Towards An Effective Negotiation Modeling: Investigating Transboundary Disputes With Cases of Lower Possibilities

Victor Chang[1]+
Ben S. Liu[2]
D. Sudharshan[3]
Qianwen Ariel Xu[1,4]

1. Artificial Intelligence and InformationSystems Research Group, School of Computing, Engineering and Digital Technologies
Teesside University
Middlesbrough TS1 3BX, UK
+ Correspondence Author

2. School of Business
Quinnipiac University
Hamden, CT 06518, USA

3. Gatton College of Business and Economics
University of Kentucky
Lexington, KY 40506, USA

4. International Business School Suzhou
Xi'an Jiaotong-Liverpool University
Suzhou 215123, China

Emails: victorchang.research@gmail.com/V.Chang@tees.ac.uk,
Ben.Liu@quinnipiac.edu, dsudharshan72@gmail.com and iamarielxu@163.com

ABSTRACT

Existing literature in group-decision making proposed various rules of aggregating individuals' opinions to group outcomes. With anonymity maintained, this paper can model round-robin assessments by a group with individuals updating their assessments every round in a Bayesian manner as per Bordley (1983, 1986, 2009). Utilizing the properties of the finite Markov Chain process, the analysis shows (a) the conditions for a group consensus to converge, (b) the maximum number of rounds before such convergence occurs, and (c) the consensus assessment. The resulting dynamic model is tested to show that it also captures the results of several empirical studies. We apply them to the negotiation for the transboundary dispute and our simulations demonstrate the convergence of three different cases of lower possibilities, which support transboundary cases and resolutions. We also develop algorithms based on Fuzzy Delphi (Murray et al. 1985, Ishikawa et al. 1993) and Grey Delphi Methods (Ma et al., 2011) to predict the probability and likely outcomes of the transboundary dispute between China and India, which is one of the cases with low probability. Upon 1,000 simulations under volatile international relations, the development of the convergence demonstrates the integrated Delphi Method is more suitable for predicting volatile situations.

[Keywords: Group decision making, Bayesian updating, finite Markov Chain process, dynamic assessment model, Fuzzy Delphi and Grey Delphi Methods, Transboundary disputes, negotiation with lower possibilities]

INTRODUCTION

Existing literature in group-decision making proposed various rules of aggregating individuals' opinions to group outcomes. For example, the general social-decision scheme model (Davis 1973), the industrial market response model (Choffray & Lilien, 1980), weighted linear/probability model (Aldrich et al., 1984), and multi-attribute dyadic choice model (Curry, Menasco & Van Ark 1991; Bordley & Pollock, 2009) all aim at studying the appropriateness of using certain final rules to aggregate individuals' opinions to determine group outcomes. In each of these models, it is assumed that a group exchanges information and after this exchange, it comes to a decision. The above models propose mechanisms by which a group decision is made after the group has exchanged information. In each of these models, the final aggregation rules can be derived or given after group members exchange information and realign their preferences. Therefore, the process of how the exchanged information changes individual preferences and thereby influences the final decision is lacking. Thus, the process of how these final aggregation rules are formed is not entirely known.

This paper focuses on the process by which a group arrives at a joint assessment of the entity under consideration, related to the negotiation aspect and transboundary dispute of peace engineering. This is important as it allows us to classify different types of negotiation outcomes in a group setting, calculate the percentage of probabilities and determine the suitable strategies for each case, all of which will be explained in our computational analysis. In addition, we select the transboundary dispute in that this paper aims at analyzing cases of lower possibilities. We model a group as engaging in several rounds of discussions, attempting to arrive at a group assessment of an entity. All individuals in the group update their evaluations based on weighting the input from each group member. (Each individual's updating is modeled as in Bordley, 1983 and 1986.) For the process modeled, this paper shows the conditions under which a consensus group assessment can arrive. It also provides a formula to calculate the number of rounds of discussion necessary for the convergence of assessments. Further, it shows the conditions under which the consensus (if one is arrived at) will be on par with, higher than, or lower than the average of the initial individual assessments.

The group decision model discussed in this paper is shown to capture several empirically observed properties reported in the group decision-making literature. Specifically, it achieves the results on the conditions of conformity or nonconformity of Back (1951), Deutsch and Gerard (1955), Asch (1956), Harvey and Consalvi (1960), Darley (1966), Wilder and Allen (1977), and Hogg and Turner (1987). The model developed here can also provide a possible theoretical representation of the Delphi process (e.g., Dalkey and Helmer, 1963; Dalkey, 1968; Dalkey, 1969; Rowe and Wright, 1999) when the other party can estimate differential weights to each individual. Wilson, Lilien, and Wilson (1991) posit seven group decision rules for decision contexts differing in terms of the risk associated with the buying task. We show that these conditions translate to specific parametric representations in our model and that these representations, in turn, lead to the corresponding decision rules posited and tested in Wilson, Lilien, and Wilson (1991). We develop algorithms on top of that and apply them for negotiation and transboundary dispute cases and resolution. Our simulations demonstrate the convergence of three different cases of lower possibilities, which support transboundary cases and resolutions. We also present an example of using Fuzzy Delphi (Murray et al. 1985, Ishikawa et al. 1993) and Grey Delphi Methods (Ma et al., 2011) to predict the probability and likely outcomes of the transboundary dispute between China and India, which is one of the cases with low probability as the starting point. Upon 1,000 simulations under stable international relations, the development of the convergence demonstrates the integrated uses of Fuzzy Delphi and Grey Delphi Methods are more suitable for predicting and analyzing volatile situations. Our paper contributes to the existing literature on the negotiations with low probability as our model has been proven suitable for investigating such kinds of negotiations.

LITERATURE REVIEW

Group decision making is an important social interaction process. Due to its importance and complexity, this process has been the subject of considerable research in a wide variety of social sciences (Hare, 1976; Beck and Fisch, 2000). The major issues investigated in the past have focused on (a) the decision outcome and (b) the group process itself. The stream of works investigating the decision outcome has proposed various rules according to which individuals' inputs are aggregated to group decision outcomes. For example, Davis (1973) proposed a general model of a social decision scheme, which dictates how individual preferences will be combined to create a group choice.

Based on this general model, a variety of social decision schemes have been proposed and tested. These include such schemes as proportionality, equiprobability, majority, plurality, compromise or arithmetic mean, truth wins, highest expected value, extreme member or most (least) risk wins, neutral member, and risk-supported (conservative-supported or caution-supported) wins. The use of these schemes depends upon the characteristics of the alternatives as well as individual preferences. For example, the equiprobability model has performed well in tasks with high uncertainty or ambiguity (members do not know the probabilities with which various outcomes will occur if an alternative is selected). While plurality schemes are associated more with moderately uncertain tasks, majority schemes are related to low-uncertainty tasks (Davis 1973).

Aldrich et al. (1984) and Wittink et al. (1982), respectively, proposed a Weighted Linear or Weighted Probability Model, which can be used to generalize ordinary least square and linear regression in group-decision research. Investigation of the decision outcome has established that individual attitudes are changed in systematic ways by group discussion (Bordley, 1983; Jones et al., 2006).

In marketing and consumer research, Choffray and Lilien (1980), Eliashberg et al. (1986), Corfman and Lehmann (1987) and Rao and Steckel (1991) have proposed mathematical models that combine individual judgments into group choice. Of these, only Rao and Steckel's (1991) model seeks to capture the group polarization effect in its formulation. In the tradition of Fleming (1952) and Harsanyi (1955), the axiomatic model developed by Rao and Steckel (1991) is a variation of the simple weighted linear model, with a constant term and constraints on coefficients. They report two small-scale experiments, one involving the group's assessment of a member of hypothetical faculty candidates in marketing, and the other assessing actual restaurants. Rao and Steckel (1991) claim that the modification[1] referred to here was the linear model to capture the group polarization effect in their experiments. According to the polarization, the model seems to be appropriate. An excellent review of the research on mathematical models of group choice can be found in Corfman and Gupta (1993), Kalton and Flores-Cervantes (2003) and Jones et al. (2006).

Studies of group processes have, on the other hand, dealt with issues of the antecedents and consequences of individual attitude change in group discussions. Festinger's (1954) theory of social comparison process posits that an individual, faced with uncertain questions, may assess one's opinion based on a "social reality" of a social consensus. The studies of Sherif (1935), MacNeil and Sherif (1976), Deutsch and Gerard (1955) uncovered the presence of normative and informational social influences that change individual attitudes. Work by Baron and Roper (1976), Cotton and Baron (1980), and Myers (1970, 1976, 1982) have shown that either normative or social influence is sufficient, but neither is necessary for attitude change to occur. Studies on conformity have led to the following results (Baron, Kerr, and Miller 1992, p66):

“People conform more when:
1. The judgment or opinion issue is difficult (Deutsch and Gerard 1955): social comparison needs are stronger under uncertainty.
2. They face a unanimous group consensus (Asch 1956; Wilder and Allen 1977): all those people can’t be wrong and isolated deviates are most likely to be rejected and punished.
3. They value or admire the group (Back 1951): rejection from one’s friends is particularly threatening.
4. Their conformity or deviation will be easily identifiable (and therefore subject to social rewards and punishments; Deutsch and Gerard 1955).
5. They are scared (Darley 1966): fear increases dependency needs by undermining confidence.

People conform less when:

[1] referred to was the modification to the linear model to capture the group polarization effect in their experiments.

1. They have great confidence in their expertise on the issue: informational social influence is low.
2. They have high social status (Harvey and Consalvi 1960);
3. They are strongly committed to their initial view (Deutsch and Gerard, 1955).
4. They do not like or respect the source of social influence (Hogg and Turner, 1987)."

In marketing, Wilson, Lilien, and Wilson (1991), using a field study method, studied the predictive abilities of seven group choice models under each of six decision contexts. The group choice models they studied are (i) autocracy, (ii) weighted probability, (iii) equiprobability, (iv) majority, (v) voting, (vi) preference perturbation, and (vii) unanimity models. The six decision contexts they studied are combinations of low, moderate, high perceived risk conditions, the two buying task conditions of a new task and modified rebuy. They hypothesized a contingency model choosing the best model(s) for each of the six conditions. In 49% of the 104 procurement decisions studied, they found that their contingency model was accurate. After presenting our model, in our discussion section, we will show the connections between our model and the seven group choice models discussed in Wilson, Lilien, and Wilson (1991).

In the literature in marketing and consumer research cited earlier, the papers on group choice try to capture the effects of risky shift. Two articles besides the Rao and Steckel (1991) stand out. These are Bordley (1983) and Eliashberg and Winkler (1981). Bordley(1983) developed a model of Bayesian updating of individuals' attitudes based on group discussion and shows that this model predicts that the group polarization effect exists. In Bordley's model, the risky shift effect arises from the information-pooling aspects of group decision making. In the Eliashberg and Winkler (1981) model, in contrast, the hazardous shift effect resulted from the pooling of individual utilities.

Bordley (1983, 1986) presents a model wherein an individual "*i*" is assumed to enter a group discussion with a prior subjective probability for an event. The individual *i* also attaches a differential weight to each individual in the group. Individual *i* is the recipient of the prior probabilities of all the other group members. Based on the weights attached by individual i to all the group members and their respective priors, individual *i*'s prior subjective probability is updated.

The information that changes (or updates) *i*'s prior is based on a weighted pooling of the priors of all the other participants. Thus an individual's updating is based on the so-called information-pooling aspects of this group decision-making process.

We use the term group assessment in a broad sense, possibly referring to a group's assignment of a probability of occurrence, a group's overall attitude, or "belief" about an event or a group utility. We are not modeling the components of an assessment. Rather, we focus on how individual updating of assessments leads to a group assessment. (Bordley, on the other hand, models updating the probability of occurrence of an event.) Therefore, an assumption is that the group assessment is based on individuals updating their knowledge of the same objective of the assessment.

Bordley's model is elegant. It proves, under a set of assumptions, that each individual's updating would exhibit the risky-shift effect. Since every individual move in the same direction; therefore, the group's average would also move from the starting average in the same direction. Thus, the whole group would exhibit group polarization. Each individual could have a different posterior (or updated) assessment. In a group decision-making context, the entire group has to arrive at a decision. Bordley’s model does not address the issue of determining the group decision. It does, however, provide an updating model for individual assessments after group discussion.

Rao and Steckel (1991) model a group's assessment (assessment in their model is the group preference for individual candidates) as a weighted linear model of individual assessment with an intercept term. The parameters of their model have to be estimated by obtaining both individual assessments and a group assessment. They do not, however, provide a model for the updating of individual assessments.

Our paper builds a model of group assessment based on individual updating as per Bordley. It also examines the consequences of such a model on a group decision and applies to transboundary disputes. Thus it combines the respective intents of Bordley (1983) and Rao and Steckel (1991). The resulting dynamic model is shown to capture the results of the empirical studies reported in

Boje and Murnighan (1982), Rao and Steckel (1991), and on the convergence in Delphi studies (e.g., Dalkey and Helmer, 1963; Rowe and Wright 1999). In the area of transboundary disputes, related literature is as follows. First, Lowi (1993) describes the dispute between Jewish, Palestinians, and Arabs between the 1960s and 1990s. These include the dispute between land, water and resources. All the demands increased over a period of time. Israel, West Bank and Jordon could reach equilibrium to share water supply and demands. Fischhendler (2008) investigate the same topic analyzing the Israeli-Jordanian water agreement. Despite wars and disputes before the 1960s, both sides acknowledged that it was the only resource they could have in the 1990s. They had different periods of negotiations and tried to persuade the other sides about the advantages of working to reach win-win situations. The 1994 Water Treaty was an achievement of their agreement and both countries used a framework to comply. Second, Wolf and Newton (2008) present the Water dispute resolution between India and Pakistan near the Indus River and tributaries. The World Bank plays a vital role in ensuring that both sides negotiated and came to final agreements between 1951 and 1960. Both countries reached an agreement through a strong third-party peacemaker to ensure both sides could understand the benefits of mutual collaboration.

Third, Wolf and Newton (2007) explain the transboundary dispute between Canada and the US between 1905 and 1909. The UK negotiated on behalf of Canada under the British colony. Both sides were keen to get an agreement and repaired their relations after the Independence war in the US and the invasion of Canada. Hence, the negotiation did not last long to reach a general consensus. Similarly, the shared ownership of water resources and quality, negotiation started and completed between the 1970s and 1980s, and the air pollution issue was discussed and agreed upon in 1991. Their model was since both sides had mutual interests and would like to develop them for mutual gains further, they could reach common goals more easily. Last, a number of studies also try to provide some methods or models for international mediations with low success probability, including transboundary disputes. Bercovitch et al. (1991) examine the contextual and process variables which affect international mediation outcomes. They identify several factors that impact the effectiveness of international mediation, including dispute intensity, mediator strategies, dispute issues, the power disparity between the disputants and so on. In their study, 284 mediation cases were examined and only 22% of them have achieved partial success or success. In terms of the territory or security issues, the probability of successful mediation was 23% and 27%, respectively. Santmire et al. (1998) simulate a hostage crisis negotiation among India, Pakistan and Sikhs to evaluate the impact of grouping decision-makers by the level of cognitive complexity on the process of crisis decision-making and crisis outcomes. They find that both ultrahomogeneous groups and heterogeneous groups are less likely to achieve agreements than homogeneous groups.

In addition, it is difficult for negotiation groups in all decision-making environments to be homogeneous. International crises are particularly dangerous because they can quickly escalate to violence and war. Therefore, the success rate of negotiations on transboundary disputes is lower. Since water resources contribute to the main motivation for transboundary disputes, Zeitoun and Mirumachi (2008) study the factors and elements for water resource disputes. They make a list of recommendations for any policymaker to reach a consensus or have the possibility to work together. Those recommendations can be used for inputs of our research analysis. Moreover, with the intention of providing suggestions for resolving an international transboundary natural resource conflict, Madani et al. (2014) develop a Negotiation Support System which promotes the fair allocation of the sea and its resources by allowing for simultaneous evaluation of utility and areal shares of the countries under different possible division methods.

Belton et al. (2019) proposed a template of the progressive six-step process of conducting a successful Delphi study. During their investigation, they summarized several approaches that were used to determine the consensus criteria and found that little research specified a threshold for consensus in advance. Therefore, in our research, we set a suitable threshold for consensus based on the existing literature related to our research target, the transboundary disputes. In addition, although they suggested that three rounds should be sufficient for a consensus to achieve, they consider that the appropriate number of rounds should ideally be determined by looking for a pattern of stability, which is in line with our opinion as well.

Finally, the traditional Delphi method has some disadvantages, such as low convergence expert opinions, high execution costs, the possibility of filtering out particular expert opinions, and the requirement for anonymity, which could be violated in real-world practices. Therefore, we search the literature and identify both the Fuzzy Delphi Method and Grey Delphi Method that can improve the traditional Delphi Method. These two methods are briefly described as follows: Murray et al. (1985) proposed the Fuzzy Delphi Method integrating the traditional Delphi Method and the Fuzzy Theory to improve the vagueness of the Delphi Method. The use of fuzzy numbers permits the appropriate representation of the subjective preferences in decision making and the fuzzy logic is still applied in recent studies. For example, to support companies to integrate sustainability into strategic decision-making, Calabrese et al. (2019) proposed a Fuzzy AHP method, providing practical support for strategic decision-makers in the prioritization of the most relevant sustainability aspects. Companies with limited resources can benefit from the proposed phased structure of the fuzzy AHP method because it helps them identify the most relevant issues so that companies can decide to solve the most important topics first, and then return to the evaluation for the remaining topics in the future. Grey Delphi Method (Ma et al., 2011) is the integration of the Grey System Theory and Delphi Method, which uses grey whitening weight function based on Delphi questionnaires to select evaluation indicators. Thus, it can improve or relax the conditions required on the Delphi method by allowing their weights to the other individuals to be adapted for a Delphi method. Because of the above, we decide to develop algorithms based on these two methods for analysis and predicting uncertain situations as a computational example to demonstrate the advantages of the integrated method over the traditional Delphi method. More details are provided in the Algorithms subsection under the Computational Example section.

THE MODEL

It is assumed that "*n*" individuals are involved in a group discussion aimed at arriving at a group assessment. For example, a panel of experts may be involved in a Delphi process to forecast the probability of occurrence of an event (say a technological breakthrough) or another interactive (round-robin) group decision process (Dalkey and Helmer, 1963; Boje and Murnighan, 1982; Martino, 1993). We make improvements by allowing each group member to know there is an event to take part, and each group member votes his or her decision at a separate period, so that they cannot meet each other and know the identities of other experts, and only can make the decision by themselves. The anonymity of the identities and process are all significant, in which the anonymity of the identities is the default condition and our process was carried out on this basis. This can prevent each group member from influencing each other and make the final outcome neutral and fair. The assessment outcome here would be the group's estimate of the probability of the event taking place. Each individual's estimate is conveyed to all the group members after each round. Each individual is asked to provide a fresh probability estimate upon receipt of other group members' estimates from the previous round (or trial). Each individual i is assumed to attach different weights to the estimate of each member of the group. Individual i's estimate is assumed to be updated in a Bayesian manner as per Bordley (1983). Our analysis examines the convergence of both the limiting (or the converged) value and the convergence rate of this dynamic process.

Let $L_i^{(R)}$ be the ith individual's assessment at round R, $w_{ij} \geq 0$ be the weight which the ith individual assigns to the jth individual. Following Bordley (1983) and assuming $\sum_{j=1}^{n} w_{ij} = 1 \ \forall \ i$,

$$L_i^{(R)} = \sum_{j=1}^{n} w_{ij} L_j^{(R-1)} \quad \text{where} \quad R \geq 1, \quad \text{and} \quad L_i^{(R)} = \log\left(\frac{P_i^{(R)}(E)}{1 - P_i^{(R)}(E)}\right) \qquad (1)$$

where $P_i^{(R)}(E)$ is the probability assessment of alternative or event E by an individual i for round R. Assume that w_{ij} is independent of the rounds. In our model, all $P_i^{(R)}$ are calculated except the initial value $P_i^{(0)}$ which are assumed to be measured.

Let $\mathbf{W} = (w_{ij})$ be the weight matrix whose element at i th row and j th column is w_{ij}. Let $\mathbf{L}^{(R)}$ be the *nx1* column vector at the R th round and $\mathbf{L}^{(0)}$ be the initial assessment column vector with $L_i^{(0)} \geq 0$ for all i [2]. The assessments for all individuals at each round are given by:

$\mathbf{L}^{(1)} = \mathbf{W}\mathbf{L}^{(0)}$

$\mathbf{L}^{(2)} = \mathbf{W}\mathbf{L}^{(1)} = \mathbf{W}(\mathbf{W}\mathbf{L}^{(0)}) = \mathbf{W}^2\mathbf{L}^{(0)}$

and in general,

$$\mathbf{L}^{(R)} = \mathbf{W}\mathbf{L}^{(R-1)} = \mathbf{W}(\mathbf{W}\mathbf{L}^{(R-2)}) = \ldots = \mathbf{W}^R\mathbf{L}^{(0)}. \quad (2)$$

In order to discuss $\mathbf{L}^{(R)}$, each individual's assessment at the R th round, it is sufficient to discuss $\mathbf{W}^R$. We now apply some actual results on Markov chains (Chung, 1967; Iosifescu, 1980) to our discussion. Viewing the n individuals as n states and the weight matrix $\mathbf{W}$ as the transition matrix of a step in a Markov chain (the proof of the existence of such a chain will be given later), w_{ij} is the probability that the Markov chain will, when in state i, next make a transition into state j.

Let $w_{ij}^{(R)}$ be the element at i'th row j'th column of matrix $\mathbf{W}^R$. Based on the assumptions we have made about ***W***, the following definitions apply.

W is a *stochastic matrix* if

$$w_{ij} \geq 0 \text{ for all } 1 \leq i, j \leq n \text{ and } \sum_{j=1}^{n} w_{ij} = 1 \text{ for all } i = 1, 2, \ldots, n. \quad (3)$$

[2] Note that when $P_i^{(0)} \geqslant 0.5$, $L_i^{(0)}$ will be $\geqslant 0$. In the case of $P_i^{(0)} < 0.5$, to satisfy the initial condition of $L_i^{(0)} \geqslant 0$, the formula can be modified by adding a positive constant value K in front of the log term to shift $L_i^{(0)}$ above zero. The results will remain the same. **It is worth noting that the condition $L_i^{(0)} \geqslant 0$ is only considered in Run 0.** After Run 0, $P_i^{(R)}$ can be any value between 0 and 1 and $L_i^{(R)}$ can be any value deriving from Equation 1, the log transformation formula, using the initial values provided in Run 0.

State j is said to be ***accessible*** by state i if there exists a positive integer R_{ij} such that $w_{ij}^{(R_{ij})} > 0$.

When state i and state j are accessible to each other, then the two states ***communicate***. All states communicate with one another if there exist n^2 positive integers R_{ij} ($1 \le i, j \le n$) such that

$$w_{ij}^{(R_{ij})} > 0 \text{ for } 1 \le i, j \le n. \quad (4)$$

When state i is accessible by itself, then $w_{ii}^{(R_{ii})} > 0$ for some positive integer R_{ii}. In general, R_{ii} is not unique. Let d_i be the largest common divisor of all such R_{ii}'s, then d_i is called the period of state i. State i is ***aperiodic*** if

$$d_i = 1. \quad (5)$$

ANALYSIS

Two theorems are presented in this section. The proofs of these theorems are provided in the Appendix. Theorem 1 establishes the existence and a unique group consensus decision in the limit and gives its value. In doing so, the formula for calculating the limiting assessment is also derived. Theorem 2 establishes an upper limit to the number of rounds of assessments necessary for the range between the highest assessment and the lowest assessment (across individuals) to be apart by a small real number δ. The convergence criterion used is that the gap between the highest and lowest assessments at any round be less than or equal to δ, where δ is a fraction of the difference between the starting values of the highest and lowest assessments. The consensus-value L^* will be between the highest and lowest assessments for any round in all circumstances. Thus the closer the highest and lowest assessments are at any round, the closer is the round at which consensus will be achieved.

In this section, we also show the conditions under which L^* is less than, equal to, or greater than the mean of the initial assessments.

Theorem 1: If conditions (3), (4) and (5) hold, then $\lim_{R\to\infty} L_i^{(R)}$ exists and the limit is independent of i. More precisely,

$$\lim_{R\to\infty} L_1^{(R)} = \lim_{R\to\infty} L_2^{(R)} = \ldots = \lim_{R\to\infty} L_n^{(R)}.$$

In order to prove the theorem, it is sufficient to show that $\mathbf{W}^R$ converges to a matrix whose elements in the same column are equal. Hence, if we can show that there exists a Markov chain having n states and transition matrix $\mathbf{W}$ and this Markov chain is *ergodic*, then the proof is done. The detailed steps of proof for Theorem 1 are given in the Appendix, which produces a similar result as that of DeGroot's axiomatic approach (1974). It is also similar to the result of the proof provided by Kemeny and Snell (1963).

Link to Contextual Conditions

If for an individual (i^*), the contextual conditions (from the literature on conformity, summarized at the end of the fifth paragraph under the Literature Review section) under which the individual is less likely to conform are true, i.e., the individual has greater confidence on his/her expertise on the issue, was high in social status, is stronger committed to his/her initial view, or does not like or respect the source of social influence, then $w_{i^*i^*}$ is likely to be 1 and $w_{i^*j^*}$ =0. If only a single individual in the group meets these contextual conditions, then convergence will occur to this individual's assessment. On the other hand, if two or more individuals meet these contextual conditions, then our convergence conditions (4) and (5) will be violated, and thus convergence will not occur. Correspondingly, the contextual conditions (from the literature on conformity, summarized at the end of the fifth paragraph under the Literature Review section) of more conformity imply $w_{ij}>0 \ \forall i,j$ which meet our convergence conditions (3), (4), and (5). For these contextual conditions, i.e., issue difficulty, higher uncertainty, unanimity amongst others, admiration of others, identifiability of deviants, fear, and thus lower self-confidence, our model would indicate that convergence will occur - thus capturing the conditions for conformity. Thus, our model captures the empirical findings and conceptual thinking in conformity literature.

Define $\alpha(\mathbf{W}) = \min_{1 \le i, j \le n} \sum_{k=1}^{n} \min(w_{ik}, w_{jk})$.

Assume there exists an integer $R_0 \ge 1$ such that $\alpha(\mathbf{W}^{R_0}) > 0$. (6)

Theorem 2: If conditions (3) and (6) hold, then the maximum number of rounds R^* required for the proportion $\frac{\overline{L}^{(R)} - \underline{L}^{(R)}}{\overline{L}^{(0)} - \underline{L}^{(0)}}$ to be less than or equal to a given positive number δ is the smallest integer R satisfying the following inequality

$$(1 - \alpha(\mathbf{W}^{R_0}))^{[R/R_0]} \le \delta \quad (7)$$

where $[R / R_0]$ is the largest integer that does not exceed R / R_0, or round towards zero. (The proof is provided in the Appendix.)

In the following propositions, we show the conditions under which L^* takes different relative values with respect to the mean of the initial assessments. In specific, Propositions 1 and 2 show the sufficient conditions for the neutral shift effect. Propositions 3 and 4 show the sufficient conditions for the cautious-shift and risky-shift effects, respectively. [Endnote [i]]

Let $\overline{\overline{L}}^{(0)}$ be the mean value of $L_1^{(0)}, L_2^{(0)}, \ldots, L_n^{(0)}$. We want to see the conditions under which the convergence value is greater than or less than or equal to $\overline{\overline{L}}^{(0)}$.

Proposition 1: Suppose conditions (3), (4), and (5) hold, and in addition, if $\sum_{i=1}^{n} w_{ij} = 1$, i.e., $\mathbf{W}$ is a double stochastic matrix, then $L_i^{(R)} \to L^* = \overline{\overline{L}}^{(0)}$ as $R \to \infty$ for all i.

Corollary of Proposition 1: If w_{ij} is equal to $\frac{1}{n}$ for all $1 \le i, j \le n$, then

$L_i^{(R)} = \overline{\overline{L}}^{(0)}$ for $R \ge 1$ and $1 \le i \le n$.

Proposition 2: Suppose conditions (3), (4), and (5) hold, and in addition, if w_{ij} is proportional to

$\frac{L_j^{(0)}}{\sum_{k=1}^{n} L_k^{(0)}}$ and not all $L_j^{(0)}$ are identical, then $L_i^{(R)} \to L^* > \overline{\overline{L}}^{(0)}$ as $R \to \infty$ for all i.

Recall from (1), $L_i^{(R)} = \sum_{j=1}^{n} w_{ij} L_j^{(R-1)}$ and $L_i^{(R)} = \log(\frac{P_i^{(R)}(E)}{1 - P_i^{(R)}(E)})$. Thus, if the individuals assign weights in proportion to the assessments observed in the first round, then this condition holds. (There may be other conditions that also lead to this relationship.)

Proposition 3: Suppose conditions (3), (4), and (5) hold, and in addition, if w_{ij} is proportional to

$1 - \frac{L_j^{(0)}}{\sum_{k=1}^{n} L_k^{(0)}}$ and not all $L_j^{(0)}$ are identical, then $L_i^{(R)} \to L^* < \overline{\overline{L}}^{(0)}$ as $R \to \infty$ for all i.

The proofs of these three propositions are also provided in the Appendix.

COMPUTATIONAL EXAMPLE

In the following, we present a computational example of a group decision with individual updating as per our model. The initial values $L_i^{(0)}$ (based on the corresponding initial estimates $P_i^{(0)}$), and the individual weights w_{ij} are shown in Table 1. While other sets of initial values can be chosen, the set of initial values for L were recommended by experts to cover a larger range of variation across different individuals participating in the simulation for group dynamics. The 100 represents those who hold maximum value, 50 as midground, 30 as medium-low and 10 as low out of a 100-point scale.

Figure 1 shows a plot of the updated values over the number of rounds (or trials). In this example, the number of rounds to convergence is around 20. In Dalkey and Helmer's (1963) study, convergence occurs after the fifth round. Their graph of updated estimates is reproduced in Figure 2. The results of our model appear to show a similar pattern as some of those exhibited by Dalkey and Helmer (1963). For example, (1) if the data is looked at prior to convergence, spurious coalitions seem to occur (i.e., the values of subsets of individuals become equal and then subsequently diverge before finally converging), (2) convergence occurs, (3) individual values do not change monotonically towards the convergence value. For our example, based on equation (7), the theoretically calculated number of rounds to convergence is 20. Although this may appear to be higher than the number of runs exhibited in that for Dalkey and Helmer, it shows a similar pattern as seen from Figures 1 and 2.

Table 1: Starting Values and Weights for Computational Example

Weight Attached by *i* to *j* , w_{ij}	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0.2	0.1	0.05	0.65
$i = 2$	0.2	0.6	0.1	0.1
$i = 3$	0.07	0	0.93	0
$i = 4$	0.01	0.49	0.1	0.4
Starting Value, $L_j^{(0)}$	10	100	50	30

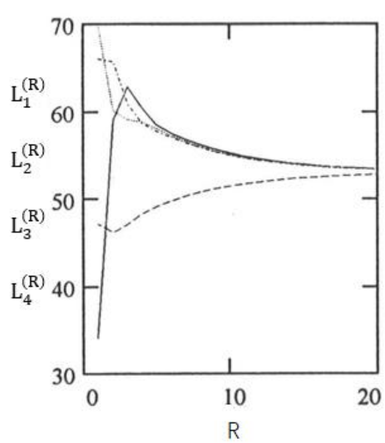

Figure 1: Plot of Convergence Individual Values over Rounds (R)

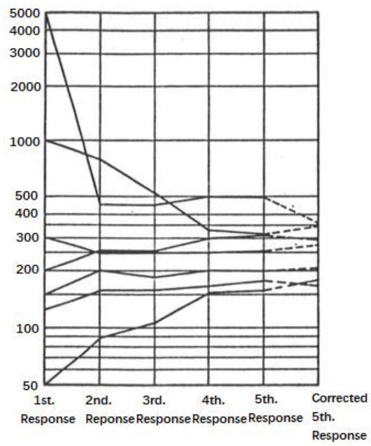

Figure 2: Successive Estimates of Bomb Requirements
Reproduced from Dalkey and Helmer (1963)

After introducing the computational example above, we apply them for the transboundary dispute in the following section and explain how our work is relevant. Our simulations support the convergence of three different cases and probabilities, with low (0.2 equivalent to 40% chance), medium (0.35 equivalent to 70% chance) and high (0.45 equivalent to 90% chance) probabilities.

Algorithms based on Fuzzy Delphi and Grey Delphi Methods for analysis and predicting uncertain situations

As explained at the end of the Literature Review section, we provide in this subsection more details about developing algorithms based on Fuzzy and Grey Delphi Methods for analysis and predicting uncertain situations to demonstrate the advantages of the integrated method over the traditional Delphi method. While the Delphi method requires anonymity, our approach adopts both the Fuzzy Delphi Method and Grey Delphi Method that can improve the traditional Delphi Method. The traditional Delphi Method has some disadvantages, such as low convergence expert opinions, high execution costs, the possibility of filtering out particular expert opinions and so on (Woudenberg, 1991). Murray et al. (1985) proposed the concept of integrating the traditional Delphi Method and the Fuzzy Theory to improve the vagueness of the Delphi Method. A membership degree is used to establish the membership function of each participant. Ishikawa et al. (1993) further introduced the Fuzzy Theory into the Delphi Method and developed max-min and fuzzy integration algorithms to predict the prevalence of computers in the future. But the limitation of this method is that it is only applicable to predict time series data. Hsu et al. (2010) further improved the Fuzzy Delphi Method by blending expert systems and decision systems partly together.

Grey Delphi Method (Ma et al., 2011) is the integration of the Grey System Theory and Delphi Method, which uses grey whitening weight function based on Delphi questionnaires to select evaluation indicators. Thus, our research made improvements or relaxed conditions required on the Delphi method by allowing their weights to the other individuals to be adapted for a Delphi method. During negotiations, certain votes can be anonymous, but some have to be open or allow a third-party with higher authority (e.g., a powerful third country or international court) for arbitration or to judge. Therefore, only the third-party knows about the offers from both negotiating parties. Every private matter can be kept confidential. The initial values are for information only. For example, if the exchange rate between GBP and USD is 1 to 1.29 on the day of decision making. If every investor has one million to invest between the US and the UK, then the initial information is required. However, how each investor decides on the process can remain anonymous.

Therefore, our analysis and predictive modeling are based on the combination of Fuzzy Delphi and Grey Delphi Methods. Apart from anonymity, there are additional benefits. First, the computational powers offered by the combined methods can improve the performance (Hsu et al., 2010; Sun et al., 2015). These allow more simulations than the original Dalkey and Helmer's (1963) version, such as running up to 20 rounds of negotiations. Second, the most up-to-date data and information can be used as the input, thus allowing more accurate analysis and predictions on uncertainties (Ma et al., 2011; Sun et al., 2015). This allows our method to simulate possible outcomes based on the most up-to-date data and information we have. One of our research contributions in developing simulations further is to run more than five rounds and see if the outcomes may vary with respect to time. Third, in the traditional Delphi method, when experts are evaluating plans or evaluating performance, for some indicators that are complex or difficult to define, the uncertainty and ambiguity of experts' subjective thinking will be significantly increased, thereby reducing the objectivity and rationality of the results. In contrast, the Fuzzy Delphi and Gray Delphi methods can deal with uncertain, incomplete, or hazy data (Hsu et al., 2010; Liu et al., 2004; Ishikawa et al., 1993; Murray et al., 1985). This allows that in our method, we are able to study negotiating issues with a quite lower probability of success, e.g., the transboundary dispute between China and India in our paper, with the experts' opinions expressed sufficiently without distortion and the results, can be more objective. We also perform up to 1,000 simulations for disputes under peaceful and stable international relations shown in Figures 3a and 3b demonstrated by the integrated use of Fuzzy Delphi and Grey Delphi Methods.

Our simulations support the convergence of three different cases, with low (0.2), medium (0.35) and high (0.45) probabilities. As shown in Figure 3a, there are three possible outcomes. The first possible outcome is presented by a green line that the probability of reaching the convergence is close to 0.5. The green dotted line is higher than 0.6, but it is coming under 0.5 after five rounds of convergence to reach an agreement. The second possible outcome is presented by a black line that the probability of reaching convergence is below 0.4. Two black lines, one from the top and the other from the bottom, meet and converge after five rounds of convergence. The third possible outcome is represented by red lines. Two red lines merge at and before five rounds of convergence, with the probability of just above 0.2. Even the probability of success is lower, and they come to an agreement. Figure 3b shows that after running 1000 simulations, the first two outcomes stay. In other words, they are more likely to work together in long and medium terms. Simulation results do not support the third possible outcome since they converge at the probability of 0.20 as they are more likely to break apart later on.

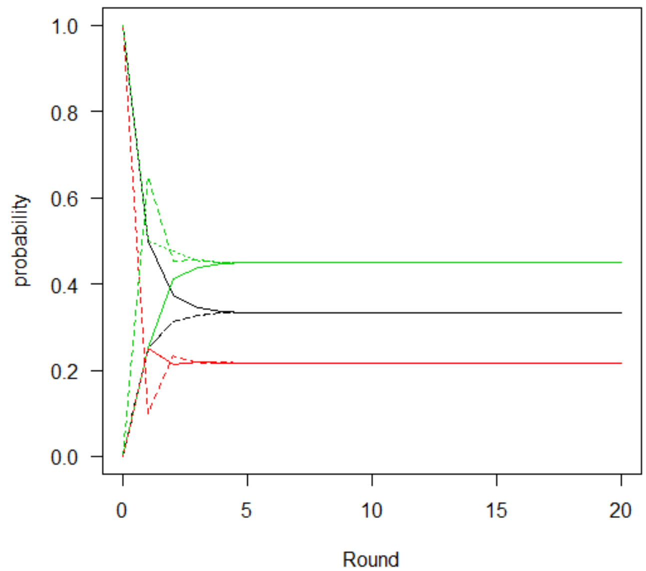

Figure 3a): Three possible outcomes after convergence

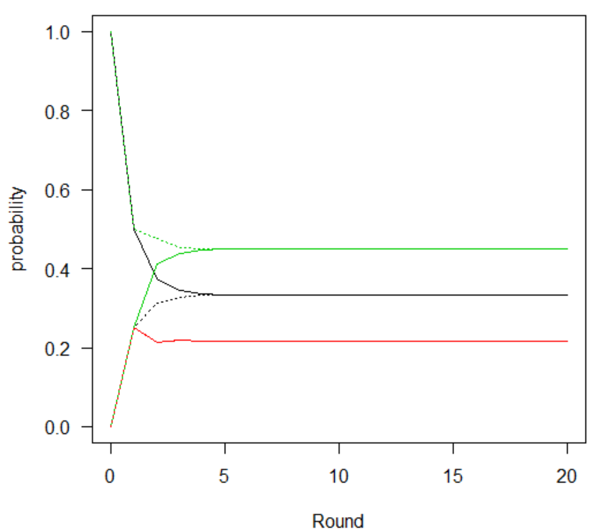

Figure 3b): Three possible outcomes of convergence after running 1,000 times

Implications

The interpretations and implications of our computational results are as follows. First, suppose two sides have similar points of view on the key agenda. In that case, there is a higher possibility to work together; they are more likely to persist in coming to a decision and not abandon it either temporarily or permanently. They are more likely to work and stay together for a longer-term. Second, for two sides that have differences at the beginning but eventually merge and can work together like the black lines, they should find things in common and work together in the medium term. When the simulation goes on for more rounds with the possibility of having stricter criteria imposed on each other, the alliance will have tendencies to get apart. Third, suppose two sides have a huge difference initially and even agree at or before the fifth round of convergence. In that case, it will be difficult to maintain a long-term relationship, as we found after running large-scale simulations. Or in other words, only one side will have to be more dominant to maintain such a collaboration. These implications from Figures 3a and 3b can be useful in negotiations and also mergers and acquisitions.

For example, in merger and acquisition, case 1 (green) can be the best scenario for two merged firms to work well together. This applies to the India and Pakistan water dispute resolution that both countries reached an agreement after major rounds and intervention of the World Bank. Similarly, the US and Canada transboundary disputes between 1905 and 1910, and the environmental quality disputes between 1970 and 1991 at two different periods, went under case 1. Not all types of discussion had smooth processes. However, it was important that both sides needed to see the positive sides and what they could get from negotiations. Seeking things in common, exchanging with the other party without disadvantaging your own side, and the greater benefits of working together could attract both sides to come to certain terms and conditions before reaching agreements.

Case 3 (red) can be adopted if one firm has fully agreed to work under the direction of a more dominant firm so that the possibility of breaking apart can be minimized. This can also be applied in negotiations. For example, if each party can take a step backward, then case two (black) is the most suitable. If party A is more dominant and has more terms on offer with more benefits to party B, then case 1 (green) will be the most suitable scenario. In contrast, if the negotiations do not go on well despite common grounds, case 3 (red) is often put up as the alternative. When both sides agree, they need to undergo the first turbulent period in Fig 3a). If both parties can work out common grounds and follow them carefully, they can still pertain to possible successful collaboration. The best example is the disputed agreement between Israel and Jordon. After nearly ten years of wars and disagreement, they could reach consensus through mutual benefits and the efforts of peace diplomats from their mutual friends. Both Israel and Jordon went through case 3 with a lower probability for agreement. Through another ten years or so, they renegotiated. This time their situation was similar to case 2. They understood each side's strengths and weaknesses and what they aimed for. After several rounds of negotiation with the help of diplomats of their mutual friends, they reached the agreement. In the Israel-Jordon case, they had case 3 and case 2 at different transboundary dispute resolution stages.

Implications from Figures 4a and 4b can be useful in negotiations caused by extreme uncertainties. When the cause of damage was considered from one side by the other party, it could cause the negotiations to be negative for some time. Compared to the original concept, this can be an area of our contribution as follows. First, predicted outcomes for each round of negotiation could be presented based on the circumstances and the needs of each side. Second, we also computed the probability of predicted outcomes for each round. Due to COVID-19, it can make the probability much lower and do not achieve successful negotiations in the short term.

While this computational example presents a specific case, a similar pattern can be obtained for a wide variety of starting conditions and weights, such as traffic control progress in different countries. We have not conducted a simulation exercise to relate the types of patterns to the distribution of initial weights, assessments, and the number of group members because there is no accepted set of typical and exhaustive conditions. Simulations, in this case, do not have ecological validity. On the other hand, the mechanism may be tested directly. Given the initial starting conditions, our mechanism can be used to calculate the expected convergent value and the number of rounds to convergence for a group decision context.

DISCUSSION

The main contribution of this paper was the development of a model of multiple round group decision making. This model, as discussed, could capture the effects observed in several major empirical studies. Individual assessments were modeled as being dynamically updated based on a weighted function of the assessments of the others. We have arrived at a mechanism to calculate the consensus value (if one exists) in such a process, given starting values. Thus, if the participating individuals provided their weights for the other individuals and also provided their respective starting assessments, the group-decision value could be calculated. Based on the example calculation we have shown and its apparent similarity to Dalkey and Helmer's (1963) data and our experimental study involving 48 groups, this model could have normative implications.

Our model allowed the calculation of the outcome based on starting conditions, the expected probability of success, and the predicted outcomes for each negotiation round. It could be argued that resorting to a purely hardwired process may be too strong. We were also of the opinion that the process of group interactions itself had inherent benefits. Our model could still be useful. For example, it identified the minimum number of rounds required for convergence. A group might feel frustrated if it does not seem to reach convergence in the early stages of decision-making. Calculations based on our model would then be helpful, in such a case, to reassure the group that if they persevere, they will reach a consensus.

The autocracy, weighted probability, equiprobability, and unanimity models studied by Wilson, Lilien, and Wilson (1991) were all models that may be used when a group converges to a consensus. The autocracy model of Wilson, Lilien, and Wilson could correspond to the condition in our model where $w_{ii^*} = 1 \; \forall \; i$, and $w_{ij} = 0 \; \forall \; i, \forall \; j \neq i^*$. From our analysis, a group with such weights would converge to the assessment (or choice) of the individual i^* in one round as in the autocracy model (a more general version of this conditions is $w_{i^*i^*} = 1$, $w_{ii} \neq 1$, $\forall\ i \neq i^*$.)

The weighted probability model is equivalent to the condition that $w_{ij} = k_j\ \forall\ i,j$. If $k_j = k_i\ \forall\ i,j$, then it is equivalent to the equiprobability model. Here again, convergence could occur in one round to a value in our model, which would be identical to that obtained from the weighted probability (or equiprobability) model.

For the unanimity model to be valid, the group's assessments (or choice) should converge. This consensus value (after discussion) should be obtained by the unanimity model. In other words, if the convergence conditions are satisfied by the weights of the group members, then one of the above models (as appropriate) would be correct from a predictive standpoint. If these conditions are not satisfied, then the other models, namely, voting, preference perturbation, and majority rules, may be applied. We thus expect that under the conditions where Wilson, Lilien, and Wilson (1991) found the voting, preference perturbation, and majority rules to apply, respectively, convergence would not have occurred. The flip side of this comment is that if it is found that convergence is unlikely (based on calculations using our model), then one of these three rules should be adopted.

Our model also provides a theoretical understanding to obtaining the pivot point in Rao and Steckel's (1991) group polarization model if the constant ϕ is known [Endnote [ii]]. Individuals' assessments, post-group discussion depend on the sign of the difference in directions from the original (or starting) assessments and the pivot point. We also show that group polarization can occur in either direction. Propositions 3 and 4 show the sufficient conditions for this effect to occur. Thus, our analysis determines whether group polarization will occur by measuring only the weights attached by the participants to each other and their starting individual assessments.

Our model allows us to determine if a risky-shift at the group level will occur or not. That is if the group's consensus estimate/decision will be in a more risky direction than the mean of the starting estimates/decisions. We have also applied how our research provides added values and simulation results to illustrate the complexity in transboundary disputes of peace engineering, including the Water Dispute Resolution between Israel and Jordon, between India and Pakistan and between Canada and the US. Thus this model provides an explanation for cases when risky-shift may not be observed.

Our model also provides a formal representation (mathematical model) of the effects obtained in the Boje and Murnighan (1982) study. Boje and Murnighan (1982) investigated the accuracy of estimates by three experimental groups. The first group interacted by providing estimates in a round-robin fashion with face-to-face feedback. The second group worked as the first except that their interactions were written down and no verbal interactions were allowed. The third group had no interactions. In other words, individuals in this group modified their estimates simply based on their own cognition and had no additional input from the outside.

Boje and Murnighan (1982) found that the mean of the estimates of the individuals in the face-to-face and written feedback groups became less accurate[3] With more rounds (or trials). The mean of the individuals in the "isolated" group, on the other hand, improved accuracy. Thus, a group decision process seems to shift individual estimates and a simple average of the group-updated individual estimates deteriorates accuracy with more rounds of updating. Our model mirrors this round-robin process, except that a group means is not calculated. Instead, a consensus value or limiting value is arrived at to represent the group decision. This decision may be quite different from the group average, as shown in propositions 2 and 3.

Boje and Murnighan's work captures a "group think" phenomenon - a subset of which is the risky shift effect. However, the Boje and Murnighan paper does not show that group decision making is inferior to individual decision making. Their results do not show whether the individual estimates converged over trials or not. Nor do their results indicate whether the group's decision would be a simple mean of individuals' estimates. The divergence between the accuracy levels of

---

[3]. Two Almanac and two subjective likelihood problems were used and thus, "true" or correct values were available to Boje and Murnighan.

the interactive groups and the isolated group implies that individuals update their estimates by assigning different weights to at least two individuals (one of whom may be themselves, otherwise all of them would have identical updated estimates equal to the mean of the priors, and thus would converge and their accuracies would be no different than for the isolated group) and that they update at each round (or trial). Our model captures these effects. Based on our model, it is implied that to test the Boje and Murnighan conjecture, the trials would have to be carried on until the estimates converged. Otherwise, implications for the use of group interactions or individual averaging inputs may not be correct.

Our simulations demonstrate the convergence of three different cases, which support transboundary cases and resolutions. They have a low starting probability (such as current Taiwan-China relations) and, in fact, should be compared to some benchmark. Normally, a probability is defined as low, medium and high depending on whether it is (respectively) less than, equal to, or greater than 0.5. But when studying issues where two negotiating parties strongly agree with each other, e.g., transboundary dispute, the negotiation success rate is lower and the threshold may become very low (Bercovitch et al., 1991). Therefore, we adjusted the threshold as low (0.2), medium (0.35) and high (0.45) probabilities, which can be corresponding to 40%, 70% and 90% success rates for cases with less hostile situations hence a higher probability of success. The three sets of values actually mean similar things respectively, failure (0.2 equivalent to 40% <50%), continued for the next rounds (0.35 equivalent to 70% > 50%) and success (0.45 equivalent to 90% >>50%).

In today's uncertain and rapidly changing world, analysis and forecasting on uncertain and lower probability may have higher values. We are looking at cases with low probability as the starting point. If putting transboundary dispute asides, such examples include current US-China relations, US-Iran relations and so on. In transboundary disputes, currently, the transboundary disputes between China and India and injuries and deaths between soldiers on both sides between June and July 2020 fall into this situation. Upon 1,000 simulations under volatile international relations, the convergence will develop it in the way that Figures 4a and 4b demonstrate the integrated uses of Fuzzy Delphi and Grey Delphi Methods is more suitable for predicting and analyzing volatile situations. Two sets of predictive modeling outcomes are as follows.

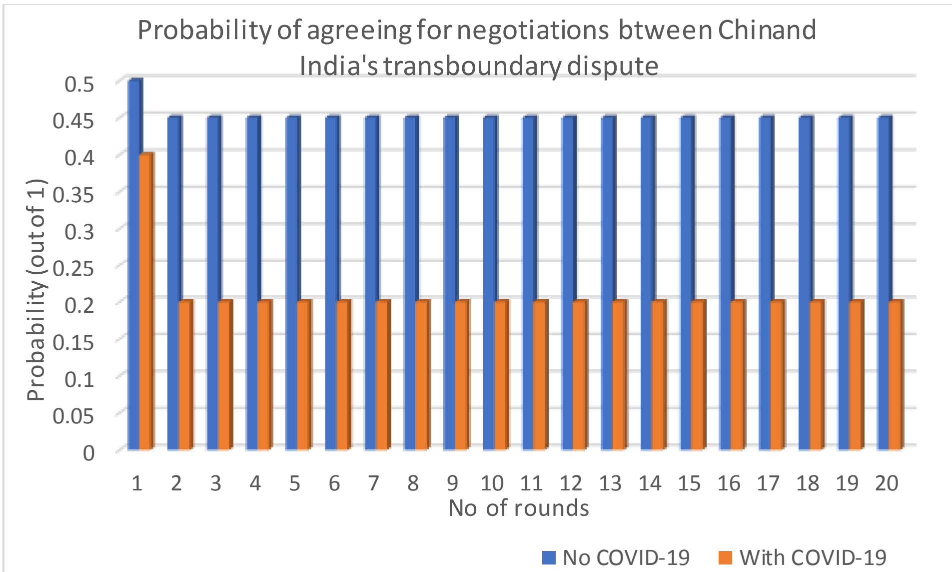

Figure 4 a): Probability for meaningful negotiations between China and India with/without COVID-19.

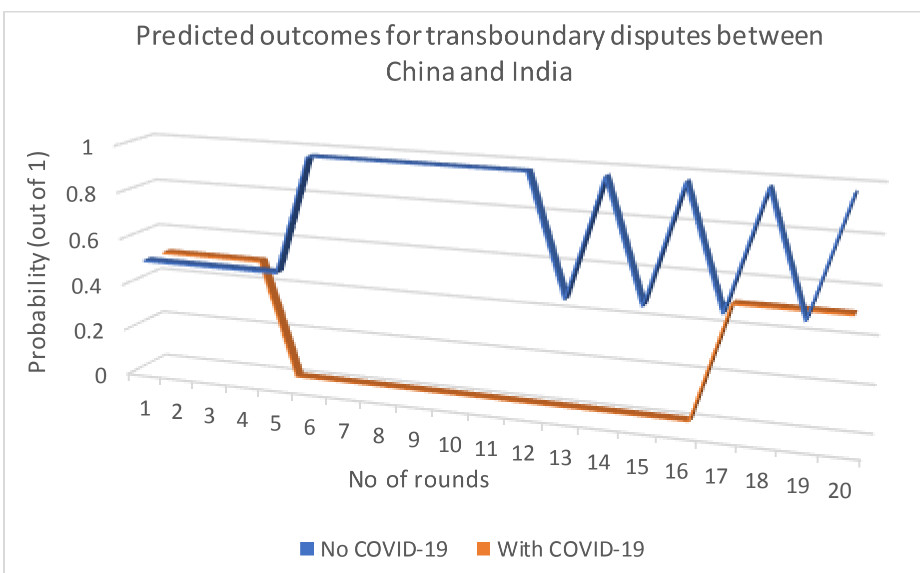

Figure 4 b): The predicted outcomes for transboundary disputes between China and India

Figure 4a shows the probability of agreeable negotiations between China and India. With and without COVID-19, the probability is very similar to the green line and red line in Figures 3a and 3b, respectively. Figure 4b shows the probability of the outcomes of negotiation. 1 means success, 0.5 means continue to the next round and 0 means failure. Each round is regarded as a different time period. The negotiations may need to be continued since this can be a long process and new events or accidents may trigger the need to renegotiate. At least this is true between China and India in recent years with rationale as follows. The diplomatic relations between the two countries had improved significantly between the Year 2016 and 2019 with the state visits between leaders of the two countries and international events hosted by the two countries. It indicates that the probability of reaching certain agreements can be high. Therefore, after a few rounds of negotiation, the predicted outcomes show 1 for the next few rounds. It means the transboundary dispute came to a temporary end due to terms and conditions agreed by both sides. After a period of time, the negotiation can restart due to the end of the previous agreements or the rise of new accidents. However, with COVID-19, the predicted outcomes show very different scenarios. China and India are likely not to reach any agreements for some time. The government and the general public in India have felt that the health and economic crisis have been related to China (Roy et al., 2020). With the increasing tension between these two countries, it is unlikely this transboundary dispute can be resolved in the short term. Until the wide availability of vaccines and improved terms and conditions, re-negotiations may start at some point.

COVID-19 is a turning point that makes a higher probability of reaching agreements to a lower likelihood. Not only have we applied for Dalkey and Helmer's work in 1963, but we also use the blended Fuzzy Delphi and Grey Delphi Methods. The difference is that we apply it to challenging cases that a lower probability of success is expected, and the method can be used to predict the likely outcomes with or without crucial additional variables (such as COVID-19). On top of this, our method also allows us to predict the likely outcomes beyond five rounds, limiting the original Delphi method.

While our model provides a structure that captures the effects observed in several empirical studies (as shown above and in our discussion of the contextual conformity conditions on pages 9 to 10), many empirical questions remain to be answered. Three of these are, for example, (1) What are the counterpart application domains to the various weight conditions here? (2) What will be individuals' satisfaction levels with (a) intermediate group decisions (b) final group decisions, and (c) model calculated group decisions? (3) How are the weights, as in our model, related to the social influence factors of Corfman and Lehmann (1987)?

Theoretically, an essential set of issues that need to be studied revolves around the dynamics of weights attached to individuals' assessments. Do these weights change? If so, (1) How? (2) What will the impact be on the convergence (and its rate) of assessments? While it is entirely reasonable to recognize the possibility that weights may change overruns of interaction as part of learning behavior, assuming a fixed weights matrix has been a major limitation for the axiomatic modeling approach that sacrifices more realistic detailed behaviors for simplicity and plausibility of closed-form solution. The original Markov Chain model also carries a similar limitation; hence, our model also is subjected to the same limitation in this respect.

CONCLUSION

In summary, we have developed a model that provides a link between how an individual's assessment is updated (Bordley, 1983) and the assessment arrived at by a group (Rao and Steckel, 1991). This model captured the effects seen in several major articles in the area and provides a theoretical mechanism for understanding the pivot point in the Rao and Steckel (1991) model. Additionally, we used the blended Fuzzy Delphi and Grey Delphi Methods to make further improvements to the traditional Delphi model with additional contributions. First, we could predict the outcomes of each round for transboundary disputes under peaceful and stable conditions. Second, our paper contributes to the existing literature on the negotiations with low probability as our model has been demonstrated to be suitable for predicting and analyzing such kinds of negotiations. Third, we could take transboundary disputes under volatile and uncertain conditions for our analysis and compared the differences, such as the impact to negotiations with and without COVID-19.

Our model could be applied in various areas, such as transboundary disputes in international relations as well as in such marketing settings as decision-making in organizational buying centers and negotiations involving multiple parties in distribution channels and industrial marketing decisions. With ongoing improvements, our research outputs can be used to demonstrate transboundary disputes of peace engineering, particularly the use of simulations to illustrate the complexity and probability of successful negotiations.

Acknowledgment

We thank Bob Bordley, Gary Lilien, Keith Murnighan, Vithala Rao, Brian Ratchford, Jose Rosa and Heichi Chan for their comments on an earlier draft of this paper. We would like to thank Prof. Zhenzong Cao from Xiamen University for his contributions at an early stage of this work, which was started when Sudharshan, Liu and Cao were at the University of Illinois at Urbana Champaign to the modeling of this paper. His current contact information cannot be established.

Funding

This research is partly supported by VC Research (VCR 0000019).

REFERENCES

Aldrich, J. H., Nelson, F. D., Adler, E. S., 1984. Linear probability, logit, and probit models. (No. 45). Sage.

Asch, S.E., 1956. Studies of Independence and Submission to Group Pressure: I. On Minority of one Against a Unanimous Majority. Psychological Monographs 70 (9), 1.

Back, KW.,1951. Influence Through Social Communication. Journal of Abnormal and Social Psychology 46, 9-23.

Baron, R.S., Roper, G., 1976. Reaffirmation of Social Comparison View of Choice Shifts: Averaging and Extremity Effects in an Autokinetic Situation. Journal of Personality and Social Psychology 33, 521-530.

Baron, R.S., Kerr, N., Miller, N., 1992. Group Process, Group Decision, Group Action. Buckingham, UK: Open University Press.

Beck, D., Fisch, R., 2000. Argumentation and emotional processes in group decision-making: Illustration of a multilevel interaction process analysis approach. Group Processes & Intergroup Relations 3(2), 183-201.

Belton, I., MacDonald, A., Wright, G., Hamlin, I., 2019. Improving the practical application of the Delphi method in group-based judgment: A six-step prescription for a well-founded and defensible process. Technological Forecasting and Social Change 147, 72-82. https://doi.org/10.1016/j.techfore.2019.07.002.

Bercovitch, J., Anagnoson, J.T., Wille, D.L., 1991. Some Conceptual Issues and Empirical Trends in the Study of Successful Mediation in International Relations. Journal of Peace Research 28 (1), 7-17.

Boje, D.M., Murnighan, J.K., 1982. Group Confidence Pressures in Iterative Decisions. Management Science 28 (10), 1187-1196.

Bordley, R.F., 1983. A Bayesian Model of Group Polarization. Organizational Behavior and Human Performance 32, 262-274.

Bordley, R.F., 1986. Linear combination of forecasts with an intercept: A Bayesian approach. Journal of Forecasting 5 (4), 243-249.

Bordley, R.F., Bier, V., 2009, Updating Beliefs about Variables given new information on how those Variables Relate. European Journal of Operational Research 193 (1), 184-194.

Bordley, R.F., Pollock, S., 2009. A Decision-Analytic Approach to Reliability-Based Design Optimization. Operations Research 57 (5), 1262-1270.

Calabrese, A., Costa, R., Levialdi, N., Menichini, T., 2019. Integrating sustainability into strategic decision-making: A fuzzy AHP method for the selection of relevant sustainability issues. Technological Forecasting and Social Change 139, 155-168. https://doi.org/10.1016/j.techfore.2018.11.005.

Choffray, J-M., Lilien, G.L., 1980. Marketing Planning for New Industrial Products. New York: John Wiley & Sons, 136-141.

Chung, K.L., 1967. Markov Chains with Stationary Transition Probabilities. 2nd Ed., New York, NY: Springer-Verlag New York Inc., p7.

Corfman, K.P., Gupta, S.,1993. Mathematical Models of Group Choice and Negotiations. in J. Eliashberg and G.L. Lilien, Eds., Handbooks in Operational Research and Management Science: Marketing 5, 83-142. Elsevier Science Publishers BV.

Corfman, K.P., Lehmann, D.R., 1987. Models of Cooperative Group Decision-Making and Relative Influence: An Experimental Investigation of Family Purchase Decisions. Journal of Consumer Research 14, June, 1-13.

Cotton, J.L., Baron, R.S., 1980. Anonymity, Persuasive Arguments and Choice Shifts. Social Psychology Quarterly 43 (4), 391-404.

Curry, D., Menasco, M., Van Ark, J., 1991. Multiattribute Dyadic Choice: Models and Tests. Journal of Marketing Research 28 (3), 259-267. doi:10.2307/3172862.

Dalkey, N., Helmer, O., 1963. An Experimental Application of The Delphi Method to The Use of Experts. Management Science 9 (3), 458-467.

Dalkey, N., 1968. Predicting the Future. Rand Corporation, P-3948, October.

Dalkey, N., 1969. An Experimental Study of Group Opinion: The Delphi Method. Futures 1, 408-420.

Darley, J.M., 1966. Fear and Social Comparison as Determinants of Conformity Behavior. Journal of Personality and Social Psychology 4 (1), 73-78.

Davis, J.H., 1973. Group decision and social interaction: A theory of social decision schemes. Psychological Review 80 (2), 97–125.

DeGroot, M., 1974. Reaching a consensus. Journal of the American Statistical Association 69, 118–121.

Deutsch, M., Gerard, H.B., 1955. A Study of Normative and Informational Social Influence Upon Individual Judgment. Journal of Abnormal and Social Psychology 51, 629-636.

Eliashberg, J., LaTour, S.A., Rangaswamy, A., Stern, L.W., 1986. Assessing the Predictive Accuracy of Two Utility-Based Theories in a Marketing Channel Negotiation Context. Journal of Marketing Research 23, May, 101-110.

Eliashberg, J., Winkler, R., 1981. Risk Sharing and Group Decision Making. Management Science 27, 11.

Festinger, L., 1954. A Theory of Social Comparison Processes. Human Relations 7, 117-140.

Fischhendler, I., 2008. Ambiguity in transboundary environmental dispute resolution: The Israeli—Jordanian water agreement. Journal of Peace Research 45 (1), 91-109.

Fleming, M., 1952. A Cardinal Concept of Welfare. Quarterly Journal of Economics 66 (August), 366-384.

Hare, A.P., 1976. Handbook of Small Group Research. New York: Free Press.

Harsanyi, J.C., 1955. Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility. Journal of Political Economy 63 (August), 309-321.

Harvey, O.J., Consalvi, C., 1960. Status and Conformity to Pressures: in Informal Groups. Journal of Abnormal and Social Psychology 56, 934-941.

Hogg, M.A., Turner, J.C., 1987. Social Identity and Conformity: A Theory of Referent Informational Influence. in W. Doise and S. Moscovici (eds) Current Issues in European Social Psychology 2. Cambridge: Cambridge University Press.

Hsu, Y.L., Lee, C.H., Kreng, VB, 2010. The application of Fuzzy Delphi Method and Fuzzy AHP in lubricant regenerative technology selection. Expert Systems with Applications 37 (1), 419-425.

Ishikawa, A., Amagasa, M., Shiga, T., Tomizawa, G., Tatsuta, R., Mieno, H., 1993. The max-min Delphi method and fuzzy Delphi method via fuzzy integration. Fuzzy sets and systems 55 (3), 241-253.

Iosifescu, M., 1980. Finite Markov Processes and Their Applications. New York, NY: John Wiley & Sons, 57-58.

Jones, A. M., Koolman, X., Rice, N., 2006. Healthrelated non‐response in the British Household Panel Survey and European Community Household Panel: using inverse‐probability‐weighted estimators in non‐linear models. Journal of the Royal Statistical Society: Series A (Statistics in Society) 169 (3), 543-569.

Kalton, G., Flores-Cervantes, I., 2003. Weighting methods. Journal of official statistics 19 (2), 81.

Kemeny, J. G., Snell, J. L., 1963. Finite Markov Chains. Princeton: VanNostrand. Kemeny Finite Markov Chains.

Liu, S. F., Dang, Y. G., Fang, Z. G., 2004. Grey system theory and its application. Beijing: Science Press.

Lowi, M. R., 1993. Bridging the divide: transboundary resource disputes and the case of West Bank water. International security 18 (1), 113-138.

Ma, Z., Shao, C., Ma, S., Ye, Z., 2011. Constructing road safety performance indicators using fuzzy delphi method and grey delphi method. Expert Systems with Applications 38 (3), 1509-1514.

MacNeil, M.K., Sherif, M., 1976. Normal Change Over Subject Generations as a Function of Arbitrariness of Prescribed Norms. Journal of Personality and Social Psychology 34, 762-773.

Madani, K., Rouhani, O. M., Mirchi, A., Gholizadeh, S., 2014. A negotiation support system for resolving an international trans-boundary natural resource conflict. Environmental Modelling & Software 51, 240–249. doi:10.1016/j.envsoft.2013.09.029

Martino, J.P., 1993. Technological Forecasting for Decision Making. 3rd ed., New York: North Holland.

Murray, T. J., Pipino, L. L., Van Gigch, J. P., 1985. A pilot study of fuzzy set modification of Delphi. Human Systems Management 5 (1), 76-80.

Myers, D.G., 1982. Polarizing Effects of Social Interaction. In H. Brandstatter, J.H. Davis and G. Stocker-Kreichgauer (eds) Group Decision Making, New York: Academic Press.

Myers, D.G., Bishop, G.D., 1970. Discussion Effects on Racial Attitudes. Science 169, 778-779.

Myers, D.G., Lamm, H., 1976. The Group Polarization Phenomenon. Psychological Bulletin 83, 602-627.

Rao, V.R., Steckel, J.H., 1991. A Polarization Model for Describing Group Preferences. Journal of Consumer Research 18 (June), 108-118.

Rowe, G., Wright, G., 1999. The Delphi technique as a forecasting tool: issues and analysis. International Journal of Forecasting 15, 353–375.

Roy, D., Tripathy, S., Kar, S. K., Sharma, N., Verma, S. K., Kaushal, V., 2020. Study of knowledge, attitude, anxiety & perceived mental healthcare need in Indian population during COVID-19 pandemic. Asian Journal of Psychiatry, 102083.

Santmire, T., Wilkenfeld, J., Kraus, S., Holley, K., Santmire, T., Gleditsch, K., 1998. The Impact of Cognitive Diversity on Crisis Negotiations. Political Psychology 19 (4), 721-748.

Sherif, M., 1935. A Study of Some Social Factors in Perception. Archives of Psychology 27, 187.

Sun, B., Jiang, J., Zheng, F., Zhao, W., Liaw, B. Y., Ruan, H., Han, Z., Zhang, W., 2015. Practical state of health estimation of power batteries based on Delphi method and grey relational grade analysis. Journal of Power Sources 282, 146-157.

Wilder, D.A., Allen, V.L., 1977. Social Support, Extreme Social Support and Conformity. Representative Research in Social Psychology 8, 33-41.

Wilson, E.J., Lilien, G.L. Wilson, D.T., 1991. Developing and Testing a Contingency Paradigm of Group Choice in Organizational Buying. Journal of Marketing Research 28 (4), November, 452-466.

Wittink, D. R., Krishnamurthi, L., Nutter, J. B., 1982. Comparing derived importance weights across attributes. Journal of Consumer Research 10 (1), 471-474.

Wolf, A. T., Newton, J. T., 2007. Case Study of Transboundary Dispute Resolution: Canada and the United States of America. https://transboundarywaters.science.oregonstate.edu/sites/transboundarywaters.science.oregonstate.edu/files/Database/ResearchProjects/casestudies/ijc.pdf.

Wolf, A. T., Newton, J. T., 2008. Case study of transboundary dispute resolution: The Indus water treaty. Department of Geosciences, Oregon State University: Corvallis, OR, USA.

Woudenberg, F.,1991. An Evaluation of Delphi. Technological Forecasting and Social Change 40, 131-150.

Zeitoun, M., Mirumachi, N., 2008. Transboundary water interaction I: Reconsidering conflict and cooperation. International Environmental Agreements: Politics, Law and Economics 8 (4), 297.

Additional Readings

Chang, V., Valverde, R., Ramachandran, M., Li, C. S., 2020. Toward Business Integrity Modeling and Analysis Framework for Risk Measurement and Analysis. Applied Sciences 10 (9), 3145.

Abdel-Basset, M., Mohamed, M., Chang, V., Smarandache, F., 2019. IoT and its impact on the electronics market: A powerful decision support system for helping customers in choosing the best product. Symmetry 11 (5), 611.

Chang, V., Abdel-Basset, M., Ramachandran, M., 2019. Towards a reuse strategic decision pattern framework–from theories to practices. Information Systems Frontiers, 21 (1), 27-44.

APPENDIX & ENDNOTES

Due to the submission system of the journal website cannot convert the formulas inside the Appendix and Endnotes for proper display, they are appended in a separate file in pdf format such that the formulas are properly displayed there. Sorry for the inconvenience caused by such a minor technical issue.