 282 |
| Table [8.3] | Transfer functions of product flow rate with steam | 283 |
| Table [8.4] | Transfer functions of product flow rate with feed | 284 |
| Table [8.5] | Wiener-Hopf controller : Off-line design programs | 300 |
| Table [8.6] | Possible combinations of Computer operation modes | 311 |
| Table [9.1.1] | Variables Measured On-Line from the Plant | 330 |
| Table [9.1.2]a | Sample of data logged from the plant: steady-state conditions, EX5112 | 332 |
| Table [9.1.2]b | Sample of data logged from the plant: steady-state conditions, EX4212 | 332 |
| Table [9.1.3] | Calculated parameters from steady-state condition | 333 |
| Table [9.4.1] | A comparison of experimental results of freon/water evaporator with the mathematical model. | 359 |

Plates

| | | |
|-----------|--|----|
| Plate (1) | Front view of the double-effect evaporator | 51 |
| Plate (2) | Rear view of the double-effect evaporator | 52 |

PRINCIPAL SYMBOLS

| | |
|----------------|---|
| B e | $= \frac{c}{p} \frac{A}{l}$ |
| c p | Heat capacity of water. |
| p | Density of water |
| T | Temperature |
| t | Time |
| x | Linear distance |
| U L | Heat transfer coefficient of the liquid phase |
| U B | Heat transfer coefficient of the boiling region |
| T b | Boiling temperature |
| T o | Inlet temperature to the 1st. effect tubes |
| \bar{T} | Steady-state component |
| ~ T | Time varying component |
| * T | Laplace transformed quantity |
| r | time delay |
| c pc | Heat capacity of copper |
| W [*] | Mass of tubes copper |
| MV | vapour mass |
| ML | Liquid mass |
| M | Total mass of flow |
| L c | Latent heat of condensation |

| | |
|------------------|--|
| L_v | Latent heat of evaporation |
| V_s | Volume of steam chest |
| S_s | Steam input flow-rate |
| S_c | Steam condensate flow-rate |
| S_o | Steam condensate flow-rate into the drainage |
| P_s | Pressure of the steam in the shell |
| P_a | Atomospheric pressure |
| C_d | Friction coefficient |
| v | Velocity |
| R | The field of real numbers |
| C | The field of complex numbers |
| $R[z]$ | The field of rational functions in z |
| $\deg A$ | Polynomial degree of A |
| P | The plant matrix |
| F | The feedback matrix |
| S | The sensitivity function |
| C | The controller matrix |
| T_c | The closed-loop transfer matrix |
| $\det A$ | Determinant of matrix A |
| $\text{Tr } [A]$ | Trace of matrix A |
| \bar{y} | Output vector |
| \bar{u} | input vector |

\bar{n} instruments noise vector

\bar{e} error signal vector

\bar{r} Controller output vector

O_n The nxn zero matrix

I_n The nxn identity matrix

r.m.s Radius mean square

\bigcup_i^n Union of all sets $i \rightarrow n$

\bigcap_i^n Intersection of all sets $i \rightarrow n$

iff if and only iff

$P(j\omega)_*$ =Adjoint $P(j\omega) = P^T(j\bar{\omega})$

CHAPTER (1)

INTRODUCTION

1.1 Introduction

Control systems play a significant role in improving the quantity and quality of manufactured products. They constitute the basic elements of automation systems and as manufacturing activities become more and more automated, the importance of using increasingly more refined control technology becomes evident. The most important technological advance that is currently taking place is a revolution in the processing of information. This will have a major impact on industrial plant control. This revolution is based on the introduction of digital computers and microprocessors.

Automatic control systems, once operating and producing an acceptable end product, continue to operate without the errors that often result from inattention by production workers, and normal human errors; therefore great savings have been realized. Many operating systems can justify the purchase and installation expenses simply on the basis of the economics realized by reduction of rejected, out-of-tolerance end product.

Automatic control systems can be adjusted to produce an end product to closer tolerances than is consistently available using human operators; therefore, improved quality of end product is frequently achieved. Many systems must be automated because the

required quality of the end product is simply not economically possible using human operators.

Automatic control is needed for processes that operate in extremes of isolation, e.g. on the moon or on the bottom of the ocean, in total darkness, where the process reacts at extremes of speed, e.g. the control of high speed military aircraft; where the degree of complexity of control is excessive e.g. in transcontinental natural gas pipeline flow; and where certain control features are too critical to trust to human capabilities.

Automatic control also has a wide domestic applications. With the high aspiration for a better quality of life in human society convenient access to resources in a hostile environment is a major objective. Simple examples of this are the availability of air conditioning systems, home heating, vending machines, and many remotely controlled servomechanisms designed primarily for convenience.

Process control, or more specifically fluid process control involves the control of plants which manufacture materials and services: oil, chemicals, paper, metals, concrete, power, and the like. These processes are smoothly non-linear, highly uncertain, and of very high order. They can be mathematically modelled only in gross approximation. Some plant may involve thousands of measurements and actuators and hundreds of control loops. Some plant complexes occupy square miles of land. Even more complex than the plant is its disturbance environment and its control objectives.

New considerations have made control design more critical. As the raw materials are perceived to be more expensive,

processes become more complex and interrelated to recycle energy and low grade product. These processes call for more refined control.

Process control is a multidisciplinary activity involving many scientific areas, and it is connected with the following broad areas:

- 1- The application of scientific knowledge: that is, principles from the various pure sciences, such as physics, chemistry, biology, and mathematics, which have been discovered and published.

- 2- The application of engineering principles; that is the practical application of experience and empirical knowledge as compared to purely theoretical knowledge.

- 3- These two broad fields both contribute to a new field called cybernetics.

Engineers and scientists in the field of control have as their goal the empirical and mathematical determination of the time-based characteristics of various processes, both controlled and uncontrolled, for which automatic control is deemed desirable. Furthermore, they strive to discover the basic laws of nature that govern biological and other feedback systems, or as it has recently been called the science of cybernetics, and to apply these basic principles to the control of processes. They implement various theories and techniques, observe their effects on process control systems, and attempt to refine the basic theories and techniques based upon the results.

All of this, has called for the development and application of theories of automatic control. To a very large extent, the field of automatic control theory deals with the processing of the error signal, that is the difference between a measurement and its desired values, and design of the controller. The balance of its area of concern is with the overall system dynamics, additional processing needed for a feedback signal, selection of points in the process where the feedback signal is to be measured, position where the command is to be applied, and interactions among various portions of the total system, which may be individually, and simultaneously, under the influence of many controllers.

The only techniques available for determining these complex interrelationships and for design of compensation and control circuits involve higher mathematics. Since most of these relationships and variables are time-dependent, the major mathematical tools are calculus, statistical theory, and more recently matrix algebra.

Whatever the justification for automatic control systems, one extremely important consideration is, and always has been, the cost of automation. Many recent technological advances have resulted in the development of new hardware which is suitable for use in process control systems, e.g. microcomputers, in significant reduction in the cost of computers and some other hardware items used for process control, and in increased capability for many process control systems hardware items.

In recent years, the advent of cheap microprocessors has made possible the interconnection of a number of distributed

controllers into a form of network. The network usually is arranged in a hierarchical fashion, and is generally called hierarchical control. Applying process computers and microcomputers, not only can the function of analogue equipment and hardwired logic devices be replaced, and extended but also quite new methods can be realized for improving the performance of hierarchical process control. These advanced methods for process control are characterized by more sophisticated, better adjusted control algorithms, forecasting of process variables, estimation of variables not directly measurable, computer aided design of algorithms and adaptive or selftuning algorithms. The basis of these advanced methods is mathematical models of the process and their signals. Therefore, mathematical modelling techniques are indispensable tools for the application of advanced control theory to the control process problem.

1.2 Mathematical Modelling

The activity called mathematical modelling is crucial in control engineering design or analysis problems because many subsequent decisions are based on results derived from the mathematical model of the system. The objective of the investigation will strongly influence the complexity of the mathematical model and the methods used in obtaining it.

Nearly all the systems which an engineer may work with are dynamic, i.e. the systems are usually changing with time so that there is no "status quo" or lasting steady-state. Many of the important facets of a design will be based on steady-state

considerations, as is normally the case in continuous process design, but the device or system will fail if it cannot withstand transient peak loads, respond quickly enough to a changing input, or operate without violent oscillations when disturbed. The technique of dynamic analysis using a mathematical model can predict such problems before a system is built. However, no real physical system behaves dynamically exactly according to any of the equations of a mathematical model, which describes the idealised elements of the system.

Water boiling in a container may be modelled as an infinite number of eddy currents and chaotic random movements with micro-temperature differences, or it may be looked at as a homogenous body of even temperature distribution; in both cases it is never behaving exactly in the ideal form as described. The ideal elements are mathematical fictions which approximate the behaviour of certain physical processes only under specialized conditions. If several of the ideal elements are combined, however, an ideal dynamic system is formed which may represent the behaviour of a real physical system over a wide range of conditions. The combination of ideal elements which is intended to represent the behaviour of a physical system is called the model of the system. The degree to which the behaviour of the ideal model corresponds to the behaviour of the physical system represented by the model is a function of the experience, skill, and engineering judgment of the modeller. Usually, the model must represent a compromise between excessive complexity and reaching the degree of accuracy required in the predicted behaviour of the physical system.

The importance of this modelling process cannot be overemphasized. Here is where the engineer must make assumptions about a real world situation to make it mathematically tractable and yield a solution with a reasonable effort. The usefulness of the solution is directly dependent on how closely the model represents the behaviour of the physical system, while being soluble with reasonable effort.

One of control engineer's most important jobs is to establish a mathematical model of the system to be analysed. This involves the identification and idealization of individual system components as well as identification and realization of their interconnection since most real systems are extremely complex when viewed in detail, an exact analysis of any system is hardly ever possible. Therefore, simplifying assumptions concerning the properties of the system components must be made, thus reducing the system to an idealized version whose behaviour approximates the significant behaviour of the physical system. In many cases, these assumptions involve neglecting effects which are clearly negligible. Many of these effects are in fact neglected as a matter of course without a clear statement of the implied assumption. For instance, the energy losses in a number of connecting pipes of a process measurement instrument are ordinarily not considered. There are, however, situations in which energy losses might have a serious effect. If the energy losses were such that they caused a significant change in the energy balance, quite different results would be observed than would be predicted in an analysis which neglected these losses.

These examples concern factors which are completely negligible in many situations of interest. However, in most situations there are slightly more important effects which will still frequently be neglected to define a problem so that it can be handled mathematically without inordinate complexity. It is this type of assumption which is most often faced. Sometimes these effects will be neglected in a first attack on a problem so that the engineer can get a quick evaluation of the predominant effects. Then, in a more precise analysis, the more complex factors will be considered. In other cases, the complicating effects are considered sufficiently small that the essential behaviour of the system is exhibited by the simplified version. In many cases, the importance of a complicating factor can not be determined without recourse to experiment.

The degree to which the behaviour of the model represents that of the actual system clearly depends greatly on the validity of the various assumptions made in arriving at the model. The experience, and judgment of the engineer are thus crucial factors in successful modelling.

A mathematical analysis is useless, however clever and complete, if it is based on a model which does not closely represent the behaviour of the physical system under consideration. On the other hand, undue complexity in formulation of system models is also to be avoided. First, the cost of completing the analysis of a system model increases very rapidly with its degree of complexity, and second, the use of an excessively complicated model will often obscure, with a mass of nonessential detail, predominant effects. The best model for any system is one that provides, with

minimum complexity, the information necessary for the purpose in hand, control action.

The control process system is usually required to be robust, have a good integrity, be able to cope with all the probable disturbances, hazard free, and of good tracking performance. To achieve these arduous requirements knowledge of the plant behaviour over a wide range of operating conditions is an important prerequisite. This is often achieved by means of a mathematical model.

In order to make the important physical assumptions needed by a control theory and to incorporate these into a mathematical model compatible with the plant and which makes engineering sense, there is a need to have good knowledge and judgment about the behaviour of the system. Here, the control designer needs some feeling for the plant operation, physical construction, and other experience developed from practical operations.

Mathematical theory and physical justification interact in a very critical way in the formulation of the model. Since any formulation of an engineering problem is tentative, if some of the consequences of the formulation prove untrue or unacceptable, then the problem must be reformulated. If any intermediate step in the mathematical argument represents a physical untruth then the results obtained by rigorous argument from an apparently sound starting point will nevertheless be wrong. Also, if in the problem a mathematical difficulty arises, the physics would very often suggest a solution to the mathematical difficulties which are occurring.

In summary, the approach taken to the dynamical analysis of the problem at hand, modelling and control of chemical plant, can be enumerated as follows :

- 1- defining the system and its components,
- 2- formulating the mathematical model,
- 3- determining the system equations,
- 4- solving for the designed output,
- 5- checking the solution by simulation against experimental results,
- 6- designing the controller, and
- 7- testing the controller.

1.3 Control System

Hitherto, the controversy about the application of modern and advanced theories of multivariable control systems to industrial and chemical processes has not been resolved. Many questions have arisen about the practical importance of these complicated methods in the process industries. The arguments have even gone to the extreme of whether these complicated expensive methods of the modern control theory are implementable.

This type of argument has almost become a natural phenomenon during the process of development of all sorts of human cognizance. Control science and engineering as any other field of knowledge, has faced this necessary critical scrutiny. For example, it is similar to the corpuscular theory versus the wave theory in the physics of light, or the quantum theory supported by Bohr versus

the relativity of Einstein, though the similarity is only equal in the wide critical discussion given to the field.

Control theory and its application have passed through different phases of development. For example, after the 1960's the state space approach has emerged as a new completely different tool for analysing and synthesising control systems. In this, it is completely different to what was the previous practice. The Laplace transform and hence the transform function technique in the s-domain was the backbone of the field and almost all the design tools were provided in that domain. Such authors as Nyquist, Bode, Truxels, Ziegler and Nichol, Evans, developed much used techniques available to the control system designer, and indeed their methods are still among the most powerful techniques in the design of control systems.

Through the centuries control theory, or in general terms, cybernetics, as many other fields of knowledge, has gone through a series of phases of development. From art to science, from heuristic methods to more mathematical rigour, and from special cases to generalization. The early reports on the field roughly started from about the late eighteenth century, with very little progress up to the beginning of the twentieth century. The progress was empirical in nature. Good examples of the early era are Watt's steam governor, followed by work of Airy on the problem of the clockwork drives used to regulate the speed of astronomical telescopes, [Airy, 1840]. Airy's work was based on the theory of celestial mechanics. Unfortunately, Airy's work was rather intricate and difficult to follow; it therefore did not become

widely known, and the subject remained shrouded in mystery to engineers grappling with the problem of fluctuating engine speeds.

The problem of the speed oscillation, or hunting of governed steam engines became a very serious one and so attracted the attention of a number of outstandingly able engineers and physicists. The controller applied was a proportional type of controller which, however, turned out to be unstable when applied to steam engines. Without current theory and the insight it affords, early experiments had to rely entirely on art and intuition. Accordingly, progress in obtaining solutions to early problems was slow.

Unfortunately it appears even now that there are many process problems which are treated in that fashion; this time not for the lack of rigorous theory but from the complications and complexity of these theories.

An exception to the early heuristic techniques to regulate the steam governors and a formal introduction to the theory of automatic control is a paper by Maxwell, [1868], which represents the beginning of theoretical development.

From the end of the 19th century up to the second world war there was a tremendous development in technology and engineering. Advances in large-scale power generation and transmission, the birth of the aeronautics industry, telephone systems, rapid growth in the processing and chemical industries and the advent of communications and electronics engineering occurred. All these were among the radical changes of that time. Such technological progress created an increasing demand for instruments and regulators. Some of these instruments and regulators for

pneumatic processes and hydraulic and electric systems founded during and before the 2nd World war are still applied.

The use of theoretical methods was limited mostly to the solution of ordinary differential equations and to the application of the Routh Hurwitz stability criteria. The application of such theoretical methods was limited to simple and low-order systems as far as manual computation was possible.

During the 1950's many technological development, many predicted in science fiction a few years before, have become a reality. Nuclear energy, digital computers, long distance communication television transmission and other phase related systems, guided weapon systems, radar tracking, to mention a few, all of these had been made possible and utilizable through the advances of control theory and application. Some times the theory has lead to important innovative technological advancement. This is shown clearly in the field of control in the work of Wiener [1961], by his basic theory of the unified feedback control applications which has extended to many important systems. At other times the important practical problem of Watt's governor has led to the development of powerful control system theory.

The late 1950's marked the rediscovery of the state-space point of view and the adoption of an entirely new approach to control problem solution. Originally developed in the field of classical mechanics by Poincare', Lyapunov, Gibbs and others, sixty to ninety years earlier, the method was found to have great utility in the analyses and design of control systems, [La Salle,1961], [Pontryagin et al,1962], [Zadeh et al,1963].

Towards the 1970's Rosenbrock, MacFarlane and others extended the frequency-domain methods to the multivariable case, the study of system equivalence and the generalization of the ideas of controllability and observability originally developed in the state-space by Gilbert and Kalman, and the frequency domain method regained ground.

With the application of all these ideas in state-space, frequency domain and more recently the generalized algebraic approach initiated by Wolovich [1974], and the geometrical approach appeared in the work of Wonham, [1974], to the control area, there has been a growing interest in the purely theoretical. In fact, the mathematical level of sophistication has reached a stage where some recent literature treats completely abstract and non real systems. This is a far cry from the humble beginnings of the control area with Watt's experimentally developed governor. To continue to perform an engineering function, the control engineer working with new and elegant theory must keep in mind the goal of application, and this has created a division between the worker in the field of control theory and control application, [Foss,1973]

To resolve this problem an advanced theory that deals immediately with the problems found in industrial processes, [Rosenbrock,1971,1962], and at the same time gives the control system designer insight into the problem is needed. A multivariable modern Wiener-Hopf approach, introduced by Youla et al, [1976], offers the possibility of a valuable synthesis method which is well adapted to handle engineering descriptions and constraints.

This work of modern Wiener-Hopf design can be traced since the early work of Newton, [1952], [1955]. Two major papers in

the control literature by Newton, [1952,1955], have made a landmark in feedback control theory. They represent the first workable approach to the treatment of the control problem as one of constrained optimization and stand as the basis of what is now called the Linear Quadratic Optimal Control.

Before Newton, early approaches to control system design were trial and error methods. Wiener was among the first to use analytical methods for control system design during World war 2. However, Newton was the first to recognize the differences between filter design, which was the main thrust of Wiener's work, and feedback control system design. These differences include the following notions :

- 1- The designer of a feedback control system often has no choice in picking some of the components of the system. These components were called fixed elements by Newton. Today the term plant or controlled process is often used to denote the same unalterable parts of the system. In filter design, the designer usually has complete freedom to choose the components of the filter.
- 2- In a feedback control problem, the usual performance specification is that the output must be an instantaneous replica of the desired output with no delay. In filter design, the restriction of zero delay is usually relaxed.
- 3- In control system design, disturbance can enter the loop at any point. In filter design, it is usually assumed that disturbances enter at the input as additive noise.

4- In control system design, the elements used often have highly variable characteristics, particularly at the high power or output end of the system. Thus the mathematical models used are quite inaccurate.

Intentional use of feedback is one of the major ways to ensure reliable performance can be achieved despite the use of unreliable models [Horowitz,1963]. Unfortunately, feedback introduces the stability problem. In filter design, the components are usually very accurately described by their mathematical models and, except for active filters, the filter designer need not be concerned about stability.

5- In most applications, the effect of saturation of amplitude limiting is one of the key features that lead to performance degradation. In contrast, filters are usually subject to large input signals and so saturation effects are of secondary importance.

6- The cost of a feedback control system increases quite rapidly as its bandwidth increases because output power requirements tend to increase with the cube of the bandwidth. This problem does not arise in filter design because the signals being processed are at a relatively low level even at the filter output.

Newton's recognition of the differences indicated above served as the basis for his major contributions to the theory. Of primary importance was his introduction of saturation constraints into control system analysis and design and his formulation of the bandwidth minimization problem as a way to minimize the cost of the

control system. Both these ideas required that the control problem be formulated as a constrained optimization problem which was solved by using the Lagrangian approach. As a consequence of Newton's work, it then became possible to determine the ultimate performance that could be achieved given a set of specifications for the control system.

What followed Newton's work is now the main stream of modern control theory. The key step was taken by Kalman who worked in the same laboratory as Newton at M.I.T. shortly after Newton's papers appeared in the literature. By introducing the state-space representation of linear systems, Kalman reformulated the Wiener-Lee filter problem. This lead to the Kalman-Bucy filter which could be implemented directly on a computer because of its recursive behaviour. Moreover, Kalman also stated the linear, quadratic optimal control problem which he solved. Combining these two problems yielded the linear quadratic Gaussian (LQG) control problem which is the basis for much of modern control systems design [Anderson et al,1979].

It is thus clear that the linear, quadratic, Gaussian control problem is a direct generalization of Newton's work. In particular, solution of the LQG problem yields a design in which the attempt is made to minimize errors, i.e the difference between the desired and actual responses. At the same time, a penalty is imposed if excessive control effort is used. The degree to which a control effort is penalized is determined by the relative weight assigned to errors in comparison to control excursions. But the relative weights are nothing more than the Lagrange multipliers that arise in the course of the solution of the saturation problem

as posed by Newton, by the Lagrange method of constrained optimisations.

Newton's formulation of the control system design problem was derived from practical experience and was an attempt to incorporate directly the actual physical constraints that limit performance into the design procedure, [Newton et al, 1967].

The work of modern Wiener-Hopf controllers developed by Youla, et al, [1976], is in essence very similar to the work of Newton. The contribution of Youla et al of the multivariable Wiener-Hopf approach has offered the possibility of a valuable synthesis technique which addresses immediately the problem of engineering description and constraints. This design method in its breadth, its incorporation of input noise, load disturbance and feedforward compensation and over all its freedom in handling plants without any restriction, asymptotically unstable, non minimum phase, interaction of the plant variables etc. has great promise, not least in its ability to handle constraints imposed by plant saturation and measurement noise. It also gives a solution to the standard optimal control problem in certain cases when the LQG approach in the time domain breaks down.

The Wiener-Hopf controller with its powerful features for addressing the process control problem has laid fundamental ground to approach the distinctive problem of process control. However, the complexity of the computational aspects of the Wiener-Hopf controller may make it an unattractive option without powerful (off-line) computational facilities.

As a result of this work another synthesis method with powerful engineering features has emerged. The new method is

based on the modern Wiener-Hopf controller structure but requires less computational effort and has a unique feature that the designer can specify the closed-loop poles of the multivariable system. The method enables a designer to shape the sensitivity function as required.

1.4 The Chemical Plant

In brief, automatic control theory is a coherent body of knowledge that acts as a bridge between mathematical abstraction and physical reality of plant control.

This work explores the implementation and synthesis of an advanced controller to an industrial process, a pilot plant climbing film evaporator. The research is to evaluate the difficulties, cost, and the return on such an application to the real world of engineering.

A distributed-parameter mathematical model of the plant has been developed in the time domain and this has been further transformed into a transfer function form.

Two advanced controllers have been developed and applied to the plant mathematical model. A distributed-computer network has been interfaced with the plant for advanced control implementation. A computer aided-synthesis package has emerged from the study on the advanced controllers.

A Modern Wiener-Hopf controller was chosen in that it is classical in dealing with transfer functions, modern in that it is easily extended to generalized algebraic treatment, and

optimal in the engineering sense. It makes the most sensible approach to process industry, by tackling the interactions and treats stochastic as well as deterministic systems.

The most important objective in process control is to keep the quality of the product within a specified tolerance during the manufacturing process. Hence, disturbance rejection, noise filtration, recovery to set-point changes are very important problems to address. A design method which addresses these issues directly with appealing engineering conditions in the design is through the Modern Wiener-Hopf design approach.

CHAPTER (2)

LITERATURE REVIEW

2.1 Mathematical Modelling

The first stage in almost every advanced control system design is the development of a process model. With the complexity now required of a control system by most plant engineers, knowledge of the plant behaviour over a wide range of operating conditions is an important prerequisite to build a control system that is robust, has good integrity, is of high reliability, and is capable to cope with the most probable disturbances.

In the present case of the double effect evaporator the requirements are for a linear model of a rather complex, non linear, distributed-parameter, multivariable industrial process. A number of possibilities exist, ranging from methods based on deterministic physical models to those based on the identification of black box models.

However, a good mathematical model to represent a plant faithfully may be expensive compared to the sketchy data which can be obtained from identifying the plant through the many techniques of process identification. Some of these identification methods have been reviewed by Gustavvson [1975] and a number of case studies as well as the theoretical advances are presented by Eykhoff [1973].

Numerous studies of the mathematical modelling of heat exchangers and evaporators have been carried out. The interest in modelling has varied from obtaining data for improvement of design and operation to automatic control.

Due to the importance of heat exchangers and evaporators in most of the field of the processing industry a vast literature investigating the behaviour of these unit operation has been published, a substantial review is given by Eckart [1974]. The studies have involved the steady-state and dynamic behavior, heat transfer coefficient, changes of physical properties and geometrical variations of the unit operation.

Most of the work has concentrated on the methods of predicting heat transfer coefficient by different means and many examples of these methods have appeared in Kern [1960], McAdams [1954], and Kreith [1972], and differences of accuracy of these methods are reviewed by Webb [1977].

For the climbing film evaporator now under investigation Webb has used a correlation formula fitted with his experiment data. The formula obtained is reported to be not valid for a range of operations and the accuracy is generally not sufficient.

However, the results given in the literature for computing heat transfer coefficients are usually rather complex due to the fact that they are normally given as function of design geometry, flow dynamics, physical properties and position within the heat transfer equipment.

For the same equipment, later, a simple formula to predict the steady-state heat transfer coefficient will be developed.

Dynamics

On the dynamic simulation of an evaporator empirical and theoretical approaches have appeared in the literature, [Johnson, 1960]. The interest has ranged from methods of finding detailed mathematical behaviour, [Koralkov, 1970], [Koppel, 1966]; methods of reducing the order of the models obtained, as well as methods of linearization, and approaches through early lumping of the system. A careful review of these different approaches and the results obtained by each have been presented by Webb. They varied from complex models, indeed, not suitable for control application to those very simple models which suffer from poor accuracy.

A generalized approach for the modelling of a multi-effect evaporator for purposes of control and estimation was developed by a team from the department of Chemical Engineering, Univ. of Alberta, Canada. This work is extensively reported by Seborg and Fisher [1976]. A tenth-order nonlinear, dynamic model of a double-effect pilot plant evaporator was derived and then linearized to produce a fifth-order state-space model. They have reported that comparison with experimental data was satisfactory. The approach they followed was to assume a lumped-parameter system and this might be quite justifiable for pool type of evaporators.

For the evaporator investigated in this study, different models have been developed for the purpose of Kalman filter applications, Payne [1976], Coleby [1974]. Payne derived a

comprehensive 18th order model of the double effect evaporator using an analytical technique based on the physical laws governing the system. This model was then reduced to fourth order by assuming that the vapour phase of the evaporator has constant temperature. Coleby assumed that the vapour phase dynamics can be described by algebraic equations since the time constant associated with the vapour phase dynamics is small compared to those relating to the heat transfer dynamics. In order to obtain a differential equation describing the rate of change of the liquid flow on the shell side of the various evaporator units, Coleby differentiated an algebraic equation and then applied the chain rule. The resulting seventh order model required the introduction of a number of parameters before it gave satisfactory results.

Webb [1977] derived for the same evaporator eleven algebraic equations and seven differential equations describing the transient and steady-state behaviour exhibited by the double effect evaporator. The main purpose of the models was for implementation of an on-line kalman filter. However, the effect on accuracy of lumping the system variables is quite apparent in the model.

In this study the approach taken addresses the distributed parameter nature of the plant and at the same time is simple to analyse and gives insight into the dynamics of the plant. It is hoped that through a distributed-parameter model more accuracy could be achieved as well as more insight into the physical assumptions and their validation.

2.2 Process Control

A brief general review of the application of the modern control theory of multivariable systems to industrial processes will now be given. The survey will cover points of tangency, divergence and overlap between the field practitioners and the theorists, and the different points of view, and what can and has been achieved by the advanced theory and what is still lacking.

Specifying the distinctive problem in the field of industrial process control a new approach is suggested that might help in resolving the points of differences, a method that addresses immediately the intrinsic problem of process control.

The review discusses the main general design methods now available to the multivariable control system designer and how these methods have been applied to the problem of industrial process control. The survey ranges from the economic viability of these complex methods, the need to control stable process up to what is the difficulty felt by workers applying them in process control systems. Indeed, the review does not cover the huge and tremendous literature of modern control theory, only those methods which have been used or suggested for the design of real practical industrial processes, or have shown to have potential applicability to control in the process industry will be discussed.

In recent years, several applications of modern control theory have been proposed for use in the process industry. However, despite the obvious theoretical advantages of these advanced strategies their implementation in an on-line situation generally requires considerable computational effort. An alternative approach is the extension of the well proven frequency domain design methods to multivariable cases; this offers the advantage of easy

implementation and the possibility of on-line adjustment or tuning of the controller. Although these frequency domain techniques are theoretically less attractive than optimal controllers resulting from state-space analysis, from a practical point of view their simplicity, high stability and low noise sensitivity may more than compensate for this.

The availability of faster and more powerful computers has opened new possibilities for control systems design and implementation on complex chemical plants. The use of state-space techniques for control of laboratory scale equipment has been reported by Fisher et al [1976], and others Jutta, et al [1977], Kershenbaum, et al [1978], but from an engineering point of view, there is a good deal to be said for design procedures which are based on extensions of classical frequency response methods. Specifically, because they do not require a complete model of the plant, these methods generally involve an order of magnitude less effort in the modelling stages than controllers based on state-space techniques. On the other hand they are generally restricted to linear analysis.

In all cases, the implementation is an application of a linear multivariable controller which is based on a linearized process model. However, there are few examples in the literature of the application of such design methods on industrial process plants, Buckey [1978], Padmanabhan, et al [1977], Schvldt, et al [1971], Olson [1976], Church [1976], Fisher et al [1976].

Bryant [1976], has described his experience with the application of modern control theory to a steel tandem cold-rolling mill. He suggested that to obtain a proper appreciation of process

control research it was necessary to invest some time in acquiring some understanding of the process, and its physical characteristics. The main difficulties felt by Bryant were the importance of understanding the process and of developing models suitable for control design. This rather neglected area often requires, from his experience, ingenuity and research ability of a high order. In defining the disturbances that affect the plant, he expects to find the most important of the many known disturbances that affect the regulation. The main points which came out as a consequence of the work are as follows:

- 1- The process equations arising mainly as a result of delay and reflection effects do not fit into state-space or rational function descriptions,

- 2- The classical approach by appealing to physical judgment of the plant and to engineering experience, resulted in structures which varied from simple to complex but all of them showed lack of appreciation of the best multivariable control structure to be employed.

Bryant followed a hybrid type of approach, making use of the advanced theories as well as the engineering experience gained from the plant, this according to Bryant appeared to work and solve the problem.

Shinnar [1976], in his work discussed his experience of the application of modern control theory and the effective methodology of how to use computers in chemical process.

He found the approach of optimal control to be unuseful, as it led to unworkable control algorithms, and less good performance than expected or claimed. He tested every optimal controller structure available at that time. They did not give any better results than the standard simple methods e.g. classical three terms PID control. This failure may be due to the lack of knowledge of the structure of successful control design. Also sometimes there are conflicts between different requirements of controller design. These may include reconciling of regulation, smooth set-point changes smoothly, fast asymptotic stability, insensitivity to model inaccuracies and to changes in process conditions, and the ability to design with a minimum amount of information.

Trevath [1976], has described his experiences with process control in the chemical industry with the following remarks in preference of classical multiloop control over advanced modern multivariable control:

- 1- It is easy for the operator to identify in case of failure, in a minimum time, and cure the failure or understand it. In multivariable control the failure may propagate to other loops,
- 2- Maintenance personnel may disconnect critical components of the multiloop system before switching the assigned loops to manual, and
- 3- they are easier in tuning than the multivariable.

There are still many difficulties which have no straight answers: making the specification to design the control system, what are the sources of disturbances which need to be

controlled etc. Two pieces of general advice given to the designer by Trevath, were to find if the process can be controlled by a simple controller, and secondly, feedforward is a good simple solution especially when the disturbance can be measured. Otherwise lead-lag compensation can be useful.

Barklew, [1976], has commented on the state of the art of modern process control on petroleum refineries. The four major processes of distillation, reaction, combustion, and blending were discussed. The need, specification, and problems associated with each were identified.

Barklew reported that refiners were more and more installing computers at an ever increasing rate. They were making them work, but apparently in ad-hoc way. He thinks that most of the benefits were derived from the information and scheduling function of on-line computers rather from the use of advanced control applications and that the general argument for the benefits of feedforward application in distillation columns were doubtful.

Athans [1976], has criticized the theory of adaptive control. In adaptive control where parameter or structure identifications are involved, the stumbling blocks were associated with the real-time computation needed. The powerful method of maximum likelihood for identification may require a great deal of real-time computation, and the iterative nature of the estimation algorithm may be unattractive for some applications. Recursive estimation like the extended Kalman filter algorithm may be

characterized by poor convergence properties and even divergence if the non-linear filtering algorithm is tuned improperly.

A session at the 1981 JACC in Charlottesville, [Johnson 1981], was devoted to the development of two test examples for adaptive control. The two candidate processes were derived by two panellists from the aerospace and the chemical process control domains. The purpose was to distill those process characteristics suggesting application of adaptive control and to provide practically meaningful test examples for comparison of various adaptive schemes. The session conceived by the organisers due to the feeling that despite valiant attempts at theoretical unification, the literature remains a confusing mass of (sometimes) conflicting suggestions and opinions based on wildly different, mostly academic simulations.

Foss and Monopolic (contributors in the above session) from the experience with the chemical reactor example they set out to solve, concluded the following:

- 1- a higher order model was needed to represent the plant well,
- 2- Delays in both the state and input variables, the impossibilities of measuring state variables, and non-minimum phase behavior, all complicated an adaptive control candidate.

An application of digital parameter adaptive control to an air-conditioning plant was carried out by Schumann, [1982]. That example has suggested for practical application, the control

behaviour of the basic multivariable parameter-adaptive controllers may not be satisfactory especially with respect to the adaption variations of control and process output signals during the first adaption phase after closing the loop and after changes of the operating point. So, to improve the control behaviour of the parameter adaptive controllers, additional measure were taken such as: automatic on-line switch-off of the parameter estimation to prevent the so called bursting of parameter estimates, (see [Schumann et al,1981]).

Mann, [1983], discussed his experience of digital control application on a rotary dryer in the sugar industry. In order to utilize pulp, which is a by-product of the extraction of sugar from beet, it must be dried. This is usually done with a rotary dryer. Generally, the dry substance percentage of the dried bulb is controlled manually; this, however, was not sufficient. The aim was to improve the quality of control and thereby to reduce the fuel consumption. To reach this goal, first a linear, time invariant, discrete-time mathematical model of the dynamic behaviour of the drying process was identified off-line. A correlation analysis method with least-squares parameter estimation was used. Based on the identified model, the parameters of the control algorithms of a digital multicascade control system were designed with the aid of a computer. The control behaviour was simulated and compared with the control behaviour achieved by an optimal state feedback control system. Although, the latter one gave a better control performance, the cascaded control system was implemented on

a process control computer. This was done for the following reasons:

1- Since the transformation of the identified models, describing the input-output behaviour, into a state-space model leads to state variables which have no physical meaning and thus cannot be measured, they have to be constructed by an observer such as, Luenberger, Kalman, etc.,

2- the non-minimum phase behaviour of the process made the optimum design unsuccessful, and

3- subsequent change of the controller parameters by the operators must be possible and that can not be done with the state-space optimum design.

The work of Buchalt and Kummel, [1981], was on a pilot plant double effect evaporator. A multivariable self-tuning regulator was applied to the control of the plant. The selftuning regulator consisted of a recursive least square algorithm combined with a single step optimal control strategy which minimizes a quadratic criterion.

$$J = E [y'(t+1)Q_1 y(t+1) + u'Q_2 u(t)]$$

where Q_1 and Q_2 are symmetric weight matrices. E denotes expected value. Q_2 the weight on control variables, make it possible to control the experimental process without excessive control effort.

The summary of problems with the parameter-self tuning controller is as follows:

- 1- A period of 100 samples was necessary before the model parameters became stationary,
- 2- during this, for some time, the system was unstable,
- 3- it was not successful for the start-up procedure,
- 4- if the plant was left on manual for 1/2 hr. and then switched to the control it took 180 samples until the system became stationary.

The performance of a self tuning-regulator was compared with an optimal regulator based on the physical model and with control criteria similar to the multivariable self-tuning regulator, comments on the experiment were as follows:

- 1- The experiments carried out do not prove stability of the selftuning control systems,
- 2- since no general method is available to secure stability of a multivariable selftuning regulator, a simulation analysis of the entire system was performed before the test on the pilot plant was carried out, and
- 3- sometimes the self tuning regulators use much more control action than the corresponding optimal strategies. This is one reason why self tuning regulators sometimes do not model the dynamics properly.

The corresponding optimal control strategies based on a physical model showed better performance than self-tuning regulators. Models based on physical properties could be used, contrary to the inaccurate model produced by the self-tuning

regulators, to construct a Kalman filter and to perform sensitivity analysis.

Bruun and Kummel [1979], have evaluated the application of classical multiloop, dynamic feedforward, modal, and optimal control methods on a double effect evaporator. A fifth order state-space model is used in the theoretical design. The criteria used in evaluating the attainment of a control method are the soundness of the principles, and its implementation.

Multiloop

the classical multiloop method is a direct adaption of single-loop methods on multivariable systems. Controlled and control variables were paired one-to-one and the parameters of each loop were determined using frequency response and root locus analysis techniques.

Two basic design problems in the multivariable case can be immediately pointed out:

- 1- The best controller configuration, and
- 2- the controllers parameters.

There is no direct guidance concerning the configuration problem and therefore the designer must use a combination of process knowledge, intuition and block diagram and sensitivity analysis to extract the most promising configurations.

Feedforward

The goal here is the rejection of measurable external disturbances. The problems which faces the designer are:

1- to specify the feedforward control variables, but only if there are more control variables available than there are controlled variables to be feedforward controlled,

2- whether the disturbance is measurable or observers are needed to construct the measurement.

Modal Control

Modal control is, in the widest sense, a method whereby the system closed-loop eigenvalues can be placed in any desired position using proportional state-feedback. This is normally applied by placing the system eigenvalues as far to the left in the complex plane as the control variable amplitudes permit.

It is evident that the method focus its attention on the speed of response of the system. However it lacks the means to address specific offset properties of controlled variables, unless integral action is included. The specification for the design needs intuition from the designer, such as :

1- where the eigenvalues are to be shifted to,

2- which control variables are to be used to realize the shifts, and

3- which measurements should be used to reconstruct the modes..

Optimal Control

The principle of optimal control is to manipulate the control variables as a function of the state variables in such a way

that some of the time response functions of the system variables are optimized. The standard stationary linear deterministic formulation was employed where the control system is given by

$$\dot{x} = Ax + Bu, \quad x(0) = x_0$$

$$u = Kx$$

and the optimization problem is to determine the feedback controller K which minimizes the quadratic performance function,

$$\int_0^{\infty} [x^T R_1 x + u^T R_2 u] dt$$

It is well known that there exists a unique K which minimizes the above objective for all x_0 provided that certain conditions are met, see [Newell et al, 1972]. Since the above optimization formulation is directed towards initial condition responses its applicability to load rejection regulator problems may be questionable.

The design problem when using the optimal control approach is to interpret the control system design goals to the element values in the weighting matrices R_1 and R_2 . This is a rather diffuse affair. For stability the system must be stabilizable and detectable [Kwakernaak et al, 1972].

A great advantage in the actual design phase is that all the loops are designed together. One therefore avoids the problem of multiple design paths that are inherent in sequential

methods, and there is no configuration problem since the whole feedback gain matrix is utilized.

On the negative side is the fact that all the state-variables must be measured, unless a suitable observer or estimator like the Kalman filter can be constructed. On failure of sensors the system integrity is lost and the system can become unstable, [Rosenbrock, 1971].

Fisher and Kuon [1977] have compared the performance of different controllers obtained by Direct Nyquist Array (DNA), Inverse Nyquist Array (INV), and the characteristic loci methods. These were compared with controllers obtained from optimal state feedback and the classical multiloop. The findings of the work are as follows; all controllers gave good results. The best was the multiloop one. The frequency domain design techniques give more insight into the system performance. The general design objectives were :

- 1- closed loop stability,
- 2- servo control; accurate set-point tracking, non interaction,
- 3- regulatory control; insensitivity to disturbance or noise, insensitivity to internal parameter variation and modal uncertainties, non interaction,
- 4- integrity ; overall system reliability, fail soft characteristic, and
- 5- specific application dependent characteristics, over damped system response.

The findings of the work can be summarised as follows :

- 1- The interaction is not removed completely but became very small,
- 2- optimal controllers are slightly more oscillatory and noisier than responses obtained in the other studies but nevertheless have provided a valid comparison. The optimal feedback controller requires state feedback, since all of the state variables of the evaporator cannot be conveniently measured directly, the use of state feedback controllers requires the addition of a state estimator such as the Kalman filter or the Luenberger observer. This obviously complicates both the design procedure and the implementation. The dynamic elements included in the controller are all first order and hence relatively easy and straightforward to implement on a computer control system,
- 3- Of the frequency domain methods the characteristic loci approach is the most general, accurate and powerful but provide less insight into iterative interactive design of the controller than INA and DNA methods, and
- 4- When the design objectives include a high degree of diagonal dominance, i.e reduced interaction, the CL, INA, and DNA methods result in basically the same controller.

Thus this section can be closed by the following comment: The output state-feedback in the wide sense is still the most accepted controller structure. Generally the goals of these methods to be completely achieved may become impossible to satisfy. The goals may be summarised as follows:

- 1- Adequate disturbance rejection,
- 2- quick response to set-point changes,
- 3- insensitive to model and measurement errors,
- 4- avoidance of controller saturation or excessive control action,
- 5- suitability over a wide range of operating conditions,
- 6- minimum process information requirements, and
- 7- high integrity, and stability in the face of instrument failure.

A superlative controller which simultaneously achieves all requirements with inexpensive implementation, indeed, is the top mark against which all controllers must be measured. This has led to a design approach which is often based on interactive graphics, allowing successive evaluation of many alternatives, such as Direct Nyquist Array, Inverse Nyquist Array, Characteristics Loci etc.

2.3 Economic Viability of Process Control

Industry will only invest effort and expenditure in a project when there is a sound economic justification that, so far as possible, supports the decision to invest. There is not much published work on the methods of the economic evaluation of such application of advanced control systems. However, the general view is that advanced process control systems are more sophisticated, more complex, but better adapted to the processes, the question arises of the economic viability of the higher costs for hardware, software and skilled personnel, balanced against possible benefits

Some examples of these few economic evaluations of applications of advanced control are the work of Astrom et al, Fank and Smith, Stout, Richalet et al, and Gorden and Spencer.

Astrom and colleagues, [1975], improved the control of a paper machine and an ore-crusher by self-tuning controllers to obtain a better economy and quality of production. The ore-crusher throughput was increased by 10%. Richalet et al [1977], reported on returns of £100,000- £150,000 a year by smaller variations of the controlled variables obtained through process identification and succeeding computer aided design control algorithm for a PVC plant, a distillation column and a cracking tower. Stout, [1973], gave a summary of published economic improvements by direct digital control with process computers in paper, glass, and chemical industries for the period 1966-1972. Benefits of 1-10% could be gained. Shreve and Brink [1977] brought to attention the application of modern control to the production of soap. As soap technology changed, continuous alkaline saponification was introduced. Computer control has allowed an automated plant for continuous saponification by NaOH of oils and fats to produce in 2 hours the same amount of soap,

(more than 300 ton/day) made in two or five days by traditional batch methods.

Hence, with the new trend in design of industrial process systems, it is advisable to consider the use of automation in these processes. Advanced control theory is then needed to tackle the problem there involved: high multivariable interaction, unstable and asymptotically unstable, large and multiple pure time delays, and the strong nonlinearity in the processes.

2.4 SYNOPSIS

Adaptive and Optimal Control

Adaptive controls are methods which compare the present position with the corresponding ideal position derived from a perfect model and try to control the trajectories as perfectly as possible; it is generally an impossibility in the process industry. Modelling of chemical processes with most of their state variables unknown or not measurable requires more than can be offered by Luenberger's observer theory. Precise knowledge of the process, noise free measurements, values of parameters and methods for reducing the order of the observer are all prerequisite for efficient implementation.

Optimal control has certain theoretical properties of merit. Its application is limited due to the difficulties of phase shift, noise amplification and conditional stability, [Rosenbrock, 1971]. The fact that most of the state variables are not

measurable makes the linear quadratic Gaussian approach impractical for the majority of problems in the process industry. The stability problem associated with parameters adaption for chemical processes has not been fully resolved. The magnitude of slow drifting disturbances as well as uncharacterized noises relative to that of control action is manyfold larger than those encountered in the aerospace industry. This gives rise to the instability problem when the parameters of models or the controller are changed significantly in the adaptive procedure.

Multivariable Frequency Methods

These methods are mainly an extension of the Nyquist method to the multivariable case. They are computer-aided based design methods. The designer has to use his experience and engineering judgment in solving the problem iteratively. Generally, there are no structured rules to guide the designer, he must rely on his experiences. Being in the frequency domain the methods have gained ground amongst the control systems engineers. However, there are limitation in performance of these methods. In decoupling control, or the non-interacting approach if a parameter changes, the design is no longer non-interacting and suffers from extreme sensitivity to model errors. Difficulties and ill-conditioning problems arise in decoupling when dead-time or non-minimum phase occurs in the transfer function matrix. In the former case, a controller with a prediction element e^{-Ts} may arise, while in the latter case the controller will contain an unstable element,

[MacFarlane, 1972]. Another disadvantage of exact decoupling is controller complexity. However, Rosenbrock [1974] has suggested the idea of diagonal dominance which is less restrictive than the decoupling approach. The characteristic loci design method by MacFarlane and Belletrutti [1973], diagonal dominance is not required in this design method, also it is helpful in obtaining a system of high integrity. Application of this design method does yield a feasible multivariable controller, but not without a great deal of experience and engineering intuition.

In general, the advent of modern computers has a strong impact on the feasibility of the application of modern control theory. Digital computers have many immediate advantages over analogue-control means. Now, it is possible through these cheap digital computers to organize the various tasks of the control system on different hierarchical structured levels: direct control, monitoring, optimization, coordination, etc. with the following general advantages :

- 1- Tuning of the controller is much easier and cheaper,
- 2- many control strategies can be implemented using the same hardware, and
- 3- many tasks can be done by the microprocessor, e.g. changes of set-point etc.

The rapidly changing pattern of energy cost has created complete chaos in the process industry. New methods of design to achieve maximum economy and new means of operation to cut down on labour costs have emerged. As well as the energy issue other

stringent requirements are being imposed. The following are some of these requirements:

- 1- tighter quality specifications,
- 2- greater process reliability and integrity,
- 3- higher production rates, better usage of capacity, less reject material, fewer breakdowns.
- 4- strict environment regulation,
- 5- increasing complexity of process, and
- 6- higher costs in energy and raw materials.

Fortunately, this has happened in concurrence with the introduction of the cheap signal processing electronic systems. This has made possible the application of the advanced control theory to attain difficult goals in the process industry, once in the past impossible, can now be achieved by the application of the tested theories of control.

CHAPTER 3

EXPERIMENTAL SET-UP

3.1 Introduction

The chemical plant used in this study was a double effect evaporator using steam to evaporate water. The evaporator was linked to a dual processor computer system. There was an interface for analogue to digital and digital to analogue conversion. This piece of equipment known as HADIOS is controlled through a Honeywell-316 minicomputer. This computer was linked to a Motorola M6800 microcomputer. The measurements taken from the evaporator were 'conditioned' prior to transmission along cables running from the pilot plant room up to the computer room.

3.2 The double Effect Evaporator

The double effect evaporator was manufactured and commissioned by the Kestener Evaporator and Engineering Co. Ltd. The plant was designed for work either as a single or a double effect evaporator, with the additional option of running under vacuum conditions. A detailed description of the engineering construction is provided in Table [3.1]

TABLE 3.1 - PHYSICAL DIMENSIONS OF THE DOUBLE EFFECT

EVAPORATOR HEAT EXCHANGERS

| HEAT EXCHANGER DIMENSIONS | PREHEATER | FIRST EFFECT | SECOND EFFECT | CONDENSER |
|---------------------------------------|-------------------------|--------------------------|--------------------------|--------------------------|
| Exchanger Type | 2:2 | 1:1 | 1:1 | 1:1 |
| Number of Tubes | 4 | 4 | 7 | 6 |
| Tube Material | Copper | Copper | Copper | Copper |
| Tube Length (M) | 1.416 | 2.590 | 1.065 | 2.197 |
| Tube O.D. (MM) | 22.2 | 22.2 | 28.6 | 28.7 |
| Wall Thickness (MM) | 1.65 | 3.20 | 3.20 | 3.30 |
| Mass Tubes (kg) | 4.9102 | 8.1922 | 8.9222 | 1.49496 |
| Tubes Internal Vol. (M ³) | 1.5933.10 ⁻³ | 2.9185.10 ⁻³ | 3.9468.10 ⁻³ | 6.6131.10 ⁻³ |
| Shell Length (M) | 1.397 | 2.590 | 1.065 | 2.197 |
| Wall Thickness (MM) | 3.40 | 4.76 | 6.81 | 4.76 |
| Shell I.D. (MM) | 101.6 | 101.6 | 152.4 | 155.7 |
| Shell Volume (M ³) | 7.7790.10 ⁻³ | 1.69384.10 ⁻² | 1.51004.10 ⁻² | 4.17654.10 ⁻² |
| Mass of Shell (kg) | 22.3038 | 36.4427 | 13.4521 | 37.6069 |
| Shell Material | Mild Steel | Mild Steel | Mild Steel | Mild Steel |
| Heat Transfer Area (M ²) | 0.39555 | 0.72376 | 0.70632 | 1.18341 |

Various operating methods can be used giving further variations on the basic operating methods.

The first effect is of the climbing type and can be used for both organic and inorganic solutions subject to the following manufacturer's limitations:

(a) When dealing with inorganic salt solutions the concentration must not be taken beyond the saturation point.

(b) With all types of liquor the percentage of evaporation should not exceed 50-60% , that is to say, only 50 to 60% of the feed liquor must be evaporated as water.

The second effect is of the forced circulation type and can, therefore, be used for concentration of salt solutions beyond the saturation point. In addition it can be used for concentration of organic and inorganic solutions up to any strength of concentration while avoiding excessive viscosity and boiling point elevation.

When operating as a double effect evaporator the plant can be used as a whole for both salting and non-salting liquors. When handling salting solutions, feed rate and concentrations must be generally adjusted such that saturation is not reached in the first effect unit.

In normal operation, as a salting type evaporator, it is essential that a minimum level of liquor above the top tube plate of the calandria is maintained in the separator. The minimum height depends mainly on whether the evaporator is working under vacuum or non-vacuum.

The circulating pump of the forced-circulation unit maintains a constant circulation of liquor at high velocity through

the calandria tubes, this high velocity ensuring rapid heat-transfer and that the tubes remain clean.

The rate of circulation through the tubes is such that the temperature rise of the liquor in passing through the calandria is limited to only a few degrees. The level of liquor in the separator above the top tube plate imposes a static head on the liquor in the calandria and this, coupled with the limited temperature rise makes it impossible for the liquor to boil inside the tubes. Evaporation does not take place until the hot liquor enters the separator where vapour is formed and released. Avoidance of crystal formation in the calandria tubes is of the utmost importance. Thus it is important to ensure that;

- i) the required working level in the separator is maintained,
- ii) the circulating pump is kept in full operation at the correct speed,
- iii) steam traps and automatic vents should be by-passed during the initial start-up,
- iv) the steam input should be increased gradually to the required level.

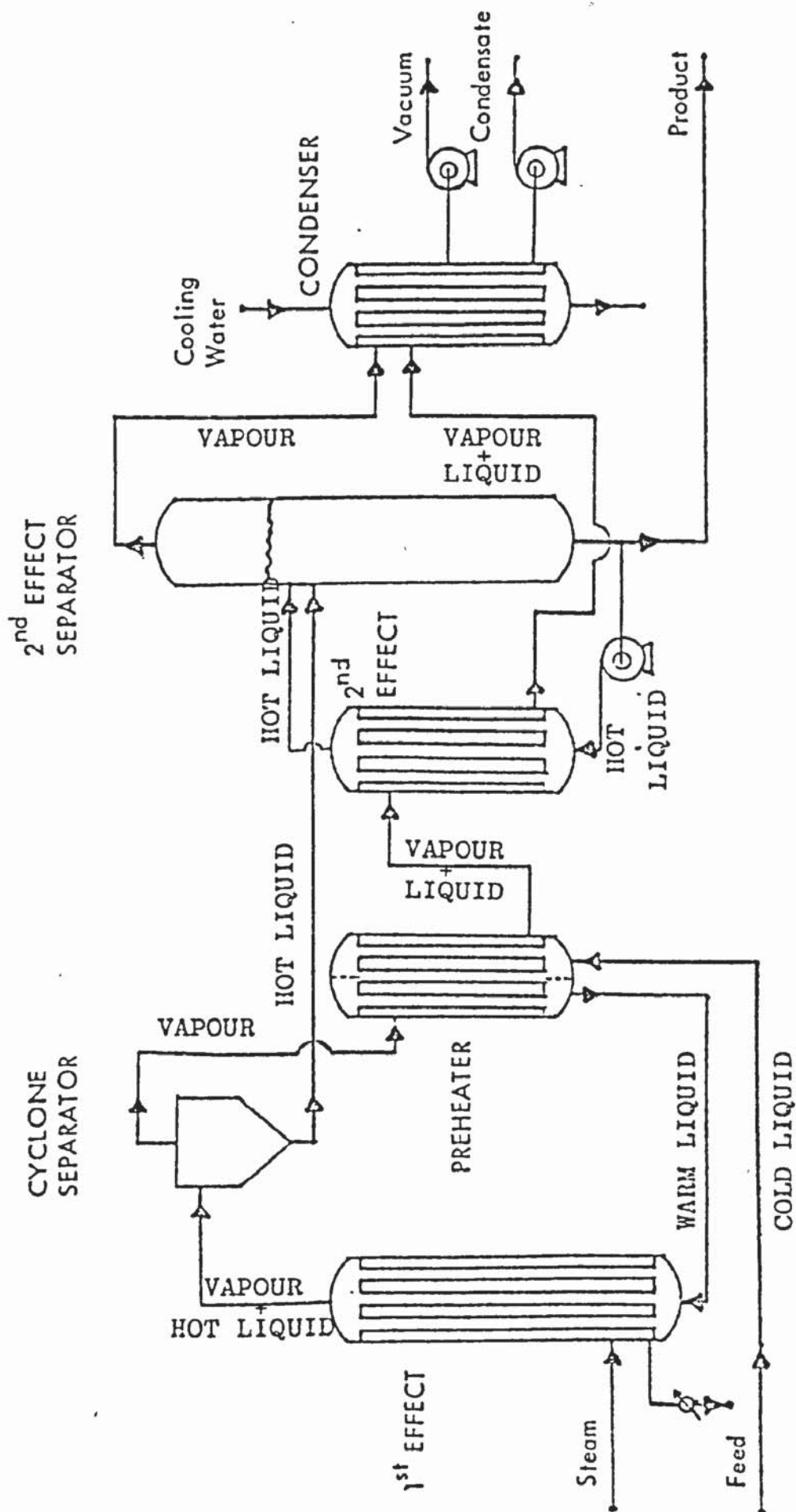
The capacity of the plant when working as a double effect evaporator is about 136.36 Kg/hr of water. The first or second effects whilst working on their own, will be capable of evaporating up to 68.18 Kg/hr of water. The first effect is suitable for working at a steam pressure up to 60 psig.

During this research two pneumatic valves were installed and some other changes made to facilitate automatic control.

3.3 PROCESS DESCRIPTION

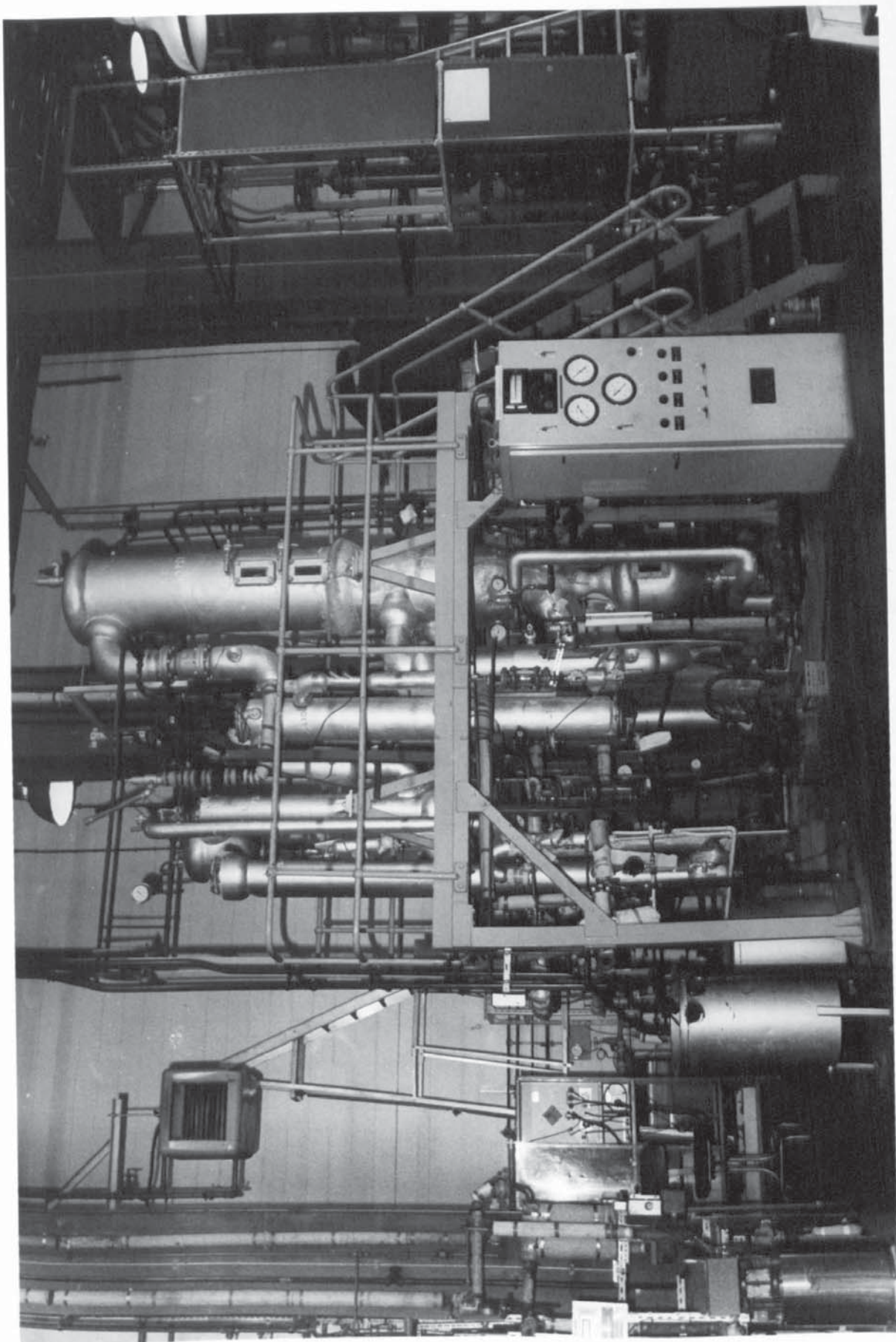
The evaporator process consists of two stages, a climbing film first effect and a forced circulation second effect. A schematic diagram of the evaporator is shown in fig. [3.1], with plates 1 and 2 showing front and rear views of the plant.

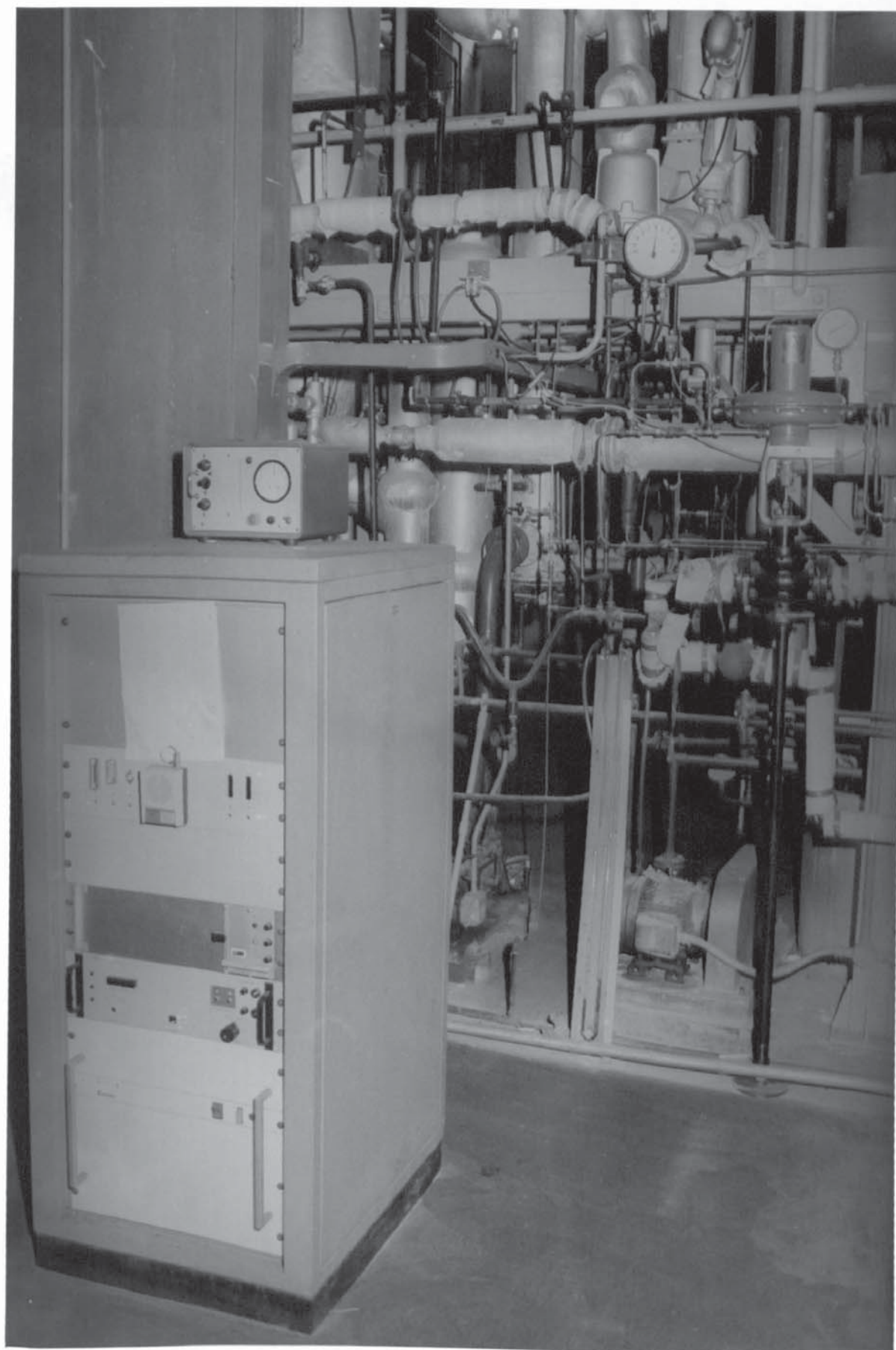
The liquid to be fed to the evaporator is first of all pumped by a centrifugal type pump to two header tanks situated eight meters above the plant. It is then gravity fed to the tube side of the preheater unit where it is heated before passing on to the tube side of the first effect. Heat transfer to the tube contents of the first effect is by condensation of steam onto the tube walls. The steam supply to the first effect shell is from an external steam supply. The evaporation process which takes place inside the tubes of the first effect is rather complex and can be best visualised, by dividing it into three regions. On entering the tubes, liquid is first of all heated to its boiling point, following which there is



Fig[3.1] DOUBLE EFFECT EVAPORATOR

PROCESS DESCRIPTION





a region of nucleate boiling and slug flow. Finally comes the region where the vapour flows up the centre of the tube dragging a liquid film up the tube walls. In this region the film appears to climb up the walls and it is this physical feature which gives the name to this evaporator unit. In the climbing film region evaporation is greatly promoted by the fact that the only phase in contact with the tube walls is the liquid phase. The vapour/liquid mixture leaving the first effect is separated by means of a tangential cyclone separator into a liquid stream which passes directly to the second effect separator and a vapour stream which passes firstly to the shell side of the preheater and then to the second effect shell. Condensation of a proportion of the vapour takes place in both units.

Liquid from the base of the second effect separator is pumped through the tubes of the second effect calandria by a centrifugal type pump. The heated liquid then passes back to the separator. Since the vapour space in the second effect separator is subject to the pull of the vacuum pump, flash boiling occurs as the heated liquid from the tubes of the second effect enters the separator.

Vapour from the second effect separator and vapour/liquid from the second effect calandria shell enters the shellside of the water-cooled, vertical, shell and tube condenser, where all the remaining vapour is then condensed and then drawn off from the base of this unit by a positive displacement rotary pump (mono pump).

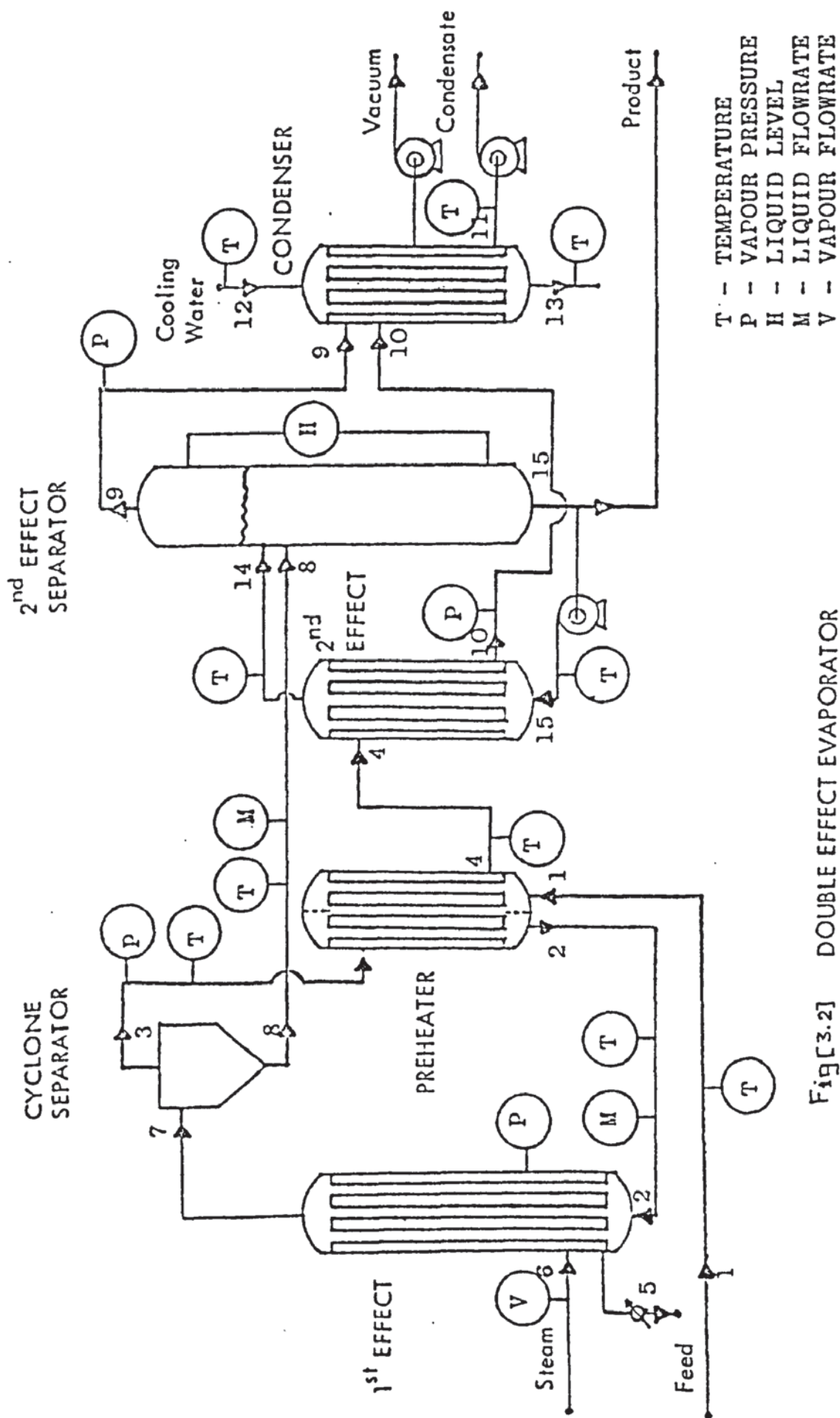
When the evaporator is operated under vacuum conditions, the vacuum pump drains on the shell-side of the condenser.

3.4 Process Instrumentation

On the double effect evaporator eighteen different process variables can be measured by four different types of transducer. With the exception of the turbine flowmeter all of the transducers provide analogue signals in the form of voltages or currents. The output from the turbine flowmeter is in the form of a TTL compatible pulsed voltage, the frequency of which can be monitored by use of the counter input subinterface of the HADIOS data logger. The positioning and function of each transducer is shown schematically in fig. [3.2].

Temperature is measured by NiCr/NiAl thermocouples, using an isothermal (0°C) reference chamber on the cell junction. This isothermal reference chamber is incorporated into the data logger remote cabinet which is situated adjacent to the evaporator.

Pressure measurement, both absolute and differential, is by strain gauge transducer of the Wheatstone bridge type.



PROCESS INSTRUMENTATION

Three different types of flowmeters are used to make on-line measurements of flowrate. The feed flowrate to the first effect tubes is measured by a turbine flowmeter with a rotameter as an off line reading for calibration purposes, whereas flow from the base of the cyclone is measured by an orifice plate connected to a differential pressure/voltage transducer and the steam flowrate to the process is measured by another orifice plate connected to a differential pressure/current transducer. The flowrate of cooling water to the condenser tubes is measured off-line by a rotameter.

3.5 Pneumatic valves

The plant was considered better to be best manipulated by two inputs to the process, steam flowrate and feed flowrate. Other options include steam pressure on the shells, feed input temperature and concentration. Different possibilities were considered of how to implement this control within the limits on the total cost of equipment purchase, and installation.

Two pneumatic valves of different flow range were chosen rather than motorised valves for the following reasons:

- 1- The total cost, with special engineering arrangements on the valves, was in favour of the pneumatic choice.
- 2- The pneumatic valves have faster response than the motorised ones.
- 3- The pneumatic valves were immediately available at the time required.

A point deserving mention here, is that the cost of the interfacing of pneumatic valves with the computer using an electric current to air pressure transducer is high, and if every valve was to be connected using such a unit then the total cost would be high enough to force examinations of other arrangements.

Fortunately, the system responses or time constants of the pneumatic valves involved allowed the use of one transducer unit for electric current to air pressure conversion to supply the two pneumatic valves on a time-sharing basis, as depicted by figure [3.3].

With the help of solenoid valves, mounted on each air pressure line after the current to pressure transducer and adjacent to the pneumatic valve, the air pressure can be addressed to any valve by opening the solenoid valve that controls the air to that valve. The hardware logic for opening the solenoid valves is designed and built in the department and can be directly addressed by the computer data bus. The sixteen bit data link is divided in the usual prescribed manner, 10 bits for digital to analogue data, and four bits as valve address.

The frequency of switching between the solenoid valves can be set as required within certain physical constraints. Mainly, the time taken by the pneumatic valve to settle at any required level is important in adjusting the switching time of the solenoid valves. Too frequent switching may not give enough time for the pneumatic valves to complete their travel. In brief, the time constants of the pneumatic valves are a deciding factor in choosing the switching time for the solenoid valves. At the moment, this switching time is set for less than 3 seconds, i.e., every solenoid

valve will stay open to the air line for 3 seconds, meanwhile other valves are disabled from the pneumatic line and they are left in the final state they have reached.

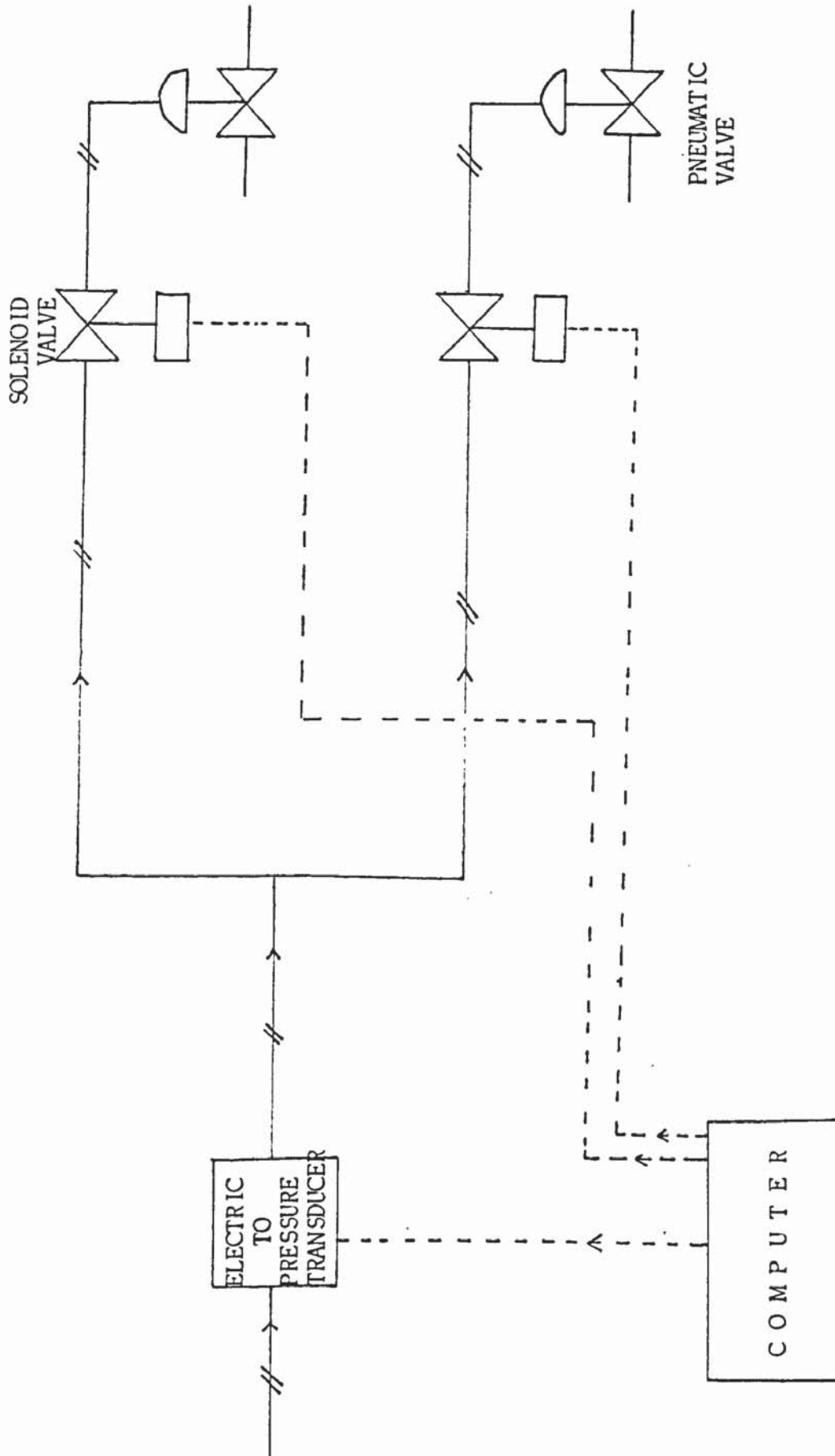


Fig. 3.3: Arrangement of the pneumatic valves

3.6 Computer Data-Network

3.6.1 Introduction

One of the main advantages of linking a digital computer to a chemical process is that the computer is capable of logging and sorting numerous measurements at high speed. Furthermore, the measurements can be processed immediately, so that the results of the computation can be made available to the process end of the system. The nature of the information returned to the process depends upon the sophistication of the on-line hardware and computer programs, ranging from direct digital control to simple printer messages for a process operator.

The two computers, a Honeywell minicomputer and a Motorola 6800 microcomputer system and the plant were connected in a star topology. The organisation of the data flow is shown in figure [3.4]. The system is comprised of the following units:

- 1- A Honeywell 316 digital computer and associated peripherals.
- 2- A Honeywell analogue digital input output system
(HADIOS)
- 3- A Motorola 6800 8-bit microprocessor.
- 4- The plant electronic interface.

3.6.2 THE HONEYWELL MINICOMPUTER SYSTEM

The Honeywell 316 computer was designed for both batch scientific applications and real time on line operations. In the second mode it was very useful during research into the online control of the plant, as well as other communications work. The huge investment of the department in developing various facilities and services for the minicomputer (Webb [1977], Shafii [1983]) has made the minicomputer a good user-friendly environment for purposes of on-line operations.

The Honeywell minicomputer is a 32 K, 16 bit word machine with a memory cycle of 1.6 μ s and I/O handling at a maximum word rate of 9600 baud.

3.6.3 Honeywell Peripheral Equipment

The Honeywell system at the moment supports the following peripherals:

- 1- A high speed paper tape reader operating at 200 character/sec.
- 2- A high speed paper tape punch operating at 75 character/sec.
- 3- A magnetic tape cassette unit, used both for input and output, at a rate of 375 bytes/s.
- 4- An ASR teletype operating at 10 char./sec.
- 5- An AED 3100 P floppy disc storage unit.
- 6- A Tektronix 4010-1 video display unit (VDU).

This VDU is capable of both character and graphic display and in the alphanumeric output mode operates at a rate of 300 baud. It is possible to use the VDU both in the computer room and adjacent to the double effect evaporator. Attached to the VDU is a hardcopy unit, which produces permanent copies of the display if required.

7- More recently a more advanced Newbury 8005 terminal has replaced the Tektronix 4010-1. The Newbury 8005 is a powerful terminal operating in four different modes of operation. It can operate in alphanumeric I/O, vector mode, graphic mode and graphical input mode. Local display and editing functions are provided to enhance page formatting and labelling. The terminal emulates the Tektronix VDU but with a poorer graphical resolution. Besides the Newbury 8005 VDU a desk top dot matrix serial impact printer, Newbury 8501, is linked to the VDU through its own RS232 port. Hard copies of graphics or programs can be obtained through this terminal

3.6.4 The Honeywell Analogue Digital I/O system (HADIOS)

This unit is the interface between the Honeywell digital minicomputer and the outside world. It provides a flexible method of interfacing the H316 computer to a wide range of I/O devices in on-line applications. HADIOS consists basically of a controller, connected to the computer input-output data and control lines, which generates subsidiary data, addresses and controls for up to fifteen different subinterfaces. These subinterfaces can be digital or analogue and input or output. The modular construction of HADIOS allows systems to be simply configured to meet the particular requirements of each application.

The following standard subinterfaces are included in the HADIOS system used in this research:

-i- High level analogue inputs. This interface consists of a single channel analogue to digital converter (ADC), with a maximum conversion rate of 40 kHz, connected to three sixteen channel multiplier subinterfaces. Thus a maximum of 48 separate analogue signals can be processed in turn by the ADC, the input signal (0-5 volts) being converted to a binary integer with ten bit resolution, i.e. 0 to 1023.

-ii- High speed counter input.

This is a subinterface which receives square pulses (0 & 1) and the contents of the counter change with each pulse. The operation of the counter is controlled by the computer software so it is possible to preset the counter to any value between 0 - 225 every cycle of operation. The counter can also operate in an interrupt mode; it interrupts the minicomputer whenever it has reached a preassigned value or it can operate in a free running mode.

-iii- A sixteen bit input port. This port accepts in its 16 lines 'logic' 0 or 'logic 1'.

-iv- A sixteen bit output port. Six bits are used for addressing and the remaining 10 carry data for conversion to an analogue signal.

3.7 The Motorola M6800 Microprocessor System.

This is a microprocessor from the 1970's generation. It is an 8-bit nMOS microprocessor which has, like 8080, become virtually an industry standard type.

Unlike other processors of its day the 6800 had a single power supply rail which simplified systems design. A further advantage was that only 5 chips were needed to form a complete 6800 system, whereas other types such as the 8080 needed many more chips for a working system.

As a system a 6800 is quite simple to use and its set of instructions, whilst very effective, are easy to understand. By using two accumulators a 6800 can achieve quite high throughput and its memory mapped input makes systems design relatively easy. These features have made the 6800 a very popular processor, perhaps second only to the 8080/8085, among 8-bit microprocessor users.

Applications of the 6800 range from the simple system controllers to small general purpose computer systems. The 6800 has become outclassed by some of the later designs, but being well established it is still extremely useful.

Unlike its competitors the 6800 has two 8-bit accumulator registers which give it great versatility when carrying out data handling and arithmetic operations.

Double length (16-bit) arithmetic is relatively easy with the 6800. There is an 8-bit condition code or status register with flag bits for carry, sign, zero, overflow and interrupt mask.

A 16-bit program counter register allows address access to 64k bytes of memory, while a 16-bit stack pointer supports subroutine or interrupt operations and for creating data stacks within the memory. Stacks may be of any length up to 64k and

multiple stacks may be set up by storing the stack pointer contents to memory for those stacks not currently in use. The 6800 has no general purpose registers in the CPU, but it can use any memory location as a general purpose register. There are also many instructions for operating directly on memory data without working through the accumulators.

Input and output are memory mapped so that the 6800 will treat all I/O devices as memory locations. This can make for a very flexible I/O operation, but external logic is needed at the I/O peripherals for decoding the memory address signals. (For full details see M6800 manual)

The basic Motorola M6800 microcomputer family comprised of five parts:

- 1- The M6800 microprocessor unit.
- 2- MC6830 masked programed read only memory (1 kbyte)
- 3- MC6810 Static random access memory (RAM) (128 bytes)
- 4- MC6820 Peripheral interface adapter (PIA) for parallel data I/O.
- 5- MC6850 asynchronous communications interface adapter for serial data I/O.

3.7.2 Peripheral Devices.

The system supports the following devices interfaced to the M6800 through the various interface protocol devices.

- 1- A VDU/TTY unit

2- A Newbury 8510 bidirectional printer connected to the VDU to give hard copy prints at 100 cps.

3- A FD-8 floppy disc memory system comprises a single disk drive and a controller.

3.8 Remote Signal Conditioning.

The various signals produced by the sensors, and transducers are usually not in a form compatible with the computer interface network, HADIOS. A cabinet which contains the additional electronics to condition the signals coming from the plant instruments and the signals sent from the computer to the plant actuator is situated near the pilot plant.

In this cabinet there is a facility to adjust the amplifier of the instrument signals during the calibration of these instruments. The switching time of pneumatic valves which governs the opening and closing of the solenoid valves on the air lines going to the pneumatic valve can also be adjusted from there.

3.9 Conclusion

The systems described are connected together in such a way as to provide for the safe operation of the plant using a variety of control methods in order to test both modelling and control of the plant.



CHAPTER (4)

MATHEMATICAL MODELLING

4.1 Scope:

A study of the mathematical modelling of the pilot plant double effect evaporator has been carried out with the objective of controlling the process by means of a multilevel on-line mini/micro computer.

The process is a very complex one to define mathematically. Many phenomena are taking place and the quantitative contribution of each to the whole process is poorly defined, the problem arises from the difficulty of defining the physiochemical processes themselves.

A distributed-parameter model has been developed with the aim of gaining more insight into the process and to test how far the different assumptions about the process are valid. The parameters of the dynamic process are assumed to be functions of the steady-state conditions, and the parameters were evaluated with a minimum of the plant variable measurements. The model was

linearized and transformed into the complex-plane through Laplace transformations.

4.2 The process physics:

4.2.1 Introduction:

Climbing film evaporators have been used for many years in the chemical industry, but owing to their complexity and the interaction between the heat transfer and the hydrodynamic characteristics occurring in such equipment, systematic and theoretical studies on this subject have left much to be desired.

The mechanisms of convective boiling are quite different from those occurring in pool boiling. These mechanisms are explained for an upward flow in a vertical tube with reference to Fig. [4.1]. The following conditions may be assumed to occur in different parts of the tube, [Coulson,1978], [Hubbard,1960], [Taitel,1976], [Hewitt,1970].

(i) subcooled-single-phase region.

It is assumed here that the liquid enters the tube below saturation and must be heated to saturation before boiling occurs. Thus the tube has a single-phase heating region whose heat transfer coefficient is given by the normal single-phase equations

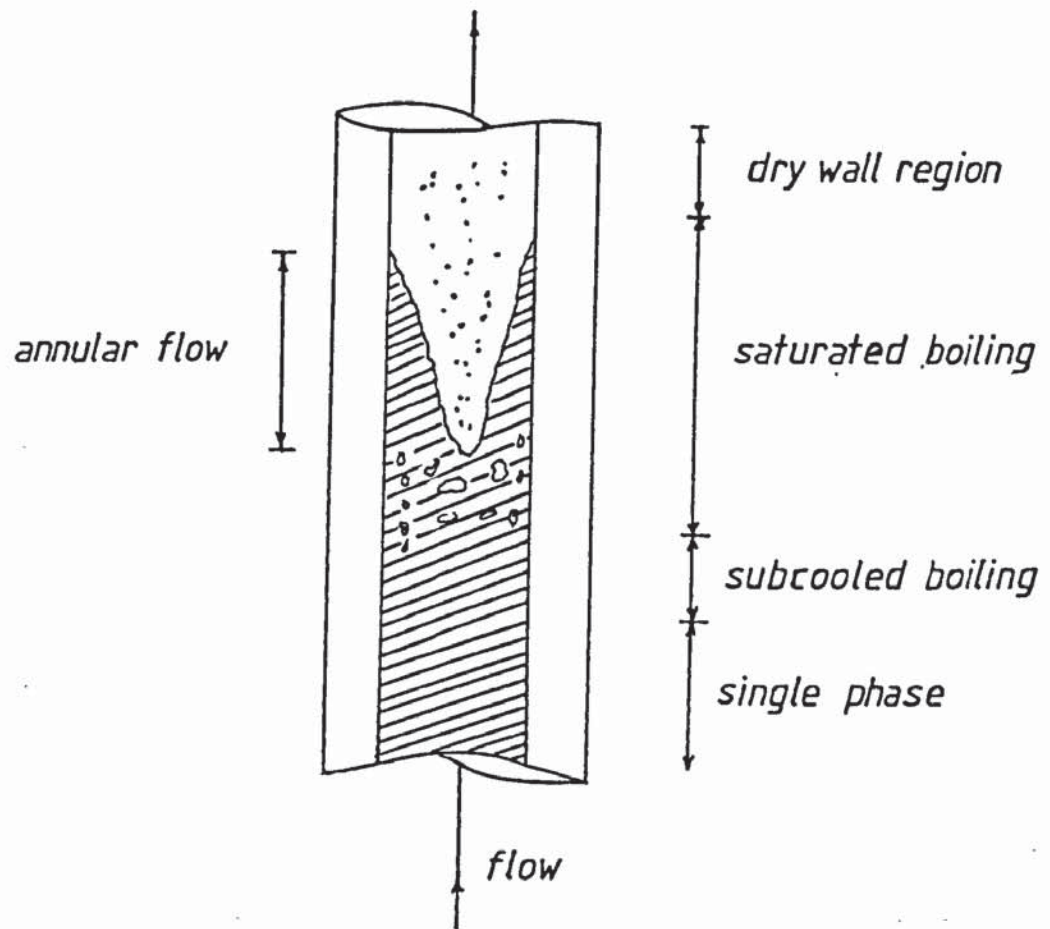


Fig. 4.1: The flow pattern along a climbing film evaporator.

(ii) subcooled-boiling region.

The liquid near the wall exceeds saturation before the bulk of the liquid reaches this condition. When the wall superheat is sufficient to produce bubbles, subcooled nucleate boiling can occur.

(iii) saturated boiling region.

When the bulk of the liquid has been heated to saturation, saturated boiling occurs, accompanied by net generation of vapour. The liquid and vapour flowing together can give rise to a number of different flow patterns. For long tubes, the final flow pattern reached in the saturated boiling region is known as annular flow and is characterized by films of liquid on the wall with droplets of liquid flowing in the high-velocity gas core.

(iv) dry-wall region.

This region occurs after the liquid film has been evaporated. There may still, however, be a considerable quantity of liquid flowing in the form of droplets. The heat-transfer coefficient usually increases along the tube from the subcooled single-phase region [Bliss,1940], [Bonilla,1941], [McMiller,1954], [McAdams,1954], through to the end of the saturated boiling region.

In the dry-wall region, however, the heat transfer drops drastically; since the heat is transferred to the gas phase which has a low thermal conductivity.

Therefore for boiling in a tube, there is a contribution from nucleate boiling arising from bubble formation together with forced convection boiling due to the high velocity liquid-vapour mixture. Such a system is inherently complex since many parameters influence these two basic processes in different ways.

4.2.2 Non Equilibrium State:

Basic Assumptions:

From the practical experience developed from the plant, some general assumptions about the plant behaviour can be made.

- 1) The temperature of the system in the shell side is constant along the tube-wall. This kind of behaviour is observed when pure steam is condensed on the outside of the tubes, when the shell fluid is perfectly mixed as in a continuous-stirred-tank reactor, or when the flow rate on one side of the evaporator is so high that for practical purposes its temperature can be considered as being independent of the distance from the inlet.

- 2) The fluid motion inside the tube is plug flow.
- 3) There is no axial or radial conduction in the fluid.
- 4) The resistance of the tubes wall to the heat transfer is almost zero.
- 5) There is no significant accumulation of energy in the condensate film.
- 6) Also the liquid is assumed to be incompressible.
- 7) Where vapour and liquid phases exist together, the temperature of the mixture is taken to be that of the saturated vapour at the operating pressure.
- 8) The shell volumes are constant i.e. no condensate hold-up
- 9) Separation in the cyclone separator is isothermal and there is no liquid hold-up or condensation at this point in the system.
- 10) The specific heat of the liquid is constant.

In addition to the above assumptions, the thermodynamic correlations shown in Appendix (C) and in program

PARAM2 (Appendix E) are considered to be applicable in the dynamic state.

The assumptions of rectangular fluid velocity, and temperature profiles, should be quite accurate for turbulent flow conditions, (see Doglous [1978]).

Axial conduction in the tube wall should be small since the wall is thin (giving a high thermal resistance) and also since there is no intentional heat sink provided to drain off the axial heat flow. In the fluid, axial conduction is relatively small because of the poor conductivity of most fluids and the existence of the "low resistance" heat flow path offered by the fluid transport "convection term".

It is assumed that there are two distinct flow patterns inside the tube, Fig [4.2]. The first part of the flow is a single phase flow and no formation of vapour takes place. As the liquid passes along the tube more heat energy is transferred from the steam, and at $x = L_1$ the liquid has reached the saturation temperature. Formation of bubbles of vapour starts, and a film of liquid starts to develop along the inside of the tube with a core of vapour dragging up the film of water.

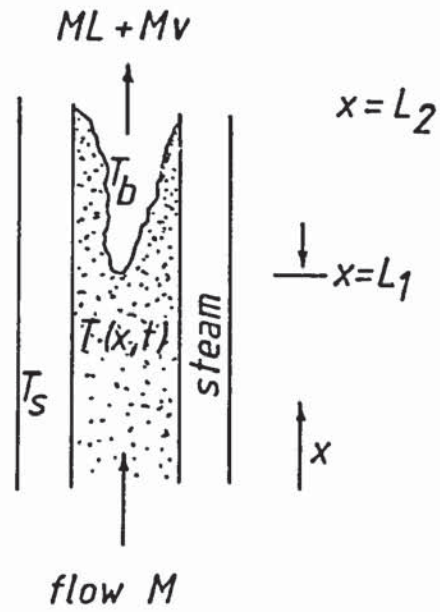


Fig. 4.2: The two main flow regions.

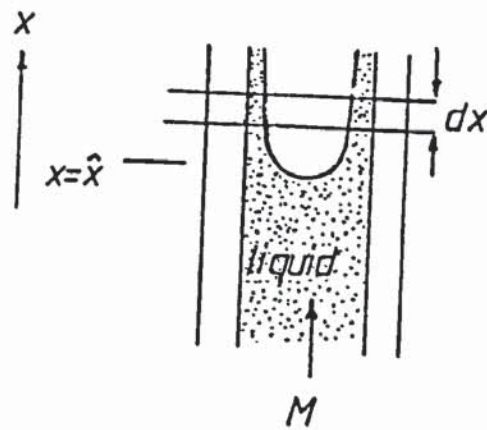


Fig. 4.3: Section of the boiling region.

4.3 LIQUID PHASE REGION

Considering an infinitesimal element of the system of length dx in the range $0 < x \leq L_1$, an energy balance on this element will reveal the behaviour of single phase flow in a differential scale.

Accumulation of energy in the element dx in time dt

$$= \frac{\partial}{\partial t} \left[\frac{c}{\rho} A T(x,t) dx \right]$$

This accumulation is due to the imbalance between the input and output of energy to the element dx .

The difference in energy, between input and output is

$$= \frac{c}{\rho} A v T(x,t) - \frac{c}{\rho} A v T(x+dx,t) + 2\pi r U(T-T_s) dx$$

Since $T(x,t)$ is assumed to be sufficiently regular in time, then

$$\frac{c}{\rho} A v T(x,t) - \frac{c}{\rho} A v T(x+dx,t) = - \frac{\frac{\partial}{\partial t} [c A v T(x,t)]}{\frac{\partial}{\partial x}} dx$$

Evoke the assumptions above, (constant density, constant area and constant heat capacity),

$$\frac{\partial [\rho c_p A T(x,t)]}{\partial t} dx = \rho c_p A \left(\frac{\partial T}{\partial t} \right) dx$$

$$= - \rho c_p A v \left(\frac{\partial T}{\partial x} \right) dx + 2 \pi r U_L dx (T - T_s)$$

Here also it is assumed that, the liquid velocity, v , is a constant with respect to x but it is varying with time. Rearranging the above expression

$$\rho c_p A \left(\frac{\partial T}{\partial t} \right) + \rho c_p A v(t) \left(\frac{\partial T}{\partial x} \right) = 2 \pi r U_L (T - T_s)$$

This is a non-linear, first order hyperbolic differential equation valid in the range $0 < x \leq L$. (The non-linearity is apparent from the term $v(t) (\partial T / \partial x)$).

$$\left(\frac{\partial T}{\partial t} \right) + v(t) \left(\frac{\partial T}{\partial x} \right) = h (T - T_s) \quad (4.1)$$

where $h = \frac{2 \pi r U_L}{\rho c_p A}$

The transient model is obtained from the general equation derived above.

In control applications the interest is, usually, to determine the transfer function that relates changes in the outlet response to changes at the inlet. It is therefore more convenient to split the problem into two parts, a transient solution

and a steady-state solution which is, of course, independent of time. To do this the solution is written as

$$T(x,t) = \bar{T}(x) + \tilde{T}(x,t)$$

and
$$v(x,t) = \bar{v}(x) + \tilde{v}(x,t)$$

$\bar{T}(x)$ and $\bar{v}(x)$ represent the steady-state distribution of temperature and velocity respectively, and are independent of time by definition. $\tilde{T}(x,t)$ and $\tilde{v}(x,t)$ are the deviation or incremental distribution of temperature and velocity respectively, and represent the difference between the total solution and the steady-distribution. $\bar{T}(x)$ and $\bar{v}(x)$ shall be called the initial steady-state distribution so that the deviation distribution has an initial value of zero. Accordingly equation. (4.1) becomes

$$\frac{\partial}{\partial t} [\bar{T}(x) + \tilde{T}(x,t)] + [\bar{v} + \tilde{v}(t)] \frac{\partial}{\partial x} [\bar{T}(x) + \tilde{T}(x,t)] = h(\bar{T}_s + \tilde{T}_s - \bar{T} - \tilde{T})$$

or

$$\frac{\partial \tilde{T}}{\partial t} + \bar{v} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \tilde{T}}{\partial x} + \tilde{v} \frac{\partial \bar{T}}{\partial x} + \tilde{v} \frac{\partial \tilde{T}}{\partial x} = h[(\bar{T}_s - \bar{T}(x))] + h(\tilde{T}_s - \tilde{T}) \quad (4.2)$$

The only non-linear term in equation (4.2) is $\tilde{v}(\partial \tilde{T} / \partial x)$. If it is assumed that the time varying component in each variable is small then their multiplication can be neglected. This will completely linearize the plant model.

The plant can be regarded as linear only if the input signals are sufficiently small. The assumption of time invariance is valid, for example while the plant is run steadily at one operating point.

Equation (4.2) can be separated into steady-state equation and a non-equilibrium one.

$$\bar{v} (\partial \bar{T} / \partial x) = h (\bar{T}_s - \bar{T}) \quad (\text{steady-state equation})$$

The solution of the above equation with boundary condition $T(0) = T_o$ is as follows;

$$\bar{T}(x) = \bar{T}_s + (\bar{T}_o - \bar{T}_s) e^{-kx} \quad 0 < x \leq L1 \quad (4.3)$$

where $k = h / \bar{v}$

The transient part of equation (4.2) is

$$(\partial \tilde{T} / \partial t) + \bar{v} (\partial \tilde{T} / \partial x) + \tilde{v} (\partial \bar{T} / \partial x) = h (\tilde{T}_s - \tilde{T}) \quad (4.4)$$

From the steady-state equation $(\partial \bar{T} / \partial x)$ can be found. The Laplace transform of equation (4.4), after using the initial condition

$\tilde{T}(x, 0^+) = 0$, becomes:

$$s \tilde{T}^*(x, s) + \bar{v} (d \tilde{T}^*(x, s) / dx) + \tilde{v}(s) (d \bar{T} / dx) = h (\tilde{T}_s^* - \tilde{T}^*(x, s)) \quad (4.5)$$

Equation (4.5) is now an ordinary differential equation in x . To determine the total solution a boundary condition is needed. Note that the initial conditions have been used in finding the Laplace expression of the partial differential equation.

The most suitable boundary condition to be used is to evoke assumption (7): at $x = L_1$ the solution is at its saturation temperature, $T_b^* = T^*(L_1, s)$. After imposing this boundary condition in the solution of equation (4.5), (see Appendix A), the following result can be obtained:

$$T^*(x, s) = B1_e \frac{T^*}{s} + B2_e M + B3_e T_o^* \quad (4.6)$$

where,

$$B1_e = kv (1 - e^{-rs}) / (s + kv)$$

$$B2_e = G_{TL} g_{ss} (e^{-rs} - 1) / s$$

$$B3_e = g_{ss} e^{-rs}$$

$$kv = k \cdot \bar{v}, \quad k = 2 \pi r U_L / M c_p$$

$$G_{TL} = k (\bar{T}_s - \bar{T}_o) c_p \bar{v} / M c_p$$

$$g_{ss} = e^{-kL_1}$$

Equation (4.6) contains the following three input

excitations; T_s^* due to the steam input, M due to the perturbation in

the feed flow-rate and T_o^* the inlet temperature of the feed.

4.4 The two phase region:

In this region, vapour generation starts and a liquid film of diminishing thickness starts moving towards the tube outlet. A basic assumption to simplify the dynamic analysis of this region is that the mass transfer is taking place fast compared to the heat transfer mechanism.

Hence it is expected that the following relations may hold :

The total mass flow-rate, $M = M_L(x) + M_v(x)$

where M_L is the liquid-phase mass and M_v is the vapour-phase mass and both are functions of position x .

$$M_L(x) + M_v(x) = M = M_L(x + \Delta x) + M_v(x + \Delta x)$$

$$0 = M_L(x + \Delta x) - M_L(x) + M_v(x + \Delta x) - M_v(x)$$

$$0 = (\partial M_L / \partial x) dx + (\partial M_v / \partial x) dx$$

therefore

$$-\partial M_L / \partial x = + \partial M_v / \partial x = \text{a constant} = \dot{m}_v \quad (4.7)$$

The minus sign in equation (4.7) indicates that the decrease in the liquid phase is met with an increase in the vapour mass. Appendix A shows many other interesting relations taking place under the assumption specified above.

Now an overall heat energy balance for the system can be derived as follows:

Considering the amount of heat accumulated in the tube copper-wall in time dt ; (only varying variables are considered)

$$\int_0^L w' c_{pc} \frac{dT_s}{dt} dx = \int_0^L \tilde{S}_c L_c dx - \int_0^{Ll} [Bl_e (\partial \tilde{T} / \partial t) + Bl'_v (\partial \tilde{T} / \partial x)] dx - \int_{Ll}^L \bar{M}_v L_v dx \quad (4.8)$$

Accumulation on the tube wall = steam heat - heat transfer to the liquid phase - heat transferred to the tube phase

Ll is the total length of the tube,

\tilde{S}_c is the mass rate of steam condensation per unit length.

\bar{M}_v is the mass rate of evaporation per unit length.

w' is the total mass of tube copper.

c_{pc} is the specific heat of copper.

w' is the mass rate of copper per unit length.

Equation (4.8) can be easily integrated with respect to space if the temperature $T(x,t)$ is explicitly known. Usually this distribution of $T(x,t)$ is rather a complicated function of time and space making the direct integration of equation (4.8) an

unattractive approach. But from the physical background of this term, a better alternative to simplify the expression may be found.

Now, consider the basic equation (4.4)

$$dQ = \left(\frac{\partial (c_p A_e T)}{\partial x} \right) dx + \left(\frac{\partial (c_p A_e T)}{\partial t} \right) dt$$

= increment in heat transfer (A)

Consider a general function u such that

$$du = M dx + N dy + p dz$$

$$\text{if } \left(\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \right), \left(\frac{\partial N}{\partial z} = \frac{\partial p}{\partial y} \right), \left(\frac{\partial M}{\partial z} = \frac{\partial p}{\partial x} \right) \quad (B)$$

This is a sufficient and a necessary condition for u to be exact and hence is path independent.

In equation (A), c and p are assumed to be independent in x and t. The application of condition (B) on equation (A) will pose whether

$$\frac{\partial^2 T}{\partial x \partial t} = \frac{\partial^2 T}{\partial t \partial x} \quad (C)$$

Condition (C) is satisfied, i.e. $\frac{\partial^2 T}{\partial x \partial t} = \frac{\partial^2 T}{\partial t \partial x}$

if the following conditions are satisfied by T:

The first and second partial derivatives of $T = T(x, t)$ are continuous at (L_1, t_0) .

Now,

$$\frac{\partial^2 T}{\partial t \partial x} = \frac{\partial^2 T}{\partial x \partial t}$$

The temperature T at the point $(L1, t)$ satisfies condition (B). $L1$ is the point where boiling starts and t is the time.

In conclusion, Q is path independent and only depends on the terminal points.

$$\int_{Q_i}^{Q_o} dQ = Q_f - Q_i$$

therefore

$$\begin{aligned} \int_0^{L1} [B_e (\partial \tilde{T} / \partial t) + B_e \tilde{V} (\partial \bar{T} / \partial x) + B_e \bar{V} (\partial \tilde{T} / \partial x)] dx \\ = Q(L1) - Q(0) \end{aligned}$$

= total heat transferred to the single phase flow

$$= \tilde{M}_p c_b (\bar{T}_o - \bar{T}_b) + \bar{M}_p c_b (\tilde{T}_o - \tilde{T}_b)$$

This is due to the contribution of \tilde{v} and \tilde{T} . After activating the assumptions that m_v , S_c and w_c are uniform and constant with respect to x , equation (4.8) becomes:

$$W_c \frac{d\tilde{T}}{dt} = \tilde{S}_c L - \tilde{M}_c (\bar{T} - \bar{T}) - \bar{M}_c (\tilde{T} - \bar{T}) - \tilde{M}_v L \quad (4.9)$$

where W_c is the total copper mass of the tube,
 \tilde{S}_c is the total steam condensate along the tube
 \tilde{M}_v is the total vapour mass generated.
 L is the latent heat of evaporation.
 L_c is the latent heat of condensation.

Taking the Laplace transform of equation (4.9) and rearranging it

$$S = \tilde{M}_c / L (\bar{T} - \bar{T}) + \bar{M}_c / L (\tilde{T} - \bar{T}) + \tilde{M}_v L / L + W_c \frac{d\tilde{T}}{dt} \quad (4.10)$$

Another useful relation in this analysis can be found by considering the boiling two-phase region again, ($L_1 < x < L$). Evoking the assumption that mass transfer dynamic is very fast, a quasi-steady-state heat balance at that region can be assumed to obey the following formula:

$$\tilde{M}_v L_v = 2 \pi r U_b (L - L_1) (\tilde{T}_s - \tilde{T}_b) \quad (4.11)$$

Also note that \tilde{T}_b is not increasing in this region, the whole heat supplied is to change the liquid into vapour at the saturated temperature.

Taking the Laplace transform of equation (4.11) and rearranging,

$$Mv = (2 \pi r U_b / L_v) (L - L_1) (\tilde{T}_s^* - \tilde{T}_b^*) \quad (4.12)$$

4.5 Steam Chest:

The accumulation of the steam mass in the shell can be obtained by considering the mass balance at the shell side.

$$V_s (d\tilde{\rho}/dt) = \tilde{S}_s - \tilde{S}_c - \tilde{S}_o \quad (4.13)$$

where V_s is the volume of the steam chest

\tilde{S}_s is the perturbation in the steam input flow

rate, and

\tilde{S}_o is varying component of the steam

condensate output to the drainage.

Equations (4.6), (4.10), (4.12), and (4.13) are the main basic equations to represent the dynamics of the climbing-film evaporator. With the help of some other auxiliary equations a total solution for the system equations to different excitations in the plant can be found. The above equations have considered the non-equilibrium state but the steady-state solution is easy to find, (see Appendix A).

S_o is generally affected by the pressure difference across the exit valve of S_o .

$$S_o = c_d (P_s - \bar{P}_a)^{1/2}$$

where c_d is the friction coefficient, P_s is the steam chest pressure

and \bar{P}_a is the external atmospheric pressure. S_o is composed of two

components, the steady-state and the time varying components.

$$S_o = \bar{S}_o + \tilde{S}_o = c_d [(\bar{P}_s - \bar{P}_a) (1 + \tilde{P}_s / (\bar{P}_s - \bar{P}_a))]^{1/2}$$

This is expanded and approximated to the first place by assuming

that $\tilde{P}_s / (\bar{P}_s - \bar{P}_a)$ is small enough for the approximation to be

valid.

$$s_o \approx \bar{s}_o (1 + \tilde{P}_s / 2(\bar{P}_s - \bar{P}_a))$$

After taking the Laplace transform,

$$s_o^* = (\bar{s}_o . P_s^*) / 2(\bar{P}_s - \bar{P}_a) \quad (4.14)$$

In the vapour state, the pressure, P_s , and T_s are usually related by various relations. For the sake of this study these relations are not needed in a complicated form but some simple correlation in the operation range will suffice.

A correlation between pressure and temperature of saturated water vapour is obtained using data reported by Perry [1974].

$$T_s = 71.204 + 2.1 P_s - 0.015 P_s^2 \quad (4.15)$$

where P is in the range

$$10 \leq P_s \leq 50 \quad \text{pressures is in psia,}$$

and T in $^{\circ}\text{C}$.

The British units of pressure have been used here to agree with the units of pressure gauges installed on the plant.

$$\text{and} \quad T = 24 \ln P + 32.87 \quad (4.16)$$

where P is in the range

$$0 < P \leq 10 \quad \text{pressures is in psia and } T \text{ is } ^\circ \text{C.}$$

For the vapour mass density the following correlation is suggested

$$\rho = (41.193 + 37.091 P) 10^{-3} \quad (4.17)$$

P is in psia and

ρ is in Kg/m^3

The Latent heat of steam condensation

$$L_c = 2501.64 - 2.407 T_s \quad \text{KJ/Kg} \quad (4.18)$$

The Latent heat for evaporation

$$L_v = 2501.64 - 2.407 T_b \quad \text{KJ/Kg} \quad (4.19)$$

Now, using the correlation of equation (4.15) in the perturbation form the following relation between the steam pressure and temperature can be found;

$$\tilde{T}_s = 2.1 \tilde{P}_s,$$

and taking the Laplace transform

$$\begin{matrix} * & * \\ T = 2.1 P \end{matrix} \quad (4.20)$$

Similarly, to relate the steam pressure and density in the transient condition it can be found that

$$\rho = 37.091 \times 10^{-3} P \quad (4.21)$$

Substituting equations (4.20), (4.21) into equation (4.13)

$$(a \frac{1}{p} \frac{1}{2.1} s) T = \begin{matrix} * & * & * & * \\ S & - S & - S & S \\ s & s & c & o \end{matrix} \quad (4.22)$$

$$\text{where} \quad a = \frac{V}{p} \times 37.09 \times 10^{-3}$$

By now the main basic equations to represent the dynamics of the climbing film evaporator have been derived. A total solution for the system equations for different input excitation to the plant can be found. To facilitate the algebraic manipulation of all the quantities involved, the signal-flow graph technique is adopted. The advantage of such a representation is that the solution of the equations has a simple interpretation in terms of the structure of the graph. Any problem involving simultaneous linear relationships among a number of variables can be formulated directly as a signal-flow graph and solved directly by inspection of the graph.

Equations (4.6), (4.10), (4.12), (4.13), (4.14), and (4.22) comprise the signal-flow graph in Fig. [4.4].

From the signal-flow graph, Fig [4.4], the combinational transfer functions relating the plant input excitations; steam flow rate, perturbation in infeed flow rate and disturbances in the inlet temperature of the tube, to the climbing film outlet temperature and concentration can be found from the graph by inspection.

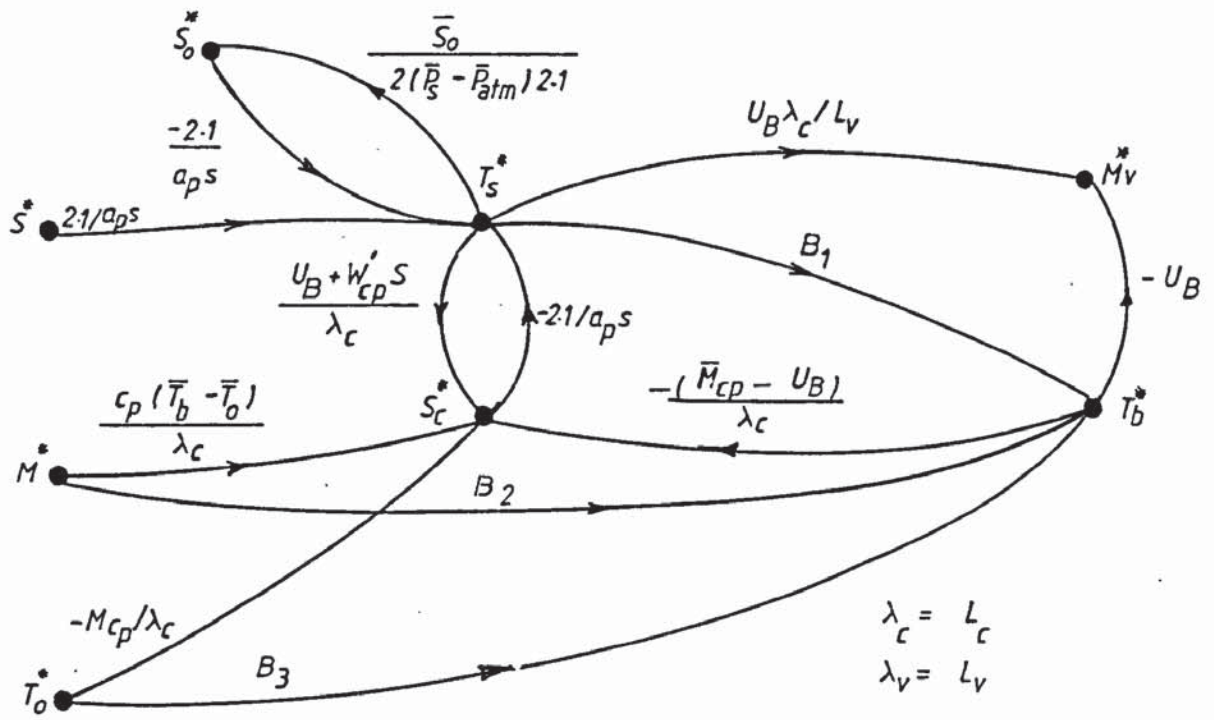


Fig. 4.4: The signal-flow graph.

Now, using Mason's rule:

there are three feedback loops, with the following gains :

$$\text{Loop (1)} \quad L1P = - \frac{\bar{S} \times 2.1}{2 a s (\bar{P} - \bar{P}) 2.1} = - \frac{\bar{S}}{2 a s (\bar{P} - \bar{P})}$$

$$\text{Loop (2)} \quad L2P = - \frac{2.1 (U + W^c c)}{L a s}$$

$$\text{Loop (3)} \quad L3P = \frac{2.1}{a s} B1 \frac{M c + U}{L c}$$

The characteristic equation of the system is

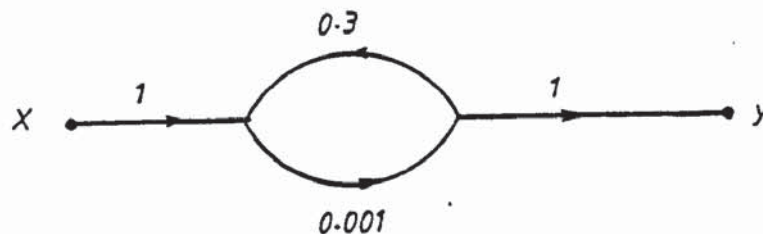
$$= 1 - L1P - L2P - L3P$$

(Note all the loops, L1P, L2P, and L3P are touching each other).

Some simplifications of the signal-flow graph can be useful in obtaining non-complicated transfer functions but taking in consideration that there are no general rules in doing these simplifications. However, what can be suggested is that while the gain of self looping can be neglected if it is comparatively small, neglecting the branch gains may jeopardize the system analysis because the branches usually have direct effect on the system controllability, observability and reachability. Hence care was taken when dealing with the signal-flow graph simplification.

The assumption that the self-loop gains can be neglected if they are small enough, can be justified by the fact that the signal is left to travel through the branch but the output from the low gain loop is then highly attenuated. A trivial simple example is used to demonstrate these effects.

Example :



Applying Mason's rule :

$$\Delta = 1 - 0.3 \times 0.001 = 0.9997$$

$$y/x = 0.001/0.9997 = 0.0010003$$

Neglecting the self-loop gain

$$\Delta = 1$$

$$y/x = 0.001/1 = 0.001$$

On the other hand, if we thought of cancelling the branch gain, 0.001, then the system becomes uncontrollable,

unreachable, and unobservable. Thus there is no relation between x and y or vice versa.

No formal proof has been tried here to find the necessary and sufficient conditions to make these simplifications. But, the experimental results based on the empirical approach above have shown good performance. However, this work can be extended and linked to the problem of interaction decoupling, dynamic dominance, controllability, and observability. Also it can be used to show where the control effort can best be put into the system.

The signal-flow graph keeps the designer in contact with the physical problem, and this gives him more insight into how the system is interacting. For example, analogue computer circuits can be directly built from the signal-flow graph.

Going back to Fig. (4.4), and applying the above simplification it can be found that loop (3), L3P, has a very low gain due to the term

$$\left(\frac{M_c}{p} \frac{1}{c} + \frac{U}{B} \frac{1}{c} \right).$$

Ignore this loop, L3P, and carry out the analysis, (see Appendix A), then

$$\frac{T^*}{b} \frac{1}{S} = \frac{K_v - K_v g \cdot e^{-rs}}{G_{FE} (T_{FE} s + 1) (s + kv)}$$

$$\frac{ML}{S} = \frac{\frac{-U_s - U_{Kv} g e^{-rs}}{B \quad B \quad ss}}{\frac{G_{FE} (T_{FE} s + 1) (s + Kv)}{FE \quad FE}}$$

*
(ML is the -ve of Mv)

$$\frac{T}{M_b} = \frac{\frac{-Kv (\bar{T}_s - \bar{T}_o) g (1 - e^{-rs})}{s \quad o \quad ss}}{SM}$$

$$\frac{c_p (\bar{T}_b - \bar{T}_o) kv (1 - g_{ss} e^{-rs})}{-}$$

$$\frac{L_{cFE} G_{FE} (T_{FE} s + 1) (s + kv)}{c \quad FE \quad FE}$$

$$ML = M - Mv$$

$$\frac{ML}{M} = 1 - \frac{Mv}{M}$$

$$\frac{Mv}{M} = \frac{\frac{-c_p (\bar{T}_b - \bar{T}_o) U (s + kv g_{ss} e^{-rs})}{B \quad o \quad B \quad ss}}{\frac{L_{cFE} G_{FE} (T_{FE} s + 1) (s + kv)}{c \quad FE \quad FE}} - \left[\frac{U_{Kv} (\bar{T}_s - \bar{T}_o) g (e^{-rs} - 1)}{B \quad s \quad o \quad ss} \right]$$

$$\frac{\dot{M}L}{\dot{T}_o} = \frac{U g e^{-rs}}{B s s} - \frac{M c U (s + k v g e^{-rs})}{p B s s} - \frac{L G (T s + 1) (s + k v)}{c F E F E}$$

$$\frac{\dot{T}_b}{\dot{T}_o} = \frac{g e^{-rs}}{s s} + \frac{M c (k v - k v g e^{-rs})}{p s s} + \frac{L G (T s + 1) (s + k v)}{c F E F E}$$

4.6 Modelling of The Preheater and the Second Effect :

By analogous analysis as shown above, similar relations have been derived in Appendix (A) for the preheater, the second effect and the separator.

4.6.1 The Preheater :

The preheater is modelled as an ordinary tube/shell heat exchanger with the main excitation to the process as follows :

- 1) M^* , the perturbation of feed flowrate.
- 2) M_v^* , the vapour mass rate coming and generated from the first effect.
- 3) P_b^* , the vapour pressure of M_v^* .
- 4) T_i^* , the inlet temperature to the preheater tube.

Since there is no evaporation or reaction taking place inside the preheater, the only output considered from the preheater is the outlet temperature T_o^* .

$$T_o^* = B_{lp}^* \times 2.1 \times P_{pb}^* + B_{2p}^* M + B_{3p}^* T_i^*$$

where

$$P_{pb}^* = \frac{\bar{M}_{vP}^*}{2\alpha(\bar{P}_b - \bar{P}_{pb})} - \frac{M_v^*}{\alpha_p(s + T_{pp})} - \frac{c_p(\bar{T}_o - \bar{T}_i)^* M}{L\alpha_c} - \frac{\bar{M}_c(T_o^* - T_i^*)}{\alpha_v L(s + T_{pp})}$$

4.6.2 The Second Effect:

This has a similar operation as the preheater. The input excitations to the process are the flow rate and the pressure of the vapour coming from the preheater shell; the pump circulation flow rate; and the inlet temperature of the liquid to the second effect tube.

$$T_{Eo}^* = B_{sl}^* \times 2.1 \times P_{sE}^* + B_{s2}^* m + B_{s3}^* T_{Ei}^*$$

$$P_{sE}^* = \frac{g_{sl}^* P_{bp}}{(s + T_{sE})} - \frac{g_{s2}^* m}{(s + T_{sE})} - \frac{g_{sl}^* (T_{Eo}^* - T_{Ei}^*)}{(s + T_{sE})}$$

For full details refer to Appendix (A).

4.7 Summary:

A distributed parameter model of the plant which consists of hyperbolic partial-differential equations has been developed to represent the plant behaviour in a linear fashion. After inserting the appropriate assumptions derived from the plant constraints and the physiochemical properties anticipated in the process, clear mathematical relations have been derived.

Heat losses to the external environment have been ignored and hence neglected from the model, since the plant is well insulated.

The heat transfer coefficients are usually a central problem in the study of heat exchangers and evaporators. However, this study is not directly involved in finding what process variables affect the heat transfer coefficient, but, rather concerned with finding suitable means to evaluate the coefficient.

The signal-flow graph technique has been used to solve the set of equations relating the system variables. The technique shows potential capability to simplify the unnecessarily complicated expressions but indeed more theoretical work is needed to explore these capabilities.

CHAPTER (5)

Modern Wiener-Hopf Multivariable Controller

5.1 Introduction:

Optimal state-space feedback control and optimal feedback filtering theory had such a great success when applied to aerospace problems. This has naturally led to attempts to apply these techniques to a wide range of (earth-bound) industrial processes. It soon became clear that they were less than immediately applicable in many such cases. Principally, because the plant models available were not sufficiently accurate, and the performance indices required to stipulate the desired controlled plant behaviour were much less obvious in form than in the aerospace context. Moreover, the controller which resulted from a direct application of state-space optimal control and optimal filtering synthesis techniques was in general a complicated one. In fact, if it incorporated a full Kalman-Bucy filter, it would have a dynamical complexity equal to that of the plant it was controlling. This is because the filter essentially consisted of a plant model with feedback around it.

In the process industry, what is usually available is a restricted amount of data about the plant. Therefore methods of control design which give the designer a direct way to treat these data is more attractive. The sophisticated state-space optimal control method proved difficult to use by industrial engineers brought up on frequency-response ideas who essentially used a mixture of physical insight and straight forward techniques. For these reasons an interest in frequency-response methods slowly began to revive in the multivariable case, [Kalman,1964], [Rosenbrock,1966], [Rosenbrock,1969], [Mayne,1973].

This has led to the development of frequency-response methods in multivariable control system synthesis (opposed to design) methods which avoid complications. The multivariable technique design methods depend heavily on design experience because there are no easy fast general rules to guide the designer in these interactive methods. This has inspired Youla et al [1976] to extend the work of Newton et al [1967] of defining the problem in a rigorous mathematical form, where the exact solution is the answer to the design problem. Usually this rigorous mathematical formulation is formed by a proposition of minimizing a certain control function. The choice of such control functions in the best possible fashion is a difficult task for the designer.

Generally the solution of the optimal problem does not include the physical constraints in the problem. The resultant

controller may be unnecessarily complicated to achieve the objectives of process industry, that is regulation of disturbances, plant noise, and parameters variation.

The aim in developing a synthesis technique, is to formulate a well defined problem with mathematical solution, preferably expressible in terms of a workable and robust computer algorithm. For a design technique the need is different. The experienced engineer requires a set of manipulates and interpretive tools which will enable him to build up, modify, and assess a controller design put together on the basis of physical reasoning within the guidelines laid down by his engineering experience.

Youla, Jabr and Bongiorner [1976], have extended the least-square Wiener-Hopf minimization to the optimal problem by defining an engineering index, to be minimized. This index includes the physical constraints of the problem and gives the designer a direct way to interpret his results on physical grounds in a way usually available also through the frequency response methods.

This work, follows closely the work of Youla et al [1976], but will extend the technique to a wider class of algebraic domains. The work is based on abstract algebraic formulation which will show the generality of the synthesis of modern Wiener-Hopf approach.

Recent years have witnessed a growing cognizance of the intrinsic presence of algebra in systems theory [Youla,1976], [Sain,1975], [Sain,1976], and this recognition has led not just to further understanding of problems already solved but to unforeseen solutions of problems, unsolved by the older, less formal methods.

Preliminaries and Notation:

Throughout this work we let $R[z]$ = a commutative ring with identity and no zeros divisions,

F =quotient field of $R = \{ a/b: a \text{ belongs to } R, b \text{ belongs to } R/o \}$

Since we will be dealing with multi-input/multi-output systems, we introduce the set $R^{n \times m}$, which is the set of $n \times m$ matrices whose elements all belong to R . Similarly, $F^{n \times m}$ denotes the set of $n \times m$ matrices whose elements all belong to F . It is easy to see that both $R^{n \times m}$ and $F^{n \times m}$ are rings under the usual definitions of addition and multiplication, and are noncommutative if $n \geq 2$.

R and C denote the fields of real and complex numbers respectively. Capital letters, A, B, C, \dots denote real matrices. If $p(z)$ is a polynomial, $\deg p :=$ degree of p . Let $K = (K_+ \cup K_-)$ be a disjoint portion of the complex plane, where K_- is

symmetric with respect to the real axis and contains at least one real point. Here \mathcal{H}_- represents the physically stable poles and \mathcal{H}_+ the undesired unstable poles of the complex plane judged by the position of the poles of the transfer functions. Note that \mathcal{H}_- is quite arbitrary and not just the open left half plane, or the open unit circle.

A rational matrix $A(z)$ can be written uniquely as the sum of two strictly proper rational matrices $A_+(z)$ and $A_-(z)$ and a polynomial matrix $A_p(z)$,

$$A(z) = A_+(z) + A_-(z) + A_p(z)$$

where all the poles of $A_+(z)$ and $A_-(z)$ are all within the region \mathcal{H}_+ and \mathcal{H}_- respectively, such factorizations can be done e.g. using partial fraction expansion. A rational matrix $A(z)$ having all its poles within \mathcal{H}_- can then said to be stable with respect to \mathcal{H}_- .

For any set A , $A^{n \times n}$ denotes the class of all $n \times n$ arrays with elements in A . Also define \mathcal{H}_p to denote the class of all proper rational functions with coefficients in R , and $\mathcal{H}_{p,0}$ to

denote the class of all strictly proper rational functions with coefficients in R .

Later in this work, certain polynomials associated with linear transformations play an important role. These are the set of all polynomials in a single indeterminate z with coefficients in R , has the structure of a ring, in particular of a principal ideal domain under the usual rules of polynomial addition and multiplication.

Definition:

Every element P of $\mathcal{H}^{n \times m}$ can be written as BA^{-1}

where B belongs to $\mathcal{H}^{n \times m}$, A belongs to $\mathcal{H}^{m \times m}$ and $\det A \neq 0$; such a pair $\{A, B\}$ is referred to as a left fractional representation of P .

5.2 General Specification:

The actual chemical process unit of the double effect evaporator by its nature is a non-minimum phase, asymptotically unstable process and subject to many disturbances from the measurement transducers, fluctuation in the pressure of the main steam line non-steady input flowrate and the random heat losses from the process.

The process considered is modelled as a set of distributed-parameter hyperbolic partial-differential equations, linearized about a steady-state operating condition.

The control specification requirements are that the output concentration of the product and its temperature are required to be maintained at a specified level irrespective of all disturbances present in the plant. The plant should track control commands of temperature and concentration requirement in a suitable manner. At the same time, the plant should be prevented from being driven beyond its operational and safety limits.

The control system to be designed should be capable of accomplishing, at least, all of the above required performance specifications.

5.3 Wiener-Hopf Controller:

For long, the problem of process control has been to design a controller that deals immediately with the interactions inherent in chemical processes. The disturbance rejection, and noise in the instruments besides the changes in the inputs to the various units operations from process to another present the main problem in chemical processes control. Also, the quality of the

product in its commercial specification and the economy required in achieving this form of final product are important requirements.

Optimal Wiener-hopf controller based on engineering criteria offers a method which deals directly with the interaction problem in chemical processes. The distinctive feature of the method, which is a synthesis method, it deals with input noise, load disturbances and measurement noise specified in terms of spectral densities, and the feedforward compensation can also be considered. The basic idea behind the method is a minimization in an index function into which enter both the spread in the response and the effect of the noise.

The proposed design method, although described for the rational transfer function case, applies to any algebra of transfer functions within which algorithms can be provided for evaluation on a digital computer.

5.3.1 Summary of the Philosophy of Modern Wiener-Hopf Design method:

The work of Wiener on detecting signals buried in noise and the mathematical and physical foundations following that work are well known to merit further description here, but for clarity of the consequent derivation of the modern Wiener-Hopf

design method for controllers the following lines will be useful to highlight the analysis.

Consider the diagram structure in Fig. [5.1].

$$e(t) = i(t) - y(t) \quad (5.1)$$

$$e^2(t) = i^2(t) - 2y(t) i(t) + y^2(t) \quad (5.2)$$

$$y(t) = \int_{-\infty}^{\infty} w(t-r) v(r) dr \quad (5.3)$$

Where $W(s)$ is the unknown transfer function of the control system. It is required that $W(s)$ is chosen such that $e(t)$, the error between the system output and the required ideal output, is minimum. Now squaring equation (5.3):

$$y^2(t) = \left\{ \int_{-\infty}^{\infty} w(t-r_1) v(r_1) dr_1 \right\} \left[\int_{-\infty}^{\infty} w(t-r_2) v(r_2) dr_2 \right] \quad (5.4)$$

Since t is a dummy variable of integration, substituting (5.3) and (5.4) into (5.2):

$$\overline{e}^2 = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T e^2(t) dt \quad (5.5)$$

Putting (5.5) into (5.2) and using

$$h_{vv}(r) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T v(t) v(t+r) dt$$

$$h_{vi}(r) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T v(t) i(t+r) dt$$

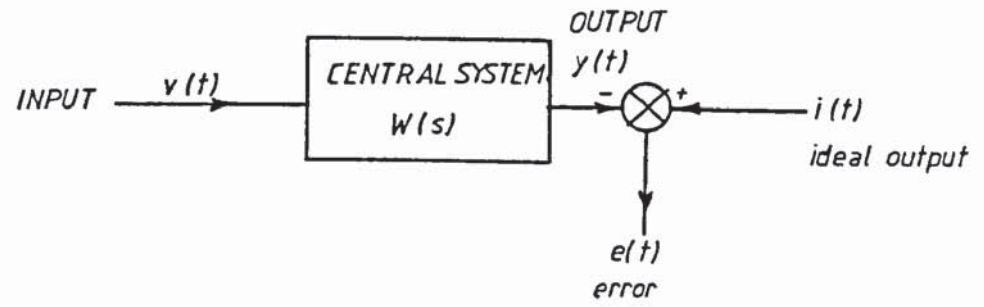


Fig. 5.1: Filter $W(s)$ to minimize the error $e(t)$.

where h_{vv} and h_{vi} are the autocorrelation of the input and cross-correlation of the input and output respectively.

After substitution ,

$$\bar{e}^2(t) = h_{ii}(0) - 2 \int_{-\infty}^{\infty} w(r)_1 h_{vi}(r)_1 dr + \int_{-\infty}^{\infty} w(r)_1 dr \int_{-\infty}^{\infty} w(r)_2 h_{vv}(r-r)_2 dr_2 \quad (5.6)$$

Now $w(r)$ is a weighting function so as to minimize the error, $e(t)$. $w(r)$ should be found to minimize the error mean square.

Now assume that $w(t)$ has a solution

$$w(t) = w_m(t) + kw_k(t) \quad (5.7)$$

where $w_m(t)$ is the optimal solution and k is a parameter which we may vary to test whether $w_m(t)$ is the optimal solution.

Substitute equation (5.7) into (5.6) and solve for the optimum solution with respect to a varying k .

$$d(\bar{e}^2(t)) / dt = 0$$

$$\begin{aligned} &= -2 \int_{-\infty}^{\infty} dr_1 w_k(r)_1 h_{vi}(r)_1 + \int_{-\infty}^{\infty} w_m(r)_1 dr_1 \int_{-\infty}^{\infty} w_k(r)_2 h_{vv}(r-r)_2 dr_2 \\ &+ \int_{-\infty}^{\infty} w_k(r)_1 dr_1 \int_{-\infty}^{\infty} w_m(r)_2 h_{vv}(r-r)_2 dr_2 \end{aligned}$$

$$+ 2 k \int_{-\infty}^{\infty} w_k(r_1) dr_1 \int_{-\infty}^{\infty} w_k(r_2) h_{vv}(r_1 - r_2) dr_2 \quad (5.8)$$

Since the autocorrelation function of stationary signals is even

$$h_{vv}(r_1 - r_2) = h_{vv}(r_2 - r_1)$$

We can interchange the variable and order of the following integration

$$\int_{-\infty}^{\infty} w_m(r_1) dr_1 \int_{-\infty}^{\infty} w_k(r_2) h_{vv}(r_1 - r_2) dr_2 = \int_{-\infty}^{\infty} w_k(r_1) dr_1 \int_{-\infty}^{\infty} w_m(r_2) h_{vv}(r_1 - r_2) dr_2$$

Using equation (5.9) and letting $\frac{d^2 \bar{e}}{dt^2} \Big|_{k=0} = 0$,

equation (5.8) becomes:

$$2 \int_{-\infty}^{\infty} w_k(r_1) dr_1 \left[\int_{-\infty}^{\infty} w_m(r_2) h_{vv}(r_1 - r_2) dr_2 - h_{vi}(r_1) \right] = 0 \quad (5.10)a$$

Since $w_k(r_1)$ is an arbitrary function and it is

zero for $r_1 < 0$, then

$$\int_{-\infty}^{\infty} w_m(r_2) h_{vv}(r_1 - r_2) dr_2 - h_{vi}(r_1) = 0 \quad (5.10)b$$

To show that w_m is the required minimum solution

test for $\frac{d^2 \bar{e}(t)}{dt^2} > 0$

Equation (5.10) contains the solution $w_m(t)$ in an implicit form of an integral equation. This integral equation is known as the Wiener-Hopf equation [Wiener, 1949].

Unfortunately, integral equations are usually somewhat difficult to solve. Wiener [1949] has derived the spectral factorization technique to solve such integral equations, indeed, the technique is a noble mathematical tool.

Youla et al [1976] have applied the same technique to shape the sensitivity function of the control system. As will be shown later in this chapter, the mean square error which is to be minimized in the control system is a function of the sensitivity variable. The optimal sensitivity is defined as the solution for the sensitivity function which minimizes the controller error, and gives the optimal control.

5.3.2 Modern Optimal Wiener-Hopf Controller:

Now, by referring to Fig. [5.2] and assuming P , P_o , F , and F_o belong to \mathcal{H}_p^C , the class of all proper rational functions with coefficients in $\mathcal{H}^{n \times m}$. More specifically $P(z)$ belongs to $\mathcal{H}^{n \times m}$, $F(z^{-1})$ belongs to $\mathcal{H}^{n \times m}$, and $C(z)$ belongs to $\mathcal{H}^{n \times m}$. Hence the multiplication F.P.C spans $\mathcal{H}^{n \times m}$.

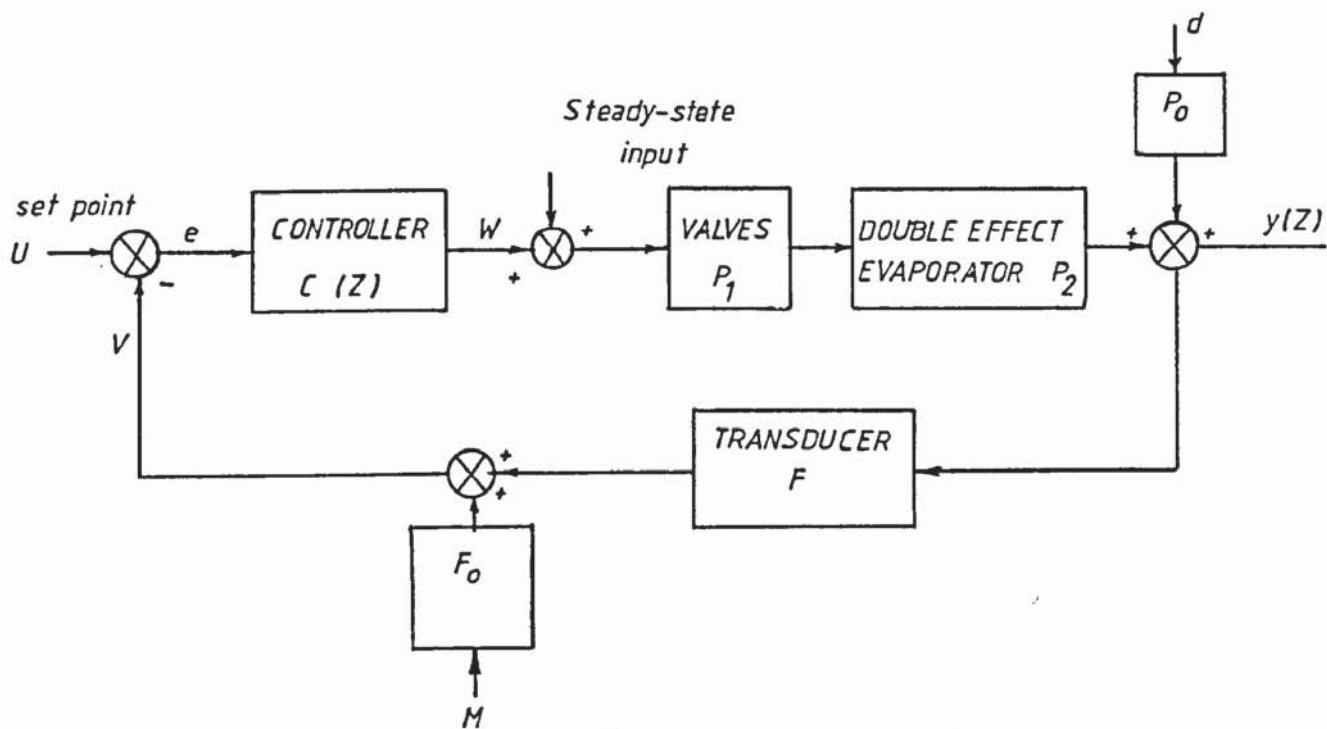


Fig. 5.2: Multivariable feedback control structure.

From Fig [5.2]

$$\bar{Y}(z) = P(z) \bar{W}(z) + P_o(z) \bar{d} \quad (5.11)$$

$$\bar{V}(z) = F(z) \bar{Y}(z) + F_o \bar{M}$$

where \bar{W} , \bar{d} , \bar{Y} , and \bar{M} are the controller vector output, load distribution input, system output vector and the instruments noise vector. We consider input, errors, and output to be functions defined on \mathbb{H}^T (typically $\mathbb{H}^T = \mathbb{R}_+$ for the continuous time case, $\mathbb{H}^T = \mathbb{N}$, set of all natural numbers, for the discrete case).

Straightforward analysis of the equations above yields:

$$\bar{Y}(z) = PR_o(\bar{U} - F_o \bar{M}) + (P_o - PR_o \bar{P}_d) \bar{d} \quad (5.12)$$

$$P_d = F_o P_o$$

$$\bar{W}(z) = R_o(\bar{U} - F_o \bar{M} - P_d \bar{d}) \quad (5.13)$$

$$\bar{e}(z) = (I_n - PR) \bar{U} + PR(F_o \bar{M}) - (P_o - PR F_o P_o) \bar{d} \quad (5.14)$$

where $RI = CS$

$$S = (I_n + F_o P_o C)^{-1} \quad (5.15)$$

and \bar{U} and I_n are the set-point vector, and the identity matrix of dimension n , respectively.

In the absence of load disturbance and measurement noise

$$\begin{aligned}\bar{Y} &= P R I \bar{U} = P C S \bar{U} \\ &= T \bar{U}\end{aligned}$$

where T is the closed-loop transfer matrix and S is the sensitivity matrix.

For unity feedback $F = I$

$$\bar{e} = S (\bar{U} - P \bar{d}) + (I_n - S) F \bar{M} \quad (5.16)$$

Now assume that the error vector composes of two error vectors \bar{e}_1

and \bar{e}_2 such that

$$\bar{e} = \bar{e}_1 + \bar{e}_2 \quad \text{where}$$

$$\bar{e}_1 = S (\bar{U} - P \bar{d}) \quad (5.17)$$

$$\bar{e}_2 = (I_n - S) F \bar{M}$$

It is now clear that \bar{e}_1 is connected directly to

the steady-state error, accuracy and load disturbance and \bar{e}_2 is due

to the contribution of instrument noise. Now for acceptable performance we need to design S and $(I - S)$ to be proper, i.e. in n

the set \mathcal{H}_p , and arbitrarily small in the range of frequencies of interest.

Lemma (1):

If a plant, feedback compensator and controller are free of unstable hidden modes, the closed loop of Fig. [5.2] is asymptotically stable if and only if the polynomial

$$h(z) = \left(\underset{P}{h}(z) \underset{C}{h}(z) \underset{F}{h}(z) \right) / \det(S(z))$$

is a strict Hurwitz polynomial,

where $\underset{A}{h}$ is a monic polynomial

$$\underset{A}{h} = \prod_{i=1}^M (z - z_i)^{d_i}$$

is the characteristic denominator of $A(z)$, and $d(\cdot)$ is the McMillan degree [Youla, 1981]. The following properties of $d(h)$ are practically obvious by inspection:

1) $d(kh) = d(h)$, k an arbitrary non-zero constant

2) $d(\underset{1}{h} + \underset{2}{h}) \leq d(\underset{1}{h}) + d(\underset{2}{h})$

3) If $\underset{1}{h}(z)$ and $\underset{2}{h}(z)$ have no common poles,

$$d(\underset{1}{h} + \underset{2}{h}) = d(\underset{1}{h}) + d(\underset{2}{h})$$

4) $d(\underset{1}{h} \cdot \underset{2}{h}) \leq d(\underset{1}{h}) + d(\underset{2}{h})$

5) $d(h) = d(h)^{-1}$ provided $h(z) \neq 0$

6) $d(h) = 0$ if and only if $h(z) = \text{constant}$

Definition:

The degree, $\bar{d}(h)$, of $h(z)$ equals the totality of its pole, infinity included and counting multiplicity.

Definition:

The degree, $\bar{d}(A; z_o)$, of $z = z_o$ as a pole of $A(z)$ is the largest multiplicity it possesses as a pole of any minor of $A(z)$.

It may be still impossible to find a controller $C(z)$ to stabilize the closed-loop. Such is the case if and only if for some finite $z = z_o$, z_o belongs to \mathcal{H}^+

$$\bar{d}(F P; z_o) < \bar{d}(P; z_o) + \bar{d}(F; z_o)$$

Definition:

The plant $P(z)$ and feedback compensator $F(z)$ form an admissible pair if each is individually free of unstable hidden modes and

$$\begin{matrix} + & + & + \\ h & = & h & (z) & h & (z) \\ PF & & F & & P \end{matrix}$$

where $h = \begin{matrix} - & + \\ h & h \end{matrix}$

where \bar{h} contains the zeros in \mathcal{H}^- and h the zeros in \mathcal{H}^+ .

Lemma (2): (for proof see Youla [1976])

There exists a controller stabilizing the given plant and feedback compensator in the closed loop configuration of Fig. [5.2] if and only if the pair is admissible, FP .

Now let the spectral densities of \bar{U} , \bar{d} , and \bar{M} be denoted by G_u , G_d , and G_M , respectively.

Define the following performance indices:

$$2\pi j E_t = \text{Tr} \int_{-j\infty}^{j\infty} \langle \bar{e}^{-1}(z) \bar{e}(z) \rangle dz \quad (5.18)a$$

This is the usual quadratic measure of steady-state response,

Similarly, if $P_s(z)$ represents the transfer matrix

coupling the plant input $\bar{r}(z)$ to those "sensitive" plant modes which must be specially guarded against excessive dynamic excursion,

$$2\pi j E_s = \int_{-j\infty}^{j\infty} \langle P_s(z) \bar{r}(z) \cdot \bar{r}^*(z) P_s^*(z) \rangle dz \quad (5.18)b$$

This is a sensible practical consideration to prevent the actuators to be driven into saturation or in other words to prevent making high demand input energy that is exceeding the capability of the input valves [Newton, et al, 1957].

Hence,

$$E = E_t + k E_s \quad (5.19)$$

k is Lagrangian positive multiplier, serves as a weighted cost combining both factors. The optimal controller is designed such that to minimize E .

From the consideration of the two performance indices and after substituting equations (5.14) and (5.5) we can see that the only variable we have freedom of design is R_1 , because the other parameters are of type "fixed elements" or are fixed by the plant given. Thus, the following lemma plays an obvious and indispensable role.

Lemma (3):

Let the given plant and feedback compensator form an admissible pair with transfer matrix description $P(z)$, $F(z)$.

$$\text{Let } F(z)P(z) = \begin{matrix} -1 \\ A(z)B(z) \end{matrix} = \begin{matrix} -1 \\ B_1(z)A_1(z) \end{matrix} \quad (5.20)$$

where the pairs A, B and B_1, A_1 constitute any left-right coprime polynomial decomposition of $F(z)P(z)$, respectively, then there exists matrices $X(z)$ and $Y(z)$ such that

$$A X + B Y = I_n \quad (5.21)$$

and this is the famous Bezout identity. Hence ,

1) The closed loop of Fig [5.2] is asymptotically stable if, and only if

$$R_1 = H.A \quad (5.22)$$

$$H = Y + A_1 K \quad (5.23)$$

where $K(z)$ has coefficients in $F^{m \times n}$ and is analytic in \mathcal{H}_+

K also should satisfy the constraint

$$\det [X - B_1 K] \neq 0 \quad (5.24)$$

2) The stability controller C , resulted from choice of admissible K , possesses the transfer matrix

$$C = (Y + A_1 K) (X - B_1 K)^{-1} \quad (5.25)$$

Another useful relation, from $AB_1 = BA_1$, is that

the Bezout identity becomes:

$$A_1(X - B_1 K) + B_1(Y + A_1 K) = I_n \quad (5.26)$$

In view of this lemma, the natural way to attack the problem of minimizing E is to vary over all $(m \times n)$ real rational matrices $K(z)$ analytic in \mathbb{C}_+ and which satisfies restriction (5.24).

Now, use of the standard variational argument [Weston, et al, 1972] examining the increment in E produced by the perturbation δR_1 leads directly to the Wiener-Hopf equation.

Since

$$\begin{aligned} R_1 &= (Y + A_1 K) A_1, \\ \delta H &= A_1 (\delta K) A_1 \end{aligned} \quad (5.27)$$

then the Wiener-Hopf equation is as follows

$$\bar{\Phi} - A_{1*} (P_{**} + KQ) A_1 K (A G A_1)^* = \Delta \quad (5.28)$$

where

$$\bar{\Phi} = A_{1*} P_{**} (G_u + P_{od} G_{pd}) A_1^* - A_{1*} (P_{**} + k Q) Y (A G A_1)^*$$

and Δ is analytic in \mathcal{H}_+ .

Youla et al [1976] have defined the physical assumptions underlying the entire optimization scheme and have proved that if Φ possesses a real rational matrix solution K analytic in \mathcal{H}_+ which satisfies (5.24) and has a finite associated cost E , then this K is optimal. From there, the optimal design can be carried out in the following manner:

1) Construct two square matrices; V, N , with coefficient in \mathbb{R} and analytic together with their inverses in \mathcal{H}_+ such that

$$A_{l^*} (P_{l^*} + K Q_l) A_l^* = V_{l^*} V_l^* \quad (5.29)$$

$$Q_l = P_{s^*} P_s \quad (5.30)$$

and $A G_{l^*} A_l^* = N_{l^*} N_l^* \quad (5.31)$

where $G = G_m + P_d G_d P_{d^*} + G_{mL} \quad (5.32)$

and $G_{mL} = F_o G_m F_{o^*} \quad (5.33)$

where G_u , G_d , and G_m are the spectral densities of \bar{U} , \bar{d} , and \bar{M} ,

respectively. \bar{U} , \bar{d} , and \bar{M} are independent processes, stochastic and are either zero-mean second-order stationary or shape-deterministic or a sum of both with rational spectral densities.

2) Let
$$J = A_{l*}^T P (G_u + P G_d P) A_{o*}^T$$

and choose any two real polynomial matrices X and Y such that

$$A X + B Y = I_n$$

3) The transfer matrix of the optimal controller is given by

$$C = (Y + A_1 K) (X - B_1 K)^{-1} \quad (5.34)$$

where

$$K = V^{-1} \left(\{V_{*}^{-1} J N_{*}^{-1}\} + \{V_{*}^{-1} A_1^{-1} Y N_{*}^{-1}\} \right) N_{*}^{-1} - A_1^{-1} Y \quad (5.35)$$

or more suitably;

$$C = H_o (A_o N - F P H_o)^{-1} \quad (5.36)$$

where

$$H_o = A_o V_l^{-1} \left(\{V_*^{-1} J N_*^{-1}\}_+ + \{V_l^{-1} Y N\}_- \right) \quad (5.37)$$

where $\{ \}_+$ and $\{ \}_-$ indicate the polynomial part of the Laurent expansion associated with all the poles in \mathcal{C}_- and \mathcal{C}_+ , respectively.

Two useful and important corollaries may follow as:

Corollary 1:

1) Suppose F.P is analytic in \mathcal{C}_+ then

$$C_o = H_o (N_o - F P H_o)^{-1} \quad (5.38)$$

$$H_o = V_r^{-1} \{V_{r^*}^{-1} J_r N_{r^*}^{-1}\}_+ \quad (5.39)$$

where

$$V_{r^*} V_r = (P_* P + k Q) \quad (5.40)$$

$$G = \begin{matrix} N & N \\ r & r^* \end{matrix} \quad (5.41)$$

$$J = \begin{matrix} P & (G + P & G & P) \\ r & * & u & o & d & * \end{matrix} \quad (5.42)$$

V and N are square, real with coefficient in $R^{n \times m}$ and analytic together with their inverses in \mathbb{C} (V_r, N_r, J_r are reduced quantities).

Proof:

The analyticity of F.P in \mathbb{C} implies

$$\det A = \det A_1 = h_{FP} = 1$$

Thus $A_r N_r = N_r, V_r A_r = V_r$

$$\{V A^{-1} Y N\} = \emptyset$$

and the rest follows by direct substitution, Q.E.D.

Under the conditions of the corollary, the feedback sensors and the plant are asymptotically stable and the resulting simplification, as evidenced in formulae (5.38) - (5.42), is striking. Note in particular that the polynomial factors A, A_1 and Y are no longer needed.

Corollary (2):

If $P(z)$ is square and analytic together with its inverse in \mathbb{C} , $F = I$ (unity feedback), let $k = \emptyset$ (no saturation constraint), G and $G_u + P G_o d o$ are diagonal matrices, then the optimal controller $C(z)$ satisfies the non-interaction condition

$$P.C = \text{diagonal matrix}$$

Proof:

Clearly, from equations (5.38) - (5.42) and the stipulated assumptions, it follows that $V_r = P$, and N is diagonal and

$$J_r = (G_u + P G_o d o^*) V_r$$

Thus,

$$V_{r^*}^{-1} J_{r^*}^{-1} N^* = (G_u + P G_o d o^*) N_{r^*}^{-1},$$

$$P H_o = V_r H_o = \{ (G_u + P G_o d o^*) N_{r^*}^{-1} \}^+$$

and
$$P.C = P H_o (N_r - P H_o)^{-1}$$

is also diagonal, Q.E.D.

Comments:

The task of the optimal controller is to optimally reset $y(t)$ to a new level dictated by $u(t)$. This must be accomplished despite the presence of load disturbance, measurement noise, non-ideal sensor dynamics, and zero steady-state error requirements.

The performance indices chosen as the control candidate are of immediate engineering value and give the designer direct access to the problem through shaping the sensitivity or the inverse of the return-difference matrix S .

The specification of the input vectors in their statistical form, indeed, is an advantageous feature over other methods, since in the process industry it is more realistic to find non-deterministic input functions. The part of the cost that reflects loop accuracy has been input directly in $u(t) - y(t)$ and does not necessarily involve all the state variables. This is a real hindrance in most of the state-space approaches where the optimal solution necessitate the construction of observers or Kalman filters to provide the measurements. In the Wiener-Hopf design method, nevertheless, these other variables are kept within bounds by an appropriate weighting of the saturation constraints, E_s . This in fact means that it is no longer necessary for the system to be complete state observable.

Although in the analysis the controller guarantees closed-loop stability in advance, yet, the system performance in the time domain is not predicted. However, there are some suboptimal controllers which exist and give satisfactory time-domain performance, if the optimal one does not function satisfactorily.

By and large, the design method does not leave the designer in the usual difficult position of trial-and-error. It is truly a synthesis method, but the computational burden is quite formidable. However, the recent introduction of APA [Nawari,1983] has greatly simplified the computational problem of polynomial matrix manipulation. The design method is now fully automated.

It is quite appropriate to mention in these comments the important work of Rosenbrock [1970], [1973], [1974] and Kalman [1965] in the theory of linear systems much of the above algebra has emerged from their pioneering work. The so-called Smith McMillan form was used by Rosenbrock in his treatment of multivariable zeros. He gave a particular comprehensive treatment of the multivariable zero problem as a part of his important and pioneering work on an algebraic theory of linear dynamical systems. In his work he made a systematic use of a particular polynomial matrix representation, the system matrix equivalence.

These studies by Kalman and Rosenbrock showed the great power and relevance of algebraic theories for fundamental studies of multivariable control systems.

Finally, the work of Youla [1961] in finding the conditions for the coexistence of spectral factorization solution to real polynomial matrices and the algorithms to find the solution have made the Modern-Wiener Hopf design method conceivable.

The application of the above theorems and ideas to the double effect evaporator will be shown in Chapters (8) and (9). However, the results there showed further work was needed. The following Chapter shows part of that research.

CHAPTER (6)

MULTIVARIABLE CLOSED-LOOP POLES ALLOCATION AND
MINIMAL SENSITIVITY : A Synthesis Approach

6.1 Introduction

The modern Wiener-Hopf controller with its powerful features for tackling the distinctive process control problem provides the fundamental ground to reach a solution. The complexity of the computation may, however, make it an unattractive option without powerful off-line computational facilities. There is also a limitation of the plant matrix must not contain poles on the infinite frequency axis. This last limitation has been found in this study during the application of multivariable Wiener-Hopf controllers to the double effect evaporator.

Another control synthesis method which also immediately deals with the process control problems has emerged from this study. The controller has powerful engineering features and there is less computational requirement than in the Wiener-Hopf controller.

6.1.1 Abstract :

A least-square optimal controller which guarantees stability in advance for a given plant is presented. There are no restrictions on the plant matrix; which may be non-minimum phase, stable, unstable, or with an infinite number of zeros. The controller has the distinctive unique feature that the closed-loop poles or the characteristic equation of the closed loop of the multivariable system can be specified in advance to achieve a certain sensitivity function.

The controller is capable of rejecting stochastic as well as deterministic disturbances and has satisfactory tracking performance. The method is particularly applicable when the modern Wiener-Hopf design fails, especially if the plant gives rise to a non positive-definite matrix.

The method has engineering features of immediate appeal. The designer can specify the poles of the closed-loop system in advance and has the freedom to shape the sensitivity of the system for any required range of frequencies desired. Also, the method makes it possible that the zeros of the closed-loop system can be specified in advance through choosing a suitable form of Smith-McMillan closed loop matrix [Rosenbrock,1973]. In this last case the freedom in shaping the sensitivity may not be attainable.

The design method is generally developed for Multi-Input/Multi-Output (MIMO) systems; but trivially, it is directly applicable to Single-Input/Single-Output (SISO) systems. In this case a wider range of options are possible, increasing the designer

scope of choice and degree of freedom. Each choice, indeed, has a consequence on the final system behavior.

Software packages, using the library APA, have been developed to handle the synthesis method. The method is fully developed in computer algorithm with graphic simulation facilities.

6.2 Synthesis of The Controller

Now, consider the construction of a feedback multivariable system shown in Fig. [6.1].

Here C , P and F are polynomial matrices, with coefficients in $R^{m,n}$, $R^{n,m}$, $R^{m,n}$, respectively. It will be assumed that P and F are known and given.

Proposition (1)

Given a plant matrix $P(z)$, and a feedback sensor matrix $F(z)$ then there exists a controller matrix $C(z)$ such that

$PC(I+PC)^{-1}$ has the following properties :

- (i) $PC(I+PC)^{-1}$ is stable,
- (ii) $PC(I+PC)^{-1}$ has any required characteristic equation,
- (iii) $(I+PC)^{-1}$ is minimum for any range of operating frequencies, (minimum in least-square sense).

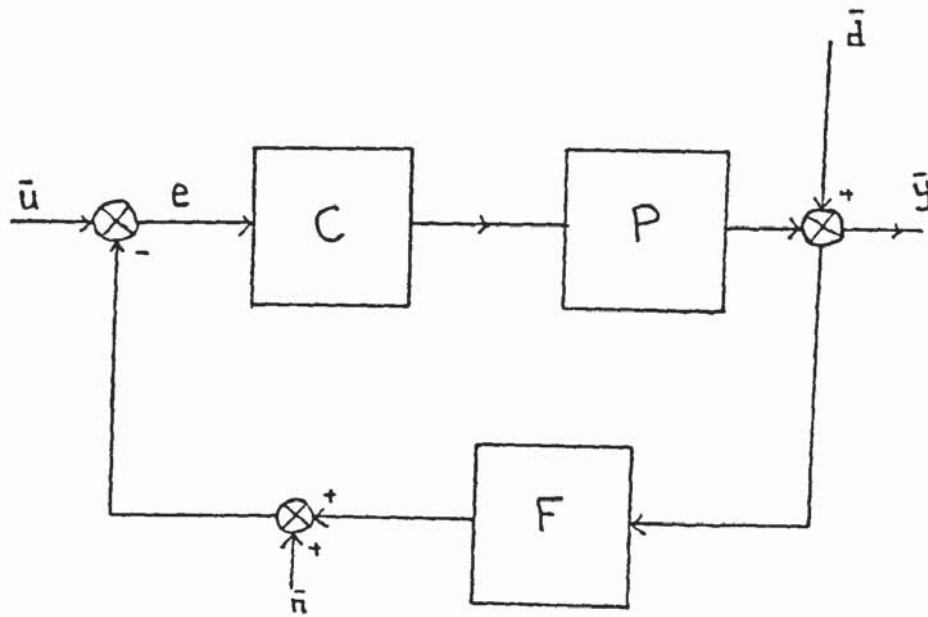


Fig. 6.1: Multivariable feedback control structure.

Proposition (2)

For any given F , and P there exists a polynomial matrix $K(z)$ such that $\det K$ is the system closed-loop characteristic equation.

$$\text{If } K = \begin{bmatrix} K_1 & K_2 \\ K_1^{-1} & K_2^{-1} \end{bmatrix},$$

(for the moment neglect the question of coprimeness of K_1 and K_2).

then,

- (i) $\det K_1$ is the system closed-loop characteristic equation ,
- (ii) K_2 determines the closed-loop system sensitivity matrix and the system closed-loop zeros, and
- (iii) The controller C is given by

$$C = (Y + AK_1)(X - BK_1)^{-1}$$

where $\{A_1, B_1\}$ are the right coprime matrix fraction description of FP . X and Y satisfy the Bezout Identity,

$$AX + BY = I$$

6.2.1 Analysis

For any given FP there exists a matrix factorization such that

$$\begin{aligned} FP &= A \begin{matrix} -1 \\ B \end{matrix} \\ &= B_1(A_1) \end{aligned}$$

where A and B are left coprime, and B_1, A_1 are right coprime matrices. It follows that there exist polynomial matrices X and Y such that

$$AX + BY = I$$

The coprimeness of A and B guarantees the existence of such X and Y . This is known as the Bezout Identity.

There exists a polynomial matrix K such that the above closed-loop system can be made stable. The poles of the closed system can be chosen arbitrarily with any degree of multiplicity. However, these closed-loop poles must be such that the limits of the plant input actuators are not exceeded, otherwise saturation at the plant input may occur. That gives rise to non-linear elements in the closed-loop system. Such non-linear elements can jeopardize the design and ultimately the system behavior. The basic assumption underlying this synthesis method is that the system is linear.

The controller C is assumed to have Youla's modern Wiener-Hopf controller structure,

$$C = (Y + AK) (X - BK)^{-1} \quad (6.3)$$

The output vector \bar{y} of the closed-loop system of fig. [6.1] is

$$\bar{y} = PC (I + FPC)^{-1} \bar{u}$$

$$(I + FPC) = (X + PY) (X - BK)^{-1} \quad (6.1)$$

Therefore,

$$\det (I + FPC) = \det [(X + A^{-1} BY) (X - BK)^{-1}]$$

from equation (6.1)

$$(I + FPC) = (X + A^{-1} BY) (X - BK)^{-1}$$

$$A (I + FPC) = (AX + BY) (X - BK)^{-1}$$

Applying the Bezout identity ,

$$AX + BY = I$$

therefore,

$$A (I + FPC) = (X - BK)^{-1}$$

and

$$(I + FPC) = A^{-1} (X - BK)^{-1}$$

$$= S^{-1} \quad (6.2)$$

where $(I + FPC)$ is the return-difference matrix or the inverse of the sensitivity matrix of the system.

Then,

$$\det (I + FPC) = (\det A)^{-1} \cdot \det (X - BK)^{-1} \quad (6.4)$$

In general, the stability of the closed-loop system is absolutely essential. In principle, the closed-loop stability can be investigated using the closed-loop characteristic polynomial. More precisely, defining the matrix return-difference

$$S^{-1} = (I + FPC)$$

it can be shown that

$$\frac{\text{closed-loop characteristic equation (C.L.C.E)}}{\text{open-loop characteristic equation (O.L.C.E)}} = \det(I+FPC)$$

[Hsu,1968], [McMorran,1970], [Popov,1964].

This means that the ratio of the closed-loop characteristic polynomial to the open-loop characteristic polynomial is identically equal to the return-difference determinant. This result is a natural generalization of the classical formula, forming the basis of the well known Nyquist and root-locus analysis and design techniques. This is also true in the multivariable case.

Another useful and important identity is the following

$$\det \begin{pmatrix} I_m + P_{12} P_2 \end{pmatrix} = \det \begin{pmatrix} I_n + P_2 P_{21} \end{pmatrix}$$

valid for any $m \times n$ matrix P_1 and $n \times m$ matrix P_2 . Making use of important relation of the det of the return-difference matrix and

the open and closed loop characteristic equations, we have from equation (6.4)

$$\det (I + FPC) = [\det A]^{-1} [\det (X - BK)]^{-1}$$

Making use of the relation

$$\det A^{-1} = 1 / \det A$$

$$\frac{\text{C.L.C.E}}{\text{O.L.C.E}} = \frac{\det (X - BK)^{-1}}{\det A}$$

$$= \det (I + FPC)$$

Now, factorize K into K_2 and K_1 such that ;

$$K = K_2 K_1^{-1}$$

neglecting for the moment the question of coprimeness of K_1 and K_2 .

$$\begin{aligned} \text{Then } (X - BK) &= (X - BK_2 K_1^{-1}) \\ &= K_1 (XK_1^{-1} - BK_2) \end{aligned}$$

therefore,

$$\det (X - BK)^{-1} = (\det K_1) \cdot \det (XK_1^{-1} - BK_2)^{-1}$$

$$\frac{\text{C.L.C.E}}{\text{O.L.C.E}} = \frac{\det K_1 \cdot \det (XK_1 - BK_2)^{-1}}{\det A}$$

$$= \frac{\det K_1}{\det A \cdot \det (XK_1 - BK_2)} \quad (6.5)$$

Relation (6.5) is a fundamental and an important relation. It is valid for any linear time-invariant system single variable or multivariable.

In the case of a single variable system $\det (K)_1$ is the actual scalar polynomial K . For if $P(z)$ is 1×1 matrix then $\det P = P$.

$$\frac{\text{C.L.C.E}}{\text{O.L.C.E}} = \frac{K_1}{A \cdot (XK_1 - BK_2)} \quad (6.5)b$$

The open loop characteristic equation of the feedback system fig [6.1] is composed of the poles of the plant and the controller.

When $F = I$, $\det A$ is the plant characteristic equation.

$$FP = A^{-1} B, \text{ or when } F = I$$

$$P = A^{-1} B$$

Hence, $\det A$ is the plant, P , characteristic equation.

In the controller structure, C, equation (6.3)

$$C = (Y + AK) (X - BK)^{-1} \quad (6.3)$$

consider $(X - BK)^{-1}$

$$\begin{aligned} (X - BK)^{-1} &= (X - BK K^{-1} K)^{-1} \\ &= [(XK - BK) K^{-1}]^{-1} \\ &= K (XK - BK)^{-1} \end{aligned}$$

$$\begin{aligned} \text{Therefore, } C &= (Y + AK) K (XK - BK)^{-1} \\ &= (YK + AK) (XK - BK)^{-1} \end{aligned} \quad (6.6)$$

Equation (6.6) is another important equation.

The controller characteristic equation is determined by

$$\det (XK - BK)$$

In fact, this completes the proof that $\det K$ determines the closed loop characteristic equation.

6.3 Some Relations in The SISO case

In the SISO case, the polynomial matrices become polynomial scalars, in this case many interesting relations can be formulated.

Equation (6.5) relating the closed-loop characteristic equation with the open-loop characteristic equation can be reduced to the form

$$\frac{\text{C.L.C.E}}{\text{O.L.C.E}} = \frac{K_1}{A_1(XK_1 - BK_2)} \quad (6.5)$$

where now K_1 is the closed-loop characteristic equation which can be chosen, almost, at will.

The plant transfer function is

$$P = \frac{B}{A} = \frac{B_1}{A_1}$$

and the controller

$$C = \frac{(YK_1 + AK_2)}{A_1(XK_1 - BK_2)} = \frac{N_c}{D_c} \quad (6.7)$$

where N_c and D_c are the controller numerator and denominator polynomials respectively.

The closed-loop transfer function will be

$$T_c = \frac{PC}{(1 + PC)} = \frac{\frac{(N/D)_c \cdot (B/A)}{c}}{[1 + \frac{(N/D)_c \cdot (B/A)}{c}]}$$

$$T_c = \frac{\frac{BN}{c}}{\frac{AD}{c} + \frac{NB}{c}} \quad (6.8)$$

$$= \frac{\frac{BN}{c}}{K_1} \quad (6.9)$$

The sensitivity function S will take the form

$$S = \frac{XA - ABK_2}{K_1} \quad (6.10)$$

6.4 Sensitivity Analysis

From the feedback structure of fig [6.1]

$$\bar{y} = PCS (\bar{u} - \bar{d} - \bar{n}) \quad (6.11)$$

$$\bar{r} = CS (\bar{u} - \bar{d} - \bar{n}) \quad (6.12)$$

$$\bar{e} = S (\bar{u} - \bar{d}) + (I - S) \bar{n} \quad (6.13)$$

where $S = (I + FPC)^{-1}$

From these relations various requirements of the control system can be discussed and analysed. For example, the error vector \bar{e} is now composed of two components. One part subsumes steady-state error, accuracy, and load disturbance while the second, $(I - S) \bar{n}$, is the error produced by measurement noise. Now it is clear that making the error vector, \bar{e} , as small as possible, or actually zero, will give rise to two conflicting requirements in equation (6.13).

Making the error vector $\bar{e} \approx 0$ then it is needed S and

$(I - S)$ both to be ≈ 0 . The impossibility of making both S , and $(I - S)$ arbitrarily small over any frequency band is partly intrinsic and partly conditioned by the plant restrictions [Bode, 1945]. This fundamental conflict is inevitable and responsible for a great deal of the difficulty surrounding practical analytic feedback design.

Now, good transient response and sensitivity require that

$$S(j\omega) \approx 0 \quad \text{in a range of } \omega,$$

n

or qualitatively,

$C(j\omega) \rightarrow \infty$ in that range of ω . The real danger of losing effective control comes when $CS \approx \infty$, in an intermediate range of frequencies, $(I - S(j\omega)) \rightarrow I$, and the plant matrix bandwidth, which is usually confined to some low-frequency interval, tends to 0.

n

In light of this argument and especially when connecting this with equation (6.12), the r.m.s. value of \bar{r} can easily exceed the saturation level of the plant because of the extremely wide-band nature of the sensors noise vector, \bar{n} .

Nevertheless, a remedy for this difficulty caused by the feedback of measurement instrument noise can be found out. By the availability of powerful on-line computational facilities the sensor noise, \bar{n} can be filtered out and smoothed prior to the feedback of these measurements. Eventually, approximate noise-free measurements can be achieved for feedback. This leaves us with the task of dealing only with S rather than both S and $(I-S)$.

The inverse of the return-difference matrix is the sensitivity function S . Minimizing the sensitivity function guarantees good tracking and better disturbance rejection. From equation (6.2)

$$\begin{aligned} S &= (X - BK)A \\ &= (X - BK \begin{matrix} -1 \\ 2 \end{matrix} \begin{matrix} K \\ 1 \end{matrix})A \end{aligned} \quad (6.14)$$

The optimum solution can be obtained if $S = 0$ in a range of frequencies.

From expression (6.14), $A \neq 0$, otherwise $A^{-1} \rightarrow \infty$ which means the matrix FP is not proper (an unusual practical case). However, if $A = 0$ that would not create a difficulty but in fact

might increase the freedom in designing matrix K . Because if $A = 0$ then $S = 0$, which is the required solution, (hence choose K_2 to be anything!).

For an ideal case, it is required that $S = 0$, but for most practical situations a minimum solution for S can be found.

6.4.1 Ideal Case : $S = 0$

For $S = 0$ in the range of frequencies of interest, then we have

$$X - B_1 K_2^{-1} K_1 = 0 \quad (6.15)$$

$$X K_1 - B_1 K_2 = 0 \quad (6.16)$$

Therefore, $K_2 = B_1^{-1} X K_1$, in that range of frequencies of operating freq (6.17)

Also, from (6.6)

$$\det (X K_1 - B_1 K_2) = \text{Characteristic equation of the controller.}$$

Using the least-square analysis or any similar technique on equation (6.17) the polynomial coefficients of every element in K_2 can be found by assuming reasonable polynomial degrees of the elements of K . Equation (6.16) might give some guidance in determining the

degree of polynomial matrix K_2 . For instance, let us assume that

$\deg (XK_1) = \deg (BK_2)$, then the following may be assumed;

$$\deg (K_2) = \deg (XK_1) - \deg (B_1)$$

although this may generate higher degrees in K_2 than required.

6.4.2 Minimal Solution : $S \rightarrow O$

Now to find a minimum solution for

$$S = (X - BK_2 K_1^{-1})A$$

X , K_1 , and A are fixed the minimization can only be obtained by

manipulating K_2 , assuming that K_1 has been chosen as a diagonal

polynomial matrix such that $\det K_1$ represents the stable

characteristic equation of the closed-loop system.

The problem now is to minimize

$$\|S\| = \|XA - BK_2 K_1^{-1} A\| \quad (6.18)$$

$$S = XA - BK_2 K_1^{-1} A$$

In the previous analysis, section (6.4.1), in effect we have

minimized SA by fitting K_2 to $XK_2 - BK_2 = 0$.

| | | |
|-------|-------|----|
| 1 | | 1 |
| k | | k |
| 11 | . . . | 1m |
| 2 | | 2 |
| k | | k |
| 11 | | 1m |
| <hr/> | | |
| . | | . |
| . | - - - | . |
| . | | . |
| <hr/> | | |
| 0 | | 0 |
| k | | k |
| 11 | | 1m |
| 1 | | 1 |
| k | | k |
| 11 | . . . | 1m |
| . | | . |
| . | | . |
| . | | . |
| de | | de |
| k | | k |
| 11 | | 1m |

$$K(z)^{-1} = Z \bar{K}$$

The problem of finding $K(z)^{-1}$ now reduces to the determination of the coefficient matrix \bar{K} .

$$\text{Therefore, } B_1 K(z)^{-1} A = B_1 Z \bar{K} A$$

Further

$$B_1 Z \bar{K} A \text{ can be written as}$$

$$= B_1 Z \bar{K}$$

where \bar{K}_1 is a new constant matrix obtained from arranging $Z \bar{K}_1^{-1} A$ and Z_1 is an augmented Z but still with a block diagonal structure.

Substitute the above in the sensitivity expression ,S,

$$S = XA - B Z_1 \bar{K}_1$$

Now, it is required to minimize $||S||^2$ for a range of frequencies of interest.

$$||S|| = ||XA - B Z_1 \bar{K}_1|| \quad (6.19)$$

\bar{K}_1 can be determined in a least-square sense. Then \bar{K} can be

obtained from \bar{K}_1 .

6.5 Options Available to The Designer in SISO Case:

An application of these SISO options has been shown by Hassan [1983]. The method is applied to the single variable control of the climbing film evaporator, as well as other examples containing unstable non-minimum phase transfer functions. These have been solved and demonstrated. Some strategies are as follows :

Option (1)

- (i) Choose K_1 as the required closed-loop C.E.
- (ii) Choose the controller poles, D_c .
- (iii) Then fit for K_2 from equation (6.7)

$$D_c = X K_1 - B K_2$$

or in other words find a polynomial K_2 that

will satisfy the above equation.

Option (2)

- (i) Choose K_1 , as required.
- (ii) Choose D_c .
- (iii) From $K_1 = D_c A + B N_c$ find a polynomial N_c that will fit this equation, (eq. 6.9)

Option (3)

- (i) Choose K_1
- (ii) Minimize the sensitivity function to find K_2 from

the following equation

$$S = XA - A B K_1 / K_2$$

Each of the above options has its own advantages, depending on the control problem the designer can choose the appropriate option.

6.6 Alternative Solution For Matrix K

6.6.1 Option (1)

A number of ways are available to find a suitable K matrix. Indeed, each option has some limitations and certain advantages depending on the control problem at hand. The designer can start with the option which he feels is simpler. If a satisfactory solution is not reached then it would be better to use one of the standard methods of Least-Square minimization applied directly to equation (6.20).

In the following two methods of determining matrix K are discussed.

Demonstration of the Coprimeness of X and B1

If it can be proved that X and B1 are coprime then K_1 and K_2

can be found such that

$$X K_1 + B1 K_2 = I$$

In the sensitivity design the above expression is to be minimized for a range of frequencies. Now if the above equation is post-multiplied by a small constant element matrix, then matrix tends to any required limit.

$$X \begin{matrix} K' \\ 1 \end{matrix} + B_1 \begin{matrix} K' \\ 2 \end{matrix} = I_e$$

where I_e is a constant diagonal matrix of very small elements and

$$\begin{matrix} K' \\ 1 \end{matrix} = K \begin{matrix} I \\ 1 \end{matrix} e$$

$$\begin{matrix} K' \\ 2 \end{matrix} = K \begin{matrix} I \\ 2 \end{matrix} e$$

But, this time there is no control on the choice of K_1 to be stable and of prescribed closed-loop properties, because $\det K_1$ represent the closed-loop system characteristic equation.

Proof of the coprimeness of X and B₁.

From $AX + BY = I$

$$X_1 A_1 + Y_1 B_1 = I$$

$$\text{and} \quad A^{-1} B = B_1 A_1^{-1}$$

$$A = B A_1 B_1^{-1}$$

$$\text{therefore,} \quad B A_1 B_1^{-1} X + BY = I$$

$$\text{multiply by } B^{-1} \quad A_1 B_1^{-1} X + Y = B^{-1} I$$

$$B_1^{-1} X + A_1^{-1} Y = A_1^{-1} B^{-1} = B_1^{-1} A^{-1}$$

$$X + B_1 A_1^{-1} Y = A^{-1}$$

$$X A + B_1 A_1^{-1} Y A = I$$

$$X A + B_1 A_1^{-1} Y A = I \quad (6.20)$$

This completes the proof.

A and $A_1^{-1} Y A$ are any matrices. It is not necessary that A and $A_1^{-1} Y A$ are the only matrices to satisfy the above identity, any K_1 and K_2 matrices may exist to satisfy the identity

$$X K_1 - B_1 K_2 = I$$

So, K_1 can be taken as A , hence the closed-loop system poles are the

plant poles and $K_2 = -A_1^{-1} Y A$. But, usually in control design the

closed-loop poles are required to be modified to satisfy certain criteria set by the customer.

However, some final adjustments may be needed in the choice of K_1 and K_2 because of another important constraint considered

above, that is $\det (X K_1 - B_1 K_2) =$ the poles of the controller.

6.6.2 Option (2)

Consider the following:

$$S = XA - B_1 K_2 K_1^{-1} A$$

$$C = (YK_1 + AK_2) (XK_1 - BK_2)$$

From the proof of the identity, equation (6.20),

$$XA + B_1 A_1^{-1} Y A = I$$

now take $A_1^{-1} Y = -K_2 K_1^{-1}$

$$XA - B_1 K_2 K_1^{-1} A = I = S \text{ for all frequencies.}$$

The controller C will be, after taking $A_1^{-1} Y = K_2 K_1^{-1}$

$$C = (0) (XK_1 - BK_2) = 0$$

No controller is needed to make the sensitivity function $S = I$.

This is expected since it is known that $\det K_1$ determines the

closed-loop system, and in this case $\det K_1 = \det A_1 =$ the open-loop characteristic equation.

6.6.3 Option (3)

$$S = XA - B_1 K_1 K_2 A$$

let S_e be $S(j\omega)$ which is a minimum function in a range of ω .

$$S_e = XA - B_1 K_1 K_2 A$$

$$\text{and } I = XA + B_1 A_1 Y_A$$

$$\text{therefore, } XA = I - B_1 A_1 Y_A$$

$$S_e = I - B_1 A_1 Y_A - B_1 K_1 K_2 A$$

$$B_1 K_1 K_2 A = [I - B_1 A_1 Y_A - S_e]$$

$$K_1 K_2 = B_1 [I - B_1 A_1 Y_A - S_e] A$$

On an extreme case of $S = 0$ for almost all $j\omega$

$$K_1 K_2 = [B_1 A - A_1 Y]$$

$$\text{From } B_1^{-1} X + A_1^{-1} Y = B_1^{-1} A^{-1}$$

$$\text{therefore, } K_{21}^{-1} = B_1^{-1} X$$

So, the closed-loop characteristic equation

$$\det K_1 = \det B_1;$$

thus, $\det B_1$ should be of desirable characteristics. But, before

$$\text{embarking on this condition we have to notice that } K_{21}^{-1} = B_1^{-1} X$$

will make the denominator of the controller matrix equal to 0.

However, it is almost impossible to make $S = 0$ for all $j\omega$, hence

$$K_{21}^{-1} = B_1^{-1} X - B_1^{-1} S A_e^{-1}$$

$$\text{let } B_1^{-1} = B_1' / \det B_1, \quad A_e^{-1} = A' / \det A, \quad S_e = S' / \det S$$

$$K_{21}^{-1} = (B_1' X \det S_e \det A - B_1' S' A') / (\det B_1 \det S_e \det A)$$

$$\text{But } \det K_1 = \det S_e$$

therefore, $\det B_1$ and $\det A$ should cancel out by the rest of the expression. Unless $\det B_1 = 1 / \det A$ then each element in $[B' X \det$

$S_e \det A - B_l S^* A^*$ should contains $\det A \det B_l$. So, the design

algorithm can take the following steps :

- (i) choose $\det S_e$,
- (ii) adjust S^* such that $\det A \det B_l$ cancels out,
- (iii) finally check that $S^* / \det S_e = S_e$ is minimal for the
chosen range of w .

6.8 Summary

The design theory has made possible the following :

- 1- There are no restrictions on the plant transfer function matrix. It can be rectangular, unstable, improper, and non-minimum phase.
- 2- The method incorporates a wide classes of load disturbances.
- 3- The optimal controller guarantees a dynamically stable closed-loop design possessing proper least-square sensitivity matrix in any desired range of frequencies.
- 4- The stability margin of the optimal design is ascertainable in advance, in fact, the required closed loop poles are chosen at will.

Software programs have been developed to handle the control synthesis method for the SISO and MIMO cases. These programs are shown in Appendix (H), and some examples of their applications were given by Hassan [1983].

CHAPTER (7)

COMPUTER-AIDED CONTROL SYSTEM

DESIGN and SYNTHESIS PACKAGE

7.1 INTRODUCTION

Computer-Aided Control System Design, (CACSD), has begun to emerge as an indispensable tool for the control system engineers. A CACSD capability not only frees the engineer from routine and mundane tasks but also provides a vehicle whereby complex algorithms for control methodologies are made available to and usable by those unfamiliar with the myriad of details that make CACSD software efficient. A good CACSD system draws on expertise from many disciplines including aspects of computer science, engineering, applied mathematics and numerical analysis, as well as control system engineering. The need for such breadth of input is partially responsible for the paucity of high quality CACSD software today.

Control aided design packages are now available in many large control centres. These are covering the most familiar techniques in the frequency domain, e.g. inverse Nyquist array, direct Nyquist array, Characteristic loci, and dyadic methods, to mention a few. Also methods using design techniques methods based on the state-space formulation have been included in many packages;

almost the Kalman filter is available every where in industry or academic institutions.

Nevertheless, a wide gap in CACSD still exists in the growing area of control theory synthesis methods. In recent years algebraic system theory has played a profound role in the understanding of many of the important phenomena that underly linear system theory in the various finite spaces. The link between state space analysis and the frequency-domain has been brought into focus and more insight has been gained in the study of dynamics systems.

Although algebraic synthesis methods have shown powerful characteristics in control systems synthesis, the computational facilities in terms of efficient algorithms, and computer software packages are not available. The tremendous symbolic manipulations required in these synthesis techniques may have been one of the reasons why the appearance of such computer packages has been delayed. Though, LISP is a powerful computer language for handling symbolic computations, yet it is not widely known in engineering disciplines.

In this research work, an extended Wiener-Hopf controller is synthesised based on one of the algebraic methods, as described in chapter (5). Indeed, without powerful software to aid the designer the synthesis techniques would have not been feasible, and therefore also most of the multivariable control design methods.

A huge software mathematical algorithms has been derived to build such a package for computer-aided control system

synthesis. The package handles algebraic operations with polynomial matrices defined over a principal ideal domain, conforming a ring.

APA, Algebraic Polynomial Algorithm, is designed to fill partially the gap in the availability of methods for symbolic manipulation of control systems design and synthesis. This is a first step towards developing digital computer packages to handle algebraic systems.

APA covers the areas of manipulations of polynomial matrices with rational form, spectral factorization of polynomial matrices, decomposition of left-right coprime matrices, symbolic summation of rational polynomials, addition and multiplication of polynomial matrices, polynomial matrix inversions, determinant of polynomial matrices, multiplication of para-Hermitian polynomial matrices, and discrete and continuous systems simulation, a few examples of possible operations to mention.

FORTRAN -77, the high level computer language, has been used to construct this package, due to its wide application, recognition and portability, especially in engineering disciplines. FORTRAN -77 has reasonably flexible control commands, and efficient structured programs can be written and tested.

The library provides a good tool for helping systems engineers in dealing with large problems defined by the modern techniques of matrix fraction description, minimal design problems arising in the design of linear multivariable control systems. Examples are minimal order L-integral inverses, exact model matching, dynamic decoupling, dynamic observers, and similar

problems. The package has direct application to the polynomial approach in control systems design and synthesis.

7.2 APA Notations :

In developing APA to deal with polynomials defined over principal ideal domains, it is necessary only to operate in the field $R^{m,n}$ of real numbers with stack pointers operating in the ring of integer numbers.

Definition :

A principal ideal domain is an integral domain A with the property that, for any subset S of A such that $a+b$ belongs to S whenever a and b belong to S such that

$$S = \{ ab : b \text{ belongs to } A \},$$

i.e, every ideal subset S of A is generated by a single element a belonging to S . The ring of integers and the operator ring $F(z^{-1})$ are examples of principal ideal domains. So also is the class of simple polynomials of integer powers.

7.2.1. Data Structure

The operations on polynomial matrices, as mentioned above, can be divided into two, those on the field of rational numbers, and those on the ring of integers.

The rational number data are held in 3 dimensional matrices of size equal to the space spanned by the polynomial elements, as follows :

i) $F(I,J,0:K)$ is the 3-dimensional matrix, where integer I points to the rows and integer J points to the columns, where the polynomial element resides. Integer K points to the cells containing the coefficients of the polynomial element. The coefficients go from lower degree to higher degree in the polynomial, with the integer 0 always points to the constant term, which is free from indeterminate.

ii) another two dimensional matrix, $D(I,J)$, is needed to act as a stack pointer to hold the degree of each polynomial element. As the polynomials are in a principal ideal domain, their degrees are integer numbers, and the matrix can therefore be of type integer.

ALGORITHMS

7.3 Spectral Factorization of Polynomial Matrices

Spectral factorization is an important technique in solving integral equations of Wiener-Hopf type. These types of equations appear in many engineering problems. Typical problems are those of image processing, filter design, and detection of signals buried in noise.

The most feasible method of solution of Wiener-Hopf equations is using the spectral factorization technique devised by

Wiener [1949]. The integral function must satisfy many conditions for a solution to be obtainable, see Youla [1961,1978] for conditions and existence of a solution. Generally, it is not straight forward to find the spectral factorization of a function.

Investigations have been carried out to develop computer algorithms which can be easily implemented in digital computers to solve the problem of spectral factorization. A robust well conditioned digital computer algorithm is needed for the solution of almost all such numerical problems. Various algorithms to solve the spectral factorization problem have emerged and appeared in the literature [Anderson,1967], [Davis,1963], [Riddle,et al,1966], [Wong,1961], [Matyka,et al,1967],[Strintzis,1972], [Ekstrom,1976]. This is because the Wiener-Hopf equations appear in a number of stationary processes, queueing theory, computer time service, etc. Many of the algorithms which have appeared in the literature may not be directly useful for computer implementation, but in simple systems, some of these algorithms may be very helpful to an engineer using a hand calculator or a home computer. Some elegant computer implementable algorithms have been developed and these have been reported by Youla [1978] and Tuel [1968]. The technique used here follows those found by Youla [1978].

Generally, the need for spectral factorization occurs when a matrix $F(z)$ belongs to $R^{r,m}[z]$ of rank L and produces a real positive definite para-Hermitian matrix $G(z)$ such that

$$F(z) \cdot F^{\star -1}(z) = G(z).$$

If $G(z)$ satisfies the above conditions then it has a Hurwitz polynomial matrix $B(z)$, left spectral factor associated with $F(z)$, such that

$$G(z) = B(z) \cdot B^*(z)$$

Algorithm

Since we have defined the polynomial matrices in the ring $R^{r,m}[z^{-1}]$, the multiplication $F(z) \cdot F^*(z^{-1})$ produces a symmetric polynomial around the constant term of the polynomial.

$$F^*(z^{-1}) \cdot F(z) = G_n^* z^{-n} + \dots + G_1^* z^{-1} + G_0 + G_1 z + \dots + G_n z^n$$

$$G_n \neq \emptyset$$

where $G_i^* = \text{transpose of } G_i$.

The Toeplitz block matrix formed by G_i has elements

$$T_i = \begin{bmatrix} G_0 & G_1^* & \dots & G_n^* \\ G_1 & G_0 & \dots & G_{n-1}^* \\ \dots & \dots & \dots & \dots \\ G_i & G_{i-1} & \dots & G_0 \end{bmatrix}$$

$i = 0, \dots, j$ since for $j > n$ $G_i = \emptyset$.

Affecting the Cholesky factorization in the Toeplitz matrix above we get,

$$T = L \cdot L^*,$$

where L is lower triangular matrix and L^* is L transposed.

Lemma

If the matrix T is symmetric and positive definite, then there exists a unique lower-triangular matrix L with positive diagonal such that $T = L \cdot L^T$. This is known as Cholesky factorization.

Due to the structure of T the factors L can be obtained recursively by adjoining a new block row to L_{i-1} . It is also possible to reduce the amount of computation and storage by taking advantage of the symmetry and only calculating the elements of L .

To avoid using unnecessary storage space, when a block column is not used any more to find new additional blocks, it is cleared and the existing blocks are renumbered, thus vacating the last block row for the new cycle. Therefore, all the calculations can be done within an $r(n+1) \times r(n+1)$ matrix.

Let $p = r \cdot n$, $q = r \cdot (n+1)$

and denote $G_{j,k}$, $j, k = 1, 2, \dots, q$ the elements of T_n .

Define a $q \times q$ lower triangular matrix L with elements $L_{j,k}$, initially

set to zero. Start by computing Cholesky factorization of the block G at bottom right of T .

$$\begin{bmatrix} G_{p+1,p+1} & \dots & G_{p+1,q} \\ \vdots & \ddots & \vdots \\ G_{q,p+1} & \dots & G_{qq} \end{bmatrix} = \begin{bmatrix} L_{p+1,p+1} & \dots & 0 \\ L_{p+2,p+1} & L_{p+2,p+2} & \dots \\ \vdots & \vdots & \ddots \\ L_{q,p+1} & \dots & L_{qq} \end{bmatrix} \begin{bmatrix} L_{p+1,p+1} & \dots & L_{p+1,q} \\ \vdots & \ddots & \vdots \\ 0 & \dots & L_{qq} \end{bmatrix}$$

For the 1st. column we have

$$L_{p+1,p+1} = \sqrt{g_{p+1,p+1}}$$

$$L_{j,p+1} = b_{j,p+1} / L_{p+1,p+1} \quad j=p+1, \dots, q$$

For the 2nd column

$$L_{p+2,p+2} = \sqrt{g_{p+2,p+2} - L_{p+2,p+1}^2}, (L_{p+2,p+1} = L_{p+1,p+2})$$

$$L_{j,p+2} = (g_{j,p+2} - L_{j,p+1} \cdot L_{p+2,p+1}) / L_{p+2,p+2}, j=p+3, \dots, q$$

and so on until we get

$$L_{qq} = \sqrt{g_{qq}^2 - L_{qq,q+1}^2 - L_{qq,q+2}^2 - \dots - L_{qq,q-1}^2}$$

Now shift the elements of L upward and to the left

$$L_{jk} = L_{j+r,k+r} \quad j,k = 1, 2, \dots, p \quad j \Rightarrow k$$

and start computing the new last r rows of L successively, column by column, we get

$$L_{j,p-r+1} = g_{j,p-r+1} / L_{p-r+1,p-r+1}$$

$$L_{j,p-r+2} = (g_{j,p-r+2} - L_{j,p-r+1} \cdot L_{p-r+2,p-r+1}) / L_{p-r+2,p-r+2}$$

$$L_{jp} = (g_{jp} - L_{jp,p-r+1} \cdot L_{p,p-r+1} - \dots - L_{j,p-1} \cdot L_{p,p-r}) / L_{pp}$$

for $j = p+1, \dots, q$ and the block at bottom right is again obtained by performing the Cholesky factorization with g_{jk} now replaced by

$$g_{jk} - L_{j,p-r+1} \cdot L_{k,p-r+1} - \dots - L_{j,p} \cdot L_{kp} \quad \text{for } j, k = p+1, \dots, q.$$

This is continued until the difference of the traces of the last two Cholesky factors are sufficiently small in absolute value.

7.4 Elementary Operations On Polynomial Matrices

Elementary operations on matrices are an indispensable tool in handling the algebraic methods of control system designs. Usually these involve the determination of matrix fraction description (MFD) such as left and right coprime matrices, greatest common divisors of matrices, Smith-McMillan forms, inversion of polynomial matrices, etc. The technique of elementary column and row operations on matrices in the Euclidean n -space is well known and defined. These operations merely depend on the idea of linear dependence and linear independence of vectors in the field F of real and complex numbers. In algebraic systems the idea of linear independence and dependence has a generalization over fields. Hence, these fields may be field of real numbers, rational numbers,

complex or even rational functions where most of the operations are defined here.

7.4.1 Fields and Rings

Fields in the wide sense of algebra are defined as follows:

Assume F is a set consisting of elements f_1, f_2, \dots , if the

following axioms are satisfied then F is a field;

Given any pair of elements (f_i, f_j) in F , then

- i) addition is commutative, for all f_i, f_j in F then $f_i + f_j = f_j + f_i$

$$f_j = f_j + f_i$$

- ii) addition is associative, for all f_i, f_j, f_k in F then $f_i + (f_j + f_k) = (f_i + f_j) + f_k$

$$(f_j + f_k) = (f_i + f_j) + f_k$$

- iii) There exists an element \emptyset such that, (called the additive identity) $f_i + \emptyset = f_i$ for all f_i in F , and \emptyset

belongs to F

- iv) An additive inverse exists in F such that

$$f_i + (-f_i) = \emptyset \text{ for all } f_i \text{ belongs to } F$$

- v) $f_i + f_j$ is uniquely defined and $(f_i + f_j)$ belongs to F ,

an element of F

Given any f_i in F then :

$$1) \quad f_i \cdot f_j = f_j \cdot f_i \quad \text{for all } f_i \text{ belongs to } F \quad i, j=0, 1, \dots$$

multiplication is commutative.

$$2) \quad \text{Multiplication is associative. For all } f_i, f_j, f_k \text{ then}$$

$$f_i (f_j \cdot f_k) = (f_i \cdot f_j) f_k$$

$$3) \quad \text{There exists a multiplicative identity in } F \text{ denoted by}$$

$$1 \text{ such that } f_i \cdot 1 = 1 \cdot f_i = f_i$$

$$4) \quad \text{For every element } f_i \neq 0 \text{ there exists an element } f_i^{-1}$$

called the multiplicative inverse such that

$$f_i \cdot f_i^{-1} = f_i^{-1} \cdot f_i = 1$$

$$5) \quad f_i \cdot f_j \text{ is unique and also is an element in } F; (f_i \cdot f_j)$$

belongs to F .

$$6) \quad \text{Multiplication is distributive;}$$

$$f_i (f_j + f_k) = f_i \cdot f_j + f_i \cdot f_k \quad \text{for all } f_i \text{ belongs to } F$$

$$, n=i, j, k, \dots$$

The set of all real numbers, denoted by R , and the set of all complex numbers, denoted by C are two examples of fields with the usual rules of addition and multiplication. Another example of a field is the set of rational functions $f_i = n_i(z)/d_i(z)$ where

$d_i(z) \neq 0$ and n_i and d_i are polynomials of finite degree with coefficients in the field R .

A set of elements f_i which obey the above axioms except one or more is a ring. For example the set of integer numbers is a commutative ring with identity. Note that the set of integer numbers has no multiplicative inverse. The set of all polynomials of finite degree (not rational polynomials) with coefficient in R is another example of commutative ring with identity.

In the following analysis it will be assumed that F is a field obeying all of the above axioms. This is essential because later the linear dependence and linear independence of vectors over the field of rational function will be discussed.

Example

$$A = \begin{bmatrix} x+3 & x+2 \\ 2 & 2 \\ x^2+7x+12x & x^2+6x+8 \end{bmatrix}$$

Now, vector A is singular and hence there is linear dependence in A . The linear dependence is not quite apparent if we had restricted ourselves in the field R of real numbers. But, it is clear that the second column, say, $P_2(x)$ is linearly dependent on the first column $P_1(x)$, say, and the proportionality factor is not a real number but this time it is a rational function.

$$P_2(x) = \frac{(x+2)}{(x+3)} P_1(x)$$

Now, rational functions form a field and with coefficients drawn from this field the basic facts for Euclidean n-space can be carried over. With this in mind, it is concluded that the elementary row and column operations for polynomial matrices have the form :

- 1- Interchange of any two columns (rows),
- 2- Addition to any column or row of a polynomial multiple of any other column or row, and
- 3- Scaling any column or row by any non-zero real or complex number.

These elementary operations can be represented by elementary matrices, postmultiplication by which corresponds to elementary column operation, while premultiplication yields elementary row operations. Elementary matrices are strongly non-singular in that their determinants are independent of x (unimodular). Normally non-singular matrices means for almost all x the det of A is independent of x .

7.4.2 Greatest Common Divisors and Coprime Matrices

Theorem: Column (row) Hermite Form.

Any F , $p \times m$ polynomial matrix of rank r can be reduced by elementary column operations i.e. by postmultiplying by a unimodular matrix to a lower or upper quasi-triangular form in which

- 1- If $m > r$, the last $m-r$ columns are identically zero;
- 2- In row j , $1 \leq j \leq r$, the diagonal element is monic and of higher degree than any (nonzero) element above it;
- 3- In row j , $1 \leq j \leq r$ if the diagonal element is unity, then all elements $F(i,j)$ are zero and
- 4- If $p > r$, no particular statements can be made about the elements in the last $p-r$ rows and the first r columns.

The Hermite forms are obtained by using only column (or only row) operations.

7.4.3 Relatively Prime Matrices (coprime matrices)

Two polynomial matrices with the same number of rows are relatively left prime or are left coprime if all their greatest common left divisor (gclid) are unimodular. Similarly, for any two polynomial matrices with the same number of columns are relatively right prime or right coprime if all their greatest common right divisors (gcrd) are unimodular.

7.4.4 Unimodular Matrices

Let $U(z)$ be any $n \times n$ matrix and let $\det(U(z)) \neq 0$. If $\det(U(z))$ is a non-zero constant, independent of z , then $U(z)$ is called unimodular matrix. Note that the inverse of such a matrix is also unimodular.

Greatest Common Divisors :

A greatest common right divisor (gcdr) of two matrices $\{A, B\}$ with the same number of columns is any matrix $R(z)$ with the following properties :

1- $R(z)$ is a right divisor of $\{A, B\}$ i.e. there exist

polynomial matrices $\bar{A}(z), \bar{B}(z)$ such that

$$A(z) = \bar{A}(z) \cdot R \quad , \quad B(z) = \bar{B}(z) \cdot R$$

2- If $R_1(z)$ is any other right divisor of A and B then R_1

is a right divisor of R , i.e there exists a polynomial matrix W such that

$$R(z) = W \cdot R_1$$

Lemma

Given $m \times m$ and $p \times m$ polynomial matrices B and A form the matrix $[B \ A]$ and find elementary row operations or equivalently find a unimodular matrix $U(z)$ such that at least p of the bottom rows on the right hand side are identically zero.

$$\begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} B \\ A \end{bmatrix} = \begin{bmatrix} R(z) \\ 0 \end{bmatrix}$$

gcd's are not unique: by carrying out the elementary operations in different orders we may get different matrices $U(z)$ and hence different (gcd). However, any two gcd, R_1 and R_2 , say, must be

related (by definition) as

$$R_1 = W_{12} R_2, \quad R_2 = W_{21} R_1.$$

since it can be written that

$$R_1 = W_{12} W_{21} R_1$$

it follows that:

- i- if R_1 is non-singular, then the $W_i(z)$, $i=1,2$, must be unimodular, and hence the gcd R_2 is nonsingular,
- ii- if gcd is unimodular, then all gcd must be unimodular.

Lemma :

If $[B' \ A']'$ has full column rank, then all gcd of $[A \ B]$ must be non-singular and can differ only by unimodular factor.

Remark :

We recall that a polynomial matrix will have full column rank if no nontrivial combination of its columns, with either rational or polynomial coefficients, is identically (i.e. for all z) equal to zero. Therefore the rank will be full for almost all z . If the rank is full for all z , the matrix (not necessarily square) will be called irreducible.

Lemma : simple Bezout identity.

A and B will be right coprime if and only if there exist polynomial matrices X and Y such that

$$XA + YB = I.$$

A and B will be left coprime if and only if there exist polynomial matrices X_1 and Y_1 such that

$$A_1X_1 + B_1Y_1 = I$$

Proof:

Given any A and B we can write any gcd as

$$R = \overline{X}A + \overline{Y}B.$$

But if A and B are coprime, then R must be unimodular so that R^{-1} will be a polynomial.

$$I = XA + YB$$

where $X = R^{-1} \overline{X}$, $Y = R^{-1} \overline{Y}$

Conversely, suppose there exist X and Y such that

$$XA + YB = I \quad \text{holds and let } R(z) \text{ be any gcd}$$

$$A = \bar{A} R, \quad B = \bar{B} R$$

Then we can write

$$I = [X\bar{A} + Y\bar{B}] R$$

$$R^{-1} = X\bar{A} + Y\bar{B} \quad \text{a polynomial matrix.}$$

Therefore $R(z)$ must be unimodular or equivalently $\{A, B\}$ must be right coprime.

Lemma :

A and B will be right coprime iff $[B^* \ A^*]^*$ has full rank for every z (i.e. iff $[B^* \ A^*]^*$ is irreducible).

Lemma :

Latent-roots characterization of relative primeness .

$A(z)$ and $B(z)$ will be (right) coprime iff they have no common latent vectors and associated common latent roots.

Proof:

We can have a non-zero solution $P(z)$ to

$$\begin{bmatrix} B(z) \\ A(z) \end{bmatrix} P(z) = 0$$

iff $[B' \ A']'$ does not have full column rank for some z .

Remarks

If $[A \ B]$ are right coprime then the matrix $[B' \ A']'$ will be irreducible. Conversely, if a full column rank matrix $F(z)$ is irreducible, note that it can be partitioned perhaps after some row operations, such that $F'(z) = [B' \ A']'$, $B(z)$ is nonsingular and AB^{-1} is an irreducible right matrix fraction description, (MFD).

The construction described earlier for finding gcd has some interesting special features when B is nonsingular. We recall that the basic operation is to find a unimodular matrix $U(z)$ such that

$$\begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} B \\ A \end{bmatrix} = \begin{bmatrix} R \\ 0 \end{bmatrix}$$

Lemma :

When B is invertable (nonsingular), then :

a) U_{22} will be nonsingular,

b) $AB^{-1} = -U_{22}^{-1} U_{21}$

c) $\{U_{21}, U_{22}\}$ will be left coprime i.e. $U_{22}^{-1} U_{21}$ will be

irreducible left MFD. Note AB^{-1} not necessarily right coprime.

d) if $\{A, B\}$ are coprime then $\deg(\det B) = \deg(\det U)$
22

and $R=I$.

In other words, from a right MFD, AB^{-1} , we can obtain an irreducible left MFD in the process of finding a gcd of the elements of the right MFD. All the above discussion is applicable for gcd and left coprime with the trivial changes.

Proof of result C above :

Result (c) follows from $U(z)$ is unimodular, so that any subset of its rows is irreducible.

Summary

To summarize the main results shown in this section they are stated again here in a compact form.

1- A matrix $U(z)$ $n \times n$ is said to be unimodular if it is non-singular and its determinant is a non-zero constant and is independent of z . The inverse of unimodular matrix is also a unimodular.

2- Matrices $A(z)$ and $B(z)$ if they have the same number of rows the composite matrix $[A(z) \ B(z)]$ can be reduced to a quasi-lower triangular matrix $[L(z) \ 0]$ by elementary operations or equivalently postmultiplying $[A(z) \ B(z)]$ by a unimodular matrix. $L(z)$ is then the greatest common left divisor of $A(z)$ and $B(z)$.

Similarly if $A(z)$ and $B(z)$ have the same number of columns

$\begin{bmatrix} A(z) \\ B(z) \end{bmatrix}$ can be reduced to upper right triangular form $[R(z) \ 0]$ and $R(z)$ is the greatest common right divisor of $A(z)$ and $B(z)$.

3- An $m \times n$, $G(z)$ matrix with $m \leq n$ can be reduced to quasi-lower triangular matrix $[L(z) \ 0]$ by postmultiplying G by a unimodular matrix $U(z)$.

$$[L(z) \ 0] = G(z) \cdot U(z) = \begin{bmatrix} L_{11}(z) & 0 & \dots & 0 \\ L_{21}(z) & L_{22}(z) & \dots & 0 \\ \dots & \dots & \dots & \dots \\ L_{m1}(z) & \dots & \dots & L_{mm}(z) \ 0 \dots 0 \end{bmatrix}$$

such that :

i) the degrees of the polynomials $L_{i1}, L_{i2}, \dots, L_{ii-1}$ are all strictly less than the degree of the polynomial L_{ii} for all i .

ii) If L_{ii} is a constant independent of z , then the elements L_{i1}, \dots, L_{ii-1} are all zero, when $m > n$, similar results will hold when premultiplying $G(z)$, and the L_{ij} will have the same properties above.

7.4.5 Computer Algorithm

From the above theoretical analysis it has been shown possible to find left and right coprime matrices during the process of extracting greatest common right and left divisors, respectively.

Given that A belongs to $R^{m,m}[z]$ and B belongs to $R^{m,p}[z]$

the algorithm produces L (the gcd, L belongs to $R^{n,m}[z]$) along with two pairs of right coprime polynomial matrices A_r, B_r, A_l, B_l such

that

$$A \cdot A_l + B \cdot B_l = L$$

$$A_r + B_r = 0$$

Note that $\{A, B\}$ need not be coprime. Also, a least common right multiple is found, $A_r = -B_r$.

1- Form $F = [A \ B]$,

2- Reduce F by elementary operations to $[L \ 0]$, or postmultiply F by

$$\text{unimodular matrix } U(z) = \begin{bmatrix} A_l & A_r \\ B_l & B_r \end{bmatrix}$$

3- To get $U(z)$ associate with F above a unity matrix. $U = I_{n+p}$ and

perform the same operations on F and U. When F is reduced to $[L \ 0]$, U is carrying the required unimodular matrix.

4- In $[A \ B]$ start the elementary column operations by :

- i- find the minimum polynomial degree in the first row,
- ii- bring this polynomial to the diagonal position,
- iii- subtract multiples of this element from the other elements in the row,
- iv- repeat steps (i),(ii),(iii) above until all the elements in the row are zero except the diagonal element, and
- v- move to the next row and start from the diagonal element and repeat the above steps.

7.5 Inversion of Polynomial Matrices

In modern Wiener-Hopf controller design finding the inverse of polynomial matrices is part of the computation in the method. The following algorithm generalizes the well known method for obtaining the inverse of a real matrix in the field R^{mxn} , field of all real numbers. The algorithm is implementable in a digital computer.

Algorithm

The inverse of a non-singular matrix $F(z)$ or a proper matrix $F(z)$ can be obtained by elementary column or row operations together with other non-elementary column or row operations. If

$F(z)$ is proper, then $\lim_{z \rightarrow \infty} F(z) < \infty$ as $z \rightarrow \infty$. This condition guarantees the existence of an inverse, but if $F(z)$ is strictly proper, $\lim_{z \rightarrow \infty} F(z) = 0$ as $z \rightarrow \infty$. This test is not conclusive and the inverse of $F(z)$ may or may not exist. Hence, the fact that $F(z)$ is proper is a necessary condition for $F(z)$ to be invertible.

The algorithm for finding the inverse of $F(z)$, generally, is to bring F to a lower triangular matrix by elementary column or row operations. This is followed by non-elementary column or row operations to bring the lower triangular form to a diagonal matrix. The non-elementary operation is a combination of all possible algebraic operation to bring the lower triangular matrix into a diagonal matrix. It can be as follows: in matrix F subtract f_{kn} times the k th column from f_{kn} times the n th column thereby making all the off-diagonal polynomials in row k zero. Carry out these steps for all rows k . This finally produces a diagonal matrix.

Performing the same operations on the identity matrix, we arrive at a polynomial matrix V .

Before performing the non-elementary operations and at the stage when F is lower triangular a polynomial matrix $F_1(z)$ is constructed. $F_1(z)$ is a diagonal matrix and the diagonal is constructed such that $F_1(1,1) = 1$, $F_1(2,2) = L(1,1)$, $F_1(i,i) = L(1,1) L(2,2) \dots L(i-1,i-1)$

where $L(i,i)$ are the diagonal polynomial elements of the lower triangular matrix that has resulted from $F(z)$ after the elementary column (or row) operations.

Now, the inverse of $F(z)$ is obtained by multiplying the matrix V times F_1 . The determinant of F is also obtained from the lower triangular matrix by multiplying the diagonal elements together. Determinant F is the common denominator of the inverse of F .

$$F^{-1} = V \cdot F_1 / \det F$$

For example, take a 4x4 matrix, by elementary operations, F has been transformed to a lower triangular and the identity matrix now has become an ordinary polynomial matrix due to the elementary operations, say,

$$F \rightarrow L = \begin{bmatrix} x_1 & 0 & 0 & 0 \\ z_1 & x_2 & 0 & 0 \\ z_2 & z_3 & x_3 & 0 \\ z_4 & z_5 & z_6 & x_4 \end{bmatrix}$$

where x_i are the diagonal elements of the lower triangular matrix L , and z is a polynomial element.

Now, L is required to be reduced into a diagonal matrix,
i.e. all $z_i = 0$

$$L \rightarrow D = \begin{bmatrix} x & x & x & x & & 0 & 0 & 0 \\ & 1 & 2 & 3 & 4 & & & \\ 0 & & & x & x & x & 0 & 0 \\ & & & 2 & 3 & 4 & & \\ 0 & 0 & & & x & x & & 0 \\ & & & & 3 & 4 & & \\ 0 & 0 & 0 & & & & & x \\ & & & & & & & 4 \end{bmatrix}$$

The inverse now is obtained by dividing the first column of V by the first diagonal element in D, the second column of V divided by the second diagonal element of D and so on the jth column of V is divided by D(j,j).

Alternatively, to make the algorithm efficient and easy to program, the following simple algebraic manipulations are needed.

$$F^{-1} = \begin{bmatrix} \frac{V(1,1)}{x \ x \ x} & \frac{V(1,2)}{x \ x \ x} & \frac{V(1,3)}{x \ x} & \frac{V(1,4)}{x} \\ & & & 4 \\ & & & 4 \\ \frac{V(2,1)}{x \ x \ x} & \frac{V(2,2)}{x \ x \ x} & \frac{V(2,3)}{x \ x} & \frac{V(2,4)}{x} \\ & & & 4 \\ & & & 4 \\ \frac{V(3,1)}{x \ x \ x} & \frac{V(3,2)}{x \ x \ x} & \frac{V(3,3)}{x \ x} & \frac{V(3,4)}{x} \\ & & & 4 \\ & & & 4 \end{bmatrix}$$

$$\left[\begin{array}{c|c|c|c} \hline V(4,1) & V(4,2) & V(4,3) & V(4,4) \\ \hline x \ x \ x & x \ x \ x & x \ x & x \\ 1 \ 2 \ 3 & 2 \ 3 \ 4 & 3 \ 4 & 4 \\ \hline \end{array} \right]$$

taking $(x \ x \ x \ x)(x \ x \ x)(x \ x)(x)$ as a common factor then we can
 $1 \ 2 \ 3 \ 4 \quad 2 \ 3 \ 4 \quad 3 \ 4 \quad 4$

write the inverse as

$$\left[\begin{array}{cccc} (x \ x \ x)(x \ x)(x) & 0 & 0 & 0 \\ 1 \ 3 \ 4 & 3 \ 4 & 4 & \\ 0 & (x \ x \ x \ x)(x \ x)x & 0 & 0 \\ & 1 \ 2 \ 3 \ 4 & 3 \ 4 & 4 \\ 0 & 0 & (x \ x \ x \ x)(x \ x \ x)x & 0 \\ & & 1 \ 2 \ 3 \ 4 & 2 \ 3 \ 4 \ 4 \\ 0 & 0 & 0 & (x \ x \ x \ x)(x \ x \ x)x \ x \\ & & & 1 \ 2 \ 3 \ 4 \quad 2 \ 3 \ 4 \quad 3 \ 4 \end{array} \right]$$

All the terms in the first column are multiplied by

$$(x \ x)(x \ x)(x) \\ 2 \ 3 \ 4 \quad 3 \ 4 \quad 4$$

All the terms in the second column are multiplied by

$$(x \ x \ x \ x)(x \ x)(x) \\ 1 \ 2 \ 3 \ 4 \quad 3 \ 4 \quad 4$$

All the terms in the third column are multiplied by

$$(x \ x \ x \ x)(x \ x \ x)(x) \\ 1 \ 2 \ 3 \ 4 \quad 2 \ 3 \ 4 \quad 4$$

All the terms in the 4th column are multiplied by

$$(x \ x \ x \ x)(x \ x \ x)(x \ x) \\ 1 \ 2 \ 3 \ 4 \quad 2 \ 3 \ 4 \quad 3 \ 4$$

Now cancellation between the terms takes place to make the common denominator minimum, i.e. prime with the other terms of the matrix.

$$\text{INVERSE} = \frac{1}{\begin{matrix} (x & x & x & x) & (x & x & x) & (x & x) & (x) \\ 1 & 2 & 3 & 4 & 2 & 3 & 4 & 3 & 4 & 4 \end{matrix}} *$$

$$\begin{bmatrix} \begin{matrix} (x & x & x) & (x & x) & (x) \\ 2 & 3 & 4 & 3 & 4 & 4 \end{matrix} & 0 & 0 & 0 \\ 0 & \begin{matrix} (x & x & x & x) & (x & x) & (x) \\ 1 & 2 & 3 & 4 & 3 & 4 & 4 \end{matrix} & 0 & 0 \\ 0 & 0 & \begin{matrix} (x & x & x & x) & (x & x & x) & (x) \\ 1 & 2 & 3 & 4 & 2 & 3 & 4 & 4 \end{matrix} & 0 \\ 0 & 0 & 0 & \begin{matrix} (x & x & x & x) & (x & x & x) & (x & x) \\ 1 & 2 & 3 & 4 & 2 & 3 & 4 & 3 & 4 \end{matrix} \end{bmatrix} \cdot V$$

$$\text{Inverse F} = \frac{1}{\begin{matrix} x & x & x & x \\ 1 & 2 & 3 & 4 \end{matrix}} \begin{bmatrix} 1 & & & \\ & x & & \\ & 1 & x & x \\ & & 1 & 2 & x & x & x \\ & & & & 1 & 2 & 3 \end{bmatrix} \cdot V$$

and generally,

$$F^{-1} = \frac{1}{\begin{matrix} x & x & \dots & x \\ 1 & 2 & & i \end{matrix}} \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & x & & & 0 \\ & 1 & x & x & \\ & & 1 & 2 & \dots \\ & & & & \dots & x & x & \dots & x \\ 0 & \dots & 0 & & & 1 & 2 & & i-1 \end{bmatrix} \cdot V$$

where x_i are the diagonal elements of the lower triangular matrix L .

It can be seen $x_1 x_2 \dots x_i$ is the determinant obtained in the usual

way of multiplying the diagonal elements, all together, of the lower triangular matrix. The final form of the inverse is in an ordinary multiplication form of polynomial matrices which is much easier to program in a computer.

A general prove of the algorithm can be shown by the mathematical induction technique.

7.6 Partial Fraction

Partial fraction expansion is an indispensable technique in various engineering and mathematical systems. The most common application of the technique is found in Laplace transformations and Laplace transformation inverse methods. In control systems design it is quite unusual not to find a problem where partial fraction expansion is not needed. Partial fraction expansion will be needed in this study to solve the transformation problem, mentioned in chapter (8), of the Laplace domain into a ring z of polynomials.

7.6.1 Algorithm Theory

A general procedure which can evaluate proper Laplace transform expressions into a partial fraction form is described below. Some basic definitions in the complex theory in connection with the algorithm will be mentioned. The algorithm produces a partial fraction of a transfer function, $G(s)$, by calculating the residues of the poles of $G(s)$.

$$G(s) = \frac{N(s)}{D(s)} = \frac{G_1}{s+a_1} + \frac{G_2}{s+a_2} + \dots + \frac{G_n}{s+a_n}$$

where a_i , $i=1, \dots, n$ may be real or complex.

There are two mild conditions on $G(s)$ for it to be amenable to the algorithm:

1- $G(s)$ is required to have no non-zero multiple poles.

There may be any multiples of zero poles,

2- $G(s)$ is strictly proper. $\lim_{s \rightarrow \infty} G(s) \rightarrow 0$

Condition (1) above may be lifted for a practical problem. When multiple non-zero poles actually exist in the physical system being studied, they may be represented by two poles displaced by a slight amount, without appreciably affecting the results.

e.g. $s^2 + 2as + (a^2 - e^2) = (s+a+e)(s+a-e)$

The second restriction is usually satisfied for physical systems which are properly modeled. It is a criterion for dynamic systems.

7.6.2 Infinite Series for Functions of a complex Variable:

Since derivatives of all orders exist for a function $G(s)$ that is analytic at a point s_0 , in the complex plane, then the analytic function $G(s)$ may be expanded about this point in a Taylor series.

$$G(s) = \sum_{n=0}^{\infty} \frac{G^{(n)}(s_0)}{n!} (s - s_0)^n$$

This Taylor series has special characteristic in the complex plane:

- 1- It is unique, so any method of generating a power series is permissible,
- 2- it converges for all values of s within a circle of convergency $|s - s_0|$ centred at s_0 within which the function is analytic at all points. Thus no test for convergence is needed in the complex variable case.

Use of the Cauchy integral formula proves that the remainder terms of the series approach zero as the number of terms approaches infinity.

The Laurent series is useful for expanding about s_0 a function $G(s)$ which is analytic on the boundaries and within the annulus formed by two concentric circles C_1 and C_2 about the point s_0 . The function $G(s)$ may or may not be analytic at the point s_0 .

$$G(s) = \sum_{n=0}^{\infty} a_n (s-s_0)^n + \sum_{n=1}^{\infty} b_n / (s-s_0)^n$$

where $a_n = \frac{1}{2\pi j} \oint_{C_1} G(s) ds / (s-s_0)^{n+1} \quad n=0,1,2,\dots$

$$b_n = \frac{1}{2\pi j} \oint_{C_1} G(s) ds / (s-s_0)^{-n+1} \quad n = 1,2,3,\dots$$

- i- If $b_i = 0$ for $i \geq n+1$, then the principal part of Laurent series has a finite number of terms, and the isolated singular point s_0 is called a pole of order n of the function $G(s)$.
- ii- When the principal part of Laurent series has an infinite number of terms the point s_0 is called an essential singular point of the function $G(s)$.

7.6.3 Residue Theorem :

Suppose that $G(s)$ has a pole of order n and has the following Laurent series expansion

$$G(s) = \frac{b_1}{(s-s_0)} + \frac{b_2}{(s-s_0)^2} + \dots + \frac{b_n}{(s-s_0)^n} + \sum_{m=0}^{\infty} a_m (s-s_0)^m \quad (A)$$

If this series is multiplied by the factor $(s-s_0)^n$ the resulting series $F(s)$ is

$$F(s) = b_1 (s-s_0)^{n-1} + b_2 (s-s_0)^{n-2} + \dots + b_n + \sum_{m=0}^{\infty} a_m (s-s_0)^{m+n} \quad (B)$$

The series (B) is valid in a neighborhood of s_0 , including the point s_0 itself; the series is a convergent power series and is

analytic at s_0 ; thus $F(s_0) = \lim_{s \rightarrow s_0} [(s-s_0)^n G(s)] = b_n$, $b_n \neq 0$

Recognizing that the series (B) is a Taylor series expansion of $F(s)$ about the point s_0 the coefficient b_1 may be written

$$b_1 = F^{(n-1)}(s_0)/(n-1)!$$

where $F^{(n-1)}$ is the $(n-1)$ th derivative of $F(s)$ w.r.t. s , evaluated at $s=s_0$.

The coefficient b_1 in the Laurent series expansion of a function is defined to be the residue of the function at the isolated singular point s_0 . If $G(s)$ has a simple pole at a point $s=a$, the corresponding Laurent series is of the form

$$G(s) = \frac{b_1}{(s-a)} + \sum_{m=0}^{\infty} a_m s^m \quad \text{where } b_1 \neq 0.$$

Multiplying both sides by $(s-a)$, we have

$$(s-a)G(s) = b_1 + (s-a) \sum_{m=0}^{\infty} a_m s^m.$$

If we let s approaches a , the right hand side approaches b_1 and we obtain

$$\text{Residue of } G(a) = \frac{b_1}{1} = \lim_{s \rightarrow a} (s-a)G(s) \quad \text{as } s \rightarrow a.$$

In most of our application we are given $G(s)$ as a ratio of two polynomials in s .

$$G(s) = N(s) / D(s).$$

If $G(s)$ has a simple pole at $s=a$ and $N(s)$ and $D(s)$ are analytic at $s=a$, $N(a) \neq 0$ and $D(s)$ has a simple zero at $s=a$.

7.6.4 Computer Algorithm

- 1- Given $G(s) = N(s)/D(s)$, $N(s)$ of order m and $D(s)$ of order p .
- 2- Zeros of $N(s)$ and $D(s)$ are found by Newton-Raphson method.
- 3- Then $G(s)$ can be written in the following form

$$G(s) = \frac{K \prod_{i=1}^q (s - a_i)}{s^u \prod_{i=1}^p (s - b_i)}$$

where $q=m$, $p+u=n$ and K is the root locus or open loop gain.

The corresponding partial fraction expansion of $G(s)$ may be written as

$$G(s) = \sum_{i=1}^u \frac{\alpha_i}{s} + \sum_{i=1}^p \frac{\beta_i}{(s - b_i)}$$

where α_i and β_i by the residue theorem above

$$\beta_i = \frac{K \prod_{i=1}^q (b_i - a_i)}{b_i^u \prod_{\substack{j=1 \\ j \neq i}}^p (b_i - b_j)}$$

$$\alpha_i = K d_{pq}^{-1} d_{iu} - \sum_{j=1}^p \frac{b_j^{i-1}}{j} \beta_j, \quad 1 \leq i \leq u$$

$$d_{ij} \text{ is the Kronecker } \delta \text{ function} \quad d_{ij} = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$$

a_i and b_i , the zeros of $N(s)$ and $D(s)$ respectively may be real or complex.

7.7 Multiplication of Polynomial Matrices

In the multiplication of polynomial matrices defined over principal ideal domains, it has been found in the development of APA that the occurrence of the coefficients of multiplication is recursive. The result of polynomial multiplication can be generated from the coefficients only. The following example demonstrates this fact.

Example

A and B are scalar polynomials of max. degree q, then if

$$G = A.B$$

the kth coefficient of G is produced according to the formula below;

$$G(k) = \sum_{i=0}^k A(i).B(k-i)$$

where $G(k)$ is the k's coefficient in polynomial G

$A(i)$ is the i's coefficient in polynomial A

$B(k-i)$ is the (k-i)'s coefficient in polynomial B

The formula above is also contained within another loop to give the complete result of multiplication.

$$G = \sum_{k=0}^q \sum_{i=0}^k A(i).B(k-i)$$

where q is the max. degree in A and B.

A formal proof of the results above can be obtained by mathematical induction.

7.8 Multiplication of a Polynomial Matrix by Its Transpose Inverted Indeterminate

In the design of modern Wiener-Hopf controller, in the z^{-1} domain, or more generally over the ring of delay operators, it is often necessary to perform a multiplication of a polynomial matrix by its transpose inverted indeterminate, i.e., a polynomial matrix $F(z^{-1})$ is multiplied by $F^*(z)$ is met. This multiplication has certain features, and use will be made of them in developing an algorithm for such multiplications.

The output of multiplying $F(z^{-1}) \cdot F^*(z)$ is a function of z and z^{-1} ;

$$G(z, z^{-1}) = F^*(z) \cdot F(z^{-1})$$

One of the noticable features of $G(z, z^{-1})$, is that it is a linear sum of two functions.

$$G(z, z^{-1}) = G_1(z) + G_2(z^{-1}) + \text{constant}$$

Here all of F, G, G_1 , and G_2 are polynomial matrices.

Another property is that the coefficients in G_1 are the symmetric transpose of those in G_2 .

In fact, these two important features have helped in developing a simple program to carry out the multiplication

$$F'(z) \cdot F(z)^{-1}.$$

The multiplication is carried out, first, as

$F'(z)^{-1} \cdot F(z)^{-1}$, and then $F'(z)$ is retained. Another point of interest to note is that :

$$\begin{aligned} F(z)^{-1} \cdot F(z) &= G'(z)^{-1}, z \\ &= G_1'(z)^{-1} + G_2'(z) + \text{constant} \end{aligned}$$

7.9 Summation of Rational Polynomial Matrices

In operation with transfer functions, there is always much algebraic symbolic manipulation of rational functions. Carrying out such operations manually consumes a lot of time and considerable effort. This activity is subject to human error. Thus if these operations can be automated then that will lift an unnecessary burden from the designer.

The time and accuracy may be gained by this automation is considerable.

Algorithm

$$\frac{N(s)}{D(s)} = \frac{P_1(s)}{Q_1(s)} + \frac{P_2(s)}{Q_2(s)} + \dots + \frac{P_r(s)}{Q_r(s)}$$

This can be considered as a sequence of nested summations:

$$[\dots [\frac{P_1(s)}{Q_1(s)} + \frac{P_2(s)}{Q_2(s)}] + \frac{P_3(s)}{Q_3(s)}] + \dots + \frac{P_r(s)}{Q_r(s)}$$

The summation can be evaluated by performing the indicated operations sequentially beginning with the operations contained in the inner brackets and working outward. If the definition

$$\frac{N_1(s)}{D_1(s)} = \frac{P_1(s)}{Q_1(s)}$$

is made, a typical step in the evaluation is given by

$$\frac{N_{k+1}(s)}{D_{k+1}(s)} = \frac{N_k(s)}{D_k(s)} + \frac{P_{k+1}(s)}{Q_{k+1}(s)}$$

$$= \frac{N_k(s)Q_{k+1}(s) + P_{k+1}(s)D_k(s)}{D_k(s)Q_{k+1}(s)} \quad k=1,2,\dots,r$$

where in each step the indicated polynomial multiplication and additions are performed to yield a resulting ratio of two simple polynomials.

7.10 Solution of The Bezout Identity

Given two polynomial matrices $A(z)$ and $B(z)$, if A and B are left coprime then there exists X and Y such that

$$AX + BY = I$$

where I is the identity matrix, and X and Y are polynomial matrices.

The identity above can be written as

$$[A \ B] \begin{bmatrix} X \\ Y \end{bmatrix} = I$$

The matrix $[A \ B]$ can be reduced by elementary operations to $[I \ 0]$ where I is the identity matrix. The reduction of $[A \ B]$ is carried out in the usual way with operation image on the companion matrix, the identity matrix. A and B being left coprime guarantee the existence of X and Y .

7.11 Time Response of Transfer Functions

A set of programs have been written to simulate transfer functions, in the s-domain, to anticipate their time responses. The program basically implements the controller-realization type of realization of transfer functions. They have different facilities to suit various requirements. Some of the programs have the facility to include pure time delay functions,

e^{-ts} , in the numerator and the denominator, others to include integral multiples of a pure time delay, e^{-ts} , e^{-2ts} , e^{-3ts} , etc, and to include transfer functions with different time delays, e^{-t_1s} , e^{-t_2s} , $e^{-(t_1s+t_2s)}$, etc.

The basic algorithm structure is as follows :

Consider the following transfer function

$$\frac{y}{x} = \frac{a_0 + a_1 s + a_2 s^2 + \dots + a_r s^r}{b_0 + b_1 s + b_2 s^2 + \dots + b_q s^q}$$

For a realizable system $q \geq r$.

Divide by s^q

$$\frac{y}{x} = \frac{a_0 s^{-q} + a_1 s^{-q+1} + \dots + a_r s^{-q+r}}{b_0 s^{-q} + b_1 s^{-q+1} + \dots + b_q} = E$$

$$x = E(b_q + \dots + b_1 s^{-(q+1)} + b_0 s^{-q})$$

$$E b_q = x - E(b_{q-1} s + b_{q-2} s^2 + \dots + b_1 s^{-(q+1)} + b_0 s^{-q})$$

$$y = E(a_0 s^{-q} + a_1 s^{-(q+1)} + \dots + a_r s^{-(q+r)})$$

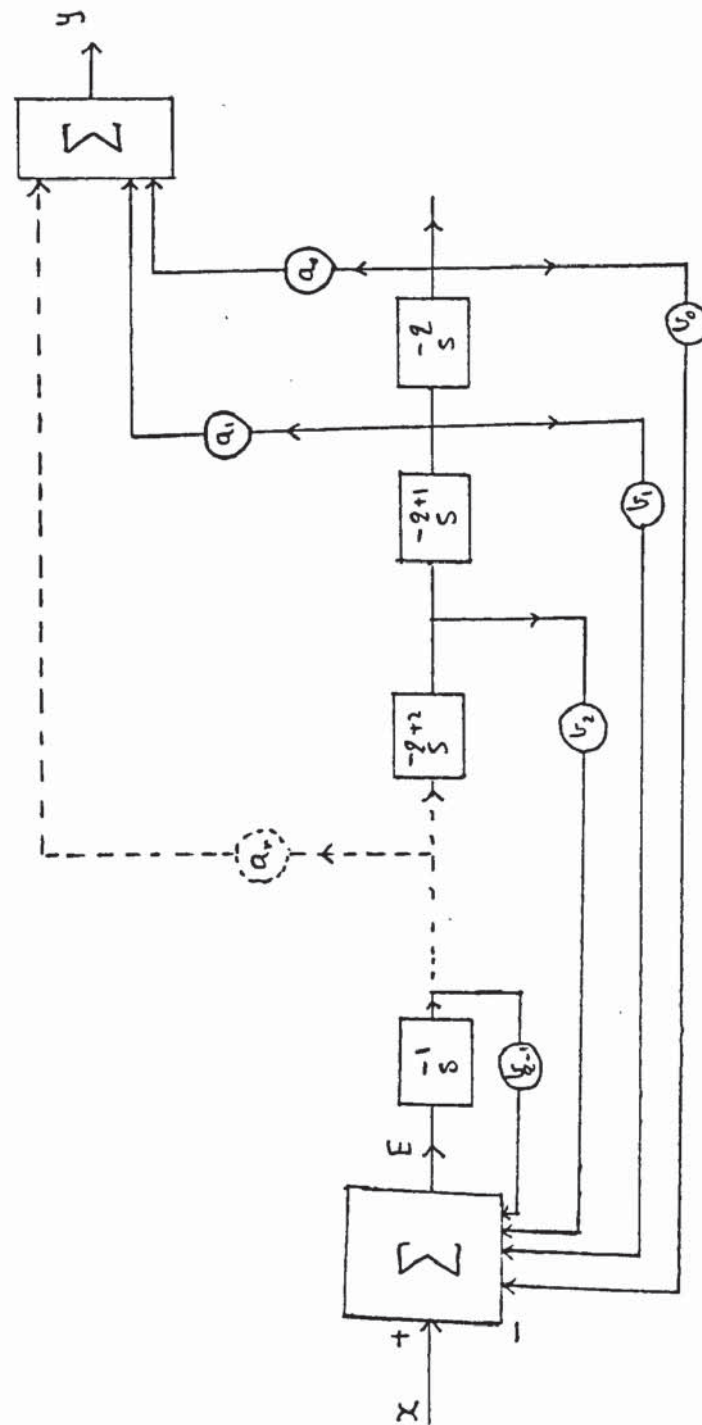


Fig. 7.1: Controller-realization of transfer functions.

Each element of a and b in the diagram above fig [7.1] is divided by b, (e.g. $\frac{a}{q} = \frac{a}{r} \cdot \frac{r}{q}$), or in other words make the

denominator in the transfer functions monic.

s^{-1} represents an integrator in the s-domain ; so,

s^{-2} is double integration, and so forth . The structure above, fig. [7.1], has been implemented in a digital computer software with a step wise input function at x.

7.12 EXAMPLES

The following are some simple test data for the library. The results obtained were almost exact. These data and examples were kept in separate testing programs; they carried the name TESTn where n is the number of the test TEST1, TEST2, etc.

$P_{dn} = 7$

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$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 1 & 1 \\ 4 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 \\ 6 & -1 & 0 & 0 \\ 7 & 1 & 0 & 0 \\ 8 & -1 & 0 & 0 \\ 9 & -1 & 0 & 0 \\ 10 & 1 & 0 & 0 \\ 11 & 0 & 0 & 0 \end{bmatrix}$$

| THE DETERMINANT | | = | | | 2 - 2z | | |
|-----------------|---|-------------|--------------|---|--------|---|--|
| 17 | 1 | 2.000000000 | 0 | 1 | 1 | 0 | |
| 18 | 2 | 0.000000000 | 1 | 1 | 0 | 0 | |
| 19 | 0 | | | | | | |
| 20 | 1 | | | | | | |
| 21 | 1 | | | | | | |
| 22 | 1 | 0.000000000 | 1.000000000 | | | | |
| 23 | 1 | 0.000000000 | 1.000000000 | | | | |
| 24 | 1 | 2.000000000 | -1.000000000 | | | | |
| 25 | 1 | 1.000000000 | 1.000000000 | | | | |
| 26 | 2 | 1.000000000 | 1.000000000 | | | | |
| 27 | 2 | 1.000000000 | 1.000000000 | | | | |
| 28 | 2 | 1.000000000 | 1.000000000 | | | | |
| 29 | 2 | 1.000000000 | 1.000000000 | | | | |
| 30 | 2 | 1.000000000 | 1.000000000 | | | | |
| 31 | 3 | 1.000000000 | 1.000000000 | | | | |
| 32 | 3 | 1.000000000 | 1.000000000 | | | | |
| 33 | 3 | 1.000000000 | 1.000000000 | | | | |
| 34 | 3 | 1.000000000 | 1.000000000 | | | | |
| 35 | 3 | 1.000000000 | 1.000000000 | | | | |
| 36 | 3 | 1.000000000 | 1.000000000 | | | | |
| 37 | 3 | 1.000000000 | 1.000000000 | | | | |
| 38 | 3 | 1.000000000 | 1.000000000 | | | | |

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THE LIBRARY COMPENDIUM

1- RREDUCE (FF,NROW,NCOLUMN,DD,VV,DV)

This subroutine performs the elementary column operations on FF. On exit, FF is quasi-lower triangular and VV in entry to the subroutine is the identity matrix and on exit it contains two right coprime matrices.

NCOLUMN is usually greater than or equal to NROW otherwise simple adjustments in the subroutine may be needed. Since NCOLUMN is greater than NROW, the dimension of VV is NCOLUMN x NCOLUMN.

The right coprime pair of FF are obtained from VV.

FF is built up from two matrices A and B such that

$$P = A^{-1} B, \quad A \text{ is non singular}$$

A is mxm

B is mxp

$$FF = [A \ B] \text{ so it is } mx(m+p).$$

The greatest right common divisor, L, is mxm

$$[A \ B] \begin{bmatrix} A_l & A_r \\ B_l & B_r \end{bmatrix} = [L \ 0]$$

W is $(m+p) \times (m+p)$

Ar is $m \times p$ (p starts from m+1)

Br is $p \times p$ (p starts from m+1)

$$P = (-Ar)Br^{-1}$$

m = NROW

m+p = NCOLUMN , (NCOLUMN \geq NROW as mentioned above)

DD and DV are integer matrices pointing to the polynomials degrees of each element in FF and VV respectively.

2- CREDUCE (FF,NROW,NCOLUMN,DD,VV,DV)

This subroutine performs elementary row operations on FF to reduce it into a quasi-upper triangular matrix. NROW and NCOLUMN are the row and column dimensions of FF. In this algorithm NROW is usually greater than NCOLUMN. VV on entry is the identity matrix of dimension NROWxNROW, on exit VV contains two left coprime matrices.

DD and DV are integer matrices pointing to the polynomial degree of each element in FF and VV respectively.

FF is composed of two matrices A and B such that:

$$P = AB^{-1} \quad B \text{ is non-singular}$$

$$F = \begin{bmatrix} B \\ A \end{bmatrix}$$

B is mxm

A is pxm

$$\begin{bmatrix} B2 & A2 \\ B1 & A1 \end{bmatrix} \begin{bmatrix} B \\ A \end{bmatrix} = \begin{bmatrix} U \\ 0 \end{bmatrix}$$

The greatest common divisor U is mxm

$$AB^{-1} = A1(-B1)^{-1}$$

where A1 and B1 are the left coprime pairs describing P.

A1 is pxp

B1 is pxm

3- RCHANG (FF,NROW,NCOLUMN,DD,VV,DV,IRR)

This subroutine is used within subroutine RREDUCE and REDUC1. It searches for the minimum polynomial degree along a column or a row, depending on the value of IRR, and brings this minimum polynomial to the diagonal position in matrix FF. IRR is an integer. If the elementary operations on FF is to be on a column wise then IRR points to the row where the column operation is on.

When RCHANG finds the minimum polynomial degree it brings that element to the diagonal position (IRR,IRR) and the two columns are interchanged, that is the original column which was containing the minimum polynomial degree and the column IRR. The same column interchange will take place in W; but, indeed, without searching for a minimum polynomial.

FF is the real coefficient matrix where the elementary column operation is taking place. VV was the identity matrix on entry and carries the unimodular matrix on exit from RREDUCE.

NROW and NCOLUMN are the row and column dimensions of FF respectively and VV is NCOLUMNxNCOLUMN usually NCOLUMN = > NROW.

DD and DV are the respective polynomial degrees of FF and VV and are of type integer. When column

interchange takes place on FF and VV, DD and DV are adjusted accordingly.

4- OCHANG (FF,NROW,NCOLUMN,DD,VV,DV,JRR)

OCHANG performs row interchange within matrix FF and the associated matrix VV. The subroutine is used within subroutine CREDUCE. It searches for a minimum polynomial degree along column JRR. When the subroutine finds the minimum degree polynomial, interchange of rows will take place. The row which contains that minimum polynomial is interchanged with the row JRR. This will bring the minimum polynomial degree to the diagonal element position, (JRR,JRR). The corresponding rows in VV are also interchanged but irrespective of any polynomial degree position.

The subroutine is called in subroutine CREDUCE whenever an element along column JRR is less in degree than the degree of the diagonal element (JRR,JRR). During the rows interchange the subroutine keeps track of the various element degree in FF and VV. DD and DV are the integer matrices which point to the various polynomial degrees in FF and VV respectively.

NROW and NCOLUMN are the row and column dimensions of FF, and VV is (NROW,NCOLUMN). The subroutine is implementable when $NROW \geq NCOLUMN$.

5- MULSPECT (ZZ,NROW,DD,AA)

MULSPECT is to perform spectral factorization in matrix ZZ. ZZ is such that :

i) square matrix NROWxNROW

ii) Para-Hermitian, i.e $ZZ(z^{-1}) = ZZ^*(z)$

iii) is non-singular,

iv) of rank NROW (almost every where)

v) positive definite for every real w, i.e.

$$ZZ(e^{iw}) > 0.$$

DD is an integer matrix which carries the polynomial degree of elements ZZ. The result of factorization goes to matrix AA. On exit AA carries the final result and DA is pointing to the degrees of the polynomial elements in AA.

ZZ is usually formed as a result of operation by subroutine POLMUL. If WV is due to a multiplication of the form;

$$ZZ = V.V^* \quad (\text{left factorization})$$

the result $ZZ = A.A^*$, A is the output.

But if ZZ has come due to

$$ZZ = V.V^*$$

then firstly, ZZ is transposed and the result A is transposed to give the final result, of right

factorization. The output from subroutine POLMUL is a multiplication of the following form:

$$ZZ = V \underset{*}{.} V$$

6- SPECSC (DEN1,ND1,AL)

This is the spectral factorization of a scalar polynomial. DEN1 holds the polynomial coefficients. As explained in the theory part, the polynomial which needs to be factorized has the following form;

$$Z = a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z + a_0$$

DEN1 holds the coefficient from a_0 to a_n . ND1 is the degree of the polynomial. The result of factorization goes to AL and it has the same degree as the original matrix.

AL and DEN1 are real matrices of dimensions (0:20).

ND1 is an integer, it is the degree of AL and also equal to the number of coefficients in DEN1 minus 1.

7- POLMUL (FF,FFTZ,ZZ,DDT,DD,DDZ,NROW,NCOLUMN)

This subroutine performs a special type of matrices multiplication. It carries out the multiplication of $FF(z^{-1})$ by $FF^*(z)$ that is the transpose of FF with the inversion of the polynomial variable z. The final result of this multiplication is of two parts;

$$FFTZ.FF = G(z^{-1}) + G^*(z)$$

The first part of the result is G and is a function of z^{-1} and the second part is G^* , a function of z. We need not to keep both of the results since they contain the same information. So, matrix ZZ holds the first part which is a function of z. The second part can be obtained by the transpose of ZZ. Note, the order of the multiplication FFTZ.FF. More details of this multiplication are shown in the theoretical part of the algorithm.

FF is NROWxNCOLUMN, FFTZ is NCOLUMNxNROW and ZZ is NCOLUMNxNCOLUMN. DD,DDT and DDZ are the respective polynomial degrees of FF, FFTZ and ZZ, respectively. They are of type integer.

FF is (NROW,NCOLUMN,0:25)

FFTZ is (NCOLUMN,NROW,0:25)

ZZ is (NCOLUMN,NCOLUMN,0:30)

ZZ is used directly in subroutine MULSPECT in producing the spectral factorization of (FFTZ.FF). This is the right factorization. To get the left factorization transpose ZZ before entering subroutine MULSPECT and again transpose the final result from MULSPECT to get the spectral factorization of (FF.FFTZ).

8- TRANPOSE (FF,FFT,DD,DDT,NROW,NCOLUMN)

This is a simple subroutine which transpose the polynomial matrix $FF(z^{-1})$ to $FFT(z^{-1})$. FF and FFT are of type real, DD and DDT are two integer matrices holding the degrees of the polynomial elements of FF and FFT respectively.

FF is (NROW,NCOLUMN,0:25)

FFT is (NCOLUMN,NROW,0:25)

DD is (NROW,NCOLUMN)

DDT is (NCOLUMN,NROW)

9- TRANFO (FFT,FFTZ,DDT,NROW,NCOLUMN)

This is another simple subroutine which is used in conjunction with subroutine TRANPOSE. TRANFO transforms the transpose of $FF(Z^{-1})$, i.e. $FFT(Z^{-1})$ to $FFT(Z^{-1})$. The polynomial degrees of the elements of $FFT(Z^{-1})$ of $FFT(Z)$ are the same but they have different coefficients.

FFT has dimension (NCOLUMN, NROW, 0:)

FFTZ is (NCOLUMN, NROW, 0:)

10- CONJTRA (FF,FFZ,DD,DDZ,NROW,NCOLUMN)

This subroutine combines the effect of both subroutines TRANSPOSE + TRANFO. In effect CONJTRA transforms the polynomial matrix FF into its transpose inverse of variable, $FF(Z^{-1})$ to $FF^*(Z)$.

FFZ is the transposed inverse of variable of FF, DD and DDZ are integer matrices holding the degree of polynomial elements of FF and FFZ respectively.

FF is (NROW,NCOLUMN,0:20)

FFZ is (NCOLUMN,NROW,0:20)

DD is (NROW,NCOLUMN)

DDZ is (NCOLUMN,NROW)

11- POLPOL (FF,F1,GG,DD,DF,DG,NROW,NCOLUMN,NCOLUMNF1)

POLPOL executes the multiplication of two polynomial matrices, FF & F1 in the following order :

$$GG = FF:F1$$

FF, F1 and GG are all of type real and they have the given below dimensions:

| | |
|----|--------------------------|
| FF | (NROW,NCOLUMN,0:25) |
| F1 | (NCOLUMN,NCOLUMNF1,0:25) |
| GG | (NROW,NCOLUMNF1,0:30) |

The number of columns of FF should be equal to the number of rows of F1 otherwise the multiplication is not defined. GG holds the final result of the multiplication. DD, DF, and DG are integer matrices pointing to the respective degrees of polynomial elements in FF, F1, and GG respectively.

As well as the multiplication of polynomial matrices the subroutine can perform scalar multiplication by putting NROW = NCOLUMN = NCOLUMNF1 = 1 and put the required scalars to be multiplied in FF and F1 in a one dimension form. This time the order of multiplication is immaterial.

12- DIAGONAL (FF,DF,DF1,DEN1,NB,NROW)

This subroutine is used in multiplying the diagonal elements of the reduced matrix FF. It produces another diagonal matrix which is put again in FF. The row elements of FF are obtained by multiplying all the diagonal elements of FF except for the element which will be substituted. In other words the row element f_{ii} of FF is

found by multiplying $f_{jj} f_{j+1,j+1} \dots f_{NLOW,NLOW}$ $j = i$.

Those new elements occupy the diagonal of matrix FF.

An outcome of this process of multiplication is that the determinant of FF is calculated and kept in DEN1. Its degree is kept in NB. The value of FF is completely changed on exit from this subroutine. DF1 is the integer matrix pointing to the degrees of the polynomials in the new FF. DF is the integer matrix showing the polynomial degrees of FF on entry to the subroutine. DIAGONAL is used with subroutine INVERH to find the inverse of polynomial matrix FF. But subroutine INVERH produces unnecessary redundant poles and zeros hence the resulting output inverse is of high degree in the numerator matrix and in the denominator, that is the determinant.

The arrays have the following dimensions:

FF — (NROW,NROW,0:6)

DF — (NROW NROW)

DF1 — (NROW)

DEN1 (0:25)

13- DETDIAG (FF,DD,DEN1,ND1,NROW)

This subroutine multiplies the diagonal elements of the polynomial matrix FF. The result goes to DEN1. Usually, this subroutine is used in finding the determinant of polynomial matrices. It is called after the reduction of FF by REDUC1 to a lower triangular polynomial matrix. The polynomial degree of the determinant is kept in ND1.

FF and DEN1 are matrices of type real, holding the coefficients of the polynomials. DD is an integer matrix showing the degrees of the polynomial in FF.

FF(NROW,NROW,0:25)

DEN1(0:30)

DD(NROW,NROW)

Note: It is not unusual that the determinants are found for square matrices so in this subroutine FF is a square matrix.

This subroutine is used in subroutine INVERT in finding the inverse of polynomial matrices and in subroutine DET in obtaining the determinant of polynomial matrices.

14- DIAINV (FF,FF1,DD,DF1,NROW)

This is an special purpose subroutine and is called within subroutine INVERT. After the reduction of FF by REDUC1 to a lower diagonal form this subroutine DIAINV, picks up the polynomial diagonal elements of FF and through a special calculation process it produces another polynomial diagonal matrix FF1. Then FF1 is used in finding the inverse of FF by multiplying it by another matrix, which has been produced from REDUC1 and OFFDIG.

$$FF^{-1} = VV.FF1$$

DIAINV is one of the family of subroutines which are called from subroutine INVERT. The matrices dimensions are as follows:

FF(NROW,NROW,0:25)

FF1(NROW,NROW,0:30)

DD and DF1 (NROW,NROW)

Note that all the matrices here are square matrices because mainly this subroutine is used in finding the inverse of polynomial matrices and for the inverse to exist FF must be square.

15- REDUCL (FF,NROW,DD,VV,DV,II,JJ)

REDUCL is used to reduce a polynomial matrix FF to a lower triangular matrix through elementary column operations. In this aspect it is very similar to subroutine RREDUCE. The only difference is that the column and row operations in REDUCL are controlled outside the subroutine body and the columns and rows are called as arguments, II and JJ.

On entry VV is the identity matrix but since the operation on FF is simultaneously taking place on VV on exit VV is a unimodular polynomial matrix. VV is generated within REDUCL.

Subroutine REDUCL is used when finding inverses and determinants of polynomial matrices. Although REDUCL can operate on any matrices dimension by controlling II, JJ, here the matrices are dimensioned in a square fashion as follows:

FF(NROW,NROW,0:25), DD(NROW,NROW)

VV(NROW NROW 0:30), D(NROW,NROW)

On exit the value of FF is changed. As mentioned above II and JJ are controlling the row and column limits of execution, and also the order of execution i.e the looping through rows or columns.

DD and DV are integer matrices pointing to
the polynomial degrees of FF and VV respectively.

16- OFFDIG (FF,VV,DD,DV,NROW)

OFFDIG is a subroutine used within subroutine INVERT. It performs non-elementary operations on FF and the associated matrix with FF, matrix W. This subroutine is called after exit from subroutine REDUC1. On exit from REDUC1 matrix FF is of lower triangular form. OFFDIG will take this lower triangular matrix FF, and execute non-elementary operations to reduce FF into a diagonal matrix. At the same time these operations are imaged on W.

DD and DV are integer matrices pointing to the polynomial degrees of FF and VV respectively. The matrices have the following dimensions :

FF(NROW,NROW,0:30)

VV(NROW,NROW,0:30)

17- INVERT (FF,NROW,DEN1,NB,DD,VO,DVO,VV,DV,DF1)

INVERT is to invert a polynomial matrix FF and the inversion result goes to matrix VO and the determinant of FF goes to DEN1.

INVERT calls other subroutines within its procedure body. It calls REDUC1, DETDIAG, DIAINV, OFFDIG and POLPOL.

FF is a square matrix which is required to be inverted and it is of dimension (NROW,NROW,0:25).

VO is the adjoint of FF (numerator matrix of FF^{-1}) and it is of dimension (NROW,NROW,0:30).

DEN1 is the polynomial determinant of FF and it is of dimension (0:30).

NB is an integer and is the polynomial degree of DEN1, determinant of FF.

DF1, DVO, DD and DV are integer matrices holding the respective polynomial degrees of elements FF1, FF, VO and VV respectively. DF1 and FF1 are produced by subroutine DIAINV which is called within INVERT.

The subroutine does not test whether the matrix FF is invertible or not. If the value of the determinant is zero this implies matrix FF is singular and the inverse does not exist, however the program does not crash in that case.

18- POMSMU (Z1,Z2,D1,D2,GG,DG,NROW,NCOLUMN)

This subroutine carries out the multiplication of a polynomial matrix Z2 by scalar polynomial Z1. Z1 multiplies every element of Z2. The resultant matrix of this multiplication is GG. POMSMU is useful when addition of two rational polynomial matrices arises.

D2 and DG are integers matrices pointing to the degree of various polynomial elements in Z2 and GG respectively. D1 is an integer showing the degree of the scalar polynomial, Z1.

Z1 (0:20)

Z2 (NROW,NCOLUMN,0:20)

GG (NROW,NCOLUMN,0:20)

D1 an integer

D2 (NROW,NROW)

DG (NROW,NCOLUMN)

19- DET (FF,NROW,DD,VV,DV,DEN1,NB)

DET finds the polynomial determinant of matrix FF and puts the result in DEN1. NB is the degree of DEN1.

DET calls other subroutines in finding the determinant. It calls subroutines REDUC1, DETDIAG, and RCHANG.

REDUC1 is described above, it reduces polynomial matrix FF through elementary column operations into a lower triangular matrix. The determinant of FF is then found by multiplying all the diagonal elements of the reduced matrix FF.

VV is the usual companion matrix to FF. It has no role in this subroutine DET but it is there as a dummy argument for subroutines REDUC, RCHANG and DETDIAG.

Matrix FF must be a square matrix of dimension NROWxNROW. If FF is singular then all the elements in DEN1 will be zeros. DD and DV are integer matrixes holding the degree of polynomial elements of FF and W respectively.

FF (NROW,NROW,0:25)

VV (NROW,NROW,0:30)

DD (NROW,NROW)

DV (NROW,NROW)

DEN1 (Ø:3Ø)

20- POLYADD (Z1,Z2,D1,D2,NROW,NCOLUMN,VV,DV)

POLYADD is to add algebraically two polynomial matrices Z1 and Z2 and to keep the result in VV. Z1 and Z2 may be any polynomial matrices with different degrees; but, indeed for addition of matrices to exist the number of columns in Z1 and Z2 should be equal, as well as the number of rows. D1 and D2 are integer matrices holding the respective polynomial degrees of Z1 and Z2. DV is also an integer matrix holding the degree of VV, the resultant matrix of addition.

Addition of matrices is commutative, so the order of Z1 and Z2 in the procedure name is immaterial, but that D1 and D2 should follow the order of Z1 and Z2.

Z1 (NROW,NCOLUMN,0:20)

Z2 (NROW,NCOLUMN,0:20)

D1 (NROW,NCOLUMN)

D2 (NROW,NCOLUMN)

VV (NROW,NCOLUMN,0:20)

DV (NROW,NCOLUMN)

21- READP (NROW,NCOLUMN,DD,FF)

READP inputs the initial components of polynomial matrix FF. It first reads the dimension of matrix FF, NROW and NCOLUMN. This is followed by reading in DD, the degree of every polynomial element in FF. After the degrees have been read the coefficients of polynomial matrix FF are read element by element and row by row. That is, it reads all the element coefficients in the first row then followed by the second row and so forth. The files which contain the data should be connected to channel 40. The data is read in list-directed format.

FF (NROW,NCOLUMN,0:20),

DD (NROW,NCOLUMN)

24- READS (ND,DEN1)

READS is a subroutine to read in the degree and coefficients of scalar polynomials. It first reads ND, the degree of the polynomial. Then this is followed by reading the coefficients in array DEN1. Usually the coefficients are fed starting from the constant term in the polynomial, the constant not necessarily be a non-zero

constant, followed by the first degree term coefficient, up to the highest degree ND.

ND is an integer

DEN1 (0:20) real.

The files containing the data should be connected to channel 40. The data is read in list-directed format.

24- WRITP (NROW,NCOLUMN,DD,FF)

WRITP is similar to READP, but this time WRITP writes out the elements in matrix FF. The subroutine first writes the degree of each of the polynomial elements in FF then the position of the element in matrix FF followed by the coefficients in that element. The coefficients are printed starting from the constant terms in the polynomial element, including zero as a constant, and up to the highest order terms. The output goes out through channel 40, so channel 40 should be connected to the file required to hold the output.

DD (NROW,NCOLUMN)

FF (NROW,NCOLUMN,0:20)

25- WRITS (ND,DEN1)

Subroutine WRITS is to write out to a data file the coefficients of scalar polynomials. The subroutine first writes the degree of the polynomial DEN1 from the constant term and upwards with the degree. The

subroutine uses channel 40 to send the output to the data file.

26- SUMRAT (R,P,Q,M,N,N1,D1,DD1,DN1)

SUMRAT is a subroutine to sum algebraically rational polynomials. P is the numerator matrix and Q is the denominator matrix. R is the number of rational terms to be added. M and N are integer matrices which point to the polynomial degree of P and Q respectively. The final result of summation is kept in a rational form, with N1 as the numerator and D1 as the denominator with DN1 and DD1 integers equal to the degree of N1 and D1 respectively.

$$\frac{N1}{D1} = \frac{P1}{Q1} + \frac{P2}{Q2} + \dots + \frac{Pr}{Qr}$$

P (8,0:8)

Q (8,0:8)

N1 (0:30)

D1 (0:30)

M (10)

N (10)

27- ROOT (DEN1,ND1,XZ,YZ)

ROOT finds the roots of a scalar polynomial DEN1 and sends the result to XZ and YZ. The roots may be real or complex. The real part of the root is kept in matrix XZ and the imaginary part is held in matrix YZ.

DEN1 holds the coefficients of the scalar polynomials in the usual data structure. ND1 is the degree of polynomial DEN1 and of course is an integer.

ROOT is the only subroutine which utilizes another subroutine from the NAG library. It calls subroutines C02AEF, X01AAF, and X02AAF. This should be acknowledged here. But to make APA independent of any other rights a Newton-Raphson algorithm for finding roots of polynomials should be written.

DEN1 (0:ND1), XZ(ND1), YZ(ND1)

PROGRAM DISCR2

DISCR2 is an interactive FORTRAN-77 program to simulate the dynamics of discrete scalar systems. The program allows for various types of input function to the discrete system, step functions, ramp, and parabolic. The transfer function of the discrete form should be put into the following form

$$\frac{Y(z)}{U(z)} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n}}{1 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_m z^{-m}}$$

There is no restriction on m and n, may be greater, less or equal to n.

The input to the feedback array is the denominator of the transfer function and the feeding of data starts from b_m to b_1 . Care should be taken when $m < n$ then the first $(m-n)$ positions in the feedback array are zeros.

The coefficients of the polynomial in the numerator and denominator are all of type real, and when some of them are zero, zero should be fed as the input of the coefficient.

The final result of the simulation, $Y(z)$, is sent to a file connected to channel 40 at the beginning of the program. Other programs can read these data and plot them graphically in various ways.

Another different program is used to simulate multivariable discrete systems. This should be used in multi-input multi-output cases.

PROGRAM PARTIAL

This is an interactive FORTRAN-77 program, which finds the residues of rational functions defined over the complex field. The rational function should beforehand be expressed as zeros and poles plus a constant gain of type real. After feeding these values the program lists out the values of residue of each pole including poles at the origin. The program is limited to handling non multiple poles but poles at the origin can be of any integer multiplicity. The program being interactive, it is self explanatory.

The output data goes to an assigned data file to channel 30.

$$\frac{K \prod_{i=1}^q (s+a_i)}{s^u \prod_{i=1}^p (s+b_i)} = \sum_{i=1}^u \frac{L_i}{s^i} + \sum_{i=1}^p \frac{A_i}{(s+b_i)}$$

Partial fractions are needed in many applications. In this work, the transformation from the s-domain to the z-domain uses the partial fraction form as straight forward technique. In the design of Wiener-Hopf optimal controllers usually it is necessary to express some polynomial matrices in partial fraction form to separate

the residues on $|z| < 1$ from those on $|z| \geq 1$, where $|z|$ represents the unit circle in the z -domain.

Epilogue

APA stands for Algebraic Polynomial Algorithms. The library consists of about 30 subroutines which handle symbolically matrices and scalar polynomial quantities. The library is defined over a commutative ring of operations. The ring is not commutative in multiplication in $R^{m,n}$, and the polynomials coefficients are in the field $R^{m,n}$ of all real numbers. The polynomials treated are finite and constituted a principal ideal domain.

APA has been designed to fill partially the gap in symbolic manipulation of control systems design. The polynomial algebraic approach has shown powerful capabilities in tackling linear systems analysis and synthesis problem. The vast computational arduousness has hindered so far the wide application of this important method.

FORTRAN-77 has been used to construct this library, due to its wide application, recognition, and portability, especially in the engineering disciplines.

CHAPTER (8)

REAL-TIME IMPLEMENTATION OF CONTROLLERS

8.1 Introduction

By the late 1950's stored-program digital computers had become reliable and widely available engineering devices. This was a necessary prerequisite for the next developments in automatic control systems analysis and design as opposed to the analogue means practised for a long time. The advantages of these digital machine over other methods are now well acknowledged in the literature of applied control. It is now reasonable to attempt much deeper and more comprehensive studies in automatic control theory and applications since the computing power and versatility of the big scientific machines have made the lengthy and intricate calculations involved a practical proposition.

At the same time, the development of small and reliable special purpose digital computers has offered the possibility of the implementation of more ambitious control schemes using information-processing devices of unprecedented computing speed and flexibility. It is then a natural step in the complicated case of the process industry, to utilize these cheap, powerful machines in handling the huge information needed on a process plant,

at the same time performing the strict scheduling tasks specified in the capital process industry.

With the continual improvements in computer hardware from microprocessors and digital signal transducers to mainframe computers and sophisticated data transmission techniques, there has been an increased use of computers for the control of industrial processes. This has led to the design and implementation of distributed computer control for the realization of control tasks. These complex systems have four general characteristics which serve to represent a large class of control systems important in engineering applications:

- (i) Microprocessors and analogue based controllers are used for first, or lower level control functions.
- (ii) Minicomputers and large mainframes are used for higher level control functions,
- (iii) The structure is physically dispersed implying the need for a computer-to-computer communication system,
- (iv) Centralized operator interfaces have access to all measured or calculated process variables, control variables, and system status indications.

Primarily as a result of the low cost of microprocessors, memory, and peripheral interface chips, there has been a large increase in the use of these devices in industrial

control systems. Not only has there been a development of microprocessor-based controllers to mimic the functions of analogue controllers, but these new controllers have significant advantages over conventional ones. The range of functions which are available, the ease of altering the control algorithm, and the compatibility with other computer-based industrial control devices are much increased. However, if there is a need for very short response times, then it is necessary to use either a dedicated microprocessor-based controller having responsibility for only one or two loops, or an analogue based controller.

While there are a large number of relatively simple control and signal processing functions appropriate for microprocessor or analogue based devices in an industrial process environment, there remain many complex control tasks which are appropriate for execution on a minicomputer or large mainframe system. Control can be taken to have a wide meaning, including the entire set of decision-making functions in an industrial process system. Such control tasks as optimization, adaption, scheduling, data base management, event logging, and management reporting are all more suitable for execution on a higher level computer system. A determining characteristic of these higher level control functions is that they do not have the stringent response time required of the direct control functions. Therefore, a large degree of operating system overhead costs may be tolerated at this level in order to

have efficient allocation of computer system resources among the competing tasks.

The addition of a high speed computer-to-computer communication network to a collection of autonomous control computers allows the realization of truly concurrent computation in a control system. The communication system is responsible for process data, program, command, and status information transfer among the control computers and devices.

The management parameters, and the structure of the communication system play a critical role in the overall performance of a distributed system. Consequently, the design of a distributed system must carefully translate the needs of the specific application into a communication topology, level and location of redundancy, and a set of operating rules, i.e. protocol, for the communication system. Commonly used structures include the loop, star, global bus, or hierarchical topology. The concepts, methodologies, software, hardware and structure of distributed computer system have been discussed by Weitzman [1980] and Bowen et al [1980]. Each structure can be accessed in a number of ways with each structure or access mode yielding different performance characteristics [Kahne et al,1979], [Shaffi,1983]. As a simple example, a star system has a reliability measure that depends critically on the reliability of the central switching unit. On the other hand, a bidirectional loop need not necessarily fail upon

failure of a single interface. Similarly, the communication delay of a loop structure can be expected to be higher than the delay of a shared global bus because of the additional delay imposed by information being routed through device interfaces along the loop.

Even though there are significant problems and questions to be addressed in the design of a communication system for a distributed control application, the potential benefits are quite appealing from the view point of modularity, ease of expansion, reliability, and performance.

The operator interface is an essential component of a control system in large industrial process plants. The need for the display of process information in a timely and appropriate manner is critical to achieving plant efficiency, safety, security, and integrated control. A computer control system with distributed intelligence and an associated computer-to-computer communication system can provide an economically attractive solution using a centralized display and command capability. This will have access to all measured and calculated process variables, controllers inputs and outputs, as well as system states indicators. Not only can a single unit of this type be incorporated in a distributed system, but additional operator interfaces of varying complexity can be conveniently located with only minimal modification or extension of the original system. In this way, the management reporting function can be approached in the same manner and with the same hardware as

the primary operator interface. Appropriate software must be provided for the computation and display functions required of the management supervisory system. The use of multiple access points to either a centralized or distributed data base describing the state of the control and management system presents considerable data base management and security problems.

By far the most important step in process control implementation, is the provision of a suitable process model to find a controller that is reliable, and satisfies performance requirements. The process model should be in compatible form with the method of control design adapted. The management programs can then be added, making use of data stored by the controller programs.

8.2 The s-Domain Model:

The mathematical model has originally been developed in the time domain and has resulted in a number of hyperbolic partial differential equations, together with a set of time varying algebraic equations. The Laplace-transform has been used to reduce the system of partial differential equations and other time-dependent quantities to equations involving a unified algebraic operator, i.e. the s operator. In this way the distributive nature of the plant parameters has been directly addressed and transformed into a set of transfer functions

relating various variables of the plant to one algebraic indeterminate s .

The resulting system of transfer functions contain transcendental terms in s , more explicitly, terms such as e^{-ts} which represent pure time delays in the process. This is a pertinent feature of distributed parameter systems. These pure time delays are difficult to analyse and synthesize to obtain specified controller transfer functions, by frequency design methods [Marshall 1979]. This has meant that it is required to express these transcendental delays in forms more amenable to the design tools.

Without scrutinizing too much into e^{-ts} , an apparent difficulty is that the phase is a linear function of the frequency, or the Nyquist plot of such a function, e^{-ts} , is a spiral around the origin of the Nyquist-plane.

Many techniques have been suggested to find linear expressions for the pure-time delay functions. These techniques vary from expanding the e^{-ts} term using Taylor's series to finding approximate rational functions to represent faithfully e^{-ts} . The most famous method of approximating pure time delay functions in the frequency-domain is the Pade' approximation. This approximation is based around finding the most suitable coefficients in a Maclaurin's Series [BAKER, 1981].

In as much as these methods are approximate approaches, in some circumstances they may give rise to weak instability, or poor convergence. Thus, care should be taken when applying these techniques, whether a first order lag is enough to represent the pure time delay or a higher order Pade' approximation is needed. However, higher order terms may also increase the overall order of the system.

The most elegant way is to transform the transfer function which is in the s -domain into another domain, where it is possible to transform the pure time delay, or in general, the transcendental terms, with the other rational terms into a new indeterminate variable which linearizes the whole expression in rational forms. In other words, it is required to constitute a principal integral domain by the new transformation.

The requirement is that the transformation or mapping should enable the visualization, in a simple way, of the behaviour of transcendental transfer function, and on that range of mapping the transfer function should have a finite number of poles and zeros. Then, the new transfer function is of rational form, and the denominators and numerators are integral polynomials.

The mapping, preferably, should be bijective, i.e. the mapping from, say, domain s to range z is one-to-one (injective), and every element in z is at least an image of element

of s or s is onto z (surjective). Alternatively, at least the mapping should be injective, otherwise some elements in s can not be mapped into z . (Note, in this discussion s covers the extended complex plane $s = +\infty$). Also, if the mapping will enable us to operate in a finite plane this will be an advantageous factor in studying regions at $s = \infty$ in the finite z -domain. However, this advantage should not be over-emphasized here, (for treatment of s at ∞ see Kalaith [1980]).

By and large, there are infinite number of transformation functions, which achieve, to a satisfactory degree, the above requirements. It is, however, a formidable task to satisfy all of the requirements as stated above. Some engineering intuition is needed to choose which conditions to relax in order to bring down the mathematical problem to one capable of solution.

Hence, if the following mapping has been chosen, on the basis that the transcendental terms are directly transformed to integral terms:

$$z^{-1} = e^{-Ts} \quad (8.1)$$

where T is a parameter carefully chosen, (later it will be called the sampling period). This mapping satisfies most of the above requirements. The infinite open left-half s -plane is mapped onto the open unit circle, $|z| < 1$. The extended right-half s -plane

is mapped onto the range outside the unit circle, $|z| \geq 1$, and the transformation of e^{-Ts} has the required properties on the domain. Nevertheless, an important requirement is still left unsatisfied. That is the hyperbolic functions have not disappeared from the transfer function as could have been expected when equation (8.1) was chosen as the mapping function. From equation (8.1) it is apparent that

$$s = -\frac{1}{T} \ln z \quad (8.2)$$

If equation (8.2) is directly substituted for s in the transfer function the resulting function in the z -domain will be far more difficult to analyse than in the s -plane. However, this problem can be overcome by the application of approximation based on engineering understanding and will be demonstrated in applying the method. It will be shown how this mapping is quite useful, important, and practical.

Now, it will be demonstrated how such an engineering approach to the real physical problem will involve the use of equation (8.2) to get a rational integral polynomial transfer function in z , using the conformal mapping $z^{-1} = e^{-Ts}$ with all its other properties.

8.2.1 Rings of Delay Operators and Conformal Mapping:

Now, a close look to equation (8.1) reveals that

z^{-1} has replaced the Laplace-transform of a time delay function. Because, if the Laplace-transform of $u(t)$ is $U(s)$ then

$$Lu(t-T) = U(s) \cdot e^{-Ts} \quad (8.3)$$

and

$$Lu(t-T) = U(s) \cdot z^{-1}$$

where z^{-1} is now acting as a delay operator. This algebraic operator, z^{-1} , has a geometric interpretation in terms of the conformal mapping, equation (8.1). It is required to find out whether this operator, z^{-1} , constitutes a ring or not, because, if so, then it is possible to operate in a principal integral domain of z^{-1} .

Now, relation (8.3) suggests the following definition:

Definition :

Let U denote the linear space consisting of all R -valued functions defined on R , the field of real numbers, with support bounded on the left (i.e. for each u belongs to U , there is

a t belongs to R such that $U(t) = \emptyset$ for all $t < t$. Let $z^{-1} : U \rightarrow U$

denote the T -second delay operator of U defined by :

$$z^{-1} u(t) = u(t-T), \quad u \text{ belongs to } U.$$

Let $R[z^{-1}]$ denote the set of all finite sums $\sum_n a_n z^{-n}$ where a_n

belongs to R and $(z^{-n} u)(t) = u(t-nT)$, u belongs to U . With the usual operation of polynomial addition and multiplication, $R[z^{-1}]$ is a ring of delay operators. The geometric properties of this delay operator is shown above.

The use of this algebraic definition of delay operators, when linked with the physical problem, will formulate the problem over a ring of delay operators. This is indeed very useful because the ring is a principal integral domain.

It is of particular interest to note that when $R[z^{-1}]$ is an integral domain, there is a ring extension of $R[z^{-1}]$ which is a field. Integral domains are commutative rings that do not contain division of zero.

An integral domain $R[z^{-1}]$ can be viewed as a subring of a field $Q(R[z^{-1}])$, called the quotient field of $R[z^{-1}]$ the field Q is equal to the set of all formal ratios a/b where a and b belong to $R[z^{-1}]$, $b \neq 0$, with addition and multiplication defined by

$$a_1/b_1 + a_2/b_2 = (a_1b_2 + a_2b_1)/(b_1b_2)$$

$$\left(\frac{a_1}{b_1}\right) \cdot \left(\frac{a_2}{b_2}\right) = (a_1 a_2) / (b_1 b_2)$$

Now the quotient field of $R[z^{-1}]$ is the field of rational functions in z^{-1} .

Hence, from this, it is apparent that taking this algebraic advantage of the above delay operator the original s-domain expression becomes a rational transfer function in z^{-1} .

The treatment above has another direct advantage when analysing hybrid systems of continuous time behaviour interfaced with discrete ones. As has been mentioned earlier, the physical plant is driven by a digital computer through a zero-order hold and sample interface. By its nature, the computer is a discrete event system, and is usually mathematically modelled by delay operators. The plant by contrast, is continuous in time. So, the delay operator, by operating on the plant continuous transfer function, produces a rational transfer function in z^{-1} . This is a very useful means to unify the analysis of these hybrid structures of continuous-time plant with discrete-time machines.

In fact, the discrete-time nature of the zero-order hold and sampler has become a suitable buffer to decompose

the mathematical difficulty of substituting s equal to $1/T \ln z$ in the s -domain model, as will be shown in the following subsection.

8.2.2 Zero-Order Hold and Sampler :

The electronic interface between the plant and the digital computer is a system of digital to analogue and analogue to digital converters, as well as time multiplexing electronic switches for channel selection and economy in the number of cables running between the computer and the plant, as has been explained in chapter (3) when the HADIOS electronic interface was described.

When the computer reads in the analogue data coming from the plant, or sends data out, it reads or sends the data through the channels sequentially at a specified time-period, called the sampling time, and this is the sampler effect. When the computer sends out data to the plant the data goes first to electronic registers. In the register, the data will be kept constant until a new set of data has arrived in the next cycle of sampling. This mechanism is modelled as a zero-order hold. (See Fig.[8.1]a,b .

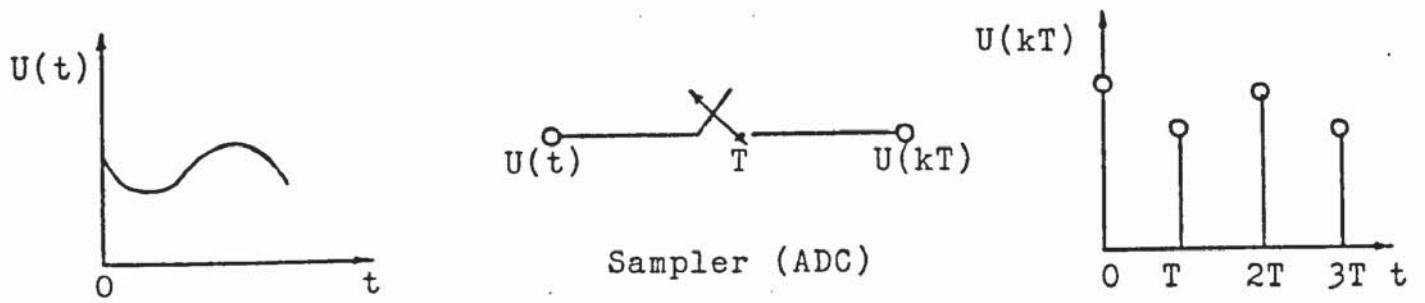


Fig. 8.1 a

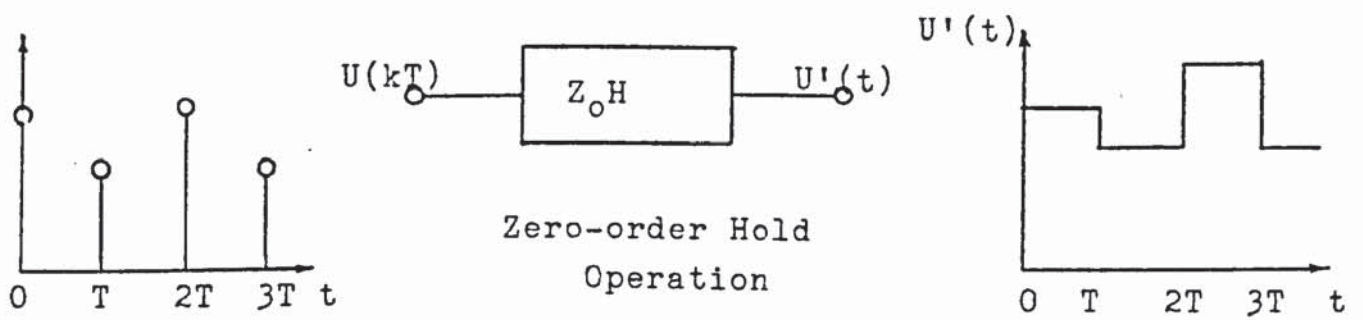


Fig. 8.1 b

In short, a zero-order hold (ZOH) is a device such that given a uniform sequence of numbers $u_0, u_1, u_2, u_3, \dots$ a stair

step output $u(t)$ is produced such that

$$u(t) = u(kT), \quad kT \leq (k+1)T$$

The staircase waveform can be thought of as composed of many pulses each pulse of width T sec. made up of a positive going step plus a delayed negative pulse; Fig [8.2] shows the hypothetical decomposition of the staircase waveform

The continuous-time response of the plant transfer function, $P(s)$, to the first step of magnitude u_0 above is

$$y(t) = u_0 \mathcal{L}^{-1} [P(s)/s]$$

where \mathcal{L}^{-1} is the inverse Laplace transform.

Due to the sampling effect, $y(t)$ will be as a sequence of train impulses.

This sequence of values $y^*(nT)$ may be represented by a train of impulses as shown in Fig [8.3].

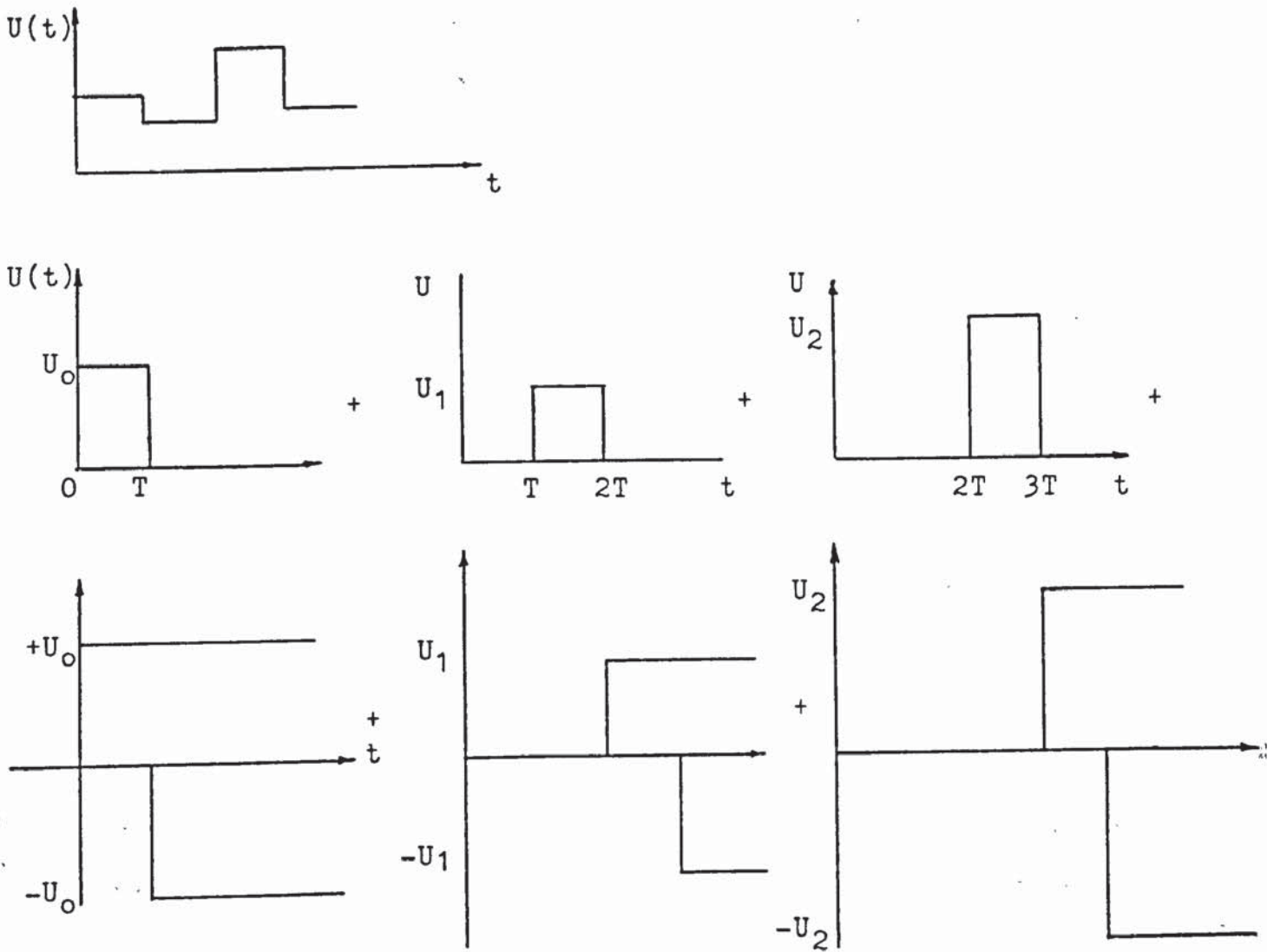


Fig. 8.2: The decomposition of the staircase waveform into many pulses.

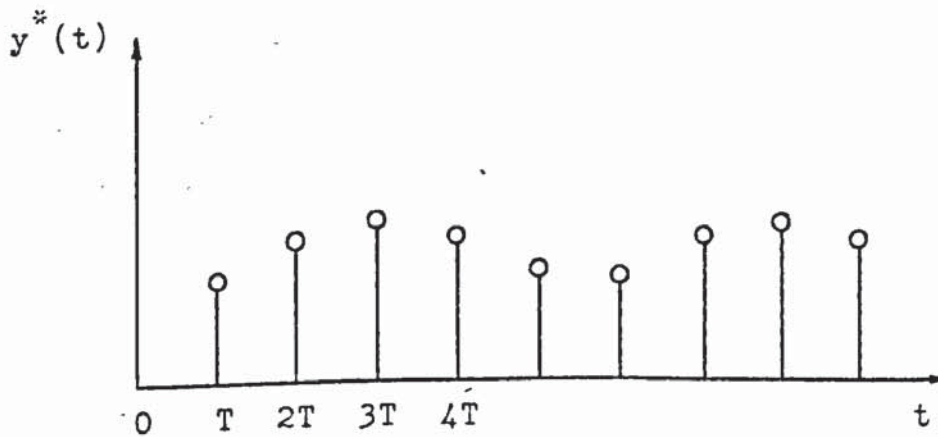


Fig. 8.3: Train of impulses.

At any given time $t = nT$, the impulse is $y(nT) \delta(t - nT)$ where $\delta(t)$ is the delta function where

$$\delta(t - a) = \begin{cases} 0 & t \neq a \\ \infty & t = a \end{cases},$$

and the train of impulses may be represented by the infinite summation :

$$y^*(t) = \sum_{n=0}^{\infty} y(nT) \delta(t - nT) \quad n = 0, 1, 2, \dots \quad (8.4)$$

the term $y^*(t)$ is called the sampled signal, which represents the continuous signal at the instants $t = nT$.

Indeed, the representation of a continuous signal by impulses of varying amplitude is a mathematical convenience which simplifies much of the analysis of sampled-data systems. Whether $y^*(t)$ represents a continuous signal $y(t)$ at every T seconds, or if $y^*(t)$ is actually a sampled or pulsed signal at regular intervals of T seconds, the impulse representation is the mechanism of the sampler in Figs.[8.2]a,b. The switch of an ideal sampler closes instantaneously every T seconds, and its output at the sampling instant nT is $y(nT)$.

Now the Laplace transform of equation (8.4) is

$$* \\ Y(s) = \mathcal{L}y^*(t) = y(0)\mathcal{L}[\delta(t)] + y(T)\mathcal{L}[\delta(t-T)] + y(2T)\mathcal{L}[\delta(t-2T)] + \dots$$

$$= \sum_{n=0}^{\infty} y(nT) L[\delta(t-nT)] = \sum_{n=0}^{\infty} y(nT) e^{-nTs} \quad (8.5)$$

From our definition of mapping $z = e^{Ts}$ equation (8.5) can be transformed to the z-domain

$$Z^* Y(s) = Y(z^{-1}) = \sum_{n=0}^{\infty} y(nT) z^{-n} \quad (8.6)$$

where the operator Z signifies the z-transform.

The notation $Y(z^{-1})$ does not signify $Y(s)$ with s replaced by z , but rather $Y[s = 1/T \ln z]$.

Alternatively, from our definition of the delay

operator z^{-1} ,

$$\delta(t - nT) = \delta(t) z^{-n}$$

Equation (8.4) becomes

$$y^*(t) = \sum_{n=1}^{\infty} y(nT) \delta(t) z^{-n} \quad (8.7)$$

Since $\delta(t) = 0$ whenever $t \neq nT$, equation (8.10) can be written as

$$y^*(t) = \sum_{n=0}^{\infty} y(nT) z^{-n} \quad (8.8)$$

Definition (8.2) :

Given a function $f : Z \rightarrow R$, (Z is the set of integer numbers), with $f(t)$ equal to zero for $t < 0$, we define the formal Z transform of f to be the formal power series $F(z^{-1}) = \sum_{n=0}^{\infty} f(nT) z^{-n}$.

By definition (8.2), equation (8.8) becomes :

$$y^*(t) = Y(z^{-1}) = \sum_{n=0}^{\infty} y(nT) z^{-n}$$

or $Z y(t) = y^*(t) = Y(z^{-1})$

Now going back to the equation describing the continuous-time response of the plant $P(s)$ to the first step of magnitude u is

$$y(t) = u \int_0^t [P(s)/s] ds$$

and the z -transform of the sampled sequence y_n is

$$Y(z^{-1}) = u \sum_{n=0}^{\infty} z^{-n} \int_0^t [P(s)/s] ds$$

The response due to the negative going step is exactly the same as the above; except it is negative and delayed by one sample period, T sec., so that the total z -domain response to the u may be calculated by superposition to be

$$Y(z^{-1}) = u \sum_{n=0}^{\infty} z^{-n} \int_0^t [P(s)/s] ds - z^{-1} \sum_{n=0}^{\infty} z^{-n} \int_0^t [P(s)/s] ds$$

where z^{-1} represents the delay by one sample period T sec. as given by Def. (8.1). Extending this idea to the second gate function, we get the response to the first two step inputs to be

$$\begin{aligned} Y(z^{-1}) &= u_0 (1-z^{-1}) ZL^{-1} [P(s)/s] + u_1 (z^{-1}-z^{-2}) ZL^{-1} [P(s)/s] \\ &= (u_0 + u_1 z^{-1}) (1-z^{-1}) ZL^{-1} [P(s)/s] \end{aligned}$$

Now for the whole series representing $u(t)$, the transform of the sampled output is

$$Y(z^{-1}) = \left[\sum_{n=0}^{\infty} u_n z^{-n} \right] \cdot (1-z^{-1}) ZL^{-1} [P(s)/s] \quad (8.9)$$

From above $\sum_{k=0}^{\infty} u_k z^{-k}$ is the z-transform of

$u^*(t)$, say, $u(z^{-1})$

therefore,

$$Y(z^{-1})/u(z^{-1}) = (1-z^{-1}) ZL^{-1} [P(s)/s] \quad (8.10)$$

When $P(s)$ contains the transcendental terms, e^{-rs} , the z-transformation then will completely solve the problem by making the system now defined over a ring of delay operators, i.e. into a rational form of integral domain polynomials.

It is expected that this procedure may give more accurate results than following the route of approximating the pure-time delay function, using Pade' approximation; and then getting the z-transform of the resulting s-domain rational transfer function expansion. Besides the better accuracy that can be obtained, the order of the system, yielded by equation (8.10), will be lower than what have been obtained through Pade' approximation.

The only approximation apparent in the method above is the rate of convergence of the infinite series of the z-transform,

$$\sum_{n=0}^{\infty} y(nk) \cdot z^{-n} \text{ as } n \rightarrow \infty.$$

It is worth mentioning here that z-transforms may represent an infinite number of possible time functions; because if

$F(z^{-1})$ is the z-transform of $f(t)$ then any function of time which coincides with $f(t)$ at the sampling instants $0, T, 2T, \dots$ has the same z-transform as $f(t)$. This is the fact that the mapping $z^{-1} = e^{-Ts}$ is onto.

In Chapter (7) a computer algorithm has been developed to solve equation (8.10) to find the formal z-transform of a Laplace function. The method is programmed on the computer such that given a transform function in the s-domain an equivalent z-domain transfer function can be obtained.

Now, it is clear to see how the idea of delay operator, z^{-1} , is connected to the conformal mapping e^{-Ts} to transform a system into a ring of delay operators with a quotient field Q . Hence, the designer is, merely, dealing with integral domain quantities rather than transcendental functions in the Laplace-domain.

8.3 Model Transformation :

In general, the mathematical modelling activity will yield a model for a plant or a process in the time domain as time-differential equations, integral equations, or integral-differential equations. For processes characterized by time-invariant-linear properties the equivalent Laplace transform of the time domain model can be found and hence, the system can be transformed into a transfer function form as a function of s , the Laplace indeterminate. The Laplace transform describes the system in a way equivalent to the continuous-time form.

In today's technology, most of the real physical systems are connected to digital computers or, in general, connected to digital machines. These physical processes are continuous in time, while the digital machines are discrete event machines, hence discrete in the time-domain. The combination of these continuous

systems with discrete event machines has necessitated the use of unified analytical approach tools to handle the integrated system. These tools are meant to cater for the two different behaviours taking place simultaneously, the continuous and the discrete events.

So far, two main approaches are widely used :

- 1) Either to find the equivalent mathematical model of the time-discrete system in the continuous-time domain, or
- 2) To represent the time-continuous system by a discrete time model.

Each of the above approaches has mathematical tools to analyse fully the problem and find an answer. But, indeed, the transformation from one domain to another is not unique, and each transformation from discrete to continuous or continuous to discrete has the limitation that it must transform all the respective properties of the system to the transformed domain. These limitations are widely discussed, [Kuo,1980], [Hamming,1977], [Bolton 1981] to merit any further description here.

In the problem of the control of continuous-time processes, if a digital compensator in a form of a digital computer, or other form, is required, the problem is best solved by transforming the whole problem into the time-discrete form. Then a digital compensator can be found directly. Otherwise, the compensator is found in the continuous form and then the equivalent

discrete form of the compensator is found by a suitable transformation. The word suitable, is stressed here because as has been mentioned earlier the transformation may incur or remove some of the original properties of the compensator [Sydenham,1982].

The control problem which is addressed now, is a continuous process, with its mathematical model transformed into the Laplace-domain. A digital compensator is one way to improve the performance of the process. It has been designed and then implemented in a digital computer.

Logically the suitable approach to follow here, is to transform the continuous-Laplace model of the process into the discrete form of the z^{-1} -domain, by the general method which has been mentioned in section 8.2. An interactive computer algorithm has been developed to automate this process of transformation from continuous to discrete or vice versa, (see Chapter (7)); hence, it is quite easy to move from a domain to another.

Tables [8.1, 8.2, 8.3, 8.4] show the various elements of the transfer matrix of the climbing film evaporator.

The algorithm of transformation consists of three programs as follows:

1) Program ONROOTS: to find the zeros of the numerator and denominator of the transfer function whether they are real or complex.

2) Program PARTIAL: to find the partial fraction of the transfer function by calculating the residue of each pole in the function, then find the equivalent poles of the z^{-1} -domain or the s -domain (see Chapter (7)),

3) Program SUMRATIO: to carry out the addition of the rational terms of the partial fraction form.

Example :

Consider the transformation of the transfer function of the outlet temperature of the climbing film evaporator to perturbation in the feed flowrate from the s -domain to the z^{-1} -domain.

$$\frac{T_b^*}{M^*} = \frac{- (6.058 \times 10^{-3} + 0.353s + 0.5224s^2)}{s(s + 0.02)(s + 0.05)(s + 0.577)} + \frac{e^{-5s} (6.058 \times 10^{-3} + 0.3258s + 0.522s^2)}{s(s + 0.02)(s + 0.05)(s + 0.577)}$$

To find the z -transform by the method described in section (8.2), multiply the transfer function by $1/s$. Now consider the first part of the numerator,

$$- (6.058 \times 10^{-3} + 0.353s + 0.5224s^2)$$

- 1) Program ONROOTS produces the zeros of this polynomial as
gain $k = -0.5224$
zeros at -0.6586 , -0.0176

- 2) Program PARTIAL finds the residue as follows:

$$\begin{array}{r} 119.786 \quad 260.62 \quad 0.244 \quad 140.59 \quad 10.494 \\ \hline s + 0.02 \quad s+0.05 \quad s+0.577 \quad s \quad \frac{2}{s} \end{array}$$

- 3) The equivalent z^{-1} transform for a sampling period of 12 seconds

$$\begin{aligned} = (1-z^{-1}) & \left[\frac{119.786}{1-0.787z^{-1}} - \frac{260.62}{1-0.549z^{-1}} + \frac{0.244}{1-0.0011z^{-1}} + \frac{140.59}{1-z^{-1}} - \right. \\ & \left. \frac{125.928z^{-1}}{(1-z^{-1})^2} \right] \quad (A) \end{aligned}$$

Now, consider the 2nd part of the numerator (pure time delay part)

$$e^{-5s} (6.058 \times 10^{-3} + 0.3258s + 0.5224s^2)$$

For a sampling period of 12 seconds:

1) Program CNROOTS has produced the zeros of the polynomial part as:

gain $k = 0.5224$

zeros at -0.0192 ; -0.6044

2) Program PARTIAL has found the residues as follows :

$$\frac{-36.54}{s+0.02} + \frac{225.686}{s+0.05} - \frac{0.082}{s+0.577} - \frac{189.065}{s} + \frac{10.506}{s^2}$$

3) The equivalent z^{-1} transform by formula [8.10] is as follows :

$$= (1-z^{-1}) \left[\frac{-31.767z^{-1}}{1-0.787z^{-1}} + \frac{159.041z^{-1}}{1-0.549z^{-1}} - \frac{1.445 \times 10^{-3} z^{-1}}{1-0.0011z^{-1}} - \frac{189.06z^{-1}}{1-z^{-1}} + \frac{73.538z^{-1} + 52.534z^{-2}}{(1-z^{-1})^2} \right] \quad (B)$$

Adding the terms of the same pole in equation (A)

and (B) above,

$$= (1-z^{-1}) \left[\frac{119.786-31.767z^{-1}}{1-0.787z^{-1}} + \frac{159.041-260.62z^{-1}}{1-0.549z^{-1}} + \frac{0.244-1.44 \times 10^{-3} z^{-1}}{1-0.0011z^{-1}} + \frac{140.59-189.06z^{-1}}{1-z^{-1}} \right] + \left[\frac{-52.39z^{-1} + 52.534z^{-2}}{1-z^{-1}} \right]$$

4) Program SUMRATIO is used to find the sum of the above expression as

$$T_b/M = \frac{-22.396z^{-1} + 31.9699z^{-2} - 6.3088z^{-3} - 3.3207z^{-4} + 5.325 \times 10^{-3}z^{-5}}{1.0 - 2.337z^{-1} + 1.77z^{-2} - 0.433z^{-3} + 4.32 \times 10^{-4}z^{-4}}$$

Similarly, expressions for the various transfer functions of the double effect evaporator can be obtained.

The behaviour in the s-domain compared with the equivalent z^{-1} -domain model for the different transfer functions is shown in Figs [8.4], [8.5], and [8.6]

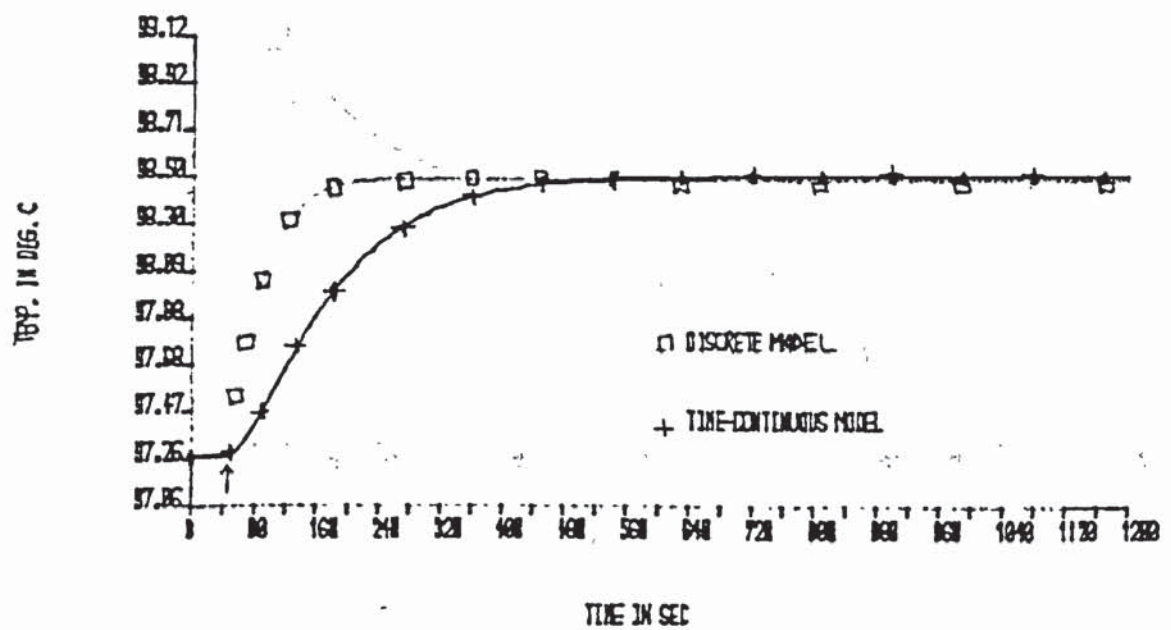


Fig. 8.4: The behaviour of the model in the s-domain and z-domain (steam perturbations)

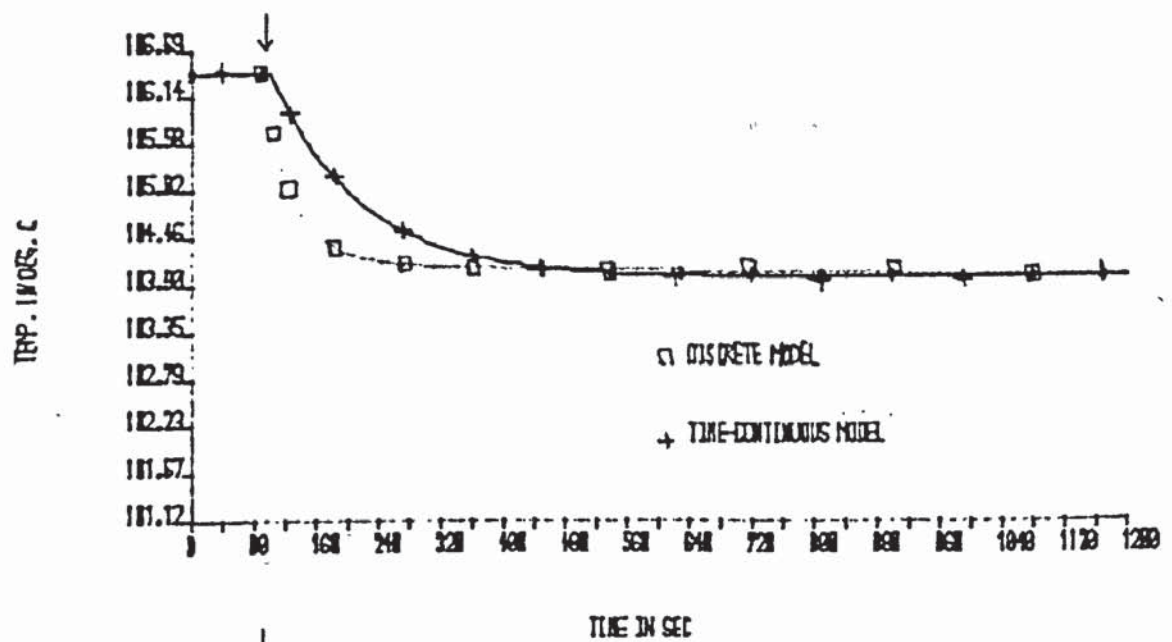


Fig. 8.5: The behaviour of the model in the s-domain and z-domain (feed flow rate perturbations).

s-domain

z^{-1} -domain

$$\frac{T}{S} = \frac{0.6489 - 0.391e^{-5s}}{(s+0.022)(s+0.05)(s+0.565)}$$

$$\frac{48.1592z^{-1} - 4.1707z^{-2} - 1.5628z^{-3} - 0.166}{1 - 1.317z^{-1} + 0.423z^{-2} - 4.642 \times 10^{-4}z^{-4}}$$

Table [8.1]

$$\frac{T}{M} = \frac{-(6.058 \times 10^{-3}s^3 + 0.353s^2 + 0.5224s) + e^{-(6.058 \times 10^{-3}s^3 + 0.3258s^2 + 0.5224s)}}{s(s+0.02)(s+0.05)(s+0.577)}$$

$$\frac{-22.396z^{-1} + 31.9699z^{-2} - 6.3088z^{-3} - 3.3207z^{-4} + 5.323 \times 10^{-3}z^{-5}}{1.0 - 2.337z^{-1} + 1.77z^{-2} - 0.433z^{-3} + 4.32 \times 10^{-4}z^{-4}}$$

Table [8.2]

$$* * \frac{(1.2 \times 10^{-4} - 2 \times 10^{-3} s + .225 s^2) e^{-15s} - (1.7 \times 10^{-4} s + 6.5 \times 10^{-4})}{(s + 0.1)(s + 0.5653)(s + 0.02)(s + 0.05)}$$

$$\frac{-0.764 + 3.1745z^{-1} - 3.882z^{-2} + 1.311z^{-3} + 0.101z^{-4} + 8.6595 \times 10^{-4} z^{-5}}{1 - 1.619z^{-1} + 0.8198z^{-2} - 0.1278z^{-3} + 1.396 \times 10^{-4} z^{-4}}$$

Table [8.3]

$$* * \quad \frac{(s^2 \times 10^2 + 3.015 \times 10^2 s + 4.082 \times 10^4) + e (1.791 \times 10^{-4} + 2.87 \times 10^{-4} s)}{ML/M = (s + 0.02)(s + 0.05)(s + 0.577)}$$

$$\frac{0.447z^{-1} - 0.35z^{-2} + 6.992 \times 10^{-4} z^{-3}}{1 - 1.337z^{-1} + 0.433z^{-2} - 4.321 \times 10^{-4} z^{-3}}$$

Table [8.4]

8.4 Wiener-Hopf Controller Algorithm:

From the general theory of modern Wiener-Hopf optimal controller, it has been shown in Chapter (3) that the computational effort is formidable, apart from the question of handling, computationally, symbolic quantities by engineering oriented higher-level computer -languages such as FORTRAN, BASIC, or a like. An effort has been made to tackle this hindrance and as a result of this effort APA, the library described in Chapter (7) and in a departmental manual [Nawari,1983], has emerged. It contains almost all of the subroutines necessary to automate the Wiener-Hopf optimal controller, and similar algebraic controllers. In general, the package is capable of manipulating polynomial matrices

algebraically, and to simulate discrete and continuous systems which are either single variable or multivariable, with on-line graphical output.

8.4.1 Optimal Wiener-Hopf Controller :

Now, given the plant transfer matrix $P(z)^{-1}$ and feedback matrix $F(z)^{-1}$, using the program COPRIME pairs of matrices can be found such that,

$$\begin{aligned} F(z)^{-1} P(z)^{-1} &= A(z)^{-1} B(z)^{-1} \\ &= B_1(z)^{-1} A_1(z)^{-1} \end{aligned}$$

Where A and B are left coprime; and B_1 and A_1 are right coprime.

The next step is to use program BEZOUT to find two matrices X and Y such that X and Y are the solution of the left-Bezout identity.

$$A(z)^{-1} X(z)^{-1} + B(z)^{-1} Y(z)^{-1} = I$$

Program BEZOUT is capable of solving the above identity, if it is given the coprime polynomial matrices A and B ,

then it will produce matrices $X(z^{-1})$ and $Y(z^{-1})$ such that the identity holds.

Next, using program SPECT1, the spectral factorization of

$$A_{l*} (P P + k P P) A_{s* s l} = V V_{*}$$

is found, in fact, a family of $V(z^{-1})$ is obtained for different values of the lagrange multiplier k .

A similar program to SPECT1 is SPECT2. It is used to evaluate the spectral factorization of

$$A G A_{*} = N N_{*}$$

where

$$G = G_u + F_o G_m F_o^{*} + P_d G_d P_d^{*}$$

The data required here are the statistical distribution of the signals of load disturbance, measuring instruments noise, and set-point changes.

Now construct matrix J by using program IIJJ such that

$$J = A_{l*} P_{*} (G_u + P_d G_d P_d^{*}) A_{*}$$

Then K can be assembled as follows

$$K = V \begin{pmatrix} -1 & -1 & -1 & -1 & -1 & -1 \\ \{V & J & N\} & + & \{V & A & Y & N\} &)N & - & A & Y \\ * & * & - & 1 & + & 1 \end{pmatrix}$$

or, use program HOWIEN to find H

$$H = A V \begin{pmatrix} -1 & -1 & -1 \\ \{V & J & N\} & + & \{V A & Y & N\} &) \\ 0 & 1 & * & * & - & 1 & + \end{pmatrix}$$

Then, the optimal controller is found from program CWIEN as

$$C = (Y + A K) (X - B K)^{-1}$$

or

$$C = H \begin{pmatrix} -1 & -1 \\ (A & N - P H) \end{pmatrix}$$

The performance of C, or in general, the behaviour of the closed-loop system is tested by program [MULDISC] for different $V(z)^{-1}$ being obtained from different lagrange multipliers k.

The closed-loop response is given by matrix

$$T = P (Y + A K) A$$

Note, in all of the cases of different V , $\det(X - B K)$ is checked not to be equal to zero, otherwise the controller C_1 is unrealizable.

In summary, the steps of the design of the optimal Wiener-Hopf can be summarized in the following steps : (see flowchart Fig. [8.7]).

$$1) \quad F(z)^{-1} P(z)^{-1} = A(z)^{-1} B(z)^{-1} \\ = B_1(z)^{-1} A_1(z)^{-1}$$

Find A_1, B_1, A , and B .

$$2) \quad A(z)^{-1} X(z)^{-1} + B(z)^{-1} Y(z)^{-1} = I$$

Find X and Y .

3) Factorize

$$(i) \quad A_1^* (P_1 P_1 + k P_1 P_1) A_1 = V_1 V_1^*$$

k is the lagrange multiplier.

$$(ii) \quad A_1 G A_1^* = N_1 N_1^*$$

$$G = G_u + F_o G_o F_o + P_d G_d P_d^*$$

$$4) \text{ Construct } J = A_1^* P_1 (G_u + P_d G_d P_d^*) A_1$$

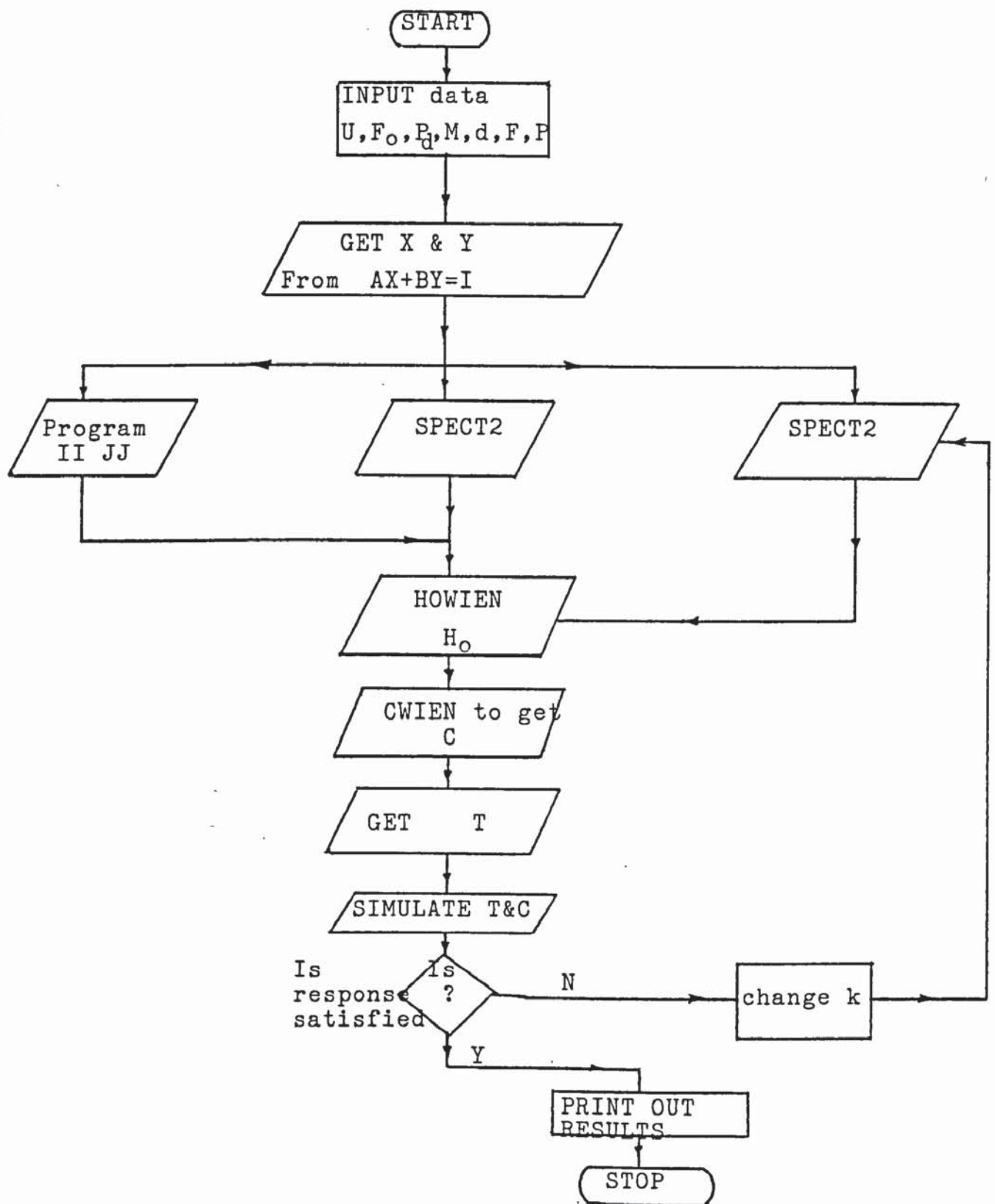


Fig. 8.7: Flow chart showing the steps in synthesizing Wiener-Hopf controller.

5) Find the rational matrix, K, such that

$$K = V \begin{matrix} -1 & -1 & -1 & -1 & -1 & -1 \\ \{ V & J & N \} & + \{ V & A & Y & N \} \end{matrix} \begin{matrix} N \\ -A \\ Y \end{matrix}$$

or

$$H = A V \begin{matrix} -1 & -1 & -1 & -1 \\ \{ V & J & N \} & + \{ V & A & Y & N \} \end{matrix}$$

6) Now find the optimal controller matrix from

$$C = (Y + A K) (X - B K)$$

or

$$C = H (A N - P H)$$

7) Then find the closed-loop transfer matrix to simulate the behaviour of the final system.

$$T = P (Y + A K) A$$

Note, steps (5-7) are repeated for different V, this due to the lagrange multiplier, k, which generates different V.

8.4.2 Initial Simplification for the Programs :

Some assumptions have been made to the above equations so as to simplify the programming aspects of the problem. However, these simplifications do not limit the procedure which has been mentioned. The basic assumptions made for simplification are as follows :

- 1) The plant disturbance vector is neglected i.e. $\bar{d} = 0$.
- 2) The sensor noise coming from the feedback loop is assumed to be within 5% of the power of the measured signal at steady-state, and the noise is assumed to be white.
- 3) There is no saturation effect due to the plant input. This means that for the whole range of $\bar{r}(s)$, the input to the plant, the plant will remain linear. Therefore P is taken as I .
 s
- 4) Feedback is unity I .

These assumptions are quite general, except for the first assumption. Nevertheless, taking into account the disturbance matrix the only action needed is to increase the data input to program SPECT2.

The effect of these assumptions on the programs is described as follows :

- 1) Program SPECT1 :

This program evaluates the expression

$$A_{1*} (P_{*} P_{*} + k P_{s*} P_{s*}) A_{1*} ,$$

and then finds the spectral factorization of this quantity.

The above expression can be expanded as follows :

$$A_{1*} P_{*} P_{*} A_{1*} + A_{1*} k A_{1*} = V V_{*}$$

where $P_s = I_s$.

This expression can still be more reduced :

Since $P_{1*} = B_{1*} A_{1*}^{-1}$

therefore $A_{1*} P_{*} P_{*} A_{1*} + A_{1*} k A_{1*} = A_{1*} (B_{1*} A_{1*}^{-1})_{*} B_{1*} A_{1*}^{-1} + A_{1*} k A_{1*}$

and making use of the identity

$$(B_{1*} A_{1*}^{-1})_{*} = A_{1*}^{-1} B_{1*}$$

therefore $V V_{*} = B_{1*} B_{1*} + A_{1*} k A_{1*}$

Now, this has become a simple expression and it has got no polynomial denominator. More important, it is in compatible form for the subroutine POLMUL to carry out the special multiplication of $B_{1*} B_1$, because

$$B_{1*} (z^{-1}) B_1 (z^{-1}) = B_1 (z) B_{1*} (z^{-1})$$

2) Program SPECT2 :

The ultimate objective of this program is to find the spectral factorization N , of

$$A G A^* = N N^*$$

The program performs the following :

- (i) evaluates G
- (ii) calculates $A G A^*$
- (iii) finds the spectral factorization N , of $A G A^*$.

In this program assumption (i) above is invoked and hence G is composed only of the set-point changes and the instrument noise vector. The disturbance vector is neglected.

Thus
$$G = \bar{u} \bar{u}^* + \bar{m} \bar{m}^*$$

where \bar{u} and \bar{m} are the input and instrument, noise vectors, respectively. F has been taken as I . The input vector \bar{u} is assumed to be changing in a stepwise fashion, since this is probably the most important changes in the process industry.

For a 2×1 input vector, \bar{u} , as this is the case in this study, this will take the form

$$\bar{u} = \frac{1}{(1-z)} \begin{bmatrix} u_1 \\ 1 \\ u_2 \\ 2 \end{bmatrix} \quad \text{where } u_1 \text{ and } u_2 \text{ are}$$

constant quantities.

$$\text{therefore } \bar{u} \bar{u}^* = -z / (1-z) \begin{bmatrix} 2 & & & \\ u_1 & 0 & & \\ 1 & & 2 & \\ 0 & & u_2 & 2 \end{bmatrix}$$

Note that, the off-diagonal elements are zero, because u_i , $i = 1, \dots, m$ are assumed to be statistically independent processes i.e.

$$\bar{u}_i \bar{u}_j = 0 \quad i \neq j$$

The noise vector \bar{m} is assumed to be white,

Hence

$$\bar{m} \bar{m}^* = \begin{bmatrix} 2 & & \\ m & 0 & \\ 1 & & \\ 0 & m & \\ & 2 & \\ & & 2 \end{bmatrix}$$

where m_i , $i = 1, \dots, n$ are constant values.

Now,

$$A G A^* = A \bar{u} \bar{u}^* A^* + A \bar{m} \bar{m}^* A^*$$

The first term in the above expression has a polynomial denominator, which has come from $\bar{u} \bar{u}^*$; while the second term has a constant denominator, without loss of generality can be taken as unity. However, in general we make a common denominator for the whole expression. The most important fact to be noticed is that the expression is in the standard form for subroutine POLMUL to carry out the special multiplication (see Chapter (7), $(A \cdot \bar{u}) \cdot (\bar{u}^* A^*)$, etc.

The denominator polynomial is naturally in the standard form for subroutine SPECSC to find its spectral factorization.

3) Program IIJJ :

This program evaluates the expression

$$A \begin{matrix} P \\ 1^* \end{matrix} (G + \begin{matrix} P \\ * \end{matrix} \begin{matrix} G \\ u \end{matrix} \begin{matrix} P \\ o \end{matrix} \begin{matrix} A \\ d \end{matrix} \begin{matrix} \\ o \end{matrix} \begin{matrix} \\ * \end{matrix})$$

From the above assumptions $G_d = 0$

Let

$$J = A \begin{matrix} P \\ 1^* \end{matrix} \begin{matrix} G \\ * \end{matrix} \begin{matrix} A \\ u \end{matrix} \begin{matrix} \\ * \end{matrix}$$

where
$$G_u = \bar{u} \bar{u}^*$$

(Note that G_u usually does not take the form $\bar{u} \bar{u}^*$ because \bar{u} is a mx1

vector and A, A_1 , and P are multiarrays. $\bar{u} \bar{u}^*$ always generates a 1x1

matrix.

$$J = A_1 P \bar{u} \bar{u}^* A$$

This expression is not in a compatible form for subroutine POLPOL to carry out ordinary multiplication of polynomial

matrices. This is because, A_1, P, \bar{u} , and A are functions of

the indeterminate z while \bar{u} is a function of z^{-1} .

But, due to the assumption assumed

earlier, that \bar{u} is stepwise vector, then $\bar{u} \bar{u}^*$ can also be expressed

in the indeterminate z . In fact,

$$\bar{u} \bar{u}^* = -z / (1 - z^2) \begin{bmatrix} 2 & & \\ u & 0 & \\ 1 & & 2 \\ 0 & u & 2 \end{bmatrix}$$

Now the whole expression of J is expressed in z .

So, subroutine POLPOL is used to evaluate

$$J = A_1 P (\bar{u} \bar{u}^*) A$$

$$= A_1 A_1 B (\bar{u} \bar{u}^*) A$$

$$J = B_1 (\bar{u} \bar{u}^*) A$$

The only polynomial denominator in the expression of J is $(1 - z^2)$, which is due to the term $\bar{u} \bar{u}^*$.

4) Program HOWIEN :

This program is to evaluate a rather lengthy expression, though it is simple and straight forward.

$$H = A \underset{0}{V} \underset{1}^{-1} \left(\{ \underset{*}{V} \underset{*}{J} \underset{*}{N} \} + \{ \underset{1}{V} \underset{1}{A} \underset{+}{Y} \underset{+}{N} \} \right)$$

All the operations involved are simple polynomial matrices multiplication and inverses. Now, separate H into three separate expressions as follows :

(i) Expression $\underset{*}{V} \underset{*}^{-1} \underset{*}{J} \underset{*}^{-1} \underset{*}{N}$:

The denominator of this expression composes of $[(\det \underset{*}{V})(\det \underset{*}{J})(\det \underset{*}{N})]$.

Also note, the numerator of this expression is multiplied by the polynomial $(\underset{*}{uLs})$, where $\underset{*}{uLs}$ stands for the denominator of the spectral factorization of N. This is due to the inversion of $\underset{*}{N}$ in this expression.

(ii) Expression $\underset{1}{V} \underset{1}{A} \underset{1}{Y} \underset{1}{N}$:

Now, the denominator of this expression composes of $[(\det A) (uLs)]_1$.

(iii) Expression $A V^{-1}_1$:

The program evaluates each of the above three expressions and output the result of each evaluation separately. Since it is extremely difficult to evaluate H directly on-line, due to the lack of mathematical algorithms to evaluate partial fraction of polynomial matrices, a remedy is suggested to this hindrance as follows :

a) use the programs ONROOTS and PARTIAL to find the partial fraction of the polynomial matrix, $(V^{-1}_1 J^{-1}_N uLs)$ element by element and take only the terms with poles in C .

b) similarly, use the two above programs to find the partial fraction of the polynomial matrix $(V^{-1}_1 A^{-1}_1 Y N)$, also, element by element and take only the terms with poles in C .

c) Finally, add the results in (a), and (b) and multiply the result by the polynomial matrix (A^{-1}_1) , this will complete the computation of H .

5) Program CWIEN :

This program is to carry out the final calculation of the controller C.

$$C = H \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix} (A \begin{pmatrix} N & -P \\ 1 & 0 \end{pmatrix} H)$$

6) Program CLMATIX :

This is a similar program to CWIEN, above, but it calculates the closed-loop transfer matrix of the system.

$$T = PC (I + PC)^{-1}$$

7) Program MULDISC :

This is a general program to simulate multivariable discrete-linear systems. It is used to simulate T and to anticipate the behaviour of the system under the controller C.

Table (8.5) shows a summary of each program function.

1) Program SPECT1 :

$$A \begin{pmatrix} P & P + k P & P \end{pmatrix} A = V V$$

$$1^* \quad * \quad \quad s^* s \quad 1 \quad *$$

2) Program SPECT2 :

$$G = G + F \quad G \quad F + P \quad G \quad P$$

$$u \quad o \quad m \quad o^* \quad d \quad d \quad d^*$$

3) Program IIJJ :

$$J = A \quad P \quad (G + P \quad G \quad P) \quad A$$

$$1^* \quad * \quad u \quad o \quad d \quad o^* \quad *$$

4) Program HOWIEN :

$$H = A \quad V \quad \begin{pmatrix} -1 & -1 & -1 & -1 \\ \{ V & J N \} & + \{ V A & Y N \} \end{pmatrix}$$

$$o \quad 1 \quad * \quad * \quad - \quad 1 \quad +$$

5) Program CWIEN :

$$C = H \quad \begin{pmatrix} -1 & -1 \\ A \quad N & - PH \end{pmatrix}$$

$$o \quad \quad o$$

6) Program CLMATLX :

$$T = PC \quad (I + PC)^{-1}$$

7) Program MULDISC :

Multivariable discrete simulation

Table [8.5]

For cases when the plant matrix and feedback matrix, FP, constitute an analytic matrix function in the C set, ⁺ the optimal controller algorithm can be modified as follows :

$$H = \begin{matrix} & -1 & -1 & -1 \\ & V & J & N \\ \begin{matrix} \cdot \\ o \end{matrix} & r & r^* & r^* \end{matrix} \begin{matrix} \\ \\ - \end{matrix}$$

and $C = \begin{matrix} & -1 \\ & N & - FPH \\ \begin{matrix} \cdot \\ o \end{matrix} & r & o \end{matrix}$

where $\begin{matrix} V & V \\ r^* & r \end{matrix} = \begin{matrix} P & P + K P & P \\ * & s^* & s \end{matrix}$

$$\begin{matrix} N & N \\ r & r^* \end{matrix} = G$$

$$G = \begin{matrix} G & + F & G & F & + P & G & P \\ u & o & u & o^* & o & d & o^* \end{matrix},$$

and $J = \begin{matrix} (G & + P & G & P) & P \\ r & u & d & d & o^* \end{matrix}$

Thus, we see that the analyticity of F.P has reduced considerably the amount of computation required. Especially, matrices like A, A_1, B, B_1, X and Y are no longer needed.

The programs to carry out the above algorithms are shown in Nawari [1983].

This completes the program to find or synthesis the modern Wiener-Hopf controller. The library, APA, has been reinforced by the necessary subroutines to carry out this huge task,

otherwise such important types of controller could not readily be implementable.

Prior to the real-time implementation of C in the computer, simulation of the behaviour of the closed-loop system,

$T(z^{-1})$, for step set-points changes is considered. After the designer is satisfied with the responses of $T(z^{-1})$ then the real-time implementation of C can be realized.

The next step of the implementation exercise is the realization of the controller, C, in the system hardware by suitable arrangements of the on-line computational facilities, in this case a Honeywell 316H minicomputer and a Motorola M6800 microprocessor.

8.5 The On-Line Computational Facilities:

The on-line computational facilities available to control the double effect evaporator, or to act as the digital compensators consist of the Honeywell 316 minicomputer, the Motorola M6800 microprocessor, and other electronics and hardware to interface the plant process with the computers system. This has been described in chapter (3).

In the following sections it will be shown how the digital compensators are designed and constructed. This will involve the description of how the system hardware and the communication software are organised. This involves local network communication programs, as well as other software available, facilities provided and the hierarchical structure of the control system.

8.5.1 System Hardware Organisation :

The basic idea is to design a hardware arrangement in a simple lucid way and be sufficient for

- (i) either computer to be able to send an interrupt request to the other.
- (ii) 16-bit digital data to be transferred between the computers (two 8-bit data bytes in the case of the M6800).
- (iii) simple and transparent software to be available for all levels of communications.

The resulting hardware organisation for the linked distributed system is shown in Fig. [3.4].

The organisation is unique in the sense that the M6800 accesses the process plant via the HADIOS and has to interrupt the H316 processing to do so. Thus, the H316 behaves as the front-end of the microcomputer.

In effect, HADIOS plays as the electronic hardware interface between the various digital machines and the plant process. The HADIOS subinterfaces used on the H316 side are a digital input (DGINB), a digital output (DGOB) and the alarm inputs. On the M6800 side, only a single chip has been used. The programmable nature of the PIA makes it very suitable for such an application.

8.5.2 The Communication Protocol :

The communication protocol has been designed to meet both the system objective and specific subinterface/adaptor requirements. All the interface devices are TTL-compatible which means the output data lines (PB0-PB7) of the PIA B side can be wired directly to the input lines (9 through 16) [N.B. In the H316, the bits are numbered 1-16 from left to right] of DGIB. Likewise, the input data lines (PA0-PA7) of the PIA A side can be wired directly to the output data lines (9-16) of DGOB. Integer formatted data words can therefore be transferred between the computers via these I/O ports.

The M6800 needs to interrupt the H316 to perform the following :

- 1) scan the process variables.
- 2) output control settings.
- 3) send data bytes to the H316 for further processing, (special codes, further computation, output to paper tape or graphical display).

8.5.3 Communication Software :

While each computer has sufficient software to operate individually in real-time mode, extra software is required to handle the communication protocol for the distributed system.

This is provided by assembly language executive programs; HADIOS Executive Rev 3, and M6800 Executive residing in each computer. For detail of these assembly programs see [Shaffi, 1983].

8.5.3.a HADIOS Executive Revision 03 :

Over the years, various packages have been developed for the H316 minicomputer and in particular, attention is focused on the development of the BASIC-16 Interpreter to permit interactive real-time use and graphical display of results. To support these applications, many versions of the HADIOS Executive have been written, in DAP-16 assembly language, to operate the various HADIOS devices. The user's BASIC program may then call subroutines in the HADIOS Executive or the graphics library to meet particular requirements.

The HADIOS Executive Rev. 03 is a considerably modified version, revised by [Shaffi,1983], of the previous versions. It is written in H316 Assembly Language (DAP-16 MOD2) and contains subroutines callable from BASIC programs level, and all the interrupt service routines. In this way, the details and low-level operations of HADIOS devices remain hidden from the general user who would be programming mainly in BASIC and FORTRAN. The function of the executive is as follows :

- a) regular access of HADIOS hardware at the required frequency from either computer,
- b) handle and service all interrupts to the H316 minicomputer,
- c) to provide a real-time base for the calculations of flowrates from counter inputs,
- d) to provide an idling loop to wait for clock interrupts,
- e) the M6800 user may exclusively use the H316 graphics and data processing software,
- f) either computer can control settings to the process plant at any desired time,

g) to terminate H316 scanning of HADIOS devices when the required number of scans has been done,

h) to handle error conditions while not disrupting any microprocessor operations.

[for HADIOS subroutines see Appendix F.]

Limitations of the HADIOS Executive:

The sampling frequency; $A(0)$, (see Appendix F.), is fixed on entry and remains constant throughout an on-line run, whereas more intelligent executives would include a facility for variable sampling speeds in response, or in adaption, to rapidly changing plant operating conditions.

Another limitation is that it is only possible to scan a single group of sequentially arranged analogue channels. If one is to scan channels 0-7 and channels 12 to 15 in one scanning routine then the user has no choice but to scan channels 0 to 15, inclusively. This means some time is wasted in scanning the irrelevant channels.

8.5.3.b The 6800 Executive :

The M6800 Executive is written in M6800 Assembly Language and followed similar lines to the HADIOS Executive.

The M6800 Executive can provide the following functions :

- a) to enable the M6800 to access HADIOS hardware at the required frequency,
- b) to handle all interrupt requests to the M6800 CPU,
- c) to provide an idling loop to wait for the M6800 scanning interrupt,
- d) to enable the M6800 user to output control settings to the process plant,
- e) to terminate M6800 scanning when the required number of scans has been done,
- f) to handle error conditions while not disrupting the H316 operations.

The functions are achievable by a combination of subroutines callable from SD-BASIC, and the interrupt service routines, together with the user's SD-BASIC program.

The M6800 Executive is normally stored as a file on floppy disk. For real-time use, the user merely appends it to an application SD-BASIC program.

As in the HADIOS Executive package, there are basically four subroutines in the M6800 Executive. (See Appendix (F).

Limitations of the M6800 Executive :

Since M6800 has the ability to access HADIOS all the limitations mentioned there also affect the performance of M6800 Executive. Other constraints reflect the fact that the M6800 is linked to the H316-HADIOS hardware via a single PIA chip. The M6800 user has no access to the plant without using some H316 machine time.

8.5.4 On Line Graphics:

The H316 Tektronix Graphic Package:

During this work, it has been found useful for the results of on-line processing to be displayed on-line in a graphical format. The graphics library routines have been grouped together to form a single segment subroutine (5) in the HADIOS Executive package.

A steering subroutine called GRAPH (see Appendix G.) allows the user to access the different graphical operation from a single call.

8.5.5 Summary of the communication software:

In summary, the system provides the user with four options or mode of operations. Table [8.6] shows all possible combinations of operation modes..

Table [8.6]

| H316 | M6800 |
|----------|----------|
| on-line | on-line |
| on-line | off-line |
| off-line | on-line |
| off-line | off-line |

Note, that the term ON or OFF line is with respect to the process plant.

The user can use high level computer languages so that programs are relatively easy to write and modify. The HADIOS Executive enables the user to specify the sampling frequency, the devices and number of scans required. The system is flexible enough for a single user to sit down at a terminal, in the immediate environment of the process plant, and conduct calibration or control experiments.

The BASIC-16 programs can call subroutines which originally were written in FORTRAN-4 or H316 Assembly Language code stored as machine code. This enables fast execution and processing of plant information.

Real-time use of the M6800 has been largely confined to the use of SD-BASIC. This is because SD-BASIC, like most BASICs, is a familiar language to most users and has many extended features suitable for such tasks. An SD-BASIC program is interruptable; it can read or write into memory locations (for instance, taking an interrupt status flag) via its POKE and PEEK statements, manipulate bits via its AND, OR, SHIFT, XOR or COM functions, call on M6800 Assembly Language subroutines, and communicate with data files stored on the floppy disks. In fact, a user may not find it necessary to interface assembly language subroutines to the BASIC program at all. This is generally true for most microcomputer BASIC languages, since microprocessor software is usually more closely related to the hardware than others. To usefully exploit these features, the user must also have a reasonable understanding of the hardware building blocks of the microcomputer system.

SD-BASIC on the M6800 microcomputers, although it is a compiled form of high level language, retains the interactive features of interpreter-based BASIC languages due to the operating system (DOS) which enables fast editing and compilation processes.

Hence, in all, the linked distributed system provides the following, general facilities:

- 1) interactive BASIC-16 programs in the H316 system, can access the plant through HADIOS with an option for on-line graphic.
- 2) the disk-based SD-BASIC programs, in the M6800 microprocessor system, can access the HADIOS hardware and the H316 graphics facility.
- 3) each processor controls its own real-time clock (hence, two independent sampling frequencies are possible).
- 4) either processor can be taken out of real-time mode and used off-line without affecting the other.

Finally, a Benchmark test to compare the speed of execution of the M6800 SD-BASIC; and H316 BASIC, and FORTRAN programs was carried out by Shaffi, [Shaffi,1983]. The following observations were noted :

- 1) SD-BASIC runs 2 to 2.5 times faster than BASIC-16 but about 8 times slower than FORTRAN code implemented on the H316.

2) in SD-BASIC, the execution time increases by about 30% if floating-point arithmetic operand is used, while in H316 this has a negligible effect.

These observations may be useful, for instance, if lengthy involved algorithms are to be implemented on-line, and the above points may help as guide-lines to resources distribution.

8.6 The Computers Hierarchical Structure and Control Task Distribution :

In the light of the above facilities the control system was organised in the following manner: The task of a control system can, logically, be divided into three main tasks:

- 1) to take in information,
- 2) to take decision in light of the information it has been given, and
- 3) to execute the decision.

This can best be illustrated in Fig [8.9]

It shows the area of where each task is taking place.

The information part consists of sensors measurements and set-point changes requirements. These channels of information are fed to the decision making mechanism which is the

compensator. The information need not be fed directly to the compensator, where sometimes the information may be filtered and smoothed from noise prior to its usage,

The compensator works on the basis of the information it has been fed to generate a decision, which we hope to be an optimal one. This decision is then sent for execution. The execution mechanism is usually in a form of actuators, which are interfaced electrically or mechanically with the compensator and the plant demanding the decision.

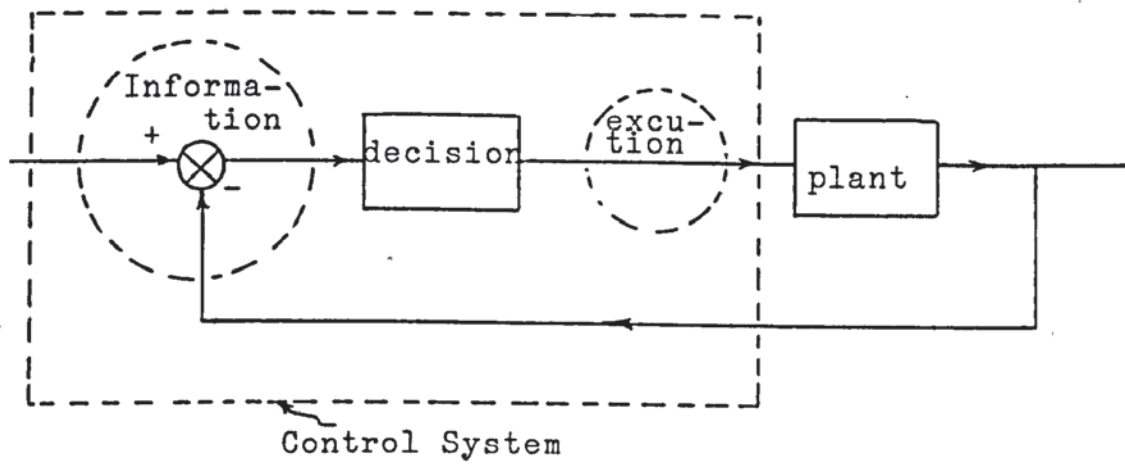


Fig. 8.8: Control task distribution.

In a hierarchical structure, a decision may be sent to other compensators before finally it reaches the actuators as a command. The cycle may repeat itself as many times as the number of compensators, until finally it reaches the actuators, (see Fig.[8.9]).

In Fig [8.9], the compensator 1 is considered to be at a higher level, while compensator 2 is taken as a lower level of the organisation of the controller. Now, it is apparent that the task of a large controller organisation can be considered as a collection of sets of type task, where set task is composed of three elements: information, decision, and execution.

In other words, let C be a collection of controllers c_i , $i = 1, \dots, n$ where n is the total number of controllers in a hierarchical system. Then

$$C = \bigcup_{i=1}^n c_i$$

with the assumption that

$$\bigcap_{i=1}^n c_i = \emptyset \quad (\text{the null set})$$

where every c_i is

$$c_i : \{ x \mid \text{information, decision, execution} \}$$

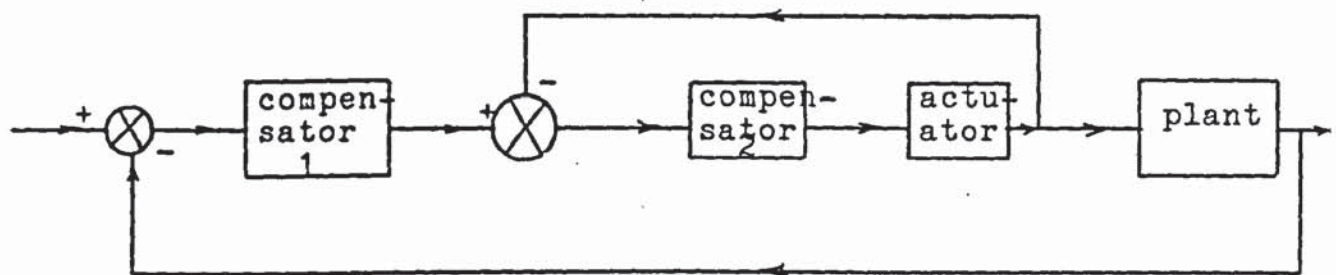


Fig. 8.9: Hierarchical structure of compensators

8.6.1 Control Task Distribution:

The task of the hierarchical control system was divided between the H316 minicomputer and the M6800 microprocessor. At the lower level, the microprocessor has full command on the plant pneumatic valves; and at the higher level, the minicomputer has the duty of performing the necessary optimization control and communicating the optimal set-point changes to the plant through the microprocessor. Measurements from the plant can be made using a variety of possible protocols. Indeed, the minicomputer can have direct access to the plant measurements through the HADIOS system, or it can obtain these information from the microprocessor at any specified intervals; in this case the microprocessor M6800 is receiving the measurements from the plant, still through the HADIOS system, (stealing a cycle from H316 minicomputer).

Hierarchical control structure has been adopted here, due to the different levels of control tasks required. At the higher level is the calculation of the optimal settings of the Wiener-Hopf optimal controller. The output of this controller should be sent to the pneumatic valves. This is not sent directly, another set of controllers are needed to derive and regulate the valves according to the incoming command of the Wiener-Hopf controller, and this is the lower level of the control task.

The Honeywell H316 minicomputer has been assigned the work of performing the higher level task of the control system mainly due to the ability of the BASIC-16 to call subroutines written in FORTRAN IV. If the Wiener-Hopf controller matrix C is implemented using FORTRAN language then this will be executed in the fastest mode available in the on-line computer network.

The Motorola M6800 microcomputer is chosen to handle the control effort requirements submitted by the H316 minicomputer; and then, to act on the pneumatic valves to bring them into the required level. Thus, the microprocessor is, in fact, acting as a servomechanism for the valves system. In effect, the lower level loop of the control system is characterized as a fast loop response, hence most of the time of the micro M6800 is spent in doing the following :

- i) receiving set-points changes from the H316 controller,
- ii) measuring the flows on-line for purposes of regulation,
- iii) regulating the valves flow loop,
- iv) setting the pneumatic valves accordingly.

To achieve this, the sampling period in the M6800 control level needs to be very short compared to the plant time constants. In fact, this is another reason why the Honeywell has been chosen to be at the higher level, since, the high rate of sampling frequencies required by the valves loops, will not give the

minicomputer ample time to perform the on-line graphics, and the handling of the communication traffic of the overall system. This is due to the architecture of the network; that all the communication in the system is monitored by the Honeywell H316 system. If on-line graphics is not required then there is no reason why the organisation of the hierarchical system should not be reversed. That is, the Honeywell H316 would handle the valves loops as well as the communication processing, and the Motorola M6800 would perform the processing of the optimal control action.

Fig [8.10] depicts the switching time forms for the two pneumatic valves. As has been shown in Chapter (3) the pneumatic valves have been arranged in a certain manner to use the rather expensive piece of equipment, the electric to pressure transducers I/P, economically. Otherwise, each valve would have its own I/P connected to the computer. Also, economy was achieved in installation and maintenance of the pneumatic system. Thus, one electric to pressure transducer (I/P) was installed to operate the two valves.

From Fig [3.3], it is clear that the electric/pressure transducer changes the air pressure in the main line feeding the two valves. Now, it is the duty of the solenoid valves to direct the air flow in each branch allowing the pneumatic valve to travel down or up admitting a certain amount of flow to pass.

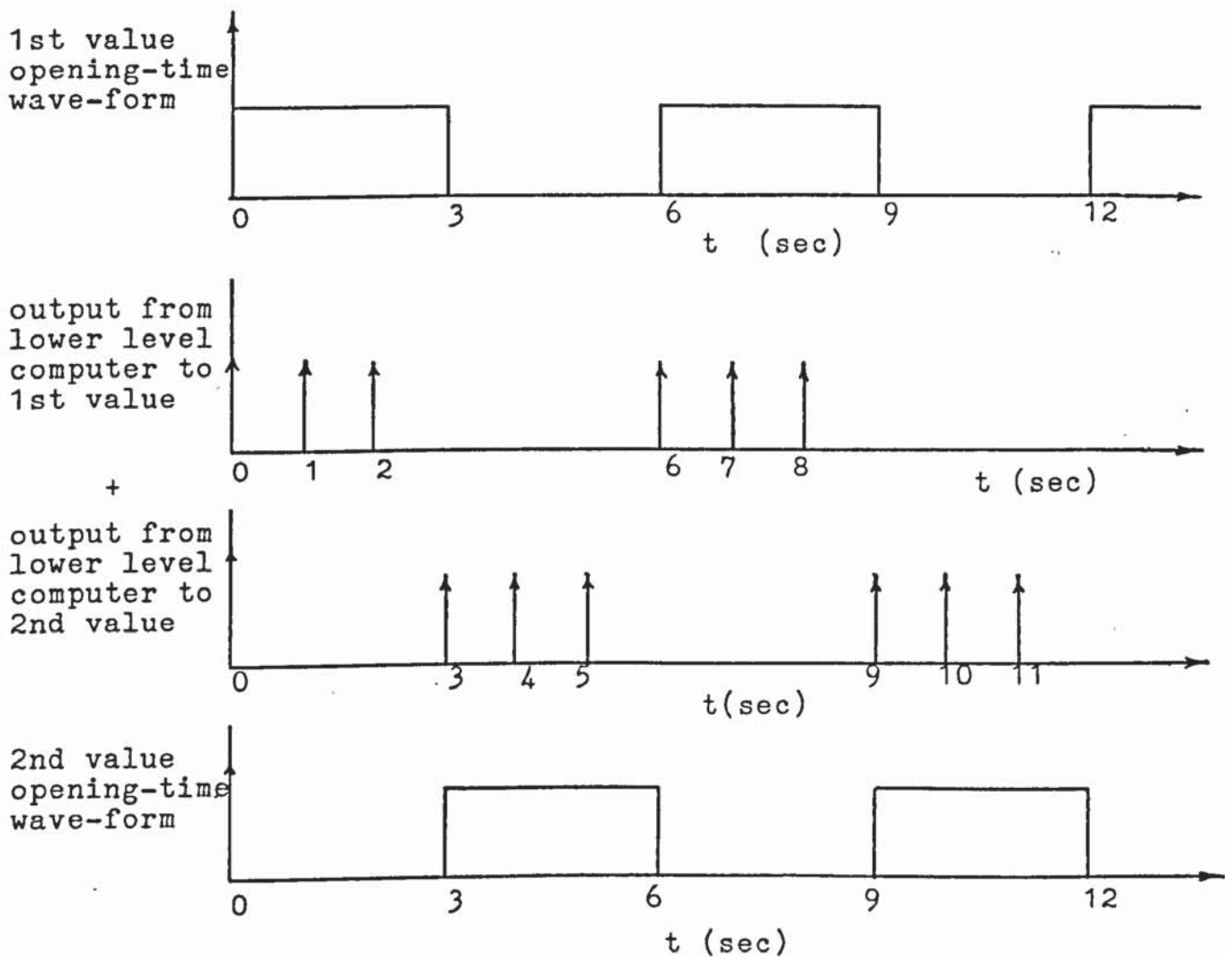


Fig. 8.10: Switching time forms for the solenoid valves.

Usually the two solenoid valves stay in close or shut-off state, unless they are envisaged by the lower level computer. The signal comes from the computer in the 16-bit data bus, where the ten right most bits are carrying the digital values of the required valve setting, and the next four bits are carrying the address to the solenoid valve, which is required to open the branch to the main air line.

8.7 The Digital Compensators: Real-Time Programs:

At this stage, after the performance of the system has been tested, off-line, by simulation; the real-time implementation of the digital compensator becomes feasible. Especially results shown by simulation on the signal level at the output of the controller are very important from a practical point of view. Since, if this signal level is higher than the maximum expected input signal to the plant, saturation will result and an unexpected behaviour may take place due to the loss of linearity.

Two higher-level language programs have been written to process the control system algorithm or the digital compensator function. Each of the programs resides in one of the computers.

8.7.1 Honeywell 316H Digital Compensator:

An interactive on-line program written in BASIC-16, is used to execute a general multivariable discrete compensator, and provide graphical output.

The operator feeds the 316H with the required information; such as, sampling period, discrete compensator transfer matrix parameters, variables to be plotted on-line, variables to be controlled, channels to be scanned. The functions of the program can be summarized as follows:

- (i) It samples the process every T sec.
H
- (ii) It calculates controller O/I, and sends the result to the M6800 through array D.
- (iii) It plots graphically, on-line, the time response of a chosen number of process variables.
- (iv) It allows the M6800 to sample the process every T sec.,
M
through the HADIOS system.

8.7.2 Motorola M6800 Microcomputer Compensator :

Another interactive program written in SD-BASIC, resides in the M6800; and performs the following functions:

- (i) It accepts measurement data of steam-flow-rate, and feed flowrate from the 316 minicomputer every T_M sec.,
- (ii) It accepts data from the 316H optimal controller as set-points to the flow rate every T_H sec.
- (iii) It drives the pneumatic valves according to the set-point changes sent by the 316H.

The program asks for the sampling period, which is usually shorter than that of the 316H minicomputer ($T_H > T_M$); the PID controller parameters of the valves, or any other type of controller as required. The on-line print-out of data on the microprocessor VDU, is a limited facility because if the sampling period, T_M , is short then there may not be enough time for displaying data on the VDU.

The 316H samples the process variables every T_H sec., and calculates the optimal controller O/P. Then, it sends this output to the M6800 through the communication channels by means of the array D.

At the same time, the M6800 samples the steam and feed flowrates every T_M Sec. At every T_H Sec., the M6800 receives new values for the valves settings from the 316H minicomputer. The M6800 accesses the valves sequentially in a cyclic manner: the first

three seconds it controls valve 1 and the next three seconds it controls valve 2, and so on as depicted in Fig [8.10]. The 3 seconds period, here, defines the time taken by the solenoid valves to change state, and this period can be hardware adjusted to any required setting.

8.8 General Remarks :

The philosophy of implementation which has been followed, here, is towards a general linear control application, and not limited to a specific application of a certain type of controller.

The method for transforming a linear system mentioned in section (8.2) is generally applicable in a linear transformation space.

The arrangement of the computers and the implementation of the controllers are modular, and any future modification or amendment is straightforward.

Each computer, the 316H and the M6800, can be interrupted without failure in the control system, because the valves will stay in the final state they had been at before the interruption of any of the computers occurred. This is due to the

fact that the digital data sent out as control values stay in hardware registers and these registers usually do not change their state unless the programs are running on the computers. Obviously, switching-off completely, the electric supply from the computer, may damage the state of these registers, an unexpected test in this study.

The controller software in the higher level computer, can accommodate all types of linear multivariable transfer matrices and hence it covers a wide class of controllers.

The APA package, used to calculate the Wiener-Hopf controller, in section (8.5), is a general algebraic symbolic-handling subroutines and are by no means confined to calculate this type of controllers.

CHAPTER (9)

RESULTS ANALYSIS AND DISCUSSION

In this chapter the results which have been obtained from the study in its various phases will be presented with a discussion. Where it is reasonable a comparison with similar results in the literature will be made. The main subjects in the study to be covered are the mathematical modelling of the process, both steady-state and dynamic; the library APA and other software; and the control aspects of the problem, including the Wiener-Hopf controller and linear systems control.

9.1 MATHEMATICAL MODELLING

9.1.1 Data logging

Real time experiments have been carried out on the double effect evaporator to test the derived model and validate of the assumptions underlying its derivation. The experimental arrangements were described in chapter (3). The experiments were designed for open loop conditions with the main manipulated variables being the steam flow rate and the feed flowrate. The valves were manually set for these experiments.

A set of experimental results was obtained for different operating conditions on the plant. In each experiment the plant was left running at a steady-state operating condition while the on-line computer, in this case the Honeywell 316, was logging the data. After around 80-120 seconds, when the operator was satisfied that the plant was in steady-state condition, then one of the valves, for steam or feed, was disturbed in a step increase or decrease around 10-20% of the steady-state flow. The plant was then left for around 30 min. until a new steady-state condition had been reached. Meanwhile, the computer was logging the data from the plant, with a sampling period of 12 sec., for the variables in table [9.1.1]. Simultaneously, a number of variables could be displayed on-line graphically on the VDU while a hard copy of the logging of all the variables was punched out on a paper tape.

Each run was kept on a separate paper tape and was then transferred into the main-frame computer storage area, ICL 1904s, HARRIS 800. The files were kept under generic names e.g. EX5112, EX4212, etc (indicating experiment, experiment no., date, month). The steady-state set of data for each experiment was kept in a separate file given names like ST5112, ST4212, etc. Each file showed the result of one experiment.

A number of programs were written, (SIMULA, DYNOSIM, PLOT, see Appendix H), to implement the model and at the same time to display the results of the model beside the experimental ones, in graphical form on-line and on the CALCOMP system as hard copy on graph papers.

UA. EC. P2140

SEQ: 31150

QS: <UA. EC. P2140>GLOSSA. QXT. 3

TB=T(6) OUTLET TEMP. FROM FE
TO=T(3) OUTLET TEMP. FROM PREHEATER
TEI=T(4) INLET TEMP. TO SE
TBP=T(8) PREHEATER SHELL TEMP.
TBC=T(9) CYCLONE TEMP.
TED=T(5) SE OUTLET TEMP.
TI=T(2) INLET TEMP. TO PREHEATER
TCSD=T(7) CONDENSER SHELL OUTLET
TCI=T(1) CONDENSER TUBE OUTLET
TCO=T(0) CONDENSER TUBE INLET
S=STEAM FLOW RATE KG/S
ML=P(11)= PRODUCT FLOW RATE KG/S
M=B(56) FEED FLOW RATE KG/S
PB=B(12) VAPOR PRESS. IN CYCLONE PSIA
PE=B(15) SE SHELL PRESS (PSIA)
PS=B(14) FE SHELL PRESS
PPB =VAPOR PRESS. IN PREHEATER

NUMBER OF PAGES 1

TABLE [9.1.1]

Before the calculation of the dynamic model, another set of programs, (PARAMA, PARAM2, PARA, STEADY), were used to calculate the parameters of the system, like UL, UB, Ll, from the steady-state data files. These parameters were functions of the process variables, and were different for every run.

A sample of the steady-state data is shown in tables [9.1.2]a,b. From here the parameters are calculated as shown in table [9.1.3]. These calculated parameters are fed to each corresponding model, e.g. responses of temperatures, vapour due to steam or feed flowrate perturbations. The results of the mathematical model in comparison with the respective experimental ones are shown in Figs. [9.1.1,2,3].

9.1.2 Results from the Model

Some of the assumptions stated in deriving the first order hyperbolic equations were that the fluid motion was plug flow, that there was no axial or radial conduction in the fluid, that there was no resistance of the wall to the heat transfer, and that there was no significant accumulation of energy in the condensate film. These assumptions besides others were stated to simplify the mathematical analysis of the problem but at the same time retaining the basic physical features of the process. From the results of the model shown beside the experimental ones, figs.[9.1.1,2,3], it is clear that the model has well tracked the plant. It has behaved satisfactorily and consistently at different operating conditions of the plant. The


```
*****I*****
*****I*****
*****I*****
***** Pdn = 7 2299-MNI Terminal: 58 29 JUN 84 13:54:46 *****
*****I*****
*****I*****
```

```
1 TB = 97.6938
2 TG = 80.7427
3 TEI = 100.779
4 TBP = 104.772
5 TBC = 103.309
6 TEO = 99.9412
7 TI = 13.5221
8 TCSO = 31.2500
9 TCO = 30.5956
10 TCI = 12.8897
11 S = 0.03178
12 ML = 0.04186
13 M = 0.05090
14 PB = 0.49465
15 PE = 0.03955
16 PS = 19.9169
17
```

Table [9.1.2]a Sample of data logged from the plant:
steady-state conditions, EX5112

```
18
19
20
21 TB = 96.5000
22 TG = 79.2353
23 TEI = 100.397
24 TBP = 103.088
25 TBC = 105.250
26 TEO = 99.4706
27 TI = 13.2353
28 TCSO = 26.2574
29 TCO = 26.3676
30 TCI = 12.6324
31 S = 0.02090
32 ML = 0.04223
33 M = 0.07150
34 PB = 0.10741
35 PE = 0.00000
36 PS = 23.5816
37
```

Table [9.1.2]B Sample of data logged from the plant:
steady-state conditions, EX4212

```
38
39
40
EOF .
```

```
*****I*****
*****I*****
*****I*****
***** No. of pages 1 2299-MNI Terminal: 58 29 JUN 84 13:54:53 *****
*****I*****
*****I*****
```


***** Pdn = 7 2299-MNI Terminal: 56 23 MAR 84 15:50:25 *****

LISTING OF 2299MNI *RS4212 LAST UPDATED ON 14-DEC-82 AT 20:08:41

| | | |
|----|--------------|-----|
| 1 | 0.916368E-03 | UB |
| 2 | 0.193214E-01 | KV |
| 3 | 0.644437 | GSS |
| 4 | 22.7403 | Tb |
| 5 | 0.135931E-02 | GFE |
| 6 | 1.24618 | TFE |
| 7 | 2189 68 | Lc |
| 8 | 2269 36 | Lv |
| 9 | 129.604 | Ts |
| 10 | 96.5000 | Tb |
| 11 | 78.2353 | To |
| 12 | 0.715000E-01 | M |
| 13 | 0.422300E-01 | m |
| 14 | 0.209000E-01 | S |
| 15 | 1 | |

***** No. of pages 1 2299-MNI Terminal: 56 23 MAR 84 15:50:27 *****

TABLE [9.1.3]

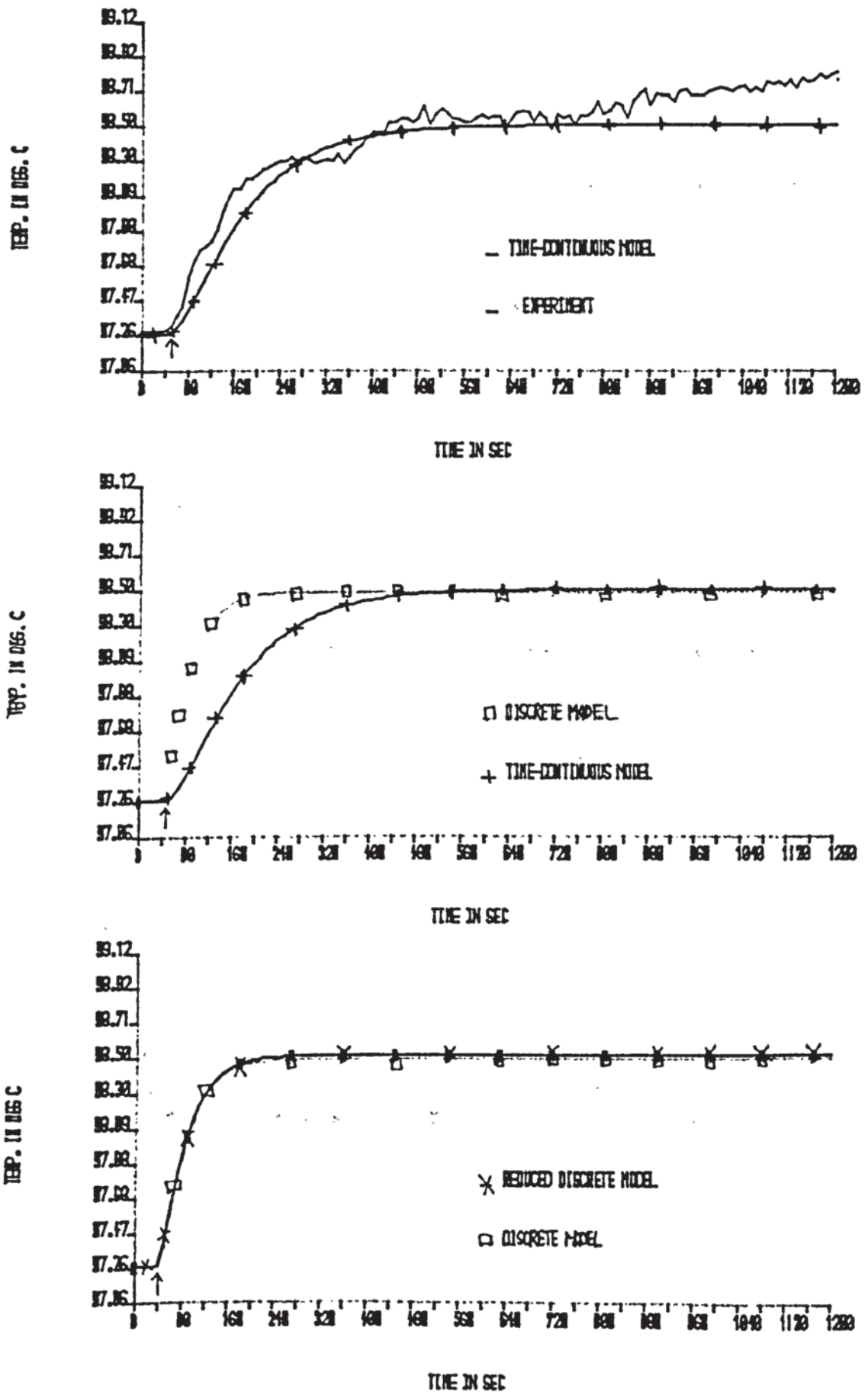


Fig. [9.1.1] a Response of the Evaporator outlet temperature to step increase insteam flow rate
(↑) (start of the experiment)

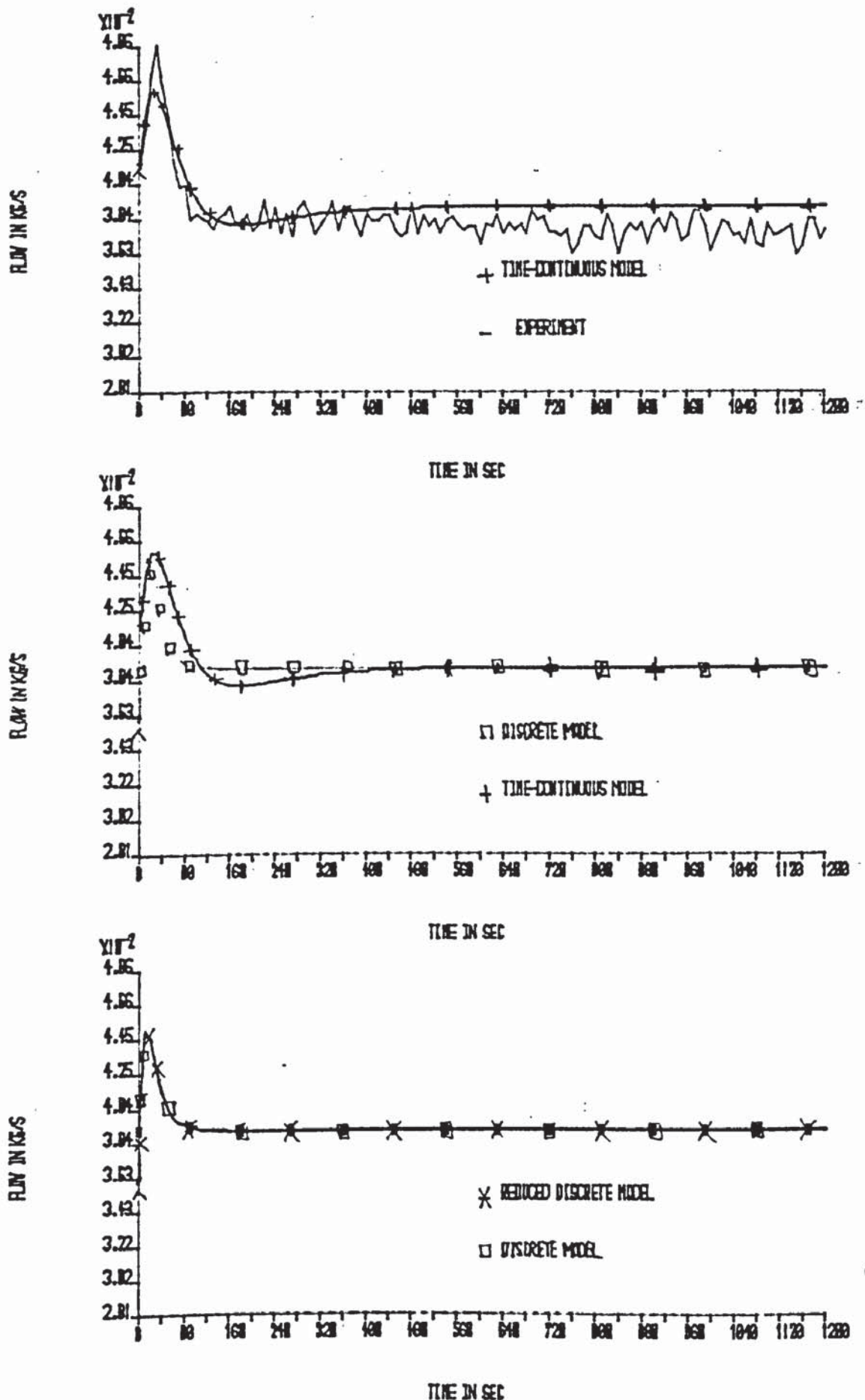


Fig. [9.1.1] b Response of the evaporator product to step increase in steam flow rate
(↑) (start of the experiment)

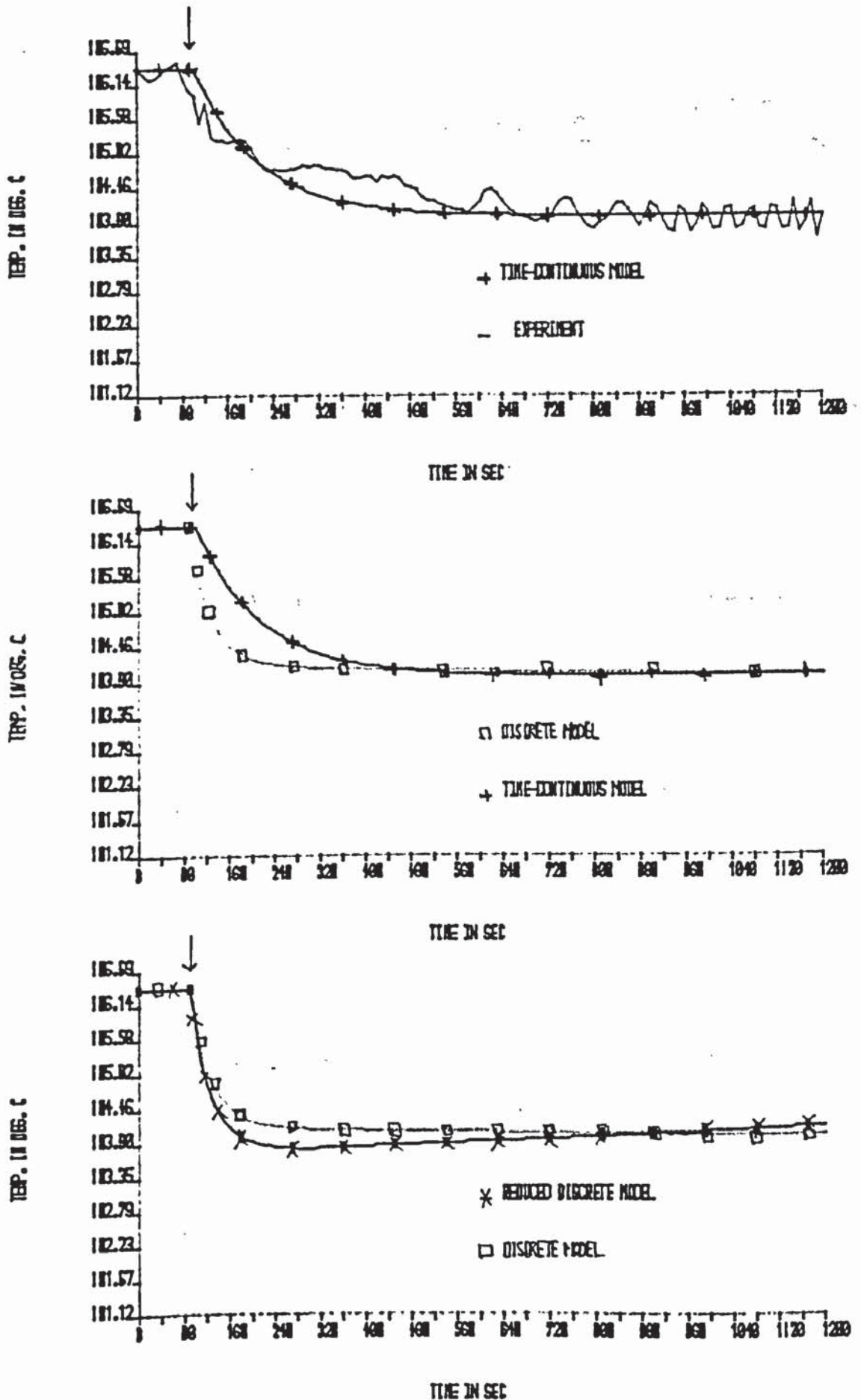


Fig. [9.1.2]a Response of the evaporator outlet temperature to step increase in feed flow rate (↑) (start of the experiment)

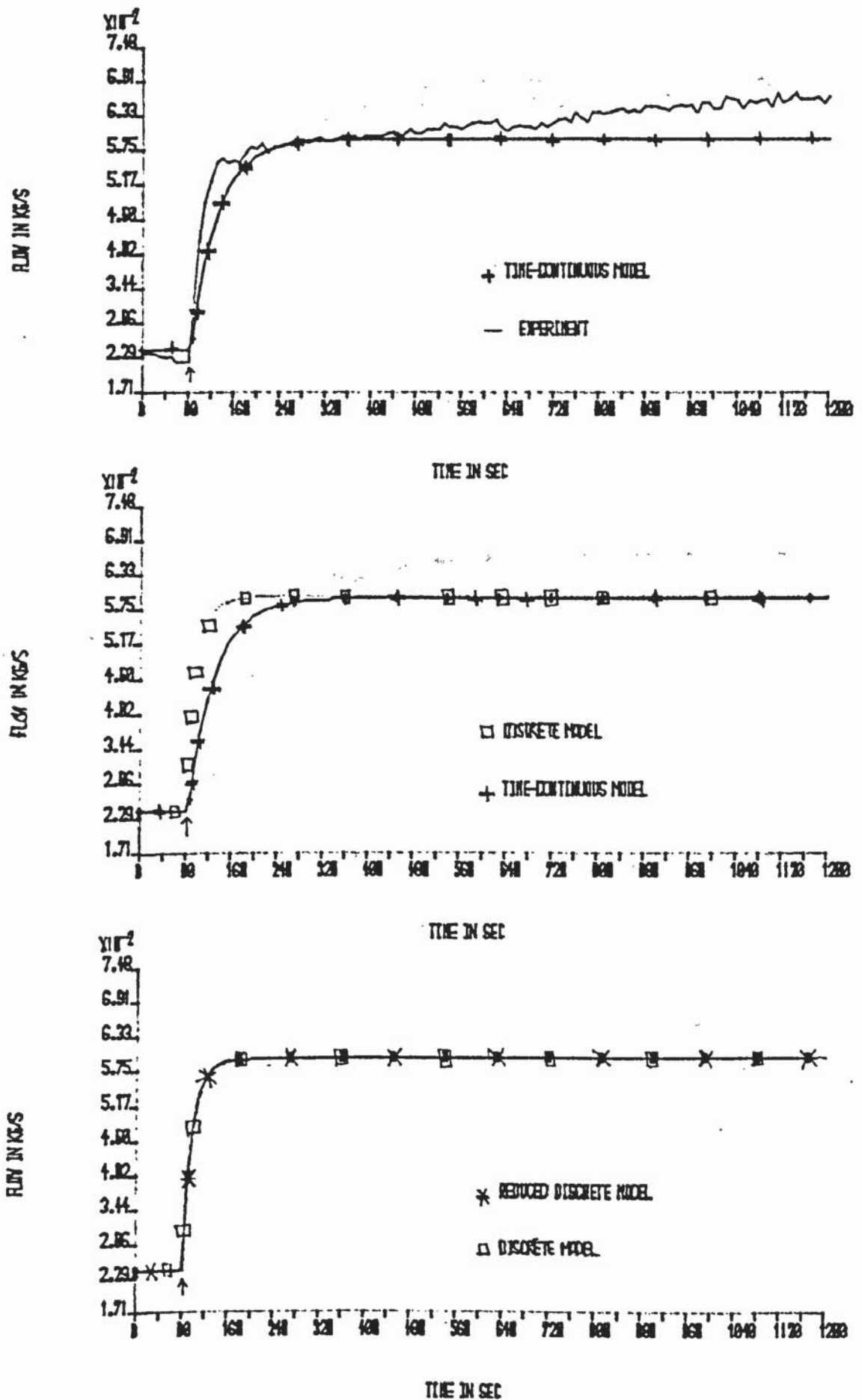


Fig. [9.1.2]b Response of the evaporator product to step increase in feed flow rate
(↑) (start of the experiment)

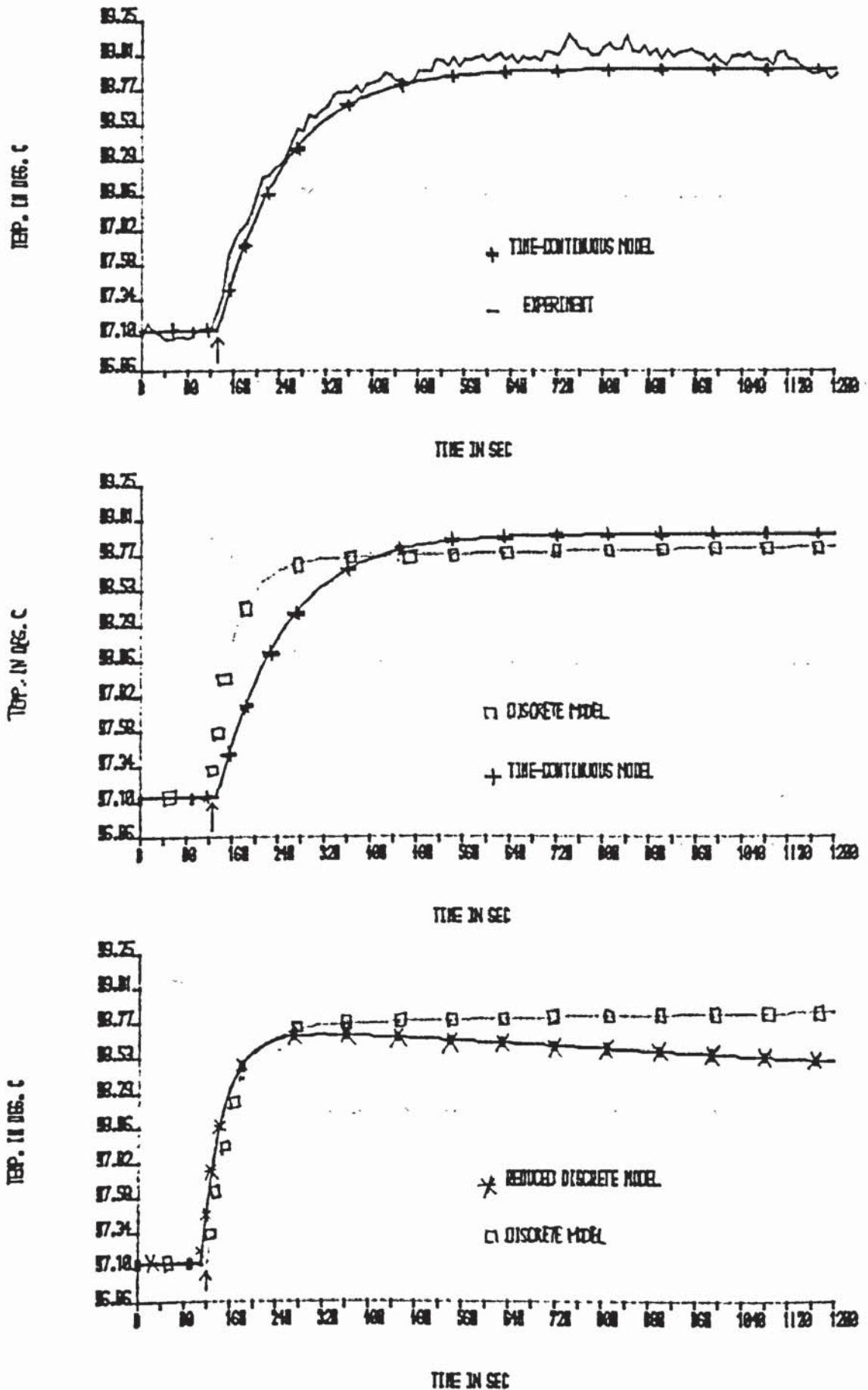


Fig. [9.1.3]a Response of the evaporator outlet temperature to step decrease in feed flow rate (↑) (start of the experiment)

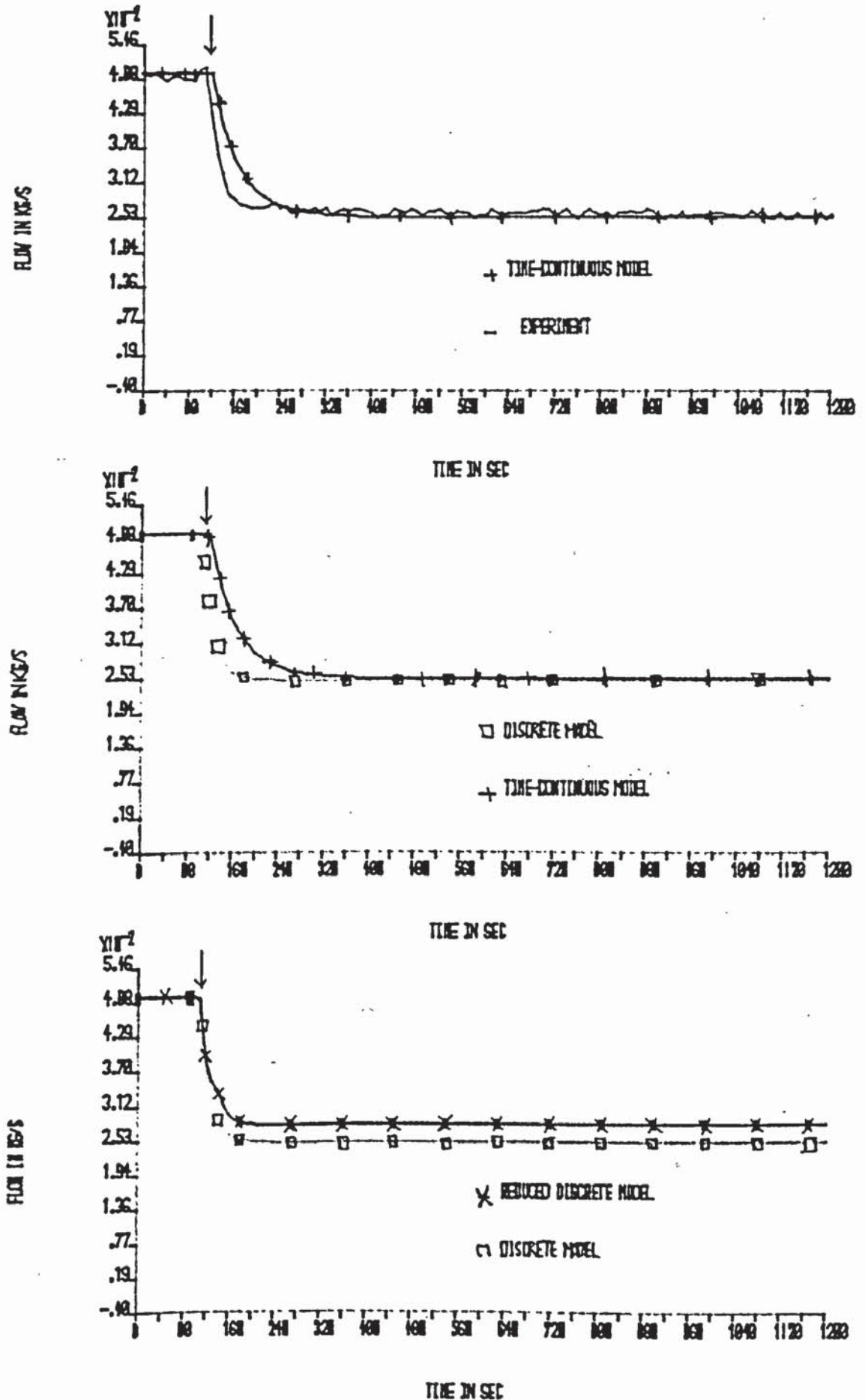


Fig. [9.1.3] b Response of the evaporator product to step decrease in feed flow rate
(↑) (start of the experiment)

model has stood robustly against changes in set-points of up to + 50% . This also may imply the linearity of the plant operation in a wide range, or in terms of the model, second order and higher orders in the Taylor series expansion of the system equations are negligible (see section 9.1.2.1).

Apparently, the rather "heavy" assumption in developing the model, that the steam on the shell has an equal rate of condensation per unit length along the tube is almost successful and the results which have been obtained are significant. This has been investigated further in section (9.1.5). This last assumption directly affects the estimate of the subcooled region and the pure time delays in the system. Also, the assumption of constant wall temperature was found to be reasonable. This kind of behavior can be expected when pure vapour is condensing on the outside of the tubes, when the shell fluid is perfectly mixed as in a continuous-stirred-tank reactor, or when the flow rate on one side of the exchanger is so high that for practical purposes its temperature can be considered as being independent of the distance from the inlet.

Neglecting the radial temperature variation in the tube wall was reasonable for a thin wall made of a good thermal conductor e.g. copper, while the assumption of rectangular fluid velocity and temperature profile, had been quite accurate for turbulent flow conditions. It was also assumed that the fluid was incompressible and that axial heat conduction in both the wall and the fluid could be neglected. Axial conduction in the tube wall should be small since the wall is thin, giving a high thermal

resistance. In the fluid, axial conduction, the diffusion term, was neglected because of the poor conductivity of most fluids and the existence of the good heat flow passage offered by the fluid transport term.

The assumption that hydraulic or mass transfer dynamics are fast compared with the heat transfer dynamics in the evaporation region seems to be a quite practical assumption. Therefore, quasi-steady state equations would suffice in cases with these fast dynamic modes in coexistence with relatively slow dynamic modes, (see Appendix A and chapter (4)).

9.1.2.1 The Heat Transfer Coefficient

Since this study is concerned with the control of the plant as a final objective, the mathematical model is based on that; hence, it is not unexpected that the heat transfer coefficient will not be a major issue for investigation, as it would be if the objective were better equipment design by improving the heat transfer coefficient. However, in the model that has been developed, the evaluation of the heat transfer coefficient is not directly needed. The evaluation of it was based on algebraic equation. The heat transfer coefficient was assumed as a variable, U , and then the necessary equations to find U were solved. The expression obtained for U was a function of a number of variables. These variables were directly and easily measured. Hence, the accuracy in getting a satisfactory value for U depends on the overall assumptions presented to get the mathematical model.

The pattern of the flow behavior assumed inside the climbing film unit, fig [4.1] seems to be of practical usage but the implicit discontinuity in the heat transfer coefficient at L_1 should not be overlooked. In the light of the results obtained, it may be assumed that the average heat transfer along the whole tube is roughly in order of magnitude equal to the heat transfer coefficient at the boiling region.

The heat losses were discarded from the model on the basis that the plant was well insulated and the effect of heat losses was negligible. The model behavior has, to a good extent, proved this consideration, which was in fact the way the plant was insulated. Inclusion of the heat losses in the model would have complicated, to a degree, the analysis. The heat loss mechanism would be visualized through the known mechanisms of radiation, convection, and conduction. In practice some assumptions can be made by which the dominant mode is assigned. The difficulty in finding the eigen-values associated with heat losses dynamics and the resulting analysis may be complex enough to blur the final result. From an operational point of view, it is of prime importance to effectively insulate the plant, to operate economically in commercial applications.

The applicability of the mathematical model over a wide range of set-point changes may be attributed to the assumption that the plant parameters were not sensitive to high level input vector. Some sort of feedback or self organization may have been taking place with a net result that the parameters variations are negligible, see Appendices A and B.

9.1.2.2 Modelling of The Parameters Variation

A model of how the parameters may be affected by set-point changes can be conceived as depicted in fig (9.1.4).

The plant parameters are dependent on the sets of input variables u , and output variables v . The dependence on the plant geometry is to be constant and not to vary with set-point changes.

The dependence of parameters on u and v is assumed to be a linear combination of all possible u and v , (this linear relations can be obtained by Taylor's theorem), u and v are not necessarily linear, they can assume any behavior.

Now, let x be a column matrix containing the parameter elements of the process, and has real coefficients in R , the field of real numbers, H, G , and F are real.

As mentioned above, vectors u and v may contain non linear functions e.g. $u = [u_1, 1, u_3, u_4, \sin u_5, \log u_6, \dots]$ and $v = [v_1, v_2, \sin v_3, \dots]$. The effect of the input vector u on v and x is through the real matrices H and F and consists of the linear

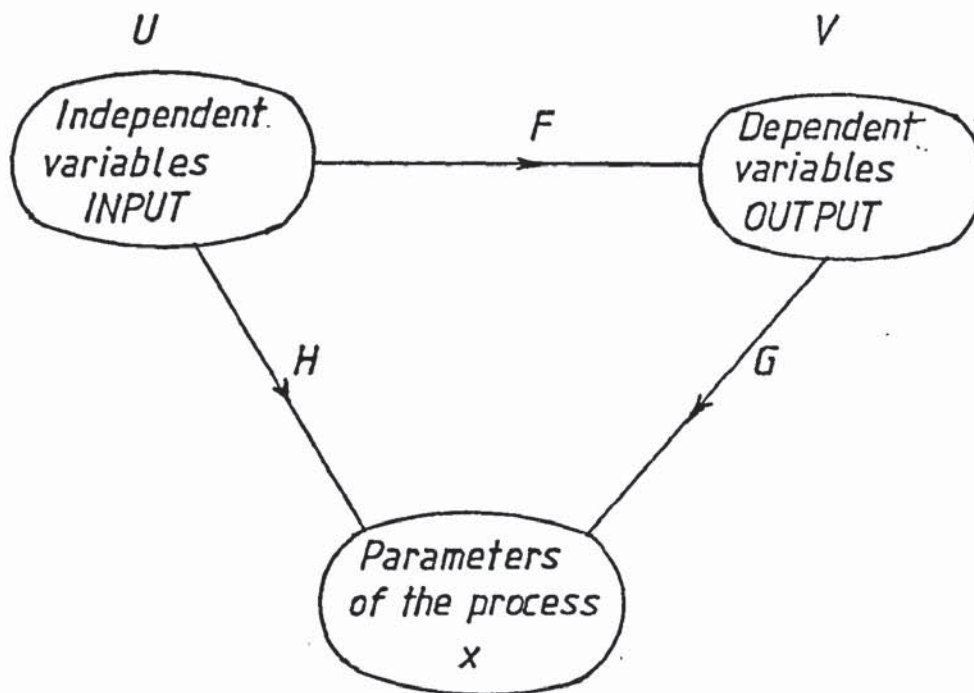


Fig. 9.1.4: A model for plant variables interactions.

combination of elements u_i , $i = 1, 2, \dots, m$. Similarly, vector v

affects vector x through the real matrix G .

From fig (9.1.4) it can be immediately written :

$$x = H.u + G.v \quad (9.1)$$

$$v = F.u$$

$$X = H.u + GF.u$$

$$X = (H + GF)u \quad (9.2)$$

Equation (9.2) can be expanded by a Taylor series around a steady-state condition origin, which without loss of generality this can be assumed as zero, but x must be analytic at that point.

$$x = B_0 + B_1 u + B_2 u^2 + B_3 u^3 + B_4 u^4 + \dots \quad (9.3)$$

where B_i is a real matrix and belongs to $R^{n \times m}$.

Now suppose the set-point change has taken place in such a way that

$$u = u + \Delta u, \text{ and } x = x + \Delta x$$

Equation (9.3) becomes :

$$(x + \Delta x) = B_0 + B_1 (u + \Delta u) + B_2 (u + \Delta u)^2 + B_3 (u + \Delta u)^3 + \dots \quad (9.4)$$

After higher order terms of Δu have been neglected x becomes

$$\Delta x = (B_0 + B_1 u + 3B_2 u^2 + 4B_3 u^3 + \dots) \Delta u \quad (9.5)$$

Now, substitute from equation (9.3) into equation (9.5) and multiply by u .

$$\begin{aligned} u \cdot \Delta x &= (x - B_0 \\ &\quad + x - B_0 - B_1 u \\ &\quad + x - B_0 - B_1 u - B_2 u^2 \\ &\quad + \dots \\ &\quad + \dots) \Delta u \end{aligned}$$

If the Taylor series in (9.3) is truncated after n terms then;

$$u \cdot \Delta x = (nx - nB_0 - (n-1)B_1 u - (n-2)B_2 u^2 - (n-3)B_3 u^3 - \dots) \Delta u \quad (9.6)$$

If n is large enough, (i.e. $n \rightarrow \infty$) which is a good condition to improve the accuracy in (9.3), then equation (9.6) becomes;

$$\begin{aligned} u \cdot \Delta x &\approx (nx - nB_0 - nB_1 u - nB_2 u^2 - nB_3 u^3 - \dots) \Delta u \\ u \cdot \Delta x &\approx (nx - nx) \Delta u \quad (9.7) \\ &= 0 \quad (9.7) \end{aligned}$$

$(B \approx_n B_n)$ since $B_n \rightarrow 0$ from Taylor series expansion theorem. Alternatively since the term B_n has $1/n!$ as a coefficient, $n.B_n$ will give $1/(n-1)!$ so if $n \rightarrow \infty$, $1/(n-1) \rightarrow 0$)

Hence, for small Δu the change in the parameters can be taken as zero and that the plant has constant parameters within a domain of the steady-state operating conditions.

9.1.3 The Interaction of The Plant Variables

The plant has shown dominant behavior among some of its variables, this is clear from the experimental data that has been obtained.

For example, the feed flowrate change has a dominant effect on both the outlet temperature of the preheater unit and, an obvious effect, on the liquid concentration output from the climbing film unit. The effect of the changes of the feed flowrate on the outlet temperature was not as dominant as with the outlet temperature of the preheater and this is expected since the available energy for heat transfer in the steam is far much higher than the heat content of vapour in the preheater shell. Hence, if feed flow rate was suddenly increased the preheater shell might not have enough energy to regulate the temperature.

At the same instant, the climbing film shell has a higher energy content so changes in the feed flow rate within the steady-state region have not produced as much changes as with the outlet temperature of the preheater. The feed flow rate has almost no direct effect on the dynamics of the second effect unit except through the changes it makes on T and ML .

b

On the other hand, the steam flow rate changes on the climbing film unit have shown negligible effect on the outlet temperature of the preheater in the transient state. In the long run as the amount of vapour coming out from the first-effect increased, the outlet temperature of the preheater also increased.

This is because the vapour generated by the first-effect, after being separated from the liquid phase by the cyclone enters the preheater shell. This mechanism might sometimes drive the plant into an asymptotic instability if the plant was left on open-loop. As the amount of vapour in the preheater shell increased and at the same time there was no effective heat transfer in the second effect, the temperature in the preheater tubes would start to increase to a point almost near the boiling point of the liquid. So the liquid on entry to the first-effect had already reached the saturation point, and due to the high availability of energy in the first-effect shell the climbing film could not establish a liquid/phase region. Vigorous evaporation may take place and this excess of vapour can go back again to the preheater shell to drive the outlet temperature even higher, (from the mathematical model this behavior is not immediately clear because mainly the model was described for a transient stable operating conditions). Under asymptotic unstable conditions the operating conditions can change very much and consequently the parameters of the model may need new evaluation at the runaway conditions. If there is enough heat transfer in the second-effect unit the vapour may be drawn from the preheater shell to the second-effect and that can reduce the effect of runaway-state in the plant. That is the case when the second-effect is under vacuum. Under atmospheric pressure the runaway condition could happen due to the fact that the separator liquid-temperature is almost equal to the outlet temperature from the first-effect. Hence, it would be expected that very little heat transfer might

take place in the second-effect unit, due to the low temperature gradient between the shell and tubes in the second effect.

Though, the runaway situation is hazardous to the plant; but, from an economical point of view it will be desirable to operate near the runaway region. Because, in that case the amount of steam needed to affect the required evaporation is at a minimum. Indeed, tied control is needed to operate the plant under these conditions. The price to be paid for an uncontrollable situation might be the total loss of the climbing film unit. This is due to the fact that the vapour pressure inside the tubes can continue to rise to a point exceeding the equipment design limit; especially, if the steam driving pressure upstream is much higher than the equipment pressure design limit. To secure this point then it is preferable to have a regulator on the steam main line feeding the plant to keep the maximum steam pressure under the equipment pressure design limit. The other disadvantage of operating near the runaway region is that the solids in the liquid start to build scaling inside the tubes wall. So, concentration in the first effect should be kept within the equipment manufacturer design limit, in this case not to exceed 50-60%.

The number of transfer units in the climbing film,

e^{-kL_1} , is small, hence, the effect of T_o was unnoticeable. From

equation (4.3), T_o is highly attenuated by this term, e^{-kL_1} , which expresses the number of transfer units.

9.1.4 The Equivalent Tube

In the pilot plant, the climbing film and the second effect were constructed from a number of tubes inside a shell. In deriving the mathematical model these bundles of tubes were considered as a single tube inside the shell. Such an assumption is very difficult to justify in a theoretical analysis: to establish on what basis can the bundle be represented in a single tube representation.

Let us follow the simple following analysis of the problem:

Define the following symbols;

Q = total flow into the tubes

n = number of tubes inside the shell

A = total cross-sectional area of the tubes

a = the cross-sectional area of a single tube

d = diameter of a single tube

D = equivalent diameter.

L = length of calandria.

Now, for the flow Q to pass through the single equivalent tube, a cross sectional area equal to the total cross-sectional areas should be available to allow the same flow, Q , to pass through the equivalent tube,

$$A = \frac{n \cdot \pi d^2}{4} = \frac{D^2 \pi}{4}$$

therefore, $D = d\sqrt{n}$ (9.8)

But, at the same time the same surface area available for heat transfer by the bundle of tubes should be available by this equivalent single tube.

Total surface area available by the bundle is

$$= \frac{n \cdot \pi \cdot d \cdot L}{2}$$

This should be equal to the surface area of the equivalent

$$\text{tube} = \frac{D \cdot \pi \cdot L}{2}$$

$$\frac{D \cdot \pi \cdot L}{2} = \frac{n \cdot \pi \cdot d \cdot L}{2}$$

$$\text{therefore; } D = nd \quad (9.9)$$

The discrepancy between equations (9.8) and (9.9) is now apparent: in finding the geometrical dimensions of the equivalent tube, two different diameters have been found when the physical criteria of heat transfer and mass flow rate are allowed. A sizable difference of \sqrt{n} is found when the analysis is based on these criteria, (because usually $n > 2$). If n is, say, 9 then the difference between the two diameters of equal surface area and equal cross-sectional area is 3 times in linear distance.

On account of these differences, another apparent similarity criterion may be investigated. Since it is required that the flow in the single equivalent tube be similar to the one in the bundle of tubes the Reynold number is at our disposal to establish this similarity.

It is a well known fact that for two flows to be dynamically similar they should have an equal Reynold number.

Now, the Reynold number of a single tube in the bundle

$$= \frac{Q \cdot 4 \cdot d}{n \cdot \pi \cdot d \cdot d \cdot u} = \frac{4 \cdot Q}{n \cdot \pi \cdot u \cdot d}$$

The Reynold number for the equivalent single tube is

$$= \frac{Q \cdot 4 \cdot D}{D \cdot D \cdot \pi \cdot u} = \frac{4 \cdot Q}{D \cdot \pi \cdot u}$$

$$\text{Now, if } \frac{4 \cdot Q}{n \cdot \pi \cdot u \cdot d} = \frac{4 \cdot Q}{D \cdot \pi \cdot u}$$

$$\text{Then } D = nd \quad (9.10)$$

This last result of equation (9.10) suggests that if the assumption of dynamic similarity is taken then the equivalent single tube diameter is the same as the one obtained by the proposition of equal surface area available for heat transfer.

Nevertheless, dynamic similarity does not completely solve the problem, since other different hydraulic phenomena arise to render or prohibit this similarity to exist.

For example, the pressure drop in each tube may not be the same and there is no general procedure by which the pressure drop in each case can be calculated for comparison. Experimentally, the pressure produced by a bundle of small diameter tubes may be greater than the pressure produced by a single large tube, however, there was no data available to verify this.

Nevertheless, when the formula in equation (9.10) was used in evaluating the model parameters the results were not consistent, except when the two different criteria had been used again. So, when evaluating the parameters concerned with the flow then the equal cross-sectional area equivalence had to be used. On the other hand, in the evaluation of the parameters connected with the heat transfer the equal surface area was used. Satisfactory results have been obtained when these two different criteria have been applied to evaluate the parameters.

9.1.5 Comparison of The Steady-State Results

The steady-state results of the model are further compared with the data available by Kerjsa et al [1967].

The data of Kerjsa came out from a study on the stability of two-phase flow in boilers. The set-up of his experiment was as follows:

- (1) a boiler section as a shell and tube heat exchanger.
- (2) Freon-113 was the test fluid flowing inside the inner tube.
- (3) hot water flowed countercurrent to the Freon flow on the annulus between the inner tube and the outer jacket.

The objective of his work was to obtain frequency response data for a single tube forced-flow boiler over a range of heat fluxes and exit vapour qualities.

Steady-state experimental data of the water/Freon system were used in the steady-state model derived in this study for the steam/water evaporator to obtain another isolated set of parameters. The isolated set chosen to be predicted by the steady-state model was the subcooled length, L_1 , and the vapour quality.

In chapter (4), mathematical modelling, the tube steady-state temperature distribution was found to be:

$$\bar{T}(x) = \bar{T}_s + (\bar{T}_o - \bar{T}_s) e^{-kx}$$

The energy balance equation in the shell/tube evaporator at liquid phase is

$$\int_0^{L_1} \bar{T}_s dx = 2\pi r U \int_0^{L_1} (\bar{T}_o - \bar{T}(x)) dx \quad (9.11)$$

Integrating the above equation after inserting the previous equation, in the range $0 < x < L_1$

$$\bar{T}_s L_1 = 2\pi r U \left(\bar{T}_o - \bar{T}_s \right) \left(1 - e^{-kL_1} \right) \quad (9.12)$$

$$= \dot{M}_c \left(\bar{T}_p - \bar{T}_o \right) (1 - q_{ss}) \quad (9.13)$$

An important point to be mentioned here is that the rate of condensation was assumed to be equal in the liquid phase and in the boiling phase, although the heat transfer coefficient was different for each flow regime. This assumption of constant condensation rate was necessary otherwise the analysis of the mass balance on the shell side would have been difficult. Accordingly,

the total amount of the condensate can be taken as the linear summation of the condensate along the tube.

$$\text{Total amount of condensate} = \bar{S}L \quad (9.14)$$

where L is the total length of the tube.

At steady-state the rate of condensation is equal to the rate of steam input, since it was assumed that there was no condensate hold up in the shell.

$$s = \frac{S}{L} \quad (9.15)$$

where S = rate of steam input.

Substitute (9.15) into (9.13)

$$\frac{L_c S L_1}{L} = \frac{M_c (\bar{T}_p - \bar{T}_o) (1 - g_{ss})}{s} \quad (9.16)$$

Equation (9.16) intimates that the ratio of the subcooled length to the tube total length is equal to the ratio of the thermal energy taken by the liquid phase to the total thermal energy input. Hence, an explicit expression to evaluate the subcooled length is obtained. This also has made possible the estimation of the steady-state pure time delay in the system, $\tau = L_1 / v$.

The experimental data of the Freon /water system were used in equation (9.16) to estimate the subcooled length L_1 . This was later

compared with the subcooled lengths obtained independently by Kerjsa.

Outlet Vapour Quality

The steady-state mass balance equation in the two phase region were found to be, (see Appendix A)

$$\frac{\partial \bar{m}}{\partial x} + \bar{a} \bar{m} = 0 \quad (9.17)$$

$$\frac{\partial \bar{M}V}{\partial x} - \bar{a} \bar{m} = 0 \quad (9.18)$$

where \bar{a} is the coefficient of evaporation per unit length. It represents the ratio of liquid mass evaporated per unit length of the evaporating tube. This will be assumed a constant of the system. We may suggest that every evaporator probably has this constant characteristic, which may be taken as a measure of comparison between different evaporators construction.

The above two equations are integrated with the following boundary condition; that is:

$$\text{at } x = 0, \bar{m} = \bar{M} \quad (\text{total feed mass flow rate})$$

$$\bar{m}(x) = \bar{M} e^{-ax} \quad 0 < x \leq L-L_1$$

$$\text{and } \bar{M}V(x) = \bar{M} (1 - e^{-ax})$$

These equations were used to evaluate the Freon exist quality.

Since it has been assumed that \bar{a} is a property of a certain evaporator and is a constant of the equipment then it can be evaluated once and used in any subsequent calculations. \bar{a} is assumed to be dependent on the operating conditions.

An example will show how the data in Kerjsa is used in our model to predict Kerjsa experimental results.

For the first experimental run of Kerjsa, (see table [9.1.4]), Freon has the following specifications:

$$M = 0.076 \text{ Kg/s}$$

$$\bar{T}_o = 297 \text{ deg K}$$

$$\bar{T}_b = 327 \text{ deg K}$$

and the water system in the shell

$$S = 0.0876 \text{ Kg/s}$$

$$\bar{T}_{si} = 371 \text{ deg K}$$

$$\bar{T}_{so} = 362 \text{ deg K}$$

TABLE (9.1.4)

| Run | M | S _s | T _o | T _b | T _{si} | T _{so} | Subcool Length | | Vapour Quality | |
|-----|-------|----------------|----------------|----------------|-----------------|-----------------|----------------|-------------------------|------------------|------------|
| | | | | | | | Expt Kerjsa | Model L ₁ | Expt Kerjsa % | Model % |
| 1 | 0.076 | 0.0876 | 297 | 327 | 371 | 362 | 0.831 | 0.84 | 12 | 12 |
| 2 | 0.083 | 0.088 | 299 | 328 | 373 | 363 | 0.831 | 0.796 | 13 | 13.6 |
| 3 | 0.056 | 0.088 | 295 | 325 | 372 | 362 | 0.559 | 0.556 | 29 | 36.4 |
| 4 | 0.036 | 0.086 | 293 | 322 | 373 | 363 | 0.203 | 0.35 | 50 | 50.9 |
| 6 | 0.081 | 0.085 | 298 | 330 | 384 | 371 | 0.686 | 0.683 | 23 | 25.2 |
| 7 | 0.057 | 0.091 | 297 | 327 | 384 | 372 | 0.406 | 0.456 | 37 | 44 |
| 10 | 0.057 | 0.088 | 296 | 326 | 394 | 383 | 0.229 | 0.51 | 31 | 39.7 |
| 11 | 0.075 | 0.085 | 298 | 333 | 394 | 379 | 0.508 | 0.599 | 31 | 32.7 |

A comparison of experimental results of freon/water evaporator with the mathematical model.

where \bar{T}_{si} is the water inlet temperature and \bar{T}_{so} is the water outlet temperature.

From equation (9.16)

$$\frac{\text{subcooled length}}{\text{tube total length}} = \frac{\text{total thermal energy in liquid-phase}}{\text{total thermal energy in the system}}$$

$$L1 = \frac{M_c (T_p - T_o)}{S_c (T_{si} - T_{so})} \cdot L$$

$$\frac{0.076 \times 0.32 \times 30.0}{0.0876 \times 1 \times 9} \times 0.91 = 0.84 \text{ m}$$

$$\frac{\bar{m}}{\bar{M}} = e^{-aL2} = 0.88$$

$$L2 = L - L1$$

$$\bar{a} \cdot L2 = 0.128$$

$$\bar{a} = 1.28 \text{ (per meter)}$$

\bar{a} has been defined as the evaporation constant.

For run (2), data in table (9.1.4), L1 and Freon vapour quality can be found.

data of liquid Freon :

$$M = 0.083 \text{ Kg/s}$$

$$\bar{T}_o = 299 \text{ deg K}$$

$$\bar{T}_b = 328 \text{ deg K}$$

data of water :

$$S = 0.088 \text{ Kg/s}$$

$$\bar{T}_{si} = 373 \text{ deg K}$$

$$\bar{T}_{so} = 363 \text{ deg K}$$

$$L1 = \frac{0.083 \times 0.32 \times 29 \times 0.91}{0.088 \times 1 \times 10} = 0.796 \text{ m}$$

$$\text{Freon exit quality \%} = (1 - e^{-a.L2}) \times 100$$

$$= (1 - e^{-1.28 \times 1.14}) \times 100$$

$$= 13.6 \%$$

Table [9.1.4] shows the comparison of the subcooled length and vapour quality predicted by the model with the corresponding values obtained independently by Kerjsa. The result is comparatively good and satisfactory. The subcooled length and the vapour quality have been chosen as an example to show the validation of the steady-state model. These two parameters are very important, since they give the estimate of the pure time delay of the system and the amount of vapour coming out from the system. The time-delay and vapour quality are very significant factors in the study of the dynamics of the evaporator developed in chapter (4).

No doubt, despite the differences between the two experiments, steam/water and water/Freon, the mathematical model has shown satisfactory behavior. The wide validation of the model may show the effort worthwhile of developing mathematical models from basic physical laws rather than the specific on-line statistical identification. However, the above statement can not be preceded quickly and concluded from two tests. This is only to high light this possible fact and to encourage the test of other published data on evaporators.

9.1.6 Vapour Pressure Surges

Indeed, it may be expensive to derive mathematical models of processes from fundamental physiochemical laws, but still it may be more expensive to identify the process on-line for purposes of control. Mathematical models derived from physical laws almost have the potentiality to show further facts, to

explain some peculiar behavior, or to suggest design changes needed to improve the response of a system.

For instance, changing measuring instruments in a process may create the need for identifying the plant again, while this is not necessary if the model has been obtained from physical laws. The characteristics of the measuring instruments are crucial in the on-line identification; but, this is not so in the mathematical modelling. This has been clearly shown by the problem faced when a turbine flow meter in the double-effect evaporator was replaced by an orifice-plate pressure transducer system, (see section 9.5.1). The output responses were completely different for the two instruments, so if the model was obtained by on-line identification method then the control system based on that model would have jeopardized the process operation. It was then only necessary to model the characteristic of the new instruments and to add to it the process model.

So far, the mathematical model of the double-effect evaporator has behaved satisfactorily and in good comparison with the experimental data, as depicted in figs [9.1.1,2,3]. A point worth mentioning here is that the non-minimum phase behaviour of the response of the liquid output from the climbing film to the steam input perturbations is not a process response rather than an instrumental dominant response in the initial instant, (see fig [9.1.3]).

In fact, the cyclone and its associated pipings, plus the orifice plate were initially neglected on the basis that they were very fast in dynamic responses compared with the plant

responses. Nevertheless, this was not so in the effect of perturbations of the steam input on the liquid product flow.

A sudden increase in the steam flowrate caused a sudden rise in the vapour and the vapour pressure. This was due to the fast response of the evaporation process at this stage of the tube. Naturally, this would be followed by an immediate decrease of the liquid-product flowrate, because the evaporated mass is on the expense of the liquid phase. This is expected, since the excess amount of vapour should come from the liquid phase, by conservation of mass. The sudden rise of vapour pressure P_b at the cyclone,

following the abrupt increase in vapour coming out from the first-effect, has caused some kind of pressure surge at the orifice-plate, upper stream. This momentarily high pressure caused an increase of liquid flowrate through the orifice plate. After a period of time the vapour pressure was settled and the flow in the orifice-plate tended to steady-state showing the actual flow of liquid product from the first-effect.

The measured response at the orifice has included in effect both the response of liquid m , and vapour pressure P_b superimposed on m . Due to the steam increase the amount of liquid product should have decreased; however, the sudden rise of P_b was much higher and hence this increase in P_b was appeared at the orifice-plate indicating a rise in the amount of liquid flowrate. Process wise, this is very illusive because actually we are interested in what have come out from the evaporator due to the disturbance of the steam.

The alternatives available to overcome the above problem may be to model those part which had been neglected in the first place, and to reconsider their model as part of the process model. The second option is to construct an on-line software low-pass filter. So, the measurements of the orifice-plates first passes through the low frequency filter prior to its usage in the control decision process. The effect of the high frequencies caused by the vapour pressure surges is to be filtered out by software. The second option is thought to be expensive in terms of development of the filter and its tuning, besides a high order system may be resulted due to this filter. Extra on-line time will be needed by the software to execute the filter function. More over, the filtering aspect of the problem may not be favoured because the instrument in fact measures an actual response of the flow. The vapour pressure has caused more liquid to flow through the orifice-plate; hence, it was recording actual flow. So, it may be better if it is consider as part of the first-effect dynamics.

Therefore, the first option will be considered: to model the effects of the vapour pressure surges on the product flow rate response, and to consider the cyclone and the orifice-plates as part of the climbing film evaporator.

Referring to fig [9.1.5], which shows the part of the equipment needs modelling, the major mechanism which may be involved in this section of the equipment are the inertance of the pipes between the cyclone and the orifice-plate beside the resistance of the pipes and the orifice

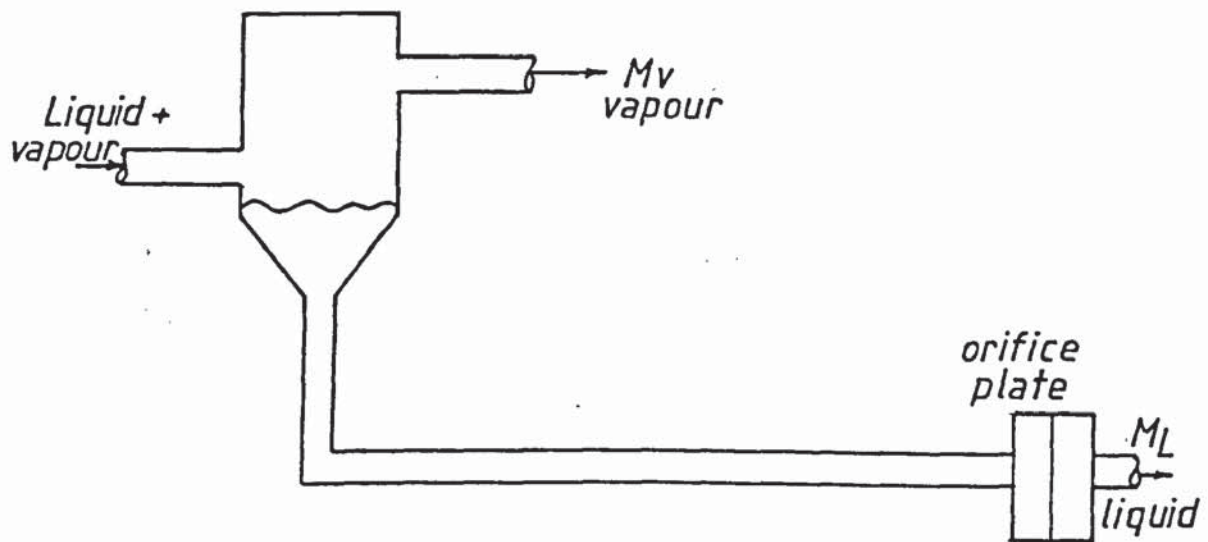


Fig. 9.1.5: The cyclone and the orifice-plate.

plate to the flow. The pressure P_b may be assumed as accelerating the fluid inside the tubes, then the tubes may be considered as having an inertance of

$$I' = \rho L/A$$

where ρ is the liquid density inside the tube

L is the length of pipes

A is the cross-sectional area of the pipes

Then, the pressure drop across the pipe can be given by

$$P = I' \frac{dQ}{dt} = I \frac{d(ML)}{dt}$$

where $\frac{dQ}{dt}$ is the volumetric product flowrate, $I = I'/\rho$

The resistance of the orifice-plate is considered more dominant than the pipes resistance. The resistance of the orifice may be estimated by a number of empirical formulae. But for the purpose here, a simple relation from which an estimation of the orifice-plate resistance can be found. Consider fig [9.1.6], the flow ML through the orifice-plate can be considered as obeying the design formula

$$ML = C_o \sqrt{\Delta P}$$

where ΔP is the pressure difference across the orifice-plate, $P_1 - P_a$, where the down stream pressure is assumed to be the atmospheric pressure.

Now,

$$ML = \bar{ML} + \tilde{ML}$$

$$P_1 = \bar{P}_1 + \tilde{P}_1$$

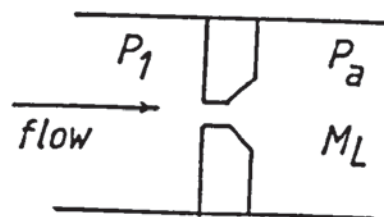


Fig. 9.1.6: Orifice-plate section.

where the $(-)$ denotes the steady-state part of the variable and the (\sim) denotes the transient part.

$$\begin{aligned}\bar{M}_L + \tilde{M}_L &= C_o \sqrt{\bar{P}_1 + \tilde{P}_1 - \bar{P}_a} \\ &= C_o \sqrt{(\bar{P}_1 - \bar{P}_a) \left(1 + \frac{\tilde{P}_1}{(\bar{P}_1 - \bar{P}_a)}\right)}\end{aligned}$$

$\tilde{P}_1/(\bar{P}_1 - \bar{P}_a)$ is a ratio < 1 ; hence, if the following is expanded we get:

$$\begin{aligned}\bar{M}_L + \tilde{M}_L &= C_o \sqrt{(\bar{P}_1 - \bar{P}_a) \left(1 + \frac{\tilde{P}_1}{2(\bar{P}_1 - \bar{P}_a)}\right)} \\ &= \bar{M}_L \left(1 + \frac{\tilde{P}_1}{2(\bar{P}_1 - \bar{P}_a)}\right)\end{aligned}$$

where again, P_1 is the upstream pressure at the orifice plate.

$$\tilde{M}_L = \frac{\bar{M}_L}{2(\bar{P}_1 - \bar{P}_a)} \tilde{P}_1$$

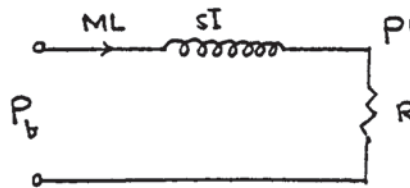
$$\tilde{M}_L = \frac{\tilde{P}_1}{R}$$

where R is the orifice resistance

$$R = \frac{2(P_1 - P)}{a \cdot ML}$$

So, the steady-state measurements of \bar{P}_1 and \bar{ML} will give an estimate of the resistance of the orifice to different flow rates conditions.

The electrical analogy of the flow inertance and resistance by an equivalent circuit diagram may be as follows :



Where the vapour pressure corresponds to a driving voltage, the inertance corresponds to an electric inductance, and the flow resistance to ohmic electric resistance. Indeed, the flow rate will correspond to the current in the circuit.

From the circuit diagram, and after transforming into Laplace quantities;

$$ML(s) = \frac{P(s)}{sI + R} = \frac{P(s)}{I(s + R/I)}$$

Therefore, the laplace transform function between the pressure

surge $P(s)$ and the product flow rate ML is

$$\frac{\overset{*}{ML}}{\underset{b}{P}} = \frac{1}{I(s+R/I)}$$

Now it is needed to relate this formula to the steam perturbation, $\overset{*}{S}$. From practical experience, the relation between the steam perturbation and the pressure perturbation inside the shell is almost a delta function for all practical cases. Hence,

$$\frac{\overset{*}{P(s)}}{\underset{b}{S(s)}} = K'$$

a constant, which can be found by experimental measurement.

So,

$$\frac{\overset{*}{ML}}{\underset{S}{*}} = \frac{K'}{I(s+R/I)} = \frac{K}{(s+R/I)}$$

where $K = K'/I$

These parameters were found to be, in a nominal operating conditions, as follows:

the inertance of the tube, $I = L/A$

$$= 1.13 \times 10^{-4} \text{ m}$$

$$\text{The resistance } R = 1.1 \times 10^{-3} \text{ m s}$$

(Note I/R is a time constant from the circuit diagram and has the dimension of time.)

After the addition of this new effect to the original model of $\dot{M}L/S$ the new complete model that represents the effect of steam perturbations on the product flow rate is as follows:

The perturbation on
the liquid product

$$= \text{effect of the vapour pressure at the cyclone} + \text{the effect of the steam on the cyclone}$$

The results of the model against the real time experimental results are compared and shown in fig [9.1.3]. The results obtained have almost justified the assumptions taken in modelling the cyclone and the pipings including the orifice-plate. This behaviour had not been reported in previous work in the pilot plant, might be, due to the fact that the product flow-rate was not measured. The turbine flow meter had caused some problems, and was replaced by a system of orifice-plate connected to a differential pressure/electric transducer interfaced to the HADIOS system. The

turbine flow meter being a minature device was not capable to work in conditions of scaling and rust. The previous experience with it has shown it was not reliable at tall. That was the main reason to replace it by an orifice-plate system, designed as shown in Appendix (D). The orifice plate has proved to be of high reliability, fast responsivity, better detectivity, and not to be easily affected by rust or scaling.

9.1.7 Measurements Aspect

To make this study possible, reliable measurements should be obtained from the rig. The transducers should be chosen in such a way so as to satisfy the accuracy and the precision required. The entire system performance has depended directly upon the accuracy of each piece of equipment that has made up the system. Therefore, although very accurate measurement equipment is expensive, going cheap on this can jeopardize the successful performance of an entire automated process control system. Not only must the system sensors be of good accuracy, but each piece of equipment subsequently used to process the signal from the sensor must also be of adequate accuracy. The subsequent signal handling equipment can rapidly deteriorate the initial accuracy from the sensor e.g. high noise amplification, poor impedance matching in the transmission lines.

In any particular system, other types of errors may be as important as basic accuracy and precision : resolution,

sensitivity, detectivity, hysteresis, linearity, dead-time, dynamic errors, stability, and others.

Therefore, utmost consideration must be given to the design of measurement, signal processing, and amplification circuits used with sensor outputs. This has been well demonstrated by the experience of the measurement of the low flow product by a turbine flow meter and later by an orifice plate, where the first has failed to show an important dynamic mode of the plant.

Also, it will be explained later in section (9.5.1), the problems that has been met when the feed flow rate was measured by a turbine flow meter interfaced with the computer through a counter. This time, the computer interface has shown an enigma in the measurement link, which was due to a peculiar behavior of the counter and counter software interface. This last problem has deteriorated a PI controller algorithm implementation in the hierarchical system.

9.2 THE MODEL TRANSFORMATION

The mathematical model which has resulted from this study was in a form of a transfer function matrix in the Laplace domain. The model included, amongst its terms, transcendental functions in the Laplace variable, s ,. Usually, these terms appear whenever there is a distributed contribution in

the modelled system. The most frequent transcendental term to appear is the pure time delay function, in the form of e^{-ts} .

The familiar practice in control engineering when tackling such type of transcendental functions is to apply Pade's method of approximation to the transcendental terms.

However, more options may now be included to handle transfer systems with pure time delays; for instance, Markov parameter method, and the latest technique of transforming the Laplace domain into delay operators defined over a ring, [Kamen,1978]

9.2.1 Markov Parameters

This method can be briefly explained as follows :

Construct the system transfer matrix, say, $T(s, e^{-ts})$ in the Laplace domain from

$$\dot{x} = A(e^{-ts})x + B(e^{-ts})u$$

$$y = C(e^{-ts})x$$

The one-sided Laplace transform of the histero-differential equation

$$T(s, e^{-ts}) = C(e^{-ts})(sI - A(e^{-ts}))B(e^{-ts})$$

Now expanding $(sI - A(e^{-ts}))^{-1}$ into a power series in s^{-1} ,

$$T(s, e^{-ts}) = \sum_{i=1}^{\infty} C(e^{-ts}) A^{i-1}(e^{-ts}) B(e^{-ts}) s^{-i}$$

Thus, $T(s, e^{-ts})$ can be determined from the sequence $(J_1(z^{-1}),$

$J_2(z^{-1}), \dots)$, where

$$J_i(z^{-1}) = C(z^{-1}) A(z^{-1}) B(z^{-1})$$

the Markov parameters.

The transfer matrix is expanded in a power series of s^{-1} , where the coefficients of s^{-1} terms are matrices of constants or in pure time delays function, e^{-ts} . These coefficients are the Markov parameters of the system. Then a Hankel matrix is constructed from these Markov parameters, by using Kamen algorithm [Kamen, 1978].

A minimal realization of the system in $A(z^{-1})$, $B(z^{-1})$, $C(z^{-1})$ can be found, after transforming the pure time delay functions, e^{-ts} , into a delay operator defined by $z^{-1} = e^{-ts}$.

However, the method involves many computations which then may or may not yield a minimal realization to $T(s, e^{-ts})$. Minimum realization is important to avoid any hidden unstable modes. Minimum realization indicates that the system is controllable and observable, [Kailath, 1980].

Nevertheless, the above technique still leaves a major problem that the system now after being transformed into a ring of delay operators has extended the ring $R[z^{-1}]$ to $R[z^{-1}, D]$ which consists of delay differential operators of the form

$$\sum_i \sum_j a_{ij} z^{-i} D^j \text{ acting on some unspecified space of functions.}$$

$$\frac{dx}{dt} = A(z^{-1})x + B(z^{-1})u$$

$$y = C(z^{-1})x$$

Techniques to design practical controllers from this extended ring, $R[z^{-1}, D]$ are not yet available. It was suggested that approximating the differential operator $D = \frac{d}{dt}$ into the delay operator may simplify the problem and confine it in the ring $R[z^{-1}]$. But, in conclusion this was not successful. The approximation methods that have been used to define D in $R[z^{-1}]$ are as follows:

- 1) Backwards difference approximation to d/dt

$$\sim \frac{1-z^{-1}}{t}$$

- 2) forward difference approximation $\sim \frac{z-1}{t}$

- 3) central difference approximation $\sim (z^{-1} - z)/2t$

- 4) four point central difference

$$\frac{(1+3z^{-1} - 3z^{-2} - z^{-3})}{6t}$$

None of the above techniques was successful: instability, high order systems and poor sensitivity performance to changes in the sampling interval are features of the test, (see Nawari [1982]).

9.2.2 Pade` Approximation

Pade` approximation is the most familiar technique found in the control design literature described to tackle and to approximate the pure time delay functions found in a transfer function. This method was implemented to approximate the pure time delays in the mathematical model, but the results obtained there have three main features (see Appendix A):

(i) sometimes the final transfer function is so weakly stable that the system might exhibit light oscillations,

(ii) the resulting order of the transfer function has increased, especially if second-order or higher Pade` approximation is used,

(iii) if the transfer function has included multiples pure time delays; e^{-t_1s} , e^{-t_2s} , ... etc, and the transfer function was a matrix rather than a single variable then Pade` approximation might not be the choice to solve the problem.

By and large, the technique has been developed in chapter (8) was remarkably successful. The transfer functions were easily transformed into a ring of delay operators by virtue of the real practical hardware aspect of the problem, (computer/process interface). This has simplified to a reasonable degree, the algorithm handling the transformation. Since physical equipment or process plants are usually interfaced to digital computers through

zero-order hold and sampler, this may certainly extend the technique to cover a wide range of applications.

Also, the technique has avoided any approximation to substitute for linear terms in s , the Laplace variable, as usually the case in the z -transforms technique. For instance, if a transfer function has s and e^{-ts} terms, by the usual z -transform, s will be equal to $\ln z)/t$ if e^{-ts} is taken to be z^{-1} . Thus, $\ln z)/t$ need to be approximated to a linear function in z . So, what has taken place in chapter (8), strictly speaking, was not a z -transformation, although it was similar and analogous to it in many aspects.

9.3 THE MODERN WIENER-HOPF CONTROLLER:

The steps followed in the computation of the modern Wiener-Hopf controller, as shown in Chapter (8), will be discussed in this section. The results and the related data files are shown in Appendix (H).

It has been mentioned previously, in this report, that the amount of computation involved in the Wiener-Hopf controller is formidable. Complicated symbolic handling subroutines are necessary to speed up the solution of the problem. The package APA has been designed to address such problems, and is applicable in general to algebraic polynomial linear systems. While the package is complete and represent an initial step in this field, yet, to reach maturity some numerical problems have to be solved by developing advanced mathematical algorithms to achieve better accuracy and convergence.

A family of programs have been written to calculate the Wiener-Hopf controller, the programs have utilized the library APA, (see chapter (8)). Following the steps in the Wiener-Hopf controller algorithms, the following outputs from each step are observed, when the process is applied to the plant model (chapter (8)):

9.3.1 Left-Right Factorization:

The algorithm of left-coprime factorization has been used to get the left coprime description of the plant transfer matrix, P .

The first step is to put the plant transfer matrix, P , in the following form:

$$P = \underset{C}{B} \underset{C}{A}^{-1}$$

where $\underset{C}{A}$ is a diagonal matrix consisting of the least common denominator of the columns of P , and $\underset{C}{B}$ is the matrix of the numerators of P when written in terms of the least common denominator. Note that the above setting is not necessarily right-coprime.

From the data given on matrix P the elements of matrix $\underset{C}{A}$ are as follows:

$$\underset{C}{A} (1,1) = 1 - 1.619z^{-1} + 0.8198z^{-2} - 0.1278z^{-3} + 1.3978 \times 10^{-4}z^{-4}$$

$$\underset{C}{A} (1,2) = 0.0$$

$$\underset{C}{A} (2,1) = 0.0$$

$$\underset{C}{A} (2,2) = 1 - 2.337z^{-1} + 1.77z^{-2} - 0.433z^{-3} + 4.32 \times 10^{-4}z^{-4}$$

and

$$B_c(1,1) = (1 - 0.301z^{-1}) (48.1592z^{-1} - 4.1707z^{-2} - 1.66 \times 10^{-2} z^{-3})$$

$$B_c(1,2) = -22.396z^{-1} + 31.9699z^{-2} - 6.309z^{-3}$$

$$B_c(2,1) = -0.764 + 3.174z^{-1} - 3.882z^{-2} + 1.311z^{-3} + 0.101z^{-4} + 8.6595 \times 10^{-5} z^{-5}$$

$$B_c(2,2) = (1 - z^{-1}) (0.447z^{-1} - 0.35z^{-2} + 6.992 \times 10^{-4} z^{-3})$$

The left factorization of matrix P,

$$P = \begin{pmatrix} A \\ L \end{pmatrix}^{-1} \cdot \begin{pmatrix} B \\ L \end{pmatrix}$$

can be obtained after the matrix PL has been constructed in the following way;

$$PL = \begin{bmatrix} A \\ C \\ B \\ C \end{bmatrix}$$

$$PL(1,1) = 1 - 1.619z^{-1} + 0.8198z^{-2} - 0.1298z^{-3} + 1.396 \times 10^{-4} z^{-4}$$

$$PL(1,2) = 0.0$$

$$PL(2,1) = 0.0$$

$$PL(2,2) = 1 - 2.337z^{-1} + 1.77z^{-2} - 0.433z^{-3} + 4.32 \times 10^{-4} z^{-4}$$

$$PL(3,1) = 48.159z^{-1} - 18.667z^{-2} - 0.307z^{-3} + 0.4538z^{-4} + 4.996 \times 10^{-3} z^{-5}$$

$$PL(3,2) = -22.396z^{-1} + 31.9699z^{-2} - 6.309z^{-3} - 3.32z^{-4} + 5.32 \times 10^{-3} z^{-5}$$

$$PL(4,1) = -0.764 + 3.174z^{-1} - 3.882z^{-2} + 1.311z^{-3} + 0.101z^{-4} + 8.6595 \times 10^{-5} z^{-5}$$

$$PL(4,2) = 0.447z^{-1} + 0.797z^{-2} + 0.3506z^{-3} - 6.992 \times 10^{-4} z^{-4}$$

The above matrix is kept in a data file [PLPR] to be later supplied to program COPRIME.

The right coprime description of matrix P can also be obtained in the following manner:

Decompose matrix P into:

$$P = A_r^{-1} \cdot B_r$$

Where A_r consists of the least common denominators of the rows of P,

and B_r is the matrix of numerators of P when written in terms of the

least common denominators. The set (A_r^{-1}, B_r) is not necessarily left-coprime.

$$A_r(1,1) = (1 - 0.768z^{-1})(1 - 0.549z^{-1})(1 - 0.0011z^{-1})$$

$$(1 - 0.787z^{-1})(1 - z^{-1})$$

$$A_r(1,2) = 0.0$$

$$A_r(2,1) = 0.0$$

$$A_r(2,2) = (1 - 0.768z^{-1})(1 - 0.549z^{-1})(1 - 0.0011z^{-1})$$

$$(1 - 0.301z^{-1})(1 - 0.787z^{-1})$$

$$B_r(1,1) = (1 - 0.787z^{-1})(1 - z^{-1})(48.159z^{-1} - 4.171z^{-2}$$

$$- 1.5628z^{-3} - 1.66 \times 10^{-4} z^{-4})$$

$$B_{r(1,2)} = (1 - 0.768z^{-1})(-22.396z^{-1} + 31.9699z^{-2} - 6.3088z^{-3}$$

$$- 3.32z^{-4} + 5.325 \times 10^{-5} z^{-5})$$

$$B_{r(2,1)} = (1 - 0.787z^{-1})(-0.764 + 3.1745z^{-1} - 3.882z^{-2}$$

$$+ 1.311z^{-3} + 0.101z^{-4} + 8.6595 \times 10^{-4} z^{-4})$$

$$B_{r(2,2)} = (1 - 0.768z^{-1})(1 - 0.301z^{-1})(0.447z^{-1} - 0.35z^{-2}$$

$$+ 6.992 \times 10^{-4} z^{-3})$$

According to the algorithm of right coprime factorization, in Chapter (7), the right-coprime factorization of P can be obtained by elementary operation on the columns of matrix

$$PR = \begin{bmatrix} A & B \\ r & r \end{bmatrix}$$

The data of matrix PR is kept in file [PLPR] to be supplied to program COPRIME to get the right-coprime factorization.

The next step in calculating the modern Wiener-Hopf controller is to solve the Bezout identity for A_L and B_L :

$$A_L X + B_L Y = I$$

to find polynomial matrices $X(z)$, and $Y(z)$ such that the identity holds.

The result of the left-factorization from the data given in matrix PL is shown in data file PLPR (Appendix (H)). It can be noticed that the results are higher in degree and magnitude

than expected, and an apparent instability which have taken place can be observed from the exaggerated values of the polynomials coefficients. Also the polynomial degree of each element is too high to be of practical use.

This generated instability and high degree may be attributed to many factors as it will be suggested and tested in the following sub-sections.

As the algorithm being similar to Gaussian elimination in nature, may suffer from numerical problems similar to the ordinary method of Gaussian elimination of constant elements matrix, in the field of real numbers. Nevertheless here polynomial elements are eliminated rather than constant elements matrix. The problem in Gaussian elimination happens when the pivoting element is comparatively small to the other elements in the matrix. Excessive magnification may easily take place and then propagate in the algorithm ruining the final result. This can be overcome by a suitable rearrangement of rows and columns.

To see how far the relative sizes of polynomial coefficients in the matrix may affect the final result, the comparatively small coefficients have been chosen by inspection to be neglected and eliminated from the original data. This modified data is kept in a separate file.

The algorithm this time being supplied by the modified data produced better results than those with the original data. The results are stable and meaningful, (see file PLP in Appendix H). Although this may not be a rigorous test, it has shown, the effect of the wide variation of magnitude of the polynomials coefficients. A point not to be overlooked here is that the elimination of a small coefficient in the polynomial should not affect significantly the accuracy of the original data. However, these comparatively small coefficients have appeared as the coefficient of the highest term in each polynomial.

Although the results which have been obtained through modifying the original data by eliminating the comparatively small coefficients are stable, the resulting polynomials still have higher degree than that practically expected.

However, direct analogy of the numerical instability in the Gaussian elimination of constant matrices with the elimination by elementary operations on polynomial matrices may not be directly valid, but the initial test of the size of the pivoting element shows some similarity. The numerical measure of $|R| < a_{ii}$ of the stability of Gaussian elimination in the case of constant matrices, is not directly applicable in the polynomial matrices case. This is because in the polynomial case the pivoting and diagonal elements are not constant numbers but rather polynomial terms.

A different property for the stability of elementary operations in polynomial matrices may be suggested as follows: the relative distribution of the zeros of the polynomial in the complex plane might have significant effect in the numeric stability of elementary elimination procedure of polynomial matrices. This is naturally the case in the study of the problem of stiffness. From a mathematical point of view the problem of stiffness arises whenever the ratio of the largest to the smallest eigenvalue is very large. Physically, stiffness occurs when the problem contains widely disparate time or length scales. Many problems arising from parabolic partial differential equations using finite differences turn out to be equivalent to solving a stiff system of ordinary differential equations. Stiffness is closely connected with the problem of ill-conditioning. An ill-conditioned system is one in which the solution is very sensitive to small changes in either the coefficient matrices or the known vector.

In light of this, the relative magnitudes of the poles of the original system have to be considered. Noticeably, the poles of the instrument dynamics have the largest eigenvalues, or, in poles term, have the smallest poles with relatively fast dynamics. So, it was decided that the dynamics of the instruments should be neglected from the plant model, matrix $P(z^{-1})$. The modified data are kept in file [PLPRNEG]. This time after the new data have been fed to the program COPRIME the results of left and

right coprime description have been very much improved. These new results can be seen in data file [PLPRNEG].

Comparison of the results which have been obtained in files [PLPR], and [PLPRNEG] has shown emphatically that the neglect of the instrument dynamics has a most prominent effect on the algorithm performance. On the other hand, the criterion of conditioning the zeros of the original plant matrix by neglecting the very small z-domain zeros in PL and PR has not improved the result as would be expected. This test is shown in data file PNEG.

9.3.1.1 The Effect of Instruments Poles:

Now, it is clear that the instruments dynamics have adverse effect on the results of the computational algorithms of the controller. So, feedback system design may be simplified by neglecting the sensor and actuator dynamics, which have been shown to be merely parasitics. Thus, it is of practical importance to investigate the effects of such "parasitics", especially when feedback gains are high.

More generally, if the desired feedback system band-width is much lower than the actuator and sensor band-width, then the actuator and sensor modes can be neglected as inferior. However, when the system band-width is close to the actuator-sensor band-width, then it may be advisable to include the actuator-sensor

dynamics in the open-loop model. This also will have the effect of increasing the complexity of the feedback design and cause numerical difficulties as it has been shown in section [9.3.1]. Fortunately, the plant band-width is much lower than the sensors band-width so the dynamic effects of these parasitics can safely be neglected. Otherwise, high frequency modes of a feedback loop with parasitics may adversely affect system stability.

9.3.2 The Solution of The Bezout Identity:

In the previous section it was shown how the variation of poles and zeros in a polynomial matrix will have an unpredictable effect on the final result of the elementary operations. These points will further be considered when solving the Bezout identity

$$AX + BY = I$$

by elementary operations.

The pair of the left-coprime matrices A and B
 $\begin{matrix} & & L & L \end{matrix}$

which describe the plant matrix have been obtained from data file PLPRNEG and used to find polynomial matrices X and Y such that

$$\begin{matrix} A & X & + & B & Y & = & I \\ L & & & L & & & \end{matrix}$$

A_L and B_L have been fed to the program BEZOUT to

solve for X and Y . The results are shown in data file ABL. From the results obtained for X and Y it is clear that the polynomial degrees of X and Y are too high to be of any practical use. However, a test has been carried out by the execution of program BEZTEST to verify whether the obtained X and Y will satisfy the identity $AX + BY = I$. Generally, the test suffices if $AX + BY = I$ generates a unimodular matrix rather than the identity matrix I .

$$AX + BY = U$$

where U is a polynomial unimodular matrix.

The results of the test of data ABL is not satisfactory; this is shown in file ABXY.

A number of tests have been carried out to seek a method by which the data in A_L and B_L can be conditioned such that a reasonable solution for the Bezout identity can be obtained. As with the factorization program, the data have been conditioned by neglecting the comparatively small coefficients of the high degree terms in the polynomials. For example, consider the following polynomial

$$a_0 + \dots + a_n z^{-n}$$

The coefficients a_i are considered from the highest degree term, a_n and descending to a_0 . Therefore, an a_{n-1} is tested if it is in a suitable order of magnitude with the rest of the terms a_{n-i-1} , $i = 1 \rightarrow n-1$ to sieze the elimination of a_{n-i} . This criterion has been applied to the data of A_L and B_L in file ABLC-14. The data of file ABLC-14 is further supplied to BEZOUT to solve for X and Y. These results have been tested for the identity $(AX + BY)$. A reasonable result has been produced in the low degree terms; while the high degree terms have shown a poor result, shown in data file ABCXY1. A relatively much better result has been obtained with the same data above upon neglecting less small coefficient terms than the above data, (see files ABLC-16 and ABCXY-6).

An alternative data modification method has been sought by using the difference in the magnitudes of the coefficients of the polynomials in each column of A_L and B_L . The magnitudes of the coefficients in the second column of each matrix are smaller by many order of magnitudes than that of the ones in the first column in each of A_L and B_L , or, when combining the two matrices in one matrix $[A_L \ B_L]$ the second and fourth columns are smaller in magnitude than the first and third columns. Scaling A_L and B_L by a unimodular matrix will not affect their coprimeness, since, if U is unimodular then UA_L and UB_L are coprime and still represent the plant matrix P.

$$P = \begin{matrix} & -1 & & -1 \\ A & B & = & (UA) & (UB) \\ L & L & & L & L \end{matrix}$$

Now, the matrix U can be chosen in a suitable manner to adjust for the relative differences in magnitude of the polynomial coefficients.

A set of unimodular matrices U of constant elements are chosen to scale the data in file ABLCL-16. The effect of the different unimodular matrices on the results of X and Y can be observed in files ABLCL-40, ABLCL-41, ABLCL-42, and ABLCL-43, with the following unimodular matrices respectively,

$$\begin{bmatrix} 1 & 0 \\ 0 & 10^2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 10^5 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 10^4 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 5.0 \times 10^3 \end{bmatrix}.$$

Although the unimodular matrices for ABLCL-41 and ABLCL-42 are different, they generated similar X and Y matrices, and this is the case for ABLCL-40 and ABLCL-43. The results in ABLCL-40 and ABLCL-43 for X and Y are not satisfactory; they are of very high order in degree and have large coefficients magnitudes. At the same time the results been obtained from files ABLCL-41 and ABLCL-42 are very good, and the test for the Bezout identity is shown in file ABXYC3. Hence, if the results in files ABLCL-16, ABLCL-40, ABLCL-42, and ABLCL-43 are compared then it is clear that scaling the data by unimodular matrices has a direct effect on the computed result. The effect is variable, this is apparent from application of a wide

variation of U with different results in every case. For files ABLC-16, ABLCl-41, and ABLCl-42 U has the values

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 10^5 \end{bmatrix}, \text{ and } \begin{bmatrix} 1 & 0 \\ 0 & 10^4 \end{bmatrix}$$

respectively, with very reasonable results have been obtained. On the other hand, files ABLCl-40 and ABLCl-43 have unimodular matrices:

$$U = \begin{bmatrix} 1 & 0 \\ 0 & 10^2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 5 \times 10^3 \end{bmatrix}$$

respectively, with unsatisfactory results in X and Y .

However, in other cases changing the range of the unimodular matrix may have no effect on the generated matrices X and Y . This can clearly be shown in the data files ABLC-51, ABLC-52, ABLC-53, and ABLC-54. These data are from A and B after the high L L

degree small coefficients have been neglected. The unimodular matrix covers a wide range of values but in all the cases the matrices X and Y have not changed. The identity test of these data is shown in file ABXYC4-50 which is a very reasonable result.

It should be noted that in all of the cases above the test of the Bezout identity is of the form

$$\begin{matrix} A & X & + & B & Y & = & U \\ L & & & L & & & \end{matrix}$$

where U is unimodular.

So, to get the required X and Y it is only needed to multiply from the right by U^{-1} , a typical example is shown in file GDASH.

It is generally required that the polynomial degree of X and Y to be as small as possible. For example, a minimal degree of X and Y will simplify the computation of the Wiener-Hopf controller and other similar controllers.

9.3.2.1 The Question of Minimum Polynomial Degree:

In the elementary operations on polynomial matrices to get matrix fraction description, (MFD), there is no guarantee that the algorithm will produce the minimum possible polynomial degree in the MFD. Although a matrix fraction description may be coprime and irreducible it is still not a unique

*

representation of the system matrix. Irreducible MFD is not unique, because if $N.D^{-1}$ is irreducible, and so is $N.W(D.W)^{-1}$ for any unimodular $W(s)$, therefore greatest common divisors are also not unique. It is expected that the minimal order of any state-space realization of a given transfer function will be equal to the degree of any irreducible MFD of the transfer function.

*
Recently, Feinstein et al have suggested a solution for which the rows of X may have minimal possible degree. (Feinstein, J, Bar-ness, Y, "The solution of the matrix Polynomial Equation $AX + BY = C$ ", IEEE Tran. on Autom. Contr. Vol AC - 29, No.1, Jan 1984

Thus, a means to extract suitable unimodular matrices from a coprime MFD will be useful when designing the controller. However, extracting unimodular matrices will not change the degree of the characteristic equation of the reduced transfer function; it will merely order the elements of the pair of coprime matrices. Nevertheless, there is no structured way to extract these unimodular matrices.

A similar argument is also be valid when solving the Bezout identity

$$AX + BY = I$$

The general solution of X and Y can be written in the following form

$$X = X_o - B_1 Q$$

$$Y = Y_o + A_1 Q$$

Where X_o and Y_o are particular solution of the above identity. A_1

and B_1 are the right coprime matrices satisfying

$$A_1 B_1 = B_1 A_1$$

Q is an arbitrary polynomial matrix.

To prove the above general solution of X and Y consider the following:

Since X_o and Y_o are solutions

$$A(X - X_o) + B(Y - Y_o) = 0 \quad (9.3.1)$$

Therefore, we have to find the general solution of equation (9.3.1), which is to say, all polynomial matrices M, such that

$$[A \ B] M = 0$$

Hence any M can be written as

$$M = M_1 Q$$

where M_1 is a full rank matrix and Q is arbitrary. Partitioning M_1 compatibly with $[A \ B]$, we get

$$M_1 = \begin{bmatrix} -B & 1 \\ A & 1 \end{bmatrix} \quad (\text{which is full rank})$$

so

$$X - X_o = -B Q$$

$$Y - Y_o = A Q$$

and the claim follows.

So, now it is apparent that the solution obtained from the Bezout identity for X and Y is not minimum in degree.

But, if matrix B_1 could be divided into X_o in such

a way that

$$X_o = B_1 U + V$$

with $\deg V_1 < \deg B_1$ then

$$X = V_1 - B_1 (Q - U)_1$$

the minimum-degree solution X, Y with respect to X becomes

$$X = V_1$$

$$Y = Y_o + A_1 U_1$$

as putting $Q = U_1$.

Nevertheless, the solution needs not be unique in this case.

9.3.3 The Spectral Factorization:

The next step in the control algorithm is to find the spectral factorization of the expressions:

$$(i) \quad A_1^* (P_1 P_1 + k) A_1$$

$$(ii) \quad P_1^* P_1 + k$$

When the program, FPANA, has been fed with the required data of P_1, A_1 , and k , (file FPAND), the result from the program was unsatisfactory. The program reported "division by zero matrix is not positive definite". Even, small changes in the accuracy of calculation and the data did not change the result. The next action was to revise the conditions of existence of the

spectral factorization. The given data have been found to satisfy the conditions of existence of spectral factorization as laid out in the literature. Some doubt has, however, been raised towards the condition that $(P P)^*$ be positive semi-definite for a finite w on

$|z| < 1$. The plant matrix $P(z)^{-1}$ has got a pole at $z = 1$, the most likely pole that makes $P(jw) \rightarrow 0$ for a finite set of w on $z =$

1. On trial, this pole, $(1-z)^{-1}$ has been extremely moved to $(1 -$

$0.9 z)^{-1}$, (this in the s -domain corresponds to changing an integrator $1/s$ to a phase lag $1/(s+0.01)$), but $P P^*$ still is not

positive definite. It is apparent that a fundamentally difficult behaviour is taking place.

Referring to the original theory for finding spectral factorization of polynomial matrices, which was developed by Youla [1961], the factorization problem can be restated as follows:

The solution of the Wiener-Hopf integral equation

$$\int_0^{\infty} K(t-r) W(r) dr = e(t) \quad t > 0$$

where $K(t)$ is the covariance matrix of the noise, $e(t)$ is a deterministic column vector function prescribed in advance by the known datum, and $W(r)$ is the unknown column vector of filter weighting functions $W_1(r), W_2(r), \dots, W_n(r)$.

$$k(t) = 1 / 2\pi j \int_{-j\infty}^{+j\infty} G(s) e^{st} ds$$

where

$$G(s) = \int_{-\infty}^{\infty} k(t) e^{-st} dt$$

where s is the usual complex plane.

To solve the integral equation above by Wiener technique it suffices to exhibit a factorization of $G(s)$ of the form

$$G(s) = H^*(-s) H(s)$$

For the above spectral factorization to exist the following conditions should be satisfied:

- i) $G(s)$ is real, $\overline{G(s)} = G(\overline{s})$.
- ii) $G(s)$ is para-Hermitian, $G^T(-s) = G(s)$.
- iii) $G(s)$ is of normal rank, almost everywhere.
- iv) $G(jw)$ is positive semi-definite for every finite real w .

The equivalent of these conditions on the z -domain may be written as follows:

- i) $G(z)$ is real
- ii) $G(z)$ is para-Hermitian, $G(z) = G^T(z^{-1})$
- iii) $G(z)$ is of normal rank (almost everywhere)

iv) $G(e^{jw})$ is positive semi-definite for all w , $0 \leq w < 2\pi$

More recently Youla et al [1978] have regrouped these conditions as follows:

$$1) G(z) = G^*(z) \quad \text{Para-Hermitian}$$

$$2) G(e^{jw}) \geq 0 \quad \text{for real } w$$

$$3) \det(G(z)) \neq 0$$

In the last condition the domain of z is not specified, i.e. $|z| \leq 1$ or $|z| < 1$.

With the conditions above, $H(z)$, the spectral factor of $G(z)$, should satisfy the following properties:

- i) $H(z)$ is analytic with its inverse in $|z| > 1$
- ii) $H(z)$ is real.

In most of the cases the first condition can be met, and that was satisfied by (P P). In fact, the failure of the

algorithm is attributed to the second condition above, $G(e^{jw}) \geq 0$.

The third condition is related to the second condition, as will be explained in the discussion below.

The condition that $G(jw) \geq 0$, be satisfied, to produce factors such as $H(jw)$ above is not valid from a theoretical point of view. It is that, $G(jw)$ should be positive-definite, $G(jw) > 0$, to produce such a factorization H .

A fundamental theorem can be stated here concerning the existence or otherwise of the above factorization.

Theorem:

G is Hermitian positive definite if and only if it satisfies: there exists a non-singular matrix H such that $G = H H^*$

Informally, if this condition holds, then

$$x^* G x = x^* H^* H x = \|Hx\|^2$$

This is certainly positive, or at least non-negative; the crucial question is whether $Hx = 0$. Since H is non-singular, this can happen only if $x = 0$. Therefore this implies the statement:

$$x^* G x > 0 \quad \text{for all nonzero vectors } x.$$

The "only if" part in the theorem stands for the statement

If there exist matrix H such that

$$H^* H = G$$

then G is positive definite.

Each of the following tests is necessary and sufficient condition for the real symmetric G to be positive definite.

- *
- 1) $x^T G x > 0$ for all nonzero vector x .
 - 2) all the eigenvalues of G satisfy $g_i > 0$
 - 3) all the submatrices G_k have positive determinants
 - 4) all the pivots (without row exchange) satisfy $d_i > 0$.

Masani [1966] has proved the following theorem:

Theorem:

If G is square ($n \times n$) matrix function defined in the interval $[-\pi, \pi]$, Hermitian, non-negative, and integrable has a Fourier series expansion, as well as the following:

- a) $G(-j\omega) = G^T(j\omega)$
- b) $G(j\omega)$ is of full rank n
- c) $\int_{-\pi}^{\pi} \ln \det G(j\omega) d\omega > -\infty$

Then $G(\omega)$ can be written as

$$G(j\omega) = H(e^{j\omega}) H^*(e^{j\omega})$$

(Note $(*)$ denotes the matrix adjoint, and always

$$G^*(j\omega) = \text{Adjoint } G(j\omega) = G^T(j\bar{\omega})$$

where the generating function H is square $(n \times n)$ matrix defined on the circle $c = \{|z| = 1\}$, with the properties:

1) H has a Fourier series expansion in non-negative powers of e^{jw} only,

$$H(e^{jw}) \quad \text{or} \quad H(jw) = \sum_{k=0}^{\infty} A_k e^{ikw}$$

The moving average coefficients $\{A_k\}$, (non recursive), are given

by

$$A_k = 1 / 2\pi \int_{-\pi}^{\pi} H(jw) e^{-ikw} dw$$

2) H admits of a holomorphic extension to the inner disk D

$$D = \{|z| < 1\}, \text{ given by } H(z) = \sum_{k=0}^{\infty} A_k z^k$$

3) The extension $H(z)$ is optimal, that is $\det H(z) \neq 0$ in D .

The above three properties determine $H(jw)$ uniquely up to a post-multiplication by any unitary matrix, or more generally, unimodular matrix.

Later, Wilson [1972] has imposed extra conditions to get $H(jw)$. He stated that A_0 ($= H(0)$) to be real, upper triangular, and with positive diagonal coefficients.

The condition A_0 to be positive diagonal is not enough to satisfy condition (3) above. Diagonal A_0 should be

positive definite, otherwise positive zero elements in the diagonal are allowed. Zero elements in the diagonal of A will make $\det A = 0$

which implies A^{-1} does not exist. If A is singular condition

(3) above will be void.

The last conditions stated above, eventually, require $G(j\omega)$ to be positive definite to have spectral factorization $H(j\omega)$.

However, problems in finding spectral factorization of positive semi-definite matrices have been persistent in three different famous algorithms, although the condition of $G(j\omega)$ to be positive definite and non-singular in $|z| \leq 1$ has not been discussed.

The algorithm derived by Youla et al [1978] is very similar to Cholesky factorization method, where obviously any matrix G to be amenable for Cholesky factorization must be positive definite. Tuel [1968] has developed an algorithm based on solving a set of non-linear algebraic equations similar to a set of equations if the problem was set in Kalman filtering fashion. Nevertheless, in the algorithm if one, or more, of the roots were on the unit circle $|z| = 1$, that is $G(j\omega)$ is not strictly positive definite, then the algorithm would not produce meaningful result.

Moreover, if $G(jw)$ is positive definite but is singular for some finite w in the domain $|z| \leq 1$ the factorization algorithm has a very poor convergence. This is clear in the example given by Youla [1978].

Now it is clear why a spectral factorization for semi-positive definite matrices is not attainable. However, even if the polynomial matrix is positive definite then the distribution of zeros of the matrix may have an effect on the accuracy and convergence of the algorithm. Knowledge of wider conditions under which the iterative solution converges would obviously be a useful contribution.

9.4 APA:

The symbolic manipulation was certainly cumbersome using FORTRAN-77 such work could now be done using LISP, the symbolic handling oriented high level language, or other such languages now widely available. Much use of HARRIS FORTRAN-77 structured language has been made to make the programs efficient, modular and robust to a reasonable degree. Although it was rather difficult to handle such huge polynomial matrices symbolically, the flexibility of the HARRIS FORTRAN-77 implementation has facilitated the venture. The HARRIS FORTRAN-77 implementation contains

important control features such as LOOP, WHILE, FOR, EXIT which are not usually found in standard FORTRAN-77. But indeed now many new BASIC based languages include such flexible structured features.

The package will help control system designers even in the simple case of designing SISO systems defined in transfer functions form. Generally, the multiplication, summation, expansion, partial expansion, to mention a few, of polynomials is tedious and arduous work when handled manually. For example, a designer working in PID controllers spent a week to calculate a certain rational transfer function, a few minutes were taken to solve the same problem using the package which showed also an error in the manual computation that had taken a week.

9.5 THE DIGITAL CONTROL AND THE HIERARCHICAL STRUCTURE

No doubt by now, the technology has crossed the point where the advantages of digital controller or microprocessor-based controllers are considered over corresponding analogue type of controllers. The following facts are firmly established:

(1) It is easier to make changes to the implemented control algorithm, compared to the usual classical analogue three terms controllers. These may completely disappear in the near future from the process industry, with the complexity and reliability now acquired by the new advanced technology. In the past classical PID controllers have found wide application in the process industries, since most of the control activity was centred around simple loops and the process had been designed from the beginning to operate safely in steady-state with very sluggish dynamic responses, though as a result the design may be expensive and relatively uneconomical.

The wide options given by digital controllers have made easier the implementation of algorithms of high complexity.

(2) Computer systems possess relative robustness in the electronic properties over the analogue controller, (there is no software drift). On the other hand, software may suffer from a lack of integrity. The integrity of software still depends on high skill and experience of the system designer and programmer.

(3) There is the possibility of information processing and handling to any level of sophistication, limited only by the computer power installed.

(4) Extra functions are available through the computers for scheduling routines, on-line graphics to display in various form the information from the plant, and other functions.

The hierarchical structure, used in this work, and its performance will be discussed in the light of the above expectations.

The hierarchical digital computer-control structure has been tested using a universal on-line multivariable controller. A universal modular controller, described in chapter (8), accepts any time-invariant controller parameters and performs the required algorithm. The algorithm defined by a ring of delay operators resulted in purely simple algebraic addition and multiplication. These operators take a fraction of the machine time, contrary to the usual problem that is faced when the algorithm is defined on the time domain, where integration and differentiation are required in the algorithm. In this case much consideration is given to the problem of integration convergence and the time required to perform it during short sampling periods. The recursive feature of the universal linear controller has made it efficient in terms of execution and memory space.

The test has shown that the software can successfully perform the various synchronization and other objectives of the hierarchical structure.

9.5.1 The Distributive Feature of The Digital Control

In the experiments on the distributed structure, the minicomputer, H316, was assigned for the supervisory job as well as performing the higher level control of the process and the display of data graphically on-line. The microprocessor, M6800, was to receive data from the minicomputer, calculate the new valve positions required and to send signals to the pneumatic valves accordingly. The data transfer between the two computers were in digital code form, while the data between the plant and the computers were in analogue form. However, the experimental set-up of the computers and the plant was not ideal or the one expected in industry. The set-up was that the micro, M6800, and the minicomputer, 316H, were at the same control room, far away from the rig. Hence, the analogue signals have to travel a long distance between the control room and the plant. This was rather an inconvenient arrangement, since the analogue signals were susceptible to noise pick-up, attenuation, etc. A better arrangement could have been obtained if the HADIOS part and the micro were as near to the plant as possible, this arrangement is usually achieved in industry. It is better to have the digital signals to travel long distances rather than the analogue ones.

Security, which means accuracy, stability, and reliable signal transmission, is another hallmark of digital systems. They can exceed with ease the high standards achieved by analogue hardware.

Noise pick-up and attenuation of signals are well known properties of transmission lines. Usually analogue signals carrying measurement information are of a magnitude comparable with

the magnitudes introduced by the above two properties. Hence the accuracy is highly affected, or some times may be lost. There is no such limit on digital signal transmission. Digital signals are transmitted as bits of binary digits, and the receiver needs only to distinguish between the 1 (presence of a signal) and 0 (absence of a signal). Generally, the 1 is represented by 5 volts and the 0 by a 0 volt, the 5 volt level is hundred times possible noise level, and the probability of confusing the 0 and 1 states becomes vanishingly small. Consequently, the accuracy of signal transmission depends only on the number of binary digits used to describe the signal. Since digital signals can be processed readily at rates exceeding 10^6 /s, a large number of bits can be used to describe the signal amplitude at any given time, even if they are sent down the line sequentially. For example, any decimal number from 0 to 1,048,575 can be sent using only 20 bits, which exceeds an accuracy of 1 part in 10^6 . If the bit transmission rate is only 200 000/s, 10 000 such numbers can be transmitted each second. If some of the bits are used as labels, many different kinds of digital signals can be sent on the same line, or multiplexed. The number of signal types, the accuracy, and the speed of transmission can all be traded off in a digital system to achieve the desired results.

Since the binary signal bits start off far above noise level, the transmitted signal is not subject to drift or error as long as the bits are not greatly attenuated by long lines. Even attenuated bits can be restored to their original waveshape by simple amplifiers or repeaters. Furthermore, some bits can be

devoted to a code which checks each word received for errors and even requests a retransmission if one is found or in application it reports an error in data transfer. Consequently, the reliability and stability of digital signals are far greater than that of their analogue counterparts.

The geographical constraints, a usual phenomenon in chemical process plants, together with advanced transmission technologies will make possible data transfer rates of many millions of bits per second within a local network. These high transmission rates, possible through fibre optics coupled with a reliable access control scheme, permit a large number of devices to share a common physical interconnection link with minimum interference to one another, while allowing large blocks of data to be transferred with simple error-recovery procedures and data management protocols. Thus, a local network is expected to transport information at high speed data transfer among a group of services terminals, typically industrial system terminals and peripheral, controllers, offices etc, via a common interconnecting medium.

The philosophy adopted in this study to construct the local network was based on star topology, fig. [9.5.2]. The star topology, which derives its name from the radial or star like connection of the various nodes to a centralized controller or computer, is implemented in point to point communication scheme that enables each node to exchange data with the central node. The control node acts as a high speed switch to establish direct connections between pairs of attached nodes.

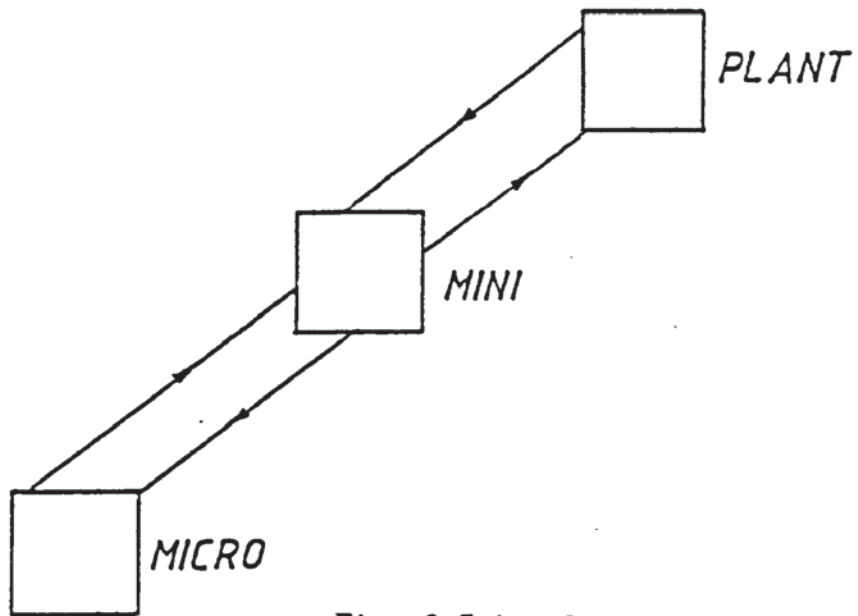


Fig. 9.5.1: Star topology.

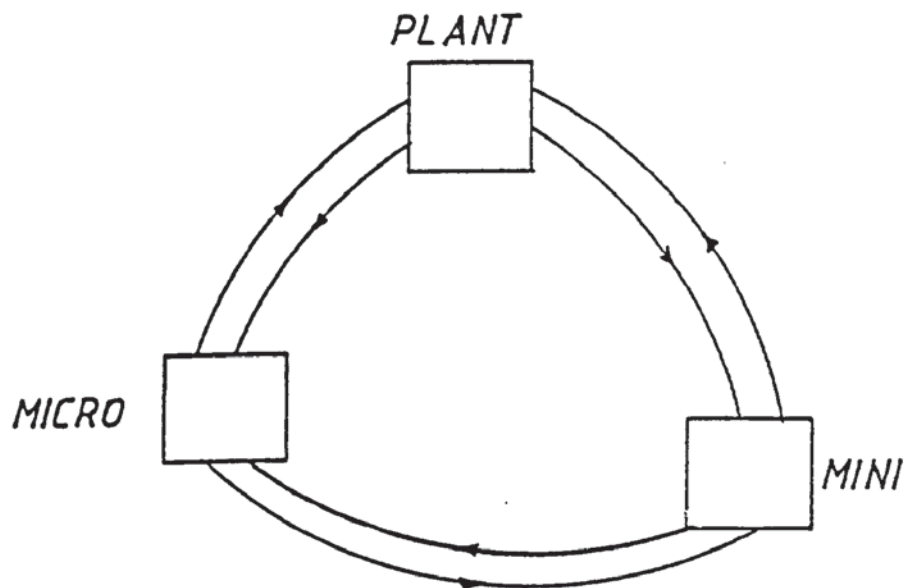


Fig. 9.5.2: Ring topology.

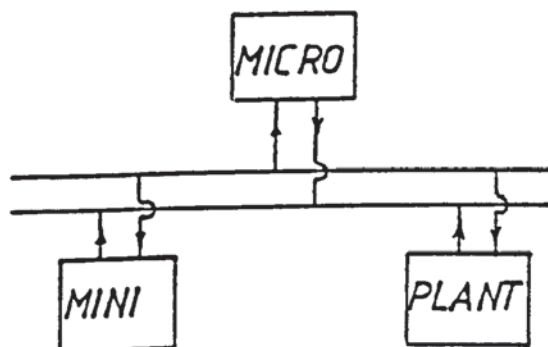


Fig. 9.5.3: Bus topology.

This may not be the best topology since there is limitation in speed of data transfer, and failure in the control node will cut-off completely the communication. The reason for choosing this structure was that the star connection would not need extra hardware, and the cost of software then might probably be less than other possible topologies. For example, the ring connection, fig. [9.5.2], may need extra hardware to handle the A/D interface.

The other possible topology is the bus which provides bidirectional transmission facility, fig. [9.5.3], to which all nodes are attached. Each node is tapped into the bus and copies the message as it passes that point in the cable. With this scheme, any node can begin transmitting data whenever it detects that the bus is idle. The node continues to monitor the bus for interference from another node that may have begun transmitting at about the same time. Any collision should be detected by all transmitting nodes, causing those nodes to halt transmission for a short random time period before attempting to transmit again.

This topology was not favoured because extra hardware and intelligence was needed to interface the plant, priorities for alarm interrupts by the plant might not be acknowledged in sufficient time, if for example, the micro and the mini were locked in conversation. However, if the usage of the ring or the bus is relatively light, and if exchanges are brief, this may work quite well. Nevertheless, as the traffic begins to pile up or the exchanges get longer, when there are many nodes to communicate, the system can arrive at the condition where most of the time is

spent in resolving contentions and most of the devices are in the wait state.

Finally, the digital format makes for compatibility with other computers of the same or different hierarchical level. This feature alone makes it easier to implement advanced control capability. Because a digital controller can take on a wider variety and greater number of functions than its analogue counterpart, the assignment of tasks between it and a supervisory computer can more closely approach optimum.

The experiments with the hierarchical structure were informative on the performance of the organization. The data handling between the two computers was almost perfectly reliable. Each computer managed to complete its task within the sampling periods specified to each computer. The H316 minicomputer was sampling data from the plant every 12 seconds and displaying results graphically on-line. At the same time it was processing the high level control task and sending the control data to the microcomputer M6800. Meanwhile, the microprocessor was sampling the plant, two input variables, feed flowrate and steam input flowrate, every 1 second. During the period of 1 second the micro has to make the necessary calculations, and to regulate the flow in the two lines according to the incoming command from the minicomputer. While the micro was doing this, it had to steal a machine cycle from the mini to communicate to the plant. However, the sampling periods, given above, were not limiting conditions, but rather a comfortable arrangement found by experience. For instance, in 12 second sampling period the mini computer has sufficient time to execute a

controller of 2×2 with polynomials of degree 11, a higher degree than most practical control design should yield. In fact, most of the 12 seconds the minicomputer was idling. The system was reliable, but some general precautions had to be taken when the computers were in operation :

(i) To avoid any power supplies switching in the vicinity of the computers and the electronic cabinet of the pilot plant. The package was not, yet, safe against these power supply spikes. It could easily "go corrupt". What actually was happening inside the computer was not known, but most probably the hardware of the computer power supply was leaking high frequency voltages, which sweep through the computers hardware registers and control circuitary disturbing the normal operation of the computers.

Certainly, noise in the mains should be completely attenuated at the computer power supply regulator.

(ii) Before running the programs in the two computers it was advisable to disconnect, initially, the counters input leads. This was to disable any hanging signals; because, the microcomputer, H316, has an initial sampling at $t=0$, during this time the counter does not show the real flow.

The counter works according to the following arrangement:

The counter was initially preset to any value between 0-127. As pulses come from the turbine flow meter the counter starts to increase; but, when the counter reaches 127 it sends an interrupt and the numbers of interrupts are stacked in the computer memory. So, if the counters are preset to a value near to

127 and the measured flow is fast then at $t=0$ the computer may be immediately faced by a counter interrupt. This will set the computer control to an unpredictable behaviour.

Thus, before executing the programs in the H316 computer the counter is initially disconnected, but immediately after the programs are executed the counter is connected back to the computer. By this action the critical condition can be avoided; indeed, as long as the connection back of the counters is done in less or greater than 12 sec, the sampling period.

This last point also brings about another important consideration when the measurements by the counters are made in the interrupt mode.

When the rate of flow is very fast compared with the scanning interval, or in other words if there are many counter interrupts during scanning interval a conflict of data handling through the communication channels between the mini H316 and the micro M6800 may arise. For instance, if an instant occurs at which the micro wants to scan at the same time, or nearly at the same time as the counter wants to raise an interrupt, the micro may miss this interrupt and hence will not preset the counter to the chosen preset value (variable A(11)). The counter will go on increasing beyond the maximum preset value, causing hazardous condition at the lower level controller.

Making the preset value a small value or possibly 0 may alleviate the problem if the flow rate is low enough. It is still necessary that less than $3/4$ of the full scale of the counter should be the state of the counter at the sampling instant.

An index to predict this problem can easily be derived as follows:

If the scanning period is T sec., and the input flow rate is I Hz then the number of interrupts, N, during that period of T sec will be

$$N = \frac{I \times T}{127}$$

N is a real number, the integer part represents the complete number of scans which have taken place. The fraction part of N represents the state of the counter at the next sampling instant. For trouble free measurements this fraction, empirically, should be less than 0.75

Example

Let T = 5 sec. , I = 100 Hz.

counter preset value = 0

$$N = \frac{5 \times 100}{127} = 3.937$$

During 5 sec. the counter has raised 3 interrupts and is near to raising the fourth just when the sampling period has finished and a new one is to start. At this instant if the micro wants to scan the mini, the mini may miss out this interrupt and hence it will not initialize the counter to the preset value. The counter will go on counting, exceeding 127.

A simple remedy to this may be by increasing or decreasing the sampling period. For the above mentioned example let

T = 4 sec. , I = 100 Hz.

$$\text{therefore, } N = \frac{4 \times 100}{127} = 4.139$$

or if $T = 6 \text{ sec.}$ $I = 100 \text{ Hz.}$

$$\text{then } N = \frac{6 \times 100}{127} = 4.7244$$

However to operate safely, an initial estimate of the range of the possible flow is established and then the above formula is used to test the safety margin.

It is to be emphasized that erroneous measurements of the flow will ruin the control action.

9.5.2 The Control Task Distribution Model

A worthwhile point to be mentioned under this section is on the model of the control task developed in chapter (8). If that model is allowed to grow it will have the topology shown in fig. [9.5.4].

From fig. [9.5.4] if the supervisor has failed then it would be expected that the lower levels of the structure could stay working on the basis of the last information they have received from the supervisory, or the one higher in hierarchy; unless the failure had resulted in sending random or noise information from the higher to the lower level. Means of securing the system could be studied under the signal flow graph theory to get a robust, and reliable control system, by making the necessary

and sufficient interconnections, or the most safe connections. Definitely, this structure should not be confused with the local network, because different control system structures can be implemented in a certain local network topology. Thus, the above structure can be implemented on a bus, star, or a ring network, since the control structures are connected logically and they have no effect on the local network topology.

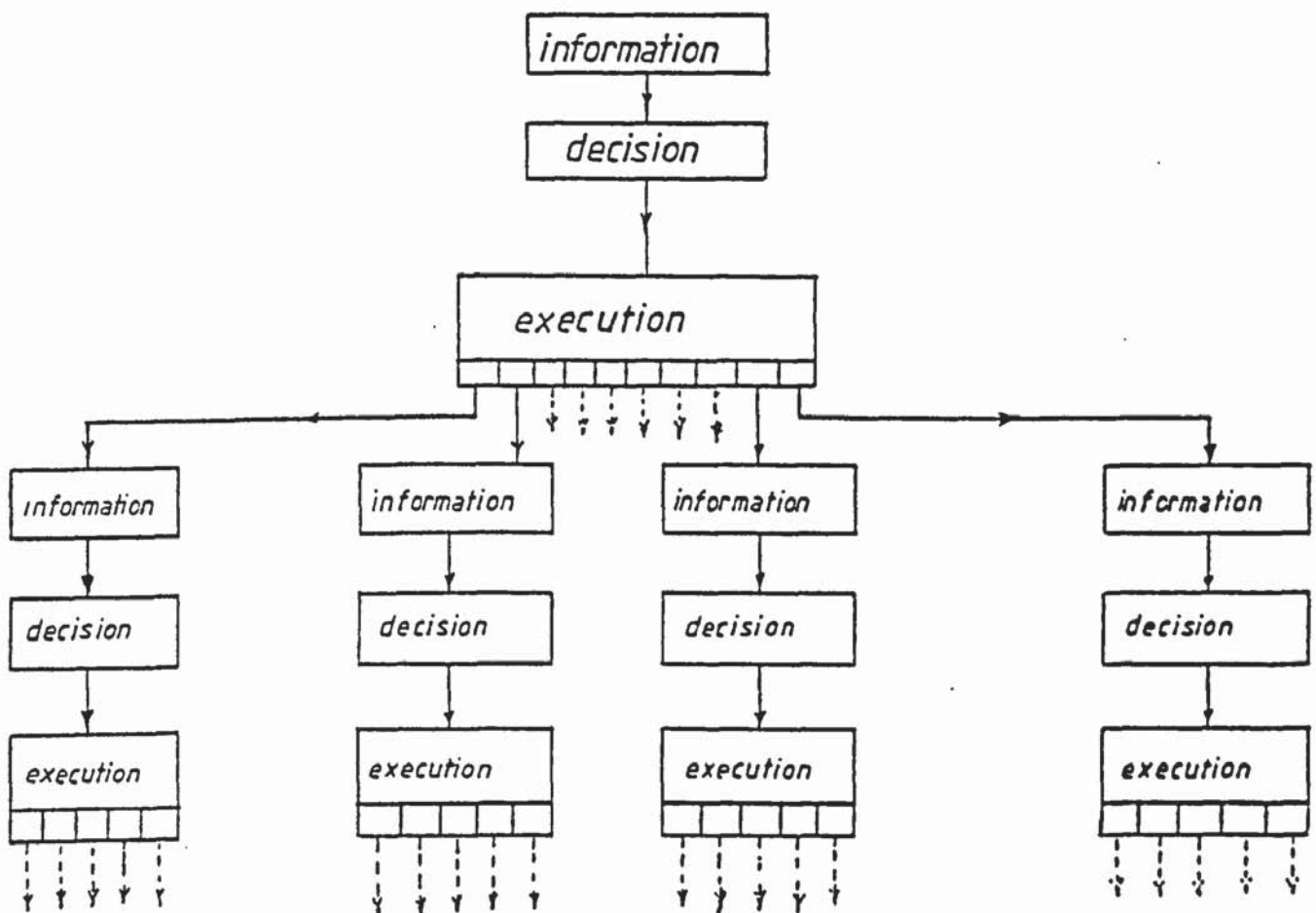


Fig. 9.5.4: A model of hierarchical control structure.

The model of information, decision, and execution in the hierarchical structure may be useful in studying the reliability and integrity of equipment or any controlled environment organization; in engineering systems, military organization, political structure or in management in general.

Nevertheless, to establish the most optimum links in a hierarchical structure is still a mathematical puzzle.

9.5.3 Process Cybernetics

The number of possible measurements made on a plant can be a large number. Some of these measurements can be used to construct measurements of other variables if the direct measurements involve complexity in hardware, and cost, or are not possible. Some of the measurements might be redundant and not present useful information in the various levels of control. Installing instruments sporadically on a plant then may not prove to be of purpose, or they cannot at least serve the purpose of control. This is not to say that control system designers have solved the problem regarding the optimum position on a plant to install a measuring device. The point is however, to emphasize the need for collaboration of control engineers and plant designers during the early design stages and the intensive test on the pilot scale before scaling up. Otherwise, instruments and other components, needed by the control system later may be expensive.

These typical problems were not faced in this research, since the double effect evaporator was well and heavily instrumented for various aspects of research.

Also, with early consideration of the control system on a plant, over design of the plant can be avoided. Full utility of the equipment and its operation at its full capacity can be achieved.

On the control side, the most important requirement is the quality control of the products from the process. Quality control usually implies regulatory type of control objectives. To keep a uniform homogenous quality product, despite any disturbances in the process, is of prime importance in industrial processes. The tracking control or servomechanism objectives are rarely specified in the control process design, except in cases of changing rates of production, or on the trivial case of local loops control, which is usually the lowest level in a hierarchical control strategy.

A controller, generally, contains some pieces of knowledge about the plant, and it can be classified as an intelligent machine. When it is fed with information it is able to make and execute appropriate decisions. The amount of knowledge built inside a controller should not include redundant descriptions. This will facilitate the speed of decision taking with a minimum usage of memory. The last two properties can be tested mathematically by ensuring that the controller states satisfy Kalman's conditions of observability and controllability.

However, whether the controller states represent the minimum amount of knowledge still remains difficult to answer. A quantitative method to measure the knowledge minimality will be of great advantage to control engineers and mathematical modellers. The problem is more difficult because the design method by which a controller is found for a process affects directly the minimality of the controller. Also, this may suggest the following proposition: from the large set of controllers $\{ C \}$, all of them accomplish and attain the specifications requirements, there is one and only one controller from the set $\{ C \}$ which is minimal in the sense of plant knowledge contents.

The knowledge built in the controller is obtained from the plant mathematical model. Hence, the complexity level of the model have a direct effect on the type of the controller. So, this will pose the problem of what level of complexity is needed in the model; however, a simple answer is not immediately available.

CHAPTER (10)

CONCLUSIONS AND FUTURE WORK

10.1 MATHEMATICAL MODELLING

The distributed parameter model of the double-effect evaporator has yielded a set of non-linear partial differential equations of hyperbolic type. Successive transformation and linearization of the set have successfully yielded linear transfer functions in a field conformed by a ring of delay operators. Hence the powerful theories of systems control can be applied to a set of linear partial differential equations. The ring of operators has made possible the mathematical operations in the frequency domain, or the equivalent state-space. Thus, the distributed nature of the process has been preserved, and early discretization or lumping has been avoided, which might otherwise have increased the order of the system. Contrary to similar studies, heat transfer coefficients have been eliminated algebraically from the model estimation through a fundamental assumption of perfect steam mixing in the steam chest, which is practically the case. The results of the method are well compared with results obtained by other workers by means of correlation or empirical formulae. Also the vapour dynamics have been carefully considered from a macroscopic point of view and the assumptions used have been derived from practical experience and observations, so

that they are valid for the purposes of this study, as a dynamic model for control.

The results predicted from the evolved model have compared satisfactorily with a set of experimental data logged from the plant. The dynamic and steady-state behaviours of the model have been tested over a range of operating conditions, and the model has proved to be robust and of good security. Qualitatively, the more varied the conditions under which a model is tested successfully the higher the security. The following points have been carefully considered, during the development of the model:

- (1) that the model should be internally consistent,
- (2) dependent on only a few parameters, and
- (3) demonstrably capable of prediction.

However, prediction has been constructed in a more general sense than merely using data from past observation to predict the values which future observations will take. Out of a set of data applying to the system being modelled the model can predict, using information derived from only one part of the set, the values appearing in the other part. The absence of information from the subset to be predicted is crucial to the test.

10.1.1 Towards The Future

The signal-flow graph technique has been used to solve the set of equations relating the system variables. The technique has shown potential capability to simplify the final solutions, through appealing to approximation, and to avoid unnecessarily complicated expressions. More theoretical work is

needed to explore these capabilities. This may lead to a quantitative answer as to which variables are best suited to control the plant and which variables have to be controlled. This is now done by intuition. The technique of the signal-flow graph can be extended and linked to the problem of interaction decoupling, dynamic dominance, controllability, observability, and where the best control effort can be put in the system. The technique keeps the designer in contact with the physical problem and this may give more insight into how the system is interacting.

As shown in this study, the model has been linearised for the obvious reasons of mathematical analysis. Generally, in engineering there may be uncertainty about the physical justification of linearization. It is probable that the nature of the non-linearities and their mutual relations are less important than the ultimate differential system in which they appear. In fact, when dealing with approximate solutions certain results can be achieved, in the engineering sense, only by simplifying some aspects of the mathematical procedure. This always requires justification, and, indeed, general results in this aspect will help as guide lines to the development of the model.

Of course, there still remains open the philosophical question of how to judge the level of accuracy of mathematical modelling to provide for faithfully representing the reality to the required accuracy. Until now, the most usual practice is to plot experimental results against those predicted by a model or models so that a visual-inspection comparison can be held. As an alternative method, two models intended to perform the same function, could be compared according to the sizes of the

errors generated in a given variable at a given point in time. This however, still does not tell how much better, over-all, a model is within the time span. Generally, in system dynamics studies a model usually has two parts : a transient part and a steady-state part. The behaviour of the model in these two regions is observed and then the performance is judged by inspection.

There is at present no quantitative method that can give a measure of the performance of models in the time domain or the frequency domain. Certainly, such a quantitative device, would give guidance and a means of reference to mathematical models engineers. However, a fundamental limit will exist in all models since the plant data generally contain noise, so that the model can be improved in the accuracy of its predictions only up to the point where the predictive error derives solely from the noise.

10.2 SYSTEM SOFTWARE

Chapter (7) has detailed the development of the library APA as an initial step towards building symbolic manipulation of polynomial matrices in the area of computer-aided control system design (CACSD). In this aspect APA is different from most well known packages in CACSD.

The package, except for the spectral factorization subroutines, is classed as a direct method, which means that it is not iterative. Therefore, there are no conditions for convergence, or for the algebraic size of errors. There are, however, the errors due to the round-off and the finite word-length

computational procedures so the results obtained are fairly accurate, only limited by the precision of the computer.

The concept of a field and a ring reveal the fundamental likeness of a great variety of collections of mathematical objects. These abstract concepts also suggest ways of extending over present knowledge to new areas, and they often enable the reduction of the proofs of many theorems to the proof of one. For example, starting with the abstract definition of a field then a result can be proved about fields in general, this result will hold true for the rational number field, the complex field, a field of matrices, a field of rational polynomials, and, in fact, for every field that exists. Abstract concepts are thus seen to be powerful tools in engineering systems. So it is quite useful to formulate engineering problems in well defined mathematical objects.

The fact that the package is algebraic symbolic means that there is a difficult task for a programmer in testing for various possible hidden ill-conditioned states which can arise during the computation process.

Although the package is implemented in HARRIS version of FORTRAN-77, the subroutines in their structured format will be transparent for portability and compatibility. The usefulness of the package is increased because it is capable of being used on a wide set of design problems.

The package, APA, in its present state requires further development : 1) different helpful error reports are absent,

and their insertion in the subroutines would help the user when a failure is detected or ill-conditioned data are met, and

2) The building of macros from the subroutines to form command functions for user interaction, with on-line graphics will fill a gap left now in the package.

The problem of conditioning the data prior to their usage has posed the question of what aspects of the data can be conditioned and what measures of relative stability are needed when dealing with polynomial matrices. However, it is apparent that by neglecting the fast dynamics of the measuring instruments, the computational result of evaluating the matrix fraction description of the plant matrix has emphatically improved. But, when solving the Bezout identity, for example, there has been no clear way to condition the input data.

10.3 MODEL TRANSFORMATION

In chapter (6) it has been shown clearly how the idea of delay operators is connected to the conformal mapping, e^{-ts} , to transform a system with transcendental functions to a ring of delay operators, with a quotient field of rational polynomials. Thus, the designer is able to deal only with integral domain quantities rather than the transcendental functions in the Laplace-domain.

This procedure has given more accurate results than would have occurred if the route of approximation for the pure

time delay function had been using Pade' approximation, and getting a z-transform of the resulting s-domain rational transfer function. As well as the better accuracy that could be obtained, the order of the system yielded by the formal transformation method is indeed lower than the order which could have been obtained through Pade' approximation or similar approximation methods.

The delay operator, z^{-1} , has given the facility for implementing a universal on-line controller algorithm for linear-time-invariant systems. The algorithm is very simple and involves only simple algebraic multiplication and addition. There are no integration or differentiation algorithms. So, the problems of stability, suitable step size of integration, step optimization etc are not found here. The universal algorithm can be executed very quickly with minimum storage requirements. The performance of the algorithm has been satisfactory in both the dynamic and steady-states.

10.4 WIENER-HOPF CONTROLLER

For long the main problem of process control has been in designing controllers that address the interaction phenomenon which is an inherent characteristic of chemical processes, rather than trying to decouple the process interaction as attempted by most of the existing techniques. Another main problem in chemical process control is dealing with disturbance, and noise rejection as well as changes in the input of various units

operation. This is especially important in keeping the quality of the product to its specifications.

The modern optimal Wiener-Hopf controller is based on appealing engineering features : disturbance rejection, regulation with the consideration of noise, and input vectors in spectral density distribution form, i.e the general statistics expected at the plant input variables. Also, restrictions on the plant matrix are generally less restrictive than required by other techniques.

The fact that the controller is based on synthesis techniques, indeed, makes it more attractive over design methods.

However, the limitation on the application of the Wiener-Hopf controller founded in this study may make its application prospects rather limited to those plants with no poles or zeros at $|z| = 1$, or even in the neighborhood of $|z| = 1$. One of the main findings of this work is that it is important to have a

positive definite matrix from the plant matrix, $P(z)^{-1} P(z)$, if ever a solution through modern Wiener-Hopf technique is sought. In fact,

if the matrix $P(z)^{-1} P(z)$ is not positive definite the spectral factorization of the matrix to determine the new zeros and poles of the closed-loop system may not be obtained. The tremendous amount of computation in the problem with symbolic handling is another hindrance facing a control system designer. Although APA is developed to address such problems, ill-conditioned data may still need special treatment before the package can be used. Occasionally, these special treatments are not available and more

investigation is needed. This is evident in the case of getting the matrix fraction description of the plant rational matrix with a wide distribution of poles, zeros, and gain constants.

Although the Wiener-Hopf synthesis method has not yielded a controller for the plant under investigation, that does not detract its impact on process control and modern control theory.

10.5 Closed Loop Poles Allocation and Minimal Sensitivity (CLOPAMS)

10.5.1 Conclusion

The modern Wiener-Hopf controller with its powerful features for addressing distinctive process control problems has laid the fundamental ground to reach a solution. However, the complexity of the computation aspect of the Wiener-Hopf controller may make it an unattractive option without powerful off-line computational facilities specially in the software area.

Another synthesis method, CLOPAMS, with powerful engineering features has emerged from this study which is simpler in computation. The new method is least-square optimal which guarantees in advance stability of a given plant. There are no restrictions on the plant matrix; it can be stable, unstable, non-minimum phase, or with an infinite number of zeros. The controller has a distinctive unique feature which is that the closed-loop poles or the characteristic equation of the closed-loop of the multivariable system can be specified in advance. This analytical

feature allows a chosen characteristic equation to have a predictable sensitivity behaviour, saving any unnecessary trial-and-error effort to fit certain specifications.

The synthesis theory is based on the capability of the controller for rejecting stochastic and deterministic disturbances and of their satisfactory tracking performance. Hence, the method has immediate appeal for its engineering features; since the designer can specify the poles of the closed-loop system in advance and has the freedom of shaping the sensitivity of the system for any required range of frequencies desired. The method is developed for MIMO systems; but trivially, it is directly applicable to SISO systems. In the case of SISO systems a wider range of options are possible, increasing the designers scope of choice and degree of freedom.

The ability of the analytical design methods to determine once and for all whether a given set of specifications can be met is one of its chief advantages over the conventional trial-and-error design procedures. However, one disadvantage of the analytical design theories that offsets to some degree their enormous advantages is the relatively complicated algebra that must be employed in working out all but the simplest problem.

10.5.2 Future Work

Indeed, besides the general issues of finding minimal polynomials for X and Y , the components of the Bezout identity, an efficient computer algorithm for finding the matrix

fraction description is required. Also, the choice of suitable polynomial degrees for matrix K in the sensitivity expression is by no means obvious. Reasonable theoretical work is needed to find a basis for defining the degree of K . The degree of it has a direct effect on the final form of the zeros of the closed-loop system. Consequently the degree of K will play a role in determining the final behaviour of the control system.

Also, the problem of aliasing may have an adverse effect on the minimization of the sensitivity within a range of required frequencies. Wide application of the method on process control problem will finally establish the feasibility of the method.

10.6 THE HIERARCHICAL STRUCTURE

The experiments with the hierarchical structure have been conclusive as far as the performance of the organization is concerned. It has been shown possible to distribute the control task among the two computers. On the high level the minicomputer, H316, has to perform the high level control e.g. the optimization algorithm. At the same time, the M6800 microprocessor handles the requirements of the high level to the plant, and looks after the valves setting at the lower level of the hierarchy.

The structure has been based on a star topology of local area network. The data handling between the two computers and the plant is almost reliable. Each computer has managed to furnish its task within the sampling period specified for each

computer. The hierarchically distributed system has shown possible, with the advent of cheap and powerful microprocessors, handling of complex function controllers can be implemented with ease.

10.6.1 Future Trends

The software has been concentrated around the control aspect of the problem. However, under advanced environment of operating system and programming language, different complicated tasks can be accomplished in large industrial organization.

So, the future trend would be expected towards some kind of standard operating system for industry to make extension, replacement, modification, etc as a routine job; such a trend would reduce the overall cost. Also, advanced high level language may extend the facilities and performance of distributed systems e.g. The ADA programming language.

The introduction of advanced operating systems in the micro/minicomputers system like the UNIX system and its similar versions may make the activity of writing software relatively inexpensive. The popularity and wide field tests of UNIX, indeed, will prove against any special tailored software. Some versions of UNIX support different communication networks like Ethernet, Ring, etc, with the powerful command to handle and manipulate files with the facility of data base management. These special features of UNIX with its introduction in the 8, 16, 32 bits computer will make possible and feasible the wide application of distributed computer control systems. The UNIX system is rapidly becoming a defacto standard as the operating system for 16 and 32

bit-microcomputers. The adaption of this operating system by the process control community offers several substantial advantages, a portable software environment (editors, file system, etc), freedom to choose among a variety of high level languages for software application, and computer hardware vendor independence.

A further word on the distributed system might be in order. The future trend goes towards distributed systems a universal standard software is needed to make the system cost effective besides the obvious benefits of ease of maintenance, replacement, development, and general reliability.

ADA : a programming language for numerical applications, systems programming applications, and applications with real-time and concurrent execution requirements, may be a feasible approach in the coming future for the above problem. All the facilities of shared variables, message passing, initiation, execution, synchronization and communication among tasks will collectively make possible the implementation of quite complex distributed systems, with more critical requirements and specifications to cater for the overall reliability of the process control which is usually scattered across square miles of land. Tasks in ADA are concurrently executable modules which are more like packages than subprograms. Each task can provide a set of resources to other tasks by means of a task specification that is separate from the implementation. The process of building large programs from modular components which are concurrently executable is supported in ADA by means of language mechanisms which are similar to those for sequential programming.