ARTICLE IN PRESS
Journal of Mathematical Economics xxx (xxxx) xxx
Contents lists available at ScienceDirect
Journal of Mathematical Economics
journal homepage: www.elsevier.com/locate/jmateco

# Competitive pricing and advertising with spillover

Xingyi Liu
*Aston Business School, Aston University, Birmingham B4 7ET, United Kingdom*

ARTICLE INFO

*Article history:*
Received 11 April 2021
Received in revised form 12 January 2022
Accepted 19 February 2022
Available online xxxx
Manuscript handled by Editor Carmen Beviá

*Keywords:*
Limited attention
Spillover
Horizontal attribute
Advertising

ABSTRACT

We study firms' advertising strategies when they face attention-limited consumers, who pay more attention to a horizontal attribute when it is more heavily advertised. Under competition, one firm's advertising can affect a consumer's valuation for competing products, which we term as the spillover effect. We show that competing firms may only advertise the horizontal attribute when the spillover is weak. Moreover, competing firms may advertise less than a monopolist.


## 1. Introduction

Firms often advertise specific attributes of a product in addition to its availability and price. There is, however, a large variation in the amount of information advertised across product categories and advertising channels.[1] To study firms' incentives in advertising specific attributes, we propose a model in which an increase in advertising implies that consumers pay closer attention to the advertised attributes of a product.[2] Moreover, we allow a firm's advertising to also induce consumers to pay closer attention to the same attributes of competing products — something we call a spillover effect.[3]

Specifically, when paying closer attention to the advertised attribute, a consumer's valuation becomes more dispersed according to a mean-preserving spread. This increases the perceived differentiation among firms. If the market is fully covered, firms then have strong incentives to increase such differentiation by inducing more dispersion, as shown by, for instance, Anderson and Renault (2009) and Zhu and Dukes (2017). However, if the market is only partially covered, the marginal consumer with a low match value for the attribute will stop buying the product, when induced to pay closer attention. Advertising then leads to a loss of market coverage.

Thus, a monopolist only advertises when the marginal consumer between buying and not buying has a high match value. With competition, due to lower competitive prices, such a marginal consumer is more likely to have a low match value, which reduces the incentives of competing firms to advertise. However, with competition, there is also a second type of marginal consumer who is indifferent between competing firms. Advertising can induce such a consumer with high match value to strictly favour one firm. This generates additional incentives for competing firms to advertise, the strength of which depends on the degree of the spillover. With strong spillover, the relative preference of a marginal consumer between firms can be hardly influenced by advertising, and hence, competing firms advertise less. With weak spillover, such a consumer can be easily induced to favour the advertising firm, which leads competing firms to advertise more. The results shed light on differences in the information content between advertising channels and product categories that differ in their targeted audience (e.g., TV vs. Magazine) or the degree of spillover (e.g., homogeneous vs. differentiated products).

The model builds on the literature of limited attention. Different from earlier papers (e.g., Van Zandt, 2004; Haan and Moraga-Gonzalez, 2011) that assume firms advertise to compete for consumers' attention in order to be noticed, we focus on consumers' attention towards specific product attributes. Several recent papers propose alternative models of attention allocation across product attributes. For instance, Bordalo et al. (2013) assume that the attribute with a valuation that differs more

*E-mail address:* x.liu29@aston.ac.uk.

[1] For instance, Abernethy and Franke (1996) show that advertising of services presents less information than that of electronics, while television advertising presents less information than magazines; Anderson et al. (2013) show that large branded manufacturers or branded manufacturers with large competitors advertise less information.

[2] It has long been recognized that a targeted attribute in advertisement is likely to affect brand evaluation (Gardner, 1983) and brand preference (Chakravarti and Janiszewski, 2004). Consumers also engage more with advertisements that provide more product information (Lee et al., 2018).

[3] The existence of and factors that influence such spillover have been studied by, e.g., Sahni (2016) for restaurants, Anderson and Simester (2013) for apparel retailers, and Magee (2013), Ieva et al. (2018), and Jones et al. (2005) for different advertising channels.

https://doi.org/10.1016/j.jmateco.2022.102660
0304-4068/

from the average attracts more attention, and Koszegi and Szeidl (2013) assume that the attribute that has a wider range of valuations attracts more attention. Instead, we assume that consumers pay closer attention to the attribute when it is more heavily advertised. This is similar to the attention-expansion approach mentioned by Zhu and Dukes (2017), but they assume full market coverage while we focus on partial market coverage to study the tradeoff between product differentiation and market coverage.

Our paper is also related to the literature on horizontal information disclosure. See, for instance, Lewis and Sappington (1994) and Johnson and Myatt (2006) for the case of monopoly, Anderson and Renault (2006) and Branco et al. (2016) for monopoly with consumer search, and Sun (2011), Hotz and Xiao (2013), and Janssen and Teteryatnikova (2016) for cases of competition. In this strand of literature, a consumer fully takes into account the disclosed or advertised information, whereas our approach allows a more flexible impact of advertising on the consumer's decision making. The attention approach also provides a natural interpretation for the spillover, which is only considered by a few papers in specific settings. For example, Meurer and Stahl (1994) assume negatively correlated match values for the two products, and Anderson and Renault (2009) consider either no spillover or full spillover by specifying the types of information disclosed. In our setup, we allow for a full spectrum of spillover effect.

The paper proceeds as follows: We present our setup in Section 2 and analyse both cases of monopoly and duopoly in Section 3. We then generalize the analysis to more than two firms in Section 4 and conclude with some avenues for future research in Section 5. All proofs are included in Appendix A.

## 2. The setup

We consider a duopoly, firm 1 and 2, selling to a representative consumer, who demands only one unit of a product. We normalize the value of the outside option to zero, if the consumer does not purchase anything. The purchase decision depends on the consumer's subjective value (referred to as simply the 'valuation' until we discuss consumer surplus in Section 3.2.4) for the product of firm $i$, $i = 1, 2$, given by,

$$v_i = \mu + \alpha_i v_i^h.$$

The first term $\mu$ is the intrinsic value/quality of the product, assumed to be the same for the two firms. In the second term, $v_i^h$ ($i = 1, 2$) is the match value of the horizontal attribute for firm $i$'s product. This formulation follows from Fehr and Rangel (2011) and has the following interpretation: the consumer always pays full attention to quality, but may only pay partial attention to the horizontal attribute, as measured by $\alpha_i$. We assume that $\alpha_i \in [\underline{\alpha}, 1]$, where $\underline{\alpha}$ is the attention paid to the attribute without advertising.

We assume that $v_i^h$ ($i = 1, 2$) are independently drawn from a continuously differentiable distribution $H(v^h)$ on the support $(-\infty, \infty)$ with zero mean and variance $\sigma^h$. The corresponding density function is $h(v^h)$, assumed to be log-concave, single-peaked and symmetric around zero. Most commonly used density functions such as Normal and Logistic satisfy this assumption, with a full support on the real line capturing incomplete market coverage conveniently. We will also refer to the example of uniform distribution to illustrate some of our results, as it allows us to have explicit solutions in some cases.[4]

The distribution and density function of $v_i$ are then given by

$$F(v_i; \alpha_i) = H(\frac{v_i - \mu}{\alpha_i}), \text{ and } f(v_i; \alpha_i) = \frac{1}{\alpha_i} h(\frac{v_i - \mu}{\alpha_i}).$$

It is easy to see that $F(v_i; \alpha_i)$ and $f(v_i; \alpha_i)$ retain the property of log-concavity and symmetry with respect to $\mu$. An increase in $\alpha_i$ leads to a mean-preserving spread of $F(v_i; \alpha_i)$.[5] In the terminology of Johnson and Myatt (2006), an increase in $\alpha_i$ leads to a rotation of $F(v_i; \alpha_i)$ around $\mu$.

A firm can attract attention to the horizontal attribute via advertising. Specifically, by advertising the horizontal attribute at intensity $s_i \in [0, 1 - \underline{\alpha}]$, firm $i$ induces

$$\alpha_i = \underline{\alpha} + (1 - \omega)s_i + \omega s_j, \text{ for } j \neq i \text{ and } \omega \in [0, 1/2]. \quad (1)$$

The parameter $\omega$ captures the spillover effect that the consumer's valuation of product $i$ may be affected by firm $j$'s advertising.[6] When $\omega = 1/2$, the consumer's valuation of a product is equally affected by the advertising of both firms, which we interpret as strong attention spillover. When $\omega = 0$, the consumer's valuation of product $i$ is only influenced by firm $i$'s advertising, which we interpret as no attention spillover.

We consider the firms choose their prices and advertising intensities simultaneously, then the consumer makes a purchase decision observing all prices and horizontal match values. We look for a symmetric equilibrium where firms choose the same advertising intensity and price.

As the consumer's decision is straightforward, we focus on the firms. For given $p_j$ and $s_j, j \neq i$, the demand for firm $i$ when it chooses $p_i$ and $s_i$ is given by

$$D_i(p_i, p_j; \alpha_i, \alpha_j) = \int_{p_i}^{\infty} F(v_i - p_i + p_j; \alpha_j) dF(v_i; \alpha_i),$$
$$\text{for } i, j \in \{1, 2\} \text{ and } i \neq j.$$

That is, the consumer purchases form firm $i$, when its valuation is higher than the price ($v_i > p_i$) and higher than that of firm $j$ ($v_i - p_i > v_j - p_j$). The profit of firm $i$ is then given by

$$\pi_i(p_i, p_j; \alpha_i, \alpha_j) = p_i D_i(p_i, p_j; \alpha_i, \alpha_j) - C(s_i),$$

where $C(s_i)$ is the cost of advertising with an intensity of $s_i$, assumed to be increasing and sufficiently convex in $s_i$ with $C'(0) = 0$ and $C'(s_i) > 0$ for $s_i > 0$.[7]

To characterize the symmetric equilibrium, we write the profit function explicitly in terms of $s_i$ and $s_j$ as $\pi_i(p_i, p_j; s_i, s_j)$. Suppose firm $j$ plays the candidate equilibrium strategy $(p^*, s^*)$, we consider the best reply of firm $i$. As shown by Caplin and Nalebuff (1991), log-concavity of $F(v_i; \alpha_i)$ implies that the demand function for firm $i$ is log-concave in $p_i$. Hence, the profit function of firm $i$ is unimodal in price and, for each $s_i$, there is a unique interior profit maximizing price $p_i(s_i)$, satisfying $\partial \pi_i(p_i(s_i), p^*; s_i, s^*) / \partial p_i = 0$. Hence, we can rewrite the problem of firm $i$ as choosing only $s_i$ to maximize its profit at the corresponding optimal price, given by,

$$\pi(s_i, s^*) \triangleq \pi_i(p_i(s_i), p^*; s_i, s^*) = p_i(s_i) D_i(p_i(s_i), p^*; s_i, s^*) - C(s_i).$$

For sufficiently convex $C(s_i)$, this profit function is concave in $s_i$, and the best-replying advertising intensity, $s_i^*$, is uniquely determined by $s_i^* = 0$ if $\pi'(s_i, s^*) \leq 0$ at $s_i = 0$, $s_i^* = 1 - \underline{\alpha}$ if

[4] Note that in this case $H(v^h)$ does not have full support on the real line, so additional assumption is needed to have incomplete market coverage, see Section 3.2.4 for more details.

[5] This differentiates our approach from Aköz et al. (2020), where product review manipulation shifts the distribution and changes the expected value of a product.

[6] One interpretation is as follows: the consumer pays attention to a fixed amount of advertising messages when evaluating a firm's product. As a firm increases the amount of messages with the attribute information, the consumer pays more attention to the attribute. When there is spillover, the consumer pays attention to a mixture of advertising messages from this firm and also the competing firm.

[7] This is a natural assumption, given that attracting attention is costly. This also differs from the literature on information disclosure, where disclosing more information is not necessarily more costly. See, for instance, Renault (2015).

$\pi'(s_i, s^*) \geq 0$ at $s_i = 1 - \underline{\alpha}$, and the solution to $\pi'(s_i, s^*) = 0$ otherwise.

Thus, for $(p^*, s^*)$ to be a symmetric equilibrium, the price $p^*$ must satisfy

$$\frac{\partial \pi_i(p^*, p^*; s^*, s^*)}{\partial p_i} = \frac{\partial \pi_j(p^*, p^*; s^*, s^*)}{\partial p_j} = 0.$$

That is, $p^*$ is the competitive price when both firms advertise at intensity $s^*$. The existence and uniqueness of such a price is well-established in the literature for log-concave density functions.[8] And, the equilibrium advertising intensity $s^*$ depends on $\pi'(s^*, s^*)$, given by

$$\pi'(s^*, s^*) = \frac{d\pi_i(p_i(s^*), p^*; s^*, s^*)}{ds_i} = p^* \frac{\partial D_i(p^*, p^*; s^*, s^*)}{\partial s_i} - C'(s^*),$$

which is decreasing in $s^*$ for sufficiently convex $C(s_i)$.[9] Hence, the symmetric equilibrium advertising intensity is uniquely determined by $s^* = 0$ if $\pi'(0, 0) \leq 0$, $s^* = 1 - \underline{\alpha}$ if $\pi'(1 - \underline{\alpha}, 1 - \underline{\alpha}) \geq 0$, and the solution to $\pi'(s^*, s^*) = 0$ otherwise.[10]

Our main analysis focuses on whether a positive advertising equilibrium with $s^* > 0$ exists, and we suppose $C''(s_i)$ is large enough such that fully advertising the horizontal attribute (i.e., $s^* = 1 - \underline{\alpha}$) is not an equilibrium. Given that $C'(0) = 0$, a positive advertising equilibrium exists if firms have incentives to advertise the horizontal attribute when neither firm does, i.e., if $\frac{\partial D_i}{\partial s_i}|_{s_i = s_j = 0} > 0$. In the following, we will identify conditions under which this occurs.

**Remark.** We show in the Online Appendix that our main results hold for a slightly different formulation,

$$\alpha_i = \underline{\alpha} + s_i + \omega s_j, \tag{2}$$

where $\omega \in [0, 1]$ measures the degree of spillover. With this formulation, the degree of spillover has a direct effect on the equilibrium price. By muting this channel, our model provides a clean analysis on the effect of spillover on advertising intensity (see Proposition 3). In addition, with this alternative formulation, the impact of spillover on attention depends on the rival firm's level of advertising, so the consumer always pays more attention as the spillover becomes stronger, whereas the reverse could be true in our model (1) as the impact of spillover depends on the relative levels of advertising.

## 3. Advertising to attention-limited consumers

### 3.1. The monopoly benchmark

Before proceeding to the equilibrium with competition, we establish the standalone incentive to advertise the horizontal attribute for a monopolist. The demand function of the monopolist with advertising intensity $s_m$ and price $p_m$ is

$$D_m(p_m; s_m) = 1 - F(p_m; \underline{\alpha} + s_m),$$

which clearly satisfies

$$\frac{\partial D_m}{\partial s_m} \lesseqgtr 0 \Leftrightarrow p_m \lesseqgtr \mu.$$

It immediately follows that:

**Proposition 1.** *The monopolist advertises the horizontal attribute if*

$$\mu f(\mu; \underline{\alpha}) < 1/2. \tag{3}$$

*Otherwise, it does not advertise the horizontal attribute.*

Condition (3) is satisfied if either the quality $\mu$ is relatively low or the dispersion of the valuation ($\sigma^h$ or $\underline{\alpha}$) is relatively high such that the monopoly price is higher than $\mu$. In this case, the marginal consumer who is indifferent between buying and not buying has a positive match value, and hence advertising the horizontal attribute further increases the perceived match value and increases demand. When the condition is reversed, the marginal consumer has a negative match value and advertising only lowers demand, which stifles the incentives to advertise.

### 3.2. Competitive advertising and attention spillover

In the presence of competition, a firm not only cares about the marginal consumer who is indifferent between buying and not buying, but also the marginal consumer who is indifferent between buying its product and the opponent's.[11] On one hand, competition drives down prices, which means the former type of marginal consumer is more likely to have a negative match value, and this reduces firms' incentives to advertise. On the other hand, the existence of the latter type of marginal consumer means that a firm may have more incentives to advertise to attract such a marginal consumer, especially when his/her match value is positive. We show in this section that the strength of the latter effect depends crucially on the degree of attention spillover, and so do the incentives to advertise the horizontal attribute.

#### 3.2.1. Strong attention spillover

We start with strong attention spillover, i.e., $\omega = 1/2$. In this case, we have

$$\alpha_1 = \alpha_2 = \alpha = \underline{\alpha} + \frac{1}{2}(s_1 + s_2).$$

The demand for each firm, at equal prices, is then given by

$$D_i(s_1, s_2) = \int_p^\infty F(v; \alpha) dF(v; \alpha) = \frac{1}{2}[1 - F^2(p; \underline{\alpha} + \frac{s_1 + s_2}{2})].$$

This implies that

$$\frac{\partial D_i(s_1, s_2)}{\partial s_i} \lesseqgtr 0 \Leftrightarrow p \lesseqgtr \mu.$$

Thus, whether firms advertise the horizontal attribute or not depends on whether the competitive price is above or below $\mu$, when neither firm advertises. Specifically, we have:

**Lemma 1.** *With strong attention spillover ($\omega = 1/2$), firms advertise the horizontal attribute if*

$$\mu[f(\mu; \underline{\alpha}) + 2\int_\mu^\infty f^2(v; \underline{\alpha}) dv] < \frac{3}{4}. \tag{4}$$

*Otherwise, no firm advertises the horizontal attribute. Moreover, (4) is more stringent than (3), i.e. competing firms are less likely to advertise the horizontal attribute than a monopolist.*

Similar to Proposition 1, Condition (4) is satisfied when $\mu$ is small or when $\sigma^h$ or $\underline{\alpha}$ is high such that the marginal consumer between buying or not buying has a positive match value for both firms. The second part follows because, under strong spillover, advertising more on the horizontal attribute does not affect the decision of a marginal consumer who is indifferent between the

[8] See, for instance, Zhou (2017) for a recent analysis in a different context.

[9] The second equality follows from the Envelop Theorem.

[10] We cannot rule out the possibility of asymmetric equilibria, especially when the advertising cost function is not too convex. In fact, without cost of attracting attention, a firm's profit would be quasi-convex in the level of attention paid to the horizontal attribute, which leads to either no horizontal advertising or full horizontal advertising (see also the discussion in Johnson and Myatt, 2006 on design costs). Thus, asymmetric equilibrium is likely to exist in such cases. We provide an example in the Online Appendix, but a full analysis on the existence and properties of asymmetric equilibrium is beyond the scope of this paper.

[11] We thank an anonymous referee for suggesting this distinction between the two types of marginal consumers.

two firms, as the consumer is always equally attentive to the attribute of both products. Thus, the only effect of competition is a lower price, which lowers the incentives to advertise. This differs from the literature, e.g. Anderson and Renault (2009) and Zhu and Dukes (2017), showing that symmetric firms have strong incentives to increase the dispersion of the consumer's valuations. The reason is two-fold: on one hand, the market is fully covered in their models, so firms need not worry about losing consumers to the outside option; on the other hand, they consider sequential choice of advertising and pricing, and hence firms advertise strategically to raise equilibrium price.

#### 3.2.2. No attention spillover

Now consider the case of no spillover with $\omega = 0$, when advertising also affects the marginal consumer who would be indifferent between the two firms. When the competitive price is above $\mu$, such a consumer has a positive match value, and thus a firm always benefits from advertising by only increasing the consumer's valuation for its own product. When the competitive price is below $\mu$, there is a probability that such a marginal consumer has a negative match value, and advertising could reduce demand. However, as long as the competitive price is not too low, this probability is low, and competing firms advertise more under no spillover compared to strong spillover.

To see this, starting from a position where both firms advertise the horizontal attribute with the same intensity $s$, define $v_s$ as

$$\frac{\partial D_i}{\partial s_i}|_{p=v_s} = \int_{v_s}^{\infty} F(v;\underline{\alpha}+s)\frac{\partial f(v;\underline{\alpha}+s)}{\partial \alpha}dv = 0, \tag{5}$$

we can show that

**Lemma 2.** *There exists a unique $v_s < \mu$ such that Eq. (5) holds. Furthermore, $\partial D_i/\partial s_i < 0$ if $p < v_s$ and $\partial D_i/\partial s_i > 0$ if $p > v_s$.*

This follows exactly from the above discussion that, firms are now able to use advertising to affect competition between firms, but not just against the outside option. This generates incentives for firms to advertise, even if the marginal consumer between buying and not buying has a negative match value, i.e., when $v_s < p < \mu$. Let $v_0$ be the solution to Eq. (5) at $s = 0$, i.e., the rotation point of $D_i$ when neither firm advertises, we have:

**Lemma 3.** *With no attention spillover ($\omega = 0$), firms advertise the horizontal attribute if*

$$v_0 < \frac{1}{2}\frac{1-F^2(v_0,\underline{\alpha})}{F(v_0,\underline{\alpha})f(v_0,\underline{\alpha})+\int_{v_0}^{\infty} f^2(v,\underline{\alpha})dv}. \tag{6}$$

Condition (6) is equivalent to say that the competitive price $p_0$ is higher than $v_0$ when neither firm advertises. Since $v_0 < \mu$, Lemma 3 implies that firms are more likely to advertise the horizontal attribute when there is no spillover compared to strong spillover. Condition (6) is satisfied if $v_0$ is small, which occurs when the match value is sufficiently dispersed.[12] The following examples further illustrate this result.

*Example: The Laplace Distribution* Suppose $h(v^h)$ follows a Laplace distribution with mean zero and scale parameter $b$, then $v$ follows a Laplace distribution with mean $\mu$ and scale parameter $\underline{\alpha}b$, that is

$$f(v;\alpha) = \begin{cases} \frac{1}{2\alpha b}e^{\frac{v-\mu}{\alpha b}} & \text{if } v \le \mu, \\ \frac{1}{2\alpha b}e^{-\frac{v-\mu}{\alpha b}} & \text{if } v \ge \mu, \end{cases} \quad \text{and}$$

$$F(v;\alpha) = \begin{cases} \frac{1}{2}e^{\frac{v-\mu}{\alpha b}} & \text{if } v \le \mu, \\ 1-\frac{1}{2}e^{-\frac{v-\mu}{\alpha b}} & \text{if } v \ge \mu. \end{cases}$$

The rotation point $v_0$ is determined by

$$\int_{v_0}^{\infty} F(v;\underline{\alpha})\frac{\partial f(v;\underline{\alpha})}{\partial \alpha}dv = 0.$$

After some simplification, this reduces to

$$v_0 = \mu - \frac{\underline{\alpha}b}{2}.$$

The price $p_0$ is determined by

$$L(p) = \frac{1}{2}[1-F^2(p;\underline{\alpha})] - p[F(p;\underline{\alpha})f(p;\underline{\alpha}) + \int_p^{\infty} f^2(v,\underline{\alpha})dv] = 0.$$

Condition (6) is satisfied if $L(\mu - \frac{\underline{\alpha}b}{2}) > 0$, which simplifies to

$$\frac{\underline{\alpha}b}{\mu} > constant \approx 0.49.$$

This condition is clearly satisfied when the mean, $\mu$, is small or when the dispersion, $\underline{\alpha}b$, is large.

*Example: The Uniform Distribution* Let $H(v_h)$ be the uniform distribution on the interval $[-b, b]$, so $F(v;\alpha) = \frac{v+\underline{\alpha}b-\mu}{2\underline{\alpha}b}$, which is the uniform distribution on the interval $[\mu - \underline{\alpha}b, \mu + \underline{\alpha}b]$. As noted, $F(v;\alpha)$ is not continuously differentiable in this case, as the support of $v$ changes when $\alpha$ changes. For a given price $p$ and $\alpha_i = \underline{\alpha} + s_i > \alpha_j = \underline{\alpha}$, we have

$$D_i(p;\alpha_i) = \int_p^{\mu+\underline{\alpha}b} \frac{1}{2\alpha_i b}\frac{v+\underline{\alpha}b-\mu}{2\underline{\alpha}b}dv + \frac{\alpha_i-\underline{\alpha}}{2\alpha_i}.$$

Then, it is straightforward to show that

$$\lim_{s_i\to 0}\frac{\partial D_i(p;\alpha_i)}{\partial s_i} = \lim_{\alpha_i\to\underline{\alpha}}\frac{\partial D_i(p;\alpha_i)}{\partial \alpha_i} = \frac{(p+\underline{\alpha}b-\mu)^2}{8b^2\underline{\alpha}^3} > 0.$$

Therefore, with uniform distribution, firms always advertise positively when there is no spillover. (In other words, we have $v_0 = -\infty$ and thus always $p_0 > v_0$).

#### 3.2.3. The general case

We are now ready to characterize the equilibrium for the full model:

**Proposition 2.** *In the symmetric equilibrium: No firm advertises the horizontal attribute if*

$$v_0 \ge \frac{1}{2}\frac{1-F^2(v_0,\underline{\alpha})}{F(v_0,\underline{\alpha})f(v_0,\underline{\alpha})+\int_{v_0}^{\infty} f^2(v,\underline{\alpha})dv};$$

*if*

$$v_0 < \frac{1}{2}\frac{1-F^2(v_0,\underline{\alpha})}{F(v_0,\underline{\alpha})f(v_0,\underline{\alpha})+\int_{v_0}^{\infty} f^2(v,\underline{\alpha})dv} \quad \textit{and}$$

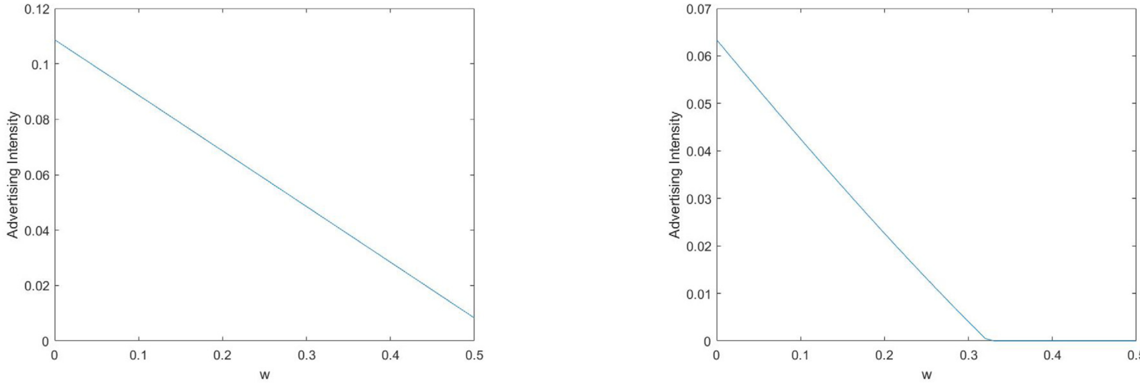
$$\mu[f(\mu;\underline{\alpha}) + 2\int_{\mu}^{\infty} f^2(v;\underline{\alpha})dv] \ge \frac{3}{4},$$

*there exists a $\hat{\omega} \in (0, 1/2]$ such that firms only advertise positively the horizontal attribute when $\omega < \hat{\omega}$; if*

$$\mu[f(\mu;\underline{\alpha}) + 2\int_{\mu}^{\infty} f^2(v;\underline{\alpha})dv] < \frac{3}{4},$$

*firms always advertise positively the horizontal attribute.*

[12] If $D_i(p_i, p_j; \alpha, \alpha)$ is log-supermodular in $p_i$ and $p_j$ and log-supermodular in $p_i$ and $\alpha$, $p_0$ is increasing in $\alpha$, and hence for large enough $\alpha$ we must have Condition (6) satisfied. In fact, the Laplace distribution satisfies both log-supermodularity. For more on the comparative statics with log-supermodular functions, readers can refer to Athey (2002).




(a) $\underline{\alpha} = 0.8$ (b) $\underline{\alpha} = 0.4$

**Fig. 1.** Advertising intensity.

The Proposition follows straightforwardly from Lemmas 1 and 3. If the quality is low and/or the dispersion of the match value is high, the competitive price is high, which means the marginal consumer has a positive match value, and firms always advertise the horizontal attribute. If the quality is high and/or the dispersion of the match value is low, the competitive price is low, meaning that the probability of a marginal consumer having a negative match value is high, and firms do not advertise the horizontal attribute at all. In the intermediate range, a threshold equilibrium exists where firms advertise the horizontal attribute only if the spillover is weak enough for the firm to attract marginal consumers between the two firms. Furthermore, in this range, when the spillover gets weaker, it becomes easier for a firm to attract such a marginal consumer. This strengthens its incentives to advertise, as shown by the following:

**Proposition 3.** *When firms advertise positively, the equilibrium intensity of horizontal advertising is decreasing in $\omega$.*

Propositions 2 and 3 demonstrate that competition can reduce horizontal advertising in the market when $p_0 < \mu < p_m$: a monopolist prices high and always advertise the horizontal attribute, but competing firms price low and do not advertise when the spillover is strong. For instance, this could occur with a regulation imposing a common format of information display. Propositions 2 and 3 also imply that in a threshold equilibrium, firms may fall into a prisoner's dilemma. This occurs when attention spillover gets weaker and firms start to advertise the horizontal attribute. However, as long as the equilibrium price remains below $\mu$, firms would mutually prefer less horizontal advertising. Thus, firms may have incentives to increase the level of spillover by, for instance, engaging in generic advertising rather than promoting only one's own brand.

*3.2.4. Example: The uniform distribution*

We provide further analysis of the uniform distribution example to illustrate some of our results. We start with the equilibrium price. For given $s_i = s_j = s$ and $p_j = p$, if firm $i$ charges $p_i > p$, its demand is given by

$$D_i(p_i, p; s, s) = \int_{p_i}^{\mu+(\underline{\alpha}+s)b} \frac{v - p_i + p + (\underline{\alpha}+s)b - \mu}{2(\underline{\alpha}+s)b} \times \frac{1}{2(\underline{\alpha}+s)b} dv;$$

if firm $i$ charges $p_i < p$, its demand is given by

$$D_i(p_i, p; s, s) = \int_{p_i}^{\mu+(\underline{\alpha}+s)b-p+p_i} \frac{v - p_i + p + (\underline{\alpha}+s)b - \mu}{2(\underline{\alpha}+s)b} \times \frac{1}{2(\underline{\alpha}+s)b} dv + \frac{p - p_i}{2(\underline{\alpha}+s)b}.$$

We have

$$\lim_{p_i \to p^-} \frac{\partial D_i}{\partial p_i} = \lim_{p_i \to p^+} \frac{\partial D_i}{\partial p_i} = -\frac{1}{2(\underline{\alpha}+s)b}.$$

Thus, the competitive price $p$ satisfies

$$D(p, p; s, s) - p\frac{1}{2(\underline{\alpha}+s)b} = 0,$$

which gives us

$$p = \mu - 3(\underline{\alpha}+s)b + 2\sqrt{3((\underline{\alpha}+s)b)^2 - \mu(\underline{\alpha}+s)b}.$$

To ensure that the competitive price is interior, i.e. $p > \mu - (\underline{\alpha} + s)b$, for any $s \geq 0$, we need

$$\underline{\alpha}b > 0.5\mu,$$

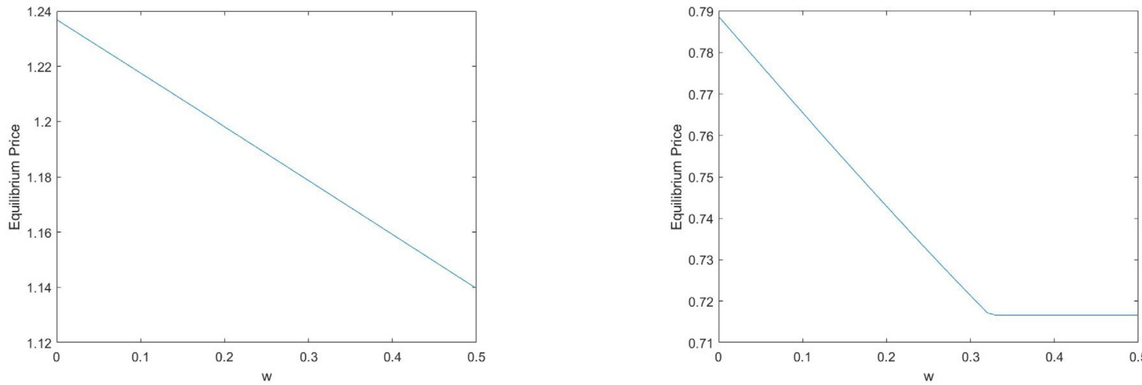
which we assume for the following analysis of this section. Furthermore, if $\underline{\alpha}b > 4\mu/3$, we have $p > \mu$ when $s = 0$.

Regarding the incentive to advertise the horizontal attribute, as discussed in Section 3.2.2, with uniform distribution, firms always advertise positively when there is no attention spillover. Therefore, we have two types of equilibrium:

1. A threshold equilibrium where firms advertise positively the horizontal attribute when attention spillover is weak.
2. An advertising equilibrium where firms always advertise the horizontal attribute.

A threshold equilibrium exists when the dispersion of the match value is relatively low such that the competitive price when no firm advertises is below $\mu$, which arises when $\underline{\alpha}b < 4\mu/3$, and an advertising equilibrium exists when $\underline{\alpha}b > 4\mu/3$. This is illustrated in Fig. 1.[13] On the left panel, we have $\underline{\alpha} = 0.8$ and firms always advertise positively; on the right panel, we have $\underline{\alpha} = 0.4$ and firms only advertise positively if $w < \hat{w} \approx 0.323$. Fig. 1 also demonstrates that the advertising intensity is decreasing with the level of spillover in both types of equilibrium. Furthermore, notice that from Proposition 1, a monopolist would

[13] We set $\mu = 1$, $b = 2$ and $C(s_i) = s_i^2$.




(a) $\underline{\alpha} = 0.8$ (b) $\underline{\alpha} = 0.4$

**Fig. 2.** Equilibrium price.

not advertise the horizontal attribute when $\underline{\alpha} = 0.4$ as $\underline{\alpha}b < \mu$. However, when the spillover is weak enough, competing firm do advertise.

As the spillover gets stronger, firms advertise less and the consumer's valuation becomes less dispersed. This tends to lower the equilibrium price as shown in Fig. 2.

This means that the consumer may be better off when the spillover gets stronger, even if this means that the consumer's purchase decision is more biased due to paying less attention. As well known in behavioural welfare economics, we can measure consumer surplus on different grounds. One way is to measure with the subjective value, that is

$$CS_{sub} = \max\{\mu + \alpha_i v_i^h - p, \mu + \alpha_j v_j^h - p, 0\},$$

where the consumer purchases the product with the higher subjective value and obtains a utility equal to that. Alternatively, we can measure with the experience value, whereas the consumer's purchase decision is still determined by the subjective value. That is,

$$CS_{ex} = \mu + v_i^h - p,$$

when product $i$ maximizes the subjective value. In this case, the consumer experiences the full value of the horizontal attribute in contrast to partial subjective value. On the equilibrium path, the product that has a higher subjective value also has a higher experience value, but the consumer may purchase too little or too much compared to the purchase decision based on the experience value.

As shown in Figs. 3 and 4, when the spillover gets stronger, consumer welfare is higher when measured by the experience value but could be lower when measured by the subjective value. The reason is that, the experience value is not affected by attention paid to the horizontal attribute, and the consumer benefits from lower prices under stronger spillover. On the contrary, the subjective value becomes less dispersed when the consumer pays less attention as the spillover becomes stronger. Thus, even if firms are charging lower prices, the consumer could lose from obtaining lower subjective values.

## 4. More than two firms

We briefly consider in the section a generalization of the analysis to more than two firms. We maintain the modelling framework and consider $N$ firms, each choosing its advertising intensity and price. To incorporate spillover, we assume that the attention paid to the horizontal attribute of firm $i$, $\alpha_i$, is determined by

$$\alpha_i = \underline{\alpha} + (1-\omega)s_i + \frac{\omega}{N-1}\sum_{j\neq i} s_j,$$

for $\omega \in [0, \frac{N-1}{N}]$, i.e., there is no spillover if $\omega = 0$ and strong spillover if $\omega = \frac{N-1}{N}$. The demand for firm $i$ is given by

$$D_i(p_i, p_{-i}; s_i, s_{-i}) = \int_{p_i}^{\infty} \prod_{j\neq i} F(v_i - p_i + p_j; \alpha_j) dF(v_i; \alpha_i), \text{ for } j \neq i,$$

where $p_{-i}$ and $s_{-i}$ are the profiles of prices and advertising intensities of firms other than firm $i$. The result of Lemma 1 under strong spillover can be readily generalized to many firms. To see this, with strong spillover, the demand of firm $i$ at equal prices is given by

$$D_i(s_i, s_{-i}) = \int_p^{\infty} F^{N-1}(v; \alpha) dF(v; \alpha) = \frac{1}{N}(1 - F^N(p, \alpha)),$$

where $\alpha = \underline{\alpha} + \frac{1}{N}\sum_{i=1}^N s_i$ is the average attention paid to the horizontal attribute. Clearly, $F^N(p, \alpha)$ decreases with $\alpha$ when $p > \mu$, and increases with $\alpha$ otherwise. Hence, firms only advertise when the competitive price with no advertising ($\alpha = \underline{\alpha}$) is above $\mu$. Furthermore, since the competitive price decreases with the number of firms,[14] competition reduces horizontal advertising under strong spillover.

The result under weak spillover is slightly different when there are many firms. We consider no spillover ($\omega = 0$), a positive advertising equilibrium exists in this case if

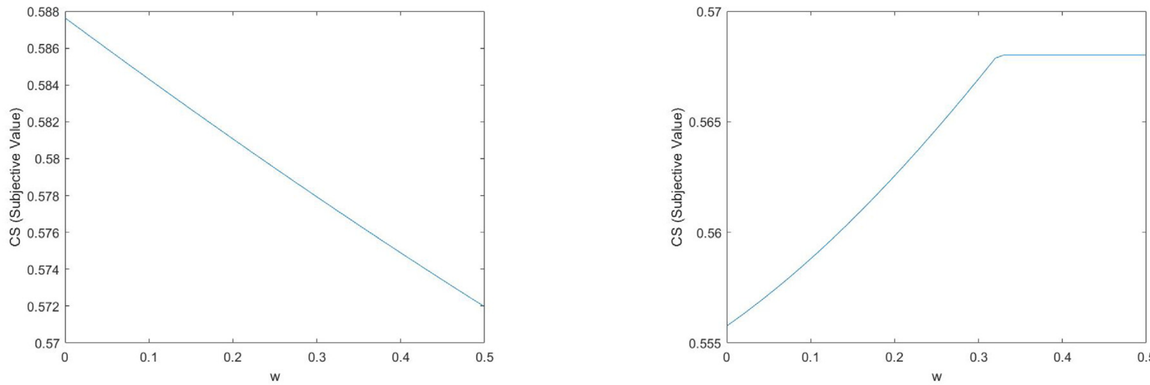
$$\frac{\partial D_i}{\partial s_i} = \int_p^{\infty} F^{N-1}(v; \underline{\alpha}) \frac{\partial f(v; \underline{\alpha})}{\partial \alpha} dv > 0,$$

where $p$ is the competitive price when no firm advertises. When $N = 2$, we have seen that a rotation point exists and the above condition is violated when the competitive price is below the rotation point. However, this is no longer the case when $N > 2$. In fact, we can show that:

**Proposition 4.** *With no spillover, there exists an $N^*$ such that $\partial D_i / \partial s_i > 0$ for any $p$ if $N > N^*$.*

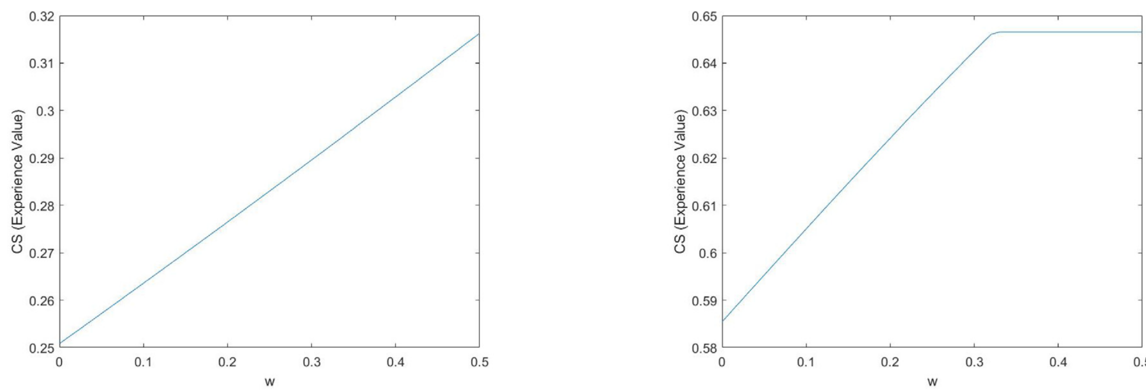
Mathematically, as $N$ increases, the integrand in $\partial D_i/\partial s_i$ shifts relatively more weight to the positive part of $\partial f / \partial \alpha$, and $\partial D_i/\partial s_i$

14 This is a known result for log-concave density functions and shown, for instance, by Zhou (2017) in a different context.




(a) $\underline{\alpha} = 0.8$ (b) $\underline{\alpha} = 0.4$

**Fig. 3.** Consumer surplus: Subjective value.




(a) $\underline{\alpha} = 0.8$ (b) $\underline{\alpha} = 0.4$

**Fig. 4.** Consumer surplus: Experience value.

eventually becomes positive for any $p$.[15] Intuitively, as the number of firms increases, it becomes relatively more likely that a marginal consumer between firm $i$ and its competitors has a positive match value, hence, advertising is more likely to benefit the firm. In this sense, competition increases horizontal advertising under no spillover.

For intermediate levels of spillover, our insights from the duopoly analysis still hold in the following sense. Let $v_0^N(\omega)$ be the highest value such that $\partial D_i(\omega)/\partial s_i = 0$ at $p = v_0^N(\omega)$,[16] and let $v_0^N(\omega) = -\infty$ if a solution to $\partial D_i(\omega)/\partial s_i = 0$ does not exist. We can show that, as $\omega$ decreases from $(N-1)/N$ to 0, $v_0^N(\omega)$ decreases from $\mu$ to $v_0^N(0) < \mu$. So, if the competitive price without advertising is higher than $\mu$, firms always advertise positively; whereas if it locates between $v_0^N(0)$ and $\mu$, a threshold equilibrium arises where firms only advertise positively if the spillover is weak.

## 5. Concluding remarks

We conclude with some avenues for future research. Our analysis fits the situation when firms choose their prices and advertising strategies simultaneously, for instance, when firms decide to advertise their prices. When firms do not advertise or cannot commit to their prices, we need to consider the additional strategic effect of advertising on price competition. We also focus on the role of advertising in attracting and manipulating a consumer's attention and thus omit the role of advertising in providing information about the availability of a product. It would be interesting to study the incentives to advertise horizontal attributes when firms compete for both the attention towards their products and the attention towards specific attributes.

## Acknowledgements

I thank the co-editor and two anonymous referees for invaluable comments that have greatly improved the paper. I am also grateful to Patrick Rey, Wilfried Sand-Zantman, Zhijun Chen, Régis Renault, Mike Riordan for insightful suggestions and participants from various conferences and seminars for useful comments. All remaining errors are mine.

## Appendix A

### A.1. Proof of Proposition 1

The profit of the monopolist with an attention level $\underline{\alpha} + s_m$ and price $p_m$ is given by

$$\pi_m(p_m; s_m) = p_m[1 - F(p_m; \underline{\alpha} + s_m)] - C(s_m).$$

[15] For instance, this is the case when $N = 3$ in the example of Laplace distribution with $\underline{\alpha} = 0.5$, $\mu = 2$ and $b = 2$.

[16] There could exist multiple solutions to $\partial D_i(\omega)/\partial s_i = 0$. In such cases, we consider the largest one. So that we have $\partial D_i(\omega)/\partial s_i > 0$ for any $p > v_0^N(\omega)$.

Positive advertising occurs if $\partial\pi_m/\partial s_m > 0$ at $s_m = 0$. This is satisfied when $p_m(0) > \mu$. Notice that the first order condition with respect to $p_m$ at $s_m = 0$ is

$$\frac{\partial\pi_m}{\partial p_m} = 1 - F(p_m; \underline{\alpha}) - p_m f(p_m; \underline{\alpha}).$$

The optimal price is higher than $\mu$ if $\partial\pi_m/\partial p_m$ is positive at $p_m = \mu$, which is equivalent to

$$\mu f(\mu; \underline{\alpha}) < 1/2,$$

by using the fact that $F(\mu; \underline{\alpha}) = 1/2$.

### A.2. Proof of Lemma 1

Given $p_j$ and $s_j$, the profit of firm $i$ is

$$\pi_i(p_i, p_j; s_i, s_j) = p_i \int_{p_i}^{\infty} F(v_i - p_i + p_j; \underline{\alpha} + \frac{s_i + s_j}{2}) \times dF(v_i; \underline{\alpha} + \frac{s_i + s_j}{2}) - C(s_i).$$

The first order conditions are

$$\begin{aligned} \tfrac{\partial\pi_i}{\partial p_i} = 0 \quad &= \textstyle\int_{p_i}^{\infty} F(v_i - p_i + p_j; \alpha) dF(v_i; \alpha) \\ &\quad - p_i[F(p_j, \alpha) f(p_i, \alpha) \\ &\quad + \textstyle\int_{p_i}^{\infty} f(v_i - p_i + p_j, \alpha) f(v_i, \alpha) dv_i]; \\ \tfrac{\partial\pi_i}{\partial s_i} = 0 \quad &= p_i \textstyle\int_{p_i}^{\infty} [\frac{1}{2} \frac{\partial F(v_i - p_i + p_j, \alpha)}{\partial \alpha_j} f(v_i, \alpha) \\ &\quad + F(v_i - p_i + p_j, \alpha) \frac{1}{2} \frac{\partial f(v_i, \alpha)}{\partial s_i}] dv_i - C'(s_i). \end{aligned}$$

In a symmetric equilibrium $(p^*, s^*)$, the FOC with respect to $s_i$ simplifies to

$$\frac{1}{2} p^* \int_{p^*}^{\infty} [\frac{\partial F(v_i; \alpha)}{\partial\alpha} f(v_i; \alpha) + F(v_i; \alpha) \frac{\partial f(v_i; \alpha)}{\partial\alpha}] dv_i = C'(s^*).$$

A sufficient and necessary condition for $s^* > 0$ is then

$$\int_{p_0}^{\infty} [\frac{\partial F(v_i; \underline{\alpha})}{\partial\alpha} f(v_i; \underline{\alpha}) + F(v_i; \underline{\alpha}) \frac{\partial f(v_i; \underline{\alpha})}{\partial\alpha}] dv_i > 0,$$

where $p_0$ is the equilibrium price corresponding to $s_1 = s_2 = 0$. This is equivalent to say that $p_0$ is above the rotation point of $D_i$, i.e. $p_0 > \mu$, which is satisfied if $\frac{\partial\pi_i}{\partial p_i}|_{p_i = p_j = \mu} > 0$, that is,

$$\frac{1}{2}[1 - F^2(\mu; \underline{\alpha})] - \mu[F(\mu; \underline{\alpha}) f(\mu; \underline{\alpha}) + \int_{\mu}^{\infty} f^2(v, \underline{\alpha}) dv] > 0.$$

which simplifies to

$$\mu[f(\mu; \underline{\alpha}) + 2\int_{\mu}^{\infty} f^2(v; \underline{\alpha}) dv] < \frac{3}{4}.$$

For the second part, it suffices to show that competition always reduces equilibrium prices, which is implied by log-concavity of $f(v; \alpha)$. Specifically, the monopoly prices without horizontal advertising is determined by

$$p^m = \frac{1 - F(p^m; \underline{\alpha})}{f(p^m; \underline{\alpha})}.$$

Under competition, when both firms do not advertise, the first order condition of price (from the proof of Lemma 1) is

$$0 = \int_{p_i}^{\infty} F(v_i - p_i + p_j; \underline{\alpha}) dF(v_i; \underline{\alpha}) - p_i[F(p_j, \underline{\alpha}) f(p_i, \underline{\alpha}) + \int_{p_i}^{\infty} f(v_i - p_i + p_j, \underline{\alpha}) f(v_i, \underline{\alpha}) dv_i].$$

The symmetric equilibrium price then satisfies

$$p^c = \frac{1}{2} \frac{1 - F^2(p^c; \underline{\alpha})}{F(p^c, \underline{\alpha}) f(p^c; \underline{\alpha}) + \int_{p^c}^{\infty} f^2(v_i; \underline{\alpha}) dv_i}.$$

A sufficient condition for $p^c < p^m$ is that for any $p$,

$$\frac{1}{2} \frac{1 - F^2(p; \underline{\alpha})}{F(p, \underline{\alpha}) f(p; \underline{\alpha}) + \int_{p}^{\infty} f^2(v_i; \underline{\alpha}) dv_i} < \frac{1 - F(p; \underline{\alpha})}{f(p; \underline{\alpha})},$$

which simplifies to

$$\underbrace{\int_{p}^{\infty} f^2(v_i; \underline{\alpha}) dv_i}_{A} - \underbrace{\int_{p}^{\infty} f(v_i)[f(p) - f(v_i)] dv_i}_{B} > 0.$$

First, notice that as $p \to \infty$, $A - B \to 0$. Furthermore, we have

$$\frac{\partial A}{\partial p} = -f^2(p; \underline{\alpha}),$$

and

$$\frac{\partial B}{\partial p} = \int_{p}^{\infty} f(v_i; \underline{\alpha}) f'(p; \underline{\alpha}) dv_i = f'(p; \underline{\alpha})[1 - F(p; \underline{\alpha})].$$

Thus,

$$\frac{\partial(A - B)}{\partial p} = -[f^2(p; \underline{\alpha}) + f'(p; \underline{\alpha})[1 - F(p; \underline{\alpha})]] < 0.$$

The last inequality follows because, when $f(v; \alpha)$ is log-concave, the hazard rate $\frac{f}{1-F}(v; \alpha)$ is increasing in $v$, i.e.

$$f'(v; \alpha)[1 - F(v; \alpha)] + f^2(v; \alpha) > 0.$$

Therefore, we must have $A - B > 0$ for any $p$.

### A.3. Proof of Lemma 2

We can rewrite the demand of firm $i$ as

$$D_i = \int_{p}^{\infty} F(v; \underline{\alpha} + s_j) f(v; \underline{\alpha} + s_i) dv = 1 - F(p; \underline{\alpha} + s_i) F(p; \underline{\alpha} + s_j) - \int_{p}^{\infty} F(v; \underline{\alpha} + s_i) dF(v; \underline{\alpha} + s_j).$$

Thus,

$$\begin{aligned} \frac{\partial D_i}{\partial s_i} &= \int_{p}^{\infty} F(v; \underline{\alpha} + s_j) \frac{\partial f(v; \underline{\alpha} + s_i)}{\partial\alpha} dv \\ &= -\frac{\partial F(p; \underline{\alpha} + s_i)}{\partial\alpha} F(p; \underline{\alpha} + s_j) \\ &\quad - \int_{p}^{\infty} \frac{\partial F(v; \underline{\alpha} + s_i)}{\partial\alpha} dF(v; \underline{\alpha} + s_j). \end{aligned}$$

Since $\frac{\partial F(v; \alpha)}{\partial\alpha} \le 0$ for all $v \ge \mu$, it is easy to see that $\frac{\partial D_i}{\partial s_i} > 0$ for all $p \ge \mu$.

When $p < \mu$, we have

$$\frac{\partial^2 D_i}{\partial s_i \partial p} = -\frac{\partial f(p; \underline{\alpha} + s_i)}{\partial s_i} F(p; \underline{\alpha} + s_j).$$

When $f(v; \underline{\alpha} + s_i)$ is log-concave, there exists a $\tilde{v} < \mu$ such that $\frac{\partial f(v; \underline{\alpha} + s_i)}{\partial s_i} < 0$ when $\tilde{v} < v < \mu$ and $\frac{\partial f(v; \underline{\alpha} + s_i)}{\partial s_i} > 0$ when $v < \tilde{v}$. To see this, for any $\alpha$, we have $f(v; \alpha) = \frac{1}{\alpha} h(\frac{v - \mu}{\alpha})$. Due to symmetry, we consider $v \in (-\infty, \mu]$ and we have

$$\frac{\partial f(v; \alpha)}{\partial\alpha} = -\frac{1}{\alpha^2} h(\frac{v - \mu}{\alpha}) - \frac{v - \mu}{\alpha^3} h'(\frac{v - \mu}{\alpha}),$$

which is positive if

$$\frac{-\frac{v - \mu}{\alpha} h'(\frac{v - \mu}{\alpha})}{h(\frac{v - \mu}{\alpha})} > 1.$$

Notice that, for $v < \mu$, $-\frac{v - \mu}{\alpha}$ is positive and decreasing; $\frac{h'}{h}(\frac{v - \mu}{\alpha})$ is also positive and decreasing (due to log-concavity). Thus, the left-hand side of the above inequality is decreasing in $v$ and equal to 0 for $v = \mu$. Therefore, $\partial f(v;\alpha)/\partial\alpha$ is either always negative, or positive for low value of $v$ and negative for high value of $v$. However, since $\int_{-\infty}^{\mu} f(v;\alpha)dv = 1/2$, and thus

$$\int_{-\infty}^{\mu} \frac{\partial f(v;\alpha)}{\partial\alpha} = 0,$$

there must exist a $\tilde{v} < \mu$ such that $\frac{\partial f(v;\alpha)}{\partial\alpha} > (<)0$ if $v < (>)\tilde{v}$.

Therefore, for $p \in (-\infty, \mu)$, $\frac{\partial D_i}{\partial s_i}$ first decreases and then increases in $p$. Moreover, we have $\lim_{p\to-\infty} \frac{\partial D_i}{\partial s_i} = 0$ and $\frac{\partial D_i}{\partial s_i}|_{p=\mu} > 0$, hence, there exists a $v_s \in (-\infty, \mu)$ such that $\frac{\partial D_i}{\partial s_i}|_{p=v_s} = 0$, and $\frac{\partial D_i}{\partial s_i} > 0$ when $p > v_s$ and $\frac{\partial D_i}{\partial s_i} < 0$ when $p < v_s$.

### A.4. Proof of Proposition 2

Whether a positive advertising equilibrium exists depends on whether the price $p_0$ is above or below the rotation point, when neither firm advertises the horizontal attribute. We prove the result by showing that the rotation point is increasing in the level of spillover $\omega$. Recall that

$$D_i(\omega) = \int_p^{\infty} F(v;\underline{\alpha} + (1-\omega)s_j + \omega s_i)dF(v;\underline{\alpha} + (1-\omega)s_i + \omega s_j).$$

Thus,

$$\begin{aligned}\frac{\partial D_i}{\partial s_i}(\omega) = \int_p^{\infty} [&\omega \frac{\partial F(v;\underline{\alpha} + (1-\omega)s_j + \omega s_i)}{\partial\alpha} \\ &\times f(v;\underline{\alpha} + (1-\omega)s_i + \omega s_j) + (1-\omega) \\ &\times F(v;\underline{\alpha} + (1-\omega)s_j + \omega s_i) \\ &\times \frac{\partial f(v;\underline{\alpha} + (1-\omega)s_i + \omega s_j)}{\partial\alpha}]dv,\end{aligned}$$

which, when $s_i = s_j = 0$, simplifies to

$$\frac{\partial D_i}{\partial s_i}(\omega) = \int_p^{\infty} [\omega \frac{\partial F(v;\underline{\alpha})}{\partial\alpha} f(v;\underline{\alpha}) + (1-\omega)F(v;\underline{\alpha})\frac{\partial f(v;\underline{\alpha})}{\partial\alpha}]dv. \quad (7)$$

We first show in a few steps that, for each $\omega \in [0, 1/2)$, there exists a $v(\omega) < \mu$ such that

$$\frac{\partial D_i(\omega)}{\partial s_i} \lesseqgtr 0 \Leftrightarrow p \lesseqgtr v(\omega).$$

*Step 1:* For all $p \geq \mu$, $\frac{\partial D_i(\omega)}{\partial s_i} > 0$.

We have $\int_p^{\infty} \frac{\partial F(v;\underline{\alpha})}{\partial\alpha} f(v;\underline{\alpha})dv < 0$ for all $p \geq \mu$ as $\partial F(v;\alpha)/\partial\alpha < 0$; and $\int_p^{\infty} F(v;\underline{\alpha})\frac{\partial f(v;\underline{\alpha})}{\partial\alpha}dv > 0$ for $p \geq \mu$ from the proof of Lemma 2. Thus,

$$\begin{aligned}\frac{\partial D_i(\omega)}{\partial s_i} &\geq \tfrac{1}{2}\int_p^{\infty}[\tfrac{\partial F(v;\underline{\alpha})}{\partial\alpha} f(v;\underline{\alpha}) + F(v;\underline{\alpha})\tfrac{\partial f(v;\underline{\alpha})}{\partial\alpha}]dv \\ &= \tfrac{\partial}{\partial\alpha}\tfrac{1}{2}\int_p^{\infty} F(v;\underline{\alpha})f(v;\underline{\alpha})dv \\ &= \tfrac{\partial}{\partial\alpha}\tfrac{1}{4}[1 - F^2(p;\underline{\alpha})], \\ &> 0,\end{aligned}$$

where the first inequality follows as $\partial D_i(\omega)/\partial s_i$ puts more weight on the positive part and less weight on the negative part for $\omega < 1/2$, and the last inequality follows as $F(v;\alpha)$ is rotation ordered with rotation point $\mu$.

*Step 2:* There is a unique $v^* \in (-\infty, \mu)$ such that

$$K(v^*) = \omega \frac{\partial F(v^*;\underline{\alpha})}{\partial\alpha} f(v^*;\underline{\alpha}) + (1-\omega)F(v^*;\underline{\alpha})\frac{\partial f(v^*;\underline{\alpha})}{\partial\alpha} = 0.$$

First, notice that $\frac{\partial F(v;\underline{\alpha})}{\partial\alpha} > 0$ for all $v < \mu$. The proof of Lemma 2 shows that there exists a $\tilde{v} < \mu$ such that $\frac{\partial f(v;\underline{\alpha})}{\partial\alpha} > 0$ for $v < \tilde{v}$. Thus, $K(v) > 0$ for $v < \tilde{v}$. Now we focus on $v \in [\tilde{v}, \mu)$. Define a function $G(v)$, for $v \in [\tilde{v}, \mu)$ as

$$G(v) = \frac{-F(v;\underline{\alpha})\frac{\partial f(v;\underline{\alpha})}{\partial\alpha}}{\frac{\partial F(v;\underline{\alpha})}{\partial\alpha} f(v;\underline{\alpha}) - F(v;\underline{\alpha})\frac{\partial f(v;\underline{\alpha})}{\partial\alpha}} = \frac{1}{1 - \frac{\frac{\partial F(v;\underline{\alpha})}{\partial\alpha} f(v;\underline{\alpha})}{F(v;\underline{\alpha})\frac{\partial f(v;\underline{\alpha})}{\partial\alpha}}}.$$

By assumption, $F(v;\alpha)$ is log-concave, and thus $\frac{f(v;\alpha)}{F(v;\alpha)}$ is positive and decreasing in $v$. Furthermore, we have

$$\begin{aligned}-\frac{\partial F(v;\alpha)/\partial\alpha}{\partial f(v;\alpha)/\partial\alpha} &= -\frac{\frac{v-\mu}{\alpha^2}h(\frac{v-\mu}{\alpha})}{\frac{1}{\alpha^2}h(\frac{v-\mu}{\alpha}) + \frac{v-\mu}{\alpha^3}h'(\frac{v-\mu}{\alpha})} \\ &= -\frac{1}{\frac{1}{v-\mu} + \frac{1}{\alpha}\frac{h'(\frac{v-\mu}{\alpha})}{h(\frac{v-\mu}{\alpha})}},\end{aligned}$$

which is also positive and decreasing in $v$ due to (1) $\frac{1}{v-\mu}$ is decreasing in $v$ and (2) $h'/h$ is decreasing in $v$ as $h$ is log-concave.

Thus, $G(v)$ is increasing in $v$ with $G(\mu) = 1$ and $G(\tilde{v}) = 0$. Hence, there exists a unique $v^* \in [\tilde{v}, \mu)$ such that $G(v^*) = \omega$. This means that $K(v) > 0$ for $v < v^*$ and $K(v) < 0$ for $v^* < v < \mu$.

*Step 3:* There exists a rotation point $v(\omega)$.

Now it is straightforward to see that, for $p \in (-\infty, \mu)$, $\frac{\partial D_i(\omega)}{\partial s_i}$ first decreases and then increases with $p$, with $\frac{\partial D_i(\omega)}{\partial s_i}|_{p\to-\infty} = 0$ and $\frac{\partial D_i(\omega)}{\partial s_i}|_{p=\mu} > 0$. Thus, there exists a unique $v(\omega)$ such that $\frac{\partial D_i(\omega)}{\partial s_i}|_{p=v(\omega)} = 0$, and

$$\frac{\partial D_i(\omega)}{\partial s_i} \lesseqgtr 0 \Leftrightarrow p \lesseqgtr v(\omega).$$

Now we are ready to show that $v(\omega)$ is increasing in $\omega$. This is from the observation that, for any given $p$,

$$\frac{\partial \frac{\partial D_i}{\partial s_i}}{\partial\omega} = \int_p^{\infty} \frac{\partial F(v;\underline{\alpha})}{\partial\alpha} f(v;\underline{\alpha})dv - \int_p^{\infty} \frac{\partial f(v;\underline{\alpha})}{\partial\alpha} F(v;\underline{\alpha})dv < 0.$$

The last inequality follows from (1) the first integration is negative for any $p$ (due to symmetry of $f$) and (2) the second integration is positive at $v(\omega)$ (as Eq. (7) is equal to zero at $v(\omega)$). Thus, if $\frac{\partial D_i(\omega_1)}{\partial s_i} \leq 0$ for all $p \leq v(\omega_1)$, then $\frac{\partial D_i(\omega_2)}{\partial s_i} < 0$ for all $p \leq v(\omega_1)$ if $\omega_2 > \omega_1$. Thus, we must have $v(\omega_2) > v(\omega_1)$.

Therefore, we have $dv(\omega)/d\omega > 0$, with $v(1/2) = \mu$ and $v(0) = v_0$.

Thus, if $p_0 \leq v_0$, no firm advertises the horizontal attribute for any level of spillover, which occurs when

$$v_0 \geq \frac{1}{2}\frac{1 - F^2(v_0,\underline{\alpha})}{F(v_0,\underline{\alpha})f(v_0,\underline{\alpha}) + \int_{v_0}^{\infty} f^2(v,\underline{\alpha})dv};$$

If $v_0 < p_0 \leq \mu$, which occurs when

$$v_0 < \frac{1}{2}\frac{1 - F^2(v_0,\underline{\alpha})}{F(v_0,\underline{\alpha})f(v_0,\underline{\alpha}) + \int_{v_0}^{\infty} f^2(v,\underline{\alpha})dv} \text{ and}$$

$$\mu[f(\mu;\underline{\alpha}) + 2\int_{\mu}^{\infty} f^2(v;\underline{\alpha})dv] \geq \frac{3}{4},$$

then firms advertise the horizontal attribute for $\omega$ low enough such that $v(\omega) < p_0$; If $p_0 > \mu$, firms always advertise the horizontal attribute and this occurs when

$$\mu[f(\mu;\underline{\alpha}) + 2\int_{\mu}^{\infty} f^2(v;\underline{\alpha})dv] < \frac{3}{4}.$$

### A.5. Proof of Proposition 3

Consider a symmetric equilibrium with $p_1 = p_2 = p^*$ and $s_1 = s_2 = s^*$, which must satisfy the first order conditions:

$$\begin{aligned} J(p^*, s^*; \omega) = \quad & p^* \int_{p^*}^{\infty} \omega \tfrac{\partial F(v^*;\alpha)}{\partial \alpha} f(v^*; \alpha) \\ & + (1-\omega) F(v^*; \alpha) \tfrac{\partial f(v^*;\alpha)}{\partial \alpha} dv - C'(s^*) = 0, \\ L(p^*, s^*; \omega) = \quad & \int_{p^*}^{\infty} F(v; \alpha) dF(v; \alpha) - p^*[F(p^*; \alpha) f(p^*; \alpha) \\ & + \int_{p^*}^{\infty} f^2(v; \alpha) dv] = 0, \end{aligned}$$

where $\alpha = \underline{\alpha} + (1-\omega)s^* + \omega s^* = \underline{\alpha} + s^*$. Take total differentiation of the two FOCs with respect to $\omega$, we obtain

$$\begin{aligned} \tfrac{\partial J}{\partial s}\tfrac{ds^*}{d\omega} + \tfrac{\partial J}{\partial p}\tfrac{dp^*}{d\omega} &= -\tfrac{\partial J}{\partial \omega}, \\ \tfrac{\partial L}{\partial s}\tfrac{ds^*}{d\omega} + \tfrac{\partial L}{\partial p}\tfrac{dp^*}{d\omega} &= -\tfrac{\partial L}{\partial \omega}. \end{aligned}$$

Thus,

$$\frac{ds^*}{d\omega} = \begin{vmatrix} -\frac{\partial J}{\partial \omega} & \frac{\partial J}{\partial p} \\ -\frac{\partial L}{\partial \omega} & \frac{\partial L}{\partial p} \end{vmatrix} \Bigg/ \begin{vmatrix} \frac{\partial J}{\partial s} & \frac{\partial J}{\partial p} \\ \frac{\partial L}{\partial s} & \frac{\partial L}{\partial p} \end{vmatrix},$$

The denominator is positive when $(s^*, p^*)$ is a maximum. Moreover, we have $\partial L/\partial \omega = 0$ when $s_i = s_j = s^*$.[17] We also have $\partial L/\partial p < 0$ due to concavity of profit in the price. Thus, $ds^*/d\omega$ has the same sign as $\partial J/\partial \omega$ and we have

$$\frac{\partial J}{\partial \omega} = \int_p^{\infty} \frac{\partial F(v;\alpha)}{\partial \alpha} f(v;\alpha) dv - \int_p^{\infty} \frac{\partial f(v;\alpha)}{\partial \alpha} F(v;\alpha) dv,$$

which is negative following similar arguments as in the proof of Proposition 2.

### A.6. More than two firms

We first prove the result that firms always advertise positively under no spillover when competition is sufficiently intense.

**Proposition 4.** With no spillover, there exists an $N^*$ such that $\partial D_i/\partial s_i > 0$ for any $p$ if $N > N^*$.

**Proof.** We have

$$\frac{\partial D_i}{\partial s_i} = \int_p^{\infty} F^{N-1}(v; \underline{\alpha}) \frac{\partial f(v; \underline{\alpha})}{\partial \alpha} dv,$$

when no firm advertises. Similar argument as in the two firms case implies that $\partial D_i/\partial s_i$ is always positive for $p \geq \mu$. So we focus on $p < \mu$. From the proof of Lemma 2, there exists $\tilde{v} < \mu < 2\mu - \tilde{v}$ such that $\partial f/\partial \alpha > 0$ for $v < \tilde{v}$ and $v > 2\mu - \tilde{v}$, and $\partial f/\partial \alpha < 0$ for $\tilde{v} < v < 2\mu - \tilde{v}$. Hence, $\partial D_i/\partial s_i$ achieves a minimum at $v = \tilde{v}$. Then, we have

$$\begin{aligned} \frac{\partial D_i}{\partial s_i}\Big|_{p=\tilde{v}} = F^{N-1}(2\mu - \tilde{v}; \underline{\alpha}) \Bigg[ & \int_{\tilde{v}}^{2\mu-\tilde{v}} \Big(\frac{F(v; \underline{\alpha})}{F(2\mu - \tilde{v}; \underline{\alpha})}\Big)^{N-1} \\ & \times \frac{\partial f(v; \underline{\alpha})}{\partial \alpha} dv \\ & + \int_{2\mu-\tilde{v}}^{\infty} \Big(\frac{F(v; \underline{\alpha})}{F(2\mu - \tilde{v}; \underline{\alpha})}\Big)^{N-1} \frac{\partial f(v; \underline{\alpha})}{\partial \alpha} dv \Bigg]. \end{aligned}$$

We focus on the two terms in the square bracket, which determine the sign of $\partial D_i/\partial s_i$. The first term is negative and increasing, and approaches zero as $N$ increases; the second term is positive and increasing, and approaches infinity as $N$ increases. Hence, there must exist an $N^*$ such that $\partial D_i/\partial s_i > 0$ at $p = \tilde{v}$ if $N > N^*$, which means we must have $\partial D_i/\partial s_i > 0$ for any $p$. □

[17] This is where our formulation on the spillover effect simplifies our analysis. For instance, if we assume $\alpha_i = \underline{\alpha} + s_i + \omega s_j$, then the direct effect of $\omega$ on price is not zero.

This means that, unlike the two firms case, a rotation point may fail to exist for sufficiently weak spillover. Specifically, for any $\omega$, we have

$$\begin{aligned} D_i(\omega) = & \int_p^{\infty} \prod_{j \neq i} F(v; \underline{\alpha} + (1-\omega)s_j + \frac{\omega}{N-1}(s_i + \sum_{k \neq i,j} s_k)) \\ & \times\ dF(v; \underline{\alpha} + (1-\omega)s_i + \frac{\omega}{N-1} \sum_{j \neq i} s_j). \end{aligned}$$

When no firm advertises, we obtain

$$\begin{aligned} \frac{\partial D_i(\omega)}{\partial s_i} = & \int_p^{\infty} \Big[\ \omega F^{N-2}(v; \underline{\alpha}) \frac{\partial F(v; \underline{\alpha})}{\partial \alpha} f(v; \underline{\alpha}) \\ & + (1-\omega) F^{N-1}(v; \underline{\alpha}) \frac{\partial f(v; \underline{\alpha})}{\partial \alpha}\ \Big] dv. \end{aligned}$$

Clearly, from the discussion above, if $\omega$ is sufficiently small, we have $\partial D_i(\omega)/\partial s_i > 0$ for any $p$ if $N$ is large. However, when $\frac{\partial D_i(\omega)}{\partial s_i}|_{p=\tilde{v}} < 0$, there exists a $v_0^N(\omega) \in (\tilde{v}, \mu)$ such that $\frac{\partial D_i(\omega)}{\partial s_i}|_{p=v_0^N(\omega)} = 0$. This is also the highest $v_0^N(\omega)$ that satisfies the equation, due to monotonicity of $\partial D_i/\partial s_i$ on the interval $[\tilde{v}, \mu]$. Furthermore, if we have $\partial D_i/\partial s_i > 0$ for $p > v_0^N(\omega)$ for a given $N$, similar arguments as in the above proof imply that we must have $\partial D_i/\partial s_i > 0$ for $p > v_0^N(\omega)$ for any higher $N$. Thus, $v_0^N(\omega)$ must be decreasing in $N$. When $\partial D_i(\omega)/\partial s_i > 0$ for any $p$, we can interpret it as $v_0^N(\omega) = -\infty$.

On the other hand, we have

$$\begin{aligned} \frac{\partial(\partial D_i(\omega)/\partial s_i)}{\partial \omega} = & \int_p^{\infty} F^{N-2}(v; \underline{\alpha}) \frac{\partial F(v; \underline{\alpha})}{\partial \alpha} f(v; \underline{\alpha}) dv \\ & - \int_p^{\infty} F^{N-1}(v; \underline{\alpha}) \frac{\partial f(v; \underline{\alpha})}{\partial \alpha} dv. \end{aligned}$$

Similar arguments as the proof of Proposition 2 imply that the first integration is always negative, and the second integration is positive at $v_0^N(\omega)$ when it is finite. Hence, the above term is negative, which means that the rotation point must be increasing in $\omega$.

## Appendix B. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jmateco.2022.102660.

## References

Abernethy, A.M., Franke, G.R., 1996. The information content of advertising: A meta-analysis. J. Advert. 25 (2), 1–17.
Aköz, K.K., Arbatli, C.E., Çelik, L., 2020. Manipulation through biased product reviews. J. Ind. Econ. 68 (4), 591–639.
Anderson, S.P., Ciliberto, F., Liaukonyte, J., 2013. Information content of advertising: Empirical evidence from the OTC analgesic industry. Int. J. Ind. Organ. 31 (5), 355–367.
Anderson, S.P., Renault, R., 2006. Advertising content. Amer. Econ. Rev. 96 (1), 93–113.
Anderson, S., Renault, R., 2009. Comparative advertising: Disclosing horizontal match information. Rand J. Econ. 40 (3), 558–581.
Anderson, E., Simester, D., 2013. Advertising in a competitive market: The role of product standards, customer learning, and switching costs. J. Mar. Res. 50 (4), 489–504.
Athey, S., 2002. Monotone comparative statics under uncertainty. Q. J. Econ. 117 (1), 187–223.
Bordalo, P., Gennaioli, N., Shleifer, A., 2013. Salience and consumer choice. J. Polit. Econ. 121 (5), 803–843.
Branco, F., Sun, M., Villas-Boas, J.M., 2016. Too much information? Information provision and search costs. Mark. Sci. 35 (4), 605–618.
Caplin, A., Nalebuff, B., 1991. Aggregation and imperfect competition: On the existence of equilibrium. Econometrica 25–59.
Chakravarti, A., Janiszewski, C., 2004. The influence of generic advertising on brand preferences. J. Consum. Res. 30 (4), 487–502.

Fehr, E., Rangel, A., 2011. Neuroeconomic foundations of economic choice–Recent advances. J. Econ. Perspect. 25 (4), 3–30.

Gardner, M., 1983. Advertising effects on attributes recalled and criteria used for brand evaluations. J. Consum. Res. 10 (3), 310–318.

Haan, M., Moraga-Gonzalez, J., 2011. Advertising for attention in a consumer search model. Econ. J. 121 (May), 552–579.

Hotz, J., Xiao, M., 2013. Strategic information disclosure: The case of multi-attribute products with heterogeneous consumers. Econ. Inq. 51 (1), 865–881.

Ieva, M., Ziliani, C., Gázquez-Abad, J.C., D'attoma, I., 2018. Online versus offline promotional communication: Evaluating the effect of medium on customer response. J. Advert. Res. 58 (3), 338–348.

Janssen, M., Teteryatnikova, M., 2016. Competition and disclosure of comparative or non-comparative horizontal product information. J. Ind. Econ. 64 (4), 589–620.

Johnson, J., Myatt, D., 2006. On the simple economics of advertising, marketing and product design. Am. Econ. Rev. 96 (3), 756–784.

Jones, M.Y., Pentecost, R., Requena, G., 2005. Memory for advertising and information content: Comparing the printed page to the computer screen. Psychol. Mark. 22 (8), 623–648.

Koszegi, B., Szeidl, A., 2013. A model of focusing in economic choice. Q. J. Econ. 128 (1), 53–104.

Lee, D., Hosanagar, K., Nair, H.S., 2018. Advertising content and consumer engagement on social media: Evidence from Facebook. Manage. Sci. 64 (11), 5105–5131.

Lewis, T.R., Sappington, D.E.M., 1994. Supplying information to facilitate price discrimination. Internat. Econom. Rev. 309–327.

Magee, R.G., 2013. Can a print publication be equally effective online? Testing the effect of medium type on marketing communications. Mark. Lett. 24 (1), 85–95.

Meurer, M., Stahl, D.O., 1994. Informative adverising and product match. Int. J. Ind. Organ. 12, 1–19.

Renault, R., 2015. Advertising in markets. In: Handbook of Media Economics, vol. 1, Elsevier, pp. 121–204.

Sahni, N.S., 2016. Advertising spillovers: Evidence from online field experiments and implications for returns on advertising. J. Mar. Res. 53 (4), 459–478.

Sun, M., 2011. Disclosing multiple product attributes. J. Econ. Manag. Strategy 20 (1), 195–224.

Van Zandt, T., 2004. Information overload in a network of targeted communication. Rand J. Econ. 35 (3), 542–560.

Zhou, J., 2017. Competitive bundling. Econometrica 85 (1), 145–172.

Zhu, Y., Dukes, A., 2017. Prominent attributes under limited attention. Mark. Sci. 36 (5), 645–812.